The Telltale Heartbeat: Detection and Characterization of Eccentric Orbiting Planets via Tides on Their Host Star

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Abstract

We present an analytic description of tides raised on a star by a small orbiting body. In particular, we highlight the disproportionate effect of eccentricity and thus the scope for using these tides to detect and characterize the orbits of exoplanets and brown dwarfs. The tidal distortions of the star produced by an eccentric orbit are, in comparison to a circular orbit, much richer in detail and potentially visible from any viewing angle. The magnitude of these variations is much larger than that in a circular orbit of the same semimajor axis. These variations are visible in both photometric and spectroscopic data and dominate other regular sources of phase variability (e.g., reflection and Doppler beaming) over a particularly interesting portion of parameter space. These tidal signatures will be a useful tool for planet detection on their own and, used in concert with other methods, provide powerful constraints on planetary and stellar properties.

Key words: asteroseismology – methods: analytical – planetary systems – planets and satellites: detection – stars: kinematics and dynamics – stars: oscillations

1. Introduction

In the coming month, it is likely that as many as 100 new exoplanets will be found. We cannot tell you their mass, their size, or where they will be in the sky. We can, however, guess at which angle they will be viewed from: most detected exoplanets have been seen in edge-on systems, with the planet passing through or close to our line of sight to the star.

This is because the two mechanisms by which we have discovered the vast majority of planets, the transit method and the radial velocity (RV) method, both depend strongly on the angle from which the system is viewed. The former searches for planets partially eclipsing their host star (Henry et al. 2000; Charbonneau et al. 2000), causing the star to appear dimmer. The latter looks for changes in the motion of a star along our line of sight, visible in the Doppler shift of the light emitted, as it orbits around the center of mass of the system (Mayor & Queloz 1995). However, as instruments, statistical tools, and our understanding of planetary systems improve, other methods for deducing the presence or properties of exoplanets are becoming more useful—see, e.g., Wright & Gaudi (2013) and Fischer et al. (2014) for recent reviews.

Tidal luminosity modulations (often called ellipsoidal variations in the literature; Morris 1985) are one of these methods, whereby a planet tidally distorts its host star slightly. As the planet orbits, the distortions follow the planet’s motion. Picture the star as a poorly thrown lemon: as it tumbles through the air, it appears to vary in size and shape, small and round to large and elongated. When it is seen side-on, it looks larger, and when seen end-on, smaller. Thus, the luminosity appears to change as it turns from side-on (brighter) to end-on (dimmer). These regularly rotating bulges, drawn by a planet on a circular orbit, lead to the star sinusoidally changing its brightness.

This photometric effect is usually small, and though it has been identified in the light curves of exoplanet hosts, it has only recently become feasible to consider it as a primary method to detect new candidate exoplanets (Faigler & Mazeh 2011; Jackson et al. 2012; Placek et al. 2014; Knuth et al. 2017).

However, the classic formulation of ellipsoidal variations assumes a circular orbit.

A better analytic understanding of the effect of eccentricity is important because such planetary systems are common, including among massive close companions, or hot Jupiters (Kane et al. 2012; Kipping 2014; Van Eylen & Albrecht 2015; Winn & Fabrycky 2015). Even planets with extreme eccentricities (e > 0.9) have been found (Naef et al. 2001; Husnoo et al. 2012; Kane et al. 2016; Bonomo et al. 2017). While these highly eccentric systems have controversial implications for planet formation, we will show that they are a blessing for the detection of tidal signatures.

In this paper, we will present an analytic model of tidal luminosity variability and how this can be used to find and characterize exoplanets. These tides modulate both the light from and the apparent velocity of their host star. We will particularly highlight the idea that significantly eccentric systems create visible signals regardless of viewing angle, allowing planets in face-on orbits to be identified as readily as if they were edge-on.

2. Raising Tides

We can split the tidal effects of an orbiting body on its host star into three regimes.

1. Circular equilibrium tides—Quasi-static deformation of the star due to the planet’s gravitational influence over a circular orbit.
2. Eccentric equilibrium tides—Variations in the quasi-static tide over an eccentric orbit, which produce a characteristic “heartbeat” signature.
3. Dynamical tides—Stellar oscillations excited by the tidal transfer of energy from the orbit to normal modes in the star.

Equilibrium tides (or ellipsoidal variations; Morris 1985) have been studied in detail in the context of planet–star interactions and identified in a handful of confirmed planetary
systems (Welsh et al. 2010; Mislis & Hodgkin 2012; Borkovits et al. 2014), but they have seen scant use as a primary method to detect new exoplanets.

Eccentric equilibrium tides will be the main focus of this paper. For high orbital eccentricity, these produce a characteristic cardiogram-like phase-variability curve, explaining the shorthand for the well-studied binary "heartbeat" stars (Welsh et al. 2011; Thompson et al. 2012; Shporer et al. 2016; Fuller 2017) and even a candidate "heartbeat planet" (de Wit et al. 2017). It would not be unfair to call these tides “time-varying” or “eccentric” corrections to circular-orbit ellipsoidal variations; however, by virtue of this time variability, they hold considerably more information. For massive, eccentric, and close bodies, the equilibrium tide can be large and remain visible in and out of the orbital plane. Kane & Gelino (2012) posited an approximate equation for the flux change caused by tides over an eccentric orbit, but here we derive the correct form analytically and also find the velocity variations it causes on the surface. Throughout the body of this paper, we focus on the quasi-static, equilibrium tide limit (e.g., Goldreich & Nicholson 1989).

Dynamical tides are an interesting and potentially under-explored extension of the study of the tidal effects of planets and brown dwarfs on their host star. They have been studied in detail in the field of compact objects, particularly with reference to tidal capture and dissipation of orbital energy (Press & Teukolsky 1977; Lee & Ostriker 1986; Fuller & Lai 2011), and are often included in consideration of heartbeat stars. In planetary systems, where the mass of the perturbing body is typically several orders of magnitude less than the star’s, they have previously been ignored. While we focus on equilibrium tides in the main part of this paper, we discuss dynamical tides briefly in Appendix A.

Numerical methods to solve these equations exist in both the field of binary stars (Wilson & Devinney 1971; Orosz & Hauschildt 2000; Prša & Zwitter 2005) and the more specific case of a planetary companion (see Gai & Knut 2018 for a summary and comparison of models). These models mostly expand on the analytic formula set out in Kopal (1959). However, in the case of two bodies with an extreme mass ratio, this calculation can be significantly simplified. We derive this result from scratch, including the full effect of eccentricity, giving simple directly calculable results that will hugely reduce the computational cost of fitting data to a physical model.

First, we will set out the basic theory behind the excitation of tides by an external potential and briefly summarize some of its results. For many readers, the fine detail of tidal perturbations may be of less interest, and we suggest they skip directly to Section 2.3, where we relate this to elliptical orbits of a small perturber. Many readers may even wish to move directly to Section 4, where we map out the observable consequences of our results in the context of exoplanets and recap the expressions necessary to model them.

Small variations in stellar properties, be they the movement of stellar material or perturbations to the pressure, potential, or density, can be expressed in the same form. Under the assumption that the star is spherically symmetric and in hydrostatic equilibrium, and that the system is adiabatic, these can be shown (see Christensen-Dalsgaard 2002 for a review) to separate into radial, angular, and time-dependent components:

\[ \propto f_n(r)Y_l^m(\theta, \phi)e^{-i\omega t}. \quad (1) \]

\[ \nu \]

Figure 1. Real component of the spherical harmonics, \( Y_l^m(\theta, \phi) \), with positive values in blue and negative in red. Spheres are shown from an oblique viewing angle with the azimuthal angle \( \phi = 0 \) marked by a black dot (and \( \phi \) increasing to the right). Modes only exist for \(|m| < l\), and note that only harmonics where \( l + m \) is even are excited by tides (these are highlighted in black). Negative values of \( m \) are not shown; for even \( m \), these are identical, and for odd \( m \), there is a sign inversion. The variation with \( \phi \) goes as the cosine, and to picture the imaginary part, one could simply substitute the sine of the angle (noting that there is no imaginary component for \( m = 0 \)). All profiles shown are normalized to their own maximum value, though their actual amplitudes vary.

Here \( Y_l^m \) is the spherical harmonic with degree \( l \) and azimuthal order \( m \), and \( f_n \) is some function describing the radial variations, which cannot be simply expressed and must be found via numerical models, though the meaning of the radial wavenumber \( n \) is sufficiently simple: \( |m| \) is the number of nodes (radii \( r \)) where \( f_n = 0 \) in the radial direction.

For geometrical intuition, \( 2m \) is the number of nodes (angular positions where \( Re\{Y_l^m\} = 0 \)) along the star’s equator, and \( 2l \) is the maximal number of nodes around any great circle. See Figure 1 for a visualization of low-order modes. The full deformation of the star can be built up from the linear combination of these wave modes.

2.1. Time Evolution of Perturbations

We can express (i) the gravitational potential due to an orbiting body and (ii) perturbations to the surface of the star as the sum of the individual wave modes. Then we can calculate the force acting on a given wave mode due to gravity from item (i) and match it to the evolution of this same mode of the perturbation from item (ii).

For this section, we will use spherical coordinates, \( r = (r, \theta, \phi) \). The position of a gravitating body (which we shall call a planet, but the same analysis applies to stars or compact objects) with mass \( M_p \) orbiting a star centered at \( r = 0 \)
can be described by \( D(t) = (D(t), \frac{\Phi}{\pi}) \). Note that the choice \( \theta = \frac{\pi}{2} \) sets the orbit in the equatorial plane, and that we are free to set the orientation of \( \phi \) and so have chosen to align \( \phi = 0 \) with the position of the planet at periapsis.

Following the notation of Fuller & Lai (2011), we can find the gravitational potential as a sum over spherical harmonics (a fuller derivation of this can be found in Jackson 1998, Section 3.6). At some position in the star, \( r \), the gravitational potential due to an orbiting body is described by

\[
U(r, t) = -\frac{GM_p}{|D - r|} = -\frac{GM_p}{|D|} \sum_{l, m} \frac{W_{lm} R_{nlm}}{D(t)^{l+1}} e^{-i m \phi(t)} Y_{lm}^m(\theta, \phi),
\]

(2)

where, in the second equality, we have expressed the potential in spherical harmonics. The numeric factor, \( W_{lm} \), is 0 for odd \( l + m \) and

\[
W_{lm} = (-1)^{l+m} \frac{4\pi}{2l+1} \frac{(l-m)!(l+m)!}{2^{l-m}(l+m)!} \]  

\[
\]  

(3)

otherwise. In other words, only modes with even \( l + m \) appear in the gravitational potential. This representation of \( U(r, t) \) allows us to pick out the effect of gravity on any particular mode by taking individual terms, \( U_{nlm} \), from the above series.

