On theory of regular accelerating Universe in Riemann-Cartan spacetime

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Isotropic cosmology built in the Riemann-Cartan spacetime is investigated. Properties of homogeneous isotropic cosmological models filled with usual gravitating matter and scalar fields are studied in the beginning of cosmological expansion near the limiting energy density. It is shown that cosmological models are regular not only with respect to the Hubble parameter and the energy density but also with respect to the torsion and curvature tensors.

Keywords: cosmological singularity; torsion; limiting energy density.

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1. Introduction

The problem of the beginning of the Universe in time – the problem of cosmological singularity (PCS) – and the problem of invisible components of gravitating matter, dark energy and dark matter, remain as the most principal problems of relativistic cosmology built on the base of general relativity theory (GR). Many attempts were undertaken with the purpose to solve these problems in the framework of GR and existent candidates to quantum gravitation theory (string theory/M-theory, loop quantum gravity) as well as different generalizations of GR. Radical ideas connected with the notions of strings, additional spacetime dimensions, spacetime quantization etc. were used. Various hypothetical fields and particles with unusual properties as possible candidates to dark energy and dark matter were discussed in literature. It should be noted that many existent generalizations of Einsteinian gravitation theory are based on ad hoc hypotheses and do not have solid theoretical foundation.

At the same time there is the gravitation theory built on the base of generally accepted field-theoretical principles including the local gauge invariance principle which is a natural generalization of GR and which offers opportunities to solve its principal problems: the Poincaré gauge theory of gravity (PGTG) – the gravitation theory in 4-dimensional physical spacetime with the structure of Riemann-Cartan...
continuum $U_4$. The formation of PGTG is inseparably connected with works $^1,^2$, from which follows that gravitation theory in Riemann-Cartan spacetime is a necessary generalization of GR by including the Lorentz group (the group of tetrad Lorentz transformations) to gauge group corresponding to gravitational interaction $^a$. The simplest PGTG is Einstein-Cartan theory of gravity based on gravitational Lagrangian in the form of scalar curvature of $U_4$ $^4$. In the case of spinless matter gravitational equations of Einstein-Cartan theory are identical to Einstein gravitational equations of GR, and in the case of spinning matter the Einstein-Cartan theory leads to linear relation between spacetime torsion and spin momentum of gravitating matter. Because of this fact the opinion that the torsion is generated by spin momentum of gravitating matter is widely held in literature. However, by taking into account that the torsion tensor plays the role of gravitational field strength corresponding to subgroup of spacetime translations connected directly with energy-momentum tensor, we can conclude that this fact discusses the degenerate character of Einstein-Cartan theory. The situation comes to normal by including to gravitational Lagrangian similarly to the theory of Yang-Mills fields terms quadratic in gauge gravitational field strengths - the curvature and torsion tensors $^5,^6$.

By using sufficiently general expression of gravitational Lagrangian of PGTG including both a scalar curvature and quadratic terms in curvature ($F_{\alpha\beta\mu\nu}$) and torsion ($S_{\alpha\mu\nu}$) tensors with indefinite parameters isotropic cosmology was built and investigated in a number of papers (see Refs. 7-15 and references herein). It was shown that gravitational interaction in the frame of homogeneous isotropic models (HIM) by certain restrictions on indefinite parameters is changed in comparison with GR and can be repulsive by certain conditions that allow solving the PCS and also to explain the acceleration of cosmological expansion at present epoch without using the notion of dark energy. From cosmological equations deduced in Ref.$^{14}$ follows that gravitational repulsion effect takes place at extreme conditions (extremely high energy densities and pressures) at the beginning of cosmological expansion by virtue of existence of limiting (maximum) energy density. As a result all cosmological solutions for HIM filled with gravitating matter satisfying standard energy conditions are regular, and they contain the compression stage before cosmological expansion stage. Gravitational repulsion effect appears also when energy density in HIM is very small that leads to cosmological solutions for accelerating Universe. Repulsion effect in this case has the vacuum origin because the physical spacetime in the vacuum has the structure of Riemann-Cartan continuum with de Sitter metrics and non-vanishing torsion $^{13}$. The change of gravitational interaction in the frame of PGTG in comparison with GR is connected with more complicated structure of physical spacetime, namely with spacetime torsion.

