Contrasting $N \rightarrow \Delta$ and $N \rightarrow N$
parity-violating asymmetries in nuclei

P. Amore†, R. Cenni♣, T.W. Donnelly♮, A. Molinari♠
†College of William and Mary,
Williamsburg, VA 23185, USA
♣Dipartimento di Fisica,
Università di Genova and INFN, sezione di Genova, Genova, Italy
♮Center for Theoretical Physics,
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology,
Cambridge, MA 02139-4307, USA
♠Dipartimento di Fisica Teorica,
Università di Torino and INFN, sezione di Torino, Torino, Italy

Abstract

Several perspectives that can be opened by studying the electroexcitation of the delta via parity-violating electron scattering from nuclei are examined, working within the context of the relativistic Fermi gas model. A strong enhancement of the left-right asymmetry in the delta sector compared with that in the quasi-elastic regime is found and the potential to find clear signatures for the axial-vector contributions of the nucleon-to-delta transition to the asymmetry identified at specific low momentum transfer kinematics. Possibilities of probing the deformation of the delta are also explored, and using both the proton and nuclei as targets, the ability to study the asymmetry on neutrons is studied.

PACS: 25.30.Rw, 14.20.Gk, 24.10.Jv, 24.30.Gd, 13.40.Gp, 12.15.Mm
Keywords: Parity violation; Inclusive electron scattering; Relativistic Fermi Gas.

I. INTRODUCTION

In this paper we study the asymmetry $A$ measured in the parity-violating nuclear scattering of longitudinally polarized electrons in the region of excitation where the $\Delta(1232)$ dominates. This theme has already been explored in the case of the single proton [1]; here we focus on scattering from nuclei and work within the context of the relativistic Fermi gas (RFG) model. Moreover, since the features that characterize the asymmetry in the region of the $\Delta$ peak emerge more transparently when comparisons are made with the same observable at the quasi-elastic peak (QEP), we shall explore both domains in parallel. In
particular, we shall focus on the energy behavior of $\mathcal{A}$ as one goes from the QEP domain to the $N \rightarrow \Delta$ region for a few values both of the momentum $q$ transferred by the electron to the nucleus and of the electron scattering angle $\theta$. We shall see that a large increase of the many-body content in $\mathcal{A}$ occurs in the $\Delta$ sector as compared with the QEP region for small values of $\theta$.

A study of $\mathcal{A}$ requires knowledge of the response functions, both parity-conserving (PC, electromagnetic) and parity-violating (PV, weak neutral current), and accordingly we proceed within the context of the RFG model via the polarization propagator method employing the $\gamma N\Delta$ vertices of Devenish et al. [2]. We thus recover the PC $N \rightarrow \Delta$ longitudinal and transverse responses obtained previously in [3], and in addition we obtain the axial $N \rightarrow \Delta$ response. It turns out that the asymmetry offers some hope of disentangling — at least in the $\Delta$ region — the otherwise quite elusive nuclear axial response.

The results referred to above are obtained initially by viewing the $\Delta$ as a stable particle. It is worth pointing out that in this scheme when the $\Delta$ mass $m_{\Delta}$ approaches the nucleon mass $m_N$ the $N \rightarrow \Delta$ RFG responses evolve into the corresponding $N \rightarrow N$ quasi-elastic RFG responses except, of course, for appropriate changes in the form factors. These formal relationships between the two sets of responses are of relevance in connection with nuclear $y$-scaling, as we shall illustrate later.

To explore the impact of the finite lifetime of the $\Delta$ on our findings in a few instances we then compute the asymmetry, ascribing to the $\Delta$ a width $\Gamma$ and folding the RFG responses obtained at zero width with a phenomenological $\Delta$ distribution. While our results for the asymmetry turn out be relatively unaffected by $\Gamma$, it is also obviously apparent that an appropriate width is required to account satisfactorily for the actual response functions.

A further theme addressed in the present work relates to the longitudinal PC $N \rightarrow \Delta$ response: this is contributed to by a presently poorly known Coulomb operator, as well as by the magnetic operator driven by relativity and Fermi motion [4]. To assess the importance of the latter versus the former in the RFG framework we proceed as previously for its counterpart in the QEP sector and reduce the longitudinal $N \rightarrow \Delta$ response, so deriving a kind of transverse Coulomb sum rule in the $\Delta$ domain which saturates at large momentum transfer ($\approx 2.5$ GeV/c) to a value set by the Fermi momentum $k_F$.

In connection with the asymmetry, such PV studies may provide new insight into specific aspects of nuclear dynamics. Accordingly we assess the feasibility of actually measuring the asymmetry in the $N \rightarrow \Delta$ domain by estimating the attainable precision when detecting $\mathcal{A}$ at kinematics relevant for TJNAF. Notably this precision turns out to be close to, if not better than, the one that can be reached in the region of the QEP [5].

Given the feasibility of such measurements the question arises: Is there an advantage to exploring the $N \rightarrow \Delta$ asymmetry in complex nuclei rather than only in the proton? We search for an answer to this question by working out the RFG predictions in the limit of vanishing $k_F$. We find that in the $\Delta$ sector the RFG asymmetry displays a minor $k_F$ dependence, as far as its magnitude is concerned, remaining essentially equal to the single-particle asymmetry over a wide range of $k_F$, no matter what the momentum transfer or scattering angle $\theta$ are. However, this is not seen to be the case in the QEP sector where the Fermi motion and the isoscalar-isovector competition conspire to yield in the asymmetry an interesting and channel-dependent $k_F$ dependence.

The present study, being confined to the RFG model, does not take into account $NN$ or
correlations and MEC contributions for the various responses. It should also be added that we have neglected the non-resonant (background) pionic contribution to the physics in the Δ sector, whose relevance should still be assessed. Finally, in this work we had to reckon with the poor knowledge presently available on most of the $N \to \Delta$ form factors. Faut de mieux for the axial sector we have relied on the one hand on the Adler scaling hypothesis \cite{7} for the electric form factor and, on the other, on the constituent quark model, which appears to predict an axial magnetic form factor that is substantially smaller than the electric one \cite{8}. In the vector sector the electric and Coulomb form factors are poorly known: here we adopt the parametrization recently suggested in \cite{3} to gain a first orientation on their role in the $N \to \Delta$ responses.

