Tests of the universality of free fall for strongly self-gravitating bodies with radio pulsars

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Abstract
In this paper, we review tests of the strong equivalence principle (SEP) derived from pulsar–white dwarf binary data. The extreme difference in the binding energy between both components and the precise measurement of the orbital motion provided by pulsar timing allow the only current precision SEP tests for strongly self-gravitating bodies. We start by highlighting why such tests are conceptually important. We then review previous work where limits on SEP violation are obtained with an ensemble of wide binary systems with small eccentricity orbits. Then, we propose a new SEP violation test based on the measurement of the variation of the orbital eccentricity ($\dot{e}$). This new method has the following advantages: (a) unlike previous methods it is not based on probabilistic considerations, (b) it can make a direct detection of SEP violation and (c) the measurement of $\dot{e}$ is not contaminated by any known external effects, which implies that this SEP test is only restricted by the measurement precision of $\dot{e}$. In the final part of the review, we conceptually compare the SEP test with the test for dipolar radiation damping, a phenomenon closely related to SEP violation, and speculate on future prospects by new types of tests in globular clusters and future triple systems.

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(Some figures may appear in colour only in the online journal)

1. Introduction
The strong equivalence principle (SEP) extends the weak equivalence principle (WEP) to the universality of free fall (UFF) of self-gravitating bodies. General relativity (GR) assumes the validity of WEP and SEP, i.e. in GR the world line of a body is independent of its chemical composition and gravitational binding energy. Therefore, a detection of SEP violation would falsify GR. On the other hand, alternative theories of gravity generally violate the SEP. This is also the case for most alternative metric theories, such as the Jordan–Fierz–Brans–Dicke (JFBD) theory [32, 23, 7] or the more general scalar–tensor theories of gravity [11], which
satisfy the WEP as a consequence of their postulate of universal coupling between matter and gravity ([61], see also review by Damour in this issue).1

SEP violation could have observable consequences even in the weak, quasi-stationary gravitational fields in our Solar System [39, 40], in particular a ‘polarization’ of the Earth–Moon system by the external Solar field [41]. Detecting such a polarization is one of the main motivations of the Lunar laser ranging (LLR) experiment, which is described in other articles in this issue.

In view of the smallness of self-gravity of Solar System bodies, the LLR experiment says nothing about strong-field aspects of gravity, like the gravitational properties of extremely compact objects like neutron stars or black holes. In the presence of such bodies, alternative theories of gravity that pass Solar System tests can still produce various observable phenomena not predicted by GR, among these the radiation of dipolar gravitational waves and strong-field SEP violation (section 2). The former has not yet been detected in the most sensitive binary pulsar experiments [37, 28], which given the current experimental precision already provide extremely stringent tests of alternative theories of gravity. In the near future, the detailed study of the gravitational wave signal emitted by mergers of compact objects will also provide independent tests of the radiative properties of gravity (e.g., [62]).

For strong-field SEP violation, the best current limits come from millisecond pulsar–white dwarf (MSP–WD) systems with wide orbits. If there is a violation of UFF by neutron stars, then the gravitational field of the Milky Way would polarize the binary orbit [16]. Such tests of the UFF of strongly self-gravitating masses are the subject of this work. In comparison with the LLR tests, they have two disadvantages, one of them being the much weaker polarizing external field ($|g| \sim 2 \times 10^{-10} \text{ m s}^{-2}$ compared with the acceleration of the Solar gravitational field at the Earth, $\sim 6 \times 10^{-3} \text{ m s}^{-2}$) and the precision of the ranging, which is of the order of 10 m for the best pulsar experiments ($10^{-2} \text{ m}$ for the LLR). This is almost completely compensated by the gravitational binding energy of neutron stars $E_{\text{grav}}$, which is a large fraction of the total inertial mass energy: $\varepsilon_{\text{grav}} = E_{\text{grav}}/M_{I}c^{2} \sim -0.15$; this is more than eight orders of magnitude larger than that of Earth ($\varepsilon_{\text{grav},\oplus} \sim -5 \times 10^{-10}$). This results in experiments with comparable limits on SEP violation, which are nonetheless complementary since they probe different regimes of binding energy.

In section 2, we summarize some of the theoretical foundations of SEP violation for strongly self-gravitating bodies. In section 3, we review the Damour–Schäfer test, which yields the best current limits on SEP violation by strongly self-gravitating bodies, if applied to a whole population of small-eccentricity systems.

In section 4, we suggest new pulsar timing experiments that avoid the probabilistic considerations of present tests and have the potential to detect SEP violation; these attempt to directly measure its effects, in particular the variation of the orbital eccentricity $\dot{e}$. It is shown that this measurement is not contaminated by external effects and, because of this, the limits on SEP violation are only restricted by the precision of the measurement of $\dot{e}$. Our simulations show that such direct tests will very soon surpass the best current limits on SEP violation for strongly gravitating bodies.

Section 5 provides a short discussion on the complementarity of SEP tests and tests for dipolar radiation damping in constraining alternative theories of gravity. Finally, in section 6, we briefly discuss the possibilities of the SEP test proposed (section 4) when the test binaries are accelerating in a field much stronger than that of the Galaxy, like that of a globular cluster or an additional outer companion. Such systems would likely offer very significant gains in

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1 In fact, Willy Scherrer was the first to propose a scalar–tensor theory of gravitation. See [29] for details on the history of scalar–tensor theories.
the power of SEP tests, essentially providing an experiment with fields and velocities very similar to those present in the LLR experiment, but with ‘proof masses’ with a $\sim 10^8$ larger binding energy—essentially a combination of the best features of both types of experiments.

