Anti-Kibble-Zurek behavior of a noisy transverse-field XY chain and its quantum simulation with two-level systems

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We study the dynamics of a transverse-field XY chain driven across quantum critical points by noisy control fields. We characterize the defect density as a function of the quench time and the noise strength, and demonstrate that the defect productions for three quench protocols with different scaling exponents exhibit the anti-Kibble-Zurek behavior, whereby slower driving results in more defects. The protocols are quenching through the boundary line between paramagnetic and ferromagnetic phases, quenching across the isolated multicritical point and along the gapless line, respectively. We also show that the optimal quench time to minimize defects scales as a universal power law of the noise strength in all the three cases. Furthermore, by using quantum simulation of the quench dynamics in the spin system with well-designed Landau-Zener crossings in pseudo-momentum space, we propose an experimentally feasible scheme to test the predicted anti-Kibble-Zurek behavior of this noisy transverse-field XY chain with two-level systems under controllable fluctuations.

I. INTRODUCTION

Kibble-Zurek mechanism (KZM) provides an elegant theoretical framework for exploring the critical dynamics of phase transitions in systems ranging from cosmology to condensed matter.1–3 The dynamics induced by a quench across a critical point with a control parameter is generally nonadiabatic due to the critical slowing down, which results in the production of topological defects. A key prediction of KZM is that the density of defects $n_0$ follows a universal power law as a function of the quench time $\tau$: $n_0 \propto \tau^{-\beta}$, where the scaling exponent $\beta > 0$ determined by the critical exponents of the phase transition and the dimensionality of the system. KZM for classical continuous phase transitions has been verified in many systems, such as cold atomic gases,4,5 ion crystals,6,7 and superconductors.8 There has been significant theoretical work on extension of KZM for quantum phase transitions,9–11 which are zero temperature transitions driven by Heisenberg quantum fluctuations rather than thermal fluctuations.12 For instance, by studying KZM in the one-dimensional transverse-field Ising model, which is one of the paradigmatic models to study quantum phase transitions, it was found that the density of defects scales as the square root of the quench time with the scaling exponent $\beta = 1/2$.13 However, the experimental tests of KZM in quantum phase transitions are still scarce since controlling the time evolution of systems cross quantum critical points is notoriously difficult.14

Landau-Zener transition (LZT), occurring when a two-level system sweeps through its anticrossing point, has served over decades as a textbook paradigm of quantum dynamics of some non-equilibrium physical systems.15–18 Recently, LZT has been extensively studied both theoretically and experimentally in, e.g., superconducting qubits19–21 and optical lattices.22–24 It was shown that the dynamics of LZT can be intuitively described in terms of KZM of the topological defect formation in non-equilibrium quantum phase transition.25–27 The correspondence between the two physical situations provide a promising way for proof-of-concept quantum simulation of KZM in quantum regime by using LZT in two-level systems, which has been experimentally demonstrated in an optical interferometer.28 Moreover, quantum simulation of the critical dynamics in the transverse-field Ising model by a set of independent Landau-Zener crossings in pseudo-momentum space has been realized in a semiconductor electron charge qubit29,30 a superconducting qubit31 and a single trapped ion32. The LZT there can be engineered well and probed with high accuracy and thus KZM of defect production in the Ising model with the scaling exponent $\beta = 1/2$ has been successfully observed in these artificial two-level systems33–35.

While KZM has been verified to be broadly applicable, a conflicting observation was reported in a recent experiment of ferroelectric phase transition: slower quenches generate more defects when approaching the adiabatic limit.36 This behavior is opposite to that predicted by the standard KZM and is termed as anti-Kibble-Zurek (anti-KZ) behavior. The quench dynamics of a transverse-field Ising chain coupled to a dissipative thermal bath has been theoretically studied in Refs.37,38,39, which show that the defect production can exhibit the anti-KZ behavior due to the emergence of thermal defects. Recently, by studying the crossing of the quantum critical point in a thermally isolated Ising chain driven by a noisy transverse field, it was demonstrated that noise contributions can also give rise to anti-KZ behavior when they dominate the dynamics.39 A natural question is
whether the anti-KZ behavior can exhibit in other quantum spin models with different scaling exponents under noisy control fields. In the experimental aspect, it would be of great value to set a stage for quantum simulation of such anti-KZ behavior in two-level systems with Landau-Zener crossings, noting that quantum spin models can only be realized in some special situations without controllable noisy fields, which may prevent the test of the anti-KZ behavior.

