Quantum model of Fraunhofer diffraction for polarized pure states and connection with the Huygens-Fresnel principle

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A quantum model of Fraunhofer diffraction by an aperture is presented. The diaphragm is considered as a device for measuring the three spatial coordinates of the particles passing through the aperture. This position measurement is described by a matrix similar to the S-matrix of the scattering theory. Due to the measurement of the longitudinal coordinate, the wavelets emission involved in the Huygens-Fresnel principle can occur from several neighboring wavefronts instead of just one. These wavefronts contribute with different weights forming a distribution whose width $\Delta z$ can be fitted to data from measurement of the diffracted wave intensity. The latter undergoes a damping which increases with $\Delta z$ and with the diffraction angle. Whatever $\Delta z$, the values at large angles are plausible contrary to those of the scalar theories of wave optics. The position measurement modifies the polarization states. For an incident photon in an elliptically polarized pure state, diffraction can rotate the ellipse axes by an angle which is a function of the diffraction angle and which can be fitted to the data from measurement of the polarization of the photon detected beyond the diaphragm.

Keywords: Fraunhofer diffraction, position measurement, S-matrix, Huygens-Fresnel principle, large diffraction angles, diffracted light polarization

I. INTRODUCTION

The use of quantum mechanics in diffraction theory has been the subject of many studies. Since the first quantum theory of Fraunhofer diffraction by a grating [1], several models have emerged, using the formalism of path integrals [2-4], the calculation of trajectories in the framework of hidden variables theories [5, 6] or the resolution of the wave equation combined with the use of the Kirchhoff integral [7]. In more recent studies, various topics are discussed such as the effects of diffraction on the transmission of information in quantum optical systems [8], the role of the quantum behavior of the diaphragm electrons in diffraction of light by a small hole [9], the interactions between the quantum states of different modes in diffracted Gaussian beams [10], the connection between orbital angular momentum transfer and helicity in the diffraction of light [11].

However, one question does not seem to have received much attention: the possibility of starting from the postulates of quantum mechanics to treat diffraction by a diaphragm as the consequence of a measurement of the position of the particle associated with the wave as it passes through the aperture. The first model based on this approach relate to the measurement of one transverse coordinate and provides the same predictions as those of wave optics for the case of Fraunhofer diffraction with slits [12]. Afterwards, some aspects of this model were discussed [13]. More recently, quantum trajectories has been used to describe the motion of the particle after the measurement of one transverse coordinate in a model giving predictions for Fraunhofer and Fresnel diffractions by a slit [14]. There does not seem to have been any other publications on this issue so far.

In the model presented below, we start from the observation that the detection of a particle in the far field region beyond a diaphragm provides a measurement of its momentum. Then, we assume that the distribution of this momentum results from a measurement of the three spatial coordinates of the particle during its passage through the aperture and that this position measurement has an effect on the polarization if the particle has spin. The change in momentum and polarization is described by a "diffraction matrix" similar to the S-matrix of the scattering theory [15]. Although this model only applies to the far field, it nevertheless provides specific predictions about the Huygens-Fresnel principle, the diffraction at large angles and, in the case of light, the polarization of the photons detected beyond the diaphragm.

We present the model in Sec. II. Next, some predictions regarding intensity and polarization measurements are described in Sec. III. Finally, we conclude in Sec. IV.

II. QUANTUM MODEL OF FRAUNHOFER DIFFRACTION BY AN APERTURE

A. Measurement of quantities related to the detected particles.

1. Experimental setup and first assumptions. The model applies for an experimental setup with the following characteristics (Fig. I). The diaphragm is a plane assumed to be of zero thickness and perfectly opaque. The
aperture, of finite area, can be of any shape and possibly formed of several parts. The origin of the laboratory frame of reference \((O; x, y, z)\) is located at the aperture and the \((Ox,Oy)\) plane is that of the diaphragm. The source is located on the \(z\) axis and can emit any type of particle. Detectors placed beyond the diaphragm measure the local counting rate and possibly the polarization. The position of a detection point is denoted by its radius-vector \(\textbf{d}\).

It is assumed that there is neither creation nor annihilation of particles during the process of diffraction, so that the source creates a one-particle state which evolves remaining a one-particle state until the time of detection beyond the diaphragm. The distances source-diaphragm and diaphragm-detectors are large enough for the aperture viewed from the detectors to be considered point-like and for the particle to be considered as a free particle before crossing the aperture and before its detection. For simplicity, we suppose that the incident wave is a monochromatic plane wave whose wave vector \(\textbf{k}\) is in the direction of the \(z\) axis.

b. Measurement of the momentum of the detected particles. From the above assumptions, we can assign the momentum \(\hbar \textbf{k}_0\) to the incident particle and a momentum \(\hbar \textbf{k}\) such that:

\[\textbf{k} \simeq \frac{(k/d)}{\textbf{d}}\]  

(1)

to the particle detected at point of radius-vector \(\textbf{d}\), provided that the modulus \(k\) is measured. However, no significant difference between the wavelength of the diffracted wave and that of the incident wave is observed in diffraction experiments with a diaphragm. Hence:

\[k \simeq k_0,\]  

(2)

which is in accordance with kinematics because the particle transfers a very small part of its energy to the diaphragm. So it is not required to determine \(k\) by a special measurement. Furthermore, from experiment, the part of the diffracted wave returning from the aperture to the region where the source is located is probably extremely weak if not zero. So we assume that the particle associated with the diffracted wave is always such that:

\[k_z > 0.\]  

(3)

The relations (1), (2) and (3) imply that it is possible to measure the momentum probability density function (p.d.f.) of the particle after its passage through the aperture on condition of being in diffraction at infinity. The measurement can be performed, for example, by arranging detectors on a hemisphere of center \(O\) and radius \(d\) in the half-space \(z > 0\). The radius must be such that \(\Delta \ll d\), where \(\Delta\) is the size of the aperture, otherwise (1) cannot be used. The criterion for the Fraunhofer diffraction, that is: \(\Delta^2/(\lambda d) \ll 1\) [16–18], is then satisfied if \(d\) is large enough, whatever the value of \(\lambda/\Delta\).

c. Measurement of the polarization of the detected particles. The polarization measuring device (analyzer for photons, Stern and Gerlach apparatus for atoms, etc...) is placed in front of the detector which is located, given \(\textbf{k}\), in the direction of the momentum \(\hbar \textbf{k}\) of the detected particle. The measurement therefore gives the probabilities of the eigenvalues of the spin component on a quantization axis \(Z[\textbf{k}]\) which must be chosen with respect to a coordinate system \(\{x[\textbf{k}], y[\textbf{k}], z[\textbf{k}]\}\) attached to the detected particle. Finally, it is possible to measure, on a particle of spin \(s\), the probability of finding the result \(\sigma\) for its spin component on a \(Z[\textbf{k}]\) axis if the measurement of its momentum gives the result \(\hbar \textbf{k}\). It is therefore a conditional probability.

By convention, the coordinate system attached to the incident particle is the laboratory frame of reference (Fig. 1) whose \(z \equiv \hat{z}[\textbf{k}_0]\) axis is in the direction of the momentum \(\hbar \textbf{k}_0\). For the detected particle, we choose the coordinate system obtained from the laboratory frame of reference by the rotation \(\mathcal{R}(\phi, \theta, 0)\) where the Euler angles \(\phi\) and \(\theta\) are defined according to the \(z\)-\(y\)-\(z\) convention and are respectively the azimuth and the polar angle of \(\textbf{k}\), so that:

\[\textbf{k} = \mathcal{R}(\phi, \theta, 0) \textbf{k}_0 , \quad z[\textbf{k}] \parallel \textbf{k}.\]  

(4)

The zero value of the third Euler angle defines a choice of the directions of the \(x[\textbf{k}]\) and \(y[\textbf{k}]\) axes in the transverse plane to \(\textbf{k}\) such that the coordinate system attached to the detected particle in the case \(\phi = \theta = 0\) is coincident with the laboratory frame of reference.

Two very different cases arise concerning the quantization axis. For a particle of non-zero mass, this axis can be chosen in any direction. There is then an infinite number of possible \(Z[\textbf{k}]\) axes for each vector \(\textbf{k}\). On the other hand, for a particle of zero mass, the quantization axis must be in the direction of the momentum because the only spin component eigenstates are the helicity states [13]. There is then only one possibility which is \(Z[\textbf{k}] = z[\textbf{k}]\), according to the above convention.

