The LFV decays $B_{d,s}^0 \rightarrow e\mu(e\tau, \mu\tau)$ with one neutral singlet scalar

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Abstract

Taking account of the constraint from radiative two body decays $l_i \rightarrow l_j\gamma$, we investigate the Lepton Flavor Violation decays $B_q^0 \rightarrow \bar{l}llk$ in the framework of the minimal extension of the Standard Model with one neutral singlet scalar. The couplings $C_{e\mu}$, $C_{e\tau}$ and $C_{\mu\tau}$ between the different generation leptons and scalar $S^0$ are constrained by the current bounds of $l_i \rightarrow l_j\gamma$. The numerical results show that the theoretical prediction of $B_q^0 \rightarrow \bar{l}llk$ strongly depend on the couplings $C_{qb}$ ($q = d$ or $s$) between down type quarks and new scalar. The contributions from couplings $C_{uc}$, $C_{ut}$ and $C_{ct}$ between up type quark and new scalar are less dominant.

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I. INTRODUCTION

Rare decays are of great importance in searching for New Physics (NP) beyond the Standard Model (SM), and the Lepton Flavor Violating (LFV) decays are particularly appealing cause they are suppressed in the Standard Model (SM), and their detection would be a manifest signal of NP. The LFV decays are discussed in various NP models, such as grand unified models [1–3], supersymmetric models with and without R-parity [4, 5], models with heavy sterile fermions [6–9] and extra $Z^0$ boson [10, 11], left-right symmetry models [12, 13] etc. Most of the current experimental focuses in searching for the LFV decays are lepton decays, $l_i \to l_j \gamma$, $l_i \to 3l_j$ and the $\mu - e$ conversion in nuclei. The LFV decays of hadrons are of great importance as well as the leptonic decays [14, 15].

In literature, the LFV processes are associated with the lepton nonuniversality effect in semileptonic decays and $b \to s ll$ transitions. These processes have been investigated in various models beyond the SM, such as supersymmetric models [17], models extended with extra gauge $Z'$ boson [16], heavy singlet Dirac neutrinos [18] or leptoquarks [19, 20] and the Pati-Salam model [21]. Current experimental upper bounds on LFV decays of $B_{s,d}^0$ are listed in TABLE. [22]. The experimental data on the LFV decays $B_{s,d}^0 \to e^\pm \tau^\mp$ and $B_{s,d}^0 \to \mu^\pm \tau^\mp$ are absent. The theoretical prediction on the branching fractions of $B_{d,s}^0 \to e^\pm \mu^\mp$ in these models can be greatly enhanced, even up to $10^{-11}$, which are very promising detected in near future. In literatures, the branching ratios of $B_{d,s}^0 \to e^\pm \tau^\mp$ and $B_{d,s}^0 \to \mu^\pm \tau^\mp$ can also be enhanced close to $B_{d,s}^0 \to e^\pm \mu^\mp$ [19, 20]. Recently, based on a sample of proton-proton collision data corresponding to an integrated luminosity of $3 fb^{-1}$, the LHCb experiment gives the following upper limits at 95% CL [23],

$$Br(B_d^0 \to e\mu) < 1.3 \times 10^{-9}, Br(B_s^0 \to e\mu) < 6.3 \times 10^{-9},$$

| TABLE I: Current limits of LFV decays of $B_{s,d}^0$. |
|-----------------------------------------------|
| Decay                                     | Bound               | Decay                                     | Bound               |
| $B_d^0 \to e^\pm \mu^\mp$                 | $< 2.8 \times 10^{-9}$ | $B_s^0 \to e^\pm \mu^\mp$                | $< 1.1 \times 10^{-8}$ |
| $B_d^0 \to e^\pm \tau^\mp$                | $< 2.8 \times 10^{-5}$ | $B_s^0 \to e^\pm \tau^\mp$               | -                   |
| $B_d^0 \to \mu^\pm \tau^\mp$              | $< 2.2 \times 10^{-5}$ | $B_s^0 \to \mu^\pm \tau^\mp$             | -                   |
which substitutes for the previous results [24].

In this paper, we study the LFV decays of $B_{d,s}^0$ in a minimal extension of SM with NP featuring extra scalar. The scalar is predicted by many extensions of SM and not observed yet even though many searches have been devoted to find it at the experiment. For simplicity, the couples of the neutral scalar with charged fermions are studied. We investigate the LFV decays of $B_{d,s}^0$ in a function of the couplings between the neutral scalar and quarks. We have considered the loop contributions for reason to understand the different contributions from tree-level diagrams and loop diagrams. It shows loop contributions is about two orders of magnitude below tree-level contributions for $B_d^0$ and one order of magnitude below tree-level contributions for $B_s^0$.

The paper is organized as follows. In Section [II], we provide a simple formalism for the description of the newly introduced scalar and give the analytic expression of LFV decays of $B_q^0 \to \bar{l}l_k$ in detail. The numerical results are presented in Section [III] and the conclusion is drawn in Section [IV].

II. FORMALISM

In this section, we give the description of the minimal extension of the SM. In general, the minimal extension of the SM involves only one scalar, one vector or one fermion. In following we will consider the extension of the SM with one neutral singlet scalar $S^0$. We will add the neutral singlet scalar $S^0$ as a minimal extension of SM, that is, we are not concerned which models predict the new particle, but only want to investigate the observable phenomenon of this SM extension.

For simplicity and consideration of couplings like SM Higgs-fermion-fermion interactions, the couplings of ‘new’ scalar and left-handed fermions are assumed equal to that of the scalar and right-handed fermions. The interactions between the different generation up type quarks, down type quarks or charged leptons and the neutral scalar $S^0$ take the following structure,

\[
C_{u^i u^j} \bar{u}^i P_{L/R} u^j S^0, u^i \neq u^j, \\
C_{d^i d^j} \bar{d}^i P_{L/R} d^j S^0, d^i \neq d^j, \\
C_{e^i e^j} \bar{e}^i P_{L/R} e^j S^0, e^i \neq e^j,
\] (1)
and the couplings $C_{u^iu^i}$, $C_{d^id^j}$ and $C_{e^ie^j}$ are real numbers. The interactions between the same generation quarks or leptons are neglected and so the interactions between the gauge vectors or Higgs and the new scalar $S^0$ for simplicity. From Eq. (1), one can see that the LFV decay originate from the interactions between different generation leptons and the new scalar $S^0$. The interactions between different generation quarks and the new scalar $S^0$ can contribute to the LFV decays $B^0_q \rightarrow \bar{t}_l l_k$ in quark sector.

