Pricing Decision for a Closed-Loop Supply Chain with Technology Licensing under Collection and Remanufacturing Cost Disruptions

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**Abstract:** Closed-loop supply chain (CLSC) management faces collection and remanufacturing cost disruption challenges. This study explores a CLSC system wherein original equipment manufacturers (OEMs) license the third-party remanufacturer (TPR) to bear the remanufacturing activities and investigate pricing decisions in the CLSC, while considering collection and remanufacturing cost disruptions. To obtain the optimal pricing strategy, we develop game theory models under the disruptions of both centralized and decentralized CLSCs. Based on theoretical and numerical analyses, we obtain the following results: (1) Whether or not disruption events occur, the centralized supply chain can better encourage consumers to participate in the collection of used products than a decentralized supply chain; (2) when collection disruption in a large positive region or the remanufacturing cost disruption in a large negative region occurs, OEM and TPR profits will greatly increase, and the OEM will raise the licensing fee to extract more profit from the remanufacturing activity; (3) a certain robust region exists for the retail price and wholesale price when the supply chain faces disruption increase; (4) when the supply chain faces the disruptions, it has great influence on the OEM’s licensing fee but little on the TPR’s acquisition price. The main contributions of the study include: (1) We considered the impacts of both technology licensing and collection and remanufacturing cost disruption; (2) we developed game theory models to determine the optimal manufacturing and remanufacturing quantities, and pricing strategy under the disruptions; (3) based on theoretical and numerical analyses, we presented some interesting and important insights. The results of this paper could provide useful guidelines for supply chain members on how to effectively control costs to obtain more profit by adjusting prices and selecting a better operation mode for the closed-loop supply chain.

**Keywords:** closed-loop supply chain; remanufacturing; third party remanufacture; technology licensing; game theory; optimization

1. Introduction

In the past decade, the shortage of global resources and environmental pollution has become increasingly serious. Countries worldwide are attempting to achieve sustainable development and operate a circular economy (Subramanian and Subramanyam, 2012). In 2016, the State Council of China issued the implementation plan of the extended producer responsibility system. The system clearly defined that the extended producer responsibility system should be implemented in four categories of products: electronics, automobiles, batteries, and packaging products. The manufacturer should bear the environmental...
responsibility for the full life cycle of its products, including design, circulation, recycling, waste disposal, and so on. Under the policy’s guidance, manufacturers achieve considerable economic and environmental benefits by implementing the operation and management of a closed-loop supply chain (CLSC). For example, Xerox’s financial statements demonstrate that the recycling and remanufacturing of copiers saved about $200 million and reduced manufacturing costs of new products by 40–65% [1]. Three strategies for recycling the used products were thus proposed: (1) directly by the manufacturers, (2) by retailers, and (3) through third-party remanufacturers [2].

Choosing proper strategies for recycling used products is crucial [3]. Issuing technology licensing to a third-party remanufacturer (TRP) is one such strategy. Some original equipment manufacturers (OEMs) do not participate in remanufacturing activities as they instead focus on manufacturing new products. To adopt sustainable manufacturing practices and take social responsibility, OEMs can entrust a third-party remanufacturer to recycle and remanufacture by means of technology licensing [4]. Technology licensing has been widely adopted by companies to protect their intellectual assets and improve companies’ profitability and efficiency [5]. For instance, Apple entrusted Foxconn Group to recycle and reprocess used Apple mobile phones [6]. Qi et al. [7] and Li et al. [8] found that compared with a non-licensing situation, the technology licensing policy can improve social welfare and increase patent holders’ profits.

The CLSC faces high uncertainty risk because of a series of factors such as the unpredictable availability of recycling products, consumer awareness of environmental protection, and product characteristics. This uncertainty can cause disruptions in the collection and remanufacturing processes. For remanufactured products, the supply of raw materials or parts originates from the collection of used products; hence, the disruption of collection quantities has a great influence on the remanufacturing cost. For example, the 2011 Tohoku earthquake in Japan caused a shortage of electronic components such as semiconductor chips. Because of the sharp rise in remanufacturing costs, downstream remanufacturers were forced to reduce production volume or to find alternative manufacturers. Thus, the problems caused by uncertainty about the quantity and quality of used products, remanufacturing cost, and demand have gained increasing attention. For the traditional supply chain, Wu, Chen, and Zhang [9] found that the dual-channel perishable product supply chain has a certain robustness region under market demand and production cost disruption. For the closed-loop supply chain, Han, Yang, and Hou [10] studied the influence of both the remanufacturing production and market demand disruptions on the motivation of manufacturers to license the third-party remanufacturer to conduct remanufacturing activities.

A remanufacturer under technology licensing naturally faces collection uncertainty and disruptions. For example, Caterpillar, one of the world’s largest remanufacturers, relies on mature technology and entrusts agent manufacturers in different regions to conduct remanufacturing activities all over the world. They suspended the operation of some plants in the regions affected by COVID-19 due to a shortage of parts in 2020. The negative impact on the operational performance and financial situation of the whole enterprise is substantial [11]. However, to the best of our knowledge, there is no research in CLSC literature considering the impacts of both technology licensing and collection and remanufacturing cost disruption.

Hence, we consider the impacts of both technology licensing and collection and remanufacturing cost disruption, and develop game theory models to determine the optimal manufacturing and remanufacturing quantities, and optimal pricing strategy under the disruptions. This study examines the impacts of both collection and remanufacturing cost disruptions on the pricing decisions in a CLSC, where the OEM licenses a TPR for remanufacturing. Further, we analytically discuss the impact of different disruption cases on the optimal pricing strategies and chain members’ profits. In this study, the following questions are addressed:

(1) How do collection and remanufacturing cost disruptions affect the decisions of chain members?
(2) How does the OEM adjust the licensing behavior to respond to different disruption cases?

The remainder of this paper is organized as follows. Section 2 briefly discusses the relevant literature. We specify the basic assumptions and notation in Section 3. In Section 4, we develop and address centralized and decentralized game theory models with and without disruptions and obtain equilibrium strategies for each disruption case. We compare different disruption cases and analyze the managerial implications of collection and remanufacturing cost disruptions on the optimal pricing decisions in Section 5. In Section 6, we present numerical examples. Finally, the paper is concluded in Section 7, and future research directions are provided.

2. Literature Review

2.1. Reverse Logistics Channel for Closed-Loop Supply Chains

Many studies have explored the reverse logistics channel for collecting and remanufacturing in a closed-loop supply chain. For collecting processes, Savaskan et al. [12] proposed three collection modes, retailer collection, manufacturer collection, and third-party collection, and analyzed the optimal pricing strategies of chain members. In their study, retailer collection was the best choice for the manufacturer. Xiong and Liang [13], Wen and Dong [14], and Yan [15] discussed the advantages and disadvantages of the three recycling models and obtained the optimal recycling channel structure from the perspective of consumer awareness of environmental protection, corporate social responsibility, and remanufacturing the RL process. Liu and Chen [16] constructed profit decision models under three different recovery strategies: manufacturer independent recovery, manufacturer joint retailer recovery and manufacturer integrated third-party recovery under the assumption of corporate social responsibility investment. Zhao et al. [17] structured a dual-channel recycling model. Through the analysis of the recycling competition model, the optimal selection of recycling channels and the optimal pricing of products are obtained. If the manufacturer aims for maximum recycling quantities of the used products, a dual-channel recycling model should be selected. Wei et al. [18] proposed that the product cycle should be considered in the dual-channel recycling model. Thus, he studied the two-stage closed-loop supply chain model with two recycling channels in a dynamic environment and found the optimal strategy for profit and recycling rate maximization. Huang et al. [19] considered the closed-loop supply chain pricing decision model under a mixed recycling model of retailers and third-parties, compared it with a previous single-channel recycling model, and finally proved that the dual-channel recycling model is better than the single-channel recycling model. Bulmus et al. [20] considered the optimal pricing model of recycling and remanufacturing when the OEM and the third-party independent remanufacturer compete and found that the optimal recycling price of the OEM is only related to its own recycling structure and not to the third-party independent remanufacturer.

