Quick-MESS: A Fast Statistical Tool for Exoplanet Imaging Surveys

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ABSTRACT. Several tools have been developed in the past few years for the statistical analysis of the exoplanet search surveys, mostly using a combination of Monte Carlo simulations or a Bayesian approach. Here we present Quick-MESS, a grid-based, non-Monte Carlo tool aimed to perform statistical analyses on results from direct imaging surveys, as well as help with the planning of these surveys. Quick-MESS uses the (expected) contrast curves for direct imaging surveys to assess for each target the probability that a planet of a given mass and semimajor axis can be detected. By using a grid-based approach, Quick-MESS is typically more than an order of magnitude faster than tools based on Monte Carlo sampling of the planet distribution. In addition, Quick-MESS is extremely flexible, enabling the study of a large range of parameter space for the mass and semimajor axes distributions without the need of resimulating the planet distribution. In order to show examples of the capabilities of Quick-MESS, we present the analysis of the Gemini Deep Planet Survey and the predictions for upcoming surveys with extreme-AO instruments.

1. INTRODUCTION

More than a decade of extensive searches have led to a sample of over 850 confirmed exoplanets1 and thousands more planet candidates (Batalha et al. 2012), almost entirely identified through indirect detection techniques. Such a large number of discoveries allows accurate statistical analyses to address questions related to the distribution of their properties, such as the mass, orbital period and eccentricity (Lineweaver & Grether 2003; Cumming et al. 2008), as well as the relevance of the host star characteristics (mass, metallicity) on the final frequency and distribution of planetary systems (see Fischer & Valenti 2005; Santos et al. 2004; Johnson et al. 2007). Since the most successful techniques (radial velocity and transit) have focused on the inner (≤5 AU) environment of main sequence solar-type stars, most of the available information on the frequency of planets concerns this phase space. Radial velocity (RV) surveys report that ∼19% of nearby Sun-like stars harbor planets within 20 AU (Cumming et al. 2008), and recent Kepler Space Telescope results indicate the frequency of stars with planets to be ∼30% (Borucki et al. 2011).

Since both the transit and the radial velocity techniques are biased towards planets in relatively close orbits, orbital separations larger than ∼5 AU are currently not well sampled. Direct imaging surveys, which are typically more sensitive to planets at larger orbital separations, can fill this gap. However, to predict the expected planet fractions, the distributions derived from the radial velocity and transit surveys need to be extrapolated.

Several tools have been developed in the past few years for the statistical analysis of direct imaging (DI) planet search surveys (see, e.g., Chauvin et al. 2010; Lafrenière et al. 2008; Nielsen et al. 2008; Bonavita et al. 2012), which try to assess at what confidence planet distributions obtained from RV surveys can be extrapolated to estimate the planet distributions to the orbital separations, where DI is more sensitive.

Bonavita et al. (2012) (hereafter, B12) describes the Multipurpose Exoplanet Simulation System (MESS), an IDL simulation code specifically designed to perform statistical analysis of direct imaging surveys.2

The MESS combines the properties of the target stars (mass, luminosity, distance, age, etc.) with the assumptions on the planet parameter distributions (PPD) to generate a synthetic planet population (SPP) and the measured/estimated detection limits of the instrument under scrutiny, to estimate the probability of detecting a companion. This is then used to validate the original assumption on the PPD, in light of the observed or predicted detection limits. One can then explore different sets of PPD, the goal being to constrain those that are compatible with the observations. In case of planned observations, this information can be used to determine which kinds of constraints on the PPD the new instrument will be able to provide, and, given a certain PPD, what the expected detection rate would be.

In this article, we present a novel way to perform statistical analyses of DI surveys, called Quick-MESS, where the standard Monte Carlo approach is replaced by grid-based sampling of the orbital parameters, which leads to a substantial reduction in the

1 See http://exoplanets.eu, 2013 January 4.

2 Available for download at www.messthecode.com.
required computational time. The main features of the code are described in § 2. In § 3, we present the two main applications of the code, and in § 4, we provide a summary.

