Cosmic acceleration driven by mirage inhomogeneities

Christophe Galfard\(^1\), Cristiano Germani\(^1\) and Alex Kehagias\(^2\)

\(^1\) DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK
\(^2\) Physics Division, National Technical University of Athens, 15780 Zografou Campus, Athens, Greece

E-mail: C.Galfard@damtp.cam.ac.uk, C.Germani@damtp.cam.ac.uk and kehagias@central.ntua.gr

Received 24 December 2005, in final form 30 January 2006
Published 28 February 2006
Online at stacks.iop.org/CQG/23/1999

Abstract

A cosmological model based on an inhomogeneous D3-brane moving in an AdS\(_5\) \(\times\) S\(_5\) bulk is introduced. Although there are no special points in the bulk, the brane universe has a centre and is isotropic around it. The model has an accelerating expansion and its effective cosmological constant is inversely proportional to the distance from the centre, giving a possible geometrical origin for the smallness of a present-day cosmological constant. Besides, if our model is considered as an alternative of early-time acceleration, it is shown that the early stage accelerating phase ends in a dust-dominated FRW homogeneous universe. Mirage-driven acceleration thus provides a dark matter component for the brane universe final state. We finally show that the model fulfils the current constraints on inhomogeneities.

PACS numbers: 04.50.+h, 11.25.Wx, 98.80.–k

1. Introduction

The motion of a D-brane in some gravitational background follows a classical geodesic, so its induced metric is time dependent. The motion of the brane leads to a time-dependent geometry, which is interpreted as a cosmological evolution by a brane observer. This has been called mirage cosmology \([1]\) and technically, there are two steps in determining the resulting cosmological evolution. To start with, the brane motion is determined by solving the world-volume field equations, as follows from the Dirac–Born–Infield (DBI) action, for the scalar fields which describe the position of the brane in the bulk. Then, the induced metric on the brane is specified, now becoming an implicit function of time, giving a cosmological evolution for the brane. This cosmological evolution can be reinterpreted in terms of cosmological ‘mirage’ energy densities on the brane via a Friedman-like equation.
Various combinations of probe branes on simple backgrounds have been analysed [1], such as stacks of Dp-branes at and out of extremality (black Dp-branes), with and without additional constant antisymmetric background tensors. They provide a cosmological evolution on the probe brane which can be simulated by various types of mirage matter on the brane. There is no a priori reason for the mirage energy–momentum tensor to satisfy the energy conditions as it does not describe real matter on the brane. What drives the cosmological evolution is not some form of matter or energy, but the brane motion in the background geometry. Indeed, in general, for a Dp-brane moving in the background of other Dp'-branes, expansion on the world-volume of the probe brane was found to correspond to an equation of state for the mirage pressure and energy of the form $p = w\rho$ with $w > 1$, violated by all forms of matter. It should be stressed, however, that there are many exotic types of mirage matter including $w$ values that are outside the range $|w| \leq 1$ required by four-dimensional causality.

Another peculiarity of this framework is that individual densities of dilute mirage matter can be negative (without spoiling the overall positivity at late times). This is linked to the fact that in this type of cosmology, the initial singularity is an artefact of the low-energy description. This can be seen, for example, by studying brane motion in simple spaces like AdS$_5 \times S^5$ which are globally non-singular. The induced cosmological evolution of a brane moving in such a space has a typical expansion profile due to radiation and an initial singularity (from the four-dimensional point of view). At the initial singularity the brane universe joins a collection of parallel similar branes and there is (non-Abelian) symmetry enhancement. The effective field theory breaks down and this gives rise to the singularity.

The next obvious question is how ‘real’ matter/energy densities on the brane affect its geodesic motion and the induced cosmological evolution. This has been studied in the mirage-cosmology context by turning on electromagnetic energy on the brane. An important point here is that the cosmological evolution is not driven by four-dimensional gravity on the brane.

Motivated by the homogeneity of the D-brane backgrounds (see however [2]), a silent feature of the mirage-cosmology scenario is that the resulting brane cosmology is isotropic and homogeneous. This is due to the fact that the scalars which describe the position of the brane in the bulk geometry are taken to only be time dependent. This corresponds to a rigidly moving 3-brane in the bulk and, to our knowledge, on such backgrounds, the mirage matter alone cannot account for the acceleration of the universe’s expansion [3]. We here consider a brane moving in such a way that its motion depends not only on time but also on some other space directions. In this case, the induced metric on the brane is neither isotropic nor homogeneous. Inhomogeneous cosmologies are well studied in the literature and it has been shown that the current data cannot rule them out (see [8] for a review). Moreover, it has been suggested in [9] that late-time cosmology cannot be approximately homogeneous. However, the recent proposals that inhomogeneities in a matter-dominated universe, on either super-horizon [6] or sub-horizon scales [7], may influence the expansion rate at late times have been criticized [7, 10–12]. In fact, it has been shown that sub-horizon [11] and super-horizon [7] perturbations [10, 12] are not viable candidates for explaining the accelerated expansion of the universe. This is simply because as long as energy conditions are valid, inhomogeneous models can never lead to accelerated expansion [12]. It should be noted that inhomogeneous cosmological models in the context of string theory have previously been discussed [14].

