An optimal control method for fluid structure interaction systems via adjoint boundary pressure

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Abstract. In recent year, in spite of the computational complexity, Fluid-structure interaction (FSI) problems have been widely studied due to their applicability in science and engineering. Fluid-structure interaction systems consist of one or more solid structures that deform by interacting with a surrounding fluid flow. FSI simulations evaluate the tensional state of the mechanical component and take into account the effects of the solid deformations on the motion of the interior fluids. The inverse FSI problem can be described as the achievement of a certain objective by changing some design parameters such as forces, boundary conditions and geometrical domain shapes.

In this paper we would like to study the inverse FSI problem by using an optimal control approach. In particular we propose a pressure boundary optimal control method based on Lagrangian multipliers and adjoint variables. The objective is the minimization of a solid domain displacement matching functional obtained by finding the optimal pressure on the inlet boundary. The optimality system is derived from the first order necessary conditions by taking the Fréchet derivatives of the Lagrangian with respect to all the variables involved. The optimal solution is then obtained through a standard steepest descent algorithm applied to the optimality system. The approach presented in this work is general and could be used to assess other objective functionals and controls. In order to support the proposed approach we perform a few numerical tests where the fluid pressure on the domain inlet controls the displacement that occurs in a well defined region of the solid domain.

1. Introduction

Optimization has always been used to improve the performance of engineering devices. Nowadays several approaches to optimization are available, such as single and multi-objective, adjoint or sensitivities based method, evolutionary algorithms and many others. In literature there are several works dealing with this subject, for a quick review the interested reader can see [1, 2, 3, 4, 5] and references therein. Fluid-Structure Interaction (FSI) problems can be defined as a system where the fluid flow changes the tensional state of a solid structure that is left free to move and the solid deformation has an important effect on the fluid flow. Examples of this type of systems are quite common in engineering, like wind turbines, man-made drones and in the study of biological systems such as hemodynamics. In literature these topics are investigated deeply and the interested reader can see [6, 7, 8, 9, 10, 11, 12, 13, 14]. Many attempts to apply optimization techniques to FSI problems can be found, for example in [15], where the authors...
propose a solution method for the problem of optimizing a non-linear aeroelasticity system in a steady-state flow by using a sensitivity method. The work in [16] deals with general shape optimization methods based on design sensitivity analysis.

In this work we refer to adjoint based methods, which have been proven to be a good approach for the optimal control of complex problems in which Computational Fluid Dynamics simulations can be performed on the system of interest, [15, 16]. Moreover these methods have a solid mathematical background and the existence of local optimal solutions can be proven for many interesting cases, [16, 17]. We are interested in a monolithic approach to the solution of the FSI system leading to a stable and well defined solution in a finite element setting [18, 19]. Inside this framework we study a pressure boundary optimal control problem applied to the monolithic FSI system. The objective of the control is the matching of a displacement field in a particular region of the solid domain. This is accomplished by changing the pressure on the fluid inlet boundary which alters the fluid flow profile and thus deforms the shape. This method can be useful in many industrially relevant applications. For example, given a pipe containing a pressurized liquid and the maximum deformation that the pipe can sustain without damaging, it is necessary to know in advance the highest admissible pressure that guarantees both the preservation of the pipe as well as a high liquid flow rate. A possible application in the biomedical field concerns stents, which are metal or plastic tube inserted into arteries. Since their flexibility is limited by the material properties, it can be useful to know what range of inner fluid pressure can be tolerated without damaging the stent itself. In the next section we derive the optimality system which consists of the state, the adjoint system and the control equation. In order to solve the optimality system we propose a simple steepest descent algorithm. In Section 3 we report some numerical results obtained by the implementation of the optimal control algorithm in a finite element parallel code designed for multiphysics simulations.

