Improved Landau gauge fixing and the suppression of finite-volume effects of the lattice gluon propagator

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For the gluon propagator of pure SU(2) lattice gauge theory in the Landau gauge we investigate the effect of Gribov copies and finite-volume effects. Concerning gauge fixing, we enlarge the accessible gauge orbits by adding nonperiodic $\mathbb{Z}(2)$ gauge transformations and systematically employ the simulated annealing algorithm. Strategies to keep all $\mathbb{Z}(2)$ sectors under control within reasonable CPU time are discussed. We demonstrate that the finite-volume effects in the infrared regime become ameliorated. Reaching a physical volume of about $(6.5 \text{ fm})^4$, we find that the propagator, calculated with the indicated improvements, becomes flat in the region of smallest momenta. There are first signs in four dimensions of a decrease towards vanishing momentum.

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I. INTRODUCTION

In recent years the infrared behavior of gauge-variant Green’s functions of Yang-Mills theories has increasingly attracted interest. This fact is mainly related to the existence of the Landau (or Coulomb) gauge confinement scenarios proposed by Gribov [1] and Zwanziger [2] on one hand and by Kugo and Ojima [3] on the other. The interest was stimulated by the practical progress achieved over the years within the Dyson-Schwinger equation (DSE) approach as pursued by Alkofer, von Smekal and others (for an intermediate review see [4]). Lattice gauge theory is able to check these scenarios from first principles. For example, one can compare lattice results with analytic and numerical solutions of the (truncated) hierarchy of DSE, however within the limitations of finite lattice discretisation and - even more important in this respect - of finite-volume effects. One crucial test concerns the proposed infrared vanishing (diverging) of the Landau gauge gluon (ghost) propagator. The closely related behavior of the two propagators is intimately connected with an infrared fixed point [5, 6] of the momentum subtraction (MOM) scheme [7] running QCD coupling (see also e.g. [8]). So far, only in two and three dimensions it was possible to reach the expected asymptotics in an unambiguous manner [9, 10, 11]. In four dimensions, for SU(2) as well as SU(3) lattice gauge theory, the ultimate decrease of the gluon propagator towards vanishing momentum has not yet been established. This paper is devoted to this question but restricted to the SU(2) case.

A possible pattern of finite-volume deviations from the far-infrared behavior of the gluon and ghost propagators has been pointed out thanks to the formulation and solution of the DSE in a compact space-time [12]. The sobering message is that really infrared results can be expected only on lattices of linear sizes $L = O(10 \text{ fm})$. However, in the DSE approach the Gribov ambiguity is assumed not
to play a relevant role, such that something comparable about the gauge-fixing vulnerability of the propagators cannot be learned from DSE solutions. Nevertheless, the restriction to the fundamental modular region might also considerably change the structure of the DSE at finite volume [2].

In the present paper we study the question to what extent the finite-volume effects observed in lattice calculations can be related to the existence of Gribov copies and can be cured (for presently accessible volumes) by a better treatment of the Gribov ambiguity, i.e. systematically pursuing a restriction to the fundamental modular region. The common hope is that in the limit of infinitely large volume Gribov copy effects become negligible. If this is true, then the random choice of an arbitrary gauge copy in the Gribov region (which is statistically equivalent to an average over all of them) should be the physically adequate solution [13].

In paper [14] it has been noted that enlarging the gauge orbits by nonperiodic \( \mathbb{Z}(2) \) gauge transformations (called “\( \mathbb{Z}(2) \) flips”) generically leads to larger values of the gauge functional \( F \). In this paper we continue to explore this approach. Furthermore, within the traditional, continuous part of the gauge-fixing problem, we systematically employ the simulated annealing algorithm. Testing these two modifications, we find that in the range of linear lattice sizes between \( L \approx 2 \) fm and 6.5 fm the choice among Gribov copies, and therefore the optimization of the gauge-fixing method, is still important. Our paper represents a systematic extension of the previous work, where the \( \mathbb{Z}(2) \) flips have been studied for the first time [14]. Besides being much less volume dependent, the gluon propagator in the extended Landau gauge is found flattened for momenta \( p < 0.5 \) GeV, and there are first indications for a decrease towards the infrared limit.

