Sudakov Electroweak effects in transversely polarized beams

Paolo Ciafaloni
INFN - Sezione di Lecce,
Via per Arnesano, I-73100 Lecce, Italy
E-mail: Paolo.Ciafaloni@le.infn.it

Denis Comelli
INFN - Sezione di Ferrara,
Via Paradiso 12, I-35131 Ferrara, Italy
E-mail: comelli@fe.infn.it

Antonio Vergine
INFN - Sezione di Ferrara,
Via Paradiso 12, I-35131 Ferrara, Italy
E-mail: vergine@fe.infn.it

Abstract
We study Standard Model electroweak radiative corrections for fully inclusive observables with polarized fermionic beams. Our calculations are relevant in view of the possibility for Next Generation Linear colliders of having transversely and/or longitudinally polarized beams. The case of initial transverse polarization is particularly interesting because of the interplay of infrared/collinear logarithms of different origins, related both to the nonabelian SU(2) and abelian U(1) sectors. The Standard model effects turn out to be in the 10% range at the TeV scale, therefore particularly relevant in order to disentangle possible New Physics effects.

1 Introduction
Energy-growing electroweak corrections in the Standard Model have received recently a lot of attention in the literature, being relevant for LHC physics [1], for Next generation of Linear Colliders (NLCs) [2] and for ultrahigh energy cosmic rays [3]. The presence of double logs (log M^2 s where M is the weak scale) in one loop electroweak corrections has been noticed in [4]. One loop effects are typically of the order of 10-20% at the energy scale of 1 TeV, so that the subject of higher orders and/or resummation of large logarithms has to be addressed. Since two of us [5] made the observation that double and single logs that appear in the 1 loop expressions are tied to the infrared structure of the theory, all order resummation has been considered at various levels: Leading Log (LL) [6], Next to Leading Log (NLL) [7] and so on. Moreover many fixed order analyses at the one [8] and two loop [9] level have been performed. Although logarithms of infrared-collinear origin also emerge in unbroken theories like QCD and QED, the presence of symmetry breaking in the electroweak sector produces large differences with respect to the unbroken gauge theories. For instance, large double logs of infrared origin are present even in fully inclusive observables, in contrast to what happens in QED and QCD [10]. This in turn implies that the hierarchy of log series is no longer the same as in QCD, and splitting functions of new kind have to be defined when studying the analogous of DGLAP equations for EW corrections [11].

In this paper we consider electro-weak corrections to inclusive observables in the case of a transverse polarized electron beam. We have in mind in particular the possibility of having transversely polarized electron beams at NLC [12]. We find that Standard Model corrections are big enough that one should take them into account when considering possible New Physics effect in transversely polarized beams [13]. The analysis of the electroweak IR singularities for the fermionic sector in presence of polarized initial states is interesting since the final result comes from the interplay of two distinct effects. On one hand, left fermions are free nonabelian charges and this produces uncanceled double logs in inclusive quantities; this phenomenon, related to the SU(2) sector, has been called “Bloch-Nordsieck violation” [10]. On the other hand, a transversely polarized fermion is a coherent superposition of left and right fermions, which have different gauge charges; this produces a different but related effect which is present also in a purely abelian U(1) theory [15]. The interplay between SU(2) and U(1) effects is analyzed in detail in section 3.

The paper is organized as follows: in next section we set up the basic formalism for longitudinally and/or transversely polarized initial beams; this formalism is borrowed largely from [14, 15, 16]. In section 3 we analyze the above cross section with the formalism of the overlap matrix which give us the rules [14] to resum in a straightforward way the double log IR series. In section 4 we analyze the phenomenological implications related to initial beams polarization in the process e^+e^- → q̅q + X at high energy.

2 Cross section with polarized initial beams
We consider the inclusive process e^+e^- → q̅q + X, where X includes γ, Z, W^± radiation and where we sum over final quark flavors (all the fermions are supposed massless). The initial electron momentum is along the positive z axis. The
initial beam is transversely polarized, the initial electron being polarized along the positive x axis and the positron in the opposite direction. θ and φ are the usual polar and azimuthal angles of the outgoing quark momentum in this frame. To describe the initial spin states in e+e− we choose the helicity states as basis which is convenient at high energies. We indicate the helicity amplitudes with \( M_{i_1 i_2} \), where \( i_1 (i_2) \) is the helicity of the initial electron (positron), while \( f_1 (f_2) \) corresponds to the final quark (antiquark).

