Dynamics of Public Opinion Evolution with Asymmetric Cognitive Bias

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Abstract—In this paper, we propose a public opinion model with incorporation of asymmetric cognitive bias: confirmation bias and negativity bias. We then investigate the generic modeling guidance of capturing asymmetric confirmation bias and negativity bias. A numerical examples is provided to demonstrate the correctness of asymmetric cognitive bias model.

Index Terms—Asymmetric confirmation bias, asymmetric negativity bias, social networks.

I. INTRODUCTION

While opinion evolution models have always been an active research area, recently with the wide use of social media [1], in conjunction with automated news generation with the help of artificial intelligence technologies [2], it has gained a vital importance in studying misinformation spread and polarization. In this regard, confirmation bias plays a key role. Confirmation bias broadly refers to cognitive bias towards favoring information sources that affirm existing opinion [3]. It is well understood that confirmation bias helps create “echo chambers” within networks, in which misinformation and polarization thrive, see e.g., [4], [5]. Recently, Abdelzaher et al. in [6] and Xu et al. in [1] reveal the significant influence of consumer preferences for outlying content on opinion polarization in the modern era of information overload. Meanwhile, Lamberson and Stuart in [7] suggest that negative information, which is far away from expectations, is more “outlying.” Motivated by these discoveries, the negativity bias, which refers to a tendency to be more attentive and/or responsive to a unit of negative information than to a unit of positive information [7], should not be ignored in the study of information spread in social networks or public opinion evolution.

In recent years, the Hegselmann-Krause model [1], [8], [9] and the like-minded social influence [10]—[12] are widely employed to capture confirmation bias. However, the considered models therein cannot fully capture the asymmetric bias, which hinders their applications in many real social problems where humans hold asymmetric confirmation bias. Motivated by the problems, we first propose a public opinion evolution model which explicitly takes asymmetric confirmation bias and negativity bias into account. We then investigate the generic modeling of asymmetric cognitive bias.

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II. PRELIMINARIES

A. Notation

The social system is composed of \( n \) individuals in a social network. The interaction among individuals is modeled by a digraph \( \mathcal{G} = (\mathcal{V}, E) \), where \( \mathcal{V} = \{v_1, \ldots, v_n\} \) is the set of vertices representing the individuals, and \( E \subseteq \mathcal{V} \times \mathcal{V} \) is the set of edges of the digraph \( \mathcal{G} \) representing the influence structure.

B. Public Opinion Evolution Model

We propose the following opinion evolution model (adopted from [10]) with asymmetric cognitive bias: confirmation bias and negativity bias.

\[
x_i(k + 1) = \alpha_i(k) s_i + \sum_{j \in \mathcal{V}} c_{ij}(x(k)) x_j(k), \quad i \in \mathcal{V}.
\]  

Here we clarify the notations and variables:

- \( x_i(k) \in [-1, 1] \) is individual \( v_i \)’s opinion, \( s_i \in [-1, 1] \) is her subconscious bias, which is based on inherent personal characteristics (e.g., socio-economic conditions where the individual grew up and/or lives in [13]).

- The state-dependent influence weight \( c_{ij}(x(k)) \geq 0 \) is proposed to capture individual \( v_i \)’s conjunctive confirmation bias and negativity bias, which is written as

\[
c_{ij}(x(k)) = (1 - \beta_i)(\overline{x}_i(k), x_j(k)) + \beta_i \underline{x}_i(k), x_j(k), \quad i \in \mathcal{V}
\]

where \( \beta_i \in [0, 1], \underline{x}_i(k), x_j(k) \) is used to describe confirmation bias, while \( \overline{x}_i(k), x_j(k) \) describes negativity bias with \( \overline{x}_i(k) \) denoting individual \( v_i \)’s sensed expectation from her neighbors, defined as the mean of her neighbors’ opinions, i.e.,

\[
\overline{x}_i(k) = \frac{1}{\sum_{i \in \mathcal{V}} w_{ii}} \sum_{j \in \mathcal{V}} w_{ij} x_j(k).
\]

