A Unified Framework for Reinforcement Learning

Jicheng Shi, Yingzhao Lian and Colin N. Jones

Abstract—Reinforcement learning has shown strong potential in learning optimal control strategy by modeling policy and/or value function. Even though policy and value function forms duality regarding the Bellman equation, there is no structure unifies this two branches. In this paper, we propose to use an convex optimization layer to combine these two branches which enables universal compatibility with all reinforcement learning algorithm without modification of the model structure. Design and training issues will be explained and validated by both linear and nonlinear control.

Index Terms—Reinforcement learning, differential of convex optimization problem, amortized reparametrization

I. INTRODUCTION

Reinforcement learning, as an unsupervised learning method, learns the optimal control strategy by trial and error. Its algorithms models optimal policy and/or control policy, where Q-learning [1] and policy gradient [2] act as backbones of these two branches respectively. Fusion of these two branches are called actor-critic algorithm. Even though optimal policy and optimal value function function show strong duality regarding the Bellman equation [3], no model has ever achieved unified structure to the best of our knowledge, such as [4]–[6].

In order to combine these two branches, we consider a new differentiable convex optimization layer [7], [8], whose gradient is calculated by sensitivity analysis of its KKT system. This layer has found strong connection to reinforcement learning in [9]–[11], where the authors applied this layer to achieve safe reinforcement learning and update model for nonlinear model predictive control (MPC). Instead of solving a specific problem, this paper consider a general reinforcement scheme and propose a unified reinforcement learning framework, our contribution are concluded as follows:

- A unified framework for reinforcement learning which show universal compatibility with any reinforcement learning algorithms.
- An amortized reparametrization method that adapts the proposed frameworks to policy gradient algorithms.
- Practical issues that enable more efficient and stable training.

The remainder of the paper is outlined as follows. In Section II the motivation is discussed with respect to the convex optimization layer. In Section III we define the unified framework with lifting, and demonstrate its effectiveness in Q-learning and Amortized Reparameterization based Policy Gradient. We then demonstrate its extension in other RL algorithms with

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B. Convex Optimization Layer

As a replacement of Deep Neural Networks (DNN) which is generally used in RL [12], the convex optimization layer gives the output in the forward pass. In the backward pass, the gradient with respect to the parameters need to be computed.

For the sake of clarity, we elaborate the convex optimization layer with a standard quadratic programming (QP), please refer to [8] for a general formulation. Suppose a QP is:

\[
\begin{align*}
\min_z & \quad \frac{1}{2} z^T Q z + q^T z \\
\text{s.t.} & \quad A z \leq b, E z = f
\end{align*}
\]

The KKT conditions for the QP are:

\[
\begin{align*}
Q z^* + q + A^T \lambda^* + E^T \nu^* &= 0 \\
D(\lambda^*) (A z^* - b) &= 0 \\
E z^* - f &= 0
\end{align*}
\]

where \( z^*, \nu^*, \lambda^* \) are the optimal primal and dual variables, \( D(\cdot) \) builds a diagonal matrix from a vector. Then the differentials of KKT conditions can be computed as:

\[
\begin{bmatrix}
Q & A^T & E^T \\
D(\lambda^*) \! A & D(A z^* - b) & 0 \\
E & 0 & 0
\end{bmatrix}
\begin{bmatrix}
dz \\
d\lambda \\
d\nu
\end{bmatrix}
= -
\begin{bmatrix}
dQ z^* + dq + dA \lambda^* + dE \nu^* \\
D(\lambda^*) dA z^* - D(\lambda^*) db \\
dE z^* - df
\end{bmatrix}
\]

By the above equation, we can compute the derivatives of \( z^* \) with respect to the parameters \((Q,q,A,b,E,f)\) in QP. For example, the solution \( dz \) of (7) gives the result of \( \frac{dL}{d z} \) if we set \( dQ = I \) and the differentials of other parameters to 0. Then if there is a loss function \( L(z^*) \), the gradient is computed as \( \frac{\partial L(z^*)}{\partial z^*} \frac{dz}{d q} \).

