Strong dissipation regime in nuclear disintegrations

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Abstract. The coupling of collective degrees of freedom with the microscopic ones causes dissipation and a modification of the adiabatic potential. The term dissipation usually refers to exchange of energy (either linear or angular momentum) by all kind of damping from collective motion to intrinsic heat. A measure of the dissipated energy can be obtained solving the time dependent pairing equations. In this presentation, a generalization of the time dependent pairing equations is presented by including the Landau-Zener effect in the superfluid model. These new equations allows a mixing of seniority one configurations that allows us to obtain a ground state at the end of the process. An application concerning the $^{14}$C emission is offered and its fine structure is explained. These new equations are furthermore used to evidence a dynamical pair breaking effect that could explain the fine odd-even effect in cold fission. Finally, the time dependent pairing equations are used to deduce a model for non-adiabatic cranking inertia.

1. Introduction
In a mean field approximation, the equations of motions take into account the two-body collisions to the extend to which the two-body collisions are incorporated in the mean field. In principle, the time-dependent HF approximation exactly treats the residual interactions only if the mean field is allowed to break all symmetries. Such an approach leads to huge computational difficulties and usually the mean field is constrained to have some symmetries. In such circumstances, two levels associated to the same symmetry cannot intersect and the level slippage is not allowed. That leads to the unpleasant feature that at the end of a nuclear process the systems cannot reach the ground state. In order to overcome this problem new approaches were developed leading to the extended mean field model [1] and to the stochastic HF equations, discussed in detail in Refs. [2, 3]. Alternatively, this problem was partially solved by taking into account the pairing interaction. The Time-Dependent Pairing Equations (TDPE) provide the possibility for the nucleon pair level slippage [4] and represent a tool to estimate the dissipation [5]. The dissipation is identified as the excitation derived from coupled channel equations, as in Refs. [6, 7, 8]. In Ref. [9] the dissipation was calculated for fission processes. For closed shell regions of fission fragments and for symmetric fission the results agree with experimental data. A deep connection with the Landau-Zener transitions is included in the time dependent pairing equations: pairs undergo Landau-Zener transitions on virtual levels with coupling strengths given by the magnitude of the gap. A problem appears in solving these equations in the case of seniority one nuclear systems: the unpaired nucleon located on the blocked level will remain on the same level during the deformation of the system. Due to this phenomenon, the system cannot end in its ground state after a disintegration. In this presentation, the first part addresses a generalization of the time dependent pairing equations by including the Landau-Zener effect in
the superfluid model [10, 11]. These new equations allows a mixing of seniority one configurations that allows us to obtain a ground state at the end of the process. An application concerning the $^{14}$C emission is offered and its fine structure is explained. Furthermore, the same formalism is used to evidence a dynamical pair breaking effect that could explain the fine odd-even effect in cold fission [12]. Finally, the time dependent pairing equations are used to deduce a model for non-adiabatic cranking inertia [13].

2. The Landau-Zener effect

Single particle levels are functions of the deformation parameters that define the mean field. Levels characterized by the same quantum numbers associated with the same symmetries of the system cannot intersect and exhibit avoided level crossing regions. The transition probability of a nucleon from one adiabatic level to another is strongly enhanced in an avoided crossing region. This promotion mechanism is known as the Landau-Zener effect [14]. Using the time-dependent Schrödinger equation, the following system of coupled channel equations is obtained [15]:

$$\dot{c}_j = \frac{1}{i\hbar} \sum_{j=1}^{n} c_j(t) h_{ij}(t) \exp \left(-\frac{1}{\hbar} \int_{0}^{t} (\epsilon_j(\tau) - \epsilon_i(\tau)) d\tau\right)$$

(1)

where $c_j$ is the amplitude of the diabatic state $j$ in the total wave function, $\epsilon_j$ is its single particle energy and $h_{ij}$ is the interaction energy in an avoided levels crossing region between two state $i$ and $j$. The probability to find the the unpaired nucleon on the level $i$ is $p_i = |c_i|^2$. These equations have already been used to explain the resonant fine structure in the fission cross section [16, 17, 18], the resonant-like structure of the inelastic cross section in heavy ion collisions [19, 20], the fine structure in cluster emission [21, 22] and in $\alpha$ decay [23].

