Neutron Electric Dipole Moment
in
SUSY SU(5) GUT

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Abstract

The electric dipole moment(EDM) of the neutron is discussed in the SUSY SU(5) GUT model. In this model, it is possible to violate CP in different mode in the Higgs sector from the standard model or the minimal supersymmetric standard model(MSSM) because of the different structure of Higgs multiplets. The neutron EDM’s value generated by this new mode in this model is estimated larger than one in the standard model, and as same order as one in the MSSM.
1 Brief Review and Our Motivation

The Electric Dipole Moment (EDM) of an elementary particle is generated by the violation of CP [1]. In the standard model [1],[2], the neutron EDM’s value \( (d_n) \) is theoretically evaluated as:

\[
d_n = 10^{-31} \sim 10^{-34} \text{(e cm)}.
\] (1)

On the other hand, the experimental upper limit on the neutron EDM is:

\[
|d_n| < 6.03 \times 10^{-26} \text{(e cm)} ,
\] (2)

at present [3]. However, this experimental limit is expected to be improved in the near future. In several extensions of the standard model, it is well-known that the neutron EDM’s value is expected or evaluated to be larger than one in the original standard model. Thus, once the neutron EDM is observed larger than the upper limit (Eq.1) in the standard model, the reality of the other models would be increased. In this paper, we estimate the neutron EDM in the SUSY SU(5) GUT model as one of the extensions of the standard model. Prior to the authors, the neutron EDM’s value in this model is already evaluated by A. Romanio and A. Strumia [5]. However, in their estimation [5], the CP violating phase only in the quark sector of the mass matrix is taken into account, and the phase in the Higgs sector is essentially neglected. In their evaluation [5], therefore, there is not so much difference of the neutron EDM’s value between in the SUSY SU(5) GUT model and in the standard model, because only the same mechanism to violate CP with the standard model is regarded. Generally speaking, according to the Kobayashi-Maskawa theorem [4], a mass matrix consisted of 3×3 or more components can have the complex phase to violate CP. Thus, we introduce the complex phase into the Higgs sector of the mass matrix, because the Higgs/Higgsino superfields have five components in this model, and the existence of this new phase is one of the features of the SUSY SU(5) GUT model in contrast to the standard model or MSSM. When trying to estimate the neutron EDM’s value in the SUSY SU(5) GUT model, this new mechanism to violate CP should be taken into consideration. In the following sections in this paper, all the estimations are carried out with including this new mechanism to violate CP symmetry.
2 Estimation of Neutron EDM

In this section, we present a sequence of the derivation from the general theory of SUSY SU(5) GUT to the estimation of the neutron EDM’s value.

2.1 Introduction of CP violation to Higgs sector

In the SUSY SU(5) GUT model, the superpotential is given as follows [6], [7]:

\[ V_S = \lambda_1 \left( \frac{1}{3} \Sigma^a_b \Sigma^b_c \Sigma^c_{-a} + \frac{1}{2} m \Sigma^a_b \Sigma^b_c \right) + \lambda_2 \bar{H}_a (\Sigma^a_b + 3m' \delta^a_b) H^b + f_{ijk} \varepsilon_{uvwab} H^u X^{vw}_j X^b_k + g_{jk} \bar{H}_a X^{ab}_j Y_{kb} + h.c. \]

(3)

where, \( a \) and \( b \) are SU(5)’s, \( j \) and \( k \) are flavors’ indices, respectively. The Higgs supermultiplets; \( H^a \) is a 5-representation and \( \bar{H}_a \) is a \( \bar{5} \)-representation. The matter supermultiplets; \( X^{ab}_j \) are the representation of 10’s, and \( Y_{ja} \) are the \( \bar{5} \)’s. Note that \( X^{ab}_j = -X^{ba}_j \).

