Supernova Neutrinos and the LSND Evidence for Neutrino Oscillations

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The observation of the $\bar{\nu}_e$ energy spectrum from a supernova burst can provide constraints on neutrino oscillations. We derive formulas for adiabatic oscillations of supernova antineutrinos for a variety of 3 and 4-neutrino mixing schemes and mass hierarchies which are consistent with the LSND evidence for $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations. Finally, we explore the constraints on these models and LSND given by the supernova SN1987A $\bar{\nu}_e$’s observed by the Kamiokande-2 and IMB-3 detectors.

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I. INTRODUCTION

In recent years, the treatment of neutrino transport in the environment of a core-collapse supernova (SN) explosion has improved to the point of making realistic predictions on the observables for neutrinos reaching the Earth [1, 2, 3, 4]. Of particular interest for this paper are the average energies at the neutrinospheres, i.e., the surfaces of last scattering for the neutrinos, estimated to be $10-13$ MeV for $\nu_e$, $14-17$ MeV for $\bar{\nu}_e$, $23-27$ MeV for $\nu_\mu,\tau$, $\bar{\nu}_\mu,\tau$ [2, 4].

The differences in temperatures between the various neutrino flavors can be qualitatively understood. Heavy-lepton neutrinos can interact only via neutral-current (NC) processes, the main contribution to their transport opacity coming from neutrino-nucleon scattering, which dominates over neutrino-electron scattering. In addition to this same NC contribution, the transport opacity for $\nu_e$’s and $\bar{\nu}_e$’s depends also on the charged-current (CC) absorptions $\nu_e + n \rightarrow p + e^-$ and $\bar{\nu}_e + p \rightarrow n + e^+$, respectively. Therefore, the $\nu_e$- and $\bar{\nu}_e$-spheres are located at larger radii with respect to the other neutrinospheres, that is at lower densities and lower temperatures. Moreover, in a neutron-rich environment, $\nu_e + n \rightarrow p + e^-$ dominates over $\bar{\nu}_e + p \rightarrow n + e^+$, the emergent $\nu_e$’s originate further outside the center of the star compared to $\bar{\nu}_e$’s, therefore at lower temperatures. The total energy released in a SN explosion is approximately equipartitioned between the different neutrino and antineutrino flavors [3].

The above predictions can be confronted with the observation of the supernova $\nu_e$ energy spectrum detected on Earth. Neutrino oscillations are expected to modify the spectrum since $\langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_e,\bar{\nu}_e} \rangle$. The energy dependence of the neutrino cross-section in the detector material, approximately $\sigma_{\nu_e,p} \propto (E_{\bar{\nu}_e} - 1.29\text{MeV})^2$ [3], helps in making the $\bar{\nu}_e$ energy spectrum distortion a sensitive experimental probe to neutrino oscillations. This is because higher energy neutrinos interact significantly more than lower energy ones.

II. ADIABATIC OSCILLATIONS AND NEUTRINO MIXING SCHEMES

A. $\bar{\nu}_e$ energy spectrum and the permutation factor

In the presence of neutrino oscillations, the $\bar{\nu}_e$ flux reaching the Earth, $F_{\bar{\nu}_e}$, can be different from the primary flux at the neutrinosphere, $F_{\bar{\nu}_e}^0$. We will assume that, at production, the energy of active antineutrinos is equally divided into the three active flavors, i.e., that $\int_0^{\infty} dE \bar{\nu}_e E_{\bar{\nu}_e} F_{\bar{\nu}_e}^0$ has the same numerical value for $\alpha = e, \mu, \tau$. Moreover, we will also consider neutrino mixing models where the three active neutrino species are augmented by a fourth sterile neutrino with no standard weak couplings: in those cases, we will assume that the sterile component is negligible at production.

