Realization of density-dependent Peierls phases to engineer quantized gauge fields coupled to ultracold matter

Frederik Görg, Kilian Sandholzer, Joaquín Minguzzi, Rémi Desbuquois, Michael Messer and Tilman Esslinger

Gauge fields that appear in models of high-energy and condensed-matter physics are dynamical quantum degrees of freedom due to their coupling to matter fields. Since the dynamics of these strongly correlated systems is hard to compute, it was proposed to implement this basic coupling mechanism in quantum simulation platforms with the ultimate goal to emulate lattice gauge theories. Here, we realize the fundamental ingredient for a density-dependent gauge field acting on ultracold fermions in an optical lattice by engineering non-trivial Peierls phases that depend on the site occupations. We propose and implement a Floquet scheme that relies on breaking time-reversal symmetry by driving the lattice simultaneously at two frequencies that are resonant with the on-site interactions. This induces density-assisted tunnelling processes that are controllable in amplitude and phase. We demonstrate techniques in a Hubbard dimer to quantify the amplitude and to directly measure the Peierls phase with respect to the single-particle hopping. The tunnel coupling features two distinct regimes as a function of the modulation amplitudes, which can be characterized by a $\mathbb{Z}_2$-invariant. Moreover, we provide a full tomography of the winding structure of the Peierls phase around a Dirac point that appears in the driving parameter space.
To directly measure the matrix elements of the tunnelling amplitude \( t_{ij}^{(0)} \) and the gauge field operator \( \hat{A}_{\text{RF}} \), we project the Hamiltonian (1) onto a link \( (k) \) by realizing individual Hubbard dimers (Fig. 1a). We introduce an asymmetry between the two sites of the dimer with a static energy bias \( \Delta_0 \) such that we can selectively address the density-assisted tunnelling processes \( t_{eg,R}^{(0)} \) and \( t_{el,L}^{(0)} \) with the drive. If \( U \) is much larger than both \( \Delta_0 \) and the static tunnelling \( t \), the ground state for static double wells occupied by two atoms in states \( \uparrow \) and \( \downarrow \), respectively, is given by the singlet \( |s\rangle = (|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle) / \sqrt{2} \). When driving the system resonantly such that \( U \approx \hbar \omega + \Delta_0 \), the singlet is coupled to the double occupancy state \( |d\rangle = |0, 1\rangle \) via \( \gamma^{(0)} t_{el}^{(0)} \) by absorbing \( l \) photons from the drive (Fig. 1b and Supplementary Fig. 1). The effective Hamiltonian in this two-level system can be written as

\[
\hat{H}_{\text{eff}}^{(i)} = h^{(i)} \cdot \sigma + \delta^{(i)} / 2 \mathbb{1}^{2 \times 2}
\]

where \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) is the vector of the Pauli spin matrices and

\[
h^{(i)} = (-\mathbb{1} / 2) t_{el}^{(i)} \cos(\psi^{(i)}) + \mathbb{1} / 2 t_{el}^{(i)} \sin(\psi^{(i)}) / 2 \delta^{(i)} / 2
\]

where \( \delta^{(i)} = U - \hbar \omega - \Delta_0 \) is the detuning from the \( l \)th-order resonance (for the measurement of \( t_{el}^{(i)} \) at \( U \approx \hbar \omega - \Delta_0 \), see Supplementary Fig. 7). To directly measure \( \psi^{(i)} \) we perform an interference measurement, in which the single-particle phase \( \psi^{(i)} \) acts as a reference (for our parameters \( \psi^{(0)} \approx 0 \), Supplementary Fig. 3). Instead of comparing \( \psi^{(i)} \) to the phase acquired by single atoms in spatially separated dimers (Fig. 1a), we can conduct an interference measurement within each doubly-occupied dimer by switching the state \( \uparrow \) to a third internal state labelled \( \rightarrow \) via a radiofrequency (RF) pulse (Fig. 1b). The interaction between the spin states \( \uparrow \) and \( \downarrow \) can be set to \( U = \Delta_0 \ll \hbar \omega \), such that the atoms experience the effective Hamiltonian in equation (2) with \( l = 0 \), which contains the single-particle tunnelling \( t_{el}^{(0)} \). On the Bloch sphere, the vector \( h^{(0)} \) representing the Hamiltonian \( H_{\text{eff}}^{(0)} \) is pointing along the \( x \) axis for \( \delta^{(0)} = 0 \), while the resonant Hamiltonian \( h^{(0)} \) is rotated around the \( z \) axis by an angle \( \psi^{(0)} \) (Fig. 1c). To characterize any quantum state \( |\varphi\rangle \), we can measure for both combinations of spins the fraction of double occupancies \( D = \langle |\delta^{(0)}\rangle^2 \rangle^2 \) and singlets \( S = \langle |\delta^{(0)}\rangle^2 \rangle^2 \). Here, \( \langle \ldots \rangle \) denotes the average over the inhomogeneous distribution of \( \Delta_0 \) in different dimers resulting from the underlying harmonic trapping potential.

In the experiment, we use a harmonically confined cloud of \( N = 41(4) \times 10^4 \) ultracold fermionic \( ^{40}\text{K} \) atoms in a balanced mixture of two initial internal states \( \uparrow \) and \( \downarrow \), which are loaded into a dimerized, three-dimensional (3D) optical lattice (Fig. 1a). The sites constituting the dimers are connected with a static tunnelling amplitude of \( \hbar t = 260(30) \text{ Hz} \) and are offset in energy by \( \Delta_0 / \hbar = 660(20) \text{ Hz} \).

