Kondo Effects in Quantum Dots at Large Bias

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Recently, the issue of whether the Kondo problem in quantum dots at large bias is a weak-coupling problem or not has been raised. In this paper, we revisit this problem by carefully analyzing a corresponding model in the solvable limit — the Emery-Kivelson line, where various crossover energy scales can be easily identified. We then try to extract the scaling behavior of this problem from various physical correlation functions within the spirit of “poor man’s scaling.” Our conclusions support some recent suggestions made by Coleman \textit{et al.} [Phys. Rev. Lett. \textbf{86}, 4088 (2001)], which are obtained by perturbative analysis: The voltage acts as a cutoff of the renormalization group flow for only half of the impurity so that the low-temperature physics is controlled by a strong-coupling fixed point. But the low-temperature response functions in general show one-channel Kondo behaviors with two-channel Kondo behaviors occurring only through proximity to a quantum critical point.

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I. INTRODUCTION

The Kondo problem is one of the best studied many-body problems in condensed matter physics. Due to advances of nanotechnology, the Kondo effect in quantum dot systems predicted by theories was observed in recent experiments. Although the quantum dot is intrinsically a multilevel system, as the energy is much lower than the single-particle level spacing, the system can be described by a single-impurity Anderson model with the level spacing playing the role of cutoff in this model. Moreover, in the Coulomb blockade regime with an odd number of electrons, it can be mapped onto a two-lead Kondo model with the following Hamiltonian: \[ H = \sum_{\alpha k \sigma} \varepsilon_{\alpha k} c_{\alpha k \sigma}^\dagger c_{\alpha k \sigma} + H_{\text{refl}} + H_{\text{trans}}, \]

\[ H_{\text{refl}} = J_R \sum_{k, k', \sigma, \sigma'} \left( c_{R k \sigma}^\dagger \tilde{\sigma}_{\sigma}^R c_{R k' \sigma'}^\dagger \cdot \vec{S} + (R \rightarrow L), \right) \]

\[ H_{\text{trans}} = J_{LR} \sum_{k, k', \sigma, \sigma'} \left( c_{L k \sigma}^\dagger \tilde{\sigma}_{\sigma}^L c_{R k' \sigma'}^\dagger \cdot \vec{S} + (R \leftrightarrow L) \right) \]

Here, \( c_{\alpha k}^\dagger \) creates an electron in lead \( \alpha \in \{L, R\} \) with momentum \( k \) and spin \( \sigma \), and \( J_L, J_R, \) and \( J_{LR} (= J_{RL}) \) are positive (antiferromagnetic) Kondo coupling constants between electrons and the dot (\( \vec{S} \)). Energies \( \varepsilon_{\alpha k} = \varepsilon_k - eV_{\alpha} \), where \( V_{\alpha} = \pm V/2 \) are the potentials of the left- and right-hand leads. Besides, derived from an Anderson model via a Schrieffer-Wolff transformation, the coupling constants obey \( |J_{LR}|^2 = J_L J_R \).

Within the spirit of “poor man’s scaling,” a recent paper by Kaminski \textit{et al.} shows that even in the present nonequilibrium case, the low-temperature properties can still be characterized by a single crossover energy scale \( T_K \) which is identified as the Kondo temperature of this problem. Through extending the information gained from perturbative renormalization group (RG) equations, Kaminski \textit{et al.}, use the ordinary one-channel Kondo fixed point to extract low-temperature transport properties for \( V \ll T_K \). However, for \( V \geq T_K \), the RG-improved perturbative calculation indicates that the conductance will saturate at a value much smaller than that in the unitary limit. This signals that the coupling between different leads \( J_{LR} \) stops growing for energy smaller than \( V \). One then wonders whether the above-mentioned one-channel Kondo (1CK) fixed point can still be used to describe the low-temperature physics in the latter case. Recently, to gain further insight into this problem, Coleman \textit{et al.} have done a perturbative calculation of the impurity magnetic susceptibility. The scaling behaviors for various couplings they obtained are basically the same as that obtained in Ref. in the high-temperature region. However, for \( T < V \), the flows of \( J_{L,R} \) and \( J_{LR} \) exhibit different behaviors: \( J_{LR} \) stops growing at the energy scale \( V \) while \( J_{L,R} \) continue to grow toward strong coupling with a new Kondo temperature \( T_K^* \). They conjecture that the physics at \( T < T_K^* \) is described by a 2CK fixed point. (See also Ref. 4.)

