A New Framework for the Performance Analysis of Wireless Communications under Hoyt (Nakagami-$q$) Fading

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Abstract

We introduce a new approach that allows to analyze the performance of a wireless link under Hoyt (Nakagami-$q$) fading in a very simple way. By exploiting the close relationship between the Hoyt distribution and the Rice $I_e$-function, we demonstrate that the squared Hoyt distribution can be constructed from a conditional exponential distribution. As a consequence, many performance metrics for Hoyt fading can be calculated by leveraging well-known results for Rayleigh fading by just performing a finite-range integral. We use this framework to obtain new results for some information and communication-theoretic metrics in Hoyt fading channels:

When analyzing the secrecy capacity of wireless Hoyt fading links we show that the outage secrecy capacity is mainly dominated by the distribution of the SNR at Bob; conversely, it becomes independent of the distribution of the SNR at Eve for sufficiently large values of the average SNR at Bob.

Then, we investigate the outage probability in Hoyt fading channels in the presence of background noise and arbitrarily distributed interference; we observe how the distribution of the interference impacts the outage probability for low and medium values of the SINR, while it is the distribution of the desired link which has a dominant effect for large values of the SINR.

Finally, when studying the channel capacity for different transmission policies, we observe that the capacity loss compared to Rayleigh fading is under 0.15 bps/Hz for $q > 0.5$ when using optimum power and rate adaptation, or optimum rate adaptation with constant power.

Index Terms

Hoyt Fading, Rice $I_e$-Function, Channel Capacity, Secrecy Capacity, Outage Probability.
I. INTRODUCTION

Among the plethora of distributions used to model the random fluctuations of the signal amplitude when transmitted through a wireless link, Rayleigh distribution is perhaps the most popular model when there is no direct line-of-sight (LOS) between the transmit and receive ends. In this case, the channel response is characterized by means of a zero-mean circularly-symmetric complex Gaussian random variable (RV). Besides having a physical interpretation, explained by the consideration of multiple scatterers with none of them being dominant, one of the advantages of such fading model relies on its comparatively simple analytic manipulation, as the received signal-to-noise ratio (SNR) is exponentially distributed.

A natural extension of this model is known as Hoyt [1] or Nakagami-\(q\) fading [2], used to model short-term variations of radio signals resulting from the addition of scattered waves which can be described as a complex Gaussian RV at the receiver where the in-phase and quadrature components have zero mean and different variances, or equivalently, where the in-phase and quadrature components are correlated. This distribution is commonly used to model signal fading due to strong ionospheric scintillation in satellite communications [3] or in general those fading conditions more severe than Rayleigh, and it includes both Rayleigh fading and one-sided Gaussian fading as special cases.

Both Rayleigh and Hoyt fading have been extensively investigated in the last few decades [4]; however, while the derivation of information and communication-theoretic performance metrics such as channel capacity and outage probability (OP) is usually tractable mathematically for the Rayleigh case, it becomes way more complicated to analyze the very same scenario when assuming Hoyt fading. For example, while the Shannon capacity of adaptive transmission techniques with diversity combining in Rayleigh channels has been known for years ever since the work by Alouini and Goldsmith [5], the equivalent expressions for Hoyt fading channels have the form of complicated infinite series expressions [6], [7], [8] even for single-antenna receivers.

Another clear example of such inconvenience arises when analyzing the OP with co-channel interference and background noise: the solution for a Rayleigh-distributed fading link with arbitrarily distributed interference is expressed directly in terms of the moment-generating function (MGF) of the aggregate interference [9]. Conversely, the analytical characterization of the OP of a Hoyt-distributed fading link with arbitrarily distributed interference and background noise requires the numerical computation of an inverse Laplace transform [9].

Dozens of papers have been published in the last years with the aim of analyzing very diverse scenarios where Hoyt fading is considered, for the sake of extending already known results for Rayleigh fading to this more general
situation. Compared to Rayleigh, Hoyt fading has an additional degree of freedom by allowing that the in-phase and quadrature components of the complex Gaussian random variable that models the channel gain have different variances. However, despite the relationship that can be inferred between both distributions, most analyses in the literature for Hoyt fading do not exploit this connection and usually require for tedious and complicated derivations. To the best of our knowledge, there is no standard procedure in the literature that takes advantage of the relationship between both distributions, and therefore the calculations for Hoyt fading usually have to be done from scratch.

In this paper, we derive a new, simple but powerful approach for the performance analysis of wireless communication systems under Hoyt fading. By exploiting the fact that the cdf of a squared Hoyt distributed random variable is a weighted Rice $I_e$-function, we demonstrate that the squared Hoyt distribution can be constructed from a conditional exponential distribution. Thus, we show that many performance metrics for Hoyt fading can readily be obtained by leveraging previously known results for Rayleigh fading, and computing a finite-range integral.

In some cases, the finite-range integral has analytical solution and hence, the expressions for Hoyt fading are of similar complexity to those obtained for Rayleigh scenarios. Otherwise, the results for Hoyt fading have the form of a finite-range integral with constant integration limits, over the performance metric of interest for the Rayleigh case. Therefore, their numerical computation is simpler than other alternatives that require the evaluation of infinite series or inverse Laplace transforms. As an additional advantage, the presented framework also permits to obtain upper and lower bounds of different performance metrics in a simple way. Using this general procedure, we provide new analytical results for three scenarios of interest in information and communication theory:

- We investigate the physical layer security of a wireless link in the presence of an eavesdropper, where both the desired and wiretap links are affected by Hoyt fading. Known analytical results are available for different scenarios such as Rayleigh [10], Nakagami-$m$ [11], Rician [12], or Two-Wave with Diffuse Power [13] fading models. However, to the best of our knowledge, there are no results in the literature for the physical layer security in Hoyt fading channels.

- We proceed to evaluate the OP of a Hoyt-faded wireless link affected by arbitrarily distributed co-channel interference and background noise. Specifically, we show that the OP in this general scenario will be given in terms of a finite-range integral over the MGF of the aggregate interference, admitting a very simple evaluation for well-known general distributions such as $\eta - \mu$ or $\kappa - \mu$ [14].

- We calculate the Shannon capacity of adaptive transmission techniques in Hoyt fading channels, thus extending the results given in [5]. Unlike [6], [7], [8], no infinite series expressions need to be evaluated.
The rest of this paper is organized as follows. In Section II the Rice $I_e$-function is reviewed and its connection with the Hoyt distribution is exploited to develop a new framework to analyze Hoyt fading channels performance metrics. This framework is used in Sections III to V to obtain analytical results in the aforementioned scenarios. Numerical results are presented in Section VI, whereas the main conclusions are outlined in Section VII.

