Symmetries at ultrahigh energies and searches for neutrino oscillations.

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Abstract
Motivated by the possibility that new (gauge) symmetries which are broken at the grand- (string-) unification scale give rise to texture zeros in the fermion mass matrices which are at the origin of the hierarchy of masses and mixings we explore the effect of such zeros on the neutrino spectrum of SUSY-GUT models. We find that the quadratic-seesaw spectrum on which most expectations are focused is neither the only nor the most interesting possibility. Cases of strong $\nu_\mu - \nu_\tau$ or $\nu_e - \nu_\tau$ mixing are present for a specific texture structure of the Yukawa matrices and experimental evidence can thus throw some light on the latter. In contrast if the quadratic-seesaw scenario should be confirmed very little could be said about the symmetries of the Yukawa sector.

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1 Introduction.

One of the main deficiencies of the Standard Model is the lack of understanding the structure of the Yukawa couplings $Y_{ij}$ in generation space $(i, j = 1, 2, 3)$. In particular, the six quark and lepton masses plus the three mixing angles and the CP violating phase of the quark-mixing matrix are not sufficient to fully specify the entries of the mass matrices $M_{u,d,e} = Y_{u,d,e} < v_{u,d} > / \sqrt{2}$, where the indices $u, d, e$ refer to the up- and down quarks and the charged leptons. In an attempt to keep the number of parameters which enter in the latter minimal, various authors have proposed (L-R) symmetric Yukawa matrices containing two or three zero entries, so-called “textures”, which, when multiplied by diagonal matrices of phases, describe successfully the mass spectra, and predict relations between the quark masses and the mixing angles [1-4].

Recently, the possibility of having L-R symmetric textures that can correctly parametrise the up- and down-quark mass matrices with a maximum number of five zeros at the grand-unification scale $M_G \approx 10^{16}$ GeV has received special attention within the context of the minimal supersymmetric standard model (MSSM) [2-7] and a list of five phenomenologically consistent solutions (I), (II), (III), (IV) and (V) were presented in ref. [4]. For convenience they have been parametrised in powers of the Cabibbo angle $\lambda \simeq 0.22$ and the coefficients $\alpha, \beta, \gamma, \delta$ and $\alpha', \beta', \gamma'$ whose values are given in Table 1:

\[ Y_u = \begin{pmatrix} 0 & \alpha \lambda^6 & \delta \lambda^4 \\ \alpha \lambda^6 & \beta \lambda^4 & \gamma \lambda^2 \\ \delta \lambda^4 & \gamma \lambda^2 & 1 \end{pmatrix}, \quad (1) \]

and

\[ Y_d = \begin{pmatrix} 0 & \alpha' \lambda^4 & 0 \\ \alpha' \lambda^4 & \beta' \lambda^3 & \gamma' \lambda^3 \\ 0 & \gamma' \lambda^3 & 1 \end{pmatrix}. \quad (2) \]

While a number of candidate models beyond the Standard Model (SM) are (L-R) symmetric, -among others some grand-unified (GUT) models that are based on the $SO(10)$ gauge group-, the origin of the texture zeros which are responsible for the well-known mass-mixing relations in the quark sector is not yet fully understood. They are thought to be the relics of new

\footnote{When counting the zeros of a symmetric matrix only the entries above or below the diagonal are considered.}
fundamental symmetries which are broken at or below the grand/string uni-
ification scale. On the other hand, in L-R symmetric models it is natural
to have righthanded neutrino states \( N_i \) and therefore Dirac neutrino mass
terms: \( M^D_\nu N_i^c \nu_j \). The righthanded neutrinos acquire normally large Major-
rana masses through radiative corrections [8-10] or due to nonrenormalisable
terms [11] which originate from supergravity and string theory [12]:
\[
M_R = \mathcal{C} \frac{< H > < H >}{M_S} Y_R,
\]
where \( M_S \sim 10^{18} \text{ GeV} \) is the string unification scale and the parameter \( \mathcal{C} \sim 1 - 10^{-3} \) is characteristic of large-radii orbifold compactification. Since
the Higgs field \( H \) acquires a vacuum expectation value at \( M_G \), typically the scale of the matrix \( M_R \) lies in the intermediate mass range:
\[
R \equiv \mathcal{C} \frac{< H > < H >}{M_S} \approx 10^{11} - 10^{14} \text{ GeV}.
\]
In ref. [7] it was shown that in addition to the scale \( R \), also the texture
structure of \( Y_R \) plays a crucial role in the determination of the mass spectrum
of the three superlight neutrino flavours: \( \nu_e, \nu_\mu, \nu_\tau \) and their mixing.

