Minimal Lepton Flavour Violation and Leptogenesis 
with exclusively low-energy CP Violation

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Abstract

We study the implications of a successful leptogenesis within the framework of Minimal Lepton Flavour Violation combined with radiative resonant leptogenesis and the PMNS matrix being the only source of CP violation, which can be obtained provided flavour effects are taken into account. We find that the right amount of the baryon asymmetry of the universe can be generated in this framework with three quasi-degenerate heavy Majorana Neutrinos under the conditions of a normal hierarchy of the light neutrino masses, a non-vanishing Majorana phase, $\sin(\theta_{13}) \gtrsim 0.13$ and $m_{\nu,\text{lightest}} \lesssim 0.04$ eV. If this is fulfilled, we find strong correlations among ratios of charged LFV processes.

1 Preliminaries

Leptogenesis [1] is an extremely successful mechanism to generate the observed matter-antimatter asymmetry of the universe. While the common belief in the past years was that distinguishing flavours in the Boltzmann equations is not necessary, it turned out recently [2–11] that flavour effects can be relevant for the generation of the baryon asymmetry of the universe (BAU).

For a long time, one thought that leptonic low-energy CP violation does not automatically imply a non-vanishing BAU through leptogenesis. This however does not universally hold when ranges of the Majorana scale are considered in which flavour effects play a role [6, 7, 10, 12–14]. Leptogenesis without high-energy CP violation has been found to be successful in frameworks with hierarchical heavy right-handed Majorana neutrinos [12–14]. But also in the case of resonant leptogenesis [4, 15–17], the BAU can be accommodated in the presence of exclusively low-energy CP violation which has been shown for a mass spectrum with two [14] and with three [10] quasi-degenerate heavy
right-handed Majorana neutrinos. The analysis for three quasi-degenerate heavy right-handed Majorana neutrinos which corresponds to the Minimal Lepton Flavour Violation scenario and has been performed in the un-flavoured regime by [21], has been presented in [10] allowing for CP violation at low and high energies with the BAU generated by radiative resonant leptogenesis (RRL) [10, 18–20].

In the present paper, we concentrate on the limit of no high-energy CP violation within this framework. We show that for a successful leptogenesis clear conditions have to be fulfilled.

Applying these constraints we find strong correlations among low-energy lepton flavour violating (LFV) decays, which are weak in the presence of high-energy CP violation [10]. Similar predictivity for ratios of LFV decays has been observed in the single-flavour case in the limit of large Majorana masses [21].

\section{Minimal Lepton Flavour Violation}

The existence of neutrino masses implies that lepton flavour is not conserved. However, from non-observation of LFV processes such as $\mu \rightarrow e \gamma$ we know that those interactions have to be highly suppressed. Extensions of the Standard Model (SM) that implement LFV should keep such processes automatically small and allow for new-physics particles with moderate masses. In the quark sector, where the situation of flavour-changing transitions is quite similar, these issues can nicely be accommodated with the Minimal Flavour Violation (MFV) hypothesis [22, 23]. How this mechanism could be established in the lepton sector was proposed by [24]. Analogously to the quark sector, Minimal Lepton Flavour Violation (MLFV) can be formulated as an effective field theory in which the lepton Yukawa couplings $Y_E, Y_\nu$ are the only sources of flavour violation.

\begin{equation}
\mathcal{L}_Y = -\bar{e}_R Y_E \phi^\dagger L_L - \bar{\nu}_R Y_\nu \phi L_L + h.c.
\end{equation}

