ORBITAL INSPIRAL INTO A MASSIVE BLACK HOLE IN A GALACTIC CENTER

TAL ALEXANDER AND CLOVIS HOPMAN
Faculty of Physics, The Weizmann Institute of Science, POB 26, Rehovot 76100, Israel
Accepted to ApJ Lett.

ABSTRACT

A massive black hole (MBH) in a galactic center drives a flow of stars into nearly radial orbits to replace those it destroyed. Stars whose orbits cross the event horizon \( r_S \) or the tidal disruption radius \( r_t \) are promptly destroyed in an orbital period \( P \). Stars with orbital periapse \( r_p \) slightly larger than the sink radius \( q \equiv \max(r_S, r_t) \) may slowly spiral in due to dissipative interactions with the MBH, e.g. gravitational wave emission, tidal heating or accretion disk drag, with observable consequences and implications for the MBH growth rate. Unlike prompt destruction, the inspiral time is typically \( \gg P \). This time is limited by the same scattering process that initially deflected the star into its eccentric orbit, since it can deflect it again to a wider orbit where dissipation is inefficient. The ratio between slow and prompt event rates is therefore much smaller than that implied by the ratio of cross-sections, \( \sim r_p/q \), and so only prompt disruption contributes significantly to the mass of the MBH. Conversely, most stars that scatter off the MBH survive the extreme tidal interaction ("tidal scattering"). We derive general expressions for the inspiral event rate and the mean number of inspiraling stars, and show that the survival probability of tidally scattered stars is \( \sim 1 \), and that the number of tidally heated stars ("squeezars") and gravity wave emitting stars in the Galactic Center is \( \sim 0.1 \)–1.

Subject headings: black hole physics—Galaxy: center—stellar dynamics—gravitational waves

1. INTRODUCTION

Evidence for the presence of massive black holes in the centers of most galaxies (e.g. Gebhardt et al. 2003), together with recent and anticipated advances in observing capabilities, have focused interest on the observational implications of star-MBH interactions such as tidal disruption (Frank & Rees 1976; Frank 1978; Lightman & Shapiro 1977; Syer & Ulmer 1999; Magorrian & Tremaine 1999), tidal scattering (Alexander & Livio 2001), gravitational wave (GW) emission (e.g. Hills & Bender 1995; Sigurdsson & Rees 1997; Freitag 2001, 2003), interaction with a massive accretion disk surrounding the MBH (Ostriker 1983; Syer, Clarke & Rees 1991; Vil'kovskij & Czerny 2002), and tidal capture and tidal heating (Novikov et al. 1992; Alexander & Morris 2003, AM03). Much effort has been devoted to the study of tidal disruption, which plays an important role in the growth of low-mass MBHs (Murphy, Cohn & Durisen 1991; Freitag & Benz 2002) and can provide a signature for the existence of MBH in galactic nuclei by the emission of tidal flares.

Dynamical analyses indicate that most of the stars scattered into radial orbits originate at the MBH radius of influence, \( r_h \), where the enclosed stellar mass roughly equals the MBH mass \( m \) and the scattering is roughly isotropic. Event horizon crossing or tidal disruption is prompt; the stars plunge toward the MBH with a cross-section that scales as \( \sim r_p \) (Hills 1975) and reach it in less than the initial orbital period, \( P_h \), irrespective of orbital periapse, as long as \( r_p < q \). Thus, the star is destroyed in a short time, e.g. \( P_0(r_h) \sim 10^5 \) yr in the Galactic Center (GC).

In contrast, the other processes listed above proceed gradually over an inspiral time \( t_0 \gg P_0 \), as a small fraction of the orbital energy is dissipated every peri-passage. The inspiral time typically rises steeply with the periapse. In order for the extracted orbital energy to heat the disk, tidally heat the star or power a high luminosity of gravity waves, the star has first to decay into a short period orbit.

