A PHYSICAL LIMIT TO THE MAGNETIC FIELDS OF T TAURI STARS

PEDRO N. SAAR

University of Maryland, Laboratory for Millimeter-Wave Astronomy, College Park, MD 20742; saar@astro.umd.edu

Received 1998 July 28; accepted 1998 November 13; published 1998 December 9

ABSTRACT

Recent estimates of magnetic field strengths in T Tauri stars yield values of $B = 1–4$ kG. In this Letter, I present an upper limit to the photospheric values of $B$ by computing the equipartition values for different surface gravities and effective temperatures. The values of $B$ derived from the observations exceed this limit, and I examine the possible causes for this discrepancy.

Subject headings: stars: atmospheres — stars: late-type — stars: magnetic fields — stars: pre–main-sequence

1. INTRODUCTION

Magnetic fields are believed to play a fundamental role in the structure and evolution of T Tauri stars. Unfortunately, their detection and measurement in these objects is a very difficult task.

Early attempts to detect magnetic fields in T Tauri stars through Zeeman polarization were unsuccessful (Johnstone & Penston 1986, 1987), and only recently are such detections forthcoming through the measurement of Zeeman broadening of photospheric lines in a handful of objects (Basri, Marcy, & Valenti 1992; Guenther 1997). These measurements yield disk-averaged, photospheric field strengths of order a few kG.

These observations do not provide a direct measurement of $B$; instead, what is measured is the difference in broadening of magnetically sensitive and insensitive lines (Robinson 1980; Marcy 1982; Saar 1988). To find the value of $B$, one has to fit the data with a stellar atmosphere model and vary $B$ and $f$—the fraction of stellar surface that is magnetized—until a good match is found. The result of this matching procedure is the product $fB$, and independent determination of $B$ and $f$ is much more problematic and sensitive to modeling assumptions (see, e.g., Solanki 1992 and references therein). Furthermore, when equivalent widths are used in lieu of measuring the line broadening, the deduced $fB$ is also very sensitive to the input model atmosphere (Basri et al. 1992; Basri & Marcy 1994).

The purpose of this Letter is to present a physical constraint on the maximum value of $fB$; this upper limit on the magnetic field strength is important to estimate $B$ from the $fB$ values obtained from the data and may help to uncover potential errors in the modeling assumptions.

The basic physics is rather well known and has been successfully applied to the study of the Sun. Magnetic fields are excluded from closed circulation patterns when the field lines are perpendicular to the axis of rotation of the fluid (see, e.g., Parker 1979, and references therein); as a result of this general principle, the solar convection sweeps the field into the down-draft regions of the eddies. Convective heat transport is strongly inhibited within the field (Biermann 1941), and, through the so-called superadiabatic effect (Parker 1978), the downflowing gas further cools and evacuates the magnetized gas; as a result, the external gas pressure is larger than that inside the tube, and the flux tube shrinks until the total internal pressure (thermal plus magnetic) balances the external pressure. Thus, the solar magnetic field is broken up and compressed into highly evacuated flux tubes.

Although we cannot resolve features on the surface of other stars, it is a reasonable inference that, in complete analogy with the Sun, the magnetic field in stars with outer convection zones—or convective throughout, as in the case of T Tauri stars—is concentrated into discrete flux tubes, which are in pressure equilibrium with the unmagnetized gas. If this is true, then an upper limit on $B$ can be derived if the pressure of the unmagnetized gas is known and the gas pressure inside the flux tubes is ignored. These arguments have been extensively applied to active dwarfs (see, e.g., Saar 1996), but so far they have never been used to study the magnetic fields in T Tauri stars.

In § 2, I derive an expression for the maximum equipartition value of $B$ as a function of the stellar and atmospheric parameters, and in § 3 I compare the results from § 2 with current observational estimates. My conclusions follow in § 4.

2. EQUIPARTITION FIELDS

Consider a straight flux tube that is initially perpendicular to the photospheric surface at some fiducial height. The condition of lateral pressure equilibrium (i.e., the flux tube neither expands nor contracts) is given by

$$\frac{B^2}{8\pi} = \Delta P$$

where $\Delta P$ is the difference between the external and internal nonmagnetic pressures, $P_i$ and $P_e$, respectively, and the external medium is unmagnetized. At any height, an upper limit to $B$ is given by the equipartition field $\propto (P_e)^{1/2}$, and because temperature and pressure decrease with height in the photosphere, the maximum detectable field is the equipartition field at the optical depth at which the continuum is formed ($\tau = 2/3$):

$$B_{eq} = \sqrt[1/2]{8\pi P_e(\tau = 2/3)}.$$  

In principle, $P_e$ includes the contributions from both thermal

Note that this argument does not apply to stars with outer radiative zones, where there are no known mechanisms to isolate and concentrate the field.
pressure and ram pressure due to convective motions \((P_r\text{ and }P_t\text{ respectively})\) because in a convective atmosphere the magnetic field outside dark spots is concentrated where the convective downdrafts are located (see, e.g., Stein, Brandenburg, & Nordlund 1992). However, it is easy to show that \(P_t \gg P_r\) and the argument is the following.

