Elementary teachers’ experience of engaging with Teaching Through Problem Solving using Lesson Study

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Received: 22 September 2021 / Revised: 14 January 2022 / Accepted: 2 April 2022 / Published online: 13 May 2022
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Abstract

For many decades, problem solving has been a focus of elementary mathematics education reforms. Despite this, in many education systems, the prevalent approach to mathematics problem solving treats it as an isolated activity instead of an integral part of teaching and learning. In this study, two mathematics teacher educators introduced 19 Irish elementary teachers to an alternative problem solving approach, namely Teaching Through Problem Solving (TTP), using Lesson Study (LS) as the professional development model. The findings suggest that the opportunity to experience TTP first-hand within their schools supported teachers in appreciating the affordances of various TTP practices. In particular, teachers reported changes in their beliefs regarding problem solving practice alongside developing problem posing knowledge. Of particular note was teachers’ contention that engaging with TTP practices through LS facilitated them to appreciate their students’ problem solving potential to the fullest extent. However, the planning implications of the TTP approach presented as a persistent barrier.

Keywords Teaching through Problem Solving (TTP) · Problem solving · Lesson Study (LS) · Elementary teachers · Professional development

Introduction

A fundamental goal of mathematics education is to develop students’ ability to engage in mathematical problem solving. Despite curricular emphasis internationally on problem solving, many teachers are uncertain how to harness students’ problem solving potential (Cheeseman, 2018). While many problem solving programmes focus on providing students with step-by-step supports through modelling,
heuristics, and other structures (Polya, 1957), Goldenberg et al. (2001) suggest that the most effective approach to developing students’ problem solving ability is by providing them with frequent opportunities over a prolonged period to solve worthwhile open-ended problems that are challenging yet accessible to all. This viewpoint is in close alignment with reform mathematics perspectives that promote conceptual understanding, where students actively construct their knowledge and relate new ideas to prior knowledge, creating a web of connected knowledge (Hiebert, 2003; Lester, 2013; Takahashi, 2006; Watanabe, 2001).

There is consensus in the mathematics education community that problem solving should not be taught as an isolated topic focused solely on developing problem solving skills and strategies or presented as an end-of-chapter activity (Takahashi, 2006, 2016; Takahashi et al., 2013). Instead, problem solving should be integrated across the curriculum as a fundamental part of mathematics teaching and learning (Cai & Lester, 2010; Takahashi, 2016).

A ‘Teaching Through Problem Solving’ (TTP) approach, a problem solving style of instruction that originated in elementary education in Japan, meets these criteria treating problem solving as a core practice rather than an ‘add-on’ to mathematics instruction.

**Teaching Through Problem Solving (TTP)**

Teaching Through Problem Solving (TTP) is considered a powerful means of promoting mathematical understanding as a by-product of solving problems, where the teacher presents students with a specially designed problem that targets certain mathematics content (Stacey, 2018; Takahashi et al., 2013). The lesson implementation starts with the teacher presenting a problem and ensuring that students understand what is required. Students then solve the problem either individually or in groups, inventing their approaches. At this stage, the teacher does not model or suggest a solution procedure. Instead, they take on the role of facilitator, providing support to students only at the right time (Hiebert, 2003; Lester, 2013; Takahashi, 2006). As students solve the problem, the teacher circulates, observes the range of student strategies, and identifies work that illustrates desired features. However, the problem solving lesson does not end when the students find a solution. The subsequent sharing phase, called Neriage (polishing ideas), is considered by Japanese teachers to be the heart of the lesson rather than its culmination. During Neriage, the teacher purposefully selects students to share their strategies, compares various approaches, and introduces increasingly sophisticated solution methods. Effective questioning is central to this process, alongside careful recording of the multiple solutions on the board. The teacher concludes the lesson by formalising and consolidating the lesson’s main points. This process promotes learning for all students (Hiebert, 2003; Stacey, 2018; Takahashi, 2016; Takahashi et al., 2013; Watanabe, 2001).

The TTP approach assumes that students develop, extend, and enrich their understandings as they confront problematic situations using existing knowledge. Therefore, TTP fosters the symbiotic relationship between conceptual understanding and
problem solving, as conceptual understanding is required to solve challenging problems and make sense of new ideas by connecting them with existing knowledge. Equally, problem solving promotes conceptual understanding through the active construction of knowledge (Hiebert, 2003; Lambdin, 2003; Takahashi, 2006). Consequently, students simultaneously develop more profound understandings of the mathematics content while cultivating problem solving skills (Kapur, 2010; Stacey, 2018).

Relevant research affirms that teachers acknowledge the merits of this approach (Sullivan et al., 2014) and most students report positive experiences (Russo & Minas, 2020). The process is considered to make students’ thinking and learning visible (Ingram et al., 2020). Engagement in TTP has resulted in teachers becoming more aware of and confident in their students’ problem solving abilities and subsequently expecting more from them (Crespo & Featherstone, 2006; Sakshaug & Wohlhuter, 2010).

Demands of TTP

Adopting a TTP approach challenges pre-existing beliefs and poses additional knowledge demands for elementary teachers, both content and pedagogical (Takahashi, 2008).

Research has consistently reported a relationship between teacher beliefs and the instructional techniques used, with evidence of more rule-based, teacher-directed strategies used by teachers with traditional mathematics beliefs (Stipek et al., 2001; Swan, 2006; Thompson, 1985). These teachers tend to address problem solving separately from concept and skill development and possess a simplistic view of problem solving as translating a problem into abstract mathematical terms to solve it. Consequently, such teachers ‘are very concerned about developing skilfulness in translating (so-called) real-world problems into mathematical representations and vice versa’ (Lester, 2013, p. 254). Early studies of problem solving practice reported direct instructional techniques where the teacher would model how to solve the problem followed by students practicing similar problems (Chapman, 2015; Hiebert, 2003; Lester, 2013). This naïve conception of problem solving is reflected in many textbook problems that simply require students to apply previously learned routine procedures to solve problems that are merely thinly disguised number operations (Lester, 2013; Singer & Voica, 2013). Hence, the TTP approach requires a significant shift for teachers who previously considered problem solving as an extra activity conducted after the new mathematics concepts are introduced (Lester, 2013; Takahashi et al., 2013) or whose personal experience of problem solving was confined to applying routine procedures to word problems (Sakshaug & Wohlhuter, 2010).

Alongside beliefs, teachers’ knowledge influences their problem solving practices. Teachers require a deep understanding of the nature of problem solving, in particular viewing problem solving as a process (Chapman, 2015). To be able to understand the stages problem solvers go through and appreciate what
successful problem solving involves, teachers benefit from experiencing solving problems from the problem solver’s perspective (Chapman, 2015; Lester, 2013). It is also essential that teachers understand what constitutes a worthwhile problem when selecting or posing problems (Cai, 2003; Chapman, 2015; Lester, 2013; O’Shea & Leavy, 2013). This requires an understanding that problems are ‘mathematical tasks for which the student does not have an obvious way to solve it’ (Chapman, 2015, p. 22). Teachers need to appreciate the variety of problem characteristics that contribute to the richness of a problem, e.g. problem structures and cognitive demand (Klein & Leiken, 2020; O’Shea & Leavy, 2013). Such understandings are extensive, and rather than invest heavily in the time taken to construct their mathematics problems, teachers use pre-made textbook problems or make cosmetic changes to make cosmetic changes to these (Koichu et al., 2013). In TTP, due consideration must also be given to the problem characteristics that best support students in strengthening existing understandings and experiencing new learning of the target concept, process, or skill (Cai, 2003; Takahashi, 2008). Specialised content knowledge is also crucial for teachers to accurately predict and interpret various solution strategies and misconceptions/errors, to determine the validity of alternative approaches and the source of errors, to sequence student approaches, and to synthesise approaches and new learning during the TTP lesson (Ball et al., 2008; Cai, 2003; Leavy & Hourigan, 2018).