Novikov et al. (1992) estimate that tidal capture by a MBH occurs for orbits with \( r_p/r_t < b_0 \sim 3 \). It then follows that stars are scattered into tidal capture orbits at a rate \( b_0 \sim 1 \)–2 times faster than that for prompt tidal disruption orbits. The orbital energy the star has to lose to circularize far exceeds its own binding energy, so it is likely that it will ultimately be disrupted (Rees 1988; AM03). This has led several authors (Frank & Rees 1976; Novikov et al. 1992; Magorrian & Tremaine 1999) to suggest that slow tidal inspiral may be at least as important as prompt disruption for feeding the MBH and for producing tidal flares. Simulations (Murphy et al. 1991; Freitag & Benz 2002) indicate that prompt tidal disruptions supply between \( \sim 0.15 \) to 0.65 of the total mass of a low mass MBH (\( m \lesssim 10^7 M_\odot \)) in a low-density galactic nuclear core. If the contribution of inspiraling stars were indeed as high as implied by the ratio of the cross-sections, this would have far-reaching implications: stars could supply most or even all of the MBH mass, thereby establishing a direct link between \( m \) and stellar dynamics on a scale of \( r_h \).

However, a small initial periapse does not in itself guarantee ultimate disruption. The star must also have enough time to complete its orbital decay. In this Letter we revisit the questions: what is the time available for orbital decay, and what is the inspiral event rate?

2. CALCULATIONS

2.1. Scattering into inspiraling orbits

We follow the analysis of Miralda-Escudé & Gould (2000, MG00) of the infall of a single mass population of stellar BHs into a MBH by diffusion into the loss-cone due to 2-body encounters in a Keplerian potential. Stars scattered from a volume \( dV \) to a periapse \( r_p \) will spiral into the MBH in a time \( t_0(a, r_p) \), determined by the dissipation process. The scattering rate per volume of
stars into orbits with periapse \(< r_p \), d\( \Gamma(\leq r_p)/dV \), can be estimated for the steady state distribution function (DF) (MG00 Eq. 28). The total rate, \( \Gamma(\leq r_p) = \int dV \, d\Gamma(\leq r_p)/dV \), is obtained by integrating over the volume between an inner boundary \( r_0 \) where the stellar cusp is truncated (e.g. by stellar collisions) to an outer boundary \( r_c \). The outer boundary is set by requiring that \( t_0 \leq t_p \), where \( t_p(a, r_p) = r_p/(dr_p/dt) \) is the orbit-averaged time for a change of order unity in the periapse by diffusion in velocity space due to many small angle deflections.

Typically, \( t_0 \) rises with a while \( t_p \) falls (Eq. 2), and so there is a critical semi-major axis \( a_c(r_p) \) such that \( t_0(a_c, r_p)/t_p(a_c, r_p) = 1 \). Stars originating from an orbit with \( a_0 > a_c \) do not have time, on average, to complete the inspiral. This simple condition is actually too restrictive because \( a(t) \) shrinks with time. Since stars are scattered off the orbit at a rate of \( \sim t_p^{-1} \), the Poisson probability for avoiding this is \( w = \exp(-s) \), where

\[
s(a_c, r_p) = \int_0^{t_0(a_c, r_p)} dt/t_p[a(t), r_p] .
\]  

The critical semi-major axis is obtained by solving \( s = 1 \) for \( a_c(r_p) \). Formally, \( \Gamma(\leq r_p) \) should be evaluated by the weighted integral \( dV \, w \, d\Gamma(\leq r_p)/dV \) over all space. However \( w \) falls off exponentially, and so \( \Gamma(\leq r_p) \) is well approximated by taking \( w = 1 \) and \( r_c/(r_p) = 3a_c/2 \), the time-averaged radius on an \( e \to 1 \) Keplerian orbit.

