Robust $L_2$-$L_\infty$ Filtering for LPV Systems with Distributed Delays

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Abstract: The linear parameter-varying (LPV) systems with distributed delays are studied by the robust filter design problem. Firstly, the parameter-dependent Lyapunov theory is used to propose the $L_2$-$L_\infty$ performance criterion of time-delay correlation. Introducing the projection to understand the coupling between the Lyapunov parameter matrix and the system parameter matrix, and obtaining the $L_2$-$L_\infty$ performance criterion which is easier to solve. Then the sufficient conditions for the existence of the filter are obtained. The parameterized linear matrix inequality (PLMIs) with infinite dimension is converted to LMIs with finite dimension by using approximate basis function and network technology. Finally, the feasibility of this method is verified by numerical simulation.

1. Introduction
Filtering is a state vector used to reconstruct the system through information input and output or a certain linear combination used to estimate the state vectors of the system. In recent years, robust $L_2$-$L_\infty$ filtering has received extensive attention [1-3]. $L_2$-$L_\infty$ filtering assumes that the input signal energy and the output signal peak are both bounded, so it is also called energy-peak filtering. The linear parameter variation (LPV) system, as a typical time-variant control system, has been widely used in the systems of aircraft, communication, and industry [4-6]. However, the study on LPV systems with time-delay is still limited compared with that of LPV systems without time-delay. Reference [7] studies the $H_\infty$ and $L_2$-$L_\infty$ gain control of time-delayed LPV systems. Reference [8] designs a new $L_2$-$L_\infty$ controller. But the research on robust filtering of time-delayed LPV systems is still insufficient.

This paper mainly studies robust $L_2$-$L_\infty$ filtering with a linear parameter variation system featuring distributed time delay. The sufficient conditions for the filtering error system to achieve asymptotic stability and acquire the performance index with $L_2$-$L_\infty$ are derived based on the parametric linear matrix inequations. Then, this paper applies the projection lemma to eliminate the coupling between the system parameter matrix and the parameter dependent Lyapunov function matrix. The approximate basis function and network technology are used to numerically simulate the filter, so as to verify the feasibility and effectiveness of the designed filter.

2. Problem description
The LPV system with distributed time delay is as follows:
\[\begin{align*}
\dot{x}(t) &= A(\rho)x(t) + A_{\rho}(\rho)x(t - d(\rho)) + A_{\dot{\rho}}(\rho)\int_{-h}^{t} x(\alpha)d\alpha + B(\rho)w(t) \\
y(t) &= C(\rho)x(t) + C_{\rho}(\rho)x(t - d(\rho)) + C_{\dot{\rho}}(\rho)\int_{-h}^{t} x(\alpha)d\alpha + D(\rho)w(t) \\
z(t) &= L(\rho)x(t)
\end{align*}\]

\(x(t) \in \mathbb{R}^n\) is the system state variable; \(y(t) \in \mathbb{R}^m\) is the measured output signal; \(z(t) \in \mathbb{R}^p\) is the signal to be estimated; \(\omega(t) \in L_2[0, \infty)\). \(d(\rho)\) is the time-varying delay of the system \((0 \leq d(\rho) \leq d)\); \(h\) is the constant for describing the phenomenon of distributed time delay. \(A(\cdot), A_{\rho}(\cdot), B(\cdot), C(\cdot), C_{\rho}(\cdot), D(\cdot), \rho(\cdot), L(\cdot)\) and \(d(\cdot)\) are the functions of \(t\) (hereinafter referred to as \(\rho\)).

The formal filter is designed as:
\[\begin{align*}
\dot{x}_f(t) &= A_f(\rho)x_f(t) + B_f(\rho)y(t) \\
z_f(t) &= C_f(\rho)x_f(t)
\end{align*}\]

