High twist contribution to the longitudinal structure function $F_L$ at high $x$

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Abstract

We performed NLO QCD fit of combined SLAC/BCDMS/NMC DIS data at high $x$. Model independent x-shape of high twist contribution to structure function $F_L$ is extracted. Twist-4 contribution is found to be in fair agreement with the predictions of infrared renormalon model. Twist-6 contribution exhibits weak tendency to negative values, although in the whole is compatible with zero within errors.

1. It is well known that on the basis of the operator product expansion the deep inelastic scattering (DIS) cross sections can be split into the leading twist (LT) and the higher twist (HT) contributions. To the moment the LT contribution is fairly well understood both from the theoretical and experimental points of view. The HT contribution is not so well explored. The theoretical investigations of HT meet with the difficulties because in the region where HT are most important the perturbative QCD calculations cannot be applied. There are only semi-qualitative phenomenological models for HT calculation in literature. These models are often based on the phenomenological considerations and contain adjusted parameters, which are to be determined from the experimental data. Unfortunately, relevant experimental data, especially on the longitudinal structure function $F_L$, are sparse, come from different experiments and therefore are difficult for interpretation. There are estimations of twist-4 contribution to the structure function $F_2$ [1, 2, 3] obtained from the combined fit to SLAC/BCDMS and SLAC/BCDMS/NMC data [4, 6]. Extraction of HT contribution to $F_3$ was obtained in [8] from the fit to CCFR data [10]. These estimations are model independent, i.e. do not imply any x-dependence of HT, and hence a phenomenological formula can be easily fitted to them. As to the experimental data on HT contribution to $F_L$, to the moment they are available only from the QCD motivated fits to world data on the structure function $R = \sigma_L/\sigma_T$. The first fit of this kind was presented in [11]. This fit was recently renewed in [13] with the inclusion of new data from the experiments E-143 and E-140X. The world data on $R$ were also analyzed using a QCD based model with the account of HT contribution [14]. As far some models predict the HT contribution to the structure function $F_L$, the comparison of these models with the data requires extraction of the HT contribution to $F_L$ from the data on the HT contribution to $R$ and $F_2$. This causes the problems with interpolation between data points and error propagation. In addition, the HT contributions to $R$, obtained in all these fits are model dependent, i.e. a priori suppose a certain x-dependence of HT.

2. In this paper we present the results of the analysis of DIS data aiming to obtain the estimation of the model independent HT contribution to $F_L$. The work is the continuation of our previous analysis [3], where estimation of the HT contribution to $F_2$ was obtained. Our consideration is limited by the region of $x > 0.3$, where the non-singlet approximation is valid. A data cut $x < 0.75$ was made to leave the region where deuteron effects are small. At the beginning the ansatz used in this work is essentially the same as in [3]. We fitted the
data within NLO QCD with the inclusion of target mass correction (TMC) [15] and twist-4 contribution in factorized form:

\[ F^{(P,D),HT}_2(x, Q^2) = F^{(P,D),TMC}_2(x, Q^2) \left[ 1 + \frac{h^{(P,D)}_2(x)}{Q^2} \right], \]

\[ F^{(P,D),TMC}_2(x, Q^2) = \frac{x^2}{\tau^{3/2}} F^{(P,D),LT}_2(\xi, Q^2) + 6 \frac{M^2 x^3}{Q^2 \tau^2} \int_1^\xi dz \frac{F^{(P,D),LT}_2(z, Q^2)}{z^2}, \]

where \( F^{(P,D),LT}_2(x, Q^2) \) are the LT terms,

\[ \xi = \frac{2x}{1 + \sqrt{\tau}}, \quad \tau = 1 + \frac{4M^2 x^2}{Q^2}, \]

\( M \) is nucleon mass, \( x \) and \( Q^2 \) are regular lepton scattering variables. The LT structure functions of proton and neutron were parametrized at the initial value of \( Q^2_0 = 9 \text{GeV}^2 \) as follows:

\[ F^p_2(x, Q_0) = A_p x^{a_p} (1 - x)^{b_p} \frac{2}{N_p}, \]

\[ F^n_2(x, Q_0) = A_n x^{a_n} (1 - x)^{b_n} \frac{1}{N_n}. \]

Here conventional normalization factors \( N_p \) and \( N_n \) are

\[ N_{p,n} = \int_0^1 dx x^{a_{p,n}-1} (1 - x)^{b_{p,n}}. \]

These distributions were evolved in NLO QCD approximation within \( \overline{MS} \) factorization scheme. The functions \( h^{(P,D)}_2(x) \) were parametrized in the model independent way: their values at \( x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 \) were fitted, between these points the functions were linearly interpolated.

