On the extended loop calculus

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Abstract

Some features of extended loops are considered. In particular, the behaviour under diffeomorphism transformations of the wavefunctions with support on the extended loop space are studied. The basis of a method to obtain analytical expressions of diffeomorphism invariants via extended loops are settled. Applications to knot theory and quantum gravity are considered.

1 Introduction

Extended loops arise as generalizations of ordinary loops [1]. The set of all extended loops can be viewed as a manifold. This manifold can be endowed with a (local) infinite dimensional Lie group structure that contains the usual group of loops as a discrete subgroup. In the same way that in the case of ordinary loops, any Lie-algebra valued connection theory can be transcribed to the language of extended loops. As the group of extended loops has a more rich mathematical structure than the conventional loop space, several benefits at the calculation and regularization levels are exhibited by the extended loop representation. These advantages has been proved to be relevant in the study of the space of states of quantum gravity [2, 3, 4].

The purpose of this talk is to give a primary approach to the extended loop calculus, specially attending its applications to knot theory and quantum gravity. We analyze with some detail the mechanism of operation of the generator of diffeomorphism transformations on the extended loop wavefunctions and settle the basis of a systematic method to obtain analytic expressions of knot invariants using extended loops. The significance of the method (that

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provide analytical expression of knot invariants in a generic way) for the case of quantum gravity are discussed.

The next section contains several preliminary remarks concerning extended loops. In Sect. 3 the properties of the wavefunctions defined in the extended loop space are considered. In Sect. 4 the mechanism of operation of the generator of diffeomorphism transformations over the extended loop wavefunctions is analyzed. In Sect. 5 a prescription to build up “families” of extended knot invariants is developed and applied in some simple cases. The conclusions are in Sect. 6.

2 Preliminaries

The use of extended loops requires a new methodology. In some sense this new way to look loops is present at the level of ordinary loops. In fact, it was the intention to solve some problems of the conventional loop representation that had lead to the discovery of the extended loop group. In order to fix a point of departure towards the extended loop world we sketch briefly the history of the primary steps.

Holonomies are the basic quantities that underlies any loop representation of a Lie-algebra valued connection theory. They are given by the ordered exponential of a line integral of the connection along a close curve \( \gamma \):

\[
H_A(\gamma) := P \exp \oint_\gamma dy^a A_a(y) = 1 + \oint_\gamma dy^a A_a(y) + \cdots + \oint_\gamma dy^a \int_y^{y_r} dy^{a_{r-1}} \cdots \int_y^{y_1} dy_1 A_{a_1}(y_1) \cdots A_{a_r}(y_r) + \cdots
\]

We write

\[
\oint_\gamma dy^a A_a(y) = \oint_\gamma dy^a \int d^3x \delta(x-y) A_a(x) = \int d^3x A_a(x) \oint_\gamma dy^a \delta(x-y) \\
\equiv \int d^3x A_a(x) X^a(x, \gamma) \equiv A_{ax} X^{ax}(\gamma)
\]

\(X^{ax}(\gamma)\) is the multitangent field of rank one associated with the loop \( \gamma \). This field is a loop dependent distribution that admits a direct geometrical interpretation: it fixes the tangent at the point \( x \). Notice that in (2) the indices were grouped into pairs and a generalized Einstein convention was used. We write the paired indices with Greek letters, \( \mu := ax \). This notation is in agreement with the behaviour of the multitangents under general coordinate transformations\(^1\). Repeating the procedure with the other terms in (1) we get

\(^1\)The multitangents behave as generalized multitensors with respect to the Greek indices.
\[ H_A(\gamma) = 1 + A_{\mu_1} X^{\mu_1}(\gamma) + \cdots + A_{\mu_1 \ldots \mu_r} X^{\mu_1 \ldots \mu_r}(\gamma) + \cdots := A_\mu X^\mu(\gamma) \]  

\( X^{\mu_1 \ldots \mu_r}(\gamma) \) is the multitangent field of rank \( r(\mu) = r \) and it is given by the ordered integration along the loop of the product of \( r \) delta functions. In general a bold Greek index represents an ordered set of paired indices \( \mu := \mu_1 \ldots \mu_r \) and repeated bold Greek indices indicates a summation from rank zero to infinity. By this procedure one is capable to write the holonomy in a very compact and useful form. In particular one sees that all the information about \( \gamma \) needed to construct the holonomy is contained in the multitangents of all ranks. As far as the loop representation is concerned ordinary loops can then be viewed as entirely equivalent to the infinite string of multitangent fields:

\[ \gamma \leftrightarrow X(\gamma) := (1, X^{\mu_1}(\gamma) + \cdots + X^{\mu_1 \ldots \mu_r}(\gamma) + \cdots) \]  