In particular, it is interesting to examine in greater detail the case where,
as a result of an extra symmetry at the Planck scale, which may as well be
at the origin of the perturbative structure of the quark Yukawa matrices,
some of the entries of the symmetric matrix \( M_R \), which for simplicity we
assume to be real,
\[
M_R = \begin{pmatrix}
R_1 & R_4 & R_5 \\
R_4 & R_2 & R_6 \\
R_5 & R_6 & R_3
\end{pmatrix},
\]
are zero while the others are of order \( R \). The underlying idea is that some of
the higher order operators of equ. (3) are forbidden when \( H \) becomes charged
under an extra symmetry which is not family blind. On the other hand, there
is evidence that the presence of different powers of \( \lambda \) at specific places in
the quark Yukawa matrices, is due to the breaking of such a \( U(1) \) symmetry
through higher order operators containing heavy Higgs fields and singlets
[13]. One may therefore hope to find a common origin of the mass- and mixing-
hierarchy in the quark and lepton sector.

If one imposes that \( M_R \) is a nonsingular matrix, the seesaw mechanism
guarantees the existence of three light neutrinos, which are obtained upon
diagonalisation of the reduced mass matrix:
\[
M^\nu_{eff} \simeq M^D_\nu M_R^{-1} M^D_\nu^T.
\]
Then, if the connection of the lepton to the quark sector, \textit{i.e.} to the Yukawa textures in eqs.(1,2), is made via the successful mass relations of grand unification [14]:

\[ M^D_\nu = M_u, \] 

and

\[ M_e = M_d, \]

where the (2,2) entry of \( M_e \) has to be multiplied by a factor minus three to account for discrepancies in the mass relations of the first two generations, it is possible to determine the neutrino spectrum for different sets of textures and look for experimental tests. As for the quark sector, we require the presence of as many texture zeros in \( M_R \) as this is compatible with its nonsingularity. The corresponding four- and three-zero textures are shown in Table 2.

In order to discuss the properties of the neutrino spectrum case by case, it is useful to write \( M^\text{eff}_\nu \) in terms of the parameters of eqs.(1-4). The elements of the reduced matrix

\[ m_{ij}^2 = \frac{m^2}{\Delta} m_{ij}, \]

with:

\[ m_{11} = \delta^2 r_3 z^4 + \alpha^2 r_2 z^6 \]
\[ m_{12} = m_{21} = \gamma \delta r_3 z^5 + (\beta \delta + \alpha \gamma) r_6 z^4 + \alpha \beta r_2 z^5 + \alpha^2 r_4 z^6 \]
\[ m_{13} = m_{31} = \delta r_3 z^2 + (\alpha + \gamma \delta) r_6 z^3 + (\alpha \gamma r_2 + \delta^2 r_5) z^4 \]
\[ m_{22} = \gamma^2 r_3 z^2 + 2 \beta \gamma r_6 z^5 + \beta^2 r_2 z^4 + 2 \alpha \beta r_5 z^5 + 2 \alpha^2 r_4 z^6 \]
\[ m_{23} = m_{32} = \gamma r_3 z + (\gamma^2 + \beta) r_6 z^2 + (\beta \gamma r_2 + \alpha r_5 + \gamma \delta r_5) z^3 + (\beta \delta + \alpha r) r_4 z^4 \]
\[ m_{33} = r_3 + 2 \gamma r_6 z + (\gamma^2 r_2 + 2 \delta r_5) z^2 + 2 \gamma \delta r_4 z^3 + \delta^2 r_1 z^4, \]

