Decentralized Connectivity-Preserving Deployment of Large-Scale Robot Swarms

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Abstract—We present a decentralized and scalable approach for deployment of a robot swarm. Our approach tackles scenarios in which the swarm must reach multiple spatially distributed targets, and enforce the constraint that the robot network cannot be split. The basic idea behind our work is to construct a logical tree topology over the physical network formed by the robots. The logical tree acts as a backbone used by robots to enforce connectivity constraints. We study and compare two algorithms to form the logical tree: outwards and inwards. These algorithms differ in the order in which the robots join the tree: the outwards algorithm starts at the tree root and grows towards the targets, while the inwards algorithm proceeds in the opposite manner. Both algorithms perform periodic reconfiguration, to prevent suboptimal topologies from halting the growth of the tree. Our contributions are (i) The formulation of the two algorithms; (ii) A comparison of the algorithms in extensive physics-based simulations; (iii) A validation of our findings through real-robot experiments.

I. INTRODUCTION

Swarm robotics [1] is a branch of collective robotics that studies decentralized solutions for the problem of coordinating large teams of robots. Robot swarms are a promising technology for large-scale scenarios, in which performing spatially distributed tasks would entail prohibitive costs for single-robot solutions [1]. Typical examples include planetary exploration [2], deep underground mining [3], ocean restoration, and agriculture.

A common aspect in these scenarios is the necessity to maintain a coherent state across the swarm. Many basic coordination problems can be solved assuming low-bandwidth, occasional communication or even no communication. However, global connectivity is an asset when information must be exchanged in a timely manner, either to optimize a global performance function, or to aggregate data in a sink. Task allocation scenarios with stringent space and time constraints, such as warehouse organization and search-and-rescue operations [4] are prime examples of this category of problems. In these scenarios, it is desirable for the robot network to allow both short-range and long-range information exchange.

In this paper, we tackle the problem of deploying a robot network in a decentralized fashion, under the constraint that long-range information exchange must be possible at any time during a mission. We assume that the robots must reach a number of distant locations. While navigating to these locations, the robots must spread without splitting the network topology in disconnected components. The robots must achieve a final configuration in which data can flow between any two target locations, using the robots as relays.

It is important to notice that it is not required for all of the robots to take part in the final topology. Rather, it is desirable that as few robots as possible are engaged in connectivity maintenance, as this would free any extra robot for other tasks or to act as occasional replacement for damaged robot in the topology. In constrast, the robots that are part of the final topology must form a persistent communication backbone that can be used by any robot when necessary.

This aspect sets apart our work from existing research on connectivity maintenance, which generally requires all robots to be part of the connected topology. The literature on this topic can be broadly divided in two classes: algorithms in which the robots must attain a final, static structure to maximize coverage [5], and algorithms in which global connectivity is enforced while navigating to a specific location as a single unit (flocking) [6]. Our work, in contrast, aims to create a dynamic, decentralized communication infrastructure that connects specific locations and uses as few robots as possible.

Our approach assumes that the robots are initially deployed in a compact, connected cluster. The robots then form a logical tree over the physical network topology. By growing the tree over time, the distribution of the robots progressively and dynamically extends to reach the target locations. The final configuration is a star-like topology, in which data can flow between any two target locations.

The main contributions of this work are:

1) The formalization of two algorithms to form and grow logical tree topologies that connect multiple target locations;
2) A comparative study of the algorithms, based on extensive physics-based simulations;
3) The validation of our findings through a large set of real-robot experiments.

