**Abstract**—We propose an unconventional structure of an electron gun to produce an annular beam for the frequency-tunable gyrotrons. The proposed gun takes advantage of the nonadiabatic electron motion during the acceleration of electrons by placing an emitter on the concave region of the cathode with a relatively weak electric field. The commonly employed adiabatic theory fails to predict the simulated transverse velocity spread, which suggests that the nonadiabatic effect plays an important role during the acceleration of electrons. Simulations are carried out by the EGUN code and verified with the commercial Computer Simulation Technology (CST) Particle Studio. From the change in the kinetic energy and the magnetic moment, we can define a section of the nonadiabatic electron motion. The rapid change in the beam-quality parameters within the nonadiabatic section is associated with the significant change in the electric field near the cathode. By considering the nonadiabatic electron motion, simulations predict appropriate parameters, namely, a pitch factor of 1.5 and a transverse velocity spread of 2.8%, over a wide range of the magnetic field (7.4–8.0 T) and the beam voltage (12–22 kV) with a high structural tolerance on the cathode geometry. The promising results enable the development of frequency-tunable gyrotrons.

**Index Terms**—Frequency-tunable gyrotron, magnetron injection gun (MIG), terahertz (THz) wave.

**I. INTRODUCTION**

Gyrotrons based on the mechanism of the electron cyclotron maser (ECM) instability [1]–[4] are capable of producing high-power, coherent, terahertz (THz) waves as compared with the classical vacuum electronic tubes. Through decades of development, frequency-tunable gyrotrons receive more and more attention [5]–[8] because of the frequency-sensitive applications, such as the enhancement of the nuclear magnetic resonance by dynamic nuclear polarization (DNP-NMR) and the measurement of positronium hyperfine splitting (Ps-HFS). The beam–wave synchronism for the gyrotrons can be expressed as

$$\omega \equiv k_z v_z + s \Omega_c$$  \hspace{1cm} (1)

where \(\omega\) and \(k_z\) are the angular frequency and the propagation constant of the electromagnetic wave, \(v_z\) is the electron axial velocity, which is associated with an applied beam voltage \(V_b\), \(s\) is the cyclotron harmonic number, and \(\Omega_c (= eB_0/m_e\gamma_0)\) is the relativistic electron cyclotron frequency, which is related to a magnetic field \(B_0\) and relativistic gamma factor \(\gamma_0\). \(e\) and \(m_e\) are the charge and rest mass of an electron. Equation (1) states that the frequency tunability of the gyrotrons [9]–[11] is attainable by varying either \(V_b\) or \(B_0\), or both. Therefore, high-performance electron sources with the stable beam quality over wide ranges of beam voltage and magnetic field deserve in-depth studies.

Beam-quality parameters of an electron beam refer to the pitch factor \((\alpha \equiv v_\perp/v_z)\) and the velocity spread \((\Delta v/v)\). Usually, the gyrotrons require a high pitch factor \((\alpha \geq 1)\) and a low velocity spread \((<10\%)\). Different types of electron sources have been proposed [12]–[15] to generate a high-quality beam for different applications. For example, a magnetron injection gun (MIG) [16]–[19] with a cone-shaped cathode and anodes is one of the commonly used electron sources for gyrotrons. The electric field between the electrodes accelerates electrons to the maximal kinetic energy. After the acceleration process, the electron motion obey the invariant of magnetic moment \([p_\perp^2 + B = \text{const.}]\) and the kinetic energy of electrons reaches a constant \([p_z^2 + p_\perp^2 = (1 + \alpha^2)p_z^2 = \text{const.}]\). These relations imply the change in the pitch factors as

$$a_2^2 = a_1^2 \frac{f}{1 + a_1^2(1 - f)}$$  \hspace{1cm} (2)

where \(f (= B_2/B_1)\) is a compression ratio of the magnetic field at an interaction section to the field near an emitter surface. The axial and transverse velocity spreads have a relation as \(\Delta v_z/v_z = a_2^2(\Delta v_\perp/v_\perp)\). The transverse velocity spread is related to the surface roughness and the thermal deformations of an emitter [20]. Except for the intrinsic velocity spreads, the fluctuations of the static electric and magnetic fields induce the velocity spread [21]. It reads

$$\frac{\Delta v_\perp}{v_\perp} = \frac{\Delta E_c}{E_c} - \frac{3}{2}\frac{\Delta B_c}{B_c}.$$  \hspace{1cm} (3)

Equations (2) and (3) predict the beam-quality parameters when the electrons are fully accelerated to the maximal kinetic energy. However, this study shows that the
beam-quality parameters predicted from (2) and (3) would be significantly affected by the acceleration process. In other words, the nonadiabatic effect between the electrodes deserves careful investigation.