3. Generalization of the Landau-Zener effect in superfluid systems

An effect analogous to the Landau-Zener one can be obtained by generalizing the TDPE for the case of seniority one nuclear systems. The problem will be explored in the simplest possible way: a monopole pairing force, and a sufficiently weak pairing such that the nucleons are not redistributed to change significantly the mean field potential. Using quasiparticle creation and annihilation operators $a_k^\dagger$ and $a_k^-$

$$a_{k(j)} = u_{k(j)} a_k - v_{k(j)} a_k^\dagger; \quad a_{k(j)}^\dagger = u_{k(j)} a_k^\dagger + v_{k(j)} a_k;$$

(2)

it is possible to construct some interactions that help us to promote the nucleon from one diabatic level to another. Note that these operators are associated to each blocked level $j$. Here $a_k^\dagger$ and $a_k$ denote operators for creating and destroying a particle in the state $k$, respectively. The state characterized by a bar signifies the time-reversed partner of a pair. The parameters $v_k$ and $u_k$ are the occupation and vacancy amplitudes, respectively. Because only the relative phase between the parameters $u_k$ and $v_k$ matters, in the following, $u_k$ is considered to be a real quantity and $v_k$ a complex one. The two situations plotted in Fig. 1 can be modeled. In the plot 1(a), the single particle follows the diabatic level $\epsilon_j$ while in 1(b) it remains on the adiabatic one $\epsilon_i$. The interaction able to promote the unpaired nucleon from one adiabatic level to another must be given by some products of operators of the type (2).

In order to obtain the equations of motion, we shall start from the variational principle taking the following energy functional

$$\delta L = \delta < \varphi \mid H - i\hbar \frac{\partial}{\partial t} + H' \mid \varphi >,$$

(3)
and assuming the many-body state formally expanded as a superposition of $n$ time dependent BCS seniority one diabatic wave functions

$$|\varphi(t)\rangle = \sum_{m} c_{m}(t) a_{m}^{+} \prod_{l \neq m} \left( u_{l(m)}(t) + v_{l(m)}(t) a_{l}^{+} a_{l}^{\dagger} \right) |0\rangle .$$

(4)

Sometimes, this energy functional is called Lagrangian [4]. The functional contains several terms. The first one is the many body Hamiltonian with pairing residual interactions

$$H(t) = \sum_{k>0} \epsilon_{k} [q(t)](a_{k}^{+} a_{k} + a_{k}^{+} a_{k}^{\dagger}) - G \sum_{k,i>0} a_{k}^{+} a_{k}^{\dagger} a_{i} a_{i}^{\dagger} .$$

(5)

The residual interactions between diabatic levels characterized by the same quantum numbers that are responsible for the Landau-Zener effect are assumed on the form:

$$H'(t) = \sum_{i,j \neq i} h_{ij}(t) a_{i}^{+} a_{j}^{\dagger} a_{j} a_{i}^{\dagger} \Pi_{k \neq i,j} \alpha_{k(i)} a_{k} a_{k}^{\dagger}$$

$$= \sum_{i,j \neq i} h_{ij}(t)(u_{i(j)} a_{i}^{+} - v_{i(j)} a_{i}^{\dagger})(u_{j(i)} a_{j} + v_{j(i)} a_{j}^{\dagger}) \Pi_{k \neq i,j} \alpha_{k(i)} a_{k} a_{k}^{\dagger},$$

(6)

The sum runs over diabatic levels $i$ and $j$.

