QCD color interactions between two quarks

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Abstract
We study the QCD color interactions between static two heavy quarks at zero temperature in a quenched $SU(3)$ lattice gauge simulation: in addition to the standard singlet $q\bar{q}$ potentials, we calculate octet $q\bar{q}$ potentials, symmetric and antisymmetric $qq$ potentials. It is shown that the antisymmetric $qq$ channel behaves as a linearly rising potential at large quark separations. We further find that the $q\bar{q}$ octet and $qq$ symmetric channels have the complex dependence on the distance; at short distances they are repulsive forces, while at large distances, they show linearly rising feature. Ratio of string tensions between $q\bar{q}$ singlet and $qq$ antisymmetric potentials is described in terms of the Casimir factor.
I. INTRODUCTION

Nonperturbative study of color QCD forces between static quarks is important for understanding quark confinement and hadron phenomenology. The behavior of a heavy-quark potential that is defined by the closed gauge-invariant Wilson loop, has been studied extensively in lattice simulations \[1\]; the heavy-quark potential is a linearly rising potential, which can be explained with the Coulomb term and the linear term with the string tension. In most previous studies of heavy-quark potentials, a color singlet channel, which yields a physical potential, has been investigated. There are, however, several other color channels between two quarks. According to the \[SU(3)\] color decomposition for a quark-antiquark sector \(q\bar{q}, 3 \otimes \bar{3} = 1 \oplus 8\). The octet channel is significant for the extensive analysis of \(J/\psi\) photoproduction \[2, 3\]. Furthermore, we have \(3 \otimes 3 = \bar{3} \oplus 6\) for a quark-quark sector \(qq\), and in particular, the antisymmetric \(qq\) channel plays an essential role in the phenomenology of penta-quark hadrons \[4, 5, 6\], in which the existence of a highly correlated diquark is assumed \[5\]. Since the usual Wilson loop cannot give the color-decomposed potentials separately, the lattice study along this line was considered as very difficult.

In order to understand quark-gluon plasma (QGP) physics \[7\], the finite-temperature behavior of the QCD color forces between two static heavy quarks is studied in lattice simulations. At finite temperature, one expects that the linearly confining color force is weakened due to the screening effect in QGP. Quarks confined in hadrons are expected to move freely in the deconfinement phase after the QGP phase transition. This can be understood in the nonperturbative behavior of the screened heavy-quark potentials defined by a Polyakov line correlator, the form of which is reduced to a Yukawa-type potential with screening masses rather than the linearly rising potential. We therefore may expect the mass shift and/or suppression of heavy quarkoniums as an indicator of the QGP phase transition \[8, 9\]. However, recent heavy-ion collision experiments and the lattice QCD simulations \[10, 11\] have indicated that this picture is too simple and QGP is a more complex system. Thus, in recent finite-temperature lattice simulations, the singlet and octet \(qq\) potentials and the symmetric and antisymmetric \(qq\) potentials have been investigated \[12, 13, 14, 15\]. This is a progress comparing with previous lattice QCD studies, where the screened heavy-quark potential had focused on only a color-averaged channel. On the other hand, the nonperturbative behavior of the \(SU(3)\) color-decomposed channels at zero temperature has
not been investigated.

Recently, using $SU(2)$ lattice gauge simulation, Greensite, Olejnik and Zwanziger studied the color confinement mechanism at zero temperature and found that the Coulomb singlet heavy-quark potential in Coulomb gauge represents a strong linearly rising potential [16, 17], and this was later verified in $SU(3)$ lattice gauge simulation [18]. The Coulomb singlet heavy-quark potential is calculated from partial-length Polyakov lines (PPL) in Coulomb gauge, not from the closed gauge-invariant Wilson loop. PPL is a Polyakov line with a restricted temporal extension [16, 17, 19]. Thus, the combinations of the PPL correlators may enable us to carry out lattice calculations in the color-decomposed channels at zero temperature.

In this letter, we study the behavior of the color-decomposed $q\bar{q}$ and $qq$ potentials at zero temperature in the quenched $SU(3)$ lattice gauge simulation. Applying the PPL correlator method [16, 17] to the color-decomposed potentials, we carry out the $SU(3)$ lattice calculation of the octet, symmetric and antisymmetric potentials. We will present the first nonperturbative study of the antisymmetric $qq$ potential at zero temperature.

II. PARTIAL-LENGTH POLYAKOV LINE

In this section, we give color decomposed correlators between two static heavy quarks and summarize how to fix the gauge on the lattice.

