Effect of partial magnetic field on thermo gravitational convection in an inclined cavity

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Abstract: The study presented gives a succinct overview of the effect of partially applied magnetic field on natural convective heat transfer in a differentially heated inclined cavity. The left and right sidewalls are differentially heated such that their temperatures are maintained (at $T_l$ and $T_c$ respectively), while the top and bottom walls have been assumed to be adiabatic, such that heat transfer through those walls can be neglected. The results are supposed to elucidate the effect of changing Rayleigh number (Ra), Hartmann number (Ha), and cavity inclination angle ($\phi$) on the thermo-fluid phenomena. The analysis is carried out numerically utilizing finite volume based computing code. The results have been demonstrated in the form of streamlines, isotherms, and average Nusselt number. From the analysis, it is found that the flow structure and associated heat transfer characteristics are severely influenced by the studied range of governing parameters.

Keywords: Partial magnetic field; Inclined cavity; Natural convection; Heat transfer.

1. Introduction

The application of the magnetic field alters the thermo-fluid phenomena and this can be a very useful technique taking into an opportunity for improving the thermal performance in many engineering processes. The magnetic field induces the Lorentz force in a fluid in motion which is further affected by the thermal gradient and other flow conditions. Furthermore, fluid flow parameters and the type of fluid used, introduction of porous media, or externally driven flow along with the application of magnetic field, and various boundary layer conditions can greatly affect the thermal performance and hence the convective heat transfer efficacy, which leads us again to the practical importance of this field of study. Fundamentals on the above subject topics can be found in Refs. [1,2].

The effect of the magnetic field on buoyancy-driven flow has been studied before in different forms by multiple authors. Detailed research in this area has been conducted by Giwa et al.[3] where the effect of several variables and changing flow parameters have been reviewed. The inclination of the magnetic field, the introduction of porous media, nanofluids, thermal and concentration boundary conditions, Brownian motion as well as some other variable conditions were briefly studied and their effects were discussed. Thermo-magnetic convective heat transfer in a confined cavity has been studied by several researchers considering uniform magnetic fields [2], line dipole [4], and inclined magnetic fields [5]. In other class of work, the magnetohydrodynamics (MHD) convection has been investigated considering partial magnetic fields [6–8] instead of whole domain magnetic fields.
Buoyancy-induced convective phenomena in a cavity subjected to a partial magnetic field have been analyzed by Jalil et al. [6]. The influence of the non-uniform magnetic field using permanent magnets of varying strength, and hence the transition from natural convection to thermomagnetic convection was studied by Szabo et al. [7]. Very recently, Geridonmez and Oztop [9] studied natural convection in a differentially heated porous cavity where a partial magnetic field was applied horizontally. They concluded that depending on the impact area of the partial magnetic field, the fluid flow is affected. A further conclusion was that at higher values of Rayleigh number, the influence of the magnetic field is more pronounced (due to an increase in Lorentz force experienced). Magneto-hydrodynamic convection in a corner heated cavity has been studied by Mondal et al. [10].

In the aforementioned literature survey, some of the researchers have focused on the technique of using partial magnetic fields, while most of the researchers have used the whole domain magnetic field to control the thermal performance during buoyancy-induced convection. With this introduction on the importance of the study of magnetohydrodynamics (MHD) the present work is formulated considering a differentially heated inclined cavity under the impact of a partially applied magnetic field. The purpose of this study is to highlight and contrast the effect of the magnetic field versus simple buoyancy-driven convective flow and to further analyze the conditions which favor a faster convective heat transfer rate.

