How to identify the structure of near-threshold states from the line shape

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We revisit the compositeness theorem proposed by Weinberg in an effective field theory (EFT) and explore criteria which are sensitive to the structure of S-wave threshold states. On a general basis, we show that the wave function renormalization constant $Z$, which is the probability of finding an elementary component in the wave function of a threshold state, can be explicitly introduced in the description of the threshold state. As an application of this EFT method, we describe the near-threshold line shape of the $D^{*0}D^0$ invariant mass spectrum in $B \to D^{*0}D^0 K$ and determine a nonvanishing value of $Z$. It suggests that the $X(3872)$ as a candidate of the $D^{*0}D^0$ molecule may still contain a small $cc$ core. This elementary component, on the one hand, explains its production in the $B$ meson decay via a short-distance mechanism, and on the other hand, is correlated with the $D^{*0}D^0$ threshold enhancement observed in the $D^{*0}D^0$ invariant mass distributions.

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I. INTRODUCTION

Recently the observations of many new resonances, namely the so-called $XYZ$ states, have initiated intensive studies of their properties in both experiment and theory. An interesting feature about most of these new resonances is that their masses are generally close to $S$-wave two-particle thresholds and their coupling to the corresponding $S$-wave two-particle channel is important. For example, the famous $X(3872)$ is one of the earliest observed state correlated to the $D^{*0}D^0$ threshold. We use the notation $D^{*0}D^0$ to denote both $D^{*0}D^0$ and $D^{*0}D^0$ states. The structure of these near-threshold resonances are still under debating and the existing theoretical interpretations are also various which include proposals for treating them as either conventional quark-antiquark states or QCD exotics such as tetraquarks, hybrid states, dynamically generated states or molecular states. In some scenarios, they are treated as mixing states of the above mentioned configurations. It is worth mentioning that the recent experimental signals for charged quarkonium states $Z_b(10610)$ and $Z_b(10650)$ [1] and their analogues in the charmonium sector $Z_c(3900)$ [2–4] and $Z_c(4020/4025)$ [5, 6] which appear to be strongly correlated to the thresholds of either $B^{(*)}$ or $D^{(*)}$ pairs. Since most of those newly observed states are in the vicinity of an $S$-wave open threshold, a theoretical method to distinguish whether such a near-threshold state is an elementary state of overall color singlet or a composite state consisting of open channel hadrons as constituents is thus crucial for our understanding of their nature. Although such an issue has been explored since long time ago and by many theorists (e.g. one can refer to early literature of Refs. [7–10] and recent review [11] and references therein), our knowledge about such non-perturbative phenomena is still far from complete.

The aim of this present work is to develop an effective field theory approach to identify the structure of the near threshold states and explore its applications on the structures of the recently discovered $XYZ$ states. In particular, we shall study the $X(3872)$ which, since its first observation by Belle collaboration in $B^\pm \to K^{\pm}\pi^+\pi^-J/\psi$ [11], has initiated tremendous interests in both experiment and theory. Nevertheless, due to the small mass difference between the measured mass of the $X(3872)$ and the $D^{*0}D^0$ threshold, it makes the $X(3872)$ the best candidate for an $S$-wave $D^{*0}D^0$ molecule [13, 19]. However, it has also been recognized that the structure of the $X(3872)$ could be rather profound because its large production rates in the $B$-factories and at Tevatron seem to favor a compact structure in its wave function rather than a loosely bound molecular state [20, 22]. Taking both the production and decay properties into account, it seems reasonable to identify the $X(3872)$ to be a mixing state between the $J^{PC} = 1^{++}$ $cc$ component and the $D^{*0}D^0$ component [20, 25]. This scenario can also explain why the $\chi_{c1}c\bar{c}$ state around 3950 MeV predicted by the single-channel theory is missing in experiment [28]. It is worth noting the recent Lattice QCD result...
that a candidate for the $X(3872)$ about $11 \pm 7$ MeV below the $D^0D^{*0}$ threshold was identified in a lattice simulation with $m_\pi = 266(4)$ MeV \[24\]. It was also shown that the pion mass dependence of the binding energy can provide important information on the structure of the $X(3872)$ \[25–27\].

