Proton pairing in neutron stars from chiral effective field theory

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(Dated: September 27, 2017)

We study the $^1S_0$ proton pairing gap in beta-equilibrated neutron star matter within the framework of chiral effective field theory. We focus on the role of three-body forces, which strongly modify the effective proton-proton spin-singlet interaction in dense matter. We find that three-body forces generically reduce both the size of the pairing gap and the maximum density at which proton pairing may occur. The pairing gap is computed within BCS theory, and model uncertainties are estimated by varying the nuclear potential and the choice of single-particle spectrum in the gap equation. We find that a second-order perturbative treatment of the single-particle spectrum suppresses the proton $^1S_0$ pairing gap relative to the use of a free spectrum. We estimate the critical temperature for the onset of proton superconductivity to be $T_c = (3.7 - 6.0) \times 10^9$ K, which is consistent with previous theoretical results in the literature and marginally within the range deduced from a recent Bayesian analysis of neutron star cooling observations.

PACS numbers: 21.30.-x, 21.65.Ef.

I. INTRODUCTION

Neutron superfluidity and proton superconductivity play an important role in the physics of neutron stars. The dilute gas of neutrons in the inner crust of a neutron star is expected to pair in the spin-singlet channel, resulting in a neutron superfluid whose vortices provide a large angular momentum reservoir critical for the production of pulsar glitches \cite{1,2,3,4}. At the higher densities present in the core of neutron stars, neutrons may be paired in the spin-triplet channel, leading to additional cooling processes involving pair formation/breaking that can impact the early thermal evolution of neutron stars \cite{5,6,7}. Neutron star cooling may also be affected by the presence of superconducting protons in neutron star cores \cite{14,15}, though the critical temperature is expected to be much larger than that for neutron superfluidity and consequently would impact the cooling curve at earlier timescales. Well below the critical temperature for neutron superfluidity and proton superconductivity, neutron emission involving neutrons or protons is highly suppressed due to the minimum gap energy required to break a Cooper pair.

Unlike electron pairing in condensed matter systems, accurate estimates for nuclear pairing gaps in the various spin and isospin channels are challenging due to uncertainties in strong interaction physics, especially poorly constrained nuclear many-body forces that become increasingly important at high densities. For this reason neutron spin-singlet pairing in pure neutron matter has been the most widely studied pairing channel, with recent work focusing on the role of three-body forces \cite{16,17} and long- and short-range correlations \cite{18,19,20} in the BCS approximation. Quantum Monte Carlo calculations \cite{21}, on the other hand, can explore neutron pairing in the strong superfluid regime and connections to ultracold Fermi gases at unitarity. In nearly all cases, however, lattice effects and the presence of nuclear clusters in the neutron star crust are neglected in microscopic many-body calculations of the neutron $^1S_0$ pairing gap. Spin-triplet pairing of neutrons in the neutron star core is anticipated from the strong attraction in the $^3P_2 - ^3F_2$ partial-wave channel observed in nucleon-nucleon (NN) elastic scattering \cite{22}. However, many-body effects such as screening, short-range correlations, and three-body forces play a more substantial role, and there is currently much uncertainty in estimates of the spin-triplet pairing gap (for a recent review, see Ref. \cite{23}).

Previous works \cite{24,25,26,27,28} studying proton pairing in neutron star cores have employed a variety of NN interaction models and many-body methods. The peak in the proton pairing gap was found to vary between $\Delta \simeq 0.4 - 0.9$ MeV and to occur around normal nuclear densities $n_0 \simeq 0.16$ fm$^{-3}$, though the density of protons is one or two orders of magnitude less and set by the condition of beta equilibrium. More recently \cite{29} a three-body force based on $\pi$ and $\rho$ meson exchange was included in the solution of the BCS gap equation and found to reduce by half the maximum value of the proton pairing gap compared to the inclusion of two-body forces alone. The two-body force employed in Ref. \cite{29} was the Argonne $\nu_{1S0}$ potential, which includes explicit one-pion exchange at large distances but treats the medium- and short-range parts of the NN potential in terms of parametrized phenomenological functions.

In the present study we focus on a microscopic description of proton pairing in neutron star cores employing a set of two- and three-body nuclear forces \cite{30,31} derived in the framework of chiral effective field theory \cite{32,33}. Previous works employing these potentials have shown that they provide a good description of nuclear matter saturation \cite{34,35,36,37,38,39,40}, the liquid-gas phase transition \cite{41,42}, nucleon-nucleus optical potentials \cite{43,44} and Fermi liquid parameters \cite{45}. In addition the derived nuclear equation of state (EOS) is consistent with other studies \cite{46,47} employing different chiral nuclear forces and
many-body methods. The present work will be important for developing consistent modeling of the equation of state and nucleonic pairing needed for neutron star cooling calculations.

