A Capsule Network-based Embedding Model for Knowledge Graph Completion and Search Personalization

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ABSTRACT

In this paper, we introduce an embedding model, named CapsE, exploring a capsule network to model relationship triples \((subject, relation, object)\). Our CapsE represents each triple as a 3-column matrix where each column vector represents the embedding of an element in the triple. This 3-column matrix is then fed to a convolution layer where multiple filters are operated to generate different feature maps. These feature maps are used to construct capsules in the first capsule layer. Capsule layers are connected via dynamic routing mechanism. The last capsule layer consists of only one capsule to produce a vector output. The length of this vector output is used to measure the plausibility of the triple. Our proposed CapsE obtains state-of-the-art link prediction results for knowledge graph completion on two benchmark datasets: WN18RR and FB15k-237, and outperforms strong search personalization baselines on SEARCH17 dataset.

1 INTRODUCTION

Knowledge graphs (KGs) containing relationship triples \((subject, relation, object)\), denoted as \((s, r, o)\), are the useful resources for many NLP and information retrieval applications (Wang et al., 2017). However, large knowledge graphs, even containing billions of triples, are still incomplete, i.e., missing a lot of valid triples (West et al., 2014). Therefore, much research efforts have focused on the knowledge graph completion or link prediction task which aims to predict missing triples in KGs, i.e., predicting whether a triple not in KGs is likely to be valid or not (Bordes et al., 2011, 2013; Socher et al., 2013). To this end, many embedding models have been proposed to learn vector representations for entities (i.e., subject/head entity and object/tail entity) and relations in KGs, and obtained state-of-the-art link prediction results as summarized by Nickel et al. (2016a) and Nguyen (2017). These embedding models score triples \((s, r, o)\), such that valid triples have higher plausibility scores than invalid ones (Bordes et al., 2011, 2013; Socher et al., 2013). For example, in the context of KGs, the score for \((Melbourne, cityOf, Australia)\) is higher than the score for \((Melbourne, cityOf, United Kingdom)\).

Triple modeling is applied not only to the KG completion, but also for other tasks which can be formulated as a triple-based prediction problem. An example is in search personalization, one would aim to tailor search results to each specific user based on the user’s personal interests and preferences (Teevan et al., 2005, 2009; Bennett et al., 2012; Harvey et al., 2013; Liu, 2015). Here the triples can be formulated as \((submitted\ query, user\ profile, returned\ document)\) and used to re-rank documents returned to a user given an input query, by employing an existing KG embedding method such as TransE (Bordes et al., 2013), as proposed in Vu et al. (2017). Previous studies have shown the effectiveness of modeling triple for either KG completion or search personalization. However, there has been no single study investigating the performance on both these tasks.
In addition to conventional embedding models such as TransE (Bordes et al., 2013), DISTMULT (Yang et al., 2015) and ComplEx (Trouillon et al., 2016), recent research has raised interest in applying deep neural network to triple-based prediction problems. For example, Nguyen et al. (2018) proposed ConvKB, a convolutional neural network (CNN)-based model for KG completion, and achieved the state-of-the-art results. However, CNN fails to effectively preserve the information about the relative positions of the entities in the triple. This can refer to a problem in computer vision where the relative positions of elements such as eyes, nose, mouth in a facial image are important and should be captured properly.

To preserve relative position information, Sabour et al. (2017) introduced capsule networks (CapsNet) that employ capsules to capture the elements and uses an iterative dynamic routing process to specify appropriate connections from capsules in a layer to those in the next layer. Hence CapsNet could encode the intrinsic spatial relationship between a part and a whole constituting viewpoint invariant knowledge that automatically generalizes to novel viewpoints. These motivate us to investigate whether CapsNet can effectively model the triples and obtain good performance on triple-based prediction problems.

To that end, we introduce the CapsNet-based embedding model – a single method that can perform for both two tasks: KG completion and search personalization. In our proposed method, $v_s$, $v_r$, and $v_o$ are unique $k$-dimensional embeddings of $s$, $r$ and $o$, respectively. The embedding triple $[v_s, v_r, v_o]$ of $(s, r, o)$ is fed to the convolution layer where different filters of the same $1 \times 3$ shape are repeatedly operated over every row of the matrix to produce different $k$-dimensional feature maps. Entries at each same dimension from all feature maps are then grouped into a capsule. As a result, $k$ capsules are constructed in the first capsule layer where each capsule can represent transitional characteristics among the same dimensional entries of $v_s$, $v_r$ and $v_o$. Lower capsule layers are connected with higher capsule layers via the dynamic routing process. Next we construct only one capsule in the last capsule layer and then measure the length of vector output of this capsule as a score for the triple. Finally we use this score to infer whether the triple $(s, r, o)$ is valid or not. We term our proposed model CaspE.