III. Unified Reinforcement Learning Framework

In this section, a general convex optimization layer is first constructed with lifting. The corresponding Q-learning and policy gradient with amortized reparametrization will be introduced. Then we detail the potential of the unified framework by an example of RL algorithm.

A. Unified Framework with Lifting

In order to model the complex detail of optimal policy, we use lifting to expand the interpretation volume of the convex optimization layer, define \( z = f_\beta(x) \) with \( f_\beta : \mathbb{R}^n \to \mathbb{R}^k \) differentiable with respect parameter \( \beta \) and \( k > n \). Then we solve the optimization problem in a higher dimensional space as

\[
\min_z \frac{1}{2} \mu^T Q \mu + q^T \mu \\
\text{s.t.} & \quad A z + B \mu = b, C z \leq D,
\]

where \( \mu \in \mathbb{R}^k \) is embedded in a higher dimensional space then control input \( u \in \mathbb{R}^m \). We fix a linear map \( K : \mathbb{R}^k \to \mathbb{R}^m \) to map the high dimensional solution back to control inputs. For the sake of compactness, all the parameter, including \( \beta, A, B, Q, q, b, C, D \), are dubbed \( \theta \).

In the following section, we will introduce how to achieve both Q-learning and policy gradient accordingly.

B. Q-learning

Q learning does gradient descent regarding the temporal difference:

\[
T = I(x_i, u_i) + V_\theta(x_{i+1}) - Q_\theta(x_i, u_i).
\]

Then Q-learning updates as \( \theta \leftarrow \theta - \alpha T \nabla_\theta Q_\theta \), where the value function and the Q-fucntion regarding [8] are defined as:

\[
\begin{align*}
V_\theta(x) &= -\min_{\mu} \frac{1}{2} \mu^T Q \mu + q^T \mu \\
\text{s.t.} & \quad A z + B \mu = b, C z \leq D, \\
f_\beta(x) &= z \\
Q_\theta(x, u) &= -\min_{\mu} \frac{1}{2} \mu^T Q \mu + q^T \mu \\
\text{s.t.} & \quad A z + B \mu + u = b, C z \leq D, \\
f_\beta(x) &= z, K \mu = u
\end{align*}
\]

Remark 1: Q-learning by the learning framework is automatically applicable in continuous action space. Note deep Q-learning (DQN) is adapted specifically for environments with discrete action spaces [12], while the continuous one is achieved by other invariant RL algorithms, like Deep Deterministic Policy Gradient [4].

C. Amoritzed Reparametrization based Policy Gradient

Policy gradient does gradient ascent regarding the expected return, \( J(\pi_\theta) = \mathbb{E}_{\tau \sim \pi_\theta} (R(\tau)) \):

\[
\theta \leftarrow \theta + \alpha \nabla_\theta J(\pi_\theta)
\]

Notice that the policy defined in (8) is deterministic as \( \pi_\theta(x) = K \arg\min_\mu \frac{1}{2} \mu^T Q \mu + q^T \mu \).

\[
\begin{align*}
\pi_\theta(x) &= K \arg\min_\mu \frac{1}{2} \mu^T Q \mu + q^T \mu \\
\text{s.t.} & \quad A z + B \mu = b, C z \leq D, \\
f_\beta(x) &= z.
\end{align*}
\]

However, a stochastic policy gradient is required for implement policy gradient algorithm. Unlike [13] where the convex optimization layer is perturbed, we proposed to perturb the optimal decision variable with an amortized reparametrization trick [14] as

\[
\tilde{\pi}_\theta(x) \sim \mathcal{N}(\pi_\theta(x), \Sigma)
\]

\[
\tilde{\pi}_\theta(x) = \pi_\theta(x) + \epsilon
\]

\[
\Sigma = L^T L, \epsilon \sim \mathcal{N}(0, I).
\]