In addition, through the comparison of different recycling models, some studies explored the impact on the recycling price of waste products. Abbey et al. [21] found that the price of the remanufactured products under the recycling mode of third-party enterprises is higher than that of the products under the recycling mode of the retailers. In addition to the single recycling model, some scholars study the collection strategies under the mixed recycling model, especially cooperation or competition between manufacturers and third parties. Örsemir et al. [22] considered the problem in which an OEM and an independent remanufacturer (IR) decide the production quantity under the competitive mode and found that the remanufacturing activities undertaken by the remanufacturer are more conducive to improving the ecological environment and social welfare.

2.2. Closed-Loop Supply Chains with Technology Licensing

Some cases wherein manufacturers and remanufacturers simultaneously participate in the remanufacturing process through a cooperative way, such as technology licensing, have also been referred to in some studies. Arora and Ceccagnoli [23] explored the influ-
encing factors and internal operation mechanism of technology licensing through empirical research. Hong et al. [24] considered a two-stage CLSC model and analyzed the impact of technology licensing on production and recycling decisions and found that franchise contracts are more conducive to increasing consumer surplus. Ali Sabbaghi and Ata Allah Taleizadeh [25] studied the three most common technology licenses in CLSCs and found that technology licenses affect welfare and financial concepts. The above studies investigate cases where OEMs and remanufacturers undertake remanufacturing activities in a competitive or cooperative manner. Zou et al. [6] studied the strategy choice of the third-party remanufacturing mode adopted by manufacturers, and found that the strategy is related to the degree of consumers’ acceptance of remanufactured products. When the degree of consumers’ acceptance of remanufactured products is high, the outsourcing mode is better than the licensing mode. In contrast, the licensing mode is the best strategy. Huang and Wang [26] considered the influence of different recycling channels on the pricing and recycling strategy of CLSCs with technology licensing. Huang and Wang [27] studied the impact of retailer information sharing on the decisions of manufacturers and third-party remanufacturers in the supply chain and found that the retailer’s sharing of demand forecast information is beneficial to the manufacturer, but it has a negative impact on the retailer itself. However, a limitation of these studies is the assumption that the market environment is static, and scarce attention has been focused on licensing activity under a disruption scenario (See Table 1). In real-life cases, some changes might may in the collection and remanufacturing processes because of unexpected economic and environmental crises; in addition, demand disruptions commonly occur in a CLSC. Therefore, we investigate the optimal pricing and production decisions in a CLSC with technology licensing under disruptions.

Table 1. The scope of research in CLSCs with technology licensing.

| Reference  | Scope                              | Limitations                                      |
|------------|------------------------------------|-------------------------------------------------|
| [23]       | Operation mechanism                | Assume that the market environment is static and licensing activity is undisturbed |
| [24,25]    | Consumer surplus or welfare and financial concepts |                                               |
| [6,26,27]  | Strategy choice                    |                                                |

2.3. Closed-Loop Supply Chains with Disruptions

Disruption risk is a significant topic in supply chain management. Many researchers have examined its impact on channel coordination. Qi et al. [28] proposed demand disruptions in a two-period supply chain and derived that wholesale quantity discount policies can coordinate both centralized and decentralized supply chains. Zhao and Zhu [29] discussed the coordination of a recycler and a remanufacturer in a closed-loop supply chain with uncertain demand. Gao and You [30] constructed an uncertain model of sustainable supply chain optimization to solve the multiple uncertainties of product demand. Han and Kang [31] explored the influence of the uncertainty of recycling quantity on the acquisition price of recyclers and remanufacturers. However, the above studies only considered one type of disruption factor. In addition, the supply of products is often uncertain. Uncertainty regarding the quantity or quality of raw materials can lead to the disruption of manufacturing cost. Han et al. [32] found that the retailer’s recycle model is the best with a positive disruption, and the manufacturer’s recycle model is the best in a stable environment or with a negative disruption in a CLSC with remanufacturing cost disruption. Wu et al. [33] also established a supply chain model with two competitive retailers, considering the impact of remanufacturing cost disruption on pricing decisions. An improved revenue-sharing contract is designed to coordinate supply chain profits effectively. The above studies discussed the impact of cost disruption on the supply chain due to the uncertainty of the recycling process. However, disruptions are known to occur at the same time. Teunter and Flapper [34] considered the relationship between product demand and remanufacturing
cost and examined the optimal strategy under static and uncertain environments. Besides the case of manufacturer recycling or retailer recycling, some OEMs actually license TPRs to recycle and remanufacture. Therefore, our work considers a CLSC mode with technology licensing where the OEM licenses the TPR to remanufacture and explore the impact of simultaneous collection and remanufacturing cost disruptions. In contrast, prior work on remanufacturing under technology licensing assumed that the production and collection process is in a stable environment. In fact, as we know, the effect of occurrence of emergencies varies. Hence, our model captures both aspects, where the OEM licenses the TPR to remanufacture in the presence of collection and remanufacturing cost disruptions. Our work explores the optimal pricing decisions in a CLSC with technology licensing under both collection and remanufacturing cost disruptions.

To sum up, the main contribution of this study lies in the following aspects: First, we investigated the optimal pricing and production decisions in a CLSC with technology licensing under disruptions. We considered the impacts of both technology licensing and collection and remanufacturing cost disruption. Second, we developed game theory models to determine the optimal manufacturing and remanufacturing quantities, and pricing strategy under the disruptions, i.e., the OEM authorized the TPR for remanufacturing in the presence of collection and remanufacturing cost disruptions. We addressed centralized and decentralized game theory models with/without disruptions, and obtained equilibrium strategies for each disruption case. Furthermore, based on theoretical and numerical analyses, we presented some interesting and important insights. The findings gained through this work could provide useful guidelines for supply chain members on how to effectively control costs to get more profit by adjusting price and to choose a better operation mode for closed-loop supply chain.

3. Problem Description and Model Assumptions

Based on the previous analysis, in this section, we consider a CLSC where the OEM licenses TPR to collect used products and produce remanufactured products. The OEM is responsible for the fabrication of new products. The OEM and TPR sell new and remanufactured products, respectively, to retailers at the same wholesale price. For instance, Eastman Kodak Company receives single-use cameras from large retailers that also develop film for customers. On average, 76% of the weight of a disposed camera is reused in the production of a new one [12]. The OEM gains the remanufacturing profits from the TPR by charging a licensing fee (Figure 1).

![Figure 1. CLSC model frame with remanufacturing.](image-url)
of technology licensing fee to obtain the OEM’s technology licensing and then recycle the waste products from consumers at a certain acquisition price.

For the sake of clarity, the relevant notations are provided in Table 2.

