Sliding mode control of wastewater treatment process with activated sludge under extreme weather events

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Abstract. The present climate change engender an increase of extreme weather events such as intensive rainfall, flash floods, droughts, prolonged periods of deviations from established average daily temperatures. These extreme weather events negative impact on sustainability of wastewater treatment process with activated sludge. The paper aim is to propose a sliding mode control of wastewater treatment process with activated sludge under extreme weather events is proposed. The principles of the binary control theory are used. The control design is carried out with direct use of the nonlinear model, described by three ordinary differential equations with uncertain parameters. The parameters are varied in the known intervals. These uncertainties are predetermined by the nature and intensity of the extreme weather events. The model of the sliding mode control is developed with respect to an auxiliary input variable in order to obtain the smooth signal of the dilution rate, which is need in the biodegradation processes. The good robustness of closed loop system with the designed sliding mode control in regard to various disturbances is proved through simulation investigations in MATLAB using Simulink.

1. Introduction
In recent years, with permanently established the climate change, there has been a steady increase in the frequency and intensity of extreme weather events such as extreme temperatures (positive and negative), intensive rainfall, flash floods, droughts, prolonged periods of deviations from established average daily temperatures and others [1, 2]. It is well known that these extreme weather events negative impact on sustainability of wastewater treatment process with activated sludge [3, 4]. The biological wastewater treatment process is commonly based on activated sludge. It includes different aerobic types of microorganisms (biomass), which are responsible for the removal of the organic pollutants [5, 6]. The activated sludge process is mostly used in order to treat the municipal and industrial wastewater. In most countries, the legislation imposes strict environmental regulations on the quality of treated water [7, 8]. It is therefore necessary to design appropriate control algorithms that ensure the effective stabilization of activated sludge wastewater treatment processes with respect to uncertainties and disturbances caused by extreme weather events [9, 10].

The activated sludge wastewater treatment processes is usually described by a system from ordinary differential equations with uncertain parameters. For these reason it is needed the developing of sophisticated control algorithms based on nonlinear models [11, 12].

It is known that the sliding mode control is effectively used in the stabilization of nonlinear and uncertain processes [13]. The guaranteed system invariance to parameter uncertainties and external disturbances is the main advantage of the sliding mode control [14]. However, the control signal is discontinuous in time, which leads to chattering phenomenon. In practice, such control is hard to realize therefore various
techniques for chattering attenuation are proposed (boundary layers, auxiliary input variable, binary control, input-dependent sliding surface) [15, 16].

The paper aim is to propose a sliding mode control on based on the binary control theory for asymptotic output stabilization of wastewater treatment process with activated sludge with respect to uncertainties and disturbances caused by extreme weather events. The control design is carried out with direct use of nonlinear model with interval parameters. The model of this sliding mode control is developed with respect to an auxiliary variable in order to obtain the smooth input signal.

2. A mathematical model of the wastewater treatment process with activated sludge under extreme weather events

The activated sludge wastewater treatment process is generally carried out in system consisting of an aeration tank with biomass (bioreactor) and a secondary precipitator (clarifier) (Fig. 1). It is assumed that hydraulic characteristics of aeration tank are those of a continuously stirred tank bioreactor with cell recycle. Ideal conditions are assumed to prevail in the settler and no biochemical reaction is taken place [6]. Here, the aeration flow, $Q_{air}$ is used as the control input to the bioreactor in order to track a prescribed by the upper control level trajectory of the dissolved oxygen, $C$ [12].

![Figure 1. Schematic description of a activated sludge wastewater treatment equipment](image)

In this study, the wastewater treatment process with activated sludge is investigated by nonlinear model with uncertain parameters. This mathematical model is described by three ordinary differential equations with interval parameters as follow [11]:

$$\frac{dX(t)}{dt} = \mu(S(t), C(t))X(t) + D_{in}(X_{in} - X(t)) + D_r(X_r - X(t))$$  \hspace{1cm} (1)

$$\frac{dS(t)}{dt} = -k_1\mu(S(t), C(t))X(t) + D_{in}(S_{in} - S(t))$$ \hspace{1cm} (2)

$$\frac{dC(t)}{dt} = -k_2\mu(S(t), C(t))X(t) + D_{in}(C_{in} - C(t)) + K_{la}(t)(C_s - C(t))$$ \hspace{1cm} (3)

