We introduce electron-like and hole-like distribution functions, which determine the currents and the fluctuation spectra of the currents measured at a normal-conductor—superconductor heterostructure. These distribution functions are expressed with the help of newly defined partial densities of states for hetero-structures. Voltage measurements using a weakly coupled contact on such a structure show the absence of a contact resistance to the superconducting reservoir and illustrate how the interface to the superconductor acts as an Andreev mirror. We also discuss the current-current correlations measured at two normal contacts and argue that the appearance of positive correlations is a purely mesoscopic effect, which vanishes in the limit of a large number of channels and in the average over an ensemble.

PACS numbers: 73.20.At, 74.50.+r, 72.70.+m, 73.23.-b

I. INTRODUCTION

The properties of a phase-coherent normal conductor can be drastically changed by the presence of a nearby superconductor (N−S system). At the interface to the superconductor electrons with energies smaller than the superconducting gap are Andreev reflected and scattered back as holes, thus inducing superconducting behavior in the normal conductor. This process is known under the name proximity effect and has during the last few years extensively been studied experimentally and theoretically. In particular, Guéron et al. succeeded in measuring the opening of a gap in the local density of states of a normal conductor in the neighborhood of a superconductor using tunneling into a spatially extended contact. In the theoretical treatment, most often, the influence of the superconductor on a disordered normal conductor is studied using a semiclassical Green’s function technique. We investigate here phase-coherent N−S structures in a fully quantum-mechanical framework and show the influence of the superconductor on measurements done at a normal contact, which is weakly coupled to the hetero-structure. We give a quantum-mechanical expression of the particle and hole distribution functions of a structure that can be either ballistic, containing few scatterers (clean) or disordered and treated in the ensemble average. These expressions are based on newly defined partial densities of states for hybrid structures. One of the most striking effects is that the correlations of currents at two normal contacts can become positive due to the bosonic correlations induced in the normal conductor by the nearby superconductor. We present a geometry that leads to positive correlations and argue that such positive correlations are truly mesoscopic and will vanish if one goes to the limit of a large number of channels or performs averages over disorder.

II. THE EFFECTIVE SCATTERING MATRIX

We consider an N−S hetero-structure consisting of a normal conducting part connected to a superconductor. The superconductor represents one terminal whereas the normal conducting part is connected to N normal conducting terminals. In Fig. 1 such a structure with N = 2 normal contacts is drawn schematically. One of the normal contacts is only weakly coupled to the rest of the system just next to the interface to the superconductor. Such a setup permits to compare local properties of a normal wire connected to two normal reservoirs with the properties of a normal wire connected to one normal and one superconducting reservoir. An additional normal contact on a heterostructure has also been used by Mortensen et al. to investigate dephasing, thus generalizing Böttiker’s model based on dephasing reservoirs for heterostructures.

At applied potentials and temperatures much smaller than the gap of the superconductor, we are interested...
in the scattering properties of electrons injected through one of the normal contacts of an \( N - S \) hetero-structure as e. g. the one shown in Fig. 1. The normal part can be disordered and is described by the self-consistently calculated equilibrium electrostatic potential \( U(x) \). The Hamiltonian describing the normal part of the heterostructure can be written as a \( 2 \times 2 \) blockmatrix \( \hat{H} \), which acts in the combined particle and hole space (Nambu space),

\[
\hat{H} = \left( \begin{array}{cc} -\frac{\hbar^2}{2m} \nabla^2 + q^e U^e(x) & 0 \\ 0 & \frac{\hbar^2}{2m} \nabla^2 + q^h U^h(x) \end{array} \right). \tag{1}
\]

Here, we introduced the electron charge \( q^e = e \), the hole charge \( q^h = -e \) and the electron and hole potentials \( U^e(x) = U^h(x) = U(x) \).

Let us first consider the case, where the superconductor is replaced by a normal conductor. The Green’s function of the disordered part containing the coupling to the \( N + 1 \) contacts and the scattering matrix describing the scattering of particles between the \( N + 1 \) contacts can then be written in Nambu space as \( 2 \times 2 \) block-diagonal matrices \[16],

\[
\hat{G} = (E - \hat{H} + i\pi \hat{\Gamma})^{-1}, \tag{2}
\]
\[
\hat{U} = 1 - 2\pi i \hat{W}^\dagger \hat{G} \hat{W}. \tag{3}
\]

We extended the coupling matrix \( \hat{W} \) for electrons to a coupling matrix for electrons and holes

\[
\hat{W} = \begin{pmatrix} W & 0 \\ 0 & W^* \end{pmatrix}, \tag{4}
\]

and \( \hat{\Gamma} = \hat{W} \hat{W}^\dagger \). The Green’s function \( \hat{G} \) and the scattering matrix \( \hat{U} \) are block-diagonal matrices. In a purely normal-conducting system there is no mixing between particle and hole space.

