On the new and old physics in the interaction of a radiating electron with the extreme electromagnetic field

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We show that an all-optical configuration of the laser-electron collision in the λ3 configuration based on 10 PW-class lasers presents a viable platform for reaching the range of parameters where a perturbative QED in strong external electromagnetic field breaks. This case is contingently referred to as a case of the nonperturbative QED; and this range of parameters is the intriguing goal from an experimental point of view because of a possible manifestation of a new physics of the interaction of a highly radiating particle with a strong electromagnetic field. We show that the strong field region can be reached by the electrons having the initial energy higher than 50 GeV. Our theoretical considerations are in agreement with three-dimensional particle-in-cell simulations. While increasing of the electron energy raises the number of electrons experiencing the strong field region, the observable signature of photon emission radiative correction in the strong field is expected to fade out when the electron energy surpasses the optimal value. This threshold of electron energy is identified and the parameters for achieving the nonperturbative limit of QED are provided.

The collision of laser-accelerated electrons with an intense laser pulse presents an all-optical configuration enabling the study of quantum electrodynamics (QED) effects such as photon emission and electron-positron pair generation [1–7]. Further increasing of laser intensities and energies of laser-accelerated electrons will bring us to the regime where the current perturbative theoretical framework of QED becomes inapplicable [8–10].

The latter point demands clarification from a terminological point of view. A number of results obtained previously in the field of dynamics of photons and strongly radiated electrons in a strong electromagnetic (EM) field are based on the Furry picture [11]. The total quantum EM field \( \mathbf{A} \) is represented as a sum of two fields: \( \mathbf{\hat{A}} = \mathbf{A} + \mathbf{\hat{A}}_1 \). Here \( \mathbf{\hat{I}} \) is the identity operator and \( \mathbf{A} \) is a four-potential of an external strong EM field. It is the classical field obeying the classical Maxwell equations. It is exactly (nonpertubatively) taken into account for ‘unperturbed’ dynamics of real and virtual electrons and positrons. Relatively weak excitations over the unperturbed classic field are described by the quantum field \( \mathbf{\hat{A}}_1 \). Its presumably weak interaction with electrons and positrons in the strong field \( \mathbf{A} \) is described by the term \( -j \cdot \mathbf{\hat{A}}_1 \) in a Lagrangian, where \( j \) is the quantum electric four-current. The Furry picture implies using the usual QED perturbation approach relative to this interaction, and, in particular, the perturbation approach (with the re-normalization) for any virtual photons, electrons and positrons whose energies are much larger than the energies of real in-coming and out-going photons, electrons and positrons taking part in the process. The success of the latter procedure is provided by the smallness of the fine structure constant, \( \alpha = e^2/(\hbar c) \). Here \( e \) stands for the elementary electric charge, \( c \) speed of light, and \( \hbar \) is the reduced Planck constant. This approach with its limitations are recently reviewed in detail in Ref. [12]. It appears that this approach cannot be applied in the case when energies of the real particles taking part in a process become very large, regardless of the smallness of \( \alpha \). The formulation of this limitation will be considered in the next paragraph. However, we may say now that it originates from the fact that cascading creation of virtual electrons, positrons and photons with energies comparable with the energies of the real particles, when the energies become sufficiently large, destroys convergence of the perturbation theory considered above. The range of the parameters, where such convergence breaks, will be called as a domain of ‘nonperturbative QED’.

