Bound states and transmission antiresonances in parabolically
confined cross structures: influence of weak magnetic fields

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Abstract

The ballistic conductance through a device consisting of quantum wires,
to which two stubs are attached laterally, is calculated assuming parabolic
confining potentials of frequencies \( \omega_w \) for the wires and \( \omega_s \) for the stubs. As
a function of the ratio \( \omega_w/\omega_s \) the conductance shows nearly periodic minima
associated with quasibound states forming in the stubbed region. Applying
a magnetic field \( B \) normal to the plane of the device changes the symmetry
of the wavefunctions with respect to the center of the wires and leads to new
quasibound states in the stubs. The presence of the magnetic field can also
lead to a second kind of state, trapped mainly in the wires by the corners of
the confining potentials, that yields conductance minima as well. In either
case, these bound states form for weak \( B \) and strong confining frequencies
and thus are not edge states. Finally, we show experimental evidence for the
presence of these quasi-bound states.
I. INTRODUCTION

Technological advances in microfabrication techniques now allow the manufacture of semiconductor structures that have dimensions smaller than the elastic and inelastic mean scattering lengths. In such *mesoscopic* structures, the electronic transport is *ballistic* [1], and the conductance is governed by the fact that the electrons behave like quantum mechanical waves. This is particularly true at low temperatures.

The wavelike behavior of electrons in such structures has led to the study of devices that are analogous to those used in microwave technology. The simplest such device is the quantum wire (QW), which can be thought of as an electronic waveguide. Some more complicated structures involve having QWs cross to form junctions, attaching finite branches to the QWs so that they become corrugated, and connecting the QWs to the electronic equivalent of a resonant cavity. These closely related structures have generated experimental and theoretical interest [2]–[9]. In particular, resonant tunneling and quasibound states in stub and cross structures have been focused on because in these systems the electrons are not bound classically by any potential barriers. In addition, they are unusual in that the presence of quasibound states can lead to resonant reflection instead of transmission, i.e., transmission antiresonances.

A special type of structure in this class is the electron stub tuner (EST), in which the length of the stub, laterally attached to the QW, would be controlled by an independent gate. If the width of the QW and the stub are such that both allow only a single propagating mode for a given incident energy, then the conductance $G$ is a periodic function of the stub length $c$, with $G$ oscillating between 0 and 1, in units of $2e^2/h$, making it potentially useful as a type of transistor [10]. The conductance minima that result can be attributed to destructive interference between the electron waves in the wire and those reflected from the stub. A more sophisticated device is the double electron stub tuner (DEST) depicted in Fig. 1 (a). If the length of the DEST is kept fixed while it is being made asymmetric by suitably synchronized gate voltages, a conductance output, nearly square wave in form, can
be achieved as a function of the degree of asymmetry with potential uses in analog-to-digital converters [11].

In this paper we consider a few important issues with regard to these devices. First of all, in previous theoretical work on cavity and stub structures, the confining potentials were always assumed to be infinite square well in nature [2]-[12]. However, it is well known that for very narrow QWs a parabolic potential is more appropriate [13]. In such a case the width of the electronic wavefunctions and thus the device dimensions are not well defined. After briefly presenting the formalism in Sec. II, we will present the zero field results for a DEST, which show a nearly periodic conductance as a function of the ratio $\omega_w/\omega_s$ even under these circumstances. Some of these results will be contrasted with those obtained assuming a square confinement.

Secondly, “true” ESTs and DESTs, with independent gates controlling the stub lengths, have yet to be fabricated. Thus far, experiments have only been done on cavity structures in which the conductance was studied as a function of a gate voltage $V_g$ that affected several of the device dimensions simultaneously making it difficult to interpret the results definitively as resulting from the interference effects mentioned above. However, $G$ has been measured in these structures as a function of a weak perpendicular magnetic field $B$ as well for fixed $V_g$ [14]. The results show minima in $G$ as a function of $B$. The values of $B$ and the confining frequencies are such that edge states do not occur and so much of the previous work on quantum dots is not applicable. As we shall show in section III, such minima can arise in DESTs when electrons are reflected resonantly from quasi-bound states in the stubs. We further show that new quasibound states are created when $B \neq 0$ that are not present in zero field. This happens because $B$ changes the symmetry of the wavefunctions at zero field with respect to the transverse direction $y$ and leads to new couplings between the wire and stub wavefunctions. In Sec. IV, we show results for actual experiments and interpret them qualitatively in terms of those of Sec. III. Conclusions follow in Sec. V.
II. FORMULATION OF THE TRANSMISSION PROBLEM

In this paper we consider parabolic confinements. For a parabolic one along the \( y \) axis in the wire (\( w \)) and stub (\( s \)) regions, i.e., we take

\[
V_{w,s}(y) = m^*\omega_{w,s}^2 y^2/2, \tag{1}
\]

and/or the presence of an applied magnetic field, \( B = (0,0,B) \). The parabolic confinement in a DEST is depicted three-dimensionally in Fig. 1 (b). The narrower parabolas, defined by the frequency \( \omega_w \), represent the two parts of the QW; the wider parabola, defined by \( \omega_s < \omega_w \), represents the stubbed region. The confinement in the stubs along \( x \) is achieved essentially through the difference in stub and wire potentials, \( \Delta V(y) = V_w(y) - V_s(y) \), so that only a finite potential barrier is created. As a result, the electronic wavefunction in the stub regions will not go to zero at the boundaries and thus it can spill over into the QW. This is a feature our present model shares with saddle potentials that is absent in infinite square-well models used in past calculations.