The present paper is devoted to analysis of properties of HIM for accelerating Universe at extreme conditions in the beginning of cosmological expansion. The

$^a$The bibliography of works dedicated to PGTG is given in $^3$. 

principal relations of isotropic cosmology built in the frame of PGTG are given in
Section 2.

2. Principal Relations of Isotropic Cosmology in PGTG

In the frame of PGTG any HIM is described by means of three functions of time:
the scale factor of Robertson-Walker metrics $R$ and two torsion functions $S_1$ and $S_2$
determining non-vanishing components of torsion tensor $S_{\lambda \mu \nu} = -S_{\lambda \nu \mu}$: $S_{10}^1 = S_{30}^3 = S_1(t)$, $S_{123} = S_{231} = S_{312} = S_2(t) \frac{R^3 r^2}{\sqrt{1 - kr^2}} \sin \theta$, where spatial
spherical coordinates are used. Then non-vanishing components of curvature tensor
are determined by means of the following functions $A_i$ ($i = 1, 2, 3, 4$):

$$
A_1 = \dot{H} - 2\dot{S}_1 + H(H - 2S_1),
$$
$$
A_2 = \frac{k}{R^2} + (H - 2S_1)^2 - S_2^2,
$$
$$
A_3 = 2(H - 2S_1)S_2,
$$
$$
A_4 = \dot{S}_2 + HS_2,
$$

where $H = \dot{R}/R$ is the Hubble parameter and a dot denotes the differentiation with
respect to time.

Isotropic cosmology was built by using the following expression of gravitational
Lagrangian (definitions and notations of $^{10}$ are used below):

$$
\mathcal{L}_g = f_0 F + F^{\alpha \beta \mu \nu}(f_1 F_{\alpha \beta \mu \nu} + f_2 F_{\alpha \nu \beta \mu} + f_3 F_{\mu \nu \alpha \beta}) + F^{\mu \nu}(f_4 F_{\mu \nu} + f_5 F_{\nu \mu}) + f_6 F^2 + S^{\alpha \mu \nu}(a_1 S_{\alpha \mu \nu} + a_2 S_{\nu \mu \alpha}) + a_3 S^{\alpha \mu \nu} S_{\beta \mu \nu}.
$$

(1)

The Lagrangian $\mathcal{L}_g$ includes the parameter $f_0 = (16\pi G)^{-1}$ ($G$ is Newton’s gravita-
tional constant, the light velocity $c = 1$) and a number of indefinite parameters: $f_i$
($i = 1, 2, \ldots, 6$) and $a_k$ ($k = 1, 2, 3$). Gravitational equations of PGTG corresponding
to gravitational Lagrangian $\mathcal{L}_g$ allow one to obtain cosmological equations general-
izing Friedmann cosmological equations of GR and equations for torsion functions
given in general form in Ref. 13. These equations contain five indefinite parameters:

$$
a = 2a_1 + a_2 + 3a_3, \quad b = a_2 - a_1,
$$
$$
f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6,
$$
$$
q_1 = f_2 - 2f_3 + f_4 + f_5 + 6f_6, \quad q_2 = 2f_1 - f_2,
$$

and their mathematical structure and physical consequences depend essentially on
restrictions on these parameters. As it was discussed in Ref. 14, the simplest HIM
possessing important physical properties take place if $a = 0$ and $q_2 = 0$. The first
restriction $a = 0$ leads to excluding of higher derivatives of the scale factor $R$ from
cosmological equations $^5$, and the appearance of limiting energy density in the case
of HIM with two torsion functions is connected directly with the second restriction
$q_2 = 0$ $^{14}$. We also give the main relations of isotropic cosmology by using these
restrictions on indefinite parameters.
Cosmological equations generalizing Friedmann cosmological equations of GR take the following form:

\[
\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = \frac{1}{6f_0Z} \left[ \rho - 6bS_2 + \frac{\alpha}{4} (\rho - 3p - 12bS_2^2) \right],
\]