Here, however, we will show that in mirage cosmology, inhomogeneities may indeed lead to accelerated cosmic expansion either at early or late times. In particular, inhomogeneities compatible with SO(3) symmetry may lead to an accelerating expansion phase for a probe
D3-brane moving in the near horizon limit of a background of a stack of D3-branes. When this phase occurs at early times, it may provide a mechanism for inflation. In that case, exit of the inflation is achieved when inhomogeneities are diluted enough due to inflation. We will see that what remains of these inhomogeneities is similar to pressureless matter. Thus, in this scenario, mirage matter left over at the end of the inflationary phase may be a dark matter component. On the other hand, when the accelerating phase is at late times, mirage cosmology can provide an alternative to \( \Lambda \text{CDM} \).

It should also be noted that our model does not have any initial singularity so that it naturally fits the homogeneity of the CMB spectra. In fact, in the absence of big-bang-like singularities, all parts of the universe may have been in causal contact in the far past. Moreover, we also show that the observed small-scale homogeneities (in the range of 100–140 Mpc \([17]\)) can be implemented in our model and related to the best fit cosmological constant of \([5]\).

2. Set-up

In general, a probe D-brane light enough for its back-reaction to be neglected will follow a geodesic as it moves in some string-theory background. Due to its motion, the induced world-volume metric on the probe brane becomes a function of time so that, from the brane ‘residents’ point of view, a changing (expanding or contracting) universe is experienced. This is the basic idea behind mirage cosmology. The simplest case corresponds to a D3-brane moving in a static spherically symmetric gravitational background with a dilaton as well as a RR background \( C(r) = C_{0\ldots3}(r) \) with a self-dual field strength. The probe brane will in general start moving in this background along a geodesic and its dynamics is governed by the DBI action. In the case of maximal supersymmetry, the DBI action is

\[
S = T_3 \int d^4 \xi \, e^{-\phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + (2\pi \alpha') F_{\alpha\beta} - B_{\alpha\beta})} + T_3 \int d^4 \xi \, \hat{C}_4 + \text{anomaly terms},
\]

where world-volume fermions have been ignored and \( \xi^\mu (\mu = 0, 1, 2, 3) \) parametrize the D3-brane world-volume. The embedded data are given by

\[
\hat{G}_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta},
\]

etc. For simplicity we will assume that the \( B \)-field is zero and there is no electromagnetic field on the brane.

In particular, we will consider here a probe D3-brane moving in the near-horizon limit of a stack of D3-branes with metric

\[
ds^2 = \frac{r^2}{L^2} \left(-dr^2 + d\rho^2 + \rho^2 d\Omega_5^2\right) + \frac{L^2}{r^3} \, dr^2 + L^2 \, d\Omega_5^2,
\]

where we have chosen spherical coordinates for the brane directions. This is just an AdS\(_5\) \( \times \) S\(_5\) spacetime and \( L \) is the AdS\(_5\) length defined by \( L^4 = 4\pi g_s N \), where \( g_s \) is the string coupling and \( N \) is the D3-brane charge. The RR field is

\[
C_{0\ldots3} = \frac{r^4}{L^4}
\]

and the dilaton \( \phi \) is constant. We will regard our universe as a D3-brane moving in this background.

We now consider an inhomogeneous trajectory for the brane motion \( r = r(t, \rho) \). The case of the homogeneous trajectory has been studied before \([1]\). No matter is included on the
brane and only geometrical quantities related to the chosen embedding will play a role. The induced metric is then
\[ ds_{\text{ind}}^2 = -\left( \frac{r^2}{L^2} - \frac{r^2 L^2}{r^2} \right) dt^2 + \left( \frac{r^2}{L^2} + \frac{r^2 L^2}{r^2} \right) d\rho^2 + 2 \frac{r r'}{r^2} L^2 d\rho dt + \frac{r^2}{L^2} \rho^2 d\Omega^2, \] (5)
where \( r = r(t, \rho) \) and a dot denotes derivative with respect to time \((\dot{t} = \partial_t)\), whereas a prime denotes derivative with respect to the brane-radial distance \((\dot{\rho} = \partial_\rho)\).

