Revisiting the Examination Hypothesis with Query Specific Position Bias

Sreenivas Gollapudi  
Search Labs, Microsoft Research  
sreenig@microsoft.com

Rina Panigrahy  
Microsoft Research  
rina@microsoft.com

ABSTRACT
Click through rates (CTR) offer useful user feedback that can be used to infer the relevance of search results for queries. However it is not very meaningful to look at the raw click through rate of a search result because the likelihood of a result being clicked depends not only on its relevance but also the position in which it is displayed. One model of the browsing behavior, the Examination Hypothesis [16, 5, 6], states that each position has a certain probability of being examined and is then clicked based on the relevance of the search snippets. This is based on eye tracking studies [3, 8] which suggest that users are less likely to view results in lower positions. A position dependent variation in the probability of examining a document is referred to as position bias. Our main observation in this study is that the position bias tends to differ with the kind of information the user is looking for. This makes the position bias query specific.

In this study, we present a model for analyzing a query specific position bias from the click data and use these biases to derive position independent relevance values of search results. Our model is based on the assumption that for a given query, the positional click through rate of a document is proportional to the product of its relevance and a query specific position bias. We compare our model with the vanilla examination hypothesis model (EH) on a set of queries obtained from search logs of a commercial search engine. We also compare it with the User Browsing Model (UBM) [6] which extends the cascade model of Craswell et al [5] by incorporating multiple clicks in a query session. We show that the our model, although much simpler to implement, consistently outperforms both EH and UBM on well-used measures such as relative error and cross entropy.

1. INTRODUCTION
Click logs contain valuable user feedback that can be used to infer the relevance of search results for queries (see [1, 12, 13] and references within). One important measure is the click through rate of a search result which is the fraction of impressions of that result in clicks. However it is not very meaningful to look at the raw click through rate of a search result because the likelihood of a result being clicked depends not only on its relevance but also the position in which it is displayed. One model of the browsing behavior, the Examination Hypothesis [16, 5, 6], states that each position has a certain probability of being examined and is then clicked based on the relevance of the search snippets. This is based on eye tracking studies [3, 8] which suggest that users are less likely to view results in higher ranks. Such position bias values can be used to correct the observed click through rates at different positions to obtain a better estimate of the relevance of the document. This raises the question of how one should estimate the effect of the position bias. One method to estimate the position bias is to simply compute the aggregate click through rates in each position for a given query. Such curves typically show a decreasing click through rate from higher to lower positions except for, in some cases, a small increase at the last position on the result page.

However, analyzing the click through rate curve aggregated over all queries may not be useful to estimate the position bias as these values may differ with each query. For example, Broder [2] classified queries into three main categories, viz., informational, navigational, and transactional. An informational query reflects an intent to acquire some information that is assumed to be present on one or more web pages. A navigational query, on the other hand, is issued with an immediate intent to reach a particular site. For example, the query cnn probably targets the site http://www.cnn.com and hence can be deemed navigational. Moreover, the user expects this result to be shown in one of the top positions in the result page. On the other hand, a query like voice recognition could be used to target a good collection of sites on the subject and therefore the user is more inclined to more results including those in the lower positions on the page. This behavior would naturally result in a navigational query having a different click through rate curve from an informational query (see Figure 1). Further, this suggests that the position bias depends on the query.

It may be argued that the difference in the click through rate of a search result which is the fraction of impressions of that result in clicks. However it is not very meaningful to look at the raw click through rate of a search result because the likelihood of a result being clicked depends not only on its relevance but also the position in which it is displayed. One model of the browsing behavior, the Examination Hypothesis [16, 5, 6], states that each position has a certain probability of being examined and is then clicked based on the relevance of the search snippets. This is based on eye tracking studies [3, 8] which suggest that users are less likely to view results in higher ranks. Such position bias values can be used to correct the observed click through rates at different positions to obtain a better estimate of the relevance of the document. This raises the question of how one should estimate the effect of the position bias. One method to estimate the position bias is to simply compute the aggregate click through rates in each position for a given query. Such curves typically show a decreasing click through rate from higher to lower positions except for, in some cases, a small increase at the last position on the result page.