where $X(t) = X$ is the biomass concentration (activated sludge concentration), [mg.l$^{-1}$]; $S(t) = S$ is the substrate concentration (organic pollutant concentration), [mg.l$^{-1}$]; $C(t) = C$ is the dissolved oxygen concentration, [mg.l$^{-1}$]; $X_{in}$ is the influent biomass concentration, [mg.l$^{-1}$]; $S_{in}$ is the influent substrate concentration, [mg.l$^{-1}$]; $C_{in}$ is the influent oxygen concentration, [mg.l$^{-1}$]; $X_{r}$ is the recycle biomass concentration, [mg.l$^{-1}$]; $C_{s}$ is the saturated oxygen concentration, [mg.l$^{-1}$]; $D_{in}$ is the influent dilution rate, [h$^{-1}$]; $D_{ef}$ is the effluent dilution rate ($D_{ef} = D_{in}$), [h$^{-1}$]; $D_r$ is the recycle dilution rate, [h$^{-1}$]; $\mu(S(t), C(t))$ is the specific growth rate of activated sludge, described by nonlinear function, [h$^{-1}$]; $k_1$ is the mass of influent substrate/biomass produced; $k_2$ is the mass of dissolved oxygen/biomass produced; $K_{la}(t)$ is the overall volumetric oxygen transfer rate, [h$^{-1}$].
The function $K_{La}(t)$ describes the oxygen transfer. It depends on the aeration actuating system and the sludge conditions of the aeration tank. Here, $K_{La}(t)$ is assumed linear with respect to air flow rate:

$$K_{La}(t) = \alpha Q_{air}(t) + \beta,$$

where $Q_{air}$ is the air flow rate [h$^{-1}$]; $\alpha = 3.34$; $\beta = 3.54$ [h$^{-1}$].

In this study, the output stabilizing feedback of the wastewater treatment process with activated sludge (1)-(4) is designed under the following assumptions:

A1. The state variables - the biomass, substrate and dissolved oxygen concentrations are positive, bounded and differentiable functions of time:

$$0 \leq X^{\min} \leq X(t) \leq X^{\max}, \quad 0 \leq S^{\min} \leq S(t) \leq S^{\max}, \quad 0 \leq C^{\min} \leq C(t) \leq C^{\max} < C_S,$$

where $X^{\min}, X^{\max}, S^{\min}, S^{\max}, C^{\min}$ and $C^{\max}$ are known constants.

A2. The control input – the air flow rate $Q_{air}(t)$ is a differentiable function of time. It belongs to given interval, depending on technological and economical requirements:

$$0 \leq Q^{\min}_{air} \leq Q_{air}(t) \leq Q^{\max}_{air},$$

where $Q^{\min}_{air}$ and $Q^{\max}_{air}$ are known constants.

A3. The specific growth rate of the activated sludge $\mu(t)$ is known and differentiable function of time. For biological reasons, this function is positive and bounded. It is nonlinear function with respect to some state variables and process parameters (in particular, the kinetic coefficients):

$$\mu(t) = \mu(S(t), C(t), p) > 0,$$

where $p = [p_1, ..., p_m]$ is the parameter vector of the kinetic coefficients. The kinetic coefficients vary under the influence of extreme weather events. These parameters are uncertain, but bounded within intervals. On the basis of expert knowledge and practical experience these variation intervals can be defined:

$$0 < p^{\min} \leq p \leq p^{\max},$$

where $p^{\min}$ and $p^{\max}$ are vectors of the known constants.

A4. It is known that, the system external disturbance - the influent substrate (influent organic pollutant) concentration varies due to different extreme weather events. In practice, it can be specified the admissible boundaries of variation of the external disturbance:

$$0 < S^{\min}_{in} \leq S_{in} \leq S^{\max}_{in},$$

where $S^{\min}_{in}$ and $S^{\max}_{in}$ are known constants; $S^{\max}_{in}$ is the maximum admissible value of the substrate concentration (the organic pollutant in the bioreactor) $S(t)$

A5. The state variable – the biomass concentration $X(t)$ is not directly measured.

A6. The state variables – the substrate concentration $S(t)$ and the dissolved oxygen concentration $C(t)$ are measurable.

A7. The dissolved oxygen concentration $C(t)$ is the output variable of the considered wastewater treatment process with activated sludge, $y(t) = C(t)$.

