Evaluation of Simple Shear Test Geometries for Constitutive Characterization using Virtual Experiments

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Abstract. In-plane simple shear tests have become commonplace in the fracture characterization of automotive sheet metals but have received less attention for constitutive characterization. Unlike tensile tests, simple shear tests do not have any tensile instability and remain in a state of plane stress until fracture. From plastic work equivalence, an isotropic hardening model can be readily constructed from the tensile and shear test data without inverse finite-element analysis. The success of the methodology hinges upon the shear specimen geometry and how the local strains in the gage region are measured using digital image correlation (DIC). In this study, finite-element simulations of seven shear test geometries were evaluated for an isotropic material in a series of virtual experiments by varying the input hardening response. The data from the simulations was extracted from the surface as if DIC was employed and used to determine the hardening behavior in comparison with the exact solution. Shear geometries without a notch eccentricity in the gage region appear to be best suited for characterizing low hardening materials with an error of less than 1% in the stress response for an n-value of 0.02. Conversely, for higher hardening materials corresponding to an n-value of 0.20 or greater, the geometries with a notch eccentricity performed best.

1. Introduction

The ever-present need for accurate constitutive characterization of sheet metals to representative strain levels found in automotive forming and crash simulations has led to a variety of experimental and numerical techniques to analyze tensile tests past the onset of diffuse necking. The use of inverse finite-element analysis (FEA) of tensile tests using a hybrid experimental-numerical approach has become common [1] but the results are dependent on the simulation parameters such as element type, mesh size and yield function. Biaxial bulge [2] and shear-based tests [3-4] are attractive alternatives to inverse modelling since the hardening behavior can be experimentally measured to larger strain levels than the tensile test. In-plane shear tests are of particular interest to the automotive sector since the shear fracture strain can also be measured to support the calibration of a stress-state dependent fracture model.

Numerous in-plane simple shear test geometries have been proposed within the literature [3, 5-7] but there has been little guidance on which geometry to select based upon the expected hardening response. Yin et al. [8] previously compared the suitability of three different shear specimens for constitutive characterization but did not cover a range of hardening behaviors. Roth and Mohr [7] considered the effects of material ductility and hardening behavior to develop three optimized geometries for shear
fracture characterization but did not assess their performance for constitutive characterization. The design of an in-plane shear specimen must balance the need for a sufficiently large shear zone in the gage region to initiate shear fracture, which is beneficial for constitutive characterization, with the need to induce an inhomogeneous strain gradient to avoid premature tensile failure near the notch edges. The choice of appropriate shear fracture geometry is ultimately specific to the material of interest and several geometries may need to be considered as suggested by Roth and Mohr [7]. Fortunately, the performance of shear geometries for constitutive characterization can be analyzed in a more general manner so that when an analyst selects a geometry for fracture, meaningful constitutive data can also be obtained.

The objective of the current study is to evaluate the suitability of seven simple shear test geometries for constitutive characterization across a range of hardening behaviors using virtual experiments. The force and strain data can be extracted from the numerical models to mimic the measured force and surface strains available to an analyst performing shear tests with digital image correlation (DIC) strain measurement. The shear data can then be analyzed like the experiments to calculate the shear stress and the work conjugate equivalent stress-strain response using the so-called shear conversion method of Rahmaan et al. [9]. The performance of each shear geometry will be shown to be contingent upon the expected hardening behavior, with different geometries recommended for high versus low hardening materials.

2. Geometries
The choice of shear geometry was restricted to designs with a single gage region and uniform thickness that can be tested on a universal tensile test frame. Seven shear specimens were considered for the virtual experiments and divided into two categories based on whether there is an eccentricity or offset between the notches in the gage section. The ASTM standard specimen [5], the ‘Modified ASTM’ specimen proposed by Merklein and Biasutti [6] and the ‘V-notch’ specimen used in Achani et al. [10] do not have an eccentricity and are depicted in Figure 1 (a-c) respectively. The ASTM sample was modified by Merklein and Biasutti [6] as shown in Figure 1(b) to facilitate reverse loading under shear. The V-notch specimen in Figure 1(c) was employed by Achani et al. [10] for evaluating calibrated yield surfaces of aluminum extrusions and subsequently by Granum et al. [11] for fracture modelling.

![Figure 1. Dimensions of the gage section for the specimens having no eccentricity or offset. (a) ASTM B831 specimen [5], (b) Modified ASTM specimen from Merklein and Biasutti [6] and (c) V-notch specimen from Achani et al. [10]. Figures not to scale. All dimensions are in millimetres.](image)

Peirs et al. [3] developed the so-called mini-shear specimen with an eccentricity of 1.0 mm between the circular notches as shown in Figure 2 (a) to maintain a simple shear stress state during deformation. The geometry was designed using a high hardening model material of a titanium alloy with an $n$-value of 0.47. The three shear geometries of Roth and Mohr [7] shown in Figure 2 (b-d) were developed using the mini-shear geometry as a baseline. Three distinct notch shapes and eccentricities were proposed based upon the expected hardening and ductility and were classified as low, moderate and high-ductility specimens or LD, MD and HD, respectively. The notch shapes of the three specimens developed by Roth and Mohr [7] are not circular like the mini-shear but rather exhibit a functional dependence on the
notch width and the coordinate frame. The parametric equation of the notches is not described here for brevity.

