Propagative oscillations in co-directional polariton waveguide couplers

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We report on novel exciton-polariton routing devices created to study and purposely guide light-matter particles in their condensate phase. In a co-directional coupling device, two waveguides are connected by a partially etched section which facilitates tunable coupling of the adjacent channels. This evanescent coupling of the two macroscopic wavefunctions in each waveguide reveals itself in real space oscillations of the condensate. This Josephson-like oscillation has only been observed in coupled polariton traps so far. Here, we report on a similar coupling behavior in a controllable, propagative waveguide-based design. By designing the gap width and channel length, the exit port of the polariton flow can be chosen. This co-directional polariton device is a passive and scalable coupler element that can serve in compact, next generation logic architectures.

Photonic circuits rely on a variety of fiber-based optical elements for their functionality, which allow easy routing and filtering of the signals; the main drawback of purely photonic schemes for logic operations, however, is a lack of self interaction for very efficient switching. The remarkable advances in exciton-polariton physics are a result of the progressing control of high-quality microcavities, in which quantum well excitons and cavity photons couple strongly to form new hybrid light-matter eigenstates. Polaritons exhibit a condensate regime at higher densities with emission properties similar to those of a traditional laser, without having to rely on population inversion. This macroscopic quantum state, or quantum fluid of light, can propagate over macroscopic distances for high-quality samples. Furthermore, polaritons can be excited, confined and thereby guided in waveguide structure. Propitiously, the excitonic fraction of the polariton condensate is responsible for the observation of strong nonlinear interaction effects, the photonic fraction allows for typical photonic benefits like a fast propagation velocity. Due to the combination of these two aspects, a variety of next generation devices based on polaritons can be envisioned. Especially the possibility to use polaritons as information carriers in logic architectures has been addressed theoretically and experimentally, in proof of concept devices.

Quite recently, these ideas have been rekindled by room temperature experiments demonstrating coherent polariton propagation in perovskite waveguides and a room-temperature organic polariton transistor. Passive routing elements are essential in polariton logic architectures to make full use of the system as a low power consumption coherent light source. Basic routing effects have been achieved and predicted for polaritons, which show some functionality but are mainly based on active optical control. To this end, we demonstrate a new polariton device in a co-directional router, harnessing a Josephson-like oscillation effect in real space, which could feasibly be scaled and does not need active external control. Josephson oscillations occur when two quantum states are coupled by a transmissive barrier and were first demonstrated in superconductors. Similar effects have been observed in atomic Bose Einstein condensates, for which the interaction between the particles is crucial to observe different interaction dependent regimes of coupling. For exciton-polaritons this effect has first been observed in a naturally occurring disorder double potential well formed during sample growth and later in a dimer micropillar arrangement, even achieving a regime where the interaction plays a crucial role in the time dynamics of these zero-dimensional systems.

Our new coupler device consists of extended one-dimensional channels which allow observation of the oscillations in the spatial domain. We use a new, specifically tailored fabrication technique where the top mirrors between the waveguides are partially etched to realize controlled coupling and thus oscillatory exchange of the

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polariton population. This technique allows routing of polaritons depending on controllable device parameters and the resulting tunable coupling between the waveguides.

In this work, dedicated to a proof of concept, we use high-quality GaAs-based microcavities, benefiting from mature fabrication techniques. For the co-directional couplers with waveguide coupling achieved by partially etched mirrors we use molecular beam epitaxially grown microcavities featuring 27 distributed Bragg reflector (DBR) pairs of AlAs/Al0.2Ga0.8As in the bottom and 23 in the top DBR. Three stacks of four GaAs quantum wells with a width of 7 nm are placed in the AlAs λ/2 cavity at the anti-nodes of the electric field. The vacuum Rabi splitting determined by white light reflection measurements is 13.9 meV. The quality factor $Q \approx 5000$ was determined at low power excitation.