Comparing to the work [3] we added to the analysis the NMC data [6] (30 points on proton and 30 points on deuterium targets) The number of data for each experiment and target are presented in Table I. We accounted for point-to-point correlations of data due to systematic errors analogously to our previous papers [7, 3, 9]. Systematic errors were convoluted into correlation matrix

\[ C_{ij} = \delta_{ij} \sigma_i \sigma_j + f_i f_j (s^K_i \cdot s^K_j), \]

where vectors \( s^K_i \) contain systematic errors, index \( K \) runs through data subsets which are uncorrelated between each other, \( i \) and \( j \) run through data points of these data subsets. Minimized functional has the form

\[ \chi^2 = \sum_{K,i,j} (f_i/\xi_K - y_i) E_{ij} (f_j/\xi_K - y_j), \]

where \( E_{ij} \) is the inverse of \( C_{ij} \). Dimension of \( s^K_i \) differs for various data set, the concrete numbers for each experiment are present in Table I. The factors \( \xi_K \) were introduced to allow for renormalization of some data sets. In our analysis these factors were released for the old SLAC experiments in view of possible normalization errors discussed in [12]. As for E-140,
Table 1: The number of data points (NDP) and the number of independent systematic errors (NSE) for the analysed data sets.

| Experiment | NDP(proton) | NDP(deuterium) | NSE |
|------------|-------------|----------------|-----|
| BCDMS      | 223         | 162            | 9   |
| E-49A      | 47          | 47             | 3   |
| E-49B      | 109         | 102            | 3   |
| E-61       | 6           | 6              | 3   |
| E-87       | 90          | 90             | 3   |
| E-89A      | 66          | 59             | 3   |
| E-89B      | 70          | 59             | 3   |
| E-139      | –           | 16             | 3   |
| E-140      | –           | 31             | 4   |
| NMC        | 30          | 30             | 13  |
| TOTAL      | 641         | 602            | 47  |

BCDMS and NMC data subsets, we fixed these factors at 1 and accounted for their published normalization errors in the general correlation matrix.

At the first stage of the analysis we reduced all $F_2$ data including BCDMS and NMC to the common value of $R$. The results of the fit within this ansatz are presented in Table II (column 2). For the comparison at column 1 we also presented the results of analogous fit from from [3], performed without the NMC data. Addition of the NMC data increased the value of $\alpha_s(M_Z)$, but within one standard deviation. In the whole the figures from column 2 are compatible with the results of earlier analysis [3].

The next step of our analysis was to change the form of HT contribution from factorized to an additive one:

$$F_2^{(P,D),HT}(x, Q^2) = F_2^{(P,D),TMC}(x, Q^2) + H_2^{(P,D)}(x) \frac{1 \text{ GeV}^2}{Q^2}$$

with $H_2^{(P,D)}(x)$ parametrized in the model independent form analogously to $h_2^{(P,D)}(x)$. We preferred to switch to this form because for the factorized parametrization HT term contains latent log factor and twist-6 contribution originating from $F_2^{LT}(x, Q)$ and target mass corrections respectively. In addition, this form is more convenient for comparison with some model predictions. The results of the fit with additive HT parametrization are presented in column 1 of Table III and Fig.1. For the comparison we also pictured in Fig.1 the value of $F_2^{TM C}(x, Q^2) \cdot h_2(x)$ for the fit from column 2 of Table II with factorized form of HT, calculated at $Q^2 = 2.5 \text{ GeV}^2$. One can see that the switching of the form leads to small decrease of HT terms at high $x$. Besides, their errors became smaller, meanwhile errors of the other parameters increased. We connect this effect with that the additive HT form is not so constrained as the factorized one. The change of the value of HT contribution is amplified due to large correlations of the HT terms with other parameters, especially with $\alpha_s$ (see discussion below).

