Numerical validation of the electromagnetic gyrokinetic code NEMORB on global axisymmetric modes

A. Biancalani, A. Bottino, Ph. Lauber and D. Zarzoso

Max-Planck-Institut für Plasmaphysik, 85748 Garching, Germany

www.ipp.mpg.de/~biancala

Received 27 November 2013, revised 25 February 2014
Accepted for publication 4 March 2014
Published 1 October 2014

Abstract

In this paper, we report on a comparison of collisionless simulations on global modes (i.e. low poloidal mode number) with the gyrokinetic particle-in-cell code NEMORB against analytical theory and the gyrokinetic semilagrangian code GYSELA. Only axisymmetric modes, i.e. with toroidal mode number \( n = 0 \), are considered, and flat equilibrium profiles. Benchmarks are performed for geodesic acoustic modes against local analytical theory. In the presence of energetic ions, benchmarks of NEMORB are performed against GYSELA. The models of adiabatic versus trapped-kinetic versus fully kinetic-electrons and of electrostatic versus electromagnetic at very low beta are compared. Scalings of Alfvén modes are also presented.

Keywords: geodesic acoustic modes, GAM, validation, axisymmetric, EGAM

(Some figures may appear in colour only in the online journal)

1. Introduction

Understanding the dynamics of plasma instabilities in tokamaks is important because they are deleterious for the stability itself and because their interaction with energetic particles (EP) can redistribute the population of EP and reduce the efficiency of external plasma heating. Here we use the gyrokinetic (GK) code NEMORB to study collisionless simulations of global (i.e. small poloidal mode number) axisymmetric (toroidal mode number \( n = 0 \)) modes in tokamaks, both in the absence and in the presence of EP. We consider flat equilibrium profiles, and compare with local analytical theory and with the GK code GYSELA. The code NEMORB [1] is a global GK particle-in-cell (PIC) code, derived as the multispecies, electromagnetic version of the code ORB5 [2], which was written for studies of turbulence in tokamak plasmas. The code GYSELA is a global flux-driven electrostatic semilagrangian GK code [3,4]. The interest of analysing axisymmetric modes like geodesic acoustic modes (GAMs) [5] in NEMORB is twofold. First, GAMs have been found to play a role in the regulation of turbulence in ASDEX Upgrade [6]. In addition, energetic-ion driven GAMs (EGAMs) [7,8] have been observed to significantly modify and modulate the turbulence in GK simulations [9].

This first section of this paper is an introduction and description of motivations for our work.

In the second section we provide the model equations of NEMORB.

In the third section, we focus on simulations of GAMs. They are modes resulting from the coupling of the \( m = 0 n = 0 \) potential perturbations and \( m = 1 n = 0 \) density perturbations, present in tokamak plasmas due to geodesic curvature, and with mainly electrostatic polarization. A GAM is observable in numerical simulations by initializing a zonal perturbation in ion density with a radial gradient. The resulting radial electric field creates an \( E \times B \) drift along the poloidal direction, which starts oscillating due to the nonhomogeneity given by the tokamak curvature. This oscillation decays due to Landau damping leaving a zonal radial electric field which produces a residual zonal flow. The frequency and damping of the linear GAM oscillation is calculated at each radial position analytically [5,8,10–16] and compared with numerical results of NEMORB (see also [17,18]). We perform this comparison for flat density and temperature profiles, and nearly flat q-profiles, as a benchmark of NEMORB against local analytical theory. For the same simulations, residual flow comparisons with analytical theory are also made. Results obtained with the electrostatic version of NEMORB by using different models to treat the electrons are compared (adiabatic versus kinetic versus hybrid) and electromagnetic simulations at very low beta are performed, as a benchmark for electromagnetic version of NEMORB.

In the fourth section, we add an EP (in particular energetic deuterium) population. In this case the frequency and damping rate of GAMs is modified and a new branch named energetic-particle driven GAM (EGAM) [7,8] appears together with...
GAMs, with different frequency and growth/damping rate. Due to the similarity in spatial structure, the problem of how classifying GAMs and EGAMs arises: we define for this particular EP shifted-Maxwellian distribution function, EGAMs as those whose growth rate increases with EP concentration. Benchmarks of NEMORB on EGAMs against GYSELA for flat equilibrium profiles are performed.

In the fifth section, we focus on modes with mainly Alfvén polarization. We show first comparisons of numerical versus analytical scalings for Alfvén waves observed with NEMORB with \( n = 0 \).

Finally, in the sixth section we give conclusions and describe our future work.