Sample processing was done via a specially developed reactive ion etching (RIE) process. The first step consists of an electron beam exposure of a polymethyl methacrylate photoresist and subsequent development. Later, a metal layer of calibrated thickness is evaporated on the sample followed by a lift-off process. After the lift-off process the sample is etched. Due to protection by the predefined metal layers, the sample is only etched at the exposed positions, which allows the fabrication of the waveguide structures. Due to the proximity of the structures and the anisotropic etching behaviour of RIE, the etching rate between the waveguides is slower, leaving a certain number of mirror pairs untouched and resulting in a rising flank at the etch edges. These left-over mirror pairs between the waveguides facilitate evanescent, photonic coupling. The area around the defined waveguide structures is nearly etched through the bottom DBR and therefore facilitates strong photonic confinement.

![FIG. 1: (a) Device schematic with indicated laser excitation (red), incoupler region (orange) and the coupling region (blue) along the x axis. (b) Top view SEM image of a co-directional polariton coupler. (c) Zoom-in highlighting the coupling region (blue) and the gap between the two waveguides. Here, the cavity and a varying number of mirror pairs are still intact. (d) Waveguide dispersion in propagation direction below and (e) above polariton condensation threshold at the input port region (orange).](image)

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![FIG. 2: (a) Energy-resolved real space photoluminescence signal for a coupler device with a gap size of 200 nm and a coupler region length of 100 µm. Polaritons are injected non-resonantly in the lower incoupler and exhibit a distinct oscillatory behavior between the two waveguides in the coupler region (right of the black line). (b) Polariton photoluminescence distribution plotted for the lower (negative) and the upper waveguide (positive) showing the oscillation as well as the damping that is expected for a dissipative system.](image)

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dimensional confinement. These modes have been fitted with an approach from Ref.\textsuperscript{10} This allows to extract a detuning of -23 meV with an exciton energy of 1.609 eV, corresponding to photonic and excitonic fractions of 92.5 \% and 7.5 \%, respectively.

To move on, an investigation of the non-linear regime in this structure, is represented in Fig. 1(e), showing polaritons above threshold being expelled from the small laser pump spot\textsuperscript{10} at a wave vector $k_x \approx 1.9 \mu$m\textsuperscript{-1}. A detailed analysis of the emission intensity and energy as a function of the excitation power revealing a lasing threshold behavior and a continuous blueshift of 4 meV is presented in the supplementary materials, confirming polariton interaction effects\textsuperscript{32}.

Let us now report on the main subject of the work, namely the oscillations in co-directional couplers. To this end, Fig. 2 (a) depicts a logarithmic color-coded real space image of the energy-resolved photoluminescence from two adjacent waveguides at an injection power of 1 mW, corresponding to the propagating condensate into the coupling region of the two coupled waveguides. Since the excitation occurs above condensation threshold, the ballistic polaritons have a well-defined frequency. Thus we focus on spatial dynamics along propagation axis $x$ considering a monochromatic case. In the low-intensity limit the dynamics can be modelled via two coupled equations for amplitudes $A_{1,2}$ in the coupled waveguides

$$i \frac{\partial}{\partial x} A_1(x) = -\kappa A_1(x) - J A_2(x),$$

$$i \frac{\partial}{\partial x} A_2(x) = -\kappa A_2(x) - J A_1(x).$$

where $J \equiv J' + iJ''$ is the complex coupling constant which is governed by the width and depth of the gap between the coupled waveguides. The complex wavevector $\kappa = k + i\alpha$ characterizes the propagation and damping of guided modes in separated waveguides (in the following $\alpha > J'' > 0$).

In order to find the super-modes of the coupler we look for a solution of Eqs. (1) and (2) in the form $A_{1,2}(x) = \alpha_{1,2} e^{i\beta x}$. By diagonalizing the system one finds two eigenmodes, namely symmetric $a_1 = a_2$ and antisymmetric $a_1 = -a_2$ one, with the propagation constants $\beta_+ = \kappa + J' + iJ''$ and $\beta_- = \kappa - J' - iJ''$, respectively. It is worth mentioning that, due to the imaginary parts of the coupling constants, the dampings $\text{Im}(\beta_{\pm}) = \alpha \pm J''$ of these two modes is different. More precisely, the antisymmetric mode has smaller propagation losses.