Section II will give an introduction to the necessary technical details. In Sec. III we discuss steps towards an optimal gauge-fixing strategy. The Gribov copy effects at finite volumes are pointed out in Sec. IV. In Sec. V all our results, obtained on various lattices with the respective optimal strategy, are put together and we summarize our findings.

II. GENERAL SETUP: EXTENSION OF THE LANDAU GAUGE

Like many other investigators of the SU(2) gluon propagator we compute it with Monte Carlo (MC) techniques on a lattice with periodic boundary conditions. The standard Wilson single-plaquette action and the lattice definition for the gauge potentials

\[
A_{\mu}(x + \hat{\mu}/2) = A_{\mu}(x + \hat{\mu}/2) \quad g^b = \frac{1}{2i a g_0} (U_{x\mu} - U_{x\mu}^\dagger)
\]

are adopted. In order to fix the Landau gauge for each lattice gauge field \( \{U\} \) generated by means of a MC procedure, the gauge functional

\[
F[g] = \frac{1}{2} \sum_{x,\mu} \text{tr} \left( g(x) U_{x\mu} g^\dagger(x + \hat{\mu}) \right)
\]

is iteratively maximized with respect to a gauge transformation \( g(x) \) which is usually taken as a periodic field, too.

In order to approach the global maximum (related to the fundamental modular region) as close as possible, we are using the simulated annealing (SA) algorithm [15], in combination with subsequent standard overrelaxation (OR). The latter is applied in the final stage of the gauge-fixing procedure in order to finalize the transformation to any required precision of the transversality condition \( \partial_{\mu} A_{\mu} = 0 \).

A decade ago, SA has been shown to be very efficient, when dealing with the maximally Abelian gauge (MAG) [16, 17]. In the latter case typically a huge number of local extrema of the gauge functional is observed. The effectiveness of the SA algorithm in the case of the Landau gauge remained quite unclear for a long time. It was practically used for this gauge in the first study of the ghost propagator [18]. In Ref. [19], also for the Landau gauge, a comparison with other algorithms was carried out. This comparative study came to the conclusion that SA might not provide a real advantage. Today, the state of the art is that SA is practiced in a hybrid form, mixed with microcanonical update steps. It is repeatedly started from random gauge transformations \( g(x) \) and ends with OR, producing each
time one gauge copy in the Gribov region. In a recent, more thorough investigation \[20\] this version of the SA algorithm was seen to become superior, with growing lattice size, to the repeated application of the pure OR algorithm. The efficiency was quantified by the ability to produce a better (narrower) distribution of copies (local extrema) within less or equal CPU time. The results of this study will be published elsewhere \[21\].

The SA algorithm, in the present context, generates a field of gauge transformations \( g(x) \) by MC iterations with a statistical weight proportional to \( \exp (F[g]/T) \). The “temperature” \( T \) is a technical parameter which is gradually decreased in order to maximize the gauge functional \( F[g] \). In the beginning, \( T \) has to be chosen sufficiently large in order to allow traversing the configuration space of \( g(x) \) fields in large steps. It has been checked that an initial value \( T_{\text{ini}} = 1.5 \) is high enough. After each quasiequilibrium sweep, including both heatbath and microcanonical updates, \( T \) has been decreased with equal step size until \( g(x) \) is uniquely captured in one basin of attraction. The criterion of success is that during the following OR the violation of transversality decreases in a monotonous manner for almost all applications of the compound algorithm. This condition is reasonably satisfied for a final lower temperature value \( T_{\text{fin}} = 0.01 \) \[20\]. The number of temperature steps was chosen of the order \( O(10^5) \).

