OPTIMUM PRICING STRATEGY FOR COMPLEMENTARY PRODUCTS WITH RESERVATION PRICE IN A SUPPLY CHAIN MODEL

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Abstract. This paper describes a two-echelon supply chain model with two manufacturers and one common retailer. Two types of complementary products are produced by two manufacturers, and the common retailer buys products separately using a reservation price and bundles them for sale. The demands of manufacturers and retailer are assumed to be stochastic in nature. When the retailer orders for products, any one of manufacturers agrees to allow those products, and the rest of the manufacturers have to provide the same amount. The profits of two manufacturers and the retailer are maximized by using Stackelberg game policy. By applying a game theoretical approach, several analytical solutions are obtained. For some cases, this model obtains quasi-closed-form solutions, for others, it finds closed-form solutions. Some numerical examples, sensitivity analysis, managerial insights, and graphical illustrations are given to illustrate the model.

1. Introduction. Supply chain management (SCM) is to control a whole system, which may contain two players as two-echelon, three players as three-echelon, or multiple players as multi-echelon. The aim of the supply chain is to reduce the total cost of the whole system or to gain more profit. In literature, Goyal [7] first introduced SCM with single-vendor single-buyer concept. Banerjee [1] extended Goyal’s [7] model with cost reduction policy for the whole system under lot-for-lot (LFL) policy. Goyal [8] wrote a note on Banerjee’s [1] model considering single-setup-multi-delivery (SSMD) policy to reduce more costs by replacing LFL policy. Traditionally, it is common that all supply chain members are with equal power, but nowadays, each player of supply chain having the same power, is not possible always. Based on this situation, the basic supply chain strategy may not be adopted everywhere. Recent studies reviles that many exiting supply chains have the same problem like one player of SCM is more dominant than other players and he becomes leader and other players are follower. If SCM contains leader and follower i.e.,

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unequal power within the players, the basic traditional classical optimization cannot used for solving those types of research models. Those models can be solved by game theoretic approach. The coordination strategy between manufacturer and retailer has been studied by several researchers in the field of supply chain management. According to El-Ansary and Stern [4], both the manufacturer and the retailer have the power as the ability of one channel member to control decision variables in the marketing strategy of another member in a given channel at a different level of distribution (Pan et al. [13]). In many local and global supply chains, not all members (manufacturers and retailers) have equal power or capacity, one may be more dominant in nature. Thus, the leader-follower game policy arises within the supply chain strategy. Generally, for a small or local market, they have equal power, while players in larger cases, they have different powers and are always trying to obtain more benefit from a coordination policy, sometimes using cooperation or sometimes using non-cooperation. According to Ertek and Griffin [5], the retailer (Walmart and Tesco) has a more dominant role than the manufacturer in the supply chain. In some supply chains, manufacturers (for instance, Intel and Microsoft) play a more dominant role than the downstream members of the supply chain. A real world example is the announcement of Tesco to enter into the Indian grocery market following the entry of Walmart (Wu et al. [20]).

All these basic models considered single-type of basic product. Nowadays, many researches considered substitutable products, even though very few researcher considered complementary products. Ample research has been conducted in area for substitutable products, but not for complementary products, this research includes Choi [3], Hsieh and Wu [9], Yao et al. [21], and Wu et al. [20]. Substitutable product means when customer needs a product which is not available, but to fulfil his demand, he may buy a product to fulfil his demand whereas the complementary products means one product cannot be used without other product. The basic example of the complementary product is toothpaste and toothbrush. This situation involves basic traditional competition where customers choose products based on performance or marketing strategies of the industries. In contrast, Yue et al. [22], introduced the concept of complementary products that arise when customers have to buy more than one product at the same time to obtain full utility of all products. Recently, these complementary products have gained research interest as several complementary products are used in daily life, such as tooth brush and tooth paste. Complementary products can be considered as “perfect complements” which indicate that one cannot be used without the another. Some good examples of complementary products are a computer and its operating system, bed and mattress, computer hardware and software, pencil and eraser, and digital versatile disk (DVD) player and DVD.

Some complementary products do not perfectly complement to each other, this means that the non-perfect complement products can be used separately, like a dryer and washer. Some complementary products can be used as a base product with another that acts as a complementary product. For instance, an operating system acts as a base product, and any implicational software such as Microsoft Office, Excel, Power Point as the complementary product. In the literature, very few researchers have considered the pricing strategies of complementary products. Gabszewicz et al. [6] wrote the price competitions of complementary products. Zhao et al. [23] discussed about the pricing strategy of substitutable products considering uncertainties. Mukhopadhyay et al. [12] developed a Stackelberg model of
pricing policy for complementary products, where two firms produces two complementary products and makes a bundle and finally sells bundle products based on the asymmetric information sharing. Wei et al. [18] discussed about pricing strategy of complementary products with firms different market powers. They solved the model with three separate game strategies and compare the profits. But, they did not consider reservation price of products. Wei et al. [19] extended Wei et al.’s [18] model with warranty price and different game policies. However, each model assumed deterministic demand pattern based on selling-price. Regarding complementary products, two recent studies by Wei et al. [18] and Wei et al. [19] both assume deterministic demand pattern, whereas our model assumes a stochastic demand pattern with the reservation price. The demand for these types of complementary products may not be deterministic always. Therefore, this model assumes a stochastic demand pattern for manufacturer 1, manufacturer 2, and the common retailer. See Table 1 for author contributions.

Cárdenas-Barrón and Sana [2] extended a two-echelon supply chain model with sales teams initiative dependent demand. They proved that pricing policy in a real market makes a significant contribution in any supply chain coordination. Based on that study, Modak et al. [11] developed a supply chain coordination model with duopolistic retailers for perfect quality products, whereas Sarkar [14] discussed a supply chain coordination model with pricing discounts from supplier to retailer for a maximum range of order quantities of fixed lifetime products.

Differing from other studies in the field of complementary products, this model considers a two-echelon supply chain model which contains two manufacturers and a common retailer having stochastic selling-price-dependent demand i.e., this present study considers a two-echelon supply chain model with two duopolistic manufacturers and a common retailer, where two manufacturers produce two types of complementary products and the retailer reserves complementary products using a reservation price [10], and makes bundles with the complementary products, and sells those bundles. Stackelberg game approach with non-cooperative and cooperative strategies is employed to obtain a global optimum solution to the model. Several managerial insights are established based on profit and analytical derivations. Rest part of the paper is designed as follows: problem definition, notation, and assumptions are given in Section 2. Mathematical model is described in Section

### Table 1. Comparison between the contributions of different authors.

| Author(s)          | SCM | Competitive price study | Reservation price | Game approach | Stochastic demand |
|-------------------|-----|-------------------------|-------------------|---------------|-------------------|
| Choi [3]          | √   |                         |                   |               |                   |
| Yue et al. [22]   |     | √                       |                   |               |                   |
| Mukhopadhyay et al. [12] |   |                          |                   |               |                   |
| Wei et al. [18]   | √   | √                       |                   |               |                   |
| Cárdenas-Barrón and Sana [2] | √ | √                       |                   |               |                   |
| Sarkar [14]       |     | √                       |                   |               |                   |
| McCardie et al. [10] |   |                          |                   |               |                   |
| This Model        | √   | √                       | √                 | √             | √                 |
3. Numerical examples are illustrated in Section 4. Finally, conclusions and future extensions are given in Section 5.

2. Problem definition, assumptions, and notation. This section consists of problem definition, assumptions, and notation of the model.

2.1. Problem definition. There are two manufacturers and one common retailer. Two manufacturers produce two separate types of complementary products, and the retailer makes a bundle of those products and sells them. The problem is that there are some cooperative and non-cooperative strategies between them. The model derives those optimum strategies with maximum profits. The demands of manufacturers and the retailer are assumed as random based on prices. The model assumes, when the retailer places an order, any of the manufacturers can respond first, he and the retailer become leaders, and the other manufacturer becomes follower. Thus, the general joint total cost policy of the supply chain model does not apply here. Some game strategy with leader and follower is needed to solve this model. Based on the sequence of the game policy, it can be considered that Stackelberg game policy is most appropriate for this model. Thus, the model uses Stackelberg game policy to obtain the optimal profit of the whole supply chain.

2.2. Assumptions. The following assumptions are considered to develop this model.

1. A two-echelon supply chain model is considered with two manufacturers and one common retailer. Two manufacturers make two separate, but complementary types of products, and the common retailer forms bundles and sells them.
2. Demand for the retailer is assumed to be stochastic for bundle products, and the demand of the duopolistic manufacturers is also stochastic in nature. The demands are also dependent on reservation price (see for instance McCardle et al. [10]).
3. When the retailer places an order, anyone within the manufacturers respond first, then that manufacturer and the retailer become the leader and the other manufacturer becomes follower. Thus, the general cost strategy of the supply chain model does not apply here. The model needs any game strategy with leader and follower (see for instance Cárdenas-Barrón and Sana [2] and Modak et al. [11]).
4. Due to the leader and follower strategy, the model uses Stackelberg game strategy to decide the optimum price and maximized profit.
5. There is no information asymmetry in this model, so all information of the retailer and manufacturers is available to each other (see for instance Mukhopadhyay et al. [12]).
6. The model assumes that there is a potential market for the products with known market size $M$.
7. There are no shortages for the retailer or manufacturers.
8. The production rate is constant and is always greater than the demand for both the manufacturers’ case.

