THE IMPACT OF GALAXY GEOMETRY AND MASS EVOLUTION ON THE SURVIVAL OF STAR CLUSTERS

JUAN P. MADRID, JARROD R. HURLEY, AND MARIE MARTIG
Center for Astrophysics and Supercomputing, Swinburne University of Technology, Hawthorn, VIC 3122, Australia

Received 2013 May 15; accepted 2014 February 1; published 2014 March 10

ABSTRACT

Direct N-body simulations of globular clusters in a realistic Milky-Way-like potential are carried out using the code NBODY6 to determine the impact of the host galaxy disk mass and geometry on the survival of star clusters. A relation between disk mass and star-cluster dissolution timescale is derived. These N-body models show that doubling the mass of the disk from $5 \times 10^{10} M_\odot$ to $10 \times 10^{10} M_\odot$ halves the dissolution time of a satellite star cluster orbiting the host galaxy at 6 kpc from the galactic center. Different geometries in a disk of identical mass can determine either the survival or dissolution of a star cluster orbiting within the inner 6 kpc of the galactic center. Furthermore, disk geometry has measurable effects on the mass loss of star clusters up to 15 kpc from the galactic center. N-body simulations performed with a fine output time step show that at each disk crossing the outer layers of star clusters experiences an increase in velocity dispersion of $\sim$5% of the average velocity dispersion in the outer section of star clusters. This leads to an enhancement of mass loss—a clearly discernable effect of disk shocking. By running models with different inclinations, we determine that star clusters with an orbit that is perpendicular to the Galactic plane have larger mass loss rates than do clusters that evolve in the Galactic plane or in an inclined orbit.

Key words: galaxies: dwarf – galaxies: evolution – galaxies: star clusters: general – stars: evolution

Online-only material: color figure

1. INTRODUCTION

In the current paradigm of galaxy formation, smaller structures merge into larger ones from the big bang up to the present day (White & Rees 1978). Galaxies grow through two main processes: the hierarchical merging with smaller galaxies and the accretion of fresh gas fueling new star formation. These different mechanisms contribute to the growth of the disk, bulge, and halo.

As the first significant stellar structures to form, globular clusters witness the entire evolution of their host galaxy as satellite systems. Globular clusters are believed to follow galaxies during galaxy mergers and close encounters (e.g., West et al. 2004). The gravitational potential of their host galaxy has a direct influence on the survival of globular clusters: they lose stars through tidal stripping and disk shocking. In turn, these lost stars contribute to the buildup of the galaxy’s stellar halo. The evolution of host galaxy and globular clusters are clearly connected.

The aim of this work is to use numerical simulations to derive the importance of the host galaxy disk mass and size on the evolution and survival of star clusters. How are satellite stellar systems affected by disks and bulges of changing mass and with different geometries? Reciprocally, how do satellite stellar systems contribute, through their dissolution, to the formation of the halo of the host galaxy?

D’Onghia et al. (2010) show that halo and disk shocking efficiently deplete the satellite population of dark matter halos within 30 kpc of the Milky Way center. The results of that study cannot be directly applied to globular clusters because of their much higher densities compared with dark-matter halos.

In their important paper, Gnedin & Ostriker (1997) model the dynamical evolution of the Galactic globular cluster system using a Fokker–Planck code. These authors build on earlier analytical work by Aguilar et al. (1988) and Kundic & Ostriker (1995), among others. Gnedin & Ostriker (1997) give analytical expressions to estimate the impact of disk shocking and also call for numerical simulations to be carried out.

With the recent progress in computational capacity, large N-body simulations can now be carried out to determine the physical mechanisms that govern the dynamical evolution of globular clusters in a galactic potential. Recently, Renaud & Gieles (2013) found that after a merger event of the host galaxy, the mass loss rate of a satellite star cluster increases. It is interesting that Renaud & Gieles (2013) also find that even if the tidal forces reach a maximum during the merger itself, they are too short-lived to have a significant impact on the long-term survival of star clusters.

Previous work on the mass loss of star clusters have focused on internal dynamical effects and stellar evolution. Vesperini & Heggie (1997) carried out numerical simulations of star clusters and determined their mass loss rates throughout a Hubble time. Baumgardt & Makino (2003) established an analytical dissolution timescale for star clusters and showed that one-third of the cluster mass is lost because of stellar evolution alone. A recent review of N-body studies can be found in Portegies Zwart et al. (2010).