To evaluate the transmission through the device depicted in Fig. 1, we solve Schrödinger’s equation on a mesh, using an iterative matrix method. We summarize the essentials of the method below and refer to [15] for all the mathematical details. The general situation is one in which the QWs, which are connected to the stub structure, extend outward to \( \pm \infty \) along the \( y \) direction. The problem is solved on a square lattice of constant \( a \). Along the \( x \) direction, the system must be cutoff after a finite number of lattice sites, say \( M \). Thus, the situation is one in which the parabolic QWs that are depicted are in fact enclosed within a larger waveguide that is bounded by infinite potential barriers. The region of interest, containing the actual stub structure, can be broken down into a series of slices along the \( y \) direction. The discrete form of the Hamiltonian relates quantum mechanical amplitudes between adjacent slices. Keeping only terms up to first order in the derivative, this has the form:

\[
(E_F - H_j)\psi_j + H_{j,j+1}\psi_{j-1} + H_{j,j-1}\psi_{j+1} = 0, \tag{2}
\]
where $\psi_j$ is a $M$-dimensional column vector containing the amplitudes of the $j$th strip. The matrices $H_j$ represent the Hamiltonians for the individual slices.

By approximating the derivative, the kinetic energy terms of the Hamiltonian get mapped onto a tight-binding model with $t = -\hbar^2/2m^*a^2$ representing the nearest-neighbor hopping. To include the effects of the confining potential, one adds to the on-site energies, which occur along the diagonals of $H_j$, the terms $v_{j,m}$, which represent the potential on site $(j, m)$ in units of $t$. Parabolic confinement can be modelled easily using Eq. 1 for $v(l, m)$, with $y = a(m - (M + 1/2))$, so that $y = 0$ occurs at the center of each slice. The matrices $H_{j,j+1}$ and $H_{j,j-1}$ give the inter-strip coupling and are related by $H_{j,j+1} = H^*_{j,j-1}$. We use the gauge in which these diagonal matrices are given by $H_{j,j+1}(l, l) = -t \exp(2\pi i \beta l)$ where $\beta = Ba^2/\phi_0$ is the magnetic flux per unit cell and $\phi_0 = \hbar/e$. In this gauge, the magnetic field points along the $z$ direction. Equation (2) can be used to derive a transfer matrix which allows us to translate across the system and thus calculate the transmission coefficients which enter the Landauer-Büttiker formula to give the conductance. Transfer matrices however are notoriously unstable due to exponentially growing and decaying evanescent modes. This problem was overcome in Ref. 15 by performing some clever matrix manipulations and turning the process of translating across the system into an iterative procedure, rather than multiplying transfer matrices together. It has been found that the method gives results equivalent to those of the recursive Green’s function technique [15], which is the most common approach to this type of problem. Its advantage over the latter is that it is conceptually simpler and easier to implement. Once the procedure is complete, one obtains the transmission coefficients, $t_{nm}$, and reflection coefficients, $r_{nm}$, for the individual modes. Given these, the amplitudes of the wavefunctions at specific values of $x$ and $y$ can be obtained by a progressive backward substitution.

The total transmission $T$ is given by

$$T = \sum_{nm} T_{nm} = \sum_{nm} |t_{nm}|^2 v_n/v_m,$$

where $v_n$ and $v_m$ correspond to the velocities of the transmitted and incident modes re-
spectively and the sum is over *propagating* channels only. The conductance \( G \) then at zero temperature is given by the Landauer-Büttiker formula: \( G = (2e^2/h)T \).

As one reduces the lattice spacing in the discrete model, so one is in the limit where \( a \ll \lambda_F \), \( \lambda_F \) being the Fermi wavelength, the results that are obtained eventually can be mapped to those of the continuous case. Since we work in the regime of small \( a \), it will be convenient to make reference to the modes that occur in the continuous case. The \( n \)th channel wavenumber \( \alpha_n \) in the wire is

\[
\alpha_n = \frac{\Omega_w}{\omega_w} \sqrt{\frac{2m^*}{\hbar^2} [E_F - (n + 1/2)\hbar\Omega_w]}
\]

Similarly, in the stub region, the wavenumber \( \gamma_m \) takes the form:

\[
\gamma_m = \frac{\Omega_s}{\omega_s} \sqrt{\frac{2m^*}{\hbar^2} [E_F - (m + 1/2)\hbar\Omega_s]}
\]

Here \( \Omega_{s,w}^2 = \omega_{s,w}^2 + \omega_c^2 \) and \( \omega_c = |e|B/m^* \) is the cyclotron frequency. The modes \( \phi_n \) along the \( y \)-axis depend on whether the waves are traveling in the positive \( \exp(\pm i\alpha_n x) \) or negative \( \exp(-i\alpha_n x) \) \( x \)-direction. We thus have wire modes, \( \phi_n^{w\pm}(y) = \varphi_n^{w}(y \mp (\hbar\omega_c/m^*\Omega_w^2)\alpha_n) \), and stub modes, \( \phi_m^{s\pm}(y) = \varphi_m^{s}(y \mp (\hbar\omega_c/m^*\Omega_s^2)\gamma_m) \), where \( \varphi_j^{w}(y) \) is the \( j \)th harmonic oscillator (HO) wave function. Notice that for \( B = 0 \), we have \( \varphi^{+}(y) = \varphi^{-}(y) \).