Teachers should also be knowledgeable regarding appropriate problem solving instruction. It is common for teachers to teach for problem solving (i.e., focusing on developing students’ problem solving skills and strategies). Teachers adopting a TTP approach engage in reform classroom practices that reflect a constructivist-oriented approach to problem solving instruction where the teacher guides students to work collaboratively to construct meaning, deciding when and how to support students without removing their autonomy (Chapman, 2015; Hiebert, 2003; Lester, 2013). Teachers ought to be aware of the various relevant models of problem solving, including Polya’s (1957) model that supports teaching for problem solving (Understand the problem-Devise a plan-Carry out the plan-Look back) alongside models that support TTP (e.g., Launch-Explore-Summarise) (Lester, 2013; Sullivan et al., 2021). While knowledge of heuristics and strategies may support teachers’ problem solving practices, there is consensus that teaching heuristics and strategies or teaching about problem solving does not significantly improve students’ problem solving ability. Teachers require a thorough knowledge of their students as problem solvers, for example, being aware of their abilities and factors that hinder their success, including language (Chapman, 2015). Knowledge of content and student, alongside content and teaching (Ball et al., 2008), is essential during TTP planning when predicting student approaches and errors. Such knowledge is also crucial during TTP implementation when determining the validity of alternative approaches, identifying the source of errors (Explore phase), sequencing student approaches, and synthesising the range of approaches and new learning effectively (Summarise phase) (Cai, 2003; Leavy & Hourigan, 2018).
Supports for teachers

Given the extensive demands of TTP, adopting this approach is arduous in terms of the planning time required to problem pose, predict approaches, and design questions and resources (Lester, 2013; Sullivan et al., 2010; Takahashi, 2008). Consequently, it is necessary to support teachers who adopt a TTP approach (Hiebert, 2003). Professional development must facilitate them to experience the approach themselves as learners and then provide classroom implementation opportunities that incorporate collaborative planning and reflection when trialling the approach (Watanabe, 2001). In Japan, a common form of professional development to promote, develop, and refine TTP implementation among teachers and test potential problems for TTP is Japanese Lesson Study (LS) (Stacey, 2018; Takahashi et al., 2013). Another valuable support is access to a repository of worthwhile problems. In Japan, government-authorised textbooks and teacher manuals provide a sequence of lessons with rich well-tested problems to introduce new concepts. They also detail alternative strategies used by students and highlight the key mathematical aspects of these strategies (Takahashi, 2016; Takahashi et al., 2013).

Teachers’ reservations about TTP

Despite the acknowledged benefits of TTP for students, some teachers report reluctance to employ TTP, identifying a range of obstacles. These include limited mathematics content knowledge or pedagogical content knowledge (Charalambous, 2008; Sakshaug & Wohlhuter, 2010) and a lack of access to resources or time to develop or modify appropriate resources (Ingram et al., 2020; Russo & Hopkins, 2019; Sullivan et al., 2015). Other barriers for teachers with limited experience of TTP include giving up control, struggling to support students without directing them, and a tendency to demonstrate how to solve the problem (Cheeseman, 2018; Crespo & Featherstone, 2006; Klein & Leiken, 2020; Takahashi et al., 2013). Resistance to TTP is also associated with some teachers’ perception that this approach would lead to student disengagement and hence be unsuitable for lower-performing students (Ingram et al., 2020; Russo & Hopkins, 2019; Sullivan et al., 2010).

Problem solving practices in Irish elementary mathematics education

Within the Irish context, problem solving is a central tenet of elementary mathematics curriculum documents (Department of Education and Science (DES), 1999) with recommendations that problem solving should be integral to students’ mathematical learning. However, research reveals a mismatch between intended and implemented problem solving practices (Dooley et al., 2014; Dunphy et al., 2014), where classroom practices reflect a narrow approach limited to problem solving as an ‘add on’, only applied after mathematical procedures had been learned and where problems are predominantly sourced from dedicated sections of textbooks (Department of Education and Skills (DES), 2011; Dooley et al., 2014; National Council for Curriculum and Assessment (NCCA), 2016; O’Shea & Leavy, 2013). Regarding the
attained curriculum, Irish students have underperformed in mathematical problem solving, relative to other skills, in national and international assessments (NCCA, 2016; Shiel et al., 2014). Consensus exists that there is scope for improvement of problem solving practices, with ongoing calls for Irish primary teachers to receive support through school-based professional development models alongside creating a repository of quality problems (DES, 2011; Dooley et al., 2014; NCCA, 2016).

Lesson Study (LS) as a professional development model

Reform mathematics practices, such as TTP, challenge many elementary teachers’ beliefs, knowledge, practices, and cultural norms, particularly if they have not experienced the approach themselves as learners. To support teachers in enacting reform approaches, they require opportunities to engage in extended and targeted professional development involving collaborative and practice-centred experiences (Dudley et al., 2019; Murata et al., 2012; Takahashi et al., 2013). Lesson Study (LS) possesses the characteristics of effective professional development as it embeds “…teachers’ learning in their everyday work…increasing the likelihood that their learning will be meaningful” (Fernandez et al., 2003, p. 171).

In Japan, LS was developed in the 1980s to support teachers to use more student-centred practices. LS is a school-based, collaborative, reflective, iterative, and research-based form of professional development (Dudley et al., 2019; Murata et al., 2012). In Japan, LS is an integral part of teaching and is typically conducted as part of a school-wide project focused on addressing an identified teaching–learning challenge (Takahashi & McDougal, 2016). It involves a group of qualified teachers, generally within a single school, working together as part of a LS group to examine and better understand effective teaching practices. Within the four phases of the LS cycle, the LS group works collaboratively to study and plan a research lesson that addresses a pre-established goal before implementing (teach) and reflecting (observe, analyse and revise) on the impact of the lesson activities on students’ learning.

LS has become an increasingly popular professional development model outside of Japan in the last two decades. In these educational contexts, it is necessary to find a balance between fidelity to LS as originally envisaged and developing a LS approach that fits the cultural context of a country’s education system (Takahashi & McDougal, 2016).

Relevant research examining the impact of LS on qualified primary mathematics teachers reports many benefits. Several studies reveal that teachers demonstrated transformed beliefs regarding effective pedagogy and increased self-efficacy in their use due to engaging in LS (Cajkler et al., 2015; Dudley et al., 2019; Fernandez, 2005; Gutierez, 2016). Enhancements in participating teachers’ knowledge have also been reported (Cajkler et al., 2015; Dudley et al., 2019; Fernandez, 2005; Gutierez, 2016; Murata et al., 2012). Other gains recounted include improvements in practice with a greater focus on students (Cajkler et al., 2015; Dudley et al., 2019; Flanagan, 2021).
Elementary teachers’ experience of engaging with Teaching…

Context of this study

A cluster of urban schools, coordinated by their local Education Centre, engaged in an initiative to enhance teachers’ mathematics problem solving practices. The co-ordinator of the initiative approached the researchers, both mathematics teacher educators (MTEs), seeking a relevant professional development opportunity. Aware of the challenges of problem solving practice within the Irish context, the MTEs proposed an alternative perspective on problem solving: the Teaching Through Problem Solving (TTP) approach. Given Cai’s (2003) recommendation that teachers can best learn to teach through problem solving by teaching and reflecting as opposed to taking more courses, the MTEs identified LS as the best fit in terms of a supportive professional development model, as it is collaborative, experiential, and school-based (Dudley et al., 2019; Murata et al., 2012; Takahashi et al., 2013). Consequently, LS would promote teachers to work collaboratively to understand the TTP approach, plan TTP practices for their educational context, observe what it looks like in practice, and assess the impact on their students’ thinking (Takahashi et al., 2013). In particular, the MTEs believed that the LS phases and practices would naturally support TTP structures, emphasizing task selection and anticipating students’ solutions. Given Lester’s (2013) assertion that each problem solving experience a teacher engages in can potentially alter their knowledge for teaching problem solving, the MTEs sought to explore teachers’ perceptions of the impact of engaging with TTP through LS on their beliefs regarding problem solving and their knowledge for teaching problem solving.

Research questions

This paper examines two research questions:

Research question 1: What are elementary teachers’ reported problem solving practices prior to engaging in LS?  
Research question 2: What are elementary teachers’ perceptions of what they learned from engaging with TTP through LS?

