Quantum Cosmology and Tachyons

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Abstract

We discuss the relevance of the classical and quantum rolling tachyon inflation in the frame of the standard, $p$-adic and adelic minisuperspace quantum cosmology. The field theory of tachyon matter proposed by Sen in a zero-dimensional version suggested by Kar leads to a model of a particle moving in a constant external field with quadratic damping. We calculate the exact quantum propagator of the model, as well as, the vacuum states and conditions necessary to construct an adelic generalization.

1 Introduction

The main task of quantum cosmology [1] is to describe the evolution of the universe in a very early stage. Usually one takes the universe is described by a complex-valued wave function. Since quantum cosmology is related to the Planck scale phenomena it is logical to consider various geometries (in particular nonarchimedean [2] and noncommutative [3] ones) and parametrizations of the space-time coordinates: real, $p$-adic, or even adelic [4]. In this article, we will generally maintain space-time coordinates and matter fields to be real and $p$-adic.

Supernova Ia observations show that the expansion of the Universe is accelerating [5], contrary to Friedmann-Robertson-Walker (FRW) cosmological models, with non-relativistic matter and radiation. Also, cosmic microwave background (CMB) radiation data are suggesting that the expansion of our Universe seems to be in an accelerated state which is referred to as the “dark energy“ effect. The cosmological constant as the vacuum energy can be responsible for this evolution by providing a negative pressure. A need for understanding these new and rather surprising facts, including (cold) “dark matter“, has motivated numerous authors to reconsider
different inflation scenarios. Despite some evident problems [6] such as a non-sufficiently long period of inflation, tachyon-driven scenarios [7, 8, 9] remain highly interesting for study.

There have been a number of attempts to understand this description of the early Universe via (classical) nonlocal cosmological models, first of all via $p$-adic inflation models [10, 11], which are represented by nonlocal $p$-adic string theory coupled to gravity. For these models, some rolling inflationary solutions were constructed and compared with CMB observations. Another example is the investigation of the $p$-adic inflation near a maximum of the nonlocal potential when non-local derivative operators are included in the inflaton Lagrangian. It was found that higher-order derivative operators can support a (sufficiently) prolonged phase of slow-roll inflation [12].

The $p$-Adic approach in HEP is mostly motivated by the following reasons: (i) the field of rational numbers $\mathbb{Q}$, which contains all observational and experimental numerical data, is a dense subfield not only in the field of real numbers $\mathbb{R}$ but also in the fields of $p$-adic numbers $\mathbb{Q}_p$ ($p$ is a prime number); (ii) there is a plausible analysis within and over $\mathbb{Q}_p$ as well as that one related to $\mathbb{R}$; (iii) general mathematical methods and fundamental physical laws should be invariant under an interchange of the number fields $\mathbb{R}$ and $\mathbb{Q}_p$; (iv) there is a quantum gravity uncertainty while measuring distances around the Planck length, which restricts the priority of Archimedean geometry based on real numbers and gives rise to the employment of non-Archimedean geometry related to $p$-adic numbers; (v) it seems to be quite reasonable to extend compact Archimedean geometries by the non-Archimedean ones when integrating over geometries in the path integral method; and (vi) adelic quantum mechanics applied to quantum cosmology provides realization of all the above statements.

In the unified form, adelic quantum mechanics contains ordinary and all $p$-adic quantum mechanics. As there is not an appropriate $p$-adic Schrödinger equation, there is also no $p$-adic generalization of the Wheeler-De Witt equation. Instead of the differential approach, Feynman’s path integral method is exploited [14]. $p$-Adic gravity and the wave function of the universe were considered [15] as an idea of the fluctuating number fields at the Planck scale. Like in adelic quantum mechanics, the adelic eigenfunction of the universe is a product of the corresponding eigenfunctions of real and all $p$-adic cases. It was shown that in the framework of this procedure one obtains an adelic wave function for the de Sitter minisuperspace model. However, the adelic generalization with the Hartle-Hawking $p$-adic prescription does not work well when minisuperspace has more than one dimension, in particular, when matter fields are taken into consideration. The solution of
this problem was found by treating minisuperspace cosmological models as models of adelic quantum mechanics. It is a strong motivation to study a class of exactly solvable quantum mechanical models and apply them in the frame of quantum cosmology. For the review and detailed discussion see [2] [16]. The nonarchimedean and noncommutative cosmological quantum models with extra dimensions and an accelerating phase have been considered [17], as well as the relevant models and techniques in pure quantum mechanical context [18] [19] [20] [21].

