Homodyne Bell’s inequalities for entangled mesoscopic superposition states

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I. INTRODUCTION

Quantum mechanical superposition principle brings unexpected and counterintuitive consequences when applied to macroscopic systems. Presumably the most famous example is Schrödinger’s cat, which remains half-alive and half-dead while entangled with a decaying radioactive atom. Recent experimental advances have opened up new possibilities to study the superposition principle beyond purely microscopic domain. It is now possible to produce in a laboratory states of the type:

$$ |\mathcal{K} \rangle = \frac{1}{\sqrt{2}} (|\uparrow \rangle \otimes |\alpha \rangle + |\downarrow \rangle \otimes |\alpha \rangle) $$

(1)

where $|\uparrow \rangle$, $|\downarrow \rangle$ are two orthogonal states of a spin-1/2 system, and $|\alpha \rangle$, $|\alpha \rangle$ are two distinguishable coherent wave packets of a harmonic oscillator. Such states have been generated for a trapped ion and a microwave cavity field entangled with an atom. They can be considered as mesoscopic equivalents of the example used by Schrödinger in his original argument. The entangled states defined in Eq. (1) are closely related to the issue of generating and detecting coherence between classically distinguishable states.

In this paper we show how states described by Eq. (1) can be used to test incompatibility of quantum mechanics with local realism. Specifically, we derive Bell’s inequalities which are violated by the Schrödinger cat states. These inequalities are based on the continuous measurements of position and momentum observables for the harmonic oscillator subsystem, and standard projections for the spin-1/2 subsystem. An interesting feature of our proposal is that detection of continuous variables having a well-defined classical analog allows one to investigate the macroscopic limit of violating Bell’s inequalities by the entangled states given by Eq. (1). We demonstrate that in the limit of large wave packet amplitudes a substantial violation of Bell’s inequalities is possible, provided ideal noise-free detection and lack of decoherence.

We also perform a general analysis of the proposed scheme, including imperfect detection and dissipation, which gives a quantitative description of the disappearance of nonlocal phenomena in the presence of these deleterious effects. In particular, our analysis shows that the violation of Bell’s inequalities in the proposed scheme vanishes at the same rate the visibility of interference between the two distinct wave packets given by the states $|\alpha \rangle$ and $|\alpha \rangle$. This result illustrates the close link between nonlocality and quantum coherence. As it will be clear from the following calculations, all these features are universal, i.e. they are independent of the particular form of the wave packets involved in the superposition. We also point out that in order to demonstrate the violation of Bell’s inequality, only one of the two measurements applied to the harmonic oscillator subsystem needs to have microscopic resolution, whereas the second one is relatively insensitive to losses. Thus, our scheme provides another example of a situation described by Yurke and Stoler, whose proposal for observing the violation of local realism employed a combination of sensitive and insensitive detectors.

For concreteness, we shall consider here a quantum optical realization of the entangled states $|\mathcal{K} \rangle$. In the case of a radiation mode, which will serve in this paper as a physical realization of the harmonic oscillator subsystem, position and momentum correspond to a pair of quadratures, which can be measured with the help of homodyne detection. The imperfect measurement of quadratures can be described by an efficiency parameter $\eta$. This parameter can be straightforwardly generalized to include interaction of the electromagnetic field with an external environment, which serves as a standard model for decoherence. The quantum optical context will be used here just to fix the notation, and our calculations retain
validity for an arbitrary physical realization of a spin-1/2 particle entangled with a harmonic oscillator system.

This paper is organized as follows. First, in Sec. II we present a simple heuristic idea behind the construction of Bell’s inequalities for mesoscopic superposition states. This idea is elucidated in quantitative terms in Sec. III. Sec. IV discusses the violation of Bell’s inequality including the realistic case of losses, and Sec. V briefly reviews some of the experimental aspects. Finally, Sec. VI concludes the paper.

