ANALYSIS OF THE 1S AND 2S STATES OF THE $\Lambda_Q$ AND $\Xi_Q$ WITH THE QCD SUM RULES

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Abstract

In this article, we study the ground states and the first radial excited states of the spin-parity $J^P = \frac{1}{2}^+$ flavor antitriplet heavy baryon states $\Lambda_Q$ and $\Xi_Q$ by carrying out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way. We observe that the higher dimensional vacuum condensates play an important role, and obtain very stable QCD sum rules with variations of the Borel parameters for the heavy baryon states for the first time. The predicted masses $6.08 \pm 0.09$ GeV, $2.78 \pm 0.08$ GeV and $2.96 \pm 0.09$ GeV for the first radial excited states $\Lambda_b(2S)$, $\Lambda_c(2S)$ and $\Xi_c(2S)$ respectively are in excellent agreement with the experimental data and support assigning the $\Lambda_b(6072)$, $\Lambda_c(2765)$ and $\Xi_c(2980/2970)$ to be the first radial excited states of the $\Lambda_b$, $\Lambda_c$ and $\Xi_c$, respectively, the predicted mass $6.24 \pm 0.07$ GeV for the $\Xi_b(2S)$ can be confronted to the experimental data in the future.

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1 Introduction

Recently, the CMS collaboration observed a broad excess of events in the region of $6040 - 6100$ MeV in the $\Lambda_0^b\pi^+\pi^-$ invariant mass spectrum based on a data sample corresponding to an integrated luminosity of up to $140$ fb$^{-1}$ [1]. If it is fitted with a single Breit-Wigner function, the obtained mass and width are $M = 6073 \pm 5$ MeV and $\Gamma = 55 \pm 11$ MeV, respectively. Subsequently, the LHCb collaboration observed a new excited baryon state in the $\Lambda_0^b\pi^+\pi^-$ invariant mass spectrum with high significance using a data sample corresponding to an integrated luminosity of $9$ fb$^{-1}$. The measured mass and natural width are $M = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV and $\Gamma = 72 \pm 11 \pm 2$ MeV, respectively, which are consistent with the first radial excitation of the $\Lambda_0^b$ baryon, the $\Lambda_0^b(2S)$ resonance [2]. The $\Lambda_b(6072)$ can be assigned to be the $\Lambda_0^b(2S)$ state [3], or assigned to be the lowest $\rho$-mode excitation in $\Lambda_b$ family [4].

In 2001, at the charm sector, the CLEO collaboration observed the $\Sigma_c^+(2765)$ or $\Sigma_c^+(2765)$ in the $\Lambda_c^+\pi^-\pi^+$ invariant mass spectrum using a 13.7fb$^{-1}$ data sample recorded by the CLEO detector at CESR [5]. The Belle collaboration determined the isospin of the $\Lambda_c^+(2765)$ or $\Sigma_c^+(2765)$ to be zero using a 980fb$^{-1}$ data sample in the $e^+e^-$ annihilation around $\sqrt{s} = 10.6$ GeV, and established it to be a $\Lambda_c$ resonance [6]. The $\Lambda_c(2765)$ can be assigned to be the $\Lambda_c(2S)$ state [7,8], however, there are several other possible assignments [9].

In 2006, the Belle collaboration reported the first observation of two charmed strange baryon states that decay into the final state $\Lambda_c^+K^-\pi^+$, the broader one has a mass of
2QCD sum rules for the $\Lambda$ present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

We interpolate the spin-parity conjunction matrix. The article is arranged as follows: we derive the QCD sum rules for the masses and the pole residues of the heavy baryon states $\Lambda $ + $J$, where $\Lambda $ collaboration confirmed the $\Xi $ possible assignments of the $\Lambda $ to be the $\Xi $.