**Fig. 1** | **Experimental set-up and driving scheme.** a, Lattice potential in the \( x-z \) plane consisting of individual dimers with an energy bias \( \Delta_0 \). The lattice position is sinusoidally modulated in the \( x \) direction at two frequencies \( \omega/(2\pi) \) and \( 2\omega/(2\pi) \) using a piezoelectric actuator (not shown). If the on-site interaction \( U \) is tuned close to a resonance \( U = \hbar \omega + \Delta_0 \), atoms pick up a phase \( \psi^{(i)} \) in a density-assisted tunnelling process \( t_{el}^{(i)} \) compared to a single-particle hopping process \( t_{el}^{(i)} \) with the drive. If \( \psi^{(i)} \) is rotated around the \( \pi \) axis at two frequencies \( \omega \), the singlet is coupled to the double well for the two-frequency driving scheme with \( K_i \). We introduce an asymmetry between the state \( \uparrow \) and \( \downarrow \) and singlets \( D_i \) is anti-aligned with the \( x \) axis, while \( h^{(i)} \) is rotated around the \( z \) axis by an angle \( \psi^{(i)} \). b, Schematic of the effective Hamiltonian in the double wells. The double occupancy state \( |d\rangle = |0, 1\rangle \) is coupled to the singlet state \( |s\rangle \) via a density-assisted tunneling process induced by multi-photon processes of the resonant drive. Using a RF pulse, it is possible to switch to a third internal state \( \rightarrow \), for which the tunnel coupling is equivalent to the single-particle hopping amplitude \( t_{el}^{(i)} \). c, Visualization of the effective two-level system in b on a Bloch sphere. For \( U = \Delta_0 \ll \hbar \omega \), the off-resonant Hamiltonian represented by the vector \( h^{(0)} \) is anti-aligned with the \( x \) axis, while \( h^{(0)} \) is rotated around the \( z \) axis by an angle \( \psi^{(0)} \). d, Time-dependent energy offset \( \Delta(x) \) between the two sites of the double well for the two-frequency driving scheme with \( K_i = 1 \) and \( K_j = 0.5 \). The common phase \( \phi_i \) leads to a mere shift of the waveform in time, while the relative phase \( \phi_i \) explicitly breaks time-reversal symmetry for \( \phi_i \neq 0, \pi \). The waveforms are offset for clarity.
Using a suitable loading procedure, 56(2)% of the atoms occupy dimers that are populated by two opposite spins (see Methods). The interaction $U/\hbar$ between atoms in states $\uparrow$ and $\downarrow$ can be tuned in a range between 5 and 10 kHz using a magnetic Feshbach resonance. The drive consists of a time-periodic modulation of the lattice position at two frequencies $\omega_l/(2\pi) = 2.75$ kHz and $2\omega_l/(2\pi) = 5.5$ kHz, which in a co-moving frame corresponds to a modulation of the energy offset $\Delta_m(\tau) = \Delta_\uparrow + \Delta_l(\tau)$ within the dimers\(^1\)). In the presence of a time-dependent part

$$\Delta_l(\tau) = \hbar \omega_0 K_1 \cos(\omega l \tau + \phi_1) + 2\hbar \omega_0 K_2 \cos(2\omega l \tau + 2\phi_1 + \phi_2) \tag{4}$$

Here, $K_1$ and $K_2$ are the dimensionless driving amplitudes and $\phi_1$ is a common phase, which shifts the waveform in time without changing its shape (Fig. 1d). It can be set to zero by choosing an appropriate origin of time. In contrast, the relative phase $\phi_2$ explicitly breaks TRS for $\phi_2 \neq 0, \pi$. Therefore, it will both affect the absolute value $|t_{\text{el}}^{(l)}|$ and, crucially, lead to a non-trivial phase $\psi_l^{(l)}$ that cannot be eliminated by a suitable gauge choice.

To derive the effective tunnelling matrix element for our driving scheme, we perform a high-frequency expansion in a rotating frame (see ref. \(^1\) and Supplementary Information) and find

$$t_{\text{el}}^{(l)} = t_0 e^{-i\phi_2} \sum_{m} J_{-2m+4}(\Delta_\uparrow) J_m(\Delta_l) e^{-im\phi_1} \tag{5}$$

to lowest order, where $J_m$ is the $m$th-order Bessel function. The effective tunnel coupling is given by the interference of all multi-photon processes in which $m$ photons are absorbed from the $2\omega$ drive and $2m - l$ photons are re-emitted into the $\omega_l$ drive, such that the total energy added to the system is $\hbar \omega_0 = U - \Delta_\uparrow$. In the experiment, we investigate the case $l = 2$, for which the leading terms of
the sum can be written as $t^{(2)}_{\alpha} = t_{\alpha}^2 + \beta^2 e^{-i\phi}$, where $t^{(2)}_\alpha$, $\beta^2 > 0$ depend on $K_1$ and $K_2$ and we fixed the gauge such that $\phi_\alpha = 0$. It can be seen that if $\phi_\alpha = 0$ or $\pi$ such that TRS is not broken, the tunnelling matrix element is real. Furthermore, if $\alpha^2 = \beta^2$ and $\phi_\alpha = \pi$, the tunnelling amplitude vanishes. Away from this singular point, $|t^{(2)}_{\alpha}|$ increases linearly with $|\alpha^2 - \beta^2|$ and $\phi_\alpha$ and is therefore forming a Dirac point in this generalized parameter space. At the same time, the Peierls phase $\psi^{(2)}$ has a vortex structure around the singularity.

In our experiment, we measure both the absolute value of the effective tunnelling on the resonance $l=2$ (see equation (5)) and its phase compared to the single-particle tunnelling $l=0$. To quantify $|t^{(2)}_{\alpha}|$ without a bias resulting from the inhomogeneity of the harmonic trap, we perform a Landau–Zener type measurement. Starting from a singlet state in the static system at $U/h = 5.41(7)$ kHz, we first ramp up the modulation in 5.45 ms while being detuned from the resonance and subsequently sweep the interactions over the avoided crossing at $U/h = 7.9(1)$ kHz in 20 ms (Fig. 2a). If the size of the gap at the resonance given by $2 \sqrt{2} |t^{(2)}_{\alpha}|$ is large enough, we adiabatically follow the Floquet eigenstate and convert $|s\rangle$ to $|0, \uparrow\rangle$ (ref. 116). According to the Landau–Zener formula, the measured double occupancy fraction after the interaction sweep will be given by $\mathcal{D} = \mathcal{D}_{\text{max}}[1 - \exp(-\Gamma^2)]$ with $\Gamma = |t^{(2)}_{\alpha}|/(\kappa \cdot t)$, where $\mathcal{D}_{\text{max}} = 0.56(2)$ is the maximum value of $\mathcal{D}$ given by the initial preparation. The sensitivity of the measurement is characterized by $\kappa$, where $\kappa = 0.15(2)$ for our interaction ramp speed. To confirm the dependence of $\mathcal{D}$ on $|t^{(2)}_{\alpha}|$, we benchmark our gap measurement by driving only at a single frequency $\omega/(2\pi)$ or $2\omega/(2\pi)$. For $K_2 = 0$ or $K_1 = 0$, $|t^{(2)}_{\alpha}|$ reduces to $\mathcal{J}[K_1]$ or $\mathcal{J}(K_2)$, respectively. Figure 2b shows that the transfer fraction to the double occupancy state first increases with the magnitude of the effective tunnelling before it saturates for a gap size that corresponds to $|t^{(2)}_{\alpha}| \approx 0.2t$. This gives us a high sensitivity for small absolute values of the effective tunnelling.

