Modelling and optimizing engineering network systems

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Abstract. This paper is connected with the necessity of problem orientation of invariant methods of mathematical modelling and numerical optimization on formation of effective engineering network systems with cluster structure both at the stage of their functioning and development. In this regard, several problems are considered. Optimization modelling of engineering network system efficiency with cluster structure is carried out. Numerical optimization of engineering network system resource efficiency is demonstrated. Numerical optimization of structural and resource (combined) efficiency of the engineering network system is carried out. The results of the paper can be useful in choosing an effective implementation of the complex components of engineering networks on the basis of a computational experiment, to carry out a cluster structuring of the engineering network system of the network system by private indicators and integrated assessments of the efficiency of the functioning of its constituent objects.

1. Introduction

Efficiency of functioning in modern conditions of engineering networks as an organizational structure is achieved through optimal cluster structuring. The aggregation nature of such systems leads to the need for mathematical modeling of the integral properties of its constituent objects, and the cluster-network nature - to the need for their effective grouping based on numerical methods of structural-parametric optimization [1-3]. In most cases, the known methods, being invariant, do not take into account a number of features that arise during a comprehensive study of scientific problems of formation of effective engineering network systems: the need to take into account when grouping objects into clusters of alternative approaches to the formation of boundary conditions, the possibility of achieving the efficiency of a network system with a cluster structure, on the one hand, by reducing the set of objects, and on the other hand - the redistribution of a limited resource. In this regard, the paper proposes an approach to modeling engineering networks [4,5].

2. Optimization modeling of engineering network system efficiency with cluster structure

It is proposed to consider the formulation of the main problems connected with optimization modeling of engineering network system efficiency: structural, resource (parametric), structural and resource (combined) [6, 7]. The first problem is to reduce the engineering network structure in order to equalize the efficiency of objects \( O_i \) \( (i = 1, 7) \) so as to raise the level of lower score \( Y_i^M \), at the same time, it does not significantly reduce the overall level of assessments. To solve this problem, the
following integration mechanism is used: in lower-level clusters, a subset of objects is allocated \( O_t (t = 1, T) \) with low integral estimates \( Y_t (t = 1, T) \), which can be absorbed by the objects-leaders \( O_l (l = 1, L) \) with a high level of integrated assessment \( Y_l (l = 1, L) \). The new object, which appeared as a result of the merger, is characterized by an integral evaluation \( Y_{\text{new}} \). The formula for obtaining an integral estimate of a new object, for example, can be presented as follows: \( Y_{\text{new}} = c_1 Y_1 + c_2 Y_t \), \( c_1 + c_2 = 1 \), \( c_1, c_2 \geq 0 \). This mechanism is transformed into an optimization statement: it is necessary to provide such absorption by each of the objects \( O_t \) one of the objects \( O_l \), to increase the level of the lower assessment and at the same time to \( Y^M \) the total decrease in the level of integrated assessment of the leading objects was minimal. This statement corresponds to the bicriteria optimization model with Boolean variables and function \([8, 9]\):

\[
x_b = \begin{cases} 1, & \text{if object } O_t \text{ absorbs object } O_l, \\ 0, & \text{otherwise,} \end{cases}
\]

\[
\sum_{t=1}^{T} \sum_{l=1}^{L} (Y_t - Y_{\text{new}}) \rightarrow \min, \quad \min \left( \sum_{t=1}^{T} Y_t x_b \right) \rightarrow \max
\]

and restrictions:

\[
\sum_{t=1}^{T} x_b = 1, t = 1, T, \quad \sum_{l=1}^{L} x_b = 1, l = 1, L, \quad x_b \in \{0,1\}, t = 1, T, l = 1, L.
\]

Note that since the evaluations \( Y_t \) in the framework of this model are constants, not variable values, the objective function (1) can be rewritten as \([10, 11]\):

\[
\sum_{t=1}^{T} \sum_{l=1}^{L} Y_t x_b \rightarrow \max.
\]

To continue the integration process within the reduced network with the number of objects \( I_i = I - T \) a new object group-leaders are allocated \( O_i (i = 1, I) \) and a new set of objects is formed \( O_i (t = 1, T), T_i = I \) and the problem (1)-(2) is solved again. When continuing the procedure of reduction of the engineering network, it is possible to absorb several objects with a low level of efficiency by the object-leader.

The next problem is to select a two-stage mechanism for allocating \( R \) resources: in the first stage between clusters, and then between the objects of the optimized engineering network. The distribution of the resource \( R \) between clusters \( R_m (m = 1, M) \) it is advisable to carry out on the principle of reverse priorities, taking into account the total needs of the cluster objects and the priority of each cluster installed in the system, and within the cluster to divide the resource into supporting \( R_1^m \) and developing \( R_2^m \). For the distribution of the second type of resource, an optimization approach is used, focused on the possibility of increasing the level of resource efficiency of the object \( O_m (t_m = 1, T_m), T_i = I \) according to the j-th indicator, the closest to the average value of this indicator for the cluster \( R_m = 1 \) of the leading objects. To do this, we introduce the proximity coefficient of the value of the j-th index of the object \( O_m (t_m = 1, T_m), m = 1, M \) to values of this indicator at objects-leaders: \( a_{m,j} = \frac{y_{m,j}}{\bar{y}_{m,j}} \), where \( \bar{y}_{m,j} \) - the average value of the j-th index for cluster objects 1, and Boolean variables \([12,13]\)
If object $O_i$ is allocated developing resource to improve the efficiency of the \( j \)-th indicator,
\[
x_{i,j} = \begin{cases} 1, & \text{if object } O_i \text{ is allocated developing resource} \\ 0, & \text{otherwise, } t_m = 1, T_m, j = 1, J. \end{cases}
\] (4)