In this paper, we consider the dynamics of a transverse-field XY chain driven across quantum critical points by noisy control fields. We numerically calculate the defect density as a function of the quench time and the noise strength, and find that the defect productions in three quench protocols with different scaling exponents exhibit the anti-KZ behavior, i.e., slower driving results in more defects. The three protocols are quenching through the boundary line between paramagnetic and ferromagnetic phase, quenching across the isolated multicritical point and along the gapless line, which have the Kibble-Zurek scaling exponents $\beta = 1/2, 1/6, 1/3$ under the noise-free driving, respectively. We also show that the optimal quench time to minimize defects scales as a universal power law of the noise strength in all three cases. Furthermore, by using quantum simulation of the quench dynamics in the spin system with well-designed Landau-Zener crossings, we then propose an experimentally feasible scheme to test the predicted anti-KZ behavior in this noisy transverse-field XY chain with two-level systems under controllable fluctuations in the control field. The driving protocols of the Landau-Zener crossings in two-level systems and the required parameter regions for observing the anti-KZ behavior in the three cases are presented.

The paper is organized as follows. In Section II, we illustrate that the defect productions of the transverse-field XY chain driven across quantum critical points by noisy control fields exhibit the anti-KZ behavior. In Section III, we propose to test the predicted anti-KZ behavior by using quantum simulation of the quench dynamics with well-designed LZT in two-level systems. Finally, a short conclusion is given in Sec. IV.

II. ANTI-KZ BEHAVIOR OF A NOISY TRANSVERSE-FIELD XY CHAIN

We begin with the spin-1/2 quantum XY chain under a uniform transverse field (homogeneous for each spin), which is one of the other exactly solvable spin models apart from the quantum Ising chain. The Hamiltonian of the transverse-field XY chain with nearest neighbor interaction is given by

$$H = -\frac{J}{2} \sum_{n=1}^{N} \left[ (1 + \gamma)\sigma_n^x \sigma_{n+1}^x + (1 - \gamma)\sigma_n^y \sigma_{n+1}^y \right] - h \sum_{n=1}^{N} \sigma_n^z. \quad (2)$$

The system reduces to the isotropic XY chain for $\gamma = 0$ and the Ising chain for $\gamma = 1$. This Hamiltonian can be exactly diagonalized by using the Jordan-Wigner transformation, which maps a system of spin-1/2 to a system of spinless free fermions $\hat{c}^\dagger$. The Jordan-Wigner transformation of spins to fermions is given by $\sigma_n^\pm = \exp\left(\pm i\pi \sum_{m=1}^{n-1} c_m^\dagger c_m \right)c_n$ and $\sigma_n^z = 2c_n^\dagger c_n - 1$, where $\sigma_n^\pm = \sigma_x \pm i\sigma_y$. In the fermionic language, the XY model Hamiltonian can be rewritten as

$$H = -J \sum_{l=1}^{N} \left[ (c_{l}^\dagger c_{l+1} + c_{l+1}^\dagger c_{l}) + \gamma(c_{l}^\dagger c_{l+1}^\dagger + c_{l+1} c_{l}) \right] - h \sum_{l=1}^{N} \left( 2c_l^\dagger c_{l} - 1 \right). \quad (3)$$

Under the periodic boundary condition $\sigma_{N+1}^n = \sigma_n^1$ with even $N$ requires that $c_{N+1} = -c_1$ and after the Fourier transformation with $c_n = e^{-i\pi/4} \sum_{k \in [-\pi, \pi]} (e^{ink} c_k)$, one can obtain $H = \sum_{k \in [0, \pi]} \Psi_k^\dagger \mathcal{H}(k) \Psi_k$, where $\Psi_k^\dagger = (c_k^\dagger, c_{-k})$ and the Hamiltonian density in the pseudomomentum space

$$\mathcal{H}(k) = -2 [\hat{\sigma}_z (J \cos k + h) + \hat{\sigma}_x (J \gamma \sin k)]. \quad (4)$$