The second novel feature of our gauge-fixing procedure compared to the standard ones is the application of \( \mathbb{Z}(2) \) flip transformations, the essence of which is an extension of the gauge orbits for any MC generated lattice configuration. We will abbreviate the extended gauge-fixing method as the FSA (flip-SA) algorithm. There is room for its realization under various strategies (see below) that can be chosen in order to save computing time. The flip transformation was first considered in the context of Landau gauge fixing in Ref. \[14\]. For \( SU(2) \) gauge theory, each flip transformation consists of a simultaneous \( \mathbb{Z}(2) \) flip of all links \( U_\nu(x) \rightarrow -U_\nu(x) \) throughout a 3D hyperplane at a given value of the coordinate \( x_\nu \). This is just a particular case of a gauge transformation which is not periodic but periodic modulo \( \mathbb{Z}(2) \),

\[
g(x + L\hat{\nu}) = z_\nu g(x), \quad z_\nu = \pm 1 \in \mathbb{Z}(2) . \tag{3}
\]

It is obvious that the above transformation of the gauge field leaves the gauge field action as well as the path integral measure invariant (note that this symmetry is unbroken in the confinement phase). This would not be true anymore in a gauge theory with a fundamental matter field. Therefore, the \( \mathbb{Z}(2) \) flip transformation cannot be applied to such models.

With respect to the flip transformation all gauge copies of one given field configuration relative to the initial gauge can be split into \( 2^4 = 16 \) sectors for \( SU(2) \) gauge fields (\( 3^4 = 81 \) sectors for \( SU(3) \)). Within each of these sectors - all being present in the path integral measure - different gauge copies are connected by continuous, strictly periodic gauge transformations. With this new element, our gauge-fixing procedure consists of two steps: the first one is to choose the best out of the 16 flip sectors and the second one with the help of SA is to find the gauge copy with the highest value of the gauge functional while staying within the given sector. In practice, both steps are performed in an intertwined manner, because the decision which is the “best” sector in principle requires knowing the best copy of each sector. It is immediately clear that this procedure allows to find higher local maxima of the gauge functional \[24\] than the traditional gauge-fixing procedures. The latter by default choose for a given configuration only one flip sector, and in most of the cases only one copy in this sector. The sector taken is usually the one randomly selected by the MC update algorithm. It is equivalent to averaging over all flip sectors and therein over copies within the so-called Gribov region.

Obviously the two prescriptions to fix the Landau gauge, the traditional one and the new one, are not equivalent. Indeed, for some modest lattice volumes it has been shown in Ref. \[14\] that they give rise to different results for the gluon as well as the ghost propagators. In the present paper for the gluon propagator we want to present some numerical evidence that the results converge to each other in the large volume limit. The ghost propagator under this extended Landau gauge fixing will be addressed in a future publication.

The computations presented in this work have been done at rather strong coupling, at \( \beta \equiv 4/g_0^2 = 2.20 \). The reason for this choice was to get access to a comparatively large physical volume. We fix the scale taking the string tension as \( \sigma = (440 \text{ MeV})^2 \) and adopting the lattice value \( \sqrt{\sigma a} = 0.469 \) found in Ref. \[22\]. Thus, our largest lattice size \( 32^4 \) has a physical size of about \( (6.5 \text{ fm})^4 \). In order to
\[
\langle F_{ns}(nc) \rangle - F_0
\]

for \(16^4\) and \(24^4\) the numbers of investigated MC configurations are 60 and 46, respectively. The inverse coupling is \(\beta = 4/g_0^2 = 2.20\).

**III. THE QUEST FOR AN OPTIMAL GAUGE-FIXING STRATEGY**

As a first step we have searched for an optimal strategy to find the best gauge copy for each lattice size. On \(16^4\) (and \(24^4\)) lattices we have produced ensembles of 60 (46) MC configurations. For each configuration we created with the help of SA 5 gauge copies as local maxima of the gauge functional \(F\) within each of the 16 flip sectors, i.e. in total 80 gauge copies per MC field configuration. In a production run we would like to get along with considerably less copies per MC configuration. This will become particularly important for \(SU(3)\), where one has to deal with \(3^4 = 81\) different \(\mathbb{Z}(3)\) sectors.