Our motivation here is that by analyzing the compositeness relation proposed by Weinberg in the effective field theory, we can establish the relation between experimental observable and the wave function renormalization constant $Z$ such that the hadron structure information encoded in $Z$ can be probed via the measurement of some of those sensitive observables. Specifically, we will show that the line shape of $D^{*0}\bar{D}^0 \rightarrow X(3872)(\rightarrow D^{*0}\bar{D}^0)K$ could be useful for shedding important light on the structure of the $X(3872)$ \[52\].

II. A BRIEF REVIEW OF WEINBERG’S COMPOSITENESS THEOREM

To proceed, we first give a short review on how to evaluate the coupling constant between a near-threshold state and its two-particle channel by Weinberg \[8, 9\]. Without losing generality, a total Hamiltonian $H$ of interest can be split into a free part $H_0$ and an interaction part $V$ to an open channel near threshold:

$$H = H_0 + V.$$  \(1\)

The eigenstates of the free part $H_0$ include the continuum states $|\alpha\rangle$ and the possible discrete bare elementary particle states $|n\rangle$, with

$$H_0|\alpha\rangle = E(\alpha)|\alpha\rangle, \quad \langle \beta|\alpha\rangle = \delta(\beta - \alpha),$$

$$H_0|n\rangle = E_n|n\rangle, \quad \langle \alpha|n\rangle = 0, \quad \langle m|n\rangle = \delta_{m,n},$$  \(2\)

where the energies are defined relative to the two-particle threshold throughout this paper. The completeness relation for the eigenstates of $H_0$ reads

$$1 = \sum_n |n\rangle\langle n| + \int d\alpha |\langle \alpha|\rangle|^2,$$  \(3\)

A physical bound state $|d\rangle$ is a normalized eignestate of the total Hamiltonian $H$, with

$$H|d\rangle = -B|d\rangle, \quad \langle d|d\rangle = 1,$$  \(4\)

where $B > 0$ is the bounding energy. We call $|d\rangle$ as a physical bound state in such a sense that it has the open channel two particles as constituents in its wavefunction and its mass is below the two-particle threshold or equally $B > 0$. With the completeness relation in Eq. \(3\) and the normalization of $|d\rangle$ we can have

$$1 = Z + \int d\alpha |\langle \alpha|d\rangle|^2, \quad Z \equiv \sum_n |\langle n|d\rangle|^2$$  \(5\)

where $Z$ is the probability of finding an elementary state in the physical bound state. Hence $Z = 0$ indicates that the physical bound state is purely composite, while $0 < Z < 1$ indicates that there also exists an elementary component inside the physical state. The determination of the value of $Z$ would thus enable us to distinguish a pure composite state from a mixture of a composite and elementary configuration.

With the relation $|d\rangle = [H - H_0]^{-1}V|d\rangle$ and Eqs. \(2\) and \(4\), we can obtain

$$\langle \alpha|d\rangle = \langle \alpha|[H - H_0]^{-1}V|d\rangle,$$

$$= -\frac{\langle \alpha|V|d\rangle}{E(\alpha) + B}.$$  \(6\)

Then, Eq. \(5\) can be written as

$$1 - Z = \int d\alpha \frac{|\langle \alpha|V|d\rangle|^2}{(E(\alpha) + B)^2}.$$  \(7\)
For small $B$, the above integral nearly diverges, it can then be approximately evaluated by restricting $|\alpha\rangle$ to low-energy two-particle state. If the coupling between $|d\rangle$ and the two-particle state is an $S$-wave coupling, we can then replace $|\langle\alpha|V|d\rangle|$ by $g$, and replace the $\alpha$ integral with

$$da = \frac{4\pi p^2 dp}{(2\pi)^3} = \frac{\mu^{3/2}}{\sqrt{2\pi^2}} E^{1/2} dE, \quad E = p^2/2\mu,$$

(8)

where $\mu$ is the reduced mass of the two constituents. After these replacements we then obtain the effective coupling constant

$$g^2 = \frac{2\pi \sqrt{2\mu B}}{\mu^2} (1 - Z),$$

(9)

which encodes the structure information of the composite system [34].

