A General Analysis of Non-Gaussianity from Isocurvature Perturbations

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Abstract

Light scalars may be ubiquitous in nature, and their quantum fluctuations can produce large non-Gaussianity in the cosmic microwave background temperature anisotropy. The non-Gaussianity may be accompanied with a small admixture of isocurvature perturbations, which often have correlations with the curvature perturbations. We present a general method to calculate the non-Gaussianity in the adiabatic and isocurvature perturbations with and without correlations, and see how it works in several explicit examples. We also show that they leave distinct signatures on the bispectrum of the cosmic microwave background temperature fluctuations.
1 Introduction

The inflationary paradigm has received strong observational support especially from the WMAP observation of the cosmic microwave background (CMB) [1]; in a simple class of the single-field inflation models, density fluctuations produced by an inflaton are known to be nearly scale-invariant, adiabatic and Gaussian, and these properties are found to be in a good agreement with the observation.

In the standard picture, the inflationary expansion is driven by a scalar field $\phi$, the inflaton, which slow-rolls on a very flat scalar potential. During inflation the inflaton $\phi$ acquires quantum fluctuations, which result in slight differences in the subsequent evolution at different places in the Universe. After inflation those differences turn into spatial inhomogeneities in the energy density. The nearly scale-invariant, adiabatic and Gaussian density perturbations are often regarded as the standard lore of the inflation theory. However, we would like to emphasize that the lore is based on a simple but crude assumption that it is only the inflaton that acquires sizable quantum fluctuations during inflation. Its apparent success does not necessarily mean that such a non-trivial condition is commonly met in the landscape of the inflation theory.

In fact, there are many flat directions in a supersymmetric (SUSY) theory and the string theory, and it may be even natural to expect that some of them remain light during inflation. If this is the case, such light scalars acquire quantum fluctuations, which may leave their traces in the CMB anisotropy such as isocurvature perturbations. This is indeed the case if those light scalars participate in the production of dark matter and/or baryon asymmetry of the Universe. For instance, the QCD axion [2], which was proposed to solve the strong CP problem, is a plausible candidate for the cold dark matter (CDM), and it is known that the axion generally acquires quantum fluctuations leading to the the CDM isocurvature perturbations. Also there are baryogenesis scenarios that contain light scalars. In the Affleck-Dine (AD) mechanism [3], for instance, a phase component of the AD field remains flat in most inflation models, leading to the baryonic isocurvature perturbations [4, 5, 6, 7]. Although the observed density perturbation is almost adiabatic and no sizable isocurvature perturbation has been discovered so far, a small admixture of the isocurvature perturbations is still allowed.

Non-Gaussianities have recently attracted much attention since Yadav and Wandelt claimed an evidence of the significant non-Gaussianity in the CMB anisotropy data [8]. On the other hand, the latest WMAP five-year result was shown to be consistent with the vanishing non-Gaussianity [1], although the likelihood distribution of the WMAP result seems to favor some amount of non-Gaussianity. Also there are active studies searching for the non-Gaussianity [9], and it is not settled yet whether the non-Gaussianity exists or not. At the present stage, it is fair to say that the observations are consistent with the nearly scale invariant and pure adiabatic perturbations with Gaussian statistics, while there is a hint of non-Gaussianity at the two sigma level.

Suppose that there is indeed significant amount of non-Gaussianity. Since it is known that the slow-roll inflation with a canonical kinetic term generally predicts a negligible amount of non-Gaussianity [10, 11, 12, 13], we need to go beyond the simplest class of
inflation models. A simple and even plausible way is to introduce additional light scalars. In the curvaton [14, 15, 16] and/or ungaussiton [17] scenarios, those light scalars can generate sizable non-Gaussianity [18, 19, 20]. In the presence of additional light scalars with quantum fluctuations, it is generically expected that isocurvature perturbation may arise. In particular, the non-Gaussianity hinted by the recent observations may originate from a small admixture of isocurvature perturbations [15, 21, 22].

We recently presented a formulation on non-Gaussianity in the isocurvature perturbations, and studied in detail how it exhibits itself in the CMB temperature anisotropy [23]. In Ref. [23], we found that the non-Gaussianity in the isocurvature perturbations leave distinctive signatures in the CMB; the non-Gaussianity is enhanced at large scales. Such features will enable us to distinguish the non-Gaussianity in isocurvature perturbations from that mainly in the adiabatic perturbation. As an example we considered the non-Gaussianity in the CDM isocurvature perturbations [23] and the baryonic isocurvature perturbations [24].

In this paper we extend our previous study in order to include possible correlations between the curvature and the isocurvature perturbations. We will give explicit examples in which there actually exist such correlations. Furthermore, as we did in the previous paper, we will present how the CMB temperature anisotropies are affected by the presence of the non-Gaussianities in the isocurvature perturbations correlated with the curvature perturbations.

This paper is organized as follows. In Sec. 2 we extend our formalism to include correlation of adiabatic and isocurvature perturbations. In Sec. 3 this formalism is applied to some explicit models. We study features in the bispectrum of CMB anisotropy in Sec. 4. Sec. 5 is devoted to discussion and conclusions.

2 Formalism

In this section, we extend the formalism developed in [23], where the formulation to calculate the non-Gaussianity of the isocurvature perturbation was provided, to include more general case that isocurvature perturbations have correlations with adiabatic one. To be definite, we consider CDM isocurvature perturbation, but an extension to other types of the isocurvature perturbations is straightforward.

2.1 Non-linear isocurvature perturbations and constraints

We write the perturbed spacetime metric as

\[ ds^2 = -N^2 dt^2 + a^2(t)e^{2\psi} \gamma_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt), \]

where \( N \) is the lapse function, \( \beta_i \) the shift vector, \( \gamma_{ij} \) the spatial metric, \( a(t) \) the background scale factor, and \( \psi \) the curvature perturbation. On sufficiently large spatial scales, the curvature perturbation \( \psi \) on an arbitrary slicing at \( t = t_f \) is expressed by [25]

\[ \psi(t_f, \vec{x}) = N(t_f, t_i; \vec{x}) - \log \frac{a(t_f)}{a(t_i)}. \]
where the initial slicing at \( t = t_i \) is chosen in such a way that the curvature perturbations vanish (flat slicing). Here \( N(t_f, t_i; \vec{x}) \) is the local e-folding number, given by the integral of the local expansion along the worldline \( \vec{x} = \text{const.} \) from \( t = t_i \) to \( t = t_f \). The curvature perturbation evaluated on the uniform-density slicing, where the total energy density is spatially uniform, is denoted by \( \zeta \). Similarly we define the quantity \( \zeta_i \) as the curvature perturbation evaluated on the slice where the energy density of the \( i \)-th component becomes uniform (\( \delta \rho_i(\vec{x}) = 0 \)). Hereafter we take the uniform density slicing at \( t = t_f \) in the last radiation dominated epoch before relevant cosmological scales enter the horizon.

In the radiation dominated epoch the curvature perturbation is approximately given by
\[
\zeta \approx \zeta_r,
\]
where \( \zeta_r \) denotes the curvature perturbation on a slice where the energy density of the total radiation is spatially uniform.

Let us assume that CDM has isocurvature fluctuation \( S \). We allow CDM to be composed of multiple particle species each of which can have different origin. Therefore, for instance, some components of CDM may have isocurvature fluctuations while the remaining ones do not.

We define the CDM isocurvature perturbation as
\[
S \equiv 3(\zeta_{\text{CDM}} - \zeta_r),
\]
(3)
Here \( \zeta_{\text{CDM}} \) is the curvature perturbation on a slice where the CDM density becomes spatially uniform. Then the total curvature perturbation in the matter dominated era is given by
\[
\zeta^{\text{MD}} = \zeta + \frac{1}{3} S,
\]
(4)
where \( \zeta \) is the total curvature perturbation in the radiation dominated era.