The one loop Feynman diagrams contributing to the LFV decay $B^0_d \rightarrow e^+\mu^-$ are presented in FIG.1, FIG.2, FIG.3, and FIG.4, and other LFV decays of $B^0_d$ and $B^0_s$ can be discussed in a similar way. Utilizing the notation of ref. [25], the effective Hamiltonian of LFV decays of $B^0_q (q = d, s)$ is given by

$$H_{\text{eff}} = \frac{1}{16\pi^2} \sum_{X,Y=L,R} \left( C_{SXY} \mathcal{O}_{SXY} + C_{VXY} \mathcal{O}_{VXY} + C_{TX} \mathcal{O}_{TX} \right),$$

(2)

where $C_{SXY}$, $C_{VXY}$ and $C_{TX}$ are the Wilson coefficients. The relevant operators, including the scalar, vector and tensor operators, are given by,

$$\mathcal{O}_{SXY} = (\bar{b}P_X q)(\bar{l}_l P_Y l_k), \quad \mathcal{O}_{VXY} = (\bar{b}\gamma^\mu P_X q)(\bar{l}_l \gamma_\mu P_Y l_k), \quad \mathcal{O}_{TX} = (\bar{b}\sigma^{\mu\nu} P_X q)(\bar{l}_l \sigma_{\mu\nu} l_k),$$

(3)

where $l_l$ and $l_k$ are leptons, $P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$. For $B^0_d \rightarrow e^+\mu^-$, symbols in Eq. (3) are $q=d$, $l_l=\mu$ and $l_k=e$. It is impossible to get a antisymmetric combination made up of $p^\mu$ by exchanging the index $\mu \leftrightarrow \nu$, so the tensor current $\langle 0 | \bar{b} \sigma^{\mu\nu} q | B^0_q (p) \rangle$ vanishes. The expectation values of the matrix elements are derived as

$$\langle 0 | \bar{b}\gamma^\mu P_{L/R} q | B^0_q (p) \rangle = \pm \frac{i}{2} b^\mu f_{B^0_q}, \quad \langle 0 | \bar{b} P_{L/R} q | B^0_q (p) \rangle = \pm \frac{i M_{B^0_q}^2 f_{B^0_q}^2}{2(m_b + m_q)};$$

(4)

where $f_{B^0_q}$ is the decay constant of $B^0_q$, $M_{B^0_q}$ is the mass of $B^0_q$.

Tree level diagrams contributing to $B^0_d \rightarrow e^+\mu^-$ is presented in FIG.1. Using the equations in Eq. (4), the relevant Wilson coefficient is calculated by,

$$C_{SLL} = \frac{i C_{db} C_{e\mu}}{m_{S^0}^2 - M_{B^0_q}^2}, \quad C_{SLR} = C_{SRL} = C_{SRR} = C_{SLL},$$

where $m_{S^0}$ is the mass of neutral scalar $S^0$. One can see that only coefficient $C_{db}$ contributes to $B^0_d \rightarrow e^+\mu^-$ in quark sector. Two point diagrams contributing to $B^0_d \rightarrow e^+\mu^-$ are presented in FIG.2. The relevant Wilson coefficient is calculated by,
FIG. 1: Tree level diagrams contribute to $B_d^0 \to e^+ \mu^-$. 

FIG. 2: Two point diagrams contribute to $B_d^0 \to e^+ \mu^-$. 

FIG. 3: Penguin diagrams contribute to $B_d^0 \to e^+ \mu^-$. 

5
The Wilson coefficients for other two point diagrams are listed in APPENDIX A. Coefficients $C_{ds}, C_{db}$ and $C_{sb}$ contribute to $B^0_d \rightarrow e^+\mu^-$ in quark sector. Penguin diagrams contributing to $B^0_d \rightarrow e^+\mu^-$ are presented in FIG. 3 and the corresponding Wilson coefficients are listed in APPENDIX B. Coefficients $C_{ds}, C_{db}, C_{sb}, C_{uc}, C_{ut}$ and $C_{ct}$ contribute to the $B^0_d \rightarrow e^+\mu^-$ in quark sector at penguin diagram level. It is noted worthwhile that coefficients $C_{uc}, C_{ut}$ and $C_{ct}$ contribute to the LFV decays only at this level. Box diagrams contributing to $B^0_d \rightarrow e^+\mu^-$ are presented in FIG. 4 and the corresponding Wilson coefficients are listed in APPENDIX C. Coefficients $C_{ds}, C_{db}$ and $C_{sb}$ contribute to the $B^0_d \rightarrow e^+\mu^-$ in quark sector at box diagram level. All integrals in Wilson coefficients can be calculated by Package-X [26], which deals with analytic calculation and symbolic manipulation of one-loop Feynman integrals.
III. NUMERICAL ANALYSIS

Every amplitude $\mathcal{M}$ in FIG [1], FIG [2], FIG [3] and FIG [4] is composed of the scalar, pseudoscalar, vector and axial-vector current, i.e.,

$$ (4\pi)^2 \mathcal{M} = F_S \bar{l}_l l_k + F_P \bar{l}_l \gamma^\mu l_k + F_V p_\mu \bar{l}_l \gamma^\mu l_k + F_A p_\mu \bar{l}_l \gamma^\mu \gamma^5 l_k, \tag{5} $$

where $l_l = \mu$ and $l_k = e$ for $B^0_d \to e^+\mu^-$, and the form factors $F_S$, $F_P$, $F_V$ and $F_A$ are combinations of Wilson coefficients,

$$ F_S = \frac{i M_{B^0_d}^2 f_{B^0_q}}{4(m_b + m_q)} (C_{SLL} + C_{SLR} - C_{SRR} - C_{SRL}), $$

$$ F_P = \frac{i M_{B^0_d}^2 f_{B^0_q}}{4(m_b + m_q)} (-C_{SLL} + C_{SLR} - C_{SRR} + C_{SRL}), $$

$$ F_V = -\frac{i f_{B^0_q}}{4} (C_{VLL} + C_{VLR} - C_{VRR} - C_{VRL}), $$

$$ F_A = -\frac{i f_{B^0_q}}{4} (-C_{VLL} + C_{VLR} - C_{VRR} + C_{VRL}). $$

From Eq. (5) one can easily calculate the squared amplitude,

$$ |\mathcal{M}|^2 = \frac{1}{128\pi^3} \left( |F_S|^2 (M_{B^0_d}^2 - (m_k + m_l)^2) + |F_P|^2 (M_{B^0_d}^2 - (m_l - m_k)^2) 
+ |F_V|^2 (M_{B^0_d}^2 (m_k - m_l)^2 - (m_k - m_l)^2) + |F_A|^2 (M_{B^0_d}^2 (m_k + m_l)^2 
- (m_k - m_l)^2) + 2 Re (F_S F_V^* (m_l - m_k) (M_{B^0_d}^2 + (m_k + m_l)^2) 
+ 2 Re (F_P F_A^*) (m_l + m_k) (M_{B^0_d}^2 - (m_k - m_l)^2) \right). $$

The analytic expression of the branching ratio of $B^0_q \to \bar{l}_l l_k$ is given by,

$$ Br(B^0_q \to \bar{l}_l l_k) = \frac{\tau_{B^0_q}}{16\pi M_{B^0_q}} \sqrt{1 - \frac{(m_k + m_l)}{M_{B^0_q}^2}^2} \sqrt{1 - \frac{(m_k - m_l)}{M_{B^0_q}^2}^2} |\mathcal{M}|^2, \tag{6} $$

where $\tau_{B^0_q}$ is the life time of $B^0_q$.