Intuitively, the different parts of a triple are routed to different capsules. In another word, each capsule has an individual perspective or focus on the triple. In summary, our main contributions from this paper are as follows:

- We propose a new embedding model CapsE based on the capsule network (CapsNet) (Sabour et al., 2017) for modeling relationship triples. To our best of knowledge, our work is the first attempt at applying CapsNet for knowledge graph completion and search personalization.
- We evaluate our proposed CapsE for knowledge graph completion on two benchmark datasets: WN18RR (Dettmers et al., 2018) and FB15k-237 (Toutanova & Chen, 2015), and show that CapsE obtains new state-of-the-art link prediction results on FB15k-237 and achieves competitive results compared to other strong models such as KBGAN, ConvE and DISTMULT on WN18RR.
- We restate the prospective strategy of expanding the triple embedding models to improve the ranking quality of the search personalization systems. We adapt our model to search personalization and evaluate on SEARCH17 (Vu et al., 2017) – a dataset of the web search query logs. Experimental results show that our CapsE achieves the new state-of-the-art results with significant improvements over strong baselines.

2 RELATED WORK

The well-known TransE model (Bordes et al., 2013) employs a transitional characteristic to model relationships between entities. Assuming that if $(s, r, o)$ is a valid fact, the embedding of head entity $s$ plus the embedding of relation $r$ should be close to the embedding of tail entity $o$, i.e., $v_s + v_r \approx v_o$. This transitional characteristic also implies the relationships among the same dimensional entries of $v_s$, $v_r$ and $v_o$. Other transition-based models extend TransE to additionally use projection vectors or matrices to translate embeddings of $s$ and $o$ into the vector space of $r$, such as: TransH (Wang et al., 2014), TransR (Lin et al., 2015b), TransD (Ji et al., 2015) and TranSparse. Furthermore, DISTMULT (Yang et al., 2015) and ComplEx (Trouillon et al., 2016) use a tri-linear dot product to
compute the score for each triple. Moreover, ConvKB \cite{Nguyen2018} applies convolutional neural network, in which feature maps are concatenated into a single feature vector which is then computed with a weight vector via a dot product to produce the score for the input triple. ConvKB is the most closely related model to our CapsE. See an overview of embedding models for KG completion in \cite{Nguyen2017}.

For search tasks, unlike classical methods, personalized search systems utilize the historical interactions between the user and the search system, such as submitted queries and clicked documents to tailor returned results to the need of that user \cite{Teevan2005, Teevan2009}. That historical information can be used to build the \textit{user profile}, which is crucial to an effective search personalization system. Widely used approaches consist of two separated steps: (1) building the user profile from the interactions between the user and the search system; and then (2) learning a ranking function to \textit{re-rank} the search results using the user profile \cite{Bennett2012, White2013, Harvey2013, Vu2015}. The general goal is to re-rank the documents returned by the search system in such a way that the more relevant documents are ranked higher. In this case, apart from the user profile, dozens of other features have been proposed as the input of a learning-to-rank algorithm \cite{Bennett2012, White2013}. Alternatively, \cite{Vu2017} modeled the potential user-oriented relationship between the submitted query and the returned document by applying TransE to reward higher scores for more relevant documents (e.g., clicked documents). They achieved better performances than the standard ranker as well as competitive search personalization baselines \cite{Teevan2011, Bennett2012, Vu2015}.

3 The Proposed CapsE

Let $G$ be a collection of valid factual triples in the form of (subject, relation, object) denoted as $(s, r, o)$. Embedding models aim to define a \textit{score function} giving a score for each triple, such that valid triples receive higher scores than invalid triples.