We consider a class of models in which \( \zeta \) and \( S \) originate from the quantum fluctuations \( \{ \delta \phi_a \} \) of light scalar fields \( \{ \phi_a \} \) during inflation. Note that the inflaton is also included in \( \{ \phi_a \} \). We can expand \( \zeta \) and \( S \) in terms of \( \delta \phi_a \) as
\[
\zeta = N_a \delta \phi_a + \frac{1}{2} N_{ab} \delta \phi_a \delta \phi_b + \ldots,
\]
(5)
\[
S = S_a \delta \phi_a + \frac{1}{2} S_{ab} \delta \phi_a \delta \phi_b + \ldots,
\]
(6)
where \( N_a \equiv \partial N/\partial \phi_a \) and \( S_{a} \equiv \partial S/\partial \phi_a \) and summation over the repeated indices \( a, b, \ldots \) is implicitly taken. Here we truncate the expansion at the second order and neglect higher order terms. For simplicity, we assume that the masses of \( \{ \phi_a \} \) are negligibly small during and after inflation, and the fluctuations are independent to each other. Then the correlation functions are given by the following form,
\[
\langle \delta \phi^a_{k_1} \delta \phi^b_{k_2} \rangle = (2\pi)^3 \delta(k_1 + k_2) P_{\delta \phi}(k_1) \delta^{ab}
\]
(7)
\(^1\) The sign of \( S \) given by Eq. (3) is opposite to the one in [20].
\(^2\) The evolution of \( \{ \phi_a \} \) is assumed to be smooth enough so that such expansions are justified.
\(^3\) Precisely speaking, \( \{ \phi_a \} \) denote the field values during inflation, and they may take different values at the onset of oscillations. If the potentials of \( \{ \phi_a \} \) are significantly deviated from quadratic one, such differences can be important for evaluating the non-Gaussianity [19, 29]. In this paper we neglect such effects.
with
\[ P_{\delta \phi}(k) \simeq \frac{H_{\text{inf}}^2}{2k^3}, \]  
where \( k \) denotes the comoving wavenumber, and \( H_{\text{inf}} \) is the Hubble parameter during inflation. For later use, we also define the following:
\[ \Delta_{\delta \phi}^2 \equiv \frac{k^3}{2\pi^2} P_{\delta \phi}(k) \simeq \left( \frac{H_{\text{inf}}}{2\pi} \right)^2. \]  

Before formulating the bispectra from non-Gaussianities in the isocurvature perturbations, here we define the auto/cross-correlation functions (and their spectra) of the primordial curvature and isocurvature perturbations, \( \zeta \) and \( S \). We define:
\[ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle \equiv (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_{\zeta}(k_1), \]  
\[ \langle \zeta_{\vec{k}_1} S_{\vec{k}_2} \rangle \equiv (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_{\zeta S}(k_1), \]  
\[ \langle S_{\vec{k}_1} S_{\vec{k}_2} \rangle \equiv (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_S(k_1). \]

Substituting (5) and (6) into these equations, we obtain
\[ P_{\zeta}(k) = N_a N_a P_{\delta \phi}(k) + \frac{1}{2} N_{ab} N_{ab} \int \frac{d^3 \vec{k}'}{(2\pi)^3} P_{\delta \phi}(k') P_{\delta \phi}(|\vec{k} - \vec{k}'|), \]  
\[ P_{\zeta S}(k) = N_a S_a P_{\delta \phi}(k) + \frac{1}{2} N_{ab} S_{ab} \int \frac{d^3 \vec{k}'}{(2\pi)^3} P_{\delta \phi}(k') P_{\delta \phi}(|\vec{k} - \vec{k}'|), \]  
\[ P_S(k) = S_a S_a P_{\delta \phi}(k) + \frac{1}{2} S_{ab} S_{ab} \int \frac{d^3 \vec{k}'}{(2\pi)^3} P_{\delta \phi}(k') P_{\delta \phi}(|\vec{k} - \vec{k}'|). \]

After performing the integration, the spectra \( P_{\zeta}, P_{\zeta S} \) and \( P_S \) can be expressed as
\[ P_{\zeta}(k) \simeq |N_a N_a + N_{ab} N_{ab} \Delta_{\delta \phi}^2 \ln(kL)| P_{\delta \phi}(k), \]  
\[ P_{\zeta S}(k) \simeq |N_a S_a + N_{ab} S_{ab} \Delta_{\delta \phi}^2 \ln(kL)| P_{\delta \phi}(k), \]  
\[ P_S(k) \simeq |S_a S_a + S_{ab} S_{ab} \Delta_{\delta \phi}^2 \ln(kL)| P_{\delta \phi}(k). \]

Here we have introduced an infrared cutoff \( L \), which is taken to be of order of the present Hubble horizon scale [27, 28].

We define a cross-correlation coefficient by \( \gamma \),
\[ \gamma \equiv \frac{-P_{\zeta S}(k_0)}{\sqrt{P_{\zeta}(k_0) P_S(k_0)}}. \]

Uncorrelated isocurvature perturbation corresponds to \( \gamma = 0 \) and totally (anti-)correlated one is \( \gamma = (-1) \). The initial condition for the structure formation is almost adiabatic, and the amplitude of isocurvature perturbations is now constrained from various cosmological observations. For the uncorrelated case, the WMAP5 constraint is [1]
\[ \frac{P_S(k_0)}{P_{\zeta}(k_0)} \lesssim 0.190, \]
while for the totally anti-correlated case, the constraint is

\[
P_S(k_0) = \left( \frac{P_S(k_0)}{P_S(k_0)} \right)^2 \lesssim 0.0111. \tag{21}
\]

As we will see, these constraints give upper bounds on the non-linearity parameters associated with the isocurvature perturbations.

## 2.2 Non-Gaussianity from isocurvature perturbations

Isocurvature perturbations must have negligible contribution to the power spectrum of the total curvature perturbation from observations. Therefore we can approximately obtain

\[
\langle \zeta^{\text{MD}}_{k_1} \zeta^{\text{MD}}_{k_2} \rangle \approx \langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta(k_1 + k_2) P_\zeta(k_1), \tag{22}
\]

where

\[
P_\zeta(k) = \alpha_\zeta P_{\delta\phi}(k), \tag{23}
\]

and

\[
\alpha_\zeta = N_a N_a + N_{ab} N_{ab} \Delta_{\delta\phi}^2 \ln(kL). \tag{24}
\]

Meanwhile, the isocurvature perturbation may significantly contribute to the three-point function of $\zeta^{\text{MD}}$. We define the bispectrum of $\zeta^{\text{MD}}$, $B_\zeta^{\text{MD}}$ by the following equation,

\[
\langle \zeta^{\text{MD}}_{k_1} \zeta^{\text{MD}}_{k_2} \zeta^{\text{MD}}_{k_3} \rangle \equiv (2\pi)^3 \delta(k_1 + k_2 + k_3) B_\zeta^{\text{MD}}(k_1, k_2, k_3). \tag{25}
\]

This contains four kind of terms, like $\langle \zeta \zeta \zeta \rangle$, $\langle \zeta \zeta S \rangle$, $\langle \zeta SS \rangle$ and $\langle SSS \rangle$. For example, focusing on the first contribution from $\langle \zeta \zeta \zeta \rangle$, the bispectrum includes the following terms,

\[
B_\zeta^{\text{MD}}(k_1, k_2, k_3) \supset N_a N_b N_{ab} [P_{\delta\phi}(k_1) P_{\delta\phi}(k_2) + 2 \text{ perms.}] \\
+ N_{ab} N_{bc} N_{ca} \int \frac{d^3 \vec{k}}{(2\pi)^3} P_{\delta\phi}(k') P_{\delta\phi}(|\vec{k}_1 - \vec{k}'|) P_{\delta\phi}(|\vec{k}_2 - \vec{k}'|), \tag{26}
\]

and similar expressions hold for other three contributions. Performing the integration, we can express $B_\zeta^{\text{MD}}$ in terms of the four combinations of the bispectrum of $\zeta$ and $S$,