III. RESULTS

A. Zero field

In previous theoretical work on ESTs, with stub length \( c \) and width \( b \), a periodic conductance output has been obtained, as a function of \( c \), for infinite square-well confinement; the period \( \delta c \) is given by

\[
\delta c = \frac{\pi}{\sqrt{2m^*E_F/\hbar^2 - (\pi/b)^2}} = \lambda_s/2
\]

when only one mode is allowed in the QW and stub regions. Equation (13) is a restatement of the condition for destructive interference, \( k_s\delta c = \pi \), since \( \lambda_s = 2\pi/k_s \) is the electronic
wavelength along the stub. Notice that the period increases as $b$ is made smaller. For a symmetric DEST [10], this period is doubled, so that $\delta c = \lambda_s$.

An interesting question is whether or not the conductance remains periodic if the confinement is instead parabolic, particularly when considering that in this case the stub length is no longer well defined. In the pertinent literature it is quite common to use the classical turning points to define an effective halfwidth $W_{\text{eff}}$ of the parabolic well through $E_F = m^* \omega_s^2 W_{\text{eff}}^2 / 2$.

Taking $\omega = \omega_s$ gives an effective stub length

$$c_{\text{eff}} = 2W_{\text{eff}} = 2\sqrt{\frac{2E_F}{m^* \omega_s}}. \tag{7}$$

If the DEST in the parabolic case behaves in a manner similar to that of past calculations, one might expect then that the conductance $G$ of a DEST to be a periodic function of $1/\omega_s$ for fixed $E_F$. As we show in Fig. 2 (a), this is in fact the case. We plot $G$ as a function of $\omega_w/\omega_s$ for fixed $\hbar \omega_w = 6.39$ meV and $E_F = 9$ meV so that there is one propagating mode in the connecting quantum wires. The width of the stub is $b = 400 \text{Å}$ (solid curve) and $b = 350 \text{Å}$ (dashed curve). When $b$ decreases the period increases; this is consistent with the results for infinite square-well confinement as expressed in Eq. (7).

The transmission minima displayed in Fig. 2 (a) can be considered to occur as a result of destructive interference. An alternate but complementary point of view is that they occur as a result of resonant reflection from quasi-bound states in the stubbed cavity. They are transmission antiresonances. This is illustrated in Fig. 2 (b), where $|\psi(x, y)|$ is plotted as a function of $x$ and $y$. To generate this plot, we have set $E_F = 9$ meV, $b = 400 \text{Å}$ and set $\hbar \omega_s = 2.89$ meV so that $\omega_w/\omega_s = 2.21$. The picture corresponds to the first transmission minimum in the solid curve in Fig. 2 (a). A standing wave corresponding to a quasibound state is apparent in the cavity region between the arrows along the $x$ axis.

Further insight into the antiresonances is obtained as follows. Since only one mode is occupied in the quantum wires, the full wavefunction $\phi(x, y)$ goes as the $n = 0$ HO wavefunction, $\varphi_0^{\omega_w}(y)$ for a set value of $x$. What is interesting is that the standing wave in the cavity region, despite being obtained by summing over the contributions of many HO
wavefunctions, can be associated with the $n = 2$ HO wavefunction $\varphi_{n=2}(y)$. In particular, if we set $x = x_o$, where $x_o$ represents the center of the stub (200 Å in this case), then $\psi(x_o, y)$ can be fit almost perfectly by using $\varphi_{n=2}(y)$ alone. While this is not true away from $x = x_o$, $\psi(x, y)$ in the stub region keeps the basic $n = 2$ HO form and thus it remains even with respect to $y = 0$, the center of the quantum wires. Consequently, the conductance minima or antiresonances can be attributed to an even-even coupling between the $n = 0$ state in the wire and the $n = 2$ state bound in the stubbed cavity or DEST. The other minima in the solid curve of Fig. 2 can similarly be associated with an even-even coupling between the $n = 0$ and $n = 4, 6, \text{ etc.}$ states. Coupling between the even, in the wire, and odd, in the stub, HO states does not occur because they are orthogonal to each other.

So far the results are similar to those obtained for a square-well confinement. The main difference between them is that the evanescent modes in the connecting QW’s in the square-well case decay very slowly. Thus, a long exponential “tail” is left in the wave-function in the exiting QW, even if there is 100 percent reflection of the incident propagating mode. This would be a major liability in the fabrication of an operating device, since the presence of the “tail” may result in resonant tunneling rather than resonant reflection thus making it difficult to produce a device that actually produces the desired effect. No such tail is apparent in the figure. The fast decay of the evanescent modes in the case of a parabolic potential is related to the wavenumbers given by Eqs. (4) and (5) rather than by Eq. (6).

B. Finite field

1. Offset or field

We now consider a finite but weak magnetic field $B$. By weak we mean a field that is not strong enough to push the wavefunctions completely over to one side. We are not in the edge state regime. The use of the term “weak” is appropriate to the experimental situation described in Sec. IV, where the dimensions of the experimental samples were several hundred to a few thousand Å, which is our motivation. For a QW with a $c_{eff}$ of a few hundred Å,
one expects $\omega_c \ll \omega_w$ for $B < 1T$. In addition, this regime has been much less explored than the edge-state regime. For simplicity we neglect the Zeeman splitting.