Let us now consider the process \( e^+ e^- \to q \bar{q} + X \). We denote the longitudinal and transverse components of the \( e^- (e^+) \) polarizations by \( P_L (P_L) \) and \( P_T (P_T) \) and we assume that the two transverse polarization vectors are parallel up to a sign*. For instance for a “natural” transverse polarization, where the electron and positron polarizations are opposite, we have \( P_T = 1, P_T = -1 \). For unpolarized beams we have \( P_L = P_L = P_T = P_T = 0 \). Finally, if both beams are polarized left, we have \( P_L = P_L = 1 \). Then we obtain the averaged squared amplitude [14, 15]:

\[
|M|^2 = \sum_{f_1 f_2} \left\{ \frac{(1 - P_L)(1 - P_L)}{4} |M_{f_1 f_2}^{1, -1}|^2 + \frac{(1 + P_L)(1 + P_L)}{4} |M_{f_1 f_2}^{1, 1}|^2 + \frac{P_T P_T}{2} \text{Re} \{ e^{2 i \phi} M_{f_1 f_2}^{1, 1} M_{f_1 f_2}^{1, -1} \} \right\} \tag{1}
\]

### 3 Overlap formulation

Let us now rewrite eqn. (1) in the language of the overlap matrix, \( \hat{O} = SS^\dagger \), \( S \) being the S-matrix. We define \( \hat{O} \) (see fig. 1) as an operator with four isospin indices (see [14] for details). All the final phase space factors that are relevant for cross section calculations are included, so that:

\[
\hat{O}_{i_1 i_2}^{t_1 t_2} (s, \theta, \phi) = \sum_{t, Y} O^{(t, Y)}(\theta, \phi) \hat{P}_{i_1 i_2}^{(t)} \hat{P}_{i_1 i_2}^{(t)}
\]

Here \( \hat{P}^{(t)} \) are 4 indices operators in isospin space, whose expressions are given in our previous works [17] and [18], while \( O^{(t, Y)}(\theta, \phi) \) are the coefficients of the overlap with defined total t-channel isospin \( t \) and total t-channel hypercharge \( Y \). We can then rewrite eqn. (1) in the form

\[
\frac{d\sigma}{d\phi d\cos \theta} = \frac{N_c N_f}{256 \pi^2 s} \left\{ (1 - P_L)(1 - P_L)O^{(0, 0)}_{RR} + (1 + P_L)(1 + P_L)(O^{(0, 0)}_{LL} \pm O^{(1, 0)}_{LL}) + (1 \pm 1)P_T P_T O^{(2, 0)}_{LR} y_L^2 y_R^2 \cos 2\phi \right\} \tag{2}
\]

where \( N_f \) is the number of families, \( N_c \) the number of colors and the + (-) sign is relative to \( \sigma_{e^- e^+} (\sigma_{e^- e^-}) \) where we have explicitly written the cross section dependence on the initial beam polarization. By inserting the Standard Model weak couplings, we find the following tree level values of the various coefficients:

\[
O^{(t=0, Y=0)}_{LL} = \left( \frac{3}{16} g_{\tau}^4 + \frac{1}{2} g_{\tau}^4 y_L^2 \sum_f y_{f L}^2 \right) (1 + \cos \theta)^2 + \frac{1}{2} g_{\tau}^4 y_L^2 \sum_f y_{f R}^2 (1 - \cos \theta)^2 \tag{3a}
\]

*for a pure initial state \(|P_L|^2 + |P_T|^2 = 1\), while for a partially polarized beam \(|P_L|^2 + |P_T|^2 < 1\); see [14]
corrections to the differential cross section $d\sigma$ for various values of $\phi$ as a function of the c.m. energy; the $\phi = \pi/4$ case corresponds to the unpolarized cross section case, while the QCD line is relative to the one loop $\alpha_S(\sqrt{s})/\pi$ contribution.

$$O_{LL}^{(t=1,Y=0)} = \left( -\frac{1}{16} g^4 + \frac{1}{2} g^2 g_{tL}^2 \sum f \left( y_{eL}^2 + y_{fR}^2(1 - \cos \theta)^2 \right) \right) (1 + \cos \theta)^2 + \frac{1}{2} g^4 y_{tL}^2 \sum f y_{fR}^2 (1 - \cos \theta)^2$$

$$O_{LR}^{(t=1,Y=y_L-y_R)} = g^4 y_{eL} y_{fR} \sum f \left( y_{fR}^2 + y_{fL}^2 \right) \sin^2 \theta$$

$$O_{RR}^{(t=0,Y=0)} = g^4 y_{eL}^2 \sum f y_{fR}^2 (1 + \cos \theta)^2 + g^4 y_{fR}^2 \sum f y_{fL}^2 (1 - \cos \theta)^2$$

