Development of Novel Methods for Compensation of Stress-strain Curves

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In this study, we analyze in detail the stress variation due to friction between the top surface of a cylindrical sample and the anvil surface by carrying out FEM analysis. We propose pragmatic and simple approaches to compensate the stress-strain curve for both Rastegaev-geometry samples and arbitrary cylindrical samples. By using these equations, the compensation process becomes easy and convenient, and the compensated flow stresses are reliable for the compression process of 5065B aluminum alloy.

KEY WORDS: hot forging process; friction coefficient; simulation; finite element analysis; friction correction.

1. Introduction

Obtaining the intrinsic stress-strain response of workpieces without the influence of external factors is always a key problem for researchers in the field of metalworking. Precise characterization of flow behavior, when combined with proper processing-microstructure relationships and failure criteria (e.g., a processing map), can help in defining and optimizing the processing conditions so as to produce defect-free parts with controlled dimensions, microstructures, and mechanical properties.1–3) There are many external factors that influence the mechanical response of materials during compression processing, such as adiabatic heating, the compression method, and interfacial friction between the sample and jig.

2. Formulation

The input energy during deformation is reported to be partially converted to heat because of adiabatic heating, which results in softening and a decrease in flow stress compared with ideal isothermal deformation conditions. The temperature rise due to adiabatic heating during deformation is usually calculated as

\[ \Delta T = \frac{\eta}{\rho c} \int_{0}^{1} \sigma d\varepsilon = \frac{\eta}{\rho c} W, \]

where \( \Delta T \) is the temperature increase due to the work done on the sample, \( \rho \) is the material density, \( c \) is the heat capacity of the material, and \( \eta \) is the heat efficiency. For a given strain rate and strain level, the flow stress after adiabatic correction is generally given by

\[ \sigma = A\dot{\varepsilon}^n \exp\left(\frac{Q}{RT}\right) \]

or

\[ \log(\sigma) = E + \frac{Q}{RT}, \]

where \( E = \log(A\dot{\varepsilon}^n) \), \( n \) is the strain rate sensitivity, \( Q \) is the thermal activation energy, \( R \) is the universal gas constant, \( T \) is the experimental temperature, and \( A \) is a constant. In this expression, although \( Q \) is assumed to be independent of \( T \), numerous studies have shown that \( T \) depends on \( Q \) because the deformation mechanism varies with temperature.5) Therefore, the stress calculated by Eq. (2) probably leads to large errors in most cases. For this reason, Y. Li et al.5) proposed a new method to compensate the stress-strain curves in which Eq. (2) is not established anymore and the relationship between \( \log(\sigma) \) and \( 1/T \) is fit by a spline curve based on the variation of \( Q \) with \( T \). Extrapolating the compensated stress along this curve gives much-improved results.

In addition, the heat efficiency \( \eta \) in Eq. (1) is generally taken for convenience to be a constant in the range of 0.90–0.96. Actually, the heat-dissipation time is closely related to strain rate: the longer the deformation time (lower strain rate), the lower the value of \( \eta \). It has been suggested that \( \eta \) varies linearly with \( \log \dot{\varepsilon} \), so that \( \eta = 0 \) at 0.001 s\(^{-1}\) and \( \eta < 0.95 \) at 1.0 s\(^{-1}\) or greater. Following this suggestion, we express \( \eta \) as

\[ \eta = (0.316)\log \dot{\varepsilon} + 0.95. \]

If a cylindrical sample is compressed in a stroke-rate-controlling process, the strain rate as a function of strain for the compression process involving the cylindrical sample is