We assume here a simple power-law stellar DF, \( n_r \propto r^{-\alpha} \). In the limit \( e \to 1 \) (MG00 Eqs. 15–18, 21)

\[
t_p(a) = A_\alpha \left( \frac{m}{M_*} \right)^2 \frac{P(a)}{N_h \log A_1} \left( \frac{r_p}{a} \right)^{3-\alpha} ,
\]  

where \( N_h \) is the number of stars within \( r_h \), \( A_1 = \Lambda(r_p/r_c)^{1/4} \), \( \Lambda = m/M \) and

\[
A_\alpha = \frac{15}{2^\alpha (3-\alpha)(22-5\alpha)} .
\]

Note that \( r_p/a = 1 - e = \theta^2 \), where \( \theta \) is the opening angle of the loss cone at \( r = a \), so Eq. (2) is similar to the estimate \( t_{\text{scatter}}(a) \theta^2 \) of Sigurdsson & Rees (1997).

The volume contributing to \( \Gamma(\leq r_p) \) includes points at distance \( r \) far from the MBH, where \( t_0 > t_p \) for an orbit with periapse \( r_p \), but where \( t_0 \leq t_p \) is possible for \( r^* < r_p \) due to higher dissipation at smaller periapse. We generalize \( \Gamma(\leq r_p) \) to account for this by solving \( s(r, r^*_p) = 1 \) (Eq. 1) for \( r^*_p(r) \), and setting \( r_m(r) = \min[r_p, r^*_p(r)] \). The scattering rate is then (MG00 Eqs. 15–18, 28),

\[
\Gamma_m(\leq r_p) = \frac{M_*^2 N_h^2}{P_h} \int_{r^*_p}^{r_p} \left[ \frac{\log \Lambda}{\log(r/r_m)} - \frac{1}{4} \right] \left( \frac{r}{r_h} \right)^\gamma dr/r_h ,
\]

where \( \gamma = 7/2 - 2\alpha \), \( P_h = P(r_p) \), \( r_q = r_c(q) \) is the distance for prompt infall, and

\[
B_\alpha \equiv \frac{4}{15} \left( \frac{\pi}{2} (3-\alpha)^2 (10\alpha - 1) - \frac{1}{4} \right) (\alpha - 1/2)! \quad (\alpha > 1/2).
\]

Thus, the rate of successful inspiral events is

\[
\Gamma_\alpha(\leq r_p) = \Gamma_m(\leq r_p) - \Gamma_m(\leq q).
\]  

The rate for prompt infall, \( \Gamma_p(\leq r_p) \), is that at which stars are deflected into orbits with periapse \( r_p \) and reach \( r < r_p \) at least once, but do not necessarily finish the inspiral. \( \Gamma_p(\leq r_p) \) thus includes horizon crossing, tidal disruption, inspiral and tidal scattering. The prompt infall time \( t_0 \approx \tilde{t}_0 \) does not depend on \( r_p \) and so \( r_m = r_p \) at all \( r \) for \( \alpha < 3 \); Eqs. 2, 15). The rate \( \Gamma_p(\leq r_p) = \Gamma_m(\leq r_p) \) is then

\[
\Gamma_p(\leq r_p) = \frac{B_\alpha}{\Gamma_p(\leq r_p)} \left[ N_h^2 / P_h \right] \times \left\{ \log \Lambda (r_p/r_h)^{1+\gamma} \left[ (1+\gamma) \log \frac{r}{r_p} - \frac{(r/r_h)^{1+\gamma}}{4(1+\gamma)} \right] \right\} ,
\]

where \( E(x) = \int_{-\infty}^x dt e^{-t}/t \) is the exponential integral.

2.2. Inspiral and infall timescales

Tidal heating.—We apply these results to “Hot Squeezers” (HS, AM03), tidally heated stars that dissipate the heat on the surface. We denote by a tilde dimensionless quantities in units of \( G = M_* = R_* = 1 \) (\( M_* \), \( R_* \) the initial stellar mass and radius). In these units \( \tilde{r}_\gamma \approx \tilde{R}(\tilde{t}) \tilde{m}^{1/3} \). The HS inspiral time in terms of \( t \equiv \tilde{r}_*/\tilde{r}_h(0) \) is

\[
\tilde{t}_0 = \frac{(2\pi)^2}{m^{-2/3} b^{2/3} \tilde{P}_h^{1/3} / T^{(b/3)^2}} (\tilde{m} \gg 1) ,
\]

where \( T \) is the tidal coupling coefficient. Two models are used to estimate the tidal heating: normal mode expansion (Press & Teukolsky 1977) and the affine stellar model (Carter & Luminet 1985). For the former, \( T \) is the leading multipole term, which is evaluated numerically for a solar model (Alexander & Kumar 2001). For the latter, we use the analytic approximation of Novikov et al. (1992).