The neutrino flux reaching the Earth is:

$$F_{\bar{\nu}_e} = (p_{\mu\rightarrow e} + p_{\tau\rightarrow e}) F_{\bar{\nu}_e}^0 + p_{e\rightarrow e} F_{\bar{\nu}_e}^0$$

$$\propto (p F_{\bar{\nu}_e}^0 + (1-p) F_{\bar{\nu}_e}^0)$$ (1)
where we have defined the permutation factor $p$ as:

$$p = \frac{p_{\mu\rightarrow e} + p_{\tau\rightarrow e}}{p_{\mu\rightarrow e} + p_{\tau\rightarrow e} + p_{e\rightarrow e}}$$

and $p_{\mu,\tau,e\rightarrow e}$ are the probabilities for a $\bar{\nu}_\mu$, $\bar{\nu}_\tau$, $\bar{\nu}_e$ respectively at the neutrinosphere to oscillate into a $\bar{\nu}_e$. In Eqs. [3][4] we have assumed that $p$ is energy-independent (as will be justified later), and that $\langle E_{\bar{\nu}_\mu} \rangle = \langle E_{\bar{\nu}_e} \rangle$. In Eq. [4], we neglect the (energy-independent) proportionality factor since we will not deal with event rates, but only with neutrino energy distributions.

B. Neutrino propagation in the adiabatic approximation

In vacuum, the Hamiltonian that governs neutrino propagation is diagonal in the mass eigenstate basis $|\nu_i\rangle$:

$$(H_0)_{ij} = \langle \nu_i | H_0 | \nu_j \rangle = E_i \delta_{ij}$$

If the neutrinos all have the same relativistic momentum $p$, their energies $E_i$ differ only by a term proportional to their squared-mass differences, since $E_i \simeq p + m_i^2/2p$. If $U$ is the unitary mixing matrix that relates the flavor eigenstates $|\nu_\alpha\rangle$ to the mass eigenstates via $|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle$, the elements of the vacuum Hamiltonian in the flavor basis are given by [4]:

$$(H_0)_{\alpha\beta} = U_{\alpha i}^* U_{\beta j} \frac{m_i^2}{2p}$$

where we have neglected the contribution $p\delta_{\alpha\beta}$ in $(H_0)_{\alpha\beta}$, which is irrelevant for neutrino oscillations.

In matter, $\bar{\nu}_e$’s undergo coherent CC forward-scattering from electrons, and all active flavor antineutrinos coherent NC forward-scattering from electrons, protons, and neutrons in the medium. These processes give rise to an interaction potential $V = V_W + V_Z$, which is diagonal in the flavor basis and proportional to the matter density $\rho$:

$$(V)_{\alpha\beta} = A_\alpha \frac{G_F \rho}{m_N} \delta_{\alpha\beta}$$

where $A_\alpha$ is a proportionality constant, in general different for $\alpha = e, \mu, \tau, \text{or } s$, $G_F$ the Fermi constant, and $m_N$ the nucleon mass. The relevant Hamiltonian for neutrino propagation in matter is therefore $H \equiv H_0 + V$.

At the neutrinosphere, the density $\rho$ is so high ($\sim 10^{12} g/cm^3$) that the interaction potential dominates over the vacuum Hamiltonian, so that the propagation eigenstates coincide with the flavor eigenstates. As the propagation eigenstates free-stream outwards, toward regions of lower density, their flavor composition changes, ultimately reaching the flavor composition of the mass eigenstates in the vacuum. Given that the neutrinos escape the SN as mass eigenstates, no further flavor oscillations occur on their path to the Earth.

More specifically, making use of the adiabatic approximation and of the fact that no energy-level crossing is permitted, the flavor eigenstate at the neutrinosphere with the maximum interaction potential reaches Earth as the mass eigenstate with the biggest neutrino mass. In general, the energy level order is maintained throughout the neutrino propagation in the SN ejecta.