By \(\langle F_{ns}(nc) \rangle\) let us denote the MC ensemble average over the maximized functional values \(F\) taken from all 16 sectors \((ns = 16)\) or a random subset of \(ns < 16\) flip sectors and from the best of \(nc \leq 5\) gauge-fixed copies. These copies are created sequentially, starting from new random periodic copies, in each of the \(ns\) chosen sectors and the best one is stored. The average \(\langle F_{16}(5) \rangle\) corresponds to the largest accessible (best) functional values. Representing the largest affordable computing effort it will serve as a reference value. Table I shows the values for the different cases. One sees that the functional values become larger, when all 16 flip sectors are taken into account. The data clearly indicate that (for the given volume) it is more important to scan all 16 sectors than to search for the best copy in one (randomly chosen) sector. But the improvement is much less dramatic for the larger lattice size \(24^4\) than for the \(16^4\) lattice. The reference values for the functional are very close for the two lattice sizes in contrast to the cases \(ns = 1\) of one randomly chosen sector. Moreover, we see that 5 random copies already seem to be optimal for both the lattice sizes. In Table II we show additionally the deviations or distances \(\Delta_{ns, ns'}(nc, nc') = \langle F_{ns}(nc) - F_{ns'}(nc') \rangle\) between would-be runs with different numbers \(ns\) and \(nc\). The \(\Delta\)-values have quite small statistical errors since the differences are always computed configuration by configuration.

From this work as well as from our earlier experience we know that the functional \(F\) and the gluon propagator at small momenta are anticorrelated (more detailed description of this anticorrelation will be published elsewhere). We wish to emphasize that substantial decrease of \(\Delta\) with increasing volume shown in Table II does not imply that the effect of improved gauge fixing on the propagator decreases also that much.

Notice that the values in Table II fall monotonously from comparison A to comparison E for both the lattice sizes. For \(24^4\) the variation covers only one order of magnitude compared with two orders for \(16^4\). The variation of the best copy results \((nc, nc' = 5)\) comparing the best sector \((ns = 16)\) with the first random sector \((ns' = 1)\) (comparison A) shows the sectors to differ much more strongly from

| \(ns\) | \(nc\) | \(\langle F_{ns}(nc) - F_0 \rangle\) for \(16^4\) | \(\langle F_{ns}(nc) - F_0 \rangle\) for \(24^4\) |
|-------|-------|--------------------------------|--------------------------------|
| 1     | 1     | \(1(8) \cdot 10^{-5}\)          | \(25(4) \cdot 10^{-5}\)       |
| 1     | 5     | \(6(8) \cdot 10^{-5}\)          | \(31(4) \cdot 10^{-5}\)       |
| 16    | 1     | \(32(9) \cdot 10^{-5}\)         | \(36(4) \cdot 10^{-5}\)       |
| 16    | 2     | \(33(9) \cdot 10^{-5}\)         | \(38(4) \cdot 10^{-5}\)       |
| 16    | 3     | \(34(9) \cdot 10^{-5}\)         | \(38(4) \cdot 10^{-5}\)       |
| 16    | 4     | \(34(9) \cdot 10^{-5}\)         | \(39(4) \cdot 10^{-5}\)       |
| 16    | 5     | \(34(9) \cdot 10^{-5}\)         | \(39(4) \cdot 10^{-5}\)       |

**TABLE I:** The average gauge functionals \(\langle F_{ns}(nc) \rangle\) as explained in the text and subtracted with \(F_0 = 0.82800\). For the lattice sizes \(16^4\) and \(24^4\) the numbers of investigated MC configurations are 60 and 46, respectively.
each other on the smaller lattice than on the larger one. This indicates that, concerning the gauge functional, the rôle of the flip sectors is weakening with increasing volume. On the other hand the variation between different copies within the same random flip sectors (case B) or within the best sectors (C, D, E) becomes stronger the larger the lattice is. Therefore, in order to distinguish the best sector we certainly need to generate more gauge copies per sector the larger the lattice volume is. How many copies are required within a given sector depends on the deviation from the reference value one considers to be tolerable (compare with cases C, D, E).

