QCD sum rules as a tool for investigation of the baryon properties at finite densities

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Introduction. Speaking about the properties of nucleons in nuclear matter, we have in mind, e.g.:

1. Potential energy of a nucleon in the medium.
2. Neutron-proton mass splitting in isotope-symmetric matter.
3. Parameters which describe the interaction of nucleons with external fields: axial coupling constant $g_A$ and magnetic moments $\mu_N$.
4. Structure functions of deep inelastic scattering.

Turning to the strange baryons we can add:

5. Properties of a strange baryon in nuclear matter.
6. The system of strange baryons ("strange matter").

Approach of traditional nuclear physics to description of properties of baryons in nuclear matter is based on conception of $NN$ interactions. The problem is that small internucleon distances, where the nucleons cannot be considered as structureless point particles appear to be of crucial importance. Thus the whole approach becomes complicated and not well defined.

However, while $NN$ interactions are complicated at small distances, the strong interactions are not. Indeed, due to asymptotic freedom of QCD, the latter are the perturbative interactions between quarks and gluons. The peculiarity of QCD is the finite value of the vacuum condensates of quark and gluon fields $\langle 0 | \bar{q}q | 0 \rangle$, $\langle 0 | \frac{1}{\pi} \alpha_s \pi G^2 | 0 \rangle$, etc. This means that in the ground state of QCD there are finite densities of quark–antiquark and gluon fields.

Shifman et al. [1] suggested the QCD sum rules (SR) for the description of characteristics of free mesons. The method was based on the features of QCD, mentioned above. Later it was expanded by Ioffe [2] to the case of free baryons. Characteristics of free nucleons were expressed through the values of QCD condensates.

In 1988 Drukarev and Levin [3] used the SR method for investigation of the properties of nucleons in nuclear matter. In [3] the first steps were made to express the potential energy of the nucleon through in-medium values of QCD condensates. This paper was followed by a number of works of Petersburg (Leningrad) Nuclear Physics Institute — (PNPI) group [4]–[9]. In 1991 the Maryland University group joined this field of investigations [10]. Also a number of papers on meson properties in nuclear matter was published later.

QCD sum rules in vacuum [1, 3]. They are based on dispersion relation

$$\Pi_0(q^2) = \frac{1}{\pi} \int \frac{\text{Im} \Pi_0(k^2) dk^2}{k^2 - q^2}$$  \hspace{1cm} (1)
for the function $\Pi_0(q^2)$ which describes the propagation of the system carrying the quantum numbers of the nucleon (proton). Equation (1) is considered at $q^2 \to -\infty$ where the system can be treated just as three quarks with perturbative interactions between themselves and with quarks and gluons of vacuum. At $q^2 \to -\infty$ $\Pi_0(q^2)$ can be presented as power series

$$
\Pi_0(q^2) = \sum_{n=0}^{2} a_n q^{2n} \ln q^2 + \sum_{n=0}^{\infty} c_n q^{-2n}
$$

known as operator expansion. The coefficients $a_n, c_n$ are related to expectation values of certain QCD operators. As to the right-hand side (r.h.s.) of Eq.(1), the spectral density $\text{Im} \Pi_0(k^2)$ is related to observable spectrum of the system. The usual approach is to single out the lowest laying state (proton), approximating the higher states by continuum:

$$
\text{Im} \Pi_0(k^2) = \lambda^2 \delta(k^2 - m^2) + \theta(k^2 - W^2) \Delta \Pi_0(-k^2) \, .
$$

This is known as ”pole + continuum” model. Parameters $m$ and $\lambda^2$ which describe the position of the lowest laying pole and the residue are characteristics of proton. Continuum threshold $W^2$ is the parameter of the model: the cut with physical threshold and unknown spectral density is replaced by that with unknown value of the threshold $W^2$ and fixed spectral density $\Delta \Pi_0(-k^2)$. The special mathematical ansatz, the Borel transform (inversed Laplace transform) increases the role of lower laying states. A function of $q^2$ transforms into the one of Borel mass $M^2$, e.g.