The approach in our work is purely numerical and different in nature to Gnedin & Ostriker (1997), given that no assumptions or explicit expressions to treat the impact of the disk are used. N-body models of star clusters were run where the properties of the star cluster remain identical but the mass and geometry of the galactic disk change. The effect of the disk on the star cluster is computed as a part of the numerical calculations of the gravitational force experienced by each star.

The above approach is carried out with the code NBODY6 used for the study of the dynamics of star clusters through N-body simulations (Aarseth 1999). NBODY6 now includes a detailed model of the host galaxy where star clusters evolve as satellites (Aarseth 2003). The current setup of NBODY6 includes the tools to model a Milky-Way-type galaxy with three distinct components: disk, bulge, and halo. The gravitational force for
Table 1
Published Values for Mass and Structural Parameters of the Galactic Disk

| Reference                  | Disk Mass (\(M_\odot\)) | \(a\) | \(b\)  |
|----------------------------|--------------------------|-------|--------|
| Bullock & Johnston (2005)  | 1.0 \times 10^{11}       | 6.5   | 0.26   |
| Gómez et al. (2010)        | 7.5 \times 10^{10}       | 5.4   | 0.30   |
| Paczynski (1990)           | 8.1 \times 10^{10}       | 3.7   | 0.20   |
| Peharribia et al. (2010)   | 7.5 \times 10^{10}       | 3.5   | 0.3    |
| Read et al. (2006)         | 5.0 \times 10^{10}       | 4.0   | 0.50   |

Notes. This table gives published values for the mass and structural parameters of the disk for Milky-Way-type galaxies. The values of \(a\) and \(b\) correspond to the disk scale length and disk scale height.

The initial mass function (IMF) of the simulated star clusters is given previously by Kroupa et al. (1993). This IMF defines a distribution of stellar masses using the quantity \(\xi(m) dm\), which is the number of stars between the masses \(m\) and \(m + dm\). The explicit expression for \(\xi(m)\) is the following broken power law:

\[
\xi(m) \propto m^{-\alpha_1} \text{ for } 0.08 \leq m/M_\odot < 0.5 , \quad \text{and} \quad \xi(m) \propto m^{-\alpha_2} \text{ for } 0.5 \leq m/M_\odot .
\]

2. MODELS SETUP

In the current framework of \(\text{NBODY6}\), the different components of the galactic model are static in time. The mass of the disk is assumed to be constant over time. In \(\text{NBODY6}\), the disk component of the galaxy is modeled following the prescriptions of Miyamoto & Nagai (1975):

\[
\Phi(r, z) = \frac{GM_{\text{DISK}}}{\sqrt{r^2 + (a + \sqrt{z^2 + b^2})^2}},
\]

where \(G\) is the gravitational constant and \(M_{\text{DISK}}\) is the mass of the disk. The geometry of the disk can be easily modified by changing the parameters \(a\) (scale length) and \(b\) (scale height). Different values adopted in previous work for these two parameters and the disk mass are given in Table 1. While different values of these scale parameters are explored in this work, the most often assumed values are \(a = 4\) kpc and \(b = 0.5\) kpc. Also, a commonly used value for the mass of the bulge is \(M_{\text{BULGE}} = 1.5 \times 10^{10} M_\odot\) (Xue et al., 2008), which we model as a point mass. We model the galactic halo as a logarithmic potential that gives the entire galaxy a rotational velocity of 220 km s\(^{-1}\) at 8.5 kpc from the galactic center (Aarseth 2003).

The initial setup of the simulated star clusters is analogous to the simulations presented in Madrid et al. (2012). Briefly, the star clusters start with \(N = 100,000\) stars, an initial mass of \(6.4 \times 10^4 M_\odot\), a half-mass radius of 6.2 pc, and a spatial distribution that assumes a Plummer sphere (Plummer 1911). The initial mass function (IMF) of the simulated star clusters is the one defined by Kroupa (2001), which extends the range given previously by Kroupa et al. (1993). This IMF defines a distribution of stellar masses using the quantity \(\xi(m) dm\), which is the number of stars between the masses \(m\) and \(m + dm\). The explicit expression for \(\xi(m)\) is the following broken power law:

\[
\xi(m) \propto m^{-\alpha_1} , \quad \text{for } 0.08 \leq m/M_\odot < 0.5 \quad \text{and} \quad \xi(m) \propto m^{-\alpha_2} \text{ for } 0.5 \leq m/M_\odot .
\]

