INFLUENCE OF THE MAGNETIC COUPLING PROCESS ON ADVECTION-DOMINATED ACCRETION FLOWS AROUND BLACK HOLES

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ABSTRACT

A large-scale closed magnetic field can transfer angular momentum and energy between a black hole (BH) and its surrounding accretion flow. We investigate the effects of this magnetic coupling (MC) process on the dynamics of a hot accretion flow (e.g., an advection-dominated accretion flow). The energy and angular momentum fluxes transported by the magnetic field are derived with an equivalent-circuit approach. For a rapidly rotating BH, it is found that the radial velocity and the electron temperature of the accretion flow decrease, whereas the ion temperature and the surface density increase. The significance of the MC effects depends on the value of the viscosity parameter \( \alpha \). The effects are obvious for \( \alpha = 0.3 \) but nearly ignorable for \( \alpha = 0.1 \). For a BH with specific angular momentum \( a_* = 0.9 \) and \( \alpha = 0.3 \), we find that for reasonable parameters the radiative efficiency of a hot accretion flow can be increased by \( \sim 30\% \).

Subject headings: accretion, accretion disks — black hole physics — magnetic fields

1. INTRODUCTION

As a variant of the Blandford-Znajek process (Blandford & Znajek 1977), the magnetic coupling (MC) between a central rotating black hole (BH) and its surrounding accretion disk has received much attention (e.g., Blandford 1999; Li & Paczyński 2000; Li 2002; Wang et al. 2002). By virtue of the large-scale closed magnetic field lines that connect the BH and its surrounding disk, the MC process conveys energy and angular momentum between the BH and the disk. Li (2002) showed that in the case of a standard Shakura-Sunyaev disk (SSD) (Shakura & Sunyaev 1973; Novikov & Thorne 1973; Page & Thorne 1974), the MC process may change the local radiative flux significantly.

Apart from SSDs, advection-dominated accretion flows (ADAFs) represent another important model (Narayan & Yi 1994, 1995; Abramowicz et al. 1995; for reviews, see Narayan et al. 1998; Kato et al. 1998) that has been applied to a number of accreting BH systems and successfully explains their spectral characteristics (see Narayan 2005 and Yuan 2007 for recent reviews). In ADAF models, the thickness of the accretion flow is of the same order as the radius, that is, \( H \sim r \), which means that a large-scale field is more easily formed in an ADAF than in a SSD (Tout & Pringle 1996; Livio et al. 1999). So, it is interesting to investigate the influences of the MC process on an ADAF. Very recently, Ye et al. (2007) discussed this problem based on a self-similar ADAF solution. In this paper, we investigate the influences of the MC process on ADAFs through global solutions.

To properly assess the dynamical effects of large-scale magnetic fields on the accretion flow, one needs to obtain the fluids and the fields at the same time by solving the trans-field equation, which is a nontrivial nonlinear partial differential equation with singular surfaces and free functions (Uzdensky 2004, 2005). An alternative way is through MHD simulations (e.g., Hawley 2000; Hawley & Balbus 2002; Koide 2003; DeVilliers et al. 2003; McKinney & Gammie 2004; Hirose et al. 2004). However, both of these approaches are complicated. Lai (1998) and Lee (1999a, 1999b) adopted a phenomenological approach to research the magnetic coupling between a neutron star and its surrounding slim disk. They specified an Ansatz for the magnetic fields and then numerically solved the basic equations of the accretion flow. In their model the disk is geometrically thin, \( \partial / \partial \tau \sim 1/r \ll 1/H \sim \partial / \partial \varphi \), and so the expressions for the electromagnetic forces can be greatly simplified by omitting terms in \( \partial / \partial r \). However, a magnetically coupled ADAF, or MCADAF, is thick, and the expression for the electromagnetic force is complex. Here, for simplicity we treat the MC process as a source of energy and angular momentum without considering the radial and vertical components of the electromagnetic force in the momentum equations.

We derive the energy and angular momentum fluxes in the Kerr metric by using the approach of equivalent circuits (Macdonald & Thorne 1982). But, for simplicity, the pseudo-Newtonian potential for a rotating black hole given by Mukhopadhyay (2002) is adopted when we solve the equations of the accretion flow.

