Can there be any new physics in $b \to d$ penguins

Anjan K. Giri$^1$, Rukmani Mohanta$^{1,2}$

$^1$Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel
$^2$School of Physics, University of Hyderabad, Hyderabad - 500 046, India

Abstract

We analyze the possibility of observing new physics effects in the $b \to d$ penguin amplitudes. For this purpose, we consider the decay mode $B_d^0 \to K^0 \bar{K}^0$, which has only $b \to d$ penguin contributions. Using the QCD factorization approach, we find very tiny CP violating effects in the standard model for this process. Furthermore, we show that the minimal supersymmetric standard model with $LR$ mass insertion and R-parity violating supersymmetric model can provide substantial CP violation effects. Observation of sizable CP violation in this mode would be a clear signal of new physics.
1 Introduction

Recently there have been a lot of interests to look for new physics effect beyond the standard model (SM). The recent measurement of the indirect CP violating parameter $S_{\phi K_S}$ in the decay mode $B_d^0 \to \phi K_S$, which is a pure $b \to s \bar{s} s$ penguin induced process, may provide the first indication of new physics beyond the SM [1]. Within the SM, the mixing induced CP asymmetry in the $B_d^0 \to \phi K_S$ mode is expected to be equal to that of $B_d^0 \to \psi K_S$ [2] within a correction of $O(\lambda^2)$. The most recent updated data on $S_{\phi K^0}$ by BABAR [3] agrees within one standard deviation with the value of $S_{\psi K_S}$ whereas, the Belle data [4] has about $2\sigma$ deviation. Therefore, the presence of new physics (NP) in this mode has not yet been ruled out from the available data. In principle, the new physics can affect either the $B_d^0 - B_d^0$ mixing or the decay amplitudes. Since the new physics effect in the mixing can affect equally to both the cases the above deviation may be attributed to the decay amplitude of $B_d^0 \to \phi K_S$ or more generally to the $b \to s$ penguin amplitudes. The next obvious question is: Do the $b \to d$ penguin amplitudes also have significant new physics contribution. The present data does not provide any conclusive answer to it. The obvious example is the $B_d^0 \to \pi \pi$ processes, which receive contribution from $b \to u$ tree and from $b \to d$ penguin diagrams. The charge averaged branching ratios of all the three processes $B_d^0 \to \pi^+ \pi^-$, $B_d^0 \to \pi^0 \pi^0$ and $B_d^0 \to \pi^+ \pi^0$ [5] and the CP violating parameters $C_{\pi\pi}$ and $S_{\pi\pi}$ in $B_d^0 \to \pi^+ \pi^-$ process [6] have already been measured. The present situation is: the measured branching ratio for the color allowed process $B_d^0 \to \pi^+ \pi^-$ is about two times smaller than the QCD factorization calculation and the measured $B_d^0 \to \pi^0 \pi^0$ color suppressed branching ratio is about six times larger than the corresponding QCD factorization calculations [7]. Thus the discrepancy between the theoretical and the measured quantities imply the following two consequences.

- The QCD factorization may not be a very successful theory for the charmless $B$ decays.
- There may also be significant new physics effect in the $b \to d$ penguins as speculated in $b \to s$ penguins.

Recently Buras et al. [8] have shown that the observed $B \to \pi \pi$ data can be explained in the standard model if one includes the large nonfactorizable contributions. In this paper we would like to look into the second possibility i.e., the existence of any new physics in $b \to d$ penguin amplitudes and indeed if it does, could it be detectable. For this purpose, we consider the decay mode $B_d^0 \to K^0 \bar{K}^0$ which has only $b \to d$ penguin contribution. The significance of this decay mode is that it originates from $b \to d \bar{s} s$ penguins with dominant
contributions coming from the QCD penguins. If one assumes that the penguin topology is dominated by internal top quark, the CP asymmetry parameters would vanish in the SM. However, as pointed out by Fleischer [9], contribution from penguins with internal up and charm quark exchanges are expected to yield nonzero CP asymmetry in $B_d^0 \to K^0 \bar{K}^0$ mode. Thus, the study of CP asymmetries in this mode may provide an interesting testing ground to explore new physics effects. The CP averaged branching ratio has recently been measured by the BABAR collaboration [10]

$$B(B_d^0 \to K^0 \bar{K}^0) = (1.19^{+0.40}_{-0.33} \pm 0.13) \times 10^{-6},$$

which agrees with the SM predictions [7]. Although, the measured branching ratio does not provide any indication for a possible new physics effect, the measurements of CP violation parameters in near future will certainly establish/rule out the possibility of new physics in the $b \to d$ penguin amplitudes. This decay mode has recently been analyzed by Fleischer and Recksiegel [11] in the SM. They have shown that this channel may be characterized through a surface in the observable space from which one can extract the relevant information. In this paper we consider the impact of new physics effect on the CP violation parameters. We show that the minimal supersymmetric model with $LR$ mass insertion and the supersymmetric model with R-parity violation can provide significant CP violation effect, the observation of which would be a clear signal of new physics.

The paper is organized as follows. In section 2, we discuss the basic formalism of CP violation. Using QCD factorization approach, we estimate the CP averaged branching ratio and CP violating parameters in the SM. The effects of new physics on the CP violating parameters are discussed in section 3. The contributions arising from minimal supersymmetric standard model with mass insertion approximation and from R-parity violating supersymmetric model are discussed in subsections 3.1 and 3.2 respectively. Our conclusion is presented in section 4.

