Interstellar Extinction

\[ \langle \text{Extinction} \rangle = \langle \text{Absorption} \rangle + \langle \text{Scattering} \rangle \]

Evidence of extinction

(a) Dark clouds in photographs
(b) Statistically star clusters brightness \(\leftrightarrow\) size
   e.g., dimmer \(\leftrightarrow\) smaller, but Trumpler in 1930s
   found clusters appear further fainter
(c) Star count
Fig. 1.—Comparison of the distances of 100 open star clusters determined from apparent magnitudes and spectral types (abscissae) with those determined from angular diameters (ordinates). The large dots refer to clusters with well-determined photometric distances, the small dots to clusters with less certain data (half weight). The asterisks and crosses represent group means. If no general space absorption were present, the clusters should fall along the dotted straight line; the dotted curve gives the relation between the two distance measures for a general absorption of 0.7 mag/kpc.
Star Count

Prediction of a uniform galaxy

Assumptions:
(i) stars uniformly distributed: \( D \) stars pc\(^{-3}\)
(ii) our galaxy infinite in extent
(iii) no extinction

(In reality, none of the above is true!)
Total number of stars out to $r$

$$N(r) = \omega D \int_0^\infty r'^2 \, dr' = \frac{1}{3} \omega D \, r^3$$

If all stars have absolute magnitude $M$ (i.e., same intrinsic brightness --- another untrue assumption), since

$$m - M = 5 \log r_{pc} - 5$$

$$r_{pc} = 10^{0.2(m-M)+1}$$

So $N(r) = 10^{0.6 \, m - C}$, where $C = C(D, \omega, M) \rightarrow N(m) \propto 10^{0.6 \, m}$

$\therefore 10^{0.6} \approx 4$, so the number of stars increases 4 times as we go 1 mag fainter.

This is logically unlikely, because if we integrate over $m$, the sky would have been blazingly bright (Olbers’ paradox).
Olbers’ Paradox --- Why is the night sky dark?

The paradox can be argued away in the case of the Galaxy by its finite size, but the same paradox exists also for the Universe → expansion of the Universe

The star count result was recognized by Kapteyn → Kapteyn Universe: star density falls as the distance increases

Extinction effect: If w/o absorption we observe \( m \) mag, then with \( a(r) \) mag absorption at \( r \), we would observe \( m + a(r) \)

Without extinction: \( \log r = 0.2(m - M) + 1 \)
So the apparent distance $r'$ ($> r$)

$$\log r' = 0.2 \left[ m + a(r) - M \right] + 1 = 0.2 \left( m - M \right) + 1 + 0.2 a(r)$$

$$= \log r + 0.2 a(r)$$

$$\rightarrow r' = 10^{0.2 a(r)} r$$

So dimming of 1.5 mag
$\rightarrow$ overestimate of distance by $2 \times$
$\rightarrow$ underestimate space stellar density by $8 \times$

Both the star density falling off and extinction should be taken into account $\rightarrow$ Galactic structure

Galactic poles: minimal extinction

Galactic disk: extinction significant $\sim 1 \text{ mag kpc}^{-1}$
In general, \( m_\lambda - M_\lambda = 5 \log r_{pc} - 5 + A_\lambda \)

Because \( A_\lambda = -2.5 \log \frac{F_\lambda}{F_{\lambda,0}} \)

\( F_{\lambda,0} \): flux that would have been observed w/o extinction

and \( F_\lambda = F_{\lambda,0} e^{-\tau_\lambda} \)

\[ 
\rightarrow A_\lambda = -2.5 \log(e^{-\tau_\lambda}) \equiv 1.086 \tau_\lambda \equiv 1.086 N_d \sigma_\lambda Q_{ext} 
\]

\( N_d \): # of dust grains cm\(^{-2}\)

\( \sigma_\lambda \): geometric cross section \((= \pi a^2)\)

\( Q_{ext} \): [dimensionless] ‘extinction efficiency factor’, \( Q_{ext} = Q_{ext}(\lambda) \)

\[ 
= \text{[optical cross section]} / \text{[geometric cross section]} 
\]

**Note:** \( A_\lambda \leftrightarrow \lambda \); extinction at different wavelengths \( \rightarrow \text{reddening} \)
Why dust? (what causes 1 mag kpc$^{-1}$?)

