Summary of Improvement and Research Based on Robust Principal Component Analysis Model

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Abstract. Moving object detection play an important role in the application of computer vision. In recent years, the proposal and improvement of robust principal component analysis has broad application prospects in intelligent video surveillance and other fields. In order to enable domestic and foreign researchers to deeply explore and apply the RPCA, this paper systematically reviews it. This paper summarizes the latest research progress, summarizes various RPCA models at home and abroad, and theoretically analyses their advantages and disadvantages. In this paper, different improved models are applied to video sequences of different scenes, and comparative experiments are carried out. Overall, the current improved algorithms can effectively remedy the shortcomings of the original RPCA method and improve the accuracy of moving object detection. However, some limitations of RPCA need to be further studied.

1. Introduction
With the continuous development of information processing and data acquisition technology, the era of big data is coming. Massive data floods into people's vision, affecting people's lives. As an important information carrier, video and image are widely used in many fields such as image processing and traffic monitoring. However, the processing of these videos and images with important information often faces great challenges. On the one hand, as the number of videos and images increases exponentially, the larger the size of the data, the higher the dimension, which makes the analysis and processing of large-scale data very difficult. On the other hand, in these large-scale data, there are often vacancies, large errors, damage, occlusion and other issues. What is more challenging is that these videos and images themselves are more complex, such as dynamic background, changing illumination, bad weather, slow movement of foreground targets, and so on, which will also make data analysis and processing more difficult.

Fortunately, research and analysis have found that large-scale data often have greater correlation and redundancy[1]. Therefore, how to make full use of the high-dimensional data, and then effectively mine the useful information contained in it, is a widely concerned research topic in the field of image processing.

2. Proposal and solution of the original RPCA

2.1. Proposal and Basic Principles of Robust Principal Component Analysis Model
In 2000, Oliver et al. first proposed the method of principal component analysis (PCA) for background modeling[3]. The main idea is to assume that the original high-dimensional data is latent in a
Wright et al. first proposed the following optimization model: low-dimensional linear subspace, which can reduce dimensionality by eliminating the noise and redundancy of related variables. Specifically, the first step is to pull each frame size of an nm-dimensional matrix into a column vector of a x b = m. Then, the n column vectors drawn from n pictures are arranged in the form of an n x nm dimensional matrix, that is, X \in R^{nm}. The classical PCA finds the potential low rank matrix L through matrix X, and then obtains the constrained optimization model (2.1) as follows:

$$\min_{L,E} \|E\|_F \quad \text{s.t.} \quad \text{rank}(L) \leq r, X = L + E.$$  \hspace{1cm} (1)

For the problem (1), PCA mainly deals with the denoising and dimensionality reduction of Gaussian noise, but in practical problems, not all noises obey the Gaussian distribution. Based on this, in 2009, Wright et al. first proposed the following optimization model:

$$\min_{L,E} \text{rank}(L) + \lambda \|E\|_F \quad \text{s.t.} \quad X = L + E.$$  \hspace{1cm} (2)

In 2011, Cands et al. applied the model to the background modeling of surveillance video, and called the convex optimization model (2) PCA model. In order to characterize noise items, Zhou et al. proposed the RPCA model\cite{2}. Model (2) is a non-linear, non-convex and non-continuous combinatorial optimization problem. It will lead to NP-hard problems. To solve the problem, some scholars have proved that using the kernel norm approximation of the rank function is an effective method. Specific convex optimization model is as follows:

$$\min_{L,E} \|L\|_* + \lambda \|E\|_F \quad \text{s.t.} \quad X = L + E.$$  \hspace{1cm} (3)

### 2.2. Basic Solution of Robust Principal Component Analysis Model

After RPCA model was proposed, scholars proposed a series of numerical algorithms for solving the model, including Iterative Soft Threshold Method, Accelerated Proximal Gradient Method, Dual Method, Augmented Lagrange Multipliers Method, and Alternating Direction Method of Multiplier. The basic algorithm, which accelerates the convergence speed of the near-end gradient algorithm, has a lower number of iterations, so it has been widely used. In 2010, literature proposed to solve RPCA model by using the generalized Lagrange multiplier algorithm (ALM). ALM algorithm has faster calculation speed and higher efficiency, and it can achieve higher solution accuracy, while requiring lower storage space. Therefore, it is most widely used. The augmented Lagrange function of the original RPCA (4) is as follows:

$$L(L,E,Y,\mu) = \|L\|_* + \lambda \|E\|_F + \langle Y, X - L - E \rangle + \frac{\mu}{2} \|X - L - E\|_F^2.$$  \hspace{1cm} (4)