The policy gradient is calculated accordingly as:

\[
\nabla_\theta J(\pi_\theta) = \mathbb{E}_{\tau \sim \pi_\theta} \left( \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(u_t|x_t) R(\tau) \right).
\]

\[
\begin{align*}
\min_{\mathcal{Q}} & \quad \frac{1}{2} z^T Q z + q^T z \\
\text{s.t.} & \quad A z \leq b, E z = f
\end{align*}
\]
The amortized reparameterization convert a deterministic control policy into a stochastic one while remain differentiability of the original problem, meanwhile if the matrix $L$ will adapt automatically to enable an adaptive search. Note that apart from the Gaussian policy in \cite{13}, different distributions, such as gamma distribution, can be deployed in stochastic policy for a better exploration \cite{15}

D. Conclusion

We conclude this section by fusing all the components into one uni-body. As described above, all the key components in RL, value function, Q-function and policy function are defined by the convex optimization programming in \cite{9} \cite{10} \cite{12} with the methods to compute the gradients. Then we can construct any critic-based and actor-based RL algorithms, and actor-critic RL algorithms.

Actor-critic RL is considered as the different combinations of actors and critics. Here, we detail an example of it, the Advantage Actor Critic (A2C) algorithm to demonstrate the potential of our unified RL framework, while it can be well deployed in other algorithms such as Deep Deterministic Policy Gradient (DDPG) \cite{6}, Soft Actor Critic (SAC) \cite{16}.

In A2C, a policy gradient is given as \cite{15}:

$$\nabla_{\theta} J(\pi_\theta) = \mathbb{E}_{\tau \sim \pi_\theta} \left[ \sum_{t=0}^{T} \nabla_{\theta} \log \pi_\theta(x_t|u_t) \Phi_{\theta,t} \right] \tag{15}$$

where $\Phi_{\theta,t} = A_\theta(x_t, u_t)$, which denotes the advantage function in \cite{2} and indicates how much better the action $u_t$ is. A2C estimates it as $\sum_{i=0}^{k-1} i (x_{t+i}, u_{t+i}) + V_{\theta}(x_{t+k}) - V_{\theta}(x_t)$.

The updating of A2C contains two parts. The “actor” part executes the gradient ascent, the same as \cite{11}. The term $\nabla_{\theta} \log \pi_\theta(x_t|u_t)$ is computed as described in \cite{III-C}. The term $\Phi_t$ is computed by the value of value function $V_{\theta}(x_t)$. The “critic” part executes the gradient descent to minimize the value of the Advantage function $A_\theta(x_t, u_t)$:

$$\theta \leftarrow \theta - \alpha A_\theta \nabla_{\theta} V_{\theta}$$

Pseudocode for A2C by our unified framework is shown in Algorithm \cite{III-D}.

Remark 2: Different from the algorithm in \cite{5}, we only use one convex optimization layer to estimate the policy and value function. In the step 17, different updating methods with some hyper-parameters can be used to tune the learning results. For example, a soft updating with the “critic” gradient executed as:

$$\hat{\theta} \leftarrow \theta - \alpha_c A_\theta \nabla_{\theta} V_{\theta}$$

$$\theta \leftarrow \tau \hat{\theta} + (1 - \tau) \theta$$

in order to stabilize training \cite{16}.

IV. PRACTICAL ISSUES IN TRAINING

In this section, we will talk about a few issues to enable a better training performance.

A. Lifting Selection

The lifting $f_\beta$ encodes more information of the states by mapping it to a higher dimensional space. Without a proper initialization of the parameter $\beta$, the training will be difficult because it learns both the optimization problem and the lifting together. As suggested in \cite{11}, the lifting function can be perceived as a model of the system, which implies that we can use system identification to initialize the parameters in this model. We take a nonlinear system for example, regarding the idea of Koopman operator \cite{17}, \cite{18}, one can apply the unsupervised learning algorithm in \cite{19} to initialize $A, B$ with the Koopman operator finite approximation while initializing $f_\beta$ as the Gaussian process feature map of the Koopman operator.

In practice, state $x$ may not be measured directly, it might be encapsulated by some redundant information captured by sensors, such as pictures in autonomous vehicle. Based on the idea of transfer learning \cite{20}, one can initialized the lifting as some pretrained neural networks, such as ResNet \cite{21}.

