Mathematical modeling of damage function when attacking file server

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Abstract. The development of information technologies in Russia and the prospects for their further improvement allow us to identify a stable trend of expansion of both functions of the corresponding automated information systems (AIS) and the spheres of their application. At the same time, many threats to information processes in the AIS are expanding, which in turn stimulates the development of adequate means and systems for ensuring the information security of the AS and methods for assessing their protection. It is necessary to assess the ability of the system to continue its normal functioning under the conditions of permanent destructive influences and to resist them, to adapt the functioning algorithms to new conditions and to organize functional restoration or to ensure functioning with a gradual process of degradation, possibly without losing the most significant “critical” information functions. The analysis and evaluation of reliability are needed to be transformed into the analysis and evaluation of survivability. Survivability can be considered as the ability of the information system to store and restore the performance of basic functions in a given volume and for a given time in the case of a change in the system structure and / or algorithms and the conditions of its functioning due to adverse effects. One of the system survivability indicators is the reserve of survivability (S-survivability) that is the critical number of defects reduced by a unit. The authors will consider defect as a unit of measurement of damage to the information system by adverse impact. U is denoted as the critical number of defects, then S = 1-U is the index of S-survivability. The article gives the definition of an analytical formula for the function of damage and risk.

1. Introduction
In the simplest cases, many future events can be considered finite and the risk is represented by a probability distribution on a finite space of elementary events [1].

2. Materials and methods.
The damage from the information security implementation threat is determined by the content of the destructive action performed during the threat implementation with respect to the protected user information, systemic or applied software, and is generally a random value. Therefore, as a rule, the average value of damage is assessed. Knowledge of the distribution law makes it possible to estimate possible deviations in the value of damage from its mean value. However, in practice, the distribution law of the damage extent is not usually applied [2-4]. Contemporary science has not developed clear
algorithms for a priori specification of the distribution law of a random variable. The distribution law can only be identified based on available statistics by testing the hypothesis of the correspondence between the random distributions of a random variable of some model known in advance. Therefore, let us define the risk as a combination of the probability of damage and the severity of this damage. The damage component formed by the malicious impact on the resources of the attacked file server can be illustrated in Figure 1, which shows probability density $f(t)$ of the success of the attack [5]. The probability of the attack will be as follows: $P\left(t_0 \pm \frac{\Delta t}{2}\right) = f(t_0)(\Delta t)$, or through the accumulated probability:

$$P\left(t_0 \pm \frac{\Delta t}{2}\right) = F(t_0 + \frac{\Delta t}{2}) - F(t_0 - \frac{\Delta t}{2})$$

Figure 1. The model of the attack process on the file server.

Thus, the lost or acquired benefit over the time interval will be in the standard form:

$$\bar{u}(t) = \int_{t_0}^{t_0+\tau} \bar{\omega}(t) d(t)$$
**Figure 2.** The component of damage caused by the profit lost during the attack [6-9]
The damage is defined as the lost profit - that is, the integral of the utility function for the period from $t_0$ - the beginning of the attack, to $T_{av}$ - the average lifetime of the component: $u(t_0) = \int_{t_0}^{T_{av}} \omega(t) dt$

$$u(t_0) = \int_{t_0}^{T_{av}} \left[ 1 - e^{-\left(\frac{T_{av}-t}{\tau_3}\right)} \right] \frac{1}{\tau_3} dt = \left[ \frac{t-T_{av}}{\tau_3} = x \right] \frac{t - T_{av}}{\tau_3} + \tau_3 x \cdot dt = \tau_3 dx$$

$$\tau_3 \int_{0}^{T_{av}-t_0} (1 - e^{-\frac{T_{av}}{\tau_3}}) \frac{1}{\beta} d\left(\frac{t-T_{av}}{\tau_3}\right) = \tau_3 \int_{0}^{T_{av}-t_0} (1 - e^{x}) \frac{1}{\beta} dx = \left[ e^{x} = z \right] \frac{dx}{dz} = \ln(z)$$

