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Conceptual design of externally posttensioned curved structures for live loads

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Abstract
Bending moments due to permanent loads can be strongly reduced in a generally shaped structure by introducing a set of additional forces through an external posttensioning system. The response of these structures under live loads has not been studied yet; therefore, this work investigates the behavior of these structural systems under load distributions that differ from the permanent ones. First, a definition of a moment-based efficiency is proposed. Next, a parametric analysis is carried out to evaluate the performance of these structural systems by varying the two main design variables and other relevant parameters (strut connection and orientation, discretization, efficiency definition, etc.). Finally, the methodology is applied to a case of study where several posttensioned alternatives are compared to the standard one (i.e., without posttensioning) to prove the benefits of adopting external posttensioning in terms of material-saving.

KEYWORDS
external posttensioning, funicular structures, live loads, material-efficiency, parametric design

1 | INTRODUCTION

Bending moments occurring in a generally shaped structure can be strongly reduced by using an external posttensioning system. Prestressing allows redirecting the internal forces to achieve an optimal distribution and therefore reduce bending moments to the minimum. This design approach results in solutions that combine efficiency in terms of material, and flexibility in terms of architectural geometric definition.

The concept of prestressing became reality through the development of a new material (high-strength steel) and novel construction techniques during the first decades of the 20th century.1–4 Nevertheless, the historical development of external posttensioning is even more recent, and most applications are related to bridges which are straight in elevation5; however, prestressed tendons can generate different types of bridges depending on their position with respect to the deck.6 Recently, external posttensioning has been applied in a number of singular roof structures, which would also be the main purpose of the structures covered by this paper.

The procedure developed by authors to define the layout of a posttensioning system to convert any two-dimensional curve into an axial-only solution under permanent loads is based on graphic statics,7,8 and for reader’s convenience can be found in other recent works.9–12 It is out of the scope of this paper to describe the graphical construction and only the main concepts are explained here.
The graphical procedure starts from a segmental discretization of the original geometry and the choice of the direction of the struts that, at each vertex, connect the original structure to the external posttensioning system.

Given a specific geometry and a set of applied permanent loads, the geometrical definition of the posttensioning system is indeterminate. In fact, there is an infinite number of posttensioning layouts that convert a specific geometry into a bending-free one.

The design space of solutions can be explored through two design variables, also known as “indeterminacies”. The first design variable or first indeterminacy is the magnitude of the forces (to be added to the applied ones) that provoke bending-free equilibrium in the original structure; in fact, the direction of the struts is arbitrarily fixed, but the magnitude of the strut forces is variable.

Figure 1 shows two alternatives obtained varying the first indeterminacy. In this case, the struts are perpendicular to the original geometry. The solution becomes unique if the value of one of the forces in the struts or, equivalently, in one of the elements of the original geometry is fixed. Conceptually, this indeterminacy corresponds to the fact that a geometry that only exhibits axial forces under a certain set of loads, will behave in the same way for any set of loads that is obtained by scaling the previous one. In such a way, an infinite set of axial-only geometries match a single loading distribution, and an infinite set of loads correspond to a single axial-only geometry.

In this work, the value of the first indeterminacy represents the ratio between the internal force (compression) in the first element of the original structure and the resultant of the applied permanent loads.

The second design variable or second indeterminacy is related to the posttensioning system layout that generates the additional forces defined previously. This step of the method can be assimilated to the classical problem of finding a funicular geometry for a given set of loads, in this case, the additional forces introduced through the struts.

Figure 2 shows two different solutions, that is, two different posttensioning layouts, that introduce the same additional forces into the structure. Fixing one parameter, for example, the maximum value of posttensioning force, or defining the starting inclination of the first posttensioning segment, yields a unique solution; in other words, there is a one-to-one correspondence between force and geometry of the external posttensioning system.

In this work, the value of the second indeterminacy represents the ratio between the horizontal reaction at the first support and the resultant of the permanent loads applied (for radial or vertical struts) or, directly,
10% of the prestressing force (for constant prestressing solutions).

As main conclusion, given a generally shaped original geometry and a specific permanent load distribution, there are infinite layouts of the posttensioning system to generate the required additional forces that reduce bending. All the possible solutions can be explored by varying the two indeterminacies related to the magnitudes of the additional forces (first design variable) and the forces within the posttensioning system (second design variable).

The material-efficiency of these bending-free structures under permanent loads has been studied in other works, but their behavior for other load distributions has not been analyzed yet.

Therefore, this paper focuses on the behavior of externally posttensioned structures under live loads. The structural performance of these systems is evaluated with a parameter defined as efficiency, whose meaning is presented in this work.