Methodology

Participants

The MTEs worked with 19 elementary teachers (16 female, three male) from eight urban schools. Schools were paired to create four LS groups on the basis of the grade taught by participating class teachers, e.g. Grade 3 teacher from school 1 paired with Grade 4 teacher from school 2. Each LS group generally consisted of 4–5 teachers, with a minimum of two teachers from each school, along with the two MTEs. For most teachers, LS and TTP were new practices being implemented...
concurrently. However, given the acknowledged overlap between the features of the TTP and LS approaches, for example, the focus on problem posing and predicting student strategies, the researchers were confident that the content and structure were compatible. Also, in Japan, LS is commonly used to promote TTP implementation among teachers (Stacey, 2018; Takahashi et al., 2013).

All ethical obligations were adhered to throughout the research process, and the study received ethical approval from the researchers’ institutional board. Of the 19 participating LS teachers invited to partake in the research study, 16 provided informed consent to use their data for research purposes.

**LS process**

Over eight weeks, the MTEs worked with teachers, guiding each LS group through the four LS phases involving study, design, implementation, and reflection of a research lesson that focused on TTP while assuming the role of ‘knowledgeable others’ (Dudley et al., 2019; Hourigan & Leavy, 2021; Takahashi & McDougal, 2016). An overview of the timeline and summary of each LS phase is presented in Table 1.

**LS phase 1: Study**

This initial study phase involved a one-day workshop. The process and benefits of LS as a school-based form of professional development were discussed in the morning session and the afternoon component was spent focusing on the characteristics of TTP. Teachers experienced the TPP approach first-hand by engaging in the various lesson stages. For example, they solved a problem (growing pattern problem) themselves in pairs and shared their strategies. They also predicted children’s approaches to the problem and possible misconceptions and watched the video cases of TTP classroom practice for this problem. Particular focus was placed on the importance of problem selection and prediction of student strategies before the lesson implementation and the Neriage stage of the lesson. Teachers also discussed readings related to

| Table 1 Lesson Study phases and timeline |
|------------------------------------------|
| **Lesson Study Phases**                  | **Organisation of participants** | **Week 1** | **Week 2** | **Week 3** | **Week 4** | **Week 5** | **Week 6** | **Week 7** | **Week 8** |
| Phase 1: Study                          | Presentation to all Lesson Study participants |            |            |            |            |            |            |            |            |
| Phase 2: Planning                       | Lesson Study group meetings (2 per group) |            |            |            |            |            |            |            |            |
| Phase 3: Implementation                 | Research lessons implemented (teach 1 & 2) |            |            |            |            |            |            |            |            |
| Phase 4: Reflection                     | Lesson Study group reflection and presentation |            |            |            |            |            |            |            |            |

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LS practices (e.g., Lewis & Tsuchida, 1998) and TTP (e.g., Takahashi, 2008). At the end of the workshop, members of each LS group were asked to communicate among themselves and the MTEs, before the planning phase, to decide the specific mathematics focus of their LS group’s TTP lesson (Table 1).

**LS phase 2: Planning**

The planning phase was four weeks in duration and included two 1½ hour face-to-face planning sessions (i.e. planning meetings 1 and 2) between the MTEs and each LS group (Table 1). Meetings took place in one of the LS group’s schools. At the start of the first planning meeting, time was dedicated to Takahashi’s (2008) work focusing on the importance of problem selection and prediction of student strategies to plan the Neriage stage of the TTP lesson. The research lesson plan structure was also introduced. Ertle et al.’s (2001) four column lesson plan template was used. It was considered particularly compatible with the TTP approach, given the explicit attention to expected student response and the teacher’s response to student activity/response.

The planning then moved onto the content focus of each LS group’s TTP research lesson. LS groups selected TTP research lessons focusing on number (group A), growing patterns (group B), money (group C), and 3D shapes (group D). Across the planning phase, teachers invested substantial time extensively discussing the TTP lesson goals in terms of target mathematics content, developing or modifying a problem to address these goals, and exploring considerations for the various lesson stages. Drawing on Takahashi’s (2008) article, it was re-emphasised that no strategies would be explicitly taught before students engaged with the problem. While one LS group modified an existing problem (group B) (Hourigan & Leavy, 2015), the other three LS groups posed an original problem. To promote optimum teacher readiness to lead the Neriage stage, each LS group was encouraged to solve the problem themselves in various ways considering possible student strategies and their level of mathematical complexity, thus identifying the most appropriate sequence of sharing solutions.

**LS phase 3: Implementation**

The *implementation* phase involved one teacher in each LS group teaching the research lesson (teach 1) in their school. The remaining group members and MTEs observed and recorded students’ responses. Each LS group and the MTEs met immediately for a post-lesson discussion to evaluate the research lesson. The MTEs presented teachers with a series of focus questions: What were your observations of student learning? Were the goals of the lesson achieved? Did the problem support students in developing the appropriate understandings? Were there any strategies/errors that we had not predicted? How did the Neriage stage work? What aspects of the lesson plan should be reconsidered based on this evidence? Where appropriate, the MTEs drew teachers’ attention to particular lesson aspects they had not noticed. Subsequently, each LS group revised their research lesson in response to the observations, reflections, and discussion. The revised lesson was retaught 7–10 days later by a second group member from the paired LS group school (teach 2) (Table 1).
The post-lesson discussion for teach 2 focused mainly on the impact of changes made after the first implementation on student learning, differences between the two classes, and further changes to the lesson.

**LS phase 4: Reflection**

While reflection occurred after both lesson implementations, the final reflection involved all teachers from the eight schools coming together for a half-day meeting in the local Education Centre to share their research lessons, experiences, and learning (Table 1). Each LS group made a presentation, identifying their research lesson’s content focus and sequence of activity. Artefacts (research lesson plan, materials, student work samples, photos) were used to support observations, reflections, and lesson modifications. During this meeting, teachers also reflected privately and in groups on their initial thoughts and experience of both LS and TTP, the benefits of participation, the challenges they faced, and they provided suggestions for future practice.

**Data collection**

The study was a collective case study (Stake, 1995). Each LS group constituted a case; thus, the analysis was structured around four cases. Data collection was closely aligned with and ran concurrent to the LS process. Table 2 details the links between the LS phases and the data collection process.

The principal data sources (Table 2) included both MTEs’ fieldnotes (phase (P) 1–4), and reflections (P1–4), alongside email correspondence (P1–4), individual teacher reflections (P1, 2, 4) (see reflection tasks in Table 3), and LS documentation including various drafts of lesson plans (P2–4) and group presentations (P4). Fieldnotes refer to all notes taken by MTEs when working with the LS groups, for example, during the study session, planning meetings, lesson implementations, post-lesson discussions, and the final reflection session.

The researchers were aware of the limitations of self-report data and the potential mismatch between one’s perceptions and reality. Furthermore, data in the form of opinions, attitudes, and beliefs may contain a certain degree of bias. However, this

| Data source | Phase 1 Study | Phase 2 Planning | Phase 3 Implementation | Phase 4 Reflection |
|-------------|---------------|------------------|------------------------|-------------------|
| MTE field notes | X | X | X | X |
| MTE reflections | X | X | X | X |
| Documentation (lesson plans, presentation) | | X | X | X |
| Teacher reflection | X | X | | X |
| Email-correspondence | X | X | X | X |

*MTE mathematics teacher educator*
paper intentionally focuses solely on the teachers’ perceived learning in order to represent their ‘lived experience’ of TTP. Despite this, measures were taken to assure the trustworthiness and rigour of this qualitative study. The researchers engaged with the study over a prolonged period and collected data for each case (LS group) at every LS phase (Table 2). All transcripts reflected verbatim accounts of participants’ opinions and reflections. At regular intervals during the study, research meetings interrogated the researchers’ understandings, comparing participating teachers’ observations and reflections to promote meaning-making (Creswell, 2009; Suter, 2012).

