Q-values of the Superallowed $\beta$-Emitters $^{26}\text{Al}^m$, $^{42}\text{Sc}$ and $^{46}\text{V}$ and their impact on $V_{ud}$ and the Unitarity of the CKM Matrix

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(Dated: March 30, 2022)

The $\beta$-decay $Q_{EC}$ values of the superallowed beta emitters $^{26}\text{Al}^m$, $^{42}\text{Sc}$ and $^{46}\text{V}$ have been measured with a Penning trap to a relative precision of better than $8 \times 10^{-9}$. Our result for $^{46}\text{V}$, 7052.72(31) keV, confirms a recent measurement that differed significantly from the previously accepted reaction-based $Q_{EC}$ value. However, our results for $^{26}\text{Al}^m$ and $^{42}\text{Sc}$, 4232.83(13) keV and 6426.13(21) keV, are consistent with previous reaction-based values. By eliminating the possibility of a systematic difference between the two techniques, this result demonstrates that no significant shift in the deduced value of $V_{ud}$ should be anticipated.

PACS numbers: 21.10.Dr, 23.40.Bw, 23.40.-s, 23.40.Hc, 27.30.+t, 27.40.+z

A recent critical survey of superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$-decays \cite{1} presented a remarkably consistent picture, from which it was possible to obtain precise values and demanding limits on a number of fundamental weak-interaction parameters \cite{1,2}. In particular, the value of the up-down element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix was determined from the superallowed data to be $V_{ud} = 0.9738(4)$. Since $V_{ud}$ is a key component of the most demanding available test of the unitarity of the CKM matrix, the precision and reliability of the value for $V_{ud}$ has a direct impact on the search for physics beyond the standard model.

Shortly after the survey was published, a new $Q_{EC}$-value measurement was reported by Savard et al. \cite{3} for the superallowed $\beta$-decay of $^{46}\text{V}$. This was the first time that the $Q_{EC}$ value for any of the nine most-precisely known superallowed transitions had been measured with an on-line Penning trap. All previous results had been obtained from reaction $Q$ values: generally from $(p,\alpha)$ or $(^3\text{He},t)$ reactions, or from a combination of $(p,\gamma)$ and $(n,\gamma)$ reactions. Stuningly, the Penning-trap result differed significantly from the survey result and left the $F_I$ value for the $^{46}\text{V}$ transition anomalously high with respect to the $F_I$ values for the other superallowed transitions. There was understandable concern that this could be signaling a previously undetected systematic error in all the reaction measurements, which, when corrected, might lead to a significant shift in $V_{ud}$ from the value obtained in the survey.

Since systematic changes of only a few hundred eV in the $Q_{EC}$ values could have an appreciable effect on $V_{ud}$, this concern prompted a careful study \cite{4} of whether such systematic errors could be excluded in past measurements of $(p,\gamma)$ and $(n,\gamma)$ reaction $Q$ values. The study’s authors concluded that systematic effects up to at least 200 eV could not be excluded, and they proposed that a Penning-trap measurement of the superallowed transition from $^{26}\text{Al}^m$ would provide an excellent case to test for systematics since the corresponding reaction-based $Q$ value was particularly soundly based.

We report here Penning-trap measurements of the $Q_{EC}$ values for three superallowed $\beta$ transitions. The first, the decay of $^{46}\text{V}$, was chosen to confirm (or not) the recent unexpected Penning-trap result \cite{3}. The second, $^{26}\text{Al}^m$, is the case proposed \cite{4} as a test for systematic effects; and the third, $^{42}\text{Sc}$, is another case in which high-quality $(p,\gamma)$ and $(n,\gamma)$ reaction measurements have previously been performed. The measurements were specifically aimed at establishing whether undetected systematic effects were present in earlier measurements and whether a significant change in $V_{ud}$ might be anticipated as a result.

All ions of interest were produced at the IGISOL facility \cite{5}. We produced $^{46}\text{V}$ and $^{26}\text{Al}^m$ via $(p,n)$-reactions, with 20- and 15-MeV proton beams incident on enriched $^{46}\text{Ti}$ and $^{26}\text{Mg}$ targets respectively. For $^{42}\text{Sc}$, we used a $^3\text{He}$ beam of 20 MeV on $^{nat}\text{Ca}$. In these bombardments, not only were the superallowed emitters of interest produced in the primary reactions but ions from the target material itself – the $\beta$-decay daughters of these emitters – were also released by elastic scattering of the cyclotron beam. The recoil ions were slowed down and thermalized in the gas cell of an ion guide filled with 150 mbar of helium \cite{5}. These were then transported by gas flow and electric fields through a differentially pumped electrode system into a high-vacuum region, accelerated to

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30 keV and passed through a 55° dipole magnet for a coarse mass selection with resolving power of 300-500.

