Supersymmetric traversable wormholes

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Abstract

We study a class of traversable wormhole solutions in pure gauged $N=2$ supergravity with and without electromagnetic fields, which are locally isometric under $\text{SO}(2,1) \times \text{SO}(1,1)$. The model allows for 1/2-BPS wormhole solutions whose corresponding globally defined Killing spinors are presented. The wormholes connect two asymptotic, locally AdS\textsubscript{4} regions and depend on certain electric and magnetic charge parameters and, implicitly, on the range of the compact coordinate around the throat.
1 Introduction

The term “wormhole” was first introduced in a paper by Fuller and Wheeler [1], where credit was given to Weyl for the idea of having a non-simply connected space-time. The idea is, however, most often associated with the work by Einstein and Rosen [2], where non-singular coordinate patches of the Schwarzschild and the Reissner-Nordström solutions were studied. It is important to realize that these two types of wormholes are physically very different. The wormhole that exists in eternal black hole solutions is non-traversable, precluding its use for transferring information. The non-simply connected nature of the space-time advocated by Weyl, on the other hand, is inspired by the idea of having electromagnetic field lines without a source. The electromagnetic fields can then be used as a classical communication channel.

Despite its interesting features, wormholes have been widely regarded as a science fiction character. As discussed in [3, 4], this is due to the fact that the null-energy condition has to be violated at the throat of a spherically symmetric, static wormhole. Hence, for asymptotically flat space-times there is not much hope for wormholes to exist in a physically sensible situation. The situation changes in asymptotically AdS space-times. When the four-dimensional space-time is Einstein, and its conformal boundary has positive scalar curvature with a space-time that is everywhere regular, then the boundary cannot have more than one connected component. If the boundary has negative scalar curvature, it is possible to construct a Euclidean wormhole by identifications in global AdS [5]. Some of these identifications have been analized in Euclidean AdS and several arguments have been given against the stability of these wormhole space-times [6]. A standard one is that conformally coupled scalar fields living on a conformal boundary of negative curvature will have an action that is unbounded from below. This is indeed correct when the boundary is in the conformal class of $H^2 \times \mathbb{R}$. However, when the boundary is itself an AdS space-time this argument no longer applies for conformally coupled scalar fields, as their masses are always above the Breitenlohner-Freedman bound [7, 8]. This observation motivated the work in [9], where a class of geometrically non-trivial solutions with two, possibly warped, AdS$_3$ boundaries was constructed. It is therefore of interest to study various aspects of the stability of these traversable wormholes. As a first step in that direction this paper addresses the question whether these wormhole solution are compatible with supersymmetry.

It is interesting to note that there is a widespread belief in the literature against the existence of traversable wormholes under physically sensible energy conditions. This is primarily based on the analysis of [10], where it is claimed that a four-dimensional space-time cannot have a wormhole with a minimal $S^2$ when the null-energy condition holds. However, the wormhole studied in this paper has a minimal $S^1$ and therefore the analysis of [10] does not apply. Indeed, the matter content that we use does satisfy the null-energy condition.

A wormhole has a non-trivial topology. When the non-contractible cycle has minimal length, one is dealing with a wormhole throat. As we shall see, this can already be achievable at the level of a locally AdS$_4$ space-time. In the coordinates that we use this is implemented by the identification along a Killing vector. After the identification is imposed, the space-time is no
longer globally AdS$_4$ but a Lorentzian wormhole of constant curvature. The same phenomenon exists for the solutions discussed in [9], which contain non-trivial electromagnetic fields. These solutions also describes wormholes upon introducing a non-contractible cycle.

As we will demonstrate in this paper both types of wormholes can only be partially supersymmetric. In the constant-curvature case, half of the supersymmetries are no longer globally defined, whereas in the presence of electromagnetic fields, only half of the Killing spinors will exist, irrespective of whether we have a non-contractible cycle or not. The reason for the latter is that the Killing spinors do not depend on the coordinate for which the identification is imposed.

It is worth mentioning that wormhole geometries in asymptotically AdS space-times have received attention in connection with holography (see e.g. [11]). The presence of multiple boundaries would then create the possibility of couplings between different CFTs. It had already been noted earlier that the interaction between two CFTs opens a throat in the bulk that causally connects the two boundaries [12]. However, most of these settings require a non-local interactions between the boundaries for the wormhole throat to open, a feature that is not present in the construction of this paper.

The outline of the paper is as follows. Gauged $N = 2$ supergravity is introduced in section 2, followed by a discussion of the wormhole solutions in section 3. In section 4 we then consider the possibility of supersymmetric wormholes by proving that the integrability condition for the existence of Killing spinors is satisfied. Subsequently we present an explicitly construction of the Killing spinors in section 4 and show that they are globally defined and fully compatible with the global features of the background. Some geometric aspects of supersymmetric wormholes are discussed in 5. Our conclusions are presented in section 6.

2 The supergravity model

In this section we present various features of pure $N = 2$ supergravity with electrically charged gravitini. As is well known, supersymmetry will then imply the presence of a cosmological term whose coefficient is proportional to the square of the gravitino charge. This theory was originally constructed in terms of the physical fields [13, 14], whose supersymmetry transformations only close under commutation up to equations of motion. Subsequently two alternative constructions were presented based on the superconformal multiplet calculus [15, 16, 17]. The physical degrees of freedom of this theory are described by the vierbein field $e_\mu^a$, electrically charged gravitini $\psi_\mu$, and a photon field $A_\mu$. In addition we employ a spin-connection field $\omega_{\mu}^{ab}$ associated with (local) Lorentz transformations, which is not an independent field. The gravitational coupling constant has been absorbed in the fields, and the gravitino charge is equal to $q$. The gravitino fields act as the gauge fields associated with local supersymmetry.\[1]