2. SEP violation and strongly self-gravitating bodies

2.1. Beyond the weak-field approximation

In general, an alternative theory of gravity is expected to violate the SEP, in the sense that the ratio between the gravitational mass $M_G$ of a self-gravitating body to its inertial mass $M_I$ will admit an expansion of the type

$$\frac{M_G}{M_I} \equiv 1 + \Delta = 1 + \eta_1 \epsilon_{grav} + \eta_2 \epsilon_{grav}^2 + \cdots.$$  \hspace{1cm} (1)

At the first post-Newtonian level $\eta_1$ parametrizes the weak-field violation of SEP (the Nordtvedt effect). In the parameterized post-Newtonian (PPN) framework, this Nordtvedt parameter is given as a combination of different PPN parameters (see [61] for details on the PPN formalism):

$$\eta_1 \equiv \eta_N = 4\beta - \gamma - 3 - \frac{10}{3} \xi - \alpha_1 + \frac{2}{3} \alpha_2 - \frac{2}{3} \zeta_1 - \frac{1}{3} \zeta_2.$$  \hspace{1cm} (2)

The parameter $\eta_N$ is well constrained by LLR experiments in the Solar System (see contributions on LLR in this review). In view of the fact that $\epsilon_{grav,\odot} \sim -5 \times 10^{-10}$, $\epsilon_{grav,\text{Moon}} \sim -2 \times 10^{-11}$ and $\epsilon_{grav,\odot} \sim -10^{-6}$, it is clear that higher order deviations from the SEP cannot be tested in LLR experiments, or any other experiment in the Solar System, in the foreseeable future. To test a violation of SEP that might occur beyond the weak-field regime, one needs strongly self-gravitating bodies. Currently, in nature, the best objects for such tests are neutron stars observed as active radio pulsars ($\epsilon_{grav} \sim -0.15$).\(^2\)

Since beyond the weak field approximation there is no general PPN formalism available, discussions of gravity tests in this regime are done in various theory-specific frameworks. An example for a very detailed investigation of higher order/strong-field deviations from GR, within the family of (well-defined) scalar–tensor theories of gravity, are the frameworks developed by Damour and Esposito-Farèse. Specifically,

(A) The four-parameter framework $T_0(\gamma, \beta; \epsilon, \zeta)$ of [13], which defines the second post-Newtonian extension of the original (Eddington) PPN framework $T_0(\gamma, \beta)$. In this framework

$$\Delta_0 \approx -\frac{1}{2}(4\beta - \gamma - 3) c_A + (\frac{4}{3} + \zeta) b_A,$$  \hspace{1cm} (3)

where the compactness $c_A \approx -2\epsilon_{grav,\odot}$ and $b_A \approx c_A^2$.

(B) The two-parameter class of bi-scalar–tensor theories $T_2(\beta', \beta'')$ introduced in [11]. Within this framework, one has no Nordtvedt effect in the Solar System, since $\eta_N = 0$. On the second post-Newtonian level, one has $\epsilon = \beta'$, $\zeta = 0$. And $\beta''$ parametrizes contributions beyond the second post-Newtonian level.

(C) The two-parameter class of mono-scalar–tensor theories $T_1(\alpha_0, \beta_0)$ of [12, 14], which for certain values of $\beta_0$ exhibits significant strong field deviations from GR, and a corresponding violation of SEP for neutron stars.

For details, we refer the reader to [10] and references therein. In the following, we will have a closer look at case C, as this class of scalar–tensor theories of gravity illustrates quite impressively a violation of SEP that can only be measured with strongly self-gravitating

\(^2\) For gravity theories that predict a no-hair theorem for black holes, neutron stars are in fact the most compact objects suitable for strong-field SEP violation tests (see, e.g., [15, 43, 1]).
bodies. We would like to note, however, that this particular case of scalar–tensor theories of gravity resembles just one example of how nonlinearities in the gravitational interaction could drive gravity away from GR in the strong fields of compact masses and lead to a strong-field violation of SEP.

2.2. Strong field effects and the violation of SEP

Damour and Esposito-Farèse [12] found that scalar–tensor theories, which pass the weak-field tests in the Solar System, could still exhibit large, strong-field-induced deviations in systems involving neutron stars (spontaneous scalarization). This has been studied extensively in a two-parameter space of theories, i.e. \( T_1(\alpha_0, \beta_0) \), defined by the coupling function, which is a quadratic polynomial in the scalar field \( \phi \):

\[
a(\phi) = \alpha_0(\phi - \phi_0) + \frac{\beta_0(\phi - \phi_0)^2}{2} \quad [12, 14, 15].
\]

The parameter \( \alpha_0 \) defines the linear matter-scalar coupling constant and \( \beta_0 \) is the quadratic coupling of matter to two scalar particles, while higher order vertices are neglected. In this sense, this is a natural extension of JFBD gravity.

In the presence of such non-perturbative strong-field deviations away from GR, we can have a situation where the effective coupling strength of the neutron star \( \alpha_A \) is of order unity, even if the scalar-matter coupling \( \alpha_0 \) is unobservably small in the Solar System\(^3\). Such an effect leads to a violation of SEP that requires test systems, which contain a neutron star. The structure dependence of the effective gravitational constant \( G_{AB} \) has the consequence that the pulsar does not fall in the same way as its companion in the gravitational field of a third body, which in our case is the Galaxy. One finds for a pulsar with a weakly self-gravitating companion, since \( \alpha_0 \ll 1 \), that [11]

\[
\Delta_p - \Delta_c \simeq \alpha_0(\alpha_p - \alpha_c) \simeq \alpha_0(\alpha_p - \alpha_0).
\]

While \( |\alpha_0| < 0.003 \) by the Cassini experiment [4], \( \alpha_p \) can be of order unity for neutron stars, as outlined above. Although the effect is greatly suppressed by a small factor \( \alpha_0 \), it could in the presence of non-perturbative strong-field deviations be many orders of magnitude stronger than in the Solar System (factor \( \alpha_p/\alpha_0 \)), and therefore still become visible in binary pulsar timing experiments.