Note that here and hereafter the Pauli matrices $\hat{\sigma}_x, \hat{\sigma}_z$ are conventionally used to write the $2 \times 2$ Hamiltonian of each independent $k$-mode ($[-k, k]$ pair), which is distinct from the spin components $\sigma_n^1$ in Eqs. (1) and (2).

The energy spectrum of the system can be obtained by diagonalizing Eq. (4), and thus the critical points or lines between different quantum phases can be found by minimizing the energy gap. Figure 2 depicts the phase diagram of the transverse-field XY chain in the space spanned by the parameters $h/J$ and $J_{x}/J_{y}$. There are a quantum paramagnetic phase denoted by PM and two ferromagnetic long-ranged phases ordering along $x$ and $y$ directions denoted by FMx and FMy, respectively. As shown in Fig. 1 there are three kinds of phase phase boundaries: an Ising critical line on the horizontal axis $\gamma = 0$ (i.e. $J_x = J_y$), a multicritical point located at $\gamma = 0, h/J = 1$, and two gapless lines along $h/J = \pm 1$. Quantum phase transition occurs when the system is driven by one or more parameters to cross the boundary line or point on the phase diagram.
To study the quench dynamics of the transverse-field XY chain, we can use the Hamiltonian decoupling into a sum of independent terms $H(t) = \sum_{k \in [0, \pi]} \mathcal{H}(k,t)$, where each $k$-mode Hamiltonian $\mathcal{H}(k,t)$ given by Eq. (4) operates on a two-dimensional Hilbert space. The time evolution of a generic state is governed by the Schrödinger equation $i \frac{d}{dt} |\psi(t)\rangle = \mathcal{H}(k,t)|\psi(t)\rangle$. This projection of the spin Hamiltonian to the $2 \times 2$ Hilbert space has effectively reduced the quantum many-body problem to the problem of an array of decoupled two-level systems.

For the convenience of experimental implementation, we choose only one of the parameters $h, J_x, J_y$ and $\gamma$ as linearly quenched in time and the rest fixed in a specific quench protocol. As shown in Fig. 1, we consider three different quench protocols for the system driven across the phase boundaries: (1) The transverse quench with only $h(t)$ is varied in time, which is quenching through the boundary line between paramagnetic and ferromagnetic phase twice; (2) The anisotropic quench across the isolated multicritical point, in which case only the parameter $J_x(t)$ is varying and $h = 2J_y$ is set to ensure passing through the multicritical point; (3) The quench along the gapless line, which requires that $h/J = 1$ and only $\gamma(t)$ is varied. For the linear quench in the absence of dissipative thermal bath or noise fluctuations, the density of defects [see Eq. (11)] formed in three quench protocols follows the Kibble-Zurek power law as a function of the quench time with the scaling exponents $\beta = 1/2, 1/6, 1/3$ respectively. In the following, we consider the three quench protocols in the transverse-field XY chain under noisy control fields and demonstrate the exhibition of anti-KZ behavior.

We now present a general framework for the description of the noise fluctuations denoted by $\eta(t)$ in the control fields. The total quench parameter is written as

$$f_j(t) = f_j^{(0)}(t) + \eta(t),$$

where $f_j^{(0)}(t) \propto t/\tau_j$ denote perfect control parameter linearly varying in time with quench time $\tau_j$ and the subscript $j = 1, 2, 3$ respectively represent the three quench protocols. Here $\eta(t)$ is white Gaussian noise with zero mean and the second moment $\langle \eta(t)\eta(t') \rangle = W^2 \delta(t-t')$, with $W^2$ being the strength of noise fluctuation (here $\eta$ is dimensionless and $W^2$ has units of time). Note that white noise is a good approximation to ubiquitous colored noise with exponentially decaying correlations. We set $W$ as a small value for the stochastic Schrödinger equation, which ensures the validity of noise-average density matrix technique.[18] We consider the system Hamiltonian containing two parts in a general form