Extended loops arise as generalizations of the multitangents in order to include more general fields. In general an extended loop will be given by the string

\[ X := (X, X^{\mu_1} + \cdots + X^{\mu_1 \ldots \mu_r} + \cdots) \equiv (X^\mu; \ r(\mu) = 0, \ldots, \infty) \]  

where \( X \) is a real number and the \( X^{\mu_1 \ldots \mu_r} \) satisfy several properties inherited from the multitangents. These properties are:

i. Coordinate transformations properties: \( X^{\mu_1 \ldots \mu_r} \) behaves as a vector density with respect to each “contravariant” index.

ii. The differential constraint: the divergence of a multivector of rank \( r \) generates a multivector of rank \( r - 1 \) (this property makes holonomies formally covariant under gauge transformations).

iii. The algebraic constraint: the linear combination \( X^{\alpha \beta} \) of multivectors of rank \( r(\alpha) + r(\beta) \) where all the indices are permuted preserving the relative order of the set \( \alpha \) and \( \beta \) split into the product \( \frac{1}{r} X^{\alpha} X^{\beta} \) (this property is related to the existence of an “order” for the contravariant indices of the multitangents -originated by the ordered integration of the distributions along the loop-).

The set \( \{X\} \) of all extended loops constitutes a vector space that can be endowed with the structure of a (local) infinite dimensional Lie group by means of the following composition law:

\[ [X_1 \times X_2]^\mu := \delta^\mu_{\pi \theta} X_1^{\pi} X_2^{\theta} \]  

where\(^{[2]}\)

\[^{[2]}\text{We use the convention } X^{\mu_1 \ldots \mu_0} \equiv 1.
\]

\[^{[3]}\text{\( \delta^\mu_{\nu_i} := \delta^{\mu_a}_{\nu_i} \delta(x_i - y_i), \) with } \mu_i = a_i x_i \text{ and } \nu_i = b_i y_i.\]
The $\delta$-matrix is a very useful device to compact sums of multivector fields. The above definition of the extended group product is the compact version of the following sum:

$$[X_1 \times X_2]_{\mu_1 \cdots \mu_r} = \sum_{k=0}^{r} X_1^{\mu_1 \cdots \mu_k} X_2^{\mu_{k+1} \cdots \mu_r}$$

where now $X^{\mu_{j+1} \cdots \mu_{j}} = X$ gives the component of rank zero of the extended loop. Notice that the $\delta$-matrix allows to write an extended loop in the following way

$$X := \delta_\nu X^\nu$$

The set of “covariant” multivectors $\delta_\nu$ can be viewed as a basis of the vector space $\{X\}$. In order to connect extended with ordinary loops we use the following properties of the multitangents:

$$X(\gamma_1) \times X(\gamma_2) = X(\gamma_1 \gamma_2)$$

$$X^\mu(\overline{\gamma}) = X^{\overline{\mu}(\gamma)}$$

where $\gamma_1 \gamma_2$ is the group product in the nonparametric loop space, $\overline{\gamma}$ is the rerouted loop and $\overline{\mu} := (-1)^{(\mu)} \mu^{-1}$ with $\mu^{-1} := \mu_r \cdots \mu_1$.

Extended loops and ordinary loops are then closely related by the multitangent fields. This relationship is also put into manifest at the level of the representations: through the use of the multitangents, the holonomy admits a direct generalization to the extended loop space. An extended holonomy will be defined by the series

$$H_A(X) := A_\mu X^\mu$$

and they allow to formally represent any Lie-algebra valued connection theory in the extended loop space in close resemblance with the case of ordinary loops.