The presence of a superconductor mixes now these two spaces. Incoming particles can be reflected as holes and vice versa. We assume that the interface between the normal conductor and the superconductor is perfectly transparent. All disorder is contained in the normal part of the system. The scattering between particles and holes at the interface is at the Fermi energy (the chemical potential of the condensate in the superconductor) described by the Andreev-reflection matrix \[4]

\[
R_A = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}. \tag{5}
\]

This is a \( 2 \times 2 \) block-matrix with each block being a diagonal matrix of the size of the number of channels at the interface. Putting the scattering matrix \( \hat{U} \) of the normal region together with the Andreev reflection matrix gives the effective scattering matrix

\[
\hat{S} = \begin{pmatrix} S^{ee} & S^{eh} \\ S^{he} & S^{hh} \end{pmatrix}, \tag{6}
\]

describing the scattering between the \( N \) normal contacts. We now divide the scattering matrix

\[
\hat{\bar{U}} = \begin{pmatrix} \bar{U}_{00} & \bar{U}_{01} \\ \bar{U}_{10} & \bar{U}_{11} \end{pmatrix} \tag{7}
\]
in such a way that the index 0 denotes the electron and hole channels at the superconducting interface and the index 1 denotes all channels of all \( N \) normal contacts. The effective scattering matrix can then be written in the following compact form,

\[
\hat{S} = \bar{U}_{11} + \bar{U}_{10}(1 - R_A \bar{U}_{00})^{-1} R_A \bar{U}_{01}. \tag{8}
\]

This effective scattering matrix is now not anymore diagonal, but contains sub-matrices \( S^{ee} \) and \( S^{hh} \) which describe scattering of incoming particles which are reflected as holes and incoming holes reflected as particles. It can be shown that we can also express this effective scattering matrix in the same form as in Eqs. \[2] and \[3],

\[
\hat{S} = 1 - 2\pi i \hat{W}_1^\dagger \hat{G}_{eff} \hat{W}_1, \tag{9}
\]
\[
\hat{G}_{eff} = (E - \hat{H}_{eff} + i\pi \hat{\Gamma}_{1})^{-1} \tag{10}
\]

by introducing the effective Hamiltonian \[17]

\[
\hat{H}_{eff} = \begin{pmatrix} H & -\pi W_0 W_0^\dagger \\ -\pi (W_0 W_0^\dagger)^* & -H^* \end{pmatrix}, \tag{11}
\]

and where we divided the coupling matrix \( W \) up into a part \( W_0 \) describing the coupling to the superconducting contact and a part \( W_1 \) describing the coupling to all normal contacts. We now define the fundamental local partial densities of states for an \( N - S \) hybrid structure in analogy to the definition for normal systems in \[18] as

\[
\nu(\alpha, \nu, \beta_{\lambda}) = -\frac{1}{4\pi i} \text{Tr} \left\{ \left( \begin{array}{c} s^{\lambda \lambda}_{\alpha \beta} \end{array} \right)^\dagger \frac{\delta s^{\mu \lambda}_{\alpha \beta}}{q^\nu \delta U^\nu(x)} \right\}.
\]

(12)

Here, the indices \( \alpha \) and \( \beta \) denote the normal contacts of the system and the indices \( \mu, \nu \) and \( \lambda \) distinguish between electrons and holes. As an example, \( \nu(2_h, x_c, 1_c) \) gives the electronic density at point \( x \) of an electron that entered the system through contact 1 and left the system as a hole in contact 2. The usefulness of such a definition will become clear soon and is also shown in connection with charge fluctuations of a heterostructure in \[19\]. We proceed by defining the electron like injectivity of a contact \( \beta \) as

\[
\nu(x_c, \beta_{\lambda}) = \sum_{\alpha, \mu} \nu(\alpha, x_c, \beta_{\lambda}) \tag{13}
\]
and the hole-like injectivity as

$$\nu(x_h, \beta_e) = \sum_{\alpha, \mu} \nu(\alpha, x_h, \beta_e). \quad (14)$$

The electron-like injectivity gives the electronic density inside the conductor of an incoming electron from the reservoir and the hole-like injectivity gives the hole density of an incoming electron. Expressed with the help of the effective Green’s function $G_{\text{eff}}$ these densities are

$$\nu(x_e, \beta_e) = \langle x | G_{\text{eff}}^{ee}(G_{\text{eff}}^{ee})^\dagger | x \rangle, \quad (15)$$

$$\nu(x_h, \beta_e) = \langle x | G_{\text{eff}}^{he}(G_{\text{eff}}^{he})^\dagger | x \rangle. \quad (16)$$