B. Diffraction operator.

a. Measurement of the position of the incident particles. Since it is possible to measure the momentum p.d.f. and the polarisation of the particles associated with the diffracted wave at infinity, we can consider the

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FIG. 1. Experimental setup and laboratory frame of reference (right-handed coordinate system).
construction of a quantum model whose purpose is to provide the expressions of these quantities. The model proposed here is based on the assumption that each incident particle undergoes a position measurement as it passes through the aperture. The detection of a particle beyond the diaphragm can indeed be considered as a proof that it has effectively passed through the aperture and that it has therefore been localized at this place during a short period of time with a precision of the order of the size of the aperture [19]. For simplicity, we consider that the localization occurs instantaneously. We then assume that the source emits a particle at time \( t_0 \), that this particle passes the aperture at time \( t_1 \) and that it is detected at time \( t_2 \). The time \( t_1 \) can then be interpreted as the time of the change of state caused by a position quantum measurement carried out by the diaphragm and the purpose of the model is to build a diffraction operator which describes this change of state.

b. Using S-matrix theory formalism. The quantum state of the particle of spin \( s \) at time \( t \) is assumed to be a pure state denoted \( |\psi^{(s)}(t)\rangle \). Since the particle is free just before arriving on the diaphragm and just before being detected, the initial and final states are described with a good approximation by stationary asymptotic states. Given the choice of a monochromatic incident wave of wave vector \( \mathbf{k}_0 \) and given (2), these asymptotic states are energy eigenstates of eigenvalue \( \hbar \omega_0 (\omega_0 \equiv \omega(\mathbf{k}_0)) \) and are such that:

\[
\begin{align*}
|\psi^{(s)}(t_1)\rangle &\approx \exp[-i\omega_0(t_1-t_0)] |\varphi^{(s)}_{\text{in}}\rangle, \\
|\psi^{(s)}(t_2)\rangle &\approx \exp[-i\omega_0(t_2-t_1)] |\varphi^{(s)}_{\text{out}}\rangle,
\end{align*}
\]

(5)

where \( |\varphi^{(s)}_{\text{in}}\rangle \) and \( |\varphi^{(s)}_{\text{out}}\rangle \) are time-independent states. Since time dependence only appears in global phase factors, knowing the exact values of \( t_0, t_1 \) and \( t_2 \) is not essential and, as in the S-matrix theory, we consider a diffraction operator \( \hat{D}^{(s)} \) which projects the initial time-independent state on the final time-independent state (called "initial state" and "final state" in the following).

The change of state is expressed by:

\[
|\varphi^{(s)}_{\text{out}}\rangle = \left[ N^{(s)}(s) \right]^{-1/2} \hat{D}^{(s)} |\varphi^{(s)}_{\text{in}}\rangle,
\]

(6)

where \( N^{(s)}(s) \) is the normalization factor:

\[
N^{(s)}(s) \equiv \left\langle \varphi^{(s)}_{\text{in}} | \hat{D}^{(s)^\dagger} \hat{D}^{(s)} | \varphi^{(s)}_{\text{in}} \right\rangle.
\]

(7)

All the information on the "particle-diaphragm interaction" is contained in the matrix elements of the diffraction operator from which we can get the transition amplitudes between the initial state and the final momentum and spin component eigenstates. Since we only consider one-particle states, these eigenstates are represented by the state vectors:

\[
\begin{align*}
s = 0 : \quad &\hat{a}^\dagger(\mathbf{k}) |\psi\rangle = |\mathbf{k}\rangle, \\
\quad &s \neq 0 : \quad \hat{a}^\dagger(\mathbf{k}, \sigma) |\psi\rangle = |\mathbf{k}\rangle \otimes |\sigma\rangle_{Z[\mathbf{k}]},
\end{align*}
\]

(8)

where \( |\psi\rangle \) is the vacuum state, \( \hat{a}^\dagger(\mathbf{k}, \sigma)_{Z[\mathbf{k}]} \) is the creation operator of a particle of momentum \( \hbar \mathbf{k} \) and spin component \( \sigma \) on the quantization axis \( Z[\mathbf{k}] \) and \( |\sigma\rangle_{Z[\mathbf{k}]} \) is the eigenstate of spin component \( \sigma \) on \( Z[\mathbf{k}] \). The initial state is given by:

\[
|\varphi^{(s)}_{\text{in}}\rangle = \begin{cases} |\mathbf{k}_0\rangle & \text{if } s = 0 \\ |\mathbf{k}_0\rangle \otimes |\chi^{(s)}_{\text{in}}\rangle & \text{if } s \neq 0, \end{cases}
\]

(9)

where \( |\chi^{(s)}_{\text{in}}\rangle \) is the initial state of spin polarization prepared with the amplitudes \( Z[\mathbf{k}_0]\langle \sigma | \chi^{(s)}_{\text{in}} \rangle \).

c. Structure of the diffraction operator. From (6) and (9), the non-normalized final state for a particle without spin is expressed by:

\[
\hat{D}^{(0)} |\varphi^{(0)}_{\text{in}}\rangle = |\mathbf{k}_0\rangle = \int d^3k \langle \mathbf{k} | \hat{D}^{(0)} |\mathbf{k}_0\rangle.
\]

(10)

To generalize this expression to the case of a particle of non-zero spin, we rely on the following observation. For the photon, the quantization axis is in the direction of the momentum and the eigenvalue zero of the spin component is impossible [13]. Therefore, the change in the direction of the momentum of the photon due to diffraction causes a modification of its spin polarization so that this impossibility of the eigenvalue zero is preserved. This suggests that the change in the photon polarization is determined, at least in part, by the change in its momentum. We assume that it is also the case for any other particle. The change in polarization corresponds to a re-arrangement of the spin component wave functions and therefore results from the action of a unitary rotation operator. So we are led to assume that if the measurement of the momentum of the detected particle gives the result \( \hbar \mathbf{k} \) then the probabilities of the results of a simultaneous measurement of the spin component correspond to a polarization state which depends on \( \mathbf{k} \) in the form:

\[
|\chi^{(s)}_{\text{out}}(\mathbf{k})\rangle = \hat{R}^{(s)}(\alpha_1(\mathbf{k}), \alpha_2(\mathbf{k}), \alpha_3(\mathbf{k})) |\chi^{(s)}_{\text{in}}\rangle,
\]

(11)

where \( \hat{R}^{(s)}(\alpha_1(\mathbf{k}), \alpha_2(\mathbf{k}), \alpha_3(\mathbf{k})) \) is the operator of the spin rotation associated with the momentum transfer \( \hbar \mathbf{k}_0 \rightarrow \hbar \mathbf{k} \). The Euler angles \( \alpha_j(\mathbf{k}) \) are three parameters of the model. They are functions, not known a priori, of \( \mathbf{k} \) and \( \mathbf{k}_0 \) and possibly depend on other quantities as for example the spin of the particle: \( \alpha_j(\mathbf{k}) \equiv \alpha_j(\mathbf{k}, \mathbf{k}_0, s, \ldots) \), \( j = 1, 2, 3 \). The state \( |\chi^{(s)}_{\text{out}}(\mathbf{k})\rangle \) is in some way the "conditional state" of polarization associated with the momentum eigenstate \( |\mathbf{k}\rangle \).

An additional assumption is needed to generalize (10). For a spinless particle, the position and momentum wave functions are Fourier transforms of each other. In the case of diffraction with a diaphragm, the shape of the final momentum distribution is therefore determined by the shape of the aperture. We assume that this determination is the same if the particle has spin, so that the
final momentum distribution of a particle with spin is the same as that of a spinless particle which would have the same energy. There do not seem to be any experimental facts invalidating this assumption.

The easiest way to generalize (10) taking into account (9), (11) and the additional assumption above is to express the action of $\hat{D}^{(s)}$ on the initial state in the following form (we use the notation $\hat{R}^{(s)}(k)$ instead of $\hat{R}^{(s)}[\alpha_{1}(k),\alpha_{2}(k),\alpha_{3}(k)]$ for simplicity and we insert the identity operator $\sum_{\sigma} |\sigma\rangle z[k]|\sigma\rangle$):

$$s \neq 0:\quad \hat{D}^{(s)}|\varphi^{(s)}_{\text{in}}\rangle = \hat{D}^{(0)}|\chi^{(s)}_{\text{in}}\rangle$$

$$= \int d^{3}k|k\rangle \langle k| \hat{\Delta}^{(0)}|k_{0}\rangle \otimes |\chi^{(s)}_{\text{out}}(k)\rangle$$

$$= \int d^{3}k|k\rangle \langle k| \hat{\Delta}^{(0)}|k_{0}\rangle \otimes \sum_{\sigma} |\sigma\rangle z[k]|\sigma\rangle \langle \hat{R}^{(s)}(k)|\chi^{(s)}_{\text{in}}\rangle. \quad (12)$$

From (6), (9), (10) and (12), the final state is a linear combination of the momentum and spin component eigenstates given by (8) and the diffraction operator is:

$$\hat{D}^{(s)} = \begin{cases} \hat{D}^{(0)} & \text{if } s = 0 \\ \int d^{3}k|k\rangle \langle k| \hat{\Delta}^{(0)} \otimes \hat{R}^{(s)}(k) & \text{if } s \neq 0. \end{cases} \quad (13)$$

The operator $\hat{D}^{(0)}$ will be called "momentum part" of the diffraction operator $\hat{D}^{(s)}$.

d. General expressions of the final amplitudes and probabilities. From (11) and since $\hat{R}^{(s)}(k)$ is unitary:

$$\langle \chi^{(s)}_{\text{out}}(k)\rangle \langle \chi^{(s)}_{\text{out}}(k)\rangle = \langle \chi^{(s)}_{\text{in}}\rangle \langle \chi^{(s)}_{\text{in}}\rangle = 1. \quad (14)$$

From (7) into which we substitute (10) (if $s = 0$) or (12) (if $s \neq 0$) and given (14), we find that the normalization factor is independent of the spin:

$$\forall s:\quad N^{(s)} = N = \int d^{3}k|k\rangle \langle k| \hat{\Delta}^{(0)}|k_{0}\rangle \otimes |\chi^{(s)}_{\text{in}}\rangle. \quad (15)$$

The probability amplitude to detect a particle of zero spin with momentum $\hbar k$ is obtained by substituting (10) into (6). Given (3) and (15), this leads to:

$$\langle k|\varphi^{(0)}_{\text{out}}\rangle = N^{-1/2} \langle k| \hat{\Delta}^{(0)}|k_{0}\rangle. \quad (16)$$