The interaction of an electron with the EM field is characterized by the Lorentz invariant parameter \( \chi_e = \sqrt{-(F_{\mu\nu}p_\nu)^2/(m_eE_S)} \) where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the EM field tensor, \( A_\nu \) is the four-potential, \( p_\nu \) is the four-momentum of the electron, \( E_S = m_e^2c^3/(\hbar c) \approx 1.33 \times 10^{16} \text{ V/cm} \) is the QED critical field and \( m_e \) is the electron mass [13–15]. For example, \( \chi_e \approx 0.1 \) has been reached in experiments presenting evidence of radiation reaction in the collision of a high-intensity laser beam with laser-wakefield accelerated electrons in the counter-propagating setup [16–17]. While for \( \chi_e \ll 1 \) the quantum effects on the electron-laser interaction are negligible, they become important as \( \chi_e \gtrsim 1 \) [2–4, 5, 16]. In this case, the probabilistic photon emission as well as the recoil and straggling effects should be taken into account [17]. Meanwhile, according to Ritus-Narozhny conjecture, the perturbative QED theoretical framework in a strong EM field fails due to the large radiative corrections.
When $\alpha \chi^2 \approx 1$ which corresponds to $\chi_e \approx 1600$. Therefore, several schemes have been proposed to experimentally probe this regime of QED. However, reaching the highest values of $\chi_e$ is possible only if the radiative losses are mitigated. This can be done by reducing the interaction time between the laser and the EM field. In the limit $\chi_e \gg 1$, the characteristic radiation time between the emission of two photons is $\sim \gamma, \hbar / (\alpha \chi^2 m_e c^2)$ where $\gamma$ is the relativistic Lorentz factor of the electron. If one considers the optical lasers, the spatial extent of the interaction is usually characterized by the laser wavelength $\lambda \approx 1 \mu m$, thus reaching $\alpha \chi^2 \approx 1$ seems to be hard using 100 GeV-class electron bunch as the interaction time is ten times longer than the characteristic radiation time [18]. Therefore other schemes reducing the interaction time have been put forward. The concept suggested in Ref. [19] is based on the generation of isolated ultra-intense 150 as pulse by the reflection of the optical laser pulse from a plasma mirror [20–23] and its collision with a counterpropagating 125 GeV electron beam. The radiation losses of counter-propagating electrons can be also mitigated by the appropriate choice of the collision angle [24] or by employing a solid target [25]. Another approach considers the collision of two 125 GeV tightly compressed and focused electron bunches having longitudinal dimension < 100 nm [18]. It has been also shown that the limit of nonperturbative QED can be approached in the collision of $\sim$TeV electrons with crystal atoms aligned along the symmetry direction [20].

Here we consider the interaction of the 100 GeV-class electron beam with tightly-focused azimuthally polarized optical laser pulse in the $\lambda^3$ configuration. Contrary to linear or circular polarization, the azimuthal one provides an order of magnitude larger area of strong-field region (doughnut-shaped) and therefore it affects more particles. We show both analytically and numerically by means of full-scale 3D Particle-In-Cell (PIC) simulations that even though the laser pulse duration is an order of magnitude greater than the characteristic radiation time, using an optical 10 PW-class laser allows approaching the nonperturbative limit of QED. The effect of photon emission correction due to the electron mass modification is discussed. As this effect would manifest itself only if electrons with large $\chi_e$ radiate, we expect that the signature of such a radiative correction on the final electron distribution starts to fade out once the electron initial energy surpasses the optimal value.

The key factor for achieving high $\chi_e$ with near-future 10 PW-class laser systems is to provide a strong EM pulse within a tiny spatio-temporal region. Therefore we have considered the tightly focused azimuthally polarized laser beam expressed in high-order expansions of a parameter $\epsilon = \lambda / (\pi w_0)$, where $w_0$ is the waist radius of the Gaussian beam [27].

\[
B_r (x, y, z, t) = E_0 e^{-r^2/w^2} \left[ \epsilon \rho C_2 + \epsilon^3 \left( -\frac{\rho C_3}{2} + \rho^3 C_4 - \frac{\rho^5 C_5}{4} \right) + \epsilon^5 \left( -\frac{3 \rho C_4}{8} - \frac{3 \rho^3 C_5}{8} + \frac{17 \rho^5 C_6}{16} - \frac{3 \rho^7 C_7}{8} + \frac{\rho^9 C_8}{32} \right) \right],
\]

\[
B_z (x, y, z, t) = E_0 e^{-r^2/w^2} \left[ \epsilon^2 \left( S_2 - \rho^2 S_3 \right) + \epsilon^4 \left( \frac{S_3}{2} + \rho^2 S_4 - \frac{5 \rho^4 S_5}{4} + \frac{\rho^6 S_6}{4} \right) \right],
\]

