Crossover from non-Fermi liquid to Fermi liquid behavior in heavy fermion systems

P. Schlottmann
Department of Physics, Florida State University, Tallahassee, FL 32306, USA
E-mail: schlottmann@physics.fsu.edu

Abstract. Many heavy fermion systems display a crossover from Fermi liquid to non-Fermi liquid behavior due to a nearby quantum critical point (QCP). Some of the features are universal and do not depend on the nature of the QCP and the tuning mechanism. A simple one-band model of heavy electrons is considered to describe the crossover. The nesting of the spherical Fermi surfaces of an electron and a hole pocket separated by a nesting vector $Q$ and the interaction between electrons gives rise to itinerant antiferromagnetism. The order can be gradually suppressed by mismatching the nesting and a QCP is obtained as $T_N$ tends to zero. We review our results on the specific heat, the quasi-particle linewidth, the electrical resistivity, the amplitudes of de Haas-van Alphen oscillations and the dynamical spin susceptibility.

1. Introduction
Landau’s Fermi liquid (FL) theory has been successful in describing the low energy properties of most normal metals. Numerous U, Ce and Yb based heavy fermion systems [1]-[3] display deviations from FL behavior, which manifest themselves as, e.g., a log($T$)-dependence in the specific heat over $T$, $C/T$, a singular behavior at low $T$ of the magnetic susceptibility, $\chi$, and a power-law dependence of the resistivity, $\rho$, with an exponent close to one. These deviations from FL are known as non-Fermi liquid (NFL) behavior. The breakdown of the FL can be tuned by alloying (chemical pressure), hydrostatic pressure or the magnetic field. In most cases the systems are close to the onset of antiferromagnetism (AF) and the NFL behavior is attributed to a quantum critical point (QCP) [4]-[14]. Some of the properties are quite universal and independent of the type of QCP. A model that is simple enough so that actual calculations can be performed can then provide insights even for more complex physical situations.

Recently we studied the pre-critical region of a heavy electron band with two parabolic pockets, one electron-like and the other hole-like, separated by a wave vector $Q$ using (i) a field-theoretical multiplicative renormalization group (RG) approach [9] and (ii) the Wilsonian RG that eliminate the fast degrees of freedom close to an ultraviolet cutoff and rewrite the Hamiltonian in terms of renormalized slow variables [12]. The interaction is the remaining repulsion between heavy quasiparticles after the heavy particles have been formed in the sense of a Fermi liquid and is assumed to be weak. The interaction between the electrons induces itinerant AF or charge density waves (CDW) due to the nesting of the Fermi surfaces of the two pockets. For perfect nesting (electron-hole symmetry) an arbitrarily small interaction is sufficient for a ground state with long-range order. The degree of nesting is controlled by the
mismatch parameter, \( \delta = \frac{1}{2}[k_{F1} - k_{F2}]v_F \) \([k_{F1} \text{ (} k_{F2} \text{)}\) is the Fermi momentum of the electron (hole) pocket]. In this way the ordering temperature can be tuned to zero, leading to a QCP.

In this paper we summarize our main results. In the paramagnetic phase the effective mass, \( m^* \), (specific heat over \( T \), \( C/T \)) and the magnetic susceptibility increase logarithmically as \( T \) is lowered and diverge at the critical point signaling the breakdown of the FL [9, 12]. There is a crossover from the \( -\ln(T) \) dependence of \( C/T \) to constant \( \gamma \) as \( T \) is lowered if the QCP is not perfectly tuned, in agreement with experiments on numerous systems. The quasi-particle linewidth shows a crossover from NFL \((\sim T)\) to FL \((\sim T^2)\) behavior with increasing nesting mismatch and decreasing temperature [15]. The electrical resistivity [16], the dynamical susceptibility [17] and the amplitudes of the de Haas-van Alphen oscillations [18] have also been studied.