After some calculations, as detailed in Ref. [11], a new set of pairing equations which account for configuration mixing results:

$$i\hbar \dot{p}_{l(m)} = \kappa_{l(m)} \Delta_{m} - \gamma \Delta_{m} ,$$

(7)

$$i\hbar \dot{\kappa}_{l(m)} = \left( 2 \rho_{l(m)} - 1 \right) \Delta_{m} + 2 \kappa_{l(m)} \epsilon_{l} - 2G \rho_{l(m)} \kappa_{l(m)} ,$$

(8)

$$i\hbar \dot{p}_{m} = \sum_{j \neq m} h_{mj}(s_{mj} - s_{jm}) ,$$

(9)

$$i\hbar \dot{s}_{jm} = s_{jm} \left\{ -\frac{G}{\gamma} \left( |\Delta_{m}|^{2} - |\Delta_{j}|^{2} \right) + (\epsilon_{m}(t) - \epsilon_{j}(t)) + G(\sum_{k \neq m} \rho_{k(m)}^{2} \sum_{k \neq j} \rho_{k(j)}^{2}) - \frac{1}{2} \kappa_{l(k)}^{*} \kappa_{l(k)}(\Delta_{m}^{*} \kappa_{l(k)}^{*} + \Delta_{m} \kappa_{l(k)}) \frac{\rho_{l(m)}^{2}}{|\kappa_{l(m)}|^{2}} - 1 \right\}$$

$$+ \frac{1}{2} \sum_{k \neq j} \left( \Delta_{j}^{*} \kappa_{l(j)}^{*} + \Delta_{j} \kappa_{l(j)} \right) \frac{\rho_{l(j)}^{2}}{|\kappa_{l(j)}|^{2}} - 1 \right\}$$

$$+ \sum_{i \neq m,j} \left[ h_{mi}(t)s_{ij} - h_{ij}(t)s_{im} \right] + h_{mij}(t)(p_{j} - p_{m}) .$$

(10)
Here, the following notations are used:

\[
\Delta_m = G \sum_{k \neq m} \kappa_k; \quad \Delta^*_m = G \sum_{k \neq m} \kappa^*_k; \\
\kappa_k = u_k v_k; \quad \rho_k = |v_k|^2; \quad p_m = |c_m|^2; \quad S_{jm} = c^*_j c_m.
\]

(11)

where \( \rho_k \) are the single particle densities, \( \kappa_k \) are the pairing moment components, and \( p_m \) denotes the probability to have an unpaired nucleon on the level \( m \). \( \rho_k \) and \( p_m \) are real quantities while \( \kappa_k \) and \( S_{jm} \) are complex ones. In analogy with the pairing moment components \( \kappa_k \), \( S_{jm} \) can be called as unpairing moment components, having the property \( |S_{jm}|^2 = p_j p_m \). Whenever the upper limit \( n \) is specified for a sum, it is implicitly assumed that the operation is realized on the \( n \) possible diabatic states of the unpaired nucleon. In this paper, the sum over pairs generally runs over the index \( k \). When the single-particle sum over \( k \) is realized only for one partner of each reversed pair the result is multiplied with a factor 2. The index \( k \) runs over a workspace that allows the pairing force to operate only within a finite number of active levels around the Fermi energy. If the blocked levels are eliminated, the system (7)-(8) reduces to:

\[
i\hbar \dot{\rho}_l = \kappa_l \Delta^* - \kappa^*_l \Delta, \\
i\hbar \dot{\kappa}_l = (2\rho_l - 1)\Delta - 2\kappa_l [\epsilon_l(t) - \lambda_l(t) - 2G\rho_l \kappa_l],
\]

(12)

that is, the well known TDPE [4, 5]. The label \( m \) is removed from \( \Delta_m \) because this quantity is now a sum over the remaining pairwise occupied levels. On the other hand if the pairing is neglected, the third equation of the system (9) can be written

\[
i\hbar (\dot{c}_m c^*_m + \dot{c}^*_m c_m) = \sum_{j \neq m} h_{mj} \left( c_j c^*_m + c^*_j c_m \right).
\]

(13)

Introducing explicitly the time dependence of the amplitudes \( c_m \)

\[
c_m(t) = c_{0m}(t) \exp\left(-\frac{i}{\hbar} \int_0^t \epsilon_m(\tau) d\tau\right),
\]

(14)

the next relation is obtained:

\[
i\hbar(\dot{c}_{0m} c^*_0 + \dot{c}^*_0 m c_{0m}) = \sum_{j \neq m} h_{mj} \left[ c_{0j} c^*_0 m \exp\left(-\frac{i}{\hbar} \int_0^t (\epsilon_j - \epsilon_m) d\tau\right) \right. \\
- \left. c^*_{0j} c_{0m} \exp\left(\frac{i}{\hbar} \int_0^t (\epsilon_j - \epsilon_m) d\tau\right) \right].
\]

(15)

The last relation gives also an equivalent form of the Landau-Zener equations for single particle systems. So, the Landau-Zener equation for single particle systems (without residual interactions) and the TDPE for quasiparticles are two particular cases of the coupled channel equations (7)-(10). So, this system represents a generalization of the TDPE in the case of seniority one nuclear systems.

