Transverse Oscillating Waves and the Effects of Capacitive Coupling in Long Intrinsic Josephson Junctions

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In this paper we investigate the excitation of longitudinal and transverse plasma waves in intrinsic Josephson junctions. We consider the outermost branch of IV characteristic (IVc) in current biased case and try to find the conditions in which plasma waves can be excited. We change the parameters of the system and get the corresponding breakpoint current at which the plasma waves start to initiate. We present specifically the modes containing only transverse waves where we can have radiation. As a result we find the range of parameters that the system can radiate.

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One of the topics in high Tc superconductors that has recently attracted a great interests in the literature is the observed terahertz wave emission from the layered structured high-temperature superconductors.[1] It's widely believed that cuprate high-temperature superconductors like Bi2Sr2CaCu2O8 (BSCCO) has a layered structure which leads to an intrinsic Josephson junction (IJJ) effect. [2] Applying current in z-direction we get AC Josephson oscillations that works in terahertz range of frequencies. Moreover, the parameters in intrinsic Josephson junctions are controlled by the atomic crystal structure rather than by leads and amorphous dielectric layer in artificial JJs. This results in the existence of different mechanisms of coupling among IJJs e.g. phonon coupling, capacitive coupling, inductive coupling, charge imbalance effect.[3] In this paper we consider inductive-capacitive coupling model where we can have longitudinal and transverse oscillations. Radiation from this system has already been investigated in literature [3, 8]. In this paper we investigate properties of the transverse and longitudinal plasma waves excited by a constant bias current in the multi-Josephson junctions. We restrict ourselves to the two-dimensional IJJ's, for simplicity, and consider that the length of the system in y-direction is small comparing to the magnetic penetration depth parallel to the ab-plane of the crystal. Therefore we assume the system to be homogeneous in y-direction. The layers are stacked along the z-direction (c-axis). We consider that the length of the junctions in x-direction Lx is comparable to the magnetic penetration depth and the phase-differences of junctions along the x-direction varies accordingly. It has been established that the junctions in these multi-junction systems are coupled both capacitively and inductively with each other. We show that the plasma waves inside the junctions are excited parametrically by the Josephson oscillations arising from the bias current.

The couplings can be incorporated into the dynamics of the phase-differences in IJJs, using the generalized Josephson relations. [4, 5, 6] i dz idt = 2eV y,i,i+1(x) − αC∇(2)V y,i,i+1(x) and ddy = 2μd(μyH y,i,i+1(x) − αL∇(2)H y,i,i−1(x)). Where the parameters αC and αL are respectively the capacitive and inductive coupling constants given by αC = e2d2 and αL = e2d2 with s, d, e, μ and λab being, respectively, the thickness of the superconducting layer and insulating layer, the dielectric constant of the insulating layers, the charge screening length of the superconducting layers and the magnetic penetration depth parallel to the ab-plane of the crystal.

The equation of motion for the gauge-invariant phase-differences ϕ(x, t) in dimension-less form is [4, 5, 6, 7, 8]:

\[
λ^2(\tilde{L}^{-1}ij)\frac{\partial^2\varphi_i}{\partial x^2} - (\tilde{C}^{-1})ij\frac{\partial^2\varphi_j}{\partial t^2} = \sin(\varphi_i) + \beta\varphi_i - I/I_c
\]

where \(\tilde{C} = 1 - α_C\nabla^2\) and \(\tilde{L} = 1 - α_L\nabla^2\) are respectively the capacitance and inductance of the junction and \(\nabla^2\) is the abbreviation of the second rank difference. This equation has been obtained from conservation relation where the total dimension-less current, \(\frac{I}{I_c}\), can be written in terms of the transverse current, \(\frac{d}{d} \frac{\partial H_z}{\partial x}\), displacement current, \(\frac{d}{d} \frac{\partial V_z}{\partial t}\), Ohmic current, \(\beta\varphi_i\), and the Josephson current, \(\sin(\varphi_i)\).

In this model we consider that an electron in the position \(x\) of \(i\)th layer can only jump to the same position on the next layer i.e. \(J_{i, i+1} = \sin(\varphi_i) + \beta\varphi_i\). We call this as local hopping approximation which is an appropriate approximation in cases of slow variation of phase difference in x direction. In this paper we use this approximation and neglect the fast varying solutions.