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\[1\] World indices $\mu, \nu, \ldots$, and tangent-space indices $a, b, \ldots$, both run from 0 to 3. The gamma matrices satisfy $\{\gamma_a, \gamma_b\} = 2 \eta_{ab} \mathbf{1}$, where the tangent-space metric equals $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$. Furthermore $\gamma^5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3$. In four space-time dimensions the charge-conjugation matrix $C$ is anti-symmetric and gamma matrices $\gamma_a$ satisfy $C \gamma_a C^{-1} = -\gamma_a^T$. **
The $N = 2$ supersymmetry transformations are described by two Majorana spinor parameters distinguished by an index $i = 1, 2$, and are decomposed in terms of their chiral components. The reason is that $N = 2$ supersymmetry in four space-time dimensions has a chiral R-symmetry group $SU(2) \times U(1)$. Therefore it makes sense to consider a doublet of positive-chirality spinor parameters denoted by $\epsilon^i$ and a similar doublet of negative-chirality parameters $\epsilon_i$, which each transform according under R-symmetry. As it turns out the electromagnetic gauge transformations correspond to an abelian subgroup of the $SU(2)$ R-symmetry group. We denote the generator of this subgroup by $t^i_j$, which is thus an anti-hermitian traceless matrix. The fact that we are dealing with Majorana spinors implies that the Dirac conjugate of a chiral spinor is proportional to the conjugate must carry a lower $SU(2)$ index, and $C \bar{\epsilon}^T = \epsilon$, where $C$ is the charge-conjugation matrix and the superscript $T$ indicates that we have taken the transpose.

Obviously these spinorial properties are carried over to the gravitino fields, where we again distinguish two chiral doublets satisfying

$$
\gamma_5 \psi_\mu^i = +\psi_\mu^i, \quad \gamma_5 \psi_\mu_i = -\psi_\mu_i.
$$  \(2.1\)

The results given below were taken from [17], where a large class of $N = 2$ theories was presented. Here we consider the following supergravity Lagrangian (up to terms quartic in the gravitini),

$$
\mathcal{L} = -\frac{1}{2} e R(\omega, e) - \frac{1}{8} e F(A)_{\mu\nu} F(A)^{\mu\nu}
- \frac{1}{2} e \left[ \bar{\psi}_\mu^i \gamma^{\mu\rho} \bar{D}_{\nu} \psi_\rho_i - \psi_\mu_i \bar{\gamma}^{\mu\rho} D_{\nu} \bar{\psi}_\rho^i \right]
+ \frac{1}{2} F(A)^{\rho\sigma} \left[ 0_{ij} \bar{\psi}_\mu^i \gamma^{[\mu} \gamma^{\rho\sigma} \gamma^{\nu]} \psi_\nu^j + \epsilon^{ij} \bar{\psi}_\mu i \gamma^{[\mu} \gamma^{\rho\sigma} \gamma^{\nu]} \psi_\nu j \right]
+ \frac{1}{2} \sqrt{2} q e \left[ \psi_{ik} k_j \bar{\psi}_\mu^i \gamma^{\mu\nu} \psi_\nu^j + e^{ik} t^{jk} \bar{\psi}_\mu i \gamma^{\mu\nu} \psi_\nu j \right] + 6 q^2 e , \quad 2.2
$$

where $F(A)_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $e = \det(e_\mu^a)$. The derivative of the gravitino fields is covariant with respect to local Lorentz and electromagnetic gauge transformations, and reads

$$
D_\mu \psi_\nu^i = \left( \partial_\mu - \frac{1}{2} \omega_\mu^{ab} \gamma_{ab} \right) \psi_\nu^i - \frac{1}{2} \sqrt{2} q A_\mu t^i_j \psi_\nu^j ,
D_\mu \psi_\nu i = \left( \partial_\mu - \frac{1}{2} \omega_\mu^{ab} \gamma_{ab} \right) \psi_\nu i - \frac{1}{2} \sqrt{2} q A_\mu t^i_j \psi_\nu j , \quad 2.3
$$

where $\omega_\mu^{ab}$ is the spin connection whose definition will be discussed momentarily. The matrices $\frac{1}{2} \gamma_{ab} = \frac{1}{4} [\gamma_a, \gamma_b]$ are the Lorentz group generators in the spinor representation. As mentioned already, $t^i_j$ is the anti-hermitian traceless generator of the electromagnetic gauge transformations, which is an abelian subgroup of SU(2). It is normalized to $t^i_j t_i^j = 2$, where $t^i_j$ denotes the complex conjugate of $t^i_j$. This implies the convenient identities,

$$
t^i_j t^j_k = \delta^i_k = -t^i_k t^j_k , \quad \varepsilon_{ijk} t^k_j = \varepsilon_{jik} t^i_j , \quad t^i_j \equiv (t^i_j)^* = \varepsilon_{ijk} \varepsilon^{jl} t^l_j . \quad 2.4
$$

These identities do not lead to a unique choice for $t^i_j$; this is consistent with the fact that the matrix can be redefined by applying a uniform chiral $SU(2)$ field redefinition on the spinors.
The spin connection $\omega^a_{\mu b}$ is derived from the supercovariant torsion constraint,

$$\mathcal{D}(\omega)_{\mu} e^a_{\nu} - \mathcal{D}(\omega)_{\nu} e^a_{\mu} = \frac{1}{2} \left[ \bar{\psi}_{\mu i} \gamma^a \psi_{\nu i} + \bar{\psi}_{\nu i} \gamma^a \psi_{\mu i} \right],$$  

(2.5)

where the Lorentz covariant derivative reads $\mathcal{D}(\omega)_{\mu} e^a_{\nu} = \partial_{\mu} e^a_{\nu} - \omega^a_{\mu b} e^b_{\nu}$. This constraint can be solved algebraically and leads to,

$$\omega^a_{\mu b} = \frac{1}{2} e^c_{\mu} \left( \Omega^{a b c} - \Omega^{b a c} - \Omega^{c a b} \right),$$  

(2.6)

where the $\Omega_{a b c}$ are the objects of anholonomy. The affine connection equals $\Gamma^a_{\mu \nu} = e_a^\rho \mathcal{D}_\mu (\omega) e^{\nu a}$, and ensures the validity of the vielbein postulate. In the absence of torsion, where the right-hand side of (2.5) vanishes, we have

$$\Omega_{a b c} = e_a^\mu e_b^\nu \left( \partial_{\mu} e^c_{\nu} - \partial_{\nu} e^c_{\mu} \right).$$  

(2.7)

The corresponding expression for the affine connection is then equal to the Christoffel connection.

