Holographic Equipartition and Gravitational Collapse

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Abstract: It is argued in the literature that gravity is an emergent phenomenon and is a statistical tendency for a gravitational system to attain the maximal entropy state which maintains the holographic principle. In this paper, we show that gravitational collapse is due to the same mechanism as the expansion of the universe that the departure of the surface degrees of freedom and the bulk degrees of freedom in a region of space drives the evolution of a gravitational system. We also argue that the ordinary thermal radiation can be taken as a signal for the existence of some kind of “confined” bulk degrees of freedom which prevent a gravitational system from collapse. When the mount of “confined” bulk degrees of freedom is not adequate, the collapse occurs until a new equilibrium is attained. Our results provide a new paradigm to understand the gravitational collapse.
1 Introduction

There is a strong evidence in the study of spacetime thermodynamics to suggest that spacetime has internal degrees of freedom, that is to say gravity and spacetime geometry are emergent phenomena like fluid mechanics or elasticity\cite{1, 2}. Since gravity is the most universal force interact between every two objects, the change in perspective of gravity has important implications for every dynamic processes in our universe, such as the cosmic expansion or the stellar evolution.

This perspective is of great significance and has far reaching implications. In 2010, Erik P. Verlinde proposed an idea that gravity can be explained as an entropic force\cite{3}. It is really an amazing idea, it depicts the gravity in a clear physical picture based on some general assumptions, viz., the spacetime has an microscopic structure although we don’t know the underlying discrete degrees of freedom, the gravitational force is caused by changes in the entropy associated with the locations of material bodies, and inertia is a consequence of the fact that a material body in rest will stay in rest because there are no entropy gradients. That is to say, gravity is just a statistical tendency to return to a maximal entropy state.

Though some remarkable achievement for the origin of gravity had been accomplished in Verlinde’s entropic interpretation of gravity, some essential aspects of this frame are still obscure, e.g., (a) The entropic force $F = T \frac{\Delta S}{\Delta x}$, it requires a non zero temperature to have a non vanishing entropic force, this means the origin of the entropic force is closely related to where the temperature comes from. In Verlinde’s paper \cite{3}, the holographic screen plays an important role in deriving the dynamic equation of gravity and in discussing the emergence of spacetime. However, there is no explicit explanation of if we can endow the holographic screen a temperature and an entropy or not. Horizons have a temperature proportional to surface gravity which can be identified by the Euclidean continuation and an entropy measured by its area. It seems thermodynamic quantities always associate with the horizon, when it comes to the holographic screen, we definitely need an explanation which will give the entropic interpretation of gravity a solid foundation? (b) The equipartition law is a key assumption in deriving Newton’s law and Einstein’s field equation. It arises the following
questions, what’s the implication of being used to spacetime thermodynamics? can this law applied to any gravitational system? or what is the context of its application? (c) In [3], holographic screen is viewed as the boundary that separate the spacetime into two parts, on one side there is spacetime emergent, on the other side nothing yet! It is of no sense to separate a large room into two parts with a suppositional screen, and tell someone that one part exists and the other does not. What is a real edge that separate the emergent spacetime region from which has not emerged yet? Here, we just point some obscure issues that should be further investigated, we hope to discuss it elsewhere.

Meanwhile, Padmanabhan made a similar discussion about gravity [4], he emphasized that the equipartition law of energy for the horizon degrees of freedom play an important role in discussing the emergence of gravity, namely the holographic equipartition. Those two works [3, 4] are complementary, some obscure aspects about the entropic interpretation of gravity stated above could be explained by the ideas in [4]. Take the first issue (a) for instance, for a gravitating system enclosed by a holographic screen, we can construct local Rindler horizons for every observers placed on the holographic screen who will experience an acceleration $a$, then, we can attribute an Unruh temperature $T = \frac{\hbar a}{2\pi k_Bc}$ to it. That is to say, we can construct local Rindler horizon for any small patch of the holographic screen which is associate with the observers who will experience an acceleration produced by the gravitational body. This explanation fullfills the logical gap in endow the holographic screen a temperature and an entropy.

Recently, in [5], the accelerated expansion of the universe is interpreted as the emergence of space and cosmology driven by the demand of the holographic principle. Specifically, the holographic principle can be interpreted by the equality of the number of degrees of freedom in a bulk region of space and the number of degrees of freedom on the boundary surface, the difference between the surface degrees of freedom and the bulk degrees of freedom drives the accelerated expansion of the universe.