However, analyzing the click through rate curve aggregated over all queries may not be useful to estimate the position bias as these values may differ with each query. For example, Broder [2] classified queries into three main categories, viz., informational, navigational, and transactional. An informational query reflects an intent to acquire some information that is assumed to be present on one or more web pages. A navigational query, on the other hand, is issued with an immediate intent to reach a particular site. For example, the query cnn probably targets the site http://www.cnn.com and hence can be deemed navigational. Moreover, the user expects this result to be shown in one of the top positions in the result page. On the other hand, a query like voice recognition could be used to target a good collection of sites on the subject and therefore the user is more inclined to more results including those in the lower positions on the page. This behavior would naturally result in a navigational query having a different click through rate curve from an informational query (see Figure 1). Further, this suggests that the position bias depends on the query.

It may be argued that the difference in the click through

1We note that click through rate measure need to be combined with other measures like dwell time as the clicks reflect the quality of the snippet rather than the document. Since this study focuses on click through rates, we interchangeably use the term document even though it may refer to the search snippet.
Figure 1: Click through rate curves over positions 1 through 10 for a navigational query, informational query. This shows that the click through rate drops differently for different queries and suggests that the examination probabilities for lower positions may depend on the query.

rate curves for navigational and informational queries arises not from a difference in position bias, but due to the sharper drop in relevance of search results for navigational queries. In this study, we present a model for analyzing a query specific position bias from the click data and use these biases to derive position independent relevance scores for search results. We note that our model by allowing for the examination to be query specific, subsumes the case of query independent position biases. Our work differs from the earlier works based on Examination Hypothesis in that the position bias parameter is allowed to be query dependent.

1.1 Contributions of this Study

Our model is based on an extension of the Examination Hypothesis and states that for a given query, the click through rate of a document at a particular position is proportional to the product of its relevance (referred to as goodness) and query specific position bias. Based on this model, we learn the relevance and position bias parameters for different documents and queries. We evaluate the accuracy of the predicted CTR by comparing it with the CTR values predicted by the vanilla examination hypothesis and the user browsing model (UBM) of Dupret and Piwowarski.

We also conduct a cumulative analysis of the derived position bias curves for the different queries and derive a single parametrized equation to capture the general shape of the position bias curve. The parameter value can then be used to determine the nature of the query as navigational or informational. One of the primary drawbacks of any click-based approach for inferring relevance is the sparsity of the underlying data as a large number of documents are never clicked for a query. We show how to address this issue by inferring the goodness values for unclicked documents through clicks on similar queries.

2. RELATED WORK

Several research works have exploited the use of user clicks as feedback in the context of ads and search results. Others have used clicks in conjunction with dwell time and other implicit measures.

Radlinski and Joachims propose a method to learn user preferences for search results by artificially introducing a small amount of randomization in the order of presentation of the results; their idea was to perform flips systematically in the system, until it converges to the correct ranking. In the context of search advertisements, Richardson et al. show how to estimate the CTRs for new ads by looking at the number of clicks it receives in different positions. Similar to our model, they assume the CTR is proportional to the product of the quality of the ad and a position bias. However, unlike our model, their position bias parameters are query independent. Joachims demonstrates how click logs can be used to produce training data in optimizing ranking SVMs. In another study based on a user behavior, Joachims et al. suggest several rules for inferring user preferences on search results based on click logs. For example, one rule ‘CLICK > SKIP ABOVE’ means if a user has skipped several search results and then clicks on a later result, this should be interpreted as the user preference for the clicked document is greater than for those skipped above it. Agichtein et al. show how to combine click information based on similar rules along with other user behavior parameters such as dwell time and search result information such as snippets to predict user preferences. Our model, on the other hand, incorporates the CTR values into a system of linear equations to obtain relevance and position bias parameters. Fox et al. study the relationship between implicit measures such as clicks, dwell time and explicit human judgments. Craswell et al. evaluate several models for explaining the effect of position bias on click through rate including one where the click through rate is proportional to the product of relevance and query independent position bias. They also propose a cascade model where the click through rate of a document at a position is discounted based on the presence of relevant documents in higher positions. Dupret and Piwowarski present a variant of the cascade model to predict user clicks in the presence of position bias. Specifically, their model estimates the probability of examination of a document given the rank of the document and the rank of the last clicked document. Guo et al. propose a click chain model which is based on the assumption that a document in position \( i \) is examined depending on the relevance of the document in the position \( i-1 \). We will briefly describe these click models next. Before we do so, we will note the main difference in our work from the earlier works based on the Examination Hypothesis and the Cascade Models is that the position bias parameter is allowed to be query dependent.

