On The Value at Risk Using Bayesian Mixture Laplace Autoregressive Approach for Modelling the Islamic Stock Risk Investment

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Abstract. Stocks are known as the financial instruments traded in the capital market which have a high level of risk. Their risks are indicated by their uncertainty of their return which have to be accepted by investors in the future. The higher the risk to be faced, the higher the return would be gained. Therefore, the measurements need to be made against the risk. Value at Risk (VaR) as the most popular risk measurement method, is frequently ignored when the pattern of return is not uni-modal Normal. The calculation of the risks using VaR method with the Normal Mixture Autoregressive (MNAR) approach has been considered. This paper proposes VaR method couple with the Mixture Laplace Autoregressive (MLAR) that would be implemented for analysing the first three biggest capitalization Islamic stock return in JII, namely PT. Astra International Tbk (ASII), PT. Telekomunikasi Indonesia Tbk (TLMK), and PT. Unilever Indonesia Tbk (UNVR). Parameter estimation is performed by employing Bayesian Markov Chain Monte Carlo (MCMC) approaches.

1. Introduction

Stocks are type of security that signifies ownership in a corporation and represents a claim on part of the corporation's assets and earnings. Stocks are known as the financial instruments traded in the capital market which have a high level of risk. Their risks are indicated by their uncertainty of their return which have to be accepted by investors in the future. The higher the risk to be faced, the higher the return would be gained. Therefore, the measurements need to be made against the risk. By knowing the risk, the investment policy can be made more scalable. Important treatment that should be made in risk management is to identify the cause of the risk. The popular and most commonly used by investors and commercial banks to determine the extent and occurrence ratio of potential losses in their institutional portfolios is Value at risk (VaR). The accuracy of the calculation of VaR is indispensable when determining the capital will be issued by the company. Thus, the risks which will faced by the company is getting smaller and losses can be handled.

Forecasting the return which will occur in the future is required in determining the likelihood of the risk occurring in the future. Stock return data is data in the form of time series. One method of forecasting time series data is ARIMA Box-Jenkins. Box-Jenkins ARIMA time series is linear methods are developed based on assumptions residual Normal distribution. Box-Jenkins ARIMA method would be appropriate if the observation of time series data is dependent or related to each other. One of the ARIMA models which characterize the condition of the stock return is Autoregressive models. Autoregressive Model is a model that describes the dependent variable is affected by the dependent
variable itself in the periods and times previously. This model requires that residuals follow Normal distribution with zero mean and a certain variance. But in reality, many time series data are not stationary in mean and tend to be multimodal. In addition, many time series data are heteroscedasticity which provide marginal patterns and leptokurtic, so the assumption of Normality is violated [1]. Thus, [1] generalize the model of Gaussian Mixture Transition Distribution (GMTD) which was introduced by [2] for the case of non-linear became Mixture Autoregressive (MAR) model. The MAR model consists of a mixture of $K$ Gaussian AR components. It is able to model time series with conditional multimodal distributions and heteroscedasticity. Not only that, but also this model can capture features better than other alternative models.

Nguyen and McLachan have been able to prove on the research which has been conducted using linear mixture of experts that by using Laplace distribution, the results obtained will be more robust than using the Normal distribution [3]. Based on the statement, [4] introduced a Mixture Laplace Autoregressive (MLAR) which consists of a mixture of $K$ Laplace AR components as an alternative of MAR. The results of the analysis conducted by Nguyen on the data calcium imaging of zebrafish brain showed that the results from the model established by MLAR been able to capture the pattern of actual conditions. This paper proposes VaR method couple with the Mixture Laplace Autoregressive (MLAR) that would be implemented for analysing the first three biggest capitalization Islamic stock return in Jakarta Islamic Index (JII), namely PT. Astra International Tbk (ASII), PT. Telekomunikasi Indonesia Tbk (TLMK), and PT. Unilever Indonesia Tbk (UNVR). Data obtained from the official website of yahoo finance in the form of daily data closing stock price (close price) from January 2010 to July 2016. Stock of the three companies were chosen because the company has the largest market capitalization, which represent the daily market trading, even capable of being a mover index in forming JII and JCI (Joint Stock Price Index) at the di Bursa Efek Indonesia (BEI). In addition, the stocks were the active and liquid stocks which meet the JII criteria during the last 6 years and one of the market leader in the industrial sector, so that it can be used as a benchmark. Parameter estimation is performed by employing Bayesian Markov Chain Monte Carlo (MCMC) approaches.