2. Model equations of NEMORB

NEMORB is a global nonlinear GK \( \delta f \) PIC code [1], derived as the multi-species electromagnetic version of ORB5 [2]. The Lagrangian formulation that is used, is based on the GK Vlasov–Maxwell equations of Hahm, Brizard and Sugama [19–21]. The Lagrangian and Hamiltonian are given by:

\[
L = \left( eA + p_c \right) \cdot \dot{R} + \frac{m^2}{\epsilon} \mu \dot{\theta} - H, \tag{1}
\]

\[
H = m \frac{V^2}{2} + m\mu B + eJ_0\Phi + C(\Phi^2),
\]

where \( \mu = \frac{v_i^2}{2} / m \), \( mU = p_c - eJ_0A_i \) and \( U = \frac{\partial H}{\partial p_z} \). Here \( R, p_c, \mu, U, \theta \) are the particle coordinates, \( U \) is the parallel velocity of the particle and \( J_0 \) is the gyroaverage operator. The equations of motion of NEMORB are the Euler–Lagrange equations:

\[
\dot{R} = \frac{\partial H}{\partial p_z B_i^*} = \frac{1}{eB B_i^*} F \cdot \nabla H, \quad \dot{p}_z = -\frac{B^*}{B_i^*} \cdot \nabla H. \tag{2}
\]

Replacing the Hamiltonian in the Euler–Lagrange equations we obtain:

\[
\dot{R} = \left( \frac{p_c}{m} - \frac{e}{m} J_0 A_i \right) B_i^* + \frac{1}{eB B_i^*} F \cdot \nabla H, \tag{3}
\]

\[
\dot{p}_z = -\frac{m B^*}{B_i^*} \cdot \left( \mu \nabla B + \frac{e}{m} \nabla J_0 \Psi \right), \tag{4}
\]

\[
\Psi = \Phi - \frac{p_c}{m} A_i, \tag{5}
\]

where \( B^* = \nabla \times A^* \), \( A^* \equiv A + (p_c/e) b \) and \( F = \epsilon \cdot B \). GK Vlasov equations are solved for ions, drift-kinetic equations for electrons. The Vlasov equation is:

\[
\frac{df}{d\tau} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial R} \cdot \dot{R} + \frac{\partial f}{\partial p_z} \dot{p}_z = C(f) + S, \tag{6}
\]

where \( C(f) \) is the collision operator, and \( S \) is the source term. In the simulations described in this paper, we have collisionless dynamics \( (C(f) = 0) \) without sources \( (S = 0) \), and energy and momentum conservation can be proved via GK field theory [22].

Finally the linearized GK Poisson equation (in the long wavelength approximation) and parallel Ampère’s law are:

\[
- \nabla_\perp \left( \sum_{\text{species}} \frac{m \rho_\perp}{e B^2} \right) \nabla_\perp \Phi = \sum_{\text{species}} \delta n, \tag{7}
\]

\[
\left( \sum_{\text{species}} \frac{\mu \rho_\perp e^2}{m} A_i \right) - \nabla^2 A_i = \mu_0 \sum_{\text{species}} \delta j_\parallel. \tag{8}
\]

The boundary conditions on the fields are unicity condition (solution does not depend on magnetic angle \( \chi \), where \( \chi = (1/q) \int_0^s B \cdot \varphi / B \cdot (d\theta') \) and 0 radial derivative at radius \( s = 0 \) and \( \phi = A_1 = 0 \) at radius \( s = 1 \). Regarding particles, they are reflected with \( \chi = -\chi \) when they exit the flux surface with \( s = 1 \).

Ideal MHD equilibria (via the CHEASE code [23]) can be used as an input. NEMORB is a multi-ion species code, which means that impurities or EP can be initialized as initial state and let evolve like a separate species. NEMORB is also massively parallelized.

3. Geodesic acoustic modes (GAMs)

3.1. Equilibrium and simulation parameters

We choose an analytical tokamak equilibrium with circular flux surfaces and high aspect ratio \( (\epsilon = a/R = 0.1) \), toroidal major radius \( R_0 = 1.3 \) m, and toroidal magnetic field \( B_{\text{tor}} = 1.9 \) T. We consider flat temperature and density profiles. Very low values of shear are considered \( (s \sim 10^{-2}) \), so that the \( q \)-profiles are nearly flat, and our simulations can be compared with local analytical theory. We initialize a deuterium charge density perturbation of the form \( \sin(\pi r/a) \), which has a radial gradient but is independent of the poloidal and toroidal angle, and we let it evolve linearly.

Typical simulations have a spatial grid of \( (s, \chi, \phi) = 64 \times 64 \times 4 \) and a time step of 10–100 \( \Omega_\parallel^{-1} \), with \( 10^9–10^{10} \) markers. The length of a typical simulation is \( 2 \times 10^5 \Omega_\parallel^{-1} \), corresponding to 2000–20 000 time steps. GAMs oscillations are observed, and we measure the radial electric field amplitude, frequency and damping rate.

An example of the evolution in time of the amplitude of the radial electric field (i.e. the radial gradient of the flux-surface-averaged scalar potential) is shown in figure 1, measured at a location with radius \( s \simeq 0.5 \). In the same figure, we plot the function \( \cos(\omega_{\text{GAM}} \times t) \exp(-\gamma_{\text{GAM}} \times t) \), where we take \( \omega_{\text{GAM}} \) from analytic theory of [10] and \( \gamma_{\text{GAM}} \) from [15]. We find a good fit of the analytical and the numerical radial electric field evolution in time, shown there in a time range after the relaxation of the initial perturbation, which takes a few GAM oscillations.