These observations suggest a strategy to keep the total number of gauge copies as low as possible, that have to be generated in order to guarantee a certain prescribed closeness of the average best gauge functional to the reference case. Since for the smaller lattice sizes the functional values of $F$ for different gauge copies generated within the best sector are scattered very closely to the maximal value in that sector, we try to identify the best sector by gauge-fixing not more than one gauge copy per sector. Actually this becomes difficult or even impossible for a larger volume. After the best sector has been figured out, we could generate a few more gauge copies for this particular sector only. In order to increase the probability not to misidentify the best sector, compared to making only one gauge-fixing attempt in all sectors, it is reasonable to perform a few more gauge fixings in a few sectors that have already been recognized as good pretenders of being the best sector.

In shorthand, we denote as “16 + 4” a strategy, where we first fix one gauge copy in all 16 sectors, and then fix a second, independent copy in the 4 best-candidate sectors, those with the highest ranking gauge functional values of the gauge copy found first. Taking again our data for the $24^4$ lattice with 80 gauge copies per configuration as the reference case to compare with, we checked the reliability of such an improved strategy. We get a difference $\langle F_{16}(5) - F_{16+4} \rangle = 1.9(2) \cdot 10^{-5}$, i.e. almost the closeness to the reference case that was obtained with two gauge copies in all sectors, although now, in the “16 + 4” strategy, a second copy has been fixed in only 4 out of 16 sectors. As a compromise between the quality and the need to limit the CPU time we have in practice chosen a strategy with “16 + 4 * 2” copies, i.e. in four selected sectors not one but two more gauge copies are created. On our test ensemble of 46 primary Monte Carlo configurations we get a difference from the reference value $\langle F_{16}(5) - F_{16+4*2} \rangle = 1.4(2) \cdot 10^{-5}$.

We have attempted to apply the same “16 + 4 * 2” strategy to $32^4$ lattices as well. We have observed that for this lattice size the best sectors are not so clearly distinguishable from the other sectors with generically lower values of the gauge functional. For this reason we decided to produce additionally 16 copies, one per sector. Thus we generated in total 40 copies per MC configurations on this lattice (“16 * 2 + 4 * 2”), instead of 80.

For $12^4$ lattices we have blindly generated 5 copies in each sector (“16 * 5”), and for $8^4$ lattices just 3 copies in each sector (“16 * 3”). We found confirmation of the features observed for $16^4$ and $24^4$ lattices as discussed in the beginning of this section.

Our produced ensembles of gauge-fixed field configurations are quoted in Table III together with the strategy used in each case. $\langle F^{bc} \rangle$ is the average gauge functional for the best copy (bc) found first. Taking again our data for the 24 configurations we get a difference from the reference value $\langle F^{bc} - F^{nc} \rangle$ means the difference between the values achieved with the preferential strategy (based always on access to all 16

| ns | nc | ns' | nc' | $\Delta_{ns,ns'}(nc, nc')$ for $16^4$ | $\Delta_{ns,ns'}(nc, nc')$ for $24^4$ |
|----|----|-----|-----|--------------------------------|--------------------------------|
| A  | 16 | 5   | 5   | 2.8(1) \cdot 10^{-4}           | 8.5(4) \cdot 10^{-5}           |
| B  | 1  | 5   | 1   | 4.6(4) \cdot 10^{-3}           | 5.3(2) \cdot 10^{-5}           |
| C  | 16 | 5   | 16  | 1.3(1) \cdot 10^{-5}           | 2.9(2) \cdot 10^{-5}           |
| D  | 16 | 5   | 16  | 4.9(8) \cdot 10^{-6}           | 1.5(1) \cdot 10^{-5}           |
| E  | 16 | 5   | 16  | 2.5(5) \cdot 10^{-6}           | 7.6(7) \cdot 10^{-6}           |