2. Literature

2.1 Mixture Laplace Autoregressive

An Autoregressive model or AR($p$) is when a value from a time series is regressed on previous values from that same time series. Equation of AR($p$) the model can be written
\[
\hat{y}_t = \phi_1 \hat{y}_{t-1} + \cdots + \phi_p \hat{y}_{t-p} + a_t; \quad \hat{y}_t = y_t - \mu, \tag{1}
\]
where $\phi_p$ is a parameter autoregressive $p$th-lag and $a_t$ is residual of $t$th-observation.

[1] was introduced Mixture Autoregressive (MAR) methods using a mixture of $K$ Gaussian components to equation
\[
F(y_t | \mathcal{F}_{t-1}; \theta) = \sum_{i=1}^{K} \pi_i \Phi(y_t; \phi_{i0} + \sum_{j=1}^{p} \phi_i y_{t-j}, \sigma_i), \tag{2}
\]
where $\Phi$ is Normal density function, $\pi_i > 0$, $\sum_{i=1}^{K} \pi_i = 1$, $\theta = (\phi_{i0}, \phi_i, \cdots, \phi_{ip}) \in R^{p+1}$, and $\sigma_i > 0$ for $i = 1, 2, \ldots, K$.

[4] introduced a Mixture Laplace Autoregressive (MLAR) which consists of a mixture of $K$ Laplace AR components as an alternative of MAR. $Y_t$ arises from a $K$ component MLAR model of order $p$ (i.e. an MLAR($K, p$) model) if $Y_t | \mathcal{F}_{t-1}; \theta$ has a density of the form
\[
F(y_t | \mathcal{F}_{t-1}; \theta) = \sum_{i=1}^{K} \pi_i \mathcal{A}(y_t; \phi_{i0} + \sum_{j=1}^{p} \phi_i y_{t-j}, \sigma_i), \tag{3}
\]
where
\[ \lambda(y; \mu, \sigma) = \left(\sigma \sqrt{2}\right)^{-1} \exp\left(-\sqrt{2} \left| x - \mu \right| / \sigma \right) \]  \hspace{1cm} (4)

is the Laplace density function with mean \( \mu \) and variance \( \sigma^2 \), and

\[ \mathbf{\theta} = (\pi_1, \pi_2, \ldots, \pi_{k-1}, \phi_1^T, \phi_2^T, \ldots, \phi_{d-1}^T, \sigma_1, \sigma_2, \ldots, \sigma_k)^T \]  \hspace{1cm} (5)

is the model parameter vector. MLAR model residual also has a Laplace Distribution [3].

MLAR model has conditional mean and conditional variance

\[ E(Y_i | \mathcal{F}_{i-1}) = \sum_{j=1}^{k} \pi_j \phi_j^T y_{(i-1)} \]  \hspace{1cm} (6)

and

\[ \text{Var}(Y_i | \mathcal{F}_{i-1}) = \sum_{j=1}^{k} \pi_j \sigma_j^2 + \sum_{j=1}^{k} \pi_j \left[ \phi_j^T Y_{(i-1)} \right]^2 + \left[ \sum_{j=1}^{k} \pi_j \phi_j^T y_{(i-1)} \right]^2. \]  \hspace{1cm} (7)

where \( y_{(i-1)} = (y_{1-1}, \ldots, y_{p-1})^T \). Given a known or estimated parameter vector, Eqs. (6) and (7) can be used to make predictions of \( y_i \) based on \( \mathcal{F}_{i-1} \).

2.2 Parameter Estimate

Bayesian models developed from Bayes methods. The basis of this method is the Bayes theorem which states the posterior distribution can be written

\[ p(\theta | y) \propto f(y | \theta) p(\theta) \]  \hspace{1cm} (8)

Prior distribution is an important part of Bayesian inference and represent information about the parameters that are not sure which, combined with the probability distribution of the new data to generate a posterior distribution, then it is used for the conclusion and future decisions involving the prior distribution. The type of prior distributions is informative Prior, Conjugate and Non-conjugate prior, proper and non-proper prior, as well as Pseudo prior.

Markov Chain Monte Carlo (MCMC) is a general method which is done by determining the values \( \theta \) of distribution approach and then \( \theta \) values corrected so it closer to the target posterior distribution \( p(\theta | y) \).

