Sum-Rate Optimal Relay Selection and Power Control in Multi-Hop Networks

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Abstract

In this paper, we focus on the achievable sum-rate optimization problem of a multi-user, multi-hop relay network. We analyze the joint relay selection and power control in the presence of interference such that the achievable sum-rate is maximized. First, we evaluate the achievable sum-rate under five relay selection strategies when the transmit power is fixed. We show that the dynamic programming based max-min relay selection with the objective of maximizing the minimum signal-to-noise-ratio results in the highest achievable sum-rate gain for larger networks. Next, we combine the relay selection problem using the max-min relay selection and the power control problem using a tight lower bound approximation and propose a novel iterative algorithm, which maximizes the achievable sum-rate. We also provide a comprehensive comparison of the proposed algorithm with respect to existing resource allocation techniques, and observe that our proposed algorithm provides significant sum-rate gains. Finally, we prove that for the special case of two-user networks, binary power allocation is optimum for at least two transmitting nodes. Extensive numerical examples are provided to illustrate the accuracy of our results.

Index Terms

Achievable sum-rate maximization, relay selection, power control, multi-user, multi-hop.
I. INTRODUCTION

Relay networks have attracted interest in future wireless communication as they can improve the network performance by increasing the network capacity as well as extend the coverage area. In multi-hop relay networks, several relay nodes cooperate to establish the communication between the source node and its intended destination node. In a multi-user, multi-hop relay network, the interference and channel fading are two major factors that affect the network throughput. As such, the relay selection as well as the transmission power control is critical in gaining advantages in terms of performance, complexity and overhead. Therefore, much research has focused on the joint relay selection and power control in cooperative relay networks [1]. With increasing focus on the data communication, the maximization of overall achievable sum-rate has gain more attention in the recent literature [2].

In relay networks literature, the multi-hop relay selection problem has been mainly analyzed for single-user relay networks with the objective of minimizing the outage probability [3]–[6]. A dynamic programming based solution to obtain the optimal relay assignment in a decode-and-forward (DF) relay network is considered in [3]. Several sub-optimal relay selection strategies are also proposed in the literature to benefit from the partial channel state information. In [4], the authors propose a simple hop-by-hop relay selection strategy. This is later extended in [5], where the authors propose an ad-hoc relay selection strategy and a block-by-block relay selection strategy to achieve the full diversity and to improve the performance, respectively. Taking a step further, dynamic programming based relay selection strategies are considered in amplify-and-forward (AF) relay networks by approximating the effective signal-to-noise-ratio (SNR) of a given path [6], [7]. In terms of throughput optimization, most research has focused on the simple dual-hop relay networks where both hops can be easily combined together to compute the end-to-end achievable rate [8]–[10]. Taking a different approach, the capacity of a multi-hop relay network with a single relay node in each hop is considered in [11], [12]. Taking a step further, in [13], the authors analyze the achievable rate under the optimal relay selection and four sub-optimal relay selection strategies for a single-user DF relay network consists of multiple hops and multiple relay nodes in each hop.

With multiple source-destination (S-D) pairs, the relay selection problem becomes much more complicated, especially in DF relay networks due to the potential interference and the limitation of one relay node serving one S-D pair to minimize the synchronization requirements. For
example, in an ad-hoc sensor network, if idle nodes serve as relays, then they may cause severe interference to other transmissions in the area [14]. In the literature, this problem has been mainly analyzed with considerations to outage probability thus maximizing the minimum signal-to-interference-plus-noise-ratio (SINR) of all user pairs [15]–[17]. In [17], the authors propose a dynamic programming based efficient algorithm to obtain the optimal relay assignment of DF relay networks when the objective is maximizing the minimum SINR. Multi-user relay selection problem with the focus of maximizing the overall network sum-rate has been mostly analyzed for AF relay networks [18]–[20] while such analysis of DF relay networks are limited to dual-hop relay networks. In [2], [21], [22], orthogonal channels are adopted so that different S-D pairs do not interfere with each other. When there is no interference among S-D pairs, the relay selection problem of a dual-hop network simplifies to an assignment problem which can be solved using the well known Hungarian algorithm. On the other hand, in [23]–[25], the authors consider different approaches to estimate the interference in the second hop and then reformulate the relay selection problem as an updated assignment problem.

With respect to power control in multi-user relay networks, the most of the research has focused on AF relay networks [22], [26]–[31]. The early work of power control with respect to DF relaying consider selection of transmit powers from a code book given the average power constraints instead of considering the optimum power allocation [32]. Power control with respect to the minimization of the total energy consumption has been analyzed in [26], [33] where as in [34], the authors consider the power control problem of a two-way relay network with respect to the bit-error-rate (BER) performance. Power control with respect to the achievable sum-rate maximization is considered in [26] for AF relay networks, where the authors use geometric programming in the high SINR region. By taking a difference approach, in [31], the authors consider the distributed power control in relay networks with multiple S-D pairs and a single relay node and propose an algorithm based on the non cooperative game theory. Recently, most of the transmit power control consider SNR matching where the transmit power of each transmitting node in a given source-relay-destination (S-R-D) path is selected such that the SNR of each receiving node in that given path is equal. This simple power control approach provides the optimum power allocation when there is no interference between multiple S-D pairs, which can be achieved by considering orthogonal transmission between different S-D pairs [21], [30]. This approach is extended to two-way relaying networks in [27]–[29] for relay networks with a single-user pair and multiple relay nodes. It is shown that the optimum power control is achieved
when SNR is matched in both directions. Transmit power control in the presence of interference is considered in [1], where the authors reformulate the power control problem as a concave optimization problem for a given relay assignment by approximating the interference using its lower bound. In [25], the authors prove that optimum power allocation of two-user dual-hop relay networks can be obtained analytically.