In Fig. 3 (a), we again plot $G$ as a function $\omega_w/\omega_s$ for fixed $E = 9$ meV, $\hbar \omega_w = 6.39$ meV and $b = 400\,\AA$, for three different situations, the upper two curves offset by $G = 1$ and $G = 2$, respectively, for clarity. The bottom curve is the same as the solid curve in Fig. 2 (a). For the middle curve, we have put in a small offset, $d = 20\,\AA$, so that the DEST is now asymmetric, with potential $V_{\text{DEST}}(y) \to m^* \omega_s^2(y + d)^2/2$. We see that with the asymmetry the antiresonances that occur in the symmetric case are now shifted down slightly. Secondly, and more importantly, a whole new set of antiresonances occur in between the original minima. These occur due to the the breaking of symmetry of the wave functions, allowing the even $n = 0$ QW state to now couple with the odd states ($n = 1, 3, 5, \cdots$) trapped in the DEST. Very similar behavior has been noted in the case of square-well confinement. The upper curve is for a symmetric DEST, but now in the presence of a finite magnetic field, $B = 0.3\,\text{T}$. We see that the presence of the magnetic field produces much the same result as the asymmetry- the shifting of the original antiresonances, and the appearance of the new set of minima at virtually the same locations. In Fig. 3 (b) we plot $|\psi(x, y)|$ for $\omega_w/\omega_s = 4.935$ and $d = 20\,\AA$ ($G \sim 0$ for these parameters). Here, the antiresonance wavefunction has six lobes, indicating the coupling of a $n = 5$ odd state in the DEST with the $n = 0$ even state in the QW in this case. The corresponding wave function in the presence of a magnetic field is shown in Fig. 3 (c) for $\omega_w/\omega_s = 4.896$ and $B = 0.3T$. The state shown in this picture is almost indistinguishable from the previous one. Interestingly, the most significant difference between the two pictures occurs in the incident waves. In the finite field case a standing wave appears that is quite similar to the one evident in Fig. 2 (b). In the asymmetric case, the waves have a more irregular appearance. One obtains similar results for the other even-odd antiresonances.

Given these results, we conclude the coupling between even and odd states in the presence of a magnetic field occurs here because, when $B$ is finite, the symmetry about $y = 0$ is broken. Noting that the wavefunction in Fig. 3 (c) appears almost completely symmetric about
$y = 0$, it is obvious that the presence of edge states is not required for this coupling to take place. In fact, it can occur for arbitrarily small $B$. However, the smaller $B$ is, the narrower the even-odd antiresonances that occur in Fig. 3(a) become. Another important point is that the position of the antiresonances depends on the value of the magnetic field. For example, the first antiresonance, which corresponds to a $n = 0$ QW-$n = 1$ DEST coupling, occurs at $\omega_w/\omega_s = 1.1164$ for $B = 0.11T$, $\omega_w/\omega_s = 1.17$ for $B = 0.29T$ and $\omega_w/\omega_s = 1.18$ for $B = 0.46T$. This shifting of the resonance as a function of $B$ for different choices of $\omega_w/\omega_s$ can be understood, at least in part, in terms of the lining up of the energy level of the bound state of the cavity, $E_{\text{bound}}$, with that of the incident electrons, $E_F$, which is necessary for a resonance effect to occur. From our previous discussion about fitting the wavefunction in the DEST, it is apparent that the energy level structure of the quasibound states is tied to $\Omega_s$. A larger (smaller) value of $\omega_w/\omega_s$ means that $\omega_s$ is smaller (larger), thus a larger (smaller) value of $B$ is required to ensure that $\Omega_s$ remains at the value that lines up the Fermi level with the bound state level. This argument, however, is somewhat oversimplified in that the bound state energy is not determined by $\Omega_s$ alone. The bound states are confined along both the $x$ and $y$ directions and so the $x$ confinement must necessarily contribute to the energy of the $n = 1$ bound state, so that we should have $E_{\text{bound}} = 3\hbar\Omega_s/2 + E_x$. However, as the confinement along $x$ is incomplete and the system is open, the contribution $E_x$ is difficult to quantify, at least analytically. Importantly, as $B$ changes $\Omega_s$, the confining potential in the stub along $x$ is also being altered, thus complicating the physical picture. As a result, the value of $\Omega_s$ for which antiresonance occurs is slightly different for different values of $B$.

The lining up of QW and DEST energy levels is also the likely explanation of the observed downward shift in both the finite $B$ and finite $d$ cases.

2. Offset and field

In Fig. 4 (a) we plot $G$ vs $B$ for fixed $\omega_w/\omega_s = 4.9$ that corresponds to the $n = 0-n = 5$ antiresonance. The solid curve corresponds to the DEST being symmetric. The broad minimum at about $\sim 0.28$ corresponds to the antiresonance in question. It is interesting to see what happens when a finite $B$ and a finite offset are present at the same time, as individually
they appear to have similar effects. The dashed and dotted curves correspond to \( d = 20 \text{Å} \) and \( d = 40 \text{Å} \), respectively. Oddly, the conductance minima become shallower for increased \( d \), as if the magnetic field and asymmetry are canceling each other out. Importantly, essentially the same curves are generated if we replace \( d \) with \( -d \). A clue to this behavior can be seen in Fig. 3 (a). While the antiresonances occur at essentially the same spots, the lineshapes are different, with \( G = 1 \) followed by \( G = 0 \) in the case of finite \( d \), and almost exactly the mirror opposite for finite \( B \). In either case, the lineshapes are asymmetric, that is, they are of Fano type. The occurrence of Fano antiresonances in stub structures has been the subject of several papers, typically using simple qualitative models [5]-[7] (stub and wire both treated as being purely one dimensional). Stub structures, unlike say a double barrier problem, yield both transmission poles in the complex energy plane, the real part of which is associated with the energy of the quasibound states and yield unit transmission, and transmission zeroes (the antiresonances). If the pole and the zero do not occur at the same location in energy, one obtains the asymmetric Fano lineshape. This gives a \( G = 1 \) peak followed by a \( G = 0 \) minimum when \( E_{\text{pole}} < E_{\text{zero}} \), and visa versa when \( E_{\text{pole}} > E_{\text{zero}} \). Figure 3 (a), however, shows the antiresonances as a function of \( \omega_w/\omega_s \), which we remind the reader is a measure of stub length for fixed \( \omega_w \).

