Monte Carlo study of the depth-dependent fluence perturbation in parallel-plate ionization chambers in electron beams

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Purpose: The electron fluence inside a parallel-plate ionization chamber positioned in a water phantom and exposed to a clinical electron beam deviates from the unperturbed fluence in water in absence of the chamber. One reason for the fluence perturbation is the well-known “inscattering effect,” whose physical cause is the lack of electron scattering in the gas-filled cavity. Correction factors determined to correct for this effect have long been recommended. However, more recent Monte Carlo calculations have led to some doubt about the range of validity of these corrections. Therefore, the aim of the present study is to reanalyze the development of the fluence perturbation with depth and to review the function of the guard rings.

Methods: Spatially resolved Monte Carlo simulations of the dose profiles within gas-filled cavities with various radii in clinical electron beams have been performed in order to determine the radial variation of the fluence perturbation in a coin-shaped cavity, to study the influences of the radius of the collecting electrode and of the width of the guard ring upon the indicated value of the ionization chamber formed by the cavity, and to investigate the development of the perturbation as a function of the depth in an electron-irradiated phantom. The simulations were performed for a primary electron energy of 6 MeV.

Results: The Monte Carlo simulations clearly demonstrated a surprisingly large in- and outward electron transport across the lateral cavity boundary. This results in a strong influence of the depth-dependent development of the electron field in the surrounding medium upon the chamber reading. In the buildup region of the depth-dose curve, the in–out balance of the electron fluence is positive and shows the well-known dose oscillation near the cavity/water boundary. At the depth of the dose maximum the in–out balance is equilibrated, and in the falling part of the depth-dose curve it is negative, as shown here the first time. The influences of both the collecting electrode radius and the width of the guard ring are reflecting the deep radial penetration of the electron transport processes into the gas-filled cavities and the need for appropriate corrections of the chamber reading. New values for these corrections have been established in two forms, one converting the indicated value into the absorbed dose to water in the front plane of the chamber, the other converting it into the absorbed dose to water at the depth of the effective point of measurement of the chamber. In the Appendix, the in–out imbalance of electron transport across the lateral cavity boundary is demonstrated in the approximation of classical small-angle multiple scattering theory.

Conclusions: The in–out electron transport imbalance at the lateral boundaries of parallel-plate chambers in electron beams has been studied with Monte Carlo simulation over a range of depth in water, and new correction factors, covering all depths and implementing the effective point of measurement concept, have been developed. © 2014 American Association of Physicists in Medicine.

Key words: Monte Carlo simulations, electron dosimetry, parallel-plate chambers, perturbation corrections
1. INTRODUCTION

When a parallel-plate ionization chamber is placed in a water phantom exposed to an electron beam, the fluence of primary and secondary electrons at points within the gas-filled volume of the chamber deviates from that at the corresponding points of the displaced volume of the phantom material. This perturbation, compared to ideal Bragg–Gray conditions, is due to the strong reduction of the energy losses and multiple scattering of the electrons in the gas-filled cavity compared with the energy losses and multiple scattering in the same cavity if it were filled with the surrounding phantom material. The result is a disturbance of the transport of electrons into and out of the cavity in comparison with a cavity filled with the phantom material.

These physical effects have already been discussed in ICRU Report 35. This report strongly influenced the common understanding of plane-parallel chambers’ behavior in electron beams, and all present dosimetry protocols can be traced back to the principles summarized in this report. Moreover, the construction of modern parallel-plate ion chambers with respect to the dimensions of the cavity and guard rings is based on the recommendations given there.

Fig. 1, based on an experiment performed by Svensson, illustrates the physical characteristics of parallel-plate chambers according to ICRU Report 35. At the lateral boundary surface between air cavity and water, the figure shows an oscillation of the dose profile measured in the bottom plane of the cavity, and across the collecting electrode C an approximately homogeneous dose profile is seen at all depths. This oscillation, as theoretically explained by Harder, is due to the fact that multiple scattering of electrons is negligible in the air-filled cavity while uncorrected multiple scattering occurs within the adjacent water. Thereby more electrons are scattered into than out of the cavity, and the short hand term “inscattering effect” has been coined to describe this positive balance of the in-bound and out-bound electron transport.

To make the chamber signal insensitive to the in–out electron transport imbalance, modern parallel-plate chambers are equipped with a wide guard ring, thereby attempting to keep the region of fluence perturbation at a safe distance from the collecting volume. According to the recommendations of the IAEA TRS-398 dosimetry protocol, the guard ring width should not be smaller than 1.5 times the cavity height, and a chamber design satisfying this requirement is considered “well-guarded.” Based on these considerations, all present dosimetry protocols recommend to use for electron dosimetry well-guarded parallel-plate ion chambers, and for these chambers the perturbation correction for the inscattering effect, $p_{\text{cav}}$ [IAEA TRS-398 (Ref. 2)] and $p_{\text{fl}}$ [AAPM TG-51 (Ref. 3)], is assumed to be unity. For chambers only equipped with narrow guard rings like the PTW-Markus chamber, a notable perturbation of the signal by the inscattering effect exists, and the necessary correction factor has been studied experimentally and by Monte Carlo simulation.

In the experimental study performed by Van der Plaetsen the correct value of the chamber signal was assumed to be supplied by a chamber with an “ideal” guard ring, and accordingly, the TRS-398 protocol as well as the German standard DIN 6800-2 (2008) (Ref. 5) give a fluence perturbation correction $p_{\text{cav}}$ for the Markus chamber for the reference depth $z_{\text{ref}}$ as function of the beam quality specifier $R_{50}$

$$\left( p_{\text{cav}} \right)_{R_{50}} = 1 - 0.037 \cdot e^{-0.27 \cdot R_{50}}. \quad (1)$$

The relevant experiments summarized in IAEA TRS-381 (Ref. 12) were all performed at the depth of the dose maximum, and the mean electron energies at these depths varied from 3.0 to 20 MeV.