\[ \dot{\varepsilon} = \dot{\varepsilon}_0 \exp(\varepsilon) \],

where \( \dot{\varepsilon}_0 \) denotes the initial strain rate in the compression process, and
where \( v \) denotes the stroke rate and \( H_0 \) denotes the original cylindrical sample height. Therefore, to analyze the stress-strain curves in the stroke-rate-controlling process for future use, correction into a strain-rate-controlling process is necessary. Because strain rate grows exponentially with strain level, this results in an increase in stress according to Eq. (2) when compared with the strain-rate-controlling process. In general, the correction into a strain-rate controlling process should be extrapolated from relationship between strain rate and stress Eq. (2). However, over a wide strain-rate range, strain rate sensitivity is generally not a constant for most compression processes, because it is widely known that the processing map–power dissipation coefficient is derived at this point. Similar to the compensation of stress for the adiabatic heating effect, the relationship between \( \sigma \) and \( \epsilon \) at specific strain levels can be fit again by a spline curve as for the compensation for adiabatic heating; new stress should therefore be extrapolated along this curve. Recently, Y. P. Li et al.\(^7\) even conducted a compression test of IHS38MSV steel to support this idea and found that the stress (after the above compensation) agrees well with the prediction. To analyze the stress-strain curve for the corresponding strain-rate-controlling compression processes, a series of simulations were performed for samples with various initial heights (same diameter). In this paper, we propose a pragmatic and simple approach to compensate the stress-strain curve for the corresponding strain-rate-controlling process.

The interfacial friction between sample surface and jig also plays an important role during compression tests. G. W. Rowe\(^9\) analyzed the influence of interfacial friction between a workpiece and a tool and found that, without the influence of friction for a circular disc, the relationship between the average axial pressure and stress is

\[
\sigma = \sigma (1 + 2m \frac{r}{3h}), \quad \text{........................... (6)}
\]

where \( r \) and \( h \) are the instantaneous radius and height, respectively, of the billet, \( \sigma \) and \( \epsilon \) are the equivalent axial stresses without and under frictional conditions, respectively, and \( m \) is the shear (or Tresca) friction coefficient. G. E. Dieter proposed a Coulomb stress distribution at both end-surfaces of the cylindrical sample,\(^9\) and found that the average axial stress (mean height of the friction hill) with respect to the stress without friction is

\[
\sigma = \frac{C^2 \sigma}{2[\exp(C) - C - 1]}, \quad \text{........................... (7)}
\]

where

\[
C = 2\mu / h. \quad \text{.......................... (8)}
\]

In the above equation, \( \mu \) denotes the Tresca friction coefficient. The relationship between the Tresca friction coefficient \( m \) and the Coulomb friction coefficient \( \mu \) is

\[
m = \sqrt{3}\mu. \quad \text{.......................... (9)}
\]

It should be noted that Eqs. (6) and (7) are based on the assumption that there is no barreling at the edges of the sample, and that the thickness is small enough so that the average axial stress \( \sigma \) may be taken as constant throughout the sample thickness. However, in an actual cylindrical hot-working process, the sample thickness is generally greater than its radius. In addition, the stress is considered to vary greatly as a function of sample height. Therefore, these two theories are not applicable to an actual compressive test.

A more precise equation for compensating the stress-strain curve is therefore required for a cylindrical compression test because the stress distributions both within the sample and at the interface between the sample and the anvil vary with the height/radius ratio and with the friction condition. In our previous study,\(^10\) a relatively precise identification of the friction coefficient of a cylindrical sample was conducted by using DEFORM-3D finite element method (FEM) software. For a cylindrical sample with Rastegaev geometry (initial height/diameter = 1.5), the relationship between \( P = \frac{R}{R_H} \), a parameter indicating the degree in geometrical change of the sample during compression, and \( m \) is expressed as

\[
a_1 + b_1 m + c_1 m^2 = P \quad \text{.......................... (10)}
\]

where \( a_1, b_1, \) and \( c_1 \) are functions of the true strain \( \epsilon \). As compared to other theories,\(^11,12\) this method is more precise under both high-strain levels and high-friction-coefficient conditions. In the current study, we used the DEFORM-3D FEM software to simulate the evolutionary behavior of flow stress in a sample under compression by adjusting the average Tresca friction coefficient \( m \). To analyze the variation of flow stress with respect to the initial height/diameter ratio of sample, a series of simulations were performed for samples with various initial heights (same diameter). In this paper, we propose a pragmatic and simple approach to compensate the stress-strain curve. To evaluate this new compensation method, we compensate stress-strain curves obtained in compression tests of Al5056 Al-5Mg alloy at ambient temperature for different lubrication conditions.

3. Experimental

3.1. Finite Element Analysis

Models of the aforementioned samples were generated using DEFORM-3D v6.13. To reduce the calculation time, one-eighth of the cylindrical sample with a mesh number of 30,000 was considered in the FEM model for analysis. The values of the Tresca friction coefficients \( m \) selected for each simulation were 0 for perfect sliding, and 0.1–1, in increments of 0.1, for restriction condition between the sample top surfaces and the anvils. FEM analysis was carried out to a true strain of about 1.8 (nominal strain of approximately 85%).

