Modelling dark current and hot pixels in imaging sensors

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Abstract: A Gaussian mixture model with a structured covariance matrix was used to analyse image data recorded by a digital sensor under darkness to model the effects of temperature and duration of exposure on the expected value and on the variance of the sensor dark current, separately for ordinary and possibly defective pixels. The model accounts for two components of variance within each latent type: random noise in each image and lack of uniformity within the sensor; both components are allowed to depend on experimental conditions. The results seem to indicate that the dependence of the expected value of dark current on duration of exposure and temperature cannot be represented by a simple parametric model. The latent class model detects the presence of at least two types of hot pixels. If we order the latent classes in decreasing order of the class weights, the corresponding expected values and variances increase. The covariance structure that emerges from our analysis has an important implication: the sign and the relative size of pixels deviations from uniformity are invariant to experimental conditions.

Key words: dark current, hot pixels, dark frames, Gaussian mixtures, components of variance, latent class models

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1 Introduction

Digital sensors used both in ordinary cameras and in scientific imaging suffer from several anomalies known as dark current, hot pixels and thermal noise, see for instance Hochederz et al. (2014) and Rauscher et al. (2011) for an accurate description based on physical models. In short, dark current denotes a signal which is detected even if no light is hitting the sensor and is known to increase with duration of exposure at a rate depending on temperature. During long exposures, a very small minority of pixels may show a very large signal even in perfect darkness; these are usually called hot pixels because they show up bright in a dark image; hot pixels may be seen as being occasionally defective; see Dunlap et al. (2012) for a hypothetical model of
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hot pixel distribution. In addition, the amount of light recorded by a given pixel is affected by quantization errors due to analog-to-digital conversion and by additional random fluctuations known as thermal noise, whose effects are known to increase with temperature and duration of exposure.

All of these limitations may be almost negligible when imaging daylight scenes where the signal-to-noise ratio is usually large, but they become a serious problem in scientific applications, like astronomical imaging, when the amount of perceivable light from far away galaxies may be close to that of the background sky. Anomalies of digital sensors may also impact on other emerging technological areas, such as biometrics, where sensor ageing results in spiky shot noise pixels (see Kauba and Andreas (2017), Fridrich (2013) and Bergmüller et al. (2014)). The usual way to cope with these problems in astronomical imaging is to obtain an estimate of the expected value of the dark current and of the position of hot pixels by averaging a set of images taken under the same conditions of duration and temperature but in complete darkness, see for instance Berry and Burnell (2000, Chapter 6). For an overview of the literature on more sophisticated methods for coping with dark current, hot pixels and noise, see, for instance, Burger et al. (2011), Chen et al. (2015), Widenhorn et al. (2010).

This article is an attempt to formulate a flexible statistical model that describes the effect of temperature and duration of exposure on dark current, hot pixels and thermal noise. The objective in Hanselaer et al. (2014) is similar, however their models are applied to the overall averages (over pixels and images acquired under constant conditions) as depending on the duration of exposure and temperature. Their approach does not take into account that the effect of experimental conditions on ordinary and hot pixels may be very different. Here, instead, the possible presence of sub-populations of pixels is explicitly taken into account by a Gaussian mixture model (Fraley and Raftery, 2002) and the issue of how many latent components to consider is examined in some depth; in addition, it is assumed that experimental conditions may affect both the mean and the variance components. Gaussian mixture models have been used in the analysis of astronomical images by Švihlák (2009), to model the distribution of wavelets coefficients, thus his normal distributions are univariate. A simple statistical procedure to detect hot pixels has been proposed in Leung et al. (2009): essentially, for each pixel, they compute a measure of discrepancy between its signal and the average on a suitable set of its neighbours. Hot pixels are detected by setting a threshold on the distribution of such discrepancies.

In this article, special attention was given to the covariance structure; background knowledge suggests the presence of independent error terms in the value recorded by each pixel in each image together with pixel specific random effects, at least in images taken under the same experimental conditions. A reviewer suggestion that observations taken under different experimental conditions might also be correlated was confirmed by a preliminary exploratory analysis indicating that at least among ordinary pixels (which account for over 98.5% of the total) correlations across experimental conditions may be substantial and follow a very specific pattern which will be discussed in detail.