**II. NONADIABATIC ELECTRON GUN**

Compared with the MIGs, a nonadiabatic electron gun has a cathode surface, which is perpendicular to the z-axis (i.e., the direction of the applied magnetostatic field) [13], [22]. Thus, the electrons are extracted parallel to the magnetic field and then passed through an inhomogeneous electric field between the electrodes. The velocity spread owing to the intrinsic properties of an emitter would be significantly reduced by considering the nonadiabatic electron motion [23]–[25]. We take advantage of the nonadiabatic motion during the acceleration of electrons to design an electron gun and even to improve an operating range. The proposed diode-type electron gun is composed of an anode and a cathode with an emitter on the concave surface. Such a configuration is different from the traditional nonadiabatic electron sources.

Fig. 1 shows the structure of the proposed electron gun (gray) overlaid with the magnetic field profile (black), the annular beam trajectories (red), and the equipotential lines (blue in the inset) calculated at a beam voltage of 16 kV. Unlike the traditional MIGs, the proposed electron gun has a relatively weak electric field on the emitter surface. The weak electric field results in the longer acceleration process.

Simulations are carried out by a 2-D particle trajectory program (EGUN code) [26] and verified with a 3-D commercial suite (Computer Simulation Technology (CST) Particle Studio). The optimized structure of the electron gun has an inclination angle of the emitter ($\theta_e$) equal to 84° and a suitable distance between the electrodes to avoid the high-voltage breakdown. The annular emitter with a width of 1.0 mm and a radius of 3.5 mm suggests a cathode loading ($J_b$) from 0.9 to 4.5 A/cm² at the desired beam current ($I_b$) from 0.2 to 1.0 A. The maximal cathode loading is smaller than the space-charge-limited current density derived from Langmuir’s equations [27], which ensures the lifetime of the emitters [28], [29]. In this case, the proposed electron gun is operated at the temperature-limited regime. In addition, we specially design a notch at the center of the cathode with a depth of 2.5 mm to align the equipotential lines along the emitter surface smoothly.

Fig. 2(a) shows the magnetic moment ($\mu \equiv 0.5 \gamma_0 m_0 \gamma^2 / B$) and the kinetic energy ($K \equiv (\gamma_0 - 1)m_0 c^2$) as the functions of the axial position $z$ of the electrons. The increasing kinetic energy indicates a section of the “nonadiabatic” electron motion, while a constant kinetic energy at $z > 3$ cm implies a section of the “adiabatic” motion. In addition, the change in the magnetic moment shows a similar profile with subtle periodic features owing to a nonzero transverse magnetic field during the acceleration of electrons. It is worth noting that the turning point at $z = 0.9$ cm is related to the rapid change in the electric field between the electrodes. When electrons travel through a region around this turning point, the beam-quality parameters are significantly changed, as shown in Fig. 2(b).

Fig. 2(b) shows the transverse velocity spread and the pitch factor as the functions of the axial position $z$ of electrons. By considering the change in the magnetic moment, the rapid change in the beam-quality parameters can be separated into two phases around a turning point at $z \sim 0.9$ cm. For the nonadiabatic electron motion in Phase 1, the transverse velocity spread rises from 24% to 32%, while the pitch factor decreases from 0.095 to 0.05. On the other hand, in Phase 2, the transverse velocity spread declines rapidly from 32% to 2.8%, while the pitch factor rises from 0.05 to 0.12. For the adiabatic electron motion at $z > 3$ cm, behaviors...
of the beam-quality parameters follow the predictions based on (2) and (3) (i.e., the gradually increasing pitch factor and a constant transverse velocity spread). However, these two equations are unable to explain the change in the beam-quality parameters during the nonadiabatic process ($z < 3\text{ cm}$).

Table I shows a comparison of the beam-quality parameters between the adiabatic calculations based on (2) and (3) and the EGUN code simulations. By considering the deviations in the electrostatic and magnetostatic fields near the cathode, the calculated transverse velocity spread is 24%, which shows a huge difference from the simulated result with a very low spread of 2.8%. The difference between the transverse velocity spreads suggests that the nonadiabatic effect plays a crucial role in the acceleration of electrons and affects the final beam-quality parameters. To study the nonadiabatic effect during the acceleration process, we start from the simulation and compare with the fundamental equations of a single electron motion.