The tidal energy deposited in the star per peri-passage is \( \Delta \tilde{E} = T/b^6 = \text{const} \) (AM03). The orbital evolution is (Eqs. 1–2, AM03 Eq. 4)

\[
\tilde{\tilde{a}}(t) = \tilde{a}_0 \left( 1 - t/\tilde{t}_0 \right)^2 ,
\]

\[
\tilde{\tilde{t}}_p(\tilde{a}(t)) = \tilde{t}_p(\tilde{a}_0)(1 - t/\tilde{t}_0)^{2\alpha-5} ,
\]

\[
s = \tilde{t}_0 / 2(3-\alpha) \tilde{t}_p(\tilde{a}_0) .
\]

The critical semi-major axis is

\[
\tilde{\tilde{a}}_c = \frac{2(3-\alpha) \tilde{A}_0 \tilde{T}^{(b/3)} \tilde{m}^{1/3}}{\log \tilde{A}_1 \tilde{b}^{2/3} \tilde{N}_h} \tilde{\tilde{r}} ,
\]

“Cold Squeezers” (CS, AM03) dissipate the tidal energy in their bulk and expand at constant effective temperature. The CS orbital evolution is calculated below numerically.

Gravitational waves.—The time for inspiral by GW emission \( (b > q/r_h) \) is (Peters 1964)

\[
\tilde{t}_0 \approx \frac{4 \sqrt{25}}{85(2\pi)^{3/4} \tilde{b}^{7/2} \tilde{P}_h^{1/3}} \left( e \to 1, \tilde{m} \gg 1 \right) ,
\]

where \( \tilde{e} \) is the speed of light. Inspiring GW emitters, like HS, follow \( da/dt \propto \sqrt{\tilde{a}} \), so Eqs. (9)–(11) apply and

\[
\tilde{\tilde{a}}_c = \frac{85(3-\alpha) \tilde{A}_0 \tilde{m}^{8/3}}{6 \sqrt{25} \log \tilde{A}_1 \tilde{b}^{2/3} \tilde{N}_h} \tilde{\tilde{r}} .
\]

Prompt infall.—The mean infall time is \( \tilde{t}_0 = \tilde{P}_h/4 \), and

\[
\tilde{\tilde{a}}_c = \frac{4 \tilde{A}_0 \tilde{m}^{7/3}}{\log \tilde{A}_1 \tilde{N}_h \tilde{r}_h} \tilde{\tilde{r}} .
\]
3. RESULTS

We apply these results to the GC by modeling it as a power-law DF with \( m = 2.6 \times 10^6 M_\odot \), \( r_h = 1.8 \) pc and \( \alpha = 1.8 \), based on the empirically derived mass model of Schodel et al. (2002). We assume a single mass population and \( N_h = \bar{m} \). We consider 3 simple cases. (1) 1 \( M_\odot \) stars, a high mean mass motivated by theoretical arguments and observational evidence for a “top heavy” initial mass function in the GC (e.g. Morris 1993; Figer et al. 1999). This model is used to test tidal inspiral and tidal scattering. (2) 0.1 \( M_\odot \) main-sequence (MS) stars, which are the most resilient against tidal disruption (Freitag & Benz 2002). This model is used to estimate GW inspiral from MS stars. (3) 0.6 \( M_\odot \) stars. Of these, 10% are white dwarfs (WD) with \( R_s = 0.01 R_\odot \), as is expected in old, bulge-like stellar populations. The rest are MS stars, whose tidal disruption radius is too large for efficient GW emission. This model is used to estimate the rate of GW inspiral by WDs, for comparison with previous works.