### Data analysis

The MTEs’ role as participant researchers was considered a strength of the research given that they possessed unique insights into the research context. A grounded theory approach was adopted, where the theory emerges from the data analysis process rather than starting with a theory to be confirmed or refuted (Glaser, 1978; Strauss & Corbin, 1998). Data were examined focusing on evidence of participants’ problem solving practices prior to LS and their perceptions of their learning as a result of engaging with TTP through LS. A systematic process of data analysis was adopted. Initially, raw data were organised into natural units of related data under various codes, e.g. resistance, traditional approach, ignorance, language, planning, fear of student response, relevance, and underestimation. Through successive examinations of the relationship between existing units, codes were amalgamated (Creswell, 2009). Progressive drafts resulted in the firming up of several themes. Triangulation was used to establish consistency across multiple data sources. While the first theme, *Vast divide between prevalent problem solving practices and TTP*, addresses research question 1, it is considered an overarching theme, given the impact of teachers’ established problem solving understandings and practices on their receptiveness to and experience of TTP. The remaining five themes (Seeing is believing: the value of practice centred experiences; A gained appreciation of the relevance and value of

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**Table 3 Teacher reflection focus questions**

| Lesson Study phase | Focus questions |
|-------------------|-----------------|
| Phase 1: Study phase | Describe your experiences of teaching problem solving to date in your teaching career. Describe the successes. Describe the challenges. |
| Phase 2: Planning phase | What aspects of TTP have been beneficial to date? What aspects of TTP have been challenging to date? Additional comments. |
| Phase 4: Reflection phase | What were your initial thoughts on TTP? What was your experience of TTP? What are your thoughts about TTP now? What did you learn from engaging in TTP? Does TTP have a role in your problem solving practice? |
TTP practices; Enhanced problem posing understandings; Awakening to students’ problem solving potential; and Reservations regarding TTP) represent a generalised model of teachers’ perceived learning due to engaging with TTP through LS, thus addressing research question 2. Although one of the researchers was responsible for the initial coding, both researchers met regularly during the analysis to discuss and interrogate the established codes and to agree on themes. This process served to counteract personal bias (Suter, 2012).

As teacher reflections were anonymised, it was not possible to track teachers across LS phases. Consequently, teacher reflection data are labelled as phase and instrument only. For example, ‘P2, teacher reflection’ communicates that the data were collected during LS phase 2 through teacher reflection. However, the remaining data are labelled according to phase, instrument, and source, e.g. ‘P3, fieldnotes: group B’. While phase 4 data reflect teachers’ perceptions after engaging fully with the TTP approach, data from the earlier phases reflect teachers’ evolving perceptions at a particular point in their unfolding TTP experience.

**Discussion of findings**

The findings draw on the analysis of the data collected across the LS phases and address the research questions. Within the confines of this paper, illustrative quotes are presented to provide insights into each theme. An additional layer of analysis was completed to ensure a balanced representation of teachers’ views in reporting findings. This process confirmed that the findings represent the views of teachers across LS groups, for example, within the first theme presented (Vast divide), the eight quotes used came from eight different teacher reflections. Equally, the six fieldnote excerpts selected represent six different teachers’ views across the four LS groups. Furthermore, in the second theme (Seeing is believing), the five quotes presented were sourced from five different participating teachers’ reflections and the six fieldnote excerpts included are from six different teachers across the four LS groups. Subsequent examination of the perceptions of those teachers not included in the reporting of findings confirmed that their perspectives were represented within the quotes used. Hence, the researchers are confident that the findings represent the views of teachers across all LS groups. For each theme, sources of evidence that informed the presented conclusions will be outlined.

**Vast divide between prevalent problem solving practices and TTP**

This overarching theme addresses the research question ‘What were elementary teachers’ reported problem solving practices prior to engaging in LS?’.

At the start of the initiative, within the study session (fieldnotes), all teachers identified mathematics problem solving as a problem of practice. The desire to develop problem solving practices was also apparent in some teachers’ reflections (phase 1 (P1), N=8):
I am anxious about it. Problem solving is an area of great difficulty throughout our school (P1, teacher reflection).

During both study and planning phase discussions, across all LS groups, teachers’ reports suggested the almost exclusive use of a teaching for problem solving approach, with no awareness of the Teaching Through Problem Solving (TTP) approach; a finding also evidenced in both teacher reflections (P1, N=7) and email correspondence:

Unfamiliar, not what I am used to. I have no experience of this kind of problem solving. This new approach is the reverse way to what I have used for problem solving (P1, teacher reflection)

Being introduced to new methods of teaching problem solving and trying different approaches is both exciting and challenging (P1: email correspondence)

Teachers’ descriptions of their problem solving classroom practices in both teacher reflections (P1, N=8) and study session discussions (fieldnotes) suggested a naïve conception of problem solving, using heuristics such as the ‘RUDE (read, underline, draw a picture, estimate) strategy’ (P1, fieldnotes) to support students in decoding and solving the problem:

In general, the problem solving approach described by teachers is textbook-led, where concepts are taught context free first and the problems at the end of the chapter are completed afterwards (P1, reflection: MTE2)

This approach was confirmed as widespread across all LS groups within the planning meetings (fieldnotes).

In terms of problem solving instruction, a teacher-directed approach was reported by some teachers within teacher reflections (P1, N=5), where the teacher focused on a particular strategy and modelled its use by solving the problem:

I tend to introduce the problem, ensure everyone understands the language and what is being asked. I discuss the various strategies that children could use to solve the problem. Sometimes I demonstrate the approach. Then children practice similar problems … (P1: Teacher reflection)

However, it was evident within the planning meetings, that this traditional approach to problem solving was prevalent among the teachers in all LS groups. During the study session (field notes and teacher reflections (P1, N=7)), there was a sense that problem solving was an add-on as opposed to an integral part of mathematics teaching and learning. Again, within the planning meetings, discussions across all four LS groups verified this:

Challenge: Time to focus on problems not just computation (P1: Teacher reflection).

From our discussions with the various LS groups’ first planning meeting, text-based teaching seems to be resulting in many teachers teaching concepts context-free initially and then matching the concept with the relevant problems afterwards (P2, reflection: MTE1)
However, while phase 1 teacher reflections suggested that a small number of participating teachers ($N=4$) possessed broader problem solving understandings, subsequently during the planning meetings, there was ample evidence (field notes) of problem-posing knowledge and the use of constructivist-oriented approaches that would support the TTP approach among some participating teachers in each of the LS groups:

Challenge: Spend more time on meaningful problems and give them opportunities and time to engage in activities, rather than go too soon into tricks, rhymes etc (P1, Teacher reflection).

The class are already used to sharing strategies and explaining where they went wrong (P2, fieldnotes, Group B)

Teacher: The problem needs to have multiple entry points (P2, fieldnotes: Group C)

While a few teachers reported problem posing practices, in most cases, this consisted of cosmetic adjustments to textbook problems. Overall, despite evidence of some promising practices, the data evidenced predominantly traditional problem solving views and practices among participating teachers, with potential for further broadening of various aspects of their knowledge for teaching problem solving including what constitutes a worthwhile problem, the role of problem posing within problem solving, and problem solving instruction. Within phase 1 teacher reflections, when reporting ‘challenges’ to problem solving practices (Table 3), a small number of responses ($N=3$) supported these conclusions:

Differences in teachers’ knowledge (P1: Teacher reflection).

Need to challenge current classroom practices (P1: Teacher reflection).

However, from the outset, all participating teachers consistently demonstrated robust knowledge of their students as problem solvers, evidenced in phase 1 teacher reflections ($N=10$) and planning meeting discussions (P2, fieldnotes). However, in these early phases, teachers generally portrayed a deficit view, focusing almost exclusively on the various challenges impacting their students’ problem solving abilities. While all teachers agreed that the language of problems was inhibiting student engagement, other common barriers reported included student motivation and perseverance:

They often have difficulties accessing the problem – they don’t know what it is asking them (P2, fieldnotes: Group C)

Sourcing problems that are relevant to their lives. I need to change every problem to reference soccer so the children are interested (P1: teacher reflection)

Our children deal poorly with struggle and are slow to consider alternative strategies (P2, fieldnotes: Group D)

Despite showcasing a strong awareness of their students’ problem solving difficulties, teachers initially demonstrated a lack of appreciation of the benefits accrued from predicting students’ approaches and misconceptions relating
to problem solving. While it came to the researchers’ attention during the study phase, its prevalence became apparent during the initial planning meeting, as its necessity and purpose was raised in three of the LS groups:

What are the benefits of predicting the children’s responses? (P1, fieldnotes).
I don’t think we can predict- we will have to wait and see (P2, fieldnotes: Group A).

