Charged Holostars

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June 16, 2003 (v1)
May 1, 2004 (v3)

Abstract

In a recent paper the so called holographic solution, in short holostar, was introduced as a new spherically symmetric solution of Einstein field equations. This paper extends the holostar solution to the charged case.

A charged holostar is an exact solution of the Einstein field equations with zero cosmological constant and an interior string type matter. It has properties similar to the known Reissner-Nordström (RN) black hole solution. The exterior metrics and fields of both solutions are equal. In contrast to a RN black hole, a charged holostar has a singularity free interior matter distribution $\rho = 1/(8\pi r^2)$ with an overall string equation of state: $P_r = -\rho$ and $P_\perp = 0$. Similar to the uncharged holostar solution, the charged holostar has a spherical boundary membrane consisting out of tangential pressure, but no mass-energy. The boundary is situated roughly a Planck coordinate distance outside of the outer horizon of the RN-solution.

The geometric properties of the charged holostar solution are very conveniently described in terms of the so called geometric mass $M_g = M + r_0/2$. $r_0$ is a Planck size correction to the gravitational mass $M$ with $r_0 \approx 2r_{Pl}$. The geometric mass of a charged holostar is always larger than its charge in natural units. For a large holostar this condition is practically identical to the classical condition $M \ge Q$. Whereas RN solutions with $M < Q$ are possible in principle, and are excluded from the physically acceptable solution space by the cosmic censorship hypothesis, a charged holostar with $M_g > Q$ doesn’t exist.

The total exterior charge $Q$ of a holostar can be attributed to the charge of its massive interior particles. $Q$ is derived by the proper integral over the interior charge density. The interior mass-energy density $\rho$ splits into an electromagnetic contribution $\rho_{em}$ and a "matter" contribution. Both contributions are proportional to $1/r^2$. Yet the total interior mass-energy density and the total principal pressures are exactly equal to the uncharged case. The same is true for the interior metric.

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The ratio of electro-magnetic to total energy density \(\rho_{\text{em}}/\rho = 4\pi Q^2/A\) is constant throughout the whole interior. It is related to the dimensionless ratio of the exterior conserved quantities \(Q^2\) and \(A\) (or alternatively \(Q/M_g\)). An extremely charged holostar has a surface area \(A = 4\pi Q^2\), so that its interior energy density consists entirely of electromagnetic energy. The tangential pressure in the membrane of an extremely charged holostar vanishes, so that the energy-density is continuous at the boundary.

Similar to the uncharged holostar, the charged holostar solution admits charged "particle-like" solutions with nearly zero gravitational mass, but with appreciable charge in natural units. A charged particle with mass and charge comparable to the electron, however with zero spin, is a genuine solution.

The equations for a zero mass extremely charged holostar are extrapolated to the rotating case. Under the assumption, that the electron can be identified with an extremely charged holostar of minimal mass, a scaling law for the fundamental area \(r_0^2\) is conjectured, according to which \(r_0^2 \approx 4\hbar\) at the Planck-energy and \(r_0^2 \approx \sqrt{3/4}\ 4\hbar\) at the low energy scale.

A large holostar can be regarded as the classical analogue of a large loop quantum gravity (LQG) spin-network state, if one identifies the links of the LQG spin-network state with the interior massless particles of the classical holostar solution. The Barbero-Immirzi parameter is determined to be equal to \(\gamma = \sigma/(\pi\sqrt{3})\), where \(\sigma\) is the mean entropy per ultra-relativistic particle. \(\gamma\) is larger by a factor of \(\approx 4.8\) than the LQG-result. An explanation for the discrepancy is given.

An (approximate) entropy conservation law for self gravitating systems in general relativity is proposed.

1 Introduction

In [9] a new class of solutions to the spherically symmetric field equations of general relativity was derived. The solutions are characterized by an interior non-zero string type matter-density and a boundary membrane with zero energy density, but non-zero tangential pressure. The interior string type pressure generally is anisotropic.

One of the new solutions, the so called "holographic" solution, in short holostar, was discussed in greater detail in [11, 10]. It’s interior equation of state is that of a spherically symmetric string vacuum, bounded by a two-dimensional membrane. The membrane’s pressure is exactly equal to the pressure attributed to the (fictitious) membrane of a black hole according to the membrane paradigm [15]. Therefore the exterior properties of the holostar are guaranteed to be virtually indistinguishable from the properties of a black hole. The holostar’s entropy and temperature are equal to the Hawking result up to a constant factor [10]. Yet the holostar has no event-horizon and to singularities. It appears to be an amazingly self-consistent model for the most compact, self-consistent static solution of the Einstein field equations that is not a black hole.
So far only uncharged solutions were discussed. In this paper I attempt to generalize the previous results to the case of a charged self gravitating body. Although the practical applicability of the charged holostar solution is expected to be limited, as self gravitating objects of astrophysical interest are assumed to be essentially uncharged, the charged holostar solution is of considerable interest from a theoretical point of view: From the study of black holes it is well known that the properties of the charged black hole solutions are in many respects similar to the properties of the spinning black hole solutions. In gravitational collapse of large stars one expects that a highly, almost maximally spinning black hole is formed. Observational evidence for a black hole with high angular momentum \(a/M \approx 0.95\) is given in [5]. Therefore a spinning holostar solution is of considerable interest and high astrophysical relevance. It might be possible to infer some of the properties of a spinning holostar from the charged solution.

2 Field equations for a spherically symmetric charged system

The approach taken in this paper is similar to the route taken in [9]. As a basis for the derivation the well known Schwarzschild coordinate system in the \( (+ - - -) \) sign-convention with units \( c = G = 1 \) is used. Without loss of generality the metric of a static spherically symmetric space-time, charged or uncharged, can be expressed in the following form:

\[
ds^2 = B(r)dt^2 - A(r)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2
\]  

The charge distribution \( \rho_c \), the electromagnetic energy density \( \rho_{em} \) and the "matter fields", i.e. the (non-electromagnetic) mass-density \( \rho_m \) and its principal pressures \( P_{r,m} \) and \( P_{\theta,m} \), are spherically symmetric. They only depend on the radial coordinate value \( r \). Whenever appropriate the functional dependence of the relevant quantities on \( r \) will not be written out explicitly.

The stress-energy tensor \( T_{\mu}^\nu \) for a charged holostar is given by the sum of a "matter-" and an electromagnetic-component.\(^1\) When the term "matter" component is used throughout this paper, I mean the components of the stress-energy tensor of the self gravitating object which are not electromagnetic in origin. With this distinction the matter component of the stress-energy tensor can be expressed as:

\(^1\)Note, that according to common knowledge charge is always associated with mass-energy. Therefore the distinction between a "matter" and an electromagnetic component in the stress energy tensor should not be viewed as a statement that the two components are necessarily physically separable, but rather as a mathematical idealization which is useful for the calculations.
\[
(T_{\mu}^{\nu})_m = \begin{pmatrix}
\rho_m(r) & -P_r m(r) & -P_{\theta m}(r) & -P_{\phi m}(r)
\end{pmatrix}
\] (2)

Its trace \(T_m\), which is in general non-zero, is given by:

\[
T_m = (T_{\mu}^{\nu})_m = \rho_m - P_r m - 2P_{\theta m}
\] (3)

For a spherically symmetric system the electromagnetic component of the stress-energy tensor is given by the following, traceless tensor:

\[
(T_{\mu}^{\nu})_{em} = \begin{pmatrix}
\frac{E^2(r)}{8\pi} & \frac{E^2(r)}{8\pi} & \frac{E^2(r)}{8\pi}
\end{pmatrix}
\] (4)

\(E(r)\) is the electromagnetic\(^2\) field strength at radial position \(r\), in natural units.

Note that the interior string-type stress-energy tensor of the uncharged holostar solution with

\[
T_{\mu}^{\nu} = diag(\rho, \rho, 0, 0)
\]

i.e. with \(P_r = -\rho\) and \(P_{\theta} = P_{\phi} = 0\), can be constructed from the sum of a vacuum contribution

\[
(T_{\mu}^{\nu})_{vac} = \rho_{vac} diag(1, 1, 1, 1)
\]

and an electromagnetic contribution

\[
(T_{\mu}^{\nu})_{em} = \rho_{em} diag(1, 1, -1, -1)
\]

if the vacuum energy-density \(\rho_{vac}\) and the electromagnetic energy density \(\rho_{em} = E^2/(8\pi)\) are equal. We will see later, that the total interior stress energy-tensor of the charged holostar solution is identical to the uncharged case. The charged and the uncharged solution only differ in their exterior fields.

\(^2\)Because of spherical symmetry we are dealing with an electro-static problem. Therefore “electromagnetic” in the context of this paper always means “electrostatic”
The electromagnetic energy density must not necessarily be associated with a net charge. The electromagnetic energy density depends on the fields squared, therefore it is possible to have $E^2 \neq 0$, even if $Q = 0$.

In the case of a static spherically symmetric charge distribution one can express the magnitude of the radially symmetric electric field $E(r)$ in terms of the total charge $Q(r)$ enclosed in a concentric region bounded by the radial coordinate $r$:

$$E(r) = \frac{Q(r)}{r^2}$$

For the Reissner-Nordström solution the electromagnetic field tensor, with $E(r)$ given by equation (5), is known to be valid outside the horizon, i.e. in the vacuum region where $Q = Q(r_h)$ is constant. In this case it is easy to evaluate $Q$ by a Gaussian flux integral. Let us make the assumption, that equations (4) and (5) also hold inside a spherically symmetric self-gravitating body, and that $Q(r)$ is given by the proper integral over the interior charge distribution:

$$Q(r) = \int_0^r \rho_c(r) dV = \int_0^r \rho_c(r) 4\pi r^2 \sqrt{\mathcal{A}} dr$$

$\rho_c$ is the interior charge density. It should not be confused with the energy density $\rho_{em} = E^2 / (8\pi)$ of the electromagnetic field.

Whenever the stress-energy tensor of the space-time is known, it is convenient to write the field equations in the following form:

$$R_{\mu\nu} = -8\pi \left( T_{\mu\nu} - \frac{T}{2} \right)$$

with

$$T_{\mu\nu} = (T_{\mu\nu})_m + (T_{\mu\nu})_{em}$$

The components of the Ricci-tensor on the left side of equation (7) can be calculated from the metric coefficients and their first and second derivatives. The actual expressions can be found in any textbook. The right side of equation (7) is easily evaluated from the expressions for $(T)_m$ and $(T)_{em}$ given above. Only the diagonal components of the field equations give non-zero expressions.

Because charge is a relativistic invariant, the integral must be taken over the proper volume element. It is not possible to integrate over the improper (flat) spherical volume element, such as in the determination of the gravitational mass.

see for example [10, p. 300] or [3] p. 128, 226]
The equation for $R_{\varphi\varphi}$ is a trivial multiple of the equation for $R_{\theta\theta}$. Thus we are left with three equations:

$$R_{tt} = -\frac{B''}{2A} + \frac{B'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) - \frac{B'}{rA} = -4\pi B(\rho_m + P_{r_m} + 2P_{\theta m} + \frac{Q^2}{4\pi r^2}) \quad (8)$$

$$R_{rr} = \frac{B''}{2B} - \frac{B'}{4B} \left( \frac{A'}{A} + \frac{B'}{B} \right) - \frac{A'}{rA} = -4\pi A(\rho_m + P_{r_m} - 2P_{\theta m} - \frac{Q^2}{4\pi r^2}) \quad (9)$$

$$R_{\theta\theta} = -1 - \frac{r}{2A} \left( \frac{A'}{A} - \frac{B'}{B} \right) + \frac{1}{A} = -4\pi r^2(\rho_m - P_{r_m} + \frac{Q^2}{4\pi r^4}) \quad (10)$$

The field equations for the charged case can be deduced from the field equations of the uncharged case, simply by making the following replacements:

$$\rho \rightarrow \rho_{tot} = \rho_m + \frac{E^2}{8\pi} = \rho_m + \frac{Q^2}{8\pi r^4} \quad (11)$$

$$P_r \rightarrow P_{r_{tot}} = P_{r_m} - \frac{E^2}{8\pi} = P_{r_m} - \frac{Q^2}{8\pi r^4} \quad (12)$$

$$P_\theta \rightarrow P_{\theta_{tot}} = P_{\theta_m} + \frac{E^2}{8\pi} = P_{\theta_m} + \frac{Q^2}{8\pi r^4} \quad (13)$$

Keep in mind, that with the notation in equations (11, 12, 13), $\rho_m$, $P_{r_m}$ and $P_{\theta_m}$ exclusively refer to the matter-contribution, excluding the electro-magnetic contribution. For the gravitational field, or rather for the metric, only the total mass/energy-density and the total principal pressures are relevant. Whenever the total values are referenced, they will appear without subscript throughout this paper.

### 3 General properties of the solution

By multiplying equations (8, 9, 10) with the metric coefficients $A$ and $B$ respectively and summing up, the following expression results:

$$R_{00}A + R_{11}B = -\frac{B}{r} \left( \frac{A'}{A} + \frac{B'}{B} \right) = -8\pi AB(\rho_m + P_{r_m}) \quad (14)$$

which can be reformulated as:
Equation (15) is identical to the uncharged case.

For a spherically symmetric electric field we have

\[ \rho_{em} + P_{r_{em}} = 0 \]  

(16)

Therefore the electromagnetic field does not contribute to \( \rho + P_r \), so that the sum of the total energy density and total radial pressure is equal to the sum of the matter-components only:

\[ \rho + P_r = \rho_m + P_{r_m} \]  

(17)

With equation (16) the term \( B^\prime / B \) can be eliminated in equation (10), giving the following differential equation for the radial metric coefficient \( A \):

\[ \left( \frac{r}{A} \right)^\prime = 1 - \frac{8\pi r^2}{8\pi r^4} \left( \rho_m + \frac{Q^2}{8\pi r^4} \right) \]  

(18)

Equation (18) differs from the respective formula for the uncharged case by adding the term \( Q^2 / (8\pi r^4) \) to the mass-energy density of the matter contribution, \( \rho_m \). The added term is nothing else than the electromagnetic energy density \( \rho_{em} = E^2 / (8\pi) \).