In this paper, we shall revisit this problem by studying the model in the solvable limit — the generalized Emery-Kivelson line. This enables us to follow the RG flow all the way from the perturbative regime down to the low-temperature region, and the concept of universality guarantees that we can obtain correct scaling at low temperature. Moreover, through studying various correlation functions in details, we can see how the impurity is screened (or unscreened) due to the presence of finite bias. This approach is complementary to the perturbative analysis in Ref. 4, which is supposed to be valid at high temperature. Combined with the results obtained from the perturbation theory, we reach a complete picture about the behavior of the Hamiltonian at large bias: The effect of the cotunneling term \( H_{\text{trans}} \) is to generate a new energy scale \( \Gamma_{LR} \) even in the channel-symmetric case \( J_L = J_R \). As \( \Gamma_{LR} \ll T_K \), there exists...
a range of temperature $\Gamma_{LR} \ll T \ll T_K$, where the uniform magnetic susceptibility exhibits the 2CK behavior, i.e., the logarithmic temperature dependence. On the contrary, it will show the 1CK behavior as $\Gamma_{LR} \approx T_K$. This is the fundamental difference between the present case and the ordinary 2CK fixed point. Previous studies on ordinary 2CK problems revealed that the presence of an unscreened Majorana fermion lies at the heart of the 2CK properties, which is the origin of the logarithmic temperature dependence in various susceptibilities. In our case, the logarithmic temperature dependence is due to the partial screening of a Majorana fermion. Although the low-temperature behavior of the uniform susceptibility is similar to that in the ordinary channel-asymmetric 2CK problem $(J_L \neq J_R$ and $H_{trans} = 0$) and there are two characteristic energy scales in both cases, the origins are distinct. In the ordinary channel-asymmetric 2CK problem, the new energy scale in addition to the Kondo problem, the new energy scale in addition to the Kondo temperature is generated from the channel-asymmetry, i.e., $J_L \neq J_R$, whereas in our case it arises from $H_{trans}$ and still exists even $J_L = J_R$.

The rest of the paper is organized as follows. We introduce the solvable model in Sec. II. In Sec. III, the impurity Green functions and the impurity contributions to the uniform magnetic susceptibility are calculated. The last section is devoted to a discussion and conclusions of our results.

II. THE SOLVABLE MODEL

After changing the notation $P(L) \rightarrow 1(2)$, we start with the following Hamiltonian:

$$H = H_0 + H_1 + H_2,$$

where

$$H_0 = \sum_{\alpha, \sigma} \int dx : \psi_{\alpha \sigma}^\dagger (i \partial_x - V_0) \psi_{\alpha \sigma} :,$$

$$H_1 = \hat{S}_\uparrow \sum_{\alpha} \lambda_{\alpha \parallel} \frac{\sigma_3}{2} \psi_{\alpha 0} + \left( \hat{S}_\uparrow \sum_{\alpha} \frac{\lambda_{\alpha \parallel}}{2} \psi_{\alpha \sigma}^\dagger \psi_{\alpha 0} + H.c. \right),$$

$$H_2 = \lambda_{LR} \hat{S}_\uparrow \sum_{\alpha} \frac{\sigma_3}{2} \psi_{\alpha 0} + \frac{\lambda_{LR \perp}}{2} \left[ \hat{S}_\uparrow \psi_{\alpha \uparrow}^\dagger \psi_{\alpha \downarrow} \right] + H.c.$$ (2)

Here $\alpha = 1, 2$, $\sigma = \uparrow, \downarrow$, and $V_2 = -V_1 = V/2$. For simplicity, we set $\epsilon = 1$ and the Fermi velocity $v_F = 1$ and : $\cdots :$ is the normal ordering with respect to the Fermi surface in the absence of $V$. (Here we assume that Fermi velocities on both leads are equal.)