II. INTEGRAL REPRESENTATION OF THE SQUARED HOYT DISTRIBUTION

A. The Rice $I_e$-function

Let $k$ and $x$ be non-negative real numbers with $0 \leq k \leq 1$, the Rice $I_e$-function is defined as [15]

$$I_e(k, x) = \int_0^x e^{-t} I_0(kt) dt \tag{1}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind and zero order.

The Rice $I_e$-function admits different infinite series representations [16] [17], and it is not considered a tabulated function, in the sense that it is not included as a built-in function in standard mathematical software packages such as Matlab or Mathematica. However, after the appropriate change of notation this function can be written in compact form, as [18]

$$I_e(k, x) = \frac{1}{\sqrt{1 - k^2}} \left[ Q(\sqrt{ax}, \sqrt{bx}) - Q(\sqrt{bx}, \sqrt{ax}) \right] \tag{2}$$

or equivalently,

$$I_e(k, x) = \frac{1}{\sqrt{1 - k^2}} \left[ 2Q(\sqrt{ax}, \sqrt{bx}) - e^{-x}I_0(kx) - 1 \right], \tag{3}$$

where $a = 1 + \sqrt{1 - k^2}$, $b = 1 - \sqrt{1 - k^2}$ and

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} e^{-\frac{\alpha^2 + \beta^2}{2}} I_0(\alpha t) dt \tag{4}$$

is the first order Marcum $Q$-function.

Since both the modified Bessel function $I_0$ and the Marcum $Q$-function are tabulated functions, (2) and (3) can be considered as closed-form representations of the Rice $I_e$-function. However, subsequent manipulations of these expressions are generally complicated and in many situations it may be preferable to express the Rice $I_e$-function in integral form. Replacing $I_0$ in (1) by its integral representation, namely [19, eq. (8.431.5)]

$$I_0(z) = \frac{1}{\pi} \int_0^\pi e^{z \cos \theta} d\theta, \tag{5}$$

after some manipulation we can write [16]

$$I_e(k, x) = \frac{1}{\sqrt{1 - k^2}} - \frac{1}{\pi} \int_0^\pi \frac{e^{-x(1-k \cos \theta)}}{1 - k \cos \theta} d\theta, \tag{6}$$
which has the important advantages with respect to (1), as the integration limits do not depend on the arguments of the defined function, and the integrand is given in terms of elementary functions. However, for reasons that will become clear in the next subsection, a much more convenient representation of the Rice $I_e$-function for the purpose of this work is the one provided in the following proposition.

**Proposition 1:** The Rice $I_e$-function can be written in integral form as

$$I_e(k, x) = \frac{1}{\sqrt{1 - k^2}} \left[ 1 - \frac{1}{\pi} \int_0^\pi \exp \left( -x \frac{1 - k^2}{1 - k \cos \theta} \right) d\theta \right]. \quad (7)$$

**Proof:** Although not specifically stated in this form, this result follows from the identities provided by Pawula in [18, eqs. (2a)-(2d)] [20, eqs. (7)-(10)]. In particular, from [20, eqs. (9)-(10)] the following identity holds:

$$W \int_0^\pi \frac{e^{-U - V \cos \theta}}{U - V \cos \theta} d\theta = \int_0^\pi \exp \left( -\frac{W^2}{U - V \cos \theta} \right) d\theta, \quad (8)$$

where $W = \sqrt{U^2 - V^2}$. By identifying $U = x$ and $V/U = k$, and with the help of (6), the desired expression is obtained. \[\square\]

**B. An Integral Representation of the SNR Distribution in Hoyt Fading**

Under Hoyt fading, the SNR at the receiver side $\gamma$ follows a squared Hoyt distribution, and its pdf is given by

$$f(x) = \frac{1 + q^2}{2q\bar{\gamma}} \exp \left[ -\frac{(1 + q^2)^2 x}{4q^2\bar{\gamma}} \right] I_0 \left( \frac{(1 - q^4)x}{4q^2\bar{\gamma}} \right), \quad (9)$$

where $\bar{\gamma}$ is the average SNR and $q$ is the Hoyt shape parameter ranging from 0 to 1.

When $q = 1$, the Hoyt fading channel collapses to the Rayleigh fading case, for which the receive SNR has an exponential distribution with pdf

$$f(x) = \frac{1}{\bar{\gamma}} e^{-x/\bar{\gamma}}. \quad (10)$$

In the following lemma, we present an integral form of the pdf given in (9). We will show that it permits to analyze many important performance metrics of wireless links under Hoyt fading using previously known results for Rayleigh fading.

**Lemma 1:** Let $X|\theta$ be an exponentially distributed random variable, conditioned on $\theta$, with pdf

$$f(x|\theta) = \frac{1}{\gamma(\theta, q)} e^{-x/\gamma(\theta, q)}, \quad (11)$$

where $\theta$ is a random variable uniformly distributed between 0 and $\pi$, and

$$\gamma(\theta, q) \triangleq \bar{\gamma} \left( 1 - \frac{1 - q^2}{1 + q^2 \cos \theta} \right), \quad (12)$$
is the average of $X|\theta$. Then, the unconditional random variable $X$, with pdf
\[
f(x) = \frac{1}{\pi} \int_0^\pi \frac{1}{\gamma(\theta, q)} e^{-x/\gamma(\theta, q)} d\theta,
\] (13)
has a squared Hoyt distribution with average $\bar{\gamma}$ and parameter $q$, i.e., (13) is an alternative expression for the pdf given in (9). The cdf of $X$ will be given by
\[
F(x) = 1 - \frac{1}{\pi} \int_0^\pi e^{-x/\gamma(\theta, q)} d\theta.
\] (14)

**Proof:** The cdf of the SNR at the receiver side under Hoyt fading can be calculated as
\[
F(x) = \int_0^x \frac{1 + q^2}{2q^2} \exp \left[ -\frac{(1 + q^2)^2 t}{4q^2\bar{\gamma}} \right] I_0 \left( \frac{(1 - q^2)t}{4q^2\bar{\gamma}} \right) dt,
\] (15)
which can be written using the definition of the Rice $I_e$-function in (11) as
\[
F(x) = \frac{2q}{1 + q^2} I_e \left( \frac{1 - q^2}{1 + q^2}, \frac{(1 + q^2)^2}{4q^2\bar{\gamma}} x \right).
\] (16)

Using the alternative definition for the Rice $I_e$-function in (7), the cdf of the SNR can be written after some algebraic manipulation as in (14). Finally, by taking the derivative of (14), the desired pdf in (13) is obtained.