Now we express the displacement at some position as the sum of individual modes,

\[
\xi(r, t) = \sum_{n,l,m} \xi_{nlm}(r, t) = \sum_{n,l, m} a_{nlm}(t) \xi_{nlm}(r),
\]

(4)

where \( a_{nlm} \) has dimensions of length and encodes the time dependence of the mode amplitude, and \( \xi_{nlm} \) is a dimensionless vector quantity obeying

\[
\tilde{\xi}_{nlm}(r) = [\xi_{r,nlm}(r)\hat{r} + r\xi_{\perp,nlm}(r) \nabla] Y_{lm}^m(\theta, \phi)
\]

(5)

and normalized such that

\[
\int_V \rho \xi_{nlm}^* \xi_{nlm} d^3r = M
\]

(6)

for a star of mass \( M \), with density \( \rho(r) \) and volume \( V \) (note that this choice of normalization differs from that in Fuller & Lai 2011).

Each mode in the star behaves as a driven simple harmonic oscillator; with the driving force coming from the perturbing gravitational potential expressed in Equation (2) (and assuming there is no coupling between modes), the displacement obeys

\[
\ddot{\xi}_{nlm} + \omega_{nlm}^2 \xi_{nlm} = \ddot{a}_{nlm} \xi_{nlm} + \omega_{nlm}^2 a_{nlm} \xi_{nlm} = -\nabla U_{nlm}.
\]

(7)

Taking the scalar product of Equation (7) with \( \xi_{nlm}^* \), multiplying by the density, and integrating over the volume gives us the complex time-dependent amplitude

\[
\ddot{a}_{nlm} + \omega_{nlm}^2 a_{nlm} = W_{lm} Q_{nlm} \frac{GM_p}{R^2} \left( \frac{R}{D} \right)^{l+1} e^{-im\phi}.
\]

(8)

where the dimensionless tidal coupling coefficient is

\[
Q_{nlm} = \frac{R^{l+1}}{M} \int_V \rho \xi_{nlm}^* \cdot \nabla (r^2 Y_{lm}^m) d^3r,
\]

(9)

\( M \) is the mass of the star, and \( R \) is its radius.

This is the equation of a forced harmonic oscillator. The amplitude of perturbations will tend to oscillate around some equilibrium value. The equilibrium value itself evolves with time due to the planet’s motion and thus the changing gravitational potential. Note that, in a system with a planetary companion, the equilibrium state will always have some nonzero \( a_{nlm} \) to account for the small perturbation of the planet’s gravity (even for circular orbits). Ignoring this perturbation to the initial state will give an oscillation around the true result (Kumar et al. 1995).

The values of \( \xi_{nlm}, \omega_{nlm}, \) and \( Q_{nlm} \) all depend on the assumed density profile of the star. These can be found by detailed stellar modeling with software such as MESA or from simpler models such as a polytropic density profile.

It may initially surprise the reader that none of these stellar parameters depend on \( m \). This is because taking the complex conjugate of spherical harmonics is equivalent to changing the sign of \( m \); thus, integrating over the dot product causes these terms to cancel.

As we are only interested in radial deformations at the surface of the star (or, more accurately, at the photosphere, a small correction we do not make here), we need only use the radial component of the displacement at the surface, \( \xi_{r,nlm}(R) \).

Thus, any mode has a characteristic displacement at the surface of the star,

\[
\delta R_{nlm}(t) = \xi_{r,nlm}(R) a_{nlm}(t),
\]

(10)

which will then be modulated by the angular dependence that comes from the spherical harmonics. We can now define the fractional change in radius of the star, at an angle \((\theta, \phi)\), as

\[
\epsilon(t, \theta, \phi) = \frac{\delta R}{R} = \frac{1}{R} \sum_{n,l,m} \text{Re} \{ Y_{lm}^m(\theta, \phi) \delta R_{nlm}(t) \}.
\]

(11)

2.2. Equilibrium Tides

Each mode in the star has a characteristic frequency, \( \omega_{nlm} \). If the orbital frequency of the perturbing body is close to this frequency, there can be a strong coupling between the orbit and the tide, and a large amount of energy can be transferred into oscillations.

However, if the frequency of the orbit is much smaller than that of the mode, there is almost no coupling between the tide and the orbit, and thus effectively no energy transferred (Kumar et al. 1995). This is because the system adapts to a change in gravitational potential on a timescale much shorter than that over which the potential changes. The tide is effectively in quasi-static equilibrium throughout the orbit, and the acceleration term in Equation (8), \( \ddot{a}_{nlm} \), can be taken to be approximately zero.

The characteristic frequency of an elliptical orbit (at periapsis distance \( r_{peri} \), as this is where most energy is transferred and the frequency is highest) is

\[
\omega_{peri}^2 \approx \frac{GM}{r_{peri}^3}.
\]

(12)
This must be compared to the frequency of stellar normal modes, which we write as a dimensionless numeric factor $\bar{\omega}_{nl}$ multiplied by the natural frequency of the star,

$$\omega^2_{nl} = \bar{\omega}_{nl}^2 \frac{GM}{R^3}. \quad (13)$$

The exact values of $\bar{\omega}_{nl}$ depend on the density model. In an $n_{\text{poly}} = 3$ polytrope, a reasonable approximation to a Sun-like star, the $n = 0$ mode (the $f$-mode) has a value of roughly 3. Higher-frequency modes, with $n > 0$ (or $p$-modes, where pressure is the restoring force), have large values of $\bar{\omega}_{nl}$. Lower-frequency modes, with $n < 0$ (or $g$-modes, with gravity as the restoring force), can exist but do not generally propagate to the surface of the star (though in stars with thin convective envelopes, they may be detectable at the photosphere).

Thus, the condition for the tide to remain in quasi-static equilibrium throughout the orbit,

$$\frac{\omega_{\text{peri}}}{\omega_{nl}} \approx \frac{1}{\bar{\omega}_{nl}} \left( \frac{R}{r_{\text{peri}}} \right)^3 \ll 1, \quad (14)$$

is generically true of $f$- and $p$-modes for all but the most extreme orbits ($r_{\text{peri}} \sim 2R$) and lowest-frequency oscillations. However, it is possible for $\omega_{\text{peri}} \gtrsim \omega_{nl}$ for a wide range of $g$-modes and orbital parameters.

As we are concerned primarily with surface deformations, for the rest of this work, we can reliably approximate the system as remaining in (time-evolving) equilibrium, i.e., setting the acceleration term in Equation (8) equal to zero. Hence,

$$a_{nlm}(t) = \frac{W_{lm} Q_{nl}}{\bar{\omega}_{nl}^2} \frac{GM_p}{R^2} \left( \frac{R}{D} \right)^{l+1} e^{-im\phi} \quad (15).$$

The only dependence of $\delta R_{nlm}$ on radial wavenumber $n$ is within the dimensionless quantities derived from stellar model assumptions, so we can separate out the total effect of all radial mode displacements into a single parameter,

$$\beta_l = \frac{1}{\bar{\omega}_{nl}} \sum_n \frac{Q_{nl} \bar{\omega}_{nl}(R)}{\bar{\omega}_{nl}^2} \left( \frac{R}{D} \right)^{l+1} e^{-im\phi}, \quad (16)$$

and thus express the displacement associated with a mode as

$$\delta R_{lm}(t) = \left[ \frac{M_p}{M} \left( \frac{R}{D(t)} \right)^{l+1} \beta_l W_{lm} e^{-im\phi(t)} \right] R, \quad (17)$$

where the term in brackets is dimensionless.

When the potential term is dominated by a particular wavenumber $l$, this allows us to rewrite Equation (11) in terms of the potential,

$$\epsilon = -\frac{\beta_l U}{R g_0}, \quad (18)$$

where

$$g_0 = \frac{GM}{R^2}. \quad (19)$$

is the surface gravity of the unperturbed star. Thus, $\beta_l$ can be calculated explicitly (see, for example, Generozov et al. 2018). However, the above form can also be compared with the theory of equipotentials (Goldreich & Nicholson 1989), where surfaces of constant density and pressure follow surfaces of constant potential, and hence the outer surface of the star can be found by finding the gravitational perturbations at the stellar surface. Such an approach, in the limit of the star being tidally locked to the planet, yields $\beta_l = 1$, regardless of internal stellar structure.

This is likely to be a good approximation and lower limit for any star whose mass is mostly confined to a small central region (as is the case for stars with a polytropic index of 1 or above). This simple argument implicitly makes the Cowling approximation, i.e., the assumption that first-order perturbations to the potential have small effects compared to perturbations to other fluid properties, and there may be regimes where it is of interest to relax this assumption and calculate $\beta_l$ more explicitly.

One consequence of this simple form of radial deviations is that for a given $m$, the time evolution of $\delta R_{lm}$ is independent, save multiplicative constants, of the wavenumbers $l$ and $n$, the stellar model imposed, or the size or mass of the star and planet.

We test the assumption of quasi-static equilibrium in Figure 2, which shows the time evolution of one mode ($l = 2$ and $m = 0$, picked for visual clarity but representative of all modes) and the error associated with approximating this evolution by Equation (17). Perhaps surprisingly, the $f$-mode is not the dominant contributor to the perturbation, and the lowest-$n$ $p$-modes also contribute. Only the lowest-frequency $g$-modes show any signs of oscillatory behavior (i.e., have significant energy transferred to the mode after periapsis). Examining the errors, our approximation holds true to within a fraction of a percent for all modes of interest ($f$- and $p$-modes). There is a
clear trend of increasing error with decreasing frequency, with
significant error only occurring for g-modes with very small
amplitudes.

Thus, Equation (17) is both an excellent approximation to
the evolution of tidal perturbations and a marked simplification
for computation. In Appendix A, we will explore in more detail
the dynamical tides excited in the low-frequency g-modes.
Throughout this work, however, we can safely work
under the assumption that each mode is in equilibrium, varying
only with the orbit of the perturber.

2.3. Eccentric Orbits

The time evolution of Equation (17) depends on the exact
orbit of the perturbing body. It is clear that for a circular orbit,
there is a constant radial deformation that corresponds to the
standard equilibrium tide. However, in this work, we are
primarily concerned with also capturing the “heartbeat” by
including the effects of eccentricity in a simple description of
tides.

