Triple helix vs. skyrmion lattice in two-dimensional non-centrosymmetric magnets

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It is commonly assumed that a lattice of skyrmions, emerging in two-dimensional non-centrosymmetric magnets in external magnetic fields, can be represented as a sum of three magnetic helices. In order to test this assumption we compare two approaches to a description of regular skyrmion structure. We construct (i) a lattice of Belavin-Polyakov-like skyrmions within the stereographic projection method, and (ii) a deformed triple helix defined with the use of elliptic functions. The estimates for the energy density and magnetic profiles show that these two ansatzes are nearly identical at zero temperature for intermediate magnetic fields. However at higher magnetic fields, near the transition to topologically trivial uniform phase, the stereographic projection method is preferable, particularly, for the description of disordered skyrmion liquid phase. We suggest to explore the intensities of the secondary Bragg peaks to obtain the additional information about the magnetic profile of individual skyrmions. We estimate these intensities to be several percents of the main Bragg peak at high magnetic fields.

I. INTRODUCTION

Topologically protected states of matter attract the attention of researchers from various fields of science. One of the well-known example of topologically protected objects are skyrmions. Despite the fact that the first appearance of skyrmions is associated with particle physics1–3, the study of magnetic skyrmions has become a rapidly developing field of condensed matter physics over the last decade. Most discussed magnetic skyrmions are nanoscale vortex-like configurations. A relatively small size of skyrmions makes them promising objects for the developing of new types of data storage devices.4,5

According to the Hobart-Derrick theorem6,7 topological arguments alone are not enough to stabilize skyrmions, while additional conditions are needed to fix a skyrmion size. Usually, a single skyrmion or a unordered set of skyrmions can be stabilized in a finite sample: a disc or a track (nanoribbon).8 In this case, the stability of skyrmions is provided by the dipole-dipole interaction and surface effects. For an infinite system the stabilization of skyrmions is achieved in non-centrosymmetric magnets, where the combination of the Dzyaloshinskii-Moriya interaction (DMI)9 and an applied magnetic field lead to an existence of long-period modulated magnetic phases, so that single skyrmions appear as elements of a so-called skyrmion crystal (SkX).10 Probably the best studied class of non-centrosymmetric magnets is $B20$ compounds, including MnSi, etc.11

Experimental investigations of such compounds show that the skyrmion phase in the bulk (also called as A-phase) exists at finite temperatures, slightly below the critical one, $T_c$. Thermal fluctuations are expected to play a crucial role in the stability of A-phase.11 This phase is observed at moderate magnetic fields, with its phase boundary far away from the critical (saturation) field. The intensity maps of neutron scattering experiments show a hexagonal pattern of Bragg peaks in the A-phase region. It allows to interpret the A-phase spin configuration in two ways: either as a hexagonal skyrmion superlattice or as a sum of three simple helices with wave-vectors directed at an angle of 120 degrees relative to each other.11 These two descriptions are not equivalent and may be distinguished in experiments, but the corresponding difference may be hidden by the experimental specifics and thermal modulation of the local magnetization.12 The latter reason makes thin films investigations more preferable, where the A-phase is more stable and exists at $T \approx 1 K$.11

It is known that the correspondence between long-period modulated phases (like a helix) and phases with a finite soliton density may be exact. One such example happens in one spatial dimension, where skyrmions are kinks in the sine-Gordon model.11,19 A one-dimensional magnet with uniaxial anisotropy, DMI and an external field is described by the sine-Gordon model with the Lifshitz invariants. This model has been exactly solved by Dzyaloshinskii as a modified helical configuration in terms of Jacobi elliptic functions.19 As an alternative (dual) description of this solution, one can consider a lattice of kinks.17–19

The two-dimensional case is more difficult for modeling. Due to non-linearity, the triple helix anzatz as a sum of three helices is not an exact solution for the ground state at $T = 0$. Moreover, one can propose several ways to construct a “triple helix” configuration. The simplest way, usually found in literature (see, e.g.11,12), is a sum of ordinary (non-modified) helices.21

Recently we showed22 that the stereographic projection method provides very good estimate of the ground state energy, while the shape of the individual skyrmions remains nearly invariant under pressure from its neighbors. The advantage of the latter method is its flexibility what concerns the the positions and sizes of individual skyrmions. One can particularly employ this way of description for the skyrmion liquid state reported in23 at some magnetic fields.