The complex phase parameter \( \theta \) to violate CP in the Higgs sector is introduced as follows:

\[ 3\lambda_2 m' H \bar{H} \rightarrow 3\lambda_2 m' H e^{i\theta} \bar{H} \],

(4)

where, \( H \) stands for the Higgs/Higgsino super-multiplets of the left-handed chiral superfield. Please take care of the fact that the phase \( \theta \) is not regarded as the CP-violating phase itself, but regarded as an introduced relative phase parameter between \( m' \) and the vacuum expectation values (VEVs) of \( \Sigma \) to allow the violation of CP in the Higgs sector. In SUSY SU(5) GUT model, the actual CP-violating phase in the Higgs sector is almost automatically fixed around its maximum, to keep the order of the Higgino mass in the usual region, in contrast to the ambiguity of such phase in other models like MSSM. For more detail, see the following section 3.
2.2 How to break SUSY SU(5) down

As we know, most of the symmetries in this model, like SUSY, are broken in our real world; the sequence to break the symmetries down is as follows:

1. The original SU(5) symmetry is breaking to SU(3) × SU(2) × U(1) with the energy scale getting lower by the Higgs mechanism of the heavy Higgs bosons in 24-representation.

2. Moreover, the supersymmetry is broken spontaneously by adding the soft breaking terms as:

\[ m_0^2 \phi_i^* \phi_i + (m_0 A_0 W_3 + B_0 \mu_0 h_1 h_2 + h.c.) + m_{1/2} \bar{\lambda}_j \lambda_j , \tag{5} \]

where, \( \phi \)’s are the scalar fields, and \( W_3 \) stands for the trilinear scalar terms, \( h_1 \) and \( h_2 \) are the two Higgs doublets, \( \lambda \)’s are the gauge fermions. \( m_0 \) is the universal scalar mass, \( A_0 \) is the universal trilinear coupling constant, \( B_0 \) is the universal bilinear coupling constant, and \( m_{1/2} \) is the universal gaugino mass.

3. Furthermore, SU(2) symmetry is spontaneously breaking as usual:

\[ SU(2) \times U(1) \rightarrow U(1)_{em} . \tag{6} \]

The observations of the neutron EDM are held in this energy region.

2.3 Representations of particle states

One of the general features of the quantum theory, as the result of the mixture among the particles with the same quantum numbers, non-diagonal elements arise in the mass matrix of bare particles. In order to coincide the representations with the physical particles, the mass matrix should be diagonalized.

The gaugino and Higgsino is mixed as the charginos and the neutralinos. As the results of the mixture, the mass matrix of the bare charginos is:

\[ M_C = \begin{pmatrix} m_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \sin \beta & \lambda_2 b \end{pmatrix} , \tag{7} \]
and of the bare neutralinos is:

$$M_N = \begin{pmatrix}
    m_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\
    0 & m_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\
    -m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & 0 \\
    m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & -\lambda_2 b & 0 \\
\end{pmatrix}, \tag{8}
$$

where, $m_1$ and $m_2$ stand for the gaugino masses, $m_W$ and $m_Z$ are W and Z boson masses, respectively. $b = 3m - 3m'e^{i\theta}$; $m$ is the vacuum expectation value (VEV) of heavy Higgs bosons in the 24-representation. $\tan \beta$ is defined as $\tan \beta = v_2/v_1$; $v_1$ and $v_2$ are the VEVs of the Higgs scaler fields 5 and 5, respectively. These mass matrices are diagonalized as:

$$U^* M_C V = \text{diag}(|m_{C_1}|, |m_{C_2}|), \tag{9}$$

$$N^T M_N N = \text{diag}(|m_{N_1}|, |m_{N_2}|, |m_{N_3}|, |m_{N_4}|). \tag{10}$$

Note that the representations of these diagonalized physical states as the charginos and neutralinos are used all through this paper, instead of the bare fields of the Higgsino and gaugino.