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The model was applied to a set of experimental data derived from images taken under complete darkness with a monochromatic Atik 314L+, a good quality charge-coupled device (CCD) designed for astronomical imaging. The results of the analysis, among other things indicate the following: (a) it is possible to detect two categories of hot pixels, where the less frequent ones are also the most deviant; (b) although the expected values within each latent class increase with duration of exposure and temperature, no simple parametric function seems to fit adequately; (c) experimental conditions affect both variance components, though in a different manner.

The article is organized as follows: after describing the data structure in Section 2 and the model in Section 3, the results of the analysis are presented in Section 4, followed by a short discussion.

2 The data

The experiments were performed on a monochromatic Atik 314L+ CCD camera having an array of $1392 \times 1040$ square pixels of 6.45 micron with a 16-bit analog-to-digital converter. In principle, after taking an image, the output recorded by each pixel is an integer in the range $[0, 2^{16} - 1]$; however the internal processor adds an offset of about 263 to prevent negative values.

Because exposures between 5 and 10 minutes are usual in astronomical imaging, duration of exposures were chosen to be 3, 300 and 600 seconds. Considering that the camera can be cooled up to $27^\circ$C below ambient temperature, images were taken at $-10^\circ$C, $0^\circ$C and $10^\circ$C. By combining duration of exposure and temperature, there are 9 different experimental conditions. In addition, a sequence of images was taken at $-10^\circ$C using the minimum allowed duration because, under these conditions, the dark current can be expected to be almost negligible.

For each of the 10 different experimental conditions, always under complete darkness, a sequence of 30 images was recorded by setting temperature, duration and number of exposures on a specific software. The original data matrix contains 300 images taken on each of the $1447680$ pixels; because this is a rather large dataset for being easily handled on an ordinary computer, a random sample consisting of $100000$ pixels and 10 images under each experimental conditions were selected without replacement to avoid induced correlations. The resulting data may be arranged into a matrix of $100000$ observations and 100 variables.

Formally, for each experimental condition $e = 1, \ldots, 10$, determined by temperature and duration of exposure, $r$ images were recorded in complete darkness under constant conditions on a collection of $n$ pixels which may belong to $K$ different latent types. What is observed is the digitized signal recorded by each pixel in each image.

The data used in this article together with a set of MATLAB functions that may be used to compute the maximum likelihood estimate of the model described in the next section is available at http://www.statmod.org/smij/archive.html.
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3 The statistical model

Let $y_{iej}$, $i = 1, \ldots, n$, $e = 1, \ldots, 10$, $j = 1, \ldots, r$, denote the value recorded by the $i$th pixel under experimental condition $e$ in the $j$th image of a sequence. Though the $y_{iej}$ originate from an analogue-to-digital converter and are thus integers, they may be treated as approximately continuous measurements. Let $y_i$ be the vector with elements $y_{iej}$ arranged in lexicographic order of the $e$, $j$ indices.

To account for the fact that the sensor may be composed of distinct sub-populations of pixels which behave differently, it is assumed that there are $K$ latent classes. Let $G_k$, $k = 1, \ldots, K$, denote the assumption that a given pixel belongs to the $k$th latent population, and assume that

$$ y_i \mid G_k \sim N(\mu_k \otimes 1_r, \Sigma_k), \quad k = 1, \ldots, K, $$

(3.1)

where $\otimes$ denotes the Kronecker product, $\mu_k$ is a vector of size 10, with one element for each experimental condition, $1_r$ is a vector of $r$ 1s and $\Sigma_k$ is a square matrix of size 100.

3.1 The covariance matrix

Though the parameters of unconstrained Gaussian mixtures are identifiable (see Yakowitz and Spragins, 1968, Proposition 2), a preliminary investigation was used to find a parsimonious covariance structure which takes into account the specific nature of the measurements.