$$\tau_3 \int_{1}^{e} \frac{1}{\beta} \frac{dz}{z} = \tau_3 \int_{J}^{1} z(1 - z) \frac{1}{\beta} dz = \tau_3 \left[ B_{y} \left( 0, \frac{1}{\beta_3} + 1 \right) - B \left( 0, \frac{1}{\beta_3} + 1 \right) \right]$$

where $B_{y}$ is determined through an incomplete beta function as:

$$B_{y}(a, \beta) = \int_{0}^{y} e^{x-1} (1 - z)^{\beta-1} dz, \quad y = e^{\frac{T_{av}-t_0}{\tau_3}} \quad (4)$$

To find the accurate risk of occurrence of a negative event at the time, it is appropriate to use the following illustration (figure 3).

**Figure 3.** Example of sampling probability density

where: $P(t_0 \pm \frac{\Delta t}{2})$ is the required probability and the area of the selected segment;

$f(t)$ is the probability density;

$\Delta t$ is sampling interval;

$$\int_{0}^{\infty} f(t) dt = 1.$$  

Hence the probability that an event occurs in interval $a (t_0 \pm \frac{\Delta t}{2})$ at proximately $\Delta t$ (for small ones) can be defined as follows:

$$P \left( t_0 \pm \frac{\Delta t}{2} \right) \equiv f(t_0) (\Delta t) \quad (5)$$

where $\Delta t \ll \frac{1}{2} f_{\text{max}}$

Then the risk is: $\text{Risk} \left( t_0 \pm \frac{\Delta t}{2} \right) = u(t_0) f(t_0) (\Delta t)$.

To find the probability of occurrence of a negative event in interval $(0, t_0)$, let us use figure 4 below.
Figure 4. Finding the accumulated probability

where \( F(t < t_0) \) is the required probability of the area of the selected segment.

The probability that the event will occur in interval \((0, t_0)\) is equal to:
\[
F(t < t_0) = \int_0^{t_0} f(t) \, dt. \tag{6}
\]

The damage must be averaged: \( u(t < t_0) = \frac{u(0) + u(t_0)}{2} \).

Consequently, the risk will be averaged:
\[
\text{Risk}(t < t_0) \approx \frac{u(0) + u(t_0)}{2} F(t < t_0). \tag{7}
\]

A more accurate estimate of the risk can be given by means of integral averaging in interval \((0, t_0)\):
\[
\text{Risk}(t < t_0) = F(t < t_0) \left[ \frac{1}{t(t_0)} \int_0^{t_0} u(t) \, dt \right]. \tag{8}
\]

The search for the probability of an event in interval \((t_1, t_2)\) is illustrated by the following figure (Figure 5):

Figure 5. Finding the interval probability

where \( F(t_1 < t < t_2) = \int_{t_1}^{t_2} f(t) \, dt \) is the required probability and the area of the selected segment.

The risk that an event will occur in time period \( t_1 < t < t_2 \) will be equal to:
\[
\text{Risk}(t_1 < t < t_2) = \left[ \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} u(t) \, dt \right] \int_{t_1}^{t_2} f(t) \, dt. \tag{9}
\]

Approximately:
\[
\text{Risk}(t_1 < t < t_2) \approx \frac{u(0) + u(t_0)}{2} F(t_1 < t < t_2). \tag{10}
\]

Next, let us consider the risk assessment, for which it is possible to use Fig. 3, which simultaneously shows the probability density of successful attack \( f(t) \) and productivity function \( \omega(t) \) in response interval \( t_r \) of the attack at moment \((t - t_r)\) with lost profit \( \overline{V} (t - t_r, t) \). In this case, taking
into account the previously stated methodological recommendations, it is possible to obtain an analytical expression of risk in time $t$:

$$\text{Risk}_{t} = \Delta t f_{X_t} - t V_{U X_t Y},$$

where $\Delta t = \frac{1}{n} f_{\text{max}}$ is the discretization step of $f(t)$ and $V$.

