CP violating supersymmetric contributions to the electroweak $\rho$ parameter

Sin Kyu Kang$^1$ and Jung Dae Kim$^{1,2}$

$^1$ School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea
$^2$ Physics Department and IPAP, Yonsei University, Seoul 120-749, Korea

Abstract

Effects of CP violation on the supersymmetric electroweak correction to the $\rho$ parameter are investigated. To avoid the EDM constraints, we require that $\arg(\mu) < 10^{-2}$ and the non-universal trilinear couplings $A_f = (0, 0, A_0)$ and also assume that gluinos are heavier than 400 GeV. The CP phase $\phi_t = \arg(A_0)$ leads to large enhancement of the relative mass splittings between $\tilde{t}_2$ and $\tilde{b}_L(\tilde{t}_1)$, which in turn reduces the one-loop contribution of the stop and sbottom to $\Delta\rho$. For small $\tan\beta$, such a CP violating effect is prominent. We also study how much the two-loop gluon and gluino contributions are affected by the CP phase. Possible contributions to the $\rho$ parameter arising from the Higgs sector with CP violation are discussed.

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The minimal supersymmetric standard model (MSSM) is the best motivated extensions of the standard model (SM). As is well known, in the MSSM, there are many new sources of CP violation beyond the Cabibbo-Kobayashi-Maskawa (CKM) phase, arising from the complex soft supersymmetry (SUSY) breaking terms, i.e., the Majorana gaugino masses $M_i$, the trilinear scalar $A$ terms, the bilinear scalar $B$ terms, as well as from the $\mu$ parameter which is the bilinear mixing of the two Higgs chiral superfields in the superpotential. While those supersymmetric CP violating (CPV) phases could give large contributions to the neutron/electron electric dipole moments (EDMs), they are strongly constrained by the current experimental measurements of the neutron/electron EDMs, which for the neutron is $d_n < 1.1 \times 10^{-25} \text{e cm}$ [1] and for the electron is $d_e < 4.3 \times 10^{-27} \text{e cm}$ [2]. To resolve this problem, several possibilities have been suggested. The first one is to make all phases small ($O(10^{-2})$) [3]. But, such small phases would require a significant amount of fine-tuning and are thus undesirable. The second is to take the SUSY spectrum heavy in the several TeV range which lies outside the reach of the accelerators [4]. Another option suggested recently is to use the cancellations among the various contributions to the neutron/electron EDMs, allowing for large CPV phases and a supersymmetric spectrum that is still within the reach of the accelerators [5]. Finally, an interesting possibility is to take a slightly non-universal scenario for the trilinear couplings $A_f$ [6,7]. As shown in Ref. [7], requiring that $\arg(\mu) < 10^{-2}$ and $A_f = (0, 0, A_0)$, and assuming that gluinos are heavier than about 400 GeV [5], one may significantly reduce the size of the neutron/electron EDMs due to Weinberg’s three-gluon operator [8] well below the current experimental limit.

Another CP violation associated with the Higgs boson sector in the MSSM can come from the one-loop corrections of the MSSM Higgs potential through soft CPV Yukawa interactions involving squarks, i.e., $A$ terms. As shown in Ref. [9], an immediate consequence of Higgs sector CP violation in the MSSM is the generation of mixing mass terms between the CP even and CP odd Higgs fields. Thanks to those scalar-pseudoscalar mixing mass terms, the small tree-level mass difference between the heaviest Higgs boson and the CP-odd scalar may be lifted considerably. The phenomenological consequences of Higgs sector CP violation
have been studied \cite{9,10}.

