Performance comparison of single-stage mixed-refrigerant Joule–Thomson cycle and reverse Brayton cycle for cooling 80 to 120 K temperature-distributed heat loads

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Abstract. Thermodynamic performance comparison of single-stage mixed-refrigerant Joule–Thomson cycle (MJTR) and pure refrigerant reverse Brayton cycle (RBC) for cooling 80 to 120 K temperature-distributed heat loads was conducted in this paper. Nitrogen under various liquefaction pressures was employed as the heat load. The research was conducted under nonideal conditions by exergy analysis methods. Exergy efficiency and volumetric cooling capacity are two main evaluation parameters. Exergy loss distribution in each process of refrigeration cycle was also investigated. The exergy efficiency and volumetric cooling capacity of MJTR were obviously superior to RBC in 90 to 120 K temperature zone, but still inferior to RBC at 80 K. The performance degradation of MJTR was caused by two main reasons: The high fraction of neon resulted in large entropy generation and exergy loss in throttling process. Larger duty and \textit{WLMTD} lead to larger exergy losses in recuperator.

1. Introduction

Mixed-refrigerant Joule-Thomson cycle (MJTR) and Reverse Brayton cycle (RBC) are two widely-used cryogenic processes for gas liquefaction. Based on these two methods, various gas liquefaction systems were developed with different process configurations.

The thermodynamic performance of MJTR was wildly investigated, as Alexeev et al. [1], Gong et al. [2]. MJTR with temperature distributed heat loads was utilized in gas liquefaction, especially in natural gas liquefaction. Gong et al. [3-4] investigated the MJTR for low pressure methane liquefaction. The heat capacity matching of warm and cold streams could be improved by mixture composition optimization to get a temperature distribution. The pinch points of the recuperator were determined by the mixture composition [5]. Venkatarthnam [6] reported the thermodynamic analysis of MJTR for nitrogen and natural gas liquefaction, with exergy loss distribution studied. The MRC processes in LNG (liquefied natural gas) industry are also developed on the base of MJTR, as reviewed by Lim et al. [7].

RBC with temperature distributed heat loads is the basis of nitrogen expansion processes for LNG production, as reported by He et al. [8]. The industrial gas (air, nitrogen, et al.) liquefaction processes like Claude [9] cycle are also closely related with RBC. Although the performance comparison of
MJTR and RBC in natural gas liquefaction has been widely investigated, the same research in lower temperature zone for nitrogen and air liquefaction has been rarely reported.

The main task of this paper is the thermodynamic analysis and comparison of MJTR and RBC in the temperature zone for low pressure $N_2$ liquefaction, which could be a significant reference for the design of gas liquefaction system. The advantages and disadvantages of two cycles were investigated. Several new parameters were introduced in the quantitative performance evaluation of the recuperative process and throttling process.

2. Analysis models

2.1. Cycle configuration

Among the various cycle configurations of MJTR and RBC, the single-stage cycle is the simplest one, which was analyzed in this paper. Single-stage cycle has been proven to be the most efficient cycle configuration for MJTR, as illustrated in [1-2]. The investigation of single-stage cycle could be a basic reference for the design of MJTR or RBC gas liquefaction processes.

Compressor (CP), after cooler (AC), recuperator (HX) and J-T element (JT) are the four main components of single-stage MJTR, as Cycle M in Fig. 1. Evaporator might not be necessary for gas liquefaction systems, it was not considered in this paper for simplification. The warm stream (high pressure stream) and feed nitrogen were cooled by cold stream (low pressure stream) in recuperator. Then the warm stream was throttled at J-T to provide cooling capacity. Single-stage RBC was similar with single-stage MJTR, while J-T element and mixed-refrigerant were replaced by an expander (EP) and pure gas, as Cycle R. Nitrogen liquefaction under various pressures was the temperature distributed heat load, including sensible heat and latent heat. For sensible heat, the cooling capacity of MJTR/RBC was utilized to cool down the feed nitrogen at different temperatures from ambient to saturation point (like 300 K to 90 K), which was temperature distributed along the recuperator (HX).