The mass-separated ion beam was then transferred to the JYFLTRAP setup. This consists, first, of a radiofrequency quadrupole (RFQ) cooler, which is used to improve the quality of the beam and bunch it for efficient injection into the Penning-trap system. The latter consists of two cylindrical traps housed inside the same superconducting 7-T magnet. The first trap is filled with helium buffer gas to allow for purification of the ion sample. With successive magnetron dipole excitation and mass-selective quadrupole excitation, a mass resolving power of up to a few×10^3 can be achieved in this first trap, which is enough to resolve the isomeric and ground states in ^{26}\text{Al} and ^{42}\text{Sc}.

After purification, the ion ensemble was injected into the second Penning trap for the actual mass measurement. A dipole excitation was used to establish a magnetron orbit with a fixed frequency and amplitude. Then, the ions were exposed to a radiofrequency quadrupole electric field for a given time. The amplitude of the RF electric field was tuned so that, when the frequency corresponded to the cyclotron frequency of the ion of interest, the whole magnetron motion was converted to cyclotron motion. After the quadrupole excitation, the ions were extracted from the trap and their time-of-flight to the micro channel plate detector recorded. The frequency corresponding to the shortest time-of-flight is the true cyclotron frequency. To locate the precise resonance frequency, we scanned the frequency and recorded the time-of-flight over a range that spanned the resonance. Examples of these frequency scans appear in Fig. 1.

The $Q_{EC}$ value of each ion of interest was obtained directly from the frequency ratio of the mother and the daughter nuclei. The cyclotron frequency measurements were interleaved: first we recorded a frequency scan for the daughter, then for the mother, then for the daughter and so on. This way, the slow drift of the magnetic field, mostly due to drifts in the room temperature, could be treated properly by interpolation of the reference frequency to the instant of measurement for the ion of interest. In the cases of $^{26}$Al and $^{42}$Sc we also measured the resonance frequencies of the nearby high-spin, long-lived states to check for consistency.

For each measurement, data were collected in several sets. Each set comprised $\sim$10 pairs of parent-daughter frequency scans taken under the same conditions. Between sets, the excitation time was changed. Each of the resonance curves was fitted with a realistic function, described in Ref. 4, which yielded values for the resonant frequency and its statistical uncertainty.

For $^{46}$V, a total of 40 resonances were obtained with $^{46}$Ti as a reference ion; these were grouped in three sets with excitation times of 700, 500 and 300 ms. For $^{42}$Sc we used $^{42}$Ca as a reference and obtained 52 resonances in 5 different sets covering three different excitation times, 300, 400 and 600 ms (see Fig. 2). As a consistency check for $^{42}$Sc, we also measured the $Q_{EC}$ value of $^{42}$Sc, referenced both to $^{42}$Ca and to $^{42}$Sc.

The $^{26}$Al measurement followed the same pattern as for $^{42}$Sc. The resonances of $^{26}$Al and $^{26}$Mg were both measured with respect to the ground state of $^{26}$Mg and, in addition, we measured the excitation energy of $^{26}$Al directly by using $^{26}$Al(gs) as a reference. In each of these three ratio measurements, excitation times of 200, 300 and 400 ms were used. As a further consistency check, the frequency ratios for $^{26}$Al and $^{26}$Mg were also obtained with $^{25}$Mg as the reference ion; however, in this latter case only a 200-ms excitation time was used and relatively few resonances were obtained. Our final measured frequency ratios for all cases are given in table I.

With the frequency ratios thus determined, we derived the $Q_{EC}$ value between mother-daughter pairs from the following equation:

$$Q_{EC} = m_m - m_d = \left(\frac{\nu_{d}}{\nu_{m}} - 1\right) m_d ,$$

(1)

where $m_m$ and $m_d$ are the masses of the singly charged mother and daughter ions and $\nu_{m}$ is its frequency ratio. In our experiment, all measured ions were singly charged and the mass excess values for the reference ions, $m_d$, were obtained from Ref. 10. Since the term inside parenthesis is small ($<10^{-3}$), the uncertainty contribution from $m_d$ to the $Q_{EC}$ value is negligible. The final

![FIG. 1: Examples of the time-of-flight resonances measured for each of the superallowed beta emitters. The solid curves are fitted functions (see text).](image-url)
0.6 detected ions per bunch, a value that corresponds to
numbers of detected ions are then extrapolated back to
the number of ions per bunch that were detected. Each
from successive sets of scans in our
average of the results from each relevant set. The results
tios (and derived
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Keep the number of ions stored simultaneously in the trap
can cause shifts in the resonance frequency. There are
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Reference frequencies, and have relatively small impact
bers were derived from the observed scattering in the
ions with
and 34 mHz to the individual frequency uncertainties for
5
exist, so we quadratically added an uncertainty of 47, 20

As was done in our earlier measurement of

Excitation time (see legend). The uncertainties given for each
set contain only statistical uncertainties from fitting and those
arising from short-term magnetic field fluctuations.