Following S. Kar’s [22] idea on the possibility of the examination of zero dimensional theory of the field theory of (real) tachyon matter, and motivated by successes and shortcomings of classical $p$-adic inflation, we consider real and $p$-adic aspects of a relevant model with quadratic damping. We calculated the corresponding propagator and considered vacuum states for $p$-adic and adelic tachyons.

2 Quantum Cosmology

According to the standard cosmological model, in the very beginning the universe was very small, dense, hot and started to expand. This initial period of evolution should be described by quantum cosmology. In the path integral approach to quantum cosmology over the field of real numbers $R$, the starting point is the idea that the amplitude to go from one state with intrinsic metric $h'_{ij}$, and matter configuration $\phi'$ on an initial hypersurface $\Sigma'$, to another state with metric $h''_{ij}$, and matter configuration $\phi''$ on a final hypersurface $\Sigma''$, is given by a functional integral of the form

$$\langle h''_{ij}, \phi'', \Sigma''|h'_{ij}, \phi', \Sigma'\rangle = \int Dg_{\mu\nu} D\Phi e^{-S[g_{\mu\nu}, \Phi]},$$

over all four-geometries $g_{\mu\nu}$, and matter configurations $\Phi$, which interpolate between the initial and final configurations. In this expression $S[g_{\mu\nu}, \Phi]$ is an Einstein-Hilbert action for the gravitational and matter fields (which can be massless, minimally or conformally coupled with gravity). This expression stays valid in the $p$-adic case too, because of its form invariance under change of real to the $p$-adic number fields.

Among many cosmological models, there is one very important type of models, the so-called de Sitter models. de Sitter models are models with the cosmological constant $\Lambda$ and without matter fields. Models of this type are exactly soluble models and because of that, they play a role similar to a linear harmonic oscillator in ordinary quantum mechanics. The general
form of the metric for these models is \[ ds^2 = \sigma^2[-N^2 dt^2 + a^2(t)d\Omega_{D-1}^2], \] (2)
where \( d\Omega_{D-1}^2 \) denotes the metric on the unit \((D-1)\)-sphere, \( \sigma^{D-2} = 8\pi G/V^{D-1}(D-1)(D-2) \), \( V^{D-1} \) is the volume of the unit \((D-1)\)-sphere. In the \( D = 3 \) case, this model is related to the multiple sphere configuration and wormhole solutions. \( \nu \)-Adic (\( \nu = \infty \) for the real, and \( \nu = p \) in the \( p \)-adic cases) classical action for this model is
\[
\bar{S}_\nu(a''; N; a', 0) = \frac{1}{2\sqrt{\lambda}} \left[ N\sqrt{\lambda} + \lambda \left( \frac{2a''a'}{\sinh(N\sqrt{\lambda})} - \frac{a'^2 + a''^2}{\tanh(N\sqrt{\lambda})} \right) \right].
\] (3)
Let us note that \( a \) denotes a scale factor and \( \lambda \) denotes here the appropriately rescaled cosmological constant \( \Lambda \), i.e. \( \lambda = \sigma^2 \Lambda \). This model was investigated in all aspects (\( p \)-adic, real and adelic) in Ref. [2]. Especially, for this model, the adelic wave function (which unifies the wave function over the field of real numbers and wave functions over the field of \( p \)-adic numbers), is in the form
\[
\Psi(a) = \Psi_\infty(a) \prod_p \Psi_p(a_p),
\] (4)
where \( \Psi_\infty(a) \) is a standard wave function and \( \Psi_p(a_p) \) are \( p \)-adic wave functions. It is very important that only for finite numbers of \( p \), \( p \)-adic wave functions can be different from \( \Omega \) function which is defined by the \( \Omega(|x|_p) = 1 \), for \(|x|_p \leq 1 \) and \( \Omega(|x|_p) = 0 \), for \(|x|_p > 1 \).

At this place we indicate a considerable similarity of the action (3) for the de Sitter model in 2+1 dimensions with the action (14) for the tachyon field in the zero dimensional model, i.e. “quadratically damped particle under gravity“.

3 \( p \)-Adic inflation

Cosmological inflation has become an integral part of the standard model of the universe. It provides important clues for structure formation in the universe and is capable of removing the shortcomings of the standard cosmology. Gibbons [7] has emphasized the cosmological implication of tachyonic condensate rolling towards its ground state. The tachyonic matter might provide an explanation for inflation at the early epochs and could contribute to a new form of dark matter at later times.