4. Single particle model

To solve the TDPE (similar to TDHFB equations), only the variations of the single-particle energies \( \epsilon_k \) are needed. The simplest way to obtain the evolutions of single-particle energies is to consider a time-dependent single particle potential in which the nucleons move independently. The basis [24, 25] for the two-center oscillators can be used for various ranges of models which are more or less phenomenological ones [26, 27]. On the other hand, they are different ways to obtain the single-particle energies for a two-center Woods-Saxon potential. Other recipes are given in Ref. [28] where the potentials are expanded in terms of harmonic oscillators.
functions. A microscopic potential must be constructed to be consistent within our nuclear shape parametrization. The simplest way it to use a semi-phenomenological Woods-Saxon potential. In order to take into account nuclear deformations going over to separate shapes and obtain two separated fragments, a two-center shell model with a Woods-Saxon potential was used:

\[ V_0(\rho, z) = -\frac{V_c}{1 + \exp \left( \frac{\Delta(\rho, z)}{a} \right)} \tag{16} \]

where \( \Delta(\rho, z) \) represents the distance between a point \((\rho, z)\) and the nuclear surface. This distance is measured only along the normal direction on the surface and it is negative if the point \((\rho, z)\) is located in the interior of the nucleus. \( V_c \) is the depth of the potential while \( a \) is the diffuseness parameter. In our work, the depth is \( V_c = V_{0c}[1 \pm \kappa(N_0 - Z_0)/N_0 + Z_0] \) with plus sign for protons and minus sign for neutrons, \( V_{0c} = 51 \text{ MeV} \), \( a = 0.67 \text{ fm} \), \( \kappa = 0.67 \). Here \( A_0, N_0 \) and \( Z_0 \) represent the mass number, the neutron number and the charge number of the parent, respectively. This parametrization, referred as the Blomqvist-Walhlborn one, is adopted because it provides the same radius constant \( r_0 \) for the mean field and the pairing field. That ensures a consistency of the shapes of the two fields at hyperdeformations, i.e., two tangent ellipsoids. The Hamiltonian is obtained by adding the spin-orbit and the Coulomb terms to the Woods-Saxon potential. The eigenvalues are obtained by diagonalization of the Hamiltonian in the semi-symmetric harmonic two center basis. The two center shell model represents a valuable instrument to investigate the role of individual orbitals for the treatment of a wide variety of nuclear processes, like cold fission [29, 30, 31, 32], formation of superheavy elements [33, 34] or supersymmetric disintegration processes, pertaining to cluster- [35, 36] and alpha-decays [23].

5. Cluster radioactivity

A fine structure in the \(^{14}\text{C}\) radioactivity of the \(^{223}\text{Ra}\) was observed in 1989 [37]. In the first experiment, the results indicate that 15±3 \% of \(^{14}\text{C}\) decays are transitions on the ground state of the daughter, while 81±6\% are transitions on the first excited state. In Ref. [38], using the M3Y potential, it was evidenced that the preformation probability must be more favorable for the excited state than for the ground state with a factor of 180. Such a value cannot be accounted from theoretical models [39] without taking into account for dynamical ingredients. This is the main reason why the fine structure phenomenon was selected to validate our equations.