II. THE $\Delta$-HOLE POLARIZATION PROPAGATOR IN THE RFG

In this section we pave the way to the calculation of the $N \to \Delta$ response functions within the framework of the symmetric RFG. These have been already computed in the vector sector in \cite{3}; here we provide as well the expression for the axial $N \to \Delta$ response, whose relevance for the asymmetry has been already alluded to in the Introduction. As an alternative to the approach of \cite{3} we perform this task by employing the method of the Δ-hole polarization propagator $\Pi_{\mu\nu}$. Specific components of the imaginary part of the latter yield, the response function for which we are looking. The polarization propagator is defined as follows:

$$\Pi_{\mu\nu}(q) = -i \int \frac{d^4 p}{(2\pi)^4} tr_{\text{spin}} tr_{\text{isospin}} \left[ G(p) \Gamma_{N\Delta}(q) S_{\alpha\beta}(p + q) \Gamma_{\Delta N}(q) \right] = i \int \frac{d^4 p}{(2\pi)^4} \frac{1}{4E(p)E_\Delta(p + q)} \left[ \frac{1}{p_0 + \omega - E_\Delta(p + q) + i\epsilon} \frac{\theta(k_F - |p|)}{p_0 - E(p) - i\epsilon} \right. \\
\times \left. tr \left[ (\slashed{p} + m_N) \Gamma_{N\Delta}^\alpha(q) (\slashed{p} + \slashed{q} + m_\Delta) \right] \times \left( g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{2}{3} \frac{(p_\alpha + q_\alpha)(p_\beta + q_\beta)}{m_\Delta^2} - \frac{\gamma_\alpha(p_\beta + q_\beta) - \gamma_\beta(p_\alpha + q_\alpha)}{3m_\Delta} \right) \gamma^0 \Gamma_{\Delta N}^{\beta\nu\dagger}(q) \gamma^0 \right] \mathcal{I},$$

(2.1)

where

$$\slashed{p} \equiv \gamma^0 E(p) - \slashed{\gamma} \cdot \vec{p}$$

(2.2)

for the nucleon and

$$\slashed{\phi} \equiv \gamma^0 E_\Delta(p) - \slashed{\gamma} \cdot \vec{p}$$

(2.3)

for the Δ, and $\mathcal{I}$ is the isospin trace

$$\mathcal{I} = tr_{\text{isospin}} T_{N\Delta} T_{N\Delta}^\dagger = \frac{4}{3},$$

(2.4)

$T_{N\Delta}$ being the isospin $N \to \Delta$ transition operator. In (2.1) the nucleon and the Δ propagators respectively read
and

\[ S^{\alpha\beta}(k) = (-) \frac{k + m_\Delta}{k^2 - m_\Delta^2 + i\epsilon} \left( g^{\alpha\beta} - \frac{1}{3} \gamma^\alpha \gamma^\beta - \frac{2 k^\alpha k^\beta}{3 m_\Delta^2} - \frac{\gamma^\alpha k^\beta - \gamma^\beta k^\alpha}{3 m_\Delta} \right) \]

\[ = (-) \frac{1}{2E(k)} \left( \frac{k + m_\Delta}{k_0 - E(k) + i\epsilon} - \frac{\tilde{k} + m_\Delta}{k_0 + E(k) - i\epsilon} \right) \times \left[ g^{\alpha\beta} - \frac{1}{3} \gamma^\alpha \gamma^\beta - \frac{2 k^\alpha k^\beta}{3 m_\Delta^2} - \frac{\gamma^\alpha k^\beta - \gamma^\beta k^\alpha}{3 m_\Delta} \right], \quad (2.6) \]

where \( \tilde{k} \equiv -\gamma_0 E(k) - \gamma \cdot \vec{k}. \)

Now the energy integration in (2.1) is easily performed, yielding

\[ \Pi^{\mu\nu}(q) = -\frac{1}{(2\pi)^3} \int d^3p \frac{m_N^2 f^{\mu\nu}(p, q)}{4E(p)E_\Delta(p + q)} \frac{\theta(k_F - |p|)}{\omega + E(p) - E_\Delta(p + q) + i\epsilon} \quad (2.7) \]

in terms of the dimensionless single-nucleon second-rank \( N - \Delta \) tensor

\[ f^{\mu\nu}(p, q) \equiv \frac{1}{m_N^2} \text{tr} \left[ (\not{p} + m_N) \Gamma^{\mu\nu}_{N\Delta}(q)(\not{p} + \not{q} + m_\Delta) \right. \]

\[ \left. \left( g^{\alpha\beta} - \frac{1}{3} \gamma^\alpha \gamma^\beta - \frac{2 (p_\alpha + q_\alpha)(p_\beta + q_\beta)}{3 m_\Delta^2} - \frac{\gamma_\alpha(p_\beta + q_\beta) - \gamma_\beta(p_\alpha + q_\alpha)}{3 m_\Delta} \right) \gamma^0 \Gamma^{\nu\beta}_{N\Delta}(q)^\dagger \gamma^0 \right]. \]

The latter can be conveniently expressed in terms of the independent tensors

\[ \xi^{\mu\nu}_a \equiv g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} = g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\tau} \quad (2.8) \]

\[ \xi^{\mu\nu}_b \equiv \frac{1}{m_N^2} \left( p^\mu - \frac{q \cdot p}{q^2} q^\mu \right) \left( q^\nu - \frac{q \cdot q}{q^2} q^\nu \right) = \left( \eta^\mu + \frac{\kappa \cdot \eta}{\tau} \kappa^\mu \right) \left( \eta^\nu + \frac{\kappa \cdot \eta}{\tau} \kappa^\nu \right) \quad (2.9) \]

\[ \xi^{\mu\nu}_c \equiv i e^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{m_N^2} = i e^{\mu\nu\alpha\beta} \eta_\alpha \kappa_\beta, \quad (2.10) \]

as follows

\[ f^{\mu\nu} = -w_1 \xi^{\mu\nu}_a + w_2 \xi^{\mu\nu}_b + \tilde{w}_3 \xi^{\mu\nu}_c, \quad (2.11) \]

where a tilde has been placed on the term associated with the weak neutral current. Note that the \( \xi^{\mu\nu} \) are orthogonal to \( q^\mu \), i.e. \( q_\mu \xi^{\mu\nu}_{a,b,c} = 0 \). Also in (2.8), (2.9) and (2.10) the two independent four-vectors of our problem, namely that of the nucleon inside the FG (\( p^\mu \))
and the one carried by the gauge boson \((q^\mu)\), have been expressed in dimensionless forms according to

\[
\eta^\mu \equiv (\epsilon, \bar{\eta}) \equiv \left( \frac{E(p)}{m_N}, \frac{\vec{p}}{m_N} \right) \tag{2.12}
\]

and

\[
\kappa^\mu \equiv (\lambda, \bar{\kappa}) \equiv \left( \frac{\omega}{2m_N}, \frac{\vec{q}}{2m_N} \right) \tag{2.13}
\]

where furthermore \(\tau \equiv \kappa^2 - \lambda^2\).