Markov Chain algorithm that has been found and is useful in many multi-dimensional problem is the Gibbs sampler also called alternating conditional sampling, defined in sub vector of \( \theta \). Let parameter \( \theta \) has been divided into \( d \) components or sub vector, \((\theta_1, \theta_2, \ldots, \theta_d)\). Each iteration of the Gibbs sampler cycle through sub vector of \( \theta \), draw each part depends on the value of other \( \theta \). At each iteration \( t \), a sequence of \( d \) sub vector selected and, in turn, each \( \theta_j \) was given a sample of the conditional distribution of all other components of \( \theta \):

\[ p(\theta_j | \theta_{-j}^{t-1}, y). \]  \hspace{1cm} (9)

where \( \theta_{-j}^{t-1} \) represents all components of \( \theta \), except for the actual value of \( \theta_j \):

\[ \theta_{-j}^{t-1} = (\theta_1^{t-1}, \theta_2^{t-1}, \ldots, \theta_{j-1}^{t-1}, \theta_{j+1}^{t-1}, \ldots, \theta_d^{t-1}). \]  \hspace{1cm} (10)

then, each sub vector \( \theta_j \) is updated in conditional on a final value of the other components of \( \theta \), where iteration value of the components ready to be updated and iterating \( t-1 \) the values of the other components.

2.3 Best Model Selection

Assume, in general, that the distribution of the data, \( y = (y_1, y_2, \ldots, y_p) \), depends on a \( p \)-dimensional parameter vector \( \theta \). From a frequentist point of view, model assessment is based on the deviance, the difference in the log-likelihoods between the fitted and the saturated model. The saturated
model refers to the model with as many parameters as observations, which yields a perfect fit to the data. By analogy, [6] suggested examining the posterior distribution of the classical deviance defined by

\[ D(\theta) = -2 \ln f(y | \theta) + 2 \ln g(y) \]  \hspace{1cm} (11)

for Bayesian model selection. [6] proposed comparing plots and potential summaries such as the posterior mean of \( D(\theta) \), and [7] followed these suggestions in the development of DIC as a model choice criterion. By Re-evaluated \( DIC = \bar{D} + p_\theta \), then obtained equation

\[ DIC = \bar{D} + 2p_\theta, \]  \hspace{1cm} (12)

2.4 Value at Risk (VaR)

Value at Risk (VaR) or also called Quantile Risk Metrics describe the estimation of the maximum loss that may occur in the bank's portfolio due to market risk in a certain time period and in a certain degree of statistical confidence. According to [8], VaR is the dominant methodology to predict exactly how much money is at risk every day in the financial markets. Based on the definition of VaR, when talking about the VaR must not be separated in terms of risk. Risk is a combination of probability of occurrence with the consequences or repercussions.

VaR can be calculated analytically, on the basis of Normal linear, historical simulation or monte carlo simulation. VaR can be defined mathematically as the value of the loss at a given confidence level \((1 - \alpha)\) and it is equivalent to decreasing the quantile of the probability distribution of the random variable, so

\[ P(X < x_\alpha) = \alpha \]  \hspace{1cm} (13)

where \( x_\alpha \) is -VaR

If distribution function \( F(x) \) of \( X \) is known then the quantile corresponding to any given value of \( \alpha \) can be calculated as

\[ x_\alpha = F^{-1}(\alpha) \]  \hspace{1cm} (14)

then

\[ \text{VaR} = -x_\alpha = -\left(F^{-1}(\alpha)\right). \]  \hspace{1cm} (15)

Normal Linear VaR calculation method is a method that assumes that the distribution of returns is Normal, and portfolios must be linear. Therefore

\[ P(X < x_\alpha) = P\left(\frac{X - \mu}{\sigma} < \frac{x_\alpha - \mu}{\sigma}\right) = P\left(Z < \frac{x_\alpha - \mu}{\sigma}\right) = \alpha, \]  \hspace{1cm} (16)

where \( Z \) is a Normal standard variable. So

\[ \frac{x_\alpha - \mu}{\sigma} = \Phi^{-1}(\alpha), \]  \hspace{1cm} (17)

With symmetry Normal distribution function,

\[ \Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha). \]  \hspace{1cm} (18)

Therefore, with substituting \( \text{VaR}_\alpha = -x_\alpha \) to equation (18) then

\[ \text{VaR} = \Phi^{-1}(1-\alpha)\sigma - \mu. \]  \hspace{1cm} (19)

Thus, the equation used to calculate the VaR on the horizon \( h \) is writable

\[ \text{VaR}_{h,\alpha} = \Phi^{-1}(1-\alpha)\sigma_h - \mu_h. \]  \hspace{1cm} (20)