\[
B_\zeta^{\text{MD}}(k_1, k_2, k_3) \simeq \left( \beta_{\zeta \zeta \zeta} + \frac{1}{3} \beta_{\zeta \zeta S} + \frac{1}{9} \beta_{\zeta SS} + \frac{1}{27} \beta_{SSS} \right) [P_{\delta\phi}(k_1) P_{\delta\phi}(k_2) + 2 \text{ perms.}], \tag{27}
\]

where each term on the right hand side ( RHS) is given by

\[
\beta_{\zeta \zeta \zeta} = N_a N_b N_{ab} + N_{ab} N_{bc} N_{ca} \Delta_{\delta\phi}^2 \ln(k_b L), \tag{28}
\]

\[
\beta_{\zeta \zeta S} = N_a N_b S_{ab} + 2 N_{ab} N_a S_b + 3 N_{ab} N_{bc} S_{ca} \Delta_{\delta\phi}^2 \ln(k_b L), \tag{29}
\]

\[
\beta_{\zeta SS} = S_a S_b N_{ab} + 2 S_{ab} S_a N_b + 3 S_{ab} S_{bc} N_{ca} \Delta_{\delta\phi}^2 \ln(k_b L), \tag{30}
\]

\[
\beta_{SSS} = S_a S_b S_{ab} + S_{ab} S_{bc} S_{ca} \Delta_{\delta\phi}^2 \ln(k_b L), \tag{31}
\]
in a squeezed configuration that one of the three wave vectors is much smaller than the other two (e.g., \( k_1 \ll k_2, k_3 \)) and we have defined \( k_b \equiv \min\{k_1, k_2, k_3\} \).

In many literatures concerning the non-Gaussianity of the primordial fluctuations, where the adiabaticity is implicitly assumed, the magnitude of the non-Gaussianity of the curvature perturbation is conventionally parametrized by the so-called non-linearity parameter \( f_{\text{NL}} \) which is defined by the ratio of the bispectrum to the square of the power spectrum. Since a single parameter is not enough to parametrize the non-Gaussianity in the presence of both the adiabatic and isocurvature perturbations because of their different effects on matter spectrum or temperature anisotropy of the CMB, we define four types of non-linearity parameters as following,

\[
\begin{align*}
6 \frac{f_{\text{(adi)}}}{5} f_{\text{NL}} &= \alpha_\zeta^{-2} \beta_\zeta \zeta, \\
6 \frac{f_{\text{(cor1)}}}{5} f_{\text{NL}} &= \frac{1}{3} \alpha_\zeta^{-2} \beta_\zeta S, \\
6 \frac{f_{\text{(cor2)}}}{5} f_{\text{NL}} &= \frac{1}{9} \alpha_\zeta^{-2} \beta_\zeta SS, \\
6 \frac{f_{\text{(iso)}}}{5} f_{\text{NL}} &= \frac{1}{27} \alpha_\zeta^{-2} \beta_{SSS},
\end{align*}
\]  

(32)

These formulae reproduce the known results for the curvaton, ungaussiton and axion, if only one of these non-linearity parameters exists, as we will see. However, in general, all the four non-linearity parameters can be concomitant and their effects on the temperature anisotropy have not been investigated in the previous literatures.

\section{Applications}

In this section, as an application of the formalism given in the previous section, we consider some simple models where the inflaton \( \phi \) and another light field \( \sigma \) contributes to the adiabatic and isocurvature perturbations. We neglect the non-Gaussianity generated by the inflaton itself (i.e. we set \( N_{\phi\phi} = 0 \)).

\subsection{Simple examples}

First we demonstrate that our formulae given in the previous section correctly reproduce known results for some simple models.

\subsubsection{Curvaton model with no CDM isocurvature perturbation}

If all the CDM is generated after the decay of the curvaton, no isocurvature perturbations are generated. Assuming \( N_\sigma \gg N_\phi \), i.e., the curvature perturbations are dominantly generated by the curvaton, we have

\[
6 \frac{f_{\text{(adi)}}}{5} f_{\text{NL}} = \frac{1}{N_\sigma} [N_\sigma^2 N_{\sigma\sigma} + N_{\sigma\sigma}^3 \Delta_{\sigma\sigma}^2 \ln(k_bL)].
\]  

(33)

This is the standard result for the curvaton model \[19\]. But in general cases, curvaton models predict the existence of correlated CDM isocurvature perturbation, and it can significantly modify the feature of non-Gaussianity in the CMB anisotropy, as we will see later.
3.1.2 Ungaussiton model with no CDM isocurvature perturbation

Similar to the curvaton case, but $\sigma$ is assumed to only affect the bispectrum of the curvature perturbation, and have only small effect on the power spectrum. In the limit $N_\sigma \ll N_\phi$, there are no isocurvature perturbations if all the CDM is generated from the inflaton decay products. In this case we obtain

$$ \frac{6}{5} f_{NL}^{(adi)} = \frac{1}{N_\phi^4} [N_\sigma^2 N_{\sigma\sigma} + N_{\sigma\sigma}^3 \Delta_{\delta\sigma}^2 \ln(k_b L)], $$

(34)

as shown in [17, 20].

3.1.3 Axion

Let us assume that $\sigma$ is stable and contributes to some fraction of the CDM. The axion ($a$) [2] is one of the well motivated candidates of such a scalar field. Since the axion has only negligible energy density in the radiation dominated phase, we have $N_\sigma \sim 0 \ll S_\sigma$ and $\beta_{\zeta\zeta} \approx \beta_{\zeta S} \approx 0$, corresponding to an uncorrelated isocurvature perturbation. Thus we obtain

$$ \frac{6}{5} f_{NL}^{(iso)} = \frac{1}{27N_\phi^4} [S_\sigma^2 S_{\sigma\sigma} + S_{\sigma\sigma}^3 \Delta_{\delta\sigma}^2 \ln(k_b L)]. $$

(35)

This coincides with our previous result [23].

3.2 General curvaton model

Let us consider a case that both radiation and CDM originate from the decay of both the inflaton ($\phi$) and the curvaton ($\sigma$). The radiation and the CDM produced from the inflaton have the same fluctuations, i.e. the energy density of the radiation becomes spatially uniform on the slice where that of the CDM becomes spatially uniform. We denote the curvature perturbation on this slice by $\zeta_\phi$. Similarly, we define $\zeta_\sigma$ as the curvature perturbation on a slicing on which the radiation and the CDM produced by the curvaton become spatially uniform.

3.2.1 CDM from the inflaton and direct decay of the curvaton

Let us first consider the case that some fraction of the CDM are produced directly from the decay of the curvaton. We assume that the universe is dominated by the radiation at the curvaton decays, and so we can approximate the total curvature perturbation as $\zeta \simeq \zeta_\sigma$. Also we assume that the curvaton decays instantaneously when $H = \Gamma_\sigma$, where $\Gamma_\sigma$ is decay rate of the curvaton. Then taking a uniform density slicing just before the curvaton decays, we have a relation given by

$$ \rho_\phi^x(x) + f \rho_\sigma^x(x) = \rho_r^x(x), $$

(36)

The following discussions do not depend on which of them dominates the total curvature perturbation, but conveniently we call $\sigma$ ‘curvaton’ in both cases.
where $f$ is the fraction of the curvaton energy density that transfers to the radiation. Because $\rho_\phi$ and $\rho_\sigma$ have different origin, they are inhomogeneous in general on this slice. Using equations $\rho_\phi = e^{4(\zeta_\phi - \zeta)} \bar{\rho}_\phi$, $\rho_\sigma = e^{3(\zeta_\sigma - \zeta)} \bar{\rho}_\sigma$, $\rho_r = \bar{\rho}_r e^{4(\zeta_r - \zeta)}$ and $\zeta \simeq \zeta_r$, Eq. (36) can be written as

\[
(1 - \epsilon_r)e^{4(\zeta_\phi - \zeta_r)} + \epsilon_r e^{3(\zeta_\sigma - \zeta_r)} = 1, \tag{37}
\]

where $\epsilon_r \equiv f_\sigma / \bar{\rho}_\sigma |_{\text{decay}}$ denotes the fraction of the radiation produced from the curvaton. Hence $1 - \epsilon_r$ is the fraction of the radiation from the inflaton.