2. Mathematical formulation
In this section we discuss the mathematical formulation of the FSI problem together with the optimality system. We briefly recall some notations about functional spaces used in this paper, for a detailed description one can consult [20, 21]. We use standard notation for the Sobolev spaces with norm \( \| \cdot \|_s(\Omega) = L^2(\Omega) \) and \( \| \cdot \|_0 = \| \cdot \| \). Let \( H^s(\Omega) \) be the space of all functions in \( H^s(\Omega) \) that vanish on the boundary of the bounded open set \( \Omega \) and \( H^{-s}(\Omega) \) be the dual space of \( H^0(\Omega) \). The trace space for the functions in \( H^1(\Omega) \) is denoted by \( H^{1/2}(\Gamma) \).

Let us consider a domain \( \Omega \subset \mathbb{R}^N \) which consists of a structure domain \( \Omega_s \) and a fluid domain \( \Omega_f \) so that \( \Omega = \Omega_s \cup \Omega_f \). The deformed solid domain \( \Omega_s(1) \) is expressed through the solid displacement field \( l \) as

\[
\Omega_s(1) = \{ x \in \mathbb{R}^3 \mid x = x_0 + 1 \},
\]

where the vector \( x_0 \) defines the initial solid domain \( \Omega_s(0) \). For our steady state FSI problem we consider the interaction of a Newtonian fluid, whose behavior is described by the Navier-Stokes equations, with an hyperelastic incompressible St.Venant-Kirchhoff material. The mathematical model in strong form of the FSI problem is then the following

\[
\begin{align*}
\nabla \cdot v &= 0 \quad \text{on} \quad \Omega_f, \\
\rho_f (v \cdot \nabla)v - \nabla \cdot T &= 0 \quad \text{on} \quad \Omega_f, \\
\nabla \cdot S(1) &= 0 \quad \text{on} \quad \Omega_s,
\end{align*}
\]

with \( v \) being the fluid velocity field and \( \rho_f \) its density. The constitutive relations for the fluid stress tensor \( T \) in the Newtonian incompressible case and for the solid Cauchy stress tensor \( S \) as
read
\[ T(p_f, v_f) := -p_f I + \mu_f (\nabla v + \nabla v^T), \]
\[ S(l) := \lambda_s (\nabla \cdot l) I + \mu_s \nabla l, \]
where \( p_f \) is the fluid pressure, \( \mu_f \) the dynamic viscosity of the fluid while \( \lambda_s \) and \( \mu_s \) are the solid Lamé parameters. In the following we study only the incompressible case that occurs when \( \lambda_s \to \infty \). In this case we introduce the solid pressure \( p_s = -\lambda_s (\nabla \cdot l) \) and write
\[ S(l) := -p_s I + \mu_s \nabla l, \]
where the displacement \( l \) is subject to the incompressible constraint \( \nabla \cdot l = 0 \). To complete the system (2-4) it is necessary to define the appropriate boundary and interface conditions, which are
\[ v = v_0 \quad \text{on} \quad \Omega_{fd}, \]
\[ l = l_0 \quad \text{on} \quad \Omega_{sd}, \]
\[ T \cdot n = 0 \quad \text{on} \quad \Omega_{fn}, \]
\[ S \cdot n = 0 \quad \text{on} \quad \Omega_{sn}, \]
\[ T \cdot n = S \cdot n \quad \text{on} \quad \Gamma_i, \]
\[ v = 0 \quad \text{on} \quad \Gamma_i, \]
where \( \Omega_{fd} \) and \( \Omega_{sd} \) are the surfaces where Dirichlet boundary conditions are imposed for the fluid velocity and solid displacement, \( \Omega_{fn} \) and \( \Omega_{sn} \) are the surfaces where standard homogeneous outflow boundary conditions are imposed for both the fields. By \( \Gamma_i \) we denote the interface between the solid and fluid domain, \( \Gamma_i = \Omega_s \cap \Omega_f \). When using a monolithic approach for the numerical solution of the FSI system the conditions (8) are automatically set, including the equilibrium of normal stresses that has to be imposed on the interface. However it is very common to solve FSI problems by using dedicated solvers for the solid and fluid domain. This solving approach is called segregated and has the drawback that the coupling conditions are not implicitly part of the solver but have to be imposed with an iterative loop.