Now we will show that Eq. (9) can be reproduced in effective field theory if $|d\rangle$ is a purely composite state or equally $Z = 0$. As an example, we will use the effective field theory for the nucleon-nucleon ($NN$) interactions. An effective theory for the $NN$ interactions was developed since 1990’s [28, 29]. It was proposed in Ref. [30, 31] that for the large scattering length system as in the $S$-wave $NN$ scattering, the power divergence subtraction (PDS) scheme should be used. By treating the small three momentum $p$ of the nucleon as the expansion parameter, the leading amplitude is at the order of $p^{-1}$ and reads [30, 31]:

$$A_{-1} = \frac{-C_0}{1 + \frac{C_0\mu}{2\pi}(\Lambda_{PDS} - \sqrt{-2\mu E - i\epsilon})},$$

(10)

where $-C_0$ is the coefficient of the leading contact term in the Lagrangian and $\Lambda_{PDS}$ is the dimensional regularization parameter; $\mu$ is the reduced mass of the $NN$ system and $E = p^2/2\mu$ is the total kinetic energy of the $NN$ system in the center of mass (c.m.) frame. Although Eq. (10) contains the parameter $\Lambda_{PDS}$ which can be arbitrary, $C_0$ should also depend on this arbitrary scale thus the leading order amplitude does not depend on the arbitrary scale. Namely, the leading order amplitude should not be scheme-dependent. Setting $\Lambda_{PDS} = 0$, Eq. (10) simply recovers the result obtained in the minimal subtraction (MS) scheme.

Notice that in Ref. [30, 31] the $S$-matrix is related to the scattering amplitude $A$ via

$$S = 1 + \frac{\mu p}{\pi} A,$$

(11)

for the $S$-wave scattering. If Eq. (10) has a bound state pole at $E = -B$, [53] such as the deuteron, then

$$1 + \frac{C_0\mu}{2\pi}(\Lambda_{PDS} - \sqrt{-2\mu B}) = 0, \quad \Rightarrow \quad C_0 = \frac{2\pi}{\mu(\Lambda_{PDS} - \sqrt{-2\mu B})}.$$

(12)

We can then expand the denominator of $A_{-1}$ around $E = -B$ and obtain

$$A_{-1} = \frac{\delta}{E + B + \Sigma(E)},$$

(13)

where $\delta = -2\pi\sqrt{2\mu B}/\mu^2$ is the residual of the bound state pole and

$$\Sigma(E) = -\frac{\sqrt{2\mu B}}{\mu} \sqrt{-2\mu E - i\epsilon} + B - E.$$

(14)

Comparing with Eq. (9), one can find that

$$\delta = -g^2(Z = 0),$$

(15)

where $g^2(Z = 0)$ denotes the value of $g^2$ when $Z = 0$. The minus sign can be understood as the Feynman rule for the deuteron-nucleon-nucleon vertex as $ig$. Therefore, we have shown that the leading order $^3S_1$ $n$-$p$ scattering amplitude can be written as

$$A_{-1} = \frac{-g^2(Z = 0)}{E + B + \Sigma(E)}.$$

(16)

Since we have known that deuteron is composite, it is not surprising that we should set $Z = 0$ in Eq. (10).
III. THE COMPOSITENESS THEOREM IN THE EFT

Now we will turn to study such a case that $|d\rangle$ is not a composite state, i.e. $Z \neq 0$. A naive extension from Eq. (16) to $A_{-1} = \frac{g^2}{E + B + \Sigma(E)}$ for $Z \neq 0$ would give a non-unitary $S$-matrix except for the case at $Z = 0$. Here, we will use the effective field theory approach to obtain the right solution. Consider a bare state $|B\rangle$ with bare mass $-B_0$ and coupling $g_0$ to the two-particle state. If $|B\rangle$ is near the two-particle threshold, then the leading two-particle scattering amplitude can be obtained by summing the Feynman diagrams in Fig. 1. Near threshold, the momenta of these two particles are non-relativistic. Therefore, the loop integral in Fig. 1 can be done the same way as that in Ref. [30, 31]. With the minimal subtraction (MS) scheme, the result of the loop integral can be written as

$$I_{\text{MS}} \equiv \int \frac{d^D \ell}{(2\pi)^D} \frac{i}{\ell^0 - \ell^2/(2m_1) + i\epsilon} \cdot \frac{i}{E - \ell^0 - \ell^2/(2m_2) + i\epsilon} = i\frac{\mu}{2\pi} \sqrt{-2\mu E - i\epsilon}. \quad (17)$$