In the same way, we obtain the similar equation to Eq. (37) for the CDM as

\[
(1 - \epsilon_{CDM})e^{3(\zeta_\phi - \zeta_{CDM})} + \epsilon_{CDM} e^{3(\zeta_\sigma - \zeta_{CDM})} = 1, \tag{38}
\]

where $\epsilon_{CDM} \equiv (1 - f) \bar{\rho}_\sigma / \bar{\rho}_{CDM} |_{\text{decay}}$ denotes the fraction of the CDM produced from the curvaton. Eqs. (37) and (38) give the fully non-linear relations between $(\zeta_r, \zeta_{CDM})$ and $(\zeta_\phi, \zeta_\sigma)$ under the sudden decay approximation.

Expanding the perturbation variables in Eqs. (37) and (38) up to the second order, we can explicitly express $\zeta_r$ and $\zeta_{CDM}$ in terms of $\zeta_\phi$ and $\zeta_\sigma$ as

\[
\zeta_r = (1 - R)\zeta_\phi + R \zeta_\sigma + \frac{1}{2} R(1 - R)(3 + R)(\zeta_\phi - \zeta_\sigma)^2, \tag{39}
\]

\[
\zeta_{CDM} = (1 - \epsilon_{CDM})\zeta_\phi + \epsilon_{CDM} \zeta_\sigma + \frac{3}{2} \epsilon_{CDM} (1 - \epsilon_{CDM})(\zeta_\phi - \zeta_\sigma)^2, \tag{40}
\]

where we have introduced a new parameter $R$ defined by

\[
R = \frac{3\epsilon_r}{4 - \epsilon_r}. \tag{41}
\]

Meanwhile, the curvaton energy density on the uniform density slice can be written as $\rho_\sigma(\vec{x}) = e^{3(\zeta_\sigma - \zeta)} \bar{\rho}_\sigma$. If we take this slice just after the curvaton starts its oscillation, the universe is dominated by the radiation produced from the inflaton and hence $\zeta = \zeta_\phi$. Denoting the density contrast of the curvaton energy density on the uniform density slicing by $\delta_\sigma \equiv (\rho_\sigma(\vec{x}) - \bar{\rho}_\sigma) / \bar{\rho}_\sigma$, we find

\[
\zeta_\sigma = \zeta_\phi + \frac{1}{3} \log(1 + \delta_\sigma). \tag{42}
\]

5 Precisely speaking, when the $\sigma$ decays into radiation, there is a subtlety in the definition of a slicing on which the energy density of $\sigma$ is spatially uniform. The equations in the text are valid if we interpret the relation between $\rho_\sigma$ on the uniform density slicing and its background value as the definition of $\zeta_\sigma$. This is because the energy density of $\sigma$ on the uniform density slicing is well defined even at moment $\sigma$ decays.

6 We assume that vacuum expectation value of the curvaton during inflation is much smaller than $M_{\text{pl}}$, where $M_{\text{pl}} = 2.4 \times 10^{18}$ GeV is the reduced Planck scale. In this case, the fraction of the curvaton energy density at the time when the curvaton begins its oscillations is roughly given by $\rho_\sigma / (M_{\text{pl}}^2 m_\sigma^2) \simeq \sigma^2 / M_{\text{pl}}^2 \ll 1$. Hence the curvaton is subdominant. Note also that it does not matter whether the inflaton has already decayed or not when the curvaton starts its oscillations.
Up to the second order in the curvaton fluctuation, the above equation can be written as

$$\zeta_\sigma = \zeta_\phi + \frac{1}{3} \left( \delta_\sigma - \frac{1}{2} \delta_\sigma^2 \right).$$

(43)

Then, using the standard formula,

$$\zeta_\phi = \frac{1}{M_{pl}^2 V_\phi} \delta \phi + \frac{1}{2 M_{pl}^2} \left( 1 - \frac{V V_{\phi \phi}}{V^2_\phi} \right) (\delta \phi)^2,$$

(44)

where \( V \) is the potential of the inflaton, \( V_\phi \) and \( V_{\phi \phi} \) are its first and second derivative with respective to \( \phi \). Noting that \( \delta_\sigma = 2 \delta \sigma / \sigma + (\delta \sigma)^2 / \sigma^2 \), we can express \( \zeta_\tau \) and \( \zeta_\sigma \) in terms of \( \delta \phi \) and \( \delta \sigma \) as

$$\zeta_\tau = \frac{1}{M_{pl}^2 V_\phi} \delta \phi + \frac{1}{2 M_{pl}^2} \left( 1 - \frac{V V_{\phi \phi}}{V^2_\phi} \right) (\delta \phi)^2 + \frac{2 R}{3 \sigma} \delta \sigma + \frac{R}{9 \sigma^2} (3 - 4 R - 2 R^2) (\delta \sigma)^2;$$

(45)

$$\zeta_{CDM} = \frac{1}{M_{pl}^2 V_\phi} \delta \phi + \frac{1}{2 M_{pl}^2} \left( 1 - \frac{V V_{\phi \phi}}{V^2_\phi} \right) (\delta \phi)^2$$

$$+ \frac{2 \epsilon_{CDM}}{3 \sigma} \delta \sigma + \frac{\epsilon_{CDM}(1 - 2 \epsilon_{CDM})}{3 \sigma^2} (\delta \sigma)^2.$$

(46)

As expected, no cross terms of \( \delta \phi \) and \( \delta \sigma \) appear in the final expressions. From these results, we can immediately read the expansion coefficients as

$$N_\phi = \frac{1}{M_{pl}^2 V_\phi}, \quad N_\sigma = \frac{2 R}{3 \sigma},$$

(47)

$$N_{\phi \phi} = \frac{1}{M_{pl}^2} \left( 1 - \frac{V V_{\phi \phi}}{V^2_\phi} \right), \quad N_{\sigma \sigma} = \frac{2 R}{9 \sigma^2} (3 - 4 R - 2 R^2),$$

(48)

$$S_\phi = S_{\phi \phi} = 0,$$

(49)

$$S_\sigma = 2 (\epsilon_{CDM} - R) \frac{1}{\sigma}, \quad S_{\sigma \sigma} = 2 \left[ \epsilon_{CDM}(1 - 2 \epsilon_{CDM}) - \frac{R}{3} (3 - 4 R - 2 R^2) \right] \frac{1}{\sigma^2}.$$

(50)

We can see that except for the trivial limit \( R = 0 \) and \( \epsilon_{CDM} = 0 \), isocurvature perturbation vanishes only when \( R = 1 \) and \( \epsilon_{CDM} = 1 \), corresponding to the case that the curvaton dominates the universe before it decays and all CDM is generated by the curvaton decay.
itself. Using this result, we can derive the following relations,

\[
\frac{6}{5} f_{NL}^{(adi)} = \frac{1}{2(1+p)^2 R^2} (3 - 4R - 2R^2),
\]

\[
\frac{6}{5} f_{NL}^{(cor1)} = \frac{3}{2(1+p)^2 R^2} \left[ \epsilon_{CDM} (1 - 2\epsilon_{CDM}) + \left(\frac{2}{3} \epsilon_{CDM} - R \right) (3 - 4R - 2R^2) \right],
\]

\[
\frac{6}{5} f_{NL}^{(cor2)} = \frac{3}{2(1+p)^2 R^5} (\epsilon_{CDM} - R) \times \left[ \epsilon_{CDM} \left( 3 - \frac{4R^2}{3} - 2R^3 - 4\epsilon_{CDM} \right) - R(3 - 4R - 2R^2) \right],
\]

\[
\frac{6}{5} f_{NL}^{(iso)} = \frac{3}{2(1+p)^2 R^4} (\epsilon_{CDM} - R)^2 \left[ \epsilon_{CDM} (1 - 2\epsilon_{CDM}) - \frac{R}{3} (3 - 4R - 2R^2) \right],
\]

when the fluctuation of the curvaton is dominated by the linear part, that is, \(\delta_\sigma \sim 2\delta \sigma / \sigma\). Here \(p \equiv N_\phi^2 / N_\sigma^2 = 9\sigma^2 V^2 / (4M^4_p V^2 R^2)\) represents the ratio of the inflaton contribution to the total curvaton perturbation and that of the curvaton. The limit \(p = 0\) corresponds to the standard curvaton scenario. In deriving these results, we have neglected the non-Gaussianity from the inflaton fluctuation because it gives \(f_{NL}\) a value of order of the slow-roll parameters which are much smaller than unity [10, 11, 12, 13].

