Does Standard Cosmology Express Cosmological Principle Faithfully?

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In 1+1 dimensional case, Einstein equation cannot give us any information on the evolution of the universe because the Einstein tensor of the system is identically zero. We study such a 1+1 dimensional cosmology and find the metric of it according to cosmological principle and special relativity, but the results contradict the usual expression of cosmological principle of standard cosmology. So we doubt in 1+3 dimensional case, cosmological principle is expressed faithfully by standard cosmology.

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I. A 1+1 DIMENSIONAL MODEL

Consider a one dimensional infinitely long system consisting of uniformly placed galaxies, see FIG.1. Suppose the system is expanding uniformly, i.e., from any galaxies (such as $O$), we will see that the two galaxies ($A$ and $B$) mostly nearest to us are receding away from us at equal speeds, and the distances between us and this two neighbors are equal.

In standard cosmology, the scale factor is scale independent, i.e., if on the scale of $|OB|$, the scale factor of the system is $a(t)$, then on the scale of $|OC|$, the scale factor is also $a(t)$. So the physical length of $|OB|$ is half of $|OC|$ and the metric describing the system is

$$ds^2 = -dt^2 + a^2(t)dx^2$$

However, for a one-dimensional gravitation system, Einstein equation cannot give us anything about its dynamical evolutions, note $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0$. So if we insists eq(1) is the only correct metric ansatz of the system illustrated in FIG.1, then we have no way to determine the function form of $a(t)$.

The reason that standard cosmology insists eq(1) is, the density of the system is not function of the position co-ordinate $x$. We doubt this statement faithfully expresses the requirements of cosmological principle. Let us consider the following series

$$v_B = v; \quad v_C = \frac{v + v}{1 + v^2}; \quad v_D = \frac{v + v_C}{1 + v \cdot v_C}; \quad \ldots \quad v_X = \frac{v + v_{X-1}}{1 + v \cdot v_{X-1}};$$

$$|AB| = 2a$$

From which we get

$$v_X = \frac{(1 + v)^X - (1 - v)^X}{(1 + v)^X + (1 - v)^X};$$

$$|OX| \sim a \sum_{N = 0}^{\infty} \sqrt{1 - v^2_N}$$

$$= l \int_0^x dx \sqrt{1 - v^2_x}$$

$$= \frac{4a}{\ln(1 + v)} \left[\arctg\left(\frac{1 + v}{1 - v}\right) - \frac{\pi}{4}\right],$$

FIG. 1: One dimensional infinitely long system consists of uniformly placed galaxies.

In eqs(2)-(5), $v$ is the relative recessing speed between two nearest galaxies, $a$ is the physical distance between them, it can also be considered as the scale factor on the smallest scales,

$$a = v \cdot t,$$

where $t$ is understood as the time from the system being created (the distance between any two galaxies is zero) to the epochs we observe it.

In our models, we will not consider dark energies. But we assume that the relative recessing velocity between any two nearest galaxies is a time-independent constance. (One reason for our assumption is, if the system is at rest initially, it will not collapse at self-gravitations from symmetry analysis, what
matters here is parity symmetry. So when the system is expanding but cosmological principle is always kept, it will not decelerate because the parity symmetry is not broken by expansions.

Let

$$\sigma = \frac{1}{2} \ln \frac{1 + v}{1 - v}$$  \hspace{1cm} (7)

we can write down the metric of our one dimensional cosmology in FIG. 1 as

$$ds^2 = -dt^2 + \frac{4v^2t^2}{(e^{\sigma x} + e^{-\sigma x})^2}dx^2$$  \hspace{1cm} (8)

In this metric space, physical co-ordinate $x_{ph}$ is related to $x$ by

$$x_{ph} = \frac{2vt}{\sigma}(\arctg[e^{\sigma x}] - \frac{\pi}{4}).$$  \hspace{1cm} (9)

Note in eqs(8) and (9), $x$ is a pure number of no-dimensions. Before a length unit is assigned, the difference of it has no meaning of any distance lengths. But if we let

$$dx_{pr} = dx \cdot vt$$  \hspace{1cm} (10)

then $dx_{pr}$ can be naturally interpreted as the proper length of line element between points $(t, x) \sim (t, x + dx)$ at time $t$. Using co-ordinate $\{t, x_{pr}\}$, the metric eq(8) can be written in the following form

$$ds^2 = -dt^2 + \frac{4}{(e^{\frac{2vt}{\sigma}} + e^{-\frac{2vt}{\sigma}})^2}dx_{pr}^2$$  \hspace{1cm} (11)

Because we are so deeply affected by Friedmann-Robertson-Walker metric and only familiar with only-time-dependent (or although both time- and position-dependent in the non-flat universes but the two are separated) scale factors, we will mostly use eq(8) in this paper. It is worth noting that the $g_{00}$ component of eq(8) has different dimension from $g_{11}$. Let us put this in mind so that when dimension problems appear, correct interpretation can be given.