Fig. 1 Cycle configurations of single-stage MJTR and pure gas RBC for nitrogen liquefaction

2.2. Exergy analysis model

The exergy analysis was conducted under nonideal conditions with extrinsic irreversibilities, including those caused by non-isentropic compression and expansion process, temperature difference and flow resistance. Other irreversibilities like heat leak were neglected. The calculation of mixtures was under the condition of phase equilibrium. Detailed calculation conditions are listed in Section 2.3. Exergy losses in each component of the cycle were calculated. The components with large exergy losses are the key point for enhancement. The analysis methods in the research of Wang et al. [10] were adopted in this paper. Exergy efficiency $(\eta, \text{kW kW}^{-1})$ of the whole cycle for temperature-distributed heat loads is the ratio of exergy gained by heat loads to exergy inputted (compressor power consumption $W$, kW) in system, as Eqs. (1):

$$\eta = \frac{\text{Exergy}_{\text{gain}}}{\text{Exergy}_{\text{input}}} = \frac{\Delta E_{\text{load}}}{W} = 1 - \frac{\sum I_j}{W}$$

(1)

The exergy gained by heat loads (nitrogen liquefaction) is the exergy difference of feed nitrogen at the inlet and outlet of HX, as Eqs. (2). Wherein $\Delta E_{\text{load}}$ (kW) is regarded as the cooling capacity ($Q_c$, kW) of MJTR or RBC in this paper. Thus volumetric cooling capacity ($q_v$, kJ m$^{-3}$) is the ratio of $Q_c$ and the compressor suction volumetric flow rate ($G_{v,suction}$, m$^3$ s$^{-1}$).

$$Q_c = \Delta E_{\text{load}} = E_{N_2,\text{out}} - E_{N_2,\text{in}} = m_{N_2} \left[ \left( h_{N_2,\text{out}} - h_{N_2,\text{in}} \right) - T_0 \left( s_{N_2,\text{out}} - s_{N_2,\text{in}} \right) \right]$$

(2)
\( q_v = \frac{Q_c}{G_v_{suction}} \) \hspace{1cm} (3)

c. Exergy losses \((I_i, \text{kW})\) in compressor (CP), after cooler (AC), recuperator (HX), J-T element (JT) and expander (EP) are calculated by Eqs. (4) to (8):

\[
I_{CP} = W - (E_{out} - E_{in}) = T_0 (S_{out} - S_{in}) \tag{4}
\]

\[
I_{AC} = E_{in} - E_{out} = Q_0 + T_0 (S_{out} - S_{in}) \tag{5}
\]

\[
I_{HX} = (E_{cold,in} - E_{cold,out}) - \left[ (E_{warm,out} - E_{warm,in}) + (E_{N_2, out} - E_{N_2, in}) \right] \tag{6}
\]

\[
I_{JT} = E_{in} - E_{out} = T_0 (S_{out} - S_{in}) \tag{7}
\]

\[
I_{EP} = (E_{in} - E_{out}) - W_{\text{expansion}} \tag{8}
\]

The exergy loss fraction \((\Pi_i, \text{kW kW}^{-1})\) of each process could be expressed as:

\[
\Pi_j = I_i/W \tag{9}
\]

2.3. Simulation conditions

Exergy efficiency \((\eta)\) and volumetric cooling capacity \((q_v)\) are two evaluation parameters in this simulation. For MJTR, the optimization was mainly focused on the mixture composition, under typical optimal pressures that were suggested in previous research, as Gong et al. [2-4]. Operating pressures were optimized further on the base of optimal composition. The optimization of pure gas RBC was mainly focused on the operating pressures. RBC could reach the highest \(\eta\) at a certain pressure ratio. As the \(q_v\) of MJTR and RBC with the highest \(\eta\) was satisfying as well; thus \(q_v\) was not optimized particularly. The objective functions are Eqs. (10) to (12), with the only optimization target of \(\eta\).

\[
\eta = f(y_1, y_2, \ldots, y_n, p_h, p_l, m_{N_2}) \tag{10}
\]

\[
\sum_{1}^{n} y_j = 1 \quad y_j > 0 \quad (j = 1, 2, \ldots, n) \tag{11}
\]

\[
p_{l, \text{min}} \leq p_l < p_h \leq p_{h, \text{max}} \tag{12}
\]