In Ref.[17], we study the masses and pole residues of the $\Lambda $ baryon states, if we carry out the operator product expansion up to the vacuum condensates of dimension 6, we have to choose the continuum threshold parameters as

The mass spectrum of the single heavy baryon states has been studied intensively in various theoretical models [3, 4, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18]. In the QCD sum rules for the single heavy baryon states with the $\Lambda $-type currents $\gamma $, respectively, the mass gaps between the ground states and the first radial excited states are less than 0.5 GeV, which are significantly lower than the amount that is expected by the 3-dimensional harmonic oscillator model. In the QCD sum rules for the single heavy baryon states, if we carry out the operator product expansion up to the vacuum condensates of dimension 6, we have to choose the continuum threshold parameters as $\sqrt{s_0} = M_{gr} + 0.6 \sim 0.8$ GeV or $0.7 \sim 0.9$ GeV to reproduce the experimental data [15, 16, 17, 18], where the subscript $gr$ stands for the ground states. The energy gaps $0.6 \sim 0.8$ GeV and $0.7 \sim 0.9$ GeV are much larger than the physical energy gap 0.5 GeV, the contributions of the first radial excited states are included in. The heavy baryon states, which have one heavy quark and two light quarks, play an important role in understanding the dynamics of light quarks in the presence of one heavy quark, also in understanding of the confinement mechanism and the heavy quark symmetry.

At the hadron side of the correlation functions in the QCD sum rules for the heavy baryon states, there are one heavy quark propagator and two light quark propagators. If the heavy quark line emits a gluon, each light quark line contributes a quark-antiquark pair, we obtain quark-gluon operators of dimension 10. In previous works, the operator product expansion is carried out up to the vacuum condensate of dimension 6 [13, 14, 15, 16, 17, 18]. In Ref.[17], we study the masses and pole residues of the $1/2^\pm$ flavor antitriplet heavy baryon states ($\Lambda _c^+, \Xi _c^+, \Xi _c^0$) and ($\Lambda _b^0, \Xi _b^0, \Xi _b^-$) by subtracting the contributions from the corresponding $1/2^+$ heavy baryon states with the QCD sum rules. Now we revisit our previous work by calculating the vacuum condensates up to dimension 10, and extend our previous work to study the first radial excited states $\Lambda Q(2S)$ and $\Xi Q(2S)$, and make possible assignments of the $\Lambda _b(6072)$, $\Lambda _c(2765)$ and $\Xi _c(2980)$.

The article is arranged as follows: we derive the QCD sum rules for the masses and the pole residues of the heavy baryon states $\Lambda Q(1S, 2S)$ and $\Xi Q(1S, 2S)$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

## 2 QCD sum rules for the $\Lambda Q(1S, 2S)$ and $\Xi Q(1S, 2S)$

We interpolate the spin-parity $J^P = 1/2^+$ flavor antitriplet heavy baryon states $\Lambda Q$, $\Lambda Q(2S)$, $\Xi Q$ and $\Xi Q(2S)$ with the $\Lambda$-type currents $J_\Lambda (x)$ and $J_\Xi (x)$, respectively,

$$
J_\Lambda (x) = \epsilon^{ijk}u^T_i(x)C\gamma_5d_j(x)Q_k(x), \\
J_\Xi (x) = \epsilon^{ijk}q^T_i(x)C\gamma_5s_j(x)Q_k(x),
$$

where $Q = c, b, q = u, d$, the $i$, $j$ and $k$ are color indexes, and the $C$ is the charge conjugation matrix.

We can interpolate the corresponding spin-parity $J^P = 1/2^-$ flavor antitriplet heavy baryon states with the $\Lambda$-type currents $i\gamma_5J_\Lambda (x)$ and $i\gamma_5J_\Xi (x)$ without introducing the

2978.5 ± 2.1 ± 2.0 MeV and a width of 43.5 ± 7.5 ± 7.0 MeV [10]. Subsequently, the BaBar collaboration confirmed the $\Xi _c(2980)$ or $\Xi _c(2970)$ [11]. The $\Xi _c(2980/2970)$ can be assigned to be the $\Xi _c(2S)$ state [7, 8], however, there are several other possible assignments [9].
relative P-wave explicitly, because multiplying $i\gamma_5$ to the currents $J_A(x)$ and $J_\Xi(x)$ changes their parity \[20\]. Now let us write down the correlation functions,

$$
\Pi(p) = i \int d^4xe^{ip\cdot x} \langle 0| T\{ J(x)\bar{J}(0) \} |0\rangle ,
$$

where $J(x) = J_A(x)$ and $J_\Xi(x)$.