Thus, let us look for risk as the product of the probability of the occurrence of damage by its magnitude:

$$\text{Risk}_{t_0 \pm \frac{\Delta t}{2}} = u(t_0) f(t_0) \Delta t.$$\quad (11)

The damage function is as follows:

$$u(t_0) = \tau_z B_{0, \beta_z + 1} - B_{0, \beta_z + 1}$$\quad (12)

The beta distributions function of the second kind:

$$f(t) = \frac{1}{B(\alpha, \beta)} (t+1)^{\alpha - 1} (t+1)^{\beta - 1}.$$\quad (13)

Let us consider the risk to the system as a random variable. The main characteristics necessary to assess the risk, as a random variable given by a certain law distribution, are:

1) mathematical expectation (the first initial moment);
2) variance (second central moment);
3) sampling interval;
4) standard deviation;
5) the range at a given level of risk.

Let us define the sampling interval. For this it is necessary to find the maximum of the distribution function.

Let us find the derivative of the distribution function:

$$f'(t) = -\frac{1}{B(\alpha, \beta)} (t+1)^{\alpha - 2} (t+1)^{-\alpha - \beta + 1} ((t-1) t - \alpha + 1) = -\frac{t^{\alpha - 2} (t+1)^{-\alpha - \beta + 1} (t^2 - t - \alpha + 1)}{B(\alpha, \beta)}.$$\quad (14)

From the extremes of the function, let us remove points $t = 0$ and $t = -1$, since it does not fall within the time interval of interest. Thus, one is only satisfied with point $t = \frac{\alpha - 1}{\beta - 1}$.

Let us find the maximum of the function:

$$f_{\text{max}} = \frac{1}{B(\alpha, \beta)} \left( \frac{\alpha - 1}{\beta - 1} \right)^{\alpha - 1} (\alpha + \beta - 2).$$\quad (15)

Therefore, the sampling interval is:

$$\Delta t = \frac{B(\alpha, \beta)}{n(\alpha - 1)}.$$\quad (16)

Let us find the mathematical expectation of risk:

$$M = \frac{\int_0^T \text{Risk}(t) dt}{\int_0^T \text{Risk}(t) dt}.$$\quad (17)

$$\int_0^T \text{Risk}(t) dt = \tau_z \int_0^T \frac{1}{B(\alpha, \beta)} (t+1)^{\alpha - 1} (t+1)^{\beta - 1} [B_{0, \beta_z + 1} - B_{0, \beta_z + 1}] dt =$$

$$\frac{\tau_z \Delta t}{B(\alpha, \beta)} \left[ B_{0, \beta_z + 1} (1+t)^{\alpha + \beta - 2} - B_{0, \beta_z + 1} \right] =$$

$$\int_0^T \frac{\tau}{(1+t)^{\alpha + \beta - 2}} dt = \left[ t = -x \right] = (1-x) \int_0^T x^\alpha (1-x)^{\alpha - \beta + 2} dx =$$

$$= -(1-\alpha) B_{0, \beta_z + 1} (\alpha + 1 - \alpha - \beta + 3).$$\quad (18)

$$\int_0^T \text{Risk}(t) dt = \frac{\tau \Delta t}{B(\alpha, \beta)} \left[ \frac{\tau}{B(\alpha, \beta)} (1+t)^{\alpha + \beta - 2} + (1-\alpha) B_{0, \beta_z + 1} \right]$$\quad (19)

$$\int_0^T \frac{\tau x^\alpha (1-x)^{\alpha - \beta + 2} dx}{(1+t)^{\alpha + \beta - 2}} = -(1-\alpha) B_{0, \beta_z + 1} (\alpha + 1 - \alpha - \beta + 3).$$\quad (20)
\[
\int_0^{T_{av}} \Delta t \left[ B_y \left( \frac{1}{\beta_3} + 1 \right) - B \left( \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(\alpha, \beta) (1 + t)^{\alpha + \beta - 2}} \, dt = \frac{\tau_{av} \int_0^{T_{av}} \text{Risk}(t) \, dt}{\int_0^{T_{av}} \text{Risk}(t) \, dt} = \frac{\tau_{av} B_y \left( \frac{1}{\beta_3} + 1 \right)}{B(\alpha, \beta) \left( 1 + t \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3) \]
\]