\[
\dot{H} - 2\dot{S}_1 + H(H - 2S_1) = -\frac{1}{12f_0Z} \left[ \rho + 3p - \frac{\alpha}{2} (\rho - 3p - 12bS_2^2) \right],
\]

where \( \rho \) is the energy density, \( p \) is the pressure, the parameter \( \alpha = \frac{f}{f_0} (f > 0) \) has inverse dimension of energy density and \( Z = 1 + \alpha (\rho - 3p - 12bS_2^2) \). Cosmological equations (2)-(3) determine the curvature functions \( A_1 \) and \( A_2 \) as functions of matter parameters \( \rho \) and \( p \) and of the torsion function \( S_2 \) obtained from gravitational equations for HIM. The torsion function \( S_1 \) is determined by the following way:

\[
\dot{S}_1 = -\frac{\alpha}{4Z} [\dot{\rho} - 3\dot{p} + 12f_0\omega HS_2^2 - 12(2b - \omega f_0)S_2\dot{S}_2],
\]

where dimensionless parameter \( \omega = \frac{2f - q}{f} \neq 0 \) is introduced. The torsion function \( S_2^2 \) depends on energy density and pressure as

\[
S_2^2 = \frac{\rho - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12bc(1 - \omega/4)},
\]

where \( X = 1 + \omega(f_0^2/b^2)[1 - (b/f_0) - 2(1 - \omega/4)\alpha(\rho + 3p)] \). In order to investigate inflationary cosmological models, we will consider HIM filled besides usual gravitating matter with energy density \( \rho_m > 0 \) and pressure \( p_m \geq 0 \) also by scalar field \( \phi \) with potential \( V = V(\phi) \). By neglecting the interaction between these two components of gravitating matter, the total energy density \( \rho \) and pressure \( p \) are as follows:

\[
\rho = \frac{1}{2} \dot{\phi}^2 + V + \rho_m \quad (\rho > 0), \quad p = \frac{1}{2} \dot{\phi}^2 - V + p_m.
\]

By minimal coupling with gravitation the equations for gravitating matter take the usual form as in GR:

\[
\dot{\rho}_m + 3H (\rho_m + p_m) = 0,
\]

\[
\ddot{\phi} + 3H \dot{\phi} = -\frac{\partial V}{\partial \phi}.
\]

By certain restrictions on indefinite parameters cosmological equations (2)-(3) take at asymptotics, when energy density is sufficiently small, the form of Friedmann cosmological equations of GR with effective cosmological constant induced by the torsion function \( S_2 \) and describe accelerating Universe in accordance with standard \( \Lambda \)CDM-model of GR. In order to investigate HIM at extreme conditions in the beginning of cosmological expansion, we transform Eqs. (2)-(8) to dimensionless
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form by introducing dimensionless units for all variables and parameter $b$ entering these equations and denoted by means of $\tilde{\cdot}$ as follows:

\[
\begin{align*}
t &\rightarrow \tilde{t} = t/\sqrt{\alpha f_0}, \\
R &\rightarrow \tilde{R} = R/\sqrt{\alpha f_0}, \\
\rho &\rightarrow \tilde{\rho} = \omega \rho, \\
\phi &\rightarrow \tilde{\phi} = \phi/\sqrt{\alpha f_0}, \\
b &\rightarrow \tilde{b} = b/f_0,
\end{align*}
\]

where the differentiation with respect to dimensionless time $\tilde{t}$ is denoted by means of the prime. Obviously Eqs. (6)-(8) conserve their form by this transformation, namely

\[
\begin{align*}
\tilde{\rho} &= \frac{1}{2} \tilde{\phi}'' + \tilde{V} + \tilde{\rho}_m, \\
\tilde{\rho}_m + 3\tilde{H} (\tilde{\rho}_m + \tilde{p}_m) &= 0, \\
\tilde{\phi}'' + 3\tilde{H} \tilde{\phi}' &= -\frac{\partial \tilde{V}}{\partial \tilde{\phi}}.
\end{align*}
\]