The first step to obtain the optimality system with the Lagrange multipliers method is to choose an objective functional for the control problem. In this work we study a solid domain displacement matching problem where the control is the pressure on the fluid inlet boundary. The objective or cost functional can be defined as follows
\[ J(l, p) = \frac{1}{2} \int_{\Omega_d} w(1-l_d)^2 d\Omega + \int_{\Gamma_c} \beta p_c^2 d\Gamma, \]
The non-negative function \( w \) is a weight function of the coordinate \( x \) that can be used to impose the objective on a specific part of the domain and \( l_d \) is the desired value of the solid displacement. The functional is completed by a regularization term involving the regularization parameter \( \beta \), which is needed to obtain a control function \( p_c \) in the space of square integrable functions \( L^2(\Omega_c) \).

If a too high value of \( \beta \) is chosen the control becomes too smooth and the objective cannot be achieved well, while a lack of regularization leads to convergence issues in the numerical solution of the problem. To obtain the optimality system we write the full constrained Lagrangian of the problem which is composed of the cost functional and state equations multiplied by the
appropriate Lagrangian multipliers
\[ \mathcal{L}(p, \mathbf{v}, l, p_a, \mathbf{v}_a, \mathbf{\hat{l}}, s_a, \beta_a) = \mathcal{J}(l, p) + \int_{\Omega_f} (\nabla \cdot \mathbf{v}) p_a \, d\Omega + \int_{\Omega_s} (\nabla \cdot \mathbf{v}) p_a \, d\Omega \\
+ \int_{\Gamma_c} (\mathbf{v}_a \cdot \mathbf{n}) p \, d\Gamma + \int_{\Omega_f} [\rho_f (\mathbf{v} \cdot \nabla \mathbf{v}) + \nabla p - \nabla \cdot (\mu_f \nabla \mathbf{v})] \cdot \mathbf{v}_a \, d\Omega \\
+ \int_{\Omega_s} \nabla \cdot [\mu_s \nabla l - \nabla p] \cdot \mathbf{v}_a \, d\Omega + \frac{1}{2} \int_{\Omega_a} \nabla^2 \mathbf{l} \cdot \mathbf{\hat{l}} \, d\Omega \\
+ \int_{\Gamma_i} \delta_a \cdot \left[ (1 - l) + \frac{\mathbf{v}}{h} \right] \, d\Gamma + \int_{\Omega_s} \beta_a \cdot \left[ \mathbf{v} - h(1 - \mathbf{\hat{l}}) \right] \, d\Omega. \]

In (10) we have introduced the auxiliary mesh displacement \( \mathbf{\hat{l}} \), defined only over the solid domain and solution of a Laplace operator. The velocity field \( \mathbf{v} \) has been extended to the whole domain \( \Omega \) as
\[ \mathbf{v} = \begin{cases} h(1 - \mathbf{\hat{l}}) & \text{on } \Omega_s, \\ \mathbf{v} & \text{solution of (2)-(3) on } \Omega_f, \end{cases} \]
with \( h \) being a positive constant. It is clear that at steady state conditions \( \Omega_s(1) = \Omega_s(\mathbf{\hat{l}}) \), with \( \mathbf{v} = 0 \) in the solid. By taking the Fréchet derivatives of the Lagrangian with respect to the adjoint variables the weak form of the state system (2-4) is obtained, together with the correct boundary and interface conditions. When the derivatives are taken with respect to the state variables, after some term rearrangement, the adjoint system in weak form reads
\[ \int_{\Gamma_c} (\rho \beta + \mathbf{v}_a \cdot \mathbf{n}) \delta p \, d\Gamma - \int_{\Omega_f} (\nabla \cdot \mathbf{v}_a) \delta p \, d\Omega - \int_{\Omega_s} (\nabla \cdot \mathbf{v}_a) \delta p \, d\Omega = 0 \quad \forall \delta p \in L^2(\Omega), \]
\[ \int_{\Omega_f} [\rho_f (\delta \mathbf{v} \cdot \nabla) \mathbf{v} \cdot \mathbf{v}_a + \rho_f (\mathbf{v} \cdot \nabla) \delta \mathbf{v} \cdot \mathbf{v}_a + \mu_f \nabla \delta \mathbf{v} \cdot \nabla \mathbf{v}_a] \, d\Omega + \int_{\Omega_f} (\nabla \cdot \delta \mathbf{v}) p_a \, d\Omega = 0 \]
\[ \int_{\Omega_s} [\mu_s \nabla \delta \mathbf{v} \cdot \nabla \mathbf{v}_a] \, d\Omega + \int_{\Omega_s} (\nabla \cdot \delta \mathbf{v}) p_a \, d\Omega + \int_{\Omega_d} w(1 - l_d) \delta \mathbf{v} \, d\Omega = 0 \quad \forall \delta \mathbf{v} \in H^1_{\Gamma_{id},\Gamma_{sd}}(\Omega). \]