are polynomials in \( \lambda^2 \equiv z \simeq 0.05 \) and \( \Delta \equiv det M_R \). As in ref. [4], we denote by \( r_i = 1, \ldots, 6 \) the minors of the matrix \( M_R \) which are obtained by omitting the row and column containing the corresponding \( R_i \) entry, \textit{e.g.}, \( r_3 = R_1 R_2 - R_4^2 \).

When the minor \( r_3 \neq 0 \) it sets the mass scale for the whole matrix and the third-generation neutrino:

\[ m_{\nu_3} = \frac{m^2}{R}, \]

which becomes the hot dark-matter candidate, and the hierarchy in the neutrino spectrum follows an analogous pattern as in the quark sector. This implies that the heaviest (lightest) fermion belongs to the third (first) generation and that the first-to-second generation mixing: \( |V_{\nu_e - \mu}| \sim \lambda/3 \) prevails.
On the other hand, when $r_3 = 0$, i.e., for the textures $M^{(b)}_R$ and $M^{(d)}_R$ in Table 2, this natural pattern is in some cases broken due to the appearance of a zero in the $m_{33}$ entry of $M^{eff}_\nu$ [7].

The mass eigenvalues of the matrix $M^{eff}_\nu$ can be determined perturbatively from the characteristic equation:

$$x^3 - r_3 f x^2 + z^4 r_4^* g x - \Delta_\nu = 0,$$

where by $\Delta_\nu$ we have denoted the determinant of $m_{ij}$, and $r_4^* = r_2 r_3 - r_6^2$. Then $f$ and $g$ are polynomials in $z$ which become of order one when all nonzero entries of $M_R$ are of order $R$:

$$f = 1 + 2 \gamma a_6 z + [(1 + a_2) \gamma^2 + 2 \delta a_5] z^2 + 2 \gamma (\delta a_4 + \beta a_6) z^3$$
$$+ [(1 + a_1) \delta^2 + \beta^2 a_2 + 2 \alpha \gamma a_5] z^4 + 2 \alpha (\beta a_4 + \alpha a_6) z^5$$
$$+ (a_1 + a_2) \alpha^2 z^6$$

$$g = 1 - 2 a_1^* (\alpha \gamma^2 - \alpha \beta - \beta \gamma \delta - 2 \gamma^3) z$$
$$+ O(z^2) + \ldots + O(z^8),$$

with $a_i \equiv r_i / r_3$ and $a_1^* = (r_3 r_4 - r_5 r_6) / r_1^*$. Notice that $r_3 \neq 0$ holds for the textures $M^{(a)}_R$ and $M^{(c)}_R$ while $r_1^* \neq 0$ holds for the textures $M^{(a)}_R$ and $M^{(d)}_R$, Table 2.

We start our discussion with the first Majorana texture $M^{(a)}_R$ for which $f$ and $g$ are functions of order one. Redefining next $x \to x / R^2$ and using the fact that:

$$\Delta_\nu \simeq \kappa^2 z^{12} \cdot R^6$$
$$\kappa = \alpha^2 + \beta \delta^2 - 2 \alpha \gamma \delta \simeq O(1),$$

equ.(12) reduces to:

$$x^3 - x^2 + z^4 x - z^{12} = 0,$$

which is the same for the five different types (I) - (V) of the $Y_{u,d}$ textures, given in eqs.(1,2) and Table 1. One obtains an entirely model-independent neutrino-mass spectrum:

$$m_{\nu_1} \simeq \frac{m_1^2}{R} z^8 \quad m_{\nu_2} \simeq \frac{m_1^2}{R} z^4 \quad m_{\nu_3} \simeq \frac{m_1^2}{R}.$$