The “flipping” of the Fano shaped antiresonance also occurs with respect to energy and this is shown in Fig. 4(b), where \( G \) vs. \( E \) is plotted for fixed \( \omega_w/\omega_s = 4.9 \). Once again, the minima here correspond to the \( n = 0 - n = 5 \) antiresonance. The solid curve corresponds to \( B = 0.28 \text{T} \) and \( d = 0 \), while the dashed curve is for \( B = 0 \) and \( d = 20 \text{Å} \). Note that the conductance minima occur at slightly different locations. The dotted curve has both \( B = 0.28 \text{T} \) and \( d = 20 \text{Å} \), which shows the hybrid lineshape, the result of the “competition” between the two sources of symmetry breaking. In the region of the minimum, this third curve looks somewhat like an average between the other two curves. We note that the conductance maximum follows the minimum in the combined curve, like the finite \( B \) only curve. We note that the minimum is much wider for the finite \( B \) only curve than for the \( d \)
only curve, indicating that the finite $B$ is producing a stronger effect in comparison to the finite $d$ in this case, and is essentially winning out. Again referring back to Fig. 3 (a), we note that the “flipping” effect does not occur when the field $B$ is turned on for the even-even antiresonances, presumably because we consider a relatively weak field $B$.

3. Two conductance minima

In Fig. 5 (a) we again plot $G$ vs $B$. However, unlike the previous example, two transmission minima are apparent for each of the curves shown here. The solid, dashed and dotted curves correspond to $\omega_w/\omega_s = 3.0, 2.91,$ and $2.85$, respectively. In Fig. 5 (b), $|\psi(x, y)|$ is plotted as a function of $x$ and $y$ for the first minimum in the $\omega_w/\omega_s = 3.0$ curve, which occurs at $B = 0.27$ T. Unlike the previous wave function plots, we are looking directly from above and higher amplitudes are represented by darker shading. The incident electron waves are traveling from the top to the bottom in this picture. The quasibound state in this case has four lobes along the length of the stub and thus represents coupling between $n = 0$ and $n = 3$ states and is yet another example of the even-odd coupling phenomenon we have already pointed out. More interesting is the wave function that corresponds to the second minimum at $B = 0.67$ T, which is plotted in Fig. 5(c). Here the wave function again has four lobes, but in this case there are two each in both the $x$ and $y$ directions. The quasibound state shown here does not arise from confinement by the stubs, but is held in place by the corners formed by the intersection points of the stub and wire potentials. Quasibound states of this type were first found to occur theoretically in intersecting quantum wires in a situation analogous to having stubs of infinite length by Schult, Ravenhall and Wyld [2]. They pointed out two such “intersection” states, the lower energy state consisting of one large lobe in the intersection region, occurring below the of the first propagating mode of the quantum wires, and a four-lobed excited state having the same odd symmetry of the state we see here.

In the curve for $\omega_w/\omega_s = 3.0$, the two minima have a relatively large spacing in $B$. When $\omega_w/\omega_s = 2.91$, the minima are quite closer to each other, with the lower minimum occurring at a higher value of $B$, while the second one remains fixed. In fact, this is as close to each
other as the minima get and they never merge for any value of $\omega_w/\omega_s$. This is a situation akin to an anticrossing from band structure theory. The wave functions for these two minima are shown in figures 5 (d) and (e). These wave functions are virtual mirror images of each other and appear to be hybrids of the stub-confined and intersection-confined states shown in the previous two panels.

For $\omega_w/\omega_s = 2.85$, the second minimum occurs at $B = 0.8$ T a somewhat higher value of $B$ than the previous two cases, while the first minimum occurs at $B = 0.57$ T. The wave function corresponding to the first minimum of this curve is shown in Fig. 5 (f). It is virtually a mirror reflection of the intersection-confined wavefunction shown in Fig. 5 (c). The wave function for the second minimum in this case is shown in Fig. 5 (g) and again has the hybrid form.

As is evident from our results, the relative positions of the two minima depend quite sensitively on $\omega_w/\omega_s$. It should be pointed out that, when $\omega_w/\omega_s$ is increased 3.05, the lower conductance minimum no longer occurs leaving only the intersection-confined state at approximately the same position as it is for $\omega_w/\omega_s = 3.0$. On the other hand, if $\omega_w/\omega_s$ is decreased further below 2.85, the position of the lower minimum, which now corresponds to the intersection-confined state, occurs at lower and lower values of $B$, but it shifts less significantly than the second minimum which occurs at increasingly higher $B$ values. That the intersection-confined state is less sensitive to changes in $\omega_w/\omega_s$ is not surprising, since its presence should not depend too strongly on stub length. On the other hand, the reason why there is a shift at all in its position, when $\omega_w/\omega_s$ is changed, is because while we are changing the stub length, we are also changing the confinement at the corners as well in our model.

