Determining of optimal number of beams in direct machine parameter optimization-based intensity modulated radiotherapy for head and neck cases

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ABSTRACT

This paper aims to introduce an algorithm called “sensitivity-based beam number selection (SBBNS)” for fully automated and case-specific determination of an optimal number of equispaced beams in intensity-modulated radiotherapy (IMRT). We tested the algorithm in five head and neck cases of varying complexity. We used direct machine parameter optimization method coupled with Auto Plan feature available in Pinnacle TPS (Version 9.10.0) for optimization. The Pearson correlation test shows a correlation of 0.88 between predicted and actual optimal number of beams, which indicates that SBBNS method is capable of predicting optimal number of beams for head and neck cases with reasonable accuracy. The major advantage of the algorithm is that it intrinsically takes into account various case- and machine-specific factors for the determination of optimal number. The study demonstrates that the algorithm can be effectively applied to IMRT scenarios to determine case specific and optimal number of beams for head and neck cases.

Key words: Dose to organs at risk; inverse planning in intensity-modulated radiotherapy; Linac radiotherapy; orientation of radiotherapy beams

Introduction

The selection of optimal number of beams has been of interest since the advent of 3D conformal radiation therapy. Currently, the beam number selection (BNS) process in intensity-modulated radiotherapy (IMRT) is based on the experience of the treatment planners or by a trial-and-error approach. Over the past decade, many researchers have attempted to automate the beam placement process in IMRT. It is well established that optimization of beam angles in IMRT is useful in generating better treatment plans.1,2

Currently, there exist a number of beam angle optimization (BAO) algorithms which give case-specific solution to the beam angle problem.3–8 However, the use of BAO algorithm in routine clinical practice is not yet widespread. Many clinics still adopt the method of placing a sufficient number of equispaced beams, which has been found to produce clinically acceptable dose distribution in many anatomic sites such as head and neck. However, research has shown that manual specification of a number of equispaced beams, which is far-off from the optimal value, can either affect the plan’s quality or its delivery efficiency.9,10 This result has triggered attempts to determine an appropriate number of equispaced beams in IMRT either by studying the effect of beam numbers in the quality of IMRT plan11,12 or by approaching the problem from fundamental theoretical viewpoints.13,14

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One common observation from these studies is that the optimal number of beams that strike a correct balance between plan quality and delivery efficiency is highly dependent on the case complexity. However, till date, user does not have a systematic and case-specific way to choose the number of equispaced beams in IMRT; thereby increasing inter-user variability in plan quality. Moreover, in complex clinical situations, it is not easy to decide an appropriate number of beams without many trial-and-error approaches potentially involving some backtracking steps. Apart from being an independent problem, BNS has also become an important step before optimizing the beam angles because many of the current BAO algorithms require the input of an appropriate number of beams for a given plan.

Our motivation is to arrive at a systematic method for determining the number of beams so that a computer program can be used to automatically determine the number of beams within a reasonable amount of time. To this end, we propose an algorithm for fully automated and case-specific selection of optimal number of beams in IMRT.

Materials and Methods

Sensitivity-based beam number selection

In single-criteria optimization, composite objective function is an overall estimate of the plan quality. A smaller value of the objective function is an indication of better plan quality. In this work, first, we experimentally show that the sensitivity of organ’s at-risk (OAR) objective function to the prescribed target minimum dose is inversely proportional to the total number of beams. This behavior is exploited in the proposed algorithm for BNS. We term this approach sensitivity-based BNS (SBBNS), the patent application of which can be found here. If less number of beams are used, composite objective function of OAR objectives will be more sensitive to an increase in the target minimum dose whereas if more number of beams are used, composite objective function of OAR objectives will be less sensitive to an increase in the target minimum dose. In other words, the objective sensitivity of OAR objectives is inversely proportional to the number of beams used in the plan. SBBNS algorithm uses this behavior for predicting optimal beam number as described in the following sections. To invoke SBBNS algorithm, the user is required to set up a reference condition for each case. The reference condition mainly involves as follows:

1. An equispaced beam geometry as specified in equation 1
2. The number of equispaced beams would be seven in all the cases.

Table 1 lists the other reference conditions to be adopted before invoking SBBNS algorithm. Seven beams are used in the reference condition as it allows sampling a good portion of the target volume and other normal organs over 360° rotation. Moreover, seven beams in a plan do not prolong the dose calculation and optimization significantly.