In this paper, we examine different descriptions of
skyrmion lattice state in two dimensions at zero temperature. In Section IV, we describe the stereographic approach for the skyrmion crystal construction. In Section V, we remind a general form of the magnetic helix for systems with DMI and magnetic field in terms of the additional elliptic parameter. With this generalization, we construct the triple helix ansatz in section VI at $T = 0$ with normalization conditions for the local magnetization. In Section VII we compare the modeling by Skyrmion crystal and triple helix with respect to density of classical energy, the period of the spatial modulation, and intensities of higher-order Bragg peaks. Our final remarks are presented in Section VIII.

II. SKYRMION CRYSTAL

We consider the two-dimensional system characterized by magnetization $\mathbf{S}(\mathbf{r})$. At zero temperature the magnetization is saturated and can be normalized, $S^2 = 1$. The classical energy density

$$\mathcal{E} = \frac{1}{2}C\partial_{\mu}S^i\partial_{\mu}S^i - D\epsilon_{\mu ij}S^i\partial_{\mu}S^j + B(1 - S^3),$$

where $\mu = 1, 2$ and $i = 1, 2, 3$. The first term corresponds to the ferromagnetic exchange, the second one is DMI, and the last one is the Zeeman energy related to an external magnetic field. One can check in the latter case that one skyrmion corresponds to

$$\bar{\kappa} = \bar{\kappa}(y)$$

and antiholomorphic function is a solution of

$$f(\bar{z}) = 0,$$

with the phase $\alpha$ is eventually determined by the sign of DMI, and a singularity-free function $\kappa(z\bar{z})$ depends smoothly on the distance from the skyrmion’s center.

The equation for $\kappa$ is quite nonlinear and can be solved only numerically. Since $\kappa$ has the dimension of length, we choose to consider a dimensionless function $\tilde{\kappa}(y) = (\kappa(0))^{-1} \kappa(y\kappa(0)^2)$ with the property $\tilde{\kappa}(0) = 1$. One could then solve the equation for $\tilde{\kappa}(y)$ for different boundary conditions. Our primary interest is to find $\tilde{\kappa}(y)$ on a disc of finite radius which mimics the case of SkX where one skyrmion is surrounded by its neighbors. The pressure exerted by this type of environment is modeled by changing the size of a disc. We found that the function $\tilde{\kappa}(y)$ is nearly invariant against changes of disc radius, in contrast to the value of the dimensionless residue, $\kappa(0)/L$. One hence can model multi-skyrmion configurations by the sum

$$f(z, \bar{z}) = \sum_f F\left(\frac{|z - z_0|^2}{z_0^2}\right),$$

where

$$F\left(\frac{z}{z_0}\right) \equiv \frac{z_0}{z} \tilde{\kappa}_\infty\left(\frac{z}{z_0}\right)^2,$$

with $\kappa_\infty$ is the solution on the disc of infinite radius, and $|z_0|$ in this formula is the skyrmion’s size. Both DMI and a magnetic field bring characteristic scales into the model, that results in the interaction between skyrmions, which is the main difference between the model (II) and BP model. This interaction should be taken into account in calculation of the energy density for SkX. Because of strong non-linear effects of the model, the interaction between skyrmions includes not only the pairwise (repulsive) interaction, defined for two skyrmions as $U_2(z_0, a) = \mathcal{E}[f_1 + f_2] - \mathcal{E}[f_1] - \mathcal{E}[f_2]$. We observed that the triple interaction, $U_3$, is also significant (attractive) and should be considered when discussing the stabilization of SkX.

The formula (5) allows us to consider the most interesting case of densely packed SkX with the hexagonal arrangement of skyrmions. The energy per unit cell in this case is given by

$$E_{cell}(z_0, a) = \mathcal{E}[f_1] + 3U_2(z_0, a) + U_3(z_0, a).$$

Eq. (7) has two parameters: the unit cell parameter of SkX, $a$, and the radius of a skyrmion, $|z_0|$. In this case we choose the most interesting case of densely packed SkX with the hexagonal arrangement of skyrmions. The energy per unit cell in this case is given by

$$E_{cell}(z_0, a) = \mathcal{E}[f_1] + 3U_2(z_0, a) + U_3(z_0, a).$$

In Fig. 2 and compare it with the triple helix configuration.
III. SINGLE HELIX

The well known expression for single helix configuration in magnets with DMI is given by:
\[ S = \hat{c}\cos \alpha + \hat{b}\cos (kR + \beta) + \hat{a}\sin (kR + \beta) \sin \alpha, \quad (8) \]
where \( \hat{a}, \hat{b}, \hat{c} \) are unit vectors with \( \hat{a} = \hat{b} \times \hat{c} \), \( k \) is the helix propagation vector and \( \alpha \) is the cone angle. Eq. (8) is the starting point for analysis of all helical states: conical, cycloidal, etc. The main question of such an analysis is the choice of \( k, \alpha, \hat{a}, \hat{b}, \hat{c} \), the values of \( k \) and \( \theta \). All these parameters are determined by particular form of the Hamiltonian, crystal symmetries, etc.