We denote $v_s, v_r$ and $v_o$ as the $k$-dimensional embeddings of $s, r$ and $o$, respectively. In our proposed CapsE, each embedding triple $[v_s, v_r, v_o] \in \mathbb{R}^{k \times 3}$, and $A_{i,:} \in \mathbb{R}^{1 \times 3}$ denotes the $i$-th row of $A$. We use a filter $\omega \in \mathbb{R}^{1 \times 3}$ operated on the convolution layer. This filter $\omega$ is repeatedly operated over every row of $A$ to generate a feature map $q = [q_1, q_2, ..., q_k] \in \mathbb{R}^k$ as follows:

$$q_i = g(\omega \cdot A_{i,:} + b)$$

where $\cdot$ denotes a dot product, $b \in \mathbb{R}$ is a bias term and $g$ is a non-linear activation function.

Our model uses different filters $\mathcal{G} \subset \mathbb{R}^{1 \times 3}$ to generate different feature maps. We denote $\Omega$ as the set of filters and $N = |\Omega|$ as the number of filters, thus we have $N$ $k$-dimensional feature maps.

We use the non-linear squashing function and the dynamic routing process \cite{Sabour2017} to build our capsule layers. In particular, we construct $L$ capsule layers in CapsE. Each layer $l \in \{1, 2, ..., L\}$ consists of $n_l$ capsules, for which each capsule $i \in \{1, 2, ..., n_l\}$ produces a vector output $u_i^{(l)} \in \mathbb{R}^{d_l \times 1}$, where $d_l$ is the number of neurons within capsules in layer $l$. Vector outputs $u_i^{(l)}$ of capsules $i$ in layer $l$ are multiplied by associated weight matrices $W_{ij}^{(l)} \in \mathbb{R}^{d_{l+1} \times d_l}$ to produce “prediction vectors” $u_{ij}^{(l)} \in \mathbb{R}^{d_{l+1} \times 1}$ which are summed to produce a vector input $s_j^{(l+1)} \in \mathbb{R}^{d_{l+1} \times 1}$ to capsule $j$ in layer $(l + 1)$. This capsule $j$ then performs the non-linear squashing function to produce a vector output $u_j^{(l+1)} \in \mathbb{R}^{d_{l+1} \times 1}$:

$$u_{ij}^{(l)} = W_{ij}^{(l)} u_i^{(l)}$$

$$s_j^{(l+1)} = \sum_i c_{ij}^{(l)} u_{ij}^{(l)}$$

$$u_j^{(l+1)} = \text{squash}(s_j^{(l+1)})$$

where squash $(s) = (a + \frac{1}{a})^{-1} \times s$ with $a = ||s||$, and $c_{ij}^{(l)}$ are coupling coefficients that are determined by the iterative dynamic routing process as presented in Algorithm 1. This ensures that the
orientation of the vector is unchanged, and the length of the vector output of a capsule is below 1. In addition, $c_{ij}^{(l)}$ helps to route the vector output $u_{i}^{(l)}$ of capsule $i$ in layer $l$ to an appropriate capsule $j$ in layer $(l + 1)$.

Algorithm 1: The iterative dynamic routing process as in [Sabour et al., 2017].

For constructing the first capsule layer in CapsE, we group entries at each same dimension from all feature maps into a capsule. Such that capsules can encode transitional characteristics among same dimensional entries of $v_s$, $v_r$ and $v_o$. Hence, we construct $k$ capsules ($n_1 = k$) in this layer, in which each capsule has a vector output $\in \mathbb{R}^{N \times 1}$ ($d_1 = N$). Furthermore, in the last capsule layer, we only construct one capsule whose the length of the vector output is used as the score for the triple.

We illustrate our proposed model in Figure 1 where embedding size: $k = 4$, number of filters: $N = 5$, number of capsule layers: $L = 2$, number of capsules in layer 1: $n_1 = k$, number of neurons within the capsules in layer 1: $d_1 = N$, number of capsules in layer 2: $n_2 = 1$ and number of neurons within the capsule in layer 2: $d_2 = 2$. The length of vector output $u_{1}^{(2)}$ is used as the score for the input triple.

