Value of color charges and structure of gauge bosons

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Abstract

The values of color and anticolor charges are proposed. The structure of gluons is predicted relative to their color and anticolor charges. It is shown that the gauge bosons of lower order theories can be used as it is for higher order theories. Various mass relations between the gauge bosons are also given.

1 Introduction

Since the birth of Quantum Chromodynamics (QCD), no attention was paid to find out the numerical value of the color charges. The color combinations were given in the literature with the help of SU(3) on the analogy of quark combinations and there was not clear definition for color and anticolor. It was observed that the gluons cannot carry a combination of a color and anticolor charge, because this combination violate the group property [1]. Gilani [1] pointed out that gluons will carry either color charges or anticolor charges. He claimed, with the help of set theory, that there will be six colored gluons, one color singlet gluon and one massless neutral gluon. He also claimed that seven gluons will be massive and only one massless gluon. Relations between color charges and anticolor charges are also defined in Ref. [1].

The paper is organized as follows: In sec. 2 the values of the color charges are predicted and obtained the values of respective anticolor charges from color charges as defined in Ref. [1]. Charged gluons structure is predicted in Sec. 3. Charged gluons masses are predicted in Sec. 4.
representation of the structure of electroweak gauge bosons and gluons is presented in Sec. 5. Position of massive gluon is explained in Sec. 6. Section 7 is devoted to Higgs. Relation between electroweak gauge bosons and QCD gauge bosons (gluons) is discussed in Sec. 8. Expected Higgs mass is predicted in Sec. 9 by using the existing value of $W$-boson mass ($m_W$). Quark color charges and some decays of mesons and baryons are presented in Sec. 10 Finally, the results are summarized in Sec. 11.

2 Can we give values to color charges?

The electroweak charges are two i.e. $+1, -1$. The question is: Is there any mathematical relation which has such a solution that we obtain a solution set as $\{+1, -1\}$? The equation is

$$x^2 = 1$$

$$x = \{+1, -1\}$$

This shows that the electroweak charges are the result of square roots of unity. In QCD, there are three charges and if we take the cube roots of unity, we obtain

$$x^3 = 1$$

$$x = \left\{+1, -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right\}$$

that is, we can predict here that the color charges are equivalent to cube roots of unity and we define

$$r \equiv +1$$

$$g \equiv -\frac{1}{2} + \frac{i\sqrt{3}}{2}$$

$$b \equiv -\frac{1}{2} - \frac{i\sqrt{3}}{2}$$

On the other hand, the anticolor charges can be obtain from the color charges and we obtain

$$\bar{b} \equiv r + g = +\frac{1}{2} + \frac{i\sqrt{3}}{2}$$
\[ \bar{g} \equiv b + r = + \frac{1}{2} - i \frac{\sqrt{3}}{2} \]
\[ \bar{r} \equiv g + b = -1 \]

The sum of all the color charges is equal to zero and the same is true for anticolor charges.

\[ r + g + b = 0 \]
\[ \bar{r} + \bar{g} + \bar{b} = 0 \]

The color and anticolor charges (Eqs. (5–10)) are plotted in Fig. 1. All the distances \( oi (i = r, g, b, \bar{b}, \bar{g}, \bar{r}) \), \( r\bar{b}, bg, g\bar{r}, \bar{r}b, bg \) and \( \bar{g}r \) are equal to unity.

### 3 Structure of charged gluons

In electroweak theory, the charged vector bosons \( W^+ \) and \( W^- \) carry only +1 and −1 charge respectively. In QCD, as in electroweak theory, color charged gluons carry color charges contrary to color-anticolor charge, this was pointed out in a recent study \[1\]. In electroweak, there are only two charges and hence we have only two charged vector gauge bosons i.e. \( W^+, W^- \). In QCD, there are three color charges and their corresponding three anticolor charges as defined in Eqs. (5–10). So, the color and anticolor charges gave us three colored and three anticolored vector gauge bosons (i.e. gluons). Let us suppose that a gluon \( g_+ \) be associated to positive charge and a gluon \( g_- \) be associated to negative charge in QCD. So, we define the various color and anticolor charged gluons with respect to their charges as

\[ g_r \equiv +g_+ \]
\[ g_g \equiv \left( -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) g_- \]
\[ g_b \equiv \left( -\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) g_- \]
\[ g_{\bar{b}} \equiv \left( +\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) g_+ \]
\[ g_{\bar{g}} \equiv \left( +\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) g_+ \]
\[ g_{\bar{r}} \equiv -g_- \]
We give the positive or negative charge to gluons only seeing the sign of the real part, see for example Eqs. (12–15). If you are not convinced at this stage why we give the same charge to the hybrid gluons [Eqs. (12–15)] as the sign their real part, you will be convinced after doing exercise of decay processes in Sec. 10.

