Pair Production of small Black Holes in Heterotic String Theories

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Abstract

We study pair production of small BPS BH’s in heterotic strings compactified on tori and in the FHSV model. After recalling the identification of small BH’s in the perturbative BPS spectrum, we compute the tree-level amplitudes for processes initiated by massless vector bosons or gravitons. We then analyze the resulting cross sections in terms of energy and angular distributions. Finally, we briefly comment on scenarios with large extra dimensions and on generalizations of our results to non-BPS, non-extremal and rotating BH’s.
Introduction

Understanding black hole physics is a challenge to any quantum theory of gravity. The possibility that black holes be (pair) produced in high energy collisions is a fascinating possibility [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] not without some worries [11, 12]. A very massive particle should behave as a small black hole when its Compton length $\lambda_C = \frac{\hbar}{M_c}$ is smaller than the Schwarzschild radius $R_S = \frac{2G_NM}{c^2}$ i.e. when its mass is larger than Planck mass $M > M_{pl} = \sqrt{\frac{\hbar c}{G_N}} = 1.2209 \times 10^{19} GeV/c^2$.

In perturbative string theory there is an infinite tower of very massive states. After turning on interactions, most of them become unstable. Some are long-lived [13, 14]. Some remain stable since they are ‘extremal’ and there is no multi-particle state with lower mass and the same conserved charges they can decay into. Among ‘extremal’ states some preserve a fraction of the original supersymmetry and are commonly called BPS states [15, 16, 17, 18].

In toroidal compactifications of the heterotic string it is very easy to identify perturbative states of this kind [19, 20, 21, 22, 23, 24, 25, 26]. They correspond to setting the L-moving oscillators in their (supersymmetric) ground state. KK momenta, windings and gauge charges can be included compatibly with level matching. At the classical level, 1/2 BPS states correspond to charged ‘extremal’ black-hole solutions of $N = 4$ supergravity coupled to $N_v$ vector fields [27, 28, 29, 30, 31]. The relevant solutions carry only ‘electric’ charges and display a null singularity since their ADM mass (in the Einstein frame) vanishes at the boundary of the $[SO(6, N_v)/SO(6) \times SO(N_v)] \times [SL(2)/O(2)]$ moduli space. At the quantum level $R^2$ corrections can modify the picture. Although 1/2 BPS states with a ‘single’ charge do not receive quantum corrections to their mass and degeneracy and thus remain singular, 1/2 BPS states with two charges receive higher derivative corrections [32, 33] that call for Wald’s entropy formula [34] that generalizes Bekenstein-Hawking’s entropy formula. The resulting finite non-zero ‘area’ of the ‘stretched’ horizon precisely reproduces the microscopic degeneracy of the perturbative string spectrum [30, 31, 35, 36, 37, 27, 38]. Such states are thus good candidates for small BPS black holes whose dynamical properties can be reliably studied in scattering processes. Due to their charge and 1/2 BPS property, they can be produced in pairs at least.

Our aim is to compute the cross section for pair production of ‘spherically symmetric’ (scalar) 1/2 BPS small BH’s with two-charges in high energy collisions of gravitons or gauge bosons. BPS BH’s are very special in that their temperature vanishes and they behave pretty much like very massive particles, with a large number of degenerate microstates, accounting for their entropy. To set the stage for a full-fledged heterotic string computation at tree level, we first describe a field theory toy model for pair production.
of charged massive scalars [39]. Since the pair-produced small BH’s are BPS and stable they do not emit Hawking radiation. We analyze angular and energy distributions of the heterotic string process with the field theory toy model in mind and comment on scenari with Large Extra Dimensions. We then consider a simple case with supersymmetry broken to $\mathcal{N} = 2$ that enjoys particular non-renormalization properties, the FHSV model [40]. Finally we briefly comment on the case of non-BPS, non-‘extremal’ or rotating BH’s. A realistic description of macroscopic BH’s with finite horizon area, even in the classical limit, should involve wrapped branes and KK monopoles that are dual to bound-states of D-branes in Type II or Type I strings [41].

The plan of the paper is as follows. In Sect. 1 we recall the partition function of perturbative 1/2 BPS states in toroidal compactifications of heterotic strings. Moreover, we analyze three types of 1/2 BPS states depending on their ‘electric’ charges and discuss the formula for their microscopic entropy. In Sect. 2 we present the field theoretic cross section for pair production of massive charged scalars. The heterotic string computation is described in Sect. 3. We describe both processes initiated by vector bosons and by gravitons. We give general formulae for the amplitudes and then specialize to the simplest non-trivial case for illustrative purposes. In Sect. 4 we write down explicit expressions for the cross sections and comment on angular and energy distributions as well as on scenari with Large Extra Dimensions. In Sect. 5 we consider analogous processes in the FSHV model [40]. We conclude in Sect. 6 with a summary of our results and some comments on (small) non-BPS, non-extremal and rotating BH’s.

1 BPS partition function in Heterotic String

The perturbative spectrum of heterotic strings compactified on tori contains massless, 1/2 BPS, and long multiplets. 1/4 BPS states with both ‘electric’ and ‘magnetic’ charges are intrinsically non-perturbative in the ‘heterotic duality frame’ and arise from bound states of fundamental strings with K-K monopoles or NS5-branes (‘H-monopoles’). In dual descriptions, e.g. Type II on $K3 \times T^2$, some 1/4 BPS states admit a perturbative description at special points in the moduli space. For simplicity, we will consider the component with maximal possible rank of the gauge group ($r = 6 + 6 + 16 = 28$) in the $\mathcal{N} = 4$ context. Rank reduction is also possible in the so-called heterotic CHL models [42], dual to Type I models with a quantized $B$ or otherwise [43, 44, 45, 46, 47].

In $D = 10$, as a result of the GSO projection, the one-loop partition function for heterotic strings reads [48]

$$Z_{Spin(32)/Z_2} = \frac{\theta^4_4 - \theta^4_4 - \theta^4_4 - \theta^4_4}{2 \eta^{12}} \times \frac{\tilde{\theta}^{16}_4 + \tilde{\theta}^{16}_4 + \tilde{\theta}^{16}_4}{2 \eta^{24}}$$

(1)
\[
Z_{E(8) \times E(8)} = \frac{\theta_3^4 - \theta_4^4}{2\eta^{12}} \times \frac{[\bar{\theta}_3^8 + \bar{\theta}_1^8 + \bar{\theta}_4^8 + \bar{\theta}_2^8]^2}{4\eta^{24}}
\]

(2)

where \(\eta\) is Dedekind’s function and \(\theta_{\alpha}\) with \(\alpha = 1, 2, 3, 4\) are Jacobi functions. The partition function vanishes thanks to Jacobi’s identity, which accounts for space-time supersymmetry in the L-moving sector. Modular invariance results after inclusion of the bosonic and (super)ghost zero-modes, producing a factor \(V/\text{Im}\tau^4\), that nicely combines with the modular invariant measure \(\frac{d^2\tau}{\text{Im}\tau^2}\).

It is convenient to express \(Z\) in terms of the characters of the \(SO(8)\) L-moving current algebra at level \(\kappa = 1\) (Little Group for massless states in \(D = 10\)) and of the R-moving current algebras. To this end, the characters of the four conjugacy classes of \(SO(2n)\) (vector \(V\), spinor \(S\), co-spinor \(C\), and singlet \(O\)) read \([49, 50]\)

\[
O_m = \theta_n^3 - \theta_2^3 + \eta_n^2, \quad V_m = \theta_n^3 - \theta_4^3 + \eta_n^2, \quad S_m = \theta_2^n + i\theta_1^n, \quad C_m = \theta_2^n - i\theta_1^n
\]

Then one finds

\[
Z_{D=10}^H = \frac{Q\bar{G}}{|\eta^8|^2}
\]

(4)

where \(Q = V_8 - S_8 = (8_v - 8_h)q^{1/3} + \text{massive}\) is the super-character introduced in \([51]\)

and \(G = O_{32} + S_{32} = q^{-2/3} + 496q^{1/3} + ...\) for \(Spin(32)/Z_2\) or \(G = E_8^2 \equiv [O_{16} + S_{16}]^2 = q^{-2/3} + (248 + 248)q^{1/3} + ...\) for \(E(8) \times E(8)\). As usual we set \(q = e^{2\pi i\tau}\).

1.1 Perturbative BPS states in toroidal compactifications

After toroidal compactification, the two heterotic strings are continuously connected by Wilson lines breaking the gauge group to a common sub-group of both \(SO(32)\) and \(E(8) \times E(8)\). Setting henceforth \(\alpha' = 2\) for notational convenience, the one-loop partition function in \(D = 4\) reads \([3]\)

\[
Z_T^H = \sum_{m,n,r} q^{\frac{1}{2}|p_L|^2} q^{\frac{1}{2}|p_R|^2} \frac{Q}{\eta^{24}}
\]

(5)

where

\[
p_L = [(E^t)^{-1}(m + A'r + (B + \frac{1}{2}A'A)n) + \frac{1}{2}En; 0]
\]

(6)

are the 6 central charges of the \(\mathcal{N} = 4\) superalgebra, the 6 graviphotons couple to, and

\[
p_R = [(E^t)^{-1}(m + A'r + (B + \frac{1}{2}A'A)n) - \frac{1}{2}En; (r + An)]
\]

(7)

While \(m = (m_1, ..., m_6)\) and \(n = (n^1, ..., n^6)\) are unrestricted 6-plexes of integers, the 16-uples \(r = (r_1, ..., r_{16})\) belong to an even self-dual lattice, i.e. \(\Gamma_{E_8} \oplus \Gamma_{E_8}\) or \(\Gamma_{Spin_{32}/Z_2}\).
are 22 ‘matter’ charges, the vector bosons in $\mathcal{N} = 4$ vector multiplets couple to. The moduli space $\mathcal{M} = SO(6, 22)/SO(6) \times SO(22)$ is parameterized by the internal metric $G_{ij}$ or rather by the 6-bein $\hat{E}^i_i$, for which $G_{ij} = \delta_{ij} \hat{E}^i_i \hat{E}^j_j$, the anti-symmetric tensor $B_{ij}$ and the Wilson lines $A^a_i a = 1, \ldots, 16$ [52]. The remaining $SL(2)/SO(2)$ is spanned by the dilaton and axion, that belong in the $\mathcal{N} = 4$ supergravity multiplet.

The level matching condition reads

$$|p_L|^2 + 2(N_L - \delta_L) = M^2 = |p_R|^2 + 2(N_R - 1)$$

(8)

where $\delta_L$ denotes the ground-state energy in the L-moving (supersymmetric) sector ($\delta^N_{LS} = 1/2, \delta^R_{LS} = 0$). For 1/2 BPS states $N_L = \delta_L$ and one has

$$1/2 \text{ BPS} \quad \rightarrow \quad Q^2 \geq -2$$

(9)

where

$$Q^2 = \eta_{AB} Q^A Q^B = |p_L|^2 - |p_R|^2 = 2nm - |r|^2 = 2(N_R - 1)$$

(10)

is the $SO(6, 22)$ invariant norm of the 28-dimensional ‘electric’ charge vector $Q = (n, m, r)$ with $\eta = (\sigma_1 \otimes 1_{6 \times 6}) \oplus (-1_{16 \times 16})$. Perturbative states with no ‘magnetic’ charges ($P = 0$) and $Q^2 < -2$ are necessarily non BPS, while states with $Q^2 \geq -2$ may be either BPS or non-BPS.