Thus, the Feynman amplitude for Fig. 1 reads

$$A = -\frac{g_0^2}{E + B_0 - g_0^2 \frac{\mu}{2\pi} \sqrt{-2\mu E - i\epsilon}} \quad (18)$$

As mentioned above that a physical bound state $|d\rangle$ corresponds to a pole at $E = -B$, we then have

$$B_0 - g_0^2 \frac{\mu}{2\pi} \sqrt{2\mu B} \equiv B, \quad \Rightarrow \quad B_0 = B + g_0^2 \frac{\mu}{2\pi} \sqrt{2\mu B}. \quad (19)$$

Then, the amplitude can be written as

$$A = \frac{\delta'}{E + B + \Sigma'(E)}, \quad (20)$$

where $\delta' = -\frac{g_0^2}{1 + g_0^2 \mu^2/(2\pi \sqrt{2\mu})}$ is the residual of the bound state pole, and

$$\Sigma'(E) = \delta'[\frac{\mu}{2\pi} \sqrt{-2\mu E - i\epsilon} + \frac{\mu \sqrt{2\mu B}}{4\pi B} (E - B)]. \quad (21)$$

Since $\delta'$ is the residual of the bound state pole, it naturally leads to the connection of $\delta' \equiv -g^2$ where $g^2$ is defined in Eq. (9) as the effective coupling constant. Note that the $Z$ dependence is embedded in Eq. (9).

Then, by setting $Z = 0$, the amplitude in Eq. (20) will be reduced to Eq. (13). One thus recognizes that the formulation of the EFT can be implemented by the Weinberg’s compositeness condition and extended to accommodate the situation of $0 \leq Z \leq 1$, i.e. to evaluate the size of an elementary component inside the threshold state.

For $0 \leq Z \leq 1$ the leading order amplitude for the two-particle scattering can be written as

$$A = -\frac{g^2}{E + B + \Sigma'(E)}, \quad (22)$$
where $\tilde{\Sigma}'(E)$ can now be written as

$$
\tilde{\Sigma}'(E) = -g^2\frac{\mu}{2\pi} \sqrt{-2\mu E - i\epsilon + \frac{\mu\sqrt{2\mu B}}{4\pi B}(E - B)}.
$$

(23)

One can easily check that the amplitude given in Eq. (22) satisfies the unitary condition. Actually, the same solution as Eq. (22) was obtained by Weinberg fifty years ago, but with a different approach [9]. With $\delta' = -g^2$ and Eq. (11), we obtain

$$
g_0^2 = \frac{2\pi\sqrt{2\mu B}}{\mu^2} \frac{1 - Z}{Z}.
$$

(24)

Combining Eq. (19) and (24) together we obtain

$$
B_0 = \frac{2 - Z}{Z} B.
$$

(25)

One can see that $B_0$ has the same order as $B$, this verifies the necessity to sum the diagrams in Fig. 1. The limit $B_0 \to \infty$ corresponds to $Z \to 0$ which is consistent with the condition discussed in [8]. Also, Eq. (24) defines the wave function renormalization constant of $|B\rangle$, i.e. $Z = 1/\left[1 + g_0^2\mu^2/(2\pi\sqrt{2\mu B})\right]$, which is the same as the result in Ref. [32].

It should be interesting to examine the behavior of the tree diagram amplitude in Fig. 1 near threshold. The tree diagram amplitude reads

$$
A_{\text{tree}} = -\frac{g_0^2}{E + B_0}.
$$

(26)

In the limit $Z \to 0$, we have $B_0 \to \infty$. Hence the energy dependence in the denominator of $A_{\text{tree}}$ can be safely neglected near threshold. With Eqs. (24) and (25), $A_{\text{tree}}$ can be approximated as

$$
A_{\text{tree}} = -\frac{2\pi}{\mu\sqrt{2\mu B}} = -C_0(\Lambda_{\text{PDS}} = 0),
$$

(27)

where $C_0(\Lambda_{\text{PDS}} = 0)$ can be obtained from Eq. (12) by setting $\Lambda_{\text{PDS}} = 0$. Equation (27) is just the equivalence of the four-Fermi theory and Yukawa theory found in Ref. [10]. One can see that due to the existence of the bound state the coefficient of the leading contact term can be enhanced to the order of $(2\mu B)^{-1/2} \sim O(p^{-1})$. Hence in such a case all the bubble diagrams of leading contact term are equally important and should be resummed at the leading order.