To directly measure the Peierls phase $\psi^{(2)}$, we implement a scheme similar to a Ramsey experiment. We design our protocol for the two-level system depicted in Fig. 1c, in which we use the near- and off-resonant Hamiltonians represented by vectors $\mathbf{h}^{(0)}$ and $\mathbf{h}^{(0)}$ as distinct rotation axes. Instead of scanning the evolution time of the state as in a typical Ramsey sequence, we vary the
Fig. 4 | Dirac point and Peierls phase vortex. a. Double occupancy fraction when ramping across the $U=2\hbar\omega + \Delta_\text{r}$ resonance as a function of the relative phase $\phi_\text{r}$ and driving amplitudes $K_1K_2$. The amplitudes are parametrized along the black line in Fig. 3a. The gap is closing at a singular point in parameter space. b. Cut through a at $K_1=1.03(1)$. The solid line is a Lorentzian fit to the data (left axis), from which we extract the relative phase where the gap closes to be $\phi_\text{r}=1.03(6)\pi$. The red dashed line shows the theoretical value of the gap (right axis), which depends linearly on the relative phase around the gap closing. c. Tomography of $\psi^{(2)}$ as a function of the relative phase $\phi_\text{r}$ and the driving amplitudes $K_1K_2$ showing a vortex structure around the Dirac point. d,e. Cuts through c for fixed values of $K_1$ (see legends). Dashed lines show the theoretical value of $\psi^{(2)}$ according to equation (5), taking into account the experimental phase offset of $-0.15(4)\pi$ (grey horizontal line). Data points and error bars in a,b denote the mean and standard deviation of five individual measurements. Mean values in c-e are derived from a sinusoidal fit to the Ramsey fringes and errors denote the standard deviation obtained from a resampling method (see Methods). Data at $\phi_\text{r}=2\pi$ are duplicated from $\phi_\text{r}=0$.

initial phase between $h^{(2)}$ and $h^{(0)}$ by changing the common phase $\phi_\text{c}$. The angle between the two rotation axes, which determines the interference fringes for the populations of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, is given by $\psi^{(2)} = -2\phi_\text{c} + \psi^{(2)}(K_1, K_2, \phi_\text{r})$. In addition to $\phi_\text{c}$, it contains the non-trivial part of the Peierls phase $\psi^{(2)}_\text{r}$ given by the amplitudes and relative phase of the two-frequency modulation (equation (5)). By varying $\phi_\text{c}$, we can extract the Peierls phase $\psi^{(2)}_\text{r}$ from the phase of the resulting fringes.

More precisely, we first prepare an eigenstate of the Hamiltonian $H^{(0)}_\text{eff}$ (equation (2)) for $\delta^{(2)}=0$, which is given by $(e^{i\psi^{(2)}_\text{r}})^{-1/\sqrt{2}}$. This is achieved by ramping up the drive within 5.45 ms away from the resonance followed by a sweep of the interactions on resonance to $U/h=6.23(8)$ kHz within 100 ms (Fig. 2c). After that, we project the system onto the off-resonant Hamiltonian $H^{(0)}_\text{eff}$ at $\delta^{(0)}=0$. The quench is achieved by applying an RF pulse that lasts 9.5 s and converts 95(4)\% of the $\downarrow$ states to spin $\rightarrow$. If $H^{(0)}_\text{eff}$ is not (anti-)parallel to $H^{(2)}_\text{eff}$, the state will start to rotate around the new Hamiltonian, leading to oscillations of the singlet and double occupancy fractions (Fig. 2c). When fixing the evolution time to the point where the Bloch vector has rotated by an angle of $\pi/2$ around $H^{(0)}_\text{eff}$, we observe Ramsey fringes for the observables as a function of $\phi_\text{c}$ given by $D(\psi_\text{c}, \phi_\text{c}) = \frac{1\pm \sin(-2\phi_\text{c} + \psi^{(2)}_\text{r})}{2}$, from which we can directly extract $\psi^{(2)}_\text{r}$ (Fig. 2d). Due to the evolution of the state during the initial preparation of the eigenstate of $H^{(2)}_\text{eff}$, we measure an overall phase offset of the Ramsey fringes, which we determine in independent measurements to be $-0.15(4)\pi$ (see Methods). For all data shown in the following, the tunnelling phase was extracted from the Ramsey fringes for $D$. We begin our investigation of the effective tunnel coupling induced by the two-frequency drive by mapping out the transition for which $|t^{(2)}_\text{eff}| = 0$ in the $K_1K_2$ parameter space. As discussed above, this occurs at the TRS point $\psi_\text{r}=\pi$ and to lowest order for $\alpha^{(2)}=\beta^{(2)}$, that is $J(K_1)J(K_2) = J_\text{r}(K_1)J_\text{r}(K_2)$. Figure 3a shows the result of the gap measurement in $K_1K_2$ parameter space following the experimental protocol in Fig. 2a,b. The gap shows a clear minimum along the diagonal, separating two distinct regions with large values of $|t^{(2)}_\text{eff}|$. The gap closing nicely follows the theoretical prediction derived from equation (5) without free parameters (Supplementary Fig. 2). Although the double occupancy fraction goes to almost zero for small values of both $K_1$ and $K_2$, the minimum is less pronounced if both amplitudes are high. In this region, the two-level approximation in equation (2) breaks down and the singlet is transformed into the other double occupancy state $|\uparrow\downarrow\rangle$ during the interaction sweep. This is demonstrated by a full numerical simulation of the gap measurement protocol (Supplementary Fig. 5). The region in which the gap closes separates two distinct regions in parameter space, which are characterized by a $\mathbb{Z}_2$-invariant (see Supplementary Information). When going from top left to bottom right in Fig. 3a, the parameter $\alpha^{(2)}-\beta^{(2)}$ and thus the tunnelling matrix element $t^{(2)}_\text{eff}$ change from negative to positive values. At the phase transition, $\alpha^{(2)}-\beta^{(2)}=0$ such that $|t^{(2)}_\text{eff}| = 0$ (see Fig. 3b), which shows a cut along the black line indicated in Fig. 3a. The sign change of the effective tunnelling amplitude can be demonstrated by measuring its phase across the transition line, which exhibits a sharp jump by $\pi$ for the critical values of $K_1$ and $K_2$ where the gap closes (Fig. 3c). This in turn proves that the gap fully closes, as the tunnelling amplitude is continuous in the modulation parameters. After mapping out the gap closing in the parameter space of the driving amplitudes, we also investigate the influence of the relative modulation phase $\phi_\text{r}$. To this end, we always fix the parametrization in the $K_1K_2$ space to be along the black line in Fig. 3a. If we expand the tunnel coupling around the point where the gap closes up to linear order in $K_1=K_1\text{crit}$ and $\phi_\text{r} = \phi_\text{r}-\pi$, we find...
This is a Dirac Hamiltonian in the driving parameters, which only affects the density-assisted tunneling processes, while the single-particle hopping remains trivial.

