Statistical Analysis of CFRP Mechanical Properties using B-Basis Based on Weibull and ANOVA Distribution Analysis

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Abstract. In this paper, the mechanical properties of composite materials are subjected to statistical analysis. This study aimed to determine the value of B-Basis strength parameters of carbon composite material with UD-0⁰, UD-90⁰, fabric-0⁰, fabric-90⁰ from the results of tensile and compression tests. The maximum normal residual test was used for outlier identification and k-sample Anderson’s darling test to determine whether parameters can be processed as one large group of data. The scope of this study focused on evaluating B-Basis values using two methods: Weibull and ANOVA. The results showed that Weibull distribution method could be applied to analysis the data and produced a confidence level of 95%. ANOVA method however, could not be applied to specific data which were the tensile strength of UD 0⁰ with a non-tabbing treatment. This was due to the inhomogeneous of the data.

1. Introduction
During the structural design process, engineers needs reliable mechanical properties data of the materials. In the aerospace industries dealing with composite materials, the confidence level of the mechanical properties data should be higher than 90% [1]. A-Basis or B-Basis, strength allowables as design values to reduce the probability of failure or reducing test and risks associated with the use of composites in aerospace structures [2]. The method will be statistically better if the test data is not too small, so the results of the analysis can be generalized to obtain design allowables [3]. Therefore statistical analysis should be used to analyse the data. B-Basis statistical method is the common practice in the aerospace industries.

In this paper, B-Basis method was used to analyse the experimental data of composite materials under tensile and compression tests. There are two types of specimens that will be analysed. First was the specimens with tab and the second was the specimens without tab (non-tab). The research will analyse the better specimens between these two.

Two methodologies were used within B-Basis method: Weibull and ANOVA. Weibull distribution is widely used to analyse the strength of composite fiber reinforced polymer (FRP). In general, the two-parameter Weibull distribution is used although the three-parameter Weibull distribution may provide better data characteristics [4]. Both stiffness and strength parameters of the composite materials can be statistically evaluated. However, the confidence level of the data is better if the dataset has many sample
variations from each batch [5]. Therefore, in this research, the mechanical properties of the composite materials were taken from different batch. The mechanical properties of carbon fiber reinforced plastic (CFRP) composites were analysed. The mechanical properties were taken from tensile and compression tests. First, the Maximum Normed Residual (MNR) tests were carried out to find the outliers in the data and removed it from the analysis. Finally, B-Basis analysis can be carried out in order to get the mechanical properties data with a confidence level higher than 90% [6].

2. Research Methodology

2.1. The Maximum Normed Residual Test

The Maximum Normed Residual Test (MNR) is a test for identification of outliers in a set of data \( x_1, x_2, \ldots, x_n \). The MNR can be calculated as:

\[
MNR = \max_i \left| \frac{x_i - \bar{x}}{s} \right|, \quad i = 1, 2, \ldots, n
\]

(1)

Where \( \bar{x} \) denoted as the sample mean and “s” as standard deviation. This number is compared to the critical value (CV) and then calculated for sample size \( n \) as:

\[
CV = \frac{n - 1}{\sqrt{n}} \sqrt{\frac{t^2}{n - 2 + t^2}}, \quad \text{where: } t = 1 - \frac{\alpha}{2n}
\]

(2)

\( \alpha \) is the significance level and recommended \( \alpha = 0.05 \). If the MNR is lower than CV (MNR < CV), then it can be concluded that no outlier was detected. If detected then it can be concluded that there was outlier and if that is detected then the sample must be removed from the calculation and the MNR test must be repeated [6].

2.2. K-sample Sample Anderson Darling Test

The k-sample Anderson Darling test is a nonparametric statistical procedure that refutes hypothesis about the contribution of two or more valid or homogeneous data groups. Hypothesis (H_0) is accepted if the ADK value is lower than the critical value (ADC). ADK can be calculated as follows by hypothesis [1], where H_0: group data from homogeneous populations and H_1: group data from inhomogeneous populations.

\[
ADK = \frac{n - 1}{n^2 (k - 1)} \sum_{i=1}^{k} \left\{ \frac{1}{n} \sum_{j=1}^{L} h_j \left( \frac{(nH_j - nF_j)^2}{H_j \left( n - H_j \right) - \frac{nh}{4}} \right) \right\}
\]

(3)

where [6]:
\( n \) = is the number of of combined samples,
\( k \) = is the number of batches,
\( L \) = is the number of observations,
\( h_j \) = is the number of values in the combined samples equal to \( z_j \),
\( H_j \) = is the number of values in the combined samples less than \( z_j \) plus one half the number of values in the combined samples equal to \( z_j \),
\( F_{ij} \) = is the number of values in the i-th group which are less than \( z_j \) plus one half the number of values in this group which are equal to \( z_j \).