$$H(t) = H^{(0)}(t) + \eta(t)V,$$

where $H^{(0)}(t)$ denotes the ideal quench Hamiltonian to describe the prescheduled evolution of the driven system, and $\eta(t)V$ denotes the fluctuation of the control fields which modify the driving process as an effectively open quantum dynamics. Defining the stochastic wave function $|\psi_{\eta}(t)\rangle$, the stochastic Schrödinger equation is applied to describe the interplay of these two factors in one noise realization (let $h = 1$):

$$i \frac{d}{dt} |\psi_{\eta}(t)\rangle = [H^{(0)}(t) + \eta(t)V]|\psi_{\eta}(t)\rangle.$$

The stochastic density matrix $\rho_{\eta} = |\psi_{\eta}\rangle\langle\psi_{\eta}|$ is a function of $\eta(t)$, whose equation of motion can be derived from the dual pair of the stochastic Schrödinger equation and is given by

$$\frac{d}{dt} \rho_{\eta}(t) = -i[H^{(0)}(t), \rho_{\eta}(t)] - i[V, \eta(t)\rho_{\eta}(t)].$$

We assume that all noise realizations are independent, which is a practical situation in realistic experiments. This allows us to implement an average over noise realizations in the stochastic process. We denote the noise-averaged density matrix by $\rho(t) = \langle \rho_{\eta}(t) \rangle$, which is a solution of the master equation

$$\frac{d}{dt} \rho(t) = -i[H^{(0)}(t), \rho(t)] - i[V, \eta(t)\rho(t)]\rangle.$$

By using Novikov’s theorem[19] for the considered Gaussian noises, one can find $\langle \eta(t)\rho_{\eta}(t) \rangle = -iW^2[V, \rho(t)]/2$ and obtain the nonperturbative exact master equation given by

$$\frac{d}{dt} \rho(t) = -i[H^{(0)}(t), \rho(t)] - \frac{W^2}{2}[V, [V, \rho(t)]]\rangle.$$
FIG. 2: (Color online) The anti-KZ behavior of the defect productions in three quench protocols. (a,c,e) For the three protocols \( j = 1, 2, 3 \), respectively, the defect density \( n_w(\tau_j) \) as a function of the quench time for different noise strength \( W \). (b,d,f) The corresponding noise-induced defect density \( \delta n = n_w - n_0 \) in the three cases. The quantity of \( W \) is marked by legend. The three quench protocols are the transverse quench \( (j = 1) \) with the linear fitting scaling exponent \( \beta_{fit} = 0.488 \) in the noise-free limit \( W = 0 \), the anisotropic quench across the multicritical point \( (j = 2) \) with \( \beta_{fit} = 0.172 \), and the quench along the gapless line \( (j = 3) \) with \( \beta_{fit} = 0.332 \). The larger \( W \) causes larger divergence from the power law prediction in KZM and the systems exhibit the anti-KZ behavior, whereby slower driving results in more defects and is remarkable in the long quench time limit. \( N_k = 500 \) is set in the simulations.

This equation is directly related to the detection in realistic experiments and thus can be used to perform simulations of the three quench protocols under noisy control fields with the strength parameter \( W \).

The definition of the density of defects in the transverse field XY chain after the quench is straightforward, similar to the case for the Ising mode. For each \( k \)-mode in a specific quench protocol, one can numerically simulate the time evolution and find the noise-averaged density matrix \( \rho_k(\tau_j) \) at the end of quench (with the quench time \( \tau_j \)). In the basis of adiabatic instantaneous eigenstate \( \{|G_k(\tau_j),|E_k(\tau_j)\}\}, one has \( p_k = \langle E_k(\tau_j)|\rho_k(\tau_j)|E_k(\tau_j)\rangle \) to measure the probability in the excited state \( |E_k(\tau_j)\rangle \) of each \( k \)-mode. For the whole XY chain, in other equivalence words, all \( k \)-modes contribute to the density of defects \( n_W \) (the subscript \( W \) denotes the presence of noises) as reasonably defined by

\[
n_W = \frac{1}{N_k} \sum_{k \in [0,\pi]} p_k, \quad (11)
\]

where \( N_k \) is the number of \( k \)-modes used in the summation, and it is consistent of \( n_0 \) for noise-free case with \( W = 0 \).

