Phase-dependent cross sections of deuteron-triton fusion in dichromatic intense fields with high-frequency limit

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Abstract We investigate the influence of strong dichromatic laser fields (i.e., $1\omega - 2\omega$ and $1\omega - 3\omega$) with high-frequency limit on the cross sections of deuteron-triton (DT) fusion in Kramers-Henneberger (KH) frame. We focus on the transitions of phase-dependent effects depending on a dimensionless quantity $n_d$, which equals the ratio of the quiver oscillation amplitude to the geometrical touching radius of the deuteron and triton as defined in our previous research. Theoretical calculations show that the angle-dependent as well as phase-dependent Coulomb barrier penetrabilities can be enhanced in dichromatic intense fields, and the corresponding angle-averaged penetrabilities and the fusion cross sections increase significantly compared with field-free case. Then, we investigate the effects of the two beams with different intensities. Moreover, we find that there are twice shifts of the peak values for the phase-dependent cross sections with the increase of $n_d$. The reason for the first shift is the angle-dependent effects for sub-barrier fusion, while the second shift is due to the accumulation of over-barrier fusion, these mechanisms are analyzed in detail in this paper.

1 Introduction

The application of strong laser technology in atomic physics and nuclear physics has attracted more and more attentions. Intense lasers can be applied to atomic ionization [1–4], charged-particle acceleration [5–7], nuclear nonlinear optics [8], excite the isomeric $^{229}\text{Th}$ nuclear state based on laser-driven electron recollision [9] and also provide a new way for manipulating nuclear processes.

It is found that intense lasers can induce resonance internal conversion, i.e., Karpeshin considered mechanisms of isomer pumping via a laser-radiation-induced resonance conversion and of isomer energy triggering in a resonance laser radiation field [10]. And intense lasers can accelerate nuclear fission processes [11,12], especially in the aspect of increasing $\alpha$-decay rates [13–15] by modifying the Coulomb potential barrier. However, there is no conclusive result on whether laser-induced enhancement of the decay rate is considerable or not because there is no successful experiment to prove these theories [16,17]. In particular, since nuclear fusion processes are mainly associated with light nuclei, laser manipulation will be more effective because of the relatively large charge-mass ratio compared with that in the heavy nuclei processes. Queisser and Schützhold studied whether the tunneling probability could be enhanced by an additional X-ray free electron laser (XFEL) using a Floquet scattering method [18]. They find that the XFEL frequencies and field strengths required for this dynamical assistance mechanism should come within reach of present-day or near-future technology.

Based on these, we investigated deuteron-triton (DT) fusion cross sections in the presence of electromagnetic fields with high-intensity and high-frequency [19]. With the help of the Kramers-Henneberger (KH) transformation [20,21], we have shown that the corresponding Coulomb barrier penetrabilities increase significantly due to the depression of the time-averaged potential barrier. As a result, we have found that DT fusion cross sections can be enhanced depending effectively on a dimensionless quantity $n_d$, which equals the ratio of the quiver oscillation amplitude to the geometrical touching radius of the deuteron and triton. Recently, it is shown that intense low-frequency laser fields, such as those in the near-infrared regime for the majority of intense laser fields.
facilities around the world, can also enhance the fusion probabilities [22–25].

The above researches are focus on the effects of intense monochromatic laser fields on fusion processes. Similarly, dichromatic laser fields display important novel features that can’t be seen with monochromatic laser fields driving. Especially the phase-dependent effects, which have been widely studied in atomic ionization [26–32].

Our goal is to study DT fusion cross sections in the presence of high-intensity dichromatic laser fields (i.e., 1ω − 2ω and 1ω − 3ω) with high-frequency limit, especially the transitions of phase-dependent effects.

The paper is organized as follows. We introduce our model in Sect. 2, the shape of the time-dependent dichromatic laser fields and the quiver oscillations of DT system in the presence of these fields are shown in this part. We analyze the conditions for the applicability of time-averaged scheme and provide the time-averaged two-body interaction potential between the DT system, including the short-range attractive nuclear potential and long-range repulsive Coulomb potential in Sect. 3. Our results on the Coulomb barrier penetrabilities and DT fusion cross sections are given in Sect. 4 where we also show the results of the two beams with different intensities and twice shifts of the peak values for the phase-dependent cross sections, and analyze the reason combined with the potential in Sect. 3. Finally, we present our conclusions in Sect. 5.