Then the optimization model for the objects of the \( m \)-th cluster has the following form:
\[
\sum_{i=1}^{T_m} \sum_{j=1}^{J} d_{i,j} x_{i,j} \rightarrow \max, \quad \sum_{i=1}^{T_m} \sum_{j=1}^{J} r_{i,j} x_{i,j} \leq R^2_m, \quad \sum_{j=1}^{J} x_{i,j} = 1, \quad x_{i,j} \in \{0,1\}, \quad t_m = 1, T_m, \quad j = 1, J. \] (5)

The objective function (1) determines the contenders for the resource \( R^2_m \), as close as possible in terms of efficiency to the objects-leaders, and the restriction (5) characterizes the condition under which the total need for the resource of objects \( O_i \) to increase the level of the \( j \)-th indicator \( r_{i,j} \) must not exceed the resource allocated for the \( m \)-th cluster \( R^2_m \). The limit (3) means that each object is initially allocated an additional resource to increase only one metric. Note that the problem (5) belongs to the class of multiple choice [14, 15] backpack problems (Multiple-choice Knapsack Problem).

If you enter a binary estimate of the fulfillment of the second condition by each object
\[
c_{i,j} = \begin{cases} 1, & \text{if object } O_i \text{ has an approximation perspective} \\ 0, & \text{otherwise, } t_m = 1, T_m, j = 1, J. \end{cases}
\] (6)

and Boolean variables
\[
x_{i} = \begin{cases} 1, & \text{if object } O_i \text{ is included in the first group} \\ 0, & \text{otherwise, } t_m = 1, T_m. \end{cases}
\] (7)

The optimization model that provides both conditions is as follows:
\[
\sum_{i=1}^{T_m} x_{i} \rightarrow \min, \quad \sum_{i=1}^{T_m} c_{i} x_{i} \geq 1, \quad j = 1, J, \quad x_{i} \in \{0,1\}, \quad t_m = 1, T_m, \] (8)

where the target function determines the fulfillment of the first condition, and the restriction - the second.

The set of optimization models allows using the developed algorithmic support to support decision-making by the control center to form an effective network system with a cluster structure [16, 17].

Let us consider possible algorithms for solving the problem (1)-(2). First of all, we note that the consideration of two objective functions in the problem (1)-(2) is justified for several reasons. First, criterion (3) aims to merge the worst objects of the set \( O_t (t=1, T) \) with the worst objects of the set \( O_l (l=1, L) \), and criterion (3.2), on the contrary, is focused on merging the worst objects from the set \( O_t (t=1, T) \) with the best objects from the set \( O_l (l=1, L) \). The introduction of two criteria would allow a compromise solution [18, 19]. The solution of the bicriteria problem is a set of effective points, among which the expert can choose the most appropriate solution from the point of view of practical implementation [20].
3. Numerical optimization of engineering network system resource efficiency

Obviously, the problem (4)-(5) is solvable only if \( \sum_{i=1}^{r_m} \min_{i \leq j \leq J} r_{w,i} \leq R_m^2 \). In addition, this problem is justified only if \( R_m^2 < \sum_{i=1}^{r_m} \max_{i \leq j \leq J} r_{w,i} \), otherwise the solution is trivial: \( x_{a,k} = 1 \), if \( k = \arg \max_{i \leq j \leq J} a_{w,i} \); \( x_{a,j} = 0 \) for \( j \neq k \). Note that the problem (4)-(5) is close to the model of the General block problem of the backpack (Multiple-choice Knapsack Problem - MCKP). There are \( m \) classes in this problem \( N_1, N_2, \ldots, N_m \) items to be packed in a backpack with a capacity of \( W \). Each item of class \( j \) from class \( N_1 \) has a value \( p_{j} \) and weight \( w_{j} \). The problem is to choose exactly one item from each class in such a way that the total value of the items was maximal, and the total weight of the backpack did not exceed some given value \( W \). Thus, the MCKP problem can be formulated as:

\[
\sum_{i=1}^{m} \sum_{j \in N_i} p_{i,j} x_{i,j} \rightarrow \text{max}, \quad \sum_{j \in N_i} x_{i,j} = 1, i = 1, m, \quad \sum_{i=1}^{m} \sum_{j \in N_i} w_{i,j} \leq W, \quad x_{i,j} \in \{0,1\}; \quad i = m; \quad j \in N_i. \quad (9)
\]

