The Origin of The Stellar Initial Mass Function

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Abstract. Observations and theories of the stellar initial mass function are reviewed. The universality and large total mass range of the power law portion of the IMF suggest a connection between stellar mass and cloud structure. The fractal patterns and formation times of young clusters suggest a further connection to turbulence. There is also a similarity between the characteristic mass at the lower end of the power law and the minimum self-gravitating mass in a typical star-forming cloud. All of this suggests that the power law part of the IMF comes from star formation in self-gravitating cloud pieces that are formed by compressible turbulence. Timing constraints involving cloud destruction and competition for gas might limit the upper stellar mass to several hundred suns. This upper limit has to be less than the mass of a clump that has a dynamical time equal to several times the dynamical time of the characteristic mass. The smallest stars and brown dwarfs may not come directly from cloud clumps, which are not self-gravitating at this mass in the cloudy phase, but from collapsed fragments or other pieces connected with the formation of more massive stars.

1. Introduction

The stellar initial mass function (IMF) is an important ingredient for studies of stellar evolution in clusters and galaxies, and it offers a clue to the physical processes involved with star formation. But since the time when the first IMF was extracted from field stars corrected for age (Salpeter 1955), there has been no general understanding of its origin, and few direct observations of stellar mass functions in their initial form, i.e., in very young clusters.

Fortunately, the observational situation is improving with infrared surveys down to sub-stellar masses in nearby young clusters (Comerón, et al. 1993; Strom, Kepner, & Strom 1995; Lada & Lada 1995; Luhman, et al. 1998; Luhman & Reike 1998, 1999; Najita, Tiede & Carr 2000) and with ground- and space-based observations of distant massive clusters (e.g., Massey & Hunter 1998; Grillmair et al. 1998; Selman et al. 1999; Sirianni et al. 2000; see reviews in Massey 1998 and Brown 1998).

The theory of the IMF is making progress too. There are numerical simulations that reproduce the IMF by following the motion and accretion of protostars in a model cluster (Bonnell et al. 1997) or the evolution of protostar interactions (Price & Podsiadlowski 1995), and there are simulations based on sam-
pling from fractal clouds (Elmegreen 1997, 1999a, 2000a,c; Sánchez & Parravano 1999). There are also analytical models of the IMF that include many physical processes involved with star formation (e.g., Silk 1995; Nakano, Hasegawa, & Norman 1995; Adams & Fatuzzo 1996; Myers 2000a).

Several reviews of the IMF are in the conference series *The Stellar Initial Mass Function*, edited by G. Gilmore and D. Howell (Cambridge University Press, 1998). Cayrel (1990) has an earlier review the theory of the IMF, and Leitherer (1999) has a recent summary of IMF observations in starburst galaxies. Other reviews are in Elmegreen (1998; 1999c; 2000d).

This paper discusses the basic classes of IMF models and various observational constraints for them. These constraints suggest that the IMF is approximately uniform in diverse environments, the star formation process is relatively rapid and possibly independent for each star, even in clusters, and that stars sample the IMF randomly in each cloud. The recent observation of IMF-like mass functions in pre-stellar condensations suggests further that the IMF may be decided in the gaseous phase, and not be part of the collapse process during which this gas gets converted into stars. These observations limit the physical processes that can be involved in determining the IMF.

2. Review of Theories

2.1. Four Types of Models

Theories of the IMF may be categorized into several distinct types. Some begin with a model for the formation of a single star and then vary the parameters to get a range of stellar masses. These variations can be random, as in the models by Larson (1973), Elmegreen & Mathieu (1983), Zinnecker (1984), and Adams & Fatuzzo (1996), or they can depend on time (Silk 1977) or position (Padoan et al. 1997). The statistical aspect of the random models is realistic, but the physical models for star formation in these pictures are often simplistic.

Other theories begin with a model for clustered star formation and consider protostar interactions (Silk & Takahashi 1979; Bastien 1981; Yoshii & Saio 1985; Lejeune & Bastien 1986; Allen & Bastien 1996; Price & Podsadlowski 1995; Murray & Lin 1996) or parameter variations for single stars induced by multiple winds (Silk 1995). The good thing about these models is that most stars are born in clusters, but the interaction theories are often simplified or unphysical. The older theories, for example, assumed the clumps would stick after a collision, but if unbound clumps are transient density structures in a compressibly turbulent gas, or if the clumps are gravitationally bound but move at the larger virial speed of a whole cloud core, then mutual collisions should destroy them. Most stars seem to form too quickly and to have relative speeds that are too low (Belloche et al. 2000, this conference) to interact like this anyway.

A third class of models considers clustered star formation with competitive accretion (Larson 1978; Tohline 1980; Bonnell et al. 1997; Myers 2000a). In these models, protostars move around in a uniform sea of gas, which is the cloud core, and accrete it as they go. Accretion is a well-accepted process of star formation, but the idea of a near-uniform reservoir of gas for shared accretion contradicts the observation that individual or binary stars form in very dense and separate clumps (i.e., in the pre-stellar condensations: Motte, André, &
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Neri 1998; Testi & Sargent 1998; Belloche et al. 2000; Tachihara et al. 2000). The interclump medium is at a much lower density than the clumps anyway, and would not be a good reservoir for clump growth if the only relative speed is from clump motions. Supersonic interclump motions from compressible turbulence should change clump masses much more rapidly than clump motions. In any case, most stars probably just get some variable fraction of the mass of the dense clump in which they are born, perhaps between 3% and 30%, and that is all there is to accretion. The accretion rate does not matter if the reservoir of gas is limited to the clump, and the accretion is not competitive if the clumps have a small volume filling factor, like a few percent, which is observed to be the case in the studies by Motte et al. (1998) and Testi & Sargent (1998). In addition, the range for the mass fraction of the clump that gets into a star is probably much too small to explain the whole IMF: most of the IMF has to come from the range of protostellar clump masses.