Throughout this paper, we will show results for a simple test
case, a closely orbiting eccentric hot Jupiter, with parameters
that lie within the observed distribution of exoplanets (Winn &
Fabrycky 2015 ) and host stars (Boyajian et al. 2013 ). In all of
our figures and numerical calculations, we shall use an example
planet with $M_p = 1.2 M_\odot$, $R_p = 1.5 R_\odot$, $M_* = 5 M_\odot \approx 0.005 M_\odot$, $a = 10 R_\odot \approx 0.05 au$, and $e = 0.25$.

The planet will orbit in a Keplerian potential (assuming
$M_p \ll M$) obeying

$$D(t) = a(1 - ec_\odot(t)),$$

$$\sqrt{1 - e} \ t_\odot = \sqrt{1 + e} \ t_\odot, \quad (20)$$

and

$$t(\eta) = \sqrt{\frac{a^3}{GM}} (\frac{\eta}{\epsilon} - c_\odot). \quad (21)$$

For the sake of conserving paper and pixels, we use the
shorthand $\cos \ x \to c_x$ (and, similarly, $\sin \ x \to s_x$; $\tan \ x \to t_x$).

The parameterization via eccentric anomaly $\eta$ is useful for
expressing the orbit simply, and $\eta(t)$ can be easily found
numerically for a given $\tau$ (Binney & Tremaine 2008).

Figure 3 shows the time evolution of all of the $l = 2$ and 3
modes. The $l = 2$ modes dominate, and we will only consider
these for further calculation (this will not affect our results
beyond a 10% level, though higher-$l$ modes may be of interest
for future study).

By limiting ourselves to $l = 2$ modes and thus dropping the
subscript on $\beta$, we express the radial displacements (via
Equations (11) and (17)) very simply in terms of two
dimensionless parameters:

$$\alpha = \frac{\beta M_p}{4 M} \left( \frac{R}{a} \right)^3, \quad (23)$$

which gives us the amplitude of the surface variations
(neglecting $m$-dependent terms), and

$$\gamma(t) = \alpha (1 - e c_\odot)^{-3}, \quad (24)$$

which gives the time dependence of the amplitude of the
perturbation.

There is a second time-dependent component of the tides
that depends simply on the planet’s orbit at a given
$\Phi(t)$. In other words, for an observer sitting on the planet
itself, only the magnitude of the deformation, $\gamma$, would appear
to change. For an observer at a fixed viewing angle, we must
also include the time dependence of $\Phi$, though we may simplify
the calculations by defining a new equatorial angle

$$\psi(\phi, t) = \phi - \Phi(t). \quad (25)$$

Figure 4 shows a sketch of these coordinates.

Incorporating the spherical harmonics and time evolution of
the orbit, we can write the fractional radial displacement as

$$c(t, \theta, \psi) = \gamma (A s_2^2 c_2^2 + B s_3^2 c_3^2 + C c_2^2), \quad (26)$$

where the constants $A$, $B$, and $C$ depend on the mode in
question. Table 1 summarizes the values of these constants for
the $l = 2$ modes.

For small $\gamma$ (true for all systems of interest in this work), this
is, to first order, the equation of a perfect ellipsoid, with axes of
length $(1 + \gamma A)$ in the direction pointing toward the planet,
length $(1 + \gamma B)$ perpendicular to this direction in the
equatorial plane, and length $(1 + \gamma C)$ out of the plane. Also note that $A + B + C = 0$ for all $l = 2$ modes, implying that this is a volume-
conserving deformation.

Examining the modes individually, we see that $m = 0$ is a
spherically symmetric equatorial bulge squeezed at the poles
(i.e., an oblate “pancake” ellipsoid). The $m = \pm 2$ modes follow
the planet’s motion, and their surfaces are elongated along the
axis pointing toward the planet, diminished perpendicular to this
direction in the orbital plane, and unchanged out of the plane
(i.e., a triaxial “surfboard” ellipsoid). The sum of all of the $l = 2$
mode gives a large displacement pointing toward the planet and
a symmetric squeezing perpendicular to this direction (i.e., a prolate “lemon” ellipsoid).

As $\gamma$ varies over an orbit, the magnitude of these deformations changes, but their general form does not. Simple inspection shows that values of $\gamma$, and hence the distortions to the star, are largest when the planet is closest to the star and drop off rapidly with distance. Even for a circular orbit (and hence a constant $\gamma$), there is still a time dependence in $\epsilon$ due to the ellipsoid following the planet’s motion (i.e., the time evolution of $\Phi$). This expression inherently captures all behavior of equilibrium tides for both circular and eccentric orbits.

Note that while the peak of the distorted surface moves with the planet, the material on the surface only moves very small distances tangentially. The star is not rotating. Material is primarily displaced radially, and, much like a wave in the ocean, the rising and falling swell of the stellar surface appears to be a moving wave.

From Equation (26), we can also find the RV of any point on the surface:

$$\delta v_r(t, \theta, \psi) = R \dot{v}_r(t, \theta, \psi).$$

From Equation (21), it can be shown that

$$\dot{\psi} = -\Phi = -\frac{\sqrt{1 - e^2}}{1 - ec_{\eta}} \dot{\eta},$$

and from Equation (22), we find

$$\dot{\eta} = \frac{GM}{a^3} (1 - ec_{\eta})^{-1}.$$

Thus,

$$\delta v_r = -\kappa(t)[2\sqrt{1 - e^2} s_{\Phi} s_{\theta} (A - B) + 3\kappa_0 (A s_{\Phi}^2 e_c^2 + B s_{\Phi}^2 d_{c}^2 + C d_{c}^2)],$$

where everything in brackets is dimensionless and of order unity and

$$\kappa(t) = \gamma(1 - ec_{\eta})^{-2} \frac{GM}{a^3} R$$

contains the dimensionality and time dependence of the velocity (although again, $\psi$ also has a time dependence). The horizontal velocity will be important for calculations of the inferred velocity of the stellar surface, but we save calculation of this for Section 3.3.

We now have all the tools we will need to calculate the luminosity and RV changes due to eccentric equilibrium tides.

3. Observing Heartbeat Planets

There are two directly observable features that stem from these small tidal deformations of a star.

1. In the light curve—As the star deforms, the gravity at the surface changes, changing the hydrostatic equilibrium state. This leads to changes in temperature on the surface and associated changes in flux. The changes in surface area, projected along the line of sight, also modulate the luminosity of the star.

2. In RVs—The movement of the surface along our line of sight can be seen in the Doppler shift of absorption lines in the stellar spectrum.

There is also the strong possibility of this periodic behavior being detectable in the power spectrum (the measure of energy in oscillations of different frequencies) of the light curve. The signals, as we will show, are not sinusoidal; hence, there will not be a sharp peak but rather excitation of higher-order harmonics (Esteves et al. 2013; Armstrong & Rein 2015; Cowan et al. 2017; see also the power spectra of heartbeat stars, e.g., Fuller 2017). We will not explore power spectra further in this paper but shall save a more focused analysis of the power spectral signatures of a planet on an eccentric orbit for future work (Penoyre & Sandford 2018).
For the remainder of this paper, we will limit ourselves to only modes with $l = 2$ and show how the light curve and RV profile of a heartbeat star can be calculated. Throughout, we will consider observations of the star as viewed from an angle, $v(v)$, as shown in Figure 4. Simple sketches of the orbits at the moment of periapse are shown in Figure 5. Figure 6 then shows representations of what the star itself would look like at periapse from a range of viewing angles. The tidal effects are exaggerated to make the deformation of the star visible. The color of each star has no physical significance, but the same color will be used for each projection over the next series of plots.

For the remainder of this paper, we will limit ourselves to only modes with $l = 2$ and show how the light curve and RV profile of a heartbeat star can be calculated. Throughout, we will consider observations of the star as viewed from an angle, $v(v)$, as shown in Figure 4. Simple sketches of the orbits at the moment of periapse are shown in Figure 5. Figure 6 then shows representations of what the star itself would look like at periapse from a range of viewing angles. The tidal effects are exaggerated to make them visible, but we can build an intuition about the geometry of the tidal deformation and its effect on the apparent brightness of the star. Simply put, the star is "lemon"-shaped, stretched along the axis directed toward the planet (and squeezed perpendicular to this direction). The regions pulled furthest from equilibrium experience the most gravity darkening (explained in more detail in the next section), and where the stellar material has been compressed, it radiates more brightly.

3.1. Coordinates and Projected Surface Integrals

All of the observables we are interested in in this paper can be expressed as scalar quantities (such as flux or line-of-sight velocity) integrated over a projected surface area. As the deformations are fully expressible as linear combinations of spherical harmonics, these integrals can be solved analytically. Here we derive the relevant integrals but save their detailed
Throughout this section, we consider only mildly aspherical ellipsoids (i.e., assume $\epsilon \ll 1$).

For the choice of coordinates, it is convenient to use the angle $\psi = \phi - \Phi$ and align the $x$-axis with $\psi = 0$ and the $z$-axis with $\theta = 0$. Thus, we recover conventional spherical coordinates in a frame aligned with the planet (hence, the frame of reference is rotating, though we will only ever be interested in properties of the system at a given $t$). A sketch of these coordinate systems is shown in Figure 4.

Consider an infinitesimal area element on the surface of the (deformed) star,

$$dA = R^2 s_0 d\theta d\psi,$$  \hspace{1cm} (32)

where $R' = R(1 + \epsilon)$.

For an unperturbed star, we have the area element

$$dA_0 = R^2 s_0 d\theta d\psi,$$  \hspace{1cm} (33)
and the unperturbed area of the star thus obeys
\[ A_0 = \int dA_0 = 4\pi R^2. \]  
(34)

Any part of the surface of the star we observe will have an apparent two-dimensional projected area,
\[ da = (\hat{n} \cdot \hat{l}) dA, \]  
(35)
where \( \hat{n} \) is the unit normal to the surface and \( \hat{l} \) is the direction of the line of sight.

From a viewing angle of \( (\theta_\nu, \phi_\nu) \), where \( \phi_\nu \) is relative to periastron of the planet, the line-of-sight vector in Cartesian coordinates is
\[ \hat{l} = \begin{pmatrix} s_\phi \psi \sin \theta_\nu \\ s_\phi \psi \cos \theta_\nu \\ c_\theta_\nu \end{pmatrix}, \]  
(36)
where \( \psi_\nu = \phi_\nu - \Phi(t) \).