To extract $F_L$ we replaced data on $F_2$ by the data on differential cross sections and
Table 2: The results of the fits with factorized parametrization of HT. The parameters $\xi$ describe the renormalization of the old SLAC data, $h_{2D}^{P,(3,4,5,6,7,8)}$ are the fitted values of the HT contribution at $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$. For the description of the columns see text.

|         | column 1 | column 2 |
|---------|----------|----------|
| $A_p$   | 0.516 ± 0.022 | 0.514 ± 0.021 |
| $a_p$   | 0.765 ± 0.028 | 0.766 ± 0.028 |
| $b_p$   | 3.692 ± 0.032 | 3.690 ± 0.032 |
| $A_n$   | 4.8 ± 4.1     | 4.8 ± 3.9     |
| $a_n$   | 0.118 ± 0.097 | 0.119 ± 0.095 |
| $b_n$   | 3.51 ± 0.11   | 3.51 ± 0.11   |
| $\alpha_s(M_Z)$ | 0.1180 ± 0.0017 | 0.1187 ± 0.0016 |
| $h_{2,3}^P$ | −0.120 ± 0.017 | −0.122 ± 0.017 |
| $h_{2,4}^P$ | −0.046 ± 0.025 | −0.054 ± 0.025 |
| $h_{2,5}^P$ | 0.059 ± 0.043 | 0.043 ± 0.042 |
| $h_{2,6}^P$ | 0.392 ± 0.076 | 0.363 ± 0.074 |
| $h_{2,7}^P$ | 0.82 ± 0.13 | 0.77 ± 0.12 |
| $h_{2,8}^P$ | 1.54 ± 0.25 | 1.47 ± 0.24 |
| $h_{2,3}^D$ | −0.123 ± 0.018 | −0.125 ± 0.018 |
| $h_{2,4}^D$ | −0.003 ± 0.026 | −0.012 ± 0.025 |
| $h_{2,5}^D$ | 0.162 ± 0.043 | 0.145 ± 0.042 |
| $h_{2,6}^D$ | 0.439 ± 0.076 | 0.410 ± 0.073 |
| $h_{2,7}^D$ | 0.79 ± 0.12 | 0.75 ± 0.11 |
| $h_{2,8}^D$ | 1.87 ± 0.26 | 1.81 ± 0.25 |
| $\xi_{P,49A}$ | 1.016 ± 0.018 | 1.016 ± 0.016 |
| $\xi_{D,49A}$ | 1.006 ± 0.017 | 1.008 ± 0.015 |
| $\xi_{P,49B}$ | 1.028 ± 0.018 | 1.028 ± 0.015 |
| $\xi_{D,49B}$ | 1.012 ± 0.017 | 1.013 ± 0.015 |
| $\xi_{P,61}$ | 1.021 ± 0.021 | 1.021 ± 0.019 |
| $\xi_{D,61}$ | 1.004 ± 0.019 | 1.006 ± 0.018 |
| $\xi_{P,87}$ | 1.025 ± 0.017 | 1.025 ± 0.015 |
| $\xi_{D,87}$ | 1.012 ± 0.017 | 1.013 ± 0.015 |
| $\xi_{P,89A}$ | 1.028 ± 0.021 | 1.029 ± 0.019 |
| $\xi_{D,89A}$ | 1.004 ± 0.021 | 1.006 ± 0.019 |
| $\xi_{P,89B}$ | 1.022 ± 0.017 | 1.021 ± 0.015 |
| $\xi_{D,89B}$ | 1.007 ± 0.017 | 1.008 ± 0.015 |
| $\xi_{D,139}$ | 1.009 ± 0.017 | 1.010 ± 0.015 |
| $\chi^2/NDP$ | 1178.9/1183 | 1258.4/1243 |
fitted them using the formula

\[ \frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2(s - M^2)}{Q^4} \left[ \left( 1 - y - \frac{(Mxy)^2}{Q^2} \right) F_2^{HT}(x, Q) + \right] 
\]

\[ + \left( 1 - \frac{m_l^2}{Q^2} \right) \frac{y^2}{2} \left( F_2^{HT}(x, Q) - F_L^{HT}(x, Q) \right), \]

where \( s \) is total c.m.s. energy, \( m_l \) is scattered lepton mass and \( y \) - lepton scattering variable. With the test purposes we first performed the fit with \( F_L \) form motivated by \( R_{1990} \) parametrization [11]:

\[ F_{L}^{(P,D),HT}(x, Q) = F_{L}^{(P,D),HT}(x, Q) \left[ 1 - \frac{1 + 4M^2x^2/Q^2}{1 + R(x, Q)} \right], \]

\[ R(x, Q) = \frac{b_1}{2\ln(Q/0.2)} \left[ 1 + \frac{12Q^2}{Q^2 + 1} \cdot \frac{0.125^2}{0.125^2 + x^2} \right] + b_2 \frac{1 GeV^2}{Q^2} + b_3 \frac{1 GeV^4}{Q^4 + 0.3^2} \]

with fitted parameters \( b_{1,2,3} \) and \( Q \) measured in GeV. This parametrization is constructed from the term with log \( Q \)-behaviour, which mimic LT contribution and HT terms with power \( Q \)-behaviour. The resulting values of the parameters \( b_1 = 0.100 \pm 0.042, b_3 = 0.46 \pm 0.11, \)

\( b_3 = -0.14 \pm 0.18 \) are in agreement with the values obtained in [11] (\( b_1 = 0.064, b_3 = 0.57, \)

\( b_3 = -0.35 \)). In our fit twist-6 contribution to \( R(x, Q) \) is compatible with zero. We can note in this connection, that the correlations between the parameters is large (see Table IV) and as a consequence the data used in the fit have limited potential both in separation of twist-4/twist-6 and log/power contributions to \( R \).

To achieve more precise determination of HT contribution to \( F_L \) one can substitute for the LT contribution a perturbative QCD formula [16] instead of phenomenological log term. In the leading order on \( \alpha_s \) and for nonsinglet case it looks like

\[ F_{L}^{(P,D),LT}(x, Q) = \frac{\alpha_s(Q)}{2\pi} \frac{8}{3} x^2 \int_x^1 \frac{dz}{z^3} F_{L}^{(P,D),LT}(z, Q). \]
Table 3: The results of the fits with additive parametrization of HT. Figures in parenthesis are global correlation coefficients. The parameters $\xi$ describe the renormalization of the old SLAC data, $H_{2,(3,4,5,6,7,8)}^{P,D}$ and $H_{L,(3,4,5,6,7,8)}^{P,D}$ are the fitted values of the HT contribution at $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$. For the description of the columns see text.