The paper is organized as follows. In Section II we describe the method employed to solve the BCS gap equation. We also detail the treatment of the proton-proton effective interaction and the proton single-particle potential in neutron-rich matter from chiral effective field theory. In Section III we present results for the density-dependent $^1S_0$ proton pairing gap at the Fermi surface in beta-equilibrated nuclear matter. Theoretical uncertainties are estimated by varying the resolution scale of the nuclear potential, the order in the chiral expansion, and the treatment of the single-particle dispersion relation. We conclude with a summary and outlook.

II. BCS GAP EQUATION

The $^1S_0$ pairing gap for a given baryon number density can be obtained in the BCS approximation by solving the gap equation

$$\Delta(k) = -\frac{1}{2} \sum_{k'} V_{\text{eff}}(k, k') \frac{\Delta(k')}{\sqrt{(e_{k'} - \mu)^2 + \Delta^2(k')}}$$

$$= -\frac{1}{\pi} \int dk' k'^2 \frac{V_{\text{eff}}(k, k') \Delta(k')}{\sqrt{(e_{k'} - \mu)^2 + \Delta^2(k')}} ,$$

(1)

where $\Delta(k)$ is the pairing gap for the momentum $k$, $V_{\text{eff}}(k, k')$ is the effective potential between two incoming particles with relative momentum $k$ and outgoing relative momentum $k'$. The single-particle energy as a function of momentum $k$ is denoted by $e_k$, and $\mu$ is the chemical potential for protons at a given density. Many BCS calculations in nuclear matter employ an effective mass approximation

$$e_k = \frac{k^2}{2M^*} + U,$$

(2)

where $U$ depends on the density but is independent of the momentum $k$. From Eq. (2), the gap equation is then approximated by substituting

$$e_k - \mu \approx \frac{1}{2M^*}(k^2 - k_F^2).$$

(3)

The above approximation assumes that the single-particle energy is nearly quadratic in $k$ near the Fermi momentum $k_F$. In this case the numerical solution for Eq. (1) can be obtained from a generalized matrix eigenvalue solution [49]. In practice one applies an adaptive mesh point scheme that depends on the effective mass for a given Fermi momentum.

We also employ the modified Brodyen method [50, 51] to compare the numerical solutions. We find that both methods agree within 1 keV when we use the effective mass approximation in Eq. (2). The numerical solution to the generalized matrix eigenvalue problem, however, is not applicable when we use a general single-particle energy spectrum instead of the effective mass approximation. Thus, we implement the Brodyen method to obtain the BCS solution in this work.

The single-particle energy spectrum plays an important role in determining the solution to the gap equation, and in the present work we consider three approximations to estimate the associated theoretical uncertainty. First, we assume a free particle spectrum given by $e_k^{(0)} = k^2/2M$. Second, we compute the proton single-particle energy in the Hartree-Fock approximation

$$e_k^{(1)} = k^2/2M + \Sigma^{(1)}(k),$$

(4)

where the first-order contribution $\Sigma^{(1)}(k)$ to the nucleon self energy is shown diagrammatically in Fig. 1(a). Third, we compute the single-particle energy self-consistently at second-order in perturbation theory

$$e_k^{(2)} = k^2/2M + \Sigma^{(1)}(k) + \text{Re}\Sigma^{(2)}(e_k^{(2)}, k),$$

(5)

where $\Sigma^{(2)}(e_k, k)$ is represented by the sum of diagrams (b) and (c) in Fig. 1.

The effective NN interaction (represented by the wavy lines in Fig. 1) is constructed by adding to the bare chiral two-body force a medium-dependent NN interaction derived from the next-to-next-to-leading order (N2LO) chiral three-body force. The medium-dependent interaction $V_{NN}^{(\text{med})}$ is obtained by averaging one state over the filled Fermi sea of noninteracting protons and neutrons in asymmetric nuclear matter [52] (for additional details see Refs. [10, 55]) and therefore depends on both the density and composition, namely, the proton fraction. The double-wavy line in Fig. 1(a) represents the fact that in the first-order Hartree-Fock contribution to the nucleon self-energy, there is an additional symmetry factor of $\frac{1}{2}$ for the medium-dependent potential, namely $V_{\text{eff}}^{HF} = V_{NN} + \frac{1}{2} V_{NN}^{(\text{med})}$.