It may be expected that observations from the same pixel taken under constant conditions have the same variances and are equally correlated, even within the same latent class, because the lack of uniformity in the sensor induces pixel specific random effects; no prior knowledge is available on how the lack of uniformity in the sensor interacts with experimental conditions. To formalize the problem, assume that

$$ y_i - \mu_k \otimes 1_r = \epsilon_i \otimes 1_r + \epsilon_i, $$

where the vectors $\epsilon_i$, $\epsilon_i$ have elements $\epsilon_{ie}$ and $\epsilon_{iej}$ which denote, respectively, the pixel and the image-specific error components. Let

$$ \text{Var}(\epsilon_{iej} \mid G_k) = \sigma_{ke}^2, \quad \text{Var}(\epsilon_{ie} \mid G_k) = \tau_{ke}^2; $$

assume also that the elements of $\epsilon_i$ are mutually independent and also independent from the elements of $\epsilon_i$.

Concerning the relation between the elements of $\epsilon_i$, two alternative hypothetical models were considered:

- $H_1$: when experimental conditions are changed, the sensor is re-settled so that observations from the same pixel under different experimental conditions are independent, which implies that $\text{E}(\epsilon_{ie} \epsilon_{ie'} \mid G_k) = 0$;
• $H_M$: the sign and the relative size of the deviations of each pixel from uniformity is invariant to experimental conditions, which implies that $E(\varepsilon_{ie} \varepsilon_{ie'} \mid G_k) = \tau_{ke} \tau_{ke'}$.

Under $H_I$ we expect the correlation matrix to be block diagonal with a block for each experimental condition, while $H_M$ implies that the covariance matrix within latent class $k = 1, \ldots, K$ should have the form

$$\Sigma_k = \text{diag}(\sigma^2_k) \otimes I_r + (\tau_k \tau_k') \otimes J_r,$$

(3.2)

where $I_r$ denotes an identity matrix of size $r$ and $J_r$ is a matrix of 1s of the same size. The elements of $\sigma^2_k$ measure the image-specific uncertainty, while those of $\tau_k$ measure the lack of uniformity in the sensor. In the given context, $\Sigma_k$ is of size $100 \times 100$ but, because of the assumed structure, it depends only on 20 parameters.

An informal assessment of $H_M$ against $H_I$ was obtained by inspecting the non-parametric estimate of the covariance matrix within a subset of pixels whose overall average did not exceed 287 (about 99.08% of the total), which are unlikely to be defective. A small portion of this matrix for four different experimental conditions and two images selected at random within each sequence of 10 is displayed in Table 1. The fact that correlations in the $2 \times 2$ off-diagonal blocks are not substantially different from those in the blocks along the main diagonal seems to rule out $H_I$.

Additional evidence in favour of $H_M$ was provided by the following procedure: (a) start from the $100 \times 100$ estimate of the covariance matrix and compute the average within each $10 \times 10$ block, by ignoring the diagonal elements appearing in the blocks along the main diagonal, which are affected by image-specific variances; (b) arrange the elements of the resulting $10 \times 10$ matrix into a vector by moving along the rows. If $H_M$ was true, the log of this vector should, approximately, follow a linear model with the row effects equal to the corresponding columns effects and no interaction. The exponential of the residual variance after fitting this model equals 1.21; because the averaged covariances are all positive with an overall average of about 16.36, the assumed structure seems to fit reasonably well. Further evidence in favour of $H_M$ will be examined in the final section.

Table 1  Portion of the non-parametric estimate of the covariance matrix, under four experimental conditions and for two randomly selected images within each sequence of 10