|       | 1            | 2            | 3            |
|-------|--------------|--------------|--------------|
| $A_p$ | 0.478 ± 0.021| 0.485 ± 0.023| 0.486 ± 0.022|
| $a_p$ | 0.830 ± 0.034| 0.816 ± 0.037| 0.817 ± 0.035|
| $b_p$ | 3.798 ± 0.043| 3.791 ± 0.047| 3.792 ± 0.045|
| $A_n$ | 4.8 ± 4.5    | 4.9 ± 4.8    | 4.7 ± 4.7    |
| $a_n$ | 0.12 ± 0.11  | 0.12 ± 0.13  | 0.12 ± 0.12  |
| $b_n$ | 3.58 ± 0.14  | 3.59 ± 0.15  | 3.58 ± 0.14  |
| $\alpha_s(M_Z)$ | 0.1190 ± 0.0021 | 0.1170 ± 0.0021 | 0.1170 ± 0.0021 |
| $H_{2,3}^P$ | -0.0374 ± 0.0067 | -0.0303 ± 0.0068 | -0.0273 ± 0.0067 |
| $H_{2,4}^P$ | -0.0159 ± 0.0085 | -0.0031 ± 0.0084 | -0.0059 ± 0.0082 |
| $H_{2,5}^P$ | 0.0080 ± 0.0099 | 0.0176 ± 0.0096 | 0.0189 ± 0.0095 |
| $H_{2,6}^P$ | 0.0416 ± 0.0096 | 0.0497 ± 0.0095 | 0.0494 ± 0.0092 |
| $H_{2,7}^P$ | 0.0404 ± 0.0073 | 0.0529 ± 0.0077 | 0.0501 ± 0.0073 |
| $H_{2,8}^P$ | 0.0323 ± 0.0057 | 0.0376 ± 0.0081 | 0.0400 ± 0.0070 |
| $H_{2,3}^D$ | -0.0348 ± 0.0068 | -0.0205 ± 0.0066 | -0.0221 ± 0.0065 |
| $H_{2,4}^D$ | -0.0029 ± 0.0071 | 0.0057 ± 0.0069 | 0.0067 ± 0.0068 |
| $H_{2,5}^D$ | 0.0195 ± 0.0074 | 0.0274 ± 0.0071 | 0.0269 ± 0.0070 |
| $H_{2,6}^D$ | 0.0330 ± 0.0069 | 0.0381 ± 0.0068 | 0.0380 ± 0.0067 |
| $H_{2,7}^D$ | 0.0301 ± 0.0050 | 0.0363 ± 0.0051 | 0.0372 ± 0.0050 |
| $H_{2,8}^D$ | 0.0278 ± 0.0041 | 0.0354 ± 0.0061 | 0.0341 ± 0.0053 |
| $H_{L,3}^P$ | - | 0.045 ± 0.027 | - |
| $H_{L,4}^P$ | - | 0.036 ± 0.024 | - |
| $H_{L,5}^P$ | - | -0.009 ± 0.023 | - |
| $H_{L,6}^P$ | - | -0.012 ± 0.017 | - |
| $H_{L,7}^P$ | - | 0.020 ± 0.013 | - |
| $H_{L,8}^P$ | - | 0.004 ± 0.019 | - |
| $H_{L,3}^D$ | - | 0.106 ± 0.016 | 0.095 ± 0.014 |
| $H_{L,4}^D$ | - | 0.044 ± 0.013 | 0.049 ± 0.012 |
| $H_{L,5}^D$ | - | 0.0343 ± 0.0089 | 0.031 ± 0.0087 |
| $H_{L,6}^D$ | - | 0.009 ± 0.010 | 0.0068 ± 0.0094 |
| $H_{L,7}^D$ | - | 0.0161 ± 0.0078 | 0.0195 ± 0.0069 |
| $H_{L,8}^D$ | - | 0.021 ± 0.018 | 0.016 ± 0.013 |
| $\xi_{P,49A}$ | 1.013 ± 0.016 | 1.017 ± 0.016 | 1.017 ± 0.016 |
| $\xi_{D,49A}$ | 1.005 ± 0.015 | 1.007 ± 0.015 | 1.009 ± 0.015 |
| $\xi_{P,49B}$ | 1.023 ± 0.015 | 1.039 ± 0.016 | 1.031 ± 0.015 |
| $\xi_{D,49B}$ | 1.010 ± 0.015 | 1.011 ± 0.015 | 1.014 ± 0.015 |
| $\xi_{P,61}$ | 1.017 ± 0.019 | 1.026 ± 0.019 | 1.028 ± 0.019 |
| $\xi_{D,61}$ | 1.004 ± 0.018 | 1.018 ± 0.018 | 1.018 ± 0.018 |
| $\xi_{P,87}$ | 1.019 ± 0.015 | 1.029 ± 0.016 | 1.025 ± 0.015 |
| $\xi_{D,87}$ | 1.008 ± 0.015 | 1.007 ± 0.015 | 1.011 ± 0.015 |
| $\xi_{P,89A}$ | 1.029 ± 0.019 | 1.057 ± 0.023 | 1.041 ± 0.020 |
| $\xi_{D,89A}$ | 1.005 ± 0.019 | 1.011 ± 0.020 | 1.011 ± 0.020 |
| $\xi_{P,89B}$ | 1.016 ± 0.015 | 1.024 ± 0.016 | 1.021 ± 0.015 |
| $\xi_{D,89B}$ | 1.003 ± 0.015 | 1.004 ± 0.015 | 1.007 ± 0.015 |
| $\chi^2/NDP$ | 1274.3/1223 | 1248.0/1243 | 1255.6/1243 |
Table 4: Correlation coefficients for the parameters of the fit motivated by $R_{1990}$ parametrization.

|   | $b_1$ | $b_2$ | $b_3$ |
|---|-------|-------|-------|
| $b_1$ | 1.00  | -0.61 | 0.38  |
| $b_2$ | -0.61 | 1.00  | -0.87 |
| $b_3$ | 0.38  | -0.87 | 1.00  |

Figure 2: The values of $H_2(x)$ from the fits with model independent form of HT contribution to $F_L$ (empty circles – unconstrained fit, full circles – the fit with the constraint $H_{P}^{P}(x) = H_{P}^{D}(x)$) For the better view the points are shifted to left/right along x-axis. The curves are predictions of IRR model for $Q^2 = 2$ GeV$^2$.