A partial-length Polyakov line (PPL) can be defined as [16, 17]

$$L(\vec{x}, T) = \prod_{t=1}^{T} U_0(\vec{x}, t), \quad T = 1, 2, \cdots, N_t.$$  (1)

Here $U_0(\vec{x}, t) = \exp(ia g A_0(\vec{x}, t))$ is a $SU(3)$ link variable in the temporal direction and $a, g, A_0(\vec{x}, t)$ and $N_t$ represent the lattice cutoff, the gauge coupling, the time component of gauge potential and the temporal-lattice size. A PPL correlator in the color-singlet channel is given by

$$G_1(R, T) = \frac{1}{3} \langle Tr[L(R, T)L^\dagger(0, T)] \rangle,$$  (2)

where $R$ stands for $|\vec{x}|$. From Eq. (2) one evaluate a color-singlet potential on the lattice,

$$V(R, T) = \log \left[ \frac{G_1(R, T)}{G_1(R, T + a)} \right].$$  (3)

For the smallest temporal lattice extension, i.e., $T = 0$, we define

$$V(R, 0) = - \log[G_1(R, 1)].$$  (4)
Greensite et al. argued that $V(R,0)$ in Coulomb gauge corresponds to a color-Coulomb potential [16, 17]. The $V(R,T)$ at $T \to \infty$ corresponds to a physical potential, which is usually calculated from the Wilson loops at $T \to \infty$. These two potentials are expected to satisfy Zwanziger's inequality, $V_{\text{phys}}(R) \leq V_{\text{coul}}(R)$ [21].

Now we extend this PPL correlator method to the other $SU(3)$ color-decomposed potentials [12]. A color-octet correlator for $q\bar{q}$ is given by

$$G_8(R,T) = \frac{1}{8} \langle TrL(R,T)TrL^\dagger(0,T) \rangle - \frac{1}{24} \langle TrL(R,T)L^\dagger(0,T) \rangle,$$

and $qq$ correlators in the symmetric and antisymmetric channels are described as

$$G_6(R,T) = \frac{3}{4} \langle TrL(R,T)TrL(0,T) \rangle + \frac{3}{4} \langle TrL(R,T)L(0,T) \rangle,$$

$$G_{\bar{3}}(R,T) = \frac{3}{2} \langle TrL(R,T)TrL(0,T) \rangle - \frac{3}{2} \langle TrL(R,T)L(0,T) \rangle.$$

In the same way as described in Eqs. (3) and (4), we will obtain the color-decomposed potentials in each color channel using the above correlators. The four potentials between two quarks are classified in terms of the Casimir factor on color $SU(3)$ group in the fundamental representation: $C_1^{q\bar{q}} = -\frac{4}{3}$ for a singlet, $C_8^{q\bar{q}} = \frac{1}{6}$ for an octet, $C_6^{qq} = \frac{1}{3}$ for symmetric, $C_{\bar{3}}^{qq} = -\frac{2}{3}$ for antisymmetric; i.e., the $q\bar{q}$ singlet and $qq$ antisymmetric channels yield an attractive force, whereas the $q\bar{q}$ octet and $qq$ symmetric channels give a repulsive force.

Since the color-decomposed potentials defined by the PPL correlators do not have gauge-invariant forms, we must fix the gauge. We use the Coulomb gauge realized on the lattice as

$$\text{Max} \sum_{\vec{x}} \sum_{i=1}^{3} \text{ReTr} U_i^\dagger(\vec{x},t),$$

by repeating the following gauge rotations:

$$U_i(\vec{x},t) \to U_i^{\omega}(\vec{x},t) = \omega^\dagger(\vec{x},t)U_i(\vec{x},t)\omega(\vec{x} + \hat{i},t),$$

where $\omega \in SU(3)$ is a gauge rotation matrix and $U_i(\vec{x},t)$ are spatial lattice link variables. Thus each lattice configuration can be gauge fixed iteratively [22].

The Coulomb gauge fixing does not fix a gauge completely, and one can still perform a time-dependent gauge rotation on the Coulomb-gauge fixed links,

$$U_i(\vec{x},t) \to \omega^\dagger(t)U_i(\vec{x},t)\omega(t),$$

$$U_0(\vec{x},t) \to \omega^\dagger(t)U_0(\vec{x},t)\omega(t + 1).$$
Thus, $TrLTrL^\dagger$ and $TrLL$ constructed by PPL, are not invariant under the transformation of Eq. (10). Therefore, when performing numerical simulations for the octet and two $qq$ correlators including $TrLTrL^\dagger$ and $TrLL$, we additionally implement a global temporal-gauge fixing on the Coulomb-gauge fixed configuration as

$$\text{Max} \frac{1}{V} \sum_{\vec{x},t} \text{Re}TrU_0^\dagger(\vec{x},t) \text{ under Eq. (10)},$$

where $V = N_xN_yN_z$ is a spatial lattice volume and $N_{x,y,z}$ is the spatial lattice size in each direction. Note that this gauge fixing does not affect intrinsic Coulomb gauge features.