2.1 Current Click Models

Two important click models which have been later extended in many works on click models are the examination hypothesis and the cascade model.

Examination Hypothesis: Richardson proposed this model based on the simple assumption that clicks on documents in different positions are only dependent on the relevance of the document and the likelihood of examining a document in that position. They assume that the probability of examining the a document at a position depends only on the position and independent of the query and the document. Thus \( c_p(d,j) = g_p(d)p(j) \), where \( p(j) \) is the position bias of position \( j \).

Cascade Model: This model, proposed by Craswell et al., assumes that the user examines the search results top down
and clicks when he finds a relevant document. The probability of clicking depends on the relevance of the document and the query. This model also assumes that the user stops scanning documents after the first click in the query session. Thus, the probability of a document \( d \) getting clicked in position \( j \) is 

\[
c_{d}(d, j) = c_{q}(d, j)g_{q}(d)
\]  

(1)

where \( c_{q}(d, j) \) is the probability that an impression of document \( d \) at position \( j \) is clicked. All prior works based on this hypothesis assume that \( c_{q}(d, j) = p(j) \) and depends only on the position and independent of the query and the document. Note that \( c_{q}(d, j) \) can also be viewed as the click through rate on a document \( d \) in position \( j \). Thus \( c_{q}(d, j) \) can be estimated from the click logs as 

\[
c_{q}(d, j) = a_{q}(d, j)/m_{q}(d, j).
\]

We define the position bias, \( p_{q}(d, j) \), as the ratio of the probability of examining a document in position \( j \) to the probability at position 1.

**Definition 3.1 (Position Bias).** For a given query \( q \), the position bias for a document \( d \) at position \( j \) is defined as

\[
p_{q}(d, j) = c_{q}(d, j)/c_{q}(d, 1).
\]

Next we define the goodness of a search result \( d \) for a query \( q \) as follows.

**Definition 3.2 (Goodness).** We define the goodness (relevance) of document \( d \), denoted by \( g_{q}(d) \), to be the probability that document \( d \) is clicked when shown in position 1 for query \( q \), i.e., 

\[
g_{q}(d) = c_{q}(d, 1).
\]

**Remark 3.3.** Note that our definition of goodness only seems to measure the relevance of the search result snippet rather than the relevance of the document \( d \). Although this may be a simplification in this study, ideally one needs to combine click through information with other user behavior such as dwell time to capture the relevance of the document.

The above definition of goodness removes the effect of the position from the click through rate of a document (snippet) and reflects the true relevance of a document that is independent of the position at which it is shown. Having defined the important concepts in our study, we will now state the basic assumption on which our model is based.

**Hypothesis 3.4 (Document Independence).** The position bias \( p_{q}(d, j) \) depends only on the position \( j \) and query \( q \) and is independent of the document \( d \).

Therefore, we will drop the dependence on \( d \) and denote the bias at position \( j \) as \( p_{q}(j) \). Furthermore, by definition, \( p_{q}(1) = 1 \) and each entry in the query log will give us the equation

\[
c_{q}(d, j) = g_{q}(d)p_{q}(j).
\]

(2)

For a fixed query \( q \), we will implicitly drop the \( q \) from the subscript for convenience and use 

\[
c(d, j) = g(d)p(j).
\]

We note that similar models based on product of relevance and position bias have been used in prior work \cite{13, 14}. However, the main difference in our work is that the position bias parameter \( p(j) \) is allowed to depend on the query whereas earlier works assumed them to be global constants independent of the query.