In this paper, we consider a multi-hop DF relay network with multiple S-D pairs where each hop consists of multiple relay nodes and analyze the joint relay selection and power control problem to optimize the achievable sum-rate. The contributions of this paper are listed as follows.

- We consider five relay selection strategies that has been proposed for maximization of minimum SNR and analyze their suitability when the objective is achievable sum-rate maximization. This contribution is presented in Section III. We show that the dynamic programming based relay selection strategy with the objective of maximizing the minimum SINR achieves better sum-rate performance compared to other sub-optimal algorithms with the sum-rate maximization objective.

- As the main contribution, the joint relay selection and power control problem is considered for a general multi-user, multi-hop relay network and a sub-optimal algorithm that uses the dynamic programming based max-min relay selection and the tight lower bound approximation based power control is proposed. This result is presented in Algorithm 1. Furthermore, the performance of our proposed sub-optimal algorithm is compared against the existing resource allocation techniques, revealing that the proposed algorithm has better achievable sum-rate performance compared to the existing techniques.

- Under the special case of two-user multi-hop relay networks, we prove that the optimum power allocation such that the achievable sum-rate is maximized can be found analytically. This is achieved when at least two transmitting nodes transmit with binary power allocation. This contribution is presented in Theorem 1.

The rest of the paper is organized as follows. In Section II, we provide the system model and the optimization problem formulation for a multi-user, multi-hop DF relay network with multiple relay nodes in each hop. Next, the relay selection problem is analyzed in Section III while the proposed solution and the sub-optimal algorithm is given in Section IV with numerical examples in Section V. A special case of two-user network is analyzed with respect to power control in Section VI and finally, the conclusions are given in Section VII.
II. SYSTEM MODEL AND OPTIMIZATION PROBLEM FORMULATION

We consider a multi-user wireless relay network consists of $N$ S-D pairs as illustrated in Fig. 1. Source nodes $(s_1, s_2, ..., s_N)$ send information to corresponding destination nodes $(d_1, d_2, ..., d_N)$. The communication is assisted by a multi-hop relay network consists of $L$ hops with $M$ DF relays in each hop where $M \geq N$. As commonly used in the literature, we assume that each S-D pair is assisted by only one relay in each hop and each relay assists at most one S-D pair to minimize the synchronization requirements, to avoid too much processing complexity in any single relay and to minimize power consumption in the network [15], [35]. As such, we select $N$ relays from $M$ available relays in each hop and denote the relay selected for S-D pair $i$ in hop $l$ as $r_{i,l}$. Similar to [3]–[6], we assume that each node operates in half-duplex mode with a maximum transmit power of $P$ for each transmission and that the transmission are scheduled so that the cross-hop interference can be neglected.

We model the channel gain between transmitter $i$ and receiver $j$ in hop $l$ as a random variable denoted by $h[i,j,l]$. In general, this includes small scale fading, path loss and shadowing. For such a network, the received signal at node $j$ in hop $l$ can be written as,

$$ y[j,l] = \sum_{i=1}^{N} h[r_{i,l-1}, j, l] x[r_{i,l-1}, l - 1] + n[j,l] \tag{1} $$
where \( x[r_{i,l-1}, l-1] \) is the information symbol transmitted by node \( r_{i,l-1} \) in hop \( l-1 \), \( E\{|x[r_{i,l-1}, l-1]|^2\} = P[r_{i,l-1}, l-1] \) with \( P[r_{i,l-1}, l-1] \) denoting the transmitted power of node \( r_{i,l-1} \) in hop \( l-1 \) and \( n[j,l] \) is the additive white Gaussian noise at node \( j \) in hop \( l \) with mean zero and variance \( \sigma^2 \). Therefore, the received SINR at the selected relay corresponding to S-D pair \( i \) in hop \( l \) can be expressed in the form of,

\[
\gamma[i, l] = \frac{P[r_{i,l-1}, l-1]|h[r_{i,l-1}, r_{i,l}, l]|^2}{\sigma^2 + \sum_{j \neq i}^N P[r_{j,l-1}, l-1]|h[r_{j,l-1}, r_{i,l}, l]|^2},
\]

where \( r_{i,0} = r_{i,L} = i \). In DF relay networks, the end-to-end received SINR of a given S-D pair depends on the minimum SINR over all the hops. As such, the achievable rate for S-D pair \( i \) can be expressed as,

\[
R_i = \log_2 \left( 1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i, l]\} \right),
\]

for a given relay assignment and power allocation. Next, we formulate the achievable sum-rate optimization problem based on the joint relay selection and power control as,

\[
\max_{r_{i,l}, P[r_{i,l}, l]} \sum_{i=1}^N \log_2 \left( 1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i, l]\} \right)
\]

\[\text{s.t. } 0 \leq P[r_{i,l}, l] \leq P \forall l, i, \text{ where } l \in \{0, \ldots, L-1\}, \]

\[r_{i,l} \neq r_{j,l} \forall l, i \neq j, \text{ where } l \in \{1, \ldots, L-1\}, \]

\[r_{i,l} \in \{1, 2, \ldots, M\} \forall l, i, \text{ where } l \in \{1, \ldots, L-1\}, \]

where \( \gamma[i, l] \) is a function of \( r_{i,l} \) and \( P[r_{i,l}, l] \) as given in (2). The optimization problem in (4) is non-convex. This, combined with the integer nature of \( r_{i,l} \) and the availability of multiple hops makes this an extremely hard problem to solve for a general multi-user, multi-hop relay network. As such, we approach this optimization problem in two steps.