In what follows we shall study the diffeomorphism transformation properties of the wavefunctions in the extended loop representation.

\[4\] This notation is due to C. Di Bartolo.  
\[5\] Extended holonomies are affected by convergence problems that question the gauge invariance of the representation. There exist several alternatives to solve this problem, as the one presented by C. Di Bartolo [6] in this volume. See also [3].
3 Extended knot invariants

Linear extended loop wavefunctions are written in the following general form

$$\psi(X) = \psi_\mu X^\mu$$  \hspace{1cm} (13)

where the propagators $\psi_\mu$ characterizes completely the state $\psi$. Any extended loop wavefunction generates a loop wavefunction when the multivectors are particularized to the multitangents:

$$\psi(X) \rightarrow \psi(\gamma) = \psi_\mu X^\mu(\gamma)$$  \hspace{1cm} (14)

The converse in not true in general. As it was mentioned, the components of an extended loop behave as multivector densities under general coordinate transformations. Using this fact\(^6\) it is possible to derive the following transformation law for the extended loop wavefunction under infinitesimal coordinate transformations $x'^a = x^a + \eta^a(x)$:

$$\psi(X') = \psi(X) + \eta^a C_{ax} \psi(X)$$  \hspace{1cm} (15)

where the generator of diffeomorphisms is given by a linear expression in the functional derivatives of the multivector fields\(^7\):

$$C_{ax} \psi(X) := [\mathcal{F}_{ab}(x) \times X^{(bx)}][\delta_{\delta X^\mu} \psi(X)] \equiv \psi_{\alpha\beta} \mathcal{F}_{ab}(x) X^{(bx\beta)}c$$  \hspace{1cm} (16)

In the above expression,

$$\mathcal{F}_{ab}^{\alpha_1...\alpha_r}(x) := \epsilon_{abc} \left[-\delta_{\nu_1...\nu_r}^{\alpha_1...\alpha_r} g^{c\nu_1\nu_2} + \delta_{\nu_1\nu_2}^{\alpha_1...\alpha_r} \epsilon^{c\nu_1\nu_2}\right],$$  \hspace{1cm} (17)

$$g^{c\nu_1} := \epsilon^{cbk} \partial_k \delta(x - y),$$  \hspace{1cm} (18)

and

$$\epsilon^{c\nu_1\nu_2} := \epsilon^{cb_1b_2} \delta(x - y_1)\delta(x - y_2)$$  \hspace{1cm} (19)

In the last expressions a mixed notation of Greek and paired indices were used. In the case that extended loops are particularized to ordinary loops we can write from \((16)\)

$$C_{ax} \psi[X(\gamma)] = \psi_{\alpha\pi\theta} \mathcal{F}_{ab}(x) X^{\theta bx \pi}(\gamma)$$

$$= \int_\gamma dy^b \delta(x - y) \psi_{\alpha\pi\theta} \mathcal{F}_{ab}(y) X^{\pi}(\gamma^y) X^{\theta \gamma^y}$$

$$= \int_\gamma dy^b \delta(x - y) \psi_\mu [\mathcal{F}_{ab}(x) \times X(\gamma^y) \times X(\gamma^y)]^\mu$$  \hspace{1cm} (20)

\(^6\)Another possibility is by translating the generator of diffeomorphisms from the space of connections to the space of extended loops, see \cite{[2].}

\(^7\)Notice that the action of the diffeomorphism operator reduces to a shift of the argument of linear wavefunctions.
where $\gamma^y_o$ is the portion of the loop from the origin $o$ to the point $y$. Introducing the identity $1 = \mathbf{X}(\gamma^y_o) \times \mathbf{X}(\gamma^y_o)$ and using the cyclicity of the set $\mu^\gamma_o$ we get from (20)

$$ C_{ax} \psi(\gamma) = \oint \gamma^b \delta(x - y) \psi_\mu [\mathbf{X}(\gamma^y_o) \times \mathcal{F}_{ab}(y) \times \mathbf{X}(\gamma^y_o) \times \mathbf{X}(\gamma^y_o)]^\mu $$

$$ \equiv \oint \gamma^b \delta(x - y) \Delta_{ab}(\gamma^y) \psi(\gamma) \quad (21) $$

where

$$ \Delta_{ab}(\gamma^y) \mathbf{X}(\gamma) := [\mathbf{X}(\gamma^y_o) \times \mathcal{F}_{ab}(y) \times \mathbf{X}(\gamma^y_o)] \times \mathbf{X}(\gamma) \quad (22) $$