We insert a complete set of intermediate baryon states with the same quantum numbers as the current operators $J_A(x)$, $i\gamma_5 J_A(x)$, $J_\Xi(x)$ and $i\gamma_5 J_\Xi(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation \[21, 22\]. After isolating the pole terms of the ground states and the first radial excited states, we obtain the following results,

$$
\Pi(p) = \lambda^2_{\pm} \frac{\not{p} + M_+}{M_+^2 - p^2} + \lambda^2_{2S,\pm} \frac{\not{p} + M_{2S,\pm}}{M_{2S,\pm}^2 - p^2} + \lambda^2_{2S,-} \frac{\not{p} - M_{2S,-}}{M_{2S,-}^2 - p^2} + \cdots ,
$$

where the $M_\pm$ and $M_{2S,\pm}$ are the masses of the ground states and the first radial excited states with the parity $\pm$ respectively, and the $\lambda_\pm$ and $\lambda_{2S,\pm}$ are the corresponding pole residues defined by $\langle 0| J(0) |B_{\pm/2S,\pm}(p) \rangle = \lambda_{\pm/2S,\pm}, B = \Lambda_Q$ and $\Xi_Q$.

We rewrite the correlation functions as

$$
\Pi(p) = \not{p} \Pi_1(p^2) + \Pi_0(p^2) ,
$$

according to the Lorentz covariance, and obtain the hadronic spectral densities through dispersion relation,

$$
\rho_{H,1}(s) = \lim_{\epsilon \to 0} \frac{\text{Im} \Pi_1(s + i\epsilon)}{\pi} ,
\rho_{H,0}(s) = \lim_{\epsilon \to 0} \frac{\text{Im} \Pi_0(s + i\epsilon)}{\pi} ,
$$

where we add the subscript $H$ to denote the hadron side of the correlation functions.

Now we carry out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way, and take into account the vacuum condensates which are quark-gluon operators of the order $O(\alpha_s^k)$ with $k \leq 1$. Again, we obtain the corresponding QCD spectral densities through dispersion relation,

$$
\rho_{QCD,1}(s) = \lim_{\epsilon \to 0} \frac{\text{Im} \Pi_1(s + i\epsilon)}{\pi} ,
\rho_{QCD,0}(s) = \lim_{\epsilon \to 0} \frac{\text{Im} \Pi_0(s + i\epsilon)}{\pi} ,
$$

where we add the subscripts $QCD$ to denote the QCD side of the correlation functions.

Then we introduce the weight function $\exp\left(-\frac{s}{T^2}\right)$ and obtain two QCD sum rules,

$$
2M_+ \lambda^2_+ \exp\left(-\frac{M^2_+}{T^2}\right) = \int_{m_Q^2}^{s_0} ds \left[ \sqrt{s} \rho_{H,1}(s) + \rho_{H,0}(s) \right] \exp\left(-\frac{s}{T^2}\right) ,
= \int_{m_Q^2}^{s_0} ds \left[ \sqrt{s} \rho_{QCD,1}(s) + \rho_{QCD,0}(s) \right] \exp\left(-\frac{s}{T^2}\right) ,
$$
\[ 2M_+ \lambda_+^2 \exp \left( -\frac{M_+^2}{T^2} \right) + 2M_{2S,+} \lambda_{2S,+}^2 \exp \left( -\frac{M_{2S,+}^2}{T^2} \right) \]

\[ = \int_{m_Q^2}^{s_0^s} ds \left[ \sqrt{s} \rho_{H,1}(s) + \rho_{H,0}(s) \right] \exp \left( -\frac{s}{T^2} \right), \]

\[ = \int_{m_Q^2}^{s_0^s} ds \left[ \sqrt{s} \rho_{QCD,1}(s) + \rho_{QCD,0}(s) \right] \exp \left( -\frac{s}{T^2} \right), \quad (8) \]

where \( \rho_{QCD,1}(s) = \rho_{\Lambda,1}(s), \rho_{\Xi,1}(s), \rho_{QCD,0}(s) = \rho_{\Lambda,0}(s), \rho_{\Xi,0}(s), \)

\[ \rho_{\Lambda,1}(s) = \rho_{\Xi,1}(s) \left|_{m_s \rightarrow 0, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s}g \sigma Gs \rangle \rightarrow \langle \bar{q}g \sigma Gq \rangle} \right., \]

\[ \rho_{\Lambda,0}(s) = \rho_{\Xi,0}(s) \left|_{m_s \rightarrow 0, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s}g \sigma Gs \rangle \rightarrow \langle \bar{q}g \sigma Gq \rangle} \right.. \quad (9) \]

\[ \rho_{\Xi,1}(s) = \frac{3}{128\pi^4} \int_{x_i}^{1} dx x (1 - x)^2 (s - m_Q^2)^2 + \frac{m_s[(\bar{s}s) - 2\langle \bar{q}q \rangle]}{16\pi^2} \int_{x_i}^{1} dx x \]