The beam angles in equispaced beam geometry are given by

$$\theta_i = \frac{360}{N} (i - 1) + \theta_1 \mod 360$$

(1)

where $\theta_1$ is the gantry angle of beam $i$, $N$ is the number of beams, and $\theta_1$ is the starting gantry angle or gantry angle of the first beam.

Once the reference conditions are set, now the user can invoke the algorithm for BNS. The algorithm essentially involves two steps:

Step 1: SBBNS algorithm takes as input the clinical dose-volume objectives and calculates the objective sensitivity with respect to a predefined change in the prescribed target dose.

Step 2: The estimated sensitivity of the objective function is used to calculate optimal beam number for the given case.

A detailed account on steps 1 and 2 is given below:

Step 1: Computation of objective sensitivity

SBBNS algorithm is implemented on the following quadratic dose-volume based objective function given by:

$$F = \sum_{i=1}^{n} [W_i f(D_i, V_i)]$$

(2)

where $r$ represents the $r^{th}$ objective function component (OFC), $D_i$ and $V_i$ represent the dose and volume parameters for the $r^{th}$ OFC, respectively, and $W_i$ is the importance or penalty factor for the $r^{th}$ OFC. $f(D_i, V_i)$ is the numerical value of the $r^{th}$ OFC.

Essentially, an OFC represents a quadratic difference between user-specified clinical goal and obtained dose at
each voxel multiplied by the importance factor assigned to the voxel belonging to a region of interest. In the SBBNS method, we define \( D \) and \( V \) as the clinical goal (i.e. the “real objective”) for the \( i \)th objective component.

We selected target minimum dose objective (in Auto Plan) of the planning target volume (PTV) as the reference to gauge the sensitivity of objective function. The following steps are used to determine the sensitivity of objective function:

1. The direct machine parameter optimization (DMPO) plan is first optimized using the initial parameters \( D \), \( V \), and \( W \), to obtain the objective function \( F_{\text{normal}} \). The plan obtained in this stage is plan A.

2. The second DMPO plan is generated in which the target dose of the PTV is made more stringent than the prescription goal for the PTV. For example, if the prescription requires only 95% of the PTV to receive a 6300 cGy dose, the second DMPO plan might be generated by requiring 95% of the PTV to receive 6800 cGy. The current plan is optimized, and the objective function obtained in this step is denoted as \( F_{\text{overconstrained}} \). The plan obtained in this stage is plan B.

3. The objective sensitivity \( S \) is measured by the change in the composite objective function of all OAR objectives included in the optimization between plan A and plan B obtained for a unit objective weight \( (W) \) given by

\[
S = \sqrt{\frac{F_{\text{overconstrained}} - F_{\text{normal}}}{l}}
\]  

(3)

The square root function is used because DMPO objective function is essentially a quadratic function over dose and volume parameters. Here, \( l \) is a linearization factor applied to linearize the numerator value, whose value is decided based on \( F_{\text{normal}} \). The value of \( l \) is decided in such a way that \( \frac{F_{\text{normal}}}{l} \) falls in between 0.1 and 1. The same linearization factor will be applied to \( F_{\text{overconstrained}} \) term as shown in equation 3. For instance, if \( F_{\text{normal}} \) is 0.01, \( l \) is set to 1; if \( F_{\text{normal}} \) is 0.001, \( l \) is set to 10; if \( F_{\text{normal}} \) is 0.0001, \( l \) is set to 0.01; if \( F_{\text{normal}} \) is 0.00001, \( l \) is set to 1000, and so on. Here, \( l \) denotes an arbitrary integer.

Essentially, the sensitivity of objective function (equation 3) measures how much the objectives of surrounding critical normal structures are compromised for any change in the prescribed PTV (target) dose. The overconstraining of target minimum dose results in sacrificing the sparing of other structures, as a result of which, \( F_{\text{overconstrained}} \) is always greater than \( F_{\text{normal}} \). Such sensitivity-based approaches had been used earlier for automated determination of IMRT objective function parameters\(^{[16][17]}\) and selection of optimal beam angles in IMRT.\(^{[18]}\) We verified the relationship between objective sensitivity and number of beams in a chosen head and neck anatomy as shown in Figure 1. It is clear from this figure that the objective sensitivity is inversely proportional to the number of beams.