Possibilities:

1. **Scattering by free electrons --- Thomson scattering**

   \[
   \sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{mc^2} \right) \approx 6.6 \times 10^{-25} \text{ (cm}^2\text{) for } \nu < 10^{20} \text{ Hz}
   \]

   Since \( A_v = 1.086 \bar{n} \sigma \ell \)

   \[
   1 = 1.086 \bar{n} \cdot 6.6 \times 10^{-25} \cdot 3 \times 10^{21} \text{ cm}
   \]

   \[
   \rightarrow \bar{n} \approx 500 \text{ cm}^{-3}
   \]

2. **Scattering by bound charges --- Rayleigh scattering?**

   \[
   \sigma_R \sim \sigma_T \left( \frac{\nu}{\nu_0} \right)^4 \text{ cm}^2 \quad (\nu \ll \nu_0)
   \]

   \[
   \sim \sigma_T \frac{\nu^4}{(\nu^2 - \nu_0^2)^2} \text{ cm}^2 \quad (\nu < \nu_0)
   \]

   Both \( \sigma_R < \sigma_T \)

   \[
   \bar{n} \approx 10 - 100 \times 10^4
   \]
(3) Absorption by solid particles?

For particle radius ~ wavelength, $Q_e \sim 1$

$$A_v = 1.086 \, \bar{n} \, \sigma \, \ell$$

Size of grains

$$1 \approx \bar{n} \pi (5 \times 10^{-5})^2 \cdot 3 \times 10^{21}$$

$$\rightarrow \bar{n} \approx 4 \times 10^{-14} \, cm^{-3}$$

Volume mass density

$$\frac{4}{3} \pi a^3 \bar{n} \rho \sim 4 \times 10^{-26} (g \, cm^{-3}) \rightarrow 1\% \, of \, Oort \, limit$$

Note: wavelength dependence

Extinction $Q \sim \lambda^{-1}$

Thomson $\sim \lambda^0$

Rayleigh $\sim \lambda^{-4}$

ρ(material) ~ 2 g cm⁻³
Oort Limit

--- mass in the plane by star counting along the height

\( \rho(z) \): (total) mass density; \( v(z) \): velocity dispersion of stars

Poisson eq.

\[
\Delta \phi = -\frac{dg_z(z)}{dz} = 4\pi G \rho(z)
\]

\( \rho_{\text{ISM}} \lesssim 6 \times 10^{-24} \text{ g cm}^{-3} \)

\( \sim \) about 2-3 H atoms cm\(^{-3}\), assuming He/H \( \sim 10\% \) by number

So, a volume mass density of \( 4 \times 10^{-26} \text{ g cm}^{-3} \) is ok, and if dust is responsible for the extinction, this implies a gas-to-dust ratio of \( \sim 100 \)
The grains appear to be loose (porous) conglomerations of smaller specks of material, which stuck together after bumping into each other.
Selective Extinction --- the wavelength dependence of extinction

Choose 2 stars of the same spectral types and luminosity classes. Observe their magnitude difference $\Delta m$ between $\lambda_1$ and $\lambda_2$

$\Delta m$ is caused by (1) different distances, and (2) extinction by intervening dust grains

OB stars are good choices because they can be seen at large distances and their spectra are relatively simple.

Observed at 2 $\lambda$s: $\Delta m_{\lambda_1} - \Delta m_{\lambda_2}$

distance dependence canceled out,

$$\Delta m_{\lambda_1} - \Delta m_{\lambda_2} = \Delta (A_{\lambda_1} - A_{\lambda_2})$$
\[ E_{\lambda_1-\lambda_2} = (m_{\lambda_1} - m_{\lambda_2}) - (m_{\lambda_1} - m_{\lambda_2})_0 \] (color difference/excess)
\[ = (m_{\lambda_1} - m_{\lambda_1,0}) - (m_{\lambda_2} - m_{\lambda_2,0}) = A_{\lambda_1} - A_{\lambda_2} \]

For example, \( \lambda_1 = 4405 \text{ Å} \) (B band), \( \lambda_2 = 5470 \text{ Å} \) (V band)

\[ E_{B-V} \text{ [color excess]} = \text{[measured color]} - \text{[intrinsic color]} \]

Traditionally shorter minus longer, e.g., \( E(B - V), E(I - K), E(U - B), E(K - W1) \)

\[ E_{B-V} = (B - V) - (B - V)_0 = A_B - A_V \]

The intrinsic colors of stars of different types are known.
The ‘normalized’ extinction (extinction/reddening law)

\[ F(\lambda) = \frac{A_\lambda - A_V}{A_B - A_V} = \frac{E_{\lambda-V}}{E_{B-V}} \]