The simulated compression processes were performed using a true strain-rate-controlling process of \( 10^{-3} \text{ s}^{-1} \). In addition, the heat efficiency was chosen to be 0 in the simulation process so that adiabatic heating (temperature rise) would not occur inside the compressed sample (it leads to softening of the flow stress in the deformation process).\(^4\) All of the above settings for the simulation process are based on the fact that the compensation of stress both from adiabatic heating and from the stroke-rate effect can be performed independently, as we mentioned above.

3.2. Experimental Procedure

Extruded 5056B Al-5Mg alloy was used in the current study. Cylindrical specimens, 8 mm in diameter and 4, 6, 8,
12, and 16 mm in height, were cut by electro-discharge machining (EDM). Spiral grooves with 0.1-mm deep were formed on the flat-end surface of the specimen so that the profile of the sample surface could be easily recognized after deformation. Compression tests were performed to a true strain level of approximately 1.0 (approximately 65%) at ambient temperature in a computer-aided hot-forging simulator (Thermecmaster-Z) to a true strain level of approximately 1.2 (approximately 70%). The strain rate was set to $10^{-3}$ s$^{-1}$. To control the friction coefficient we conducted compression tests with the following lubrication conditions: no lubrication, plastic rubber, graphitic sheet, and compression lubrication by MoS$_2$ spray between the sample top surface and the anvil. Before compression testing, the grooved top surface of sample was painted red so that the original top profiles could be easily recognized before calculating the friction coefficient.

4. Results and Discussion
4.1. Finite Element Results
4.1.1. Rastegaev Geometry
Let the dimensionless parameters $\sigma_z$ and $\sigma$ represent the stress with and without the influence of friction, respectively, during the compression process. Therefore, the ratio $\sigma_z/\sigma$ is a parameter expressing the extent to which friction influences the stress of materials. For a sample with Rastegaev geometry, Fig. 1 shows the nominal strain dependencies of $\sigma_z/\sigma$ for each simulating compression process with various Tresca friction coefficients $m$. The results show that the influence of friction on the flow stress of materials is very small for strain values less than approximately 0.6; however, at high strain levels, the stress increases sharply because of the friction. Oh et al.\textsuperscript{13) studied the cylindrical compression of a Ti alloy sample with Rastegaev geometry to a true strain level of approximately 0.6–0.7 using DEFORM FEM software, and their findings suggest that stress variations due to the influence of friction are below 5% for strain values below 0.6–0.7, which is in good agreement with the current results. However, they did not report any results for higher strain levels. The results of our current study indicate that, at a high strain level, the influence of friction is extremely large. In fact, numerous compression tests were conducted using a high strain level to further refine the microstructure or improve the mechanical properties of the final products. Therefore, analyzing the influence of friction at both high and low strain levels is important.

A plot of $\sigma_z/\sigma$ as a function of average Tresca friction coefficient $m$ and for different strain levels is shown in Fig. 2. We find that $m$ is linear to $\sigma_z/\sigma$, where the slopes $K$ of these lines are closely related to the nominal strain levels. From the above-mentioned results and for a specific strain level, the relationship between the ratio $\sigma_z/\sigma$ and $m$ is

$$\frac{\sigma_z}{\sigma} = 1 + K(\varepsilon)m,$$

where the slope $K(\varepsilon)$ of the line depends on the strain:

$$K(\varepsilon) = f(\varepsilon).$$

The plot of the slope $K(\varepsilon)$, calculated as a function of true strain, is shown in Fig. 3. We find that the curve is well fit using the following third-order spline equation:

$$K = f(\varepsilon) = a + b\varepsilon + c\varepsilon^2 + d\varepsilon^3,$$

where $a$, $b$, $c$, and $d$ are constants determined by the fitting process, and suitable values are listed in Table 1. By substituting Eq. (13) into Eq. (11), the following equation for correcting the stress-strain curve for a sample with Rastegaev geometry is obtained:

Fig. 1. True strain dependencies of $\sigma_z/\sigma$ for various Tresca friction coefficients.

