Analysis of individual contribution of two compression Waves in vertical vibration of water-saturated soils

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Abstract. Based on the Biot’s dynamic consolidation equation, the individual contributions of these two compression waves in vertical vibration of water-saturated soils are analyzed. The equation belong to displacements of solid skeleton is established by using the decoupling of dynamic equation which is combined with boundary conditions to derive an analytic solution about displacements of solid skeleton under the excitation of the individual contributions of these two waves. And then, displacements of pore fluid and pressure of pore water under the excitation of the individual contributions of these two waves are obtained on the basis of that analytic solution. The attenuation of these two types of compression waves influenced by pore permeability is examined. And it should be mentioned that the contribution of the second compression wave is the main consideration when permeability coefficient is large. On the contrary, the contribution of different compression wave should be considered on the different soil column height when permeability coefficient is small.

1. Preface

It is known that the research on compression wave propagation in saturated soils began in the 1950s, and a propagation theory about fluid-saturated porous medium wave has been set up by Biot from 1956 to 1962 which successfully predicted the existence of two compression waves and a shear wave[1-4] corresponding to dynamic excitations in water-saturated soils. The three kinds of body wave predicted by Biot’s theory had been confirmed by experiments until 20 years later[5]. Some interesting issues have been studied from different angles by many scholars at home and abroad after Biot. Since 1957, water-saturated porous medium has been regarded as a kind of mixture. And the consolidation equation of water-saturated porous medium has been deduced[6,7] on the basis of mixture theory put forward by a group of mechanics, actually, there is no essential difference between mixture theory and Biot’s consolidation theory. The known results based on Biot’s dynamic consolidation theory basically consider the combination of two compression waves, on the contrary, it did not have a consideration about the individual contributions of these two waves. Based on the assumption that two types of waves function alone put forward by Jun Yang, some regularity understandings of vertical vibration problem in a special case for the individual contributions of two waves in the saturated soil column are obtained by linear decomposition method which is compared with that of the problem in a case for the joint of two compression waves and it pointed out that the attenuation regular pattern of two compression waves
when permeability coefficient is small, and it did not have an analysis when permeability coefficient is large[8].

Based on the generally adopted Biot’s dynamic consolidation equation for engineering in this paper, an analytic solution of vibration problem in a case for vertical dynamic loading in the saturated soil column is obtained by strict mathematical method. It will focus discussion on what is the influence of the permeability coefficient on the velocity of two compression waves and it pointed out that the permeability coefficient has a great influence on the velocity of two compression waves. It should be mentioned that, the contribution of the second compression wave is the main consideration when permeability coefficient is large, on the contrary, the contribution of different compression wave should be considered on the different soil column height when permeability coefficient is large.

The calculation model considered in this study is shown in Fig. 1. Soil layer is assumed to be a two-phase saturated medium of homogeneous isotropic in this study, the length of soil layer is taken as L, it is the rigid base of impermeable at the bottom of column and the surface of soil layer is free face. This model correspond to a soil layer is subjected to the simple harmonic vibration force whose circular frequency is ω and amplitude is σ0. Take a unit width of a water-saturated soil column on which both sides there is no horizontal displacement and seepage occurs only in vertical direction.

In the context of the Biot’s dynamic consolidation equation, the vertical movement equation for this one dimensional problem under axial symmetry cylindrical coordinate system can be written as

\[
(\lambda + 2G) \frac{\partial^2 u_z}{\partial z^2} - \alpha \frac{\partial p_t}{\partial z} = \rho \ddot{u}_z + \rho_i \ddot{w}_z
\]

(1)

The motion equation of fluid can be given as

\[-\frac{\partial p_t}{\partial z} = \frac{1}{k_d} \ddot{w}_z + \rho_i \ddot{u}_z + \frac{\rho_i}{n} \ddot{w}_z
\]

(2)

The soil seepage equation in a row can be written as

\[M \frac{\partial \dot{w}_z}{\partial z} + \alpha M \frac{\partial \ddot{u}_z}{\partial z} = -\dot{p}_t
\]

(3)

Where \(u_z\) and \(w_z\) are axial displacements of solid skeleton and pore fluid with respect to the solid phase; \(\rho, \rho_s, \rho_f\) are mass densities of two-phase saturated medium, solid skeleton and fluid, respectively; \(\rho = (1-n)\rho_s + n \rho_f\), \(n\) is porosity; \(\lambda, G\) are the Lamé’s constants; \(p_f\) is excess pore water pressure; \(k_d'\) is the dynamic permeability coefficient of saturated soil, \(k_d' = k_d/(\rho_f g)\), \(g\) is the acceleration of gravity; \(1/M = (\alpha-n)/K_s + n/K_f\), \(\alpha = 1-K_b/K_s, K_s, K_f, K_b\) are the bulk modulus of solid grains, fluid and solid skeleton.