We should also note that there is a hidden hypothesis in [3] that the maximal entropy state is the state that saturates the holographic entropy bound [6–9]. Using the ideas in [4] and [5], it is stated that the maximal entropy state maintain the holographic principle, viz., the number of degrees of freedom in the boundary is equal to the number of degrees of freedom in the bulk defined by the holographic equipartition, that is $N_{bulk} = \frac{|E|}{(1/2)k_BT} = N_{sur}$. Form this perspective of gravity, we can generalize the observation in [5] and suspect that the dynamical evolution of every process in our universe related to gravity is governed by the tendency to saturate the holographic entropy bound. Combining the ideas of [3] and [4], it is declared that gravity is emergent, it is a statistical tendency to achieve to the maximal entropy state that maintain the holographic principle.

In this paper, we will study the stellar evolution in the spirit of this paradigm. In section 2, we review the derivation of Friedmann equation from the novel idea that the expansion of cosmic space is driven by the demand of the holographic principle. Our main content is in section 3, we show that the gravitational collapse is due to the same mechanism as the expansion of universe. It is also argued that the “confined” degrees of freedom play an important role in prevent a star form collapse. Section 4 is for conclusion and discussion. The natural units with $c = 1, \hbar = 1,$ and $k_B = 1$ is used, unless otherwise
2 Expansion of Cosmic Space and the Holographic Equipartition

Viewed in the perspective of spacetime thermodynamics, the Friedmann equation of FRW universe is an equation of state, since the Friedmann equation can be derived from the first law of thermodynamics with some general assumptions in various gravity theories\cite{10–12}, such as Einstein gravity, Gauss-Bonnet gravity, Lovelock gravity, scalar-tensor gravity and $f(R)$ gravity, or even in the Hořava-Lifshitz gravity\cite{13}. Recently, another approach for reproducing the Friedmann equation is provided in \cite{5}, which extends our understanding of emergence of space and cosmology dramatically.

Our work is closely related to the ideas in \cite{5}, we shall give a brief review on it. The author argued that the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space causes the accelerated expansion of the universe, the Friedmann equation can be reproduced through a simple equation

$$\frac{dV}{dt} = L_P^2 \left( N_{\text{sur}} - N_{\text{bulk}} \right), \quad (2.1)$$

where

$$N_{\text{sur}} = \frac{4\pi}{L_P^2 H^2}, \quad (2.2)$$

is the number of degrees of freedom on the spherical surface of Hubble radius $H^{-1}$that one degree of freedom is attribute to one Planck area $L_P^2 = \frac{\hbar G}{c^3}$,

$$N_{\text{bulk}} = \frac{|E|}{\frac{1}{2} k_B T} \quad \text{is the effective number of degrees of freedom which are in equipartition at the horizon temperature} \quad T = \frac{H}{2\pi}, \quad \text{and} \quad V = \frac{4\pi}{3 H^3} \quad \text{is the Hubble volume in Planck units} \quad \text{and} \quad t \quad \text{is the cosmic time in Planck units correspond to the geodesic observers.} \quad \text{The effective energy} \quad |E|\text{contained inside the Hubble volume can be taken as} \quad |(\rho + 3p)| V, \quad \text{then}$$

$$N_{\text{bulk}} = - \frac{2(\rho + 3p)}{k_B T} V = - \frac{16\pi^2 (\rho + 3p)}{3 H^4}. \quad (2.3)$$

Substituting Eq.(2.2) and Eq.(2.3) in Eq.(2.1), it simplifies to

$$\frac{\ddot{a}}{a} = - \frac{4\pi G}{3} (\rho + 3p), \quad (2.4)$$

which is just the acceleration equation for the dynamical evolution of the universe. Substitute the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

into the above equation (2.4), and integrate using the de Sitter boundary condition at late times, we finally obtain

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho, \quad (2.5)$$

this is the standard Friedmann equation that governs the evolution of the universe, the integration constant $k$ can be regarded as a cosmological constant.
3 Gravitational Collapse and the Holographic Equipartition

The evolution of a star had been extensively studied in the framework of general relativity. Generally speaking, a star of a mass under the Chandrasekhar limit will finally evolve into a stable white dwarf star, a star weighted between the Chandrasekhar limit and the Tolman-Oppenheimer-Volkoff (TOV) limit will become a neutron star, and those with masses exceeding the TOV limit will evolving into a black hole inevitably (see [14] for example). Traditionally, gravitational collapse is viewed as a inward falling process of a star due to its own gravity, form the point of view of spacetime thermodynamics, it is a thermodynamic process for a system to attain an equilibrium state. In this section, we will discuss the reason of the gravitational collapse from the perspective of spacetime thermodynamics and its relation to holographic equipartition. Here we only study the spherical system for simplicity.