In § 2, we describe the MCADAF model and calculate the energy and angular momentum fluxes transferred by the magnetic field. In § 3, we write down the basic equations describing an MCADAF. The numerical results are presented in § 4, and § 5 is devoted to a summary and discussion. Throughout, the geometric units \( c = G = 1 \) are used.

2. MCADAF MODEL

We assume that the ADAF is stationary and axisymmetric. It extends from the outer edge of the accretion disk, \( r_{\text{out}} \), to the BH horizon \( r_H \). There are two kinds of magnetic field in this model, that is, the large-scale closed magnetic field that connects the BH with the ADAF and a small-scale tangled magnetic field, with the former contributing to the MC process and the latter to the viscosity. We that assume these two kinds of fields work independently. Unless otherwise mentioned, when we refer to “the magnetic field” we mean the large-scale closed one hereafter. The region between the BH and the ADAF is assumed to be ideally conducting and force-free.
The field lines are supposed to be distributed in the range \((r_{\text{in}}, r_{\text{out}})\) on the disk and \((0, \theta_0)\) on the horizon. Because of our lack of knowledge about the magnetic field around the BH, we assume that the field threading the BH is constant, that is, \(B_H(\theta) = \text{const}\). The field threading the ADAF is assumed to decrease with \(r\) following a power-law form, but within the marginally stable orbit \((\leq r_{\text{ms}})\) the radial velocity of the accretion flow increases much faster, and thus the field is likely to strengthen with radius. Given this consideration, we assume the field has the following distribution:

\[
B_r(r) = B_0 F(r) = \begin{cases} 
B_0 \exp (r/r_p - 1), & \text{if } r_H < r \leq r_p, \\
B_0 (r/r_p)^{-\eta m}, & \text{if } r < r_p \leq r_{\text{ex}}.
\end{cases}
\]

Here \(r_H = M[1 - (1 - a^2)^{1/2}]\) denotes the radius of the BH horizon, \(a_0\) is the dimensionless spin parameter of the BH, and \(r_p\) is parameterized as \(r_p = r_H + \lambda (r_{\text{ms}} - r_H)\).

Morslinski et al. (1997) estimated \(B_H\) by assuming balance between the magnetic pressure and the ram pressure of the falling material, that is, \(B_\Psi^2/8\pi \sim \rho \sim M_P/(4\pi r_H^3)\). Since the ram pressure can be larger than the magnetic pressure, we introduce a parameter \(c_B\) to indicate the strength of the magnetic field threading the horizon, as

\[
B_H = c_B \sqrt{2M/r_H}, \quad 0 \leq c_B \leq 1.
\]

In the remainder of the derivation in this subsection, Boyer-Lindquist coordinates are used. Assume that all the field lines threading the BH connect with the disk; then, from conservation of magnetic flux we have

\[
\Psi = \int B_H(\rho \varpi) d\theta d\phi = \int B_\Psi \left(\frac{\varpi}{\sqrt{\Delta}}\right)_D dr d\phi,
\]

where the subscripts \(H\) and \(D\) are used to indicate quantities on the horizon and on the equatorial plane of the disk \((\theta = \pi/2)\), respectively. The Boyer-Lindquist coordinates are given by

\[
\Sigma^2 = (r^2 + a^2M^2)^2 - a^2M^2 \Delta \sin^2 \theta, \\
\rho^2 = r^2 + a^2M^2 \cos^2 \theta, \\
\Delta = r^2 + a^2M^2 - 2Mr, \quad \varpi = (\Sigma/\rho) \sin \theta.
\]

Since \(\Delta = 0\) at \(r = r_H\), the lower boundary of the integration interval in the second equality above is set to \(r_{\text{in}} + \delta r\), where \(\delta r\) is a small quantity and taken to be \(\delta r = 0.01\).\(^1\) Substituting equation (1) into equation (3), we obtain

\[
B_0 = \frac{\int B_H(\rho \varpi) d\theta d\phi}{\int F(r)(\rho \varpi/\sqrt{\Delta})_D dr d\phi} = \frac{2Mr_H(1 - \cos \theta_0)B_H}{\int F(r)(\rho \varpi/\sqrt{\Delta})_D dr d\phi} = 2r_H k(a_0, n) B_H / M.
\]