2 Model

In the presence of dichromatic laser fields (i.e., 1ω − 2ω), the time-dependent Schrödinger equation of DT fusion in the velocity gauge and in the center-of-mass frame is

\[ i\hbar \frac{\partial}{\partial t} \Psi(t, \vec{r}) = \left( \frac{\vec{p}^2}{2m} + V(t, \vec{r}) \right) \Psi(t, \vec{r}), \]

(1)

where \( m = m_d m_t / (m_d + m_t) \) is the reduced mass, \( m_d \) and \( m_t \) are masses of deuteron and triton, \( q = e(Z_d A_t - Z_t A_d) / (A_d + A_t) = e/5 \) is an effective charge, where \( Z_d = 1 \) and \( Z_t = 1 \) are charge numbers of deuteron and triton, \( A_d = 2 \) and \( A_t = 3 \) are mass numbers of deuteron and triton. \( \vec{A}(t, \vec{r}) \) is the vector potential for the time-dependent dichromatic intense field. When the wave length of the laser field is greater than the characteristic size of the nuclei, the dipole approximation can be used (i.e., \( kr \) is negligible), so the vector potential depends only on time. \( V(\vec{r}) \) is the Coulomb potential with rectangular potential well.

Applying the unitary KH transformation

\[ \Omega(t) = \exp \left( i \frac{\hbar}{\hbar} \int_{-\infty}^{t} \left( \frac{q^2}{m} \vec{A}^2(\tau) \right) d\tau \right), \]

(2)

the wavefunction under the KH framework, denoted as \( \Psi_{kh} = \Omega(t) \Psi \), has the same total probability as \( \Psi(t, \vec{r}) \) due to \( \Omega^2(t) \Omega(t) = 1 \). The time-dependent Schrödinger equation can be written as

\[ i\hbar \frac{\partial}{\partial t} \Psi_{kh}(t, \vec{r}_{kh}) = \left( \frac{\vec{p}_{kh}^2}{2m} + V_{kh}(t, \vec{r}_{kh}) \right) \Psi_{kh}(t, \vec{r}_{kh}), \]

(3)

where the time-dependent potential is found to be

\[ V_{kh}(t, \vec{r}_{kh}) = -\theta \left( \frac{1 - \frac{1}{r_n}}{r_n} \right) U_0 \]

\[ + \theta \left( \frac{1}{r_n} \right) V_0 \left( \frac{r_n}{r_{kh}(t)} \right), \]

(4)

with the height of the Coulomb barrier \( e^2 / 4 \pi \varepsilon_0 r_n^2 \) denoted as \( V_0 \). Here, The coordinate operator is \( \vec{r}_{kh}(t) = \vec{r} - \vec{r}_c(t) \), where \( \vec{r} \) is relative displacement vector of deuteron and triton, and \( \vec{r}_c(t) \) is the quiver motion of a free nucleus in the dichromatic laser fields. For a DT collision, the geometrical touching radius \( r_n = 1.44(A_d^{1/3} + A_t^{1/3}) \) fm and \( V_0 \) are approximately 3.89 fm and 0.37 MeV, respectively. While the nuclear potential well \(-U_0\) is approximately \(-30 \) to \(-40 \) MeV.

Supposing that the fields are a laser field and its second harmonic, which are linearly polarized along the z-axis, i.e., \( \vec{E}(t) = \vec{e} \cdot E_0 (\sin \omega t + \delta \sin (2 \omega t + \phi)) \), where \( \delta = E_1 / E_0 \), \( E_0 \) and \( E_1 \) are the amplitudes of the laser electric field and its second harmonic, respectively. Then we can get \( \vec{r}_c(t) = \vec{e} r_c (\sin \omega t + \delta \sin (2 \omega t + \phi) / 4) \) with \( r_c = e \sqrt{2e \mu_0 / 5m \omega^2} \), where \( e \) is the relative phase. Let us introduce a dimensionless quantity \( n_d = r_c / r_n = 4.89 \times 10^{-6} \sqrt{T} / (\hbar \omega)^2 \), where the units of \( I \) and \( \hbar \omega \) are \( W/cm^2 \) and \( eV \), respectively. The ratio of \( r_c \) to \( r_n \) determines how external electromagnetic fields manipulate nuclear fusion processes. The position and momentum operators maintain the commutation relation \( [\vec{p}_{kh}(t), \vec{p}_{kh}^\dagger(t)] = i\hbar \delta^{ij} \).

