Comment on “System-environment coupling derived by Maxwell’s boundary conditions from the weak to the ultrastrong light-matter coupling regime”

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In a recent work [1], Bamba and Ogawa developed a microscopic model describing the field of a photonic cavity coupled to a matter exciton-like resonance. One of the results they obtain studying such a model is that, in the ultrastrong coupling regime, usually safe approaches can give wrong results for the dissipation rates of the polaritonic excitations. In particular the dissipation rates calculated applying the rotating wave approximation on the system-environment coupling qualitatively differ from the ones calculated using a microscopic theory based on the quantum electrodynamics for dielectric media. Here I show that this result is an artifact, caused by an inconsistent application of the rotating wave approximation and by a questionable parameter choice.

In this comment I will show that such a discrepancy is an artifact. Point (I) is the consequence of an inconsistent application of the RWA while point (II) is the consequence of a questionable parameter choice. If such mistakes are corrected the two approaches give fully consistent results.

About point (I), in Ref. [1] the following formula is used for the dissipation rate of the m\textsuperscript{th} polaritonic mode in the RWA

\[
\kappa_{jm}^{\text{RWA}} = |\langle j | \omega_{jm} | m \rangle|^2 \kappa_m, \quad (1)
\]

where \(j = L, U\) indexes the two polaritonic branches. This result is found starting from the system-environment coupling Hamiltonian

\[
H_{S-E} = \sum_m d \omega \frac{i}{\hbar} \sqrt{\frac{\kappa_m}{2\pi}} (\alpha(\omega)^{\dagger}a_m - a_m^\dagger \alpha(\omega)), \quad (2)
\]

and expressing the cavity photon operators as a function of the polaritonic operators

\[
a_m = \sum_{j=L,U} (\omega_{jm} P_{jm} - y_{jm} P_{jm}^\dagger). \quad (3)
\]

Neglecting the resulting anti-resonant terms, that is considering \(y_{jm} = 0\), Eq. (1) is obtained. The problem with this procedure is that, in order to respect bosonic commutation relations, the coefficients in Eq. (2) have to respect the normalisation condition

\[
\sum_{j=L,U} |\omega_{jm}|^2 - |y_{jm}|^2 = 1. \quad (4)
\]

To simply neglect the \(y_{jm}\) coefficients amounts to consider non-normalised, and thus non-bosonic polaritonic operators. The problem can be solved renormalising the Hopfield coefficients after having put the antiresonant terms to zero. This gives the dissipation rate

\[
\kappa_{jm}^{\text{RWA}} = \frac{|\omega_{jm}|^2}{\sum_{j=L,U} |\omega_{jm}|^2 \kappa_m}. \quad (5)
\]

In Fig. 1 (b) there is a comparison between the MBC results from Ref. [1] and the results obtained by the normalised RWA in Eq. (5). As we can see problem (I) has...
been completely solved. The dissipation rates obtained using the two approaches are consistent, although still inverted.

About point (II) Bamba and Ogawa consider for simplicity their parameter $\Lambda(\omega)$ to be frequency independent over the frequency range of interest. As $\Lambda(\omega) = \eta(\omega)\omega/c$ this implies that the mirror’s permittivity $\eta(\omega)$ is proportional to $\omega^{-1}$. This seems a physically unjustified assumption especially given that, as the mirror in their model is infinitely thin, it would seem more natural to assume it is metallic, having thus a permittivity roughly proportional to $\omega^{-2}$. Taking into account this different frequency dependency, the Eq. (26) of Ref. [1]

$$\kappa_{\text{MBC}}(\Omega) \simeq \frac{\kappa_0}{1 + (\Omega/\omega_{\text{ex}})^2}, \quad (6)$$

giving the loss rates using the MBC approach becomes instead

$$\tilde{\kappa}_{\text{MBC}}(\Omega) \simeq \frac{\kappa_0}{1 + (\omega_{\text{ex}}/\Omega)^2}. \quad (7)$$

In Fig. 1 (c) we can see that the loss rates calculated using MBC with metallic mirrors from Eq. (7) and the ones using the normalised RWA formula from Eq. (5) are in very good agreement, and that also the problem highlighted in point (II) has thus disappeared.

The problem of understanding the reliability of the RWA phenomenological approach to system-environment coupling in the USC regime remains open. The very good agreement between the results obtained using the metallic MBC in Eq. (7) and the RWA with frequency independent loss rates in Eq. (5) is not well understood, and it is at this point unclear if some form of generality does hold, or if in the USC regime a degree of microscopic modeling is necessary to obtain quantitative results. Still this comment shows that the model developed in Ref. [1] does not disprove the applicability of the RWA approach but, on the contrary, it gives a first microscopic justification of its validity, at the very least for the particular system considered.

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