Utilizing the theoretical framework, three quench protocols can be analyzed by substituting specific \( H_j^{(0)}(t) \) and \( V_j(t) \) into Eq. (10). We first consider the transverse quench \( (j = 1) \), in which case only the parameter \( h(t) \) is time-dependent, as shown in Fig. 1. To describe the effect of noise in the control field \( h \), the Hamiltonian for each \( k \)-mode \( H_1(k) \) can be separated into the determined and stochastic parts as \( H_1(k) = H_1^{(0)}(k) + V_1 \), where

\[
H_1^{(0)}(k) = -2[(J_x + J_y) \cos k + h(t)]\hat{\sigma}_z - 2[(J_x - J_y) \sin k]\hat{\sigma}_x, \quad V_1 = -2\hat{\sigma}_z, \quad (12)
\]

Here \( h(t) = f_1(t) = v_h t \) with the quench velocity \( v_h \) drives the system from left PM-phase region through middle FM-phase part to the right PM-phase region as shown in Fig. 4. In our numerical simulations, \( h(t) \) is set to vary from \( h_{min} = -5/3 \) to \( h_{max} = 5/3 \), with the other two independent parameters being fixed as \( J_x = 1 \) and \( J_y = -1/3 \). For the entire process of the quench time \( \tau_1 \), we have \( v_h = (h_{max} - h_{min})/\tau_1 = 10/(3\tau_1) \) and the whole evolution time \( t \in [-\tau_2/2, \tau_2/2] \).

Figure 2(a) illustrates the numerical results of the defect density \( n_W \) as a function of the quench time \( \tau_1 \) in the transverse quench protocol. When the driving field is free from noises with \( W = 0 \), the results of \( n_0 \) agree well with the fitting exponent \( \beta_{fit} = 0.488 \) agrees well with the
Theoretical prediction of Kibble-Zurek scaling exponent $\beta = 1/2$ in the thermodynamical and long quench time limits.\textsuperscript{13,15} Note that the deviation of $\beta_{\text{fit}}$ (about 2%) here comes from the finite size of $k$-modes $N_k$ ($N_k = 500$ is set) and the finite quench time $\tau_1$ in our simulations. We have numerically confirmed that this deviation can be decreased with the increasing of $N_k$ and $\tau_1$. For short quench time, the defect density $n_W$ as a function of the quench time $\tau_1$ is close to the scaling form in the noise-free case. As the strength of noise fluctuation grows (increasing $W$), the corresponding defect density increases compared to the results under noise-free condition, and the deviation becomes more significant for longer quench time, where the dynamics is dominated by the noise-induced non-adiabatic effects. Then the power-law scaling form of $n_W(\tau_1)$ fails and the system exhibits the anti-KZ behavior, i.e., slower driving (larger $\tau_1$) results in more defects. Finally $n_W$ is completely governed by the anti-KZ contribution in the limit of very long quench time.

The physics of the anti-KZ behavior can be further interpreted as follows: The defect production of the noise-free part of the quench system is due to the critical slowing down in KZM. In contrast, the noisy fluctuations in the control fields allow the absorption of energy to generate excitations in the system, which accumulate during the evolution with the increasing of the quench time. Thus, the resulting defect production is determined by the two independent mechanisms. For the relatively weak noises considered in our work, the noises contribute negligible excitations for the short quench time and then the scaling of the defect production is still effectively governed by the Kibble-Zurek predictions. When the quench time becomes large enough, the accumulation of noise-induced excitations dominates and the system enters the anti-KZ regime.