$Q_{EC}$ value (or, where appropriate, the excitation energy,
$E_{ex}$) for each pair was obtained from the weighted av-
erage of the results from each relevant set. The results
from successive sets of scans in our $^{42}$Sc measurement are
shown in Fig. 4.

In obtaining a final uncertainty on the frequency ra-
tios (and derived $Q_{EC}$ values), we considered more than
just the statistical uncertainties in the fitted resonance
frequencies. Although the slow, linear drift of the mag-
netic field was accounted for by the interpolation process
already described, short-term field fluctuations may also
exist, so we quadratically added an uncertainty of 47, 20
and 34 mHz to the individual frequency uncertainties for
ions with $A=46, 42, 26$, respectively. These num-bers were derived from the observed scattering in the
reference frequencies, and have relatively small impact
on the final uncertainties.

An important source of systematic uncertainty is the number of ions stored simultaneously in the trap, which
can cause shifts in the resonance frequency. There are
two ways to deal with this effect. The first is simply to
keep the number of ions stored simultaneously in the trap
small. We took this approach in the cases of $^{46}$V and in
$^{42}$Sc, for which we filtered the data during analysis,
only including time-of-flight results from bunches that in-
cluded 2 ions or less. We took account of any possible re-
mainning systematic shift due to this non-ideal countrate
by including an additional uncertainty of $\sqrt{2}\times 0.008$ Hz,
as was done in our earlier measurement of $^{62}$Ga.

The second way of dealing with the count-rate effect is
to divide the data into different groups depending on
the number of ions per bunch that were detected. Each
group is fitted separately and a resonance frequency ob-
tained. The resonant frequencies obtained for the various
numbers of detected ions are then extrapolated back to
0.6 detected ions per bunch, a value that corresponds to
1 ion stored per bunch corrected for the known detection
efficiency of 60%\(^{12}\). Only in the case of $^{26}$Al\(^{m}\) did we
have high enough statistics to allow us to analyze the
data following this procedure.

Finally, because most of our measurements were of
mass doublets, each partner having the same mass num-
ber, mass-dependent systematic effects are expected to
cancel out in these cases. Only in the case where $^{25}$Mg
was used as a reference for $A=26$ ions, was it necessary
for us to add an additional uncertainty of $1\times 10^{-8}$ to the
derived results.

Our results for the $Q_{EC}$ (or $E_{ex}$) values are given in
Table II for each measured doublet. Also given are the
derived mass excesses for each identified ion. Our data
for both $A=26$ and $A=42$ allow us to obtain the superal-
lowed $Q_{EC}$ values by two routes: via the direct doublet
measurement and via the combination of the other two
doublets involving the non-0$^+$ state in the mother nu-
cleus. In both cases the two routes led to statistically
consistent results, and it is their average that we quote
for our final $Q_{EC}$ values.

We note as well that for both $A=26$ and $A=42$, we
obtain the excitation energies of the isomeric states in
the mother nuclei. In the case of $^{26}$Al\(^{m}\) we obtain this
energy via three different routes, giving an average result
of 228.27(13) keV. This compares very favorably with the
accepted value\(^{13}\) of 228.305(13), which is based on
$\gamma$-ray measurements. For $^{42}$Sc\(^{m}\) the two paths we have available yield an average excitation energy of 616.61(22),
which also is in tolerable agreement with 616.28(6), the
accepted value\(^{13}\). These results provide a gratifying
confirmation of the consistency of our measurements.

There are three important conclusions we can draw
from our $Q_{EC}$-value results. First, our result for the su-
perallowed $Q_{EC}$ value for $^{46}$V, 7052.72(31) keV, confirms
the recent Savard et al.\(^3\) measurement of 7052.90(40)
keV, and disagrees with the previously accepted value
of 7050.71(89) keV, a survey result\(^1\) principally based on
a 30-year-old ($^4$He,t) $Q$-value measurement by Vonach et al.\(^{14}\).