The proton fraction is determined by enforcing beta equilibrium, which requires computing the proton and neutron chemical potentials from the equation of state of asymmetric nuclear matter [48, 54]. The electrons are
treated as a relativistic gas of noninteracting Fermions, and the nuclear equation of state is computed consistently at second order in perturbation theory.

We employ chiral nucleon-nucleon interactions at next-to-next-to-leading order (N2LO) and next-to-next-to-leading order (N3LO) in the chiral power counting. For values of the momentum-space cutoff $\Lambda \lesssim 500$ MeV, nucleon-nucleon potentials generally exhibit good convergence properties in many-body perturbation theory. In the present work we therefore consider two values of the cutoff ($\Lambda = 450$ MeV and 500 MeV) at N2LO and three values of the cutoff ($\Lambda = 414$ MeV, $\Lambda = 450$ MeV and 500 MeV) at N3LO \cite{31}. We note that the value $\Lambda = 414$ MeV is not the result of fine tuning but instead corresponds to the relative momentum for nucleon-nucleon scattering at a lab energy of $E_{\text{lab}} = 350$ MeV, the maximum energy at which modern nucleon-nucleon potentials are fitted to phase shifts. In all cases we include also the N2LO chiral three-body force whose low-energy constants $c_D$ and $c_E$ are fitted to reproduce the binding energies of $A = 3$ nuclei and the beta-decay lifetime of $^3\text{He}$ \cite{33,34}.

### III. RESULTS

In Fig. 2 we show the equation of state of beta equilibrated nuclear matter calculated from the five chiral nuclear forces employed in the present work. We first compute the equation of state for isospin-asymmetric nuclear matter at second order in perturbation theory:

$$\mathcal{E}^{(1)} = \frac{1}{2} \sum_{12} n_1 n_2 \langle 12 \left| \left( V_{NN} + V_{NN}^{(\text{med})} / 3 \right) \right| 12 \rangle, \quad (6)$$

FIG. 2: (Color online) Equation of state of nuclear matter in beta equilibrium from the chiral two- and three-nuclear forces used in this work.

FIG. 3: (Color online) Proton fraction as a function of density for beta-equilibrated nuclear matter for $n \geq 0.5n_0$. Results are shown for the five density-dependent nuclear interactions at N2LO and N3LO.

$$\mathcal{E}^{(2)} = -\frac{1}{4} \sum_{1234} \langle 12 \left| \left[ \mathcal{V} \right]_{\text{eff}} \right| 34 \rangle^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2}, \quad (7)$$

where $\mathcal{E} = E/V$ is the energy density, $n_j = \theta \left( k_f - |\vec{p}_j| \right)$ is the zero-temperature distribution function, $\bar{n}_j = 1 - n_j$, $\mathcal{V} = V - \mathcal{P}_{12} V$ is the antisymmetrized $NN$-potential with $\mathcal{P}_{12}$ the exchange-operator in spin-, isospin- and momentum-space. The density-dependent $NN$-potential derived from the N2LO chiral three-body force is denoted by $V_{NN}^{(\text{med})}$ and $V_{\text{eff}} = V_{NN} + V_{NN}^{(\text{med})}$.

The proton and neutron chemical potentials can then be evaluated as

$$\mu_p = \frac{\partial \mathcal{E}}{\partial n_p} \bigg|_{n_n}, \quad \mu_n = \frac{\partial \mathcal{E}}{\partial n_n} \bigg|_{n_p}, \quad (8)$$

where $n_p$ is the proton number density and $n_n$ is the neutron number density. The electron density is set by charge neutrality, and beta equilibrium is then found by enforcing $\mu_n = \mu_p + \mu_e$. As observed in Ref. \cite{34} the energy per particle from the two N2LO chiral potentials is systematically larger than that from the three N3LO potentials, and this difference grows as the density increases. The equations of state from the N2LO potentials are stiffer than those from the N3LO potentials, which we anticipate will correlate with a smaller pairing gap at N2LO. Beyond $n = 2n_0$ a description of the nuclear equation of state based on chiral effective field theory is likely unreliable for the low-momentum perturbative potentials considered in the present work. We anticipate a corresponding increase in the $^1S_0$ proton pairing gap uncertainty band beyond $n \gtrsim n_0$.