Select state vector as \(\xi(t) = [x^T(t) \quad x_f^T(t)]^T\), and filter error output as \(e(t) = z(t) - z_f(t)\). Combine Equations (1) and (2) and apply the idea of state augmentation to obtain the filtering error system:
\[\begin{align*}
\dot{\xi}(t) &= \overline{A}(\rho)\xi(t) + \overline{A}_{\rho}(\rho)H\dot{\xi}(t - d(\rho)) + \overline{A}_{\dot{\rho}}(\rho)\int_{-h}^{t} \xi(\alpha)d\alpha + \overline{B}(\rho)\omega(t) \\
e(t) &= \overline{L}(\rho)\xi(t)
\end{align*}\]

in which,
\[\overline{A}(\rho) = \begin{bmatrix} A(\rho) & 0 \\ B_f(\rho)C(\rho) & A_{\rho}(\rho) \end{bmatrix}, \quad \overline{A}_{\rho}(\rho) = \begin{bmatrix} A_{\rho}(\rho) \\ B_f(\rho)C_{\rho}(\rho) \end{bmatrix}, \quad \overline{A}_{\dot{\rho}}(\rho) = \begin{bmatrix} A_{\dot{\rho}}(\rho) \\ B_f(\rho)C_{\dot{\rho}}(\rho) \end{bmatrix}, \quad \overline{B}(\rho) = \begin{bmatrix} B(\rho) \\ B_f(\rho)D(\rho) \end{bmatrix}, \quad \overline{L}(\rho) = \begin{bmatrix} L(\rho) & -C_f(\rho) \end{bmatrix}, \quad H = [I \quad 0].
\]

Lemma 1 [9]:
For an arbitrary matrix with \(W > 0\), scalar \(b > a\), and vector function of \(\omega:[a,b] \rightarrow \mathbb{R}^n\), the following integral inequalities hold.
\[\int_{a}^{b} w(s)ds^T W \int_{a}^{b} w(s)ds \leq (b - a)^2 \int_{a}^{b} w^T(s)Ww(s)ds\]

Lemma 2 [10]:
For a given arbitrary matrix with \(R = R^T \geq 0\) and scalar \(\tau > 0\), the following inequalities hold.
\[\tau \int_{t - \tau}^{t} \dot{\xi}^T(s)R\dot{\xi}(s)ds \leq \begin{bmatrix} \tau & 0 \\ 0 & -\tau \end{bmatrix} \begin{bmatrix} \dot{\xi}(t) \\ \dot{\xi}(t - \tau) \end{bmatrix} \begin{bmatrix} \tau & 0 \\ 0 & -\tau \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t - \tau) \end{bmatrix}\]

Lemma 3(Projection Lemma):
For a given symmetric matrix \((\psi \in \mathbb{R}^{m \times m})\) and two matrices: \(P\) and \(Q\) with the dimension of \(m\), if \(X\) exist, then the following LMI holds.
\[\psi + P^T XQ + Q^T X^T P < 0\]

If and only if the projection inequality for \(X\) holds, \(\psi + P^T XQ + Q^T X^T P > 0\)

\(N_P\) and \(N_Q\) represent the arbitrary bases on the null space of \(P\) and \(Q\), respectively.

This paper aims to design a filter with the shape (2) for the system (1), which can make the following two conditions true:
(1) When \(w(t) \equiv 0\), the filtering error system (3) is asymptotically stable.
(2) For non-zero interference \(w(t) \in L_2[0, \infty)\), the filtering error system (3) satisfies the following
$L_2-L_{\infty}$ performance indicators:

$$\|e(t)\|_\infty < \gamma \|w(t)\|_2$$

3. Performance analysis of the filtering error system $L_2-L_{\infty}$

**Theorem 1:** Based on the filtering error system (3), if there exist normal numbers of $h$ and $d$, a positive definite matrix $P(\rho)$, and matrices with appropriate dimensions, $Q_1 > 0$, $Q_2 > 0$, $R > 0$ and $R > 0$, which makes the following inequality groups true:

$$\Phi = \begin{bmatrix}
\Phi_{11} & P(\rho)A(\rho) + H^T S & 0 & P(\rho)A(\rho) & P(\rho)B(\rho) & d\bar{\gamma}(\rho)H^T S \\
* & - (1 - \sum_{i=1}^{n} (\tau_i \frac{\partial d(\rho)}{\partial \rho_i}))Q_1 - S & 0 & 0 & 0 & d\bar{\gamma}(\rho)H^T S \\
* & * & - Q_2 & 0 & 0 & 0 \\
* & * & * & - h^2 R & 0 & d\bar{\gamma}(\rho)H^T S \\
* & * & * & * & - I & dB(\rho)H^T S \\
* & * & * & * & * & - S
\end{bmatrix} < 0 \tag{4}$$

$$\begin{bmatrix}
P(\rho) & \bar{L}(\rho) \\
* & \gamma \Gamma
\end{bmatrix} > 0 \tag{5}$$

Then, the filtering error system is asymptotically stable and satisfies the $L_2-L_{\infty}$ with the noise suppression level of $\gamma$.