2.3. Notation. The following notation are used to develop the model.
Decision variables

- \( Q \): order quantity (units)
- \( P_i \): selling-price of product \( j, j = 1, 2 \) ($/unit)
- \( P_r \): selling-price of the bundle product ($/unit)

Random variables

- \( D_{m_i} \): demand for product \( j, j = 1, 2 \) (units)
- \( D_r \): demand for the bundle product (units)

Parameters

- \( C_i \): manufacturing cost of product \( j, j = 1, 2 \) ($/unit)
- \( h_{m_i} \): holding cost of product \( j \) per unit per unit time, \( j = 1, 2 \) ($/unit/unit time)
- \( h_r \): holding cost of the bundle product per unit per unit time ($/unit/unit time)
- \( S_{m_i} \): setup cost per setup of product \( j, j = 1, 2 \) (units)
- \( K_{m_i} \): production rate of product \( j, j = 1, 2 \) (units)
- \( M \): known market size (units)
- \( A \): ordering cost per order of the retailer ($/order)
- \( I_{m_i} \): inventory of manufacturer \( i \) at \( t \in [0, t_{m_i}], i = 1, 2 \)
- \( I_{m_i}^{T_{m_i}} \): inventory of manufacturer \( i \) at \( t \in [t_{m_i}, T_{m_i}], i = 1, 2 \)
- \( AP_{m_i} \): expected average profit of manufacturer \( i, i = 1, 2 \)
- \( AP_r \): expected average profit of the retailer
- \( t_{m_i} \): time required for maximum inventory of manufacturer \( i, i = 1, 2 \)
- \( T_{m_i} \): cycle time of manufacturer \( i, i = 1, 2 \)
- \( R_{m_i}^a \): lower limit of reservation price of manufacturer \( i, i = 1, 2 \)
- \( R_{m_i}^b \): upper limit of reservation price of manufacturer \( i, i = 1, 2 \)

3. **Mathematical model.** This section consists of model formulation, profit of manufacturer 1, profit of manufacturer 2, profit of the common retailer, and solution methodology.

3.1. **Model formulation.** There are two manufacturers, who produce two basic types of products that are complementary to each other, like toothpaste and toothbrush; DVD players and DVD; etc. The products are sold to a common retailer who bundles and sells them. The model assumes those products as product 1 and product 2. The manufacturers buy raw materials at \( C_1 \) and \( C_2 \). The finished products are sold by the manufacturer to the retailer at \( P_1 \) and \( P_2 \). The demand of each product 1 and product 2 is specified by the reservation price \( R_1 \) and \( R_2 \), respectively. The model assumes that there is a potential market for the products with known market size \( M \). The assumption of market size \( M \) is reasonable as products are basic products and there are old available past data. However, uncertainty occurs due to the variability in reservation prices. The distribution of reservation prices for product 1 and product 2, is assumed to be uniform between \( R_{m_1}^a \) and \( R_{m_1}^b \) for product 1 and \( R_{m_2}^a \) and \( R_{m_2}^b \) for product 2. The model assumes without any loss of generality \( R_{m_1}^a = 0 \) and \( R_{m_1}^b = 1 \). Thus, it can be obtained \( R_{m_2}^a \geq 0, C_1 \leq 1 \) and \( C_2 \leq R_{m_2}^b \). Finally, the model assumes \( R_{m_2}^b \leq 1 \).
The demand of product 1 from the manufacturer 1 is given by

\[ D_{m1} = M \int_{P_1}^{1} \frac{1}{1-P_1} dx_1 = M(1 - P_1) \]  

(1)

when the reservation price lies between [0,1]. The demand of product 2 from manufacturer 2 is based on reservation price (see for instance McCardle et al. [10]) as

\[ D_{m2} = M \int_{P_2}^{R_b^2} \frac{1}{R_b^2 - R_a^2} dx_2 = M \left( R_b^2 - P_2 \right) \]  

(2)

when the reservation price lies between \([R_a^2, R_b^2]\). Based on the assumptions, models of manufacturer 1, manufacturer 2, and the common retailer are explained in Section 3.2, 3.3, and 3.4, respectively.

3.2. Profit of manufacturer 1. By assumptions 7 and 8, there is no shortage, and production is constant, thus, the model of manufacturer 1 considers the basic economic production quantity model. Thus, the governing differential equation of the present inventory position of manufacturer 1 is given by

\[ \frac{dI_{m11}(t)}{dt} = K_{m1} - D_{m1}, \ 0 \leq t \leq t_{m1} \]

with initial condition \( I_{m11}(0) = 0 \)  

(3)

\[ \frac{dI_{m12}(t)}{dt} = -D_{m1}, \ t_{m1} \leq t \leq T_{m1} \]

with initial condition \( I_{m12}(T_{m1}) = 0 \)  

(4)

The respective solutions of (3) and (4) are given by

\[ I_{m11}(t) = (K_{m1} - D_{m1})t, \ 0 \leq t \leq t_{m1} \]  

(5)

\[ I_{m12}(t) = D_{m1}(T_{m1} - t), \ t_{m1} \leq t \leq T_{m1} \]  

(6)

Basically, these equations represent the inventory position at time \( t \). Due to the continuity condition from (5) and (6),

\[ I_{m11}(t_{m1}) = I_{m12}(t_{m1}) \]

which gives

\[ t_{m1} = \frac{T_{m1}D_{m1}}{K_{m1}} \]

This is the time by which the maximum inventory for product 1 are produced by manufacturer 1.

The setup cost for manufacturer 1 is

\[ \frac{S_{m1}}{T_{m1}} = S_{m1} \frac{D_{m1}}{Q} \]

The holding cost for manufacturer 1 is

\[ h_{m1} = \frac{h_{m1}}{T_{m1}} \left[ \int_0^{t_{m1}} I_{m11}(t) dt + \int_{t_{m1}}^{T_{m1}} I_{m12}(t) dt \right] \]

\[ = \frac{h_{m1}Q}{2} \left[ 1 - \frac{D_{m1}}{K_{m1}} \right] \]
Thus, the average profit for manufacturer 1 is
\[ AP_{m_1}(P_1, Q) = (P_1 - C_1)M(1 - P_1) - \frac{S_{m_1}M(1 - P_1)}{Q} - \frac{Qh_{m_1}}{2} \left( 1 - M \left( 1 - \frac{P_1}{K_{m_1}} \right) \right) \] (7)

Now, the profit of manufacturer 2 can be calculated by assuming a stochastic demand pattern.

3.3. Profit for manufacturer 2. The governing differential equation of the present status of the inventory of manufacturer 2 are
\[ \frac{dI_{m_{21}}(t)}{dt} = K_{m_2} - D_{m_2}, \ 0 \leq t \leq t_{m_2} \]
with initial condition \( I_{m_{21}}(0) = 0 \) (8)
\[ \frac{dI_{m_{22}}(t)}{dt} = -D_{m_2}, \ t_{m_2} \leq t \leq T_{m_2} \]
with initial condition \( I_{m_{22}}(T_{m_2}) = 0 \) (9)

After solving the above differential equations, the present inventory conditions at time \( t \) can be written as
\[ I_{m_{21}}(t) = (K_{m_2} - D_{m_2})t, \ 0 \leq t \leq t_{m_2} \] (10)
and
\[ I_{m_{22}}(t) = D_{m_2}(T_{m_2} - t), \ t_{m_2} \leq t \leq T_{m_2} \] (11)

From the continuity condition from (10) and (11) at time \( t_{m_2} \), one can write
\[ I_{m_{21}}(t_{m_2}) = I_{m_{22}}(t_{m_2}) \]
which determines the maximum inventory produced by manufacturer 2.

The setup cost for manufacturer 2 is
\[ = \frac{S_{m_2}}{T_{m_2}} = \frac{S_{m_2}D_{m_2}}{Q} \]
The holding cost for manufacturer 2 is
\[ = \frac{h_{m_2}}{T_{m_2}} \left[ \int_0^{t_{m_2}} I_{m_{21}}(t)dt + \int_{t_{m_2}}^{T_{m_2}} I_{m_{22}}(t)dt \right] \]
\[ = \frac{h_{m_2}Q}{2} \left[ 1 - \frac{D_{m_2}}{K_{m_2}} \right] \]

Thus, the average profit of the manufacturer 2 is given by
\[ AP_{m_2}(P_2, Q) = (P_2 - C_2)M \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right) - \frac{S_{m_2}M \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right)}{Q} - \frac{Qh_{m_2}}{2} \left( 1 - \frac{M \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right)}{K_{m_2}} \right) \] (12)
3.4. **Profit for the common retailer.** Now, the profit of the common retailer can be calculated. The retailer has the following demand for bundle products (refer to McCardle et al. [10]).

\[ D_r = \int_{\max\{P_r-1,R_r^2\}}^{R_r^2} \left( \int_{\max\{P_r-x,0\}}^{1} dx_1 \right) \frac{M}{R_r^2 - R_r^2} dx_2 \]

\[ = \frac{M(2 - 2P_r + R_r^2 + R_r^2)}{2} \]  

(13)

This demand is for bundle products by customers to the retailer. Thus, the profit for the retailer is

\[ AP_r = (P_r - P_1 - P_2) \frac{M(2 - 2P_r + R_r^2 + R_r^2)}{2} - \frac{AM(2 - 2P_r + R_2^a + R_2^b)}{2Q} - \frac{h_r Q}{2} \]

(14)

3.5. **Solution methodology.** Based on the different powers and strategies of the supply chain members, there are several cases for which the global maximum solution can be obtained. As on the order quantities of the common retailer, anyone of the manufacturers has to agree with the same order quantities. If manufacturer 2 accepts first, the model assumes the fact as Case 1 and if manufacturer 1 accepts first, then it is assumed as Case 2.