We are interested in the 2D spatial case, so \( k \) lies in a plane. Parametrizing the basis as \( \hat{a} = (-\sin \varphi, \cos \varphi, 0) \), \( \hat{b} = (-\sin \theta \cos \varphi, \cos \theta \sin \varphi, \sin \theta) \) and \( \hat{c} = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta) \), one can show for arbitrary DMI, that in the 2D case the vector \( \hat{c} \) lies in a plane, \( \theta = \pi/2 \), and the cone angle collapses, \( \alpha = \pi/2 \). It means that the spin configuration becomes
\[ S_\varphi = \begin{pmatrix} \sin \varphi \sin (k_\varphi R + \beta) \\ -\cos \varphi \sin (k_\varphi R + \beta) \\ \cos (k_\varphi R + \beta) \end{pmatrix}. \quad (9) \]
The angle \( \varphi \) defines the plane of magnetization rotation and in turn determines the direction of \( k_\varphi \) for particular form of DMI. In this paper we use the relation
\[ k_\varphi = k(\cos \varphi, \sin \varphi, 0), \]
appropriate for our 2D model \([1]\). This case is realized in the case of cubic symmetry of crystal (B20 compounds for example), where Dzyaloshinskii vector is parallel to bonds. Different types of crystal symmetries could lead to different forms of DMI, and the relation between \( k_\varphi \) and \( \varphi \) could be different.

Actually in the presence of an external magnetic field perpendicular to the plane, Eq. (6) is not exact solution of the model \([10]\). The well-known fact is that in uni-axial magnets with DMI the simple helix also transforms to the chiral soliton lattice (CSL) \([2]\). If spins are modulated over \( \hat{x} \) direction and lie in the perpendicular plane, \( S = (0, \sin (\phi(x) + \beta), \cos (\phi(x) + \beta)) \), then the energy \([2]\) takes the form
\[ E = \frac{1}{2} (\partial_x \phi(x))^2 - \partial_x \phi(x) + b(1 - \cos (\phi(x) + \beta)) \], \quad (10) \]
with the resulting Euler-Lagrange equation:
\[ \partial_x^2 \phi(x) = -b \sin (\phi(x) + \beta). \quad (11) \]
this is the sine-Gordon equation having the quasi-periodic solution:
\[ \phi_0(x) = 2 \text{ am} \left( \sqrt{b \pi m} \left| m \right| \right) - \beta \quad (12) \]
with the elliptic parameter \( m \).

We expect that in the presence of additional small terms in (10) the solution (12) might be no longer exact. In this case we can still use (12) as a general model form of deformed helix with one control parameter, \( m \):
\[ S_\varphi = \begin{pmatrix} \sin \varphi \sin \left( 2 \text{ am} \left( \frac{K(m)}{\pi} k_\varphi R \right) m + \beta \right) \\ -\cos \varphi \sin \left( 2 \text{ am} \left( \frac{K(m)}{\pi} k_\varphi R \right) m + \beta \right) \\ \cos \left( 2 \text{ am} \left( \frac{K(m)}{\pi} k_\varphi R \right) m + \beta \right) \end{pmatrix}. \quad (13) \]
where \( K(m) \) is a complete elliptic integral of the first kind. This expression is the extension of Eq. (9) with the same spatial period, and additional “degree of ellipticity”. It coincides with Eq. (9) at \( m = 0 \).

IV. TRIPLE HELIX

In the literature one can find a statement that SkX state can be modeled by the sum of three helices with zero sum of helix propagation vectors. In particular, it was argued\([11]\) that thermal fluctuations stabilize the superposition of three helices at high temperatures in three-dimensional case. Moreover it has been shown in\([12]\) that second Bragg peaks in neutron scattering can be mostly attributed to the result of double scattering, and they have insignificant intensities in comparison with the first Bragg peaks.