4 Masses of charged gluons

As in electroweak theory, the masses of charged vector gauge bosons are equal. On same analogy we suppose that $m_{g_+} = m_{g_-} = m_g$. Therefore,

\begin{align*}
m_{g_r} & = |m_{g_+}| = m_g \\
m_{g_g} & = \left( -\frac{1}{2} + i\frac{\sqrt{3}}{2} \right) m_{g_-} = m_g \\
m_{g_b} & = \left( -\frac{1}{2} - i\frac{\sqrt{3}}{2} \right) m_{g_-} = m_g \\
m_{g_b} & = \left( +\frac{1}{2} + i\frac{\sqrt{3}}{2} \right) m_{g_+} = m_g \\
m_{g_g} & = \left( +\frac{1}{2} - i\frac{\sqrt{3}}{2} \right) m_{g_+} = m_g \\
m_{g_r} & = |m_{g_-}| = m_g
\end{align*}

Equations (17–22) give the masses of gluons and we have found that the masses of color charged gluons and anticolor charged gluons are equal, i.e. $m_{g_r} = m_{g_g} = m_{g_b} = m_{g_b} = m_{g_g} = m_{g_r} = m_g$.

5 Pictorial representation of electroweak gauge bosons and gluons

The electroweak gauge bosons are $\gamma$, $W^+$, $W^-$ and $Z^0$. Among these four gauge bosons, only photon ($\gamma$) is massless while the remaining three are massive. The gauge bosons are predicted only by set theory but not by special unitary groups i.e. SU(2) or SU(3) [1]. The electroweak charges can be described by Eq. (1) and we obtain a set of its roots as given in Eq.
A set has proper and improper subsets. In this case, we have two component set, so we have two proper subsets and two improper subsets, i.e.,

$$\{+1, -1\} \Rightarrow \{\{\}, \{+1\}, \{-1\}, \{+1, -1\}\}$$

The subsets \{+1\}, \{-1\} are proper subsets of the set of charges while \{\}, \{+1, -1\} are improper subsets. If we plot all these, then we fix the empty subset at the origin and \{+1\} on the x-axis at +1 position and \{-1\} at the -1 position while the \{+1, -1\} gets the position above the empty subset along the z-axis. We have shown the electroweak gauge bosons in Fig. 2. How the \(Z^0\) vector gauge boson gets the position above the photon (\(\gamma\)), we will explain it in one of the next sections.

It was claimed that the gluons does not obey the SU(3) but can be predicted by set theory [1]. The value of the color charges is explained in section 2 and plotted in Fig. 1. We have sketched a pattern of the gluons with respect to their charges as shown in Fig. 3.

6 Position of the massive neutral gluon

The massive color-singlet (neutral) gluon \(G_0\) is placed over the massless gluon as shown in Fig. 4, just like the neutral vector boson \(Z^0\) is placed over the photon (\(\gamma\)), see Fig. 2. The question is, how ? and why ?

