Optimal control of pulse compensators of oscillatory phenomena in a network oil and gas pipeline system

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Abstract. Within the framework of oil and gas engineering, the problem of optimal control of pulse compensators that counteract harmful oscillatory phenomena in a continuous medium during transportation via network gas-hydraulic carriers is considered. Powerful compressor units that create high pressure in the carrier of a continuous medium, to a large extent contribute to the formation of undesirable oscillatory phenomena (pulsations) that occur at the output of these compressors. These ripples are transmitted to the network carrier environment, which significantly reduces the efficiency of compressor units and even causes accidents in the networks of gas and hydraulic carriers. The latter means that the software engineering of the oil and gas industry should include research in the direction of improving the reliability of operation of compressor units and gas-hydraulic carriers. In the presented study, the mathematical description of the oscillatory process of a continuous medium is carried out by formalisms of a differential-difference system of hyperbolic equations with distributed parameters on a graph. At the same time, the mathematical model contains a fairly accurate mathematical description of controlled pulse compensators. The problem of controlling pulse compensators of an oscillatory process is considered as the problem of a point control action on a controlled differential-difference system at the places where continuous medium vibration dampers are connected to a network carrier. This is a characteristic feature of the presented study, which is quite often used in practice when engineering the processes of transporting various kinds of continuous media through network oil and gas carriers. The study essentially uses the conjugate state and the conjugate system for a differential-difference system - the relations determining the optimal point control are obtained. The results of the work are applicable in the framework of oil and gas engineering to the study of issues of stabilization and parametric optimization.

1. Introduction
When designing and subsequent operation of pipeline networks, questions arise related to the creation of reliable structures that counteract the oscillatory effects generated by powerful compressor units, which are the main equipment of the technological part of the pipeline system. It is these installations
that contribute to the appearance of harmful pulsations that are transmitted to the pipeline system and initiate the vibration of the system. The methods of influencing these oscillatory phenomena to eliminate them in their diversity are based on the use of various types of pulsation dampers directly at the outlet of the compressor unit or at the initial section of the pipeline system [1]. With all the significance of the results available at the moment, it should be noted that the calculation of pulsation dampers is focused on a certain (set) frequency and does not take into account the change in pulsation frequencies over time in linear fragments of the pipeline system. Below is an approach that takes into account the change in the characteristics of vibrations in the fragments of the pipeline system over time. The latter predetermined the emergence of controlled pulse compensators. The results presented below are based on the results of the work on optimal control of differential systems with distributed parameters on graphs [2] (see Also in the monograph [3], the study is carried out on the example of transportation of a pulsating gas flow (oil and gas medium) along a pipeline line having a certain number of linear fragments, the state of the gas flow is observed in discrete time.

2. Notations and basic concepts
A pipeline line of length \( \ell \), which is the carrier of the gas flow, is interpreted by us as the simplest oriented graph (network) \( \Gamma \) containing \( R \) edges \( \gamma_j, j = 1, J \) (linear fragments of the network), respectively parametrized by segments \([ (j-1)h, jh] \), \( h = \ell/J \), \( j = 1, J \), and nodes \( \xi_s, s = 0, 1, ..., J \).

Thus \( \Gamma = \sum_{j=1}^{J} \gamma_j, \xi_0, \xi_J \) - boundary nodes, \( \partial \Gamma = \{ \xi_0, \xi_J \} \) - set of boundary nodes, \( \xi_s, s = 1, J - 1 \) - internal nodes, \( \xi = \bigcup_{s=1}^{J} \xi_s \) - set of internal nodes. The nodes of the set \( \xi \) are the places where gas flow vibration dampers are connected to the network. Each pulse damper is structurally a cylinder with a piston built into the pipeline system, which is driven according to a certain (set) law and this determines the external impact on the system.

The studied fluctuations of the gas flow are described by the differential-difference equation

\[
\frac{1}{\tau} [y(k+1) - 2y(k) + y(k-1)] = \frac{d}{dx} \left( a(x) \frac{dy(k)}{dx} \right) + f(k), \quad k = 1, 2, ..., M - 1,
\]

on the set \( \Gamma_0 = \Gamma \setminus \bigcup \partial \Gamma \), where \( y(k) := y(x; k), f(k) := f(x; k), k = 0, 1, 2, ..., M \). At the points of the set of connection of pulse dampers to the network, the following conditions occur

\[
y(k)_{|_{x=\beta \gamma_j}} = y(k)_{|_{x=\beta \gamma_{j+1}}},
\]

\[
a(x) \frac{dy(k)}{dx} \bigg|_{x=\beta \gamma_j} + g(k)_{|_{x=\beta \gamma_j}} = a(x) \frac{dy(k)}{dx} \bigg|_{x=\beta \gamma_{j+1}}.
\]