III. NUMERICAL ANALYSIS

In the numerical analysis, we will adopt following values for parameters of meson $B^0_{d,s}$,

$$ m_{B^0_d} = 5.279 GeV, f_{B^0_d} = 0.190 GeV, \tau_{B^0_d} = 1.520 \times 10^{-12} s, $$

$$ m_{B^0_s} = 5.366 GeV, f_{B^0_s} = 0.277 GeV, \tau_{B^0_s} = 1.509 \times 10^{-12} s. $$

7
The experimental bounds on LFV decays, such as radiative two body decays \( l_i \rightarrow l_j \gamma \), leptonic three body decays \( l_i \rightarrow 3l_j \) and \( \mu - e \) conversion in nuclei, can give strong constraints on the coefficients \( C_{e\mu}, C_{e\tau} \) and \( C_{\mu\tau} \). In the following we will use LFV decays \( l_i \rightarrow l_j \gamma \) to constrain the coefficients \( C_{e\mu}, C_{e\tau} \) and \( C_{\mu\tau} \).

The scalar \( S^0 \) mediated diagrams for \( \mu \to e\gamma \) are shown in FIG.5. Taking account of the gauge invariance, and assuming the photon is on shell and transverse, the amplitude for \( \mu \to e\gamma \) is given by

\[
M(\mu \to e\gamma) = e^{\mu*} \bar{u}_e(p_e)[i\gamma^\nu \sigma_{\mu\nu}(A + B\gamma_5)]u_\mu(p_\mu).
\]

Then, the analytic expression of decay width is calculated by,

\[
\Gamma(\mu \to e\gamma) = \frac{(m_\mu^2 - m_e^2)^3}{8\pi m_\mu^3}(|A|^2 + |B|^2),
\]

where A is given by

\[
A = \frac{ie}{16\pi^2}C_{e\tau}C_{\mu\tau}(m_\mu(C_{e\tau}C_{\mu\tau}(m_\mu(C_{e\tau}C_{\mu\tau}))
+ m_\mu(C_{12}(m_\mu^2, 0, m_\mu^2, m_{S\tau}, m_\tau, m_\tau) + C_{11}(m_\mu^2, 0, m_\mu^2, m_{S\tau}, m_\tau, m_\tau)) + (m_\mu + m_\tau)
\times C_2(m_\mu^2, 0, m_\mu^2, m_{S\tau}, m_\tau, m_\tau) + (m_\mu + m_\mu)C_1(m_\mu^2, 0, m_\mu^2, m_{S\tau}, m_\tau, m_\tau))}
\]

and B equals zero. Actually, only FIG.5(b) contributes to the decay width cause the amplitudes in FIG.5(a) and FIG.5(c) are proportional to \( e^{\nu*} \bar{u}_e(p_e)\gamma_\nu u_\mu(p_\mu) \) or \( e^{\nu*} \bar{u}_e(p_e)\gamma_\nu \gamma_5 u_\mu(p_\mu) \).

The decay width for \( \tau \to e(\mu)\gamma \) can be formulated in a similar way. The integrals can also be calculated through the Package-X \[26\].

From Eq.(7), one can see that the experimental bound of \( \mu \to e\gamma \) can give constraint on coefficients \( C_{e\tau}C_{\mu\tau} \). For \( \tau \to e\gamma \) and \( \tau \to \mu\gamma \), the coefficients \( C_{e\mu}C_{\mu\tau} \) and \( C_{e\mu}C_{e\tau} \) can be
constrained. Assuming the mass of the extra scalar $m_{S^0}=13$ TeV and taking account of the current limits of LFV decays $l_i \to l_j \gamma$ listed in TABLE II, one can get the following values,

| Decay         | Bound        | Decay         | Bound        |
|---------------|--------------|---------------|--------------|
| $\mu \to e \gamma$ | $4.2 \times 10^{-13}$ | $\tau \to e \gamma$ | $3.3 \times 10^{-8}$ |
| $\tau \to \mu \gamma$ | $4.4 \times 10^{-8}$ | | |

$C_{e\tau}C_{\mu\tau} \sim 4 \times 10^{-5}$, $C_{e\mu}C_{\mu\tau} \sim 6$, $C_{e\mu}C_{e\tau} \sim 60$, and easily calculate the result,

$$C_{e\mu} \sim 3000, C_{e\tau} \sim 0.02, C_{\mu\tau} \sim 0.002.$$  \hfill (8)

If not special specified, values in Eq. (8) are used as default in investigating the LFV decays of $B^0_{d,s}$.

![Graph](image.png)

FIG. 6: (a) Br($B^0_d \to e^+ \mu^-$) (solid line), Br($B^0_d \to e^+ \tau^-$) (dash line) and Br($B^0_d \to \mu^+ \tau^-$) (dot line) vs coefficient Log[$C$]; (b) Br($B^0_s \to e^+ \mu^-$) (solid line), Br ($B^0_s \to e^+ \tau^-$) (dash line) and Br ($B^0_s \to \mu^+ \tau^-$) (dot line) vs coefficient Log[$C$]. $C_{ds} = C_{db} = C_{sb} = C_{uc} = C_{ut} = C_{ct} = C$ is assumed.

In general case, we discuss the behavior of LFV decays of $B^0_{d,s}$ when all coefficients are universal. Taking $C_{ds} = C_{db} = C_{sb} = C_{uc} = C_{ut} = C_{ct} = C$, $m_{S^0} = 13$ TeV, we plot the theoretical prediction of Br($B^0_d \to e^+ \mu^-$) (solid line), Br ($B^0_d \to e^+ \tau^-$) (dash line) and Br
\( (B^0_d \rightarrow \mu^+\tau^-) \) (dot line) vs coefficient \( \log[C] \) in Fig.6 (a) and the theoretical prediction of \( \text{Br}(B^0_s \rightarrow e^+\mu^-) \) (solid line), \( \text{Br}(B^0_s \rightarrow e^+\tau^-) \) (dash line) and \( \text{Br}(B^0_s \rightarrow \mu^+\tau^-) \) (dot line) vs coefficient \( \log[C] \) in Fig.6 (b). It shows that a linear relationship is displayed between LFV decays of \( B^0_d,s \) and \( \log[C] \), and this displays the great dependence of LFV decays of \( B^0_d,s \) on coefficient \( C \). When coefficient \( C \sim 10^9 \), the prediction of \( \text{Br}(B^0_d,s \rightarrow e^+\mu^-) \) is very close to the current limit in TABLE I. The prediction of LFV decays with outgoing \( \tau \) lepton are far below the current limits.

Next, we investigate the LFV decays of \( B^0_d,s \) in two cases: (I) Only the interactions between \( S^0 \) and down type quarks are considered, the interactions between \( S^0 \) and up type quarks are ignoring; (II) Only the interactions between \( S^0 \) and up type quarks are considered, the interactions between \( S^0 \) and down type quarks are ignoring. We also investigate the individual contributions from six coupling coefficients between quarks and new scalar, for example, by ignoring the solid lines in FIG.7(a) and FIG.8(a), then the rest three lines in FIG.7(a) and three lines in FIG.8(a) are the six individual contributions from six coupling coefficients.