**Case 1. Manufacturer 2 and the common retailer as leader and manufacturer 1 as follower.** Because there is a collaborative supply chain and asymmetry of power for manufacturers and retailer, they maximize their own profit as follows:

**Case 1.1. Non-cooperation between leaders (manufacturer 2 and retailer) and the follower (manufacturer 1).** The model assumes when the retailer places an order, manufacturer 2 accepts the order first. Based on the joint decision of the retailer and manufacturer 2, manufacturer 1 has to provide the same amount that manufacturer 2 offers to the retailer. Based on this assumption, Stackelberg game policy is used. For this case, manufacturer 2 and retailer are leaders, and manufacturer 1 is the follower. Instead of a leader-follower condition, they maximize their own profits, indicating the non-cooperative relationship between them. Therefore, the optimization process starts with manufacturer 1.

**Theorem 1.** The profit of manufacturer 1 contains the global maximum at \( Q^* = \sqrt{\frac{2S_m D_m}{h_m (1 - \frac{D_m}{S_m})}} \) and \( P_1^* = \frac{1}{4Q^* C_1} [2Q^* K_1 + 2Q^* K_1 C_1 + \frac{S_m (K_m - D_m + 2M_m D_m)}{K_m - D_m}] \)

if it satisfies

(i) \( \frac{2S_m D_m}{Q^*} > 0 \) or \( \frac{2S_m D_m}{Q^*} \left( -2M - \frac{MS_m}{Q^*} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_m}{2K_m} \frac{\partial Q^*}{\partial P_1^*} \right) \)

\[ - \left( \frac{MS_m}{Q^*} + \frac{Mh_m}{2K_m} \right)^2 > 0 \]  

and \( \frac{2S_m D_m}{Q^*} \left( -2M - \frac{MS_m}{Q^*} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_m}{2K_m} \frac{\partial Q^*}{\partial P_1^*} \right) \left( \frac{MS_m}{Q^*} + \frac{Mh_m}{2K_m} \right)^2 < 0 \)  

or (ii) if \( \frac{2S_m D_m}{Q^*} < 0 \) or \( \frac{2S_m D_m}{Q^*} \left( -2M - \frac{MS_m}{Q^*} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_m}{2K_m} \frac{\partial Q^*}{\partial P_1^*} \right) \)
The profit of manufacturer 1 contains the global maximum at

\[ P_1^* = \frac{1}{4Q^*K_m} \left[ 2Q^*K_m + 2Q^*K_mC_1 + \frac{S_{m_1}(K_m - D_{m_1} + 2K_mD_m)}{K_m - D_{m_1}} \right] \]

if it satisfies

(i) \[ \frac{2S_{m_1}D_{m_1}}{Q^*} > 0 \] or \[ \frac{2S_{m_1}D_{m_1}}{Q^*} \left( -2M - \frac{MS_{m_1}}{Q^*} \frac{\partial Q^*}{\partial P_{1^*}} + \frac{Mh_{m_1}}{2K_m} \frac{\partial Q^*}{\partial P_{1^*}} \right) \]

and

\[ \frac{2S_{m_1}D_{m_1}}{Q^*} \left( -2M - \frac{MS_{m_1}}{Q^*} \frac{\partial Q^*}{\partial P_{1^*}} + \frac{Mh_{m_1}}{2K_m} \frac{\partial Q^*}{\partial P_{1^*}} \right) > 0 \]

and

\[ \left( \frac{MS_{m_1}}{Q^*} + \frac{Mh_{m_1}}{2K_m} \right)^2 < 0 \]
Proof. The proof and the theorem are the similar as Theorem 1 of the non-cooperative strategy. Thus, the model uses the same optimum values as

- Theorem 7. The profit of manufacturer 2 is the global maximum at

\[ P^*_2 = \frac{1}{2} \left[ \frac{R^b}{Q^*} + \frac{2 + R^2 + R^2_2}{4} + (P^*_1 + P^*_2) \right] \text{ if it satisfies } \frac{2M}{R^2_2 - R^2_1} > 0 \text{ and } R^b_2 - R^2_1 < 4. \]

Proof. See Appendix D for the proof of this theorem.

Now, within two manufacturers, if manufacturer 1 accepts the order first, based on that the model considers the fact as Case 2.

**Case 2. Manufacturer 1 and the common retailer as leader and manufacturer 2 as follower:** Within the leaders, there may arise the cooperative or non-cooperative strategy based on their own profit or joint profit. Thus, two subcases may occur.

**Case 2.1. Non-cooperation between leaders (manufacturer 1 and retailer) and the follower (manufacturer 2).** In this case, manufacturer 1 responds as soon as the retailer orders for product. Manufacturer 2 has to follow the decision of manufacturer 1 and retailer, who consider their own profits, i.e., they follow the non-cooperative policy.

**Theorem 6.** The profit of manufacturer 2 is the global maximum at

\[ Q^* = \sqrt{\frac{2S_{m_2}D_{m_2}}{Q^*}} \quad \text{and} \quad P^*_2 = \frac{1}{2} \left[ \frac{R^b_2}{Q^*} + \frac{S_{m_2}}{Q^*} + \frac{Q^* h_{m_2}}{2K_{m_2}} \right] \]

if it satisfies (i) \( \frac{4S_{m_2}D_{m_2}}{Q^*} > 0 \)

- or (ii) if \( \frac{4S_{m_2}D_{m_2}}{Q^*} < 0 \).
The profit of manufacturer 2 is the global maximum at 
\[ P_2^* = \frac{1}{2} \left[ \frac{A}{Q^2} + 2 + \frac{Q^2}{2} + (P_1^* + P_2^*) \right]. \]

**Proof.** See Appendix F for the proof of this theorem. \(\square\)

**Theorem 8.** The profit of the common retailer is the global maximum at 
\[ Q^* = \frac{1}{2} \left( P_2^* - 2 + \frac{Q^2}{2} + (P_1^* + P_2^*) \right) \]

if it satisfies
\[ 4S_m D_m = Q^2 \left( \begin{array}{c} 2M \frac{D_m}{R_2 - R_2^*} - MS_m \frac{\partial Q^*}{\partial P_1} + \frac{Mh_m}{2Km_m (R_2 - R_2^*)} \partial Q^* \partial P_2 \end{array} \right) \]

or
\[ 4S_m D_m = Q^2 \left( \begin{array}{c} 2M \frac{D_m}{R_2 - R_2^*} - MS_m \frac{\partial Q^*}{\partial P_1} + \frac{Mh_m}{2Km_m (R_2 - R_2^*)} \partial Q^* \partial P_2 \end{array} \right) \]

and \[ Q = \sqrt{\frac{2S_m D_m}{h_m (1 - \frac{Q^2}{R_2})}}. \]

The profit equation of the retailer and manufacturer 1 under a cooperative strategy is as follows:
\[ AP_{m1}(P_1, P_r, Q) = (P_1 - C_1) D_{m1} - S_{m1} D_{m1} - \frac{Q h_m}{2} \left( 1 - \frac{D_{m1}}{K_{m1}} \right) \]

\[ - \left( \frac{AD_{12}}{Q^2} - h_r Q + \frac{P_r - P_1 - P_2}{2} \right) \]

**Theorem 10.** The profits of manufacturer 1 and the common retailer are global maximum at 
\[ P_1^* = \frac{1}{2} \left[ 1 + C_1 + \frac{S_{m1}}{Q^2} - Q^2 h_m - \frac{2R_2 + R_2^* - 2P_r^*}{2K_{m1}} \right] \]

and
\[ P_r^* = \frac{1}{2} \left[ 1 + C_1 + \frac{S_{m1}}{Q^2} - Q^2 h_m - \frac{2R_2 + R_2^* - 2P_r^*}{2K_{m1}} \right]. \]

**Proof.** See Appendix H for the proof of this theorem. \(\square\)
Table 2. Input data

| Player      | Manufacturer 1 $M = 1500$ | Manufacturer 2 $M = 1500$ | Retailer $M = 1500$ |
|-------------|---------------------------|---------------------------|---------------------|
| Setup cost  | $S_{m_1} = 20$           | $S_{m_2} = 20$           | $A = 1$             |
| Holding cost| $h_{m_1} = 0.015$        | $h_{m_2} = 0.015$        | $h_r = 0.01$        |
| Production rate | $K_{m_1} = 2000$       | $K_{m_2} = 2000$        | $-$                 |
| Purchasing cost | $C_1 = 0.25$            | $C_2 = 0.15$            | $-$                 |
| Reservation interval | [0, 1]           | [0.1, 0.9]             | [0.1, 0.9]          |

$-$ indicates that the parameter is not available for this case.