The rest of this paper is organized as follows. In Sec. II we formalize the problem statement. In Sec. III we present our methodology. In Sec. IV we report an evaluation of the algorithms. In Sec. V we discuss related work. The paper is concluded in Sec. VI.
II. PROBLEM STATEMENT

A. Robot Dynamics

We consider \( N \) robots with linear discrete dynamics
\[
    x_i(t+1) = Ax_i(t) + Bu_i(t)
\]
where \( x_i(t) \in \mathbb{R}^{2M} \) is the state of robot \( i \) at time \( t \), \( u_i(t) \in \mathbb{R}^{2M} \) is the control signal, and \( A, B \in \mathbb{R}^{2M \times 2M} \). The state \( x_i(t) \) is defined as \([p_i(t), v_i(t)]\), where \( p_i(t) \in \mathbb{R}^M \) designates the position of robot \( i \) and \( v_i(t) \in \mathbb{R}^M \) its velocity. State and controls are subject to the convex constraints
\[
    \forall t \geq 0 \quad x_i(t) \in X_i \quad u_i(t) \in U_i.
\]
In this work we focus on 2-dimensional navigation \((M = 2)\).

B. Robot Communication

We assume that the robots are capable of situated communication. This is a communication modality in which robots broadcast data within a limited range \( C \), and upon receiving data, a robot is able to estimate the relative position of the data sender with respect to its own local reference frame.

We define the communication graph \( G_C = (\mathcal{V}, \mathcal{E}_C) \), where \( \mathcal{V} \) is the set of robots \( \{1, \ldots, N\} \), and \( \mathcal{E}_C \subseteq \mathcal{V} \times \mathcal{V} \) is the set of edges connecting the robots. An edge \((i,j)\) between two robots exists at time \( t \) if their distance is within their communication range \( C \), i.e., \( \| p_i(t) - p_j(t) \| \leq C \).

Definition 1 (Graph connectivity): A graph is connected if there exists a path between any two nodes.

Graph connectivity can be verified through well-known concepts in spectral graph theory. From the definition of the graph adjacency matrix
\[
    A_{ij} = \begin{cases} 
    1 & \text{if } (i,j) \in \mathcal{E}_C \\
    0 & \text{otherwise}
    \end{cases}
\]
and of the graph degree matrix
\[
    D_{ij} = \begin{cases} 
    \sum_k A_{ik} & \text{if } i = j \\
    0 & \text{otherwise}
    \end{cases}
\]
we can derive the Laplacian matrix \( L = D - A \). The graph is connected if and only if the second smallest eigenvalue of \( L \) is greater than 0. For this reason, this eigenvalue is called algebraic connectivity or Fiedler value [7]. We will employ algebraic connectivity as a performance measure in the experiments of Sec. IV.

C. Objectives

The objective of this work can be stated as follows: we aim to create a progressive deployment strategy that can reach an arbitrary number of geographically distant tasks while satisfying connectivity constraints. In particular, the final configuration of the network topology must allow communication between any two target locations.

III. APPROACH

A. Roles

In both algorithms, we assume that the robots are initially deployed in a fully connected cluster. Subsequently, the robots must form a tree by dynamically assuming a specific role in the process.

In both tree-forming algorithms, the robots can have four possible roles: root, worker, connector, or spare. The root robot corresponds to the tree root, and at any time during the execution only one robot can assume this role. The worker robots are the tree leaves, and they correspond to robots that must reach the target locations, forcing the tree to grow progressively. The connector robots dynamically join the tree to support its growth, leaving the pool of available spare robots.

B. High-Level Behavior Specification

The algorithms can be formalized through a high-level state machine that encodes the behavior of every robot, as depicted in Fig. 1.

Every robot starts in state INIT. We assume that a process that assigns the role of worker to the robots closest to the targets has been already executed, through, e.g., a task allocation algorithm or a gradient-based algorithm. In addition, a random robot is assumed assigned the role of root. The other robots are initially spare.

The START TREE state is triggered by the root, which propagates a signal throughout the robot network. This state signifies that a new tree must be created. As the message propagates throughout the network, the robots estimate their distance from the root. This is possible because of situated communication—every robot can estimate a relative vector to each of its immediate neighbors.