The simple sum of three helices \([9]\):
\[ S_{3q} = S_{\varphi=0} + S_{\varphi=2\pi/3} + S_{\varphi=4\pi/3} + S_0 \hat{e}_z \quad (14) \]
has a different magnitude from point to point, i.e. \( |S_{3q}(R)| \neq \text{const} \). For the A-phase of 3D compounds, the possibility of this variation can be explained by a closeness to the critical point where the magnitude of magnetization could vary significantly. But in the planar case of our interest at \( T = 0 \), one should expect the constraint \( |S| = 1 \). Below we consider several ways to obtain the normalized triple helix configuration.

A. Triple helix in the stereographic projection method

As discussed above, the stereographic projection automatically provides the low-temperature normalization constraint \( |S| = 1 \), which is convenient for a discussion of multi-skyrmion configurations. It is tempting to use the method also for construction of a multiple-helix configuration.

One can easily verify that the single helix \([9]\) is represented by the function:
\[ f_\varphi = i e^{i \varphi} \tan \left( \frac{k_\varphi R + \beta}{2} \right). \quad (15) \]
This function has a striped structure of zeros and poles lines. At first glance, the sum of three helices of the form (15) with different $k_\varphi$ (and $\beta$) appears to be a good choice for description of two-dimensional lattice of skyrmions. One observes that (i) a sum of two functions $f_{3\varphi} = f_{\varphi=4\pi/3} + f_{\varphi=2\pi/3}$ corresponds to a lattice with rhomboid primitive cell and (ii) arbitrary $\beta$ in (15) corresponds to simply shifting the origin, $R = 0$. The singularity lines of this configuration are shown by black lines in Fig. 1 and dotted lines correspond to $|f| = 1$, i.e. to places where magnetization lies in a plane. The addition of the third helix, i.e. considering

$$f_{3\varphi} = f_{\varphi=0} + f_{\varphi=4\pi/3} + f_{\varphi=2\pi/3}$$

makes the choice of $\beta$ not harmless, as can be seen in Fig. 1. Depending on $\beta = \pi$ or $\beta = 0$, two different configurations of singularity lines appear, corresponding to different topological charge $Q$ per rhombic unit cell: for the triangular case with $Q = 2$, and for the kagomé case with $Q = 3$.

Our calculation shows also that this way for the construction of the triple helix leads to the higher energy density, as compared both to the SkX ansatz from Sec. II and to the “triple helix” considered in the next subsection. Therefore, we do not discuss Eq. (16) in the rest of the paper.

### B. Normalized sum of three deformed helices

As discussed in Sec. II a magnetic field deforms a helix configuration into the more optimal configuration, called as a deformed helix or chiral soliton lattice, Eq. (13). It seems then only natural to use a more general combination of three such deformed helices (13) instead of simple expression (14). To be able to compare the energies of different configurations, we should normalize the resulting magnetization :

$$\hat{S}_{3\varphi} = \frac{S_{\varphi=0} + S_{\varphi=4\pi/3} + S_{\varphi=2\pi/3}}{S_{\varphi=0} + S_{\varphi=4\pi/3} + S_{\varphi=2\pi/3}}$$

We call this expression the deformed triple helix (DTH) below.

The expression (17) has three variational parameters for energy minimization: a pitch of helices $k$, the elliptical parameter $m$ and the additional magnetization perpendicular to the plane $S_0$. In terms of the resulting SkX structure, the pitch $k$ defines the cell parameter of SkX, while both $m$ and $S_0$ determine the radius and shape of individual skyrmions. The energy density found for such an optimal configuration from Eq.(2) is plotted as a function of magnetic field in Fig. 2. In this Figure we show also the energy found for SkX ansatz (5) and for the single deformed helix (13) with optimal parameters. It is seen that at a low external magnetic field $b_{cr1} \lesssim 0.25$ CSL configuration (13) is energetically favorable, and SkX is advantageous in the intermediate region $b \in (0.25, 0.8)$. In its turn, SkX is destroyed by a magnetic field at $b_{cr2} \approx 0.8$, when the uniform configuration delivers the energy minimum. This calculation is in a good agreement with previous work [10,19].

We also note here that the energy difference, $\delta \rho$, between configuration (17) with $m = 0$ and the one with optimal value of $m$ is not significant, it is $\delta \rho \approx 0.005$ at smaller $b \approx 0.3$ while $\delta \rho$ tends to zero near $b \approx 0.75$.