Table 3. Optimum results of Example 1

| Case | $Q^*$ units | $P_1^*$ $/\text{unit}$ | $P_2^*$ $/\text{unit}$ | $P_r^*$ $/\text{unit}$ | $AP_{m_1}^*$ $/\text{year}$ |
|------|-------------|------------------------|------------------------|------------------------|-----------------------------|
| 1.1  | 1433.14     | 0.63                   | 0.53                   | 1.33                   | 195.39                      |
| 1.2  | 1433.14     | 0.63                   | 0.44                   | 1.29                   | 195.39                      |
| 2.1  | 1690.88     | 0.63                   | 0.53                   | 1.33                   | 195.18                      |
| 2.2  | 1690.88     | 0.51                   | 0.53                   | 1.27                   | $-$                         |

| Case | $AP_{m_2}^*$ $/\text{year}$ | $AP_r^*$ $/\text{year}$ | $AP_{m_1r}^*$ $/\text{year}$ | $AP_{m_2r}^*$ $/\text{year}$ |
|------|-----------------------------|------------------------|-----------------------------|-----------------------------|
| 1.1  | 246.92                      | 36.37                  | $-$                         | $-$                         |
| 1.2  | $-$                         | $-$                    | $-$                         | 294.18                      |
| 2.1  | 247.15                      | 35.90                  | $-$                         | $-$                         |
| 2.2  | 247.15                      | $-$                    | 245.87                      | $-$                         |

$-$ indicates that the average profit is not available for this case.

4. Numerical examples. As all models belong to complementary products covered the area of the firm, not the two-echelon supply chain, thus all values cannot be taken from any model directly. Almost in all models of complementary products, the profit functions are written with assuming the initial fixed cost like setup cost, holding cost, and others are zero. In reality, it is almost impossible to consider all these basic costs are zero. Thus, the model considers the values for each case in all cases.

4.1. Numerical experiments. There are two examples as Example 1 and Example 2.

Example 1. The input data for Example 1 is given in Table 2 and the optimal results are given in Table 3.

Example 2. The data is taken from McCardle et al. [10] and it is given in Table 4. The optimal results are given in Table 5.

It is found that this model obtains more profit than the exiting model [10]. Based on [10], the profit was $28.125, whereas this model obtains more than the profit from
Figure 1. Graphical representation for Case 1.1, total profit of manufacturer 1 versus selling-price and lot size

Figure 2. Graphical representation for Case 1.1, total profit of manufacturer 2 versus selling-price

existing literature for the whole supply chain for both cases. The sensitivity analysis is given to analyze the effect the key parameters of this model.

4.2. Sensitivity analysis. The optimal values of the different parameters change significantly with changes (−50%, −25%, +25%, +50%) in the following table. On the basis of the sensitivity analysis of different parameters, the following features are observed.

1. If market size increases the profits of manufacturers and the common retailer increase. The retailer’s profit is more insightful than those of manufacturer 1 and manufacturer 2. Profit of manufacturer 1 is less sensitive than the retailer’s profit, and manufacturer 2’s profit is the least sensitive within all of them. With any change in market size in positive or negative direction, profits
of all players are almost equally sensitive percentages of setup cost, holding cost, and purchasing cost increase, profits of manufacturer 1 and manufacturer 2 decrease. The sensitivity analysis results, shown in the Table 2, indicates that, for manufacturer 1 and manufacturer 2, the changes in setup cost and holding cost are almost similar. Among all key parameters of manufacturer 1 and manufacturer 2, purchasing cost is most sensitive in both cases. It is more sensitive in manufacturer 1 than manufacturer 2. With the increasing of ordering cost and holding cost, the profit of the retailer is decreasing. Comparing between the ordering cost and holding cost of the common retailer, it is found that holding cost is more sensitive than ordering cost. See Table 6 for its sensitivity analysis.
2. In Case 1.2, the supply chain players follow the cooperative strategy. With the increasing value of market size manufacturer 1’s profit individually and the profit of manufacturer 2 and retailer increases jointly. The joint profit of manufacturer 2 and retailer is less sensitive than manufacturer 1 for the cooperative strategy. With the increasing percentage of setup cost, holding cost, and purchasing cost, the profit of manufacturer 1 and manufacturer 2 decreases, which is same as Case 1.1. Among all cost parameters, purchasing
cost is most sensitive than others. Setup cost for manufacturer 1 is less sensitive than the setup cost of manufacturer 2, whereas the vice-versa is true for holding cost. For retailer, with the increasing of ordering cost and holding cost, the profit is decreasing as similar as in Case 1.1. In between ordering cost and holding cost of retailer, holding cost is more sensitive than ordering cost, which is similar as the previous Case 1.1. See Table 7 for its sensitivity analysis.

3. For Case 2.1, due to increase of market size, profits of manufacturer 1, manufacturer 2, and retailer increases and within this three profits, retailer’s profit
As on Case 1.2, the supply chain players also follow cooperative strategy. If ordering cost, holding cost, setup cost, and purchasing cost of manufacturers and retailer increase, total profit for each case decreases. Among all costs, purchasing cost of manufacturer 1 is the most sensitive and ordering cost of the retailer is the least sensitive with respect to other costs. Hence, ordering is more sensitive than other manufacturers. Therefore, for longtime business, retailer is gainer more than others. Thus, the retailer always try to maintain the supply chain as well as both the manufacturers follow the same. See Table 8 for its sensitivity analysis.
Table 4. Input data from McCardle et al. [10]

| Player  | Manufacturer 1 | Manufacturer 2 | Retailer |
|---------|----------------|----------------|----------|
| Market size (units) | | | |
| Setup cost ($/setup) | $S_{m1} = 0$ | $S_{m2} = 0$ | $A = 0$ |
| Holding cost ($/unit/year) | $h_{m1} = 0$ | $h_{m2} = 0$ | $h_r = 0$ |
| Production rate (units/year) | $K_{m1} = 0$ | $K_{m2} = 0$ | $-$ |
| Purchasing cost ($/unit) | $C_1 = 0.25$ | $C_2 = 0.25$ | $-$ |
| Reservation interval | $[0, 1]$ | $[0, 0.9]$ | $[0, 0.9]$ |

$-$ indicates that the parameter is not available for this case.

Table 5. Optimum results of Example 2

| Case | $Q^*$ units | $P^*_1$ $/\text{unit}$ | $P^*_2$ $/\text{unit}$ | $P^*_r$ $/\text{unit}$ | $AP^*_m$ $/\text{year}$ | $AP^*_r$ $/\text{year}$ |
|------|-------------|------------------------|------------------------|------------------------|--------------------------|--------------------------|
| 1.1  | 300         | 0.625                  | 0.625                  | 1.375                  | 14.0625                  |                          |
| 1.2  | 300         | 0.625                  | 0.541667              | 1.33                    | 14.0625                  |                          |
| 2.1  | 300         | 0.625                  | 0.625                  | 1.375                  | 14.0625                  |                          |
| 2.2  | 300         | 0.625                  | 0.541667              | 1.33333                |                          |                          |

| Case | $AP^*_m$ $/\text{year}$ | $AP^*_r$ $/\text{year}$ | $AP^*_m_r$ $/\text{year}$ | $AP^*_m_2r$ $/\text{year}$ |
|------|--------------------------|--------------------------|---------------------------|---------------------------|
| 1.1  | 14.0625                  | 1.5625                   | $-$                       | $-$                       |
| 1.2  | $-$                      | $-$                      | $-$                       | 16.1458                  |
| 2.1  | 14.0625                  | 1.5625                   | $-$                       | $-$                       |
| 2.2  | 14.0625                  | $-$                      | 16.1458                  | $-$                       |

$-$ indicates that the average profit is not available for this case.

cost may increase until a certain limit for a long run business as its effect is very small, but the purchasing cost should not increase. As the purchasing cost of raw materials of product 1 and raw materials of product 2 both are the most sensitive; thus, manufacturers should reduce the purchasing cost by changing raw materials or using other discount policies for reducing the purchasing cost. See Table 9 for its sensitivity analysis.

