Squeezed States in the de Sitter Vacuum

Martin B. Einhorn* and Finn Larsen†

Michigan Center for Theoretical Physics, Randall Laboratory of Physics

The University of Michigan, Ann Arbor, MI 48109-1120

Abstract

We discuss the treatment of squeezed states as excitations in the Euclidean vacuum of de Sitter space. A comparison with the treatment of these states as candidate no-particle states, or alpha-vacua, shows important differences already in the free theory. At the interacting level alpha-vacua are inconsistent, but squeezed state excitations seem perfectly acceptable. Indeed, matrix elements can be renormalized in the excited states using precisely the standard local counterterms of the Euclidean vacuum. Implications for inflationary scenarios in cosmology are discussed.

*meinhorn@umich.edu
†larsenf@umich.edu
1 Introduction

Some years ago it was understood that there exists a two parameter set of de Sitter invariant states that may be regarded as candidate no-particle states, or vacua, for a scalar field in de Sitter space [1, 2, 3]. These so-called alpha-vacua are alternatives to the Euclidean or Bunch-Davies vacuum [4] usually considered appropriate for cosmological inflation. Recently there has been much work on the interpretation of these alpha-vacua as ambiguities in the low energy theory parametrizing physics beyond the Planck scale [5, 6]. Alpha-vacua are also important in considerations regarding de Sitter holography [7, 8].

In a previous paper [9] (hereinafter referred to as I), we argued that the choice of de Sitter invariant vacuum is in fact ambiguous only for free field theory, since the Feynman rules for interacting fields lead to ill-defined loop diagrams for all but the Euclidean vacuum. Similarly objections have been raised to the nonlocal counterterms that alpha-vacua would require [10]. Against these concerns stands the fact that alpha-vacua clearly resemble squeezed states in quantum optics [11, 12], so the question naturally arises as to whether one could not regard such states as excited states in the Euclidean vacuum. Just as in quantum optics, it must be that the problems encountered in I can be avoided. Moreover, from this point of view one does not expect non-local counterterms, since transition amplitudes involving excited states should be rendered finite by the same local counterterms that renormalize vacuum amplitudes.

The purpose of this paper is to resolve the obvious tension between these various results and expectations. The first step towards this goal will be to discuss some key differences, present already in the free theory, between the treatment of squeezed state excitations of the Euclidean vacuum and the interpretation of these states as vacua in their own right. For example, we will argue that the time-ordered two point correlators are in fact different in these two situations. Another (related) difference is that the notorious antipodal singularities of the two point correlators are associated with sources, when the state is treated as an excitation, but not when it is interpreted as a non-standard vacuum. These features of the free theory lead to the suspicion that squeezed states might be perfectly viable as excited states in the Euclidean vacuum, even if they are unacceptable as vacua.¹

The true test of these ideas is the full interacting theory. In section 4 we explain how the Feynman rules can be formulated for squeezed states, treated as excited states in the Euclidean vacuum. According to these computational rules, the divergences will be those of the Euclidean vacuum, removable by the standard local counterterms, as expected. Our treatment could easily be extended to other excited states, such as to coherent states. It is interesting that, for consistency reasons alone, one can rule out the interpretation of the alpha-vacua as no-particle states, yet admit such states as excited states of the unique Euclidean vacuum. Our results illustrate some of the powerful constraints resulting from going beyond free field theory to consider interacting fields.

The outline of the paper is as follows: in the next section, we give a short review of squeezed states and alpha-vacua. In section 3 we consider the free correlators in the two

¹While this work was underway, a paper appeared [13] which makes the same distinction as we, and also reaches the conclusion that excited states are viable. Unfortunately, we disagree on both the Feynman rules and the renormalization counterterms. In fact, the prescription for loop diagrams given in [13] seems to be precisely the one criticized in I as being mathematically nonsensical.
situations and, in section 4, we proceed to the interacting theory and discuss Feynman rules and renormalization. Finally, in section 5, we conclude by discussing implications our result for the Hadamard condition of quantum field theory in curved space, and for cosmology.