3.1. Survival probability of tidally scattered stars

Tidal inspiral is complementary to tidal scattering (Alexander & Livio 2001), where stars narrowly escape tidal disruption by being scattered to wider orbits, after suffering extreme tidal distortion, spin-up, mixing and mass-loss that may affect their evolution and appearance. Such stars eventually comprise a few percent of the population within \( r_h \). We now show that their probability of survival is \( \sim 1 \) by comparing the inspiral and prompt infall rates. Table 1 lists the tidal inspiral rate \( \Gamma_i(<b_{0i}r_i) = \rho_i \) for 1 \( M_\odot \) HSs and CSs with the tidal heating estimated using either normal mode expansion or the affine model (counting numerically for the fact that \( \bar{f}(\bar{R} = b) < t_0 \) for small \( b \)). The stellar cusp is truncated at \( r_0 \sim 0.02 \) pc, the radius where stellar collisions destroy MS stars (Alexander 1999). The prompt disruption rate for this GC model is \( \Gamma_p(<r_i) \sim 9 \times 10^{-5} \) yr\(^{-1} \), consistent with previous estimates, \( \Gamma_p(<r_i) = 5 \times 10^{-5} \) yr\(^{-1} \) (Syer & Ulmer 1999) and \( \Gamma_p(<r_i) \sim \text{few} \times 10^{-5} \) yr\(^{-1} \) (Alexander 1999). However, the squeezar inspiral (tidal capture) event rate is only \( \sim 0.05 \) of the prompt disruption rate, and not \( \lesssim 2 \) times larger, as naively implied by the ratio of the cross-sections.

The probability of successful inspiral for stars with periapse between \( r_i \) and \( r_p \) is

\[
P_i(<r_p) = \frac{\Gamma_i(<r_p)}{\Gamma_p(<r_p) - \Gamma_p(<r_i)},
\]

while the tidal scattering survival probability is \( P_s = 1 - P_i \).

Our squezzar models have \( P_s(<r_i) < 0.8 \) to \( P_s(<b_{0i}r_i) < 0.9 \). We thus confirm that \( P_s < 1 \), as was anticipated from general arguments (Alexander & Livio 2001). The MBH’s Brownian motion, neglected here, may further increase \( P_s \) for loosely bound \( (b > b_0) \) tidally scattered stars.

A tidal scattering event is deemed “strong” if \( r_p \) is within some adopted limit. Since \( P_s \sim 1 \), the tidal scattering and prompt disruption rates are related. The diffusive cross-section, modeled here, roughly scales as \( r_p^3 \), where \( \delta \sim (9 - 4\alpha)/(8 - 2\alpha) = 0.4 \) for \( \alpha = 1.8 \) (Eqs. 4, 15). Unbound stars with isotropic velocities have \( \delta = 1 \) (Hills 1975). Since most tidally scattered stars originate between the diffusive and the full loss cone (isotropic) regimes, where \( E \sim 0 \) (Lightman & Shapiro 1977), realistically \( \delta \sim 0.4 - 1 \).

| Process | \( r_0 \) | \( \Gamma_i \) | \( n \) | \( b_0 \) | \( L_i \) | \( \bar{P}_i \) |
|---------|---------|-----------|-----|-----|-----|-----|
| HS (S)  | 2(–2)  | 3(–6)    | 0.2 | 2.1 | 170 L_\odot | 4(3) |
| HS (A)  | 2(–2)  | 5(–6)    | 0.4 | 3.6 | 150 L_\odot | 6(3) |
| CS (S)  | 2(–2)  | 4(–6)    | 0.2 | 2.8 | 200 L_\odot | 5(3) |
| CS (A)  | 2(–2)  | 7(–6)    | 0.4 | 4.4 | 170 L_\odot | 7(3) |

MS GW 6(–3) 2(–7) 0.2 7.5 20(35) erg/s 1(1) 1(1)