This leads us to a fourth class of theories, in which much of the IMF is attributed to cloud or clump mass functions. In the older versions of this theory, the star mass scaled with some power of the clump mass, and the clump mass spectrum observed in CO surveys was used to give the star mass spectrum (Zinnecker 1989; Nakano, Hasegawa, & Norman 1995). The advantage of this model is that the structure of clouds can be observed directly. However, the interpretation of this structure in terms of mass functions for gas is very difficult (see Sect. 4 below). A second problem is that the origin of the clumpy structure in the gas is not well understood, so this solution for the IMF merely replaces one unknown with another. Moreover, if cloud structure dominates the IMF, then the star formation process may hardly affect it. That leads to the disappointing possibility that a correct and successful theory of the IMF slope will not, in the end, reveal much about star formation. It will tell us primarily about clump formation and the origin of cloud structure. A new class of theories based only on sampling cloud structure (Sect. 5) has this aspect.

2.2. A Characteristic Mass for Star Formation

The IMF has essentially three measurable parameters: the average slope of a power law part at intermediate to high mass, a different average slope at low mass, down to and below the brown dwarf mass, and an intermediate slope and characteristic mass that separates these two regimes. The intermediate and high mass slope has received the most attention by theoreticians, and models discussed above can usually reproduce it with enough free parameters. The slope at low mass is only recently known and only one theory so far has been proposed to explain it (Elmegreen 2000a; see also the discussion in Myers 2000b).

The characteristic mass of a star at the border between these two power law parts, which is $\sim 0.3 - 1 \, M_\odot$, would seem to be relatively easy to explain. There are very few characteristic masses in the interstellar medium, which mostly shows power-law behavior over a wide range of scales. Indeed, there are only two recent models that get this mass. One suggests that cloud pieces collapse to much smaller objects first, and that accretion continues until the star begins to limit its own mass. This self-limitation defines a characteristic mass, and models give it about the right value following Deuterium burning in the protostar (Larson 1982; Shu et al. 1987; Nakano et al. 1995; Adams & Fatuzzo 1996). Note
that in these models, the characteristic mass is not actually the mass at the threshold for Deuterium burning, which is much less, $0.018\,M_\odot$ (D'Antona & Mazzitelli 1994). An advantage of this model is that the characteristic mass is determined partly by the star itself and is not overly sensitive to properties of the surrounding cloud. Of course, higher pressure environments should have higher density cloud cores, and the higher pressures inside of them could resist the budding wind more at an early stage, thereby allowing more massive stars to form. Higher temperature clouds could accrete mass at a greater rate, holding back the wind too. But self-limitation should dominate these cloud processes if pre-stellar winds are strong, and this implies that the IMF might have a universal quality, independent of environment.

The other theory for a characteristic mass in the IMF is that it is linked to the smallest possible self-gravitating mass in a cloud (Larson 1992; Elmegreen 1997, 1999a, 2000a,c). Clouds consist of structures on a wide range of masses, from far below the smallest star to far above the largest star. Diffuse molecular clouds are known to contain structures down to $10^{-4}\,M_\odot$ (Heithausen et al. 1998), and self-gravitating clouds may have structures down to $10^{-3}\,M_\odot$ (Langer et al. 1995). Some of the clumps seen with molecular emission lines could be the result of velocity crowding (Pichardo et al. 2000) and not be real physical objects, but the tiny clumps in 100 $\mu$ IRAS maps of diffuse clouds could not have this origin because IRAS integrates over the whole line of sight. The smallest cloud clumps even in molecular clouds are probably not self-gravitating (Bertoldi & McKee 1992; Falgarone, Puget, & Pérault 1992), in which case they are not likely to form protostars. To become unstable, these smallest clumps have to be compressed by pressures that are much larger than anything likely to occur in a local star-forming region. Because of this, the origin of the stars that form from clumps less massive than the smallest self-gravitating piece in a cloud should differ from that of the more massive stars.

The smallest self-gravitating mass in a cloud is about the thermal Jeans mass or Bonner-Ebert mass, which depends primarily on the thermal temperature and the total pressure: $\mu_J \sim 0.3\left(\frac{T}{10\,K}\right)^2\left(\frac{P}{10^6\,\text{K}\,\text{cm}^{-3}}\right)^{-1/2}$. Cloud pieces much smaller than this should not begin the star formation process unless an excursion in $P$ causes $\mu_J$ to decrease locally. For typical cloud parameters, using the total turbulent and magnetic pressure for $P$ in this expression ($P_{\text{total}} \sim 10^6\,\text{K}\,\text{cm}^{-3}$), the thermal Jeans mass is at the inflection point of the IMF. The sharp rise in star counts with decreasing star mass, going from massive to intermediate mass stars, ends at this point. It is followed by an IMF that is flat or slightly falling toward lower masses on a log-log plot (e.g., Luhman & Rieke 1998, 1999; Muench, Lada & Lada 2000; Hillenbrand & Carpenter 2000; Kaas & Bontemps 2000).

Sometimes a mathematical formula cannot be used to connect the parameters in a cloud, such as density, pressure and temperature, with a final stellar mass that forms after a long chain of events. Cloud conditions are turbulent and chaotic, perhaps like atmospheric clouds on the Earth. Then the approximate initial conditions for star formation may not completely specify the outcome of cloud evolution. It might be no more possible to predict the final stellar mass from initial cloud conditions than to predict rainfall amounts in New York from the humidity and temperature of the same air when it is in Chicago two days
earlier. The divergence in outcomes of two infinitesimally close initial conditions in a turbulent medium is well known. This divergence problem should be particularly troublesome for theories of the IMF that give a functional form to a star’s mass based on cloud parameters and then vary the parameters.