On this basis, the role of the design variables or indeterminacies on material-efficiency is investigated through a parametric analysis with the goal of providing guidance on the most appropriate decisions during the conceptual design stage of such structures. Finally, this paper illustrates the application of the system to a case of study based on the structure of the Terminal 2E of the Charles de Gaulle airport in Paris to demonstrate the potential of the design approach.

2 MATERIAL-EFFICIENCY OF EXTERNALLY POSTTENSIONED CURVED STRUCTURES

2.1 Analytical solution for the moment-based efficiency of circular shapes

The first approach for the analytical study of the efficiency of externally posttensioned curved structures follows the research by Ruiz-Teran and Aparicio on under-deck cable-stayed bridges. In these structures, the deck is a straight beam and a posttensioning system is added below the deck, strengthening and stiffening it. The study focuses on cases with single, multiple or infinite struts, identifying the parameters governing the behavior of these structures and providing a definition of the efficiency based on the maximum bending moments resisted by the deck with and without the under-deck cable system. Under-deck cable-stayed bridges are conceptually close to externally posttensioned curved structures, with a similar behavior under permanent loads, where displacements are zero at the points where posttensioning is applied.

From a dimensional analysis, Ruiz-Teran and Aparicio established the following Equation (1) to determine a certain response for under-deck cable-stayed bridges:

\[ F[R^*], \left( \frac{Q}{E \cdot L^2} \right), \left( \frac{q}{E \cdot L} \right), (\chi) = 0 \quad (1) \]

where \( R^* \) is a dimensionless monomial depending on the response analyzed, \( Q \) is the magnitude of the point load, \( q \) is the magnitude of the distributed load, \( E \) is the Young’s Modulus of the deck, \( L \) is the span length and \( \chi \) is a dimensionless parameter that depends on the geometry of the structure and the ratios between the stiffness of the deck and the cables. This expression does not consider the deformation of the struts, which are supposed rigid.

The parameter \( \chi \) is defined as:

\[ \chi = \chi_1 + \frac{\chi_2}{\chi_A} = \frac{E \cdot I}{E_{SC} \cdot A_{SC} \cdot L^2} \cdot g_1 \left( \frac{L_s}{L}, n \right) + \frac{I}{A \cdot L^2} \cdot g_A \left( \frac{L_s}{L}, n \right) \quad (2) \]

where \( I \) is the second moment of area of the deck, \( E_{SC} \) is the Young’s Modulus of the cables, \( A_{SC} \) is the area of the cables, \( A \) is the area of the deck, \( g_1 \) and \( g_A \) are two functions depending on the geometry of the structure, \( L_s \) is the length of the middle strut and \( n \) is the number of struts.

A simplification of (2) can be made if the axial stiffness of the deck is much bigger than its bending stiffness. In that case, the second term in the definition of \( \chi \) (2) becomes negligible.

Based on the parameter \( \chi \), Ruiz-Teran and Aparicio proposed defining the efficiency \( \xi_M \) as the portion of the bending moment provoked by the live load that is resisted by the posttensioning system. The same definition, comparing the situations with and without external posttensioning, has been adopted in this paper as:

\[ \xi_M = \frac{M_0 - M_f}{M_0} \quad (3) \]

where \( M_0 \) is the maximum bending moment without the cable system and \( M_f \) is the bending moment in the structure with the cable system. For the analyses in this paper, the efficiency is measured at the point where the absolute maximum bending moment occurs for the structure without the cable system.
With this definition of the efficiency Ruiz-Teran and Aparicio\textsuperscript{13} show that, for the case of an under-deck cable-stayed bridge with a single strut and a point load applied at midspan, the efficiency can be expressed as:

\[
\xi_M = \frac{1}{1 + \frac{1}{2} \sqrt{\frac{2}{\pi}} C_1 \chi(4)}
\]

A similar study to\textsuperscript{13} has been conducted on one of the structures covered by this paper, specifically, on a circular arch discretized in four segments with a point load applied at midspan as depicted in Figure 3 left. In a first approach, the axial stiffness of the arch is considered to be large enough to neglect the term \(\chi I / \chi A\) in the definition of \(\chi(2)\).

The efficiency obtained in this case is given by the following expression:

\[
\xi_M = \frac{1}{1 + \frac{EI}{E A L^2} \left( \alpha^2 + 2 \right) \left( \tan(\alpha) + 1 \right) \cos^2(\beta) \left( \tan(\beta)^2 + \frac{1}{2} - \tan(\alpha) \frac{\tan(\beta) + \sqrt{2}}{\tan(\alpha) \sec(\alpha) + \sqrt{2}} \right)^2}
\]

where \(\alpha\) and \(\beta\) are the angles of the two pairs of cables with the horizontal plane and \(L\) is the length of one of the discretized segments of the arch. The form of this expression is very similar to those deduced by Ruiz-Teran and Aparicio\textsuperscript{13} and the second term of the denominator can be identified as the factor \(\chi_I\).