This finding evidences teachers’ relatively limited knowledge for teaching problem solving, given that this practice is fundamental to TTP and constructivist-oriented approaches to problem solving instruction.

**Perceived impacts of engaging with TTP through LS**

In response to the research question ‘What are elementary teachers’ perceptions of what they learned from engaging with TTP through LS?’, thematic data analysis identified 5 predominant themes, namely, *Seeing is believing: the value of practice centred experiences; A gained appreciation of the relevance and value of TTP practices; Enhanced problem posing understandings; Awakening to students’ problem solving potential; and Reservations regarding TTP.*

**Seeing is believing: the value of practice centred experiences**

Teachers engaged with TTP during the study phase as both learners and teachers when solving the problem. They were also involved in predicting and analysing student responses when viewing the video cases, and engaged in extensive reading, discussion, and planning for their selected TTP problem within the planning phase. Nevertheless, teachers reported reservations about the relevance of TTP for their context within both phase 2 teacher reflections (N=5) as well as within the planning meeting discourse of all LS groups. Teachers’ keen awareness of their students’ problem solving challenges, coupled with the vast divide between the nature of their prior problem solving practices and the TTP approach, resulted in teachers communicating concern regarding students’ possible reaction during the planning phase:

I am worried about the problem. I am concerned that if the problem is too complex the children won’t respond to it (P2, fieldnotes: Group B)
The fear that the children will not understand the lesson objective. Will they engage? (P2, Teacher reflection)

Acknowledging their apprehension regarding students’ reactions to TTP, from the outset, all participating teachers communicated a willingness to trial TTP practices:

Exciting to be part of. Eager to see how it will pan out and the learning that will be taken from it (P1, teacher reflection)
They should be ‘let off’ (P2, fieldnotes: Group A).

It was only within the implementation phase, when teachers received the opportunity to meaningfully observe the TTP approach in their everyday work context,
with their students, that they explicitly demonstrated an appreciation for the value of TTP practices. It was evident from teacher commentary across all LS groups’ post-lesson discussions (fieldnotes) as well as in teacher reflections (P4, \( N=10 \)) that observing first-hand the high levels of student engagement alongside students’ capacity to engage in desirable problem solving strategies and demonstrate sought-after dispositions had affected this change:

Class teacher: They engaged the whole time because it was interesting to them. The problem is core in terms of motivation. It determines their willingness to persevere. Otherwise, it won’t work whether they have the skills or not (P3, fieldnotes: Group C)

LS group member: The problem context worked really well. The children were all eager and persevered. It facilitated all to enter at their own level, coming up with ideas and using their prior knowledge to solve the problem. Working in pairs and the concrete materials were very supportive. It’s something I’d never have done before (P3, fieldnotes: Group A)

Although all teachers showcased robust knowledge of their students’ problem solving abilities prior to engaging in TTP, albeit with a tendency to focus on their difficulties and factors that inhibited them, teachers’ contributions during post-lesson discussions (fieldnotes) alongside teacher reflections (P4, \( N=9 \)) indicate that observing TTP in action supported them in developing an appreciation of value of the respective TTP practices, particularly the role of prediction and observation of students’ strategies/misconceptions in making the students’ thinking more visible:

You see the students through the process (P3, fieldnotes: Group C)

It’s rare we have time to think, to break the problem down, to watch and understand children’s ways of thinking/solving. It’s really beneficial to get a chance to re-evaluate the teaching methods, to edit the lesson, to re-teach (P4, teacher reflection)

Analysis of the range of data sources across the phases suggests that it was the opportunity to experience TTP in practice in their classrooms that provided the ‘proof of concept’:

I thought it wasn’t realistic but bringing it down to your own classroom it is relevant (P4, teacher reflection).

Hence from the teachers’ perspective, they witnessed the affordances of TTP practices in the implementation phase of the LS process.

A gained appreciation of the relevance and value of TTP practices

While during the early LS phases, teachers’ reporting suggested a view of problem solving as teaching to problem solve, data from both fieldnotes (phases 3 and 4) and teacher reflections (phase 4) demonstrate that all teachers broadened their understanding of problem solving as a result of engaging with TTP:
Interesting to turn lessons on their head and give students the chance to think, plan and come up with possible strategies and solutions (P4, Teacher reflection)

On witnessing the affordances of TTP first-hand in their own classrooms, within both teacher reflections (P4, N=12) and LS group presentations, the teachers consistently reported valuing these new practices:

I just thought the whole way of teaching was a good way, an effective way of teaching. Sharing and exploring more than one way of solving is vital (P4, teacher reflection)

There is a place for it in the classroom. I will use aspects of it going forward (P4, fieldnotes: Group C)

In fact, teachers’ support for this problem solving approach was apparent in phase 3 during the initial post-lesson discussions. It was particularly notable when a visitor outside of the LS group who observed teach 1 challenged the approach, recommending the explicit teaching of strategies prior to engagement. A LS group member’s reply evidenced the group’s belief that TTP naturally exposes students to the relevant learning: ‘Sharing and questioning will allow students to learn more efficient strategies [other LS group members nodding in agreement]’ (P3, fieldnotes; Group A).

In turn, within phase 4 teacher reflections, teachers consistently acknowledged that engaging with TTP through LS had challenged their understandings about what constitutes effective problem solving instruction (N=12). In both teacher reflections (P4, N=14) and all LS group presentations, teachers reported an increased appreciation of the benefits of adopting a constructivist-oriented approach to problem solving instruction. Equally for some, this was accompanied by an acknowledgement of a heightened awareness of the limitations of their previous practice:

Really made me re-think problem solving lesson structures. I tend to spoon-feed them …over-scaffold, a lot of teacher talk. … I need to find a balance… (P4, teacher reflection)

Less is more, one problem can be the basis for an entire lesson (P4, teacher reflection)

What was unexpected, was that some teachers (P4, N=8) reported that engaging with TTP through LS resulted in them developing an increased appreciation of the value of problem solving and the need for more regular opportunities for students to engage in problem solving:

I’ve come to realise that problem solving is critical and it should be focused on more often. I feel that with regular exposure to problems they’ll come to love being problem solvers (P4, teacher reflection)

Enhanced problem posing understandings

In the early phases of LS, few teachers demonstrated familiarity with problem characteristics (P2 teacher reflection, N=5). However, there was growth in teachers’ understandings of what constitutes a worthwhile problem and its role within TTP
within all LS groups’ post-lesson discussions and presentations (fieldnotes) and teacher reflections (P4, N=10):

I have a deepened understanding of how to evaluate a problem (P2, teacher reflection)
It’s essential to find or create a good problem with multiple strategies and/or solutions as a springboard for a topic. It has to be relevant and interesting for the kids (P4, teacher reflection)

As early as the planning phase, a small group of teachers’ reflections (N=2) suggested an understanding that problem posing is an important aspect of problem solving that merits significant attention:

It was extremely helpful to problem solve the problem (P2, teacher reflection)

However, during subsequent phases, this realisation became more mainstream, evident within all LS groups’ post-lesson discussions and presentations (fieldnotes) and teacher reflections (P4, N=12):

During the first planning meeting, I was surprised and a bit anxious that we would never get to having created a problem. In hindsight, this was time well spent as the problem was crucial (P4, teacher reflection)
I learned the problem is key. We don’t spend enough time picking the problem (P4, fieldnotes: Group C).

Alongside this, in all LS groups’ dialogues during the post-lesson discussion and presentations (fieldnotes) and teacher reflections (P4, N=15), teachers consistently demonstrated an enhanced awareness of the interdependence between the quality of the problem and students’ problem solving behaviours:

Better perseverance if the problem is of interest to them (P4, teacher reflection)
It was an eye-opener to me, relevance is crucial, when the problem context is relevant to them, they are motivated to engage and can solve problems at an appropriate level…They all wanted to present (P3, fieldnotes: Group C)

The findings suggest that engaging with TTP through LS facilitated participating teachers to develop an enhanced understanding of the importance of problem posing and in identifying the features of a good mathematics problem, thus developing their future problem posing capacity. In essence, the opportunity to observe the TTP practices in their classrooms stimulated an enhanced appreciation for the value of meticulous attention to detail in TTP planning.