For a characteristic mass inside the cloud, the problem with turbulent chaos should not be too bad. The thermal Jeans mass or Bonner-Ebert mass should be accurate enough for most purposes if it is applied to a cloud structure that currently has the assumed temperature and pressure. The result gives a local minimum for a self-gravitating clump mass, and it provides a starting point for a theory on the IMF inflection. The individual stellar masses are conceptually far removed from this cloud mass, considering the additional processes that must come into play before a star forms. So the connection between \( M_J \) and any star mass, even at the inflection point, remains vague. An easy way out is to suppose that \( M_J \) has meaning only for cloud processes, and that once a self-gravitating cloud piece forms, all of the additional complications regarding collapse and star formation produce only a modest range for the conversion factor from the mass of that cloud piece into a star mass. Then the mass of the piece would still determine the average mass of the star that forms in it, perhaps to within a factor of \( \sim 3 \). The distribution function for star mass inside a cloud piece of a given mass should be viewed as another property of star formation, like the IMF, but not necessarily equal to the final IMF, which also contains information about the origin and evolution of the cloud pieces (Elmegreen 2000a).

Another problem with the \( M_J \) limit is that stars form at much lower masses too, so \( M_J \) is not a hard barrier to star formation. This may not mean that smaller cloud pieces turn independently into stars. The smaller stars may arise from fragmentation inside \( M_J \) pieces (more on this below).

Magnetic forces and turbulent motions should increase the self-gravitating mass above \( M_J \), and a decrease is possible at the center of a converging flow (Hunter & Fleck 1982). If the total pressure used in the expression for \( M_J \) includes these turbulent motions, then the result should still be a useful lower limit to self-gravitating cloud structure.

A few years ago there was a third model for a characteristic mass in star formation, the mass of an optically thick core at the limit of strong self-gravity (Rees 1976; Yoshii & Saio 1985). This mass is very small, \( \sim 10^{-3} M_\odot \), so models that used it had to rely on coalescence or continued accretion to achieve final stellar masses. In that case, the final characteristic mass will be more closely related to the other characteristic masses given above than to the opacity-limited mass.

3. Observational Constraints

3.1. IMF Uniformity

The IMF is approximately uniform in space and time, as illustrated by its similar form in galactic clusters (Scalo 1998), OB associations (Massey 1998), old globular clusters (de Marchi & Paresce 1997; Chabrier & Mera 1997; Pulone, de Marchi, & Paresce 1999; De Marchi, Paresce, & Pulone 2000; Paresce & De Marchi, 2000), halo stars (Nissen et al. 1994), and bulge stars (Holtzman et al. 1998). The near-uniformity of the IMF has also been demonstrated by a
similar ratio of iron to oxygen abundance, which is a measure of the ratio of low-mass to high-mass stars, in elliptical galaxies (Wyse 1998), the intracluster medium (Renzini, et al. 1993; Wyse 1997, 1998; but see Loewenstein & Mushotzky 1996), and QSO Ly$\alpha$ absorption systems (Lu et al. 1996; Wyse 1998). The IMF is also independent of metallicity (Freedman 1985; Massey, Johnson & DeGioia-Eastwood 1995a).

The IMF is usually a power law at intermediate to large mass, with a slope between $\sim -1$ and $\sim -1.5$ on a log-log plot. At low mass the IMF flattens to a slope of $\sim 0.5$ to $-0.5$. A compilation of IMF slopes from a variety of clusters was made by Scalo (1998) and is reproduced here in Figure 1. Each point represents an average mass for the observed part of the cluster versus the slope of the IMF centered on this average mass. Deviations of $\pm 0.5$ from point to point around the Salpeter slope of $\sim -1.35$ are present. These could be statistical fluctuations, considering the small number of stars that are usually measured.

For example, Figure 2 shows a similar plot that is based on a random sampling model for cloud pieces in a fractal cloud, as described below (Elmegreen 1999a). Each point represents the IMF from the brightest 200 members of a model cluster, and there are 100 clusters of various total masses contributing to the plot. More massive clusters have bigger most massive stars, and so populate the right hand part of this diagram, as in the real observations. The scatter in the models is about the same as the scatter in the observations. The other three panels in Figure 2 show the model IMFs that are represented by the crosses in the

Figure 1. IMF slopes in different clusters as a function of the average log mass, in $M_\odot$, from Scalo (1998). The Salpeter value of $-1.35$ is shown by a dashed line. Solid squares are for clusters in the Milky Way, and open squares are for the Large Magellanic Cloud.
Figure 2. (top left) IMF slopes in 100 models plotted as a function of the average logarithm of the mass. Each IMF slope is fit using 200 stars. The three values indicated by crosses have their complete IMFs shown in the other panels, with the fitted portions of these IMFs indicated by the offset lines.

top-left panel. Both the steep and the shallow IMFs look statistically significant in these plots, but they are only random variations around the average slope, which is the Salpeter value in the model.

