Mathematical simulation of surface heating during plasma spraying

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Abstract. A mathematical model of temperature distribution over the flat ‘coating-substrate’ system section during plasma spraying, taking into account a plasma gun travel and coating buildup has been developed. It has been shown that the temperature value in the near-surface layer of the sprayed coating during the plasma gun passage can significantly exceed the temperature values in underlayers.

1. Introduction

The creation of promising aviation gas turbine engines having more advanced technical characteristics is closely connected with the increase of the temperature of the gas flow at the inlet of the turbine, as a result of which increasingly high demands are made for the parts of the hot gas path of the turbine. In view of the fact that the potential of the modern refractories is virtually exhausted, one of the most efficient methods of lowering the temperature loading on the surface of the structural material is application of special coatings, possessing high indices of thermal stability, erosion resistance and providing the effective protection of the parts surface from intercrystalline high-temperature corrosion. Owing to the possibility of combination of high values of physico-mechanical and service properties during formation of the meso-ordered structure, plasma heat-resistant coatings are the best prospect. However, in the process of applying these coatings, the surface of the construction material can be subjected to considerable thermal actions on the part of the plasma jet, as a result of which the problem of temperature distribution in the system ‘coating-substrate’ is highly relevant [1-10].

2. Statement of the problem of the mathematical model of surface heating

When defining the temperature distribution during plasma spraying, the averaged influence of the heat flux on the system ‘coating-substrate’ [2] was taken into consideration. In this statement of the problem, the heat obtained by the substrate can be represented as a sum of several constituents:

\[ Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 - Q_6 - Q_7, \]  

where \( Q_1 \) is the heat released owing to a conversion of kinetic energy of the particles into thermal energy during the impact on the substrate; \( Q_2 \) is the heat obtained by the substrate from the plasma jet; \( Q_3 \) is the heat released during crystallization of the molten particles; \( Q_4 \) is the heat released during cooling down of the particles; \( Q_5 \) is the heat released during exothermic reactions in the sprayed materials; \( Q_6 \) is the heat transmitted by the surface owing to convective heat exchange; \( Q_7 \) is the heat withdrawn owing to the heat conduction inside the substrate material.
When considering some small volume of the surface of the coating and representing the process of coating in the form of the model with a continuous build-up of the layer, using the heat-balance equation, the boundary condition for the sprayed surface can be represented as:

$$\lambda_2 \frac{\partial T_2}{\partial z} = q(t) - \alpha_z (T_z - T_a), \quad q(t) = q_0 f(t), \text{ when } z = \delta(t),$$  \hspace{1cm} (2)

where $q(t)$ is the greatest heat flux on the jet axis, $\alpha_z$ is the heat exchange factor, $T_z$ is the coating temperature, $T_a$ is the ambient temperature, $z$ is the coordinate denoting the thickness in the ‘coating-substrate’ system, and $\delta(t)$ is the sprayed-surface coordinate.

The heat of the plasma jet is transmitted to the part through a heating spot with diameter $d$, which, in general case, differs from the diameter of the spraying spot. Heat flux density $q$ owing to uneven distribution over the heating spot (the amount of heat introduced through the spraying spot surface per time unit) can be described by means of a certain distribution law. As a rule, the density of the thermal energy, introduced by the heated gas and heated particles, is recorded with the use of different laws of distribution. However, when the plasma gun is adjusted in a certain way, so that the maximums of heat flux density coincide, the summation thermal action can be described by a single law of distribution with averaged parameters determined based on the experiments.

Owing to the fact that transmission and transfer of heat in the plasma jet depend on a large number of random factors, it is possible to consider that the density of the heat flux complies with the normal law of distribution. This assumption has found its experimental verification.

From the practical standpoint, an opportunity of considering the periodical influence of the plasma jet on the system ‘coating-substrate’ when calculating the temperature distribution is of special interest. The distribution of the heat flux of the plasma jet throughout the area of the heating spot in case of high capacity of the heat flux and the significant speed of the plasma gun travel can be recorded by means of the normal law of distribution:

$$q(r) = q_0 \exp(-kr^2), \text{ when } r = (x-Vt)^2 + y^2,$$ \hspace{1cm} (3)

where $k$ is the factor of concentration of the jet heat flux, and $V$ is the speed of the plasma gun travel.