In the same way, we can define electron- and hole-like emissivities of a contact $\alpha$, $\nu(\alpha, x_d)$, by summing the local partial density $\nu(\alpha, x_d, \beta_0)$ over the incoming contacts $\beta_0$. In the presence of a magnetic field $B$ we have the following symmetry relations,

$$\nu_B(x_\mu, 1_\nu) = \nu_B(1_\nu, x_\mu), \quad (17)$$

$$\nu_B(x_e, 1_e) = \nu_B(x_h, 1_h), \quad (18)$$

$$\nu_B(x_h, 1_e) = \nu_B(x_e, 1_h). \quad (19)$$

The local density of states at a point $x$ is the sum

$$\nu(x) = \frac{1}{2} \sum_{\beta, \mu, \lambda} \nu(x_\mu, \beta_\lambda)$$

$$= -\frac{1}{2\pi} \text{Im} \{ G_{\text{eff}}^{ee}(x, x) + G_{\text{eff}}^{he}(x, x) \}. \quad (20)$$

The electronic and hole-like injectivity can also be expressed with the help of scattering states. In the presence of superconductivity, the wavefunction $\psi$ of an incoming electron consists of two components, one being the electron wavefunction $\psi^e$ and one being the hole wavefunction $\psi^h$. The amplitude of the scattering state $\psi_{\beta n}$ describing an incoming electron in channel $n$ of contact $\beta$ is then at a point $x$

$$\psi_{\beta n}(x) = \begin{pmatrix} \psi^e_{\beta n}(x) \\ \psi^h_{\beta n}(x) \end{pmatrix}. \quad (21)$$

The injectivities are proportional to the absolute squares of the electron and hole wavefunction,

$$\nu(x_e, \beta_e) = \sum_{n \in \beta} \frac{1}{\hbar t_{\beta n}} |\psi^e_{\beta n}(x)|^2, \quad (22)$$

$$\nu(x_h, \beta_e) = \sum_{n \in \beta} \frac{1}{\hbar t_{\beta n}} |\psi^h_{\beta n}(x)|^2. \quad (23)$$

With the help of the symmetry relations \([17] - [19]\) the emissivities can be expressed using the scattering states of the Hamiltonian with the reversed magnetic field.

### III. THE DISTRIBUTION FUNCTIONS

We consider now an $N - S$ structure to which on the normal conductor a local tunneling contact is added as shown in figure \([1]\). We have therefore a setup with three contacts, two of which are normal and one superconducting contact. At small temperatures and applied potentials $\mu_\alpha$ measured relative to the electro-chemical potential of the superconductor the average current at a contact $\alpha$ is \([20]\)

$$\langle I_\alpha \rangle = \sum_\mu \langle I_\mu_\alpha \rangle$$

$$= \sum_\mu \frac{q_\mu}{\hbar} \int_0^\infty dE \sum_{\beta \nu} T^{\mu \nu}_{\alpha \beta} \left[ f^\mu_\alpha(E) - f^\nu_\beta(E) \right]. \quad (24)$$

The indices $\mu$ and $\nu$ distinguish between electrons and holes and $\alpha$ and $\beta$ label the normal contacts. The normal and Andreev transmission and reflection probabilities $T^{\mu \nu}_{\alpha \beta}$ are calculated from the effective scattering matrix $\hat{S}$,

$$T^{\mu \nu}_{\alpha \beta} = \text{Tr} \left[ s^{\mu \nu}_\alpha (s^{\nu \mu}_\alpha)^\dagger \right]. \quad (25)$$

The distribution functions of electrons and holes $f^{\nu/h}_\alpha(E)$ injected from a normal reservoir $\alpha$ are

$$f^{\nu}_\alpha(E) = f_0(\mu_\alpha - E) \quad \text{and} \quad f^{h}_\alpha(E) = f_0(-\mu_\alpha - E) \quad (26)$$

with the Fermi distribution function $f_0(\epsilon) = (\exp(-\epsilon/kT) + 1)^{-1}$. Note, that in Eq. \(24\) we integrate only from zero, the electro-chemical potential of the superconductor, to infinity in order to avoid double counting. In the absence of a superconductor (i.e. replacing the Andreev reflection by normal reflection) the current formula \(24\) reduces to the well known equation for purely normal systems \([2]\).

