Thermodynamic analysis of volume flow rate ratio on the performance of a NH₃/CO₂ cascade refrigeration system

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Abstract. NH₃/CO₂ cascade refrigeration system is the most potential and promising selection in low temperature applications. To meet the cooling demands of CRS at various working conditions, the capacity regulation of the screw refrigeration compressor is needed. This paper specifically studied the impacts volume flow rate ratio of high temperature cycle (HTC) and low temperature cycle (LTC) on the system performance. The simulation model of CRS including energy and exergy analysis was established. Based on the obtained results with the models, it can be found that there exists an optimum volume flow ratio that makes the maximum COP and exergy efficiency and the optimum volume flow ratio value is different under various working conditions. When the evaporation temperature is -55℃, the optimum volume rate ratio is about 2.4 and corresponding maximum COP and exergy efficiency of CRS is 1.031 and 0.346, respectively. The optimum condensing temperature of LTC is -9.75℃. When the volume flow ratio is increasing, which means the motor frequency or rotational speed of HTC compressor increases while the rotational speed of LTC compressor is fixed, the optimum intermediate temperature decreases and the discharge temperature of HTC compressor increases proportionally. The cooling capacity could be enhanced by increasing the volume flow ratio. The obtained results are helpful for the screw refrigeration compressor regulation in CRS under various working conditions and cooling demands.

1. Introduction

The cascade refrigeration system (CRS) which consists of two different refrigeration cycles was introduced to meet the demand for low temperature refrigeration which is required in the temperature range from -100℃ to -30℃. A NH₃/CO₂ cascade refrigeration system uses ammonia and carbon dioxide as refrigerants in high and low temperature circuits, respectively. Both ammonia and carbon dioxide are natural refrigerant and many investigations of the NH₃/CO₂ cascade refrigeration system are attracting attention.

In the process of introducing and designing a CRS, one of the most important issues is to find an optimal intermediate temperature, which is the condensation temperature of low temperature cycle (LTC) or the evaporation temperature of a high temperature cycle (HTC), to obtain the best efficiency of the system [1]. Many researches have been conducted on the optimum condensing temperature of LTC for various cascade systems. Lee et al. theoretically studied the NH₃/CO₂ cascade refrigeration system to determine the optimal condensing temperature under given various design parameters. The useful correlations that yield the optimal condensing temperature of the CRS are proposed[2]. Dopazo et al. also investigated a NH₃/CO₂ cascade refrigeration system with numerical analysis to find optimal
conditions[3]. Hansaem Park et al. researched the optimal intermediate temperature of a cascade refrigeration system with R134a and R410A cascade refrigeration system by thermodynamic analysis. Theoretical analysis was targeted for establishing a mathematical model to enable the prediction of optimal intermediate temperatures that make the system operate with its maximum efficiency. For actual operation of cascade refrigeration system, working conditions including the condensing temperature and evaporation temperature. An evaporative condenser or air-cooled condenser is often used in CRS. The outdoor air temperature will have an effect on the condensation pressure of cascade refrigeration system. Therefore, it is necessary to regulate the working parameters to ensure a good performance of CRS. Reciprocating or screw compressors are the main type utilized in NH3/CO2 CRS. For screw refrigeration compressor, sling valve regulation and variable frequency regulation are two capacity control methods. Although many theoretical and experimental studies have been conducted to investigate the optimum intermediate temperature for CRS, these studies do not provide much attention to the volume flow rates of HTC and LTC compressor and its influence on the system performance. Especially, when the screw refrigeration compressor used in CRS is regulated by variable frequency method, which caused a changing of volume flow rate ratio of HTC compressor and LTC compressor. For rolling piston refrigeration compressor used in air conditioner and two-stage air source heat pump, the selection of the cylinder volume ratio of the two compressor is one of the most important tasks, which will not only influence the initial cost but also have a significant impact on the operation cost. Ko et al. investigated the performance of a two-stage vapour compressor under ARI conditions. Three different cylinder volume ratios, 0.59, 0.68 and 0.82, were tested. The results show that the performance of the system with a cylinder volume ratio of 0.59 is superior to those with other ratios[4]. Baek et al. experimentally measured the heating performances of the vapor injection CO2 systems including the flash tank vapor injection and sub-cooler vapor injection cycles, by adjusting the cylinder volume ratio of the twin rotary compressor at the ambient temperature of −15 °C. It was pointed that the optimum cylinder volume ratio is 0.7 in order to achieve the maximum COP with the designed heating capacity[5].

