$W^+ W^- H$ production through bottom quarks fusion at hadron colliders

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Phenomenology 2021 Symposium, University of Pittsburgh, May 26, 2021
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Motivation: \( b \bar{b} \rightarrow W^+ W^- H \)

- Higgs sector in SM is not well explored, in particular \( HHH \), \( HHHH \) and \( VVHH \) couplings are still not well measured.
- Few processes can probe the \( VVHH \) coupling.
  - VBF mechanism for HH production

At HL-LHC the bound could be \( 0.55 < \kappa_{V_2H_2} < 1.65 \) at 95% confidence level. But the bound comes from both coupling \( WWHH \) and \( ZZHH \).
Motivation

- Higgs-strahlung: HHV (V=W, Z) production

At the HL-LHC the bound will be quite weak:

\[-9 < \kappa_{V_2 H_2} < 11.\]

- VVH (V=W, Z) production

We can probe two VVHH couplings separately.
Motivation

- The $b\bar{b}$ contribution is sizeable. One should probe it in QCD regime.
- One can study the polarization dependence of physical observables which will be very useful for background suppression.

| $pp \rightarrow WWH$(LO) | $gg$ | $q\bar{q}$ | $b\bar{b}$ |
|-------------------------|------|------------|------------|
| $\sigma(fb)$ at 14TeV   | 0.29 | 8.66       | 0.25       |
| $\sigma(fb)$ at 27TeV   | 1.34 | 23.0       | 1.31       |
| $\sigma(fb)$ at 100TeV  | 17.4 | 126.8      | 20.6       |
Feynman diagrams:
Feynman Diagrams:

- Total number of diagrams:
  - LO: 20 diagrams
  - NLO: Pentagon + Box + Triangle + Self Energy diagrams. Total 121 NLO diagrams.

- The trick is to calculate the minimum no. of diagrams, called *prototype diagrams* and then map the rest of the diagrams to those prototype diagrams.
  - LO prototype diagrams are 10
  - Loop-level prototype diagrams are 30.
\[ M \sim g_w^3 M_{LO} + g_s^2 g_w^3 M_{NLO} + O(g_s^4) \]
\[ |M|^2 \sim \alpha_w^3 |M_{LO}|^2 + \alpha_s \alpha_w^3 \text{Re}(M_{LO} M_{NLO}^*) + O(\alpha_s^2) \]
Techniques to compute amplitudes:

- We compute helicity amplitudes by using spinor helicity formalism at the matrix element level.

- We use four-dimensional helicity (FDH) scheme to compute the amplitudes where all the $\gamma$-matrices, momentums and spinors are taken in 4-dimensions.

- In one-loop amplitude, individual one-loop Feynman diagram will give rise to tensor integrals containing powers of the loop momentum in the numerator.

- We use an in-house routine $OVReduce$, based on Oldenborgh-Vermaseren reduction techniques to reduce tensor integrals in terms of scalar integrals.

- We use the 'OneLOop' package for scalar integrals computation.
QCD renormalizes the fermion mass.

Higgs vertex will be renormalized due to mass involved in coupling.

The Coupling strength of $Hf\bar{f}$ vertex is $-\frac{ig}{2} \frac{m_f}{m_W}$.

Counterterm for $Hf\bar{f}$ vertex is $-\frac{ig}{2} \frac{\delta m_f}{m_W}$. Where

$$\delta m_f = -\frac{\alpha_s}{4\pi} C_F \frac{6}{\epsilon}.$$
UV divergence : Self-energy CT diagrams

- Counterterm for self energy diagram : $-i(\not{p}\delta Z_2 - m_f\delta Z_m)$
- $\delta Z_2 = -\frac{\alpha_s}{4\pi} C_F \frac{2}{\epsilon}$ and $\delta Z_m = -\frac{\alpha_s}{4\pi} C_F \frac{8}{\epsilon}$. 
Infrared divergence:

- IR or "mass singularities" arises from two kinds of singularities called the *collinear* and *soft* singularity. Singularities appear as $\sim \ln(m/Q)$. Where $m$ is the mass of the particle and $Q$ is a large scale.