Given the configuration of the field, we can derive the energy and angular momentum flux in the MC process by using a modified equivalent-circuit approach (Wang et al. 2002). Consider a loop to correspond to two adjacent flux surfaces (characterized by the magnetic fluxes \(\Psi\) and \(\Psi + \Delta \Psi\); then the electromotive forces due to the rotation of the BH and the disk are expressed as

\[
\Delta \epsilon_D = (\Delta \Psi/2\pi) \Omega_H, \quad \Delta \epsilon_H = (\Delta \Psi/2\pi) \Omega, \\
\Delta \Psi = 2\pi (\rho \varpi \rho_H) \Delta \theta B_H.
\]

The minus sign in the expression for \(\Delta \epsilon_D\) arises from the direction of the flux. The parameter \(\Omega\) is the angular velocity of the ADAF, and \(\Omega_H = a_0/(2r_H)\) is the angular velocity of the BH horizon.

The equivalent surface resistivity of the BH horizon is \(4\pi\) (Macdonald & Thorne 1982; Thorne et al. 1986), while the surface resistivity of the disk is \(\sim 1/(\Omega r) = 4\pi \eta \sigma / H\), where \(\eta = 1/(4\pi \sigma)\) is the diffusivity of the magnetic field. As in many papers (e.g., Lubow et al. 1994; Lovelace et al. 1995; Soria et al. 1997), we take \(\eta\) to be of the same order as the Shakura-Sunyaev (1973) kinematic \(\alpha\)-viscosity coefficient, that is, \(\eta \sim \nu = \alpha c_s H\). The resistances of the annulus on the horizon and the disk are thus

\[
\Delta Z_H = 4\pi \rho_H \Delta \theta / 2\pi \sigma_H, \\
\Delta Z_D = 1 / (H \sigma / 2\pi \sigma_D) = 2\alpha c_s \Delta r / \sigma_D,
\]

and the current in the loop is

\[
I = \frac{\Delta \epsilon_H + \Delta \epsilon_D}{\Delta Z_H + \Delta Z_D} = \frac{\Delta \Psi}{\Delta \Psi \Omega_H} \frac{\Omega_H - \Omega}{\Delta \Psi \Omega_H (1 + \xi)} = \frac{1}{1 + \xi [\csc^2 \theta - 1 + \sqrt{1 - a^2}]} MB_H,
\]

where

\[
\xi = \frac{\Delta Z_D}{\Delta Z_H} = \frac{\alpha c_s \rho_H/2\pi \sigma_D}{\rho_H/2\pi \sigma_D} \frac{\Delta \Psi}{\Delta \Psi \Omega_H}, \quad \beta_{\text{HD}} = 1 - a^2 / \Omega_H.
\]

In order to obtain \(\xi\) and \(I\), we have to find the value of \(\Delta \Psi / \Delta \theta\), which is related to the mapping between the angular coordinate on the horizon and the radial coordinate on the disk, that is, \(\theta(r)\). According to the conservation of magnetic flux between the two adjacent flux surfaces,

\[
d \Psi = B_H 2\pi (\varpi \rho)_H d\theta = -B_\Psi 2\pi (\varpi \rho / \sqrt{\Delta})_D dr.
\]

Substituting equations (1) and (5) into the equation above, we have

\[
M \frac{d \cos \theta}{dr} = k(a_0, n) \sqrt{r^4 + a^2M^2} F(r) / M \sqrt{r^2 + a^2M^2 - 2Mr} \equiv G(a_0, r, n).
\]

Integrating equation (11), we obtain the mapping relation:

\[
\cos \theta = \cos \theta_0 + \int_{r_H}^{r} G(a_0, r, n) dr.
\]

