Photon statistics of semiconductor microcavity polaritons

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Abstract. Quantum statistical properties of the coherently driven semiconductor microcavity are studied. The unique quantum solution for the two-mode system of coupled excitons and photons was obtained in the adiabatic approximation. This solution predicts both bunching and antibunching regimes for the photonic field in the domain of parameters where the single mode polariton approximation fails.

1. Introduction

Investigation of the quantum properties of light interacting with nonlinear medium represents a fundamental task of modern quantum optics and many-body physics. In the problem of such kind statistical properties of light are of special interest. Nowadays engineering of photon sources possessing non-classical statistics is essential in many practically important applications. The main principle utilized in such devises is a government the light properties by the interaction with the medium which is usually considered as classical in the sense that its fluctuations are negligible. However the influence of fluctuations of the purely quantum medium represents an intriguing problem. The appropriate candidates for such quantum media are Bose-Einstein condensates. These exotic states of matter were observed both in atomic and solid-state semiconductor structures. While atomic condensates are obtained at rather low temperatures the exciton polariton condensates formed in semiconductor microcavities with a wide band gap (like GaN and ZnO) exist even at room temperatures. Quantum well excitons are distinguished by the strong nonlinearity evoked by the Coulumb interaction. Thus a rich dynamical and statistical behaviour triggered by quantum fluctuations are expected in microcavity system. So, semiconductor systems supporting interaction of quantum well excitons with a microcavity mode represent a robust platform for engineering of light sources with unusual statistics.

Here we study the system consisted of a 0D semiconductor microcavity with embedded quantum well irradiated by a coherent laser field which frequency is close both to photon and to exciton resonances of the structure. We perform an analysis of the influence of quantum fluctuations on the properties of the photonic mode paying special attention to nonlinear interaction between the excitons. Analytical solutions for this problem were found using generalized $P$-function [1] in the adiabatic limit [2]. It has been also demonstrated that a presence of quantum noise leads to metastability of the steady-state solution of the lower branch of bistability loop [3]. Non-classical statistics (both antibunching and bunching regimes) of photons were predicted.
2. The model

The system we are working with may arise in a variety of contexts. It consists of a single photonic mode interacting with a nonlinear medium. For concreteness we consider a semiconductor microcavity of a small radius which fundamental mode frequency $\omega_{ph}$ is tuned closely to the exciton resonance $\omega_{ex}$ of the embedded quantum well. Since the light is tightly confined inside the pillar both excitonic and photonic modes are devoid of spatial degrees of freedom. It implies that, unlike in planar semiconductor microcavities, the exciton wavevector $k$ normal to the structure growth axis has the fixed value, say $k = 0$. To sustain the nontrivial steady-state population of both photonic and excitonic modes an external quasi-resonant laser driving is applied. Thus the Hamiltonian of the two coupled modes under coherent driving reads

$$H_c = h\omega_{ph}\hat{\phi}^\dagger\hat{\phi} + h\omega_{ex}\hat{\chi}^\dagger\hat{\chi} + h\omega_k\left(\hat{\chi}^\dagger\hat{\phi} + \hat{\phi}^\dagger\hat{\chi}\right) + h\alpha\hat{\chi}^\dagger\hat{\phi}^\dagger \hat{\phi} + i\hbar\left(E\hat{\phi}^\dagger - E^*\hat{\phi}\right).$$

Here $\hat{\phi}$ ($\hat{\phi}^\dagger$) and $\hat{\chi}$ ($\hat{\chi}^\dagger$) annihilates (creates) photon and exciton respectively. The quantity $\omega_k$ corresponds to the strength of the coherent coupling between the excitonic and the photonic modes and corresponds to the half of Rabi splitting. The quantity $\alpha$ sets the strength of two-body exciton-exciton interaction. The coherent CW driving $E$ is spatially homogeneous and intense. So we take it as a classical force directly exciting the photon field, $E = E_c e^{i\omega t}$ with the frequency $\omega$ which is close but not necessary equals to the photonic resonance of the micropillar. Thus, the value of modulus square $|E|^2$ is proportional to the intensity of the driving laser.