Formally, we define the score function $f$ for the triple $(s, r, o)$ as follows:

$$f(s, r, o) = ||\text{capsnet}(g([v_s, v_r, v_o] * \Omega))||$$

where the set of filters $\Omega$ is shared parameters in the convolution layer; * denotes a convolution operator; and capsnet denotes a capsule network operator. We use the Adam optimizer (Kingma
Ba, 2014) to train CapsE by minimizing the loss function (Trouillon et al., 2016; Nguyen et al., 2018) as follows:

\[
\mathcal{L} = \sum_{(s, r, o) \in \{G \cup G'\}} \log (1 + \exp (-t_{s, r, o} \cdot f(s, r, o)))
\]

in which, 

\[
t_{s, r, o} = \begin{cases} 
1 & \text{for } (s, r, o) \in G \\
-1 & \text{for } (s, r, o) \in G'
\end{cases}
\]

here \(G\) and \(G'\) are collections of valid and invalid triples, respectively. \(G'\) is generated by corrupting valid triples in \(G\).

4 EXPERIMENTS

In this section, we evaluate our proposed model on two tasks: knowledge graph completion and search personalization.

4.1 KNOWLEDGE GRAPH COMPLETION

In the knowledge graph completion, also referred to as link prediction task (Bordes et al., 2013), the goal is to predict a missing entity given a relation and another entity, i.e., inferring a head entity \(s\) given \((r, o)\) or inferring a tail entity \(o\) given \((s, r)\). The results are calculated based on ranking the scores produced by the score function \(f\) on test triples.

4.1.1 DATASETS

We use two recent benchmark datasets: WN18RR (Dettmers et al., 2018) and FB15k-237 (Toutanova & Chen, 2015) that are correspondingly subsets of two common datasets WN18 and FB15k extracted from knowledge graphs WordNet and Freebase (Bordes et al., 2013). As noted by Toutanova & Chen (2015), WN18 and FB15k are relatively easy because they contain many reversible relations. Knowing relations are reversible allows us to easily predict the majority of test triples, e.g., the state-of-the-art results on both WN18 and FB15k are obtained by using a simple reversal rule-based strategy as shown by Dettmers et al. (2018). Therefore, WN18RR and FB15k-237 are created to avoid this reversible relation problem in WN18 and FB15k, so that the prediction task becomes more realistic and hence more challenging. Table 1 presents the statistics of WN18RR and FB15k-237.

| Dataset    | #E  | #R  | #Triples in train/valid/test |
|------------|-----|-----|-----------------------------|
| WN18RR     | 40,943 | 11  | 86,835 3,034 3,134          |
| FB15k-237  | 14,541 | 237 | 272,115 17,535 20,466       |

Table 1: Statistics of the experimental datasets. #E is the number of entities. #R is the number of relations.

4.1.2 EVALUATION PROTOCOL

Following Bordes et al. (2013), for each valid test triple \((s, r, o)\), we replace either \(s\) or \(o\) by each of all other entities to create a set of corrupted triples. We use the “Filtered” setting protocol (Bordes et al., 2013), i.e., not taking any corrupted triples that appear in the KG into accounts. We rank the valid test triple and corrupted triples in descending order of their scores. We employ evaluation metrics: mean reciprocal rank (MRR), Hits@1 and Hits@10 (i.e., the proportion of the valid test triples ranking in top 10 predictions). Higher MRR, Hits@1 and Hits@10 indicates better link prediction performance.

4.1.3 TRAINING PROTOCOL

We use the common Bernoulli trick (Wang et al., 2014; Lin et al., 2015b) when sampling invalid triples. We also use the same pre-trained entity and relation embeddings provided by Nguyen et al. (2018) to initialize the embeddings in CapsE (i.e. the embedding sizes are \(k = 50\) for WN18RR...
and $k = 100$ for FB15k-237). To learn our model parameters, we use Adam (Kingma & Ba, 2014) and select its initial learning rate $\eta \in \{5e^{-6}, 1e^{-5}, 5e^{-5}, 1e^{-4}, 5e^{-4}\}$. We use ReLU (Nair & Hinton, 2010) as the activation function $g$. We set the batch size to 256, the number of iterations in the dynamic routing algorithm $m = 3$, the number of capsule layers $L = 2$, and the number of neurons within the capsule in the second capsule layer $d_2 = 10$. We select the number of filters $N \in \{50, 100, 200, 400, 500\}$. The learning process of our model is terminated after 200 epochs. We use outputs from the last epoch for evaluation. MRR, Hits@1 and Hits@10 scores on the test set are computed for the model obtaining the highest Hits@10 on the validation set. The highest Hits@10 scores on the validation set are obtained when using $N = 100$, and the initial learning rate at $5e^{-5}$ on WN18RR; and $N = 400$, and the initial learning rate at $1e^{-4}$ on FB15k-237.