By comparing (10) and (13), Lemma 1 states that a Hoyt fading channel can be viewed as a finite-range integral of Rayleigh fading channels with continuously varying averages. Note that the factor that multiplies $\cos \theta$ in (12) coincides with the squared third eccentricity $\epsilon$ of the ellipse represented by the underlying complex Gaussian random variable of the Hoyt distribution [21], i.e. $\epsilon = \frac{1 - q^2}{1 + q^2}$. When $q = 1$, then (12) reduces to $\bar{\gamma}$ which corresponds to the Rayleigh case, i.e. a circularly symmetric random variable.

As a direct implication of Lemma 1 any performance metric of single-channel reception in Hoyt fading that can be obtained by averaging over the SNR pdf can be calculated from existing results for Rayleigh fading, by performing a finite-range integral. Since most performance metrics of interest for Rayleigh fading in the literature are usually given in closed-form, the proposed approach allows for easily extending the results to Hoyt fading in a very simple manner. This is formally stated in the following lemma.

**Lemma 2:** Let $h(\gamma)$ be a performance metric depending on the instantaneous SNR $\gamma$, and let $\overline{h}_R(\gamma)$ be the metric in Rayleigh fading with average SNR $\bar{\gamma}$ obtained by averaging over an interval of the pdf of the SNR, i.e.,
\[
\overline{h}_R(\gamma) = \int_a^b h(x) \frac{1}{\gamma} e^{-x/\gamma} dx,
\] (17)
with $0 \leq a < b \leq \infty$. Then, the metric in Hoyt fading with average SNR $\bar{\gamma}$, denoted as $\overline{h}_H(\bar{\gamma})$, can be calculated as
\[
\overline{h}_H(\bar{\gamma}) = \frac{1}{\pi} \int_0^\pi \overline{h}_R(\gamma(\theta, q)) d\theta.
\] (18)
Proof: The metric $h_H(\gamma)$ is obtained as

$$h_H(\gamma) = \int_a^b h(x)f(x)dx.$$  

(19)

where $f(x)$ is the pdf of a squared Hoyt random variable given in (9) or, equivalently, (13). Thus, we can write

$$h_H(\gamma) = \int_a^b h(x)\frac{1}{\pi} \int_0^\pi \frac{1}{\gamma(\theta,q)}e^{-x/\gamma(\theta,q)}d\theta dx,$$

(20)

and reversing the order of integration yields

$$h_H(\gamma) = \frac{1}{\pi} \int_0^\pi \left[ \int_a^b h(x)\frac{1}{\gamma(\theta,q)}e^{-x/\gamma(\theta,q)}dx \right] d\theta.$$  

(21)

By recognizing that the integral between brackets is actually $h_R(\gamma(\theta,q))$, (18) is finally obtained.

Lemma 2 provides a very simple approach for the performance analysis of Hoyt fading channels.

An interesting consequence of lemma 2 is the following corollary.

**Corollary 1:** The MGF of a squared Hoyt random variable of average $\overline{\gamma}$ and shape parameter $q$ can be written as

$$\phi(s) = \frac{1}{\pi} \int_0^\pi \frac{1}{1 - \gamma(\theta,q)s}d\theta.$$  

(22)

Proof: This result follows directly from Lemma 2 and the fact that the MGF of an exponentially distributed random variable of average $\overline{\gamma}$ is given by $(1 - \overline{\gamma}s)^{-1}$.

The interest of (22) relies on the fact that it actually provides an alternative demonstration of the integral representation of the pdf of a squared Hoyt random variable given in (13). Indeed, because of the way it has been constructed, it is clear that (22) is the MGF of a random variable which pdf is given by (13). On the other hand, the integral in (22) can be solved in closed-form, using [19, eq. (3.613.1)], yielding

$$\phi(s) = \left[ 1 - 2\overline{\gamma} + \frac{q^2(2\overline{\gamma})^2}{(1 + q^2)^2} \right]^{-1/2},$$

(23)

which is the well known MGF of a squared Hoyt random variable [4]. Therefore, from the uniqueness theorem of the MGF, (9) and (13) are actually the same pdf.

Another benefit of using the proposed framework relies in the fact that the calculations are based on an integration involving a bounded trigonometric function; hence, this permits to find simple upper and lower bounds of the performance metrics. These bounds can be found by taking into account that symbol error rate performance metrics are usually convex decreasing functions with respect to the SNR, whereas channel capacity metrics are typically concave increasing functions. The following proposition establishes a sufficient condition to determine the monotonicity and convexity of some important average performance metric functions.
Proposition 2: Let \( h(\gamma) \) be a performance metric depending on the instantaneous SNR \( \gamma \) and let \( \overline{h}_R(\overline{\gamma}) \) be defined as in Lemma 1. If \( h(\gamma) \) is a decreasing convex (increasing concave) function of \( \gamma \) in \([0, \infty)\), then \( \overline{h}_R(\overline{\gamma}) \) is a decreasing convex (increasing concave) function of \( \overline{\gamma} \).

Proof: If \( h(\gamma) \) is a decreasing convex function then the first and second order derivatives of \( h(\gamma) \) verify
\[ h'(\gamma) \leq 0, \quad h''(\gamma) \geq 0. \]
By a simple change of variables in (17), considering the interval \([0, \infty)\), we can write
\[ \overline{h}_R(\overline{\gamma}) = \int_{0}^{\infty} h(\gamma x)e^{-x}dx, \] (24)
and its first and second order derivatives verify
\[ \overline{h}'_R(\overline{\gamma}) = \int_{0}^{\infty} h'(\gamma x)x e^{-x}dx < 0, \] (25)
\[ \overline{h}''_R(\overline{\gamma}) = \int_{0}^{\infty} h''(\gamma x)x^2 e^{-x}dx > 0. \] (26)
Therefore, \( \overline{h}_R(\overline{\gamma}) \) is a decreasing convex function of \( \overline{\gamma} \).

Analogously, if \( h(\gamma) \) is an increasing concave function, then the first and second order derivatives of \( h(\gamma) \) verify
\[ h'(\gamma) \geq 0, \quad h''(\gamma) \leq 0. \] Thus, \( \overline{h}_R(\overline{\gamma}) \) is an increasing concave function.