Our calculations show that for the ADAF, the ratio of the height to the radius around \(r_{\text{in}}\) is \(\lesssim 0.3\), and thus we assume \(\theta_0 = 0.4\pi\), so that \(c_{\text{in}} \approx 0.3\). Substituting equation (11) into equation (9), we have

\[
\xi = \frac{2\alpha c_s G(a_0, r, n)^{-1}}{\sqrt{1 + a^2M^2 - 2Mr} / \sqrt{r^2 + a^2M^2 - 2Mr} (2 \csc^2 \theta - 1 + \sqrt{1 - a^2})}.
\]
Since the current $I$ on the BH horizon feels the Ampère force, the BH exerts a net torque on the magnetic flux tube,

$$
\Delta T_{MC} = \mathcal{R} B_H I / \rho \Delta \Omega = (\Delta \Psi / 2 \pi) I
$$

$$
= \frac{4a_s(1 - \beta_{HD}) G(a_s, r, \eta) r_H}{(1 + \xi)(2 \csc^2 \theta - 1 + \sqrt{1 - a_s^2})} MB_H^2 \Delta r.
$$

From the second equality it is easy to find that this torque is equal to that exerted on the disk by the same flux tube or, equivalently speaking, that angular momentum flows between the BH and the disk. The angular momentum flux can be written as

$$
H_{MC} = \frac{1}{4\pi r} \frac{\Delta T_{MC}}{\Delta r}.
$$

The power transmitted to the disk through the tube is given by

$$
\Delta P_{MC} = I \Delta \epsilon_D + I^2 \Delta Z_D
$$

$$
= 4\pi r H_{MC} \Omega \Delta r + 4\pi r H_{MC} (\Omega - \Omega) \Delta r
$$

$$
= \Delta P_{MW} + \Delta Q_{ohm},
$$

where

$$
\Omega = \frac{\Omega_H \Delta Z_D + \Omega \Delta Z_H}{\Delta Z_H + \Delta Z_D}
$$

is the angular velocity of the magnetic field lines, $\Delta P_{MW} \equiv 4\pi r H_{MC} (\Omega - \Omega) \Delta r$ is the rate of mechanical work done by the electromagnetic torque on the disk, and $\Delta Q_{ohm} \equiv 4\pi r H_{MC} (\Omega - \Omega) \Delta r$ is the rate of ohmic heating in the disk. It is easy to calculate the power dissipated on the BH’s stretched horizon intersecting with the flux tube:

$$
\Delta Q_{BH} = I^2 \Delta Z_H,
$$

which increases the irreducible mass of the BH.

3. BASIC EQUATIONS OF THE ACCRETION FLOW

We assume that the energy and angular momentum transferred by the MC process are deposited into the accretion flow homogeneously.
in the vertical direction. The height-averaged basic equations describing the MCADAF can then be written as

\[ \dot{M} = -4\pi r \rho H \nu = \text{const}, \]

\[ d v dr = (\Omega^2 - \Omega_k^2) v - \frac{1}{\rho} \frac{dp}{dr}, \]

\[ \dot{M} \frac{d}{dr}(\Omega r^2) + 4\pi r H_{\text{MC}} = -\frac{d}{dr}(4\pi r^3 \tau_{\nu r} \nu H), \]

\[ \rho v T_i \frac{d s_i}{dr} = (1 - \delta) \left( q_{\text{vis}} + \frac{Q_{\text{ohm}}}{2H} \right) - q_e, \]

\[ \rho v T_e \frac{d s_e}{dr} = \delta \left( q_{\text{vis}} + \frac{Q_{\text{ohm}}}{2H} \right) + q_e - q^-. \]