Some people may ask, why not redefine the co-ordinate $x$ so that the $x_{ph}$ has simple linear dependence on it? Yes, we can do that way, but we should note after the re-definition, the relation between $x_{pr}$ and $x$ will change correspondingly, which will introduce corresponding complexities, so we will not re-define the co-ordinate $x$ at this time. Physically, if we re-define $x$ to $\tilde{x}$ so that the metric eq(8) has the form $ds^2 = -dt^2 + t^2 d\tilde{x}^2$, then we should note in equal length of line elements $(t, \tilde{x} - \frac{1}{2}d\tilde{x}) \sim (t, \tilde{x} + \frac{1}{2}d\tilde{x})$ and $(t, \tilde{x}' - \frac{1}{2}d\tilde{x}') \sim (t, \tilde{x}' + \frac{1}{2}d\tilde{x}')$, we will not find equal number of galaxies, as long as $|x| \neq |\tilde{x}|$.

Obviously, eqs(8) or (11) contradicts the standard cosmological results eq(1) remarkably. Standard cosmology insists that cosmological principle requires the density of the system is not function of the position co-ordinate $x$, so get its metric ansatz eq(1). While we insists that cosmological does not require so, it only requires that on any galaxies, observers will measure that his two nearest neighbors are equally far away from him and recessing at equal velocities. The density of the system can be functions of the position co-ordinate, as long as wherever the origin is chosen, the metric function has the same form. Although we do not think the generalization from 1+1 dimension to 1+3 dimension is trivial, we think this is an evidence that standard cosmology may not express cosmological principle faithfully. We will put aside debates at this moment and focus on the fact if we generalize the metric eq(8) or (11) into three dimensions, the prediction is consistent with observations or not. We will answer the main criticisms from standard cosmologists in [15].

II. GENERALIZATION TO 1+3 DIMENSIONS

In generalizing eq(8) to three dimensional case we have two methods, i.e.,

$$ds^2 = -dt^2 + \frac{v^2t^2}{\cos^2 \sigma r}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$  \hspace{1cm} (12)

or

$$ds^2 = -dt^2 + \frac{v^2t^2}{\cos^2 \sigma x}dx^2 + \frac{v^2t^2}{\cos^2 \sigma y}dy^2 + \frac{v^2t^2}{\cos^2 \sigma z}dz^2$$  \hspace{1cm} (13)

It is impossible to get eq(12) from eq(13), because eq(13) describes a cubic lattice system, while eq(12) describes a spherical symmetric system. The most remarkable difference between eq(12) and the usual Friedmann-Robertson-Walker metric are, (i) the maximum symmetric subspace of metric space eq(12) is 2-spheres, while that of standard cosmology is homogeneous 3-balls; (ii) eq(12) contains no unknown function such as standard cosmology’s scale factor, the evolution of the universe are completely prescribed by one parameter $v$. We will explain these differences in [15].

As the first step, let us verify that the metric eq(12) indeed describing a homogeneous, isotropic system. Using Einstein equation $G_{\mu\nu} = -8\pi GT_{\mu\nu}$ we calculate the energy momentum tensor of our cosmology in the following

$$-8\pi GT_{\mu\nu} = \text{diag} \left\{ -4\sigma(-1 + e^{2\sigma r}) + \sigma^2(-10 + 2e^{2\sigma r} + e^{4\sigma r})r - 12e^{2\sigma r}rv^2 \right\} \frac{4e^{2\sigma r}t^2v^2}{(1 + e^{2\sigma r})^2},$$

$$r\left[\sigma(-1 + e^{2\sigma r}) + 2\sigma(e^{2\sigma r}t^2r + 4e^{2\sigma r}rv^2) \right] \frac{1 + e^{2\sigma r}}{(1 + e^{2\sigma r})^2},$$

