Characterising the effect of global and local geometric imperfections on the numerical performance of a brace member

M S Hassan¹, J Goggins¹ and S Salawdeh²
¹College of Engineering & Informatics, National University of Ireland, Galway, Ireland
²Department of Civil Engineering, An-Najah National University, Nablus, Palestine

Email: m.hassan3@nuigalway.ie

Abstract. A numerical imperfection study is carried out on a hot rolled tubular brace member under displacement controlled amplitudes. An appropriate range of global and local imperfections is used in the finite element analyses to evaluate the initial-post buckling compressive strength, lateral storey drift, energy dissipation and mid-length lateral deformation of the brace member. The purpose of this study is to assess the impact of the geometrical imperfection on the numerical performance, and to determine an amplitude range that can be used unequivocally for numerical modelling of brace members. It is shown that the amplitude of global imperfections has an effect on the initial response, whereas the amplitude of local imperfections has influence on the resistance capacity of the brace member at higher ductility level. Based on the results, a refined range of amplitude of global and local imperfections is proposed. This range is found to have a good agreement with design standards. In addition, an already established equation to find lateral deformation is compared to results from the analyses and found that the equation with some modification can be used accurately in design. In this paper, a modification factor is proposed in the equation to find the lateral deformation to account for the imperfection amplitude in the numerical analyses of brace members.

1. Introduction

The current knowledge on both the distribution and magnitude of cross-sectional and member imperfection is significantly insufficient for replication of physical configuration into a numerical prototype. Also, the guidelines in the design standards and in the product certificates only cover a conservative range of the upper limit for amplitudes of imperfection to be used in the analysis. One of the reasons of such physical absence is the complexity of the imperfection associated with the rolling and fabrication process which have excluded their definite characterisation [1]. Consequently, numerical techniques have adopted a variety of forms and amplitudes to account for imperfections. However, those forms are entirely based either on numerical studies or on the mathematical formulation rather than physical tests. An alternative solution to the problem is the probabilistic treatment of imperfection proposed by Schaffer and Pekož [2].

In advanced modelling of braces, such as physical theory based models, geometrical imperfections categorised longitudinally into two or more elements to form half-sine wave along the member length. Such models were developed by Salawdeh and Goggins [3] and Uriz et al [4]. However, imperfection in a similar manner cannot be modelled in the finite element (FE), as they do not represent an explicit
approach to model imperfection. In such models, the behaviour of imperfection is characterised by very small spaced Eigenvectors representing the deformed values of the buckling mode. Alternatively, these deformations could be applied to the nodes directly into the desired form of imperfection to the model. A model of this latter type was developed by Nip et al. [5].

Thus, this paper attempts to highlight the way different imperfection types and their interaction affect the compressive response of the hollow member by means of finite element analyses. This is carried out to find a suitable range of amplitudes to be used for the forthcoming research. This study is part of a larger research project focusing on the inelastic behaviour of the tubular bracing members in concentrically braced frames by Broderick et al. [6]. The FE analyses are carried out on a series of geometrical imperfections incorporated in an axially compressed brace member. These imperfections are categorised into (i) global imperfection consisting of half-sine wave and named as Method 1, (ii) equivalent lateral load-global imperfection as Method 2, and (iii) local imperfection as Method 3.

2. Finite element modelling of SHS brace member

A commercial finite element package ABAQUS [7] is used to construct the model of a brace member subjected to a displacement controlled axial loading. The displacement amplitudes are implemented into a nonlinear static solver that accounts for large geometrical nonlinearity during the analysis based on the Newton-Raphson technique. The symmetry of the cross section has been considered herein, such that only half of the section was modelled.

2.1 Brace properties

The studied brace member is a 40x40x3-1250 mm SHS formed from hot rolled carbon steel of grade S355 J2H, which is classified as Class 1 according to the slenderness criteria set by EC3 Part 1-1 [8]. The normalised global slenderness, \( \bar{\lambda} \) is 0.6, which is defined by EC3 Part 1-1 [8] as \( \bar{\lambda} = \frac{f_y}{A/N_{cr}} \), where \( f_y \) is the yield strength, \( A \) is the gross cross sectional area, and \( N_{cr} \) is the elastic critical buckling load. This value is less than the upper limit on the global slenderness \( \bar{\lambda} \) specified in EC8 [9] of 2.0 for bracing member in the concentrically braced frames. The monotonic tensile mechanical properties of the material are given in Table 1.