This is illustrated in Tab. 1 for three neutrinos in the row labelled “Normal (1+1+1)”, where we have taken $A_\gamma > A_\beta > A_\alpha$ and $m_3 > m_2 > m_1$. For example, the probability for a $\bar{\nu}_\alpha$ to emerge from the SN environment as a $\bar{\nu}_\beta$ is given by:

$$p_{\alpha \rightarrow \beta} = |\langle \bar{\nu}_\beta | U^{\text{evol}} | \bar{\nu}_\alpha \rangle|^2 = |\langle U_{\beta i} | U^{\text{evol}} | \bar{\nu}_\alpha \rangle|^2 = |U_{\beta i}^*\delta_{i1}|^2 = |U_{\beta 1}|^2$$

| Model | Hierarchy | Propagation |
|-------|-----------|-------------|
| Normal (1+1+1) | $m_3 > m_2 > m_1$ | $\bar{\nu}_3 \rightarrow \bar{\nu}_1$ |
| Normal (1+1) | $m_2 \gg m_1$ | $\bar{\nu}_2 \rightarrow \bar{\nu}_1$ |
| LSND-inverted (1+1) | $m_1 \gg m_2$ | $\bar{\nu}_1 \rightarrow \bar{\nu}_2$ |
| Normal (2+1) | $m_3 > m_2 \gg m_1$ | $\bar{\nu}_3 \rightarrow \bar{\nu}_2$ |
| LSND-inverted (2+1) | $m_1 > m_3 \gg m_2$ | $\bar{\nu}_1 \rightarrow \bar{\nu}_2$ |
| Normal (2+2) | $m_3 > m_2 \gg m_1 > m_0$ | $\bar{\nu}_3 \rightarrow \bar{\nu}_2$ |
| LSND-inverted (2+2) | $m_1 > m_3 \gg m_2 > m_0$ | $\bar{\nu}_1 \rightarrow \bar{\nu}_2$ |
| Normal (3+1) | $m_4 \gg m_3 > m_2 > m_1$ | $\bar{\nu}_4 \rightarrow \bar{\nu}_1$ |
| LSND-inverted (3+1) | $m_3 > m_2 > m_1 \gg m_4$ | $\bar{\nu}_4 \rightarrow \bar{\nu}_1$ |
where $U_{\text{evol}}$ is the adiabatic evolution operator. In Eq.\ref{eq:Uevol}, we have used Tab.\ref{tab:masses} to get:

$$\langle \bar{\nu}_i | U_{\text{evol}} | \bar{\nu}_\alpha \rangle = \delta_{i,1} \tag{7}$$

This result can be immediately generalized to any number of antineutrino generations. Also, as long as the adiabatic approximation is satisfied, the formula does not depend on the specific dynamics for the neutrino propagation, for example on the number and position in the SN environment of MSW-resonances. We will comment more on the validity of the adiabatic approximation in the next section.

In this paper, we consider three or four flavor components, including a sterile one. At tree-level, the proportionality factors $A_{\nu_i}$ in the interaction potential for neutral matter are \cite{10}:

$$A = \begin{cases} 
(1 - 3 Y_e)/\sqrt{2}, & \text{for } \bar{\nu}_e \\
(1 - Y_e)/\sqrt{2}, & \text{for } \bar{\nu}_\mu, \bar{\nu}_\tau \\
0, & \text{for } \bar{\nu}_s
\end{cases} \tag{8}$$

where $Y_e$ is the electron fraction per nucleon. Following the assumptions of \cite{10} \cite{11}, we use $Y_e \simeq (1 + (E_{\nu_i}/(E_{\nu_s}))^{-1} > 1/3$ at the neutrinosphere. Considering also one-loop electroweak radiative corrections, a difference in the $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ interaction potentials of magnitude $(A_\mu - A_\tau)/A_\mu \sim 10^{-4}$ appears due to the difference in the charged lepton masses \cite{12} \cite{13}. At the neutrinosphere, this second-order effect in the interaction potential dominates over the vacuum Hamiltonian terms (as long as $|m_1^2 - m_2^2| < 10 \text{ eV}^2$ for all $i, j$), and removes the $\bar{\nu}_\mu - \bar{\nu}_\tau$ degeneracy. Therefore, for the antineutrino channel considered here, we take:

$$A_\mu > A_\tau > A_s > A_e \tag{9}$$

For the neutrino channel, one should substitute $A \to -A$ in Eq.\ref{eq:Uevol} and the order in Eq.\ref{eq:adiabatic} would be inverted.