The tangential pressure \( P_{\theta_m} \) can be calculated via one of the equations (8, 9):

\[ P_{\theta_m} + \frac{E^2}{8\pi} = \frac{1}{8\pi AB} \left( \frac{B''}{2} + \frac{B'}{r} \right) - \left( \frac{\rho_m + P_{r_m}}{4} \right) \left( \frac{rB'}{B} + 2 \right) \]  

(19)

Alternatively the tangential pressure can be derived from the continuity equation. In the general case \( AB \neq 1 \) this is usually computationally much less involved, especially if the total stress energy tensor, i.e. the sum of electromagnetic and matter contributions, is known and has a simple form.

\[ P_\theta = P_{\theta_m} + \frac{E^2}{8\pi} = P_r + \frac{rP'_r}{2} + \frac{rB'}{B} (\rho + P_r) \]  

(20)

Whenever \( \rho_m, P_{r_m} \) and the electromagnetic field \( E \) are known, the metric coefficients \( A \) and \( B \) and \( P_{\theta_m} \) can be determined by the following procedure:

- Integrate equation (18) to obtain the radial metric coefficient \( A \); in order to do this \( \rho_m \) and \( E^2 / (8\pi) \) must be known.
Determine $B$ by integrating equation (15); the integration requires knowledge of $\rho_m$ and $P_{rm}$ ($A$ has been obtained in the first step).

Determine the tangential pressure by equation (19) or (20); this requires knowledge of $E$, $\rho_m$, $P_{rm}$ (and of $A$ and $B$, which were obtained in the previous two steps).

On the other hand, if the metric is known, the matter-fields $\rho_m$, $P_{rm}$ and $P_{\theta m}$ can be determined by differentiation of the metric coefficients. This, however, requires a prior knowledge of the electromagnetic energy-density $E^2/(8\pi)$. We find:

\[
\rho = \rho_m + \frac{E^2}{8\pi} \left( 1 - \left( \frac{r}{A} \right)' \right) \tag{21}
\]

\[
P_r + \rho = P_{rm} + \rho_m = \frac{(\ln AB)'}{8\pi r A} \tag{22}
\]

\[
P_\theta = P_{\theta m} + \frac{E^2}{8\pi} = \text{19 or 20} \tag{23}
\]

Equations (21, 22, 23) carry a very important message: $\rho$, $P_r$ and $P_\theta$, which appear on the left side of the above equations, are the diagonal components of the total stress-energy tensor for a static charged spherically symmetric system, including the electromagnetic contribution. The values of $\rho$, $P_r$ and $P_\theta$ can be determined exclusively from the metric: First $\rho$ is determined via equation (21). Knowing $\rho$ one obtains $P_r$ from equation (22) and last $P_\theta$ via equation (23). In order to determine the total stress-energy tensor of a spherically symmetric, charged self gravitating object we therefore don’t need to know anything about its electromagnetic field. How the electromagnetic field contributes to the total mass-energy of the space-time, be it just a fraction, be it zero or large, is irrelevant. In order to determine the total stress-energy tensor only the metric coefficients $A$ and $B$ need be known. In order to determine the metric nothing else than the total stress-energy tensor is required. This property doesn’t come unexpected. It is exactly what is required from a universal theory of gravitation that treats all forms of mass-energy equal.

However, whenever we care to distinguish between the "matter contribution", and the "electromagnetic contribution", we must know the electro-magnetic energy density $E^2/(8\pi)$.

4 Integration of the field equations for a charged holostar

Whenever the sources of the gravitational field (matter and electromagnetic) are known, the integration of the field equations in order to obtain the metric is
straight forward. In this paper we are not interested in the general solution to the field equations of a spherically symmetric charged system with an arbitrary source-distribution, but rather in the charged extension of the holostar solution.

The holostar-solution is characterized by the property $AB = 1$ throughout the whole space-time. I will assume that this property remains valid for the charged case.\(^5\) Setting $AB = 1$ allows the following simplification:

\[
\frac{A'}{A} + \frac{B'}{B} = (\ln AB)' = 0 \quad (24)
\]

from which

\[
\rho + P_r = \rho_m + P_{\theta m} = 0 \quad (25)
\]

follows.

The field equations are reduced to the simple problem of solving the following two equations:

\[
(rB)' = 1 - 8\pi r^2 \left( \rho_m + \frac{Q^2}{8\pi r^4} \right) \quad (26)
\]

\[
8\pi \left( P_{\theta m} + \frac{Q^2}{8\pi r^4} \right) = \frac{B''}{2} + \frac{B}{r} \quad (27)
\]

They differ from the respective equations of the uncharged case (with $AB = 1$) only in that the energy-density of the electromagnetic field $E^2/(8\pi)$ is added to the mass-density $\rho_m$ and to the tangential pressure $P_{\theta m}$ of the "matter" fields.

We are interested in a solution, where the sources of the fields (electromagnetic and matter) are confined to a region $r \leq r_h$, i.e. a solution that is situated in an exterior spherically symmetric electro-vac space-time. The exterior solution therefore must be given by the well-known Reissner-Nordström solution:

\[
B_e(r) = \frac{1}{A_e(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \left( 1 - \frac{r_+}{r} \right) \left( 1 - \frac{r_-}{r} \right) \quad (28)
\]

with

\[
r_{\pm} = M \pm \sqrt{M^2 - Q^2} \quad (29)
\]

\(^5\)Note that $AB = 1$ is fulfilled everywhere in the (charged) Reissner-Nordström space-time, except at the central singularity.
\[ Q \] is the total charge of the spherically symmetric Reissner-Nordström solution, \( M \) its total gravitational mass.

The exterior electro-vac-solution has to be fitted to the interior solution in such a way, that the metric remains continuous at the boundary \( r_h \) of the interior matter/charge distribution.

We now have to construct an appropriate interior solution, for which the uncharged holostar-solution is to serve as a guideline. The (uncharged) holostar solution in [11] is singled out from all other possible interior solutions by the following properties:

- \( A(r < r_h) \propto r \)
  The radial interior metric coefficient \( A \) is proportional to \( r \). Only such an interior metric leads to the prediction, that in thermodynamic equilibrium the number of ultra-relativistic particles is proportional to the proper surface area of the object. i.e. the condition \( A \propto r \) ensures, that the interior solution is compatible with the holographic principle and the Hawking entropy and -temperature laws.

- \( \rho = 1/(8\pi r^2) \)
  Related to the above condition is the requirement, that the interior mass-energy density \( \rho \) is fixed to \( \rho = 1/(8\pi r^2) \). If one assumes that the Einstein field equations with zero cosmological constant are valid, this condition is equivalent to the first.

- \( \int T \sqrt{-g} dV = M \)
  The improper integral over the trace of the stress-energy tensor \( T \), taken over the whole space-like volume, is exactly equal to the gravitating mass \( M \) of the holostar.

- \( \int 2P_\theta \sqrt{-g} dV \approx M \)
  Alternatively or complementary to the above condition: The ”mass-energy content” of the membrane of the uncharged holostar, i.e. the improper integral over the two non-zero principal pressure components in the membrane, is (nearly) equal to its gravitating mass \( M \).

- \( P_\theta = 0 \)
  The interior tangential pressure \( P_\theta \) is exactly zero. In combination with \( \rho = -P_r \) this means, that the interior matter has a string equation of state.

It is not a priori clear, how these properties are to be generalized to the case of a charged (or a rotating) holostar. Yet it is quite obvious that the essential requirement responsible for the remarkable properties and self-consistency of
the holostar solution is, that the interior metric be subject to the following condition:

\[ \left( \frac{r}{A} \right)' = 1 - 8\pi r^2 \rho = 0 \]  \hspace{1cm} (30)

i.e. \((rB)' = 0\) in the case of \(AB = 1\).

Henceforth I assume that the condition \((r/A)' = 0\) holds for the interior of the charged holostar. If this is so, the interior metrics of the charged and uncharged holostar must be identical, except possibly for a different integration constant \(r_0\):

\[ A = \frac{r}{r_0} = \frac{1}{B} \]  \hspace{1cm} (31)

With the above condition the total interior mass-energy density of the charged holostar is uniquely determined:

\[ \rho = \rho_m + \frac{E^2}{8\pi} = \frac{1}{8\pi r^2} \]  \hspace{1cm} (32)

The only difference to the uncharged case is, that the interior mass density of the uncharged solution, \(\rho\), must be replaced by the total mass-energy density, which now consists of a matter-contribution and an electromagnetic contribution.

It is reasonable to assume, that the matter-contribution and the electromagnetic contribution to the interior mass-energy follow a inverse square law in \(r\), i.e. both are proportional to the total mass-energy density with the same constant factor throughout the whole interior. At least this is the simplest assumption compatible with the requirement that the total mass-energy density should scale with \(1/r^2\). We arrive at the following ansatz for the interior mass-density (matter-contribution):

\[ \rho_m = \frac{c}{8\pi r^2} (1 - \theta) \]  \hspace{1cm} (33)

\(\theta = \theta(r - r_h)\) is the Heavyside-step functional. To save space the argument of the \(\theta\)-functional will not be shown explicitly. Whenever a Heavyside \(\theta\)- or Dirac \(\delta\)-functional is referenced in this paper, its argument will always be given by \((r - r_h)\). \(c \leq 1\) is an arbitrary constant, not to be confused with the velocity of light.

With the additional assumption \(AB = 1\) we get \(\rho_m + P_m = 0\). This relation between mass-density and radial pressure is not only true for the matter-part,
but is also trivially fulfilled for the electromagnetic contribution\footnote{A spherically symmetric electric field always has $\rho = -P_r = P_\theta = P_\phi$.}, and therefore for the total mass-density and radial pressure.

With the above ansatz for the matter-contribution the energy-density of the electromagnetic field in the holostar’s interior must come out as:

$$\rho_{em} = \frac{E^2}{8\pi} = \frac{1 - c}{8\pi r^2}$$  \hspace{1cm} (34)

$c$ will be less than 1, because the electromagnetic energy contribution is always positive.

We now have to make an ansatz for the charge-distribution, i.e. for the sources of the electromagnetic field, which is consistent with equation (34). Principally there are two physically distinct choices: The charge could be associated with some of the holostar’s interior matter, or with the membrane (or a combination of both).

The first impulse would be to associate the charge entirely with the membrane. Such an association appears to be indicated by the membrane paradigm for a charged black hole \cite{15}. Furthermore, if the holostar’s charge could be attributed exclusively to its boundary, one could view this as another independent verification - or clarification - of the holographic principle.

If the total charge were assembled in the membrane there would be no interior electromagnetic field. The interior energy density will be "normal uncharged" matter. In this case the interior of a charged holostar would be exactly equal to the interior of an uncharged holostar.

Despite the apparent attractiveness of the above approach, in this paper I take the position that the charge of the holostar must be attributed to its interior particles and that the membrane is essentially uncharged. This appears as the most natural choice:

We know that charged particles exist. We don’t know of any fundamental physical principle that forbids charged particles to enter the holostar’s interior. Therefore a charge-free interior space-time, with all of the holostar’s charge being placed at the membrane, doesn’t appear physically acceptable.

If the charge is due to the interior particles, cannot yet the membrane carry a non-zero net-charge? In principle it could. However, two arguments stand against such a possibility:

According to our present knowledge charge is always associated with mass-energy. We don’t know of any charged particle with zero rest mass. If the membrane were charged, we should expect an appreciable mass-energy density
situated within the membrane.\footnote{This will be even then the case, if charged particles with zero rest-mass would exist. Such particles would have to move with the speed of light within the membrane, and therefore would carry energy. The lowest energy possible will be comparable to the energy of a photon with a wavelength equal to the proper circumference of the membrane. Therefore a membrane containing charged particles would have to contain mass-energy.} But the membrane of an uncharged holostar consists out of pure tangential pressure and the mass-energy density $\rho$ within the membrane is zero, at least in the uncharged case. There is evidence, that this property of the membrane should also be valid for the charged case:

If the membrane had a non-zero energy density, the metric wouldn’t be continuous. The metric coefficient $A$ is determined by the integral over the total mass-energy density $\rho$, which consists of a "matter term", $\rho_m$, and an electromagnetic term $E^2/(8\pi)$. If the membrane carries a (finite) net charge $\Delta Q$, the electromagnetic term to the total energy density $\rho$ will only contain a finite step-discontinuity.\footnote{If $Q$ is the total charge of the interior particles and $\Delta Q \neq 0$ the non-zero net-charge of the membrane, the electric field at the inner side of the membrane is given by $Q/r^2_h$, at its outside by $(Q + \Delta Q)/r^2_h$. The energy density of the electric field will jump at the membrane from $Q^2/(8\pi r^4_h)$ to $(Q + \Delta Q)^2/(8\pi r^4_h)$, which is finite.} The integral over a finite step-discontinuity is continuous. There is no problem with a discontinuity in the metric so far. The problem, however, lies in the matter-term: We have assumed, that charge is always associated with mass (or more generally with particles, which have a non-zero energy even if their rest-mass is zero). If this is true, any non-zero charge-density in the membrane, the electric field at the inner side of the membrane is given by $Q/r^2_h$, at its outside by $(Q + \Delta Q)/r^2_h$. The energy density of the electric field will jump at the membrane from $Q^2/(8\pi r^4_h)$ to $(Q + \Delta Q)^2/(8\pi r^4_h)$, which is finite.