\[ - \frac{m_s[(\bar{s}g_s \sigma Gs) - 3\langle \bar{q}g_s \sigma Gq \rangle]}{96\pi^2} \delta(s - m_Q^2) + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle}{6} \delta(s - m_Q^2) \]

\[ - \frac{m_s^4 [\langle \bar{s}g_s \sigma Gs \rangle + 2\langle \bar{q}g_s \sigma Gq \rangle]}{24T^2} \left( 1 + \frac{s}{T^2} \right) \delta(s - m_Q^2) \]

\[ + \frac{m_Q^4 [\langle \bar{s}g_s \sigma Gs \rangle + 2\langle \bar{q}g_s \sigma Gq \rangle]}{96T^5} \delta(s - m_Q^2) + \frac{1}{128\pi^2} \alpha_s GG \int_{x_i}^{1} dx \frac{1}{x^2} \]

\[ - \frac{m_s^2 [\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle]}{384\pi^2} \alpha_s GG \int_{x_i}^{1} dx \frac{1}{x^2} - \frac{m_Q^2 [\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle]}{288T^4} \alpha_s GG \int_{x_i}^{1} dx \frac{1}{x^2} \]

\[ - \frac{m_s^2 [\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle]}{108T^6} \alpha_s GG \int_{x_i}^{1} dx \frac{1}{x^2} \delta(s - m_Q^2), \quad (10) \]
\[ \rho_{\Xi,0}(s) = \frac{3}{128\pi^4} \int_{x_i}^1 dx (1-x)^2 (s - \bar{m}_Q^2)^2 + \frac{m_s[(\bar{s}s) - 2(\bar{q}q)]}{16\pi^2} \int_{x_i}^1 dx \\
- \frac{m_s[(sg_\sigma G_s) - 3(\bar{q}g_\sigma G_q)]}{96\pi^2} \delta(s - m_Q^2) + \frac{(\bar{s}s)(\bar{q}q)}{6}\delta(s - m_Q^2) \\
- \frac{m_Q^2[(\bar{s}s)(\bar{q}g_\sigma G_q) + (\bar{q}g_\sigma G_s)(\bar{q}q)]}{24T^2} \\
+ \frac{m_Q^2(\bar{q}g_\sigma G_q)(\bar{s}g_\sigma G_s)}{48T^6} \left( -1 + \frac{s}{2T^2} \right) \delta(s - m_Q^2) \\
- \frac{m_Q^2(\bar{q}g_\sigma G_q)(\bar{s}g_\sigma G_s)}{384\pi^2} \frac{\alpha_sGG}{\pi} \int_{x_i}^1 dx \frac{(1-x)^2}{x^3} + \frac{1}{128\pi^2} \frac{\alpha_sGG}{\pi} \int_{x_i}^1 dx \frac{(1-x)^2}{x^2} \\
+ \frac{1}{128\pi^2} \frac{\alpha_sGG}{\pi} \int_{x_i}^1 dx - \frac{m_s m_Q^2[(\bar{s}s) - 2(\bar{q}q)]}{288T^4} \frac{\alpha_sGG}{\pi} \int_{x_i}^1 dx \frac{1}{x^3} \\
+ \frac{m_s[(\bar{s}s) - 2(\bar{q}q)]}{96T^2} \frac{\alpha_sGG}{\pi} \int_{x_i}^1 dx \frac{1}{x^2} - \frac{m_Q^2(\bar{s}s)(\bar{q}q)\pi^2}{108T^6} \frac{\alpha_sGG}{\pi} \delta(s - m_Q^2) \\
+ \frac{(\bar{s}s)(\bar{q}q)\pi^2}{36T^4} \frac{\alpha_sGG}{\pi} \delta(s - m_Q^2), \tag{11} \]

\( x_i = \frac{m_Q^2}{s} \), the \( T^2 \) is the Borel parameter, the \( s_0 \) and \( s'_0 \) are the continuum threshold parameters.

We derive the QCD sum rules in Eq. (7) in regard to \( T^2 \), then eliminate the pole residues \( \lambda_\perp \) and obtain the masses of the ground states \( \Lambda_Q \) and \( \Xi_Q \),

\[ M_+^2 = \frac{d}{ds} \left[ \int_{m_Q^2}^{s_0} ds \left[ \sqrt{s} \rho_{QCD,1}(s) + \rho_{QCD,0}(s) \right] \exp \left( -\frac{s}{T^2} \right) \right]. \tag{12} \]

Thereafter, we will refer to the QCD sum rules in Eq. (7) and Eq. (12) as QCDSR I.