2. **Mathematical modeling and numerical technique**

A schematic figure of the model is shown in Figure 1. The geometry is a square cavity (of side $H$) containing air as the working fluid under the effect of a partially applied uniform magnetic field. The cavity is inclined to the horizontal direction and the inclination angle is represented by $\phi$. The left sidewall is the hot wall (at a temperature $T_h$) which is isothermally heated, while the right sidewall is cold (at a temperature $T_c$). The upper and lower walls of the cavity are kept adiabatic. The magnetic field is perpendicular to the sidewalls over a length of $0.5H$ (positioned at the center of the sidewall) and the intensity of the field has been varied (which is reflected by changing the Hartmann number). Here, the magnetic field along the $x$ direction is generated due to the influence of electric field or voltage applied perpendicular to the $x$-$y$ plane. In this study, the external magnetic field is acting perpendicular to the sidewalls and partially. As the cavity inclination angle is changing with an angle $\phi$ with respect to the horizontal direction, magnetic field remains perpendicular to the sidewalls. However, the gravity force acts into sin and cos components of the buoyancy force (which are appeared in the momentum equations).

![Figure 1. Schematic diagram of the computational domain.](image)

The flow field is considered as two-dimensional (the perpendicular direction is considered infinitely long). The flow is assumed to be laminar, steady, and incompressible while Boussinesq
approximations are taken to hold good. The working fluid (of Pr = 0.71) is assumed to be electrically conducting. The dimensionless governing equations for the problem are as follows:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + Pr Ra \theta \sin \phi
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \lambda_a Ha^2 Pr V + Pr Ra \theta \cos \phi
\]

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}
\]

where,

\[
Pr = \frac{v}{\alpha} ;\; Ra = \frac{g \beta (T_s - T_a) H^3}{\nu^2} Pr ;\; Ha = BH \sqrt{\frac{\sigma}{\mu}}
\]

The non-dimensional governing equations have been constructed from the following scale factors:

\[
(X, Y) = (x, y) / H ; \; (U, V) = (u, v) H / \alpha \; ; \; \theta = (T - T_a) / (T_s - T_a) ;\; P = (p - p_a) H^2 / \rho \alpha^2
\]

The partial magnetic fields were adopted through the term \( \lambda_a \) by setting 1 and 0 for the active and inactive zones, respectively. The dimensionless partial differential equations (1)–(4) are solved numerically using an in-house computing code adopting the FVM through ADI sweep, TDMA solver, using the SIMPLE algorithm [11]. The converged solution is obtained when continuity mass-defect becomes less than 10^{-10}. The same code has been used in our earlier works and validated extensively under the different problem of published literature [2,12–18]. In the present simulation, a uniformly distributed 200x200 grid size is selected after conducting the grid independency study.

The boundary conditions are implemented by setting \( \theta = 1 \) and \( \theta = 0 \) respectively at the left and right walls, \( \partial \theta / \partial Y = 0 \) at the top and bottom walls, and zero-velocity \( (U = V = 0) \) for all the walls.

### 3. Results and discussion

The present work investigated the thermo-magnetic natural convective heat transfer in an inclined cavity subjected to a magnetic field applied partially. The effects of various changing parameters are depicted here. The Rayleigh number \( (Ra = 10^3, 10^4, 10^5, 10^6) \), Hartmann number \( (Ha = 0, 30, 50, 70) \), and cavity inclination angle \( (\phi = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 150^\circ, 180^\circ) \) have been changed individually and the effect is observed for each variation while the other two parameters remained constant. The results are visualized through streamlines, isotherms, and average Nusselt number (Nu) values.

