Heavy-light meson spectrum with and without NRQCD

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Results for the spectrum of S and P-wave charmed mesons are obtained in the quenched approximation from a tadpole-improved anisotropic gauge field action and a D\textsuperscript{234} quark action. This is compared to the spectrum obtained from an NRQCD charm quark and a D\textsuperscript{234} light antiquark. NRQCD results for bottom mesons are also discussed.

1. MOTIVATION

If the charm quark is sufficiently heavy, then lattice NRQCD is an efficient computational method for obtaining the spectrum of mesons with a single charm quark. Recent studies of the S and P-wave meson spectrum have determined the contributions from first, second and third order in the inverse charm mass expansion.\cite{2} Second and third order contributions were found to be statistically insignificant for the P-wave mesons, but the S-wave spin splittings ($D^* - D$ and $D^*_s - D_s$) acquire substantial corrections at both second and third order. It is difficult to make a firm statement about convergence of the expansion from these findings alone.

In the present work, the existing results are compared to a new study, where the NRQCD charm quark is replaced by a relativistic charm quark using a D\textsuperscript{234} fermion action.\cite{3} The new simulations use the same set of gauge field configurations and the same light antiquark propagators as were used in Ref. \cite{2}, so the only differences are the heavy quark action (NRQCD versus D\textsuperscript{234}) and the operators used to create or destroy the various meson states.

The S-wave masses have been discussed many times in the literature (see Refs. \cite{1, 2, 4} and references therein), but the P-waves have only recently been extracted from lattice data: the limit of an infinitely heavy quark was studied by Michael and Peisa\cite{5}, a subset of the P-wave charmed masses were found by Boyle using the clover action (he also extrapolated to the bottom region)\cite{6}, and two groups have used NRQCD to study the charmed and bottom meson spectra\cite{2, 4, 7, 8}.

2. LATTICE CHOICES AND METHOD

The gauge fields of Ref. \cite{2}, also used for the present study, are on $10^3 \times 30$ anisotropic lattices with $(a_s/a_t)_{\text{bare}} = 2$. The gauge action is classically improved by including rectangular plaquettes, and tadpole improved using the mean link in Landau gauge. The coupling is fixed at $\beta = 2.1$, and the renormalized anisotropy is found to be $a_s/a_t = 1.96(2)$.

The light antiquark fields are also taken from Ref. \cite{2}. They are derived from a D\textsuperscript{234} action\cite{3} which removes leading and next-to-leading classical errors and is tadpole improved. Two hopping parameters ($\kappa = 0.23$ and 0.24) are used, corresponding to $m_\pi/m_\rho = 0.815(3)$ and 0.517(8). The physical $\rho$ meson mass leads to $a_t = 0.1075(23)$ fm, and the physical kaon mass implies $\kappa_{\text{strange}} = 0.2356(3)$. Dirichlet time boundaries are used for the light antiquark.

In Ref. \cite{2} the heavy quark was described by NRQCD, and new results are presented here where a heavy D\textsuperscript{234} quark is used with $\kappa = 0.182$, which is in the vicinity of charm. By interpolating the NRQCD results to this same heavy quark mass, a direct comparison of NRQCD and D\textsuperscript{234} results can be made.

With and without NRQCD, heavy-light meson masses come from 2000 quenched configura-
The quantum numbers are dictated by $\Omega$ as presented in Table 1. At the source, the parameters $(c_s, n_s)$ are set to (0.15,10) for NRQCD and (1,15) for D234. The sink operator is local in all simulations.

### 3. S-WAVE HEAVY-LIGHT MASSES

The S-wave hyperfine splitting for D234 is shown in Fig. 4 in lattice units, for the two available light antiquark hopping parameters. Also shown are the NRQCD results from Ref. 2 after interpolation of the $^1S_0$ kinetic mass to match the D234 value of the $^1S_0$ meson mass. This interpolation was done independently at each order in the NRQCD expansion, so the $^1S_0$ mass maintains a common value in each case. From Fig. 4, lowest order NRQCD is seen to give smaller hyperfine splittings than D234. The addition of $O(1/M^2)$ terms tends to increase the NRQCD results, but $O(1/M^3)$ contributions are larger in magnitude than the preceding order so the status of the NRQCD expansion for this observable is unclear.