The considered system is open in the sense that it interacts with the environment which acts as a sink for both photon and exciton fields balancing the driving gain. Such an interaction also leads to the rise of noise according to the driving-dissipative theorem. Accounting of this noise implies treating the environment as a thermal baths that is why we address the noise of such kind as "thermal". Since the current experiments with exciton-polariton supporting microstructures are carried out under rather low temperatures (about units of Kelvins for GaAs-based systems, for instance), the level of thermal noise is rather low. Actually, the noise strength is proportional to the thermal occupation of the bath which exponentially falls as the ratio $\omega/kT$ increases. So we neglect by the effect of thermal noise since $\omega_{ex} - \omega_{ph} \gg kT$. We also exclude the factor of noise originating from the apparatus such as fluctuations of the driving intensity [4]. Thus, the only source of noise that we account for is the so-called quantum noise which influence is expected to be amplified by the strong nonlinear interactions in the exciton subsystem.

Nevertheless, the impact of the environment can not be completely neglected since it evokes fast damping of both modes. The presence of the damping affects essentially dynamical and steady-state properties of the system. We treat losses of the exciton-photon system in the Born-Markov approximation which leads to the following master equation for the density operator:

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}\left[\hat{H}_c, \rho\right] + \gamma_{ph}\left(\hat{\phi}^\dagger \hat{\phi}^\dagger \hat{\phi} - \hat{\phi}^\dagger \hat{\phi} \hat{\phi}^\dagger\hat{\phi}\right) + \gamma_{ex}\left(\hat{\chi}^\dagger \hat{\chi}^\dagger \hat{\chi} - \hat{\chi}^\dagger \hat{\chi} \hat{\chi}^\dagger\hat{\chi}\right),$$

where $\gamma_{ph}$ and $\gamma_{ex}$ are the damping rates for photonic and excitonic modes respectively.

The semiclassical treatment of the equations (2) and (1) implies the mean field description. It operates with the average values instead of operators, $\chi = \langle \hat{\chi} \rangle$ and $\phi = \langle \hat{\phi} \rangle$, and postulates factorization of the averaged products as the products of averaged, in particular $\langle \hat{\chi}^\dagger \hat{\chi} \rangle \to |\chi|^2 \chi$. The corresponding dynamical equations in the frame rotating with the driving frequency are

$$\frac{\partial}{\partial t} \phi = -\left(i(\Delta - \Omega) + \gamma_{ph}\right)\phi + E_c - i\omega_k \chi,$$

$$\frac{\partial}{\partial t} \chi = -\left(-i(\Delta + \Omega) + \gamma_{ex}\right)\chi - i\omega_k \phi - 2i\alpha |\chi|^2 \chi,$$

(3a)  
(3b)
where $\Delta = (\omega_{ph} - \omega_{ex})/2$ is a half of the exciton-photon detuning and $\Omega = \omega_{d} - (\omega_{ph} - \omega_{ex})/2$ is detuning of the driving field from the central frequency between the exciton and the photon resonances.

The system (3) is known to sustain a couple of stable steady-state solutions, $\partial \chi / \partial t = 0$, $\partial \phi / \partial t = 0$ [5], reflecting the bistable optical response of the microcavity which manifests itself in a hysteretic behaviour of the output light intensity [3] – see figure 1(a). Originating from competition between the losses and the positive nonlinear feedback of the microcavity excitons [4] the optical bistability occurs within the definite range of the driving field intensities.

The existence of the bistable regime is determined by both the exciton-photon detuning and the frequency of the driving field [5, 6]. The bistability domain on the parameter plane of $\Delta$ and $\Omega$ is shown on the figure 1(b) – see [7]. Color of the filling corresponds to the value of the threshold driving intensity $I^0$ above which the upper stable solution exists (see the panel (b)).