Critical values of ADC can be calculated as below:
\[ ADC = 1 + \sigma_n \left[ 1.645 + \frac{0.678}{\sqrt{k-1}} - \frac{0.362}{k-1} \right] \] (4)

### 2.3. The Analysis of Variance (ANOVA)

The analysis of variance can be used for the calculation for the B-Basis value in the case of negative result of the k-sample ADK. The Analysis of Variance can be used only if the following assumptions are fulfilled [1] [6]:

- The data from each batch are normally distributed
- The within-batch/group variance is the same from batch to batch
- The batch/group are normally distributed.

One assumption for applying the ANOVA method is the equality of batch / group variants. In the Levene's test, the data must be transformed with the following equation. To perform this test, from the transformed data:

\[ w_{ij} = |x_{ij} - \bar{x}_i| \] (5)

Where \( \bar{x}_i \) is the median of the \( n_i \) values in the \( i \)-th group/batch. Then perform an F-Test on these transformed data. The F-Test is used for testing of batch mean (or variance) equality. In order to test the hypothesis if the sample batches have the same mean, the statistics must be calculated [1]:

\[ w_{ij} = |x_{ij} - \bar{x}_i| \] (5)

where \( \bar{x}_i \) is the mean of \( n_i \) values in the \( i \)-th group/batch and \( \bar{x} \) is the mean of all \( n \) observations. For the calculations of the B-Basis value, following calculations must be performed [1] [6]:

\[ n^* = \sum_{i=1}^{k} n_i^2 / n \] (7)

\[ n_i = (n - n^*) / (k - 1) \] (8)

\[ \bar{x} = \frac{\sum_{i=1}^{k} n_i \bar{x}_i}{n} \] (9)

\[ MSB = \frac{\sum_{i=1}^{k} n_i (\bar{x}_i - \bar{x})^2}{k - 1} \] (10)

\[ MSE = \frac{1}{n-k} \sum_{i=1}^{k} (n-k)s_i^2 \] (11)

Where \( n_i \) is the number of samples in the \( i \)-th, \( n \) denotes the total number of samples across \( k \) batches or groups. Furthermore, it is necessary to calculation the population standard deviation:

\[ S = \sqrt{\frac{MSB}{n} + \left( \frac{n-1}{n} \right) MSE} \] (12)

and also the ratio of mean squares defined as:

\[ u = \frac{MSB}{MSE} \] (13)

The tolerance limit factor can be calculated as follows:

\[ T = \frac{k_o - \frac{k}{\sqrt{n}} + (k_i - k_o)w}{1 - \frac{1}{\sqrt{n}}} \] (14)
where [1][6]:

\[
w = \sqrt{\frac{u}{u+n-1}}
\]

(15)

The parameters \(k_0\) and \(k_1\) are tolerance factors for simple random from a normal distribution with sample size \(n\) and size \(k\), respectively. Mentioned can be found in Tables One Sided B-Basis Tolerance Limits Factor [1]. The B-Basis value is:

\[
B = \bar{x} - T.S
\]

(16)

3. Result and Discussion

Based on the processing of data analysis on tabbing and non tabbing treatments, the strength and modulus results obtained with the MNR test showed that outliers were not detected. Furthermore, k-sample Anderson Darling test of all parameters tested was conducted to test the hypothesis with Levene’s test for equality of variance showed that the parameters of the batch or group data are from a homogeneous populations. Therefore, a B-Basis values can be obtained using statistical methods.

Table 1. B-Basis Values for Composite Material Test Result (Tabbing)

| Sample size | T-U\(^a\) 0 | T-U 90 | T-F\(^a\) 0 | T-F 90 | C-U\(^c\) 0 | C-U 90 | C-F\(^d\) 0 | C-F 90 |
|-------------|-------------|--------|-------------|--------|-------------|--------|-------------|--------|
| Mean        | 2930.91     | 51.88  | 917.30      | 821.15 | 1074.62     | 160.73 | 657.10      | 560.35 |
| Std. Dev    | 186.12      | 6.21   | 80.39       | 78.24  | 130.72      | 21.70  | 70.87       | 111.19 |
| Minimum     | 2537.86     | 40.46  | 634.27      | 647.38 | 813.75      | 119.40 | 501.66      | 257.27 |
| Maximum     | 3209.59     | 66.18  | 1042.08     | 976.90 | 1314.62     | 200.89 | 847.16      | 703.15 |