Robots receiving a “start tree” signal switch to SELECT PARENT. In this state, each robot must identify a new parent to attach to. The selection of a new parent aims to create the shortest possible paths between the root robot and the worker robots, i.e., the leaf nodes in the tree. The specifics of this
state are different in the outwards and inwards algorithms, and are explained in Sec. III-D and Sec. III-E. At the end of this state, a robot is part of two trees—the one from the previous iteration of the algorithm (excluding the very first iteration), and a new one that reflects the new parent.

Once every robot has selected a new parent, the robots switch to the Grow Tree state, in which the robots forget the tree from the previous iteration and spare robots are accepted to join an edge. The algorithms differ in the implementation of this state, and details are reported in Sec. III-D and Sec. III-E.

Once the growth state is complete, the robots switch to the Select Root state. As the tree grows, the initial choice of the root robot (which is random) or an uneven distribution of target locations might render the tree topology suboptimal. By selecting a new root, the swarm can balance the tree branches, thus fostering even growth over time. The design of this state is illustrated in Sec. III-C.

Finally, the new assigned root switches to state Start Tree and broadcasts a new “start tree” signal.

In Fig. 1, certain state transitions are marked with diamonds. These transitions, which we call barriers, are special in that they correspond to “wait states” in which the robots must stay until a certain condition is verified for every robot. The specific implementation of these conditions depends on the algorithms. However, the general principle is that the root aggregates the information necessary to evaluate a certain condition, and then broadcasts a “go” signal throughout the tree. The “go” signal triggers a state transition in the robots that receive it.

C. Selection of a New Root

The purpose of selecting a new root is to balance the tree, which fosters better growth and compensates for an uneven distribution of target locations. In addition, balancing the tree has positive effects on the scalability of our algorithms. Every state in our algorithms involves some form of diffusion/aggregation process across the tree, with a time complexity that is linear with the depth of the tree. By balancing the tree, we also shorten its depth, thus lowering the time for diffusion/aggregation processes to complete.

These considerations suggest that the best location for the root is as close as possible to the centroid of the distribution of robots. The selection of a new root occurs at the end of a tree configuration loop, but the data upon which the process depends is collected in state Select Parent, when the robots select a new parent.

The algorithm provides an estimate of the centroid in the root reference frame by adding up each robot contribution from the leaves to the root. The algorithm is formalized in Alg. 1. An intuitive explanation of this algorithm proceeds as follows. Since each robot only knows its relative position to other robots, it must send to its parent an accumulation vector \( q_i \), which aggregates its contributions and that of all its descendants in the tree, according to its own reference frame. Fig. 2 reports an example with three robots, where robot 0 is the root, robot 2 is a worker, and robot 1 is a connector.

To perform the final calculation of the centroid, Alg. 1 needs the number of robots in the swarm. A tree-based distributed algorithm to count the number of robots currently committed in the tree is reported in Alg. 2. This algorithm requires the robots to aggregate a partial count, denoted with \( c_i \), from the tree leaves to the root.

In our implementation, both Alg. 1 and Alg. 2 are executed in parallel in state Select Parent. In select root, the current root compares its position and the position of its neighbors to the centroid estimate (all are expressed in its reference frame). If the current root is the closest to the centroid, it remains the root and restarts a new tree loop. Otherwise, it designates a new root and sends the centroid vector and the angle to the new root. When the new root receives this message, it sends an acknowledgement message to the old root, and then it expresses the centroid in its own reference frame. The process is repeated until the root is the closest robot to the centroid estimate.