In order to solve the time dependent pairing equations, the single-particle level schemes for neutrons and protons must be computed along the minimal action trajectory. It is known that \(^{223}\text{Ra}\) has the spin \( \frac{3}{2} \) emerging from \( 1i_{11/2} \). Adiabatically, the unpaired neutron reaches the \( 2g_{9/2} \) level of the daughter \(^{209}\text{Pb}\). As also evidenced in Refs. [15, 22], the fine structure in the \(^{14}\text{C}\) radioactivity can be understood by an enhanced transition probability of the unpaired neutron from the adiabatic level \( \Omega = 3/2 \) emerging from \( 1i_{11/2} \) to the adiabatic level with the same spin projection \( \Omega \) that emerges from \( 1j_{15/2} \), in terms of the Landau-Zener effect. The involved levels were selected and the time dependent pairing equations were solved as indicated in Ref. [11]. The branching ratio \( r \) between the partial half-life for transitions to the ground state of the daughter and the partial half-live to the first excited state was calculated. The experimental values of \( r \) range between 5.4 and 5.9. Our theoretical value is \( r = 5.48 \), which is in an excellent agreement with experimental data.

6. New dynamical pair breaking effect

The dynamical pair breaking effect emerges from a new set of coupled channel equations deduced for the time-dependent probability to find the system in a seniority-one state or in a seniority-two one. In such an approximation, if the interactions produced in the avoided crossing regions or those due to the Coriolis coupling are not taken into consideration, other perturbations between
two different states are not possible. In the following, the calculations are restricted only for perturbations produced in the avoided crossing regions. This approximation does not affect the essential features of the model but leads to a considerable simplification of the mathematical apparatus.

Using quasiparticle annihilation and creation operators, it is possible to construct some interactions able to break a Cooper pair when the system traverses a avoided crossing region. The three situations plotted in Fig. 2 can be modeled within products of such creation and annihilation operators acting on Bogoliubov wave functions. In the plot 2(a), the Cooper pairs remain on the adiabatic levels \( \epsilon_i \) and \( \epsilon_j \) after the passage through the avoided crossing region, in Fig. 2 (b) the pair destruction is illustrated, while in Fig. 2 (c) two fermions generate a pair after the passage through an avoided crossing region. Formally, to describe these three situations, an interaction in the avoided crossing can be postulated as follows:

\[
H'(t) = \sum_{i,j \neq i} h_{ij} [q(t)] \left[ \alpha_{i(0)} \alpha_{j(0)}^+ \prod_{k \neq i,j} \alpha_{k(0)} a_{k+} a_{k-} + \alpha_{i(0)}^+ \alpha_{j(0)} \prod_{k \neq i,j} \alpha_{k(0)}^+ a_{k+} a_{k-} \right],
\]

where \( h_{ij} \) is the interaction between the levels. Acting on a suited Bogoliubov wave function, the product over \( k \) transforms the seniority-two configuration in the seniority-zero one in the case of the first term in the left hand of Eq. (17), and vice-versa in the case of the second term. If the product \( \alpha_{i(0)} \alpha_{j(0)}^+ \) acts on a seniority-zero function, then it annihilates a pair and creates two unpaired fermions in states \( i \) and \( j \). If the product \( \alpha_{i(0)}^+ \alpha_{j(0)}^+ \) acts on a seniority-two wave function, then it creates a pair distributed on both orbitals \( i \) and \( j \). In order to obtain the equations of motion, we shall start from the variational principle taking the energy functional (3) and by assuming the many-body state formally expanded as a superposition of time dependent BCS seniority-zero and seniority-two adiabatic wave functions

\[
| \varphi(t) > = \left[ c_0(t) \prod_k \left( u_{k(0)}(t) + v_{k(0)}(t) a_{k+} a_{k-}^+ \right) + \sum_{j \neq j} c_{jl}(t) a_{j+} a_{l+}^+ \prod_{k \neq j,l} \left( u_{k(jl)}(t) + v_{k(jl)}(t) a_{k+} a_{k-}^+ \right) \right] | 0 >,
\]

where \( c_0 \) and \( c_{jl} \) are amplitudes of the two kind of configurations. To minimize, the energy functional is derived with respect the independent variables \( v_{k(0)}, v_{k(jl)}, c_0, c_{jl} \), together with their complex conjugates, and the resulting equations are set to zero. Eventually, eight coupled-channel equations are obtained:

\[
i \hbar \hat{\rho}_{k(0)} = \kappa_{k(0)} \Delta_0^* - \kappa_{k(0)}^* \Delta_0;
\]
i\hbar \dot{\rho}_{k(\gamma)} = \kappa_{k(\gamma)}^* \Delta^*_{\gamma} - \kappa_{k(\gamma)} \Delta_{\gamma};
i\hbar \dot{k}_{k(0)} = \left(2\rho_{k(0)} - 1\right) \Delta_0 + 2\kappa_{k(0)} \epsilon_k - 2G\rho_{k(0)} \kappa_{k(0)};
i\hbar \dot{k}_{k(\gamma)} = \left(2\rho_{k(\gamma)} - 1\right) \Delta_\gamma + 2\kappa_{k(\gamma)} \epsilon_k - 2G\rho_{k(\gamma)} \kappa_{k(\gamma)};
i\hbar \dot{P}_0 = \sum_{\gamma} \hbar \gamma (S_{0\gamma}^* - S_{0\gamma});
i\hbar \dot{P}_\gamma = \hbar \gamma (S_{0\gamma} - S_{0\gamma}^*);
i\hbar \dot{S}_{0\gamma} = S_{0\gamma}(E_0 - E_\gamma) + S_{0\gamma} \left(\sum_{k \neq \gamma} T_{k(\gamma)} - \sum_k T_{k(0)}\right) + \sum_\beta \hbar \beta S_{\beta\gamma} + \hbar \gamma (P_\gamma - P_0);
i\hbar \dot{S}_{\beta\gamma} = S_{\beta\gamma}(E_\beta - E_\gamma) + S_{\beta\gamma} \left(\sum_{k \neq \beta} T_{k(\gamma)} - \sum_{k \neq \gamma} T_{k(\beta)}\right) + \hbar \beta S_{\beta\gamma} + \hbar \gamma S_{\gamma}^*; (19)

\text{where the partial derivatives with respect the time are denoted by a dot.}\ \text{If (}\gamma\text{) denotes the indexes}\ (j, l)\ \text{and (}\beta\text{) the indexes}\ (m, n),\ \text{the sums are restricted by the conditions}\ j \neq l, m \neq n, m \neq j,\ \text{and}\ n \neq l.}\ E_\gamma\ \text{are exactly the expected values of the Hamiltonian (5) for the seniority-zero or seniority-two configurations:}

E_0 = \frac{2}{G} \sum_k \rho_{k(0)} \epsilon_k - \frac{|\Delta_0|^2}{G} - \frac{1}{G} \sum_k \rho_{k(0)}^2;

E_{jl} = \frac{2}{G} \sum_{k \neq j, l} \rho_{k(jl)} \epsilon_k - \frac{|\Delta_{jl}|^2}{G} - \frac{1}{G} \sum_k \rho_{k(jl)}^2 + \epsilon_j + \epsilon_l; (20)

\text{and}\ T_{k(\gamma)}\ \text{are energy terms associated to single-particle states:}

T_{k(\gamma)} = 2\rho_{k(\gamma)} (\epsilon_k - \lambda_\gamma) - 2G\rho_{k(\gamma)}^2 + \frac{\kappa_{k(\gamma)} \Delta^*_{\gamma} + \kappa_{k(\gamma)} \Delta_{\gamma}}{2} \left(\frac{\rho_{k(\gamma)}^2}{|\kappa_{k(\gamma)}|^2} - 1\right).

\text{The following notations are used in Eqs. (19):}

\Delta_0 = G \sum_k \kappa_{k(0)}; \quad \Delta_{jl} = G \sum_{k \neq j, l} \kappa_{k(jl)};

\kappa_{k(\gamma)} = u_{k(\gamma)} v_{k(\gamma)}; \quad \rho_{k(\gamma)} = |v_{k(\gamma)}|^2; \quad P_\gamma = |c_\gamma|^2; \quad S_{\gamma\beta} = c_\gamma^* c_\beta^*; (21)