Now we present the aforementioned bounds in the next lemmas:

Lemma 3: Let \( \overline{h}_R(\overline{\gamma}) \) and \( \overline{h}_H(\overline{\gamma}) \) be functions obtained by averaging a given function \( h(\gamma) \) in, respectively, Rayleigh and Hoyt fading channels, where \( \overline{\gamma} \) is the average SNR, and let \( \overline{h}_R(\overline{\gamma}) \) be a decreasing convex function. Then, the following inequality holds:
\[ \overline{h}_R(\overline{\gamma}) \leq \overline{h}_H(\overline{\gamma}) \leq \overline{h}_R\left(\frac{2q^2}{1 + q^2}\overline{\gamma}\right), \] (27)

Lemma 4: Let \( \overline{h}_R(\overline{\gamma}) \) and \( \overline{h}_H(\overline{\gamma}) \) be functions obtained by averaging a given function \( h(\gamma) \) in, respectively, Rayleigh and Hoyt fading channels, where \( \overline{\gamma} \) is the average SNR, and let \( \overline{h}_R(\overline{\gamma}) \) be a concave increasing function. Then, the following inequality holds:
\[ \overline{h}_R\left(\frac{2q^2}{1 + q^2}\overline{\gamma}\right) \leq \overline{h}_H(\overline{\gamma}) \leq \overline{h}_R(\overline{\gamma}). \] (28)

Proof: Let us first demonstrate (27): As \( \overline{h}_R(\overline{\gamma}) \) is a decreasing function of \( \overline{\gamma} \) and the lowest value of \( \gamma(\theta, q) \) is obtained for \( \theta = 0 \), an upper bound of \( \overline{h}_H(\overline{\gamma}) \) can be found as
\[ \overline{h}_H(\overline{\gamma}) = \frac{1}{\pi} \int_{0}^{\pi} \overline{h}_R(\gamma(\theta, q))d\theta \leq \frac{1}{\pi} \int_{0}^{\pi} \overline{h}_R(\gamma(0, q))d\theta \]
\[ = \overline{h}_R(\gamma(0, q)) = \overline{h}_R\left(\frac{2q^2}{1 + q^2}\overline{\gamma}\right). \] (29)
A lower bound of $\overline{h}_H(\gamma)$ can be found from Jensen’s inequality and taking into account that $\overline{h}_R(\gamma)$ is convex:

$$\overline{h}_R(\gamma) = \overline{h}_R\left(\frac{1}{\pi} \int_0^\pi \gamma(\theta, q) d\theta\right) \leq \frac{1}{\pi} \int_0^\pi \overline{h}_R(\gamma(\theta, q)) d\theta = \overline{h}_H(\gamma)$$

(30)

On the other hand, (28) can be obtained analogously when $\overline{h}_R(\gamma)$ is a concave increasing function.

The bounds in Lemmas 3 and 4 state that performance in Hoyt fading, for a given average SNR, will be bounded between that of Rayleigh fading with the same average SNR and that of Rayleigh fading when the average SNR is scaled by a factor $2q^2/(1 + q^2)$. Note also that the derived bounds are asymptotically exact as $q \to 1$.

We have introduced a general framework for the analysis of wireless communication systems affected by Hoyt fading. In the following sections, we use this approach to derive new results for difference performance metric of interest.

III. Secrecy Capacity

A. Problem Definition

We consider the problem in which two legitimate peers, say Alice and Bob, wish to communicate over a wireless link in the presence of an eavesdropper, say Eve, that observes their transmission through a different link. Let us denote as $\gamma_b$ the instantaneous SNR at the receiver for the link between Alice and Bob, and let $\gamma_e$ be the instantaneous SNR at the eavesdropper for the wiretap link between Alice and Eve.

Unlike the classical setup for the Gaussian wiretap channel [22], it is known that fading provides an additional layer of security to the communication between Alice and Bob [10], [23], allowing for a secure transmission even when Eve experiences a better SNR than the legitimate receiver Bob.

According to the information-theoretic formulation in [10], the secrecy capacity in this scenario is defined as

$$C_S = C_B - C_E,$$

(31)

where $C_B$ is the capacity of the main channel

$$C_B = \log (1 + \gamma_b),$$

(32)

and $C_E$ is the capacity of the eavesdropper channel

$$C_E = \log (1 + \gamma_e).$$

(33)
For the sake of simplicity, we assumed a normalized bandwidth \( B = 1 \) in the previous capacity definitions. Since channel capacity is by definition a non-negative metric, the secrecy capacity for a given realization of the fading links is therefore given by

\[
C_S = \begin{cases} 
\log (1 + \gamma_b) - \log (1 + \gamma_e) & , \gamma_b \geq \gamma_e \\
0 & , \text{o.w.}
\end{cases}
\]

(34)

In [10], [23], the physical layer security of the communication between Alice and Bob in the presence of Eve was characterized in terms of several performance metrics of interest, assuming that both wireless links undergo Rayleigh fading. Specifically, closed-form expressions were derived for the probability of strictly positive secrecy capacity \( P(C_S > 0) \), and for the outage probability of the secrecy capacity \( P(C_S < R_S) \), where \( R_S \) is defined as the threshold rate under which secure communication cannot be achieved. As these expressions will be used in the forthcoming analysis, we reproduce them for the readers’ convenience

\[
P(C_S > 0) = \frac{\bar{\gamma}_b}{\bar{\gamma}_b + \bar{\gamma}_e},
\]

(35)

\[
P(C_S < R_S) = 1 - \frac{\bar{\gamma}_b}{\bar{\gamma}_b + 2R_S \bar{\gamma}_e} \exp \left( -\frac{2R_S - 1}{\bar{\gamma}_b} \right),
\]

(36)

where \( \bar{\gamma}_b \) and \( \bar{\gamma}_e \) are the average SNRs at Bob and Eve, respectively.

We note that \( P(C_S > 0) = 1 - P(C_S < R_S)_{R_S=0} \); hence, the probability of strictly positive secrecy capacity will be considered as a particular case of the outage secrecy capacity.

**B. Secrecy Capacity Analysis**

Let us consider the scenario where the wireless links experience a more severe fading than Rayleigh, say Hoyt, where \( q_b \) and \( q_e \) represent the Hoyt shape parameters for the desired and eavesdropper links, respectively. We also define the eccentricities associated with both Hoyt distributions as \( \epsilon_b = \frac{1-q_b^2}{1+q_b^2} \) and \( \epsilon_e = \frac{1-q_e^2}{1+q_e^2} \).