Awakening to students’ problem solving potential

In the final LS phases, teachers consistently reported that engaging with TTP through LS provided the opportunity to see the students through the process, thus supporting them in examining their students’ capabilities more closely. Across post-lesson discussions and presentations (fieldnotes) and teacher reflections (P4, N=14), teachers acknowledged that engagement in core TTP practices, including problem posing, prediction of
students’ strategies during planning, and careful observation of approaches during the implementation phase, facilitated them to uncover the true extent of their students’ problem solving abilities, heightening their awareness of students’ proficiency in using a range of approaches:

Class teacher: While they took a while to warm up, I am most happy that they failed, tried again and succeeded. They all participated. Some found a pattern, others used trial and error. Others worked backward- opening the cube in different ways. They said afterward ‘That was the best maths class ever’ (P3, fieldnotes: Group D)

I was surprised with what they could do. I have learned the importance of not teaching strategies first. I need to pull back and let the children solve the problems their own way and leave discussing strategies to the end (P4, teacher reflection)

In three LS groups, class teachers acknowledged in the post-lesson discussion (fieldnotes) that engaging with TTP had resulted in them realising their previous underestimation of [some or all] of their students’ problem solving abilities. Teacher reflections (P4, N=8) and LS group presentations (fieldnotes) also acknowledged this reality:

I underestimated my kids, which is awful. The children surprised me with the way they approached the problem. In the future I need to focus on what they can do as much as what might hinder them…they are more able than we may think (P4, reflection)

In all LS groups, teachers reported that their heightened appreciation of students’ problem solving capacities promoted them to use a more constructivist-orientated approach in the future:

I learned to trust the students to problem solve, less scaffolding. Children can be let off to explore without so much teacher intervention (P3, fieldnotes: Group D)

Some teachers (N=3) also acknowledged the affective benefits of TTP on students:

I know the students enjoyed sharing their different strategies…it was great for their confidence (P4, teacher reflection)

Interestingly, in contrast with teachers’ initial reservations, their experiential and school-based participation in TTP through LS resulted in a lessening of concern regarding the suitability of TTP practices for their students. Hence, this practice-based model supported teachers in appreciating the full extent of their students’ capacities as problem solvers.

Reservations regarding TTP

When introduced to the concept of TTP in the study session, one teacher quickly addressed the time implications:

It is unrealistic in the everyday classroom environment. Time is the issue. We don’t have 2 hours to prep a problem geared at the various needs (P1, fieldnotes)
Subsequently, across the initiative, during both planning meetings, the reflection session and individual reflections (P4, N = 14), acknowledgements of the affordances of TTP practices were accompanied by questioning of its sustainability due to the excessive planning commitment involved:

It would be hard to maintain this level of planning in advance of the lesson required to ensure a successful outcome (P4, teacher reflection)

Given the extensive time dedicated to problem posing, solving, prediction, and design of questions as well as selection or creation of materials both during and between planning meetings, there was agreement in the reflection session (field-notes) and in teacher reflections (P4, N = 10) that while TTP practices were valuable, in the absence of suitable support materials for teachers, adjustments were essential to promote implementation:

There is definitely a role for TTP in the classroom, however the level of planning involved would have to be reduced to make it feasible (P4, teacher reflection)

The TTP approach is very effective but the level of planning involved is unrealistic with an already overcrowded curriculum. However, elements of it can be used within the classroom (P4, teacher reflection)

A few teachers (N = 3) had hesitations beyond the time demands, believing the success of TTP is contingent on ‘a number of criteria…’ (P4, teacher reflection):

A whole-school approach is needed, it should be taught from junior infants (P4, teacher reflection)
I still have worries about TTP. We found it difficult to decide a topic initially. It lends itself to certain areas. It worked well for shape and space (P4, teacher reflection)

Conclusions

The reported problem solving practice reflects those portrayed in the literature (NCCA, 2016; O’Shea & Leavy, 2013) and could be aptly described as ‘pendulum swings between emphases on basic skills and problem solving’ (Lesh & Zawojewski, 2007 in Takahashi et al., 2013, p. 239). Teachers’ accounts depicted problem solving as an ‘add on’ occurring on an ad hoc basis after concepts were taught (Dooley et al., 2014; Takahashi et al., 2013), suggesting a simplistic view of problem solving (Singer & Voica, 2013; Swan, 2006). Hence, in reality there was a vast divide between teachers’ problem solving practices and TTP. Alongside traditional beliefs and problem solving practices (Stipek et al., 2001; Swan, 2006; Thompson, 1985), many teachers demonstrated limited insight regarding what constitutes a worthwhile problem (Klein & Lieken, 2020) or the critical role of problem posing in problem solving (Cai, 2003; Takahashi, 2008; Watson & Ohtani, 2015). Teachers’ reports suggested most were not actively problem posing, with reported practices limited to cosmetic changes to the problem context (Koichu et al., 2013). Equally, teachers
demonstrated a lack of awareness of alternative approaches to teaching for problem solving (Chapman, 2015) alongside limited appreciation among most of the affordances of a more child-centred approach to problem solving instruction (Hiebert, 2003; Lester, 2013; Swan, 2006). Conversely, there was evidence that some teachers held relevant problem posing knowledge and utilised practices compatible with the TTP approach.

All teachers displayed relatively strong understandings of their students as problem solvers from the outset; however, they initially focused almost exclusively on factors impacting students’ limited problem solving capacity (Chapman, 2015). Teachers’ perceptions of their students’ problem solving abilities alongside the vast divide between teachers’ problem solving practice and the proposed TTP approach resulted in teachers being initially concerned regarding students’ response to TTP. This finding supports studies that reported resistance by teachers to the use of challenging tasks due to fears that students would not be able to manage (Ingram et al., 2020; Russo & Hopkins, 2019; Sullivan et al., 2010). Equally, teachers communicated disquiet from the study phase regarding the time investment required to adopt the TTP approach, a finding common in similar studies (Ingram et al., 2020; Russo & Hopkins, 2019; Sullivan et al., 2015). Hence, the transition to TTP was uneasy for most teachers, given the significant shift it represented in terms of moving beyond a teaching to problem solve approach alongside the range of teacher demands (Takahashi et al., 2013).

Nevertheless, despite initial reservations, all teachers reported that engagement with TTP through LS affected their problem solving beliefs and understandings. What was particularly notable was that they reported an awakening to students’ problem solving potential. During LS’s implementation and reflection stages, all teachers acknowledged that seeing was believing concerning the benefits of TTP for their students (Kapur, 2010; Stacey, 2018). In particular, they recognised students’ positive response (Russo & Minas, 2020) enacted in high levels of engagement, perseverance in finding a solution, and the utilisation of a range of different strategies. These behaviours were in stark contrast to teachers’ reports in the study phase. Teachers acknowledged that students had more potential to solve problems autonomously than they initially envisaged. This finding supports previous studies where teachers reported that allowing students to engage with challenging tasks independently made students’ thinking more visible (Crespo & Featherstone, 2006; Ingram et al., 2020; Sakshand & Wohluter, 2010). It also reflects Sakshaug and Wohlhuter’s (2010) findings of teachers’ tendency to underestimate students’ potential to solve problems. Interestingly, at the end of LS, concern regarding the appropriateness of the TTP approach for students was no longer cited by teachers. This finding contrasts with previous studies that report teacher resistance due to fears that students will become disengaged due to the unsuitability of the approach (challenging tasks) for lower-performing students (Ingram et al., 2020; Russo & Hopkins, 2019; Sullivan et al., 2015). Hence, engaging with TTP through LS supported teachers in developing an appreciation of their students’ potential as problem solvers.

Teachers reported enhanced problem posing understandings, consisting of newfound awareness of the connections between the quality of the problem, the approach to problem solving instruction, and student response (Chapman, 2015;
They acknowledged that they had learned the importance of the problem in determining the quality of learning and affecting student engagement, motivation and perseverance, and willingness to share strategies (Cai, 2003; Watson & Oktani, 2015). These findings reflect previous research reporting that engagement in LS facilitated teachers to enhance their teacher knowledge (Cajkler et al., 2015; Dudley et al., 2019; Gutierez, 2016).