To sum up, it is essential to investigate the volume flow ratio in CRS and its impacts on the system performance. This work employs thermodynamic energy and exergy analysis to investigate the effect of volume flow ratio of HTC and LTC compressor on the performance characteristics. A theoretical simulation model is established based on energy and exergy balance equations and the effects of volume flow ratio on the optimum intermediate temperature, COP, exergy efficiency are investigated.

2. Theoretical model of CRS

2.1. System description
A schematic diagram of the NH3/CO2 cascade refrigeration system is illustrated in figure 1. The CRS is constituted by two single stage refrigeration circuits, HTC and LTC. These two circuits are connected by a cascade heat exchanger, which serves as the condenser of LTC and evaporator of HTC. The LTC with CO2 as the refrigerant is used for cooling and the HTC with NH3 as the refrigerant is used to condensate the CO2 of the LTC. The evaporator absorbs a refrigerated load \( Q_e \) from the cold space at \( T_{cl} \) to the evaporating temperature \( T_e \). The heat absorbed by the evaporator of the LTC plus the work input to the LTC compressor equals the heat absorbed by the evaporator of the HTC. The condenser of HTC rejects a heat of \( Q_c \) from the condenser at condensation temperature \( T_{c} \) to ambient. Temperature difference in cascade heat exchanger represents the difference between the condensation temperature of LTC and the evaporation temperature of HTC. The corresponding temperature-entropy diagram is shown in figure 2.

2.2. Mathematical model
In order to simplify the thermodynamic analysis, the following assumptions have been made in the analysis:
(a) Refrigerants at evaporator outlet and condenser outlet is saturated state.
(b) Pressure drops and heat losses in linking pipes and system components are negligible.
(c) All components are assumed to be a steady-state and steady-flow process.
(d) The temperature difference in cascade heat exchanger and the temperature difference between cold space and evaporation temperature are both 5°C.

Figure 1. Schematic diagram of the NH$_3$/CO$_2$ cascade refrigeration system.

Taking into account the assumptions previously made, the theoretical model is established based on the principle of CRS and the mass, energy and exergy balances are given by equations (1)-(3), respectively.

$$\sum_{in} m = \sum_{out} m$$

$$Q - W = \sum_{out} m \cdot h - \sum_{in} m \cdot h$$

$$ED = \sum_{out} \left(1 - \frac{T_i}{T_f}\right) \cdot Q_j - W + \sum_{in} m \cdot e - \sum_{out} m \cdot e$$

The volume flow ratio is defined as the ratio of refrigerant volume flow rate of HTC compressor and that of LTC compressor.

$$n_v = V_{HTC} / V_{LTC}$$

The mass flow rate can be obtained and is the product of volume flow rate and suction state density. The exergy analysis method is necessary and more accurate to evaluate the intensity of the irreversibility in individual components. Generally, the physical exergy of every state point in a cycle can be expressed as:

$$e = m[(h - h_0) - T_s(s - s_0)]$$

Where $h_0$ and $s_0$ is the enthalpy and entropy of the refrigerant at ambient temperature $T_s$, respectively. The compression is adiabatic and the isentropic efficiency of compressors of HTC and LTC is related to compression ratio. Different empirical correlations to calculate the compressor’s isentropic efficiency can be found from open literatures. The isentropic efficiencies of both compressors are determined by a simple empirical form from reference[3].

$$\eta_i = 0.874 - 0.0135 \cdot e$$
The enthalpy of state 2 and state 6 were calculated based on isentropic efficiency as follows.

\[ h_2 = \frac{h_{2s} - h_1}{\eta_{\text{LTC,comp}}} + h_1 \]  
\[ h_6 = \frac{h_{6s} - h_5}{\eta_{\text{HTC,comp}}} + h_5 \]

The mechanical and electrical efficiency are also considered in the analysis. For LTC compressor, the value of mechanical and electrical efficiency is 0.9 and 0.95 and 0.9 for both efficiencies for the NH\(_3\) compressor[3]. Therefore, the overall compressor’s efficiency was determined by the equation

\[ \eta_{\text{comp}} = \eta_s \cdot \eta_{\text{mec}} \cdot \eta_{\text{elec}} \]

Specific equations for each component of CRS are summarized in table 1.

