Output Prediction of waste concrete based on GM (1, 1)

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Abstract: In order to promote efficient use of waste concrete in Chongqing, the building area estimation method is employed to estimate the waste concrete output from 2012 to 2018, and the gray model is used to predict the waste concrete output from 2019 to 2023. By Matlab programming, the waste concrete output in Chongqing will reach 3181.14t in 2023, which is 1.44 times that in 2012. The results show that the gray model can accurately forecast the output of waste concrete in Chongqing in the short and medium term.

1. Introduction
With urbanization advancing, construction and demolition projects continuously generate numerous construction waste every year. In 2012, the amount of construction waste generated in China was 1.546 billion tons, accounting for 30% to 40% of the total urban waste [1]. The composition of construction waste is complex, and waste concrete makes up 50% of the total, which is a substantial component [2]. However, such a huge amount of waste concrete is generally piled up in the open air or buried, which not only takes up massive land and contaminates water and soil, but also brings about considerable resource wasting [3]. Meanwhile, due to the vigorous development of the shed reform in Chongqing recently, the waste concrete output continues to increase over time, and problems of resource wasting and environmental pollution have become more and more austere.

Presently, there is no authoritative calculation rule to make statistical analyses of construction waste outputs. Generally speaking, main sources of building waste consist of construction, decoration and demolition. Because decoration generally does not change the main structures, the amount of waste concrete produced by decoration is negligible. Therefore, this paper starts with construction and demolition to estimate and forecast the waste concrete output in Chongqing.

2. Estimation of waste concrete output
Zhao J [4] estimated the output of construction waste based on three angles: building area, construction material consumption and the output ratio of urban population. Due to the insufficient accuracy of the latter two methods, this paper uses the building area estimation method for
In general, every 10,000m² of building area will generate 550t of waste concrete [3]. Therefore, the output of waste concrete during construction = the output of waste concrete produced per unit area × the area of completed buildings per year. Demolition is another major origin of waste concrete. Zhao W [5] pointed out 1.0-1.35t of construction waste was generated for every 1m² of the building, of which waste concrete accounts for about 50% of construction waste. According to the current situation in Chongqing, this paper adopt 1.2t/m² for the calculation of construction waste, and thereby 0.6t/m² (the conversion factor) for the calculation of waste concrete. Additionally, due to no special statistical result of building demolition area presently, 10% of the annual building construction area is taken as the annual building demolition area [6]. Hence, the amount of waste concrete generated by demolition = annual building demolition area × construction waste output per unit area × conversion factor.

To sum up, annual waste concrete outputs = waste concrete outputs generated by construction + waste concrete outputs generated by building demolition. In light of Chongqing Statistical Yearbook of 2012-2018, the waste concrete estimates are achieved (as shown in Table 1).

### Table 1. The estimation of waste concrete output in Chongqing from 2012 to 2018.

| Year | Completed area (10000m²) | Construction waste concrete (10kt) | Construction area (10000m²) | Demolition area (10000m²) | Demolition waste concrete (10kt) | Annual waste concrete (10kt) |
|------|--------------------------|-----------------------------------|-----------------------------|---------------------------|---------------------------------|-------------------------------|
| 2012 | 11602                    | 638.11                            | 26270                       | 2627                      | 1576.20                         | 2214.31                      |
| 2013 | 12240                    | 673.20                            | 29885                       | 2988.5                    | 1793.10                         | 2466.30                      |
| 2014 | 12816                    | 704.88                            | 32887                       | 3288.7                    | 1973.22                         | 2678.10                      |
| 2015 | 13543                    | 744.87                            | 32802                       | 3280.2                    | 1968.12                         | 2712.99                      |
| 2016 | 13752                    | 756.36                            | 32077                       | 3207.7                    | 1924.62                         | 2680.98                      |
| 2017 | 13448                    | 739.64                            | 33210                       | 3321                      | 1992.60                         | 2732.24                      |
| 2018 | 13780                    | 757.90                            | 35140                       | 3514                      | 2108.40                         | 2866.30                      |

### 3. Forecast of waste concrete output

For predicting the output of waste concrete, on one hand, you can obtain the future trend of the waste concrete output; on the other hand, you can formulate effective countermeasures accordingly to reduce the waste concrete output and improve the utilization efficiency. These can provide basic data for government and construction waste recovery enterprises.