The simulation was conducted under the following conditions. In order to simplify the calculation model, the adiabatic efficiency of compressors and expanders were 75% and 70% respectively. For RBC, the variations of operating pressures in different cases were small, so as the adiabatic efficiencies. For MJTR, compressor adiabatic efficiencies could also be almost constant with different mixture composition and pressure ratios, according to the experimental investigation [13]. The minimum approaches (the minimum temperature difference) in heat exchangers were 3 K; both the pressure drops along the high or low pressure streams were 0.5 bar. The maximum discharge pressure and minimum suction pressure of compressors were 20 bar and 3 bar respectively. The discharge temperature in MJTR was constrained to less than 383 K. Liquid in the compressor and expander was avoided. The expansion work was fully recovered. The ambient temperature, feed nitrogen temperature and the outlet temperature of the after cooler were 300 K. The mixture composition in MJTR was assumed to be constant along the cycle. Heat leak was ignored. Pure helium was adopted in RBC. The properties of mixtures were calculated by Peng–Robinson equation of state [11] and van der Waals mixing rule [12].

3. Analysis and comparison

The key operating parameters of optimized single-stage MJTR and RBC were listed in Table 1 and 2. All pressures in this paper are absolute pressure value. The mixture compositions are mole fraction.

| Table 1 – Mixture composition and operating parameters of MJTR from 80 to 120 K. |
|------------------|------------------|------------------|------------------|------------------|------------------|
| Items            | 120 K | 110 K | 100 K | 90 K | 80 K |
| Ne (% mole)      | 0.00  | 0.00  | 0.00  | 6.73 | 20.07 |
3.1. Comparison of each component in MJTR and RBC

3.1.1. Compression process. The exergy losses in compressor ($\Pi_{CP}$) of MJTR and RBC at various refrigeration temperatures were similar and nearly changeless, which were 20.22% to 20.95% and 19.85% to 22.15% respectively. The difference of $\Pi_{AC}$ between various refrigeration temperatures was obviously smaller than that of $\Pi_{HIX}$ and $\Pi_{HT}$. The large adiabatic exponent of He in RBC lead to high discharge temperature ($T_{AC}$) of RBC (411.4 to 434.3K), which was much higher than MJTR (less than 383 K). A high $T_{AC}$ leads to a large exergy loss in after cooler ($\Pi_{AC}$) of RBC (18.33% to 19.72%), which was obviously larger than that of MJTR (10.54% to 13.05%).

3.1.2. Recuperative process. As illustrated in previous investigations [4], the heat capacity (including sensible heat and latent heat) match of warm and cold streams has significant effect on the performance of recuperator. A better heat capacity match could lead to a small $WLMTD$ (heat load weighted logarithmic mean temperature difference), and a small $\Pi_{HIX}$ as consequence.

Several evaluation parameters for the recuperator were calculated by Eqs. (13) to (15), with results in Fig. 2; wherein $\eta_{HIX}$ is the ratio of exergy gained by warm stream to exergy released by cold stream, as Eqs. (13):

$$\eta_{HIX} = \frac{E_{W0} - E_{W1}}{E_{L0} - E_{L1}} = \frac{\Delta E_{L} - I_{HIX}}{\Delta E_{L}}$$

(13)
The heat exchanger $UA$ (W K$^{-1}$) was also important for system design, which determines the recuperator size. The heat transfer duty of the recuperator has significant effect on $I_{H_{HX}}$. As the investigation in [11], larger recuperator heat transfer duty could result in a larger $I_{H_{HX}}$.

$$\tau_{UA} = \frac{Q_c}{UA_{H_{HX}}}$$

$$k_{H_{HX}} = \frac{Q_{H_{HX}}}{Q_c} = \frac{WLMTD \cdot UA_{H_{HX}}}{Q_c} = \frac{WLMTD}{\tau_{UA}}$$

Wherein $\tau_{UA}$ was the ratio of cooling capacity ($Q_c$) to recuperator $UA$ ($UA_{H_{HX}}$); $k_{H_{HX}}$ was the ratio of recuperator heat transfer duty ($Q_{H_{HX}}$) to $Q_c$. A system with larger $\tau_{UA}$ requires smaller recuperator to achieve a certain $Q_c$, which could reduce the system size.

The $I_{H_{HX}}$ of MJTR got larger with the decreasing refrigeration temperature ($T_c$, equal to the nitrogen liquefaction temperature in this paper). The larger $WLMTD$ and $k_{H_{HX}}$ were two main reasons, especially for 80 K MJTR. For one, as the fraction of neon and nitrogen in 80 K mixture was high, excessive middle-high-boiling components (mainly propane) were adopted to reduce discharge temperature (the usage of high boiling components like iC$_3$H$_6$ was limited to avoid liquid in the compressor). Thus the temperature difference in middle-high temperature zone was enlarged, leading to a larger $WLMTD$. Then, the $k_{H_{HX}}$ (reflecting the heat transfer duty) of 80 K MJTR was relative large (27.60), leading to a large $I_{H_{HX}}$, which could enlarge the $I_{H_{HX}}$.