$$
\hat{B} \frac{1}{k^2 + q^2} = \exp \left( -\frac{k^2}{M^2} \right) \, .
$$

The model for the left-hand side (l.h.s.) of Eq.(1) becomes increasingly true at large values of $M^2$. The one for r.h.s. works better at small $M^2$. The basic assumption of the method is that there is certain region of the values of $M^2$ in which both r.h.s. and l.h.s. of Eq.(1) approximate the true function $\Pi_0(q^2)$ well enough. Then the parameters $m, \lambda^2$ and $W^2$ can be expressed through the values of QCD condensates. Ioffe [2] found that the value of $m$ depends mainly on the condensate $\langle \bar{q}q \rangle$. Thus the picture of formation of the proton mass turned out to be very simple. It appears due to the exchange by quarks between our probe system and the quark-antiquark pairs of QCD vacuum.

**QCD sum rules in nuclear matter. Calculation of potential energy [3, 4, 6].** The generalization of the SR method to the case of finite densities is not straightforward. The spectrum of the function $\Pi(q)$ is more complicated now. One should single-out the singularities connected with the baryon but not with the medium itself. This can be done by the special choice of variables. Neglecting the Fermi motion of the nucleons of the matter, we can fix the pair energies $S$ of our probe hadron and that of the matter. Presenting the QCD SR for the function $\Pi(q) = \Pi(q^2, s)$ we can single-out the singularities connected with the probe hadron until we limit ourselves to its pair interactions with the nucleons of the matter. In this approach ”pole+continuum” model, employed for vacuum can be used. This choice of variables insures the condition $q_0 \to \infty$ at $q^2 \to -\infty$ which is necessary for the operator expansion of the function $\Pi(q)$. Another problem comes since each term of operator expansion corresponds, in the general case, to infinite number of condensates. Due to the presence of logarithmic loops, several lowest order terms of operator expansion contain, however, finite number of the condensates.

In the papers [3, 4, 6] QCD SR in nuclear matter were presented as Borel transformed dispersion relations for the difference of the functions $\Pi(q)$ in matter and in vacuum. The shifts of the parameters $m, \lambda^2$ and $W^2$ caused by interaction with the matter ($\Delta m, \Delta \lambda^2$ and $\Delta W^2$) were expressed through in-medium values of QCD condensates.

On the other hand the shift of the position of the nucleon pole in external field is $\Delta m = U$ with $U$ standing for the potential energy of the nucleon. It was found that the value of $U$ is determined mainly by the averaged values of the quark operators $\bar{q}\gamma_0q$ and $\bar{q}q$. The condensate $\langle M | \bar{q}\gamma_0q | M \rangle$, which vanishes in vacuum, is just the density of baryon number in the system. The expectation value $\langle M | \bar{q}q | M \rangle$ is the density of quark-antiquark pairs. Thus we come to a simple picture of formation of the potential energy.
One can generalize the approach for the case of neutron--proton mass splitting in symmetric matter. The latter can contribute by its valence quarks \( \langle M|\bar{q}q_0|q_0\rangle \) and by modification of its sea of quark-antiquark pairs \( \langle M|\bar{q}q|N\rangle \) -- (0)\( |\bar{q}q_0|\).