Figure 2. Dimensions of the gage section for the geometries having notch eccentricity. (a) Mini-shear specimen of Peirs et al. [3], (b) Low-ductility (LD), (c) Moderate-ductility (MD) and (d) High-ductility (HD) shear specimens proposed by Roth and Mohr [7]. All dimensions are in millimetres.

3. Virtual Experiments

3.1. Materials

The model material for the virtual shear experiments was assumed to have the elastic properties of steel, a sheet thickness of 1.0 mm, and obey the Swift’s power-law hardening model

\[ \sigma = K(\varepsilon_0 + \varepsilon^p)^n \]  

(1)

where \( K \), \( \varepsilon_0 \) and \( n \) are the coefficients of the Swift model and \( \varepsilon^p \) is the equivalent plastic strain. The values of \( K \) and \( \varepsilon_0 \) were taken as 1200 MPa and 0.001, respectively. To cover a broad range of hardening behaviors typical of automotive steels, three hardening exponents of 0.02, 0.10 and 0.20 were selected with the corresponding hardening responses shown in Figure 3. The \( n \)-value in Swift’s law with \( \varepsilon_0 = 0.001 \) is approximately equal to the strain at the onset of diffuse necking in a tensile test according to the Considère criterion.

Figure 3. Input hardening response for the three hardening exponents considered.
3.2. Finite-element modelling of the simple shear tests

The seven simple shear geometries were modelled in LS-DYNA using fully integrated shell elements (ELFORM=16) with three through-thickness integration points using explicit time integration due to the plane stress nature of shear deformation. The second-order Jaumann rate was used in LS-DYNA by invoking the objective stress update (OSU) option. An approximately uniform mesh size of 0.05 mm was maintained in the gage region and selected from a mesh convergence study based upon the local plastic strain. A representative mesh pattern near the gage region is shown for the mini-shear specimen in Figure 4(a). All shear geometries were modelled as if they were clamped within the grips of a tensile test frame and subjected to a displacement-controlled boundary condition from one end with a cross-head velocity of 0.003 mm/s, as depicted schematically in Figure 4(b) for the mini-shear specimen.

![Figure 4](image_url)

**Figure 4.** (a) Pattern of the mesh in the gage region for the mini-shear specimen, and (b) schematic of the boundary conditions applied to the blank in the numerical model of mini-shear

Yielding and plastic flow of the material was assumed to adhere to the isotropic von Mises yield criterion (MAT_24 in LS-DYNA). The shear simulations were terminated once the force reached its peak value so that the conversion of the shear stress to an equivalent stress would not predict an erroneous softening response.

3.3. Methodology to determine the isotropic hardening behaviour to large strains using shear tests

The experimental methodology of Rahmaan *et al.* [9] was used to convert the shear test data to the equivalent stress and plastic strain through plastic work equivalence. The applied shear stress, \( \tau \), was calculated from the reaction force, \( F \), initial gage length, \( l \), and initial thickness, \( t \), for each geometry as

\[
\tau = \frac{F}{lt}
\]  

(2)

The in-plane normal and shear strains of all the elements inside a square box of size 0.2 × 0.2 mm\(^2\) at the center of the gage section was then extracted, as shown in Figure 5, to imitate the DIC measurement procedure of Rahmaan *et al.* [9]. Subsequently, the maximum in-plane shear strain, \( \varepsilon_{12}^{\text{max}} \), of the elements were calculated and then averaged.

\[
\varepsilon_{12}^{\text{max}} = \sqrt{\left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)^2 + \varepsilon_{12}^2}
\]  

(3)

The plastic work from the input hardening curve and the applied shear stress were then calculated using
\[ w_{eq}^p = \int \sigma_{eq} d\varepsilon_{eq}^p; \quad w_{shear}^p = \int 2\tau d\varepsilon_{12}^{\text{max}-p} \] (4)

\[ \varepsilon_{12}^{\text{max}-p} = \varepsilon_{12}^{\text{max}} - \frac{\tau}{2G} \] (5)

in which \( \sigma_{eq} \) is the equivalent stress, \( d\varepsilon_{eq}^p \) is the work-conjugate equivalent plastic strain increment, and \( G \) is the shear modulus, respectively. The ratio of the shear stress to equivalent tensile stress, or so-called shear stress ratio, \( T \), was calculated at the plastic work corresponding to the onset of diffuse necking in the uniaxial tensile test, \( w_n^p \),

\[ T = \frac{\tau}{\sigma_{eq}}(w_n^p) \] (6)