We introduce the notations \( \tau = \frac{1}{T^2}, D^n = \left( -\frac{d}{d\tau} \right)^n \), and use the subscripts 1 and 2 to represent the ground states \( \Lambda_Q, \Xi_Q \), and the first radially excited states \( \Lambda_Q(2S), \Xi_Q(2S) \), respectively for simplicity.

\[ \bar{\lambda}_1^2 \exp(-\tau M_1^2) + \bar{\lambda}_2^2 \exp(-\tau M_2^2) = \Pi_{QCD}(\tau), \tag{13} \]

where \( \bar{\lambda}_1^2 = 2M_+^2, \bar{\lambda}_2^2 = 2M_{2S}^2, \bar{\lambda}_{2S, +}^2 \), we introduce the subscript \( QCD \) to denote the QCD representation of the correlation functions below the continuum thresholds \( s'_0 \).

Firstly, let us derive the QCD sum rules in Eq. (13) with respect to \( \tau \) to obtain,

\[ \bar{\lambda}_1^2 M_1^2 \exp(-\tau M_1^2) + \bar{\lambda}_2^2 M_2^2 \exp(-\tau M_2^2) = D \Pi_{QCD}(\tau). \tag{14} \]

From Eqs. (13)-(14), we can obtain the QCD sum rules,

\[ \bar{\lambda}_1^2 \exp(-\tau M_1^2) = \frac{D - M_1^2}{M_1^2 - M_j^2} \Pi_{QCD}(\tau), \tag{15} \]
where the sub-indexes \( i \neq j \). Then let us derive the QCD sum rules in Eq.(15) with respect to \( \tau \) to obtain

\[
M_i^2 = \frac{(D^2 - M_j^2 D) \Pi'_{QCD}(\tau)}{(D - M_j^2) \Pi'_{QCD}(\tau)},
\]

\[
M_i^4 = \frac{(D^3 - M_j^2 D^2) \Pi'_{QCD}(\tau)}{(D - M_j^2) \Pi'_{QCD}(\tau)}.
\]

The squared masses \( M_i^2 \) satisfy the equation,

\[
M_i^4 - bM_i^2 + c = 0,
\]

where

\[
b = \frac{D^3 \otimes D^0 - D^2 \otimes D}{D^2 \otimes D^0 - D \otimes D},
\]

\[
c = \frac{D^3 \otimes D - D^2 \otimes D^2}{D^2 \otimes D^0 - D \otimes D},
\]

\[
D^j \otimes D^k = D^j \Pi'_{QCD}(\tau) D^k \Pi'_{QCD}(\tau),
\]

the indexes \( i = 1, 2 \) and \( j, k = 0, 1, 2, 3 \). Finally we solve the equation in Eq.(17) analytically to obtain two solutions [26, 27],

\[
M_1^2 = \frac{b - \sqrt{b^2 - 4c}}{2},
\]

\[
M_2^2 = \frac{b + \sqrt{b^2 - 4c}}{2}.
\]

From the QCD sum rules in Eqs.(19)-(20), we can obtain both the masses of the ground states and the first radial excited states, the ground state masses from the QCD sum rules in Eq.(19) suffer from additional uncertainties from the first radial excited states \( \Lambda_Q(2S) \) and \( \Xi_Q(2S) \), and we neglect the QCD sum rules in Eq.(19). Thereafter, we will refer to the QCD sum rules in Eq.(15) and Eq.(20) as the QCDSR II.

3 Numerical results and discussions

At the QCD side, we take the vacuum condensates to be the standard values \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^2 \), \( \langle s\bar{s} \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle \), \( \langle \bar{q}g, Gg \rangle = m_0^2 \langle \bar{q}q \rangle \), \( \langle \bar{s}g, Gs \rangle = m_0^2 \langle s\bar{s} \rangle \), \( m_0^2 = (0.8 \pm 0.1) \text{GeV}^2 \), \( \langle \bar{q}Gq \rangle = (0.33 \text{GeV})^4 \) at the energy scale \( \mu = 1 \text{GeV} \) [21, 22, 23], and take the \( \overline{MS} \) masses \( m_c(m_c) = (1.275 \pm 0.025) \text{GeV} \), \( m_b(m_b) = (4.18 \pm 0.03) \text{GeV} \) and \( m_s(\mu = 2 \text{GeV}) = (0.095 \pm 0.005) \text{GeV} \) from the Particle Data Group [24]. Moreover, we take into account the energy-scale dependence of the quark condensates, mixed quark
condensates and $\overline{MS}$ masses according to the renormalization group equation,