Figure 4a shows the measurement of the gap near the Dirac point located at $K_{\text{ext}}$ and $\phi = \pi$. It demonstrates that $|\psi_{\text{dirac}}^{(2)}|$ has a clear minimum at the singularity and increases away from it. The gap closes at $\phi = 1.03(6)\pi$, as expected from theory (see analytical results in Fig. 4b and Supplementary Fig. 4 and a numerical simulation of the gap measurement in Supplementary Fig. 6). In addition, Fig. 4c shows a full tomography of the tunnelling phase $\psi^{(2)}_{1,2}$ around the Dirac point. It has a vortex structure and the phase increases by $2\pi$ when going clockwise around the singularity. For high values of $K_{1} > K_{\text{ext}}$, $\psi^{(2)}_{1,2}$ only changes little as a function of the relative phase (Fig. 4d). In this regime, the driving component at $\omega$ is dominant, which corresponds to the lower right corner in Fig. 3a. Around the critical point $K_{1} = K_{\text{ext}}$ and $\phi = \pi$, the tunneling phase is very sensitive to the exact driving parameters and suddenly jumps from 0 to $\pi$ when lowering $K_{1}$ (Fig. 3c). For $K_{1} < K_{\text{ext}}$, we enter the upper left region in Fig. 3a and suddenly observe a running phase $\psi^{(2)}$, which means that the state vector winds once around the Bloch sphere when $\phi$ is swept from 0 to $2\pi$ (Fig. 4c).

In future experiments, the full control over both the amplitude and Peierls phase of the density-dependent tunneling matrix element demonstrated in this work can be further extended by introducing a temporal or spatial dependence for the driving parameters, which maps the Dirac point into another parameter space. In addition, it is straightforward to couple the individual dimers by allowing for tunneling in all directions in order to study the intriguing interplay between the interaction-induced gauge field and the atomic density. Recent experiments have shown that driven many-body systems can be well understood in the effective Hamiltonian picture and that problems associated with interacting Floquet systems such as heating can be mitigated in certain lattice geometries. In higher dimensions, a variety of phenomena related to density-dependent gauge fields could be studied, such as anyonic statistics in one dimension or flux attachment. Finally, the symmetry between atoms in different internal states can be broken by using a spin-selective drive, such that distinct gauge and matter particles can be identified. As shown in the Supplementary Information, using such a driving scheme involving two modulation frequencies enables us to engineer both a dynamical $Z_{2}$ gauge field $\mathcal{A}_{(\delta)} = \text{diag}(0, \pi)$, which only requires real-valued tunneling matrix elements as well as a $Z_{2}$ gauge field $\mathcal{A}_{(\delta)} = \text{diag}(0, 2\pi/3, 3\pi/3)$.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, statements of code and data availability and associated accession codes are available at [https://doi.org/10.1038/s41567-019-01654-5](https://doi.org/10.1038/s41567-019-01654-5).

Received: 14 December 2018; Accepted: 28 June 2019; Published online: 19 August 2019