In the following we are going to get some information about the system of equations (1). It’s well known that in the case of short Josephson junctions the IVc has a multi branch structure corresponding to different configurations of the rotating and oscillating states among the junctions in z direction. It has been shown that neglecting the effects of boundary layers, the number of branches is equal to the number of the Josephson junctions.[2] But what about the long Josephson junction case? Does it lead to the same number of branches? In order to answer this question, let us first solve the equation of motion for one Josephson junction. The result of this solution is...
presented in Fig. (1). To obtain this result we have considered 100 points along the junction (x direction). Then using fourth order Rounge-Kuta algorithm we have solved the system of equations in time. This result shows another kind of branch structure which is well known and corresponds to different distributions of phase along the junction. In general we can distinguish two types of solutions: corresponding to flux-flow mode and oscillating distributions. It’s clear from this result that if we start from outermost branch we reach first the oscillating mode and then to the flux-flow mode. To clarify the situation we specify some regions of the IVc by letters (A), (B), (C), (D), (E), and (F) respectively and present, as left inset, the FFT analysis of the time dependence of \( \frac{V(x=0.02)-V(0.01)}{0.01} \) in units of Josephson frequency and as right inset, the distribution of voltage along the junction. We can see three, two and one flux-flow mode states at the regions (C), (D) and (F) respectively and standing oscillating waves at the regions (B) and (D). Although the number of fluxons can be seen directly from the distribution of voltage, we can also find it from the number of peaks in unit range of the FFT results i.e. \( N_{\text{peaks}} = 2N_{\text{fluxon-anti-fluxon}} \).

Let us now consider the multi layered structure and find the number of branches in IVc in cases of Long Josephson junctions. From above discussion we find that there are at least two mechanisms of branching in IVc i.e. (1) different configurations of the rotating and oscillating states among the junctions in z direction and (2) different distributions of phase along the junction in x direction. Should we consider both effects the number branches exceeds the number of junctions which is apparently not in accordance with experimental results. In the following we see that in the case of high numbers of junctions the number of branches tends to the number of junctions. We show this by noting that at large number of junctions, upon creation of longitudinal plasma wave at the breakpoint current the system immediately transits to another branch containing less resistive junctions. Therefore, should the system starts from an oscillating mode with longitudinal waves, the system does not reach other resonance regions corresponding to some other transverse modes and so the additional branches due to inductive coupling in x-direction does not appear. But what if the system in the outermost branch starts from a state with only transverse mode (without longitudinal mode) i.e. (n, 0)? It’s clear that, in this case, upon decreasing the current, the system continues to be in this state until it reaches to another resonance region which contains longitudinal plasma wave and then the system jumps to another branch corresponding to less number of resistive states.

From Fig. (1) we learned that the flux-flow modes appear far from the breakpoint current. Additionally we know that in multi Josephson junction case, upon creation of longitudinal plasma waves the system jumps to another state with less number of resistive junctions. In the following, we neglect the flux-flow modes and consider only the region close to breakpoint current of outermost branch in IVc. Later in this paper we can see that, if we consider the longitudinal coupling, the system in fact hardly reaches to the flux-flow regions and therefore the flux-flow modes play no effective role in multi Josephson junctions. As a result we start from points (A) and (B) in Fig. (1) and investigate their characteristics. Point (A) represents breakpoint current and the system at this current enters into the parametric resonance region. Throughout this paper we consider this point only and discuss about its features. In Fig. (1) we see that, the region (B) which we call radiation branch, has close characteristics to the point (A) and also it appears after the breakpoint current. In the following we discuss that this branch might be observable in IVc.

By the fact that on the outermost branch all of the junctions are in the resistive (rotating) state, we neglect the effect of boundaries (large number of junctions) and consider that initially, phases of the junctions are the same. We consider also that this phase does not depend on the position \( x \) i.e. \( \varphi_1(x, t) = \phi(t) \). Then we test the stability of such a state by adding a small term to the phase i.e. \( \varphi_1(x, t) = \phi(t) + \delta_1(x, t) \). Linearizing the equation with respect to this term and then making Fourier transformation with respect to \( x \) and \( i \) leads

\[
\frac{1}{C(k_z)} \frac{d^2\delta(k_z, k_z)}{d\tau^2} + \beta \frac{d\delta(k_z, k_z)}{d\tau} + (\varepsilon \frac{n^2}{L(k_z)}) \cos(\phi(\tau)) \delta = 0
\]

(2)
of ing solutions. Additionally, one can see that bigger values of $n$ shows breakpoint current versus parameter of the system. Fig. 2, 3, 4 we show curve are observable only. We note also that increasing $\alpha = 0.2$ is also an integer number. Due to absence of external magnetic field the derivative of $L$ we get $k_z = 2m \pi /N$. Where $m$ is also an integer number and $N$ is the number of junctions. $n$ and $m$ represent the transverse and longitudinal wave numbers respectively.

We found that this initial condition is stable at the big currents and unstable bellow some critical current. In Figs. 2, 3, 4 we show maximum of breakpoint current for different modes ($n, m$) where $n$ and $m$ are taking the values 0, 1, 2. Where $n_{max} = 2$ means that the number of Josephson junctions is 2 and $n_{max} = 2$ means that we are neglecting fast varying solutions. Additionally one can see that bigger values of $n$ appear at smaller $\beta$. Therefore, to see the results corresponding to bigger $n$ we need to choose smaller $\beta$ which is out of scope of this paper, because, we are neglecting fast varying solutions. In Figs. 2, 3, 4 we show especially the modes that longitudinal waves are absent i.e. the oscillation that can lead to radiation. We note that the parts of these curves that coincide with the black curve are observable only. We note also that increasing the number of junctions smears the $\beta$-dependence of the breakpoint current.