Here \( \dot{M} \) is the accretion rate, \( (p/\rho)^{1/2} \equiv c_s \) is the isothermal sound speed, and \( \rho = \rho_{\text{gas}}/\beta_i = \rho_{\text{gas}} c_s^2/\beta_i \) is the total of the pressure of the tangled magnetic field and the gas pressure, with \( \beta_i \) the ratio of the gas to the total pressure, which is fixed at its "typical" value \( \beta_i = 0.9; T \) is the temperature, and \( s \) is the entropy. The subscripts "i" and "e" indicate quantities for ions and electrons, respectively. The quantity \( \tau_{\nu r} = -\alpha \rho \) is the \( r \nu \)-component of the viscous stress tensor, adopting the \( \alpha \)-prescription (Shakura & Sunyaev 1973), \( H = c_s/\Omega_k \) is the vertical scale height, \( \Omega_k \) is the Keplerian angular velocity calculated using the pseudo-Newtonian potential given by Mukhopadhyay (2002), \( \delta \) describes the fraction of the total energy that directly heats the electrons and is set to \( \delta = 0.3 \) following the detailed modeling result for the supermassive black hole in our Galactic center (Yuan et al. 2003). \( q_{\text{vis}} = r \tau_{\nu r} (d\Omega/dr) \) is the heating rate due to the viscosity, \( q_{\text{vis}} \) represents the rate of energy transfer per unit volume from ions to electrons through Coulomb collisions, \( q^- \) is the rate of cooling of the electrons (which consists of bremsstrahlung, synchrotron, and Comptonization; Narayan & Yi 1995; Manmoto et al. 1997), and \( Q_{\text{ohm}} = \Delta Q_{\text{ohm}} / 4\pi r^2 \Delta r = H_{\text{MC}} (\Omega_F - \Omega) \) is the rate of ohmic dissipation per unit area of the disk.

Adopting the no-torque boundary condition at the horizon, we integrate equation (21) from \( r_H \) to \( r \) and obtain the conservation equation for the angular momentum,

\[ l + \frac{\alpha c_{\text{vis}}^2}{v} = \frac{1}{M} T_{\text{MC}}(r) = N_0, \]
where
\[ T_{MC}(r) = \int_{r_{out}}^{r} \frac{\partial T_{MC}}{\partial r} dr, \]  
\[ N_0 = l_0 + \frac{1}{M} T_{MC}(r_H) = \text{const} \]  
with \( l_0 \) being the angular momentum per unit mass swallowed by the BH. The three terms on left-hand side of equation (24) correspond to the advected angular momentum, viscous torque, and magnetic torque due to the field lines in the range from \( r \) to \( r_{out} \). One more relation comes from the equation of state,
\[ p_{\text{gas}} = k(T_i/\mu_i + T_e/\mu_e)/m_u, \]  
where \( \mu \) is the mean molecular weight, \( k \) is Boltzmann's constant, and \( m_u \) is the atomic mass unit.

Thus we have a set of six equations including one integral, two algebraic, and three differential equations, that is, equations (19), (20), (22)–(24), and (27), for six unknown quantities, \( H \) (or \( c_s \)), \( n, \rho, \Omega, T_i \), and \( T_e \). This set of equations can be solved with three outer boundary conditions and the adoption of some parameter values (we specify these below). However, there is some difficulty in obtaining \( \Omega \) from the integral equation, namely, equation (24), so we use the first-order approximation \( T_{MC}(r) \approx T_{MC}(r + \Delta r) - \frac{\partial T_{MC}}{\partial r} |_{r} dr \).

### 4. NUMERICAL RESULTS

We adopt \( M = 10 M_\odot \) and \( r_{out} = 10^3 M \) in this paper. Regarding the outer boundary of the ADAF, the ion temperature \( T_i \) should be of the same order as the virial temperature, and the electron temperature \( T_e \) should be somewhat lower than \( T_i \) because of the radiation of the electrons. The angular velocity of the accreting flow should be sub-Keplerian (Narayan et al. 1998). So, we impose the boundary conditions \( T_i = 2 \times 10^5 \) K, \( T_e = 1 \times 10^5 \) K, and \( v/c_s = 0.3 \). At last, by adjusting the eigenvalue of the problem, \( N_0 \), we can obtain the global transonic solution, that is, a solution that passes through the sonic point smoothly.

The free parameters of our MCADAF model include \( a_s, c_B, \lambda, n, \alpha, \) and \( \dot{m} \), where \( \dot{m} = M/M_{\text{Edd}} \) with the Eddington accretion rate \( M_{\text{Edd}} = 1.39 \times 10^{19} M_\odot \text{g s}^{-1} \). The first parameter describes the spin of the BH, the next three are associated with the magnetic field, and the last two describe the ADAF.