B-Basis Values

| Weibull\(^e\) | 2581 | 38   | 774   | 669   | 814   | 119   | 503   | 378   |
| ANOVA        | 2480 | 44   | 852   | 699   | 779   | 100   | 523   | 347   |
| a Tensile UD. |      |      | b Tensile Fabric. |      | c Compression UD. |      | d Compression Fabric. |      |

Table 2. B-Basis Values for Composite Material Test Result (Non Tabbing)

| Sample size | T-U\(^a\) 0 | T-U 90 | T-F\(^a\) 0 | T-F 90 | C-U\(^c\) 0 | C-U 90 | C-F\(^d\) 0 | C-F 90 |
|-------------|-------------|--------|-------------|--------|-------------|--------|-------------|--------|
| Mean        | 1966.17     | 66.27  | 859.53      | 734.29 | 813.25      | 163.57 | 703.40      | 695.63 |
| Std. Dev    | 807.67      | 10.33  | 190.38      | 123.33 | 153.21      | 21.63  | 50.57       | 100.3 |
| Minimum     | 266.07      | 29.61  | 444.64      | 182.99 | 496.99      | 118.61 | 600.45      | 463.97 |
| Maximum     | 2737.50     | 79.62  | 1036.63     | 897.26 | 1086.48     | 198.62 | 917.52      | 920.49 |

B-Basis Values

| Weibull\(^e\) | 699  | 51   | 597   | 551   | 541   | 124   | 545   | 501   |
| ANOVA        | -175 | 49   | 480   | 554   | 410   | 103   | 645   | 455   |
| a Tensile UD. |      |      | b Tensile Fabric. |      | c Compression UD. |      | d Compression Fabric. |      |

\(^a\) Tensile UD. \(^b\) Tensile Fabric. \(^c\) Compression UD. \(^d\) Compression Fabric. \(^e\) Weibull MLE.
Table 1 and 2 showed the tab specimens is much more reliable than the data provided by non-tab specimens. The tab specimens follow well the Weibull distribution; while the non-tab specimens produced negative values of the strength using ANOVA method which is impossible to happen.

4. Conclusion
Tabbing and non-tabbing specimens give significant differences or the variations in tabbing gives differences to the tensile and compressive strength of UD and Fabric composites. The results of data processing and analysis. comparison of three analyzes that have been done show negative values in the tensile strength UD 0° specimens with non-tab specimens. This is due to the inhomogeneous distribution of test results for non-tab specimens. Comparison of the three analytical methods. it can be concluded that the tab-specimens provide better tensile and compressive strength These should be taken into account for experimental engineers doing with composite materials tests.

Further research should include a larger sample for each batch or group, so that a higher confidence level of the mechanical properties data can be produced statistically.

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References
[1] MIL-HDBK-17, 2002. Military Handbook-MIL-HDBK-17-1F: Composite Materials Handbook; Volume 1. United State: U.S. Departement of Defense.
[2] Abdi, F., Clarkson, E., Godines, C., & DorMohammadi, S., 2016. A-B Basis Allowable Test Reduction Approach and Composite Generic Basis Strength Values. In 18th AIAA Non-Deterministic Approaches Conference, p. 0951. https://doi.org/10.2514/6.2016-0951
[3] Nam, K., Park, K. J., Shin, S., Kim, S. J., & Choi, I.-H. 2015. Estimation of Composite Laminate Design Allowables Using the Statistical Characteristics of Lamina Level Test Data. International Journal of Aeronautical and Space Sciences, vol.16 no.3, pp.360–69. https://doi.org/10.5139/ijass.2015.16.3.360
[4] Alqam, M., Bennett, R. M. & Zureick, A. H., 2002. Three-Parameter vs Two-Parameter Weibull Distribution for Pultruded Composites Material Composites. Composites Structures 58, pp. 497-503.
[5] Bek, L., Kottner, R., Krystek, J. & Las, V., 2016. Statistical Based Approach to Material Identification of Composite Materials. Srm, Czech Republic , 54th International Conference on Experimental Stress Analysis (EAN 2016).
[6] Bek, L., Kottner, R., Krystek, J. & Las, V., 2017. Calculation of B-Basis Values from Composite Material Strength Parameters Obtained from Measurement of Non-Identical Batches. Novy Smokovec, Slovakia , 54th International Conference on Experimental Stress Analysis (EAN 2017).