Truncating the perturbative expansion in Eqs. (45) and (46) at the linear order, we find

\[
\frac{P_S}{P_{\zeta}} = \frac{S_\sigma}{N_\phi^2 + N_\sigma^2} = \frac{9(\epsilon_{CDM} - R)^2}{(1+p) R^2},
\]

\[
\gamma = -\frac{P_{\zeta S}}{\sqrt{P_{\zeta} P_S}} = -\frac{1}{\sqrt{1+p}}.
\]

Note that the cross-correlation parameter \(\gamma\) is reduced to \(-1\) in the standard curvaton scenario.

Let us suppose \(p = 0\) for simplicity and consider the case of the large non-Gaussianity \(f_{NL} \gtrsim 10\). To have the large \(f_{NL}\), \(R\) must be much smaller than unity. From the isocurvature constraints, \(\epsilon_{CDM}\) must be very close to \(R\) and hence \(\epsilon_{CDM}\) is also a small quantity. Then the term \(\beta_{\zeta \zeta \zeta}\) is the largest and the second largest term \(\beta_{\zeta \zeta S}\) is suppressed by \(\epsilon_{CDM}\) compared to \(\beta_{\zeta \zeta \zeta}\). Other terms are more suppressed by the power of \(\epsilon_{CDM}\). Hence the leading contribution to \(f_{NL}\) from the isocurvature perturbation comes from the term \(\beta_{\zeta \zeta S}\).

### 3.2.2 CDM from the inflaton and thermal bath after the curvaton decay

Let us next consider a case that in addition to the CDM (denoted by X) produced from the inflaton decay, CDM (denoted by Y) are also produced thermally during the radiation dominated era after the curvaton decay. We assume that all the curvaton energy density decays into the radiation.
Similar to the previous case, taking the uniform density slice just before curvaton decays, we obtain

\[(1 - \epsilon_r)e^{4(\zeta_\phi - \zeta_r)} + \epsilon_r e^{3(\zeta_\sigma - \zeta_r)} = 1.\]  

(57)

On the other hand, taking the uniform density slice after the total CDM is created, we have the following relation,

\[\epsilon_X e^{3(\zeta_X - \zeta_{CDM})} + \epsilon_Y e^{3(\zeta_Y - \zeta_{CDM})} = 1,\]  

(58)

where \(\epsilon_X / \epsilon_Y\) is the fraction of \(X/Y\) in the CDM. Because CDM is assumed to be composed of \(X\) and \(Y\), \(\epsilon_X + \epsilon_Y = 1\) must be satisfied. \(X\) is produced from the decay product of the inflaton. Hence \(\zeta_X = \zeta_\phi\). Meanwhile, \(Y\) is produced from the radiation which originate from both the inflaton and the curvaton. Hence \(\zeta_Y\) is equal to the curvature perturbation on the slice where the total radiation energy density becomes homogeneous, \(\zeta_Y = \zeta_r\). Eqs. (58) and (57) give the fully non-linear relation in the case of the thermally produced CDM. Expanding these equations up to the second order, we obtain

\[\zeta_r = (1 - R)\zeta_\phi + R\zeta_\sigma + \frac{1}{2}R(1 - R)(3 + R)(\zeta_\phi - \zeta_\sigma)^2,\]  

(59)

\[\zeta_{CDM} = \zeta_\phi - R(1 - \epsilon_X)(\zeta_\phi - \zeta_\sigma) + \frac{1}{2}R(1 - \epsilon_X)[3R\epsilon_X + (1 - R)(3 + R)](\zeta_\phi - \zeta_\sigma)^2.\]  

(60)

By doing the same procedures as in the previous subsection, we arrive at

\[S_\sigma = -\frac{2R\epsilon_X}{\sigma}, \quad S_{\sigma\sigma} = -\frac{2R\epsilon_X}{3} \left[3 - 4R - 2R^2 - 6R(1 - \epsilon_X)\right] \frac{1}{\sigma^2}.\]  

(61)

From this we can immediately see that the isocurvature perturbation vanishes if \(\epsilon_X = 0\), i.e., all CDM is generated thermally after the curvaton decays, as expected. Other quantities (\(N_\phi, N_\sigma, \ldots\)) are same as those obtained in the previous subsection. Thus we can calculate non-linearity parameters as

\[
\frac{6}{5} f^{(\text{adi})}_{NL} = \frac{1}{2(1 + p)^2R}(3 - 4R - 2R^2),
\]

(62)

\[
\frac{6}{5} f^{(\text{cor1})}_{NL} = -\frac{3\epsilon_X}{2(1 + p)^2R} \left[3 - 4R - 2R^2 - 2R(1 - \epsilon_X)\right],
\]

(63)

\[
\frac{6}{5} f^{(\text{cor2})}_{NL} = \frac{3\epsilon_X^2}{2(1 + p)^2R} \left[3 - 4R - 2R^2 - 4R(1 - \epsilon_X)\right],
\]

(64)

\[
\frac{6}{5} f^{(\text{iso})}_{NL} = -\frac{\epsilon_X^2}{2(1 + p)^2R} \left[3 - 4R - 2R^2 - 6R(1 - \epsilon_X)\right],
\]

(65)

\[
\frac{P_S}{P_\zeta} = \frac{9\epsilon_X^2}{1 + p},
\]

(66)

\[
\gamma = -\frac{1}{\sqrt{1 + p}}.
\]

(67)
As the simplest case, let us assume $p = 0$. Then, from the isocurvature constraints, we have a upper bound on $\epsilon_X$ as

$$\epsilon_X \lesssim 0.035.$$  \hfill (68)

We find $\epsilon_X$ must be much smaller than unity. In this case, the leading contribution to the non-Gaussianity from the isocurvature perturbation comes from $f_{NL}^{(cor1)}$ because $f_{NL}^{(cor2)}$ and $f_{NL}^{(iso)}$ are more suppressed by the power of $\epsilon_X$.

Through the analyses in the previous and this subsection, we have found that in both cases the bispectrum of the adiabatic perturbation dominantly contribute to the non-Gaussianity while the leading contribution from the isocurvature perturbation comes from $f_{NL}^{(cor1)}$ which are suppressed by the small parameter. But this does not necessarily mean the isocurvature contribution is irrelevant. Since $f_{NL}^{(cor1)}$ represents the contribution from the non-Gaussian fluctuation of the correlated isocurvature perturbation between CDM and photon, it has more drastic effects on large-scale CMB anisotropy. Actually it was pointed out in [23] that an isocurvature type non-Gaussianity of the large scale temperature anisotropy can be about 100 times larger than the usual adiabatic case, even if $f_{NL}^{(adi)} = f_{NL}^{(iso)}$. Thus it may be the case that the non-Gaussianity in the curvaton model is dominated by the isocurvature type, while satisfying the constraint on the magnitude of the isocurvature fluctuation from the power spectrum. We need numerical calculation to quantify the consequences of such mixed adiabatic and isocurvature non-Gaussianity.

### 3.2.3 A ‘realistic’ example

Here some particle physics motivated examples of the curvaton model are exhibited, and we show explicitly that it is natural to expect the existence of a correlated isocurvature perturbation in realistic curvaton models, if a large non-Gaussianity is generated by the curvaton.