## 2 CP violating parameters in the Standard Model

We first present a very general treatment of the CP violating parameters. The time dependent CP asymmetry for $B_d^0 \to K^0 \bar{K}^0$ can be described by

$$a_{KK}(t) = C_{KK} \cos \Delta M_{B_d} t + S_{KK} \sin \Delta M_{B_d} t ,$$

(2)
where
\[ C_{KK} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad S_{KK} = -\frac{2\text{Im}(\lambda)}{1 + |\lambda|^2}. \] (3)

In the above expression \( \lambda \) corresponds to
\[ \lambda = \frac{q}{p} A(B_d^0 \rightarrow K^0\bar{K}^0), \] (4)
where, \( q \) and \( p \) are the mixing parameters defined as
\[ \frac{q}{p} = \frac{\sqrt{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}{M_{12} - \frac{i}{2}\Gamma_{12}}. \] (5)

The off diagonal element of the mass matrix is given by the matrix element of the \( \Delta B = 2 \) transition as
\[ \langle B_d^0|\mathcal{H}_{\Delta B=2}|\bar{B}_d^0 \rangle = M_{12} - \frac{i}{2}\Gamma_{12}. \] (6)

In the standard model, the box diagrams are dominated by the \( W \)-boson and \textit{top} quark in the loop, as a result of which, one obtains (ignoring terms of \( O(\Gamma_{12}/M_{12}) \))
\[ \frac{q}{p} = \frac{V_{tb}V_{td}}{V_{tb}V_{td}^*} \approx e^{-2i\beta}. \] (7)

The amplitude for the decay mode \( B_d^0 \rightarrow K^0\bar{K}^0 \), which receives dominant contribution in the SM from QCD penguins with \textit{top} quark in the loop can be written as
\[ A(\bar{B}_d^0 \rightarrow K^0\bar{K}^0) = V_{tb}V_{td}^* P_t, \] (8)
where \( V_{ij} \) are the CKM matrix elements which provide the weak phase information and \( P_t \) is the penguin amplitude arising from the matrix elements of the four quark operators of the effective Hamiltonian. The amplitude for the corresponding CP conjugate process is given as
\[ A(B_d^0 \rightarrow K^0\bar{K}^0) = V_{tb}^*V_{td} P_t. \] (9)

Thus one gets
\[ \lambda = \left(\frac{V_{tb}V_{td}}{V_{tb}V_{td}^*} \right) \left(\frac{V_{ib}V_{id}^*}{V_{ib}V_{id}} \right) = 1, \] (10)
and hence
\[ C_{KK} = S_{KK} = 0. \] (11)

Thus if the measured CP violating asymmetries in \( B^0 \rightarrow K^0\bar{K}^0 \) deviates significantly from zero it would be a clear signal of new physics. However, the decay amplitude also receives
some contribution from the internal up and charm quarks in the loop. Therefore, the CP violating parameters may not be zero identically. Now including the effects of \(u, c, t\) quarks in the loop and using CKM unitarity one can write the decay amplitude as

\[
A(\bar{B}_d^0 \rightarrow K^0 \bar{K}^0) = \lambda_u P_{ut} + \lambda_c P_{ct} = \lambda_u P_{ut} \left[ 1 - r e^{i(\delta + \gamma)} \right],
\]

where \(\lambda_q = V_{qb} V_{qd}^*\), \(P_{qt} = P_q - P_t\) are the penguin amplitudes, \(\delta = \delta_{ct} - \delta_{ut}\) is the relative strong phase between them and \(\gamma\) is the weak phase. The parameter \(r\) is defined as

\[
r = \frac{1}{R_b} \left| \frac{P_{ct}}{P_{ut}} \right|,
\]

where \(R_b = \left( 1 - \frac{\lambda_u^2}{2} \right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\rho^2 + \eta^2} \). (13)

Thus one obtains the CP asymmetries as

\[
S_{KK} = -\frac{\sin 2\alpha + 2r \cos \delta \sin(2\beta + \gamma) - r^2 \sin 2\beta}{1 + r^2 - 2r \cos \delta \cos \gamma},
\]

\[
C_{KK} = -\frac{2r \sin \delta \sin \gamma}{1 + r^2 - 2r \cos \delta \cos \gamma},
\]

where \(\alpha\), \(\beta\) and \(\gamma\) are the three angles of the unitarity triangle. Thus, to know the precise value of the CP violating asymmetries we must know the values of \(r\) and \(\delta\). The CP averaged branching ratio for the process is given as

\[
\mathcal{B}(B_d^0 \rightarrow K^0 \bar{K}^0) = \frac{1}{2} \left[ \text{Br}(B_d^0 \rightarrow K^0 \bar{K}^0) + \text{Br}(\bar{B}_d^0 \rightarrow K^0 \bar{K}^0) \right],
\]

where the individual branching ratios are given as

\[
\text{Br}(B_d^0 \rightarrow K^0 \bar{K}^0) = \frac{\tau_{B_d^0} |P_{c.m.}|}{8\pi m_B^2} \left| A(B_d^0 \rightarrow K^0 \bar{K}^0) \right|^2.
\]

We now use the QCD factorization approach to calculate the branching ratio and CP asymmetry parameters. The effective Hamiltonian describing the process under consideration is

\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{qb} V_{qd}^* \sum_{j=3}^{10} C_j O_j + C_g O_g,
\]

where \(q = u, c\). \(O_3, \ldots, O_6\) and \(O_7, \ldots, O_{10}\) are the standard model QCD and electroweak penguin operators respectively, and \(O_g\) is the gluonic magnetic penguin operator. The values of the Wilson coefficients at the scale \(\mu \approx m_b\) in the NDR scheme are given in Ref. [12] as

\[
\begin{align*}
C_3 &= 0.014, & C_4 &= -0.035, & C_5 &= 0.009, & C_6 &= -0.041, & C_7 &= -0.002\alpha \\
C_8 &= 0.054\alpha, & C_9 &= -1.292\alpha, & C_{10} &= 0.263\alpha, & C_g &= -0.143.
\end{align*}
\]
In the QCD factorization approach [7], the hadronic matrix elements can be represented in the form

$$\langle K^0 \bar{K}^0 | O_i | B_d^0 \rangle = \langle K^0 \bar{K}^0 | O_i | B_d^0 \rangle_{\text{fact}} \left[ 1 + \sum r_n \alpha_s^n + O(\Lambda_{\text{QCD}}/m_b) \right], \quad (19)$$

where \(\langle K^0 \bar{K}^0 | O_i | B_d^0 \rangle_{\text{fact}}\) denotes the naive factorization result and \(\Lambda_{\text{QCD}} \sim 225\) MeV is the strong interaction scale. The second and third terms in the square bracket represent higher order \(\alpha_s\) and \(\Lambda_{\text{QCD}}/m_b\) corrections to the hadronic matrix elements.

