Dynamic characteristics for system of bearing on the top of first story column based on geometry nonlinearity

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Abstract: According to the serial system of bearing with column, the natural frequency of serial system is investigated, and the effects of axial load and different rubber bearings are discussed. Taking into account the effects of the cross-section rotation, shear distortion and compression axial force, the mathematical model for free vibration of the system is established. The differential quadrature element method is employed for the discrete processing in governing equations and boundary conditions. The natural frequency of serial system with clamped-free boundary conditions is solved numerically, and the figures of natural frequency with compression axial force are presented. It is shown that the axial force will reduce the lower order natural frequencies significantly, and in certain case of total height, the equivalent bending stiffness of rubber bearings reduces the natural frequencies to different extent.

1 Introduction
The effect of isolation building has been fully verified in the earthquake, and the isolation structure has been widely used in countries all over the world [1]. Recently, the main application of isolation technology is base isolation, which the isolation layer locates on the top of the foundation. The serially connected isolation system is a complex structural system evolving on the base isolation system, which the rubber bearing connected with the basement column, shown in Figure 1. The superstructure and substructure will affect the vibration characteristics of serial system of bearing with column. The early studies of rubber bearing were based on the Haringx theory by Gent [2]. Kelly studied the horizontal stiffness, vertical stiffness and tension buckling of rubber bearings, based on the theory of beam [3, 4]. Zhou et al. [5, 6] set up the analytic model of the serial system, deduced the practical calculation formulas and analyzed the stability. Ma et al. [7] proposed an iteration-free algorithm for the dynamic response analysis of structures with isolators on the top of the columns, and studied the influences of $P - \Delta$ effects on the responses of the substructure. However, there is less report on the dynamic performance of the serial system of bearing with column. Taking into account the effects of the cross-section rotation, shear distortion and compression axial force, the natural frequency of serial system of bearing with column was studied less. But the moment of inertia, shear deformation and axial force will change the natural frequency of serial system of bearing with column.

In this paper, the motion equations and the frequency equations of the serial system are presented. By the extended differential quadrature method [8-10], the equations and boundary conditions are discretized. And the natural frequency of serial system with clamped-free boundary conditions is solved. The dynamic characteristics of the system and the effects of the equivalent bending stiffness of bearings are discussed. The results provide a basis for the following study on the dynamic behavior and random vibration of the serial system of bearing with column.
2 Governing differential equation
Consider the serial system (shown in Fig.1) made of two segments column and bearing. It is assumed that the bearing: (1) is made of a homogenous linear material [11, 12, 13] with cross-section area of rubber bearing $A^{(2)}$ and moment of inertia $I^{(2)}$, equivalent compression Young’s modulus of bearing $Ecv^{(2)}$, equivalent flexural modulus of bearing $Ebv^{(2)}$; (2) its centroidal axis is a straight line; (3) is loaded axially at the ends along its centroidal $y$-axis with a constant load $P$. Cantilever basement column is connected with rubber bearing fixedly at length $h$. Here, it is presumed that the deformed central axis is still in $Oxy$ plane as shown in Figruie. 2.

The transverse and bending equations of equilibrium of the serial system differential element shown in Figure 2 are the following:
The superscript $^1$ is indicated to the element of basement column; $^2$ is represented to the element of rubber bearing.

The related boundary conditions of a clamped-glided become:

$$\theta (0) = 0, \quad \theta (H) = 0, \quad w (0) = 0$$

(2)

With the continuity condition between the rubber bearing and cantilever basement column:

At $y = h$, $\theta^1 = \theta^2$, $w^1 = w^2$,

$$N^1 = N^2, \quad M^1 = M^2, \quad Q^1 = Q^2$$

(3)

### 3 Eigenvalue equation

Set harmonic vibration for $w^e = W^e (y) e^{i \omega t}$, $\Theta^e = \Theta^e (y) e^{i \omega t}$, representing deflection and rotational angle, respectively. Taking it into (1), it is rewritten as:

$$\begin{align*}
\kappa^2 A_1^e G^e (\frac{\partial \Theta^e}{\partial y} - \frac{\partial w^e}{\partial y}) + \rho \frac{\partial \Theta^e}{\partial t} + \rho^1 A^1 \frac{\partial^2 w^e}{\partial t^2} &= 0, \\
0 < y < h; \\
E^1 I^2 \frac{\partial^2 \Theta^e}{\partial y^2} + \kappa^1 A_1^e G^e (\frac{\partial \Theta^e}{\partial y} - \frac{\partial w^e}{\partial y}) + P^1 \rho \frac{\partial \Theta^e}{\partial t} + \rho^1 I^1 \frac{\partial^2 \Theta^e}{\partial t^2} &= 0, \\
0 < y < h; \\
A^2 \frac{\partial^2 \Theta^e}{\partial y^2} + \kappa^2 A_1^e G^e (\frac{\partial \Theta^e}{\partial y} - \frac{\partial w^e}{\partial y}) + P^2 \rho \frac{\partial \Theta^e}{\partial t} + \rho^2 I^2 \frac{\partial^2 \Theta^e}{\partial t^2} &= 0, \\
h < y < H; \\
E^2 I^2 \frac{\partial^2 \Theta^e}{\partial y^2} + \kappa^2 A_1^e G^e (\frac{\partial \Theta^e}{\partial y} - \frac{\partial w^e}{\partial y}) + P^2 \rho \frac{\partial \Theta^e}{\partial t} + \rho^2 I^2 \frac{\partial^2 \Theta^e}{\partial t^2} &= 0,
\end{align*}$$

