What Have We Learned About and From $F_L(x, Q^2)$ at HERA?

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Abstract

Recently the H1 collaboration has published a “determination” of the structure function $F_L(x, Q^2)$ at low $x$. I address the question of how reliable this determination really is. I argue that it is in fact a consistency check of a given theoretical approach rather than a real determination of the value of $F_L(x, Q^2)$, but potentially a very useful one. I compare the consistency of different approaches, and indeed find that a LO–in–$\alpha_s$ calculation of structure functions is completely ruled out. I also find that the “determined” values of $F_L(x, Q^2)$ are surprisingly stable under changes in theoretical approach but, when working consistently within a well-defined theoretical framework, the values of $F_L(x, Q^2)$ implied are somewhat lower than previously quoted.
1. Introduction

The recent measurements of $F_2(x, Q^2)$ at HERA [1][2] have provided data on a structure function at far lower values of $x$ than any previous experiments. However, these measurements are not of the structure function directly. Rather they are of the differential cross-section

$$\frac{d^2\sigma}{dx dQ^2} \equiv \frac{2\pi\alpha^2}{xQ^4} \left[ (2(1-y)+y^2)F_2(x, Q^2) - y^2 F_L(x, Q^2) \right].$$

(1.1)

$Q^2$ is the squared four-momentum transferred in the lepton–proton scattering, $x$ is the scaling variable and $y = Q^2/sx$ where $s$ is the centre of mass energy squared. At HERA $s = 90200\text{GeV}^2$. However, $F_L(x, Q^2)$ is expected to be much smaller than $F_2(x, Q^2)$ for most of the parameter space in which measurements take place (it must be smaller than $F_2(x, Q^2)$ at all $x$ and $Q^2$), and within this parameter space $y$ is nearly always small (0.25 or less). Hence, the measurement is usually effectively of $F_2(x, Q^2)$.

However, at the fringe of parameter space, i.e. the highest $Q^2$ values for the lowest $x$ measurements, the values of $y$ can be somewhat larger, reaching $\approx 0.7$ at their maximum. For values of $y$ larger than about 0.25 the fact that $F_L(x, Q^2)$ is expected to have a non-zero value starts affecting the value of $F_2(x, Q^2)$ extracted from the measurement of the differential cross-section by about 1%. This increases to about $10-20\%$ at the highest $y$ values of 0.7 (clearly depending on the value of $F_L(x, Q^2)$ used). Hence in this region of parameter space, within the typical quoted errors, the HERA collaborations cannot be claiming to measure $F_2(x, Q^2)$ directly. It would perhaps be simplest to release data in this region of parameter space in the form of the differential cross-section rather than in terms of structure functions. However, both H1 and ZEUS choose to take the values of $F_L(x, Q^2)$ from some theoretical prediction, and extract the consequent “measured” values of $F_2(x, Q^2)$. Both collaborations obtain their values of $F_L(x, Q^2)$ from fits to the data using the usual approach of solving renormalization group equations for parton distributions and combining with coefficient functions for parton scattering with the calculations done to NLO–in-$\alpha_s$. In [1] $F_L(x, Q^2)$ is calculated using the GRV parameterization of partons [3], while in both of [2] it is calculated using the parton distributions obtained from their own iterative fit to $F_2(x, Q^2)$.

Thus, both collaborations produce values of $F_2(x, Q^2)$ which take into account the values of $F_L(x, Q^2)$ in an approximate manner. However, it is very important to remember that the values of $F_2(x, Q^2)$ quoted in [1] and [2] are not really measurements of $F_2(x, Q^2)$ in the high $y$ region, and any attempt to fit the HERA data using some theoretical model should take this into account. The values of $F_2(x, Q^2)$ used in a fit should really be those obtained from the measured differential cross-section and the prediction for $F_L(x, Q^2)$ consistent with the particular theoretical approach (it is difficult to imagine a genuine theoretical approach which does not calculate $F_L(x, Q^2)$ alongside $F_2(x, Q^2)$). Alternatively, the fit could be directly to the cross-section using the calculated values of both $F_2(x, Q^2)$ and $F_L(x, Q^2)$. (The two approaches are identical if the error on $F_2(x, Q^2)$ in the
former does not change with the predicted $F_L(x, Q^2)$, but is equal to its value when $F_L(x, Q^2) = 0$.) For any “conventional” NLO–in–$\alpha_s$ approach the differences will not be all that large, but when using different approaches, such as those involving summation of leading logs in $(1/x)$, or even a LO–in–$\alpha_s$ calculation, the differences in the predictions for $F_L(x, Q^2)$ can be large enough that the extracted values of $F_2(x, Q^2)$ move by amounts greater than their errors on the fringes of parameter space. Moreover, the direction of movement tends to be the same for every point, so ignoring this effect can have a sizeable effect on the fit. Unfortunately the consistent manner of comparing theory to $F_2(x, Q^2)$ values described above seems not to be done in practically all fits to data.

Recently the H1 collaboration took a rather different approach to the treatment of their high $y$ data [4]. Rather than adopt the procedure outlined above, they fitted the data for $F_2(x, Q^2)$ in the region of low $y$ using NLO–in–$\alpha_s$ QCD, used this fit to extrapolate $F_2(x, Q^2)$ into the high $y$ region, and from the difference between this extrapolation and the measured cross–section obtained a “measurement” of $F_L(x, Q^2)$. This determination of the value of $F_L(x, Q^2)$ relies only on assuming the correctness of the NLO–in–$\alpha_s$ fit for $F_2(x, Q^2)$. But if one is to assume this correctness then it seems perverse indeed not to assume the correctness of the NLO–in–$\alpha_s$ $F_L(x, Q^2)$ that is predicted as a result. Hence, if the theoretical approach is correct, $F_L(x, Q^2)$ is already determined. The difference between the measured cross–section and the extrapolated $F_2(x, Q^2)$ then provides nothing more than a consistency check: if it disagrees with the predicted $F_L(x, Q^2)$ it suggests that the theory is wrong, if it agrees then the theory is not necessarily wrong. This is all that one can conclude. There may well be other theoretical approaches which fit the low $y$ data equally well (or better) but which have different extrapolations and/or predictions for $F_L(x, Q^2)$. Indeed, in the small $x$ region this is likely since higher orders in $\alpha_s$ are accompanied by higher order terms in $\ln(1/x)$, and these leading $\ln(1/x)$ terms may well introduce important corrections to the standard approach. If the extrapolations and predictions match for these other approaches, then the predicted $F_L(x, Q^2)$ is just as likely to be correct, but is no more a real measurement than any other matching case.