While all teachers acknowledged the benefits of the TTP approach for students (Cai & Lester, 2010; Sullivan et al., 2014; Takahashi, 2016), the majority confirmed their perception of the relevance and value of various TTP practices (Hiebert, 2003; Lambdin, 2003; Takahashi, 2006). They referenced the benefits of giving more attention to the problem, allowing students the opportunity to independently solve, and promoting the sharing of strategies and pledged to incorporate these in their problem solving practices going forward. Many verified that the experience had triggered them to question their previous problem solving beliefs and practices (Chapman, 2015; Lester, 2013; Takahashi et al., 2013). This study supports previous research reporting that LS challenged teachers’ beliefs regarding the characteristics of effective pedagogy (Cajkler et al., 2015; Dudley et al., 2019; Fernandez, 2005; Gutierez, 2016). However, teachers communicated reservations regarding TTP, refraining from committing to TTP in its entirety, highlighting that the time commitment required for successful implementation on an ongoing basis was unrealistic. Therefore, teachers’ issues with what they perceived to be the excessive resource implications of TTP practices remained constant across the initiative. This finding supports previous studies that report teachers were resistant to engaging their students with ‘challenging tasks’ provided by researchers due to the time commitment required to plan adequately (Ingram et al., 2020; Russo & Hopkins, 2019; Sullivan et al., 2015).

Unlike previous studies, teachers in this study did not perceive weak mathematics content or pedagogical content knowledge as a barrier to implementing TTP (Charalambous, 2008; Sakshaug & Wohlhuter, 2010). However, it should be noted that the collaborative nature of LS may have hidden the knowledge demands for an individual teacher working alone when engaging in the ‘Anticipate’ element of TTP particularly in the absence of appropriate supports such as a bank of suitable problems.

The findings suggest that LS played a crucial role in promoting reported changes, serving both as a supportive professional development model (Stacey, 2018; Takahashi et al., 2013) and as a catalyst, providing teachers with the opportunity to engage in a collaborative, practice-centred experience over an extended period (Dudley et al., 2019; Watanabe, 2001). The various features of the LS process provided teachers with opportunities to engage with, interrogate, and reflect upon key TTP practices. Reported developments in understandings and beliefs were closely tied to meaningful opportunities to witness first-hand the affordances of the TTP approach in their classrooms with their students (Dudley et al., 2019; Fernandez et al., 2003; Takahashi et al., 2013). We suggest that the use of traditional ‘one-off’ professional development models to introduce TTP, combined with the lack of support during the implementation phase, would most likely result in teachers maintaining their initial views about the unsuitability of TTP practices for their students.
In terms of study limitations, given that all data were collected during the LS phases, the findings do not reflect the impact on teachers’ problem solving classroom practice in the medium to long term. Equally, while acknowledging the limitations of self-report data, there was no sense that the teachers were trying to please the MTEs, as they were forthright when invited to identify issues. Also, all data collected through teacher reflection was anonymous. The relatively small number of participating teachers means that the findings are not generalisable. However, they do add weight to the body of relevant research. This study also contributes to the field as it documents potential challenges associated with implementing TTP for the first time. It also suggests that despite TTP being at odds with their problem solving practice and arduous, the opportunity to experience the impact of the TTP approach with students through LS positively affected teachers’ problem solving understandings and beliefs and their commitment to incorporating TTP practices in their future practice. Hence, this study showcases the potential role of collaborative, school-based professional development in supporting the implementation of upcoming reform proposals (Dooley et al., 2014; NCCA, 2016, 2017, 2020), in challenging existing beliefs and practices and fostering opportunities for teachers to work collaboratively to trial reform teaching practices over an extended period (Cajkler et al., 2015; Dudley et al., 2019). Equally, this study confirms and extends previous studies that identify time as an immense barrier to TTP. Given teachers’ positivity regarding the impact of the TTP approach, their consistent acknowledgement of the unsustainability of the unreasonable planning demands associated with TTP strengthens previous calls for the development of quality support materials in order to avoid resistance to TTP (Clarke et al., 2014; Takahashi, 2016).

The researchers are aware that while the reported changes in teachers’ problem solving beliefs and understandings are a necessary first step, for significant and lasting change to occur, classroom practice must change (Sakshaug & Wohlhuter, 2010). While it was intended that the MTEs would work alongside interested teachers and schools to engage further in TTP in the school term immediately following this research and initial contact had been made, plans had to be postponed due to the commencement of the COVID 19 pandemic. The MTEs are hopeful that it will be possible to pick up momentum again and move this initiative to its natural next stage. Future research will examine these teachers’ perceptions of TTP after further engagement and evaluate the effects of more regular opportunities to engage in TTP on teachers’ problem solving practices. Another possible focus is teachers’ receptiveness to TTP when quality support materials are available.

In practical terms, in order for teachers to fully embrace TTP practices, thus facilitating their students to avail of the many benefits accrued from engagement, teachers require access to professional development (such as LS) that incorporates collaboration and classroom implementation at a local level. However, quality school-based professional development alone is not enough. In reality, a TTP approach cannot be sustained unless teachers receive access to quality TTP resources alongside formal collaboration time.

Acknowledgements The authors acknowledge the participating teachers’ time and contribution to this research study.
Funding  This work was supported by the Supporting Social Inclusion and Regeneration in Limerick’s Programme Innovation and Development Fund.

Declarations

Ethical approval  We have received ethical approval for the research presented in this manuscript from Mary Immaculate College Research Ethical Committee (MIREC).

Consent for publication  The manuscript has only been submitted to Mathematics Education Research Journal. All authors have approved the manuscript submission. We also acknowledge that the submitted work is original and the content of the manuscript has not been published or submitted for publication elsewhere.

Informed consent  Informed consent has been received for all data included in this study. Of the 19 participating teachers, 16 provided informed consent.

Conflict of interest  The authors declare no competing interests.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education., 59(5), 389–408.
Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. Lester (Ed.), Research and issues in teaching mathematics through problem solving (pp. 241–255). National Council for Curriculum and Assessment.
Cai, J., & Lester, F. (2010). Why is teaching through problem solving important to children learning? National Council of Teachers of Mathematics.
Cajkler, W., Wood, P., Norton, J., Pedder, D., & Xu, H. (2015). Teacher perspectives about lesson study in secondary school departments: A collaborative vehicle for professional learning and practice development. Research Papers in Education, 30(2), 192–213.
Chapman, O. (2015). Mathematics teachers’ knowledge for teaching problem solving. LUMAT International Journal on Math Science and Technology Education, 3(1), 19–36.
Charalambous, C. Y. (2008). Mathematical knowledge for teaching and the unfolding of tasks in mathematics lessons: Integrating two lines of research. In O. Figueras, J. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), Proceedings of the 32nd conference of the International Group for the Psychology of Mathematics Education (pp. 281–288). PME.
Cheeseman, J. (2018). Teachers’ perceptions of obstacles to incorporating a problem solving style of mathematics into their teaching, In J. Hunter, P. Perger, & L. Darragh (Eds.), Making waves, opening spaces (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia) (pp. 210–217). MERGA.
Clarke, D., Cheeseman, J., Roche, A., & Van Der Schans, S. (2014). Teaching strategies for building student persistence on challenging tasks: Insights emerging from two approaches to teacher professional learning. Mathematics Teacher Education and Development, 16(2), 46–70.
Creswell, J. W. (2009). Research design: Qualitative, quantitative and mixed methods approaches (3rd ed). Sage.
Crespo, S., & Featherstone, H. (2006). Teacher learning in mathematics teacher groups: One math problem at a time. In K. Lynch-Davis & R. L. Rider (Eds.), The work of mathematics teacher educators: Continuing the conversation (pp. 97–115). Association of Mathematics Teacher Educators.
Department of Education and Science (DES). (1999). Primary school mathematics curriculum. The Stationery Office.
Department of Education and Skills (DES). (2011). Literacy and numeracy for learning and life: The national strategy to improve literacy and numeracy among children and young people 2011–2020. The Stationery Office.

Springer
Dooley, T., Dunphy, E., & Shiel, G. (2014). Mathematics in early childhood and primary education. Research report 18. National Council for Curriculum and Assessment.