In order to additionally explain the smallness of neutrino masses with the help of the see-saw mechanism, the MFV hypothesis in the lepton sector requires lepton number violation at some high scale, and three heavy right-handed Majorana neutrinos being introduced,

\begin{equation}
\mathcal{L}_M = -\frac{1}{2} \bar{\nu}_R \nu_R M_R + h.c.
\end{equation}

with the Majorana mass matrix having a trivial structure $M_R = M_R \mathbb{1}$. For a different definition of MLFV see [25]. The branching ratios of LFV decays like $\mu \rightarrow e \gamma$ proceeding at a scale $\Lambda_{\text{LFV}}$ in the framework of MLFV are given by [24]

\begin{equation}
B(l_i \rightarrow l_j \gamma) = 384\pi^2 e^2 \nu^2 M^2_{\Lambda_{\text{LFV}}} |(Y_\nu^\dagger Y_\nu)_{ij}|^2 |C|^2.
\end{equation}
Here $C$ summarizes the Wilson coefficients of the relevant operators involved that can be calculated for a specified model. As the Wilson coefficients are naturally of order one we will set those coefficients to unity here. As $C$ is independent from the external lepton flavours, this dependence is canceled when ratios of $l_i \rightarrow l_j \gamma$ decays are considered. In particular one has

$$R_{\text{LFV}} = \frac{B(l_i \rightarrow l_j \gamma)}{B(l_m \rightarrow l_n \gamma)} = \frac{|(Y^\dagger_{\nu_i} Y_{\nu})_{ij}|^2}{|(Y^\dagger_{\nu_i} Y_{\nu})_{mn}|^2}. \quad (4)$$

### 3 Radiative Resonant Leptogenesis

Since radiative corrections spoil the degeneracy of the Majorana masses [10, 21], we combine the MLFV hypothesis with a choice of a scale at which the Majorana masses are exactly degenerate as it was done in the analysis of [10]. A natural selection for the degeneracy scale is the GUT scale $\Lambda_{\text{GUT}}$,

$$M_R(\Lambda_{\text{GUT}}) = M_\nu I. \quad (5)$$

The mass splittings at the Majorana scale that are necessary for leptogenesis are then induced radiatively which can be described by Renormalization Group Equations (RGE) and are approximately

$$\frac{\Delta M}{M} \sim Y_\nu Y^\dagger_\nu \ln \left( \frac{M}{\Lambda_{\text{GUT}}} \right). \quad (6)$$

For quasi-degenerate heavy right-handed Majorana neutrinos with mass splittings comparable to their decay widths, the CP asymmetries relevant for thermal leptogenesis are resonantly enhanced. The radiatively generated mass splittings (6) automatically fulfill the condition of resonant leptogenesis [15–17]. For a more detailed description of *radiative resonant leptogenesis* and mass splittings due to the RGE of the SM and MSSM see [10].

Thermal leptogenesis [1] requires CP and lepton number violating out-of-equilibrium decays of heavy right-handed neutrinos. These decays produce a lepton asymmetry which is turned into a baryon asymmetry by sphaleron processes. The CP asymmetry due to the Majorana neutrino $N_i$ and lepton flavour $l$, defined as

$$\varepsilon^l_i = \frac{\Gamma(N_i \rightarrow L_l \phi) - \Gamma(N_i \rightarrow \bar{L}_l \bar{\phi})}{\sum_l \left[ \Gamma(N_i \rightarrow L_l \phi) + \Gamma(N_i \rightarrow \bar{L}_l \bar{\phi}) \right]}, \quad (7)$$

that enters the baryon asymmetry is given by

$$\varepsilon^l_i = \frac{1}{(Y_{\nu} Y^\dagger_{\nu})_{ii}} \sum_j \text{Im}((Y_{\nu} Y^\dagger_{\nu})_{ij}(Y_{\nu} Y^\dagger_{\nu})_{il}) \cdot g(M^2_i, M^2_j, \Gamma^2_j). \quad (8)$$
with the full expression of \( g(M_i^2, M_j^2, \Gamma_j^2) \) given in [16]. For a successful leptogenesis at the Majorana scale in addition to \( \Delta M \neq 0 \), the Yukawa matrices have to contain complex phases.