Among them, $\Phi_{11} = P\overline{A}(\rho) + \overline{A}(\rho)P + H^T (Q_1 + Q_2 + hR + S)H + \sum_{i=1}^{n} (\tau_i \frac{\partial P(\rho)}{\partial \rho_i})$

**Proof:**

Select the Lyapunov functional as follows:

$$V(t) = \sum_{i=1}^{n} V_i(t), \quad V_1(t) = \xi^T(t)P(\rho)\xi(t), \quad V_2(t) = \int_{-d(\rho)}^{d(\rho)} \xi^T(\alpha)H^T Q_1 H\xi(\alpha)d\alpha,$$

$$V_3(t) = \int_{-d(\rho)}^{d(\rho)} \int_{-d(\rho)}^{d(\rho)} \xi^T(\alpha)H^T Q_2 H\xi(\alpha)d\alpha,$$

$$V_4(t) = \int_{-d(\rho)}^{d(\rho)} \int_{-d(\rho)}^{d(\rho)} \xi^T(\alpha)H^T R H\xi(\alpha)d\alpha,$$

$$V_5(t) = \int_{0}^{h} \int_{-d(\rho)}^{d(\rho)} \xi^T(\alpha)H^T S H\xi(\alpha)d\alpha$$

Derive the above functional:

$$\dot{V}_1(t) = \dot{\xi}^T(t)P(\rho)\dot{\xi}(t) + \xi^T(t)P(\rho)\dot{\xi}(t) + \xi^T(t)P(\rho)\dot{\xi}(t),$$

$$\dot{V}_2(t) = \xi^T(t)H^T Q_1 H\xi(t) - (1 - \sum_{i=1}^{n} (\tau_i \frac{\partial d(\rho)}{\partial \rho_i}))\xi^T(t - d(\rho))H^T Q_1 H\xi(t - d(\rho)),$$

$$\dot{V}_3(t) = \xi^T(t)H^T Q_2 H\xi(t) - h^2 \xi^T(t - h)H^T Q_2 H\xi(t - h),$$

$$\dot{V}_4(t) = h^2 \xi^T(t)H^T R H\xi(t) - \int_{-h}^{h} \xi^T(\alpha)H^T R H\xi(\alpha)d\alpha,$$

$$\dot{V}_5(t) = h^2 \xi^T(t)H^T S H\xi(t) - \int_{-h}^{h} \xi^T(\alpha)H^T S H\xi(\alpha)d\alpha.$$

According to Jensen Inequation (Lemma 2), it can be obtained that:

$$-d\int_{-h}^{h} \xi^T(\alpha)H^T S H\xi(\alpha)d\alpha \leq \left[\begin{array}{c}
\xi(t) \\
\xi(t - d(\rho))
\end{array}\right]^T \left[
\begin{array}{cc}
\bar{L}(\rho) & * \\
* & \gamma \Gamma
\end{array}\right] \left[
\begin{array}{c}
\xi(t) \\
\xi(t - d(\rho))
\end{array}\right]$$

Apply Lemma 1, then:

$$\int_{-h}^{h} \xi^T(\alpha)R\xi(\alpha)d\alpha \leq h^2 \left(\int_{-h}^{h} \xi(\alpha)d\alpha\right)^2 R\left(\int_{-h}^{h} \xi(\alpha)d\alpha\right)$$

