Randomly Interacting Bosons, Mean-Fields and $L = 0$ Ground States

R. Bijker, A. Frank

1ICN-UNAM, AP 70-543, 04510 México, DF, México
2CCF-UNAM, AP 139-B, Cuernavaca, Morelos, México

Random interactions are used to investigate to what extent the low-lying behavior of even-even nuclei depend on particular nucleon-nucleon interactions. The surprising results that were obtained for the interacting boson model, i.e. the dominance of ground states with $L = 0$ and the occurrence of both vibrational and rotational structure, are interpreted and explained in terms of a mean-field analysis.

1 Introduction

In empirical studies of medium and heavy even-even nuclei very regular features have been observed, such as the tripartite classification of nuclear structure into seniority, anharmonic vibrator and rotor regions. In each of these three regimes, the energy systematics is extremely robust, and the transitions between different regions occur very rapidly, typically with the addition or removal of only one or two pairs of nucleons. Traditionally, this regular behavior has been interpreted as a consequence of particular nucleon-nucleon interactions, such as an attractive pairing force in semimagic nuclei and an attractive neutron-proton quadrupole-quadrupole interaction for deformed nuclei.

It came as a surprise, therefore, that recent shell model studies of even-even nuclei with two-body random interactions (TBRE) displayed a marked statistical preference for ground states with $L = 0$, energy gaps, and other signals of ordered behavior. This work has sparked a large number of investigations, both in fermion systems and in boson systems, in order to further explore and to explain these remarkable and unexpected results.

These robust features suggest that there exists an underlying simplicity of low-energy nuclear structure never before appreciated. In this contribution, we discuss the results of a study of the systematics of collective levels in the framework of the interacting boson model (IBM) with random interactions, and present a possible explanation in terms of a mean-field analysis.

2 Randomly interacting bosons

In the IBM, collective excitations in nuclei are described in terms of a system of $N$ interacting monopole and quadrupole bosons. We consider the most
Figure 1: Percentages of ground states with $L = 0$ and $L = 2$ in the IBM with random one- and two-body interactions calculated exactly for 1,000 runs (solid lines) and in mean-field approximation (dashed lines).

general one- and two-body Hamiltonian. The two one-body and seven two-body matrix elements are chosen independently from a Gaussian distribution of random numbers with zero mean and width $\sigma$, such that the ensemble is invariant under orthogonal basis transformations. For each set of randomly generated one- and two-body matrix elements we calculate the entire energy spectrum and analyze the results.

In Fig. 1 we show the percentages of $L = 0$ and $L = 2$ ground states as a function of the total number of bosons $N$ (solid lines). We see a clear dominance of ground states with $L = 0$ with ~ 60-75%. Both for $L = 0$ and $L = 2$ there are large oscillations with $N$. For $N = 3n$ (a multiple of 3) we see an enhancement for $L = 0$ and a decrease for $L = 2$. The sum of the two hardly depends on the number of bosons.

For the cases with a $L = 0$ ground state, we show in Fig. 2 the probability distribution $P(R)$ of the energy ratio for $N = 16$ (solid line)

$$ R = \frac{|E_{4_1} - E_{0_1}|}{|E_{2_1} - E_{0_1}|}. $$

(1)

There are two very pronounced peaks, right at the vibrational value of $R = 2$ and at the rotational value of $R = 10/3$, a clear indication of the occurrence of vibrational and rotational structure. This has been confirmed by a simultaneous study of the quadrupole transitions between the levels.
These are surprising results in the sense that, according to the conventional ideas in the field, the occurrence of $L = 0$ ground states and the existence of vibrational and rotational bands are due to very specific forms of the interactions. The study of the IBM with random interactions seems to indicate that this may not be the entire story. However, the above results were obtained from numerical studies. It would be very interesting to gain a better understanding as to why this happens. What is the origin of the regular features which arise from random interactions? In this respect, there is a relevant quote by E.P. Wigner (as communicated to us by M. Moshinsky): ‘I am happy to learn that the computer understands the problem, but I would like to understand it too’.

A first attempt in this direction was made for the vibron model, which has many of the same qualitative features as the IBM, but has a much simpler mathematical structure. We showed that the emergence of regular features from the vibron model with random interactions is related to the existence of three different geometric shapes. In the next section, we carry out a similar mean-field study of the IBM.

3 Mean-field analysis

The connection between the IBM, potential energy surfaces, equilibrium configurations and geometric shapes, can be studied by means of coherent states.
The coherent state for the IBM can be written as a condensate of a deformed boson which is a superposition of a scalar and a quadrupole boson,

\[ |N, \alpha \rangle = \frac{1}{\sqrt{N!}} \left( \sqrt{1 - \alpha^2} s^d + \alpha d^d_0 \right)^N |0\rangle , \]

with \(-1 < \alpha \leq 1\). The potential energy surface is then given by the expectation value of the Hamiltonian in the coherent state

\[ E_N(\alpha) = \langle N, \alpha | H | N, \alpha \rangle = a_4 \alpha^4 + a_3 \alpha^3 \sqrt{1 - \alpha^2} + a_2 \alpha^2 + a_0 , \]

where the coefficients \(a_i\) are linear combinations of the parameters of the Hamiltonian which in turn are taken as independent random numbers.

For random interactions, we expect the trial wave function of Eq. (2) and the energy surface of Eq. (3) to provide information on the distribution of shapes that the model can acquire. The equilibrium configuration is characterized by the value of \(\alpha = \alpha_0\) for which the energy surface \(E_N(\alpha)\) has its minimum value. For a given Hamiltonian, the value of \(\alpha_0\) depends on the coefficients \(a_4, a_3\) and \(a_2\). The distribution of shapes for an ensemble of Hamiltonians then depends on the joint probability distribution \(P(a_4, a_3, a_2)\).