The neutrino Yukawa matrix \( Y_\nu \) can be parametrized according to [26]

\[
Y_\nu = \frac{i}{\nu} \sqrt{M_R} R \sqrt{m_\nu} U_\nu^\dagger, 
\]

where \( m_\nu = \text{diag}(m_1, m_2, m_3) \) are the light-neutrino masses, \( M_R = \text{diag}(M_1, M_2, M_3) \) the masses of the heavy right-handed Majorana neutrinos, \( U_\nu \) is the PMNS matrix and \( R \) an orthogonal complex matrix that encodes three physically relevant complex parameters [10]. Apart from radiative corrections, \( R \) provides all high-energy CP violation. If \( R \) is real, this corresponds to the limit of no high-energy CP violation. In this case the only complex phases in \( Y_\nu \) are those of the PMNS matrix.

### 4 Flavour Effects in Leptogenesis Scenarios

There have been several investigations recently of the importance of flavour effects [2, 4, 5, 7–10]. Below some temperature, the interactions associated with the \( \mu \) and \( \tau \) charged lepton Yukawa couplings are much faster than the expansion of the universe and so are in equilibrium. This is the case for \( T \sim M_\nu \lesssim 10^9 - 10^{12} \text{ GeV} \). The interactions of the \( \tau \) Yukawa coupling are in equilibrium below about \( 10^{12} \text{ GeV} \), where two flavours then can be distinguished, followed by the ones of the \( \mu \) Yukawa coupling below approximatively \( 10^9 \text{ GeV} \), where three distinguishable flavours exist. Also processes that wash out the lepton asymmetries are flavour dependent. In these regimes below \( 10^9 - 10^{12} \text{ GeV} \), flavour specific solutions to the Boltzmann equations are required.

An order-of-magnitude estimate of solutions for the BAU \( \eta_B \) of the Boltzmann equations in the strong washout regime in the single-flavour treatment is given by [4]

\[
\eta_B \simeq -10^{-2} \sum_{i=1}^{3} e^{-(M_i - M_1)/M_1} \frac{1}{K} \sum_{l=e,\mu,\tau} \varepsilon_{l,i}, 
\]

and an estimate including flavour effects by [4]

\[
\eta_B \simeq -10^{-2} \sum_{i=1}^{3} \sum_{l=e,\mu,\tau} e^{-(M_i - M_1)/M_1} \varepsilon_{l,i} K_i^l \frac{K_i^l}{K_i^l K_i}, 
\]

with

\[
K_i^l = \frac{\Gamma(N_i \rightarrow L_l \phi) + \Gamma(N_i \rightarrow \bar{L}_l \bar{\phi})}{H(T = M_i)} 
\]
\[ K = \sum_i K_i, \quad K_i = \sum_{l=e,\mu,\tau} K^l_i, \quad K^l = \sum_{i=1}^3 K^l_i, \quad H(T = M_i) \simeq 17 \frac{M^2_i}{M_{Pl}}, \] (13)

Here \( M_{Pl} = 1.22 \times 10^{19} \) GeV and \( K^l_i \) is the washout factor due to the inverse decay of the Majorana neutrino \( N_i \) and the lepton flavour \( l \). In the strong washout regime the following conditions for the single-flavour (14) and the three-flavour (15) case have to be fulfilled:

\[ K = K_1 + K_2 + K_3 \gtrsim 50, \] (14)

\[ K^l_i \gtrsim 1. \] (15)

Similar estimates can be found in [8,17].

In the past, the common belief was that for a successful leptogenesis high-energy CP violation is required since the complex phases from the PMNS matrix do not offer a sufficient amount of CP violation. This statement is not true when flavour effects are considered. A flavour-specific treatment of the washout and the CP asymmetries can accommodate the BAU of the right order of magnitude [10,12–14]. When flavours have to be treated specifically, the un-summed terms in the CP asymmetry (8), \( (Y^\nu_l)(Y^\nu_l)^{\dagger}_{lj} \), can become important. Summing over the flavours as in the estimate (10), it follows from (9) that the PMNS matrix cancels due to its unitarity (apart from radiative corrections between the GUT and the Majorana scale) while this is not the case in (11), where the CP asymmetry of a single flavour is weighted separately due to the \( K \) factors. This gives a notion how the PMNS phases come into play when flavours are treated specifically.