Table 2. Parameters and decision variables.

| Symbol | Definition |
|--------|------------|
| \( w \) | The unit wholesale price |
| \( p \) | The unit retail price |
| \( \delta \) | The unit acquisition price for the collected product |
| \( f \) | The licensing fee |
| \( q_n \) | The quantity of new products |
| \( q_r \) | The quantity of remanufactured products |
| \( c_n \) | The unit cost of manufacturing new products |
| \( c_r \) | The unit cost of remanufacturing returned products |
| \( \alpha \) | The market size |
| \( \beta \) | Sensitivity of consumers to the retail price |
| \( \Delta \) | Cost savings per unit of product in the remanufacturing, \( \Delta = c_n - c_r \) |
| \( \Delta_u \) | Collection disruption |
| \( \Delta_r \) | Remanufacturing cost disruption |
| \( \lambda_1 \) | The unit inventory cost |
| \( \lambda_2 \) | The unit shortage cost |

Note: New products and remanufactured products are indicated by subscripts \( n \) and \( r \), respectively.

The following modelling assumptions were used in constructing the model.

1. The market demand is \( D(p) = \alpha - \beta p \) for \( \alpha > 0 \), \( \beta > 0 \), and \( \alpha > \beta c_n \). This assumption is widely cited in the supply chain management research literature [12].
2. The acquisition quantity of used products is \( G(\delta) = u + v\delta \), where \( u \) and \( v \) represent the acquisition quantity when the acquisition price \( \delta = 0 \) and the sensitivity of consumers to the acquisition price, respectively. This assumption is similar to the research of Bakal and Akcali [38].
3. All collected products can be remanufactured successfully and resold. Additionally, we can determine that the quantity of remanufactured products is equivalent to the acquisition quantity of used products (i.e., \( q_r = G(\delta) \)), and the quantity of new products can be calculated as \( q_n = D(p) - G(\delta) \).
4. No difference exists between new and remanufactured products in terms of quality, feature, packaging, and price [39]. With the enhancement of consumers’ awareness of environmental protection, an increasing number of consumers are willing to accept the same price of new products, considering that remanufactured products are more conducive to energy conservation and environmental protection [40], especially in the remanufacturing practice of papermaking and some electronic products [41].
5. The unit cost of the remanufactured product is lower than the unit cost of the new product, for \( c_r < c_n \), and \( c_n - c_r = \Delta \), where \( \Delta \) represents cost savings per unit of product in the remanufacturing process and on the assumption of \( \Delta > r \). This assumption is analogous to research into making the remanufacturing process profitable [42].
6. The manufacturer is the leader, and the market information is completely symmetrical. Each member of the supply chain is risk neutral and expects the maximum profit as the decision-making goal.
4. Model Development

4.1. The CLSC Model without Disruptions

4.1.1. Centralized CLSC Model without Disruptions

In the centralized model, when no disruptions occur, the CLSC system is considered as a whole. It aims to maximize the overall benefits of the CLSC, and the total profit function of the centralized CLSC is determined by the following:

\[
\pi^c = (p - c_n)q_n + (\Delta - \delta)q_r = (p - c_n)(\alpha - \beta p) + (\Delta - \delta)(u + v\delta),
\]

(1)

In a centralized CLSC with no disruptions, the total profit function is \(\pi^c\) jointly concave in \(p\) and \(\delta\), taking the first-order partial derivatives of \(\pi^c\) with respect to \(p\) and \(\delta\), and letting the derivative be zero, we have

\[
\frac{\partial \pi^c}{\partial p} = \alpha + \beta w - 2\beta p, \quad (2)
\]

\[
\frac{\partial \pi^c}{\partial \delta} = v\Delta - u - 2v\delta, \quad (3)
\]

By solving the equation system of Equations (2) and (3), we obtain the optimal retail price and acquisition price and \(p^c = \frac{\alpha + \epsilon + \beta}{2\beta}, \delta^c = \frac{\Delta - u}{2\beta}\). Substituting \(p^c = \frac{\alpha + \epsilon + \beta}{2\beta}\) and \(\delta^c = \frac{\Delta - u}{2\beta}\) into \(q_n\) and \(q_r\), we derive \(q_n^{c*} = \frac{\alpha - p - \epsilon - \beta - v\Delta}{2\epsilon}\) and \(q_r^{c*} = \frac{u + v\Delta}{2\epsilon}\). Substituting the values of the parameters back into Equation (1), the supply chain’s total profit is

\[
\pi^c = (\alpha + \epsilon + \beta)\Delta + v\beta^2 + v(\alpha^2 - 2\epsilon + \beta + 2u\Delta). 
\]

4.1.2. Decentralized CLSC Model without Disruptions

In a decentralized CLSC, it is considered that the OEM, who acts as the channel leader, first determines the optimal wholesale price and technology licensing fee to maximize their own profits and then the retailer and the TPR decide the retail price and acquisition, respectively. We can obtain the optimal solutions by backward induction. Based on the previous assumptions and analysis, the profit function of the retailer, the OEM, and the TPR are formulated as follows:

\[
\pi_R = (p - w)(q_n + q_r) = (p - w)(\alpha - \beta p), \quad (4)
\]

\[
\pi_M = (w - c_n)q_n + f q_r = (w - c_n)(\alpha - \beta p - u - v\delta) + f(u + v\delta), \quad (5)
\]

And:

\[
\pi_T = (w - c_r - \delta - f) q_r = (w - c_r - \delta - f)(u + v\delta), \quad (6)
\]

Taking the second-order derivatives of Equations (4) and (6) with respect to and \(p\delta\), respectively. We can obtain \(\frac{\partial^2 \pi_R}{\partial p^2} = -2\beta < 0\) and \(\frac{\partial^2 \pi_T}{\partial \delta^2} = - 2v < 0\). Thus, \(\pi_R\) and \(\pi_T\) are concave in \(p\) and \(\delta\), respectively. Taking the first-order partial derivatives of \(\pi_R\) and \(\pi_T\) with respect to \(p\) and \(\delta\), respectively, and letting the derivative be zero, we have the following:

\[
\frac{\partial \pi_R}{\partial p} = \alpha + \beta w - 2\beta p, \quad (7)
\]

and

\[
\frac{\partial \pi_T}{\partial \delta} = v(w - f - c_r) - u - 2v\delta, \quad (8)
\]

Taking the second-order partial derivatives of \(\pi_M\) with respect to \(f\) and \(w\), we have the Hessian matrix as follows:

\[
\begin{pmatrix}
\frac{\partial^2 \pi_M}{\partial f^2} & \frac{\partial^2 \pi_M}{\partial f \partial w} \\
\frac{\partial^2 \pi_M}{\partial w \partial f} & \frac{\partial^2 \pi_M}{\partial w^2}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\alpha^2 & \alpha \beta \delta + \frac{\beta^2}{2}\Delta \\
\alpha \beta \delta + \frac{\beta^2}{2}\Delta & (\alpha - \beta p)^2 + f^2
\end{pmatrix}
\]

