We study a higher dimensional cosmology with phantom field associated with a negative kinetic term. Assuming that the universe possesses the phantom field defined in $D$ dimensional spacetime, we investigate in detail the solutions involved in the higher dimensional phantom cosmology, to explicitly predict photon size and phantom field strength at present in nature. To be specific, we find that the photon size decreases drastically at the early stage of the universe after the Big Bang. Next we explicitly demonstrate the dependences of the photon size, universe size and phantom field strength on the spacetime dimensionality $D$. We observe that the size of the universe undergoes stiff explosion with different types of slope depending on $D$. Moreover the scale factor of the universe at present is shown to approach to a saturated value, which is independent of $D$ and is the same as that in the $D = 4$ Friedmann-Robertson-Walker cosmology. The photon size and phantom field strength in the greater dimensionality are also shown to be larger and lower than those in the smaller one, respectively. Next the photon size at present $b_*$ in $D = 5$ is numerically shown to be extremely small, namely $b_* = 6.08 \times 10^{-216} \text{ cm}$, comparing to $b_* = 1.56 \times 10^{-63} \text{ cm}$ in $D = 10$. In contrast, the phantom field strength at present $\sigma_*$ is shown to be relatively large $\sigma_* = 4.72 \times 10^{24} (\text{dyne})^{1/2}$ in $D = 5$, comparing to $\sigma_* = 1.30 \times 10^{22} (\text{dyne})^{1/2}$ in $D = 10$.

Keywords: photon size; higher dimensional cosmology; phantom field; universe size

I. INTRODUCTION

As it is well known, a particle in the string theory is supposed to be an extended object \cite{1,2}. Via the string version of the Hawking-Penrose singularity theorem (HPST) \cite{3}, the stringy cosmology in $D$ dimensional total spacetime was investigated \cite{4,5} with a success that, with the extended string particle, one describes precisely the motion types of stringy congruence in terms of the universe expansion rate after the Big Bang. Moreover, in the stringy HPST, one has an advantage that the degrees of freedom of the rotation and shear of stringy congruence are introduced naturally in the early universe. Next in phantom cosmology \cite{6-16}, the universe expansion up to the age of the universe has been studied with various phantom field potentials. In particular, a higher dimensional phantom cosmology (HDPC) has been exploited to investigate an exact cosmological solution for the scale factor of the universe \cite{9}. In the HDPC, the universe has been shown to undergo a continuous transition from deceleration to acceleration at some finite time. This transition time could be proposed as recent acceleration of the universe. Moreover, a phantom field with an exponential potential has been considered to study a solution for the phantom cosmology together with the Hubble parameter \cite{13}. Next the inflationary Big Bang cosmology has been developed into a precision astrophysics by recent cosmological observations such as cosmic microwave radiation \cite{17} and supernova data \cite{18,19}. The observations have drawn astrophysics community attention to the origin of dark energy \cite{20,21}. The phantom cosmology model is also one of the approaches for investigating the dark energy \cite{6,23,24}.

The idea of extra dimensions, tracing back to the pioneering works of Kaluza and Klein, has been exploited in various higher dimensional models such as the string theory \cite{1,2}, the Randall-Sundrum cosmology \cite{25}, the HDPC \cite{9}, the stringy HPST \cite{4,5} and the higher dimensional electroweak model \cite{26} for instance. Motivated by the idea, in this paper we will consider the HDPC in $D$ dimensional total spacetime, in order to investigate explicitly the photon size and phantom field strength at present and their dependences on $D$. In addition, in $D$ dimensions we will discuss the evolution of the universe size from the Big Bang to the present epoch, to demonstrate the dependence of its size on the spacetime dimensionality $D$.

The paper is organized as follows. In Sec. II, we will present the set up of the HDPC with the dimensionality $D$, to study in detail the solutions for the differential equations associated with the HDPC. In Sec. III, we will investigate phenomenology in the HDPC. Explicitly we will evaluate the photon size at present in the $D$ dimensional spacetime. Next we will study the universe and phantom field evolutions from the Big Bang to the epoch at present. Sec. IV includes conclusions.