The initial and boundary conditions are determined by the relations

\[
y(0) = \phi(x), \quad y(1) = \psi(x),
\]

\[
y(k)_{|_{x=\ell}} = 0, \quad k = 0, 1, 2, ..., M,
\]

respectively. Note that in the internal nodes \( \xi_s \) (\( s = 1, J - 1 \)), flows \( g(k)_{|_{x=\beta \gamma_j}} \) (\( j = 1, J \)) from pulse dampers are added to the flows \( a(x) \frac{dy(k)}{dx} \bigg|_{x=\beta \gamma_j} \) in the pipeline system; the function \( a(x) \) determines the wave properties of the oscillating medium, the function \( f(k) \) describes the effects of the environment surrounding the pipeline. Note also that the boundary conditions (5) can be
inhomogeneous, then it is not difficult to obtain homogeneous boundary conditions (5) using standard substitutions and translating the inhomogeneities to the right side of equation (1) [2].

**Definition 1.** The solution of the differential-difference equation (1) with conditions (2), (3) is called the functions \( y(k) \ (k = 1, 2, \ldots, M) \) belonging to the space

\[
Y(0, \ell) = C[0, \ell] \cap C^1([0, \ell] \setminus \xi) \cap C^2((0, \ell) \setminus \xi)
\]

and satisfying the initial (4) and boundary (5) conditions.

The following statements take place.

**Theorem 1.** Let \( \varphi(x), \psi(x) \in C[0, \ell], \ a(x) \in C^1([0, \ell] \setminus \xi), \ f(k) \in C[0, \ell], \ k = 0, 1, 2, \ldots, M, \ M \geq 2 \). The solution of the differential-difference system of equations (1)-(5) in space \( Y(0, \ell) \) exists and is unique.

Thus, the differential-difference system of equations (1)-(5) is a time-discrete mathematical description of the oscillatory process of a gas flow in a pipeline line.

**Theorem 2.** When the conditions of Theorem 1 are met, the solution \( y(k) \ (k = 0, 1, 2, \ldots, M) \) of the differential-difference system (1)-(5) continuously depends on the initial data \( \varphi(x), \psi(x) \) and \( f(k), \ k = 0, 1, 2, \ldots, M \), for any \( M \geq 2 \).

The proof of the statements of theorems 1 and 2 is similar to those given in [2].

### 3. Main result

Let us consider the problem of controlling pulse compensators in a pipeline system, investigating the problem of controlling the differential-difference system (1)-(5), assuming that \( \varphi(x), \psi(x) \in C[0, \ell], \ a(x) \in C^1([0, \ell] \setminus \xi), \ f(k) \in C[0, \ell], \ k = 0, 1, 2, \ldots, M \). The control actions on the system (1)-(5) are the numerical values \( \nu(k) = g(k) \bigg|_{x = jh} \), \( j = 1, \ldots, J \), \( k = 1, \ldots, M \), pulse dampers flow values

\(
\nu(k) = \{u(k)_1, u(k)_2, \ldots, u(k)_J\}, \)

next denote the permissible set of values of such flows: \( U \) - the set of Euclidean space \( \mathbb{R}^J \). The system of relations (1), (3), using the properties of \( \delta \)-functions, we will write more compactly. Adding a term \( g(k) \bigg|_{x = jh} \) to the pipe in the vicinity of the flow point \( x = jh \) from the pulse suppressor (ratio (3)) means that a term \( \nu(k)\delta'(x - jh) \) will be added to the right side of ratio (1), then ratios (1), (3) will be replaced by the ratio

\[
\frac{1}{\tau} [y(k + 1) - 2y(k) + y(k - 1)] - \frac{dy}{dx} = f(k) + \sum_{j=1}^{J} \nu(k)\delta'(x - jh), \quad k = 1, 2, \ldots, M - 1,
\]

and we come to the problem (4)-(6) for functions \( y(k;\nu(k)) := y(x; k;\nu(k)) \ (k = 1, 2, \ldots, M) \) belonging to the space \( Y(0, \ell) \) and satisfying the conditions (4), (5). These functions depend on the flow \( \nu(k) = \{u(k)_1, u(k)_2, \ldots, u(k)_J\} \) of pulse dampers and describe the state of the system (4)-(6) for each fixed \( \nu(k) \). Let's define the observation \( y(k;\nu(k)) \) by a linear operator \( C: Y(0, \ell) \rightarrow C[0, \ell] \), i.e.

\[
w(k;\nu(k)) := w(x, k;\nu(k)) = Cy(k;\nu(k)).
\]

As follows from the statements of Theorems 1 and 2, the mapping \( \nu(k) \rightarrow y(k;\nu(k)) \) of space \( U \) to space \( Y(0, \ell) \) is continuous for any \( k = 1, 2, \ldots, M \), \( M \geq 2 \).