The UV ‘bump’ \(1/\lambda \sim 4.6\)
\[ \Rightarrow \lambda \sim 2175 \text{ Å} \]

In the visible
\[ F(\lambda) \sim 1/\lambda \]

F(V) = 0
F(B) = +1

Find
\[ A_B/A_V=? \]
Draine Fig 21.1

\[ \lambda > 2 \mu m \]

- C-H 3.4\mu m
- silicate 10\mu m
- 2175\AA feature
- aromatic carbon?
Table 21.1 Extinction for Standard Photometric Bands for $R_V = 3.1$

| Band | $\lambda$ (\textmu{}m) | $A_{\lambda}/A_{IC}$ | Band | $\lambda$ (\textmu{}m) | $A_{\lambda}/A_{IC}$ |
|------|-----------------|-----------------|------|-----------------|-----------------|
| $M$  | 4.75            | 0.0573          | $i$  | 0.7480          | 1.125           |
| $L'$ | 3.80            | 0.0842          | $R_C$| 0.6492          | 1.419           |
| $L$  | 3.45            | 0.101           | $R_J$| 0.6415          | 1.442           |
| $K$  | 2.19            | 0.212           | $r$  | 0.6165          | 1.531           |
| $H$  | 1.65            | 0.315           | $V$  | 0.5470          | 1.805           |
| $J$  | 1.22            | 0.489           | $g$  | 0.4685          | 2.238           |
| $z$  | 0.893           | 0.830           | $B$  | 0.4405          | 2.396           |
| $I_J$| 0.8655          | 0.879           | $U$  | 0.3635          | 2.813           |
| $I_C$| 0.8020          | 1.000           | $u$  | 0.3550          | 2.867           |

Draine

Riek & Lebofsky (1985)

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Table 1  Interstellar extinction and $A(\lambda)/A(J)$, where $J \approx 1.25 \mu m^a$

| $\lambda$ (\mu m) | $A(\lambda)/A(J)$ | $\lambda$ (\mu m) | $R_V = 3.1$ | $R_V = 5.0$ | $\lambda$ (\mu m) | $A(\lambda)/A(J)$ |
|------------------|------------------|------------------|-------------|-------------|------------------|------------------|
| 250\(^b\)       | 0.0015           | 5                | 0.095       | 0.095       | 0.24            | 9.03            | 5.13            |
| 100              | 0.0041           | 3.4              | 0.182       | 0.182       | 0.218           | 11.29           | 6.03            |
| 60               | 0.0071           | 2.2              | 0.382       | 0.382       | 0.20            | 10.08           | 5.32            |
| 35               | 0.013            | 1.65             | 0.624       | 0.624       | 0.18            | 8.93            | 4.66            |
| 25               | 0.048            | 1.25             | 1.00        | 1.00        | 0.15            | 9.44            | 4.57            |
| 20               | 0.075            | 0.9              | 1.70        | 1.70        | 0.13            | 11.09           | 4.89            |
| 18               | 0.083            | 0.7              | 2.66        | 2.43        | 0.12            | 12.71           | 5.32            |
| 15               | 0.053            | 0.55             | 3.55        | 3.06        | 0.091\(^c\)    | 17.2            | ---             |
| 12               | 0.098            | 0.44             | 4.70        | 3.67        | 0.073           | 19.1            | ---             |
| 10               | 0.192            | 0.365            | 5.53        | 4.07        | 0.041           | 9.15            | ---             |
| 9.7              | 0.208            | 0.33             | 5.87        | 4.12        | 0.023           | 7.31            | ---             |
| 9.0              | 0.157            | 0.28             | 6.90        | 4.34        | 0.004           | 3.39            | ---             |
| 7                | 0.070            | 0.26             | 7.63        | 4.59        | 0.002           | 1.35            | ---             |

\(^a\) $A(\lambda)/A(J)$ is the same for $\lambda > 0.9 \mu m$ for all lines of sight, to within present errors. To estimate $A(\lambda)/N(H)$, multiply tabulated entry for $R_V = 3.1$ by $1.51 \times 10^{-22} \text{ cm}^2 \text{ (H atom)}^{-1}$. Except as noted below, entries are calculated from CCM89. Other values of $R_V$ can be determined from that paper.

\(^b\) For $\lambda > 250 \mu m$, multiply entry for 250 $\mu m$ by $(250 \mu m)/\lambda$.