### 4. Learning the Goodness and Position Bias Parameters

In this section we show how to compute the values \( g(d) \) and \( p(j) \) for a given query based on the above model. Note
that different document, position pairs in the click log associated with a given query give us a system of equations $c(d, j) = g(d)p(j)$ that can be used to learn the latent variables $g(d)$ and $p(j)$. Note that the number of variables in this system of equations is equal to the number of distinct documents, say $m$, plus the number of distinct positions, say $n$. We may be able to solve these system of equations for the variables as long as the number of equations is at least the number of variables. However, the number of solutions may be more than the number of variables in which case the system is over constrained. In such a case, we can solve for $g(d)$ and $p(j)$ in such a way that best fit our equations so as to minimize the cumulative error between the left and the right side of the equations, using some kind of a norm. One method to measure the error in the fit is to use the $L_2$-norm, i.e., $\|c(d, j) - g(d)p(j)\|_2$. However, instead of looking at the absolute difference as stated above, it is more appropriate to look at the percentage difference since the difference between CTR values of 0.4 and 0.5 is not the same as the difference between 0.001 and 0.1001. The basic equation stated as Equation 2 can be easily modified as

$$\log c(d, j) = \log g(d) + \log p(j).$$

Let us denote $\log g(d)$, $\log p(j)$, $\log c(d, j)$ by $\hat{g}_d$, $\hat{p}_j$, and $\hat{c}_{d,j}$, respectively. Let $E$ denote the set of all query, document, position combinations in click log. We get the following system of equations over the set of entries $E_q \in E$ in the click log for a given query.

$$\forall (d, j) \in E_q \hat{g}_d + \hat{p}_j = \hat{c}_{d,j} \quad (4)$$

$$\hat{p}_1 = 0$$

We write this in matrix notation $Ax = b$ where $x = (\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_m, \hat{p}_1, \hat{p}_2, \ldots, \hat{p}_n)$ represents the goodness values of the $m$ documents and the position biases at all the $n$ positions. We solve for the best fit solution $x$ that minimizes $\|Ax - b\|_2 = \hat{p}_1^2 + \sum_{(d, j) \in E_q}(\hat{g}_d + \hat{p}_j - \hat{c}_{d,j})^2$. The solution is given by $x = (A'A)^{-1}A'b$.

**4.1 Invertibility of $A'A$ and graph connectivity**

Note that finding the best fit solution $x$ requires that $A'A$ be invertible. To understand when $A'A$ is invertible, for a given query we look at the bipartite graph $B$ (see Figure 2) with the $m$ documents $d$ on left side and the $n$ positions $j$ on the right side, and place an edge if the document $d$ has appeared in position $j$ which means that there is an equation corresponding to $\hat{g}_d$ and $\hat{p}_j$ in Equations 4. We are essentially deducing $\hat{g}_d$ and $\hat{p}_j$ values by looking at paths in this bipartite graph that connect different positions and documents. But if the graph is disconnected we cannot compare documents or positions in different connected components. Indeed we show that if this graph is disconnected then $A'A$ is not invertible and vice versa.

**Claim 4.1.** $A'A$ is invertible if and only if the underlying graph $B$ is connected.

**Proof.** If the graph is connected, $A$ is full rank. This is because, since $\hat{p}_1 = 1$, we can solve for all $\hat{g}_d$ for all documents that are adjacent to position 1 in graph $B$. Further, whenever we have a value for a node, we can derive the values of all its neighbors in $B$. Since the graph is connected, every node is reachable from position 1. So $A$ has full rank implying that $A'A$ is full ranked and therefore invertible.

If the graph is disconnected, consider any component which does not contain position 1. We will argue that the system of equations for this component is not full rank. This is $Ax = Ax'$ for a solution vector $x$ with certain $\hat{g}_d$ and $\hat{p}_j$ values for nodes in the component, and the solution vector $x'$ with values $\hat{g}_d - \alpha$ and $\hat{p}_j + \alpha$, for any $\alpha$. Therefore, $A$ is not full rank as we can have many solutions with same left hand side, implying $A'A$ is not invertible.