**Step 2: Determination of optimal beam number (Step 2)**

It is evident from Figure 1 that the OAR objective sensitivity will be lower for a given case if there is enough number of beams to ensure the normal tissue sparing when prescribed target dose is increased as mentioned before. Similarly, the OAR objective sensitivity will be higher for the same case if there is not enough number of beams to ensure the normal tissue sparing when prescribed target dose is increased. Stated differently, if the objective sensitivity is found to be higher, the number of beams required to produce an optimal dose distribution for target volume and OARs would be more. Hence, the required number of beams for a given case is proportional to the sensitivity of the objective function obtained under the reference beam geometry condition. Let \( N_{\text{optimal}} \) be the optimal number of beams for a given case, then

\[
N_{\text{optimal}} = k.S
\]  

(4)

where \( k \) is a beam number constant, which is calculated under reference beam geometry condition using a reference case for which the optimal beam number is known. By manipulating equation 4, one can obtain \( k \) as given below:

\[
k = \frac{N_{\text{optimal}}}{S}
\]  

(5)

where \( N_{\text{optimal}} \) denotes the known optimal beam number for a reference case under reference beam geometry condition.

In this work, we chose a moderately complex head and neck case as a reference and applied the reference conditions mentioned before to calculate beam number constant \( k \). To find out \( N_{\text{optimal}} \) for this case, objective function saturation curve is plotted [Figure 2a] for different beam numbers
starting from 3 to 11 and an optimal beam number is chosen for the reference case. From Figure 2a, it is evident that at beam number seven the objective function approximately saturates. Meanwhile, $S$ was calculated for this case as explained in step 1.

By substituting these values in equation 5, one can get the value of beam number constant ($k$) as below:

$$k = \frac{N^{\text{known}}_{\text{Optimal}}}{S} = \frac{7}{0.382} = 18.32$$  \hspace{1cm} (6)

Using the above value of $k$ in equation 4, it gives the optimal number of beams.

$$N_{\text{optimal}} = 18.32 S$$  \hspace{1cm} (7)

Equation 7 gives the final expression for the optimal beam number required regarding sensitivity of the objective function ($S$) obtained for a given head and neck case using equation 3. The whole process involved in SBBNS algorithm is illustrated in Figure 3.

**Results**

We used five head and neck cases for which optimal number of beams is to be found. The cases chosen in the study have a wide range of complexity in terms of tumor geometry, normal tissue locations, and dose and dose-volume objectives. Table 2 gives the typical clinical objectives for the head and neck cases. To validate the proposed algorithm, we independently determined the optimal number of beams for all the five cases using objective function saturation curves. An objective function saturation curve is a graphical illustration of how the

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**Figure 2**: Objective function saturation curves obtained for head and neck - reference case (a), head and neck - Case 1 (b), head and neck - Case 2 (c), head and neck - Case 3 (d), head and neck - Case 4 (e) and head and neck - Case 5, (f) corresponding to beam numbers 3–11

**Figure 3**: The flowchart of sensitivity-based beam number selection algorithm
composite objective function changes with respect to a number of equispaced beams (3–11 equispaced beams) used in the optimization for the same clinical objectives set for PTVs and OARs. In the saturation curve, the optimal beam number is chosen to be the point (in the graph) at which the objective function has approximately started to saturate. We considered the starting point of the saturation as optimal because adding more beams in the optimization beyond the saturation point has been found to be only increasing the total monitor unit (MU) of the plan without improving the plan quality. Quantitatively, the saturation point is considered to be a beam number (in the saturation plot) beyond which the reduction in composite objective value is not more than 10%.

We used Auto Plan feature available in Pinnacle TPS Version 9.10.0 (Philips Radiation Oncology Systems, Fitchburg, WI) for all the DMPO optimizations, which automatically sets appropriate objective function parameters (dose, volume, and weight) for each objective. The number of segments input to DMPO algorithm before optimization was set as sixty in all the optimizations involving different equispaced beam numbers. Figure 2a–f show the objective function saturation curves obtained for the head and neck cases. As mentioned earlier, the optimal beam numbers for each case is located by taking input from the saturation curves. At the same time, we used the proposed SBBNS algorithm to find the optimal beam number for the same cases using equation 7. Table 3 compares the optimal number of beams obtained using the saturations curves and the proposed SBBNS algorithm. We rounded off the number predicted by SBBNS to nearest whole number. It is evident from Table 3 that the beam number predicted using SBBNS and saturation curves are in complete agreement for three cases (case 1, 3, and 5). SBBNS has predicted two additional beams for Case 2 and one additional beam for Case 4. The Pearson correlation test shows a correlation of about 0.9 between \( S \) and \( N_{optimal}^{predicted} \) (obtained from saturation curves) and a correlation of about 0.88 between \( N_{optimal}^{predicted} \) (obtained from saturation curves) and \( N_{optimal}^{computed} \) (computed using equation 7). This result indicates that SBBNS method is capable of predicting optimal number of beams for head and neck cases with reasonable accuracy.