The curvature associated with the spin connection equals

$$R^a_{\mu \nu b}(\omega) = \partial_{\mu} \omega^a_{\nu b} - \partial_{\nu} \omega^a_{\mu b} - \omega^a_{\mu c} \omega^c_{\nu b} + \omega^a_{\nu c} \omega^c_{\mu b},$$  

(2.8)

which satisfies the Bianchi identity $\mathcal{D}(\omega)_{[\mu} R^a_{\nu \rho]}(\omega) = 0$. After converting the tangent-space indices in $R^a_{\mu \nu b}(\omega)$ to world indices, it will be equal to the Riemann tensor, up to terms quadratic in the gravitino fields that originate from the right-hand side in (2.5). Its contractions,

$$R^a_{\mu}(e, \omega) = e^b_{\nu} R^a_{\mu \nu b}(\omega), \quad R(e, \omega) = e^\mu_{\nu} e^\nu_{\nu} R^a_{\mu \nu b}(\omega),$$  

(2.9)

yield the Ricci tensor and scalar, up to gravitino terms. Substituting the solution of (2.5) into $R(e, \omega)$ yields the Ricci scalar up to terms quartic in the gravitino fields.

Let us now list the supersymmetry transformation rules,

$$\delta e^a_{\mu} = \bar{\epsilon}^i \gamma^a \psi_{\mu i} + \bar{\epsilon}_i \gamma^a \psi^i_{\mu},$$

$$\delta \psi^i_{\mu} = 2 \mathcal{D}_\mu \epsilon^i - \frac{1}{4} F(A)_{\rho \sigma} \gamma^a \gamma^b \psi^i_{\mu} \epsilon^j_{\nu} \epsilon_{\nu k} + \sqrt{2} q \epsilon^i_{\nu} t^k_{\mu} \gamma_{\nu k},$$

$$\delta A_{\mu} = 2 \left( \bar{\epsilon}^i \gamma^j \psi_{\nu j} + \bar{\epsilon}_j \gamma^j \psi^i_{\nu} \right),$$  

(2.10)

where in the gravitino transformations we suppressed terms cubic in the gravitino fields. The covariant derivatives of the supersymmetry parameters are given by

$$\mathcal{D}_\mu \epsilon^i = \left( \partial_{\mu} - \frac{1}{4} \omega^a_{\mu b} \gamma^b \right) \epsilon^i - \frac{1}{2} \sqrt{2} q \ A_{\mu} t^i_{\nu j} \epsilon^j,$$

$$\mathcal{D}_\mu \epsilon_i = \left( \partial_{\mu} - \frac{1}{4} \omega^a_{\mu b} \gamma^b \right) \epsilon_i - \frac{1}{2} \sqrt{2} q \ A_{\mu} t^i_{\nu j} \epsilon_j.$$  

(2.11)

The Lagrangian (2.2) is invariant under space-time diffeomorphisms, supersymmetry, local Lorentz transformations, and electromagnetic gauge transformations, whose infinitesimal transformations will close under commutation. Of particular interest is the commutator of two supersymmetry transformations, which closes into the diffeomorphism with parameter $\xi^\mu$, the local
Lorentz transformations and the electromagnetic gauge transformations, but only modulo the gravitino field equations,
\[ [\delta(\epsilon_1), \delta(\epsilon_2)] = \xi^\mu \hat{D}_\mu + \delta_L(\epsilon) + \delta(\Lambda), \] (2.12)
where the derivative is fully covariant with respect to all the symmetries. This implies that there is a contribution from \( \xi^\mu \) times each of the connections that contribute. The explicit variations are simply additional and they do not involve the connections. The parameters of the various infinitesimal transformations on the right-hand side are given by
\[ \xi^\mu = 2 \bar{\epsilon}_2^i \gamma^\mu \epsilon_{1i} + \text{h.c.}, \]
\[ \varepsilon^{ab} = \varepsilon_{ij} \bar{\epsilon}_1^i \epsilon_2^j F^{ab+} + \text{h.c.}, \]
\[ \Lambda = 4 \varepsilon_{ij} \bar{\epsilon}_2^i \epsilon_1^j + \text{h.c.}, \] (2.13)
where the first term proportional to \( \xi^\mu \) denotes a supercovariant translation, i.e. a general coordinate transformation with parameter \( \xi^\mu \), suitably combined with field-dependent gauge transformations so that the result is supercovariant.

We will be interested in solutions that have full or partial supersymmetry. The fully supersymmetric solution is well known and we will briefly refer to it at the end of this section. There exist many solutions with partial supersymmetry. Well-known examples are, for instance, the extremal Reissner-Nordström black holes solutions, which are invariant under half the supersymmetries [18]. However, the main objective of this paper is to analyze the possible supersymmetry of wormhole solutions belonging to the class constructed in [9].