\[
E_\theta (x, y, z, t) = E_0 e^{-r^2/w^2} \left[ \epsilon \rho C_2 + \epsilon^3 \left( \frac{\rho C_3}{2} + \frac{\rho^3 C_4}{2} - \frac{\rho^5 C_5}{4} \right) + \epsilon^5 \left( \frac{3 \rho C_4}{8} + \frac{3 \rho^3 C_5}{8} + \frac{3 \rho^5 C_6}{16} - \frac{\rho^7 C_7}{4} + \frac{\rho^9 C_8}{32} \right) \right],
\]

where for $n = 2, 3, ...$

\[
C_n = \left( \frac{w_0}{w} \right)^n \cos (\varphi + n \varphi_G),
\]

and

\[
S_n = \left( \frac{w_0}{w} \right)^n \sin (\varphi + n \varphi_G).
\]

The radial distance from the beam axis is $r = \sqrt{y^2 + z^2}$, $\rho = r/w_0$, $w = w_0 \sqrt{1 + (x/x_R)^2}$, $x_R = k w_0^2 / 2$ is the Rayleigh range, $k = 2\pi / \lambda$ is the wavenumber, $\varphi = \varphi_0 + \omega_0 t - k x - k r^2 / 2 R$, $\omega_0$ is the laser frequency, $R = x + x_R^2 / x$, $\varphi_0$ is the initial phase and $\varphi_G = \arctan (x/x_R)$ is the Gouy phase. We assume that the laser pulse is propagating along the positive $x$-axis. For $\varphi_0 = 0$, the radial magnetic $B_r$ and azimuthal electric $E_\theta$ fields reach their maxima at time $t = 0$ s in the focal plane $x = \mu m$, while the longitudinal magnetic field $B_z$ is zero here, see Fig. 1. The peak power (with respect to the peak intensity) of a tightly focused beam is given by [27]

\[
P = \frac{\pi w_0^2}{2} \frac{B_z^2}{\mu_0} \left( \frac{\epsilon}{2} \right)^2 \left[ 1 + 3 \left( \frac{\epsilon}{2} \right)^2 + 9 \left( \frac{\epsilon}{4} \right)^4 \right].
\]

The laser field is characterized by the parameter $a_0 = e E_0 / (m_e \omega_0 c)$, where $E_0$ is the amplitude of the electric field. Assuming $\gamma_e \gg 1$, the maximum $\chi_e$ that can be
achieved in a head-on collision is $2\gamma_e E_0/E_S$. However, it is not guaranteed that the maximum value of $\chi_e$ parameter will be achieved by the radiating particle as this depends on laser parameters and electron initial energy. Therefore, below we estimate the ratio of the initial electron number, that achieves the laser pulse center without emitting a photon.

In the case of the azimuthally polarized laser beam, the $\chi_e$ parameter is maximized when the electron interacts with the maximum of the counterpropagating EM field created by the radial magnetic $B_r$ and azimuthal electric $E_\theta$ fields. These transverse fields generate a doughnut-shaped intensity structure. Since the EM field structure of the azimuthally polarized laser pulse is cylindrically symmetric, we analyze the interaction with an electron in 1D geometry taking advantage of the approach described in Ref. [28]. However, in the tightly focused laser beam, the azimuthal component of the field is always stronger than the radial one and they reach their maxima at different radial distances. To take this effect into account, we consider the average strength of the transverse electromagnetic field

$$ a_{0\perp} = \frac{e G_\perp}{m_e \omega_0 c}, \quad (7) $$

where $G_\perp = \max(|B_r + E_\theta|)/2$. The maximum expected value of the corresponding $\chi_e$ parameter is, therefore,

$$ \chi_{e\perp} = 2\gamma_e \frac{a_{0\perp}}{a_S}, \quad (8) $$

where $a_S$ corresponds to the QED critical field.

We assume that the electron interacts with a laser pulse having a Gaussian temporal envelope of full-width-at-half-maximum (FWHM) duration $\tau$ in laser intensity.