By solving the equation system of these second-order derivatives, we obtain the optimal solutions of \(f^*\) and \(w^*\) and the optimal profit of the decentralized CLSC:

\[
\pi^d = (\alpha + \epsilon + \beta)\Delta + v\beta^2 + v(\alpha^2 - 2\epsilon + \beta + 2u\Delta). 
\]
\[ H_M = \begin{bmatrix} \frac{\partial^2 \pi_M}{\partial w^2} & \frac{\partial^2 \pi_M}{\partial w \partial f} \\ \frac{\partial^2 \pi_M}{\partial f^2} & \frac{\partial^2 \pi_M}{\partial f^2} \end{bmatrix} = \begin{bmatrix} -v - \beta & v \\ v & -v \end{bmatrix} \] since \(-v - \beta < 0\) and \(|H_M| = \beta v > 0\), \(\pi_M\) is jointly concave in \(f\) and \(w\). Taking the first-order partial derivatives of with \(\pi_M\) respect to \(f\) and \(w\), respectively, and letting the derivatives be zero, by solving the following equation system \(\begin{cases} \frac{\partial \pi_M}{\partial f} = 0 \\ \frac{\partial \pi_M}{\partial w} = 0 \end{cases}\), we obtain the optimal price strategies as follows:

\[ w^* = \frac{\alpha + c_n \beta}{2\beta}, \quad f^* = \frac{\nu \alpha - v c_n \beta + \beta u}{2\nu \beta}, \] (9) (10)

Substituting Equations (9) and (10) into Equations (7) and (8), we can derive the optimal retail price and acquisition price as follows:

\[ p^* = \frac{3\alpha + c_n \beta}{4\beta}, \] (11)

\[ \delta^* = \frac{\Delta v - 3u}{4v}, \] (12)

Furthermore, the optimal quantities of new and remanufactured products can be obtained and simplified as follows:

\[ q_n^* = \frac{\alpha - u - c_n \beta - \Delta v}{4}, \] (13)

\[ q_r^* = u + \frac{\Delta v - 3u}{4}, \] (14)

4.2. The CLSC Model with Disruptions

When disruption events occur, they will cause collection and remanufacturing cost disruptions in a CLSC. The quantity of remanufactured products will be disrupted to \(\tilde{q} = u + \Delta_u + v \delta\) (Huang and Wang, 2018 [4]), and the remanufacturing cost will be disrupted to \(\tilde{\lambda} = c_r + \Delta_r\) (Wu and Han, 2016 [42]). We use the notation with a tilde (or \(\sim\)) to represent the disruption. To be specific, assume that \(u + \Delta_u > 0\) and \(\Delta - \Delta_r > 0\). In addition, the production plan will be changed by the occurrence of disruption, which will cause the increased inventory cost and shortage cost. We define \(\lambda_1\) as the unit inventory cost of an increased product and \(\lambda_2\) as the unit shortage cost of a decreased product, where \(\lambda_1 < \lambda_n\) and \(\lambda_2 > \lambda_n\). Moreover, we let \(\lambda_{n1}\) and \(\lambda_{n2}\) denote the unit inventory cost of an increased new product and the unit shortage cost of a decreased new product, respectively. Then, let \(\lambda_{r1}\) and \(\lambda_{r2}\) denote the unit inventory cost of an increased remanufactured product and the unit shortage cost of a decreased remanufactured product, respectively.

4.2.1. Centralized CLSC Model with Disruptions

In the centralized model, the entire CLSC system is regarded as a whole, with only one decision maker. The total profit function of the centralized CLSC is determined in three cases under disruption as follows:

\[ \tilde{\pi}^c = \begin{cases} (p - c_n)(\alpha - \beta p) + (\Delta - \Delta_r - \delta)(u + \Delta_u + v \delta) - \lambda_{n1}(\tilde{q}_n - q_n^*)^+ - \lambda_{r1}(q_r^* - \tilde{q}_r)^+, & \text{when } \tilde{q}_n > q_n^* \text{ and } \tilde{q}_r < q_r^* \\ (p - c_n)(\alpha - \beta p) + (\Delta - \Delta_r - \delta)(u + \Delta_u + v \delta) - \lambda_{n2}(q_n^* - \tilde{q}_n)^+ - \lambda_{r2}(q_r^* - \tilde{q}_r)^+, & \text{when } \tilde{q}_n = q_n^* \text{ and } \tilde{q}_r = q_r^* \\ (p - c_n)(\alpha - \beta p) + (\Delta - \Delta_r - \delta)(u + \Delta_u + v \delta) - \lambda_{n2}(\tilde{q}_n - q_n^*)^+ - \lambda_{r1}(q_r^* - \tilde{q}_r)^+, & \text{when } \tilde{q}_n(q_n^* \text{ and } \tilde{q}_r)q_r^* \end{cases}, \] (15)
\( \tilde{q}_n \) and \( \tilde{q}_r \) represent the demand quantity of new products and remanufactured products under disruptions, respectively. Because shortage and inventory cannot occur at the same time, and the total market demand is constant, when \( \tilde{q}_r > q^*_r \) then \( \tilde{q}_n < q^*_n \), or when \( \tilde{q}_n > q^*_n \) then \( \tilde{q}_r < q^*_r \). To facilitate the analysis, according to the different ranges of disruption, the occurrence of disruptions can be divided into three situations for discussion. The disruption ranges in different situations are shown in Table 2. The range of values in Table 3 is obtained by comparing the demand of new products and remanufactured products in a disrupted environment with demand in a stable environment. The demand for new products and remanufactured products in a disrupted environment is obtained by substituting the optimal price under the disruptions into the demand function.

| Case       | The Optimal Quantities          | The Disruption Cases |
|------------|---------------------------------|----------------------|
| \( \tilde{q}_n > q^*_n \) | \( \tilde{q}_r = q^*_r + \frac{\Delta u - \frac{v(\Delta - \Delta n - \Delta r - \lambda n - \lambda r)}{2}}{\beta} \) | \( \Delta u > \beta \lambda n \) |
| \( \tilde{q}_n < q^*_n \) | \( \tilde{q}_r = q^*_r + \frac{\Delta u - \frac{v(\Delta + \lambda n + \lambda r)}{2}}{\beta} \) | \( \Delta u < - (\lambda n + \lambda r) \) |
| \( \tilde{q}_n < q^*_n \) | \( \tilde{q}_r = q^*_r + \frac{\Delta u + \frac{v(\Delta + \lambda n + \lambda r)}{4} - \frac{\Delta n}{2}}{\beta} \) | \( \Delta u < - \beta \lambda n \) |
| \( \tilde{q}_n > q^*_n \) | \( \tilde{q}_r = q^*_r + \frac{\Delta u + \frac{v(\Delta - \lambda n - \lambda r)}{2}}{\beta} \) | \( \Delta u > (\lambda n + \lambda r) \) |
| \( \tilde{q}_n = q^*_n \) | \( \tilde{q}_r = q^*_r + \frac{\Delta u + \frac{v(\Delta - \lambda n - \lambda r)}{2}}{\beta} \) | \( - (\lambda n + \lambda r) \leq \Delta u \leq (\lambda n + \lambda r) \ |
| \( \tilde{q}_n = q^*_n \) | \( \tilde{q}_r = q^*_r + \frac{\Delta u + \frac{v(\Delta + \lambda n + \lambda r)}{2}}{\beta} \) | \( - \beta \lambda n \leq \Delta u \leq \beta \lambda n \ |