The normal vector will depend on the perturbations to the stellar surface. We will make use of the fact that to first order, the \( l = 2 \) modes are perfect ellipsoids. Thus, we express the Cartesian coordinates of any point on the surface as
\[ r = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (1 + A\gamma)s_\theta c_\phi \\ (1 + B\gamma)s_\phi \psi c_\theta \\ (1 + C\gamma)c_\theta \end{pmatrix}, \]  
(37)
where \( A, B, \) and \( C \) depend on the modes considered and can be read off Table 1. For the sum of all \( l = 2 \) modes, \( A = 4, B = -2, \) and \( C = -2 \) (giving a prolate “lemon” ellipsoid).

We can define the function
\[ f(t, r) = \left( \frac{x}{1 + A\gamma} \right)^2 + \left( \frac{y}{1 + B\gamma} \right)^2 + \left( \frac{z}{1 + C\gamma} \right)^2, \]  
(38)
and it can be seen that on the surface, this is a constant
\[ f(t, r) = R^2. \]  
(39)

The normal vector is perpendicular to this surface; hence, it must point in the direction of the gradient of this function, i.e.,
\[ \mathbf{n} = \nabla f = 2 \begin{pmatrix} x(1 + A\gamma)^{-2} \\ y(1 + B\gamma)^{-2} \\ z(1 + C\gamma)^{-2} \end{pmatrix} = 2R \begin{pmatrix} s_\theta c_\phi (1 + A\gamma)^{-1} \\ s_\phi \psi c_\theta (1 + B\gamma)^{-1} \\ c_\theta (1 + C\gamma)^{-1} \end{pmatrix}. \]  
(40)

To normalize, we compute
\[ n = \sqrt{\mathbf{n} \cdot \mathbf{n}} = 2R \left( \frac{s_\theta c_\phi}{1 + A\gamma} \right)^2 \left( \frac{s_\phi \psi c_\theta}{1 + B\gamma} \right)^2 \left( \frac{c_\theta}{1 + C\gamma} \right)^2. \]  
(41)

These expressions can be used directly, but as \( \gamma \) and \( \epsilon \) are both \( \ll 1 \), we will expand to first order. This gives
\[ \hat{n} = \frac{1}{n} \mathbf{n} = \frac{(1 + \epsilon - A\gamma)s_\theta c_\phi}{(1 + \epsilon - B\gamma)s_\phi \psi c_\theta} + O(2). \]  
(42)

For an unperturbed star, the surface normal is simply in the radial direction; hence, the projected area element is
\[ da_0 = \mu dA_0, \]  
(43)
where
\[ \mu = s_\theta s_\phi \psi c_\theta - c_\phi, \]  
(44)
and the total projected area is
\[ a_0 = \int_{\text{visible area}} da_0 = \pi R^2. \]  
(45)
Throughout the rest of this section, we will implicitly drop terms of second order or higher. Thus, for a perturbed star,
\[ (\hat{n} \cdot \hat{l}) = (1 + \epsilon)\mu - h, \]  
(46)
where
\[ h = \gamma(As_\theta s_\phi \psi c_\theta + Bs_\phi \psi c_\phi c_\theta + Cc_\phi c_\theta). \]  
(47)

To first order, the projected area element is
\[ da = ((1 + 3\epsilon)\mu - h) dA_0. \]  
(48)

To calculate the effect of perturbations, we will make use of the results:
\[ \int_{\text{visible area}} e_\nu \mu^n dA_0 = 2\pi R^2 e_\nu \int_0^1 3\mu^2 - 1 - \mu^n d\mu, \]  
(49)
and
\[ \int_{\text{visible area}} h\mu^n dA_0 = 2\pi R^2 e_\nu \int_0^1 \mu^{n+1} d\mu. \]  
(50)
Here \( e_\nu \) is the fractional surface deviation along the line of sight and is equal to
\[ e_\nu = \epsilon(t, \theta_\nu, \psi(t, \phi_\nu)). \]  
(51)
These results are derived in Appendix B.

The total surface area of the perturbed star is equal to
\[ A = \int_{\text{whole area}} (1 + 2\epsilon) dA_0 = A_0; \]  
(52)
thus, these perturbations do not change the total area of the star (or the volume).

Depending on viewing angle, the projected area does change:
\[ \frac{a}{a_0} = \frac{\int ((1 + 3\epsilon)\mu - h) dA_0}{\pi R^2} = 1 + \frac{1}{\pi R^2} \left( 3 \int e_\mu dA_0 - \int h dA_0 \right) = 1 - \frac{1}{4} e_\nu. \]  
(53)
Note that here and for the rest of this section, the limits of integration can be assumed to be over the visible area of the star.

### 3.2. Heartbeat Luminosity

There are two factors that contribute to the change in the star’s effective luminosity over a single orbit: the change in area and the change in flux. While we have already calculated the former, the behavior of both can be encapsulated in a single integral, as the total observed luminosity of the star is the flux...
integrated over the entire (perturbed) visible surface. Note that the total luminosity of the whole star is not necessarily changing, only the luminosity we infer by assuming it to be an isotropically radiating point source.

What we see as the flux from the star is effectively the flux at a certain depth, the point at which photons can travel unimpeded out of the star without being scattered or absorbed. This means that at more oblique angles, the photons that reach us have originated from larger radii (and traveled further through the stellar atmosphere) from a cooler, darker region of the photosphere. This process is called limb darkening and leads to a drop in the apparent brightness toward the edge of the photosphere. This process is called limb darkening and through the stellar atmosphere we have originated from larger radii (This means that at more oblique angles, the photons that reach us have originated from larger radii (and traveled further through the stellar atmosphere) from a cooler, darker region of the photosphere. This process is called limb darkening and leads to a drop in the apparent brightness toward the edge of the stellar disk. The Eddington limb-darkening profile, a simple analytic parameterization derived from the principles of radiative transfer (see, e.g., Rutten 2003, Section 4.2), models this well:

$$\lambda = \frac{I}{I_0} = \frac{2}{5} \left( 1 + \frac{3(\mathbf{n} \cdot \hat{r})}{2} \right).$$  \hspace{1cm} (54)

Here $I$ is the apparent intensity and $I_0$ is the intensity at the center of the projected stellar disk (where $\mathbf{n} \cdot \hat{r} = 1$). This profile is not widely used in the exoplanet literature because, while it gives a good description of the intensity profile over the surface of a Sun-like star (with no free parameters), it fails at the very edge (Espinoza & Jordán 2015). Accurate modeling of the outermost edge of the intensity profile is important for transit in-gresses/egresses but is of less relevance for calculations involving the whole star.

We can see that for the unperturbed star,$$
\lambda_0 = \frac{2}{5} \left( 1 + \frac{3\mu}{2} \right),$$

and using Equation (46), the perturbed star has a limb-darkening profile

$$\lambda = \lambda_0 + \frac{3}{5}(\mu \epsilon - h).$$  \hspace{1cm} (56)

The flux at the surface is modified not only by limb darkening but also by the thermal properties of the surface reacting to the changing potential. This phenomenon is called gravity darkening and is most easily understood as a perturbation to hydrostatic equilibrium and hence to pressure and temperature. Regions of the star experiencing increased gravitational force will heat up and radiate more brightly, and material experiencing a reduced gravitational force will become cooler and darker.

According to Von Zeipel’s theorem, the radiative flux is proportional to the local effective gravity,

$$F \propto g_e,$$  \hspace{1cm} (57)

where the effective gravity must account for the perturbing presence of the planet and the slight displacement of the surface. Assuming the change in flux is small,

$$F = F_0 \left( 1 + \frac{\delta g_e}{g_e} \right),$$  \hspace{1cm} (58)

where $F_0$ is the flux of the unperturbed star.

For small perturbations,

$$g_e = \frac{GM}{R^2(1 + \epsilon)^2} + \frac{\partial U}{\partial r},$$  \hspace{1cm} (59)

and thus

$$\frac{\delta g_e}{g_e} = -2\epsilon + \frac{1}{g_0} \frac{\partial U}{\partial r}.$$  \hspace{1cm} (60)

where $g_0$ is the surface gravity of the unperturbed star, given by Equation (19).

As we are only interested in terms with $l = 2$,

$$\frac{\partial U}{\partial r} = \frac{2U}{R}$$  \hspace{1cm} (61)

at the surface. We can rewrite this using Equation (18), giving

$$F = F_0(1 - 2(1 + \beta^{-1})\epsilon).$$  \hspace{1cm} (62)

Putting these pieces together, the observed luminosity from an infinitesimal area element of the star is

$$dL = F\lambda(\mathbf{n} \cdot \hat{r})dA.$$  \hspace{1cm} (63)

For the unperturbed star, this gives

$$dL_0 = F_0\lambda_0 \mu dA_0,$$  \hspace{1cm} (64)

while to first order for the perturbed star, we have

$$dL = dL_0 - \frac{2}{5} F_0(3(\beta^{-1} - 1)\mu^2 \epsilon + (2\beta^{-1} - 1)\mu \epsilon + (1 + 3\mu)h)dA_0.$$  \hspace{1cm} (65)

Thus, using Equations (49) and (50), the total luminosity of the star obeys

$$\frac{L}{L_0} = \frac{\int dL}{\int dL_0} = 1 - \frac{49 + 16\beta^{-1}}{40} \epsilon_c.$$  \hspace{1cm} (66)

To present this in terms of only first order, we can also express this luminosity variation in the conservative case where $\beta = 1$ as\(^4\)

$$\frac{\delta L}{L} = \frac{L - L_0}{L} = -\frac{13}{8} \epsilon_c.$$  \hspace{1cm} (67)

Having computed these luminosity changes for our example planet (with $M = 1.2M_\odot$, $M_p = 0.005M_\odot$, $R = 1.5R_\odot$, $a = 10R_\odot$, and $e = 0.25$) assuming $\beta = 1$, we present its light curves in Figure 7. There are many features to these curves, and we will highlight only a few here, saving a more detailed discussion for Section 4. Two exciting details to note, however, are that the effect is moderately large causing modulations of the light curve of tens of parts per million and that it is visible from almost all angles.