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II. SET UP OF HDPC FORMALISM IN $D$ DIMENSIONS

In this section, we formulate the solutions for the HDPC where the extra dimensions are introduced to allow the internal manifold. The HDPC in $D$ dimensional total spacetime is now described by the Lagrangian of the form

$$\mathcal{L} = -\sqrt{-g} \left( \frac{1}{2} R + \frac{1}{2} \sigma \alpha g^{M N} \partial_{M} \sigma \partial_{N} \sigma - \Lambda \right),$$

(2.1)

where $\sigma$ is the phantom field and $\alpha$ is shorthand defined as $\alpha = 8\pi G/c^4$. Here $M$ and $N$ are $D$ dimensional indices and $\Lambda$ is the cosmological constant. In the HDPC with the $D$ dimensional total spacetime, we have $(D-4)$ dimensional internal extra manifold, as in the string theory. Note that the string as an extended object is described in this internal extra manifold. Inspired by this, in the HDPC the extra manifold will be treated as that in which the photon is also described. In this way, we can have the string-like photon.

Variation of the Lagrangian with respect to the metric yields the Einstein equations

$$R_{M N} = \alpha T_{M N} - \frac{1}{D-2} \alpha g_{M N} T + \frac{2}{D-2} g_{M N} \Lambda,$$

(2.2)

where the energy-stress tensor is given by

$$T_{M N} = -\partial_{M} \sigma \partial_{N} \sigma + \frac{1}{2} g_{M N} \partial_{Q} \sigma \partial^{Q} \sigma.$$

(2.3)

We take an ansatz for the metric

$$g_{M N} = \text{diag} (-1, a^2(t), b^2(t)),$$

(2.4)

where $a(t)$ is the scale factor of the three dimensional universe and $b(t)$ is the scale factor of the photon described in the extra $(D-4)$ dimensions. Note that the extra dimensions are associated with the internal torus space $[27]$.

Making use of (2.2), we find differential equations with the overdots denoting the derivatives with respect to time,

$$\begin{align*}
3 \frac{\ddot{a}}{a} + (D-4) \frac{\ddot{b}}{b} &= \frac{2}{D-2} \Lambda + \alpha \dot{\sigma}^2, \\
\frac{\ddot{a}}{a} + 2 \frac{\dot{a}^2}{a^2} + (D-4) \frac{\dot{a}}{a} \cdot \frac{\dot{b}}{b} &= \frac{2}{D-2} \Lambda, \\
\frac{\ddot{b}}{b} + (D-5) \frac{\dot{b}^2}{b^2} + 3 \frac{\dot{a}}{a} \cdot \frac{\dot{b}}{b} &= \frac{2}{D-2} \Lambda, \\
\ddot{\sigma} + \left(3 \frac{\ddot{a}}{a} + (D-4) \frac{\ddot{b}}{b}\right) \dot{\sigma} &= 0.
\end{align*}$$

(2.5)

whose solutions are given by

$$\begin{align*}
a(t) &= a_0 \exp \left[ \left( \frac{2}{(D-1)(D-2)} \right)^{1/2} \sqrt{\alpha \Lambda t} + \frac{n(D-4)}{D-1} \left( \frac{D-2}{2(D-1)} \right)^{1/2} \left( 1 - \exp \left[ - \left( \frac{2(D-1)}{D-2} \right)^{1/2} \sqrt{\alpha \Lambda t} \right] \right) \right], \\
b(t) &= b_0 \exp \left[ \left( \frac{2}{(D-1)(D-2)} \right)^{1/2} \sqrt{\alpha \Lambda t} - \frac{3n}{D-1} \left( \frac{D-2}{2(D-1)} \right)^{1/2} \left( 1 - \exp \left[ - \left( \frac{2(D-1)}{D-2} \right)^{1/2} \sqrt{\alpha \Lambda t} \right] \right) \right], \\
\sigma(t) &= \sigma_0 \exp \left[ - \left( \frac{2(D-1)}{D-2} \right)^{1/2} \sqrt{\alpha \Lambda t} \right].
\end{align*}$$

(2.6)

Here $a_0$, $b_0$ and $\sigma_0$ are the initial values of $a$, $b$ and $\sigma$ at $t = 0$, respectively, and $n$ is a model parameter to be fixed later. Explicitly, $\sigma_0$ is given by

$$\sigma_0 = \frac{n}{D-1} \left( \frac{3(D-2)(D-4)}{2\alpha} \right)^{1/2}. $$

(2.7)

Note that for the case of $D = 10$ the above formalism becomes the phantom cosmology in Ref. [9].
III. PHENOMENOLOGY IN HDPC

In this section, we proceed to study further novel stringy photon phenomenology using the HDPC. To do this, we take an ansatz that, after the Planck time \( t_{\text{Planck}} = 5.39 \times 10^{-44} \) second near the Big Bang, the sizes of the universe and photon are the same each other so that one can choose \( a_0 \) and \( b_0 \) to become the Planck length \[25\]

\[ a_0 = b_0 = t_{\text{Planck}} = 1.62 \times 10^{-33} \text{ cm}. \] (3.1)