A possible way to probe SUSY is to search for the virtual effects of SUSY particles which would be sizable enough to be detected in the present experiments. In most experimentally accessible processes, the heavy SUSY particles decouple from the low energy electroweak observables. However, similar to the SM, if there happens large "custodial" $SU(2)_V$ breaking, the electroweak observables which are parameterized by the $\rho$ parameter \cite{11}, defined as the ratio of the strengths of neutral and charged currents at vanishing momentum transfer, can severely be affected. In the MSSM, the main potential source of $SU(2)_V$ breaking is the splitting between the stop and sbottom masses. In addition, there can be large splitting within a supersymmetric multiplet, which also affects the $\rho$ parameter. From the numerical analysis, it is well known that the leading contribution of the squark loops, in particular stop and sbottom loops, to the $\rho$ parameter can reach at the level of a few $10^{-3}$ which lies within the range of the experimental sensitivity \cite{12}. There are also small additional contributions coming from the exchange of the additional Higgs bosons, the charginos and neutralinos. Some discussions of supersymmetric contributions to the $\rho$ parameter already exist in the literature \cite{12,13}. Moreover, higher order corrections have also been studied so as to treat the SUSY loop contributions to the electroweak observables at the same level of accuracy as the SM contribution \cite{14}. However, so far, the analyses have been done within the context of the CP conserving (CPC) MSSM. Since new CPV phases may affect the SUSY spectrum and generate additional couplings which do not appear in the CPC MSSM, it would be quite interesting to study the effects of the CPV SUSY phases on the electroweak observables via the $\rho$ parameter.

In this respect, the purpose of this letter is to examine how much the $\rho$ parameter can be affected by the non-vanishing CP phases in the MSSM. In this analysis, we will impose the universal conditions on the soft SUSY breaking terms which provide three complex parameters, i.e., universal gaugino mass $m_{1/2}$, the trilinear soft term $A_0$ and the bilinear soft term $B_0$. In addition, there is a complex mass parameter $\mu$. However, not all phases of the four complex parameters are physical \cite{15}. It is always possible to make a phase
transformation on the gaugino fields so as to make $m_{1/2}$ real. From minimization conditions on the Higgs potential, one can make the phase of $B_0$ equal to the phase difference of the two Higgs doublets in the MSSM. As a result, there are only two independent CPV phases in the MSSM with universal soft SUSY breaking terms, which are usually chosen to be $\arg(A_0)$ and $\arg(\mu)$. To satisfy the EDM constraints, we will simply take $\arg(\mu)$ to be zero and the trilinear coupling of the Higgs bosons to the squarks of the first and second generation to be much smaller than the one of the third generation, as mentioned above, and assume that the gluino mass to be larger than 400 GeV. Thanks to this non-vanishing phase $\arg(A_0)$, as will be shown later, the stop spectrum and their mixing angle are changed, which lead in turn to the shift of the contributions to $\Delta \rho$. Also, since the neutral Higgs boson mass eigenstates in the CPV MSSM are mixtures of CP even and CP odd states, new gauge-Higgs couplings are induced at the tree level through those mixings. These couplings can generate the additional radiative corrections to the gauge boson self-energies, and may thus affect the $\rho$ parameter. However, the contributions coming from the gaugino and higgsino exchanges will not be affected by the non-vanishing CPV phase $\arg(A_0)$ in the leading order because they are affected by only the CPV phase $\arg(\mu)$ which is neglected in this analysis. Thus, we shall not consider those contributions. In addition, we shall study how much the dominant two-loop contributions via gluon and gluino exchanges can be shifted by the effect of the CPV phases. The numerical calculation will be done with the help of the program package FEYNHIGGS [16] which is modified so as to calculate the CPV SUSY contributions to the $\rho$ parameter.

The $\rho$ parameter is represented by

$$\rho = \frac{1}{1 - \Delta \rho}, \quad \Delta \rho = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2},$$

where $\Pi_{WW(ZZ)}(0)$ denotes the transverse parts of the $W/Z$-boson self-energies at zero momentum transfer. In the SM, the main contribution comes from the large splitting between the top quark and the bottom quark masses and is given by

$$\Delta \rho_0^{SM} = \frac{N_c G_F}{8 \sqrt{2} \pi^2} F_0(m_t^2, m_b^2),$$

where $F_0$ is a function of the top and bottom quark masses. This contribution is modified in the CPV MSSM due to the non-vanishing phase $\arg(A_0)$.
where \( N_c \) is the color factor and the function \( F_0 \) is given by

\[
F_0(x, y) = x + y - \frac{2xy}{x-y} \ln \frac{x}{y}.
\]  

(3)

In the limit of large mass splitting it becomes proportional to the heavy quark mass squared, i.e., \( F_0(m_t^2, m_b^2) \approx m_t^2 \). Including QCD corrections, the SM contribution of \( \Delta \rho \) is \( \Delta \rho_{SM}^F = -\Delta \rho_0^{SM} \left( \frac{2\alpha_s}{3\pi} \right) (1 + \pi^2/3) \) [17]. For \( \alpha_s(M_Z) \approx 0.12 \), the QCD correction reduces the one-loop result by about 10% and shifts \( m_t \) upwards by about 10 GeV [18].