The bare mass matrix ($6 \times 6$) of the u-type squarks is given as:

$$M_U^2 = \begin{pmatrix}
    m_{\tilde{q}_L}^2 + D_L 1 + m_\tilde{u}^2 & m_U (A_\tilde{q}^* m_0 + \lambda_2 b (v_2/v_1)) \\
    m_U (A_\tilde{q}^* m_0 + \lambda_2 b (v_2/v_1)) & m_{\tilde{u}}^2 + D_R 1 + m_\tilde{u}^2 \\
\end{pmatrix}, \tag{11}$$

and of the d-type squarks is:

$$M_D^2 = \begin{pmatrix}
    m_{\tilde{q}_L}^2 + D_L 1 + m_\tilde{d}^2 & m_D (A_\tilde{q}^* m_0 + \lambda_2 b (v_1/v_2)) \\
    m_D (A_\tilde{q}^* m_0 + \lambda_2 b (v_1/v_2)) & m_{\tilde{d}}^2 + D_R 1 + m_\tilde{d}^2 \\
\end{pmatrix}, \tag{12}$$

where,

$$D_L = m_Z^2 (T_3 - Q_{U,D} \sin^2 \theta_W) \cos 2\beta, \tag{13}$$

$$D_R = m_Z^2 Q_{U,D} \sin^2 \theta_W \cos 2\beta. \tag{14}$$

$Q_{U,D}$ stands for the electric charge of the u,d-type quark, respectively. $m_{U,D}$ denotes the u,d-type ordinal quark mass matrix ($3 \times 3$), respectively. $m_{\tilde{q}_L}$ and $m_{\tilde{q}_R}$ are the matrices of the soft breaking mass parameters of the squarks. $A_\tilde{q}$ is
the trilinear coupling constants matrix. Both of the squark mass matrices $6 \times 6$ are diagonalized as:

$$D\tilde{u}^\dagger M\tilde{u}D\tilde{u} = \text{diag}(|\tilde{u}_{11}|, |\tilde{u}_{12}|, |\tilde{u}_{13}|, |\tilde{u}_{21}|, |\tilde{u}_{22}|, |\tilde{u}_{23}|), \quad (15)$$

$$D\tilde{d}^\dagger M\tilde{d}D\tilde{d} = \text{diag}(|\tilde{d}_{11}|, |\tilde{d}_{12}|, |\tilde{d}_{13}|, |\tilde{d}_{21}|, |\tilde{d}_{22}|, |\tilde{d}_{23}|). \quad (16)$$

These definitions of the squark states are used all through this paper, like of the charginos and neutralinos. Note that we neglect all the non-diagonal elements of $A\tilde{q}$, $m\tilde{q}_{L,R}$ and $m_{U,D}$, because certainly these elements in the squark mass matrices can occur another phase to violate CP, however, this additional effect has already estimated by Inui et al. [9]; The EDM’s value generated by this additional phase is quantitatively quite smaller than one by the phase in the Higgs sector we discuss.

2.4 EDM formulae

In order to estimate the EDM’s value of a quark current, the shapes of one loop graphs shown in Fig.1 are regarded. The diagrams including the virtual loop of the chargino, neutralino or the gluino can generate the CP-odd contributions to the current amplitude. Therefore, the contributions of these three kinds of the diagrams are taken into account. The graphs including the virtual loop of the heavy gauginos (i.e. X-ino and Y-ino) and the heavy Higgsinos are also possible to contribute CP-oddly, however, they are neglected. Roughly speaking, since the EDM’s values are proportional to the inverse of the virtual particle masses; the EDM’s values of such graphs including heavy virtual particles are extremely small compared to the graphs of the charginos, neutralinos, or the gluino.

For the purpose of calculating the loop correction to a current of the quark with spin 1/2, the current amplitude is decomposed according to the form factors as follows:

$$\langle q_f(p')|j^\mu(q)|q_i(p)\rangle \quad (17)$$

$$= \bar{q}_f(p') \left[ \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}q_\nu F_2(q^2) + \gamma_5\sigma^{\mu\nu}q_\nu F_3(q^2) \right.$$  

$$+ \left. \left( \frac{q^2}{2m} \gamma^\mu - q^\mu \right) \gamma_5 F_A(q^2) + \cdots \right] q_i(p),$$
where, $j_\mu(q)$ is the current with four-momentum transfer $q = p' - p$. $F_3(q^2)$ is the CP-odd form factor.