WD GW 4(–4) 2(–7) 0.04 25.7 20(36) erg/s 1(2) 1(2)

Table 1: Inspiral in the Galactic Center

3.2. Inspiral in the Galactic Center

The observable implications of having on average \( \bar{\tau} \) inspiraling stars near the MBH can be estimated by considering the properties of the leading (shortest period) star. The mean number is \( \bar{m}(<b_{0i}r_i) \equiv \int r_i^{b_{0i}} \Gamma_p(t_0) (d\Gamma_p/dr_p) \), where \( (t_0) \sim (2 + \gamma) t_0 (r_p, r_i)/(3 + \gamma) \) is the r-averaged value of \( t_0 \) (\( t_0 \propto \sqrt{\alpha} \) and Eq. 4). The mean inspiral time \( t_i \equiv \Gamma_i(<b_{0i}r_i) / \Gamma_i(<b_{0i}r_i) \) and the corresponding averaged \( \sqrt{\alpha} \), also define \( \bar{\tau}_0 \) and \( \bar{P}_0 \) for “typical” HSs or GW emitters. For simplicity, we adopt this estimate also for CSs, although their \( t_0(a_0) \) is different. The leading star completes on average \( \bar{f}_1(t_0(\bar{\tau}_0) = \bar{\tau}(\bar{\tau} + 1) \) of its inspiral time in \( N = \bar{m}(\bar{\tau} + 2) t_0(\bar{\tau} + 2) \) orbits (AM03). Since \( \bar{f}_1 = \bar{m}/(\bar{\tau} + 1)^2 \) and \( \bar{P}_1 = \bar{P}_0(\bar{\tau} + 1)^3 \), the total extracted orbital energy is \( \Delta E_1 = (\bar{m}/2\bar{m}_0)(\bar{\tau} + 2) \) and the total luminosity is \( \bar{L}_i(\bar{f}_1) \sim \Delta E_1/N \bar{P}_1 = (\bar{m}/a_0 t_0)(\bar{\tau} + 1)^3 \). Typically, \( \bar{L}_i \gg \bar{L}_s \) (the intrinsic stellar luminosity) even for small \( \bar{\tau} \).

Table 1 lists \( \bar{\tau}, \bar{L}_i, \bar{P}_1 \) for tidal and GW inspiral. We find that the GC contains on average \( \sim 0.1 - 1 \), squezzars, and that on average the leading squezzar is \( \sim 5 \) brighter than its normal bolometric magnitude. The rate of WD inspiral derived here, \( \Gamma_i \sim 2 \times 10^{-7} \) yr\(^{-1} \), agrees with the estimates of Sigurdsson & Rees (1997) and Freitag (2003), but the rate of GW inspiral by MS stars, \( \Gamma_i \sim 2 \times 10^{-7} \) yr\(^{-1} \), is less than 0.1 of that estimated by Freitag (2003). The source of this discrepancy is unclear.

4. DISCUSSION AND SUMMARY

Different dissipation mechanisms may lead to the same outcome: orbital decay around a MBH in the presence of two-body perturbations. In this Letter we derive general expressions for estimating the inspiral event rate for any given inspiral time \( t_i(a_0, r_p) \) and orbital evolution \( a(t) \). Inspiral is a race between orbital energy extraction and two-body scattering. The probability that a star will reach the final, observationally interesting stage of a short period orbit is small unless it starts out on a tight enough orbit. Since there are fewer stars close to the MBH, inspiral events are much rarer than prompt disruption events.

We applied these results to the GC, using simple single-mass models to represent the stellar cluster. These reproduce the prompt disruption and the WD inspiral event
rates that were independently estimated by previous studies. We find that (1) The survival probability of tidally scattered stars is \(0.8 - 0.9\). (2) The rate of tidal scattering scales with the prompt tidal disruption rate as \(\Gamma_p(<r_t)/(r_p/r_t)^\delta \sim 0.4 - 1\) with \(\delta \sim 0.4 - 1\). (3) The contribution of slow tidal inspiral in the GC to the total tidal disruption rate is only \(\sim 5\%\), and not 100–200\% as proposed by previous studies. (4) The GC contains on average \(\sim 0.1-1\) squersars at any given time, with a tidal bolometric luminosity \(\gtrsim 150L_\star\) on \(\sim 5 \times 10^3\) yr orbits. (5) There are on average \(\sim 0.1\) GW emitters in the GC at any given time.