Without loss of generality, for a isotropic spherical body, the spacetime can be described by the metric

$$ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2.$$  \hspace{1cm} (3.1)

Further, we suppose that the energy-momentum tensor of the matter in the star has the form of a perfect fluid, viz.,

$$T_{ab} = \left( \rho + p \right) U_a U_b + p g_{ab},$$

where \(U_a\) denotes the four-velocity of the fluid, and \(\rho, p\) are the energy density and pressure, respectively.

Under the framework of general relativity, the structure of a spherically symmetric star of isotropic material in static gravitational equilibrium is constrained by the Tolman-Oppenheimer-Volkoff equation

$$\frac{dp}{dr} = -\frac{(\rho + p)[Gm(r) + 4\pi Gr^3p]}{r^2[1 - \frac{2Gm(r)}{r}]},$$ \hspace{1cm} (3.2)

where we have defined

$$m(r) = 4\pi \int_0^r \rho(x)x^2 dx.$$  

The condition \(m(0) = 0\) should be satisfied, which is needed to ensure that \(e^{2\beta}\) is finite.

The above equation can also be written in the form

$$\frac{dm}{dr} = 4\pi r^2 \rho(r),$$

which also satisfies the relation that

$$m(r) = \frac{1}{2G} (r - re^{-2\beta}).$$ \hspace{1cm} (3.3)

We will see \(m(r)\) is likely to a notion of quasilocal mass that it’s value equals to the Komar mass when the gravitational system evolute to a final equilibrium state. We should also note that the proper integrated mass contained in a sphere of radius \(r\) is

$$M(r) = 4\pi \int_0^r \rho(x)[1 - \frac{2m(x)}{x}]^{-\frac{1}{2}} x^2 dx,$$

which is bigger than \(m(r)\), and the difference between these two quantities can be interpreted as the gravitational binding energy.
Using the relation of eq. (3.3), the metric (3.1) can be expressed as
\[ ds^2 = -e^{2\alpha}dt^2 + \left[1 - \frac{2Gm(r)}{r}\right]^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 . \] (3.4)

In rest of this section, we shall study the evolution of the isotropic spherical body under the framework of spacetime thermodynamics. For the calculation of it’s Unruh temperature, we need to generalize the definition of surface gravity. Fortunately, the (generalized) surface gravity definition in [15] which has the meaning of the “redshifted” 4-acceleration of the fiducial observer (FIDO) happen to meet our requirements. This definition has advantages that can be applied not only to event horizons, but also to FIDOs skimming along the boundary of a gravitational system.

Using this proper definition, the surface gravity of the gravitational system described by the metric (3.1) can be calculated as
\[ \kappa = e^{\alpha - \beta}\alpha' \] (3.5)
here the \( \alpha' \) denote \( d/dr \). Then, the corresponding Unruh temperature is
\[ T_U = \frac{\kappa}{2\pi k_B} = \frac{e^{\alpha - \beta}\alpha'}{2\pi k_B} . \] (3.6)

The active gravitational mass for a stationary spacetime is called Komar mass, for the system we discussed above, the Komar mass acquires the form
\[ M_K(r) = r^2\kappa = r^2e^{\alpha - \beta}\alpha' . \] (3.7)
This could be thought as a generalized form of Newton’s law of gravitation in the circumstance we setted above.

According to [5], the number of degrees of freedom on the spherical surface of a gravitational system of radius \( R \) is
\[ N_{\text{sur}} = \frac{4\pi R^2}{L_P^2} , \] (3.8)
and the effective number of degrees of freedom which are in equipartition at the Unruh temperature \( T_U \) expressed in eq. (3.6) is
\[ N_{\text{bulk}} = \frac{M_K(R)}{\frac{1}{2}k_B T_U} = \frac{4\pi M_K(R)}{\kappa} = \frac{4\pi R^2}{L_P^2} , \] (3.9)
where eq. (3.7) has been used and we reintroduce the Planch length \( L_P \) in last equality. We can find that \( N_{\text{sur}} = N_{\text{bulk}} \) is always hold. This result agrees with the statement in [16, 17] that in any static spacetime the equality of \( N_{\text{sur}} \) and \( N_{\text{bulk}} \) is maintained.