4.3. Managerial insights from the comparative studies.

1. Figure 11 indicates the manufacturer 2’s profit versus selling-price of product 2 under cooperative and non-cooperative strategy. From Figure 11, there are several insights can be drawn like the cooperative strategy is more beneficial than non-cooperative strategy, but there is a upper limit of selling-price of product 2; it means if the selling-price of product 2 increases more than the upper limit per product, then the non-cooperative strategy is more dominant than the cooperative strategy. The main reason for using supply chain is to gain more profit and there are several studies, where the researchers proved
Table 6. Sensitivity analysis for Case 1.1

| Parameter | \(AP_{m_1}\) (in %) | \(AP_{m_2}\) (in %) | \(AP_r\) (in %) |
|-----------|----------------------|----------------------|------------------|
| -50%      | -52.76               | -52.18               | -59.85           |
| -25%      | -26.38               | -26.09               | -29.93           |
| \(M\) +25%| +26.38               | +26.09               | +29.93           |
| +50%      | +52.75               | +52.18               | +59.85           |

| Parameter | \(AP_{m_1}\) (in %) | \(AP_{m_2}\) (in %) | \(AP_r\) (in %) |
|-----------|----------------------|----------------------|------------------|
| -50%      | +1.99                | -50%                 | +1.97            |
| -25%      | +0.99                | -25%                 | +0.98            |
| +25%      | -0.99                | +25%                 | -0.98            |
| +50%      | -1.98                | +50%                 | -1.95            |

| Parameter | \(AP_{m_1}\) (in %) | \(AP_{m_2}\) (in %) | \(AP_r\) (in %) |
|-----------|----------------------|----------------------|------------------|
| -50%      | +2.33                | -50%                 | +1.42            |
| -25%      | +1.06                | -25%                 | +1.42            |
| +25%      | -0.94                | +25%                 | -0.71            |
| +50%      | -1.78                | +50%                 | -1.42            |

| Parameter | \(AP_{m_1}\) (in %) | \(AP_{m_2}\) (in %) | \(AP_r\) (in %) |
|-----------|----------------------|----------------------|------------------|
| -50%      | +38.63               | -50%                 | +22.18           |
| -25%      | +18.54               | -25%                 | +10.82           |
| +25%      | -17.04               | +25%                 | -10.29           |
| +50%      | -32.58               | +50%                 | -20.04           |

that always cooperative strategy is beneficial than non-cooperative strategy. But, for this study, the model obtains a limit of selling-price of product 2, after that no can conclude that the cooperative strategy is always beneficial than non-cooperative strategy, which is clearly obtained by the comparative study of Figure 11. The 2nd important outcomes from Figure 11 is that the maximum price of product 2 should not exceed 1st limit point per product for cooperative strategy and 2nd limit point per product for non-cooperative strategy as after that manufacturer 2 will face loss which is clearly shown from Figure 11. Thus, from Figure 11, it is found that the product 2’s selling-price can vary within a certain fixed closed-interval for cooperative strategy and another certain fixed closed-interval for non-cooperative strategy, which is only from the view point of manufacturer 2.

2. For Case 1, total supply chain profit is less for non-cooperative strategy and more for cooperative strategy, respectively. Thus, anyone can judge to stay with cooperative policy, not with non-cooperative strategy.

3. Figure 12 indicates the common retailer’s profit versus selling-price of bundle product under cooperative and non-cooperative strategy. From Figure 12, it is found that the cooperative strategy is more beneficial than non-cooperative strategy. Both the profit functions show strictly concave functions within
Table 7. Sensitivity analysis for Case 1.2

| Parameter | $AP_{m_1}$ change (in %) | $AP_{m_2r}$ change (in %) |
|-----------|--------------------------|----------------------------|
| $-50\%$  | -52.15                   | -53.04                     |
| $-25\%$  | -26.24                   | -26.52                     |
| $M$       | $+25\%$                  | +26.49                     |
|           |                          | +26.52                     |
|           | $+50\%$                  | +53.18                     |
|           |                          | +53.04                     |

| Parameter | $AP_{m_1}$ change (in %) | $AP_{m_2r}$ change (in %) |
|-----------|--------------------------|----------------------------|
| $S_{m_1}$ | $-50\%$                  | -50%                       |
|           | $-25\%$                  | -25%                       |
|           | $+25\%$                  | +25%                       |
|           | $+50\%$                  | +50%                       |
| $h_{m_1}$ | $-50\%$                  | -50%                       |
|           | $-25\%$                  | -25%                       |
|           | $+25\%$                  | +25%                       |
|           | $+50\%$                  | +50%                       |
| $C_1$     | $-50\%$                  | -50%                       |
|           | $-25\%$                  | -25%                       |
|           | $+25\%$                  | +25%                       |
|           | $+50\%$                  | +50%                       |

4. Figure 13 indicates the manufacturer 1’s profit versus selling-price of product 1 under cooperative and non-cooperative strategy. From Figure 13, there are several insights can be drawn like the cooperative strategy is more beneficial than non-cooperative strategy, but there is a limit of selling-price of product 1; it means if the selling-price of product 1 will increase more than a certain price per product, then the non-cooperative strategy is dominant over the cooperative strategy. The main reason for using supply chain is to gain more and there are several studies, where the researchers proved that always cooperative strategy is beneficial than non-cooperative strategy. But, for this study, the model obtains a limit of selling-price of product 1, after that no can conclude cooperative strategy is beneficial than non-cooperative strategy.
Table 8. Sensitivity analysis for Case 2.1

| Parameters | \( AP_{m_1} \) (in %) | \( AP_{m_2} \) (in %) | \( AP_{r} \) (in %) |
|------------|-----------------|-----------------|-----------------|
| \(-50\%\)  | 53.25           | 52.57           | -61.77          |
| \(-25\%\)  | 26.62           | 26.28           | -30.89          |
| \(+25\%\)  | 26.62           | 26.28           | +30.89          |
| \(+50\%\)  | 53.25           | 52.57           | +61.77          |

| Parameter   | \( AP_{r} \) (in %) |
|-------------|---------------------|
| \(-50\%\)  | +0.21               |
| \(-25\%\)  | +0.11               |
| \(+25\%\)  | -0.11               |
| \(+50\%\)  | -0.21               |

| Parameter   | \( AP_{m_1} \) (in %) |
|-------------|---------------------|
| \(-50\%\)  | +1.70               |
| \(-25\%\)  | +0.85               |
| \(+25\%\)  | -0.84               |
| \(+50\%\)  | -1.69               |

| Parameter   | \( AP_{m_2} \) (in %) |
|-------------|---------------------|
| \(-50\%\)  | +2.34               |
| \(-25\%\)  | +1.17               |
| \(+25\%\)  | -1.17               |
| \(+50\%\)  | -2.34               |

| Parameter   | \( AP_{r} \) (in %) |
|-------------|---------------------|
| \(-50\%\)  | +11.77              |
| \(-25\%\)  | +5.89               |
| \(+25\%\)  | -5.89               |
| \(+50\%\)  | -11.77              |

forever at any circumstances. The 2nd important outcomes from Figure 13 is that the maximum price of product 1 should not exceed a fixed certain price per product for cooperative strategy and another fixed certain price per product for non-cooperative strategy as after that manufacturer 1 will face loss which is clearly shown from Figure 13. Thus, from Figure 13, it is found that the product 1’s selling-price can vary within a certain closed interval for cooperative strategy and another certain closed interval for non-cooperative strategy, which is only from the view point of manufacturer 1.

5. For Case 2, total supply chain profits for non-cooperative strategy is less than the cost from cooperative strategy. Thus, anyone can judge to stay with cooperative policy, not with non-cooperative strategy. It is found that the maximum total supply chain profit is found for Case 2.2 i.e. cooperative policy with retailer and manufacturer 1 as leader and manufacturer 2 as follower. Thus, retailer always prefers manufacturer 1 should reply first, then the profit of the whole supply chain will always be maximum.

6. Figure 14 indicates the common retailer’s profit versus selling-price of bundle product under cooperative and non-cooperative strategy. From Figure 14, there are several insights can be drawn like the cooperative strategy is more beneficial than non-cooperative strategy. Both the profit functions show
strictly concave functions within certain limit. An important outcomes from Figure 14 is that the maximum price of bundle product should not exceed a certain limit of selling-price per product for cooperative strategy and 2nd certain limit as selling-price per product for non-cooperative strategy as after that the common retailer will face loss as on Figure 14. Thus, from Figure 14, it is found that the bundle product’s selling-price can vary within a certain interval of selling-price for cooperative strategy and another certain interval of selling-price for non-cooperative strategy which is only from the view point of the common retailer.

5. Conclusions. The model derived a two-echelon supply chain model with unequal powers for supply chain players. Two manufacturers produced two different types of complementary products and sold them to the common retailer. The retailer bundled those complementary products and sold the bundles to customers. As on the order quantities of the retailer, at least one of the manufacturers agreed first with the lot size ordered by the retailer, and another manufacturer had to agree with the same order quantities. Stackelberg game policy is utilized to obtain the maximum profit. An analytical derivation was considered to develop the model. There were several theorems to determine the global maximum profit of
Figure 11. Comparative studies of cooperation and non-cooperation for the selling-price of product 2 of manufacturer 2 in Case 1. Blue ink of the graphical representation indicates under cooperative strategy and the red ink of the graphical representation indicates under noncooperative strategy.

Figure 12. Comparative studies of cooperation and non-cooperation for the selling-price of bundle product of retailer in Case 1. Blue ink of the graphical representation indicates under cooperative strategy and the red ink of the graphical representation indicates under noncooperative strategy.

supply chain players of the supply chain. The model obtained some closed-form and some quasi-closed-form solutions. The numerical study proved the global optimum profits for the players. From the numerical study, it was found that the total supply chain profit was more for this model from the exiting literature [20]. Generally, the coordination within the supply chain players always give more profit than the non-coordination policy, but this model obtained that there were some limitations for those cases, the non-coordination strategy was more dominant than the coordination policy. The model obtained some limits for selling-price s of product 1, product 2, and bundle product, if the selling-price is more that the limit, the whole supply chain may face some loss instead of profit. It can be concluded that
retailer would prefer manufacturer 1 for accepting order first as the more supply chain profit was obtained for the coordination within retailer and manufacturer 1 as leader and manufacturer 2 as follower. It may be possible that the retailer may allow some loans to make products for manufacturer 1 or any advanced payment policy may be used for manufacturer 1 by which the total supply chain profit can be increased. That would be an interesting research direction, which was indicated by this model. The model was considered, when the retailer only sold bundled complementary products. This is the limitation of the model. Further extension can be done by assuming that the common retailer would sell complementary products in bundles and individually. Another extension of this model is to consider uncertainties within the model, i.e., the market size is assumed as known, which may be fuzzy for different products. The shipment from manufacturers to retailer can be
used by single-setup-multi-delivery policy [21]. The setup cost of both manufacturers can be reduced by using continuous investment [22] and with deterioration [23] of complementary products. That would contribute a lot for another extensions of this model. This is the ongoing future research for this model.