Steep IMF slopes are sometimes found in lower density regions, including the local field (Garmany, Conti, & Chiosi 1982; Humphreys & McElroy 1984; Scalo 1986; Blaha & Humphreys 1989; Basu & Rana 1992; Kroupa, Tout, & Gilmore 1993; Parker et al. 1998) and some low-density parts of clusters (J.K. Hill et al. 1994; R.S. Hill et al. 1995; Ali & DePoy 1995). This difference may result from mass segregation in clusters, or from differential drift of the low and high-mass stars away from their points of origin. For example, the IMF is known to be shallower in dense cluster cores than along the cluster periphery (Sagar, et al. 1986; Jones & Walker 1988; Sagar & Bhatt 1989; Sagar & Richtler 1991; Pandey, Mahra, & Sagar 1992; Vazquez et al. 1996; Fischer et al. 1998; Kontizas et al. 1998; Hillenbrand & Hartmann 1998). There is no satisfactory explanation for this segregation of high-mass stars to the center; normal two-body interactions do not seem to work fast enough to do this (Subramaniam, Sagar, & Bhatt 1993; Fischer et al. 1998; Bonnell & Davies 1998; although see Giersz & Heggie 1996). Competitive accretion might do it because gas accretion provides a drag on stellar orbits, and this drag is larger for stars that accrete more, causing them to migrate closer to the center of the cluster (Bonnell et al. 1997). However, if the accreting gas has random speeds like the protostars and is not perfectly static, then the orbital energy densities of the
protostars and the accreting gas will be the same. In this case, there should be no additional mass segregation with competitive accretion over and above the root-N decrease in center-of-mass energy that arises simply from the presence of more sub-clumps in more massive stars. That is, random sampling of turbulent gas by any mechanism will give a slight mass segregation all by itself because the average momentum per unit mass of a cloud piece relative to the cloud center is smaller if that cloud piece contains a larger number of random gas elements, i.e., if it is more massive. Differential drift can also give the field stars a steep IMF, because low-mass stars live longer and drift further from their formation sites than high-mass stars.

A very steep IMF slope of $\sim -4$ (compared to the Salpeter slope of $-1.35$) was found in extreme field regions far from known clusters and associations in the LMC and local field (Massey et al. 1995b). There are simple explanations for this extreme steepening based on cloud destruction reviewed below, but the observations need to be confirmed before the models can be carried much further.

Perhaps the most interesting of the proposed deviations from a universal IMF is a shift toward higher mass stars in starburst regions. This shift could take the form of a more shallow IMF in the intermediate to high-mass range, or it could be a parallel shift of the whole IMF toward higher mass without a change in the shape. The main reason the IMF looks different in starbursts is the large ratio of luminous to dynamical mass (Rieke et al. 1980, 1993; Kronberg, Biermann, & Schwab 1985; Wright et al. 1988). Confirmations of this proposal have come from galactic evolution models (Doane & Matthews 1993), spectroscopic line ratios (Doyon, Joseph, & Wright 1994; Smith et al. 1995), and infrared excesses (Smith, Herter & Haynes 1998). However, Devereux (1989) and Satyapal et al. (1995, 1997) lowered the extinction correction for M82, and this makes the IMF there more normal. Other recent studies involving evolutionary models (Schaerer 1996), multiwavelength spectroscopy and broad-band infrared photometry (Calzetti 1997), and emission line spectroscopy (Staňińska & Leitherer 1996) give normal IMFs too. A general problem with large IMF shifts in starburst galaxies is that they should produce unobserved red populations of stars after the turnoff age reaches the stellar lifetime at the truncation mass (Charlot et al. 1993). The ISM should also develop too high an oxygen abundance compared to iron because of the overabundance of high-mass stars (Wang & Silk 1993).

Other indirect observations have led to theoretical predictions of IMF changes. Fabian (1994) proposed that the IMF is biased toward low mass stars in galaxy cluster cooling flows, and Elmegreen (1999b) proposed a similar bias in ultracold molecular gas, both because of a low thermal Jeans mass that is expected in these regions. In the first case, the high pressure lowers the Jeans mass and in the second case the low temperature does. Larson (1998) and Bromm, Coppi, & Larson (1999) proposed that the IMF shifted toward higher mass in the early Universe, and that this would help explain the G-dwarf problem (i.e., the presence of metals in the oldest Galactic halo stars), the high temperature and high metal abundance of intracluster gas, and the large luminosities of young elliptical galaxies.

A theory of the IMF should be flexible enough to accommodate changes in different environments. What actually changes is not yet known, however. Of
the three aspects of the IMF mentioned above, i.e., the high and low-mass slopes and the characteristic mass, the latter alone may cause most of the observed changes. The characteristic mass for a star depends on the physical properties of star formation, and perhaps the upper and lower mass limits depend on these properties too, so we might expect differences in extreme environments that could shift the characteristic mass either up or down without changing the slopes much. In normal regions, however, the general similarity of the IMF from place to place and over time suggests that the basic process by which the gas partitions itself into stars is somewhat universal.

3.2. Independent Formation of Stars even in Dense Clusters

The observed correlations between cluster density and maximum star mass (Testi, Palla & Natta 1999), and between cluster mass and maximum star mass (Larson 1982), give the appearance of collective effects during star formation, as if stars influence each other, perhaps by radiative effects or coagulation in dense cluster cores. However, the actual observations are no more correlated than what is expected from random sampling (Elmegreen 1983, 1997, 2000c; Schroeder & Comins 1988; Massey & Hunter 1998; Selman et al. 1999; Bonnell & Clarke 1999), so there is no real evidence that any young stars care about their neighbors, except possibly for binary stars.

A purely statistical IMF in a Salpeter power law leads to a correlation between cluster mass (in the power-law range of the IMF) and maximum star mass of the form (Elmegreen 2000c):

$$M_{\text{cluster}} \sim 3 \times 10^3 \left( \frac{M_{\text{max}}}{100 \, \text{M}_\odot} \right)^{1.35} \text{M}_\odot.$$  

This means that bigger clusters form bigger most-massive stars simply because they sample further out into the tail of the IMF. The correlation between cluster density and maximum star mass found by Testi, Palla & Natta (1999) could, in principle, be the result of IMF sampling, because most of their clusters have about the same radius, 0.2 pc, so cluster density is proportional to mass (see also Bonnell & Clarke 1999). There is no explanation for this constant radius, and it is a surprising result given that CO cloud radii generally correlate with the inverse square roots of their masses (Larson 1981).