2 On Vacua and Squeezed States

Let us begin by recalling the notion of a vacuum in curved space. We adopt the notation of I, mostly in common with 3. The quantum field for a free scalar is written in terms of a mode expansion

$$\phi(x) = \sum_n [a_n u_n(x) + a_n^\dagger u_n^*(x)] ,$$

and the corresponding specification of the “no-particle” or “vacuum” state

$$a_n |\text{vac}\rangle = 0 .$$

As usual, the subscript $n$ denotes all quantum numbers on which the mode depends; Fock states are built up by applications of the $a_n^\dagger$, and the Hilbert space is defined by their completion. The standard choice of the functions $\{u_n\}$ is variously known as the Euclidean vacuum or the Bunch-Davies vacuum. Numerous arguments in favor of the standard choice are given in the literature, e.g. 14, 15. Since we already reviewed these arguments in I we shall not do so again, apart from noting that the Euclidean vacuum also is the vacuum singled out by the considerations of I that interactions be introduced consistently.

Alternate definitions of vacua are associated with different choices for the mode functions. Since each choice is a complete set of modes, new choices may be expressed in terms of the Euclidean modes $\{u_n\}$ as a Bogoliubov transformation which, for our purposes, can be taken of the form

$$\tilde{u}_n(x) = u_n(x) \cosh \alpha_n + u_n^*(x)e^{i\beta_n} \sinh \alpha_n .$$

The corresponding field may be expanded as

$$\phi(x) = \sum_n [\tilde{a}_n \tilde{u}_n(x) + \tilde{a}_n^\dagger \tilde{u}_n^*(x)] ,$$

where the associated Bogoliubov transform of the operators is

$$\tilde{a}_n = a_n \cosh \alpha_n - a_n^\dagger e^{-i\beta_n} \sinh \alpha_n ,$$

with corresponding state

$$\tilde{a}_n |\alpha, \beta\rangle = 0 ,$$

for all $n$.

In order to interpret the candidate vacuum states as excited states note that the Bogoliubov transformation eq. 5 may be implemented by a unitary transformation 11, 12

$$S(\xi) a_n S(\xi)^\dagger = \tilde{a}_n ,$$

where $\xi_n \equiv e^{-i\beta_n} \alpha_n$ and

$$S(\xi) \equiv \exp \sum_n \frac{1}{2} [\xi_n a_n^{12} - \xi_n^* a_n^{21}] ,$$
The operator $S$ can be rewritten as

$$S(\xi) = \exp \sum_n \left[ \frac{1}{2} a_n^{\dagger} e^{-i\beta_n} \tanh \alpha_n \right] \exp \sum_n \left[ -\frac{1}{2} \left( a_n a_n^{\dagger} + a_n^{\dagger} a_n \right) \ln(\cosh \alpha_n) \right]$$

$$\times \exp \sum_n \left[ -\frac{1}{2} a_n^{\dagger} e^{i\beta_n} \tanh \alpha_n \right] , \quad (9)$$

so that the state eq. (6) may be represented in terms of the original quanta as

$$\left| \alpha, \beta \right> = S(\xi) \left| 0 \right> = \exp \sum_n \left[ -\frac{1}{2} \ln(\cosh \alpha_n) \right] \exp \sum_n \left[ \frac{1}{2} \tanh \alpha_n e^{-i\beta_n} a_n^{\dagger} \right] \left| 0 \right> \quad (10)$$

This formula is of central importance, since it shows that the vacuum defined by $\xi$ can in general be represented as a state in the Euclidean theory.

One of the nice properties of the Euclidean modes $\{u_n\}$ is that they respect the de Sitter invariance of the background. The alternate vacua $\{\tilde{u}_n\}$ break this symmetry except when $\alpha_n \equiv \alpha$ and $\beta_n \equiv \beta$ are independent of mode-number $n$. The family of de Sitter invariant vacua parametrized by $\alpha, \beta$ are those found by Mottola \cite{2} and Allen \cite{3}. In most of our considerations we will keep the general $n$-dependence and refer to alpha-vacua, when we interpret these states as no-particle states of the system, and alpha-states (or sometimes squeezed states) when we treat them as excited states in the Euclidean vacuum. The de Sitter invariant alpha-vacua, independent of $n$, are the MA-vacua.