The hierarchy implied by eq.(16) is of the quadratic-seesaw type, i.e., the neutrino masses scale as the up-quark masses squared.
For the Majorana texture $M^{(c)}_R$ the neutrino mass spectrum is again universal, i.e. independent of the particular structure of quark Yukawa matrices. In this case equ.(12) reduces to:

$$x^3 - x^2 + z^5 x - z^{12} = 0 , \quad (17)$$

and gives rise to a distorted seesaw spectrum:

$$m_{\nu_1} \simeq \frac{m^2_t}{R} z^7 \quad m_{\nu_2} \simeq \frac{m^2_t}{R} z^5 \quad m_{\nu_3} \simeq \frac{m^2_t}{R} , \quad (18)$$

On the other hand for the remaining textures $M^{(b)}_R$ and $M^{(d)}_R$ the universality is broken. The characteristic equation assumes in general the form:

$$x^3 - z^n x^2 + z^m x - z^{12} = 0 , \quad (19)$$

where $m = 4$ for $M^{(d)}_R$, and $m > 5$ for $M^{(b)}_R$, while, $n = 1$ for $M^{(d)}_R$ and $Y^{(IV)}_{u,d}$, $n = 2$ for $M^{(b)}_R$ and $Y^{(IV)/(V)}_{u,d}$, $n = 4$ for $M^{(d)}_R$ and $Y^{(VII)}_{u,d}$ and for $M^{(b)}_R$ and $Y^{(I)}_{u,d}$, and $n = 6$ for $M^{(d)}_R$ and $Y^{(I)}_{u,d}$. The resulting neutrino masses and mixings are shown in Table 3. For simplicity we have assumed that there are no extra CP-violating phases in the lepton sector so that the lepton-mixing matrix is given by:

$$V_l = U_{\nu} U_P U_e^{-1} , \quad (20)$$

where $U_{\nu}$ and $U_e$ are the matrices diagonalising $M^{eff}_{\nu}$ and $M_e$ respectively while

$$U_P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix} , \quad (21)$$

is the matrix relating the basis where $M^{eff}_{\nu}$ is diagonal to the basis where $M_e$ is real. Written in powers of $\lambda$ and to lowest order,

$$U_e = \begin{pmatrix} 1 - \lambda^2/18 & -\lambda/3 & \gamma/\lambda^4/3 \\ \lambda/3 & 1 - \lambda^2/18 & -\gamma/\lambda^3 \\ \gamma/\lambda^3 & \gamma/\lambda^3 & 1 \end{pmatrix} . \quad (22)$$

The results of Table 3 confirm our previous statement that for the texture $M^{(a)}_R$, which is almost proportional to the unit matrix, and $M^{(c)}_R$ for which the minor $r_3$ is not zero the usual hierarchy of fermion masses and
mixing elements is encountered also in the light-neutrino sector. The mass eigenstates $\nu_1$, $\nu_2$, $\nu_3$ obey the quadratic-seesaw relation of eq.(16) or the distorted seesaw relation of eq.(18), and the mixing between the different neutrino flavours is given by:

$$\sin^2 2\theta_{e-\mu} \simeq 0.02 \quad \sin^2 2\theta_{\mu-\tau} \simeq 10^{-2} - 10^{-3} \quad \sin^2 2\theta_{e-\tau} \simeq 2 \times 10^{-5} - 10^{-10}. \quad (23)$$

Unfortunately, as long as the precise value of the entries in the righthanded neutrino mass matrix is not specified these results represent only an order of magnitude estimate. In any case this most attractive scenario that can incorporate the small-angle solution to the solar-neutrino problem through $\nu_e \rightarrow \nu_\mu$ transitions and the tau neutrino as a candidate of hot dark matter (HDM) will be for sure tested by the CHORUS and NOMAD $\nu_\mu \rightarrow \nu_\tau$ oscillation experiments. With respect to previous findings that a rather “unsophisticated” Majorana-mass sector, i.e., one that contains no particular symmetry or one that is proportional to the unity, would ideally lead to a quadratic-seesaw scenario \footnote{The only exception to this rule is found for the combined texture choice $M_1^{(c)}$ with $Y_u^{(III)}$. In this case the lightest state is the $\nu_\mu$ followed by the electron neutrino.}, it is interesting to add the new cases involving mostly but not exclusively the texture $M_1^{(c)}$. Therefore if this scenario should receive experimental confirmation it will become impossible to draw any conclusion on the structure of the heavy Majorana sector.