Therefore, given a specific neutrino mass and mixing model, the permutation factor can be easily evaluated in the adiabatic approximation, and its numerical value does not depend on the neutrino energy. We will comment on possible energy-dependent Earth matter effects in the next section. In practice, one proceeds backwards: given a certain measured value of $p$, it is possible to constrain possible models for neutrino oscillations. This approach is used for example in \cite{13} to constrain models explaining the solar and atmospheric neutrino data; in this paper, we focus on 3 and 4-neutrino models explaining the LSND data.

### C. Possible mixing schemes

The results for the $\bar{\nu}_\mu, \bar{\nu}_\tau, \bar{\nu}_e \to \bar{\nu}_s$ adiabatic oscillation probabilities, the permutation factor $p$, and the LSND oscillation amplitude $\sin^2 2\theta$ as a function of the mixing parameters and $p$ for the eight possible mass and mixing schemes considered below are given in Tab.\ref{tab:masses}. The mass hierarchy and the adiabatic propagation of the neutrino eigenstates for these mixing schemes are depicted in Tab.\ref{tab:masses}.

The simplest possible mixing scheme is a $(1+1)$ model explaining only $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND oscillations in vacuum, and not the atmospheric or solar oscillations:

$$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \end{pmatrix} \tag{10}$$

where the mixing angle $\theta$ can assume any value in the range $0 < \theta < \pi/4$.

We consider a $(2+1)$ model motivated, for example, by CPT-violating scenarios (see, e.g. \cite{14}), in which atmospheric and LSND oscillations in the antineutrino channel are obtained via the mixing \cite{15}:

$$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2} \alpha - \frac{\sqrt{3}}{2} \beta \\ \alpha + \frac{\sqrt{3}}{2} \beta \\ 0 - \frac{1}{2} \beta \end{pmatrix} \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix} \tag{11}$$

The matrix in Eq.\ref{eq:21} is chosen to ensure large $\bar{\nu}_\mu \to \bar{\nu}_\tau$ mixing for atmospheric neutrinos ($\sin^2 2\theta_{\text{atm}} = 3/4$), while the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ mixing is fixed by the parameter $\alpha (\sin^2 2\theta_{\text{LSND}} = 4\alpha^2)$.

The most popular models which explain the solar, atmospheric and LSND signatures (and the null results obtained by other experiments) via neutrino oscillations invoke the existence of a sterile neutrino $\bar{\nu}_s$. One example of a $(2+2)$ model is the following, which is taken from \cite{16}:

$$\begin{pmatrix} \bar{\nu}_s \\ \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & \beta & \beta \\ \beta & -\beta & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \bar{\nu}_0 \\ \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix} \tag{12}$$

where one pair of nearly degenerate mass eigenstates has maximal $\nu_e \to \nu_s$ mixing for solar neutrinos and the other pair has maximal $\nu_\mu \to \nu_\tau$ mixing for atmospheric neutrinos. Small inter-doublet mixings through the $\beta$ parameter accomodates the LSND result ($\sin^2 2\theta_{\text{LSND}} = 8\beta^2$).