Figure 1 shows the profile of \( \xi (\equiv \Delta Z_H/\Delta Z_h) \) when \( a_s = 0.9, \) \( n = 3, \lambda = 1 \) (corresponding to \( r_p = r_{\text{rms}} \)), \( c_B = 1, \alpha = 0.3, \) and \( \dot{m} = 0.01 \). As can be seen, the resistance of the disk is small compared
with the resistance on the stretched horizon. In particular, at the outer boundary the resistance of the disk is completely negligible. Since the distribution of the magnetic field we assumed is not smooth, there is a break at \( r_{\text{ms}} \). Figure 2 shows the curves of \( \Omega, \Omega_F, \Omega_H \), and \( \Omega_F - \Omega \) for the same parameters as in Figure 1. From this figure we find that \( \Omega_F \) always lies between \( \Omega_H \) and \( \Omega \), which agrees with equation (17). We also find that the angular velocity of the magnetic field lines relative to the disk, that is, \( \Omega_F / \Omega \), achieves a maximum at some radius between the inner and outer boundaries. This is a natural result because \( \Omega_F / \Omega \) is proportional to the product of the two factors \( \Delta Z_{\text{OH}} / (\Delta Z_{\text{OH}} + \Delta Z_{\text{DP}}) \) and \( \Omega_H / \Omega \) (see eq. [17]), which approach zero at the outer and inner boundaries, respectively. At the innermost region, \( \Omega_F > \Omega \). This unphysical result arises because we did not use exact general relativity.

From Figure 3, we can get some idea of the partitioning of the magnetically extracted rotational energy of the BH. In this figure, we show the curves of the rate of mechanical work due to electromagnetic torque and three kinds of heating rate per unit area. The solid line represents \( Q_{\text{ohm}} \), the long-dashed line is for \( P_{\text{MW}} / 4\pi r^2 = H_{\text{MC}}/r \), the short-dashed line is for the ohmic dissipation on the BH’s stretched horizon, \( Q_{\text{BH}} = \Delta Q_{\text{BH}} / 4\pi r^2 \), and the dotted line is for the viscous heating rate per unit area, \( Q_{\text{vis}} = 2Hq_{\text{vis}} \). As the figure shows, \( Q_{\text{ohm}} \) dominates over \( P_{\text{MW}} \) in the outer region of the disk, while the latter dominates in the inner region. Comparing \( Q_{\text{ohm}} + P_{\text{MW}} \) with \( Q_{\text{BH}} \), we find that the efficiency of extracting energy from the BH to the disk is very small except in the inner region. Moreover, it can be seen that the MC power is small, in contrast to the viscous heating rate.

The profiles of the radial Mach number, surface density \( \Sigma \), \( T_e \) and \( T_i \), specific angular momentum \( l \), and advection factor \( f = q_{\text{adv}} / (q_{\text{adv}} + q_{\text{vis}} + q_{\text{MC}}/H) \) of the accretion flow for different values of \( c_B \) are shown in Figure 4. The solid, dotted, and dashed lines correspond to \( c_B \)-values of 1, 0.5, and 0, respectively.

From Figure 4, we find that when the MC process is present, the sonic point moves inward, \( T_e \) decreases, and \( \Sigma \) and \( T_i \) increase, while \( l \) and \( f \) decrease in the outer region and increase in the inner region. These effects can be understood as follows: As the spin of the BH is very fast \( (a \sim 0.9) \), angular momentum and energy are transferred from the BH to the disk. The energy flux raises the temperature of the ions. The angular momentum flux hinders the infall of the accreting material. Thus, the sonic point moves inward and the surface density increases. Since the optical depth is proportional to the surface density, the Compton cooling rate goes up, and consequently the temperature of the electrons decreases. Compared with the case without the MC process, the \( r_{\phi} \)-component of the viscous stress tensor in an MCADAF around a fast-rotating BH is a bit larger because of the higher pressure \( p \) in the outer region.
region, so the angular momentum is transferred more efficiently and the specific angular momentum there is smaller. But at small radii, the increase of the angular momentum due to the MC process dominates over the decrease due to the viscous torque, so that the specific angular momentum can even increase, as can be seen from equation (21). Similarly, in the outer region of the disk the radiative cooling rate becomes higher and \( f \) decreases, while in the inner region \( f \) goes up because the heating rate due to the MC process increases more quickly than the radiative cooling rate does. In addition, as the magnetic field threading the BH becomes stronger, the influences of the MC process become more significant. However, as Figure 3 shows, the contribution of the MC process is of comparatively minor importance, so its overall influence is small.