However, it has been discussed that the rule according to which the guard ring width should not be smaller than 1.5 times the cavity height might be insufficient to completely avoid the inscattering effect, since for a fraction of the electrons, increasing with depth, the directions of flight form rather large angles with the original beam direction. Moreover with increasing depth, energy loss and multiple scattering of the electrons are accompanied by a third effect, the reduction of the electron fluence due to range straggling of the electrons, which has an additional influence on the in–out balance of the electron fluence at the lateral boundary surface of the cavity. Actually, we have to consider three typical depth regions, namely, (a) shallow depths where the fluence of electrons in the region lateral from the cavity increases with increasing depth due to multiple electron scattering [Fig. 2(a)], (b) the region of the depth-dose maximum where the fluence in the lateral region shows little change with depth [Fig. 2(b)], and (c) the region of large depths where the fluence in the lateral region falls with increasing depth due to range straggling [Fig. 2(c)]. Evidently, these effects will affect the in–out balance of the electron fluence at the lateral cavity boundary. Thus, we arrive at the insight that
Monte Carlo simulation of the penetration of a divergent electron beam with energy $E_0 = 6$ MeV into a water phantom containing an embedded cavity (filled with water of density 1.293 mg/cm$^3$) of 0.4 cm thickness and 2 cm radius with its front face at 5, 14, and 26 mm depth. For the Monte Carlo simulations, the cavity was divided into slices of 1 mm height. The curves show the transverse profiles of the absorbed dose to water within the four cavity slices (a) at a shallow depth, (b) near the depth-dose maximum and, (c) near the half-value depth $R_{50}$. The oscillations of the profiles near the lateral cavity boundary, most expressed at the bottom of the cavity, are largest at shallow depth but mostly disappear at large depth, where they are replaced by S-shaped curve wings. The statistical uncertainty of the Monte Carlo results is smaller than the symbol width.

the experiments by Van der Plaetsen et al.,$^9$ performed under the conditions typical for the dose maximum, do not exactly represent the conditions of shallow and large depths where the depth-dependent fluence or dose gradient in the phantom material is positive or negative. Moreover, this raises the issue that the cavity correction factor stated in recent protocols to hold at $d_{ref}$ was regarded as equal to the correction experimentally determined at $d_{max}$; this may become a significant effect for 18 MeV electrons and higher energies where $d_{ref}$ is in the falling portion of the depth-dose curve.

The consequence of these considerations is to acknowledge the need for a depth-dependent correction of the indicated value of the chamber which also accounts for the impact of these fluence gradients upon the in–out imbalance of the electron fluence at the lateral cavity boundary. In order to develop this correction, we will use spatially resolved Monte Carlo simulations inside the air gap of parallel-plate ionization chambers to analyze the magnitude of the fluence perturbations. On this basis, we will develop a new depth-dependent cavity correction factor for the signals of parallel-plate chambers in clinical electron beams. An alternative way to account for these fluence perturbations is a shift of the effective point of measurement (EPOM) as already proposed and experimentally verified by Roos et al.$^{13}$ The influence of an EPOM shift is also investigated here by spatially resolved Monte Carlo simulations of the dose distribution in gas-filled cavities.

The main purpose of this study has not particularly been to provide an updated cavity correction for the Markus chamber, today a still respectable, but already historical design of a parallel-plate chamber for electron dosimetry. Rather, the central aim has been to investigate the surprisingly large but not well known influence of the wide angular distribution and of range straggling of the electron beam at depths beyond the dose maximum upon the fluence imbalance at the boundary surface of a flat, gas-filled cavity in general. Insofar, the present study is a continuation of our previous work about the perturbation corrections of parallel-plate chambers.$^{14,15}$ The results will be applicable not only to the Markus chamber but also to the more recent designs of flat ionization chambers for electron beam dosimetry such as the Roos, Exradin A10, and the Advanced Markus chamber. Similarly to the experimental study performed by Johansson,$^8$ we only consider wall-less cavities, i.e., the impact of the chamber wall always present in real ionization chambers will not be investigated.
2. METHODS

The Monte Carlo simulations were performed with the code system EGSnrc\textsuperscript{16,17} (release V4 2.4.0) applying the user code egs-chamber\textsuperscript{18} (release 1.21). All geometries were modeled with the egs++ geometry package.\textsuperscript{19} To investigate the inscattering effect of gas-filled cavities, Monte Carlo calculations were performed for coin-shaped cavities with radius $a$ placed in a water phantom with their entrance plane at depth $z$. The cavity height $\xi = 0.2$ cm was chosen comparable to the heights of commercially available parallel-plate chambers used for clinical electron dosimetry. In order to provide spatial dose resolution within the gas-filled cavity and the surrounding water, the cavity itself and also the surrounding water layer were subdivided in cylindrical scoring zones with variable radius $r$ [see Fig. 3(f)]. The dose was scored within these zones, and for a zone extending from $r$ to $r + \Delta r$, the resulting dose $D(r)$ was understood as the mean value over cavity height $\xi$ and zone width $\Delta r$. Depending on $r$, the value $\Delta r$ varied between 0.01 and 0.1 cm, resulting in 13 cylindrical zones for the smallest cavity with $a = 0.3$ cm and 28 for a cavity with $a = 1.3$ cm.