The IMF itself, aside from the maximum star mass, appears to be relatively independent of cluster density over a factor of 200 in density (Hunter, et al. 1997; Massey & Hunter 1998; Luhman & Rieke 1998).

Another reason one might get the impression that star neighbors influence final star mass is that the Orion cluster has its most massive star in the center, in a very crowded field of other stars. Perhaps this star got to be so massive because of some influence from all of the other stars. However, stars as massive as the O5 star in Orion appear all over the 30 Doradus cluster in the LMC and not just in the center, as shown in Figure 3. These peripheral O-type stars may have been born in the cluster core and drifted out into this surrounding region over time, but until this is demonstrated by a comparison of stellar ages, we cannot conclude that all massive stars are born in dense cluster centers. There is an excess concentration of massive stars in the center of this cluster, but no
Figure 3. Positions of massive stars in the R136 field of 30 Dor taken from the WFPC2 camera. The positions are given in units of camera pixels, where the pixel size is 0.0455". The data were kindly provided by D. Hunter from the publication Massey & Hunter 1998.

There is some evidence that collisions might be important in dense clusters, but no evidence yet that the IMF will be affected by this. The collision time between pre-stellar condensations in the Ophiuchus core region seems to be fairly short compared to the total star formation time. The angular filling factor for Ophiuchus condensations in cores A and B of the survey by Motte et al. (1998) is about 10%. This means that the number of half-orbits that a condensation has to undergo before colliding physically with another condensation is the inverse of this filling factor divided by the gravitational focussing factor, $1 + 2GM/(Rv^2) \sim 7$, for protostellar mass $M \sim 0.3 \, M_\odot$, radius $R \sim 0.01 \, \text{pc}$, and mutual speed $v \sim 0.2 \, \text{km s}^{-1}$. These numbers imply that pre-stellar condensations should collide after only about one crossing time, which is fast enough for coalescence to be important. However, the density of a pre-stellar condensation is enormous, $\sim 3 \times 10^7 \, \text{H}_2 \, \text{cm}^{-3}$, so the internal dynamical time of each one is only $\sim 0.01 \, \text{Myr}$, which is one-tenth of their mutual collision time. Thus the observed pre-stellar condensations should collapse significantly before they interact, leaving only rotationally-supported disks at the presently observed size of $\sim 0.01 \, \text{pc}$. These disks will probably interact in the manner discussed above, but the stars inside of them should be too small to coalesce. Thus the IMF will probably not be affected by direct stellar coalescence, even in a dense region like the core of Ophiuchus. The nature of protostellar disks could be affected by cluster density, however.

If the star-formation time in a dense core were much longer than a crossing time, say $\sim 100$ crossing times, then stellar coalescence might be more important. But there is no evidence for such prolonged star formation. On the contrary, the available evidence seems to suggest that star formation is very rapid. This
comes from direct observations of young-star ages in embedded clusters, from the high fraction of clouds with young stars and the rapid dispersal of gas after star formation, and from the observation of hierarchical structure in young star fields, which indicates very little star-star mixing (Elmegreen 2000b). There is also a correlation between the duration of star formation and the size of the region (Elmegreen & Efremov 1996; Efremov & Elmegreen 1998), which is essentially the same as that between the turbulent crossing time for gas and the region size. This correlation indicates that the duration of star formation is always about the turbulent crossing time for a wide range of scales.

3.3. The IMF as a Statistical Ensemble

We have discussed above how the most massive star in a cluster appears to come from IMF sampling only; more massive clusters produce more massive stars primarily for this reason. If this is really a statistical effect, then sometimes a low-mass cloud will produce a high-mass star – a result that seems counterintuitive. However, the interstellar gas is often structured in a self-similar and hierarchical way, so massive clouds contain lower-mass sub-clouds, and these contain even lower mass sub-sub-clouds, and so on. When viewed from a great distance, a whole young star field may appear to be forming in a giant cloud complex, but when viewed from nearby, stars of various masses should be seen forming in sub-parts of the cloud having various smaller masses, in an overall random fashion. In this sense, some of the smaller subpieces, perhaps the size of the Taurus clouds, should be forming massive stars, unlike the Taurus region itself. But these subpieces are parts of larger clouds too. Which of the various cloud masses and sub-cloud masses should be counted in this statistical interpretation?

The main point is that as far as we can tell, a star of any mass seems to be able to form in a cloud of any mass, provided there is enough material to make the star. This means that the summed IMF from 10 clouds with only $10^4 \, M_\odot$ each should be the same as the IMF from a single cloud with $10^5 \, M_\odot$. The problem here is not with the star formation part, but with defining cloud mass in a fractal distribution of density. Indeed, the average IMF in a whole galaxy is indistinguishable from the IMF in a single cluster. Whole-galaxy IMFs come from the color-magnitude diagrams of nearby dwarfs (Greggio et al. 1993; Marconi et al. 1995; Holtzman et al. 1997; Grillmair et al. 1998), from the equivalent widths of Hα in galaxies (Kennicutt, Tamblyn & Congdon 1994; Bresolin & Kennicutt 1997), and from the iron/oxygen ratio, as discussed above. This equivalence means that the summed IMFs from many small clouds and clusters is similar in form to the IMF from any one of them. This can only occur if stars of all masses have equal probabilities of forming in clouds of all masses (Elmegreen 2000d).

