Abstract.

The Nucleon-Nucleon One Meson Exchange Potential, its wave functions and related Meson Exchange Currents are analyzed for point-like nucleons. The leading $N_c$ contributions generate a local and energy independent potential which presents $1/r^3$ singularities, requiring renormalization. We show how invoking suitable boundary conditions, neutron-proton phase shifts and deuteron properties become largely insensitive to the nucleon substructure and to the vector mesons. Actually, reasonable agreement with low energy data for realistic values of the coupling constants (e.g. SU(3) values) is found. The analysis along similar lines for the Meson Exchange Currents suggests that this renormalization scheme implies tremendous simplifications while complying with exact gauge invariance at any stage of the calculation.

Keywords: NN interaction, One Boson Exchange, Renormalization, Strong form factors, Large N_c, Chiral symmetry, Gauge invariance.

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INTRODUCTION

The original idea of Yukawa that NN interaction at long distances is due to One Pion Exchange (OPE) was verified quantitatively by the Nijmegen benchmarking partial wave analysis for NN scattering in the elastic region with $\chi^2$/DOF $\sim 1$: a partial wave and energy dependent square well potential was considered for distances below 1.4-1.8 fm [1] while the neutral and charged pion masses could be determined from the fit to their currently accepted PDG values assuming OPE above such distances. The verification of other meson exchanges is less straightforward from NN elastic scattering since the shortest resolution distance probed at the pion production threshold is about $\lambda \approx \hbar/\sqrt{M_Nm_\pi} \sim 0.5$fm and effectively short distance interactions admit a variety of parameterizations and forms [2, 3, 4] which show up quantitatively when nucleons are placed off-shell by the presence of a third particle. Already in the simplest case of a photon as the additional particle either in the initial or final state (see [5] and references therein) Gauge invariance relates meson exchange potentials and currents (MEC’s) but also requires that Hamiltonian eigenstates are used to compute electroweak matrix elements. Clearly, one should not expect to know the longitudinal currents any better than potentials. Ambiguities in transverse, i.e. non-minimal coupling, terms of the current just reflect the composite character of Nucleons as well as their finite size. In the OBE potential, strong form factors are added to mimic this finite size [6], which is about 0.6fm. In the case of gauge invariance, the inclusion of a form-factor introduced by hand, i.e., not computed consistently within meson theory, implies a kind of non-locality in the interaction which could be made gauge invariant by introducing link operators between two points, thereby generating a path dependence, for which no obvious resolution has been found yet. Note that purely phenomenological potentials not based entirely on the Meson Exchange picture are inherently ambiguous. If the corresponding NN wave functions are combined with MEC’s conflicting results with gauge invariance are eventually produced. Since the finite size of the nucleon is comparable to the minimal resolution probed in NN scattering, we do not expect to see the difference between a point-like nucleon and an extended one at sufficiently low energies. Recently [7] we have suggested replacing strong form factors in the NN potential by renormalization conditions on low energy scattering properties. In the lowest partial waves we have shown that after renormalization finite nucleon effects parameterized as strong form factors are indeed marginal. We suggest using renormalization ideas for potentials, wave functions and currents computed consistently. In the present contribution we review our findings [7] and apply them to analyze the radiative neutron capture, $n + p \rightarrow d + \gamma$, a nuclear reaction where the MEC are known to be essential [5].
TABLE 1. Deuteron properties and low energy parameters in the $3S_1-3D_1$ channel for OBE potentials including $\pi, \sigma, \rho, \omega$ mesons [7] as well as axial-vector meson $a_1$. We use the same numbers and notation as in Ref. [7]. Here AVMD means taking $m_{a_1} = \sqrt{m_{\rho}} \simeq 1107\text{MeV}$ and PDG taking $m_{a_1} = 1230\text{MeV}$. See also Ref. [14] and references therein.