3. A HEAVIER OR LIGHTER DISK

Independent models of globular clusters are run with different masses for the host galaxy disk. The mass of the disk is made to vary between \(1 \times 10^{10} M_\odot\) and \(10 \times 10^{10} M_\odot\) by incremental steps. The first step is of \(1.5 \times 10^{10} M_\odot\) while subsequent steps are of \(2.5 \times 10^{10} M_\odot\). This range of values covers the different masses that a disk has during its evolution according to galaxy formation theory (e.g., Leitner 2012). These disk masses are also consistent with values published in the literature and are shown in Table 1. This series of models are run at the same galactocentric distance of 6 kpc, and with the same properties; the only parameter that changes in the simulations is the disk mass. The geometry of the disk is kept constant with \(a = 4\) kpc and \(b = 0.5\) kpc.

The total mass of simulated star clusters versus time in simulations with different disk masses is plotted in Figure 1. A natural result of these simulations is that a more massive disk enhances the mass loss rate of an orbiting star cluster owing to a stronger tidal field. An enhanced mass loss rate implies a shortened dissolution time. The star cluster that orbits a light disk with a mass of \(1 \times 10^{10} M_\odot\) has a remaining mass of \(1.2 \times 10^4 M_\odot\), or 19\% of its initial mass, after a Hubble time of evolution. In contrast, a star cluster that evolves within a heavy disk with a mass of \(10 \times 10^{10} M_\odot\) is completely dissolved after 8.2 Gyr of evolution. As expected, disks with masses between...
these two examples define intermediate regimes of mass loss, as shown in Figure 1.

By introducing the half-mass relaxation time $t_{th}$, also computed by NBODY6, we can derive a simple relation between the number of relaxations a star cluster undergoes before dissolution and the mass of the disk. The half-mass relaxation time is given by

$$t_{th} = \frac{0.14 N}{\ln \Lambda} \sqrt{\frac{r_{hm}^3}{GM}},$$

where $\Lambda = 0.4N$ is the argument of the Coulomb logarithm, $N$ is the number of stars, and $r_{hm}$ the half-mass radius (Spitzer & Hart 1971; Binney & Tremaine 1987). Figure 2 shows the number of relaxation times ($t/t_{th}$) versus mass of the disk at the time when the cluster has only 10% of its initial mass left. This quantity ($10\% M_0$) is preferred over the time it takes for the cluster to completely dissolve since numerical simulations become noisy for low $N$ owing to small number statistics. The best-fitting linear relation between dissolution time, relaxation time, and disk mass is plotted in Figure 2 and is given by the following:

$$t/t_{th} = -5 \times \frac{M_{\text{DISK}}}{10^{10} M_\odot} + 61.$$  (5)

We find that doubling the mass of the disk from $5 \times 10^{10} M_\odot$ to $10 \times 10^{10} M_\odot$ leads to the dissolution of the star cluster in half the time.

4. DISK GEOMETRY

The current setup of NBODY6, based on the formula of Miyamoto & Nagai (1975), allows the geometry of the disk to be modified by changing the values of the parameters $a$ and $b$ of Equation (1). Table 1 shows that the mass and structural parameters of the Galactic disk take a range of values in different studies. Eighteen models of star clusters where the disk has different scale parameters were carried out to evaluate the impact of disk geometry on the survival of star clusters. In these models the mass of the disk is kept constant. Selected parameters of models executed for this section are listed in Table 2.