**Algorithm 1** Distributed centroid estimation algorithm executed by robot \( i \): \( a_i \) denotes an accumulator value; \( q_i \) denotes the contribution of robot \( i \) to the estimation algorithm; \( c_i \) and \( d_i \) denote the number of robots in the swarm estimated by robot \( i \) and the tree depth of robot \( i \), respectively; and \( p_i \) is the vector from robot \( i \) to its parent.

```plaintext
1: \( a_i = 0 \)
2: for all child \( j \) do
3: \( q_i = q_i + q_j \) in \( i \)'s reference frame
4: \( a_i = a_i + q_j \)
5: end for
6: if robot \( i \) has a parent then
7: \( q_i = a_i - (c_i - d_i + 1) \cdot p_i \) nb descendants
8: end if
9: if robot \( i \) is the root then
10: \( q_i = a_i / c_i \) robot count
11: end if
```

**Algorithm 2** Tree-based count algorithm for robot \( i \). The depth of robot \( i \) in the tree is denoted as \( d_i \). The depth of the tree root is set to 1. The count calculated by robot \( j \) is denoted as \( c_j \).

```plaintext
1: switch number of children do
2: case 0
3: return \( d_i \)
4: case 1
5: return \( c_i \)
6: default
7: return \( \sum_{neighbors} (c_j - d_i) + d_i \)
8: end switch
```
D. The Outwards Algorithm

The intuition behind the outwards algorithm is to build a logical spanning tree over the entire robot network. The process starts at the root, and robots join the tree progressively.

In state SELECT PARENT, robot \( i \) considers its neighbors as potential candidates. Viable candidates are non-workers already in the tree within communication range. Among these, the robot selects the closest robot. The robot commits to the tree and starts broadcasting its parent id, which indicates to the parent robot that robot \( i \) is a child and that \( i \) is a connector. Each connector maintains its list of children and checks for obstructions of line-of-sight with respect to its parent. If a robot cannot receive data from its selected parent, it selects another parent and updates its data.

In state GROW TREE, the robots undergo two main phases: first, they discard the information about the old tree; second, they prune tree branches that contain no workers. To establish whether a branch contains a worker, when a worker selects a parent (state SELECT PARENT), the latter propagates this information upstream towards the root.

The branches not containing a worker are considered “useless” and the robots that are part of them take the spare role. To disband a useless branch, spare robots abandon it starting from the leaves. The leaves curl the branch back towards the root, and upon entering in contact with another branch might decide to join it. The logic for spares to join a branch is explained in Sec. III-F.

E. The Inwards Algorithm

The intuition behind the inwards algorithm is that the robots join the tree starting from the workers towards the root. Growth is therefore directed, and the final topology is a sparse tree, in that only a subset of the robots takes part in it. The spare robots, in contrast to the outwards algorithm, do not form branches; rather, they disperse along the tree and select a robot to use as reference.

In state SELECT PARENT, viable candidates for parent selection are non-workers in the tree or robots not in the tree which are at a distance smaller than the communication range \( C \). Among these, a robot selects a neighbor with the smallest distance to the root. When the robot \( i \) commits to the tree, it broadcasts its parent id, which indicates to the parent robot that robot \( i \) is a child and that \( i \) is a connector. In the inwards algorithm, by definition, all branches are useful because they all terminate with a worker as leaf node.

In state GROW TREE, spare robots attempt to join a branch. The logic for branch joining is the same as in the outwards algorithm, and it is explained in Sec. III-F.
F. Spare Management

The diagram in Fig. 5 describes the part of the GROW TREE state that concerns the interaction between spare robots and non-spare robots (i.e., connectors, workers, and root).

Non-spare robots enter the NO NEED state when they have no need for a spare robot. They exit this state either if the distance to their parent becomes smaller than the safe communication range \( S \), or if at least one of their children’s state is the NEED state. In the NEED state, each robot continuously checks if it is in an edge selected by a spare robot, or if their parent is in the AWAIT state. If one of these conditions is fulfilled, the robot transitions to the AWAIT state. In the AWAIT state, the robot waits for the insertion of a spare robot either in one of its edges or upstream in the tree.

Spare robots enter the WAIT state and look for an edge to extend. They transition to the EXTEND EDGE state or the ADJUST POSITION state after performing a search for edges in need among their neighbors. In the ADJUST POSITION state, spare robots rotate around their parent if they are within the safe radius or move towards their parent in a straight line otherwise. In the EXTEND EDGE state, spare robots head for the middle of the edge to be extended.