The procedure to arrive at a solvable model is given in the following. First we have to bosonize the Hamiltonian [3]. To deal with Klein factors carefully, we employ bosonization formulas on a finite length:

$$\psi_{\alpha \sigma}(x) = \frac{1}{\sqrt{2 \pi a_0}} F_{\alpha \sigma} \exp \{-i \Delta_L (\hat{N}_{\alpha \sigma} - P_0/2)\} \times \exp \{-i \sqrt{4 \pi a_0} \phi_{\alpha \sigma}\},$$ (3)

where $\hat{N}_{\alpha \sigma}$ is the number of $\psi_{\alpha \sigma}$ fermions and $\Delta_L = 2 \pi / L$. Here $L$ is the system size and $a_0$ is a short-distance cutoff. $P_0$ takes care of the boundary conditions of fermion fields. $F_{\alpha \sigma}$’s are Klein factors, which satisfy the commutation relations $[F_{\alpha \sigma}, \hat{N}_{\alpha' \sigma'}] = \delta_{\alpha \alpha'} \delta_{\sigma \sigma'} F_{\alpha \sigma}$, $\{F_{\alpha \sigma}, F_{\alpha' \sigma'}^\dagger\} = 2 \delta_{\alpha \alpha'} \delta_{\sigma \sigma'}$, and $\{F_{\alpha \sigma}, F_{\alpha' \sigma'}^\dagger\} = 0$. Next four bosonic fields are introduced by

$$\phi_c = \frac{1}{2} (\phi_{1 \uparrow} + \phi_{1 \downarrow} + \phi_{2 \uparrow} + \phi_{2 \downarrow}),$$

$$\phi_s = \frac{1}{2} (\phi_{1 \uparrow} - \phi_{1 \downarrow} + \phi_{2 \uparrow} - \phi_{2 \downarrow}),$$

$$\phi_f = \frac{1}{2} (\phi_{1 \uparrow} + \phi_{1 \downarrow} - \phi_{2 \uparrow} - \phi_{2 \downarrow}),$$

$$\phi_{sf} = \frac{1}{2} (\phi_{1 \uparrow} - \phi_{1 \downarrow} - \phi_{2 \uparrow} + \phi_{2 \downarrow}).$$ (4)

and the corresponding transformation on $\hat{N}_{\alpha \sigma}$ is

$$\hat{N}_c = \frac{1}{2} (\hat{N}_{1 \uparrow} + \hat{N}_{1 \downarrow} + \hat{N}_{2 \uparrow} + \hat{N}_{2 \downarrow}),$$

$$\hat{N}_s = \frac{1}{2} (\hat{N}_{1 \uparrow} - \hat{N}_{1 \downarrow} + \hat{N}_{2 \uparrow} - \hat{N}_{2 \downarrow}),$$

$$\hat{N}_f = \frac{1}{2} (\hat{N}_{1 \uparrow} + \hat{N}_{1 \downarrow} - \hat{N}_{2 \uparrow} - \hat{N}_{2 \downarrow}),$$

$$\hat{N}_{sf} = \frac{1}{2} (\hat{N}_{1 \uparrow} - \hat{N}_{1 \downarrow} - \hat{N}_{2 \uparrow} + \hat{N}_{2 \downarrow}).$$ (5)

Here $\hat{N}_m \in \mathbb{Z} + P/2$ for $m = c, s, f, sf$ and $\sum_m \hat{N}_m = 0$ mod 2. $P = 0.1$ for the total number of fermions being even and odd integers, respectively. After plugging these into the Hamiltonian [3], we perform the Emery-Kivelson (EK) transformation: $U = \exp \{i \sqrt{4 \pi} \phi_{\alpha \sigma}(0)\}$. To proceed, we also introduce four more Klein factors $F_m$ with $m = c, s, f, sf$, which satisfy the commutation relation: $[F_m, \hat{N}_m'] = \delta_{m m'} F_{m'}$. With the help of Eq. [6], the identification between $F_{\alpha \sigma}$ and $F_m$ can be found. What we need is the following ones: $F_{1 \uparrow} F_{1 \uparrow}$, $F_{1 \downarrow} F_{2 \uparrow}$, $F_{1 \uparrow} F_{2 \uparrow}$, $F_{1 \downarrow} F_{2 \uparrow}$, and $F_{1 \uparrow} F_{1 \downarrow} = F_f F_s$. Then, the EK-transformed Hamiltonian is refermionized by the following formulas:

$$d^\dagger = F_s \hat{S}_\uparrow^+, \quad d = F_s \hat{S}_\uparrow^-,$$

$$\hat{S}_\uparrow = d^\dagger d - 1/2,$$ (6)

and

$$\Psi_m(x) = \frac{1}{\sqrt{2 \pi a_0}} F_{m \uparrow} \exp \{-i \Delta_L (\hat{N}_m - 1/2)\} \times \exp \{-i \sqrt{4 \pi a_0} \phi_m\}, \quad m = f, sf,$$

$$\Psi_s(x) = \frac{1}{\sqrt{2 \pi a_0}} F_{s \uparrow} e^{i \pi d^\dagger d} \exp \{-i \Delta_L (\hat{S}_\uparrow - 1/2)\} \times \exp \{-i \sqrt{4 \pi a_0} \phi_s\},$$ (7)
we obtain

$$\xi_m^1 = \frac{\Psi_m + \Psi_m^\dagger}{\sqrt{2}}, \quad \xi_m^2 = \frac{\Psi_m - \Psi_m^\dagger}{\sqrt{2}}, \quad m = f, sf,$$

$$a = \frac{d + d^\dagger}{\sqrt{2}}, \quad b = \frac{d - d^\dagger}{\sqrt{2}}. \quad (8)$$

Now, by taking $L \to \infty$ and ignoring terms of $O(1/L)$, we obtain

$$H' \equiv UHU^\dagger = H_0 + H_f + H_{sf} + H_{int} + \text{const},$$

where

$$H_0 = \sum_{q > 0} g_{bq}^\dagger b_q + \int dx : \Psi_i^\dagger i\partial_x \Psi_s(x) :,$$

$$H_f = \int dx \left( \sum_{\alpha=1,2} \frac{i}{2} \xi_{f\alpha}^i \partial_x \xi_{s\alpha}^i + i\nu \xi_{f1}^i \xi_{s1}^i \right)$$

$$- i\sqrt{2\Gamma_{LR}} a \xi_{s1}^2 (0),$$

$$H_{sf} = \int dx \sum_{\alpha=1,2} \frac{i}{2} \xi_{s\alpha}^f \partial_x \xi_{sf\alpha}^i - i\sqrt{2\Gamma_f} b \xi_{sf1}^2 (0)$$

$$+ i\sqrt{2\Gamma_f} a \xi_{sf1}^2 (0),$$

$$H_{int} = ab \{ i\delta \lambda_1 : \Psi_i^\dagger \Psi_s^\dagger (0) - \delta \lambda_2 \xi_{sf1}^2 (0)$$

$$+ \delta \lambda_3 \xi_{sf1}^2 (0) \} . \quad (9)$$

Here $b_{bq} = \sqrt{2q/L} \int dx e^{iqx} \phi_c(x)$ ($q > 0$). $\Gamma_1 = \lambda_{1i}^2/(4\pi a_0)$ with $i = \pm, LR$. $\delta \lambda_1 = \lambda_{1+} - \lambda_{1-}$, $\delta \lambda_2 = \lambda_{2+} - \lambda_{2-}$, and $\delta \lambda_3 = \lambda_{LR}$. $\lambda_{2i} = (\lambda_{1i} + \lambda_{2i})/2$ with $i = \pm, \parallel$. Notice that for symmetric dots, $\Gamma_+ = 0 = \delta \lambda_2$. The charge sector is completely decoupled from the impurities and we will not consider it hereafter. From Eq. (9), we see that the EK line corresponds to $\delta \lambda_1 = 0$ ($i = 1, 2, 3$) and its Hamiltonian $H_f + H_{sf}$ can be solved exactly.

### III. PHYSICAL OBSERVABLES

Now we are in a position to compute impurity contributions to the uniform magnetic susceptibility, which is defined by $\chi_{imp} = \lim_{L \to \infty} (\partial M/\partial h)|_{h=0} = L\chi_0/L$ where $\chi_0$ is the Pauli susceptibility of bulk electrons and $M = g(\hat{S}_T)$ is the magnetization. (We consider the case with gyromagnetic ratios on the impurity site and bulk electrons being equal and denote it by $g$.) From Eq. (8) and the operator product expansion (OPE) of $\Psi_i^\dagger (z + ia) \Psi_s (z)$, we have $\hat{S}_T = \int dx : \Psi_i^\dagger \Psi_s (x) : + (P + 1)/2$. The last term does not depend on the magnetic field $h$ and we ignore it.