The ratio \(P_t/P_r\) can be written as

\[
\frac{P_t}{P_r} = \frac{1}{3} \gamma \left(\frac{\bar{v}}{v_s}\right)^2, \tag{3}
\]

where \(\gamma\) is the ratio of the specific heat capacities, \(\bar{v}\) is the average speed of a (turbulent) convective element, and \(v_s\) is the adiabatic speed of sound. Although recent simulations of compressible convection (Cattaneo & Malagoli 1992) show that horizontal surface flows can be intermittently transonic, the downflows are subsonic, and, because \(\gamma \leq 5/3\), it follows that \(P_t/P_r \leq 5/9\). A more stringent limit on this ratio is given by estimating \(\bar{v}/v_s\) using the mixing-length theory of convection, which gives (see, e.g., Cox & Giuli 1968, eq. [14.64])

\[
\frac{\bar{v}}{v_s} = 0.4 \left(\frac{L_*}{1.4 L_S}\right)^{1/3} \left(\frac{T_{\text{eff}}}{4000 \text{ K}}\right)^{-1/2} \left(\frac{\mu}{1.7}\right)^{1/2}, \tag{4}
\]

where \(L_*\), \(T_{\text{eff}}\) and \(\mu\) are, respectively, the stellar luminosity, effective temperature, and photospheric mean molecular weight. Therefore, for the stellar parameters of a typical T Tauri star (TTS), \(P_t/P_r = 0.2\), and by neglecting \(P_t\) the error introduced in \(B_{eq}\) is \(\approx 10\%\) much smaller than the observational errors (see below). Henceforth I will neglect the contribution of ram pressure\(^2\) to \(B_{eq}\) and use \(P_t = P_r\).

The value of \(P_t(\tau = 2/3)\) follows from the condition of hydrostatic equilibrium, and to a very good degree of approximation (generally, better than a factor of 2 or 3; see, e.g., Cox & Giuli 1968), one can write

\[
P_t(\tau = 2/3) = \frac{2}{3} \frac{g}{\kappa}, \tag{5}
\]

where \(\kappa\) is the Rosseland mean opacity. The gas temperature \(T\) at \(\tau = 2/3\) is assumed to be the effective temperature of the star \(T_{\text{eff}}\), and, because \(\kappa\) is a function of density \(\rho\) and \(T\), equation (5) is an implicit equation for \(\rho(T = T_{\text{eff}})\) for a given \(T_{\text{eff}}\) and \(g\).

\(^2\) The same result, essentially, applies to the Sun, where the Maxwell stresses at the surface are an order of magnitude larger than the Reynolds stresses.

---

**TABLE 1**

Comparison between \(B_{eq}\) from Equation (6) and Detailed Stellar Atmospheres

| Spectral Type | log \(g\) | \(B_{eq}^*\) (kG) | \(\Delta^b\) | Atmospheric Model |
|--------------|----------|-----------------|-------------|------------------|
| Sun          | 4.4      | 1.2             | 0.08        | Guenther et al. 1992 |
| K5           | 4.5      | 1.8             | 0.06        | Allard & Hauschildt 1995 |
| M1           | 4.0      | 1.5             | -0.13       | Allard & Hauschildt 1995 |
| M5           | 4.0      | 1.5             | -0.36       | Allard & Hauschildt 1995 |
| M5           | 5.0      | 4.8             | -0.52       | Allard & Hauschildt 1995 |

\(^*\) Computed from eq. (6).

\(^b\) Fractional difference; the equipartition field computed from a detailed atmospheric model is given by \(B_{eq}(1 + \Delta)\).

---

I have computed \(B_{eq}\) for different values of \(T_{\text{eff}}\) and \(g\) by solving equation (5) using the Alexander & Ferguson (1994) Rosseland mean opacities and using equation (6). To gauge the magnitudes of the errors introduced by the approximation used to derive equation (5), I have also computed \(B_{eq}\) by obtaining \(P_t(T = T_{\text{eff}})\) from the detailed stellar-atmosphere models by Guenther et al. (1992) and Allard & Hauschildt (1995). The comparison between the two sets of \(B_{eq}\) so obtained is presented in Table 1.