2.2 Element and mesh size

The 4 nodes doubly curve shell element with reduced integration, designated S4R, is used to mesh the model. This element deforms explicitly in shear and provides less computational cost compared to the other quadratic and cubic elements. In addition, this element has been used in previous similar studies for modelling of tubular members [5, 10].

Nip et al. [5] carried out a detailed mesh study prior to simulation, and found that a variation of strain output in the highly localised strain region was small beyond a mesh size of 32 elements per face. In addition, this size of element possesses an aspect ratio close to unity. Thus, it was found suitable to be used at the mid-length and ends of the models, where the local buckling was likely to occur. The extend of the refined mesh at the regions of the local buckling was set to 1.5 times the larger dimension of the cross section. In other regions, coarser mesh was used to reduce computational cost.

2.3 Boundary conditions

The boundary conditions are applied to both ends of the brace, such that they are restrained in all directions (six degree of freedom at each node) except the loaded end. In addition, restraint in the y-direction was imposed along the length of the member due to the symmetry of the cross-section as shown in Figure 1.
Table 1. Monotonic tensile engineering material properties [5].

| Model            | Young’s Modulus, E (N/mm²) | Yield Strength, f_y (N/mm²) | Ultimate tensile strength, f_u (N/mm²) |
|------------------|----------------------------|-----------------------------|---------------------------------------|
| 40x40x3-CS-HR    | 219610                     | 478                         | 555                                   |

![Figure 1. Symmetry condition and end restraints of the brace model](image)

2.4 Geometrical imperfection
A detailed literature review was carried out on the types and amplitudes of imperfection that have been used previously for the prediction of load-displacement behaviour of brace members. For global imperfection, two approaches are widely used in numerical models, (i) a half-sine wave profile is used to derive the initial geometric shape of the brace to capture its out-of-straightness (henceforth, known as Method 1) (see, for example, [3,4,11-13]) and (ii) the application of an equivalent notional lateral load normal to the plane of the buckling at the mid length of the un-deformed member (henceforth, known as Method 2) (see, for example, [5]). In the first approach, the nodal coordinates of the brace represent the unloaded imperfect brace member, using a half sine wave imperfection over its length (Figure 2a). However, in the latter approach, the nodal point loads at the mid-length as shown in Figure 2b, represents an initial imperfection in the brace member.

Long, thin plate buckles locally into half sine waves under uniform axial compression, therefore, it was assumed appropriate to model the brace with local imperfection by using a sine function, which is given as:

\[ \omega_2 \cdot \sin \left( \frac{x\pi}{L} \right) \]  

Where, \( \omega_2 \) represents the amplitudes of local imperfection, x’s are the nodal coordinates of the models and L is the overall length of the brace model. The equated imperfections are incorporated into the numerical model by defining the nodal coordinates into their imperfect configuration, which results in a ‘flowing fish’ pattern of imperfections over the brace length as in the Figure 2c.

In order to characterise the varying effect of global imperfections across a range of amplitudes, a suitable range of global amplitudes, \( \omega_1 \), from L/100 to L/2000 for initial mid-length out-of-straightness was selected. This range represents a broad scale of amplitudes compared to the tolerance limit of L/500 prescribed in the European (EN 10210-2 2006) [14], North American (ASTM A501 2005) [15] and Australian (AS 1163 1991) [16] standards for hot-rolled sections. Furthermore, the wall thickness, t, tolerance in those standards for hot rolled section is given as ±10% of the section thickness. Thus, a range of t/5 to t/200 was chosen for local amplitudes, \( \omega_2 \), to account local imperfection in brace models.

3. Discussion of results
3.1 Initial and post buckling compressive strength
The initial buckling loads (\( F_{cr} \)) correspond to the types of imperfections with varying amplitudes are presented in Table 2. These values are normalised to the un-factored design strengths calculated according with EC3 [8] using buckling curve a and AISC [17] design standards. The results are tabulated
in the order of decreasing geometrical imperfection. Similar arrangements are used in the figures throughout this study.