The NRQCD results are shown in physical units in Fig. 5. Whereas Fig. 5 compared the different NRQCD orders by choosing a common physical $^1S_0$ mass, Fig. 5 shows the NRQCD expansion when the bare charm quark mass is held fixed. Neither plot gives a compelling defense for a convergent $1/M_{charm}$ expansion, but convergence is not disproven by such plots either. The $D_s - D$ splitting receives insignificant corrections from second and third orders, but corrections to the spin splittings are significant. Experimentally the $D_s - D^+$ splitting is 104 MeV, and therefore agrees with the NRQCD determination, but the spin splittings are $D^{*+} - D^+ = 141$ MeV and $D_s^* - D_s = 144$ MeV. With or without NRQCD, the quenched prediction is smaller than experiment.

Ref. 2 also provides results for the bottom mesons, and finds that the S-wave bottom masses are completely dominated by leading order in NRQCD so convergence is not disputed. As for charm, the spin splittings are smaller than experiment.
4. P-WAVE HEAVY-LIGHT MASSES

In Ref. [2], nonleading corrections to P-wave heavy-light masses were found to be insignificant for both bottom and charm mesons. This leads one to expect agreement between D234 and NRQCD determinations of these P-wave masses. Fig. 3 shows all four of the P-wave masses in lattice units for both of the available light antiquark hopping parameters. D234 results are new, and the NRQCD results are interpolations of the data from Ref. [2] so that the $^1S_0$ kinetic mass matches the D234 $^1S_0$ mass. Only statistical uncertainties are shown.

According to Fig. 3, the P-wave masses do not depend upon whether the charm quark is described by NRQCD or D234. One rather dramatic exception seems occur for the $^1P_1$ with the heavier antiquark, but this systematic error is understood: the D234 calculation has no true plateau in this case. The plateau-finding algorithm of Ref. [2] has been carried over to the present work, and it chooses the best plateau by maximizing the quality factor,

$$Q \equiv \frac{\Gamma(N/2 - 1, x^2/2)}{\Gamma(N/2 - 1, 0)},$$  \hspace{1cm} (4)$$

where

$$\Gamma(a, x) = \int_x^\infty dt \, t^{a-1} \exp(-t)$$  \hspace{1cm} (5)$$

and $N$ is the number of timesteps in the proposed plateau region. The D234 $^1P_1$ meson becomes quite noisy at early Euclidean times such that no clear plateau is evident, so the method of maximum $Q$ chooses a “plateau” which begins too near the source. This leads to the erroneously large mass shown in Fig. 3. Except for this one correlator, Fig. 3 presents a clear agreement between results with and without NRQCD.

The $^3P_0$ and $^3P_2$ masses for charm and bottom are displayed in Figs. 4 and 5 from the NRQCD results of Ref. [2], along with NRQCD results from Ref. [8], the clover results of Ref. [6], and a variety of model calculations [9]. A disagreement between the two lattice NRQCD calculations for $B_2^* - B_0^*$ is evident; Ref. [2] notes that no choice of plateau region allows those data to attain the large splitting of Ref. [8]. The lattice results of Ref. [4] cannot resolve the discrepancy. The non-lattice models are more easily distinguished from one another for charm than for bottom, and the lattice result clearly favours a small $D_2^* - D_0^*$ splitting (substantially less than 100 MeV).

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LATTICE QCD:
present work

MODELS:
Godfrey,Kokoski
Dai,Huang,Jin
Gupta,Johnson
Lahde,Nyfalt,Riska
Isgur
Ebert,Galkin,Faustov

(a) D_{2s}^*-D_{0s}^* and D_{s2}^*-D_{s0}^*

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(b) B_{2s}^*-B_{0s}^* and B_{s2}^*-B_{s0}^*

Figure 4. Splittings between the $^3P_2$ and $^3P_0$ masses for the $D^{**}$ and $B^{**}$ systems. Open(solid) symbols involve an s(u,d) quark.

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Figure 5. S-P splittings for heavy-light mesons. Open(solid) symbols involve an s(u,d) quark.

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