II. HEURISTIC CONSIDERATIONS

We shall start from giving a simple heuristic argument which motivated us to formulate Bell’s inequalities for Schrödinger cat states. For concreteness, we shall assume that the states $|\alpha\rangle$ and $|-\alpha\rangle$ describe two Gaussian wave packets in a harmonic potential centered around dimensionless positions $\sqrt{2}\alpha$ and $-\sqrt{2}\alpha$, respectively, with zero average momentum and ground state widths. From the following discussion it will be clear that the violation of Bell’s inequalities in our scheme is completely insensitive to the specific form of the wave packets. The states $|\alpha\rangle$ and $|-\alpha\rangle$ have the scalar product equal to $\langle -\alpha | \alpha \rangle = \exp(-2\alpha^2)$, and for sufficiently large $\alpha$ they can be considered as approximately orthogonal. In this case, one can establish a formal analogy [12] between the state $|K\rangle$ and the singlet state of two spin-1/2 particles used in original Bell’s argument, based on the correspondence: $|\alpha\rangle \rightarrow |\downarrow\rangle$ and $|-\alpha\rangle \rightarrow -|\uparrow\rangle$. Following this analogy, in order to violate Bell’s inequalities one should be able to perform two noncommuting measurements in the subspace spanned by $|\alpha\rangle$ and $|-\alpha\rangle$, which would correspond to projecting the spin onto two different directions. As the first measurement on the harmonic oscillator subsystem, let us simply choose the projection in the basis $\{ |\alpha\rangle, |-\alpha\rangle \}$. This measurement can be effectively accomplished by the measurement of position: if the distance between the centers of the wave packets is much larger than their spatial extent, the sign of the position variable almost unambiguously discriminates between the states $|\alpha\rangle$ and $|-\alpha\rangle$. As the second measurement on the harmonic oscillator subsystem we shall take the projection in the basis of the superpositions:

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{N_{\pm}}}(|\alpha\rangle \pm |-\alpha\rangle),$$

where $N_{\pm} = 2(1 \pm e^{-2\alpha^2})$ are the normalization constants. As depicted in Fig. 1, these two superpositions generate distinct interference patterns in the momentum distribution: location of the maxima for the state $|\Psi_{+}\rangle$ corresponds to the minima for the state $|\Psi_{-}\rangle$, and vice versa. Consequently, we can approximately discriminate between the superpositions $|\Psi_{+}\rangle$ and $|\Psi_{-}\rangle$ by checking whether the result of the momentum measurement falls within the vicinity of the interference fringes either for the state $|\Psi_{+}\rangle$ or $|\Psi_{-}\rangle$. Of course, such discrimination is imperfect as the momentum distributions partially overlap; nevertheless, we shall demonstrate that the error rate involved is low enough to enable the violation of Bell’s inequalities.

![Plot of the momentum distributions for the superpositions $|\Psi_{+}\rangle$ (solid line) and $|\Psi_{-}\rangle$ (dashed line), assuming perfect noise-free measurement and $\alpha = 6$. Location of maxima for one state corresponds to the location of minima for the other one.](image)

III. QUANTITATIVE ANALYSIS

Let us now discuss the idea sketched above in quantitative terms. As a concrete physical realization, we will take $|\alpha\rangle$ and $|-\alpha\rangle$ to be two coherent states of a single radiation mode, with $\alpha$ assumed to be real. The optical analog of the position and momentum observables is a pair of canonically conjugated quadratures defined in general as $\hat{x}_\theta = (e^{i\theta}\hat{a}^\dagger + e^{-i\theta}\hat{a})/\sqrt{2}$. Two values of the phase: $\theta = 0$ and $\theta = \pi/2$ correspond to the position and the momentum operators, respectively. The standard technique for measuring quadratures is homodyne detection, described by the positive operator-valued measure [13]:

$$\hat{H}(x; \theta) = \frac{1}{\sqrt{\pi(1-\eta)}} \exp \left( -\frac{(x/\sqrt{\eta} - \hat{x}_\theta)^2}{1/\eta - 1} \right),$$