In order to fix \(w_1, w_2\) and \(\tilde{w}_3\), the vertex functions \(\Gamma^\alpha_{N\Delta}\), whose matrix elements between the Rarita-Schwinger spinor \(u^\alpha_\Delta\) and the Dirac nucleon spinor \(u\) yield the \(N \rightarrow \Delta\) current, namely

\[
\langle \Delta | J^\mu(0) | N \rangle \equiv J^\mu(q) = \bar{u}^\alpha_\Delta(p + q) \Gamma^\mu_{\Delta N} u(p) ,
\]

are required. These we take from the work of Devenish et al. \[2\]; they have the following expressions

\[
\begin{align*}
\Gamma^\mu_{M(V)} &= -\frac{3}{2} \frac{\mu + 1}{Q_+} e^\mu(pq) \tag{2.15} \\
\Gamma^\mu_{E(V)} &= -\frac{3}{2} \frac{\mu + 1}{Q_+} i e^\mu(pq) e^\sigma(pq) \gamma_5 \tag{2.16} \\
\Gamma^\mu_{C(V)} &= -\frac{3}{2} \frac{\mu + 1}{Q_+} 2q_5 \left( q^2 p^\mu - p \cdot q q^\mu \right) \gamma_5 \tag{2.17} \\
\Gamma^\mu_{M(A)} &= -i \frac{3}{2} \frac{\mu + 1}{Q_+} 2e^\sigma(pq) e^\mu(pq) \tag{2.18} \\
\Gamma^\mu_{E(A)} &= -i \frac{3}{2} \frac{\mu - 1}{Q_+} 2e^\sigma(pq) e^\mu(pq) \tag{2.19} \\
\Gamma^\mu_{C(A)} &= i \frac{3}{2} \frac{\mu - 1}{Q_+} 2q_5 \left( q^2 p^\mu - p \cdot q q^\mu \right) \tag{2.20}
\end{align*}
\]

with \(\mu \equiv m_\Delta/m_N\) and

\[
Q_\pm \equiv (m_\Delta \pm m_N)^2 - q_5^2 = m_N^2 \left[ (\mu \pm 1)^2 + 4\tau \right] .
\]

In \((2.15-2.20)\) the shorthand notation \(e^{3\alpha}(pq) \equiv e^{3\sigma\mu\nu} p_\mu q_\nu\) has been used.

Then by associating \(N \rightarrow \Delta\) form factors, to be denoted \(G_E^{(v)}(\tau)\) and \(G_M^{(a)}(\tau)\) in the vector and axial sectors respectively, to each of the vertices \((2.15-2.20)\) and by performing the spin traces, one finally obtains

\[
\begin{align*}
\omega_1(\tau) &= -\frac{1}{16} \left( 3G_E^{(v)}(\tau)^2 + G_M^{(a)}(\tau)^2 \right) \left( \mu + 1 \right)^2 \left[ (\mu - 1)^2 + 4\tau \right] \tag{2.21} \\
\omega_2(\tau) &= \frac{\tau(\mu + 1)^2}{(\mu + 1)^2 + 4\tau} \left[ 3G_E^{(v)}(\tau)^2 + G_M^{(a)}(\tau)^2 + \frac{4\tau}{\mu^2} G_C^{(v)}(\tau)^2 \right] \tag{2.22} \\
\bar{w}_3(\tau) &= \frac{1}{4} (\mu^2 - 1) \left( 3G_E^{(v)}(\tau) G_M^{(a)}(\tau) + G_E^{(v)}(\tau) G_M^{(a)}(\tau) \right) \tag{2.23}
\end{align*}
\]

Worth noticing in the above formula is the disappearance of the axial Coulomb multipole which thus does not contribute to the single-nucleon \(N \rightarrow \Delta\) tensor, as should be the case.
III. THE $N - \Delta$ SYMMETRIC RFG RESPONSES

In the previous section we have set up all of the ingredients required to compute the $N \rightarrow \Delta$ responses. These obtain, through appropriate specifications of the Lorentz indices, according to the formula

\[
\mathcal{R}^{\mu\nu}(\lambda, \kappa) = -\frac{V}{\pi} \text{Im} \Pi^{\mu\nu}(\lambda, \kappa) = -A \frac{3\pi^2}{2m_N^3 \eta_F^3} \text{Im} \Pi^{\mu\nu}(\lambda, \kappa),
\]

where $A$ is the number of nucleons enclosed in a volume $V$ and the dimensionless Fermi momentum $\eta_F = k_F/m_N$ has been introduced.

Now if the $\Delta$ is assumed to be a stable particle on its mass-shell then the three-dimensional integration over the nucleon’s momentum in the imaginary part of (2.7) can be easily converted into a one-dimensional integration over the energy $\epsilon = \sqrt{1 + \eta^2}$ by exploiting the energy-conserving delta function. One thus gets

\[
\mathcal{R}^{\mu\nu}(\lambda, \kappa) = -\frac{3N}{4m_N \eta_F^3} \int_{\tilde{\gamma}_-}^{\epsilon_F} d\epsilon f^{\mu\nu}(p, q)|_{\theta_0},
\]

where the angle between $\vec{\eta}$ and $\vec{\kappa}$ in the single-nucleon tensor of the $N \rightarrow \Delta$ sector is fixed by the energy conservation to be

\[
\cos \theta_0 = \frac{\lambda \epsilon - \tau \rho}{\kappa \eta},
\]

with

\[
\rho = 1 + \frac{\mu^2 - 1}{4\tau}.
\]

Moreover the upper limit of integration in (3.1) is set by the dimensionless Fermi energy $\epsilon_F = \sqrt{1 + \eta_F^2}$, whereas the lower limit

\[
\tilde{\gamma}_- \equiv \kappa \sqrt{\frac{1}{\tau} + \rho^2 - \lambda \rho}
\]

extends to the $\Delta$ domain the $\gamma_-$ of the QEP sector [4] to which indeed it reduces when $\rho \rightarrow 1$ ($\mu \rightarrow 1$).

The energy integration in (3.2) can be carried out without difficulty and one gets for the longitudinal, transverse and the axial channels the expressions

\[
R^L(\kappa, \lambda) = C \int_{\tilde{\gamma}_-}^{\epsilon_F} d\epsilon \, f^{00}(\epsilon, \theta_0) = C \, \xi_F \, (1 - \psi^2) \, U_L(\kappa, \lambda),
\]

\[
R^T(\kappa, \lambda) = C \int_{\tilde{\gamma}_-}^{\epsilon_F} d\epsilon \, \left( f^{11}(\epsilon, \theta_0) + f^{22}(\epsilon, \theta_0) \right) = C \, \xi_F \, (1 - \psi^2) \, U_T(\kappa, \lambda),
\]

\[
\tilde{R}^T_{VA}(\kappa, \lambda) = -iC \int_{\tilde{\gamma}_-}^{\epsilon_F} d\epsilon \, f^{12}(\epsilon, \theta_0) = C \, \xi_F \, (1 - \psi^2) \, U_{T^*}(\kappa, \lambda),
\]

where
\[ C = \frac{3 A}{4 m_N \kappa \eta_F^3} \] (3.9)

and the indices on the left hand side of (3.8) refer to the coupling of the vector current of the lepton with the axial current of the hadron.