G. Robot Motion

The integrity of the tree over time is ensured by constraining the robots’ motion. We enforce the constraints by expressing the robot motion as a sum of virtual potential forces (we omit time dependency for brevity of notation):

\[
u_i = \begin{cases} 
 u_{i, \text{tree, old}} + u_{i, \text{tree, new}} + f_i(d_{i, \text{parent}}^p)(u_i^\text{target} + u_i^\text{avoid}) & \text{if } d_{i,j} \leq E \\
 0 & \text{otherwise}
\end{cases}
\]

In these equations \( d_{i,j} = \| p_i - p_j \| \), \( E < C \) is the emergency range beyond which a robot is dangerously distant from its parent, and

- \( u_{i, \text{tree, old}} \) and \( u_{i, \text{tree, new}} \) indicate the interaction law between robots \((i, j)\) in a parent-child relationship, in either the old or the new tree. We use the control law

\[
u_i^\text{tree} = \frac{\epsilon}{d_{i,j}} \left( \left( \frac{\delta}{d_{i,j}} \right)^2 - \left( \frac{\delta}{d_{i,j}} \right)^4 \right)
\]

where \( \delta = E \) and \( \epsilon \) are parameters to set at design time.

- \( u_i^\text{target} \) is a control law that attracts a robot to a target, promoting tree growth. For workers, this is a force that points the assigned target location \( l_i \) and calculated with

\[
u_i^\text{target} = \tau \frac{l_i - p_i}{\| l_i - p_i \|}
\]

where \( \tau \) is a design parameter. Workers propagate to their parents the calculated \( u_i^\text{target} \), and connectors apply it in turn.

- \( u_i^\text{avoid} \) is a repulsive force for obstacle avoidance between neighbors not in a parent-child relationship.

- \( f_i(d_{i, \text{parent}}^p) \) is a function defined as follows:

\[
f_i(d_{i, \text{parent}}^p) = \begin{cases} 
 1 & \text{if } d_{i, \text{parent}}^p \leq S \\
 0 & \text{otherwise}
\end{cases}
\]

where \( d_{i, \text{parent}}^p \) is the distance between a robot and its parent and \( S < E \) is the safe communication range. Through this function, a robot can ignore navigation to target and obstacle avoidance to perform emergency maneuvers when the distance to its parent becomes unsafe.

IV. Evaluation

A. Parameter Setting

The dynamics and the performance of our algorithms depends on the design parameters reported in Table I. To set their value, we used a genetic algorithm. We ran multiple instances of the optimization process for both inwards and outwards, and Table I reports the best values we found.

Every instance of the optimization was executed for 100 generations. We set this number as a reasonable margin after observing that, across instances, after about 50 generations the optimization process would find a plateau beyond which no improvement was found.

Every generation consisted of trials in which 9 Khepera IV robots\(^1\) were placed in the arena in a tight cluster. We configured two types of trials:

- 2 target locations on a circle with a radius of 2.3 m at 180° from each other;
- 3 targets on a circle with a radius of 1.6 m at 120° from each other.

We ran the trials in the ARGoS multi-robot simulator [8], and maximized a two-step performance function. The first step (performance 0 to 1) promoted connectivity maintenance by penalizing the time spent with disconnected robots; the second step (performance 1 to 2) was activated when no disconnections occurred, and higher values corresponded to lower times to reach the targets.

B. Simulated Experiments

We tested the performance of the algorithms by varying three parameters: the target radius, the redundancy factor, and number of targets. We placed multiple targets on a circle with equal angles between each other. The target radius is the radius of the circle. We chose radii of 3, 6 and 9 meters corresponding to small, medium and large scales. The redundancy factor is the factor by which we multiply the minimum required number of robots needed to reach all the targets given our communication range. We tested the values of 2, 3 and 4 for this parameter. The number of targets was 2, 3, and 4. The largest configuration we considered involved 94 robots. Each scenario was executed with 50 different random seeds. We ran all the experiments for both algorithms with and without activating line-of-sight obstructions in the communication models of ARGoS, to assess its importance.

