Neutrino Sector with Majorana Mass Terms and Friedberg-Lee Symmetry

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Abstract

We examine a recently proposed symmetry/condition by Friedberg and Lee in the framework where three right-handed neutrinos are added to the spectrum of the three-family Minimal Standard Model. It is found that the right-handed neutrinos are very special, with respect to this symmetry. In the symmetry limit the neutrinos are massless and that may be a hint about why they are light. Imposed as a condition and not as a full symmetry, we find that one of the three right-handed neutrinos simply decouples (has only gravitational interactions) and that there is a massless interacting neutrino. The possible relation of the model to the see-saw mechanism is briefly discussed.

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1 Introduction

The question of the origin of the quark and lepton masses is considered to be one of the most important ones in physics. However, in spite of a huge amount of effort during the past three decades, we do not understand the pattern of the fermion masses and mixings. This area of research is badly in need of new ideas.

Recently, Friedberg and Lee have in a series of papers ([1] - [3]) addressed this question, by proposing a translational symmetry for fermion fields (see below). We find that their proposal merits further thought and, therefore, in this short paper we take a closer look at it in the framework of the three-family Standard Model with the addition of three right-handed neutrinos. We extend the Friedberg-Lee analysis by adding Majorana mass terms for the right-handed neutrinos.

The plan of this article is as follows. First, we briefly recapitulate the Friedberg-Lee proposition and then consider some alternative approaches to implementing it, in the framework of the Standard Model. We find that the right-handed neutrino sector is very special, due to the fact that these objects do not interact with the gauge fields. Consequently, we examine the case of neutrinos in more detail and present our conclusions in the last section.

2 The Friedberg-Lee Proposition

Friedberg and Lee take the fermion mass terms, in the Lagrangian, to be of the form

\[ L_{\text{mass}} = f_j M_{jk} f_k \] (1)

where, for example, for the up-type quarks the \( f_j, j = 1 - 3 \), represent three fermion fields with the quantum numbers of the up-type quarks and \( M \) is the three-by-three Hermitian mass matrix of these quarks. This mass term is required to be invariant under

\[ f_j \rightarrow f_j + \eta_j \theta \] (2)

where \( \eta_j \) are constants, to be determined, and \( \theta \) is an anticommuting (Grassmann) parameter, \( \theta^2 = 0 \), which is independent of space and time.

There are four different \( L_{\text{mass}} \) terms in the Lagrangian, corresponding respectively to up-type quarks, down-type quarks, charged leptons and neutrinos. Therefore, there are, in the Friedberg-Lee scheme four different \( \theta \)'s, as well as four sets of \( \eta \)'s.

The Friedberg-Lee requirement yields that \( M_{jk} \eta_k = 0 \). In the symmetry limit, i.e., when this relation is to hold for arbitrary \( \eta \), the mass matrix would identically vanish. Therefore, the condition is imposed for a set of \( \eta \)'s, to be determined. This implies that \( M \) has a zero eigenvalue, i.e., \( m_u = 0 \), for the case of up-type quarks. Indeed the up-quark mass is much smaller than the charm
and the top quark masses and, therefore, one could imagine that it were actually zero, in the lowest order in some more elaborate scheme. Similarly, the down quark, the electron and one of the neutrinos are massless. These results are again interesting as $m_e/m_\mu$ and $m_d/m_s$ are small numbers. With one massless fermion in each sector at the departure point, Friedberg and Lee have invested substantial effort into the needed modifications for getting a realistic model. See references [1] - [3].

3 Possible Approaches in the Standard Model

In the Standard Model the situation looks somewhat more complicated because the left-handed and right-handed fermions have a priori nothing to do with one another. One could say that they are born in different worlds and meet each other through the Higgs field, the mass matrices being their coupling constants (up to an overall factor) to the Higgs field. These matrices are in general not Hermitian. We shall now consider how these matters may affect the Friedberg-Lee conditions.

Consider the Lagrangian of the Standard Model which symbolically can be written in the form

$$L = L(F, G) + L(F, H) + V(H) + L(G) + L(H, G)$$

Here $F$, $G$ and $H$ denote the fermion, gauge and Higgs fields respectively. $L(F, G)$ gives the kinetic terms of the fermions and their interactions with the gauge bosons and $L(F, H)$ contains the fermion mass terms. The last three terms have been written down for completeness but will not directly concern us here.

The fermion mass terms, for quarks and charged leptons, are of the form

$$L_{mass}^{SM} = \bar{f}_j L M^\text{Dir}_{jk} f_k + h.c.$$ (4)

In the literature, these are called Dirac mass terms, being the only possible terms for these charged particles.