The EDM($d_f$) of the quark is given by

$$ d_f = -\frac{e}{2m}F_3(0). $$

(18)

By making use of this formula, the neutron EDM($d_n$) is written as:

$$ d_n = \frac{4}{3}d_d - \frac{1}{3}d_u, $$

(19)

where, $d_d$ means the d-quark EDM, as $d_u$ is the u-quark EDM.

As the results of the loop calculation, $F_3(q^2 \to 0)$ is written as:

$$ F_3(0) = \sum_{i,k} \frac{m_i}{(4\pi)^2 m_k^2} \text{Im}(F_{ik}G_{ik}^*) \left\{ Q_i A\left(\frac{m_i^2}{m_k^2}\right) + Q_k B\left(\frac{m_i^2}{m_k^2}\right) \right\}, $$

(20)

$$ A(r) = \frac{1}{2(1-r)^2} \left( 3 - r + \frac{2 \ln r}{1 - r} \right) $$

(21)

$$ B(r) = \frac{1}{2(1-r)^2} \left( 1 + r + \frac{2r \ln r}{1 - r} \right) $$

(22)

where, $Q_k = Q_f - Q_i$. Moreover, the chromoelectric dipole moment(CDM) is also taken into account. The CDM is defined as the factor $d^{CDM}$ of the effective operator arise in the QCD Laglangian:
\[ \mathcal{L}_{CDM} = -\frac{i}{2} d_{CDM}^\alpha \bar{q} \sigma_{\mu\nu} \gamma_5 T^a q G^{\mu\nu} \]  \hspace{1cm} (23)

where, \( T^a \)'s stand for the generators of SU(3). The CDM contributions to the neutron EDM is as follows:

\[ d_n = \frac{1}{3} e \left( \frac{4}{3} d_{d}^{CDM} + \frac{2}{3} d_{u}^{CDM} \right) , \hspace{1cm} (24) \]

For more details of the EDM and CDM formulae, see Appendix B.

### 3 Evaluation of parameters

In order to evaluate the soft breaking mass parameter, the correction of the renormalization group flow at one loop level is discussed in this section. The explicit forms of the renormalization group equations(RGEs) are shown in appendix A. In order to solve the RGEs, we assume the following conditions:

i) The gauge coupling constants are bended at \( M_{SUSY} \simeq 1\) (TeV), and unified at \( M_{SU(5)} \simeq 10^{16}\) (GeV)\(^{[12]}\).

ii) Yukawa coupling constants of the top and bottom quarks are given to coincide with their values at electroweak scale \( M_Z \) independent of given \( \tan \beta \).

iii) The spontaneous SUSY breaking scale \( M_X \) is fixed to \( 6.0 \times 10^{10}\) (GeV). Note that this condition is \( ad-hoc \) for the pure SUSY SU(5) GUT, because this is a consequence of the super Higgs effect in hidden sector based on N=1 supergravity grand unified theory\(^{[5]}\). Also this is mere a formal boundary condition, indeed the values of \( m_{\tilde{q}} \) and \( A_{\tilde{q}} \) are almost independent of \( M_X \).

iv) The masses of the supersymmetric scalar particles are set equal to \( m_0 \) at energy scale \( M_X \).

v) \( B_0 = (A_0 - 1)m_0 \) is assumed. This is also \( ad-hoc \) condition for the pure SUSY SU(5) GUT adopted from the minimal supergravity case.

vi) Once an assumed \( B_0 \) at \( M_X \) is given, the parameters like \( m_{h1} \) or \( m_{h2} \) (mass parameter of two Higgs doublet) at \( M_Z \) scale can be obtained by solving
the RGEs directly. On the other hand, to break the electro-weak symmetry
down as usual, two conditions:

\[ 2B\mu = -(m_{h_1}^2 + m_{h_2}^2 + 2\mu^2) \sin 2\beta , \]
\[ \mu^2 = -\frac{m_Z^2}{2} + \frac{m_{h_2}^2 - m_{h_1}^2 \tan^2 \beta}{\tan^2 \beta - 1} \]

are required. As the results of these two equations, \( B \) and \( \mu \) at \( M_Z \) are
derived. However, this value of \( B \) at \( M_Z \) should be coincident with another
estimation. To match both two \( B \) values at \( M_Z \), the initial \( B_0 \) at \( M_X \) is
tuned recursively and decided by numerical analysis. Thus, \( B_0 \) (and also
\( A_0 \)) is come to a decision.