Hence, the procedure adopted by the H1 collaboration is not in itself a real measurement of $F_L(x, Q^2)$. However, it is potentially a very useful way of discriminating between different theoretical approaches. The consistency of the predicted $F_L(x, Q^2)$ and the “measured” $F_L(x, Q^2)$ for a given approach is a very non–trivial check on the theory since, if the free parameters in the theory are already tied down by a fit for high $y$ data, then it is a parameter free check on a particular relationship between $F_2(x, Q^2)$ and $F_L(x, Q^2)$. Hence, in this paper I will present the approach used by H1 in the manner of a consistency check. I will do this for four different types of approach: a fit analogous to that of H1 in [4], which I will demonstrate is not actually a useful NLO–in–$\alpha_s$ fit: a more correct NLO–in–$\alpha_s$ fit: a LO–in–$\alpha_s$ fit: and a fit using the LORSC approach which includes leading $\ln(1/x)$ terms[5]. I will also include more data at high $y$ than that in [4]. I will find that the “measured” $F_L(x, Q^2)$ is very similar in each of the three latter approaches, but
lower than in the first one. However, this similarity would start to disappear at higher \( y \). I also find that the data with its current errors is not yet sufficient to rule out any approach other than the LO–in–\( \alpha_s \) fit, which it does rule out very clearly. One can, if one wants, interpret the “measurements” of \( F_L(x, Q^2) \) within the two approaches satisfying consistency to be the likely values of \( F_L(x, Q^2) \), but this is not really fundamentally different from quoting instead the values of \( F_L(x, Q^2) \) predicted within either of these approaches.

2. Comparison of Different Theoretical Approaches.

I compare the four different approaches outlined above. First I discuss the data to be used and the accuracy of \( F_L(x, Q^2) \) achieved. I define a rescaled differential cross–section

\[
\tilde{\sigma}(x, Q^2) = \frac{Q^4 x}{2\pi\alpha^2} \frac{1}{(2(1 - y) + y^2)} \frac{d^2\sigma}{dxdQ^2}.
\]

(2.1)

Hence I may write

\[
\tilde{\sigma}(x, Q^2) = F_2(x, Q^2) - Y^{-1} \cdot F_L(x, Q^2), \quad Y = \frac{(2(1 - y) + y^2)}{y^2}.
\]

(2.2)

This is clearly a useful definition since in the limit \( F_L(x, Q^2) \to 0 \) or \( y \to 0 \), \( \tilde{\sigma} = F_2(x, Q^2) \). Hence it allows a simple parameterization of the effect of non–zero \( F_L(x, Q^2) \) on the extracted \( F_2(x, Q^2) \). From (2.2) it is clear that denoting \( F_2(x, Q^2) \) obtained from the the fit by \( F_2^p(x, Q^2) \), the “measured” value of \( F_L(x, Q^2) \), denoted by \( F_2^m(x, Q^2) \), is given by

\[
F_2^m(x, Q^2) = Y(F_2^p(x, Q^2) - \tilde{\sigma}(x, Q^2)),
\]

(2.3)

and that the error in \( F_2^m(x, Q^2) \) is

\[
\Delta F_2^m(x, Q^2) = Y \cdot \left[ (\Delta \tilde{\sigma}(x, Q^2))^2 + (\Delta F_2^p(x, Q^2))^2 \right]^{\frac{1}{2}},
\]

(2.4)

where \( \Delta \tilde{\sigma}(x, Q^2) \gg \Delta F_2^p(x, Q^2) \) in practice. Hence, the error on \( F_2^m(x, Q^2) \) depends linearly on the error on the measurement of the cross–section and roughly quadratically on \( y^{-1} \). Although this means that, if one wishes to make a sensible consistency check for \( F_2^m(x, Q^2) \) (by comparing to the predicted \( F_2^p(x, Q^2) \)), it is necessary to have high \( y \), a relatively small decrease in \( y \) can be countered by a large decrease in the error on the measurement of \( \tilde{\sigma}(x, Q^2) \). For example, an error of 5% on the measurement of \( \tilde{\sigma}(x, Q^2) \) at \( y = 0.6 \) leads to a more accurate determination of \( F_2^m(x, Q^2) \) than an error of 10% on the measurement of \( \tilde{\sigma}(x, Q^2) \) at \( y = 0.7 \). Examining the full range of data in [1] and [2] there are a number of points which give an accuracy of measurement of \( F_2^m(x, Q^2) \) which is comparable to the 6 points in [4]. Taking a cut on the error produced for \( F_2^m(x, Q^2) \) to be 0.3 and a lower cut on \( Q^2 \) of 5GeV\(^2\) (in order not to be too close to the charm
threshold), I find that there is one point in [1], and 8 points in the latter of [2]. Of these additional points, 5 of the ZEUS points have $y$ comparable to the 0.7 in [4], while the other 3 ZEUS points and the H1 point have somewhat smaller $y$ but better accuracy in measurement. The full set of data points to be used is shown in table 1.

Now that the relevant data are defined, let me consider the fits. First I note that the heavy quark thresholds are treated rather differently than in [4]. Rather than using the NLO–in–$\alpha_s$ fixed flavour scheme, where the charm contribution to the structure function is entirely generated by coefficient functions convoluted with light parton distributions, I simply change the number of active quark flavours discontinuously at $m_h^2 = Q^2$, and treat the heavy quarks as massless above this. I take $m_c^2 = 2.75\text{GeV}^2$ in order to give a good description of the charm data[6][7], and choose $m_b^2 = 20\text{GeV}^2$. Neither of the above approaches for the treatment of charm is entirely satisfactory since the former does not sum leading logarithms in $Q^2/m_c^2$ while the latter does not treat the threshold in a correct manner. In practice the former is more accurate for $Q^2 < 10\text{GeV}^2$ and the latter more accurate for $Q^2 > 20\text{GeV}^2$. For $10\text{GeV}^2 < Q^2 < 20\text{GeV}^2$ the approaches are of roughly equal validity.