If only one waveguide is excited (with an amplitude $\alpha$), the analytical solution for the mode dynamics can be easily found as

$$A_1(x) = A e^{i(kx-\alpha x)} \times (\cos(J'x) \cosh(J''x) - i \sin(J'x) \sinh(J''x)), \quad (3)$$

$$A_2(x) = A e^{i(kx-\alpha x)} \times (i \sin(J'x) \cosh(J''x) - \cos(J'x) \sinh(J''x)). \quad (4)$$

In the limit of a vanishing imaginary part of the coupling coefficient ($J'' = 0$), the solution has the form of damped oscillations $|A_1(x)|^2 = A^2 e^{-2\alpha x} \cos^2(J'x)$, where polaritons oscillate between the two channels with a spatial period $\pi/J'$. The physical origin of the oscillations is the beating of two (symmetric and antisymmetric) eigenmodes.

However, due to the etching of the Bragg mirror be-

![FIG. 3: Propagation dynamics of polaritons calculated within the model Eqs. (3) & (4) for coupled waveguides with separation gap sizes of (a) 200 nm, (b) 300 nm, (c) 400 nm, and (d) 500 nm. For small gap sizes (a) and (b) a clear oscillatory behaviour is observed. The insets on the right hand sides show the respective distribution in momentum space (on the parameter plane $k_y$ and $k_x$). Antisymmetric and symmetric modes of the coupler are visible.](image-url)
tween two channels, the effect of local losses becomes comparable with the polariton tunnelling dynamics and thus the imaginary part of the coupling cannot be neglected. For a non-zero imaginary part \( J'' > 0 \), the symmetric mode decays faster and at propagation distances of the order of \( 1/J'' \) it becomes much less intensive than the antisymmetric mode. This suppresses the mode beating at large propagation distances where \( A_1 = -A_2 \) and thus the transverse structure of the field is antisymmetric.

To underpin this rather qualitative analysis with a more thorough theoretical study, we performed numerical calculations in the frame of the mean-field model for 2D intracavity photons coupled strongly to the quantum well exciton.\(^\text{(e)}\) This is a widely accepted approach for exciton-polariton dynamics in microcavities where the required waveguiding geometry can be accounted by inducing an appropriate potential for photons. Neglecting polarization effects one obtains two coupled Schrödinger equations for the photonic field \( E \) and coherent excitons \( \Psi \) given as

\[
\partial_t E - \frac{i\hbar}{2m_c} \nabla^2_{x,y} E + iV(x, y)E + [\gamma - i(\omega_p + \delta)] E = i\Omega_R\Psi + E_p(x, y)e^{ik_p x},
\]

\[
\partial_t \Psi - \frac{i\hbar}{2m^*_c} \nabla^2_{x,y} \Psi + [\gamma - i\omega_c] \Psi = i\Omega_R E.
\]

The complex amplitudes are obtained through a standard averaging procedure of the related creation or annihilation operators. \( \gamma \) denotes the cavity photon and exciton damping constants, which are assumed to be equal in this model. The effective photon mass in the planar region is given by \( m_c = 36.13 \times 10^{-6}m_e \) where \( m_e \) is the free electron mass. The effective mass of excitons is \( m^*_c \approx 10^5m_e \). \( \Omega_R \) is the Rabi frequency which defines the Rabi splitting \( 2\Omega_R = 13.9\text{meV} \). The photon - exciton detuning is given by the parameter \( \delta = \omega_0 - \omega_c \), where \( \omega_c \) is the cavity resonance frequency and \( \omega_0 \) is the exciton resonance.

The external photonic potential \( V(x, y) \) defines the waveguide geometry induced by etching of the Bragg mirror. In order to account for dissipative effects, we assume that the potential has a non-zero imaginary contribution. In our modelling, the waveguide profile is given by a super-Gauss \( V(y) = (V_{re} - iV_{im})(1 - \exp(-y^2/s^2)) \) with \( hV_{re} = 120\text{meV} \) and \( hV_{im} = 8\text{meV} \). Since we are interested in the propagation dynamics of polaritons with a well-defined frequency it is sufficient to consider the case of coherent excitation by a pump beam \( E_p(x, y) \) with a frequency \( \omega \) and with a momentum \( k_p \), such that the parameter \( \omega_0 = \omega - \omega_c \) describes the detuning of the pump frequency \( \omega \) from the excitonic resonance.