**Figure 2.** Brace model derived using three methods: (a) Half sinusoidal profile used to derive the initial geometric shape (Method 1), (b) Concentrated nodal point load at mid-length corresponding to the equivalent lateral load imperfection (Method 2), (c) Initial sinusoidal local imperfection profile (flowing fish pattern imperfection) (Method 3).

When the initial global out-of-straightness of the brace member has a magnitude $\omega_1 \leq L/500$, the initial buckling loads obtained from the FE models that employ Method 1 to capture initial global imperfections is between 2 and 8% higher than estimates from EC3 [8] and 5-11% higher than predictions from AISC [17]. On the other hand, the estimates from the codes overestimated the initial buckling capacity of between 5 and 15% for models where higher imperfection is used. As expected, the buckling capacity of the member reduces with increase in initial out-of-straightness. From Table 2, it is evident that lower initial buckling capacities are obtained for a given amplitude of the initial out-of-straightness when the imperfections are applied using a notional lateral load at mid-length (i.e. Method 2), rather than creating a geometric model of the unloaded member incorporating its out-of-straightness (i.e. Method 1).

The initial buckling capacity of the braces obtained using Method 2 is affected by both the residual stresses and deformed geometry induced by the equivalent lateral load, whereas no residual stresses are present in Method 1. Consequently, the local deformation induced by the equivalent lateral load applied locally to the brace surface at mid-length has a significant effect on overall brace buckling capacity, especially, when $\omega_1 \geq L/250$. Based on the FE results, the tolerance of $L/500$ specified by international standards for the initial out-of-straightness seems appropriate when considering the initial buckling capacity of brace members. However, engineers should be aware of the impact that various methods of applying global imperfection have on the buckling capacity of the brace. In reality, the real physical behaviour of brace lies between Method 1 and Method 2.

In figures 3 and 4, the buckling resistance ($F_{cr}$) normalised by the yield strength of the section ($F_{y}A$) is plotted against normalised axial displacement ($\delta_{c}$) to yield displacement ($\delta_{y}$). This facilitates the direct comparison of the response at buckling strength during the entire loading history. Models with initial lower out-of-straightness shows a clear interaction of global-local effect by a bell-shaped transition curve in the post-buckling range, which indicates that the local buckling was initiated at higher strain rates. However, such transition effect is suppressed in braces where higher imperfection is present. This is due to the inelastic strain being concentrated over a large critical region, that affect the strain rate in the critical region of the models with higher amplitudes of imperfection. This was proven by monitoring the strain rate at locally buckled region around the mid-length of the brace models.
Table 2. Comparison of the initial buckling load for braces subject to axial compression loading and with various amplitudes of out-of-straightness, $\omega_1$.

| Amplitude $\omega_1$ | Initial imperfection (mm) | Method-1 | Method-2 |
|----------------------|---------------------------|----------|----------|
|                      | $F_{cr}$ | $F_{cr}/F_{EC3-a}$ | $F_{cr}/F_{AISC}$ | $F_{cr}$ | $F_{cr}/F_{EC3-a}$ | $F_{cr}/F_{AISC}$ |
| L/100                | 12.5    | 160     | 0.85     | 0.87     | 68      | 0.36     | 0.37     |
| L/175                | 7.14    | 172     | 0.91     | 0.94     | 121     | 0.64     | 0.66     |
| L/250                | 5       | 180     | 0.95     | 0.98     | 160     | 0.84     | 0.87     |
| L/500                | 2.5     | 193     | 1.02     | 1.05     | 170     | 0.90     | 0.92     |
| L/1000               | 1.2     | 200     | 1.05     | 1.09     | 184     | 0.97     | 1.00     |
| L/1500               | 0.833   | 203     | 1.07     | 1.11     | 186     | 0.98     | 1.01     |
| L/2000               | 0.625   | 204     | 1.08     | 1.11     | 194     | 1.02     | 1.06     |

$a^*$ represents buckling curve $a$ of EC3 used for calculation of buckling strength
$\omega_1^*$ imperfection assumed at the mid-length from the face of an equivalent perfect straight member

Upon local buckling, the local post-buckling capacities of the models with method-1 reduce further regardless of the influence of global amplitudes of imperfection. However, a fluctuation is observed in results of models with method-2, due to the same reason outlined in this section previously.