When a bosonic field configuration is fully or partially supersymmetric, it implies that all or some of the supersymmetry transformations of the fermions are vanishing. The transformations that vanish are characterized by certain spinorial parameters that are known as generalized Killing spinors. The only spinors that we are dealing with in this particular case are the gravitini, so we have to simply analyse their supersymmetry transformation, which amounts to deriving possible solutions for \( \epsilon^i \) and \( \epsilon_i \) of the equations,
\[ 2 \mathcal{D}_\mu \epsilon^i - \frac{1}{4} F(A)_{\rho\sigma} \gamma^{\rho\sigma} \epsilon_j = 0, \]
\[ 2 \mathcal{D}_\mu \epsilon_i - \frac{1}{4} F(A)_{\rho\sigma} \gamma^{\rho\sigma} \epsilon^j = 0. \] (2.14)
It is convenient to first consider an integrability condition for these differential equations, which follows by applying a second derivative \( \mathcal{D}_\nu \) and anti-symmetrizing over the indices \( \mu \) and \( \nu \). The resulting equations take the following form,
\[ \Xi_{\mu i} \equiv \mathcal{D}_\nu \delta \psi^i - \mathcal{D}_\nu \delta \psi^i = \left[ R(\omega)_{\mu\nu} \gamma_{ab} + 4 q^2 \gamma_{ab} + \frac{1}{2} F_{\rho\sigma} \gamma^{\rho\sigma} \gamma_{[\mu} \gamma^{\nu]} \right] \epsilon^i \\
+ \frac{1}{2} \sqrt{2} q \left[ 4 F_{\mu\nu} - F_{\rho\sigma} \gamma^{\rho\sigma} \gamma_{[\mu} \gamma^{\nu]} \right] t^i_j \epsilon^j = 0, \]
\[ \Xi_{\mu i} \equiv \mathcal{D}_\nu \delta \psi_{ji} - \mathcal{D}_\nu \delta \psi_{ji} = \left[ R(\omega)_{\mu\nu} \gamma_{ab} - 4 q^2 \gamma_{ab} + \frac{1}{8} F_{\rho\sigma} \gamma^{\rho\sigma} \gamma_{[\mu} \gamma^{\nu]} \right] t^i_j \epsilon^j + \left( \nabla_{[\mu} F_{\rho\sigma} \right) \gamma^{\rho\sigma} \gamma_{[\nu]} \epsilon^i = 0. \] (2.15)
where the covariant derivative $\nabla_\mu$ contains only the Christoffel connection.

To analyze the above equations it is convenient to switch from two Majorana spinors to a single Dirac spinor. To do so, one first chooses, without loss of generality, the charge matrix $t^i_j$ to be equal to diag$(i, -i)$. Subsequently one defines

$$\chi \equiv \epsilon_1 + \epsilon_2, \quad \Xi_{\mu\nu} \equiv \Xi_{\mu\nu}^1 + \Xi_{\mu\nu}^2.$$  \hspace{1cm} (2.16)

Now $\chi$ is no longer a Majorana spinor, because under charge conjugations it will lead to another independent spinor $\epsilon_1 + \epsilon_2$. Since the two spinors are related by charge conjugation, it suffices to only consider the quantities $\Xi_{\mu\nu}$, which constitute six different $4 \times 4$ matrices acting on the 4-component Dirac spinor $\chi$, defined by (2.16).

With these redefinitions the equations Killing spinor equation (2.14) and the integrability condition (2.15) reads as follows,

$$2 D_\mu \chi + \frac{1}{2} F(A)_{\rho\sigma} \gamma^{\rho\sigma} \gamma_\mu \gamma^5 \chi - \sqrt{2} i q \gamma_\mu \gamma^5 \chi = 0,$$  \hspace{1cm} (2.17)

$$\Xi_{\mu\nu} = \left[ R(\omega)_{\mu\nu}^{ab} \gamma_{ab} - 4 q^2 \gamma_{\mu\nu} + \frac{1}{2} F_{\rho\sigma} F_{\lambda\tau} \gamma^{\rho\sigma} \gamma_{\mu} \gamma^{\lambda\tau} \gamma_{\nu} \right] \chi$$

$$+ \frac{1}{2} \sqrt{2} i q \left[ 4 F_{\mu\nu} - F_{\rho\sigma} \gamma^{\rho\sigma} \gamma_{\mu\nu} - \gamma_{\mu} F_{\rho\sigma} \gamma^{\rho\sigma} \gamma_{\nu} \right] \chi - (\nabla_{\mu} F_{\rho\sigma}) \gamma^{\rho\sigma} \gamma_{\nu} \gamma^5 \chi = 0,$$

The covariant derivative of $\chi$ follows from (2.11),

$$D_\mu \chi = (\partial_\mu - \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} - \frac{1}{2} \sqrt{2} i q A_\mu) \chi.$$  \hspace{1cm} (2.18)

We recall that all fermionic fields have been suppressed on the right-hand side of the equations (2.14) and (2.18), because we will be dealing with purely bosonic backgrounds when exploring the possible supersymmetry of wormhole solutions.

The maximally supersymmetric solution has vanishing $A_\mu$, so that the integrability relation then takes the form $R(\omega)_{\mu\nu}^{ab} = 4 q^2 e_\mu^{[a} e_{\nu]}$. This equation implies that the supersymmetric field configuration is just an anti-de Sitter space-time with AdS radius $\ell$ given by

$$\ell^{-1} = \sqrt{2} |q|.$$  \hspace{1cm} (2.19)

In the following sections we will consider a class of wormhole solutions that can be partially supersymmetric. Their possible supersymmetry will be investigated by analyzing the equations (2.17).