1.2 1/2 BPS states with $Q^2 = -2, 0$

1/2 BPS states with $Q^2 = -2$ have $N_R = 0$ and are the only states that can lead to gauge symmetry enhancement [54, 55, 30, 31]. Since $Q = (n, m, r) \neq 0$, these states are generically massive as $M^2_{\text{BPS}} = |p_L|^2 \geq 0$. For special choices of the moduli, the conditions $p_L = 0$ can be satisfied giving rise to massless non-abelian vector multiplets. In particular states with $nm = -1$, i.e. $r = 0$ may become massless at self-dual points. States with $r^2 = 2$ and $n = m = 0$, corresponding to the 480 ‘charged’ vector bosons and gauginos in $D = 10$, remain massless in the absence of Wilson lines. States with $r^2 = 2$ and $n, m$ not all zero, still with $nm = 0$, are associated to generalized KK excitations.

1/2 BPS states with $Q^2 = 0$ have $N_R = 1$. For $n = m = r = 0$, they correspond to the massless moduli and their superpartners. For any other choice of $Q = (n, m, r) \neq 0$ with $Q^2 = 0$ one gets massive states.

For a given set of charges $(n, m, r)$ with $Q^2 = -2 (N_R = 0)$ there is only one state or rather multiplet, after including superpartners with non-zero spin,

$$d^{\mathcal{N}=4}_{1/2\text{BPS}}(Q) = 1 \quad \text{for} \quad Q^2 = -2$$

(11)

We thank Sergio Ferrara for enlightening discussions on the orbits of $\mathcal{N} = 4$ states.
For $Q^2 = 0$ ($N_R = 1$) the ‘degeneracy’ is finite: one spin 2 multiplet ($2 \times [24_B - 24_F]$ states including CPT conjugates) and 21 spin 1 multiplets ($2 \times [8_B - 8_F]$ states). We do not expect these states to correspond to smooth classical solutions even after inclusion of higher derivative corrections [30, 31, 37, 38].

For a square torus without Wilson lines and antisymmetric tensor, $G_{ij} = R^2 \delta_{ij}$, $B_{ij} = 0$ and $A_i^a = 0$, the $(8 + 8) \times (8 + 16 + 480)$ massless states correspond to taking the massless ground states for both Left and Right movers

$$Z_{m=0} = (8_v - 8_s)(8_v + 496_{Adj}) = 4032B - 4032F$$

the minus sign accounts for the different statistic of bosons and fermions i.e. $Z$ is rather a Witten index $I_W = \text{tr}(-)^F(q\bar{q})^H$ than a genuine partition function.

1.3 1/2 BPS states with $Q^2 \geq 2$

1/2 BPS states with $Q^2 \geq 2$ are always massive inside the moduli space, since $M_{BPS}^2 = |p_L|^2 = |p_R|^2 + Q^2 M_s^2 \geq 2M_s^2$ with $M_s = \sqrt{2/\alpha'}$. BPS states with $r \neq 0$ are ‘charged’ wrt the ‘visible’ gauge group, already present in $D = 10$. For fixed charges, masses are moduli dependent. Keeping $M_{Pl}$ i.e. $G_N$ fixed

$$M_s = (2\pi)^3 g_s^{(4)} M_{Pl} \quad \text{where} \quad g_s^{(4)} = g_s^{(10)} \sqrt{\frac{2}{\hat{V}_{T6}}}$$

with $\hat{V}_{T6}$ the volume of the six-dimensional internal torus in string length units $\ell_s = \sqrt{\alpha'}/2$. As already mentioned $M_{BPS}$ is extremized at the boundary of moduli space [56, 57] where $M_{BPS} = 0$ since $g_s^{(4)} \rightarrow 0$. Keeping $g_s^{(4)}$ fixed and non-zero, the mass is extremized at points where $p_R = 0$ and $M_{BPS} = (2\pi)^3 g_s^{(4)} M_{Pl} \sqrt{2N_R - 2}$, which can be kept hierarchically smaller than $M_{Pl}$ for extremely small $g_s^{(4)}$.

The large degeneracy of BPS states with fixed charges is related to the exponential growth with $N_R$ of the number of states for the transverse R-moving bosonic oscillators. Neglecting spin, one indeed finds [48]

$$d_{1/2BPS}^{N=4}(N_R) \approx e^{4\pi\sqrt{N_R}}$$

for $N_R >> 1$.

As mentioned in the Introduction, although the ADM mass vanishes at the boundary of moduli space and the mass at the horizon classically vanishes [58, 59, 60], including

\[3\]In $D = 10$, one has $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4(g_s^{(10)})^2$. 

5
higher derivative corrections makes the BH solutions with \( Q^2 > 0 \) become smooth and acquire a non-vanishing area that, for \( Q^2 >> 1 \) reproduces the microscopic entropy

\[
S_{BH} \approx 4\pi \sqrt{\frac{1}{2} Q^2}
\]

resulting from the exponential degeneracy of string states.

\( 1/2 \) BPS multiplets with \( Q^2 \geq 2 \) may include states with higher spin, \( i.e. J > 2 \). For \( J_{\text{hws}} = J + 1 \), so that \( J_{\text{lws}} = J - 1 \) (for \( J \neq 0 \)), these multiplets contain \((2J+1)(8_B - 8_F)\) complex charged states. The multiplicities of states with a given spin coincide with the dimensions of representations of \( Sp(4) \) that rotates the 4 real supercharges acting as raising and as many as lowering operators. The above degeneracy is computed for fixed mass and conserved internal charges but without fixing the spin of the state or multiplet. One can refine the analysis and compute the character valued partition function that yields the degeneracy for fixed spin \( J \). The result crucially depends on whether \( J \approx J_{\text{Max}} = N_R \) (at fixed \( N_R \)) or not [61, 62].

2 Pair Production of Scalars in Field Theory

For later comparison with the heterotic string results and to fix the notation, we now briefly analyze pair production of charged scalars \( \phi \) of mass \( M \) in the collisions of vector bosons and gravitons.

Since any theory at tree level can be viewed as the truncation of a supersymmetric theory, transition amplitudes are formally supersymmetric [63]. In particular, all 4-point amplitudes we consider are MHV (Maximally Helicity Violating) in the formal limit \( M \to 0 \). Moreover, at least at tree level [64, 65, 66] and in some very special case beyond [67, 68], (super)gravity amplitudes can be expressed as squares of (color-ordered) gauge theory amplitudes. In turn, each term in a gauge theory amplitude factorizes into a part which depends on the charges or other gauge quantum numbers and a part which depends on the spins and other kinematical variables. For fixed group theory structure, the amplitude must be gauge invariant.

For instance, in scalar QED the amplitude for Compton scattering or pair production/annihilation reads

\[
\mathcal{A}_{\gamma \phi} = -2q_e^2 (\tilde{a}_2 \tilde{a}_3)
\]

where

\[
\tilde{a}_i = a_i - \frac{p_1 a_i}{p_1 k_i}
\]

are manifestly gauge invariant combinations of the (incoming) photon polarizations \( a_i \).
\(i = 2, 3\). For given helicity \(\lambda\), \(a^{(\lambda)}(k)\) satisfies:

\[
k \cdot a^{(\lambda)}(k) = 0, \quad a^{(\lambda)}(k) \cdot a^{(\lambda')}(k) = \delta_{\lambda, -\lambda'},
\]

where \(k_2\) and \(k_3\) denote the 4-momenta of the photons \((k_i^2 = 0)\) and \(p_1\) and \(p_4\) denote the 4-momenta of the scalars \((p_i^2 = M^2)\).

In (super)gravity the situation seems daunting at first look. The Lagrangian relevant for the process consists of many terms obtained by expanding in the weak field limit around flat Minkowski space-time \(g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa g_{\mu\nu}\), where \(\kappa = \sqrt{8\pi G_N}\), with \(G_N\) Newton’s constant, up to \(o(h^3)\) (linearized gravity). In addition one has to keep the terms involving the interactions between the scalar field \((\phi)\) and the graviton \((h_{\mu\nu})\). Quite remarkably the gravitational scattering amplitude is essentially the square of the scattering amplitude in scalar QED \[39\]. We will find the same result in the string theory approach. This should not come as a surprise in view of the KLT relations between open and closed string amplitudes \[69\].

Either by brute force computation of the relevant Feynmann diagrams or by exploiting KLT-like relations with gauge theory amplitudes, the resulting transition amplitude reads \[39\]

\[
\mathcal{M}_{h\phi} = \frac{\kappa^2}{2g^4} F A^2_{\gamma\phi}
\]

where

\[
F = \frac{(p_1k_2)(p_1k_3)}{k_2k_3}
\]

is an ubiquitous kinematical factor.

Linearized general coordinate invariance allows the decomposition of the graviton spin-2 polarizations \(h^{(2\lambda)}_{\mu\nu}\) into a product of two spin-1 polarizations

\[
h^{(2\lambda)}_{\mu\nu} = a^{(\lambda)}_{\mu} a^{(\lambda)}_{\nu} = h^{(2\lambda)}_{\nu\mu}
\]

so that the polarization tensor \(h^{(2\lambda)}_{\mu\nu}\) is symmetric and satisfies:

\[
k^\mu h^{(2\lambda)}_{\mu\nu} = h^{(2\lambda)}_{\nu\mu}k^\nu = h^\mu_{\mu} = 0,
\]

with the graviton 4-momentum \(k\).

Defining

\[
f = \frac{M^2}{2F} = \frac{M^2}{p_1 k_2} - \frac{M^2}{p_1 k_3}
\]

\[^4\text{For compactness, we sometimes use the notation } p_2 \text{ and } p_3 \text{ for } k_2 \text{ and } k_3, \text{ i.e. in the argument of the } \delta \text{ function of momentum conservation.}\]

\[^5\text{As customary in String Theory, we use mostly plus signature. This entails various sign changes with respect to Field Theory formulae in } e.g. \[39\]. \text{ Moreover } \kappa_{Ref}^{[39]} = 2\kappa_{\text{standard}}.\]
one finds (for Compton scattering)

\[ \mathcal{M}_{\lambda_1,\lambda_2}^{h\phi} = 2\kappa_\gamma^2 F(\delta_{\lambda_1,\lambda_2} + f)^2 = 2\kappa_\gamma^2 F[(1 + f)^2\delta_{\lambda_1,\lambda_2} + f^2\sigma_{\lambda_1,\lambda_2}] \]  

(24)

where \( \delta_{\lambda_1,\lambda_2} \) is helicity preserving and \( \sigma_{\lambda_1,\lambda_2} \) is helicity flipping. Note that for \( M = 0 \) \((f = 0)\) only the helicity preserving amplitude survives. If both gravitons are incoming, as in pair production processes, helicity flipping and preserving amplitudes get exchanged.