In the limit $\chi_e \gg 1$, the average energy of the electron in the center of the laser pulse of a FWHM duration $\tau = T/2$, where $T$ is the laser period, can be approximated as

$$ \mathcal{E}_e \approx (1 - 16/63)^{W_\gamma \tau/\sqrt{2\ln 2} \mathcal{E}_e}, \quad (9) $$

where $\mathcal{E}_e$ is the initial electron energy, and

$$ W_\gamma \approx \frac{3^{2/3}28\Gamma(\frac{2}{3})}{54h\mathcal{E}_e} \alpha m_e^2 c^4 \lambda e_{\perp}^{2/3} \quad (10) $$

is the rate of single-photon emission per unit time for the Compton process and $\Gamma(x)$ is the Gamma function [15]. The factor $16/63$ presents the average energy of the emitted photon [15] and the exponent defines the number of emitted photons. Due to the relationship between the Gaussian and flat-top temporal envelope, the effective amplitude is $a_{0\perp}/\sqrt{2}$ resulting in the effective $\chi_e$ parameter $\sqrt{2}\gamma_e a_{0\perp}/a_S$ and the full pulse length $\tau$ gets a factor of $2/\sqrt{2\ln 2}$. Therefore the time required for an electron to reach the center of the laser pulse is $\tau/\sqrt{2\ln 2}$.

If such an electron radiates, it does not reach the maximum $\chi_e$ parameter in the center of the laser pulse. Therefore $\mathcal{E}_e/\mathcal{E}_e$ approximates the ratio of the number of electrons that achieve the laser pulse center without emitting a photon (and thus reach the maximum $\chi_e$) to the total number of electrons. To maximize this ratio for a fixed interaction duration $\tau$, the rate $W_\gamma \propto a_{0\perp}/\gamma_e^{1/3}$ has to be reduced while keeping $(2\gamma_e a_{0\perp}/a_S)^{2/3} \approx 1$ in order to achieve the nonperturbative limit of QED.

In the following, we consider the $\lambda^3$ configuration of the laser-electron collision [29]. The spatio-temporal parameters of the azimuthally polarized laser pulse are as follows: $\lambda = 0.8 \, \mu m$, duration $\tau = T/2$ and $w_0 = 0.424A$. The initial electron energy $\mathcal{E}_e$ is in the range 50–200 GeV. The results for the above-mentioned parameters are shown in

FIG. 1. The field isosurfaces $|E_\theta(x, y, z, t)|$ and $|B_z(x, y, z, t)|$ of the laser pulse at $t = 0$ s for the initial phase $\varphi_0 = 0$. The focal plane is at $x = 0$ µm. The laser pulse propagates in the positive $x$-direction.

FIG. 2. The ratio of electron number reaching $2\gamma_e^{2/3} \approx 1$ as a function of their initial energy $\mathcal{E}_e$ for laser parameters mentioned in the text. The black solid line represents the expected value obtained from Eq. (9). The bullets represent the results from 3D PIC simulations. The second line shows the corresponding $a_{0\perp}$ (right axis) and its color the required power $P$. 

In the limit $\lambda^3 = 0.8 \, \mu m$, duration $\tau = T/2$ and $w_0 = 0.424A$. The initial electron energy $\mathcal{E}_e$ is in the range 50–200 GeV. The results for the above-mentioned parameters are shown in
Fig. 2. The black solid line represents the simple analytic model estimating the ratio of the number of electrons that do not emit before reaching the laser pulse center to their total number. The corresponding $a_{0,z}$ is depicted by the second line colored according to the laser peak power $P$. For e.g. 10 PW laser, the nonperturbative limit of QED is reached in the head-on collision with 140 GeV electrons. In this case, 1/3 of electrons interacting with the region of a strong transverse electromagnetic field $a_{0,z} \approx 960$ reach $\alpha \chi^2_{\perp} \approx 1$. The energy of laser-wakefield accelerated electrons scales as $E_e [\text{GeV}] \propto 10 \times P [\text{PW}]$, see Ref. [20] and references cited therein. The combination of 10 PW-class lasers thus provides the all-optical scheme for probing the nonperturbative limit of QED.