The transformation of the torsion function $S_2$ defined by (5) to dimensionless form according to (9) gives:

\[
\tilde{S}_2^2 = \frac{\tilde{\rho} - 3\tilde{\rho}_m}{2\tilde{b}} + \frac{1}{2}\frac{1}{\tilde{b}}(1 + \sqrt{X}) - \frac{\omega}{2}
\]

where

\[
X = 1 + \frac{1}{b}(1 - 1) - 2(1 - \omega/4)\frac{1}{b^2}(\tilde{\rho} + 3\tilde{\rho}_m).
\]

By using (10) we can transform the torsion function $S_1$ determined by (4) to the following dimensionless form:

\[
\tilde{S}_1 = -\frac{3}{4bZ}(\tilde{H} \tilde{D} + \tilde{E}),
\]

where

\[
\begin{align*}
\tilde{D} &= \frac{1}{2} \left(3 \frac{d\tilde{p}_m}{d\tilde{\rho}_m} - 1\right) (\tilde{\rho}_m + \tilde{p}_m) + \frac{1}{3} (\tilde{\rho}_m - 3\tilde{\rho}_m) + \frac{2}{3} \tilde{\phi}'' + \frac{4}{3} \tilde{V} - \frac{\omega}{6(1 - \omega/4)} \sqrt{X} \\
&\quad + \frac{1 - (\omega/2\tilde{b})}{2\sqrt{X}} \left(3 \frac{d\tilde{p}_m}{d\tilde{\rho}_m} + 1\right) (\tilde{\rho}_m + \tilde{p}_m) + \frac{4}{3} \tilde{\phi}_m^2 + \omega(1 - \tilde{b}/2) \frac{3(1 - \omega/4)}{3(1 - \omega/4)} \\
\tilde{E} &= \left(1 + \frac{1 - (\omega/2\tilde{b})}{\sqrt{X}}\right) \frac{\partial \tilde{V}}{\partial \tilde{\phi}} + Z = -\frac{\omega}{4} + \frac{b/2(1 + \sqrt{X})}{1 - \omega/4}
\end{align*}
\]

Dimensionless form of cosmological equations (2)-(3) obtained by multiplying these equations on $(\alpha f_0/\omega)$ and by using (11)-(14) is the following:

\[
\begin{align*}
\frac{k}{\tilde{R}^2} &+ \left[\tilde{H} (1 + \frac{3}{2bZ} \tilde{D} + \frac{3}{2bZ} \tilde{E})\right]^2 = \frac{1}{Z} \left[\tilde{\rho} + (1/2)(\frac{Z}{b} - 1)\right] \\
\left[\tilde{\rho} - 3\tilde{p} + \omega \frac{1 - (b/2)(1 + \sqrt{X})}{1 - \omega/4}\right] + \omega \frac{1 - (b/2)(1 + \sqrt{X})}{4(1 - \omega/4)^2} \\
\end{align*}
\]
\[
(\tilde{H}' + \tilde{H}^2) \left(1 + \frac{3}{2bZ} \tilde{D}' \right) + \frac{3}{2bZ} \left[ \tilde{H}(\tilde{D}' - \frac{Z'}{Z} \tilde{D} + \tilde{E}) + E' - \frac{Z'}{Z} \tilde{E} \right] \\
= -\frac{1}{2Z} \left[ \tilde{\rho} + 3\tilde{p} - \omega \left( \frac{1 - (b/2)(1 + \sqrt{X})^2}{2(1 - \omega/4)^2} \right) \right].
\]

By using the obtained dimensionless form of relations (10)-(16), we will analyze below the behavior of cosmological solutions for accelerating Universe at the beginning of cosmological expansion.

3. Regular Properties of Cosmological Models of Accelerating Universe

Cosmological equations (15)-(16) contain \(\sqrt{X}\), and the condition \(X \geq 0\) leads to principal constraint for admissible energy densities and pressures. In the case of models without scalar field, the equality \(X = 0\) determines a limiting energy density, near by which the gravitational interaction is repulsive. In the case of models containing also scalar field the equality \(X = 0\) determines in space of matter parameters \((\rho_m, \phi, \phi')\) a limiting \(L\)-surface ensuring the existence of limiting energy density, which is different for various solutions. Now we will analyze properties of cosmological solutions when \(X \ll 1\).