Due to re-parametrization invariance, there is a gauge freedom, which may be fixed by choosing the static gauge, \( x^a = \xi^a \), so that in this gauge, the total Lagrangian is
\[ \mathcal{L} = \frac{r^4}{4\pi T_5} = \frac{r^4}{L^4} \rho^2 \sqrt{1 + \frac{r^2 L^4}{r^4} - \frac{r^2 L^4}{r^4} - \frac{r^4}{L^4}}. \] (6)
In the following we will consider the ‘non-relativistic’ approximations:
\[ \frac{r^2 L^4}{r^4} \ll 1, \quad \frac{r^2 L^4}{r^4} \ll 1. \] (7)
In this case the field equations \( \delta \mathcal{L} = 0 \) reduce to
\[ \ddot{r} = \frac{1}{\rho^2} \dot{\rho}(\rho^2 r') = 0. \] (8)
As the bulk is regular, the only possible brane singularity occurs when the brane shrinks to a point, i.e. when \( r = 0 \). The most general solution without singularity is
\[ r = \frac{\mu}{2} (t - t_0)^2 + \frac{\mu}{6} \rho^2 + r_0, \] (9)
as one can check by calculating all the curvature invariants. Here, \( r_0(>0) \) corresponds to the position of the centre of the brane at some fiducial time \( t_0 \) and \( \mu \) is a mass parameter. We also would like to stress that this solution does not have a homogeneous limit, and thus belongs to a new branch of solutions.

Note that the above discussion is valid in the non-relativistic approximation (7). Using (9), it is easy to verify that a sufficient condition for the validity of this approximation is
\[ \mu \ll \frac{r_0^3}{L^2}. \] (10)

3. Accelerating universe

In this section, we will show that the expansion rate of geodesic congruences is accelerated with a natural ending, the end state corresponding to pressureless mirage matter. Depending on when mirage effects dominate conventional Einstein gravity, inhomogeneous mirage matter may lead to either early- or late-time cosmological acceleration. In particular, it may provide an alternative to inflation, or to the late-time cosmic expansion we observe today. We may employ the present model for inflationary cosmology if mirage matter is dominating at early times. Then, not only our universe is accelerated at early times, but also it does not have a cosmological singularity, avoiding the existence of a particle horizon. In our model, in fact, all parts of the universe are connected to each other by a timelike curve and are separated by a finite amount of proper time. This does not occur for non-inflating singular cosmologies like the FRW model. In our case, therefore, the homogeneity of the CMB spectra is natural.
3.1. Light-cone acceleration

To simplify the problem, we shall approximate the induced metric (5) by

$$ds^2 \simeq \frac{r^2}{L^2} (d\eta^2),$$  \hspace{1cm} (11)

where $d\eta^2$ is the four-dimensional Minkowski metric. For our purpose, (11) will be a good approximation to (5) if the relative discrepancy of the Christoffel and the second variations of the determinant of the metrics are very small compared to 1.\footnote{i.e. if ‘i’ corresponds to the induced metric (5) and ‘c’ to the conformal one (11), we want $\frac{\sqrt{\det g} - \sqrt{\det c}}{\sqrt{\det c}} \ll 1$ and, similarly, we require that $\frac{\partial^2 \sqrt{\det g}}{\partial \sqrt{\det c}^2} \ll 1$, where $\det g$ is the determinant of the corresponding metric.} One can check that these conditions are met assuming the non-relativistic approximation (10). The 4-velocity of a null radial geodesic in the metric (11) is then easily found to be

$$k^\mu = \left[ \frac{F(\rho + t)}{(\mu \rho^2 + 3 \mu (t - t_0)^2 + 6r_0)^2}, \frac{F(\rho + t)}{(\mu \rho^2 + 3 \mu (t - t_0)^2 + 6r_0)^2}, 0, 0 \right],$$  \hspace{1cm} (12)

where $F(\rho + t)$ is a non-determined function that depends on the parametrization chosen for the null geodesic.

Defining the expansion scalar as $\theta = \nabla_\alpha k^\alpha$, the acceleration parameter $Q_n$ of a null-congruence is \[13\]

$$Q_n = k^\alpha \nabla_\alpha \theta + \frac{1}{2} \theta^2 = 96 \mu^2 F(\rho + t)^2 \left[ \frac{t - (t_0) + \frac{r_0}{\mu(t - t_0)}}{[\rho^2 + 3(t - t_0)^2 + 6r_0]^6} \right].$$  \hspace{1cm} (13)