The shape derivatives with respect to the fluid and solid domain have been taken into account and simplified. Since \( \Omega_d \) and \( \Omega_e \) are fixed and the system is solved by using a monolithic approach, then the shape derivative contributions appear only in the equation for the adjoint displacement \( \mathbf{\hat{l}} \). We do not need the solution of the \( \mathbf{\hat{l}} \) adjoint equation to compute the pressure boundary control \( p \) and therefore it may be neglected.

Finally, when considering the surface terms in (12) we obtain the following relation for the control pressure \( p_c \) on the controlled surface \( \Gamma_c \)
\[ p_c = p = -\frac{\mathbf{v}_a \cdot \mathbf{n}}{\beta}. \]

In order to recover the strong form of the adjoint system it is necessary to perform some integrations by parts on the terms where the variations of the state variables are differentiated. After performing the integration by parts, we recover the adjoint state \((\mathbf{v}_a^f, \mathbf{v}_a^s, p_a) \in H^1_{\partial \Omega_f - \Gamma_i}(\Omega^*) \cap \dot{H}^1(\Omega^*) \times H^1_{\partial \Omega^* - \Gamma_i}(\Omega^*) \cap \dot{H}^1(\Omega^*) \times L^2(\Omega^*) \cap \dot{H}^1(\Omega^*)\), by solving
\[ \nabla \cdot \mathbf{v}_a^f = 0, \]
\[ -\rho_l (\nabla \mathbf{v})^T \mathbf{v}_a^f + \rho_l [\mathbf{v} \cdot \nabla] \mathbf{v}_a^f + \nabla p_a - \nabla \cdot (\mu_l \nabla \mathbf{v}_a^f) = w(1 - l_d), \]
\[ \nabla \cdot \mathbf{v}_a^s = 0, \]
\[ \nabla \cdot \mathbf{S}(\mathbf{v}_a^s) = 0. \]
with boundary conditions defined as
\[ v^i_a = v^f_a \quad \text{on} \quad \Gamma_i, \]
\[ S(v^i_a) \cdot n = T(v^f_a) \cdot n \quad \text{on} \quad \Gamma_i, \]
\[ \mu (\nabla v_a) \cdot n = -(v \cdot n) v_a, \quad p_a = 0 \quad \text{on} \quad \Gamma^f_i, \]
\[ v_a = 0 \quad \text{on} \quad \Gamma_{fd} \cup \Gamma_{sd}. \]

It is worth noticing the duality between (19) and (8). In fact if a Dirichlet boundary condition for a state variable is set then the corresponding adjoint variable must satisfy the same type of condition in homogeneous form. The adjoint velocity \( v_a \) must be continuous and different from zero on the interface since the source term \( w (1-L) \) acts in the solid region and the information has to propagate towards the control boundary \( \Gamma \) which is part of the fluid domain. The equilibrium conditions on the interface are automatically satisfied due to the monolithic approach.