The uncertainties in these estimates can be addressed by more detailed modeling of the stellar cluster, which should include a realistic, multi-mass stellar DF and take into account mass segregation. Other dynamical processes not considered here, such as resonant scattering (Rauch & Tremaine 1996), deviations from spherical symmetry (Magorrian & Tremaine 1999), chaotic orbits in triaxial systems (Poon & Merritt 2002), or the effects of massive perturbers (Zhao et al. 2002) may increase the inspiral event rates. The MBH’s Brownian motion will not have a large effect, as typical inspiral orbits originate well within \(r_h\), where the stars follow the MBH (Reid et al. 2003).

We thank M. Freitag and M. Morris for helpful discussions. TA is supported by ISF grant 295/02-1, Minerva grant 8484, and a grant by Sir H. Djangoly, CBE, of London, UK.

REFERENCES

Alexander, T. 1999, ApJ, 520, 137
Alexander, T., & Kumar, P. 2001, ApJ, 549, 948
Alexander, T., & Livio, M. 2001, ApJ, 560, L143
Alexander, T., & Morris, M. 2003, ApJL, in press (AM03)
Carter, B., & Luminet, J.-P. 1985, MNRAS, 212, 23
Figer, D. F., Kim, S. S., Morris, M., Serabyn, E., Rich, M., & McLean, I. S. 1999, ApJ, 525, 750
Frank, J. 1978, MNRAS, 184, 87
Frank, J. & Rees, M. J. 1976, MNRAS, 176, 633
Freitag, M. 2001, Class. Quant. Grav., 18, 4033
Freitag, M. 2002, ApJL, 583, L21
Freitag, M., & Benz, W. 2002, A&A, 394, 345
Gebhardt, K., et al. 2003, ApJ, 583, 92
Hills, J. G., 1975, Nat, 254, 295
Hills, D., & Bender, P. L., 1997, ApJ, 445, L7
Lightman, A. P., & Shapiro, S. L. 1977, ApJ, 211, 244
Magorrian, J, & Tremaine, S. 1999, MNRAS, 309, 447
Miralda-Escudé, J., & Gould, A. 2000, ApJ, 545, 847 (MG00)
Morris, M., 1993, ApJ, 408, 496
Murphy, B. W., Cohn, H. N., & Durisen, R. H., 1991, ApJ, 370, 60
Novikov, I. D., Petchik, C. J., & Polnarev, A. G. 1992, MNRAS, 255, 276
Ostriker, J. P. 1983, ApJ, 273, 99
Peters, P. C., 1964, Phys. Rev., 136, 1224
Poon, M. Y. & Merritt, D. 2002, (astro-ph/0212581)
Press, W. H., & Teukolsky, S. A., 1977, ApJ, 213, 183
Rauch, K. P., & Tremaine, S. 1996, New Astron., 1, 149
Rees, M. J. 1988, Nature, 333, 523
Reid M. J., Menten, K. M., Genzel, R., Ott, T., Schödel, R., & Eckart, A. 2003, ApJ, 587, 208
Schödel, R., et al. 2002, Nature, 419, 694
Sigurdsson, S., & Rees, M. J. 1997,MNRAS, 284, 318
Syer, D., Clarke, C. J., & Rees, M. J. 1991, MNRAS, 250, 505
Syer, D., Ulmer, A. 1999, MNRAS, 306, 35
Vilkoviskij, E. Y., & Czerny, B., 2002, A&A, 387, 804
Zhao, H.-S., Haehnelt, M. G., & Rees, M. J. 2002, New Astron., 7, 385