Recent experimental results \cite{17} show that pure $\nu_e \to \nu_s$ solar oscillations are excluded at high significance. We therefore consider a more general $(2+2)$ scenario, in which solar neutrinos can undergo any combination of $\nu_e \to \nu_s$ and $\nu_\mu \to \nu_\tau$ oscillations, while atmospheric neutrinos can undergo any combination of $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_s$ oscillations. We follow the procedure in \cite{18} to obtain this more general mixing starting from Eq.\ref{eq:22} by substituting the $(\bar{\nu}_s, \bar{\nu}_\tau)$ states with the rotated states $(\bar{\nu}_s', \bar{\nu}_\tau')$:

$$\begin{pmatrix} \bar{\nu}_s' \\ \bar{\nu}_\tau' \end{pmatrix} = \begin{pmatrix} \cos \varphi_s & \sin \varphi_s \\ -\sin \varphi_s & \cos \varphi_s \end{pmatrix} \begin{pmatrix} \bar{\nu}_s \\ \bar{\nu}_\tau \end{pmatrix} \tag{13}$$
where the rotation angle $\varphi_s$ fixes the sterile component in the atmospheric doublet ($0 < \varphi_s < \pi/2$). Eq. (13) then becomes:

$$
\begin{pmatrix}
\bar{\nu}_s \\
\bar{\nu}_e \\
\bar{\nu}_\mu \\
\bar{\nu}_\tau
\end{pmatrix} =
\begin{pmatrix}
\cos \varphi_s & \cos \varphi_s & \sin \varphi_s & -\sin \varphi_s \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\beta & -\beta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\sin \frac{\varphi_s}{\sqrt{2}} & \sin \frac{\varphi_s}{\sqrt{2}} & -\cos \frac{\varphi_s}{\sqrt{2}} & -\cos \frac{\varphi_s}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\bar{\nu}_0 \\
\bar{\nu}_1 \\
\bar{\nu}_2 \\
\bar{\nu}_3
\end{pmatrix}
$$

(14)

which contains Eq. (12) in the specific case $\varphi_s = 0$. We note that the LSND oscillation amplitude formula $\sin^2 2\varphi_{LSND} = 8\beta^2$ holds also for the more general case of Eq. (13), and that our results are independent of the value of $\varphi_s$ (see Tab. 1).

Another possible 4-neutrino model has a (3 + 1) hierarchy; as an example for this model, here we consider the following mixing, which is also taken from Eq. (13):

$$
\begin{pmatrix}
\bar{\nu}_e \\
\bar{\nu}_\mu \\
\bar{\nu}_\tau \\
\bar{\nu}_s
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & \gamma \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \delta \\
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{2}(\delta-\gamma) & -\frac{1}{2}(\delta-\gamma) & 1
\end{pmatrix}
\begin{pmatrix}
\bar{\nu}_1 \\
\bar{\nu}_2 \\
\bar{\nu}_3 \\
\bar{\nu}_4
\end{pmatrix}
$$

(15)

where the solar and atmospheric oscillations are approximately described by oscillations of three active neutrinos, and the LSND result by a coupling of $\bar{\nu}_\mu$ and $\bar{\nu}_e$ through small mixings with $\bar{\nu}_s$ that has a mass eigenvalue widely separated from the others ($\sin^2 2\varphi_{LSND} = 4\gamma^2\delta^2$). For the (3 + 1) scenario, the constraint given by the permutation probability $p$ is not sufficient to determine the LSND oscillation amplitude $\sin^2 2\varphi_{LSND}$. Therefore, the constraint on $|U_{\mu 4}|^2 = \delta^2$ given by the CDHS and Super-K experiments will also be used, as explained later.

We should note that the mixing matrices defined in Eqs. (10) [13] are approximations in the sense that the matrices are unitary only up to order $O(\alpha, \beta, \gamma, \delta)$. These are the parameters in the mixings responsible for LSND-type oscillations, which we let float for our analysis, but we know they are small.