TABLE II: Distances $\Delta_{ns,ns'}(nc, nc') = \langle F_{ns}(nc) - F_{ns'}(nc') \rangle$ as defined in the text. The statistics and the inverse coupling are the same as quoted in Table I.
TABLE III: Lattice sizes, statistics, gauge-fixing strategy employed and the data on average values of the gauge functional \( F \). The meaning of \( F_{bc} \), \( F_{fc} \) and of \( F_{OR}^{fc} \) is explained in the text.

| \( L \) | \# | strategy | \( \langle F_{bc} \rangle \) | \( \langle F_{bc} - F_{fc} \rangle \) | \( \langle F_{OR}^{fc} \rangle \) |
|---|---|---|---|---|---|
| 8 | 200 | “16 * 3” | 0.82721(23) | 0.00298(7) | 0.82365(25) |
| 12 | 200 | “16 * 5” | 0.82817(10) | 0.00077(2) | 0.82715(11) |
| 16 | 60 | “16 * 5” | 0.82834(9) | 0.00028(1) | 0.82715(11) |
| 16 | 180 | “16 + 4 * 2” | 0.82834(8) | 0.000244(6) | 0.82779(5) |
| 24 | 46 | “16 * 5” | 0.82839(4) | 0.000085(4) | 0.82779(5) |
| 24 | 300 | “16 + 4 * 2” | 0.82843(2) | 0.000132(2) | 0.82805(3) |
| 32 | 247 | “16 * 2 + 4 * 2” | 0.82843(1) | 0.000075(1) | 0.82815(1) |

IV. THE GLUON PROPAGATOR: GRIBOV COPY AND FINITE-VOLUME EFFECTS

The gluon propagator is defined by

\[
D^{ab}_{\mu\nu}(p) = \langle \tilde{A}^a_{\mu}(k) \tilde{A}^b_{\nu}(-k) \rangle = \left( \delta_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2} \right) \delta^{ab} D(p),
\]

where \( \tilde{A}(k) \) represents the Fourier transform of the gauge potentials according to Eq. (1) after having fixed the gauge. The momentum \( p \) is given by \( p_{\mu} = (2/a) \sin(\pi k_{\mu}/L) \), \( k_{\mu} \in (-L/2, L/2] \). For \( p \neq 0 \), one gets

\[
D(p) = \frac{1}{9} \sum_{a=1}^{3} \sum_{\mu=1}^{4} D^{aa}_{\mu\mu}(p),
\]

whereas at \( p = 0 \) the “zero momentum propagator” \( D(0) \) is defined as

\[
D(0) = \frac{1}{12} \sum_{a=1}^{3} \sum_{\mu=1}^{4} D^{aa}_{\mu\mu}(p = 0).
\]

In order to compare with standard methods employed by other authors we have carried out our own analysis with standard overrelaxation (OR) without \( Z(2) \) flips and restricting always to the first gauge copy. The corresponding findings together with our \( bc \)-FSA results obtained with the “16 * 2 + 4 * 2” strategy on the largest lattice \( 32^4 \) are plotted in Fig. 1. We have convinced ourselves that the OR results for the \( 24^4 \) lattice are in perfect agreement with those recently obtained for a \( 22^4 \) lattice and the same \( \beta = 2.20 \) in Ref. [10]. A deviation of our FSA results in the infrared \( (p < 0.4 \text{ GeV}) \) towards lower values of \( D(p) \) becomes clearly visible.

As one might expect, due to the bias towards a larger gauge functional in the case of the FSA algorithm (compared with the OR algorithm) not only the expectation value of the gluon propagator becomes suppressed at low momenta, but also the statistical fluctuations of the gluon propagator become reduced. The effect is most clearly seen for the zero momentum propagator \( D(0) \).

In comparison to the finite-size dependence showing up after gauge fixing with standard OR our new FSA method provides results very stable against varying lattice size. This is demonstrated in Fig. 2 collecting our main results. All data points nicely fall onto a universal curve. Indeed, comparing
FIG. 1: The lattice gluon propagator versus momentum for $\beta = 2.20$ and various lattice sizes obtained by OR in comparison with FSA results for $32^4$.