Nowadays, the main methods for predicting the annual output of construction waste include multiple linear regression, autoregressive prediction, BP neural network prediction, and gray model prediction. The multiple linear regression method can predict the construction waste output based on the independent variables (such as urban population, annual building construction area, urban GDP, etc.), but the calculation is large, and it is easy to cause multicollinearity and result in distortion of the results [7]. The autoregressive prediction method can use its own time sequence to make predictions, but the prediction accuracy of data that is greatly affected by social factors is undesirable. The BP neural network prediction method is suitable for data with complex internal structure, and its network layer is difficult to construct. Notwithstanding, the GM (1,1) model is the core of the gray prediction, which can be modeled and predicted based on a small amount of known information, and is particularly suitable for data sequence of samples with less data and showing a significant increase or decrease trend. Proved by practice, its precision is higher. The model is alike to a differential and difference equation, and has mathematical properties such as exponential, difference, and differential. The parameters and structure of the model can be selected from a small amount of data information based on known parameter changes, which breaks the limitations of large sample data modeling [8].
3.1. Establishment of gray model

The GM (1,1) model is a prediction model of sequence generated on the basis of accumulation. The specific steps are as follows:

(1) Suppose that the time series \( x^{(0)}(i) = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\} \) has \( n \) observed values \( x^{(0)}(i) (i = 1, 2, \cdots, n) \), which are the original data to be predicted, and all are non-negative numbers, and then perform an accumulation process on the original data to get a new sequence \( x^{(1)}(t) = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\} \). Some of it can be defined as follows:

\[
x^{(1)}(k) = \sum_{i=0}^{k} x^{(0)}(i), k = 1, 2, \cdots, n
\]

Compared with the original sequence, the new one effectively weakens the randomness and volatility of the data, and the stability is improved to a certain extent.

(2) Set up differential equations to indicate the changing trend of the new sequence:

\[
dx^{(1)}(t)/dt + ax^{(1)}(t) = u
\]

Where \( a \) and \( u \) are identification parameters, which can be obtained by least square fitting:

\[
\begin{bmatrix}
a \\
u
\end{bmatrix} = \left(B^T B\right)^{-1} B^T Y_N
\]

Where \( Y_N = [x^{(0)}(2), \cdots, x^{(0)}(n)]^T \).

(3) Construct data matrix \( B \):

\[
B = \begin{bmatrix}
-1/2[x^{(1)}(1) + x^{(1)}(2)] & 1 \\
-1/2[x^{(1)}(2) + x^{(1)}(3)] & 1 \\
\vdots & \vdots \\
-1/2[x^{(1)}(N-1) + x^{(1)}(N)] & 1
\end{bmatrix}
\]

(4) Derive the prediction model:

\[
x^{(1)}(t+1) = [x^{(0)}(1) - u/a] e^{-at} + u/a
\]

3.2. Prediction for prospective outputs

3.2.1. Data analysis. It can be seen from Table 1 that the original data of the annual output of waste concrete is: \( x^{(0)}(t) = \{2214.31, 2466.3, 2678.1, 2712.985, 2680.98, 2732.24, 2866.3\} \), and the first cumulative data sequence is obtained as follows: \( x^{(1)}(t) = \{2214.31, 4680.61, 7358.71, 10071.695, 12752.675, 15484.915, 18351.215\} \), which can be obtained by calculation:

\[
Y_N = [2466.3, 2678.1, 2712.985, 2680.98, 2732.24, 2866.3]^T
\]

Substitute \( Y_N \) and \( B \) into formula (3), and then with the matrix calculation we can find that: \( a = -0.0225, u = 2461.9 \). Substitute them into equation (5), then the prediction model is:

\[
x^{(1)}(t+1) = 111632.0878 \times e^{0.0225t} - 109417.7778
\]
3.2.2. Model check. (1) Residual test. Substitute $t = 0, 1, \cdots, 6$ into equation (6) to get the cumulative value of the output of construction waste from 2012 to 2018: $\hat{x}^{(1)}(t) = \{2214.31, 4754.50, 7352.50, 10009.61, 12727.18, 15506.59, 18349.25\}$; and then calculate the predicted values $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)\,,$ absolute error values $\hat{e}^{(0)}(t) = x^{(0)}(t) - \hat{x}^{(0)}(t)$ and relative error values $q(t) = \hat{e}^{(0)}(t) / x^{(0)}(t)\,.$ The results are shown in Table 2. From the data in Table 2, the average relative error is calculated to be 0.38%, indicating high model accuracy.