In the heavy quark limit the decay amplitude for the \(B_d^0 \rightarrow K^0 \bar{K}^0\) process, arising from the penguin diagrams is given as

$$A(B_d^0 \rightarrow K^0 \bar{K}^0) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qd} \left[ a_4^q - \frac{a_{10}^q}{2} + a_{10a}^q + r_x \left( a_6^q - \frac{a_8^q}{2} + a_8^q \right) \right] X, \quad (20)$$

where \(X\) is the factorized matrix element. Using the form factors and decay constants defined as [13]

$$\langle K^0(p_K) | \bar{s} \gamma^\mu b | \bar{B}_d^0(p_B) \rangle = \left[ (p_B + p_K)^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu \right] F_1(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^\mu F_0(q^2),$$

$$\langle \bar{K}^0(q) | \bar{d} \gamma^\mu \gamma_5 s | 0 \rangle = -i f_K q^\mu, \quad (21)$$

we obtain

$$X = \langle K^0(p_K) | \bar{s} \gamma_\mu (1 - \gamma_5) b | \bar{B}_d^0(p_B) \rangle \langle \bar{K}^0(q) | \bar{d} \gamma^\mu (1 - \gamma_5) s | 0 \rangle$$

$$= -i f_K F_0(m_K^2) \left( m_B^2 - m_K^2 \right). \quad (22)$$

The coefficients \(a^q\)'s which contain next to leading order (NLO) and hard scattering corrections are given as [14, 15]

$$a_4^q = C_4 + \frac{C_3}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} \left[ C_3 \left[ 2 F_K + G_K(s_d) + G_K(s_b) \right] \right]$$

$$+ C_1 G_K(s_q) + (C_4 + C_6) \sum_{f=u} b G_K(s_f) + C_9 G_{K,g},$$

$$a_6^q = C_6 + \frac{C_5}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} \left[ C_3 \left[ G'_K(s_d) + G'_K(s_b) \right] + C_4 G'_K(s_q) \right]$$

$$+ (C_4 + C_6) \sum_{f=u} b G'_K(s_f) + C_9 G'_{K,g},$$
\[ a_8^q = C_8 + \frac{C_7}{3}, \]
\[ a_{8a}^q = \alpha_s \frac{C_F}{4\pi N} \left[ (C_8 + C_{10}) \frac{3}{2} \sum_{f=u}^{b} e_f G'_K(s_f) + C_9 \frac{3}{2} [e_d G'_K(s_d) + e_b G'_K(s_b)] \right], \]
\[ a_{10}^q = C_{10} + \frac{C_9}{N} + \alpha_s \frac{C_F}{4\pi N} C_F F_K, \]
\[ a_{10a}^q = \alpha_s \frac{C_F}{4\pi N} \left[ (C_8 + C_{10}) \frac{3}{2} \sum_{f=u}^{b} e_f G_K(s_f) + C_9 \frac{3}{2} [e_d G_K(s_d) + e_b G_K(s_b)] \right], \quad (23) \]

where \( q \) takes the values \( u \) and \( c \), \( N = 3 \), is the number of colors, \( C_F = (N^2 - 1)/2N \). The internal quark mass in the penguin diagrams enters as \( s_f = m_f^2/m_b^2 \). The other parameters in (23) are given as

\[ F_K = -12 \ln \frac{\mu}{m_b} - 18 + f^I_K + f^{II}_K, \]
\[ f^I_K = \int_0^1 dx \ g(x) \phi_K(x), \quad g(x) = 3 \frac{1 - 2x}{1 - x} \ln x - 3i\pi, \]
\[ f^{II}_K = \frac{4\pi^2}{N} \frac{f_K f_B}{F_0^{B-K}(0)m_B^2} \int_0^1 \frac{dz}{z} \phi_B(z) \int_0^1 \frac{dx}{x} \phi_K(x) \int_0^1 \frac{dy}{y} \phi_K(y), \]
\[ G_{K,g} = -\int_0^1 \frac{dx}{1 - x} \frac{2}{1 - x} \phi_K(x), \]
\[ G_K(s) = \frac{2}{3} - \frac{4}{3} \ln \frac{\mu}{m_b} + 4 \int_0^1 dx \ \phi_K(x) \]
\[ \times \int_0^1 du \ u(1-u) \ln [s - u(1-u)(1-x) - i\epsilon], \]
\[ G'_K(g) = -\int_0^1 \frac{dx}{2} \frac{3}{2} \phi^0_K(x) = -\frac{3}{2}, \]
\[ G'_K(s) = \frac{1}{3} - \ln \frac{\mu}{m_b} + 3 \int_0^1 dx \ \phi^0_K(x) \]
\[ \times \int_0^1 du \ u(1-u) \ln [s - u(1-u)(1-x) - i\epsilon]. \quad (24) \]

The light cone distribution amplitudes (LCDA’s) at twist two order are given as

\[ \phi_B(x) = N_B x^2 (1-x)^2 \exp \left( -\frac{m_B^2 x^2}{2\omega_B^2} \right), \]
\[ \phi_K(x) = 6x(1-x), \quad \phi^0_K(x) = 1, \quad (25) \]

where \( N_B \) is the normalization factor satisfying \( \int_0^1 dx \phi_B(x) = 1 \) and \( \omega_B = 0.4 \) GeV. The quark masses appear in \( G(s) \) are pole masses and we have used the following values (in GeV) in our analysis

\[ m_u = m_d = m_s = 0, \quad m_c = 1.4 \quad m_b = 4.8. \]
\( r_\chi = 2 m_K^2 / (m_b - m_s) (m_s + m_d) \) denotes the chiral enhancement factor. It should be noted that the quark masses in the chiral enhancement factor are running quark masses and we have used their values at the \( b \) quark mass scale as \( m_b(m_b) = 4.4 \) GeV, \( m_s(m_b) = 90 \) MeV and \( m_d(m_b) = 6.4 \) MeV.