The above responses, just as happens in the QEP domain [5], display a common factor \( C \xi_F (1 - \psi^2) \) and therefore scale, i.e., depend only upon a single variable

\[
\psi = \sqrt{\frac{1}{\xi_F} \left( \kappa \sqrt{\frac{1}{\tau} + \rho^2} - \lambda \rho - 1 \right)} \quad \left\{ \begin{array}{ll} +1 & , \quad \lambda > \lambda_0 \\ -1 & , \quad \lambda < \lambda_0 \end{array} \right. \quad (3.10)
\]

where

\[
\lambda_0 = \frac{1}{2} \left( \sqrt{1 + 4 \kappa^2 \rho^2} - 1 \right). \quad (3.11)
\]

Again (3.10) reduces to the scaling variable of the QEP domain when \( \rho \to 1 \) \((\mu \to 1)\) and carries the physical significance of the minimum energy (in units of the Fermi kinetic energy \( \xi_F = \epsilon_F - 1 \)) a nucleon should have to respond to the external field.

The common factor discussed above reflects the many-body physics of the RFG. The \( U \) factors in (3.6), (3.7) and (3.8) relate instead mostly (but not only) to the single-nucleon physics. They read

\[
U_L(\kappa, \lambda) = \frac{\kappa^2}{\tau} \left[ (1 + \tau \rho^2) w_2(\tau) - w_1(\tau) + w_2(\tau) \ D_L(\kappa, \lambda) \right] \quad (3.12)
\]

\[
U_T(\kappa, \lambda) = 2 w_1(\tau) + w_2(\tau) \ D_T(\kappa, \lambda) \quad (3.13)
\]

\[
\tilde{U}_T'(\kappa, \lambda) = 2 \sqrt{\tau (1 + \tau \rho^2)} \ \tilde{w}_3(\tau) \left[ 1 + D_{T'}(\kappa, \lambda) \right] \quad (3.14)
\]

and feel the impact of the medium on the single-nucleon physics through the quantities \( D_L \), \( D_T \) and \( D_{T'} \). Indeed the longitudinal \( D_L \) and the transverse \( D_T \) simply express the mean square value of the nucleon transverse momentum in the medium, i.e.

\[
D_L(\kappa, \lambda) = D_T(\kappa, \lambda) = \frac{1}{\epsilon_F - \tilde{\gamma}_-} \int_{\tilde{\gamma}_-}^{\epsilon_F} d\epsilon \ \eta_\perp^2 \\
= \frac{\tau}{\kappa^2} \left[ (\lambda \rho + 1)^2 + (\lambda \rho + 1) (1 + \psi^2) \ \xi_F + \frac{1}{3} (1 + \psi^2 + \psi^4) \ \xi_F^2 \right] - (1 + \tau \rho^2), \quad (3.15)
\]

whereas the axial \( D_{T'} \) turns out to be closely related to the mean transverse kinetic energy of the nucleon inside the RFG according to

\[
D_{T'} = \frac{1}{\epsilon_F - \tilde{\gamma}_-} \int_{\tilde{\gamma}_-}^{\epsilon_F} d\epsilon \ \left( \sqrt{1 + \eta_\perp^2 \frac{\tau}{1 + \tau \rho^2} - 1} \right) \\
= \frac{1}{\kappa} \sqrt{\frac{\tau}{1 + \tau \rho^2} \left[ 1 + \xi_F (1 + \psi^2) + \lambda \rho \right] - 1}. \quad (3.16)
\]
We refer the reader to [10] for a more thorough discussion of the D’s; here we simply recall that they vanish, as should be the case, when \( \eta_F \to 0 \) and that for \( \rho \to 1 \) they evolve into the corresponding quantities for the QEP [11].

From the above discussion it thus emerges that the form factors substitutions

\[
3G_E^{(v)}(\tau)^2 + G_M^{(v)}(\tau)^2 \to G_M(\tau)^2
\]

and

\[
(2\tau G_C^{(v)}(\tau))^2 \to G_E(\tau)^2,
\]

with \( G_{E,M}(\tau) \) being the nucleon’s electric and magnetic form factors, allow one naturally to recover the RFG responses in the QEP from those in the \( N \to \Delta \) domain by letting \( \rho \to 1 \).

Of course in carrying out this procedure the factor \( (\mu_2^2 - 1) \) in \( \tilde{w}_3(\tau) \) should be disregarded.

This supports the feasibility of devising a universal dividing factor as was done in the QEP to reduce the nuclear responses over an energy range encompassing an extended region of the spectrum of nucleon excitation by exploiting form factors substitutions of the type (3.17) and (3.18). This might help when attempting to analyze nuclear y-scaling at higher energies.

As mentioned in the Introduction in order to compute the RFG \( N \to \Delta \) responses we still have to face the issue of the \( N \to \Delta \) form factors. Here we simply provide the expressions we have employed in our calculation. They are

\[
G_M^{(v)}(\tau) = \frac{G_M^{(v)}(0)}{(1 + \lambda_M^V \tau)^2} \frac{1}{\sqrt{1 + \tau}}
\]

\[
G_E^{(v)}(\tau) = \frac{G_E^{(v)}(0)}{(1 + \lambda_E^V \tau)^2} \frac{1}{\sqrt{1 + \tau}}
\]

\[
G_C^{(v)}(\tau) = \frac{G_C^{(v)}(0)}{(1 + \lambda_C^V \tau)^2} \frac{1}{\sqrt{1 + \tau}}
\]

in the vector sector and

\[
G_M^{(a)}(\tau) = \frac{G_M^{(a)}(0)}{(1 + \lambda_M^A \tau)^2}
\]

\[
G_E^{(a)}(\tau) = \frac{G_E^{(a)}(0)}{(1 + \lambda_E^A \tau)^2}
\]

\[
G_C^{(a)}(\tau) = \frac{G_C^{(a)}(0)}{(1 + \lambda_C^A \tau)^2}
\]

in the axial one. Below we quote the values of the parameters entering into the above formulas used here:

\[
G_M^{(v)}(0) = 2.97 \quad , \quad G_E^{(v)}(0) = -0.03 \quad , \quad G_C^{(v)}(0) = -0.44
\]

\[
G_M^{(a)}(0) = 0 \quad , \quad G_E^{(a)}(0) = 2.22 \quad , \quad G_C^{(a)}(0) = 0
\]

\[
\lambda_M^V = 4.97 \quad , \quad \lambda_E^V = 4.97 \quad , \quad \lambda_C^V = 4.97
\]

\[
\lambda_M^A = 3.53 \quad , \quad \lambda_E^A = 3.53 \quad , \quad \lambda_C^A = 3.53
\]

A few comments are worth making at this junction:
FIG. 1. Comparison of the $G_M^{(v)}(\tau)$ with the pure dipole form (dashed curve). Their ratio is displayed in the right panel.