The response function to superconductivity (of the FFLO type with one quasi-particle in each pocket) diverges as \( T_N \) is approached [19], but the dominating correlations are AF. NFL behavior, AF order and superconductivity in the neighborhood of a QCP have been observed in CePd\(_2\)Si\(_2\) and CeIn\(_3\) under pressure [20]. We have also investigated the renormalization of the electron-phonon coupling, the softening of the phonon with wave vector \( Q \) and the consequences of this softening on the thermal expansion [21].

2. Two-pocket model

A strong interaction between electrons gives rise to heavy fermion bands. In the spirit of the FL theory, there are weak remaining interactions between the heavy quasi-particles left after the heavy particles are formed. The heavy electron band is described by two pockets, one electron-like and the other one hole-like, separated by a wavevector \( Q \) [9, 12]

\[
H_0 = \sum_{k\sigma} \left[ \epsilon_1(k) \, c_{1k\sigma}^\dagger c_{1k\sigma} + \epsilon_2(k) \, c_{2k\sigma}^\dagger c_{2k\sigma} \right],
\]

where \( k \) is measured from the center of each pocket, and assumed to be small compared to the nesting vector \( Q \). Here \( \epsilon_1(k) = v_F(k - k_{F1}) \) and \( \epsilon_2(k) = v_F(k_{F2} - k) \), and for simplicity we assume that the Fermi velocity is the same for both pockets.

The weak remaining interactions between quasi-particles are given by [9, 12]

\[
H_{12} = V \sum_{kk'q_\sigma q_{\sigma'}} c_{1k+q_\sigma c_{1k\sigma}}^\dagger c_{2k'-q_{\sigma'} c_{2k'\sigma'}} + U \sum_{kk'q_\sigma q_{\sigma'}} c_{1k+q_\sigma c_{1k\sigma}}^\dagger c_{2k'-q_{\sigma'} c_{2k'\sigma'}},
\]

where \( V \) and \( U \) represent the interaction strength for small \((|q| \ll |Q|)\) and large \((\text{of the order of } Q)\) momentum transfer between the pockets, respectively. The limit of the Hubbard model is obtained by choosing \( V = U \). Here interactions between electrons in the same pocket are neglected.

The leading order corrections to the vertex are the bubble diagrams with antiparallel propagator lines (zero-sound type), which are logarithmic in the external energy \( \omega \). Assuming that \( \omega \) is small compared to the cutoff energy \( D \), and that the density of states for electrons and holes is constant, \( \rho_F \), we have

\[
\tilde{V} = V/(1 - \rho_F V \xi), \quad 2\bar{U} - \tilde{V} = (2U - V)/[1 + \rho_F (2U - V) \xi],
\]

where \( \xi = \ln[D/(|\omega| + 2T + \delta)] \) [12]. A divergent vertex indicates strong coupling and signals an instability [9, 12].

Within the logarithmic approximation the linear response to a staggered magnetic field, \( \chi_S(Q, \omega) \), and to a CDW, \( \chi_c(Q, \omega) \), are given by [9]

\[
\chi_S(Q, \omega) = 2\xi \rho_F \tilde{V}/V, \quad \chi_c(Q, \omega) = 2\xi \rho_F (2\bar{U} - \tilde{V})/(2U - V),
\]
which are closely related to the vertices in Eq. (3). Hence, if \( V > 0 \) a spin density wave is possible with a Néel temperature \( T_N = \frac{1}{2} D \exp[-(\rho_F V)^{-1}] - \frac{1}{2} \delta \), and if \( 2U < V \) a CDW can be formed at \( T_c = \frac{1}{2} D \exp\{-[\rho_F(V - 2U)]^{-1}\} - \frac{1}{2} \delta \). The condition for a QCP is \( T_N = 0 \) or \( T_c = 0 \), and if \( T_N < 0 \) and \( T_c < 0 \) long range order has not developed. Thus, for sufficiently large Fermi surface mismatch the renormalization does not lead to an instability [12]. The QCP is an unstable fixed point and can only be reached by perfectly tuning the system [9].