2.3. SEP violation and orbital motion of binary pulsars

In the previous section, we have seen that one could expect violations of SEP in gravity regimes where the Solar System tests are insensitive, and therefore one has to utilize test systems that contain strongly self-gravitating bodies. Currently, binary pulsars are the best probes for testing such kind of strong field gravity effects [10]. Before we discuss in details the SEP tests that can be conducted in timing binary pulsars, we need to outline the orbital dynamics of a binary system that falls freely in the gravitational field of our Galaxy in the presence of a SEP violation.

In case of a violation of SEP, the equations for the relative motion \( \mathbf{R}(t) \) are given by

\[
\ddot{\mathbf{R}} = -\frac{\mathcal{G} M}{R^3} \mathbf{R} + \mathbf{A}_{\text{PN}} + \mathbf{A}_{\Delta},
\]

where \( M \) is the total mass of the binary and \( \mathcal{G} \equiv G_{\text{pc}} \) is the effective gravitational constant between the two bodies [16]. The post-Newtonian contributions are denoted by \( \mathbf{A}_{\text{PN}} \) and the

\(^3\) The quantity \( \alpha_A \equiv \frac{\partial \ln M_A}{\partial \phi} \) measures the effective strength of the coupling between a self-gravitating body \( A \), with the total mass \( M_A \) and the scalar field \( \phi \). It is equivalent to the negative ratio of the total scalar charge to the total mass. For a weakly self-gravitating body, \( \alpha_A \simeq \alpha_0 \).
additional acceleration caused by the SEP violation \( \mathbf{A}_\Delta \) is given to leading order by

\[
\mathbf{A}_\Delta = (\Delta_p - \Delta_c) \mathbf{g},
\]

where \( \Delta_c \) is defined by the ratio between ‘gravitational’ and ‘inertial’ mass as given in equation (1), and \( \mathbf{g} \) is the external acceleration caused by the gravitational field of the Galaxy. The secular changes to the binary system with orbital frequency \( n_b \) and eccentricity \( e \) caused by a violation of SEP are \[16\]

\[
\langle dh_b / dt \rangle = 0, \quad \langle de / dt \rangle = f \times \mathbf{l} + \dot{\omega}_{PN} \hat{\mathbf{k}} \times \mathbf{e}, \quad \langle d\mathbf{l} / dt \rangle = f \times \mathbf{e},
\]

where

\[
e \equiv e \hat{\mathbf{a}}, \quad \mathbf{l} \equiv \sqrt{1 - e^2} \hat{\mathbf{k}}, \quad \mathbf{f} \equiv \frac{3}{2Y_O} (\Delta_p - \Delta_c) \mathbf{g},
\]

and \( Y_O \equiv (G M_{n_b})^{1/3} \) is a measure for the relative orbital velocity between the pulsar and its companion. The unit vector \( \hat{\mathbf{a}} \) points toward periastron and the unit vector \( \hat{\mathbf{k}} \) is parallel to the orbital angular momentum. The post-Newtonian precession of periastron is given by

\[
\dot{\omega}_{PN} = 3F (Y_O/c)^2 \frac{\mathbf{e}}{1 - e^2} n_b,
\]

where \( F \) is a theory-dependent factor \((F = 1 \text{ in GR})\). As a result of equations (7), a violation of the UFF would lead to a change in the observed orbital eccentricity

\[
\dot{e} = \langle de / dt \rangle \cdot \hat{\mathbf{a}} = \sqrt{1 - e^2} (\mathbf{b} \cdot \mathbf{f}), \quad (\hat{\mathbf{b}} \equiv \hat{\mathbf{k}} \times \hat{\mathbf{a}})
\]

and a change in the angle \( i \) between the line-of-sight direction to the pulsar, i.e. \( \mathbf{K}_0 \), and \( \hat{\mathbf{k}} \): \[11\]

\[
\frac{d\cos i}{dt} = \frac{d}{dt} \left( \frac{\mathbf{K}_0 \cdot \mathbf{l}}{\sqrt{1 - e^2}} \right) = \frac{e}{\sqrt{1 - e^2}} (\mathbf{K}_0 \cdot \mathbf{b}) (\hat{\mathbf{b}} \cdot \mathbf{f}).
\]

In binary pulsar timing experiments, this change in \( i \) becomes apparent as a change in the timing parameter \( x \), which is the projected semi-major axis of the pulsar orbit given by \( x = a_p \sin i / c \).

As shown in [16], for very small eccentricities, the equations of motion (7) essentially decouple. As a consequence, the orbital plane remains fixed and the evolution of the eccentricity can be written as

\[
e(t) = e_{PN}(t) + e_{\Delta}, \quad e_{\Delta} \equiv f_L / \dot{\omega}_{PN}.
\]

The vector \( e_{PN}(t) \) has a fixed length and is turning in the orbital plane with the angular velocity \( \dot{\omega}_{PN} \). The polarization of the orbit due to the violation of SEP is represented by the constant eccentricity vector \( e_{\Delta} \), which points into the direction of the projection of \( \mathbf{f} \) into the orbital plane (denoted by \( f_L \)). Figure 1 illustrates the time evolution of the orbital eccentricity. Thus, in the presence of a SEP violation, the observed eccentricity oscillates between a minimum \((|e_{PN} - e_{\Delta}|)\) and a maximum \((|e_{PN} + e_{\Delta}|)\) eccentricity with a period of \( 2\pi / \dot{\omega}_{PN} \).