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Appendix A.

Proof. The profit of manufacturer 1 can be written as

\[ AP_{m_1}(P_1, Q) = (P_1 - C_1)D_{m_1} - \frac{S_{m_1}D_{m_1}}{2} \left(1 - \frac{D_{m_1}}{K_{m_1}}\right) - \frac{Qh_{m_1}}{2} \left(1 - \frac{D_{m_1}}{K_{m_1}}\right) \]

To maximize the profit of manufacturer 1, by taking the first order derivative of the profit equation of manufacturer 1 with respect to \(Q\), one can obtain

\[ \frac{\partial AP_{m_1}}{\partial Q} = \frac{S_{m_1}D_{m_1}}{Q^2} - \frac{h_{m_1}}{2} \left(1 - \frac{D_{m_1}}{K_{m_1}}\right) \]

Using necessary condition of profit maximization, it gives

\[ Q^* = \sqrt{-\frac{2S_{m_1}D_{m_1}}{h_{m_1} \left(1 - \frac{D_{m_1}}{K_{m_1}}\right)}} \]

This is the optimal lot size that the retailer and manufacturer 2 finalize, and manufacturer 1 has to produce.

Taking the derivative of profit equation of manufacturer 1 with respect to \(P_1\), one obtains

\[ \frac{\partial AP_{m_1}}{\partial P_1} = -(P_1 - C_1)M + M(1 - P_1) + \frac{S_{m_1}M}{Q^*} + \frac{Q^*h_{m_1}M}{2K_{m_1}} \]

Using the necessary condition of profit maximization gives

\[ P_1^* = \frac{1}{4Q^*K_{m_1}} \left[2Q^*K_{m_1} + 2Q^*K_{m_1}C_1 + \frac{S_{m_1}(K_{m_2} - D_{m_2} + 2K_{m_2}D_{m_1})}{K_{m_1} - D_{m_1}}\right] \]

This is the optimal selling-price at which the manufacturer sells product to the retailer. [For simplification see Appendix A.]

From the sufficient condition of profit maximization, one has
\[
\frac{\partial^2 AP_{m_1}}{\partial Q^2} = \frac{2S_{m_1}D_{m_1}}{Q^{*2}} > 0,
\]
\[
\frac{\partial^2 AP_{m_1}}{\partial Q \partial P_1^*} = \frac{MS_{m_1}}{Q^{*2}} + \frac{Mh_{m_1}}{2K_{m_1}},
\]
\[
\frac{\partial^2 AP_{m_1}}{\partial P_1^{*2}} = -2M + \left[ \frac{MS_{m_1}}{Q^{*2}} - \frac{Mh_{m_1}}{2K_{m_1}} \right] \left( \frac{2S_{m_1}M(1-P_1^*)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)} \right)^{-1/2}
\times \frac{MS_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right) + M^2S_{m_1}(1-P_1^*)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)^2}
\]
\[
\frac{\partial^2 AP_{m_1}}{\partial Q^2} \frac{\partial^2 AP_{m_1}}{\partial P_1^{*2}} - \left( \frac{\partial^2 AP_{m_1}}{\partial Q^2 \partial P_1^*} \right)^2 = \frac{2S_{m_1}D_{m_1}}{Q^{*3}} \left( -2M + \left[ \frac{MS_{m_1}}{Q^{*2}} - \frac{Mh_{m_1}}{2K_{m_1}} \right] \left( \frac{2S_{m_1}M(1-P_1^*)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)} \right)^{-1/2}
\times \frac{MS_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right) + M^2S_{m_1}(1-P_1^*)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)^2} \right)^2
\]

The optimal selling price \(P_1^*\) of product 1 must satisfy the above equation. When it is satisfied, the optimum profit of the follower (as manufacturer 1) for the optimum lot size and selling-price of products can be obtained, and manufacturer 2 optimizes its profits. Now, for the maximum profit of manufacturer 2, the theorem is as follows:

Therefore, the profit of manufacturer 1 contains the global maximum at \(Q^* = \sqrt{\frac{2S_{m_1}D_{m_1}}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)}}\) and \(P_1^* = \frac{1}{4Q^*K_{m_1}} [2Q^*K_{m_1} + 2Q^*K_{m_1}C_1 + S_{m_1}\{K_{m_1} - D_{m_1} + 2K_{m_1}D_{m_1}\}]

if it satisfies

(i) \(\frac{2S_{m_1}D_{m_1}}{Q^{*3}} > 0\) or \(\frac{2S_{m_1}D_{m_1}}{Q^{*3}} \left( -2M + \frac{MS_{m_1}}{Q^{*2}} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_{m_1}}{2K_{m_1}} \frac{\partial Q^*}{\partial P_1^*} \right) - \left( \frac{MS_{m_1}}{Q^{*2}} + \frac{Mh_{m_1}}{2K_{m_1}} \right)^2 \right) > 0

and \(\frac{2S_{m_1}D_{m_1}}{Q^{*3}} \left( -2M + \frac{MS_{m_1}}{Q^{*2}} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_{m_1}}{2K_{m_1}} \frac{\partial Q^*}{\partial P_1^*} \right) - \left( \frac{MS_{m_1}}{Q^{*2}} + \frac{Mh_{m_1}}{2K_{m_1}} \right)^2 \right) < 0

or (ii) \(\frac{2S_{m_1}D_{m_1}}{Q^{*3}} \left( -2M + \frac{MS_{m_1}}{Q^{*2}} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_{m_1}}{2K_{m_1}} \frac{\partial Q^*}{\partial P_1^*} \right) - \left( \frac{MS_{m_1}}{Q^{*2}} + \frac{Mh_{m_1}}{2K_{m_1}} \right)^2 \right) < 0

and \(\frac{2S_{m_1}D_{m_1}}{Q^{*3}} \left( -2M + \frac{MS_{m_1}}{Q^{*2}} \frac{\partial Q^*}{\partial P_1^*} + \frac{Mh_{m_1}}{2K_{m_1}} \frac{\partial Q^*}{\partial P_1^*} \right) - \left( \frac{MS_{m_1}}{Q^{*2}} + \frac{Mh_{m_1}}{2K_{m_1}} \right)^2 \right) > 0;\) otherwise, there is no global maximum.

\[
Q = \sqrt{\frac{2S_{m_1}M(1-P_1)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)}}
\]
\[
\frac{\partial Q}{\partial P_1} = -\left( \frac{2S_{m_1}M(1-P_1)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)} \right)^{-1/2} \frac{MS_{m_1} \left( 1 - \frac{M(1-P_1)}{K_{m_1}} \right) + M^2S_{m_1}(1-P_1)}{h_{m_1} \left( 1 - \frac{M(1-P_1^*)}{K_{m_1}} \right)^2} \]
and \( \frac{\partial AP_{m1}}{\partial Q} = -\frac{S_{m1}}{Q^2} M(1 - P_1) + \frac{h_{m1}}{2} \left( 1 - \frac{M(1 - P_1)}{K_{m1}} \right) \)

The expression for the second principal minor is as follows:

\[
\frac{\partial^2 AP_{m1}}{\partial Q^2} \frac{\partial^2 AP_{m1}}{\partial P_1^2} - \left( \frac{\partial^2 AP_{m1}}{\partial Q \partial P_1} \right)^2 = \frac{2S_{m1}D_{m1}}{Q^3} \left( -2M - \frac{MS_{m1}}{Q^2} \frac{\partial Q}{\partial P_1} + \frac{Mh_{m1}}{2K_{m1}} \frac{\partial Q}{\partial P_1} \right) - \left( \frac{MS_{m1}}{Q^2} + \frac{Mh_{m1}}{2K_{m1}} \right)^2
\]

\[\square\]

**Appendix B.**

**Proof.** The profit equation of manufacturer 2 can be written as

\[ AP_{m2}(P_2, Q) = (P_2 - C_2)D_{m2} - \frac{S_{m2}D_{m2}}{Q} \left( 1 - \frac{D_{m2}}{K_{m2}} \right) \]

As the lot size is already at the optimal level, substituting the value of \( Q \), it gives

\[ AP_{m2}(P_2) = MP_2 \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right) - MC_2 \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right) - \frac{MS_{m2}}{Q^*} \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right) - \frac{Q^*h_{m2}}{2K_{m2}} \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right) \]

To obtain the optimum selling-price for manufacturer 2, by taking derivatives two times with respect to \( P_2 \), one can obtain

\[ \frac{dAP_{m2}}{dP_2} = M \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right) - \frac{MP_2}{R_2^b - R_2^a} + \frac{MC_2}{R_2^b - R_2^a} + \frac{MS_{m2}}{Q^*} \frac{\partial Q^*}{\partial P_2} \frac{R_2^b - R_2^a}{2K_{m2}(R_2^b - R_2^a)} \]

\[ \frac{d^2 AP_{m2}}{dP_2^2} = -\frac{2M}{R_2^b - R_2^a} + \frac{MS_{m2}}{Q^*} \frac{\partial Q^*}{\partial P_2} + \frac{Mh_{m2}}{2K_{m2}} \frac{\partial Q^*}{\partial P_2} \]

From the necessary condition of profit maximization, some simplification results in

\[ P_2^* = \frac{1}{2} \left[ R_2^b + C_2 + \frac{S_{m2}}{Q^*} - \frac{h_{m2}Q^*}{2K_{m2}} \right] \]

This is the optimum selling-price of product 2, at which the retailer has to buy product from manufacturer 2.