From $\dot{V}(t)$, we can get:

$$\dot{V}(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \leq \eta^T(t)\Phi\eta(t)$$
Among them: \( \eta^T(t) = [\xi^T(t) \quad \xi^T(t - d(\rho))H^T \quad \xi^T(t - h)H^T \quad \int_{t-h}^t x(\alpha)d\alpha)^T H^T \quad w^T(t)] \),

\[
\Phi = \begin{bmatrix}
\phi_{11} & P(\rho)\bar{A}_d(\rho) + H^T S & 0 & 0 & 0 \\
-1 & -Q_2 & 0 & 0 & 0 \\
* & * & -h^{-1}R & 0 & 0 \\
* & * & * & 0 & 0 \\
* & * & * & * & 0 \\
\end{bmatrix} + d^2 \begin{bmatrix}
\bar{A}_d^T(\rho) \\
\bar{A}_h^T(\rho) \\
\bar{A}_h^T(\rho) \\
\bar{B}_h^T(\rho) \\
\bar{B}_h^T(\rho) \\
\end{bmatrix} H^T SH \\
\begin{bmatrix}
P(\rho)A_h(\rho) \\
P(\rho\bar{A}_h(\rho)) \\
P(\rho\bar{A}_h(\rho)) \\
P(\rho\bar{A}_h(\rho)) \\
P(\rho\bar{A}_h(\rho)) \\
\end{bmatrix}
\]

Define the following performance indicators:

\[
J = V(t) - \int_0^T w^T(s)w(s)ds
\]

Under the zero initial condition, \( V(t) = \int_0^T \dot{V}(s)ds \). For any non-zero \( w(t) \in L_2[0, \infty) \ (t > 0) \), the following formula holds:

\[
J = \int_0^T \left[ \dot{V}(s) - w^T(s)w(s) \right] ds = \int_0^T \eta^T(t)\Phi(t)\eta(t)ds < 0
\]

and so, \( V(t) < \int_0^T w^T(s)w(s)ds \)

From Schur complement lemma, we can get that Equation (5) is equivalent to \( \bar{L}_d^T(\rho)\bar{L}_d(\rho) < \gamma^2 P(\rho) \), then \( \|e(t)\|_2 < \gamma \|w(t)\|_2 \).

In summary, Equations (4) and (5) can ensure that the filtering error system (3) is asymptotically stable and satisfies the given performance index. Thus, Theorem 1 is proved.

4. Filter design of filtering error system \( L_2-L_\infty \)

**Theorem 2:** Based on the filtering error system (3) (\( d > 0 \)), the sufficient condition for the existence of a stable system meeting \( L_2-L_\infty \) performance indicators is the presence of the positive definite symmetric matrices \( P(\rho), Q_i, R, S \) and the general matrix \( W \), which satisfy the Equations (5) and (6):

\[
\begin{bmatrix}
-W - W^T & P(\rho) + W^T \bar{A}_d(\rho) & W^T \bar{A}_h(\rho) & W^T \bar{A}_h(\rho) & W^T \bar{B}(\rho) & W^T \\
* & -P(\rho) + \Omega & H^T S & 0 & 0 & 0 \\
* & * & \phi_{22} & 0 & 0 & 0 \\
* & * & * & -Q_2 & 0 & 0 \\
* & * & * & * & -I & 0 \\
* & * & * & * & * & -P(\rho) \\
* & * & * & * & * & -S \\
\end{bmatrix} < 0
\]

Among them: \( \phi_{22} = -(1 - \sum_{i-1}^t (\tau_i \frac{\partial d(\rho)}{\partial \rho_i}))Q_i - S \), \( \Omega = H^T (Q_i + Q_2 + hR + S)H + \sum_{i=1}^\infty (\tau_i \frac{\partial P(\rho)}{\partial \rho_i}) \)

**Proof:**

Apply Lemma 3, the above formula is equivalent to the following formula

\[
\psi + P^T XQ + Q^T X^T P < 0
\]

In which: \( P = [-I, \bar{A}(\rho), \bar{A}_d(\rho), \bar{A}_h(\rho), \bar{B}(\rho), I], Q = [I, 0, 0, 0, 0, 0] \)
Based on the filtering error system (3), ensure the asymptotic stability of the system, meaning that the performance criteria for decoupling are obtained. From Schur complement lemma, we can get that Formula (7) is equivalent to the first 5×5 sub-parameter matrix inequation of Equation (6). Equation (7) contains Equation (8), so the matrix is defined as:

\[
\Lambda(\rho) = \begin{bmatrix} N_p^T & 0 \\ 0 & I \end{bmatrix}
\]