The above argument is based on the assumption, that the two-dimensional membrane is "real", i.e. that it truly has no or negligible radial extension. If the membrane is spread out over a radial coordinate region of roughly Planck size, a non-zero net charge of the membrane might be acceptable. In fact, even for a purely two-dimensional membrane the relative change of the metric coefficients at the position of the membrane is very small, if the charged membrane consists of particles such as the electron or proton with small mass to charge ratio $m/e$ in natural units. (For an electron $m/e \approx 10^{-22}$). If we find a discontinuity $\Delta B/B \leq 1$ in the metric coefficients acceptable, we are led to the condition $Q < (r_0/2) (e/m) \sqrt{r_h/r_0}$. In natural units $r_0/2 \approx 1$. Because of the large value of $e/m$ the above condition is easily fulfilled for any "small" holostar with $r_h < 10^{10}r_0$. For large holostars we get a limit for the number of charged elementary particles ($Q = en_e$) in the membrane: $n_e < 1/\sqrt{2}(m_P/m) N^{3/2}$, with $N$ being the total number of particles within the holostar. Therefore the number of charged particles in the membrane grows only as the fourth root of the total number of particles, or as the square-root of the holostar’s gravitational radius or mass.

\footnote{The other argument is based on the small ratio of gravitational to electromagnetic force, at least for a large holostar. A significantly charged membrane
would produce a high electric field, which will expel identically charged particles from the membrane. This will almost instantly neutralize the membrane, unless leaving the membrane requires a lot of work. This work would have to be done against the gravitational field, whose gradient is quite weak for a large holostar, especially in the interior direction.\textsuperscript{10} It is quite improbable that the (gravitational) work required to leave the membrane would turn out greater than the (electromagnetic) energy that is released when the charged particles are expelled from the membrane exclusively into the direction of the exterior space-time.

A third argument might come from string theory. The membranes in string theory (D-branes) usually have non-zero pressure, but zero mass-energy density.\textsuperscript{11}

We therefore should associate the holostar’s charge with its interior particles. Charge is a relativistic invariant, therefore we are forced to associate the charge density with the number-density of the particles, and not with their mass-density.\textsuperscript{12} The most natural assumption is, that the ratio of charged particles to the total number of particles is constant throughout the whole interior.

The problem with the above assumption is, that the number densities of massive and massless particles are different in the holostar. A choice has to be made, whether to associate charge with massive or with massless particles. But this choice has already been made: So far as no massless charged particle is known to exist, we should associate the charge with the number-density of the massive particles, which scales as $n_m \propto 1/r^{5/2}$ if no particles are created or destroyed.\textsuperscript{13} We find:

\textsuperscript{10}At the inside of the membrane of an arbitrary holostar the proper gravitational acceleration $g_i$, measured by an observer momentarily at rest with respect to the membrane, is proportional to $1/r^{3/2}$, which is negligible for a large holostar. The tidal forces are almost unnoticeable. Although just outside of the membrane the proper gravitational acceleration $g_o$ is large compared to the inside acceleration ($g_o \propto 1/r^{1/2}$), the value of $g_o$ can become arbitrarily low for large holostars. The ratio of $g_o$ to $g_i$ at the membrane is given by $r_h/r_0 - 1$. Furthermore $g_o$ decreases very rapidly outside the membrane.

\textsuperscript{11}This has to do with the fact, that the strings are merely attached to the membrane, but there shouldn’t be any string-segments lying within the membrane. In string theory it is the string-segments which carry mass-energy, whereas the string end-points only “carry” pressure.

\textsuperscript{12}Contrary to charge, mass is not a relativistic invariant. Therefore the ratio of charge-to-mass-density depends on the interior motion of the particles, which is unsatisfactory.

\textsuperscript{13}This dependence is derived in [11], where it is shown that the volume of a geodesically moving spherical shell of massive particles evolves with $r^{5/2}$. Assuming a constant number of particles in the shell, the number-density evolves as the inverse volume. However, the negative radial pressure in the holostar space-time leads to particle creation/destruction, as the shell expands/contracts, so that in general the assumption of a constant particle number in the shell might not be valid. For charged particles, however, the situation is different. Due to charge-conservation only uncharged particles (or particle pairs) can be created by the pressure. Charge conservation in combination with charge-quantization then requires, that the difference of positively and negatively charged particles must remain constant in the shell. The charge density only depends on this difference, so that the net number-density of charged particles always evolves as $n_c \propto n_+ - n_- \propto 1/r^{5/2}$.\textsuperscript{14}
\[ \rho_c = \frac{\kappa}{8\pi r^2} \sqrt{\frac{r_0}{r}} (1 - \theta) \]  

(35)

The factors of proportionality were arranged such, that \( \kappa \) is a dimensionless constant. Its value will be determined later.

The energy density of the electromagnetic field can be derived from the charge density. In the spherically symmetric case the electromagnetic field strength \( E(r) \) can be calculated just as in the Newtonian case, via equations (5, 6).

Using equation (35) for the charge density, we arrive at the following result for the charge function \( Q(r) \):

\[ Q(r) = \frac{\kappa}{2} r (1 - \theta) + \frac{\kappa}{2} r_h \theta \]  

(36)

The total charge \( Q(r) \) grows linearly with \( r \) inside the holostar (quite similar to the radial metric coefficient), and remains constant outside.

From (36) the electromagnetic field, and thus the field’s energy density follows:

\[ \frac{E^2(r)}{8\pi} = \frac{\left( \frac{\kappa}{2} \right)^2}{8\pi r^2} (1 - \theta) + \frac{\left( \frac{\kappa}{2} r_h \right)^2}{8\pi r^2} \theta \]  

(37)

As required, the electromagnetic contribution to the interior mass-energy density follows an inverse square law and is strictly proportional to the matter contribution. This result is non-trivial. If we had naively set the charge-density proportional to the mass-density, we would have found a \( 1/r \)-dependence of the interior electromagnetic energy density, due to the factor \( \sqrt{A} \) in the proper volume element \( dV \). Only by setting the interior charge-density proportional to the number-density of the massive particles, and by determining the charge \( Q(r) \) by integrating over the proper volume element, and not the improper volume element \( \tilde{dV} = 4\pi r^2 dr \), could this result be achieved.

The total charge of the holostar, which can be measured by an asymptotic observer in the exterior space-time by means of a Gaussian flux integral, is given by:

\[ Q = Q(r_h) = \frac{\kappa}{2} r_h \]  

(38)

This allows us to express \( \kappa \) in terms of the total charge \( Q \) and the position of the membrane \( r_h \):

\[ \frac{\kappa}{2} = \frac{Q}{r_h} \]  

(39)
Therefore the electromagnetic and "matter" contributions to the interior energy density are given in terms of the external parameters $Q$ and $r_h$ as follows:

$$\rho_{em}(r \leq r_h) = \frac{Q^2}{r_h^2} \frac{1}{8\pi r^2}$$

(40)

and

$$\rho_m(r \leq r_h) = \left(1 - \frac{Q^2}{r_h^2}\right) \frac{1}{8\pi r^2}$$

(41)

The respective values of the radial pressure are just the negative of the respective energy densities, i.e. $P_{rm} = -\rho_m$ and $P_{rem} = -\rho_{em}$.

The metric must be continuous at the boundary, $r_h$. Therefore the exterior Reissner Nordström metric must match the interior metric $B = 1/A = r_0/r$ at the position of the membrane.

This condition can be expressed as follows:

$$\frac{r_0}{r_h} = 1 - \frac{2M}{r_h} + \frac{Q^2}{r_h^2}$$

(42)

which can be solved for $r_h$:

$$r_{h\pm} = M + \frac{r_0}{2} \pm \sqrt{(M + \frac{r_0}{2})^2 - Q^2}$$

(43)

Equation (43) only has a real valued solution for

$$|Q| \leq M + \frac{r_0}{2}$$

(44)

For all practical purposes $r_{h-}$ in equation (43) appears to be irrelevant:

Whenever $|Q| \leq M$ we are forced to take $r_{h+}$: If we would match the interior and exterior metric at $r_{h-}$, the exterior metric will undergo a sign change, i.e. $B$ will necessarily become negative in some space-time region outside the holostar.\textsuperscript{14} This is not desirable. We would like to construct a charged solution without event horizon.

If interior and exterior metric are matched at $r_{h+}$, the membrane is the global minimum of the metric coefficient $B$, whenever $|Q| \leq M$. As $B(r_h) = r_0/r_h$

\textsuperscript{14}It is easy to see, that for $|Q| \leq M$ the (global) minimum of the exterior metric $B(r_{min}) = 1 - M^2/Q^2$ is negative. Furthermore it can be shown with a little bit of algebra, that $r_{h-} < r_{min}$, whenever $|Q| \leq M$. Therefore if interior and exterior metrics are matched at $r_{h-}$, the exterior metric will necessarily go through its minimum, which is negative whenever $|Q| < M$.  

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is always positive, $B$ will never undergo a sign change when the exterior and interior metric are matched at $r_{h+}$, and $|Q| \leq M$.

In contrast to the Reissner-Nordström solution, however, $|Q|$ can be larger than $M$, albeit just by a small Planck sized amount. Whenever the mass is in the range $M < |Q| \leq M + r_0/2$ it is possible to take either $r_{h+}$ or $r_{h-}$. There is no problem with a sign-change in $B$, because interior and exterior metric are always positive for $|Q| > M$ on the whole positive $r$-axis.\(^{15}\)

Having two solutions is not desirable. In order to get a unique solution in the range $M < |Q| \leq M + r_0/2$, the root in equation (43) should be set to 0. Doing this we get $r_h = |Q| = M + r_0/2$, which corresponds the most extremely charged holostar possible for a given mass $M$. The choice $|Q| = M + r_0/2$ is attractive for another reason: The interior and exterior metrics match smoothly not only with respect to the metric coefficients, but also with respect to their first derivatives.

For an extremely charged holostar with $r_h = |Q| = M + r_0/2$ the membrane is not positioned at the global minimum of $B$. The minimum of $B$ lies outside the membrane, at $r_{min} = Q^2/M = r_h(1 + r_0/(2M))$. For a large holostar the position of the minimum is almost identical with the position of the membrane. However, whenever $M \ll r_0$, such as for an electron, $r_{min}$ can lie several orders of magnitude outside the membrane. Note also, that for an extremely charged holostar the membrane is situated at $r_h = |Q|$, which is roughly one tenth of the planck length, if the charge $Q$ is set equal to the electron charge $e$\(^{16}\).

If we know $r_0$, $M$ and $Q$, all quantities of the charged holostar-solution, interior or exterior, are determined. The gravitational mass $M$ and the charge $Q$ of the charged holostar can be measured by an observer in the exterior spacetime. The only unknown quantity is $r_0$, which is expected to be very small, roughly equal to the Planck-length.

## 5 Some properties of the charged holostar solution

In this section some properties of the charged holostar solution are compiled for further reference.

The expressions for a charged holostar are very similar to the respective expressions of a RN black hole, if we replace the gravitational mass $M$ with $M + r_0/2$. Let us therefore define the geometric mass

\[
M_g = M + \frac{r_0}{2} \tag{45}
\]

---

\(^{15}\)The global minimum of the exterior metric is positive for $|Q| > M$. The interior metric is always positive (assuming $r_0 > 0$).

\(^{16}\)\(e^2/h = \frac{e}{\alpha} \rightarrow e = \sqrt{\alpha \sqrt{h}} = \sqrt{\alpha r_{Pl}} \approx 0.085 r_{Pl}\).
and express all relations in terms of the geometric mass, whenever appropriate.

5.1 Metric

The metric of the charged holostar solution can be expressed in the following compact form:

\[ B = \frac{1}{A} = \frac{r_0}{r}(1 - \theta) + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\theta \]  

(46)

with

\[ r_h = M_g + \sqrt{M_g^2 - Q^2} \]  

(47)

With the above expression for \( r_h \), the position of the membrane will always lie between the radius defined by the charge and the radius defined by the mass of the holostar:

\[ |Q| \leq r_h \leq 2M_g \]  

(48)

5.2 Energy-density and radial pressure

The total mass-density and radial pressure of the charged holostar solution are given by:

\[ \rho = \frac{1}{8\pi r^2}(1 - \theta) + \frac{Q^2}{8\pi r^4}\theta \]  

(49)

The total radial pressure is opposite to the mass-density:

\[ P_r = -\rho = -\frac{1}{8\pi r^2}(1 - \theta) - \frac{Q^2}{8\pi r^4}\theta \]  

(50)

Outside of the holostar the total energy-density and radial pressure are solely due to the electromagnetic field. The interior mass-density, which is equal to \( 1/(8\pi r^2) \), splits into a "electromagnetic" and a "matter" part. The same is true for the interior radial pressure. The matter part carries a fraction \( c \) of the total interior mass-density, \( \rho_m = c\rho \), with \( c \) given by:

\[ c = 1 - \frac{Q^2}{r_h^2} = 2\left(1 - \frac{M_g}{r_h}\right) = \frac{2}{r_h\sqrt{M_g^2 - Q^2}} \]  

(51)
The other fraction, $Q^2/\pi r_h^2$, is carried by the electromagnetic field.

For $Q^2 = r_h^2 = M_\delta^2$, i.e. for an extremely charged holostar, the total mass-density and the total radial pressure are continuous at the position $r_h$ of the membrane.

### 5.3 Tangential pressure

The tangential pressure can be determined via equations (19, 20). The easiest way is to determine the total tangential pressure via equation (20), using the total energy-density and radial pressure given in equations (49, 50). We find:

$$P_\theta = \frac{c}{16\pi r_h} \delta + \frac{Q^2}{8\pi r^4} \theta$$

with $c$ given by equation (51).

Similar to the uncharged holostar, the total interior tangential pressure $P_\theta(r < r_h)$ is zero. There is a $\delta$-function of tangential pressure at the membrane, which is non-zero, except for an extremely charged holostar with $r_h = |Q| = M + r_0/2$.\(^\text{17}\) The tangential pressure outside of the holostar is solely due to the exterior electromagnetic field.

Although the total interior tangential pressure is zero, the respective electromagnetic and matter parts are generally non-zero. They exactly cancel each other. The electromagnetic part of the interior tangential pressure is positive and given by:

$$P_{\theta \text{em}}(r < r_h) = \frac{1 - c}{8\pi r^2}$$

The ”matter contribution” to the interior tangential pressure is always negative and given by:

$$P_{\theta \text{m}}(r < r_h) = -\frac{1 - c}{8\pi r^2}$$

For the special value $c = 1/2$, i.e. when the interior energy-density is distributed equally between the electromagnetic field and the remaining ”matter field”, the matter contribution to the interior stress-energy tensor is a genuine vacuum stress-energy tensor, with $\rho = -P_r = -P_\theta = -P_\phi$.