In the disordered (paramagnetic) phase the \( \gamma \)-coefficient of the specific heat is given by the effective thermal mass [9, 12]

\[
m'(T)/m = \gamma/\gamma_0 = 1 + \frac{\xi \rho_F^2}{4T^2} \left[ 3V\tilde{V} + (2U - V)(2\tilde{U} - \tilde{V}) \right],
\]

(5)

where \( \gamma_0 \) refers to the non-interacting system. Here we kept only the leading logarithmic contributions, and \( \xi \) is to be taken with \( \omega = 0 \).

The \( T \)-dependence of \( C/T \) as a function of \( \ln(T) \) is shown in Fig. 1. Here \( \delta_0 = 0.07 \) corresponds approximately to the critical mismatch (the exact critical mismatch is a little smaller). For the perfectly tuned QCP, \( C/T \) increases logarithmically as \( T \) is lowered and diverges at the critical point signaling the break-down of the Fermi liquid [9, 12]. If \( \delta > \delta_0 \) there is a crossover from the logarithmic dependence (NFL) to a constant \( C/T \) (FL) as \( T \) is lowered [15]. The crossover temperature depends on the \( \delta - \delta_0 \) and is discussed below.

![Figure 1. Enhancement of the thermal mass as a function of \( \ln(T) \) for \( V\rho_F = U\rho_F = 0.2 \), \( D = 10 \), and several mismatch parameters \( \delta \). \( \delta_0 \approx 0.07 \) is approximately the critical mismatch. Note the crossover from NFL to FL for \( \delta > \delta_0 \) as \( T \) is lowered [15].

3. Quasi-particle linewidth

In an FL the damping of the quasi-particles is proportional to \( T^2 \), while the nesting condition changes this behavior to a quasi-linear dependence in \( T \). The linewidth \( \Gamma \) is calculated following a procedure outlined by Viroztek and Ruvalds [22] in the context of high-\( T_c \) superconductivity. In the disordered phase \( \Gamma \) is given by the imaginary part of the electron self-energy, which can be expressed as a convolution of a staggered susceptibility \( \chi''_S(\omega/2T) \) with a fermion Green’s function [15],

\[
\Gamma_{NFL}(\omega, T) = \frac{1}{2} T \int dx \left[ \coth(x) - \tanh(x - \frac{\omega}{2T}) \right] \chi''_S(x) \left[ 3|\tilde{V}|^2 + |2\tilde{U} - \tilde{V}|^2 \right] \rho_F,
\]

(6)

\[
\chi''_S(\omega/2T) \approx \frac{\rho_F}{2} \sum_{s=\pm 1} \text{Im} \psi \left( \frac{1}{2} + \frac{\Gamma_{NFL}}{2\pi T} + i\frac{\omega - 2s(\delta - \delta_0)}{4\pi T} \right),
\]

(7)

where \( \text{Im} \psi \) is the imaginary part of the digamma function, \( \omega \) is the external frequency, and \( \delta_0 \) is the nesting mismatch corresponding to the QCP. The frequency in the vertices is \( 2T|x| + |\omega|/2 \).
and we use the analytic continuation of the vertex functions, i.e. $i\pi/2$ is added to $\xi$. The frequency of $\Gamma_{NFL}$ in $\text{Im}\, \psi$ is $2T|x|$. The selfconsistent solution of Eqs. (6) and (7) yields the quasi-particle NFL linewidth as a function of $\omega$ and $T$ [15].

There is also a FL contribution to the quasi-particle linewidth given by [15]

$$\Gamma_{FL}(\omega, T) = \frac{\pi}{8}[\omega^2 + (\pi T)^2][3V^2 + (2U - V^2)]\rho_F^3,$$

which is added to $F_{NFL}$ assuming that Matthiessen’s rule is valid. The vertices in $\Gamma_{FL}$ are not dressed, since this contribution does not arise from the nesting condition.

