Theta term instead of the Higgs field in Electroweak theory

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We consider the electroweak theory without fundamental scalar field. The topological excitation of the $SU(2) \times U(1)$ theory (the monopole) plays the role of the Higgs field, it carries the $SU(2) \times U(1)$ topological charge due to the theta–term of the special type.

In this paper we suggest a new formulation of Wainberg - Salam model on the lattice. The nice feature of our approach is that we have no fundamental Higgs field in the initial Lagrangian (there been a number of attempts to eliminate the fundamental Higgs field from electroweak Lagrangian [1,2]). The Higgs field appears as topological monopole excitation; due to $\theta$–term with $\theta = \pi$ the $U(1)$ monopole acquires the $SU(2) \times U(1)$ charge and become the scalar electrically charged field (see also [3,4]). The theory may contain phase in which $U(1)$ monopoles are condensed and they play the role of the Higgs field in the electroweak theory.

The Villain action of the $U(1)$ model with the theta-term has the form:

$$ S = \sum_{plaq} \beta |d\theta + 2\pi M|^2 - \frac{i}{4\pi} (d\theta + 2\pi M)^* (d\theta + 2\pi M). $$  (1)

Here $\theta$ is $U(1)$ gauge field, $j =* dM$ is the monopole worldline, $*$ is the duality transformation, $M$ is the Villain variable. Thus we have

$$ S = \sum_{plaq} \beta |d\theta + 2\pi M|^2 + i(\theta, j) + i\pi (M,^* M). $$  (2)

Note that monopole currents $j$ interact with the gauge field $\theta$ via the Wilson loop, $i(\theta, j)$. Thus monopoles carry the $U(1)$ charge. The partition function of the theory has the form

$$ Z = \int_{-\pi}^{\pi} D\theta \sum_{j=sdM}^{\infty} \exp(-\sum_{plaq} \beta |d\theta + 2\pi M|^2 - i(\theta, j) - i\pi (M,^* M)) $$

$$ = \int_{-\infty}^{\infty} D\theta \sum_{j=0}^{\infty} \exp(-\sum_{plaq} \beta |d\theta|^2 - U(j) - i(\theta, j)) $$

(3)

Here $U(j)$ is a nonlocal potential. Next we can transform the sum over the monopoles into the integral over the scalar field.

$$ Z = \int_{-\infty}^{\infty} D\theta D\Phi \exp(-\sum_{plaq} \beta |d\theta|^2 - V(|\Phi|) $$

$$ - \sum_{xy}^\infty (\Phi_x e^{i\theta_{xy}} - \Phi_y^*) (\Phi_x e^{-i\theta_{xy}} - \Phi_y) $$

(4)

Here $V(r)$ is a nonlocal potential.

This example illustrates how the scalar electrically charged field appears from the theory containing monopoles. Our aim now is to generalize this construction to the nonabelian theory.

First we consider the continuum electroweak theory. The fermion part of the action looks like (the fermions are supposed to be massless)

$$ S = S_{left_{leptons}} + S_{right_{leptons}} $$

$$ S_{left_{leptons}} = S_{left_{leptons}} + S_{right_{leptons}} $$

$$ S_{right_{leptons}} = \int L(\bar{\partial}^\mu + iA^\mu_{SU(2)} + iB^\mu_{U(1)}) \gamma_\mu Ld^4x $$

$$ S_{left_{leptons}} = \int R(\bar{\partial}^\mu + 2iB^\mu_{U(1)}) \gamma_\mu Rd^4x $$

The physical variables depend upon parallel transporters along the closed lines in the appro-
appropriate representations. Thus the additional symmetry takes place:

\[ A^i \rightarrow A^i + (A^j t^j / \sqrt{\text{tr}(A^j t^j)^2}) \pi (*v)^i \]

\[ B^i \rightarrow B^i - \pi (*v)^i \]  

Here \( v \) is the 3-dimensional hypersurface, \( t^i_v \) is the unity vector in the direction \( *v \).

\[ v^{ijk} = \int_v \delta(x - x(a, b, c)) dx^i \Lambda dx^j \Lambda dx^k \]  

\[ t^i = \epsilon^{ijkl} (dx^k / da)(dx^l / db)(dx^j / dc). \]

This additional symmetry appears since we consider the “compact” continuum theory in which the singular gauge fields are not forbidden. The compact continuum nonabelian fields are discussed in ref. \[5\] as the limit of lattice gauge fields. The lattice analogue of this symmetry is

\[ U \rightarrow -U; \quad \theta \rightarrow \theta + \pi \]  

for \( U \in SU(2), \theta \in U(1). \)

Figure 1. Polyakov line and confinement - deconfinement phase transition. Dots “without \( S^* \)” represent the results for the theory without theta-term.