We proceed to consider the second quench protocol ($j = 2$), the anisotropic quench across the multicritical point with the control field $J_x(t)$ and fixed $h = 2J_y$, as shown in Fig. 1. In this case, the Hamiltonian for each $k$-mode can be written as $H_2(k) = H_2^{(0)}(k) + V_2$, where

$$H_2^{(0)}(k) = -2\{(J_x(t) + J_y)\cos(k) + h\}6\zeta)$$
$$V_2 = -2\{(\sin(k)6\zeta) + (\cos(k)6\zeta)\}.$$  \hspace{1cm} (13)$$

Here the linearly driving field $J_x(t) = J_{x}^{(0)}(t) = v_x t$ with the quench velocity $v_x$. Similar to the first protocol mentioned above, in our numerical simulations, we choose $h = 2$ and $J_y = 1$, and let $J_x$ ramp from $-1$ to $3$ with the overall quench time $\tau_2$. Under this condition, the system is initially in the FM phase and then driven through the multicritical point into the FMx phase. Consequently, the quench velocity $v_x = 4/\tau_2$, and the evolution progress is $t \in [-\tau_2/4, 3\tau_2/4]$. Figure 2(c) shows the numerical results of the defect density $n_W(\tau_2)$ in this quench protocol. In the noise-free limit with $W = 0$, we find that $n_0 \propto \tau^{-\beta_{\text{fit}}}$ with the fitting exponent $\beta_{\text{fit}} = 0.172$ agrees with the theoretical prediction $\beta = 1/6$. When the dynamics is dominated by the noise effects for increasing the noise strength and (or) quench time, this power-law scaling again fails and the system exhibits the anti-KZ behavior.

For the last quench protocol along the gapless line protocol ($j = 3$), the parameter $\gamma = f_3^{(0)} = (J_x - J_y)/J$ is used as the control field to linearly drive the system, which takes the form $\gamma(t) = \nu_v t = (\nu_v/\tau_3) t$ in the evolution progress $t \in [-\tau_3/2, \tau_3/2]$. The other parameters are set as $h = J = J_x + J_y = 1$. Therefore, the Hamiltonian for each $k$-mode in this case is $H_3(k) = H_3^{(0)}(k) + V_3$, where the two parts

$$H_3^{(0)}(k) = -2\{J\cos(k) + h\}6\zeta)$$
$$V_3 = -2J\sin(k)6\zeta).$$  \hspace{1cm} (14)$$

The numerical results of the defect density $n_W(\tau_3)$ in this quench protocol are shown in Fig. 2(e). For $W = 0$, we find that $n_0 \propto \tau^{-\beta_{\text{fit}}}$ with the fitting scaling exponent $\beta_{\text{fit}} = 0.332$ consistent with the theoretical prediction $\beta = 1/3$. Increasing the noise strength $W$, the system interverts the anti-KZ regime, with more defects formed for longer quench time. Hence, we come to the conclusion that when the noises presents in the control fields, the anti-KZ behavior exists in all the three quench protocols in the transverse-field XY chain with different noise-free Kibble-Zurek scaling exponents.

Due to the exhibition of the anti-KZ behavior in the quench when the noise presents, it is imperative to find an optimal quench time $\tau_{\text{opt}}$ to minimize the defects. Under the condition of finite quench time $\tau_j$, the optimal control
in the annealing of quantum simulator give a challenge that defect (excitation) density is produced as less as possible. For our numerical results, we focus on the region $3.0 \leq \ln(\tau_j) \leq 5.5$, since the experimental systems only maintain their quantum coherent characteristic in finite ramp time. As the defect density in the absence of noises $n_0 \approx c \tau_j^{-\beta}$, where the prefactor $c$ is predicted by KZM and depends on a specific protocol. One can argue that in the limit of small noise and finite quench time, the total density of defect $n_W = n_0 + \delta n$, where the noise-induced part $\delta n$ is given by