where $\eta$ is the detection efficiency. More generally, the parameter $\eta$ can include dissipation of the electromagnetic field generated by an interaction with an environment. The perfect noise-free measurement of quadratures is obtained in the limit $\eta = 1$, whereas $\eta < 1$ describes non-ideal detection. Such non-ideal detection corresponds to a blurred measurement of position and momentum with finite resolution equal to $\sqrt{(1/\eta - 1)/2}$, expressed in the canonical dimensionless units of the harmonic oscillator.

The states $|\Psi_{\pm}\rangle$ defined in Eq. (2) generate the following interference patterns for $\theta = \pi/2$:
\[
\langle \Psi_{\pm} | \hat{H}(x; \frac{\pi}{2}) | \Psi_{\pm} \rangle = \frac{2}{\sqrt{\pi} N_{\pm}} e^{-x^2} [1 \pm e^{-2(1-\eta)x^2} \cos(\sqrt{8\eta}x)].
\]

(4)

It is easily seen that within the Gaussian envelope given by the factor \(e^{-x^2}\), the spacing between the interference fringes is given by \(T = \pi/\sqrt{2\eta}\). For the state \(|\Psi_{+}\rangle\) the interference pattern has maxima for integer multiples of \(T\), whereas the interference maxima for the state \(|\Psi_{-}\rangle\) are shifted by \(T/2\).

With the above definitions in hand, we can now specify the measurement scheme which leads to the violation of Bell’s inequalities for Schrödinger cat states. The party measuring the spin-1/2 subsystem performs homodyne detection with the realizations of the quantum optical analog of the position measurement scheme. In this case, the continuous outcome \(x\) of the homodyne detection needs to be converted into its sign, as we effectively perform the measurement of the following symmetries of the operators \(\hat{C}_0\) and \(\hat{C}_{\pi/2}\):

\[
\langle \alpha|\hat{C}_0|\alpha\rangle = \langle -\alpha|\hat{C}_0|-\alpha\rangle
\]

\[
\langle \alpha|\hat{C}_{\pi/2}|\alpha\rangle = \langle -\alpha|\hat{C}_{\pi/2}|-\alpha\rangle.
\]

(10)

With the help of these identities, one easily obtains the Bell combination expressed in terms of the matrix elements of the operators \(\hat{C}_0\) and \(\hat{C}_{\pi/2}\):

\[
S = E(a, 0) + E(a, \pi/2) + E(a', 0) - E(a', \pi/2).
\]

(9)

For local hidden variable theories, the absolute value of this combination is bounded by \(|S| \leq 2\). Explicit calculation of the combination \(S\) in our scheme is simplified by the following symmetries of the operators \(\hat{C}_0\) and \(\hat{C}_{\pi/2}\):

\[
\langle \alpha|\hat{C}_0|\alpha\rangle = \text{erf} (\sqrt{2\eta}a)
\]

(12)

and the off-diagonal element of the operator \(\hat{C}_{\pi/2}\):

\[
\langle \alpha|\hat{C}_{\pi/2}|-\alpha\rangle = -e^{-2\eta} + \frac{2}{\sqrt{\pi}} e^{-2\eta}(1-\eta) \times \sum_{n=-\infty}^{\infty} \int_{(n-1/4)T}^{(n+1/4)T} e^{-x^2} \cos(\sqrt{8\eta}ax)dx.
\]

(13)