## 5 Leptogenesis as a Constraint of LFV Processes

Since the size of \( B(l_i \rightarrow l_j \gamma) \) is governed by the size of the ratio \( M^2_\nu/\Lambda^4_{\text{LFV}} \) in the context of MLFV, there is in principle a rich spectrum of possibilities for the size of such effects. The question is whether a successful generation of the BAU by radiative resonant leptogenesis provides a strong correlation with LFV processes. The observed value of the BAU is given by [27]

\[ \eta_B = \frac{n_B}{n_\gamma} = (6.10 \pm 0.21) \cdot 10^{-10}. \] (16)

Even if the baryon asymmetry is just a single number, it provides an important relation between the early-universe cosmology and the SM of particle physics and its extensions. Successful leptogenesis could constrain the large amount of possibilities for \( l_i \rightarrow l_j \gamma \) or ratios of such decays. However, the more parameters are contained in a given theory, the less restrictive is the constraint that stems from the baryon asymmetry.
5.1 Low- and high-energy CP Violation

Recently, two analyses were performed that implemented leptogenesis in the framework of MLFV and included CP violation at low and high energies. In [21] the BAU was generated in a framework similar to RRL. In this analysis, flavour effects were not considered. In this case, the BAU provides a lower mass bound on the Majorana scale of $\sim 10^{12}$ GeV. In this framework a relation among LFV processes could be obtained:

$$B(\mu \rightarrow e\gamma) < B(\tau \rightarrow \mu\gamma).$$  \hspace{1cm} (17)

In [10] with the inclusion of flavour effects in the relevant regimes, the BAU of the right order of magnitude could be accommodated in the framework of RRL with RGE of the SM and MSSM basically independent from the Majorana scale. Due to this fact, it turned out that there are only very weak correlations among leptogenesis and charged LFV processes and the relation (17) could not be confirmed. In contrast to MFV in the quark sector, predictivity seems to be almost lost in MFV in the lepton sector when high-energy CP violation is present and flavour effects are considered.

5.2 Low-Energy CP Violation alone

![Figure 1: The BAU $\eta_B$ versus the Majorana scale up to $10^9$ GeV. The black points correspond to the three-flavour estimate, the light-blue points to the single-flavour solution. The two black lines mark where $\eta_B$ is of the right order of magnitude.](image)

In this section we consider the framework of MLFV and RRL in the three-flavour regime ($M_\nu < 10^9$ GeV) with RGE for the SM and investigate conditions for a successful leptogenesis without high-energy CP violation.

For the PMNS matrix that describes leptonic low-energy mixing and CP violation, we
use the convention

\[ U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot V \]  

where \( c_{ij} = \cos(\theta_{ij}) \), \( V = \text{diag}(e^{ia}, e^{i\beta}, 1) \), \( \alpha \) and \( \beta \) denote the Majorana phases and \( \delta \) denotes the Dirac phase. \( 0 \leq \alpha, \beta \leq \pi \) and \( 0 \leq \delta \leq 2\pi \) are the physically relevant ranges for the PMNS phases. PMNS matrix elements enter the CP asymmetries according to \( (8) \) and \( (9) \). Further we consider the following ranges: \( 10^5 \text{ GeV} < M_\nu < 10^9 \text{ GeV} \) in which the three-flavour estimate of \( \eta_B \) \( (11) \) can be applied, \( 0 \leq \sin(\theta_{13}) \leq 0.2 \) and \( 0 \leq m_{\nu,\text{lightest}} \leq 0.2 \text{ eV} \). We use \( c_{23} = s_{23} = 1/\sqrt{2} \) and \( \theta_{12} = 33^\circ \) and for the light neutrinos we have the low energy values

\[ \Delta m^2_{\text{sol}} = m^2_2 - m^2_1 = 8.0 \cdot 10^{-5} \text{ eV}^2 \quad (19) \]

\[ \Delta m^2_{\text{atm}} = |m^2_3 - m^2_2| = 2.5 \cdot 10^{-3} \text{ eV}^2 \quad (20) \]

with \( m_{\nu,\text{lightest}} = m_1(m_3) \) for normal (inverted) hierarchy, respectively.