First, we consider the relay selection problem for a given transmit power allocation \( P[r_{i,l}, l] \forall i, l \) where \( l \in \{0, \ldots, L-1\} \) as,

\[
\max_{r_{i,l}, \ldots, r_{N,l}} \sum_{i=1}^N \log_2 \left( 1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i, l]\} \right)
\]

\[\text{s.t. } r_{i,l} \neq r_{j,l} \forall l, i \neq j, \text{ where } l \in \{1, \ldots, L-1\}, \]

\[r_{i,l} \in \{1, 2, \ldots, M\} \forall l, i, \text{ where } l \in \{1, \ldots, L-1\}. \]
Next, we consider the power control problem for a given relay assignment \([r_{1,l}, ..., r_{N,l}], \forall l\) where \(l \in \{1, ..., L - 1\}\) as,

\[
\max_{P[r_{i,l},l]} \sum_{i=1}^{N} \log_2 \left(1 + \min_{l \in \{1, ..., L\}} \{\gamma[i,l]\}\right)
\]

\[s.t \quad 0 \leq P[r_{i,l},l] \leq P, \quad \forall i, l, \text{ where } l \in \{0, ..., L - 1\}. \tag{6}\]

It is important to note that, for a general multi-user, multi-hop relay network, solving each optimization problem in (5) and (6) separately is still a challenging problem [23], [26]. In the following, we first consider the optimization problems in (5) and (6), separately and then propose an iterative algorithm that combines the proposed solutions in order to provide a novel joint solution.

III. Relay Selection

In this section we focus on the optimization problem formulated in (5). We note that the optimum relay assignment would involve selecting \(N\) non-overlapping S-R-D paths such that each user has one distinct path and the achievable sum-rate of all users are maximized. For a relay network with \(M\) relays and \(L\) hops there are \(M^{L-1}\) possible S-R-D path combinations for each user and \(\prod_{i=0}^{N-1} (M - i)^{L-1}\) possible paths for \(N\) users [15]. The achievable sum-rate depends on the combination of \(N\) SINR values instead of the effective SINR of each user. As a result, for a given relay assignment in a given hop, the best path and its respective \(N\) SINR values depend on the relay selection in past and future hops. Therefore, unlike the single-user network, we cannot use dynamic programming based approach (ex: Viterbi algorithm) to find the optimum relay assignment in a multi-user relay network. As such, the optimal relay selection involves exponential complexity and different sub-optimal relay selection strategies with low complexity have been considered in the literature. Therefore, we consider four sub-optimal relay selection strategies used in literature namely, the hop-by-hop relay selection [4], ad-hoc relay selection, block-by-block relay selection [5] and sliding window based relay selection [6]. In addition, we evaluate the achievable sum-rate obtained by the optimal relay selection when the objective is maximizing the minimum SINR [17]. These strategies are detailed in the following.

A. Hop-by-Hop Relay Selection

Under this strategy, the relay selection in each hop is performed independently such that the achievable sum-rate of the current hop is maximized when the signals are transmitted from the
relays selected in the previous hop. Therefore, with this strategy, there is no sum-rate optimization involved in the last hop where the destination nodes are fixed. Since, the hop-by-hop relay selection cannot achieve full-diversity, we next consider the ad-hoc relay selection.

B. Ad-hoc Relay Selection

Under this strategy, the hop-by-hop relay selection is extended by combining the last two hops together to achieve full diversity. Therefore, while the first \( L - 2 \) relays for each user are selected similar to the hop-by-hop relay selection, the last relay for each user is selected such that the achievable sum-rate of the last two hops is maximized.

C. Block-by-Block Relay Selection

Next, we consider the block-by-block relay selection to improve the performance further. Under this strategy, \( L \) hops are divided into non-overlapping blocks of \( w \) hops and the relays are selected such that the achievable sum-rate of each block is maximized. With this strategy, it is important to ensure that the block size is selected such that the last block would be greater than one. Otherwise, there will be no sum-rate optimization involved with the last hop where the destination nodes are fixed.

D. Sliding Window Based Relay Selection

Taking a step further, we consider the sliding window based relay selection to remove the dependency of block size on the number of hops and to improve the performance further. Under this strategy, we consider a sliding window of \( w \) hops to determine the relay selection in the first hop of the window. For example, we start by considering the first \( w \) hops and find the relay selection such that the achievable sum-rate in those \( w \) hops is maximized. However, we only fix the relays selected in the first hop. Next, we consider \( w \) hops from the second hop to \( w + 1 \) and fix the relays selected for the second hop. We continue this until relay selection is fixed for first \( L - w \) hops. Then we consider the last \( w \) hops and fix the relays selected for all of them.

E. Max-Min Relay Selection

Finally, we consider the max-min relay selection that is used to minimize the outage probability. Under this strategy, we consider the optimal relay selection when the objective is maximizing the minimum SINR across all users. We use the dynamic programming based algorithm, which
has a linear complexity with respect to $L$, proposed in [17]. Once the relay selection is completed, we compute the achievable sum-rate according to the objective function in (5).

We first note that when the objective is maximizing the minimum SINR, the final objective value depends only on the value of the objective function for each block of hops. As such, it is shown that the performance improves with the hop-by-hop relay selection, ad-hoc relay selection, block-by-block relay selection and sliding window based relay selection, respectively [6]. However, when the objective is the achievable sum-rate maximization, the final effective sum-rate does not only depend on the achievable sum-rate of each block of hops. It need to be computed based on the effective minimum SINR for each user. As a result, we cannot guarantee that any of these sub-optimal relay selection strategies are always better than the others [36]. Therefore, in the following example, we compare the average performance of the five relay selection strategies considered in this paper.