As in (2), the first part of \(\chi_I\) in (5) depends on the mechanical properties of the arch and the posttensioning system, whereas the second part only depends on the geometry of the structure and thus can be identified with the \(g_I\) function.

The same analysis can be developed for the structure subjected to an asymmetric load as shown in Figure 3 right. The efficiency shows again the same dependency on the parameters mentioned above.

\[
\xi_M = \frac{1}{3 + 2 \sqrt{2}} \left( \frac{1}{1 + \chi_I} \right)
\]

The impact of the mechanical properties of the structure can be verified through Equation (5). As also stated by Ruiz-Teran and Aparicio\textsuperscript{13} the efficiency increases if the flexural stiffness of the original geometry is reduced or if the axial stiffness of the posttensioning system enlarges.

It is also possible to study the influence of the geometric function \(g_I\) on the efficiency. If the factor corresponding to the mechanical properties of the structure is kept constant, the efficiency varies with the two indeterminacies as shown in Figure 4a).

The blue area on the graphs in Figure 4 represents the domain of valid solutions, where the posttensioning system acts in tension under permanent loads and the arch in compression. The red area represents the invalid domain, that therefore should not be considered.

It can be observed that the efficiency decreases as the first indeterminacy grows. In the direction of the second indeterminacy a maximum in the efficiency can be found. This may suggest that there are certain optimal geometrical configurations regarding the efficiency of the structure. On the other hand, the values that minimize the efficiency correspond to the two creases appearing in Figure 4 and relate to the situations in which the struts and the cables become almost parallel.

However, taking a constant value for the mechanical factor in the expression of the efficiency (5) might not be an appropriate assumption. This would imply adopting constant section properties for all the geometric configurations attainable by changing the indeterminacies. It must be taken into account that the variation of the
indeterminacies will affect the forces within the elements of the structure caused by the permanent loads. Thus, for the rest of the analyses in this paper, it has been deemed reasonable to take the area of the cross-section of the elements proportional to the axial forces that they withstand under permanent loads.

In this case, the influence of the second indeterminacy on the efficiency is reduced, and the first indeterminacy prevails, as shown in Figure 4b).

If the analysis is extended to account for the axial deformability of the elements in the arch, the equation for the factor $\chi$ gets much more complex. The expression of the efficiency (7) in this case includes terms of $\chi I/A$ cube and square, which are attributed to the arch behavior of the structure:

$$\xi_M = \frac{\Delta L I g_1 + \Delta L^2 I g_2 + \Delta L^3 I g_3 + g_4}{\Delta L I g_1 + \Delta L^2 I g_5 + \Delta L^3 I g_6 + g_4 + \frac{EA L^5}{EI}_{AC} g_7 + \frac{EA L^2}{EA}_{AC} g_8 + \frac{EA}{L^3}_{AC} g_9}$$

(7)

where $g_i$ are functions depending only on the geometry of the structure.

Figure 5 compares the efficiency of the arch in Figure 3 left considering or not the axial deformation of the original structure, showing that the difference between both solutions grows as the first indeterminacy increases. The areas of the elements are taken proportional to the axial forces they withstand under permanent loads. Thus, higher values of the first indeterminacy imply bigger cross-sections, for which the ratio $I/A$ is also higher. As a consequence, neglecting the axial deformability of the original geometry might not be adequate for high values of the first indeterminacies or bigger cross-sections, but it provides a fair estimation of the efficiency for a wide range of structures. In addition, considering the deformability of the arch diminishes the efficiency of the structure due to the lower stiffness of the global system.

2.2 | Strain energy-based efficiency

In addition to the moment-based definition of the efficiency which focuses only on the bending moment at the most solicited point of the structure and to get a more global overview of the structural behavior, another
definition of the efficiency $\xi_D$ based on the strain energy may be proposed:

$$\xi_D = \frac{D_0 - D_f}{D_0}$$

(8)

where $D_0$ and $D_f$ are respectively the strain energy in the structure without and with the posttensioning system.