The peculiarity of the problem (4)-(5) is that all classes contain the same elements (a set of performance indicators of the network system), whereas in the block problem of the backpack (6)-(78) classes \( N_i \) they are disjoint and consist of various objects (which, however, does not have a significant impact on the essence of the developed algorithms). Block "knapsack" problem is NP-complete. At the moment several exact algorithms have been developed to find the solution of the block problem. For example, the method of branches and boundaries can be developed for it, and as an evaluation problem the continuous analog of the original formulation is solved, i.e. the block problem of the backpack in which the restriction (9) is replaced by:

\[
0 \leq x_{i,j} \leq 1; \quad i = 1, m; \quad j \in N_i \quad (10)
\]

In this study, an approximate method based on the use of genetic algorithms is proposed to solve the problem (4)-(5). First of all, we note that if in the initial formulation of the problem (3.4)-(3.5) there are numbers \( l \) and \( k \) such that \( a_{l,j} < a_{k,j} \) \& \( r_{l,j} \geq r_{k,j} \), that option \( t_{l,j} \) dominated by option \( t_{k,j} \), and, therefore, the variable \( x_{a,j} \) the optimal solution is 0 (because the object \( O_{l,a} \), using fewer resources may be more important to increase the rate of \( k \) than the figure \( l \)).

Figure 1 shows (for example) all possible options for improving one of the indicators for the object \( O_{l,a} \) for each of the indicators \( j = 1, J \) (on the abscissa axis, the need for a resource is postponed \( r_{l,j} \) to increase the level of the \( j \)-th indicator, on the ordinate axis – the value of the corresponding increase). Non-dominated (Pareto-optimal) variants are highlighted in black, one of which should be selected as a result of solving the problem. Denote by \( N_{l,a} \) subset of indices of

![Figure 1. A variety of options to improve one from the performance indicators for the facility \( O_{l,a} \).](image-url)
indicators $j$, which correspond to non-dominated variants $I_{m,j}$ improving the level of efficiency of the object $O_{m}$. Let's order the elements of each set $N_{m,j}$ in descending order $a_{m,j}$. (For example, if $N_{m} = (3,1,5)$ – this means that $a_{m,1} \geq a_{m,2} \geq a_{m,3}$). In this case, the elements of sets $N_{m,j}$ also will be automatically sorted in descending order $r_{m,j}$ (By virtue of the $N_{m}$ only non-dominated options if $a_{m,j} \geq a_{m,k}$, to and $r_{m,j} \geq r_{m,k}$).

### 4. Results

To carry out a computational experiment in solving the bicriteria problem, a program is developed to find the Pareto-optimal solution of the problem by a complete search of possible options (in the case of a small dimension), as well as the approximation of the Pareto-optimal set, solving the problem by the method of successive concessions (for a given number of iterations) or by a genetic algorithm. When choosing to solve the problem of genetic algorithm, it is possible to use the methods of implementation of multi-criteria genetic algorithm.

A comparative analysis of the approximations of the Pareto-optimal set obtained by these algorithms was made on the basis of two criteria: efficiency and representativeness of the obtained solutions. The efficiency of the approximations was estimated on the basis of the proportion of non-dominated points found by each of the methods separately, in the total set of non-dominated points that fell into the final combined approximation of the Pareto-optimal set obtained by the results of solving the problem by all five methods. The more non-dominated points are found by the algorithm, the better the approximation is constructed by it.

To assess the representativeness of the approximations, a criterion for the uniformity of the distribution of the solution points was developed, based on the division of the approximation points into clusters. After solving the problem, each of the studied methods in the criterion space (in the case of two criteria – on the plane) calculated the area of a rectangular region containing all the points of approximation. The boundaries of the region were determined by the minimum and maximum values of the approximation points for each of the criteria.

After solving the multicriteria problem, each of the methods under study calculated the number of clusters in a rectangular region containing all the approximation points. Then the number of clusters containing at least one of the approximation points was determined. On the basis of the statistics collected during the computational experiment, the average number of clusters containing at least one approximation point found by each of the algorithms was calculated. Large values of the criterion correspond to the best representativeness of the points of the constructed approximation.

In figure 2 we can see the values of the criteria for the problem.

![Figure 2](image.png)

**Figure 2.** The values of the criteria (9) as Pareto-optimal points.

It is seen that the solutions have significant differences. For example, it is possible that the total value of the integrated indicators of the network structure objects after the merger will be 259, but the worst figure will be 2. In another embodiment, the total value of all integrated indicators of objects of the network structure after the merger will be only 212, but the figure of the worst object will be 9 units (which may allow to save the network from outsiders who do not meet the specified requirements). The final decision on the choice of the option of reduction of the engineering network should be made by the head-expert, taking into account the mutual location of the objects of the network structure and the practical possibility of the appropriate Association.
5. Conclusion
Optimization models of extreme statements of problems of formation of the effective systems of engineering networks having cluster structure are formed and their features demanding problem orientation of numerical methods of discrete programming are characterized. The combined algorithms of numerical optimization of structural and resource efficiency of engineering network systems, providing the account of features of statements of canonical problems of discrete programming in relation to the investigated class of complex systems are offered. A computational experiment was carried out to demonstrate the efficiency of the proposed algorithms.

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