For $c = 1/2$ the position of the membrane is given by:

\(^{17}\)That the membrane vanishes for an extremely charged holostar could already have been deduced from the fact, that the total radial pressure (and energy-density) are continuous at the boundary of the source distribution, $r_h$, for an extremely charged holostar.
The value \( c = 2/3 \) is interesting as well. It is known that for \( Q^2 \geq 3/4M^2 \) the heat capacity of a Reissner-Nordström black hole becomes positive. For the charged holostar this happens whenever \( c \leq 2/3 \), i.e. when the electromagnetic contribution becomes larger than 1/3 of the total (local) interior energy density. In this case we have \( r_h = 3/2M = \sqrt{3} |Q| \).

### 5.4 Integrated energy-densities

For the uncharged holostar solution the ”stress-energy” content of the membrane was (almost) equal to the gravitating mass \( M \) of the holostar, whereas the integral over the trace of the stress-energy tensor was exactly equal to the gravitating mass. We would like to find out, whether the charged case differs from the uncharged case in this respect.

The integrated energy-densities obtained in this section are ”normalized” to the position of the asymptotic exterior observer at spatial infinity. An asymptotic observer will perform the usual integral over the proper spatial volume element, which in spherical coordinates is given by \( dV = 4\pi r^2 \sqrt{A} dr \). However he will correct the (local) energy \( dE = \rho dV \) of any thin spherical shell by the gravitational redshift factor of the shell with respect to his position, i.e. by \( \sqrt{B} \). For the charged as well as for the uncharged holostar we have \( AB = 1 \) throughout the whole space-time. Thus the red-shift corrected proper integral over the energy-density is simply given by the (improper) integral over the flat spherical volume element \( 4\pi r^2 dr \).

The integral over the two tangential pressure components is given by:

\[
\int_{r_h - \epsilon}^{r_h + \epsilon} 2P_\theta 4\pi r^2 dr = \int_{r_h - \epsilon}^{r_h + \epsilon} \frac{1 - Q^2} {8\pi r_h} \delta(r - r_h) 4\pi r^2 dr + \int_{r_h}^{\infty} \frac{Q^2} {4\pi r^4} 4\pi r^2 dr = \frac{r_h}{2} \left( 1 - \frac{Q^2}{r_h^2} \right) + \frac{Q^2}{2r_h} \quad \text{(55)}
\]

The integral consists of a contribution from the membrane and a contribution from the exterior electromagnetic field. The holostar’s interior doesn’t contribute, because the total tangential pressure in the interior of the charged holostar is zero.

\( ^{18}E = \int \sqrt{B} dE = \int \rho \sqrt{AB} 4\pi r^2 dr = \int \rho 4\pi r^2 dr \), whenever \( AB = 1 \).
The result of equation (55) is in some respect similar to the uncharged case: For the uncharged holostar the integral over the two tangential pressure components (which are non-zero only in the membrane) also gives \( \frac{r_h}{2} \).

For an uncharged holostar \( \frac{r_h}{2} \) is nearly equal to its gravitational mass, i.e. \( \frac{r_h}{2} = M(1 + \frac{r_0}{r_h}) \). Contrary to the uncharged holostar solution neither the integral over the membrane alone, nor the integral over the whole space-time, yields a quantity that is equal or proportional to the gravitating mass \( M \) of the charged holostar. In general for a charged holostar \( \frac{r_h}{2} \neq M \).

This is not overly disconcerting. Neither the energy-density nor the principal pressures of a space-time are Lorentz-invariant quantities. Their individual values depend on the coordinate system. But the trace of the stress-energy tensor is coordinate-independent. We therefore should be rather interested in how the integral over the trace of the stress-energy tensor comes out for the charged holostar. We find:

\[
T = \rho - P_r - 2P_\theta = \frac{1}{4\pi r^2}(1 - \theta) - \frac{1 - \frac{Q^2}{r_h^2}}{8\pi r_h} \delta
\]

Note that the trace over the stress-energy tensor gives a quite accurate account on the location and "strength" of the sources of the gravitational field. Outside of the holostar, where there are no sources, \( T \) is zero. Inside the holostar \( T \) is proportional to the energy-density of the fields associated with the interior sources.

Integrating \( T \) defined in equation (56) over the whole space-time gives the following result:

\[
\int_0^\infty T 4\pi r^2 dr = r_h \left( \frac{r_h}{2} - \frac{1}{1 - \theta} \right) = M + \frac{r_0}{2} \tag{57}
\]

The above integral doesn't include the negative point mass \( M_0 = -\frac{r_0}{2} \) at the center of the holostar. If we include the negative point mass, or - which is the preferred procedure (see for example [11]) - if we start the integration not at the unphysical region \( r = 0 \), but rather at \( r = \frac{r_0}{2} \), the integral over \( T \), carried out over the whole physically meaningful space-time, is exactly equal to the gravitating mass \( M \) of the charged holostar:

\[
\int T\sqrt{AB} 4\pi r^2 dr = \int T\sqrt{-g} dV = M \tag{58}
\]

The total electromagnetic energy of the space-time, \( E_{em} \), as evaluated by an asymptotic observer at spatial infinity, is given by the improper integral over the electromagnetic energy density \( E^2/(8\pi) \). We find:
\[ E_{em} = \int_{0}^{\infty} \frac{E^2}{8\pi} 4\pi r^2 \, dr = \int_{0}^{r_h} \frac{Q^2}{2r_h^2} \, dr + \int_{r_h}^{\infty} \frac{Q^2}{2r^2} \, dr = \frac{Q^2}{r_h} \quad (59) \]

The electromagnetic energy splits into two terms, an interior part and an exterior part, which are given by the above two integrals. Both integrals are exactly equal. Therefore the electromagnetic energy is distributed equally over the interior and the exterior space-time. This is different from the "matter" part of the energy, which is exclusively situated in the interior space-time.

5.5 How are the external parameters \( M \) and \( Q \) related to the internal energy distribution?

In this section we will analyze how the interior energy distribution, i.e. the constant local ratio of electromagnetic energy density, \( \frac{E^2}{(8\pi)} \), to the energy density of the (remaining) matter, \( \rho_m = \frac{c}{(8\pi r^2)} \), relates to the exterior parameters \( M \) and \( Q \).

For the following discussion it is convenient to define the (modified) exterior mass to charge ratio:

\[ \xi = \frac{M_g}{Q} = \frac{M}{Q} \left( 1 + \frac{r_0}{2M} \right) \in [1, \infty) \quad (60) \]

This ratio is always greater than 1. For an extreme holostar \( (Q = M_g) \) the ratio is unity, whereas for an uncharged holostar \( \xi \to \infty \).

The following abbreviations are useful:

\[ \kappa(\xi) = \left( 1 \pm \sqrt{1 - \frac{1}{\xi^2}} \right) \in [1, 2] \quad (61) \]

\[ \lambda\left( \frac{r_0}{M} \right) = \left( 1 + \frac{r_0}{2M} \right) \in (1, \infty) \quad (62) \]

\( \kappa \) is a quantity that more or less characterizes how heavily charged the holostar is. For an extremely charged holostar we find \( \kappa = 1 \), whereas an uncharged holostar is characterized by \( \kappa = 2 \).

\( \lambda \) is a measure how "heavy" the holostar is with respect to the Planck-mass. Under the assumption that \( r_0 \approx r_{Pl} \), a large heavy holostar with \( M \gg r_0 \) has \( \lambda \simeq 1 \), whereas a light holostar with \( M \ll r_0 \) has very high \( \lambda \).

With \( \lambda \) the modified mass-to-charge ratio \( \xi \) can be expressed in terms of the actual mass-to-charge ratio which can be measured by the asymptotic observer.
\[ \xi = \frac{M}{Q} \lambda \]  

For a large holostar \((\lambda \simeq 1)\) the modified mass-to-charge ratio is nearly equal to the actual ratio.

With the above defined quantities the radial position of the membrane, \(r_h\), can be expressed as follows:

\[ r_h = M \kappa \lambda = Q \xi \kappa \]  

For an extreme holostar \((\xi = \kappa = 1)\) one gets \(r_h = Q\): The membrane is situated exactly at the radius defined by the charge (in natural units). Whenever the extreme holostar is large \((\lambda \simeq 1)\) we find \(M \simeq Q\). This result is quite similar to the relation \(M = Q = r_+\) for the classical extreme Reissner-Nordström solution. \((r_+ \text{ is the position of the outer horizon of the Reissner-Nordström solution}).

For an uncharged or nearly uncharged holostar \((\kappa \to 2)\) we find \(r_h \approx 2M\), analogous to the relation \((r_+ = 2M)\) for the Schwarzschild solution.

The coefficient \(c\), which determines the relative contributions of matter and electric field to the interior energy distribution, can be expressed in terms of \(\xi\) as well. We find:

\[ c = 2 \frac{1 - \xi}{(\xi \pm \sqrt{\xi^2 - 1})} = 2 \frac{1 - \sqrt{\kappa(2 - \kappa)}}{\kappa} \]  

As could have been expected, for an extremely charged holostar \((\xi \to 1 \text{ or } \kappa \to 2)\) the matter-contribution to the interior energy density becomes arbitrary small with respect to the total energy density. An extremely charged holostar therefore consists of pure electromagnetic energy.

The total electromagnetic energy is given by:

\[ E_{em} = \frac{Q^2}{r_h} = \frac{|Q|}{\xi \kappa} \]  

For an extreme holostar \((\xi = \kappa = 1)\) the total electromagnetic energy is equal to the absolute value of the charge \(|Q|\).

With the above definitions the interior ratio of the "matter" part of the energy density \(\rho_m\) to the electromagnetic contribution to the energy density \(E^2/(8\pi)\) is given by:

\[ x_i = \frac{\rho_m}{\rho_{em}} = \frac{c}{1 - c} = \frac{M^2}{Q^2} 2\lambda^2 \kappa (\kappa - 1) \]  

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For a weakly charged holostar $\kappa \to 2$. In this case the above formula is simplified:

$$x_i \simeq 4\xi^2 = \left(\frac{2M}{Q}\right)^2 \lambda^2 = \left(\frac{2M}{Q}\right)^2 \left(1 + \frac{r_0}{2M}\right)^2$$  \hspace{1cm} (68)

If the holostar is large, i.e. $\lambda \simeq 1$, $x_i$ is roughly given by:

$$x_i \simeq \left(\frac{2M}{Q}\right)^2$$  \hspace{1cm} (69)

The interior ratio of matter energy to electromagnetic energy is proportional to the square of the exterior ratio of gravitating mass to charge.

It is quite remarkable, that the quantities on the left hand side of equation (69), which describe the local distribution of energy in the holostar’s interior, go in linearly, whereas the quantities on the right hand side, which describe the global distribution of mass and charge in the exterior space-time, go in squared.$^{19}$

For an extremely charged holostar ($\kappa = 1$) the ratio $x_i$ goes to zero. This is as expected. The higher the charge, the more the internal energy density will be dominated by the charge, i.e. by electromagnetic energy, and not by the remaining matter. For an extremely charged holostar the interior energy-density is entirely electromagnetic.

So far the interior ratio of the energy-densities were expressed in terms of the dimensionless ratio $Q/M$, or rather $Q/M_g$. However, general relativity is a geometric theory. The mass $M$ is not a genuine geometric, but rather a derived quantity.$^{20}$ Yet for all black hole solutions there is a one-to-one correspondence between the gravitational mass $M$ and the horizon area $A$, which is related to the entropy by the Hawking formula $S = A/(4\hbar)$. Therefore $M$ and $A$ are interchangeable. A similar result holds for the holostar solution. From a geometric point of view it is more natural to interpret $A$ as the fundamental variable. In this respect it is quite remarkable, that the ratio of electro-magnetic to total energy density can be expressed in a very simple way in terms of the dimensionless ratio $Q^2/A$:

$^{19}$This might be interpreted as another hint, that for the quantities measurable by an exterior observer, such as $M$ and $Q$, not the quantities themselves, but rather their squares are fundamental. In geometric units $c = G = 1$ both $M$ and $Q$ have dimensions of length (or mass/energy). Their squares have dimension of area. Area has the same dimension as angular momentum in geometric units, which is quantized in units of $\hbar$. In quantum gravity area is quantized in terms of the spin variables of the $SU(2)$ connection.

$^{20}$Note also, that the interior quantities, i.e. the energy-densities, have dimensions of inverse area in natural units $c = G = 1$. Equation (69) relates the interior energy-densities (units: $1/\hbar$) to the squares of the exterior quantities (units: $\hbar$), which hints at some sort of duality correspondence between these quantities.

In the context of the holostar solution this can be seen quite clearly in the appearance of $M_g = M + r_0/2$ instead of $M$ in various equations.
\[ \frac{\rho_{em}}{\rho} = \frac{Q^2}{r_h^2} = \frac{4\pi Q^2}{A} \]  

(70)

with \( A = 4\pi r_h^2 \). For an extremely charged holostar it is easy to see that \( Q = M_g \) so that \( r_h = M_g = Q \) and therefore \( A = 4\pi Q^2 \). Thus for an extremely charged holostar the interior electro-magnetic energy-density is identical to the total energy-density, meaning that the interior energy-density is consists exclusively out of electro-magnetic energy.