We have benchmarked our analytical results against 3D PIC simulations of laser-electron collision in the code SMILEI in which photon emission and Breit-Wheeler electron-positron pair creation are implemented via the Monte-Carlo method [31]. The simulation box dimensions are $4 \lambda \times 20 \lambda \times 20 \lambda$ resolved with $512 \times 2560 \times 2560$ cells. The EM fields given in Eqs. (1)–(3) are numerically evaluated at the simulation box boundary for the whole duration of the laser pulse and they propagate towards the center of the simulation box by using the finite-difference time-domain Maxwell solver [31]. The tightly focused laser pulse propagates in the positive $x$-direction and collides head-on with $10^9$ electrons distributed in a $\lambda/5$ thick disc of a radius $\lambda$ initially located at a such distance $x$ that it reaches the focal plane $x = 0 \mu\text{m}$ at $t = 0 \text{s}$. The electron initial energy $E_e$ is in the range 50–200 GeV. The parameters of the laser pulse are the same as in the previous text. Since $a_{0} \gg 1$, the condition for local constant field approximation is satisfied [15, 32]. From the simulation data one can obtain the $\chi_e$ parameter of each particle and thus identify the ones that approach the nonperturbative limit of QED. Their ratio is obtained by comparing the number of particles that $\chi_e$ reach at least $\alpha \chi^2_{\perp} = 0.95$ with the number of all particles originating in the same region. As can be seen in Fig. 2 the simple analytic estimate (black line) well approximates the results obtained in 3D simulation (bullets) across the whole studied energy range.

We see in Fig. 2 that the higher the initial electron energy, the more particles approach the nonperturbative limit of QED. As in Refs. [18, 19, 25], we have used the model of the photon emission rate which has been developed within the perturbative theory in order to perform the predictive simulations for reaching the nonperturbative limit of QED. Therefore, applying this approach for $\alpha \chi^2_{\perp} < 1$ would increase the validity of the predictions. At the nonperturbative limit $\alpha \chi^2_{\perp} = 1$, the exact theory taking into account all the radiative corrections to electron and photon motion is needed [8, 33]. For example, it is known that the existence of photon emission in a strong EM field is connected with the radiative effect leading to the electron mass increase [33]. This electron mass shift may consequently affect the photon emission rate. However, the exact calculation of the corresponding photon emission rate correction is beyond the scope of this paper.

Using the approach described in Ref. [28] one can calculate the average final electron energy while neglecting the radiative corrections. This value might serve as a reference for further theoretical and experimental discussions considering the radiative corrections. If the electron mass modification would not be neglected in the calculation of photon emission rate, we expect that the modified rate would result in a final electron energy distribution that differs from the one calculated by Eq. (10). Therefore it is interesting to inspect the question what are the optimal parameters of laser-electron collision for observing the effect of a modified photon emission rate? We note, that the effect of mass modification on photon emission rate has been previously considered in Ref. [18] within the perturbative QED framework. In this cited paper, the magnitude of a correction to the photon emission rate resulting from the change of the electron mass was estimated by considering the corresponding change in $\chi_e$ parameter.

In general, the effect of a correction in a photon emission rate on the final electron energy can only manifest itself if electrons having large $\chi_e$ parameter radiate. This poses contradictory requirements on the laser-electron interaction as the maximum $\chi_e$ can be reached by the electron only if photon emission is mitigated. In other words, there is a trade-off between using high electron energy to reach high $\chi_e$ and maximizing the amount of emitted energy. If the energy of the colliding electron is relatively low, it will radiate all its energy before reaching the high value of $\chi_e$. The higher values of $\chi_e$ can be achieved with the increasing initial electron energy, however, the photon emission rate is proportional to $1/\gamma_e^{1/3}$, thus less amount of its initial energy would be emitted. And finally, if the energy of the colliding electron is high enough, it will reach the center of the laser pulse without emission and therefore achieve the maximum value of $\chi_e$. Therefore, we need to optimize the electron and laser parameters in order to maximize the energy emitted by the electrons with a high $\chi_e$ parameter.