1. in the axial sector we only retain the electric $N \rightarrow \Delta$ form factor, the largest one according to the constituent quark model [8];
2. its value at the origin has been fixed in accord with a scaling law [7];
3. the square root in the denominator of (3.19), (3.20) and (3.21) yields a decrease of about 30% at $\tau = 1$ of the vector form factors with respect to pure dipole behavior (see Fig. 1).

We are now ready to display our results for both the PC and PV RFG responses in the nucleon and $\Delta$ sectors. We shall consider three different kinematical domains. Indeed it is well-known that the quasi-elastic and the $\Delta$ responses occur in the $(\lambda, \kappa)$ plane in the domains defined by the curves

$$
\lambda_{1,2}^{(N)} = \frac{1}{2} \left\{ \sqrt{(2\kappa \pm \eta_F)^2 + 1} - \sqrt{1 + \eta_F^2} \right\}
$$

and

$$
\lambda_{1,2}^{(\Delta)} = \frac{1}{2} \left\{ \sqrt{(2\kappa \pm \eta_F)^2 + \mu^2} - \sqrt{1 + \eta_F^2} \right\}
$$

These are displayed in Fig. 2 together with the light-front. From the figure the three domains referred to above are apparent, namely:
1. the region where the $N$ and $\Delta$ response region do not overlap and the $\Delta$ region is cut by the light-front, occurring for $\kappa_{N\Delta} \leq \kappa \leq \kappa_{N\Delta}^+$, where

$$\kappa_{N\Delta}^\pm = \frac{\mu^2 - 1}{4} (\eta_F \pm \eta_F) \, ; \quad (3.31)$$

2. the region where the two domains do not overlap and the $\Delta$ region is not cut by the light-front, occurring for $\kappa_{N\Delta}^+ \leq \kappa \leq \kappa_{int}$, where

$$\kappa_{int} = \frac{\mu^2 - 1}{8\eta_F} \, ; \quad (3.32)$$

3. the region where the two domains overlap, with both lying below the light-front, occurring for $\kappa \geq \kappa_{int}$.

Our results for the PC responses are displayed in Figs. 3, 4 and 5 for $k_F = 220$ MeV/c, roughly corresponding to the density of $^{12}$C. Henceforth we use $q = 350, 520$ and 1000 MeV/c, which are representative of the three regions referred to above. One sees that the transverse contribution in the $\Delta$ region is dominant in all three cases considered. In the QEP the transverse contribution gradually takes over the longitudinal one as $q$ increases. Finally at $q = 350$ MeV/c the linear behavior of the QEP response functions for small $\omega$ reflects the Pauli blocking.

We now turn to a consideration of the PV responses. The longitudinal and transverse ones are obtained according to

$$\tilde{R}_{AV}^{L,T} = \beta^{I=0} R_{L,T}^N(I = 0) + \beta^{I=1} R_{L,T}^N(I = 1) \, , \quad (3.33)$$

where

$$\beta^{I=0} = -2 \sin^2 \theta_w \, , \quad \beta^{I=1} = (1 - 2 \sin^2 \theta_w) \, , \quad (3.34)$$

$I$ is the isospin quantum number and, as in (3.8), the indices refer to the coupling of the axial current of the lepton with the vector current of the hadron. These are displayed in Fig. 6, 7 and 8. From the figures the dominance of the transverse contribution in the $\Delta$ sector is again apparent, just as in the PC case. Now, however, the axial contribution is appreciable, the more so the smaller is $q$. The same happens in the QEP. The longitudinal channel turns out to be small in both sectors, but for different reasons. In the $\Delta$ sector $\tilde{R}_{AV}^L$ is small because so is $G^{(v)}_C(\tau)$ at moderate values of $\tau$, which likewise also implies that the magnetic contribution to the longitudinal channel is small. In the QEP sector instead the smallness of $R^L$ stems from the competition between its isoscalar and isovector components implied by (3.33) and (3.34). The same argument clearly does not apply to $R^T$ because in this case the isoscalar contribution is quenched by the smallness of the isoscalar magnetic moment of the nucleon.

The responses computed in this section have so far been obtained viewing the $\Delta$ as an elementary particle. We evaluate the impact of the width of the $\Delta$ by following [3]: we write
FIG. 2. The $N \rightarrow \Delta$ and $N \rightarrow N$ response regions in the $(\lambda, \kappa)$ plane for $\eta_F = 0.28$. For $\kappa \leq \kappa_{N\Delta}^(-)$ the $\Delta$ cannot be excited by space-like photons (the response region lies entirely in the time-like domain); for $\kappa_{N\Delta}^(-) \leq \kappa \leq \kappa_{N\Delta}^($ only part of the response region is accessible (the light-cone lies inside the response region); for $\kappa_{N\Delta}^($ $\leq \kappa \leq \kappa_{int}$ the response regions of the $\Delta$ and nucleon are still separated; for $\kappa \geq \kappa_{int}$ the response regions of the $\Delta$ and nucleon overlap.

FIG. 3. Longitudinal (solid) and transverse (dashed) RFG responses at $q = 350$ MeV/c. The Fermi momentum is $k_F = 220$ MeV/c, here and in all the following figures.
FIG. 4. Longitudinal (solid) and transverse (dashed) RFG responses at $q = 520$ MeV/c.

