Gravitational lensing of STU black holes

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May 7, 2014

Abstract

In this paper we study gravitational lensing by STU black holes. We considered extremal limit of two special cases of zero-charged and one-charged black holes, and obtain the deflection angle.

Keywords: Gravitational lensing; STU black hole.

1 Introduction

Strong and weak gravitational lensing is one of the interesting subjects of recent studies in theoretical physics, cosmology and astrophysics [17]. The investigation of gravitational lensing is powerful method to probe the extra dimensions. In that case the gravitational lensing by squashed Kaluza-Klein black holes studied in Refs. [8, 9] and developed to charged squashed Kaluza-Klein black hole by the Ref. [10].

Also there are interesting relation between the gravitational lensing and dark energy. So in the Ref. [11] the effect of phantom scalar field (as a candidate for dark energy) on the gravitational lensing has been studied. In the similar way one can study effect of Chaplygin gas [12, 13] on the gravitational lensing.

Now, aim of this paper is studying the gravitational lensing by so-called STU black hole [14-20]. The STU black hole exist in special case of $D = 5, \mathcal{N} = 2$ gauged supergravity theory which is dual to the $\mathcal{N} = 4$ SYM theory with finite chemical potential. In this background, generally there are three electric charges. In this paper we assume that two of them will be zero and discussed about one-charged black hole.

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2 Gravitational lensing

For a given generic black hole background of the form,
\[ ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2, \]  
(1)

it is useful to define the following quantity [21],
\[ X = u \sqrt{\frac{B(r)}{C(r)\left[\frac{C(r)}{A(r)} - u^2\right]}}. \]  
(2)

Therefore, the deflection angle is given by,
\[ \alpha = -\pi + 2 \int_{r_m}^{\infty} Xdr, \]  
(3)

where,
\[ u = \sqrt{\frac{C(r_m)}{A(r_m)}}, \]  
(4)

is the impact parameter and \( r_m \) is closest distance between traveling photon and the black hole. Now, if we assume a source at coordinates \((r_S, \phi_S)\) and an observer at \((r_O, \phi_O)\), then the total azimuthal shift experienced by the photon is given by,
\[ \Phi_- = \int_{r_m}^{r_O} Xdr + \int_{r_m}^{r_S} Xdr. \]  
(5)

Also it is possible that the photon directly goes from the source to the observer far from the black hole. In that case the azimuthal shift is given by,
\[ \Phi_+ = \int_{r_S}^{r_O} Xdr. \]  
(6)

Therefore, lens equation will be \( \Phi = \Phi_- \) if \( \Delta\phi > \Phi_\pm \) and \( \Phi = \Phi_+ \) if \( \Delta\phi \leq \Phi_\pm \), where
\[ \Delta\phi = \phi_O - \phi_S + 2n\pi. \]  
(7)

Also, the angle \( \theta \) where the observer detects the photon is given by the following expression,
\[ \theta = \sin^{-1}\left( u \sqrt{\frac{A(r_O)}{C(r_O)}} \right). \]  
(8)
3 STU black hole

STU black hole is described by the following solution,

$$ds^2 = -\frac{f}{\mathcal{H}^3}dt^2 + \mathcal{H}^{\frac{1}{3}}\left(\frac{dr^2}{f} + \frac{r^2}{R^2}d\Omega^2\right),$$

where,

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{R^2}\mathcal{H},$$

$$\mathcal{H} = \prod_{i=1}^{3} H_i,$$

$$H_i = 1 + \frac{q_i}{r^2}, \quad i = 1, 2, 3,$$

$$A_i^i = \sqrt{\frac{q_i}{q_i} + \frac{\mu q_i}{r^2}}(1 - H_i^{-1}),$$

and $R$ is the constant AdS radius which relates to the coupling constant via $R = 1/g$ (also, coupling constant relates to the cosmological constant via $\Lambda = -6g^2$), and $r$ is the radial coordinate along the black hole. The black hole horizon specified by $r = r_h$ which is obtained from $f = 0$. Also $\mu$ is called non-extremality parameter. So, for the extremal limit one can assume $\mu = 0$. The Hawking temperature of STU black hole is given by,

$$T = \frac{r_h}{2\pi R^2} \frac{2 + \frac{1}{r_h^3} \sum_{i=1}^{3} q_i - \frac{1}{r_h^3} \Pi_{i=1}^{3} q_i}{\sqrt{\Pi_{i=1}^{3} (1 + \frac{q_i}{r_h^2})}}.$$  