\text{where}\ \rho_{k(\gamma)}\ \text{are single-particle densities and}\ \kappa_{k(\gamma)}\ \text{are pairing moment components.}\ P_\gamma\ \text{denote the probabilities to find the system in the configurations}\ \gamma.\ S_{\gamma\beta}\ \text{are moment components between two configurations}\ \gamma\ \text{and}\ \beta\ \text{and have the property}\ |S_{\gamma\beta}|^2 = P_\gamma P_\beta.\ \Delta_\gamma\ \text{is the gap parameter.}\ \text{The values of}\ \rho_{k(\gamma)}\ \text{and}\ P_\gamma\ \text{are reals.}\ \text{The particle number conservation conditions}\ 2 \sum_k \rho_{k(0)} = 2N, 2 \sum_{k \neq \gamma} \rho_{k(\gamma)} = 2N - 2\ \text{and}\ P_0 + \sum_\gamma P_\gamma = 1\ \text{are fulfilled by Eqs. (19).}

7. Application to fission processes

A direct application of the system (19) is related to the pair breaking and the odd-even structure in fission fragment yields. In fission, it is considered that the paired configuration is preserved until an interaction breaks some pairs in combination with the existence of a sufficient high excitation energy. The odd-even structure in fission is explained usually within statistical arguments, as for example in Refs. [40]. Alternatively, the probability to break a pair can be
determined dynamically by taking into account only the interaction available in the avoided crossing regions within the present model. To solve the pair breaking equations (19), the variations of single-particle energies $\epsilon_k$ together with perturbations $h_{ij}$ must be supplied. These quantities are obtained within the two center Woods-Saxon model along the minimal action fission path [41]. We used a method initiated in Ref. [42] which minimizes numerically the action integral. Alternatively, it is also possible to solve the associated Euler-Lagrange equations of motion [43, 44]. The results [12] are summarized in Fig. 3 for different constant values of the inter-nuclear velocity $\dot{R}$ ranging from $10^4$ to $10^6$ m/s. The results exhibit a clear decrease of $P_{\text{odd}}$ as function of $E^*$. It is interesting to note that at zero excitation energy, the probability to find the system in a seniority-two state is practically one. In cold fission, at very low excitation energies of the fragments, the odd-even yields are always larger than the even-even ones [45, 46]. The even-even fragmentation dominates at larger excitation energies of the fragments, above 4-6 MeV. It is a very strange phenomenon because in cold processes the system doesn’t possess enough energy to break a pair and because the penetrability is hindered for odd-systems due to the specialization energies associated to the two unpaired nucleons. If one assumes that the odd-even effect in the fission fragments distribution is strongly correlated to the seniority-two state probability, this phenomenon can be alternatively explained by solving the coupled-channel system of time-dependent pair breaking equations as evidenced above. In this work, only the $\Omega=1/2$ subspace of the proton level diagram is treated, but the same formalism can be applied to other subspaces.

8. Effective mass
A semi-adiabatic cranking approximation is derived [13] for even-even systems to investigate the general features of the cranking mass parameters when the nucleus is internally excited. It is assumed that the system deforms slowly in time. Therefore, the matrix elements of the time-derivative of the wave functions are small. Because these matrix elements are responsible for quasi-particle excitations, our approximation allows to consider the system in a seniority-zero state during fission. In other words, the contributions to the inertia originating from seniority-two configurations are considered negligible. At each deformation, instantaneous values of the single-particle densities and of the pairing moment components are deduced. The semi-adiabatic effective mass is evaluated using these values. In the above approximation, the semi-adiabatic
cranking formula can be obtained for the elements of the effective mass tensor $B_{\nu\mu}$:

$$
B_{\nu\mu} = 2\hbar^2 \sum_{m,n\neq m} \frac{(E_{mn} - E_0) | \frac{\kappa_m}{\sqrt{\epsilon_m}} |^2 \frac{\partial H}{\partial q_\nu} | n \rangle \langle m |}{(E_{mn} - \sum_{k\neq m,n} T_k - E_0 + \sum_k T_k)^2 (\epsilon_m - \epsilon_n)^2}
$$

$$
+ 2\hbar^2 \sum_{m} \frac{(E_{mm} - E_0) (\frac{\epsilon_m}{\rho_m} \frac{\partial \rho_m}{\partial q_\nu} - \frac{\kappa_m}{\rho_m} \frac{\partial \kappa_m}{\partial q_\nu})}{(E_{mm} - \sum_{k\neq m} T_k + T_m - E_0 + \sum_k T_k)^2} \frac{\epsilon_m}{\rho_m} \frac{\partial \rho_m}{\partial R} \frac{\kappa_m}{\rho_m} \frac{\partial \kappa_m}{\partial R}
$$

where the values of $\rho_k$ and $\kappa_k$ are solutions of the time dependent pairing equations and the index $\nu$ is associated to the collective coordinate $q_\nu$. $E_0$, $E_{jj}$ and $E_{jj}$ are exactly the expected values of the Hamiltonian (5) for seniority zero and seniority-two configurations, respectively.