In the limit of vanishing high-energy CP violation, the matrix \( R \) of \( (9) \) has to be real and we set it equal to unity at the GUT scale since at this scale the heavy right-handed Majorana neutrinos are exactly degenerate.

\[ R(\Lambda_{\text{GUT}}) = \mathbb{1} \quad (21) \]

Then the only complex phases that enter the neutrino Yukawa matrix \( Y_\nu \) and so the BAU apart from radiative corrections are the one of the PMNS matrix. It has been found that

![Figure 2: The BAU \( \eta_B \) versus \( \sin(\theta_{13}) \) and the lightest neutrino mass \( m_\nu \). In our scenario with exclusively low-energy CP violation, successful leptogenesis requires \( 0.13 \lesssim \sin(\theta_{13}) \) and \( m_\nu \lesssim 0.04 \text{ eV} \).](image)

it is possible to obtain the baryon asymmetry generated by RRL with only low-energy
CP violation of the right order of magnitude in the regime where the flavour-dependent estimate \( (11) \) is valid \([10]\). This can also be seen in Figure \( \text{1} \) showing that the BAU can be accommodated properly independent from the Majorana scale with the estimate \( (11) \), whereas with a single-flavour treatment in this regime, the right size of the BAU cannot be obtained (light-blue points).

In the scenario presented here with exclusively low-energy CP violation, we find that successful leptogenesis implies the lower bound

\[
\sin (\theta_{13}) \gtrsim 0.13. \tag{22}
\]

For the lightest neutrino mass we obtain the upper bound

\[
m_\nu \lesssim 0.04 \text{ eV} \tag{23}
\]

(see Figure \( \text{2} \)). These bounds can be tested experimentally providing information on the viability of this framework.

If not stated differently, all plots presented here correspond to normal hierarchy in the light neutrino masses.

For a scenario with two quasi-degenerate heavy right-handed neutrinos, it has been found \([14]\) that it is possible to generate the BAU of the right order of magnitude for all hierarchies of the light neutrino masses.

In this setup with three quasi-degenerate heavy right-handed neutrinos, we find that for inverted hierarchy, it is not possible to generate the BAU of the right order of magnitude with low-energy CP violation alone and RGE for the SM. This is shown in Figure \( \text{3} \).

\[\begin{align*}
\text{Figure 3: The BAU } & \eta_B \text{ versus } \sin (\theta_{13}) \text{ for } \text{inverted hierarchy} \text{ of the light neutrino masses with exclusively low-energy CP violation. It is not possible to obtain the BAU of the right size with inverted hierarchy.}
\end{align*}\]
Figure 4: Left Plot: The baryon asymmetry $\eta_B$ (left) plotted over the Dirac phase $\delta$ for $\delta$ being the only source of CP violation which is not sufficient to obtain the right order of magnitude of the BAU indicated by the black line in the left plot. The range $\pi \leq \delta \leq 2\pi$ corresponds to negative values of $\eta_B$. Right Plot: The baryon asymmetry $\eta_B$ with the Majorana phase $\beta$ being the only source of CP violation.

5.2.1 CP Violation governed by a single PMNS phase

Now we want to investigate whether single CP violating phases could be sufficient to generate the BAU. If the Dirac phase $\delta$ is the only complex phase involved (see left plot of Figure 4), we find that it is not possible to fulfill the leptogenesis constraint which can be explained by the suppression by $\sin(\theta_{13})$ of the corresponding PMNS entries. This implies that the observation of CP violating neutrino oscillations alone is not sufficient to ensure successful leptogenesis in this framework.

However it is possible to successfully generate the BAU with a single Majorana phase $\alpha$ or $\beta$. This is depicted in the right plot of Figure 4 where $\beta \neq 0$, $\alpha = \delta = 0$ and $R(\Lambda_{\text{GUT}}) = 1$. This corresponds to the results of [14] where in the resonant case a strong sensitivity on the Majorana phases has been observed.