2. QUICK-MESS

The Quick-MESS (hereafter QMESS) code presented here does not use a Monte Carlo approach, as used by most of the statistical tools used so far for the analysis of exoplanet surveys results (see, e.g., B12; Chauvin et al. 2010; Lafrenière et al. 2008). Instead, it takes a grid-based approach consisting of several steps that allow the determination of the probability of detecting a planet in the considered parameter space, given a set of assumptions on the distribution of the planet orbital parameter and the mass-luminosity function. The main steps of the code, explained in detail in § 2.1 and 2.2, can be summarized as follows:

1. Evaluate the distribution of planets with a certain normalized separation, s, as a function of the orbital parameters (as discussed in § 2.1) and integrate and normalize it to get f(s, e), the distribution of planets as a function of the eccentricity e and the normalized separation s, where s is defined as s = R/a with R equal to the physical separation and a equal to the planet’s semimajor axis.

2. Multiply f(s, e) for the eccentricity distribution f(e).

3. Use the planetary evolutionary models (e.g., Baraffe et al. 2003) to estimate for each target in the studied sample the minimum detectable planetary mass $M_{\text{lim}}$ as a function of the projected separation $\rho$, given the contrast limits of the instrument.

4. For each value of semimajor axis on an uniform grid, use the distance of the star to convert the normalized separation s into the physical projected separation $\rho$, thus obtaining f($\rho$, e) from f(s, e).

5. For each value of the planetary mass ($M_p$) over a uniform grid, find the range of projected separations where $M_p > M_{\text{lim}}$ and integrate $f(\rho, e)$ over this interval to obtain the distribution of detectable planets with a given value of mass and semimajor axis f($M_p$, a).

6. Finally, multiply f($M_p$, a) with the required mass and semimajor axis distributions [f($M_p$) and f(a), respectively] to obtain the detection probability map $g(M_p, a) = f(M_p, a)f(M_p)f(a)$.

7. In the case of a survey, integrate g($M_p$, a) over the whole range of masses and semimajor axis in order to obtain $P(M_{\text{min}} \leq M_p \leq M_{\text{max}}, a_{\text{min}} \leq a \leq a_{\text{max}})$, the probability of detecting a planet in the considered parameter space.

2.1. Evaluation of the Projection Probability

The distribution of planets that, given a certain combination of orbital parameters, can be found at a certain position on their projected orbit is assessed by calculating the orbit of the planet in normalized separation $s(\phi) = R(\phi)/a$ (where $a$ is the semimajor axis of the orbit, and $R$ is the radius vector that, together with the true anomaly $v$, gives the polar coordinates of the planet on the orbit) as a function of the orbital phase $\varphi$ over a finely sampled grid of $0 \leq \phi \leq 1$ and for a range of eccentricities $0 \leq e < 1$. For a given instant in time $t$, with $T_0$ being the time of periastron passage and $p$ being the orbital period of the planet, the orbital phase $\varphi$ is defined as $\phi = \frac{t - T_0}{p}$, which is used to calculate the mean anomaly $M$ as in equation (1). The eccentric anomaly $E$ is then calculated using equation (2), where $E_0$ and $M_0$ are introduced as approximations of $E$ and $M$ and the whole calculation is repeated until the final result is stable (see Heintz 1978). Finally, the true anomaly $v$ is evaluated using equation (3):

$$M = \left(\frac{t - T_0}{p}\right)2\pi = 2\pi\phi = E - e \sin E \quad (1)$$

$$E_0 = M + e \sin M + \frac{e^2}{2} \sin 2M$$

$$M_0 = E_0 - e \sin E_0$$

$$E = E_0 + (M - M_0)/(1 - e \cos E_0) \quad (2)$$

$$\tan v/2 = \sqrt{(1 + e)/(1 - e)} \tan E/2. \quad (3)$$

Given $\nu$, we can define the normalized separation $s$:

$$s = R/a = \cos(v + \omega) \sec(\theta - \Omega). \quad (4)$$

In equation (4), $\omega$ is the argument of periapsis, $\Omega$ is the longitude of the node, and the angle $\theta$ together with the projected separation $\rho$ is used to define the coordinates of the planet on the projected orbit. The angle $(\theta - \Omega)$ is defined as:

$$\tan(\theta - \Omega) = \tan(v + \omega) \cos i, \quad (5)$$

where $i$ is the inclination of the orbit.