The boundary conditions for the model considered are as follows:
The surface of column is stress-free and free draining
\[ \sigma_x(L) = -\sigma_0 e^{ikx}, \quad p_x(L) = 0 \]
Vertical displacement is zero at the bottom of the rigid foundation, and it does not drain
\[ u_z(0) = 0, w_z(0) = 0 \]

2. Equation solution
This model corresponds to a soil layer subjected to the steady-state harmonic vibration whose circular frequency is \( \omega \). With the joint of Eq. (2), Eq. (1) can be written as
\[ A_1 \frac{\partial^2 u_z}{\partial z^2} + B_1 u_z = w_z \]
where
\[ A_1 = (\lambda + 2G) / \left( \frac{\alpha \rho \omega^2}{n} - \rho_i \omega^2 - \frac{\alpha \omega}{k_d} \right), \quad B_1 = (\rho \omega^2 - \alpha \rho_i \omega^2) / \left( \frac{\alpha \rho \omega^2}{n} - \rho_i \omega^2 - \frac{\alpha \omega}{k_d} \right) \]

With the joint of Eq. (3) and Eq. (4), Eq. (2) can be written as
\[ \frac{\partial^4 u_z}{\partial z^4} + A_2 \frac{\partial^2 u_z}{\partial z^2} + B_2 u_z = 0 \]
where
\[ A_2 = (B_1 M + A_1 \frac{\rho_i \omega^2}{n} - \frac{1}{k_d} \omega^2 + \frac{A_1 i \omega}{A M}) / (A M), \quad B_2 = (\rho \omega^2 + B_1 \frac{\rho_i \omega^2}{n} - \frac{B_1 i \omega}{k_d}) / (A M) \]
The effective solution of Eq. (5) can be given as
\[ u_z = C_1 e^{\beta_1 z} + C_2 e^{\beta_2 z} + C_3 e^{-\beta_1 z} + C_4 e^{-\beta_2 z} \]
in which
\[ \beta_{1,2} = \sqrt{(-A_2 \pm \sqrt{A_2^2 - 4B_2}) / 2} \]

With the joint of Eq. (6), Eq. (4) can be written as
\[ w_z = C_1 A_1 \beta_1^2 e^{\beta_1 z} + C_2 A_1 \beta_2^2 e^{\beta_2 z} + C_3 A_1 \beta_1^2 e^{-\beta_1 z} + C_4 A_1 \beta_2^2 e^{-\beta_2 z} + C_1 B_1 e^{\beta_1 z} + C_2 B_1 e^{\beta_2 z} + C_3 B_1 e^{-\beta_1 z} + C_4 B_1 e^{-\beta_2 z} \]
With the joint of the following boundary condition:
\[ u_z = C_1 \text{sh}(\beta_1 z) + C_2 \text{sh}(\beta_2 z) \]
\[ w_z = C_1 (A_1 \beta_1^2 + B_1) \text{sh}(\beta_1 z) + C_2 (A_1 \beta_2^2 + B_2) \text{sh}(\beta_2 z) \]
\[ p_x = -MC_1 (A_1 \beta_1^2 + B_1) \beta_1 + \alpha MC_1 \beta_1 \text{ch}(\beta_1 z) - [MC_1 (A_1 \beta_2^2 + B_2) \beta_2 + \alpha MC_2 \beta_2] \text{ch}(\beta_2 z) \]
and \( p_x(L) = 0 \)
\[
C_i = \frac{[(A_i \beta_1^2 + B_i) \beta_1 + \alpha \beta_1] \text{ch}(\beta_1 L)}{[(A_i \beta_2^2 + B_i) \beta_2 + \alpha \beta_2] \text{ch}(\beta_2 L)} C_i
\]  