In order to compute the cross section, one needs

\[ |\mathcal{M}_{\lambda_1,\lambda_2}^{h\phi}|^2 = 4\kappa_\gamma^4 F^2 \{(1 + f)^4 + f^4\} \delta_{\lambda_1,\lambda_2} + 2f^2(1 + f)^2\sigma_{\lambda_1,\lambda_2} \]  

(25)

Averaging over helicities of the incoming gravitons one gets

\[ \langle |\mathcal{M}_{\lambda_1,\lambda_2}^{h\phi}|^2 \rangle = 2\kappa_\gamma^4 F^2 \{(1 + f)^4 + f^4\} + 2f^2(1 + f)^2 \]  

(26)

In the CM, the kinematics for incoming momenta reads

\[ k_2 = (E, k) \quad k_3 = (E, -k) \quad p_1 = (-E, \bar{p}) \quad p_4 = (-E, -\bar{p}) \]  

(27)

with \( k_2 + k_3 + p_1 + p_4 = 0, |k| = E \) and \( |\bar{p}| = \sqrt{E^2 - M^2} \). As a result

\[ s = -(k_2 + k_3)^2 = 4E^2 \quad t = -(k_2 + p_1)^2 = M^2 - 2E^2 - 2k \cdot \bar{p} \quad u = -(k_3 + p_1)^2 = M^2 - 2E^2 + 2k \cdot \bar{p} \]  

(28)

Moreover, indicating by \( \theta \) the scattering angle so that \( k \cdot \bar{p} = |k||\bar{p}| \cos \theta \), one gets

\[ F = \frac{(p_1k_2)(p_1k_3)}{k_2k_3} = -\frac{1}{2}[E^2 \sin^2 \theta + M^2 \cos^2 \theta] \quad , \quad f = \frac{M^2}{2F} = -\frac{M^2}{E^2 \sin^2 \theta + M^2 \cos^2 \theta} \]  

(29)

Setting \( \eta \equiv E/M \geq 1 \) (threshold for the process) one eventually finds

\[ \frac{d\sigma}{d\Omega} = \frac{\kappa_\gamma^4 M^2}{32(4\pi)^2 \eta^3 |\eta^2 \sin^2 \theta + \cos^2 \theta|^2} \{2 - 4|\eta^2 \sin^2 \theta + \cos^2 \theta| \} \]

\[ + 4|\eta^2 \sin^2 \theta + \cos^2 \theta|^2 - 2|\eta^2 \sin^2 \theta + \cos^2 \theta|^3 + \frac{1}{2}[|\eta^2 \sin^2 \theta + \cos^2 \theta|^4] \} \]  

(30)

Integrating over the solid angle yields the total cross section

\[ \sigma = \frac{\pi M^2 \sqrt{\eta^2 - 1}}{2 M^4 \pi^2 \eta^3} \left[ \frac{1}{\eta^2} + \frac{1 - 4\eta^2}{2\eta^3 \sqrt{\eta^2 - 1}} \log \left( \frac{\eta + \sqrt{\eta^2 - 1}}{\eta - \sqrt{\eta^2 - 1}} \right) + \frac{4\eta^4}{15} - \frac{6\eta^2}{5} + \frac{103}{30} \right] \]  

(31)

that displays the characteristic growth with the square of the energy at large \( E \), justifiable on purely dimensional grounds \[2, 3\]. If one were to interpret the charged massive scalars as small BPS BH’s of opposite charge, even a very crude estimate for the cross-section of pair production in (super)gravitons collisions would require the inclusion of degeneracy.
factors $d_{BH}(Q) \approx \exp S_{BH}(Q)$ and possibly of form factors. At CM energies of the order of some TeV’s the above process has a vanishingly small cross-section, unless the fundamental scale of gravity be much lower than $M_{Pl}^{(4)}$ \cite{70, 71}. Yet similar processes are expected to take place even at LHC after replacing the gravitons with gluons or quarks and the massive complex scalars with stable not necessarily BPS BH’s charged wrt the Standard Model as can appear in superstring (flux) compactification.

In the following we will address the problem in the largely simplified, yet tractable, context of heterotic compactifications on tori and simple orbifolds.

3 Pair Production Amplitudes for Heterotic Strings

We have previously seen that string states that correspond to small BH’s with two charges can be pair produced in graviton or gauge boson collisions at very high energies. Let us proceed and compute the tree-level amplitude for these processes. For simplicity we will mostly focus on the subspace of moduli space with zero Wilson lines, where a distinction between a ‘visible’ and a ‘hidden’ gauge groups is possible. In our conventions, the former corresponds the non abelian gauge bosons already present in $D = 10$. The latter corresponds to the mixed components of the metric and anti-symmetric tensor with generically abelian symmetry. At a generic point in the moduli space of toroidal compactifications such a distinction makes little or no sense, since the various vectors can mix with one another. It becomes meaningful again in phenomenologically more interesting cases with lower or no supersymmetry.

The amplitudes for charged scalar BH pair production in vector boson or graviton collisions are given by

$$A_{vv\rightarrow \Phi \bar{\Phi}} = \langle V_v V_v V_v V_{\Phi} \rangle$$

(32)

and

$$M_{hh\rightarrow \Phi \bar{\Phi}} = \langle V_h V_h V_h V_{\Phi} \rangle$$

(33)

where $V_v$, $V_h$ and $V_{\Phi}$ are vertex operators for vector bosons, gravitons and small BH’s.

3.1 Vertex operators

Up to normalization factors, to be discussed momentarily, in the canonical superghost picture, the gauge boson vertex operator is \cite{48}

$$V_v = a_\mu e^{-\phi} \psi_\mu J^a e^{ik \cdot X}$$

(34)
with \( k^2 = k \cdot a = 0 \), the graviton vertex operator is

\[
V_h = h_{\mu\nu} e^{-\varphi} \psi^\mu \bar{\partial} X^\nu e^{ik \cdot X} \tag{35}
\]

with \( h_{\mu\nu} = h_{\nu\mu} \) and \( k^2 = k^\mu h_{\mu\nu} = h_{\mu\nu} = 0 \), and the two-charge massive scalar vertex is

\[
V_\Phi = \Phi_i^{(N_R)} e^{-\varphi} \psi^i \bar{\partial} X^a e^{ip X} \tag{36}
\]

where \( p^2 = -M^2 = -|p_L|^2 \) and

\[
\Phi_i^{(N_R)} = \Phi_{i,j_1...j_n} \bar{\partial}^j_1 X_R^{j_1} ... \bar{\partial}^j_n X_R^{j_n} \tag{37}
\]

is a polynomial of degree \( N_R = \sum_{\ell} \ell \) in the derivatives of the R-moving bosonic coordinates. As already said, \( N_R \) is determined by level matching to be \( N_R = 1 + nm - \frac{1}{2} |r|^2 = 1 + \frac{1}{2} Q^2 \). Similar arguments apply to the super-partners of the scalar BH’s under consideration. For simplicity we will mostly focus on spherically symmetric small BH’s, whereby \( j_r \) label internal coordinates \( X^i_R \) with \( i = 1, ..., 6 \) or R-moving (non-abelian) currents \( \bar{J}^a \), with \( a = 1, ..., \text{dim} G \). BPS states or rather multiplets with higher spins require inclusion of \( \bar{\partial} X^a_R \) and will be briefly considered at a later stage together with non-BPS and non-extremal BH’s.

Due to super-ghost number violation at tree level, one needs the vertex operators for vector bosons and gravitons with super-ghost number \( q = 0 \), that read

\[
V_v^{(0)} = a_\mu (\partial X^\mu + i (k \cdot \bar{\psi} \psi^\mu)) J^a e^{ik \cdot X} \tag{38}
\]

and

\[
V_h^{(0)} = h_{\mu\nu} (\partial X^\mu + i (k \cdot \bar{\psi} \psi^\mu)) \bar{\partial} X^\nu e^{ik \cdot X} \tag{39}
\]

In order to get the right dependence on \( g_{YM} \) and \( G_N \) in the formal field theory limit \( \alpha' \to 0 \) one has to dress the vertex operators with the normalization factors

\[
N_v = N_\Phi = g_{YM} \sqrt{\frac{2}{\alpha'}} \quad N_h = \frac{4\sqrt{\pi \kappa}}{\alpha'M_{Pl}} \tag{40}
\]

where \( \kappa \) is the level of the current algebra, \( M_{Pl} = (2\pi)^{-3} (g_s^{(4)})^{-1} \sqrt{2/\alpha'} \) and

\[
g_{YM} = \frac{g_s^{(4)}}{\sqrt{\kappa}} = g_s^{(10)} \sqrt{\frac{2}{\kappa\delta_TV_T}} \tag{41}
\]

and include a factor

\[
N_{\text{sphere}} = \frac{(2\pi)^{4\delta} (\sum_i p_i) \delta_TV_T}{(g_s^{(10)})^2 (\alpha'/2)^2} \tag{42}
\]

for the sphere.
3.2 Correlation functions

Apart from the space-time bosonic zero-modes that implement momentum conservation and the internal bosonic zero-modes that produce a factor $V_T$, all world-sheet correlation factorize into Left- and Right- movers:

$$W(z_i, \bar{z}_i) = W_L(z_i)W_R(\bar{z}_i)$$

Neglecting the bosonic ghosts and setting $P_{L,i} = (p_i, \pm p_L)$, for the L-movers one has

$$W_L(z_i) = \langle e^{-\phi} \psi^i e^{ip_L i X_L} a_2 (\partial X - i \psi k_2 \psi) e^{ik_2 X_L} a_3 (\partial X - i \psi k_3 \psi) e^{ik_3 X_L} e^{-\phi} \psi^j e^{ip_L j X_L} \rangle$$

that further factorizes as $W_L(z_i) = W_L^{int}(z_i)W_L^{s-t}(z_i)$ into an internal part

$$W_L^{int}(z_i) = \langle e^{-\phi} \psi^i e^{ip_L i X_L}(z_1) e^{-\phi} \psi^j e^{-ip_L j X_L}(z_4) \rangle = \delta^{ij} z_{14}^{-1} |p_L|^2$$

with manifest $SO(6)$ R-symmetry and a space-time part

$$W_L^{s-t}(z_i) = \langle e^{ip_X} L(z_1) a_2 (\partial X - i \psi k_2 \psi) e^{ik_2 X_L}(z_2) a_3 (\partial X - i \psi k_3 \psi) e^{ik_3 X_L}(z_3) e^{ip_X}(z_4) \rangle$$

Setting $W_L^{s-t}(z_i) = B_L^{s-t}(z_i) + C_L^{s-t}(z_i)$ one has

$$B_L^{s-t}(z_i) = \langle e^{ip_X} L(z_1) a_2 \cdot \partial X e^{ik_2 X_L}(z_2) a_3 \cdot \partial X e^{ik_3 X_L}(z_3) e^{ip_X}(z_4) \rangle$$

$$= - \left( \frac{a_2 a_3}{z_{23}^2} + \sum_{r \neq 2} \frac{a_2 p_r}{z_{2r}^2} \sum_{s \neq 3} \frac{a_3 p_s}{z_{3s}^2} \right) I_L(p_i, z_i)$$

$$C_L^{s-t}(z_i) = \langle e^{ip_X} L(z_1) i k_2 \cdot \psi a_2 \cdot \psi e^{ik_2 X_L}(z_2) i k_3 \cdot \psi a_3 \cdot \psi e^{ik_3 X_L}(z_3) e^{ip_X}(z_4) \rangle$$

$$= \frac{1}{2} \frac{1}{z_{23}^2} \left[ (a_2 a_3 (p_2 p_3) - (a_2 p_3) (p_2 a_3) \right] I_L(p_i, z_i)$$

where $I_L(p_i, z_i) = \prod_{i < j} z_{ij}^{p_i p_j}$ is the L-mover Koba-Nielsen factor.

For the R-movers one has to compute correlation functions of the form

$$W_R(\bar{z}_i) = \langle \Phi_{R}^{(1)} e^{ip_X(\bar{z}_1)} e^{iK_2 X_R(\bar{z}_2)} e^{iK_3 X_R(\bar{z}_3)} \Phi_{R}^{(2)} e^{ip_X(\bar{z}_4)} \rangle$$

where $\bar{p}_i = (p_i, \pm p_R)$ satisfy $|p_R|^2 = p_i^2 + |p_R|^2 = -M^2 + |p_L|^2 - 2(N_R - 1)$, while $K_i = (k_i, 0)$ satisfy $k_i^2 = k_i^2 = 0$. Depending on the incoming particles the indices $M, N$ run over space-time $(\mu, \ldots)$, ‘hidden’ $(i, \ldots)$ or ‘visible’ $(a, \ldots)$ gauge group respectively.