IV. EXPERIMENTAL EVIDENCE FOR QUASI-BOUND STATES

In this section, we present experimental results which lend support to our theoretical analysis and provide evidence for the presence of quasibound states in a DEST device and
the appearance of new transmission minima under the influence of a magnetic field applied perpendicular to the device plane. Some preliminary results and details of sample fabrication and experimental measurement technique have been reported earlier [14]. The DEST device was fabricated using Schottky gates to define device geometry from a high-mobility ($\mu = 110 m^2/V s$ at 4.2K) and low-electron-density ($n = 3.1 \times 10^{15} m^{-2}$) AlGaAs/GaAs modulation-doped (Si) heterostructure grown by MBE and is shown in the inset of Fig. 6. The Fermi energy of the 2DEG was measured to be $E_F = 8.50$ meV. The lithographic dimensions of the device were: $a = b = 2500 \, \AA$, $c = 8500 \, \AA$, and $l = 1500 \, \AA$, respectively, $l$ being the length of the connecting wires. Figure 6 shows the conductance $G$ of the device in the absence of magnetic field measured as a function of gate voltage $V_g$ at 70 mK. This temperature is a small fraction of $E_F$ to be considered essentially zero. As $V_g$ is made more negative, the device dimensions $a$, $b$, and $c$ all decrease at the same time due to depletion. From measurements of the quantized conductance plateaus of a single quantum wire with lithographic width the same as that of the DEST wires, it was found that at $V_g = -500 mV$ the Fermi level lies just below the bottom of the second ($n = 1$) wire subband, and the corresponding wire width is $400 \, \AA$, so that for $V_g$ ( $-500$ mV one could say that transport is in the fundamental mode of the connecting quantum wires and only the lowest ($n = 0$) wire subband is occupied. Assuming the depletion at the stub edges is the same as that at the wire edges as the gate voltage is decreased, a rough estimate of the DEST dimensions at $V_g = -500$ mV could be obtained: $a = b = 400 \, \AA$, $c = 6400 \, \AA$. Though the estimate is rough, we can safely expect quite a few DEST subbands to be occupied. Since the Fermi level is the same across the device and the Fermi energy does not change with $V_g$, a decrease in $V_g$ accompanied by corresponding reduction of device dimensions means a decrease in the effective wire width and stub length as derived from the definition of classical turning points and given by Eq. (7). One could then say that the effective wire and stub confining frequencies increase as the gate voltage is made more negative. Since the depletion at the gate edges ( $\sim 2.9 \, \AA/mV$) is the same for the wires and the stub, a change in $V_g$ over a small range brings about little relative change in the stub length. However, for the wires, because
of the much shorter dimension, the relative change in the wire width is quite important as $V_g$ is swept. Considering the $V_g$ range between $-500mV$ and pinch-off, one could then possibly consider the stub confining frequency $\omega_s$ to stay practically constant, while the wire confining frequency $\omega_w$ to increase rapidly with decreasing $V_g$. In Fig. 6, therefore, decreasing $V_g$ would mean increasing the ratio $\omega_w/\omega_s$. It would also mean sweeping the Fermi level down across the stub subband levels given by $\omega_s$. In Fig. 6, the conductance $G$ shows two prominent minima and three maxima for $V_g$ less than $-500mV$. The observed minima can be attributed to an even-even coupling between the $n = 0$ state in the wire and the $n =$ even quasi-bound states in the stubbed cavity or DEST, as the Fermi level sweeps down the stub energy level structure. This analysis is in line with the theoretical prediction of the previous section and the observed minima can be considered as an experimental support of the theoretical analysis illustrated in Fig. 2(a). Note that in the present device geometry as $V_g$ changes, the stub width changes as well. The observed minima are thus expected to be much broader than the theoretically predicted ones for a constant stub width. Moreover, the stub shape may also depend somewhat on the gate voltage. The shallowness of the minima can be attributed to asymmetry and/or defects [17], while values of the maxima less than $2e^2/h$ can be attributed to backscattering at the wire entrance and/or impurities. For $V_g$ larger than $-500mV$, transport in the wire and in the stub since $a \approx b$ is multimode. The resulting enhanced mixing between different modes will result in a more irregular $G$-curve and may cause the regular oscillations observed below $V_g = -500mV$ to be gradually washed out as seen in Fig. 6. Based on the above analysis, we could index (n) the minima and maxima of Fig. 6. The indexing is shown by arrows. Using the known value of EF and the above indexing, we get, for $V_g = -500mV$, $\hbar \omega_w = 5.67$ meV and $\hbar \omega_s = 1.030$ meV, giving $\omega_w/\omega_s = 5.50$. This value is close to that used to generate Fig. 4(a). Note also that at this gate voltage $a = b = 400\AA$. Figure 7 shows how the conductance maximum (index 5) of the DEST at $V_g = -500mV$, changes under the influence of a magnetic field applied perpendicular to the plane of the device. We have added to Fig. 7, for comparison purposes, the theoretical curves of Fig. 4(a) which correspond to $b = 400\AA$. As the field is
increased, experimental $G$ decreases and goes through a pronounced dip which corresponds to a transmission minimum. The minimum in $G$ occurs at $B = 0.29$ T, a value that is not strong enough to produce edge states. The experimentally observed minimum follows remarkably well the B-dependence predicted by theory and may be understood in terms of the formation of a new quasi-bound state due to even-odd coupling induced by a weak magnetic field as discussed above. The shallowness of the observed dip may be due to asymmetry of the experimental DEST as illustrated by the theoretical curves THA20 and THA40, respectively. The fabrication of a perfectly symmetric DEST is a matter of chance and can not be priori guaranteed. The presence of disorder may be playing a role as well.