The results in Table 1 show that for spectral type K5 or earlier, the approximation used in equation (5) results in an underestimate of the field by less than 10%. On the other hand, for later spectral types, the field is overestimated by at most a factor of 2, but this error decreases with decreasing gravity. The reason for this larger discrepancy at cooler temperatures is the increase in opacity due to \(H_2O\) once the gas temperature drops below 3500 K (Alexander & Ferguson 1994), which results in a strong nonmonotonic behavior of \(\kappa\) with optical depth in the outer layers.

Therefore, for the typical gravities of T Tauri stars (log \(g \leq 4.0\)) and the spectral types currently accessible to Zeman-broadening measurements (spectral type K7 or earlier), the equipartition fields predicted from equation (6) are too small by at most 10% when compared with detailed atmospheric models. These errors are much smaller than the error bars for the measured \(B'\)s (see Fig. 1 below).

It is important to keep in mind that the measured field depends on the geometrical depth at which the emission originates. Because flux tubes are strongly evacuated, the optical depth inside a magnetized region is smaller than in the sur-
Also, a good approximation to the fields outside spots in other surrounding atmosphere. If the flux tube is slender enough, it will be mostly transparent to the radiation crossing its walls, and, because of its lower density, it may appear as a bright feature against the continuum from its surroundings, as do the network fields in the Sun. In the solar case, the enhanced temperature compensates for the lower optical depth, and the values of $B$ measured in these bright points compare well with $B_{eq}(T = T_{ecl})$. If the Sun is any guidance, then $B_{eq}(T = T_{ecl})$ should be also a good approximation to the fields outside spots in other stars.

On the other hand, larger features like sunspots are much more evacuated and cooler than slender flux tubes, and at a fixed geometrical height the optical depth is much smaller inside the spot. Thus, one can see deeper into a sunspot, and they indeed appear as dimples on the Sun’s surface when seen near the limb—a phenomenon known as the Wilson depression. Because the continuum from a spot originates at a larger geometrical depth, the external pressure there is larger, and, therefore, the fields measured inside sunspots are larger than those from slender features like network fields. In the Sun, the Wilson depression can amount to a few scale heights, and thus the measured fields in sunspots are $\sim 2B_{eq}(T = T_{ecl})$, i.e., of order 3 kG (although, sometimes, values as high as 5 kG have been measured). In complete analogy with the Sun, one should not expect $B_{eq}(T = T_{ecl})$ to be a useful limit—within a factor of a few—to the fields in stellar dark spots. The equipartition argument still applies; however, to derive a useful limit on $B$, it is necessary to know the detailed thermal structure of the spot, which is a problem that still lacks a full solution even for the Sun.

3. COMPARISON WITH CURRENT MEASUREMENTS

At present, only two sets of magnetic-field measurements for T Tauri stars are available; the measurements by Basri et al. (1992) and those of Guenther (1997). These observations measure the broadening of Fe i lines in the wavelength range 5000–7000 Å; at these wavelengths, a dark spot with temperature $T_{ecl} - \Delta T$ that covers a fraction $f$ of the surface of a star with $T_{ecl}$ contributes a fraction $fB_{eq}(T_{ecl} - \Delta T)/(1 - f)B_{eq}(T_{ecl})$ of the total emission, where $B_{eq}$ is the Planck function. Using the typical values $f = 0.3$, $\Delta T = 1000$ K, and $T_{ecl} = 4000$ K (see, e.g., Bouvier et al. 1993), one finds that starspots contribute less than 7% to the light in the range 5000–7000 Å. Therefore, current measurements are insensitive to the magnetic fields in the center of dark spots, where $B$ can be larger than $B_{eq}$ at $T = T_{ecl}$.

Figure 1 is a comparison between the lower limits to $B$ derived from the observations (because $f \leq 1$) and $B_{eq}$ calculated from equation (6) as a function of $T_{ecl}$ and different surface gravities. Also shown is the value of $B_{eq}$ for the Sun; comparison of this value with the one derived from equation (6) for a G2 V star gives a measure of the typical errors (Table 1) in the constant-$g$ loci in Figure 1 for the spectral types currently accessible to Zeeman-broadening measurements (spectral type K7 or earlier). Note that these errors are much smaller than the observational error bars.