We recall that for a null geodesic vector $k^\alpha$, the following geometrical condition holds:

$$k^\alpha \nabla_\alpha \theta + \frac{1}{2} \theta^2 = -R_{\alpha\beta} k^\alpha k^\beta + \omega_{\alpha\beta},$$  \hspace{1cm} (14)

where the vorticity $\omega_{\alpha\beta}$ and shear $\sigma_{\alpha\beta}$ are defined as

$$\omega_{\alpha\beta} = \nabla_{[\alpha} k_{\beta]}, \hspace{1cm} \sigma_{\alpha\beta} = \hat{h}_{[\alpha} \hat{h}_{\beta]} \nabla_{[\alpha} k_{\beta]} - \frac{1}{2} \theta^2,$$  \hspace{1cm} (15)

in terms of the 2-metric $\hat{h}_{\alpha\beta}$, which satisfies $\hat{h}_{\alpha\beta} k^\alpha = 0$. One can show that the 4-velocity (12) has zero vorticity and shear for the lowest order induced metric (11). So in order to have acceleration ($Q_n > 0$), the null energy conditions $R_{\alpha\beta} k^\alpha k^\beta \geq 0$ must be violated. Indeed, for the metric (11), $R_{\alpha\beta} k^\alpha k^\beta = -Q_n$, so that when the universe’s light-cone expansion accelerates, the null energy conditions are violated. However, since no physical matter has really been used, there is no violation of any fundamental physical law. There is no reason to expect mirage matter to satisfy any energy condition, since it does not correspond to any form of real matter. This has also been stressed before \[1\], where mirage matter violating causality bounds was found to drive cosmological expansion.

It is now easy to see that the light cone is accelerating ($Q_n > 0$) for $\rho + \frac{r_0}{\mu(t - t_0)} > (t - t_0)$ and that the acceleration naturally ends when $\rho + \frac{r_0}{\mu(t - t_0)} = t - t_0$. Thus, the larger the radial distance, the longer the accelerating era.

We would like to stress that the acceleration is only indirectly due to inhomogeneities. In fact, as we showed, the only contribution to the Raychaudhuri equation (13) is coming from the negative mirage energy density of the brane inhomogeneities.
3.2. Timelike congruences

For a timelike congruence, the geodesic equation is \( u^\alpha \nabla_\alpha u^\beta = 0 \), where the vector \( u^\alpha \) is normalized,

\[
    u^\alpha u_\alpha = -1. \tag{16}
\]

For a radial motion, we have that \( u^\alpha = (u^t(t, \rho), u^\rho(t, \rho), 0, 0) \) and equation (16) is satisfied for

\[
    u^t = -\sqrt{\frac{L^2}{r^2} + (u^\rho)^2}. \tag{17}
\]

As it is difficult to solve the above geodesic equation, some approximations must be used to get analytical results. Let us consider the following quasi-homogeneous congruence:

\[
    u^\rho \approx \frac{\alpha}{r} \tag{18}
\]

so that the time component of \( u^\alpha \) can be approximated by

\[
    u^t = -\frac{L}{r} \left[ 1 + \frac{r^2}{2L^2} \right], \tag{19}
\]

The geodesic motion turns out to be given, to leading order, by a single equation for \( u^\rho \),

\[
    \dot{v} = \frac{r' L}{r^2}, \tag{20}
\]

and its solution with initial condition \( v(t = t_0, \rho) = 0 \) is

\[
    v = \frac{\alpha(t - t_0)}{2\beta[(t - t_0)^2 + \beta]} + \frac{\alpha}{2\beta^{3/2}} \arctan\left( \frac{t - t_0}{\beta^{1/2}} \right). \tag{21}
\]

The parameters \( \alpha, \beta \) are given by

\[
    \alpha = \frac{4\rho L}{3\mu}, \quad \beta = \frac{\rho^2}{3} + \frac{2r_0}{\mu}, \tag{22}
\]

in terms of which we may express the solution in equation (9) as

\[
    r = \frac{\mu}{2} ((t - t_0)^2 + \beta). \tag{23}
\]

Since, to us, the universe looks homogeneous, isotropic and accelerated, we may also consider our position as being very far from its centre. According to condition (10), we can consider \( \mu \) small and/or \( r_0 \) large so that \( \beta \) is large. In the \( \beta \gg (t - t_0)^2 \) limit we have

\[
    v = \frac{\alpha(t - t_0)}{\beta^2} + O\left( \frac{1}{\beta^3} \right) \tag{24}
\]

and condition (18) is fulfilled for

\[
    (t - t_0) \ll \frac{\rho}{2} + \frac{3r_0}{\mu \rho}. \tag{25}
\]

Considering units such that our position is at \( \rho > 1 \) and assuming \( t/t_0 > 1 \), we find that a sufficient condition for the validity of our approximation actually is

\[
    \beta \gg (t - t_0)^2. \tag{26}
\]

We may now calculate the acceleration parameter \( Q_t \). For timelike geodesics, it is

\[
    Q_t = \dot{\theta} + \frac{1}{2} \theta^2.
\]
In the large $\beta$ limit, we find that $Q_t$ is given by

$$Q_t = \frac{32 L^2}{3 \mu^2 \beta^3} + \mathcal{O} \left(\frac{1}{\beta^2}\right).$$  (27)

