The stochastic resonance (SR), which describes the phenomenon that a noise of finite strength optimizes the response signal to a weak periodic external force in a nonlinear system, has attracted much attention for the past two decades. Conventional SR phenomena are generally known to require an energy barrier between two stable states, a weak external periodic force, and noise. However, the SR behavior was also exhibited without any external periodic force, in an Ising-Cycle system and excitable system such as FitzHugh-Nagumo model. Subsequently such coherence resonance or autonomous SR has also been observed in systems with delay and inertia. In the latter cases, SR phenomenon can still be understood by the time-scale matching argument since the delay or the inertia provides an external time scale. In the former, on the other hand, such an external time scale apparently does not exist, and direct noise dependencies of the activation time and the excursion time have been proposed as an explanation of the coherence resonance. Recently, the fully frustrated Josephson-Junction ladder has been proposed to give a good physical realization of the standard two-state SR system with any degrees of freedom. In the presence of the external current which is periodic in time and staggered in space, the SR behavior and other rich physics have been demonstrated theoretically in the ladder system. Furthermore, recent progress in the fabrication technique makes such Josephson-Junction system available for experimental study.

In this work we study fully frustrated Josephson-Junction ladders driven by uniform constant currents, paying particular attention to the possibility of the autonomous SR phenomenon. Note that this system is bistable, possess two stable states; this is in sharp contrast with the usual excitable system, characterized by a single stable state. Numerical integration of the coupled equations of motion, established within the resistively shunted junction (RSJ) model, shows the existence of the critical current beyond which oscillations emerge between the two ground states. Below the critical current, such oscillations are suppressed by the lattice potential; here the addition of noise currents, relevant at finite temperatures, induces again oscillations, giving rise to SR behavior. In particular, the signal-to-noise ratio (SNR) is found to display array-enhanced stochastic resonance. It is suggested that such behavior may be observed experimentally through the measurement of the staggered voltage.

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where $y (= 1, 2)$ is the leg index of the $i$th grain. Henceforth, we write the current, the temperature, and the time in units of $I_c, -I_c, 2e$, and $-2eR_1$, respectively.

In the absence of external currents, the fully frustrated $(f = 1 = 2)$ ladder is well known to have two degenerate ground states, where a vortex is located on every two plaquettes (Note that the length $L$ of a fully frustrated ladder should be an even number.) These states can be characterized by the staggered magnetization

$$m = \frac{1}{L} \sum_X (1)^X n_X \frac{1}{2};$$

where $n_X$ is the vorticity on plaquette $X$. The vorticity is given by the plaquette sum $n_X = (1 = 2)_{x} \left( I_{ij} A_{ij} \right)_{x} + 1 = 2$, where the gauge-invariant phase difference is defined modulo 2 in the range $(-\pi; \pi)$. Here uniformly applied currents in the $y$ direction drive vortices to the $x$ direction, generating a cw of the vortex array along the ladder. Due to the spatial periodicity of the vortex array, the cw in the $x$ direction results in the alternation between the two ground states, which can be manifested by the oscillation of the staggered magnetization. In contrast to a homogenous thin wire, the array system has a lattice potential which tends to suppress the cw of a vortex array. Accordingly, oscillations of the staggered magnetization should appear only when the external currents exceed the critical value $I_c = \frac{5}{2}$. The time evolution of the staggered magnetization at zero temperature, obtained from direct integration of Eq. (5) and presented in Fig. 2 for various values of $I$, indeed confirms the existence of the critical value: For small currents ($I < I_c$), the staggered magnetization remains constant, indicating that the ladder is pinned by the lattice potential to one of the two ground states. Above the critical value $I_c$, there arise oscillations of the staggered magnetization. Note also that the oscillation frequency increases with currents, reflecting that the vortex array oscilates faster at larger currents.

To probe the possibility of autonomous SR here, we consider the average rate of the change in the staggered magnetization

$$v(t) = m \frac{m(t)}{t} \frac{m(t)}{t};$$

which measures the transition rate between the ground states. The presence of periodic oscillations between the ground states is manifested by the peak in the power spectrum $S_v(\omega)$ at the corresponding frequency. We then compute the SNR:

$$R = 10 \log_{10} \frac{S}{N};$$

where the signal $S_v(\omega)$ is the peak value of the power spectrum (at the peak frequency $\nu_p$). The background noise level $N$ is estimated by the power spectrum averaged over the interval of frequency much higher than $\nu_p$, where the power spectrum is at. Equation (5) has thus been integrated directly with the time steps $t = 0.05$, from which $v(t)$ has been computed. We have also reduced $t$ to 0.1, to obtain the same results with the same accuracy, and followed an annealing schedule with the equilibration time 1000 at each temperature. The data have been averaged over 200 to 5000 independent runs, depending on the system size. For convenience, $t$ has been set to be 0.25. We have also considered smaller values of $t$, and found that the overall results remain qualitatively the same although higher noise levels make it more formidable to perform precise numerical analysis. The periodic boundary conditions have been imposed along the $x$ direction.