If $s \neq 0$, the probability amplitude to detect a particle of momentum $\hbar k$ and spin component $\sigma$ on the $Z[k]$ axis is obtained by substituting (12) into (9). Given (15) and (10), this leads to:

$$\langle k| z[k]|\sigma\rangle \langle \varphi^{(s)}_{\text{out}}\rangle = \langle k| \varphi^{(0)}_{\text{out}}\rangle z[k] \langle \sigma| \chi^{(s)}_{\text{out}}(k)\rangle. \quad (17)$$

The p.d.f. to detect a particle of zero spin with momentum $\hbar k$ is:

$$f^{(0)}_{k}(k) = \langle \langle k| \varphi^{(0)}_{\text{out}}\rangle \rangle. \quad (18)$$

The joint probability function to detect a particle of spin $s \neq 0$ with momentum $\hbar k$ and spin component $\sigma$ on the $Z[k]$ axis is expressed, according to the definition of the conditional probability and from (17), by:

$$f_{k}^{(s)}(k,|\sigma|Z[k]) = f_{k}^{(s)}(k) \int_{|\sigma|Z[k]} P^{(s)}_{Z[k]|K=k}(|\sigma|Z[k])$$

$$= \langle \langle k| \varphi^{(0)}_{\text{out}}\rangle \rangle^{2} |Z[k]| \langle \sigma| \chi^{(s)}_{\text{out}}(k)\rangle. \quad (19)$$

where $f_{k}^{(s)}(k)$ is the p.d.f. to detect, without polarization measurement, a particle of spin $s$ with momentum $\hbar k$ and $P^{(s)}_{Z[k]|K=k}(|\sigma|Z[k])$ is the conditional probability to detect a particle with spin component $\sigma$ on the $Z[k]$ axis if its momentum is $\hbar k$.

If $s \neq 0$, $f_{k}^{(s)}(k)$ is the marginal p.d.f. obtained by summing (19) over $\sigma$. Given (14) and (18), this leads to $f_{k}^{(s)}(k) = f_{k}^{(0)}(k)$. Hence, given (10) and (18):

$$\forall s:\quad f_{k}^{(s)}(k) = f_{k}^{(0)}(k) = N^{-1} \langle \langle k| \hat{\Delta}^{(0)}|k_{0}\rangle \rangle, \quad (20)$$

which expresses that the momentum p.d.f. of the detected particle without polarization measurement is independent from its spin and initial polarization. Moreover, substituting (10) into (19) and given (20), we get:

$$P^{(s)}_{Z[k]|K=k}(|\sigma|Z[k]) = |Z[k]| \langle \sigma| \chi^{(s)}_{\text{out}}(k)\rangle. \quad (21)$$

The experimentally accessible quantities are those given by (20) and (21). To calculate them, we therefore need to express the matrix elements $\langle \langle k| \hat{\Delta}^{(0)}|k_{0}\rangle \rangle$ and the amplitudes $Z[k] \langle \sigma| \chi^{(s)}_{\text{out}}(k)\rangle$. This is the subject of the next two subsections.

C. Momentum part of the diffraction operator

In this subsection, we first deal with the case of non-relativistic particles. We will then show that the developed formalism can be transposed to the case of photons.

a. Position measurement and Huygens-Fresnel principle. At the time $t_{1}$ of the position measurement, the position wave function of the incident particle undergoes a filtering so that the new position wave function is non-zero only if the transverse coordinates ($x, y$) correspond to the aperture $A$ of the diaphragm (postulate of wave function reduction). It can be considered that the transmission of the incident wave is the same over the entire area of the aperture so that the initial position wave function is simply transversely truncated. This type of filtering is both a transverse filtering and a uniform filtering.
It is obtained by making the following projector:
\[ \hat{P}_T^A \equiv \int_A dx dy \, |xy⟩⟨xy| \]  
(22)
- called "transverse filtering projector" - act on the initial state. However, this projector is defined in the space of states associated with the transverse motion of the particle whereas the latter moves in the 3D space. It is therefore essential to generalize \( \hat{P}_T^A \) by an operator acting in the space of states associated with the 3D motion of the particle. The simplest way is to define the projector:
\[ \hat{P}_T^A,\Delta z \equiv \hat{P}_T^A \otimes \hat{P}_L^{\Delta z}, \]  
(23)
where:
\[ \hat{P}_L^{\Delta z} \equiv \int_{-\Delta z/2}^{+\Delta z/2} dz \, |z⟩⟨z| \]  
(24)
is a "longitudinal filtering projector" whose action can be interpreted as a projection corresponding to a measurement of \( z \) giving a result inside the interval \([-\Delta z/2, +\Delta z/2]\) centered at \( z = 0 \) which is the longitudinal coordinate of the diaphragm. This measurement is then associated with a longitudinal filtering of the initial position wave function.

In summary, the action of \( \hat{P}_T^A,\Delta z \) corresponds to a measurement of the three coordinates \((x, y, z)\) with an accuracy of the order of the size of the aperture \( A \) for the transverse coordinates and an accuracy of the order of \( \Delta z \) for the longitudinal coordinate. In this context, \( \Delta z \) is a parameter of the model. Besides the case where \( \Delta z \) is finite, there are the following limit cases corresponding to two opposite situations:

- if \( \Delta z \to \infty \), the projector \( \hat{P}_L^{\Delta z} \) tends to the identity operator. The action of the projector \( \hat{P}_T^A,\Delta z \) then corresponds to a measurement of \((x, y)\) (transverse filtering, accuracy given by the size of the aperture \( A \)) without measurement of \( z \) (no longitudinal filtering, no accuracy).

- if \( \Delta z \to 0 \) \((\Delta z \neq 0)\), the action of \( \hat{P}_T^A,\Delta z \) localizes the probability of presence of the particle in the region of the wavefront at the aperture and consequently its longitudinal coordinate is \( z = 0 \) with almost infinite accuracy. This localization of the probability of presence occurs at time \( t_1 \). Therefore, at any time \( t > t_1 \), the diffracted wave has been emitted from a volume including the wavefront at the aperture and its close vicinity. We are then close to a situation consistent with the Huygens-Fresnel principle. This case seems more plausible than the case \( \Delta z \to \infty \) for which such compatibility does not appear.

If \( \Delta z = 0 \), a perfect compatibility with the Huygens-Fresnel principle would be obtained provided that the p.d.f. of the longitudinal coordinate of the particle just after the localization of the wave function at time \( t_1 \) is equal to the Dirac distribution \( δ(z) \). However, it is not possible to obtain this result using \( \hat{P}_T^A,\Delta z \) because the integral of the right-hand side is zero if \( \Delta z = 0 \). Nevertheless, given the good agreement between the measurements performed so far and the predictions of the classical theories based on the Huygens-Fresnel principle, this is worth looking for a way to treat this limit case. Fortunately, it turns out that this is possible provided, however, that the notion of projector is generalized.

b. Position filtering operator. From \( (23) \), we have: \( \hat{P}_T^A,\Delta z \ket{k_0} = \hat{P}_T^A \ket{k_{0x} k_{0y}} \otimes \hat{P}_L^{\Delta z} \ket{k_{0z}} \). So from \( (24) \), the p.d.f. corresponding to the probability to obtain a result within the interval \([z, z + dz]\) when measuring the longitudinal coordinate is proportional to:
\[ |⟨z | \hat{P}_L^{\Delta z} | k_{0z}⟩|^2 = \begin{cases} (2\pi)^{-1} & \text{if } z \in [-\Delta z/2, +\Delta z/2] \\ 0 & \text{if } z \notin [-\Delta z/2, +\Delta z/2] \end{cases} \]  
(25)
It is not possible to obtain a p.d.f. from this function if \( \Delta z = 0 \). However, a p.d.f. equal to \( δ(z) \) can be obtained if we replace the projector \( \hat{P}_L^{\Delta z} \) by a filtering operator \( \tilde{F}_L^{\Delta z} \) defined as:
\[ \tilde{F}_L^{\Delta z} \equiv \int dz \, \sqrt{\delta_L^{\Delta z}(z)} \, |z⟩⟨z|, \]  
(26)
where \( \delta_L^{\Delta z}(z) \) is a positive function such that:
\[ \int dz \, \delta_L^{\Delta z}(z) = 1, \quad \lim_{\Delta z \to 0} \delta_L^{\Delta z}(z) = δ(z), \]  
(27)
and such that its integral outside the interval \([-\Delta z/2, +\Delta z/2]\) is negligible. From \( (26) \):
\[ |⟨z | \tilde{F}_L^{\Delta z} | k_{0z}⟩|^2 = |⟨z | k_{0z}⟩|^2 \delta_L^{\Delta z}(z) = (2\pi)^{-1} \delta_L^{\Delta z}(z). \]  
(28)
Given \( (27) \), \( |⟨z | \tilde{F}_L^{\Delta z} | k_{0z}⟩|^2 \) is proportional to \( δ(z) \) if \( \Delta z = 0 \). The p.d.f. obtained after normalization is then equal to \( δ(z) \). From the first equality of \( (28) \), we can interpret \( \delta_L^{\Delta z}(z) \) as the weight with which the filtering operator selects the result \( z \) from the value at \( z \) of the position wave function in the initial state \( |k_{0z}⟩ \). This weight, as a function of \( z \), will be called longitudinal position filtering function.