Using equation (4), $f(s, e, \omega, i)$, the distribution of planets found at a given separation, given $e$, $\omega$, and $i$, can be calculated. Since $\omega$ and $i$ are assumed to be uniformly distributed, they can be integrated over in order to give $f(s, e)$, the distribution of planets with a given normalized separation and eccentricity$^3$:

$$f(s, e) = \int_0^{360} \int_0^1 f(s, e, \omega, \cos i) d\omega d\cos i. \quad (6)$$

For QMESS, this is done on a fine grid in $\omega$ and $\cos i$, with $0 \leq \omega < 360$ and $0 \leq \cos i \leq 1$ and step sizes of $\Delta \omega = 1^\circ$ and $\Delta \cos i = 0.1$.

$^3$Note that the dependence on the longitude of the node, $\Omega$, is removed using equation (5).
\[ \Delta \cos \iota = 0.01, \text{ respectively. This map is computationally} \]
intensive but only needs to be calculated once. The map used

for QMESS, consisting of 1000 steps in \( e \) and 2000 steps in \( s \),

is shown in Figure 1. Note that \( f(s, e) \) is assumed to be uniformly

distributed in \( e \).

Note that although \( f(s, e) \) is, strictly speaking, a probability
density function (representing the fraction of planets with a cer-
tain combination of eccentricity \( e \) and normalized separation \( s \)),

we will for simplicity from now on refer to it as “projection probability.”

### 2.2. Calculating the Detection Probability

The projection probability \( f(s, e) \) obtained in § 2.1 is neces-
sary to evaluate the probability of finding a planet of mass \( M_p \)

and separation \( a \), which is the final goal of QMESS. Together

with \( f(s, e) \), a set of distributions for the planet eccentricity

\( [f(e) = \frac{dN}{de}] \), mass \( [f(M_p) = \frac{dN}{dM_p}] \), and semimajor axis

\( [f(a) = \frac{dN}{da}] \) is also needed. We assume that \( f(e), f(M_p), \)

and \( f(a) \) are independent. The implication of this assumption

have been discussed by, e.g., Cumming et al. (2008) and Hogg

et al. (2010). In order to account for a nonuniform eccentricity
distribution, \( f(s, e) \) is multiplied by \( f(e) \); although this slightly
decreases the flexibility later on, it significantly reduces the

memory usage.

The next step consists of converting the instrument detection

limit (expressed in minimum planet/star contrast detectable in

the chosen band for a given target, as a function of the projected

separation \( \rho \)) into a minimum planet mass \( (M_{\text{lim}}) \) versus

projected separation limit. To do this, a set of mass-luminosity

models are used (e.g., Baraffe et al. 2003; Burrows et al.

2003), assuming that the planet and the star are coeval. The

uncertainties introduced by this approach are discussed in detail

in B12.

Subsequently, the code generates a uniform grid of masses,

\( M_{\text{lim}} < M_p < M_{\text{max}} \), and semimajor axes, \( a_{\text{lim}} < a < a_{\text{max}} \),

and evaluates the distribution of detectable planets \( f(M_p, a) \).

First, for each semimajor axis, we use the distance \( d \) of the star
to evaluate the projected separation \( \rho = \frac{a}{d} \) and obtain \( f(\rho, e) \) for
each value of \( a \) on the grid. Then, for each value of \( M_p \) the
values of \( \rho_{\text{lim}} \) and \( \rho_{\text{max}} \) are evaluated from the detection limits,
such as \( M_p \geq M_{\text{lim}} \) for \( \rho_{\text{lim}} \leq \rho \leq \rho_{\text{max}} \). Note that QMESS

assumes that the contrast curve is smooth and has only one (local)
minimum, i.e., that the contrast curve crosses a given level of

\( M_p \) no more than 2 times. This assumption should hold for most

cases, although a local bias from a (bright) nearby companion
could affect the results.

The distribution of detectable planets as a function of \( M_p \)

and \( a \), \( f(M_p, a) \), is then calculated as:

\[
 f(M_p, a) = \int_0^1 \int_{\rho_{\text{lim}}(M_p, a)}^{\rho_{\text{max}}(M_p, a)} f(\rho, e) d\rho d e. \tag{7}
\]

The limits on \( \rho (\rho_{\text{lim}} \text{ and } \rho_{\text{max}}) \) are defined by the minimum

and maximum separation at which a planet is detectable given

the contrast curve.

The distribution \( f(M_p, a) \) is then stored for each target.

Note that \( M_p \) and \( a \) are uniformly distributed in \( f(M_p, a) \),

although \( f(M_p, a) \) itself is not normalized. The (expected) dis-

tribution for semimajor axes \( f(a) \) and planet mass \( f(M_p) \), all

normalized, are then folded into the \( f(M_p, a) \) to provide

\( g(M_p, a) = f(M_p, a) f(a) f(M_p) \), a new distribution of detect-
able planets, now taking into account the observed/predicted
distribution of planets.