Let us discuss next the more interesting cases of strong mixing that follow from different mass patterns. We find two cases of strong $\nu_\mu \rightarrow \nu_\tau$ mixing, both containing the texture $M_1^{(d)}$, namely $M_1 \sim Y_u^{(I)} M_1^{(d)} Y_u^{(I)}$ and $M_2 \sim Y_u^{(III)} M_1^{(d)} Y_u^{(III)}$, and two almost identical cases of strong $\nu_e \rightarrow \nu_\mu$ mixing where the texture $M_3^{(b)}$ is involved: $M_3 \sim Y_u^{(II),(IV)} M_1^{(d)} Y_u^{(II),(IV)}$. The spectrum of $M_1$ and $M_2$ contains a very light $\nu_e$ and two mass-degenerate states which are linear combinations of $\nu_\mu$ and $\nu_\tau$ with:

$$\sin^2 2\theta_{e-\mu} \simeq 0.02 \quad \sin^2 2\theta_{\mu-\tau} \simeq 1 \quad \sin^2 2\theta_{e-\tau} \simeq 10^{-5}. \quad (24)$$

A scenario with any of the standard neutrinos being a HDM candidate is here ruled out but an explanation of the atmospheric-neutrino deficit becomes an interesting possibility. In the spectrum of $M_3$ the lightest (heaviest) state is a $\nu_\mu$ ($\nu_\tau$) while $\nu_2$ is a linear combination of $\nu_e$ and $\nu_\tau$ with:

$$\sin^2 2\theta_{e-\mu} \simeq 0.02 \quad \sin^2 2\theta_{\mu-\tau} \simeq 10^{-2} \quad \sin^2 2\theta_{e-\tau} \simeq 1. \quad (25)$$
Here again the usual HDM scenario is excluded while the possibility of explaining the solar-neutrino problem through a $\nu_e \rightarrow \nu_\tau$ transition could be tested in the future.

In the cases, $M_4 \sim Y_u^{(III)} - 1 M_R^{(c)} - 1 Y_u^{(III)}$ and $M_5 \sim Y_u^{(IV)} - 1 M_R^{(d)} - 1 Y_u^{(IV)}$, the hierarchy between $\nu_e$ and $\nu_\mu$ is flipped with respect to the usual seesaw spectrum but without alteration of the mixing angles in eq.(23). The case $M_6 \sim Y_u^{(I)} - 1 M_R^{(b)} - 1 Y_u^{(I)}$ is an interesting example of a scenario with $\nu_\mu$ as the heaviest neutrino (while $\nu_\tau$ is the second heaviest state) and vanishing $\nu_e \rightarrow \nu_\tau$ mixing. For the $\nu_\mu$ to become a HDM candidate the Majorana mass scale should be: $R \sim z^3 \times 10^{12}$ GeV which is typical of radiatively generated righthanded neutrino masses in nonsupersymmetric GUTs.