|                  | 0.01°, −10°C | 3°, 0°C | 300°, −10°C | 3°, 10°C |
|------------------|--------------|---------|-------------|----------|
| 0.01°, −10°C     | 267.2        | 1.4     | 1.6         | 1.9      | 2.3      | 3.2      | 7.0      | 7.2      |
| 3°, 0°C          | 1.4          | 268.2   | 2.1         | 0.9      | 2.5      | 5.7      | 8.3      | 9.1      |
| 300°, −10°C      | 1.6          | 2.1     | 302.0       | 4.1      | 4.7      | 4.0      | 17.9     | 16.8     |
| 3°, 10°C         | 1.9          | 0.9     | 4.1         | 304.1    | 4.4      | 4.6      | 19.1     | 18.7     |
|                  | 2.3          | 2.5     | 4.7         | 4.4      | 293.4    | 10.1     | 16.3     | 15.6     |
|                  | 3.2          | 5.7     | 4.0         | 4.6      | 10.1     | 295.9    | 14.9     | 15.0     |
|                  | 7.0          | 8.3     | 17.9        | 19.1     | 16.3     | 14.9     | 471.8    | 82.7     |
|                  | 7.2          | 8.1     | 16.8        | 18.7     | 15.6     | 15.0     | 82.7     | 468.6    |
3.2 Regression models

The dependence of the mean and the variance components on experimental conditions may be formulated in a flexible way within the general assumption that the elements of $\mu_k$, $\sigma_k$, $\tau_k$, $k = 1, \ldots K$, are suitable functions of duration and temperature. To model the dependence of the two variance components on covariates, a log link may be used to ensure that estimates of variance components are non-negative (Aitkin, 1987),

$$\sigma_{ke} = \exp(z'e\alpha_k), \quad \tau_{ke} = \exp(z'e\gamma_k),$$

where $\alpha_k$ and $\gamma_k$ are both of size 3, $z_e = (1 t_e d_e)'$ with $t_e, d_e$ denoting temperature and duration in the $e$th experiment.

The dependence of $\mu_{ke}$ on temperature and duration of exposure was investigated in some detail by comparing a few parametric models against a non-parametric one. The notion that dark current increases linearly with the duration of exposure is generally accepted in the literature, see for instance Hochedez et al. (2014), p. 2. The analysis by Hanselaer et al. (2014) seems to support this property, however their conclusions are based on the behaviour of the overall averages, including hot pixels. It is also well known that the rate of growth of dark current increases with temperature. For instance, Hanselaer et al. (2014) used a forth degree polynomial which, however, seems to be more an attempt at fitting than interpreting the phenomenon.

The assumption of normality will be submitted to some scrutiny in the actual application where it emerges that the assumption seems to hold with satisfactory accuracy for ordinary pixels but may fail when pixels start to saturate, that is receiving an amount of signal (dark current) close to their maximum capacity. This happens to some degree also to ordinary pixel at $10^\circ$C and long exposures.

3.3 The likelihood function and the EM algorithm

Let $\ell_{ki}$ denote the log-likelihood for the $i$th pixel conditional on $G_k$,

$$\ell_{ki} = -\frac{1}{2} \left[ \log |\Sigma_k| + (y_i - \mu_k \otimes 1_r)'\Sigma_k^{-1}(y_i - \mu_k \otimes 1_r) - r \log(\pi) \right],$$

where the expression for $\Sigma_k$ is given in (3.2). Though an explicit inverse of $\Sigma_k$ could, in principle, be derived by symbolic computation, the resulting expression would span several pages of code, while numerical computation requires less than 0.01 seconds on an average computer.

Let $\pi$ be the $K \times 1$ vector of prior probabilities and $\theta$ be the vector whose elements are the logits of $\pi$ with respect to the first entry. Formally we may write

$$\pi = \exp(G\theta)/(1_k \exp(G\theta)),$$
where $G$ is an identity matrix without the first column. Under the assumption that observations on different pixels are independent conditionally on $G_k$, the manifest log likelihood may be written as

$$L(\theta, \beta, \alpha, \gamma) = \sum_i \log \left( \sum_k \pi_k \exp(\ell_{ki}) \right).$$

Let $q_{ki} = \exp(\ell_{ki})$, $q_i$ denote the vector with elements $q_{ki}$; with these notations, we may write $L(\theta, \beta, \alpha, \gamma) = \sum_i \log(\pi' q_i)$.