There is a troubling trend in Figure 1, which is most obvious for the Guenther (1997) data. The field strengths derived by this author are above the main-sequence values of $B_{eq}$. In other words, the only way for these objects to have magnetic fields as strong as implied by the data is for them to be more compact (log $g \approx 4.5$) than a main-sequence star of the same spectral type. Obviously, this is impossible; moreover, log $g \approx 3.5$ for classical TTs such as those in the Guenther (1997) data set and, also, the absorption lines form at $\tau < 2/3$.

Furthermore, since I have assumed $f = 1$ in deriving a value of $B$ from the measured $B_{eq}$, these values, again, are lower limits to the field implied by the observations. Therefore, the measurements of Guenther (1997) seem to overestimate $B$ by at least a factor of greater than $2(2/3)r^{1/3}$ in the case of LkCa 15 and a factor greater than $2.5(2/3)r^{1/2}$ for T Tau, respectively.

The measurements of Basri et al. (1992) for TAP 35 seem compatible with the expected $B_{eq}$, in particular because they assumed log $g = 4.0$ in their models. However, if $f < 0.65$, their measurements would imply a value of $B$ larger than the equipartition field in a main-sequence star of the same spectral type. Moreover, they found that if log $g = 3.5$ instead of log $g = 4.0$, then a stronger field is required to match the data; this is inconsistent with the dependence of $B_{eq}$ on $g$ at fixed $T_{ecl}$ (Fig. 1). Therefore, there are also problems with the Basri et al. (1992) measurements, and the implied fields are larger than $B_{eq}$.

Another perspective on this inconsistency is presented in Figure 2. This figure is a plot of $B_{eq}$ as a function of $T_{ecl}$ along the pre-main-sequence isochrones of D’Antona & Mazzitelli (1994); also shown are the paths in the $(T_{ecl}, B_{eq})$-plane traced by two stars with $M = 1.5M_\odot$ and $M = 0.4M_\odot$, respectively, as they evolve toward the zero-age main sequence. These results show that, for a given value of the stellar mass, $B_{eq}$ increases with age; therefore, if one takes $\approx 10$ Myr as the age limit for the T Tauri phase, the results in Figure 2 imply that in TTs of spectral type K7 or earlier, $B \approx 1.5$ kG outside dark spots.

4. CONCLUSIONS

The equipartition field $B_{eq}$ at any level in a stellar atmosphere is an upper limit to the magnetic field strength in a flux tube...
in pressure equilibrium with the surrounding, nonmagnetized gas. I have computed $B_{eq}$ for the physical regime appropriate to T Tauri stars and shown that the field strengths derived from current Zeeman-broadening measurements exceed this upper limit by large factors and therefore are unphysical.

What are the causes for this discrepancy? Because the derivation of $B$ from the measured line widths requires detailed modeling of the emergent line profiles, the fact that current models yield $B > B_{eq}$ indicates that some piece of physics is missing. To be fair to the Zeeman-broadening technique, when the Zeeman components are resolved this method can be very precise (see, e.g., the recent measurements of $B$ in Eri by Valenti, Marcy, & Basri 1995), whereas Basri et al. (1992) based their analysis on the equivalent widths of the lines (due to the faintness of TTSs and the high signal-to-noise ratio required by this method), and Guenther (1997) used an auto-correlation analysis without benefit of a detailed atmospheric model. However, it is hard to imagine how the use of equivalent widths rather than line profiles can explain the finding by Basri et al. (1992) that a lower surface gravity requires a larger magnetic field to match the data, when the field is supposed to decrease with decreasing gravity (see eq. [6]). Therefore, the source of the discrepancy is, most likely, the input physics of the models.

In particular, current models for TTSs use the same atmospheric structure for the magnetic and nonmagnetic parts of the photosphere, whereas at any given height the gas pressure in a flux tube has to be lower than that of its surroundings for pressure equilibrium to obtain. Because the measured field depends on the geometrical height at which the lines are formed and the flux tubes must be highly evacuated, using the same atmospheric structure for the magnetized and quiet regions of the photosphere is not correct and is the most likely origin of the discrepancy. Therefore, this and other refinements (see Landolfi, Degl’Innocenti, & Degl’Innocenti 1989) must be incorporated to derive a more accurate estimate of $B$ in TTSs. In addition, Valenti et al. (1995) showed that the Zeeman-broadening method is much more immune to the details of the model atmosphere when infrared lines are used. Infrared measurements should be pursued also, and one hopes that the discrepancy with the results presented here will disappear.