For \( \phi = 0 \) and \( \pi / 2 \), we show the shape of two cycles of the time-dependent dichromatic electric fields and the quiver oscillations of DT system in these fields for \( n_d = 1, 11 \) and 18 in Fig. 1. It can be seen from Fig. 1a, b that the maximum instantaneous field is the least when \( \phi = 0 \), this maximum value of 1.76 \( E_0 \) is obtained twice per cycle, while the maximum instantaneous field is the greatest when \( \phi = \pi / 2 \), this maximum value of 2 \( E_0 \) is obtained once per cycle. Figure 1c, d are the quiver oscillations accordingly, and the range are about \([-1.10 n_d, 1.10 n_d] \) for \( \phi = 0 \) and \([-1.25 n_d, 0.75 n_d] \) for \( \phi = \pi / 2 \), respectively. We choose the laser parameters for \( n_d = 1, 11 \) and 18 according to the Fig. 1 of our previous study [19], when the laser frequency is 1 keV, the corresponding laser intensities for \( n_d = 1, 11 \) and 18 are \( 4.16 \times 10^{22} W/cm^2, 5.40 \times 10^{24} W/cm^2 \) and \( 1.50 \times 10^{25} W/cm^2 \), respectively. These intensities are higher than current XFEL, but considering the rapid devel-
opment of laser, we should take a positive attitude towards the realization of higher intensity lasers.

3 Time-averaged potential

Equation (3) indicates that all time dependence of the problem is shifted into the potential function, and \( V_{kh}(t, \tilde{r}_{kh}) \) is just a two-body Coulomb potential dressed by a time-dependent harmonic oscillation origin \( \tilde{r}_{0}(t) \) along the polarization direction \( \hat{e}_{z} \) with a quiver oscillation amplitude \( r_{0} \). In this case, the space can be divided into two parts: the inner region denoted by a capsule-like region swept by the nuclear potential well \(-U_{0}, \) i.e., \( D_{in} = \{ \vec{r}|r_{kh}(t) \leq r_{in}, \exists t \in [0, 2\pi/\omega) \} \), where \( r_{in} \) is the boundary of capsule-like region. And the outer region denoted by \( D_{out} = R^{3}/D_{in} \).

\( V_{kh}(t, \tilde{r}_{kh}) \) in KH frame can be expanded in terms of Fourier series:

\[
V_{kh}(t, \tilde{r}_{kh}) = \sum_{n=-\infty}^{\infty} V_{n}(\theta, r; \phi, n_{d}) e^{-int}, \tag{5}\]

where the expansion coefficient is

\[
V_{n}(\theta, r; \phi, n_{d}) = \frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} V_{kh}(t, \tilde{r}_{kh}) e^{int} dt \tag{6}.
\]

The amplitudes of \( V_{n} \) (i.e., \( n = 0, 1, 2 \)) for \( n_{d} = 1 \) are shown in Fig. 2. It’s shown that the static component dominates over the rest Fourier components at \( n_{d} = 1 \). We also check for the \( n_{d} \) up to 20 and find that the situations are similar.

Even though these high-order terms are rather small, they might still alter the particle’s scattering process in the combined Coulomb repulsive force and laser oscillating field through some possible resonances. In the scattering process, there are two intrinsic time-scales: one is characterized by the laser frequency, the other is the Coulomb interaction time duration.