An observation like this rules out many IMF models in which stars influence each other. For example, if cloud destruction by massive stars halts star formation, and if larger clouds need more massive stars for their destruction (because of their greater self-binding), then star formation will proceed in a low-mass cloud, building up more and more massive stars until a particular star mass is reached, and then stop without making any more massive stars. Elsewhere in a more massive cloud, the IMF will proceed further, making stars of the same
mass range as the lower mass cloud, but also higher mass stars before cloud destruction. In a scenario like this, the summed IMF will be steeper than the IMF in either cluster because there are a lot of low-mass clouds that will add only to the low-mass part of the total IMF.

Also in a model in which star mass increases as more and more stars form, perhaps as a result of enhanced heating from all the stars, we would have a similar result that only massive clusters can produce high mass stars. But then the sum of the IMFs from low-mass clusters and high mass clusters would be steeper than either one, contradicting the observation that these IMFs are the same.

The steepening of the IMF in the extreme field regions studied by Massey, et al. (1995b) could be an example where the summed IMF is in fact steeper than each component. What differs about these regions is their extremely low pressures. Perhaps clouds are more easily disrupted by star formation at such low pressures: the observed IMF slope in these field regions is consistent with cloud destruction by ionization because the destructive power of OB stars increases with star mass in the correct fashion (Elmegreen 1999a). The contribution of these few field stars to the total IMF in a galaxy is small, however.

The IMF in the general field is also somewhat steeper than in clusters (cf. Sect. 3.1.). This is observed directly, before correcting for mass segregation or differential drift. Perhaps this implies there is a self-limitation of stellar mass by other stars, but at the present time, the field and galaxy-wide IMFs are not well enough known to give any definitive insight into this possibility.

4. On the Meaning of Mass Spectra in a Fractal Medium

Clump-finding algorithms do not consider nested structures like the hierarchical fractals of real interstellar clouds. Algorithms like these find only those structures within a factor of $\sim 3 - 10$ of the telescope beam size (Verschuur 1993; Elmegreen & Falgarone 1996). Larger structures are resolved out and ignored, while smaller structures are not observable. These algorithms give a false impression of what cloud structure is like. We know from power spectra of emission maps of clouds that there is structure on a much wider range of scales than the clump sizes that come from contour drawings of the same region (Stutzki, et al. 1998). Yet the mass spectra made from these clump-finding algorithms look reasonable: they often span a factor of $\sim 100$ in mass and define a nice power law. However, even this mass power law gives a false impression. The mass scales with about the 2.3 power of the size of a region (this power is presumably the fractal dimension; Elmegreen & Falgarone 1996; Heithausen et al. 1998), and so a size spectrum with only a narrow range around the telescope beam size can turn into a mass spectrum with a fairly wide range. Yet, the mass spectrum is just as biased by the beam as the size spectrum because they both come from the same clump identifications that ignore the scale-free fractal structure.

In hierarchical clouds, the mass spectrum of structures of all types is $n(M) d\log M \propto M^{-1}d\log M$ (Fleck 1996). This spectrum has equal mass in equal logarithmic intervals of mass. Hierarchies give this because the levels in the hierarchy are logarithmically spaced. For example, one big piece divides into 4 smaller pieces, and each of these divides into 4 more, giving 16 total, and
then 64, etc.. The same mass is counted again and again in each level. Such perfect hierarchical structure may not apply to real interstellar clouds, but cloud structure is hierarchical (Scalo 1985), and the ideal case is conceptually useful.

Multiple counting of mass for hierarchical clouds is bad if the sum of the clump masses must equal the cloud mass, but it is good if all we want is a probability distribution function for clump mass. If structures at any level in an ideal hierarchy are randomly chosen, then the probability distribution function for mass $M$ is $M^{-1}d\log M \equiv M^{-2}dM$. Mass spectra of clumps found by contour-drawing methods, which tend to find clumps only near the beam size as discussed above, differ from mass spectra of randomly sampled pieces in a self-similar hierarchy of structures, presumably because of the different ways the structures are defined.

An example of a clump mass spectrum that multiply counts pieces and also gets a slope close to the expected value of $-2$ was given by Heithausen, et al. (1998). They studied a region in the Polaris Spur using three telescopes with different beam sizes. One of the clumps found with the largest beam was observed again with a smaller beam, and one of the small clumps observed with this smaller beam was observed a third time with an even smaller beam. All of the structures were put on the same mass spectrum and the resulting slope was $-1.85$, much steeper than the usual $\sim -1.5$ for cloud pieces.

Star clusters and OB associations are the result of star formation in locally dense regions of interstellar gas. These dense regions cluster together on a wide range of scales because of the fractal nature of the gas, and this causes the stars to cluster in the same way (see review in Elmegreen et al. 2000e). Clusters and OB associations sample from all over the hierarchy of gas structures, and because of this, end up with an approximately $M^{-1}d\log M$ spectrum (Kennicutt, Edgar, & Hodge 1989; Battinelli et al. 1994; Comeron & Torra 1996; Elmegreen & Efremov 1997; Feinstein 1997; McKee & Williams 1997; Oey & Clarke 1998). Such a hierarchical clustering of stars offers a good example of how mass structures can be counted without the telescope beam bias that is present in contouring algorithms. Clusters randomly sample the fractal hierarchy of gas, and have the expected $\sim M^{-1}d\log M$ distribution.

5. A Random Sampling Model for the Initial Mass Function

Stars also sample mass from the hierarchy of gas structures, and would presumably have an $M^{-1}d\log M$ spectrum like the clusters except the stars compete with each other for gas. Once a star forms in part of a hierarchy, the mass remaining in higher levels of the hierarchy no longer has that gas available to make another star. So any other star that forms later at a higher level, which would normally have more mass than the first star because it includes many lower level clumps, would find itself short one of these clumps (because that clump is now an independent star), and as a result, this next star would have a slightly lower mass than its initial share. This systematic lowering of the masses of all late-forming stars will steepen the IMF a little if the late-forming stars are slightly more massive than the first stars, on average. It does not violate the premise that stars of all masses form in clouds of all masses (Sect. 3.3.), because the steepening process occurs during the conversion of a hierarchical cloud
into stars, whereas the comparison between galactic and cluster IMFs discussed above is done after the cluster formation process is mostly over.