Then,

\[
N_p^T \psi N_p < 0 \\
N_p^T \psi N_q < 0
\]

From Schur complement lemma, we can get that Formula (7) is equivalent to the first 5×5 sub-parameter matrix inequation of Equation (6). Equation (7) contains Equation (8), so the matrix is defined as:

\[
\Lambda(\rho) = \begin{bmatrix} N_p^T & 0 \\ 0 & I \end{bmatrix}
\]

Congruently transform Inequation (6) using Matrix (9), so that we can get Equation (4) in Theorem 1. In this way, Equation (6) can ensure the asymptotic stability of the system, meaning that the performance criteria for decoupling are obtained.

**Theorem 3**: Based on the filtering error system (3) \((d > 0)\), the sufficient condition for the existence of a stable system meeting \(L_2-L_\infty\) performance indicators is the presence of the positive definite symmetric matrices \(P(\rho), Q, S\) and the general matrix \(R, F, U, \bar{A}_f(\rho), \bar{B}_f(\rho), \bar{C}_f(\rho)\), which satisfy the Equations (10) and (11):

\[
\begin{bmatrix} \bar{P}_{11}(\rho) & \bar{P}_{12}(\rho) & L(\rho) \\ * & \bar{P}_{22}(\rho) & -\bar{C}_f(\rho) \end{bmatrix} > 0
\]

\[
\begin{bmatrix} -R - R^T & -F - U^T & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & R^T & U^T & dS \\ * & -U - U^T & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & F^T & U^T & 0 \\ * & * & \Xi_{33} & \Xi_{34} & S & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{P}_{11}(\rho) & -\bar{P}_{12}(\rho) & 0 \\ * & * & * & * & * & * & * & * & -\bar{P}_{22}(\rho) & 0 \\ * & * & * & * & * & * & * & * & * & -S \end{bmatrix} < 0
\]

In which: \(\Xi_{13} = R^T \bar{A}_f(\rho) + \bar{B}_f(\rho) \bar{C}(\rho) + \bar{P}_{11}(\rho)\), \(\Xi_{14} = \bar{A}_f(\rho) + \bar{P}_{12}(\rho)\).
$\Xi_{15} = R^T A_3(\rho) + \overline{B}_3(\rho) C_3(\rho)$, $\Xi_{16} = R^T A_4(\rho) + \overline{B}_4(\rho) C_4(\rho)$, $\Xi_{17} = R^T B(\rho) + \overline{B}_f(\rho) D(\rho)$, $\Xi_{23} = F^T A_3(\rho) + \overline{B}_3(\rho) C_3(\rho) + \overline{P}_{11}(\rho)$, $\Xi_{24} = \overline{A}_4(\rho) + P_{22}(\rho) \overline{B}_f(\rho) C_f(\rho)$, $\Xi_{25} = F^T A_4(\rho) + \overline{B}_4(\rho) C_4(\rho)$, $\Xi_{26} = F^T A_4(\rho) + \overline{B}_f(\rho) C_f(\rho)$, $\Xi_{27} = F^T B(\rho) + \overline{B}_f(\rho) D(\rho)$, $\Xi_{33} = -\overline{P}_{11}(\rho) + Q_1 + Q_2 + h R + S + \sum_{i=1}^{34} \left( \tau_i \frac{\partial \overline{P}_{11}(\rho)}{\partial \rho_i} \right)$, $\Xi_{34} = -\overline{P}_{12}(\rho) + \sum_{i=1}^{34} \left( \tau_i \frac{\partial \overline{P}_{12}(\rho)}{\partial \rho_i} \right)$, $\Xi_{44} = -\overline{P}_{22}(\rho) + \sum_{i=1}^{34} \left( \tau_i \frac{\partial \overline{P}_{22}(\rho)}{\partial \rho_i} \right)$, $\Xi_{55} = -(1 - \sum_{i=1}^{34} \left( \tau_i \frac{\partial d(\rho)}{\partial \rho_i} \right)) Q_3 - S$