We next examine the effects of noise above the critical current. Figure 3 shows the power spectrum at various temperatures in the ladder of length $L = 50$ above the critical current ($I = 0.25$). The power spectrum exhibits a sharp peak even at zero temperature, arising from the current-induced oscillation. As the temperature is raised, however, the signal decreases monotonically, reflecting that the thermal noise disturbs the coherent motion of
The vortex array. Accordingly, the SNR displays monotonic decrease with the temperature, as shown in Fig. 4.

We next reduce the current $I$ to 0.2, below $I_0$. Unlike at zero temperature, there are no oscillations between the two ground states at finite temperatures, as manifested by the peak of $S_{\nu}$ (!) in Fig. 4. The signal $S$ increases as the temperature is raised from zero while further increase of the temperature tends to suppress the peak, eventually forcing the power spectrum to be white. The SNR plotted in Fig. 4 for $I = 0.2$ and lower values clearly displays autonomous SR, i.e., SR without periodic driving. Here it should be stressed that the SR observed above does not describe the direct response to the external driving. As shown in Fig. 5, the peak frequency, which is proportional to the mean square velocity of the vortex array, increases monotonically with the tem-
The autonomous SR in the fully frustrated ladder is rather peculiar, arising from the combination of the external uniform currents and the spatial periodicity of the ground state inherent in the system. Although currents smaller than \( I_0 \) do not induce the vortex array to move, they add an overall gradient to the lattice potential along the \( x \) direction. At lower temperatures, the additional noise currents assist vortices to hop to neighboring plaquettes while the potential gradient generated by the external currents makes hopping in one direction more favorable than that in the other. Consequently, the vortex array is encouraged to move in one direction, leading to oscillations of the transition rate \( v(t) \). These oscillations grow with the temperature, which gives the enhancement of the signal. At higher temperatures, however, the lattice potential gradient can be neglected, and the random nature of the noise disturbs the coherent signal, weakening the signal. In this manner, the two different roles of the noise produces the autonomous SR behavior in the system. A power spectrum also reveals the nature of the autonomous SR behavior in the system. Above the critical current, the power spectrum exhibits a coherent peak at the unfrustrated temperature, as demonstrated in Figure 8 (b). For \( I = 0 \) the coherent SR behavior of the staggered magnetization \( m(t) \) is evident.

![Figure 7](image1.png)

**Figure 7**: Peak frequency \( \omega_p \) as a function of the temperature in a ladder of length \( L = 50 \) for \( I = 0.2 \).

![Figure 8](image2.png)

**Figure 8**: Power spectrum of the staggered magnetization \( m(t) \) (in arbitrary units) at various temperatures \( T = 0.01; 0.02; 0.03; 0.04; 0.06; 0.08; 0.12; 0.16; 0.24 \) and \( 0.24 \) (from left) in a ladder of length \( L = 50 \) for \( (a) I = 0.25 \) and \( (b) I = 0.2 \).
FIG. 9: Coherence measure versus temperature in a ladder of length $L = 50$ for (a) $I = 0.25$ and (b) $I = 0.02$.

FIG. 10: SNR versus temperature for various sizes and $I = 0.02$.

FIG. 11: SNR for the staggered voltage as a function of the temperature in a ladder of length $L = 50$ for $I = 0.2$.

ders. However, the enhancement should not persist with the length since the ladder does not evolve long-range order in the thermodynamic limit. Indeed the SNR in Fig. 11 eventually saturates for $L > 30$, appearing independent of the length. While array-enhanced coherence was first reported in the array system exhibiting conventional SR, recent studies have also revealed the possibility of array-enhanced coherence resonance in coupled excitable systems. It is remarkable that similar array-enhancement of autonomous SR can also be observed in the Josephson-junction ladder, which possesses two stable states like the conventional SR system (but in the absence of periodic driving).

Finally, we discuss how to observe the autonomous SR in experiment. As a candidate for the measurable quantity characterizing the autonomous SR, we suggest the average staggered voltage:

$$V_s(t) = \frac{1}{L} \sum_x (1)^x V_x(t);$$

(9)

where $V_x(t)$ is the rung voltage at position $x$ averaged over the interval $t$ around time $t$. The rigid motion of the vortex array at low temperatures should give rise to periodic oscillations of $V_s(t)$, similarly to the transition rate $v(t)$. The SNR for the staggered voltage $V_s(t)$, denoted by $R_s$, is plotted in Fig. 11, where SR behavior similar to that of $v(t)$ is shown. We thus suggest that the autonomous SR phenomena in the ladder should be observed through the measurable of the staggered voltage.

In summary, we have investigated fully frustrated Josephson-junction ladders, driven by uniform constant currents, with regard to the possibility of stochastic resonance. While the system under large currents displays oscillations between the two ground states, the lattice potential suppresses such oscillations for all currents. Still the addition of noise currents, relevant at finite temperatures, induces again oscillations, giving rise to
stochastic resonance behavior. In particular the signal-to-noise ratio has been found to display array-enhanced stochastic resonance phenomena. It has also been suggested that such behavior may be observed experimentally through the measurement of the staggered voltage. In view of this, the fully frustrated Josephson-junction ladder makes a good physical realization of the system with many degrees of freedom, adequate for the study of the autonomous as well as the conventional stochastic resonance phenomena.

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