In order to avoid the calculation of stopping power ratios $s_{w,\text{gas}}$, all spatially resolved doses for radii $r$ inside the cavity were understood as absorbed doses $D_{\text{cav}}(r)$ to “low-density water” (LDW), i.e., water with the density of air but with a density correction corresponding to normal density water, and all doses for radii $r$ outside the cavity were absorbed doses to water, $D_w(r)$. Wang and Rogers\textsuperscript{20} have shown that for electron energies below 30 keV there is only a small fluence perturbation due to material differences between air and low-density water. From the surface to the depth $R_{50}$, this perturbation results in a small depth dependence of the ratio $D_{\text{LDW}}/(D_{w} s_{w,\text{gas}})$ of less than 0.2%. For spatially resolved calculations, we have as well proved in a preliminary study that within statistical uncertainty limits of 0.2%, the spatially resolved dose distributions were the same for air or LDW filling when the former were multiplied by the mass stopping power ratios. The small depth dependencies due to the material differences mentioned above are neglected here. Total perturbation correction factors $p$ for gas-filled cavities at depth $z$ were calculated as the ratio

\begin{equation}
    p_{\text{cav}} = \frac{D_w(z)}{D_{\text{cav}}},
\end{equation}

where $D_w(z)$ is the dose to water and $D_{\text{cav}}$ is the average of $D_{\text{cav}}(r)$ over the whole cavity (within the radial region of the collecting electrode) with the cavity’s entrance plane positioned at depth $z$. The impact of a guard ring on the cavity dose $D_{\text{cav}}$ and therefore on $p_{\text{cav}}$ can be calculated from the spatially resolved dose calculations by integrating the dose not over the whole cavity radius $0 \leq r \leq a$ but over the interval $0 \leq r \leq r_C$, where $r_C$ is the radius of that part of the electric field which causes ion charge collection upon the central electrode. For brevity, $r_C$ will be denoted as the “collection electrode radius,” and $a - r_C$ as the “guard ring width.”

For all dose calculations, a divergent electron beam was incident on a cubic water phantom ($30 \times 30 \times 30$ cm\textsuperscript{3}), the field size was $10 \times 10$ cm\textsuperscript{2} at the source-to-surface distance 100 cm. Since scattering effects are largest for low electron energies, all calculations were performed with a spectrum of a clinical linear accelerator of nominal energy 6 MeV [Varian Clinac (Ref. 21)] whose 50% range was $R_{50} = 2.63$ cm and whose reference depth was $z_{\text{ref}} = 1.48$ cm. To calculate the perturbation corrections $p_{\text{cav}}$ according to Eq. (2), the highly resolved depth-dose curve in water was calculated within cylindrical water voxels with radius $r = 0.5$ cm and height $h = 0.002$ cm.

In preliminary simulations, the influence of different cutoff/threshold energies upon the photon and electron transport was investigated. The impact of decreasing the cutoff energy from 10 keV for photons and 521 keV for electrons to 1 and 512 keV was <0.1% for the spatially resolved simulations, but the simulation times were increased by a factor of 4. Therefore, it was decided to perform all simulations with cutoff/threshold energies of 10 keV for photons and 521 keV for electrons. Except the bremsstrahlung cross section data [NIST instead of Bethe–Heitler bremsstrahlung cross sections (Ref. 17)] all transport options within the EGSnrc system were set to their defaults.

In order to determine the in–out imbalance of electron transport between the cavity and the surrounding water due to differences in scattering and range straggling, the different geometries shown in Fig. 3 were realized. Geometry (a) is the simplest one, where the cavity is placed in the water phantom at depth $z$, irradiated with a clinical electron spectrum of primary energy $E_0$. The radius $a$ of the cavity was varied in the range from 0.3 cm, complying with the radius of the Markus chamber, to 1.3 cm, close to the radius of the Roos chamber. The height of all cavities was 0.2 cm. In geometry (b), a very thin slab of water ($\Delta z = 0.0001$ cm) in front of the cavity was introduced with the cutoff energy for electron transport, ECUT, set larger than the primary electron energy $E_0$, so that all electrons bound to enter the cavity directly through the front surface were stopped in front of the cavity.

Fig. 3. (a)–(e) Simulation geometries: the cavity filled with low-density water (white) is surrounded by water (gray). The thick black lines symbolize thin slabs of water ($\Delta z = 0.0001$ cm) with cutoff energy $ECUT > E_0$ (energy of primary electrons). (f) Top view of the simulation geometry. To get spatial information about the dose deposition, the cavity and the surrounding slab of water is divided into cylindrical scoring zones of width $\Delta r$ varying from 0.01 to 0.1 cm.
In this geometry, the electrons could only enter the cavity via the side or rear surface. In geometry (c), an additional ECUT > $E_0$ region was placed behind the cavity. In that case no electron can enter the cavity through the rear surface, and also all electrons coming in through the lateral surface and being backscattered at the rear surface are missing. In geometry (d), an ECUT > $E_0$ region was introduced at depth $z$ for all radii outside the cavity so that no electron from the outside region can reach the cavity through the lateral or rear surfaces. Geometry (e) is the same as in (d) but with an ECUT region placed at the rear surface to prevent any backscattering from the material behind the cavity.

3. RESULTS

3.A. Depth dependence of the electron fluence perturbation at the gas–water boundary

The electron transport phenomena occurring at the gas–water boundary of a parallel-plate ionization chamber, obtained by Monte Carlo simulation of the geometries defined in Fig. 3, are illustrated in Fig. 4. The simulations were performed for a cavity of radius $a = 0.3$ cm and height 0.2 cm filled with LDW and placed at depth 0.5 cm within the water phantom, i.e., in the dose buildup region of an electron beam with $E_0 = 6$ MeV.

The dose profile (a) corresponds to the real geometry, with no ECUT > $E_0$ regions present. The oscillation of the transverse profile, already mentioned in the Introduction, is visible at the gas–water boundary surface, and its origin will be explained below. When a thin slab of water with ECUT > $E_0$ is introduced in front of the entrance surface, the resulting profile (b) represents the dose within the cavity due to electrons entering it through the lateral or the rear boundary surfaces (“inscattered” electrons). With an additional ECUT region behind the cavity, the dose profile (c) is obtained whose values are slightly smaller than those in geometry (b) because electrons now cannot enter the cavity from the rear, and electrons coming from the side and backscattered at the rear boundary surface are missing. In geometries (d) and (e), the region with ECUT > $E_0$ covers the whole field except the front surface of the cavity. The dose profile outside the cavity is now due to electrons that have entered the cavity through the front surface, leaving it mainly through the lateral boundary surface (“outscattered” electrons). In profile (d), the doses within the cavity are somewhat larger compared to profile (e) because (d) contains electrons backscattered at the rear surface.