In order to determine the permutation factor for the mixing models, we also need to specify the neutrino mass hierarchy. In this paper, we consider for each mixing model both the cases of a “normal” and a “LSND-inverted” mass hierarchies. By “normal” hierarchy, here we mean that $m_i > m_j$ for $i > j$, where $m_i$ is the mass eigenvalue for the $|\nu_i\rangle$ state. We define the “LSND-inverted” hierarchies as the ones obtained substituting $\Delta m_{LSND} \rightarrow -\Delta m_{LSND}$ in the normal hierarchies, without changing the hierarchy of the eventual solar and atmospheric splittings (see Tab. 1); $\Delta m_{LSND}$ is the neutrino mass difference responsible for LSND oscillations.

A common feature to all the mixing schemes is apparent in Tab. 1. In the adiabatic approximation, normal mass hierarchies predict small permutation factors, while an almost complete permutation would be present for LSND-inverted hierarchies.

Given the specific neutrino mixing models considered here, we can now partially address the question whether the adiabatic approximation is applicable in this context. At a resonance, where the non-adiabaticity is maximal, this is a good approximation if the width of the resonance region is large compared with the local neutrino oscillation length. The width of the resonance is, in turn, determined by the characteristic length scale of the radial matter density variations at the resonance. While there are reliable models for the matter density profile of the progenitor star, there are still uncertainties on the profile seen by neutrinos in their free-streaming propagation.

It is now thought that neutrino heating of the proto-neutron star mantle drives the supernova explosion, which would happen with a $\sim 1s$ delay after the creation of the shock-wave, ultimately responsible for the explosion; during this delay, the shock-wave would be stalled at a radius of $\sim 200$ km from the neutron star, corresponding to a density $\rho \sim 10^9 - 10^{10}$ g/cm$^3$ [4]. Therefore, the density profile in the proximity of the stalled shock-wave, which is difficult to model reliably, is a potential site for non-adiabatic oscillations.

In Fig. 1, we show the energy splittings between the local neutrino energy eigenvalues $E_i$, as a function of matter density, for all eight neutrino models considered here.

| Model | Mixing | $p_{\mu \rightarrow e}$ | $p_{\tau \rightarrow e}$ | $p_{e \rightarrow e}$ | $p$ | $\sin^2 2\varphi_{LSND}$ |
|-------|--------|----------------|----------------|----------------|----|----------------|
| Normal (1+1) | Eq. (10) | $\sin^2 \varphi$ | 0 | $\cos^2 \varphi$ | $\sin^2 \varphi$ | $\sin^2 2\varphi = 4p(1-p)$ |
| LSND-inverted (1+1) | Eq. (10) | $\cos^2 \varphi$ | 0 | $\sin^2 \varphi$ | $\cos^2 \varphi$ | $\sin^2 2\varphi = 4p(1-p)$ |
| Normal (2+1) | Eq. (11) | $\frac{1}{2}\alpha^2$ | $\frac{1}{2}\alpha^2$ | 1 | $\alpha^2/(1 + \alpha^2)$ | $4\alpha^2 = 4p(1-p)$ |
| LSND-inverted (2+1) | Eq. (11) | 1 | $\frac{1}{2}\alpha^2$ | $\frac{1}{2}\alpha^2$ | $(1 + \frac{1}{2}\alpha^2)/(1 + \alpha^2)$ | $4\alpha^2 = 4p(1-p)/(p - \frac{3}{4})$ |
| Normal (2+2) | Eq. (14) | $\beta^2$ | $\beta^2$ | $\frac{1}{2}$ | $4\beta^2/(1 + \beta^2)$ | $8\beta^2 = 2p(1-p)$ |
| LSND-inverted (2+2) | Eq. (14) | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $1/(1 + \beta^2)$ | $8\beta^2 = 8(1-p)/p$ |
| Normal (3+1) | Eq. (15) | $\gamma^2$ | 0 | $\frac{1}{2}$ | $2\gamma^2/(1 + 2\gamma^2)$ | $4\gamma^2 \delta^2 = 2\delta^2 p/(1-p)$ |
| LSND-inverted (3+1) | Eq. (15) | 0 | $\frac{1}{2}$ | $\gamma^2$ | $1/(1 + 2\gamma^2)$ | $4\gamma^2 \delta^2 = 2\delta^2 (1-p)/p$ |