IV. VIOLATION OF BELL’S INEQUALITY

In order to discuss the violation of Bell’s inequality for the combination \(S\), let us first perform maximization over the unit vectors \(a\) and \(a'\) along which the spin-1/2 system is measured. An easy calculation shows that the maximum value of \(S\) reads:
\[ S_{\text{max}} = 2\sqrt{\langle \alpha|\hat{C}\alpha \rangle^2 + |\langle \alpha|\hat{C}\pi/2| - \alpha \rangle|^2} \]  
(14)

and that it is obtained for the following directions of the spin measurements:

\[ a_x = -a'_x = \frac{2}{S_{\text{max}}} \text{Re} \langle \alpha|\hat{C}\pi/2| - \alpha \rangle \]

\[ a_y = -a'_y = \frac{2}{S_{\text{max}}} \text{Im} \langle \alpha|\hat{C}\pi/2| - \alpha \rangle \]

\[ a_z = a'_z = \frac{2}{S_{\text{max}}} \langle \alpha|\hat{C}\alpha \rangle. \]  
(15)

In Fig. 2 we plot \( S_{\text{max}} \) as a function of \( \alpha \) for several values of the detection efficiency \( \eta \). In the case of perfect detection of quadratures, the value of \( S_{\text{max}} \) tends with increasing \( \alpha \) to a constant value, equal about 2.37. This result clearly contradicts predictions of local hidden variable theories. In the case of non-ideal measurement of quadratures, the violation of Bell’s inequality can be still observed for sufficiently high efficiency \( \eta \) of homodyne detection, but this effect vanishes with the increasing coherent state amplitude \( \alpha \).

![Graph showing \( S_{\text{max}} \) as a function of \( \alpha \) for different \( \eta \) values.]

FIG. 2. Maximum violation of Bell’s inequality for Schrödinger cat states as a function of the coherent state amplitude \( \alpha \). The graphs correspond to measurements performed using a homodyne detector having several different efficiencies.

In order to understand the behavior of \( S_{\text{max}} \) in simple terms, we will now perform an approximate analysis of the expression (14) valid for large \( \alpha \). Under an additional condition \( \sqrt{\eta \alpha} \gg 1 \), which means that homodyne detection with the phase \( \theta = 0 \) is capable of discriminating between the wave packets \( |\alpha\rangle \) and \( |-\alpha\rangle \), we have:

\[ \langle \alpha|\hat{C}0|\alpha \rangle \approx 1. \]  
(16)

The approximately constant value of this matrix element means that the outcome of the homodyne measurement performed for \( \theta = 0 \) is insensitive to the detection efficiency.

Approximation of the matrix element \( \langle \alpha|\hat{C}\pi/2| - \alpha \rangle \) given in Eq. (13) is slightly more intricate. Large \( \alpha \) means that the spacing \( T \) between the interference fringes observed in the momentum distribution is small compared to the extent of the wave packets. Consequently, we can assume that the Gaussian envelope multiplying the integrand in Eq. (13) is constant over each of the integration intervals, which allows us to evaluate the integrals analytically. Furthermore, in this regime the sum over \( n \) of the remaining Gaussian factors can be approximated by an integral. Thus we obtain:

\[ \sum_{n=-\infty}^{\infty} e^{-x^2} \cos(\sqrt{\eta \alpha}x)dx \]

\[ \approx \frac{1}{\pi} \sum_{n=-\infty}^{\infty} Te^{-(nT)^2} \approx \frac{1}{\sqrt{\pi}}. \]  
(17)

This expression yields the following approximate formula for the off-diagonal matrix element of the operator \( \hat{C}\pi/2 \):

\[ \langle \alpha|\hat{C}\pi/2| - \alpha \rangle \approx \frac{2}{\pi} \exp[-2\alpha^2(1 - \eta)], \]  
(18)

where we have made use of the assumption \( \sqrt{\eta \alpha} \gg 1 \) in order to eliminate the first term from Eq. (13). This matrix element depends critically on the efficiency of the homodyne detector.