Having fixed a spot on the surface of spraying with coordinates $x = y = 0$, let us write the expression for determination of the density of the heat flux during a single passage of the plasma gun through this point:

$$q(r) = q_0 \exp(-kV^2t^2).$$ \hspace{1cm} (4)

In the process of spraying, the plasma gun passes over this point with time interval $t_1$; the duration of the influence of the plasma jet on this spot is defined by equation $t_2 = d_s / V$, where $d_s$ is the spraying spot diameter. For time $t_2$, the process of the coating build-up takes place at speed $d\delta / dt$. In this case, the heat flux density has the following view:

$$q(t) = \sum_{n=1}^{N} q_n(t) - \left\{ q[t - (n-1)t_1] - \eta[t - (n-1)t_1 - t_2] \right\},$$ \hspace{1cm} (5)

$$q_n(t) = q_0 \exp\left\{ -kV^2 [t - (n-1)t_1 - 0.5t_2] \right\},$$ \hspace{1cm} (6)

where $\eta(t)$ is the asymmetric unit function; index $n$ corresponds to the number of plasma gun passages over this point; and the concentration factor of the heat flux of the jet is related to spraying spot diameter $d$ by relationship $d = 2(\ln 20)^{1/2} k^{1/2}$.

The coating build-up rate and the process of its thickness increase can be presented by expressions:

$$\frac{d\delta}{dt} = V_z(t) \left\{ \eta\left[t - (n-1)t_1\right] - \eta\left[t - (n-1)t_1 - t_2\right]\right\},$$ \hspace{1cm} (7)
\[ \delta(t) = (n-1)\delta_0 + \int_{t_1}^{t} V_\varepsilon(\tau) d\tau, \quad t \in [(n-1)t_1, (n-1)t_1 + t_2] \]

where \( \delta_0 = \int_{0}^{t_1} V_\varepsilon(\tau) d\tau \) is the coating thickness obtained per single plasma gun passage, and \( V_\varepsilon(t) \) is the coating build-up rate per single plasma gun passage over the surface spot under consideration.

### 3. A mathematical model of surface heating

The mathematical model of the heat problem during spraying the coating onto a substrate surface of a plate-like shape has the following view in case of the averaged thermophysical parameters:

\[
\begin{align*}
\frac{\partial T_2}{\partial t} &= a_2^2 \frac{\partial^2 T_2}{\partial z^2}, \quad 0 < z \leq \delta(t); \\
\frac{\partial T_1}{\partial t} &= a_1^2 \frac{\partial^2 T_1}{\partial z^2}, \quad -h < z \leq 0; \\
\lambda_i \frac{\partial T_i}{\partial z} &= q(t) - a_z(T_2 - T_c), \quad z = \delta(t); \\
\lambda_i \frac{\partial T_i}{\partial z} &= a_i(T_1 - T_c), \quad z = -h; \\
\lambda_i \frac{\partial T_i}{\partial z} &= \lambda_2 \frac{\partial T_2}{\partial z}, \quad T_i = T_2, \quad z = 0; \\
T_i(z, 0) &= T_0,
\end{align*}
\]

where \( T_i \) is the substrate temperature, \( T_0 \) is the initial temperature of the system, \( a_i \) and \( \lambda_i \) are temperature conductivity and heat conductivity factors of the substrate \((i=1)\) and the coating \((i=2)\), respectively.

For the convenience of solving the boundary value problem (9-14), let us introduce dimensionless variables:

\[
\xi = \frac{z}{h}, \quad \zeta = \frac{\delta}{h}, \quad \zeta_0 = \frac{V d_n}{hV}, \quad Fo = \frac{a_i^2 t}{h^2}, \quad \Theta_i = \frac{T_i - T_0}{T_0 - T_c}, i = 1, 2;
\]

and parameters:

\[
Bi_i = \frac{a_i h}{\lambda_i}; \quad Bi_z = \frac{a_z h}{\lambda_2}; \quad k_i = \frac{\lambda_i}{\lambda_2}; \quad k_z = \frac{a_z^2}{a_2^2}; \quad \beta = kh; \quad Ki_h = \frac{hq_0}{\lambda_2(T_0 - T_c)},
\]

\[
Fo_1 = \frac{a_1^2 t_1}{h^2}; \quad Fo_2 = \frac{a_2^2 t_2}{h^2}; \quad Pe_1 = \frac{hV}{a_i}; \quad Pe_2 = \frac{hV}{a_i},
\]

where \( h \) is the coating thickness.