In the particular case of the circular arch in four segments, neglecting the axial deformability of the original structure and subjected to a symmetric live load, the strain energy-based and moment-based efficiency are perfectly equivalent and given by expression (5). However, if the load case is not symmetric, the two efficiencies show different values. For instance, for the load case in Figure 3 right, the strain energy-based efficiency is given by a more complex function of the factor $\chi_I$:

$$\xi_D = \frac{1}{13 + 8\sqrt{2}} \cdot \frac{1 + \frac{\chi_I}{2}}{1 + \chi_I}$$

(9)

Moreover, if the axial deformability of the original geometry is considered or if the arch is discretized in a different number of segments, the results from both definitions differ as well. These differences are evident in the analyses in Section 3.

Next section is addressed to explore how the efficiencies defined before are affected by several parameters. The following analyses in this paper always consider the axial deformability of the original structure, but neglect that of the struts.

3 | PARAMETRIC ANALYSIS

3.1 | Basis of the analysis

The analytical definitions of the efficiency described in Section 2 proved the relevance of the two indeterminacies/design variables on the global material-efficiency of these structural typologies. Therefore, this section presents a parametric analysis aimed to understand better how these two design variables govern the behavior of externally posttensioned curved structures.

As discussed in the previous section, the cross-sections of the elements do not remain constant, in fact, they change depending on the axial forces acting on the different elements under permanent loads. For the analyses in this section, the cross-section of the original geometry is a steel hollow pipe with a diameter-to-thickness ratio of 0.05. The area is chosen to withstand a utilization of 50% under permanent loads. The area of the posttensioning is the same for the whole structure taken to obtain a maximum utilization of 50% on the most loaded element under permanent loads.

The parametric study investigates: i) the influence of the indeterminacies/design variables, ii) the effect of using pinned or bending-connected struts, iii) the impact of the strut orientation: perpendicular, vertical or oriented to achieve a constant force in the posttensioning system, and, finally, iv) the behavior under symmetric and asymmetric live loads.

Three different structures have been studied: a circular arch, a basket arch and a free-form geometry, all spanning 30 m. The circular arch has been analyzed for a discretization in six and twelve segments to evaluate the effect of a finer discretization.

As mentioned before, the struts are considered as infinitely rigid. The effects of this assumption will be commented later in this section. The analyses have been conducted using an updated version of EXOEQUILIBRIUM,

The tool is implemented in the environment provided by Rhinoceros-Grasshopper

and uses graphic statics to design an external posttensioning system that achieves funicular behavior for a given original geometry under permanent loads. The form and forces of the resulting structure are parametrically controlled through the two indeterminacies explained in Section 1. In its last version, the software allows calculating the internal forces provoked by live loads using the stiffness method. It is worth mentioning that this tool can be freely obtained contacting the authors.

3.2 | Influence of the design variables

Figure 6 shows the results of the parametric analysis in terms of the moment-based efficiency ($\gamma$-axis) as defined in (3) of the circular arch discretized in six segments with pinned struts under symmetric loads.

The graph illustrates that the influence of the first indeterminacy clearly prevails over the second, as it was observed in the previous section for the analytical solution of the circular arch discretized in four segments.

The plot of Figure 7 presents the moment-based efficiency for the same circular arch under an asymmetric load. Again, the first indeterminacy has a stronger influence than the second. The efficiency is much lower than for the symmetric load case, even obtaining negative values. A negative value of the efficiency represents a situation where the bending moment in the structure with posttensioning is higher than the moment in the isolated arch at the point where the maximum bending moment happens for the case without posttensioning. Nevertheless, the bending moment might be reduced by the addition of the posttensioning at other points of the structure.
3.3 Influence of the efficiency definition

The previous example has proved that the moment-based definition of the efficiency does not always provide a global overview of the behavior of the structure and it may be incomplete for a reliable evaluation of the structural performance. Thus, in this section, the approach based on the strain-energy from (8) is proposed, leading to the results shown in Figure 8.

The strain energy-based efficiency for the case of symmetric load, Figure 8a), is very similar to the one obtained with the moment-based definition of the efficiency. Therefore, it can be concluded that the trends are the same. However, in this case, the values of the efficiency are slightly higher. The graph corresponding to asymmetric loads, Figure 8b), shows that the values of the efficiency are again very small in comparison to the case of symmetric loads, but they remain always positive.

3.4 Influence of the connection between the struts and the original geometry

The previous plots were based on a hinged connection between struts and original geometry. Figure 9 shows the results of the moment-based efficiency of a 6-segment circular arch with bending-connected (i.e., clamped) struts.

The shape of the plot is considerably different from the pinned case (Figures 6 and 7). For bending-connected struts, the influence of the first indeterminacy is lower, and the most relevant characteristic of the graph is the fold that appears when the second indeterminacy
approaches zero, where the highest and the lowest values of the efficiency happen. Following the fold in the direction of the first indeterminacy the efficiency decreases. The lower values of the efficiency correspond to the smallest eccentricities between the arch and the posttensioning system. This can be explained by the fact that, as the first indeterminacy increases, the directions of the elements of the arch and of the posttensioning system become more parallel and the global eccentricity reduces, which can be observed through the corresponding force diagram. The peak in efficiency under symmetric loads corresponds to a configuration resembling a bi-articulated frame.