Figure 1. (a) The bistability map on the parameter plane of $\Delta$ and $\Omega$. Shaded region corresponds to the existence of bistability. Color of the filling corresponds to the lower threshold of the driving field intensity within the bistability loop (see panel (b)). The parameters are $h\alpha = 0.001$ meV, $h\omega_k = 2.5$ meV, $\gamma_{ex} = 0.01$ ps$^{-1}$ and $\gamma_{ph} = 0.1$ ps$^{-1}$.

(b) Population of the photonic mode predicted within the mean field approach (blue curve) and by the unique quantum solution (4) (red dashed curve) for $\Delta = 3.1$ ps$^{-1}$ and $\Omega = -4.85$ ps$^{-1}$ (blue star on the panel (a)). The green dotted line corresponds to the second order coherence function of the photon field for zero delay.

There are two domains where the bistability exists. The shape of lower boundaries of these domains can be explained with the use of polariton states which are linear eigenstates of the Hamiltonian (1) arising from the strong coupling between excitons and photons. Dispersions of the two polariton branches are $\omega_{LP,UP} = \pm \sqrt{g^2 + \Delta^2}$, where LP (UP) denotes the lower (upper) polariton state – blue solid lines in figure 1(a). The bistability occurs if the driving frequency is blue detuned in respect to the polariton resonance. Actually, the increase of the driving intensity leads to the blue shifting of the energy of the state until it locks in resonance with the driving. This results in switching from the lower to the upper state of the bistability loop. So, the bistability domains lie above both polaritonic branches and are shifted upwards on the amount of frequency which is proportional to the level of losses, see [3].

In the context of mean field theory the upper and the lower states of the bistability loop are dynamically stable [7]. It implies that these states live infinitely long in the presence of small perturbations of the amplitudes of the exciton and the photon fields. However, generally, this statement can't be extended to the presence of noise. If the noise, either purely quantum or originating
from the apparatus, is strong, it forces the system to switch from one of the stable branches to the other. These stochastic switches are clearly observed experimentally [4,8,9].

Besides stochastic dynamics, the statistical properties of light reemitted by the microcavity can be also affected by quantum noise. Within the mean field approach both excitonic and photonics modes are supposed to be in the coherent state since they inherit its property from the coherent driving field. This assumption seems reasonable under high level of pumping when quantum fluctuations are negligible. However it should be verified for an arbitrary intensity of the coherent driving and both within and out of bistability existence domain shown in figure 1(b).

Both of previously mentioned phenomena have been already treated in the context of dissipative phase transition in a microcavity polariton system [10]. Experimental studies of the statistics of the microcavity radiation under quasiresonant driving reveal [8,9] both random switching between bistability branches and non-classical second-order correlations of the emitted photonic field. The obtained results demonstrate good coincidence with theoretical predictions. Note that the developed approaches treated a single mode problem when the driving is tuned closely to the low polariton branch and the upper polariton state can be carefully omitted. In this paper we, on the contrary, does not restrict ourselves with the lower polariton branch and use a complete two-component model of the coupled excitonic and photonic fields.

3. Quantum statistics of light in semiconductor microcavity

To get access into quantum properties of the exciton-photon system beyond mean field we use an approach of positive $P$-distribution developed by P. Drummond [1]. Converting the master equation (2) into partial differential equation for the distribution we get the Fokker-Planck equation for the $P$-function. The latter appears when expressing the density operator $\rho$ in the basis of coherent states of the photonic and the excitonic fields, see [2,7] for more details. In order to make the diffusion matrix positively definite the phase space should be doubled [1]. The obtained Fokker-Planck equation is written for the quasiprobability Glauber-Sudarshan $P$-function which phase space dimension is eight. This equation can be solved with the method of potentials. However to satisfy the condition of potentiality a so-called adiabatic approximation is necessary. It implies that excitonic and photonic modes possess greatly unequal dumping rates. Although the values of $\gamma_{ph}$ and $\gamma_{ex}$ vary in a wide range, the condition $\gamma_{ph} \gg \gamma_{ex}$ is satisfied for a typical micropillar structure. In this case the strongly dissipative photonic mode can be adiabatically eliminated that leads to a reduced Fokker-Planck equation yielding the following steady-state solution for the quasi-probability distribution in the steady-state:

$$P_{ss} = \chi^{-(i\gamma/\alpha + 2)} \chi^+(\chi^+)^{i\gamma/(\alpha - 2)} e^{2\chi^+ \chi^-(\sigma E_i/\chi + \sigma E^*_i/\chi)},$$

where

$$Y = \gamma_{ex} + \omega_R \gamma_{ph} \left(\gamma_{ph}^2 + (\Delta - \Omega)^2\right) - i\left(\Delta + \Omega + (\Delta - \Omega)\omega_R^2 \left(\gamma_{ph}^2 + (\Delta - \Omega)^2\right)\right)$$

and

$$\sigma = \omega_R^2 \gamma_{ph} \left(\gamma_{ph}^2 + (\Delta - \Omega)^2\right).$$

Note that $\chi$ and $\chi^+$ have a meaning of $c$-numbers, while the asterisk demotes complex conjugate algebraic numbers. Solution (4) gives the values of normally ordered quantum averages $G^{(mn)} = \left<\chi^m \chi^{n*}\right>$ which are found as moments of the $P$-distribution:

$$G^{(mn)} = -\frac{\left<\sigma E_i^m\sigma E_i^n\right>}{\Gamma(\sigma E_i^m)\Gamma(i\gamma/\alpha)\Gamma(-i\gamma/\alpha)\pi F_2\left(\frac{m + i\gamma/\alpha, n - i\gamma/\alpha, 2\sigma^2}{\sigma E_i^m}\right)}$$

where $\Gamma$ is Gamma function and $\pi F_2$ is hypergeometric function.

Steady-state quasiprobability distribution (4) together with expression (5) provides complete information about the excitonic subsystem. However the exciton statistics is hardly accessible.
experimentally. The valuable information about the system state is usually obtained from the statistical properties of the emitted light. In particular, we are interested in the second order correlation function of the photonic field \( \langle \hat{b}^\dagger(t)\hat{b}(t+\tau)\rangle \). Early we claim that \( \gamma_{ph} \gg \gamma_{ex} \). This condition guarantees that photonic field fluctuations are slaved by the excitonic mode [11]. Thus we express the value of the photonic field amplitude (in \( c \)-numbers) as
\[
\phi = \frac{E_d \left( \gamma_{ph} - i(\Delta - \Omega) \right)}{\left( \Delta - \Omega \right)^2 + \gamma_{ph}^2} - i \omega_k \left( \gamma_{ph} - \Delta - \Omega \right) / \left( \Delta - \Omega \right)^2 + \gamma_{ph}^2 \chi
\]
and then obtain the following expression for the second order coherence function for zero delay \( \tau = 0 \),
\[
g_{ph}^{(2)}(0) = \frac{E_d^4 - E_d^2 \omega_k^2 \left( G^{(00)} + G^{(20)} - 4G^{(11)} \right) - 2i \omega_k E_d^2 \left( G^{(01)} - G^{(10)} \right) + 2i \omega_k^3 E_d \left( G^{(21)} - G^{(12)} \right) + \omega_k^4 G^{(22)}}{\left( E_d^2 + 2i \omega_k E_d \text{Im}[G^{(00)}] + \omega_k^2 G^{(11)} \right) + \omega_k^2 G^{(22)}}. \quad (7)
\]

Behaviour of the second order correlation function of the fluctuating field in the presence of Kerr-like nonlinearity was investigated previously [2]. That results should be also valid for the considered two mode model provided that driving is tuned closely to the lower (or upper) polariton resonance thereby justifying single mode approximation, see [8,10]. The behaviour of \( g_{ph}^{(2)}(0) \) is determined by the stochastic dynamics of the field affected by the quantum noise. When the driving intensity is within the bistability cycle, quantum fluctuations initiate chaotic jumps between the two metastable states. The characteristic time scale of these jumps can be determined from the Liouvillian gap and crucially depends on the system parameters of the system [10]. The presence of such stochastic switching of the field intensity affects significantly the mean photon number and the statistical properties of the emitted light.