Representing $\Phi_{R} = \Phi_{R_1 \ldots R_n} \bar{\beta}^{I_1} X_R^{I_1} \ldots \bar{\beta}^{I_n} X_R^{I_n}$ with $N_R = \sum_i \ell_i$ in exponential form

$$\bar{\beta}^{I_1} X_R^{I_1} \ldots \bar{\beta}^{I_n} X_R^{I_n} = \left[ \frac{\partial}{\partial \beta^{(\ell_1)}_{I_1}} \ldots \frac{\partial}{\partial \beta^{(\ell_n)}_{I_n}} \exp \sum_k \beta^{(t)}_{I_1} \bar{\beta}^{I_1} X_R^{I_1} \right]_{\beta^{(t)}_{I_1} = 0}$$

\[\text{Overall numerical and } g_s \text{ dependent factors will be reinstated at the end.}\]
one gets

\[ \mathcal{W}_R(z_i) = \bar{z}_{14}^{\text{|p}_R|^2} \mathcal{I}_R(p_i, \bar{z}_i) \left[ \Phi_{(1)}^{(1)} \left( \frac{\partial}{\partial \beta_{(1)}} \right)^{\nu} \cdots \frac{\partial}{\partial b^M} \frac{\partial}{\partial b^N} \Phi_{(4)}^{(4)} \left( \frac{\partial}{\partial \beta_{(4)}} \right)^{\nu} \right]_{N_R} \]

\[ \mathcal{U}_R(z_i) \exp \left( \sum_{n,m} (-)^n (n + m - 1)! \frac{\beta_{n}^{(1)} \cdot \beta_{m}^{(4)}}{z_{14}^{n+m}} \right) \]

where, similarly to \( \mathcal{I}_L(p_i, z_i) \),

\[ \mathcal{I}_R(p_i, \bar{z}_i) = \prod_{i<j} z_{ij}^{p_{ij}} \]

is the R-mover Koba-Nielsen factor and

\[ \mathcal{U}_R(z_i) = \left( \frac{b_2 b_3}{z_{23}} + \left( \sum_n \frac{(-)^n n! b_2 \cdot \beta_{n}^{(1)}}{z_{14}^{n+1}} + \sum_m \frac{m! b_2 \cdot \beta_{m}^{(4)}}{z_{24}^{m+1}} + i \sum_{i \neq 2} \frac{b_2 \cdot \text{p}_{R,i}}{z_{2i}} \right) \right) \times \left( \sum_n \frac{(-)^n n! b_3 \cdot \beta_{n}^{(1)}}{z_{13}^{n+1}} + \sum_m \frac{m! b_3 \cdot \beta_{m}^{(4)}}{z_{34}^{m+1}} + i \sum_{j \neq 3} \frac{b_3 \cdot \text{p}_{R,j}}{z_{3j}} \right) \]

Factoring out \( \bar{z}_{23} \bar{z}_{14}^{-2N_R} \), one eventually finds

\[ \mathcal{W}_R(z_i) = \langle \Phi_1 | \Phi_4 \rangle (\text{p}_R) \mathcal{U}_R(p_R, b_2, b_3; \bar{z}) \bar{z}_{23}^{-2} \bar{z}_{14}^{-2N_R - 1} - |\text{p}_R|^2 \prod_{i<j} z_{ij}^{p_{ij}} \]

that nicely fit with the \( \bar{z}_{14}^{-\text{|p}_L|^2} \mathcal{I}_L(z_i, p_i) \) L-moving factor, since \( |\text{p}_L|^2 = 2(N_R - 1) + |\text{p}_R|^2 \). The function \( \mathcal{U}_R(p_R, b_2, b_3; \bar{z}) \) of the cross ratio \( \bar{z} = \bar{z}_{12} \bar{z}_{34}/\bar{z}_{13} \bar{z}_{24} \) depends on the choice of colliding particles, while \( \langle \Phi_1 | \Phi_4 \rangle (\text{p}_R) \) denote the ‘overlap’ of the pair of small BH states.

### 3.3 Integration and amplitudes

Including the bosonic ghost correlator

\[ |\langle c(z_1) c(z_2) c(z_4) \rangle|^2 = |z_{12}|^2 |z_{14}|^2 |z_{24}|^2 \]

and setting \( z_1 \to \infty, z_2 \to 1, z_3 = z \) and \( z_4 \to 0 \), one eventually gets amplitudes of the form

\[ \mathcal{A}(p_i) = g_{YM}^2 (2\pi)^4 \delta(\Sigma_i p_i) \delta^{ij} \int d^2z \ |z|^{2k_1 p_1} |1 - z|^{2k_2 k_3 - 4} \mathcal{E}_L(z) \mathcal{E}_R(\bar{z}) \]

where

\[ \mathcal{E}_L = (a_2 a_3)(k_2 k_3 - 1) - (a_2 k_3)(p_4 a_3) \frac{1 - z}{z} + (a_2 p_4)(k_2 a_3)(1 - z) - (a_2 p_4)(p_4 a_3) \frac{(1 - z)^2}{z} \]
and $\mathcal{E}_R$ depends on the incoming particles. All the necessary integrals are of the form [72, 73]

\[
I(a, n, b, m) = \int d^2z|z|^a|1 - z|^b z^n(1 - z)^m
\]

\[
= \frac{\Gamma(-1 - (a + b)/2)\Gamma(1 + n + a/2)\Gamma(1 + m + b/2)}{\Gamma(-a/2)\Gamma(-b/2)\Gamma(2 + n + m + (a + b)/2)}
\]

(58)

and lead to the Shapiro-Virasoro-like form factor

\[
\mathcal{F}_{SV} = \frac{\Gamma(1 + k_2k_3)\Gamma(k_2p_1)\Gamma(k_2p_4)}{\Gamma(2 - k_2k_3)\Gamma(-k_2p_1)\Gamma(-k_2p_4)}
\]

(59)

up to rational expressions in the kinematical variables.

In the following, for illustrative purposes, we will specialize to the case $N_R = 2$. Generalization to higher level is tedious but straightforward using the above procedure for the scalar product of ‘internal’ R-moving states.

### 3.4 2 Vectors - 2 small BH amplitude: mutually neutral case

We first consider the case in which the two charged small BH’s are neutral wrt to the incoming gauge bosons i.e. they are charged wrt a ‘hidden’ gauge group not the ‘visible’ one. The tree-level amplitude for this process reads

\[
\mathcal{A}_{\nu\nu\rightarrow\Phi\Phi}(p_i) = g_s^2\int d^2z_3 \langle c\bar{c}V^{ij}_{\Phi}(-1)(z_1, z_1)c\bar{c}V^{a(0)}_\nu(z_2, z_2)V^{b(0)}_\nu(z_3, z_3)c\bar{c}V^{kl(-1)}_{\Phi}(z_4, z_4)\rangle
\]

(60)

where

\[
V^{ij}_{\Phi}(-1) = \psi^i e^{-\varphi} e^{ip_{\nu}X_{\nu}} \bar{\partial}^2 X^{j}_R e^{ip_{R}X_{R}} e^{ip_{-}X}
\]

(61)

describes a small BPS BH with mass $M^2 = |p_{\nu}|^2 = |p_R|^2 + 2 (N_R = 2)$. $\mathcal{A}_{\nu\nu\rightarrow\Phi\Phi}$ can be decomposed as

\[
\mathcal{A}_{\nu\nu\rightarrow\Phi\Phi}(p_i) = g_s^2\int d^2z_3|z_{12}|^2|z_{14}|^2|z_{24}|^2\langle e^{-\varphi(z_1)} e^{-\varphi(z_4)} \psi^i(z_1) \psi^k(z_4) \rangle
\]

(62)

The R-moving contribution $\mathcal{E}_R$ consists in the internal boson correlator

\[
\langle \bar{\partial}^2 X^{j}_R e^{ip_{R}X_{R}}(z_1) \bar{\partial}^2 X^{k}_R e^{ip_{R}X_{R}}(z_4) \rangle = \left( 6 \frac{\delta^{jl}}{z_{14}^2} - \frac{p^l_{R}p^j_{R}}{z_{14}^4} \right) z_{14}^{-|p_R|^2}
\]

(63)
and in the current correlator
\[ \langle \overline{J}^a(z_2) J^b(z_3) \rangle = \frac{\delta^{ab}}{z_{23}^{2}} \]  

(64)

Reinstating normalization factors, one finally gets
\[ \mathcal{A}^{i,a,b,kl}_{vv \rightarrow \Phi \Phi}(p_i) = \frac{g_{YM}^2}{M_s^2} (2\pi)^4 \delta(\sum p_i) (\tilde{a}_2 \tilde{a}_3) \mathcal{F}_{SV} F \delta^{ab} (6 \delta^{ji} - p^i_R p^j_R) \]  

(65)

where the kinematical factor \( F \) and the Shapiro-Virasoro-like form factor \( \mathcal{F}_{SV} \) are defined in (20) and (59) while \( \tilde{a}_{i\mu} \) are manifestly gauge invariant polarizations defined in Eq (17).

It is worth noticing that in the ‘formal’ field theory limit \( \alpha' \rightarrow 0 \) (\( M_s \rightarrow \infty \)) with fixed \( M \), \( \mathcal{F}_{SV} \rightarrow 1 \) and \( \mathcal{A}^{i,a,b,kl}_{vv \rightarrow \Phi \Phi}(p_i) \) reproduces the supergravity result. To lowest order the process is indeed mediated by graviton exchange that is suppressed by a factor of \( g_{YM}^2/M_s^2 \sim 1/M_P^2 \).

For heterotic strings, one can replace \( \partial^2 X^k \) in one or both vertex operators with \( \partial X^i \partial X^j \). These states mix with each other and contribute to the degeneracy of (very) small BH’s with \( N_R = 2 \).

### 3.5 2 Vectors - 2 small BH amplitude: mutually charged case

We now consider the more interesting but slightly more involved case in which the two small BH’s are charged wrt the ‘visible’ gauge group of the two incoming vector bosons. The tree-level amplitude for this process reads
\[ \mathcal{A}^{ika,b,c,jld}_{vv \rightarrow \Phi \Phi}(p_i) = g_s^2 \int d^2 z_3 \langle c\bar{c} V_{\Phi}^{ika}(-1) (z_1, \bar{z}_1) c\bar{c} V_v^{b}(0) (z_2, \bar{z}_2) V_v^{c}(0) (z_3, \bar{z}_3) c\bar{c} V_{\Phi}^{jld}(-1) (z_4, \bar{z}_4) \rangle \]  

(66)

where
\[ V_{\Phi}^{ika(-1)} = \psi^i e^{-\varphi} e^{ip_L X_L} \partial X^k_{R} \bar{J}^{a} e^{ip_R X_R} e^{ip_X} \]  

(67)

In the R-moving sector, in addition to the elementary two-point function
\[ \langle \partial X^k_R \bar{\partial} X^l_R \rangle = -\frac{\delta^{kl}}{z_{14}^2} \]  

(68)

one needs the correlator of four currents given by
\[ \langle \bar{J}^{a_1}(\bar{z}_1) J^{a_2}(\bar{z}_2) \bar{J}^{a_3}(\bar{z}_3) J^{a_4}(\bar{z}_4) \rangle = \left[ \begin{array}{c} T_{12} T_{34} \bar{z}_{12} \bar{z}_{24} + T_{13} T_{24} \bar{z}_{13} \bar{z}_{24} + T_{14} T_{23} \bar{z}_{14} \bar{z}_{23} \\ \bar{z}_{12} \bar{z}_{24} \bar{z}_{13} \bar{z}_{24} \bar{z}_{14} \bar{z}_{23} \end{array} \right] \]  

(69)
where $T_{ij} = \text{Tr}(T_a T_{aj})$ and $T_{[ij][kl]} = \text{Tr}([T_a, T_{aj}][T_{bk}, T_{al}])$.