From the above observation, we can say that the predicted response corresponding to the lower amplitudes of imperfection, i.e. when $\omega_1 \leq L/500$, represents the true buckling behaviour of brace under axial compression loading that meet the limits specified by international standards for initial out-of-straightness.

The initial elastic compressive strength, $F_e$, obtained from the local analyses of the models consisting of sinusoidal wave imperfection is presented in Table 3. The elastic response of the imperfect models under axial compression is similar to that for a member under tensile loading and is independent of the influence of initial local imperfection. In those locally imperfect models, local buckling occurs after the yield strength is exceeded.

![Figure 3](image-url)  
**Figure 3.** Normalised $F_{cr}$-$\delta_c$ curves of FE Models with half-sine wave imperfection (Method 1) with drift capacity.
Figure 4. Normalised F<sub>cr</sub>-δ<sub>c</sub> curves of FE Models with sinusoidal wave imperfection (Method 2) with drift capacity.

In figure 5, at ductility level μ ≥ 3 the compressive resistance of the brace models with local imperfection ω < t/100 is increased relative to a perfectly straight member. However, at the same ductility level the compressive strength of the models is reduced when ω ≥ t/50. It may be due to the difference of the numbers of local buckling occurred in the critical regions in the models when, t/5 > ω ≥ t/50 (i.e. simultaneous five local buckling) and ω < t/100 (seven local buckling occurs) respectively. In general, we can say that amplitudes of initial local imperfection has negligible effect on elastic response, F<sub>e</sub>, but significant effect on the resistance capacity at higher ductility level.

3.2 Energy dissipation capacity

The area under the force-displacement curves represent the energy dissipation capacity of the brace members and are presented in tables 4 and 5. These values are calculated at a ductility level of 4 and 10, and indicated as W<sub>μ=4</sub> and W<sub>μ=10</sub>. The obtained values are normalised with the elastic strain energy of the member, which is a function of the cross sectional area and yield strength.

When the dissipated energy corresponding to the braces with higher amplitude of imperfection (i.e. ω=L/100) are compared with those having lower amplitude of imperfection (i.e. ω=L/2000), an average of 35% less energy is dissipated by the braces with a higher amplitude of imperfection at two ductility levels, i.e. at μ=4 and μ=10, when modelled using Method 1. At the same ductility levels and with same amplitudes of imperfection, the brace modelled using method-2 (i.e. with an equivalent lateral load imperfection), dissipated 90% lesser energy when it contained a high level of imperfection in comparison with the model having lower amplitude of imperfection. The reasons for such a markedly varying dissipated capacity of the models in two methods are the same outlined in the previous section. It should be noted that despite two different imperfection methods, almost same dissipated energy capacity is achieved when lower amplitude of imperfections are employed in the models (Table 4).

The models with higher amplitudes of local imperfection, i.e. ω≥t/50, show on average 15% reduction in energy dissipation capacity relative to a model with no imperfection at a higher ductility level, i.e. μ=10. However, at the same ductility, an increase of 12% in energy dissipation capacity is observed in models where lower amplitudes of imperfection were used. At lower ductility level, μ=4,
the variation in energy dissipation capacity is minimal due to the difference in the numbers of occurrences of local buckling in the models, as discussed earlier.

3.3 Lateral deformation
The results of the FE models in relation to the out-of-plane deformation are plotted against axial displacement in Figure 6. The values are normalised with the brace length, L and yield displacement, δy in order to facilitate the comparison between the lateral deformation and the axial shortening. The figure shows curves that lateral deformations of braces with larger amplitudes of global imperfection commences during the elastic loading. However, for braces with smaller amplitudes (i.e. \( \omega_1 \leq L/1000 \)), such excessive deformation was initiated upon the onset of the global buckling. This response agrees with the response of the brace member concluded by Tremblay [18].

The curves are compared to the predictive equations proposed by Tremblay [18] and given in Eqs (2) and (3) for higher and lower ductility levels. These equations are a function of the applied axial displacement, \( \delta_c \), cross sectional depth, h, and brace length, L having a coefficient at the front that depend on the support conditions.