(A) Squark contributions: Since the scalar partners of the light quarks are expected to be almost degenerate in most supersymmetric theories, their contributions to \( \Delta \rho \) are negligible and thus only third generation will contribute. The stop mass matrix is given by

\[
M_t^2 = \begin{pmatrix}
  m^2_{t_L} + m^2_f + D_L & m_t m^*_L R \\
  m_t m^*_L R & m^2_{t_R} + m^2_f + D_R
\end{pmatrix}
\]  

(4)

where \( m_{LR} = A_0 - \mu^*/\tan \beta \), \( D_L = \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \cos 2 \beta M_Z^2 \) and \( D_R = \frac{2}{3} \sin^2 \theta_W \cos 2 \beta M_Z^2 \).

The stop fields \( (\tilde{t}_L, \tilde{t}_R) \) are linear combinations of the mass eigenstates \( \tilde{t}_{1,2} \) so that \( \tilde{t}_L = \cos \theta t \tilde{t}_1 - \sin \theta t e^{-i \phi t} \tilde{t}_2, \tilde{t}_R = \sin \theta t e^{i \phi t} \tilde{t}_1 + \cos \theta t \tilde{t}_2 \), where

\[
\tan 2\theta t = \frac{2m_t |m_{LR}|}{m^2_{t_L} - m^2_{t_R} + D_L - D_R}.
\]  

(5)

and the mass eigenvalues of stop fields are given by

\[
m^2_{\tilde{t}_{1,2}} = \frac{1}{2} \left[ m^2_{t_L} + m^2_{t_R} + 2m^2_f + D_L + D_R \right. \\
\left. \mp \left( (m^2_{t_L} - m^2_{t_R} + D_L - D_R)^2 + 4m^2_t |m_{LR}|^2 \right)^{1/2} \right],
\]  

(6)

\[
|m_{LR}| = |A_0 - \mu^*/\tan \beta| \\
= \sqrt{|A_0|^2 + |\mu|^2/\tan^2 \beta - 2|A_0||\mu|/\tan \beta \cos \phi_t},
\]  

(7)

where \( \phi_t \equiv \arg(A_0) \). As expected, the non-vanishing phase \( \phi_t \) will change the stop mass spectrum which can in turn lead to the shift of \( \Delta \rho \). Neglecting the mixing in \( \tilde{b} \) sector, the contribution of \( (\tilde{t}, \tilde{b}) \) to \( \Delta \rho \) is presented at one-loop order by:

\[
\Delta \rho_{\tilde{t}\tilde{b}} = \frac{3G_F}{8\sqrt{2\pi}} \left[ -\sin^2 \theta t \cos^2 \theta t F_0(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}) \\
+ \cos^2 \theta t F_0(m^2_{\tilde{t}_1}, m^2_{b_L}) + \sin^2 \theta t F_0(m^2_{\tilde{t}_2}, m^2_{b_L}) \right],
\]  