In order to solve the optimal control problem iteratively we use the steepest descent method described in Algorithm 1. When the algorithm reaches the minimum the quantity \((v_a \cdot n) / \beta\) is proportional to the functional gradient. The step length \( r \) determines how far, from the current state solution, we are moving, along the gradient direction, by changing the control. In the loop \( j \) we solve iteratively the state system by adjusting this parameter and obtain a proper decrease of the functional. When this is accomplished we save the control and iterate again on the main loop \( i \) where the adjoint system is solved. This algorithm requires several solutions of the state and adjoint systems in order to find the optimal control. However it does not need a great amount of memory which is limited to a standard CFD simulation.

We implemented this algorithm in an in-house finite element code (FEMuS) which is parallelized by using openMPI libraries and uses a multigrid solver with mesh-moving capability [22, 23, 19]. We have used standard quadratic-linear elements for the velocity and pressure solution, in order to fulfill the \( LBB \) \( inf-sup \) condition and a SUPG stabilization technique. The displacements are approximated with standard quadratic elements as well. In the following

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Algorithm 1 Description of the steepest descent algorithm.

1: function FIND OPTIMAL
2: set a state \((v^0, p^0, l^f)\) satisfying (2-4) \( \triangleright \) Setup of the state - Reference case
3: compute the functional \( J^0 \) in (9)
4: set \( r^0 = r_0 \)
5: for \( i = 1 \rightarrow i_{max} \) do
6: Solve the system (12)-(13) to obtain the adjoint state \((v^i_a, p^i_a)\)
7: for \( j = 1 \rightarrow j_{max} \) do
8: compute the control \( p^i_{c,j} = p^i_{c,j-1} - r^i_{j} (p^i_{c,j-1} - v^i_a \cdot n / \beta) \)
9: solve (2-4) for the state \((v^{i+1}, p^{i+1}, l^{f+1})\) with the new control \( p^i_{c,j} \)
10: compute the new functional \( J^{i+1,j+1} \) in (9)
11: if \( \| J^{i+1,j+1} - J^{i,j+1} \| / J^{i,j} < \text{toll} \) then
12: convergence reached \( \triangleright \) end of the algorithm
13: else if \( J^{i+1,j+1} > J^{i,j} \) then
14: set \( r^{i+1,j} = 1/2 r^{i,j}, j = j + 1 \) and go to 8 \( \triangleright \) loop on \( j \) again
15: else if \( J^{i+1,j+1} < J^{i,j} \) then
16: set \( r^{i+1,j} = r_0, i = i + 1 \) and go to 6 \( \triangleright \) loop on \( i \) again
17: end if
18: end for
19: end for
20: end function
we report the results obtained with the proposed algorithm in a few simple test cases. First we give an overview of the not controlled FSI problem, then we report the optimal solutions that have been found for different objective solid domain displacement $l_d$ and values of $\beta$ ranging from $10^{-9}$ to $10^{-11}$.

3. Numerical results
In this section we present the results obtained by applying Algorithm 1 to a two-dimensional test case. The reference geometry is shown in Figure 1 where the fluid domain is marked with $\Omega_f$ and the solid one with $\Omega_s$. The physical properties are set as $\rho_s = \rho_l = 1000 \text{Kg/m}^3$ with $\mu_l/\rho_l = 0.07 \text{Pa m}^3/\text{Kg}$ and $\mu_s = 76250 \text{Pa}$ so that the flow is not turbulent and the solid can bend easily. The fluid inlet and outlet are marked by the segments $AB$ and $EF$, with the control surface $\Gamma_c$ along the segment $AB$. On these boundaries we impose pressure boundary conditions and vanishing tangential velocity, with inlet and outlet pressure $p_{EF}$, $p_{AB}$ in the uncontrolled case and $p_{AB} = p_c$ for the controlled one. The domain is symmetric with respect to the left boundary $AF$, while the segment $BE$ lies on the interface $\Gamma_i$. The fluid flows from bottom to top on the left side of the domain and the solid bends on the right due to the high pressure values in the fluid region, as imposed by the boundary constraint $T \cdot \mathbf{n} = S \cdot \mathbf{n}$. All the boundaries are fixed with the exception of the right segment $CD$, that may move with fixed endpoints $C$ and $D$.