The characteristic Coulomb interaction time duration can be approximated by the ratio of interaction length to an average relative velocity \([19]\), i.e., \( \Delta t = \Delta l/\bar{v} \). Here, the interaction length \( \Delta l \) refers to the distance from the position where the Coulomb repulsive potential begins to take effect (for instance, we assume \( V_{c}(r_{1}) = \epsilon/10^{3} \)) to the classical turning point \( V_{c}(r_{2}) = \epsilon \), where \( \epsilon = mv_{i}^{2}/2 \) is the initial relative kinetic energy of deuteron-triton. So \( \Delta l = q_{d}q_{t}(10^{3}/\epsilon - 1/\epsilon)/(4\pi\epsilon_{0}) \), where \( q_{d} = q_{t} = e, \epsilon_{0} \) is the permittivity of vacuum. The average of the initial velocity \( v_{i} \) and the final velocity \( v_{f} = 0 \) at the turning point, gives \( \bar{v} \approx \sqrt{2\epsilon/m} = \sqrt{\epsilon/2m} \). So the interaction time duration \( \Delta t \approx 10^{3}q_{d}q_{t}\sqrt{2m}/(4\pi\epsilon_{0}\sqrt{\epsilon}) \) explicitly depending on the initial relative kinetic energy, the corresponding Coulomb interaction time durations are ranged from 7.2 femtoseconds to 0.0072 femtoseconds for the relative energies of DT ranged from 1 to 100 keV. For the laser frequency (photon energy) is larger than 1 keV as we choose in the paper, the corresponding field oscillating period is about 4.14 attoseconds, which is fast comparing with the interaction time duration, we therefore believe that the time-averaged scheme in the KH framework should be valid, i.e., in the high-frequency laser fields, the incident nucleus feels a time-averaged potential \( \bar{V}_{0}(t, \tilde{r}_{kh}) = \bar{V}_{0}((\theta, r; \phi, n_{d}) \). It is worth noting that the choice of the used criterion involving the interaction length and correspondingly the deduced interaction time duration is the one that leads to our following results.

Notice that the KH transformation \([20,21]\) is a unitary transformation which allows for shifting all time dependence of the problem in the potential function. It is perfectly valid. The main question is the validity of the approximation of replacing the time-dependent potential by its static component through a time-averaged scheme. The KH transformation and time-averaged scheme have been applied to varied systems such as atomic ionization, alpha decay and present work of DT fusion. We emphasize here that the conditions for the time-averaged schemes also varied with respect to different physical processes (see appendix A for more discussions).

The time-averaged potential in both its peak value and tunneling width is distorted in the presence of strong fields, and the effective potential, including the short-range attractive nuclear potential and long-range repulsive Coulomb potential, is dependent on \( \phi \) and \( \theta \). The corresponding time-averaged potential \( \bar{V}_{0} \) for \( n_{d} = 1, 11, \) and 18 are shown in Fig. 3, respectively. Along the polarization direction \( \hat{e}_{z} \) of \( \theta = 0, \pi \), both the peak value and the barrier width are found to decrease significantly. For \( \phi = 0 \), The angle dependence...
of time-averaged potential is symmetrical for $\theta = \pi/2$. For $\phi = \pi/2$, such symmetrical structure doesn’t exist.

4 Main results

4.1 Penetrability

Using the Wentzel–Kramers–Brillouin (WKB) approximation [42], the penetrability through the Coulomb barrier at the relative energy $\epsilon$ along the direction $\hat{r}$ can be given by

$$P(\theta; \phi, \epsilon, n_d) = e^{-\frac{1}{\hbar}} \int_{r_{\text{in}}}^{r_{\text{out}}} \sqrt{2m(V_0(\theta, r; \phi, n_d) - \epsilon)} dr,$$

(7)

where $r_{\text{in}}$ and $r_{\text{out}}$ are the inner and outer turning points, respectively. Due to the symmetry of the Hamiltonian, the penetrability is independent of the azimuth $\phi$. The penetrability explicitly depends on the inclination angle $\theta$, the incident energy $\epsilon$ and the dimensionless parameter $n_d$.