In a similar way as for \( f(s, e) \), we will refer to \( f(M_p, a) \) and

\( g(M_p, a) \) as “detection probability” or “detection map,”

although these are also defined as probability density functions.

They in fact represent the fraction of detectable planets with a
given mass, \( M_p \), and semimajor axis, \( a \), assuming \( f(a) \) and

\( f(M_p) \) as the distributions of those parameters.

This approach assumes that the distributions of mass, eccen-
	ricity and semimajor axes are not correlated. The implications

of this assumption have been widely addressed by previous

works, including Cumming et al. (2008) and Hogg et al. (2010).

Finally, \( g(M_p, a) \) is integrated over the considered range of

mass and semimajor axis to obtain the probability of detecting a

planet with \( M_{\text{min}} \leq M_p \leq M_{\text{max}} \) and \( a_{\text{min}} \leq a \leq a_{\text{max}} \) as defined

by equation (8):

\[
 P(M_{\text{min}} \leq M_p \leq M_{\text{max}}, a_{\text{min}} \leq a \leq a_{\text{max}}) = \int_{M_{\text{min}}}^{M_{\text{max}}} \int_{a_{\text{min}}}^{a_{\text{max}}} g(M_p, a) dM_p da. \tag{8}
\]

The probability \( P \) can be also used to evaluate the upper/ lower

limits on the frequency of planets in the range of mass

FIG. 1.—Projection probability, as a function of the eccentricity and normalized

separation. Note that the angular dependencies (\( \omega, \Omega, \iota \)) have been integrat-
ed out (see text).
and semimajor axis explored by the analyzed survey as a function of the assumptions made on the mass and semimajor axis distributions (see, e.g., Lafrenière et al. 2007; Vigan et al. 2012).

2.3. Optimising the Computing Efficiency

In order to make the code as fast as possible, several optimizations are made. For contrast curves with a sharp inner working angle (IWA) the calculations are performed until $M_p = M_{\lim}(IWA)$, after which the probability map will not change. The limits on $\rho, \rho_{\text{min}}$ and $\rho_{\text{max}}$ are defined by the minimum and maximum separation at which the planet is detectable, given the contrast curve. At the IWA, the contrast curve increases so rapidly that effectively $\rho_{\text{min}}$ is no longer dependent on $M_p$ for $M_p = M_{\lim}(IWA)$. Since $\rho_{\text{min}}$ does not change any more, the detection probability $f(M_p, a)$ no longer changes compared to that for $M_p = M_{\lim}$ and thus $f(M_p > M_{\lim}) = f(M_{\lim}, a)$.

If there is also an outer working angle (OWA), the code takes it into account as well.

2.4. Comparison with the Classic MESS Code

In order to test the consistency of the results, we ran a set of simulations with identical setups, using both MESS and QMESS. The sample used for this test is the one observed for the Gemini Deep Planet Survey (GDPS; Lafrenière et al. 2007), consisting of 85 young FGK stars. In both cases we used a grid of 100 values of the $M_p$ between 1 and 75 $M_{\text{Jup}}$ and 500 values of $a$ between 1 and 500 AU.

Figure 2 shows the detection probability $f(M_p, a)$, evaluated as in § 2.2, for several values of the companion mass $M_p$. The output of the classic MESS tool (open circles) is plotted over the ones from QMESS (lines). The same contours are shown for two stars in the GDPS sample. The difference between the two methods is about 5%. This is consistent with the expected ~3.2% error predicted by Poisson statistics over 1000 planets per grid point generated by MESS.

Figure 3 shows the fraction of stars with a given computing time for the 85 stars in the GDPS sample in the two test runs. The total time needed for the full run is 2.6 hr for QMESS, as opposed to 4.2 days for the same run performed with the MESS. Besides the optimizations used in the QMESS code (which explains the width of the distribution of execution times; see § 2.3), the difference in computing time between the two codes is due to the fact that, for QMESS, the projection probability map is calculated once before the runs described in § 2.1. QMESS therefore does not need to evaluate the position on the projected orbit of each generated planet, which is the most time consuming step of MESS, given the number of planets involved. For the run with the GDPS sample, MESS generates 1,000 planets for each step in the mass-semimajor axis grid. So a total of $4.25 \times 10^9$ planets ($5 \times 10^7$ per star) are generated and need to be projected for the entire MESS run.