Figure 3: The same as Fig.2 for $H_L(x)$. 
We performed the fit using the QCD expression for $F_L$ with the account of TMC

$$F_{L,\text{TMC}}^{(P,D)}(x, Q^2) = F_{L,\text{TMC}}^{(P,D),LT}(x, Q^2) + \frac{x^2}{\tau^{3/2}} F_2^{(P,D),LT}(\xi, Q^2)(1 - \tau) +$$

$$+ \frac{M^2 x^3}{Q^2 \tau^2} (6 - 2\tau) \int_\xi^1 dz \frac{F_2^{(P,D),LT}(z, Q^2)}{z^2}$$

and additive form of HT contribution to $F_L$, i.e.

$$F_{L,\text{HT}}^{(P,D)}(x, Q^2) = F_{L,\text{TMC}}^{(P,D),LT}(x, Q^2) + H_L^{(P,D)}(x) \frac{1}{Q^2} GeV^2,$$

where $H_L^{(P,D)}(x)$ is parametrized in the model independent form analogously to $H_2^{(P,D)}(x)$ and $h_2^{(P,D)}(x)$. The results of this fit are presented in column 2 of Table III and in Figs. 2, 3.

One can see that with model independent parametrization of HT contribution to $F_L$, the renormalization factors for old SLAC data noticeably increased. We remind in this connection that in the source paper \[12\] these data were renormalized using the data of dedicated SLAC-E-140 experiment. As far the latest did not reported data on proton target, renormalization of proton data was performed using bridging through SLAC-E-49B data. There is no possibility to conclusively choose between our renormalization scheme and the one used in \[12\]. More proton data are necessary to adjust old SLAC results. As to deuteron data, their normalization factors practically do not deviate from 1. The errors of $H_L^{(P,D)}(x)$, due to SLAC-E-140 deuteron data, are significantly smaller, than for $H_P^{(P,D)}(x)$. In view of large errors of the last, HT contributions to $F_L$ for proton and deuteron are compatible within the errors and we performed one more fit imposing the constraint $H_L^{(P,D)}(x) = H_L^{(D)}(x)$. The results of this fit are presented in column 3 of Table IV and in Figs. 2, 3. One can see that $\chi^2$ obtained in this fit is practically equal to the value of $\chi^2$ obtained in the fit without constraints minus the number of additional parameters. Comparison with Fig. 1 shows, that if $F_L$ is fitted, the HT contribution to $F_2$ slightly growth. The value of strong coupling constant is insensitive to the constraint imposing. Their value

$$\alpha_s(M_Z) = 0.1170 \pm 0.0021 (\text{stat} + \text{syst}),$$

correspond to

$$\Lambda^{(3)}_{\overline{MS}} = 337 \pm 29 (\text{stat} + \text{syst}) \text{ MeV},$$

obtained at $Q^2 = 2 GeV^2$ with the help of relation

$$\alpha_s(Q) = \frac{2\pi}{\beta_0 \ln(Q/\Lambda)} \left[1 - \frac{2\pi}{\beta_0 \beta} \ln(2 \ln(Q/\Lambda))\right],$$

where

$$\beta_0 = 11 - \frac{2}{3} n_f, \quad \beta = \frac{2\pi \beta_0}{51 - \frac{19}{3} n_f}$$

and $n_f = 3$.

We checked how are the analysed data sensitive to the twist-6 contribution to $F_L$. For this purpose we fitted data with $F_L(x, Q^2)$ expressed as

$$F_{L,\text{HT}}^{(P,D)}(x, Q^2) = F_{L,\text{TMC}}^{(P,D),LT}(x, Q^2) + H_L(x) \frac{1}{Q^2} GeV^2 + H_{L}^{(4)}(x) \frac{1}{Q^4} GeV^4,$$
where functions $H_L^{(4)}(x)$ and $H_L(x)$ were the same for proton and deuteron and parametrized in model independent way. The resulting behaviour of $H_L^{(4)}(x)$ is presented in Fig. 4. One can observe the trend to the negative values at highest $x$, but with poor statistical significance. The $\chi^2$ decrease in this fit is about 25 (remind that standard deviation of $\chi^2$ is $\sqrt{2 \cdot NDF}$, i.e. is about 40 in our analysis). These results are in correspondence with the estimates of twist-6 contribution to $R(x, Q)$ presented above - the fitted twist-6 contribution to $F_L$ is at the level of one standard deviation. In our studies we did not completely accounted TMC correction of the order of $O(M^4/Q^4)$ and then, in a rigorous treatment, $H_L^{(4)}$ does not correspond to a pure dynamical twist-6 effects. Meanwhile the contribution of the omitted terms to $H_L^{(4)}$ estimated using formula from [15], do not exceed 0.001 in our region of $x$ and hence can be neglected.