The discussion so far tacitly assumes that the mode numbers are discrete, which is true for certain coordinate systems. However, for the planar coordinates typical of FRW models, the notation must be refined in order to deal with continuous indices. One must replace the discrete index $n$ by the wave number $\vec{k}$, introduce wave packets in momentum space, and write the Bogoliubov transformation eq. (5) in the non-local form

$$\tilde{a}_{\vec{k}} = a_{\vec{k}} \cosh \alpha_{\vec{k}} - a_{-\vec{k}}^{\dagger} e^{-i\beta_{\vec{k}}} \sinh \alpha_{\vec{k}} . \quad (11)$$

As is familiar from Minkowski space, the treatment becomes more cumbersome in order to deal with the mathematics of distributions rather than functions. To keep formulae as simple as possible, we will retain the discrete notation with the understanding that it can be adapted to the continuous case as necessary.

It can be shown that a non-zero phase $\beta_n$ is associated with CPT violation \cite{3} so, for most applications, it is presumed that $\beta_n = 0$. To simplify the formulae here, we will also make that assumption, although it would not be difficult to extend our treatment to the general case. Accordingly, $\xi_n = \alpha_n$, and we will abbreviate the state $|\alpha, 0\rangle$ as $|\alpha\rangle$.

The overlap between the alpha-state (10) and the Euclidean vacuum is

$$\langle 0 | \alpha \rangle = \prod_n \frac{1}{\sqrt{\cosh \alpha_n}} . \quad (12)$$

The $\alpha_n = \alpha$ are $n$-independent for the MA-vacua; so the overlap vanishes, as does the overlap with any Fock state of the Euclidean theory. Formally this shows that these states $|\alpha\rangle$ are orthogonal to the Euclidean Hilbert space. If taken at face value, this means the $|\alpha\rangle$ cannot be described as excited states, but rather must be treated as vacua. Then there would be an
orthogonal Hilbert space for each $\alpha$, with the no-particle state annihilated by the appropriate $\tilde{a}_n$ and the Fock states built by application of the corresponding $\tilde{a}_n^\dagger$.

However, in our view, this formal argument fails to represent a sensible approximation to the physics. According to the modern view of renormalization [16] every local field theory should be regarded as an effective field theory below some high energy scale, the cut-off. As a result, the alpha-states, whether dependent on $n$ or not, should be treated as having finite overlap with states in the Euclidean theory and hence can be regarded as excited states. In fact, because of difficulties with defining the Feynman rules for MA-vacua, these do not really represent alternatives to the Hilbert space based on the Euclidean vacuum.

3 Correlators in Free Field Theory

We now proceed to explain how the treatment of excited states differ from that of vacua, by discussing the correlators of the free theory. Using eq. (10), Wightman functions for correlators in the states $|\alpha\rangle$ may be represented in terms of operators in the Euclidean vacuum as

$$W_\alpha(x, y) = \langle \alpha | \phi(x)\phi(y) | \alpha \rangle = \langle 0 | \tilde{\phi}(x)\tilde{\phi}(y) | 0 \rangle,$$

where $\tilde{\phi}(x) \equiv S^\dagger(\alpha)\phi(x)S(\alpha).$ (13)

To evaluate this further we use eq. (7) to write

$$\tilde{\phi}(x) = \sum_n \cosh \alpha_n \left[ a_n u_n(x) + a_n^\dagger u_n^*(x) \right] + \sum_n \sinh \alpha_n \left[ a_n u_n(\overline{x}) + a_n^\dagger u_n^*(\overline{x}) \right],$$

$$\equiv A_\alpha(x) + B_\alpha(\overline{x}).$$ (14)

where we have adopted the choice of basis introduced by Allen [3] for which $u_n^*(x) = u_n(\overline{x}),$ where $\overline{x}$ represents the antipode of $x$.