As for null geodesics, we find that these timelike geodesic congruences are accelerating ($Q_t > 0$). Considering our position in the universe to be $\rho = \rho_0 \gg \sqrt{6r_0/\mu}$, we have

$$Q_t \simeq \frac{288 L^2}{\mu^2 \rho_0^6} = \Lambda,$$  (28)

where $\Lambda$ is the cosmological constant as measured today. Thus, our relative position with respect to the centre is

$$\rho_0 \simeq \left(\frac{288 L^2}{\mu^2 \Lambda}\right)^{1/6}$$  (29)

and, therefore, the measured small cosmological constant is in principle compatible with a large distance of our position with respect to the centre of the universe. It should be stressed that a fine-tuning of $Q_t$ is needed in order to match the observed acceleration. However, in this case the smallness of the latter is geometrical and due to the large distance of our position from the centre. Still there is a fine-tuning as we have to choose the parameters $\mu_0, L$, which nevertheless is not better than the usual fine-tuning of the cosmological constant.

Clearly, the present model is not relevant for the cosmological constant problem arising from the contribution of supersymmetry breaking, phase transitions, etc in the vacuum energy. As usual, we assume that a mechanism exists which neutralizes all these vacuum-energy sources leaving only the mirage contribution to account for the observed acceleration.

One may also verify that the vorticity $\omega_{\alpha\beta}$ and the shear $\sigma_{\alpha\beta}$ given in terms of the three metric $h_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$ by

$$\omega_{\alpha\beta} = \nabla_[\alpha] u_\beta, \quad \sigma_{\alpha\beta} = h^\mu_\alpha h^\nu_\beta \nabla_\mu u_\nu - \frac{1}{3} \theta^2$$  (30)

vanish to lower order and

$$\omega^2 = \omega_{\alpha\beta} \omega^{\alpha\beta} = \mathcal{O} \left(\frac{1}{\beta^2}\right), \quad \sigma^2 = \sigma_{\alpha\beta} \sigma^{\alpha\beta} = \mathcal{O} \left(\frac{1}{\beta^2}\right).$$  (31)

In addition, a short calculation of the energy conditions reveals that

$$R_{\alpha\beta} u^\alpha u^\beta = -\frac{32 L^2}{3 \mu^2 \beta^3} + \mathcal{O} \left(\frac{1}{\beta^2}\right)$$  (32)

and, clearly, the strong energy conditions $R_{\alpha\beta} u^\alpha u^\beta \geq 0$ are violated, while the expansion accelerates. For the timelike geodesic congruence we are discussing, the Raychaudhuri equation is satisfied,

$$\dot{\theta} + \frac{1}{3} \theta^2 = -R_{\alpha\beta} u^\alpha u^\beta + \omega^2 - \sigma^2.$$  (33)

A comparison to the FRW homogeneous model can easily be made by approximating the induced metric (11) by

$$ds^2 \simeq \left((t - t_0)^2/2 + \rho_0^2/6 + r_0\right)^2 \left(-dt^2 + d\rho^2 + \rho^2 d\Omega^2_2\right).$$  (34)

around some fixed value of the radial distance $\rho = \rho_0$ with $\rho = \rho_0 + \varphi$ and $(t - t_0) \gg \varphi$. Then in ‘synchronous time’ $d\tau = \left((t - t_0)^2/2 + \rho_0^2/6 + r_0\right) dt$ one can rewrite the metric as a FRW-like one,

$$ds^2 \simeq -d\tau^2 + a(\tau)^2 \left(d\varphi^2 + \varphi^2 d\Omega^2_2\right).$$  (35)
Figure 1. ‘FRW-like’ expansion $Q = \ddot{a}/a$. Units of AdS length, i.e. $L = 1$. Decreasing curves close to $\tau = 0$ correspond to $\rho = 10, 200, 220, 240$. Values used, $t_0 = 0$, $r_0 = 1$ and $\mu = 10^{-4}$.

The expression for $a(\tau)$ is rather involved so instead of giving its explicit form, figure 1 plots the acceleration parameter $\ddot{a}/a = \partial_\tau \theta + 1/3\theta^2$, where $\theta$ is calculated for timelike geodesic observers. At large proper time, the asymptotic expansion of the acceleration is

$$\frac{\ddot{a}}{a} \sim -\frac{2}{9} \tau^{-2},$$

which is the acceleration of a dust-dominated universe in a FRW background, as we shall now show. Considered as a model for inflation, our model has certain advantages. In particular, a mirage-driven inflationary phase naturally ends when inhomogeneities are diluted enough by the inflation. Indeed, when $\mu t^3$ dominates $\mu \rho^2 + r_0$ in $r$, the metric becomes approximately homogeneous,

$$\text{d}s^2 \simeq -\text{d}\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{4/3} \text{d}x_3^2,$$

(37)

where $\tau_0 = 3 \sqrt{2L/\mu}$. At late time the spacetime looks like FRW with a scale factor $a(\tau) \sim \tau^{2/3}$, as expected in a dust-dominated universe. Moreover, the inhomogeneous mirage acceleration provides a dark matter component left over from an early-time inflationary phase.
4. Homogeneity bounds

Mirage inhomogeneities may also account for the observed accelerating expansion of the universe deduced from type Ia supernova observations and CMB WMAP data [5]. In this case, it would be enough for us to be at a distance from the centre such that \( t > t_0 + \frac{v_0}{c \mu(t-t_0)} \) to have acceleration of the light cone, thus explaining the observed cosmic acceleration. However, there are constraints coming from the homogeneity bounds of the universe.