**References**

1. Goldman, N., Juzeliunas, G., Öhberg, P. & Spielman, I. B. Light-induced gauge fields for ultracold atoms. *Rep. Prog. Phys.* 77, 126410 (2014).
2. Cooper, N. R., Dalibard, J. & Spielman, I. B. Topological bands for ultracold atoms. *Rev. Mod. Phys.* 91, 015005 (2019).
3. Cooper, N. R. Rapidly rotating atomic gases. *Adv. Phys.* 57, 539–616 (2008).
4. Lin, Y.-I., Compton, R. L., Jiménez-García, K., Porto, J. V. & Spielman, I. B. Synthetic magnetic fields for ultracold neutral atoms. *Nature* 462, 628–632 (2009).
5. Bukov, M., D’Alessio, L. & Polkovnikov, A. Universal high-frequency behavior of periodically driven systems from dynamical stabilization to Floquet engineering. *Adv. Phys.* 64, 139–226 (2015).
6. Eckardt, A. Colloquium: atomic quantum gases in periodically driven optical lattices. *Rev. Mod. Phys.* 89, 011004 (2017).
7. Aidelburger, M. et al. Experimental realization of strong effective magnetic fields in an optical lattice. *Phys. Rev. Lett.* 107, 255301 (2011).
8. Miyake, H., Siviloglou, G. A., Kennedy, C. J., Burton, W. C. & Ketterle, W. Realizing the Harper Hamiltonian with laser-assisted tunneling in optical lattices. *Phys. Rev. Lett.* 111, 185302 (2013).
9. Struck, J. et al. Engineering Ising-XY spin-models in a triangular lattice using tunable artificial gauge fields. *Nat. Phys.* 9, 738–743 (2013).
10. Jotzu, G. et al. Experimental realisation of the topological Haldane model. *Nature* 515, 237–240 (2014).
11. Cheng, T.-P. & Li, L.-F. *Gauge Theory of Elementary Particle Physics* (Oxford Univ. Press, 1991).
12. Levin, M. & Wen, X. *Colloquium: photons and electrons as emergent phenomena*. *Rev. Mod. Phys.* 77, 871–879 (2005).
13. Savary, L. & Balents, L. Quantum spin liquids: a review. *Rep. Prog. Phys.* 80, 016502 (2017).
14. Wiese, U. J. Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories. *Annu. Phys.* 525, 777–796 (2013).
15. Zoller, P., Cirac, J. I. & Reznik, R. Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices. *Rep. Prog. Phys.* 79, 014401 (2016).
16. Dalmonte, M. & Montangero, S. Lattice gauge theory simulations in the quantum information era. *Contemp. Phys.* 57, 388–412 (2016).
17. Martinez, E. A. et al. Real-time dynamics of lattice gauge theories with a few-qbit quantum computer. *Nature* 534, 516–519 (2016).
18. Edmonds, M. J., Valiente, M., Juzeliunas, G., Santos, L. & Öhberg, P. Simulating an interacting gauge theory with ultracold Bose gases. *Phys. Rev. Lett.* 110, 085301 (2013).
19. Clark, L. W. et al. Observation of density-dependent gauge fields in a Bose–Einstein condensate based on micromotion control in a shaken two-dimensional lattice. *Phys. Rev. Lett.* 121, 030402 (2018).
20. Keilmann, T., Lanzmich, S., McCulloch, I. & Roncaglia, M. Statistically induced phase transitions and anyons in 1D optical lattices. *Nat. Commun.* 2, 3367 (2011).
21. Greschner, S., Sun, G., Poletti, D. & Santos, L. Density-dependent synthetic gauge fields using periodically modulated interactions. *Phys. Rev. Lett.* 113, 215303 (2014).
22. Greschner, S. & Santos, L. Anyon hubbard model in one-dimensional optical lattices. *Phys. Rev. Lett.* 115, 055302 (2015).
23. Bermudez, A. & Porras, D. Interaction-dependent photon-assisted tunneling in optical lattices: a quantum simulator of strongly-correlated electrons and dynamical field theories. *New J. Phys.* 17, 103021 (2015).
24. Cardarelli, L., Greschner, S. & Santos, L. Engineering interactions and anyon statistics by multicolor lattice-depth modulations. *Phys. Rev. A* 94, 023615 (2016).
25. Sträter, C., Srivastava, S. C. & Eckardt, A. Floquet realization and signatures of one-dimensional anyons in an optical lattice. *Phys. Rev. Lett.* 117, 205303 (2016).
26. Barbiero, L. et al. Coupling ultracold matter to dynamical gauge fields in optical lattices: from flux-attachment to $Z_{2}$ lattice gauge theories. Preprint at arXiv:1810.01777 (2018).
27. Struck, J. et al. Tunable gauge potential for neutral and spinless particles in driven optical lattices. *Phys. Rev. Lett.* 108, 225304 (2012).
28. Stöferle, T., Moritz, H., Schori, C., Köhl, M. & Esslinger, T. Transition from a strongly interacting 1D superfluid to a Mott insulator. *Phys. Rev. Lett.* 92, 130403 (2004).
29. Jördens, R., Strohmaier, N., Günter, K. L., Moritz, H. & Esslinger, T. A Mott insulator of fermionic atoms in an optical lattice. *Nature* 455, 204–207 (2008).
30. Greif, D., Tarruell, L., Uehlinger, T., Jördens, R. & Esslinger, T. Probing nearest-neighbor correlations of ultracold fermions in an optical lattice. *Phys. Rev. Lett.* 106, 145302 (2011).
31. Chen, Y.-A. et al. Controlling correlated tunneling and superexchange interactions with ac-driven optical lattices. *Phys. Rev. Lett.* 107, 210405 (2011).
32. Ma, R. et al. Photon-assisted tunneling in a biased strongly correlated Bose gas. *Phys. Rev. Lett.* 107, 095301 (2011).
33. Meinert, F., Mark, M. J., Lauber, K., Daley, A. J. & Nägerl, H.-C. Floquet engineering of correlated tunneling in the Bose–Hubbard model with ultracold atoms. Phys. Rev. Lett. 116, 205301 (2016).
34. Desbuquois, R. et al. Controlling the Floquet state population and observing micromotion in a periodically driven two-body quantum system. Phys. Rev. A 96, 053602 (2017).
35. Görg, F. et al. Enhancement and sign change of magnetic correlations in a driven quantum many-body system. Nature 553, 481–485 (2018).
36. Meser, M. et al. Floquet dynamics in driven Fermi–Hubbard systems. Phys. Rev. Lett. 121, 233603 (2018).
37. Xu, W., Morong, W., Hui, H. Y., Scarola, V. W. & DeMarco, B. Correlated spin-flip tunneling in a Fermi lattice gas. Phys. Rev. A 98, 023623 (2018).
38. Sandholzer, K. et al. Quantum simulation meets nonequilibrium DMFT: analysis of a periodically driven, strongly correlated Fermi–Hubbard model. Preprint at https://arxiv.org/abs/1811.12926 (2018).
39. Schweizer, C. et al. Floquet approach to Z₂ lattice gauge theories with ultracold atoms in optical lattices. Nat. Phys. https://doi.org/10.1038/s41567-019-0649-7 (2019).
40. Jotzu, G. et al. Creating state-dependent lattices for ultracold fermions by magnetic gradient modulation. Phys. Rev. Lett. 115, 073602 (2015).

Acknowledgements
We thank L. Barbiero, A. Bermudez, A. Eckardt, N. Goldman, F. Grusdt, Y. Murakami, M. Rizzi, L. Santos, K. Viebahn, P. Werner and O. Zilberberg for insightful discussions, and K. Viebahn and O. Zilberberg for a careful reading of the manuscript. We acknowledge SNF (projects nos. 169320 and 182650), NCCR-QSIT, QUIC (Swiss State Secretary for Education, Research and Innovation contract no. 15.0019) and ERC advanced grant TransQ (project no. 742579) for funding.