(8)
Before examining how much the value of $\Delta \rho_{t\bar{b}}$ can be affected by the phase $\phi_t$, let us first investigate the mass splittings between the two squark mass eigenstates which may indicate to what extent the contribution of $(\tilde{t}, \tilde{b})$ to $\Delta \rho$ is deviated. In Fig. 1a (b), we present numerical estimation of the relative mass splittings between the two squark mass eigenstates, $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|/m_{\tilde{q}}^2$ and $|m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2|/m_{\tilde{q}}^2$, as a function of the phase $\phi_t$, for $\tan \beta = 1.6(20)$, $m_{\tilde{q}} = 200$ GeV, $|A_0| = 200$ GeV and $|\mu| = 100$ GeV. Here we take $0 \leq \phi_t \leq \pi$. As the phase $\phi_t$ increases, the both relative mass splittings increase. The effect of the maximal CP violation corresponding to $\phi_t = \pi$ may lead to the enhancement of the relative mass splittings by about 90% (40%) for $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|/m_{\tilde{q}}^2$ ($|m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2|/m_{\tilde{q}}^2$) compared to the CPC case. This is attributed to the fact that the absolute value of $m_{LR}$ monotonically increases with the increasing phase $\phi_t$, which makes the mass splitting between $\tilde{t}_1(\bar{b}_L)$ and $\tilde{t}_2$ larger, and the mixing angle $\theta_t$ closer to $\pi/4$. Note that the mixing angle $\theta_t$ close to $\pi/4$ are naturally obtained because the off-diagonal elements of the stop mass matrix may be larger than the difference of the diagonal entries. As one can see from Fig. 1a, in the case of small $\tan \beta$, the splitting $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|/m_{\tilde{q}}^2$ is larger than $|m_{\tilde{t}_1}^2 - m_{\tilde{b}_1}^2|/m_{\tilde{q}}^2$ for $\phi_t = 0$ (CPC case), whereas the latter becomes larger than the former for large $\phi_t$. Thus, the splitting originating from the left-right mixing of stops may be more important for the CPV case with large phase. In the case of large $\tan \beta$, $|m_{\tilde{t}_1}^2 - m_{\tilde{b}_1}^2|/m_{\tilde{q}}^2$ is larger than $|m_{\tilde{t}_1}^2 - m_{\tilde{b}_1}^2|/m_{\tilde{q}}^2$ for all $\phi_t$, and the mass splittings are weakly dependent on $\phi_t$ since the term concerned with the phase $\phi_t$ in $|m_{LR}|$ is suppressed by the large $\tan \beta$. We also observed that the mass splitting $|m_{\tilde{t}_1}^2 - m_{\tilde{b}_L}^2|/m_{\tilde{q}}^2$ is much smaller than the other two. For the same input values of the SUSY parameters, the dependence of $\Delta \rho_{t\bar{b}}$ on the phase $\phi_t$ is shown as solid lines in Fig. 2. Although the CPV case provides larger mass splitting than the CPC case, $\Delta \rho_{t\bar{b}}$ is monotonically decreased with the increasing $\phi_t$. The effect of the CPV phase reduces the value of $\Delta \rho_{t\bar{b}}$ for the CPC case by about up to 35%. The reason is that as the phase increases, the mixing angle $\theta_t$ gets close to the maximal mixing and $F_0(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$ increases faster than $F_0(m_{\tilde{b}_2}^2, m_{\tilde{b}_1}^2)$ in Eq. (8) so that they lead to destructive contribution to $\Delta \rho_{t\bar{b}}$. As $|A_0|$ increases, $\Delta \rho_{t\bar{b}}$ tends to decrease, whereas it tends to increase with the increasing $|\mu|$. We have observed that the effect of the
CPV phase is diminished as long as $m_{\tilde{q}}$ becomes larger than $|A_0|$ and $|\mu|$. This is because $m_{\tilde{t}_1} \sim m_{\tilde{t}_2} \sim m_{\tilde{b}_L}$ in the limit of large $m_{\tilde{q}}$.

The two-loop QCD correction to $\Delta \rho_{tb}$ is given by Eq. (8) in Ref. [14], and the two-loop contribution mediated by gluino exchange is also represented in Ref. [14]. Those gluon and gluino contributions add up to about 30% of the one-loop value in the CPC case.

In Fig. 3, the dependence of the dominant two-loop QCD correction and gluino contribution to the $\rho$ parameter on the phase $\phi_t$ is plotted by the dashed line and dotted line, respectively, for $\tan \beta = 1.6$ (a) and $\tan \beta = 20$ (b). As the phase $\phi_t$ increases, the two-loop gluonic SUSY contribution decreases, whereas the gluino contribution increases. The reason that the gluino contribution increases with the increasing phase $\phi_t$ is that its contribution depends inversely on the mass splittings between $\tilde{t}_1(\tilde{b}_L)$ and $\tilde{t}_2$. In particular, for small $\tan \beta$, the value of the gluino contribution becomes larger than that of the gluonic SUSY contribution at large $\phi_t$.

