Numerical investigation of the resonance behavior of flow-excited Helmholtz resonators

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Aeroacoustics has acquired added significance in the automotive industry over recent years. A low level of noise within the car cabin provides additional value to the automotive and increases security of passengers. At slow speeds, noises are primarily due to the engine and rolling of the tires. But at higher speeds and with increasing electrification of cars, flow-induced noise comes more and more to the fore. In particular, the functional gap that can be found at the tailgate of cars combined with air flowing over it poses a relevant sound source.

In this contribution, we numerically investigate the resonance behavior of the tailgate gap of a car based on experiments presented in [1]. As depicted in Fig. 1, a generic model was created based on the cross section of this gap in order to carry out harmonic acoustic simulations using the finite element method (FEM).

1 Introduction

1.1 Synthetic excitation signal

To model the acoustic source density q on the orifice of the cavity without performing CFD computations, the Corcos model is employed [2]. The Corcos model describes wall-pressure fluctuations associated with a turbulent boundary layer (TBL). This model is based on the cross spectral density (CSD) Γ of the wall-pressure at different points on the wall associated with the TBL.

\[ \Gamma(\xi_x, \xi_y, \omega) = \Phi(\omega)G(\xi_x, \xi_y, \omega) = \Phi(\omega)e^{-|\xi_x U_c/\omega| - |\xi_y \epsilon_c \omega|}e^{j \frac{\xi_y \omega}{\epsilon_c \omega}}, \tag{1} \]

where Φ is the power spectral density, ω the angular frequency, ξx the distance between the respective points in x-direction, ξy the distance between the respective points in y-direction, Uc the convective velocity, \( \epsilon_c \) the longitudinal decay rate of the coherence (in flow direction), \( \epsilon_y \) the lateral decay rate of the coherence, and \( j = \sqrt{-1} \) is the imaginary unit. G(ξx, ξy, ω) is a coherence function that determines the cross-spectral density for all points on the wall based on the power spectral density. For this investigation, it is assumed that the acoustic source density on the cavity orifice shows the same coherence characteristics as the wall pressure fluctuations ahead of the orifice.

The orifice of the generic cavity has an area of 50 mm × 6 mm. With these small dimensions and a high convective velocity of 25.6 m/s, the distances in flow direction x can be neglected. Therefore, the cross spectral density Γ reads

\[ \Gamma(\xi_x, \xi_y, \omega) = \Phi(\omega)e^{-|\xi_x U_c/\omega|}. \tag{2} \]

Since the goal is to carry out harmonic acoustic simulations without performing (CFD) computations, the power spectral density Φ(ω) in Eq. (2) is set equal to a constant Φ0. Thus one will only gain qualitative information about the pressure at the bottom of the cavity and may not judge absolute amplitudes.

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2.3 Generation of a coherent excitation signal

To transform the cross spectral density $I$ into a signal that shows the coherence characteristics given by the coherence function $G(\xi_x, \xi_y, \omega) = \exp \left(-|\xi_x c \omega / U_c|\right)$, the first step is to discretize this coherence function. Using the same discretization for the orifice as used for the FEM model yields the coherence matrix $C$. The coherence matrix is, in this case, a matrix whose element in the $(i, j)$-position is the coherence between the acoustic source density signal at position $i$ and $j$ on the orifice.

Following the approach proposed in [3], the next step is carrying out an eigenvalue decomposition $C = V D V^T$, where $D$ is a diagonal matrix of the eigenvalues $\lambda_i$ of $C$, and $V = \{v_i\}$ is the matrix of eigenvectors $v_i$, such that $C v_i = \lambda_i v_i$. Next, a complex vector $u$ of mutually independent random values $u_i = \exp(j \varphi)$ with $\varphi \sim U[-\pi, \pi]$ is created and used to compute the desired coherent signals as

$$\hat{q}(\omega) = \frac{1}{c_N} V \sqrt{D}^H u,$$

where the superscript $H$ denotes the Hermitian transpose and $c_N$ is a normalization constant to ensure that the overall root mean square of the acoustic source density on the orifice is equal for every frequency.

2.4 Damping

In order to take viscous friction and thermal losses into account, a complex mass density $\rho \in \mathbb{C}$ was applied to the neck of the cavity. Therefore, the approach proposed by Jaouen and Bécot [4] was employed. Even though this work is aimed at cylindrically perforated facings used for noise control applications, it is expected that the proposed model can be adapted to the generic cavity. To do so, the hydraulic diameter of the orifice was used and the real part of the complex density was modified such that it is equal to the density of air at 20°C.

2.5 Results

Applying both the synthetic acoustic source density and the complex mass density to the FEM model in order to carry out a harmonic acoustic simulation using the finite element software CPS++ [5] yields the transfer function depicted in Fig. 2. This transfer function describes the pressure at the bottom of the cavity relative to the acoustic source density on the cavity orifice. The maximum amplitude at a frequency of 544 Hz indicates that this frequency is the Helmholtz frequency. This value is very close the measured value of 545 Hz. Hence, the Helmholtz frequency of the generic cavity can be resolved very well with the proposed modeling strategy.

Fig. 1: Sectional view of the real and generic cavity.  
Fig. 2: Transfer function resulting from a harmonic calculation.

3 Conclusion

In this contribution, a method to generate a synthetic acoustic source density signal based on the Corcos model was presented. Furthermore, it was shown how to take damping into account for acoustic simulations of cavities. With this computationally efficient approach, the Helmholtz frequency of the generic cavity can be resolved with good agreement to measurements.

References

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