It is important to point out, the main difficulty in controlling the activated sludge wastewater treatment processes comes from the variation of the process parameters and the influent waste load due to negative impacts of natural hazards. These variations induce process state changes that lead to a reduction of the water treatment efficiency, unless the system is effectively controlled.

3. Sliding mode control design of activated sludge wastewater treatment process under extreme weather events

In this study, the problem is to propose an approach for designing of sliding mode control of the activated sludge wastewater treatment process. The principles of the binary control theory are used.

In particular, the control design idea is the output stabilization of the dissolved oxygen concentration, $y(t) = C(t)$ at a desired set-point $g = C^*$ by using the air flow rate as control action, $u(t) = D(t)$. Therefore the dissolved oxygen error (system output error) $e(t)$ can be defined as:
\[ e(t) = C(t) - C^* = y(t) - g^* \]  

In order to solve the output stabilization problem, the nonlinear system (1)-(4) is transformed into a second-order differential equation with respect to the system output. The transformation is carried out by two times differentiation of the output variable \( y(t) = C(t) \) on base of the assumptions (A1)-(A7):

\[
\frac{d^2 C}{dt^2} = -k_2 \left( X \frac{d\mu}{dt} + \mu \frac{dX}{dt} \right) - (D_{in} + \alpha Q_{air} + \beta) \frac{dC}{dt} + \alpha (C_S - C) \frac{dQ_{air}}{dt}
\]  

Further, the non-measurement variable \( X \) can be expressed from equation (3), as follows

\[
X = \frac{1}{k_2 \mu} D_{in} (C_{in} - C) + (\alpha Q_{air} + \beta) (C_S - C) - \frac{dC}{dt}
\]  

Substituting the variable \( X \) and its first derivative \( dX/dt \), respectively from (11) and (1) into (10), the nonlinear process model is rewritten in the following form:

\[
\frac{d^2 C}{dt^2} = \left( \frac{1}{\mu} \frac{d\mu}{dt} + \mu - D_{in} - D_r \right) \left( D_{in} (C_{in} - C) + (\alpha Q_{air} + \beta) (C_S - C) - \frac{dC}{dt} \right) +
\]

\[
+ k_2 \mu (D_{in} X_{in} + D_r X_r) - (D_{in} + \alpha Q_{air} + \beta) \frac{dC}{dt} + \alpha (C_S - C) \frac{dQ_{air}}{dt}
\]  

It is easy to see that the expression (10) for the second derivative of the output variable is a function of the measurement variables only, in particular \( S(t) \) and \( C(t) \). The nonlinear functions \( \mu(S(t), C(t)) \) and \( d\mu/dt \) include only the measurement variables too. Therefore, the proposed model transformation (12) eliminates the non-measurement variable from the mathematical description of the activated sludge process (1)-(4).

Taking into account (9) and (12), the following input–output differential equation of the closed loop system error \( e(t) \) is obtained:

\[
\frac{d^2 e}{dt^2} = \left( \frac{1}{\mu} \frac{d\mu}{dt} + \mu - D_{in} - D_r \right) \left( D_{in} (C_{in} - g - e) + (\alpha u + \beta) (C_S - g - e) - \frac{de}{dt} \right) +
\]

\[
+ k_2 \mu (D_{in} X_{in} + D_r X_r) - (D_{in} + \alpha u + \beta) \frac{de}{dt} + \alpha (C_S - g - e) \frac{du}{dt}
\]  

Define the new state variables \( e_1(t) = e(t), \ e_2(t) = de_1(t)/dt \) and introduce the auxiliary input variable \( v(t) = du/dt \).

For control design purposes, the nonlinear error model (13) is represented by the generalized observability canonical form [16]:

\[
\frac{de_1(t)}{dt} = e_2(t)
\]

\[
\frac{de_2(t)}{dt} = f(e_1(t), e_2(t), u(t), \mu(t)) + b(e_1(t))v(t)
\]  

where the functions \( f(t) = f(e_1(t), e_2(t), u(t), \mu(t)) \) and \( b(t) = b(e_1(t)) \) are given, as follows:

\[
f(t) = \left( \frac{1}{\mu(t)} \frac{d\mu}{dt} + \mu(t) - D_{in} - D_r \right) \left[ D_{in} (C_{in} - g - e_1(t)) + (\alpha u + \beta) (C_S - g - e_1(t)) - e_2(t) \right] +
\]

\[
+ k_2 \mu(t) (D_{in} X_{in} + D_r X_r) - (D_{in} + \alpha u + \beta) e_2(t)
\]  

\[
b(t) = \alpha (C_S - g - e_1(t)) > 0
\]
It should be noted that the functions $f(t)$ and $b(t)$ are continuous and bounded, taking into account the assumptions (A1)-(A7). The functions $\mu(t)$ and $d\mu/dt$ are nonlinear with respect to the measurable state variable $S(t)$ and the output variable $C(t)$.