In the presence of external magnetic fields, there is an additional term in the Hamiltonian, $\Delta H = -gh\hat{S}_T$, which is invariant against the EK transformation. Performing the EK transformation and renormalization successively, we obtain the Hamiltonian describing noninteracting $\Psi_s$ fermions as $H_s = \int dx : \Psi_s^\dagger (i\partial_x - gh) \Psi_s (x) :$.

The term linear in $h$ can be removed by the transformation $\Psi_s (x) \to \Psi_s (x) e^{-ihx}$. As a result, $H_s \to \int dx : \Psi_s^\dagger \partial_x \Psi_s (x) : + \text{const}$. By noticing that $: \Psi_s^\dagger \Psi_s (x) : \to : \Psi_s^\dagger \Psi_s (x) : + gh/(2\pi)$, the magnetization becomes

$$M = g \int dx : \Psi_s^\dagger \Psi_s (x) : + L\chi_0 h, \quad (10)$$

while $H_{\text{int}}$ turns into $H_{\text{int}} = H_{\text{int}} + i(gh/2\pi)\delta \lambda_1 ab$. It is straightforward to see that on the EK line the first term in Eq. (10) vanishes. This leads to $\chi_{imp} = 0$ and the leading contribution to $\chi_{imp}$ must arise from $H_{\text{int}}$.

To calculate contributions to $\chi_{imp}$ given by $H_{\text{int}}$, we use the Keldysh formula. Before plunging into the calculation, we need impurity Green functions $G_\alpha (\omega)$ and $G_b (\omega)$ on the EK line:

$$G_\alpha (\omega) = \left( \frac{1}{\omega + \nu} - \frac{2\Gamma_L (\nu - \omega - \nu + \omega)}{1 - \frac{\pi}{\omega - i\nu}} \right),$$

$$G_b (\omega) = \left( \frac{1}{\omega + \mu} - \frac{2\Gamma_+ (\mu - \omega - \mu + \omega)}{1 - \frac{\pi}{\omega - i\mu}} \right), \quad (11)$$

where $\Gamma = \Gamma_{LR} + \Gamma_-$ and $n_\alpha$ is the Fermi distribution function. Here all Green functions are defined in the Keldysh space as

$$G = \begin{pmatrix} G_R & G_K \\ 0 & G_A \end{pmatrix}.$$
Here \( \text{Re}\{f(z)\} \) means the real part of \( f(z) \). \( I_1 \) gives the result of the ordinary 2CK problem and \( \Gamma_+ \) plays the role of Kondo temperature \( T_K \) in that case without channel asymmetry. Effects of finite bias on \( \chi_{\text{imp}} \) are through the function \( I_2 \) arising from scattering in the spin-flip cotunneling channel.

Based on Eq. (13) and in terms of the asymptotic formula of digamma function \( \psi(z) = \ln z - 1/2z - 1/12z^2 + \cdots \), we can discuss the low temperature behavior of \( \chi_{\text{imp}} \). It depends on the ratio \( R \equiv \Gamma_+ / \Gamma \).

(i) \( R = O(1) \). When \( T \ll \Gamma_+, \Gamma \), we have

\[
I_1 = \ln (\Gamma_+/\Gamma) + O\left((T/\Gamma_+(\Gamma))^2\right),
\]

\[
I_2 \sim \ln f(V) + O\left((T/\epsilon_1)^2\right),
\]

with \( f(V) = \sqrt{(\Gamma_+^2 + V^2)/(\Gamma^2 + V^2)} \). Here the crossover energy \( \epsilon_1 = \Gamma_+(\Gamma) \) and \( |V| \) for \( |V| \ll \Gamma_+(\Gamma) \) and \( |V| \gg \Gamma_+(\Gamma) \), respectively. Thus, the leading behavior of \( \chi_{\text{imp}} \) is

\[
\chi_{\text{imp}}(T,V) = \frac{(g_6\lambda_4)^2}{4\pi^3} \frac{1}{\Gamma_+ - \Gamma} \left[ \ln \left( \frac{\Gamma_+}{\Gamma} \right) \right.
+ \frac{\Gamma_{\text{LR}}}{\Gamma_+ + \Gamma} \ln f(V) \bigg].
\]