We first consider the steady state obtained when the control is not active and the flow is driven by a guess boundary pressure field. From this steady state we then apply an optimal boundary
control in order to increase (case A) and decrease (case B) the target desired deformations. In the optimal control cases, in order to simplify the analysis, we set in the controlled region $\Omega_d$ a weight function $w$ as follows

$$w = \begin{cases} 1 & x \in \Omega_d, \\ 0 & \text{otherwise}, \end{cases}$$

where $\Omega_d$ denotes the red region around the middle point of the right boundary $CD$, as shown in Figure 1.

For the initial solution we impose a constant pressure profile. We set $p_{AB} = 16500 Pa$ and $p_{EF} = 15000 Pa$ for the inlet and outlet pressure, respectively. The displacement, pressure and velocity fields are shown in Figure 2. On the left of Figure 2 the fluid velocity field is reported with 15 iso-magnitude lines to better visualize the flow and the horizontal component of the displacement field in the solid region $\Omega_s$ is shown in colors. The region that undergoes the maximum displacement is near the horizontal center-line. In Figure 2 one can see clearly the x-coordinate of point-region $\Omega_d$ (0.3484) at the steady solution. By subtracting the corresponding value in the reference case, see Figure 1, one recovers the horizontal component of the displacement $l_{d,x} = 0.0484$.

In the test cases with optimal control we set the desired displacement $l_{d1} = (0.07, 0)$ (case A) and $l_{d2} = (0.03, 0)$ (case B). Therefore, when using $l_{d1}$ as objective, we aim to reach a higher solid deformation along the x-axis, by increasing the pressure at the fluid inlet. Vice versa, by choosing $l_{d2}$ as objective we intend to reduce the structure horizontal displacement by reducing the inlet pressure. On the right of the same Figure 2 the liquid pressure is shown in color, while the adjoint velocity field is reported with arrows colored according to its magnitude for the test case A, with $l_{d1}$ and $\beta = 10^{-11}$. The adjoint flow field enters the domain from the bottom fluid inlet, crosses the interface and exits from the right surface as prescribed by the boundary conditions (19). The control pressure $p_c$, on the control surface $\Gamma_c$, is then related with the adjoint velocity through the scalar equation (14).
Figure 3. Test case A. On the left velocity streamlines in the fluid and horizontal component of the displacement field in the solid, with $\beta = 10^{-11}$. On the right pressure distribution on the inlet surface, comparison between reference case (A, dotted line), test case with $\beta = 10^{-11}$ (B, solid line) and test case with $\beta = 10^{-10}$ (C, dashed line).