- \( \alpha_i(k) \geq 0 \) is the “resistance parameter” of individual \( v_i \) on her subconscious bias. To guarantee \( x_i(k) \in [-1, 1] \) for all \( k \in \mathbb{N} \) and \( \forall i \in \mathcal{V} \), it is determined in the sufficient and necessary condition:

\[
\alpha_i(k) + \sum_{j \in \mathcal{V}} c_{ij}(x(k)) x_j(k) = 1, \quad \forall i \in \mathcal{V}.
\]

III. ASYMMETRIC COGNITIVE BIAS MODELING

To simplify the presentation without loss of generality, we refer \( x_i(k), \bar{x}_i(k) \) and \( x_j(k) \) to \( x_i, \bar{x}_i \) and \( x_j \), respectively, in this section.
A. Literature Review

We first use the examples of confirmation bias to present the modeling issues of symmetric bias. In recent few years, the Hegselmann-Krause model [8], i.e.,

\[
\begin{align*}
x(k+1) &= A[x(k)]x(k), \\
[A(x(k))]_{i,j} &= \begin{cases}
0, & \text{if } x_i(k) - x_j(k) < \varepsilon_i \\
> 0, & \text{if } x_i(k) - x_j(k) \leq \varepsilon_i
\end{cases}
\end{align*}
\]  

(5)

and the state-dependent influence weights, i.e.,

\[
\zeta(x_i, x_j) = \beta_i - \gamma_i |x_i - x_j|, \quad \beta_i \geq \gamma_i \geq 0
\]

(6)

are widely used in [1], [9–12] to capture confirmation bias. However, the model (5) with asymmetric level of confidence (i.e., \(|\varepsilon_i| \neq \varepsilon_j\)) cannot fully capture the asymmetric bias while the model (6) can only capture the symmetric bias, which however hinder their applications in many real social problems where humans hold asymmetric confirmation bias. For example, we suppose the topic being discussed is “COVID-19 Is a Hoax”, then −1 and 1 correspond to individual vi completely opposing and supporting the claim, respectively. As a consequence, \(x_j \in [-1, 0]\) means the opinion of supporting position -1 while \(x_j \in [0, 1]\) means the opinion of supporting position 1. We suppose that the opinions forwarded by two individuals are \(x_1 = -0.3\) and \(x_2 = 0.5\), and an individual vi’s opinion is \(x_i = 0.1\). By that logic, and according to confirmation bias, \(v_i\) should place more weight on the opinion \(x_2 = 0.5\) than the opinion \(x_1 = -0.3\), since both \(x_2 = 0.5\) and \(x_1 = 0.1\) are supporting position 1 and the difference is their supporting degree. Yet \(|x_2 - x_1| = |x_1 - x_2| = 0.4\), which according to (6) and (5), respectively, implies \(\zeta(x_i, x_j) = \zeta(x_i, x_h)\) and \([A(x)]_{i,j} = [A(x)]_{i,h}\) if \(0.4 < \min\{\varepsilon_i, -\varepsilon_j\}\). We thus conclude that the more realistic asymmetric confirmation bias is not captured by both the models (6) and (5).

Building on DeGroot model [14], Dandekar et al. in [15] proposed the opinion polarization dynamics with biased assimilation. We now examine if the model can capture the interested asymmetric confirmation bias in a simple scenario as considered in [15], where the social network consists of only two individuals: \(v_i\) and \(v_j\). The proposed opinion dynamics in this scenario is

\[
x_i(t + 1) = \frac{\sum_{j \neq i} (x_j(t) + (x_j(t))^2 w_{ij} x_j(t))}{\sum_{j \neq i} w_{ij} x_j(t)},
\]

where \(x_i(t), x_j(t) \in [0, 1]\). If individual vi’s opinion at time t is neutral, i.e., \(x_i(t) = 0.5\). In light of the model, we have

\[
x_i(t + 1) = \frac{0.5 w_{ij} + 0.5 x_j(t)}{w_{ij}},
\]

which indicates that regardless of individual vi’s opinion \(x_i(t)\), individual vi puts the same influence weight on \(x_j(t)\) at time \(t + 1\). Therefore, we can conclude the proposed polarization dynamics cannot fully capture the symmetric confirmation bias in the simplified scenario.