Then the core of the problem is transformed into solving the following sub-problems iteratively at each step:

$$\left(L_{k+1}, E_{k+1}\right) = \arg\min_{L,E} L(L_k, E_k, Y_k, \mu_k)$$  \hspace{1cm} (5)

First, update variable L:

$$L_{k+1} = \arg\min_{L} L(L, S_k, Y_k, \mu_k) = \arg\min_{L} \|L\|_* + \frac{\mu}{2} \|L - X + \mu^{-1} S_k Y_k\|_F^2 = U_S \mu^{-1} \left[S \nu \nu^T\right].$$  \hspace{1cm} (6)

Where \((U, S, V) = \text{svd}(X - E + \mu^{-1} Y)\), \(S \mu^{-1}\) is a contraction operator.

Then, update variable E:

$$E_{k+1} = \arg\min_{E} L(L_{k+1}, E, Y_k, \mu_k) = \arg\min_{E} \lambda \|E\|_F + \frac{\mu}{2} \|E - X + \mu^{-1} S_k Y_k\|_F^2 = S \mu^{-1} \left[S - L + \mu^{-1} Y\right].$$  \hspace{1cm} (7)

### 3. The Research Status in and out China

#### 3.1. Improvements to the original RPCA model
In order to better deal with the problem of foreground-background separation, Yang et al. estimated the dense motion of the moving foreground with dense optical flow method, obtained the real value matrix M of the binary template, and proposed the Motion-Assisted Matrix Restoration model. At the same time, in order to make the model more robust to the dynamic components and noise in the background, Yang et al. also extended the MAMR model to get Robust MAMR model.

\[
\min_{B,F} \|P\|_F + \lambda \|F\|_1 + \gamma \|G\|^2_F. \quad \text{s.t. } W \circ D = W \circ (B + F + G).
\]  

(8)

In dynamic background scenes, camera jitter, disguised moving objects or illumination changes, the temporal and spatial structure of moving objects may be lost. Javed et al. proposed sparse RPCA algorithm with sparse spatial-temporal structure[7]. The model is as follows:

\[
\min_{B,F} \|P\|_F + \lambda \|F\|_1 + \gamma_1 \text{Tr}(S^T L_S S) + \gamma_2 \text{Tr}(H L_T H^T) \quad \text{s.t. } D = B + F, F = H, F = S.
\]  

(9)

The premise in RPCA is that the background is static or quasi-static, and the foreground object moves very quickly, but it is not applicable in practice. In order to overcome the shortcomings of RPCA, Li et al. proposed segmented and saliency constraints RPCA (SSC-RPCA) method, whose solution is standardized by segmented and saliency constraints[6]. The model is as follows:

\[
\min \|A\|_1 + \lambda \|E\|_{(2,1)} + \beta \sum \|L_s\|_1 + \epsilon \|S_s\|_1 \quad \text{s.t. } D = A + E.
\]  

(10)

Li et al. first established a video segmentation algorithm. Then the segmentation constraints are integrated into RPCA and the saliency estimation is expressed as a combination of convex computing and RPCA to get a new model. Unlike RPCA, SSC-RPCA performs moving target detection and significance estimation jointly, and performs experiments on the database. The results show that SSC-RPCA is superior to RPCA-based method.

### 3.2. Improvements to the TV-RPCA model

Cao et al. proposed Total Variation Regularized RPCA model, which uses total variation regularization to divide the foreground into smooth component and sparse component.

\[
\min \|P\|_F + \lambda \|M\|_1 + \lambda_2 \|E\|_1 + \lambda_3 \|D\|_q \quad \text{s.t. } O = B + M, M = F + E.
\]  

(11)

Shijila et al.[8] proposed a new method for moving object detection, called noise reduction and low-rank approximate l1-TV normalization (DNLRl1TV). The model is as follows:

\[
\min \|P\|_F + \lambda_1 \|O - B - S\|_F^2 + \lambda_2 \|S\|_1 + \epsilon \|C\|_{TV} \quad \text{s.t. } O = B + S, \text{rank}(B) \leq r, S = C.
\]  

(12)

The model achieves simultaneous denoising and mod performance. This method is superior to the most advanced method in terms of denoising ability and detection accuracy under the conditions of shadows, camera jitter, bad weather and dynamic background.