This leads to

$$
\mathcal{E}_R(\bar{z}) = T_{14} T_{23} + T_{12} T_{34} \left( 1 - \frac{2}{\bar{z}} + \frac{1}{\bar{z}^2} \right) + T_{13} T_{24} \left( 1 - 2 \bar{z} + \bar{z}^2 \right)
+ 2 T_{[12][34]} \left( 1 - \frac{1}{\bar{z}} \right) + 2 T_{[13][42]} (1 - \bar{z})
$$

that combined with $\mathcal{E}_L(z)$ and integrated yields

$$
\mathcal{A}^{\alpha a, bc, j ld}_{\nu v \to \Phi \Phi} \left( \nu_i \right) = g_s^2 (2\pi)^4 \delta(\Sigma_i p_i) \delta^{ij} \left( -\delta^{kl} - p^k_R p^l_R \right) J
$$

where

$$
J = A_0 \left( T_{12} T_{34} + T_{13} T_{24} + T_{14} T_{23} + 2 T_{[13][24]} + 2 T_{[12][34]} \right) + A_2 T_{13} T_{24}
- 2 A_1 \left( T_{13} T_{24} + T_{[13][24]} \right) - 2 A_{-1} \left( T_{12} T_{34} + T_{[12][34]} \right) + A_{-2} T_{12} T_{34}
$$

with

$$
A_n = \int d^2 z \left| z \right|^{2 k_3 p_4} \left| 1 - z \right|^{2 k_2 k_3 - 4} \bar{z}^n \mathcal{E}_L(z) = (\bar{a}_2 \bar{a}_3) \frac{\Gamma(1 + 2 k_3 k_4)}{\Gamma(2 - 2 k_3 k_4)} J_n
$$

where

$$
J_0 = - \frac{\Gamma(k_3 p_4) \Gamma(k_3 p_1)}{\Gamma(-k_3 p_4) \Gamma(-k_3 p_1)}
$$

$$
J_1 = \frac{1 + k_3 p_4}{k_3 p_1} J_0,
J_{-1} = - \frac{1 + k_3 p_1}{k_3 p_4} J_0
$$

$$
J_2 = \frac{(2 + k_3 p_4)(1 + k_3 p_4)}{(k_3 p_1 - 1) k_3 p_1} J_0,
J_{-2} = \frac{(2 + k_3 p_1)(1 + k_3 p_1)}{(k_3 p_4 - 1) k_3 p_4} J_0
$$

Reinstating normalization factors, one eventually gets

$$
\mathcal{A}^{\alpha a, bc, j ld}_{\nu v \to \Phi \Phi} \left( \nu_i \right) = \frac{g_s^2}{M_s^2} (2\pi)^4 \delta(\Sigma_i p_i) \delta^{ij} \left( \delta^{kl} + p^k_R p^l_R \right) (\bar{a}_2 \bar{a}_3) F_{n' SV} \mathcal{I}
$$

where

$$
\mathcal{I} = T_{12} T_{34} + T_{13} T_{24} + T_{14} T_{23} + 2 T_{[13][24]} + 2 T_{[12][34]}
+ \frac{2(2 + \alpha' k_3 p_4)}{\alpha' k_3 p_1} [T_{13} T_{24} + T_{[13][24]}] + \frac{2(2 + \alpha' k_3 p_1)}{\alpha' k_3 p_4} [T_{12} T_{34} + T_{[12][34]}]
+ \frac{(4 + \alpha' k_3 p_1)(2 + \alpha' k_3 p_4)}{\alpha' k_3 p_4(2 - \alpha' k_3 p_4)} T_{12} T_{34} + \frac{(4 + \alpha' k_3 p_4)(2 + \alpha' k_3 p_4)}{\alpha' k_3 p_1(2 - \alpha' k_3 p_1)} T_{13} T_{24}
$$

In the ‘formal’ field theory limit $\alpha' \to 0 \left( M_s \to \infty \right)$ with fixed $M$, $F_{n' SV} \to 1$, only terms with ‘poles’ in $\mathcal{I}$ survive and the amplitude reproduces the SYM theory result. Similarly to the previous case (BPS BH’s charged wrt to a ‘hidden’ sector), one can replace $\partial X^k J^a$ with $\partial J^a$ in one or both BPS vertex operators. This would lead to mixing and eventually account for the degeneracy of the small BH’s.
3.6 2 Gravitons - 2 small BH’s amplitude

Finally we consider small BH pair production in high energy graviton collisions. This process is practically impossible at LHC or near future accelerators but it may prove dominant in the very early universe or in models with low-scale gravity. The tree-level amplitude for the process reads

$$\mathcal{M}_{hh\to \Phi \Phi}^{ij,kl}(p_i) = g_s^2 \int d^2 z_3 \langle c \mathcal{E} V_{ij}^{(-1)}(z_1, \bar{z}_1) c \mathcal{E} V_{kl}^{(0)}(z_2, \bar{z}_2) V_{h}^{(0)}(z_3, \bar{z}_3) c \mathcal{E} V_{\Phi}^{kl(-1)}(z_4, \bar{z}_4) \rangle$$

(79)

where $V_{\Phi}^{ij(-1)}$ has been defined in Eq. $(61)$. For calculation purposes, it is convenient to ‘factorize’ graviton polarization tensors as $h_{\mu
u}^{(2)} = a_{\mu}^{(\lambda)} a_{\nu}^{(\lambda)}$, that satisfy $k^\mu h_{\mu\nu} = h_{\mu\nu} k^\nu = 0$ and $h_{\mu\mu} = 0$. The amplitude can be decomposed as

$$\mathcal{M}_{hh\to \Phi \Phi}^{ij,kl}(p_i) = g_s^2 \int d^2 z_3 \langle c \mathcal{E} V_{ij}^{(-1)}(z_1, \bar{z}_1) c \mathcal{E} V_{kl}^{(0)}(z_2, \bar{z}_2) V_{h}^{(0)}(z_3, \bar{z}_3) c \mathcal{E} V_{\Phi}^{kl(-1)}(z_4, \bar{z}_4) \rangle$$

(80)

The R-moving contribution requires

$$\mathcal{B}_{R}^{s-t} = \langle e^{ip_1 \cdot X(z_1)} a_2^e \partial X_{\mu} e^{ik_2 \cdot X(z_2)} a_3^\sigma \partial X_{\sigma} e^{ik_3 \cdot X(z_3)} e^{ip_4 \cdot X(z_4)} \rangle$$

(81)

Eventually the result is simply given by $\mathcal{B}_{R}^{s-t}(z) = \mathcal{B}_{L}^{s-t}(z)_{z\to \bar{z}}$ previously computed in Eq. $(47)$.

Combining L- and R-moving parts one has

$$\mathcal{M}_{hh\to \Phi \Phi}^{ij,kl}(p_i) = g_s^2 (2\pi)^4 \delta(\sum_i p_i) \delta^{ijkl} (6\delta^{ij} - p_{R}^i p_{R}^j) \mathcal{W}$$

(82)

where $\mathcal{W}$ can be expressed in terms of the integrals $I(a, b, m)$ in $(55)$. Setting $I(a, 0, b, 0) = I_0$, using the factorial properties of $\Gamma$ function and the compact notation

$$hh = h_{\mu\nu} h^{\mu\nu} , \quad \phi \mu = p^\mu h_{\mu\nu} p^\nu , \quad \phi \mu \phi^\mu = p^\mu h_{\mu\nu} h^{\nu\sigma} p_\sigma$$

(83)

one finds
\[
\frac{\mathcal{W}}{I_0(1 - k_2k_3)} = h_2h_3 - k_3h_2h_3k_2 - \frac{(2 - k_2k_3)(k_3h_2h_3p_4) - (k_3h_2k_3)(p_4h_3k_2)}{(k_3p_4)} - \frac{(2 - k_2k_3)(p_4h_2h_3k_2) - (k_3h_2p_4)(k_2h_3k_2)}{(k_2p_4)} + \frac{(k_2k_3)((2 - k_2k_3)(p_4h_2h_3p_4) - (p_4h_2k_3)(k_2h_3p_4))}{(k_3p_4)(k_3p_1)} + \frac{(1 - k_2k_3)(k_3h_2k_3)(p_4h_3p_4)}{(k_3p_4)^2} + \frac{2(1 - k_2k_3)(k_3h_2p_4)(p_4h_3k_2)}{(k_3p_4)(k_2p_4)} - \frac{2(1 - k_2k_3)(k_2h_3k_4)(p_4h_2p_4)}{(k_2p_4)^2(k_3p_4)} + \frac{2(1 - k_2k_3)(k_2k_3)(p_4h_3p_4)(k_3h_2p_4)}{(k_3p_4)^2(k_2p_4)} - \frac{2(1 - k_2k_3)(k_3k_3)(p_4h_3p_4)(k_3h_2p_4)}{(k_3p_4)^2(k_2p_4)}
\]

(84)

The integral \( I_0 \) is given by

\[
I_0 = \frac{\mathcal{F}_{SV} F}{k_2k_3 - 1}
\]

(85)

where \( F \) and \( \mathcal{F}_{SV} \) are the by now familiar kinematical factor and S-V form factor. One eventually gets

\[
\mathcal{M}_\phi^{ij;kl}(p_i) = \frac{16\pi}{M_{pl}^4} \left(2\pi\right)^4 \delta(\Sigma_i p_i) \mathcal{F}_{SV} F \delta^{ik} \left(6\delta^{jl} - p_k^j p_l^j\right) \left[(\tilde{h}_2\tilde{h}_3) + \mathcal{H}\right]
\]

(86)

where

\[
\tilde{h}_{ij\mu} = \left(\delta_{\mu}^\rho - \frac{k_{i\rho} p_{k_\mu}}{p_4 k_\rho}\right) \left(\delta_{\nu}^\sigma - \frac{k_{\nu\sigma} p_{k_\nu}}{p_4 k_\rho}\right) h_{\rho\sigma}
\]

(87)

is a manifestly gauge invariant quantity and

\[
\mathcal{H} = \frac{\alpha'}{2} \left\{ -(k_2k_3)(\tilde{h}_2\tilde{h}_3) + (k_2k_3)(h_2h_3) - (k_3h_2h_3k_2) \right. \\
\left. + \frac{(k_2k_3)(p_1h_2h_3p_4) - (p_1h_3k_2)(p_1h_3k_2)}{k_3p_4} + \frac{(k_2k_3)(p_1h_2h_3p_4) - (p_1h_2k_3)(p_1h_3k_2)}{k_2p_4} \right\}
\]

(88)

represent higher-derivative \( \alpha' \) corrections. In the ‘formal’ field theory limit \( \alpha' \to 0 \), \( \mathcal{F}_{SV} \to 1 \), the surviving gravitational amplitude \( \tilde{h}_2\tilde{h}_3 \) is essentially the square of the gauge theory amplitude \( \tilde{a}_2\tilde{a}_3 \).

As in the first case considered, one can replace \( \partial^2 X^k \) in one or both BPS scalar vertex operators with \( \partial X^i \partial X^j \). These states mix with each other and contribute to the degeneracy of the resulting ‘small BH’. 

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4 Cross section, Angular and Energy distribution

We are now ready to compute the cross section for pair production of small BH’s in graviton or gauge boson scattering.