The unique quantum solution (4) predicts that the averaged value of the cavity mode population, \( n_{ph} = \langle \hat{n} \rangle \), lies between two metastable states predicted by the mean field theory – figure 1(b). The discrepancy between the quantum and the mean field solutions occurs for the driving intensities which are usually within the bistability loop. However for some specific parameters this variance appears also in the region where only single solution exists and any jumps between the metastable states are impossible. This very case is shown in figure 1(b). Such behaviour of the mean photon number indicates that the coherent state approximation underling the quasiclassical equations (3) is violated and the cavity mode is not in a coherent state due to the presence of quantum noise.

The nonclassical behaviour of \( g_{ph}^{(2)}(0) \) usually appears as a bunching peak, \( g_{ph}^{(2)}(0) \gg 1 \), under parameters where the quantum and quasiclassical values of the photon density differ, see the green dotted curve in figure 1(b). The discussed results were obtained for the coupled exciton-photon system. However they are reproduced well with the single-mode polariton model provided that the driving field frequency is close to the lower dispersion branch. To demonstrate the capability of the developed two-mode approach we consider the region where the single mode approximation fails. In particular, we consider the domain around triple resonance of the cavity mode, the excitonic state and the driving frequency, \( \Delta = 0 \) and \( \Omega = 0 \). The map of the zero-delay second order coherence function \( g_{ph}^{(2)}(0) \) of the intracavity field on the parameter plane of \( \Delta \) and \( \Omega \) is shown in figure 2(a). The driving field is fixed, \( E_d = 25 \) ps\(^{-1}\). The obtained dependence has several interesting peculiarities. Firstly, the giant bunching effect, \( g_{ph}^{(2)}(0) \gg 1 \), occurs for the parameters lying close to the line determined by the relation \( \Delta = -\Omega \). Secondly, both bunching and antibunching effects are observed for the region where bistability is absent (above the white dotted line in figure 2(a)).
Figure 2. (a) The second order coherence function of the photon field for zero delay $g_{ph}^{(2)}(0)$ for various values of the driving detuning $\Omega$ and the exciton-photon detuning $\Delta$. (b) The ratio between populations of the excitonic and the photonic modes $|\chi|^2/|\phi|^2$ on the same parameter plane as in the panel (a). (c) The dependence of $g_{ph}^{(2)}(0)$ on the driving detuning $\Omega$ for the fixed values of the exciton-photon detuning, shown by the vertical dot-dashed lines in the panel (a), $\Delta = -0.75$ ps$^{-1}$ (green), $\Delta = 0$ ps$^{-1}$ (red) and $\Delta = 0.75$ ps$^{-1}$ (blue). (d) The dependence of $g_{ph}^{(2)}(0)$ on the driving amplitude $E_d$ for $\Delta = -0.75$ ps$^{-1}$ and the fixed values of the driving detuning (color crosses in the panel (a)), $\Omega \approx 0.6$ ps$^{-1}$ (blue), $\Omega \approx 0.65$ ps$^{-1}$ (red), $\Omega \approx 0.67$ ps$^{-1}$ (green) and $\Omega \approx 0.75$ ps$^{-1}$ (yellow).