In comparing the predicted and measured $F_L(x, Q^2)$ I adopt a different procedure from H1 and assume that the variation in $F^p_L(x, Q^2)$ is entirely due to variations within a given theoretical approach fitting a particular set of data. The difference between the various theoretical approaches, and/or from using different sets of data will be seen in the four different sets of results. Within a given approach the variation in $F^p_L(x, Q^2)$ is due only to letting the $\chi^2$ for the fit vary to the extent that the quality has a confidence level only a little lower than the absolute best fit. I will discuss the results of this later, but mention here that the errors are very small. Also, since I have as much faith in the NLO–in–$\alpha_s$ calculation of $F^p_L(x, Q^2)$ as of $F^p_2(x, Q^2)$, and hence am only performing a consistency check, I include all data other than those points in table 1 in the fit, and let the value of $F^p_L(x, Q^2)$ used in the extraction of $F^p_2(x, Q^2)$ for these points be that predicted by the parameters determined in the fit (i.e. the fit is iterative). Thus, the procedure differs from [4] in including the small number of points with $y > 0.35$ not in table 1, which has an extremely small effect on the fit, and also by using a particular $F^p_L(x, Q^2)$ for the points in the fit rather than letting it vary over the rather extreme range of $0 \rightarrow F_2(x, Q^2)$. Avoiding this second variation means that I do not have the large systematic error in the extrapolated $F^p_2(x, Q^2)$ which is seen in [4]. Overall the method of defining the uncertainties due to the fitting assumptions reduces them immensely compared to those in [4].

1. First I consider the analogous fit to that performed by H1. In this the fit is performed to BCDMS [8] and H1 data only. This leads to a problem often encountered (or rather ignored) by those attempting to fit small $x$ structure function data: the parameters defining the parton distributions which are determined by the best fit turn out to be completely incompatible with some perfectly respectable data which have not been included in the fit. The parameters determined by the the
fit are not included in [4]. However, performing a fit to precisely the same data I obtained a gluon distribution which is far too small at $x > 0.1$ to be consistent with the WA70 prompt photon data [9], and is even much smaller than the gluon produced by the H1 collaboration in the fit in [1]. Although there is a certain amount of uncertainty about the true accuracy of the prompt photon data, the gluon obtained is very much smaller than any possible lower limit on the data. It is also far too small to be consistent with the EMC charm data [6] using any sensible treatment of charm, and I note that the gluon required by this charm data is actually similar to that required by the WA70 prompt photon data, i.e. $\sim 2.5(1-x)^6$ at $Q^2 = 5$GeV$^2$. Since there is so little gluon at high $x$ in my fit, the momentum sum rule allows much more at small $x$. This enables the H1 data to be fit well even using a low value of $\Lambda_{QCD}$ which is preferred by the BCDMS data. Indeed, my fit chooses $\Lambda_{QCD}^{n_f=4} = 186$MeV which is very similar to the H1 $\Lambda_{QCD}^{n_f=3} = 210$MeV. This should be compared with those analyses of only H1 data which produce $\Lambda_{QCD}^{n_f=4} \approx 350$MeV when the high $x$ parton distributions are constrained differently [10].

Ignoring this very serious problem, the fit is performed as in [4], and considers only the same points as [4] for measuring $F_L(x, Q^2)$. As already mentioned, the only uncertainty in $F^p_L(x, Q^2)$ is that obtained in allowing the quality of the fit to vary. For this fit to a relatively small number of data points the variation in $F^p_L(x, Q^2)$ while maintaining a “good” quality fit is not insignificant. However, since I am not really taking this fit seriously anyway, but are only using it for illustrative purposes, I quote the error as being due to the cross–section measurement only. As one would expect, the results of this procedure are very similar to those in [4]. The fit to the small $x$ H1 data is very similar, as is the extrapolation. The values of $F^m_L(x, Q^2)$ for the 6 data points for $\tilde{\sigma}$ used in [4] are shown in table 2 along with the values “measured” in [4](denoted by $F^m_L(x, Q^2)$[4]) and the values of $F^p_L(x, Q^2)$. The results are also displayed in fig. 1, where they are compared directly with those in [4]. One can see that the results for $F^m_L(x, Q^2)$ are indeed very similar to those in [4]. The predicted $F_L(x, Q^2)$ is very slightly higher than in [4] at lowish $Q^2$ but the two converge at higher $Q^2$, which can be interpreted as the effects of the different treatment of charm in the two approaches not compensating exactly at lowish $Q^2$, but the difference disappearing at higher $Q^2$, as expected. Hence, I have very good compatibility with [4] so far.

I am now able to explore the implications of this type of fit quantitatively. I do indeed have the previously mentioned dramatic inconsistency with the prompt photon data and the EMC charm data. Also, if I simply put the parameters obtained by the previous fit into a global fit to structure function data, i.e include NMC [11], CCFR [12] and ZEUS data, then I find that the fit to ZEUS data is a little worse than that obtained form a global NLO–in–$\alpha_s$ fit, and the fit to NMC and CCFR data is very much worse. Overall, the global fit is completely uncompetitive with the fits produced by e.g. MRS or CTEQ even ignoring the particular problem of the high $x$ gluon. Hence, the type of fit performed above certainly does not lead to a correct parameterization of the parton distributions and hence extrapolation of the structure function. Thus, even assuming the
correctness of the NLO–in–$\alpha_s$ calculation of structure functions one does not really learn anything concrete about $F_L(x, Q^2)$ from the above fit, or from [4].