(I) In this case, coefficients \( C_{uc}, C_{ut}, C_{ct} \) are set zero. Taking \( C_{ds} = C_{db} = C_{sb} = C \), we plot the theoretical prediction of LFV decays of \( B^0_{d,s} \) vs coefficient \( \log[C] \) in Fig.7 (solid line). It shows the theoretical prediction of LFV decays of \( B^0_{d,s} \) is very close to the general case. Then we plot the theoretical prediction of LFV decays of \( B^0_{d,s} \) vs \( C_{ds} \) (dash line), \( C_{db} \) (dot line), \( C_{sb} \) (dash dot line) separately. It is manifest that coefficient \( C_{db} \) dominates the LFV decays \( B^0_d \) and so does coefficient \( C_{sb} \) for the LFV decays \( B^0_s \). Contributions from other coefficients are several orders of magnitudes below the condition in dominant coefficient. One can find reasons in FIG.1 that these LFV decays can appear in tree level where \( C_{db} \) or \( C_{sb} \) exist. The second coefficient dominates the LFV decays of \( B^0_d \) is \( C_{ds} \) and the coefficient \( C_{sb} \) contributes the least among three coefficients. For the LFV decays of \( B^0_s \), The second coefficient dominates the LFV decays is \( C_{sb} \) and the last is \( C_{ds} \).

(II) In this case, coefficients \( C_{ds}, C_{db}, C_{sb} \) are set zero. Taking \( C_{uc} = C_{ut} = C_{ct} = C \), we plot the theoretical prediction of LFV decays of \( B^0_{d,s} \) vs coefficient \( \log[C] \) in Fig.8 (solid line). Different from case (I), it shows the theoretical prediction of LFV decays of \( B^0_{d,s} \) is several orders of magnitude below the general case. Then we plot the theoretical prediction of LFV decays of \( B^0_{d,s} \) vs \( C_{uc} \) (dash line), \( C_{ut} \) (dot line), \( C_{ct} \) (dash dot line) separately. It displays that the theoretical prediction for LFV decays of \( B^0_d \) with three coefficients is \( Br(C_{ut}) >
FIG. 7: Branching ratios of $B_{d,s}^0$ decay: (a) $\text{Br}(B_{d}^0 \to e^+ \mu^-)$, (b) $\text{Br}(B_{d}^0 \to e^+ \tau^-)$, (c) $\text{Br}(B_{d}^0 \to \mu^+ \tau^-)$, (d) $\text{Br}(B_{s}^0 \to e^+ \mu^-)$, (e) $\text{Br}(B_{s}^0 \to e^+ \tau^-)$, and (f) $\text{Br}(B_{s}^0 \to \mu^+ \tau^-)$. Following assumptions are used: (1) $C_{ds} = C_{db} = C_{sb} = C$, $C_{uc} = C_{ut} = C_{ct} = 0$ (solid line), (2) $C_{ds} = C$, $C_{db} = C_{sb} = C_{uc} = C_{ut} = C_{ct} = 0$ (dash line), (3) $C_{db} = C$, $C_{ds} = C_{sb} = C_{uc} = C_{ut} = C_{ct} = 0$ (dot line), (4) $C_{sb} = C$, $C_{ds} = C_{db} = C_{uc} = C_{ut} = C_{ct} = 0$ (dash dot line). 

11
FIG. 8: Branching ratios of $B^0_{d,s}$ decay: (a)$\text{Br}(B^0_d \rightarrow e^+\mu^-)$, (b)$\text{Br}(B^0_d \rightarrow e^+\tau^-)$, (c)$\text{Br}(B^0_d \rightarrow \mu^+\tau^-)$, (d)$\text{Br}(B^0_s \rightarrow e^+\mu^-)$, (e)$\text{Br}(B^0_s \rightarrow e^+\tau^-)$ and (f)$\text{Br}(B^0_s \rightarrow \mu^+\tau^-)$. Following assumptions are used: (1)$C_{ds} = C_{db} = C_{sb} = 0$, $C_{uc} = C_{ut} = C_{ct} = C$ (solid line), (2)$C_{ds} = C_{db} = C_{sb} = 0$, $C_{uc} = C$, $C_{ut} = C_{ct} = 0$ (dash line), (3)$C_{ds} = C_{db} = C_{sb} = C_{uc} = 0$, $C_{ut} = C$, $C_{ct} = 0$ (dot line), (4)$C_{ds} = C_{db} = C_{sb} = C_{uc} = C_{ut} = 0$, $C_{ct} = C$ (dash dot line).
Br(C_{ct}) > Br(C_{uc}), and Br(C_{ct}) > Br(C_{ut}) > Br(C_{uc}) for LFV decays of B^0_s. This may be explained by the CKM matrix,

$$|V_{CKM}| = \begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix} = \begin{pmatrix}
0.9742 & 0.2243 & 0.00394 \\
0.218 & 0.997 & 0.0422 \\
0.0081 & 0.0394 & 1.019
\end{pmatrix}. $$

For meson B^0_d,

$$\mathcal{M}(C_{uc}) \propto |V_{ud}||V_{cb}| + |V_{ub}||V_{cd}| = 0.0419702, $$

$$\mathcal{M}(C_{ut}) \propto |V_{ud}||V_{tb}| + |V_{ub}||V_{td}| = 0.992742, $$

$$\mathcal{M}(C_{ct}) \propto |V_{cd}||V_{tb}| + |V_{cb}||V_{td}| = 0.222484, $$

$$\mathcal{M}(C_{ct}) > \mathcal{M}(C_{ut}) > \mathcal{M}(C_{uc}). \quad (9)$$

For meson B^0_s,

$$\mathcal{M}(C_{uc}) \propto |V_{us}||V_{cb}| + |V_{ub}||V_{cs}| = 0.0133936, $$

$$\mathcal{M}(C_{ut}) \propto |V_{us}||V_{tb}| + |V_{ub}||V_{ts}| = 0.228717, $$

$$\mathcal{M}(C_{ct}) \propto |V_{cs}||V_{tb}| + |V_{cb}||V_{ts}| = 1.01761, $$

$$\mathcal{M}(C_{ct}) > \mathcal{M}(C_{ut}) > \mathcal{M}(C_{uc}). \quad (10)$$

The orders listed in Eq.(9) and Eq.(10) coincide with the behavior displayed in FIG.8

IV. CONCLUSIONS

In this work, taking account of the constraints on the parameter space from LFV decays Br(l_i \rightarrow l_j \gamma), we analyze the LFV decays of B^0_q \rightarrow \bar{l}_i l_k as a function of the six coefficients C_{ds}, C_{db}, C_{sb}, C_{uc}, C_{ut} and C_{ct} in the framework with one neutral single scalar introduced. The LFV decays of B^0_d strongly depend on the magnitude of couplings between new scalar S^0 and the down type quarks, especially C_{db} and the LFV decays of B^0_s strongly depend on the magnitude of couplings C_{sb}. With one scalar S^0 introduced, the prediction on branching ratios of B^0_{d,s} \rightarrow e^\mp \tau^\pm and B^0_{d,s} \rightarrow \mu^\mp \tau^\pm are far below B^0_{d,s} \rightarrow e^\pm \mu^\mp and the later are more promising to observed in future experiment.
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Appendix A: The Wilson coefficients for two point diagrams