The $\omega$ and $T$ dependence of the selfconsistent $\Gamma_{NFL}$ can be understood from some limiting cases [15]. First, consider the perfectly tuned QCP, i.e. $\delta = \delta_0$, set $\omega = 0$ and neglect $\Gamma_{NFL}$ in the digamma function, as well as the vertex renormalizations. The integral on the right-hand side of Eq. (6) is then independent of $T$ and hence $\Gamma_{NFL} \propto T$, and not $T^2$ as for a FL. Similarly, as $T \to 0$, neglecting $\Gamma_{NFL}$ in the digamma function and the vertex renormalizations, we obtain, for $\delta = \delta_0$, that the right-hand side of Eq. (6) is proportional to $|\omega|$, which again differs from the FL behavior ($\propto \omega^2$). The vertex renormalizations yield additional logarithmic corrections, so that to logarithmic order we have approximately

$$\Gamma_{NFL} \propto [3V^2 + |2U - V|^2]\rho_F^2 \max(|\omega|, T).$$

In the presence of an instability the vertex corrections strongly enhances $\Gamma_{NFL}$. The selfconsistent solution, as a consequence of the logarithmic corrections, yields a $\Gamma_{NFL}$ that has a slightly sublinear $T$- and $|\omega|$-dependence [15].

Second, for $\delta \neq \delta_0$, neglecting again the selfconsistency and the vertex corrections, $\Gamma_{NFL}$ is exponentially activated at low $T$ and gradually crosses over to a linear $T$-dependence with increasing $T$. Hence, at low $T$ the FL contribution (proportional to $T^2$) dominates, but at higher $T$ there is NFL behavior (left panel of Fig. 2). Third, at $T = 0$, $\Gamma_{NFL}$ vanishes identically for $|\omega| < 2(\delta - \delta_0)$ and is proportional to $|\omega| - 2(\delta - \delta_0)$ at larger frequencies (right panel of Fig. 2). Hence, again the FL contribution (proportional to $\omega^2$) dominates at low energies [15]. As a consequence of this gap, the low energy behavior is FL like.

![Figure 2](image-url)

**Figure 2.** (left panel) Quasi-particle linewidth $\Gamma = \Gamma_{NFL} + \Gamma_{FL}$ as a function of $T$ for $V\rho_F = U\rho_F = 0.2$, $D = 10$ and several values of $\delta$. $\delta_0 = 0.07$ corresponds approximately to the tuned QCP. The arrows indicate the crossover from NFL to FL behavior with decreasing $T$. (right panel) Frequency dependence of $\Gamma_{NFL}$ at $T = 0$ for the same parameters. The low energy gap is responsible of the FL behavior [15].
The selfconsistent solution of $\Gamma = \Gamma_{NFL} + \Gamma_{FL}$ for zero frequency is displayed in the left panel of Fig. 2. The crossover from FL to NFL is indicated by arrows, where the two contributions are equal. The crossover region is not a precisely defined quantity, especially at intermediate temperatures. In the specific heat, the crossover from the $-\ln(T)$ dependence of $C/T$ to constant $\gamma$ agrees with that of the linewidth. Both, $T_N = -(\delta - \delta_0)/2$ (separating the AF and paramagnetic phases) and the crossover temperature (shaded area) are shown in Fig. 3. Below the dashed curve the temperature dependence of $\Gamma$ follows the FL $T^2$-behavior.

4. Resistivity

A proper definition of quasi-particles requires that their linewidth at low $T$ is small compared to their energy. This is satisfied in a FL, where the width grows proportional to $(\omega/D)^2$, with $D$ being the bandwidth. However, for the tuned QCP, the linewidth of the quasi-particles grows as fast as their energy, and the quasi-particles are not well-defined. The resistivity is then not necessarily proportional to $\Gamma_{NFL}$.