#### 3.1. Effect of Rayleigh number (Ra)

Figure 2 shows the variation of streamlines (first row) and isotherms (second row) for changing the Rayleigh number \( (Ra = 10^3, 10^4, 10^6) \) while the cavity remains horizontal \( (\phi = 0) \) and the Hartmann number constant at Ha = 50. Due to differential heating at the cavity, fluid circulation establishes in the clockwise direction (CW). The variation of streamlines distribution with Rayleigh number shows that at a low value of Ra = 10^3 (which is indicative of low buoyancy forces due to conduction dominance), the flow structure is almost symmetric, with the reduced flow strength. There are two smaller vortices placed vertically away from the mid-horizontal plane of the cavity. This happens due to the imposed magnetic field partially about the middle zone of the cavity (about 0.5H length). Also, the streamlines are more widely spaced in the middle zone for low Ra values, which is indicative of low fluid velocity in the region. Thus, fluid flows faster away from the middle of the cavity. As Ra increases, the two rotating cells towards the top and bottom wall shift and align themselves such that they move farther away from the center and at Ra = 10^6, the lower cell disappears. The flow strength obviously increases with an increasing Rayleigh number, which is reflected by the increasing magnitudes of the maximum value of streamfunction. The flow field is pushed towards the periphery as the value of Ra increases, moving away from the zone of influence of the magnetic field, with
overall flow velocity increasing due to an increase in buoyant forces as the Rayleigh number increases. Also, the streamlines are denser in the zone of the magnetic field for $Ra = 10^6$ compared to the previous cases showing that the buoyant forces are causing an increased flow velocity while the flow is receding away from the central zone of the cavity under the effect of increased Lorentz forces that is restricting fluid flow. Thus, it can be concluded that with increasing Rayleigh number, the magnitude of flow velocity increases and leads to an increase in heat transfer.

The isotherm lines at $Ra = 10^3$ are nearly all vertical with very slight changes in orientation throughout, which can be assumed to be uniform. The isotherms vary with increasing values of $Ra$ and gradually become horizontal (at $Ra = 10^6$). At lower $Ra$, the heat transfer mechanism is governed by the conduction mode, thus lesser heat transfer as indicated by average $Nu$. The temperature distribution is more affected by the increased $Ra$ of the flow as compared to the increased magnitude of Lorentz forces accompanying it. At higher $Ra$ ($> 10^5$), the convection mode of heat transfer dominates the conduction mode. Leading to the higher fluid velocity and associated heat transfer. Thus, the average $Nu$ value increases markedly with the increasing $Ra$.

![Figure 2](image)

3.2. Effect of Hartmann number ($Ha$)

Figure 3 shows the variation of streamlines and isotherms with changing the Hartmann number ($Ha = 0, 30, 70$), keeping the $Ra = 10^5$ and $\phi = 0^\circ$ fixed. The case of no magnetic field ($Ha = 0$) resents natural convection under no influence of the magnetic field. Clockwise streamlines are formed, with two small cells aligned diagonally in the middle of the square cavity. As $Ha$ value increases, the strength of the flow decreases due to increasing opposing Lorentz force that affects the flow field, as is evident from the decreasing value of the absolute magnitude of streamfunction. The alignment of the fluid flow also changes with changing the intensity of the magnetic field. At $Ha = 0$, the two central
vortices are aligned along the ‘top-left bottom-right’ diagonal. But, as the Ha increases, the fluid flow (perpendicular to the magnetic field direction) in the middle zone of the cavity is arrested-evident by the fact that the streamlines are more diffused in the middle zone, which indicates a lower fluid velocity. Hence, the fluid velocity is cut down in the active zone of the imposed magnetic field and is increased towards the periphery (denoted by denser streamlines) with increasing Ha. Also, the two rotating cells in the middle that was inclined along the ‘top-left bottom-right’ diagonal for the case of Ha = 0, become inclined along the opposite diagonal when the Hartmann number reaches an appreciable value (Ha = 70).

The isotherms, also show marked change with increasing Ha, such that, for the case of Ha = 0, the isotherms that are almost horizontal become almost vertical for when the Ha = 70. The isotherms, therefore, orient themselves perpendicular to the direction of magnetic field lines in the middle zone where the magnetic field acts as the field becomes more pronounced. The temperature gradient, therefore, becomes more horizontal as the isotherms become vertical. The heat transfer rate is somewhat compromised with increased Ha values, which is also indicated in the figure below.