(4)

If the superscript $^\ddot{e} = ^\ddot{1}$, it is indicated to the element of basement column; $^\ddot{e} = ^\ddot{2}$ is represented to the element of rubber bearing.

### 4 Dimensionless equations

The following dimensionless quantities are introduced:

$$\begin{align*}
\xi &= \frac{y}{H}, \quad \bar{W} = \frac{W}{H}, \quad \delta^e = \frac{A_1^e G^e}{A_y^e E_y^e}, \quad p = \frac{P}{A_y^2 E_y^2} \\
\beta^e &= \frac{E^e I^e}{A_y^e E_y^e H^2}, \quad R^e = \frac{I^e}{A_y^e H^2}, \quad h^e = \frac{h}{H} \\
\rho^e &= \frac{\rho^2 A^2}{\rho^1 A^1}, \quad E_1 = \frac{E^1 I^1}{E^2 I^2}, \quad \bar{\omega}^2 = \frac{\omega^2 \rho^2 A^2}{A_y^e E_y^e}
\end{align*}$$

(5)
Then, Eqs. (2), (3) and (4) can be written in terms of the non-dimensional forms as:

\[
\left\{ \begin{align*}
\left( \kappa^1 \delta^1 + p \right) \sum_{j=1}^{n} A_{ij} \Theta_j^1 - \kappa^1 \delta^1 \sum_{j=1}^{n} B_{ij} W_j^1 &= 0, \\
0 &< \xi < h_1; \\
\rho_A \sum_{j=1}^{n} A_{ij} \Theta_j^i - \kappa^2 \delta^2 \sum_{j=1}^{n} B_{ij} W_j^2 &= 0, \\
0 &< \xi < h_1; \\
\rho_A \sum_{j=1}^{n} A_{ij} \Theta_j^i - \kappa^2 \delta^2 \sum_{j=1}^{n} B_{ij} W_j^2 &= 0, \\
h_1 &< \xi < 1;
\end{align*} \right.
\]

(6)

(7)

5 Equations for DQEM

Due to the strong nonlinearity of the governing equations, it is difficult to find any analytical solution. Thus, the differential quadrature element method is used for solving the nonlinear equations. Now, by applying DQEM, Eq.(6) is discretized at n inner points. Finally, the DQ discretized form of the Eq. (6), (7) can be expressed as:

\[
\left\{ \begin{align*}
\left( \kappa^1 \delta^1 + p \right) \sum_{j=1}^{n} A_{ij} \Theta_j^i - \kappa^1 \delta^1 \sum_{j=1}^{n} B_{ij} W_j^i &= 0, \\
0 &< \xi < h_1; \\
\left( \kappa^2 \delta^2 + p \right) \sum_{j=1}^{n} A_{ij} \Theta_j^i - \kappa^2 \delta^2 \sum_{j=1}^{n} B_{ij} W_j^i &= 0, \\
h_i < \xi < 1;
\end{align*} \right.
\]

(8)
\[
\begin{align*}
\hat{W}^I = \Theta^I &= 0, \\
\check{\xi} &= 0; \\
\hat{W}^I = \tilde{W}^I, \quad \Theta^I &= \Theta^I, \quad E_j \sum_{j=1}^{n} A_{ij} \Theta_j^I &= \sum_{j=1}^{n} A_{ij} \Theta_j^I, \\
\check{\xi} &= h_j; \\
(k^3 \delta^I + p) \Theta_n^I - k^3 \delta^I \sum_{j=1}^{n} A_{nj} \hat{W}_j^I &= (k^3 \delta^I + p) \Theta_j^I - k^3 \delta^I \sum_{j=1}^{n} A_{nj} \hat{W}_j^I, \\
\check{\xi} &= h_j; \\
(k^3 \delta^I + p) \Theta_n^I - k^3 \delta^2 \sum_{j=1}^{n} A_{nj} \hat{W}_j^I &= 0, \sum_{j=1}^{n} A_{nj} \Theta_j^I &= 0, \\
\check{\xi} &= 1;
\end{align*}
\]

(9)

The Chebyshev-Gauss-Lobatto grid points in normalized interval \([0,1]\) will be employed in this work.