Clearly it is possible to relate \( \rho_{em}/\rho \) to the dimensionless ratio \( Q/M \), or rather to the modified charge to mass ratio \( Q/M_g = 1/\xi \). Using equation (64) for \( r_h \) the above relation (70) can be transformed to:

\[ \frac{\rho_{em}}{\rho} = \frac{1}{\xi^2 \kappa^2(\xi)} = \frac{1}{\kappa^2(\xi)} \left( \frac{Q}{M_g} \right)^2 \]  

(71)

The factor \( 1/\kappa^2(\xi) \) is only slightly dependent on the modified charge to mass ratio. It increases monotonically with \( \xi = Q/M_g \) and varies between 1/4 (for an uncharged holostar) and 1 (for an extremely charged holostar). For a moderately charged holostar with \( Q/M_g \ll 1 \) we have:

\[ \frac{\rho_{em}}{\rho} \rightarrow \frac{1}{4} \left( \frac{Q}{M_g} \right)^2 \ll 1 \]  

(72)

For an extremely charged holostar:

\[ \frac{\rho_{em}}{\rho} \rightarrow \left( \frac{Q}{M_g} \right)^2 \rightarrow 1 \]  

(73)

5.6 Extremely charged holostars

In this section I will briefly discuss the characteristic properties of an extremely charged holostar. We find the following relations:

\[ r_h = Q = M + \frac{r_0}{2} = M_g \]  

(74)

\[ r_0 = 2(Q - M) \]  

(75)

For an extremely charged holostar with \( M \ll Q \), a condition which is very well fulfilled by all known elementary particles, we find \( r_0 \approx 2Q \approx 2r_h \). In this particular case \( r_0 \) lies outside the membrane.
For an extreme holostar the membrane is not the global minimum of the
time coefficient of the metric $B$. The minimum lies at $r_{\text{min}}$, which is given by:

$$r_{\text{min}} = r_h \lambda = Q \frac{M_g}{M}$$  \hspace{1cm} (76)

Whenever $\lambda = M_g/M$ is large, the minimum will lie very far outside of the
membrane.

The values of $B$ at the minimum and at the membrane are given by:

$$B(r_{\text{min}}) = 1 - \frac{1}{\lambda^2} = 1 - \frac{M^2}{M_g^2}$$  \hspace{1cm} (77)

$$B(r_h) = 2(1 - \frac{1}{\lambda}) = 2(1 - \frac{M}{M_g})$$  \hspace{1cm} (78)

For a large holostar, with $\lambda \approx 1$ the time coefficient of the metric $B$ is almost
zero at the position of the membrane. The same applies to the position of the
minimum, which is very close to the membrane. For a small holostar of Planck
mass or less, i.e. with $\lambda$ large, the situation is quite different: $B(r_{\text{min}}) \simeq 1$ and
$B(r_h) \simeq 2$.

The value $\lambda = 2$ is special. In this case $r_h = r_0 = Q$. The gravitational mass
is half the charge, i.e. $M = Q/2$. The minimum of $B$ lies outside the membrane
at $r_{\text{min}} = 2Q$ with $B(r_{\text{min}}) = 3/4$. The value of $B$ at the membrane is unity,
i.e $B(r_h) = 1$.

6 Some remarks about the cosmic censorship
hypothesis

The condition $|Q| \leq M_g = M + r_0/2$ in equation \[13\] is very similar to the
condition $M \geq |Q|$ for the Reissner-Nordström (RN) solution. The requirement,
that the charge of a self gravitating body shall never exceed its gravitational
mass, is postulated by the cosmic censorship hypothesis: Whenever $Q > M$ the
RN-solution exhibits a naked singularity.

The only difference between the holostar-condition and the condition derived
from the cosmic-censorship hypothesis in a RN-spacetime is, that the Planck-
sized quantity $r_0/2$ is added to the gravitational mass $M$ in the "holostar ver-
sion" of this condition.
The holostar condition $|Q| \leq M_g$ can be traced to a somewhat different origin. The holostar has no naked singularity. There is no necessity to invoke the cosmic censorship hypothesis.

Yet the holostar solution has an interior metric coefficient $B$, which falls off with $1/r$ and becomes nearly zero at the surface of a large holostar: $B_{\text{min}} = r_0/r_h$. If the exterior vacuum metric is to match the interior (non-vacuum) metric, the exterior metric must approach zero at some coordinate value $r$ in the exterior space-time. Therefore, for a large holostar the exterior and interior metrics will be matched at a position very close the event horizon of the exterior electro-vacuum metric. It is obvious, that whenever the exterior vacuum metric has no event horizon, it will be difficult, if not impossible, to match the interior and exterior metrics.

Whereas the cosmic censorship hypothesis postulates an event horizon of the (global) vacuum space-time in order to conceal any interior singularity, the holostar requires an (almost) event horizon in the exterior vacuum space-time, in order to match the interior and exterior metrics without unbelievable fine-tuning.

7 A rough lower bound for the ”fundamental length” $r_0$

So far I have merely assumed that $r_0$ is a quantity of roughly Planck-size. This is a reasonable assumption. The Planck-length is the only universal quantity with dimension of length that can be constructed from first principles, and that at the same time is independent from the internal workings of any particular field-theory (except possibly gravity itself). In [11], $r_0$ has been determined to

21 In the purely classical treatment the holostar-solution formally has a negative point-mass singularity of roughly a Planck-mass at its center. However, this formal ”singularity” should be regarded as artifact of the classical description, which is expected to break down at the Planck-scale, anyway. The ”singularity” is completely contained to a region of Planck area/volume. Neither in the charged, nor in the uncharged case does this formal negative mass ”singularity” have any effect outside of the Planck-region: The gravitational mass integrated over a nearly Planck-volume defined by the fundamental length scale $r_0$ (or $r_0/2$) is exactly zero, if the formal negative point mass at the center is included in the integration. According to the findings of loop quantum gravity (LQG), the geometry must be considered to be discrete at the Planck-scale. It makes no sense to probe a volume bounded by an area smaller than the smallest area-eigenvalue of LQG. Therefore, taking the results of LQG seriously, the holostar solution contains no singularity at all. The effect of the negative point mass only becomes noticeable, when we probe distances smaller than the Planck-scale, or more accurately, when we probe into a space-time region bounded by an area smaller than the smallest area-eigenvalue of LQG. This, however, makes no sense.

22 In principle the time coefficient of the exterior vacuum metric could come very close to zero without ever reaching zero. If this is so, a match between exterior vacuum and interior metric might be possible. However, for a holostar of the size of the sun this would require that the exterior vacuum metric $B_e$ should approach zero to the remarkable accuracy of $B_e \approx 10^{-40}$, without actually hitting zero. This is quite improbable.
be roughly twice the Planck length from cosmological data, i.e. \( r_0 \approx 1.88 r_{Pl} \).\(^{23}\)

In \[^{10}\] further arguments were given, why \( r_0 \), or rather \( r_0^2 \), should be regarded as a universal quantity, only moderately dependent on the energy-scale. The arguments given in the above mentioned two citations, however, refer to large holostars containing a macroscopic number of interior particles. It is therefore worthwhile to find out whether the assumption of a fundamental length scale \( r_0 \) roughly equal to the Planck length is compatible with what is known about the microscopic world.

Let us take the position, that \( r_0 \) is universal, i.e. the same - or very nearly the same - quantity regardless of the size of a self-gravitating object.

Equation (44) can be expressed as a condition restraining the possible values for \( r_0 \):

\[
 r_0 \geq 2(Q - M) \tag{79}
\]

Assuming that this relation must hold universally, if we apply this relation to microscopic particles we find, that whenever the mass of a particle is small with respect to its charge, the above equation will serve as a lower bound on \( r_0 \). All of the known charged fundamental particles have an extremely small mass (expressed in natural units) with respect to their charge. Therefore they are quite ideal candidates in order to determine a lower bound for \( r_0 \). If we plug in the mass of the electron \( m_e \approx 10^{-23} r_{Pl} \) for \( M \) and the electron charge \( e = \sqrt{\alpha} \sqrt{\hbar} \approx 0.1 r_{Pl} \) for \( Q \), the mass of the electron can be utterly neglected with respect to its charge, because \( m_e \ll e \) in natural units. We get

\[
 r_0 \geq 2\sqrt{\alpha} r_{Pl} \approx 0.17 r_{Pl} \approx\tag{80}
\]

in natural units (with \( r_{Pl} = \sqrt{\hbar} \)). \( \alpha \) is the fine-structure constant.\(^{24}\)

If we take a muon, a tau or a W-boson (even a proton or any other charged, but uncolored particle), the result is very much the same. The masses of all of these particles are extremely small with respect to their charge, which is always equal to (or a multiple of) the charge of the electron. Note however, that all known charged particles have non-zero spin. Therefore the spherically symmetric charged holostar solution doesn’t apply to those particles. In the following section I will discuss some properties of a possible extension of the charged holostar solution to the spinning case, which is expected to give a better estimate of \( r_0 \).

\(^{23}\)\( r_0 \) was derived from the ratio of the average matter-density of the universe to the fourth power of the microwave-background temperature.

\(^{24}\)Note, that \( \alpha \) depends on the energy-scale, which hints that \( r_0 \), although postulated to be universal in the sense that it should not depend on the size of a self-gravitating object, might yet depend moderately on the energy-scale via the running value of \( \alpha \).
8 The electron as a charged, rotating holostar?

The electron has a charge which is roughly one tenth of the Planck-mass in natural units, whereas its gravitating mass is roughly $4 \times 10^{-23}$ of the Planck mass.\(^{25}\)

The spherically symmetric charged holostar allows solutions with large charge, but negligible mass, in natural units. Whenever $r_0/r_{Pl} < 2\sqrt{\alpha} \simeq 0.17$ one can find a solution with $Q = e$ and with arbitrarily small mass. In fact, the mass could be identical zero. Such a solution, however, has zero angular momentum.

No fundamental charged particle with spin 0 has been found so far. This leads us to the assumption, that $r_0$ should be larger than the bound in equation 80, i.e. larger than $0.17 r_{Pl}$.\(^{26}\) In fact, if the running of the coupling constants is taken into account, one gets $r_0 > 0.4 r_{Pl}$ for $\alpha \approx 1/25$ at the energy where the three coupling constants of the Supersymmetric Standard Model are unified.\(^{17, 12, 3}\) If the holostar solution is to describe charged elementary particles, angular momentum will have to be included. As long as no solutions for a spinning holostar are available, one must resort to approximate reasoning. It is well known, that charged black holes have similar properties as rotating black holes. By comparing the known formula for the Kerr-Newman and Reissner-Nordström solutions with the formula for the charged holostar, we are led to extend equation (47) in the following way:

$$r_h = M_g + \sqrt{M_g^2 - Q^2 - \frac{J(J + 1)}{M_g^2}}$$

(81)

I.e. the gravitating mass $M$ in the classical black hole formula should be replaced by the ”geometric mass” $M \rightarrow M_g = M + r_0/2$ and the angular momentum $J \rightarrow J(J + 1)$.

In the classical limit, i.e. for $M \gg m_{Pl}$, the above formula approximates the well known formula for the Kerr-Newman solution

$$r_+ = M + \sqrt{M^2 - Q^2 - \frac{J^2}{M^2}}$$

(82)

25Note that the electron charge $e$ could have been used to define a system of units similar to the Planck units. In ”charge” units, the natural length (or mass) scale is given by: $r_c = \sqrt{\alpha} r_{Pl} \simeq 0.085 r_{Pl}$, which is roughly a factor of 11 smaller than the Planck length. However, in contrast to Planck’s constant the effective charge depends on the energy scale.

26There are other arguments which create some doubt whether a fundamental spin-0 particle truly exists: According to the area-formula of quantum gravity a spin-network state with a spin-0 link has zero area, and therefore zero entropy. There is some evidence that the particles in the holostar can be identified with the links of a large quantum gravity spin-network (see section 9). A particle with zero area and entropy, however, is difficult to accept. Is this the death for the spin-0 s-electron, or even the downfall of supersymmetry? Presumably not. Charge has a similar effect as angular momentum in general relativity. Maybe loop quantum gravity must be extended to incorporate charge into the area formula, endowing a charged spin-0 particle with a non-zero area.
whereas in the limit \( J = 0 \) we find the formula \( 47 \) for the charged non-rotating holostar.

What are the conditions under which the holostar solution could be an acceptable description for an elementary particle? Elementary particles, such as the electron or neutrino, are completely described by few parameters, such as their charge(s), their angular momentum and mass. The same is true for black holes and for the holostar solution. Furthermore all elementary particles of the same kind are indistinguishable, very light (in natural units), and the lightest particles of a certain class are stable.\( ^{27} \) If we identify the holostar with a "particle" (be it fundamental or composed), equation \( 81 \) indicates that several "particles" with the same charge and angular momentum, but different masses are possible. The different mass-states can be regarded as excitations. According to equation \( 81 \) the lightest "particle" with a given charge and angular momentum is the extreme case for which the square-root in equation \( 81 \) vanishes.

If the (classical) electron is to be described by a holostar, it is reasonable to assume that it will be an extreme holostar. Besides that an extreme holostar has the smallest mass (for a given charge and angular momentum), it has other properties which make it a suitable candidate for an elementary particle: An extreme holostar has no membrane. At least this is true for the charged, non-rotating holostar. It should be true for the rotating holostar as well. Second, not only the metric is continuous at the boundary of an extreme holostar - if the membrane vanishes the metric derivatives are continuous as well. Therefore an extreme holostar should be particularly stable. From black hole physics we have learned that extreme black holes have zero temperature. A stable elementary particle quite certainly requires zero temperature, otherwise its Hawking radiation would be devastating. As the holostar is in many respects similar to a black hole, this argument also points to the extreme holostar as the suitable candidate for the description of an elementary particle.

For an extreme holostar the argument of the square-root in equation \( 81 \) is zero. This implies that \( r_0 \) can be determined whenever the values \( M, Q \) and \( J \) of an extreme rotating/charged holostar are known:

\[
\frac{r_0}{2} = \sqrt{\frac{Q^2}{2} + \sqrt{\left(\frac{Q^2}{2}\right)^2 + J(J+1)}} - M \tag{83}
\]

We can plug in the values of the electron into equation \( 83 \):

\[
\frac{r_0}{2r_{Pl}} = \sqrt{\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4} - 4 \cdot 10^{-23}}} \approx \sqrt{\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}}} \tag{84}
\]

\( ^{27} \)Or very long lived, with respect to the age of the universe.
\( \alpha \) is the fine structure constant, which at \( r \gg 10^{20} r_{Pl} \) is roughly 1/137.