3.2. Power balance check

In order to quantify the numerical errors affecting the results of our linear simulations, we use here a diagnostic that measures the power balance between the perturbed field growth of the mode and the power transferred from the particles (see [24]). The former is measured by calculating the time derivative of
the energy contained in the perturbed field for a certain wave number $k$, $dE_{\text{field}}/dt$. The latter, is calculated as $dE_k/dt = j_k \cdot E_k$. The relative error of the power balance, $\delta E$, is calculated and gives us how much the power balance is violated:

$$\delta E_{\text{rel}} = \frac{dE_{\text{field}}/dt - dE_k/dt}{dE_{\text{field, max}}/dt},$$

(9)

where $dE_{\text{field, max}}/dt$ is the maximum value of $dE_{\text{field}}/dt$ in the time interval of interest. In figure 1, the relative error of a NEMORB simulation is shown. For this specific run, the error falls within 2%.

### 3.3. Frequency

Here we show two benchmarks: the first with the scalings of the GAM frequency versus the safety factor $q$ and the second of the GAM frequency versus the electron to ion temperature ratio $\tau_e$. A plasma with $\rho^* = \rho_e/\rho_i = 1/160$ is considered. For the first benchmark, several simulations are carried out, all with $\tau_e = 1$, and each with different $q$-profile. Each $q$-profile is linear and centred at a rational surface with a particular value of $q$. The frequency measured with NEMORB for the several simulations with different $q$-profiles is found to agree well with the GK dispersion relation given in [10] (see figure 2):

$$i\Lambda = \delta W_f + \delta W_k,$$

(10)

where $\Lambda$ is the generalized inertia (defined in the appendix), $\delta W_f$ is the ideal MHD contribution to the potential energy perturbation and $\delta W_k$ is the kinetic component to $\delta W$ due to the EP compressions. In our cases we have $\delta W_f = \delta W_k = 0$. An electrostatic relation given by [8] is also shown to fit well with NEMORB results. The explicit formula given by equation (2.9) of [13] is derived for large values of $q$ and consistently it is shown to fit well in the appropriate limit. For the second benchmark, several simulations are performed, all with $q = 3$, and each with different value of $\tau_e$. The comparison with analytic theory is shown in figure 2. We obtain a good agreement for this scaling except at very low values of $\tau_e$, where further study is needed.

### 3.4. Damping rate

The damping rate of GAMs can also be measured in NEMORB and compared with analytical theory. We find that this study is more numerically demanding than the study of frequencies: for low number of markers the amplitude of electric field oscillation is observed not to decrease exponentially, due to numerical errors. Therefore we increase the number of markers up to $10^6$ and decrease the time step down to $dt = 10 \Omega_i^{-1}$. We find that the scaling of NEMORB results is in good agreement with analytic theories for $q < 2$ (see figure 3), but for larger values of $q$ finite orbit width (FOW) effects become important, and the theories not accounting for them are observed to deviate. These FOW effects are found to be difficult to estimate. In fact, the initial electric field perturbation develop during the evolution large radial gradients especially near the border $s = 1$ (see also figure 4), due probably to our boundary conditions, and this increases the value of $k_\perp \rho_i$ and affects the dynamics also at lower radii. We measure $k_\perp$ and use it in the explicit formula of Sugama and Watanabe [15], obtaining a qualitative good match in the scaling with $q$. Further comparison at $q \geq 2$ is on the way. As a next step on this field, further work is needed to increase the accuracy or decrease $k_\perp$ of our perturbation. Other theories (like Zonca et al. [11, 25]) are also going to be compared with NEMORB’s results, and expected to give same values as Sugama and Watanabe [15], like shown in [12].

### 3.5. Zonal flow residuals

The residual zonal electric field, which remains after the GAM oscillation is damped in collisionless simulations, can be measured and compared with analytical prediction. Rosenbluth and Hinton in 1998 [26] calculated the residual zonal flow $A_y = u_y(t = \infty)/u_y(t = 0)$ (with $u_E$ being the $E \times B$ drift), for small values of inverse aspect ratio $\epsilon = r/R_0$ and in the limit of radial wave-length much larger than the
Figure 2. On the left: GAM frequency for several local simulations with $\tau = 1$, centred at different values of $q$. Values given by NEMORB (black) fit well with the analytic theory of [10] (blue) and with [8] (green) at all values of $q$. Explicit formula (equation (2.9) of [13]) valid for large $q$ is also shown in blue. On the right: GAM frequency for several local simulations with $q = 3$ and different values of $\tau_e$. A good agreement is found except at very low values of $\tau_e$.

Figure 3. Damping rate for the same simulations as in figure 3. Each simulation has the $q$-profile centred at a different value. Results of NEMORB (black crosses) are compared with explicit analytical formula of Sugama and Watanabe [13] (red dashed line), with the same formula where the FOW corrections have been neglected (green dashed line) and with the dispersion relation of Zonca (blue dashed line). We can see that for these simulations the FOW effects are increasingly important the more we go into higher values of $q$.