**Proof:**

If there exist filter matrices $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$ while $P(\rho)>0$, $Q>0$, $S>0$ satisfy Inequations (5) and (6), it is necessary to partition $W$ and $P$ in Theorem 2 to maintain generality:

$$P(\rho) = \begin{bmatrix} P_{11}(\rho) & P_{12}(\rho) \\ P_{21}(\rho) & P_{22}(\rho) \end{bmatrix}, \quad W = \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix}$$

Assume $W_1$ and $W_4$ are reversible, then the matrix is defined as

$$J = \begin{bmatrix} I & 0 \\ 0 & W_4^{-1}W_3 \end{bmatrix}$$

and

$$\overline{P}(\rho) = J^T P(\rho) J = \begin{bmatrix} \overline{P}_{11}(\rho) & \overline{P}_{12}(\rho) \\ \overline{P}_{21}(\rho) & \overline{P}_{22}(\rho) \end{bmatrix} \begin{bmatrix} \overline{A}_f(\rho) & \overline{B}_f(\rho) \\ \overline{C}_f(\rho) & 0 \end{bmatrix} = \begin{bmatrix} W_1^T & 0 & A_f(\rho) & B_f(\rho) & W_4^{-1} & 0 \\ 0 & I & C_f(\rho) & 0 & 0 & I \end{bmatrix}$$

$$R = W_1, \quad F = W_2W_4^{-1}W_3, \quad U = W_3W_4^{-1}W_3$$

Congruently transform Equations (6) and (5) using $\text{diag}\{J J J J J J\}$ and $\text{diag}\{J I\}$, then substitute Equations (12), (13) and (14) to obtain Equations (11) and (10). In this way, Theorem 3 is proved.

The filter parameters can be solved by the following equations:

$$A_f(\rho) = U^{-T} \overline{A}_f(\rho), \quad B_f(\rho) = U^{-T} \overline{B}_f(\rho), \quad C_f(\rho) = \overline{C}_f(\rho)$$

**5. Numerical simulation**

Based on the distributed time-delayed LPV system like form (1), the system parameter matrix is

$$A = \begin{bmatrix} 0.1p(t) & -1 \\ -2 & -3+0.1\rho(t) \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.2\rho(t) & 0.1 \\ -0.2+0.1\rho(t) & -0.3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0.1 \\ -0.2+0.1\rho(t) & -0.3 \end{bmatrix}, \quad B = \begin{bmatrix} -0.6 \\ 0.2\rho(t) \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0.2+0.1\rho(t) \\ 0.2 & -0.1+0.1\rho(t) \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & 1+0.1\rho(t) \end{bmatrix}, \quad D = 2+0.1\rho(t), \quad L = \begin{bmatrix} 0.8+0.1\rho(t) & 0.5+0.2\rho(t) \end{bmatrix}$$

as follows:

$\rho(t) = \sin t$ is the time-varying parameter, which satisfies $\rho(t) \in [-1,1]$ and $\dot{\rho}(t) \in [-1,1]$; the time-varying delay is $d(\rho) = 0.3\rho(t)$. With the aid of approximate basis functions and network technology, then we can get following conditions:

$$Y(\rho) = \sum_{j=1}^{k} f_j(\rho)Y_j > 0$$

Select basis functions: $f_1(\rho) = 1$ and $f_2(\rho) = \rho(t)$

When $h = 0.1$, the performance index is $\gamma = 1.8914$ based on MATLAB, and the corresponding $L_2-L_\infty$ filter parameter matrix is:
The simulation results are as shown in Figures 1 and 2, in which the filtering error is \( e(t) = z(t) - z_f(t) \).

6. Conclusion

This paper mainly studies robust \( L_2-L_\infty \) filtering with a linear parameter variation system featuring distributed time delay. The sufficient conditions for the filtering error system to achieve asymptotic stability and acquire the performance index with \( L_2-L_\infty \) are derived based on the parametric linear matrix inequations. Then, this paper applies the projection lemma to eliminate the coupling between the system parameter matrix and the parameter dependent Lyapunov function matrix. The approximate basis function and network technology are used to numerically simulate the filter, so as to verify the feasibility and effectiveness of the designed filter.

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