Figure 5 shows the influence of the parameter \( \lambda \) (see just below eq. [1]). The solid and dashed lines are for \( \lambda = 1 \) and \( \lambda = 1.5 \), respectively. For purposes of comparison, the dotted lines show the case with no MC process. From this figure, one can see that the effects of increasing \( \lambda \) are similar to those of increasing \( c_B \). This is because the magnetic field in the region \( r > r_p \) strengthens as these two parameters increase. Although an increase in \( \lambda \) also leads to a decrease of the magnetic field in the region \( r_H < r < r_p \), the MC effects are small in this region, because the gravitational force there is so strong and the radial velocity is so high that the energy and angular momentum transferred by the MC process do not play any significant role.

Figure 6 shows the effects of the parameter \( n \) (see eq. [1]). The solid and dashed lines are for \( n = 4 \) and \( n = 3 \), respectively. The case with no MC process is shown by the dotted lines. From this figure we find that the MC effects are more significant for smaller \( n \). This is because the magnetic field is weaker in the outer region \( (r > r_p) \) when \( n \) is larger, which can be seen from equations (1) and (5).

We also calculated the influence of the MC process on the radiative efficiency of the ADAF. The results are shown in Figure 7. The parameters are \( \alpha = 0.9 \), \( c_B = 1.0 \), \( \lambda = 1 \), and \( n = 3 \). The quantity \( M_{\text{crit}} \) denotes the critical accretion rate of an ADAF, which is \( \sim \alpha^{-2} M_{\text{edd}} \) (see, e.g., Narayan et al. 1998). The radiative efficiency increase due to the MC process is written as \( \eta_{\text{MC}} = \eta_{\text{MCADAF}} - \eta_{\text{ADAF}} \), where \( \eta_{\text{MCADAF}} \) and \( \eta_{\text{ADAF}} \) are the efficiencies of the MCADAF and a pure ADAF, respectively. From Figure 7 we find that when \( \alpha = 0.3 \), the MC process can raise the efficiency by about one percentage point, that is, \( \sim 0.3 \eta_{\text{ADAF}} \). But when \( \alpha = 0.1 \), the effect of the MC process is very weak. The efficiency goes up for two reasons: first, the MC process transports additional energy to the ADAF; second, the angular momentum transported by the MC process decreases the radial velocity of the ADAF and thus makes the ADAF more efficient at radiating. When \( \alpha \) is smaller, the specific angular momentum of the accreting material is larger since viscosity is less efficient in moving angular momentum out, and consequently the difference between the angular velocities of the BH and the disk is smaller. Considering equations (14), (15), and (16), the angular momentum and energy transported by the MC process decrease.

In all the above discussion, the spin of the BH is very high. The angular momentum and energy are transferred from the BH to the disk. If the BH rotates slowly, the angular momentum and energy may be transferred from the disk to the BH. However, since \( H_{\text{MC}} \) is proportional to \( a_\ast \), the MC effect is not so significant as in the case when \( a_\ast = 0.9 \). In addition, there is a critical value of \( a_\ast \), at which the total energy and angular momentum transmitted by the MC process are zero and consequently \( \eta_{\text{MC}} = 0 \). In the SSD case, this value is about \( a_\ast = 0.283 \) for \( n = 3 \) (Wang et al. 2003), whereas in our MCADAF model it is about 0.172. The critical value is smaller in the MCADAF case because the angular velocity of the ADAF is sub-Keplerian.