3 Maxwell-Einstein-AdS wormholes

Following [9], we consider a class of four-dimensional space-time metrics expressed into two different functions, $f(r)$ and $h(r)$,

$$ds^2 = \frac{4 \ell^4 \, dr^2}{\sigma^2 f(r)} + h(r) \left[ - \cosh^2 \theta \, dt^2 + d\theta^2 \right] + f(r) (du + \sinh \theta dt)^2,$$  \hspace{1cm} (3.1)
where $\ell$ denotes the AdS radius. When considering supersymmetry we will also need a corresponding set of vierbeine, for which we make the following choice,

\[
\begin{align*}
    e^0 &= \sqrt{h(r)} \cosh \theta \, dt, \\
    e^1 &= \frac{1}{\sigma q^2 \sqrt{f(r)}} \, dr, \\
    e^2 &= \sqrt{h(r)} \, d\theta, \\
    e^3 &= \sqrt{f(r)} \left( du + \sinh \theta \, dt \right).
\end{align*}
\]

(3.2)

At this point we first discuss the global $\text{AdS}_4$ space-time that we just introduced at the end of the previous section. It corresponds to the following choice for parameter $\sigma$ and the functions $f(r)$ and $h(r)$,

\[
\sigma = 4, \quad f(r) = h(r) = \frac{1}{4} \ell^2 (r^2 + 1).
\]

(3.3)

Its topology is trivial because the coordinates cover the full $\mathbb{R}^4$. However, it is possible to impose identifications on surfaces that are orthogonal to $\partial_r$ so that one obtains a constant curvature wormhole. In this case the space-time is only locally $\text{AdS}_4$ and has two conformal boundaries located at $r = \pm \infty$. The relevant identification in the Lorentzian case is $u \sim u + a$, which for constant $r$ yields the three-dimensional Coussaert-Henneaux space-time [19]. This identification obviously introduces a non-contractible cycle in space-time. In the case at hand, the location of the throat is at $r = 0$, when the non-contractible circle has minimal length. The perimeter of the throat is an extra parameter of the metric that is encoded in the range of the compact coordinate $u \in [0, a]$.

Since this field configuration is a solution of the Einstein-Maxwell system with a cosmological term, it can also be a solution of pure $N = 2$ supergravity, which means that it is a solution of its bosonic field equations that follow from the Lagrangian (2.2). These combined field equations that it satisfies will therefore take the form

\[
\begin{align*}
    \partial_\mu (e F^{\mu \nu}) &= 0, \\
    R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R + \frac{1}{4} [F_{\mu \rho} F_{\nu \sigma} - \frac{1}{4} g_{\mu \rho} F_{\nu \sigma} F^{\rho \sigma}] + 6 q^2 g_{\mu \nu} &= 0,
\end{align*}
\]

(3.4)

where $\ell^{-1} = \sqrt{2 |q|}$ and $F_{\mu \nu} = 0$.

Let us now move to a more complicated metric where the functions $f(r)$ and $h(r)$ are equal to

\[
\begin{align*}
    f(r) &= \frac{2}{q^2 \sigma^2} \frac{r^4 + (6 - \sigma) r^2 + r + \sigma - 3}{r^2 + 1} - \frac{Q^2 + P^2}{r^2 + 1}, \\
    h(r) &= \frac{1}{2 q^2 \sigma} (r^2 + 1),
\end{align*}
\]

(3.5)

and construct a corresponding solution of the above equations. Here $Q$ and $P$ are electric and magnetic charge parameters that will determine the physical charges (whose definition requires to properly account for wormhole topology) and the corresponding electric and magnetic fields of the solution. These charges are induced because the second field equation (3.4) requires the presence of electric and magnetic fields, which will be given momentarily. Note, however, that we still
retain the homogeneous Maxwell equations, because the only charged sources are the gravitini, which are not included in the bosonic background solution. In addition the metric depends on two integration constants denoted by \( m \) and \( \sigma \). The parameter \( m \) is proportional to the mass of the space-time, while \( \sigma \) is related to the warping of the asymptotic region. More details can be found in [9].

The solution of (3.4) for the vector potential is given by
\[
A = \Phi (r) \left( du + \sinh \theta \, dt \right),
\]
with \( \Phi(r) \) equal to
\[
\Phi (r) = \frac{2Qr + 4P(1 - r^2)}{r^2 + 1}.
\]

It turns out that (3.6) is invariant under the isometries given below in (3.11). Obviously the vector potential \( A_\mu \) describes an electric and a magnetic field component. Its field strength in the adopted coordinate system is equal to
\[
F_{ru} = \frac{2(r^2 - 1)Q - 4rP}{(r^2 + 1)^2},
F_{rt} = \frac{2(r^2 - 1)Q - 4rP}{(r^2 + 1)^2} \sinh \theta,
F_{\theta t} = \frac{2Qr + 4P(1 - r^2)}{r^2 + 1} \cosh \theta.
\]

The possible existence of a non-contractible cycle requires that \( f(r) \) must be positive everywhere.\footnote{A straightforward analysis shows that \( f(r) \) never vanishes provided}

Asymptotically, for \( r = \pm \infty \) the space-time is locally AdS_4 with the following fall-off for the curvature tensor,
\[
R(\omega)^{ab}_{\mu \nu} = \left[ 2 \ell^{-2} + O(r^{-2}) \right] e^{[a}_{\mu} e^{b]}_{\nu}.
\]

The bosonic field configuration associated with global AdS_4 is invariant under the isometry group SO(3,2). This group is broken for the deformed functions \( f(r) \) and \( h(r) \) specified in (3.5) and the electromagnetic fields (3.8) to a subgroup generated by the following four Killing vectors,
\[
\xi_{[1]} = \partial_t,
\xi_{[2]} = \sin t \partial_\theta + \tanh \theta \cos t \partial_t + \frac{\cos t}{\cosh \theta} \partial_u,
\xi_{[3]} = \cos t \partial_\theta - \tanh \theta \sin t \partial_t - \frac{\sin t}{\cosh \theta} \partial_u,
\]
\[
X = 3(Q^2 + P^2)\ell^{-2} \leq 1,
\frac{12 + 12\sqrt{1 - X}}{1 + X + \sqrt{1 - X}} > \sigma > \frac{12 - 6\sqrt{1 - X}}{1 + X + \sqrt{1 - X}}.
\]

For these ranges of the parameters, the metric functions are everywhere positive and regular and a non-trivial wormhole space-time will exist.
The first three Killing vectors generate the group SO(2, 1), while the fourth isometry is abelian and commutes with the first three. Not surprisingly, the two functions given in (3.5) depend only on \( r \) and are therefore invariant under the four isometries. Our solution can be seen as a deformation of AdS\(_3\) embedded in a four-dimensional space. The deformation by the function \( f(r) \) breaks the SO(2, 2) \( \cong \) SO(2, 1) \( \times \) SO(2, 1) isometries to its subgroup SO(2, 1) \( \times \) SO(1, 1).