For neutrinos the situation is more complicated. Introducing right-handed neutrinos results not only in Dirac mass terms but also in Majorana mass terms

$$L^{\text{Maj}} = \bar{f}_j R M^\text{Maj}_{jk} f_k + h.c.$$ (5)

where in this equation $f$ stands for the neutrino field.

The left-handed and right-handed fermions being unrelated to each other implies that there are several options for implementing the Friedberg-Lee proposition. One could introduce it for

1. only the left-handed fermions
2. only the right-handed fermions
3. both chiralities
One could also choose some mixtures of the above. We find that the option 2
is the most attractive one. The reason being that the right-handed fermions have
less gauge interactions. In option 1, the condition would forbid all interactions
of the left-handed quarks and/or leptons. The right-handed quarks and charged
leptons do not interact with the $W$’s. Thus in option 2 “only” their interactions
with the $B$ field would be forbidden. The right-handed neutrinos are in this
respect very special as they have no interactions at all with the gauge fields.
They constitute a basis for maximal symmetry.

For the case of quarks and charged leptons we have nothing to add to the
Friedberg-Lee analysis as the assumption that the above condition be valid only
for right-handed fermions (or for that matter only for left-handed fermions) leads
to a zero mass state in each of these sectors. Therefore, from now on, we focus
our attention on the neutrino sector.

4 Friedberg-Lee Condition in $\nu_R$-sector

Consider the transformation

$$\nu_{Rj} \rightarrow \nu_{Rj} + \eta_j \theta \quad (6)$$

Here $\eta_j$, $j = 1 - 3$ are in general complex numbers, and $\theta$ carries the quantum
numbers of the right-handed neutrinos (weak isospin and hypercharge equal to
zero).

A particularly attractive feature of this transformation is that all the other
terms in the Standard Model Lagrangian are invariant under it, with arbitrary $\eta$.
More precisely, the kinetic terms for the right-handed neutrinos are not invariant
but since $\theta$ does not depend on space and time the resulting action is invariant.
Thus the above translation is a symmetry of the Standard Model excluding the
neutrino-Higgs terms in $L(F,H)$ and the Majorana mass terms. Therefore, re-
quiring the above translations to be a symmetry of the entire action immediately
yields that the neutrino masses are all zero. This is a welcome result as it perhaps
may explains why that neutrino masses are so small. They start off by being zero
in the symmetry limit. One could imagine that there is a scenario beyond the
Standard Model in which the above symmetry holds and somehow in the broken
version the $\eta$’s get fixed.

In the real world the $\eta$’s are not arbitrary as the neutrinos are not massless.
Let us consider, in somewhat more detail, what happens in the Standard Model,
with three right-handed neutrinos. The neutrino mass matrix is then of the form

$$L_{\text{mass}}^{(\nu)} = -\frac{1}{2}(\nu_L, \nu_R)M(\nu_L, \nu_R) + h.c. \quad (7)$$
where
\[ \nu_X = \begin{pmatrix} \nu_{1X} \\ \nu_{2X} \\ \nu_{3X} \end{pmatrix} \]  
\[ X = L, R. \] Here \( \mathcal{M} \) is the six-by-six neutrino mass matrix given by
\[ \mathcal{M} = \begin{pmatrix} 0 & A \\ A^T & M \end{pmatrix} \]  
\[ (9) \]
\( \mathcal{M} \) is a symmetric matrix; \( A \) and \( M \) being respectively the three-by-three Dirac and Majorana mass matrices and \( T \) stands for transpose. For simplicity we have suppressed the superscripts \( \text{Dir} \) and \( \text{Maj} \).

The Friedberg-Lee condition that \( \nu_{Rj} \to \nu_{Rj} + \eta_j \theta \) should leave the Lagrangian invariant implies
\[ A_{jk} \eta_k = 0, \quad M_{jk} \eta_k = 0 \]  
\[ (10) \]
We may now redefine the right-handed neutrino fields by
\[ \nu_R \to U \nu_R \]  
\[ (11) \]
where \( U \) is a three-by-three unitary matrix. Under this transformation the Lagrangian retains its form. All that happens is a redefinition of the mass matrices
\[ M \to M' = U^* M U^\dagger, \quad A \to A' = A U^\dagger \]  
\[ (12) \]
The Friedberg-Lee condition now reads
\[ A'_{jk} \eta'_k = 0, \quad M'_{jk} \eta'_k = 0 \]  
\[ (13) \]
where \( \eta' = U \eta \). We may choose \( U \) such that \( M' \) is diagonal. Since \( M' \) has a zero eigenvalue, by permutation of the right-handed neutrino fields (which leaves the rest of Lagrangian invariant) it may be written in the form
\[ M' = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  
\[ (14) \]
Assuming that \( M_1 \) and \( M_2 \) are nonzero, we find that the vector \( \eta' \) is proportional to \((0, 0, 1)\). This in turn implies that the third column in matrix \( A' \) is zero. Therefore it is of the form
\[ A' = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix} \]  
\[ (15) \]
A remarkable consequence of the Friedberg-Lee condition is that one of the three right-handed neutrinos simply decouples, in the sense that it has only gravitational interactions due to its kinetic energy term. We end up with three left-handed neutrinos and just two right-handed neutrinos. The mass matrix is then of the form