FIG. 5. Longitudinal (solid) and transverse (dashed) RFG responses at $q = 1000$ MeV/c. The dotted lines represent the total of the $\Delta$ and nucleon contributions.
FIG. 6. The PV $\tilde{R}_{AV}^L$ (solid), $\tilde{R}_{AV}^T$ (dashed) and $\tilde{R}_{VA}^{T'}$ (dotted) at $q = 350$ MeV/c.

$$R_{\Gamma}(q,\omega) = \int_{m_N+m_x}^{W_{max}} \frac{1}{\pi} \frac{\Gamma(W)/2}{(W-m_{\Delta})^2 + \Gamma(W)^2/4} R(q,\omega,W)dW,$$

where the integration interval goes from threshold to the maximum value allowed in the Fermi gas model, i.e. $W_{max}^2 = (E_F + \omega)^2 - (q - k_F)^2$. Our results are displayed in Figs. 4 and 10 for $q = 0.5$ GeV/c and $q = 1$ GeV/c and are obtained both with a constant width $\Gamma = 120$ MeV and with an energy-dependent width, taken from $[3]$. As expected the inclusion of the $\Delta$ width produces a broadening and, correspondingly, a decrease of the strength of the $\Delta$ peak, the more so the smaller is the momentum transfer. The energy dependence of $\Gamma$ is seen to have a modest impact and clearly it yields results quite close to those obtained with a constant $\Gamma$.

IV. ASYMMETRY

In this section we compute the asymmetry of the symmetric RFG on the basis of the response functions obtained in the previous sections. In the one gauge boson ($\gamma$ or $Z_0$) exchange approximation the asymmetry, namely the ratio between the inclusive, inelastic PV and PC cross sections, reads

$$A = \left( \frac{d^2\sigma}{d\Omega \, d\epsilon} \right)^{PV} - \left( \frac{d^2\sigma}{d\Omega \, d\epsilon} \right)^{PC} = A_0 \frac{v_L \tilde{R}_{AV}^L(q,\omega) + v_T \tilde{R}_{AV}^T(q,\omega) + v_T \tilde{R}_{VA}^{T'}(q,\omega)}{v_L R^L(q,\omega) + v_T R^T(q,\omega)},$$

where
FIG. 7. The PV $\tilde{R}_{AV}^L$ (solid), $\tilde{R}_{AV}^T$ (dashed) and $\tilde{R}_{VA}^{T'}$ (dotted) at $q = 520$ MeV/c.

FIG. 8. The PV $\tilde{R}_{AV}^L$ (solid), $\tilde{R}_{AV}^T$ (dashed) and $\tilde{R}_{VA}^{T'}$ (dotted) at $q = 1000$ MeV/c.
FIG. 9. The transverse response is displayed at $q = 500$ MeV/c in the nucleonic (dot-dashed line) and $\Delta$ sector. In the latter the dashed curve corresponds to zero width for the $\Delta$, the dotted line to a constant width of $\Gamma = 120$ MeV and the solid line to an energy-dependent width $\Gamma(s)$.

FIG. 10. The transverse response is displayed at $q = 1000$ MeV/c in the nucleonic (dot-dashed line) and $\Delta$ sector. In the latter the dashed curve corresponds to zero width for the $\Delta$, the dotted line to a constant width of $\Gamma = 120$ MeV and the solid line to an energy-dependent width $\Gamma(s)$. 
\[ A_0 = \frac{\sqrt{2}}{\pi \alpha} \left( G m_N^2 \right) \tau \approx 6.5 \times 10^{-4} , \]  

(4.2)

\( \alpha \) and \( G \) are the fine structure and Fermi coupling constants and

\[ v_L = \left( \frac{\tau}{\kappa^2} \right)^2 , \quad v_T = \frac{1}{2 \kappa^2} + \tan^2 \frac{\theta}{2} , \quad v_{T'} = \tan \frac{\theta}{2} \sqrt{\frac{\tau}{\kappa^2} + \tan^2 \frac{\theta}{2}} . \]  

(4.3)

The responses appearing in (4.1) are generic. If, however, we restrict (4.1) to the domain of the \( \Delta \), which is a pure isovector excitation of the nucleon, then the asymmetry becomes

\[ A_{N-\Delta} = A_0 \left\{ - \left( 1 - 2 \sin^2 \theta_w \right) + v_{T'} \frac{\tilde{R}_{V_A}(q, \omega)}{v_L R_L(q, \omega) + v_T R_T(q, \omega)} \right\} . \]  

(4.4)

The above formula clearly shows that if the axial \( N \to \Delta \) response can be neglected then the inelastic asymmetry, normalized to \( A_0 \) and displayed versus \( \lambda \) for fixed \( \kappa \), would be flat in the \( \Delta \) domain. Hence a departure from flatness would signal the presence of the axial response. The contribution of the latter is however suppressed with respect to the first term on the right hand side of (4.4) by the smallness of the vector coupling of the lepton to the axial current of the hadron, whose value is \((-1 + 4 \sin^2 \theta_w) \approx -0.092\).

It should, however, be observed that, unlike the case of parity-violating elastic electron scattering, here the axial contribution to the asymmetry does not vanish at forward electron scattering angles. Indeed (see [1] for details) from (4.3) it follows that when \( \theta \to 0 \) and \( \tau \to 0 \) one has

\[ \frac{v_L}{v_T} \to 0 , \]  

(4.5)

but

\[ \frac{v_{T'}}{v_T} \to \frac{\epsilon^2 - \epsilon'^2}{\epsilon^2 + \epsilon'^2} \neq 0 , \]  

(4.6)

where \( \epsilon \) and \( \epsilon' \) are the initial and final electron energies, respectively.

We thus a priori expect that, small as it might be, the best possibility of detecting the axial response in the \( \Delta \) region should be found for not too large momentum transfers and as near as possible to the light cone.

Indeed our results, displayed in the Figs 11, 12 and 13, confirm these expectations. There we display the ratio \( A/A_0 \) for the same kinematical conditions considered in calculating the responses in the previous section. Indeed it appears that the largest axial contribution occurs at \( q = 350 \text{ MeV}/c \) and \( \theta = 10^o \) where it increases the asymmetry by about 10%. On a flat background these effects should be detectable. In Figs 11, 12 and 13 results are given both with vanishing width (left panels) and with energy-dependent width \( \Gamma(s) \), taken from [3] (right panels). Only small changes to the asymmetries are observed.

The most striking result emerging from these figures relates to the large reduction of the asymmetry in the QEP with respect to the \( \Delta \) region for small electron scattering angles. The interpretation of this result is the following: when \( \theta \) is small the longitudinal contribution in the QEP makes its most pronounced contribution to the asymmetry. But this, as we have
previously remarked, is very small in the RFG framework, because of the isoscalar-isovector competition. Hence the reduced magnitude of the asymmetry in the QEP is observed. In contrast, such a competition does not exist in the Δ sector, which is purely isovector in character. Hence a large asymmetry occurs, independent of the scattering angle. This result appears to us to be noteworthy: indeed a failure in experimentally detecting it might signal of the impact of \( NN \) and \( N\Delta \) correlations.

This being the case, it is important to establish the precision that can be reached when measuring the asymmetry in the Δ region as compared to that in the QEP. In fact the two turn out to be close to each other, with perhaps the precision in the Δ sector being even larger, as demonstrated in Fig. 14, which has been computed for conditions relevant for CEBAF.