In fact, the optimal decisions can be obtained by using backward induction to solve Equation (15). The optimal solutions are as follows:

\[
\begin{aligned}
\tilde{p}^* &= \begin{cases} 
\frac{a + b(c_u - \lambda n)}{2b} \Delta u > \beta \lambda n \text{ and } \Delta r < - (\lambda n + \lambda r) \\
\frac{a + b(c_u - \lambda n)}{2b} \text{ and } \Delta u < - (\lambda n + \lambda r) \\
\frac{a + b(c_u - \lambda n)}{2b} \Delta u < - \beta \lambda n \text{ and } \Delta r > (\lambda n + \lambda r)
\end{cases} \\
\tilde{q}_n^* &= \begin{cases} 
\frac{v(\Delta - \Delta r - \lambda n + \lambda r - \lambda n - \lambda r - \Delta n)}{2\beta} \Delta u < - \beta \lambda n \text{ and } \Delta r > (\lambda n + \lambda r) \\
\frac{v(\Delta - \Delta r - \lambda n + \lambda r - \lambda n - \lambda r - \Delta n)}{2\beta} \Delta u > \beta \lambda n \text{ and } \Delta r < - (\lambda n + \lambda r) \\
\frac{a - \frac{v(\Delta - \Delta r - \lambda n + \lambda r - \lambda n - \lambda r - \Delta n)}{2}}{2\beta} \Delta u < \beta \lambda n \text{ and } \Delta r > (\lambda n + \lambda r)
\end{cases} \\
\tilde{q}_r^* &= \begin{cases} 
\frac{a + \frac{v(\Delta - \Delta r - \lambda n + \lambda r - \lambda n - \lambda r - \Delta n)}{2} + \Delta n}{2} \Delta u > \beta \lambda n \text{ and } \Delta r < - (\lambda n + \lambda r) \\
\frac{a + \frac{v(\Delta - \Delta r - \lambda n + \lambda r - \lambda n - \lambda r - \Delta n)}{2} + \Delta n}{2} \Delta u < - \beta \lambda n \text{ and } \Delta r > (\lambda n + \lambda r)
\end{cases}
\end{aligned}
\]

According to Equations (18) and (19) and the optimal quantity of new and remanufactured products in a centralized CLSC, when \( \tilde{q}_r > q^*_r \) and \( \tilde{q}_n < q^*_n \), the optimal quantity satisfies \( \tilde{q}_r = q^*_r + \frac{\Delta u - \frac{v(\Delta + \lambda n + \lambda r)}{2}}{\beta} \) and \( \tilde{q}_n = q^*_n + \frac{\Delta u - \frac{v(\Delta - \lambda n + \lambda r)}{2}}{\beta} \). Here, given \( \tilde{q}_r > q^*_r \) and \( \tilde{q}_n < q^*_n \), we have \( \frac{\Delta u - \frac{v(\Delta + \lambda n + \lambda r)}{2}}{\beta} > 0 \) and \( \frac{\Delta u - \frac{v(\Delta - \lambda n + \lambda r)}{2}}{\beta} < 0 \). Solving these equations simultaneously, we found that \( \Delta u < - \beta \lambda n \), \( \Delta r > (\lambda n + \lambda r) \). The results of other cases in Table 2 are analogous.

Substituting Equations (16)–(19) into Equation (15), we can obtain the optimal profit \( \tilde{\pi}^* \):
\[ \tilde{\pi}^* = \begin{cases} 
\frac{(a + \Delta_u)^2 \beta + v \left( a^2 - 2c_n \alpha \beta + \beta \left( c_n^2 \beta + 2u (\Delta_u - \Delta - \lambda_{\omega}) + 2\Delta u (\Delta_u - \lambda_{\omega} - \lambda_{\omega} - \lambda_{\omega}) \right) \right)}{4p} + \frac{v^2 \beta (\Delta^2 - 2\Delta u + \Delta (\Delta_u + 2(\lambda_{\omega} + \lambda_{\omega})))}{4p} 
\end{cases} \]

4.2.2. Decentralized CLSC Model with Disruptions

When disruption occurs, it will cause collection and remanufacturing cost perturbations in the decentralized CLSC. The assumptions in this model are consistent with the centralized decision-making model under the disruption events and will not be discussed in detail. When the disruption occurs, the profit function of the retailer, OEM, and TPR are formulated as follows:

\[ \tilde{\pi}_R = (p - w)(q_n + q_r) = (p - w)(\alpha - \beta p), \]  

\[ \tilde{\pi}_M = (w - c_n)\tilde{q}_n + f\tilde{q}_r - \lambda_{n1}(\tilde{q}_n - q_n^*) + \lambda_{n2}(q_n^* - \tilde{q}_n), \]  

\[ \tilde{\pi}_T = (w - c_r - \Delta_r - \delta - f)(\tilde{q}_r - q_r^*) - \lambda_{n2}(q_r^* - \tilde{q}_r) \]

Thus, the profit function of TPR can be determined:

\[ \tilde{\pi}_T = \begin{cases} 
(w - c_r - \Delta_r - \delta - f)(u + \Delta_u + v\delta) - \lambda_{r1}(\tilde{q}_r - q_r^*) & \tilde{q}_r > q_r^* \\
(w - c_r - \Delta_r - \delta - f)(u + \Delta_u + v\delta) & \tilde{q}_r = q_r^* \\
(w - c_r - \Delta_r - \delta - f)(u + \Delta_u + v\delta) - \lambda_{r2}(q_r^* - \tilde{q}_r) & \tilde{q}_r < q_r^* 
\end{cases} \]

In addition, the profit function of OEM can be determined as follows:

\[ \tilde{\pi}_M = \begin{cases} 
((w - c_n))(a - \beta p - (u + \Delta_u + v\delta)) + f(u + \Delta_u + v\delta) - \lambda_{n2}(q_n^* - \tilde{q}_n) & \tilde{q}_r > q_r^* \\
((w - c_n))(a - \beta p - (u + \Delta_u + v\delta)) + f(u + \Delta_u + v\delta) & \tilde{q}_r = q_r^* \\
((w - c_n))(a - \beta p - (u + \Delta_u + v\delta)) + f(u + \Delta_u + v\delta) - \lambda_{n1}(\tilde{q}_n - q_n^*) & \tilde{q}_r < q_r^* 
\end{cases} \]

Assuming that \( \tilde{q}_r > q_r^* \), using the second-order derivatives of Equations (21) and (24) with respect to \( p \) and \( \delta \), respectively, we obtain \( \frac{\partial^2 \tilde{\pi}_r}{\partial p^2} = -2\beta < 0 \) and \( \frac{\partial^2 \tilde{\pi}_T}{\partial \delta^2} = -2v < 0 \). Thus, \( \tilde{\pi}_R \) and \( \tilde{\pi}_T \) are concave in \( p \) and \( \delta \). The optimal retail price and \( \tilde{p}^* \) the optimal acquisition price \( \tilde{\delta}^* \) are solved as:

\[ \frac{\partial \tilde{\pi}_R}{\partial p} = \alpha + \beta w - 2\beta p = 0, \]

\[ \frac{\partial \tilde{\pi}_T}{\partial \delta} = 0 \]

Taking the second-order partial derivatives of \( \tilde{\pi}_M \) with respect to \( f \) and \( w \), we have the Hessian matrix as follows:

\[ H_M = \begin{bmatrix} \frac{\partial^2 E(U_{\pi_M})}{\partial w^2} & \frac{\partial^2 E(U_{\pi_M})}{\partial w \partial f} \\ \frac{\partial^2 E(U_{\pi_M})}{\partial f \partial w} & \frac{\partial^2 E(U_{\pi_M})}{\partial f^2} \end{bmatrix} = \begin{bmatrix} -v - \beta & v \\ v & -v \end{bmatrix} \]

Because \( -v - \beta < 0 \) and \( |H_M| = \beta v > 0 \), \( \tilde{\pi}_M \) is jointly concave in \( f \) and \( w \). Considering the first-order partial derivatives of \( \tilde{\pi}_M \) with respect to \( f \) and \( w \), respectively, and letting the derivatives be zero, by solving the following equation system \( \frac{\partial \tilde{\pi}_M}{\partial f} = 0 \) and \( \frac{\partial \tilde{\pi}_M}{\partial w} = 0 \), we can obtain the optimal price strategies as follows:

\[ \tilde{w}^* = \frac{\alpha + c_n \beta - \beta \lambda_{\omega}}{2p} \text{ and } \tilde{f}^* = \frac{\beta (u + \Delta_u) + v (a - \beta (c_n + \Delta_u + \lambda_{\omega}))}{2p} \]

Substituting \( \tilde{w}^* \) and \( \tilde{f}^* \) into Equations (26) and (27), we can obtain the optimal retail price...
and acquisition price $\tilde{p}^* = \frac{3\alpha + c_n \beta - \beta \lambda_{n2}}{4\beta}$ and $\tilde{\delta}^* = \frac{\Delta u - 3u - v \Delta_r - 3\Delta_n - v \lambda_{n2} - v \lambda_{1}}{4v}$. The other cases of $\tilde{q}_n$ and $\tilde{q}_r$ are similar.