**4.2 Handling disconnected graphs**

Even if the graph $B$ is disconnected, we can still use the system of equations to compare the goodness and position bias values within one connected component. This is achieved by measuring position bias values relative to the highest position within the component instead of position 1. To overcome the problem of disconnected graphs, we solve for the solution that assumes that the average goodness in the different connected components are about equal. This is achieved by adding the following equations to our system:

$$\forall (d, j) \in E_q \epsilon(\hat{g}_d - \mu) = 0$$

where $\mu$ is the average goodness of the documents for the query and $\epsilon$ is a small constant that tends to 0. $\epsilon$ simply gives a tiny weight to these system of equations that is essentially saying that the goodness of all the documents are equal (to $\mu$). If the bipartite graph is connected, these additional equations make no difference to the solution as $\epsilon$ tends to 0. If the graph is disconnected, it combines the solutions in each connected component in such a way as to make the average goodness in all the components as equal as possible.

**4.3 Limitations of the Model**

We briefly describe some concerns that arise from our model and describe methods to address some of these concerns.

- The Document Independence Hypothesis may not be true as people may not examine lower positions depending on whether they have already seen a good
5. EXPERIMENTAL EVALUATION

In this section we analyze the relevance and position bias values obtained by running our algorithm on a commercial search engine click data. Specifically, we adopt widely-used measures such as relative error and perplexity to measure the performance of our click prediction model. Throughout this section, we will refer to our algorithm by QSEH, the vanilla examination hypothesis by EH, and the user browsing model by UBM. The UBM model was implemented using Infer.Net [14]. We show that the our model, although much simpler to implement, outperforms EH and UBM.

Click data

We consider a click log of a commercial search engine containing queries with frequencies between 1000 and 100000 over a period of one month. We only considered entries in the log where the number of impressions for a document in a top-10 position is at least 100 and the number of clicks is non-zero. The truncation is done in order to ensure the $c_0(d, j)$ is a reasonable estimate of the click probability. The above filtering resulted in a click log, call it $Q$, containing 2.03 million entries with 128,211 unique queries and 1.3 query million distinct documents. One important characteristics that affect the performance of our algorithm is the frequency. Table 1 summarizes the distribution of query frequencies in each frequency range. It largely follows our intuition that the more frequent queries are more likely to have a search engine click data. Specifically, we adopt widely-used measures such as relative error and perplexity to measure the performance of our click prediction model. Throughout this section, we will refer to our algorithm by QSEH, the vanilla examination hypothesis by EH, and the user browsing model by UBM. The UBM model was implemented using Infer.Net [14]. We show that the our model, although much simpler to implement, outperforms EH and UBM.

Clickthrough Rate Prediction

We compute the relative error between the predicted and observed clickthrough rates for each $(q, d, j)$ triple in the test set $T$ to measure the performance of our algorithm. We compute the relative error as $|\hat{c}_q(d, j) - c_q(d, j)|/c_q(d, j)$, where $\hat{c}_q(d, j)$ is the predicted CTR from the model and $c_q(d, j)$ is the actual CTR from the click logs. A good prediction will result in a value closer to zero while a bad prediction will deviate from zero. We present the relative error over all triples in $T$ as a cumulative distribution function in Figure 3. Such a plot will illustrate the fraction of queries that fall below a certain relative error. For example, for a relative error of 25%, EH produces 48.3% queries below this error, UBM results in 46.12% queries, while QSEH results in 51.57% queries below this error - an improvement of 10.6% over UBM and 6.34% over EH. As we can observe from the figure, while EH outperforms UBM at smaller errors, the trend reverses at larger errors. In Figure 4, we present the relative error in a different way keeping the sign of the error. This figure shows that QSEH does much better in not over predicting the CTRs when compared to EH and UBM while it does marginally better than UBM when it comes to under-prediction. QSEH under predicts by an average 48.6%, EH under-predicts by an average 86.54%, and UBM under-predicts by an average 78.00%. The respective number in the case of over-prediction are 44.07%, 78.00%, and 48.95%.