**Table 1. Balance equations for each component of the CRS**

| Component          | Energy balance                                      | Exergy balance                                      |
|--------------------|-----------------------------------------------------|-----------------------------------------------------|
| LTC compressor     | \( W_{\text{comp,LTC}} = \frac{m_{\text{LTC}}(h_{2s} - h_2)}{\eta_{\text{LTC,comp}}} \) | \( ED_{\text{comp,LTC}} = T_a m_{\text{LTC}} (s_2 - s_1) \) |
| LTC expansion      | \( h_4 = h_3 \)                                     | \( ED_{\text{exp,LTC}} = T_a m_{\text{LTC}} (s_4 - s_3) \) |
| LTC evaporator     | \( Q_{\text{evap,LTC}} = m_{\text{LTC}} (h_4 - h_4) \) | \( ED_{\text{evap,LTC}} = T_a \left[ m_{\text{LTC}} (s_4 - s_4) - \frac{Q_{\text{evap,LTC}}}{T_{cl}} \right] \) |
| HTC compressor     | \( W_{\text{comp,HTC}} = \frac{m_{\text{HTC}}(h_{6s} - h_6)}{\eta_{\text{HTC,comp}}} \) | \( ED_{\text{comp,HTC}} = T_a m_{\text{HTC}} (s_6 - s_6) \) |
| Condenser          | \( Q_{\text{cond,HTC}} = m_{\text{HTC}} (h_7 - h_6) \) | \( ED_{\text{cond,HTC}} = T_a m_{\text{HTC}} (s_7 - s_6) + Q_{\text{cond,HTC}} \) |
| HTC expansion      | \( h_8 = h_7 \)                                     | \( ED_{\text{exp,HTC}} = T_a m_{\text{HTC}} (s_8 - s_7) \) |
| Cascade heat       | \( Q_{\text{che}} = m_{\text{LTC}} (h_2 - h_3) = m_{\text{HTC}} (h_3 - h_5) \) | \( ED_{\text{che}} = T_a \left[ m_{\text{LTC}} (s_2 - s_3) + m_{\text{HTC}} (s_3 - s_5) \right] \) |

The coefficient of performance (COP) was used to evaluate the CRS based on the first law of thermodynamics and can be defined as:

\[ \text{COP} = \frac{Q_{\text{evap,LTC}}}{W_{\text{comp,LTC}} + W_{\text{comp,HTC}}} \]

According to the second law of thermodynamics, exergy analysis is necessary to evaluate the maximum work capacity of the CRS. In addition, from the analysis of irreversible loss, one can know the degree of a real cycle deviating from the ideal reversible cycle, the effect of each part of the irreversible loss on the system and the component in which the maximum loss occurs. Exergy efficiency of CRS is defined as the ratio of the minimum work requirement to the actual input power. The exergy efficiency of the system is given by the equation

\[ \eta_{\text{eff}} = \frac{W_{\text{comp,LTC}} + W_{\text{comp,HTC}} - ED_{\text{total}}}{W_{\text{comp,LTC}} + W_{\text{comp,HTC}}} \]
where $W_{\text{comp,LTC}}$ and $W_{\text{comp,HTC}}$ are the input power of LTC compressor and HTC compressor, respectively, $ED_{\text{total}}$ are the total exergy destruction rate of the CRS system.

A parametric study was performed to evaluate the performance of CRS. The ambient temperature is 25°C and the temperature difference in cascade heat exchanger is regarded as 5°C. The condensing temperature of CRS is assumed to 40°C. The temperature difference between cold space and evaporation temperature is 5°C. The refrigerant volume flow rate of LTC compressor is 180 m$^3$/h according to an actual type screw compressor. When the volume flow ratio is varied, which means the volume flow rate of HTC compressor is changing which caused by frequency regulation or slide valve regulation method. The calculation flowchart of the simulation process is shown in figure 3. The needed refrigerant thermodynamic parameters were obtained by using REFPROP 9.0 and the model is simulated by using Matlab.

![Figure 3](image_url)

**Figure 3.** Calculation flowchart for optimum volume flow rate ratio and system performance.

2.3. Model validation

In order to verify the theoretical model and program, the simulation results in this research were compared with the reported data in the literature as shown in figure 3. It can be seen that there is a very good agreement between simulation results and the reported data from the literature[3, 6]. From the calculated results, the maximum difference appears to be in the condenser of HTC when the condensing temperature of LTC is -25°C and the maximum difference is 0.646%.