**Example:** Consider a two-user ($N = 2$), multi-hop relay network where the channels between nodes follow a Rayleigh distribution with zero mean and unit variance. For such a network, the gain in achievable sum-rate obtained based on different relay selection strategies compared to that of the hop-by-hop relay selection is given in Table I. We consider different $M$ and $L$ values with $w = 2, 4$ and the average received SNR of $10$ dB. In order to maintain full diversity gain, we only consider block-by-block relay selection when $L$ can be fully divided by $w$. This introduces one limitation of the block-by-block relay selection, where the block size $w$ needs to be selected depending on the number of hops $L$. From the table, we observe that when $L = 2$, the ad-hoc relay selection, block-by-block relay selection and sliding window based relay selection have same achievable sum-rate gains. When $L = 2$, all three relay selection strategies are equivalent to the optimum relay selection which considers both hops together. Thus, all three relay selection strategies result in same achievable sum-rate. Similarly, when $L = 4$ both the block-by-block relay selection and sliding window based relay selection have same achievable sum-rate gains with $w = 4$. For any other $L$, the sliding window based relay selection has better achievable sum-rate gain for a given $M$ and $w$ compared to the ad-hoc relay selection and block-by-block relay selection. Further, we can observe that the sliding window based relay selection has much higher achievable sum-rate gain compared to the block-by-block relay selection when $w = 4$. We can also observe that unlike the block-by-block relay selection, the sliding window based relay selection with $w = 4$ has less sensitivity to increasing $L$ compared to $w = 2$. Therefore,
Table I: Achievable sum-rate gain percentage compared to hop-by-hop relay selection

|                  | Sliding window w=2 | Sliding window w=4 | Block-by-block w=2 | Block-by-block w=4 | Ad-hoc | Max-Min |
|------------------|---------------------|--------------------|--------------------|--------------------|--------|---------|
| M=2, L=2        | 11.767              | -                  | 11.767             | -                  | 11.767 | 2.410   |
| M=2, L=4        | 16.303              | 28.058             | 11.694             | 28.058             | 10.010 | 6.442   |
| M=2, L=6        | 14.724              | 34.013             | 8.549              | -                  | 7.578  | 7.383   |
| M=2, L=8        | 12.408              | 37.516             | 6.773              | 23.073             | 7.139  | 7.026   |
| M=2, L=10       | 8.140               | 36.764             | 3.599              | -                  | 5.919  | 5.877   |
| M=2, L=12       | 4.981               | 36.540             | 1.784              | 13.949             | 4.732  | 5.043   |
| M=3, L=2        | 30.163              | -                  | 30.163             | -                  | 30.163 | 13.942  |
| M=3, L=4        | 28.459              | 52.483             | 24.806             | 52.483             | 21.303 | 30.744  |
| M=3, L=6        | 28.098              | 53.833             | 23.716             | -                  | 19.015 | 45.387  |
| M=3, L=8        | 25.217              | 52.207             | 19.926             | 48.881             | 17.338 | 52.996  |
| M=3, L=10       | 22.010              | 48.363             | 17.262             | -                  | 15.686 | 58.645  |
| M=3, L=12       | 18.897              | 47.343             | 14.863             | 42.048             | 14.203 | 65.579  |
| M=4, L=2        | 40.283              | -                  | 40.283             | -                  | 40.283 | 22.234  |
| M=4, L=4        | 37.763              | 68.725             | 33.768             | 68.725             | 29.983 | 49.917  |
| M=4, L=6        | 33.862              | 63.154             | 30.075             | -                  | 25.896 | 59.840  |
| M=4, L=8        | 32.566              | 62.072             | 26.411             | 59.602             | 23.585 | 68.946  |
| M=4, L=10       | 29.260              | 59.554             | 23.866             | -                  | 22.135 | 75.536  |
| M=4, L=12       | 26.786              | 57.770             | 21.395             | 51.908             | 20.963 | 81.644  |

we can conclude that increasing $w$ provides higher performance improvements for the sliding window based relay selection compared to the block-by-block relay selection.

When $M = 2$ and $w = 2$, we can also observe that the achievable sum-rate gain of the block-by-block relay selection is worse than that of the ad-hoc relay selection with increasing $L$. However, with larger $M$ block-by-block relay selection is slightly better than the ad-hoc relay selection. Similarly, when $M = 2$, the max-min relay selection has the lowest achievable sum-rate gain for any given $L$. However, with increasing $M$, it outperforms all other relay selection strategies. We can also observe that for larger $M$ and $L$ values, the simple max-min relay selection provides better achievable sum-rate gains compared to the sliding window based relay selection even with $w = 4$. In addition, we can also observe that with increasing $M$, the achievable sum-rate gain of all four strategies increases for a given $L$ and $w$. This can be explained by the improved diversity introduced by increasing $M$. On the other hand, with increasing $L$, the achievable sum-rate gain of the ad-hoc relay selection, block-by-block relay
selection and sliding window based relay selection decreases for a given $M$ and $w$ where as that of the max-min relay selection increases for a given $M$. This can be explained by the fact that the max-min relay selection considers the channel gains of all $L$ hops while the other three strategies are unaware of the future channel gains when making the relay selection decision. Therefore, the performance of other relay selection strategies deteriorate with increasing $L$. As such, even though the objective function is different, the consideration of all hops improves the achievable sum-rate obtained with the max-min relay selection.

From the above example, we realized that the simple max-min relay selection strategy, which has a linear complexity with respect to $L$, provides better achievable sum-rate performance for larger relay networks where $M > N$ and $L > 2$. As the optimal relay selection can be found via exhaustive search for smaller networks, in this paper we focus on the relay selection of larger multi-user, multi-hop relay networks. As such, we propose the use of max-min relay selection strategy to solve the relay selection problem in (5).

