LEARNING FROM DISPERSIVE EFFECTS IN THE NUCLEON POLARISABILITIES

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Static nucleon polarisabilities gauge the stiffness of the nucleon against an external electro-magnetic field, parameterising the part of the real Compton amplitude at zero energy which is not explained by the pole terms, i.e. by the successive interactions of two photons with a point-like nucleon of anomalous magnetic moment $\kappa$. Dynamical nucleon polarisabilities are the energy dependent generalisation to real Compton scattering and thus provide more information about the low energy effective degrees of freedom inside the nucleon. Here, I sketch their definition and interpretation in terms of the low energy degrees of freedom, referring to \cite{1,2} for particulars and references.

Investigating in more detail the nucleon polarisabilities at non-zero photon energy, one first subtracts the “nucleon pole” effects from the real Compton amplitude $T$, writing $\bar{T}(\omega, z) = \bar{A}_1(\omega, z) (\vec{\epsilon}'^* \cdot \vec{\epsilon}) + \bar{A}_2(\omega, z) (\vec{\epsilon}'^* \cdot \vec{K}) (\vec{\epsilon} \cdot \vec{K}')$ + ... for the two spin-independent structure amplitudes in the centre of mass (cm) frame, with the non-Born part already subtracted. $\omega$ is the cm energy of an incident (outgoing) real photon with momentum $\vec{k}$ ($\vec{k}'$) and polarisation $\vec{\epsilon}'$ ($\vec{\epsilon}$), scattering under the cm angle $\theta$ off the nucleon, with $z = \cos \theta$.

At fixed real photon energy, the structure dependent part of the amplitude is then analysed by expanding it in multipoles. In terms of the first four electric and magnetic (dipole and quadrupole) polarisabilities of definite multipolarity, $\alpha_{E1}(\omega), \beta_{M1}(\omega), \alpha_{E2}(\omega)$ and $\beta_{M2}(\omega)$, the amplitudes read

\begin{align}
\bar{A}_1(\omega, z) &= \frac{4\pi W}{M} \left[ (\alpha_{E1}(\omega) + z\beta_{M1}(\omega)) \omega^2 + \frac{1}{12} (z\alpha_{E2}(\omega) + (2z^2 - 1)\beta_{M2}(\omega)) \omega^4 + \ldots \right] \\
\bar{A}_2(\omega, z) &= -\frac{4\pi W}{M} \left[ \beta_{M1}(\omega) \omega^2 + \frac{1}{12} (-\alpha_{E2}(\omega) + 2z\beta_{M2}(\omega)) \omega^4 + \ldots \right],
\end{align}

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where \( W = \omega + \sqrt{M^2 + \omega^2} \) is the total cm energy and \( M \) the nucleon mass. The multipolarities are dis-entangled by their angular dependence. The prefactors are chosen such that at zero photon energy, the definitions of the static polarisabilities are recovered, e.g.:
\[
\bar{\alpha}_E = \alpha_E(\omega = 0), \quad \bar{\beta}_M = \beta_M(\omega = 0).
\]

Dynamical polarisabilities test the temporal response of the global, low energy excitation spectrum of the nucleon at non-zero energy. They therefore contain information about dispersive effects induced by internal relaxation mechanisms, baryonic resonances and meson production thresholds of the nucleon. This is also clearly seen in Figs. where the result of a dispersion theory analysis of the four leading dynamical polarisabilities is compared to a chiral effective field theory describing the nucleon at low energies, called Modified Small Scale Expansion MSSE. The latter is a rigorous, model-independent and systematic approach to low energy QCD. It incorporates the nucleon, the \( \Delta(1232) \) and their respective pion clouds as explicit low energy degrees of freedom, see Fig. for the dominant contributions. Short distance physics

\( \text{Figure 1. The diagrams contributing at leading one loop order in MSSE. Graphs obtained by permuting vertices or external lines are not displayed. Double line: } \Delta(1232). \)

is sub-sumed into local counter terms whose strengths are naïvely given by dimensional analysis to be of the order of \( |\delta\alpha_{E1}, \delta\beta_{M1}| \sim 1 - 4 \) which would make them higher order effects. However, fitting these two free parameters to reproduce the static values of the dipole polarisabilities shows that they are indeed much larger, \( \delta\alpha_{E1} \approx -5, \delta\beta_{M1} \approx -10 \), i.e. of leading order in accord with the MSSE power counting. It is at this point that the new dimension of the dynamical polarisabilities comes into play by allowing for a parameter-free prediction of the energy dependence after the zero energy value is fixed.