The mass parameters can be obtained from formula (22) only if the derivatives $\partial \rho_m/\partial q_\nu$ and $\partial \kappa_m/\partial q_\nu$ are supplied. If the system follows a trajectory in the configuration space and the motion is characterized by some given collective velocities, then the derivatives depend also on $\dot{q}_\nu$. Therefore, the second term in Eq. (22) depends on the history of the system and can be calculated only if the variations of $\rho_k$ and $\kappa_k$ are known. The inertia along the trajectory is

$$
B = \sum_{\nu,\mu} B_{\nu\mu} \frac{\partial q_\mu}{\partial R} \frac{\partial q_\nu}{\partial R} + 2\hbar^2 \sum_{m} \frac{(E_{mm} - E_0) | \frac{\epsilon_m}{\rho_m} \frac{\partial \rho_m}{\partial R} - \frac{\kappa_m}{\rho_m} \frac{\partial \kappa_m}{\partial R} |^2}{(E_{mm} - \sum_{k\neq m} T_k + T_m - E_0 + \sum_k T_k)^2}
$$

where one coordinate $R$ is taken as the independent variable. The other coordinates $q_\nu$ are taken to be function of $R$ [41]. From definition, the collective kinetic energy is $E_c = B R^2/2$ and the last term in Eq. (23) becomes:

$$
E_{c0} = \hbar^2 \sum_{m} \frac{(E_{mm} - E_0) | \frac{\epsilon_m}{\rho_m} \dot{\rho}_m - \frac{\kappa_m}{\rho_m} \dot{\kappa}_m |^2}{(E_{mm} - \sum_{k\neq m} T_k + T_m - E_0 + \sum_k T_k)^2}
$$

This term depends explicitly only on the derivatives with respect to time $\dot{\epsilon}_m$ and $\dot{\rho}_m$. Their expressions are given by the coupled channel system of equations (19). That means, the term (24) is practically independent on a specific collective velocity and depends only on the values of $\epsilon_m$, $\kappa_m$ and $\rho_m$, that is on the structure of the system and its intrinsic excitation and represents the minimal collective kinetic energy shared among all degrees of freedom. In other words, if the nucleus is internally excited and the derivatives of the probabilities $\dot{\kappa}$, $\dot{\rho}$ are different from zero, the system possesses a minimal collective kinetic energy.

In Ref. [13], it was shown that the effective mass decreases when the excitation energy increases. It can be also mentioned that using BCS approximation, the formula (22) gives the classical result for the cranking inertia [41]. As remarked in Ref. [47], the results given by this non-adiabatic cranking are similar to that given by the Gaussian overlap approximation [48].

9. Conclusions

In this contribution, three approaches were used to generalize the the time dependent pairing equations and to derive an excitation dependent inertia. The new formalisms are valid for any kind of mean field approximations that include a monopole pairing field. These new equations offer information about the spectroscopic amplitudes and the dissipated energies in different final channels. Using these equations, qualitative and quantitative features of the fine structure phenomenon in cluster decay were reproduced. Moreover, the odd-even effect in cold fission processes was explained. The main trends concerning the dependence of the odd-even effect in fragments yields versus the fragments excitation energy were reproduced. Finally, the values of the inertia obtained from the time dependent pairing equations exhibit a strong dependence on the intrinsic excitation energy. In general, the semi-adiabatic inertia have a similar shell structure as the adiabatic cranking model. It is worth to mention, that other successfull models describe collective excitations within the stationary coupled channel approach [49].
Acknowledgments

Work supported by CNCS UEFISCDI, project number PN-II-ID-PCE-2011-3-0068

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