vii) \( \tan \beta \) is restricted \( 2 < \tan \beta < 40 \) to avoid emerging the Landau pole
divergence of Yukawa coupling constant of the top quark.

With these conditions, several additional restrictions on other parameters are
obtained from the RGE analysis. Note that \( m_{1/2} \) implies the unified gaugino
mass at the scale \( M_X = 6.0 \times 10^{10} \text{(GeV)} \). Among the masses of each gauginos,
the following relation is realized at one loop level \([11]\):

\[ \frac{m_i(M_Z)}{\alpha_i(M_Z)} = \frac{m_{1/2}}{\alpha_i(M_X)} , \quad (i = 1, 2, 3) , \]

where \( \alpha_i \)'s are the gauge coupling constants.

Furthermore, the CP-violating phase \( \theta' \) in the Higgs sector is fixed. The parameter \( \mu \) can be regarded as the Higgsino mass, and is defined as:

\[ \mu = \lambda_2 b , \]
\[ b = -3m + 3m'e^{i\theta} . \]

As mentioned in the section 2.1, the phase \( \theta \) can be regarded as the relative
phase between \( m' \) and VEV of heavy Higgs bosons in the 24-representation;
\( m, m' \approx 10^{16} \text{(GeV)} \). Thus, the phase \( \theta \) is given as:

\[ \sin \theta = \frac{|\mu|}{\lambda_2} \frac{1}{3m} \approx 0 \]

in good approximation, because \( |\mu| \) is supposed to be TeV order as usual.

When \( \mu \) is rewritten as:

\[ \mu = |\mu| \exp(i\theta') , \]

9
the complex phase $\theta'$ of the Higgsino mass $\mu$ is:

$$\cos \theta' = \sin \theta \simeq 0,$$

therefore, $\theta' = \pi/2 - \theta \simeq \pi/2$ is obtained; the phase $\theta'$ of Higgsino as the CP-violating phase in the Higgs sector is fixed around its maximum value. Additionally, the product $\lambda_2 b$ is effectively regarded as combined one parameter, because the residual single effects of $\lambda_2$ separated from $|b|$ is quite small. Therefore, $\lambda_2$ is simply fixed as $10^{-1}$ by hand all through the estimation. This assumption on the order of $\lambda_2$ has no effect on any estimation in this paper.

### 4 Numerical Analysis and Results

The rest free parameters are $m_{1/2}$, $m_0$, and $\tan \beta$. These parameters are expected to settle in the regions as usual:

- $|m_2|$ : TeV order.
- $|m_0|$ : less than a few TeV.
- $\tan \beta : 2 \sim 40$ ($\simeq |m_t|/|m_b|$ at $M_Z$)

In Fig.2, the neutron EDM’s values are shown as functions of $m_{1/2}$: $1 \leq m_{1/2} \leq 10$(TeV) for $m_0 = 1$(TeV), $\tan \beta = 3$, 5, and 10. The flat line is the experimental upper limit of EDM. Obviously, $m_{1/2}$’s value is required to be more than 4.5(TeV) by the experimental limit.

In Fig.3, on the $m_{1/2}$ dependence of the neutron EDM’s value is drawn for $m_0 = 1$, 3, and 5(TeV). $\tan \beta = 5$ is fixed. The flat line means the experimental upper limit, as in Fig.2, $m_{1/2}$ is necessary to be larger than 5(TeV). The graphs implies that the neutron EDM’s value is not sensitive on $m_0$.