For the massless case

$$\sim 1/\epsilon, \ 1/\epsilon^2 \quad [\epsilon = (4 - D)/2]$$

- Because of light quarks and gauge bosons, most of the one-loop diagrams are IR singular.

- The real emission diagrams are also IR singular in soft and collinear regimes.

- The real emission and renormalized virtual amplitudes are both divergent in 4-dimension, but the sum of these two is finite.

- Three real emission sub-process can contribute to $\sigma^{NLO}$.
  
  1. $b\bar{b} \rightarrow W^+W^-Hg$, 2. $g\bar{b} \rightarrow W^+W^-H\bar{b}$ and 3. $bg \rightarrow W^+W^-Hb$
The real emission sub-processes starting with gluon have $t$-quark resonant diagrams which jeopardize the perturbative computations.

We use $b$-quark tagging with 100% efficiency. We exclude these two sub-processes to avoid the $t$-quark resonances.

We implemented the Catani-Saymour dipole subtraction method to remove IR singularities. The $l$-term exactly cancel the IR singularities in virtual diagrams and dipole terms $D_{ij,k}$ exactly cancel IR singularities in real emission diagrams.
SM prediction

Results: SM predictions

We took SM parameters from PDG 2016. We use CT14lo and CT14nlo PDF set for LO and NLO cross section calculation respectively. We take \( \overline{\text{MS}} \) and On-shell renormalization scheme for massless and massive fermions respectively. The following results are in the ab unit for different CMEs with the scale uncertainties.

| TeV | \( \sigma_0(\alpha^3_W) \)          | \( \sigma^{NLO}_{qcd}(\alpha_s\alpha^3_W) \) | \( RE \) |
|-----|-----------------------------------|-----------------------------------------------|--------|
| 14  | 217\,+16.1\%_{-18.9}\%          | 289\,+17.6\%_{-20.8}\%                     | 33.2\% |
| 27  | 1086\,+19.2\%_{-20.5}\%         | 1559\,+18.0\%_{-20.8}\%                    | 43.6\% |
| 100 | 15258\,+22.0\%_{-20.9}\%        | 23097\,+20.6\%_{-21.0}\%                   | 51.4\% |

The relative enhancement is defined as \( RE = \left( \frac{\sigma^{NLO}_{qcd} - \sigma_0}{\sigma_0} \right) \). We choose a dynamical scale as

\[
\mu_R = \mu_F = \mu_0 = \frac{1}{3} \left( \sqrt{p_{T,W+}^2 + M_W^2} + \sqrt{p_{T,W-}^2 + M_W^2} + \sqrt{p_{T,H}^2 + M_H^2} \right)
\]
Results: SM predictions

Polarization dependence of cross section:

| Pol. ($W^+ W^−$) | $14 \text{ TeV (ab)}$ | $100 \text{ TeV (ab)}$ |
|------------------|---------------------|---------------------|
|                  | $\sigma_0$ | $\sigma_{\text{qcd}}^{\text{NLO}}$ | $\text{RE} (%)$ | $\sigma_0$ | $\sigma_{\text{qcd}}^{\text{NLO}}$ | $\text{RE} (%)$ |
| ++               | 13       | 18       | 38.5          | 702     | 1056     | 50.4          |
| +−               | 18       | 25       | 38.9          | 965     | 1499     | 55.3          |
| +0               | 37       | 49       | 32.4          | 2568    | 3336     | 29.9          |
| −+               | 4        | 6        | 50.0          | 229     | 334      | 45.9          |
| −−               | 13       | 18       | 38.5          | 707     | 1044     | 47.7          |
| −0               | 22       | 28       | 27.3          | 1454    | 1346     | −7.4          |
| 0+               | 22       | 28       | 27.3          | 1470    | 1216     | −17.3         |
| 0−               | 37       | 49       | 32.4          | 2583    | 3151     | 22.0          |
| 00               | 51       | 67       | 31.4          | 4490    | 9748     | 117.1         |
| $\sum$           | 217      | 289      | 32.2          | 15258   | 23097    | 51.4          |

Where $+ \equiv \frac{1}{\sqrt{2}} (\epsilon_x + i\epsilon_y)$, $− \equiv \frac{1}{\sqrt{2}} (\epsilon_x − i\epsilon_y)$ and $0 \equiv \epsilon_z$.