In the case of uniform filtering (UF), the filtering function of the filtering operator \( \tilde{F}_L^{\Delta z,UF} \) is: \( \delta_L^{\Delta z,UF}(z) = (\Delta z)^{-1} \) if \( z \in [-\Delta z/2, +\Delta z/2] \) or 0 otherwise. From \( (24) \) and \( (26) \), we have: \( \tilde{F}_L^{\Delta z,UF} = (\Delta z)^{-1/2} \hat{P}_L^{\Delta z} \), which implies that the action of the two operators leads to the same state after normalization. Therefore, the filtering operator can be used in place of the usual projector in the particular case of a uniform filtering (provided that \( \Delta z \neq 0 \)). We assume that this type of operator is appropriate for the more general case of a non-uniform filtering.

In particular, the longitudinal filtering could be non-uniform for the following reason. The truncation made by the projector \( \tilde{P}_L \) is similar to that made by the projector \( \hat{P}_T^A \) involved in the transverse filtering (Eqs. \( (22) \)
and \((25)\)). However, the two filterings are probably not of the same type because the aperture is limited by a material edge in the transverse plane whereas there are no edges along the longitudinal direction. The longitudinal filtering could then be a non-uniform filtering associated with a continuous filtering function forming a peak centered around \(z = 0\) and of width \(\Delta z\). The precise shape of the filtering function is part of the assumptions of the model. This shape may matter if \(\Delta z\) is large but probably not if \(\Delta z\) is close to zero because the p.d.f. is then close to the Dirac distribution.

The aperture can be defined as the 3D region where the position wave function is temporarily localized during the position measurement. According to this definition, the interval \([-\Delta z/2, +\Delta z/2]\) can be considered as an aperture in the longitudinal direction. In the case of a uniform filtering, it represents the region where the filtering function is non-zero. In the case of a non-uniform filtering, the filtering function can be non-zero everywhere (for example if it is a Gaussian). We are then led to define more generally the aperture as the interval outside of which the integral of the filtering function is negligible.

In summary, for the longitudinal coordinate, it is assumed that the projector \(\hat{P}_L^\Delta z\) defined by \((21)\) can be replaced by a filtering operator \(\hat{F}_L^\Delta z\) of the form \((26)\). Using a similar generalization for the transverse coordinates, we assume that the projector \(\hat{F}_T^A\) given by \((22)\) can be replaced by a filtering operator of the form:

\[
\hat{F}_T^A = \int dx dy \sqrt{\delta_T^A(x, y)} \langle xy \rangle \langle xy \rangle ,
\]

where \(\delta_T^A(x, y)\) is the transverse position filtering function. Since the transverse filtering is assumed to be uniform, this function is given by:

\[
\delta_T^A(x, y) = \begin{cases} 
S(A)^{-1} & \text{if } (x, y) \in A \\
0 & \text{if } (x, y) \notin A,
\end{cases}
\]

where \(S(A)\) is the area of \(A\). Finally, given \((20)\) and \((29)\), the projector \(\hat{F}_T^A \Delta z\) defined by \((23)\) can be replaced by the filtering operator: \(\hat{F}_T^A \Delta z = \hat{F}_T^A \otimes \hat{F}_L^\Delta z = \int d^3r \sqrt{\delta_T^A(r)} \sqrt{\delta_L^\Delta z} |r\rangle \langle r|,\)

where:

\[
\delta_T^A(x, y) \equiv \delta_T^A(x, y) \delta_L^\Delta z(z).
\]

The volume \(A \equiv A \times [-\Delta z/2, +\Delta z/2]\) of transverse section \(A\) and length \(\Delta z\), centered at the origin \(O\) is called 3D aperture. The aperture \(A\) and the interval \([-\Delta z/2, +\Delta z/2]\) are respectively called transverse 2D aperture and longitudinal 1D aperture (Fig. 2).

In \((31)\), \(\Delta z\) does not depend on \(x\) and \(y\), which is an implicit assumption in the definition \((24)\). More generally, the position filtering operator is defined by:

\[
\hat{F}^A = \int d^3r \sqrt{\delta^A(r)} |r\rangle \langle r| ,
\]

where \(\delta^A(r)\) is the position filtering function whose expression can be assumed to be different from that of \(\delta_T^A \Delta z\) given by \((31)\).

c. Need to consider kinematics. From \((32)\),

\[
|\langle r \mid \hat{F}^A \mid k_0 \rangle|^2 \text{ proportional to } \delta^A(r).
\]

So the state \(\hat{F}^A \mid k_0 \rangle\) is associated with the momentum p.d.f. of the particle just after its localization at the aperture, when it begins to move away from the diaphragm. Moreover, from \((20)\), the state \(\hat{D}^{(0)} \mid k_0 \rangle\) corresponds to the momentum p.d.f. \(f_k(k)\) of the particle detected beyond the diaphragm. Since the particle is free after its passage through the aperture and before its detection, this suggests that \(\hat{D}^{(0)}\) is nothing other than \(\hat{F}^A\). However, this cannot be the case for the following reason. From \((32)\), the momentum wave function of the state \(\hat{F}^A \mid k_0 \rangle\) is expressed by:

\[
\langle k \mid \hat{F}^A \mid k_0 \rangle = (2\pi)^{-3/2} \mathcal{F}^A(k - k_0),
\]

where \(\mathcal{F}^A(k - k_0)\) is the Fourier transform of the square root of the position filtering function:

\[
\mathcal{F}^A(k - k_0) \equiv (2\pi)^{-3/2} \frac{\sqrt{\delta^A(r) \exp[-i(k - k_0) \cdot r]}}{\int d^3r \sqrt{\delta^A(r)}}.
\]

If \(\hat{D}^{(0)}\) is equal to \(\hat{F}^A\), the p.d.f. \(f_k(k)\) is obtained by substituting \((33)\) into \((20)\). Then, the widths \(\Delta k_x, \Delta k_y\) and \(\Delta k_z\) of this p.d.f. are those of the distribution associated with the Fourier transform \(\mathcal{F}^A(k - k_0)\) and are therefore related to the widths \(\Delta x, \Delta y\) and \(\Delta z\) of the 3D aperture by the uncertainty relations. However, if \(\Delta x\) for example is small enough, the relation \(\Delta k_x \Delta x \gtrsim 1\) implies that \(\Delta k_x\) can be sufficiently large so that \(|k_x| > k_0\) with
non-zero probability and therefore the relation (2) is not satisfied in such a case. But this is not possible because (2) results from kinematics and is moreover confirmed by experiment. This issue comes from the fact that the position wave function of the state \( \hat{F}^A | k_0 \rangle \) is localized in the 3D aperture \( A \) and that consequently its momentum wave function is spread out, which results in a spreading of the distribution of the momentum modulus and therefore of the energy. For (2) to be satisfied, we are led to assume that \( \hat{D}^{(0)} \) is not simply equal to \( \hat{F}^A \) but is rather of the form:

\[
\hat{D}^{(0)} = \hat{F}^{k_0} \hat{F}^A,
\]

where \( \hat{F}^{k_0} \) is an energy-momentum filtering operator whose role is to project the state \( \hat{F}^A | k_0 \rangle \) which is then a transitional virtual state - on a final state of same energy as that of the initial state.

d. Energy-momentum filtering operator. It is logical to assume that \( \hat{F}^{k_0} \) has a form similar to that of \( \hat{F}^A \) (Eq. (32)) and is associated with a domain \( \delta_0 \) of the momentum space corresponding to the vectors \( k \) compatible with kinematics. So we define:

\[
\hat{F}^{k_0} \equiv \int d^3k \sqrt{\delta \kappa_0(k)} \ | k \rangle \langle k |,
\]

where \( \delta \kappa_0(k) \) is a momentum-energy filtering function which must represent the weight with which the filtering operator selects the result \( k \) from the value at \( k_0 \) of the momentum wave function in the transitional virtual state \( \hat{F}^A | k_0 \rangle \). From (2) and (3), we are led to assume that this function is of the form:

\[
\delta \kappa_0(k) \equiv C \delta \Delta k([k] - k_0) \delta_1 \mathrm{sgn}[k_z],
\]

where \( C \) is a normalization constant that will be calculated below, \( \delta \Delta k([k] - k_0) \) is a function of the modulus of \( k \) forming a peak centered at \([k] = k_0\) and of width \( \Delta k \) close to zero (in accordance with (2)) and the Kronecker delta \( \delta_1 \mathrm{sgn}[k_z] \) ensures that \( \delta \kappa_0(k) \) is zero if \( k_z \leq 0 \) (in accordance with (3)). From (37), using the spherical coordinates, the normalization to 1 of \( \delta \kappa_0(k) \) is expressed by:

\[
1 = C \int_0^{\infty} dk k^2 \delta \Delta k(k - k_0) \int_0^{\pi} \sin \theta \delta_1 \mathrm{sgn} \cos \theta \int_0^{2\pi} d\phi.
\]

Since \( \Delta k \) is close to zero, we can replace \( \delta \Delta k(k - k_0) \) by \( \delta(k - k_0) \) in the integral over \( k \) whose value is therefore close to \( k_0^2 \). Then, (38) implies: \( C \approx k_0^{-2} (2\pi)^{-1} \). Substituting (37) with this value of \( C \) into (36), we get:

\[
\hat{F}^{k_0} \approx (2\pi)^{-1/2} k_0^{-1} \int d^3k \sqrt{\delta \Delta k([k] - k_0) \delta_1 \mathrm{sgn}[k_z]} \ | k \rangle \langle k |.
\]

We can interpret \( \hat{F}^{k_0} \) as an operator which represents an energy-momentum measurement including a measurement of the momentum modulus (in other words of the energy) giving the result \( \hbar k_0 \) with near certainty and a measurement of the momentum longitudinal component giving the result \( \hbar k_z > 0 \).