According to the general framework introduced in the previous section, the outage secrecy capacity in Hoyt fading channels is given by

\[
P(C_S < R_S) = 1 - \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \exp \left( -\frac{2R_S - 1}{\gamma_b(1-\epsilon_b \cos \theta_b)} \right) \times \frac{\gamma_b(1-\epsilon_b \cos \theta_b)}{\gamma_b(1-\epsilon_b \cos \theta_b) + 2R_S \gamma_e(1-\epsilon_e \cos \theta_e)} d\theta_e d\theta_b.
\]

(37)
We observe that the integral over $\theta_e$ can be solved, and hence we obtain

$$P(C_S < R_S) = 1 - \frac{1}{\pi} \int_{0}^{\pi} \frac{\hat{\gamma}_b(\theta)}{\hat{\gamma}_b(\theta) + 2R_S\hat{\gamma}_e} \frac{1}{\sqrt{1 - \left(\frac{\hat{\gamma}_b(\theta) + 2R_S\hat{\gamma}_e}{\hat{\gamma}_b(\theta) + 2R_S\hat{\gamma}_e}\right)^2}} d\theta,$$

where $\hat{\gamma}_b(\theta) = \hat{\gamma}_b(1 - \epsilon_b \cos \theta)$. Hence, the outage secrecy capacity is given in terms of a very simple integral form. This result is new in the literature to the best of our knowledge, and shows the strength and versatility of the proposed framework to derive new performance metrics for Hoyt fading by leveraging existing results for Rayleigh fading.

Directly from (38), the probability of strictly positive secrecy capacity can be easily obtained as

$$P(C_S > 0) = \frac{1}{\pi} \int_{0}^{\pi} \frac{\hat{\gamma}_b(\theta)}{\hat{\gamma}_b(\theta) + \gamma_e} \frac{1}{\sqrt{1 - \left(\frac{\hat{\gamma}_b(\theta) + \gamma_e}{\hat{\gamma}_b(\theta) + \gamma_e}\right)^2}} d\theta.$$

Expressions (38) and (39) admit an easy manipulation, in order to extract insights on the effect of fading severity into the secrecy capacity. One clear example arises if we assume that the eavesdropper link suffers from a more severe fading compared to the desired link: this can be achieved by setting $q_b = 1$, and seeing what is the impact of $q_e$. In this case, the integral over $\theta$ disappears, yielding to a closed-form expression for the outage secrecy capacity, and hence for $P(C_S > 0)$:

$$P(C_S < R_S) |_{q_b=1} = 1 - \frac{\hat{\gamma}_b}{\hat{\gamma}_b + 2R_S\hat{\gamma}_e} \frac{\exp \left(\frac{-2R_S-1}{\hat{\gamma}_b}\right)}{\sqrt{1 - \left(\frac{\hat{\gamma}_b + 2R_S\hat{\gamma}_e}{\hat{\gamma}_b + 2R_S\hat{\gamma}_e}\right)^2}}.$$

Comparing (36) and (40), we observe that both have similar form, and the effect of the distribution of the fading for the eavesdropper link is captured by a multiplicative term that modulates the result for the Rayleigh case. Since this additional term is always larger than one, it is clear that for a fixed value of $\hat{\gamma}_b$ and $\hat{\gamma}_e$, the outage secrecy capacity $P(C_S < R_S)$ decreases with $q$. This illustrates the fact that when Eve suffers from a more severe fading, then the probability of having a secure communication between Alice and Bob grows.

A similar conclusion can be extracted when examining the probability of strictly secrecy capacity in this particular scenario:

$$P(C_S > 0) |_{q_b=1} = \frac{\hat{\gamma}_b}{\hat{\gamma}_b + \gamma_e} \frac{1}{\sqrt{1 - \left(\frac{\gamma_e}{\hat{\gamma}_b + \gamma_e}\right)^2}}.$$

Again, the effect of considering $q_e < 1$ (i.e. a more severe fading than Rayleigh for the eavesdropper link) causes that $P(C_S > 0)$ grows as $q_e$ is reduced.
IV. OUTAGE PROBABILITY

A. Outage Probability in Noise-Limited Scenarios

The outage probability is one of the most important performance metrics in wireless communications, and it is defined as the probability that the received SNR falls below a predefined threshold $\gamma_o$. Thus, the outage probability is given by $P_{out}(\gamma_o) = F(\gamma_o)$, where $F(\cdot)$ is the cdf of the received SNR. Therefore, the outage probability under Hoyt fading can be written, from (14), as

$$P_{out}(\gamma_o) = 1 - \frac{1}{\pi} \int_0^{\pi} e^{-\gamma_o/\gamma(\theta,q)} d\theta,$$

which can be very efficiently computed, as the integrand varies smoothly for all possible values of parameter $q$. Conversely, the integrand of the integral representation of the outage probability given in [24] becomes sharply peaked when $q$ is close to 0.

It is also interesting to note that, as the outage probability in Rayleigh fading (given by $1-e^{-\gamma_o/\gamma}$) is an increasing concave function with respect to $\gamma$, the outage probability in Hoyt fading can be bounded as

$$1 - e^{-\gamma_o/\gamma} \leq P_{out}(\gamma_o) \leq 1 - e^{-(1+q^2)\gamma_o/(2q^2\gamma)}.$$

Alternatively, the outage probability can be expressed in closed-form using (2) and (16) as

$$P_{out}(\gamma_o) = Q\left(a(q)\sqrt{\frac{\gamma_o}{\gamma}}, b(q)\sqrt{\frac{\gamma_o}{\gamma}}\right) - Q\left(b(q)\sqrt{\frac{\gamma_o}{\gamma}}, a(q)\sqrt{\frac{\gamma_o}{\gamma}}\right),$$

or, equivalently, from (3) and (16),

$$P_{out}(\gamma_o) = 2Q\left(a(q)\sqrt{\frac{\gamma_o}{\gamma}}, b(q)\sqrt{\frac{\gamma_o}{\gamma}}\right) - e^{-d(q)\gamma_o/\gamma} I_0\left(c(q)\sqrt{\frac{\gamma_o}{\gamma}}\right) - 1,$$

where

$$a(q) = \frac{1 + q}{2q} \sqrt{1 + q^2}, \quad b(q) = \frac{1 - q}{2q} \sqrt{1 + q^2},$$

$$c(q) = \frac{1 - q^4}{4q^2}, \quad d(q) = \frac{(1 + q^2)^2}{4q^2},$$

which were previously derived in [25] in a slightly different way. Note, however, that (42) is easier to compute than these closed-form expressions because the Marcum Q-function is actually a two-fold integral, as the Bessel function is an integral itself. Although a finite-range integral can be used to compute the Marcum Q-function [4, eqs. (4.39), (4.42)], the integrand is sharply peaked for low values of $q$, which complicates the computation. Mathematical software packages often use truncated infinite power series [26] to compute the Marcum Q-function.
B. Outage Probability with Co-Channel Interference

In many practical scenarios, the signal of interest is affected by co-channel interference. In this case the outage probability can be defined as the probability that the signal-to-interference-plus-noise ratio (SINR) falls below a threshold level $\gamma_o$. More formally, let $X$ denote the SNR of the signal of interest and $Y$ the total interference-to-noise ratio (INR). The outage probability will thus be defined as

$$P_{\text{out}}(\gamma_o) = P(X < \gamma_o(Y + 1)) = F_X(\gamma_o(Y + 1)),$$

(46)

where $F_X(\cdot)$ is the cdf of $X$. In scenarios where the background noise can be neglected, the signal-to-interference ratio (SIR) is typically considered instead of the SINR, which usually simplifies the analysis.