At low densities the results for the nuclear equation of state shown in Fig. 2 are in better agreement for the
shown \cite{57} to increase the nuclear symmetry energy by
values \( S_e \) and its density dependence. The proton fraction in
the beta-equilibrated nuclear matter found in this work is consistent
with constraints from the most current experimental and theoretical predictions. Beyond nuclear saturation density, the theoretical uncertainty in the proton fraction increases significantly. The two N2LO chiral potentials produce the largest ground-state energy for beta-equilibrated nuclear matter and give rise to proton fractions \( Y_p = 7.5-8.5\% \) at twice saturation density. The three N3LO chiral potentials, on the other hand, predict smaller values of \( Y_p = 4-6\% \) at \( n = 2n_0 \).

In Fig. 4 we show the proton and neutron self-consistent second-order single-particle energies \( \epsilon_k^{(2)} \) as a function of the momentum \( k \) for a specific value of the proton Fermi momentum \( k_F^p = 0.4 \text{ fm}^{-1} \), corresponding to a total baryon number density of \( n \approx 0.5n_0 \). This is the density at which the neutron star inner crust transitions to homogeneous nuclear matter in the core, and as we show below it also corresponds to the density at which the proton \(^1\text{S}_0\) pairing gap is maximal. The single-particle energies are computed at second-order in perturbation theory according to Eq. (\text{7}).

The transition density was identified employing two different methods: (i) comparing the ground state energies of the homogeneous and inhomogeneous phase as a function of density in the Thomas-Fermi approximation and (ii) the thermodynamic instability method \cite{56} where the density of homogeneous matter is lowered until an instability to cluster formation appears. Given the tight range of momentum for protons and neutrons in beta-equilibrated nuclear matter at \( F = 0.4 \text{ fm}^{-1} \), as we show below it also corresponds to the density at which the proton \(^1\text{S}_0\) pairing gap is maximal. The single-particle energies are computed at second-order in perturbation theory according to Eq. (\text{7}). The first- and second-order diagrammatic contributions to the nucleon self energy have the form

FIG. 4: (Color online) Single-particle energies as a function of momentum for protons and neutrons in beta-equilibrated nuclear matter at \( k_F^p = 0.4 \text{ fm}^{-1} \). The self-consistent second-order approximation to the single-particle energy, shown in Eq. (\text{7}), is employed.

\begin{align}
\Sigma^{(1)}_t(k) &= \sum_{1} \langle \vec{k} \hat{n}_1 \bar{s}_1 t_1 | (V_{NN} + V_{NN}^{(med)}/2) | \vec{k} \hat{n}_1 \bar{s}_1 t_1 \rangle n_1, \\
\Sigma^{(2a)}_t(k, \omega) &= \frac{1}{2} \sum_{123} \frac{|\langle \vec{p}_1 \vec{p}_2 \bar{s}_1 \bar{s}_2 t_1 t_2 | V_{eff} | \vec{k} \hat{n}_2 \bar{s}_2 t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} n_1 n_2 n_3, \\
\Sigma^{(2b)}_t(k, \omega) &= \frac{1}{2} \sum_{123} \frac{|\langle \vec{h}_1 \vec{h}_3 \bar{s}_1 \bar{s}_3 t_1 t_3 | V_{eff} | \vec{k} \bar{s}_2 t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 - i\eta} n_1 n_2 n_3,
\end{align}

where \( t \) labels the isospin quantum number of the external particle.

The same approximations employed in the present work have been shown to give very good agreement with the real part of the nucleon-nucleus optical potential and its dependence on the isospin asymmetry \cite{41,42}. From Fig. 4 we see that the different nuclear potentials give very similar results for the momentum dependence of the proton single-particle energy. As expected for the case of highly neutron-rich matter, the proton single-particle potential is much more strongly attractive than the neutron single-particle potential. In fact, at the proton Fermi momentum \( k_F^p = 0.4 \text{ fm}^{-1} \) the proton chemical potential is \( \mu_p = \epsilon_p(k_F^p) \simeq -65 \text{ MeV} \).
We next turn our attention to the calculation of the proton pairing gap from Eq. (1). The pairing gap at the Fermi momentum $\Delta(k_F)$ is denoted by $\Delta_F$ here and throughout. We first neglect the presence of three-body forces, in which case the nuclear potential is independent of the density and proton fraction, and focus on the role of the single-particle potential, which we parametrize with different choices of the effective mass. In general, the effective mass depends on the density and proton fraction, but for orientation we consider the case of a constant effective mass. In Fig. 5 we show the proton $^1S_0$ pairing gap from the n3lo450 nucleon-nucleon potential as a function of the proton Fermi momentum for effective masses ranging from $M^*/M = 0.6 - 1.0$. A free proton spectrum ($M^*/M = 1.0$) gives rise to a maximum in the pairing gap of $\Delta \approx 3.5$ MeV. Even a moderate reduction in the effective mass to $M^*/M = 0.7$ leads to a decrease in the maximum of the pairing gap by a factor of 2. However, the density at which the pairing gap is maximal decreases by only 10%. The strong dependence of the maximum in the pairing gap on the effective mass can be understood from Eq. (1). A small effective mass corresponds to a strong momentum dependence of the single-particle energy around the Fermi surface. As the intermediate-state momentum in Eq. (1) varies away from the Fermi momentum, the energy denominator increases more rapidly for a small effective mass, reducing the size of the pairing gap.