### 4.1.4 Main experimental results

Table 2 compares the experimental results of our CapsE with previous up-to-date results, using the same experimental setup. On WN18RR, CapsE obtains better MRR score than KBGAN, TransE and ConvKB, and achieves better Hits@10 score than KBGAN and ConvE. On FB15k-237, CapsE reaches the new state-of-the-art scores for all MRR, Hits@1 and Hits@10. In addition, our proposed model performs better than its closely related CNN-based model ConvKB on both experimental datasets (except Hits@10 on WN18RR), especially on FB15k-237 where our CapsE gains significant improvements of $0.538 - 0.396 = 0.142$ in MRR (which is about 35% relative improvement), and $56.7\% - 51.7\% = 5\%$ absolute improvements in Hits@1 and Hits@10, respectively.

Following Bordes et al. (2013), for each relation $r$ in FB15k-237, we calculate the averaged number $n_s$ of head entities per tail entity and the averaged number $n_o$ of tail entities per head entity. If $n_s < 1.5$ and $n_o < 1.5$, $r$ is categorized one-to-one (1-1). If $n_s < 1.5$ and $n_o \geq 1.5$, $r$ is categorized one-to-many (1-M). If $n_s \geq 1.5$ and $n_o < 1.5$, $r$ is categorized many-to-one (M-1). If $n_s \geq 1.5$ and $n_o \geq 1.5$, $r$ is categorized many-to-many (M-M). We observe that 17, 26, 81 and 113 relations are labelled 1-1, 1-M, M-1 and M-M, respectively. And 0.9%, 6.3%, 20.5% and 72.3% of the test triples in FB15k-237 contain 1-1, 1-M, M-1 and M-M relations, respectively.

Figure 2 shows the Hits@10 and MRR results for predicting head and tail entities on FB15k-237 with respect to (w.r.t) relation category. CapsE works better than ConvKB in predicting entities on the “side M” of triples (e.g., predicting head entities in M-1 and M-M; and predicting tail entities in 1-M and M-M), while ConvKB performs better than CapsE in predicting entities on the “side 1” of triples (i.e., predicting head entities in 1-1 and 1-M; and predicting tail entities in 1-1 and M-1). In short, we find that our CapsE outperforms ConvKB on M-M relations (e.g. in Hits@10, $58.4 + 58.9 = 58.7$ against $47.5 + 53.6 = 50.6$), while ConvKB performs better than CapsE when predicting 1-1, 1-M and M-1 relations. Since the number of M-M triples in FB15k-237 is much

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1. We re-run the ConvKB implementation with the optimal hyper-parameters provided by Nguyen et al. (2018) to reproduce exact published scores and then extract scores with respect to each relation and each relation category.

2. NOTE that formulas of MRR and Hits@1 show a strong correlation between these two scores. So using Hits@1 does not reveal any additional information for this task.
larger than the numbers of other triples, overall the performance of CapsE is better than that of ConvKB.

We further show the Hits@10 and MRR scores regarding each relation for CapsNet and ConvKB on WN18RR in Figure 3. derivationally related form and also see are two only M-M relations, while hypernym and has part are M-1 and 1-M relations respectively. It can be seen that results shown in figures 2 and 3 are consistent. Specifically, CapsE is less accurate than ConvKB for hypernym and has part. However, it performs better than ConvKB on the M-M relations derivationally related form and also see. Results in figures 2 and 3 also indicate that our proposed CapsE would be a potential candidate for applications which contain many M-M relations such as search personalization.

We analysis how and why our CapsE works: (1) We use capsules to encapsulate both entities and relations. Therefore, the probability of relationship can be represented by the length of the vector output of the corresponding capsule. More importantly, different cases of relationship can be represented by the orientation of the vector output of the capsule. (2) Different cases of relationship in a real-world application are dominated by the “side M” of triples (e.g., predicting head entities in M-1 and M-M). For example, (Melbourne, isCityOf, Australia) and (Sydney, isCityOf, Australia) are different cases of relationship “isCityOf” given “Australia”. These two cases can be determined very well by the length and orientation of the capsule’s vector output. This is why CapsE works best on the “side M” of triples where entities do not appear frequently, but it may fail on the “side 1” of triples where entities appear more frequently. Additionally, other proposed models such as DISTMULT, ComplEx and ConvE can perform well for entities with high frequency, but not for rare entities with low frequency. Note that the number of M-M triples is much larger than the numbers of other triples in FB15k-237, but not in WN18RR (as shown in Figure 3). Therefore, our CapsE works best on FB15k-237 while it is still outperformed by some models on WN18RR.