In the above we have used the MS scheme to evaluate the loop integral. If we use the PDS scheme, then Eq. (18) will become

$$
A = -\frac{g_0^2}{E + B_0 - g_0^2\frac{\mu}{2\pi}(\sqrt{-2\mu E - i\epsilon - \Lambda_{\text{PDS}}})}.
$$

(28)

By extracting the bare mass similar to what we have done above, we find that Eqs. (22)–(24) are still hold but Eq. (25) will be changed to

$$
B_0 = \frac{2 - Z}{Z} B - \frac{1 - Z}{Z} \sqrt{2B/\mu\Lambda_{\text{PDS}}},
$$

(29)

If $Z \neq 0$ and also $Z \neq 1$, the bare mass $B_0$ determined from Eq. (29) will depend on the regularization parameter $\Lambda_{\text{PDS}}$ which can be arbitrary. It means that the determination of the bare mass will inevitably depend on the scheme as emphasized in Ref. [33]. As a consequence, one presumably need not worry too much about the physical meaning of a bare mass. In contrast, the physical observable such as Eq. (22) is scheme-independent and can be determined by measuring the line shape.
IV. STUDY OF THE $X(3872)$ IN THE EFT APPROACH

We show how to incorporate Weinberg’s compositeness theorem in the EFT which can be applied to the study of threshold states in which both elementary and molecular configurations could be present. Although most of the formulae we present above had been obtained with the quantum mechanical approach [9, 34], it is still useful to reproduce them in the EFT approach. The idea is that with the EFT, we can obtain the relevant Feynman rules for the near-threshold states. These Feynman rules can then be directly applied to processes involving such states as a more realistic prescription for their threshold behaviors. What is more important is that with the EFT approach, we can set up the power counting and study the higher order corrections systematically. Therefore the EFT approach can be applied much easier to phenomenological studies and may provide a clearer physical picture for some of those threshold states. We also mention that some of those points have been addressed or demonstrated in recent analyses of Refs. [48–50].

\[ G(E) = \frac{iZ}{E + B + \Sigma'(E) + i\Gamma/2}, \]  

(30)

where $\Gamma$ denotes the width of the $X(3872)$ which comes from the decay modes that do not proceed through its $D^{*0}\bar{D}^0$ component. Our convention is that a factor of $\sqrt{2}M_X$ has been absorbed into the field operator of the $X(3872)$. It is convenient to use this convention for the boson in the nonrelativistic formalism. Hence a boson field has the dimension of $3/2$ and the Feynman rule for an external boson should be $\sqrt{2}M$. Actually, the coupling constant $g^2$ in Eq. (9) is defined under this convention. The Feynman rule for the $XD^{*0}\bar{D}^0$ coupling is given as

\[ i\frac{g_0}{\sqrt{2}} = i\left(\frac{g^2}{2}\right)^{1/2}, \]  

(31)

where the factor $1/\sqrt{2}$ is due to the definition of the $C$-even state $\sqrt{2}(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0})$.

Near threshold, there are two small momenta, the binding momentum $\gamma = (2\mu B)^{1/2}$ and the three momentum of the charmed meson $p$. We can count these two momenta as the same order, i.e. $\gamma, p \sim O(p^2)$. One can then easily check that the elastic scattering amplitude given in Eq. (22) is at the order of $O(p^{-3})$ which is consistent with the result in Ref. [31].
Now we come to describe the line shape of $D^0 \bar{D}^0$ in $B \to X(3872)K \to D^0 \bar{D}^0 K$ in this EFT approach. For studies of the line shape with other approaches one can refer to Refs. 36–40. In particular, Refs. 36–38 describe the $D^* \bar{D}^0$ line shape with Flatté parametrization. Assuming the $X(3872)$ production via the short-distance process, Ref. 38 further addresses the question of a possible $\chi_{c1}$ charmonium admixture in the wave function of the $X(3872)$. Generally, there are two different mechanisms for the production of $X(3872)$ in the $B$ decay, i.e. the short-distant and long-distant production mechanisms. In the short-distant production mechanism the $X(3872)$ is produced directly at the short distant vertex of the $B$ decay, while in the long-distant production mechanism a $D^0 \bar{D}^0$ pair is produced first in the $B$ decay and then rescatters to the $X(3872)$. The answer to the question about which production mechanism is more important than the other would depend on the structure the $X(3872)$. As follows, instead of making assumptions on the structure of the $X(3872)$ in advance, we actually consider both these two different production mechanisms in our analysis.