For example, a large dia-magnetic but only very weakly energy dependent contribution is needed in the magnetic dipole polarisability to cancel the well known large para-magnetism coming from the strong \( M1 \rightarrow M1 \) transition between nucleon and \( \Delta \) which clearly rules this channel. As the shape of

\( \text{All dipole polarisabilities here given in the “natural units” of } 10^{-4} \text{ fm}^3. \)
\( \beta_{M1}(\omega) \) is dominated by the \( \Delta \) already below the pion production threshold \( \omega_\pi \), the good agreement between its experimentally measured value at zero energy \( (\beta_{M1}(0) = 1.5) \) and the result of Heavy Baryon Chiral Perturbation Theory HB\( \chi \)PT with only pions and nucleons as low energy degrees of freedom \( (\beta_{M1}^{\text{HB\( \chi \)PT}}(0) = 1.2) \) can be seen as accidental. The energy dependence of the dynamical polarisabilities demonstrates therefore that the underlying physics mimicked at zero energy by the counter term is in a large range insensitive to derailed dynamics at short distances. No genuinely new degrees of freedom are missed at low energies. As the \( \Delta \) has no width at leading order in MSSE, the result for \( \beta_{M1}(\omega) \) diverges close to the \( \Delta \) resonance energy \( \omega_\Delta \). For this reason, the imaginary part of \( \beta_{M1}(\omega) \) above the one pion production threshold is also ill reproduced in MSSE. Dispersion theory shows that it is clearly dominated by the non-zero width of the \( \Delta(1232) \). The cusp strength and structure at \( \omega_\pi \) is well captured in MSSE, and very well so for \( \alpha_{E1}(\omega) \).

Thanks to the contribution from the pion cloud around the \( \Delta \), the agreement for \( \alpha_{E2}(\omega) \) at low energies is good. Not surprisingly, there is no residue of the famed \( E2 \rightarrow E2 \) transition due to its smallness. No counter term is used or necessary in this channel to account for short distance physics. A nucleon resonance might be seen in the discrepancy between the DR and chiral calculations at very high momenta \( \omega > 250 \text{ MeV} \).

Albeit there is some improvement by adding the \( \Delta \pi \) continuum, nearly half of the static strength of \( \beta_{M2}(\omega) \) is still missing. Introducing a counter term to reproduce the correct static value, as was done for the dipole polarisabilities but is for the quadrupole polarisabilities inconsistent with the MSSE power counting, is of no help in the magnetic quadrupole polarisability because the slope of the dispersion theory result is much steeper than the one obtained in MSSE. Not even the physics of the pion production threshold seems to be reproduced correctly.

This analysis is an overall success for MSSE: In the régime at low energies where it is supposed to work, the agreement is surprisingly good, and the deviation around \( \omega_\Delta \) is clearly related to the fact that the “natural” power counting must be modified in order to account for the non-zero width of a resonance by summing a sub-class of diagrams.

Dynamical polarisabilities do not contain more or less information than the corresponding Compton scattering amplitudes, but as with any multipole decomposition, the facts are more readily accessible and easier to interpret. Stringent constraints for models and model-independent power countings of a low energy effective field theory of the nucleon follow from their analysis.
References

1. H. W. Grießhammer and T. R. Hemmert: Phys. Rev. C65, 045207.
2. R. Hildebrandt, H. W. Grießhammer, T. R. Hemmert and B. Pasquini, forthcoming.
Figure 2. Comparing the Dispersion Relations results (thick solid line) of the real (top four) and imaginary (bottom four) parts of the iso-scalar dynamical electric and magnetic dipole and quadrupole polarisabilities with the leading one loop order MSSE prediction. Dashed: HB$\chi$PT prediction (nucleons and pions only); dotted: $\Delta$ pole contribution added; dot-dashed: pion cloud around $\Delta$ added; thin solid: total MSSE result (counter terms for the dipole polarisabilities fixed to the static values of the dipole polarisabilities). For the imaginary parts, the MSSE and HB$\chi$PT predictions are identical at this order.