This symmetry corresponds to the center \( Z_2 \) of \( SU(2) \) group, and it can be shown \[8\] that due to this symmetry the so-called center monopoles coincide with the ordinary \( U(1) \) monopoles in the continuum. The action for the pure gauge fields should also posses this symmetry. The above mentioned symmetry prompts us the following definition of the \( SU(2) \times U(1) \) theta - term

\[ Q = 1 / 4 \pi \int R^* R dx \]  

\[ R_{ij} = \partial_i B_j - \partial_j B_i + H_{ij} + 2 \pi \epsilon_{ijkl} \Sigma_{kl}, \]  

Here \( H \) is t’Hooft tensor

\[ H_{ij} = \text{tr}(G_{ij} n) - \text{tr}(n D_i n D_j n), \]  

\[ G_{ij} = [D_i, A_j], \quad n = n^a \sigma^a. \]

\( \Sigma_{ij} \) is the antisymmetric tensor representing the 2- dimensional surface \( \Sigma \). This tensor is the analogue of the Villain variable from the lattice formulation of \( U(1) \) theory with theta-term.

The integration over \( \Sigma \) and \( n (n^a n^a = 1) \) is assumed.

\[ \Sigma^{ij} = \int_\Sigma \delta(x - x(a, b)) dx^i \Lambda dx^j \]  

We have the model with the partition function

\[ Z = \int DADBD \phi \exp(-S_{\text{fermions}} - S_{\text{gauge fields}} + iQ) \]

We found that

\[ Z = \int DADBD \phi \exp(-S_{\text{fermions}} - S_{\text{gauge fields}}) \]

\[ P \exp(i \int_j (A_i - B_i) dx_i e^{-V(j)}) \]  

Here \( V(j) \) is nonlocal potential, \( j = \delta \Sigma \) is the monopole worldline.

We can express the sum over the worldlines of the monopoles \( j \) as the integral over the scalar field

\[ Z = \int DADBD \phi \exp(S_{\text{fermions}} - S_{\text{gauge fields}}) - \int |(d + iA - iB) \phi|^2 - V(|\phi|) \]

Here \( V(|\phi|) \) is unknown potential. It means that the low energy approximation of our theory can be equivalent to the Wainberg - Salam model for
the massless fermions if the potential \( V \) is of the Higgs type.

We can express the continuum partition function in the following way:

\[
Z = \int DADBDgDj \exp(-S_{\text{fermions}} - S_{\text{gauge fields}}) \exp(\int j \cdot A^g_3) \tag{16}
\]

We use this expression to construct the lattice theory

\[
Z = \int DUD\theta DG \sum_{\delta j = 0} \exp(-S_f - S_g) \exp(\Pi_{\text{link}}((U^g)_{\text{link}}^{11}\exp(i\theta_{\text{link}})))
\]

Here

\[
S_g = \beta \sum_{\text{plaquettes}} \left((1 - 1/2trU_p\cos\theta_p)
+ (1 - \cos2\theta_p)\right)
\]

The lattice theory possesses the above considered additional symmetry \([11]\).

In numerical calculations we use the lattice \( 10^4 - 16^4 \). At large values of \( \beta \) the confinement is destroyed. It follows from the evaluation of the Creutz ratios and the Polyakov line. The confinement - deconfinement phase transition is of 1-st order. It follows from the calculation of the correlation:

\[
\rho(|x - y|) = < \sum_{x \in \text{plaq}} (trU_{\text{plaq}}\cos\theta_{\text{plaq}}) \\
\sum_{y \in \text{plaq}} (trU_{\text{plaq}}\cos\theta_{\text{plaq}}) > \\
- < \sum_{x \in \text{plaq}} (trU_{\text{plaq}}\cos\theta_{\text{plaq}}) >^2
\]

\[
\rho(r) \rightarrow \text{Const} \cdot \exp(-Mr) \tag{17}
\]

We found that \( M \) is almost independent on \( \beta \). That means that the correlation length does not tend to infinity at the point of the phase transition. Thus the theory under consideration has no continuum limit.

Our conclusions are:

1. We consider the \( SU(2) \times U(1) \) gauge theory. We found that the fermion part of Wainberg-Salam model (for massless fermions) possesses the additional \( Z_2 \) symmetry. We suppose that this symmetry plays the important role in the construction of the action for the gauge field.

2. We construct the \( SU(2) \times U(1) \) theta - term of the special type.

3. The resulting theory contains monopoles, which carry the \( SU(2) \times U(1) \) charge. The monopoles are condensed by the construction. The low energy approximation of the theory can be equivalent to the Wainberg - Salam model. The monopole field plays the role of the Higgs field.

4. We formulate a lattice version of the theory. It occurs that this version has no continuum limit. It means that we should redefine the lattice action to obtain the second order phase transition.

5. We do not discuss how the fermion masses appear in our model.

6. We do not discuss how \( M_W \) and \( \theta_W \) appear in our model.

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REFERENCES

1. H.C. Cheng, B.A. Dobrescu and C.T. Hill, Nucl.Phys. B589 (2000) 249.
2. A. Arkani-Hamed, A.G. Cohen and H. Georgi, Phys.Lett. B513 (2001) 232.
3. E. Witten, Phys. Lett. B86B (1979) 283.
4. T. Vachaspati, Nucl. Phys. B439 (1995) 79.
5. M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov and V.I. Zakharov Nucl. Phys. B592 (2001) 107.
6. B.L.G. Bakker, A.I. Veselov and M.A. Zubkov, Phys. Lett. B 497 (2001) 159.