### V. COMPARISON OF THE MODELS

We observe in Fig. 2 that the difference in two descriptions, in terms of SkX and deformed triple helix, becomes essential in the region of relatively strong magnetic fields. More details can be found in analysis of the optimal modulation vector for SkX and DTH, corresponding to inverse unit cell parameter of SkX, $(4\pi/a\sqrt{3})$, and the pitch, $k$, respectively. The results are presented in Fig. 3 it is seen that the DTH solution becomes increasingly different from SkX in the region of high magnetic fields, $b \in (0.6, 0.8)$. In this region, the SkX with increasing unit cell parameter is eventually described as a rarified...
gas of weakly interacting skyrmions, and a dissolution or melting of SkX happens at the critical field \( b = b_c \). At the same time, the DTH model predicts nearly the same value of helical pitch up to \( b \approx 0.73 \) when the uniform ferromagnetic (FM) state becomes lower in energy. Considering the density of topological charge \( p = k^2 \sqrt{3}/8\pi^2 \) as an order parameter in the skyrmion phase, one can say that the transition to the FM state in the DTH model corresponds to \( p \) abruptly changing to zero. It is instructive to compare this conclusion with SkX ansatz (16), where the energy of two skyrmions placed at the distance \( R \) from each other behave\(^{23} \) as \( E_2 \approx 2x + A\exp(-R/\ell) \), with \( x \approx b - b_{c2} \), correlation length in the FM state \( \ell = b^{-1/2} \) and \( A \approx 1 \). Minimization of the energy density, \( \sim (x + 3A\exp(-R/\ell))/R^2 \) with respect to \( R \) leads to \( p \) depicted in Fig. 2. It also leads to the dependence of topological charge \( p \sim (\ln(A/|x|))^{-2} \) and the pitch \( k \sim (\ell \ln(A/|x|))^{-1} \) in the vicinity of \( b = b_{c2} \). We show the fit by the latter dependence in Fig. 3 by the red dashed line. The dependence of \( p \) on \( b \) near \( b_{c2} \) looks qualitatively the same and we do not show it here.

Note that Fig. 2 indicates the transitions from SkX phase to helical and FM states at \( b_{c1} = 0.25 \) and \( b_{c2} = 0.8 \), respectively. According to the recent findings in\(^{23} \), additional transitions from skyrmion-solid to skyrmion-hexatic and later to skyrmion-liquid phases happen at intermediate fields in thin films of Cu$_2$OSeO$_3$ compound. If we associate the upper critical field found in\(^{23} \) at low temperatures with \( b_{c2} \), then we obtain the values for the additional transitions to be \( b = 0.54 \) and \( b = 0.64 \), respectively. Comparing these numbers with our Fig. 2 we see that deviations between our DTH and SkX description happen at higher fields, which correspond to skyrmion-liquid phase in terms of Ref.\(^{23} \). We saw that SkX modelling\(^{23} \) provided a better description at higher fields in terms of energy. We point out an additional advantage of this description in the anticipated skyrmion-liquid phase, because the SkX modelling with Eq. (15) does not require a long-range ordering in positions of skyrmions, in contrast to DTH and other regular helical structures.

### A. Elastic cross-section

The simple formula with a linear combination of three helices\(^{14} \) contains only six spatial Fourier harmonics, i.e. only six peaks in the reciprocal space at \( k = (3\phi/\pi, 0, \ldots, 5) \). This is what observed experimentally in high-temperature A-phase in bulk material\(^{11,12} \). But as we discussed above, at low temperature for thin films we should think about normalization of magnetization, and elliptical deformations\(^{17} \) should also contain higher harmonics, \( k_{x2} + k_{x2} \).

The cross-section of the elastic unpolarized neutron scattering on a magnetic structure is given by\(^{27} \)

\[
\frac{d\sigma}{d\Omega} \propto \sum_{ij} (\delta^{ij} - \hat{q}^i \hat{q}^j) \langle S^i_{\mathbf{q}} \rangle \langle S^j_{-\mathbf{q}} \rangle, \tag{18}
\]