B. Miscellaneous

In order to train the convex optimization layer, one need to ensure that $z$ is feasible. Without a priori knowledge of the feasible set, we can reformulated the problem as a soft-constrained problem. Then the optimization layer is
constructed as:
\[
\min_{\mu} \frac{1}{2} \mu^T Q \mu + q^T \mu + \rho(\varepsilon),
\]
\[\text{s.t. } Ax + B \mu = b, Cz \leq D + \varepsilon, \tag{17}\]
where $\varepsilon$ denote the slack variables which ensure the optimization problem always feasible. $\rho(\varepsilon)$ are the penalty for the constraint violation, which can be formulated in quadratic or linear way.

V. EXPERIMENTS

In this section, we will run our algorithms on linear and nonlinear systems. The OpenAI gym package is used to build system environment and pytorch is used for RL execution.

A. Linear System

We consider the real system as a linear system with box constraints,
\[
x^+ = \begin{bmatrix} 1.53 & 0.25 \\
-0.56 & -0.52 \end{bmatrix} x + \begin{bmatrix} 1.23 \\
-0.96 \end{bmatrix} u
\]
\[
\begin{bmatrix} I_2 \\
-I_2 \end{bmatrix} x_i \leq 41, \quad \begin{bmatrix} I_2 \\
-I_2 \end{bmatrix} u_i \leq 14
\]
\[\tag{18}\]
where the system is controllable and unstable.

For each new trajectory, the system starts with a random states: $[-2 2] \leq x(0) \leq [2 2]$. Along the trajectory, a negative quadratic stage cost is given as $J(x, u) = \|x\|^2 - \|u\|^2$.

The stopping criterion is: $x \leq [-2 2]$ or $x \geq [2 2]$.

In the convex optimization layer, the lifting function based on the system dynamics is used to lift 2-dimensional $x$ to 20-dimensional $z$:
\[
z = \begin{bmatrix} I \\
A \\
\vdots \\
A^9 \end{bmatrix} x + \begin{bmatrix} B & 0 & \ldots & 0 \\
AB & B & \ldots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
A^8 B & A^7 B & \ldots & B \end{bmatrix} \mu
\]
\[\tag{19}\]
where matrices $A, B$ defines the linear system dynamics as $x^+ = Ax + Bu$, $\nu = [u_0, u_1, \ldots, u_T]^T$ and the linear map $u = K\nu$ with $K = [I, I]$. We train $A, B$ by regression as the initial values, while other parameters are initialized with random values from a uniform distribution.

We execute the A2C in our unified framework with learning rate $\lambda_{\text{critic}} = 10^{-3}, \lambda_{\text{actor}} = 10^{-3}$. Figure 1 shows the evolution of the critic loss and actor rewards. Note the we don’t evaluate the actor loss because it does not measure performance in the policy gradient part.

The critic loss converges and rewards reach a high level very quickly, within hundreds updating steps, while the conventional RL algorithms by deep neural networks require far more than $10^4$ steps [24].

In order to measure the performance of the RL algorithm, we use a MPC controller with real parameters as a comparison. Figure 2 shows the difference of the trajectory reward by MPC-based RL policy and by MPC controller, w.r.t to different trajectory starting points.

B. Nonlinear System

VI. CONCLUSION

In this paper, we propose to use a convex optimization layer to combine these two branches which enables universal compatibility with all reinforcement learning algorithms without modification of the model structure. Design and training issues will be explained and validated by both linear and nonlinear control.

In this paper, a convex optimization layer is deployed to build a unified framework for the RL algorithms. We demonstrate its effectiveness in Q-learning and Amoritized Reparameterization based Policy Gradient. An algorithm application example, A2C, is proposed to show the potential of its universal compatibility. Some tricks are proposed to deal with the practical issues. We end up the paper with two training experiments respectively by linear and nonlinear control.

Future work can propose tuning methods for the algorithms for the nonlinear system.

VII. REFERENCE

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