**Discussion**

Producing a good dose distribution fundamentally requires an appropriate selection of a number of beams and their angles. It has been reported that adding more beams in IMRT beyond a point increases the MUs without any considerable improvement in dose distribution, leading to more leakage radiation, and increased critical organ dose. At the same time, having an insufficient number of beams can result in sub-optimal treatment plan. Hence, determining a suitable beam number turns out to be a valid clinical problem that requires a case-specific solution.

### Table 2: Typical dose-volume objectives used in the Auto Plan module of Pinnacle for the head and neck cases. The total number of fractions was 35

| Organ       | Objective type | Dose (cGy) | Volume (%) |
|-------------|----------------|------------|------------|
| PTV 7000    | Target dose    | 7000       | 95         |
| PTV 6300    | Target dose    | 6300       | 95         |
| PTV 5600    | Target dose    | 5600       | 95         |
| Spinal cord | Maximum dose   | 4500       | 0          |
| Brain stem  | Maximum dose   | 5400       | 0          |
| Larynx      | Mean dose      | 5000       | NA         |
| Parotids    | Mean dose      | 2600       | NA         |
| Lips        | Mean dose      | 3500       | NA         |
| Oral cavity | Mean dose      | 3800       | NA         |
| Esophagus   | Maximum DVH    | 3500       | 50         |
| Mandible    | Maximum dose   | 7000       | 0          |
| Submandibular | Mean dose   | 4000       | NA         |

PTV: Planning target volume, DVH: Dose-volume histogram, NA: Not applicable

### Table 3: The comparison of optimal beam numbers obtained using objective function saturation curves and sensitivity-based beam number selection algorithm

| Head and neck case | Optimal beam number obtained from objective function saturation curves | Objective sensitivity (S) | Beam number predicted using SBBNS algorithm | After rounding off to the nearest whole number |
|--------------------|-----------------------------------------------------------------------|---------------------------|--------------------------------------------|---------------------------------------------|
| Case 1             | 9                                                                     | 0.50                      | 9.29                                       | 9                                           |
| Case 2             | 5                                                                     | 0.35                      | 6.57                                       | 7                                           |
| Case 3             | 7                                                                     | 0.37                      | 6.82                                       | 7                                           |
| Case 4             | 10                                                                   | 0.61                      | 11.26                                      | 11                                          |
| Case 5             | 8                                                                     | 0.41                      | 7.58                                       | 8                                           |

SBBNS: Sensitivity-based beam number selection

It is to be noted that the beam numbers determined using SBBNS algorithm are tightly coupled with the user defined objectives. Inputting simple and easily achievable objectives in SBBNS will result in a relatively lesser number of beams. Likewise, inputting complex objectives will result in a larger number of beams. Hence, it is important to ensure that the clinical objectives included in the optimization are reasonable and clinically relevant to fully make use of SBBNS approach for BNS.

Auto Plan has been used just as a substitute to an expert planner in this study. In the absence of Auto Plan feature, a planner has to decide the objective function parameters to meet the clinical objectives. The clinical validation of Auto Plan can be found elsewhere. Moreover, the proposed SBBNS approach coupled with auto-contouring tools such as SPICE and automatic planning tools such as Auto Plan allows for a complete automation of the planning process once the clinical objectives are set.

The susceptibility of beam number constant value \( k = 18.32 \) to any change in the reference conditions.
mentioned in Table 1 needs to be investigated. For instance, it will be useful to see how \( k \)-value changes if the starting angle is set as a nonzero angle. However, a huge change in \( k \) is not expected because the numerator of beam number constant term (equation 6) will change correspondingly to any change in the denominator term occurring due to alterations in the reference conditions.

**Conclusion**

It is evident from the results that the algorithm is capable of predicting optimal beam number with reasonable accuracy. Although we included only head and neck geometry in this study, in theory, the method should also be applicable to other anatomic sites such as thorax and pelvis. However, a detailed investigation might be required to understand how the proposed beam number constant value changes according to anatomy.

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**Conflicts of interest**

There are no conflicts of interest.

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