The cases involving $Y_{u,d}^{(V)}$ are treated separately due to the remarkable stability that the neutrino spectrum exhibits as a function of the structure of the Majorana mass matrix. This is due to the fact that the $m_{ij}$ entries of the light neutrino mass matrix, equ.(10), are on one hand power series of $z$ with coefficients $\beta, \gamma, \delta$ (up to $z^4$) which are of order one for the up-quark texture $Y_u^{(V)}$, on the other hand they are ordered series of the minors $r_3; r_6; r_5; r_4; r_1$. Therefore the hierarchical structure of $M^{(V)}_\nu$ up to fourth order in $z$ is, independently of $M_R$,

$$M^{(V)}_\nu \sim \frac{m_{ij}^2 z^p}{R} \begin{pmatrix} z^4 & z^3 & z^2 \\ z^3 & z^2 & z \\ z^2 & z & 1 \end{pmatrix},$$  

(26)

where the power $p$ can in principle go from zero to four. In all models of this type one has a quadratic-seesaw mass spectrum, but for which the mixing between $\nu_e$ and $\nu_\mu$ turns out to be two to three times larger than usual:

$$\sin^2 2\theta_{e-\mu} \simeq 0.05,$$

(27)

the mixing between $\nu_e$ and $\nu_\tau$ exceeds by far the seesaw expectations:

$$\sin^2 2\theta_{e-\tau} \simeq 0.9 \cdot 10^{-2},$$

(28)

while the mixing between $\nu_\mu$ and $\nu_\tau$ is negligible. This anomaly is representative of the structure of the quark-Yukawa sector and has been discussed in ref.([7]).

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Table 1: The values of the parameters in equ.(45) and equ.(46) that correspond to the five distinct classes of maximally-predictive GUT models from ref.[7].

|   | (I) | (II) | (III) | (IV) | (V) |
|---|-----|------|-------|------|-----|
| $\alpha$ | $\sqrt{2}$ | 1 | 0 | $\sqrt{2}$ | 0 |
| $\beta$ | 1 | 0 | 1 | $\sqrt{3}$ | $\sqrt{2}$ |
| $\gamma$ | 0 | 1 | 0 | 1 | $1/\sqrt{2}$ |
| $\delta$ | 0 | 0 | $\sqrt{2}$ | 0 | 1 |
| $\alpha'$ | 2 | 2 | 2 | 2 | 2 |
| $\beta'$ | 2 | 2 | 2 | 2 | 2 |
| $\gamma'$ | 4 | 2 | 4 | 0 | 0 |

Table 2: Nonsingular symmetric textures with a maximum of zero entries representing the Majorana mass matrix $M_R$ of the righthanded neutrinos.

$M_R^{(a)} = \begin{pmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{pmatrix}$

$M_R^{(b)} = \begin{pmatrix} 0 & 0 & R_5 \\ 0 & R_2 & 0 \\ R_5 & 0 & 0 \end{pmatrix}$

$M_R^{(c)} = \begin{pmatrix} 0 & R_4 & 0 \\ R_4 & 0 & 0 \\ 0 & 0 & R_3 \end{pmatrix}$

$M_R^{(d)} = \begin{pmatrix} R_1 & 0 & 0 \\ 0 & 0 & R_6 \\ 0 & R_6 & 0 \end{pmatrix}$
Table 3: The neutrino spectrum for the four righthanded-neutrino textures $M_R^{(a)-(d)}$ of Table 2 and the different parametrisations of the quark-Yukawa matrices $Y_{u,d}^{(I)-(IV)}$ defined by the eqs.(1,2) and Table 1.
\[ M_{\nu}^{\text{eff}} \approx \frac{\langle v \rangle}{\sqrt{2}} Y_u^{(I)^T} \left[ \begin{array}{c} M_{R}^{(a)-1} \\
 M_{R}^{(b)-1} \\
 M_{R}^{(c)-1} \\
 M_{R}^{(d)-1} \end{array} \right] Y_u^{(I)} \]

\[
\begin{array}{l|cccc}
      & m_{\nu_1} & m_{\nu_2} & m_{\nu_3} & |V_{\nu_1-\mu}| \\
\hline
\frac{z^8}{z} & z^6 & z^7 & z^8 & \frac{m_2}{R} \\
\frac{z^4}{z} & z^3 & z^5 & z^2 & \frac{m_2}{R} \\
1 & z^3 & 1 & z^2 & \frac{m_2}{R} \\
\frac{4\lambda^3}{z} & 1 & 4\lambda^3 & 1 & \\
0 & 0 & 4\lambda^5 & 4\lambda^5 & \end{array}
\]