The dependences of $g_{ph}^{(2)}(0)$ on the driving detuning $\Omega$ for the three different values of the exciton-photon detuning, $\Delta = -0.75$ ps$^{-1}$, $\Delta = 0$ ps$^{-1}$ and $\Delta = 0.75$ ps$^{-1}$, are shown in figure 2(c). Formation of the giant bunching peaks can be attributed to a high difference between the populations of the photonic and the excitonic states. Our analysis reveals that the ratio $|\chi|^2/|\phi|^2$ maximizes under parameters corresponding to the giant bunching effect, see the panel (b) of figure 2. Since in this case the number of excitons greatly exceeds the population of the photonic mode, any fluctuation of the exciton state amplitude leads to the strong perturbation of the photon density because these subsystems are coherently coupled. This sudden inflation of the intracavity field manifests the presence of the photon bunching effect.
The antibunching behaviour, \( g_{\text{ph}}^{(2)}(0) < 1 \), of the photon statistics appears in vicinity of the giant bunching peak in the region of single metastable solution, see the panel (a) of figure 2 and the green curve in the panel (c). The behaviour of the second order coherence function in this region strongly depends on the driving field. The dependences of \( g_{\text{ph}}^{(2)}(0) \) on the amplitude \( E_d \) for various driving detunings \( \Omega \) and fixed \( \Delta = -0.75 \text{ ps}^{-1} \) are demonstrated in figure 2(d). For the small values of the driving detuning (below \( \Omega \approx 0.6 \text{ ps}^{-1} \) for the discussed parameters) the photon field demonstrates antibunching under weak driving, see the blue curve. The growth of the driving amplitude \( E_d \) brings the system into the coherent state, \( g_{\text{ph}}^{(2)}(0) \rightarrow 1 \). However, as the driving detuning approaches the region of the high disbalance between the photon and the exciton populations (figure 2(b)) the effect of bunching increases (the green and the yellow curves).

4. Conclusion
Quantum fluctuations may significantly affect both the expectation values and the statistical properties of the light emitted from the coherently driving microcavity. The recent experimental studies elucidate an intriguing physics of dissipative phase transitions with the microcavity polaritons and its stochastic dynamics driven by the quantum noise. The approaches developed early were based on the single mode approximation and thus is relevant only in the case when the driving intensity is tuned in the vicinity of low polariton branch. In this paper we used the more general approach considering a coupled exciton-photon system. The rigorous quantum solution for the quasiprobability function demonstrates nontrivial statistical properties of the photon field. In particular both the antibunching and the giant bunching effects were predicted in the region close to the upper boundary of the mean field bistability domain.

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References
[1] Drummond P D, Gardiner C W 1980 Generalized P-representations in quantum optics Journal of Physics 3 2353–2368
[2] Drummond P D, McNeil K J, Walls D F 1981 Non-equilibrium transitions in sub/second harmonic generation. Optica Acta: International Journal of Optics 25(2) 211–225
[3] Bass A, Karr J Ph, Giacobino E 2004 Optical bistability in semiconductor microcavities Physical Review A 69 023809
[4] Abbaspour H, Sallen G, Trebaol S, Morier-Genoud F, Portella-Oberli M T and Deveaud B 2015 Effect of a noisy driving field on a bistable polariton system Phys. Rev. B 92 165303
[5] Yulin A V, Egorov O A, Lederer F and Skryabin D V 2008 Dark polariton solitons in semiconductor microcavities Phys. Rev. A 78 061801
[6] Werner A, Egorov O A and Lederer F 2014 Exciton-polariton patterns in coherently pumped semiconductor microcavities Phys. Rev. B 89 245307
[7] Demirchyan S S, Khudaiberganov T A, Chestnov I Yu, Alodjants A P 2017 Quantum fluctuation in the system of exciton polaritons in semiconductor microcavities J. Opt. Technol. 84 75-81
[8] Rodriguez S R K, Casteels W, Storme F, Zambon N C, Sagnes I, Le Gratiet L, Bloch J. 2017 Probing a dissipative phase transition via dynamical optical hysteresis Phys. Rev. Lett. 118(24) 247402.
[9] Fink T, Schade A, Höfling S, Schneider C, İmamoğlu A 2017 Signatures of a dissipative phase...
transition in photon correlation measurements arXiv:1707.01837

[10] Casteels W, Fazio R and Ciuti C 2017 Critical dynamical properties of a first-order dissipative phase transition Phys. Rev. A, 95(1) 012128

[11] Gardiner C W 1986 Handbook of stochastic methods for physics, chemistry and the natural sciences Applied Optics 25 3145