We exploit the relation

\[
\Lambda = \frac{(D - 1)(D - 2)}{6} \Lambda_*,
\]

\[
\Lambda_* = 2.07 \times 10^{-56} \text{ cm}^{-2},
\] (3.2)

where \( \Lambda_* \) is the four dimensional cosmological constant. We next define a dimensionless variable \( x \) \( (0 \leq x \leq 1) \) as

\[
t = x t_*,
\]

\[
t_* = 4.28 \times 10^{17} \text{ sec},
\] (3.3)

with \( t_* \) being the age of the universe \[29\]. Inserting \( \Lambda \) in (3.2) and \( t \) in (3.3) into (2.6), we obtain

\[
a(x) = a_0 \exp \left[ 1.07 x + \frac{n(D - 4)}{D - 1} \left( \frac{D - 2}{2(D - 1)} \right)^{1/2} (1 - \exp [-1.07 x(D - 1)]) \right],
\] (3.4)

\[
b(x) = b_0 \exp \left[ 1.07 x - \frac{3n}{D - 1} \left( \frac{D - 2}{2(D - 1)} \right)^{1/2} (1 - \exp [-1.07 x(D - 1)]) \right],
\] (3.5)

\[
\sigma(x) = \sigma_0 \exp [-1.07 x(D - 1)].
\] (3.6)

Making use of the size of the present universe at \( x = 1 \) \[30\]

\[ a_* = 4.35 \times 10^{28} \text{ cm}, \] (3.7)

and (3.4), we fix the value of \( n \) to be

\[ n = 140.37 \times \frac{D - 1}{D - 4} \left( \frac{2(D - 1)}{D - 2} \right)^{1/2}. \] (3.8)

With the above value of \( n \), we obtain

\[
a(x) = a_0 \exp \left[ 1.07 x + 140.37 (1 - \exp [-1.07 x(D - 1)]) \right],
\] (3.9)

\[
b(x) = b_0 \exp \left[ 1.07 x - \frac{421.12}{D - 4} (1 - \exp [-1.07 x(D - 1)]) \right],
\] (3.10)

where \( a(x) \) and \( b(x) \) describe the evolutions of the universe and photon sizes, respectively.\(^1\)

Now we depict \( \log_{10}[a(x)/a_0] \) in Fig. 1(a) and \( \log_{10}[b(x)/b_0] \) in Fig. 1(b) in terms of the dimensionless time variable \( x \) \( (0 \leq x \leq 1) \). Here we treat the total dimensionality \( D \) as a parameter of the curves, to investigate the dependence of the scale factors \( a(x) \) and \( b(x) \) on \( D \). Fig. 1(a) shows that the curve is rising up with extreme slope at the early stage of the universe evolution. Note that, along the time evolution, the universe size in the greater dimensionality increases much stifferly than that in the smaller one. In Fig. 1(b) we find that the photon size is drastically falling down at the early stage of the photon evolution after the Big Bang. Note that the photon size curve in the greater dimensionality is much higher than that in the smaller one. Next we depict \( \log_{10}[\sigma(x)/\sigma_*] \) in Fig. 2 in terms of the dimensionless time variable \( x \). Note that all the curves in Fig. 2 decrease linearly along the time evolution. Moreover

\(^1\)Note that in the limit of \( D = 4 \) with \( b = 0 \), solving the differential equations in \[25\] yields \( a = a_0 e^{\sqrt{\frac{6}{5}} x} \) and \( \sigma = 0 \) meaning that we do not have a nontrivial solution for the phantom field in \( D = 4 \) \[29\]. Moreover in (3.8) we need to have the condition \( D \neq 4 \) in order to obtain a reasonable value of \( n \).
FIG. 1: (a) \( \log_{10}[a(x)/a_0] \) and (b) \( \log_{10}[b(x)/b_0] \) in terms of the dimensionless time variable \( x \). The curves in (a) and (b) are for \( D = 5, 6, 7, 8, 9, 10 \) from bottom to top.

the dimensionless phantom field in logarithmic scale \( \log_{10}[\sigma(x)\sqrt{\alpha}] \) in the greater dimensionality is lower than that in the smaller one.

Now it seems appropriate to comment on the phenomenology of the HDPC at present. First, putting \( x = 1 \) in (3.9) we are left with the universe size \( a_* \) at present

\[
a_* = a_0 \exp \left[ 1.07 + 140.37 (1 - \exp [-1.07(D - 1)]) \right],
\]

where \( a_0 \) is now given by (3.1). Note that according to Fig. 1(a) and (3.11), there are no significant differences among the universe sizes. In other words, all the curves of the universe sizes converge to a saturated value given by (3.7), independent of \( D \), and the value is the same as that in the \( D = 4 \) Friedmann-Robertson-Walker (FRW) cosmology. This phenomenon is due to the fact that the last term in (3.11) is predominant in these curves.