\(^1\)https://www.k-team.com/mobile-robotics-products/khepera-iv
TABLE I: Optimized Design Parameters

| Type               | Symbol | Meaning                                      | Outwards | Inwards | Unit   |
|--------------------|--------|----------------------------------------------|----------|---------|--------|
| Motion             | S      | Safe range between parent and child          | 138.93   | 135.2558 | cm     |
|                    | A      | Non-parent-child avoidance range            | 43.16    | 40.99   | cm     |
|                    | δ      | Ideal distance between parent and child      | 190      | 154.0841| cm     |
|                    | ε      | Factor gain in parent-child interaction      | 10       | 10      |        |
|                    | τ      | Magnitude of attraction to target            | 0.49     | 0.2339  |        |
| Tree Growth        | R      | Reconfiguration period                       | 38.8     | 44.0    | sec    |
|                    | I      | Information liveness period                  | 1.2      | 0.5     | sec    |
| Uncommitted Management | E    | Distance threshold for spare recruitment     | 132.09   | 132.1353| cm     |
|                    | J      | Distance threshold to switch to connector    | 9.79     | 6.6395  | cm     |

1) Simulation Time: We studied the time performance of both algorithms, and declared an experiment finished when all workers reached their targets. To compare results across different scales, we normalized the mission duration by the maximum allowed time. The maximum allowed time was computed by considering the time for a robot to reach a target from the center of the arena; this time was then multiplied by 10. The results are reported in Fig. 6. For small scales, the *outwards* algorithm outperforms the *inwards* algorithm. However, as the scale of the experiment is increased, the directed growth of the *inwards* algorithm is increasingly advantageous. In addition, with the *outwards* algorithm, some missions do not reach their targets in the allotted time limits when higher redundancy factor is employed. This is due to the increased interference that too many useless branches create in robot navigation. This effect is not prominent in the *inwards* algorithm because the robots are added to the tree only when it is necessary.

2) Disconnected Time: We studied the ability to maintain connectivity by considering the following metrics: (i) The disconnected time ratio, defined as the number of time steps (over the total experiment time) with at least a broken edge in the tree; (ii) The Fiedler value time ratio, defined as the number of time steps (over the total experiment time) with swarm-wide Fiedler value lower than $10^{-3}$. The results are reported in Fig. 7. In small-scale scenarios, in only two experiments out of 50 have positive disconnected time, and the global communication graph always stays connected. In medium-scale scenarios, larger numbers of redundant robots cause occasional line-of-sight obstructions that delay messages exchanges, but connectivity is generally maintained throughout the duration of the experiment. In large-scale scenarios, the disruptive effect of a large number of redundant robots is prominent for both algorithms. With fewer robots, the inwards algorithm is capable of maintaining global connectivity in all of the experiments, despite occasional breaking of tree edges (in less than 5% of the experiments).

C. Real-Robot Validation

To validate the simulated results simulations, we tested our algorithms with 9 Khepera IV robots. A Vicon motion capture system was used to track the position and orientation of the robots throughout the duration of the experiments, and to simulate situated communication. We employed 2 experimental scenarios: (i) 2 targets on a circle with a radius of 2.3 meters at approximately 180 degrees from each other; (ii) 3 targets on a circle with a radius of 1.6 meters at approximately 120 degrees from each other. We rescaled the distance-related parameters in Table I to fit the arena and accommodate for the small number of robots involved. We repeated these experiments 15 times for setup (i) and 10 times for setup (ii) with robots starting from the same positions and orientations for better comparison. We also performed the same experiments in simulation, with the same initial positions.