\[
x_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{i-1}{n-1} \pi \right) \right], (i = 1, 2, \cdots, n)
\]

(10)

Using replacement method to deal with the boundary conditions, Eq.(8), (9) can be written as matrix form:

\[
\begin{bmatrix}
[K_{ee}] & [K_{ei}] \\
[K_{ie}] & [K_{ii}]
\end{bmatrix}
\begin{bmatrix}
\{X_e\} \\
\{X_i\}
\end{bmatrix} = \begin{bmatrix}
\{0\} \\
\{0\}
\end{bmatrix}
\]

(11)

Where the order of matrix \([K_{ee}]\), \([K_{ei}]\), \([K_{ie}]\), \([K_{ii}]\) are (8×8), (8×(4n-8)), ((4n-8)×8), (4n-8)×(4n-8)) respectively; the dimension of column vector \(\{X_e\}\), \(\{X_i\}\) are 8 and (4n-8);

\[
\begin{align*}
\{X_e\} &= \{\hat{W}_1^1, \hat{W}_2^1, \hat{W}_n^1, \hat{W}_1^2, \hat{W}_2^2, \hat{W}_n^2, \hat{W}_1^3, \hat{W}_2^3, \hat{W}_n^3, \Theta_1^1, \Theta_1^2, \Theta_n^1, \Theta_n^2, \Theta_1^3, \Theta_1^4, \Theta_n^3, \Theta_n^4\}^T \\
\{X_i\} &= \{\tilde{W}_1^1, \cdots, W_{n-1}^1, \Theta_1^1, \cdots, \Theta_{n-1}^1, \tilde{W}_n^2, \cdots, \tilde{W}_{n-1}^2, \Theta_1^2, \cdots, \Theta_{n-1}^2, \Theta_1^3, \cdots, \Theta_{n-1}^3\}^T
\end{align*}
\]

It can be got from Eq. (11):

\[
\left( \begin{bmatrix}
[K_{ee}] - [K_{ii}] & [K_{ei}] \\
[K_{ie}] & [K_{ii}]
\end{bmatrix} \right) \{X_e\} - \check{\omega}^2 \{X_i\} = \{0\}
\]

(12)

This is the eigenvalue equation of natural frequency for the system.

6 Numerical results

The parameter of basement columns: concrete strength grade of C30, the side length of square section is 1050 mm, the height is 3.0 m; the type of rubber bearings are GZP500, GZP600, GZP700, GZP800, GZP900. The bearing parameter is shown in Table 1.


Table 1 Parameters of rubber bearings

| Model   | Height (mm) | Thickness of rubber layer (mm) | G (Gpa) |
|---------|-------------|--------------------------------|---------|
| GZP500  | 164         | 4.87                           | 0.6     |
| GZP600  | 233         | 5                              | 0.6     |
| GZP700  | 265         | 4.94                           | 0.6     |
| GZP800  | 303         | 5                              | 0.6     |

Table 2 Frequency under 10MPa vertical load

| Model   | 1   | 2   | 3   | 4   | 5   |
|---------|-----|-----|-----|-----|-----|
| GZP500  | 2.4305 | 6.9015 | 7.3128 | 12.4849 | 17.5153 |
| GZP600  | 1.4447 | 3.6932 | 4.3361 | 7.3674  | 10.3332 |
| GZP700  | 1.2742 | 3.5478 | 3.8214 | 6.42316 | 9.0001  |
| GZP800  | 0.8831 | 2.2346 | 2.6422 | 4.4280  | 6.2041  |

In numerical calculation, the inner point n is 19. Table 2 is the first five order natural frequency of serial system under vertical load. The variations of the first-mode natural frequency with the applied compressive axial load p are shown in Figure 3 for three different bearings (GZP600, 700, 800). The critical axial load is found when the first natural frequency $\bar{\omega}=0$. The buckling dimensionless loads are: for GZP600, $p=0.0569$; GZP700, $p=0.0684$; GZP800, $p=0.0779$. In the frequency steady descending segment, the first frequency decreases with the bearing models. Then, it also shows that the critical load of serial system increases with the bearing models.

Figure 3: Frequency of serially isolated system—vertical load curve.

It is known that there are more parameters of laminated rubber bearings. So the equivalent bending stiffness EI of the bearing is taken as the macroscopic parameters to analyze the first order frequency of the serial system. In Figure 4, the first order frequency decrease with the equivalent stiffness, except for P=28 Mpa. In P=28 Mpa, the curve appears catastrophe. It is shows that the bearing GZP500 and GZP600 have entered into the frequency rapid descending segment, but GZP800 is still in the steady descending segment.
7 Conclusions

The natural frequency analyses for serial system of bearing with column are presented by using the differential quadrature element method. The results show that, the natural frequency decreased with the vertical load; when the vertical load is close to the critical load, the natural frequency tends to zero; the first mode frequency reduced with the equivalent bending stiffness of the bearings.

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