We derive in this section a simple expression for the outage probability for the case when the signal of interest undergoes Hoyt fading and there is an arbitrary number of interferers. Our analysis will be quite general, as we assume that background noise is not necessarily neglected and each interferer undergoes an arbitrary fading. The main result for this scenario is presented in the following proposition.

**Proposition 3:** Let the desired signal undergo Hoyt fading, and the interfering signals experience arbitrarily distributed fading. Then, the outage probability in the presence of co-channel interference and background noise is given by

$$P_{\text{out}}(\gamma_o) = 1 - \frac{1}{\pi} \int_0^\pi e^{-\gamma_o/\gamma(\theta, q)} \phi_Y \left( \frac{-\gamma_o}{\gamma(\theta, q)} \right) d\theta,$$

(47)

where $\phi_Y$ is the MGF of $Y$.

**Proof:** As $X$ follows a squared Hoyt distribution, from (14) we have that the conditional (on $Y$) outage probability can be written as

$$P_{\text{out}}(\gamma_o)\big|_{Y=y} = 1 - \frac{1}{\pi} \int_0^\pi e^{-\gamma_o(y+1)/\gamma(\theta, q)} d\theta.$$

(48)

Averaging over $Y$, the unconditional outage probability will be

$$P_{\text{out}}(\gamma_o) = 1 - \int_0^\infty \frac{1}{\pi} \int_0^\pi e^{-\gamma_o(y+1)/\gamma(\theta, q)} d\theta f_Y(y) dy,$$

(49)

where $f_Y(\cdot)$ denotes the pdf of $Y$. By interchanging the order of integration we can rewrite (49) as

$$P_{\text{out}}(\gamma_o) = 1 - \frac{1}{\pi} \int_0^\pi e^{-\gamma_o/\gamma(\theta, q)}$$

$$\times \left[ \int_0^\infty e^{-y\gamma_o/\gamma(\theta, q)} f_Y(y) dy \right] d\theta.\]$$

(50)

By noticing that the MGF of a positive random variable $\alpha$ is defined as $\phi_\alpha(s) = \int_0^\infty e^{st} f_\alpha(t) dt$, with $f_\alpha(\cdot)$ being the pdf of $\alpha$, (47) is obtained.
Proposition (3) allows to analyze the outage probability in Hoyt fading channels with \textit{arbitrarily distributed} co-channel interference in the presence of background noise. Since the MGF is given in closed-form the for most relevant fading distributions, then (47) can be easily evaluated as a finite-range integral. The particular case where the background noise can be neglected is presented in the next corollary:

\textbf{Corollary 2:} When the total interference power is much higher that the background noise and the latter can be neglected, the outage probability is given by

\begin{equation}
P_{\text{out}}(\gamma_o) = 1 - \frac{1}{\pi} \int_0^\pi \phi_Y \left( \frac{-\gamma_o}{\gamma(\theta, q)} \right) d\theta.
\end{equation}

\textit{Proof:} When the background noise can be neglected, the outage probability can be defined as

\begin{equation}
P_{\text{out}}(\gamma_o) = P(X < \gamma_o Y) = F_X(\gamma_o Y),
\end{equation}

and by following the same steps as in Proposition 3, (51) is obtained.

When \( L \) independent interferers are considered, the MGF of \( Y \) will be

\begin{equation}
\phi_Y(s) = \prod_{i=1}^L \phi_{Y_i}(s),
\end{equation}

where \( \phi_{Y_i}(-) \) is the MGF corresponding to the \( i \)-th interferer.

In recent years, several general fading models such as \( \eta - \mu \) or \( \kappa - \mu \) have been developed, showing a better fit to experimental measurements than traditional fading models in many different environments [14]. The \( \eta - \mu \) model includes Hoyt, Nakagami-\( m \), Rayleigh and one-sided Gaussian fading as particular cases, whereas the \( \kappa - \mu \) model includes Rician, Nakagami-\( m \) and Rayleigh as particular cases. In spite of their generality, these models have a MGF that can be expressed in simple terms [27]. The MGF of \( \eta - \mu \) is given by

\begin{equation}
\phi(s) = \left( \frac{4\mu^2 h}{(2(h-H)\mu - s\gamma)(2(h+H)\mu - s\gamma)} \right)\mu,
\end{equation}

and the distribution is defined in two different formats. In format 1 we have \( H = (\eta - 1 - \eta)/4 \) and \( h = (2 + 2^{-1} + \eta)/4 \), with \( 0 < \eta < \infty \), while in format 2: \( H = \eta/(1 - \eta^2) \) and \( h = 1/(1 - \eta^2) \), with \( -1 < \eta < 1 \).

For the \( \kappa - \mu \) fading model we have

\begin{equation}
\phi(s) = \left( \frac{\mu(1 + \kappa)}{\mu(1 + \kappa) - s\gamma} \right)\mu \exp \left( \frac{\mu^2\kappa(1 + \kappa)}{\mu(1 + \kappa) - s\gamma} - \mu \kappa \right),
\end{equation}

Introducing (54) and/or (55) as factors in (53) we can consider many different and general interfering scenarios.

Note that, as we have not impose any restriction on the interferers statistics, our outage probability expression also includes the case of correlated interferers, as long as the MGF of the total interference power is known. Fortunately,
there exist closed-form expressions for the MGF of the addition of correlated signal powers of traditional fading models, such as Rayleigh, Nakagami-$m$ or Rician [9], and more general fading models such as $\eta - \mu$ [28].

V. CHANNEL CAPACITY

The channel capacity in Rayleigh fading channels was characterized in [5] for different transmission policies. Even though closed-form expressions were attained for the Rayleigh case, the channel capacity in Hoyt fading channels does not lend itself to an easy evaluation. In fact, only infinite series expressions of very complicated argument are available in the literature [6] [7] to the best of our knowledge.

Based on the closed-form expressions given in [5] for Rayleigh fading, we now provide easy-to-compute expressions for the channel capacity considering different transmission policies in Hoyt fading channels.