The leading order Feynman diagrams for these two different production mechanisms are presented in Fig. 2 for which the Feynman amplitudes can be explicitly expressed as

$$iM_a = -\frac{\mathcal{A}_{XK}}{\sqrt 2} \frac{\sqrt Z g}{E + B + \Sigma'(E) + i\Gamma/2} \vec{p}_K \cdot \epsilon^*(D^*),$$

$$iM_b = B_{DDK} \frac{\mu}{2\pi} \frac{g^2 \sqrt{-2\mu E - i\epsilon}}{E + B + \Sigma'(E) + i\Gamma/2} \vec{p}_K \cdot \epsilon^*(D^*),$$

where $p_K$ is the momentum of the $K$ meson in the rest frame of the $B$ meson, and $\epsilon(D^*)$ is the polarization vector of the outgoing $D^*$. We use $A_{XK}$ and $B_{DDK}$ to denote the first production vertices in Fig. 2, i.e. $B \to X(3872)K$ and $B \to D^0 \bar{D}^0 K$, respectively. Near the threshold of $D^0 \bar{D}^0$, we can treat $A_{XK}$ and $B_{DDK}$ as constants. Note that we have omitted the factors from the external $D^{(*)}$ mesons in Eq. (32) which can be absorbed into $A_{XK}$ and $B_{DDK}$.

It is easy to count the power of the above amplitudes and one can find $M_a \sim \mathcal{O}(p^{-3/2})$ and $M_b \sim \mathcal{O}(p^0)$. From the power counting, one may find that the short-distant production mechanism is more important than the long-distant one. However, it should be noted that $M_a$ is proportional to the factor $\sqrt{Z}$. Therefore, its contribution will be suppressed if the $X(3872)$ is dominated by a molecular component. It is interesting to note that with $Z = 0$ the term of $M_a$ will vanish, and then the production of the $X(3872)$ will only come from the long-distant production mechanism $M_b$. This feature ensures that our separation of the short-distant production mechanism from the long-distant one makes sense.

Taking into account the non-resonance production contribution, the full amplitude to describe $B \to D^0 \bar{D}^0 K$ is expressed as

$$i\mathcal{M} = iM_a + iM_b + B_{DDK} \vec{p}_K \cdot \epsilon^*(D^*) ,$$

where the term $B_{DDK} \vec{p}_K \cdot \epsilon^*(D^*)$ describes the non-resonance production which is at the same order as $iM_b$. Now we can use the above amplitude to describe the Belle and BaBar data 41, 42. The free parameters in our calculation include $\Gamma$, $B$, $Z$, $A_{XK}$ and $B_{DDK}$. However, since the data from experiment are with large error bars. To reduce the uncertainty, we fix $\Gamma$ and $B$ with the values that are determined in $X(3872) \to J/\psi X$, where $X$ denotes the light hadrons. The reason is because the data from the decay modes of $X(3872) \to J/\psi X$ are with higher statistics and there the $X(3872)$ appears as a narrow Breit-Wigner structure. We adapt the PDG 43 value $M_{X(3872)} = 3871.68$ MeV for the mass of $X(3872)$ which is the average over those measured in the decay modes of $X(3872) \to J/\psi X$. With $M_{D^0} = 2006.99$ MeV and $M_{D_s} = 1864.86$ MeV 43, we can fix the binding energy as $B = 0.17$ MeV. The width of $X(3872)$ is not settled down by the data for $X(3872) \to J/\psi X$, but the upper limit is given as $\Gamma < 1.2$ MeV. Since the width is small, we fix the non-$D^0 \bar{D}^0$ width $\Gamma = 0$ in our fitting. We have checked that the $D^0 \bar{D}^0$ line shape is not sensitive to $B$ and $\Gamma$ around the fixed values. Therefore, our fitting parameters are $A_{XK}$, $B_{DDK}$ and $Z$.