First note that $R$ is given by

\[
R \simeq \begin{cases} 
\frac{\sigma_i^2}{4M_{pl}^2} \frac{T_R}{T_\sigma} & \text{for } m_\sigma > \Gamma_\phi \\
\frac{\sigma_i^2}{4M_{pl}^2} \frac{T_{osc}}{T_\sigma} & \text{for } m_\sigma < \Gamma_\phi 
\end{cases}
\]  \hfill (69)

where $m_\sigma$ is the mass of the curvaton, $\Gamma_\phi$ is the decay rate of the inflaton, $T_\sigma$ is the decay temperature of $\sigma$, $T_R$ is the reheating temperature after inflation defined by $T_R = (10/\pi^2 g_*)^{1/4} \sqrt{T_\phi M_{pl}}$ with the effective relativistic degrees of freedom $g_*$, $T_{osc}$ is the temperature at which the curvaton begins to oscillate defined by $T_{osc} = (10/\pi^2 g_*)^{1/4} \sqrt{m_\sigma M_{pl}}$, and $\sigma_i$ is the initial amplitude of the curvaton, which is assumed to be much smaller than $M_{pl}$.

In order to generate sizable non-Gaussianity, $R$ should be around 0.1.

\footnote{Note that in order that the curvaton can generate the observed magnitude of the density perturbation, $R$ cannot be smaller than $10^{-5}$.}
In a SUSY theory, there exist many scalar fields and some of them may remain light during inflation. These light scalars are candidates for the curvaton [30]. Good examples are the right-handed sneutrino [31], flat direction in the minimal supersymmetric standard model [32, 7], scalar partner of the axion (saxion) [33, 34], and so on. We assume $p = 0$ and that the curvaton decays before the LSP freezes out $T_\sigma \gtrsim m_{\text{LSP}}/20$, where $m_{\text{LSP}}$ denotes the mass of the lightest supersymmetric particle (LSP), which is assumed to be the lightest neutralino. In that case, the LSPs are produced in thermal bath after the curvaton decays, and they can be the dark matter if their annihilation cross section takes an appropriate value ($\langle \sigma v \rangle \sim 3 \times 10^{-26} \text{ cm}^3\text{s}^{-1}$). These LSPs do not have isocurvature perturbation. However, in general, LSPs are also produced nonthermally by the gravitino decay, and gravitinos are produced at the reheating epoch after inflation [35, 36]. Thus energy density of the gravitino fluctuates in the same way as that of the inflaton, so does the LSP directly created by the gravitino decay. Thus those nonthermal LSPs have (correlated) isocurvature perturbations. This is exactly the case of Sec. 3.2.2. Here $\epsilon_X$ reads the fraction of the nonthermally produced LSP to the total LSP abundance. The gravitino number-to-entropy ratio ($Y_{3/2}$) is given by [35, 37]

$$Y_{3/2} \sim 2 \times 10^{-12} \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( 1 + \frac{m_{\tilde{g}}^2}{3m_{3/2}^2} \right),$$

(70)

where $m_{\tilde{g}}$ and $m_{3/2}$ denote the mass of the gluino and gravitino, respectively. Thus the fraction of the nonthermally produced LSP in the total dark matter abundance is estimated as

$$\epsilon_X \sim 5 \times 10^{-3} \left( \frac{m_{\text{LSP}}}{1 \text{ TeV}} \right) \left( \frac{T_R}{10^7 \text{ GeV}} \right).$$

(71)

Thus in order to satisfy the constraint (68), we have upper bound on $T_R$ as

$$T_R \lesssim 7 \times 10^7 \text{ GeV} \left( \frac{1 \text{ TeV}}{m_{\text{LSP}}} \right),$$

(72)

which is marginally consistent with an upper bound from the cosmological gravitino problem [37]. On the other hand, $T_R$ cannot be much smaller than this value, since otherwise $R$ becomes too small, yielding too large non-Gaussianity, which conflicts with observations.

Therefore, it seems natural to expect that some fraction of the LSP dark matter has different origin, which inevitably involves isocurvature fluctuations. In such a situation, an analysis usually done in many literatures assuming the existence of only an adiabatic non-Gaussianity is not valid. Instead we need to carefully study the effect of (correlated) isocurvature perturbations on the resulting non-Gaussian features in the CMB anisotropy.

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8 Since gravitinos decay well after the freezeout of the LSP, nonthermally produced LSPs by the gravitino decay remain decoupled with thermal bath.
4 CMB temperature anisotropy

In this section we investigate the effects of the non-linear isocurvature perturbation on the CMB anisotropy, following the notations used in [38, 39]. Since the adiabatic and isocurvature perturbations have quite different properties regarding their imprints on the CMB anisotropy, we need to correctly evaluate the bispectrum of the temperature anisotropy in the presence of both adiabatic and isocurvature non-Gaussianity.

From temperature anisotropies originated from the adiabatic and isocurvature perturbations $\Delta T^{(\text{adi})}(\vec{n})$ and $\Delta T^{(\text{iso})}(\vec{n})$ for a given direction $\vec{n}$, we define $a_{\ell m}$ by

$$a_{\ell m} = \int d\vec{n} \left[ \frac{\Delta T^{(\text{adi})}(\vec{n})}{T} + \frac{\Delta T^{(\text{iso})}(\vec{n})}{T} \right] Y_{\ell m}^*(\vec{n}). \quad (73)$$

Transfer functions are defined by

$$\Theta^{(\text{adi})}_\ell (\vec{k}) \equiv g^{(\text{adi})}_T(k) \zeta_\vec{k}, \quad (74)$$

and

$$\Theta^{(\text{iso})}_\ell (\vec{k}) \equiv g^{(\text{iso})}_T(k) S_\vec{k}, \quad (75)$$

where $\Theta^{(\text{adi/iso})}_\ell (\vec{k})$ is the multipole moment of CMB temperature anisotropy from the adiabatic/isocurvature perturbation:

$$\frac{\Delta T^{(\text{adi/iso})}(\vec{n})}{T} = \int \frac{d^3k}{(2\pi)^3} \sum_\ell i^{\ell}(2\ell + 1)\Theta^{(\text{adi/iso})}_\ell (\vec{k}) P_\ell (\hat{k} \cdot \vec{n}). \quad (76)$$

Here $P_\ell$'s are the Legendre polynomials. From these equations, multipole moments can be divided as $a_{\ell m} = a^{(\text{adi})}_{\ell m} + a^{(\text{iso})}_{\ell m}$ where

$$a^{(\text{adi})}_{\ell m} = 4\pi i^\ell \int \frac{d^3k}{(2\pi)^3} g^{(\text{adi})}_T(k) Y_{\ell m}^*(\vec{k}) \zeta_\vec{k}, \quad (77)$$

$$a^{(\text{iso})}_{\ell m} = 4\pi i^\ell \int \frac{d^3k}{(2\pi)^3} g^{(\text{iso})}_T(k) Y_{\ell m}^*(\vec{k}) S_\vec{k}. \quad (78)$$

The angular power spectrum of $a_{\ell m}$ is calculated as

$$\langle a_{\ell m} a^{*}_{\ell' m'} \rangle \equiv \left[ C^{(\text{adi})}_\ell + 2 C^{(\text{cor})}_\ell + C^{(\text{iso})}_\ell \right] \delta_{\ell \ell'} \delta_{m m'}. \quad (79)$$

Using (78), we obtain

$$C^{(\text{adi})}_\ell = \frac{2}{\pi} \int_0^\infty dk \, k^2 \left( g^{(\text{adi})}_T(k) \right)^2 P_\ell(k), \quad (80)$$

$$C^{(\text{cor})}_\ell = \frac{2}{\pi} \int_0^\infty dk \, k^2 g^{(\text{adi})}_T(k) g^{(\text{iso})}_T(k) P_\zeta S(k), \quad (81)$$

$$C^{(\text{iso})}_\ell = \frac{2}{\pi} \int_0^\infty dk \, k^2 \left( g^{(\text{iso})}_T(k) \right)^2 P_S(k). \quad (82)$$
Similarly, the angular bispectrum of $a_{\ell m}$ is defined by

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \equiv B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3}. \quad (83)$$

Statistical isotropy divides the angular bispectrum into the following form,

$$B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} = G_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} b_{\ell_1 \ell_2 \ell_3}. \quad (84)$$

Here $G_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \equiv \int d\vec{n} Y_{\ell_1 m_1}(\vec{n}) Y_{\ell_2 m_2}(\vec{n}) Y_{\ell_3 m_3}(\vec{n})$ (Gaunt integral) and $b_{\ell_1 \ell_2 \ell_3}$ is the reduced bispectrum, on which we will focus in the following.