IV. JOINT RELAY SELECTION AND POWER CONTROL

In section III we analyzed the relay selection problem. Therefore, in this section we first focus on the optimization problem formulated in (6) and present an iterative power control algorithm. In general, the achievable sum-rate optimization problem given in (6) is non-convex with respect to $P[r_{i,l}, l]$ [26]. Therefore, we consider the tight lower bound approximation in [37], [38] and approximate its objective function as,

$$
\sum_{i=1}^{N} \log_2 \left( 1 + \min_{l \in \{1, ..., L\}} \{\gamma[i, l]\} \right) \geq \frac{1}{\log(2)} \sum_{i=1}^{N} a_i \log \left( \min_{l \in \{1, ..., L\}} \{\gamma[i, l]\} \right) + b_i,
$$

that is tight at a chosen value $\bar{z} = [\bar{z}_1, ..., \bar{z}_N]$ when the constants $a_i$ and $b_i$ are chosen as,

$$
a_i = \frac{\bar{z}_i}{1 + \bar{z}_i}, \quad b_i = \log(1 + \bar{z}_i) - \frac{\bar{z}_i}{1 + \bar{z}_i} \log(\bar{z}_i).
$$

By selecting $\bar{z}_i$ as the end-to-end received SINR for S-D pair $i$, achieved using the initial solution or the solution achieved via the previous iteration, we can re-write the achievable sum-rate optimization problem given in (6) as,

$$
\max_{P[r_{i,l}, l], \forall i, l} \sum_{i=1}^{N} a_i \log \left( \min_{l \in \{1, ..., L\}} \{\gamma[i, l]\} \right) + b_i
$$

s.t. $0 \leq P[r_{i,l}, l] \leq P$, $\forall i, l$, where $l \in \{0, ..., L - 1\}$.
In order to convert this non-convex objective function into a concave function we use the variable transformations $P[r_{i,l}, l] = e^{q[r_{i,l}, l]}$ and $t[i] = \log \left( \min_{l \in \{1, \ldots, L\}} \{ \gamma[i, l] \} \right)$ and reformulate (8) as,

$$
\max_{q[r_{i,l}, l] \forall i,l} \sum_{i=1}^{N} a_i t[i] + b_i \\
\text{s.t.}
$$

$$
t[i] \leq q[r_{i,l}, l] + \log(\|h[r_{i,l}, r_{i,l+1}, l+1]\|^2) - \log \left( \sigma^2 + \sum_{j \neq i} e^{q[r_{j,l}, l]} \|h[r_{j,l}, r_{i,l+1}, l+1]\|^2 \right), \forall i, l,
$$

$$
q[r_{i,l}, l] \leq \log(P), \forall i, l,
$$

(9)

where $l \in \{0, \ldots, L - 1\}$. For a given relay assignment, the optimization problem (9) is concave. Therefore, in each iteration, we can compute the coefficients $a_i$ and $b_i$ based on the solution of the previous iteration and solve the above problem using any existing convex solver or by implementing a gradient decent algorithm. Since, the tight lower bound approximation considered here results in a monotonically improving objective, the sequence always converges [37], [38]. Therefore, (9) can be solved iteratively to find the optimum approximated achievable sum-rate for a given relay assignment.

Next, we combine the iterative power control solution with relay selection to focus on the joint relay selection and power control problem of a multi-user, multi-hop relay network and propose Algorithm 1 that maximizes the achievable sum-rate. At the start of Algorithm 1 we initialize all transmit powers to $P$ and the maximum achievable sum-rate $R^*$ to zero. In each iteration, we first solve the relay selection problem for a given transmit power allocation using the dynamic programming based max-min relay selection and assign the relays selected to a matrix denoted by $X$. If the resultant achievable sum-rate is higher than the maximum achievable sum-rate, we assign $X$ to the optimum relay assignment matrix denoted by $X_{opt}$. Therefore, after the first iteration, the relay selection is changed only if the resultant achievable sum-rate is higher for a different relay selection under the new transmit power allocation. Then we proceed to iteratively solve the power control problem. In the $m^{th}$ iteration, we solve the concave optimization problem in (9) and assign the solution to matrix $Q_{L \times N}^{(m)} = \{q[r_{1,l}, l], \ldots, q[r_{N,l}, l]\}$, $\forall l$, where $l \in \{0, \ldots, L - 1\}$. Then the calculated error $e$ is compared against a user defined threshold $e_{th}$. The tight lower bound approximation monotonically improves the objective function and always converges [38]. Thus, the achievable sum-rate improves within each iteration of the inner loop and the transmit power matrix is changed at iteration $n$ only if the resultant achievable sum-rate is higher for
Algorithm 1: The Proposed Iterative Joint Relay Selection and Power Control Algorithm

Input: Instantaneous channel state information (CSI), $P$

Output: Maximum achievable sum-rate $R^*$, optimum relay assignment $X_{opt}$ and power allocation $P[r_i,l], \forall i, l$

1 $n = 1$, $X_{opt} \leftarrow \{\}$, $R^* \leftarrow 0$, $P[r_i,l] \leftarrow P, \forall i \in \{1, ..., N\}, l \in \{0, ..., L - 1\}$

2 while true do

3 \[X, R^{(n)}] \leftarrow \text{solution to relay selection problem via [17, Algorithm 1]}

4 if $(R^{(n)} - R^*)/R^{(n)} > e_{th}$ then

5 $r_i,l \leftarrow X(l, i) \forall i \in \{1, ..., N\}, l \in \{1, ..., L - 1\}$

6 $R^* \leftarrow R^{(n)}$, $X_{opt} \leftarrow X$

7 $m = 1$

8 while true do

9 $Q^{(m)} \leftarrow \text{solution to problem (9)}$

10 $e \leftarrow |Q^{(m)} - Q^{(m-1)}|/|Q^{(m)}|$

11 if $e < e_{th}$ then

12 \hspace{1cm} break

13 $m \leftarrow m + 1$

end

14 $P[r_i,l] = e^{q[r_i,l]}, \forall i \in \{1, ..., N\}, l \in \{0, ..., L - 1\}$

15 $R^{(n)} \leftarrow \text{achievable sum-rate for } r_i,l \text{ and } P[r_i,l], \forall i, l$

16 if $(R^{(n)} - R^*)/R^{(n)} > e_{th}$ then

17 \hspace{1cm} $R^* \leftarrow R^{(n)}$, $n \leftarrow n + 1$

18 else

19 \hspace{1cm} break

21 end

a different transmit power allocation under the new relay assignment. Therefore, in the $n^{th}$ iteration of the outer loop, the achievable sum-rate improves monotonically until it converges to a solution. We note that the optimization problem (4) is non-convex and hence there might exist multiple local optima. Since the objective function monotonically improves in each iteration it eventually converges to one of the local solutions. We consider the proposed algorithm to be
sub-optimal as we cannot guarantee that the converged solution is the global optimum solution. In implementation, we stop the algorithm and consider it to have converged when the relative difference between the achievable sum-rate in \(n\)th iteration and \((n+1)\)th iteration is less than a user defined threshold.