In the MA-vacua, for which $\alpha_n = \alpha$ is independent of $n$, eq. (14) can be written formally as

$$\tilde{\phi}(x) = \phi(x) \cosh \alpha + \phi(\overline{x}) \sinh \alpha,$$ (15)

and so the Wightman function eq. (13) becomes

$$W_\alpha(x, y) = W_0(x, y) \cosh^2 \alpha + W_0(\overline{x}, \overline{y}) \sinh^2 \alpha + \frac{1}{2} [W_0(x, \overline{y}) + W_0(\overline{x}, y)] \sinh 2\alpha,$$

$$= W_0(x, y) \cosh^2 \alpha + W_0^*(x, y) \sinh^2 \alpha + \frac{1}{2} [W_0(x, \overline{y}) + W_0^*(x, \overline{y})] \sinh 2\alpha,$$ (17)

where, in the second line, we used $W_0(x, y) = W_0^*(y, x)$ and $W_0(\overline{x}, \overline{y}) = W_0(y, x)$ in the Allen basis. This equation is central for our interpretation of alpha-vacua: if we treat $|\alpha\rangle$ as a true vacuum, the correlator $W_\alpha(x, y)$ is simply the amplitude for creation of a particle at $y$ and annihilation at $x$. In contrast, if we treat the same formula as a statement in the Euclidean vacuum we see that the amplitude has components not only involving creation of a particle at $y$ followed by its annihilation at $x$, but also involving creation of a particle at $\overline{y}$ followed by its annihilation at either $\overline{x}$ or $x$. The apparent non-local and acausal creation and annihilation of particles is perhaps unfamiliar; however, it is not paradoxical in view of the fact that we are postulating a highly correlated background state.
The significance of this interpretation becomes clear when we consider time-ordered correlators. Allen [3] treats the $|\alpha\rangle$ as true vacua and defines

$$iG^F_{\alpha}(x, y) \equiv \langle \alpha | \mathcal{T} (\phi(x)\phi(y)) \rangle |\alpha\rangle = \Theta(x, y)W_{\alpha}(x, y) + \Theta(y, x)W_{\alpha}(y, x),$$

(18)

where the time-ordering symbol is $\Theta(x, y) \equiv (1 + \text{Sgn}(x, y))/2$, with $\text{Sgn}(x, y) \equiv 0$ if $x$ and $y$ are spacelike separated, while for timelike or lightlike separations, $\text{Sgn}(x, y) \equiv +1$ if $x > y$, or $\equiv -1$ if $x < y$. This expression can be written

$$G^F_{\alpha}(Z) = \cosh^2 \alpha G^F_0(Z) + \sinh^2 \alpha(G^F_0(Z))^* + \frac{1}{2} \sinh 2\alpha \left( G^F_0(-Z) + \text{c.c.} \right).$$

(19)

When we treat $|\alpha\rangle$ as an excited state it is more natural to introduce time-ordering according to the definition

$$\mathcal{T} \langle \alpha | \phi(x_1)\phi(x_2)\ldots\phi(x_n) \rangle |\alpha\rangle \equiv \mathcal{T} \langle 0 | \tilde{\phi}(x_1)\tilde{\phi}(x_2)\ldots\tilde{\phi}(x_n) |0\rangle.$$ (20)

The meaning of the right hand side of this expression is that the fields $\tilde{\phi}(x)$ should be expressed first as the linear combinations eq. (15), and then time-ordering is carried out with respect to the arguments of the fields $A(x)$ and $B(x)$. In the case of the MA-states, this reduces to linear combinations of the field $\phi$ itself, eq. (16).

That eq. (20) is the correct definition of time-ordering when we treat $|\alpha\rangle$ as an excited state follows directly from the physical interpretation of eqs. (15) and (16). In the context of the Fock space of the Euclidean theory, the first sum $A_{\alpha}(x)$ in eq. (15) involves the creation and annihilation of particles at the point $x$, while the second sum $B_{\alpha}(\overline{x})$ must be interpreted as the creation and annihilation of particles at the antipodal point $\overline{x}$.

Alternatively we can derive eq. (20) from the Feynman path integral (FPI) representation for the generating functional

$$\exp\{iW[J]\} = \int \mathcal{D}\phi \exp \left[ iS[\phi] + i \int dx \sqrt{g} J(x)\phi(x) \right].$$

(21)

For the Euclidean vacuum, this formal expression may be properly defined by Wick rotation from Euclidean signature, just as is normally done in Minkowski background. Accordingly, it is the generating functional for time-ordered Green’s functions in the unique Euclidean vacuum. For the alpha-states, it is clear that the corresponding generating functional should be taken as

$$\exp\{i\tilde{W}_{\alpha}[J]\} = \int \mathcal{D}\phi \exp \left[ iS[\phi] + i \int dx \sqrt{g} J(x)\phi(x) \right].$$

(22)

Since the products of fields are automatically time-ordered with respect to their arguments by the FPI, the Green’s functions so generated will correspond to the prescription given above for the right-hand side of eq. (20).