e. Matrix element of the momentum part of the diffraction operator. Substituting (32) - in which we insert the identity operator \( \int d^3k | k \rangle \langle k | \) after \( | r \rangle \langle r' | \) - and (39) into (55), and given (34), we obtain:

\[
\hat{D}^{(0)} \approx (2\pi)^{-2} k_0^{-1} \int d^3k \sqrt{\Delta \kappa([k] - k_0) \delta_1 \mathrm{sgn}[k_z]} \times \int d^3k' \hat{F}^A(k - k') | k \rangle \langle k' |.
\]

Hence, instead of (55):

\[
\langle k | \hat{D}^{(0)} | k_0 \rangle \approx (2\pi)^{-2} k_0^{-1} \times \sqrt{\Delta \kappa([k] - k_0) \delta_1 \mathrm{sgn}[k_z]} \hat{F}^A(k - k_0).
\]

f. Photons. A position filtering operator of the form (32), where the projector \( | r \rangle \langle r | \) is involved, cannot be used for the photon due to the issues raised by the construction of a position operator and by the interpretation of the position wave function in the case of a zero mass particle [20][21]. However, we can get an expression of the momentum part of the diffraction operator for a photon by replacing in (32) the projector \( | r \rangle \langle r | \) by an operator constructed from the scalar product \( \hat{E}^{-}(r,t) | \text{vac} \rangle \langle \text{vac} | \hat{E}^{(+)}(r,t) \) where \( \hat{E}^{(+)}(r,t) \) and \( \hat{E}^{-}(r,t) \) are the field operators respectively associated with the positive and negative frequency components of the free electric field which is directed in the transverse plane to \( k \). These field operators are given by [24]:

\[
\hat{E}^{-}(r,t) = \left[ \hat{E}^{(+)}(r,t) \right]^\dagger = -i \sqrt{\frac{\hbar c}{2\varepsilon_0}} (2\pi)^{-3/2} \int d^3k \sum_{l=x,y} \sqrt{k} \exp[ -i(k \cdot r - \omega t)] \epsilon_l(k) \hat{a}^\dagger (k,l|k|),
\]

where \( \omega = ck \), \( \epsilon_l(k) \) is the unitary vector of the \( l|k| \) axis of the coordinate system attached to \( k \) and \( \hat{a}^\dagger (k,l|k|) \) is the creation operator of a photon of momentum \( \hbar k \) and linearly polarized in the direction of the \( l|k| \) axis. Similarly to (8), we have:

\[
\hat{a}^\dagger (k,l|k|) | \text{vac} \rangle = | k \rangle \otimes | l \rangle | k \rangle,
\]

where \( | l \rangle _k \) is the basis state of linear polarization in the direction of the \( l|k| \) axis. From (42) and (43):

\[
\hat{E}^{-}(r,t) | \text{vac} \rangle \langle \text{vac} | \hat{E}^{(+)}(r,t) = \frac{\hbar c}{2\varepsilon_0} (2\pi)^{-3} \int d^3k \int d^3k' \sqrt{kk'} e^{i(k - k') \cdot r} e^{i(|k| - |k'|)r} | k \rangle \langle k' | \times \sum_{l=x,y} \sum_{l'=x,y} \epsilon_l^*(k) \epsilon_{l'}(k') | l \rangle | k \rangle | l' \rangle.
\]
The photon has a spin 1 and this implies that its spin projection eigenstates are equivalent to vectors of complex components in the basis \( \{ \mathbf{e}_k^{(x)}, \mathbf{e}_k^{(y)}, \mathbf{e}_k^{(z)} \} \). Moreover, the basis states \( |l\rangle_k \) are specific linear combinations of the spin projection eigenstates \( |l\rangle_k \) such that \( |l\rangle_k \) is equivalent to the real vector \( \mathbf{e}_k \). Therefore, replacing in (32) particles and for photons.

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From (35), we have:

\[ \hat{F}_A^A(k_0) \equiv \hat{F}_A^A(k_0) \hat{F}_A^A(k_0) \tag{46} \]

Then, substituting (33) and (15) into (46) and given (34) - we finally obtain:

\[ \hat{D}_\text{phot}^A(t) \simeq \begin{pmatrix} \hat{F}_A^A(k_0) \hat{F}_A^A(k_0) \end{pmatrix} \]

By calculating the matrix element \( \langle k | \hat{D}_\text{phot}^A(t) | k_0 \rangle \) from (47), we get an expression with the factor \( \exp[i(\omega - \omega_0)t] \). Now, from (2), we have: \( k \simeq k_0 \) so that \( \omega \approx \omega_0 \). Therefore, the matrix element in question does not actually depend on time and we find that its expression is nothing other than that given by (47). This relation can therefore be used both for non-relativistic particles and for photons.

Moreover, using (32), (39) and (15), we can verify that the product of operators in the right-hand sides of (35) and (14) is not commutative. This non-commutativity imposes the order in which the operators act to create the final state from the initial state. This order is related to the temporal unfolding of an irreversible process whose sequence is: initial state \( \rightarrow \) position measurement \( \rightarrow \) transitional virtual state \( \rightarrow \) energy-momentum measurement \( \rightarrow \) final state.

### D. Polarization amplitudes of the detected particles

#### a. General case.

From (11), we can express the final polarization amplitudes in terms of the initial ones as (using the notation \( \alpha_j \equiv \alpha_j(k) \) for simplicity):

\[
\langle \text{in} | \sigma \rangle \chi^{(s)}(k) = \sum_{\sigma' \sigma''} z[k_0 \sigma' | \sigma'' \rangle \chi^{(s)}(k_0) \chi^{(s)}(k) \tag{48}
\]

where two different quantization axes, \( Z[k_0] \) and \( Z[k] \), are involved. The scalar product \( Z[k] \sigma | \sigma'' \rangle \chi^{(s)}(k) \) is related to the rotation which transforms the quantization axis \( Z[k_0] \) used for the incident particle into the quantization axis \( Z[k] \) used for the detected particle. We have:

\[
Z[k] = R(\phi, \Theta, \Psi) Z[k_0],
\]

where the Euler angles \( \Phi, \Theta, \Psi \) are defined with respect to a coordinate system whose \( z \) axis is \( Z[k_0] \). The angles \( \Phi \) and \( \Theta \) depend on \( k \) and \( \phi \). The angle \( \Psi \) defines the directions of the axes in the transverse plane to \( Z[k] \) - is arbitrary and we choose \( \Psi = 0 \) for simplicity. The rotation that changes the quantization axis causes a rotation of the eigenstates with the same Euler angles because a physical system in the eigenstate \( | \sigma \rangle \) must rotate with the quantization axis so that it remains an eigenstate with the eigenvalue \( \sigma \). Hence:

\[
| \sigma \rangle \chi^{(s)}(k) = \hat{R}^{(s)}(\phi, \Theta, 0) | \sigma \rangle \chi^{(s)}(k_0). \tag{49}
\]

Using the conjugate of this expression and since the unitarity of the rotation operator implies:

\[
\hat{R}^{(s)}(\phi, \Theta, 0) = \hat{R}^{(s)}(\phi, \Theta, 0)^{-1} = \hat{R}^{(s)}(0, -\Theta, -\phi), \tag{50}
\]

we are led to:

\[
\langle Z[k] \sigma | \sigma'' \rangle = z[k_0 \sigma' | \sigma'' \rangle \chi^{(s)}(k_0) \chi^{(s)}(k) \tag{49}
\]

where \( \Theta, \Phi, \) and the \( \alpha_j \) depend on \( k(\phi, \theta, \phi) \). The matrix element of the product of the rotation operators can be calculated from the standard formula:

\[
\langle \sigma \rangle \hat{R}^{(s)}(\alpha, \beta, \gamma) \sigma' \rangle = e^{-i\alpha \beta} d^{(s)}(\beta) e^{-i\alpha' \gamma}, \tag{50}
\]
where \((d^0_{\alpha\sigma}(\beta))\) is a \((2s + 1) \times (2s + 1)\) matrix whose expression is known \(^{20}\).

b. Particles of zero mass. The amplitude \(^{19}\) cannot be expressed in a general way as a function of \(\theta\) and \(\phi\) for a particle of non-zero mass because the functions \(\Theta(\theta, \phi)\) and \(\Phi(\theta, \phi)\) are specific to the choice of the quantization axes. On the other hand, the case of a particle of zero mass is straightforward because we must then choose: \(Z[k_0] = z[k_0]\) and \(Z[k] = z[k]\), so that the change of quantization axis is given by: \(z[k] = R(\Phi, \Theta, 0) z[k_0]\). Now, from \(^{14}\): \(z[k] = R(\Phi, \Theta, 0) z[k_0]\). Therefore: \(\Phi = \phi\) and \(\Theta = \theta\). Then, using for simplicity the notation \(|\sigma\rangle_k\) instead of \(|\sigma\rangle_{z[k]}\) (since \(z[k] \parallel k\)), \(^{(49)}\) becomes:

\[
k \langle \sigma | \chi^{(s)}_{\text{out}}(k) \rangle = \sum_{\sigma'} k_0 \langle \sigma | \hat{R}^{(s)}(0, -\theta, -\phi) \times \hat{R}^{(s)}(\alpha_1, \alpha_2, \alpha_3) | \sigma' \rangle_{k_0} k_0 \langle \sigma' | \chi^{(s)}_{\text{in}} \rangle.
\]

In the rest of this subsection, we apply the model to the case of the photon.