Cosmological equation (15) leads to the following expression for the Hubble parameter:

\[
H_\pm = H_L \left[ 1 + \frac{\sqrt{X}}{(1 - (\omega/2b))^2} \frac{\partial V}{\partial \phi} \phi' \pm \frac{2bZ}{3} \left( \frac{Z}{Z'} \rho_m + \frac{1}{2} \phi'^2 + V + \frac{1}{2} (b - 1) (\rho_m - 3p_m - \phi'^2 + 4V + \omega \left(1 - (b/2)(1 + \sqrt{X})^2 \right) \right) \right] \frac{1}{1 - \omega/4} \frac{4}{1 - \omega/4} \right] - \frac{1}{2} \left( \frac{3 dp_m}{d\rho_m} - 1 \right) (\rho_m + p_m) + \frac{1}{3} (\rho_m - 3p_m) + \frac{2}{3} \phi'^2 + \frac{4}{3} V + \frac{\omega (1 - b/2)}{3(1 - \omega/4)} - X \frac{\omega b}{6(1 - \omega/4)} \right]^{-1},
\]

where

\[
H_L = -\frac{2}{(3 \frac{dp_m}{d\rho_m} + 1) (\rho_m + p_m) + 4\phi'^2}.
\]

Since cosmological equations and their solutions are considered below only in dimensionless form, the ”tilde” is omitted in this Section.
In the case of HIM without scalar field the Hubble parameter vanishes by reaching a limiting energy density, and $H_-$ - and $H_+$ -solutions describe compression and expansion stage respectively. If $X \ll 1$, the expression (17) can be written as

$$H_\pm = H_L \frac{1 + B_1 \sqrt{X} + B_2 X + \ldots}{1 + C_1 \sqrt{X} + C_2 X + \ldots}.$$  \tag{19}$$

where

$$B_1 = \frac{1}{(1 - (\omega/2b))} \frac{\partial V}{\partial \phi} \left[ \frac{\partial V}{\partial \phi} + \frac{2bZ^{(0)}}{3} \left[ \frac{1}{Z^{(0)}} (\rho_m + \frac{1}{2} \phi'^2 + V + \frac{1}{2} (\frac{Z^{(0)}}{b} - 1)(\rho_m - 3p_m - \phi'^2 + 4V + \frac{1 - (b/2)}{1 - \omega/4}) + \frac{1}{4(1 - \omega/4)^2} \frac{k}{R^2} \right] \right],$$

$$B_2 = \frac{2b}{3(1 - (\omega/2b))} \frac{\partial V}{\partial \phi} \left[ Z^{(1)} \left[ \frac{1}{Z^{(0)}} (\rho_m + \frac{1}{2} \phi'^2 + V + \frac{1}{2} (\frac{Z^{(0)}}{b} - 1)(\rho_m - 3p_m - \phi'^2 + 4V + \frac{1 - (b/2)}{1 - \omega/4}) + \frac{1}{4(1 - \omega/4)^2} \frac{k}{R^2} \right] \right],$$

$$C_1 = \frac{2}{(1 - (\omega/2b)) \left[ \left( 3 \frac{dp_m}{dp_F} + 1 \right) (\rho_m + p_m) + 4 \phi'^2 \right]} \left[ \frac{2b}{3} Z^{(0)} + \frac{1}{2} \left( 3 \frac{dp_m}{dp_F} + 1 \right) (\rho_m + p_m) + \frac{1}{3} (\rho_m - 3p_m) + \frac{2}{3} \phi'^2 + \frac{4}{3} V + \frac{\omega(1 - b/2)}{3(1 - \omega/4)} \right],$$

$$C_2 = \frac{2}{(1 - (\omega/2b)) \left[ \left( 3 \frac{dp_m}{dp_F} + 1 \right) (\rho_m + p_m) + 4 \phi'^2 \right]} \left[ \frac{2b}{3} Z^{(1)} - \frac{\omega b}{6(1 - \omega/4)} \right].$$