| Label | Bulge Mass ($M_\odot$) | Disk Mass ($M_\odot$) | $a$ (kpc) | $b$ (kpc) | $R_{GC}$ (kpc) | $t_{th}$ (Gyr) | $t_{10\%}$ (Gyr) |
|-------|------------------------|-----------------------|-----------|-----------|---------------|---------------|-----------------|
| 1     | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 0.4       | 0.5       | 4             | 0.1           | 0.3             |
| 2     | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 0.4       | 0.5       | 6             | 1.1           | 7.2             |
| 3     | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 0.4       | 0.5       | 8             | 2.6           |                 |
| 4     | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 0.4       | 0.5       | 10            | 4.5           |                 |
| 5     | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 0.4       | 0.5       | 12.5          | 6.6           |                 |
| 6     | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 0.4       | 0.5       | 15            | 8.6           |                 |
| 7     | $6.5 \times 10^{10}$   | $0$                   | 0         | ...       | 6             | ...           | 8.7             |
| 8     | $1.5 \times 10^{10}$   | $1.0 \times 10^{10}$  | 4.0       | 0.5       | 6             | 2.6           | 16.8            |
| 9     | $1.5 \times 10^{10}$   | $2.5 \times 10^{10}$  | 4.0       | 0.5       | 6             | 2.3           | 15.3            |
| 10    | $1.5 \times 10^{10}$   | $5.0 \times 10^{10}$  | 4.0       | 0.5       | 6             | 1.7           | 11.0            |
| 11    | $1.5 \times 10^{10}$   | $7.5 \times 10^{10}$  | 4.0       | 0.5       | 6             | 1.2           | 6.8             |
| 12    | $1.5 \times 10^{10}$   | $10 \times 10^{10}$   | 4.0       | 0.5       | 6             | 0.9           | 4.8             |
| 13    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 4.0       | 0.5       | 6             | 4.9           | 5.1             |
| 14    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 4.0       | 0.5       | 8             | 2.8           |                 |
| 15    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 4.0       | 0.5       | 10            | 4.0           |                 |
| 16    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 4.0       | 0.5       | 12.5          | 5.6           |                 |
| 17    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 4.0       | 0.5       | 15            | 7.2           |                 |
| 18    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 8.0       | 0.5       | 4             | 1.4           | 9.2             |
| 19    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 8.0       | 0.5       | 6             | 2.3           | 14.5            |
| 20    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 8.0       | 0.5       | 8             | 3.1           | 18.6            |
| 21    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 8.0       | 0.5       | 10            | 3.9           | 22.8            |
| 22    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 8.0       | 0.5       | 12.5          | 5.0           |                 |
| 23    | $1.5 \times 10^{10}$   | $5 \times 10^{10}$    | 8.0       | 0.5       | 15            | 6.3           |                 |

Notes. Column 1: model label; Column 2: bulge mass in solar masses; Column 3: disk mass in solar masses; Column 4: Miyamoto disk scale length $a$; Column 5: Miyamoto scale height $b$; Column 6: galactocentric distance; Column 7: half-mass relaxation time ($t_{th}$); Column 8: time when the star cluster has only 10% of the initial mass left ($t_{10\%}$).—Columns 7 and 8 are proxies for the dissolution time.

Figure 1. Total mass of simulated star clusters vs. time for models with disks of different masses. The labels give the mass of the disk in units of $10^{10} M_\odot$, the lightest disk being $1 \times 10^{10} M_\odot$. Heavier disk masses enhance mass loss rates and accelerate the dissolution of star clusters.

Figure 2. Dissolution time/relaxation time ($t/t_{th}$) vs. disk mass for star clusters orbiting at 6 kpc from the galactic center. $t/t_{th}$ is computed when the star cluster has only 10% of its initial mass left. A heavier disk shortens the number of relaxations a star cluster undergoes before dissolution.

Table 2. Parameters of Models Executed.
parameters as the value of the scale length parameter to a more centrally concentrated one similar to a prolate bulge, second set of models (labels 10 and 13–17 in Table 2) have disk than the standard value used in the second set of models. The a and Nagai scale parameters in Table 2). All together, the evolution of 1.9 million stars was simulated for this section.

The disk density profiles of the three different disk geometries are represented in Figure 3. The disk model with the Miyamoto and Nagai scale parameters \( a = 0.4 \text{ kpc} \) and \( b = 0.5 \text{ kpc} \) is represented on the top panel. In this model, the mass of the disk is highly concentrated toward the center of the galaxy with a very steep falloff: the mid-plane disk density drops from \( 3.5 \times 10^9 M_\odot \text{ kpc}^{-3} \) at \( R_{GC} = 1 \text{ kpc} \) to \( 1 \times 10^7 M_\odot \text{ kpc}^{-3} \) at \( R_{GC} = 7 \text{ kpc} \).

For each disk profile, we study the evolution of an identical star cluster at six different radii from the galactic center. Models were executed at \( R_{GC} = 4, 6, 8, 10, 12.5, \) and 15 kpc from the galactic center. In Figure 3, circles with sizes proportional to the masses of the simulated star clusters after 10 Gyr of evolution are drawn at their respective galactocentric distances. The percentage of the initial star cluster mass remaining after 10 Gyr is also given in Figure 3.

Several simulated clusters do not survive to 10 Gyr, let alone up to a Hubble time. For the first set of models, with a very concentrated disk, the simulated star cluster evolving at 4 kpc from the center of the galaxy is completely dissolved at a record time of only 334 Myr. The simulated star cluster at an orbit of 6 kpc has a remaining mass of only \( \sim 870 M_\odot \) at 10 Gyr, just \( \sim 1\% \) of its initial mass. At 8 kpc from the galactic center, the modeled star cluster has \( 28\% \) of its initial mass after 10 Gyr of evolution.