For numerical evaluation we have used the following input parameters. The value of the form factor at zero recoil is taken as \( F_0(0) = 0.38 \), and its value at \( q^2 = m_K^2 \) can be obtained using simple pole dominance ansatz [13] as \( F_0(m_K^2) = 0.383 \). The values of the decay constants are as \( f_K = 0.16 \) GeV and \( f_B = 0.19 \) GeV, the particle masses and the lifetime of \( B_0^0 \) meson \( \tau_{B^0} = 1.536 \) ps are taken from [5]. Thus we obtain the amplitude (in units of \( 10^{-2} \))

\[
A(\bar{B}_d^0 \to K^0 \bar{K}^0) = - \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* \left( 13.56 + i 4.59 \right) + V_{cb} V_{cd}^* \left( 14.98 + i 2.06 \right) \right]
\]

\[
= - \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* \left( 14.32 e^{i18.7^\circ} \right) + V_{cb} V_{cd}^* \left( 15.12 e^{i7.83^\circ} \right) \right].
\]  

(26)

We use the values of the CKM matrix elements at the 1\( \sigma \) CL in the Wolfenstein parameterization from Ref. [16] as

\[
\lambda = 0.2265^{+0.0025}_{-0.0023}, \quad A = 0.801^{+0.029}_{-0.020}, \quad \bar{\rho} = 0.189^{+0.088}_{-0.070}, \quad \bar{\eta} = 0.358^{+0.046}_{-0.042},
\]

(27)

which correspond to the angles of the CKM unitarity triangle

\[
\sin 2\alpha = -0.14^{+0.37}_{-0.41}, \quad \beta = (23.8^{+2.1}_{-2.0})^\circ, \quad \gamma = (62^{+10}_{-12})^\circ.
\]

(28)

With these input parameters we obtain the CP averaged branching ratio as

\[
\mathcal{B}(B_d^0 \to K^0 \bar{K}^0) = (9.15 \pm 0.30) \times 10^{-7},
\]

(29)

which is slightly below the central experimental value (1). Since, we have used the LCDA’s at twist two order our predicted result is slightly lower than that of Ref. [7] where they have included the twist three power corrections in the distribution amplitudes. From Eqs. (12) and (26) one can obtain

\[
r \approx 2.6 \quad \text{and} \quad \delta \approx 11^\circ.
\]

(30)

With these values we get the CP asymmetry parameters in the SM as

\[
S_{KK} = 0.061 \quad C_{KK} = -0.163.
\]

(31)
By allowing the CKM matrix elements to vary within their 1σ range as given in Eqs. (27) and (28), we obtain the correlation between $S_{KK}$ and $C_{KK}$ in the SM as shown in Figure-1, which gives the constraints

$$0.02 \leq S_{KK} \leq 0.13, \quad -0.17 \leq C_{KK} \leq -0.15.$$  \hspace{1cm} (32)

Thus, if the measured values of CP asymmetry parameters will be outside the above ranges, would be a clear sign of new physics.

Figure 1: The correlation plot between $S_{KK}$ and $C_{KK}$ for the $B^0_d \to K^0\bar{K}^0$ process in the SM, where we have used $r = 2.6$, $\delta = 11^\circ$ and the CKM parameters are varied within the range as given in Eqs. (27) and (28).

### 3 New Physics effects on the CP violating parameters

Here, we consider the effect of new physics on the CP violating parameters. Because of the new physics contributions, the CP asymmetry parameters (14) become modified. In principle, the new physics can affect either the $B^0_d - B^0_d$ mixing or the decay amplitudes. Let us first investigate its effect in the mixing phenomena. In the presence of new physics, there are additional contributions to the mixing parameters arising from the new box diagrams.
These contributions to the $\Delta B = 2$ transitions are often parametrized as \[17\]
$$
\sqrt{\frac{M_{12}^{SM}}{M_{12}^{NP}} = r_m e^{i \theta_m}}, \quad (33)
$$
where $M_{12} = M_{12}^{SM} + M_{12}^{NP}$ is the off diagonal element of the mass matrix, contains contribution both from the SM and from new physics. Hence, the ratio of the mixing parameters $q/p$ as given in Eq.(5) becomes
$$
\frac{q}{p} \sim e^{-2i(\beta + \theta_m)}. \quad (34)
$$
Thus, in the presence of new physics, the mixing induced CP asymmetry in $B_d^0 \to \psi K_S$ can be given as
$$
S_{\psi K_S} = \sin(2\beta + 2\theta_m) \quad (35)
$$
However, since the present world average on the measurement of $S_{\psi K_S} = 0.726 \pm 0.037$ [18] agrees quite well with the SM prediction $S_{\psi K_S} = 0.715_{-0.045}^{+0.055}$ [19], we do not consider the effect of NP in mixing in our analysis.

Now we consider the effect of new physics on the CP violating parameters arising from the new contribution to the standard model decay amplitude (12). In the presence of new physics the amplitude can be written as
$$
A(B_d^0 \to K^0 \bar{K}^0) = A_{SM} + A_{NP} = \lambda_u P_{ut} \left[ 1 - re^{i(\delta + \gamma)} + r_{NP} e^{i \theta_N} e^{i \delta_N} \right], \quad (36)
$$
where $r_{NP} = |A_{NP}/\lambda_u P_{ut}|$, $\theta_N$ and $\delta_N$ are the relative weak and strong phase between them. The amplitude for the corresponding CP conjugate process can be obtained by changing the sign of the weak phases.