The heat capacity of pure gas in RBC could be smaller than that of two-phase mixture in MJTR, leading to smaller $Q_{H_{HX}}$ and $k_{H_{HX}}$. As the $WLMTD$ of RBC was close to that of MJTR, $I_{H_{HX}}$ in RBC was smaller than MJTR, especially at low temperatures.

3.1.3. Throttling and expansion process. It is obvious that the entropy generation in throttling process and non-isentropic expansion process could result in exergy losses. A parameter of efficiency was adopted to evaluate the performance of throttling process and non-isentropic expansion process, based on exergy method respectively. It was the ratio of cooling capacity (in exergy) gained after an isenthalpic throttling process (AJT) for MJTR or after a non-isentropic expansion process (AEP) for RBC, to that gained after an isentropic expansion process, as Eqs. (16) and (17):

$$\eta_{JT} = \frac{e_{li} - e_{AJT}}{e_{li} - e_{isentropic}}$$

$$\eta_{EP} = \frac{e_{li} - e_{AEP}}{e_{li} - e_{isentropic}}$$

The subscripts to denote the refrigerant status are illustrated in Fig.3. It was obvious that lower $\eta_{JT}$ is accompanied with larger exergy loss fraction in throttling process ($I_{JT}$), as shown in Fig 4.
The $\eta_{JT}$ of MJTR was much higher than the $\eta_{EP}$ of RBC in 90 to 120 K temperature zone, except at 80 K. It should be indicated that the cooling capacity in gas liquefaction system is utilized along the recuperator instead of concentrated in the evaporator. Thus the calculate method of $\eta_{JT/EP}$ could not reflect the actual physical factor of gas liquefaction system exactly, which is just a model to evaluate throttling or expansion processes.

The exergy loss in throttling process ($\Pi_{JT}$) is close related with the mixture status before throttling (BJT). A large degree of subcooling or a low vapor quality was beneficial to achieve high $\eta_{JT}$, as shown in Table 3, which could also be directly illustrated by the mixture T-s diagram. The increasing degree of subcooling (4.46 to 14.01 K) in 120, 110 and 100 K MJTR brought about increasing $\eta_{JT}$ and decreasing $\Pi_{JT}$. The mixture in 80 K MJTR contained a high fraction of low-boiling component of neon (20.07%), whose normal boiling point (27.104 K) was much lower than $T_{HO}$. The mixture vapor quality before throttling ($x_{MJTR}$) was relatively higher (0.1941). The entropy generation of two phase flow was much larger than that of subcooled liquid. The throttling entropy generation of 100 to 120 K MJTR maintained at a low level, and sharply increased from 90 K to 80 K. Thus, the $\eta_{JT}$ of 80 K MJTR (63.69%) was lower than that at 100 to 120 K (above 87%). The 90 K MJTR with less neon (6.73%) and smaller $x_{MJTR}$ (0.0559) could be a contrast, whose $\eta_{JT}$ (86.05%) and $\Pi_{JT}$ (9.26%) were between those of 80 K and 100 K MJTR.

Table 3 – Throttling parameters of MJTR at different temperatures.

| Cases | 120 K | 110 K | 100 K | 90 K | 80 K |
|-------|-------|-------|-------|------|------|
| Vapor quality before throttling (kg kg$^{-1}$) | Subcooled | Subcooled | Subcooled | 0.0559 | 0.1941 |
| Dew point of high pressure mixture (K) | 124.46 | 117.75 | 114.01 | 50.21 | 34.22 |
| Temperature before throttling (K) | 120 | 110 | 100 | 90 | 80 |
| Temperature drop of throttling (K) | 8.27 | 8.68 | 6.94 | 5.03 | 3.92 |
| Degree of subcooling (K) | 4.46 | 7.75 | 14.01 | / | / |
| Entropy generation ($10^{-3}$ kJ kg$^{-1}$ K$^{-1}$) | 25.05 | 27.32 | 26.19 | 46.37 | 101.25 |

The exergy loss in expander ($\Pi_{EP}$) of RBC was stable under various temperatures, which was obviously larger than in the $\Pi_{JT}$ MJTR. However, as the expansion work of expanders could not be fully recovered in actual applications, the $\Pi_{EP}$ would be enlarged further.