In this work, the problem for asymptotic stabilization of the nonlinear system (1)-(4) is solved by defining a suitable nonlinear feedback $u(t) \equiv u(S(t), C(t), p)$. The feedback depends only on the measurable variables. This stabilizing feedback is designed on the basis the sliding mode control theory [14]. Thereby it is guaranteed the system robustness to parameter uncertainties and to external disturbances is guaranteed, too [16].

It is well known that the control input signal, air flow rate $u(t) = Q_{air}(t)$, in activated sludge processes has to be smooth. This comes from fact that these wastewater treatment processes involve living organisms besides their activation decreases in the presence of chattering phenomenon [16].

For this reason, to avoid the chattering phenomenon and to obtain a smoothed input signal in the closed-loop system, the stabilizing feedback is designed as a second order sliding mode controller. In particular, the sliding mode should be realize with respect to the auxiliary input variable:

$$v(t) = d\mu(t)/dt = dQ_{air}(t)/dt.$$  

These reflections are the main motivation to define the stabilizing feedback as follows [16]:

$$v(t) = \gamma(t) V_M(t)$$  

$$\frac{d\gamma(t)}{dt} = -\varphi(t) \text{sgn} \left[ \sigma(t) + \gamma(t) |\sigma(t)| \right],$$  

where $\gamma(t)$ is a real variable; $\varphi(t)$ is a continuous function and $\varphi(t) \geq const > 0$ with $|\gamma(t)| \leq 1$ at every $t \geq t_0$; $\sigma(t)$ is the sliding surface.

The relation of the sliding surface $\sigma(t)$ is defined as

$$\sigma(t) = c_1 e_1(t) + c_2 e_2(t),$$  

where $c_1$ and $c_2$ are the positive design parameters.

The expressions for $\gamma(t)$ and $\varphi(t)$ are determined from stability condition of closed-loop system on the basis of the binary control theory, using Lyapunov theorem [14]. Therefore, the conditions for asymptotic output stabilization of activated sludge process (10)-(11) are derived as follows:

$$V_M(t) > \sup_t \left| \tilde{c}_1 e_2(t) + \tilde{c}_2 f(t) \right|,$$  

where $\tilde{c}_1 = c_1 + \delta \beta_1 > 0$, $\tilde{c}_2 = c_2 + \delta \beta_2 > 0$; $0 < \delta < 1$; $0 < \beta_i \leq 1$, $i = 1,2$; and

$$\varphi(t) \geq \frac{2}{\delta} \sup_t \left| \tilde{c}_1 e_2(t) + \tilde{c}_2 \left[ f(t) + b(t) \gamma(t) V_M(t) \right] \right|,$$  

where $\tilde{c}_1 = c_1 + \delta \lambda \beta_1 > 0$, $\tilde{c}_2 = c_2 + \delta \lambda \beta_2 > 0$; $0 \leq \lambda \leq 1$.

The suitable choice of the control design parameters ensures the desired dynamics of the closed-loop system (15)-(19).

4. Simulation investigations and results
Several simulation investigations in MATLAB using Simulink are carried out to demonstrate the effectiveness of the designed sliding mode control to stabilize the nonlinear wastewater treatment process with activated sludge in presence of uncertain parameters and external disturbances due to under extreme weather events.

There are many nonlinear functions to describe activated sludge growth limited by the organic pollutant and dissolved oxygen. However, Monod-type kinetics are widely used to express the aerobic wastewater treatment processes. Here, simulation investigations, the specific growth rate of activated sludge is written as follow [11]:

\[
\mu(t) = \frac{\mu_{\text{max}} S(t)}{k_s + S(t)} \frac{C(t)}{k_c + C(t)},
\]

where \(\mu_{\text{max}}\) is the maximum specific growth rate \([\text{h}^{-1}]\); \(k_s\) is the saturation constant for organic pollutant concentration \([\text{mg.1}^{-1}]\); \(k_c\) is the saturation constant for dissolved oxygen \([\text{mg.1}^{-1}]\). For this reason, the vectors of the kinetics coefficients are written as \(p = [\mu_{\text{max}}, k_s, k_c]\).