The middle panel of Figure 3 shows the disk density profile for a disk model with \( a = 4 \text{ kpc} \) and \( b = 0.5 \text{ kpc} \), i.e., the same geometry as the models of Section 3. In this second set of models, the star cluster orbiting at 4 kpc is also completely dissolved before 10 Gyr.

The star cluster orbiting at 6 kpc in this more extended disk model (middle panel) has a mass of \( 7972.8 M_\odot \) after 10 Gyr of evolution, nine times the mass of the star cluster retained in a centrally peaked disk model. Clusters on orbits of 8 kpc in both models have nearly the same mass, i.e., only \( 1\% \) difference, after 10 Gyr of evolution. Similarly, for models at \( R_{GC} = 10 \text{ kpc} \) and beyond, their mass difference is clearly measurable but on the order of \( 3\% \) for clusters at \( R_{GC} = 10 \) and 12.5 kpc and 2\% for clusters at \( R_{GC} = 15 \text{ kpc} \).

A third set of models, with a more extended disk, was also studied. These models are represented in the bottom panel of Figure 3. With this disk geometry, that is \( a = 8 \text{ kpc} \) and \( b = 0.5 \text{ kpc} \), the mass of the disk is more spread out and the additive tidal effects of bulge and disk are not strong enough to completely disrupt the star cluster evolving at \( R_{GC} = 4 \text{ kpc} \). This innermost star cluster survives 10 Gyr of evolution with \( 8\% \) of its initial mass.

The aforementioned models show that disk geometry has a clear impact on the globular cluster mass function. This impact is more evident in the inner regions of the galaxy where a centrally concentrated disk (upper panel of Figure 3) adds to the tidal effects of the bulge and enhances the destruction rate of globular clusters within 6 kpc of the galactic center. At \( R_{GC} = 8 \text{ kpc} \), there is a switch or transition in the tidal effects due to the different disk geometries. Star clusters at \( R_{GC} = 10 \text{ kpc} \) and beyond have more mass on the first set of models with the centrally concentrated disk than on the second and third set of models with more extended disks. The tidal effects of the disk are still clearly measurable out to \( R_{GC} = 15 \text{ kpc} \) but are small in intensity, of the order of \( 3\%–5\% \) of the initial mass.

During 10 Gyr of evolution, most of the mass of star clusters within \( R_{GC} = 6 \text{ kpc} \) will be transferred to the host galaxy. Mass transfer from globular clusters to the host galaxy in the form of stellar tidal tails has been well documented observationally in Galactic globular clusters and modeled by Küpper et al. (2010). For example, Odenkirchen et al. (2001) described the presence of two tidal tails emerging from Palomar 5 using Sloan Digital Sky Survey data. After dissolution, the stars that formed the star cluster become part of the host galaxy stellar population.

5. HEATING AND COOLING DURING DISK CROSSINGS

Gnedin & Ostriker (1997) describe the effect of disk shocking as a shock or impulse of extra energy given to each star in the cluster as it crosses the disk (see also Spitzer 1958; Gnedin & Ostriker 1999). Stars close to the tidal boundary can be lost during disk-crossing events given that this extra energy allows them to escape from the star cluster. This section presents a close-up of the mass-loss and velocity dispersion of star clusters at each disk crossing.

A simulation with output given at more frequent intervals (every 1 Myr) was carried out to sample in detail the effects of a single disk crossing on a star cluster. The simulated star cluster is placed at 6 kpc from the galactic center with disk scale...
parameters of $a = 4$ kpc and $b = 0.5$ kpc, an initial orbital inclination of $\theta = 22.5^\circ$, and an orbital period of $\sim 150$ Myr. The height of the star cluster above the plane of the disk and the internal velocity dispersion are represented in Figure 4. The internal velocity dispersion shown in Figure 4 corresponds to the velocity dispersion of the outer 50% of the mass of the cluster; this is the section of the star cluster where individual stars experience the greatest changes in energy during each disk crossing. The velocity dispersion depicted in Figure 4 has been offset from an average level of $\sim 2$ km s$^{-1}$. For comparison, the average internal velocity dispersion of the entire star cluster is $\sim 3$ km s$^{-1}$ after 3 Gyr of evolution. The maximum height reached by a star cluster is 2 kpc from the plane of the disk.