3. The Damour–Schäfer test

In 1991, when Damour and Schäfer first wrote their paper describing the orbital dynamics of a binary pulsar under the influence of SEP violation, only four binary pulsars were known in the Galactic disk. Two of these (PSR B1913+16 and PSR B1957+20) were clearly inadequate for that test—not only because of the compactness of their orbits, but also because one of them is a double neutron star system that lacks the required amount of asymmetry in the binding

\[4\] As the gravitational field of the Galaxy at the location of the binary pulsars is weak, and since most of the mass of the Galaxy is made up of non-strongly self-gravitating bodies, any higher order contributions to \( \mathbf{A}_\Delta \) are negligible.
energy, necessary for a stringent test of SEP violation [the term \((\alpha_p - \alpha_c)\) in equation (4)]. The remaining systems were PSR B1855+09 [45] and PSR B1953+29 [6]. The latter was already deemed to be the best probe for the detection of SEP violation effects because of its orbital period of 117 days, by far the largest then known. Nevertheless, even for this system, the orbital effects predicted for allowed levels of SEP violation—in particular, the changing eccentricity \(\dot{e}\)—were too small for detection given the timing precision and baseline of that system in 1991. Another way of saying this is that at that time no interesting limits on the SEP violation could be derived from existing constraints of \(\dot{e}\). It is for this reason that Damour and Schäfer proposed a novel, indirect statistical SEP test, by assuming that the SEP violation is responsible for a maximal part of the observed small orbital eccentricities.

3.1. Small-eccentricity binary pulsars and SEP

The basic idea presented by Damour and Schäfer has been already described above, namely that the violation of the SEP introduces a polarization of the orbit of binary pulsars that is best represented by a vector addition where the observed eccentricity vector \(e(t)\) lies on a circle (see figure 1). In the extreme case, however, the vector addition leads to a near or even complete cancellation of the eccentricity vector and a SEP violation is therefore not detectable. Unfortunately, the intrinsic vector \(e_{PN}\), and therefore also its orientation relative to the SEP component vector \(e_{\Delta}\), is unknown. However, for a sufficient age of the system, one can assume that the relativistic precession of the orbit will have caused the eccentricity vector to have made many turns since the system’s birth, thereby effectively randomizing the relative orientation \(\theta \equiv \dot{\omega}_{PN} t\). In fact, the angular velocity of the periastron advance, i.e. \(\dot{\omega}_{PN}\), should be appreciably larger than the angular velocity of the rotation of the Galaxy with which \(g\) rotates in the reference frame of the binary system. As a result, the projection of the Galactic acceleration vector onto the orbit can be considered constant, and statistical arguments based on the exclusion of small \(\theta\), i.e. possible near cancellation, can be applied. In the ideal case, the masses of the system components, the inclination of the orbit and the distance to the pulsar are known. The angle describing the orientation of the orbit around the line-of-sight \(\Omega\) is generally not observable and has to be treated as an independent random variable uniformly distributed between 0 and 2\(\pi\).

As the treatment above has been derived for small-eccentricity, long-orbital period pulsars, the figure-of-merit for suitable systems considered by Damour and Schäfer was the large values of \(P_{0}/e\)—the large values of \(P_{0}\) decrease the relative orbital velocities \(V_0\), which increases the
amplitude of \( f \) (equation (8)) and the predicted change of eccentricity (equations (7)); the small value of \( e \) implies that, in a statistical sense the orbit has been little changed by such SEP violation effects.

They applied their method to the two systems with (by far) the best \( P_b^2/e \) at the time, PSR B1855+09 and PSR B1953+29. While the measurement of a Shapiro delay [49, 27] for PSR B1855+09 [45, 34] were available, providing constraints on \( \sin i \) and the component masses, evolutionary arguments were used to constrain the parameters for PSR B1953+29. Overall, Damour and Schäfer derived 90% confidence level limits of \(|\Delta p| < 5.6 \times 10^{-2}\) and \(|\Delta p| < 1.1 \times 10^{-2}\), respectively.

Six years later, the author of [59] presented an updated analysis, applying this Damour–Schäfer test to eight systems with large values for \( P_b^2/e \); this excluded the SEP violation at a level of \( 5 \times 10^{-3} \) with more than 95% confidence.

3.2. A population of small-eccentricity binary pulsars

The analysis presented in [59] has a caveat, by selecting only those systems with the best figure-of-merit \( P_b^2/e \). By this, one introduces a selection bias, as it is possible that on the one hand a significant eccentricity (which would reduce the figure-of-merit and a possible weight in the analysis) is actually the result of an SEP violation, and on the other hand, the small eccentricities selected are those where by chance \( \theta \) is small. (This was pointed out to one of us (NW) by Kenneth Nordtvedt.)

In order to take this into account, the author of [60] presented an updated analysis that included all relevant small-eccentricity binary pulsars at that time. Based on extensive Monte–Carlo simulations, a large set of simulated (cumulative) distributions of eccentricities was compared to the distribution of eccentricities observed in the population of small-eccentricity binary pulsars. In this analysis, unknown angles like \( \theta \) and \( \Omega \) were distributed uniformly between 0 and 2\( \pi \). Based on a Kuiper’s test like criterion, the number of simulated distributions, which are in agreement with the observed distribution was determined. As a result, the author of [60] found a 95%-confidence limit of \(|\Delta p| < 9 \times 10^{-3}\). This limit is weaker than the one in [59], but certainly more representative by being able to consistently include also systems with seemingly worse figure-of-merits.