The first one is on the homogeneity of the CMB spectra. In standard cosmology, an initial big-bang-like singularity prevents very distant objects from having been in causal contact in the far past. In the present model, however, there is no initial singularity. It is thus allowed to consider a homogeneous radiation put on top of an inhomogeneous spacetime. Our model should be compared to tests on homogeneity at small scales. In particular, the Luminous Red Galaxies (LRG) data [15] of the Sloan Digital Sky Survey [16] can be used as a measure of the homogeneity of the universe. The analysis suggests that the universe is homogeneous for comoving distances of the order of \( R \sim 70 h^{-1} \) Mpc. It actually reveals that our universe is homogeneous from around 100 Mpc up to distances of 140 Mpc [17, 18]. This means that within this shell, the number of luminous objects in a given volume \( V \) of a box of size \( \ell \) scales like \( \ell^3 \). Possible inhomogeneities are thus restricted and a comparison of the present model with observational data is welcome. So far, the model considered was empty of real matter. We consider the galaxies to be dust and would like to see that the mirage inhomogeneities are not inconsistent with the observed homogeneity of the universe. In effect, since for dust the energy contained in a given volume will grow with the volume, we can roughly say that the number of objects in the volume is proportional to the volume itself. Therefore, our universe will look homogeneous if the 3-volume scales like the distance to the cube up to 140 Mpc. For this, we need to write down the induced metric (11) in comoving coordinates.

This is not an easy task but we may find an approximate answer. First we need to find the coordinate transformation from \((t, \rho, \theta, \phi)\) to \((\tau, \rho, \theta, \phi)\) such that the 4-velocity \( u^\alpha = -\delta^\alpha_\tau \), where \( \tau \) is the proper time. So we impose

\[
-1 = u^\tau = t u^t + \tau u^\rho, \tag{39}
\]

and

\[
0 = u^\rho \equiv R u^\rho + R' u^\rho. \tag{40}
\]

As before we will use the large-\( \beta \) approximation. In particular, since we are interested in our position in the universe, we will consider \( \beta \sim \rho^2 / 3 \) in (22).

Equations (39, 40) are solved by

\[
R = \frac{\mu}{3} (t - t_0)^2 + \frac{\mu}{6} \rho^2 \simeq \frac{\mu}{6} \rho^2, \quad \tau = \frac{2}{3L} (t - t_0) \left[ R + \frac{\rho^2 \mu}{12} \right], \tag{41}
\]

which can be inverted, giving

\[
\rho = \sqrt{\frac{6R}{\mu}}, \quad t = L \frac{\tau}{R} + t_0. \tag{42}
\]

In \((\tau, R, \theta, \phi)\) coordinates, the metric is

\[
g_{\tau \tau} = -1, \quad g_{RR} = g_{\rho \rho} \left[ (\partial_R \rho)^2 - (\partial_R t)^2 \right] \simeq \frac{3}{2} \frac{R}{\mu L^2}, \tag{43}
\]

\[
g_{RT} = g_{\rho t} \left[ \partial_R t \partial_t \rho - \partial_R \rho \partial_t \rho \right] \simeq \frac{\tau}{R},
\]
so that the line element reads
\[ ds^2 = -d\tau^2 + \frac{3}{2\mu L^2} dR^2 + 2\frac{\tau}{R} dR d\tau + \frac{6R^3}{\mu L^2} d\Omega_2^2. \] (44)

In order to calculate the 3-volume in a box around a given point, we introduce a new radial variable \( \xi \) so that the circumference of the circles in \( d\Omega_2^2 \) is \( 2\pi \xi \):
\[ \xi = \sqrt{\frac{6}{\mu L^2} R^{3/2}} \] (45)
in terms of which (44) becomes
\[ ds^2 = -d\tau^2 + \frac{1}{9} d\xi^2 + \frac{4\tau}{3\xi^2} d\xi d\tau + \xi^2 d\Omega_2^2. \] (46)