The final expression for the mathematical expectation takes the form:
\[
M = \frac{\tau_{av} \int_0^{T_{av}} \text{Risk}(t) \, dt}{\int_0^{T_{av}} \text{Risk}(t) \, dt} = \frac{\tau_{av} B_y \left( \frac{1}{\beta_3} + 1 \right)}{B(\alpha, \beta) \left( 1 + t \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3). \tag{22}
\]

Risk dispersion:
\[
D = \frac{\int_0^{T_{av}} (M-t)^2 \text{Risk}(t) \, dt}{\int_0^{T_{av}} \text{Risk}(t) \, dt}. \tag{23}
\]

Dispersion will be determined by the formula:
\[
D = M^2 I_1 - 2M I_2 + I_3,
\]
where
\[
M = \frac{\tau_{av} B_y \left( \frac{1}{\beta_3} + 1 \right)}{B(\alpha, \beta) \left( 1 + t \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3),
\]
\[
I_1 = \frac{\tau_{av} \Delta t}{B(\alpha, \beta)} \left[ B_y \left( \frac{1}{\beta_3} + 1 \right) - B \left( \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(\alpha, \beta) (1 + t)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3); \tag{24}
\]
\[
I_2 = \frac{\tau_{av} \Delta t}{B(\alpha, \beta)} \left[ \frac{1}{B(\alpha, \beta)} \left( 1 + t \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt \right] + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3) \right] I_3 =
\]
\[
\frac{\tau_{av} \Delta t}{B(\alpha, \beta)} \left[ B_y \left( \frac{1}{\beta_3} + 1 \right) - B \left( \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(\alpha, \beta) (1 + t)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3). \tag{25}
\]

The standard deviation for risk is defined as: \( \sigma(T_{av}) = \sqrt{D} \).

Let us find the peak and mode of risk. For this, it is necessary to calculate the derivative for the risk function:
\[
\text{Risk} \left( t_0 \pm \Delta t/2 \right) = (\Delta t \tau_{av} B_y \left( \frac{1}{\beta_3} + 1 \right) - B \left( \frac{1}{\beta_3} + 1 \right)) \left[ \frac{1}{B(\alpha, \beta) (1 + t)^{\alpha + \beta - 2}} \right] \frac{1}{B(\alpha, \beta) \left( 1 + t \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3) \right] I_3 =
\]
\[
\frac{1}{B(\alpha, \beta) (1 + t)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} \left[ B_y \left( \frac{1}{\beta_3} + 1 \right) - B \left( \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(\alpha, \beta) \left( 1 + t \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3). \tag{26}
\]

Let us equate the derivative to 0 and find the risk mode:
\[
\frac{T_{av} - t_0}{\tau_{av}} \Delta t = \frac{T_{av} - t_0}{\tau_{av}} \Delta t = \left[ \frac{1}{B(\alpha, \beta) (1 + t_0)^{\alpha + \beta - 2}} \right] \left[ \frac{B_y \left( \frac{1}{\beta_3} + 1 \right)}{B(\alpha, \beta) \left( 1 + t_0 \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3) \right] I_3 =
\]
\[
\frac{1}{B(\alpha, \beta) (1 + t_0)^{\alpha + \beta - 2}} \left[ B_y \left( \frac{1}{\beta_3} + 1 \right) - B \left( \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(\alpha, \beta) \left( 1 + t_0 \right)^{\alpha + \beta - 2} \int_0^{T_{av}} \text{Risk}(t) \, dt} + (-1)^{\frac{\alpha}{2}} \left( \frac{1}{\beta_3} + 1 \right) B_{-T_{av}} (\alpha + 1, -\alpha - \beta + 3). \tag{27}
\]
\[- \frac{T_{av} - t_0}{\tau_3} \Delta \tau_3 = \frac{t_0(1 + t_0)^{\alpha + \beta - 2} - \frac{(1 + t_0)^{\alpha + \beta - 1}}{\alpha + \beta - 1}}{\alpha(1 + t_0)^{\alpha + \beta - 2}},\]