In the same spirit, an updated analysis was presented first in [52] and later in [30], using even larger samples of pulsar–WD systems, i.e. the population of all known systems that are thought to have evolved with similar extended accretion periods. For deriving the median-likelihood value of \(|\Delta p|\) for each pulsar, they use a Bayesian analysis marginalizing over the parameters with similar assumptions to those in [16, 59]. In systems like PSRs 0437−4715 and J1713+0747, where the orientation of the orbit can be measured [55, 51], posterior probability density functions were derived appropriately. From all available information, they obtained \(|\Delta p| < 5.6 \times 10^{-3}\) (95% CL). The best present limit was obtained by [30] using 27 binary systems (including additional astrometric and mass information), \(|\Delta p| < 4.6 \times 10^{-3}\) (95% CL).

4. Detecting the SEP violation directly

Many new advances in pulsar astronomy have occurred in the two decades elapsed since the publication of [16]. Three of them are especially important in this context.

(A) Continuing pulsar surveys have discovered many more binary systems, including several with significantly wider orbits than PSR B1953+29, i.e. systems more suitable for the detection of SEP violation (section 3).
Advances in the sensitivity and bandwidth of radio receivers and improvements in the time resolution of instrumentation mean that we can time the binary pulsars in these systems much more precisely than possible before.

The sheer amount of elapsed time provides much larger timing baselines.

All of these developments increase the sensitivity to the direct effects of SEP violation, in particular \( \dot{e} \) (section 4.1), to the point that it should now be possible to derive even more stringent limits on the SEP violation from them (section 4.2). This has several advantages over the statistical method described in section 3.

(i) We can actually detect a potential SEP violation by measuring a nonzero \( \dot{e} \), particularly if the same phenomenon is observed for several binary MSPs. With the statistical method reviewed in section 3, we can only estimate upper limits for the effect.

(ii) For wide pulsar–WD systems, only this hypothetical SEP violation can cause a measurable \( \dot{e} \): orbital circularization due to emission of gravitational waves or aberration effects cause a change in eccentricity that is many orders of magnitude below the current experimental precision on \( \dot{e} \); furthermore, there are no effects due to mass loss in the system or its motion (section 4.3). In this sense, this is a clean test of the validity of GR.

(iii) The clean nature of the \( \dot{e} \) test implies that the limit on the SEP violation will improve at the same rate as the precision of the measurement of \( \dot{e} \). Thus, if the latter improves continuously with time (section 4.1), the same will happen with limits on the SEP violation.

(iv) Since the test can be done with a single binary system, we do not need to assume that \( \Delta p \) is the same for all systems, as assumed in the statistical test of section 3.2. Indeed, as pointed out in [10], alternative theories of gravity predict that \( \Delta p \) is a function of the pulsar mass. Therefore, a rigorous analysis requires an accurate knowledge of the masses of the pulsars, which for many of the pulsar–WD systems are not available. In view of strong-field effects like ‘spontaneous scalarization’, the combination of binary pulsars in a generic SEP test could even be rendered meaningless.

(v) We do not need to restrict our sample to systems with small eccentricities (section 4.4).

We now discuss which of the known binary pulsars are most suitable for this particular test.

### 4.1. Figure of merit for detection of SEP violation

Our simulations indicate that for the parameters affected by the SEP violation the uncertainty provided by timing is given by

\[
\delta \dot{e} \simeq 8.0 \times \frac{\delta t}{x \sqrt{NT^3}},
\]

\[
\delta \dot{x} \simeq 5.3 \times \frac{\delta t}{\sqrt{NT^2}},
\]

where \( \bar{N} \) is the average number of TOAs per unit time (which we assumed to be uniform in our simulations), \( \delta t \) is the rms of the TOA residuals and \( T \) is the observing baseline. This also implies, in general, that \( \delta \dot{e} \simeq 1.5 \delta \dot{x}/x \). These expressions assume the absence of red timing noise. If those effects are present at measurable levels, then the improvement with time is slower [36].

Using the expressions in section 2 and the expression for the total mass \( M \) derived from the mass function (equation (3.15) in [18]), we can re-write \( \dot{e} \) as

\[
\dot{e} = \frac{3}{2} (\Delta p - \Delta c) (\hat{b} \cdot \hat{g}) \left( \frac{p}{G M_c} \right)^{1/2},
\]
where $\rho \equiv x c (1 - \rho^2) / \sin i$ is the length of the semi-latus rectum of the pulsar’s orbit and $\mathbf{\hat{b}}$ is the unit vector along its length. With the two last equations, we can calculate, in the eventuality of SEP violation, the significance of its measurement:

$$\frac{\dot{\rho}}{\delta \rho} \simeq 1.7 \times 10^{-5} \, s^{-3/2} \left( \Delta_{\rho,3} - \Delta_{\epsilon,3} \right) \left( \mathbf{\hat{b}} \cdot \mathbf{\hat{g}} \right) \left( \frac{G T_y^3 \tilde{N}_d}{\delta t_{\mu}^2} \frac{x^3 (1 - \epsilon^2)}{m_e \sin i} \right)^{1/2}, \quad (16)$$

where $m_e \equiv M_e / M_{\odot}$, $\mathbf{\hat{g}}_{10} \equiv \mathbf{g} / 10^{-10} \, m \, s^{-2}$, $\Delta_{\rho,3} \equiv \Delta_{\rho} / 10^{-3}$, $T_y$ is the timing baseline in years, $\tilde{N}_d$ is the number of TOAs per day and $\delta t_{\mu}$ is the TOA r.m.s. in $\mu$s. Furthermore, limits on viable alternative theories of gravity from pulsar–WD systems [5, 28] imply that we can assume $G / G \simeq 1$, where $G$ is Newton’s gravitational constant. If there is the SEP violation, then given enough time $T$ it will eventually be detected to high significance.