A periodic impulse given to the velocity dispersion of the outer layers of the star cluster at each disk crossing is evident in Figure 4. The amplitude of this impulse, measured from an average level before disk crossing, is of 0.11 km s$^{-1}$. This increase is 5.2% in the average velocity dispersion of the stars that make up the outer 50% of the star cluster mass. There is a time delay of $\sim 7$ Myr between the star cluster crossing the equatorial plane of the disk (i.e., $z = 0$) and the peak of the impulse in velocity dispersion. This time delay is of the same order of magnitude as the crossing time that is, in this case, $t_{\text{cross}} \sim 2$ Myr. Following its peak, the velocity dispersion of the star cluster experiences a reduction of $\sim 0.14$ km s$^{-1}$ on average. Gnedin et al. (1999) mention a “refrigeration effect” or slight reduction in the energy dispersion of a star cluster following a disk shock. Figure 4 shows that at every disk crossing, an increase is followed systematically by a decrease in velocity dispersion.

The same velocity dispersion discussed earlier and the mass of the cluster outside its tidal radius ($M_{\text{out}}$) are plotted in Figure 5. The mass outside the tidal radius $M_{\text{out}}$ plotted in Figure 5 corresponds to the mass between the tidal radius and the escape radius. The escape radius is at least twice the length of the tidal radius, as defined in Section 2 and in Madrid et al. (2012).

Figure 5 shows that at each disk crossing ($z = 0$), the mass outside the tidal radius reaches a minimum, reflecting the strength of the tidal field felt by the star cluster. Over the 10 disk crossings represented in Figure 4, the star cluster loses $\sim 2600 M_\odot$, i.e., on average $260 M_\odot$ per disk crossing. After the star cluster passes the disk, its velocity dispersion decreases and the mass outside the tidal radius increases as this region of the cluster fills up because of a weaker tidal field. $M_{\text{out}}$ reaches a maximum when the star cluster is at the furthest distance from the disk.

The stars that make up the mass outside the tidal radius come from within the tidal radius as shown in Figure 5: the mass inside the tidal radius decreases when the mass outside the tidal radius increases. For the time lapsed during the disk crossings represented in Figures 4 and 5, the tidal radius can be considered to be constant. Over longer time scales, the tidal radius is a dynamic quantity by virtue of Equation (3).

An elementary estimate for the mass loss during a disk crossing can be derived using the above data and previous theoretical work. The energy change for stars in a star cluster is $\Delta E \sim (\Delta V)^2$ (Gnedin & Ostriker 1997) and using the virial theorem $E \propto M^2$, we write that $dM/M \sim dV/V$. Using the above simulations, we can give a numerical value to the expression $dM/M \sim dV/V$. The velocity increase in the outer regions of the star cluster has a numerical value of $dV/V \sim 0.1$ (see Figure 5, top panel). The change of mass can be determined also from the simulations, at 3 Gyr, $dM/M = 299/26125 \sim 0.01$. We find thus $dM/M \sim 0.2 dV/V$, that is, the change of mass is proportional to roughly 20% of the velocity dispersion change induced by a disk crossing.

6. ORBITAL INCLINATION

For a cluster on an inclined orbit, the passage through the disk implies an enhanced, or impulsive, gravitational force owing to the proximity of the mass that constitutes the disk, as shown in the previous section. Is the transient nature of the energy shift that stars receive during disk passages more important
than a high but constant tidal field? To properly disentangle the tidal effects and those of disk shocking, two simulations with different orbital inclination were carried out. One simulated star cluster orbits the galaxy in the plane of the disk, i.e., all its initial velocity is in the $y$ direction $V = V_y (\theta = 0)$. A second simulation, with the same characteristics of the previous simulation, evolves in an orbit perpendicular to the disk ($\theta = 90^\circ$); in this case, $V = V_x$. Both simulations evolve at a galactocentric distance of 6 kpc, with a disk mass of $5 \times 10^{10} M_\odot$ and are otherwise identical to the previous simulations with $\theta = 22.5^\circ$.

The impact of orbital inclination on the mass of the star cluster is shown in Figure 6. We plot the total mass versus time taking as a reference a simulated star cluster in a regular or standard orbit with an inclination of $\theta \sim 22.5^\circ$. We also plot the total masses of the simulated star clusters with orbits on the plane of the disk ($\theta = 0$), and perpendicular to it ($\theta = 90^\circ$).