Thus the CP asymmetry parameters (3) become
$$
S_{KK}^{NP} = -\frac{X}{Y}, \quad \text{and} \quad C_{KK}^{NP} = -\frac{Z}{Y}, \quad (37)
$$
where
\begin{align*}
X &= \sin 2\alpha + 2r \cos \delta \sin(2\beta + \gamma) - r^2 \sin 2\beta + 2r_{NP} \cos \delta_N \sin(2\alpha + \theta_N) \\
&\quad - 2rr_{NP} \cos(\delta - \delta_N) \sin(\theta_N - (2\beta + \gamma)) + r_{NP}^2 \sin(2\alpha + 2\theta_N), \\
Y &= 1 + r^2 + r_{NP}^2 - 2r \cos \delta \cos \gamma + 2r_{NP} \cos \delta_N \cos \theta_N \\
&\quad - 2rr_{NP} \cos(\delta - \delta_N) \cos(\gamma - \theta_N), \\
Z &= 2 \left[ r \sin \delta \sin \gamma - r_{NP} \sin \delta_N \sin \theta_N + rr_{NP} \sin(\delta - \delta_N) \sin(\gamma - \theta_N) \right]. \quad (38)
\end{align*}
The branching ratio in the presence of new physics is given as
\[
Br(\bar{B}_d \to K^0 \bar{K}^0) = Br^{SM} \left( 1 + \left|\frac{A_{NP}}{A_{SM}}\right|^2 + 2r_{NP} \left|\frac{A_{NP}}{A_{SM}}\right| \cos \phi_N \right),
\]
where $Br^{SM}$ denotes the SM branching ratio and $\phi_N$ is the relative phase between the new physics and standard model amplitudes.

Now, we consider two beyond the standard model scenarios: the minimal supersymmetric standard model with mass insertion approximation and R-parity violating supersymmetric model and study their effects on CP violation parameters in the following subsections.

### 3.1 Contribution from minimal supersymmetric standard model with mass insertion approximation

Here, we analyze the decay process $B^0_d \to K^0 \bar{K}^0$, in the minimal supersymmetric standard model (MSSM) with mass insertion approximation. This decay mode receives supersymmetric (SUSY) contributions mainly from penguin and box diagrams containing gluino-squark, chargino-squark and charged Higgs-top loops. Here, we consider only the gluino contributions, because the chargino and charged Higgs loops are expected to be suppressed by the small electroweak gauge couplings. However, the gluino mediated FCNC contributions are of the order of strong interaction strength, which may exceed the existing limits. Therefore, it is customary to rotate the effects, so that the FCNC effects occur in the squark propagators rather than in couplings and to parameterize them in terms of dimensionless parameters. Here we work in the usual mass insertion approximation [20, 21] where the flavor mixing $i \to j$ in the down-type squarks associated with $\tilde{q}_B$ and $\tilde{q}_A$ are parametrized by $(\delta_{AB})_{ij}$, with $A, B = L, R$ and $i, j$ as the generation indices. More explicitly $(\delta_{LL})_{ij} = (V^d_L M_d^2 V^d_L)^{ij}/m_{\tilde{q}}^2$, where $M_d^2$ is the squared down squark mass matrix and $m_{\tilde{q}}$ is the average squark mass. $V_d$ is the matrix which diagonalizes the down-type quark mass matrix.

The new effective $\Delta B = 1$ Hamiltonian relevant for the $B \to K^0 \bar{K}^0$ process arising from new penguin/box diagrams with gluino-squark in the loops is given as
\[
\mathcal{H}_{\text{eff}}^{SUSY} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \sum_{i=3}^{6} \left( C_i^{NP} O_i + \tilde{C}_i^{NP} \tilde{O}_i \right) + C_g^{NP} O_g + \tilde{C}_g^{NP} \tilde{O}_g, \tag{40}
\]
where $O_i$ ($O_g$) are the QCD (magnetic) penguin operators and $C_i^{NP}, C_g^{NP}$ are the new Wilson coefficients. The operators $\tilde{O}_i$ are obtained from $O_i$ by exchanging $L \leftrightarrow R$. 

11
To evaluate the amplitude in the MSSM, we have to first determine the Wilson coefficients at the \( b \) quark mass scale. At the leading order in mass insertion approximation the new Wilson coefficients corresponding to each of the operator at the scale \( \mu \sim \tilde{m} \sim M_W \) are given as [21, 22]

\[
\begin{align*}
C_3^{NP} &\simeq -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td} m_q^2} \left( \delta_{LL}^d \right)_{13} \left[ -\frac{1}{9} B_1(x) - \frac{5}{9} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right], \\
C_4^{NP} &\simeq -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td} m_q^2} \left( \delta_{LL}^d \right)_{13} \left[ -\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right], \\
C_5^{NP} &\simeq -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td} m_q^2} \left( \delta_{LL}^d \right)_{13} \left[ \frac{10}{9} B_1(x) + \frac{1}{18} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right], \\
C_6^{NP} &\simeq -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td} m_q^2} \left( \delta_{LL}^d \right)_{13} \left[ -\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right], \\
C_7^{NP} &\simeq -\frac{2 \sqrt{2} \alpha_s \pi}{2 G_F V_{tb} V_{td} m_q^2} \left[ \left( \delta_{LL}^d \right)_{13} \left( \frac{3}{2} M_3(x) - \frac{1}{6} M_4(x) \right) + \left( \delta_{LR}^d \right)_{13} \left( \frac{m_b}{m_q} \right) \frac{1}{6} \left( 4 B_1(x) - \frac{9}{x} B_2(x) \right) \right],
\end{align*}
\]

where \( x = m_b^2/m_q^2 \). The loop functions appear in these expressions can be found in Ref. [21]. The corresponding \( \tilde{C}_i \) are obtained from \( C_i^{NP} \) by interchanging \( L \leftrightarrow R \). It should be noted that the \( \langle \delta_{LR} \rangle_{13} \) contribution is enhanced by \( (m_b/m_q) \) compared to the contribution from the SM and the \( LL \) insertion due to the chirality flip from the internal gluino propagator in the loop.