### 4.2. Current precision of direct SEP violation test

To evaluate equation (16), we must determine $(m_e \sin i)$ with some degree of precision; this requires a significant measurement of the Shapiro delay. Apart from this, $\omega$ and $\Omega$ (the longitude of periastron and position angle of the line of nodes) are necessary for determining the absolute orientation of the orbital plane in space. Furthermore, the parallax of the system $\pi_x$ is necessary for determining its distance $d$ and its location in the Galaxy, which are necessary to estimate the Galactic acceleration $\mathbf{g}$ at that location and its projection $(\mathbf{\hat{b}} \cdot \mathbf{\hat{g}})$ along the direction of the semi-latus rectum. With the exception of $\omega$ all of these parameters require high timing precision. There are only two systems for which all these parameters have been measured: PSR J0437–4715 and J1713+0747. The latter has a significantly larger $x$, which according to equation (16) increases its sensitivity to SEP violation. Thus, in what follows, we discuss this system in more detail.

Pulsar timing can in principle be used to detect very low-frequency gravitational waves [46, 21]. Currently, several large-scale projects are attempting to achieve this with precise, sustained timing of several MSPs (a pulsar timing array, or PTA, [56, 22, 19]). PSR J1713+0747 is part of the whole effort because it is detectable by all major radio telescopes and is currently one of the three pulsars with the smallest $\delta t$ known, 0.1 $\mu$s [19]. It has also been precisely timed for two decades, making its $T$ unusually large. All of this makes this system extremely sensitive to $\dot{\rho}$ caused by the SEP violation. Using a PTA pulsar for this experiment also means that the SEP violation test proposed here demands no extra time allocation—the scientific results can be obtained for free as an added benefit of ongoing effort.

We now use TOA simulations to estimate what limits on $\dot{\rho}$ can be achieved for this pulsar. We do this partly because no values and uncertainties for $\dot{\rho}$ have been published to date for this pulsar (an illustration of how unexpected a measurable $\dot{\rho}$ is, see section 4.3), but also to estimate future limits on its precision. In our simulations, we use TOA datasets with uncertainties, number, start and end times given in table 1 of [51] and table 2 of [19]; the latter dataset appears to have started at the end of 2004 and continues to the present and it completely dominates our simulated dataset. For the first 6 years, 2368 TOAs had been taken, at an average rate of 1 TOA per day; we assume in our simulations that 30 TOAs were obtained in a single session every 30 days; furthermore, we assume that this uniform rate continues at present.

With these assumptions, we obtain $\delta \dot{\rho} = 1.6 \times 10^{-18} \, s^{-1}$ as a realistic uncertainty at the end of 2012. If the measured value for $\dot{\rho}$ is consistent with zero, then given the location and orbital orientation of this binary, this would result in $\Delta_{\rho} < 2 \times 10^{-3}$ (95% CL), which is already twice as constraining as the current limits from the Damour–Schäfer test. If we improve the timing precision by a factor of 2 and keep the same timing strategy, then we would have to wait until 2030 for the precision of the test to increase by one order of magnitude. Significantly
faster progress might be achievable for MSPs with larger $x$, or with improvements in timing precision that will be provided by the Square Kilometre Array (SKA) [47].

### 4.3. Cleanest binary pulsar experiment

Using the results in [18], in particular, equation (2.4c), we see that the orbital eccentricity $e$, being adimensional, is not affected by the Doppler shift $D$ that must necessarily occur during the coordinate transformation from the reference frame of the binary to the Solar System Barycenter (SSB). Another example of such an adimensional quantity is the Shapiro delay parameter $\varsigma$ [27]. Therefore, it follows that (unlike all time-like quantities, like $P$, $P_b$ and $x$) $e$ is not affected even when there is a change in that Doppler factor, $\dot{D}$. At any moment, the observed $e$ is the ‘intrinsic’ $e$, plus a small term due to geodetic precession.

It follows from this that if the observed $\dot{e}$ is too small for detection, the same limit will also apply to the intrinsic $\dot{e}$, irrespective of $\dot{D}$. This is extremely important: as an example, the intrinsic variation of the orbital period ($\dot{P}_{\text{int}}^b$) or the variation of the projected semi-major axis, $\dot{x}$ is always ‘polluted’ by $\dot{D}$ [17], and we thus observe

$$
\left(\frac{\dot{P}_b}{\dot{P}_b}\right)_{\text{obs}} = \left(\frac{\dot{P}_b}{\dot{P}_b}\right)_{\text{int}} - \frac{\dot{D}}{D}, \quad \left(\frac{\dot{x}}{\dot{x}}\right)_{\text{obs}} = \left(\frac{\dot{x}}{\dot{x}}\right)_{\text{int}} - \frac{\dot{D}}{D}.
$$

(17)

Thus, uncertainties in the estimation of $\dot{D}$ impose a fundamental limit on the precision of the measurement of $\dot{P}_{\text{int}}^b$ and therefore ultimately limit the precision of radiative tests of gravity [58].

Another effect that changes $\dot{P}_{\text{int}}^b$ is mass loss from the binary, which has a lower limit given by the loss of rotational energy from the pulsar [17]; furthermore outgassing from the companion can in some binaries be so large that no test of GR can be made. However, such instances of steady mass loss do not affect the orbital eccentricity [31].

For very wide MSP–WD systems, tidal effects—in particular orbital circularization—are also expected to be extremely small, since both objects are extremely small compared to the size of the orbit. They behave effectively as two point masses.