2. In order to make a more reliable consistency check for the high $y$ data I now consider the implications of using precisely the same theoretical framework but including more data in the fit. In order to see the correct results of using the NLO–in–$\alpha_s$ calculation I must determine the free parameters in the calculation by fitting to all available structure function data, with some appropriate cuts, and also imposing the constraint that the charm structure function data from EMC [6] and H1 [7] is also described well. In practice I perform the fit to data in precisely the same manner as in [5] except for one minor point. The ZEUS shifted vertex data in the latter of [2] seems to be systematically larger than the data in the former. Hence I allow their normalization to be 0.98 that of the nominal vertex data, i.e. the amount allowed by the uncorrelated errors on normalization (the fit would actually prefer 0.97). Hence, the fit is almost identical to the NLO1 fit in [5], but this shift in normalization allows the $\chi^2$ to be a few points better. It is qualitatively a great deal different from the fit discussed above. The constraint on the gluon at high $x$ results in there being much less gluon at small $x$ and the value of $\Lambda_{QCD}^{n_f=4}$ rises to 315MeV in order to produce a satisfactory fit to small $x$ data. This rise in $\Lambda_{QCD}^{n_f=4}$ results in a worse fit to BCDMS data, but the total fit quality is a great deal better than that for the type of parameterization above. In the fit the normalization required by the H1 data is 0.985 and for the ZEUS data it is 0.99. The quality of the absolute best fit has a rather low confidence level (assuming Gaussian errors) of 6%. Letting this reduce to 1% results in a variation of $F_L^m(x, Q^2)$ which when inserted in (2.4) leads to negligible effect and I take the error to be that due to $\Delta \tilde{\sigma}(x, Q^2)$ alone.

Using this fit the form of $F_L^p(x, Q^2)$ is rather different at small $x$ from that obtained using the first procedure. The growth with $Q^2$ is slower than above, even taking into account the different normalization of the data, and hence in a given $x$ bin the value of $F_L^p(x, Q^2)$ becomes progressively smaller as $y$ increases. From (2.3) it is obvious that this will result in a lower value of $F_L^m(x, Q^2)$. The results for $F_L^m(x, Q^2)$, along with $F_L^p(x, Q^2)$ are shown in table 3 and displayed in fig. 2 where now I use all 15 data points. (I display the extracted points with a negative value of $F_L^m(x, Q^2)$ if this is what is obtained from the use of (2.2), even though this is forbidden in practice.) In the top figure all data with $y \approx 0.7$ is shown, while the lower figure contains the points with rather lower $y$. The measured values are indeed smaller than previously. Compared with the values in [4] $F_L^m(x, Q^2)$ is about 0.1 smaller for the lowest $x$ points increasing to more than 0.15 lower for the highest $x$ and $Q^2$ points. Hence, for some of these points the value of $F_L^m(x, Q^2)$ reduces to less than 60% of its value in [4]. By examination of fig. 2 one sees that, as well as the values of $F_L^m(x, Q^2)$ for the 6 points corresponding to those in [4] having reduced significantly, the inclusion of the other data points tends also to imply a somewhat lower value of $F_L^m(x, Q^2)$. A simple unweighted average of the 6 points in [4] gives $F_L^A(x, Q^2) = 0.48$. The same 6 points in this extraction yield $F_L^A(x, Q^2) = 0.34$, while for all 11 points with $y \approx 0.7$ I get $F_L^A(x, Q^2) = 0.27$. I claim that if one
believes the validity of the NLO–in–$\alpha_s$ calculation then the values in table 3 are the most reliable extractions of $F_L^m(x, Q^2)$ since they result from a much more realistic fit. However, taking these values results in a rather different quantitative conclusion on the value of $F_L^m(x, Q^2)$ for $y \approx 0.7$ for $0.001 > x > 0.0001$ from that reached in [4], i.e. it is $\approx 60\%$ of that value.

Also shown in table 3 and fig. 2 is the theoretical prediction $F_L^p(x, Q^2)$ for this fit. As in the previous fit, and as in [4], it is calculated using the NLO–in–$\alpha_s$ parton distributions obtained from the fit to $F_2^m(x, Q^2)$ and the order $\alpha_s$ and order $\alpha_s^2$ longitudinal coefficient functions[13]. Hence, I use coefficient functions at one higher order for the longitudinal coefficient function than for $F_2(x, Q^2)$ because they begin at one higher order. This is the correct procedure to obtain an expression consistent with the use of the two–loop coupling constant in a given renormalization scheme. $F_L^p(x, Q^2)$ is a little smaller than that in the first approach in the region shown (it is actually larger for $x > 0.001$), with the difference becoming larger at smaller $x$. This qualitative result is due to the gluon being rather flatter at small $x$ than the gluon in the former approach, which grows quickly as $x$ decreases. As one can see from fig. 2 there is reasonable agreement between the prediction and the measurement, even though there seems to a systematic trend for the values of $F_L^m(x, Q^2)$ to fall as $x$ increases while over most of the range the values of $F_L^p(x, Q^2)$ increase slowly with increasing $x$. The comparison of $F_L^m(x, Q^2)$ to $F_L^p(x, Q^2)$ yields a $\chi^2$ of 13.9 for 15 points. Thus, this consistency check gives no real reason to distrust the NLO–in–$\alpha_s$ approach. However, it may be argued that both the quality of the global fit and the comparison of $F_L^m(x, Q^2)$ with $F_L^p(x, Q^2)$ in the high $y$ region do not give overwhelming support for this approach.

3. I now consider a fit to precisely the same data but using a different theoretical framework. I use the LORSC calculation of structure functions discussed in [5], which includes all term which are of lowest order in $\alpha_s$ for all different types of term in $x$, and does so in terms of physical quantities rather than unphysical, definition–dependent parton distributions and coefficient functions (see [14] for a definition of physical anomalous dimensions). Hence it includes a summation of the so–called leading ln$(1/x)$ terms for physical quantities. It is only done to leading order because the full set of terms required for the NLO calculation is not yet known. The fit uses leading order $\Lambda_{QCD}^{n_f=4} = 100\text{MeV}$, which corresponds to $\alpha_s(M_Z^2) = 0.115$. In this fit the normalization required by the H1 data is 1.00 and for the ZEUS data it is 1.015. The quality of the absolute best fit has a higher confidence level than that of the NLO–in–$\alpha_s$ fit, i.e. 34%. Letting this reduce to 10% results in a variation of $F_2^p(x, Q^2)$ which again when inserted in (2.4) leads to negligible effect, and I take the error to be that due to $\Delta\tilde{\sigma}(x, Q^2)$ alone. If I were to let the confidence level reduce to 1%, as in the NLO–in–$\alpha_s$ fit then the uncertainty in $F_2^p(x, Q^2)$ would start to increase the error on $F_L^m(x, Q^2)$ significantly.