Author contributions
The data were measured and analysed by FG., K.S. and J.M. The theoretical framework and measurement scheme were developed by FG. All work was supervised by TE. All authors contributed to planning the experiment, discussions and preparation of the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
Supplementary information is available for this paper at https://doi.org/10.1038/s41567-019-0615-4.
Reprints and permissions information is available at www.nature.com/reprints.
Correspondence and requests for materials should be addressed to TE.
Publisher’s note: Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.
© The Author(s), under exclusive licence to Springer Nature Limited 2019
Methods

Driving scheme and effective Hamiltonian. In the following, we derive the effective Hamiltonian for a Fermi–Hubbard model, which is driven at two frequencies that are resonant with the on-site interaction \( U \). We show that this scheme is suited to control both the amplitude and Peierls phases of density assisted tunneling processes independently of single-particle hopping. For simplicity, we discuss the case of a 1D lattice. For more details and extensions of the scheme to realize dynamical \( Z_2 \) and \( Z_4 \) gauge fields see Supplementary Information.

The full time-dependent Hamiltonian can be written as \( \hat{H}(t) = \hat{H}_0 + \hat{V}(t) \), where the static part corresponds to the usual Fermi–Hubbard model

\[
\hat{H}_0 = - \sum_{j,\sigma} \left( \epsilon_{ij} c_{ij \sigma}^\dagger c_{ij \sigma} + U \sum_{\sigma} n_{ij \sigma} n_{ij \bar{\sigma}} \right)
\]

and the drive is given by

\[
\hat{V}(t) = - \sum_{j,\sigma} \left( \hat{A}_j^{\dagger}(t) c_{ij \sigma}^\dagger + \hat{A}_j(t) c_{ij \sigma} \right)
\]

in a frame that is co-moving with the shaken lattice. For the two-frequency modulation scheme, the oscillating inertial force reads

\[
f(t) = \hbar \omega_{0k} \cos(\omega t + \phi) + 2 \hbar \omega_{0k} \cos(2\omega t + 2\phi + \phi')
\]

in analogy to the site offset in the case of a double well (see equation (4) of the main text). Using Floquet theory, we can derive an effective static Hamiltonian around the resonance \( \omega = \omega_{0k} \), which yields to lowest order

\[
\hat{H}_0^{\text{eff}} = - \sum_{j,\sigma} \left( \epsilon_{ij} c_{ij \sigma}^\dagger c_{ij \sigma} + U \sum_{\sigma} n_{ij \sigma} n_{ij \bar{\sigma}} \right) + \left( \hat{A}_j^{\dagger}(t) c_{ij \sigma}^\dagger + \hat{A}_j(t) c_{ij \sigma} \right)
\]

Here, single-particle tunnelling associated with the operator

\[
\hat{A}_j^{\dagger}(t) \equiv \frac{1}{\sqrt{2}} \left( \hat{a}_j^{\dagger} \hat{b}_j - \hat{b}_j^{\dagger} \hat{a}_j \right)
\]

are the lattice depths in units of the recoil energy \( \hbar^2/2m \). All tunnelling amplitudes and phases are described by the operators

\[
\hat{V}_f \equiv \hbar \Omega \cos(\omega t + \phi) \equiv \hbar \Omega \cos(\omega t + \phi) \equiv \hbar \Omega \cos(\omega t + \phi)
\]

where \( \hat{V}_f \) is the inter-dimer tunnel coupling. To account for systematic errors from the calibration and fluctuations of the lattice depths, we include a relative error of 4.9% for \( X \) and 2.3% for \( Y \) on the lattice depths for the calculation of the tight-binding parameters. In addition, we take into account a relative uncertainty of the magnetic field of 10−5. In the final lattice configuration, the depths are given by \( V_{\text{X,Y,Z}} = 22(1), 1.00(5), 38.8(9), 29.3(7) \) \%.

The corresponding potential consists of an array of individual double wells aligned in the \( x \) direction with an intra-dimer tunnel coupling \( \hbar = 260(30) \) Hz. All dynamics between different dimers are suppressed by adjusting the inter-dimer tunnelling amplitudes in all spatial directions to be below \( \hbar = 2 \) Hz. In addition, the phase \( \theta \) in equation (13) is adjusted to 1.0050(1), which introduces a static energy bias between the two sites of the double well of \( \Delta_0/\hbar = 660(20) \) Hz. Due to the underlying harmonic confinement with trapping frequencies of \( \omega_{0x,y,z} = (12.1, 10.7, 15.1) \) Hz, an additional inhomogeneous site offset is introduced in the dimers. For a double well located at a typical distance of 20 lattice sites away from the centre of the trap, the additional tilt is on the order of \( \Delta/2 \).

Preparation of the atoms in the double wells. The preparation procedure for two distinguishable fermions in the double wells is very similar to that described in earlier work2,3. Briefly, the starting point of our experiment is a balanced mixture of atoms in the \( F = 9/2, m_F = -9/2 \) and \( F = 9/2, m_F = -7/2 \) hyperfine states of \( ^{87}\text{Rb} \) (called \( \uparrow \) and \( \rightarrow \) in the main text), which are confined in an optical harmonic trap. After evaporatively cooling the atoms, we end up with \( N = 4(4) \times 10^4 \) atoms at a temperature \( T/T_F = 0.09(2) \) \( (T_F \) is the Fermi temperature). After this, we tune the \( s \) wave scattering length between the atoms to be very strongly attractive \( a \rightarrow -\infty \) by means of a magnetic Feshbach resonance located at \( 202.1 \) G. We first load the atoms into a chequerboard lattice with \( V_{\text{X,Y,Z}} = 22(1), 1.00(5), 38.8(9), 29.3(7) \) \% within 200 ms followed by a second ramp to a deep chequerboard lattice with \( V_{\text{X,Y,Z}} = 20(1), 30.30, 30.30 \) \% for 20 ms. Due to the strong attractive interactions, \( 56.2(7) \% \) of the atoms form a doubly occupied site, while no site is occupied by more than two atoms due to Pauli blocking. The next step is to perform a Landau–Zener sweep with a RF pulse to transfer the atoms in the \( F = 9/2, m_F = -7/2 \) state to the \( F = 9/2, m_F = -5/2 \) state (called \( \downarrow \) in the main text). The interactions of the mixture in the \( m_F = -9/2 \) and \( m_F = -5/2 \) states can be tuned by a second magnetic Feshbach resonance at \( 224.2 \) G, which allows us to access strong repulsive interactions with \( a > 175(6) \) (\( a_0 \) is the Bohr radius). We adjust the scattering length to \( a = 237(14) \) \( a_0 \) and subsequently split the wells of the chequerboard lattice into two sites by ramping to the final lattice configuration with \( V_{\text{X,Y,Z}} = 22(1), 1.00(5), 38.8(9), 29.3(7) \) \% in 20 ms. Here, the atoms interact with an on-site interaction energy \( U/\hbar = 5.41(7) \) \% and this is the starting point of the experiments.