![Figure 5. Normalised load-axial displacement response of models incorporated with sinusoidal local imperfection (Method 3).](image)

| Amplitude | Initial imperfection (mm) | Fe (KN) | Fy/FEC3-a | Fu/FAISC |
|-----------|---------------------------|---------|------------|---------|
| \( t/5 \) | 0.6                        | 192     | 1.01       | 1.05    |
| \( t/10 \) | 0.3                        | 203     | 1.07       | 1.11    |
| \( t/15 \) | 0.2                        | 205     | 1.08       | 1.12    |
| \( t/25 \) | 0.12                       | 205     | 1.08       | 1.12    |
| \( t/50 \) | 0.06                       | 205     | 1.08       | 1.12    |
| \( t/100 \) | 0.03                       | 206     | 1.09       | 1.13    |
| \( t/150 \) | 0.02                       | 206     | 1.09       | 1.13    |
| \( t/200 \) | 0.015                      | 206     | 1.09       | 1.13    |
| No imperfection | 0                       | 204     | 1.08       | 1.12    |

\( \omega \): imperfection assumed from the face of an equivalent perfect straight member

Table 3. Summary of the numerical results for local analyses of braces with local imperfections over length as sinusoidal waves (flowing fish pattern imperfection).
Table 4. Summary of the dissipation capacity of the brace models with global imperfection.

| Amplitude | Method-1 | Method-2 |
|-----------|----------|----------|
|           | $W_{\mu=4}$ | $W_{\mu=10}$ | $W_{\mu=4}$ | $W_{\mu=10}$ |
| L/100     | 3.3       | 6.6       | 0.5         | 0.8         |
| L/175     | 3.7       | 7.7       | 1.8         | 3.1         |
| L/250     | 4.2       | 8.0       | 2.6         | 4.6         |
| L/500     | 4.6       | 9.0       | 3.9         | 6.8         |
| L/1000    | 5.1       | 9.4       | 4.6         | 8.3         |
| L/1500    | 5.3       | 10.0      | 4.7         | 8.8         |
| L/2000    | 5.5       | 10.1      | 5.0         | 9.0         |

Table 5. Summary of the dissipation capacity of the brace models with local imperfection.

| Amplitude | Method-3 |
|-----------|----------|
|           | $W_{\mu=4}$ | $W_{\mu=10}$ |
| t/5       | 5.8       | N/A           |
| t/10      | 6.5       | 15.1          |
| t/15      | 6.5       | 15.6          |
| t/25      | 6.5       | 16.4          |
| t/50      | 6.8       | 17.0          |
| No imperfection | 7.3 | 19.3 |
| t/100     | 7.4       | 21.4          |
| t/150     | 7.4       | 21.5          |
| t/200     | 7.4       | 21.6          |

Three different responses of the lateral deformation can be observed. Firstly, the curves running close to the higher predictive equation are those with a larger global imperfection corresponding to Method 2. Secondly, the curves running furthest away from the same equation. Thirdly, the curves running between the boundaries of the two predictive equations, and are similar to each other in shape.

\[
\Delta = 0.7 \cdot \sqrt{\delta_c \cdot (L - 2h)} \quad \text{For higher ductility level} \quad (2)
\]

\[
\Delta = 0.16 \cdot \sqrt{\delta_c \cdot L} \quad \text{For lower ductility level} \quad (3)
\]

The predictive equation for higher ductility level tends to give conservative results. The same conclusion was reported by Nip et al. [5]. This can be expected because it accounts for the lateral deformation of buckled slender braces in the post buckling region, which show larger lateral deformations at larger axial displacements compared to less slender braces.

Based on the FE results and the predictive equation, a dimension-less factor is introduced to the equation proposed by Tremblay [18]. This factor consists of a $\Gamma$ factor that depends on the adjacent ratio of the imperfection amplitude, $\omega_1$, and the brace length, L. The imperfection amplitudes $\omega_1$ can take global amplitudes, $\omega_1$, up to a maximum and minimum of L/5 to L/2000 respectively.

\[
\Delta = \Gamma \cdot \frac{\omega_1}{L} \cdot 0.7\sqrt{\delta_c \cdot (L - 2h)} \quad (4)
\]

The values for $\Gamma$ can be obtained using Equation (5). This equation is derived from a curve fitting analysis, with a correlation coefficient ($R^2$) = 1. The ratio of imperfection amplitude, $\omega_1$, and the brace length, L, has been replaced with variable $\alpha$ for equation convenience, such that:

\[
\Gamma = \frac{0.97}{\alpha^{0.96}} \quad (5)
\]
Figure 6. Normalised lateral deformation at mid-length corresponds to (a) half-sine wave imperfection and (b) equivalent load imperfection.