TABLE II: Results on the probabilities $p_{\mu, \tau, e \rightarrow e}$ for a $\bar{\nu}_{\mu, \tau, e}$ to emerge from the SN as a $\bar{\nu}_e$, the permutation factor $p$ of Eq. (14), and the LSND oscillation amplitude $\sin^2 2\varphi_{LSND}$, for the various neutrino mixing schemes considered.
For an \(n\)-neutrino model, we plot \(E_{i,i+1} \equiv E_i - E_{i+1}\), where \(i = 1, \ldots, n-1\); the eigenvalues are ordered such that \(E_1 > E_2 > \ldots > E_n\). Clearly, a resonance corresponds to a local minimum in one of the curves. As can be seen from Fig. 1, all the resonances (except the inconsequential one in Fig. 1c) lie at densities well below the stalled shock-wave density of \(\rho \sim 10^9 - 10^{10} \text{ g/cm}^3\). Therefore, the impact of level crossing between propagation eigenstates is likely to be small even where the neutrinos encounter the shock-wave.

If the SN neutrinos cross the Earth on their way to the detector, as for example happened for the SN1987A \(\bar{\nu}_e\)’s detected by the Kamiokande-2 and IMB-3 detectors, it is also necessary to evaluate the importance of Earth matter effects in the neutrino propagation. Clearly, for neutrino oscillation models where no solar splitting is involved (for example the (1 + 1) and (2 + 1) models in this paper), this effect is negligible. In the models where such a splitting is allowed (i.e. the (2 + 2) and (3 + 1) models considered here), the situation is more complicated. However, the Earth matter effects have been shown to be small even where the neutrinos encounter the shock-wave, and even when the level crossing between propagation eigenstates is likely to be inconsequential one in Fig. 1f between \(\nu_e \leftrightarrow \bar{\nu}_e\) (see text). As already mentioned, for the (3 + 1) model, the permutation factor \(p\) does not fully determine the LSND oscillation amplitude \(\sin^2 2\theta_{\text{LSND}}\) (see Tab. I). At 99% CL, SN1987A data provide no constraints on the (2 + 2) model, and a constraint which is weaker than existing bounds from the accelerator experiment KARMEN [23] and the reactor experiment Bugey [24] for the (1 + 1) and (2 + 2) models (see Fig. 2b). Therefore, these models are compatible with the SN1987A data.

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FIG. 2: 99% CL LSND allowed region and 99% CL exclusion regions for the neutrino mixing schemes considered in the text and with normal mass hierarchy. The exclusion regions are estimated as in [26]. a) shows the exclusion regions for the \((1 + 1), (2 + 1)\) and \((2 + 2)\) models, b) for the \((3 + 1)\) model. The exclusion regions refer to experimental data from the following experiments. a) Dotted line: Karmen; dashed line: Bugey; dark solid line: SN1987A for the \((2 + 2)\) model; light solid line: SN1987 for the \((1 + 1)\) model; SN1987A data provide no constraints at 99% CL for the \((2 + 1)\) model. b) Dotted line: Karmen; dashed line: Bugey, CDHS and Super-K; solid line: SN1987A, CDHS and Super-K.

| Model                | SN1987A constraint on LSND region (99% CL) |
|----------------------|--------------------------------------------|
| Normal \((1 + 1)\)   | partially excluded (Fig. 2a)               |
| LSND-inverted \((1 + 1)\) | excluded                                    |
| Normal \((2 + 1)\)   | unconstrained                               |
| LSND-inverted \((2 + 1)\) | excluded                                    |
| Normal \((2 + 2)\)   | partially excluded (Fig. 2b)               |
| LSND-inverted \((2 + 2)\) | excluded                                    |
| Normal \((3 + 1)\)   | partially excluded (Fig. 2b)               |
| LSND-inverted \((3 + 1)\) | excluded                                    |

TABLE III: Summary of the SN1987A constraints on the LSND allowed region, for the various models considered in this paper; see Fig. 2 also.