We now report the results of test case A, with objective $I_{d1}$, in Figure 3. On the left of Figure 3 are reported 15 iso-magnitude lines of the fluid velocity and the solid horizontal displacement in colors, while the x-coordinate of point-region $\Omega_d$ can be read on the ticked line. On the right of the same Figure it is shown a plot of the control pressure on the fluid inlet boundary ($y = 0$) for the reference and test case A with two different values of $\beta$. In this test case the control increases the inlet pressure, in order to minimize the objective functional (9). As a consequence the fluid velocity also increases, as can be seen on the left of this Figure, in order to satisfy the mass conservation constraint. From the plot on the right we notice that when $\beta = 10^{-10}$ (dashed line) the inlet pressure has a regular, quite parabolic profile, while with $\beta = 10^{-11}$ (solid line) becomes more irregular, moreover in both cases the inlet pressure is significantly higher than in the reference one. In Figure 4 we report some results obtained for test case B with objective $I_{d2}$. On the left are reported 15 iso-magnitude lines of the fluid velocity, the solid horizontal displacement in colors and the x-coordinate of the point-region $\Omega_d$. On the right of the same Figure the control pressure on the fluid inlet boundary is plotted for test case B, with two different values of $\beta$, and for the reference one. In order to reduce the objective functional (9) the control has to reduce the inlet pressure, as it can be seen clearly on the right. In test case B the fluid has reversed the direction of the motion, flowing in the channel from top to bottom and the velocity magnitude is significantly higher than in the reference case. This inversion occurs because the control pressure $p_c$ on the bottom is lower than the one on the top, which is fixed to a positive value (16500 Pa) by the boundary conditions. The plot on the right of Figure 4 shows that the inlet pressure reduction obtained with $\beta = 10^{-11}$ (solid line) is more pronounced than the one obtained with $\beta = 10^{-10}$ (dashed line). For a more quantitative description of the results we reported in Table 1 the objective functional (9) as computed in the reference and in the controlled cases with different $\beta$. In both test cases the functional has been reduced significantly with $\beta = 10^{-11}$ and $10^{-10}$. However, with $\beta = 10^{-9}$ the optimal control algorithm has provided a functional reduction only in the test case B. Also, in Figure 5 one can see a comparison between test case A and B. In this Figure the horizontal ($u$) and the vertical
Figure 4. Test case B. On the left velocity streamlines in the fluid and horizontal component of the displacement field in the solid, $\beta = 10^{-11}$. On the right pressure distribution on the inlet surface, comparison between reference case (A, dotted line), test case with $\beta = 10^{-11}$ (B, solid line) and test case with $\beta = 10^{-10}$ (C, dashed line).

Table 1. Objective functionals computed with different $\beta$ values for test cases with $L_{d1}$ (Test A) and $L_{d2}$ (Test B). The reference case with no control is labeled with $\beta = \infty$.

|       | $J(1,p)$ | $\infty$ | $10^{-9}$ | $10^{-10}$ | $10^{-11}$ |
|-------|----------|----------|-----------|------------|------------|
| Test A|          | 9.30e-04 | 9.30e-04  | 1.16e-05   | 1.29e-07   |
| Test B|          | 6.80e-04 | 7.59e-05  | 2.48e-05   | 1.71e-08   |

Figure 5. Test case A (on the left) and test case B (on the right). Horizontal ($u$) and vertical ($v$) velocity components over the fluid domain $\Omega_f$ and horizontal ($l_x$) and vertical ($l_y$) displacement components over the solid domain $\Omega_s$. 
(v) velocity components over the fluid domain $\Omega_f$ together with the horizontal ($l_x$) and vertical ($l_y$) displacement components over the solid domain $\Omega_s$ are reported. It is important to remark the inversion of the direction of the motion, in case B, due to the change of the control pressure.

4. Conclusions

In this work we have studied an optimal control method for a fluid structure interaction problem. The objective is the matching of a displacement field in a particular region of the solid domain by controlling the pressure on the fluid inlet boundary. In order to achieve the coupling between the solid and fluid domain we have used the FSI problem with a monolithic variational formulation that automatically takes into account the interface conditions. Furthermore, we have extended the velocity field to the solid domain to couple adjoint variables and forces on the interface. The optimality system has been obtained by setting to zero the first order Fréchet derivatives of the complete Lagrangian and this system has been solved by using a steepest descent algorithm. The results obtained show the accuracy and robustness of this approach and the possible uses of this method in many industrially relevant applications. In future works we plan to assess this pressure control to more realistic geometries to show the feasibility of this optimization approach in real complex cases. Furthermore this work can provide a good starting point for more complex FSI boundary optimal control problems, for example relying on the inlet velocity profile as control parameter. In that case the optimality condition obtained imposing the first order necessary conditions is a differential equation, which is far more difficult to be solved numerically than the algebraic one obtained considering pressure as control parameter.

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