4.2 Search personalization

Given a user, a submitted query and the documents returned by a search system for that query, our approach is to re-rank the returned documents so that the more relevant documents should be ranked higher. Following [Vu et al. (2017)], we represent the relationship between the submitted query, the user and the returned document as a (s, r, o)-like triple (query, user, document). The triple captures
how much interest a user puts on a document given a query. Therefore, we can also evaluate the effectiveness of our CapsE model for the search personalization task.

4.2.1 Dataset

We use the same dataset, named SEARCH17 (Vu et al., 2017), of query logs of 106 users collected by a large-scale web search engine. A log entity consists of a user identifier, a query, top-10 ranked documents returned by the search engine and clicked documents along with the user’s dwell time. Vu et al. (2017) constructed short-term (session-based) user profiles and use the profiles to personalize the returned results for uniformly splitting the log entries into the training, validation and test sets. This split is also for the purpose of using historical data in the training set to predict new data in the test set. They then employed the SAT criteria (Fox et al., 2005) to identify whether or not a clicked document is relevant from the query logs (i.e., a SAT click). After that, they assigned a relevant label to a returned document if it is a SAT click and also assigned irrelevant labels to the remaining top-10 documents. The rank position of the relevant labeled documents is used as the ground truth to evaluate the search performance before and after re-ranking. The dataset contains 1,584 session-based user profile, 6,632 distinct queries and 33,591 distinct documents. The training, validation and test sets consist of 5,658, 1,184 and 1,210 valid/relevant triples; and 40,239, 7,882 and 8,540 invalid/irrelevant triples, respectively.

4.2.2 Evaluation protocol

Our CapsE is used to re-rank the original list of documents produced by a search engine as follows: (1) We train our model and employ the trained model to calculate the score for each triple \((s, r, o)\). (2) We then sort the scores in the descending order to obtain a new ranked list. To evaluate the performance of our proposed model, we use two standard evaluation metrics: mean reciprocal rank (MRR) and Hits@1\(^1\). For each metric, the higher value indicates the better ranking performance.

We compare CapsE with the following baselines using the same experimental setup: (1) SE: The original rank is returned by the search engine. (2) CI (Teevan et al., 2011): This baseline uses a personalized navigation method based on previously clicking returned documents. (3) SP (Bennett et al., 2012; Vu et al., 2015): A search personalization method makes use of the short-term user profiles. (4) Following Vu et al. (2017), we use TransE as a strong baseline model for the search personalization task. Previous work shows that the well-known embedding model TransE, despite its simplicity, obtains very competitive results for the knowledge graph completion (Lin et al., 2015a; Nickel et al., 2016b; Trouillon et al., 2016; Nguyen et al., 2016a, 2018). (5) The CNN-based model ConvKB is the most closely related model to our CapsE.

4.2.3 Training protocol

Embedding initialization: We follow Vu et al. (2017) to initialize user profile, query and document embeddings for the baselines TransE and ConvKB, and our CapsE.

We train a LDA topic model (Blei et al., 2003) with 200 topics only on the relevant documents (i.e., SAT clicks) extracted from the query logs. We then use the trained LDA model to infer the probability distribution over topics for each document. We use the topic proportion vector of each document as its document embedding (i.e. \(k = 200\)). In particular, the \(z^{th}\) element \((z = 1, 2, ..., k)\) of the vector embedding for document \(d\) is: \(v_{d,z} = P(z | d)\) where \(P(z | d)\) is the probability of the topic \(z\) given the document \(d\).

We also represent each query by a probability distribution vector over topics. Let \(D_q = \{d_1, d_2, ..., d_n\}\) be the set of top \(n\) ranked documents returned for a query \(q\) (here, \(n = 10\)). The \(z^{th}\) element of the vector embedding for query \(q\) is defined as in (Vu et al., 2017): \(v_{q,z} = \sum_{i=1}^{n} \lambda_i P(z | d_i)\), where \(\lambda_i = \delta^{i-1}\) is the exponential decay function of \(i\) which is the rank of \(d_i\) in \(D_q\). And \(\delta\) is the decay hyper-parameter \((0 < \delta < 1)\). Following Vu et al. (2017), we use \(\delta = 0.8\).