The proposed factor significantly accounts the variation of the lateral deformation across the imperfection amplitudes for short brace member. At higher amplitudes it will produce the magnitude close to the unity and the equated results will be approximately in line with the results of the Tremblay Equation (2). However, the factor will reduce the magnitude of the lateral deformation when lower imperfection amplitudes are used in the equation.

4. Recommendation for geometrical imperfection for less-slender hot rolled sections
The summary of the imperfection study with the view of finding a suitable range of imperfection amplitudes for the numerical modelling of hot-rolled tubular brace member is:

- When using global half sine wave imperfection, a minimum amplitude of L/2000 is used but not greater than L/500, which is the maximum tolerance specified by the international standards.
- For notional lateral load imperfection, amplitudes L/1000 to L/2000 are recommended to be used at the mid length of the critical region, provided that the computational cost is of significance interest.
- An amplitude range of t/10 to t/50 is recommended for local imperfection in order to avoid the enhancement of the forcible resistance at higher ductility levels.
- If physically measured imperfection is available these should be utilised in the numerical model to more accurately capture the actual performance of brace member.

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References
[1] Gardner L and Nethercot D A 2004 Numerical modelling of stainless steel structural components —A consistent approach J. Struct Div ASCE. 130 1586-01.
[2] Schafer B W and Pekoz T 1998 Computational modelling of cold formed steel: characterizing geometrical imperfection and residual stresses J. Construct Steel Res. 47 193-10.
[3] Salawdeh S and Goggins J 2013 Numerical simulation for steel braces incorporating a fatigue model Eng Struct. 46 332-49.

[4] Uriz P, Filippou F C and Mahin S A 2008 Model for cyclic inelastic buckling of steel braces J. Struct Div ASCE. 34 619-28.

[5] Nip K H, Gardner L and Elghazouli A Y 2010 Cyclic testing and numerical modelling of carbon steel and stainless steel tubular bracing members Eng Struct. 32 424-41.

[6] Broderick BM, Hunt A, Mongabure P, Goggins JM, Salawdeh S, O'Reilly G, Beg D, Moze P, Sinur F and Elghazouli AY 2015 'Assessment of the seismic response of concentrically-braced steel frames' In: Taucer, Fabio, Apostolska, Roberta (Eds). Experimental Research in Earthquake Engineering, Geotechnical, Geological and Earthquake Engineering, Vol. 35. London.

[7] ABAQUS 2013 User manual, version 6.13. Hibbitt, Karlsson and Sorensen Inc.

[8] CEN 2005 Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings, EN 1993-1-1:2005/AC: 2009.

[9] EN 1998-1:2004 Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for building. European Standard EN 1998-1:2004.

[10] Haddad M 2015 Concentric tubular steel braces subjected to seismic loading: Finite element modelling J.Construct Steel Res. 104 155-66.

[11] Mamghani IHP, Usami T and Mizuno E 1996 Inelastic large deflection analysis of structural steel members under cyclic loading Eng Struct. 89 659-68.

[12] Wijesundra KK 2009 Design of Concentrically braced steel frames with RHS shape braces PhD thesis. Pavia: European Centre for Training and Research in Earthquake Engineering (EUCENTRE).

[13] Fell B V 2008 Large-Scale Testing and Simulation of Earthquake-Induced Ultra Low Cycle fatigue in Bracing Members Subjected to Cyclic Inelastic Buckling PhD thesis. Davis, UC.

[14] EN 10210-1, CEN; 2006 Hot finished structural hollow section of non-alloy and fine grain steels-Pat 1: Technical delivery conditions, European Standard.

[15] ASTM A501 2005 standard specification for hot-formed, welded and seamless carbon steel structural tubing in round and shapes.

[16] AS 1163 1991 Australian Standard Structural steel hollow sections.

[17] ANSI/AISC 360-05 An American National Standard Specification for Structural Steel Building.

[18] Tremblay R 2002 Inelastic seismic response of steel bracing members J. Construct Steel Res. 58 665-01.