The fitting results are presented in Fig. 3 and compared with the experimental data 41, 42. Notice that there is an arbitrary scaling factor between the BABAR and Belle data, we fit the ratio $A_{XK}/B_{DDK}$ and $Z$ for these two sets of data simultaneously but leave a free scale factor to be fitted by the data, respectively. This, in principle, introduces an additional parameter and leads to $\chi^2/d.o.f = 0.4$ which indicates some
correlations among the parameters. This can be improved by future experimental measurement. For the physical discussion, we only list the fitted ratio $A_{XX}/B_{DDK}$ and parameter $Z$ as follows

$$A_{XX}/B_{DDK} = -0.15 \pm 0.65 \text{ GeV}^{3/2}, \quad Z = 0.19 \pm 0.29.$$ (34)

One can see that the fitted parameters are with large uncertainties due to the large error bars with the BABAR and Belle data. In Fig. 3, we also show the contribution from different pieces of the amplitude, i.e. $iM_a$ and $iM_\ell = iM_b + B_{DDK}\vec{p}_K \cdot \bar{e}(D^*)$ as the dotted and dashed line, respectively.

Due to the relatively large uncertainties with the fitted parameters, we discuss the following possible scenarios arising from the fitting results:

- The main feature of Fig. 3 is that a small nonvanishing value of $Z$ will result in a sizeable contribution from the short-distance process, i.e. $iM_a$. This indicates that the production of the $X(3872)$ in the $B$ decay is driven by the short-distance production mechanism. Even a small component of e.g. the $c\bar{c}$ core will lead to a relatively larger production rate for the $X(3872)$ in comparison of treating it as a pure $D^{*0}\bar{D}^0$ molecule. Nevertheless, the dominance of the short-distance production mechanism seems to always produce the threshold enhancement which may bring concerns about the molecular feature of the $X(3872)$. However, this may provide a natural explanation for the sizeable production rate for the $X(3872)$ in the $B$ decay, and also explain the large isospin violations given that the compact $c\bar{c}$ component can couple strongly to the charged $D^*\bar{D}$ pair. This will give rise to enhanced isospin violation transitions into $J/\psi\rho$ via the intermediate charged and neutral $D$ meson loops as discussed in the literature. If the compact component of the $X(3872)$ is $\chi_{c1}'$, its production rate in the $B$ decay should be comparable with that of $\chi_{c1}$ [20]. Meanwhile, if the $X(3872)$ is a pure molecule, its production rate will be strongly suppressed. The recent PDG gives $\text{Br}(B^+ \to \chi_{c1}K^+) = (4.79 \pm 0.23) \times 10^{-4}$, while the production ratio of the $X(3872)$ is constrained as $\text{Br}(B^+ \to X(3872)K^+) < 3.2 \times 10^{-4}$ [43]. Thus, it is not conclusive for the structure of the $X(3872)$ based on such a measurement. We expect that a more precise measurement of the decay rate of $B \to X(3872)K$ would provide a quantitative constraint on the $X(3872)$ structure in the future.

- It is interesting to discuss the behavior of term $iM_\ell$ in the line shape of $D^{*0}\bar{D}^0$. In case that the $X(3872)$ is a pure molecule, i.e. $Z = 0$, the line shape will be determined by $iM_a$ with $iM_a = 0$. For convenience, we can express $iM_\ell$ by a more compact form

$$iM_\ell = iM_b + B_{DDK}\vec{p}_K \cdot \bar{e}(D^*) = B_{DDK}\frac{ZE + (2-Z)B}{E + B + \Sigma(E)}\vec{p}_K \cdot \bar{e}(D^*),$$ (35)
where we have set $\Gamma = 0$ as discussed above. By setting $Z = 0$ the energy dependence of $\mathcal{M}_\ell$ is similar to the so-called universal scattering amplitude for an S-wave threshold resonance in Ref. [32, 40]. Such a similarity may not be surprising because the universal scattering amplitude can be derived from a nonrelativistic quantum field theory with a contact interaction as recognized in Refs. [27, 40]. If we fix $Z = 0$, these two approaches should indeed converge as expected. However, the explicit $Z$-dependence will bring novel aspects to the line shape description.