In general, the efficiency under asymmetric loads is smaller than for the symmetric case, but it is consistently higher than the efficiency of the structure with pinned struts under the same loads.

Using the definition of the efficiency based on the strain energy, the behavior of the circular arch with bending-connected struts under live loads is represented in Figure 10.
The graphs are considerably similar to those obtained with the moment-based definition of the efficiency. The most relevant difference derives from taking into account the strain energy of the struts, which in this case are subjected to bending. The struts have been designed to have a utilization of 20% under permanent loads. The influence of the bending strain energy of the struts is very important, even provoking negative values of the efficiency, since the analyses consider the struts with an infinite flexural stiffness. Figure 11 shows the results of the strain energy-based efficiency obtained considering the flexural deformability of the struts. This analysis has been conducted using Karamba\textsuperscript{16}, a validated plug-in of Grasshopper.

When considering the flexural deformability of the struts, the negative values of the efficiency from Figure 10 disappear. These values were produced in the situations of high eccentricity of the posttensioning system with long infinitely rigid struts withstandiong high bending moments. When the flexural deformability of

**FIGURE 10**  Strain energy-based efficiency for a six-segment circular arch with external posttensioning and bending-connected struts under (a) symmetric and (b) asymmetric load

**FIGURE 11**  Strain energy-based efficiency for a six-segment circular arch with external posttensioning and bending-connected struts considering their flexural deformability under (a) symmetric and (b) asymmetric load
the struts is considered, long struts become more flexible and therefore cannot assume such high bending moments, which are redistributed within the main structure. It can be concluded that it is only reasonable to neglect the flexural deformability of the struts when these are short.

### 3.5 Influence of discretization

The plots in Figure 12 illustrate the strain energy-based efficiency for the same circular arch discretized in twelve segments (instead of six as shown in Figure 6).

In the symmetric case, Figure 12a), there are no significant differences with respect to the case with coarser discretization. For the asymmetric load case, Figure 12b), the efficiency increases in comparison with the arch discretized in six segments. The higher number of elements and struts allows for a smoother distribution of bending moments. Further refining the discretization does not affect significantly the efficiency of the system. The efficiency changes considerably if the position of the loads on the structure is modified.

### 3.6 Influence of the orientation of the struts

The previous analyses have been carried out with perpendicular struts (Figure 13, left), meaning that their axes are perpendicular to the outline of the original geometry before discretization. However, as mentioned in Section 1,
the direction of the struts is a design criterion, and different alternatives can be explored (Figure 13).

The strain energy-based efficiency that is obtained for a circular arch with pinned struts oriented in the vertical direction under symmetric load (Figure 13 center) is shown in Figure 14.

Since in this case the struts are all oriented in the same direction, the variation of the efficiency with the two indeterminacies shows only one crease, corresponding to the situation where the cables of the posttensioning system and the struts become parallel. The domain of valid solutions also changes.

The dependency of the efficiency on the two indeterminacies is different from the case of perpendicular struts, but the first indeterminacy continues to prevail over the second. Again, the efficiency under asymmetric loads is much lower.

Another approach consists in choosing the direction of the struts in such a way that the axial forces are

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**Figure 14**  Strain energy-based efficiency for a six-segment circular arch with external posttensioning and pinned vertical struts under (a) symmetric and (b) asymmetric load.

**Figure 15**  Strain energy-based efficiency for a six-segment circular arch with external posttensioning and pinned struts oriented to achieve constant axial forces under permanent loads under (a) symmetric and (b) asymmetric live load.
constant throughout the whole posttensioning system under permanent loads (Figure 13 right). Certain combinations of the two indeterminacies do not allow obtaining a solution verifying this condition as seen in Figure 15.

Once again, the domain of valid solutions changes. In this case, the second indeterminacy has a stronger influence on the efficiency of the structure. Also, the values of the efficiency obtained under asymmetric live loads for certain configurations of the posttensioning system are high in comparison with those achieved with perpendicular and vertical struts.

### 3.7 Analysis of other shapes

Figure 16 summarizes some of the analyses previously presented for a circular arch for the case of a basket arch, which is discretized in six segments. The results show the efficiency based on the strain energy. The global shape is quite similar to those obtained for the circular arch.