Clusters complete with each other for mass too, but the mass of a final cluster is taken to be the sum of the masses of its parts. That is, the cluster mass is the whole mass in a certain part of the hierarchy of structures. It includes all of the lower mass sub-clusters in its total, unlike the late-forming stars. So, the cluster mass is the whole hierarchy mass, while the star mass is the hierarchy mass minus the masses of the other stars that formed first in lower parts of the hierarchy.

This description of star formation is made for an idealized static model, whereas clumps come and go on a local dynamical time, and the conversion of gas into stars operates on a dynamical time too. However, the result is the same whether we speak of a static model or a dynamical model. In a static model, there are more small pieces than large because there is more room for the small pieces, whereas in a dynamical model, there are more small piece than large because the formation rate of the small pieces is much larger than the formation rate of the large pieces that contain them. The difference between the two models reflects one's point of view, not the mass functions for cloud structure. Thus, numerical simulations in which supersonic turbulence continuously generates fractal structure should give the same mass spectra for structures at any one time as they do for all of the structures that form over time.

The final result of gas competition for star formation depends on the relative birth order of the stars. If all stars form at the local dynamical rate, $(G\rho)^{1/2}$, then denser regions tend to form stars first. This would be the case for gravitational processes, or for kinematical processes in virialized regions including magnetic diffusion, clump collisions, and turbulence compression. In a fractal, hierarchical cloud, the density depends on the mass: $\rho \propto M^{1-3/D}$. For $D < 3$, as observed in interstellar clouds, the low-mass regions are denser and proceed toward star formation faster than the high-mass regions that contain them. For this reason, an IMF based on random sampling in fractal clouds is steeper than $M^{-1}d\log M$; i.e., low-mass stars are more competitive for the available gas than high-mass stars. Numerical simulations of this competitive process give the Salpeter IMF, i.e., $\sim M^{-1.35}d\log M$ (Elmegreen 1997, 1999a).

The scaling relations for molecular clouds suggest that the intermediate-to-high mass part of the IMF comes from a factor of $\sim 7$ in length scale. This is because $M \propto L^D$ for fractal dimension $D \sim 2.3$. This power law part of the IMF also comes from only a factor of $\sim 2.5$ in dynamical rate, because the dynamical time $\propto M^{0.2}$ (Elmegreen 2000c). What this means is that essentially all star formation happens on length and time scales that are close to the threshold for self-gravity. This conclusion is consistent with the overall rapid rate of star formation in molecular clouds, and it may also have important consequences for the formation of high-mass stars, discussed below.

The low-mass, approximately flat part of the IMF may be showing us some type of fragmentation spectrum inside each clump (Elmegreen 2000a). If $\epsilon$ is defined to be the star-to-clump mass ratio and $P(\epsilon)$ is the distribution function for $\epsilon$, then the flat or slightly falling (toward lower mass) part results if $P(\epsilon)d\log \epsilon = \text{constant}$ or $\propto \epsilon^\kappa$ with $\kappa \sim 0 - 0.5$ for all star-forming clumps, including those whose stars contribute to the power law part of the IMF.
Figure 4. IMF model with random sampling from a hierarchical cloud for four values of the mass range $R$, defined to be the ratio of the maximum fraction to the minimum fraction of the clump mass that goes into a star. The straight line has a slope of 1.3.
Models are shown in Figure 4 that were made by randomly sampling mass structures in hierarchical clouds with a sampling rate that is weighted by the local density, \( \rho^{1/2} \) (from Elmegreen 2000a). Various ranges \( (R = \epsilon_{\text{max}}/\epsilon_{\text{min}}) \) for \( \epsilon \) are given, with \( P(\epsilon)d\log \epsilon = \text{constant} \), and with \( \epsilon_{\text{max}} \) and \( \epsilon_{\text{min}} \) equal to the maximum and minimum values of \( \epsilon = M_{\text{star}}/M_{\text{clump}} \) in the distribution function \( P(\epsilon) \). If \( \epsilon \) has a large range, then the flat part of the IMF is long. The Salpeter slope always appears at intermediate-to-high mass, with a turnover to a flat slope at low mass. This flat slope is entirely the result of our assumption that \( P(\epsilon)d\log \epsilon = \text{constant} \). The important point is not the value of \( P \), for which there is no theory yet, but that the flat or shallow part of the IMF shows up only below the thermal Jeans mass, even though \( P(\epsilon) \) has been applied to all clumps regardless of mass. In this interpretation, the smallest stars or brown dwarfs come from the smallest cloud pieces that are able to collapse into stars, namely those with mass near \( M_J \), and they arise for those few cases where \( \epsilon \) is near its minimum value. The physical mechanism for their formation is unknown. They could form in the disks of low-mass stars, for example.

At the high-mass end of the IMF, the power law Salpeter function drops until there are very few massive stars in most clusters. According to Massey & Hunter (1998) and Selman et al. (1999), the high mass end of the IMF in 30 Dor has not yet reached the limit of possible stellar masses: the power law function continues to higher and higher mass until there is only one most-massive star. Generally, bigger clusters continue further, giving larger most-massive stars because they sample further out in the IMF. This makes us wonder if there is a physical limit to the mass of a star. Perhaps accretion can be optically thick, as in a disk, and in this way get around the Eddington limit. Or perhaps supermassive stars can form by coalescence (e.g., Zinnecker 1986), which is also an optically thick form of accretion. If the accretion is optically thin, however, then the Eddington limit should influence the upper end of the IMF (Norberg & Maeder 2000).