**Proposition 1.** When disruption causes the collection and remanufacturing cost perturbation, the optimal price strategies of supply chain members are as follows:

\[
\tilde{p}^* = \begin{cases} 
\frac{3\alpha + c_n \beta - \beta \lambda_{n2}}{4\beta}, & \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
\frac{3\alpha + c_n \beta}{4\beta}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
\frac{3\alpha + c_n \beta + \beta \lambda_{n1}}{4\beta}, & \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (28)

\[
\tilde{\alpha}^* = \begin{cases} 
\frac{\alpha + c_n \beta - \beta \lambda_{n2}}{2\beta}, & \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
\frac{\alpha + c_n \beta}{2\beta}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
\frac{\alpha + c_n \beta + \beta \lambda_{n1}}{2\beta}, & \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (29)

\[
\tilde{\beta}^* = \begin{cases} 
\frac{\Delta u - 3u - v \Delta_r - 3\Delta_n - v \lambda_{n2} - v \lambda_{1}}{4v}, & \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
\frac{\Delta u - 3u - v \Delta_r - 3\Delta_n}{4v}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
\frac{\Delta u - 3u - v \Delta_r - v \lambda_{n2} - v \lambda_{2}}{4v}, & \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (30)

\[
\tilde{q}^* = \begin{cases} 
\frac{\beta(u + \Delta_u) + v(\alpha - (\beta \alpha + \lambda_{n1}))}{2\beta}, & \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
\frac{\beta(u + \Delta_u) + v(\alpha - (\beta \alpha + \lambda_{n2}))}{2\beta}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
\frac{\beta(u + \Delta_u) + v(\alpha - (\beta \alpha + \lambda_{n2}))}{2\beta}, & \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (31)

5. Comparisons with Managerial Implications

In this section, the optimal results derived in Section 4 are compared and some preliminary corollaries are obtained.

**Corollary 1.** In a centralized CLSC, both the optimal retail price and optimal quantity are given as follows:

\[
\tilde{p}^{*c} = \begin{cases} 
p^{*c} - \frac{\lambda_{n2}}{2}, & \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
p^{*c}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
p^{*c} + \frac{\lambda_{n1}}{2}, & \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (32)

\[
\tilde{q}_{n}^{*c} = \begin{cases} 
q_{n}^{*c} + \frac{v(\Delta u + \lambda_{n2} + \lambda_{r1}) + \beta \lambda_{n2} - \Delta}{2}, & q_{n}^{*c}, \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
q_{n}^{*c} + \frac{\Delta u - \Delta_n}{2}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
q_{n}^{*c} + \frac{v(\Delta u - \lambda_{n1} - \lambda_{r2} - \Delta u - \beta \lambda_{n1})}{2}, & q_{n}^{*c}, \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (33)

\[
\tilde{q}_{r}^{*c} = \begin{cases} 
q_{r}^{*c} + \frac{\Delta u - v(\Delta u + \lambda_{n2} + \lambda_{r1})}{2}, & q_{r}^{*c}, \Delta_u > \beta \lambda_{n2} \text{ and } \Delta_r < - (\lambda_{n2} + \lambda_{r1}) \\
q_{r}^{*c} + \frac{\Delta u - \Delta_u}{2}, & - \beta \lambda_{n1} \leq \Delta_u \leq \beta \lambda_{n2} \text{ and } - (\lambda_{n2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{n1} + \lambda_{r2}) \\
q_{r}^{*c} + \frac{v(\Delta u - \lambda_{n1} + \Delta u - \Delta u - \beta \lambda_{n1})}{2}, & q_{r}^{*c}, \Delta_u < - \beta \lambda_{n1} \text{ and } \Delta_r > (\lambda_{n1} + \lambda_{r2})
\end{cases}
\] (34)

The proof of Corollary 1 is shown in Appendix A. (The other proofs of Corollary can also be found in Appendix A)

Corollary 1 indicates that when the collection disruption occurs in a relatively small region, a certain robust region for the retail price exists, regardless of the value of $\Delta_u$. The
occurrence of disruptions does not strictly affect the chain member’s decision. A counter-
balance and restriction between the deviation of acquisition quantity and remanufacturing
cost exists. Basically, the negative effects of one disruption are offset by the positive effects
of another disruption. In the case of \( \Delta u > \beta_1 \lambda_2 \), with an increase in \( \Delta u \), more used products
being collected means more supply quantities for remanufacturing, remanufacturing cost
decreases owing to the scale effect, and demand quantity will increase because of the lower
retail price. While, if \( \Delta u < -\beta_1 \lambda_1 \), the remanufacturing cost will increase with the decrease
in \( \Delta u \), so the demand quantity of remanufactured products will decrease owing to the
higher retail price. Moreover, the adjustment of the optimal price policies is only related to
the increased stand \( \lambda_1 \lambda_2 \).

**Corollary 2.** In a centralized CLSC, acquisition price under the large positive disruption region
of \( \Delta u \) is lower than that under small disruption region of \( \Delta u \). The acquisition price under a large
negative disruption region of \( \Delta u \) is higher than that under the small disruption region of \( \Delta u \).

Corollary 2 indicates that the optimal acquisition price under large positive disruption
region of \( \Delta u \) is lower than that under the small disruption region of \( \Delta u \). The acquisition price
under a large negative disruption region of \( \Delta u \) is higher than that under the small region
disruption region of \( \Delta u \). Since the occurrence of disruption, if \( \Delta u \) is significantly decreased,
the remanufacturing cost will be significantly increased, and the supply quantities of
the remanufactured products will also be significantly reduced at this time. To ensure
that overall profit of the supply chain will not be lost, the acquisition price needs to be
increased. In contrast, in case of the collection quantities of used products increase, because
of the decrease in remanufacturing cost, the acquisition price of used products can be
appropriately reduced.