The effective mass approximation, Eq. (2), provides an accurate parametrization of the nucleon single-particle energy at the Hartree-Fock level. However, second-order perturbative contributions to the nucleon self-energy lead to a strong momentum dependence of the effective mass that is peaked close to $M^*/M = 1$ at the Fermi surface [59], the regime where the spectrum most strongly affects the value of the pairing gap. In Fig. 6 we study the effect of different parametrizations of the nucleon single-particle energy on the density-dependent pairing gap. In all cases we include both two- and three-body forces. In the first case, shown as the dotted curve in Fig. 6, we consider a free-particle spectrum $\epsilon_k^{(0)} = k^2/2M$. The dotted vertical line stands for the Fermi momentum at the core-crust boundary of a neutron star ($n \sim 1/2n_0$, $Y_p \sim 0.03$). Comparing to Fig. 5 we see that three-body forces lead to a reduction in the maximum value of the pairing gap by a factor of four. Although the proton Fermi momentum is small, the large neutron density leads to a more strongly repulsive effective two-body proton-proton interaction as shown in Ref. [53]. Consequently the maximum proton pairing gap shown in Fig. 6 is roughly 1/3 the $^1S_0$ neutron pairing gap in neutron star inner crusts [17], where three-body forces play a much smaller role. Treating the single-particle energy in the Hartree-Fock approximation $\epsilon_k^{(1)} = k^2/2M + \Sigma^{(1)}(k)$ leads to an additional reduction in the pairing gap by about 40% as shown by the dashed line of Fig. 6. Finally, employing the self-consistent second-order single-particle energy $\epsilon_k^{(2)}$ (see Eq. (5)) in the denominator of the gap equation leads to an increase of 20% in the maximum gap size relative to the Hartree-Fock approximation. This may be understood from the fact that the second-order contribution $\Sigma^{(2)}(\epsilon_k, k)$ to the self energy on average increases the effective mass in the vicinity of the Fermi surface.

Fig. 7 shows the $^1S_0$ proton pairing gap in the presence of three body forces using the n3lo450 chiral nuclear potential. The blue dashed curves correspond to
different values of the (fixed) proton fraction $Y_p$, which ranges from $0.002 \leq Y_p \leq 0.06$ with $\Delta Y_p = 0.002$, and the solid red curve is that for nuclear matter in beta equilibrium. For a given $Y_p$ we calculate the solution to the BCS gap equation using the first-order approximation for the single-particle energies $\epsilon^{(1)}(k)$. We see that the proton fraction is an important parameter for determining the size of the pairing gap. For instance at $k_F^p = 0.4$ fm$^{-1}$, changing the proton fraction from $Y_p = 0.03$ to $Y_p = 0.04$ would increase the gap size from $\Delta_F = 0.5$ MeV to $\Delta_F = 0.75$ MeV.

We note that the nuclear potential $V_{\text{eff}}(k, k')$ depends on the proton fraction when three-body forces are included. As shown in Fig. 7 the proton pairing gap and the available pairing domain in $k_F^p$ increase as the proton fraction increases because $V_{\text{eff}}(k, k')$ depends sensitively on the proton fraction. As mentioned in Section 4 three-body forces have been considered previously in a phenomenological way to compute the proton pairing gap in beta-stable nuclear matter. In this work, three-nucleon forces consistent with the low-energy constants in the two-body force and fitted to the properties of $A = 3$ nuclei have been employed. In addition we have calculated the nuclear EOS with the same nuclear forces to determine the proton fraction.