At first sight, one would expect that the addition of dose profiles (b) and (d) should result in profile (a). However, as shown in Fig. 4, the sum of these two profiles within the cavity is about 1.5% smaller than profile (a) in the real geometry. This difference can be explained by the lack of backscattering of the inscattered electrons from the front surface of the cavity in geometry (b). Thus, the two essential components of the total dose profile are on the one hand the profile (d), exclusively due to electrons that have entered the gas from the cavity’s front side and may also be backscattered at its rear surface, and on the other hand the contribution (a)–(d) by all other electrons.

Figure 5 shows the result of this component analysis for several depths $z$. The predominant feature of (A) and (B) is the surprisingly large effect of electron transport across the lateral gas–water interface and its deep penetration toward the center of the cavity. Without the influence of this transport, i.e., for an infinitely large radius of the cavity, the ratio $D_{cav}(r)/D_w(z)$ in Fig. 5(A) would have the value 1.00 since in the almost complete absence of scattering and energy losses in the low-density water gas, the dose $D_{cav}(r)$ would equal $D_w(z)$, the dose to water in the entrance plane of the cavity. However for a real cavity radius, the “outscattering” or outbound electron transport, results in a considerable dose reduction even at the center of a cavity with 0.3 cm radius. At depth $z = 0.5$ cm, the extrapolated dose at the center of the cavity is reduced to $D_{cav}/D_w = 0.88$, at depth $z = 2.6$ cm even to the value $D_{cav}/D_w = 0.65$. Figure 5(B) shows the effect of “inscattering,” or inbound electron transport, which produces considerable dose values at the center of the cavity, namely, $D_{cav}/D_w = 0.14$ at depth $z = 0.5$ cm and $D_{cav}/D_w = 0.30$ at depth $z = 2.6$ cm. This obvious imbalance between inbound and outbound electron transport is caused by the difference between the almost complete absence of electron interaction events within the gas layer, in contrast to the
ongoing electron interactions in the bulk of water laterally from the cavity. The term “interactions” here refers to elastic scattering including backscattering, as well as energy losses, the production of secondary electrons, and even the appearance of track ends of the electrons, an important feature of the electron field in the falling region of the depth-dose curve.

This identification of the underlying physical effect as the inbound–outbound imbalance of electron transport between the cavity of a parallel-plate ionization chamber and its surrounding water medium especially influences the dose in a cylindrical cavity and the adjacent water at different depths $z$ in water. The geometries are labeled according to Fig. 3. The radius $a = 0.3$ cm of the cavity is marked by the dotted line. The statistical uncertainty of the Monte Carlo results corresponds to symbol width. The dose $D_w(z)$ was calculated at the depth of the entrance window.

As a consequence of the physical situation illustrated in Figs. 4 and 5, the inbound–outbound imbalance of electron transport between the cavity of a parallel-plate ionization chamber and its surrounding water medium especially influences the $D_{cav}(r)/D_w(z)$ ratio near the lateral gas–water boundary. The $D_{cav}(r)/D_w(z)$ ratio was therefore studied for various cavity radii from $a = 0.3$ to 1.3 cm. The left panel of Fig. 6 shows that the dose oscillation typical for the shallow depth $z = 0.5$ cm, so far obtained for cavity radius $a = 0.3$ cm (Fig. 5), regularly appears close to the cavity boundary whatever the cavity radius is. Its shape is always similar; there is merely a slight difference in the dose level reached in the region internal from the boundary, indicating a larger average inscattered dose in case of the smaller cavity radius. In analogy, the right panel of Fig. 6 shows that the “shoulder,” typical for the dose profile at the large depth $z = 2.6$ cm and already known from Fig. 5 for the cavity radius $a = 0.3$ cm, regularly appears with its steepest point at the cavity boundary whatever the cavity radius is. Again here, the levels of the dose reached in the region internal from the boundary are slightly different, indicating a larger average outscattered dose in case of the smaller cavity radius.

This obvious occurrence of the most inhomogeneous sections of the dose profiles near the gas–cavity boundary, clearly
Spatially resolved Monte Carlo simulations of the dose profiles within the LDW-filled cavity and the adjacent water at different depths \( z \) for different cavity radii. The geometries are labeled according to Fig. 3. The error bars represent the statistical uncertainty for the Monte Carlo results of all cavity radii. The vertical lines mark the cavity radius \( a \). The dose \( D_a(z) \) was calculated at the depth \( z \) of the cavity’s reference point, i.e., the depth of the entrance window.

Visible in experiments as well (Fig. 1), has lead to the idea of reducing the relative influence of these sections upon the measured value of the parallel-plate chamber by increasing the radius \( a \) of the chamber. In order to examine this idea, Fig. 7 shows the dose contributions by in- and outscattered electrons expressed as fractions of \( D_a(z) \), calculated for the (d) and (a)–(d) profiles of Fig. 5 by integration over the interval \( 0 \leq r \leq a \) for chambers with \( a = 0.3 \) and 1.3 cm at five different depths in the electron beam. As expected, the relative contributions of the in- and outscattered fractions and also the differences between them strongly decrease with increasing radius of the cavity.