Poloidal gyroradius $\rho_p = \rho q / \epsilon$:

$$A_{r, RH} = \frac{1}{1 + 1.6 q^2 / \epsilon^{1/2}}.$$  

(11)

Later on, in 2006, Xiao and Catto extended this formula for radial wave-length of the same order of magnitude as the poloidal gyroradius, and added higher-order corrections of aspect ratio $[27]$:

$$A_{r, XC} = \frac{1}{1 + q^2 \Theta / \epsilon^2}.$$  

(12)

with $\Theta$ defined as:

$$\Theta = \left(1.6 \epsilon^{3/2} + \frac{1}{5} \epsilon^2 + 0.36 \epsilon^{5/2}\right) - 2.44 \epsilon^{5/2} k_1^2 \rho_p^2.$$

In figure 4 the residual flow is shown for simulations with $q = 1.5$ and $\epsilon_0 = 0.1$. We measure it in NEMORB as the average of $V \phi(t)/\phi(t = 0)$ over the last few periods of GAM oscillation (here $\phi$ is the flux-surface-averaged scalar potential). The fluid limit is studied by repeating the simulations with gradually smaller value of $\rho^*$. The values of $L_x$ are chosen in the set $[20; 1280]$, corresponding respectively to $\rho^* \in [0.1; 0.0016]$. This limit is also called local limit in GK simulations of turbulence $[28]$. For values of $L_x$ larger than 200, corresponding to values of $\rho^*$ smaller than 0.01, the results of NEMORB are in good agreement with the fluid limit of the analytical prediction of Xiao and Catto, namely equation (12) where we neglect the finite poloidal Larmor-radius effects (expressed in the term with $k_1 \rho_p$).

On the other hand, for values of $\rho^*$ larger than 200, we still do not find a good quantitative agreement of NEMORB’s results with the analytical prediction of Xiao and Catto with finite-Larmor-radius (FLR) effects included. The analytical predictions of Xiao and Catto shown in figure 4 have been calculated by using three different values of $k_{1x}/a = 2 \pi / (k_1 a)$.

The first two values are $k_{1x}/a = 2$ and $k_{1x}/a = 4$, where $k_{1x}/a = 2$ is the value which is consistent with the radial shape of our initial perturbation in density ($\delta n \sim \sin(\pi r / a)$), but it does not fit NEMORB’s results, so we have plotted also $k_{1x}/a = 4$ for comparison. These two values correspond respectively to $k_{1x} \rho_p \in [9.42; 0.147]$. The third value of $k_{1x}/a$ used for the calculation of the prediction of Xiao and Catto is in fact a profile in $s$, derived from the value of $k_{1x} a$ measured in NEMORB as $V \phi / \phi$ over the last few periods of GAM oscillation. This profile is shown for a simulation with $L_x = 320$ in figure 4 and compared with the profile measured at $t = 0$. This simulation has a grid of $(64, 64, 4)$ points in the $(s, \chi, \phi)$ direction, a time step of $dt = 10 \Omega_{\chi}^{-1}$ and $10^7$ markers. The discrepancy we
Figure 4. Residuals of the zonal flow $A_r$ for an equilibrium with $q = 1.5, \epsilon_0 = 0.1$. On the top left, the fluid limit is studied by repeating the same simulation with increasing values of the parameter $L_x = 2/\rho^* = 2a/\rho_i$. In this limit, the results of simulations with NEMORB (black points) tend to the fluid limit of the analytical prediction of Xiao and Catto (blue dotted horizontal line). The Rosenbluth–Hinton value is also given as a red dotted horizontal line. The prediction of Xiao and Catto for finite poloidal gyroradius $\rho_p = \rho_i q/\epsilon$ are also shown, for values of $\lambda_r = 2a, \lambda_r = 4a$ and $\lambda_r$ measured by NEMORB at the end of the simulation ($\lambda_r/a \in [12.3, 5.7]$). On the top right, the profile of the residuals is shown for the simulation with $L_x = 320$. The difference of the residuals at $s = r/a = 0$ from the Xiao–Catto prediction is thought to be due to the boundary conditions. In the lower graph, the value of $k_\perp a$ measured by NEMORB is shown, at the beginning and at the end of the simulation with $L_x = 320$.

3.6. From adiabatic electrons to kinetic electrons

In the previous section, Rosenbluth–Hinton simulations with adiabatic electrons have been described. Now we want to describe the differences we find, if we treat the electron species with a kinetic model, rather than adiabatic. We have two models for this treatment: the trapped-kinetic-electrons model and the fully kinetic-electrons model. In the trapped-kinetic-electrons model, NEMORB selects at $t = 0$ the electrons which will perform trapped orbits and treats them kinetically, whereas those which will perform passing orbits are treated adiabatically. The advantage of this model is that wave–particle resonances at the bounce or precessional frequency are considered for trapped electrons, and modes like trapped electron modes are included in the model, and at the same time NEMORB does not need to resolve the parallel electron dynamics, which requires a very small time step. In the fully kinetic-electrons model, all electrons are treated kinetically, like ions. This model has the advantage that also wave–particle interaction with passing electrons are retained in the model, but the time step has to be set small enough to resolve the fast parallel electron dynamics.

In figure 5, the evolution of the radial electric field oscillation of a GAM is shown, for the three different models of electrons of NEMORB, with $q = 3, \epsilon = 0.3125$, and $L_x = 128$. As we can see, frequency does not change much in the three models, whereas damping rates are higher when trapped electrons are treated kinetically, possibly due to resonances with bounce frequencies of trapped electrons (consistently with [29]).