The spin of the BH can even be negative, that is, retrograde. From equations (14) and (15), it is easy to find that the angular momentum always flows from the ADAF to the BH when \( a_\ast < 0 \). If the resistance of the ADAF is zero, that is, \( \xi = 0 \) or \( \Omega_F = \Omega \), the energy flows in the same direction as that of the angular momentum, as can be seen from equation (16). If \( \xi \) is nonzero and large enough, ohmic dissipation in the disk may offset the loss of energy that was conveyed to the BH. According to equation (16), the critical condition is \( \Delta P_{\text{MC}} = 0 \) or, equivalently, \( \xi + \beta_{\text{HD}} = 0 \).
Our calculations show that when $a_\ast < 0$, $\xi$ is so small that $\xi + \beta_{1D} < 0$ holds for almost all cases, and the net energy flows from the ADAF to the BH. If the BH rotates rapidly, for example, $a_\ast = -0.9$, the effects of the MC process will be negative compared with the case of $a_\ast = 0.9$: the radial velocity and the temperature of the electrons increase, the sonic point moves outward, the ion temperature decreases, etc.

5. SUMMARY AND DISCUSSION

In this paper we investigated the influence of magnetic coupling on the dynamics of an ADAF. The effect of the MC process on the basic equations of the accretion flow was simplified to that of a source of angular momentum and energy. The angular momentum and energy fluxes were derived with the equivalent-circuit approach. We find that when the BH rotates fast (e.g., $a_\ast = 0.9$) and when the viscosity parameter $\alpha$ is large, $\alpha = 0.3$, for a reasonable magnetic field the MC process can mildly affect the dynamics of the ADAF, increasing the ion temperature and the density of the accretion flow and decreasing the electron temperature and radial velocity. The MC process can also raise the efficiency of the ADAF by $\sim 30\%$. But if $\alpha = 0.1$ or smaller, the influences of the MC process are much weaker and can be neglected.

In the above calculations, the strength of the magnetic field was estimated following Moderksi et al. (1997), which is equivalent to assuming $c_B \leq 1$. Obviously, uncertainties exist in this estimate. On one hand, recent MHD simulations show that the magnetic field strength near the horizon can be very high, almost 4 times as large as the equipartition value (McKinney 2005), which corresponds roughly to $c_B \approx 2$. On the other hand, our calculation requires that there exist an upper limit on the value of $c_B$. This is because if $c_B$ were too large, the transferred angular momentum from the BH to the accretion flow would be so significant that the accretion could not proceed, because of the strong centrifugal force. We find that the highest value of $c_B$ depends on the accretion rate for a given $a_\ast$. It can be $\sim 5$ if the accretion rate is very low but $\sim 1$ if the accretion rate is as high as $m \gtrsim 0.1$. Considering the above two limitations on $c_B$, the increased efficiency due to the MC process, $\eta_{MC} = 1 - \eta_{ADAF}$, can be as high as $10\%$.

Observations of the hard state of BH X-ray binaries sometimes indicate luminosities as high as $L_X \sim (0.1-0.3)L_{\text{Edd}}$, which could be high enough to explain the observed highest $L_X$. However, the magnetic field strength near the horizon can be very high, almost 4 times as large as the equipartition value (McKinney 2005), which was estimated following Moderski et al. (1997), which is equivalent to assuming $c_B \leq 1$. Obviously, uncertainties exist in this estimate. In the above calculations, the strength of the magnetic field was estimated following Moderksi et al. (1997), which is equivalent to assuming $c_B \leq 1$. Obviously, uncertainties exist in this estimate. On one hand, recent MHD simulations show that the magnetic field strength near the horizon can be very high, almost 4 times as large as the equipartition value (McKinney 2005), which corresponds roughly to $c_B \approx 2$. On the other hand, our calculation requires that there exist an upper limit on the value of $c_B$. This is because if $c_B$ were too large, the transferred angular momentum from the BH to the accretion flow would be so significant that the accretion could not proceed, because of the strong centrifugal force. We find that the highest value of $c_B$ depends on the accretion rate for a given $a_\ast$. It can be $\sim 5$ if the accretion rate is very low but $\sim 1$ if the accretion rate is as high as $m \gtrsim 0.1$. Considering the above two limitations on $c_B$, the increased efficiency due to the MC process, $\eta_{MC} = 1 - \eta_{ADAF}$, can be as high as $10\%$.

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