B. Asymmetric Cognitive Bias Conditions

The modeling challenge moving forward is How to capture the asymmetric cognitive bias? In this section, we provide the generic modeling conditions to address the challenge.

1) Asymmetric Confirmation Bias: It is well understood

- confirmation bias happens when a person gives more weight to evidence that confirms their beliefs and undervalues evidence that could disprove it [16].
- both polarization and homogeneity are the results of the conjugate effect of confirmation bias and social influence [9, 17].

motivated by which, to capture asymmetric confirmation bias, we require the influence weights \(\zeta(x_i, x_j) \geq 0\) in (2) to satisfy

\[
\zeta(x_i, x_j) > \zeta(x_i, x_d),
\]

if \(|x_j - x_i| < |x_d - x_i|, x_j \cdot x_d > 0,
\]

or \(|x_j - x_i| = |x_d - x_i|, x_i > x_j \cdot x_d,
\]

or \(|x_j - x_i| = \zeta(x_j, x_d, x_i)|x_d - x_i|, x_i > x_j < 0,
\]

\(x_i \cdot x_j > 0\) and \(0 < \zeta(x_j, x_d, x_i) < 1\),

(7a)

\[
\zeta(0, x_d) = \zeta(0, x_d), \quad \text{if } x_j = -x_d.
\]

(7b)

How the proposed condition (7) can capture the asymmetric confirmation bias are explained in the following remarks.

Remark 1: The condition \(x_i \cdot x_d > 0\) included in the first item of the condition (7a) means \(x_i > 0\) & \(x_d > 0\), or \(x_i < 0\) & \(x_d < 0\). In light of the expression, the first item indicates that for the two sensed opinions that are both supporting the position -1 or 1, the individual will put larger influence weight on the closer opinion with hers.

Remark 2: We note that the second item in the condition (7a) includes the case: \(\zeta(x_i, u_d) > \zeta(x_i, x_d), \) if \(|x_j - x_i| = |x_d - x_i|, x_i \cdot x_d > 0\) and \(x_i > x_d \cdot x_i < 0\). If \(x_i > 0\), we have implying \(x_i > 0\) and \(x_d < 0\). We here conclude that in this case, although the two opinions \(x_j\) and \(x_d\) has the same distance with individual opinion \(x_i\), i.e., \(|x_j - x_i| = |x_d - x_i|\), individual puts larger influence weight on \(u_d\), i.e., \(\zeta(x_i, x_d) > \zeta(x_i, x_d),\) since both of them support the position 1, and vice versa as \(x_i < 0\). Therefore, the second item in the condition (7a) captures the asymmetric confirmation bias when \(x_j \cdot x_i > 0\) and \(x_d \cdot x_i < 0\).

Remark 3: The second item in the condition (7a) also includes the case: \(\zeta(x_i, x_j) > \zeta(x_i, x_d),\) if \(|x_j - x_i| = |x_d - x_i|\) and \(x_j \cdot x_i > x_d \cdot x_d > 0\). Taking \(x_i < 0\) as an example, we have \(x_j < x_d < 0\), by which this case implies that although \(x_j\) and \(x_d\) has the same distance with the opinion \(x_i\) and all of them support the position -1, the individual \(v_j\) puts larger influence weight on \(x_i\) since \(x_i\) and \(x_d\) are closer to the supporting position of \(-1\) than \(x_d\).