In Figs. 15 - 17 we show the RFG asymmetries and for comparison the asymmetries found for the proton and neutron under the same kinematical conditions. In the Δ region all are very similar, as expected, since the \( N \to \Delta \) transition is isovector. In contrast, significant differences are observed for the quasi-elastic asymmetries. This can be understood using the following arguments: for scattering in the impulse approximation we have

\[
A_{nucleus} = \cos^2 \Theta A_p + \sin^2 \Theta A_n,
\]

where \( A_{p,n} \) are the individual proton and neutron asymmetries. Here one has

\[
\tan^2 \Theta = \frac{N [v_L R_L + v_T R_T]_n}{Z [v_L R_L + v_T R_T]_p}.
\]

For an \( N = Z \) nucleus, as assumed here in obtaining the RFG results, one finds in the quasi-elastic region that

\[
\tan^2 \Theta \approx \frac{\varepsilon G^2_{E_n} + \tau G^2_{M_n}}{\varepsilon G^2_{E_p} + \tau G^2_{M_p}} \approx \frac{\tau \mu_n^2}{\varepsilon + \tau \mu_p^2},
\]

and thus for large \( \varepsilon \) (small \( \theta \)) and small \( \tau \) we find that \( \tan^2 \Theta \to 0 \), whereas for large \( \varepsilon \) and/or large \( \tau \) we obtain \( \tan^2 \Theta \approx (\mu_n/\mu_p)^2 \approx 4/9 \). In (4.9) \( \varepsilon \equiv \frac{[1 + 2 (1 + \tau) \tan^2 \theta/2]}{1} \).

Using this as a rough guide it is easy to see that the p-to-n weighting of the single-nucleon asymmetries varies in the required way to explain the dots and lines in the figures. As noted in previous work [5,12], these differences are very important when attempting to isolate the various form factor dependencies in the quasi-elastic region and now we also have insight into the (different) behaviour in the Δ region.

**V. THE TRANSVERSE COULOMB SUM-RULE**

The deduction of the amount of deformation of the Δ is currently receiving a lot of attention experimentally and is of importance for the understanding of non-perturbative QCD. Accordingly in this Section we study the longitudinal response of the Δ, which clearly relates
FIG. 11. Asymmetry at $\theta = 10^0$ (solid), $\theta = 30^0$ (dashed) and $\theta = 150^0$ (dotted) for $q = 350$ MeV/c. The $\omega$ range encompasses both the QEP and the $\Delta$ domain. The left and right panels respectively refer to a vanishing width and to a finite decay width $\Gamma(s)$.

FIG. 12. Asymmetry at $\theta = 10^0$ (solid), $\theta = 30^0$ (dashed) and $\theta = 150^0$ (dotted) for $q = 520$ MeV/c. The $\omega$ range encompasses both the QEP and the $\Delta$ domain. The left and right panels respectively refer to a vanishing width and to a finite decay width $\Gamma(s)$.
FIG. 13. Asymmetry at $\theta = 10^\circ$ (solid), $\theta = 30^\circ$ (dashed) and $\theta = 150^\circ$ (dotted) for $q = 1000$ MeV/c. The $\omega$ range encompasses both the QEP and the $\Delta$ domain. The left and right panels respectively refer to a vanishing width and to a finite decay width $\Gamma(s)$.

to the above issue. This type of excitation, as already mentioned, gets two contributions: one arises directly from the Coulomb multipoles (and is small at small $\tau$) and one is induced by the Fermi motion and relativity. These two elements indeed allow the large magnetic excitation of the $\Delta$ to contribute in the longitudinal channel, the more so the larger is $\tau$.

We start by considering the latter contribution. We do so by reducing the $R_L$ of the $\Delta$, i.e. by devising a dividing factor such that it disentangles, as far as possible, the physics of the single-nucleon from that of the many-body problem. This is in complete analogy with the procedure adopted in the QEP domain. We suggest, as a convenient reducing factor, the following expression

$$H = \frac{3A}{4m_N} \frac{\kappa}{\tau} w_2(\tau) .$$

(5.1)

Then, by setting $G_C^{(v)}(\tau) \equiv 0$, we obtain for the reduced response

$$r^L(\kappa, \lambda) = \frac{\xi_F}{\eta_F} (1 - \psi^2) D^L L(\kappa, \lambda) .$$

(5.2)

Now a kind of transverse Coulomb sum rule $\Sigma_\Delta$ can be worked out by integrating (5.2) over the energy range set by (3.30). To perform this integration is useful to exploit the following integral representation for $D_L$, namely

$$D^L(\kappa, \lambda) = \frac{1}{\epsilon_F - \gamma_-} \int_{\eta_F}^{\eta_F} d\eta \int_{-1}^{+1} d\cos \theta \, \delta \left( \lambda - \frac{\epsilon_{\kappa+\eta} - \epsilon}{2} \right) \frac{\kappa \eta^4 \sin^2 \theta}{\epsilon_{\kappa+\eta}} ,$$

(5.3)
FIG. 14. Fractional precision $\delta A / A$ for the $^{12}$C in the QEP domain (left) and in the $\Delta$ domain (right). The scattering angle takes the three values: $10^0$ (dotted line), $30^0$ (solid line) and $150^0$ (dashed line). We assume the following experimental conditions: $p_e = 1$, $\mathcal{L} = 10^{38}$ cm$^{-2}$ s$^{-1}$, $T = 1000$ hr and $\Delta \Omega = 250$ msr for $\theta = 30^0$, $150^0$ and $\Delta \Omega = 16$ msr for $\theta = 10^0$. 
FIG. 15. Asymmetry at $q = 350$ MeV/c. The lines refer to a symmetric Fermi gas with $k_F = 220$ MeV/c. The points refer to the asymmetry on a free nucleon, closed (open) for proton (neutron). The solid lines and the squares correspond to $\theta = 10^0$. The dashed lines and triangles correspond to $\theta = 30^0$. The dotted lines and circles correspond to $\theta = 150^0$.