(ii) \( R \gg 1 \). When \( \Gamma \ll T \equiv \Gamma_+ \), we have

\[
\chi_{\text{imp}}(T,V) = \frac{(g_6\lambda_4)^2}{4\pi^3} (A/\Gamma_+) \ln (T_K/T),
\]

with \( T_K \approx T_K \{ 1 + [\Gamma_{\text{LR}}/2(\Gamma_+ + \Gamma)] \ln [1 + (\Gamma+/V^2)] \} \) and \( T_K \) for \( |V| \gg T \) and \( |V| \ll T \), respectively. Here \( A = 1, 1 + \Gamma_{\text{LR}}/(\Gamma_+ + \Gamma) \) for \( |V| \gg T \) and \( |V| \ll T \), respectively. \( T_K = \Gamma c \), with \( c = 2e^2/\pi \) and \( \gamma \) being the Euler constant. Since \( \Gamma_{\text{LR}}/\Gamma_+ \ll 1 \), \( T_K \) is in general with the same order as \( T_K \). For extremely low temperature \( T \ll \Gamma \), the behavior of \( \chi_{\text{imp}} \) turns into that shown by Eq. (13). It is, however, a crossover not a phase transition. Equations (13) and (16) are the main results of this paper. We shall discuss their implications now.

IV. CONCLUSIONS AND DISCUSSIONS

Combined with perturbative analysis in Ref. [4] we arrive at the following picture: In the case with \( T_K \approx \Gamma \), all couplings flow to the strong-coupling regime at low energy as \( |V| \ll \Gamma_+ \) and both \( a \) and \( b \) fermions are completely screened. It leads to 1CK behavior. This can also be understood by noticing that the two-lead Hamiltonian \( H_f + H_{sf} \) in Eq. (4) has the structure of two copies of the 2CK problem where half of the impurity is screened by \( \psi_f \) and another half of the impurity is screened by \( \psi_{sf} \), and the voltage acts only on \( \psi_f \). (For simplicity, we take \( \Gamma_- = 0 \).) For \( V \rightarrow 0 \) and \( T \ll \Gamma_{LR}, \Gamma_+ \), both channels will flow to strong couplings and the system manages itself into a single-channel Kondo problem, which can be easily seen by taking \( \Gamma_+ = \Gamma_{LR} \) and rewriting the Hamiltonian through defining a new fermion \( \psi = (1/\sqrt{2}) (\psi_1 + i \psi_2^1) \). On the other hand, for \( |V| \gg \Gamma_+ \), the flow of \( J_{LR} \) stops at the scale \( V \) while \( J_{LR} \) still continue to flow towards strong couplings without being affected by the voltage. This is the origin of the scale \( \epsilon_1 \) appearing in Eq. (14). It also leads to the fact that the Majorana fermion \( a \) cannot be completely screened and fluctuates with a scale \( \Gamma \). Therefore, we expect that the 2CK behavior emerges in some situation as shown in Eq. (16). This in part explains the stability of this 2CK problem against the perturbation \( J_{LR} \). It is, however, not exactly equivalent to an ordinary 2CK problem because the coupling \( J_{LR} \) below the scale \( V \) is not irrelevant but marginal. In other words, half of the impurity, the \( a \) fermion, can be viewed as a free fermion only at energy scales higher than \( \Gamma \). As the temperature is far below it, the impurity behaves as if it is completely screened.