It is doubtable, whether the above formula will give an altogether correct description of the electron. It would require that classical general relativity were correct right down to the Planck scale. Although we shouldn’t expect highly accurate numerical results, the above formula might allow us to make an order of magnitude estimate, which - if we are lucky - could even be correct on the tree or loop level.

Under the assumption that formula (84) is at least approximately correct for an electron, the mass \( M \) appears as a very small correction with respect to the other quantities \( Q, J \) and \( r_0 \). This is as expected. An extreme ”elementary” holostar should have \( M \approx 0 \) in Planck units.

Neglecting the mass of the electron with respect to the other quantities and using the actual value for the fine-structure constant (at low energies and zero momentum transfer), we find:

\[
 r_0 = 1.8651346 \, r_{Pl} \tag{85}
\]

Note that equation (85) can be interpreted as a scaling law for the fundamental area \( r_0^2 \). With this interpretation the fundamental area depends on the energy / distance scale via the coupling constant \( \alpha \):

\[
 \frac{r_0^2(E)}{4\hbar} = \frac{\alpha(E)}{2} + \sqrt{\left(\frac{\alpha(E)}{2}\right)^2 + \frac{3}{4}} \tag{86}
\]

Whenever \( \alpha \) is small, i.e. in the low energy range, the term under the root is dominated by the spin-term \( 3/4 \). For cosmological distances (low \( E \)) we can neglect \( \alpha \) with respect to \( \sqrt{3/4} \). Setting \( \alpha = 0 \) we find:

\[
 \frac{r_0}{2r_{Pl}} = \left(\frac{3}{4}\right)^{\frac{1}{4}} \approx 0.93 \tag{87}
\]

Therefore \( r_0 \approx 1.86 \, r_{Pl} \).

Keep in mind that formula (86) only incorporates the electro-magnetic coupling, i.e. can only be expected to be valid at the low energy regime, where the only other long range force, besides gravity, is the electro-magnetic interaction. For higher energies the other coupling constants will have to be included.

Quite interestingly \( r = \sqrt[4]{3/4} \, r_{Pl} \) is the minimum eigenvalue of the length operator in loop quantum gravity as given by [14]. This points at a connection between the (classical) holostar solution and loop quantum gravity.

\[^{28}\text{One might rather consider } \pi r_0^2 \text{ as the fundamental quantity. It is the smallest possible surface area, that a holostar can have, as } r_h = r_0/2 \text{ for an extreme holostar and for given values of } J, \, Q \text{ and } M \text{ an extreme holostar has the smallest possible area of the membrane.}\]
In [11] the scale parameter $r_0$ has been estimated to be $r_0 \approx 1.88 r_{Pl}$ by comparing the average mass-density in the universe, as derived from the WMAP data [4], to the microwave background temperature. It is truly remarkable and quite likely not a coincidence, that two quite different estimates of $r_0$, one in the regime of elementary particles, the other in the regime of cosmology, give practically the same result.\footnote{Note, that both estimates refer to low energies, where $\alpha \approx 0!$} We shouldn’t become too enthusiastic, though: The errors in the cosmological estimate are rather large, so that the almost exact correspondence could be coincidental. Nevertheless, the above finding can be interpreted as a very strong indication, that $r_0^2$ is a truly universal quantity and that the interpretation of equation (86) as scaling law is essentially correct. This interpretation is further discussed and enforced in [10].

Note also, that from the viewpoint of loop quantum gravity (LQG) it appears reasonable to assume that $r_0^2 = 4\sqrt{3/4h}$ nearly exactly at very large length scales. The smallest non-zero area-value in quantum gravity is given by: $a_0 = 8\pi\gamma h\sqrt{3/4}$. $\gamma$ is the so called Barbero-Immirzi parameter, an undetermined parameter in LQG. If we compare the smallest possible area-eigenvalue of loop quantum gravity to the smallest possible area of a holostar, $A_0 = 4\pi(r_0/2)^2$, we can determine $\gamma$ by setting both areas equal. We find: $\gamma = 1/2$.

The value of the fine-structure constant $\alpha$ depends on the energy scale. If the predictions of the Grand unified theories (GUTs) are correct, all of the coupling constants meet at a scale of roughly $10^3 r_{Pl}$, i.e. at an energy that is roughly a factor of $10^3$ below the Planck energy. In most GUT-models $\alpha_{GUT}$ is still small at this scale, usually around 1/25 (see for example [3, 17, 12]).

Although the GUTs indicate that $\alpha_{GUT}$ remains small at the Planck scale, it is instructive to find out how $r_0$ is effected in the case of large $\alpha_{GUT}$. For $\alpha_{GUT} = 1$ equation (84) gives the following value for $r_0$:

$$r_0 = \sqrt{6} r_{Pl} \approx 2.45 r_{Pl}$$

(88)

The estimate for large $\alpha_{GUT}$ is not much different from the estimate of $r_0$ for small $\alpha$. We can be quite confident that $r_0$ should lie in the range $1.86 < r_0 < 2.5$, i.e. be roughly twice the Planck length, at any conceivable energy scale.

According to the discussion in [9, 10] not $r_0$, but rather its square (multiplied by $\pi$ or $4\pi$) should be considered as fundamental parameter. $r_0^2$ has the dimension of area, which is equal to the dimension of action or angular momentum in natural units $c = G = 1$. In quantum physics action and angular momentum are quantized in units of $\hbar/2$. In loop quantum gravity the area operator is quantized (however not in integer multiples of $\hbar/2$). Therefore it doesn’t seem unreasonable to assume that $r_0^2$ might be quantized as well and possesses a non-zero minimum eigenvalue.
Any definite value for $r_0^2 = \beta \hbar$ requires a particular value for $\alpha$, which is determined by solving the following equation:

$$\frac{\beta}{4} = \frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}}$$

(89)

For $r_0 = 2r_{Pl}$, i.e. $\beta = 4$ we get the following "prediction" for $\alpha$ for a spin-half particle (at the energy-scale defined by $r_0 = 2r_{Pl}$):

$$\alpha = \frac{1}{4}$$

(90)

From equation (81) we can see, that the membrane is situated at $r_h = r_0/2$ for any "elementary" extreme holostar with zero or negligible mass. Therefore the surface area of an elementary, extreme holostar will be given by:

$$A_0 = \frac{4\pi r_h^2}{\hbar} = 4\pi \left(\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}}\right) = \pi \beta$$

(91)

Its "cross-sectional area" $\sigma = \pi r_h^2$, which - according to the Hawking formula is equal to its entropy $\sigma_s$ turns out as:

$$\sigma_s = \frac{\sigma}{\hbar} = \frac{A_0}{4\hbar} = \pi \left(\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}}\right) = \frac{\pi \beta}{4}$$

(92)

Note, that $\pi r_0^2 = \pi \beta \hbar$ is the smallest possible surface area, that any holostar can have. For $\alpha = 1/4$ and $j = 1/2$ we have $\beta = 4$, so that the area of such an elementary holostar will be $A_0 = 4\pi \hbar$ and its "cross-sectional area" $\sigma_0 = A_0/4 = \pi \hbar$.

For $\alpha = 1/4$ and $j = 0$ (which could be interpreted as a $s$-electron) we get the "prediction" $\beta = 1$, i.e. $r_0 = r_{Pl}$. In this case (no rotation) the area of the membrane would be $\pi$ and the cross-sectional area $\pi/4$ in Planck units.

9 The holostar as a quantum gravity spin-network

In this section I attempt to relate the area of a holostar with the area-eigenvalues of quantum gravity. The main motivation behind this undertaking is the observation, that the number of punctures of a large loop quantum gravity (LQG) spin network state is proportional to the area that is being "measured" by the spin-network. Usually this area has been identified with the event horizon.  

30See however the discussion in footnote 39.
of a large black hole. For the holostar we should identify this area with the holostar’s boundary area (i.e. with the membrane).\textsuperscript{31} But the boundary area of the holostar is proportional to the number of its interior particles with roughly the same factor of proportionality as the number of punctures of the associated spin-network state. Furthermore, both the particles in the holostar and the links of a spin-network state carry spin\textsuperscript{32} and for large spin-networks, as well as for large holostars, the spin $1/2$ entities dominate. This suggests a connection between the links of a large spin network state and the respective spins of the relativistic interior particles of a holostar.

The area operator in loop quantum gravity has the following so called ”full” spectrum \textsuperscript{2}:

$$A = 4\pi\gamma\hbar \sum \sqrt{2j_u(j_u + 1) + 2j_d(j_d + 1) - j_t(j_t + 1)}$$  \hspace{1cm} (93)

with

$$|j_u - j_d| \leq j_t \leq j_u + j_d$$

$\gamma$ is the Barbero-Immirzi parameter.

The $j_u$ and $j_d$ are positive half-integers and $j_t$ varies in integer steps according to the triangle summation law for angular momenta. $j_u$, $j_d$ and $j_t$ are the respective spin quantum numbers for the links of quantum gravity spin network, which ”puncture” the surface that is just being measured and thus endow the surface with a calculable quantum of area at each puncture. The ”full spectrum” of the area operator assumes, that the nodes of a quantum gravity spin network can lie within the surface. For any puncture the spins $j_u$ and $j_d$ label the links that don’t lie within the surface. $j_u$ is the spin quantum number of the link on the ”upper” side of the surface (assuming a given orientation), $j_d$ labels the link on the ”down” side, and $j_t$ stands for the link that is tangential to the surface, i.e. lies within the surface at the puncture. The spin-quantum numbers of the links of a quantum gravity spin-network can only change at the nodes. Only at a node three or more links can join.\textsuperscript{33} Links that lie completely

\textsuperscript{31}This identification appears very much preferable over the event horizon: Whereas the position of the holostar’s boundary is defined by matter, and thus respects diffeomorphism invariance, this is definitely not the case for the event horizon of a black hole, which is a fictitious ”surface” in vacuum, whose position cannot be determined by any local measurement. In fact, in order to ”know” the position of the event horizon one has to have access to the whole space-time’s future, as the event horizon ”moves” a-causally in anticipation of the matter that will eventually pass it in the future.

\textsuperscript{32}The reader might object, that the two spin-variables describe different aspects of nature and are not related. Although such a point of view is feasible, there is not much predictive power in this viewpoint, whereas the identification of the spins appears to yield consistent results.

\textsuperscript{33}Whenever there are more than three links at a node, the links can always be combined into at least three links, so the three-valent nodes can be regarded as fundamental.
within the surface (such as closed loops) don’t contribute to its area. Therefore \( j_t \) is only relevant, if there is a node within the surface.

If one assumes that all of the nodes of the spin-network lie outside the surface, we have \( j_t = 0 \) and \( j_u = j_d \). In this case we get the "reduced" area spectrum, that was first derived by Rovelli and Smolin [13]:

\[
A = 8\pi\gamma h \sum \sqrt{j(j + 1)}
\]  

(94)

The spin quantum number \( j \) runs over all the punctures.

If a correspondence between the classical holostar solution and a (large) quantum gravity spin-network is to be derived, an extremely rotating holostar appears as the best suited starting point:

The area of a large spin-network state is dominated by the spin 1/2 links [1]. The number of punctures of a large LQG-spin network therefore will be given by:

\[
N = \frac{A}{8\pi\gamma h} \sqrt{\frac{4}{3}}
\]  

(95)

Let us denote by \( J \) the "total spin" of the spin-network, i.e. the sum of the spin quantum numbers of all of the links (over all punctures). Due to the predominance of the links with spin 1/2, the "total spin" \( J \) will be roughly equal to \( N/2 \) for large \( N \).

\[
J = \sum j \approx \frac{N}{2}
\]  

(96)

If we replace \( N \) by \( J \) in equation [95], we get the following relation between area \( A \) and the "total spin" \( J \):

\[
A \approx 8\pi J h \gamma \sqrt{3}
\]  

(97)

This looks very much like the relationship between area and angular momentum of an extreme (maximally rotating) Kerr black hole

\[
A = 8\pi J h
\]  

(98)

whenever \( \gamma \approx 1/\sqrt{3} \) and if we identify the total sum of the spins of the links with the angular momentum \( J \) of the black hole.

This relationship between the sum of the spins of large LQG spin network state and the angular momentum area law for an extreme Kerr black hole was already noted by Krasnov [3]. Knowing that a spin-network can in principle
consist of one single link with macroscopic spin $J$ and area $A = 8\pi\gamma J(J + 1)$, Krasnov argued, that the Immirzi parameter must be equal to or greater than one, if spin network-states with macroscopic links exist and their spin can be identified with the angular momentum of a Kerr-black hole. If $\gamma < 1$ one could devise a spin-network state with a few large links, whose "angular momentum" exceeds that of an extreme Kerr black hole, i.e. would correspond to the unphysical Kerr-black hole solution with a naked singularity.

Here, however, we take the position that the links of the LQG spin-network states are not combined into one large link, but rather are dominated by the smallest links with $j = 1/2$ and each link contributes individually to the area and entropy in the large $N$ limit. This seems a more natural choice, if the links are to be identified with fundamental particles, which all have low spins and definite entropies. In fact, all of the fundamental particles of the Standard Model (not including the gauge bosons) are spin 1/2 fermions.

Lets make the argument more definite: Let us assume that any link of a LQG spin network can be identified with an (ultra-relativistic) interior particle of the holostar solution and that the spin of the ultra-relativistic particle is equal to the spin of the respective link. With this identification we should be able to determine the Barbero-Immirzi parameter.

First, however, we have to establish whether the reduced area formula of equation (94) can be used. There is evidence from the charged holostar solution, that an extreme rotating holostar has no membrane. If there is no membrane, there should be no nodes within the boundary surface of an extreme holostar, which justifies using the reduced spectrum (at least in the extreme case).