Support for the transmission antiresonances predicted in the last section is provided by the experimental results of Ref. [14]. The conductance $G$ of a DEST device, as a function of a perpendicular magnetic field $B$, shows a deep minimum apparent in the main Fig. 5 of Ref. [14]. The device was fabricated from high-mobility AlGaAs/GaAs modulation-doped heterostructures grown by MBE using the split-gate technique. The gate voltage $V_g$ was so adjusted that transport was in the fundamental mode in the quantum wire with the Fermi level ($E_F = 9$ meV) lying just below the second subband and transmission was unity in the absence of $B$. The experiments were performed at 70 mK, which is a small enough fraction of $E_F$ to be considered essentially zero temperature. The device dimensions under these conditions were estimated to be: wire width $w = 480 \text{ Å}$ and total DEST length $c = 6500 \text{ Å}$. This however is a very rough estimate. At any rate, we expect $\omega_w$ to be considerably larger than $\omega_s$ and thus many DEST subbands to be occupied.

The conductance minimum occurs for $B = 0.29$ T, a value that is not strong enough to produce edge states. The large experimentally observed minimum in $G$ may be understood in terms of the even-odd coupling and the formation of bound states discussed above. The minimum also shows some superimposed fine structure which may result from the presence of disorder in the quantum wire and/or in the DEST, which has been found theoretically to produce the type of jagged curve shown here [17]. In addition, we note the appearance of narrower minima superimposed on top of the main one. While these may be disorder-
induced noise, they may indicate the presence of more than one quasibound state, perhaps combinations of the intersection-confined and stub-confined states shown in Fig. 5. It is impossible to say at this point to what kind of state corresponds the main minimum in this experimental example.

The lowest point of the main dip in Fig. 6 shows about 25% transmission whereas there is no transmission ($G \approx 0$) at the corresponding minimum of the solid curve in Fig. 4. Again this may by attributable to disorder or the sample’s asymmetry as suggested by Fig. 4.

V. CONCLUSIONS

We have calculated the conductance for stubbed electron waveguides defined by a parabolic potential. In the absence of a magnetic field we find a periodic conductance output as the stubbed cavity is made longer, which is consistent with previous theoretical work done assuming infinite square well potentials. The conductance minima or antiresonances correspond to quasibound states in the stubbed regions. When the two parabolas representing the wire and stub confining potentials are displaced with respect to each other, the symmetry of the wave functions, with respect to the center of the wire, is broken and new quasibound states occur in the intersection regions. The same holds when the two parabolas are not displaced but a weak magnetic field $B$ is present because the field too breaks this symmetry thus allowing states in the cavity and wire, that were previously orthogonal, to couple. The appearance of these quasibound state is heralded by one or more dips in the conductance as a function of magnetic field. We emphasize that these dips occur in short and long stubs, i.e, whether there are just a few or many stub subbands occupied for electrons incident at the Fermi energy. Such dips have been observed experimentally in electron waveguides with stubbed cavities$^{14}$.

We have also investigated more sophisticated models for the confinement potentials, in particular models in which the transition between the quantum wire and stub regions is made gradually instead of abruptly as well as combinations of flat and parabolic confinement. We
find that for the most part the results are qualitatively similar to those of the simple double parabolic model shown here. Importantly, most quasibound states, that occur when the transition in confinement is not abrupt, tend to be variations of the hybrid type discussed in the context of Fig. 5. In addition, we find that it is much more difficult to get the conductance minima at the low values of B considered here when all potentials are defined by infinite square-well confinement. Unless there is some rounding of the potentials, as one expects in real devices, the energy level spacing is too large to permit it.

ACKNOWLEDGMENTS

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REFERENCES

[1] For review articles on the subject see C. W. J. Beenaker and H. van Houten, Solid State Phys. 44, 1 (1991), and S.E. Ulloa, A. MacKinnon, E. Castaño, and G. Kirczenow in Handbook on Semiconductors, Vol. 1, ed. P.T. Landsberg (Elsevier, Amsterdam, 1992).

[2] F. M. Peeters, Superlatt. Microstruct. 6, 217 (1989).

[3] R. L. Schult, D. G. Ravenhall, and H. W. Wyld, Phys. Rev. B 41, 12760 (1990).

[4] J. J. Palacios and C. Tejedor, Phys. Rev. B 48, 5386 (1993).

[5] W. Porod, Z. Shao, and C.S. Lent, Appl. Phys. Lett. 61, 1350 (1992).

[6] P. J. Price, Appl. Phys. Lett. 62, 289 (1993).

[7] E. Tekman and P. F. Bagwell, Phys. Rev. B 48, 2553 (1993).

[8] Z-L. Ji and K-F. Berggren, Phys. Rev. B 43, 4760 (1991).

[9] M. Leng and C.S. Lent, Phys. Rev. Lett. 71, 137 (1993).

[10] A. B. Fowler, U.S. Patent No. 4, 550,330 (Oct. 29,1985); F. Sols, M.Macucci, U. Ravaioli and K. Hess, Appl. Phys. Lett. 54, 350, (1989); S. Datta, Superlatt. Microstruct. 6, 83 (1989).