The Wilson coefficients at low energy \( C_i^{NP}(\mu \sim m_b) \), can be obtained from \( C_i^{NP}(M_W) \) by using the Renormalization Group (RG) equation as discussed in Ref. [12], as

\[
C(\mu) = U_5(\mu, M_W) C(M_W),
\]

where \( C \) is the \( 6 \times 1 \) column vector of the Wilson coefficients and \( U_5(\mu, M_W) \) is the five-flavor \( 6 \times 6 \) evolution matrix. In the next-to-leading order (NLO), \( U_5(\mu, M_W) \) is given by

\[
U_5(\mu, M_W) = \left( 1 + \frac{\alpha_s(\mu)}{4\pi} J \right) U_5^{(0)}(\mu, M_W) \left( 1 - \frac{\alpha_s(M_W)}{4\pi} J \right),
\]

where \( U_5^{(0)}(\mu, M_W) \) is the leading order (LO) evolution matrix and \( J \) denotes the NLO corrections to the evolution. The explicit forms of \( U_5(\mu, M_W) \) and \( J \) are given in Ref. [12].

Since the \( O_\mu \) contribution to the matrix element is \( \alpha_s \) order suppressed, we consider only
leading order RG effects for the coefficient $C_g^{NP}$, which is given as [23]

$$C_g^{NP}(m_b) = \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{2/21} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{2/23} .$$

(44)

For the numerical analysis, we fix the SUSY parameter as $m_{\tilde{g}} = m_{\tilde{q}} = 500$ GeV, $\alpha_s(M_W) = 0.119$, $\alpha_s(m_t) = 4.4$ GeV, $\alpha_s(m_t) = 175$ GeV, and $\alpha_s(m_t) = 0.107$. Now substituting the values of the RG evolved Wilson coefficients $C_i^{NP}(m_b)$'s in Eq. (23) we obtain the corresponding $a_i$'s and hence with Eq. (20) the amplitude. Assuming that all the mass insertion parameters $(\delta_{AB})_{13}$ have a common weak phase, we obtain the fraction of new physics amplitude as defined in Eq. (36)

$$r_{NP} \simeq 0.33 \left( |(\delta_{LL})_{13}| - |(\delta_{RR})_{13}| \right) + 465.86 \left( |(\delta_{LR})_{13}| - |(\delta_{RL})_{13}| \right) .$$

(45)

It should be noted that because of the opposite chiral structure of the currents $O_i$ and $\tilde{O}_i$, the LL and RR and also the LR and RL contributions occur with opposite sign. As seen from Eq. (45), the LR(RL) insertions have dominant effect because of the $m_{\tilde{g}}/m_b$ enhancement. We use the limits on the $(\delta_{LL})_{13}$ and $(\delta_{LR})_{13}$ mixing parameters from [24] for $x = 1$ as

$$|\langle(\delta_{LL})_{13}\rangle| \leq 0.2 \quad |\langle(\delta_{LR})_{13}\rangle| \leq 0.01$$

(46)

and assume that only one of these gives a dominant SUSY contribution. This gives $(r_{NP})_{LL} \leq 6.6 \times 10^{-2}$ and $(r_{NP})_{LR} \leq 4.66$. Since the new physics effect due to LL insertion is almost negligible it will not provide any significant effect on the CP violating observables. The correlation plot between $C_{KK}^{NP}$ and $S_{KK}^{NP}$ for LL insertion is shown in Figure-2, where we use $r = 2.6$, $\delta = 11^\circ$ as obtained from QCD factorization analysis, the central values of the CKM weak phases from (28), the relative weak phase $\theta_N = \pi$ and vary the relative strong phase $\delta_N$ between 0 and $2\pi$. In this case, because of the negligible new physics contribution one gets only tiny CP violating effects. In Figure-3, we present the correlation plot for LR insertion, where we have used $(r_{NP})_{LR} = 4.66$, $0 \leq \delta_N \leq 2\pi$ and a representative set of weak phases $\theta_N = \pi, \pi/2, \pi/3, \pi/4$. For $r$ and $\delta$, we have used the values as obtained from QCD factorization (30). As expected, in this case large CP violation can be generated.

The branching ratio (39) versus $\phi_N$ is plotted in Figure-4 for $|A_{NP}/A_{SM}| = 2.19$. One can see from Figure-4 that the observed data can be easily accommodated in minimal supersymmetric standard model with LR mass insertion.

Thus in future, if sizable CP violation effects will be observed in $B_d^0 \rightarrow K^0\bar{K}^0$ mode, the minimal supersymmetric standard model with LR mass insertion may be a strong contender of new physics to explain the data.
Figure 2: The correlation plot between $S^{NP}_{KK}$ and $C^{NP}_{KK}$ for the $B^0_d \rightarrow K^0\bar{K}^0$ process in the MSSM with only $LL$ insertion, where we have used $(r_{NP})_{LL} = 6.6 \times 10^{-2}$, the weak phase $\theta_N = \pi$, $r = 2.6$, $\delta = 11^\circ$ and varied the strong phase $\delta_N$ between 0 and $2\pi$.