With respect to field theory amplitudes for pair production of massive (charged) scalars, heterotic string amplitudes contain higher derivative correction and are dressed with Shapiro-Virasoro-like form factors defined in (59), that contain further higher-derivative corrections. Moreover, summming over final BPS states with the same charges and mass but different (unresolved) R-moving string oscillator modes enhances the result by the micro-state degeneracy factor \( d(N_R) \). For large \( N_R \), \( \log d(N_R) \approx 4\pi\sqrt{N_R} \) reproduces the ‘macro-scopic’ Wald entropy of the small BH’s.

It is easy to check that gravity mediated amplitudes, including the one with product BH’s neutral wrt the incoming vector bosons, are largely suppressed wrt the amplitudes with products BH’s charged wrt to the incoming gauge bosons. In more realistic scenari small BH’s that couple minimally to the ‘visible’ gauge group have a chance to be produced even at LHC [9].

To proceed further, let us recall that in the CM frame Mandelstam variables assume values

\[
\begin{align*}
  s &= -(k_2 + k_3)^2 = -2(k_2 k_3) = 4E^2 \\
  t &= -(k_2 + p_1)^2 = M^2 - 2(k_2 p_1) = M^2 - 2E^2(1 + \sqrt{1 - \mu^2 \cos \theta}) \\
  u &= -(k_2 + p_4)^2 = M^2 - 2(k_2 p_4) = M^2 - 2E^2(1 - \sqrt{1 - \mu^2 \cos \theta})
\end{align*}
\]

where \( \mu = M/E = 1/\eta \). Exploiting the notation \( \hat{w} = \alpha'w/4 \), the S-V form factor (59) reads

\[
F_{SV} = \frac{\Gamma(1 - \hat{s})\Gamma(\hat{M}^2 - \hat{t})\Gamma(\hat{M}^2 - \hat{u})}{\Gamma(2 + \hat{s})\Gamma(\hat{t} - \hat{M}^2)\Gamma(\hat{u} - \hat{M}^2)}
\] (92)

By using the factorial property of \( \Gamma \) function and \( \Gamma(z)\Gamma(1 - z) = \pi/\sin(\pi z) \), one gets

\[
F_{SV} = \frac{\hat{s}}{(1 + \hat{s})} \frac{\sin(\pi \hat{s})}{\pi} B(-\hat{s}, \hat{M}^2 - \hat{t})B(-\hat{s}, \hat{M}^2 - \hat{u})
\] (93)

Then, using the Mittag-Leffler expansion of \( B(u, v) \)

\[
B(u, v) = \sum_{n=0}^{\infty} \frac{\mathcal{R}_n(u)}{v + n}
\] (94)

where \( \mathcal{R}_n(u) = (-1)^n (u - 1) \ldots (u - n)/n! \), one obtains

\[
F_{SV} = \frac{\hat{s}}{(1 + \hat{s})} \frac{\sin(\pi \hat{s})}{\pi} \sum_{n=0}^{\infty} \frac{\mathcal{R}_n(\hat{s})}{(a_n + bx)} \sum_{k=0}^{\infty} \frac{\mathcal{R}_k(\hat{s})}{(a_k - bx)}
\] (95)
where $a_n = n + \hat{s}/2$, $b = (\hat{s}/2)\sqrt{1 - \mu^2}$ and $x = \cos \theta$.

4.1 Cross section for small BH’s in the ‘visible’ sector

Henceforth we will focus on pair production of small BH’s charged wrt to the ‘visible’ gauge group, whose transition amplitude is given by (77) \(\text{viz.}\)

\[
|A|^2 = \frac{g_Y^4}{M_s^4} |F_{SV}|^2 F^2 \mathcal{T}^2 (\tilde{a}_2 \tilde{a}_3)^2
\]  

(96)

where

\[
\mathcal{T} = T_{12} T_{34} + T_{13} T_{24} + T_{14} T_{23} + 2T_{[13][24]} + 2T_{[12][34]} \\
+ 2(a_1 + bx) (T_{13} T_{24} + T_{[13][24]} + \frac{2(a_1 - bx)}{(a_0 + bx)} (T_{12} T_{34} + T_{[12][34]})) \\
+ \frac{(a_2 - bx)(a_1 - bx)}{(a_1 + bx)(a_0 + bx)} T_{12} T_{34} + \frac{(a_2 + bx)(a_1 + bx)}{(a_1 - bx)(a_0 - bx)} T_{13} T_{24}
\]  

(97)

and

\[
F = \frac{1}{2} E^2 [(1 - \mu^2)x^2 - 1]
\]  

(98)

In the helicity basis, the amplitude reads

\[
|A|^2 = \frac{g_Y^4}{M_s^4} |F_{SV}|^2 F^2 \mathcal{T}^2 [(1 + f)^2 \sigma_{\lambda_1, \lambda_2} + f^2 \delta_{\lambda_1, \lambda_2}]
\]  

(99)

Averaging over helicities of the incoming vector bosons and summing over final states of the small scalar BH’s one gets

\[
\langle |A|^2 \rangle = \frac{3g_Y^4}{M_s^4} d(N_R)^2 \mathcal{F}_{SV}^2 F^2 \mathcal{T}^2 (1 + 2f + 2f^2) \\
= \frac{3g_Y^4}{4M_s^4} d(N_R)^2 \frac{\hat{s}^2 \sin^2(\pi \hat{s})}{\pi^2(1 + \hat{s})^2} \sum_{n=0}^{\infty} \frac{\mathcal{R}_n(\hat{s})}{(a_n + bx)} \left| \sum_{k=0}^{\infty} \frac{\mathcal{R}_k(\hat{s})}{(a_k - bx)} \right|^2 \mathcal{T}^2 [D(1 - x^2)^2 + H]
\]  

(100)

where $D = (E^2 - M^2)^2$ and $H = M^4$.

Setting

\[
\mathcal{T}_1 = T_{12} T_{34} + T_{13} T_{24} + T_{14} T_{23} + 2T_{[13][24]} + 2T_{[12][34]} \\
\mathcal{T}_2 = 2(T_{13} T_{24} + T_{[13][24]}) \\
\mathcal{T}_3 = 2(T_{12} T_{34} + T_{[12][34]}) \\
\mathcal{T}_4 = T_{12} T_{34} \\
\mathcal{T}_5 = T_{13} T_{24}
\]  

(101) \quad (102) \quad (103)

\footnote{Including superpartners of the scalar states would be tantamount to replacing a factor of 6 with a factor of 16 = 8_B + 8_F.}
one finds

\[
\mathcal{I}^2 = T_1^2 + \frac{(a_1 + bx)^2}{(a_0 - bx)^2} T_2^2 + \frac{(a_1 - bx)^2}{(a_0 + bx)^2} T_3^2 + \frac{(a_2 - bx)^2}{(a_1 - bx)^2} T_4^2 + \frac{(a_2 + bx)^2}{(a_1 + bx)^2} T_5^2 + 2T_1 T_2 \frac{(a_1 + bx)}{(a_0 - bx)} + 2T_1 T_3 \frac{(a_1 - bx)}{(a_0 + bx)}
\]

\[
+ 2T_1 T_4 \frac{(a_2 - bx)(a_1 - bx)}{(a_0 + bx)(a_1 - bx)} + 2T_1 T_5 \frac{(a_2 + bx)(a_1 + bx)}{(a_0 - bx)(a_1 - bx)}
\]

\[
+ 2T_2 T_3 \frac{(a_1 + bx)(a_1 - bx)}{(a_0 + bx)(a_0 - bx)} + 2T_2 T_4 \frac{(a_1 + bx)(a_1 - bx)(a_2 - bx)}{(a_0 + bx)(a_0 - bx)(a_1 + bx)}
\]

\[
+ 2T_2 T_5 \frac{(a_1 + bx)^2(a_2 + bx)}{(a_0 - bx)^2(a_1 - bx)} + 2T_3 T_4 \frac{(a_1 - bx)^2(a_2 - bx)}{(a_0 + bx)^2(a_1 + bx)}
\]

\[
+ 2T_3 T_5 \frac{(a_1 + bx)(a_1 - bx)(a_2 + bx)(a_2 - bx)}{(a_0 + bx)(a_0 - bx)(a_1 + bx)(a_1 - bx)}
\]

\[
(104)
\]

In order to average over colours one can peruse the relation \(Tr_R(T^a T^b) = \ell_R \delta^{ab}\) and its corollary \(\ell_R d_G = C_R d_R\), where \(d_G\) is the dimension of the group, \(d_R\) the dimension of the representation \(R\) and \(C_R\) its (quadratic) Casimir. For the Adjoint representations of both \(E(8)\) and \(SO(32)\) \(C_A = 30\). Setting \(\langle T \rangle_c = T/d_G\), one eventually gets

\[
\langle T_1^2 \rangle_c = \frac{12C_A^2}{d_G} + 16C_A d_G + \frac{6}{d_G} + 3, \quad \langle T_2^2 \rangle_c = \langle T_3^2 \rangle_c = \frac{4C_A^2}{d_G} + 4
\]

\[
(105)
\]

\[
\langle T_4^2 \rangle_c = \langle T_5^2 \rangle_c = 1, \quad \langle T_1 T_2 \rangle_c = \langle T_1 T_3 \rangle_c = \frac{6C_A^2}{d_G} + 8C_A d_G + \frac{4}{d_G} + 2
\]

\[
(106)
\]

\[
\langle T_1 T_4 \rangle_c = \langle T_1 T_5 \rangle_c = 2C_A d_G + \frac{2}{d_G} + 1, \quad \langle T_2 T_3 \rangle_c = \frac{2C_A^2}{d_G} + 8C_A d_G + \frac{4}{d_G}
\]

\[
(107)
\]

\[
\langle T_2 T_4 \rangle_c = \langle T_3 T_5 \rangle_c = 2C_A d_G + \frac{2}{d_G}, \quad \langle T_2 T_5 \rangle_c = \langle T_3 T_4 \rangle_c = 2, \quad \langle T_4 T_5 \rangle_c = \frac{1}{d_G}
\]

\[
(108)
\]

Reinstating normalization factors, the differential cross section becomes

\[
\frac{d\sigma}{d\Omega} = \frac{3g_Y^4 d(N_R)^2}{(8\pi)^4 M_s^2} \sqrt{1 - \mu^2} \left[ (1 - \mu^2)^2 (1 - x^2)^2 + \mu^4 \right] \langle \mathcal{I}^2 \rangle_c \sin^2(\pi \hat{s}) \left( \sum_{n=0}^{\infty} \frac{R_n(\hat{s})}{(a_n + bx)} \right)^2 \left( \sum_{k=0}^{\infty} \frac{R_k(\hat{s})}{(a_k - bx)} \right)^2
\]

\[
(109)
\]

that displays – albeit not very explicitly – the angular distribution of the products.

In order to compute the total cross section one should perform integrals of the form
\[ \int_{-1}^{1} \frac{D(1-x^2)^2 + H}{(a_n + bx)(a_n' + bx)(a_k - bx)(a_k' - bx)} \]

\[ = \int_{-1}^{1} \left\{ D \left[ \frac{C_n}{(a_n + bx)} + \frac{C_{n'}}{(a_n' + bx)} + \frac{C_k}{(a_k + bx)} + \frac{C_{k'}}{(a_k' + bx)} \right] + H \right\} \]

\[ = D \log \left[ \frac{(a_n + b)(a_n' + b)}{(a_n - b)(a_n' - b)} \right] C_n C_{n'} \frac{C_k}{(a_k + b)} C_{k'} + 2H \quad (110) \]

In a similar fashion one can compute the other integrals. We refrain from displaying the rather uninspiring results. Alternatively, one could derive the total cross section for pair production by means of the optical theorem i.e. computing the forward scattering amplitude at one loop projected along BPS states [74, 75, 76]. We will not pursue this viewpoint any further here. Let us instead discuss the energy distribution.