The posterior probabilities that pixel $i$ belongs to latent type $k$ are computed in the E-step as

$$E_{ki} = \frac{\pi_k q_{ki}}{\sum_k \pi_k q_{ki}}.$$

Then the complete log likelihood to be maximized in the M-step has the form

$$L_C(\theta, \beta, \alpha, \gamma) = \sum_i \sum_k E_{ki} \log \left[ \pi_k q_{ki} \right].$$

Because $L_C$ can be factorized as

$$L_C(\theta, \beta, \alpha, \gamma) = \sum_k \log(\pi_k) \sum_i E_{ki} + \sum_k \left[ \sum_i E_{ki} \ell_{ki} \right],$$

an explicit estimate may be derived for the prior probabilities: $\hat{\pi}_k = E_{.k}/E.$; for the second component, maximization can be applied separately for each $k$ with respect to the corresponding parameters where the expression to be maximized takes the form $L_k = \sum_i E_{ki} \ell_{ki}$. An expression for the score vector will be made available as supplementary material and exploits known expressions for differentiating the determinant and the inverse of the covariance matrix. The modified Fisher-scoring algorithm of Forcina (2017) which uses the empirical information matrix was used and seems to work well in this context.

Because the expected information matrix could not be computed with reasonable accuracy due to numerical problems arising when computing derivatives of the manifest likelihood, a non-parametric bootstrap was used to estimate standard errors.

### 4 Analysis of the data

#### 4.1 Model selection

If we take for granted the notion that there are ordinary and hot pixels, one could set $K = 2$ latent classes. However, for instance, Leung et al. (2009) claim that they
detected two different types of hot pixels, with one type behaving in a less discrepant way. A formal procedure for choosing $K$ may be based on different criteria; it has been observed that the usual Bayesian information criteria (BIC) may tend to choose a larger $K$ when the number of observations is large relative to the number of parameters. Table 2, in addition to the value of BIC, gives also that of ICL proposed by Biernacki et al. (2000) and NEC, see Celeux and Soromenho (1996). The reason for BIC and ICL to be equal up to the first 5 significant digits is that changes in the overall entropy in the posterior probabilities is almost negligible relative to the log-likelihood.

Within each fitted model, the latent classes may be ordered so that $\hat{\mu}_k \leq \hat{\mu}_{k+1}$, $k = 1, \ldots, K - 1$, with inequalities holding element-wise; thus it seems reasonable to interpret latent class 1 as consisting of ordinary pixels and latent classes 2, $\ldots, K$ as consisting of increasingly hot pixels. With $K = 2$, the proportion of hot pixels is about 0.23%; with $K = 3$, the hottest pixels are about 0.19% of the total; in addition a new category of moderately hot pixels which account for about 1.23% of the total is detected. With $K = 4$, the relative size of the latent class containing the hottest pixels is about 0.0003; the model with $K = 5$ leads to detect even more extreme collections of hot pixels, but this leads to numerical instability, probably because most of the posterior probabilities of belonging to the last latent class are too small to provide a reliable estimate of the covariance matrix. A reasonable compromise between ICL and NEC is to choose $K = 3$.

Having selected a suitable number of latent classes, an attempt to find a parsimonious parametric model for $\mu_k$, $k = 1, \ldots, K$ was attempted because various conjectures about the expected value growing linearly with duration and exponentially with temperature have appeared in the literature, see for instance Hanselaer et al. (2014). The largest parametric model containing interactions between temperature and duration (LEI) allows the intercept parameter on the log scale to depend on duration

$$\mu_c = \mu_{bi} = \beta_1 + \exp(\beta_2)d_b + \exp(\beta_{2+b} + \beta_6t_1);$$

this model, which requires 6 parameters within each latent class, was compared against a non-parametric model (NPM) which does not impose any functional restriction on the 10 elements of $\mu_k$. Because LEI is nested within NPM, model selection may be based on the likelihood ratio; this is greater than 70,000 on 12
degrees of freedom, leading to rejection. The BIC leads to the same conclusion with (2.35 against 2.36) $\times 10^7$.

### 4.2 Main results

The estimated mean values as functions of duration, separately for each temperature, are displayed in Figure 1 with a panel for each latent type. The effect of duration on ordinary pixels is almost negligible and approximately linear among hot pixels. The effect of temperature is more substantial and at $10^\circ$C the effect can be perceived even among ordinary pixels.