Using this approach the values of $F_2^p(x, Q^2)$ in the region of $x$ under consideration are about 1.015 times those in the NLO–in–$\alpha_s$ approach at $Q^2 = 6\text{GeV}^2$. Taking into account the difference in normalization of the data in the two approaches this means that when using H1 data the comparison
to data is very similar for the two approaches at this $Q^2$, while for the ZEUS data the LORSC $F_T^p(x,Q^2)$ is slightly smaller relative to data than the NLO–in–$\alpha_s$ prediction (due to the 2.5% shift in normalization of the data). The growth of $F_T^p(x,Q^2)$ with $Q^2$ is greater in the LORSC approach, even taking into account the normalization difference, and this leads to the better global fit, as discussed in [5]. This greater growth means that the values of $F_L^m(x,Q^2)$ extracted using this approach start out very similar to the NLO–in–$\alpha_s$ approach for H1 data and $Q^2$ not much above 6GeV$^2$, but steadily increase with respect to them as $Q^2$ increases. For the ZEUS data the larger normalization shift leads to the LORSC values of $F_L^m(x,Q^2)$ being slightly smaller than those for the NLO–in–$\alpha_s$ fit for lowish $Q^2$, but they increase steadily above those for the NLO–in–$\alpha_s$ fit at higher $Q^2$. The values of $F_L^m(x,Q^2)$ are shown in table 4 and fig. 3 and by comparing to table 3 and fig. 2 the trend described above can be easily observed. It results in the average value of $F_L^m(x,Q^2)$ being slightly higher in this approach than the previous one, and in the tendency of the values of $F_L^m(x,Q^2)$ to fall with increasing $x$ or $Q^2$ to reduce. The discrepancy between the extracted values for the two theoretical approaches, though not large for the points considered in this paper, would continue to increase for increasing $Q^2$.

Also shown in table 4 and fig. 3 is the theoretical prediction $F_L^p(x,Q^2)$ for this fit. As discussed in [5] it is smaller than that in the NLO–in–$\alpha_s$ approach in the region shown but the difference becomes less at smaller $x$, where the NLO–in–$\alpha_s$ curve at constant $y = 0.7$ falls as $x$ goes to 0.0001 and below, while in the LORSC approach it stays more constant. As one can see from fig. 3 there is again reasonable agreement between the prediction and the measurement, though some of the values of $F_L^m(x,Q^2)$ at smaller $x$ lie quite a long way above the $F_L^p(x,Q^2)$ curve. The comparison of $F_L^m(x,Q^2)$ to $F_L^p(x,Q^2)$ yields a $\chi^2$ of 12.4 for 15 points. This is very slightly better than in the NLO–in–$\alpha_s$ approach. Hence, this consistency check is certainly satisfactory, giving no evidence for the failure of the LORSC approach. Indeed, it is even slightly better than the NLO–in–$\alpha_s$ approach which has a rather less successful global fit to data.

4. Finally I consider the consequences of using a simple LO–in–$\alpha_s$ calculation of the structure functions. Methods of fitting the small $x$ data based simply on the LO–in–$\alpha_s$ calculations have not been uncommon in recent years, the “double asymptotic scaling” formula comes from this calculation [15], and have been advertised as being relatively successful, especially if $Q^2$ is not too low. I find that the standard approach is rather too simplistic, and in fact the LO–in–$\alpha_s$ approach is completely ruled out. If one performs a global fit to structure function data other than that used in the extraction of $F_L^m(x,Q^2)$ and using simply those values of $F_2(x,Q^2)$ advertised in [1] and [2], then the result, requiring the same normalizations as the NLO–in–$\alpha_s$ approach, is very nearly as good as for the NLO–in–$\alpha_s$ fit over the whole $x$ range. It yields a $\chi^2$ of $\sim 20$ more for $\sim 1100$ points and a confidence level of 2%, compared to the earlier 6%. However, the quality of the fit to the small $x$ data is achieved at the expense of having a fairly large coupling constant, one–loop $\alpha_s(M^2_z) = 0.124$ and a much larger small $x$ gluon than the NLO–in–$\alpha_s$. This, coupled with the
fact the the NLO corrections to $F_L(x, Q^2)$ are negative at small $x$, leads to $F_T^p(x, Q^2)$ being a great deal larger at small $x$ than in the NLO–in–$\alpha_s$ approach. Moreover, as in the NLO–in–$\alpha_s$ approach it is the small $x$ – large $y$ region where $F_T^p(x, Q^2)$ is beginning to systematically undershoot data. Hence, correcting the values of $F_T^m(x, Q^2)$ in the fit above for the correct values of $F_T^p(x, Q^2)$ leads to a worsening of the fit by $\sim 25$ (even after the fit is redone) and a consequent reduction of the confidence level to 0.5%. This effect, which is always ignored in LO–in–$\alpha_s$ fits to structure function data, clearly weakens the support for this type of approach considerably.