Second we have to know the exact number of punctures within the boundary surface. Intuitively one expects, that the number of punctures should be equal to the number of different spins, i.e. the total number of the interior particles of the holostar. However, we have to consider the possibility that there are links in the spin-network (i.e. particles), that don’t puncture the boundary. The number of these links (particles) is expected to be small: Every interior particle of an extremely rotating holostar is believed to be aligned with respect to the rotation axis and the sum of all commonly aligned spins of the interior particles is (nearly) equal to the total exterior angular momentum of the holostar (see the discussion in [10]). Therefore each spin of an interior particle will be "visible" to the exterior observer as a contribution to the exterior angular momentum of the holostar. If the spin of an interior particle (to be identified with a LQG-link) is to be "visible" for the observer outside, its spin should "puncture" the boundary area. Therefore the number of punctures should be (nearly) equal to the number of interior particles of the holostar. Furthermore, the equations of geodesic motion within the holostar space-time show, that any geodesically moving particle must eventually cross the boundary membrane, swinging back and forth between interior and exterior space-time. So in a - at least semantically correct - sense, every particle must "puncture" the membrane.
9.1 Determination of the Barbero-Immirzi parameter for a fermionic holostar

We are now ready to determine the Barbero-Immirzi parameter. Let us first consider the case of a holostar, whose (interior) particle content is dominated by spin-1/2 fermions, which appears to be the physically most relevant case.

The fermionic (spin 1/2) holostar is suggested by the current results of loop quantum gravity, according to which the area of a large spin-network state is dominated by spin 1/2 links. The identification of links with ultra-relativistic particles leads to the assumption, that the interior relativistic particles of a large holostar should consist predominantly out of spin-1/2 particles. From the viewpoint of holostar thermodynamics a holostar consisting only out of spin 1/2 fermions is the simplest possible model that works.

With this assumption the area of the holostar according to the loop quantum gravity area formula is given by:

\[ \frac{A_{\text{LQG}}}{\hbar} = 8\pi\gamma N \sqrt{\frac{3}{4}} \]  

where the number of punctures is set equal to the number of ultra-relativistic spin 1/2 particles within the holostar.

In [10] it has been shown, that the total number of particles \( N \) within the holostar is proportional to the area of the membrane and is given by:

\[ \frac{A}{\hbar} = 4\sigma N \]

\( \sigma \) is the (mean) entropy per particle. Its exact value depends on the relative number of ultra-relativistic bosonic and fermionic degrees of freedom and on the relation between the chemical potentials.

If the area determined by loop quantum gravity is to be equal to the area of the holostar given by equation (100) the Barbero-Immirzi parameter can be determined:

\[ \gamma = \frac{\sigma}{2\pi} \sqrt{\frac{4}{3}} = \frac{\sigma}{\pi \sqrt{3}} \]

This is quite compatible with the result in [10], where it was shown that the mean spin quantum number of the interior particles cannot be much larger than 1/2, otherwise the holostar would acquire a higher angular momentum than an extreme rotating Kerr black hole, simply by aligning all of the spins of its interior particles.

There is no solution for a holostar in thermodynamic equilibrium that consists exclusively out of bosons. At least one fermionic species is required!
Note that $\sigma$ is always slightly larger than $\pi$, at least in the thermodynamic models discussed in [10]. If the contribution of the bosonic degrees of freedom can be neglected, i.e. $f_B = 0$, the entropy per particle at high temperatures is given by $\sigma \simeq 3.3792$, so that the numerical value of $\gamma$ turns out as:

$$\gamma \simeq 0.621$$

(102)

The value of the Barbero-Immirzi parameter in equation (101) is larger than a factor of roughly 4.8 than the value that was determined in [1] by counting horizon-surface states:

$$\gamma_0 = \frac{\ln 2}{\pi \sqrt{3}}$$

(103)

The difference can be traced to the following reason. In [1] any spin 1/2 link of a large LQG spin-network is associated with an entropy of $\ln 2$, because there are two "area states" associated with a spin 1/2 variable.$^{36}$ This is a perfectly reasonable assumption, which however is based on the line of thought that the thermodynamics of a black hole should be completely determined by the states of its event horizon.$^{37}$ In this work the viewpoint is taken, that the entropy of a compact self gravitating body doesn’t correspond directly to

$^{36}$The analysis in [1] is much more sophisticated. Yet it turns out that the above statement approximates the true picture fairly well in case of a large spin-network state. A more accurate description is this: Any link with a given spin induces a deficit angle on the surface punctured by the link. There are only a small number of allowed deficit angles for a link with a given spin. These deficit angles can be labelled by the "magnetic quantum number" of the spin. A spin 1/2 link has two possible deficit angles, $+2\pi$ and $-2\pi$, corresponding to $m = \pm 1/2$, i.e. $\Delta \varphi = 4\pi m$. So there are two different "area-states" associated with a spin 1/2 link. A spin-1 link has three choices of deficit angle, corresponding to $m = -1, 0, 1$ etc.

$^{37}$In my opinion there is one major draw-back in the determination of the number of physically distinct horizon surface states for a given macroscopic area according to [1]. The derivation in [1] assumes that all links (with the same deficit angle) in any large spin-network state are distinguishable. This is an assumption which is difficult to accept. There is no "tag" on the links which would enable us to discern any two links with the same spin and the same deficit angle. In quantum theory we have learned, that all fundamental objects with the same set of quantum numbers are indistinguishable from each other. If there is no special marker (i.e. hidden variable) on each spin 1/2 (or spin 1) link, how can any two identical horizon surface patches punctured by a single spin 1/2 link and inducing the same deficit angle can be regarded as distinguishable? Likewise there is no physical process conceivable, which would allow us to keep track of every fundamental surface patch of a macroscopic black hole in a way such that all surface patches / punctures can be individualized. The information required just to keep a record of the individual "positions" of the surface patches constituting the event horizon of a macroscopic black hole would require a second black hole with at least the size of the black hole whose horizon surface states we would like to keep track of. Not to forget that any "measurement" of the "position" of a single surface patch accurate enough to distinguish its position from that of its neighbors will require a tremendous amount of energy: The neighboring surface patches (punctures) are within a Planck distance. The energy required to determine the "position" of just one single puncture with Planck-accuracy will effectively "erase" the positions of roughly $\sqrt{N}$ punctures, where $N$ is the total number of punctures through the event horizon of the black hole.
some abstract, distinguishable horizon surface states, but should be rather be
determined from the microscopic entropy of the principally indistinguishable
constituent particles within the interior space-time of the self gravitating body.
With this point of view the phase-space available for the fundamental quanta of
matter (=particles), which are identified with fundamental quanta of geometry
(=links of a large spin-network state), is increased. Therefore any spin-network
link of a large spin-network should be associated with an entropy $\sigma \approx \pi$, which
not only derives its value from the (two or more) horizon surface states, but
from the total interior phase space available to the "links". Equation (103) then
should be modified by replacing $\ln 2$ with $\sigma$, which is the mean thermodynamic
entropy per particle whose exact value can be determined from microscopic
statistical thermodynamics along the lines presented in [10].

It is remarkable, however, that the determination of the Barbero-Immirzi
parameter by counting the horizon surface states of a quantum gravity spin-
network and the semi-classical determination of the Barbero-Immirzi parameter
by counting particle states in the interior phase space of the space-time yields
essentially the same result. Therefore we can be quite confident, that classical
general relativity, combined with microscopic-statistical thermodynamics, truly
arises in the large $N$ limit of quantum gravity.

With the Barbero-Immirzi parameter given by equation (101) the area
of the smallest spin-network state of quantum gravity (one single half spin) can be
determined. We find:

$$\frac{A_0}{\hbar} = 8\pi\gamma \sqrt{j(j+1)} = 8\frac{\sigma}{\sqrt{3}} \sqrt{\frac{3}{4}} = 4\sigma$$

(104)

We should be able to identify this area with the area and entropy of the
holostar’s interior spin 1/2 particles. The particle’s cross-sectional area $\sigma_0$,
which is equal to its (intrinsic) entropy according to the Hawking formula, turns
out to be exactly equal to the (mean thermodynamic) entropy per particle.

$$\frac{\sigma_0}{\hbar} = \frac{A_0}{4\hbar} = \sigma$$

(105)

Therefore the total entropy of a fermionic spin 1/2 holostar is nothing else
than the sum over the Hawking-entropies of all of its interior ultra-relativistic
particles.\footnote{Note, however, that the surface area of the particles, from which the entropy is determined
via the Hawking-formula, is not the area of the (non-existent) event horizon of the particles,
but rather the area of their boundaries, i.e. the area of the membrane.}

This result is relevant with respect to the discussion in [11]. There it was
shown, that in the high temperature regime of the holostar the outward directed
gedesic acceleration and the inward directed pressure-induced acceleration ex-
actly cancel, whenever the cross-sectional area of the particles that produce the
pressure, expressed in Planck-units, is equal to the mean entropy per particle \( s \). It is quite remarkable, that loop quantum gravity - with the numerical value of the Barbero-Immirzi parameter given by equation (101) - appears to deliver exactly the value required for the cross-sectional area of the "pressure particles", in order that the holostar be truly static in the high temperature regime.

There is reason to believe, that the fermionic holostar is a good approximation to a realistic self gravitating object only at high temperatures. However, if this is true, one should be able to calculate the value of the "fine-structure constant" at the Planck energy \( \alpha_0 \) via equation (86), whenever the area of a fundamental spin 1/2 particle, \( A_0 \), is known:

\[
\frac{A_0}{\hbar} = 4\pi \frac{r_0^2}{4\hbar} = 4\pi \left( \frac{\alpha_0}{2} + \sqrt{\left( \frac{\alpha_0}{2} \right)^2 + \frac{3}{4}} \right) = 4\sigma
\]

This can be solved for \( \alpha_0 \):

\[
\alpha_0 = \frac{\sigma}{\pi} - \frac{3\pi}{4\sigma}
\]

With \( \sigma \simeq 3.3792 \) we find:

\[
\alpha_0 \simeq 0.378
\]

This is very close to the value 3/8 predicted by \( SU(5) \) for \( \sin^2 \theta_W \) at the unification energy. The electromagnetic coupling constant \( \alpha \) is related to the \( SU(2) \) coupling constant \( \alpha_2 = g^2/(4\pi) \) by the Weinberg-angle: \( \alpha = \alpha_2 \sin^2 \theta_W \). With \( \alpha_0 = 0.378 \) and \( \sin^2 \theta_W = 3/8 \) at the unification energy, we get the remarkable "prediction", that the unified \( SU(2)/SU(3) \)-coupling constant \( \alpha_{2/3} \) at the Planck energy should be nearly unity, i.e.

\[
\alpha_{GUT} \simeq \frac{0.378}{0.375} \approx 1.009
\]

9.2 Immirzi parameter for a supersymmetric holostar

Quite likely the assumptions in the previous section are too simplistic. It might not be possible to neglect the bosonic degrees of freedom completely in the holostar’s interior.\(^{39}\)

\(^{39}\)Note also, that in the simple thermodynamic model discussed in [10] it has been assumed, that all particles are ultra-relativistic. Whereas this is a reasonable assumption at high temperatures, i.e. for small holostars, the model in [10] will have to be extended to incorporate more than two particle species, including massive species to accurately describe the phenomena in the low-temperature regime.
There is another reason, why bosons might play an important role in the holostar’s interior: The holographic solution is based on Einstein’s equations with zero cosmological constant. A zero cosmological constant, or rather a zero vacuum energy, can be explained in theories with unbroken supersymmetry. This suggests, that supersymmetry might play an important role within the holostar, at least at high energies.

In this section I discuss a very simple ”supersymmetric” thermodynamic model of the holostar, which is characterized by an equal number of fermionic and bosonic degrees of freedom of the interior particles, i.e. \( f_F = f_B \).

In [10] two possibilities for a supersymmetric phase with \( f_F = f_B \) were discussed:

The ”normal” supersymmetric phase consists of a gas of equal numbers of degrees of freedom for the ultra-relativistic fermions and bosons, which are in thermal equilibrium with each other and their anti-particles. It can be shown, that in the holostar space-time the ultra-relativistic fermions must have a chemical potential proportional to the radiation temperature \( \mu_F = u_F T \), where \( u_F = \pi/\sqrt{3} \approx 1.814 \) is a positive dimensionless constant (in units \( k = 1 \)). The anti-fermions have the opposite chemical potential of the fermions, i.e. \( \mu_F = -u_F T \) and all of the bosons have zero chemical potential.

However, in [10] it was noted, that for equal fermionic and bosonic degrees of freedom there exists a second, ”abnormal” supersymmetric phase, which has nearly identical thermodynamic properties to the ”normal” gas phase consisting exclusively out of ultra-relativistic fermions and anti-fermions. If one identifies the anti-fermions in the ”normal” - exclusively fermionic - gas phase with the bosons in the ”abnormal” supersymmetric phase, the thermodynamic properties of both phases are nearly identical. The bosons in the ”abnormal” supersymmetric phase in a sense have ”disguised” themselves as the anti-particles of the fermions of the ”normal” phase. This ”abnormal” supersymmetric phase is characterized by the property, that there are only fermions and bosons (no anti-particles!) with equal degrees of freedom, i.e. \( f_F = f_B \), and that the chemical potentials of fermions and bosons are related by \( \mu_F + \mu_B = 0 \). For this unconventional supersymmetric phase we have \( u_F = -u_B \approx 1.353 \).

Whenever one specifies the relation between the number of fermionic and bosonic degrees of freedom, which in a supersymmetric context is given by \( f_F = f_B \), and the relations between the chemical potentials (\( \mu_F + \mu_B = 0 \) and \( \mu_B = 0 \) for the ”normal” supersymmetric phase; \( \mu_F + \mu_B = 0 \) and \( \mu_F > 0 \) for the ”abnormal” supersymmetric phase), the thermodynamic properties of the holostar solution are completely determined. The mean entropy per particle, \( \sigma \), the relative number- and energy-densities of fermions and anti-fermions to bosons and the respective entropies of the fermions, \( \sigma_F \), anti-fermions \( \overline{\sigma_F} \) and bosons, \( \sigma_B \), can be read off from the tables given in [10].