Thus we finally arrive to the following expression for the Bell combination:

\[ S_{\text{max}} \approx 2\sqrt{1 + \left(\frac{2}{\pi} \exp[-2\alpha^2(1 - \eta)]\right)^2}. \]  
(19)

In the case of perfect homodyne detection, the right hand side of the above formula is constant and equal to \( 2\sqrt{1 + 4/\pi^2} \). This is the asymptotic value observed in Fig. 2 in the plot of \( S_{\text{max}} \) for \( \eta = 100\% \). For imperfect homodyne detection, the violation of Bell’s inequality is damped for large \( \alpha \) by the exponential factor \( \exp[-2\alpha^2(1 - \eta)] \), which decreases with the increasing separation between the positions of the wave packets. Let us note that the calculations which led to the approximate form of the matrix elements given in Eqs. (16) and (18) are independent of the particular form of the wave packets \( |\alpha\rangle \) and \( |-\alpha\rangle \) involved in the superposition \( |\mathcal{K}\rangle \). In order to derive Eq. (18) one only needs to assume that the spatial extent of the wave packet is smaller than the separation between them. Similarly, Eq. (13) follows directly from the assumption that the wave packet envelopes in the momentum domain vary slowly on the scale of the oscillations generated by the interference term.

It is interesting to note that the violation of Bell’s inequality in the proposed scheme is directly related to the visibility of interference between the wave packets \( |\alpha\rangle \) and \( |-\alpha\rangle \). Of course, homodyne detection performed alone on the harmonic oscillator subsystem does not reveal any interference, as its reduced density matrix is just a statistical mixture of \( |\alpha\rangle \) and \( |-\alpha\rangle \). However, conditioning the homodyne measurement on a specific outcome of the
spin projection yields a clear signature of interference. A simple calculation gives the following probability distribution of obtaining the quadrature $x$ for the phase $\theta = \pi/2$ conditioned on the spin up outcome for a projection along the axis $a$:

$$p(x; \vec{\xi}) \propto e^{-x^2} \{1 + e^{-2a^2(1-\eta)} \times \left[a_x \cos(\sqrt{\eta}a x) + a_y \sin(\sqrt{\eta}a x)\right]\}.$$  

(20)

The above formula shows that the visibility of the interference fringes is proportional to the factor $\exp[-2a^2(1-\eta)]$. It is exactly the same factor which appears in the expression for the Bell combination given in Eq. (13). Thus, the better the visibility of the interference fringes is, the stronger the violation of Bell’s inequalities takes place. If $\eta$ is interpreted as the parameter describing interaction with a reservoir, it is clearly seen that the violation of Bell’s inequality and the interference visibility decay at the same rate.

When passing to the mesoscopic domain, demonstration of quantum nonlocality in our scheme requires use of a measuring apparatus which is capable of detecting interference between the components of the superposition. However, this sensitivity is important only in the homodyne measurements performed for the phase $\theta = \pi/2$. The other half of homodyne measurements can be in principle realized with a detector which does not have microscopic sensitivity. This provides another example of a situation described first by Yurke and Stoler [7], who presented a scheme for the violation of local realism employing a combination of sensitive and insensitive detectors. Analogously to their proposal, the discrimination between two amplitudes of coherent states is not sensitive to the efficiency of the detector. The second type of the measurement used by them is the determination of the photon number parity, which requires single photon resolution. In our case, the sensitive measurement has the form of homodyne detection capable of resolving interference fringes in the superpositions $|\Psi_{\pm}\rangle = (|\alpha\rangle \pm |\alpha\rangle)/\sqrt{N_{\pm}}$. Let us note that the task of distinguishing the states $|\Psi_{\pm}\rangle$ and $|\Psi_{-}\rangle$ in our scheme could in principle also be performed by measuring the photon number parity, as these two superpositions have non-zero occupation probabilities only for even or odd Fock states, respectively.