Remark 4: Taking \(x_i > 0\) as an example and considering \(0 < \zeta(x_j, x_d, x_i) < 1\), the third item in the condition (7a) implies that it is possible that \(\zeta(x_i, x_j) > \zeta(x_i, x_d),\) if \(x_i < 0,\) \(x_d < 0\) and \(x_j > 0\). This case means although both \(v_i\) and \(v_d\) support the position -1, while \(v_j\) supports the position 1, the individual \(v_i\) puts larger influence weight on \(x_j\) than \(x_d\) when the ratio of their supporting-degree differences is larger than a threshold, i.e., \(\frac{x_d - x_i}{x_j - x_i} > \frac{1}{\zeta(x_j, x_d, x_i)} > 1\).

Remark 5: The condition (7b) means that if individual's opinion is neutral, i.e., \(x_i = 0\), she will put identical influences on her sensed opinions that have the same distance with hers.
2) Asymmetric Negativity Bias: In this subsection, we present the conditions pertaining to capturing asymmetric negativity bias. Lamberson and Soroka in [7] revealed that

- negative information, which is far away from expectations, is more “outlying” (which motivates the sensed expectation $x_i(k)$ in (2) and (3)),
- the negativity bias refers to a tendency that is more attentive and/or responsive to a unit of negative information than to a unit of positive information.

Motivated by the discoveries and inspired by (7), to capture asymmetric negativity bias, we require the influence weights

$$\tau(\bar{x}_i, x_j) \geq 0$$

satisfy

$$\tau(\bar{x}_i, x_j) > \tau(\bar{x}_i, x_d),$$

if $|x_j - \bar{x}_i| > |x_d - \bar{x}_i|$, $x_j \cdot x_d > 0$,

or $|x_j - \bar{x}_i| = |x_d - \bar{x}_i|$, $\bar{x}_i \cdot x_j < \bar{x}_i \cdot x_d$, or $|x_d - \bar{x}_i| \leq \zeta(x_j, x_d, \bar{x}_i)|x_j - \bar{x}_i|$, $\bar{x}_i \cdot x_j > 0$,

$$\tau(0, x_j) = \tau(0, x_d), \text{ if } x_j = -x_d.$$ (8b)

C. Asymmetric Cognitive Bias Modeling Guidance

In this paper, we construct $g(x_i, x_j)$ and $\tau(\bar{x}_i, x_j)$ to have the following general forms:

$$g(x_i, x_j) = g_1(f_i(x_i) - f_j(x_j)) \geq 0,$$ (9)

$$\tau(\bar{x}_i, x_j) = \frac{\tau_1(f_i(\bar{x}_i) - f_j(\bar{x}_j))}{\tau_2} \geq 0.$$ (10)

We next present the sufficient and necessary conditions of the models (9) and (10) on satisfying (7) and (8), respectively, which will work as a guidance of modeling the asymmetric confirmation bias and negativity bias.

**Theorem 1:** The influence weight $g(x_i, x_j)$ given in (9) satisfies (7) if and only if

$$g_1(f_i(x_i) - f_j(x_j)) \text{ is strictly decreasing w.r.t. the distance } |f_i(x_i) - f_j(x_j)|,$$ (11a)

$$f_i(x_i) \text{ is strictly increasing w.r.t. } x_i,$$ (11b)

$$f_i(x_i) < \frac{f_i(x_j) + f_i(x_d)}{2}, \text{ if } |x_j - x_i| = |x_d - x_i|, x_j > x_d \text{ and } x_i > 0,$$ (11c)

$$f_i(x_i) > \frac{f_i(x_j) + f_i(x_d)}{2}, \text{ if } |x_j - x_i| = |x_d - x_i|, x_j < x_d \text{ and } x_i < 0,$$ (11d)

$$f_i(0) = \frac{f_i(x_j) + f_i(-x_j)}{2}.$$ (11e)