3. For the comparison with other parametrizations we calculated deuteron $R(x, Q)$ using the relation

$$ R^{D,HT}(x, Q) = \frac{F_2^{D,HT}(x, Q)}{F_2^{D,HT}(x, Q) - F_L^{D,HT}(x, Q)} \left[ 1 + \frac{4M^2x^2}{Q^2} \right] - 1 $$

and the parameters values from column 3 of Table III. The obtained values of $R$ are presented in Fig. 5 together with $R_{1990}$ [11] and $R_{1998}$ [13] parametrizations. In average all three parametrizations coincide within errors, although our curves lay higher at the edges of $x$-region and lower in the middle.

The same tendency is valid for HT+TMC contribution to $R$, evaluated as the difference between $R^{D,HT}(x, Q)$ and $R^{D,LT}(x, Q)$, where

$$ R^{D,LT}(x, Q) = \frac{F_L^{D,LT}(x, Q)}{F_2^{D,LT}(x, Q) - F_L^{D,LT}(x, Q)}. $$

This contribution is presented in Fig.6 together with the power parts of the variant b) of $R_{1990}$ and $R_{1998}$ parametrizations. Since error bands are not available for latest ones, only central values are pictured. Relative difference between log parts of $R_{1990}$ and $R_{1998}$ parametrizations and $R^{D,LT}(x, Q)$ is very large at highest $x$, although absolute difference is not significant due to smallness of these terms. In Fig.8 we present the TMC contribution to $R$, calculated as
Figure 5: One standard deviation bands for our value of $R^{D,HT}(x, Q)$ (full lines), $R_{1990}$ (dashed lines) and $R_{1998}$ (dotted lines) at $Q^2 = 2 \text{ GeV}^2$. Our value of $R$ is obtained as spline interpolation between points $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$.

Figure 6: One standard deviation bands for HT+TMC contribution to $R$ (full lines). Dashed and dotted lines correspond to power contributions to $R_{1990}$ and $R_{1998}$. Dashed-dotted line represents the power term of BRY parametrization [14].
Figure 7: One standard deviation bands for LT contribution to $R$ (full lines). Dashed and dotted lines correspond to log contributions to $R_{1990}$ and $R_{1998}$.

Figure 8: One standard deviation bands for HT contribution (full lines) and TMC contribution (dashed lines) to $R$. Lower and upper bands for TMC are practically indistinguishable.
Figure 9: The E140X data on nucleon $R(x, Q)$ (empty circles). Inner bars correspond to statistic errors, total bars – statistic and systematic combined in quadratures. Also are presented calculations of deuteron $R(x, Q)$ performed on our fit (full circles). For the better view the points are shifted to left/right along x-axis.

Table 5: Coefficients of correlations between the HT parameters and $\alpha_s(M_Z)$. Figures in parenthesis are global correlation coefficients for the HT parameters.

| x    | $H^P_2(x)$ | $H^D_2(x)$ | $H_L(x)$ |
|------|------------|------------|----------|
| 0.3  | -0.507(0.914) | -0.707(0.964) | -0.029(0.868) |
| 0.4  | -0.847(0.955) | -0.910(0.976) | 0.122(0.852)  |
| 0.5  | -0.909(0.968) | -0.944(0.987) | 0.244(0.870)  |
| 0.6  | -0.905(0.971) | -0.917(0.977) | 0.222(0.866)  |
| 0.7  | -0.871(0.972) | -0.901(0.980) | 0.072(0.894)  |
| 0.8  | -0.460(0.868) | -0.429(0.875) | 0.156(0.870)  |

$R^{D,TMC} - R^{D,LT}$, where

$$R^{D,TMC}(x, Q) = \frac{F^D_{TMC}(x, Q)}{F^D_{TMC}(x, Q) - F^D_{LTMC}(x, Q)} \left[1 + \frac{4M^2x^2}{Q^2}\right] - 1,$$

and the HT contribution, calculated as $R^{D,HT} - R^{D,TMC}$. It is seen that the TMC contribution also is significantly smaller at large $x$ than the HT one, so the latter is dominating in this region. Meanwhile we should note in this connection that as it was observed in [8, 17], account of NNLO QCD can diminish the value of HT contribution. This effect can be seen in Fig.6, where we present power part of $R$ parametrization [14], obtained with the partial account of NNLO.