Let us summarize. We have introduced two types of time-ordering: eq. (18) for the vacuum interpretation and eq. (20) for the excited states. The crucial point simply is that these, quite manifestly, are different

$$\langle \alpha | \mathcal{T} (\phi(x)\phi(y)) |\alpha\rangle \neq \mathcal{T} \langle \alpha | \phi(x)\phi(y) \rangle |\alpha\rangle.$$ (23)
To understand what the difference in time-ordering procedure means, recall that the Feynman propagator of the MA-vacua, satisfies the same equation as the Euclidean propagator, viz.,

\[
(\nabla_x^2 + m^2)G^F_\alpha(x, y) = -\delta(x, y), \text{ where } \delta(x, y) \equiv \frac{\delta(x - y)}{\sqrt{g(x)}}.
\]

(24)

Thus, as emphasized by Allen [3], the difference between the MA-propagator and the Euclidean propagator satisfies the homogeneous Klein-Gordon equation, even though the MA-propagator is singular both for \(x = y\) and \(x = \overline{y}\). It is this peculiar singularity structure that leads to difficulties for interacting fields in the MA-vacua [9].

In contrast, the two-point functions in the squeezed states is given by the right-hand-side of eq. (23) and may be denoted \(\tilde{G}^F_\alpha\). This expression corresponds to linear combinations of time-ordered products in the Euclidean vacuum in exactly the same way as the ordinary products in eq. (17). We therefore find

\[
\tilde{G}^F_\alpha(x, y) = G^F_0(x, y) \cosh^2 \alpha + \frac{1}{2} \left[ G^F_0(x, \overline{y}) + G^F_0(x, y) \right] \sinh 2\alpha.
\]

(25)

Using \(G^F_\alpha(x, y) = G^F_0(x, \overline{y})\), the four terms may combined to two,

\[
\tilde{G}^F_\alpha(x, y) = G^F_0(x, y) \cosh 2\alpha + G^F_0(x, \overline{y}) \sinh 2\alpha.
\]

(26)

This implies that, in contrast to eq. (24),

\[
(\nabla_x^2 + m^2)\tilde{G}^F_\alpha(x, y) = -\delta(x, y) \cosh 2\alpha - \delta(x, \overline{y}) \sinh 2\alpha,
\]

(27)

so that there really is particle creation and annihilation at the antipodal point in squeezed states. A source associated with the antipodal singularity is one of the ingredients needed if one wants to treat this singularity as an image as, for example, in [17].

Let us write our time ordered correlators in the conformally massless case where formulae can be made explicit. The propagator for the vacuum case

\[
iG^F(Z) = \frac{1}{8\pi^2} \left[ \frac{\cosh^2 \alpha}{Z - 1 - i\epsilon} + \frac{\sinh^2 \alpha}{Z - 1 + i\epsilon} - \frac{\sinh 2\alpha}{Z + 1} \right],
\]

(28)

where, in terms of the embedding coordinates \(X(x)\) and \(Y(y)\), \(Z = -X \cdot Y\). Also recall that, in the embedding coordinates, the antipode of \(X\) is simply \(-X\), accounting for the appearance of \(-Z\). In contrast, the time-ordered correlator for the excited state is

\[
iG^F(Z) = \frac{1}{8\pi^2} \left[ \frac{\cosh 2\alpha}{Z - 1 - i\epsilon} - \frac{\sinh 2\alpha}{Z + 1 + i\epsilon} \right].
\]

(29)

The singularity structure of eq. (28) and eq. (29) is completely different: for vacua the \(i\epsilon\) prescriptions are mixed, and the anti-podal singularity is simply the principal value; for excited states a uniform \(i\epsilon\) prescription is applied.

We should emphasize that we are not suggesting that one simply replace the MA-propagator in the \(\alpha\)-vacua with \(\tilde{G}^F_\alpha\), eq. (25). We only wish to indicate that time-ordered
field correlators in the presence of background squeezed states are different from the time-ordering of field operators in the $\alpha$-vacua. This opens the possibility that the physics of the two situations are different when one goes beyond free field theory.