c. Spin component amplitudes of the detected photons. Since the photon has a spin 1, the eigenstates of its spin component are \(|+1\rangle\), \(|0\rangle\), \(|-1\rangle\) but since it also has zero mass, the quantization axis is in the direction of its momentum and the eigenvalue zero is impossible whatever this momentum \(^{13}\). Therefore:

\[
k \langle 0 | \chi^{(1)}_{\text{out}}(k) \rangle = k_0 \langle 0 | \chi^{(1)}_{\text{in}} \rangle = 0.
\]

This relation determines the functions \(\alpha_1[k(k, \theta, \phi)]\) and \(\alpha_2[k(k, \theta, \phi)]\). Indeed, substituting it into \(^{(51)}\) applied to \(s = 1\) and \(\sigma = 0\), we obtain:

\[
0 = \sum_{\sigma' = \pm 1} k_0 \langle 0 | \hat{R}^{(1)}(0, -\theta, -\phi) \times \hat{R}^{(1)}(\alpha_1, \alpha_2, \alpha_3) | \sigma' \rangle_{k_0} k_0 \langle \sigma' | \chi^{(1)}_{\text{in}} \rangle,
\]

which must be satisfied whatever the initial state. Hence:

\[
k_0 \langle 0 | \hat{R}^{(1)}(0, -\theta, -\phi) \hat{R}^{(1)}(\alpha_1, \alpha_2, \alpha_3) | \pm 1 \rangle_{k_0} = 0.
\]

We then express the left-hand side by using \(^{(50)}\) applied to \(s = 1\) and where the matrix \((d^1_{\sigma\sigma'}(\beta))\) is given by \(^{20}\):

\[
(d^1_{\sigma\sigma'}(\beta)) = \frac{1}{2} \begin{pmatrix}
1 + \cos \beta & -\sqrt{2} \sin \beta & 1 - \cos \beta \\
\sqrt{2} \sin \beta & 2 \cos \beta & -\sqrt{2} \sin \beta \\
1 - \cos \beta & \sqrt{2} \sin \beta & 1 + \cos \beta
\end{pmatrix}.
\]

\(\text{(Note: the order of the values of } \sigma \text{ and } \sigma' \text{ is: } +1, 0, -1).\)

This leads to the equations:

\[
\begin{align*}
\sin \theta \sin(\phi - \alpha_1) &= 0, \\
\sin \theta \cos \alpha_2 \cos(\phi - \alpha_1) - \cos \theta \sin \alpha_2 &= 0.
\end{align*}
\]

The first equation implies \(\alpha_1(k) = \phi + n\pi, n = 0, 1\). Substituting into the second equation, we get:

\[
\alpha_2(k) = (-1)^n \theta + n'\pi, n' = 0, 1.
\]

If \(\phi = \theta = 0\), we then have: \(k = k_0\), which implies \(\alpha_1(k_0) = n\pi\) and \(\alpha_2(k_0) = n'\pi\). But if \(k = k_0\), there is no reason for the spin polarization state to change. Hence: \(\chi^{(s)}_{\text{out}}(k_0) = \chi^{(s)}_{\text{in}}\) and, given \(^{(11)}\): \(\hat{R}^{(1)}(\alpha_1(k_0), \alpha_2(k_0), \alpha_3(k_0))\) is equal to the identity operator, which implies: \(\alpha_1(k_0) = \alpha_2(k_0) = \alpha_3(k_0) = 0\). Therefore: \(n = n' = 0\) and we obtain:

\[
\begin{align*}
\alpha_1(k) &= \phi, \\
\alpha_2(k) &= \theta,
\end{align*}
\]

\(\alpha_3(k_0) = 0.\)

From \(^{(50)}\), \(^{(53)}\) and \(^{(57)}\), the matrix whose elements appear in the right-hand side of \(^{(51)}\) is given by:

\[
\begin{pmatrix}
\exp[-i\alpha_3(k)] & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \exp[i\alpha_3(k)]
\end{pmatrix}.
\]

Finally, from \(^{(51)}\), \(^{(57)}\) and \(^{(59)}\):

\[
k \langle \sigma | \chi^{(1)}_{\text{out}}(k) \rangle = \exp[-i\sigma \alpha_3(k)] k_0 \langle \sigma | \chi^{(1)}_{\text{in}} \rangle.
\]

\(\text{Diffraction causes a phase shift of } 2\alpha_3(k) \text{ between the amplitudes of the helicity states } |\pm 1\rangle \) and conserves the modulus of each of these amplitudes.

d. Linear polarization amplitudes of the detected photons. It is useful to express the amplitudes of linear polarization for any orientation of the analyzer. We define the coordinate system attached to the analyzer by: \(\{x[k, \gamma], y[k, \gamma], z[k]\}\), where \(x[k, \gamma]\) is the axis of maximum transmission and \(\gamma\) is the azimuth of this axis with respect to the \(x[k, \gamma]\) axis of the coordinate system attached to the particle (so that: \(\{x[k, 0], y[k, 0], z[k]\}\) = \(\{x[k], y[k], z[k]\}\)). The azimuth \(\gamma\) is also the angle of a rotation around the \(z[k]\) axis. The action of such a rotation is expressed by: \(l[k, \gamma] = R(\gamma_1, 0, \gamma_2) l[k], (l = x, y)\) where \(\gamma_1 + \gamma_2 = \gamma\). This rotation of the axes is equivalent to the reverse rotation of the physical system and is therefore associated with the operator \(\hat{R}^{(1)}(\gamma_2, 0, -\gamma_1)\). From \(^{(50)}\), \(^{(55)}\) and the property: \((d^1_{\sigma\sigma'}(0)) = \delta_{\sigma\sigma'}\), the result of the action of this rotation operator on the spin component eigenstates is the following phase change:

\[
|\sigma\rangle_{k, \gamma} = \exp(-i\sigma\gamma) |\sigma\rangle_k.
\]

The helicity states and the basis states of linear polarization in the directions of the \(l[k, \gamma]\) axes are related by \(^{24, 26}\):

\[
|\xi\rangle_{k, \gamma} = \frac{-\xi}{\sqrt{2}} \left( |x\rangle_{k, \gamma} + i |y\rangle_{k, \gamma} \right),
\]

where \(\xi = \pm 1\) is the helicity. According to \(^{(61)}\) applied to the helicity states \(|\xi\rangle_{k, \gamma}\) and \(|\xi\rangle_k\) expressed from
We use the following definitions:

\[ \zeta_0 \equiv \zeta(k_0) = \text{angle between the } x[k_0] \text{ axis and the major axis of the ellipse in the transverse plane to } k_0, \ 0 \leq \zeta_0 < \pi. \]

\[ \eta_0 = \arctan[\text{length of the minor axis}/\text{length of the major axis}], \ 0 \leq \eta_0 \leq \pi/4. \]

\[ \xi_0 = \pm 1, \text{ represents the direction of rotation of the electric field vector (provided that } \eta_0 \neq 0). \text{ The value } \xi_0 = +1 \text{ corresponds to a counterclockwise rotation if the rotation axis and the momentum of the photon are directed toward the receiver.} \]

If \( \eta_0 = 0 \), the polarization is linear along the direction defined by the angle \( \zeta_0 \). If \( \eta_0 = \pi/4 \), the polarization is circular and \( \zeta_0 \) is equal to the helicity because \( \xi_0 = \pm 1 \) becomes identical to \( \xi = \xi_0 \), \( k = k_0 \) and \( \gamma = \zeta_0 \).

\[ \zeta_0 = \zeta(k_0) = \text{angle between the } x[k_0] \text{ axis and the major axis of the ellipse in the transverse plane to } k_0, \ 0 \leq \zeta_0 < \pi. \]

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III. SOME PREDICTIONS OF THE MODEL

A. Relative intensity (polarization not measured)

a. Angular distribution of the final momentum. From (20) and (21), the p.d.f. of the final momentum if the polarization is not measured is expressed by:

\[ f_{k|k}(k) \simeq N^{-1}(2\pi)^{-1+k_0^{-2}} \times \delta^{\Delta k}(|k| - k_0) \delta_{\text{spin}}[k_z] |F^A(k - k_0)|^2. \]

Since the experimental setup directly measures the direction of \( k \), it is useful to replace the Cartesian components by the modulus and two angles giving the direction. Given \( \theta_0 \), this change of variables must be done by a one-to-one transformation defined in the half-space \( k_z > 0 \). The spherical coordinates \( k, \theta, \phi \) cannot be used because the associated transformation is not one-to-one (if \( \theta = 0, \phi \) is undefined and the Jacobian is zero). On the other hand, we can use the diffraction angles \( \theta_x \) and \( \theta_y \) which are the projections of the polar angle \( \theta \) on the planes \((x, z)\) and \((y, z)\) (Fig. 3). The new variables

\[ (k, \theta_x, \theta_y) \] are such that: \( k > 0, -\pi/2 < \theta_x < +\pi/2, -\pi/2 < \theta_y < +\pi/2 \) and the required transformation \((k_x, k_y, k_z) \leftrightarrow (k, \theta_x, \theta_y)\) is:

\[ k(k, \theta_x, \theta_y) = k \cos \theta \begin{pmatrix} \tan \theta_x \\ \tan \theta_y \\ 1 \end{pmatrix}, \]

\[ \cos \theta = \left(1 + \tan^2 \theta_x + \tan^2 \theta_y\right)^{-1/2}, \ 0 \leq \theta < \pi/2. \]