Figure 6 shows that the star cluster in a perpendicular orbit to the plane of the disk is affected by a stronger mass loss than the cluster in a less inclined orbit or the cluster in an orbit in the plane of the disk. The simulated cluster on the orbit perpendicular to the disk is fully dissolved in 12.3 Gyr. After 12 Gyr of evolution, the simulated cluster evolving in the plane of the disk is 87 times more massive than the cluster on the orbit perpendicular to the disk. That is a total mass difference of 4477 $M_\odot$.

The models discussed in this section show that the compressive shocks experienced by the star cluster at each disk crossing are a dominant factor driving its dissolution. The variable gravitational potential generated by the presence of the disk on the orbit of the star cluster plays a more important role in the dissolution of the cluster than a constant and higher tidal field experienced by a star cluster that evolves in the plane of the disk. Note that star clusters with a small inclination angle with respect to the disk will experience additional shocking because of the presence of spiral arms (Gieles et al. 2007).

7. GALAXY EVOLUTION AND THE SURVIVAL OR DISSOLUTION OF A STAR CLUSTER

At the present time, NBODY6 does not allow for a dynamic host galaxy model. However, the first steps to include a time-dependent potential have already been taken by Renaud et al. (2011). Within the current framework of NBODY6, and to investigate the fate of a star cluster whose host galaxy grows with time, different simulations of star clusters evolving with different disk masses are combined. These are the simulations presented in Section 3.

Two different pathways for the evolution of a star cluster are built. The first series of accretion events experienced by the host galaxy leads to the survival of the star cluster after a Hubble time. The second scenario, where mass accretion lead to a more massive disk than in the first case, brings the star cluster to a complete dissolution before a Hubble time.

In the first scenario, a simulated star cluster evolves during 4 Gyr as the satellite of a host galaxy with a disk mass of $1 \times 10^{10} M_\odot$. At 4 Gyr, the host galaxy undergoes an instantaneous accretion event that brings the mass of the disk to $2.5 \times 10^{10} M_\odot$. At 10 Gyr, the host galaxy experiences a second accretion event that brings the host galaxy disk to a mass of $5 \times 10^{10} M_\odot$. In this scenario, after a Hubble time, the star cluster has more mass than does a simulated star cluster that evolves for a Hubble time in a galaxy with a constant disk mass of $5 \times 10^{10} M_\odot$, as expected.

In the second scenario, the star cluster begins its evolution in a galaxy with a disk mass of $5 \times 10^{10} M_\odot$; at 4 Gyr, the disk mass increases to $7.5 \times 10^{10} M_\odot$ and at 10 Gyr, it increases again to become $10 \times 10^{10} M_\odot$. The simulated star cluster is fully dissolved before a Hubble time because of the enhanced mass loss rates induced by these series of accretion events that built a more massive disk. During this exercise, the geometrical parameters of the disk are kept constant $a = 4$ and $b = 0.5$ kpc.

The evolution of the total mass of the star cluster as a function of time in the two scenarios of accretion undergone by the host galaxy described earlier are given in Figure 7. In addition to these two accreting models, the mass evolution of a star cluster evolving around a disk with a constant mass of $5 \times 10^{10} M_\odot$ is also plotted. This exercise shows how different accretion events experienced by the host galaxy can determine the survival or dissolution of a satellite star cluster. The solid line represents the star cluster evolving on a host galaxy with a constant disk mass of $5 \times 10^{10} M_\odot$. The lower panel displays the evolution of the disk mass that increases over time with discrete accretion events.
histories can lead to different depletion rates of satellite star clusters.

The results of this section can be obtained by joining together the appropriate curves from Figure 1. For example, we reconstruct the effect of a galaxy growing in mass by joining sections of the mass loss rates of star clusters in galaxies with a constant disk mass. The results of this section are also in agreement with Equation (5) in Section 3 that yields a relation between dissolution time, relaxation time, and the disk mass of the host galaxy. This equation shows, for example, that increasing the disk mass from $1 \times 10^{10} \, M_{\odot}$ to $10 \times 10^{10} \, M_{\odot}$ accelerates the destruction time of a star cluster from $56 \, t_{\text{rh}}$ to $11 \, t_{\text{rh}}$.

We note that we do not claim that the aforementioned exercise represents a realistic mass growth history of the Milky Way. Discrete accretion events simulate what is certainly a smoothly growing galaxy. An exciting future perspective is to have upcoming versions of NBODY6 with a fully incorporated time-dependent host galaxy potential.