(11)

by using the following boundary condition:

\[
\sigma_z(L) = \lambda \frac{\partial u}{\partial z} = -\sigma_0 e^{i\omega}
\]

and the solution can be derived as

\[
C_i = \frac{\sigma_0}{\lambda \beta_i \text{ch}(\beta_i L)} \frac{(A_i \beta_1^2 + B_i) - (A_i \beta_2^2 + B_2)}{(A_i \beta_1^2 + B_1 + \alpha)}
\]  

(12)

From Eq. (8), Eq. (9) and Eq. (10), the authors come to the conclusion that the individual contributions from two types of compression waves are two added term of the equations above, respectively.

3. Analysis of the individual contributions from two types of compression waves

In this section, parameter analysis based on the theoretical results is presented to illustrate the characteristics of the individual contributions from the two compression waves. The properties, as listed in Table 1, are used in the analysis.

Table 1. Soil properties used in parameter analysis

| Property | Ks(GPa) | Kf(GPa) | ν (poisson ratio) | G(MPa) | ρf(kg/m³) | ρs(kg/m³) | ω (Hz) | L (m) | n |
|----------|---------|---------|------------------|--------|-----------|-----------|--------|-------|---|
| Value    | 36      | 2       | 0.3              | 40     | 1000      | 2700      | 5      | 15    | 0.4 |

Solid displacements in terms of individual contributions from two compression waves are taken as U1 and U2, respectively. Total displacement which is a dimension less displacement is taken as U0=uz/us when analyzing, where us=σpL/(λ+2G)

![Fig. 2. The effect of permeability coefficient on the contribution of the first compression wave](image-url)
Fig. 3. The effect of permeability coefficient on the contribution of the second compression wave.

It is easy to draw a conclusion that permeability coefficient has a big influence on the individual contribution of two compression waves from Fig. 2 and 3. Fig. 2 shows that: the real part of the first compression wave did not change when changes take place in permeability coefficient under the situation of \( k_d' < 10^{-6} \), and the imaginary part of the first compression wave did not change when changes take place in permeability coefficient under the situation of \( k_d' < 10^{-7} \). The amplitude of the contribution of the first compression wave increased with the increase of permeability coefficient when permeability coefficient is small. The reason is that saturated soil form a closed system and the contribution of the first compression wave approached to a definite value when permeability coefficient is large. Fig. 3 shows that: the contribution of the second compression wave basically approached to zero in the lower part of the soil column under the situation of \( k_d' < 10^{-7} \), the contribution of the second compression wave increased with the increase of height in the upper part of the soil column, the effect on the height of column move up with the decrease of permeability coefficient. The results are basically identical with literature.
Fig. 4 shows that: when permeability coefficient is small ($kd'<10^{-7}$), the real part of the first compression wave is obvious, the contribution of the second compression wave basically can be ignored when compared to the first one, and the imaginary part of two compression waves are both small in the lower part of soil column; Fig. 5 shows that: both the real and imaginary parts of the second compression wave have a big effect on soil column when permeability coefficient is large ($kd'>5\times10^{-7}$).

The real and imaginary parts of the first compression wave increased with the increase of the height of the soil column which are changing in a linear fashion when the first compression wave function alone, the real and imaginary parts of the second compression wave is relate to permeability coefficient which are changing in a linear fashion when permeability coefficient is large, the real part changed from positive to negative, the imaginary part increased with the increase of the height of soil column, and amplitude is also increasing when permeability coefficient is small.

4. Conclusions

Based on the generally adopted the Biot’s dynamic consolidation equation for engineering in this paper, an analytic solution of vibration problem in a case for vertical dynamic loading in the saturated soil column is obtained by strict mathematical method. It pointed out that the permeability coefficient has a great influence on the individual contribution of two compression waves.

When $kd'<10^{-7}$, the contribution of the second compression wave can be ignored in the lower part of soil column and the contribution of the second compression wave is the main consideration in the upper part of soil column. The real and imaginary parts of the first compression wave did not change when changes take place in permeability coefficient and the second compression wave is relate to permeability coefficient.

When $kd'>5\times10^{-7}$, the contribution of the second compression wave is the main consideration, while the contribution of the first compression wave can be ignored when compared to the second one.

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