To calculate the volume of a box centred at our position it is useful to use ‘Cartesian’ coordinates so that \( d\xi^2 + \frac{1}{9} d\xi^2 \) becomes \( d^2 x + d^2 y + d^2 z \) and \( \xi^2 = x^2 + y^2 + z^2 \). The line element becomes
\[ ds^2 = -d\tau^2 + \left(1 - \frac{8x^2}{9\xi^2}\right) dx^2 + \left(1 - \frac{8y^2}{9\xi^2}\right) dy^2 + \left(1 - \frac{8z^2}{9\xi^2}\right) dz^2 + \frac{4\tau}{3\xi^2} dx d\tau \]
\[ + \frac{4\tau}{3\xi^2} dy d\tau - \frac{16}{9} dx dy - \frac{16}{9} dx dz - \frac{16}{9} dy dz. \] (47)

For simplicity, we consider our spatial position in the universe to be at the point \((x_0, x_0, x_0)\), where, as discussed before, \( x_0 \) is large with respect to any other length considered. There is nothing particular about this point and it has been chosen for computational simplicity only. The volume of a box of coordinate side size \( 2x_1 \), centred at \((x_0, x_0, x_0)\), can now be estimated.

We first need to find the proper length of the box. The proper length of the side of the box is given by
\[ \ell = \int_{-x_1}^{x_1} v_\alpha dx^\alpha, \] (48)
where the 4-vector \( v_\alpha \) is tangent to one of the box’s edges and satisfies \( v_\alpha u^\alpha = 0 \) and \( u_\alpha u^\alpha = 1 \).

The volume \( V \) of the box for large \( x_0 \) is then
\[ V \propto \ell^3 \left(1 - \frac{8}{\sqrt{33} x_0}\right) = \ell^3 \left(1 - \frac{4\ell}{\sqrt[4]{11} \sqrt[3]{8} \sqrt[6]{\Lambda}}\right), \] (49)
where tracing back the coordinate transformations, we used
\[ x_0 = \frac{\mu \rho_0^3}{6\sqrt{3} L} \simeq \frac{2}{3} \sqrt[3]{\Lambda}^{-1/2}, \] (50)
where we used (29) for the last step.

To compare with observations we take \( \Lambda^{-1/2} = 2.6 \times 10^8 \) Mpc [5]. We check that the fractal dimension of our universe up to 140 Mpc away from the observer, and with a cut off at the scale of inhomogeneities taken to be at 100 Mpc [17], is within the observed homogeneity bound. Denoting the 100 Mpc cut-off values for the volume and length by \( V_c \) and \( l_c \), respectively, the fractal dimension \( D \) we are looking for is defined by
\[ D = \frac{\log V_c}{\log l_c}, \] (51)
which we find to be \( D = 3 \pm 0.03 \), i.e. within the observed bound [17].
5. Conclusions

In this paper, we introduced an inhomogeneous type of solution of a model of a D3-brane moving in an AdS$_5 \times S^5$ bulk. We assumed that all matter contributions are sub-dominant to the geometrical mirage energy contribution. Our brane is inhomogeneous and although from the bulk viewpoint there is no special point, a centre of the universe must be introduced. This centre appears due to the inhomogeneity, it is not singular and the universe is isotropic around this point. The cosmological evolution does not have an initial singularity and a cosmological singularity. This prevents us from having a horizon problem. We found that the brane is initially accelerated with increasing magnitude as the distance from the centre decreases. The acceleration naturally ends in a FRW-like dust-dominated universe at late time as the distance from the centre becomes large. The mirage matter can then be thought of as a component of the observed dark matter.

We may also consider this model for explaining the present-day observed cosmological acceleration. In this case, the cosmological constant can be understood as being geometrical, since it is inversely proportional to the distance to the centre of the universe. A small cosmological constant simply means that our position is far from the centre.

It is possible that the introduction of matter can connect the early- and late-time accelerations with a decelerated era driven by matter. However, to explain how this mechanism might occur is beyond the scope of the present paper.

A potential problem in these types of models is that objects around us (like galaxies) look homogeneously distributed. Analysis of observational data suggests that we live in a homogeneous universe up to a scale of $\sim$140 Mpc [17, 18] (below 100 Mpc, the inhomogeneities correspond to the coarse graining of the energy density). This constrains the allowed inhomogeneity for a late-time mirage acceleration and does not rule it out. It is also interesting to note that the observed cosmological constant introduces a scale at which the inhomogeneities of our model could in principle be observed. More precisely, our model predicts inhomogeneities at scales just above the observationally observable ones. It is intriguing that the very small inhomogeneities of the CMB spectra from the WMAP data are actually orthogonal to the ecliptic plane [21]. It has been suggested that this phenomenon can be explained by introducing inhomogeneities in the universe [19]. However, we need to test our model against the full CMB spectrum. This could probably be done by considering the effective four-dimensional Einstein equations obtained by rewriting the geodesic equations of the brane in terms of standard general relativity with matter sources. In this case, if these sources can be re-interpreted as coupled scalar fields, at least under some approximations, it might be possible to compare our model with standard cosmology using the techniques developed in [20] for the study of the CMB power spectrum and bispectrum on inhomogeneous background geometries sourced by coupled scalar fields.$^5$ This is, however, postponed for future research.