\[- \frac{T_{av} - t_0}{\tau} \Delta \tau = \frac{t_0 - \frac{t_0 + 1}{\alpha + \beta - 1}}{\alpha}, - \frac{(T_{av} - t_0) \Delta t}{\alpha(\alpha + \beta - 1)} = \frac{t_0 - \frac{(\alpha + \beta)t_0 - 1}{\alpha(\alpha + \beta - 1)}}{1},\]

\[-T_{av} \Delta t + t_0 \Delta t = \alpha + \beta - \frac{t_0 - \alpha(\alpha + \beta - 1)}{1} = T_{av} \Delta t - \frac{\alpha + \beta - 1}{1},\]

\[t_0 = \frac{T_{av} \Delta t (\alpha + \beta - 1) - 1}{\Delta t \alpha(\alpha + \beta - 1) - \alpha + \beta}.\] (27)

Consequently, the risk mode is: 
\[t_0 = \frac{T_{av} \Delta t (\alpha + \beta - 1) - 1}{\Delta t \alpha(\alpha + \beta - 1) - \alpha + \beta}.\]

Accordingly, the peak of risk is:
\[\text{Risk}_{\max} = \Delta \tau_3 \left[ B_y \left( 0, 1 \frac{1}{\beta_3} + 1 \right) - B \left( 0, 1 \frac{1}{\beta_3} + 1 \right) \right] \times \frac{1}{B(\alpha, \beta)} \left( \frac{\alpha - 1}{\alpha + 1} \right) \left( \frac{\Delta t}{\alpha(\alpha + \beta - 1) - \alpha - \beta} \right)^{\alpha + \beta - 2}.\] (28)

For a beta- distribution of the second kind, the initial moments are calculated as follows:
\[a_k = \int_0^{T_{av}} \frac{1}{B(\alpha, \beta)} \tau^{\alpha + k t_0 - 1} (1 + t)^{-\alpha - \beta} dt = \int_0^{T_{av}} \frac{t^{\alpha + k t_0 - 1}(1 + t)^{\alpha - \beta}}{B(t_0 - \alpha, \beta - 2 t_0)} dt = \frac{B(k t_0 - \alpha, \beta - k t_0)}{B(\alpha, \beta)}.\] (29)

Let us calculate the first initial moments:
\[a_1 = \frac{B(t_0 - \alpha, \beta - t_0)}{B(\alpha, \beta)}, a_2 = \frac{B(2 t_0 - \alpha, \beta - 2 t_0)}{B(\alpha, \beta)}, a_3 = \frac{B(3 t_0 - \alpha, \beta - 3 t_0)}{B(\alpha, \beta)}, a_4 = \frac{B(4 t_0 - \alpha, \beta - 4 t_0)}{B(\alpha, \beta)},\]