Relationships (5-8) determining \( q(t) \) and \( \delta(t) \) in designations (15-16) are transformed as follows:

\[
Ki(Fo) = Ki_0 \sum_{n=1}^{N} f_n(Fo) \left[ \eta \left[ Fo - (n-1)Fo_1 \right] - \eta \left[ Fo - (n-1)Fo_1 - Fo_2 \right] \right]; \quad (17)
\]

\[
f_n(Fo) = \exp \left[ -\beta Pe^2 \left[ Fo - (n-1)Fo_1 - \frac{1}{2}Fo_2 \right] \right]; \quad (18)
\]
Solving the boundary value problem using the differential series method [3, 4] and neglecting the terms of order of smallness $\varepsilon^{\pm k}$, $k \geq 2$, let us write the following conditions on stationary boundary $\zeta = 0$:

\[
\frac{\partial \theta}{\partial \zeta} \bigg|_{\zeta=0} = \psi(F_0) \frac{B_i}{k_2} \left(1 + \theta_1(0, F_0)\right) - \frac{k_3}{k_2} \frac{\partial \theta}{\partial F_0} \bigg|_{\zeta=0} ;
\]

\[
\psi(F_0) = \frac{K_i}{k_3} \left[ \frac{K_i - B_i \left(1 + \theta_1(0, F_0)\right)}{1 + B_i} \right] B_i \varepsilon^{\tau};
\]

The solution of the boundary value problem of thermal conductivity intended for the substrate (having stationary boundaries $\zeta = -1$, $\zeta = 0$) differs from the solution obtained in [3] by the fact that in this approach, owing to supplementary terms of the expression, the heat losses and heating of the lower layers of the coating are taken into consideration. This approach is the most important one when solving the problems with rapidly moving heat sources.

Using a series of the subsequent transformations, including the Laplace transformation by variable $F_0$, and applying the expansion theorem, let us write the expression for relatively excessive temperature:

\[
\theta_1(\zeta, F_0) = -1 + 2 \sum_{n=0}^{\infty} \frac{A_n \cos \mu_n \zeta + B_n \sin \mu_n \zeta}{D_n} \exp(-\mu_n^2 F_0) + 2 \int_{0}^{F_0} \psi(F_0 - \tau) \sum_{n=0}^{\infty} \frac{A_n \cos \mu_n \zeta + B_n \sin \mu_n \zeta}{D_n} \exp(-\mu_n^2 \tau) d\tau.
\]

Substituting solution (21) in (22) for $\psi(F_0)$, we will obtain:

\[
\theta_1(\zeta, F_0) = -1 + 2 \sum_{n=0}^{\infty} \frac{A_n \cos \mu_n \zeta + B_n \sin \mu_n \zeta}{D_n} \exp(-\mu_n^2 F_0) + 2 \int_{0}^{F_0} \frac{K_i(F_0 - \tau) \varepsilon^{\tau - 1} + B_i B_i \varepsilon^{\tau}}{1 + B_i \varepsilon^{\tau}} \theta_1(0, F_0 - \tau) \frac{B_i \varepsilon^{\tau} (F_0 - \tau)}{1 + B_i \varepsilon^{\tau}} d\tau.
\]

Let $\mu$ be the root of characteristic equation (24); $B_i = B_i/k_i$; $\overline{k}_i = k_i/k_i$.

\[
\begin{align*}
A_n &= \mu_n \left(1 + B_i \overline{k}_i \right) \sin \mu_n ; \\
B_n &= \mu_n \left(B_i \cos \mu_n + B_i \overline{k}_i \mu_n \right) + B_i B_i \left(\cos \mu_n - 1\right) ; \\
D_n &= \left[1 + B_i \overline{k}_i + 2 \overline{k}_i \mu_n^2 - B_i B_i \right] \cos \mu_n - \left[\overline{k}_i \mu_n - \left(1 + B_i \overline{k}_i + B_i + B_i \right) \mu_n^2 - B_i B_i \right] \sin \mu_n ,
\end{align*}
\]
Having inserted $\xi = 0$ in (23), we will obtain the Volterra integral equation for determining $\theta_1(0, Fo)$:

$$f(Fo) = \theta_1(0, Fo) = -1 + 2\sum_{s=1}^{n} \frac{A_s}{D_s} \exp(-\mu_s^2 Fo) + 2 \int_0^{Fo} \frac{K_i(Fo-\tau)k_i^{-1} + B_iB_i\zeta(Fo-\tau)}{1 + B_i\zeta(Fo-\tau)} d\tau \sum_{n=1}^{\infty} \frac{A_n}{D_n} \exp(-\mu_n^2 F) d\tau$$

The Volterra integral equation of the second kind (30) is solved using the method of successive approximations taking into consideration the smallness of the parameter:

$$\left| \frac{B_i\zeta(Fo)}{1 + B_i\zeta(Fo)} \right| << 1.$$

Functions $K_i(Fo)$ and $\zeta(Fo)$ included in (23), (30) are determined by relationships (17-19).