Periodic driving. The driving was implemented as in previous work2,3.5

We sinusoidally modulate the position of the retro-reflecting mirror with a piezoelectric actuator along the direction of the dimers. As a result, the entire lattice potential is moving in space and the time-dependent potential for the two-frequency drive is given by \( V(x, y, z, t) = \text{Re}(\epsilon_{ij} c_{ij \sigma}^\dagger c_{ij \sigma} + 2U n_{ij \sigma} n_{ij \bar{\sigma}}) \).

The interactions of the mixture in the \( m_F = -9/2 \) and \( m_F = -5/2 \) states can be tuned by a second magnetic Feshbach resonance at \( 224.2 \) G, which allows us to access strong repulsive interactions with \( a > 175(6) \) (\( a_0 \) is the Bohr radius). We adjust the scattering length to \( a = 237(14) \) \( a_0 \) and subsequently split the wells of the chequerboard lattice into two sites by ramping to the final lattice configuration with \( V_{\text{X,Y,Z}} = 22(1), 1.00(5), 38.8(9), 29.3(7) \) \% in 20 ms. Here, the atoms interact with an on-site interaction energy \( U/\hbar = 5.41(7) \) \% and this is the starting point of the experiments.

An experimental setup for a 1D optical lattice, which is formed by a combination of four orthogonal, retro-reflecting laser beams at a wavelength of \( \lambda = 1.064 \) nm (for more details, see earlier work7). Although the beams \( X \) and \( Y \) are effectively not interfering with any other beam due to a frequency detuning, beams \( X \) and \( Z \) are interfering with each other and are actively phase-stabilized to \( \varphi = 0.00(3) \) Hz. The resulting potential for the atoms is given by

\[
V(x, y, z) = -V_2 \cos^2(kx) + \frac{\epsilon_{ij} c_{ij \sigma}^\dagger c_{ij \sigma} + 2U n_{ij \sigma} n_{ij \bar{\sigma}}}{\hbar^2/2m}
\]

where \( k = 2\pi/\lambda \) and \( V_{\text{X,Y,Z}} \) are the lattice depths in units of the recoil energy \( \hbar^2/2m \). The resulting potential for the atoms is given by

\[
V(x, y, z) = -V_2 \cos^2(kx) + \frac{\epsilon_{ij} c_{ij \sigma}^\dagger c_{ij \sigma} + 2U n_{ij \sigma} n_{ij \bar{\sigma}}}{\hbar^2/2m}
\]
subsequently keep fixed driving amplitudes $K_i$ and $K_s$. For the gap measurements, we ramp the interactions while modulating the lattice from the initial value $U/\hbar = 5.41(7)$ kHz across the resonance to the final interaction $U/\hbar = 7.9(1)$ kHz within 20 ms. Alternatively, for the measurement of the tunnelling phase, we prepare an eigenstate of the resonantly driven double well by ramping the interactions to the resonance at $U/\hbar = 6.23(8)$ kHz within 100 ms.

**Experimental measurement of the Peierls phase.** For the measurement of the tunnelling phase, we first prepare an eigenstate on the resonance $U = 2\hbar \omega - \Delta_e$ as described above, followed by a projection onto an off-resonantly driven double well by switching the internal state of the atoms with an RF pulse. To realize the near-resonant condition, we work with a $\{\uparrow, \downarrow\}$-pair of atoms in the $m_F = -9/2$ and $m_F = 9/2$ states, which are strongly interacting. Afterwards, we switch to a $\{\uparrow, -\downarrow\}$-pair of atoms in the $m_F = -9/2$ and $m_F = 7/2$ states, which have a much weaker on-site interaction energy. Importantly, we have to simultaneously match the two resonance conditions for the interactions $U_{2\uparrow\downarrow} = \hbar \omega + \Delta_e$ and $U_{2\uparrow\downarrow} = \Delta_e$ at the same strength of the magnetic offset field. For our lattice configuration and choices of $\omega/2\pi = 2.75$ kHz and $\Delta_e/\hbar = 600(20)$ Hz, this condition is fulfilled for a magnetic field of $B_0 = 210.82(2)$ G. Here, the scattering lengths are $a_{\uparrow\uparrow} = 2.7(1)$ a.u. and $a_{\uparrow\downarrow} = 262(2)$ a.u., and the corresponding on-site interactions are $U_{\uparrow\uparrow} = 6.23(8)$ kHz and $U_{\uparrow\downarrow}/\hbar = 0.56(5)$ kHz. To switch between the two different regimes, we transfer the atoms from the $m_F = -5/2$ to the $m_F = -7/2$ state with a fidelity of 95(4)% by applying an RF pulse with a duration of 9.5 μs and a frequency of 48.692 MHz. After the interaction quench, the quantum state will start to rotate around the new off-resonant Hamiltonian on the Bloch sphere with a frequency of $2\pi f$. To measure the Ramsey fringes, we fix the evolution time to $t = h/(8 |2\pi f|)$ where the rotation angle is equal to $\pi/2$. Because $|2\pi f|$ is changing as a function of our driving parameters $K_i$, $K_s$, and $\phi$, we have to adjust the timing for each choice of parameters. We do this experimentally by projecting a pure singlet state with the RF pulse onto the off-resonant Hamiltonian, which results in coherent oscillations between the singlet and double occupancy states. From these oscillations, we extract the $\pi/2$ time for a certain set of driving parameters and interpolate between them.