Fig. 3(a)-(d) show examples of propagation dynamics in which the coupling strength is continuously weakened due to decreased wavefunction overlap via increased gap width. A clear change in the oscillation pattern is observed which is quantified by the extraction of the spatial oscillation period for the three smallest gaps as \( \sim 18\mu m, \sim 25\mu m \) and \( \sim 40\mu m \) for gaps widths 200 nm, 300 nm and 400 nm, respectively. These oscillations are governed by interference of the symmetric and the antisymmetric modes of the coupler, which is clearly visible in the two-dimensional spectrum, shown in the inset to Fig. 3(a). The insets in each panel represent the momentum space distribution of the propagating polaritons. Note that, due to pronounced dissipative effects within the gap between waveguides, the antisymmetric mode, which has the lowest overlap with the gap region, dominates the spectrum for larger gaps and, as a result, oscillations disappear [see Fig. 3(d)].

Now, in order to demonstrate the polariton dynamics in the co-directional coupler devices experimentally, we have performed energy- and time-resolved streak camera measurements using two devices with gap sizes of 200 nm and 500 nm. Using a streak camera with an overall time resolution of 10 ps, the PL has been measured up to 150 ps after the laser beam excites the structure at \( t=0 \). However, due to the fast dynamics of polaritons in this sample, the polariton propagation is only shown up to 30 ps. The respective intensity patterns are plotted in Figs. 4(a)-(c) and (d)-(f).

In Fig. 4(a) at \( t=0\) ps, we observe the laser excitation spot on the lower left input coupler from where polaritons are repulsively expelled into the coupling region. At \( t=12\) ps, the polaritons have finished the first full oscillation in the upper waveguide and are back in the lower one. Finally, after approximately 30 ps, the polariton population has dissipated after a propagation length of 100 \( \mu m \), underlining the excellent quality of the patterned microcavity structure. The oscillation frequency is \( \sim 20\mu m \) in excellent agreement with the theoretical model. Fig. 4(d)-(f) shows the temporal evolution in a system with a much larger gap of 500 nm. In this case, while there is some evanescent coupling to the upper waveguide, no pronounced oscillatory behavior is observed, again in ex-
evidenced this by a precise control of the lithography and engineering the frequency and coupling length. We pass passive polariton routing, which is easily scalable and allows co-directional routing to a predetermined exit-port via a Josephson-like oscillation effect in real space by engineering the upper arm. 

![Graph showing co-directional coupler devices](image)

FIG. 5: Co-directional coupler devices with a reduced coupling length of 20 µm and a gap size of (a) 200 nm and (b) 300 nm. By reducing the coupling, the oscillation frequency and thus the dominant output port is changed.

Therefore we have shown that this device configuration allows co-directional routing to a predetermined exit-port via a Josephson-like oscillation effect in real space by engineering the frequency and coupling length. In conclusion, we have demonstrated the possibility for passive polariton routing, which is easily scalable and integratable to large polariton based logic networks. We evidenced this by a precise control of the lithographically engineered photonic landscape, which allows for the observation of these oscillations in real space between polaritonic waveguides. Such detailed tailoring of the flow of quantum fluids of light paves the way to harness their non-linearity in next generation photonics. Furthermore, the understanding of the coupling of two polariton waveguides is the necessary foundation for larger coupled waveguide arrays, comparable to those that have been implemented in Si- photonics for the demonstration of transport in a topological defect waveguide or Floquet waveguides for the study of topologically protected bound states in photonic parity-time symmetric crystals. In this respect, our work opens a new route, to use polariton waveguides for polariton logic as well as for topological devices involving nonlinearity, gain, interactions and coherence, inherent to the polariton system.

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