![Figure 4](image_url)

**Figure 4.** Comparison of the total exergy
3. Results and discussion

Figure 5 shows the effect of the volume flow rate ratio on optimum condensation temperature of LTC. It can be seen that the optimum condensation temperature of LTC is decreasing with the volume flow rate ratio increases at given working conditions. The reason can be explained from the energy balance of cascade heat exchanger. When the suction volume rate of LTC compressor is kept constant, the suction volume rate of HTC is increasing with the increases of volume flow rate ratio \( n_V \). If the intermediate temperature remains constant, the condensation heat of LTC in cascade heat exchanger is also unchanged. However, under this circumstance, the evaporation capacity of HTC in cascade heat exchanger is increasing proportionally with the increasing of suction volume flow rate of HTC compressor since the density at the suction state of HTC compressor is constant and caused an increase of mass flow rate. Therefore, in order to ensure the energy balance in cascade heat exchanger, the intermediate temperature is decreased. In addition, the optimum condensation temperature of LTC is different for different evaporation temperature when the condensing temperature of HTC is kept constant.

![Figure 5. Effect of \( n_V \) on optimum condensation temperature of LTC.](image)

Figure 6 depicts the variation of volume flow rate ratio on input compressor power. As figure showed, the \( \text{CO}_2 \) compressor power consumption decreases with the increasing of \( n_V \). The pressure-enthalpy diagram illustrates that as the condensation temperature of LTC decreases, the compression isentropic of LTC compressor shifts to down, which causes a lower value of \((h_2-h_1)\), leading to a lower input power of the compressor. The variation of input power of HTC \( \text{NH}_3 \) compressor has an opposite trend. It increases as \( n_V \) increases. The effect of \( n_V \) on total input compressor power is illustrated in figure 7. It can be learned that the total compressor power decreases slightly at first and then increases with the increasing of \( n_V \).

The variation of discharge temperature of HTC compressor and LTC compressor with the volume flow rate ratio are shown in figure 8 and figure 9. It is observed that the discharge temperature of HTC is increasing with the volume flow rate ratio increases. The results indicate that as the volume flow rate ratio increases, the discharge temperature of HTC increase rapidly firstly and then slowly increase. At a given working conditions that the evaporation temperature of -45°C and condensing temperature of 40°C, the discharge temperature of HTC increased from 114.50°C to 170.81°C when the \( n_V \) increased from 2 to 4. When the volume flow ratio is higher, the discharge temperature of \( \text{NH}_3 \) compressor in HTC is too higher that should be controlled carefully. However, the variation of LTC compressor discharge temperature has an opposite trend. This is due to the change of optimum condensation temperature as shown in figure 5. With the increasing of \( n_V \), the optimum intermediate destruction between present study and reference.
temperature is decreasing and results in an increasing of HTC compressor pressure ratio and a decreasing of LTC compressor pressure ratio.

![Figure 6. Effect of nv on HTC and LTC input compressor power.](image1)

![Figure 7. Effect of nv on total input compressor power.](image2)

![Figure 8. Effect of nv on discharge temperature of NH₃ compressor.](image3)

![Figure 9. Effect of nv on discharge temperature of CO₂ compressor.](image4)

Figure 6 indicates the effect of volume flow ratio on the COP of the cascade refrigeration system. It can be seen that there exists an optimum volume flow rate ratio that makes the maximum COP of CRS. In addition, the optimum condensation temperature of LTC is different for different evaporation temperature if the condensation temperature of HTC is kept constant. When the evaporation temperature is -55°C, the optimum volume flow rate ratio is 2.4 and corresponding maximum COP of CRS is 1.031 and optimum condensation temperature of LTC is -9.75°C. The COP of CRS reaches a maximum value of 1.259 and 1.537 and the corresponding volume flow rate ratio is 2.8 and 3.2 at the evaporation temperature value of -45°C and -35°C.

Figure 10 indicates the effect of volume flow ratio on the COP of the cascade refrigeration system. It can be seen that there exists an optimum volume flow rate ratio that makes the maximum COP of CRS. In addition, the optimum condensation temperature of LTC is different for different evaporation temperature if the condensation temperature of HTC is kept constant. When the evaporation temperature is -55°C, the optimum volume flow rate ratio is 2.4 and corresponding maximum COP of CRS is 1.031 and optimum condensation temperature of LTC is -9.75°C. The COP of CRS reaches a maximum value of 1.259 and 1.537 and the corresponding volume flow rate ratio is 2.8 and 3.2 at the evaporation temperature value of -45°C and -35°C.