The partially screened Majorana fermions also reveal themselves in the conductance

\[
G(T,V) = G_U \frac{\bar{\Gamma}}{2\pi T} \text{Re} \left\{ \psi' \left( \frac{1}{2} + \frac{\bar{\Gamma} + iV}{2\pi T} \right) \right\},
\]

where \( \psi'(z) = d\psi/dz \) and \( G_U = (\Gamma_{LR}/\bar{\Gamma}) e^2/\pi \) is the conductance in the unitary limit. The low-temperature asymptotic behavior for \( T \ll \Gamma \) is

\[
G(T,V) = G_U \frac{\bar{\Gamma}^2}{2\pi^2 T^2 + V^2} + O \left( (T/\epsilon_2)^2 \right),
\]

with the crossover energy \( \epsilon_2 = |V| \) and \( \bar{\Gamma} \) for \( |V| \gg \bar{\Gamma} \) and \( |V| \ll \bar{\Gamma} \), respectively. From Eq. (13), we see that the conductance exhibits very different behaviors for large and small bias. For bias much smaller than the crossover energy \( \bar{\Gamma} \), the conductance will reach the unitary limit, while for \( V \gg \bar{\Gamma} \), the voltage plays the role of a crossover energy scale, and the conductance will saturate at a much smaller value than that in the unitary limit. Within the spirit of “poor man’s scaling,” we can extract scaling behaviors of the corresponding coupling: For \( V \ll \bar{\Gamma} \), \( J_{LR} \) will flow towards strong coupling and completely screen the impurity. However, for \( V \gg \bar{\Gamma} \), the RG flow will terminate at the scale \( V \) and the impurity can not be completely screened. Note that the scaling behavior of \( J_{LR} \) gained here is completely consistent with that obtained in Ref. [4]. In addition, the asymptotic behavior of Eq. (18) is similar to that appearing in Ref. [3] which reveals the existence of some kind of universality in this nonequilibrium problem.

Concerning the possible experimental realization of the above-mentioned 2CK behavior, we need to be close enough to that quantum critical point. This requires a large ratio \( R \) for the renormalized scales, which is equivalent to the condition \( J_{LR} \ll J_{LR, R} \). However, it is very difficult to achieve this goal due to the constraint on the
bare couplings, $J_{LR}^2 \approx J_L J_R$, and the logarithmic nature of the RG flow. Thus, unless we can design a dot with a very small ratio for bare interlead and intralead tunnelings, it seems that this window is too narrow to observing the 2CK behavior.

Finally, we would like to mention that similar crossover behaviors, i.e., the 2CK behavior at the intermediate-temperature regime while the ordinary 1CK behavior in the extremely low-temperature regime, have already appeared in the context of channel-asymmetric 2CK problems. Although some technical details are not completely the same, the underlying physical reasons are similar — the appearance of a new energy scale suppresses some scattering processes and changes the direction of the RG flow. We would also like to point out that a recent paper by Zvyagin discussed a similar problem but in a totally different physical context by using the Bethe ansatz. More precisely, Ref. 11 considers the low-energy properties of conduction electrons hybridized with localized 5f electrons. When the concentration of 5f electrons is low, the magnetic susceptibility exhibits a similar crossover behavior to our Eqs. (15) and (16). In that case, the new energy scale in addition to the Kondo temperature arises from the hybridization anisotropy. In a word, the most crucial difference between the models considered in Refs. 10 and 11 and the present one is that the appearance of two crossover scales in the former case is due to channel anisotropy, while in the latter case it is due to the cotunneling processes. The mechanisms to generate the new energy scale reveal their distinctions in the channel-symmetric limit. In that limit, for the models considered in Refs. 10 and 11, the new crossover scale vanishes and the low-temperature dynamics is described by the 2CK fixed point. On the contrary, for the Hamiltonian in the channel-symmetric limit, the two crossover energy scales never vanish, and depending on the relative magnitude of the two scales (a large difference induced by large bias), the low-temperature dynamics is controlled either by the 2CK or 1CK fixed points. However, it always exhibits 1CK-type behaviors when the temperature is far below both scales.

To sum up, we arrive at the following conclusions: (i) Regardless of the magnitude of bias $V$, the low-temperature physics of quantum dots coupled to two leads is controlled by a strong-coupling fixed point. (ii) This strong-coupling fixed point should exhibit the behavior of a one-channel Kondo fixed point instead of a two-channel one. (iii) There is a nearby two-channel Kondo fixed point located at the unphysical parameter space $\bar{\Gamma} = 0$ (in the sense that $\bar{\Gamma} = 0$ requires $J_{LR} = 0$). For dots with $\bar{\Gamma} \ll T_K$, it controls the physics at the range of temperature $\bar{\Gamma} \ll T \ll T_K$.

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