\(^c\) For $\lambda < 0.1 \mu m$, entries are from (107), increased by 1.15 for continuity with the CCM89 extinction value at 0.12 $\mu m$. 

Mathis (1990) 22
Total Extinction  … quantified by $A_V$ (at 5550 Å)

Ratio of total-to-selective extinction, $R_v \equiv A_v/E_{B-V}$

$A_v \leftrightarrow$ total number of grains and grain properties

Generally accepted $\langle R_v \rangle \approx 3.1 \pm 0.1$, i.e., $A_V = 3.1 \ E_{B-V}$

$N_H/E (B - V) = 5.8 \times 10^{21} \ [\text{H atoms cm}^{-2} \ \text{mag}^{-1}]$

$E (B - V)/N_H$, the dust-to-gas ratio, is almost constant along MW lines of sight.

In the solar neighborhood $n_H \approx 1 \ \text{cm}^{-3}$, so $A_V \approx 1.6 \ [d/kpc]$ (Binney & Merrifield)
◆ $A_V$ can be estimated by observing stars.

◆ The estimate is not reliable toward any particular direction or object, because clouds are patchy.

◆ In dark molecular clouds, $R_V$ can be large $\sim 5 - 7$, implying large average sizes of dust grains,

$$ a \uparrow \Rightarrow R_V \uparrow $$

◆ Grain properties: shape, size, composition, structure (core, mantle) $\leftrightarrow$ optical properties
Stellar atmosphere (gas) \( \rightarrow \) absorption lines

ISM dust (solid) \( \rightarrow \) extinction profile with no strongly marked lines or bands, except a few weak bands at 3.1 \( \mu m \) (\( H_2O \) ice) and 9.7 \( \mu m \) (silicates)
Figure 5.8 The spectrum of sources in the galactic center show strong absorption features due to dust grains along the line of sight. These are shown here on an optical depth scale. Identifications are indicated on top. Some of these features originate in the diffuse interstellar medium (e.g., the hydrocarbon [HAC] bands), while others are due to material in molecular clouds (e.g., H$_2$O, NH$_3$, and CH$_4$ ice). Some bands have probable contributions from both media. Figure courtesy of J. E. Chiar. Data from J. E. Chiar, et al., 2000, Ap. J., 537, p. 749.
Figure 4.3. The infrared spectrum of the Galactic centre, taken over a wavelength range 2.4–45 μm by the Short Wavelength Spectrometer on the Infrared Space Observatory (ISO). In addition to various emission lines from the hotter regions along this line of sight, there are strong absorptions due to material in the dust grains near 3 μm (H₂O ice), 9.7 μm and 18 μm (silicates). Some weaker features are formed, including those at 3.4 μm (hydrocarbons) and 4.3 μm (solid carbon dioxide). (Courtesy D Lutz et al 1996.)
Cross-Linked Hetero Aromatic Polymers in Interstellar Dust
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Abstract: The discovery of cross-linked hetero-aromatic polymers in interstellar dust by
instruments aboard the Stardust spacecraft would confirm the validity of the biological
grain model that was suggested from spectroscopic studies over 20 years ago. Such
structures could represent fragments of cell walls that survive 30km/s impacts onto
detector surfaces. \textit{Astrophysics and Space Science, 2000}

\textit{Escherichia coli}
大腸桿菌
So it all amounts to the efficiency factors $Q$’s

$$Q_{\text{extinction}} = Q_{\text{scattering}} + Q_{\text{absorption}}$$

$$E_{B-V} = A_B - A_V = 1.086(\tau_B - \tau_V) = 1.086\pi a^2 n_d \ell (Q_B - Q_V)$$

Scattering of EM waves by spherical particles (the simplest case) \(\rightarrow\) Mie scattering (Lorenz-Mie solution to the Maxwell’s eqs)
**Albedo** ("whiteness"): how much light is reflected without being absorbed. \( A = \frac{Q_{\text{sca}}}{Q_{\text{ext}}} \)

In astronomy, **geometric albedo** \( A_G \) (measured brightness when illumination comes from directly behind the observer), **Bond albedo** \( A_B \) (total energy reflected).

For solar-system planets,
- Earth: \( A_G = 0.43, A_B = 0.31 \),
- Venus: \( A_G = 0.69, A_B = 0.76 \),
**Scattering**

Size of particles $a$

- $2\pi a \ll \lambda \implies$ scattering $\leftrightarrow \lambda$
  
  $I_{\text{scattering}} \propto \lambda^{-4}$ (Rayleigh scattering)

  This is why a clear sky is blue.