The angle-dependent penetrability can be readily obtained by numerically calculating Eq. (7), and the results are plotted in Fig. 4. As shown in Fig. 4a, the angle-dependent penetrability exhibit different structures for different laser parameters. For $\phi = 0$, The angle-dependent penetrabilities are symmetrical for $n_d = 1$ and a local maximum penetrability for $n_d = 11$. In particular, the angle-dependent penetrability exhibits an interesting double-hollow structure for $n_d = 11$, this indicates that the penetrability can reach local maxima in the directions parallel and perpendicular to the laser polarization direction, i.e., $\theta = 0, \pi$ and $\theta = \pi/2$. For $n_d = 18$, most angles achieve $P(\theta) = 1$. But it’s obvious that such symmetrical structure doesn’t exist for $\phi = \pi/2$. Smaller-angle penetrabilities are enhanced, while larger-angle penetrabilities are depressed.

The angle-averaged penetrability can be obtained by taking an average over the solid angle, that is,

$$P_{\text{ave}}(\epsilon; \phi, n_d) = \frac{\int_0^{\pi} P(\theta; \phi, \epsilon, n_d) \sin \theta d\theta}{2}.$$

(8)

The penetrability versus the relative energy for different $n_d$ values is shown in Fig. 4b, indicating that the penetrability increases significantly with respect to the dimensionless parameter $n_d$.

4.2 DT fusion cross sections

Nuclear fusion is commonly believed to consist of three processes [43–45]. DT nuclear fusion cross-section is used to describe the probability that a nuclear reaction will occur. It’s usually given in a phenomenological Gamow form as a product of three terms

$$\sigma(\epsilon) = \frac{S(\epsilon)}{\epsilon} \exp\left(-\sqrt{\frac{\epsilon G}{\epsilon}}\right),$$

(9)

where the term $1/\epsilon$ is the geometrical cross section, which is proportional to the square of the relative motion’s de Broglie wavelength. And the term $\exp(-\sqrt{\epsilon G/\epsilon})$ is the tunneling probability through the Coulomb potential barrier, which holds as far as $\epsilon \ll \epsilon_G$. For DT fusion, the Gamow energy factor is $\epsilon_G = (e^2\sqrt{2m}/4\hbar\delta_0)^2 = 1.18$ MeV, so Eq. (9) applies very well to relative energies of $\epsilon \leq 100$ keV. Astrophysical $S$ factor describes the nuclear physics within the
Fig. 3 For $\delta = 1$, $\phi = 0$ and $\pi/2$, a to f contour plots on x-z section ($y = 0$) of the effective potential for $n_d =$ 1, 11 and 18, respectively. The blue areas represent the section of inner region $D_{in}$ where the potential value is approximately $-U_0$. g to l $V_0$ for different inclination angles $\theta$ with respect to varied $n_d$.

nuclear potential effective range. In the absence of external electromagnetic fields, the $S$ factor can be given by a fitting function

$$S(\varepsilon) = a + \frac{b}{\pi} \frac{d}{(\varepsilon - c)^2 + d^2},$$

where the parameters are found to be $a = 118.8$ keV · barn, $b = 8.647 \times 10^5$ keV² · barn, $c = 45.05$ keV and $d =$ 86.76 keV.

The laser intensities being chosen and regarded as very intense laser fields are still negligible compared to nuclear potentials. So we assumed that the laser fields do not alter the DT fusion process from the fundamental level, such as affecting the astrophysical $S$ function. Then we can get DT fusion cross-section according to Eq. (9) where the the tunneling probability can be calculated by angle-averaged penetrability Eq. (8). The results are shown in Fig. 5a. For $n_d =$ 18, the cross-section is shown to be enhanced by about 8 times of magnitude for collision energy $\varepsilon =$ 64 keV and its maximum equals 63.05 barns for $\varepsilon =$ 47 keV rather than $\varepsilon =$ 64 keV. This means that laser assisted deuteron-triton fusion can obtain bigger cross sections at smaller relative energies. Here, we exploit the time-dependent semiclassical (SC) model developed recently [24] to make some comparisons. The results are presented in Fig. 5b. We can see that the results are qualitatively consistent: The cross sections can be modulated by the relative phase $\phi$ and the trends are very similar. Quantitatively, the results of SC model are smaller than that of KH results. This might be because in our KH treatment in contrast to that of the time-dependent SC model, we only retain the zero order term while ignore the higher order terms (see Eq. (6)). Even though the amplitudes of the higher order terms are small, they might affect the cross sections to some extent considering the sensitive dependence of the tunneling probability on the effective potential barrier.