Substituting (77) and (78) into (83), we obtain

$$b_{\ell_1 \ell_2 \ell_3} = \frac{8}{\pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 \int_0^\infty dk_2 k_2^2 \int_0^\infty dk_3 \frac{k_3^2 j_1(k_1 r) j_2(k_2 r) j_3(k_3 r)}{P_{\delta\delta}(k_1) P_{\delta\delta}(k_2) + (2 \text{ perms})} \times \left[ g_{T\ell_1}^{(adi)}(k_1) g_{T\ell_2}^{(adi)}(k_2) g_{T\ell_3}^{(adi)}(k_3) \beta_{\zeta\zeta\zeta}(k_1, k_2, k_3) + \{ g_{T\ell_1}^{(adi)}(k_1) g_{T\ell_2}^{(iso)}(k_2) g_{T\ell_3}^{(iso)}(k_3) + (\text{cyclic with } \ell' \text{'s}) \} \beta_{\zeta\zeta S}(k_1, k_2, k_3) + \{ g_{T\ell_1}^{(adi)}(k_1) g_{T\ell_2}^{(iso)}(k_2) g_{T\ell_3}^{(iso)}(k_3) + (\text{cyclic with } \ell' \text{'s}) \} \beta_{\zeta S S}(k_1, k_2, k_3) + g_{T\ell_1}^{(iso)}(k_1) g_{T\ell_2}^{(iso)}(k_2) g_{T\ell_3}^{(iso)}(k_3) \beta_{S S S}(k_1, k_2, k_3) \right] \times \frac{1}{P_{\delta\delta}(k_1) P_{\delta\delta}(k_2) + (2 \text{ perms})}, \quad (85)$$

where $j_\ell(x)$ is the spherical Bessel function. Here we have assumed that the final terms in Eqs. (28)-31 dominate. Otherwise, the above expression becomes more complicated.

As we found in the general curvaton model (see Sec. 3.2), requiring the isocurvature constraints $P_S/P_\zeta \lesssim 0.01$ for totally anti-correlated case, the dominant contributions to the bispectrum come from the first two terms in Eq. (85). The first term, which is denoted by $b_{\ell_1 \ell_2 \ell_3}^{(adi)}$, can be written as

$$b_{\ell_1 \ell_2 \ell_3}^{(adi)} = \frac{8}{\pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T\ell_1}^{(adi)}(k_1) j_1(k_1 r) P_{\delta\delta}(k_1) \int_0^\infty dk_2 k_2^2 g_{T\ell_2}^{(adi)}(k_2) j_2(k_2 r) P_{\delta\delta}(k_2) \int_0^\infty dk_3 k_3^2 g_{T\ell_3}^{(adi)}(k_3) j_3(k_3 r) \beta_{\zeta\zeta\zeta} + (2 \text{ perms})$$

$$= \frac{48}{5\pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T\ell_1}^{(adi)}(k_1) j_1(k_1 r) P_{\zeta}(k_1) \int_0^\infty dk_2 k_2^2 g_{T\ell_2}^{(adi)}(k_2) j_2(k_2 r) P_{\zeta}(k_2) \int_0^\infty dk_3 k_3^2 g_{T\ell_3}^{(adi)}(k_3) j_3(k_3 r) j_{NL}^{(adi)} + (2 \text{ perms}). \quad (86)$$
The second term, which is denoted by $b_{l_1 l_2 l_3}^{(cor)}$, can be written as

$$b_{l_1 l_2 l_3}^{(cor)} = \left[ \frac{8}{\pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T l_1}^{(adi)} (k_1) j_{l_1} (k_1 r) P_{\delta \phi} (k_1) \right] \int_0^\infty dk_2 k_2^2 g_{T l_2}^{(adi)} (k_2) j_{l_2} (k_2 r) P_{\delta \phi} (k_2) \\
\times \int_0^\infty dk_3 k_3^2 g_{T l_3}^{(iso)} (k_3) j_{l_3} (k_3 r) \beta_{\zeta \zeta \zeta} S + (2 \text{ perms}) \right] + (\text{cyclic with } \ell \text{'s})$$

$$= \left[ \frac{144}{5 \pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T l_1}^{(adi)} (k_1) j_{l_1} (k_1 r) P_{\zeta} (k_1) \right] \int_0^\infty dk_2 k_2^2 g_{T l_2}^{(adi)} (k_2) j_{l_2} (k_2 r) P_{\zeta} (k_2) \\
\times \int_0^\infty dk_3 k_3^2 g_{T l_3}^{(iso)} (k_3) j_{l_3} (k_3 r) f_{NL}^{(cor)} + (2 \text{ perms}) \right] + (\text{cyclic with } \ell \text{'s}). \quad (87)$$

In deriving these equations, we have dropped logarithmic dependence on $k_1, k_2, k_3$ of $\beta_{\zeta \zeta \zeta}$ and $\beta_{\zeta S \zeta}$. The third and fourth terms, $b_{l_1 l_2 l_3}^{(cor2)}$ and $b_{l_1 l_2 l_3}^{(iso)}$, can also be written in the same way as

$$b_{l_1 l_2 l_3}^{(cor2)} = \left[ \frac{8}{\pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T l_1}^{(adi)} (k_1) j_{l_1} (k_1 r) P_{\delta \phi} (k_1) \right] \int_0^\infty dk_2 k_2^2 g_{T l_2}^{(iso)} (k_2) j_{l_2} (k_2 r) P_{\delta \phi} (k_2) \\
\times \int_0^\infty dk_3 k_3^2 g_{T l_3}^{(iso)} (k_3) j_{l_3} (k_3 r) \beta_{\zeta \zeta \zeta} S + (2 \text{ perms}) \right] + (\text{cyclic with } \ell \text{'s})$$

$$= \left[ \frac{432}{5 \pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T l_1}^{(adi)} (k_1) j_{l_1} (k_1 r) P_{\zeta} (k_1) \right] \int_0^\infty dk_2 k_2^2 g_{T l_2}^{(iso)} (k_2) j_{l_2} (k_2 r) P_{\zeta} (k_2) \\
\times \int_0^\infty dk_3 k_3^2 g_{T l_3}^{(iso)} (k_3) j_{l_3} (k_3 r) f_{NL}^{(cor2)} + (2 \text{ perms}) \right] + (\text{cyclic with } \ell \text{'s}), \quad (88)$$

$$b_{l_1 l_2 l_3}^{(iso)} = \frac{8}{\pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T l_1}^{(iso)} (k_1) j_{l_1} (k_1 r) P_{\delta \phi} (k_1) \right] \int_0^\infty dk_2 k_2^2 g_{T l_2}^{(iso)} (k_2) j_{l_2} (k_2 r) P_{\delta \phi} (k_2) \\
\times \int_0^\infty dk_3 k_3^2 g_{T l_3}^{(iso)} (k_3) j_{l_3} (k_3 r) \beta_{\zeta S \zeta} S + (2 \text{ perms})$$

$$= \frac{1296}{5 \pi^3} \int_0^\infty dr \int_0^\infty dk_1 k_1^2 g_{T l_1}^{(iso)} (k_1) j_{l_1} (k_1 r) P_{\zeta} (k_1) \right] \int_0^\infty dk_2 k_2^2 g_{T l_2}^{(iso)} (k_2) j_{l_2} (k_2 r) P_{\zeta} (k_2) \\
\times \int_0^\infty dk_3 k_3^2 g_{T l_3}^{(iso)} (k_3) j_{l_3} (k_3 r) f_{NL}^{(iso)} + (2 \text{ perms}) \right] + (\text{cyclic with } \ell \text{'s}). \quad (89)$$