As \( R_2^b - R_2^a \) is always positive, manufacturer 2 has the global optimum profit at

\[ P_2^* = \frac{1}{2} \left[ R_2^b + C_2 + \frac{S_{m2}}{Q^*} - \frac{h_{m2}Q^*}{2K_{m2}} \right] \]

Now,

\[ Q = \sqrt{\frac{2S_{m2}M \left( \frac{R_2^b - P_2}{R_2^b - R_2^a} \right)}{h_{m2} \left( 1 - \frac{M}{K_{m2}} \frac{R_2^b - P_2}{R_2^b - R_2^a} \right)}} \]
Differentiating partially with respect to $P_2$, one can find
\[
\frac{\partial Q}{\partial P_2} = -\frac{MS_{m_2}}{h_{m_2}} \left( \frac{2MS_{m_2}(R_2^b - P_2)}{h_{m_2}\{K_{m_2}(R_2^b - R_2^a) - M(R_2^b - P_2)\}} \right)^{-1/2} \left\{ K_{m_2}(R_2^b - R_2^a) - M(R_2^b - P_2) \right\} + \frac{M(R_2^b - P_2)}{2} \left\{ K_{m_2}(R_2^b - R_2^a) - M(R_2^b - P_2) \right\}^2
\]

\[\to 0\]

**Appendix C.**

**Proof.** The profit equation of the common retailer can be written as
\[
AP_r(P_r, Q) = (P_r - P_1^* - P_2^*) D_{12} - \left( \frac{AD_{12}}{Q} + h_r Q \right)
\]

As the optimum lot size is already obtained from manufacturer 1, the retailer uses the same optimum value of lot size. Thus, substituting the value of optimum lot size $Q^*$, produces
\[
AP_r(P_r) = \frac{h_r Q^*}{2} + \frac{(P_r - P_1^* - P_2^*)M(2 + R_2^a + R_2^b - 2P_r)}{2} - \frac{AM(2 + R_2^a + R_2^b - 2P_r)}{2Q^*}
\]

To maximize the profit of the common retailer, taking derivative two times with respect to $P_r$, it can be written as
\[
\frac{dAP_r(P_r)}{dP_r} = AM + \frac{M(2 + R_2^a + R_2^b - 2P_r)}{2} - M(P_r - P_1^* - P_2^*) = 0
\]
\[
\frac{d^2AP_r(P_r)}{dP_r^2} = -2M < 0
\]

From the necessary condition of optimization, the optimum value of the bundle price can be found as
\[
P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} + \frac{2 + R_2^a + R_2^b}{2} + (P_1^* + P_2^*) \right]
\]

As $M$ is always greater than zero, the retailer always has global optimum profit at $P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} + \frac{2 + R_2^a + R_2^b}{2} + (P_1^* + P_2^*) \right]$. Therefore, for each case, the model obtains the global maximum solution analytically.

**Appendix D.**

**Proof.** As manufacturer 2 and the common retailer have the same optimum lot size as manufacturer 1, substituting the value of $Q^*$ from Case 1.1 of the non-cooperative
policy, the profit equation becomes
\[
AP_{m2r}(P_2, P_r) = M(P_2 - C_2) \left( \frac{R_b^2 - P_2}{R_b^2 - R_2^2} \right) - \frac{S_{m2} M(R_b^2 - P_2)}{(R_b^2 - R_2^2)} \\
- \sqrt{\frac{2S_{m2} D_{m1}}{h_{m1}(1-D_{m1}/K_{m1})}} h_{m2} \left( 1 - \frac{M(R_b^2 - P_2)}{(R_b^2 - R_2^2)} \right) \\
- \frac{AM(2 + R_b^2 + R_b^2 - 2P_r)}{2} - h_r \sqrt{\frac{2S_{m2} D_{m1}}{h_{m1}(1-D_{m1}/K_{m1})}} \\
+ \frac{M(P_r - P_1^* - P_2)(2 + R_b^2 + R_b^2 - 2P_r)}{2}
\]

To obtain the optimum \( P_2 \) and \( P_r \), the derivatives with respect to \( P_2 \) and \( P_r \), produces
\[
\frac{\partial A_{m2r}(P_2)}{\partial P_2} = M \left( \frac{R_b^2 - P_2}{R_b^2 - R_2^2} \right) - \frac{M(P_2 - C_2)}{R_b^2 - R_2^2} + \frac{MS_{m2}}{R_b^2 - R_2^2} \\
- \frac{Q^* M h_{m2}}{R_b^2 - R_2^2} - \frac{M(2 + R_b^2 + R_b^2 - 2P_r)}{2},
\]
\[
\frac{\partial A_{m2r}(P_r)}{\partial P_r} = \frac{AM}{Q^*} + \frac{M(2 + R_b^2 + R_b^2 - 2P_r)}{2} - \frac{M(P_r - P_1^* - P_2)}{2}.
\]

From the necessary conditions of profit maximization, the optimum values of \( P_2 \) and \( P_r \) can be found by setting \( \frac{\partial A_{m2r}(P_2)}{\partial P_2} = 0 \) and \( \frac{\partial A_{m2r}(P_r)}{\partial P_r} = 0 \), which give
\[
P_2^* = \frac{1}{2} \left[ R_b^2 + C_2 + \frac{S_m}{Q^*} - Q^* h_{m2} - \frac{(R_b^2 - R_2^2)(2 + R_b^2 + R_b^2 - 2P_r^*)}{2} \right]
\]
and \( P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} + \frac{2 + R_b^2 + R_b^2}{2} + (P_1^* + P_2^*) \right] \).

From the sufficient conditions, the second order derivatives are as follows:
\[
\frac{\partial^2 A_{m2r}(P_r^*)}{\partial P_r^2} = -2M < 0
\]
\[
\frac{\partial^2 A_{m2r}(P_2^*)}{\partial P_2^2} = - \frac{2M}{R_b^2 - R_2^2} < 0
\]
\[
\frac{\partial^2 A_{m2r}}{\partial P_2^2 \partial P_r^2} = M
\]
\[
\frac{\partial^2 A_{m2r}}{\partial P_r^2 \partial P_2^2} - \left( \frac{\partial^2 A_{m2r}}{\partial P_2^2 \partial P_r^2} \right)^2 = M^2 \left( \frac{4}{R_b^2 - R_2^2} - 1 \right) > 0
\]
i.e., \( R_b^2 - R_2^2 < 4 \)

Thus, both the 1st principal minor is less than zero and 2nd principal minor is greater than zero, which implies the principal minors are alternating in sign. Therefore, the profits of manufacturer 2 and the common retailer have a global maximum at
\[
P_2^* = \frac{1}{2} \left[ R_b^2 + C_2 + \frac{S_m}{Q^*} - Q^* h_{m2} + \frac{(R_b^2 - R_2^2)(2 + R_b^2 + R_b^2 - 2P_r^*)}{2} \right]
\]
and \( P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} + \frac{2 + R_b^2 + R_b^2}{2} + (P_1^* + P_2^*) \right] \).
Appendix E.