It is when I perform the consistency check between the values of $F_T^m(x, Q^2)$ and $F_T^p(x, Q^2)$, i.e. examine those points which are most sensitive to $F_L(x, Q^2)$, that the most dramatic effects are seen. The values of $F_T^p(x, Q^2)$ are fairly similar to those in the NLO–in–$\alpha_s$ case, and hence so are the values of $F_T^m(x, Q^2)$. Also, any significant change in the extrapolated $F_T^p(x, Q^2)$ reduces the confidence level of the fit to unacceptable levels, so once again the error on $F_T^m(x, Q^2)$ is taken to be that on $\tilde{\sigma}(x, Q^2)$ alone. However, the values of $F_T^p(x, Q^2)$ are very much larger than in the previous approaches. The set of values of $F_T^m(x, Q^2)$ and $F_T^p(x, Q^2)$ are shown in table 5 and displayed in fig. 4 . It is immediately clear that the measured values undershoot the predicted values significantly. A calculation of the $\chi^2$ is not necessary to prove that an extrapolation into the region where both $F_2(x, Q^2)$ and $F_L(x, Q^2)$ make important contributions to the cross–section rules out a LO–in–$\alpha_s$ calculation of structure functions completely.

An alternative way to demonstrate this is to compare the values of $F_T^p(x, Q^2)$ obtained from the fit to the low $y$ data to the “measured” values $F_T^m(x, Q^2)$ obtained from the measurement of the cross–section and the predicted values of $F_T^p(x, Q^2)$. This is shown for two $x$ bins in fig. 5, and it is very easy to see that the large predicted $F_T^p(x, Q^2)$ combined with the measured $\tilde{\sigma}(x, Q^2)$ leads to a large upturn in the data, above the theoretical curve, as $y$ becomes larger. A very similar effect is seen in all the different $x$ bins, and is clear evidence that a LO–in–$\alpha_s$ fit is completely ruled out by high $y$ data. I note that in this LO–in–$\alpha_s$ fit a large component of the small $x$ $F_2(x, Q^2)$ comes from the steepness of the input quark. If I were to follow the reasoning behind double asymptotic scaling, i.e. that the small–$x$ inputs should be flat at low $Q^2$, and the rise generated by evolution, then not only would the quality of the global fit fall, but the value of $F_T^p(x, Q^2)$ would be even larger, and this clear discrepancy between theory and data at high $y$ would become even more pronounced. Therefore, this approach is very strongly ruled out in the region of small $x$ and large $y$. The fact that the LO–in–$\alpha_s$ approach fails badly, but acquires very important small–$x$ corrections at NLO–in–$\alpha_s$ also implies that the large small–$x$ corrections at higher orders in $\alpha_s$ should be important, although, as already mentioned, there is no overwhelming evidence for this yet.

The three different theoretical approaches used above are far from being the full set proposed to describe small $x$ physics, and this technique can be applied to others. However, perhaps rather alarmingly they are the only approaches which have been used in fits to a global range of data and
which are fully constrained (except for approximate fits using factorization–scheme–dependent, and hence incorrect methods of including leading \(\ln(1/x)\) terms within the collinear factorization framework), and considerable work would be required to use any other approach.

I also note that at rather higher \(x\) than considered in this paper there have been a number of direct measurements of \(F_L(x, Q^2)\): the SLAC hydrogen and deuterium scattering experiments [16], the SLAC E140 experiments [17], CDHSW [18], BCDMS [19], CCFR [20] and NMC [11]. This data has fairly large error bars, and much is in the region of the charm threshold. However, it seems that with a better treatment of this threshold than used in this paper then either the NLO–in–\(\alpha_s\) approach or the LORSC approach would fit the data fairly well, with the former tending to be perhaps a little high, and the latter perhaps a little low. (The \(R_{QCD}\) curve in [11] should be treated with caution since it uses the gluon from [1], which has been criticised above, and a LO–in–\(\alpha_s\) formula using four massless quarks rather than the appropriate NLO–in–\(\alpha_s\) formula with massive charm quarks.)

The very low \(Q^2\) data from these measurements gives some evidence for higher twist effects [21].

I finally note that during the preparation of this article some new preliminary data at high \(y\) has been released by H1 [22]. There is new data at \(y = 0.7\) for the same values of \(Q^2\), except for the lowest \(Q^2\) bin, which seems to improve the comparison for both the NLO–in–\(\alpha_s\) fit and the LORSC fit, i.e. the measured cross–section for \(Q^2 = 12\text{GeV}^2\) and \(20\text{GeV}^2\) increase, while that for \(25\text{GeV}^2\) goes down a little. The improvement for the LORSC fit appears to be better than that for the NLO–in–\(\alpha_s\) fit. The comparison for the LO–in–\(\alpha_s\) will get even worse. There is also some data at \(y = 0.82\) with large error bars. Here the cross–section measurements look rather low. The QCD study is presumably performed in the same manner as in [4]. A detailed study in the manner of this paper will await final data.

3. Conclusions.

In this paper I have examined the implications of the idea proposed in [4] of extrapolating a fit performed to structure function data at relatively low \(y\) into the region of high \(y\), where the longitudinal structure function starts to make an impact on the cross–section. In [4] it was assumed that believing the extrapolation of \(F_2(x, Q^2)\) into this high \(y\) region using a given theoretical framework, the difference between the extrapolation and the measured cross–section would give a measurement of \(F_2(x, Q^2)\). In this paper I have worked on a different principle, pointing out that different theoretical models will lead to different extrapolations, and also different predictions for \(F_L(x, Q^2)\). All one is really examining is whether, in the region where both structure functions contribute to the cross-section, the data on the cross–section are consistent with the theory. One way to present this is to compare the “measured” values of \(F_L(x, Q^2)\) with the predicted values.
I have first done this using the same theoretical framework as in [4] (up to a different treatment of charm mass effects) and fitting to the same “low $y$” data. This provides results almost identical to [4], which may be naively interpreted as consistency of the NLO–$\alpha_s$ structure functions at high $y$. However, I have also argued very strongly that this type of fit is very badly underconstrained, particularly concerning the gluon for $x > 0.1$, and is not useful. I have then repeated the procedure using a NLO–$\alpha_s$ global fit at low $y$ which is a great deal more constraining. I have also used points in the latter of [2], and also in [1], which give similar uncertainty on an extracted $F_L(x, Q^2)$ to those points in [4]. Even using this same theoretical framework, the conclusion concerning the measured values of $F_L(x, Q^2)$ changes somewhat, with the average value being $\sim 60\%$ those in [4]. The agreement between the “measured” and predicted values is good.