**(B) Higgs sector contributions:** There are also possible contributions arising from the Higgs sector to the $\rho$ parameter. In the CPC case, it is known [12] that the Higgs boson masses and couplings to gauge bosons are related in such a way that they lead to large cancellations in their contributions to the $\rho$ parameter, which is at most of order of $10^{-4}$. In the decoupling limit where the heavy neutral CP-even Higgs, CP-odd Higgs and the charged Higgs bosons are nearly degenerate and their couplings to gauge bosons tend to zero, while the lightest CP-even Higgs boson reaches its maximal mass value and has almost the same properties as the SM Higgs boson. Then, the contribution of the Higgs sector of the MSSM to the $\rho$ parameter is practically the same as in the SM. However, while the contribution of the SM Higgs sector gives rise to logarithmic $\log M_h/M_Z$ in the limit of large Higgs mass which may reach about $10^{-3}$ order, the MSSM contribution reaches at most $10^{-4}$ due to the upper bound on the mass of the lightest Higgs boson.

Now, let us take into account the contributions coming from the neutral Higgs sector with CP violation. The decomposition of the two Higgs doublets is given by
\[ \Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + ia_1) \end{pmatrix}, \quad \Phi_2 = e^{i\theta} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + ia_2) \end{pmatrix}, \] 

(9)

where \( v_1 \) and \( v_2 \) are the vacuum expectation values and \( \theta \) is their relative phase. As shown in Ref. [19], the Higgs quartic couplings receive significant radiative corrections from \( A_f \) terms. The vacuum expectation values \( v_1, v_2 \) and the phase \( \theta \) are determined from the minimization conditions on the Higgs potential of the MSSM. The CP odd fields are rotated to \( a_1 = \cos \beta G^0 - \sin \beta a \) and \( a_2 = \sin \beta G^0 + \cos \beta a \) so that the Higgs potential shows up a flat direction with respect to the Goldstone field \( G^0 \). The non-vanishing CPV phases in \( A_i \) and \( A_b \) can lead to the non-vanishing \( \theta \), for which the scalar-pseudoscalar mixing mass terms are generated and the neutral Higgs boson mass matrix in the weak basis \((G^0, a, \phi_1, \phi_2)\) can be written by

\[ M_0^2 = \begin{pmatrix} M_P^2 & M_{PS}^2 \\ M_{SP}^2 & M_S^2 \end{pmatrix}, \] 

(10)

where \( M_P^2 \) and \( M_S^2 \) are \( 2 \times 2 \) matrix for CPC parts in the basis \((G^0, a)\) and \((\phi_1, \phi_2)\), respectively, and \( M_{PS}^2 = (M_{SP}^2)^T \) describes the CPV mixings between \((G^0, a)\) and \((\phi_1, \phi_2)\). From the tadpole condition, the Goldstone field \( G^0 \) does not mix with the other neutral fields and thus becomes massless. Then, the neutral Higgs mass matrix \( M_0^2 \) reduces to a \( 3 \times 3 \) matrix \( M^2 \), which is spanned in the basis \((a, \phi_1, \phi_2)\). The Higgs boson mass matrix \( M^2 \) can be diagonalized with the help of an orthogonal rotation matrix \( U \): \( U^T M^2 U = diag(M_1^2, M_2^2, M_3^2) \).

Then, the contributions of the Higgs sector with CP violation to the \( \rho \) parameter can be given by

\[ \Delta \rho_H = \frac{3\alpha}{16\pi c_W^2} \sum_{i=1}^3 U_{1i}^2 \Delta \rho_H^{SM}(M_{H_i}^2) + \frac{\alpha}{16\pi s_W^2 M_W^2} \times \left( \sum_{i=1}^3 (1 - U_{1i}^2)F_0(M_{H_i}^2, M_{H_i}^2) - \frac{1}{2} \sum_{i,j} (U_{1i}U_{2j} - U_{2i}U_{1j})^2 F_0(M_{H_i}^2, M_{H_j}^2) \right) \] 