Furthermore, Fig. 8 describes the variation of the extrapolated dose in the center of the cavity as a function of the cavity’s radius \( a \) for different depths \( z \). The center dose \( D_{\text{cav}}(r = 0) \) was approximately obtained as the mean dose of the cavity within \( r \leq 0.1 \) cm. With increasing radius and therefore with decreasing influence of the in- and outscattered fractions, the relative doses in the cavity center are tending toward value 1.00, but even a radius of 2 cm is not sufficient for the doses in the center to perfectly reach this limit value. This tendency, owed to the deep radial penetration of the in- and outbound transport of electrons into the gas-filled cavity, has been the reason for the choice of a comparatively large collecting electrode radius, namely, \( r_C = 0.78 \) cm, for the Roos chamber.

The other idea to reduce the relative influence of the near-boundary sections of the disturbed dose profile upon the measured value of a parallel-plate chamber is to use the guard ring, originally devised in order to shape the electric field in the chamber, as a means of excluding from the measured value any \( D_{\text{cav}}(r) \) contributions from \( r \geq r_C \), where \( a - r_C \) is the guard ring width. The influence of a guard ring has been analyzed in Fig. 9, where the ratio \( p = D_a/D_{\text{cav}} \) has been plotted versus depth in water for \( E_0 = 6 \) MeV for a set of different guard ring widths. The most prominent feature of Fig. 9 is the difference between the ordinate scales of the two panels which are valid for \( a = 0.3 \) and 1.3 cm,
Fig. 9. Total correction factor $p = D_{w}/D_{\text{cav}}$ as a function of scaled depth $z/R_{S0}$ for two cavities with different guard ring widths and different cavity radii $a$. The cavity dose $D_{\text{cav}}$ was determined from the spatially resolved Monte Carlo simulations by volume averaging over the collecting electrode radius $r_{C}$. The guard ring width (“guard”) was $a-r_{C}$. The dose $D_{w}$ is calculated at the depth $z$ of the front face of the cavity.

again indicating the already mentioned reduction of the fluence disturbance with increasing radius of the cavity as a consequence of the concentration of the effect close to the gas–water boundary.

The modification of $D_{w}(z)/D_{\text{cav}}(r)$ associated with a variation of the guard ring width is shown by the calculated points in Fig. 9. Narrow guard rings, in the left panel examined in combination with the small cavity radius $r = 0.3$ cm, have little effect on the deviation of $D_{\text{cav}}(r)$ from $D_{w}(z)$, whereas a guard ring width of 1.11 cm (right panel) can significantly reduce this deviation. At the depth of the dose maximum, i.e., at $z = 1.4$ cm or $z/R_{S0} = 0.53$, a guard ring of 0.5 cm width just happens to yield $p = 1$. With regard to this non-negligible, but not dominating effect, the introduction of a guard ring, e.g., of width 0.4 cm as for the Roos chamber, is not the instrument by which the influence of the inbound–outbound imbalance of electron transport upon the measured value of a parallel-plate chamber can be completely eliminated. Rather, as shown in Fig. 8, the cavity radius $a$ is a more effective instrument to reduce the deviation of the average dose to the gas from $D_{w}(z)$. This is the consequence of the deep radial penetration of the inward and outward transport of electrons into the gas-filled cavity mentioned above.

3.C. The effective point of measurement

In consideration of the strong influence of the inward and outward transport of electrons into the gas-filled cavity, and therefore of the gradient of the electron fluence field in the region laterally from the gas-filled cavity, upon the measured reading of a parallel-plate ionization chamber in an electron beam (Figs. 4, 5, 6, 8, and 9), one may question the underlying idea of regarding $D_{\text{cav}}(r)$ as the measurable quantity representative of $D_{w}(z)$, the dose in the entrance plane of the cavity. Rather, it is a plausible conjecture that $D_{\text{cav}}(r)$ might be more closely linked with the dose $D_{w}(z+\Delta z)$ at a slightly larger depth $z+\Delta z$ because that dose would be subjected to the influence of the gradient of the electron fluence field in the lateral region as well. The depth $z+\Delta z$ would then play the role of the “measuring depth” in the water phantom, and a point of the chamber at downstream distance $\Delta z$ from the entrance plane would appear as the “effective point of measurement” of the chamber, to be placed at the measuring depth. The effective point of measurement has been experimentally determined for the Markus chamber already by Roos et al.13 and for the Markus chamber and the Roos chamber by Looe et al.23

This idea has been the origin of plotting in Fig. 10 the radial profiles of ratio $D_{\text{cav}}(r)/D_{w}(z+\Delta z)$ for two values of $\Delta z$ for a cavity with $a = 1.3$ cm and thickness 0.2 cm at various water depths in a 6 MeV electron beam. It is shown that the ratio $D_{\text{cav}}(r)/D_{w}(z+\Delta z)$ is noticeably modified dependent on the choice of $\Delta z$, and there may even exist an optimum value of $\Delta z$, where the mean value $D_{\text{cav}}(r)/D_{w}(z+\Delta z)$ achieves such small depth dependence that this dependence could be neglected in clinical practice.

The search for this optimum value of $\Delta z$ has been performed by comparing the ranges of the depth-dependent variation of $p$ associated with various $\Delta z$ values. The result of this search is plotted in Fig. 11 for cavities with $a = 0.3$ cm and $a = 1.3$ cm and different guard ring widths. Accordingly, the ratio $p = D_{w}(z+\Delta z)/D_{\text{cav}}(r)$ does not vary more than $\pm 0.5\%$ over all depths up to $z = R_{S0} = 2.63$ cm, i.e., $z/R_{S0} = 1$. Figure 11 shows that there exists an EPOM which results in a very similar depth dependence of the resulting perturbation correction $p$ for every supposed guard ring width. Thus, for the Markus chamber with its narrow guard ring of 0.035 cm width, the effective point of measurement would lie at $\Delta z = 0.045$ cm, which is in perfect agreement with the Monte Carlo based value given by Wang and Rogers.22 Experimental investigations on the EPOM of the Markus chamber were performed by Roos et al.13 and Looe et al.23 Regarding the uncertainties of these data, the results given by Roos, $\Delta z = 0.05$ cm, and Looe, $\Delta z = (0.04 \pm 0.01)$ cm, are also in good agreement with the EPOM shift suggested here. For $a = 0.3$ cm, the only remaining correction would be to multiply the measured values of $D_{\text{cav}}$ by 0.99 in order to obtain $D_{w}(z+\Delta z)$. On the other hand, for a chamber with $a = 1.3$ cm and guard ring width 0.3 cm, the optimum would
lie at $\Delta z = 0.0155$ cm as shown in Fig. 11(B). For the Roos chamber with $a = 1.2$ cm and guard ring width of 0.4 cm Wang and Rogers\textsuperscript{22} published a value $\Delta z = 0.018$ cm. In our previous publication considering the Roos chamber,\textsuperscript{14} a value $\Delta z$ in the same range was determined.