(11)

So, in the case of $q_i = 0$ we get,

$$r_h = \pi R^2 T,$$

(12)

and for the case of one-charged black hole ($q_1 = q$, $q_2 = q_3 = 0$) one can obtain,

$$r_h = \frac{1}{2} \sqrt{2\pi^2 R^4 T^2 \left(1 + \sqrt{1 + \frac{2q}{\pi^2 R^4 T^2}}\right) - 2q},$$

(13)

which is reduced to the equation (12) for $q = 0$.

It should be pointed out that the solution (10) written for the spherical space with curvature $k = 1$. It is also possible to consider the case of flat space with $k = 0$, so one can write [22],

$$f = -\frac{\mu}{r^2} + \frac{r^2}{R^2}\mathcal{H}.$$  

(14)
4 Uncharged black hole

First of all we consider very special case of zero-charged black hole with \( q = 0 \). Then using line element (9) in the equations (2), (3) and (4) gives the following deflection angle,

\[
\alpha = -\pi + 2R(\ln 2 + \ln R) + 2R \left( \ln \left( \frac{r_m}{\sqrt{R^2 + r_m^2}} \right) + \ln \left( \sqrt{\frac{r_m^4(1 - R^2) + r_m^2 R^2}{R^2 + r_m^2}} \right) - \ln (r_m) \right). \tag{15}
\]

Then, by using the equation (8) one can obtain the angle \( \theta \) as the following,

\[
\theta = \sin^{-1} \left( \frac{r_m^2 \sqrt{R^2 + r_m^2}}{r_O^2 \sqrt{R^2 + r_m^2}} \right). \tag{16}
\]

In the Fig. 1 we draw the deflection angle in terms of closest distance of a photon with the black hole. Interesting assumption is that the photon reach the near horizon \( (r_m \approx r_h) \), in that case one can obtain the deflection angle in terms of the black hole temperature,

\[
\alpha = \pi - R \left( \ln 2 + \ln \pi + 2 \ln R + \ln (T) - \ln \left( 1 + \pi^2 R^2 T^2 \right) \right), \tag{17}
\]

which is illustrated in The Fig. 2. It shows that the black hole temperature increases the deflection angle.

![Figure 1: Deflection angle versus \( r_m \) for \( R = 0.5 \) (dashed line), \( R = 1 \) (solid line) and \( R = 2 \) (dotted line).](image)

5 One-charged black hole

In order to find the effect of black hole charge on the deflection angle, we consider the simplest case of one-charged black hole. In that case we set \( q_1 = q \) and \( q_2 = q_3 = 0 \). Therefore we
Figure 2: Deflection angle versus $T$ for $R = 0.5$ (dashed line), $R = 1$ (solid line) and $R = 2$ (dotted line).

We find,

$$\alpha = -\pi + \frac{2R \sqrt{r_m^2 + q}}{r_m} \left( \ln 2 + 2 \ln R + \ln r_m - \ln r_m^2 + R^2 + q \right). \quad (18)$$

In the Fig. 3 we find that the black hole charge increased the deflection angle. Then the angle $\theta$ obtained as the following,

$$\theta = \sin^{-1} \left( \frac{\sqrt{r_m^2 + q \sqrt{R^2 + r_O^2 + q}}}{\sqrt{r_O^2 + q \sqrt{R^2 + r_m^2 + q}}} \right) \quad (19)$$

Figure 3: Deflection angle versus $r_m$ with $R = 1$ for $q = 1$ (solid line), $q = 2$ (dashed line) and $q = 3$ (dotted line).
6 Conclusion

In this paper we considered one-charged black hole of STU model at extremal limit to find the effect of the black hole charge on the gravitational lensing. In that case we found that the black hole charge increased the deflection angle. This work will be extended to the three-charged non-extremal black hole, however it is expected that the black hole charge increased the deflection angle.

Also it is interesting to study gravitational lensing by other important black holes. For instance there are a class of two dimensional extremal black holes [23], BTZ black holes [24], or Hořava-Lifshitz black hole [25]. Also there are new works, where some black holes extend to the hyperscaling version [26, 27], which are interesting subjects to study gravitational lensing.

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