V. NUMERICAL AND SIMULATION RESULTS

In this section, we present simulation results to evaluate the performance of the proposed sub-optimal algorithm.

We note that the joint relay selection and power control problem has not been considered in the literature for a multi-user, multi-hop relay network when the objective is maximization of the achievable sum-rate. Therefore, the performance of our proposed algorithm is compared against two commonly used reference techniques. Under the first reference technique, we perform naive relay selection where the first user selects the best path without considering the interference from other users. Then the second user selects the best path from the remaining set of relays without considering the interference from other users and so on. We use dynamic programming based optimal relay selection proposed in [3] to find the best path for each user. Then we consider power control via SINR matching. By SINR matching we simply mean that the transmit power of each transmitting node in the S-R-D path of each user is selected such that the SINR of each receiving node in that path is equal. Under the second reference technique, we perform random relay assignment where each user randomly selects a relay path without any conflict. Similarly, power control is performed via SINR matching.

We consider a multi-user multi-hop relay network where the channels between nodes follow a Rayleigh distribution with unit variance. We assume equal distance between hops with the distance between source nodes and destination nodes set to 2 km and the mean of the channel gain is calculated using the standard path-loss model [39] as,

\[
E|h_L|^2 = \left(\frac{c}{4\pi f_c}\right)^2 d_L^{-\beta},
\]

where \(c\) is the speed of light, \(f_c\) is the carrier frequency, \(d_L\) is the distance between the transmitter and the receiver of link \(L\), \(\beta\) is the path loss exponent and \(h_L\) is the instantaneous channel fading gain of link \(L\). For these examples, we set \(f_c = 1.9\) GHz and \(\beta = 3.6\) which corresponds to urban areas [40]. We also set the user defined threshold value for Algorithm 1 as \(e_{th} = 10^{-3}\) for all the simulation examples. AWGN noise variance is computed as \(\sigma^2 = kT_0B\), where \(k\) is
the Boltzmann’s constant, \( T_0 = 290 \) Kelvin is the ambient temperature and \( B = 200 \) kHz is the equivalent noise bandwidth.

Fig. 2 plots the achievable sum-rate versus \( P \) with \( N = 2, M = 4, 6 \) and \( L = 6 \), when Algorithm 1 based joint optimization is employed. From the plot, we observe that the achievable sum-rate of the proposed algorithm linearly increases with the maximum transmit power while both reference techniques have almost constant achievable sum-rate. Relay selection in these two reference techniques does not depend on the interference. Any increase in maximum transmit power increases both the received SNR and the received interference. As such, there would be no gain in the achievable sum-rate. In addition, we can also observe that the achievable sum-rate increases with \( M \). As \( M \) increases, the number of available relay combinations increases. As a result, the probability of a user selecting a relay path with larger gain and lower interference increases as well, thus increasing the achievable sum-rate. Fig. 3 plots the achievable sum-rate versus \( P \) with \( N = 2, 4, M = 6 \) and \( L = 6 \), when Algorithm 1 based joint optimization is employed. From the plot, we observe that the achievable sum-rate of the proposed algorithm increases with \( N \). As \( N \) increases, the number of summation terms in (4) increases. Even though individual rate of each user decreases due to the interference, when relay selection is performed with consideration to interference, we can increase the overall network sum-rate. However, as the naive relay selection does not consider interference when performing relay selection, the achievable sum-rate of the naive relay selection decreases with \( N \). On the other hand, the random relay selection does not depend on the number of users. As a result, the proposed algorithm improves the achievable sum-rate of multi-user relay networks in the presence of interference in comparison to existing relay selection solutions.

Fig. 4 plots the achievable sum-rate versus \( L \) with \( N = 2, M = 6 \) and \( P = 10 \) dB, when Algorithm 1 based joint optimization is employed. From the plot, we observe that the achievable sum-rate of the proposed algorithm increases with \( L \) while that of the two reference techniques slightly decreases with \( L \). As \( L \) increases, the distance between transmitting and receiving nodes of a given hop decreases. This reduces the path loss between two nodes, thus increasing the achievable sum-rate. However, as the two reference techniques make their relay selection decision purely based on the SNR, there is high probability of selecting a relay path with high interference. As the reduction in path loss increases the interference as well, the achievable sum-rate slightly decreases with \( L \) for the naive relay selection and random relay selection.

Fig. 5 plots the average computation time of Algorithm 1 and reference techniques versus
Fig. 2: Achievable sum-rate versus $P$ with $N = 2, L = 6$

Fig. 3: Achievable sum-rate versus $P$ with $M = 6, L = 6$ dB
when $N = 2, M = 6$ and $P = 10$ dB. From the plot, we observe that the computation time of the proposed algorithm increases with $L$. We also note that the computation times of the two reference techniques are significantly smaller than our proposed algorithm. As such, there is a clear trade-off between the complexity and the achievable sum-rate performance. As both relay selection strategies used in the naive relay selection and the proposed algorithm have linear complexity with respect to $L$, the difference in the computation time is due to the iterative approach considered in Algorithm 1. Therefore, we next analyze the complexity of the proposed algorithm in terms of the number of iterations for convergence.