(11)

where \( M_{H_i} \) denotes the charged Higgs boson mass and

\[ \Delta \rho_H^{SM}(M^2) = \frac{\alpha}{16\pi s_W^2 M_W^2} \left[ F_0(M^2, M_W^2) - F_0(M^2, M_Z^2) \right] + \frac{\alpha}{16\pi s_W^2} \left[ \frac{M^2}{M^2 - M_W^2} \log \frac{M^2}{M_W^2} - \frac{1}{c_W^2} \frac{M^2}{M^2 - M_Z^2} \log \frac{M^2}{M_Z^2} \right] \] 

(12)
corresponds to the SM Higgs boson contribution to $\Delta \rho$.

In Fig. 3, we present the dependence of the Higgs boson sector contribution to $\Delta \rho$ on the phase $\phi_t$ (solid lines). Similar to the one-loop contribution of stop quark to $\Delta \rho$, the one-loop contribution of Higgs sector is decreased with the increasing phase $\phi_t$. This is mainly because the relative mass splittings among three neutral Higgs bosons decrease as the phase $\phi_t$ increases as shown in Fig. 4. In the case of small $\tan \beta$, $|M_3^2 - M_1^2|/M_{H_u}^2$ (solid line) is larger than $|M_2^2 - M_1^2|/M_{H_u}^2$ (dotted line). For only small $\tan \beta$, the contribution of the Higgs sector is larger than the two-loop contributions mediated by the gluon and gluino exchanges, and those three contributions are almost the same at $\phi_t = \pi$, which increases the one-loop contribution of stop and sbottom by about $30 - 35\%$. For large $\tan \beta$, the Higgs sector contribution to $\Delta \rho$ is negligibly small.

In summary, effects of CP violation on the supersymmetric electroweak correction to the $\rho$ parameter have been investigated. To avoid the EDM constraints, we have required negligibly small arg($\mu$) and the non-universal couplings $A_f = (0, 0, A_0)$ and also assumed that the mass of gluino is larger than 400 GeV. The CP phase $\phi_t$ leads to large enhancement of the relative mass splittings between $\tilde{t}_2$ and $\tilde{b}_L(\tilde{t}_1)$, which in turn reduces the one-loop contribution of the stop and sbottom to the $\rho$ parameter. For small $\tan \beta$, such a CP violating effect is prominent. We have also studied how much the two-loop gluon and gluino contributions are affected by the CP phase. Possible contributions to the $\rho$ parameter arising from the Higgs sector with CP violation have been discussed.

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FIG. 1. The relative mass splittings between the two squark mass eigenstates, \( |m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|/m_{\tilde{q}}^2 \) (solid line) and \( |m_{\tilde{t}_2}^2 - m_{\tilde{b}_L}^2|/m_{\tilde{q}}^2 \) (dotted line) as a function of the phase \( \phi_t \) for \( \tan \beta = 1.6 \) (a) and \( \tan \beta = 20 \) (b). We take \( m_{\tilde{q}} = 200 \) GeV, \( |A_0| = 200 \) GeV, and \( |\mu| = 100 \) GeV.

FIG. 2. The contribution of stop and sbottom to \( \Delta \rho \) as a function of the phase \( \phi_t \) for the same input values of the SUSY parameters as those in Fig. 1.
FIG. 3. The contributions of the two-loop QCD correction (dashed line), the two-loop gluino (dotted line) and the Higgs boson sector (solid line) to $\Delta \rho$ as a function of the phase $\phi_t$ for $\tan \beta = 1.6$ (a) and $\tan \beta = 20$ (b). The input values of the SUSY parameters are taken to be the same as those in Fig. 1, and the gluino mass and the charged Higgs boson mass to be $m_{\tilde{g}} = 500$ GeV $M_{H_c} = 200$ GeV, respectively.
FIG. 4. The relative mass splittings among three neutral Higgs bosons: $|M_3^2 - M_2^2|/M_{H_c}^2$ (solid line) and $|M_2^2 - M_1^2|/M_{H_c}^2$ (dotted line). The input values of the SUSY parameters are taken to be the same as the case of Fig. 3. The lightest (heaviest) Higgs masses are denoted by $M_2(M_3)$. 