To answer this question, let us join the color points by straight lines which results in an equilateral triangle \(\Delta rgb\) having the length of each side \(\sqrt{3}\). Similarly, we can draw anticolor triangle \(\Delta \bar{b}\bar{g}\bar{r}\) by joining anticolor points. Anticolor triangle \(\Delta \bar{b}\bar{g}\bar{r}\) is also an equilateral triangle having length of each side of the triangle \(\sqrt{3}\) as shown in Fig. 4. Take the apex \(r\) of triangle \(\Delta rgb\) and \(\bar{r}\) of the other triangle \(\Delta \bar{b}\bar{g}\bar{r}\), and join them together, which meet on the z-axis as shown in Fig. 5. The points \(r\) and \(\bar{r}\) meet on the z-axis at the point \(z\). On the same grounds, if we take apexes \(g\) and \(\bar{g}\) of the respective triangles, which meet again at the same point \(z\) on the z-axis, similar the case will be for the apexes \(b\) and \(\bar{b}\). Now the question is, what is the value of \(z = ?\) Let us any triangle \(\Delta ozr\), the side \(|or| = 1\), \(|rz| = \sqrt{3}\) and \(|oz| = ?\). The triangle \(\Delta ozr\) is a right triangle. Applying Pathagora’s theorem, \(|oz|^2 = |rz|^2 - |or|^2 = 3 - 1 = 2\) i.e. \(z = |oz| = \sqrt{2}\). This shows that the point \(z\) is \(\sqrt{2}\) units above the origin along the z-axis.
7 Higgs: Is there any?

Following the discussion given in Sec. 6 in triangle $\triangle ozr$, the side $|or|$ gives the size of the charged gluon $i.e. W^r(g_r)$ [see Eq. (11)] and the side $|oz|$ gives the size of the color singlet gluon $(G_0)$. The remaining third side of the triangle $|zr|$ will give the size of Higgs. We get six Higgs of equal size. Among these Higgs, three carry color charge and three carry anticolor charge. The sides of the triangle $\triangle ozr$ have certain ratio between each other, i.e.

$$|or| : |oz| : |rz| = 1 : \sqrt{2} : \sqrt{3}$$

From the above ratio between the side of triangle, we can predict the masses of the color singlet gluon $(G_0)$ and the Higgs $(H_i)$ as

$$m_{G_0} = \sqrt{2}m_g = \sqrt{2}m_W,$$  \hspace{1cm} (23)

$$m_{H_i} = \sqrt{3}m_g = \sqrt{3}m_W,$$  \hspace{1cm} (24)

respectively.

8 Relation between electroweak and QCD gauge bosons

Is there any relation between electroweak and QCD theories? This question raised by many scientists who wrote articles entitled: ‘Theory of Everything’. Keeping in mind the above discussion, we conclude that QCD is nothing but a higher order electroweak theory. Electroweak theory is second order theory and QCD is third order theory because the structure of their charges obtained from Eqs. (1) and (3) respectively. The position of $r = +1$ and $\bar{r} = -1$ charges is exactly same as the respective charges in electroweak theory. Due to this similarity, we suppose that

$$g_+ = W_+, \hspace{1cm} g_- = W_-$$ \hspace{1cm} (25)

Therefore, we can modify the Eqs. (11–16) as

$$W^r = g_r \equiv +W_+, \hspace{1cm} (26)$$

$$W^g = g_g \equiv \left( -\frac{1}{2} + i\frac{\sqrt{3}}{2} \right) W_-, \hspace{1cm} (27)$$
\[ W^b = g_b \equiv \left( -\frac{1}{2} - i\frac{\sqrt{3}}{2} \right) W_-, \]  
(28)

\[ W^b = g_b \equiv \left( \frac{1}{2} + i\frac{\sqrt{3}}{2} \right) W_+, \]  
(29)

\[ W^g = g_g \equiv \left( \frac{1}{2} - i\frac{\sqrt{3}}{2} \right) W_+, \]  
(30)

\[ W^f = g_f \equiv -W_-, \]  
(31)

and their masses

\[ m_{g_r} = m_{g_b} = m_{g_g} = m_{g_b} = m_{g_f} = m_W. \]  
(32)

Also the Higgs masses are

\[ m_{H_i} = \sqrt{3}m_W, \quad i = r, g, b, \bar{b}, \bar{g}, \bar{r} \]  
(33)

and color singlet gluon \( G_0 \) mass is

\[ m_{G_0} = m_{Z_0} = \sqrt{2}m_W. \]  
(34)

If we take away the blocks of \( g_b, g_g, g_b \) and \( g_\bar{b} \) from Fig. 3 we are left with Fig. 2. Again if we take away the blocks of \( W^+ \) and \( W^- \), we are left with massless photon (\( \gamma \)) and \( Z^0 \), which serves as the gauge bosons for theory of gravity. This means that the photon will serve the purpose of massless graviton } and \( Z^0 \) plays the role of massive graviton \( G \) as predicted first time by Gilani in his recent article \[ ] . Finally, take away the massive graviton (\( Z^0 \)), we are left with massless photon at the origin which serve the purpose of Casimirion \[ ] as a gauge boson of Casimir force.