For temperatures and potential differences that are much smaller than the superconducting gap, we can neglect the energy dependence of the Andreev reflection of the superconductor, to infinity in order to avoid double counting. In the absence of a superconductor (i.e. replacing the Andreev reflection by normal reflection) the current formula \(24\) reduces to the well known equation for purely normal systems \([2]\).

For temperatures and potential differences that are much smaller than the superconducting gap, we can neglect the energy dependence of the Andreev reflection of the superconductor, to infinity in order to avoid double counting. In the absence of a superconductor (i.e. replacing the Andreev reflection by normal reflection) the current formula \(24\) reduces to the well known equation for purely normal systems \([2]\).

The current at the tunneling contact we need the transmission probabilities from and to the massive contact to first order in the coupling constant $t$,

$$T^{\mu \nu}_{1,tip} = 4\pi^2 \nu_{tip} |t|^2 \nu(x_\mu, 1_\nu), \quad (27)$$

$$T^{\mu \nu}_{1,tip} = 4\pi^2 \nu(1, x_\nu) |t|^2 \nu_{tip}. \quad (28)$$

Here, $\nu_{tip}$ is the density of states at the tip. Using these transmission probabilities we can rewrite Eq. \(24\) in the form of a two-terminal current,

$$\langle I_{tip} \rangle = \frac{e}{h} \int_0^\infty dE \langle G \rangle \left[ \langle f^{\mu}_{\text{tip}}(E) - f^{\nu}_{\text{eff}}(E) \rangle \right.$$\(29\)

$$- \langle f^{\nu}_{\text{tip}}(E) - f^{\mu}_{\text{eff}}(E) \rangle \right].$$

Here, we defined the effective distribution of electrons...
\[ f_{\alpha}^\text{eff}(E) = \frac{\nu(x, \beta_\alpha)}{\nu(x)} f^\beta_\alpha(E) \] (30)

and of holes
\[ f_{\alpha}^h(E) = \frac{\nu(x_h, \beta_\alpha)}{\nu(x)} f^h_\beta(E) \] (31)

at a point \( x \) inside the conductor. Due to the possibility of Andreev reflection, injected electrons can contribute to the distribution of holes and injected holes can contribute to the distribution of electrons. Equations (29)-(31) are central results of this paper and represent a generalization of similar formulas developed in [22] for purely normal conducting systems.

At zero temperature the distribution function of electrons and holes can be replaced by step functions, taking into account the possibility of Andreev reflection. Injected electrons can contribute to the distribution of holes and injected holes can contribute to the distribution of electrons. Equations (29)-(31) are then written as
\[ I_{\text{tip}} = \frac{e^2}{\hbar} \left( N_{\text{tip}} - T_{\text{tip},\text{tip}}^{\text{ee}} + T_{\text{tip},\text{tip}}^{\text{he}} \right) \mu_{\text{tip}} \]
where the Andreev-reflection probability is
\[ T_{\text{tip},\text{tip}}^{\text{he}} = \frac{T_{\text{tip},\text{tip}}^{\text{ee}} \mu_{\text{tip}}}{\hbar} \] (37)

If the hole-like injectivity is zero, i. e. the electrons don’t see the superconductor, the tunneling tip measures the electro-chemical potential \( \mu_1 \) of contact 1. In the following chapter we will discuss further examples.

### A. Examples

We use Eq. (25) to investigate some \( N - S \) structures where one normal contact is connected to a superconductor. The interface between the normal conductor and the superconductor is always perfectly transparent, while the normal conductor can be ballistic or contain scattering regions.

First we consider a perfect ballistic normal conductor. Since all electrons that enter the conductor propagate to the superconducting interface, they are Andreev reflected as holes, the electron- and hole-injectivities are everywhere in the ballistic region identical. We have \( \nu(x, 1_e) = \nu(x_h, 1_e) \) and therefore the measured potential at the tip \( \mu_{\text{tip}} \) is always zero, i. e. equal to the chemical potential of the superconductor.