The change of p.d.f. due to the change of variables is expressed by:

\[ f_{k, \theta_x, \theta_y}(k, \theta_x, \theta_y) = |J(k, \theta_x, \theta_y)| f_{k|k}(k(k, \theta_x, \theta_y)). \]

where \( J(k, \theta_x, \theta_y) \) is the determinant of the Jacobian of the transformation (70) which is finite and non-zero and whose calculation leads to the angular factor:

\[ \Gamma(\theta_x, \theta_y) \equiv k^{-2} |J(k, \theta_x, \theta_y)| = \frac{\cos \theta}{1 - \sin^2 \theta_x \sin^2 \theta_y}. \]
Expressing $f_{K} (k, \theta_{x}, \theta_{y})$ from (69) and substituting into (14), given (72), we get:

$$f_{K,\theta_{x},\theta_{y}} (k, \theta_{x}, \theta_{y}) \simeq N (2\pi)^{-4} k_{0}^{-2} k^{2} \overline{\delta} (k-k_{0}) \times \Gamma (\theta_{x}, \theta_{y}) \mid F^{A} [k(k, \theta_{x}, \theta_{y}) - k(k_{0}, 0, 0)] \mid^{2}. \quad (73)$$

From (2), $\Delta k$ is close to zero. We can therefore replace the function $\overline{\delta} (k-k_{0})$ by the Dirac distribution $\overline{\delta} (k-k_{0})$ and express the angular distribution of the final momentum by:

$$f_{\theta_{x},\theta_{y}} (\theta_{x}, \theta_{y}) \equiv \int_{0}^{\infty} dk' \int_{0}^{2\pi} d\theta_{x} \int_{0}^{2\pi} d\theta_{y} f_{K,\theta_{x},\theta_{y}} (k', \theta_{x}, \theta_{y}) \simeq \frac{\Gamma (\theta_{x}, \theta_{y})}{(2\pi)^{4} N} \mid F^{A} [k(k, \theta_{x}, \theta_{y}) - k(k_{0}, 0, 0)] \mid^{2}. \quad (74)$$

where we now consider for simplicity that $k$ represents both the modulus of $k_{0}$ and that of $k$. The normalization factor $N$ can be expressed by substituting (14) into (15). Using the change of variables (70) and given (2), we get:

$$N \simeq (2\pi)^{-4} \int_{-\pi/2}^{\pi/2} d\theta_{x} \int_{-\pi/2}^{\pi/2} d\theta_{y} \times \Gamma (\theta_{x}, \theta_{y}) \mid F^{A} [k(k, \theta_{x}, \theta_{y}) - k(k_{0}, 0, 0)] \mid^{2}. \quad (75)$$

From (74) and since $\Gamma (0, 0) = 1$, this leads to:

$$\left[ \frac{I (\theta_{x}, \theta_{y})}{I (0, 0)} \right]^{A}_{\text{QM}} \simeq \frac{\Gamma (\theta_{x}, \theta_{y})}{(2\pi)^{4} N} \mid F^{A} [k(k, \theta_{x}, \theta_{y}) - k(k_{0}, 0, 0)] \mid^{2}. \quad (76)$$

From (74) and since $\Gamma (0, 0) = 1$, this leads to:

$$\left[ \frac{I (\theta_{x}, \theta_{y})}{I (0, 0)} \right]^{A}_{\text{QM}} \simeq \frac{\Gamma (\theta_{x}, \theta_{y})}{(2\pi)^{4} N} \mid F^{A} [k(k, \theta_{x}, \theta_{y}) - k(k_{0}, 0, 0)] \mid^{2}. \quad (77)$$

For an aperture of the form $\mathcal{A} \equiv \mathcal{A} \times [-\Delta z/2, +\Delta z/2]$, where $\Delta z$ is independent of $(x, y)$, the position filtering function $\overline{\delta} (r)$ is equal to $\overline{\delta} (\mathcal{A}, \Delta z) (r)$ given by (31). From this and (30), the relation (31) leads to:

$$\mathcal{F}^{A} (k - k_{0}) = \mathcal{F}_{L}^{A} (k_{x}, k_{y}) \mathcal{F}_{L}^{\Delta z} (k - k_{0}) \quad (78)$$

where:

$$\mathcal{F}_{L}^{A} (k_{x}, k_{y}) \equiv (2\pi)^{-1} S (\mathcal{A})^{-1/2} \times \int_{\mathcal{A}} dx dy \exp [-i (k_{x} x + k_{y} y)]. \quad (79)$$

$$\mathcal{F}_{L}^{\Delta z} (k_{x} - k_{0}) \equiv (2\pi)^{-1/2} \times \int dz \sqrt{\delta_{L}^{\Delta z} (z)} \exp [-i(k_{z} z)}. \quad (80)$$

Substituting (78) into (77) and expressing $k(k, \theta_{x}, \theta_{y})$ from (70), we obtain:

$$\left[ \frac{I (\theta_{x}, \theta_{y})}{I (0, 0)} \right]^{A}_{\text{QM}} \simeq \frac{\Gamma (\theta_{x}, \theta_{y})}{(2\pi)^{4} N} \mid F^{A} [k(k, \theta_{x}, \theta_{y}) - k(k_{0}, 0, 0)] \mid^{2}. \quad (81)$$

where $\theta$ and $\Gamma (\theta_{x}, \theta_{y})$ are respectively given by (70) and (72), $T^{A} (k, \theta_{x}, \theta_{y})$ is the transverse diffraction term:

$$T^{A} (k, \theta_{x}, \theta_{y}) \equiv \frac{\mathcal{F}_{L}^{A} (k \cos \theta \tan \theta_{x}, k \cos \theta \tan \theta_{y})}{\mathcal{F}_{L}^{2 A} (0, 0)} \quad (82)$$

and $L^{A} (k, \theta)$ is the longitudinal diffraction term:

$$L^{A} (k, \theta) \equiv \frac{\mathcal{F}_{L}^{A} [k (\cos \theta - 1)]}{\mathcal{F}_{L}^{2 A} (0, 0)} \quad (83)$$

c. Extended application of the Huygens-Fresnel principle. The relative intensity expressed by the quantum formula (31) depends on the width $\Delta z$ of the longitudinal 1D aperture (Fig. 2). The value of $\Delta z$ can therefore be fitted to data obtained from the measurement of the intensity as a function of the defraction angle. The interpretation of the result is then as follows. The Huygens-Fresnel principle applied to a single wavefront corresponds to the case $\Delta z = 0$. The case $\Delta z > 0$ would therefore correspond to a situation where the Huygens-Fresnel principle applies to several wavefronts contributing with different weights whose distribution is the longitudinal position filtering function $\overline{\delta}_{L}^{\mathcal{A}, \Delta z} (z)$. An experimental study directly concerning the Huygens-Fresnel principle can therefore be considered.

d. Comparison with the predictions of the scalar theories of wave optics. In wave optics (WO), there are several versions of the scalar theory of diffraction which differ by their assumed boundary conditions. The best known are the theories of Fresnel-Kirchhoff (FK) and Rayleigh-Sommerfeld (RS1 and RS2). In Fraunhofer diffraction, for an initial monochromatic plane wave, the amplitude predicted by these theories at a point of radius vector $d$ beyond the diaphragm can be expressed, given (1), in the form (16-17):

$$\mathcal{U}_{\text{WO}}^{A} (d) \equiv \mathcal{U}_{\text{WO}}^{A,k} (d, \frac{k}{k}) \simeq - C_{0} \frac{i k \exp [i k (d_{0} + d)]}{2 \pi d_{0} d} \quad (84)$$

where $C_{0}$ is a constant, $d_{0}$ is the distance source-aperture and $\Omega [(k_{0}, k)]$ is the obliquity factor. The latter depends on the deflection angle $(k_{0}, k)$ which is also the polar...
angle θ (Fig. 3). The obliquity factor is specific to the theory:

\[
\Omega(\theta) = \begin{cases} 
(1 + \cos \theta)/2 & \text{(FK)} \\
\cos \theta & \text{(RS1)} \\
1 & \text{(RS2)}.
\end{cases}
\] (85)

From (11), the intensity at point of radius-vector \( \mathbf{d} \) is proportional to the intensity in the direction of \( \mathbf{k}(k, \theta_x, \theta_y) \). Hence:

\[
\left[ \frac{I(\theta_x, \theta_y)}{I(0, 0)} \right]_\text{WO}^A = \frac{|\mathcal{U}(k, \theta_x, \theta_y)|^2}{|\mathcal{U}(k, 0, 0)|^2}.
\] (86)

Expressing \( \mathbf{k}(k, \theta_x, \theta_y) \) and \( \mathbf{k}(k, 0, 0) \) from (70) and substituting into (84) then into (86), we see that \( d \) is eliminated. Then, since \( \Omega(0) = 1 \) and given (79) and (82):

\[
\left[ \frac{I(\theta_x, \theta_y)}{I(0, 0)} \right]_\text{WO}^A \approx \Omega(\theta)^2 T^A(k, \theta_x, \theta_y).
\] (87)

The comparison of formulae (81) and (82) shows that the transverse diffraction term \( T^A(k, \theta_x, \theta_y) \) is the same in the two cases. This is because the integrals in (79) and (82) are the same. The differences come from the angular factors \( \Gamma(\theta_x, \theta_y) \) and \( \Omega(\theta)^2 \) and from the presence of the longitudinal diffraction term \( L^{\Delta z}(k, \theta) \) in the quantum formula. If the angles are small, the angular factors and the longitudinal diffraction term are all close to 1 so that the quantum model gives the same result as that of wave optics. On the other hand, if the angles increase, discrepancies appear between the different predictions.