When we make a zoom in time of the GAM oscillation, when the electrons are treated kinetically, we observe a higher frequency oscillation (see figure 5, for a case with $q = 2, \epsilon = 0.323$, and $L_x = 460$). This high frequency oscillation, sometimes called ‘surface quasi-electrostatic Alfvén wave’ is the limit of kinetic Alfvén waves for $\beta$ going to zero (electrostatic model) at fixed temperature [30, 31].
3.7. From electrostatic to electromagnetic at low beta

We now show results of Rosenbluth–Hinton simulations in the case where the electrons are treated kinetically and NEMORB also solves the Ampère’s law. In this case, also magnetic perturbations are treated in the model. At this stage of code validation, we restrict our simulations to very low values of beta $\beta = 10^{-4}$, and we compare with simulations where Ampère’s law is not solved (electrostatic mode). The equilibrium is a local equilibrium with $q = 3$.

As we can see in figure 6, the benchmark of electromagnetic versus electrostatic mode of NEMORB is successful for these Rosenbluth–Hinton tests: in fact the dynamics is found to be the same. If we make a zoom in time, we find that higher frequency oscillations are present together with the GAM oscillation. These are shear-Alfvén oscillations (with toroidal mode number $n = 0$), which are initialized implicitly in the Rosenbluth–Hinton test due to finite $\epsilon$ effects (which couples different m-modes). These Alfvén modes will be described more in details in section 5.

4. Energetic-ion driven GAMs

In this section we describe results of GAMs with EP, for equilibrium profiles of the nonlinear simulations performed with GYSELA [3, 4] and described in [8]: these are flat equilibrium temperature and density profiles and a nearly flat $q$-profile. They are expected to behave as local simulations, due to the fact that the profiles are nearly flat. Results of nonlinear electrostatic simulations are shown, with electrons treated adiabatically. In previous section, NEMORB has been successfully benchmarked on GAMs against analytical theory for linear simulations falling into this limit (see also [17, 18]). In particular, in this report we show some first results of simulations with NEMORB where also an EP population is present. At this step we consider with NEMORB a bump-on-tail EP population (see [8]).

The dependence, in the linear phase, of frequency and growth/damping rates of GAMs and EGAMs on EP concentration is shown, and compared with results of GYSELA shown in [8].

4.1. Equilibrium and simulation parameters

We choose a tokamak equilibrium with circular flux surfaces and moderate aspect ratio ($R = 1$ m, $\varepsilon = a/R = 0.3125$), $B = 1.9$ T, with flat equilibrium temperature and density profiles (we choose $\tau_i = T_i/T_e = 1$ and $\rho^* = \rho_i/a = 1/64$). Very low values of shear are considered at the centre of the simulation box, where the measurements are done, so that the $q$-profile is nearly constant with value $q = 3$, and our simulations are therefore expected to match local theory (see figure 7). The radial domain is chosen in the range $0.2 < \rho < 0.8$. In the inner half of the radial domain the
value of the safety factor is always very near \( q = 3 \), with a maximum around \( s = 0.25 \) (\( s \) is the squared root of the poloidal magnetic flux, used here as radial coordinate). In the outer half of the radial domain, the safety factor increases to reach the value of \( q = 3.25 \) at the edge. Some simulations are also performed with flatter \( q \)-profile and no difference in the results is observed. We initialize at \( t = 0 \) a charge density perturbation of the form \( \sin(\pi r/a) \), which has a radial gradient but is independent of the poloidal and toroidal angle. In NEMORB, a bump-on-tail distribution function for the EP population is implemented at the initialization of our simulations, \( t = 0 \), like in equation (27) of [8]:

\[
F_{eq,h} = F_{M,h}e^{-\frac{\hat{\xi}^2}{\mu_B}} \cosh\left(\frac{\hat{\xi} \hat{\theta}}{T_h}\right),
\]

(13)

where \( F_{M,h} = \frac{n_i}{(2\pi T_h/m_v)^{3/2}} e^{\frac{\hat{\xi}^2}{\mu_B}} \). We choose the hot ion normalized temperature as \( T_h \). The parallel velocity of hot ions as \( \hat{\xi} = v_i/v_{th} = 4 \). The parallel perturbation is let evolve in time in a nonlinear electrostatic simulation with adiabatic electrons. We are interested here only in the linear phases. In order to investigate the effect of nonlinearities, we also perform a set of linear simulations, using a linearized version of NEMORB. A typical simulation has a spatial grid made of \( s = 64 \times 64 \times 4 \) points and the time step is 0.5 \( \Omega_i^{-1} \), with \( 10^7 \) markers. The length of a typical simulation is \( 2 \times 10^4 \Omega_i^{-1} \), corresponding to \( 4 \times 10^4 \) time steps. Running in parallel on 512 CPU it takes about 15 h.