An appropriate approach to obtain the resistivity is the Kubo equation. The dynamical conductivity can be expressed in terms of a memory function $M(z)$ [23],

$$\sigma(z) = (i\omega_p^2/4\pi)/[z + M(z)].$$ (10)

Here $\omega_p^2 = 4\pi e^2(N_1/m_1 + N_2/m_2)$ is the plasma frequency for the two-pocket model, and $N_1(N_2)$ and $m_1(m_2)$ are the particle (hole) density and mass, respectively. After a lengthy calculation [16] we obtained the imaginary part of the memory function. The full set of equations determining $M''(\omega)$ will not be presented here. In Fig. 4 we show the results for the resistivity $\rho_{NFL}(T) = 4\pi M''(0)/\omega_p^2$. Note that there is also a FL contribution to the resistivity, which strongly depends on impurity scattering [16].

The electrical resistivity is qualitatively similar to the quasi-particle linewidth and roughly proportional to $\Gamma_{NFL} + \Gamma_{FL}$ for zero frequency. $\rho(T)$ is slightly sublinear in $T$ for the tuned QCP and for Fermi surface mismatch larger than the critical one, the resistivity displays a crossover from NFL ($\sim T$) to FL ($\sim T^2$) behavior with decreasing temperature, in agreement with experiments [16].

At low $T$ the dynamical conductivity for the tuned QCP has strong deviations from the usual Lorentzian Drude behavior. This is the consequence of the NFL-dependence of $M''(\omega)$, which is roughly proportional to $|\omega|$, rather than to a constant (impurity scattering) as for a FL. Hence, instead of falling off as $\omega^{-2}$, $\sigma(\omega)$ decreases roughly as $\omega^{-1}$ [16].

CeRu$_{0.48}$Fe$_{1.52}$Ge$_2$ is a system with QCP for which inelastic neutron scattering has revealed a linear $T$-dependence of the linewidth of the quasi-elastic peak [24]. At low $T$ the quasi-elastic...
peak deviates from a Lorentzian (similarly to the conductivity in the present model) and when the Ru/Fe concentration is tuned away from quantum criticality, FL behavior and a Lorentzian peak are recovered.

5. Amplitudes of de Haas-van Alphen oscillations

Although the quasi-particles are not properly defined for the tuned QCP, there is a one-to-one correspondence between the excitations with those of a FL. Varying the Fermi surface mismatch parameter we can continuously interpolate between the states of the FL and the tuned QCP. Since all the states have free Fermi gas statistics, the extended Lifshitz-Kosevich equation can be applied [25].

There are two circular orbits corresponding to extremal cross-sectional areas of the Fermi surface of radii $k_{F1}$ (electrons) and $k_{F2}$ (holes), respectively, and hence two fundamental frequencies of oscillation. The amplitude of the oscillations modified by the quasi-particle linewidth is [18, 25]

$$ A_r = 2 \frac{2 \pi^2 T_r}{\hbar \omega_c} \sum_{\xi_n > 0} \exp \left[ - \frac{2 \pi r}{\hbar \omega_c} \xi_n \left( 1 + \int_{-D}^{D} \frac{d\omega}{\pi} \frac{\Gamma(\omega)}{\omega^2 + \xi_n^2} \right) \right]. $$

(11)

Here $r = 1, 2, 3, \cdots$ labels the harmonics, $\omega_c = eB/m^*$, $m^*$ is the effective heavy fermion mass before renormalization, and $\xi_n = \pi T(2n + 1)$ for $n = 0, 1, 2, \cdots$ are the fermionic Matsubara frequencies. $A_r$ is defined such that $A_r \to 1$ for $T \to 0$ if $\Gamma = 0$ (Lifshitz-Kosevich limit).