Next, we consider a normal one-channel conductor which contains a scattering region. The scattering region is characterized by a scattering matrix leading to the transmission probability \( T \) and reflection probability \( R \). In between the scattering region and the interface to the superconductor electron- and hole-injectivities are the same, so that a voltage probe measures in this region always the potential of the superconductor. To the left of the scattering region, away from the superconductor, the electron- and hole-injectivities are given by the normal-and Andreev-reflection probabilities,
\[ \nu(x, 1_e) = 1 + T_{\text{ee}} \quad \text{and} \quad \nu(x_h, 1_e) = T_{\text{he}}, \] (38)

where the Andreev-reflection probability is
\[ T_{\text{he}} = \frac{T^2}{(2 - T)^2} \] (39)

and the normal reflection is \( T_{\text{ee}} = 1 - T_{\text{he}} \). In calculating the injectivities above we neglected the fast oscillating interference terms between incoming and reflected waves. Using these injectivities, the measured potential, Eq. (37), to the left of the scatterer is
\[ \mu_{\text{tip}} = T_{\text{ee}} \mu_1 = \frac{4(1 - T)}{(2 - T)^2} \mu_1. \] (40)

The voltage drop from the normal reservoir to the left side of the scattering region is \( \mu_1 - \mu_{\text{tip}} = T_{\text{he}} \mu_1 \). If we divide this voltage drop by the current \( I = 2(e/\hbar)T_{\text{he}} \mu_1 \) flowing through the sample we get the contact resistance
\[ R_C = \frac{\mu_1 - \mu_{\text{tip}}}{eI} = \frac{\hbar}{2e^2}. \] (41)
Since one measures on the right side of the scattering region always the chemical potential of the superconductor, there is no contact resistance on the superconducting side. The total contact resistance of an \( N - S \) structure is therefore half as big as the one for a purely normal system. This is in agreement with the findings in Refs. [23] and [24].

As a last example we consider a metallic diffusive conductor in the ensemble average. The diffusive region extends from \( x = 0 \) to \( x = L \), where it is in contact with a superconductor. Its length \( L \) is much bigger than its width \( W \) so that it is justified to treat the diffusion to be effectively one-dimensional in the ensemble average. We have to find the electron- and hole injectivity. For this we have to solve the differential equations

\[
\frac{d^2}{dx^2} \nu(x_e, 1_e) = 0 \quad \text{and} \quad \frac{d^2}{dx^2} \nu(x_h, 1_e) = 0
\]

with the boundary conditions \( \nu(x = 0_e, 1_e) = \nu_0 \) and \( \nu(x = 0_h, 1_e) = 0 \), where \( \nu_0 = m^* / 2\pi \hbar^2 \) is the two dimensional density of states. In addition, due to the transparent interface and the perfect Andreev reflection we have \( \nu(x = L_e, 1_e) = \nu(x = L_h, 1_e) \) and due to particle number conservation we have \( \nu(x_e, 1_e) + \nu(x_h, 1_e) = \nu_0 \). Solving the differential equations using the boundary conditions we get the injectivities

\[
\nu(x_e, 1_e) = \nu_0 \left(1 - \frac{x}{2L}\right) \quad \text{and} \quad \nu(x_h, 1_e) = \nu_0 \frac{x}{2L}.
\]

Note here, that these densities look like the densities of a diffusive conductor of length \( 2L \) connected to two normal contacts [22]. The potential \( \mu_{\text{tip}}(x) \) measured at a position \( x \) along the diffusive conductor is

\[
\mu_{\text{tip}}(x) = \left(1 - \frac{x}{L}\right) \mu_1,
\]

which means, that there is a linear voltage drop along the diffusive conductor from \( \mu_1 \) on the left side to zero, the electro-chemical potential of the superconductor, at the interface. Thus, attaching a superconductor to a normal diffusive wire does not change the voltage measurement along the wire.

IV. CURRENT FLUCTUATIONS AND POSITIVE CORRELATIONS

In this section we investigate the time dependent current measured at a normal contact in the presence of superconductivity. The system we consider consists of several normal contacts and one superconducting contact. In order to describe the time dependent current it is useful to use the formulation of second quantization. We closely follow the lines of Ref. [21], where the theory was developed for purely normal conducting systems. The essential point in the presence of superconductivity is, that at energies which are small compared to the superconducting gap, there are no propagating quasiparticles inside the superconductor. The superconductor influences the normal part of the system only by reflecting electrons as holes and vice versa and generating a supercurrent inside the superconductor. A similar derivation of current correlations for multiterminal \( N - S \) hybrid structures has already been given by Anantram and Datta in Ref. [11]. Statistical particle counting arguments have been used by Martin in [12].