For the horizonless object we discussed above, we can argue that for FIDOs skimming along the boundary of a gravitational system will see a temperature no less than the local measured Unruh temperature since the ordinary radiation of the star could be emitted to infinity, when redshifted to infinity, we get
\[ T \geq T_U . \]
Replacing $T_U$ by $T$ in eq.(3.9) gives

$$N_B = \frac{M_K(R)}{\frac{1}{2}k_B T} \leq N_{\text{bulk}}.$$  

(3.10)

I now demonstrate the physical causes that lead to the gravitational collapse. Assume the isotropic spherical gravitational system evolve in quasi-static process, namely, the spacetime of the star is not absolute stationary, but we can still use the metric of eq.(3.1) to describe it. Using the proposed eq.(2.1) to the isotropic spherical object, viz., insert eq.(3.8) and eq.(3.10) into eq.(2.1), we have

$$\frac{dV_S}{dt} = -\frac{dV}{dt} = -L_p^2(N_{\text{sur}} - N_B) \leq 0,$$  

(3.11)

note that as the gravitational system collapse the volume of the space outside it expand, or the change of the volume of the system $\Delta V_S = -\Delta V$. The above relation (3.11) implies that the volume of the gravitational system $V_S$ will decrease as time progresses. That is:

From the perspective of spacetime thermodynamics the gravitational collapse of a system occurs because it is a non-equilibrium thermodynamic system, gravitational collapse is a necessary thermodynamic evolution process for the system to reach equilibrium state when the equilibrium temperature is just the Unruh temperature without the contribution of ordinary radiation and the holographic principle attained.

From the point view of quasilocal observer, we can replace $M_K(R)$ by $m(R)$ in eq.(3.9), then

$$\tilde{N}_{\text{bulk}} = \frac{m(R)}{\frac{1}{2}k_B T_U} = \frac{4\pi m(R)}{k_B \kappa} = \frac{\frac{1}{2}k_B \kappa^{-\frac{3}{2} - \frac{3}{2}}}{}.$$  

(3.12)

Assuming the star obeys the holographic principle, namely $N_{\text{sur}} = \tilde{N}_{\text{bulk}}$, then we have $\frac{R^2}{L_p^2} = \frac{m(R)}{k_B \kappa}$, which is consistent with the properties of a Schwarzschild black hole that $\kappa = 1/4m(R)$ and event horizon radius $R = 2m(R)$. In fact, If we demand $\frac{R^2}{L_p^2} = \frac{m(r)}{k_B \kappa}$ to hold for any $r$, using the natural units, we get $m(r) = \kappa r^2$. Moreover, by using eq.(3.3), it is easy to find that $e^\alpha = [1 - 2m(r)/r]^{1/2}$, the metric (3.4) becomes

$$ds^2 = -[1 - \frac{2Gm(r)}{r}] dt^2 + [1 - \frac{2Gm(r)}{r}]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2,$$

which is very similar to the Schwarzschild metric. We can see that as $T$ tends to $T_U$, $m(r) \rightarrow M_K(r)$. The above discussion indicate that without other interaction except gravity come into play, a gravitational system with sufficient mass is tend to evolve to a maximal entropy state that the Holographic Principle hold.

It is also interesting to inquire what mechanism in spacetime thermodynamics makes a gravitational system stay in stable. Traditionally, a gravitational system is stable when the gravitational force is counterbalanced by its internal pressure, a spherical star in equilibrium is ruled by the TOV equation (3.2). The TOV equation (3.2) is derived by solving the Einstein equation for a general time-invariant, spherically symmetric metric, and is equivalent to

$$\frac{da}{dr} = \frac{m(r) + 4\pi r^3 p}{r[r - 2m(r)]}.$$  

(3.13)
via the continuity equation $\nabla_\mu T^{\mu\nu} = 0$. In fact, eq.(3.13) is just the $(rr)$ part of Einstein equation

$$2\rho \alpha' + (1 - e^{2\beta}) = 8\pi pr^2 e^{2\beta}$$

(3.14)

by using eq.(3.3).