The central moment of the second order:
\[\mu_2 = \frac{a_3}{a_1} - \left( \frac{a_2}{a_1} \right)^2 = \left( \frac{B(3 t_0 - \alpha, \beta - 3 t_0)}{B(\alpha, \beta)} \right)^2 - \left( \frac{B(2 t_0 - \alpha, \beta - 2 t_0)}{B(\alpha, \beta)} \right)^2 = \frac{B(3 t_0 - \alpha, \beta - 3 t_0)}{B(t_0 - \alpha, \beta - t_0)} - \frac{B(2 t_0 - \alpha, \beta - 2 t_0)}{B(t_0 - \alpha, \beta - t_0)} = \frac{B(t_0 - \alpha, \beta - t_0) B(3 t_0 - \alpha, \beta - 3 t_0) - B^2(2 t_0 - \alpha, \beta - 2 t_0)}{B^2(t_0 - \alpha, \beta - t_0)}.\] (30)

The third central point is:
\[\mu_3 = \frac{a_3}{a_1} - 3 \frac{a_2 a_3}{a_1^2} + 2 \frac{a_3^2}{a_1^3} = \frac{B(4 t_0 - \alpha, \beta - 4 t_0)}{B(\alpha, \beta)} - \frac{B(2 t_0 - \alpha, \beta - 2 t_0) B(3 t_0 - \alpha, \beta - 3 t_0)}{B(\alpha, \beta)} + 3 \frac{B(2 t_0 - \alpha, \beta - 2 t_0)}{B(\alpha, \beta)} \left( \frac{B(t_0 - \alpha, \beta - t_0)}{B(\alpha, \beta)} \right)^2 = \frac{B(2 t_0 - \alpha, \beta - 2 t_0)}{B(\alpha, \beta)},\] (31)

The fourth central point is:
\[\mu_4 = \frac{a_4}{a_1} - 4 \frac{a_2 a_4}{a_1^2} + 6 \frac{a_3 a_2}{a_1^3} - 3 \frac{a_3^2}{a_1^4} = \frac{B(5 t_0 - \alpha, \beta - 5 t_0)}{B(\alpha, \beta)} - 4 \frac{B(4 t_0 - \alpha, \beta - 4 t_0) B(2 t_0 - \alpha, \beta - 2 t_0)}{B(\alpha, \beta)} + 6 \frac{B(2 t_0 - \alpha, \beta - 3 t_0) B(2 t_0 - \alpha, \beta - 2 t_0)}{B^2(\alpha, \beta)} - 3 \frac{B(2 t_0 - \alpha, \beta - 2 t_0)}{B^2(\alpha, \beta)},\] (32)

Let us find the range by the level of risk.
\[\text{Risk} \left( t_0 \pm \Delta t \frac{1}{2} \right) = \Delta t \times \tau_3 \left[ B_y \left( 0, 1 \frac{1}{\beta_3} + 1 \right) - B \left( 0, 1 \frac{1}{\beta_3} + 1 \right) \right] \times \frac{1}{B(\alpha, \beta)(1 + t_0)^{\alpha + \beta - 2}}.\] (33)
In order to find the values of damage for a given level of risk, let us compose the following equation:

\[ \text{Risk}_{\text{max}} = \Delta t \tau_3 \left[ B_y \left( 0, \frac{1}{\beta_3} + 1 \right) - B \left( 0, \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(a,\beta)} \frac{(a+\beta-2)^{\frac{a-1}{2}}}{(a+\beta-1)^{\frac{a+\beta-2}{2}}} = \Delta t \times \]

where \( k \) is the risk level factor.

Let us solve the equation:

\[
\Delta t \times \tau_3 \left[ B_y \left( 0, \frac{1}{\beta_3} + 1 \right) - B \left( 0, \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(a,\beta)} \frac{(a+\beta-2)^{\frac{a-1}{2}}}{(a+\beta-1)^{\frac{a+\beta-2}{2}}} = \Delta t \times
\]

\[
\tau_3 \left[ B_y \left( 0, \frac{1}{\beta_3} + 1 \right) - B \left( 0, \frac{1}{\beta_3} + 1 \right) \right] \frac{1}{B(a,\beta)} \frac{(a+\beta-2)^{\frac{a-1}{2}}}{(a+\beta-1)^{\frac{a+\beta-2}{2}}} = \Delta t \times
\]

\[
D = 4 - 4 \left( 1 - \left( \frac{(1 + (\text{risk})_{a+\beta-1})^{a+\beta-2}}{k(\text{risk})_{a+\beta-1})^{a+\beta-2}} \right) \right),
\]