4.2. Power balance check

Before analysing the result of the simulation, we check the conservation of energy and calculate an estimation of the numerical error, similarly to what is done also for GAMs in the absence of EP in previous section. In order to quantify the numerical errors affecting the results of our simulations, we use here a diagnostic that measures the power balance between the perturbed field growth of the mode and the power transferred from the particles [24]. The only difference with the diagnostic used for GAMs, is that here we take into account also the energy transfer with the EP. In figure 8, the relative error of a NEMORB simulation is shown. For these simulations we consider as physical the dynamics in the time phase where this relative error falls within 1%.

4.3. Frequencies and growth rates

In a typical run we observe oscillations of \( \text{Im}(\phi_{10}) \) (see figure 8), which is the imaginary part of the complex Fourier transform in \( \theta \) of the zonal component \( (n = 0) \) of the scalar potential \( \phi \) (in other words, \( \text{Im}(\phi_{10}) \) is the Fourier coefficient of \( \phi \) relative to the \( \sin(\theta) \) component). The typical period of the oscillations is of the order of the sound time \( \tau_s \sim R/c_s \). We can observe the coexistence of two modes with different frequencies, where one is damped (GAMs) and one is growing (EGAMs). In the late phase we can observe a nonlinear saturation of the EGAM. In our simulations, we filter out all perturbations with toroidal mode number \( n \neq 0 \). Therefore, even though our simulations are nonlinear, no interaction with ITG or TEM turbulence is studied at this stage. The frequency of the modes is measured by performing a Fourier transform in time of \( \text{Im}(\phi_{10}) \) at the mid radius, \( s = 0.5 \). Two main frequencies are detected (see figure 9): the higher frequency corresponds to GAMs and the lower to EGAMs. The growth rate of EGAMs is measured by performing a linear fit in logarithmic scale of the absolute value of \( \text{Im}(\phi_{10}) \).

A scan of the frequency and growth rate of GAMs and EGAMs is shown in figure 10, versus the relative EP concentration \( n_h/n_i \). Two set of simulations of NEMORB are shown: a set of nonlinear simulations, and a set of simulations performed with the linearized version of NEMORB. We can see that results of nonlinear simulations performed with NEMORB agree with those of GYSELA described in [8]. GAMs are always damped and their frequency is difficult to measure for EP concentration lower than the EGAM instability threshold. On the other hand, we find EGAM frequency difficult to measure for EP concentration lower than \( n_h/n_i = 0.05 \). Regarding the growth rates of EGAMs, we observe a very good agreement of NEMORB nonlinear simulations with GYSELA, with an instability threshold at \( n_h/n_i \sim 0.15 \) and an almost linear dependence on \( n_h/n_i \). Slightly higher values of \( \gamma \) are found right above the threshold. Regarding simulations performed with linearized version of NEMORB, we observe a general qualitative agreement with the nonlinear simulations, except for the frequency of GAMs, which is lower in linear simulations than in nonlinear simulations, and growth rates of EGAMs, which are higher in linear simulations than in nonlinear ones. This difference in the results is probably due to the fact that in the linear model we have chosen to initialize the EP distribution function as a function of constants of motion only, by neglecting the unperturbed terms in \( \partial \tilde{\gamma}/\partial t \) in the Vlasov Equation. In the nonlinear model the initial distribution function is not an equilibrium and slowly evolves during the simulation. This could lead to the excitation of EGAMs for values of \( n_h \) slightly different from the ones in the linear simulation.
Figure 8. On the left: the imaginary part of the complex Fourier transform in θ of the zonal component (n = 0) of the scalar potential φ, measured at the centre of the radial domain, s = 0.5, versus time, for a nonlinear simulation with \( n_H/n_i = 0.15 \), \( dt = 0.5 \Omega_i^{-1} \), and \( 10^7 \) markers. On the right, the relative error of the power balance for the same simulation.

Figure 9. On the left, Fourier transform in time of \( \text{Im}(\phi_{10}) \) with NEMORB (blue points), for a nonlinear simulation with \( n_H/n_i = 0.15 \), \( dt = 0.5 \Omega_i^{-1} \), and \( 10^7 \) markers. Red points are results obtained with GYSELA and described in [8]. On the right, the absolute value of \( \text{Im}(\phi_{10}) \) in logarithmic scale, versus time, with the linear fit performed to measure the growth rate, for a nonlinear simulation with \( n_H/n_i = 0.20 \), \( dt = 0.5 \Omega_i^{-1} \), and \( 10^7 \) markers.

Figure 10. On the left: frequencies of GAMs (higher frequencies) and EGAMs (lower frequencies) versus fast ion relative concentration, normalized with the theoretical value given in the absence of fast ions in figures 8, 9 and 10 of [8], \( \omega_{th,zarz} = 0.0125 \Omega_i \). Green and black points are respectively nonlinear and linear results of NEMORB, whereas red points are results obtained with GYSELA and described in [8]. On the right: EGAM growth rate versus fast ion relative concentration.
5. Shear Alfvén modes