**Fit of the Ramsey fringes.** For the fringes, we perform three independent measurements of the final double occupancy fraction for seven different values of the common phase $\phi$, between 0 and $\pi$ (Fig. 2d). To extract the tunnelling phase $\psi$, we fit the resulting double occupancy fringe with a function $D(\phi) = A \sin(2\phi + \psi) + a$, where the period is fixed to $\pi$. To estimate the error, we use a resampling method that assumes that the measurement results for each value of $\phi$ follow a normal distribution according to the measured values of the mean and standard deviation of $D$. Afterwards, we randomly sample a finite fringe for each common phase and refit the dataset. We repeat this procedure 1,500 times while additionally varying the initialization values for the fit parameters $A$ and $b$ by ±10%. The mean (+−) standard deviation of the distribution of phases fitted on the resampled data is used as an upper (lower) bound for the fitted value of $\psi$, of the measured data, which is expressed in asymmetric error bars in Figs. 3c and 4d. The same resampling method is also employed to estimate the uncertainty on the centre position of the Lorentzian fits that are used to determine the gap closing (Figs. 3b and 4b and Supplementary Fig. 7b).

**Phase offset of the Ramsey fringes.** In the measurements of the tunnelling phase, we observe an overall offset; that is, the phase of the Ramsey fringes is not vanishing for $\psi = 0$. This can be explained by the evolution of the state during the adiabatic preparation of the eigenstate of $H_{\text{eff}}^{(0)}$. In particular, the relative phase between the singlet and double occupancy states is not only given by $-2\phi + \psi$, but it has an additional dynamical phase contribution in the lab frame given by $-2\omega t$. Therefore, in addition to the slow adiabatic following to the equator of the Bloch sphere (Fig. 2c), the state vector rotates at a frequency of $\omega/2\pi$ around the $z$ axis. Even when fixing the total preparation time of the eigenstate to a multiple of the driving period, any residual detuning from the resonance will lead to a modified rotation frequency and therefore to a finite phase accumulation up to the point at which the RF pulse is applied. Because the preparation takes hundreds of driving cycles, this phase offset can be significant. Furthermore, finite frequency effects and dynamics that depend on the exact launching protocol of the drive lead to additional phase shifts. To calibrate the resulting phase offset in the experiment, we take four reference Ramsey fringes for a single frequency drive with $\omega/2\pi = 5.5$ kHz, both for positive and negative site offsets $\Delta_e$. For this single-frequency drive, the non-trivial contribution $\psi$, which allows us to directly measure the phase offset. From these measurements, we obtain an offset of $-0.15(4)$ μ rad (the uncertainty denotes the standard error).

**Detection.** The detection of the double occupancy and singlet fractions is similar to that in earlier work\(^{40,41}\). To characterize the state of the atoms, we first freeze all dynamics by quickly ramping up the tunnelling barrier in the double well within 100 μs to a $V_{XX,YY,ZZ} = \{30, 0, 30\} E_k$ cubic lattice. We can detect double occupancies both for a $\{\uparrow, \downarrow\}$-pair and a $\{\uparrow, -\downarrow\}$-pair of atoms. For this, we ramp down the magnetic field below the Feshbach resonance of the $m_F = -9/2$ and $m_F = -7/2$ states. We then selectively transfer one of the atoms forming the double occupancy from the $m_F = -9/2$ to the $m_F = -7/2$ state (or vice versa) with an RF sweep by making use of the interaction shift. We can count the number of atoms in each $m_F$ state by applying a Stern–Gerlach pulse during a time-of-flight expansion followed by absorption imaging. To detect singlets, we apply a magnetic field gradient after the lattice freeze, which leads to coherent oscillations between the singlet and triplet state. After properly adjusting the evolution time, we detect the singlet state by merging two adjacent sites by going to a $V_{XX,YY,ZZ} = \{30, 30, 0\} E_k$ chequerboard lattice. In this process, the singlet state will be adiabatically transformed to a double occupancy in the final lattice, which we can detect as outlined above.

**Theoretical treatment of the driven double well.** We perform both analytical and numerical studies of a double well subject to a two-frequency drive (for an analytical derivation of the effective Hamiltonian and tunnelling matrix element see Supplementary Information, and for details about the numerical simulation see ref. \(^{10}\)). To calculate the numerical quasi-energy spectrum and the Floquet eigenstates of the time-dependent problem (Supplementary Figs. 1a–c and 5e–h), we use a Trotter decomposition to compute the evolution operator over one modulation cycle. The content of double occupancy and singlet states for a given Floquet eigenstate $|\psi_i\rangle (i = 1, \ldots, r)$ is then given by $D = |\{\uparrow, 0 | \psi_i\rangle\rangle|^2 + |\{0, 11 | \psi_i\rangle\rangle|^2$ and $D = |\{0 | \psi_i\rangle\rangle|^2 + |\{1 | \psi_i\rangle\rangle|^2$, respectively. In addition, we perform a numerical simulation of the full gap measurement protocol described above (Supplementary Figs. 5a–c and 6). These calculations capture the full time-dependence of the system, that is, the drive at frequencies $\omega/2\pi$ and $2\omega/2\pi$ as well as the ramps of the driving amplitudes $K_i$ and $K_s$ and the interaction $U$. In detail, we initiate the system in a singlet state at $U = 5.41$ kHz and first increase the amplitudes $K_i$ and $K_s$ and the two-frequency drive within 5.45 μs to their final values at a fixed relative phase $\phi$, and $\phi = 0$. Then, $U$ is ramped to $U = 7.9$ kHz within 20 ms. To determine the double occupancy in the final state $|\psi_r\rangle$, we compute both overlaps with the double occupancy states $|\{\uparrow, 0 | \psi_r\rangle\rangle|^2$ and $|\{0, 1 | \psi_r\rangle\rangle|^2$, respectively. The results for the same driving parameters as in Fig. 3 (Fig. 4a) are shown in Supplementary Fig. 5a–c (Supplementary Fig. 6).

**Data availability**

All data files are available from the corresponding author upon request. Source data for Figs. 2–4 and Supplementary Figs. 1d and 7 are provided in the Supplementary information.

**Code availability**

The source code for the fit of the Ramsey fringes is available from the corresponding author upon request.

**References**

41. Tarruell, L., Greif, D., Uehlinger, T., Jotzu, G. & Esslinger, T. Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice. *Nature* **483**, 302–305 (2012).

42. Greif, D., Uehlinger, T., Jotzu, G., Tarruell, L. & Esslinger, T. Short-range quantum magnetism of ultracold fermions in an optical lattice. *Science* **340**, 1307–1310 (2013).