One of the simplest ways to characterize and distinguish these curves (and thus constrain planetary properties) is to find the amplitude and timing of the extrema. We can express the luminosity variations entirely in $\Phi$ (remembering that

\(^4\) The simple treatment of gravity darkening employed here glosses over some complex physics. As a lower limit on the magnitude of the signal, we can calculate $\delta L$ for a system with no gravity darkening ($F = F_0$), giving

$$\delta L = \frac{85}{176} \epsilon_c L_0.$$
The temporal evolution of flux variability seen in Equation (68) is more complicated than that for ellipsoidal variations in circular orbits (Faigler & Mazeh 2011) and differs in detail from the phenomenological models proposed in past work on exoplanet-induced tides (Kane & Gelino 2012; Placek et al. 2014).

\[ \psi_v = \phi_v - \Phi \]

Differentiating informs us that the extrema occur when

\[ \frac{1}{s_{\phi_v}} = 3c_{\psi_v} - \frac{1 + ec_{\phi_v}}{es_{\phi_v}s_{2\psi_v}}. \]

Figure 7. Theoretical variation in the luminosity of a Sun-like star (with mass 1.2M_\odot and radius 1.5R_\odot) due to tides raised by a Jupiter-like planet with a mass of 5M_j on a close, eccentric orbit with a = 10R_\odot and e = 0.25. The light curve is shown over one period, with periapsis always at T = 0. The primary (secondary) transit period is shown in dark (light) blue (note that only for \( \theta_0 = \frac{\pi}{2} \) will the transit be visible). The star is symmetric along the planes x = 0, y = 0, and z = 0 at all times, and thus the behavior shown for these angles generalizes to any observing angle.
undetectable amplitude. The appearance of the additional extrema pairs is disfavored for face-on observers at low $\theta_v$.

Equation (69) must be solved numerically. If the true anomalies of the two extrema labeled $i$ and $j$ ($\phi_i$, and $\phi_j$, and thus $\psi_{i,j}$ and $\psi_{i,j}$) can be estimated, then the ratio of the amplitudes of these extrema is

$$\frac{\delta L_i}{\delta L_j} = \left(1 + \frac{e \cos \phi_i}{1 + e \cos \phi_j}\right)^{4} \frac{s_{\phi_i} s_{2\phi_i}}{s_{\phi_j} s_{2\phi_j}}. \quad (70)$$

The flux ratio between a pair of extrema will, provided the orbital period is known, therefore encode a combination of $e$ and $\phi_v$, and if a second pair of extrema exists (as in the “heartbeat” signal, visible for high-$\theta_v$ systems), we can solve for both $e$ and $\phi_v$. In this situation, we can also find $\theta_v$ using Equation (69) and then use Equation (68) in combination with the magnitude of a single extremum ($\delta L/L$) to determine the dimensionless characteristic amplitude, $\alpha$ (Equation (23)), completely solving the system.

Even if there are only two extrema (as is the case for low-$\theta_v$ systems), the whole light curve can be numerically fitted to Equation (68), which will also yield $\theta_v$ and $\alpha$. Additionally, we note that the case where only two extrema are visible occurs at viewing angles near the poles. With some degree of approximation, one could take $s_{\phi}^2 \approx 0$ in this regime, facilitating the determination of $e$ and $\alpha$.

In short, the tides raised by a planet on an eccentric orbit are sufficiently rich in their features that even measuring a handful of their gross photometric properties suffices to characterize many properties of the orbit and perturber.

### 3.3. Heartbeat RV

The light we see from a star is the light emitted from its outer layers, so time-varying tidal displacements should be visible in the line-of-sight velocity. This is a similar argument to that presented in Arras et al. (2012), though with the specific goal of deriving simple equilibrium tide expressions valid for arbitrary eccentricity.

We have calculated the RV of a given point on the surface in Equation (30). However, to estimate the total line-of-sight velocity that would be observed due to tides, we must average the projected velocity over the whole visible surface.

The line-of-sight velocity observed at any point on the surface is $\hat{v}_r \cdot \mathbf{v}(\theta_v, \phi_v)$, the projection of the 3D velocity along the line of sight. So far, we have derived the radial component of the velocity. We will use this to calculate the radial contribution to the line-of-sight velocity ($\hat{v}_r$), and, rather than explicitly calculating the horizontal displacements, we will use the expedient approximation presented in Arras et al. (2012) that the contribution from the horizontal displacements follows

$$\hat{v}_h = \frac{l + 4}{l(l + 1)} \frac{b_l}{a_l} \hat{v}_r, \quad \text{ (71)}$$

where $a_l$ and $b_l$ are numeric constants that depend on the limb-darkening profile. As we are primarily interested in the $l = 2$ mode, we can use the results of Arras et al. (2012) that $a_2 = 0.321$ and $b_2 = 0.775$. Thus, the line-of-sight velocity due to tides, $\hat{v}_{l\text{,tides}}$, is

$$\hat{v}_{l\text{,tides}} = \hat{v}_r + \hat{v}_h = 3.41 \hat{v}_r. \quad \text{ (72)}$$

Thus, it only remains to find the contribution of the radial motion to the line-of-sight velocity,

$$\hat{v}_r = \hat{\mathbf{v}} \cdot \mathbf{v}(\theta_v, \phi_v) = -\mu_\text{\text{d}}\hat{v}_r. \quad \text{ (73)}$$

This second equality exploits the fact that $\hat{v}_r$ is small and thus all other first-order terms can be ignored, and the negative sign ensures that movement of the surface away from the observer has a positive value. Much like Equations (49) and (50), we can exploit the symmetries of the spherical harmonics to make integration of $\delta v_r$ over a spherical surface analytic:

$$\int_{\text{visible area}} \delta v_r \mu^2 dA_0 = 2\pi R^2 \delta v_r(t, \theta_v, \psi_v) \int_0^1 \frac{3\mu^2 - 1}{2} \mu^3 d\mu \quad \text{ (74)}$$

(derived in Appendix B).

Thus, the observed line-of-sight velocity is the flux-weighted average over the stellar surface,

$$\hat{v}_r = \frac{\int \delta v_r dL}{\int dL} = -\frac{\int \left(\mu^2 + \frac{3\mu^2}{2}\right) \delta v_r dA_0}{\int \left(\mu + \frac{3\mu^2}{2}\right) dA_0} = -\frac{107}{240} \delta v_r(t, \theta_v, \psi_v, \hat{v}_r). \quad \text{ (75)}$$

And finally,

$$\hat{v}_{l\text{,tides}} = -1.53 \delta v_r(t, \theta_v, \psi_v, \hat{v}_r). \quad \text{ (76)}$$

Figure 8 shows the line-of-sight velocities from a range of viewing angles (with $\beta = 1$). Here we can see the characteristic “heartbeat” form, which has significantly larger amplitude when the system is viewed edge-on. Also shown are the line-of-sight velocity profiles due to the movement of the star around the system’s center of mass (the signal used in the RV method), as calculated by

$$v_{l\text{,orbit}}(t, \theta_v, \phi_v) = \sqrt{\frac{GM^2}{aM} \frac{s_{\phi_v}(s_{\psi_v} + es_{\phi_v})}{\sqrt{1 - e^2}}}., \quad \text{ (77)}$$

taken from Lovis & Fischer (2010). To fit both on the same plot, this has been divided by a factor of 200.

Measurements of the apparent velocity of these stars will be dominated by their orbital motion. However, as discussed more in Section 4, these variations of a few ms$^{-1}$ are on the verge of observability. Spectroscopic data are theoretically cleaner than photometric data, with fewer other physical effects modulating the apparent velocity.

The velocities observed in face-on orbits are significantly smaller, though not negligible. For perfectly face-on systems, the orbital RV measurement goes to zero, leaving tidal deformation as the primary source of velocity variability.

This also demonstrates another strength of searching for tidal signatures: these planets are excellent candidates for RV follow-up, but they can be identified directly from the light curve (which is a comparatively less expensive measurement to take).
3.4. The Effect of Eccentricity

As a final note in this section, we show the effect of varying eccentricities on the luminosity and RV trends. Figure 9 shows orbits with a range of eccentricities but otherwise equal properties (including semimajor axis) to those used in previous plots, as seen from an oblique angle. Most of the signal occurs as the planet passes close to periapse ($|\Phi| < \frac{\pi}{2}$), a period of time that decreases rapidly with increasing $e$.

As eccentricity increases, the velocities and changes in luminosity increase dramatically. Higher-eccentricity systems are not plotted, as the characteristic amplitudes quickly become very large, especially the velocities (which have a very strong dependence on $e$).

4. Observational Prospects

The main aim of this paper is to survey the parameter space of photometric and kinematic variability due to stellar tidal deformation from planets on eccentric orbits. The resulting equilibrium tides, when they can be found, offer significant constraining power on properties of exoplanetary orbits. In this section, we will also discuss the possible observational promise of searching for these signals in eccentric planetary systems.

4.1. In Photometric Data

As we have shown in Section 3 (particularly Figure 7), for a reasonable example planet, the equilibrium tide can, over the
course of an eccentric orbit, cause changes in luminosity that are

1. roughly equal in magnitude regardless of viewing angle,
2. strongest at or near the moment of pericenter passage, and
3. feature-rich, holding a wealth of data about the planet and the star.

These characteristics are exciting in the context of planet detection. A signal of the amplitude of our example planet (~50 ppm) is on the limit of the photometric precision of a survey like *Kepler*. A very rough estimate for a typical *Kepler* planet and star (though there is a large variation between systems) is an achievable signal-to-noise ratio of the order of 100 ppm per orbit, and with short-period planets, there may be hundreds of observed orbits. Phase folding and other statistical techniques to combine data over many orbital periods (remembering that the error reduces with the square root of the number of observations) can thus reduce this signal-to-noise ratio to of order 1–10 ppm (Jansen & Kipping 2018). More precise instruments, such as space telescopes like *Hubble* and the forthcoming *James Webb Space Telescope* (Gardner et al. 2006), can manage significantly higher photometric precision (though each individual observation only covers a small area of the sky). Most tantalizing is the TESS survey (Ricker et al. 2015), an all-sky search for planets around nearby stars. It will survey a similar number of stars but in much closer proximity than the *Kepler* mission, so that measurements of changes in the star’s brightness can be measured with approximately twice the precision (Sullivan et al. 2015).

Summarizing the results of the above sections, for a star with mass $M$ and radius $R$ and a planet with mass $M_p$ (and radius $R_p$) orbiting with semimajor axis $a$ and eccentricity $e$, the apparent fractional change in luminosity caused by tides is described by

$$ \frac{\delta L_{\text{tide}}}{L} = \frac{13}{16} \frac{M_p}{M} \left( \frac{R}{a} \right)^3 \frac{3 \sin^2 \theta_e \cos^2 \psi_e - 1}{(1 - e \cos \eta)^3}. $$

(78)

This applies equally well to systems with arbitrary eccentricity and fully describes ellipsoidal variations due to areal deformation and gravity darkening. Here we have made the simplifying assumption of taking the parameter governing the stellar response to tides, $\beta$ (Equation (16)), to be equal to 1.