In addition to the basket arch, the free-form geometry, shown in Figure 17a, has been analyzed. The results for the strain-energy based efficiency for this free-form discretized in eight segments with pinned perpendicular struts under loads in various locations are shown in Figure 17.

The results show that the variation of the efficiency with respect to the two indeterminacies depends on the shape of the structure. Also, the domain of valid solutions changes. However, in this case, the first indeterminacy continues to prevail. The importance of the location of the loads is also appreciable in Figure 17.

### 4 CASE OF STUDY

#### 4.1 Geometrical definition and design criteria

The parametric study described in the previous section has illustrated how the performance of a generally shaped externally posttensioned structure is strongly influenced by the two indeterminacies, the strut connection and orientation, and the discretization of the base geometry. Based on these findings, this Section 4 illustrates the application of the methodology to a case of study. This study allows evaluating the effects related to
the use of an external posttensioning system on the dimensioning of the structure and on its structural performance.

The analysis is based on a comparison, in terms of material-efficiency, between a standard solution (i.e., without posttensioning) and four different posttensioned alternatives.

The geometry chosen for this case of study is the 30 m-span elliptical shape employed for the roof of the Terminal 2E of the Charles de Gaulle Airport.\(^{17}\)

The main structure is an elliptical reinforced concrete frame with a rectangular cross-section. The adopted longitudinal distance between frames is 10 m; therefore, in addition to the self-weight of the frame, a dead load of 15 kN/m and a wind live load that ranges from 5 to 10 kN/m depending on zones, according to Eurocode 1\(^{18}\) have been considered. In this simplified analysis, only wind load is considered because for this specific shape it is the governing one. However, this is not a general statement and for a general shape of the original geometry both wind and snow have to be taken into account. Concrete with characteristic strength \(f_{ck}\) of 35 MPa is used for the original geometry.

To have a reliable comparison of results, the width of the cross-section and the geometrical-reinforcement-ratio have fixed values of 0.60 m and 0.4%, respectively. The depth of the cross-section depends on the forces in the structural elements, and the minimum section satisfying ULS is taken for each alternative. This analysis uses the orientation of the struts that produces constant axial forces throughout the whole posttensioning system under permanent loads.

As mentioned in the previous sections, the geometry of the posttensioning system as well as its internal forces are not unique for a given geometry. Therefore, the designer can explore the infinite range of equilibrated solutions by varying the two indeterminacies. The forces to be added to the base geometry are related to the maximum allowable eccentricity between the concrete frame and the posttensioning system. The eccentricity can be controlled through the two indeterminacies. This distance between the two curves may be described as a percentage of the span and is a relevant parameter for evaluating the efficiency of the system. As a general rule, the smaller the distance between the concrete frame and the posttensioning system, the higher the forces in both components. In this analysis, the eccentricity maximum is fixed to 3 m (i.e., 10% of the span). Finally, the dimensioning of the posttensioning, composed of full locked cables, is based on limiting the design force at service limit state to a value lower than 45% of the Guaranteed Ultimate Tensile Strength (GUTS).

The standard solution (Alt. 0), that is, without posttensioning, where axial forces and bending moments coexist, requires a concrete cross-section of 0.60 \(\times\) 1.80 m.

A first analysis of the efficiency under live loads is done using this cross-section for the original geometry and adding the external posttensioning system with a cross-section that withstands a utilization of 25% under permanent loads. The results of this analysis are shown in Figure 18. Two pairs of values of the indeterminacies/design values meeting the criteria above and maximizing the efficiency are chosen. The dimensioning of the cross-sections of the original geometry and the posttensioning system are then refined in an iterative process that derives in the four design alternatives.

Alternatives 1 and 2 are based on locating the posttensioning always outside the main structure. The only difference between these alternatives is the connection of struts with the main structure, in one case (Alt. 1) they are pinned, while in the other case (Alt. 2) they are clamped.

Alternatives 3 and 4 are based on minimizing the posttensioning force without taking into account if the posttensioning is above or below the main structure. Again, the struts are pinned in one case (Alt. 3) and clamped in the other (Alt. 4).
## 4.2 Analysis of results

As anticipated by the values of efficiency in Figure 18, the external posttensioning does not contribute significantly in resisting the live loads. In fact, the portion of the bending moment provoked by the live load that is resisted by the posttensioning system is very low and this led to very low values of $\xi_D$. This is due to the high bending stiffness of the concrete section with respect to the axial stiffness of the posttensioning.

Figure 19 shows a summary of the results obtained for the five alternatives in terms of force and form diagrams. The force polygons (second column) reveal at a first look the magnitude of the forces to be introduced with respect to the permanent loads, and therefore they indicate the efficiency under permanent loads of each specific option. Both the efficiency under permanent and live loads should be taken into account simultaneously to achieve an optimized design.