III. SIMULATION RESULTS

We carry out $SU(3)$ lattice gauge simulations in quench approximation to calculate the color decomposed PPL correlators. The lattice configurations are generated by the heat-bath Monte Carlo technique with a plaquette Wilson gauge action, and to fix a gauge we adopt the iterative method [22].

Numerical results of the color decomposed potentials with $T = 2$ are displayed in Fig. 1. This simulation is done on the $24^3 \times 32$ lattice at $\beta = 5.90 \ (a \sim 0.124\text{fm})$ and the 200 configurations measured by every 100 steps are used here. The singlet $q\bar{q}$ and the antisymmetric $qq$ potentials rise linearly at large distances; i.e., they are an attractive confining potential. The octet $q\bar{q}$ and the symmetric $qq$ channels show repulsive forces at short distances, while at large distances they have the linearly rising attractive force rather than the large-distance repulsive force. This nonperturbative behavior in the adjoint channel is comparable with the numerical result in 3d $SU(2)$ gauge theory [23, 24].

In order to analyze the singlet $q\bar{q}$ and the antisymmetric $qq$ potentials, we employ the following fitting function,

$$V(R,T) = C + KR + A/R, \quad A = -\pi/12,$$

where $C$ is a constant and $K$ corresponds to the string tension. This fitting works well: approximately $\chi^2/ndf \sim O(1)$ for the fitting range $R=2-6$. The results are represented by solid lines in Fig. 1.

We obtain the string tension $K_3$ in the antisymmetric channel. The singlet-string tension $K_1$ approaches asymptotically the Wilson-loop string tension [16, 18], but the $T$ dependence
of $K_3$ is unknown. Here, we investigate the ratio of the string tensions obtained from the singlet $q\bar{q}$ and the antisymmetric $qq$ potentials; if the string tension itself is universal, then the difference between $K_1$ and $K_3$ can be understood in terms of the color $SU(3)$ Casimir factor; that is, $K_1/K_3 \to C^1_{qq}/C^3_{qq} = (-4/3)/(-2/3) = 2$. Those ratios calculated at $\beta = 5.85 - 6.00$ are summarized in Fig. 2. The dependence of the ratio on $\beta$ and $T$ is weak and they are consistent with the expected values. We conclude that the relation between the singlet $q\bar{q}$ and the antisymmetric $qq$ potentials is described in terms of the color $SU(3)$ Casimir in nonperturbative regions.

Since the repulsive channels have the complex dependence in the distance, we do not carry out a fitting analysis for them.

FIG. 1: Singlet and octet potentials for $q\bar{q}$ sector and symmetric and antisymmetric potentials for $qq$ sector. The solid curves represent the fitted results for the two attractive channels. The $\hat{R}$ with physical dimension is set as $\hat{R} = aR$ where the lattice cutoff $a$ is approximately 0.124 fm for this lattice simulation.

IV. SUMMARY

We have nonperturbatively studied the zero-temperature behavior of the color decomposed potentials between static two heavy quarks in the quenched $SU(3)$ lattice gauge simulation with the partial-length Polyakov line correlators.
We show that the $qq$ potential in the antisymmetric channel rises linearly at large distances and is fitted by the function that includes the Coulomb and the string-tension linear terms. Moreover, the string tensions obtained from $q\bar{q}$ singlet and $qq$ antisymmetric potentials seem to be understood in terms of the Casimir; in other words, the string tension itself is universal. As a result, not only the $q\bar{q}$ singlet channel but also the $qq$ antisymmetric channel is found to be strongly correlated in nonperturbative regions.

The octet $q\bar{q}$ and symmetric $qq$ potentials show the complex behavior as a function of the distance in contrast to the case of the attractive channels: the $q\bar{q}$ octet and $qq$ symmetric channels yield a repulsive force at short distances, while at large distances it seems to be reduced to a linear potential. The larger lattice simulation therefore may be required to deal with the long-range features in the repulsive channels.

In the case of color non-singlet channels, it is especially important to investigate the volume size effect, because the color non-singlet potentials may include divergent parts in the infinite volume limit.

Our numerical simulation has been concentrated on the two quarks system. With the PPL correlator we may deal with multiquark potentials on the lattice. The discovery of the penta-quark hadron [4] promotes the nonperturbative lattice research about the multiquark state [6, 25, 26, 27]. The $SU(3)$ lattice simulation of tetra- and penta-quark potentials also has been carried out [25, 26, 27]. The PPL correlator enables us to construct them without any assumption to make tentative forms of the multiquark potential, although the
gauge-fixing procedure is indispensable.

The color decomposed potentials at zero temperature are computed in the quenched lattice simulation, and especially the behavior of the antisymmetric $qq$ potential should be also investigated in a dynamical quark simulation. The PPL correlator (or the usual Polyakov line correlator) is applicable to the dynamical-quark lattice simulation.

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