We have investigated the effect that 3- and 4-neutrino oscillation schemes would have in modifying the energy spectrum of supernova \(\bar{\nu}_e\)'s. Throughout the paper, we apply the adiabatic approximation for the antineutrino propagation in the supernova environment and neglect Earth matter effects. Moreover, we have used our results to test the compatibility between the SN1987A data and the LSND evidence for \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) oscillations.

We have provided specific relations for the permutation factor, which gives the admixture of a higher energy flux to the original \(\bar{\nu}_e\) flux at production from \(\bar{\nu}_\mu, \bar{\nu}_\tau \rightarrow \bar{\nu}_e\) oscillations, for various neutrino mass and mixing models. The permutation factor may be measurable with good accuracy in future supernova experiments.

Based on SN1987A data only, which seem to indicate a small (if non-zero) value for the permutation factor, we are able to exclude all of the four models considered which would explain the LSND signal via a “LSND-inverted” neutrino mass hierarchy, as defined in the text. For the normal mass hierarchy schemes considered, SN1987A data do not provide any stronger constraints on the LSND allowed region for oscillations than those already obtained with reactor, accelerator and atmospheric neutrinos; additional experimental input is necessary to unambiguously discern the neutrino mass and mixing properties. Undoubtedly, the detection of supernova neutrinos by present or near-term experiments would prove very useful in this respect.
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APPENDIX A: UPPER BOUNDS ON THE PERMUTATION FACTOR FROM SN1987A DATA

In this appendix, we discuss the statistical methodology and the physics assumptions used to estimate the upper bound on the permutation factor $p$ quoted in the text, $p < 0.22$ at 99% CL. We use the same statistical methodology as in [2], that is we use the Kolmogorov-Smirnov test on the joint Kam-IMB dataset to derive the upper bound. Most of the physics assumptions are identical to those in [2].

The expected energy spectrum for the positrons, observed in the Kamiokande and IMB detectors via the reaction $\bar{\nu}_e p \rightarrow e^+ n$, is:

$$14 < \langle E_{\bar{\nu}_e} \rangle < 17 \text{ MeV}, \quad 23 < \langle E_{\bar{\nu}_e} \rangle < 27 \text{ MeV}, \text{ that is the one corresponding to } \langle E_{\bar{\nu}_e} \rangle = 14 \text{ MeV}, \langle E_{\bar{\nu}_e} \rangle = 23 \text{ MeV} \text{ (cross in Fig.3).}$$

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FIG. 3: Solid lines: isocontours for the upper bounds on the permutation factor $p$ at 99% CL obtained from SN1987A data, as a function of the average energies $\langle E_{\bar{\nu}_e} \rangle$, $\langle E_{\bar{\nu}_\mu} \rangle$. As expected, the bound becomes more stringent for supernova models in which the neutrino average energies are higher.

SN1987A data are incompatible at 99% CL with all supernova neutrino models predicting $\langle E_{\bar{\nu}_e} \rangle > 16.6$ MeV, for all values of $p$ and $\langle E_{\bar{\nu}_\mu} \rangle$. We adopt a conservative approach, and quote as the upper bound on $p$ the largest value for supernova neutrino models in the range $14 < \langle E_{\bar{\nu}_e} \rangle < 17$ MeV, $23 < \langle E_{\bar{\nu}_e} \rangle < 27$ MeV, that is the one corresponding to $\langle E_{\bar{\nu}_e} \rangle = 14$ MeV, $\langle E_{\bar{\nu}_e} \rangle = 23$ MeV (cross in Fig.3).
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