\[
\langle \overline{q}q \rangle (\mu) = \langle \overline{q}q \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2nf}},
\]

\[
\langle \overline{s}s \rangle (\mu) = \langle \overline{s}s \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2nf}},
\]

\[
\langle \overline{q}g_s\sigma Gq \rangle (\mu) = \langle \overline{q}g_s\sigma Gq \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2nf}},
\]

\[
\langle \overline{s}g_s\sigma Gs \rangle (\mu) = \langle \overline{s}g_s\sigma Gs \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2nf}},
\]

\[
m_b(\mu) = m_b(m_b) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right]^{\frac{12}{33-2nf}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2nf}},
\]

\[
m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{33-2nf}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0 t} + \frac{b_2}{b_0 t^2} (\log^2 t - \log t - 1) + b_0 b_2 \right],
\]

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2nf}{12\pi^2}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857 - 363}{128\pi^4} n_f + \frac{27}{128\pi^4} n_f^2$, $\Lambda = 213\text{ MeV}$, 296 MeV and 339 MeV for the flavors $n_f = 5$, 4 and 3, respectively [24, 25]. For the charmed baryon states $\Lambda_c(1S, 2S)$ and $\Xi_c(1S, 2S)$, we choose the flavor numbers $n_f = 4$, while for the bottom baryon states $\Lambda_b(1S, 2S)$ and $\Xi_b(1S, 2S)$, we choose the flavor numbers $n_f = 5$.

In the QCDSR I, we choose the continuum threshold parameters to be $\sqrt{s_0} = M_{gr} + 0.50 \pm 0.10\text{ GeV}$ rather than to be $M_{gr} + 0.6 \sim 0.8\text{ GeV}$ or $0.7 \sim 0.9\text{ GeV}$ as a constraint to exclude the contaminations from the first radial excited states [15, 16, 17, 18], where the subscript $gr$ denotes the ground states $\Lambda_Q$ and $\Xi_Q$. Furthermore, we choose the energy scales of the QCD spectral densities in the QCD sum rules for the $\Lambda_c$, $\Xi_c$, $\Lambda_b$ and $\Xi_b$ to be the typical energy scales $\mu = 1\text{ GeV}$, 1 GeV, 2 GeV and 1.8 GeV, respectively, where we subtract 0.2 GeV in the energy scale for the $\Xi_b$ to account for the finite mass of the $s$-quark. After trial and error, we obtain the Borel parameters $T^2$, continuum threshold parameters $s_0$, pole contributions of the ground states and perturbative contributions, which are shown explicitly in Table 1. From the Table, we can see that the pole contributions are about (40 – 60)% or (40 – 70)%, the pole dominance is satisfied. The perturbative contributions are larger than 50% except for the $\Lambda_b$, although the perturbative contribution is about (43 – 46)% in that case, the contributions of the vacuum condensates of dimension 10 are tiny, the operator product expansion is well convergent.

Now we take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the ground states of the flavor antitriplet heavy baryon states $\Lambda_Q$ and $\Xi_Q$, which are shown in Figs 1-2 and Table 2. From Table 1 and Figs 1-2, we can see that there appear rather flat platforms in the Borel windows, the
Figure 1: The masses with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$ correspond to the $\Lambda_c$, $\Xi_c$, $\Lambda_b$, $\Xi_b$, $\Lambda_c(2S)$, $\Xi_c(2S)$, $\Lambda_b(2S)$ and $\Xi_b(2S)$, respectively, the expt denotes the experimental values.
Figure 2: The pole residues with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$ correspond to the $\Lambda_c$, $\Xi_c$, $\Lambda_b$, $\Xi_b$, $\Lambda_c(2S)$, $\Xi_c(2S)$, $\Lambda_b(2S)$ and $\Xi_b(2S)$, respectively.
In Fig. 3, we plot the predicted mass of the ground state $\Lambda_c$ with variations of the Borel parameter $T^2$ by taking into account the vacuum condensates up to dimension 6, 8 and 10 respectively for the continuum threshold parameter $\sqrt{s_0} = 2.75$ GeV. From the figure, we can see that the truncation $D = 6$ fails to lead to a flat platform and fails to reproduce the experimental value of the mass of the $\Lambda_c$, while the truncations $D = 8$ and 10 both lead to very flat platforms and reproduce the experimental value. In fact, the truncations $D = 8$ and 10 make tiny difference, which indicates that the vacuum condensates of dimension 8 (10) play an important (a tiny) role. We should take into account the vacuum condensates up to dimension 10 for consistence. If we insist on taking the truncation $D = 6$, we have to choose a much larger continuum threshold parameter $\sqrt{s_0} = 3.0$ GeV, the predicted mass increases monotonically with the increase of the Borel parameter $T^2$, we can reproduce the experimental value of the mass of the $\Lambda_c$ with suitable Borel parameter but large uncertainty.