![Figure 3](image_url)

**Figure 3.** Effect of Ha on the contours of streamlines (top row), and isotherms (bottom row) for Ra = 10^5, φ = 0°.

### 3.3. Effect of cavity inclination (φ)

Figure 4 illustrates the effect of cavity inclination angle (φ = 30°, 60°, 90°, 150°) with the horizontal direction, keeping Ra = 10^5, Ha = 50 fixed. The flow field change markedly due to the influence of changing orientation of the hot and cold walls positions. As the cavity inclination increases from φ = 30° to 60°, we see that the flow field shifts towards the left slightly; and towards the middle, the streamlines come closer and become more concave. Also, it is observed that the flow velocity shows a slight increase, which essentially indicates a better heat transfer rate through convection. At φ = 90°, the streamlines change markedly, such that clockwise and anticlockwise rotating cells are separately
formed and coexist. At the position \( \phi = 90^\circ \), the hot wall is at the bottom and the cold wall is at the top and the unique streamline pattern comes as a consequence of the arrangement. The flow velocity remains almost the same \( \phi = 90^\circ \) as in the previous case \( \phi = 60^\circ \). The flow regime shows a distinct shift to the left as the angle of inclination becomes obtuse with the two core rotating cells shifting to the left side. The flow circulation also changes to an anticlockwise direction at \( \phi = 150^\circ \). The flow regime at \( \phi = 150^\circ \) is a perfect mirror image of the flow pattern at \( \phi = 30^\circ \) with a similar magnitude of the absolute maximum values of the streamfunction.

The isotherms show little variation from \( \phi = 30^\circ \) to \( \phi = 60^\circ \), the isotherms near the cold wall become slightly more concave. At \( \phi = 60^\circ \), however, they show a marked change and assume a more symmetrical structure with a high concavity of the lines towards the middle of the cold wall. The isotherms at \( \phi = 150^\circ \) is essentially a mirror image of the isotherms at \( \phi = 30^\circ \). The effective heat transfer rate for both cases is also likely to be the same.

\[
\begin{array}{cccc}
\phi = 30^\circ & \phi = 60^\circ & \phi = 90^\circ & \phi = 150^\circ \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Nu} = 3.059, |\psi_{\text{max}}| = 5.780 & \text{Nu} = 2.929, |\psi_{\text{max}}| = 6.390 & \text{Nu} = 2.899, |\psi_{\text{max}}| = 6.260 & \text{Nu} = 3.059, |\psi_{\text{max}}| = 5.780 \\
\end{array}
\]

Figure 3. Effect of \( \phi \) on the streamlines (top row) and isotherms (bottom row) contours at \( Ra = 10^5 \), \( Ha = 50 \).

3.4. Heat transfer characteristics

The average Nusselt number (Nu) is utilized to calculate the heat transfer characteristics at the heated wall as

\[
Nu = \frac{1}{Y} \left( \frac{\partial \theta}{\partial X} \right)_{Y=0} \quad dY
\]

Figure 5a depicts the variation of global heat transfer characteristics (average Nu) as one changes the Hartmann number values (that is the magnetic field strength) for \( Ra = 10^5 \) and \( 10^6 \) respectively. From the figure, it is observed that the average Nu for all the values of \( Ha \) in the case of \( Ra = 10^5 \) is greater than for the case of \( Ra = 10^6 \). This indicates that convective heat transfer is largely increased with an increase in Rayleigh number value. In other words, buoyant forces aid convection. The general trend for both the plots is that with increasing values of Hartmann number, the average Nusselt number gradually decreases. Thus, convective heat transfer becomes less predominant as the magnetic field strength is increased. This can be attributed to the Lorentz forces which reduce the flow velocity.
The curve for Ra = 10^6 has a steeper downward slope than the one for Ra = 10^5. This indicates that the effect of Lorentz force is greater in a flow condition with a higher Rayleigh number value.