Fig.4 summarizes the topographic plots of EDM’s experimental upper limit as functions of $m_0$ and $m_{1/2}$ for $\tan \beta = 3$, 5, and 10. This plot shows the two facts - $m_{1/2}$’s value should be larger than a few TeV, and $\tan \beta$ is required to be small. The EDM’s value is more sensitive on $m_{1/2}$ rather than $m_0$. 
\[ \tan \beta = 3 \]
\[ \tan \beta = 5 \]
\[ \tan \beta = 10 \]

Figure 2: Neutron EDM's values as functions of $|m_{1/2}|$ for several $\tan \beta$
Figure 3: Neutron EDM's values as functions of $|m_{1/2}|$ for several $m_0$
Figure 4: Excluded parameter regions restricted by experimental neutron EDM’s upper limit

5 Conclusion and Summary

Summary, in the SUSY SU(5) GUT model, the neutron EDM’s value is obtained by introducing the complex phase into the mass matrix of the Higgs sector. As the results, the neutron EDM’s value of this mechanism is derived significantly larger than existing estimations by the CP violation in the quark sector. According to this result, the constraints on the $m_0$, $m_{1/2}$ and $\tan \beta$ become more strict than existing evaluations, however, the allowed parameter region is still remained. Additionally, we should take care of the phases of other parameters, like $m_{1/2}$’s, because such phases can cancel the effect of the introduced phase in the Higgs sector. If such phases were possible to exist, the evaluation of the EDM’s value in this paper is regarded as the maximum, and the most strict estimation. Anyway, once the neutron EDM’s value would be observed larger than the prediction of the standard model ($\sim 10^{-32}$(e cm)), the other models including the SUSY SU(5)
GUTs (predicting large EDM) would be influential.

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A Equations of Renormalization Group

The RGEs at one loop level are as follows \cite{11}, \cite{13}:

\[
\frac{d\alpha_a}{dt} = -2b_a\alpha_a^2 , \quad (a.1)
\]

\[
\frac{dm_a}{dt} = -2b_a\alpha_a m_a , \quad (a.2)
\]

\[
\frac{d\alpha_t}{dt} = 2\alpha_t(-c_a\alpha_a + 6\alpha_t + \alpha_b) , \quad (a.3)
\]

\[
\frac{d\alpha_b}{dt} = 2\alpha_b(-c'_a + \alpha_a + \alpha_t + 6\alpha_b) , \quad (a.4)
\]

\[
\frac{dA_i}{dt} = -2c_a\alpha_a m_a/m_0 + 12\alpha_t A_i + 2\alpha_b A_b , \quad (a.5)
\]

\[
\frac{dA_{a,c,d,sL}}{dt} = -2c_a\alpha_a m_a/m_0 + 6\alpha_t A_i , \quad (a.6)
\]

\[
\frac{dA_b}{dt} = -2c'_a\alpha_a m_a/m_0 + 2\alpha_t A_i + 12\alpha_b A_b , \quad (a.7)
\]

\[
\frac{dA_{a,c,d,sL}}{dt} = -2c'_a\alpha_a m_a/m_0 + \alpha_b A_b , \quad (a.8)
\]

\[
\frac{dB}{dt} = -2c''_a\alpha_a m_a + 6\alpha_t m_0 A_i + 6\alpha_b m_0 A_b , \quad (a.9)
\]

\[
\frac{dm_{h_1}^2}{dt} = -2c^{(3)}_a\alpha_a m_a^2 + 6\alpha_b \Sigma_b^2 , \quad (a.10)
\]

\[
\frac{dm_{h_2}^2}{dt} = -2c^{(3)}_a\alpha_a m_a^2 + 6\alpha_t \Sigma_t^2 , \quad (a.11)
\]

\[
\frac{dm_{u,c,d,sL}^2}{dt} = -2c^{(4)}_a\alpha_a m_a^2 + \frac{1}{5}\alpha_1 \text{Tr}(Y m^2) , \quad (a.12)
\]