Using two further theoretical approaches, the LORSC calculation and the LO–$\alpha_s$ calculation fits to the same low $y$ data the “measured” values of $F_L(x, Q^2)$ do not change by very large amounts, although the changes would quickly increase with increasing $y$. The predictions for $F_L(x, Q^2)$ are however different to the previous case: for the LORSC approach consistency is in fact very slightly better than the NLO–$\alpha_s$ approach, but in the LO–$\alpha_s$ approach the predicted $F_L(x, Q^2)$ is far too large. Thus, the LO–$\alpha_s$ approach, and consequently (and particularly) double asymptotic scaling, is ruled out precisely in the region where it is claimed most strongly to hold, i.e. high $Q^2$ and low $x$.

Hence, within this limited study I conclude that both the NLO–$\alpha_s$ and the LORSC calculations are consistent with high $y$ cross-section data. Therefore, forgetting other possible theoretical models, the values of $F_L(x, Q^2)$ are very likely to be similar to the predictions of these approaches when they are constrained by a global fit to structure function data. By definition this means that the values are similar to those “measured” using the two approaches, but somewhat lower than those presented in [4]. An increase in the precision of measurement of the high $y$ cross-section, or an extension to slightly higher values of $y$ would be very important in differentiating between the two theoretical approaches, and potentially any others.

Alternatively, a direct measurement of $F_L(x, Q^2)$ would be even more useful. In this case there would not be any inbuilt uncertainty due to a particular (and always to some degree approximate) theoretical model, or to the measurement depending on how accurately, or correctly, unknown parameters are determined by the fit to low $y$ data (even though this is probably small). It would also eliminate the significant possibility that incorrectness in both $F_2(x, Q^2)$ and $F_L(x, Q^2)$ in a given theoretical approach could act to partially, or even largely cancel out and still lead to apparent consistency in the above approach. Hence I encourage strongly any attempt to measure $F_L(x, Q^2)$ directly using any method as a way to help discriminate strongly between different theoretical approaches to calculating small $x$ structure functions, and to obtain real data on a real physical quantity.
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Table 1
Data points with sufficiently high $y$ and low enough error on $\tilde{\sigma}$ for the error on $F_L^m(x,Q^2)$ to be less than 0.3. The $Q^2$ values are denoted in GeV$^2$.

| $Q^2$ | $x \cdot 10^4$ | $y$ | $\tilde{\sigma}$ | $\Delta \tilde{\sigma}$ | $Q^2$ | $x \cdot 10^4$ | $y$ | $\tilde{\sigma}$ | $\Delta \tilde{\sigma}$ |
|-------|----------------|-----|-------------------|---------------------|-------|----------------|-----|-------------------|---------------------|
| 8.5   | 1.35           | 0.70| 1.165             | 0.099               | 6.5   | 1.00           | 0.72| 1.092             | 0.084               |
| 12.0  | 1.90           | 0.70| 1.198             | 0.079               | 8.5   | 1.60           | 0.59| 1.248             | 0.074               |
| 15.0  | 2.38           | 0.70| 1.368             | 0.085               | 10.0  | 1.60           | 0.69| 1.225             | 0.114               |
| 20.0  | 3.17           | 0.70| 1.276             | 0.079               | 12.0  | 2.50           | 0.53| 1.233             | 0.068               |
| 25.0  | 3.96           | 0.70| 1.439             | 0.089               | 15.0  | 2.50           | 0.67| 1.424             | 0.108               |
| 25.0  | 5.00           | 0.56| 1.430             | 0.069               | 27.0  | 6.30           | 0.48| 1.407             | 0.053               |
| 35.0  | 5.54           | 0.70| 1.435             | 0.099               | 35.0  | 6.30           | 0.62| 1.459             | 0.086               |
|       |                |     |                   |                     | 60.0  | 10.0          | 0.67| 1.438             | 0.088               |

Table 2
The “measurement” $F_L^m(x,Q^2)$ and prediction $F_L^p(x,Q^2)$ of $F_L(x,Q^2)$ for the NLO–$\alpha_s$ fit performed in a manner analogous to that in [4], and for the data in [4] only. Also shown are the “measured” values $F_L^m(x,Q^2)$ in [4].

| $Q^2$ | $x \cdot 10^4$ | $F_L^p$ | $F_L^m$ | $\Delta F_L^m$ | $F_L^m$ [4] |
|-------|----------------|---------|---------|----------------|-------------|
| 8.5   | 1.35           | 0.31    | 0.51    | 0.22           | 0.51        |
| 12.0  | 1.90           | 0.34    | 0.62    | 0.18           | 0.63        |
| 15.0  | 2.38           | 0.35    | 0.33    | 0.19           | 0.35        |
| 20.0  | 3.17           | 0.36    | 0.61    | 0.17           | 0.67        |
| 25.0  | 3.96           | 0.39    | 0.30    | 0.20           | 0.33        |
| 35.0  | 5.54           | 0.38    | 0.33    | 0.22           | 0.39        |
Table 3

$F^m_L(x, Q^2)$ and $F^p_L(x, Q^2)$ for the NLO–in–$\alpha_s$ global fit.

| $Q^2$ | $x \cdot 10^4$ | $F^p_L$ | $F^m_L$ | $\Delta F^m_L$ | $Q^2$ | $x \cdot 10^4$ | $F^p_L$ | $F^m_L$ | $\Delta F^m_L$ |
|-------|---------------|---------|---------|----------------|-------|---------------|---------|---------|----------------|
| 8.5   | 1.35          | 0.24    | 0.41    | 0.22           | 6.5   | 1.00          | 0.19    | 0.39    | 0.17           |
| 12.0  | 1.90          | 0.28    | 0.51    | 0.18           | 8.5   | 1.60          | 0.23    | 0.16    | 0.25           |
| 15.0  | 2.38          | 0.30    | 0.21    | 0.19           | 10.0  | 1.60          | 0.26    | 0.34    | 0.26           |
| 20.0  | 3.17          | 0.31    | 0.50    | 0.17           | 12.0  | 2.50          | 0.270   | 0.42    | 0.29           |
| 25.0  | 3.96          | 0.35    | 0.19    | 0.20           | 15.0  | 2.50          | 0.29    | 0.03    | 0.27           |
| 25.0  | 5.00          | 0.33    | 0.02    | 0.27           | 27.0  | 6.30          | 0.32    | -0.18   | 0.30           |
| 35.0  | 5.54          | 0.38    | 0.22    | 0.22           | 35.0  | 6.30          | 0.33    | 0.05    | 0.26           |
|       |               |         |         |                | 60.0  | 10.0          | 0.32    | 0.07    | 0.22           |