To simplify we assumed that both pockets have the same linewidth, $\Gamma(\omega) = \Gamma_{NFL}(\omega) + \Gamma_{FL}(\omega)$. For $\Gamma = 0$, i.e. no interactions, $A_r$ reduces to $1/[2 \sinh(2 \pi^2 k_B T r / \hbar \omega_c)]$. For interacting electrons $A_r$ is always reduced with respect to the noninteracting system and the overall reduction of the amplitudes is largest close to the QCP [18]. Fig. 5 shows the logarithm of the oscillation amplitude for the first five harmonics as a function of $T$ for the tuned QCP, $\delta_0 = 0.07$. If $\ln(A_r)$ vs. $T$ is a straight line (except at very low $T$) the effective mass approximation (Lifshitz-Kosevich) is valid. Except for some curvature, this is approximately the case at higher $T$, but at low $T$ there are deviations from the effective mass approximation. A very low Dingle temperature is necessary to observe the fundamental frequency.

The dHvA-oscillations are periodic as a function of $B^{-1}$, and are measured over a magnetic field interval. Hence, the amplitude of oscillation cannot be associated with a given $B$. The magnetic field frequently also acts as a tuning parameter for the QCP. The present discussion of the amplitudes is only meaningful if $B$, within the regime of measurement, does not affect the tuning of the QCP [18].
6. Dynamical spin susceptibility

The dynamical susceptibility is a function of the energy and momentum transfers, $\omega$ and $\mathbf{Q} + \mathbf{q}$. We first evaluate the response function for $\mathbf{q} = 0$ in the absence of interactions to get qualitative insight, and incorporate then the interactions by summing the ladder diagrams (leading order logarithms) and inserting the quasi-particle line-width (selfenergy).

For the non-interacting system the imaginary part of the dynamical susceptibility is given by [17]

$$
\chi''_0(Q, \omega) = \frac{\pi \rho_F}{4} \left[ \tanh \left( \frac{\omega + 2\delta}{4\pi T} \right) + \tanh \left( \frac{\omega - 2\delta}{4\pi T} \right) \right],
$$

where for simplicity we used parabolic dispersions with $m_1 = m_2 = m$ and $v_{F1} = v_{F2} = v_F$. The nesting condition is $\epsilon_2(k) = 2\delta - \epsilon_1(k)$ and $\rho_F = k_{F1}m/(2\pi^2)$ is the density of states. Eq. (7) is antisymmetric in $\omega$, and at low $T$ gapped for $-2\delta \leq \omega \leq 2\delta$. The gap is the consequence of the momentum and energy conservation, which cannot be satisfied simultaneously unless $|\omega| > 2\delta$. With increasing $T$ the gap gradually closes.

We now incorporate the quasi-particle linewidth into the calculation. Eq. (7) is the bubble diagram with antiparallel propagator lines. These propagators are now broadened into Lorentzians. The momentum integrations can be carried out, yielding another Lorentzian with line-width given by the sum of the widths of the two original Lorentzians, i.e. $\Gamma_{\text{LW}} = \Gamma_{\text{NFL}}(\omega' + \omega/2, T) + \Gamma_{\text{NFL}}(\omega' - \omega/2, T)$. Here $\omega'$ is the energy integration variable for the bubble [22].

At low $T$ the $\omega'$-integration is limited to the interval $(-\omega/2, \omega/2)$. Hence, $\Gamma_{\text{LW}}$ predominantly depends on $\omega$ and not on the integration variable $\omega'$ [15, 22]. The $\omega'$-integration is then straightforward and we obtain approximately [17]

$$
\chi''_0(Q, \omega) = \frac{\rho_F}{2} \text{Im} \psi \left( \frac{1}{2} + \frac{\Gamma_{\text{LW}}}{4\pi T} \right) + \frac{\rho_F}{2} \text{Im} \psi \left( \frac{1}{2} + \frac{\Gamma_{\text{LW}}}{4\pi T} \right),
$$

with $\text{Im}$ denoting imaginary part and $\Gamma_{\text{LW}} \sim 2\Gamma_{\text{NFL}}$. Note that expression (13) is different from $\chi''_{\text{NFL}}(\omega/2T)$ in Eq. (7).