To eliminate undesirable oscillatory phenomena, it is sufficient to select the flow \( \nu(k) \) of pulse dampers in such a way that the state \( y(k;\nu(k)), \ k = 1, 2, \ldots, M \), of the system (4)-(6) becomes zero at \( [0, \ell] \). We define the minimizing functional \( \Psi(\nu) \) by the relation

\[
\Psi(\nu) := \Psi(\nu(1), \nu(2), \ldots, \nu(M)) = r \sum_{k=1}^{M} \Psi_k(\nu(k)),
\]
where $\Psi^*_i(v(k)) = PCy(k;v(k))P_{e_1}^2 + (Nv(k),v(k))_{R'}$, and $N$ is a Hermitian $J$-matrix, which is connected by a condition $(Nv(k),v(k))_{R'} \geq \varepsilon P_{e_1}v(k)P^2_{e_1}$ ($\varepsilon > 0$) $\forall v(k) \in U$, $k = 1,2,...,M$; here $P_{e_1}$ denotes the norm in $L_2(\Gamma)$ and the scalar product and the norm in $R'$, respectively. Let $U_0$ be a bounded subset $U$.

The task of optimal control of pulse compensators is to find
$$\inf_{v \in U_0} \Psi(v), \text{ where } v = \{v(k), k = 1,2,...,M\}.$$ 

Theorem 3. The problem of point control of the system (2)-(6).

Thus, we come to the problem of point control of the system (2)-(6).

4. Conclusion

The statement of Theorem 5 sets an algorithm for obtaining the optimal control $v^* \in U_0$, $\Psi(v^*) = \min_{v \in U_0} \Psi(v)$. For small $\tau$, the problem is uniquely solvable in space $Y(0,\tau)$.

Theorem 4. The system (8), (9) for small $\tau$ is uniquely solvable in space $Y(0,\tau)$.

To prove the statement, it is necessary to change the numbering in the relations (8), (9) according to the law $l = M - k$, $k = M,M-1,...,1,0$, and use the statement of Theorem 1.

Theorem 5. Provided that the statement of Theorem 3 is fulfilled, optimal control $v^* \in U_0$ is characterized by the following relations
$$\frac{1}{\tau}[y(k+1;v(k+1)) - 2y(k;v(k)) + y(k-1;v(k-1))] - \frac{d}{dx}(a(x)\frac{dp(k)}{dx}) = \frac{1}{\tau}[y(k+1;v'(k+1)) - 2y(k;v'(k)) + y(k-1;v'(k-1))] - \frac{d}{dx}(a(x)\frac{dp(k;v'(k))}{dx}) = f(k) + \sum_{j=1}^{p} v(k) \delta(x - jh), \quad k = 1,2,...,M - 1,$$

$$\sum_{j=1}^{p} p_j v_j(k)(v_j(k) - v'_j(k)) + (Nv(k),v(k) - v'(k))_{R'} \geq 0, \quad k = 0,1,...,M,$$

The proof of the statement of the theorem is similar to those presented in the monograph [3, p. 289]. The statement of Theorem 5 sets an algorithm for obtaining the optimal control action $v^* = \{v^*(k), k = 1,2,...,M\}$ and states $y(k;v^*(k)), p(k;v^*(k)), k = 1,2,...,M$ for systems (2), (6) and 8), (9), respectively.
dampers (in the mathematical model, these are the internal nodes of the graph). In the analysis of the control problem, the conjugate state and the conjugate system (8) are significantly used, (9) for the initial system (2)-(6) - the relations (statement of Theorem 5) for determining optimal control are obtained. It should be noted that the approach used can be extended to the problem of damping vibrations during the transport of continuous media with boundary conditions of a more general form than are presented in relation (5). The presented results were a reflection of the natural need for the transition to the analysis of pulse phenomena of evolutionary network-like nonequilibrium processes of continuous media transfer over 3D carriers, which was initiated by the works [3, 4]. The results can be applied in the study of stability problems of these processes [5], as well as in the study of business interaction processes in socio-economic systems [6], in the analysis of optimization [7, 8, 9] and the search for stability conditions for various dynamic systems.

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