9 Expected Higgs mass

Consider triangle \( \Delta ozr \) (see Fig. 5)

\[ |rz|^2 = |or|^2 + |oz|^2 \]

\[ |rz| = \sqrt{|or|^2 + |oz|^2} \]

\[ m_H = \sqrt{m_W^2 + m_Z^2} \]

\[ = 121.5855 \text{ GeV} \]  
(35)
where \( m_W = 80.423 \text{ GeV} \) and \( m_Z = 91.1876 \text{ GeV} \). Whereas, from Eq. (33)
\[
m_H = \sqrt{3} m_W \\
= 139.2967 \text{ GeV}
\]
From Eqs. (35) and (36), the two values for Higgs masses does not match. Equation (35) obtains the value of Higgs mass from the experimental values of \( m_W \) and \( m_Z \), while Eq. (36) gives the value of Higgs mass using only experimental value of W-mass. This scheme gives the relation between the \( m_W \) and \( m_Z \) [see Eq. (34)]. This Eq. (34) does not match the experimental value of \( m_Z \), if we take the \( W \)-boson mass as standard.

10 Quark color charges

We will not consider the quark fractional charges, like \( +\frac{2}{3} e \) for up-type quarks and \( -\frac{1}{3} e \) for down-type quarks. We cannot take at the same time two type of charges i.e. fractional charges and color charges upon the quarks. We recommend to give color charges to up-type quarks (i.e. \( u, c, t \)) and anticolor charges to down-type quarks (i.e. \( d, s, b \)). Let us see when a \( B \) meson decay into a \( \rho \)-meson
\[
\bar{B}^0 (b \bar{d}) \rightarrow W^+ \rho^- (u \bar{d}) ,
\]
\[
\rightarrow W^b \rho^\circ (u \bar{d}) ,
\]
In the above decay, a \( b \bar{d} \) quark decay into \( u \bar{b} (u \bar{d}) \) and \( W^r (W^b) \), and \( W^r \equiv W^+ \) further decay into \( l^+ \nu_l \). In the above decay we consider only green color and anticolor combination. Let us see if we take the other combination of quark colors for the decay of \( B^0 \)-meson.
\[
\bar{B}^0 (b \bar{d}) \rightarrow W^r \rho^\circ (u \bar{d}) ,
\]
\[
\rightarrow W^g \rho^\circ (u \bar{d}) ,
\]
\[
\bar{B}^0 (b \bar{d}) \rightarrow W^b \rho^\circ (u \bar{d}) ,
\]
\[
\rightarrow W^g \rho^\circ (u \bar{d}) ,
\]
where

\[ W^b = \left( -\frac{1}{2} - i\frac{\sqrt{3}}{2} \right) W^- , \]  

\[ W^g = \left( -\frac{1}{2} + i\frac{\sqrt{3}}{2} \right) W^- , \]  

and

\[ \rho^\bar{b} = \left( \frac{1}{2} - i\frac{\sqrt{3}}{2} \right) \rho^+ , \]  

\[ \rho^\bar{g} = \left( \frac{1}{2} + i\frac{\sqrt{3}}{2} \right) \rho^+ . \]  

This shows that by solving the decays like \( \bar{B}_0^0 \left( b^\bar{g} d^g \right) \rightarrow W^r \rho^x \left( u^b d^g \right) \) and \( B^0 \left( b^g d^\bar{g} \right) \rightarrow W^r \rho^x \left( \bar{u}^\bar{b} d^g \right) \), we can solve all the remaining decays easily. We cannot ignore the possibility that the up-type quarks serve as quarks and their respective down-type quarks serve as anti-quarks but at the moment we are not sure. If such a possibility exists then the quarks will be reduced to three (i.e. \( u, c, t \)) and their anti-quarks (i.e. \( \bar{u} \equiv d, \bar{c} \equiv s, \bar{t} \equiv b \)), then the life will become too much simple. We will now focus on the hybrid states like given in Eqs. (45, 46), if such states are virtual and further decay. Then

\[ \rho^\bar{b} \left( u^r d^g \right) \rightarrow W^b \rho^0 \left( d^g d^g \right) . \]  