From experimental point of view, it might be interesting to know the width of the region in IVc that there’s electromagnetic radiation and it’s dependence on the parameters of the system. The resonance width is defined by the difference between breakpoint current and jump point current at which the system goes to another branch. We have already shown that in the presence of longitudinal plasma waves the width of the resonance region is getting zero with increasing the number of junctions. In Fig. 4 we show especially the modes that longitudinal waves are absent i.e. the oscillation that can lead to radiation. We note that the parts of these curves that coincide with the black curve are observable only. We note also that increasing the number of junctions smears the $\beta$-dependence of the breakpoint current.

\[
\frac{d^2 \phi}{dt^2} + \beta \frac{d\phi}{dt} + \sin(\phi) = I/I_c
\]  

(3)

where $C(k_z) = 1 + 2 \alpha C(1 - \cos(k_z)), L(k_z) = 1 + 2 \alpha L(1 - \cos(k_z)), \epsilon = \pi \lambda^2 / L_2$ and $n$ is an integer number. Due to absence of external magnetic field the derivative of phase at the boundaries of the junctions are zero. This boundary condition leads to $k_z = n \pi$. On the other hand, considering a periodic boundary condition for $z$-direction we get $k_z = 2m \pi /N$. Where $m$ is also an integer number and $N$ is the number of junctions. $n$ and $m$ represent the transverse and longitudinal wave numbers respectively.

We found that this initial condition is stable at the big currents and unstable below some critical current which we call breakpoint current. We find the breakpoint current at different parameters of the system. Fig. 2 shows breakpoint current versus $\beta$ at different capacitive coupling parameters. The black curve shows the maximum of breakpoint current for different modes ($n, m$) where $n$ and $m$ are taking the values 0, 1, 2. Where $n_{max} = 2$ means that the number of Josephson junctions is 2 and $n_{max} = 2$ means that we are neglecting fast varying solutions. Additionally one can see that bigger values of $n$ appear at smaller $\beta$. Therefore, to see the results corresponding to bigger $n$ we need to choose smaller $\beta$ which is out of scope of this paper, because, we are neglecting fast varying solutions. In Figs. 2, 3, 4 we show especially the modes that longitudinal waves are absent i.e. the oscillation that can lead to radiation. We note that the parts of these curves that coincide with the black curve are observable only. We note also that increasing the number of junctions smears the $\beta$-dependence of the breakpoint current.

From experimental point of view, it might be interest-
that the return current doesn’t depend strongly on the mode of transverse plasma waves. This leads to the conclusion that jump point current can be obtained from the breakpoint of the modes with the longitudinal plasma waves. Therefore we can get the range of current that we have only transverse modes and can be touched from the outermost branch. We can see the result of this calculation in Fig. 5. This model predicts that if we get the branch corresponding to radiation (only transverse waves) in the return current part of the outermost branch, we can then increase the current and have radiation until the end of the branch. But one should note that the end of branch in this case is not critical current ($I = I_c$). In addition, if we include the dissipation due to the radiation power, it will lead to increase in voltage in $\text{I}V_c$ and therefore decrease in the length of the radiation branch. We note that although increasing the number of junctions has not strong effect on the peak of the $\beta$-dependence of $\Delta I_{\text{Radiation}}$ but it decreases the range of $\beta$ that $\Delta I_{\text{Radiation}}$ is nonzero. This shows that increasing the number of junctions makes the situation more difficult to get the radiation region.

Now that we know the condition for electromagnetic radiation one question arises: What is the frequency of oscillation at the breakpoint current and breakpoint region ($\omega_{\text{Jbp}}$)? Results of our numerical calculations e.g. (A) point in Fig. 4 shows that the Josephson frequency at the breakpoint in $\text{I}V_c$ follows the $(M + \frac{1}{2})\omega_{\text{Jbp}}$ relation, where $M$ is integer number. We note that this result corresponds to the breakpoint of outermost branch which is not stationary. The stationary solution which can be obtained from the original nonlinear equation and we call radiation branch, contains frequencies equal to $\frac{1}{2}M\omega_{\text{Jbp}}$.

In Fig. 6 we show the first frequency of the plasma waves ($M = 0$) in units of Josephson plasma frequency (black curves) and in units of breakpoint Josephson frequency (red curves). We can see that the frequency of plasma waves is close to half of the Josephson frequency at the breakpoint. The deviation is due to the fact that the system is current biased.

We conclude that upon decreasing the bias from the fully resistive state, the system enters into resonance between the Josephson frequency and the plasma waves. As a result we have stationary plasma waves that can contain longitudinal and transverse waves. Should the longitudinal wave exist, some of the junctions immediately switch back from the resistive into superconducting states. In case of transverse waves without longitudinal one, an additional branch, that we call the radiation branch, appears and the system starts to radiate electromagnetic power. Upon decreasing current on the radiation branch the system finally enters into the resonance regions corresponding to other modes of plasma waves that yields a transition to another state with less resistive states. Therefore in the long intrinsic Josephson junction systems where both longitudinal and transverse waves exist, the branching in $\text{I}V_c$ is mainly due to longitudinal waves and the only observable effect of transverse waves is accompanied by radiation.

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