![Figure 3. Optimal value of modulation vector for “Triple helix” and SkX for different values of \( b \). The red dashed line is the fit of SkX values of \( k \) as described in text.](image)

with \( \langle S^i_{\mathbf{q}} \rangle = \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \langle S^i(\mathbf{r}) \rangle \). For periodic structures, such as SkX and DTH one can represent the cross-section as a sum over reciprocal lattice vectors:

\[
\frac{d\sigma}{d\Omega} \propto C_0 + \sum_{m,n} C_{mn} \delta(m \mathbf{b}_1 - n \mathbf{b}_2), \tag{19}
\]

here \( \mathbf{b}_1 = k_{x0} = 0 \), \( \mathbf{b}_2 = k_{x0} = \pi/3 \) and

\[
C_{mn} = \sum_{ij} \left( \delta^{ij} - \frac{(m \mathbf{b}_1 + n \mathbf{b}_2)(m \mathbf{b}_1 + n \mathbf{b}_2)}{|m \mathbf{b}_1 + n \mathbf{b}_2|^2} \right) \times \langle S^i_{m \mathbf{b}_1 + n \mathbf{b}_2} \rangle \langle S^j_{-m \mathbf{b}_1 - n \mathbf{b}_2} \rangle. \tag{20}
\]

We are interested in relative values of intensities of higher-order Bragg peaks, \( C_{mn}/C_{10} \). In our models we find that the magnitude \( C_{mn} \) rapidly decreases with \( m, n \) so that only \( C_{11}/C_{10} \) and \( C_{20}/C_{10} \) are of order of few percents, while the other coefficients are even smaller in the whole range of magnetic field. The results of the calculation for different models of our spin texture are shown in Fig. 3. It can be seen in this plot that for magnetic fields in the range \( 0.3 < b < 0.6 \), where SkX and DTH ansatzes yield practically the same energy density, both these models give similar results for \( C_{ij}/C_{10} \). This indicates that the spin configuration described by these two approaches is nearly identical.

The situation changes in the region of higher magnetic field \( 0.65 < b < 0.8 \), when DTH ansatz fails to reproduce the expected increase in distance between skyrmions. We note that for well-separated skyrmions of certain shape within SkX description (6) the magnitude of the higher peaks \( C_{11}, C_{20} \) is defined roughly by the Fourier image of an individual skyrmion, \( \langle S^i_{\mathbf{q}} \rangle \) taken at \( \mathbf{q} = \mathbf{b}_1 + \mathbf{b}_2 \), \( \mathbf{q} = 2 \mathbf{b}_1 \), respectively. DTH ansatz, on the contrary, describes somewhat deformed triple helix even at fields \( b \approx b_{cr2} \) with insignificant admixture of higher harmonics. As a result, we see in Fig. 3 that the values of \( C_{11}/C_{10} \) and
$C_{20}/C_{10}$ predicted by SkX approach are much larger than for DTH near the melting transition, $b \approx b_{cr2}$.

According to\textsuperscript{23} (see also\textsuperscript{28}) the perfect skyrmion crystal is melted before undergoing to uniform ferromagnetic state at $b > b_{cr2}$. Our predictions for the ratio of amplitudes $C_{ij}/C_{10}$ should partly survive in the intermediate skyrmion liquid phase. Instead of the well-defined Bragg peaks one observes the concentric circles, corresponding to short range order in the isotropic state. The above intensities $C_{10}, C_{11}, C_{20}$ should then be associated with the integrated intensities near $|q| = k, |q| = k\sqrt{3}, |q| = 2k$, respectively.

At the same time, the above predictions for $C_{11}, C_{20}$ cannot be simply compared to Lorentz TEM results\textsuperscript{23} where the profile of skyrmions has been modelled by $\delta$-function, $\delta(r-r_j)$, as opposed to above Eqs.\textsuperscript{6, 17}.

VI. CONCLUSIONS

We considered two alternative approaches to construction of 2D skyrmion crystal at $T = 0$. The first one is the modification of the stereographic projection method used in the seminal paper\textsuperscript{25} for the pure $O(3)$ sigma model. The second approach is the generalization of the triple helix ansatz\textsuperscript{17}. The numerical analysis of the classical energy shows that the two approaches yield very close estimates at intermediate values of an external magnetic field, $b$, but are different at lower and higher magnetic fields, close to critical fields characterizing the transitions either to single helix or to uniform ferromagnetic phase. In perhaps more interesting region of higher magnetic fields, the distance between skyrmions grows, whereas the size of each skyrmion decreases, so that the description in terms of a set of skyrmions becomes more appropriate as compared to the sum of three helices.

In conclusion, the description in terms of a set of individual skyrmions is more adequate near the transition to the topologically trivial uniform state in 2D materials. An investigation of the secondary Bragg reflexes in the skyrmion state can give the additional information about the magnetic profile of individual skyrmions.
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