Alternatives 1 and 2, shown in the second row, result in a cross-section of 0.60x0.60 m (three times lower than the standard solution) and an external posttensioning system with a 70-mm diameter cable. Bending in the base geometry due to live loads is smaller in Alternative 2 because of the clamped connected struts and therefore the reinforcement in the cross-section can be eliminated.

Alternatives 3 and 4, illustrated in the third row, result in a concrete cross-section of $0.60 \times 0.75$ m and an external posttensioning system consisting in a cable of 35 mm diameter. The concrete section has a slightly higher area compared to Alternatives 1 and 2, but the area of the posttensioning system decreases to one half.

The third column of Figure 19 describes the interaction diagrams ($N_{Ed}$, $M_{Ed}$) for each alternative. Please consider that the diagrams are at different scales, the first one corresponding to the standard solution has a dimension which is around 10 times higher than the diagram corresponding to Alternative 1. In addition, the interaction diagrams show that the cross-section dimensions have been selected to strictly comply with the worst combination of design values ($N_{Ed}$, $M_{Ed}$).

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**Figure 19** Results of different alternatives applied to the case of study
Furthermore, the horizontal and vertical reactions due to permanent loads are shown for the five alternatives. The vertical component depends only on the permanent load acting on the original geometry (here the self-weight of the posttensioning system is neglected). The horizontal component of the reaction is strongly affected by the introduction of the posttensioning. In fact, Alternatives 1 and 2 yield a very high horizontal reaction generated by the posttensioning, which is more than three times the horizontal reaction of the standard solution. This value strongly decreases for Alternatives 3 and 4, where the value is lower than the horizontal reaction of the standard solution. These results may have a clear impact on the foundation design. Furthermore, in Alternatives 1–4, the direction of the horizontal reaction is inward while for the standard solution it is outward.

Table 1 shows an approximate cost comparison for the five alternatives. This is a simplified assessment and the costs of concrete, reinforcement, and posttensioning are considered separately to evaluate their influence for each alternative.

Results illustrate how the position of the posttensioning, which is controlled through the two indeterminacies, strongly influences the structural performance and therefore the cost of the alternatives. If the posttensioning is forced to be outside the structure (Alt. 1 and 2), the structure becomes much slender compared to alternative 0, but the material-savings are not relevant with respect to the standard structure. This is because very high values of external forces must be introduced into the structure and posttensioning becomes very expensive as a consequence. In addition, as mentioned before, a high horizontal reaction should be carefully considered in foundation design. On the other hand, Alternatives 3 and 4 result in savings of 40% of the final cost and the horizontal reaction that is around 57% of the reaction obtained for the standard solution. This economic outcome should be summed again to a slenderer and visually elegant structure.

5 | CONCLUSIONS

This paper has investigated the response of externally posttensioned curved structures under live loads. Two different definitions of structural efficiency are proposed: the first is based on bending moments and the second on strain energies. First through an analytical approach and next with a parametric study, this paper has analyzed the influence of the two design variables or indeterminacies, the effect of using pinned or bending-connected struts joining the original structure and the posttensioning and the influence of their orientation, the behavior under

| Alternatives | Struts type | Vertical reaction (kN) | Horizontal reaction (kN) | Cross-section (m²) | Length arch (m) | Concrete cost (€/m³) | Concrete cost (A) (€) | Rein. cost (€/kg) | Rein. cost (B) (€) | Posttensioning cost (€/kg) | Posttensioning cost (C) (€) | Total cost (A + B + C) (€) |
|--------------|-------------|------------------------|-------------------------|-------------------|----------------|---------------------|---------------------|----------------|----------------|-------------------------|-------------------------|--------------------------|
| 0            | Pinned      | 973                    | 556                     | 0.36              | 50            | 556                 | 0.36                | 50             | 556           | 0.36                    | 50                      | 14,654                   |
| 1            | Clamped     | 973                    | 556                     | 0.36              | 50            | 556                 | 0.36                | 50             | 556           | 0.36                    | 50                      | 14,654                   |
| 2            | Clamped     | 973                    | 556                     | 0.36              | 50            | 556                 | 0.36                | 50             | 556           | 0.36                    | 50                      | 14,654                   |
| 3            | Pinned      | 617                    | 99                      | 0.45              | 50            | 99                  | 0.45                | 50             | 99            | 0.45                    | 50                      | 10,040                   |
| 4            | Pinned      | 617                    | 99                      | 0.45              | 50            | 99                  | 0.45                | 50             | 99            | 0.45                    | 50                      | 10,040                   |
symmetric and asymmetric live loads and the impact of discretization. In summary, results show that:

- the study of the efficiency under live loads of externally posttensioned curved structures from an analytical approach provides relevant information on optimized geometrical configurations.
- structural behavior is basically governed by the first design variable or indeterminacy, and it prevails over the second. This finding is more evident when pinned struts are used;
- the trend of the strain energy-based efficiency for the case of symmetric load is very similar to the one obtained with the moment-based definition of the efficiency, therefore both definitions provide similar findings;
- the shape of the plot for bending-connected struts is considerably different from the pinned case. Furthermore, the efficiency under asymmetric loads is consistently higher than the efficiency of the structure with pinned struts under the same loads. However, results are strongly affected by the stiffness of the struts;
- a different discretization does not affect results in the symmetrical case, but efficiency increases up to a certain point in the asymmetrical case if a finer discretization is used;
- the directions of the struts have a strong influence on the efficiency of externally posttensioned curved structures, with the orientation producing constant axial forces in the posttensioning system under live loads showing the best performance among the alternatives analyzed;
- in addition to eliminating bending moments under permanent loads, external posttensioning contributes to reduce bending under live loads in curved structures;
- the case of study shows that the efficiency of the external posttensioning system under live loads is low in comparison to the efficiency for permanent loads. This is expected to be the case in most concrete structures where the bending stiffness of the original geometry is high compared to the axial stiffness of the posttensioning;
- however, the outcomes of the case of study also illustrate that, despite the introduction of higher axial forces through the posttensioning, the act of reducing bending moments allows to design more cost-effective and slender structures.

Further research on the behavior of curved externally posttensioned structures would have to address topics such as buckling of the elements, application of other types of loads like snow, seismic and other dynamic loads, geometric and material nonlinearities and consideration of additional design specifications. Incorporating these aspects could enhance the practical application of curved externally posttensioned structures.

The data that support the findings of this study are available from the corresponding author upon reasonable request.

**DATA AVAILABILITY STATEMENT**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

**NOTATION**

\( A \) \hspace{1em} \text{area of the cross-section of the original geometry}  
\( A_{SC} \) \hspace{1em} \text{area of the cross-section of the posttensioning system}  
\( D_{f} \) \hspace{1em} \text{strain energy of the structure under live loads with the posttensioning system}  
\( D_{0} \) \hspace{1em} \text{strain energy of the structure under live loads without the posttensioning system}  
\( E \) \hspace{1em} \text{Young's Modulus of the original geometry}  
\( E_{SC} \) \hspace{1em} \text{Young's Modulus of the posttensioning system}  
\( I \) \hspace{1em} \text{second moment of area of the original geometry}  
\( L \) \hspace{1em} \text{characteristic length, span or length of discretized segments}  
\( L_{s} \) \hspace{1em} \text{length of the middle strut}  
\( N_{Rd} \) \hspace{1em} \text{design value for the axial strength}  
\( M_{f} \) \hspace{1em} \text{maximum bending moment in the structure under live loads with the posttensioning system}  
\( M_{0} \) \hspace{1em} \text{maximum bending moment in the structure under live loads without the posttensioning system}  
\( M_{Rd} \) \hspace{1em} \text{design value for the bending moment strength}  
\( Q \) \hspace{1em} \text{magnitude of a point load applied to the structure}  
\( R^{*} \) \hspace{1em} \text{dimensionless monomial depending on the response of the structure under study}  
\( f_{ck} \) \hspace{1em} \text{characteristic compressive cylinder strength of concrete at 28 days}  
\( g \) \hspace{1em} \text{function depending on the geometrical characteristics of the structure}  
\( i_{1} \) \hspace{1em} \text{first indeterminacy or design variable}  
\( i_{2} \) \hspace{1em} \text{second indeterminacy or design variable}  
\( n \) \hspace{1em} \text{number of struts}  
\( q \) \hspace{1em} \text{magnitude of a distributed load applied to the structure}  
\( \alpha \) \hspace{1em} \text{first angle describing the geometry of the posttensioning system for a circular arch discretized in four segments}  
\( \beta \) \hspace{1em} \text{second angle describing the geometry of the posttensioning system for a circular arch discretized in four segments}  
\( \xi_{M} \) \hspace{1em} \text{moment-based efficiency of the structure under live loads}  

strain energy-based efficiency of the structure under live loads

χ dimensionless parameter controlling the efficiency of the structure

χ1 dimensionless parameter relating the bending stiffness of the original geometry and the axial stiffness of the posttensioning system

χA dimensionless parameter relating the axial stiffness of the original geometry and that of the posttensioning system

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