We also calculate \( \mathcal{L}_{\xi} e_{\mu}^a = \xi^\nu \partial_\nu e_{\mu}^a + \partial_\mu \xi^\nu e_{\nu}^a \) for each of the Killing vectors. As it turns out \( \mathcal{L}_{\xi} e_{\mu}^a \) vanishes on all the vierbeine for the \( \xi[1] \) and \( \xi[4] \), the non-trivial action of the other Killing vectors on the vierbeine yields

\[
\mathcal{L}_{\xi[2]} e^0 = \frac{\cos t}{\cosh \theta} e^2, \quad \mathcal{L}_{\xi[3]} e^0 = -\frac{\sin t}{\cosh \theta} e^2, \quad \mathcal{L}_{\xi[2]} e^2 = \frac{\cos t}{\cosh \theta} e^0, \quad \mathcal{L}_{\xi[3]} e^2 = -\frac{\sin t}{\cosh \theta} e^0.
\]

Hence the vierbeine are not invariant under the diffeomorphisms generated by the Killing vectors, but they are invariant under these diffeomorphisms when accompanied by tangent-space transformations that are opposite to the ones indicated above. On spinors these tangent transformations will take the form

\[
\delta[2] \psi = -\frac{\cos t}{2 \cosh \theta} \gamma^0 \gamma^2 \psi, \quad \delta[3] \psi = \frac{\sin t}{2 \cosh \theta} \gamma^0 \gamma^2 \psi.
\]

We will return to these compensating tangent-space transformation at the end of sections \( S \), where we will discuss the corresponding invariances of the Killing spinors. Note that the transformations \( \xi[1] \) and \( \xi[4] \) do not involve any compensating tangent-space transformations.

### 4 Supersymmetric wormholes

To investigate whether the wormhole solutions can be supersymmetric, one may first consider the integrability for the complex Killing spinors \( \chi \), which was presented in (2.17). In the actual calculations we use the following representation for the gamma matrices,

\[
\gamma^0 = -i \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad \gamma^2 = i \begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix},
\]

where we remind the reader of the definition \( \gamma^5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \text{diag}(\sigma_2, -\sigma_2) \).

A necessary condition for the existence of non-trivial Killing spinors is that the determinant of each of the six \( 4 \times 4 \) matrices \( \Xi_{\mu\nu} \) defined in (2.16) must vanish. As it turns out all six determinants take the form of a constant times \( (r^2 + 1)^{-6} \) times a function \( Z(r) \). This function also depends on the charges and the integration constants \( \sigma \) and \( m \) in the metric based on (3.5) so the condition for supersymmetry is that \( Z(r) \) must vanish. Explicit calculation shows that the function \( Z(r) \) has the following form,

\[
Z(r) = Z_2 r^2 + Z_1 r + Z_0, \quad (4.2)
\]

\(^3\)With these gamma matrices we can choose the charge conjugation matrix as \( S = S^{-1} = -S^T \), so that the charge conjugate of a spinor \( \psi \) is equal to \( S \bar{\psi}^* = \psi^* \).
where $Z_2$, $Z_1$ and $Z_0$ are fairly complicated expressions that contain the charges and integration constants. However, the integrability condition should hold for any value of the radial coordinate $r$. Therefore one concludes that $Z_2$, $Z_1$ and $Z_0$ should separately vanish. For $Z_2$ this leads to the equation,

$$Z_2 = \frac{\sigma^2}{q^2} (mP + 8Q - 2Q\sigma) = 0.$$  \hspace{1cm} (4.3)

Since the metric is singular when $\sigma$ vanishes, we conclude that

$$m = \frac{2Q}{P} (\sigma - 4).$$  \hspace{1cm} (4.4)

When this equation is satisfied then $Z_1$ turns out to vanish identically. Hence the only remaining condition follows from requiring that $Z_0$ must vanish,

$$Z_0 = \frac{(P^2 + Q^2)^2}{2P^4q^6} \left(2q^2\sigma^2P^2 + (\sigma - 4)^2\right) \left(-2(\sigma - 4) + P^2\sigma^2q^2\right)^2 = 0,$$  \hspace{1cm} (4.5)

where we made again use of equation (4.4). Combining the above results one obtains the conditions

$$P = \frac{1}{|q|\sigma} \sqrt{2(\sigma - 4)}, \quad m = |q|\sigma Q \sqrt{2(\sigma - 4)}.$$  \hspace{1cm} (4.6)

Supersymmetry thus implies $\sigma > 4$ which is the same result that was found in [9] by requiring holographic stability.

Now that we have solved the integrability condition for the existence of Killing spinors, let us proceed to an explicit determination of these spinors. To appreciate the possible relevance of the identification $u \sim u + a$ for supersymmetry, we determine the possible Killing spinors explicitly. To solve the Killing spinor we use the Dirac spinor $\chi$ defined in (2.16). The Killing spinor equations for $\chi$ was already given in (2.14). Substituting the expression for the bosonic covariant derivative, it reads

$$\left[\partial_\mu - \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} - \frac{1}{2} \sqrt{2} i q A_\mu + \frac{1}{8} F_{\rho\sigma} \gamma^{\rho\sigma} \gamma_5 - \frac{1}{2} \sqrt{2} i q \gamma_\mu \gamma_5\right] \chi = 0.$$  \hspace{1cm} (4.7)