We now choose opposite spins for the propagators, so that the correlation function involves a spin-flip (transversal susceptibility). The most important terms contributing to $\chi$ are the ladder diagrams in $V$, i.e. the random phase approximation (RPA) diagrams [17],

$$
\chi(Q, \omega) = \chi_0(Q, \omega)/[1 - V\chi_0(Q, \omega)].
$$

**Figure 5.** dHvA amplitudes for the first five harmonics $r$ as a function of $T$ for fixed $B$ for the tuned QCP, $\delta_0 = 0.07$, and the same parameters as before. The Lifshitz-Kosevich equation with an effective mass would correspond to a straight line. For $D = 1000$ K the magnetic field is $40$ T for $m^*/m = 20$, $m$ being the free electron mass [18].
Note that to leading order the $U$-interaction does not contribute to the transversal susceptibility (in RPA). The dissipative part of the transversal susceptibility is given by [17]

$$\chi''(Q, \omega) = \frac{\chi''_0(Q, \omega)}{[1 - V\chi'(Q, \omega)]^2 + V^2\chi''_0(Q, \omega)^2}. \quad (15)$$

The imaginary part for $\chi_0(Q, \omega)$ to be inserted in Eq. (15) is Eq. (13) and the expression for $\chi'_0(Q, \omega)$ is

$$\chi'_0(Q, \omega) = \ln(D/2\pi T) + c + \frac{\rho_F}{2} \sum p \text{Re} \psi(Z_p), \quad (16)$$

$$Z_p = \frac{1}{2} + \frac{\Gamma_{NFL} \Gamma_{FL}}{2\pi T} + i\frac{\omega + 2p\delta}{4\pi T}, \quad (17)$$

where $c$ is a constant, $p = \pm 1$ and Re denotes real part. Here $\Gamma_{NFL}$ and $\Gamma_{FL}$ are functions of $\omega$ and $T$. The additive constant $c$ arises from the cut-off in the Cauchy transformation of $\chi''_0$ and is in principle arbitrary. The constant is determined by the quantum criticality condition, i.e. $1 = V\chi'_0(Q, \omega = 0)$ for $T = 0$, and $\delta = \delta_0$. For the parameters used in this paper, we have $c = 0.038152$.

Two features should be pointed out in $\chi''(Q, \omega)/\omega$ for the critical Fermi surface mismatch $\delta_0$: (i) The quasi-elastic peak around $\omega = 0$ (Fig. 6) and (ii) the shoulder at $\omega = \pm 2\delta_0$ at low $T$ (Fig. 7). The shoulder is reminiscent of the noninteracting case, Eq. (12), and is gradually smeared with increasing temperature. The central peak, on the other hand, arises from the interaction through the denominator in Eq. (14) and is displayed in the left panel of Fig. 6 as a function of $\omega$ for several temperatures. As $\omega \rightarrow 0$ the imaginary component of the susceptibility tends to zero and also the real part, $1 - V\chi'$, becomes small, thus giving rise to the peak. The maximum, defined as $\lim_{\omega \rightarrow 0} \chi''(Q, \omega)/\omega$, is proportional to $T^{-3}$ at low $T$. The form of the resonance is approximately Lorentzian with the half-width of the peak at half of its height being proportional to $T$ (right panel of Fig. 6).

For non-critical nesting mismatch the behavior of $\chi''(Q, \omega)/\omega$ is different, since the denominator in Eq. (14) does no longer vanish at $\omega = 0$. Hence, only for the critical mismatch $\delta_0$ the peak is quasi-elastic, while for other $\delta$ the peak is inelastic. As seen in Fig. 7 the height of the peak is strongly reduced with increasing noncritical mismatch.
7. Conclusions

The nesting of a heavy electron Fermi surface can give rise to itinerant AF long-range order. The degree of nesting is controlled by a mismatch parameter. This way the ordering temperature can be tuned to zero, leading to a QCP. The QCP is an unstable fixed point and can only be reached by perfectly tuning the system. Otherwise, the RG flow will deviate to a phase with long-range order or the compound remains a heavy electron paramagnet. There is no characteristic energy scale associated with the QCP and a small perturbation to the system may give rise to a new physical situation at low energies. This fact is probably responsible for the lack of universality in NFL compounds close to the QCP.