FIG. 2: The gluon propagator obtained with FSA gauge fixing in the infrared region for various lattice sizes, all simulated at $\beta = 2.20$.

the data for different lattice sizes entering Fig. 2 one can see that the finite-volume effects for the momenta shown in the figure are indeed small. This is particularly important for the minimal nonzero (on-axis) momenta for each given lattice size which are not excluded from the plot. Notice that for
lattices $24^4$ and $32^4$ all momenta with components $k_\mu$ satisfying the condition

$$\sum_\mu k_\mu^2 - \left( \sum_\mu \frac{1}{2} k_\mu \right)^2 < 3$$

are shown. There is no significant breaking of rotational invariance for the momenta included in the figure. Only in the case of OR there is one bigger deviation from rotational invariance: on the largest lattice $32^4$ the propagator values for momenta with components $k = (0, 0, 0, 2)$ and $k = (1, 1, 1, 1)$ differ by less than 3 standard deviations. For the lattices $16^4$ and $24^4$ we have found a good agreement within both gauge-fixing algorithms.

In Figs. 1 and 2 the data obtained with FSA on the $32^4$ lattice show a tendency to decrease toward smaller values at the smallest nonzero momentum. This is the first lattice result in favor of a decreasing gluon propagator towards the infrared in four dimensions. In both figures we have also shown the values of the gluon propagator at zero momentum, $D(0)$, which has a monotonous downward volume dependence (compare also Fig. 3 [24]). However, the value of $D(p \equiv 0)$ is expected to be affected by stronger finite-volume (and Gribov ambiguity) effects than $D(p_{\text{min}} \to 0)$ [12]. We have also checked, whether our result can be seen in agreement with the expectation $D(p \to 0) = 0$. Indeed, a fit restricted to the interval $0 < p < 500$ MeV with the function

$$D(p) = p^{2\alpha} \cdot (g_0 + g_1 \cdot p^2) ,$$

worked perfect ($\chi^2/\text{d.o.f.} = 0.06$) with an exponent $\alpha = 0.09(1)$, which is in qualitative agreement with the DSE result [23] $\kappa_D = 1 + \alpha = 1.19$. Although this cannot be taken too seriously, our result gives some credit to the assumption that we are beginning to see the gluon propagator to decrease toward zero momentum. The replacement of the fc –SA algorithm (i.e. with one copy $nc = 1$ in one random flip sector $ns = 1$) by the bc –FSA algorithm (with 16 sectors under control and the preferential strategy according to Table III) leads to a systematic change of the resulting propagator which is presented in Fig. 4. The Figure shows for all lattice volumes that for fixed lattice size the relative deviation of the FSA results for the gluon propagator from the simple SA results decreases with increasing momentum going rather quickly to zero within error bars. Furthermore, for fixed physical momentum the relative deviation goes to zero with increasing volume, indicating that the two Landau gauge-fixing prescriptions (without and with flips) become equivalent in the large volume limit. On the other hand, if we compare data for the minimal momenta for every lattice we find that the respective relative deviation decreases with increasing lattice volume rather slowly indicating that...
the effect of flip sectors for the minimal momentum will be important for all accessible lattices. This is also a valid conclusion, although to a smaller extent, for the next-to-minimal momentum.

V. CONCLUSIONS

In this paper we have reinvestigated the Landau gauge gluon propagator on the lattice within $SU(2)$ pure Yang-Mills theory. Our main achievement is the use of an improved gauge-fixing prescription which takes into account $\mathbb{Z}(2)$ flip transformations equivalent to nonperiodic gauge transformations as well as the use of the simulated annealing method in combination with subsequent overrelaxation steps. Comparing with the exclusive use of standard overrelaxation without applying flips we confirm clear Gribov copy effects for the gluon propagator. But more important, we observe that finite-size effects seem to become suppressed for a gauge-fixing prescription providing copies closer to the fundamental modular region. For the first time in the 4d $SU(2)$ case on symmetric lattices we see a flattening or a signal for a turnover giving access to a limit $D(q \to 0) = 0$ in agreement with DSE predictions and confinement scenarios by Zwanziger [2] or Kugo and Ojima [3].

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