3.2 R-parity violating supersymmetric contribution

We now analyze the decay mode in the minimal supersymmetric model with R-parity violation (RPV). In the supersymmetric models there may be interactions which violate the baryon number $B$ and the lepton number $L$ generically. The simultaneous presence of both $L$ and $B$ number violating operators induce rapid proton decay, which may contradict strict experimental bound. In order to keep the proton lifetime within the experimental limit, one needs to impose additional symmetry beyond the SM gauge symmetry to force the unwanted baryon and lepton number violating interactions to vanish. In most cases this has been done by imposing an ad hoc symmetry called R-parity defined as, $R = (-1)^{(3B+L+2S)}$, where $S$ is the intrinsic spin of the particles. Thus R-parity can be used to distinguish the particles ($R = +1$) from their superpartners ($R = -1$). The conservation of R-parity implies that the supersymmetric particles must be produced in pairs and the lightest supersymmetric particle (LSP) must be stable. However, there is no compelling reason to require the conservation of R-parity. Less restrictive symmetries- conservation of baryon/lepton number alone can be
Figure 3: The correlation plot between $S^ {NP}_{KK}$ and $C^ {NP}_{KK}$ for the $B^0_d \to K^0\bar{K}^0$ process in the MSSM with LR mass insertion, where we have used $r = 2.6$, $\delta = 11^\circ$, $(r^ {NP})_{LR} = 4.66$, a set of weak phases $\theta_N = \pi, \pi/2, \pi/3, \pi/4$, and varied the strong phase $\delta_N$ between 0 and $2\pi$.

imposed to prohibit the unwanted proton decay. Extensive studies has been done to look for the direct as well as indirect evidence of R-parity violation from different processes and to put constraints on various R-parity violating couplings [25].

Here, we consider only the lepton number violating effects. The most general R-parity and lepton number violating super-potential is given as

$$W^ {\not{R}} = \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda_{ijk} L_i Q_j D^c_k,$$

where, $i, j, k$ are generation indices, $L_i$ and $Q_j$ are $SU(2)$ doublet lepton and quark superfields and $E^c_k, D^c_k$ are lepton and down type quark singlet superfields.

Thus the relevant four fermion interaction induced by the R-parity and lepton number violating model is

$$\mathcal{H}_R = -\frac{1}{2m^2_{\nu_\alpha}} \eta^{-s/\beta_0} \left[ \lambda'_{i31} \lambda'_{i22} (\bar{s}_\alpha \gamma_\mu Lb_\beta)(\bar{d}_\beta \gamma^\mu Rs_\alpha) + \lambda'_{i13} \lambda'_{i22} (\bar{s}_\alpha \gamma_\mu Rb_\beta)(\bar{d}_\beta \gamma^\mu Ls_\alpha) \\
+ \lambda'_{i32} \lambda'_{i12} (\bar{d}_\alpha \gamma_\mu Lb_\beta)(\bar{s}_\beta \gamma^\mu Rs_\alpha) + \lambda'_{i23} \lambda'_{i21} (\bar{d}_\alpha \gamma_\mu Rb_\beta)(\bar{s}_\beta \gamma^\mu Ls_\alpha) \right],$$

(48)
where \( \eta = \frac{\alpha_s(m_f)}{\alpha_s(m_b)} \) and \( \beta_0 = 11 - \frac{2}{3} n_f \). The QCD correction factor \( \eta^{-8/\beta_0} \) arises due to running from the sfermion mass scale \( m_{\tilde{f}_i} \) (100 GeV assumed) down to the \( m_b \) scale.

The amplitude for \( B^0_d \to K^0 \bar{K}^0 \) process in the RPV model is given as

\[
A_R(B^0 \to K^0 \bar{K}^0) = -\frac{1}{8m^2_{\tilde{f}_i}} \eta^{-8/\beta_0} X \left[ \frac{1}{N} \left( \lambda'_{22} \lambda'^*_{113} - \lambda'_{31} \lambda'^*_{22} \right) - \frac{2m^2_K}{(m_b - m_s)(m_d + m_s)} \left( \lambda'_{i21} \lambda'^*_{i23} - \lambda'_{i32} \lambda'^*_{i12} \right) \right],
\]

where we have kept only the leading order factorization contributions. We use the parameterization \( \lambda'_{22} \lambda'^*_{113} = -\lambda'_{31} \lambda'^*_{22} = ke^{i\theta} \) and \( \lambda'_{32} \lambda'^*_{i12} = -\lambda'_{i21} \lambda'^*_{i23} = k_1 e^{i\theta} \), assuming the same weak phase for all the RPV couplings. The limits on the couplings \( |\lambda'_{i32} \lambda'^*_{i12}| = |\lambda'_{i21} \lambda'^*_{i23}| \) are obtained from \( B^0_d \to \phi \pi \) decay in Ref. [26]

\[
k_1 = |\lambda'_{i32} \lambda'^*_{i12}| = |\lambda'_{i21} \lambda'^*_{i23}| \leq 4.0 \times 10^{-4}.
\]

In our analysis we use \( k = k_1 \leq 4.0 \times 10^{-4} \) and obtain the new physics parameter

\[
r_{NP} \leq 3.92.
\]
The correlation plot between $C_{KK}^{NP}$ and $S_{KK}^{NP}$ for the above value of $r_{NP}$ is shown in Figure-5, for some representative values of the weak phase and $0 \leq \delta^{NP} \leq 2\pi$. The values of $r$ and $\delta$ are used as derived from QCD factorization. In this case also one can get observable CP violation effects. Plotting the branching ratio (39) vs. $\phi_N$ for $|A_{NP}/A_{SM}| = 1.84$ we can see from Figure-4 that the observed branching ratio can be easily accommodated in the RPV model.