(37)

Therefore, the risk range is as follows:

\[
t_0 \in \left( 1 - \frac{(1 + (\text{risk})_{a+\beta-1})^{a+\beta-2}}{k(\text{risk})_{a+\beta-1})^{a+\beta-2}} ; 1 + \frac{(1 + (\text{risk})_{a+\beta-1})^{a+\beta-2}}{k(\text{risk})_{a+\beta-1})^{a+\beta-2}} \right).
\]

(38)

Graphically, the roots of the equation are shown in Figure 6. The resulting roots of the equation are the boundaries of damage for a given level of risk, which is specified by multiplying coefficient \( k \) by the maximum value of the risk.

Figure 6. Boundaries of damage at a given level of risk
An approximate view of the graph of function $F(t)$ is shown in Figure 7. Figure 7 shows the distribution characteristics: $t_{0.5}$ is the median of the distribution, and $t_{0.25}$, $t_{0.75}$ are the lower and upper quartiles.

Thus, distribution function $F(t)$ serves as a probabilistic characteristic of the process under consideration. By distribution function $F(t)$, one can find corresponding probability density $f(t)$ as derivative $f(t) = F'(t)$.

![Figure 7. Type of graph of distribution function $F(t)$](image)

However, it is more convenient to associate the process of “life” that the authors are investigating not with distribution function $F(t)$ but with function $S(t) = 1 - F(t)$, complementing the unit as $S(t) = 1 - P(T < t) = P(T > t)$.

Function $S(t)$ given by the relations mentioned above is called the "survival" function (survival).

The curve of survival function $S(t)$ can easily be constructed from the graph of distribution function $F(t)$. The corresponding type of graph $y = S(t)$ is shown in Figure 8.

![Figure 8. Type of graph of survival curve $S(t)$](image)

The graph of function $y = S(t)$ is called the survival curve. The value at which $S(\tau_{0.5}) = 0.5$ is called the median survival. It is easy to see that the median survival rate coincides with the median of the distribution of random variable $T$. Similar to the quantiles of the distribution of order $p$, where $0 < p < 1$. 

<1, denoted by $t_p$, quantiles of survival are used. Let us denote the survival quantile of order $p$ by $S(t_p) = p$. Then for the surviving quantiles and the corresponding distribution quantiles, relation $\tau_p = t_{1-p}$, i.e. the survivability quantile of order $p$ is a quantile of the order of $1-p$. The relation can be easily proved: let us obtain:

$$p = S(t_p) = 1 - F(t_p).$$

Consequently:

$$F(t_p) = 1 - p.$$

Thus, the value of the distribution $\tau_p$-quantile of random variable $T$ is of the order $(1 - p)$.

It is possible to estimate the chance of successful functioning of the system up to instant $t_0$:

$$\text{Chc}(t_0) = \left(1 - F(t_0)\right)V(t_0).$$

Let us calculate the benefits that the system receives until the time of attack $t_0$ begins as an integral of the utility function:

$$V(t_0) = \int_0^{t_0} \bar{\omega}(t) dt,$$

$$V(t_0) = \int_0^{t_0} \left[1 - e^{-\frac{(T_{av} - t)}{\tau_3}}\right]^{1/2} dt = \left[\frac{t - T_{av}}{\tau_3} = x \quad T_{av} - t = \tau_3 x \right] = \frac{t_0^{\tau_3}}{\tau_3} \left(1 - e^{-\frac{t_0}{\tau_3}}\right)^{1/2} d\tau_3 = \tau_3 \int_1^{e^{-\tau_3}} (1 - z)^{\frac{1}{2}} dz = \tau_3 \left[B_y \left(0, \frac{1}{\beta_3} + 1\right) - B \left(0, \frac{1}{\beta_3} + 1\right)\right],$$

where $\tau_3$ - the time of the beginning of the “sunset” of the life cycle of the component; $\beta_3$ - the slope parameter of the decline in the utility function.