is the loop derivative [4] defined in the nonparametric loop space. This result shows that the expression (16) gives the correct transformation law under diffeomorphisms for ordinary loop wavefunctions once the multivectors are specialized to the multitangents. This means that any solution of the equation

$$ C_{ax} \psi(\mathbf{X}) = 0 \quad (23) $$

can be viewed as an “extended knot invariant” (in the sense that the restriction of the domain of definition of the solution to ordinary loops will always generate a knot invariant). According to (13) and (17) we have

$$ C_{ax} \psi(\mathbf{X}) = \sum_{r=0}^{\infty} \epsilon_{abc} \psi_{\mu_1...\mu_r} [-g^{c\mu_1} X^{(bx \mu_2...\mu_r)c} + \epsilon^{c\mu_1 \mu_2} X^{(bx \mu_3...\mu_r)c}] = 0 \quad (24) $$

In the following section we analyze how (24) works.

### 4 The action of $C_{ax}$

The possibility to systematize the search of solutions of (24) is based on the following observation: for all the known analytical expressions of knot invariants, the propagators $\psi_{\mu_1...\mu_r}$ are completely expressed in terms of the two (g..) and three (h...) point propagators of the Chern-Simons theory, given by

$$ g_{\mu_1 \mu_2} \equiv \epsilon_{a_1 a_2 k} \delta_{x_1}^{x_2} := -\epsilon_{a_1 a_2 k} \frac{\partial_k}{\sqrt{2}} \delta(x_1 - x_2) \quad (25) $$

and

$$ h_{\mu_1 \mu_2 \mu_3} := \epsilon^{a_1 a_2 a_3} g_{\mu_1 a_1} g_{\mu_2 a_2} g_{\mu_3 a_3} \quad (26) $$

Moreover, the following properties are also used:

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8 The fact that $\psi_\mu$ is invariant under a cyclic permutation of the indices assures that the wavefunctions do not depend on the origin $o$ of the loops.
1. The propagators $\psi_{\mu_1...\mu_r}$ vanishes for a certain (maximum) rank $N$; that is to say, $\psi_{\mu_1...\mu_r} = 0$ for all $r > N$.

2. For $r = N$, $\psi_{\mu_1...\mu_N}$ is given exclusively by products of two point Chern-Simons propagators.

3. The minimum rank $n$ does not contain any two point Chern-Simons propagator.

The successive ranks of the wavefunction are linked by the action of the operators. Let us consider the case of $N = 6$ and $n = 4$. The general form of the wavefunctions of this type is:

$$\psi(X) = g.g.g.X^{bx...cx} + h.g.X^{bx...cx} + h.g^{**}h_x...X^{bx...cx}$$ (27)

and one obtains the following general picture for the result of the application of $C_{ax}$ onto this state:

$$C_{ax} g.g.g.X^{bx...cx} : -\epsilon_{abc}g...g$X^{(bx...cx...)}c + G_{ax bx} ...X^{(bx...)}c$$

$$C_{ax} h...g.X^{bx...cx} : -\epsilon_{abc}h...X^{(bx...cx...)}c - G_{ax bx} ...X^{(bx...)}c + H_{ax bx} ...X^{(bx...)}c$$

$$C_{ax} h...g^{**}h_x...X^{bx...} : -H_{ax bx} ...X^{(bx...)}c + I_{ax bx} ...X^{(bx...)}c$$

$G$, $H$ and $I$ are certain expressions containing $g$’s and/or $h$’s with two spatial indices fixed at the point $x$. Notice that $G \cdot X$ represents a contribution of rank five, $H \cdot X$ one of rank four and $I \cdot X$ one of rank three. As a general rule, the action of $C_{ax}$ on a rank $r$ of the wavefunction generates two types of contributions, one of rank $r$ and other of rank $r - 1$. For $r > n$ the contribution of rank $r - 1$ is always canceled by terms that appear when the operator acts on the rank $r - 1$. A chain of cancellations linking intimately the successive ranks of the wavefunction then occurs induced by the diffeomorphism generator $\mathcal{L}$. Two more punctuations can be quoted from the above result:

P1: For $n < r \leq N$ a contribution of the form $\epsilon_{abc}\psi...X^{(bx...cx...)}c$ appears. This type of contributions do not enter in the chain of cancellations and they have to vanish by means of symmetry considerations$^{[p]}$.