Finally, if GR is the correct theory of gravity, then $\dot{e}$ is given by [18]

$$
\dot{e}^{\text{GR}} = \dot{e}^{\text{GW}} + \dot{e}^{\text{A}},
$$

(18)

where the first term is caused by gravitational wave emission and the second is caused by a change in the aberration parameter that is to be expected from geodetic precession. The leading term of $\dot{e}^{\text{GW}}$ is given by [42]

$$
\dot{e}^{\text{GW}} = - \frac{304}{15} n_b^{8/3} (T_\odot m_e)^{4/3} \frac{q}{(q+1)^{1/3}} \frac{e(1+121e^2/304)}{(1-e^2)^{5/2}},
$$

(19)

where $q = m_p/m_e$ is the mass ratio and $T_\odot \equiv GM_\odot/c^3 = 4.925 \times 10^9 \mu s$ is one Solar mass in time units. We derive the second term from the equations in [18]

$$
\dot{e}^{\text{A}} = - \frac{1}{\pi} \frac{P}{x} \frac{n_b^2 T_\odot m_e}{(q+1)^2} \frac{q + 3/4}{(1-e^2)^{2/5}} J(i, \lambda, \eta),
$$

(20)

$$
J(i, \lambda, \eta) = \frac{1}{\sin^2 \lambda} \left( \sin i \cos \lambda \sin 2\eta + \cos i \sin \lambda \cos \eta \right),
$$

(21)

where $\eta$ is the longitude of the projection of the pulsar spin axis in the plane of the sky measured from ascending node and $\lambda$ is the angle between the pulsar spin axis and the line of sight from the pulsar to the Earth (this cannot be zero, otherwise there would be no pulsations).
Figure 2. Limits on $\Delta_p$ (95% CL) as a function of the longitude of the ascending node $\Omega$ for the PSR J1903+0327 binary system, using the values for $s$ and $\dot{e}$ in [25] and assuming an orbital inclination of 77.47° (the other possible inclination is 102.53°, for which we would have similar constraints, but at different values of $\Omega$). A distance of 6.4 kpc was assumed. The upper limits approaching $\sim 0.09$ at angles of $\sim 80°$ and $\sim 250°$ etc are derived from the measurement of $\dot{x}$, while the limits peaking at $\sim 70°$ and $\sim 230°$ are derived from the measurement of $\dot{e}$. The overall upper limit is given by the corresponding combination of both constraints.

For PSR J1713+0747, these terms are given, respectively, by $\dot{e}^{GW} = -8.2 \times 10^{-29}$ s$^{-1}$ and $\dot{e}^A = 8.2 \times 10^{-28}$ s$^{-1} J(i, \lambda, \eta)$. Since $J(i, \lambda, \eta)$ must be of the order of unity$^6$, this means that $\dot{e}^{GR}$ is at least nine orders of magnitude smaller than the current $\delta \dot{e}$. Given the smallness of all polluting terms, we reach the conclusion that the limits on $\Delta_p$ will improve as much as the experimental precision of $\dot{e}$.

4.4. Testing the SEP with the 1903+0327 binary system

An important advantage of these direct tests of SEP violation is that we do not need to restrict our study to binaries with small eccentricities. The binary system PSR J1903+0327 [8, 25] has good timing precision ($\delta t \simeq 1 \mu s$) and a wide ($x = 105.593$ lt-s) orbit for which $\dot{e} = (14 \pm 6) \times 10^{-17}$ s$^{-1}$ and $x_{obs} = +21(3) \times 10^{-15}$ lt-s s$^{-1}$ have been published [25]. A precise distance is not known yet, nor $\Omega$. However, we can use the parameters we know precisely (position in the sky, $\sin i$, $\omega$) and a model of the Galactic potential to estimate limits on $\Delta_p$ for any assumed $d$ and $\Omega$. These are displayed graphically in figure 2 for $d = 6.4$ kpc.

Note that we can estimate $\Delta$ limits for every value of $\Omega$ because for such eccentric systems SEP violation also causes a torque in the orbital plane (equation (11)). This would add a component $\dot{x}_{SEP}$ to the kinematic contribution expected for the system at each particular $\Omega$, $\dot{x}_{kin}$ [2, 35] and the Doppler contribution for each particular $d$ (equation (17)), such that $\dot{x}_{SEP} = \dot{x}_{kin} - \chi D/D$. These limits are not as constraining as those derived from PSR J1713+0747, but they are already interesting because they apply to a massive neutron star.

$^6$ This is a fully recycled pulse; therefore, the vast majority of its rotational angular momentum came from orbiting material. This implies that the spin and orbital angular momenta must be very closely aligned; therefore, $\eta \approx \pi/2$ and $J(\lambda, \eta) \leq 0$ (i.e. much smaller than unity). This means that for this kind of system the only likely contribution to $\dot{e}^{GR}$ comes from $\dot{e}^{GW}$, which is ten orders of magnitude smaller than the present $\delta \dot{e}$.
\( M_p = 1.667 \pm 0.021 M_\odot, 99.7\% \text{ CL} \), for which no precise constraints of \( \Delta p \) have been obtained until now (we elaborate on this in section 5). Furthermore, this limit will increase fast because (a) the \( \dot{e} \) and \( \dot{x} \) limits were obtained with only 2 years of data (from 2008 to 2010), so significant improvements will be possible in a relatively short timescale and (b) new broadband coherent dedispersion systems will yield a major improvement in the timing precision of this system.

Future infrared interferometric experiments like GRAVITY [3] will be able to measure precisely the astrometric motion of the main sequence companion to PSR J1903+0327, allowing therefore a precise, independent measurement of \( \pi_x, i \) and \( \Omega \), thus allowing for unambiguous SEP tests in this system. Moreover, if indeed the SEP is violated at a measurable level in PSR J1903+0327, the eccentric nature of this system allows a unique cross-check of this since both \( \dot{x}_{\text{SEP}} \) and \( \dot{e} \) should be influenced in a characteristic way: according to equations (10) and (11), the quantity \( \dot{x}_{\text{SEP}}/(\dot{x} \dot{e}) \) only depends on the orientation and the eccentricity of the pulsar orbit.