The amount of energy emitted by the electron during its propagation to the center of the laser pulse is on average given by $E_e - E_e^c$. We are interested in the emission of electrons with a high $\chi_e$ parameter. The parameter $\chi_e$ in Gaussian temporal envelope gradually increases from 0 to 1600. At a distance FWHM/2 = $\pi/2$ from the center of the laser pulse, the value of $\chi_e = 1600/\sqrt{2} \approx 1100$ is reached. In the following, we consider "low" ($\chi_e \leq 1100$) and "high" (1100 < $\chi_e$ ≤ 1600) values of $\chi_e$ parameter. The amount of energy emitted by electrons with "high" $\chi_e$ parameter can be obtained as the total emitted energy
$(\mathcal{E}_e - \mathcal{E}_e^\gamma)$ minus energy emitted by electrons with “low” $\chi_e$ parameters. Due to the geometrical correspondence between the Gaussian and the flat-top temporal envelope, the amount of energy emitted by electrons with “low” $\chi_e$ parameter can be calculated by only considering the corresponding contraction of the flat-top laser pulse duration. For this case, the duration of a flat-top laser envelope has to be modified by a factor of 0.25. Thus the energy delivered by the front part of the Gaussian laser pulse up to a moment of reaching $\chi_e = 1100$ corresponds to a flat-top temporal envelope of duration $\approx \tau/4\sqrt{2\ln 2}$. Therefore, the amount of energy emitted by electrons with “low” $\chi_e$ is

$$\mathcal{E}_e - \mathcal{E}_e^\text{low} \chi_e = \mathcal{E}_e - (1 - 16/63)W_\gamma \tau/4\sqrt{2\ln 2} \mathcal{E}_e. \quad (11)$$

Then

$$\Delta\mathcal{E}_e^{\text{high} \chi_e} \approx (\mathcal{E}_e - \mathcal{E}_e^\gamma) - (\mathcal{E}_e - \mathcal{E}_e^\text{low} \chi_e) \quad (12)$$

characterizes the amount of energy emitted by electrons with “high” $\chi_e$. To maximize this quantity, we define

$$f(\gamma_e) = \frac{\Delta\mathcal{E}_e^{\text{high} \chi_e}(\gamma_e)}{\mathcal{E}_e(\gamma_e)} = \frac{\mathcal{E}_e^\text{low} \chi_e(\gamma_e) - \mathcal{E}_e^\gamma(\gamma_e)}{\mathcal{E}_e(\gamma_e)} \quad (13)$$

as it represents the ratio of the initial electron energy that is radiated by electrons having a “high” $\chi_e$ parameter.

We propose to use the average final electron energy as the observable quantity which could be altered as a consequence of photon emission rate correction. To make this difference noticeable, we need to maximize the amount of energy emitted by electrons with a high value of $\chi_e$ parameter. The reason is that the radiative corrections are expected to become non-negligible for electrons with high $\chi_e$. The radiative correction of photon emission rate can only be observable in the final electron energy distribution if many electrons of such a high $\chi_e$ radiate. The optimal initial electron energy for observing the signature of photon emission correction can be found by solving $(df/d\gamma_e)|_{\alpha\chi^{2/3}_e=1}=0$. For our parameters, $f(\gamma_e)$ reaches its maximum at $\gamma_e$ that corresponds to the initial energy of approximately 80 GeV. This result indicates that the amount of energy radiated by electrons with high $\chi_e$ will be maximized if the interacting electrons will have the above-mentioned initial energy (while keeping $\alpha\chi^{2/3}_e=1$ in this case). Therefore, we expect that the effect of photon emission correction on the final electron energy will be most obvious for $\mathcal{E}_e \approx 80$ GeV. However, the exact calculation of the magnitude of the photon emission rate correction goes beyond the scope of this paper and needs further development of theoretical framework applicable in the range where the perturbative QED approach fails.

We provide the parameters for reaching the strong field region in the collision of an electron beam with a counter-propagating azimuthally polarized laser pulse. We estimate how many electrons reach the nonperturbative limit of QED characterized by $\alpha\chi^{2/3}_e \approx 1$. When the interaction in the $\chi^3$ configuration is considered, the initial electron energy required for achieving the nonperturbative QED limit needs to be higher than 50 GeV. It is shown that e.g. for 10 PW laser, the nonperturbative limit of QED is experienced by one-third of the interacting electrons having energy 140 GeV, thus the upcoming generation of 10 PW-class lasers provides a viable all-optical scheme for probing the nonperturbative QED. The theoretical considerations are benchmarked against three-dimensional particle-in-cell simulations. The effect of photon emission correction due to the electron mass modification is discussed. We expect that the signature of such a radiative correction on final electron distribution starts to fade out once the electron initial energy surpasses the optimal value.

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