All time dependence in Equation (78) is present in the eccentric anomaly $\eta$ and true anomaly $\Phi$ (where $\psi_e = \phi_e - \Phi$).

Here $\theta_e$ and $\phi_e$ are the angles from which the system is viewed in spherical coordinates (with zenith angle $0 < \theta < \pi$ and azimuthal angle $0 < \phi < 2\pi$) oriented such that planetary motion is confined to the equatorial plane ($\theta = \frac{\pi}{2}$), and the planet passes through $\phi = 0$ at periapse. Thus, $\theta_e$ and $\phi_e$ denote the position in the frame orientated relative to the planet’s orbit. These can be converted to the usual elements of inclination and argument of pericenter with $i = \theta_e$ and $\omega = \frac{\pi}{2} - \phi_e$, respectively.

From Equation (78), we can see that the maximum change in luminosity is

$$ \left| \frac{\delta L_{\text{tide}}}{L} \right|_{\text{max}} \approx \frac{2 M_p}{M} \left( \frac{R}{r_{\text{peri}}} \right)^3, $$

(79)

where $r_{\text{peri}} = a (1 - e)$ is the radius of pericenter.

This maximum ignores the effect of viewing angle, but moving from the edge-on ($\theta_e = \frac{\pi}{2}$) systems that have the largest amplitude to face-on systems ($\theta_e = 0$) only decreases the amplitude by a factor of $\approx 2 e$ (assuming $e$ is small; the dependence will be more complex for $e \to 1$).

Here we have summed over all quadrupolar ($l = 2$) modes, the lowest-order stellar harmonics excited by tides. The contribution of each higher-order mode (larger $l$) is suppressed relative to quadrupolar deformations by a factor $\approx (R/r_{\text{peri}})^{l^2}$; hence, higher-order modes are only of interest for very precise calculations.

To summarize, the maximum amplitudes of luminosity oscillations in a very eccentric orbit are comparable to the luminosity oscillations in a circular orbit of the same pericenter $r_{\text{peri}}$ but with a much richer temporal structure that is described by Equation (78). Equilibrium tides in an eccentric orbit are much stronger than those in a circular orbit of the same semimajor axis $a$.

### 4.2. In Spectroscopic Data

The variations in stellar surface velocities due to equilibrium tides raised by our example planet are on the verge of resolvability. Current-generation spectrographs manage
velocity resolution of order a few m s\(^{-1}\) (e.g., Bean et al. 2010), and those of the next generation aim to reduce this to cm s\(^{-1}\) (Pasquini et al. 2008).

As shown in Figure 8, the orbital RVs are orders of magnitude larger than those caused by tides. Any system detected displaying heartbeat features in its light curves will likely be an excellent candidate for follow-up RV measurements (unless \(\theta_v = 10^\circ\), which can be assessed by fitting models to \(\delta L/L\)). Photometric data are relatively inexpensive to obtain, and observations already exist of many systems in which this signal may be visible. Because it is comparatively expensive to obtain RV measurements, providing a selection of strong candidates via other methods is very valuable.

Summarizing the radial motions of the star’s surface due to tides, we can write the apparent RV along the line of sight as

\[
v_{\text{tides}} = 0.764 \frac{GM}{a} \frac{M_p}{M} \left( \frac{R}{a} \right)^4 \sqrt{2\left(1 - e^2\right) \sin^2 \theta_v \sin 2\psi_v + e \sin \eta (3 \sin^2 \theta_v \cos^2 \psi_v - 1)} \frac{1}{(1 - \cos \eta)^5}.
\]

From this, we can find an approximate maximum RV,

\[
|v_{\text{tides}}|_{\text{max}} \approx v_{\text{peri}} \frac{M_p}{M} \left( \frac{R}{r_{\text{peri}}} \right)^4,
\]

where

\[
v_{\text{peri}} = \sqrt{\frac{GM(1 + e)}{r_{\text{peri}}}}.
\]

is the orbital speed of the planet as it moves through pericenter (the fastest it travels in an orbit).

We have again ignored the orbital inclination in this simple estimate, but we note that the smallest amplitude (face-on) is a factor of approximately \(\frac{1}{2}\) less than the largest (edge-on). This relationship is found by assuming \(e \ll 1\), and the scaling will differ for large \(e\).

The signals due to the orbit and tides are markedly different in profile, and the total variation is simply the sum of the two individual effects. Thus, if the form of the orbit is well constrained, the tidal signal may still be visible and extractable.

### 4.3. Comparing to Known Exoplanets

We have limited our theoretical calculations to an example system, introduced in Section 2.3, containing a star with mass \(M = 1.2M_\odot\) and radius 1.5\(R_\odot\) and a planet with mass \(M_p = M_j\), whose orbit has semimajor axis \(a = 0.05\) au and eccentricity \(e = 0.25\).

It is a simple matter to extend order-of-magnitude calculations of the size of these signals for other observed exoplanets. Figure 10 shows the approximate amplitude of the observable effects of tides compared to the properties of the system. The planets shown are confirmed detections, sourced from the NASA Exoplanet Archive. Only planets with recorded \(M, R, M_p, a, e\) are shown, which biases the selection to planets either with both RV and transit measurements (or in systems for which this is true for another planet) or for which some parameters have been found via modeling.

First, it can be seen that our example planet is in no way unreasonable. Similar observed planets exist with more
Figure 11. Histograms of planetary properties. The black line shows the properties of all 578 planets (with known $M$, $R$, $a$, $e$, and $v_1$), and the colored regions show the cumulative distribution in intervals of 20%. The red and blue lines show the properties of the 355 planets discovered by transits and the 218 planets discovered by RV. The y-scaling is linear.

extreme eccentricities (Barbieri et al. 2009) and masses (Bakos et al. 2012), though significantly closer orbits are rare. Some of these higher-mass “Supiters” exhibit luminosity variations approaching the percentage level and changes in velocity approaching 100 m s$^{-1}$, all caused by tides.

The results shown here ignore the geometric dependence on viewing angle: for orbits close to circular, the amplitude of a face-on signal goes to zero, but for highly eccentric face-on orbits, the amplitude only drops proportional to $e$. Thus, tidal signatures are visible from all angles for highly eccentric planets.

By far the strongest dependence in $\delta L/L$ and $v_1$ is the ratio of stellar radius to pericenter distance ($R/\text{peri}$). Secondary dependencies can also be seen on the eccentricity ($e$), dimensionless semimajor axis ($a/R$), and planetary mass ($M_p$). These dependencies and the similarities between the scaling of effects on the luminosity and velocity can be seen directly in Equations (79) and (81).

Most exoplanet searches have targeted Sun-like stars; hence, the variation in stellar mass ($M$) is very small. Similarly, most stars with companions are of roughly stellar radius, but a significant population of giant stars has been found (via RV methods) to host exoplanets. Among these giants, there is a strong trend for larger stars to have greater tidal signatures.

Brown dwarfs are not shown here but can be expected to exhibit similar trends (in fact, the largest-mass planets here are so close in mass as to make the distinction semantic for our purposes). As the mass ratio $M_p/M$ approaches unity, some of our first-order approximations and the assumption of equilibrium tides will break down, and the interested reader would be better served by the literature on heartbeat stars (e.g., Fuller 2017). However, as long as the fractional distortions, $e$, are small, this analysis should be a reasonable approximation for the distortions of planets or low-mass stars.

Figure 11 shows the distribution of eccentricities, semimajor axes, and the strength of tidal signals. From this, we can see that a significant fraction (10%–20%) of the sample of confirmed exoplanets has tidal signatures as large as or greater than those of our example planet. The clear bimodality is simply a consequence of the bias for the transit method for close orbits (and short periods). It is also interesting to note that around half of these planets have eccentricities greater than 0.15 (the point at which heartbeat effects are roughly equal in magnitude to circular-orbit ellipsoidal variations), though this will be biased by the fact that we have selected only planets with measured values of or upper limits on $e$.

Finally, an interesting consequence of this method and its dependencies is that, if successful in finding new planets, it will have a very strong selection bias. Tidal signatures will be especially strong in eccentric hot Jupiters (and smaller planets with correspondingly tighter or more eccentric orbits). This is a fascinating population to study, particularly as these systems are likely short-lived; hence, their observation may be evidence of a transient phase in the lifetime of a planetary system (Haswell 2010; Dawson & Johnson 2018). The photometric signatures of heartbeat oscillations lack a strong viewing-angle dependence and therefore offer a potentially unique way to identify a flux-complete population of planets (albeit with strong selection effects in system properties like $R/\text{peri}$ and $M_p/M$).

4.4. Comparing to Other Observables

Observing tidal signatures is just one of many ways in which the presence of a planet and the properties of a system can be attained by observations of the host star. Here we briefly consider some of the others and the relative strength of their signals.

The two methods that have yielded the most confirmed exoplanets are the observations of change in luminosity caused by transits (partial eclipse of the star by the planet) and the RV variation as the star orbits around the system center of mass.

The flux change caused by transits, ignoring limb darkening for a simple order-of-magnitude estimate, is just the fractional area of the star covered by the planet and thus

$$\frac{\delta L_{\text{transit}}}{L} = \left(\frac{R_p}{R}\right)^2,$$

independent of orbital parameters (though many can be derived via the timing). There are very strong constraints on $\theta_0$, as only for viewing angles that are almost perfectly edge-on will the planet transit.

We have already expressed in Equation (77) the orbital velocity along the line of sight used in the RV method, assuming $M_p \ll M$ (Lovis & Fischer 2010). From this, we can find the maximal amplitude,

$$|v_{1,\text{orbit}}|_{\text{max}} \approx \frac{M_p}{M} v_{\text{peri}}.$$

As previously discussed, this strongly dominates typical velocities associated with tidal deformation.