Since the metric of a stable gravitational system is static, in the spirit of holographic paradigm, it is reasonable to suppose that the departure of the surface degrees of freedom and the bulk degrees of freedom divided equally by $\frac{1}{2} k_B T$ is balanced by certain kind of “confined” degrees of freedom $N_c$ which is invisible to the observer, that is

$$N_{\text{sur}} - N_B = N_c.$$  

(3.15)

The above equation can be rewritten as

$$N_c = N_{\text{sur}}(1 - \frac{T_U}{T}),$$

which means the amount of “confined” degrees of freedom is a part of the total degrees of freedom though some kinds of coarse graining processes and is undetectable when we use the detect “ruler” as $\frac{1}{2} k_B T$, it will “unconfined” when $T \to T_U$. This change between the confining and deconfining of degrees of freedom at $T_U$ is probable related to a phase transition (for black hole phase transitions, see [18] for example and references therein). It is the confined degrees of freedom that prevent a star from collapse. When the mount of confined degrees of freedom is not adequate, the collapse occurs until an new equilibrium is attained.

For the spherically symmetric system of isotropic material in static gravitational equilibrium, its Komar mass should satisfy some additional conditions. Insert eq.(3.10), eq.(3.7) and eq.(3.13) into eq.(3.15), we get

$$M_K(R) = \frac{1}{2}(N_{\text{sur}} - N_c)k_B T = \frac{e^\alpha}{[1 - \frac{2m(R)}{r}]^{1/2}}[m(R) + 4\pi R^3 p] = e^{\alpha + \beta}[m(R) + 4\pi R^3 p]$$

$$= 4\pi e^{\alpha + \beta} \int_0^R (\rho(x) + 3p(R))x^2 dx$$

which is the expression of Komar mass when the star is in stable. This expression can also be interpreted as a constraint between $N_c$ and $T$, once we have another relation between them from the underlying theory, we can determine $N_c$ and $T$ respectively.

Finally, in the perspective of a quasilocal observer, eq.(3.15) is replace by

$$N_{\text{sur}} - \tilde{N}_{\text{bulk}} = \tilde{N}_c,$$  

(3.16)

note that $\tilde{N}_c$ is different from $N_c$. It is easy to see that the above eq.(3.16) is equivalent to

$$M_K(R) - m(R) = \frac{1}{2} \tilde{N}_c k_B T_U.$$  

We see that the difference between $M_K(R)$ and $m(R)$ is just the “confined” energy. Compare eq.(3.16) with eq.(3.14), using the expression of eq.(3.8) and eq.(3.12), we have

$$\tilde{N}_c = \frac{16\pi^2 r^3 p e^{2\beta} + 2\pi r (e^{-\alpha - \beta} - 1)(1 - e^{2\beta})}{\alpha'} |_{r=R}.$$  

(3.17)
Substitute $\alpha'$ and $e^{2\beta}$ into eq.(3.17) using eq.(3.13) and the relation (3.3), we finally get

$$\tilde{N}_c = \left( 4\pi r^2 - 4\pi r^2 m(r) e^{-(\alpha+\beta)} \right) \left| r = R \right.,$$

which agrees with its definition (3.16) as it should be.

### 4 Conclusion and Discussion

In this article, we have studied the gravitational collapse from the framework of spacetime thermodynamics. We have argued that the appearance of ordinary thermal radiation makes the observer experience a higher temperature than it’s Unruh temperature, since it is hard for the observer to distinguish the ordinary thermal radiation from the Unruh radiation, this effect breaks the holographic equipartition, and there is a departure from the holographic principle which causes the gravitational collapse. From this point of view the ordinary thermal radiation is a signal that the gravitational system is still on the way to its final equilibrium state. Gravitational collapse is the thermodynamic process which makes a system to achieve the equilibrium state when the isolated matter of the gravitational system is surrounded by a newly formed horizon and the holographic equipartition recovered.

We have reviewed that the emergence of cosmic space is also caused by the difference between the surface degrees of freedom and the bulk degrees of freedom. This effect can be interpreted that the expansion of the universe is a response to protect the holographic principle. Besides the standard evolution of the universe, this approach also provides a novel paradigm to study other non-standard evolution process in the universe, such as the inflation[19]. It may also help us to understand the origin of the holographic dark energy[20].

Our discussion is consistent with the declaration we made in section 1 that the most stable state for a gravitational system is the one that maintain the holographic principle. In this light, the emergence of the holographic dark energy[21] and the asymptotics of our universe to a de Sitter universe is really natural.

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