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$^9$Exactly the same chain of cancellations takes place also in the case of the Hamiltonian constraint of quantum gravity.

$^{10}$For extended knot invariants these terms would involve in general a symmetric expression in the contravariant indices $bx$ and $cx$. 

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P2: The term of lower rank is responsible of the closure of the chain of cancellations in a consistent way; that is to say, the contribution of rank $n - 1$ has to vanish identically.

The systematic operation displayed by the diffeomorphism operator is essentially a consequence of the fact that $g$ and $h$ were taken as the building blocks of the propagators $\psi_\mu$. It is worth to emphasize that no other propagators than the Chern-Simons are known at present to participate in the analytical expression of the knot invariants. In what follows we are going to see that this procedure can be used to build up analytical expressions of extended knot invariants in terms of $g$ and $h$ in a generic way.

5 Extended knot families

A family of extended knot invariants is a set $\{\psi_{i}^{[N,n]}\}$ of wavefunctions with the same maximum and minimum ranks that satisfy the following properties:

F1: $C_{ax} \psi_{i}^{[N,n]} = 0$ for all $i$.

F2: $\psi_{i}^{[N,n]}(X) = \psi_{i}^{i} X^{\mu}$ with $\psi_{i}^{i}$ a cyclic propagator.

F3: $\psi_{i}^{i_{1}...\mu_{n}}$ is the same for all members of the family.

The following steps allow to construct extended knot families in a systematic way:

Step 1: For the minimum rank $n$ construct all the cyclic combinations involving only three point Chern-Simons propagators $h$ (and its contractions if necessary).

Step 2: Identify those combinations that satisfy the consistence condition P2. Each one of these combinations $\psi_{i}^{i_{1}...\mu_{n}}$ could be the origin of a family of knot invariants.

Step 3: Determine the cyclic combinations of rank $n + 1$ (where $h$'s are substituted by $g$'s) that makes

$C_{ax} \{\psi_{i}^{i_{1}...\mu_{n}} X^{\mu_{1}...\mu_{n}} + \psi_{i}^{i_{1}...\mu_{n+1}} X^{\mu_{1}...\mu_{n+1}}\} = $ terms of rank $n + 1$

Verify that the result does not include a remnant contribution of the type described in P1.

Step 4: Repeat the procedure for the successive increasing ranks until all the $h$'s were replaced by $g$'s.

\[11\] Notice that we do not dispose in this case of a symmetry argument like in P1.
We see that the procedure of construction goes from the minimum to the maximum rank. The most simple family has only one member, the Gauss invariant:

\[
\{ \psi^{[2,2]} \equiv \varphi_G \} \quad \text{with} \quad \varphi_G := g_{\mu_1 \mu_2} X^{\mu_1 \mu_2}
\]

(28)

The next family has \( N = 4 \) and \( n = 3 \). Their members have the following general form

\[
\psi^{[4,3]} = g..g..X^{\cdots} + h..X^{\cdots}
\]

(29)

We shall see how the method works in this simple case:

**Step 1:** \( h_{\mu_1 \mu_2 \mu_3} \) is the only possibility.

**Step 2:**

\[
C_{ax} h_{\mu_1 \mu_2 \mu_3} X^{\mu_1 \mu_2 \mu_3} = \\
\{ -g_{\mu_1 [ax} g_{b2]} \mu_2 + \epsilon_{abc} (\phi^{cx}_{x_1} - \phi^{cx}_{x_2}) g_{\mu_1 \mu_2} \} X^{(bx \mu_1 \mu_2)_c} \\
+ 2 \{ h_{ax bx \mu_1} - \epsilon_{abc} \phi^{cx}_{z} \phi^{dz}_{x} g_{\mu_1 dz} \} X^{(bx \mu_1)_c}
\]

(30)

The consistency condition reads in this case:

\[
I_{ax bx \mu_1} := h_{ax bx \mu_1} - \epsilon_{abc} \phi^{cx}_{z} \phi^{dz}_{x} g_{\mu_1 dz} \equiv 0
\]

(31)

that is verified directly by developing \( h_{ax bx \mu_1} \) according to (28).