\[
C_{SRR}^{(b)} = \sum_{i=u,c,t} \frac{e^2 C_{sb} C_{ep} K_{id} K_{is} m_s m_s}{2 s_w m_W^2 (m_{S0}^2 - M_{Bq}^2) (m_s^2 - m_d^2)} \left((m_d^2 - m_i^2 - 2m_W^2) B_1(m_d^2; m_W, m_i) - m_W^2 B_0(m_d^2; m_W, m_i) + A_0(m_i)\right), \quad C_{SRL}^{(b)} = C_{SRR}^{(b)}
\]

\[
C_{SLL}^{(b)} = \sum_{i=u,c,t} \frac{e^2 C_{sb} C_{ep} K_{id} K_{is}}{2 s_w m_W^2 (m_{S0}^2 - M_{Bq}^2) (m_s^2 - m_d^2)} \left(((m_i^2 - m_d^2)^2 + m_W^2 (m_i^2 - m_W^2)) \times B_0(m_d^2; m_W, m_i) - m_W^2 (m_i^2 + m_d^2) B_1(m_d^2; m_W, m_i) - (m_i^2 + 2m_W^2) \times A_0(m_i) + (2m_W^2 + m_i^2 - m_d^2) A_0(m_W)\right), \quad C_{SRL}^{(b)} = C_{SLL}^{(b)}
\]

\[
C_{SLL}^{(c)} = \frac{C_{sb} C_{sd} C_{ep}}{(m_{S0}^2 - M_{Bq}^2) (m_s^2 - m_d^2)} \left(m_b (m_s + m_b) B_1(m_b^2; m_{S0}, m_d) - A_0(m_{S0}) + (m_{S0}^2 - m_d^2 + m_b m_s - m_s m_d + m_d m_d) B_0(m_b^2; m_{S0}, m_d) + A_0(m_d)\right)
\]

\[
C_{SRL}^{(c)} = C_{SRL}^{(c)} = C_{SRR}^{(c)} = C_{SLL}^{(c)}
\]

\[
C_{SLL}^{(d)} = \sum_{i=u,c,t} \frac{e^2 C_{sd} C_{ep} K_{id} K_{is} m_s}{2 s_w m_W^2 (m_{S0}^2 - M_{Bq}^2) (m_s^2 - m_d^2)} \left(m_W^2 B_0(m_b^2; m_W, m_i) - A_0(m_i) + (2m_W^2 + m_i^2 - m_b^2) B_1(m_b^2; m_W, m_i)\right), \quad C_{SRL}^{(d)} = C_{SLL}^{(d)}
\]

\[
C_{SRL}^{(d)} = \sum_{i=u,c,t} \frac{e^2 C_{sd} C_{ep} K_{id} K_{is}}{2 s_w m_W^2 (m_{S0}^2 - M_{Bq}^2) (m_s^2 - m_d^2)} \left(((m_i^2 - m_d^2)^2 + m_W^2 (m_i^2 - m_W^2)) \times B_0(m_d^2; m_W, m_i) - m_W^2 (m_i^2 + m_d^2) B_1(m_d^2; m_W, m_i) - (m_i^2 + 2m_W^2) A_0(m_i) + m_W^2 (2m_W^2 + m_i^2 - m_d^2) B_1(m_b^2; m_W, m_i)\right), \quad C_{SRL}^{(d)} = C_{SRR}^{(d)}
\]
Appendix B: The Wilson coefficients for penguin diagrams

\[ C_{SLL}^{(e)} = \sum_{i=d,s,j=s,b} \frac{C_{di}C_{sj}C_{ij}C_{m}B_{0}(M_{Bq}^{2}; m_i, m_j) + (m_b + m_d - m_i + m_j)(m_d C_2 + m_b C_1) + (m_b (m_d + m_j) - m_d m_i - m_i m_j + m_{S0}^2) C_0}{(m_{S0}^2 - m_{Bq}^{2})} \]

\[ C_{SRL}^{(e)} = C_{SRL}^{(e)} = C_{SRL}^{(c)} = C_{SLL}^{(c)}; C_{\{0,1,2\}}^{(e)} = C_{\{0,1,2\}}^{(c)}(m_b^{2}, M_{Bq}^{2}, m_d^{2}, m_{S0}, m_i, m_j) \]

\[ C_{SLL}^{(f, Z)} = \frac{e^2 C_{db} C_{emu}}{36 s_{w}^2 c_{w}^2 m_{Z}^2(m_{S0}^2 - M_{Bq}^{2})} \left( (4 s_{w}^4 - 6 s_{w}^2) A_0(m_{Z}) + 4 s_{w}^2 m_{Z}^2(3 - 2 s_{w}^2) B_0(m_b^2, m_{Z}, m_b) + 3 m_{b} m_{d} B_0(M_{Bq}^{2}; m_b, m_d) + 4 s_{w}^2 m_{Z}^2(3 - 2 s_{w}^2) B_0(m_d^2, m_{Z}, m_d) + 6 m_{d} m_{b}^2 \times B_1(m_b^2, m_{Z}, m_d) - m_{b}^2 (8 s_{w}^4 - 18 s_{w}^2 + 9) B_1(m_d^2, m_{Z}, m_b) + m_{b}^2 (12 s_{w}^2 - 8 s_{w}^4) + m_{b} m_{d} (16 s_{w}^4 - 36 s_{w}^2 + 9) + 4 s_{w}^2 (2 s_{w}^2 - 3) (M_{Bq}^{2} - m_{d}^2) C_0^{Z} - 2 m_{Z}^2 \times (m_b - m_d) [(4 m_{d} s_{w}^{2} C_2^{Z} + m_b (3 - 2 s_{w}^{2} C_2^{Z})] \right) \]

\[ C_{SRL}^{(f, Z)} = C_{SRL}^{(f)} + \frac{e^2 C_{db} C_{emu}(4 s_{w}^2 - 3)}{12 s_{w}^2 c_{w}^2 m_{Z}^2(m_{S0}^2 - M_{Bq}^{2})} \left( m_{b}^2 B_0(m_b^2, m_{Z}, m_b) + 2 m_{b} m_{d} B_0(M_{Bq}^{2}; m_{Z}, m_{Z}, m_d) + 2 m_{Z}^2 m_{b} (m_{b} - m_d) C_2^{Z} - 2 m_{b} m_{d} m_{b}^{2} C_0^{Z} \right) \]

\[ C_{SLL}^{(f, \gamma)} = \frac{2 e^2 C_{db} C_{emu}}{9 M_{Bq}^{2} - m_{S0}^2} \left( B_0(m_b^2; 0, m_b) + B_0(m_d^2; 0, m_d) + (m_b - m_d)(m_b C_1^{\gamma} + m_d C_2^{\gamma}) \right) + (m_b - m_d)^2 - M_{Bq}^{2}) C_0^{\gamma} \]

\[ C_{SLL}^{(f, Z)} = C_{SRL}^{(f, Z)}; C_{SRL}^{(f, Z)} = C_{SRL}^{(f, \gamma)} = C_{SRL}^{(c, Z)}; C_{\{0,1,2\}}^{(f, Z)} = C_{\{0,1,2\}}^{(c, Z)}(m_b^{2}, M_{Bq}^{2}, m_d^{2}, m_{Z}, m_b, m_d) \]