It is useful to first study the Killing spinors of global $\text{AdS}_4$ in terms of the coordinates used throughout this paper, we suppress for the moment the presence of $A_\mu$ and $F_{\mu\nu}$ in (4.7). In this way we obtain the following four real Killing spinors,

$$\chi^1_{\text{AdS}} = \begin{pmatrix}
\frac{\sqrt{1 + \sqrt{1 + r^2}}}{\sqrt{1 + \sqrt{1 + r^2}}} \left[\cosh \theta/2 \cos t/2 - \sinh \theta/2 \sin t/2\right] \\
- \frac{\sqrt{-1 + \sqrt{1 + r^2}}}{\sqrt{1 + \sqrt{1 + r^2}}} \left[\cosh \theta/2 \cos t/2 - \sinh \theta/2 \sin t/2\right] \\
\frac{\sqrt{1 + \sqrt{1 + r^2}}}{\sqrt{1 + \sqrt{1 + r^2}}} \left[\cosh \theta/2 \sin t/2 - \sinh \theta/2 \cos t/2\right] \\
\frac{\sqrt{-1 + \sqrt{1 + r^2}}}{\sqrt{1 + \sqrt{1 + r^2}}} \left[\cosh \theta/2 \sin t/2 - \sinh \theta/2 \cos t/2\right]
\end{pmatrix}.$$  \hspace{1cm} (4.7)

\text{\footnotesize{\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline

$\sigma_{\pm} = \frac{1}{q^2 P^2} \left(1 \pm \sqrt{1 - 8q^2P^2}\right).$

\hline
\end{tabular}
\end{table}}}
\[ \chi_{2 \text{AdS}} = \begin{pmatrix} -\sqrt{1 + \sqrt{1 + r^2}} \left[ \sinh \theta/2 \cos t/2 + \cosh \theta/2 \sin t/2 \right] \\ \sqrt{1 + \sqrt{1 + r^2}} \left[ \sinh \theta/2 \cos t/2 + \cosh \theta/2 \sin t/2 \right] \\ \sqrt{-1 + \sqrt{1 + r^2}} \left[ \sinh \theta/2 \sin t/2 + \cosh \theta/2 \cos t/2 \right] \\ \sqrt{-1 + \sqrt{1 + r^2}} \left[ \sinh \theta/2 \sin t/2 + \cosh \theta/2 \cos t/2 \right] \end{pmatrix}, \]

\[ \chi_{3 \text{AdS}} = e^{u/2} \begin{pmatrix} \sqrt{-1 + \sqrt{1 + r^2}} \\ -\sqrt{1 + \sqrt{1 + r^2}} \\ 0 \\ 0 \end{pmatrix}, \quad \chi_{4 \text{AdS}} = e^{-u/2} \begin{pmatrix} 0 \\ 0 \\ \sqrt{-1 + \sqrt{1 + r^2}} \\ \sqrt{1 + \sqrt{1 + r^2}} \end{pmatrix}. \] (4.8)

However, we have to remember that we are constructing representations for complex Killing spinors, so that the above spinors can be multiplied by arbitrary complex normalization factors. Hence we are dealing with eight independent Killing spinors, which will indeed provide a basis for full \( N = 2 \) supersymmetry, as is expected for a global AdS\(_4\) space-time.

A noteworthy feature in the context of the present paper is that the last two Dirac spinors, \( \chi_{3 \text{AdS}} \) and \( \chi_{4 \text{AdS}} \), are incompatible with a periodic coordinate \( u \). Therefore, when dealing with a non-contractible cycle \( u \sim u + a \), half of the Killing spinors will no longer be globally defined, so that this particular field configuration must be regarded as a 1/2-BPS solution. At the same time, the equations of motion will still be locally satisfied.

At this point one can invoke the supersymmetry algebra given by (2.12), which relates the commutator of two supersymmetry transformations to the bosonic symmetries of the model. When choosing supersymmetry parameters expressed in terms of linear combinations of the Killing spinors \( \epsilon^i \), one obtains all the bosonic transformations that should be compatible with the supersymmetric background, and in particular one would obtain the Killing vectors of AdS\(_4\). However, when the Killing spinors are not all globally defined, then some of the Killing vectors of the space-time will not be globally defined either.

Let us now continue and derive the Killing spinors for the non-constant curvature wormhole with non-trivial electromagnetic fields. A lengthy analysis shows that there exist only two Dirac, Killing spinors, so that the number of Killing spinors is reduced to one half. Furthermore, these spinors do no longer depend on the coordinate \( u \), so that they are globally defined. We will give the explicit expressions momentarily. It turns out that that the first and the second component of these spinors differ by an overall function \( G(r) \), whereas the third and the fourth component differ by an overall function \( \bar{G}(r) \) that equals the complex conjugate of \( G(r) \). This function \( G(r) \) is quite complicated and takes the following form,

\[ G(r) = \frac{-1}{q^2 \sigma h(r)} \left[ \sqrt{f(r)} - 2 \sqrt{2} q h(r) \right] \left[ \sqrt{f(r)} - i\Phi(r) \right] \frac{f'(r) + i\sqrt{f(r)} \Phi'(r)}{f'(r)} . \] (4.9)
The two Dirac Killing spinors now take the form,

\[
\chi^{1\text{WH}} = \alpha(r) \begin{pmatrix}
e^{i\beta(r)} \left[ \cosh \theta/2 \cos t/2 - \sinh \theta/2 \sin t/2 \right] \\
-e^{i\beta(r)} G(r) \left[ \cosh \theta/2 \cos t/2 - \sinh \theta/2 \sin t/2 \right] \\
e^{-i\beta(r)} \left[ \cosh \theta/2 \sin t/2 - \sinh \theta/2 \cos t/2 \right] \\
e^{-i\beta(r)} \bar{G}(r) \left[ \cosh \theta/2 \sin t/2 - \sinh \theta/2 \cos t/2 \right]
\end{pmatrix},
\]