In general, the expression for the chance will be as follows:

$$\text{Chc}(t_0) = \left(1 - F(t_0)\right)V(t_0),$$

$$F'(t) = f(t).$$

Let us find $F(t)$ as an indefinite integral of the distribution function:

$$F(t) = \int \frac{1}{B(\alpha, \beta)} \frac{t^{\alpha-1}}{(1+t)^{\alpha+\beta-1}} dt = \frac{1}{B(\alpha, \beta)} \frac{\alpha+\beta-2}{\alpha-1} \frac{\Gamma(\alpha-1)}{\Gamma(\alpha)\Gamma(\beta)} t^{\alpha-1} - F(t) = \frac{\alpha+\beta-2}{\alpha-1} \frac{\Gamma(\alpha-1)}{\Gamma(\alpha)\Gamma(\beta)} \sum_{i=0}^{\alpha-1} C_n(l) t^{\alpha-1} - F(t) = \frac{\alpha+\beta-2}{\alpha-1} \frac{\Gamma(\alpha-1)}{\Gamma(\alpha)\Gamma(\beta)} \sum_{i=0}^{\alpha-1} C_n(l) t^{\alpha-1},$$

Let us find the expression for the chance:

$$\text{Chc}(t_0) = \left(1 - \frac{\sum_{i=0}^{\alpha+\beta-2} C_n(l) t^{\alpha-1}}{B(\alpha, \beta)}\right) \tau_3 \left[B_y \left(0, \frac{1}{\beta_3} + 1\right) - B \left(0, \frac{1}{\beta_3} + 1\right)\right].$$

Evaluation of the viability at time $t$ components can be written as follows:

$$\mathcal{K}_{abs}(t_0) = \text{Chc}(t_0) - \text{Risk}(t_0) = \left(1 - \frac{\sum_{i=0}^{\alpha+\beta-2} C_n(l) t^{\alpha-1}}{B(\alpha, \beta)}\right) \tau_3 \left[B_y \left(0, \frac{1}{\beta_3} + 1\right) - B \left(0, \frac{1}{\beta_3} + 1\right)\right] - \Delta t \ast \tau_3 \left[B_y \left(0, \frac{1}{\beta_3} + 1\right) - B \left(0, \frac{1}{\beta_3} + 1\right)\right] \left[\frac{1}{B(\alpha, \beta)} \frac{t_0^{\alpha-1}}{(1+t_0)^{\alpha+\beta-2}}\right],$$

$$\mathcal{K}_{rel}(t_0) = \frac{\text{Chc}(t_0)}{\text{Risk}(t_0)} = \frac{\frac{\sum_{i=0}^{\alpha+\beta-2} C_n(l) t^{\alpha-1}}{B(\alpha, \beta)}}{\Delta t \ast \tau_3 \left[B_y \left(0, \frac{1}{\beta_3} + 1\right) - B \left(0, \frac{1}{\beta_3} + 1\right)\right] \left[\frac{1}{B(\alpha, \beta)} \frac{t_0^{\alpha-1}}{(1+t_0)^{\alpha+\beta-2}}\right].$$
3. Conclusions.
Thus, the authors of the paper get a risk-model of the survival of the file server. In the course of the study, an analytical dependence for the damage on the time of the onset of the attack was derived. After this, based on this dependence, an expression for risk was derived and its mathematical analysis was carried out, namely, such parameters as mathematical expectation, variance, standard deviation, sampling interval, central moment, range of risk over the interval were found.

After that an analytical expression for the dependence of the chance on the time of the onset of the attack was found. Also, based on the risks and chances of the system, the absolute and relative viability functions depending on the time of the attack were calculated.

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