**Step 3:** The next (and last) rank includes the following (independent) cyclic combinations of two \( g \)'s:

\[
C^{1}_{\mu_1 \ldots \mu_4} = g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} + g_{\mu_1 \mu_4} g_{\mu_2 \mu_3}
\]

(32)

\[
C^{2}_{\mu_1 \ldots \mu_4} = g_{\mu_1 \mu_3} g_{\mu_2 \mu_4}
\]

(33)

The following results are obtained:

\[
C_{ax} C^{1}_{\ldots} X^{\cdots} = \{ -g_{\mu_1 [ax g_{bx}] \mu_2} + \epsilon_{abc} (\phi^{cx}_{x_1} - \phi^{cx}_{x_2}) g_{\mu_1 \mu_2} \} X^{(bx \mu_1 \mu_2)_c}
\]

(34)

\[
C_{ax} C^{2}_{\ldots} X^{\cdots} = \{ g_{\mu_1 [ax g_{bx}] \mu_2} - \epsilon_{abc} (\phi^{cx}_{x_1} - \phi^{cx}_{x_2}) g_{\mu_1 \mu_2} \} X^{(bx \mu_1 \mu_2)_c}
\]

(35)

Notice that there are not terms of the form \( \epsilon_{abc} \psi^{\ldots} X^{(bx \ldots cz)_c} \). This is due to the cyclicity of the \( C \)'s. Comparing with (30) we see that there exist two possibilities to cancel the contribution of rank three:

\[
\psi^{[4,3]}_1 = (g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} + g_{\mu_1 \mu_4} g_{\mu_2 \mu_3}) X^{\mu_1 \mu_2 \mu_3 \mu_4} - h_{\mu_1 \mu_2 \mu_3} X^{\mu_1 \mu_2 \mu_3}
\]

(36)

\[
\psi^{[4,3]}_2 = g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} X^{\mu_1 \mu_2 \mu_3 \mu_4} + h_{\mu_1 \mu_2 \mu_3} X^{\mu_1 \mu_2 \mu_3}
\]

(37)

These are the members of the family \([4, 3]\).
Notice that

\[
\psi_1^{[4,3]} + \psi_2^{[4,3]} = (g_{\mu_1\mu_2}g_{\mu_3\mu_4} + g_{\mu_1\mu_5}g_{\mu_2\mu_4} + g_{\mu_1\mu_4}g_{\mu_2\mu_3})X^{\mu_1\mu_2\mu_3\mu_4} = \frac{1}{2}(\psi_1^{[2,2]})^2
\]  

This linear expression corresponds to the square of the Gauss invariant in the general extended space.\(^{12}\) This means that from the two wave-functions, only one represents a new diffeomorphism invariant (that is identified as the second coefficient of the Alexander-Conway polynomial). This is a characteristic of the procedure of construction outlined above: in general the family \(\{\psi_i^{[N,n]}\}\) would contain the linear version of the product of invariants belonging to lower rank families. Other examples of extended knot families are considered in \(\text{[11]}\).

## 6 Conclusions

The basis of a systematic method for obtaining analytic expressions of diffeomorphism invariants (the extended knots) in term of the Chern-Simons propagators are settled. As it was shown, any extended knot would generate an ordinary knot with the only requirement of substituting the general multivectors by the multitangent fields. The construction procedure opens a new possibility to explore knots.

This new possibility to look at the world of knots is specially relevant for quantum gravity. As it is known, the quantum states of gravity in the loop representation are given by knot invariants (due to the diffeomorphism invariance of the theory). But the evaluation of the Hamiltonian constraint onto knot invariants is a very involved task. In fact, the analysis of the Hamiltonian onto loop dependent wavefunction has been traditionally based upon geometric (in contrast to analytic) properties of the loops. The lack of an effective machinery to put forward the analytical calculations and the limited knowledge of the analytic properties of knots constitute serious obstacles for the loop representation of quantum gravity. Extended loops offers a way to overcome these difficulties. For one hand, the above proposed method allows to obtain analytic expressions of knot invariants in a generic way (the Mandelstam identities can also be checked systematically in this approach). On the other, the explicit analytic evaluation of the Hamiltonian constraint can be thoroughly accomplished within the extended loop approach. These features suggest

\(^{12}\)In order to fulfill the linearity constraint, the product of invariants has to be put into correspondence with a linear expression in the multivector fields. The algebraic constraint makes this job.
that extended loops could be an effective resort to advance towards the identification of the space of states of quantum gravity.

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