Figure 3.—Fraction of the targets with a given computing time for MESS (shaded) and QMESS (unshaded) for the GDPS sample (85 stars in total).

3. APPLICATIONS

The two main purposes of QMESS are (1) a statistical tool for the analysis of a direct imaging survey (§ 3.1) and (2) a predictive tool (§ 3.2) to prepare future surveys.

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3.1. Statistical Analysis of a Survey

The main purpose of QMESS is to analyse the (non-)detection from direct imaging surveys in order to test different assumptions on the planet population and to constrain the maximum occurrence of planets for a given planet population by calculating the expected number of planets and comparing that with the results from a survey with a large target sample.

As an example of this approach, we used the GDPS sample described in § 2.4 and investigated a variety of different assumptions on the planet parameter distributions. QMESS assumes, by default, uniform distributions for both planetary mass and semimajor axis $f(M_p) \text{ and } f(a)$. Therefore, we can easily probe a wide variety of parameter distributions by folding them into the output grid. Since the properties of the planets detected via the RV and transit methods seem to
be well fitted by simple power laws (e.g., Cumming et al. 2008), we explored a grid of distributions,
\[ f(M_p) = M_p^\alpha \] and \[ f(a) = a^\beta \], by varying \( \alpha \) and \( \beta \) as well as the maximum
cutoff for the semimajor axis distribution. We used a grid of
100 steps between \( M_{\text{min}} = 1 M_{\text{jup}} \) and \( M_{\text{max}} = 75 M_{\text{jup}} \) for
\( M_p \) and a grid of 1 AU steps from \( a_{\text{min}} = 1 \) AU to the chosen
value of the cutoff for the semimajor axis.

The results are shown in Figure 4. The left panels show the
expected planet fraction (which is equivalent to the probability
\( P(M_{\text{min}} \leq M_p \leq M_{\text{max}}, a_{\text{min}} \leq a \leq a_{\text{max}}) \) evaluated as in
eq (8)) as a function of \( \alpha \) and \( \beta \), with the semimajor axis cutoff
\( (a_{\text{max}}) \) fixed to 100, 50, and 10 AU (top to bottom). The star
symbols correspond to the values reported by Cumming et al.
(2008): \( \alpha = -1.31, \beta = -0.65 \). The right panels of the same
figure show the impact of varying the semimajor axis cutoff,
while keeping the power law for the mass distribution fixed
to \( \alpha = 1.3, 0, \) and \(-1.3 \) (top to bottom). No planet was discov-
ered in the GDPS survey, but Lafrenière et al. (2007) report a
previously known brown dwarf companion around HD 130948,
with an estimated mass in the range \( 40-65 M_{\text{jup}} \) and physical
separation of \( \sim 47 \) AU. Taking into account this detection, and
using the same approach used by Lafrenière et al. (2007), we

FIG. 5.— Comparison of the expected performances of the GPI and SPHERE integral field spectrographs (see Beuzit et al. 2008; Macintosh et al. 2007). The probability \( P(M_{\text{min}} \leq M_p \leq M_{\text{max}}, a_{\text{min}} \leq a \leq a_{\text{max}}) \) as a function of the age, distance, and spectral type for both GPI and SPHERE is shown. For the planet population, we used \( f(M) \propto M^{\alpha_{0.31}}, f(a) \propto a^{\beta_{0.61}} \) and \( f(e) = \text{constant (left panels)} \) and \( f(e) = e^{-e^{2/0.3}} \) (right panels). We also assumed \( M_{\text{min}} = 1 M_{\text{jup}}, M_{\text{max}} = 75 M_{\text{jup}}, a_{\text{min}} = 1 \) AU, \( a_{\text{max}} = 100 \) AU, and \( 0 \leq e \leq 1 \). The sharp cutoff for the later spectral types is due to the magnitude limits for the adaptive optic system (\( I < 10 \) for
SPHERE and \( I < 9 \) for GPI).
estimate a frequency of brown dwarf companions (13 \( M_{\text{up}} < M_p < 75 \) \( M_{\text{up}} \)) with separations in the range 25–225 AU of 1.4\(^{+5.9}_{-1.0}\) at 95% confidence level, which is in good agreement with the value of 1.9\(^{+8.3}_{-1.2}\) found by Lafrenière et al. (2007) for the same mass and semimajor axis boundaries.