Here we can see that there are huge contributions and increments in ‘00’ polarization mode.
$p_T$-distributions:

Figure: The NLO differential cross section distribution with respect to transverse momentums ($p_T$) for 14 and 100 TeV CMEs.
Invariant Mass distributions:

Figure: The NLO differential cross section distribution with respect to invariant masses ($M_{ij/ijk}$) for 14 and 100 TeV CMEs.
Differential distributions:

**Figure:** The LO and NLO differential cross section distribution with respect to transverse momentums ($p_T$) and invariant masses ($M_{ij/ijk}$) for 100 TeV CME.
**Differential distributions:**

\[ b\bar{b} \rightarrow HW^+W^- \]

\[ d\sigma/dM \text{ [ab/bin]} \]

\[ M_{HW^+W^-,LO} \quad M_{HW^+W^-,NLO} \]

\[ \sqrt{s} = 100 \text{ TeV} \]

**Figure:** The LO and NLO differential cross section distribution with respect to invariant masses \( M_{WWH} \) for 100 TeV CME.
Anomalous coupling effects: \( \kappa \)-framework

| CME(TeV) | \( \kappa V^2 H^2 \) | \( \sigma^{LO} [ab] \) | RI | \( \sigma^{NLO} [ab] \) | RI |
|----------|-----------------|-----------------|----|-----------------|----|
| 14       | 1.0 (SM)        | 217             |     | 289             |     |
|          | 2.0             | 216 [−0.5%]     |     | 288 [−0.3%]     |     |
|          | −2.0            | 222 [+2.3%]     |     | 295 [+2.1%]     |     |
| 100      | 1.0 (SM)        | 15258           |     | 23097           |     |
|          | 2.0             | 14925 [−2.2%]   |     | 22607 [−2.1%]   |     |
|          | −2.0            | 16997 [+11.4%]   |     | 25465 [+10.3%]  |     |

Table: Effect of anomalous \( WWHH \) coupling on the total cross section at 14 and 100 TeV CMEs. Where \( RI = \frac{\sigma_{\kappa V^2 H^2} - \sigma_{SM}}{\sigma_{SM}} \).

| \( \kappa V^2 H^2 \) | \( \sigma^{LO} [ab] \) | RI  | \( \sigma^{NLO} [ab] \) | RI  |
|-----------------|-----------------|-----|-----------------|-----|
| 1.0 (SM)        | 4490            |     | 9748            |     |
| 2.0             | 4159            [−7.4%] |     | 9544            [−2.1%] |     |
| −2.0            | 6164            [+37.2%] |     | 11993           [+23.0%] |     |

Table: Effect of anomalous \( VVHH \) coupling in ‘00’ mode at 100 TeV CME.
Anomalous coupling effects

Differential distributions:

Figure: Effect of anomalous $VVHH$ coupling on the differential cross section distribution at 100 TeV CME.
**Figure:** Effect of anomalous $VVHH$ coupling on the differential cross section distribution at 100 TeV CME.
We have focused on the NLO QCD correction to $b\bar{b} \rightarrow W^+ W^- H$. This process has significant dependence on $VVHH$ coupling.

The contribution of this process to $pp \rightarrow W^+ W^- H$ is only about $10 - 15\%$ of that light quark scattering. But when both $W$-bosons are longitudinally polarized then this fraction can increase to $50\%$.

At 100 TeV the NLO corrections are about $50\%$ but the corrections are about $115\%$, when both $W$-bosons are longitudinally polarized.

Our study suggests that the measurement of the polarization of the final state $W/Z$-bosons can be a useful tool to measure the couplings of the vector bosons and Higgs boson.

Total cross section enhanced by $10\%$ and cross section in ’00’ mode enhanced by $20 - 30\%$ when we set $\kappa_{V_2H_2} = -2$.

We find that the invariant mass and the $p_T$ distributions are considerably harder for the negative values of $\kappa_{V_2H_2}$. This can also be useful to put a stronger bound on the coupling.
Thank You