A recent paper on \( p \)-adic inflation [10] gives rise to the hopes that non-local inflation can succeed where the real string theory fails. Starting from
the action of the $p$-adic string, with $m_s$ the string mass scale and $g_s$ the open string coupling constant,

$$S = \frac{m_s^4}{g_p^2} \int d^4x \left( -\frac{1}{2} \phi \frac{\partial^2 \phi}{2m_s^2} + \frac{1}{p + 1} \phi^{p+1} \right), \quad \frac{1}{g_s^2} = \frac{1}{g_p^2} p^2 - 1,$$

(5)

for the open string tachyon scalar field $\phi(x)$, it has been shown that a $p$-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential. Even though this result is constrained by $p \gg 1$ and obtained by an approximation, it is a good motivation to consider $p$-adic inflation for different tachyonic potentials. In particular, it would be interesting to study $p$-adic inflation in quantum regime and in adelic framework to overcome the constraint $p \gg 1$, with an unclear physical meaning. For some details, and further development see [11] and the references therein.

4 Classical and quantum tachyons

A. Sen proposed a field theory of tachyon matter a few years ago [24] (see also [9]). The action is given as:

$$S = -\int d^{D+1}x V(T) \sqrt{1 + \eta^{ij} \partial_i T \partial_j T}$$

(6)

where $\eta_{00} = -1$ and $\eta_{\alpha\beta} = \delta_{\alpha\beta}$, $\alpha, \beta = 1, 2, 3, ..., D$, $T(x)$ is the scalar tachyon field and $V(T)$ is the tachyon potential which unusually appears in the action as a multiplicative factor and has (from string field theory arguments) exponential dependence with respect to the tachyon field $V(T) \sim e^{-\alpha T/2}$. In this paper we will focus our attention on this type of the potential. It is very useful to understand and to investigate lower dimensional analogs of this tachyon field theory. The corresponding zero dimensional analogue of a tachyon field can be obtained by the correspondence: $x^i \rightarrow t$, $T \rightarrow x$, $V(T) \rightarrow V(x)$. The action reads

$$S = -\int dt V(x) \sqrt{1 - \dot{x}^2}.$$  

(7)

In what follows, all variables and parameters can be treated as real or $p$-adic without a formal change in the obtained forms. It is not difficult to see that action (7), with some appropriate replacement leads to the equation of
motion for a particle with mass $m$, under a constant external force, in the presence of quadratic damping:

$$m\ddot{y} + \beta \dot{y}^2 = mg.$$  

(8)

This equation of motion can be obtained from two Lagrangians [25]:

$$L(y, \dot{y}) = \left(\frac{1}{2}m\dot{y}^2 + \frac{m^2g}{2\beta}\right)e^{\frac{2\beta}{m}y}, \quad (9)$$

$$L(y, \dot{y}) = -e^{-\frac{\beta}{m}y}\sqrt{1 - \frac{\beta mg}{y^2}}. \quad (10)$$

Despite the fact that different Lagrangians can give rise to nonequivalent quantization, we will choose the form (9) that can be handled easily. The first one is better because of the presence of the square root in the second one. The general solution of the equation of motion is

$$y(t) = C_2 + \frac{m}{\beta}\ln[\cosh(\sqrt{\frac{g\beta}{m}}t + C_1)]. \quad (11)$$

For the initial and final conditions $y' = y(0)$ and $y'' = y(T)$, for the $\nu$-adic classical action we obtain

$$\bar{S}_\nu(y'', T; y', 0) = \frac{\sqrt{mg\beta}}{2 \sinh(\sqrt{\frac{g\beta}{m}}T)} \left[(e^{\frac{2\beta}{m}y'} + e^{\frac{2\beta}{m}y''}) \cosh(\sqrt{\frac{g\beta}{m}}T) - 2e^{\frac{2\beta}{m}(y'+y'')}\right]. \quad (12)$$

In the $p$-adic case, we get a constraint which arises from the investigation of the domain of a convergence analytical function which appears during the derivation of the formulae (11). This constraint is $|\dot{y}|_p \leq \frac{1}{p}\sqrt{\frac{gm}{\beta}}|_p$.

By the transformation $X = \frac{m}{\beta}e^{\frac{2\beta}{m}y}$, we can convert Lagrangian (9) in a more suitable, quadratic form

$$L(X, \dot{X}) = \frac{m\dot{X}^2}{2} + \frac{g\beta X^2}{2}. \quad (13)$$

For the conditions $X' = X(0)$, and $X'' = X(T)$, action for the classical $\nu$-adic solution $X(t)$ is

$$\bar{S}_\nu(X'', T; X', 0) = \frac{\sqrt{mg\beta}}{2 \sinh(\sqrt{\frac{g\beta}{m}}T)} \left[(X'^2 + X''^2) \cosh(\sqrt{\frac{g\beta}{m}}T) - 2X'X''\right]. \quad (14)$$
We note that this action is different from the action (3) only in one constant term. Because action (14) is quadratic one (with respect to the initial and final point), the corresponding kernel is: 