\[ C_{SLL}^{(g)} = \sum_{i,j=u,c,t} \frac{e^2 C_{ij} C_{emu} K_{g} K_{j} d m_{b}}{2 s_{w}^2 m_{W}^2(M_{Bq}^{2} - m_{S0}^2)} \left( m_{i} B_1(m_i^2; m_{W}, m_i) + m_{j} B_0(M_{Bq}^{2}; m_i, m_j) \right) + m_{i} m_{W} C_0 + (m_i^2 m_j - m_d^2 m_i - (m_i - m_j)(m_i m_j - 2 m_{W}^2)) C_1 \]

\[ C_{SRR}^{(g)} = \sum_{i,j=u,c,t} \frac{e^2 C_{ij} C_{emu} K_{g} K_{j} d m_{d}}{2 s_{w}^2 m_{W}^2(M_{S0}^2 - M_{Bq}^{2})} \left( m_{i} B_0(M_{Bq}^{2}; m_i, m_j) + m_{j} B_1(m_i^2; m_{W}, m_i) \right) - m_{i} m_{W} C_0 - (m_i^2 m_j - m_d^2 m_i - (m_i - m_j)(m_i m_j - 2 m_{W}^2)) C_2 \]

\[ C_{SLR}^{(g)} = C_{SLL}^{(g)}, C_{SRR}^{(g)} = C_{SRL}^{(g)}; C_{\{0,1,2\}}^{(g)} = C_{\{0,1,2\}}^{(c, g)}(m_b^{2}, M_{Bq}^{2}, m_d^{2}, m_{W}, m_i, m_j) \]
Appendix C: The Wilson coefficients for box diagrams

\[ C_{(h)}^{(i)} = C_{sL}C_{sR}C_{eL}C_{eR}(m_s(m_t + m_\mu)D_0 - m_d(m_\tau + m_\mu)D_1 - m_b(m_\tau + m_\mu)D_2 \]

\[-(m_d(m_\tau + m_\mu) - m_b(m_\tau + m_\mu)D_3 - m_d m_\mu D_{13} - m_b m_\mu D_{23} - m_d m_\mu D_{33}) \]

\[ C_{VLL} = C_{sL}C_{sR}C_{eL}C_{eR}D_{00} \]

\[ C_{(h)}^{(i)} = C_{SLR}^{(i)} = C_{SRR}^{(h)} = C_{VLR}^{(i)} = C_{VRR}^{(i)} \]

\[ D_{(0,1,...)} = D_{(0,1,...)}(m_d^2, M_{Bq}^2, m_e^2, m_d^2 + m_e^2 - M_{Bq}^2; m_b^2, m_\mu^2; m_s, m_{S0}, m_{S0}, m_\tau) \]

\[ C_{(j,Z)}^{(j,Z)} = \frac{e^2 C_{dL}C_{eL}}{18c_w^2}(6 - 4s_w)C_0 - 2m_d^2(s_w^2 - 3)D_0 + (M_{Bq}^2(4s_w^2 - 6) + 3m_d^2 - 2(2s_w - 3) \]

\[ \times (m_e^2 - m_\mu^2))D_3 + (2s_w^2 - 3)(m_d^2 D_{33} + 2m_d^2 D_{23} + (m_d^2 + m_\mu^2)D_{22}) + (M_{Bq}^2(4s_w^2 - 6) \]

\[ + 3m_d^2 - 2(2s_w^2 - 3)(m_e^2 - m_\mu^2))D_2 + (2s_w^2 - 3)(m_d^2 D_{11} + (m_b^2 - M_{Bq}^2 + m_\mu^2)(D_{13} \]

\[ + D_{12}) + 4D_{00}) - 3m_b m_d D_1), C_{SRL}^{(j,Z)} = C_{SLL}^{(j,Z)} \]

\[ C_{SRR}^{(j,Z)} = \frac{e^2 C_{dL}C_{eL}}{18c_w^2}(2m_d^2s_w^2D_{33} + 2D_{23}) - 4s_w C_0 - m_d^2(2s_w^2 - 3)D_0 + (4s_w^2(M_{Bq}^2 - m_e^2 \]

\[ + m_b^2) - 3m_d^2)D_3 + 4M_{Bq}^2s_w^2D_2 - 4m_e^2s_w^2D_2 - 3m_d^2D_2 + 2s_w^2((m_d^2 + m_\mu^2)D_{22} + m_b^2D_{11} \]

\[ + (m_d^2 - M_{Bq}^2 + m_\mu^2)D_{13} + D_{12}) + 4D_{00}) + 3m_b m_d D_1), C_{SRL}^{(j,Z)} = C_{SLL}^{(j,Z)} \]

\[ C_{VLL}^{(j,Z)} = \frac{e^2 C_{dL}C_{eL}m_\mu}{18c_w^2}(2m_d s_w^2(D_{23} + D_{22}) + (3m_b - 2m_b s_w^2)D_{12} + m_d(2s_w^2 - 3)D_2) \]

\[ C_{VRR}^{(j,Z)} = \frac{e^2 C_{dL}C_{eL}m_\mu}{18c_w^2}(2m_d s_w^2(D_2 + D_{22}) - 2m_b s_w^2D_{12} + m_d(2s_w^2 - 3)D_{23} - 3m_d D_{22}) \]

\[ C_{VLR}^{(j,Z)} = C_{VLL}^{(j,Z)}, C_{VRL}^{(j,Z)} = C_{VRR}^{(j,Z)}, C_{(0)} = C_{(0)}(m_e^2, m_\mu^2, M_{Bq}^2, m_{S0}, m_\mu, m_Z) \]