A. Optimal Simultaneous Power and Rate Adaptation

With optimal transmission power and rate adaptation, the channel capacity under Hoyt fading for average transmit power constraint will be

$$C_{opra} = \frac{B}{\ln 2} \frac{1}{\pi} \int_0^\pi E_1 \left( \frac{\gamma_o}{\gamma(\theta, q)} \right) d\theta,$$

where $E_1(x)$ is the exponential integral function, which is commonly a built-in function in mathematical software packages, $B$ is the channel bandwidth and $\gamma_o$ is the optimal cutoff SNR level below which data transmission is suspended, which must satisfy

$$\frac{1}{\pi} \int_0^\pi \frac{e^{-\gamma_o/\gamma(\theta, q)}}{\gamma_o} d\theta - \frac{1}{\pi} \int_0^\pi \frac{1}{\gamma(\theta, q)} E_1 \left( \frac{\gamma_o}{\gamma(\theta, q)} \right) d\theta = 1.$$

A simple lower bound for the channel capacity in this case will be

$$C_{opra} \geq \frac{B}{\ln 2} E_1 \left( \frac{(1 + q^2)\gamma_o}{2q^2\gamma} \right).$$

In this strategy, when the received SNR is below $\gamma_o$ no data is sent and an outage is considered to occur. The outage probability can be calculated using the results in Section [IV]

B. Optimum Rate Adaptation with Constant Transmit Power

The channel capacity under optimum rate adaptation with constant transmit power (average channel capacity in flat fading) will be given by

$$C_{ora} = \frac{B}{\ln 2} \frac{1}{\pi} \int_0^\pi e^{1/\gamma(\theta, q)} E_1 \left( \frac{1}{\gamma(\theta, q)} \right) d\theta.$$
A simple lower bound can be found as
\[
C_{ora} \geq \frac{B}{\ln 2} e^{(1+q^2)/(2q^2 \gamma)} E_1 \left( \frac{1+q^2}{2q^2 \gamma} \right).
\]
(60)

C. Channel Inversion with Fixed Rate

When the channel state information is available at the transmitter side, it can adapt its power to maintain a constant received SNR. This transmission technique is not effective since much of the transmit power is wasted for the compensation of very deep fades.

A more effective policy is the use of channel inversion only above a fixed fading depth \( \gamma_o \), which is referred to as the truncated channel inversion technique. In this case, the channel capacity will be
\[
C_{tfr} = B \log_2 \left( 1 + \frac{1}{\frac{1}{\pi} \frac{1}{\gamma_0} \int_0^\pi \frac{1}{\gamma(\theta,q)} E_1 \left( \frac{\gamma_0}{\gamma(\theta,q)} \right) d\theta} \right) \times (1 - P_{out}(\gamma_o)),
\]
(61)
where \( P_{out}(\gamma_o) \) is given in (42), (44) and (45). In this transmission strategy, capacity is maximized for an optimal cutoff SNR \( \gamma_0^* \) which increases as a function of \( \gamma \).

Channel capacity for this transmission policy is actually not in the form given in (17); however, it can be checked that it is a concave increasing function of \( \gamma \), although the formal proof is very long and tedious. A slightly simpler lower bound can be obtained by using (43) in (61).

VI. Numerical Results

After describing how the proposed framework for the analysis of wireless links affected by Hoyt fading can be applied to obtain new results for some information and communication-theoretic performance metrics, now it is time to discuss the main implications that arise in practical scenarios.

First, we will focus on the scenario considered in Section III; specifically, we will evaluate the effect of considering that the links between Alice and Bob (and equivalently between Alice and Eve) can suffer from different fading severities, quantified by the parameters \( q_b \) and \( q_e \). In Fig. 1 the secrecy capacity outage derived in (38) is represented as a function of the average SNR at Bob \( \bar{\gamma}_b \), for different sets of values of the Hoyt shape parameters. We assume that the normalized rate threshold value used to declare an outage is \( R_S = 0.1 \), and an average SNR at Eve \( \bar{\gamma}_e = 15 \) dB.

For a given value of \( q_b \), we observe two different effects depending on the magnitude of \( \bar{\gamma}_b \): in the low-medium SNR region, we see how a lower value of \( q_e \) (i.e. a more severe fading in the eavesdropper link) makes the
occurrence of a secrecy outage to be less likely. Hence, \( \mathcal{P}(C_S < R_S) \) decreases with \( q_e \) for a given \( \bar{\gamma}_b \); we also note how the secrecy in this region is barely affected by the value of \( q_b \).

Conversely, in the large SNR region we observe how the outage secrecy probability is mainly dominated by the fading severity of the desired link \( q_b \). In this region, it is the distribution of \( \gamma_b \) the dominant factor in the secure communication between Alice and Bob.

The probability of strictly positive secrecy capacity given in (39) is evaluated in Fig. 2, for the same set of parameter values considered in the previous figure.

We can extract similar conclusions with regard of the effects of the fading severity in the desired and eavesdropper links. For low values of \( \bar{\gamma}_b \), the secure communication is mainly determined by the distribution of \( \gamma_e \); specifically, considering \( \bar{\gamma}_b = 5 \) dB we see how the \( \mathcal{P}(C_S > 0) \) is twice larger for \( q_e = 0.1 \), compared to \( q_e = 0.5 \). We also observe how a less severe fading in the desired link (i.e. a larger value of \( q_b \)) leads to this probability to be larger.

Now, we consider the scenario analyzed in Section [IV], where the OP of wireless links in Hoyt fading is investigated in the presence of arbitrarily distributed co-channel interference and background noise. We consider that the interference can be distributed according to the general \( \eta - \mu \) and \( \kappa - \mu \) distributions [14], widely employed in the literature for modeling NLOS and LOS propagation, respectively. For the sake of simplicity in the discussion, we consider a single interferer; however, note that the analysis introduced in Section [IV] can accommodate to an arbitrary number of interferers, admitting correlation between them as well as combinations of \( \eta - \mu \) and \( \kappa - \mu \) distributions.

It is worth mentioning that a very general scenario for the OP evaluation in the presence of co-channel interference and background noise was introduced in [29], where any combination of \( \eta - \mu \) and \( \kappa - \mu \) distribution was admitted either for the desired signal or the interferers. However, we must note that the only restriction considered in [29] (integer values of the \( \mu \) parameter for the \( \eta - \mu \) distribution) excludes the specific case of Hoyt fading, since \( \mu = 0.5 \) in this case.