Fig. 6 plots the total number of iterations including both inner and outer loops of Algorithm 1 versus $L$ when $N = 2, M = 6$ and $P = 10$ dB. From the figure, we observe that as $L$ increases, the number of iterations increases as well. Since, the increment in the number of iterations is linear, we can conclude that the proposed algorithm has linear complexity with respect to the total number of iterations.

VI. SPECIAL CASE OF TWO-USER NETWORK

Let us now consider the special case of two-user networks. In [25], the authors prove that optimum power allocation of two-user dual-hop relay networks can be obtained analytically. We
Fig. 5: Average computation time versus $L$ with $N = 2, M = 6, P = 10$ dB

Fig. 6: Average number of iterations versus $L$ with $N = 2, M = 6, P = 10$ dB
take a step further and extend this result for a two-user multi-hop relay network. First, we prove the following Lemma.

**Lemma 1.** For a two user multi-hop relay network, there exists an optimum power vector which maximizes the overall achievable sum-rate, such that for each user the resulting SINRs in all the hops are equal.

*Proof.* Please refer to Appendix A.

Next, based on Lemma 1, we can develop the following theorem.

**Theorem 1.** For a multi-hop DF relay network with two users, binary power allocation is optimum for at least two transmitting nodes. Transmit powers of the remaining nodes can be found via SINR matching.

*Proof.* According to Lemma 1, at the optimum achievable sum-rate, each user can have equal SINRs in all the hops. Therefore, the achievable sum-rate optimization problem can be expressed as,

\[
\max_{P[r_0, 0], P[r_0, 1], P[r_1, 0], P[r_1, 1]} \left( 1 + \frac{P[r_0, 0][h[r_0, r_1, 1]]^2}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} \right) \left( 1 + \frac{P[r_0, 0][h[r_0, r_1, 1]]^2}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} \right)
\]

subject to

\[
\frac{P[r_0, 0][h[r_0, r_1, 1]]^2}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} = \frac{P[r_1, 0][h[r_1, r_2, l]]^2}{\sigma^2 + P[r_1, 0][h[r_1, r_2, l]]^2}, \quad \forall l \in \{2, \ldots, L\},
\]

\[
\frac{P[r_2, 0][h[r_2, r_1, 1]]^2}{\sigma^2 + P[r_2, 0][h[r_2, r_1, 1]]^2} = \frac{P[r_2, 0][h[r_2, r_1, l]]^2}{\sigma^2 + P[r_2, 0][h[r_2, r_1, l]]^2}, \quad \forall l \in \{2, \ldots, L\},
\]

\[
0 \leq P[r_1, 0], P[r_1, 1] \leq P, \quad \forall l \in \{0, \ldots, L-1\}
\]  

(11)

Since the objective function and the equality constraints are twice differentiable with respect to \(P[r_1, l], P[r_2, l] \forall l\), we can re-write (11) as an unconstrained optimization problem using the Lagrangian dual as,

\[
\max_{\lambda_0^l, \lambda_2^l} \min_{P[r_1, l], P[r_2, l], \forall l} \left[ -1 - \frac{P[r_1, 0][h[r_0, r_1, 1]]^2}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} \left( \sum_{l=1}^{L-1} \frac{\lambda_0^l}{\sigma^2 + P[r_2, 0][h[r_0, r_1, 1]]^2} + \frac{\lambda_2^l}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} \right) \right]
\]

\[
\max_{\lambda_0^l, \lambda_2^l} \min_{P[r_1, l], P[r_2, l], \forall l} \left[ -1 - \frac{P[r_1, 0][h[r_0, r_1, 1]]^2}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} \left( \sum_{l=1}^{L-1} \frac{\lambda_0^l}{\sigma^2 + P[r_2, 0][h[r_0, r_1, 1]]^2} + \frac{\lambda_2^l}{\sigma^2 + P[r_0, 0][h[r_0, r_1, 1]]^2} \right) \right]
\]

subject to

\[
0 \leq P[r_1, 0], P[r_1, 1] \leq P, \quad \forall l \in \{0, \ldots, L-1\},
\]  

(12)
and $\lambda_1^{(l)}, \lambda_2^{(l)} \forall l$ where $l \in \{1, \ldots, L-1\}$ are the Lagrangian multipliers. Note that in (12), the minimization of the negative achievable sum-rate is considered. Therefore, the maximum achievable sum-rate is achieved when the objective function of (12) is minimized. In the following, we denote the objective function of (12) by $f$, which is a variable of $P[r_{1,l}, l]$ and $P[r_{2,l}, l] \forall l$ where $l \in \{0, \ldots, L-1\}$. Since the Lagrangian multipliers are related to the equality constraints, at the optimum solution of (12) they can have any real value. Whilst not given here due to page limitations, by analyzing the first and the second derivatives of $f$ with respect to these variables, we can show that at least for two variables either the first derivative cannot be zero (i.e., they are either increasing or decreasing functions, indicating that the achievable sum-rate is maximized at the corner points) or the second derivative is not positive (indicating that any existing critical point would be a local maximum of $f$). This implies that the achievable sum-rate is maximized at the corner points for at least two of the variables out of $P[r_{1,l}, l]$ and $P[r_{2,l}, l] \forall l$ where $l \in \{0, \ldots, L-1\}$. Therefore, we can conclude that irrespective of the value of the Lagrangian multipliers, at least for two transmitting nodes binary power allocation is optimum. Since, the $2(L-1)$ equality constraints in (12) connect all $2L$ power values, the other $2(L-1)$ power values can be obtained solving those equations. This concludes the proof of Theorem 1.

As such, for a two user network, it is possible to find the optimum solution for the power control problem given in (6) with Theorem 1.