Figure 11 shows the variation of cooling capacity with volume flow rate ratio. It can be observed that the cooling capacity of the cascade refrigeration system is increasing rapidly firstly and then increases gradually as nv increases. Since the volume flow rate of LTC compressor is assumed to be constant value, the optimum intermediate temperature decreases with the increasing of nv as shown in figure 5. The pressure ratio of LTC is also reduced while the evaporation temperature is kept constant. As a result, the volumetric cooling capacity is increased. When the evaporation temperature is -55°C, the cooling capacity is increased from 178.75kW to 205.09kW and increased by about 14.73% as nv increase from 2 to 4. The cooling capacity increased more at higher evaporation temperature. It increased by about 20.09% as nv increase from 2 to 4 when evaporation temperature is -35°C. This
means that when the LTC compressor operates at a constant evaporation temperature and speed, the cooling capacity of the CRS could be increased through increasing the rotational speed of HTC compressor. As showed in figure 10 and figure 11, although the cooling capacity is continuously to increase, a maximum COP value exists during volume flow rate ratio increasing. The discharge temperature of NH₃ compressor should be carefully controlled as the \( n_V \) increases.

![Figure 10. Effect of \( n_V \) on COP.](image1)

![Figure 11. Effect of \( n_V \) on cooling capacity.](image2)

Figure 12 depicts the effect of \( n_V \) on total exergy destruction rate. It can be seen that the total exergy destruction rate has a minimum value as the volume flow ratio increases. Figure 13 illustrates that the impact of volume flow rate ratio on exergy efficiency of cascade refrigeration system. The exergy efficiency first increases and then decreases as volume flow rate ratio increases. There exists an optimum volume flow rate ratio that makes exergy efficiency maximum. When evaporation temperature is -55°C, the maximum exergy efficiency value of 0.346 is obtained and the corresponding volume flow rate ratio is about 2.4, which is consistent with the optimum value obtained previously as shown in figure 10. This means that the optimum volume flow ratio could make COP and exergy efficiency maximum instantaneously. It can be further noticed that the maximum exergy efficiency of the CRS at evaporation temperature of -45°C is higher than that of evaporation temperature of -55°C and -35°C.

![Figure 12. Effect of \( n_V \) on total exergy destruction](image3)

![Figure 13. Effect of \( n_V \) on exergy efficiency.](image4)

4. Conclusions
The simulation model of cascade refrigeration system including energy and exergy analysis was developed to study the effect of volume flow ratio of HTC compressor and LTC compressor on the system performance. Based on the obtained results with the models, it can be found that there exists an
optimum volume flow ratio that makes the maximum COP and exergy efficiency and the optimum volume flow ratio is different under various working conditions. When the evaporation temperature is -55°C, the optimum volume flow rate ratio is about 2.4 and corresponding maximum COP and exergy efficiency of CRS is 1.031 and 0.346, respectively. The optimum condensing temperature of LTC is -9.75°C. When the volume flow ratio is increasing, which means the motor frequency or rotational speed of HTC compressor increases while the rotational speed of LTC compressor is fixed, the optimum intermediate temperature decreases and the discharge temperature of HTC compressor increases rapidly and should be controlled. The cooling capacity could be enhanced by increasing the volume flow ratio. The obtained results are helpful for the screw refrigeration compressor regulation in CRS under various working conditions and cooling demands.

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Nomenclature

| Symbol | Description                  | Subscripts |
|--------|------------------------------|------------|
| $T$    | temperature [°C]             | HTC        |
| $\Delta T$ | temperature difference [°C]  | Subscripts |
| $h$    | specific enthalpy [kJ/kg]    | comp, comp |
| $\varepsilon$ | compressor pressure ratio    | che, che   |
| $m$    | refrigerant mass flow rate [kg/s] | c,cond, cond |
| $n_v$  | volume flow rate ratio       | exp, exp   |
| $ED$   | exergy destruction rate [kW] | e, evap    |
| $\eta_{II}$ | exergy efficiency           | s, s      |
| $W$    | input power of compressor [kW]| cl        |
| $V$    | refrigerant volume flow rate [m³/s] | opt |
| $LTC$  | low-temperature cycle        | a, a      |
| $\text{env}$ | environmental temperature    |           |

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