As a result the Hubble parameter can be written in the form of expansion in $\sqrt{X}$:

$$H_\pm = H_L [1 + (B_1 - C_1) \sqrt{X} + (B_2 + C_2 - B_1 C_1) X + \ldots].$$  \tag{22}$$

Any characteristics $F$ of HIM can be presented near $L$-surface similarly:

$$F_\pm = F^{(0)} + F^{(1/2)} \sqrt{X} + F^{(1)} X + \ldots.$$  \tag{23}$$
where coefficients of expansion $F^{(0)}$, $F^{(1/2)}$, $F^{(1)}$... are some functions of matter parameters. If some terms of expansion coefficients (23) contain two signs, we assume that the upper (lower) sign is related to $H_+$-solution ($H_-$-solution) respectively. The function $F$ is continuous on $L$-surface ($X = 0$), if $F^{(0)}$ is continuous. Obviously coefficients of expansion (23) in the case of the Hubble parameter are the following:

$$H^{(0)} = H_L,$$
$$H^{(1/2)} = H_L(B_1 - C_1),$$
$$H^{(1)} = H_L(B_2 + C_1^2 - C_2 - B_1 C_1).$$  

(24)

We can obtain the expansion for the torsion function $S_1$ determined by (13) in the form of expansion (23). It is essential that terms of $D$ and $E$ defined by (14), which are proportional to $(\sqrt{X})^{-1}$, are reduced mutually by virtue of formula (18). As result we have:

$$S_1 = -\frac{3}{4bZ}(H \pm D + E),$$
$$S_1 = S_1^{(0)} + S_1^{(1/2)} \sqrt{X} + ..., $$

$$S_1^{(0)} = \frac{1}{2} H_L \pm \frac{1}{2} \left[ \frac{1}{Z(0)} \left[ \rho_m + \frac{1}{2} \phi'^2 + V \right] \right]^{1/2},$$

$$S_1^{(1/2)} = \frac{1}{1 - \omega/2b} \left( \frac{\omega}{4b} H_L - S_1^{(0)} - \frac{3}{4bZ(0)} H^{(1/2)} \right) \left[ \frac{3}{4b^2 \rho_m} \right]$$

$$\left( \frac{\rho_m + p_m}{3} \right) + \frac{1}{3} (\rho_m - 3p_m) + \frac{2}{3} \phi'^2 + \frac{4}{3} V + \frac{\omega(1 - b/2)}{3(1 - \omega/4)} -$$

$$\frac{3(1 - \omega/4)}{4b^2} H^{(1)} \left[ \left( 3 \frac{d \rho_m}{d \rho_m} + 1 \right) (\rho_m + p_m) + 4 \phi'^2 \right].$$  

(25)

Unlike the Hubble parameter (22), which is continuous function at $L$-surface, the torsion function $S_1$ according to (25) undergoes a finite jump by transition from $H_-$ to $H_+$-solution. Though the expansion (23) for $F_\pm$ contains a term with $\sqrt{X}$, the expansion for derivative $F_\pm'$ is regular because the derivative of $\sqrt{X}$ does not diverge at $X = 0$ by virtue of (18), namely we have:

$$\frac{X'}{\sqrt{X}} = \frac{6}{b^2} (1 - \omega/4) (H^{(1/2)} + H^{(1)} \sqrt{X} + ...)$$

$$\left[ \left( 3 \frac{d \rho_m}{d \rho_m} + 1 \right) (\rho_m + p_m) + 4 \phi'^2 \right].$$  

(26)
As result the derivative of the Hubble parameter near $L$-surface is:

$$H'_\pm = H'_L + \frac{3}{b^2} (1 + \omega/4) H^{(1/2)2} \left[ (3 \frac{d\rho_m}{d\rho_m} + 1) (\rho_m + p_m) + 4\phi'^2 \right] + \left[ H^{(1/2)'} + \frac{9}{b^2} (1 - \omega/4) H^{(1/2)} H^{(1)} \left[ (3 \frac{d\rho_m}{d\rho_m} + 1) (\rho_m + p_m) + 4\phi'^2 \right] \right] \sqrt{X} + ... (27)$$

and for derivative $S'_1$ we obtain the following expansion:

$$S'_1 = S_1^{(0)} + S_1^{(1/2)} \sqrt{X} + ... ,$$

$$S_1^{(0)} = S_1^{(0)'} + \frac{3}{b^2} (1 - \omega/4) H^{(1/2)} \left[ (3 \frac{d\rho_m}{d\rho_m} + 1) (\rho_m + p_m) + 4\phi'^2 \right] S_1^{(1/2)}, ... (28)$$