Hence, we often assume the portfolio is expected to return the risk free rate so that \( \mu_h \), the present value of the expected return, is zero. We shall assume this in the following, unless explicitly stated otherwise. Without the drift adjustment, the Normal linear VaR formula is simply

\[ \text{VaR}_{h,\alpha} = \Phi^{-1}(1-\alpha)\sigma_h. \]  \hspace{1cm} (21)
By the symmetry of the Normal distribution function,

\[
\text{VaR}_{h,\alpha} = -\left(\Phi^{-1}(1-\alpha)\right)\sigma_h
\]

So, under the assumption of i.i.d. returns, it is only when the portfolio is expected to return the discount rate, i.e. \(\mu_t = 0\), that \([9]\]

\[
\text{VaR}_{h,\alpha} = \sqrt{h} \times \text{VaR}_{1,\alpha}
\]

Thus, \(\text{VaR}\) to calculate the Laplace distribution mix as follows

\[
\text{VaR}_{\text{mix}} = \pi_1 \sqrt{h} \times \text{VaR}_{1,\alpha} + \pi_2 \sqrt{h} \times \text{VaR}_{1,\alpha} + \ldots + \pi_K \sqrt{h} \times \text{VaR}_{1,\alpha}
\]

2.5 Return

Return is important because it is used as a measure of corporate performance and is also useful as a basis for determining the expected return and risk in the future. The formula used to determine the return is

\[
R_t = \frac{d_t - d_{t-1}}{d_{t-1}}
\]

3. Previous Research

Based on previous research conducted by \([10]\) with the same data obtained identification results return three stocks as follows

Figure 1 shows that the stock return data PT Astra International Tbk, PT Telekomunikasi Indonesia Tbk and PT Unilever Indonesia Tbk bring nature slope and kurtosis indicating abnormalities in the data return stock. In addition it was shown that the variability of return sufficient high, so that led to stock returns tails at the extreme left end or right. If the forced return using univariate Normal unimodal pattern seen their deviations.

![Figure 1. Histogram of stock return data PT Astra International Tbk, PT Telekomunikasi Indonesia Tbk and PT Unilever Indonesia Tbk](image)
Irregularities that occurs is due to the fairly extreme return data or outliers, resulting slope value is greater than zero. The number of returns is worth around zero making patterns return has a high peak is commonly called the leptokurtic. Consequently alleged that on the pattern of stock returns ASII, TLKM and UNVR to have a mixture distribution or a mixture distribution is quite clear. This is supported by results conformance testing distribution (goodness of fit).

Figure 2 provides information that return data ASII shares, TLKM and UNVR fluctuate around the mean. Thus, visually ASII return data, TLKM and UNVR can be said to have been stationary in the mean. After found that the three stock return data has been stationary in mean

4. Result

In this section, will explain the results of the comparison the best model between the MNAR and MLAR methods, estimate of the best model and VaR calculation results based on the parameters obtained from the estimated best model. Bayesian MNAR method performed again as was done by [10] but with a number of longer period. This was done to compare goodness of fit between Bayesian MNAR and MLAR model based on the value of DIC. Comparison between Bayesian MNAR and MLAR the return data PT. Astra International Tbk (ASII), PT. Telekomunikasi Indonesia Tbk (TLMK), and PT. Unilever Indonesia Tbk (UNVR) shown by Table 1.

Table 1 shows that in PT. Astra International Tbk (ASII), PT. Telekomunikasi Indonesia Tbk (TLMK), and PT. Unilever Indonesia Tbk (UNVR) best model generated using Bayesian MLAR analysis. This is indicated by the value of Bayesian MLAR DIC smaller base than Bayesian MNAR. On the results of the analysis of each share also shows that the same order, Bayesian MLAR also showed better results than the Bayesian MNAR. It shows that in this case, the method MLAR better able to model better than MNAR methods, so that the prediction results generated by Bayesian MLAR also better than MNAR Bayesian methods.
Table 1. Comparison of Bayesian MNAR and MLAR model based on the value of DIC

| Stock     | Methods        | Models                                | DIC    |
|-----------|----------------|---------------------------------------|--------|
| ASII      | Bayesian MLAR  | MLAR(2;[3],[3,6])                    | -12572.1 |
|           | Bayesian MNAR  | MLAR(3;[3],[3,6],0)                  | -14648.1 |
| TLKM      | Bayesian MLAR  | MLAR(2;[2,3],[3,4])                  | -13725.4 |
|           | Bayesian MNAR  | MLAR(2;[2],[3])                      | -13731.2 |
|           |                 | MLAR(2;[2],[4])                      | -13729.9 |
|           |                 | MLAR(2;[3],[4])                      | -13725.8 |
|           |                 | MLAR(3;[2,3],[3,4],[2,3,4])          | -16385.7 |
|           |                 | MLAR(3;[2],[3],[4])                  | -16383.6 |
| UNVR      | Bayesian MLAR  | MLAR(2;2,[11])                       | -12772.7 |
|           | Bayesian MNAR  | MLAR(3;1,2,[11],0)                  | -14934.4 |