[11] P. Debray, R. Akis, P. Vasilopoulos, and J. Blanchet, Appl. Phys. Lett. 66, 3137 (1995).

[12] H. Wu, D. W. L. Sprung, J. Martorel,1 and S. Klarsfeld, Phys. Rev. B 44, 6351 (1991).

[13] A. Kumar, S. E. Laux, and F. Stern, Appl. Phys. Lett. 54, 1270 (1989).

[14] P. Debray, J. Blanchet, R. Akis, P. Vasilopoulos, and J. Nagle, Inst. Phys. Conf. Ser. 141 835 (1995).

[15] T. Usuki, M. Saito, M. Takatsu, R. A. Kiehl, and N. Yokoyama, Phys. Rev. B 52, 8244 (1995).
[16] R. Akis, P. Vasilopoulos, and P. Debray, Phys. Rev. B 52, 2805 (1995).

[17] H. Sordan and K. Nikolic, Phys. Rev. B 52, 9007 (1995).
FIGURES

FIG. 1.  (a) A stubbed cavity of width $b$ connected to two quantum wires. (b) The confining potential in the wires and the stubbed cavity. The picture is generated with $\hbar \omega_w = 6.39 \text{ meV}$, $\hbar \omega_s = 2.8 \text{ meV}$, and $b = 300 \ \text{Å}$. The x range is from $-300 \ \text{Å}$ to $600 \ \text{Å}$ and the y range from $-800 \ \text{Å}$ to $800 \ \text{Å}$.

FIG. 2.  (a) Conductance $G$ vs $\omega_w/\omega_s$ for $b = 400 \ \text{Å}$ (solid line) and $b = 350 \ \text{Å}$ (dashed) with fixed $\omega_w = 6.39 \text{ meV}$ and $E = 9 \text{ meV}$. b) A three-dimensional plot of $|\psi(x,y)|$ vs $x$ and $y$ for $b = 400 \ \text{Å}$ and $\omega_s = 2.88 \text{ meV}$. This corresponds to the first minimum in the solid curve in (a). The two arrows on the bottom right indicate the edges of the cavity and those on the left the width $W_{\text{eff}}$ of the quantum wire.

FIG. 3.  (a) Conductance $G$ vs $\omega_w/\omega_s$ for $b = 400 \ \text{Å}$, $\omega_w = 6.39 \text{ meV}$, and $E = 9 \text{ meV}$. The bottom curve is for a symmetric DEST at $B = 0 \text{ T}$. For the middle curve, offset by $G = 1$, the DEST has been made asymmetric by a factor of $d = 20$. For the top curve, offset by $G = 2$, a $B = 0.3 \text{ T}$ has been applied. Notice the additional antiresonances that occur in the presence of finite asymmetry and magnetic field. (b) $|\psi(x,y)|$ vs $x$ and $y$ is plotted for $\omega_w/\omega_s = 4.935$ and $d = 20$. This quasibound state corresponds to the 5th minimum in the middle curve in (a). (c) As in (b) but for $B = 0.3 \text{ T}$ and $\omega_w/\omega_s = 4.896$. This state corresponds to the 5th minimum in the top curve in (a).

FIG. 4.  (a) Conductance $G$ vs $B$ for $b = 400 \text{Å}$, $\omega_w/\omega_s = 4.9$. The solid curve is for a symmetric DEST and the dashed and dotted curves for an asymmetric one with $d = 20$ and $d = 40$, respectively. (b) Conductance $G$ vs $E$ for $b = 400 \text{Å}$ and $\omega_w/\omega_s = 4.9$. The solid curve is for $B = 0.28 \text{ T}$ and $d = 0$ and the dashed one for $B = 0$ and $d = 20$. The dotted curve is for $B = 0.28 \text{ T}$ and $d = 20$.  

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FIG. 5.  (a) Conductance $G$ vs $B$. The solid, dashed, and dotted curves correspond to $\omega_w/\omega_s = 3.0, 2.91, \text{ and } 2.85$, respectively. Note that two conductance minima occur in each curve. (b) In panels (b) through (g) the wave functions corresponding to these minima are plotted vs $x$ and $y$ with darker shading corresponding to higher amplitude. Panels (b) and (c) correspond to the first and second minima, respectively, for $\omega_w/\omega_s = 3.0$; (d) and (e) correspond to $\omega_w/\omega_s = 2.91$ and (f) and (g) to $\omega_w/\omega_s = 2.85$.

FIG. 6.  Conductance $G$ as function of gate voltage $V_g$ for a nominally symmetric DEST at 70 mK. The numbers accompanied by arrows give stub subband indices. The inset shows a schematic drawing of the DEST geometry as defined by lithography. The hatched areas (G) represent Schottky gates.

FIG. 7.  Conductance $G$ as function of magnetic field $B$ applied perpendicular to device plane for the DEST shown in Fig. 6 at fixed $V_g = -500$ mV and 70 mK. The theoretical curves THS, THA20, and THA40 are reproduced from Fig. 4(a). THS : symmetric DEST, THA20 : with offset 20 Å, THA40 : with offset 40 Å. See text for details.
Fig 1a  Akis et al.
Fig. 2 (a)  Akis et al.
Fig. 2b  Akis et al.
Fig. 3(a) Akis et al.
Figure 3 (b) and 3 (c)
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Fig. 4 Akis et al.
Fig. 5 (a)  Akis et al.
Figures 5 (b), (c),(d),(e),(f),(g)
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Fig. 6 Akis et al.
Fig. 7  Akis et al.