- $2\pi a \gg \lambda \implies$ scattering $\not\leftrightarrow \lambda$ (cross section $\sim$ geometric)
  
  $I_{\text{scattering}} \propto \lambda^0$

  This is why a cloudy sky is gray.

- $2\pi a \approx \lambda$ (dust $\bar{a} = 0.3 \ \mu m$; $\lambda$ in UV/visible)
  
  $I_{\text{scattering}} \propto \lambda^{-1}$

  This is interstellar reddening --- why distant stars are red.

Usually one assumes a size distribution, e.g., a power law, $n(a) \propto a^{-\beta}$
**Index of refraction**

\[ m = n + i k \]

- \( m = \infty \)  **Dielectric**
- \( m = 1.33 \)  **Ice**
- \( m = 1.33 - 0.09i \)  **Dirty ice**
- \( m = 1.27 - 1.37i \)  **Iron**

**Real part:** ratio of vacuum speed of light to the phase speed in the medium  
**Imaginary part:** absorption

\[ x = \frac{2\pi a}{\lambda} = \text{dust size/wavelength} \]
• In Earth’s atmosphere, scattering
  \( \sim \lambda^{-4} \) for small particles
  \( \sim \lambda^0 \) for large particles

• In ISM at visible wavelength, scattering
  \( \sim \lambda^{-1} \) for particle size \( \approx \) wavelength \( \sim 0.5 \mu m \)

• For large particles, \( Q \sim 2, \)
  i.e., \( \sigma \approx 2 \) times the geometric cross section,
  because light diverges over larger extent
Figure 1. Solid line: The extinction cross-section normalized per H-atom of the diffuse neutral (95% HI, 5% HII, 10% HeI) interstellar medium from the far-infrared to the X-rays. Dotted lines: The UV extinction on the lines of sight in two extreme cases, HD 204827 (upper curve) and HD 37023 (θ1-Orionis D, lower curve). Shortward of the Lyman limit, the dotted line corresponds to the ionization state of the solar neighbourhood (80% HI, 20% HII, 5% HeI, 5% HeII). The sources of the data are given in the text.
Rayleigh Scattering by Small Particles

Dielectric sphere \( a \) \((2\pi a \ll \lambda)\)

Polarization of plane wave, \( \lambda \)

Sphere oscillates with the \( \mathbf{E} \) field, and radiates like an electric dipole.

Power radiated in all directions

\[
P = \frac{2}{3} \frac{e^2}{c^3} \ddot{x}^2 \quad \text{where } \ddot{x} \text{ is acceleration.}
\]

\[
x = x_0 e^{-j\omega t}
\]

\[
\ddot{x} = -x_0 \omega^2 e^{-j\omega t}
\]
Poynting vector

\[ S = \frac{c}{8\pi} \mathbf{E}\mathbf{B}^* = \frac{c}{4\pi} \mathbf{E}^2 \]

\[ Q_{\text{sca}} = \sigma / \pi a^2 \]

One can show ...

\[ Q_{\text{sca}} = \frac{8}{3} x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \propto \lambda^{-4} \]

\[ Q_{\text{abs}} = -4 x \text{Im} \left( \frac{m^2 - 1}{m^2 + 2} \right) \propto \lambda^{-1} \]

✓ \( x = 2\pi a / \lambda \)
✓ \( m = n + i k \)
✓ \( Q_{\text{ext}} = Q_{\text{sca}} + Q_{\text{abs}} \)
Note:

- When $m$ is real, i.e., no imaginary part $\rightarrow$ no absorption

- With the imaginary part, most extinction at small $x$ comes from absorption $\rightarrow Q_{ext}$ increases

- For pure ice, transmitted and refracted signals interfere $\rightarrow$ large scale oscillation

- If there is impurity (internal absorption) $\rightarrow$ oscillation is reduced
Fig 21.3

Polarization for 2765 stars within 1 kpc from Heiles (2000)

Polarization for 5453 stars from Heiles (2000)
Empirically, polarization peaks around V band (the “Serkowski law” (1973))

\[ p(\lambda) \approx p_{\text{max}} \exp[-K \ln^2(\lambda/\lambda_{\text{max}})] , \]

where \( \lambda_{\text{max}} \approx 5500 \text{ Å} \), and \( K \approx 1.15 \).

Polarization is caused by dust grains partially aligned by IS B field, with the shortest axis parallel to the field direction.
References

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