Fig. 2. Dependence of $\sigma_z/\sigma$ on friction coefficient $m$ at various strain levels.

Fig. 3. Slope $K$ as a function of true strain, and the result of fitting with a third-order polynomial equation for a Rastegaev-geometry sample.
where $m$ can be calculated using the methodology developed in our previous study.\textsuperscript{10} The stress without the influence of friction can be compensated by inserting the other parameters of Table 1 into Eq. (14), regardless of the experimental conditions or of the characteristics of the material.

### 4.1.2. Arbitrary Geometry

Cylindrical samples with various shapes are also used in hot compression processes (initial diameter/height ratio, $D_0/H_0$). Because of the stress varies in the direction of the sample thickness, the initial diameter/height ratio changes, causing the average axial stress to change accordingly. The compensating equation used for the sample with Rastegaev geometry is not applicable in this case, and a new relationship between the stress increment and friction must be developed on the basis of the results obtained in section 3.1. The simulation results shown in Fig. 4 indicate that for compression processes with a constant Tresca friction coefficient $m = 0.5$, the flow stress increases markedly with increasing the initial diameter/height ratio $D_0/H_0$, demonstrating a strong effect of the initial geometry of cylindrical sample in obtaining stress-strain curve. All the curves behave similarly, with $\sigma_0/\sigma$ increasing gradually at lower strain levels and drastically at higher strain levels. In addition, for a given $D_0/H_0$, the value of $\sigma_0/\sigma$ increase linearly with friction coefficient $m$ according to Eq. (11) as shown in Fig. 5. In addition, at a given strain level, an increase in $D_0/H_0$ results in an increase in slope $K$ of Eq. (11), as is also observed in Fig. 6. From above-mentioned results, $K$ is related to both the strain level and the initial diameter/height ratio $D_0/H_0$ and is expressed by

\[
K = f \left( \varepsilon, \frac{D_0}{H_0} \right) \quad \text{(15)}
\]

To obtain a precise relationship between the parameter $K$ and the initial diameter/height ratio $D_0/H_0$, we consider the $\sigma_0/\sigma$ and initial diameter/height ratio $D_0/H_0$ and plot the relationship between them at each strain level, as shown in Fig. 6. We find that for any given strain level, the relationship between these two parameters can be expressed by

\[
K = A + B \frac{D_0}{H_0} \quad \text{(16)}
\]
where \( A \) and \( B \) are constants that are related only to the strain level. From the results shown in Fig. 6, \( A \) and \( B \) are plotted as functions of the true strain in Fig. 7. Each parameter can be expressed as follows:

\[
A = a' + b' \varepsilon + c' \varepsilon^2 \\
B = a'' + b'' \varepsilon + c'' \varepsilon^2 + d'' \varepsilon^3
\]

where \( a', b', c', a'', b'', c'' \), and \( d'' \) are constants determined by fitting. The values of these constants are listed in Table 1. Therefore, the stress obtained with an arbitrary initial diameter/height ratio can be compensated by using the following equation, which is obtained by combining Eqs. (11) and (15)–(18):

\[
\sigma = \sigma_c \left[ 1 + \left( a' + b' \varepsilon + c' \varepsilon^2 + \left(a'' + b'' \varepsilon + c'' \varepsilon^2 + d'' \varepsilon^3 \right) \frac{D_o}{H_0} \right)^m \right] \cdot \\
\]  

where all parameters except for \( m \) can be obtained experimentally, and the average Tresca friction coefficient \( m \) is computed using the method described in a previous research. In the current study, we conducted simulations with initial diameter/height ratios that ranged from 2 to 4 (4 to 16 mm height and 8 mm diameter), and Eq. (19) is relatively precise in this range. In addition, the values listed in Table 1, which are obtained by the nonlinear fitting process, are assumed to be independent of the materials used and the experimental conditions. These values can be used directly in subsequent research for friction correction.