Second, inserting \( x = 1 \) into (3.10), we arrive at the photon size \( b_* \) at present

\[
b_* = b_0 \exp \left[ 1.07 - \frac{421.12}{D - 4} (1 - \exp [-1.07(D - 1)]) \right],
\]

where \( b_0 \) is given by (3.1). Note that at the present epoch there exist very significant differences among the photon sizes as shown in Fig. 1(b). This phenomenon originates from the fact that the last term in (3.12) is predominant in these curves. The predicted values of the sizes of the stringy (not point) photon \( b_* \) in (3.12) are listed in Table I in terms of the total dimensionality \( D \). In particular, the photon size prediction in \( D = 10 \) case seems to be interpreted as the string size itself [1, 2]. Note also that, in the limit of \( D = 4 \), (3.12) yields \( b_* = 0 \) consistent with the fact that we have no extra dimensions in that limit.

Third, inserting \( x = 1 \) and (2.7) into (3.6), we find the phantom field strength \( \sigma_* \) at present

\[
\sigma_* = \frac{140.37}{\sqrt{\alpha}} \left( \frac{3(D - 1)}{D - 4} \right)^{1/2} \exp [-1.07(D - 1)],
\]

with \( \frac{1}{\sqrt{\alpha}} = 6.95 \times 10^{23} \) (dyne)^{1/2} and \( D \neq 4 \). The predicted values of \( \sigma_* \) in (3.13) are listed in Table I in terms of the total dimensionality \( D \).

According to the stringy HPST [4, 5], the photon has a spin structure, due to its rotation whose magnitude is assumed to be the same as that of the rotating universe itself. Note that the universe consists of celestial objects such as galaxies, stars and planets. If we combine the results on the photon rotation from the stringy HPST with those of the HDPC, the photon has an extremely tiny size with the spin degree of freedom. The explanation about the photon spin phenomenology seems to be consistent with the corresponding experiment.

\[\text{Note that the unit of } \sigma^2 \text{ is dyne, and this phantom field strength seems to be interpreted as a source of detonation of the universe at the early stage of the time evolution as shown in Fig. 2. Moreover the phantom field also plays a role of late-time expansion in any epoch of the universe evolution.}\]
FIG. 2: $\log_{10}[\sigma(x)\sqrt{\alpha}]$ in terms of the dimensionless time variable $x$. The curves are for $D = 5, 6, 7, 8, 9, 10$ from top to bottom.

TABLE I: The present photon size $b_*$ in unit of cm, and the present phantom field strength $\sigma_*$ in unit of (dyne)$^{1/2}$ in the total dimensionality $D$.

| $D$ | $b_*$ | $\sigma_*$ | $D$ | $b_*$ | $\sigma_*$ | $D$ | $b_*$ | $\sigma_*$ |
|-----|-------|------------|-----|-------|------------|-----|-------|------------|
| 5   | $6.08 \times 10^{-216}$ | $4.72 \times 10^{24}$ | 7   | $5.12 \times 10^{-94}$ | $3.95 \times 10^{24}$ | 9   | $1.25 \times 10^{-69}$ | $4.77 \times 10^{22}$ |
| 6   | $1.69 \times 10^{-124}$ | $1.28 \times 10^{24}$ | 8   | $8.93 \times 10^{-79}$ | $1.27 \times 10^{24}$ | 10  | $1.56 \times 10^{-63}$ | $1.39 \times 10^{22}$ |

IV. CONCLUSIONS

In summary, we have assumed that the universe has the phantom field associated with a negative kinetic term in the $D$ dimensional spacetime, to find the solutions for the differential equations related with the HDPC. Making use of the solutions, we have explicitly evaluated the photon size and phantom field strength at present in nature. The formula for the scale factor of the universe has indicated that the universe size increases rapidly with different types of slope depending on $D$ at the early stage of the evolution, but after the universe age $t_* = 4.28 \times 10^{17}$ sec it approaches to the saturated value independent of $D$, which is the same as that in the $D = 4$ FRW cosmology. In contrast the photon size has decreased drastically at the early stage of the evolution after the Big Bang, and then the photon size at present $b_*$ in $D = 5$ has been shown to be extremely small, comparing to $b_*$ in $D = 10$, as shown in Table I. We also found that the phantom field strength decreases along the time evolution, and the strength in the greater dimensionality is lower than that in the smaller one. The phantom field strengths at present have been numerically predicted in terms of the dimensionality as shown in Table I. Moreover, we have given some comments on the photon spin in the framework of the stringy HPST combined with the HDPC. Finally, it is interesting to note that the effective equation of state originated from the phantom field is consistent with that of the cosmic fluid of the present superacceleration era [9, 31–33].