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & 0 & a_{11} & a_{12} \\
0 & 0 & 0 & a_{21} & a_{22} \\
0 & 0 & 0 & a_{31} & a_{32} \\
a_{11} & a_{21} & a_{31} & M_1 & 0 \\
a_{12} & a_{22} & a_{32} & 0 & M_2
\end{pmatrix}
\] (16)

To sum up, in this model there is, in addition to the non-interacting massless neutrino, a massless interacting neutrino. The masses and mixings will depend on the choice of the parameters. A natural scenario would be to have small values for magnitudes of $M_1$ and $M_2$ because in the symmetry limit these are zero. In the limit $M_1, M_2 \to 0$ one obtains three Dirac neutrinos, one of them being massless. The model may or may not violate CP symmetry depending on how the charged lepton mass matrix looks like. The if and only if condition for CP violation in this case is as for the quarks, that the determinant of the commutator of the respective mass matrices be nonzero [4].

However, since we don’t really know what is natural or not, the model also allows see-saw mechanism, which is obtained by taking the magnitudes of $M_1$ and $M_2$ to be very large. Then there will be two very heavy neutrinos that do not interact with the gauge bosons and three light ones, one of them being massless. The phenomenology of see-saw models with two right-handed neutrinos has been the subject of several studies in the literature (see, for example [5] and [6] and references cited therein).

It should be mentioned that the fermionic translations may look ”innocent” but are far more subtle than their bosonic counterparts, such as $\phi \to \phi + v$, or $A_\mu \to A_\mu + \partial_\mu \Lambda$. For example, under Lorentz transformations $\theta$ must transform as a fermionic field, i.e., $\nu_R \to S \nu_R$ and $\theta \to S \theta$, where $S$ is the appropriate transformation matrix. An important point to keep in mind is that a nonzero vacuum expectation value of a fermionic operator will break Lorentz invariance. One could imagine that the Friedberg-Lee translation is actually driven by the requirement of Lorentz invariance. However, this matter requires further study which is beyond the scope of the current paper. We would only like to add that possible mechanisms for the violation of Lorentz invariance in particle physics have been studied by many authors. For example, Coleman and Glashow [7] have suggested adding tiny renormalizable terms, that violate Lorentz invariance, into the Lagrangian of the Standard Model and have examined their phenomenological consequences for many processes, among them neutrino oscillations. The work of these authors has led to a wave of further studies. Currently, violation of Lorentz invariance is a rather popular domain of research.
5 Conclusions and Outlook

In this paper, we have examined the Friedberg-Lee proposition, that the mass terms in the Lagrangian be invariant under a prescribed translation of fermion fields. For the case of quarks and charged leptons, it does not matter whether the constraint is imposed on left-handed or the right-handed fermions. The result, in each case, is as obtained by Friedberg and Lee. In the neutrino sector, we find that the right-handed neutrinos are very special because the action is invariant under the translation, $\nu_Rj \rightarrow \nu_Rj + \eta_j \theta$, with the exception of those terms which involve neutrino masses and interactions with the Higgs particle. Adding three right-handed neutrinos, to the three-family version of the Minimal Standard Model and taking into account both the Dirac and Majorana mass terms, we find a very interesting result: the Friedberg-Lee condition forces one right-handed neutrino to decouple. The ensuing model is equivalent to introducing only two right-handed neutrinos. See-saw-like model is allowed, with two heavy neutrinos and three light neutrinos, one being massless. But the Majorana mass terms could also be very small. In this case, one could argue that the neutrino masses are small because in the symmetry limit (i.e., with $\eta$ being arbitrary) the Friedberg-Lee condition requires neutrinos to be massless. One may wonder what happens with the neutrino counting, that gives the result $N_\nu$, at the Z-peak. Doesn’t one then get too large a value for this number? The current experimental value is $N_\nu = 2.9840 \pm 0.0082$ [8]. The answer is no. It has been shown (9) that adding any number of right-handed neutrinos to the spectrum of the three family Minimal Standard Model yields $N_\nu \leq 3$.

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