V. EXPERIMENTAL PROSPECTS

An important practical aspect of experimental schemes for testing quantum nonlocality is their sensitivity to various imperfections [15]. The effect of non-ideal spin projection is relatively straightforward to describe. If we assume that in a fraction of events the measuring device returns a flipped value of the spin, then the ideal spin projections along a direction $a$ are replaced by a generalized two-element positive operator-valued measure:

$$\hat{P}_\uparrow(a) = \frac{1+\xi}{2} |\uparrow_a\rangle\langle\uparrow_a| + \frac{1-\xi}{2} |\downarrow_a\rangle\langle\downarrow_a|$$

$$\hat{P}_\downarrow(a) = \frac{1-\xi}{2} |\uparrow_a\rangle\langle\uparrow_a| + \frac{1+\xi}{2} |\downarrow_a\rangle\langle\downarrow_a|$$

(21)

where $|\uparrow_a\rangle$ and $|\downarrow_a\rangle$ are the eigenvectors of the operator $a \cdot \vec{\sigma}$ and the parameter $\xi$, bounded between 0 and 1, characterizes the efficiency of the spin measurement: for $\xi = 1$ we recover perfect spin projections, whereas $\xi = 0$ corresponds to the completely noisy limit. A simple calculation shows that within such a model of imperfect spin measurements each of the correlation functions defined in Eq. (8) is multiplied by the parameter $\xi$. Consequently, the complete Bell combination $S$ becomes rescaled by the factor $\xi$ smaller than one. Thus, in the limit $\sqrt{\eta}a \gg 1$ we obtain:

$$S_{\text{max}} \approx 2\xi \sqrt{1 + \left(\frac{a}{\pi} \exp[-2\alpha^2(1-\eta)]\right)^2}.$$  

(22)

For homodyne detection performed on the harmonic oscillator subsystem, the role played by the efficiency parameter $\eta$ is more involved. In Fig. 3 we plot the maximum value of the Bell combination $S_{\text{max}}$ as a function of the homodyne detector efficiency $\eta$ for several values of the coherent state amplitude $\alpha$, under the assumption of perfect spin projections. According to these numerical results, the lower bound for the homodyne detector efficiency enabling the violation of Bell’s inequality is about 66%. We have found that this bound, clearly seen in the plot for $\alpha = 2$, shifts to even slightly lower values with increasing $\alpha$. However, for larger $\alpha$ the strict bound becomes rather meaningless, as in its vicinity the violation of Bell’s inequality becomes negligibly small. Thus, its observation would require first huge sample of experimental data, and secondly completely perfect spin measurements.

![Graph showing the maximum value of the Bell combination as a function of detector efficiency](image-url)

FIG. 3. Maximum value of the Bell combination for several values of $\alpha$ as a function of the detection efficiency $\eta$, assuming perfect spin projections. For $\alpha = 2$ the minimum efficiency necessary to demonstrate nonlocality of the cat state is slightly above 66%. 

We shall close this section with a brief review of experimental prospects for demonstrating the violation of Bell’s inequalities using Schrödinger cat states. Among possible realizations, the physical systems which would be most likely to achieve strict locality conditions include a single radiation mode entangled with an atom [3,5], or with polarization states of a single photon [6]. The radiation mode can either be confined in a high-$Q$ cavity or travel in free space. In the latter case, the homodyne measurement is a well-established technique used widely in quantum-optical experiments, with the advantage of achieving very high detection efficiencies. In the case of the measurement of quadratures for radiation fields in a cavity, several schemes are available [16], which remain however considerably more complicated in practical realization.

VI. CONCLUSIONS

In summary, we have shown how Schrödinger cat states exhibiting quantum entanglement can be applied to test the violation of Bell’s inequalities. The discussion was based on a concrete realization of the scheme using the homodyne detection technique for a light mode. We have analyzed the effects of losses and imperfections, including the non-unit detection efficiency, and pointed out a direct link between the visibility of interference and the violation of Bell’s inequalities. We have found that when passing to the macroscopic domain, a substantial violation of Bell’s inequalities is possible in the limit of perfect noise-free detection and absence of dissipation. In a realistic case, losses destroy the nonlocal effects at the same rate as they decrease the visibility of quantum interference between the classically distinguishable wave packets in the superposition.

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