$$r\left[\sigma(-1 + e^{2\sigma r}) + 4e^{2\sigma r}r + 4e^{2\sigma r}rv^2 \right] \frac{1 + e^{2\sigma r}}{(1 + e^{2\sigma r})^2} \sin^2 \theta$$  \hspace{1cm} (14)
Note in our frame-works, the dimension of $T_{00}$ is different from that of $T_{ii}$, because the component of our metric are of different dimensions. The same problem will occur on the four velocity of an observer, see eq(15). If we use the metric of form eq(11), this will not be a problem. Superficially, our energy momentum tensor is position-dependent, which seems to violate cosmological principles. This is not the case. Let us calculate the energy density and pressures measured by an observer at position $(t, r, \theta, \phi)$, whose four velocity can be written as

$$\mathbf{u}^{(t,r,\theta,\phi)} = \frac{1}{\sqrt{N}} \{1, \frac{v_r}{v_t}, 0, 0\},$$

where $v_r = \frac{e^{\sigma r} - e^{-\sigma r}}{e^{\sigma r} + e^{-\sigma r}},$

$$N = 1 - \frac{4v_r^2}{\left(e^{2\sigma r} + e^{-2\sigma r}\right)^2}.\quad (15)$$

It is very easy to verify $g_{\mu\nu} u^\mu u^\nu = -1$. Measured by this observer, the energy density and pressure are respectively

$$8\pi G \rho = [T_{\mu\nu} u^\mu u^\nu]_{v\to 0} = 9t^{-2},$$

$$8\pi G p = [T_{\mu\nu}(g^\mu_{\nu} + u^\mu u^\nu)]_{v\to 0} = -9t^{-2}.\quad (16)$$

Some people may not understand the limit procedure in eq(16), please see the notations under eq(26). From eqs(14)+(16) we can see that, viewing from any point, we can see an isotropic but not in-homogeneous universe. The inhomogeneity originates from Lorentz contraction, it is just a kinematical effects instead a dynamical one. Obviously, if we can take photos of the universe from different places at the same time, we get the same pictures. We think this is a faithful expression of cosmological principle. While standard cosmology’s statements, the energy momentum tensor should not depends on the position co-ordinate of the universe, does not express cosmological principle faithfully.

Now let us consider super-novae in the metric space eq(12). We want to calculate their luminosity-distance v.s. red-shift relations. Let us follow the same procedures eq(11). We want to calculate their luminosity-distance using Lorentz dilating, the photons emitted in period $t$.

$$v_r = \frac{e^{\sigma r} - e^{-\sigma r}}{e^{\sigma r} + e^{-\sigma r}},\quad (17)$$

so, if the proper frequency of a photon emitted from this super-novae is $\omega_0$, the frequency measured by us is $\omega$, then the red-shift $z$ of this photon satisfy

$$(1+z) = \frac{\omega^{-1}}{\omega_0} = e^{\sigma r},\quad (18)$$

considering Lorentz dilating, the photons emitted in period $\delta t_1$ can only reach us in period $\delta t_1 e^{\sigma r}$. So we get the luminosity distance v.s. red-shift relation as

$$d_l = (1+z) \cdot \frac{2v_0 H^{-1}_0}{\sigma} \left[ \arctg(1+z) - \frac{\pi}{4} \right].\quad (19)$$

FIG. 2: The luminosity distance v.s. red-shift relation of super-novae. Red(solid) line is the prediction of this paper; Black(dot) line is the prediction of $\Lambda$CDM cosmology, in which $\Omega_{m0} = 0.27, \Omega_\Lambda = 0.73, H_0 = 71km/(s \cdot Mpc)$; Blue(dash) line is the prediction of standard CDM cosmology, in which $\Omega_{m0} = 1.0, H_0 = 71km/(s \cdot Mpc)$.

The relation between $\sigma$ and $v$ is given by eq(7). From best fitting observational results of [6], we get $v = 0.79/3000, H_0 = 60km/(s \cdot Mpc), \chi^2 = 303 (186Golden+Silver sample)$ or $v = 0.899/3000, H_0 = 60km/(s \cdot Mpc), \chi^2 = 237 (157Golden sample)$.

We illustrate the numerical results of eq(19) in FIG.2. As comparisons, we also depict the predictions of $\Lambda$CDM and sCDM cosmologies. From the figure we can easily see that the prediction of our eq(19) is very similar to that of $\Lambda$CDM cosmology. Because we consider special relativity effects on the definition of homogeneity in our theory, we call our results eqs(8), (12) and (19) in this paper as Relativity Cosmology.