In addition to the obvious threshold at \( E_{CM} = 2M \), the cross section is modulated by the Regge poles, i.e. string excitations. Their presence, drastically changes the high energy behavior wrt field theory amplitudes [48].

In the high energy limit \( \hat{s} \gg 1 \), \( \mu = M/E \sim 0 \), one finds

\[ \hat{M}^2 - \hat{t} \sim \frac{1}{2} \hat{s}(1 + x) \quad , \quad \hat{M}^2 - \hat{u} \sim \frac{1}{2} \hat{s}(1 - x) \quad (111) \]

and

\[ F^2(1 + 2f^2) \sim \frac{s^2}{64}(1 - x^2)^2 \quad (112) \]

Moreover, for fixed \( n = 0, \pm 1, \pm 2, a_n \sim b \sim \hat{s}/2 \), so that \( \mathcal{I} \sim \mathcal{I}_\infty(x) \) becomes a rational function of \( x = \cos \theta \) independent of \( s = E_{CM}^2 \).

Perusing Stirling formula \( \Gamma(z) \sim \sqrt{2\pi} z^{z-1/2} e^{-z} \) in the S-V form factor yields

\[ F_{SV} \sim \frac{\sin[\pi \hat{s}(1 + x)/2] \sin[\pi \hat{s}(1 - x)/2]}{\hat{s} \sin(\pi \hat{s}/2) \cos(\pi \hat{s}/2)} \left( \frac{2}{1 + x} \right)^{-\hat{s}(1+x)} \left( \frac{2}{1 - x} \right)^{-\hat{s}(1-x)} \quad (113) \]

If one simply takes \( x = 0 \ (\theta = \pi/2) \), the poles at \( \hat{s} = 2n \) cancel and one eventually gets

\[ F_{SV} \sim \frac{2^{-2\hat{s}}}{\hat{s}} \tan(\pi \hat{s}/2) \quad (114) \]

The exponentially suppressed Regge behaviour, related to the presence of an infinite number of string resonances, is universal in String Theory and could mark the difference with alternative scenari with low-scale gravity to which we now turn our attention.
4.2 Mass scales and Large Extra Dimensions

Following the original proposal of AADD [70, 71], there has been an enormous interest in models with Large Extra Dimension and TeV scale gravity or strings. The former predict BH production at LHC at a very high rate [1, 2, 3, 4, 5, 6]. The latter predict the usual exponential decay (Regge behavior) at high energies and the characteristic modulation by the presence of Regge poles [74, 75, 76, 77, 78]. Our results are in line with this expectation. In particular we don’t find any growth of the scattering amplitudes with energies as in the FT toy model. Moreover, the geometric cross section (area of BH horizon) that sets the order of magnitude for the production of a single BH is replaced by other dynamical quantities in pair production processes.

In perturbative heterotic strings is notoriously difficult to accommodate LED with coupling constants for gauge interactions. Quite generally, tree-level coupling constants are given by

\[ g_s^2 / \hat{V}_{int} = g_{YM}^2 \]

so that \( M_s^2 = g_{YM}^2 M_{Pl}^2 \). Barring large threshold corrections, which are anyway absent in toroidal compactifications, it seems hard if not impossible to separate the BH mass, whose lower bound for fixed charges is of order \( M_{BH} \sim M_s \sqrt{N_R} \), from the Planck scale so as to lower the threshold for the production process to accessible energies. In particular, in order to have \( M_{BH} \sim M_s \sim TeV \) one should have a implausibly small gauge coupling \( g_{YM} \sim 10^{-15} \).

The situation improves in theories with open and un-oriented strings where one has instead

\[ g_s \frac{\hat{V}_\perp}{\hat{V}_{T6}} = g_{YM}^2 \]  

with \( \hat{V}_\perp \) the volume of the internal space transverse to the D-branes, so that

\[ M_s^2 = g_{YM}^4 M_{Pl}^2 \frac{\hat{V}_{T6}^2}{\hat{V}_\perp^2} \]  

which is compatible with reasonably small \( g_{YM} \), low string scale \( i.e. \) BH masses) and large extra (transverse) directions. In this context, (small) BH’s are described by bound-states of D-branes, accommodating the ‘visible’ sector, whose mass diverges in the \( g_s \to 0 \) limit. The analysis is much more involved. So far only static properties, such as the micro-state counting, and the grey-body factor have been computed [41]. Computing dynamical properties of (small) BH’s corresponding to bound-states of D-brane looks very challenging.
5 Small BH’s in the FHSV model

The results found for toroidal compactifications can be easily and reliably generalized to special heterotic models with lower supersymmetry that are still protected against significant quantum corrections. The simplest and probably most interesting possibility is the FHSV model \[40\]. It admits both heterotic and Type II descriptions, related to one another by ‘Second Quantized Mirror Symmetry’ \[40\]. In the heterotic description, the model corresponds to a freely acting \(Z_2\) orbifold of a toroidal compactification on \(T^6 = T^4 \times T^2\) with identical Wilson lines on the two \(E_8\) factors. The \(Z_2\) orbifold generator is

\[ g = \mathcal{I}_4 \sigma v \pi_G \]  

where \(\mathcal{I}_4\) is the inversion of the four coordinates of \(T^4\), \(\sigma v\) is a non-geometric order two shift along \(T^2\) of parameter \(v = (v_L, v_R)\) and \(\pi_G\) is the exchange of the two \(E_8\)’s. Level matching requires that the shift be left-right asymmetric with

\[ v \cdot v = |v_L|^2 - |v_R|^2 = \frac{1}{2} \text{mod 1} \]  

The partition function consists of four terms

\[ \mathcal{Z} = \frac{1}{2}(\mathcal{Z}_{00} + \mathcal{Z}_{01} + \mathcal{Z}_{10} + \mathcal{Z}_{11}) \]  

In the untwisted sector one finds

\[ \mathcal{Z}_{00} = (Q_o + Q_v) \frac{\Lambda_{4,4}^+ \Lambda_{2,2}^{+\prime}}{\eta^4 |\eta^2|^2} \tilde{E}_8(q) \tilde{E}_8(q) \]  

\[ \mathcal{Z}_{01} = (Q_o - Q_v)|X_o - X_v|^2 \frac{\Lambda_{2,2}^-}{\eta^2} \tilde{E}_8(q^2) \]  

where

\[ \tilde{E}_8(q) = \sum_\alpha \frac{\theta^8(0|q)}{2\eta^8(q)} = O_{16} + S_{16} \]  

and

\[ Q_o = V_4 O_4 - S_4 S_4 \quad Q_v = O_4 V_4 - C_4 C_4 \]  

compactly represent the projection on the fermionic coordinates \[49, 50\],

\[ X_o + X_v = \frac{1}{\eta^4} \quad X_o - X_v = \frac{4\eta^2}{\theta^2_2} \]  

on the bosonic coordinates of \(T^4\) and

\[ \Lambda_{2,2}^+ = \sum_{p_L, p_R} q^{\frac{1}{2}|p_L|^2} q^{\frac{1}{2}|p_R|^2} \quad \Lambda_{2,2}^- = \sum_{p_L, p_R} (-)^{v_L p_L - v_R p_R} q^{\frac{1}{2}|p_L|^2} q^{\frac{1}{2}|p_R|^2} \]
on the bosonic coordinates of $T^2$.

In the twisted sector one finds

$$Z_{10} = 16(Q_s + Q_c)|X_s + X_c|^2 \frac{\tilde{\Lambda}_{2,2}^+}{|\eta|^2} \tilde{E}_8(\sqrt{q})$$

(126)

$$Z_{11} = 16(Q_s - Q_c)|X_s - X_c|^2 \frac{\tilde{\Lambda}_{2,2}^-}{|\eta|^2} \tilde{E}_8(-\sqrt{q})$$

(127)

where the factor 16 accounts for the number of fixed points and $[49, 50]$

$$Q_s = O_4S_4 - C_4O_4 \quad Q_c = V_4C_4 - S_4V_4$$

(128)

$$X_s + X_c = \frac{\eta^2}{\theta_4^2} \quad X_s - X_c = \frac{\eta^2}{\theta_3^2}$$

(129)

$$\tilde{\Lambda}_{2,2}^+ = \sum_{p_L,p_R} q^2|p_L + \nu_L|^2 q^2|p_R + \nu_R|^2$$

$$\tilde{\Lambda}_{2,2}^- = \sum_{p_L,p_R} (-)^{\nu_Lp_L - \nu_Rp_R} q^2|p_L + \nu_L|^2 q^2|p_R + \nu_R|^2$$

(130)

Due to the shift, there are no massless states in the twisted sector.

There are however 1/2 BPS states both in the untwisted and the twisted sector $[77]$. Setting the left-movers in their ground states yields

$$Q_o \rightarrow 2_v + 2_o - 2_s - 2_c \quad \text{vector}$$

(131)

$$Q_v \rightarrow 4_o - 2_s - 2_c \quad \text{hyper}$$

(132)

$$Q_s \rightarrow 2_o - 1_s - 1_c \quad \text{half hyper}$$

(133)

while $Q_c$ only contributes excited non-BPS states.

In the untwisted sector, the surviving 1/2 BPS states are a subset of the original BPS states in the parent $\mathcal{N} = 4$ theory.

$$Z_{1/2 \text{ BPS}}^{\text{vect,untw}} : \frac{1}{\eta^2} \{ \tilde{X}_o[\Gamma_{2,2}^+ \tilde{E}_8^{(2)+} + \Gamma_{2,2}^- \tilde{E}_8^{(2)-}] + \tilde{X}_v[\Gamma_{2,2}^+ \tilde{E}_8^{(2)+} + \Gamma_{2,2}^- \tilde{E}_8^{(2)-}] \}$$

(134)

$$Z_{1/2 \text{ BPS}}^{\text{hyp,untw}} : \frac{1}{\eta^2} \{ \tilde{X}_v[\Gamma_{2,2}^+ \tilde{E}_8^{(2)+} + \Gamma_{2,2}^- \tilde{E}_8^{(2)-}] + \tilde{X}_o[\Gamma_{2,2}^+ \tilde{E}_8^{(2)+} + \Gamma_{2,2}^- \tilde{E}_8^{(2)-}] \}$$

(135)

where

$$\Gamma_{2,2}^\pm = \frac{\Lambda_{2,2}^+ \pm \Lambda_{2,2}^-}{2\eta^2}$$

(136)

and

$$\tilde{E}_8^{(2)\pm} = \frac{1}{2}[\tilde{E}_8(q) \tilde{E}_8(q) \pm \tilde{E}_8(q^2)]$$

(137)

---

8Neglecting spin carried by the R-movers
In the twisted sector, there are new 1/2 BPS states with respect to the original BPS states in the parent $\mathcal{N} = 4$ theory.

$$Z^{1/2 \text{ BPS}}_{\text{hyp}, \text{tw}} : \frac{1}{\eta^2} \{ \tilde{X}_s[\tilde{\Gamma}_{2,2}^+ \tilde{E}_{8,t}^{(2)+} + \tilde{\Gamma}_{2,2}^- \tilde{E}_{8,t}^{(2)-}] + \tilde{X}_c[\tilde{\Gamma}_{2,2}^+ \tilde{E}_{8,t}^{(2)+} + \tilde{\Gamma}_{2,2}^- \tilde{E}_{8,t}^{(2)+}] \} \quad (138)$$

where

$$\tilde{\Gamma}_{2,2}^\pm = \frac{\tilde{\Lambda}_{2,2}^\pm}{2\eta^2} \quad (139)$$

and

$$\tilde{E}_{8,t}^{(2)\pm} = \frac{1}{2}[\tilde{E}_8(\sqrt{q}) \pm \tilde{E}_8(-\sqrt{q})] \quad (140)$$

### 5.1 Pair Production of small BH’s

Tree-level scattering amplitudes only involving states in the untwisted sector of the FHSV model are identical to those of the parent theory with $\mathcal{N} = 4$ susy. The two-derivative effective action is expected to receive no quantum corrections, thanks to $n_v = n_h$, i.e. the number of vector and hyper multiplets are always equal, everywhere in the moduli space, including points of enhanced symmetry [40]. However, twisted states contribute to higher derivative corrections to the effective action and can be pair produced at tree-level.

