Current–Carrying Cosmic String Loops Leading to Vortons

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In this article we review recent work aimed at showing explicitly the influence of electromagnetic self corrections on the dynamics of a circular vortex line endowed with a current at first order in the coupling between the current and the self–generated EM-field.

To appear in
Proceedings of the Eighth Marcel Grossmann Meeting
on General Relativity, Gravitation and Relativistic Field Theories.
22–27 June 1997, Jerusalem
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Topological defects are widespread in most extensions of the standard model of particle interactions. Among them cosmic strings could have been produced as vacuum vortex lines as the outcome of early phase transitions. A resulting network of strings would indeed have key features and leave unique traces in a variety of present day observables, ranging from the small–scale anisotropies in the CMB radiation to the lensing of distant astrophysical sources. Extensions of the simplest models of cosmic strings, as the one envisaged by Witten, involve extra degrees of freedom which are coupled to the vortex–forming Higgs field. This is the source of currents, bound to the core of the vortices, that may play a fundamental role in the dynamics of the defects and, in the case of string loops that we will consider here, may build up so strong as to compensate the natural string tension towards collapse. This observation, first pointed out by Davis and Shellard in 1989, led to the study of rotating, equilibrium particle–like vorton configurations and to the eventual excess danger implied by these remnants for the standard cosmology.

Regarding vorton formation, not any arbitrary cosmic string loop, with given characteristic “quantum” numbers, will in general end up as a vorton. The way in which one can quantify the analysis is by studying an initial configuration (circular in our present work) by means of the covariant macroscopic formalism for general strings developed by Carter, and letting this initial state evolve in order to see whether it attains stability or not. This project is presently in progress and results were already obtained both for the neutral current–carrying case and also when the effect of including electromagnetic (e.m.) self coupling is taken into account. Radiative effects are of course important for the complete analysis and are currently under investigation. Let us recapitulate briefly the microphysics setting and its connection with the macroscopic string description we use in our analysis. We consider a Witten–type bosonic superconductivity model in which the fundamental Lagrangian is invariant under the action of a $U(1) \times U(1)$ symmetry group. The first $U(1)$ is spontaneously broken through the usual Higgs mechanism in which the Higgs field $\Phi$ acquires a non–vanishing vacuum expectation value. Hence, at an energy scale $\sim m$ we are left with a network of ordinary cosmic strings with tension and energy per unit length $T \sim U \sim m^2$, as dictated by the Kibble mechanism. The Higgs field is coupled not only with its associated gauge vector but also with a second charged scalar boson $\Sigma$, the current carrier field, which in turn obeys a quartic potential. A second phase transition breaks the second $U(1)$ gauge (or global, in
the case of neutral currents) group and, at an energy scale $\sim m_\pi$, the generation of a current–carrying condensate in the vortex makes the tension no longer constant, but dependent on the magnitude of the current, with the general feature that $T \leq m^2 \leq U$, breaking therefore the degeneracy of the Nambu–Goto strings. The fact that $|\Sigma| \neq 0$ in the string results in that either electromagnetism (in the case that the associated gauge vector $A_\mu$ is the e.m. potential) or the global $U(1)$ is spontaneously broken in the core, with the resulting Goldstone bosons carrying charge up and down the string. The invariance of the Lagrangian with respect to local changes in the phase $\varphi$ of the current carrier field $\Sigma = \Sigma(x, y) \exp[i\varphi(z, t)]$ is expressed by the conservation of the Noether current $2|\Sigma|^2(\partial^\mu \varphi - eA^\mu)$, where we implicitly consider a string along the $z$ direction and where the physically relevant scalar boson amplitude cannot depend on the internal string coordinates, $a = z, t$. In order to get the total macroscopic current one just needs to integrate over the string cross section (taking $A_\mu \sim \text{const}$ in the core) to get the current $z_a = 2\Sigma(\partial_a \varphi - eA_a) \equiv 2\Sigma \varphi_a$, where $\Sigma(z, t) = \int_{\text{core}} dx dy |\Sigma|^2$. In the macroscopic string description a key rôle is played by the dynamics of the system determined by the Lagrangian $\mathcal{L}\{w\}$. From it we get the conserved particle current vector $z_a = -\partial \mathcal{L}/\partial \dot{\varphi}_a = K^{-1} \varphi_a = 2\Sigma \varphi_a$, where we define $K^{-1} = -2d\mathcal{L}/dw$ which is in turn proportional to the amplitude of the condensate $\Sigma$. From the above middle equality we get $\mathcal{L}\{w\} = -m^2 - \varphi_\mu \varphi^\mu/2K = -m^2 - w/2K$ which, for weak currents (small $w$) coincides (recall $K \to 1$ for $w \to 0$) with the generalization of the Nambu–Goto model given by Carter and Peter in 1995: $\mathcal{L}\{w\} = -m^2 - \frac{1}{2}m_\pi^2 \ln \left\{ 1 + w/m_\pi^2 \right\}$.