Note that if we learn query and document embeddings during training, the models will overfit to the data and will not work for new queries and documents. So, after the initialization process, we fix

\(^1\)We re-rank the list of top-10 documents returned by the search engine, so Hits@10 scores are same for all models.
| Method       | MRR     | Hits@1 (%) |
|--------------|---------|------------|
| SE [⋆]       | 0.559   | 38.5       |
| CI [⋆]       | 0.597   | 41.6       |
| SP [⋆]       | 0.631   | 45.2       |
| TransE [⋆]   | 0.645   | 48.1       |
| TransE (ours)| 0.669   | 50.9       |
| ConvKB       | 0.750   | +12.1%     |
| Our CapsE    | 0.766   | +14.5%     | 62.1       |

Table 3: Experimental results on the test set. [⋆] denotes the results reported in Vu et al. (2017). The subscripts denote the relative improvement over our TransE result.

Figure 4: Learning curves on the validation set with the initial learning rate at $5e^{-5}$.

(i.e., not updating) query and document embeddings during training for TransE, ConvKB and CapsE. In addition, as mentioned by Bennett et al. (2012), the more recently clicked document expresses more about the user current search interest. Hence, we make use of the user clicked documents in the training set with the temporal weighting scheme proposed by Vu et al. (2015) to initialize user profile embeddings for the three embedding models.

Hyper-parameter tuning: For our CapsE model, we set batch size to 128, the number of capsule layers to 2 ($L = 2$), and also the number of neurons within the capsule in the second capsule layer to 10 ($d_2 = 10$). The number of iterations in the dynamic routing algorithm is also set to 3 ($m = 3$). For training model, we use the Adam optimizer with the initial learning rate $\in \{5e^{-6}, 1e^{-5}, 5e^{-5}, 1e^{-4}, 5e^{-4}\}$. We also use ReLU as the activation function $g$. We select the number of filters $N \in \{50, 100, 200, 400, 500\}$. We run model up to 200 epochs and perform a grid search to choose optimal hyper-parameters on the validation set. We monitor the MRR score after each training epoch and obtain the highest MRR score on the validation set when using $N = 400$ and the initial learning rate at $5e^{-5}$.

We employ the TransE and ConvKB implementations provided by Nguyen et al. (2016b) and Nguyen et al. (2018) and then follow their training protocols to tune hyper-parameters for TransE and ConvKB, respectively. We also monitor the MRR score after each training epoch and attain the highest MRR score on the validation set when using margin = 5, $l_1$-norm and SGD learning rate at $5e^{-3}$ for TransE; and $N = 500$ and the Adam initial learning rate at $5e^{-4}$ for ConvKB.

4.2.4 Main results

To illustrate our training progress, we plot performances of CapsE on the validation set over epochs in Figure 4. We observe that the performance is improved with the increase of number of filters since capsules can encode more useful properties for a large embedding size.

Table 3 presents the experimental results of the baselines and our model. Embedding models TranE, ConvKB and CapsE produce better ranking performances than traditional learning-to-rank search personalization models CI and SP. This indicates a prospective strategy of expanding the triple embedding models to improve the ranking quality of the search personalization systems. Our CapsE model achieves the highest performances in both MRR and Hits@1 (with significant improvements over all strong baselines, i.e., $p < 0.05$ with the paired t-test). Specifically, our MRR and Hits@1 scores are higher than that of TransE (with relative improvements of 14.5% and 22% over TransE, respectively).

5 Conclusion and future work

We have shown the first successful applications of capsule networks for knowledge graph completion and search personalization, by proposing the embedding model CapsE to model relationship triples. Experimental results show that our CapsE obtains state-of-the-art results for knowledge graph completion on two benchmark datasets: WN18RR and FB15k-237. We also show the effectiveness of
our proposed model for the search personalization, in which our CapsE outperforms the competitive baselines on the dataset SEARCH17 of the web search query logs. We have also demonstrated that CapsE is capable to effectively model many-to-many relationships. We plan to adapt CapsE to other tasks where we could also formulate each task as a triple-based problem e.g., the personalized item suggestion/auto-completion in recommender systems (i.e., (query, user profile, clicked item)). Our code is available at: https://anonymous-url/.

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