Let us consider the production of two small BPS BH’s in the twisted sector. For scalar states, corresponding to spherically symmetry small BH’s, vertex operators are of the form

$$V_{\Phi,f}^\tau : = e^{-\varphi} \sigma_f S^\tau e^{i\bar{p}_L Z_L} : B_{N_R^X} \sigma_f :: B_{N_R^Z} e^{i\bar{p}_R Z_R} :: B_{N_R^J} \bar{\Psi}^u : e^{i\rho X} \quad (141)$$

where $\sigma_f$ are $\mathbb{Z}_2$ twisted fields located at the fixed point $f$, $S^\tau$ is an internal $SO(4)$ spin field of positive chirality, $\bar{\Psi}^u$ is a primary field of $E_8^{(2)}$, whose currents are twisted (i.e. half-integer modes). $B_{N_R^X}, B_{N_R^Z}, B_{N_R^J}$ are polynomials in the derivatives of the currents $J$ and of the internal coordinates $Y$, for $T^4$, and $Z$ for $T^2$. Level matching requires

$$\frac{1}{2} |\bar{p}_L|^2 = \frac{1}{2} |\bar{p}_R|^2 + \frac{3}{4} + N_R^X + N_R^J + \frac{1}{4} |r|^2 \quad (142)$$

with

$$N_R^{\text{tot}} = N_R^Z + N_R^X + N_R^J = \sum_{k=1}^{\infty} [kn_k + (k - \frac{1}{2})(n_{k-\frac{1}{2}}^X + n_{k-\frac{1}{2}}^J)] \quad (143)$$

that amounts to

$$(m + a)(n + b) = N_R^{\text{tot}} + \frac{1}{4} |r|^2 + \frac{3}{4} \quad (144)$$

with $2ab = 1/2$ (mod 1).

The amplitudes for the processes under consideration

$$\mathcal{A} = \langle V_{\Phi,f} V_{\nu/k} V_{\nu/k} V_{\Phi,f} \rangle \quad (145)$$
can be decomposed into various parts.

In addition to the ubiquitous L-moving contribution \( \mathcal{E}_L(z_i, p_i) \) that combines with

\[
\langle e^{-\phi} S^r \sigma_f e^{i\vec{p}_L z_L}(z_1) e^{-\phi} S^s \sigma_f e^{-i\vec{p}_L z_L}(z_4) \rangle = e^{rs} \delta_{f'f} z_{14}^{-2|\vec{p}_L|^2}
\] (146)

one needs the contribution of the R-movers

\[
\mathcal{E}^{EHV}_R(z_i, p_i) = \langle B_{N_R'k} \bar{f} B_{N_R'k}^r e^{i\vec{p}_R z_R} B_{N_R'k} B_{N_R'k}^r \bar{\Psi} e^{ip_1 X_R(z_1)} \bar{\Psi} e^{ip_2 X_R(z_2)} \rangle
\]

\[
\bar{\xi} \xi e^{ik_3 X_R(z_3)} \bar{B}_{N_R'k} B_{N_R'k} \bar{\Psi} e^{ip_4 X_R(z_4)} \rangle
\] (147)

Let us consider for simplicity the case of small BH’s charged wrt the ‘visible’ gauge group \( E_8 \), whose vertex operator involves a primary field \( \bar{\Psi}^u \) \((N_R' = 0)\) of dimension

\[ h_\Psi = \frac{1}{2} + \frac{1}{4} |r|^2 \] (148)

in a representation \( \mathbf{R} \), with highest weight \( |r| \), of the \( E_8 \) current algebra at level \( \kappa = 2 \).

For gauge bosons in the initial state, \( \partial \xi^{MN} \rightarrow J_{a/b} \), one needs the correlation function

\[
\hat{C}^{ab}_{\Psi v}(z_i, p_i) = \langle \bar{\Psi}^u(z_1) \bar{\Psi}^v(z_4) \rangle = \frac{N_\Psi}{z_{23}^{-2h_\Psi}} \left[ 2 \delta^{ab} \delta^u_v + [t^a, t^b]_v \left( \frac{z_{12}^{24}}{z_{13}^{24}} - \frac{z_{12}^{34}}{z_{13}^{24}} \right) + \left\{ t^a, t^b \right\}_v \left( \frac{z_{12}^{24}}{z_{13}^{24}} + \frac{z_{12}^{34}}{z_{13}^{24}} \right) \right] \mathcal{I}_R(z_i, p_i)
\] (149)

where \( N_\Psi \) is some normalization and \( \mathcal{I}_R \) is the R-mover Koba-Nielsen factor.

For gravitons in the initial state, \( \partial \xi^{MN} \rightarrow \partial X_{\mu/\nu} \), one needs instead

\[
\hat{G}^{\mu\nu}_{\Psi v}(z_i, p_i) = \langle \bar{\Psi}^u(z_1) \bar{\Psi}^v(z_4) \rangle = \frac{N_\Psi}{z_{23}^{-2h_\Psi}} \left[ \eta^{\mu\nu} + \sum_{i \neq 2} \frac{P_i^\mu}{z_{2i}^2} \frac{P_j^\nu}{z_{3j}^2} \right] \mathcal{I}_R(z_i, p_i)
\] (150)

The remaining correlation function factorizes and yields

\[
\mathcal{K}_R(z_i, \vec{p}_R) = \langle B_{N_R'k} \bar{f} B_{N_R'k}^r e^{i\vec{p}_R z_R}(z_1) B_{N_R'k} \bar{f} B_{N_R'k} e^{-i\vec{p}_R z_R}(z_4) \rangle = \frac{N_{YZ}(\vec{P}_R)}{z_{14}^{2N_R'k + 2 + 2N_R'k + |\vec{p}_R|^2}}
\] (151)

Combining with the L-mover contribution, the computation proceeds as in the toroidal case.

For small BH’s in the twisted sector of the FHSV model, the amplitude for pair production in graviton scattering is the same as in the untwisted sector, up to some (moduli dependent) normalization.
As in toroidal compactifications, pair production of small BH’s in gauge boson scattering is very sensitive to the ‘gauge’ quantum numbers of the products. For the above simple case in which the charged BH’s correspond to primary fields of the current algebra the resulting scattering amplitude is

\[
A = g_s^2(2\pi)^4\delta^4(\Sigma_i p_i)\delta f_1 f_4 e^{r i r_4} N(\tilde{p}_R)[\delta^{ab}\delta^u_v A_0 + (t^a t^b)^u_v A_1 + (t^b t^a)^u_v A_{-1}]
\]

where \(A_n(p_i, a_i)\) are defined in Eq (73). Notice that \(A_1\) and \(A_{-1}\) get exchanged under \(2 \leftrightarrow 3\) or, equivalently, \(1 \leftrightarrow 4\).

For generic small BH with the same charges \((m, n, r)\) and mass, level matching allows for descendants under the action of the current algebra. We expect this to change at most the normalization constant. After summing over all the degenerate ‘final’ states, one finds a large multiplicity due to the microscopic entropy of the small BH states.

6 Conclusions and outlook

Let us conclude with few comments and directions for future investigation.

Our results for pair production of small BH’s with two charges, corresponding to perturbative BPS states in toroidal compactifications of heterotic strings or in the FHSV model, display a certain degree of universality. The presence of Regge poles and soft UV behavior are a hallmark of any string amplitude. In more realistic string models describing e.g. collisions at LHC, one should convolute ‘partonic’ cross sections for BH production, such as the ones computed here, with the parton distributions inside the protons, that may significantly change the shape of the signal. The results also depend on the particles initiating or mediating the process, gravitons or vector bosons. Not surprisingly, for gravitons we found a huge suppression related to the smallness of Newton’s constant. In perturbative heterotic strings, scenari with LED are hard to accommodate. Anyway the relative suppression wrt processes mediated by ‘visible’ gauge bosons is \(1/M_s^2\). Needless to say we don’t find any increase of the cross section with the CM energy as in FT toy models. Although we focussed on the lowest spin scalar components of BPS BH multiplets, our analysis applies to the spin 1/2 and 1 superpartners. In any case the corresponding BH solutions have non-rotating horizons, since the broken supersymmetry parameters generating BPS BH supermultiplets vanish at the horizon.

We expect the same qualitative behavior for the production of extremal non BPS BH’s with \(N_R = 1\) and \(M^2 = |p_R|^2 = |p_L|^2 + N_L - \delta_L\), whereby the Left movers are not in their ground states. The world-sheet computations would be very similar and reliable.

\[\text{We thank Ashoke Sen for a clarifying discussion on this and related issues.}\]
Extremal BH’s – be BPS or not – cannot decay since there are no states with the same charges and lower mass. They do not emit Hawking radiation. In some cases they admit a fuzzball description \[\text{[78, 79]}\].

A quantitatively different but qualitatively similar story applies to highly excited string states which are far from the BPS bound \[\text{[13, 14]}\]. While, it is believed anyway that the string/BH correspondence principle should work even in this case \[\text{[80, 81]}\], far from extremal BH’s have properties, including their mass and entropy, that are sensitive to the string coupling and other freely adjustable parameters (moduli fields). Although the grey-body factor for near extremal BH’s has been very successfully computed more than ten years ago \[\text{[41]}\], dynamical properties of (small) non-extremal BH’s corresponding to fundamental heterotic strings have been only explored in the recent past. In particular in \[\text{[82]}\], convincing evidence was given in favour of a string/BH transition in dynamical processes of emission and absorption. Furthermore, a dynamical analysis, similar to the one performed here, elucidated the distribution in size and typical configuration of very massive closed string states as a function of the string coupling \[\text{[83, 84]}\].

Finally, we would like to briefly comment on non spherically symmetric e.g. rotating BH’s. It is well known that string excitations at level \(N\) can carry high spin. The maximal spin is \(J_{Max} \approx N\). BPS states with higher spin are described by vertex operators of the form

\[
V_s^\mathcal{H} = \mathcal{H}_{i,\mu_1...\mu_s} e^{-\nu \psi} e^{ipL X_L} \bar{\partial} X^{\mu_1} ... \bar{\partial} X^{\mu_s} e^{ipR X_R} e^{ipX}
\]

(153)

The tensor \(\mathcal{H}_{i,\mu_1...\mu_s}\) is totally symmetric by construction in the \(\mu\) indices and, in order for the state to be BRS invariant, it should be transverse \(p^\mu \mathcal{H}_{i,\mu_2...\mu_s} = 0\) and traceless. It is well possible that BPS strings with high angular momentum be in correspondence with black rings rather than BH’s \[\text{[85, 86, 87, 88]}\]. Moreover the validity of the string/BH correspondence principle may need to be reconsidered for states with angular momentum since varying the moduli states with high spin can ‘decay’ or rather transform into bound states of components with lower spin.

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