Let us emphasize this point by considering other Green’s functions. The MA-propagator eq. (18) can be written

$$iG^{F}_\alpha(x, y) = \frac{1}{2} \left[ G^{(1)}_\alpha(x, y) + i \text{Sgn}(x, y) D_\alpha(x, y) \right],$$

where the symmetric term $G^{(1)}_\alpha$ is called the Hadamard function; and $D_\alpha$, the commutator function. The various two-point functions are given in terms of their Euclidean counter-parts by

$$G^{(1)}_\alpha(x, y) = \cosh 2\alpha G^{(1)}_0(Z) + \sinh 2\alpha \left[ G^{(1)}_0(-Z) \right],$$

$$D_\alpha(x, y) = D_0(x, y).$$

If we decompose $\tilde{G}^F_\alpha$ similarly, we find that the Hadamard function is the same as for Allen’s propagator, $\tilde{G}^{(1)}_\alpha = G^{(1)}_\alpha$, but the antisymmetric part is different and given by

$$i\text{Sgn}(x, y)D_0(x, y) \cosh 2\alpha + i\text{Sgn}(x, \overline{y})D_0(x, \overline{y}) \sinh 2\alpha,$$

as expected from eq. (27). Since the imaginary part of the two-point function reflects the production of particles “on-mass-shell”, i.e. on classical geodesics, we would expect it to reflect particle creation at both points as is evident in eq. (33). This illustrates an important way in which the analytic structure of correlation functions differs in MA-vacua and in the corresponding alpha states.

For simplicity we have written our expressions in this section for the case of mode-independent $\alpha_n$. This could certainly be relaxed, but the equations would become more cumbersome. It is clear however, that the basic conclusions are independent of this idealization. For example, in the spirit of effective field theory, we could consider constant $\alpha_n$ below some large cutoff, and vanishing $\alpha_n$ above the cut-off. Then the various sources would be smeared; but the conclusion would remain that there are sources for excited states also at the antipodal points, albeit smeared ones.

4 The Interacting Theory

We now turn out attention to the interacting theory. As in I, the Feynman rules for perturbation theory may be obtained from

$$\exp\{iW[J]\} = \exp\{i \int dx \sqrt{g} \mathcal{L}_I(-i \frac{\delta}{\delta J})\} \exp\{iW_I[J]\},$$

where $W[J]$ is the generating functional of connected Green’s functions, and $\mathcal{L}_I(\phi)$ is the interaction Lagrangian density (assumed in this formula to be nonderivative). $W_I[J]$ is the
free field generating functional given by\(^2\)

\[
W_f[J] = \frac{1}{2} \int dx \sqrt{g(x)} dy \sqrt{g(y)} J(x) G^F(x, y) J(y). \tag{35}
\]

Although we shall assume without proof that the interacting theory in the Euclidean vacuum is well-defined, at least perturbatively, some comments may be in order. The reasons for our confidence in this vacuum are essentially the same as in Minkowski space. With the use of the Euclidean Feynman propagator in eq. (35), correlation functions may be defined by Wick rotation from Euclidean signature, and, correspondingly, the Feynman rules yield amplitudes whose integrands are singularity-free for Euclidean signature. As a result, the usual apparatus of perturbation theory goes through. The derivation of eq. (35) is especially straightforward in the path integral formalism, but it can also be performed in the operator formalism. One may pass from the Heisenberg picture to the interaction picture and develop the analogues of the Gell-Mann-Low formula and the Dyson expansion for vacuum expectation values (VEVs) of Green’s functions. As a result, the counterterms needed to renormalize the field theory in the Euclidean vacuum are local. Further, the Källen-Lehmann spectral representation of the two-point functions [18] may then be extended from the free to the interacting theory,

\[
G(x, y) = \int d\sigma \rho(\sigma) G(x, y; \sigma), \tag{36}
\]

where \(G(x, y; \sigma)\) is the free field two-point function for a particle of mass-squared \(\sigma\).