First we need the current operator \( I_\alpha(t) \) for the current in lead \( \alpha \). The current consists of two parts, the current carried by the electrons, \( I^e_\alpha(t) \), and the current carried by the holes, \( I^h_\alpha(t) \). The total current is the sum of these two currents which are given by

\[
\dot{I}^\alpha(t) = \frac{q\mu_0}{\hbar} \int_0^\infty dE dE' \sum_{\nu} (\hat{\mathbf{a}}^\nu_0) \langle E \rangle A_{\beta\nu,\gamma\lambda}(\alpha\mu; E, E') \hat{\mathbf{a}}^\lambda(E')
\]

\[\times \exp[i(E - E')t / \hbar].\]

with the current matrix

\[
A_{\beta\nu,\gamma\lambda}(\alpha\mu; E, E') = \delta_{\beta\gamma} \delta_{\nu\lambda} \delta(\alpha\mu) - (\delta^\mu_\alpha) \langle E \rangle a^\lambda_\gamma(E').
\]

The indices \( \alpha, \beta \) and \( \gamma \) enumerate the normal leads and the indices \( \mu, \nu, \lambda = e, h \). All energies have to be measured relative to the electro-chemical potential of the superconductor. We can now continue like in Ref. [21] to find the low-frequency spectrum of the current correlations [11].

\[
\langle \Delta I^\mu_\alpha \Delta I^\nu_\beta \rangle = \frac{2q^\mu q^\nu}{\hbar} \sum_{\delta \kappa} \int_0^\infty dE \text{Tr} \left[ A_{\gamma\lambda,\delta\kappa}(\alpha\mu) A_{\delta\kappa,\gamma\lambda}(\beta\nu) \right]
\]

\[\times f^\gamma_\nu(E) (1 - f^\delta_\lambda(E)).\]

In this expression both energy arguments in the current matrix are equal to \( E \) and the distribution functions are the ones given in Eq. (26). Eq. (47) is in fact equivalent to Eq. (1.16) of Ref. [21] if we keep in mind that, in the superconducting case, we replaced the scattering matrix between \( N + 1 \) normal contacts by an effective scattering matrix between \( 2N \) contacts (contacts for electrons and holes counted separately) labelled by pairs of indices \( \alpha\mu \). As the normal \((N + 1) \times (N + 1)\) scattering matrix, the effective \( 2N \times 2N \) scattering matrix is unitary. The integral in Eq. (47) goes only from zero to infinity because we add particle and hole excitations.

Now we can apply this formula to an \( N - S \) system which has an additional local tunneling contact, called tip (c.f. figure 1). The fluctuations of the currents at the tip are

\[
\langle (\Delta I_{\text{tip}})^2 \rangle = \langle (\Delta I_{\text{tip}}^e)^2 \rangle + \langle (\Delta I_{\text{tip}}^h)^2 \rangle + 2 \langle \Delta I_{\text{tip}}^e \Delta I_{\text{tip}}^h \rangle.
\]
We have to evaluate the current matrix using the effective scattering matrix for this system. To the lowest order in the coupling strength $t$ of the tip this gives

$$
\langle (\Delta I_{\text{tip}})^2 \rangle = \frac{2e^2}{h} 4\pi^2 \mu_{\text{tip}} |t|^2 \left[ \nu(x_e, 1_e) (\mu_1 - \mu_{\text{tip}}) + \nu(x_e, 1_h) \mu_{\text{tip}} + \nu(x_h, 1_e) \mu_1 \right].
$$

If we chose the electrochemical potential at the tip according to Eq. (43) such that there is on average zero current flowing into the tip, we can write for the current fluctuations at the tip

$$
\langle (\Delta I_{\text{tip}})^2 \rangle = 8eG_0 V \frac{\nu(x_e, 1_e)}{\nu(x)} \left( 1 - \frac{\nu(x_e, 1_e)}{\nu(x)} \right)
$$

with $G_0 = \left( e^2 / h \right) 4\pi^2 \nu_{\text{tip}} |t|^2 \nu(x)$ and the applied bias $eV = \mu_1$.

### A. Examples of current fluctuations

In a ballistic conductor electron- and hole-injectivity are identical so that the fluctuations at the tip become,

$$
\langle (\Delta I_{\text{tip}})^2 \rangle = 2eG_0 V.
$$

We compare this result with the case where the superconductor is replaced by a normal conductor found in\(^\text{[22]}\), \(\langle (\Delta I_{\text{tip}})^2 \rangle = eG_0^N V\), with the conductance \(G_0^N = \left( e^2 / h \right) 4\pi^2 \nu_{\text{tip}} |t|^2 \left[ \nu(x_e, 1_e) + \nu(x_e, 0) \right]\). Switching on superconductivity thus doubled the noise measured at the weakly coupled contact. A doubling of the fluctuations of the current at a normal ballistic wire which is connected to a superconductor is also found in\(^\text{[23]}\) and\(^\text{[24]}\).