**Proof:** We first prove the condition (11) is a sufficient condition.

**Sufficient Condition:** Without loss of generality, we let $x_i \geq x_j > 0$. It follows from (11b) that $|f_i(x_i) - f_i(x_j)| = f_i(x_i) - f_i(x_j)$. If $u_d$ decreases to $x_d > 0$, we then have $f_i(x_i) - f_i(x_d) < |f_i(x_i) - f_i(x_d)|$ and $|x_i - x_d| < |x_i - x_d|$. Considering (11a) and (9), we have $g(x_i, x_j) > g(x_i, x_d)$. If $u_d$ can increase to $x_d$ such that $x_d - x_i > x_i - x_j \geq 0$, we obtain from (11b) that $|f_i(x_i) - f_i(x_j)| < |f_i(x_j) - f_i(x_d)|$ and $|x_i - x_d| < |x_i - x_d|$. Considering (11a) and (9), we then have $g(x_i, x_j) > g(x_i, x_d)$. We thus conclude that the conjunctive conditions (11a) and (9) imply that

$$g(x_i, x_j) > g(x_i, x_d), \text{ if } |x_d - x_i| > |x_i - x_d|, x_j > 0, x_d > 0.$$ (12)

In the case of $0 > x_i \geq x_j$, following the same steps to derive (12), we have

$$g(x_i, x_j) > g(x_i, x_d), \text{ if } |x_d - x_i| > |x_i - x_d|, x_j < 0, x_d < 0.$$ (13)

The results (12) and (13) indicate that the conjunctive conditions (11a) and (11b) result in the first item in (7a).

We now consider the condition

$$|x_j - x_i| = |x_d - x_i| \text{ and } x_d x_i > x_d x_i.$$ (14)

If $x_i > 0$, (14) implies that $x_j - x_i = x_i - x_d > 0$ and $x_j > x_d$, which follows from (11b) that $|f_i(x_j) - f_i(x_d)| = f_i(x_j) - f_i(x_d) < |f_i(x_i) - f_i(x_d)| = f_i(x_i) - f_i(x_d)$. We then can obtain from (11a) and (9) that

$$g(x_i, x_j) > g(x_i, x_d), \text{ if } |x_d - x_i| = |x_i - x_j|, x_j > x_d, x_i > 0.$$ (15)

If $x_i < 0$, following the same steps to derive (15), we have

$$g(x_i, x_j) > g(x_i, x_d), \text{ if } |x_d - x_i| = |x_i - x_j|, x_j < x_d, x_i < 0.$$ (16)

The results (15) and (16) means that the conjunctive conditions (11a)–(11d) result in the second item in (7a).

Let us consider the condition

$$x_i \cdot u_d < 0 \text{ and } x_i \cdot x_d > 0.$$ (17)

If $x_i > 0$, the condition (17) implies that $x_j < 0$ and $x_d > 0$. Without loss of generality, we let $x_i < x_d - x_i$. In the light of (15) and (12), we thus have

$$g(x_i, x_j) < g(x_i, x_d), \text{ if } |x_j - x_i| = |x_i - x_d|$$

$$g(x_i, 0) > g(x_i, x_d), \text{ if } 0 < x_i < x_d - x_i,$$

which indicates that there exist $x_j < 0$ such that $|x_i - x_d| > x_i - x_i$ and $g(x_j, x_j) > g(x_i, x_d)$. Therefore, we conclude that there exists an $x_j$ such that

$$g(x_i, x_j) > g(x_i, x_d), \text{ if } |x_i - x_j| < |x_i - x_d|, x_j > 0,$$

$$x_j < 0, x_d > 0.$$ (18)

If $x_i < 0$, following the same steps to derive (18), we have conclude that there exists an $x_j$ such that

$$g(x_i, x_j) > g(x_i, x_d), \text{ if } |x_i - x_j| < |x_i - x_d|, x_i < 0,$$

$$x_j > 0, x_d < 0.$$ (19)