| $\gamma$ (fm$^{-1}$) | $\eta$ | $\alpha$ (fm$^{-1}$) | $\alpha_0$ (fm$^{-1}$) | $\alpha_1$ (fm$^{-1}$) | $\alpha_2$ (fm$^2$) | $r_0$ (fm) |
|----------------------|--------|----------------------|----------------------|----------------------|------------------|------------|
| $\pi\rho a_1$ (AVMD) | 0.02557 | 0.8946 | 1.9866 | 0.2792 | 65.3% | 0.639 | 5.467 | 1.723 | 6.621 | 1.722 |
| $\pi\rho a_1$ (PDG) | 0.02555 | 0.8937 | 1.9846 | 0.2780 | 65.8% | 0.671 | 5.463 | 1.714 | 6.607 | 1.712 |
| $\pi\sigma a_1$ (AVMD) | 0.02544 | 0.8966 | 1.9909 | 0.2788 | 59.0% | 0.5087 | 5.477 | 1.720 | 6.604 | 1.734 |
| $\pi\sigma a_1$ (PDG) | 0.02540 | 0.8951 | 1.9876 | 0.2773 | 60.1% | 0.557 | 5.470 | 1.708 | 6.588 | 1.724 |
| Nijm11 | 0.02521 | 0.8845(8) | 1.9675 | 0.2707 | 56.35% | 0.4502 | 5.418 | 1.647 | 6.505 | 1.753 |
| Reid93 | 0.02514 | 0.8846(9) | 1.9686 | 0.2703 | 56.99% | 0.4515 | 5.422 | 1.645 | 6.453 | 1.755 |
| Exp. | 0.231605 | 0.02564(9) | 0.8846(9) | 1.9754(9) | 0.2853(9) | 56.7(4) | 5.419(7) | – | – | 1.753(8) |

**MESON EXCHANGE POTENTIALS**

A useful and simplifying assumption arises from our observation that the symmetry pattern of the sum rules for the old nuclear Wigner and Serber symmetries discussed in Refs. [8, 9] largely complies to the large $N_c$ and QCD based contracted SU($4_c$) symmetry [10]. In the large $N_c$ limit with $\alpha_s N_c$ fixed, nucleons are heavy, $M_N \sim N_c$, and the definition of the NN potential $N_c$ makes sense. The tensorial spin-flavour structure was found to be [10]

$$V(r) = V_C(r) + r_1 \cdot r_2 [\sigma_1 \cdot \sigma_2 W_3(r) + S_1 W_T(r)] \sim N_c.$$  \hspace{1cm} (1)

Other operators such as spin-orbit or relativistic corrections are $O(N_c^{-1})$ and hence suppressed by a relative $1/N_c^2$ factor. While these counting rules are directly obtained from quark-gluon dynamics, quark-hadron duality and confinement requires that above the confinement scale one can saturate Eq. (1) with multiple exchanges of mesons which have a finite mass for $N_c \gg 3$ [11]. We retain one boson exchange (OBE) with $\pi, \sigma, \rho$ and $\omega$ and $a_1$ mesons. The corresponding potential reads

$$V_C(r) = \frac{g_{\pi NN}^2}{4\pi} e^{-m_{\pi} r} + \frac{g_{\omega NN}^2}{4\pi} e^{-m_{\omega} r},$$  \hspace{1cm} (2)

$$W_3(r) = \frac{g_{\pi NN}^2 m_{\pi}^2 e^{-m_{\pi} r}}{48\pi \Lambda_N^3} + \frac{f_{\rho NN}^2 m_{\rho}^2 e^{-m_{\rho} r}}{24\pi \Lambda_N^3} - \frac{g_{a_1 NN}^2 e^{-m_{a_1} r}}{6\pi} r,$$  \hspace{1cm} (3)

$$W_T(r) = \frac{g_{\pi NN}^2 m_{\pi}^2 e^{-m_{\pi} r}}{48\pi \Lambda_N^3} \left[ 1 + \frac{3}{m_{\pi} r} + \frac{3}{m_{\pi} r^2} \right] - \frac{f_{\rho NN}^2 m_{\rho}^2 e^{-m_{\rho} r}}{48\pi \Lambda_N^3} \left[ 1 + \frac{3}{m_{\rho} r} + \frac{3}{m_{\rho} r^2} \right]$$  

$$+ \frac{g_{a_1 NN}^2 e^{-m_{a_1} r}}{12\pi r} \left[ 1 + \frac{3}{m_{a_1} r} + \frac{3}{m_{a_1} r^2} \right],$$  \hspace{1cm} (4)

where $\Lambda_N = 3M_p/N_c$ and $g_{\pi NN}, g_{\omega NN}, f_{\rho NN}, g_{a_1 NN} \sim \sqrt{N_c}$ and $m_{\pi}, m_{\sigma}, m_{\rho}, m_{\omega}, m_{a_1} \sim 1.753(8)$. To leading and sub-leading order in $N_c$, one may neglect spin orbit, meson widths and relativity. The tensor force $W_T$ is singular at short distances $\sim 1/r^3$ and requires renormalization. The renormalization is carried out in coordinate space using a boundary condition at a short distance cut-off $r_c$ (see [7] for details) which makes the Hamiltonian self-adjoint for $r > r_c$. Besides being much simpler and efficient, this method allows to deal with cut-off independent potentials. In practice convergence is achieved for $r_c \sim 0.3\text{fm}$ (see e.g. left panel Fig. 3 for the asymptotic D/S ratio $\eta$).