### 4.2. Application In An Actual Compression Process

Figures 8(a) and 8(b) show the true stress-true strain curves of 5056B Al alloy samples with Rastegaev geometry and arbitrary geometry. From the results shown in Fig. 6, \( A \) and \( B \) are plotted as functions of the true strain in Fig. 7. Each parameter can be expressed as follows:

\[
A = a' + b' \varepsilon + c' \varepsilon^2 \\
B = a'' + b'' \varepsilon + c'' \varepsilon^2 + d'' \varepsilon^3
\]

where \( a', b', c', a'', b'', c'' \), and \( d'' \) are constants determined by fitting. The values of these constants are listed in Table 1. Therefore, the stress obtained with an arbitrary initial diameter/height ratio can be compensated by using the following equation, which is obtained by combining Eqs. (11) and (15)–(18):

\[
\sigma = \sigma_c \left[ 1 + \left( a' + b' \varepsilon + c' \varepsilon^2 + \left(a'' + b'' \varepsilon + c'' \varepsilon^2 + d'' \varepsilon^3 \right) \frac{D_o}{H_0} \right)^m \right] \cdot \\
\]

In Eq. (19), all parameters except for \( m \) can be obtained experimentally, and the average Tresca friction coefficient \( m \) is computed using the method described in a previous research. In the current study, we conducted simulations with initial diameter/height ratios that ranged from 2 to 4 (4 to 16 mm height and 8 mm diameter), and Eq. (19) is relatively precise in this range. In addition, the values listed in Table 1, which are obtained by the nonlinear fitting process, are assumed to be independent of the materials used and the experimental conditions. These values can be used directly in subsequent research for friction correction.

### Table 2. Geometry of the 5065B aluminum alloy cylindrical samples before and after compressions and the friction coefficient calculated using the method given in Ref. 10.

| Sample | \( H_0 \) | \( D_0 \) | \( D_o/H_0 \) | \( H \) | \( D_o \) | \( D_t \) | \( \varepsilon \) | \( m \) |
|--------|--------|--------|-------|--------|--------|--------|--------|------|
| 1#     | 6.02   | 7.97   | 4/3   | 2.41   | 12.21  | 10.98  | 0.601  | 0.19 |
| 2#     | 8.02   | 7.97   | 1     | 3.15   | 12.84  | 10.65  | 0.607  | 0.31 |
| 3#     | 12.02  | 7.97   | 2/3   | 4.01   | 14.01  | 9.2    | 0.667  | 0.52 |
| 4#     | 12.02  | 7.97   | 2/3   | 3.91   | 13.95  | 9.65   | 0.676  | 0.39 |
| 5#     | 12.01  | 7.96   | 2/3   | 4.05   | 14.01  | 10.35  | 0.663  | 0.31 |
| 6#     | 12.02  | 7.97   | 2/3   | 3.85   | 14.20  | 10.54  | 0.680  | 0.28 |
| 7#     | 12.02  | 7.97   | 2/3   | 4.21   | 13.95  | 10.48  | 0.651  | 0.26 |
| 8#     | 16.02  | 7.97   | 1/2   | 6.82   | 12.60  | 9.66   | 0.576  | 0.20 |
10, and the results are tabulated in Table 2. The friction corrections in stress-strain curves of samples with both Rastegaev geometry and arbitrary geometry were performed using Eqs. (14) and (19), respectively. Fig. 9(a) shows the results for the sample with Rastegaev geometry after friction corrections, in which the stresses from low strain levels to high strain levels are all corrected to almost the same level. Similar results were also obtained for the arbitrary-geometry sample (Fig. 9(b)), which testifies to the high reliability of the current method.

For the current study, we performed simulations for $D_o/H_0$ ranging from 2–0.5 (4 to 16 mm high and 8 mm diameter), for which Eq. (19) is thought to have relatively high precision. In addition, we believe that the values obtained by nonlinear fitting (tabulated in Table 1) do not vary with materials and experimental conditions; therefore, they may be used directly for friction correction in other studies. The geometry change due to the friction is not related to the temperature, so current results are considered to be also applicable in high temperature compression process. For a given compression test, compensating the stress-strain curve from the influence of friction and adiabatic heating effect could be carried out separately, therefore,

5. Conclusions

In the current research, we performed a series of calculations addressing the influence of the friction coefficient on the stress during compression process by means of FEM simulations. Based on the simulation results and on a systematic analysis of the variation in stress with respect to the friction coefficient, we propose equations that can be used to compensate the flow stress of materials for samples with either the Rastegaev geometry or an arbitrary geometry. We find that this method is relatively precise when applied to a 5065B sample.

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