We will consider two other effects here: relativistic beaming and the reflection of the star’s light by the planet. The former is due to the relativistic aberration of light, which causes a source radiating isotropically in its rest frame to appear brighter when it is moving toward an observer (and dimmer when moving
away). The effect can be simply described by
\[ \frac{\delta L_{\text{beam}}}{L} = \frac{4v_L \nu_{\text{orbit}}}{c}, \]  (85)
where \( c \) is the speed of light (Loeb & Gaudi 2003). More complex expressions exist that include the spectral dependence of the star’s light, but we will not consider them further here. From this, the maximal amplitude can be found:
\[ \left| \frac{\delta L_{\text{beam}}}{L} \right|_{\text{max}} \approx 4 \frac{M_p v_{\text{peri}}}{M} c, \]  (86)

The reflection of the star’s light by the planet is more complex to model and requires further assumptions about the reflective and thermodynamic properties of planetary atmospheres. The simplest model is that of a Lambert surface (often termed a Lambert sphere), which reradiates any incident energy isotropically. Thus, it scatters photons perfectly. This is a poor fit for rocky bodies, such as the Moon (and even worse for liquid surfaces, such as Earth), but passable for gas giants like Jupiter, the most relevant body in the solar system to this work. The reflection luminosity is then given by
\[ \frac{\delta L_{\text{reflect}}}{L} = A_r \frac{R_p}{a} \left( \sin \nu + (\pi - \nu) \cos \nu \right)^2 \frac{\pi (1 - e \cos \eta)^2}{(\pi (1 - e \cos \eta)^2)} \]  (87)
(Kopal 1966), where \( \nu \) is the angle between the line of sight and the direction from which the planet is illuminated (which itself is just the vector position of the planet relative to the star). Thus, \( 0 < \nu < \pi \) and \( \cos \nu = -\sin \theta_p \cos \psi_p \).

Here \( A_r \) is the geometric albedo, a measure of the reflectiveness of the planet, and can range from 0.01 to 0.5 (Millholland & Laughlin 2017). For the remainder of this paper, we will use a value of 0.1 for demonstration purposes. This ignores the luminosity of the planet itself, which can roughly be considered as a blackbody with a temperature set by the incident flux of the star. This is of most significance when the bandpass of the detector coincides with the luminosity temperature of the planet (with wavelengths in the infrared or longer). This is far from the effective temperature of most stars and therefore not a significant effect for most photometric instruments. It has been shown that there is little variation in the radii of planets more massive than Jupiter (Chen & Kipping 2017); hence, we will use \( R = R_j \) for our example planet.

Thus, the maximum fractional change in luminosity is
\[ \left| \frac{\delta L_{\text{reflect}}}{L} \right|_{\text{max}} \approx \frac{1}{10} \left( \frac{R_p}{r_{\text{peri}}} \right)^2, \]  (88)

Figure 12 shows the relative maximum amplitude of these signals compared to tides. Unsurprisingly, RV signals and transits are generally orders of magnitude greater than the variation due to tides. This is necessarily true for RV, though systems where tides dominate over transits are theoretically possible, e.g., if the companion is a compact object.

Tidal signatures have the strongest dependence on \( r_{\text{peri}} \) while all signals are larger for smaller orbits (except transits). This leads to tidal signatures dominating over reflections and beaming for the systems with the largest-amplitude signals. Their strong dependence on \( r_{\text{peri}} \) also leads to tides having the strongest dependence on eccentricity and thus are the best candidate signals to search for highly eccentric exoplanets. However, for most systems of interest, including our example planet, it is likely that both tides and reflections will have a perceptible effect, and both should be modeled.

Figure 12 ignores the geometric effect of the viewing angle (we have assumed that all systems are being viewed edge-on). If we move to face-on systems, only the signals from tides and reflections will remain, diminishing by a factor of order \( e \). All
other amplitudes scale as \( \sin \theta_i \), and thus disappear when the system is viewed face-on.

It is worth noting that these are confirmed exoplanets, many with publicly available data, for which the signatures of tides and reflections (and potentially beaming) could already be identifiable and would serve to better constrain and understand these systems.

4.5. Derivable Properties of the System

Observations of tidal signatures serve not only to identify new planetary candidates but also to characterize their systems. These features can be combined with other observables to give strong constraints on planetary and stellar parameters. The interplay of various measurements is a complex, if promising, field that we will only touch on here.

One particular point of interest for tides is that for eccentric systems, the amplitude of their effect is largest at or near periapse and thus the timing, not just the signal amplitude, is largely independent of viewing angle. In comparison, the amplitude of the signal from reflections is largest as the planet passes through the line of sight behind the star (though at very high eccentricities, the signal is stronger at periapse). Transits occur at a time defined by their orientation independent of periapse (though due to their duration, we are most likely to observe transits at or near periapse; Kipping & Sandford 2016). The RV (and thus beaming) signals go to zero as the planet passes through the line of sight (and are generally strongest near pericenter). To summarize, tidal signals offer us the possibility of a strong observable at a second independent moment in the planet’s orbit.

Assuming the heartbeat signal can be well characterized, the basic properties available to us when we have observations via multiple methods are as follows.

1. Transit + heartbeat luminosity profiles. The orientation of the system is derivable (\( \theta_i \approx \frac{\pi}{2} \)), and the degeneracy between the projected impact parameter, stellar density, and eccentricity can be broken (Seager & Mallén-Ornelas 2003; Carter et al. 2008; Sandford & Kipping 2017), either by finding \( e \) or putting constraints on \( \theta_i \). The transit observations are independent of planet mass; hence, if the system is well characterized by the transit, \( M_p \) can be easily found. Likewise, the heartbeat method is independent of the planetary radius, whereas this is derivable from the transit.

2. RV + heartbeat signals. Again, the orientation is derivable from tidal deformations, though \( \theta_i \) is no longer so tightly constrained by the primary RV signal. The RV measurements are independent of the stellar radius; hence, if other parameters are reliably derived, the heartbeat signal can be used to find \( R \). Note that we have not specified whether we have observations of heartbeat luminosity or velocity profiles. Both contain roughly the same information, and thus either can be used.

There are many other combinations of signals, not to mention the effects of uncertainties, that we do not consider here. However, it is generally safe to say that more data—e.g., tidal deformation signatures—will give new or stronger constraints on system parameters. A more detailed exploration of the combination of tidal, beaming, and reflection signals will follow in Penoyre (2018, in preparation).

Finally, we discuss briefly what can be garnered from a system for which only the photometric tidal signal is observed (ignoring here the apparent velocity changes due to tides, as, if these are observed, it is very likely we also have orbital RV measurements). Both the photometric and spectroscopic profiles are dependent on the same system parameters \((M, M_p, R, a, e, \theta_i, \text{and} \phi_i)\). The period of the orbit should be easy to derive, and from this, we can find \( \frac{M}{\bar{p}} \) exactly via Kepler’s third law.

Regardless of viewing angle, almost all features of the profiles occur near periapse \((|\theta| < \frac{\pi}{2})\), though an exact relation for this window of time is nontrivial to find. Simple intuition tells us that it will predominantly be a function of the orbital period (known) and the eccentricity, and thus we can estimate \( e \) reliably.

Using Equations (69) and (70), the timing and relative heights of the extrema in the light curve can be used to find the viewing angles, \( e \) and \( \frac{M}{\bar{p}} (\frac{a}{R})^3 \) (except in the case where only two peaks are visible, though when this is the case, we can assume \( \theta_i \) is small and solve for all other above parameters). We can also express this in terms of stellar density (thus circumventing the dependence on \( R \) and \( M \)) as \( \frac{1}{\bar{p}} \frac{M_p}{\rho_{\star}} \), where \( \bar{p} \) is the mean density of the star.

Thus, tidal signatures alone inform us of the system’s orientation and eccentricity and constrain (with degeneracies) the mass and radius of the star and the planet’s mass and semimajor axis.

4.6. Assumptions of the Model

Before we move on, we should discuss the assumptions and approximations that might lead to errors in our model and predictions.

Perhaps the largest concern is that we have ignored the rotation of the star. The former is a correction many other similar models have made (Kumar et al. 1995; Fuller & Lai 2012) and is a well-defined additional degree of freedom for mode-orbit coupling models (though we leave this for future work). Without including rotation, we also cannot fully consider the Rossiter–McLaughlin effect (the apparent velocity caused by a transiting planet eclipsing part of the surface of a rotating star; see, for example, Ohta et al. 2005), which is of interest in transiting systems, particularly due to the apparent similarity to the velocity profiles caused by tides.

Stellar activity should also be considered, though it is hard to quantify and comes in many forms. Any star will likely have oscillations driven by its own internal turbulence, and other phenomena, such as sunspots and flares, may confuse the interpretation of photometric data. Observations of the system over many orbital periods may be sufficient to separate out these effects (which happen over timescales set by the star) from the tidal effects of the planet.

We also assume that the orbit is roughly constant, at least over the period of time we are observing. This should be a reasonable approximation for most systems, and, though it is these eccentric close passages that will have the largest effect on the orbital energy, any discernable difference will happen over a period of many years, whereas the orbital period is typically just days. The relative age of these systems is still a concern, particularly as high-eccentricity planets in a system may be a sign of relative youth (Batygin et al. 2016), implying greater stellar activity.
Our simple model for gravity darkening, which dominates the change in luminosity, brushes over a complex area of physics (Claret 2012). While we would not expect the qualitative behavior to change, this could be significant in some systems. In the limiting case, where gravity darkening is negligible, the change in the luminosity of the star drops by a factor of two compared to Equation (67).

We have also only considered a single planet, or at least assumed that a single planet dominates the tidal behavior. However, in the limit of small perturbations, the signals will be approximately additive, and thus the effect of more planets could be easily added together.

5. Conclusions

To briefly summarize the results above, we have shown the following.

1. A simple model can be constructed for the small deviations to a stellar surface caused by an eccentric planetary orbit (Section 2).
2. These tides cause appreciable changes to the light curves, RV profiles, and power spectra of the host star (Section 3).
3. This is an extension of the known study of ellipsoidal variations, but we have highlighted the disproportionate effects of eccentricity. Compared to circular orbits of equal semimajor axis, the amplitude can be orders of magnitude greater; compared to circular orbits of equal pericenter, the temporal behavior of the signal can be much richer.
4. Eccentric equilibrium tides cause the light emitted by a star to vary and the surface to move. Thus, they lead to observable signals both in photometric and spectroscopic data.
5. The magnitude of these signals is not strongly dependent on viewing angle; thus, systems may be identified and understood at a range of inclinations from face-on to edge-on.
6. The magnitude of these signals is large enough to be observable via various methods and provides a new tool for planetary detection and classification (Section 4).

This means that there is already a sample of planets detected via other methods for which data already are freely available that can be further constrained by searching for heartbeat tidal signatures. At the same time, there may be many systems in existing and future data where these planets and their characteristic signal can be found. Especially for (the large majority of) systems seen at high enough inclination to make transits impossible, this may be the only method by which these planets are detectable in the stellar light curves.