While this optimization of $\Delta z$ would provide a depth-dependent variation of $p$ that might be negligible in clinical practice, this approach does not prevent determinations of the cavity correction factor $p$ with the highest possible accuracy, e.g., for $z = z_{\text{ref}}$ (in this example 1.48 cm). In Fig. 11(A) and for guard ring width 0.035 cm, this would mean $p(z_{\text{ref}}) = 0.987 \pm 0.001$. In Fig. 11(B) and for guard ring width 0.3 cm the result would be $p(z_{\text{ref}}) = 0.999 \pm 0.001$.

4. DISCUSSION

4.A. Comparison with earlier results

The well-known picture from ICRU Report 35,\textsuperscript{1} Svensson’s film-dosimetric demonstration of the oscillations of the dose profile of a 6 MeV electron beam near the gas–medium boundary shown here as Fig. 1, has raised concerns because in our calculation such oscillations were obtained at 0.5 cm depth but not at 1.4 cm depth (compare Figs. 4, 5, and 6). However, the conditions were somewhat different as we have here treated a cavity of 2 mm height, whereas Svensson’s dose values were obtained at the bottom of a 4 mm height cavity. Thus, his conditions are more closely simulated in Fig. 2 of our paper, where the oscillations have been reproduced at the bottom of a 4 mm high cavity even at 1.4 cm depth.

Depth-dependent measurements of the deviation of the $D_{\text{cav}}$ values from $D_w(z + \Delta z)$ have been performed by Laub et al.\textsuperscript{24} for a Markus chamber in a 10 MeV electron beam by comparison with a diamond detector. Although their experimental deviation varied only from $-2\%$ to $+3\%$, the calculated variation of the deviation from $-2\%$ to $+7\%$ for 6 MeV in Fig. 9 is consistent with the experimental result considering the difference in electron beam energy and of the additional uncertainty introduced by using the diamond detector as the reference.

Depth-dependent Monte Carlo calculations of the perturbation correction $p$ at electron energies up to 6 MeV have...
been performed by Lauterbach\textsuperscript{25} who found that the magnitude of the necessary correction is most effectively reduced by restricting the height of the cavity. The development of the Advanced Markus chamber was based on this investigation.

The message for the construction of guard rings to be derived from Fig. 9 is that for a cavity of 0.2 cm height and radius $a = 1.3$ cm, a guard ring width of at least 0.8 cm is required in order to completely eliminate the effect of the in–out electron transport imbalance on the measured value of $D_{\text{cav}}$ at the reference depth $z_{\text{ref}}$. The same result has been obtained experimentally at 6 MeV by Roos et al., as reported in IAEA TRS-381.\textsuperscript{12} Thus, the guard ring width of 0.4 cm chosen in the commercially available Roos chamber (PTW) is a practical solution in which a small deviation from the ideal by about 0.2\% is accepted. These numbers are all valid for the reference depth.

The impact of an EPOM shift on the resulting perturbation correction $p$ for a Markus-like cavity (cavity radius $a = 0.3$ cm, guard width 0.035 cm) was already investigated by Wang and Rogers.\textsuperscript{10} Their Monte Carlo results, showing that the depth dependence of $p$ could be minimized by an EPOM shift of 0.045 cm, are in excellent agreement with our results. The data from Wang and Rogers have been included in Fig. 11(A). It should be noted that Wang and Rogers scored the dose within the whole active volume of the cavity whereas our data for the perturbation correction $p$ are calculated from the spatially resolved simulations by integrating the dose value $D(r)$ over the radius $r$ of the active volume, i.e., $0 \leq r \leq r_c$. The good agreement of both data sets can be taken as a validation of our spatially resolved dose calculations. In summary, all available comparisons with other results have shown consistency of our Monte Carlo values with the previous experimental and computational insight into the causes affecting the measured values of $D_{\text{cav}}(r)$ for parallel-plate ionization chambers in electron beams. Particularly, the oscillations of the dose profile at shallow depths, the incomplete effect of too narrow guard rings and the turn of the sign of the ratio from about $\sim -2\%$ at shallow depths into several percent with positive sign at the larger depths, the consequence of the turn from an overshooting to an undershooting in–out imbalance of the electron transport at the lateral cavity boundary (see Fig. 5), are consistent results.

4.B. Recommended corrections

The cavity correction to be applied to the measured values of $D_{\text{cav}}(r)$ for Markus chambers at 6 MeV can therefore be directly obtained from the present Monte Carlo results and will be denoted by $p^{MC}$. One possibility is $p^{MC} = D_p(z) / D_{\text{cav}}$, i.e., to convert the measured value of the chamber into the dose at the depth of the entrance plane of the cavity. These values taken from Fig. 9 are replotted in Fig. 12. Their disadvantage is their considerable depth dependence. The other possibility is $p^{MC} = D_p(z + \Delta z) / D_{\text{cav}}$ for $\Delta z = 0.045$ cm, i.e., to convert the measured value of the chamber into the dose at the effective point of measurement of the chamber, whose values, taken from Fig. 11, have been replotted in Fig. 12. Evidently, the correction factor $p^{MC}_{\Delta z = 0.045}$ can in practice be regarded as a constant value of $0.990 \pm 0.005$, which would mean a considerable advantage for practical applications.