Following \( B^0 \) decay given in Eq. (48)

\[ \bar{B}^0 \left( b^g d^\bar{g} \right) \rightarrow W^b \rho^\bar{b} \left( u^r d^g \right) \]  

\[ \rightarrow W^b W^\bar{b} \rho^0 \left( d^g d^g \right) \]  

\[ \rightarrow Z^0 \rho^0 \left( d^g d^g \right) \]  

(47)

So, in the above decay

\[ b^g \rightarrow W^b u^r \rightarrow W^b W^\bar{b} d^g \rightarrow Z^0 d^g . \]  

(48)

For three quark baryon states, we will write the combinations as

\[ \left( \frac{u^b d^\bar{b} + u^g d^g}{\sqrt{2}} \right) d^r = (u dd)^- , \]  

\[ u^r \left( \frac{u^b d^\bar{b} + u^g d^g}{\sqrt{2}} \right) = (u ud)^+ , \]  

(49)
In the above examples, combination of type $u^rd_r^d$ or $u^ru^rd_r^d$ does not exist because of repeated anticolor or color index, respectively. We are just giving here simple examples. The rest of the states we can make like

$$\left(\frac{u^gd_g + u^bd_r}{\sqrt{2}}\right) d_r = (udd)^-, \quad (50)$$

etc. If $\Lambda_0^b \rightarrow W^+\Lambda^-$, then

$$\Lambda_0^b \left(b^g d_g^d d_r^d\right) \rightarrow W^r\Lambda_0^g \left(u^b d_g^d d_r^d\right), \quad \Lambda_0^b \left(b^b d_b^d d_r^d\right) \rightarrow W^r\Lambda_0^b \left(u^b d_g^d d_r^d\right) \quad (51)$$

where $b^g \rightarrow W^r u^b$ and $b^b \rightarrow W^r u^g$ or vice versa. Let us concentrate upon the above examples of meson decays (37–42) and baryon decays (51). If we consider down-type quarks as antiquarks of up-type quarks then in the baryon case $\Lambda_0^b$ is composed of three down-type quarks while the $\Lambda^r$ is composed of one up-type quark and two down-type quarks. Whereas the baryons are built up of either quarks or antiquarks. By keeping this view, we cannot consider down-type quarks as antiquarks of up-type quarks.

11 Conclusions

In this article, the value of the color charges are given. The structure of the colored, anticolored and color singlet gluons are proposed. Their mass relations are also given. It is shown that the gauge boson in the electroweak, QCD and gravity theories are not different but they are linked to one another. The gauge bosons of lower order theories serve the purpose for higher order theories. The proof of the claims of Ref. [1] are given in a systematic way.

Quark fractional charges (i.e. $+\frac{2}{3}e$ for up-type quarks and $-\frac{1}{3}e$ for up-type quarks) are totally rejected. Only color charges are given to up-type quarks and anticolor charges to down-type quarks. This is explained by applying to meson and baryon decays.

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References

[1] A. H. S. Gilani, Are gluons massive? [hep-ph/0404026 v2]
12 Figure Captions

1. The plot of cube roots of unity, i.e. the color and anti color charges.

2. Electroweak gauge bosons $\gamma, W^+, W^-$ and $Z^0$. Photon ($\gamma$) gets position at the origin as it is massless while the others relative to their charge positions.

3. The gluons are plotted relative to their charge positions. The massless gluon gets the position at origin.

4. The color points are joined by lines which form triangle $\Delta rgb$ and by joining anticolor points another triangle is formed $\Delta \bar{b}\bar{g}\bar{r}$. Both the triangles are equilateral triangles having length of each side $\sqrt{3}$.

5. Taking the apexes $r$ and $\bar{r}$ of the triangles $\Delta rgb$ and $\Delta \bar{b}\bar{g}\bar{r}$ respectively, which meet at the $z$ on $z$-axis.
Figure 1
Figure 2
Figure 3:
Figure 4
Figure 5