According to the condition (15) and expression (20), the function \(f(t)\) has the form:

\[
f(t) = -k_2 \left[ \frac{d\mu(t)}{dt} + \frac{\mu_{\text{max}} S(t)}{k_s + S(t)} \frac{C(t)}{k_c + C(t)} \left( \frac{\mu_{\text{max}} S(t)}{k_s + S(t)} \frac{C(t)}{k_c + C(t)} - D_{\text{in}} - D_r \right) \right] X(t) +
\]

\[
+ \frac{\mu_{\text{max}} S(t)}{k_s + S(t)} \frac{C(t)}{k_c + C(t)} \left( D_{\text{in}} X_{\text{in}} - D_r X_r \right) - \left( D_{\text{in}} + \alpha (u(t) + Q_{\text{air}}^*) + \beta \right) e_2(t),
\]

where

\[
\frac{d\mu(t)}{dt} = \frac{\mu_{\text{max}}}{k_s + S(t)} \frac{dS(t)}{dt} \frac{C(t)}{k_c + C(t)} - \frac{k_s}{k_c + C(t)} \frac{dC(t)}{dt} S(t).
\]

In this study, the following nominal values of the process parameters are assumed:

\[
\begin{align*}
\mu_{\text{max}} &= 0.35 ; & k_s &= 100 ; & k_c &= 2 ; & C_s &= 10 \\
D_{\text{in}} &= 0.2 ; & D_r &= 0.04 ; & k_1 &= 2 ; & k_2 &= 3 ; \\
S_{\text{in}} &= 200 ; & C_{\text{in}} &= 1 ; & X_{\text{in}} &= 100 ; & X_r &= 2000 .
\end{align*}
\]

For simulation investigations, the following values for the parameters of the designed sliding mode control are selected:

\[
c_1 = 0.7 ; & \quad c_2 = 1 ; & \quad \beta_1 = 1 ; & \quad \beta_2 = 1 ; & \quad \delta = 0.1 ; & \quad \lambda = 0.5
\]

The simulation results obtained from the step changes of the set point \((g = 4.5 \rightarrow 7.5 \rightarrow 3.5 \rightarrow 6)\) are presented in Fig. 2.

The good process behavior and the smooth control input \(u = Q_{\text{air}}\) (the air flow rate) could be observed. The chattering effect is removed by means of the first derivative of the control input signal \(v = du/dt = dQ_{\text{air}}/dt\).

In practice, the influent substrate concentration, \(S_{\text{in}}\) is usually varied. Figure 3 shows the system response when the value of the external disturbance, \(S_{\text{in}}\) (the influent substrate concentration) is changed, as follows: \((S_{\text{in}} = 200 \rightarrow 150 \rightarrow 250)\).

The influent substrate concentration can be varied by extreme weather events. The good features of the proposed nonlinear control, in relation to the changes of the external disturbance are evident.
The performance of the designed output stabilizing feedback is also surveyed with respect to the parameter uncertainties. Computer simulations are realized by random varying the kinetics coefficients in the admissible intervals:

\[
\begin{align*}
\mu_{\text{max}} &= 0.35 \rightarrow 0.5 \rightarrow 0.25 \rightarrow 0.45 \\
k_s &= 100 \rightarrow 70 \rightarrow 140 \rightarrow 90 \\
k_c &= 2 \rightarrow 2.5 \rightarrow 1.6 \rightarrow 1.9
\end{align*}
\]

The investigation results are presented in Fig. 4 and proved good robustness of the proposed control to the parameter uncertainties caused by extreme weather events.

5. Conclusions

The sliding mode control of wastewater treatment process with activated sludge under extreme weather events is proposed. The principles of the binary control theory are used. The control design is carried out with direct use of the nonlinear model, described by three ordinary differential equations with uncertain parameters. The model parameters are uncertain but varied in the known intervals. These uncertainties are predetermined by the nature and intensity of the extreme weather events. The model of the sliding mode control is developed with respect to an auxiliary input variable in order to obtain the smooth signal of the dilution rate, which is need in the biodegradation processes. The state variables, external disturbance, process output and control input are varied in the known intervals. The good
robustness of closed loop system with the designed sliding mode control in regard to various disturbances is proved through simulation investigations in MATLAB using Simulink.

6. References

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