The results (18) and (19) means that the conjunctive conditions (11a)–(11d) result in the third item in (7a).
**Necessary Condition:** Given the form (9), it is straightforward to verify the condition (7b) is equivalent to (11e). For the rest of proof, we consider contradiction, i.e., assuming (11a)–(11d) do not hold, the condition (7d) does not hold as well. We assume that (11) does not hold, i.e., \( g_i(f_i(x_i) - f_j(x_j)) \) is non-decreasing w.r.t. \( f_i(x_i) - f_j(x_j) \). We let \( x_i \geq x_j > 0 \), and thus have \( f_i(x_i) - f_j(x_j) = f_i(x_i) - f_i(x_j) \). If \( x_j \) decreases to \( x_j > 0 \), we then have \( f_i(x_i) - f_j(x_j) < f_i(x_i) - f_j(x_j) \) and \( |x_i - x_j| < |x_i - x_d| \). Considering (11a) and (9), we have
\[
\|f_i(x_i) - f_j(x_j)\| = \phi(x_i, x_j) \leq \phi(x_i, x_d).
\] (20)

We now consider the case that \( g_i(f_i(x_i) - f_j(x_j)) \) is strictly non-decreasing w.r.t. \( f_i(x_i) - f_j(x_j) \) and \( f_i(x_i) - f_j(x_j) \) is strictly non-increasing w.r.t. \( x_e \). Let us set \( 0 < x_d < x_j < x_i \). We thus have \( |x_i - x_d| > |x_i - x_j|, |f_i(x_i) - f_j(x_d)| \geq |f_i(x_i) - f_j(x_j)| \) and (20). Following the same analysis method, we can conclude that if the conditions (11a)–(11d) do not hold, the (7d) does not hold as well.

**Theorem 2:** \( \phi(x_i, x_j(k)) \) given in (10) satisfies (8) if and only if
\[
\|f_i(x_i) - f_j(x_j)\| = \phi(x_i, x_j) \leq \phi(x_i, x_d).
\] (20)

Proof: The proof steps completely follow those of Theorem [1] it is thus omitted.

D. Numerical Example

Theorems [1] and [2] provides the guides to construct the models of asymmetric confirmation bias and negativity bias, respectively. The models are not unique. Take the confirmation bias as an example, its models include \( \phi(x_i, x_j) = \chi_i - \gamma_i(tanh(x_i) - tanh(x_j))^2, \phi(x_i, x_j) = \chi_i - \gamma_i|x_i^2 - x_j^2| \), with \( \gamma_i > 0 \), and among many others. We now use \( \phi(x_i, x_j) = 0.6 - 0.011(tanh(x_i) - tanh(x_j))^2 \) as one numerical example to demonstrate its checkable properties, observing its values shown in Figure [1] we can verify that

- If individual \( v_i \)’s opinion is neutral, i.e., \( x_i = 0 \), she puts the identical influence weights on the opinions that have the same distance with hers.
- If individual \( v_i \) supports the position 1 with \( x_i = 0.2 \), for the two opinions 0.8 and -0.4 that have the same distance with \( x_i = 0.2 \), she puts larger influence weight on 0.8, since 0.8 and 0.2 are in the same supporting domain.
- If individual \( v_i \) supports the position 1 with \( x_i = 0.2 \), for the two opinions 0.8 and -0.1, she puts larger influence weight on -0.1, since although 0.2 and 0.8 are in the...
same supporting domain while \(-0.1\) is in the opposing domain, 0.8 has much larger distance with \(x_i = 0.2\) compared with \(-0.1\).

- If individual \(v_i\) supports the position 1 with \(x_i = 0.4\), for the two opinions 0.6 and 0.2 that have the same distance with \(x_i = 0.4\), she puts larger influence weight on 0.6, since all of them are in the same supporting domain but both 0.6 and 0.4 stick more to the supporting domain compared with 0.2.

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