Fig. 8 shows the proton pairing gap using five different chiral potentials. The dotted sections of the curves indicate the pairing gap for densities lower than that of the neutron star core-crust boundary. The large symbols on the curves indicate the values of the pairing gap at nuclear densities $n = n_0/2$ (open circle), $n_0$ (filled circle), $3n_0/2$ (open square), and $2n_0$ (filled square). We see that proton pairing is expected to vanish beyond two times nuclear saturation density, and therefore a treatment within chiral effective field theory should be valid. The chiral potentials used in this work become more repulsive as the momentum cutoff increases. In the case of the N3LO potentials the pairing gap increases as the cutoff decreases as can be seen in Fig. 8. For neutron matter and beta stable nuclear matter, it was shown that the N2LO equations of state are stiffer than at N3LO. This explains why the N3LO gaps are generically larger than the N2LO gaps.

In Fig. 9 we compare the proton pairing gap uncertainty band calculated in the present work to previous results. We find that the maximum in the pairing gap lies in the range $0.56$ MeV $< \Delta_F < 0.91$ MeV, which is consistent with previous calculations, but the maximum density at which proton pairing is expected to occur is systematically smaller than other models. This is largely caused by three-body forces and the behavior of the chiral potential $V_{\text{eff}}(k, k')$ as the proton fraction is increased in neutron star matter. This suggests that the asymmetric nuclear matter potential should also be used to calculate the $^3P_2 - ^3F_2$ neutron pairing gap, which is typically calculated in pure neutron matter.

In the weak coupling approximation, the critical temperature for the onset of pairing is given by

$$T_c \simeq 0.57 \Delta_F(T = 0).$$

We find that in the present analysis the critical temperature for proton pairing in the core of neutron stars is

$$T_c \sim (3.70 - 6.03) \times 10^9 \text{K}.$$  

Compared to the range of critical temperatures predicted in a recent study from neutron star cooling using Bayesian analysis [61], where $T_c = 7.1^{+1.6}_{-1.3} \times 10^9$ K, our prediction has a smaller central value but is consistent at the highest range.
proton single-particle potential in neutron star matter reduce the maximum size of the proton $^1S_0$ pairing gap. In particular, three-body forces reduce the maximum gap size by a factor of 3, while a self-consistent second-order treatment of the proton single-particle potential leads to an additional reduction of about 30%. Our results for the $^1S_0$ proton pairing gap have a similar range of sizes compared to previous studies. However, the maximum density at which proton pairing may exist in neutron stars is systematically smaller than previous results. This ultimately comes from the inclusion of three-body forces in our effective field theory calculation, which requires a consistent calculation of the proton fraction in beta-equilibrium matter. The three-body force leads to additional repulsion in the effective interaction and a suppression in the pairing gap as the density increases.

These results will be important for a consistent description of neutron star cooling. Proton $^1S_0$ pairing will likely not give any reduction factor for nucleon direct Urca cooling, since the pairing gap is seen to vanish well before the proton fraction reaches a value high enough for the onset of the direct URCA process. However, proton pairing will certainly give a reduction factor to the thermal conductivity, heat capacity, and neutrino emission processes involving protons. Thus the enhanced cooling processes in neutrons stars arising from Cooper-pair breaking/formation is likely to be dominated by $^3P_2$ neutron pairing in the core rather than $^1S_0$ pairing of protons.

**IV. SUMMARY**

In this work we have studied the proton $^1S_0$ pairing gap in nuclear matter at beta equilibrium using five different nuclear two- and three-body potentials derived within the framework of chiral effective field theory. Nucleon-nucleon potentials at both N2LO and N3LO were considered, together with the chiral three-body force at N2LO. In addition to the choice of nuclear potential, also the single-particle spectrum employed in the BCS gap equation is a source of theoretical uncertainty.

We find that both three-body forces and a realistic proton single-particle potential in neutron star matter reduce the maximum size of the proton $^1S_0$ pairing gap. In particular, three-body forces reduce the maximum gap size by a factor of 3, while a self-consistent second-order treatment of the proton single-particle potential leads to an additional reduction of about 30%. Our results for the $^1S_0$ proton pairing gap have a similar range of sizes compared to previous studies. However, the maximum density at which proton pairing may exist in neutron stars is systematically smaller than previous results. This ultimately comes from the inclusion of three-body forces in our effective field theory calculation, which requires a consistent calculation of the proton fraction in beta-equilibrium matter. The three-body force leads to additional repulsion in the effective interaction and a suppression in the pairing gap as the density increases.

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**Acknowledgments**

Work supported by the National Science Foundation under Grant No. PHY1652199. Portions of this research were conducted with the advanced computing resources provided by Texas A&M High Performance Research Computing.

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