5.1. Equilibrium and simulation parameters

In order to perform these first scans of shear Alfvén modes with NEMORB, we choose a tokamak equilibrium with local profiles and large aspect ratio, in order to fall into the local regime. The major and minor radii are \( R_0 = 1.667 \text{ m} \) and \( a = 0.1667 \text{ m} \), with aspect ratio \( \varepsilon = 0.1 \), circular cross section, toroidal magnetic field \( B_{\text{tor}} = 2.4 \text{ T} \). The density and temperature profiles are flat, with values \( T_e = 1 \text{ keV} \) and \( \beta_e = 10^{-4} \), and we perform simulations with different \( q \)-profiles, each of them almost flat and centred at a different value of \( q \). A typical simulation has a spatial grid of \( (s,\chi,\phi) = 64 \times 32 \times 4 \) and a time step of \( 4 \Omega_i^{-1} \), with \( 10^7 \) markers for ions and \( 10^7 \) for electrons. The length is \( 300 \Omega_i^{-1} \), corresponding to 300 time steps. The beginning of a GAM oscillation is observed in the perturbed scalar potential \( \phi \), and on top of it we measure faster oscillations in parallel vector potential \( A_{\parallel} \) (see figure 11).

5.2. Scaling with \( q \)

We measure the frequency of oscillation of the parallel component of the vector potential, \( A_{\parallel} \), for different simulations with different \( q \)-profile. Local theory of shear Alfvén wave predicts an oscillation with a frequency scaling like: \( \omega = v_A k_\parallel = v_A m/q R_0 \), where \( v_A = B/\sqrt{4\pi n m_i} \). In figure 11, the comparison is shown. We find a good qualitative agreement of NEMORB results with analytical scaling, which is the \( 1/q \) scaling (all other dependencies are kept constant in this scan). The reason why NEMORB’s measured frequencies are slightly higher than analytical prediction might be the finite \( \varepsilon \) effect, and is to be investigated more in details.

6. Conclusions and next steps

In this paper, we have performed a numerical validation of the code NEMORB on GAMs in collisionless simulations with flat equilibrium profiles. Benchmarks against local analytical theory have been described for GAMs and against the code GYSELA for EGAMs. Scalings of frequencies, damping rates and electric field residuals have been analysed. First scalings of Alfvén modes have also been shown.

In the absence of energetic particles, we have found good matching of GAM frequency against analytical theory whereas the damping rates and the residuals have been found more difficult to compare, due to the strong dependence of finite-orbit-width effects on the radial wave vector \( k_\parallel \), which evolves in global simulations leading to values different from the initial one. More detailed analysis is required in this context both on the numerical side, and on the analytical one, where we are going to compare also with other theories [11, 14]. Differences in the dynamics have been discussed as depending on the model used for treating electrons (adiabatic versus trapped-kinetic versus fully kinetic). Moreover, the first benchmark of GAMs between electromagnetic simulations at very low beta and electrostatic simulations has been showed.

In the presence of EP, we have found a good agreement between frequencies obtained with NEMORB and with GYSELA for both nonlinear and linear simulations with NEMORB. The benchmark has been performed successfully, with some divergences in the frequency of GAMs measured with the linear and nonlinear version of NEMORB, for small but finite EP concentrations, which requires further investigation. Regarding the growth rates, we have found a good quantitative agreement with nonlinear results of NEMORB and GYSELA, making our benchmark successful, and slightly higher values with the linearized version of NEMORB, which will be further investigated in the future. A comparison of NEMORB results with analytical theory, like in [32, 33], is also going to be performed, and a general parametric distribution function for EP is going to be implemented in NEMORB [34].

Finally, first scalings of shear Alfvén modes have been showed as a validation of the code in electromagnetic modes for Alfvénic fluctuations.

As further next steps, we are going to perform simulations of GAMs with realistic global tokamak equilibrium profiles, both in electrostatic and electromagnetic mode, and compare with other codes like LIGKA [35] (Linear gyrokinetic shear Alfvén). EGAMs are also going to be studied in electrostatic and electromagnetic mode, and the electromagnetic effects are going to be investigated. Alfvén modes in non-axisymmetric equilibria are going to be investigated, both in the absence or in the presence of EP.
Appendix. Generalized fishbone-like dispersion relation for GAMS

The generalized fishbone-like dispersion relation (GFLDR) [10], as written in equation (10), describes linear electromagnetic oscillations in the presence of EP, neglecting FLR effects and FOW effects, in the framework of GK theory. For our purpose of benchmarking electrostatic simulations of GAMS in the absence of EP, we can assume $\delta W_f = \delta W_e = 0$. The GFLDR can be used in this case invoking the degeneracy BAE/GAM at the shear Alfvén wave continuum accumulation point. The inertia term, $\Lambda$, is given by [10]:

$$
\Lambda^2(\omega) = \frac{\omega^2}{\omega_i^2} \left[ 1 - \frac{\omega_n}{\omega} \right] + q^2 \frac{\omega_n}{\omega_i} \left[ 1 - \frac{\omega_n}{\omega} \right] \frac{1}{F(\omega/\omega_i)} - \frac{\omega_n \tau}{\omega} G(\omega/\omega_i) - \frac{N^2(\omega/\omega_i)}{D^2(\omega/\omega_i)}.
$$

(A.1)