$$\delta n \approx n_W - c \tau_j^{-\beta}. \quad (15)$$

Note that the effective decoupling of the KZM dynamics from noise-induced effects, as interpreted previously, leads to the additive form of $n_W$. Figure 2(h,d,f) display that $\delta n$ for the three quench protocols ($j = 1, 2, 3$) is almost linearly depend on $\tau_j$ when $W \ll 1$, and thus one has

$$\delta n \approx r \tau_j, \quad (16)$$

with $r$ being the coefficient whose value depends on the noise strength. Secondly, the optimal quench time $\tau_{\text{opt}}$ can be derived approximately by minimizing $\delta n$ in Eq. [15], which is then given by

$$\tau_{\text{opt}} \propto r^{-1/(\beta+1)} = r^\alpha. \quad (17)$$

This relation is verified to be applicable for all the three quench protocols and the parameter regions we considered, as illustrated in Fig. 3. In the original KZM scenario, only the control fields without noises accounts for the production of defect and long quench time $\tau_j$ prevents defects formation. However, we have shown that the noise in the control fields, which is a more practical situation in realistic experiments, can also induce defects and it is intuitively to use shorter quench time as optimum to suppress the defect production when noise strength $W$ gets larger.

III. QUANTUM SIMULATION OF THE ANTI-KZ BEHAVIOR IN TWO-LEVEL SYSTEMS

In this section, we propose to test the predicted anti-KZ behavior by quantum simulation of the quantum dynamics with LZT in two-level systems. We first present a method to transform a generic two-level Hamiltonian with time linearly dependent term in the diagonal term into a standard LZT form. In a two-level system, the Schrödinger equation for the state vector $\left| \psi(t) \right\rangle = u_1(t) \left| e \right\rangle + u_2(t) \left| g \right\rangle$ with $\left| e \right\rangle$ and $\left| g \right\rangle$ as the diabatic basis can be written as

$$i \frac{d}{dt} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -2 \begin{pmatrix} vt + C & \Delta \\ \Delta & -vt - C \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \quad (18)$$

FIG. 4: (Color online) (a) The excitation probability $p_k$ as a function of $k$. In the transverse quench ($j = 1$), the regions with major contribution are near the points $k_c = 0, \pi$, while near $k_c = \pi$ in the quench through the multicritical point ($j = 2$) and along gapless line ($j = 3$). (b) The relative length of the suggested simulation regions (see the text) $|k_c - k|$ as a function of $\ln(\tau_j)$, where the cutoff $k_c$ is determined by the excitation probability $p_k(k) > 0.03$.

Here we assume the parameters $v, \Delta$ and $C$ are real constants. We further use the substitution

$$v_{\text{LZ}} = v/(2\Delta)^2 \quad t_{\text{LZ}} = 4\Delta(t + C/v) \quad (19)$$

into the origin two-level system Hamiltonian [18], which can then be transformed into the standard LZT form,

$$i \frac{d}{dt_{\text{LZ}}} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} v_{\text{LZ}}t_{\text{LZ}} & 1 \\ 1 & -v_{\text{LZ}}t_{\text{LZ}} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \quad (20)$$

The probability in the excited state at the end of the driving is approximately given by the Landau-Zener formula

$$P_{\text{LZ}} = \exp(-\pi/2v_{\text{LZ}}^2). \quad (22)$$

On the other hand, KZM can be used to study the dynamics in the quantum phase transition driven across the quantum critical point. The essence of KZM is the adiabatic impulse approximation [23], where the quench process is divided into three stages: adiabatic far away from the critical point, frozen state in the vicinity of the point when $[-t, t]$ with $t$ denoting the freeze-out time scale, and the restart of adiabatic process. For convention, we define the relaxation time as $\tau_{\text{rel}} = g^{-1}$, where $g$ is the energy gap between the ground state and the first excited state, and $t$ is estimated with $\tau_{\text{rel}} = \alpha t$ and $\alpha = \mathcal{O}(1)$. Let $\alpha = 1$, the impulse region is given by $(-v_{\text{LZ}}^{1/2}, v_{\text{LZ}}^{1/2})$. When the starting and ending points of $t_{\text{LZ}}$ in the Landau-Zener Hamiltonian is outside the
impulse region, it can be regarded that there is a complete LZT in this two-level system. The similarity between LZT and KZM was firstly point out in[43], one of the most prominent features is that when the system approaches the critical point, the inverse of the energy gap tends to infinity in KZM and the counterpart in LZT also increases.