**Corollary 3.** In a decentralized CLSC, the price policies in the cases with and without disruptions
satisfy the relationships as follows:

\[
\tilde{p}^* = \begin{cases} 
    p^* - \frac{\lambda_2^2}{n^2}, & \Delta u > \beta_1 \lambda_2 \text{ and } \Delta r < - (\lambda_2^2 + \lambda_1) \\
    p^*, & -\beta_1 \lambda_1 \leq \Delta u \leq \beta_1 \lambda_2 \text{ and } - (\lambda_2^2 + \lambda_1) \leq \Delta r \leq (\lambda_1 + \lambda_2) \\
    p^* + \frac{\lambda_2^2}{n^2}, & \Delta u < -\beta_1 \lambda_1 \text{ and } \Delta r > (\lambda_1 + \lambda_2) 
\end{cases}
\]

\[
\tilde{w}^* = \begin{cases} 
    w^* - \frac{\lambda_2^2}{n^2}, & \Delta u > \beta_1 \lambda_2 \text{ and } \Delta r < - (\lambda_2^2 + \lambda_1) \\
    w^*, & -\beta_1 \lambda_1 \leq \Delta u \leq \beta_1 \lambda_2 \text{ and } - (\lambda_2^2 + \lambda_1) \leq \Delta r \leq (\lambda_1 + \lambda_2) \\
    w^* + \frac{\lambda_2^2}{n^2}, & \Delta u < -\beta_1 \lambda_1 \text{ and } \Delta r > (\lambda_1 + \lambda_2) 
\end{cases}
\]

Corollary 3 indicates that the occurrence of disruption can affect the chain members’
decisions in a decentralized CLSC. In the case of \( \Delta u > \beta_1 \lambda_2 \) and \( \Delta r < - (\lambda_2^2 + \lambda_1) \),
wholesale price decreases due to the decrease in remanufacturing cost, so the retailer will
set a lower retail price because of the large margin. If \( \Delta u < -\beta_1 \lambda_1 \text{ and } \Delta r > (\lambda_1 + \lambda_2) \),
the wholesale price increases because of the increase in the unit remanufacturing cost; the
retail price increases. The wholesale price and retail price are constant in the robust region
\( -\beta_1 \lambda_1 \leq \Delta u \leq \beta_1 \lambda_2 \) and in the case without disruptions.

**Corollary 4.** The optimal prices satisfy the following relationships in cases without disruption:
\( p^* > p^e \), \( \delta^e < \delta^c \).

The optimal quantities of remanufactured products satisfy the following relationships in the
case without disruption: \( q^e_r < q^c_r \).

As described in Corollary 4, the acquisition price and supply quantity of remanufactured
products in centralized CLSC are higher than those in decentralized CLSC. The retail
price in centralized CLSC is less than that in decentralized price. A large margin is required
to reduce the retail price for remanufactured products because of the lower acquisition
price in centralized CLSC. Thus, the retail price in centralized CLSC is less than that in decentralized supply chain.

**Corollary 5.** In the same disruption region, we have \( \tilde{p}^* > \tilde{p}_c^* \) and \( \tilde{\delta}^* < \tilde{\delta}_c^* \).

Corollary 5 indicates that regardless of the retail price of the centralized CLSC being lower than that of the decentralized supply chain, the acquisition price of centralized CLSC remains higher than that of the decentralized supply chain.

From Corollaries 4 and 5, whether or not a disruption event occurs, the retail price of centralized CLSC is lower than that of the decentralized supply chain, and the acquisition price of centralized CLSC is higher than that of the decentralized supply chain. The centralized supply chain is considered more conducive to encouraging consumers to participate in the collection of used products and remanufacturing.

**Corollary 6.** In the decentralized mode, the acquisition price is negatively correlated with the collection and remanufacturing disruption costs. The technology licensing fee is positively correlated with the collection disruption and negatively correlated with remanufacturing disruption cost.

Corollary 6 indicates that, with a positive increase in collection disruption, the acquisition price will decrease and the technology licensing fee will increase. The OEM will raise the licensing fee. According to Corollary 2, under this circumstance, the TPR can collect more used products with a lower acquisition price for their own profit. Similarly, when the remanufacturing cost increases due to disruption events, the TPR will reduce the acquisition price of used products to ensure cost savings for remanufacturing.

Based on the above discussion, we obtain the following managerial implications:

1. From the perspective of decentralization, when the remanufacturing cost is reduced due to the sudden increase in the quantities of used products, the OEM should set a lower wholesale price, a higher technology licensing price, and extract more profits from the TPR. To ensure their profits, the TPR will set a lower acquisition price to control costs. The retailer’s profit depends on the wholesale and retail prices. Because the manufacturer sets a lower wholesale price, the retailer will gain more profit when the acquisition volume is positively disrupted.

2. From the perspective of a centralized CLSC, when recycling quantity is reduced to a certain extent due to the occurrence of disruption events, the profits of a centralized CLSC will be reduced. In other cases, if the supply chain wants to obtain higher profits, it needs to consider the relationships between various factors. Finally, regardless of whether disruption occurs, the centralized CLSC will decide a lower retail price and higher acquisition price than the decentralized CLSC. Thus, the centralized CLSC can more easily stimulate consumers to participate in remanufacturing activities.

### 6. Numerical Examples

To further examine the results of the above discussion and find potential patterns of change, we first focus on how the change in the disruption \( \Delta_u \) when \( \Delta_r = 0 \) holds (and \( \Delta_r \) when \( \Delta_u = 0 \) holds) affects the profit of OEM and TPR, respectively. Second, we compare and analyze the impacts of the change of the disruption \( \Delta_u \) when \( \Delta_r = 0 \) holds (or \( \Delta_r \) when \( \Delta_u = 0 \) holds) on the acquisition price and licensing fee, respectively. The relevant parameters are set according to the conditions as follows: \( a = 300, c_u = 30, c_r = 10, \beta = 5, \sigma = 2, \lambda_{t1} = 0.9, \lambda_{t2} = 1.2, \lambda_{r1} = 1, \lambda_{r2} = 1.5, u = 15, v = 15, u_1 = -0.53, \) and \( u_2 = 1.75. \)

#### 6.1. The Profits of OEM and TPR with Different \( \Delta_u \) and \( \Delta_r \)

As clearly shown in Figure 2, the profits of the OEM and TPR increase with the increase in \( \Delta_u \). In \( \Delta_u \) a large negative region, the profits of the OEM and TPR are lower than those in the small range of \( \Delta_u \). Owing to the relatively small profit of the TPR, the OEM cannot extract much profit from charging the licensing fee, and the OEM will reduce...
the licensing fee. When \( \Delta_u \) is in a large positive region, the profits of the OEM and TPR will be increasing greatly. OEM will raise the licensing fee to extract more profit from the TPR from remanufacturing.

![Figure 2](image1.png)

**Figure 2.** The profits of OEM and TPR with different \( \Delta_u \).

As clearly shown in Figure 3, when \( \Delta_u = 0 \) is satisfied, the profit of the OEM decreases with an increase in \( \Delta_r \). When \( \Delta_r \) is in a large negative region, the TPR will obtain more profit from remanufacturing, which means that the OEM can extract more profit from remanufacturing. When \( \Delta_r \) is in a large positive region, the OEM and TPR will lose much profit because of the higher remanufacturing cost.

![Figure 3](image2.png)

**Figure 3.** The profits of OEM and TPR with different \( \Delta_r \).

From Figures 2 and 3, the impact of \( \Delta_u \) and \( \Delta_r \) on the profit of the TPR is similar to that on the profit of the OEM. In the same disruption range, the impact of the change in collection disruption on the profit is more stable than that of remanufacturing cost disruption.

6.2. The Acquisition Price and the Licensing Fee with Different \( \Delta_u \) and \( \Delta_r \)

As described in Figures 4 and 5, in the same disruption range, the acquisition price will decrease with the increase in \( \Delta_u \) and \( \Delta_r \). Overall, the acquisition price under different disruption regions remains flat.