### 3.3. Improvements to the non-convex RPCA model

The approximate rank function of the kernel norm has many shortcomings, and the calculation cost is expensive. In 2015, Kang et al. proposed a non-convex function to approximate the rank function of the matrix[5].

\[
\min \log \det(I + Z^T Z) + \rho \|X - XW\|_F^2 \quad \text{s.t. } Z = W.
\]  

(13)

The model is applied to face shadow removal and other practical problems to verify the effectiveness of non-convex approximation.

### 3.4. Improvements to the Schatten p-RPCA model

In 2016, Xie et al. proposed the Weighted Schatten P-Norm Minimization model, which is a further extension of the Nuclear Norm Minimization model[4].

\[
\min_{A,E} \|A\|_{w,p} + \lambda \|E\|_1 \quad \text{s.t. } D = A + E + N, \|N\|_F \leq \eta.
\]  

(14)
Jia et al. proposed an on-line Schatten-p Quasi-norm RPCA model[9]. And reconstructed the Schatten-p Quasi-norm by introducing variational formulas:

\[
\|X\|_{SP}^p = \min \frac{p}{q} \|V\|_{Lq}^2 + \frac{p}{2} \|W\|^2_F.
\]

(15)

The online Schatten-p Quasi-norm RPCA model is as follows:

\[
\min_U \sum_{i,j} \min \left\{ \frac{1}{2} \|v_i - Uv_i - e_i\|_2^2 + \beta \|v_i\|_2^2 + \lambda_2 \|v_i\|_1 + \alpha \|f_i\|_2^2 \right\}.
\]

(16)

The OSTP algorithm has no high svd and storage cost, and has nothing to do with the size of the data sample, which can meet the practical application of real-time video processing. In addition, scholars also consider matrix decomposition methods. Numerical results show that the effect of non-convex approximation is better than convex approximation.

4. Experimental comparison

In this paper, some improved RPCA models were selected to compare experimental results and operational efficiency.

| Model (Data sets) | Time(s) | Model (Data sets) | Time(s) |
|-------------------|---------|-------------------|---------|
| MAMR (12 noisy video clips) | 208 | TV-RPCA (Highway/Shadow) | 54.81/49.61 |
| SSC-RPCA (320x240 video) | 0.02 | DNLRl1TV (Highway/Shadow) | 9.05/9.07 |
| ORPCA (Watersurface/Fountain) | 1.32/0.97 | WSP3DTV (Watersurface/Fountain) | 0.91/0.63 |

Figure 1. The comparison of the experimental results of the improved model for different data sets. The first column is the original sequence, the second column is the ground real image, and the third column is the experimental effect of the model.

5. Summary and outlook

Mobile target detection wants to accurately and efficiently detect foreground targets, but has not yet reached the current RPCA algorithm level, and needs to improve accuracy and efficiency. Many scholars pay attention to the spatio-temporal prior information of foreground moving targets based on
the traditional RPCA. With the increase of the number of video frames to be processed and the amount of computation, the RPCA algorithm has higher and higher requirements on computer memory consumption, which is one of the research priorities. Although RPCA still has defects, it has great potential.

References

[1] Candès E J., Tao T. (2009) The power of convex relaxation: nearoptimal matrix completion. J. Transactions on Information Theory. 56: 2053-2080.

[2] Zhou Z H., Li X D., Wright J. (2010) Stable principal component pursuit. C. International Symposium on Information Theory. International Symposium on Information Theory. 1518-1522.

[3] Oliver N M., Rosario B., Pentland A P. (2000) A Bayesian computer vision system for modeling human interactions. J. Trans pattern Anal mach. 22: 255-272.

[4] Xie Y., Qu Y., Tao D. (2016) Hyperspectral image restoration via iteratively regularized weighted schatten p-norm minimization. J. Transactions on Geoscience and Remote Sensing. 1-18.

[5] Kang Z., Peng C., Cheng. (2015) Log-det rank minimization with application to subspace clustering. J. Computational intelligence and neuroscience. 2015: 68.

[6] Li Y., Liu G., Liu Q. (2019) Moving object detection via segmentation and saliency constrained RPCA. J. Neurocomputing. 323: 352-362.

[7] Javed S., Mahmood A., Al-Maadeed S. (2015) Moving object detection in complex scene using spatiotemporal structured-sparse RPCA. J. Transactions on Image Processing. 28: 1007-1022.

[8] Shijila B., Tom A J. (2019) Simultaneous denoising and moving object detection using low rank approximation. J. Future Generation Computer Systems.90: 198-210.

[9] Jia X., Feng X., Wang W. (2019) Online Schatten quasi-norm minimization for robust principal component analysis. J. Information Sciences.476: 83-94.