(2) Post-difference test. The standard deviations of the original data sequence $x^{(0)}(t)$ and the absolute error sequence $\hat{e}^{(0)}(t)$ can be calculated from the original data sequence and the absolute error sequence, respectively:

\[
\hat{x}^{(0)} = \frac{1}{7} \sum_{i=1}^{7} x^{(0)}(i) = 10130.59 \quad \quad \hat{\Delta}^{(0)} = \frac{1}{7} \sum_{i=1}^{7} e^{(0)}(i) = 27.33
\]

\[
S_1 = (\sum_{i=1}^{n} [x^{(0)}(i) - \hat{x}^{(0)}]^2)/n - 1 = 46199 \quad S_2 = (\sum_{i=1}^{n} [\hat{e}^{(0)}(i) - \hat{\Delta}^{(0)}]^2)/n - 1 = 3187
\]

Table 2. The relative errors between predicted values and actual values

| Year  | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|-------|------|------|------|------|------|------|------|
| Actual Value | 2214.31 | 4754.50 | 7352.50 | 10009.61 | 12727.18 | 15506.59 | 18349.25 |
| Predicted Value | 2214.31 | 4754.50 | 7352.50 | 10009.61 | 12727.18 | 15506.59 | 18349.25 |
| Absolute Error | 0.00 | 73.89 | 6.21 | 62.09 | 25.49 | 21.68 | 1.96 |
| Relative error/% | 0.00 | 1.58 | 0.08 | 0.62 | 0.20 | 0.14 | 0.01 |

Then the variance ratio is $C = S_2 / S_1 = 0.069$ and the small error probability is set as $P = \{ | \hat{e}^{(0)}(t) - \hat{\Delta}^{(0)} | < 0.6745 S_1 \}$ \, Suppose $M = \{ | \hat{e}^{(0)}(t) - \hat{\Delta}^{(0)} | \}$ \, and then $M = \{ 27.33, 46.56, 21.12, 34.75, 1.84, 5.65, 25.37 \}$. The result is fully consistent with the occurrence of the small error probability of $P$ event, so $P = 1$. As shown in Table 3, the prediction accuracy level is Level A.

Table 3. Model accuracy level

| Forecast Accuracy Level | C   | P   |
|------------------------|-----|-----|
| Level A(Excellent)     | $<0.35$ | $>0.95$ |
| Level B(Qualified)     | $0.35 \leq C < 0.50$ | $0.80 \leq P < 0.95$ |
| Level C(Barely Qualified) | $0.50 \leq C < 0.65$ | $0.70 \leq P < 0.80$ |
| Level D(Not Qualified) | $C > 0.65$ | $P < 0.70$ |

3.2.3. Forecast of waste concrete production. As per the obtained model, the annual output of waste concrete from 2019 to 2023 is predicted, as shown in Table 4. Figure 1 shows the comparison between the predicted and actual values. It can be seen from the comparison that the predicted and actual values of waste concrete output from 2012 to 2018 are close to coincide, which signifies that the GM (1, 1) model is highly accurate in predicting waste concrete in Chongqing. From the predicted value, it illustrates that the output of waste concrete in Chongqing will proceed with increase in the next few years, breaking through $3 \times 10^7 t$ in 2021, with an annual increase of about 700,000t. The increase in the annual output of waste concrete is in need of pushing the government and relevant departments to deliberate counterplans, and also makes enterprises consider how to create proceeds in the waste concrete market with great potentials.
Table 4. Predicted annual output of waste concrete from 2019 to 2023.

| Year | 2019   | 2020   | 2021   | 2022   | 2023   |
|------|--------|--------|--------|--------|--------|
| Predicted value/10000t | 2907.34 | 2973.50 | 3041.16 | 3110.36 | 3181.14 |

Figure 1. Comparison of predicted and actual values of annual output of waste concrete.

4. Suggestions on disposal of waste concrete

4.1. Establish restraint mechanisms
Combined with the current situation, the landfill disposal cost of waste concrete and the resource tax on natural sand and gravel should be increased. It is vital to establish a reporting management mechanism, strengthen the supervision of the collection and operation of transportation enterprises, prevent illegal dumping caused by subcontracting, and effectively convert waste concrete into raw materials that can be utilized resourcefully.

4.2. Promote the formation of industrial chain
It is also crucial to implement tax reduction or exemption, cheap or interest-free loans from government, and accelerated depreciation of equipment or purchase subsidy, and to transfer the taxation charged from users of the natural aggregate and waste concrete landfill to institutions of construction waste recycling and recycled aggregate and recycled concrete production, in order to support and cultivate every link of the industrial chain, and quickly realize large-scale operation.

4.3. Perfect industry standards
Only by consummating the industry standards can we further guide and promote the development of recycling and utilization of waste concrete. Under the joint actions of the government, universities, industry associations, enterprises and other institutions, the quality standards and quality certification system of recycled products are formulated based on the successful cases at home and abroad, in order to guide the research and application of waste concrete recycling.

5. Conclusion
With the original data in Chongqing from 2012 to 2018, the GM (1, 1) model was verified to have high accuracy, which can properly predict waste concrete outputs in the next 5 years. GM is only appropriate for short-term forecast in actual operation. Therefore, it is indispensable to continuously increase and modify the data to ensure the precision of predicted values.
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