There is a way to determine if stars have an upper mass limit: from the statistics of star formation in a whole galaxy. Even though single clusters do not have enough stars to sample out to a maximum stellar mass of say, 1000 \( M_\odot \), a whole galaxy does. According to equation 1, a whole galaxy with \( 10^6 \) \( M_\odot \) of young stars should produce a most-massive star with \( \sim 7000 \) \( M_\odot \). Such stars are not observed, even as pre-main-sequence objects, so the IMF cannot continue indefinitely to higher and higher masses. Either there is a sharp cutoff just beyond the maximum mass of \( \sim 120 \sim 150 \) \( M_\odot \) that is observed in 30 Doradus, or there is a more gradual cutoff in most IMFs, with an excess of massive stars compared to this in 30 Dor. Note that internal disruption of a supermassive star by main-sequence or post-main-sequence oscillations (Maeder 1985) will not prevent a pre-main-sequence version of this star from forming; the prevention of supermassive pre-main-sequence stars has to occur in the collapse or pre-collapse phase.

Aside from the Eddington limit, another possible mass limit to a star comes from timing. As an absurd illustration of this problem, one can ask why whole GMCs don’t form single stars. If the ISM is scale free, then they should. The answer seems to be that other things happen first: lower mass stars take the gas for themselves, and cloud disruption removes the remaining gas and halts the
Figure 5. Two IMF simulations are shown. The solid line is for a model with no timing constraint in the choice of clumps for star formation, while the dotted line is with the timing constraint. Both models contain $2 \times 10^6$ stars.

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whole process. These other things are important within only a few dynamical times of the formation time of the lower mass stars. This is the duration of star formation in most regions. However, $\sim 2 - 3$ dynamical times includes the entire stellar mass range in the power law part of the IMF, as discussed above. Thus stars much more massive than $\sim 100 \, M_\odot$ do not have time to form. Any clump of this mass that is physically able to form a supermassive star has a large probability of forming other, smaller stars first.

The IMF should begin to drop suddenly for stars that form in clumps that are so massive that their dynamical times exceed $\sim 3$ times the dynamical time of an $M_J$ star. This is partly because turbulence remixes the gas during this time, preventing the large turbulent clumps from becoming uniform enough to form a single star, and it is partly because star formation from lower mass stars, which tend to form first statistically, uses up the gas and disrupts the cloud.

Figure 5 shows a simulation of this process in a computer-generated IMF with $2 \times 10^6$ stars, made by randomly sampling a hierarchical tree of gas structure with a $\rho^{1/2}$ sampling bias and no Eddington limit (from Elmegreen 2000c). The dotted line is made with an additional step in which a randomly chosen clump of mass $M$ is allowed to turn into a star only with the probability $\exp(-\tau [M] / \tau [M_J])$. Clumps that fail at this stage are sent back to the hierarchical gas tree for further sampling, and are most likely to form several lower mass stars instead. The solid line is for an identical simulation without this additional step. The dotted line falls fast enough at high mass to avoid the formation of supermassive stars. Such stars could still exist if $M_J$ increases, be-
cause according to the theory, the high mass cutoff scales with this characteristic mass. Perhaps starburst galaxies have supermassive stars.


6. Conclusions

The IMF has many properties suggestive of a connection to turbulence. It is universal, stochastic, and power-law. Star formation also has dynamic and structural properties similar to turbulence: the duration of star formation in a cloudy structure is usually only a few crossing times, regardless of scale, and starbirth positions are often fractal.

We have found that virtually any mechanism for star formation that operates on a dynamical time scale in an initially fractal cloud will produce the Salpeter IMF at intermediate to high mass. This IMF is a consequence of random sampling from the various mass structures in the cloud, and not a consequence of the star formation processes. Only the mass at the lower limit of the power law part of the IMF, and the masses at the upper and lower limits, should depend on the detailed physics of star formation. The power law part could be determined in the gas phase.

The low-mass break in the power law part of the IMF might result from a lack of self-gravity in tiny clumps. This break point should be higher in regions with higher temperatures and/or lower pressures. The flat-to-falling part of the IMF below the break may come from fragmentation or other secondary processes inside the dense clumps or disks where stars form, or from pressure fluctuations affecting the Jeans mass. That is, low-mass stars and brown dwarfs should form along with other more massive stars in the smallest cloud clumps. The lack of supermassive stars in galaxies was considered to be a constraint on the IMF. The IMF should fall off more rapidly than the Salpeter slope at high mass because of timing limitations and the Eddington limit. For the timing limit, clumps whose internal dynamical timescales are longer than several times the dynamical time of a clump with the characteristic mass, $M_J$, should not proceed directly to form a single star. Smaller stars should form in these clumps first, making the gas unavailable for the formation of super-massive stars.

Observations of variations in the IMF in different environments would be a good check on the theories. Extreme environments such as high-temperature and high-pressure starburst regions, remote or low-pressure field regions, highly compressed or triggered regions, ultracold gas, and high-pressure cluster cooling flows, might shift the characteristic mass for star formation up or down, depending on the thermal Jeans mass. If this is the only change that is observed in these regions, then models where the power-law part of the IMF comes from universal cloud geometry would be supported. If the power law slope of the observed IMF changes with environment, however, and not just as a result of mass segregation or selective cloud destruction discussed above, then the IMF would not have the universal and scale-free aspect that is expected from turbulence. Other IMF models that are more dependent on the detailed processes of star formation would be required.
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