In [18] it was argued that the analytic structure of the propagator eq. (18) renders an interacting field theory in an \(\alpha\)-vacuum ill-defined. The thesis of the present work is to argue that, in contrast, interactions can be included if the states \(|\alpha\rangle\) are regarded as excited states in the Euclidean vacuum, at least approximately for modes below some cutoff. Then one expects matrix elements of fields between excited states to be well-defined and calculable using the Feynman rules of the Euclidean theory. Moreover, in any sensible formulation, they should be renormalizable using the same counterterms as for VEVs. The fulfillment of these expectations is complicated by the fact that the definition of the excited states corresponding to the free field \(\alpha\)-states is necessarily more complicated. The correct choice will be dictated by the particular physical situation under consideration. We shall consider several possibilities.

The first possibility is to use eq. (22) as the definition for correlators in \(\alpha\)-states for an interacting theory just as for a free theory. The computational prescription is thus to compute interacting correlators in the Euclidean theory, and then form the alpha-state correlators by taking the linear combinations indicated in, e.g., eq. (25), and similarly for higher point functions. It is manifest from this prescription that the local counterterms of the Euclidean vacuum will suffice for renormalization. This definition of alpha-states and the corresponding computational rules are correct when the interaction is adiabatically switched

\(^2\)Although we write the integral in terms of coordinates, we mean the coordinate-independent integration over the entire de Sitter manifold. If global coordinates are chosen, this is manifest. Otherwise, the integral must be defined by integration over various coordinate patches. The existence of horizons for certain coordinate systems complicates the discussion, but they do not present any problems of principle. One need only replace them by other coordinates in the neighborhood of such horizons; de Sitter space is everywhere nonsingular.
on and off in the distant past and future. An important example is the conjectured dS/CFT correspondence, in which a kind of meta-S-matrix \[19\] is formally introduced, with in- and out-states defined by reference to global coordinates \[7, 8\]. This singles out particular \(\alpha\)-states as non-interacting asymptotic states on \(\mathcal{I}^+\) and \(\mathcal{I}^-\). For an interacting field theory, one would have to use the definition of alpha-states discussed here, in order to have a well-defined field theory; so the in- and out states of dS/CFT correspondence should not really be thought of as vacua but as highly-correlated excited states of the Euclidean theory.\footnote{For even dimensions, one must restore the non-CPT-invariant phase factor to the preceding formulae.} A major drawback of defining alpha-states by adiabatically switching off interactions is that this procedure makes the concept frame dependent. For example, in planar coordinates common to cosmological applications, the distant past of a particular observer is the light-cone of an apparent horizon.

A second possibility for defining the alpha-states is to apply free field definitions such as eqs. \[6\] and \[10\] directly in the interacting theory. This is similar to the definition used, for example, by Danielsson \[5\], although interactions were incidental to that work. The problem with this procedure is that the creation and annihilation operators are time-dependent. At best, then, one might employ these equations at some fixed time or, more generally, on some Cauchy surface. One may canonically quantize the theory on such a spacelike section and then interpret the system as being in such a state at that time. In that case, the transformation between \(\phi\) and \(\tilde{\phi}\), eqs. \[13\] and \[14\], must be interpreted at that time, and one must solve for the behavior of correlators at other times. Although well-defined in principle, it generally seems intractable to carry out this procedure in practice. However, for two-point functions, analyticity and de Sitter invariance are sufficient to go from correlation functions at equal times to two arbitrary spacetime points, using the Källen-Lehman representation eq. \[36\]. Stated otherwise, knowing the two-point function for all points on a spacelike surface determines it for all times. Thus, for the two-point correlators the prescription again becomes taking linear combinations, as in the free theory eqs. \[17\] and \[26\]. For applications such as the density fluctuations in the cosmic microwave background, it is in fact the two-point functions that are of primary interest so this prescription could perhaps be used to justify the sorts of calculations in refs. \[5, 6\]. However, it is important to note that it is not just the short-distance modifications that distinguish our interpretation from some of those, but rather the long-distance, on-shell structure of the states. In our framework, these states have nothing to do with “trans-Planckian” physics.

There may be other definitions of alpha-states at the interacting level, appropriate in other applications. For example, in planar coordinates, commonly used in discussion of inflation, it is common to speak of in-states defined along the null-surface at conformal time in the distant past, and one may define \(\alpha\)-in states there. For measurements involving Unruh detectors, which refer to in-in correlation functions rather than in-out correlators, these would be the relevant states to consider. Presumably one may develop a formalism for evaluating such correlators similar to the real-time formalism in finite-temperature perturbation theory \[20\].