Dudley, P., Xu, H., Vermunt, J. D., & Lang, J. (2019). Empirical evidence of the impact of lesson study on students' achievement, teachers' professional learning and on institutional and system evolution. European Journal of Education, 54, 202–217. https://doi.org/10.1111/ejed.12337

Dunphy, E., Dooley, T., Shiel, G. (2014). Mathematics in early childhood and primary education. Research report 17. National Council for Curriculum and Assessment.

Ertle, B., Chokshi, S., & Fernandez, C. (2001). Lesson planning tool. Available online: https://sarahbsd.files.wordpress.com/2014/09/lesson_planning_tool.pdf

Fernandez, C. (2005). Lesson Study: A means for elementary teachers to develop the knowledge of mathematics needed for reform-minded teaching? Mathematical Thinking and Learning, 7(4), 265–289.

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US–Japan lesson study collaboration reveals critical lenses for examining practice. Teaching and Teacher Education, 19, 171–185.

Flanagan, B. (2021). Teachers' understandings of lesson study as a professional development tool, Unpublished thesis, University of Limerick.

Glaser, B. G. (1978). Theoretical sensitivity: Advances in the methodology of grounded theory. Sociology Press.

Goldenberg, E. P., Shteingold, N., & Feurzig, N. (2001). Mathematical habits of mind for young children. In F. Lester (Ed.), Research and issues in teaching mathematics through problem solving (pp. 15–30). National Council for Curriculum and Assessment.

Gutiérrez, S. B. (2016). Building a classroom-based professional learning community through lesson study: Insights from elementary school science teachers. Professional Development in Education, 42(5), 801–817.

Hiebert, J. (2003). Signposts for teaching mathematics through problem solving. In F. Lester (Ed.), Research and issues in teaching mathematics through problem solving (pp. 53–62). National Council for Curriculum and Assessment.

Hourigan, M., & Leavy, A. (2018). The role of perceptual similarity, data context and task context when selecting attributes: Examination of considerations made by 5-6 year olds in data modelling environments. Educational Studies in Mathematics. 97(2), 163–183. https://doi.org/10.1007/s10649-017-9791-2

Ingram, N., Holmes, M., Linsell, C., Livy, S., McCormick, M., & Sullivan, P. (2020). Exploring an innovative approach to teaching mathematics through the use of challenging tasks: A New Zealand perspective. Mathematics Education Research Journal, 32(3), 497–522.

Kapur, M. (2010). Productive failure in mathematical problem solving. Instructional Science, 38, 523–550.

Klein, S., & Leikin, R. (2020). Opening mathematical problems for posing open mathematical tasks: What do teachers do and feel? Educational Studies in Mathematics, 105, 349–365.

Koichu, B., Harel, G., & Manaster, A. (2013). Ways of thinking associated with mathematics teachers’ problem posing in the context of division of fractions. Instructional Science, 41(4), 681–698.

Lambdin, D. V. (2003). Benefits of teaching through problem solving. In F. Lester (Ed.), Research and issues in teaching mathematics through problem solving (pp. 2–15). National Council for Curriculum and Assessment.

Leavy, A., & Hourigan, M. (2018). The role of perceptual similarity, data context and task context when selecting attributes: Examination of considerations made by 5-6 year olds in data modelling environments. Educational Studies in Mathematics. 97(2), 163–183. https://doi.org/10.1007/s10649-017-9791-2

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 763–804). Charlotte, NC: Information Age.

Lester, F. K., Jr. (2013). Thoughts about research on mathematical problem-solving instruction. The Mathematics Enthusiast, 10(1–2), 245–278.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How Research lessons improve Japanese Education. American Educator, 22(4), 12–17, 50–52.

Murata, A., Bofferding, L., Pothen, B. E., Taylor, M. W., & Wischnia, S. (2012). Making connections among student learning, content, and teaching: Teacher talk paths in elementary mathematics lesson study. Journal for Research in Mathematics Education, 43(5), 616–650.
National Council for Curriculum and Assessment (NCCA). (2016). Background paper and brief for the development of a new primary mathematics curriculum. NCCA.

National Council for Curriculum and Assessment (NCCA). (2017). Primary mathematics curriculum. Draft specifications. Junior infant to second class. For Consultation. NCCA.

National Council for Curriculum and Assessment (NCCA). (2020). Draft primary curriculum framework. For consultation. Primary curriculum review and development. NCCA.

O’Shea, J., & Leavy, A. M. (2013). Teaching mathematical problem-solving from an emergent constructivist perspective: the experiences of Irish primary teachers. Journal of Mathematics Teacher Education, 16(4), 293–318. https://doi.org/10.1007/s10857-013-9235-6

Polya, G. (1957). How to solve it (2nd edition). Doubleday.

Russo, J., & Hopkins, S. (2019). Teachers’ perceptions of students when observing lessons involving challenging tasks. International Journal of Science and Mathematics Education, 17(4), 759–779.

Sakshaug, L. E., & Wohlhuter, K. A. (2010). Journey toward teaching mathematics through problem solving. School Science and Mathematics, 110(8), 397–409.

Shiel, G., Kavanagh, L., & Millar, D. (2014). The national assessments of English reading and mathematics: Volume 1 performance report. Educational Research Centre.

Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. Educational Studies in Mathematics, 83(1), 9–26.

Stacey, K. (2018). Teaching Mathematics through Problem Solving. Numeros, 98, 7–18.

Stake, R. E. (1995). The art of case study research. Thousand Oaks, CA: Sage Publications.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers’ beliefs and practices related to mathematics instruction. Teaching and Teacher Education, 17(2), 213–226.

Stauss, A., & Corbin, J. (1998). Basics of qualitative research: Techniques and procedures for developing grounded theory (2nd ed). Sage.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., & Roche, A. (2015). Supporting teachers in structuring mathematics lessons involving challenging tasks. Journal of Mathematics Teacher Education, 18(2), 123–140.

Sullivan, P., Bobis, J., Downton, A., Feng, M., Hughes, S., Livy, S., McCormick, M., & Russo, J. (2021). An instructional model to support planning and teaching student centred structured inquiry lessons. Australian Primary Mathematics Classroom, 26(1), 9–13.

Sullivan, P., Clarke, D., Cheeseman, J., Mornane, A., Roche, A., Sawatzki, C., & Walker, N. (2014). Students’ willingness to engage with mathematical challenges: Implications for classroom pedagogies. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), Curriculum in focus: Research guided practice. (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia) (pp. 597–604). MERGA.

Sullivan, P., Clarke, D. M., Clarke, B., & O’Shea, H. (2010). Exploring the relationship between task, teacher actions, and student learning. PNA, 4(4), 133–142.

Suter, W. N. (2012). Introduction to educational research: A critical thinking approach (2nd ed). Sage.

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. Research in Education, 75(1), 58–70.

Takahashi, A. (2006). Characteristics of Japanese mathematics lessons. Paper presented at the APEC International Conference on Innovative Teaching Mathematics through Lesson Study, Tokyo, Japan, January 14–20. https://www.cridged.tsukuba.ac.jp/math/sympo_2006/takahashi.pdf

Takahashi, A. (2008). Beyond show and tell: neriage for teaching through problem-solving—ideas from Japanese problem-solving approaches for teaching mathematics. Paper presented at the 11th International Congress on Mathematics Education in Mexico (Section TSG 19: Research and Development in Problem Solving in Mathematics Education), Montereey, Mexico.

Takahashi, A. (2016). Recent trends in Japanese mathematics textbooks for elementary grades: Supporting teachers to teach mathematics through problem solving. Universal Journal of Educational Research, 4(2), 313–319.

Takahashi, A., Lewis, C., & Perry, R. (2013). US lesson study network to spread teaching through problem solving. International Journal for Lesson and Learning Studies, 2(3), 237–255.

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: Maximizing the impact of lesson study. ZDM: Mathematics Education, 48, 513–526.
Thompson, A. G. (1985). Teachers’ conceptions of mathematics and the teaching of problem solving. In E. A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives (pp. 281–294). Erlbaum.

Watanabe, T. (2001). Anticipating children’s thinking: A Japanese approach to instruction. National Council for Curriculum and Assessment.

Watson, A., & Ohtani, M. (2015). Task design in mathematics education: An ICMI study 22. Springer International.

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