FIG. 16. Same as fig. 15 at $q = 522$ MeV/c.
where we have set

\[ \epsilon_{\vec{\kappa}+\vec{\eta}} = \sqrt{\mu^2 + \eta^2 + 4\kappa^2 + 4\kappa\eta\cos \theta} . \]  

Indeed the \( \delta \)-function allows us to perform the \( \lambda \)-integration immediately, yielding

\[
\Sigma_{\Delta} = \frac{\xi_{F}}{\eta_{F}^{3}} \int d\lambda (1 - \psi^2) D^{L}(\kappa, \lambda) \\
= \frac{\kappa}{\eta_{F}^{3}} \int_{0}^{\eta_{F}} d\eta \frac{\eta^{4}}{\sqrt{1 + \eta^{2}}} \int_{-1}^{+1} dx \frac{1 - x^{2}}{\sqrt{\xi^{2} + \eta^{2} + 4\kappa^{2} + 4\kappa\eta x}} ,
\]

where the integral over the variable \( x \) can be analytically expressed in terms of elliptic functions. We prefer to perform this task numerically and the resulting \( \Sigma_{\Delta} \) is displayed in Fig. 18 for \( k_{F} = 220 \text{ MeV/c} \). In fact, the behaviour of \( \Sigma_{\Delta}(\kappa) \) for small and large \( \kappa \) can easily be obtained from (5.5) and one gets

\[
\Sigma_{\Delta} = \frac{4\kappa\eta_{F}^{2}}{15\zeta} \quad \text{(5.6)}
\]

and

\[
\Sigma_{\Delta} = \frac{2\eta_{F}^{2}}{15} ,
\]

respectively.
It thus appears, as expected, that $\Sigma_\Delta$ is a growing function of the density: should it be an experimentally accessible quantity, it would allow a determination of the Fermi momentum $k_F$, which would be interesting to compare with the values obtained via the width of the quasi-elastic peak or the ground-state density of nuclei.

We should now account for the contribution to $R_L$ stemming from $G_C^{(v)}(\tau)$. This we do by displaying the full $R_L$ (shown together with the separated transverse contribution) in Figs. 19 and 20 at $q = 1$ GeV and $q = 2$ GeV, respectively, and using the Coulomb $N \rightarrow \Delta$ form factor given in (3.21). We see that while the contribution of the latter remains moderate at $q = 1$ GeV/c, it grows strongly as $q$ increases further. As already observed in [3], this casts serious doubts on the reliability of the parametrization for the $G_C^{(v)}(\tau)$ adopted in the present paper. In this way experimental investigation of $A$ could help in elucidating the behaviour of $G_C^{(v)}(\tau)$.

VI. CONCLUSIONS

In this paper we have first studied the responses of nuclei to an external electromagnetic or weak neutral current field in both the quasi-elastic and $\Delta$ domains. We have addressed the formal connections between the response functions when $m_\Delta$ goes into $m_N$ in the two energy regimes: indeed they smoothly evolve from one to the other, but for an appropriate replacement of the form factors.

On the basis of the computed response functions we have next set up the left/right asymmetry as measured in the inelastic scattering of longitudinally polarized electrons off nuclei for momentum transfers up to 1 GeV. We have explored to what extent the $N \rightarrow \Delta$
FIG. 19. $R^L_{N\Delta}$ with all form factors (solid) and only magnetic (dashed) at $q = 1000 \text{ MeV}/c$.

FIG. 20. $R^L_{N\Delta}$ with all form factors (solid) and only magnetic (dashed) at $q = 2000 \text{ MeV}/c$. 
axial response function stands out from the otherwise flat energy behaviour characterizing the asymmetry in the $N \to \Delta$ sector (i.e., what would occur if the axial response vanishes and the background contributions are negligible). We have found that for modest momentum transfers and near the light-cone an effect exists and should be detectable owing to the high fractional precision attainable for measurement of the asymmetry in the $\Delta$ domain. This last outcome relates, of course, to the large cross section for electroexcitation of the $\Delta$.

The most notable feature of the $A$ found relates to the dramatic increase of its magnitude as one makes a transition from the QEP into the $N \to \Delta$ region for small electron scattering angles. Because of its size this effect should be measurable both at large, say 1 GeV/c, and at moderate, say 300–400 MeV/c, momentum transfers. In the former case nuclear interactions are not likely to disrupt the RFG predictions too much. In the latter a modification of the effect could take place, but then this might eventually help to shed light on the nature of the $NN$ and $N\Delta$ forces.

Of relevance is also our finding concerning the proton’s and neutron’s asymmetries. It turns out that they differ significantly from each other and from the RFG results depending upon the specific kinematics. Indeed, a comparison between the two as performed here allows one to identify the kinematical domains where they most differ. Hence it appears possible that, by measuring the asymmetry on an $N = Z$ nucleus, one could arrive at information that would help in disentangling the asymmetry on the neutron.

Finally we have explored the longitudinal response function of the $\Delta$, in particular the interplay between its two contributions involving magnetic and Coulomb contributions. A measurement of the separated $R^L$ in the $\Delta$ domain, while undoubtely difficult, would greatly improve our knowledge on the elusive nature of the latter.

A rich harvest of interesting physics appears indeed to wait to be unraveled in the $\Delta$ domain.

ACKNOWLEDGMENTS

This work is supported in part by DOE grant 333271 (P.A.), by the Bruno Rossi INFN-MIT exchange program (R.C. and A.M.) and by funds provided by the U.S. Department of Energy under Cooperative Research Agreement No. DE-FC02-94ER40818 (T.W.D.).
REFERENCES

[1] N.C. Mukhopadhyay, M.J. Ramsey-Musolf, S.J. Pollock, J. Liu and H.W. Hammer, Nucl. Phys. A 633 (1998) 481
[2] R.C.E. Devenish, T.S. Eisenschitz and J.G. Körner, Phys. Rev. D 14 (1976) 3063
[3] J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly and A. Molinari, Nucl. Phys. A 657 (1999) 161
[4] G. Chanfray, J. Delorme, M. Ericson and A. Molinari, Nucl. Phys. A 556 (1993) 439
[5] T.W. Donnelly, M.J. Musolf, W.M. Alberico, M.B. Barbaro, A. De Pace and A. Molinari, Nucl. Phys. A 541 (1992) 525
[6] H.F. Jones and M.D. Scadron, Annals of Physics 81 (1973) 1
[7] S.L. Adler, Ann. Phys. (N.Y.) 50 (1968), 189
[8] T.R. Hemmert, B.R. Holstein and N.C. Mukhopadhyay, Phys.Rev. D51 (1995) 158
[9] W.M. Alberico, A. Molinari, T.W. Donnelly, E.L. Kronenberg and J.W. Van Orden, Phys.Rev. D51 (1995) 158
[10] L. Alvarez-Ruso, M.B. Barbaro, T.W. Donnelly and A. Molinari, submitted for publication
[11] P. Amore, R. Cenni, T.W. Donnelly and A. Molinari, Nucl. Phys. A 615 (1997) 353
[12] E. Hadjmichael, G.I. Poulis and T.W. Donnelly, Phys.Rev. C 45 (1992) 2666
[13] R. Cenni, T.W. Donnelly and A. Molinari, Phys. Rev. C 56 (1997) 276