The next example is a normal conductor with a scatterer of transmission probability $T$. Using the electron- and hole-injectivity for such a system to the left of the scatterer and neglecting phase coherence, Eqs. (48), gives the noise spectrum,

$$
\langle (\Delta I_{\text{tip}})^2 \rangle = 2eG_0 V T^{\text{he}} (2 - T^{\text{hr}}).
$$

Again, comparing this expression to the result for a normal wire\(^\text{[22]}\) shows the appearance of a factor of two in the superconducting case and the appearance of the Andreev reflection probability $T^{\text{he}}$ instead of the normal transmission probability $T$.

The last and most interesting example is the metallic diffusive wire. Inserting the corresponding injectivities, Eqs. (45), into the expression for the current fluctuations yields

$$
\langle (\Delta I_{\text{tip}})^2 \rangle = 8eG_0 V \frac{x}{2T} \left( 1 - \frac{x}{2T} \right).
$$

This can be compared with the one obtained for a purely normal conductor, \(\langle (\Delta I_{\text{tip}})^2 \rangle = 4eGV \frac{x}{2T} (1 - \frac{x}{2T})\). Whereas the voltage measurement showed the same linear voltage drop over a diffusive region in the purely normal case as well as in the case where the diffusive part is in contact with a superconductor, the fluctuation spectrum measured at the tip shows a different behaviour. The fluctuations are maximal at the interface $x = L$ and vanish at the contact to the normal reservoir $x = 0$. This can be understood, if we think of the interface to the superconductor as a mirror thus representing the middle of a fictitious wire of length $2L$ which continues after the interface into the superconductor\(^\text{[3]}\). However, at the interface to the superconductor, the measured fluctuations are \(\langle (\Delta I_{\text{tip}})^2 \rangle = 2eG_0 V\), twice as large as the fluctuations measured in the middle of a purely normal wire. We could also compare the normal wire of length $2L$ with a wire of length $L$ connected to a superconductor. Then, on the first half of the purely normal wire ($0 < x < L$), the correlations of the purely normal and the hybrid structure differ in exactly a factor of two.

### B. Examples of current cross-correlations

In this section we investigate the correlations of the currents measured at two normal contacts of a normal-superconducting hybrid structure starting from equation\(^\text{[17]}\). The system we consider is again shown in figure 1. For purely normal systems, the correlations of fermions are due to their exclusion statistic always negative. However, the presence of a nearby superconductor induces bosonic correlations between the electrons in a normal conductor. It was therefore shown by analytic calculations in\(^\text{[11,12]}\) that the bosonic character of electron pairs can in principle lead to positive correlations of the currents at two normal contacts. In\(^\text{[1]}\) one could in fact find through numerical investigation positive correlations at two normal contacts, which were sandwiched in between a ring shaped superconductor representing the third terminal. Very recently, positive correlations were found for a system consisting of a wave splitter connected to a superconductor\(^\text{[13]}\). We show here in an analytic calculation that one can get positive correlations on a system where one of the two normal contacts is only weakly coupled. The calculation presented below sheds light on the truely mesoscopic origin of these positive correlations. The existence of positive correlations is in contrast to the recently measured negative correlations on purely normal conductors by Henny et al.\(^\text{[25]}\) and Oliver et al.\(^\text{[29]}\).

The contact 1 is held at the same potential as the superconductor whereas a bias $V$ is applied at the tip. We restrict ourselves to the case of zero temperature. Then only the Fermi distribution function \(f_2^f(E)\) of the electrons in the tip is different from zero,

$$
f_2^f(E) = \theta(eV - E),
$$

all other distribution functions vanish (One has to keep in mind that we are only interested in the distribution function for energies which are larger than the electrochemical potential of the superconductor). We have
now to evaluate the general formula for the current-correlations using these Fermi functions. We expand the effective scattering matrix for scattering between the two normal contacts in powers of the coupling energy $t$ and get the following correlations,

\[ \langle \Delta I^x_1 \Delta I^\nu_{tip} \rangle = -\alpha 4\pi^2 \nu_{tip} |t|^2 \nu(x_e, 1_e), \]  
\[ \langle \Delta I^\nu_1 \Delta I^x_{tip} \rangle = \alpha 4\pi^2 \nu_{tip} |t|^2 \nu(x_e, 1_h), \]  
\[ \langle \Delta I^x_1 \Delta I^x_{tip} \rangle = \langle \Delta I^\nu_1 \Delta I^\nu_{tip} \rangle = 0, \]  