These new possibilities for the depth-dependent correction of the indicated values $D_{\text{cav}}$ of plane-parallel ionization chambers applied in electron beam dosimetry have to be compared with the previous recommendations. Perturbation correction factors experimentally determined by various groups for a set of flat ionization chambers, all obtained at the depth of the dose maximum and for mean electron energies at this depth from 3 to 20 MeV, have been collected in IAEA TRS-381.\textsuperscript{12} Somewhat later, in IAEA TRS-398,\textsuperscript{2} the same data for the Markus chamber, now recast to be valid for the reference depth $z_{\text{ref}}$, were represented by the fitting formula for the perturbation correction factor,

\begin{equation}
(p^{\text{Markus}})_{R_{50}} = 1 - 0.037 \cdot e^{-0.27 \cdot R_{50}} \quad (R_{50} \geq 2 \text{ g/cm}^2),
\end{equation}

where $R_{50}$ characterizes the incident electron beam. For our 6 MeV electron beam with $R_{50} = 2.63$ cm in water, this formula gives the diamond point in Fig. 12, whose closeness with the present Monte Carlo calculations is within 0.2\%. However, IAEA TRS-398 does not recommend perturbation corrections for other depths.

The German standard DIN 6800-2 (Ref. 5) still uses the form of the perturbation correction

\begin{equation}
(p^{\text{Markus}})_{R_{50}} = 1 - 0.039 \cdot e^{-0.28 \cdot E_{50}^1},
\end{equation}

originally recommended in IAEA TRS-381, which for the reference depth again yields the value indicated by the diamond symbol in Fig. 12. However, DIN 6800-2 also makes a first attempt to recommend a perturbation correction for other depths by again recasting this formula, always assuming that the in–out electron transport imbalance is the same as in the reference depth. This has led to a lengthy formula not reproduced here but plotted as the dashed-dotted line in Fig. 12. It is evident that this approach now needs to be corrected...
in consideration of the increased knowledge about the depth dependence of the in–out electron transport imbalance, as we have shown above.

For the more recent chamber designs such as the Roos, Exradin A10, and Advanced Markus chambers the very small depth dependence of \( p \) associated with the optimum choice of \( \Delta z \) as shown in Fig. 11(B) warrants to neglect this depth dependence in clinical practice.

5. CONCLUSION

When an ionization chamber is placed in a water phantom and exposed to an electron beam, the fluence of primary and secondary electrons at points within the gas-filled volume of the chamber deviates from that at the corresponding points of the replaced volume of the phantom material. For a plane-parallel ionization chamber, this fluence perturbation is due to the imbalance of the in- and outbound electron transport across the gas–water boundary surface. In the present investigation, these transport phenomena have been studied by means of a spatially resolved Monte Carlo simulation, which particularly illustrated the deep radially directed penetration of the fluence perturbance into the gas volume and demonstrated the depth dependence of this perturbance. The study of the effects of constructional countermeasures such as increased widths of central collecting electrodes and guard rings showed that some corrections of the indicated values for parallel-plate chambers introduced into clinical practice are still needed. These corrections have been numerically derived for Markus- and Roos-type cavities when applied in a 6 MeV electron beam (Fig. 11). Besides, the traditional correction which converts the indicated value of the chamber into the absorbed dose to water in the entrance plane of the chamber, a correction involving a strong depth dependence, an alternative correction with almost negligible depth dependence is now proposed, based on the idea to convert the indicated value into the absorbed dose to water at the depth of the effective point of measurement of the chamber. Compared with the presently standardized perturbation corrections for plane-parallel ionization chambers, no changes at the reference depths are required, but the depth-dependent correction factor originally proposed in the German standard DIN 6800-2 (Ref. 5) needs to be revised.

APPENDIX: MATHEMATICAL MODEL OF THE ELECTRON TRANSPORT IMBALANCE AT THE LATERAL BOUNDARY OF A FLAT, GAS-FILLED CAVITY IN WATER IN TERMS OF MULTIPLE SCATTERING THEORY

A mathematical model describing the transport of electrons into and out of a flat, gas-filled cavity and the adjacent water medium, based on the Fermi–Eyges multiple scattering theory26 as summarized in ICRU Report 35,\(^1\) will be briefly described here. Although the small-angle multiple scattering theory is a mathematical instrument correctly applicable only at shallow depths in an electron beam, it is illustrative for a qualitative discussion of the origin of the fluence disturbance at the lateral boundary of a flat, gas-filled cavity.

The geometrical layout is described in Fig. 13. A water phantom is exposed to a wide parallel beam of high-energy electrons. A cavity filled with gas (low-density water) of thickness \( \xi \) and width \( 2a \), thought to be infinitely long in the direction perpendicular to the drawing plane in order to provide a 1D problem, is positioned in the phantom with its front surface at depth \( z \). The scoring plane at which the lateral profile of the electron fluence will be considered is the bottom plane of the cavity at depth \( z + \xi \).