Figure 5b represents the variation of average Nusselt number values with changing the angle of inclination of the square cavity w.r.t the horizontal. The Rayleigh number has been assumed fixed at 10^5. From Figure 5b, it is observed that the average Nusselt number values for Hartmann number 50 are always less than the corresponding values for Hartmann number 0. Thus, increased magnetic field strength decreases the convective heat transfer rate. For Hartmann number 0, the average Nu value increases very little as the inclination angle increases from horizontal to 30°. It reduces from 30° till 90°, after which it shoots up as the inclination reaches 150°. For Hartmann number 50, the figure shows a distinctly different plot. The Nusselt number increases markedly from the angle of inclination 0° to 30°. The Nusselt number value remains fairly constant until the inclination value reaches 150°. Unlike the plot at Hartmann number 0, even when the angle increases from 90° to 150°, the Nusselt number only shows a negligible increase—meaning that under the presence of the magnetic field, Nusselt number and hence heat transfer by convection is affected with increasing angle of inclination but after a certain inclination value, the convection heat transfer rate becomes somewhat stabilized and shows lesser dependency on any further inclination changes when compared to the case of Ha = 0.

Figure 4. Heat transfer characteristics with varying Ra, (a) Ha, (b) φ keeping other parameters fixed.

4. Conclusions
In this study, the impact of partial magnetic fields on the thermal convective heat transfer in an inclined differentially heated cavity is investigated numerically. The thermo-fluid phenomena are explored under different parametric influences like Ra, Ha, and φ. The following conclusions may be drawn:

a) As the Rayleigh number increases, convection mode governs the heat transfer mechanism with increased flow velocity and a higher rate of heat transfer. Flow structure as well as static temperature distribution affected severely in the zone of the imposed partially magnetic field.

b) With increasing Hartmann number, the Lorentz force counteracts the buoyancy force. The fluid velocity is markedly higher away from the zone of magnetic field compared to the fluid flow under the influence of magnetic forces, for a high value of Ha. The isotherms also align themselves in a way that the temperature gradient variation occurs in the direction of the magnetic field for such cases. Higher is the Ha, leads to the decreasing flow velocity and thus decreasing rate of heat transfer.

c) With the change in the inclination angle, fluid and temperature distribution changes significantly. At smaller angles of cavity inclination (below 30°) for an appreciable value of Ha, there is a significant effect on the average Nu (and hence the convective heat transfer rate). Further increase in the angle, the heat transfer rate decreases. The change in average Nu is minimal for changes in angle of inclination for Ha = 50 compared with Ha = 0. In the absence of a magnetic field, the Nu
value shows larger variations against changing values of angle of inclination (for inclination values greater than 30°).

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Nomenclature

| Symbol | Description |
|--------|-------------|
| $B$    | magnetic fields (Tesla, N A⁻¹ m⁻²) |
| $g$    | acceleration due to gravity (m s⁻²) |
| $H$    | height of the cavity/length scale (m) |
| $\alpha$ | thermal diffusivity (m² s⁻¹) |
| $\beta$ | thermal expansion coefficient (K⁻¹) |

Greek symbols
| Symbol | Description |
|--------|-------------|
| $\text{Ha}$ | Hartmann number |
| $L$ | length of the cavity (m) |
| $\text{Nu}$ | average Nusselt number |
| $P$ | pressure (Pa) |
| $\text{Pr}$ | Prandtl number |
| $\text{Ra}$ | Rayleigh number |
| $T$ | temperature (K) |
| $u, v$ | velocity components (m s$^{-1}$) |
| $X, Y$ | dimensionless coordinates |

| Symbol | Description |
|--------|-------------|
| $\theta$ | dimensionless temperature |
| $\phi$ | inclination angle of the cavity |
| $\lambda_m$ | magnetic field parameter |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity (m$^2$s$^{-1}$) |
| $\rho$ | density (kg m$^{-3}$) |
| $\sigma$ | electrical conductivity (µ Scm$^{-1}$) |
| $\psi$ | dimensionless stream function |

**Subscripts**
- $a$: ambient
- $c$: cold
- $h$: hot