We can take a closer look at the $Z$-dependence of $i\mathcal{M}_\ell$ which is illustrated by the line shape in Fig. 4. Note that Fig. 4 is rescaled by an arbitrary factor due to the unknown value of the line shape. One can see that the line shape is very sensitive to $Z$. If $Z = 0$, the line shape has a clear near threshold enhancement. But when $Z$ increase, the near threshold enhancement disappear quickly. The reason is that, if $Z \neq 0$ the factor $ZE$ in the numerator of Eq. (35) will play an important role in the line shape. From Eq. (35) one can also find that, for $Z = 0$, $i\mathcal{M}_\ell$ is proportional to the small bounding energy $B$. Therefore, we can conclude that if $Z$ is non-negligible, the near-threshold enhancement of the $D^{*0}\bar{D}^0$ mass spectrum in the $B$ decay will be driven by the short-distant production mechanism of the $X(3872)$, although the dominant component of the $X(3872)$ is molecule. This feature is again consistent with the success of treating the $X(3872)$ as pure molecule in the explanation of the line shape [36–40].

- One may consider to further describing the line shape measured from $X(3872) \to J/\psi\pi^+\pi^-$ in order to have a better determination of $Z$. However, since the coupling between $X(3872)$ and $J/\psi\pi^+\pi^-$ is unclear, namely, they may couple directly or through the intermediate meson loops, the inclusion of the line shape measured in $X(3872) \to J/\psi\pi^+\pi^-$ will inevitably introduce more free parameters. This will be studied in the future with the availability of more precise experimental data.

- In obtaining the above amplitudes, the MS scheme is adopted to evaluate the loop integral. It is still interesting to discuss the results when the PDS scheme is adopted. With the PDS scheme, the amplitude $\mathcal{M}_\alpha$ remains the same but the amplitude $\mathcal{M}_\ell$ will change to

$$i\mathcal{M}_\ell = B_{DDK} \frac{ZE + (2 - Z)B - (1 - Z)\sqrt{2B/\mu_0\Lambda_{PDS}}\vec{p}_K \cdot \vec{r}(D^*)}{E + B + \Sigma(E)}.$$  

If $Z = 0$, the arbitrary scale $\Lambda_{PDS}$ can be absorbed into the definition of $B_{DDK}$ to make sure that the physical amplitude $\mathcal{M}_\ell$ does not depend on this arbitrary scale. However, if $0 < Z < 1$, it seems impossible to do that due to the factor $ZE$ in the numerator. Therefore, for $0 < Z < 1$ the amplitude $\mathcal{M}_\ell$ will inevitably depend on the arbitrary scale $\Lambda_{PDS}$ if the PDS scheme is adopted. Whether this means that the PDS scheme may not be suitable for the study of the decay processes in our EFT needs to be further investigated. We note that the same problem does not occur in the two particles elastic scatterings in the EFT approach as discussed before.

V. SUMMARY

In summary, we have proposed an EFT approach with the compositeness theorem incorporated for the study of threshold states. By determining $Z$ which is the probability of finding an elementary component in the bound state via physical observables, our EFT approach can be used to identify the structure of the S-wave near-threshold states. As an example of the application, we use the EFT approach to describe the line shape of the $D^{*0}\bar{D}^0$ mass spectrum in the decay of $B \to D^{*0}\bar{D}^0K$. By fitting the data BABAR and Belle, we obtain a nonvanishing value of $Z = 0.19 \pm 0.29$. Although higher statistics data for $B \to X(3872)(\to D^{*0}\bar{D}^0)K$ are needed to reduce the uncertainty of $Z$, the study of the $Z$ dependence of the transition amplitudes suggests that a small value of $Z$ inside the $X(3872)$ would cause the threshold enhancement in the $D^{*0}\bar{D}^0$ invariant mass spectrum via the short-distance production mechanism. It alternatively implies that the $X(3872)$ is dominated by the molecular $D^{*0}\bar{D}^0$ molecular component. This scenario can naturally explain the observation of sizeable isospin violation decays of $X(3872) \to J/\psi\rho^0$ via the charged and neutral $D^*\bar{D} + c.c.$ meson loops as the leading contribution. Finally, it will be very interesting to constrain $Z$ from...
FIG. 4: The exclusive contribution from $iM_{\ell}$ to the line shape of the $D^{*0}\bar{D}^0$ spectrum in $B \to D^{*0}\bar{D}^0K$ with different $Z$. Here, we set $B = 0.5$ MeV as an illustration. The results with $B = 0.17$ MeV are similar. An arbitrary normalization is implemented.

other approaches. For example, in Ref. [47] the determined value of $Z$ is $Z = (28 - 44)\%$ which is close to our result.

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