\[
\chi^{2\text{WH}} = \alpha(r) \begin{pmatrix}
-e^{i\beta(r)} \left[ \sinh \theta/2 \cos t/2 + \cosh \theta/2 \sin t/2 \right] \\
e^{i\beta(r)} G(r) \left[ \sinh \theta/2 \cos t/2 + \cosh \theta/2 \sin t/2 \right] \\
e^{-i\beta(r)} \left[ \sinh \theta/2 \sin t/2 + \cosh \theta/2 \cos t/2 \right] \\
e^{-i\beta(r)} \bar{G}(r) \left[ \sinh \theta/2 \sin t/2 + \cosh \theta/2 \cos t/2 \right]
\end{pmatrix},
\]

where

\[
\alpha(r) = \frac{h^{1/4}(r)}{\sqrt{1 + |G(r)|^2}},
\]

\[
e^{2i\beta(r)} = (1 + \frac{i}{2} \sqrt{\sigma - 4}) \sqrt{\frac{f(r)}{h(r)} \frac{1 + |G(r)|^2}{1 + G(r)^2}}.
\]

Therefore we find that there are two independent Dirac Killing spinors (which in this case are actualy complex). This solution is therefore 1/2-BPS. As before we can invoke the supersymmetry algebra, and verify that one reproduces the Killing vectors (3.11), which will be globally defined. All this provides a non-trivial check of the correctness of our results.

We can also determine how these Killing spinors transform under the the symmetries of the bosonic field configuration. As explained at the end of section 4, these symmetries take the form of a linear combination of the isometries (3.11) and certain tangent-space transformations that act on spinors according to (3.13). The Killing spinors thus transform under both transformations. As it turns out, the tangent space transformation will cancel in this linear combination, and we are left with the following transformations,

\[
\delta[1]\chi^{\text{WH}} = \frac{i}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \chi^{\text{WH}},
\]

\[
\delta[2]\chi^{\text{WH}} = \frac{i}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \chi^{\text{WH}},
\]

\[
\delta[3]\chi^{\text{WH}} = -\frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \chi^{\text{WH}},
\]

\[
\delta[4]\chi^{\text{WH}} = 0.
\]

where

\[
\chi^{\text{WH}} = \begin{pmatrix} \chi^{1\text{WH}} \\
\chi^{2\text{WH}} \end{pmatrix}.
\]
Obviously the Killing spinors thus transform according the two-dimensional representation of \( \text{SO}(2,1) \).

## 5 Geometric aspects of supersymmetric wormholes

In the previous section we proved the existence of \( 1/2 \)-BPS wormhole solutions in \( N = 2 \) supergravity. Now we turn to a discussion of the geometric properties of these space-times.

The throat of the supersymmetric wormhole is located at the minimum of the volume of the \( t, r = \) constant surfaces. This is at the minimum of the function \( f(r) \cdot h(r) \). A plot with the time it takes for a photon to cross the whole space-time, as seen by a geodesic observer located at \( r = t = \theta = 0 \) and constant \( u \), is shown in Fig. 1.

![Figure 1: Crossing time \( \Delta t \) for a photon as a function of the charge \( Q \) and the parameter \( \sigma \). The region where the metric is regular and the wormhole is BPS corresponds to the shaded area in the lower plane. This restriction originates from the bounds given in (3.9) and the BPS conditions (4.6). The crossing time remains finite, and starts growing as one approaches the upper bound on \( Q \).](image)

An important remark is now in order. The vector \( \partial_t \), which is asymptotically time-like for \( \sigma \geq 4 \) may become space-like in the interior of the wormhole when the following inequality holds,

\[
f(r) \sinh^2 \theta - h(r) \cosh^2 \theta > 0.
\]

This would lead to an ergoregion, as happens in [20], and tends to be in contradiction with supersymmetry [21]. However, from the supersymmetry conditions (4.6) one can show in a straightforward manner that the inequality (5.15) cannot be fulfilled, so that the asymptotically timelike Killing vector \( \partial_t \) is actually timelike everywhere in the interior of the BPS wormhole geometry.

The induced metric on the surfaces at constant \( t \) and \( \theta \) is equal to

\[
ds^2 = \frac{dr^2}{q^4 \sigma^2 f(r)} + f(r) \, du^2 = d\rho^2 + R^2(\rho) \, du^2,
\]

## 13
where the second equation is obtained by going to the proper radial coordinate. The function \( R(\rho) \) defines the radius of the circles parameterized by the compact coordinate \( u \). As \( r \to \pm \infty \) one has \( \rho \to \pm \infty \) and \( R(\rho) \sim e^{\pm \rho} \), as expected due to the locally AdS asymptotics. The coordinate \( \rho \) is such that \( r = 0 \) implies \( \rho = 0 \). Fig. 2 shows the plot of the radial function \( R(\rho) \) as a function of \( \rho \). The latter runs radially on the wormhole geometry and measures the proper radial distance from the throat.

Figure 2: Embedding of the charged supersymmetric wormhole with \( Q = 10^{-1} \) and \( \sigma = 5 \) (left panel) and \( \sigma = 6 \) (right panel).

6 Conclusion

In this paper we considered supersymmetric transversable wormholes that are everywhere regular with and without electromagnetic fields. In this respect these wormholes are crucially different from black holes, which become singular in the interior of the event horizon. The supersymmetric wormholes preserve half of the supersymmetries. An interesting fact is that this situation also exists in an AdS space upon the introduction of a non-contractible cycle. In that case there exist potentially eight Killing spinors, but only half of them are globally defined, as was shown in equation (4.8). The supersymmetry algebra then implies that the Killing vectors of this space-time exhibit the same feature, namely that some of them will not be globally defined. Note, however, that the latter scenario does not involve electromagnetic fields.

The supersymmetric, charged, transversable wormholes provide a concrete physical realization of the Weyl’s idea referred to in the introduction. Non-trivial electromagnetic field lines can be supported by a geometry that is consistent with the Einstein-Maxwell system.

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