\[ D_{(0,1,...)} = D_{(0,1,...)}(m_b^2, m_e^2, m_\mu^2, m_d^2, m_d^2 + m_e^2 - M_{Bq}^2, M_{Bq}^2; m_d, m_{S0}, m_\mu, m_Z) \]
\[
C^{(j,\gamma)}_{SLL} = \frac{e^2 C_{db} C_{em}}{3} (m_d^2 (D_{33} + 2D_{23} - D_0) + (m_d^2 + m_{\mu}^2) D_{22} + 2(M_{Bq}^2 - m_e^2) D_2 - 2C_0 \\
+ 2(M_{Bq}^2 + m_d^2 - m_{\mu}^2) D_3 + m_d^2 D_{11} + (m_{\mu}^2 + m_d^2 - M_{Bq}^2) (D_{13} + D_{12}) + 4D_{00})
\]
\[
C^{(j,\gamma)}_{SLL} = \frac{e^2 C_{db} C_{em\mu}}{3} (m_d D_{23} + m_d D_{22} - m_b D_{12} + m_d D_2)
\]
\[
C^{(j,\gamma)}_{SLR} = C^{(j,\gamma)}_{SRL} = C^{(j,\gamma)}_{VLR} = C^{(j,\gamma)}_{VLL} = C^{(j,\gamma)}_{VRL} = C^{(j,\gamma)}_{VRR}
\]
\[
C_0 = C_0 (m_b^2, m_{\mu}^2, M_{Bq}^2; m_{s0}, m_{\mu}, 0)
\]
\[
D_{(0,1,...,33)} = D_{(0,1,...,33)} (m_b^2, m_{\mu}^2, m_d^2, m_e^2 + m_d^2 - M_{Bq}^2, M_{Bq}^2; m_d, m_{s0}, m_{\mu}, 0)
\]
\[
C^{(k,Z)}_{SLL} = \frac{e^2 C_{db} C_{em}}{6c_w} ((2s_w^2 - 3)(m_e^2 D_{33} + 2(M_{Bq}^2 + m_e^2 - m_{\mu}^2) D_0 + 2(M_{Bq}^2 + 2m_e^2 - m_{\mu}^2) D_3) \\
+ (2s_w^2 (3M_{Bq}^2 - m_b^2 + m_d^2 + 2m_e^2 - 2m_d^2) + 3m_b(m_b + m_d) - 9M_{Bq}^2 - 6m_e^2 \\
+ 6m_d^2) D_2 + M_{Bq}^2 (2s_w^2 - 3) D_{22} + 2(2s_w^2 - 3)(M_{Bq}^2 + m_e^2 - m_{\mu}^2) D_{23} + (M_{Bq}^2 (4s_w^2 - 6) \\
+ m_d^2 (4s_w^2 - 3) + 2(2s_w^2 - 3)(m_e^2 - m_{\mu}^2)) D_1 + (2s_w^2 - 3)(m_d^2 D_{11} + 2(M_{Bq}^2 + m_e^2 \\
- m_{\mu}^2) D_{13} + (M_{Bq}^2 + m_d^2 - m_b^2) D_{12} + 4D_{00}))
\]
\[
C^{(k,Z)}_{SRR} = C^{(k,Z)}_{SRL} = C^{(k,Z)}_{VLR} = C^{(k,Z)}_{VLL} = C^{(k,Z)}_{VRL} = C^{(k,Z)}_{VRR}
\]
\[
C^{(k,Z)}_{VLL} = C^{(k,Z)}_{VLR} = \frac{e^2 C_{db} C_{em\mu}}{6c_w} ((3m_b - 2m_b s_w^2 - 2m_d s_w^2) D_{23} - 2m_d s_w^2 D_{13} - 3m_d D_3)
\]
\[
C^{(k,Z)}_{VRR} = C^{(k,Z)}_{VRL} = \frac{e^2 C_{db} C_{em\mu}}{6c_w} ((m_d (3 - 2s_w^2) - 2m_b s_w^2) D_{23} + m_d ((3 - 2s_w^2) D_{13} + 3D_3))
\]
\[
D_{(0,...,3)} = D_{(0,...,3)} (m_d^2, m_{\mu}^2, m_{e}^2; M_{Bq}^2, m_d^2 + m_{\mu}^2 - M_{Bq}^2; m_Z, m_d, m_{s0}, m_e)
\]
\[
C^{(k,\gamma)}_{SLL} = \frac{e^2 C_{db} C_{em}}{3} (m_d^2 (D_{33} - D_0) - (M_{Bq}^2 - 2m_d^2 + m_{\mu}^2) D_{23} + 4D_{00} + m_b^2 D_{11} \\
- 2(M_{Bq}^2 + m_{\mu}^2) D_3 - (M_{Bq}^2 - m_e^2 + m_{\mu}^2) D_2 - (M_{Bq}^2 - m_d^2 - m_{\mu}^2) D_{22} \\
- (M_{Bq}^2 - m_d^2 - m_b^2) (D_{13} + D_{12}))
\]
\[
C^{(k,\gamma)}_{VLL} = \frac{e^2 C_{db} C_{em\mu}}{3} (m_d D_{23} + m_d D_{22} - m_b D_{12} + m_d D_2)
\]
\[
C^{(k,\gamma)}_{SLR} = C^{(k,\gamma)}_{SRL} = C^{(k,\gamma)}_{SRR} = C^{(k,\gamma)}_{VLR} = C^{(k,\gamma)}_{VLL} = C^{(k,\gamma)}_{VRL} = C^{(k,\gamma)}_{VRR}
\]
\[
D_{(0,...,3)} = D_{(0,...,3)} (m_b^2, m_{\mu}^2, m_{e}^2; m_d^2 + m_{\mu}^2 - M_{Bq}^2; M_{Bq}^2; m_d, m_{s0}, m_e, 0)
\]
\(C_{SLL}^{(l,Z)} = \frac{-e^2 C_{d\bar{b}C_{ep}}}{6c_w^2} (2s_w^2(m_d^2 + m_e^2 - M^2_{Bq}) D_{33} - m_b m_d D_0 + (m_d(m_b - m_b) - M^2_{Bq} + m_e \times (m_e + m_\mu) D_3) + (3 m_b(m_b + m_d) - 2 s_w^2 (m_d(m_b - m_d) + M^2_{Bq})) D_2 + 2 s_w^2 (m_b^2 D_{22}) + (m_b^2 - M^2_{Bq}) D_{23} - (M^2_{Bq} - 2 m_d^2 - m_e^2 + m_\mu^2) D_{13} + m_d(m_d - m_b) D_1 + 4 D_{00} \times m_d D_{11} + (m_b^2 + m_d^2 - M^2_{Bq}) D_{12})) \), \(C_{SLR}^{(l,Z)} = C_{SLL}^{(l,Z)} \)

\(C_{SRR}^{(l,Z)} = \frac{-e^2 C_{d\bar{b}C_{ep}}}{6c_w^2} (2s_w^2 - 3) ((M^2_{Bq} - m_d^2 - m_e^2) D_{33} + m_b m_d D_0 + (m_d(m_b - m_d) + M^2_{Bq} - m_e(m_e + m_\mu)) D_3) + (3 m_b(m_b + m_d) - 2 s_w^2 - 3)(M^2_{Bq} - m_d^2) D_2 - (2 s_w^2 - 3) \times (m_b^2 D_{22} + (m_b^2 - M^2_{Bq} + m_d^2) D_{23} + m_d(m_d - m_b) D_1) + (2 s_w^2 - 3)(-m_d^2 D_{11} + (M^2_{Bq} - 2 m_d^2 - m_e^2 + m_\mu^2) D_{13} - (m_b^2 - M^2_{Bq} + m_d^2) D_{12} - 4 D_{00}) \), \(C_{SRL}^{(l,Z)} = C_{SRR}^{(l,Z)} \)

\(C_{VLL}^{(l,Z)} = \frac{-e^2 C_{d\bar{b}C_{ep}}}{6c_w^2} (2s_w^2 - 3) ((m_d m_d - m_d(m_e + m_\mu) - m_b m_\mu) D_3) + 2 s_w^2 m_b(m_e + m_\mu) D_2 + m_b(2 s_w^2 - 3)(m_e + m_\mu) D_0 + 2 m_b m_\mu s_w^2 D_{23} - m_d(2 s_w^2 - 3)(m_e D_{13} + (m_e + m_\mu) D_1) \)

\(D_{0\{0,1,...\}} = D_{0\{0,1,...\}}(m_d^2, M^2_{Bq}; m_e^2, m_d^2 + m_e^2 - M^2_{Bq}; b, m_b; m_b, m_{S0}, m_Z, m_e) \)