The purpose of this section is to compare the properties derived from the supersymmetric thermodynamic model of the holostar with the results of loop
quantum gravity. Most likely the right way to do this is to choose the "normal" supersymmetric thermodynamic model for such a comparison. The difficulty is, that one has to know the mean spin of the fermions and bosons, which depends on the fundamental supersymmetric particle group, or rather on the relative numbers of fundamental particles with spins 0, 1/2, 1, 3/2, 2. So far there appears to be no theoretical preference for a specific supersymmetric particle group. In such a situation it might by safer to use a minimalistic approach, which incorporates only the observed spin-varieties. The only fundamental particles that are known to exist are spin 1/2 fermions (quarks and leptons) and spin-1 (gauge) bosons (γ, W±, Z, g). Quite interestingly, if one compares the "abnormal" supersymmetric model with the LQG-spin-network description under the assumption, that all the fermions have spin 1/2 and all the bosons have spin-1, one gets an astoundingly consistent picture, which seems worthwhile reporting, although it is yet not possible to decide, whether this picture bares some deeper physical significance.

For the "abnormal" supersymmetric model we can read off the mean entropy per particle σ, the ratio of the number-densities of fermions to bosons, w, and the entropies per fermion σF and boson σB from the tables given in [10]. We find:

\[
\sigma = 3.37174 \quad (110)
\]

\[
w = \frac{N_F}{N_B} = 10.8031 \quad (111)
\]

\[
\sigma_F = 3.1948 \quad (112)
\]

\[
\sigma_B = 5.2835 \quad (113)
\]

With the assumption that all of the fermions are spin-1/2 particles and all of the bosons carry spin-1 we can calculate the surface area with the loop quantum gravity area formula:

\[
\frac{A_{LG}}{\hbar} = 8\pi\gamma N \left(\frac{w}{1 + w + \sqrt{\frac{3}{4} + \frac{1}{w + 1}}\sqrt{2}}\right) = 0.913 \left(8\pi\gamma N\right) \quad (114)
\]

The number of punctures N is identified with the total number of interior particles. This appears as the most reasonable assumption. However, if the bosons preferentially assemble in the membrane, as suggested in [10], this assumption might have to be modified.

By setting the areas of equation (100) and equation (114) equal, we find the following value for the Immirzi parameter:
\[ \gamma = 0.5881 \] (115)

We are now in the position to compare the entropies of the fermions and bosons, as calculated by holostar thermodynamics and as calculated via the loop quantum gravity area formula.

According to the LQG area formula, the entropy of a spin \(1/2\)-fermion turns out as:

\[ \sigma_F = 2\pi \gamma \sqrt{3/4} = 3.200 \] (116)

This is almost equal to the entropy per fermion determined from holostar thermodynamics, \(\sigma_F = 3.195\).

The loop quantum gravity result for the entropy per boson is given by

\[ \sigma_B = 2\pi \gamma \sqrt{2} = 5.226 \] (117)

which again is close to, but not equal to the value determined from the thermodynamic model, \(\sigma_B = 5.284\).

The very simple supersymmetric model discussed here appears to give some quite consistent results. The entropies predicted by holostar thermodynamics for fermions and bosons are within 1 \% of the entropies we expect from the loop quantum gravity area formula in combination with the Hawking entropy-area law. The correspondence is not perfect. Further insights will be necessary to resolve the discrepancies in a satisfying manner.

Note, that the basic assumption on which the "abnormal" supersymmetric thermodynamic model is based, namely that at ultra-high energies and temperatures the anti-fermionic degrees of freedom become nearly indistinguishable - at least in a thermodynamic sense - from the bosonic degrees of freedom, so far has no serious theoretical justification. Therefore further insights might very well lead us to the conclusion, that the apparent correspondence between the "abnormal" supersymmetric thermodynamic model and LQG was nothing more than a curious numerical coincidence.

10 **Is entropy a (nearly) conserved quantity?**

As has been shown in [10] the entropy of a holostar is proportional to its number of constituent particles. The thermodynamic entropy per interior particle within a large holostar, \(\sigma\), is constant and slightly larger than \(\pi\). The value of \(\sigma\) is quite independent of the specifics of the thermodynamic model.
On the other hand, according to the discussion in the previous sections it makes sense to identify the microscopic constituents of the holostar (=particles) with "elementary" extreme holostars, which are the smallest and lightest possible holostars for any given value of particle angular momentum and charge. These elementary extreme holostars have a non-zero boundary area $A_0 = 4\pi r_0^2 = \pi r_0^2 = \pi \beta \hbar$. This boundary area allows one to attribute an intrinsic entropy, $\sigma_i = A/(4\hbar)$, to the particles via the Hawking entropy-area relation.

The boundary area can either be determined by the "semi-classical" expression for $r_0^2$ or by the LQG area formula, if the value of the Barbero-Immirzi-parameter is known.

Let us first discuss the intrinsic entropy of the particles determined via the semi-classical expression for the fundamental area $r_0^2$, given in equation (86). We find:

$$\sigma_i = \frac{\pi \beta}{4} = \pi \left(\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + j(j+1)}\right)$$

All fundamental particles of the Standard Model are spin-1/2 fermions. For these the intrinsic entropy at low energies ($\alpha \simeq 0$) is roughly $\sigma_i \simeq \pi \sqrt{3/4} \simeq 0.8 - 0.84 \sigma$, with $\sigma$ being the thermodynamic entropy, which lies in the range between $\sigma \in [3.16, 3.38]$ for realistic thermodynamic models.

A particle well outside the gravitational radius of the holostar is expected to have very low energy, so that its intrinsic entropy will be quite a good lower bound for its real thermodynamic entropy. Therefore we find, that the entropy of the whole system is not very much affected, whether the particle is inside the holostar, where it has a thermodynamic entropy of $\sigma$ in the relatively narrow range given before, or whether it is outside the holostar, where its entropy is at least $0.8 \sigma$.

Whenever a single elementary particle enters the holostar, the holostar’s entropy changes by the mean thermodynamic entropy per particle $\sigma$, because the number of its constituent particles has been increased by one.\footnote{The accretion process doesn’t necessarily have to be adiabatic. For non-adiabatic processes we just have to wait long enough, until the holostar has attained thermal equilibrium.} But the change in the (thermodynamic) entropy of the holostar is nearly equal to the intrinsic entropy of the particle that has entered the holostar, because its surface area is roughly $A_0 \approx 4\sigma \hbar$. Taking the intrinsic (Hawking) entropy of the particle into account, the total entropy before and after the merger is roughly equal. The same argument applies to a process, where the holostar emits a particle (via Hawking radiation) or when two holostars of arbitrary size collide and join. Note, that the difference between the entropy before and after the merger is largest, when a single particle enters a large holostar. For large holostars the thermodynamic entropies of the individual constituent particles are nearly
equal, especially if the holostars have nearly equal size. Therefore the larger any two merging holostars become, the better their total entropies should be conserved.

If we estimate the intrinsic entropy of a particle from the area assigned to the particle by the loop-quantum gravity area formula, we find that the entropy might not only be conserved approximately, but appears to be conserved strictly for the particular case of a fermionic holostar consisting only out of spin-$1/2$ particles, when we choose $\gamma = \sigma/(\pi \sqrt{3})$. However, one must keep in mind that $\gamma$ was explicitly chosen such, that $\sigma_i = A_0/(4\hbar) = \sigma$, so strict entropy conservation is not a genuine prediction in this particular case. Note also that $\gamma$ depends explicitly on $\sigma$, which is a model-dependent quantity. One would rather expect, that the Barbero-Immirzi-parameter is constant, independent of the specifics of the thermodynamic model. Therefore other values for the Barbero-Immirzi-parameter might be contemplated, such as $\gamma = 0.5$, as suggested in section 8. On the other hand, it has been speculated whether $\gamma$ might undergo a - moderate - rescaling depending on the energy scale. In fact, $\gamma$ is related to the fundamental area, so if $r_0^2$ can be regarded as a running quantity, the same might apply to $\gamma$. In such a case one would expect $\gamma = 0.5$ at the Planck energy, becoming larger for lower energies.

At the current state of knowledge the evidence is quite in favor of approximate entropy conservation, but not strong enough to postulate strict entropy conservation (which would be attractive for theoretical reasons). Yet even approximate conservation of entropy in general relativity is highly attractive, because it forbids thought-experiments of the following kind: Imagine a large black hole of galactic size or larger. Assemble a spherical shell of matter with total mass equal to the mass of the imagined black hole and place the shell close to its (imagined) gravitational radius. The mass-density of the shell can be chosen arbitrary low. The smaller the mass-density should be, the larger a black hole has to be imagined. Without (approximate) entropy conservation there appears to be no physical law that forbids us to arrange the matter in the spherically symmetric shell in an orderly fashion, so that its entropy can be chosen almost arbitrary small. Now let the shell collapse under its own gravity. When it passes its own gravitational radius, a black hole will form. When the event-horizon (or the apparent horizon) forms, an enormous amount of entropy will be generated almost on the spot. This is quite unbelievable. No physical process should produce an arbitrary large amount of entropy almost instantaneously.

If entropy is (approximately) conserved in general relativity, why then do we live in an era where quite apparently $\Delta S > 0$, i.e. the entropy increases over time? It has been speculated (see for example [7]) that this might have to do with our present time-asymmetric situation, which is characterized by the fact that we live in an expanding universe. However, as far as I know no convincing explanation has been given so far, why an expanding universe should be associated with $\Delta S > 0$, whereas in a phase of contraction the entropy should shrink. In the holostar there appears to be some sort of rudimentary explanation: In
the low density regions of a holostar, the number of photons with respect to the number of massive particles increases over time, if the motion of the outmoving massive particles is nearly geodesical, in fact $\frac{\pi_i}{\pi_m} \propto \sqrt{\tau}$ (see [11]). If we define the co-moving volume of the geodesically moving massive particles as reference, the entropy within this volume increases throughout the expansion, because there will be more and more photons per nucleon. Furthermore, according to the discussion in [11] the negative pressure in the holostar space-time has the effect to create new (massive) particles within any expanding volume, which further increases the entropy within this volume. For particles moving inward on a geodesic trajectory (i.e. the time-reversed situation) the number of particles in the co-moving volume shrinks, so contraction is associated with $\Delta S < 0$. In [11] a more detailed discussion of global and local entropy- and energy-conservation is given. There it will be shown, that the total entropy and energy within the holostar space-time is conserved. However, the total energy and entropy increase within the local Hubble-volume of an outward moving observer, whereas both quantities decrease for an inward moving observer. Increase and decrease are exactly opposite to each other at every interior space-time point, so in toto entropy and energy are conserved.

11 Discussion

The holostar solution has been generalized to the spherically symmetric charged case. Many of the characteristic properties of the uncharged holostar also apply for the charged case.

The charged holostar solution possesses a remarkable degree of self-consistency. The interior charge distribution, from which the total charge can be determined by proper integration, is proportional to the number density of its interior massive particles. This is not self-evident and depends on the subtle interplay between the metric and the interior energy density, from which the metric is derived. The exterior metric and electric field are identical to the Reissner-Nordström electro-vac solution. Similar to the uncharged holostar solution, the total gravitational mass $M$ of the space-time can be derived from a proper integral over the trace of the stress-energy-tensor, a Lorentz invariant quantity.

The charged holostar solution allows us to make some predictions, which are quite in accord with the expectations from black hole physics: A charged holostar solution with $|Q| \gg M$ is not possible. Whereas in the case of electro-vac space-times solutions with $|Q| > M$ are ruled out by the cosmic censorship hypothesis, a holostar solution with $Q$ much higher than $M$ in natural units simply doesn’t exist.

Extremely charged holostars are of particular interest. They consist exclusively out of electro-magnetic energy. The energy density is continuous and the metric is differentiable continuous across the boundary. Charged extreme
holostar solutions with negligible mass (in natural units) are among the possible solutions. Those solutions have many features in common with elementary particles.

By extending the formula describing an extremely charged holostar to the case of angular momentum (and charge), the scale parameter $r_0$ has been estimated. $r_0^2$ amounts to roughly four times the Planck area, which is quite in agreement with the experimental determination of $r_0^2$ in a macroscopic (cosmological) context. There is a theoretical preference for a value of $r_0^2 \approx 4\sqrt{3}/4$ at macroscopic scales and $r_0^2 \approx 4\sigma/\pi$ at the Planck scale, where $\sigma > \pi$ is the mean entropy per ultra-relativistic particle determined by holostar thermodynamics (see [10]). $r_0/2$ is the radial coordinate value of the membrane of an elementary extreme holostar of (nearly) zero mass.

If the holostar solution is identified with a large quantum gravity spin-network state, the Barbero-Immirzi parameter $\gamma$ can be determined. We find $\gamma = \sigma/(\pi\sqrt{3})$, when the ultra-relativistic particles of the holostar solution are identified with the links of a quantum gravity spin-network state. $\gamma$ is roughly a factor of 4.8 higher than the value determined in [1] by counting horizon surface states. The exact value of $\gamma$ depends on the particle content of the thermodynamic model (i.e. the number of fermionic and bosonic degrees of freedom with different spins) and on the respective relations between the chemical potentials.

The identification of an elementary extreme holostar of minimal mass with an elementary particle leads to the conjecture, that the entropy of a self-gravitating system in general relativity should be (approximately) conserved. The observation, that entropy increases with cosmological time appears as a consequence of our current time-asymmetric situation, i.e. that we live in the expanding sector of the universe. An argument is given, why expansion should lead to $\Delta S > 0$ and contraction to $\Delta S < 0$.

A major topic of future research will be the search for a rotating holostar, which should enable us to resolve many of the yet open questions addressed in this paper and to put the preliminary results presented here on a firmer basis. Hopefully the charged holostar solution will provide some theoretical guidance to find a charged/rotating solution.

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