In this approximation, we find that there are only three possible equilibrium configurations:

- \(\alpha_0 = 0\): \(s\)-boson condensate. This corresponds to a spherical shape which can only have \(L = 0\).
- \(0 < \alpha_0 < 1\) or \(-1 < \alpha_0 < 0\): deformed condensate with prolate or oblate symmetry, respectively. A deformed shape corresponds to a rotational band with angular momenta \(L = 0, 2, \ldots, 2N\). The ordering of the energy levels is determined by the sign of the moment of inertia

\[ E_{rot} = \frac{1}{2I_5} L(L + 1) . \]

- \(\alpha_0 = 1\): \(d\)-boson condensate. The rotational structure of a \(d\)-boson condensate is more complicated. It is characterized by the labels \(\tau, n_\Delta\) and \(L\). The boson seniority \(\tau\) is given by \(\tau = 3n_\Delta + \lambda = N, N - 2, \ldots, 1\) or 0 for \(N\) odd or even, and the values of the angular momenta are \(L = \lambda, \lambda + 1, \ldots, 2\lambda - 2, 2\lambda\). In this case, the rotational excitation energies depend on two moments of inertia

\[ E_{rot} = \frac{1}{2I_5} \tau(\tau + 3) + \frac{1}{2I_3} L(L + 1) , \]

which are associated with the spontaneously broken three- and five-dimensional rotational symmetries of the \(d\)-boson condensate.
The probability that the ground state of each of these equilibrium configurations has $L = 0$ can be estimated by evaluating simultaneously the corresponding moments of inertia (e.g. with the Thouless-Valatin prescription).

In Fig. 1 we show the percentages of $L = 0$ and $L = 2$ ground states, as calculated in the mean-field analysis (dashed lines). A comparison with the exact results (solid lines) shows an excellent agreement. The spherical and deformed condensates contribute constant amounts of 39.4 % and 23.4 %, respectively, to the $L = 0$ ground state percentage. The oscillations observed for $L = 0$ are therefore entirely due to the rotational structure of the $d$-boson condensate. The $L = 2$ ground states arise completely from the $d$-boson condensate solution. In the mean-field analysis the ground state has $L = 0$ or $L = 2$ in $\sim 77$ % of the cases. For the remaining 23 % of the cases the ground state has the maximum value of the angular momentum $L = 2N$. This percentage is almost a constant and hardly depends on $N$, in agreement with the exact results.

In Fig. 2 we show the contribution of each of the equilibrium configurations to the probability distribution of the energy ratio of Eq. (1). We see that the spherical shape (dashed line) contributes almost exclusively to the peak at $R = 2$, and similarly the deformed shape (dotted line) to the peak at $R = 10/3$, which once again confirms the vibrational and rotational character of these maxima. For $N = 16$ the contribution of the $d$-boson condensate is small.

4 Summary and conclusions

In this contribution, we have studied the properties of low-lying collective levels in the IBM with random interactions. We addressed the origin of the regular features, that had been obtained before in numerical studies, in particular the dominance of $L = 0$ ground states and the occurrence of vibrational and rotational band structures.

It was shown that a mean-field analysis of the IBM with random interactions can account for all of these features. They are related to the existence of three different equilibrium configurations or geometric shapes: a spherical shape ($\sim 39$ %), a deformed shape ($\sim 36$ %) and a condensate of quadrupole bosons ($\sim 25$ %). Since the spherical shape only has $L = 0$, and the deformed shape in about two thirds of the cases, these two solutions account for $\sim 63$ % of $L = 0$ ground states. The oscillations observed for the $L = 0$ ground state percentage can be ascribed totally to the contribution of the $d$-boson condensate. Finally, we found a one-to-one correspondence between the peaks in the probability distribution for the energy ratio and the occurrence of the spherical and deformed equilibrium configurations.
In summary, the use of mean-field techniques allows one to associate different regions of the parameter space with geometric shapes. This method bypasses the diagonalization of thousands of matrices, and provides an explanation of all regular features that have been observed in studies of the IBM with random interactions.

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References

1. N.V. Zamfir, R.F. Casten and D.S. Brenner, Phys. Rev. Lett. 72, 3480 (1994).
2. C.W. Johnson, G.F. Bertsch and D.J. Dean, Phys. Rev. Lett. 80, 2749 (1998).
3. R. Bijker, A. Frank and S. Pittel, Phys. Rev. C 60, 021302 (1999).
4. C.W. Johnson, G.F. Bertsch, D.J. Dean and I. Talmi, Phys. Rev. C 61, 014311 (2000).
5. R. Bijker, A. Frank and S. Pittel, Rev. Mex. Fís. 46 S1, 47 (2000).
6. D. Mulhall, A. Voyla and V. Zelevinsky, Phys. Rev. Lett. 85, 4016 (2000).
7. Y.M. Zhao and A. Arima, RIKEN preprint, August 2000.
8. R. Bijker and A. Frank, Phys. Rev. Lett. 84, 420 (2000).
9. R. Bijker and A. Frank, Phys. Rev. C 62, 014303 (2000).
10. D. Kusnezov, N.V. Zamfir and R.F. Casten, Phys. Rev. Lett. 85, 1396 (2000).
11. D. Kusnezov, Phys. Rev. Lett. 85, 3773 (2000).
12. R. Bijker and A. Frank, Phys. Rev. Lett, in press.
13. F. Iachello and A. Arima, The interacting boson model (Cambridge University Press, 1987).
14. R. Bijker and A. Frank, preprint.
15. J.N. Ginocchio and M. Kirson, Phys. Rev. Lett. 44, 1744 (1980).
16. A.E.L. Dieperink, O. Scholten and F. Iachello, Phys. Rev. Lett. 44, 1747 (1980).