![Figure 4](image3.png)

**Figure 4.** The acquisition price with different \( \Delta_u \).
Figure 5. The acquisition price with different $\Delta r$.

From Figures 6 and 7, the licensing fee increases with an increase in $\Delta u$. This will decrease with an increase in $\Delta r$. Overall, the licensing fee under different disruption regions changes in a large range.

Figure 6. The licensing fee with different $\Delta u$.

Figure 7. The licensing fee with different $\Delta r$.

7. Conclusions and Further Research

In this study, we take the CLSC in which the OEM licenses the TPR to remanufacture as the research object and study the factors affect the optimal pricing decision and the overall profit of the supply chain when the collection and remanufacturing cost disruptions occur simultaneously. Our research provides the following findings.

1. Whether or not disruption events occur, the centralized supply chain could better encourage consumers to participate in the collection of used products than a decentralized supply chain.

2. When collection disruption in a large positive region or the remanufacturing cost disruption in a large negative region occurs, the OEM' and TPR profits will greatly increase, and the OEM will raise the licensing fee to extract more profit from the remanufacturing activity.
3. A certain robust region exists for the retail price and wholesale price when the supply chain faces disruption increase. The OEM should raise the licensing fee to extract more profit from the remanufacturing activity and the TPR should keep the acquisition price basically unchanged when the collection disruption and remanufacturing cost disruption in a large region.

4. When the disruptions occur, they have a great influence on the OEM’s licensing fee but little on the TPR’s acquisition price.

The findings gained through this work can provide useful guidelines for supply chain members on how to effectively control costs to get more profit by adjusting price and selecting a better operation mode for a closed-loop supply chain. This paper may be extended in the following aspects. It is assumed that no difference between new and remanufactured products exists. Remanufactured products not reaching the same specifications as the new products is thus a more realistic case. Further, this paper does not consider the dual-channel supply chain structure and the case of licensing multiple TPRs to participate in remanufacturing.

For future study, we can consider more practical cases in which new and remanufactured products are treated at different prices. In addition, blockchain technology, as a distributed digital ledger technology that ensures transparency, traceability, and security, has shown promise for easing some global supply chain management problems, and has attracted practitioner attention in the supply chain domain [43–46]. Specifically, blockchain technology also aids in environmental supply chain sustainability, such as tracking and identifying further transaction information both of new products and used products, improving the recycling, etc. [46]. We also plan to integrate circular blockchain platforms for TPR and OEM, as suggested by [44].

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Appendix A

Proof of Corollary 1. Substituting \( p^* \) into Equation (16), we can obtain Equation (32). The proofs of Equations (33) and (34) are similar to Equation (32).

\[
\delta_{cs} = \begin{cases} 
\delta_{cs} - \frac{\Delta_u - \Delta_r}{2v}, & \Delta_u > \beta \lambda_{d2} \text{ and } \Delta_r < - (\lambda_{d2} + \lambda_{r1}) \\
\delta_{cs} - \frac{\Delta_u - \Delta_r}{2v}, & -\beta \lambda_{d1} \leq \Delta_u \leq \beta \lambda_{d2} \text{ and } - (\lambda_{d2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{d1} + \lambda_{r2}) \\
\delta_{cs} - \frac{\Delta_u - \Delta_r}{2v} + \frac{\lambda_{d1} + \lambda_{r2}}{2}, & \Delta_u < -\beta \lambda_{d1} \text{ and } \Delta_r > (\lambda_{d1} + \lambda_{r2}) 
\end{cases}
\]

Proof of Corollary 2. Substituting the acquisition price \( \delta^* \) without disruptions into \( \delta_{cs} \) with disruptions under centralized CLSC, we can obtain Equation (35):

\[
\delta_{cs}^{\text{central}} = \begin{cases} 
\delta_{cs}^* - \frac{\Delta_u - \Delta_r}{2v}, & \Delta_u > \beta \lambda_{d2} \text{ and } \Delta_r < - (\lambda_{d2} + \lambda_{r1}) \\
\delta_{cs}^* - \frac{\Delta_u - \Delta_r}{2v}, & -\beta \lambda_{d1} \leq \Delta_u \leq \beta \lambda_{d2} \text{ and } - (\lambda_{d2} + \lambda_{r1}) \leq \Delta_r \leq (\lambda_{d1} + \lambda_{r2}) \\
\delta_{cs}^* - \frac{\Delta_u - \Delta_r}{2v} + \frac{\lambda_{d1} + \lambda_{r2}}{2}, & \Delta_u < -\beta \lambda_{d1} \text{ and } \Delta_r > (\lambda_{d1} + \lambda_{r2}) 
\end{cases}
\]
Proof of Corollary 3. Substituting $p^*$ into $\tilde{p}^*$, we can obtain Equation (35). The proof of Equation (36) is similar to Equation (35). □

Proof of Corollary 4. Comparing the optimal strategies in centralized and decentralized decisions with no disruptions, we can obtain equations:

$$p^* - p^{c*} = \frac{\alpha - \beta c u}{4\beta} > 0,$$

$$\delta^* - \delta^{c*} = -\frac{\Delta v + u}{4v} < 0$$

$$q^*_r - q^{c*}_r = -\frac{\Delta v + u}{4} < 0 \quad \square$$

Proof of Corollary 5.

1. In the case of $\Delta_u > \beta \lambda n_2$ and $\Delta_r < -(\lambda n_2 + \lambda r_1)$, according to (28) and (16):

$$\tilde{p}^* - \tilde{p}^{c*} = \frac{\alpha - c_n \beta + \beta \lambda n_2}{4\beta} > 0,$$

The same results $\tilde{p}^* > \tilde{p}^{c*}$ can be proven in the other two cases.

2. In the case of $\Delta_u > \beta \lambda n_2$ and $\Delta_r < -(\lambda n_2 + \lambda r_1)$, according to (30) and (17), we have the following:

$$\delta^* - \delta^{c*} = -\frac{\Delta v - u + v \Delta_r - \Delta_u + v \lambda n_2 + v \lambda r_1}{4v} < 0$$

In the case of $\Delta_u < -\beta \lambda n_2$ and $\Delta_r > (\lambda n_1 + \lambda r_2)$, the following equation exists:

$$\tilde{\delta}^* - \tilde{\delta}^{c*} = -\frac{\Delta v - u + v \Delta_r - \Delta_u - v \lambda n_2 - v \lambda r_1}{4v} < 0, \quad (A7)$$

We can obtain the same result $\tilde{\delta}^* < \tilde{\delta}^{c*}$ in the case of $-\beta \lambda n_1 \leq \Delta u \leq \beta \lambda n_2$ and $-(\lambda n_2 + \lambda r_1) \leq \Delta r \leq (\lambda n_1 + \lambda r_2)$. \hfill □

Proof of Corollary 6. Using the second-order partial derivatives of $\tilde{\delta}^*$ and $\tilde{f}^*$, respectively, with respect to $\Delta_u$ and $\Delta_r$, we have the following:

$$\frac{\partial^2 \tilde{\delta}^*}{\partial \Delta_u^2} = -\frac{1}{2} > 0, \quad \frac{\partial^2 \tilde{\delta}^*}{\partial \Delta_r^2} = -\frac{1}{2} < 0, \quad \frac{\partial^2 \tilde{f}^*}{\partial \Delta_u^2} = \frac{1}{2} < 0, \quad \frac{\partial^2 \tilde{f}^*}{\partial \Delta_r^2} = \frac{1}{2} < 0 \quad \square$$

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