The shear stress ratio was then assumed to remain constant although its variation due to anisotropy and its evolution with the material frame rotation could be considered, as done by Abedini et al. [12]. The equivalent stress as a function of the plastic work for deformation levels beyond \( w_n^p \) was then determined, along with the equivalent plastic strain increment (assuming work equivalence between tension and shear),

\[ \sigma_{eq}(w^p) = \frac{\tau}{\tau}; \quad d\varepsilon_{eq}^p = \frac{dw^p}{\sigma_{eq}(w^p)} \] (7)

To evaluate the deviation between the exact hardening response utilized in the simulations and the measured or predicted stress response from the analysis of the virtual shear tests, the error in the stress response was quantified as

\[ Error(\%) = \frac{\sigma_{\text{measured}} - \sigma_{\text{input}}}{\sigma_{\text{input}}} \times 100 \] (8)

Elements within a square box of 0.2 × 0.2 mm² for average max. shear strain calculation in the gauge region

![Figure 5. Square region of interest used to extract the local strains for the shear test analysis](image)

4. Results and Discussion

4.1. Constitutive response for low hardening materials (n=0.02)

The equivalent stress-strain curves predicted by the shear specimens after the true strain at uniform elongation (2% strain) along with the errors in the stress response are summarized in Figure 6(a,b)
respectively. Note that the input hardening response is used until a plastic strain of 2% and is assumed to be available from a prior tensile test. It can be seen that the shear geometries without any notch eccentricity were able to closely approximate the actual hardening response to within 1% error. The ASTM specimen was accurate to the largest strain level of nearly 0.40 while the V-notch shear was only valid until about 0.20 strain. In contrast, all shear geometries with an eccentricity significantly overestimated the hardening response by 4-8% and therefore do not appear to be suitable for characterizing low hardening materials.

Figure 6. (a) Comparison of the input and measured hardening response of all the shear geometries evaluated for a low hardening material ($n=0.02$) and (b) error in the stress response with plastic strain

4.2. Constitutive response for moderate hardening materials ($n=0.10$)
The exact and measured hardening responses for a material with a moderate hardening rate ($n=0.10$), along with the error in stress response, are shown in Figure 7(a, b). All of the geometries produced some deviation from the exact solution, as seen in Figure 7 (b) with the V-notch and mini-shear samples outperforming the other geometries with an error of ~1% at an equivalent plastic strain of about 0.40. The shear specimens designed for fracture characterization overestimated the stress response, particularly the low-ductility geometry. Overall, the geometries without a notch eccentricity underestimated the stress response while those with an eccentricity overestimated the stress.

Figure 7. (a) Comparison of the input and measured hardening response of all the shear geometries for a moderate hardening material ($n=0.10$) and (b) error in the measured stress response with plastic strain
4.3. Constitutive response for high hardening materials \((n=0.20)\)

The hardening responses obtained from the seven shear geometries for a high hardening material are shown in Figure 8 with the accumulated error depicted in Figure 9. Overall, it can be seen that the shear geometries with a notch eccentricity are better suited for characterizing high hardening materials with the exception of the V-notch specimen. The high-ductility shear geometry limited the maximum error to less than 0.5% and closely recreated the exact solution. The V-notch, mini-shear, low-ductility and medium-ductility geometries also captured the response of the material with errors on the order of 1% at an equivalent plastic strain of 0.60 while the ASTM and modified ASTM specimens underestimated the stress by approximately 2% and 4%, respectively. Although not shown for brevity, similar trends were observed in the shear geometries when evaluated for a higher hardening exponent of 0.30.

![Figure 8](image1.png)

**Figure 8.** Comparison of the exact and measured hardening response of shear geometries evaluated for a high hardening material \((n = 0.20)\) for specimens with (a) no eccentricity and (b) notch eccentricity

![Figure 9](image2.png)

**Figure 9.** Error in the hardening response for all shear geometries for a high hardening \((n=0.20)\) material
5. Conclusions

Seven different simple shear geometries with a single gage region were adopted from the literature and evaluated for constitutive characterization of a generic steel sheet material exhibiting shear isotropy and a range of hardening behaviors. The primary conclusions that can be drawn from this study are:

- For materials that exhibit a low hardening rate ($n=0.02$), the shear specimens without a notch eccentricity are recommended for constitutive characterization. Shear geometries with an eccentricity systematically overestimated the stress response.
- The overall performance of all the shear specimens was comparable for a moderately hardening material ($n=0.10$). The V-notch and mini-shear specimens performed best with an error in the stress limited to 1% or less at an equivalent plastic strain of 0.40.
- Shear geometries with a notch eccentricity appear best suited for constitutive characterization of materials with a hardening exponent of 0.20 or higher, since the error was 1% or less at an equivalent plastic strain of 0.60. Shear geometries without an eccentricity systematically underestimated the stress response.
- The observed performance of the shear geometries is specific to the modeling assumptions in the analysis and did not consider fracture. It is possible that the best performing geometries for constitutive characterization may not be optimal for fracture characterization and will depend upon the material of interest.

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