Therefore one can say that experimental observation or non-observation of Majorana phases will decide whether the setup presented could be realized in nature.

5.2.2 LFV Processes

From equation (3) one can read that $B(\mu \rightarrow e\gamma)$ rises with increasing Majorana scale for a fixed scale of lepton number violation. In a scenario with low-energy CP violation the Majorana scale is bounded by up to which scale the flavour-dependent analysis is valid. In this setup we chose $10^9$ GeV for to be conservative. Analyses that include the two-flavoured regime $10^9$ GeV $< M_\nu < 10^{12}$ GeV have to decide whether successful leptogenesis without high-energy CP violation for larger Majorana scales is possible.

If this is not the case and the Majorana scale is bounded to be below $10^9$ GeV as in...
Figure 5: $B(\mu \to e\gamma)$ versus $M_\nu$ for the general analysis [10] including high-energy CP violation and $\Lambda_{\text{LFV}} = 1$ TeV (left) and without high-energy CP violation (right) where $\Lambda_{\text{LFV}} = 1$ TeV (red) and $\Lambda_{\text{LFV}} = 300$ GeV (light-blue). The black points indicate where the leptogenesis constraint is fulfilled. The black line corresponds to the present bound on $B(\mu \to e\gamma) < 10^{-11}$ and the lower pink line to the sensitivity of the upcoming PSI experiment $\sim 10^{-13}$.

Figure 6: $R_{\text{LFV}} = B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$ versus $\sin(\theta_{13})$ for the general analysis [10] including high-energy CP violation (left) and without high-energy CP violation (right) where the relation (17) is satisfied. The black points fulfill the leptogenesis constraint. In the scenario considered, $\Lambda_{\text{LFV}}$ could be as low as 300-500 GeV to obtain $B(\mu \to e\gamma)$ in the reach of the PSI experiment (see Figure 5).

The very important point of this analysis is that with exclusively low-energy CP violation the relation $B(\mu \to e\gamma) < B(\tau \to \mu\gamma)$ is valid which was not true in the case of CP violation at low and high energies [10] (see Figure 6). Furthermore we find $B(\tau \to e\gamma) < B(\tau \to \mu\gamma)$. Figure 6 nicely depicts how dramatic the situation changes when high-energy CP violation is excluded. The rich amount of possibilities for the ratio $R_{\text{LFV}} = B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$ that is present in the high-energy CP violation case even if it solely depends on the relevant Yukawa couplings (4), can then significantly be constrained. We find MLFV with low-energy CP violation predicts a strong correlation among these LFV decays that can be tested in the future.
6 Conclusions

Recent studies showed the relevance of the inclusion of flavour effects in the Boltzmann equations for the generation of the baryon asymmetry of the universe and the existence in the flavoured-regime CP violation at high energies is no longer a necessary requirement for successful leptogenesis.

When leptogenesis is implemented in the framework of Minimal Lepton Flavour Violation by radiative resonant leptogenesis including high-energy CP violation [10], a rich spectrum of possibilities for charged LFV processes and ratios of such processes is present matching with the leptogenesis constraint implying that correlations among leptogenesis and LFV are weak.

In this paper we analysed the framework of Minimal Lepton Flavour Violation in the limit of no CP violation at high energies, with the PMNS phases being the only complex ingredient apart from radiative corrections. We find that with radiative resonant leptogenesis with RGE of the SM, one can successfully generate the BAU in the three-flavour regime provided that:

- there is a non-vanishing Majorana phase,
- the light neutrino masses have normal hierarchy,
- the lightest neutrino mass and the PMNS angle $\theta_{13}$ fulfill $m_\nu \lesssim 0.04$ eV and $0.13 \lesssim \sin(\theta_{13})$.

When these constraints are fulfilled, we find strong correlations among LFV processes and that the rich spectrum of possibilities that is present when high-energy CP violation is included [10] can significantly be reduced. In this case MLFV turns out to be much more predictive.

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