Since the strong dichromatic laser fields (i.e., $1\omega - 2\omega$ and $1\omega - 3\omega$) are investigated, one would expect them to be one the fundamental and the other one a higher harmonic laser. In this case, the two beams would also have different intensities. Next, we investigate the effect of varying $\delta$. We give the fusion cross-section versus $\delta = E_1/E_0$ in Fig. 6 for
For $\delta = 1$ as well as $n_d = 0, 1, 11$ and 18, a angle-dependent penetrability for relative energy of $\varepsilon = 64$ keV. b angle-averaged penetrability versus relative energy

For $\delta = 1$, fusion cross-section versus relative energy for $n_d = 0, 1, 11$ and 18. a KH results, b SC results: For comparison, in the calculations we choose high-frequency limit of 100 keV different $n_d$ and relative energies $\varepsilon$, which are corresponding to the maximum cross sections. We find that the fusion cross-section changes monotonically with $\delta$, which is in the range of 0–1. For $n_d = 1$, $\varepsilon = 64$ keV the fusion cross-section increases with the increase of $\delta$ for $\phi = 0$, while it decreases with the increase of $\delta$ for $\phi = \pi/2$. But it is reversed for $n_d = 11$, $\varepsilon = 60$keV and $n_d = 18$, $\varepsilon = 47$keV. So we could modulate the fusion cross sections by modulating the phase of two beams. And the larger the $\delta$ is, the more obvious the modulation effect will be.

Contour plots of $(\sigma(\phi = \pi/2) - \sigma(\phi = 0))/\sigma(\phi = 0)$ for $n_d \in [0, 20]$ are shown in Fig. 7. Figure 7a are $1\omega - 2\omega$ fields. As shown in Fig. 3a, b, g, h, for $n_d = 1$, the difference of the potential for $\theta = \pi/2$ between $\phi = 0$ and $\phi = \pi/2$ is very small, so the angle-averaged potential is primarily decided by other $\theta$ that the weighted coefficient are smaller. It’s clear that $V_0(\phi = \pi/2, \theta = 0) > V_0(\phi = 0, \theta = 0 \text{ or } \theta = \pi) > V_0(\phi = \pi/2, \theta = \pi)$, although the penetrability is exponentially dependent on the potential, our calculations show that the indices are very small, so the penetrability is not simply dependent on the smaller angle-averaged potential. We have shown that $P_{ave}(\phi = 0) > P_{ave}(\phi = \pi/2)$ in Fig. 4b.

For $n_d = 11$, the difference of the potential for $\theta = \pi/2$ between $\phi = 0$ and $\phi = \pi/2$ becomes more and more clear, so a transition is expected. As shown in Fig. 3c, d, i, j, for $\theta = \pi/2$, $V_0(\phi = 0) > V_0(\phi = \pi/2)$, so we can see that $P_{ave}(\phi = 0) < P_{ave}(\phi = \pi/2)$ for $\varepsilon = 64$ keV in Fig. 4b. This result demonstrates that penetrability is mainly decided by the extremum of the electric fields. For fundamental frequency and second-harmonic generation lasers, the minimum and the maximum of the electric fields occur at $\phi = 0$ and $\phi = \pi/2$, respectively.

For $n_d = 18$, another transition occurs due to over-barrier effects, i.e., more and more angles achieve $P(\theta) = 1$, as shown in Fig. 3e, f, k, l. For $\theta = \pi/2$, we can see that $V_0(\phi = 0)$ and $V_0(\phi = \pi/2)$ are smaller than $\varepsilon = 64$ keV. So the angle-averaged potential is primarily decided by other $\theta$ that the weighted coefficient are smaller. Here, smaller angles have higher peaks and broader tunneling widths for
Fig. 7 For $\delta = 1$, contour plots of $(\sigma (\phi = \pi/2) - \sigma (\phi = 0))/\sigma (\phi = 0)$ for different $n_d$. a $1\omega - 2\omega$, and b $1\omega - 3\omega$.