\[
\frac{dm_{uR}^2}{dt} = -2c^{(5)}_a\alpha_a m_a^2 + \frac{4}{5}\alpha_1 \text{Tr}(Y m^2) , \quad (a.13)
\]
\[
\frac{dm_{dR}^2}{dt} = -2c_a^{(6)}\alpha_a m_a^2 + \frac{2}{3}\alpha_1 \text{Tr}(Y m^2), \quad \text{(a.14)}
\]
\[
\frac{dm_{t,bl}^2}{dt} = 2\alpha_t \Sigma_t^2 + 2\alpha_b \Sigma_b^2 - 2c_a^{(4)}\alpha_a m_a^2 + \frac{1}{5}\alpha_1 \text{Tr}(Y m^2), \quad \text{(a.15)}
\]
\[
\frac{dm_{t,R}^2}{dt} = 4\alpha_t \Sigma_t^2 - 2c_a^{(5)}\alpha_a m_a^2 + \frac{4}{5}\alpha_1 \text{Tr}(Y m^2), \quad \text{(a.16)}
\]
\[
\frac{dm_{b,R}^2}{dt} = 4\alpha_b \Sigma_b^2 - 2c_a^{(6)}\alpha_a m_a^2 + \frac{2}{5}\alpha_1 \text{Tr}(Y m^2), \quad \text{(a.17)}
\]

where, \( t = (1/4\pi) \ln(Q/M) \), \( m_a \) are the gaugino masses, and \( \alpha_a = \lambda_a^2/4\pi \) are the gauge coupling constants. The suffix \( a \) is always summed from 1 to 3 up. The optional coefficients and variables are defined as:

\[
b_a = \begin{cases} 
(-41/10, 19/6, 7) & , M_Z \leq Q \leq M_{\text{SUSY}} \\
(-33/5, -1, 3) & , M_{\text{SUSY}} \leq Q \leq M_{\text{GUT}}
\end{cases}, \quad \text{(a.18)}
\]
\[
c_a = (13/15, 3, 16/3), \quad \text{(a.19)}
\]
\[
c_a' = (7/15, 3, 16/3), \quad \text{(a.20)}
\]
\[
c_a'' = (3/5, 3, 0), \quad \text{(a.21)}
\]
\[
c_a^{(3)} = (1, 3, 0), \quad \text{(a.22)}
\]
\[
c_a^{(4)} = (1/15, 3, 16/3), \quad \text{(a.23)}
\]
\[
c_a^{(5)} = (16/15, 16/3, 2/5), \quad \text{(a.24)}
\]
\[
c_a^{(6)} = (4/15, 0, 16/3), \quad \text{(a.25)}
\]
\[
\Sigma_t^2 = (A_t^2 + m_{h_2}^2 + m_{tL}^2 + m_{tR}^2), \quad \text{(a.26)}
\]
\[
\Sigma_b^2 = (A_b^2 + m_{h_1}^2 + m_{bL}^2 + m_{bR}^2). \quad \text{(a.27)}
\]

All the Yukawa couplings except of the top and bottom quarks are neglected, because their values are too small to take into account. The exceptional two (relatively large) Yukawa couplings \( \alpha_{t,b} = \lambda_{t,b}^2/4\pi \) are defined as usual:

\[
\lambda_t = \frac{g_2}{\sqrt{2}} \frac{m_t}{m_W \sin \beta}, \quad \text{(a.28)}
\]
\[
\lambda_b = \frac{g_2}{\sqrt{2}} \frac{m_b}{m_W \cos \beta}. \quad \text{(a.29)}
\]

Furthermore, keeping the traceless conditions \( \text{Tr}(Y m^2) = m_{tL}^2 \text{Tr}(Y) = 0 \) of the SU(5) gauge are required to avoid the gravitational mixed anomaly. Since the
initial or boundary conditions are necessary to solve the RGEs, they are given at the electro-weak scale $M_Z \simeq 10^2$$(\text{GeV})$ for the gauge and Yukawa couplings, and at $M_X \simeq 6.0 \times 10^{10}$$(\text{GeV})$ scale for the other variables. Note that we set all the initial values of the soft breaking mass parameters equal to $m_0$, and an ad-hoc constraint $B_0 = (A_0 - 1)m_0$ from the minimal supergravity theory is applied.