8. FINAL REMARKS

The simulations presented in this work show that star clusters experience first hand the merger and accretion history of their host galaxy. The mass of the host galaxy disk plays an important and measurable role in the evolution of satellite star clusters, by affecting their mass loss rates and thus their structural parameters. The N-body models of star clusters have shown that different masses and geometries of the host galaxy disk can lead to different substructure within the inner 15 kpc of the galactic center. With each galaxy having a different mass growth history, there is still a lot of work yet to be conducted in order to understand how these different histories affect globular clusters. The mass and geometry of the disk directly affect the depletion rates of satellite stellar systems in a manner similar to how dark-matter halos are affected (D’Onghia et al. 2010).

Assuming a constant disk mass over a Hubble time, as it is often done, can lead to an overestimate of the dissolution rates of globular clusters and thus impact the derived globular cluster mass function.

The authors thank the referee for a constructive report that helped to improve this paper. This research made use of the NASA Astrophysics Data System Bibliographic Services and of Google. This work was performed on the GPU Supercomputer for Theoretical Astrophysical Research, a national facility operated by Swinburne University of Technology. The GPU Supercomputer for Theoretical Astrophysical Research is funded by Swinburne and the Australian Government’s Education Investment Fund. The authors give many thanks to Darren Croton, Chris Flynn, Anna Sippel (Swinburne), and Allan Duffy (Melbourne University) for asking the inquisitive questions that inspired this work.

REFERENCES

Aarseth, S. J. 1999, PASP, 111, 1333
Aarseth, S. J. 2003, Gravitational N-body Simulations: Tools and Algorithms (Cambridge Monographs on Mathematical Physics; Cambridge: Cambridge Univ. Press)
Aguilar, L., Hut, P., & Ostriker, J. P. 1988, ApJ, 335, 720
Baumgardt, H., & Makino, J. 2003, MNRAS, 340, 227
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton, NJ: Princeton Univ. Press), 514
Bullock, J. S., & Johnston, K. V. 2005, ApJ, 635, 931
D’Onghia, E., Springel, V., Hernquist, L., & Keres, D. 2010, ApJ, 709, 1138
Gieles, M., Athanassoula, E., & Portegies Zwart, S. F. 2007, MNRAS, 376, 809
Gnedin, O. Y., & Ostriker, J. P. 1997, ApJ, 474, 223
Gnedin, O. Y., & Ostriker, J. P. 1999, ApJ, 513, 626
Gnedin, O. Y., Lee, H.-M., & Ostriker, J. P. 1999, ApJ, 522, 935
Gómez, F. A., Helmi, A., Brown, A. G., & Li, Y-S. 2010, MNRAS, 408, 935
Kroupa, P. 2001, MNRAS, 322, 231
Kroupa, P., Tout, C. A., & Gilmore, G. 1993, MNRAS, 262, 545
Kundic, T., & Ostriker, J. P. 1995, ApJ, 438, 702
Küpper, A. H., Kroupa, P., Baumgardt, H., & Heggie, D. C. 2010, MNRAS, 401, 105
Leitner, S. N. 2012, ApJ, 745, 149
Madrid, J. P., Hurley, J. R., & Sippel, A. C. 2012, ApJ, 756, 167
Miyamoto, M., & Nagai, R. 1975, PASJ, 27, 533
Odenkirchen, M., Grebel, E. K., Rockosi, C. M., et al. 2001, ApJL, 548, L165
Paczynski, B. 1990, ApJ, 348, 485
Peñarrubia, J., Belokorov, V., Evans, N. W., et al. 2010, MNRAS, 408, L26
Plummer, H. C. 1911, MNRAS, 71, 460
Portegies Zwart, S. F., McMillan, S. L. W., & Gieles, M. 2010, ARA&A, 48, 431
Read, J. I., Wilkinson, M. I., Evans, N., Gilmore, G., & Klement, J. T. 2006, MNRAS, 367, 387
Renaud, F., & Gieles, M. 2013, MNRAS, 431, L83
Renaud, F., Gieles, M., & Boily, C. M. 2011, MNRAS, 418, 759
Spergel, D. N., Verde, L., Peiris, H. V., et al. 2003, ApJS, 148, 175
Spitzer, L. 1958, ApJ, 127, 17
Spitzer, L., & Hart, M. H. 1971, ApJ, 164, 399
Vesperini, E., & Heggie, D. C. 1997, MNRAS, 289, 898
West, M. J., Côté, P., Marzke, R. O., & Jordán, A. 2004, Natur, 427, 31
White, S. D. M., & Rees, M. J. 1978, MNRAS, 183, 341
Xue, X. X., Rix, H. W., Zhao, G., et al. 2008, ApJ, 684, 1143