3.1.4. Summary of the exergy loss distribution. Both for MJTR and RBC, $\Pi_{HX}$ in recuperator took the largest part of the overall exergy losses and increased from 120 to 80 K, as shown in Fig.5. The enlarging recuperator duty and $WLMTD$ lead to increasing $\Pi_{HX}$. Therefore the optimization should pay attention on enhancing the recuperative process. With the application of neon, $\Pi_{JT}$ of MJTR sharply increased at low refrigeration temperatures (80 K). The entropy generation of two-phase neon-enriched flow was a significant reason. Expander $\Pi_{EP}$ in RBC was obviously larger than $\Pi_{JT}$ of MJTR. Besides, the variation of $\Pi_{CP}$ and $\Pi_{AC}$ was relatively small under different refrigeration temperatures.
3.2. Effect of operating pressure

Operating pressures are significant parameters of MJTR and RBC. For MJTR, properly high back pressures could reduce the heat capacity difference between low and high pressure streams, compression power consumption and discharge temperature, which could improve the exergy efficiency. The volumetric cooling capacity could also be improved with enlarged refrigerant suction density. Nevertheless, the throttling effect was smaller under smaller compression ratios.

For RBC, the exergy efficiency was mainly affected by operating pressures in this paper. It was indicated that RBC could reach the highest exergy efficiency under a certain pressure ratio. Higher back pressure was also beneficial to achieve high efficiency and volumetric cooling capacity. The Exergy efficiency ($\eta$) and volumetric cooling capacity ($q_v$) of 100 K He RBC under various operating pressures are illustrated in Fig. 6. However, the highest $\eta$ and $q_v$ that RBC could achieve within the simulation limit were inferior to those of MJTR at 90 to 120 K.

![Fig. 6 Exergy efficiency and volumetric cooling capacity of 100 K He RBC under various operating pressures](image)

3.3. Overall performance comparison of RBC and MJTR

Exergy efficiency ($\eta$) and volumetric cooling capacity ($q_v$) are the two main performance evaluation parameters of MJTR and RBC, which could reflect system operating costs and construction costs respectively. The performance comparison is illustrated in Fig. 7.

It was indicated that the $\eta$ and $q_v$ of MJTR was obviously superior to those of RBC in 90 to 120 K temperature zone. MJTR could maintain a relative high $\eta$ of 32.62% to 33.23% at 100 to 120 K. With the adoption of neon, the degradation of $\eta$ occurred at 90 to 80 K. The of 80 K MJTR was only 11.87%, lower than 80 K RBC (13.19%). The $q_v$ of MJTR also decreased obviously from high to low refrigeration temperatures ($T_c$). As a contrast, the performance of RBC was relatively stable at various $T_c$, with $\eta$ of 12.12% to 13.56%. The $q_v$ of RBC even got larger at low $T_c$.

Generally, MJTR had significant advantages over RBC at the $T_c$ above 90 K (nitrogen pressure of 3.6 bar), especially above 100 K (nitrogen pressure of 7.8 bar), when neon was not necessary. While RBC was more suitable for lower $T_c$ like 80 K. It could be a reference for the thermodynamic design of nitrogen liquefiers which are based on MJTR technology.

![Fig. 7 Exergy efficiency and volumetric cooling capacity of MJTR and RBC](image)
4. Summary and discussion
The performance comparison of single-stage MJTR and RBC for cooling fixed-temperature heat loads (nitrogen liquefaction) from 80 to 120 K was conducted in this paper. Conclusions as following:

The exergy efficiency ($\eta$) and volumetric cooling capacity ($q_v$) of MJTR were superior to RBC at 90 to 120 K, but lower than those of RBC at 80 K. With the application of neon in 80 K MJTR, the large entropy generation in the throttling process of two-phase flow resulted in a relative large exergy loss in J-T element. As the exergy losses in recuperator was also large with large $WLMTD$, the performance of MJTR degraded sharply at 80 K. The performance of RBC was stable at various refrigeration temperatures. A refrigeration temperature above 90 K was recommended for MJTR nitrogen liquefaction systems. Both for MJTR and RBC, the exergy loss in recuperator took the largest percentage in the overall loss.

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