In the QCDSR II, we can borrow some ideas from the conventional charmonium states. The masses of the ground state, the first radial excited state and the second excited state of the charmonium states are $m_{J/\psi} = 3.0969$ GeV, $m_{\psi'} = 3.686097$ GeV and $m_{\psi''} = 4.039$ GeV respectively from the Particle Data Group [24], the energy gaps are $m_{\psi'} - m_{J/\psi} = 0.59$ GeV, $m_{\psi''} - m_{J/\psi} = 0.94$ GeV, we can choose the continuum threshold parameters $\sqrt{s_0} \leq M_{gr} + 0.90$ GeV tentatively to avoid contaminations from the second radial excited states. Furthermore, we choose the energy scales of the QCD spectral densities in the QCD sum rules for the $\Lambda_c(2S)$, $\Xi_c(2S)$, $\Lambda_b(2S)$ and $\Xi_b(2S)$ to be the typical energy scales $\mu = 2$ GeV, 2 GeV, 4 GeV and 3.8 GeV, respectively, again we subtract 0.2 GeV in the energy scale for the $\Xi_b(2S)$ to account for the finite mass of the $s$-quark.
Table 1: The Borel parameters $T^2$ and continuum threshold parameters $s_0(s'_0)$ for the heavy baryon states, where the "pole" stands for the pole contributions from the ground states or the ground states plus the first radial excited states, and the "perturbative" stands for the contributions from the perturbative terms.

|          | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole (%)          | perturbative (%)  |
|----------|---------------|-------------------|------------------|-------------------|
| $\Lambda_c$ | 1.4 - 1.8     | 2.75 ± 0.10       | (40 - 72)%       | (50 - 58)%        |
| $\Xi_c$    | 1.7 - 2.1     | 3.00 ± 0.10       | (42 - 71)%       | (64 - 71)%        |
| $\Lambda_b$ | 3.6 - 4.0     | 6.10 ± 0.10       | (41 - 60)%       | (43 - 46)%        |
| $\Xi_b$    | 3.8 - 4.2     | 6.30 ± 0.10       | (40 - 60)%       | (51 - 54)%        |
| $\Lambda_c$(2S) | 1.8 - 2.4     | 3.00 ± 0.10       | (41 - 74)%       | (70 - 80)%        |
| $\Xi_c$(2S) | 1.8 - 2.4     | 3.25 ± 0.10       | (54 - 84)%       | (74 - 83)%        |
| $\Lambda_b$(2S) | 4.6 - 5.0     | 6.30 ± 0.10       | (49 - 66)%       | (76 - 79)%        |
| $\Xi_b$(2S) | 5.1 - 5.5     | 6.55 ± 0.10       | (51 - 66)%       | (83 - 85)%        |

Table 2: The masses and pole residues of the heavy baryon states, where the masses of the $\Lambda_c$(3S), $\Xi_c$(3S) and $\Lambda_b$(3S) are obtained from the Regge trajectories.