\[ M_{\nu}^{\text{eff}} \approx \frac{\langle v \rangle}{\sqrt{2}} Y_u^{(II)^T} \left[ \begin{array}{c} M_{R}^{(a)-1} \\
 M_{R}^{(b)-1} \\
 M_{R}^{(c)-1} \\
 M_{R}^{(d)-1} \end{array} \right] Y_u^{(II)} \]

\[
\begin{array}{l|cccc}
      & m_{\nu_1} & m_{\nu_2} & m_{\nu_3} & |V_{\nu_1-\mu}| \\
\hline
\frac{z^8}{z} & z^6 & z^7 & z^8 & \frac{m_2}{R} \\
\frac{z^4}{z} & z^4 & z^5 & z^3 & \frac{m_2}{R} \\
1 & z^2 & 1 & z & \frac{m_2}{R} \\
\frac{4\lambda^3}{z} & 1 & 4\lambda^3 & 1 & \\
\frac{4\lambda^5}{z} & 4\lambda^5 & & & \end{array}
\]

13
\[ M_{\nu}^{eff} \simeq \frac{v}{\sqrt{2}} Y_u^{(III)^T} \left[ M_{R}^{(a)-1}; M_{R}^{(b)-1}; M_{R}^{(c)-1}; M_{R}^{(d)-1} \right] Y_u^{(III)} \]

\[
\begin{align*}
m_{\nu_1} & \quad \rightarrow \quad z^8 \quad z^6 \quad z^7 \quad z^8 \quad \times \frac{m^2}{R} \\
m_{\nu_2} & \quad \rightarrow \quad z^4 \quad z^4 \quad z^5 \quad z^2 \quad \times \frac{m^2}{R} \\
m_{\nu_3} & \quad \rightarrow \quad 1 \quad z^2 \quad 1 \quad z^2 \quad \times \frac{m^2}{R} \\
|V_{\nu_1-\mu}| & \quad \rightarrow \quad \frac{\lambda}{3} \quad \frac{\lambda}{3} \quad 1 \quad \frac{\lambda}{3} \\
|V_{\nu_2-\tau}| & \quad \rightarrow \quad 4\lambda^3 \quad 4\lambda^3 \quad \lambda^4 \quad 1 \\
|V_{\nu_1-\tau}| & \quad \rightarrow \quad \lambda^4 \quad \lambda^4 \quad 4\lambda^3 \quad \lambda^4
\end{align*}
\]

\[ M_{\nu}^{eff} \simeq \frac{v}{\sqrt{2}} Y_u^{(IV)^T} \left[ M_{R}^{(a)-1}; M_{R}^{(b)-1}; M_{R}^{(c)-1}; M_{R}^{(d)-1} \right] Y_u^{(IV)} \]

\[
\begin{align*}
m_{\nu_1} & \quad \rightarrow \quad z^8 \quad z^6 \quad z^7 \quad z^8 \quad \times \frac{m^2}{R} \\
m_{\nu_2} & \quad \rightarrow \quad z^4 \quad z^4 \quad z^5 \quad z^3 \quad \times \frac{m^2}{R} \\
m_{\nu_3} & \quad \rightarrow \quad 1 \quad z^2 \quad 1 \quad z \quad \times \frac{m^2}{R} \\
|V_{\nu_1-\mu}| & \quad \rightarrow \quad \frac{\lambda}{3} \quad 1 \quad \frac{\lambda}{3} \quad 1 \\
|V_{\nu_2-\tau}| & \quad \rightarrow \quad \lambda^2 \quad 1 \quad \lambda^2 \quad \lambda^4 \\
|V_{\nu_1-\tau}| & \quad \rightarrow \quad \lambda^4 \quad \lambda^2 \quad \lambda^4 \quad \lambda^2
\end{align*}
\]