with $\alpha = 2^2 \hbar eV$. The total correlation of the currents at contact 1 and 2 is the sum of all four terms. In the absence of a magnetic field, the correlations are proportional to the injected charge density $q(x) = \nu(x_e, 1_e) - \nu(x_h, 1_e)$,

\[ \langle \Delta I_1 \Delta I_{tip} \rangle = -2e^2 \hbar V 4\pi^2 \nu_{tip} |t|^2 q(x) \]
\[ = -2eG_0V \frac{q(x)}{p(x)} \]

with the conductance $G_0$ defined as for equation (50). This means, that if at the point $x$ the hole density of in contact 1 injected electrons is larger than the electron density, the net injected charge density becomes negative and therefore, the correlations become positive. The electron and hole injectivity are proportional to the absolute squared value of the corresponding wavefunction amplitude. For a system consisting of a barrier in the normal conducting part the wavefunction of an injected electron is

\[ \psi_{1e}(x_e) = e^{ikx} + r^{ee} e^{-ikx}, \]
\[ \psi_{1e}(x_h) = \psi^{he} e^{-ikx}. \]

Here, $r^{ee} = \sqrt{R^{ee}}$ is the normal reflection amplitude and $r^{he} = \sqrt{R^{he}}$ is the Andreev reflection amplitude. For simplicity, we chose these two amplitudes to be real. The injected charge density at a point $x$ is then

\[ q(x) = \frac{2}{\hbar e} \left\{ R^{ee} + \sqrt{R^{ee}} \cos(2kx) \right\}. \]

One sees immediately, that, since $0 \leq R^{ee} \leq 1$, one can find for every $R^{ee}$ positions where the charge density is negative and the measured correlations become positive. Note, however, that this is only the case if one respects the phase coherence of incoming and reflected wave (as it is done in the above calculation). If phase coherence is neglected, the cosine term disappears and the injected charge is always positive. (Remember that we defined the charge density $q(x)$ such that it is positive if there is a net electronic charge in the system and negative if there is a net hole charge.) Similarly, the cosine term averages out if one considers a phase-coherent conductor with $N \gg 1$ open channels. If we assume to have no interchannel scattering and that the reflection amplitudes $R_{m}^{ee} \equiv R$ are independent of the channel index $m$, the charge density at a point $x$ is

\[ q(x) = \sum_{m=1}^{N} \frac{2}{\hbar \nu_{m}} \left[ R \sqrt{R^{ee}} \cos(2k_{m}x - \delta_{m}) \right], \]

with the scattering phase shifts $\delta_{m}$. It is easily seen, that the cosine terms will cancel out for randomly distributed phase shifts, thus destroying the possibility of positive correlations. Neither does one get positive correlations for a metallic diffusive wire in the ensemble average using the above derived densities. Positive correlations are therefore, at least in the setup described here, a truly mesoscopic effect and can only be expected to be seen on one or few channel conductors that preserve the phase coherence. Up to now all examples exhibiting positive correlations were perfectly phase coherent one channel conductors in the neighborhood of a superconductor. Furthermore, it is instructive to note that Ref. [11] finds a sign change of the correlations as a function of the Aharonov-Bohm flux, which determines the phase difference at the two interfaces to the superconductor. This also points to a purely mesoscopic effect. Thus the picture which emerges is that in hybrid systems there is a mesoscopic effect of order $1/N$, which can give a positive contribution in correlations. The main part (of order 1) has the sign observed in normal conductors.

V. CONCLUSIONS

We have used the effective scattering matrix to define fundamental partial densities of states, Eq. (12), for systems containing normal and superconducting parts. These partial densities and the injectivities and emissivities constructed from them are shown to be very useful in the description of current and current fluctuations and correlations measured at the normal contacts of a hybrid structure. The current at a tunneling tip is sensitive to the effective distributions of electrons and holes inside a normal multiprobe conductor to which one superconducting reservoir is attached. The local distributions are evaluated for conductors containing a single scatterer and for disordered conductors in the ensemble average. We also gave an example illustrating the possibility of positive correlations of the currents at two normal contacts due to induced bosonic behaviour. However, we reasoned that positive correlations are a mesoscopic $(1/N)$ effect, which experimentally can only be expected to be observable on conductors containing very few open channels.

VI. ACKNOWLEDGMENT

We thank Andrew M. Martin for useful discussions. This work was supported by the Swiss National Science foundation.
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