### B Neutron EDM Formulae

The neutron EDM and CDM are given by following equations. The chargino contribution is:

\[
d_C^f/e = -\frac{\alpha_{em}}{4\pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{jik}) \frac{m_C^i}{m_{f_k}^2} \times \left[ Q_f B \left( \frac{m_C^i}{m_{f_k}^2} \right) + (Q_f - Q_{f'}) A \left( \frac{m_C^i}{m_{f_k}^2} \right) \right],
\]

(b.1)

\[
d_{f-CDM}^C = -\frac{\alpha_{em} g_s}{4\pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{jik}) \frac{m_C^i}{m_{f_k}^2} B \left( \frac{m_C^i}{m_{f_k}^2} \right),
\]

(b.2)

where, $f = u, d$ for $f' = d, u$.

\[
\Gamma_{uik}^C = \kappa_u V_{i1}^* D_{d1k} (U_{i1}^* D_{d1k} - \kappa_d U_{i2}^* D_{d2k}) ,
\]

(b.3)

\[
\Gamma_{dik}^C = \kappa_d U_{i2}^* D_{u1k} (V_{i1}^* D_{u1k} - \kappa_u V_{i2}^* D_{u2k}) ,
\]

(b.4)

and

\[
\kappa_u = \frac{m_u}{\sqrt{2m_W \sin \beta}} ,
\]

(b.5)

\[
\kappa_d = \frac{m_d}{\sqrt{2m_W \cos \beta}} .
\]

(b.6)

The neutralino contribution is:

\[
d_{f}^N/e = \frac{\alpha_{em}}{4\pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{4} \text{Im}(\Gamma_{jik}^N) \frac{m_N^i}{m_{f_k}^2} Q_f B \left( \frac{m_N^i}{m_{f_k}^2} \right),
\]

(b.7)

\[
d_{f-CDM}^N = \frac{\alpha_{em} g_s}{4\pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{4} \text{Im}(\Gamma_{jik}^N) \frac{m_N^i}{m_{f_k}^2} B \left( \frac{m_N^i}{m_{f_k}^2} \right),
\]

(b.8)
$$\Gamma^N_{f_{1k}} = \left[ -\sqrt{2}\{\tan\theta_W(Q_f - T_{3f})N_{1i} + T_{3f}N_{2i}\} D^*_{f_{1k}} + \kappa_f N_{bi} D^*_{f_{2k}} \right]$$
$$\times \left( \sqrt{2}\tan\theta_W Q_f N_{1i} D_{f_{2k}} - \kappa_f N_{bi} D_{f_{1k}} \right), \quad \text{(b.9)}$$

where, $b = 3(4)$ for $T_{3f} = -1/2(+1/2)$, respectively.

The gluino contribution is:
$$d_f^G/e = -\frac{2\alpha_s}{3\pi} \sum_{k=1}^{2} \text{Im}(\Gamma^G_{f_{jk}}) \frac{m_{\tilde{g}}}{m^2_{f_k}} Q_f B \left( \frac{m_{\tilde{g}}}{m^2_{f_k}} \right), \quad \text{(b.10)}$$
$$d_f^{G-CDM} = \frac{\alpha_s g_s}{4\pi} \sum_{k=1}^{2} \text{Im}(\Gamma^G_{f_{jk}}) \frac{m_{\tilde{g}}}{m^2_{f_k}} C \left( \frac{m_{\tilde{g}}}{m^2_{f_k}} \right) \quad \text{(b.11)}$$

where,
$$\Gamma^G_{f_{jk}} = D_{f_{2k}} D^*_{f_{1k}}, \quad \text{(b.12)}$$
$$C(r) = \frac{1}{6(r-1)^2} \left( 10r - 26 + \frac{2r \ln r}{1-r} - \frac{18 \ln r}{1-r} \right). \quad \text{(b.13)}$$

Note that all the generations of the squark are summed up in the virtual loops.
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