At depth \( z \), the electron beam is thought to have a uniform fluence profile along the \( x \)-axis. The directional distribution of the fluence of the electrons at the depth of the cavity’s front plane is characterized by \( \Theta^2(z) \), the mean square of the polar angle \( \Theta \) at depth \( z \). For low \( Z \) materials, it is sufficient to consider that \( \Theta^2(z) \) increases with \( z \) in an almost linear fashion up to a depth of about 50% of the practical range.\(^1\)

To calculate the profile of the electron fluence along the \( x \)-axis, including the lateral boundaries at \( x = \pm a \), the electron beam is subdivided into pencil beams starting at the front plane of the cavity. Their initial widths shall be zero, but their initial directional distribution shall be Gaussian, with mean square angle \( \Theta^2(z) \). In the description of the further passage of the electrons toward the bottom plane at depth \( z + \xi \), the well-known approximations valid for multiple scattering of electrons in thin layers of matter as described in ICRU Report 35 (Ref. 1) can be applied. Thus, for pencil beams originating from the front surface of the gas-filled volume and passing merely through the gas filling, i.e., with negligible multiple scattering, the mean square lateral displacement at the bottom plane \( z + \xi \) will be\(^1\)

\[
\sigma_{l}^2 = \Theta^2(z) \xi^2. \tag{A1}
\]

By contrast, for a pencil beam originating from the same plane, but outside the gas-filled volume, the mean square lateral displacement will be\(^1\)

\[
\sigma_{u}^2 = \Theta^2(z) \xi^2 + \int_{0}^{\xi} T(u)(\xi - u)^2 du = \Theta^2(z) \xi^2 + \frac{1}{3} T(z) \xi^3, \tag{A2}
\]

where \( T(u) \) is the linear scattering power of water valid for the energy spectrum of the electrons at depth \( u \); it is here...
assumed to maintain the constant value $T$ over the depth interval of the cavity. Note that $\sigma_g^2$ and $\sigma_w^2$ are mean square lateral displacements in the $x, y$ direction.

The fluence profiles resulting in plane $z + \xi$ due to the transmission of all pencil beams through the gas-filled volume, respectively, through the adjacent layer of water can be described by convolutions of the pencil beams with rectangular functions corresponding to the partial beams hitting the cavity and the adjacent water, respectively. The convolution of a rectangular function with a Gaussian kernel yields the error function, so that the resulting 1D lateral fluence profile and, neglecting changes of the stopping power ratio, absorbed dose to water profile in the $x$ direction valid for plane $z + \xi$ can be written as:

$$
\frac{D(x,z+\xi)}{D(z)} = \frac{1}{2}\left[erf\left(\frac{x+a}{\sigma_g^2}\right) - erf\left(\frac{x-a}{\sigma_g^2}\right)\right] \\
+ \frac{1}{2}\left[erf\left(\frac{x+a}{\sigma_w^2}\right) - erf\left(\frac{x-a}{\sigma_w^2}\right)\right],
$$

(A3)

The first term in Eq. (A3) describes the dose contribution by the electrons having passed the water layer lateral from the gas layer, i.e., having missed a water layer of width $2a$. Equation (A3) can be generalized by considering that the depth gradient of the dose in the water layer lateral from the gas layer due to multiple scattering and range straggling might be non-negligible, so that it will then take the more general form

$$
\frac{D(x,z+\xi)}{D(z)} = \frac{1}{2}\left[erf\left(\frac{x+a}{\sigma_g^2}\right) - erf\left(\frac{x-a}{\sigma_g^2}\right)\right] \\
+ A - \frac{1}{2}\left[erf\left(\frac{x+a}{\sigma_w^2}\right) - erf\left(\frac{x-a}{\sigma_w^2}\right)\right],
$$

(A4)

where $A > 1$ would be valid in the dose buildup region, i.e., at depths of the chamber front plane more shallow than that of the dose maximum, and $A < 1$ in the dose falloff region where range straggling prevails in the water medium lateral from the cavity. The mean square lateral displacements $\sigma_g^2$ and $\sigma_w^2$ are available from Eqs. (A1) and (A2).

It is immediately clear from considering Eq. (A4) that the first and the third term are similarly structured but with their denominators containing the slightly different parameters $\sigma_g^2$ and $\sigma_w^2$. Thus, the superposition of their lateral curve wings is expected to show a local oscillation. To give an example basically related to the Monte Carlo results shown in Fig. 2, we have numerically evaluated Eq. (A4) for 6 MeV electrons and a cavity with $a = 2.0$ cm and $\xi = 0.4$ cm, for the three cases

(a) $z = 5$ mm, $\sigma_g = 0.117$ cm, $\sigma_w = 0.141$ cm, $A = 1.06$ (i.e., in the buildup region),

(b) $z = 14$ mm, $\sigma_g = 0.195$ cm, $\sigma_w = 0.222$ cm, $A = 0.965$ (i.e., near the dose maximum),

(c) $z = 26$ mm, $\sigma_g = 0.265$ cm, $\sigma_w = 0.346$ cm, $A = 0.50$ (i.e., in the falloff region),

using the electron scattering power data of water at 4.5, 2.9, and 1.2 MeV from ICRU 35.1

As shown in Fig. 14, basically similar dose profiles have been obtained as in Figs. 1 and 2. This comprises the dose oscillation occurring (a) at small depths and (b) in the depth region of the dose maximum, as well as (c) the monotonous transition of the dose from its value in the cavity to that in the surrounding medium occurring in the falloff region of the depth-dose curve. This monotonous decrease tends toward a low asymptotic value, already coined by the range straggling which prevails in the surrounding water medium.

Admittedly, this annex uses the small-angle multiple scattering theory strictly applicable only at shallow depths, but it may serve to illustrate that the dose oscillation near the cavity’s lateral boundary is simply the consequence of the superposition

![Application of Eq. (A4) for a cavity of width $a = 2$ cm and height $\xi = 0.4$ cm for different depths $z$. The plotted radial dose profiles are valid for the bottom plane of the cavity, corresponding to slice 4 in Fig. 2. The model parameters $\sigma_g$, $\sigma_w$, and $A$ are given in the text.](image-url)
of the dose profiles of the electrons which have only traversed the gas-filled cavity and of those electrons which in the front surface plane of the cavity started with the same initial conditions but were slowed down and scattered in the surrounding water medium.

111707-13 Zink et al.: Spatial resolved Monte Carlo simulations in gas-filled cavities 111707-13

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