Our model does not consider matter contributions. In order to be tested against BBN nucleosynthesis it must be implemented with matter. Since early-time matter and BBN constraints only deal with radiation, one could imagine a consistent model introducing a small contribution of Maxwell fields on the brane and then study perturbations on it. We believe this will not spoil the acceleration as the Maxwell fields will always be sub-dominant. However, this has to be tested with numerical calculations and is therefore beyond the scope of this paper. An additional constraint would be to check our model against the scale invariant perturbation observed in the CMB, although it is not clear what a perturbation would mean in this case as there is no homogeneous background. It should be noted that another point, connected with the

$^5$ We thank the referee for pointing this out to us.
brane-matter contributions, which should be checked is the flatness of the universe. However, as long as we have not introduced any matter we cannot answer this since an estimate of the total energy density versus the critical one is lacking in this case.

Acknowledgments

C Germani and C Galfard would like to thank Misao Sasaki and Akihiro Ishibashi for useful discussions about physical observers, and Oisín MacConamhna for comments on supergravity approximations. C Germani would like to thank Pierstefano Corasaniti and Bartjan VanTent for useful discussions on observational cosmology. C Germani was supported by PPARC research grant PPA/P/S/2002/00208. This work was co-funded by the European Social Fund (75%) and National Resources (25%)—(EPEAK-B)-PYTHAGORAS.

References

[1] Kehagias A and Kiritsis E 1999 *J. High Energy Phys.* JHEP11(1999)022 (Preprint hep-th/9910174)
[2] Horowitz G T and Maeda K 2002 *Phys. Rev.* D *65* 104028 (Preprint hep-th/0201241)
[3] Apostolopoulos P S and Tetradis N 2005 *Phys. Rev.* D *71* 0435006 (Preprint hep-th/0412246)
[4] Papantonopoulou E and Zamaras V 2004 *J. High Energy Phys.* JHEP10(2004)051 (Preprint hep-th/0408227)
[5] Spergel D N et al (WMAP Collaboration) 2003 *Astrophys. J. Suppl.* 148 175 (Preprint astro-ph/0302209)
[6] Kolb E W, Matsarese N, Notari A and Riotto A 2005 *Phys. Rev.* D *71* 023524 (Preprint hep-ph/0409038 (Preprint hep-th/0503117)
[7] Rasans S 2005 *Preprint* astro-ph/0504005
[8] Krasinski A 1998 *Preprint* gr-qc/9806039
[9] Notari A 2005 *Preprint* astro-ph/0503715
[10] Geshnizjani G, Chung D J H and Afshordi N 2005 *Phys. Rev.* D *72* 023517 (Preprint astro-ph/0503553)
[11] Flanagan E E 2005 *Phys. Rev.* D *71* 103521 (Preprint hep-th/0503202)
[12] Hirata C M and Seljak U 2005 *Phys. Rev.* D *72* 083501 (Preprint astro-ph/0503582)
[13] Alikes H, Amarguiont M and Gron O 2005 *Preprint* astro-ph/0506449
[14] Siegel E R and Fry J N 2005 *Astrophys. J.* 628 L1 (Preprint astro-ph/0504421)
[15] Giovannini M 2005 *Preprint* hep-th/0505222
[16] Giovannini M 2005 *J. Cosmol. Astropart. Phys.* JCAP09(2005)009 (Preprint astro-ph/0506715)
[17] Hawking S W and Ellis G F R 1973 *The Large Scale Structure of Space-Time* (Cambridge: Cambridge University Press)
[18] Joyce M, Labini F Sylos, Gabrielli A, Montuori M and Pietronero L 2005 *Astron. Astrophys.* 443 11 (Preprint astro-ph/0501583)
[19] Moffat J W 2005 *Preprint* astro-ph/0503526
[20] Moffat J W 2005 *Preprint* astro-ph/0504004
[21] Rigopoulos G I, Shellard E P S and Tent B W van 2005 *Preprint* astro-ph/0504508
[22] Land K and Magueijo J 2005 *Mon. Not. R. Astron. Soc.* 357 994 (Preprint astro-ph/0405519)
[23] Land K and Magueijo J 2005 *Mon. Not. R. Astron. Soc.* 362 L16 (Preprint astro-ph/0407081)
[24] Land K and Magueijo J 2005 *Mon. Not. R. Astron. Soc.* 357 994 (Preprint astro-ph/0405519)
[25] Land K and Magueijo J 2005 *Mon. Not. R. Astron. Soc.* 362 L16 (Preprint astro-ph/0407081)
[26] Copi C J, Huterer D, Schwarz D J and Starkman G D 2005 *Preprint* astro-ph/0508047