4. The method for calculating the surface heating during plasma spraying

The temperature distribution in the system ‘coating-substrate’ is realized in several steps:

1) the following parameters of the technological process of the coating spraying are set:
   $h, \alpha, \lambda, \alpha, T_0, T_1, q_0, V_1, V, t_1, n$;
2) using relationships (15-16), the following dimensionless parameters and values are calculated: $Bi, \lambda, n, \beta, Pe, Fo, \zeta_0, \zeta_0$;
3) using the method of successive approximations, function $f(Fo) = \theta_1(0, Fo)$ is determined in equation (30), and function $\theta_1(0, Fo)$ is determined from expression (23);
4) using ratios (17-19) and (20), in accordance with the developed algorithm, the temperature distribution in the system ‘substrate-coating’ is calculated for various types of heat sources.

The suggested method is effective when calculating the heat density of the products such as plates during the application and increase of plasma coatings.

5. Analysis of results

The experimental researches of temperature distributions in case of the system ‘coating-substrate’ were conducted depending on the technological parameters during different durations of the process of spraying on the samples with dimensions $40 \times 40 \times 4$ mm. In the centre of these samples, at a depth of 2 mm, there was a chromel-alumel thermocouple with electrodes having a $\varnothing 0.1 \text{ mm}$ diameter. The temperature data were recorded by means of a ‘KSP-4’ recorder. Coatings were sprayed on the samples under the following process conditions: current strength is 400 A; orifice gas consumption is the following: $1.7 \times 10^{-4} \text{ m}^3/\text{s}$ (for hydrogen), $8.3 \times 10^{-4} \text{ m}^3/\text{s}$ (for argon); the material consumption is $5 \times 10^{-4} \text{ kg/s}$; the linear speed of the plasma gun travel is $11.4 \times 10^{-2} \text{ m/s}$. For the purpose of study of the influence of operating practices during spraying on the temperature distribution in the system ‘coating-substrate’, these parameters were varied in the following limits: current strength is (260-540) A, hydrogen consumption is $(0.8-2.5) \times 10^{-4} \text{ m}^3/\text{s}$, argon consumption is $(5.6-12.1) \times 10^{-4} \text{ m}^3/\text{s}$. The coating was sprayed on samples at distances of spraying equal to (0.08-0.22) m, the linear speed of the plasma gun travel varied in the range of $7.3 \times 10^{-3}$ to $21.2 \times 10^{-2} \text{ m/s}$.

The results of comparison between the obtained theoretical and experimental data confirm the adequacy and applicability of the suggested mathematical model of surface heating calculation during plasma spraying (Figure 1). Minor discrepancy between the calculated data and experimental ones is connected with the inertia of a thermocouple sensor. The temperature in the increased layer of the coating can be significantly higher than the integral temperature of the system recorded by the thermocouple.
From the analysis of the obtained data (Figure 1), it follows that the adjustment of the heating of the powder material during formation of the coating can be realised both owing to the parameters of the plasma jet [1, 2, 5-10] and by means of controlling the duration of the magnitude of the thermal spike arising during plasma gun passage over the surface area under consideration.

6. Conclusions
A mathematical model of temperature distribution over the flat ‘coating-substrate’ system section during plasma spraying, taking into account a plasma gun travel and coating buildup has been developed. It has been shown that the temperature value in the near-surface layer of the sprayed coating during the plasma gun passage can significantly exceed the temperature values in underlayers. The heating of the powder material during formation of the coating can be realised both owing to the parameters of the plasma jet and by means of controlling the duration of the magnitude of the thermal spike arising during plasma gun passage over the surface area under consideration.

![Figure 1](image.png)

**Figure 1.** Distribution of the temperature depending on the duration of spraying in the system of the Ni-Co-Cr-Al-Y (o) coating and substrate (Δ, ×) at a depth of \( z = -0.5h \): o, Δ – calculation data; × – experimental data.

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