One can immediately calculate the condensate
\[
\langle M|\bar{q}\gamma_0q|M\rangle = \sum_i n_q, \rho_i
\]
with \( n_q \) being the number of \( q \) quarks in a nucleon of the matter (\( i \) denotes proton or neutron), \( \rho_i \) stands for the density. The SR analysis provides the contribution \( \Delta, m \approx +200 \text{ MeV} \) caused by this condensate (at \( \rho_n = \rho_p = \rho/2, \rho = 0.17 \text{ Fm}^{-3} \)). The scalar condensate can be presented as
\[
\langle M|\bar{q}q|N\rangle - \langle 0|\bar{q}q|0\rangle = \rho\langle N|\bar{q}q|N\rangle + F(\rho)
\]
with the first term in r.h.s. of Eq.(6) standing for the gas approximation while \( F(\rho) \) describes the contribution of the meson cloud. Fortunately the first term can be expressed through observables since
\[
\langle N|\bar{q}q|N\rangle = \frac{2\sigma}{m_u + m_d}
\]
Here \( \sigma \) denotes pion-nucleon sigma term which can be extracted from experimental data on low energy \( \pi N \) scattering. The gas approximation provides the contribution \( \Delta, m \approx -300 \text{ MeV} \) to the potential energy. Several steps beyond the gas approximation have been made also. The function \( F(\rho) \) is determined mainly by the relatively large distances of the order of inverted Fermi momenta or larger ones. This makes some approximate calculations available. They lead to the saturation curve with reasonable values of the equilibrium density and of the binding energy.

**Other applications of the method.** One can generalize the approach for the case of neutron--proton mass splitting in symmetric matter. In QCD language this effect is caused by finite values of the difference of quark masses \( m_d - m_u \) and by the non-vanishing value of the operator \( \bar{d}d - \bar{u}u \). Both contributions were included explicitly into QCD SR analysis [11]. Neutron was found to be bound stronger than the proton with reasonable value of the mass difference.

As to parameters of interaction of the nucleons with external fields, the application of the method at finite densities is the straightforward generalization of this method in vacuum [11]. In the left-hand side of SR the quark system interacts with external field while in the right-hand side the corresponding parameter of nucleon enters the equation. The first approach to the calculation of renormalization of axial coupling constant was made in [5].

In the same way the method was applied to the calculation of the deep inelastic structure functions of nuclei. In this case the system interacts with the hard virtual photon. In our paper [9] we calculated the deviations of the structure function \( F_2 \) from that of a system of free nucleons at intermediate values of Bjorken variable \( x \). The calculated values followed typical EMC behaviour. As the next steps of application of the approach to this problem we plan to investigate cumulative aspects of the process. The method can be applied also to investigation of gluon structure function. Another interesting object is the structure function of a polarized nucleon.

One can see that the method can be applied for description of a strange baryon in the matter. All the problems considered in this section and in the previous one can be approached in the same way. Also behaviour of a baryon in the system of strange ones can be described in terms of the condensates of \( u, d \) and \( s \) quarks.

**Summary.** We made first steps in solving the problem of expressing the characteristics of baryons at finite densities through the in-medium values of the condensates. Potential energy of a nucleon in nuclear matter was expressed as the sum of the terms proportional to vector and scalar condensates. The former is positive while the latter is negative. Hence, the structure of potential energy reproduces that of quantum hadrodynamics. The saturation of the matter in our approach is provided by non-linear contribution to the scalar condensate \( \langle M|\bar{q}q|M\rangle \). We obtained at least qualitative description of neutron-proton mass splitting in nuclear matter. We described also the influence of medium on nucleon structure functions.
Note that this approach does not describe quark effects only. It describes the hadron effects, expressing them through certain quark effects. For example exchange by mesons (pairs of strongly correlated quarks) in the r.h.s. of the sum rules is expressed through exchange by pairs of uncorrelated quarks in l.h.s.

We obtained some new knowledge. We show the scalar forces to be related to $\pi N$ sigma term. In the case of isotope-breaking forces we show the scalar channel to be as important as the vector one. Thus, the method provides guide-likes for traditional nuclear physics.

All the results, described above, were obtained without fitting parameters. We did not use a controversial conception of $NN$ interaction.

Note one more point. The QCD SR method provides a unique approach to the problems, listed in the beginning of this paper. In framework of traditional nuclear physics they require different knowledge and different skill. Thus, usually they attract attention of different communities of the explorers.

The method should be improved by inclusion of more complicated in-medium condensates. Also the role of higher order terms of expansion in powers of Fermi momentum should be clarified.

Note that investigation of QCD SR stimulated other directions of research. Say, the first analysis of the function $\langle M|\bar{q}q|M\rangle$ carried out in [3] was followed by more than a dozen works on the subject.

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