\(C_{SLL}^{(l,\gamma)} = \frac{e^2 C_{d\bar{b}C_{ep}}}{3} ((m_d^2 + m_\mu^2) D_{33} - 2 C_0 + (m_d(m_b + 2 m_d) - 2 m_\mu^2) D_0 + m_b^2 D_{22} + m_d D_{11} + (m_d(m_b + 3 m_d) - m_\mu(m_e + m_\mu)) D_3 + (2 m_b^2 + m_b m_d - M^2_{Bq} + m_d^2 \times D_2 + 4 D_{00} + (m_b^2 + m_d^2 - M^2_{Bq}) (D_{23} + D_{12}) + m_d(2 m_d D_{13} + (m_b + 3 m_d) D_1)) \)

\(C_{VLL}^{(l,\gamma)} = \frac{e^2 C_{d\bar{b}C_{ep}}}{3} (m_d m_d m_d D_3 - m_b(m_e + m_\mu) D_0 + (m_d(m_e + m_\mu) - m_b m_\mu) D_3 - m_b m_d D_{23} - m_b(m_e + m_\mu) D_2 + m_d m_\mu D_{13} + m_d(m_e + m_\mu) D_1) \)

\(C_{SLR}^{(l,\gamma)} = C_{SLL}^{(l,\gamma)} = C_{SRR}^{(l,\gamma)} = C_{VLL}^{(l,\gamma)} = C_{VRL}^{(l,\gamma)} = C_{VRR}^{(l,\gamma)} = C_{SRL}^{(l,\gamma)} = C_{SLR}^{(l,\gamma)} \)

\(C_0 = C_0(m_b^2, m_e^2, m_d^2 + m_e^2 - M^2_{Bq}; m_b, 0, m_e) \)

\(D_{\{0,\ldots\}} = D_{\{0,\ldots\}}(m_d^2, m_d^2, m_e^2, m_e^2 + m_d^2 - M^2_{Bq}; m_b^2, m_b^2, m_b, m_{S0}, 0, m_e) \)
\[ C_{SLL}^{(m,Z)} = \frac{e^2 C_{dm} C_{em}}{6c_w^2} (2s_w^2(m_d^2 + m_e^2 - M_{Bq}^2)D_{33} - m_b m_d D_0 + (m_d^2 - m_d m_b - M_{Bq}^2 + m_e D_{22}) + m_b (m_e + m_\mu)) \]
\[ \times m_b (m_d + m_e) D_3 + (3m_b (m_b + m_d) - 2s_w^2 (m_d (m_b - m_d) + M_{Bq}^2)) D_2 + 2s_w^2(m_b^2 D_{22} + (m_b^2 D_{22} - 2m_d^2 + m_e^2 - m_\mu^2) D_{13} + m_d (m_d - m_b) D_1) \]
\[ + 2s_w^2(m_b D_{11} + (m_b^2 + m_d - M_{Bq}^2) D_{12} + 4D_{00})) , \quad C_{SLL}^{(m,Z)} = C_{SLL}^{(m,Z)} \]

\[ C_{SRR}^{(m,Z)} = \frac{e^2 C_{db} C_{ep}}{6c_w^2} ((2s_w^2 - 3)((m_d^2 + m_e^2 - M_{Bq}^2) D_{23} - m_b m_d D_0 + (m_d^2 - m_d m_b - M_{Bq}^2) \]
\[ + m_\mu (m_e + m_\mu)) D_3 + m_b^2 D_{22} + (2m_d^2 - M_{Bq}^2 - m_e^2 + m_\mu^2) D_{13} + (m_b^2 + m_d - M_{Bq}^2) \]
\[ \times (D_{12} + D_{13}) + m_b^2 D_{11} + 4D_{00} + m_d (m_d - m_b) D_1) + (3m_b^2 + 2m_b m_d s_w^2 \]
\[ + (2s_w^2 - 3)(M_{Bq}^2 - m_b^2)) D_2) , \quad C_{SRL}^{(m,Z)} = C_{SRR}^{(m,Z)} \]

\[ C_{VLL}^{(m,Z)} = \frac{-e^2 C_{db} C_{ep}}{6c_w^2} ((3 - 2s_w^2)(m_d m_e D_{33} - m_b (m_e + m_\mu)) D_0 + (m_b m_e - m_d (m_e + m_\mu)) \]
\[ \times D_3 - m_b m_e D_{13} - m_d (m_e + m_\mu) D_1) + 2m_b m_e s_w^2 D_{23} + 2m_b s_w^2 (m_e + m_\mu) D_2 \]

\[ C_{VLL}^{(m,Z)} = \frac{e^2 C_{db} C_{ep}}{6c_w^2} (2s_w^2(m_d m_e D_{33} + m_e D_{13} + (m_e + m_\mu) D_1) - m_b (m_e + m_\mu)) D_0 \]
\[ + (m_b (m_e + m_\mu) - m_b m_e) D_3) - m_b (2s_w^2 - 3)(m_e D_{23} + (m_e + m_\mu) D_2) \]

\[ C^{(m,Z)} = C_{VLL}^{(m,Z)} , \quad C_{VRL}^{(m,Z)} = C_{VRR}^{(m,Z)} \]

\[ D_{01,...} = D_{01,...} (m_d^2, M_{Bq}^2, m_e^2, m_d + m_e - M_{Bq}^2, m_b^2, m_b; m_b, m_{80}, m_Z, m_\mu) \]

\[ C_{SLL}^{(m,\gamma)} = \frac{e^2 C_{db} C_{ep}}{3} ((m_d^2 + m_e^2 - M_{Bq}^2) D_{33} - m_b m_d D_0 - (m_d (m_b - m_d) + M_{Bq}^2) D_2 + m_b^2 D_{22} \]
\[ + (m_d (m_d - m_b) - M_{Bq}^2 + m_\mu (m_e + m_\mu)) D_3 + (m_b^2 - M_{Bq}^2 + m_\mu^2) D_{23} + m_d (m_d \]
\[ - m_b) D_1 + m_b^2 D_{11} - (M_{Bq}^2 - 2m_b^2 + m_e^2 - m_\mu^2) D_{13} + (m_b^2 - M_{Bq}^2 + m_\mu^2) D_{12} + 4D_{00} \]

\[ C_{VLL}^{(m,\gamma)} = \frac{e^2 C_{db} C_{ep}}{3} (m_d m_e D_{33} - m_b (m_e + m_\mu)) D_0 + (m_b (m_e + m_\mu) - m_b m_e) D_3 - m_b \]
\[ \times (m_e + m_\mu) D_2 - m_b m_e D_{23} + m_d m_e D_{13} + m_d (m_e + m_\mu) D_1 \]

\[ C_{SLL}^{(m,\gamma)} = C_{SLL}^{(m,\gamma)} = C_{SRL}^{(m,\gamma)} = C_{SRR}^{(m,\gamma)} = C_{VLL}^{(m,\gamma)} = C_{VRL}^{(m,\gamma)} = C_{VRR}^{(m,\gamma)} \]

\[ D_{0,...} = D_{0,...} (m_d^2, m_{Bq}^2, m_e^2, m_d + m_e - M_{Bq}^2, m_b^2, m_b; m_b, m_{80}, 0, m_\mu) \]

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