We now move to see how non-Gaussianity from correlated isocurvature and adiabatic perturbations shows its signature on CMB bispectra $b_{l_1 l_2 l_3}$. To see this we plotted CMB bispectra $b_{l_1 l_2 l_3}$ in Fig. 1 and 2 using a modified version of the publicly available CMB code camb [30]. Here we assumed a flat scale-invariant SCDM model ($\Omega_m = 1$) and adopted a following set of cosmological parameters ($\Omega_b = 0.05, \Omega_{cdm} = 0.95, H_0 = 50$), where $\Omega_b$ and $\Omega_{cdm}$ are the energy density of baryon and CDM, and $H_0$ is the Hubble parameter in unit of km/s/Mpc. For simplicity in visualization, we have chosen following sets of ($l_1, l_2$) = (4, 6), (9, 11), (19, 21), (49, 51), (99, 101), (199, 201) and shown $b_{l_1 l_2 l_3}$ as a function of $l_3$. Fig. 1 shows bispectra separately with of their dependences on initial perturbations, $b_{l_1 l_2 l_3}^{(adi)}, b_{l_1 l_2 l_3}^{(cor1)}, b_{l_1 l_2 l_3}^{(cor2)}, b_{l_1 l_2 l_3}^{(iso)}$, where we fixed $\beta_{\zeta \zeta \zeta} = \beta_{\zeta S \zeta} = \beta_{\zeta S S} = 1$. 

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We can see from Fig. 1 that the bispectra with isocurvature, \( b_{\ell_1 \ell_2 \ell_3}^{(\text{cor1})} \), \( b_{\ell_1 \ell_2 \ell_3}^{(\text{cor2})} \), and \( b_{\ell_1 \ell_2 \ell_3}^{(\text{iso})} \), all show that their amplitudes are larger at large angular scales than that of the adiabatic bispectrum \( b_{\ell_1 \ell_2 \ell_3}^{(\text{adi})} \). This can be easily understood that the transfer function for isocurvature perturbation \( g_{\ell}^{(\text{iso})}(k) \) is large at large angular scales (\( \ell \lesssim 10 \)) than adiabatic one \( g_{\ell}^{(\text{adi})}(k) \) (See [23] for more detailed discussions). At small scales isocurvature perturbation tends to give small amplitude on CMB anisotropies. However, in some specific configurations, bispectra arising from correlations of adiabatic and isocurvature give comparable or even larger amplitudes than the pure adiabatic bispectrum at relatively small scales. Especially, \( b_{\ell_1 \ell_2 \ell_3}^{(\text{cor1})} \) is large when two of \( \ell \)'s are in regime of acoustic oscillation (e.g. \( \ell_1, \ell_2 \simeq 200 \)) and the other \( \ell \) is small (\( \ell_3 \lesssim 10 \)), where \( g_{\ell}^{(\text{adi})}, (\text{iso}) (k) \) of their largest amplitudes are picked out.

In Fig. 2 we plotted total bispectra \( b_{\ell_1 \ell_2 \ell_3} \) with realistic values of \( f_{\text{NL}}^{(\text{adi})} = 10, f_{\text{NL}}^{(\text{cor1})} = -0.9, f_{\text{NL}}^{(\text{cor2})} = 0.03, f_{\text{NL}}^{(\text{iso})} = -3 \times 10^{-4} \), which, for example, can be realized by taking \( p = 0, R = 0.1, \epsilon_X = 0.03 \) in Eqs. (63-66). We can see there are considerable differences from pure adiabatic bispectra, which we have plotted in Fig. 2 for reference. The most dominant contribution for deviating from pure adiabatic bispectra comes from the bispectra from the correlation of two adiabatic and one isocurvature initial perturbations, \( b_{\ell_1 \ell_2 \ell_3}^{(\text{cor1})} \) since \( f_{\text{NL}}^{(\text{cor1})} \) is at least order of magnitude larger than \( f_{\text{NL}}^{(\text{cor2})} \) and \( f_{\text{NL}}^{(\text{iso})} \) so as not to conflict with current constraints on isocurvature perturbations. Therefore we can conclude that the signature of non-Gaussianity from correlated isocurvature and adiabatic perturbations are found in CMB bispectra at least one of \( \ell \)'s is small, where isocurvature perturbation gives large amplitude on CMB anisotropies. Also at intermediate angular scales (\( \ell \simeq 100 \)), where bispectra shows acoustic oscillation but still have sufficient power from isocurvature perturbation, we can see the oscillation of \( b_{\ell_1 \ell_2 \ell_3} \) has different phase from that of pure adiabatic one \( b_{\ell_1 \ell_2 \ell_3}^{(\text{adi})} \), which would be a striking evidence for correlated isocurvature non-Gaussianity.

5 Discussion and conclusions

In this paper we have generalized our formalism provided in Ref. [23] for calculating non-Gaussianity, to include more general case where correlation between adiabatic and isocurvature perturbations exist, and shown that it can significantly affect the bispectrum of the CMB anisotropy. Actually in the curvaton scenario, the correlated isocurvature perturbation between CDM and radiation exists, unless all the dark matter arise after the curvaton decays. Although we have focused on the CDM isocurvature perturbation, similar mechanism can produce the baryonic isocurvature perturbation, if the baryon number is created before the curvaton decays. Therefore, the standard prediction for \( f_{\text{NL}} \) in the curvaton/ungaussiton scenario correctly characterizes the non-Gaussian properties only when no isocurvature perturbations exist. In other words, there is a chance to probe the physics in the early Universe by using non-Gaussian isocurvature perturbations, if detected.
Figure 1: CMB bispectra $b^{(\text{adi})}_{\ell_1 \ell_2 \ell_3}$ (solid red line), $b^{(\text{cor1})}_{\ell_1 \ell_2 \ell_3}$ (dashed green line) $b^{(\text{cor2})}_{\ell_1 \ell_2 \ell_3}$ (dotted blue line) and $b^{(\text{iso})}_{\ell_1 \ell_2 \ell_3}$ (dot-dashed magenta line). The thick (thin) lines correspond to positive (negative) values of bispectra. We plotted $b^{(\text{adi})}_{\ell_1 \ell_2 \ell_3}$ as a function of $\ell_3$ with fixing $(\ell_1, \ell_2) = (4, 6), (9, 11), (19, 21), (49, 51), (99, 101), (199, 201)$. Unobservable multipoles are shown in the shaded regions.

The formulations provided in this paper can be applied to broad class of models. The right-handed sneutrino ($\tilde{N}$) may be light during inflation and acquire quantum fluctuations. The decay of $\tilde{N}$ generates lepton number, as well as some fraction of the total radiation. If the fluctuation of $\tilde{N}$ significantly contributes to the curvature perturbation, it leaves the correlation between the adiabatic and baryonic isocurvature perturbations. The AD field can have similar effect. The modulated reheating scenario \[41\] also predicts mixture of adiabatic and isocurvature perturbations with correlations if the CDM/baryon is created before the inflaton decays.

It is obvious that non-Gaussianity in the cosmological perturbations provides invaluable information on the very early Universe. It would be interesting if the future detection of the non-Gaussianity tells us about the origin of CDM and/or baryon asymmetry of the Universe, through their small isocurvature perturbations.

*Note added*
While finalizing this manuscript, Ref. [42] appeared in the preprint server, which treats
Figure 2: CMB bispectra $b_{\ell_1 \ell_2 \ell_3}$ (dashed green line) with $f_{NL}^{\text{adi}} = 10, f_{NL}^{\text{cor1}} = -0.9, f_{NL}^{\text{cor2}} = 0.03, f_{NL}^{\text{iso}} = -3 \times 10^{-4}$. For reference, we also plotted $b_{\ell_1 \ell_2 \ell_3}^{\text{adi}}$ (solid red line). Same as in Fig. 1, the thick (thin) lines correspond to positive (negative) values of bispectra and unobservable multipoles are shown in the shaded regions.

similar subjects to ours.

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