5. The complementarity of SEP violation and dipolar radiation tests

As discussed in the introduction, a theory of gravity that predicts a violation of SEP is also expected to predict the emission of dipolar gravitational radiation in asymmetric binary systems, like pulsar–WD binaries [61]. By now there are several binary pulsars that provide tight constraints on the existence of dipolar gravitational radiation within scalar–tensor gravity, as well as within more general frameworks [5, 38, 28, 37]. For these theories, these current radiative tests on binary pulsars are more constraining than SEP tests [28].

Nevertheless, there are two aspects that support the importance of SEP tests with binary pulsars. First, they can be interpreted as generic, direct tests for the UFF of strongly self-gravitating bodies, independent of any specific gravity theory. Secondly, nonlinear strong-field effects, like spontaneous scalarization, could be limited to very massive neutron stars, which until now have only been observed in wide binary systems like PSR J1614−2230 \( (M_p \approx 1.97 M_\odot, P_b = 8.7 \text{ days [20]}) \) and PSR J1903+0327 \( (M_p \approx 1.68 M_\odot, P_b = 95 \text{ days [25]}) \). In these wide systems, a dipolar contribution to the gravitational wave damping could, even in future, be too small to be detectable, or could not be separable from kinematic effects [17], while the deviation from GR could still be measurable in an SEP test. In particular, as outlined in section 4.3, the \( \dot{e} \)-test is not ‘contaminated’ by external effects.

6. Future prospects: globular cluster pulsars, triple systems and mergers

The external gravitational acceleration by the Galaxy is rather small \( (\mid g \mid \sim 2 \times 10^{-10} \text{ m.s}^{-1} \text{ at the location of the Sun}) \). In a stronger external gravitational field, the SEP violation would be proportionally more prominent (equation (8)). There are numerous pulsar binaries known to exist in globular clusters [24], where the external acceleration is typically two orders of magnitude larger than in the field of the Galaxy [63, 9, 26]. Unfortunately, one cannot determine the exact location of these systems within the globular cluster as one does not have a good handle on the radial distance for these pulsars; however, this can be somewhat constrained for systems with negative period derivatives. Furthermore, the latter provide direct lower limits on \( g \) along the line of sight. If the semi-latus rectum happened to lie along the line of sight, then we would automatically have all that we need to derive an upper limit on \( \Delta p \).

An even stronger external field would arise in a hierarchical triple star system, where the pulsar is in a tight orbit with a weakly self-gravitating object, and this inner binary falls in
the gravitational field of a third companion. This would resemble the SEP test done in the
Earth–Moon–Sun system, but with a strongly self-gravitating object. In the globular cluster
M4, there is the MSP PSR B1620−26, where the inner companion to the pulsar is a \( \sim 0.3 M_\odot \)
WD and the outer companion is a \( \sim \) Jupiter mass companion [53, 48]. Unfortunately, the low-
mass outer companion yields an external acceleration \( g \), which is only a factor of a few larger
than the typical value for the acceleration in the Galactic plane, and therefore does not lead
to any interesting constraints. The situation would be quite different if the outer companion
were a \( \sim 1 M_\odot \) star, and even better if it were also a neutron star. A future detection of a
comparable-mass triple with a pulsar is not unlikely [54], particularly since we have already
discovered a binary system, PSR J1903+0327, which started its life as a hierarchical triple
system and appears to have become a binary system much later in its evolution [25]. Other
systems with similar origins might still survive as hierarchical triples.

Similarly to equation (4), in a hierarchical triple system, one would have [11]

\[
\Delta_p - \Delta_c \simeq \alpha_{ex} (\alpha_p - \alpha_c).
\]  

According to this equation, the ideal triple system combination would be a pulsar–WD
(\( \alpha_c = \alpha_0 \ll 1 \)) or pulsar–black hole (\( \alpha_c = 0 \)) system in the field of a more distant neutron star,
for which \( \alpha_{ex} \), like \( \alpha_p \), could in theory be much larger than \( \alpha_0 \). Consequently, in a hierarchical
triple system not only the external gravitational acceleration felt by the internal binary would
be much larger than for a binary pulsar falling in the field of the Galaxy, also the effective
scalar coupling of the source of the external field could be significantly larger, provided the
outer companion is a neutron star.

In such a triple system, the challenge lies in obtaining a sufficient number of higher order
derivatives in the pulsar frequency, in order to constrain the orbit and mass of the distant
companion such that any effects from the SEP violation inflicted on the inner orbit could be
separated from ‘classical’ orbital perturbations [44, 33]. This depends on the details of the
system, particularly its orbital period and the timing observations (precision, time span).

Finally, if the SEP violating interaction is only of limited range, like in the massive
Brans–Dicke theory of gravity (see [1]), then Galactic binary pulsars are insensitive to the
corresponding violation of the UFF, since the masses that cause the external gravitational field
are at large distances. In such a case, direct tests of strong field aspects of the UFF would
require a hierarchical triple system.

In about 3 years from now, it is expected that, after their upgrade, the ground-based
gravitational wave detectors will make their first detections of gravitational waves. One of
the most promising sources for these detectors are inspiralling compact binaries, consisting
of neutron stars or black holes [57]. This will not only mark the beginning of the era of
gravitational-wave astronomy, it will also provide new possibilities to test gravity, including
(indirect) tests of UFF for self-gravitating bodies via its theoretical connection to the radiative
properties of a gravity theory. Although, for certain scalar–tensor theories (including JFBD),
it has been shown that binary pulsar experiments are already more constraining than it is
expected for the advanced LIGO/VIRGO detectors [62, 15, 28], there are many theoretical
aspects where ground-based gravitational wave antennae will nicely complement binary pulsar
experiments in the near future, notably theories where the gravitational interaction is partly
mediated by very short range (\( \sim 1 \) lt-s, and less) fields.

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