We see that from mathematical point of view the limiting $L$-surface plays a special role because some physical characteristics undergo a finite jump by transition from $H_-$- to $H_+$-solution. However, it is of principal meaning that all physical characteristics including the torsion functions and curvature tensor are regular for $H_-$- and $H_+$-solutions and do not diverge by approaching to limiting $L$-surface. As it follows from our analysis the Hubble parameter, the torsion function $S_2$ and the curvature functions $A_1$ and $A_2$ determining the structure of cosmological equations (2)-(3) are continuous on limiting $L$-surface while the torsion function $S_1$, the derivative $S'_2$ and as result the curvature functions $A_3$ and $A_4$ undergo a finite jump by transition from $H_-$- to $H_+$-solution. In the case of HIM without scalar field the limiting $L$-surface is transformed into state with limiting energy density ($X = 0$) which corresponds to a bounce, and in this case according to obtained formulas (27)-(28) the derivative of the Hubble parameter and derivative $S'_1$ are continuous by transition from $H_-$- to $H_+$-solution. In the case of HIM with scalar field a bounce takes place in points of extremum surface, which we obtain from cosmological equation (15) by supposing that the Hubble parameter vanishes. In this case all physical characteristics of HIM including inflationary models are continuous at a bounce. Cosmological solutions for inflationary HIM can be found by numerical integration of eqs. (16) and (10) by choosing initial conditions for $(\rho_m, \phi, \phi')$ on extremum surface $H = 0$.  

As it is known, the PGTG based on gravitational Lagrangian (1) by certain restrictions on indefinite parameters of $L^L$ is free of such pathological objects as ghosts and tachyons that was shown by investigation of gravitational perturbations in Minkowski and Einsteinian backgrounds. Restrictions on indefinite parameters used in this paper and allowed the regularity of all HIM are compatible with corresponding restrictions excluding ghosts and tachyons which are obtained in Ref. 6: conditions $a = 0$ and $q_2 = 0$ lead to excluding of massive particles with spin-parity $0^+$ and $0^-$ respectively, and the conditions $\omega \alpha > 0$, which ensures the appearance of limiting energy density, is compatible with the presence of particles with spin-parity
2− and 1− by excluding particles 2+ and 1+ c. As it was noted in Ref. 13, if the physical spacetime in the vacuum has the structure of Riemann-Cartan continuum with de Sitter metrics and non-vanishing torsion, the particle content of PGTG has to be investigated on such background. However, this hard problem is not yet solved. It should be noted that the change of the structure of vacuum spacetime in the frame of PGTG leads to principal differences of gravitational interaction in comparison with other fundamental physical interactions (by supposing that PGTG is correct gravitation theory), and possibly the search of gravitational interaction requires not traditional approach.

4. Conclusion

The investigation of isotropic cosmology built in the framework of PGTG presented above shows that this theory allows one to solve the PCS on the base of classical consideration in four-dimensional physical spacetime. Unlike HIM with the only torsion function $S_1^{7,8}$, in the case of which the torsion and hence the curvature diverge by transition from $H_-$ to $H_+$-solution, all HIM with two torsion functions (by certain restrictions on indefinite parameters) are regular not only with respect to metrics with its time derivatives and energy density, but also with respect to the torsion and curvature tensors. Their regular behaviour is of principal meaning for consistent description of HIM in the frame of classical theory. The investigation of applicability limits of obtained physical results in the case of gravitating systems with lesser spacial symmetry is of principal interest for cosmology as well as for astrophysics. In particular, the existence of limiting energy density and gravitational repulsion effect at extreme conditions can explain the presence of massive objects in galaxies centrum.

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