|          | $M$(GeV)   | $\lambda$(10$^{-2}$GeV$^3$) | $M$(GeV)[expt] |
|----------|------------|-----------------------------|----------------|
| $\Lambda_c$ | 2.24 ± 0.09 | 1.51 ± 0.22                 | 2.28646        |
| $\Xi_c$    | 2.45 ± 0.10 | 2.21 ± 0.35                 | 2.46795        |
| $\Lambda_b$ | 5.61 ± 0.12 | 1.96 ± 0.36                 | 5.6196         |
| $\Xi_b$    | 5.79 ± 0.09 | 2.23 ± 0.35                 | 5.7919         |
| $\Lambda_c$(2S) | 2.78 ± 0.08 | 3.20 ± 0.48                 | 2.7666         |
| $\Xi_c$(2S) | 2.96 ± 0.09 | 4.48 ± 0.56                 | 2.9671         |
| $\Lambda_b$(2S) | 6.08 ± 0.09 | 6.35 ± 0.93                 | 6.0723         |
| $\Xi_b$(2S) | 6.24 ± 0.07 | 8.36 ± 1.05                 | 6.4935         |

|          | $M$(GeV)   |                                      |
|----------|------------|--------------------------------------|
| $\Lambda_c$(3S) | 3.1749                      |
| $\Xi_c$(3S)    | 3.3936                      |
| $\Lambda_b$(3S) | 6.4935                      |
After trial and error, we obtain the Borel parameters $T^2$, continuum threshold parameters $s_0$, pole contributions and perturbative contributions, which are shown explicitly in Table 1. From the Table, we can see that the pole contributions vary from 40% to 80%, the pole dominance is satisfied. The perturbative contributions are larger than 70%, the operator product expansion is well convergent.

Again we take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the first radial excited states of the flavor antitriplet heavy baryon states, which are also shown in Figs. 1-2 and Table 2. From Table 1 and Figs. 1-2, we can see that there appear rather flat platforms in the Borel windows, the uncertainties originate from the Borel parameters are rather small. The predicted masses $M_{\Lambda_b(2S)} = 6.08 \pm 0.09$ GeV, $M_{\Lambda_c(2S)} = 2.78 \pm 0.08$ GeV and $M_{\Xi_c(2S)} = 2.96 \pm 0.09$ GeV, are in excellent agreement with the experimental data $6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV, $2766.6 \pm 2.4$ MeV and $2967.1 \pm 1.4$ MeV [2, 24], and support assigning the $\Lambda_b(6072)$, $\Lambda_c(2765)$ and $\Xi_c(2980/2970)$ to be the first radial excited states of the $\Lambda_b$, $\Lambda_c$ and $\Xi_c$, respectively. The prediction $M_{\Xi_b(2S)} = 6.24 \pm 0.07$ GeV can be confronted to experimental data in the future.

If the masses of the ground states, the first radial excited states, the third radial excited states, etc of the heavy baryon states $\Lambda_Q$ and $\Xi_Q$ satisfy the Regge trajectories,

$$M_n^2 = \alpha (n - 1) + \alpha_0 ,$$

with two parameters $\alpha$ and $\alpha_0$. We take the experimental values of the masses of the ground states and the first radial excited states shown in Table 2 as input parameters to fit the $\alpha$ and $\alpha_0$, and obtain the masses of the second radial excited states, which are also shown in Table 2 as the "experimental values". From the Tables we can see that the continuum threshold parameters $\sqrt{s_0} \leq M_{\Lambda_b(3S)}$, $M_{\Lambda_c(3S)}$ and $M_{\Xi_b(3S)}$, respectively, the contaminations from the second excited states are excluded.

### 4 Conclusion

In this article, we construct the $\Lambda$-type currents to study the ground states and the first radial excited states of the spin-parity $J^P = \frac{1}{2}^+$ flavor antitriplet heavy baryon states $\Lambda_Q$ and $\Xi_Q$ by subtracting the contributions from the corresponding spin-parity $J^P = \frac{1}{2}^-$ heavy baryon states with the QCD sum rules. We carry out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way, and observe that the higher dimensional vacuum condensates play an important role, and obtain very stable QCD sum rules with variations of the Borel parameters for the ground states for the first time. Then we study the masses and pole residues of the first radial excited states in details, the predicted masses $M_{\Lambda_b(2S)} = 6.08 \pm 0.09$ GeV, $M_{\Lambda_c(2S)} = 2.78 \pm 0.08$ GeV and $M_{\Xi_c(2S)} = 2.96 \pm 0.09$ GeV are in excellent agreement with the experimental data, and support assigning the $\Lambda_b(6072)$, $\Lambda_c(2765)$ and $\Xi_c(2980/2970)$ to be the first radial excited states of the $\Lambda_b$, $\Lambda_c$ and $\Xi_c$, respectively. Finally we use the Regge trajectories to obtain the masses of the second radial excited states and observe that the continuum threshold parameters are reasonable to avoid the contaminations from the second radial excited states.
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