We investigated the quasi-particle lifetime as the QCP is approached. Landau’s FL theory predicts that for normal metals the linewidth is proportional to $\omega^2$ and $T^2$. The QCP modifies this behavior to a linear $|\omega|$ and $T$ dependence with logarithmic corrections. As a function of $T$ and the Fermi surface mismatch there is a crossover from FL to NFL behavior. This crossover is also reflected in the specific heat over temperature and the electrical resistivity, leading to a phase diagram that is in qualitative agreement with experiments. For the tuned QCP the resistivity is approximately linear in $T$.

The dynamical spin susceptibility was selfconsistently calculated using the linewidth of the quasi-particles and the random phase approximation. For the tuned QCP a Lorentzian quasi-elastic peak with a width linear in $T$ is obtained. For noncritical mismatch of the Fermi surface the peak becomes inelastic, i.e. it is shifted to $\omega \neq 0$. The dynamical susceptibility is not in agreement with the experimental observations (inelastic neutron scattering). The present model can also not account for the restructuring of the Fermi surface at $\delta_0$ for $T \to 0$ [26]. A two-band model with one dispersionless band (Anderson lattice) is necessary to capture these features.

We calculated the amplitudes of the dHvA oscillations using a modified Lifshitz-Kosevich expression. As expected the amplitudes are strongly reduced and the oscillations are difficult to observe.

The results are valid in the disordered phase for weak and intermediate coupling. However, since the renormalization group does not allow a return to a weak-coupling fixed point once the system is strongly coupled, the present approach qualitatively describes the entire precritical regime.

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References
[1] Stewart G R 2001 Rev. Mod. Phys. 73 797
[2] von Löhneysen H 1994 Physica B 206 & 207 101
[3] Maple M B et al 1994 J. Low Temp. Phys. 95 225
[4] Andraka B and Tselvik A M 1991 Phys. Rev. Lett. 67 2886
[5] Tselvik A M and Reizer M 1993 Phys. Rev. B 48 4887
[6] Millis A J 1993 Phys. Rev. B 48 7183
[7] Continentino M A 1993 Phys. Rev. B 47 11587
[8] Moriya T and Takimoto T 1995 J. Phys. Soc. Jpn. 64 960
[9] Schlottmann P 1999 Phys. Rev. B 59 12379
[10] Si Q, Rabello S, Ingersent K, and Smith J L 2001 Nature (London) 413 804
[11] Coleman P and Pépin C 2002 Physica B 312-313 383
[12] Schlottmann P 2003 Phys. Rev. B 68 125105
[13] Custers J et al 2003 Nature 424 524
[14] Gegenwart P, Si Q and Steglich F 2008 Nature Physics 4 186
[15] Schlottmann P 2006 Phys. Rev. B 73 085110
[16] Schlottmann P 2006 Phys. Rev. B 74 235103
[17] Schlottmann P 2007 Phys. Rev. B 75 205108
[18] Schlottmann P 2008 Phys. Rev. B 77 195111
[19] Schlottmann P 2000 J. Appl. Phys. 87 5140
[20] Mathur N D et al 1998 Nature (London) 394 39
    Walker I R et al 1997 Physica C 282 303
[21] Schlottmann P 2004 J. Appl. Phys. 95 7216
[22] Virosztek A and Ruvalds J 1990 Phys. Rev. B 42 4064
    Virosztek A and Ruvalds J 1999 Phys. Rev. B 59 1324
[23] Götze W and Wölfle P 1972 Phys. Rev. B 6 1226
[24] Montfrooij W et al 2003 Phys. Rev. Lett. 91 087202
[25] Wasserman A and Springford M 1996 Adv. in Phys. 45 471
[26] Paschen S et al 2004 Nature 432 881