The evolution in double log approximation is given by the weights $e^{-L_W(t(t+1)+\tan^2 \theta_W Y^2)}$ where $L_W = \frac{2\mu}{16\pi \log^2 \frac{t}{M_W}}$ and where $t$ and $Y$ are respectively the total isospin and the total hypercharge in the t channel. The effect of all orders resummation is therefore given by the following substitutions [17]:

$$O^{(t,Y)}(\theta, \phi) \rightarrow e^{-L_W(t(t+1)+\tan^2 \theta_W Y^2)} O^{(t,Y)}(\theta, \phi)$$

so that the full improved leading double logs $e^+e^-$ cross section is:

$$\frac{d\sigma_{e^+e^-}}{d\cos \theta d\phi} = \frac{N_e N_f}{256\pi^2 s} \left\{ (1 - P_L)(1 - P_L)O^{(0,0)}_{RR} + (1 + P_L)(1 + P_L)O^{(0,0)}_{LL} + e^{-2L_W} O^{(1,0)}_{LL} \right\}$$

$$+ 2P_T P_T e^{-L_W} \left( \frac{1}{2} + \tan^2 \theta_W (y_{eL}^2 - y_{eR}^2) \right) O^{(1,0)}_{LL} (\frac{1}{2} y_{eL}^2 - y_{eR}^2) \cos 2\phi$$

This expression features an interesting effect in the mixed L-R channel $O_{LR}$ where both SU(2)($\sim \frac{3}{2} L_W$) and U(1)($\sim L_W \tan^2 \theta_W (y_{eL}^2 - y_{eR}^2)$) related effects are present. In fact this channel is characterized both by a nonzero t-channel isospin $T = \frac{1}{2}$ and a nonzero hypercharge $y_{eL}^2 - y_{eR}^2$. Overall, the analysis of the electroweak mass singularities for the fermionic sector in presence of polarized initial states turns out to be quite interesting, not only for the presence of IR singularities generated by the existence of free non abelian asymptotic states, but also for the fact that the asymptotic states can be a coherent superposition of different gauge eigenstates: in this case, a transverse polarized fermion is a coherent superposition of left and right fermionic gauge charges. Such a mixing phenomenon is quite common in the EW theory: in the bosonic sector the Z and the photon are a superposition of the $B_\mu$ and the $W_\mu^3$ gauge fields, giving rise to a peculiar pattern of EW double log corrections [19]. In the scalar sector the higgs and the longitudinals goldstone bosons are a superposition of the gauge doublets $\Phi$ and $\Phi^*$ (having opposite U(1) charges), and Block-Nordsieck violation is present even in abelian theories [18].

In figs. 2, 3, 4, various combinations of the resummed differential cross section are plotted in the case of transversely polarized beams ($P_L = P_T = 0, P_T = 1, P_T = 1$). Namely, in fig. 2 the percent corrections for the integrated cross section $\int_1^{10} d\cos \theta \frac{d\sigma}{d\cos \theta}$ are plotted at different values of the $\phi$ angle as function of the c.m. energy. In fig.3 corrections to the the differential cross section $\frac{d\sigma}{d\cos \theta}$ at one TeV is plotted for different values of the azimuthal angle.

Figure 2: Resummed double log corrections to the $\theta$-integrated cross section for transversely polarized beams and for various values of $\phi$ as a function of the c.m. energy; the $\phi = \pi/4$ case correspond to the unpolarized cross section case, while the QCD line is relative to the one loop $\alpha_S(\sqrt{s})/\pi$ contribution.
An interesting observable that singles out the transverse $\phi$ dependent piece of the cross section is the asymmetry

$$A(\cos \theta, s) = \frac{\int_0^{\pi/4} d\phi \sigma - \int_{\pi/4}^{\pi/2} d\phi \sigma}{\int_0^{\pi/2} d\phi \sigma}$$  \hspace{1cm} (6)$$

which results particularly sensitive to the EW IR corrections. A plot of this quantity is given in fig. 4.

Overall, we see that EW double log resummed corrections for transversely polarized initial beams are well into the 10% range, so that this kind of corrections have to be taken into account when considering possible New Physics effects [13].

The full approach to such a kind of phenomena requires the introduction of “anomalous” structure functions which doesn’t have a clear probabilistic interpretation. As it is easy to see in fig.1, the “diagonal” overlap matrix $\hat{O}_{LL}$ can be rephrased in terms of the convolution of structure functions for left handed fermions, the “mixed” one $\hat{O}_{LR}$ requires the introduction of a third kind of mixed structure function with two different legs (left and right). Clearly the naive classical probabilistic interpretation fails to explain such a structure being generated by the quantum phenomenon of the coherent superposition of the wave functions. The full set of evolution eqs of the SM taking into account such a phenomena will be given soon in [20].

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Figure 4: EW corrections as a function of the $\theta$ angle to the Asymmetry of eq[6] when the beam energy is 1 TeV.

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