A UNIVERSAL DENSITY SLOPE – VELOCITY ANISOTROPY RELATION

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Abstract. One can solve the Jeans equation analytically for equilibrated dark matter structures, once given two pieces of input from numerical simulations. These inputs are 1) a connection between phase-space density and radius, and 2) a connection between velocity anisotropy and density slope, the $\alpha - \beta$ relation. The first (phase-space density v.s. radius) has been analysed through several different simulations, however the second ($\alpha - \beta$ relation) has not been quantified yet. We perform a large set of numerical experiments in order to quantify the slope and zero-point of the $\alpha - \beta$ relation. When combined with the assumption of phase-space being a power-law in radius this allows us to conclude that equilibrated dark matter structures indeed have zero central velocity anisotropy, central density slope of $\alpha_0 \approx -0.8$, and outer anisotropy of approximately $\beta_\infty \approx 0.5$.

1 Introduction

We have seen remarkable progress in the understanding of pure dark matter structures over the last few years. This was triggered by numerical simulations which have observed general trends in the behaviour of the radial density profile of equilibrated dark matter structures from cosmological simulations, which roughly follow an NFW profile (Navarro et al. 1996) (Moore et al. 1998) (see Diemand et al 2004 for references). General trends in the radial dependence of the velocity anisotropy has also been suggested (Cole & Lacey 1996). Recently, more complex relations have been identified, holding even for systems that do not follow the simplest radial power-law behaviour in density. These relations are first that the phase-space density, $\rho/\sigma^3$, is a power-law in radius (Taylor & Navarro 2001) and second that there is a linear relationship between the density slope and the anisotropy (Hansen & Moore 2005, hereafter HM04). A connection between the shape of the velocity distribution function and the density slope has also been suggested (Hansen et al 2005a, 2005b).

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Using the Jeans equation together with the fact that phase-space density is a power-law in radius allows one to find the density slope in the central region numerically (Taylor & Navarro 2001) and even analytically for power-law densities (Hansen 2004). Recently Dehnen & McLaughlin (2005, hereafter DM05) using both the phase-space density being a power-law in radius and also the \( \alpha - \beta \) relation, showed that one can solve the Jeans equations analytically and extract the radial dependence of density, anisotropy, mass etc.

The relationship between phase-space density and radius has been considered several times and seems to be well established, however, the other crucial ingredient in the analysis, namely the linear relationship between density slope and anisotropy has only been investigated qualitatively. We have performed a large set of simulations in order to quantify this relationship. We show that with present day simulations there does indeed appear to be a linear relation between density slope and anisotropy, which implies zero anisotropy near the density slope of approximately \(-0.8\). We combine our findings with analytical results and find that the outer anisotropy is radial and close to \(+0.5\).

2 The simulations

The \( \alpha - \beta \) relation was suggested based on a set of different numerical simulations, including spherical collapse simulations, collisions between initially isotropic NFW structures, merger between two disk galaxies, and finally structures formed in a \( \Lambda \)CDM cosmological simulation. The original relation had a large scatter, and there were several questions not fully answered, such as a possible dependence on shape or initial conditions. We have here performed controlled experiments to test exactly these points. The set of experiments are described in glorious detail in a coming publication, and we will not get into these aspects here. We will only mention, that for each simulation have we extracted all the relevant parameters in radial bins, logarithmically distributed from the softening length to beyond the region which is fully equilibrated. We calculate the radial derivative of the density (the density slope)

\[
\alpha \equiv \frac{d \ln \rho}{d \ln r},
\]

and the velocity anisotropy

\[
\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2},
\]

in each radial bin, where the \( \sigma_{t,r} \) are the one dimensional velocity dispersions in the tangential and radial directions respectively.

2.1 Dependence on shape?

One of the test we performed was the head-on collision between two initially isotropic NFW structures. We then took the resulting structures and collided these headon again. The resulting structure is prolate when observed in density
A universal $\alpha - \beta$ relation

contours. One may fear that the resulting $\alpha - \beta$ relation will depend strongly on the axis-ratios of such structures. In order to test this question, we extracted the resulting prolate structure after the first collision described above. We can now decide to collide this structure along different axes, either along the long, intermediate or short axes. We perform these 3 different collisions.