Fig. 3(a) shows a simulated pattern of the electric field based on the EGUN code. The annular emitter is located on a concave surface with a weak electric field strength of 25 kV/cm. Two beam trajectories are sketched in dashed lines for electrons emitted from the inner (black) and outer (red) sides of the emitter. The motion of the electrons influenced by the rapidly changing electric field for $z < 1.4$ cm results in the rapid change in the beam-quality parameters in Fig. 2(b). Fig. 3(b) shows the transverse (dashed lines) and axial (solid lines) electric fields experienced by electrons from the inner (black) and outer (red) sides of the emitter. The rising axial electric field is associated with the increasing kinetic energy in Fig. 2(a), while the small deviation in the axial electric field $E_z$ experienced by different electrons in Phase 1 ($z < 0.9$ cm) comes from the nonuniform field distribution near the cathode. On the other hand, the transverse electric field $E_\perp$ along the $z$-axis decreases from 3 to $-5$ kV/cm with a turning point at $z \sim 0.9$ cm, while the deviation in the transverse electric field experienced by different electrons during the entire nonadiabatic process gradually declines. In short, both electric field components slowly approach zero along the $z$-axis, which suggests that the nonadiabatic motion of electrons progressively becomes the adiabatic motion.

Fig. 3(c) shows the simulated electron velocities from the inner (black) and outer (red) beam trajectories. The increasing axial velocities are attributed to the gradually increasing kinetic energy in Fig. 2(a). Furthermore, the deviations in the transverse velocities from different trajectories around the turning point at $z \sim 0.9$ cm result in the rapid change of the transverse velocity spread in Fig. 2(b). The deviations are associated with different transverse electric fields experienced by electrons with different trajectories. In Phase 1 ($z < 0.9$ cm), the rapidly increasing transverse velocity of the inner electron is attributed to the higher transverse electric field. While in Phase 2 ($z > 0.9$ cm), the increase in the transverse velocity of the inner electron slows down due to the smaller transverse electric field. As the electrons move forward, the transverse velocity of each electron gradually converges with each other, which reduces the transverse velocity spread. The change in the transverse velocities at the turning point ($z \sim 0.9$ cm) is associated with the reversal direction of the electric field experienced by each electron. It is worth noting that the step change in the transverse velocity is also the reason for the dent of the pitch factor around the turning point in Fig. 2(b).

To demonstrate the influence of the electric field during the nonadiabatic motion of electrons, we calculate the velocities from the equations of a single electron motion. By considering the azimuthal symmetry in the cylindrical coordinates, the static electric and magnetic fields can be written as
also displayed. Fig. 4(a) shows that the larger notch radius with respect to a notch radius and cathode inclination angle is the higher pitch factor due to an enhancement of the transverse electric field near the edge of the emitter.

\[ \alpha = \frac{E_r}{\gamma_0 m_e (E_z - v_0 B_z)} \]  

According to the simulations of the nonadiabatic motion, the electron velocities \( v_z \) and \( v_r \) show a strong dependence on the axial and radial positions \( z \) and \( r \), respectively. In this case, the total derivatives of each velocity component can be reduced into a partial derivative form during the acceleration process. From the distribution of the electrostatic and magnetostatic fields between electrodes, one can deduce the electron velocities as well as the beam-quality parameters.

The electric field plays an important role on the nonadiabatic electron motion. The magnetic field in (4) and rewrite the equations into an integration form with a relation

\[ v_z^2 \approx \frac{2q}{\gamma_0 m_e} \int E_z dz \]  
\[ v_r^2 \approx \frac{2q}{\gamma_0 m_e} \int E_r dr. \]

The calculated electron velocity based on an integration of the electric field [gray lines in Fig. 3(c)] shows similar features to the change of simulated electron velocity [black and red lines in Fig. 3(c)], including the gradually increasing axial component and the step change in the transverse component. The agreement of the transverse velocity between the calculation and simulation for \( z < 3 \text{ cm} \) suggests that the rapidly changing electric field is the main reason to induce the significant change in the beam-quality parameters at \( z \approx 0.9 \text{ cm} \) in Fig. 2(b). As the motion of electrons becomes adiabatic for \( z > 1.4 \text{ cm} \), the magnetic field starts to dominate the electric motion of each electron in the transverse direction. The beam-quality parameters for \( z > 1.4 \text{ cm} \) also follow the predictions based on (2) and (3). It is the reason for the deviation of the transverse velocity between the calculation and the EGUN code simulation. In short, to study the rapid change in the beam-quality parameters during the nonadiabatic electron motion, it is important to analyze a distribution of the electric field between the electrodes.