Second, we can effectively rule out widespread system-
atic differences of more than $\sim$100 eV between reaction-
based $Q$-value measurements and those obtained on
an on-line Penning trap. In their study of past measure-
ments of $(p,\gamma)$ and $(n,\gamma)$ reaction $Q$ values near $^{26}$Al,
Hardy et al.\(^1\) derived a “best” reaction-based result for
the mass excess of $^{26}$Al of -12210.27(11) keV. By com-
paring reaction $Q$ values with much more precise off-line
Penning-trap measurements of stable nuclei in this same
mass region, the authors cited evidence for possible sys-
tematic effects in the former of $\sim$100 eV. They then de-
river a second mass excess for $^{26}$Al of -12210.21(22) keV, a
value that they state has been “adjusted for possible
systematics.” Our measurement of the $^{26}$Al mass excess –
the first one made with a Penning trap – is -12209.95(16)
keV and does not differ significantly from either of the
values presented by Hardy et al.; however, it certainly
agrees more closely with their systematics-adjusted value.
measurements already exist and new Penning-trap superallowed transitions, several precise reaction-based simply to have been wrong. For all other “well known” previously been available [14], a measurement that appears for which only a single dominant measurement had pre-
reasonably be concluded that between reaction and Penning-trap measurements, it can no evidence now of significant systematic differences be-

agreement with the previously accepted values [1] and $Q_{EC}$-values become available. With our Penning-trap results for the superallowed decay branches are indicated by bold type. The reference mass excesses were taken from Ref. [10].

| ion      | reference | #  | frequency ratio, $v_{ref}/v_{ion}$ | $Q_{EC}$ or $E_{ex}$ (keV) | mass excess (keV) |
|----------|-----------|----|------------------------------------|----------------------------|------------------|
| $^{46}$V | $^{40}$Ti | 40 | 1.0001647674(71)                   | 7052.72(31)               | -37070.68(86)   |
| $^{42}$Sc | $^{42}$Ca | 52 | 1.0001644199(52)                   | 6426.14(22)               | -32120.93(32)   |
| $^{42}$Sc | $^{42}$Ca | 29 | 1.00018201961(54)                  | 7042.73(23)               | -31504.34(33)   |
| $^{42}$Sc | $^{42}$Sc | 23 | 1.000157743(58)                    | 616.62(24)                | -31504.64(35)   |

Final superallowed $^{42}$Sc—$^{42}$Ca $Q_{EC}$ value

| $^{26}$Alm | $^{28}$Mg | 22 | 1.0001748934(64)                   | 4232.79(15)               | -11981.79(16)   |
| $^{26}$Al | $^{28}$Mg | 18 | 1.0001654660(64)                   | 4004.63(15)               | -12209.95(16)   |
| $^{26}$Alm | $^{26}$Al | 18 | 1.0000094314(64)                   | 228.30(16)                | -11982.01(17)   |

Final superallowed $^{26}$Alm—$^{26}$Mg $Q_{EC}$ value

| $^{26}$Alm | $^{25}$Mg | 3  | 1.040075606(21)                    | -                           | 11981.36(49)    |
| $^{26}$Al | $^{25}$Mg | 4  | 1.040065775(19)                    | -                           | 12210.15(45)    |
| $^{26}$Alm—$^{26}$Al using $^{25}$Mg as reference | | | | 228.79(62) |

We cannot therefore exclude systematic differences of up to $\sim 100$ eV between reaction-based and on-line trap measurements but anything significantly greater is ruled out. This conclusion is further supported by our $Q_{EC}$-value result for $^{42}$Sc, 6426.13(21) keV, which agrees well with the most precise previous result, 6425.84(17) keV, obtained from (p,$\gamma$) and (n,$\gamma$) reaction $Q$ values [1]. This leads to our third conclusion, that no significant shift in the value of $V_{ud}$ should be anticipated as more and more on-line Penning-trap measurements of the superallowed $Q_{EC}$-values become available. With our Penning-trap results for the $Q_{EC}$-values of $^{26}$Alm and $^{42}$Sc in good agreement with the previously accepted values [1] and no evidence now of significant systematic differences between reaction and Penning-trap measurements, it can reasonably be concluded that $^{46}$V was an anomalous case, for which only a single dominant measurement had previously been available [14], a measurement that appears simply to have been wrong. For all other “well known” superallowed transitions, several precise reaction-based measurements already exist and new Penning-trap $Q_{EC}$-value measurements, when they appear, can safely be averaged on an equal footing with those previous results. To date, on-line Penning-trap results are being quoted with uncertainties comparable to the best of the earlier measurements, so no large changes should be expected in the resultant averages.

Although our result for $^{46}$V confirms that there is a small anomaly in the $Ft$ value for this transition [3], if we incorporate our three new $Q_{EC}$-values and the one from ref. [3] into the 2005 survey data [1] (and include improved radiative corrections [15]) we find $V_{ud} = 0.9737(3)$, only marginally changed – and slightly improved – from the value 0.9738(4) quoted in the survey.

The work was supported by the EU under contract numbers 506065 and HPRI-CT-2001-50034 and by the Academy of Finland under the COE Programme 2000–2005 (Project No. 44875). JCH was supported by the U. S. Dept. of Energy under Grant DE-FG03-93ER40773 and by the Robert A. Welch Foundation. AJ and HP are indebted to financial support from the Academy of Finland (Projects 46351 and 202256).

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