**e. Example of comparison.** Let us consider the intensity variation in the horizontal plane \((Ox, Oz)\) for which we have: \( \theta_y = 0, \theta_x = \theta \) if \( \theta_x \geq 0, \theta_x = -\theta \) if \( \theta_x \leq 0 \). In this case, it is convenient to make the notation change: \( (\theta_x, \theta) \rightarrow (\theta, |\theta|) \), where \(-\pi/2 < \theta < +\pi/2\) (diffraction angle) and \(0 \leq |\theta| < +\pi/2\) (polar angle in the half-space \( z > 0 \)). Since \( \cos |\theta| = \cos \theta \), the relations (72) and (83) then lead to:

\[
\Gamma(\theta, 0) = \cos \theta, \quad \Omega(|\theta|) = \Omega(\theta).
\] (88)

We now consider the case of a rectangular slit \( R \) of width \( 2a \) and of height \( 2b \) centered at \( (x, y) = (0, 0) \). The expression (70) leads to:

\[
\mathcal{F}_1^R(k_x, k_y) = \frac{ab}{\pi} \sin ak_x \sin bk_y.
\] (89)

Given the notation change introduced above, the relation (71) implies: \( k_x = k \cos |\theta| \tan \theta = k \sin \theta \) and \( k_y = 0 \). Applying (89) to these values and substituting into (82), we get the well-known result:

\[
T^R(k, \theta, 0) = \left[ \frac{\sin(ak \sin \theta)}{ak \sin \theta} \right]^2.
\] (90)

Then, we suppose that the longitudinal filtering function is for example a Gaussian. In this case, the width of the longitudinal aperture depends on the standard deviation and on a threshold under which the integral of the Gaussian outside the interval \([-\Delta z(\sigma_z)/2, +\Delta z(\sigma_z)/2]\) is considered as negligible (for example, with a threshold of \(10^{-2}\), we have: \( \Delta z(\sigma_z) \approx 5.16 \sigma_z \)). Assuming that \( \delta L^{\Delta z}(\sigma_z) \) is a Gaussian centered at \( z = 0 \) and of standard deviation \( \sigma_z \), the expression (80) leads to (82):

\[
\mathcal{F}_L^{\Delta z}(\sigma_z)(k_z - k) = \left( \frac{2\pi}{\sigma_z} \right)^{1/4} \sqrt{\pi} \exp \left[ -\frac{\sigma_z^2 (k_z - k)^2}{} \right].
\] (91)

Substituting into (83), we get:

\[
L^{\Delta z}(\sigma_z)(k, |\theta|) = \exp \left[ -8 \sigma_z^2 k^2 \sin^2(\theta/2) \right].
\] (92)

Curves obtained from formulae (81) and (87) (applied with (83), (88), (90) and (92)) are shown in Fig. 4 for a case of photon diffraction.

If \( \sigma_z = 0 \), the longitudinal diffraction term is equal to 1. This corresponds to the largest values predicted by the quantum model. It is with the FK theory that the quantum model is in better agreement. However, at 90°, the FK theory predicts values that are generally non-zero, which does not seem plausible (same for the RS2 theory). The angular factors \( \Gamma(\theta, 0) = \cos \theta \) of the quantum model and \( \Omega(\theta)^2 = \cos^2 \theta \) of the RS1 theory are the only ones which account for the decrease in intensity towards zero at 90°. However, the factor \( \cos \theta \) seems more likely because it is the same as that obtained by applying the exact calculation of the diffraction by a wedge (29) to the case of two wedges of zero angle placed opposite one another to form a slit (30).

If \( \sigma_z > 0 \), the longitudinal diffraction term is strictly less than 1. So the values of the quantum model, maximum for \( \sigma_z = 0 \), undergo a damping which increases with \( |\theta| \) and \( \sigma_z \). When \( \sigma_z \) is increasing from zero, the QM curve eventually pass below the RS1 curve. Coincidently, these curves can be very close but they cannot be exactly the same everywhere because the factors \( \cos \theta \) and \( \cos^2 \theta \) are different. The value of \( \sigma_z \) can be fitted to the data. If \( \sigma_z \) is large enough, the QM curve decreases globally much more rapidly than the WO curves and a significant gap can be obtained at not too large angles. Such a result obtained experimentally would be a strong signal that the Huygens-Fresnel principle must apply to several wavefronts.

**f. Large diffraction angles.** From the above analysis, it turns out that the relative gaps between the predictions of the different models considered here are significant at large angles. Moreover, from a survey of the literature, it seems that no accurate experimental study of the diffraction in this region has been carried out so far. Since the time when the FK and RS1-2 theories were formulated (late nineteenth century), technologies in optics have made tremendous progress thanks in particular
to accurate measurements of intensity by charge-coupled devices which make it possible to achieve a sufficiently expanded dynamic range. An experimental study of this still unexplored region is therefore probably feasible at the present time.

### B. Polarization probabilities (photons)

From (21) and (67), the conditional probability to detect a photon of helicity \( \xi \) if its momentum is \( h\mathbf{k} \) is:

\[
P_{\{\xi|\mathbf{k}\}}^1(\mathbf{k}) = \left| \left\langle \mathbf{k} \left| \chi_{\text{out}}(\mathbf{k}) \right| \xi \right\rangle \right|^2,
\]

or

\[
P_{\{\xi|\mathbf{k}\}}^1(\mathbf{k}) = \left| \left\langle \mathbf{k} \left| \chi_{\text{in}}(\mathbf{k}) \right| \xi \right\rangle \right|^2.
\]

From (94), the conditional probability to detect a photon of helicity \( \xi \) if its momentum is \( h\mathbf{k} \) is:

\[
P_{\{\xi|\mathbf{k}\}}^1(\mathbf{k}) = \left| \left\langle \mathbf{k} \left| \chi_{\text{out}}(\mathbf{k}) \right| \xi \right\rangle \right|^2 = 1 - \left| k,\gamma \left\langle \mathbf{y} \right| \chi_{\text{in}}(\mathbf{k}) \right\rangle \right|^2.
\]

So the probabilities of the helicity states and consequently of the circular polarizations are conserved.

For an elliptically polarized initial state \( |\chi_{\text{in}}^{(1)}\rangle \), with major axis azimuth \( \zeta_0 \), ellipticity angle \( \eta_0 \) and handedness \( \xi_0 \) (Eq. (69)), the conditional probabilities of linear polarization in the direction defined by the angle \( \gamma \) with respect to the \( x|\mathbf{k}\rangle \) axis are expressed, from (67) and given (64), by:

\[
P_{\{\xi|\mathbf{k},\gamma\}}^1(|x|\mathbf{k},\gamma) = \left| k,\gamma \left\langle \mathbf{y} \right| \chi_{\text{out}}(\mathbf{k}) \right\rangle \right|^2 = 1 - \left| k,\gamma \left\langle \mathbf{y} \right| \chi_{\text{in}}(\mathbf{k}) \right\rangle \right|^2.
\]

\[
= \frac{1}{2} \left\{ 1 + \cos 2\eta_0 \cos 2(\zeta_0 + \alpha_3(k) - \gamma) \right\},
\]

where \( \alpha_3(k) \) is the rotation angle of the ellipse axes due to diffraction. From (94), we have (provided that \( \eta_0 \neq \pi/4 \)):

\[
\alpha_3(k) = \gamma - \zeta_0 + \frac{1}{2} \arccos \frac{2 \left| k,\gamma \left\langle \mathbf{y} \right| \chi_{\text{out}}(\mathbf{k}) \right\rangle \right|^2 - 1}{\cos 2\eta_0},
\]

In the case of a linear polarization \( (\eta_0 = 0) \), the final polarization is also linear in the direction defined by the angle \( \zeta_0 + \alpha_3(k) \) (Eq. (68)). The measurement can be carried out by rotating the polarization measuring device around \( z|\mathbf{k}\rangle \) so as to find the angle \( \gamma_1(k) \) such that

\[
\left| k,\gamma_1(k) \left\langle \mathbf{z} \right| \chi_{\text{out}}(\mathbf{k}) \right\rangle \right|^2 = 1.
\]

Then, (69) leads to:

\[
\alpha_3(k) = \gamma_1(k) - \zeta_0.
\]

### IV. CONCLUSION

It is possible to build a model of Fraunhofer diffraction by a diaphragm based exclusively on quantum mechanics by using the S-matrix formalism and the concept of quantum measurement applied to a 3D aperture. The model presented here suggests that the Huygens-Fresnel principle applies to several close wavefronts distributed along the longitudinal direction in the aperture region. These wavefronts contribute with different weights to the amplitude of the diffracted wave and the width of their distribution can be fitted to the data from measurement of the intensity as a function of the diffraction angle. If this width is large enough, a significant damping of the intensity at large angles is predicted. A direct experimental study of the Huygens-Fresnel principle...
is therefore possible. Moreover, the model account for the decrease in intensity towards zero at 90°, contrary to most of the scalar theories of wave optics, and therefore provides predictions concerning the region of large diffraction angles which is still largely unexplored. Finally, in the case of light in single-photon states and for an incident monochromatic plane wave, the transfer of momentum between the photon and the diaphragm conserves the probabilities of the circular polarizations but can cause a phase shift between the amplitudes of the associated helicity states. For an initial state elliptically polarized, the conservation of the ellipticity and of the handedness is predicted. The phase shift between the amplitudes of the helicity states corresponds to a rotation of the axes of the ellipse. The angle of this rotation depends on the diffraction angles and is not known a priori. Its values can be fitted to the data from measurements of the polarization of the photons detected beyond the diaphragm. It would therefore be possible to obtain quantitative information on how diffraction modifies the polarization of light.

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