Estimates of $\sigma_{ke}$ and $\tau_{ke}$ are displayed together in Figure 2 as function of duration, again separately for each temperature. At $-10^\circ$C and short duration, the standard error specific of individual images is slightly above 16 which is the read-out noise of this camera. Among ordinary pixels the lack of uniformity in the sensor is negligible. It is also small among moderately hot pixels, but is very sensitive to duration and temperature and in bad conditions becomes the main source of variability. The situation among very hot pixels is even more extreme, though these estimates are not sufficiently reliable because the sample size here is just about 19. Among ordinary pixels the effect of duration seems negligible on both components of variance while that of temperature seems a little more substantial. Among moderately hot pixels the effect of duration on image specific errors becomes non negligible but the stronger effect appears to be on the lack of uniformity of the sensor.

A set of quantile/quantile plots of averages among ordinary pixels conditionally on experimental conditions are displayed in Figure 3 to provide an informal critical assessment of the assumption of normality, at least for the sample averages of each set of 10 observations. Discrepancies can be detected mainly at $10^\circ$C, that is when dark current becomes substantial.
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Figure 2 Plots of $\hat{\delta}_k$ ('ise', solid lines) and $\hat{\pi}_k$ ('pse', dotted lines) versus duration, separately for each temperature

Figure 3 Quantile/quantile plot of sample averages for ordinary pixels under three different experimental conditions

5 Discussion

The quality of the data produced by imaging sensors are affected by dark current and hot pixels which introduce bias and additional noise. The data analysed in this article were acquired with a CCD device for astronomical imaging according to an experimental design aimed at studying the effect of temperature and duration of exposure. A finite mixture model fitted to the data led to detect, besides ordinary and really hot pixels, accounting for about 0.19% of the total, an intermediate category of moderately hot pixels (about 1.3% of the total) whose behaviour is deviant but not so extreme like the very hot pixels.

In the analysis it was assumed that, within each latent type, experimental conditions do not change the sign and the relative size of the random effect associated with individual pixels, they are simply re-scaled. This conjecture is supported by the
results of Burger et al. (2011): for different experimental conditions, they compute the average recorded by each pixel in 200 images and show that the ordering of pixels based on the recorded averages is not affected by experimental conditions. A similar experiment was designed with the Atik camera by taking 400 images in two different experimental conditions: 3" at 8°C and 2" at 12°C. A random sample of 100,000 pixels was selected and, for the subset of pixels which at 8°C have an average below 360 (99.982% of the total), a linear regression line was fitted to the averages at 12°C with respect to the averages at 8°C. Having averaged 400 images, we expect that the image specific error component is almost negligible so that the regression coefficient, 1.83, gives a measure of the amplification. For the same reason, we expect that the correlation coefficient between pixel averages, in the two experiments, is close to 1 and indeed it equals 0.967.

On the whole, our results indicate that both temperature and duration of exposure have a substantial effect on the mean behaviour and noise of hot pixels. However, recent developments in the acquisition and processing of astronomical images, based on applying small random shifts to the camera between images, so that a given hot pixels does not appear in the same position across images; this, combined with a suitable procedure for outlier rejection, can make hot pixels almost irrelevant.

The range of temperatures used in the experiment were limited by the fact that the Atik camera can, at most, achieve −27°C below the ambient temperature. An additional limitation in the range of experimental conditions is that, even with only 10 different experimental conditions, the whole dataset was difficult to handle; because of this, analysis was restricted to a random sample of pixels and replications.

Some of the diagnostic plots in Figure 3 indicate that the normal distribution may not provide an adequate approximation of the distribution of the response variable under certain experimental conditions. Though this happens mainly among hot pixels which account for a small minority of the observations, it may have affected certain results. A feasible alternative might be fitting mixtures of skew normal distribution (see Lee and McLachlan, 2013), but the implementation of these methods will require a considerable amount of additional work.

In recent years, CMOS technology is emerging as a possible improvement relative to CCD sensors. It is well known that also CMOS sensors suffer from dark current and hot pixels, however, to the best of our knowledge, no systematic investigation comparable to the one presented in this article has been conducted.

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