$$
\mathcal{K}_\nu(X'', T; X', 0) = \chi_\nu \left( \frac{1}{2} \sqrt{\frac{\sqrt{g\beta m}}{\sinh \left( \sqrt{\frac{g\beta m}{m}} T \right)}} \right) \left| \frac{1}{\sqrt{\sinh \left( \sqrt{\frac{g\beta m}{m}} T \right)}} \right|^{1/2} \chi_\nu \left( -\frac{1}{h} \bar{S}_\nu \right),
$$  

(15)

where $\chi_\nu$ is the adelic additive character $\mathbb{A}$. 

The necessary condition for the existence of an adelic model is the existence of a $p$-adic quantum-mechanical ground state $\Omega(|X|_p)$, i.e.

$$
\int_{|X|_p \leq 1} \mathcal{K}_p(X'', T; X', 0) dX' = \Omega(|X''|_p).
$$  

(16)

Analogously, if a system is in the state $\Omega(p^\nu|X|_p)$, then its kernel must satisfy

$$
\int_{|X'|_p \leq p^{-\nu}} \mathcal{K}_p(X'', N; X', 0) dX' = \Omega(p^\nu|X''|_p).
$$  

(17)

In case the the $p$-adic ground state is of the form of the $\delta$-function, we have to investigate conditions under which the corresponding kernel of the model satisfies the equation

$$
\int_{Q_p} \mathcal{K}_p(X'', T; X', 0) \delta(p^\nu - |X'|_p) dX' = \chi_p(ET) \delta(p^\nu - |X''|_p),
$$  

(18)

with zero energy $E = 0$. In what follows, we apply (16), (17) and (18) to our model. As a result for the $p$-adic wave functions (in the case $p \neq 2$), we get

$$
\Psi_p(X) = \Omega(|X|_p), \quad |T|_p \leq \left| \frac{m}{2\hbar} \right|_p \left| \frac{g\beta m}{4\hbar^2} \right|_p < 1
$$  

(19)

$$
\Psi_p(X) = \Omega(p^\nu|X|_p), \quad |T|_p \leq \left| \frac{m}{2\hbar} \right|_p p^{-2\nu} \left| \frac{g\beta m}{4\hbar^2} \right|_p \leq p^{3\nu}
$$  

(20)

$$
\Psi_p(X) = \delta(p^\nu - |X|_p), \quad \left| \frac{T}{2} \right|_p \leq \left| \frac{m}{h} \right|_p \left| \frac{g\beta m}{\hbar^2} \right|_p \leq p^{2-3\nu}.
$$  

(21)

The above conditions are in accordance with the conditions for the convergence of the $p$-adic analytical functions which appear in the solution of the equation of motion (11) and the classical action (12). We see there is a wide freedom in choosing the parameters of the model, such as mass of the tachyon field $m$, damping factor $\beta$, parameter $g$ related to the “strength
of the constant gravity $^*$, and cosmological constant $\Lambda$ which appears in the de Sitter $(2 + 1)$ dimensional model. A relevant physical conclusion served from these relations still needs a more realistic model with tachyon matter and with a precise form of metrics.

5 Conclusion

In spite of the very attractive features of the tachyonic inflation, first of all, the rolling tachyon condensate, this approach faces difficulties such as reheating [6]. It seems that both mechanisms, based on real tachyons - conventional reheating mechanism and quantum mechanical particle production during inflation - do not work. Recent results in nonlocal ($p$-adic) tachyon inflation [10, 11], in which a $p$-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential deserve much more attention. The classical $p$-adic models succeed with inflation where the real string theory fails. In this paper we have calculated a quantum propagator for the $p$-adic and adelic tachyons, found conditions for the existence of the vacuum state of $p$-adic and adelic tachyons, noted interesting relations with the minisuperspace closed homogenous isotropic model in $(2 + 1)$ dimensions using Einstein gravity with a cosmological constant and an antisymmetric tensor field matter source [2, 23]. We have shown that the new results can give rise to a better understanding of the $p$-adic and real quantum tachyons, their relation via Freund-Witten formula and a possible role of tachyon field as a dark matter. Our results can also be used as a basis for further investigation of ($p$-adic) quantum mechanical damped systems and corresponding wave functions of the universe in the minisuperspace models based on the tachyonic matter with different potentials. Further investigation should contribute to the better understanding of quantum rolling tachyon scenario in a real [26] and $p$-adic case.

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