In Fig.9 data of SLAC-E140X experiment on nucleon $R(x, Q)$ [18], which were not included in the fit, are compared with our $R^D(x, Q)$, calculated at the parameters values from column 3 of Table III. One can observe a good agreement of the data and our parametrization. We also compared the results of model independent analysis with a predictions of infrared renormalon model (IRR) [19]. In this model the HT $x$-dependence is connected with the LT
x-dependence. In particular for nonsinglet case twist-4 contribution is expressed as

\[
H_{2,L}^{(P,D)}(x) = A'_2 \int_x^1 dz C_{2,L}(z) F_2^{(P,D),LT}(x/z, Q),
\]

\[
C_2(z) = -\frac{4}{(1-z)_+} + 2(2 + z + 6z^2) - 9\delta(1-z) - \delta'(1-z),
\]

\[
C_L(z) = 8z^2 - 4\delta(1-z),
\]

\[
A'_2 = -\frac{2C_F}{\beta_0} [\Lambda_R]^2 e^{-C},
\]

where \( C_F = 4/3 \), \( C = -5/3 \). The normalization factor \( \Lambda_R \) can be considered as a fitted parameter, or, in other approach, set equal to the value of \( \Lambda_{QCD} \). In Figs. 2,3 we present the IRR model predictions for \( Q^2 = 2 \text{ GeV}^2 \). At this value of \( Q \) the number of active fermions \( n_f = 3 \) and we set \( \Lambda_R = \Lambda_{(3)_{MS}} = 337 \text{ MeV} \), as obtained in our analysis. Parameters of LT structure functions were taken as in column 2 of Table III. One can see that the model describes the data on \( H_L(x) \) fairly well. This in agreement with the curves, presented in Fig. 6 of Ref. [20] (remind that our value of \( \Lambda \) is about 1.3 times larger, than used in [20] to calculate these curves). At the same time there is evident discrepancy between the model and the data on \( H_2(x) \). In this connection we should note that \( H_2(x) \) is strongly correlated with other parameters, and in particular with \( \alpha_s(M_Z) \). In the Table V we present a global correlation coefficients \( \rho_m \), calculated as

\[
\rho_m = \sqrt{1 - 1/e_{mm}/c_{mm}},
\]

where \( c_{mn} \) is parameters error matrix, \( e_{mn} \) – its inverse and indices \( m, n \) runs through fitted parameters. Global coefficients characterize the extent of correlation of a parameter with all others parameters. (In the case of the fit with two parameters \( \rho_1 = \rho_2 \) and equal to the correlation coefficient between these parameters). From the Table V one can see that \( H_L \) is almost uncorrelated with \( \alpha_s(M_Z) \) and is correlated with other parameters less than \( H_2 \). This fact can be readily understood qualitatively. The total value of \( R^{HT} \) is defined from the y-dependence of cross sections and is weakly correlated with \( \alpha_s \). Then, the correlation of HT contribution to \( F_L \) can arise only due to dependence of LT contribution on \( \alpha_s \). As far, the LT contribution is small comparing with the HT one (see Figs. 7,8) this dependence does not cause significant correlations of the HT contribution and \( \alpha_s \). Due to lower values of \( \rho \), \( H_L \) is more stable in respect with the change of the ansatz and input of the fit. The change of \( \alpha_s \) towards higher values would not affect \( H_L \), but can decrease \( H_2 \).

**Conclusion**

In conclusion, we performed NLO QCD fit of the combined SLAC/BCDMS/NMC DIS data at high \( x \). Model independent x-shape of high twist contribution to structure function \( F_L \) is extracted. Twist-4 contribution is found to be in fair agreement with the predictions of infrared renormalon model. Twist-6 contribution exhibits weak tendency to negative values, although in the whole is compatible with zero within errors.

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