In another set of experiments, we repeated the above experiment for queries bucketed according to their frequencies to study the effect of query frequency on the CTR prediction. In this experiment, we estimate average relative error over all test triples for queries in the frequency bucket. The effect of the query frequency is shown in Figure 5. As the figure illustrates, in the case of QSEH, the relative error is...
stable across all query frequencies while it is higher for the both \(EH\) and \(UBM\). We note that the stable trends in the figure are for cases where there are reasonable number of queries in that particular frequency range. We can attribute the large fluctuation in values for frequency greater than 35000 to the small number of queries in any of the frequency bucket (see Table \(5\)). Finally, we note that the average relative error for \(EH\) is 39.33%, \(UBM\) is 43%, and is significantly lower for \(QSEH\) at 29.19%.

Another measure we use to test the effectiveness of our predicted CTRs is perplexity. We used the standard definition of perplexity for a test set \(U\) of query, document, position triples as

\[
P_i = 2^{-\frac{1}{|U|} \sum_{(q,d,j) \in U} c_q(d,j) \log \hat{c}_q(d,j)},
\]

where \(c_q(d,j)\) is the observed CTR at position \(j\) for query \(q\) and document \(d\) and \(\hat{c}_q(d,j)\) is the predicted CTR. This is essentially an exponential function of the cross entropy. A small value of perplexity corresponds to a good prediction and can be no smaller than 1.0. We computed the perplexity as a function of the position as well as the query frequency. In the former we group entries in \(T\) by position, and in the latter, we simply group by all the queries in a certain frequency range. Figures 6 and 7 illustrate the relative performance of \(QSEH\), \(EH\), and \(UBM\). For different query frequencies, the average perplexity of \(QSEH\) is 1.0671. The corresponding values for \(EH\) and \(UBM\) are 1.0726 and 1.0693 respectively. In the case of different positions, the corresponding values for \(QSEH\), \(EH\), and \(UBM\) are 1.1081, 1.1286, and 1.1211 respectively.

Understanding Patterns of position bias

We also consider a subset of queries, labeled \(LQ\) 10, whose largest component includes all positions 1 through 10 – these are queries where the bipartite graph \(B\) is a fully connected component. This dataset has 112,735 number of entries with 2,614 unique queries and 42,119 unique documents. We use the position bias vectors derived for fully connected components in \(LQ\) 10 to study the trend of the position bias curves over different queries. A navigational query will have small \(p(j)\) values for the lower positions and hence \(\hat{p}_j\) (\(= \log p(j)\)) that are large in magnitude. An informational query on the other hand will have \(\hat{p}_j\) values that are smaller in magni-
For a given position bias vector \( p \), we look at the entropy \( H(p) = -\sum_{j=1}^{10} p(j) \log p(j) \), where \( |p| \) is the sum of all the position bias values over all positions. The entropy is likely to be low for navigational queries and high for informational queries. We measured the distribution of \( H(p) \) over all the 2500 queries in LC10 and divided these queries into ten categories of 250 queries each obtained by sorting the \( H(p) \) values in increasing order.

We then study the aggregate behavior of the position bias curves within each of the ten categories. Figure 8 shows the median value \( m\|p\| \) of the position bias \( \hat{p} \) curves taken over each position over all queries in each category. Observe that the median curves in the different categories have more or less the same shape but different scale. It would be interesting to explain all these curves as a single parametrized curve. To this end, we scale each curve so that the median log position bias \( m\|p\| \) at the middle position 6 is set to -1. Essentially we are computing \( \text{normalized}(m\|p\|) = m\|p\|/m\|p_6\| \). The normalized curves over the ten categories are shown in Figure 9. From this figure it is apparent that the median position bias curves in the ten categories are approximately scaled versions of each other (except for the one in the first category). The different curves in Figure 9 can be approximated by a single curve by taking their median; this reads out to the vector \( \Delta = (0, -0.2952, -0.4935, -0.6792, -0.8673, -1.0000, -1.1100, -1.1939, -1.2284, -1.1818) \). The aggregate position bias curves in the different categories can be approximated by the parametrized curve \( \alpha \Delta \).