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[1] M.B. Green, J.H. Schwarz and E. Witten, Superstring Theory (Cambridge Univ. Press, Cambridge, 1987).
[2] J. Polchinski, String Theory (Cambridge University Press, 1998).
[3] S.W. Hawking and R. Penrose, Proc. Roy. Soc. Lond. A 314, 529 (1970).
[4] Y.S. Cho and S.T. Hong, Phys. Rev. D 78, 067301 (2008), arXiv:0806.2061.
[5] Y.S. Cho and S.T. Hong, Phys. Rev. D 83, 104040 (2011), arXiv:1103.0300.
[6] R.R. Caldwell, Phys. Lett. B 545, 23 (2002), astro-ph/9908168.
[7] J.E. Lidsey, Phys. Rev. D 70, 041302 (2004), gr-qc/0405055.
[8] V. Faraoni, Class. Quant. Grav. 22, 3235 (2005), gr-qc/0506095.
[9] S.T. Hong, J. Lee, T.H. Lee and P. Oh, Phys. Rev. D 78, 047503 (2008), arXiv:0801.3781.
[10] L.P. Chimento, F.P. Devecchi, M.I. Forte and G.M. Kremer, Class. Quant. Grav. 25, 085007 (2008), arXiv:0707.4455.
[11] C. Kaemikohm, B. Gumjudpai and E.N. Saridakis, Phys. Lett. B 695, 45 (2011), arXiv:1008.2182.
[12] H. Wei, Nucl. Phys. B 845, 381 (2011), arXiv:1008.4968.
[13] A.A. Andrianov, F. Cannata and A.Y. Kamenshchik, JCAP 10, 004 (2011), arXiv:1105.4515.
[14] R.J. Yang, Eur. Phys. J. C 72, 1948 (2012), arXiv:1108.0227.
[15] A.V. Astashenok, S. Nojiri, S.D. Odintsov and A.V. Yurov, Phys. Lett. B 709, 396 (2012), arXiv:1201.4056.
[16] B. Novosyadlyj, O. Sergijenko, R. Durrer and V. Pelykh, Phys. Rev. D 86, 083008 (2012), arXiv:1206.5194.
[17] D.N. Spergel et al., (WMAP Collaboration), Astroph. J. Suppl. Ser. 170, 377 (2007), astro-ph/0603449.
[18] S. Perlmutter et al., (Supernova Cosmology Project Collaboration), Astroph. J. 517, 565 (1999), astro-ph/9812133.
[19] A.G. Riess et al., Astroph. J. 659, 98 (2007), astro-ph/0611572.
[20] S. Perlmutter, M.S. Turner and M.J. White, Phys. Rev. Lett. 83, 670 (1999), astro-ph/9901052.
[21] P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003), astro-ph/0207347.
[22] T. Padmanabhan, Phys. Rep. 380, 235 (2003), arXiv:0212290.
[23] M. Szydlowski, O. Hrycyna and A. Krawiec, JCAP 06, 010 (2007), hep-th/0608219.
[24] X.M. Chen, Y. Gong and E.N. Saridakis, JCAP 04, 001 (2009), arXiv:0812.1117.
[25] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), hep-ph/9905221.
[26] S.V. Bolokhov, K.A. Bronnikov and S.G. Rubin, Phys. Rev. D 84, 044015 (2011), arXiv:1011.2828.
[27] Y. Tosa, Phys. Rev. D 30, 2054 (1984).
[28] S.M. Carroll, Spacetime and geometry (Addison Wesley, 2004).
[29] J.G. Hartnett and F.J. Oliveira, Found. Phys. Lett. 19, 519 (2006), astro-ph/0603500.
[30] C.H. Lineweaver and T.M. Davis, Scient. Am. 292, 36 (2005).
[31] S. Capozziello and V. Faraoni, Beyond Einstein Gravity (Springer, Heidelberg, 2011).
[32] V. Faraoni, Int. J. Mod. Phy. D 11, 471 (2002), astro-ph/0110067.
[33] A. Riazuelo and J.P. Uzan, Phys. Rev. D 62, 083506 (2004), astro-ph/0004156.