Table 4

$F^m_L(x, Q^2)$ and $F^p_L(x, Q^2)$ for the LORSC global fit.

| $Q^2$ | $x \cdot 10^4$ | $F^p_L$ | $F^m_L$ | $\Delta F^m_L$ | $Q^2$ | $x \cdot 10^4$ | $F^p_L$ | $F^m_L$ | $\Delta F^m_L$ |
|-------|---------------|---------|---------|----------------|-------|---------------|---------|---------|----------------|
| 8.5   | 1.35          | 0.20    | 0.43    | 0.22           | 6.5   | 1.00          | 0.19    | 0.37    | 0.17           |
| 12.0  | 1.90          | 0.22    | 0.53    | 0.18           | 8.5   | 1.60          | 0.19    | 0.12    | 0.25           |
| 15.0  | 2.38          | 0.22    | 0.25    | 0.19           | 10.0  | 1.60          | 0.21    | 0.33    | 0.26           |
| 20.0  | 3.17          | 0.22    | 0.55    | 0.17           | 12.0  | 2.50          | 0.20    | 0.37    | 0.29           |
| 25.0  | 3.96          | 0.24    | 0.24    | 0.20           | 15.0  | 2.50          | 0.22    | 0.03    | 0.27           |
| 25.0  | 5.00          | 0.23    | 0.09    | 0.27           | 27.0  | 6.30          | 0.21    | -0.18   | 0.30           |
| 35.0  | 5.54          | 0.23    | 0.30    | 0.22           | 35.0  | 6.30          | 0.22    | 0.09    | 0.26           |
|       |               |         |         |                | 60.0  | 10.0          | 0.21    | 0.13    | 0.22           |
Table 5

$F_L^m(x, Q^2)$ and $F_L^p(x, Q^2)$ for the LO–in–$\alpha_s$ global fit.

| $Q^2$ | $x \cdot 10^4$ | $F_L^p$ | $F_L^m$ | $\Delta F_L^m$ | $Q^2$ | $x \cdot 10^4$ | $F_L^p$ | $F_L^m$ | $\Delta F_L^m$ |
|-------|----------------|---------|---------|----------------|-------|----------------|---------|---------|----------------|
| 8.5   | 1.35           | 0.61    | 0.35    | 0.22           | 6.5   | 1.00           | 0.60    | 0.33    | 0.17           |
| 12.0  | 1.90           | 0.60    | 0.45    | 0.18           | 8.5   | 1.60           | 0.58    | 0.07    | 0.25           |
| 15.0  | 2.38           | 0.59    | 0.17    | 0.19           | 10.0  | 1.60           | 0.61    | 0.28    | 0.26           |
| 20.0  | 3.17           | 0.57    | 0.47    | 0.17           | 12.0  | 2.50           | 0.56    | 0.33    | 0.29           |
| 25.0  | 3.96           | 0.60    | 0.17    | 0.20           | 15.0  | 2.50           | 0.58    | -0.01   | 0.27           |
| 25.0  | 5.00           | 0.56    | 0.01    | 0.27           | 27.0  | 6.30           | 0.51    | -0.18   | 0.30           |
| 35.0  | 5.54           | 0.55    | 0.23    | 0.22           | 35.0  | 6.30           | 0.53    | 0.07    | 0.26           |
| 60.0  |                |         |         |                | 60.0  | 10.0           | 0.45    | 0.13    | 0.22           |
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Figure Captions

Fig. 1. Result of the measured values of $F_L^m(x, Q^2)$ for the six points used in [4], along with the predicted value, $F_L^p(x, Q^2)$, along a curve of constant $y = 0.7$ (the solid line), when using a NLO–in–$\alpha_s$ fit analogous to that in [4]. Also shown are the measured values of $F_L(x, Q^2)$ in [4], $F_L^m(x, Q^2)[4]$, and the curve of predicted values in this paper (dashed curve).

Fig. 2. Result of the measured values of $F_L^m(x, Q^2)$ for the 15 points satisfying the cut of 0.3 on the error, along with the predicted values, $F_L^p(x, Q^2)$, for a NLO–in–$\alpha_s$ global fit. In the top figure the points all have $y \approx 0.7$ and $F_L^p(x, Q^2)$ is along a curve of constant $y = 0.7$ (the solid line). In the lower figure the points have $y \approx 0.55$ and $F_L^p(x, Q^2)$ is along a curve of constant $y = 0.55$.

Fig. 3. Result of the measured values of $F_L^m(x, Q^2)$ for the 15 points satisfying the cut of 0.3 on the error, along with the predicted values, $F_L^p(x, Q^2)$, for a LORSC global fit.

Fig. 4. Result of the measured values of $F_L^m(x, Q^2)$ for the 15 points satisfying the cut of 0.3 on the error, along with the predicted values, $F_L^p(x, Q^2)$, for a LO–in–$\alpha_s$ global fit.

Fig. 5. The “measured” values of the structure function, $F_2^m(x, Q^2)$ (data points), compared to the theoretical values, $F_2^p(x, Q^2)$, obtained from the best fit (solid line) in two different small $x$ bins, when using the LO–in–$\alpha_s$ calculation.
FIG. 1
$F_L$ from H1 and ZEUS

FIG. 2
$F_L$ from H1 and ZEUS

FIG. 3
$F_L$ from H1 and ZEUS

FIG. 4
$F_2$ against $Q^2$ from LO-in-$\alpha_s$ fit

FIG. 5