For a semimajor axis cutoff of 10 AU, no constraints can be made on the power-law indices. For larger cutoff values, the null detection is only marginally consistent with the Cumming et al. (2008) distributions.

3.2. Predictive Mode

In addition to the analysis of a survey, QMESS can also be used to assess the performances of and to select the most suitable targets for new surveys, instruments, and/or different observing strategies.

To demonstrate the capabilities of QMESS as a predictive tool, we evaluate the probability \( P(M_{\text{min}} \leq M_p \leq M_{\text{max}}, a_{\text{min}} \leq a \leq a_{\text{max}}) \) for two of the next generation planet finders for 10 m-class telescopes that will soon be available to the community—the Spectro-Polarimetric High-contrast Exoplanet REsearch (SPHERE; e.g., Beuzit et al. 2008) at VLT and the Gemini Planet Finder (GPI, e.g., Macintosh et al. 2007) on Gemini south.

Both instruments are expected to be available in 2013 and will target young nearby stars looking for planets in wide orbits. To assess the performances, we use the estimated detection limits by Mesa et al. (2011) for SPHERE and Macintosh et al. (2007) for GPI.

Whether or not it is correct to use the results of the radial velocity surveys to analyze the results of direct imaging ones is still open to discussion. This is due to the lack of a statistically significant sample of planets in wide orbits. This approach is the most widely used so far for this kind of analysis, and its caveats and limitations have been discussed by several authors (see, e.g., Vigan et al. 2012; Nielsen & Close 2010). For this reason, rather than investigating the impact of different distributions for mass and semimajor axis, for which we use the distributions from Cumming et al. (2008) \( f(a) = a^{-0.61} \) and \( f(M_p) = M_p^{-1.3} \), we investigate the impact of two different distributions of the eccentricity \( f(e) \): (1) uniform and (2) Gaussian \( f(e) = e^{(e-\mu)^2/(2\sigma^2)} \). We also choose to fix the mean of the distribution, \( \mu \), and its variance, \( \sigma^2 \), so that \( \mu = 0 \) and \( \sigma = 0.3 \), as suggested by Hogg et al. (2010). The mass range used for the simulations is the same as the one used in § 3.1 (\( M_{\text{min}} = 1 \) M\(_{\text{Jup}}\), \( M_{\text{max}} = 75 \) M\(_{\text{Jup}}\)), and we extrapolated the semimajor axis distribution from \( a_{\text{min}} = 1 \) AU up to \( a_{\text{max}} = 100 \) AU.

To explore the dependency of the planet detection probability on the characteristics of the targeted stars (e.g., spectral type, age, and distance), we generated a set of simulations for five different stellar types (A0, F0, G0, K0, and M0), and a logarithmic grid in distance and age. Note that this simulation is not intended to represent the stellar population in the solar neighborhood, but rather to investigate the impact of differences in age, distance, and stellar type on the probability of finding a planet. The results, shown in Figure 5, indicate that the expected planet fraction depends only weakly on the eccentricity distribution. The expected performances of the two instruments appear to be fairly similar and with a high (~20%) chance of detection for planetary mass companions, especially while targeting young, nearby stars. This is mainly due to the more favorable planet/mass contrast predicted for younger systems and because closer-in targets allow a search for companions with a smaller semimajor axis, which are more likely given the assumed \( f(a) \) at a given projected separation.

4. SUMMARY

In this article we describe Quick-MESS (QMESS), a fast alternative to the classic Monte Carlo tools for the statistical analysis of exoplanet direct imaging surveys. The use of a grid-based approach reduces substantially the computing times for the analysis, while reducing spurious noise from Monte Carlo sampling, as shown in § 2.4. In this article, we demonstrated the two main purposes of QMESS and showed that, as for the MESS code, it can be used both as a statistical and predictive tool (see § 3.1 and 3.2). For the GDPS survey, we find that their null detection is marginally consistent with the distributions extrapolated from the RV results (Cumming et al. 2008). We have also shown that the eccentricity distribution only has a minor impact on the expected fraction of planet detection for the upcoming surveys, which could have a detection efficiency of up to 20% (depending on the choice of targets), provided that the distribution of planet mass and semimajor axis are the same at large (>1 AU) and small (<1 AU) separations.

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