![Figure 5: The correlation plot between $S_{KK}^{NP}$ and $C_{KK}^{NP}$ in the RPV model for $r_{NP} = 3.92$, $r = 2.6$, $\delta = 11^\circ$, $\theta_N = \pi$, $\pi/2$, $\pi/3$, $\pi/4$ and $0 \leq \delta_N \leq 2\pi$](image)

4 Conclusion

The recent measurement of the mixing induced CP asymmetry in $B_d^0 \rightarrow \phi K_S$ which has significant deviation from $\sin(2\beta)_{\psi K_S}$ may provide the first indication of new physics effects present in the $b \rightarrow s$ penguin amplitudes. In this paper, we have investigated the possibility of observing new physics effects in the $b \rightarrow d$ penguin amplitudes. We have considered the decay mode $B_d^0 \rightarrow K^0\bar{K}^0$ which proceeds through the quark level FCNC transition $b \rightarrow d \bar{s}s$, receiving contributions only from one-loop $b \rightarrow d$ penguin diagrams. If one would
assume only the top quark exchange in the penguin loop as usually done, the CP asymmetry parameters would vanish in the SM. However, contributions from penguins with internal up and charm quark exchanges are expected to yield small non-vanishing CP asymmetries. Thus, if significant CP asymmetries will be found in this channel then it would be a clear indication of new physics effects in $b \to d$ penguin amplitudes. However, as discussed in Ref. [27], the nonfactorizable long-distance charm penguins may also give significant contributions which in turn yield sizable CP asymmetries. In that case it is practically impossible to disentangle the new physics effects from the nonfactorizable charm and GIM penguins without any additional assumptions. However, very recently, it has been pointed out by Beneke et al [28] that the nonfactorizable charm penguin contributions are of higher order in $1/m_b$ expansion. Thus the observation of sizable CP asymmetry in this mode may be considered as the signal of new physics.

Using QCD factorization approach, we found the CP averaged branching ratios in the SM for $B_d^0 \to K^0\bar{K}^0$ process as $\sim 0.9 \cdot 10^{-6}$, which is slightly below the present experimental value. The CP asymmetry parameters are found to be $S_{KK} = 0.06$ and $C_{KK} = -0.16$. Allowing the CKM parameters to vary within their 1$\sigma$ limits, we obtained the allowed ranges as $0.02 \leq S_{KK} \leq 0.13$ and $-0.17 \leq C_{KK} \leq -0.15$. If the observed values would deviate significantly from the above ranges would be a clear signal of new physics. We next analyzed the decay mode in the MSSM with mass insertion approximation and found that the LR insertion has significant effects than the LL or RR insertions. In this case one can have significant CP violating asymmetries. Considering the R-parity violating supersymmetric model we found that one can also obtain significant CP violation with the present available RPV couplings. Therefore, the future experimental data on $B_d^0 \to K^0\bar{K}^0$ CP violating parameters will serve as a very good hunting ground for the existence of new physics beyond the SM and also support/rule out some of the existing new physics models.

Acknowledgments

R.M. would like to thank the HEP theory group at the Technion for the kind hospitality and AKG would like to thank Lady Davis Foundation for financial support. The work of RM was partly supported by Department of Science and Technology, Government of India, through Grant No. SR/FTP/PS-50/2001.
References

[1] T. Browder, Talk presented at the Lepton-Photon, 2003, Int. J. Mod. Phys. A 19, 965 (2004).

[2] Y. Grossman and M. P. Worah, Phys. Lett. B 395, 241 (1997); D. London and A. Soni, Phys. Lett. B 407, 61 (1997); Y. Grossman, G. Isidori and M. Worah, Phys. Rev. D 58, 057504 (1997).

[3] B Aubert et al., (BABAR Collaborations), hep-ex/0408072.

[4] Y. Sakai, (Belle Collaborations), Talk presented at International Conference on High Energy Physics, 2004.

[5] S. Eidelman et al., Review of Particle Physics, Particle Data Group, Phys. Lett. B592, 1 (2004).

[6] K. Abe et al., [Belle Collaboration], Phys. Rev. Lett. 93, 021601 (2004); Phys. Rev. D68, 012001 (2003); B. Aubert et al., [BABAR Collaboration], Phys. Rev. Lett. 89, 281802 (2002); hep-ex/0408089.

[7] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B606, 245 (2001); M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003).

[8] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. 92, 101804 (2004); hep-ph/0402112.

[9] R. Fleischer, Phys. Lett. B 341, 205 (1994).

[10] B Aubert et al., (BABAR Collaborations), hep-ex/0408080.

[11] R. Fleischer and S. Recksiegel, hep-ph/0408016.

[12] G. Buchalla, A.J. Buras, M. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[13] M. Wirbel, M. Bauer, B. Stech, Z. Phys. C 29, 637 (1985); ibid 34, 103 (1987).

[14] T. Muta, A. Sugamoto, M. Z. Yang and Y. D. Yang, Phys. Rev. D62, 094020 (2000).

[15] D. Du, H. Gong, J. Sun, D. Yang and G. Zhu, Phys. Rev. D65, 074001 (2002).
[16] J. Charles et al., hep-ph/0406184.

[17] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B 407, 307 (1997).

[18] Z. Ligeti, Talk presented at International Conference on High Energy Physics, 2004, hep-ph/0408267.

[19] A. J. Buras, hep-ph/0210291; A. J. Buras, F. Parodi and A. Stocchi, JHEP 0301, 029 (2003).

[20] L. J. Hall, V. L. Costelecky and S. Raby, Nucl. Phys. B 267, 415 (1986).

[21] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silverstini, Nucl. Phys. B 477, 321 (1996).

[22] G. L. Kane et al., Phys. Rev. D 70, 035015 (2004).

[23] X. G. He, J. Y. Leou and J. Q. Shi, Phys. Rev. D 64, 094018 (2001).

[24] P. Ko, J.-H. Park and G. Kramer, Euro. Phys. Jour. C 25, 615 (2002).

[25] For reviews on $R$-parity violation see H. Dreiner, An Introduction to Explicit $R$-Parity Violation in Perspectives on Supersymmetry, p.462-479, Ed. G.L. Kane (World Scientific); G. Bhattacharyya, Nucl. Phys. Proc. Suppl. 52 A, 83 (1997); R. Barbier, et al., hep-ph/0406039.

[26] S. Bar-Shalom, G. Eilam and Y. D. Yang, Phys. Rev. D 67, 014007 (2003).

[27] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silverstini, Phys. Lett. B 515, 33 (2001); C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70, 054015 (2004).

[28] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, hep-ph/0411171.