I wish to thank Robert Rosner for a conversation during which the idea for this Letter was conceived and Lee Mundy, Steve Stahler, and Stephen White for their careful reading of the manuscript. I am also indebted to an anonymous referee, who provided comments and criticisms that significantly improved an earlier version of this Letter. This work was supported by NSF grant AST 96-13716 to the Laboratory for Millimeter-Wave Astronomy at the University of Maryland.

REFERENCES

Alexander, D. R., & Ferguson, J. W. 1994, ApJ, 437, 879
Allard, F., & Hauschildt, P. H. 1995, ApJ, 445, 433
Babcock, H. W. 1960, ApJ, 132, 521
Basri, G., & Marcy, G. W. 1994, ApJ, 431, 844
Basri, G., Marcy, G. W., & Valenti, J. A. 1995, ApJ, 390, 622
Biermann, L. 1941, Vierteljahrsschr. Astron. Ges., 76, 194
Bouvier, J., Cabrit, S., Fernandez, M., Martin, E. L., & Matthews, J. M. 1993, A&A, 272, 176
Cattaneo, F., & Malagoli, A. 1992, in ASP Conf. Ser. 26, Cool Stars, Stellar Systems, and the Sun: Seventh Cambridge Workshop, ed. M. S. Giampapa & J. A. Bookbinder (San Francisco: ASP), 139
Cox, J. P. & Giuliani, R. T. 1968, Principles of Stellar Structure, Vol. 2 (New York: Gordon & Breach), 590
D’Antona, F., & Mazzitelli, I. 1994, ApJS, 90, 467
Donati, J.-F., Semel, M., Carter, B. D., Rees, D. E., & Collier Cameron, A. 1997, MNRAS, 291, 658
Guenther, E. W., & Emerson, J. P. 1996, A&A, 309, 777
Johnstone, R. M., & Penston, M. V. 1986, MNRAS, 219, 927
———. 1987, MNRAS, 227, 797
Kükner, M., & Rüdiger, G. 1997, A&A, 328, 253
Landolfi, M., Landi Degl’Innocenti, M., & Landi Degl’Innocenti, E. 1989, A&A, 216, 113
Marcy, G. W. 1982, PASP, 94, 989
Parker, E. N. 1978, ApJ, 221, 368
———. 1979, Cosmical Magnetic Fields (Oxford: Clarendon), chap. 16
Robinson, R. D. 1980, ApJ, 239, 961
Saar, S. H. 1988, ApJ, 324, 441
———. 1996, in IAU Symp. 176, Stellar Surface Structure, ed. K. G. Strassmeier & J. L. Linsky (Dordrecht: Kluwer), 237
Solanki, S. K. 1992, in ASP Conf. Ser. 26, Cool Stars, Stellar Systems, and the Sun: Seventh Cambridge Workshop, ed. M. S. Giampapa & J. A. Bookbinder (San Francisco: ASP), 211
Stein, R. F., Brandenburg, A., & Nordlund, Å. 1992, in ASP Conf. Ser. 26, Cool Stars, Stellar Systems, and the Sun: Seventh Cambridge Workshop, ed. M. S. Giampapa & J. A. Bookbinder (San Francisco: ASP), 148
Valenti, J. A., Marcy, G. W., & Basri, G. 1995, ApJ, 439, 939

Note added in proof.—C. M. Johns-Krull, J. A. Valenti, A. P. Hatizes, & A. Kanaan (ApJ, 510, L39 [1999]) recently used spectropolarimetry—which gives a direct measurement of $B$ that does not suffer from the shortcomings of the Zeeman-broadening technique—to measure the magnetic field in BP Tau and found $B = 2.4 \pm 0.1$ kG from the circular polarization of the He I $\lambda$5876 line. BP Tau is a K7 star (G. H. Herbig, ApJ, 214, 747 [1977]) with $T_{eff} = 4000$ K and $\log g = 4.3$ (R. P. Schiavon, C. Batalha, & B. Barbary, A&A, 301, 840 [1995]); using these values, I find $B_{eq} = 1.7$ kG, which is too small by 30%. However, the He I $\lambda$5876 line is believed to form in the accretion shock (see, e.g., S. A. Lamzin, A&A, 295, L20 [1995]), which may be located deeper than the height at which $\tau = 2/3$ in the unperturbed atmosphere. For example, if the accretion shock is located at a depth at which $\tau = 1$ in the surrounding atmosphere, the equipartition argument gives $B_{eq} = 2.1$ kG, in much better agreement with the results of Johns-Krull et al. (1999).