Overall, the agreement is good for realistic couplings and masses as expected from other sources (see Ref. [7] for a short compilation) including a natural SU(3) value for $g_{\omega NN}$ coupling. The deuteron properties and low energy parameters are shown in table 1. The $^1S_0$ phase shift is reproduced for $m_\pi \sim 500\text{MeV}$ while the $^3S_1-^3D_1$ phase shifts are plotted in Fig. 1. Space-like electromagnetic form factors in the impulse approximation [15] for $G_E(-q^2) = 2$

Other mesons such as $\eta$ are sub-leading. Due to the $U(1)$ anomaly the $\eta'$ meson appears to be heavy, but in the large $N_c$ limit one it becomes degenerate with the pion $m_{\eta'} = m_\pi + O(1/N_c)$. So, one might think that this meson would be as important as the pion itself. Actually this is not so since being an iso-scalar state it generates terms in the potential $V_C$, $V_T$ and $V_S$ which are $O(1/N_c)$ and hence do not contribute to the leading potential in Eq. (1).

$^3$ In our previous work we left out the $a_1$ meson. We use the chiral Lagrangian [12] and take $g_{a_1 NN} = (m_{a_1}/m_\pi) f_{\pi NN} = 8.4$.

$^4$ Imposing a cut-off in momentum space generates an apparent delayed convergence due to the long distances distortion of the potential, so that unexpectedly large momentum cut-offs are needed. Renormalized results, however, agree in both $r-$ and $p-$spaces [13].
FIGURE 1. np spin triplet eigen phase shifts for the total angular momentum $j = 1$ as a function of the c.m. momentum compared to an average of the Nijmegen partial wave analysis and high quality potential models [2]. See table 1 for notation. The band in the case of the $a_1$ represents the error of changing $m_{a_1}$ from the AVMD value to the PDG value.

FIGURE 2. Deuteron charge (left), magnetic (middle) and quadrupole (right) form factors as a function of transfer $q$ (in MeV). See Table 1 for notation. Data can be traced from Ref. [15] (see also references in [13]).

$$\frac{1}{1 + \frac{q^2}{m_p^2}}$$ and without MEC are plotted in Fig. 2 (see [13] for the $\pi$ case). As we see, including shorter range mesons induces moderate changes, due to the expected short distance insensitivity embodied by renormalization, despite the short distance singularity and without introducing strong meson-nucleon-nucleon vertex functions $^5$.

MESON EXCHANGE CURRENTS

Gauge invariance is easily preserved within our coordinate space approach by keeping the same boundary condition as for the potential in the large $N_c$ setup without need of new parameters (see also Ref. [18]). The simplest (purely transverse) MEC correction to the deuteron magnetic moment in the Impulse Approximation (IA) $\mu_{IA}^d = (\mu_p + \mu_n) + \frac{3}{2}(\mu_p + \mu_n + \frac{1}{2}) P_B$ is shown in Fig. 3 (middle panel) as a function of $r_c$. Likewise, we show (right panel) the neutron capture cross section. The (longitudinal) MEC contribution yields a constant shift at relatively large distances. The different short distance behaviour between transverse and longitudinal MEC’s will be elaborated elsewhere.

$^5$ We include only the OBE part of the leading $N_c$ potential but multiple meson exchanges could also be added as well as $\Delta$ degrees of freedom to comply with large $N_c$ counting rules [11]. Eq. (1) yields $V_c^t$ at leading order in $N_c$. It is worth reminding that for chiral potentials $V_{c} = \mathcal{O}(g^4_{\lambda}/(f^2_{\pi}M_{\pi}))$ is Next-to-next-to-leading order (NNLO). Actually, as noted in Refs. [9, 16, 7] the expected large $N_c$ behaviour [11] does not hold for the (Two Pion Exchange) chiral potentials even after inclusion of $\Delta$ [17]. Likewise, the Wigner symmetry pattern is not fulfilled for those chiral potentials [8, 9].
CONCLUSIONS

Self-adjointness of the two-body Hamiltonian and current conservation are simply intertwined within the renormalization with boundary conditions approach above a certain cut-off distance. Current conservation is guaranteed at any value of the cut-off as long as matrix elements are consistently evaluated with NN wave functions constructed from the meson exchange Hamiltonian. The conditions for finiteness for both deuteron, scattering and electromagnetic matrix elements of longitudinal MEC’s coincide.

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