Here the characteristic frequencies are $\omega_{not} = [(T_e c)/(e B)] \times (k \times b) \cdot (\nabla n)/n, \omega_{dT} = [(T_e c)/(e B)] (k \times b) \cdot (\nabla T)/T, \, n_i = n$ is the ion density, $\omega_{pi} = \omega_{ni} + \omega_{ti} = \sqrt{2T_i/m_i}/(q R_0)$, and $F, \, G, \, N, \, D$ are:

$$
F(x) = x(x^2 + 3/2) + (3^2 + x^2 + 1/2)Z(x),
$$

(A.2)

$$
G(x) = x(x^4 + x^2 + 2) + (x^6 + x^4 + x^2 + 3/4)Z(x),
$$

(A.3)

$$
N(x) = \left[ 1 - \frac{\omega_n}{\omega} \right] [s + (1/2 + x^2)Z(x)]
$$

$$
- \frac{\omega_n \tau}{\omega} (x(1/2 + x^2) + (1/4 + x^2)Z(x)],
$$

(A.4)

$$
D(x) = \left[ \frac{1}{\tau} \right] \left[ 1 + \frac{1}{\tau} \right] + \left( 1 - \frac{\omega_n}{\omega} \right)Z(x)
$$

$$
- \frac{\omega_n \tau}{\omega} [x + (x^2 - 1/2)Z(x)],
$$

(A.5)

where $\tau = T_i/\tau_i$ and $Z(x)$ is the plasma dispersion function:

$$
Z(x) = \pi^{-1/2} \int_{-\infty}^{\infty} e^{-y^2} (y - x) \, dy.
$$

(A.6)

The subscript $s$ stands for a particle species index ($s = i$ for ions and $s = e$ for electrons). The term proportional to $\omega^2/\omega_i^2$ in $\Lambda$ is the usual polarization current contribution, whereas the other term is due to geodesic curvature coupling.

The GFLDR in the form of equation (10) does not include FLR and FOW effects, which are proved in [13,15] to be important. As next steps for this benchmark, we are going to compare results of NEMORB with the extension of the GFLDR given in [11], and with dispersion relations given in [8,13–15].

Acknowledgments

Enlightening discussions with Bruce Scott, Zhiyong Qiu, Claudio di Troia, Fulvio Zonca and Liu Chen are kindly acknowledged. Fruitful discussions with Guoyong Fu, Garrard Conway and Patrick Simon are also kindly acknowledged. Simulations were performed on the IFERC-CSC Helios supercomputer, ORBFAST project. The authors would like to thank the referees for their helpful advices.

References

[1] Bottino A. et al 2011 Plasma Phys. Control. Fusion 53 124027
[2] Jolliet S. et al 2007 Comput. Phys. 177 409
[3] Grandgirard V. et al 2008 Commun. Nonlinear Sci. Numer. Simul. 13 81
[4] Sarazin Y. et al 2010 Nucl. Fusion 50 054004
[5] Winsor N. et al 1968 Phys. Fluids 11 2448
[6] Conway G.D. et al 2011 Phys. Rev. Lett. 106 065001
[7] Fu G. et al 2008 Phys. Rev. Lett. 101 185002
[8] Zarzoso D. et al 2012 Phys. Plasmas 19 022102
[9] Zarzoso D. et al 2013 Phys. Rev. Lett. 110 125002
[10] Zonca F., Chen L. and Santoro R.A. 1996 Plasma Phys. Control. Fusion 38 2011–28
[11] Zonca F., Chen L., Santoro R.A. and Dong J.Q. 1998 Plasma Phys. Control. Fusion 40 2009
[12] Zonca F. and Chen L. 2008 Europhys. Lett. 83 35001
[13] Sugama H. and Watanabe T.H. 2006 J. Plasma Phys. 72 825
[14] Gao Z., Itoh K., Sanuki H. and Dong J.Q. 2006 Phys. Plasmas 13 100702
[15] Sugama H. and Watanabe T.H. 2008 J. Plasma Phys. 74 139
[16] Hallatschek K. 2007 Plasma Phys. Control. Fusion 49 B137–48
[17] Angelino P. et al 2008 Phys. Plasmas 15 062306
[18] Vernay T. et al 2010 Phys. Plasmas 17 122301
[19] Hahm T.S., Lee W.W. and Brizard A. 1988 Phys. Fluids 31 1940
[20] Brizard A.J. and Hahm T.S. 2007 Rev. Modern Phys. 79 421
[21] Sugama H. 2000 Phys. Plasmas 7 466
[22] Scott B. and Smirnov J. 2010 Phys. Plasmas 17 112302
[23] Lütjens H., Bondeson A. and Sauter O. 1996 Comput. Phys. Commun. 97 219
[24] Bottino A. et al 2004 Phys. Plasmas 11 198–206
[25] Qiu Z., Chen L. and Zonca F. 2009 Plasma Phys. Control. Fusion 51 012001
[26] Rosenbluth M.N. and Hinton F.L. 1998 Phys. Rev. Lett. 80 724
[27] Xiao Y. and Catto P. 2006 Comput. Phys. 72 243
[28] Scott B. 2013 private communication
[29] Lauber Ph., Günter S., Könnies A. and Pinches S.D. 2007 J. Comput. Phys. 226 447