Almost all planets that can be discovered by this method will be prime candidates for RV observation. Thus, we can use relatively abundant and inexpensive photometric data to find promising new targets for spectroscopic observations.

As these signals depend strongly on the eccentricity of the planet and on it having high mass and a close pericenter, there will be a large sampling bias in the planets the method will find. Eccentric hot Jupiters, in particular, will give very large tidal signals. These planets are of great interest for their peculiarity compared to our own solar system and because they represent one step in the life cycle of standard hot Jupiters (Dawson & Johnson 2018). We have shown that tides can reveal such planets regardless of viewing angle, and with the imminent launch of the TESS satellite (Ricker et al. 2015), we may have the tools necessary now to find a full sample of all such nearby systems.

In short, the observational effect of eccentricity on planet- raised tides is significant. The prospects for the detection and characterization of planetary systems, unshackled from any strong dependence on viewing angle, are exciting, numerous, and may afford us, indulging in a moment of hope, illuminating new insights into extrasolar planets.

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Appendix A

A Glimpse into Planetary Asteroseismology

As discussed in Section 2 (and shown in Figure 2), only stellar oscillations with natural frequencies comparable to that of the orbital frequency at pericenter will experience significant excitation. This can be seen directly in Equation (8): since the timescale on which modes react to external changes is dictated by their natural frequencies, and the timescale the perturbations change over is dictated by the orbital frequency, high-frequency modes remain always at or near equilibrium.

Only $f$- and $p$-modes with radial wavenumber $n \geq 0$ are visible at the surface, and these have high natural frequencies. So, while these modes gain energy and increase in amplitude during pericenter passage (the main focus of this paper), as the planet moves away from periapsis, almost all of that energy is transferred back, and the mode amplitude returns to zero.

In contrast, modes that occur at the core of the star (that may not occur at all in particularly small or large stars; see, e.g., Christensen-Dalsgaard 2002) have significantly lower frequencies. Thus, gravitational perturbations from exoplanetary companions can effectively transfer energy to these modes.

An example of this is shown in Figure 13. The energy is calculated as

$$E_{nlm} = M \left( \frac{a_{nlm}^2}{2} + \frac{a_{nlm}^2 \omega_{nlm}^2}{2} \right).$$

which assumes that the system is well approximated as a simple harmonic oscillator. This holds true at all times except when the right-hand side of Equation (8) is large (i.e., true away from periapsis), and this assumption is why we see large spikes at periapse but an approximate constant over the rest of the orbit.

The parameters of the system here are the same as used in Figure 2, with the exception that the semimajor axis has been changed to $a = 9.5 R_\odot$, tuned to ensure that the orbital and mode frequencies are very close to a resonance ($\frac{\nu_m}{\nu_{orb}} = 10.05$).

Thus, energy is injected at nearly the same point in the
oscillation each orbit, and both the energy and amplitude increase with each close passage.

Notice that both the amplitude and energy of the oscillations increase over a number of orbits; later, both begin to decrease. As the orbit is close to resonant with the natural frequency, the energy injected at each periapse is almost in phase with the oscillation for the first few periods. The slight phase difference introduced in each orbit (due to the small deviation from a perfect resonance) eventually leads to the energy being injected out of phase, and the energy in the mode decreases.

The closer the orbit is to a resonance with a normal mode of the star, the more orbits occur before the injection of energy goes out of phase with the oscillation, and thus the higher the maximum attainable energy. Tidally driven modes near such a resonance may have much higher energy than we would expect to be naturally excited by processes such as convective turbulence in the star. This is the more conventional source of oscillatory energy and can be expected to transfer energy to modes while roughly obeying equipartition of energy, meaning most of the energy goes to modes with low wavenumbers.

Even though mode-orbit resonances may transfer a large amount of energy compared to what is expected in a given mode, it is still a small amount compared to the total orbital energy; thus, we do not expect it to cause significant precession of the planet’s orbit over the periods of interest. However, any small change in orbital frequency does put an upper limit on how close to resonance the system can remain, and this may be important in more detailed calculations.

These g-modes themselves are not directly visible, existing only in the core of the star. However, it is feasible that these could couple, if the frequencies fortuitously combine, into modes that do reach the surface and are thus visible (e.g., Weinberg et al. 2013). The exact physics of the coupling motion between different layers of a star is complex and beyond the scope of this paper, but it is feasible, and the ramifications are very interesting.

1. If a large amount of energy (compared to that expected from equipartition) is transferred to low-frequency modes, that energy may be coupled to oscillations visible at the surface.
2. Thus, we may observe an anomalously large amount of power in these oscillation modes.
3. These frequencies are most likely to be harmonics of the planet’s orbital frequency; thus, there may be an observable asteroseismic effect caused by orbiting planets.

This is a tantalizing proposition, one hinted at in work such as de Wit et al. (2017); however, here we only introduce the possibility and encourage others to explore it further. Without detailed calculation and arguments, we make no claim about the likelihood or feasibility of this effect beyond suggesting that it may be an interesting new regime through which to explore asteroseismology if indeed the effect proves significant.

### Appendix B

#### Integrating Over the Visible Area

In this section, we will explicitly derive Equations (49), (50), and (74), the integrals of observable quantities over the visible surface area. This is a summary of the methods presented in Dziembowski (1977, hereafter D77) and Balona & Stobie (1979) for analytic integration of quantities over spheroidal surfaces.

Most of this work has been expressed in spherical polar coordinates, θ and ψ (we have chosen to use the notation ψ rather than φ to draw more direct comparison to the expressions derived in Section 3). The limits of the integration, those that describe the visible area of the star, are relatively complex in these coordinates. Expressed in the simplest form (and assuming the north pole is visible, i.e., 0 < θ < π/2), though this is easily generalized, the integration runs around the full range of ψ, from 0 to 2π, and for each value of ψ, θ runs from zero to the horizon at θₜ (the point at which the normal is perpendicular to the line of sight, ⟨n ⋅ I⟩ = 0).

To zeroth order, for a given ψ, the horizon occurs at a polar angle, θₜ, satisfying

$$\tan(\theta_t(\psi)) = \frac{-1}{\tan \theta_v \cos(\psi - \psi_v)}.$$    \hspace{1cm} (90)

The reader may notice that we have omitted first-order components of Equation (90). This is because these corrections to the limits of integration cause negligible corrections to the computed integral. These are the limits that should be used for numerical integration, but, in the case where the function we are integrating can be expressed as linear functions of spherical harmonics, we can analytically compute these integrals directly.

This derivation exploits the properties of the spherical harmonics, defined (as throughout the rest of this work) to be

$$Y_l^m(\theta, \psi) = N_{l,m} P_l^m(\cos \theta)e^{im\psi},$$    \hspace{1cm} (91)
where \( P^n_l \) is an associated Legendre polynomial and \( N_{l,m} \) is the normalization constant, such that the integral of \( |Y^n_l|^2 \) over the unit sphere is equal to unity.

The projected area element and integration limits are much more simply expressed in coordinates, \( \theta', \psi' \), aligned such that the \( z \)-axis (where \( \theta' = 0 \)) points along the line of sight. In these coordinates,

\[
(\hat{n} \cdot \hat{I})_0 = \cos \theta',
\]

and the integration runs over \( 0 < \psi' < 2\pi \) and \( 0 < \theta' < \frac{\pi}{2} \).

It is not trivial to express functions such as \( \epsilon \) in these new coordinates, but we can use the fact that the spherical harmonics, with degree \( l \), form a complete set, and thus we can express any spherical harmonic in one coordinate system as a sum over harmonics in another, i.e.,

\[
Y^m_l (\theta, \psi) = \sum_{k=-l}^{k=l} q_{lm} Y^k_l (\theta', \psi').
\]

Thus, the integral of any spherical harmonic over the visible area becomes

\[
\int_{\text{visible area}} Y^m_l (\theta, \psi) dA_0 = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sum_{k=-l}^{k=l} q_{lm} Y^k_l (\theta', \psi') R^2 \sin \theta' d\theta' d\psi'
\]

(note that we are integrating over the unperturbed area element \( dA_0 \)). For \( k = 0 \), the integral around the azimuthal angle \( \psi' \) is zero (see Figure 1; moving around the sphere at constant latitude, there are equal, and hence canceling, positive and negative contributions).

This means that the only term that enters into the calculation is \( k = 0 \), for which we can find (see D77) the constant

\[
q_{00} = \frac{1}{N_{0,0}} Y^0_0 (\theta, \psi)
\]

(note that D77 employed unnormalized versions of the spherical harmonics; hence, the term \( N_{0,0} \) is absent).

Thus,

\[
\int_{\text{visible area}} Y^m_l (\theta, \psi) dA_0 = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sum_{k=-l}^{k=l} q_{lm} Y^k_l (\theta, \psi') R^2 \sin \theta' d\theta' d\psi'
\]

\[
= 2\pi R^2 Y^0_0 (\theta, \psi) \int_0^{\frac{\pi}{2}} P^0_l (\cos \theta') \sin \theta' d\theta'
\]

\[\text{(using } Y^0_0 (\theta', \psi') = N_{0,0} P^0_l (\cos \theta')).\]

We can express this more generally and simply by defining

\[
\mu = \cos \theta'
\]

\[\text{(note that this is exactly equivalent to the definition of } \mu \text{ given in Equation (44)) and expressing some general linear function of spherical harmonics with degree } l \text{ as}
\]

\[
g_l (\theta, \psi) = \sum c_m Y^m_l (\theta, \psi),
\]

functions of \( \mu \). Thus, we can find the general expression

\[
\int_{\text{visible area}} d^2 g_l (\theta, \psi) dA_0 = 2\pi R^2 g_l (\theta, \psi) \int_0^1 \mu^l P^l_l (\mu) d\mu,
\]

which can be easily analytically solved.

The fractional displacement of the star (\( \epsilon \); Equation (26)) and the velocity of the surface (\( \delta \); Equation (30)) are both expressible as linear combinations of spherical harmonics with \( l = 2 \). Thus, using

\[
P^2_l (\mu) = \frac{3\mu^2 - 1}{2},
\]

we can derive Equations (49) and (74). Similarly, the function \( h \) (Equation (47)) can be expressed as a combination of harmonics with \( l = 1 \), thus using

\[
P^1_l (\mu) = \mu,
\]

and by noticing that \( h(\theta, \psi) = \epsilon \), we can derive Equation (50).

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