### III. COMPARING OUR MODELS WITH STANDARD COSMOLGY

Now let us return to standard cosmology where we are taught that homogeneity and isotropy of the observed universe directly leads to the conclusion that Freimann-Robertson-Walker metric is the unique metric describing our real universe (we will put perturbations and structure formation problems in the future works).

$$ds^2 = -dt^2 + \frac{a^2(t)}{1-kr^2} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

where $k = +1, 0, -1.\quad (20)$

While in the co-moving co-ordinate the energy momentum tensor of the cosmological fluid has the form

$$T_{\mu\nu} = \text{diag}(\rho, p, p, p)$$

if no radiation and/or dark energy is involved

$$= \text{diag}(\rho(t), 0, 0, 0)\quad (21)$$
distances between us and the nearest neighbors (should be on the same line with the previous galaxies) we get results 2r, and so on. So the system has translation symmetry at a given time, i.e., the maximum symmetric subspace of the whole space-time is homogeneous 3-ball.

If special relativity is considered, then when we were put on a given galaxy O and were asked to measure the distances between us and the nearest neighbors (B), we got results, say r. But if we were asked to measure the distances between us and the next-nearest neighbor (C), we did not get 2r, we got a number less than 2r because of Lorentz contraction. In this case the maximum symmetric subspace of our physical universe is only a 2-sphere instead of a homogeneous 3-ball.

According to the results of [1], section 13.5, the general metric describing a space-time with maximum symmetric subspace of $S_2$ can only be reduced to

$$ds^2 = -dt^2 + U(t, r)dr^2 + V(t, r)(d\theta^2 + \sin^2 \theta d\phi^2)$$

(22)

instead of eq(20). Just for the same reason, we can only write the energy momentum tensor describing our real universe as

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu)$$

where $u^{(t, r, \theta, \phi)} \propto (1, v_r, 0, 0)$

(23)

instead of eq(21). If generalize our metric eq(11) into three dimensions it will just has the form of eq(22), while the appropriate energy momentum tensor eq(14), will also have the form of eq(23) correspondingly.

On the contrary, standard cosmology does not consider special relativity when define homogeneities, which will introduce problems to it. We provide one in the following. Starting from metric (20), let $k = 0$ for the moment, using Einstein equation we calculate the energy momentum tensor

$$8\pi G T_{\mu\nu} = H^2(t) \cdot \text{Dia}l\{-3, A(t), A(t)r^2, A(t)r^2 \sin^2 \theta\},$$

where

$$A(t) = a^2(t) + \frac{a^3(t)u^{\nu}u_{\nu}}{a^2}.$$  (24)

Note $T_{00}$ only depending on $t$ does not mean observers on different places will get the equal energy densities in measures. It only means that energy density measured by observers on the origin of the co-ordinate is position independent. To calculate the energy density measured by observers on different places, we have to consider observers on general positions $(t, r, \theta, \phi)$, whose four velocity are

$$u^\mu = \frac{1}{\sqrt{1 - [a(t)\dot{a}(t)r]^2}}\{1, \dot{a}(t)r, 0, 0\}$$

(25)

The energy density and pressure measured by these observers are respectively

$$8\pi G \rho = 8\pi G T_{\mu\nu}u^\mu u^\nu$$

$$= \frac{H^2(t)}{1 - [a(t)\dot{a}(t)r]^2}(-3 + A(t)\dot{a}^2(t)r^2)$$

$$8\pi G p = 8\pi G T_{\mu\nu}(g^{\mu\nu} + u^\mu u^\nu)$$

$$= \frac{5r^2a^2\dot{a}^4 - 6a\dot{a} + \dot{a}^2(-3 + 4r^2a^2\ddot{a})}{a^2(-1 + r^2a^2\ddot{a})}.$$  (26)

Obviously, without a limiting procedure like that in eq(16), eq(26) will tell us that both the energy density and pressure measured by observers at different places are not the same. However, if we take the limiting $\dot{a} \to 0$, the energy density of the cosmological fluid will become zero. We think this is a problem of standard cosmology. But our cosmological models in eqs(12)+(14)+(16) does not have this problem.

IV. CONCLUSIONS

We express our suspicions that standard cosmology expresses cosmological principle faithfully. In 1+1 dimensional case, we prove that the background metric of the universe is not Friedmann-Robertson-Walker type. We then generalize the 1+1 dimensional results into 1+3 dimensional case and explain the observed luminosity-distance v.s. red-shift relations of super-novaes naturally without introducing any concepts of dark energies. We will answer the criticisms from standard cosmologists in another extended version of this paper, [15].

Of course, the observed luminosity-distance v.s. red-shift relations of super-novaes is not the only evidence.
of dark energies, see [2–8] for experimental works and [9–13] for theoretical ones. We will study the perturbations of eqs(12)+(14) and structure formation problems in the future. The original ideal of this paper is also expressed in [14].

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