Whichever definition is used for alpha-states, the calculation of Green’s functions for excited states involves only the Feynman rules and the counterterms of the Euclidean field
theory. No non-local counterterms are required. This would be manifest in the determination of the spectral density in the Källen-Lehmann representation for the renormalized field.

5 Applications and Discussion

Our discussion has interesting implications for the Hadamard condition, a test often imposed to determine an acceptable vacuum in a curved space setting. The Hadamard theorem requires, among other things, that the leading short distance singularity in the Hadamard function $G^{(1)}$ should take its flat space value. As noted in Section 3, the leading singularity of the Hadamard function is $\cosh 2\alpha$ times its flat space value, for the alpha-vacua as well as the alpha-states. However, there are several reasons why this is not adequate to reject alpha-vacua out of hand. We have already pointed out that in order to ensure that their overlap with the Euclidean Fock space, MA-states must be cut off at some high scale, above which it might asymptote to the Euclidean vacuum. This makes the discussion of the singularity structure at short distances subtle, to say the least. Such cut-off states will formally satisfy the Hadamard condition. The problem is that, in the limit that the cutoff is removed, they do not. To our mind, that does not mean that there is no sensible low-energy physics associated with such states. Indeed, the main theme of this paper has been that alpha-vacua are acceptable if interpreted as excited states, but not if they are treated as vacua. When comparing the Hadamard function for the MA-vacua with that of the excited state (see eq. (31) et seq.), we found they were the same. So the Hadamard function does not discriminate between the two situations.

This conundrum is further aggravated by interactions, since then the quantum field suffers wave function renormalization with the consequence that the bare field satisfies canonical commutation relations (CCR) but must be cutoff, and the renormalized field does not satisfy CCR but has cutoff-independent correlation functions. In our view the ingredient needed to improve this situation is the analytic properties of correlation functions, a crucial tool in Minkowski space that has been insufficiently exploited heretofore in curved spacetime. We are encouraged in this program that one can distinguish on theoretical grounds between the treatment of the alpha-vacua as no-particle states and their interpretation as excited states of the unique Euclidean vacuum.

There is no S-matrix in de Sitter space, and we have not addressed the important issue of what are observables or how to relate the $n$-point functions to them. This question is not peculiar to considering excited states and is not the focus of this paper. We assume that whatever they are, it is sufficient to know how to calculate the $n$-point functions for VEVs. At the very least, one must entertain successive measurements by idealized Unruh detectors, as was assumed in I. This suggests that one should consider in-in matrix elements of fields, e.g., as with the two-point response function. The rules for relating in-in matrix elements to Wightman functions are more complicated than for S-matrix elements, but presumably can be extended from vacuum amplitudes to alpha-states also using the methods given in this paper.

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4 As noted previously, we disagree with [13] in this respect. Compare also ref. [10].
As discussed in Section 3, in an alpha-state the response to a source at the point $x$ is particle production both locally at the point $x$ and nonlocally at the antipodal point $\pi$. The latter sounds highly acausal and impermissible in a sensible theory. However, the situation is very much analogous to that seen in gedanken experiments of the EPR type [22]. It is indeed possible to have nonlocal, seemingly acausal effects, in the presence of highly correlated states. Of course it is contrary to normal experience to entertain such highly correlated states, because interactions almost certainly wash out such correlations. In cosmology, however, gravitational effects are out of equilibrium, and the inflaton field in particular is especially weakly interacting. If one is willing to imagine that the state of the universe has such correlations over large distances built in from the beginning, by assumption or design, then it is possible that they can be maintained until they cross the cosmological horizon and freeze out. Then the question becomes whether such highly correlated initial states represent physically acceptable or attractive alternatives for the approximate initial state just prior to the onset of inflation. Some have argued that such a situation can arise naturally in certain kinds of hybrid inflation models [23] whereas others note that such states are almost impossible to generate and maintain [24]. It certainly seems bizarre to imagine that such correlations were built in; but so much about our present understanding of the big bang seems so highly contrived that the supposition that there is such a degree of coherence would not seem to be ruled out. Of course, there are no particular reasons to prefer these squeezed states over other possible excited states. It is a matter of the pre-Big Bang physics and their consequences for the inflationary paradigm.

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