Fig. 8 shows that real-robot and simulated experiments follow analogous trends. In particular, we verified that for small-scale experiments with low redundancy factor (in these experiments it was set to 1) the *outwards* algorithm has better performance than the *inwards* algorithm.

V. RELATED WORK

Extensive literature exists on methods for connectivity preservation. Several recent works consist of motion control laws that include an estimate of the Fielder value. Yang *et al.* [9] introduced a decentralized algorithm to estimate the Fiedler value and use it to maintain connectivity while moving towards a target location. This algorithm was later refined by Sabattini *et al.* [10] and Williams *et al.* [11]. Further extensions include inter-robot collision avoidance [12] and multi-target exploration [6]. The main advantage of this family of approaches is that they allow navigation with arbitrary topologies. However, accurate decentralized computation of the Fiedler value is not easy in realistic settings in which messages might be lost due to communication interference [13]. In addition, computing the Fielder value in a decentralized manner involves network-wide power iteration methods [14], the slow convergence of which makes them suitable only for small teams of robots [11], [15]. It should also be noted that all of the above algorithms, with the exception of [12], have only been tested in simulation.

A second family of methods select a communication subgraph and aim to preserve its edges through some form of global consensus. Hsieh *et al.* [16] devised a reactive control law based on radio signal and bandwidth estimation, in which links between robots can be activated and deactivated as the topology changes over time. Michael *et al.* [17] employed distributed consensus and auctions algorithms to establish which links to activate and deactivate over time. Cornejo *et al.* [18], [19] proposed a distributed algorithm for link selection in which the robots undergo a number...
of motion rounds, during which the selected links must be preserved. Being based on achieving global consensus before any topology modification can be finalized, these algorithms are not scalable and work best when teams involve a small number of robots.

A third class of connectivity-preserving algorithms assumes that a certain structure is pre-existing. The dynamic structure is some form of logical tree, dynamically built and updated over the physical links of the robot network. Our work falls into this category. Krupke et al. [20] employed a Steiner tree as a pre-existing structure, and use spring-like virtual forces to balance connectivity and cohesiveness while reaching distant targets. A number of works, which constitute our main source of inspiration, utilized minimum spanning trees as structures to preserve. Aragues et al. [5] focused on a distributed coverage strategy with connectivity constraints, and proposed a method based on maintaining a network-wide minimum spanning tree. Analogously, Sooleymani et al. [21] proposed a distributed approach that constructs and preserves a network-wide minimum spanning tree, allowing for tree switching. Schuresko et al. [22] studied a theoretical approach for distributed and robust switching between minimum spanning trees. All these works were only demonstrated in numerical simulations. The main advantage of these methods is the ease and speed with which spanning trees can be built and updated in a distributed manner. However, as discussed in this paper, spanning trees do not scale well with the number of robots involved.

VI. CONCLUSIONS

In this paper, we presented two algorithms to construct a long-range communication backbone that connects multiple distant target locations. The algorithms are decentralized and
based on the idea of constructing a logical tree over the set of physical network links.

Through extensive experimentation, both in simulation and with real robots, we assessed the performance of the algorithms according to various experimental conditions. Our results show that, in small-scale scenarios, \textit{outwards} tree growth, corresponding to spanning tree formation, is a viable approach. However, as the scale of the environment and the number of robots involved increase, a more directed, \textit{inwards} growth from target locations towards the tree root, is a preferable approach.

Our results also show that, as the number of unnecessary robots increases, the benefit of redundancy is voided by the increased physical interherence in navigation. While a better spare robot strategy could diminish this phenomenon, our results suggest that a more progressive approach to deployment might be a better idea.

Nonetheless, the presence of a reasonable number of spare robots offers the opportunity to tackle the problem of maintaining \textit{persistent} long-range global connectivity despite individual limitations in the energy supply of individual robots. We plan to consider this scenario in future research.

In addition, possible extensions of our work include the presence of moving targets, rather than static ones, and the presence of obstacles in the environment.

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