The description of the dynamics of macroscopic strings in terms of $\mathcal{L}\{w\}$ is easily understandable, given the clear physical meaning of the phase $\varphi$. However, in what follows, an equally powerful dual formalism will be used, it being based on the master function $\Lambda\{\chi\}$, with $w = K^2 \chi$ and $\chi = z^\mu z_\mu$, where $z^\mu$ is the tangential current vector on the string worldsheet defined by $z^\mu = z^a x^a_\mu$ in terms of the current $z^a$ in the worldsheet. Both $\Lambda$ and $\mathcal{L}$ are related by a Legendre transformation $\Lambda = \mathcal{L} + K \chi$. These functions provide values for the string $U$ and $T$ depending on the signs of the state parameters $\chi$ and $w$. Hence we have $U = -\Lambda$ ($U = -\mathcal{L}$), $T = -\mathcal{L}$ ($T = -\Lambda$) for timelike (spacelike) currents $\chi < 0$ ($\chi > 0$). The validity of this description follows from the requirement of local stability which demands that the squared speeds $c^2_\pm = T/U$ and $c^2_\mp = -dT/dU$ of extrinsic and longitudinal perturbations be positive. Therefore $\mathcal{L}/\Lambda > 0 > d\mathcal{L}/d\Lambda$ in both regimes.

Electromagnetic corrections have recently been calculated by Carter (these proceedings). The result is surprisingly simple to implement. The correction enters through a modification of the string equation of state. The regularization of the divergent $A_\mu$ leads to a renormalization of $\Lambda$ of the kind $\Lambda \to \Lambda + \frac{1}{2}\lambda q^2 \chi$, where $\lambda$ is given by the current self generated potential $A^\mu = \lambda j^\mu$, with the e.m. current $j^\mu = q^a x^a_\mu$ flowing along the string and $\lambda = 2 \ln (m_\sigma \Delta)$, where $\Delta$ is an infrared cutoff scale to compensate for the asymptotically logarithmic behavior of the e.m. potential and $m_\sigma$ the ultraviolet cutoff corresponding to the effectively finite thickness of the charge condensate, i.e., the Compton wavelength of the current-carrying $m_\sigma^{-1}$. In the practical situation of a closed loop, $\Delta$ should at most be taken as the total length of the loop. We are now able to compute the variation of the
string equation of state with the e.m. self correction $\lambda q^2$, which we plot in Fig 1 (left panel) as a function of the sign-preserving square root of the state parameter $\nu = \text{Sign}(w)\sqrt{|w|}$. It is interesting to see that the inclusion of self correction allows larger currents along the string before reaching the saturation point.

Now, regarding the motion of circular vortex rings in flat space, a previous analysis of one of us shows that the variation of the string radius follows from the equation $M\sqrt{1-\dot{r}^2} = \Upsilon(r)$, where $M$ is the string’s total mass and $\Upsilon(r)$ is the self potential on which the string’s radius evolves. $\Upsilon$ itself can be written in terms of conserved quantities such as the number of carrier particles in the loop $Z$ and the topologically conserved (in the 2D worldsheet) winding number $N$ of the carrier–field’s phase $\phi$ around the loop. It is an easy task to derive the form of the ring radius in terms of these conserved quantities, $r^2 \propto Z^2(\frac{b^2 - K^2}{K^2})\chi$, where $b \equiv |N/Z|$. This tells us that the nature of the current (whether $\chi$ is positive or negative) depends on the sign of $b^2 - K^2$, where $b$ characterizes a particular current state of the string and $K$ is given by the particular macroscopic model (through its Lagrangian) including e.m. self corrections. What one finds from local stability considerations is that the range of variation of $K$ is $\lambda q^2 \leq K \leq 1 + \lambda q^2$ for $\chi \leq 0$, whereas $1 + \lambda q^2 \leq K \leq 2 + \lambda q^2$ in the $\chi \geq 0$ case. Therefore, it is only possible for $\chi$ to be positive if $b \geq 1 + \lambda q^2$, and negative otherwise. As the critical value $b_c = 1 + \lambda q^2$ was unity in the decoupled case, we see that interestingly enough e.m. corrections can modify the nature of the current for a given set of $Z$ and $N$.

We plot the variations of the self potential $\Upsilon$ with the ring’s circumference $\ell$ and the e.m. self coupling $\lambda q^2$ for $(m/m_\ast)^2 = 1 = b$ in the right panel of Fig 1. The thick curve, a ‘safe’ stable zone in parameter space is shown. For all regimes, this zone is limited to $\lambda q^2 \leq b \leq \lambda q^2 + 2$, a condition which is increasingly restrictive as $\lambda q^2$ increases, and may even forbid vorton formation altogether for a very large coupling, with interesting consequences for the vorton excess problem.

Extensive list of references is given in papers hep-ph/9609402 and hep-ph/9705204. A.G. thanks the British Council for partial financial support. This work was partially supported by EEC grants Nr PSS*0992 and CII*CT94-0004. We would like to thank C.Boehm, B.Carter and P.Peter for enlightening discussions.