### III. Simulated Results

Based on the analysis of the beam-quality parameters during the nonadiabatic electron motion, we optimize the proposed nonadiabatic electron gun with the parameters listed in Table II. Sensitivity analysis of the beam-quality parameters with respect to a notch radius and cathode inclination angle is also displayed. Fig. 4(a) shows that the larger notch radius results in the higher pitch factor due to an enhancement of the transverse electric field near the edge of the emitter. The desired pitch factor \( \alpha = 1.5 \) is obtained at a notch radius of 2.5 mm, which also gives the local minimum of the transverse velocity spread. On the other hand, Fig. 4(b) shows that the pitch factor changes slightly with respect to the inclination angle, while the transverse velocity spread grows with the increasing cathode inclination. As the inclination angle approaches 90° (i.e., the cathode surface perpendicular to the \( z \)-axis), the emitted electrons experience different patterns of the electric field. Integrations of the electric field along the beam trajectories imply that the transverse velocity of each electron diverges from each other during the nonadiabatic electron motion. On the other hand, as the inclination angle becomes smaller, the motion of the electrons has different trajectories and different patterns of the electric field, which adjusts the transverse velocity spread of an electron beam. For the proposed electron gun, the transverse velocity spread of the
Fig. 5. (a) Pitch factor and (b) transverse velocity spread as functions of the beam voltage and the magnetic field. Shaded areas mark the region of interest.

The electron beam has a low terrace of inclination angle in a range of 82°–84°.

Fig. 5 shows the simulated beam-quality parameters concerning the beam voltage \( V_b \) and the applied magnetic field \( B_0 \). The beam voltage and the magnetic field have a simple and linear relationship with the pitch factor as follows:

\[
\alpha = 1.5 + 0.135 \times (V_b - 16) - 0.63 \times (B_0 - 7.7) \quad (7)
\]

where \( B_0 \) is in the unit of Tesla and \( V_b \) is in the unit of kilovolt. The transverse velocity spread has a complex dependence on both \( B_0 \) and \( V_b \) with a relation

\[
\Delta v_{\perp} / v_{\perp} (%) = 2.6 + 0.0215 \times (V_b - 19)^2 + 0.3 \times (B_0 - 8)^2 - 0.125 \times (V_b - 19)(B_0 - 8) \quad (8)
\]

The transverse velocity spread is small and good enough for the applications of the frequency-tunable gyrotron. Adjusting either \( V_b \) from 15 to 17 kV or \( B_0 \) from 7.4 to 8.0 T will maintain a high pitch factor of 1.5 and a minimal transverse velocity spread of 2.8%. The working range of the beam voltage and the magnetic field is broad and can be further extended depending on the requirements of the gyrotron operations.

Fig. 6 shows the photograph of the proposed cathode fabricated by Spectra-Mat, Inc. This cathode has been used to study the frequency-tunable Gyro-BWO at the TE_{02} mode in 203 GHz. By considering the similar interacting cavity design proposed by Chen et al. [30], we expect to obtain a peak efficiency of 30% with a 3-dB tuning bandwidth of 8.5 GHz for a beam current of 0.5 A.

IV. CONCLUSION

In conclusion, we proposed a diode-type electron gun with an emitter on the concave surface of the cathode, in which the nonadiabatic electron motion plays an important role. The analytical adiabatic theory fails to predict the pitch factor and the velocity spread in the nonadiabatic electron motion. Instead, the authors proposed a way to estimate the velocity of an electron motion, where the axial velocity is accurately obtained by integrating the electric field along the \( z \)-axis. In addition, the significant change in the beam-quality parameters can be predicted by integrating the electric field in the radial direction. The simulations show that the proposed electron gun can produce an electron beam with the high pitch factor of 1.5 and the low transverse velocity spread of 2.8% over a very wide parameter space spanned by a magnetic field (7.4–8.0 T) and a beam voltage (12–22 kV). The design takes advantage of the nonadiabatic electron motion to adjust the deviations of each electron velocity and provide a feasible structure for the electron gun design.

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