Proof. The profit equation of manufacturer 2 is

\[ AP_{m_2}(P_2, Q) = MP_2 \left( \frac{R_b^b - P_2}{R_b^b - R_2^b} \right) - C_2 M \left( \frac{R_b^b - P_2}{R_b^b - R_2^b} \right) - \frac{M S_{m_2}}{Q} \left( \frac{R_b^b - P_2}{R_b^b - R_2^b} \right) \]

To obtain the optimum \( P_2 \) and \( Q \) for manufacturer 2, the derivative with respect to \( P_2 \) is calculated as

\[ \frac{\partial AP_{m_2}}{\partial P_2} = M \left( \frac{R_b^b - P_2}{R_b^b - R_2^b} \right) - \frac{M P_2}{R_2^b - R_2^b} + \frac{M C_2}{R_2^b - R_2^b} + \frac{M S_{m_2}}{Q(R_2^b - R_2^b)} \]

From the necessary condition of profit maximization, this expression becomes

\[ \frac{\partial AP_{m_2}}{\partial P_2} = 0 \]

Simplifying this results in the optimum value of \( P_2 \) as

\[ P_2^* = \frac{1}{2} \left[ R_2^b + C_2 + \frac{S_{m_2}}{Q} + \frac{Q h_{m_2}}{2K_{m_2}} \right] \]

which is the optimum selling-price of manufacturer 2, at which the retailer buys product 2. Applying the necessary condition of profit maximization for \( Q \), it can be written as

\[ \frac{\partial AP_{m_2}}{\partial Q} = -\frac{2 S_{m_2} D_{m_2}}{Q^2} + \frac{h_{m_2}}{K_{m_2}} \left( 1 - \frac{D_{m_2}}{K_{m_2}} \right) = 0 \]

\[ \Rightarrow Q^* = \sqrt{\frac{2 S_{m_2} D_{m_2}}{h_{m_2} \left( 1 - \frac{D_{m_2}}{K_{m_2}} \right)}} \]

Thus, the selling-price \( P_2^* \) is the optimum for manufacturer 2, and the lot size \( Q \) is optimum for manufacturer 2, as well as for manufacturer 1 and the retailer. These values of lot size and selling-price are optimum or not, which can be confirmed by the sufficient condition of profit maximization, thus, one can obtain

\[ \frac{\partial^2 AP_{m_2}(Q^*)}{\partial Q^2} > 0 \]

\[ \frac{\partial^2 AP_{m_2}(P_2^*)}{\partial P_2^{*2}} = -\frac{2 M}{R_2^b - R_2^b} + \left( \frac{M S_{m_2}}{Q^2(R_2^b - R_2^b)} + \frac{M h_{m_2}}{2K_{m_2}(R_2^b - R_2^b)} \right) \]

\[ \times \frac{MS_{m_2}}{h_{m_2} \left( K_{m_2}(R_2^b - R_2^b) - M(R_2^b - P_2^*) \right)^{1/2}} \]

\[ \times \left\{ K_{m_2}(R_2^b - R_2^b) - M(R_2^b - P_2^*) + M(R_2^b - P_2^*) \right\}^2 \]
\[
\frac{\partial AP_{m_2}}{\partial Q^* \partial P_2^*} = \frac{2S_{m_2} M}{Q^* \left(R_2^b - R_2^a\right)} + \frac{M h_{m_2}}{2K_{m_2} \left(R_2^b - R_2^a\right)}
\]

\[
\frac{\partial^2 AP_{m_2}}{\partial Q^* \partial P_2^*} - \left(\frac{\partial AP_{m_2}}{\partial Q^* \partial P_2^*}\right)^2 = \frac{4S_{m_2} D_{m_2}}{Q^* \left(R_2^b - R_2^a\right)} - \frac{2M}{Q^* \left(R_2^b - R_2^a\right)} - \frac{MS_{m_2}}{Q^* \left(R_2^b - R_2^a\right)} \frac{\partial Q^*}{\partial P_2^*}
\]

\[
= \frac{2K_{m_2} (R_2^b - R_2^a)}{M h_{m_2}} \frac{2K_{m_2} (R_2^b - R_2^a)}{M h_{m_2}} - \left(\frac{2S_{m_2} M}{Q^* \left(R_2^b - R_2^a\right)} + \frac{M h_{m_2}}{2K_{m_2} \left(R_2^b - R_2^a\right)}\right)
\]

Thus, if these conditions hold, then one can obtain global maximum solutions.  

**Appendix F.**

**Proof.** The optimum lot size \(Q\) is already known from manufacturer 2. Thus, substituting the value of \(Q\), the profit equation of manufacturer 1 becomes

\[
AP_{m_1}(P_1) = MP_1(1 - P_1) - MC_1(1 - P_1) - \frac{MS_{m_1}(1 - P_1)}{Q^*} + \frac{Q^*}{2} - \frac{M(1 - P_1)Q^*}{2K_{m_1}}
\]

To obtain the optimum selling-price of product 1, taking derivative two times with respect to \(P_1\), results in

\[
\frac{dAP_{m_1}}{dP_1} = M - 2MP_1 + MC_1 + \frac{MS_{m_1}}{Q^*} - \frac{MQ^*}{2K_{m_1}}
\]

\[
\frac{d^2 AP_{m_1}}{dP_1^2} = -2M < 0
\]

By the necessary condition of profit maximization, the optimum value of \(P_1\) is

\[
P_1^* = \frac{1}{2} \left[1 + C_1 + \frac{S_{m_1}}{Q^*} - \frac{Q^*}{2K_{m_1}}\right]
\]

Thus, the profit of manufacturer 1 is always the global maximum at

\[
P_1^* = \frac{1}{2} \left[1 + C_1 + \frac{S_{m_1}}{Q^*} - \frac{Q^*}{2K_{m_1}}\right].
\]

**Appendix G.**

**Proof.** As the retailer has the same optimum lot size as manufacturer 1, substituting the optimum value \(Q\), the retailer’s profit equation is

\[
AP_r(P_r) = -\left(\frac{AM(2 + R_2^B + R_2^a - 2P_r)}{2Q^*} + \frac{b_2 Q^*}{2}\right)
\]

\[
+ \left(P_r - P_1^* - P_2^*\right)M(2 + R_2^a + R_2^B - 2P_r)
\]

To obtain the optimum value of \(P_r\), by taking derivative two times with respect to \(P_r\), is calculated as

\[
\frac{dAP_r}{dP_r} = \frac{AM}{Q^*} - \frac{M}{2} \left(2 + R_2^a + R_2^B\right) - 2MP_r + M(P_1^* + P_2^*)
\]
\[
\frac{d^2 P_r}{dP_r^2} = -2M < 0
\]

Applying the necessary condition of profit maximization, gives

\[
P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} - \frac{2 + R_a^2 + R_b^2}{2} + (P_1^* + P_2^*) \right]
\]

It is obvious that \(M\) is greater than zero. Thus, the profit of the retailer is always the global maximum at \(P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} - \frac{2 + R_a^2 + R_b^2}{2} + (P_1^* + P_2^*) \right]. \)

**Appendix H.**

**Proof.** As manufacturer 1 and the common retailer are leaders, the optimum lot size value has to take from the manufacturer 2. Thus, substituting the optimum value of \(Q\), the profit function is

\[
AP_{m,r}(P_1, P_r) = M(P_1 - C_1)(1 - P_1) - \frac{Q^* h_{m2}}{2} \left( 1 - \frac{M(1 - P_1)}{K_{m1}} \right)
\]

\[
- \left( \frac{AM(2 + R_a^2 + R_b^2 - 2P_r)}{2Q^*} + \frac{h_r Q^*}{2} \right) - \frac{M S_{m1}(1 - P_1)}{Q^*}
\]

\[
+ \frac{(P_r - P_1 - P_2^*) M(2 + R_a^2 + R_b^2 - 2P_r)}{2}
\]

To obtain the optimum selling-price of manufacturer 1 and the common retailer, the derivative with respect to \(P_1\) and \(P_r\), results in

\[
\frac{\partial AP_{m,r}}{\partial P_1} = M(1 - P_1) - M(P_1 - C_1) + \frac{MS_{m1}}{Q^*} - \frac{MQ^* h_{m2}}{K_{m1}}
\]

\[
- \frac{M(2 + R_a^2 + R_b^2 - 2P_r)}{2}
\]

\[
\frac{\partial AP_{m,r}}{\partial P_r} = \frac{2AM}{Q^*} - M(P_r - P_1 - P_2^*) + \frac{M(2 + R_a^2 + R_b^2 - 2P_r)}{2}
\]

Using the necessary condition of profit maximization, results in

\[
\frac{\partial AP_{m,r}}{\partial P_1} = 0
\]

which gives

\[
P_1^* = \frac{1}{2} \left[ 1 + C_1 + \frac{S_{m1}}{Q^*} - \frac{Q^* h_{m1}}{2K_{m1}} - \frac{2 + R_a^2 + R_b^2 - 2P_r}{2} \right]
\]

and

\[
\frac{\partial AP_{m,r}}{\partial P_r} = 0
\]

which becomes

\[
P_r^* = \frac{1}{2} \left[ \frac{A}{Q^*} + \frac{2 + R_a^2 + R_b^2}{2} + (P_1^* + P_2^*) \right]
\]
To show the global maximum profit, the sufficient conditions have to check. For sufficient conditions,
\[ \frac{\partial^2 AP_{m1r}}{\partial P_1^*\partial P_r^*} = -2M < 0 \]
\[ \frac{\partial^2 AP_{m1r}}{\partial P_r^*\partial P_r^*} = -2M < 0 \]
\[ \frac{\partial^2 AP_{m1r}}{\partial P_1^*\partial P_r^*} = M \]
\[ \frac{\partial^2 AP_{m1r}}{\partial P_1^*\partial P_r^*} - \left( \frac{\partial^2 AP_{m1r}}{\partial P_1^*\partial P_r^*} \right)^2 = 3M^2 > 0 \]

Hence, all principal minors are alternative in sign. Therefore the profit of manufacturer 1 and common retailer has the global maximum at
\[ P_1^* = \frac{1}{2} \left[ 1 + C_1 + \frac{S_{m1}}{Q^*} - \frac{Q^* h_{m1}}{2K_{m1}} - \frac{2+R_1^* k_{m1}^2 - 2P_r^*}{2} \right] \]
\[ P_r^* = \frac{1}{2} \left[ A + \frac{2+R_1^* k_{m1}^2}{2} + (P_1^* + P_2^*) \right]. \]

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