We will start considering that both co-channel interference and background noise are present, the desired link undergoes Hoyt fading, and the interference is distributed according to the \( \eta - \mu \) or \( \kappa - \mu \) distribution with arbitrary values of their parameters. We use the format 2 for the \( \eta - \mu \) distribution. We define the average SINR in this scenario as \( \text{SINR} = \frac{\bar{\gamma}_d}{1+\bar{\gamma}_i} \), where \( \bar{\gamma}_d \) is the ratio between the desired signal average receive power and the noise power (i.e., the SNR), whereas \( \bar{\gamma}_i \) accounts for the ratio between the interferer average receive signal power at the receiver and the noise power (i.e., an interference-to-noise ratio, INR).
Fig. 3 evaluates the OP for different fading conditions for the desired and interfere links. We assume INR = 5 dB, and a SINR threshold value used to declare an outage $\gamma_o = 0$ dB. In the low SINR regime, it is possible to note the effect of the distribution of the interference in the OP: as $\mu$ is reduced, the propagation of the interfering signal experiences a more severe fade in both types of fading and hence the OP decays. Conversely, we observe that for large values of the average SINR the OP tends asymptotically to a value that is determined by the distribution of the desired link, i.e. by the parameter $q$. For a given value of SINR, the OP grows as $q$ is reduced (i.e., an outage is declared with more probability when the direct link experiences a more severe fade).

Now, we assume an interference-limited system, meaning that the background noise will be neglected. In this scenario, we define the average SIR as the ratio between the average powers of the desired and interfering signal. As in the previous case, we set the threshold value used to declare an outage as $\gamma_o = 0$ dB. In Fig. 4 the OP is represented as a function of the SIR, for different values of the parameters $q$, $\kappa$, $\eta$ and $\mu$. In the large SIR regime, it is reinforced the conclusion that the asymptotic OP is dominated by the distribution of the desired link. For low values of SIR, a similar behavior as in the previous figure is observed. However, it is noted a larger difference in the OP values when $\mu$ changes; this is due to the fact that when background noise vanishes, the differences between the different distributions of the interference become more evident.

Finally, as detailed in Section V, we study the Shannon capacity of different adaptive transmission techniques in Hoyt fading channels. Fig 5 shows the capacity per unit bandwidth of the optimal simultaneous power and rate adaptation (OPRA) policy as a function of the average SNR $\bar{\gamma}$, for different values of the Hoyt shape parameter $q$.

We observe how the capacity loss due to a more severe fading is low for values of $q > 0.5$, being under 0.15 bps/Hz in this range. In fact, it is noted how the achievable performance of OPRA policy when $q = 0.9$ is practically coincident with the Rayleigh case. Markers indicate the simple lower bounds obtained in [58]. We see how the lower bound becomes tighter as $q$ is increased, whereas the performance for the Rayleigh case serves as an upper bound.

A similar behavior is observed in Fig. 6 when analyzing the optimal rate adaptation (ORA) policy with fixed power. The tightness of the bounds and the evolution of the capacity as $q$ changes exhibit the same trend as in the previous figure. Note that in both Figs. 5 and 6 a noticeable performance gap is observed for low values of $q$, i.e. around 0.8 bps/Hz for large $\bar{\gamma}$.

In Fig. 7 we evaluate the performance of the truncated channel inversion with fixed rate (TIFR) policy. As discussed in [5], the cut-off rate in this scenario can be designed either to achieve an optimal performance, or to
satisfy a given constraint in terms of outage probability. We have assumed here the latter policy, with OP = 10^{-1}.

Clearly, the achievable performance under this policy is below the one attained with OPRA and ORA policies. We observe a similar trend as in the previous policies when changing the q parameter; however, we see how the performance gap with respect to the Rayleigh case is larger under TIFR policy. Conversely, we observe that the bounds in this case are very tight regardless of the value of q.

Now, we compare the performance of these policies for different values of q. As in [5], OPRA and ORA policies tend to have a similar asymptotic performance, being the capacity under ORA policy more degraded for low values of \( \bar{\gamma} \). As expected, lower values of q reduce the achievable performance in all policies.

VII. CONCLUSIONS

We have provided a new look at the analysis of wireless communication systems in Hoyt (Nakagami-q) fading. Unlike previous approaches in the literature, we have found a connection between the Rayleigh and Hoyt distributions that facilitates the analysis in the latter scenario.

By deriving integral expressions for the pdf and cdf of the squared Hoyt distribution, we have shown that the squared Hoyt distribution is in fact a composition of exponential distributions with continuously varying averages. Using this connection, we have readily obtained easy-to-compute finite-range integral expressions of different performance metrics in Hoyt fading channels, as well as simple upper and lower bounds which become asymptotically tight as q \( \to 1 \), by simply leveraging existing results for Rayleigh fading channels.

As a direct application, we have derived new expressions for several scenarios of interest in information and communication theory: (a) wireless information-theoretic security in Hoyt fading, (b) outage probability analysis of Hoyt fading channels with \textit{arbitrarily distributed} interference and background noise, and (c) capacity analysis of adaptive transmission policies in Hoyt fading channels.

A further implication of the results in this paper is that there is no need to reproduce complicated calculations for Hoyt fading in the cases where tractable expressions are available for the Rayleigh case. Instead, these analyses can be easily extended to Hoyt scenarios by using a straightforward finite-range integral.
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Fig. 1. Outage probability of secrecy capacity as a function of $\bar{\gamma}_b$, for different values of $q_e$ and $q_b$. Parameter values $\bar{\gamma}_e = 15$ dB and $R_S = 0.1$. 
Fig. 2. Probability of strictly positive secrecy capacity as a function of $\bar{\gamma}_b$, for different values of $q_e$ and $q_b$. Parameter value $\bar{\gamma}_e = 15$ dB.

Solid lines only indicate $q_b = 0.2$; solid lines with markers are included for $q_b = 0.8$. 
Fig. 3. Outage probability with co-channel interference and background noise as a function of the SINR, considering a Hoyt distributed desired link and interference distributed according to $\eta - \mu$ or $\kappa - \mu$ distributions. Parameter values are INR = 5 dB and $\gamma_o = 0$ dB.
Fig. 4. Outage probability with co-channel interference as a function of the SIR, considering a Hoyt distributed desired link and interference distributed according to $\eta - \mu$ or $\kappa - \mu$ distributions. Parameter value $\gamma_o = 0$ dB.
Fig. 5. Normalized capacity vs $\bar{\gamma}$ using OPRA policy, for different values of $q$. Markers indicate the lower bounds on capacity given by $(58)$, for $q = 0.5$ ('x') and $q = 0.9$ (squares).
Fig. 6. Normalized capacity vs $\bar{\gamma}$ using ORA policy, for different values of $q$. Markers indicate the lower bounds on capacity given by (60), for $q = 0.5$ ('x') and $q = 0.9$ (squares).
Fig. 7. Normalized capacity vs $\bar{\gamma}$ using TIFR policy, for different values of $q$. Markers indicate the lower bounds on capacity given by (43) and (61).
Fig. 8. Normalized capacity vs $\bar{\gamma}$ using OPRA, ORA and TIFR policies, for different values of $q$. Solid lines correspond to $q = 0.2$, solid lines and markers correspond to $q = 0.8$. 