The defect density $n_W$ can also be estimated by the integral of the transition probability $P_{LZ} \approx p_k$ over the pseudo-momentum space${}^{[42]}$

$$n_W \approx \frac{1}{\pi} \int_0^\pi P_{LZ}(k)dk,$$  

which can be measured in two-level systems by means of quantum simulation of the quench dynamics with well-designed Landau-Zener crossings, similar as the experiments for the Ising chain without noise.$^{[41][43]}$ For the three quench protocols in the noisy XY chain, the parameters in (18) for a two-level system correspond to the counterparts in the Hamiltonian of the noise-free quench $\mathcal{H}_{x}^{(0)}(j = 1, 2, 3$ for the three protocols). In addition, the noise part $\eta(t)V_j$ correspond to stochastic fluctuations of the control fields $V_j$, which can be realized by inducing the $\tilde{\sigma}_z$ or (and) $\tilde{\sigma}_x$ fluctuations with tunable strength $W$ into the two-level systems.

To simulate the transverse quench ($j = 1$) in transverse field XY chain with many independent LZT in pseudo-momentum space, one can use the mapping $v \rightarrow v_h, \Delta \rightarrow (J_x - J_y) \sin k$ and $C \rightarrow (J_x + J_y) \cos k$. This indicates the substitution in this quench protocol

$$v_{LZ} = v_h/(2J\gamma \sin k)^2,$$
$$t_{LZ} = 4J\gamma \sin k(t + J \cos k/v_h),$$

which can transform $\mathcal{H}_{x}(k)$ into the standard LZT form. For the second quench protocol ($j = 2$) of anisotropic quench through the multicritical point, following the mapping of the parameters similar as those in the first case, one can obtain the corresponding substitution

$$v_{LZ} = v_x/[2(J_y \sin 2k + h \sin k)]^2,$$
$$t_{LZ} = 4(J_y \sin 2k + h \sin k)/[t + (J_y \cos 2k + h \cos k)/v_x].$$

Similarly, for the third quench protocol ($j = 3$) along the gapless line, the mapping of the Hamiltonian in pseudo-momentum space gives the corresponding substitution

$$v_{LZ} = v_x \sin k/[2(\cos k + 1)]^2,$$
$$t_{LZ} = -4(\cos k + 1)t.$$  

To reduce the number of implementing LZT in the estimation of the predicted $n_W$, we consider the distribution of the excitation probability $p_k$ as a function of $k$. The results for the three protocols with typical quench time $\tau_j$ are plotted in Fig. 3(a). One can find that only those $k$ modes in the regions near the points $k_c = 0$ and (or) $k_c = \pm \pi$ contribute the major to the excitation formation. Therefore in experiments, one can just implement some LZT of the $k$ modes in these regions to extract the simulated defect density. For practical implementation, one may define a cutoff pseudo-momentum $k_c$ in quantum simulation, which separates the $k$ axis into two or three (for transverse quench) parts determined by the excitation probability $p_k(k) > 0.03$ here (other small values are also applicable). Figure 4(b) depicts $|k_c - k_e|$ as a function of the quench time in the three protocols, which approximately gives the length of the required simulation regions measured from the point $k_c$. The results also show that the region length monotonously decreases as increasing $\tau_j$.

IV. CONCLUSIONS

In summary, we have studied the quench dynamics of a transverse-field XY chain driven across quantum critical points by noisy control fields and demonstrated that the defect productions for three quench protocols with different scaling exponents exhibit the anti-KZ behavior. We have also shown that the optimal quench time to minimize defects scales as a universal power law of the noise strength for all the three cases. Moreover, by using quantum simulation of the quench dynamics in the spin system with well-designed Landau-Zener crossings in pseudo-momentum space, we have proposed an experimentally feasible scheme to test the predicted anti-KZ behavior.

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