VaR calculation is done using VaR formula that is based on the return distributed graphic form Laplace. Thus, it is necessary to determine VaR formula based CDF Laplace distribution. The resulting decline in VaR following formula

$$VaR_{h,\alpha} = -\left(\sigma_\alpha \left(\ln \left(4\left(-\alpha^2 + \alpha\right)\right)\right)\right)$$

Similarly, the VaR is based on the Normal curve,

$$VaR_{h,\alpha} = \sqrt{h} \times VaR_{1,\alpha}$$

Thus, VaR to calculate the Laplace distribution mix as follows

$$VaR_{\text{mix}} = \pi_1 \sqrt{h} \times VaR_{1,\alpha} + \pi_2 \sqrt{h} \times VaR_{2,\alpha} + \ldots + \pi_K \sqrt{h} \times VaR_{K,\alpha}$$

Parameter estimate of the best model from PT. Astra International Tbk (ASII), PT. Telekomunikasi Indonesia Tbk (TLMK), and PT. Unilever Indonesia Tbk (UNVR) shown by Table 2. In calculation VaR, we only need significant parameter estimate of \( \pi \) and \( \sigma \). Calculation VaR OF PT. Astra International Tbk (ASII), PT. Telekomunikasi Indonesia Tbk (TLMK), and PT. Unilever Indonesia Tbk (UNVR) shown by Table 3

Table 3 show that the 5% 1-day VaR of ASII is 3.56% of return value. With 100 million rupiah invested in the return the VaR is Rp. 100 million \( \times 0.03561 = \) Rp. 3561000. In other words, we are 95% confident that we will lose no more than Rp. 3561000 from investing in this fund over the next day. The Interpretation Applicable to the VaR others.
Table 2. Estimate Parameter of Bayesian MLAR model

| Node | ASII MLAR(3;[3],[3,6], 0) | TLKM MLAR(3;[2,3],[3,4],[2,3,4]) | UNVR MLAR(3;2,[11],[2,[11]]) |
|------|--------------------------|---------------------------------|---------------------------------|
| $\pi_1$ | 0.33320 | 0.33350 | 0.33450 |
| $\pi_2$ | 0.33280 | 0.33330 | 0.33280 |
| $\pi_3$ | 0.33400 | 0.33320 | 0.33280 |
| $\phi_{1,1}$ | - | -0.26520 | -0.09611 |
| $\phi_{1,2}$ | -0.09142 | -0.03814 | |
| $\phi_{2,3}$ | -0.09649 | -0.08405 | |
| $\phi_{3,4}$ | -0.07648 | - | |
| $\phi_{2,6}$ | -0.05720 | - | |
| $\phi_{2,11}$ | - | - | 0.05798 |
| $\phi_{3,3}$ | -0.10141 | -0.09634 | |
| $\phi_{3,4}$ | -0.08699 | -0.08622 | |
| $\phi_{3,11}$ | - | - | 0.05125 |
| $\sigma_1$ | 0.02149 | 0.01786 | 0.02053 |
| $\sigma_2$ | 0.02151 | 0.01786 | 0.02032 |
| $\sigma_3$ | 0.02133 | 0.01788 | 0.02058 |

Table 3. VaR calculation

| Stock | Long Investment | 95% CI | 99% CI |
|-------|----------------|--------|--------|
| ASII  | 1-day          | 0.03561| 0.06924|
|       | 5-days         | 0.07963| 0.15482|
|       | 20-days        | 0.15926| 0.30964|
| TLKM  | 1-day          | 0.02908| 0.05769|
|       | 5-days         | 0.06501| 0.12900|
|       | 20-days        | 0.13003| 0.25800|
| UNVR  | 1-day          | 0.03401| 0.06612|
|       | 5-days         | 0.07605| 0.14786|
|       | 20-days        | 0.15210| 0.68878|

5. Conclusion
In this paper show that analysis using MLAR approach was determined DIC value better than MNAR approach. So. estimation using MLAR more appropriate to calculate VaR. Furthermore, the higher the confidence interval is used. the VaR values generated will be greater and the longer the investment is
done. then the risk will be received by investors are also getting bigger. In other words, the possibility of losses that come will be even greater.

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