VII. Conclusion

We considered the achievable sum-rate optimization problem in a general multi-user, multi-hop relay network with multiple relay nodes in each hop. First, we investigated the suitability of five sub-optimal relay selection strategies that have been considered for single-user multi-hop relay networks, namely the hop-by-hop relay selection, ad-hoc relay selection, block-by-block relay selection, sliding window based relay selection and max-min relay selection. It is shown that the dynamic programming based max-min relay selection with the objective of maximizing the minimum SINR results in higher achievable sum-rate gain compared to other sub-optimal relay selection strategies with the objective of maximizing the achievable sum-rate. Next, we combined the max-min relay selection and the tight lower bound approximation based power control to present a novel iterative algorithm. Our proposed algorithm performs joint relay assignment and power control in a such a way that the achievable sum-rate is maximized. Further, we proved that for the special case of two-user networks, binary power allocation is optimum for at least
two transmitting nodes. Transmit power of other nodes can be obtained by considering that the received SINR of each user is equal over all the hops.

The interference management technique that treats interference as noise is close to optimal when the interference is sufficiently weak. In addition, that reduces the complexity involved with the receivers compared to successive interference cancellation. Therefore, in this work, we considered that single user decoding is performed at each receiver with interference treated as noise. However, with the discussion of the research community moving towards successive interference cancellation, a desirable extension would be to consider joint relay selection and power control in the presence of successive interference cancellation.

APPENDIX A
PROOF OF LEMMA 1

In this section we provide the proof of Lemma 1. Let $R^*$ denotes the optimum achievable sum-rate that results from the transmit power vector $[P_{r_1,l}^*, P_{r_2,l}^*]$ $\forall l \in \{0, ..., L - 1\}$ with $P_{r_1,l}^*$ and $P_{r_2,l}^*$ denoting the optimum transmit powers of nodes $r_{1,l}$ and $r_{2,l}$ in hop $l$, respectively. Let the resulting optimum SINRs for the $l$th hop and the resulting overall minimum optimum SINRs of $s_1$ and $s_2$ be denoted by $\gamma_{[1,l]}^*$, $\gamma_{[2,l]}^*$, $\gamma_1^*$ and $\gamma_2^*$, respectively.

As a result, we can write $R^* = \log_2(1 + \gamma_1^*) + \log_2(1 + \gamma_2^*)$. We start the proof by assuming that the two users do not have equal SINRs in all the hops at the same time, i.e, $\gamma_{[1,l]}^* = \gamma_1^*$ and $\gamma_{[2,l]}^* = \gamma_2^*$ for all $l \in \{1, ..., L\}$ does not happen simultaneously. In the following, we consider the two possible scenarios resulting from the above assumption.

Scenario 1 - Only one user has equal SINRs in all the hops

Without the loss of generality, let us assume that the first user has a higher SINR in the first hop such that $\gamma_{[1,1]}^* > \gamma_1^*$ and the second user has equal SINRs in all the hops. Next, we change the power values for $s_1$ and $s_2$ as $P_{r_{1,0}} = P_{r_{1,0}}^* - x_1$ and $P_{r_{2,0}} = P_{r_{2,0}}^* - y_1$ such that $\gamma_{[1,1]} = \gamma_1^*$ and $\gamma_{[2,1]} = \gamma_2^*$. Based on the values of $x_1, y_1$ and considering the fact that $\gamma_{[1,1]}^* > \gamma_1^*$, it can be shown that the new power values $P_{r_{1,0}}$ and $P_{r_{2,0}}$ falls within $0$ and $P$. Therefore, we can achieve $\gamma_{[1,1]} = \gamma_1^*$ and $\gamma_{[2,1]} = \gamma_2^*$ for the same optimum achievable sum-rate $R^*$. Likewise, We can update the transmit power values of any hop where the first user has a higher SINR without changing $R^*$ following a similar approach.
**Scenario 2 - None of the users have equal SINRs in all the hops**

Under this scenario, two users either can have their corresponding higher SINRs in the same hop or in two different hops.

Let us first consider the situation where two users have their corresponding higher SINRs in the same hop. Without loss of generality, let us assume that both users have higher SINRs in the first hop such that $\gamma[1,1]^* > \gamma_1^*$ and $\gamma[2,1]^* > \gamma_2^*$. Next, we change the power values for $s_1$ and $s_2$ as $P[r_{1,0},0] = P[r_{1,0},0]^* - x_2$ and $P[r_{2,0},0] = P[r_{2,0},0]^* - y_2$ such that $\gamma[1,1] = \gamma_1^*$ and $\gamma[2,1] = \gamma_2^*$. Similar to scenario 1, we can show that the new power values $P[r_{1,0},0]$ and $P[r_{2,0},0]$ falls within 0 and $P$. Therefore, we can achieve $\gamma[1,1] = \gamma_1^*$ and $\gamma[2,1] = \gamma_2^*$ for the same optimum achievable sum-rate $R^*$. Again, we can update the transmit power values of any hop where both users have higher SINRs in the same hop without changing $R^*$ following a similar approach.

Let us now consider the situation where two users have their corresponding higher SINRs in two different hops. Without loss of generality, let us assume that the first user has a higher SINR in the first hop and the second user has a higher SINR in the second hop such that $\gamma[1,1]^* > \gamma_1^*$ and $\gamma[2,2]^* > \gamma_2^*$. Next, we change the power value for $s_1$ as $P[r_{1,0},0] = P[r_{1,0},0]^* - x_3$ such that $\gamma[1,1] = \gamma_1^*$. Similar to scenario 1, we can show that $0 < P[r_{1,0},0] < P$. However, this would results in $\gamma[2,1] > \gamma_2^*$. Therefore, after updating the transmit power values of any hop with a higher SINR for one user, we will have a network with only one user having equal SINRs which is considered under scenario 1.

The fact that under both above scenarios, we can achieve the same achievable sum-rate $R^*$ such that $\gamma[1,l] = \gamma_1^*$ and $\gamma[2,l] = \gamma_2^*$ for all $l \in \{1,...,L\}$ completes the proof of Lemma 1.

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