$\phi = \pi/2$, so we get $P_{\text{ave}}(\phi = 0) > P_{\text{ave}}(\phi = \pi/2)$ in Fig. 4b.

5 Conclusion

In summary, we show that DT fusion cross sections can be enhanced depending on a dimensionless parameter $n_d$ in the presence of high-intensity dichromatic laser fields with high-frequency limit. For $1\omega - 2\omega$ fields, relative energy of 64 keV and $n_d = 18$, the fusion cross-section is approximately 8 times as large, and the maximum fusion cross-section can be enhanced to 63.05 barns corresponding to $\epsilon = 47$ keV, which is approximately 15 times that of the field-free case. In this situation we approximately estimate that the averaged fusion reactivity can be multiplied so that the Lawson criterion [46] might be reduced.

We find that there are twice shifts of the peak values for the phase-dependent cross sections whenever the frequency becomes sufficiently low or the intensity sufficiently high. And different strong dichromatic laser fields (i.e., $1\omega - 2\omega$ and $1\omega - 3\omega$) have different shifts.

In our model, the intense fields oscillate periodically. We know that this treatment is valid for ideal dichromatic intense lasers. In reality, typically XFEL pulses are not fully temporally coherent and have a spiky structure. This issue is not an easy task and will be discussed in detail in our future work.

However, when calculating energy balance, we ignore the power that need to create and maintain the super-strong electromagnetic fields. This issue will be discussed in detail in our future work. On the other hand, this work focuses on the high-frequency limit. Extending these discussions to the situation of relatively low-frequency is undergoing.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This is a theoretical study and there are no data associated to it.]

Appendix A

In this appendix, by comparison, we present how the average schemes are applied and what are the criteria for the averaging of atomic ionization, alpha decay and present work of DT fusion, respectively.

What we want to address is that DT fusion process is quite different to the alpha decay process as well as the atomic ionization: Even though all these models contain two essential processes of quantum tunneling and classical scattering processes, the time sequences of tunneling and classical scattering for the fusion are reversed comparing with other two models. This property makes the condition for averaged scheme of DT fusion also different.
Table 1  The model sketches and criteria for the averaging of atomic ionization, alpha decay and present work of deuteron-triton (DT) fusion, respectively

| Physical processes | Model sketches | Criteria of time-averaged approach |
|--------------------|----------------|-----------------------------------|
| Atomic ionization in intense laser fields with frequency of \( \omega \) | ![Atomic Ionization Diagram](image1) | \( \omega \gg \frac{I_p}{\hbar} [1] \), where \( I_p \) is ionization potential. According to quantum Floquet theory, details refer to Ref. [1] |
| Alpha decay process in intense laser fields with frequency of \( \omega \) | ![Alpha Decay Diagram](image2) | (1) \( \omega \gg \frac{Q_{\alpha}}{\hbar} [11] \), where \( Q_{\alpha} \) is relative energy of the emitted \( \alpha \) particle, called the Q value of the decay process. (2) \( \omega \gg \frac{2\pi}{\tau_{\alpha}} [15,16] \), where \( \tau_{\alpha} \) is the time required for \( \alpha \) particle to cross the barrier. Here, \( \tau_{\alpha} = \frac{\sqrt{2Q_{\alpha}/m_{\alpha}}}{r_{out} - r_{in}} \) where \( m_{\alpha} \) is the mass of \( \alpha \) particle. (3) \( \omega \gg \frac{2\pi}{T} [13,17] \), where \( T \) is the time the nucleus spends before decaying. This interval can vary from a few microseconds (\( \mu s \)) to years |
| DT fusion process in intense laser fields with frequency of \( \omega \) | ![DT Fusion Diagram](image3) | \( \omega \gg \frac{2\pi}{\Delta t} [19] \), where \( \Delta t \) is the Coulomb interaction time duration, which can be approximated by the ratio of interaction length to an average relative velocity. The Coulomb interaction time durations are ranged from 7.2 femtoseconds (fs) to 0.0072 femtoseconds (fs) for the relative energies of DT ranged from 1 keV to 100 keV, details refer to our manuscript |

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