We find that there is virtually no difference in the $\alpha - \beta$ relation for these 3 different simulations. We conclude from this that whereas the definition of density slope does depend slightly on the shape of the density contours, then the $\alpha - \beta$ relation is almost independent of the shape.

![A collection of various simulations, including the tests of initial conditions and shape. The red (dashed) line is a fit through the data. The thin (blue, solid) line shows the theoretical central quantities (HM05). The crossing of these two lines must be the true central values.](image)

**Fig. 1.** A collection of various simulations, including the tests of initial conditions and shape. The red (dashed) line is a fit through the data. The thin (blue, solid) line shows the theoretical central quantities (HM05). The crossing of these two lines must be the true central values.

2.2 Dependence on initial conditions?

To address the question of how strongly the $\alpha - \beta$ relation from the head-on collisions described above depend on the initial conditions, we now perform collisions between structures with different initial degree of anisotropy. First we make a repeated head-on collision between two isotropic NFW structures. Next we make repeated head-on collisions between two initially strongly radially anisotropic struc-
tures. And finally do we perform repeated head-on collisions between initially strongly tangentially anisotropic structures.

We find that the resulting \( \alpha - \beta \) relation is virtually independent of initial conditions, and we conclude that the collisions were sufficiently violent to erase the initial conditions sufficiently to conform with the \( \alpha - \beta \) relation.

3 Comparison with theory

In a recent paper, Dehnen & McLaughlin (2005) showed, that under the two assumption that phase-space density is a power-law in radius, and that there is a linear \( \alpha - \beta \) relation, that the central slope of dark matter structures must be \( \alpha_0 = -(7 + 10\beta_0)/9 \), where \( \beta_0 \) is the central anisotropy. We can now compare this result with our findings, and in figure 4 we show the DM05 result for the central values as a thin (blue) straight line. We see that the two lines cross near \( \beta = 0 \) and \( \alpha = -7/9 \), showing that the central part of dark matter structures is isotropic, and that the central density slope is indeed \( -7/9 \). The outer density slope is about \( \gamma_{\infty} = -31/9 \approx -3.44 \) [Austin et al. 2005] which when compared with our findings result in \( \beta_{\infty} \approx 0.53 \).

4 Conclusions

We have quantified the relationship between the density slope and the velocity anisotropy, the \( \alpha - \beta \) relation. We have performed a large set of simulations to investigate systematic effects related to shape and initial conditions, and we find that the \( \alpha - \beta \) relation is almost blind to both shape and initial conditions, as long as the system has been perturbed sufficiently and subsequently allowed to relax. When compared with analytical results we find that the central region is indeed isotropic with a density slope about 0.8, and that the outer asymptotic anisotropy is radial, with a magnitude of \( \beta \approx 0.5 \).

References

Austin, C. G. et al. 2005, arXiv:astro-ph/0506571.
Cole, S., & Lacey, C. 1996, MNRAS, 281, 716
Dehnen, W., & McLaughlin, D. 2005, arXiv:astro-ph/0506528 (DM05)
Diemand, J., Moore, B., & Stadel, J. 2004, MNRAS, 353, 624
Hansen, S. H 2004, MNRAS, 352, L41
Hansen, S. H., & Moore, B. 2004, arXiv:astro-ph/0411473 (HM04)
Hansen, S. H., Egli, D., Hollenstein, L., & Salzmann, C. 2005, New Astron. 10, 379
Hansen, S. H., Moore, B., Zemp, M., & Stadel, J. 2005, arXiv:astro-ph/0505420
Moore, B., Governato, F., Quinn, T., Stadel, J. & Lake G. 1998, ApJ, 499, L5
Navarro, J. F., Frenk C. S. & White, S. D. M. 1996, ApJ, 462, 563
Taylor, J. E. & Navarro, J. F. 2001, ApJ, 563, 483