Such a parametrized curve can be used to approximate the position bias vector for any query. The value of \( \alpha \) determines the extent to which the query is navigational or informational. Thus the value of \( \alpha \) obtained by computing the best fit parameter value that approximates the position bias curve for a query, can be used to classify the query as informational or navigational. Given a position bias vector \( \hat{p} \), the best fit the value of \( \alpha \) is obtained by minimizing \( \| \hat{p} - \alpha \Delta \|_2 \), which results in \( \alpha = \Delta \hat{p}/\Delta \). Table 5 shows some of the queries in LC10 with the high and low values of \( e^{-\alpha} \). Note that \( e^{-\alpha} \) corresponds to position bias (since \( p(6) = e^{\alpha \Delta} \)) at position 6 as per parametrized curve \( \alpha \Delta \).

5.1 Testing the Document Independence Hypothesis

Table 2: \( e^{-\alpha} \) for a few queries.

Recall that our model is based on the Document Independence Hypothesis \( \text{Fig. 4} \) that is, \( p_\delta(d, j) \) is independent of \( d \). In this section we show a simple method to test this hypothesis from the click data.

To test the hypothesis we look at the bipartite graph \( B \) for a query with documents on one side and positions on the other and each edge \((d, j)\) is labeled by \( \hat{c}_{dj} \). We show that cycles in this graph (see Figure 10) must satisfy a special property.

For each edge \((d, j)\) in this graph, we have a \( c(d, j) \) obtained from the query log. Let \( C = (d_1, j_1, d_2, j_2, d_3, \ldots, d_k, j_k, d_1) \) denote a cycle in this graph with alternating edges between documents \( d_1, d_2, \ldots, d_k \) and positions \( j_1, j_2, \ldots, j_k \) and connecting back at node \( d_1 \). We now show that our hypothesis implies that the sum of the \( \hat{c}_{dj} \) values \( \hat{c}_{dj} = log_c(d, j) \) on odd and even edges on the cycle are equal. This gives us a simple test for our hypothesis by computing the sum for different cycles.

\[
\text{Claim 5.1. Given a cycle } C = (d_1, j_1, d_2, j_2, d_3, ..., d_k, j_k, d_1), \\
\text{our Independence hypothesis } \text{Fig. 4} \text{ implies that } \\
\text{sum}(C) = \sum_{i=1}^{k} \hat{c}_{d_i j_i} - \sum_{i=1}^{k} \hat{c}_{d_{i+1} j_i} = 0 \text{ (where } d_{k+1} \text{ is the same as } d_1 \text{ for convenience).}
\]

\[
\text{Proof. We need to show that } \\
\sum_{i=1}^{k} \hat{c}_{d_i j_i} = \sum_{i=1}^{k} \hat{c}_{d_{i+1} j_i}.
\]

Note that \( \hat{c}_{d_i j_i} = \hat{g}_d + \hat{p}_j \). So \( \hat{c}_{d_i j_i} = \hat{g}_d + \hat{p}_j \). Similarly \( \sum_{i=1}^{k} \hat{g}_{d_{i+1}} = \hat{g}_{d_1} + \hat{p}_{j_1} \), \( \sum_{i=1}^{k} \hat{g}_{d_{i+1}} + \hat{p}_{j_i} = \sum_{i=1}^{k} \hat{g}_{d_i} + \hat{p}_{j_i} \) (since \( d_{k+1} = d_1 \)).

Clearly in practice we do not expect \( \text{sum}(C) \) to be exactly 0. In fact longer cycles are likely to have a larger error from 0. To normalize this we consider \( \text{ratio}(C) = \sqrt{\sum_{i=1}^{k} \hat{c}_{d_{i+1} j_i}^2 + \sum_{i=1}^{k} \hat{c}_{d_i j_i}^2} \). The denominator is essentially...
Figure 10: A cycle $C$ in bipartite graph of documents and positions for a given query. To test the Document Independence Hypothesis check if $\sum(c(d,j)) = \sum_{(d,j) \in \text{odd edges of } C} \hat{c}_{dj} - \sum_{(d,j) \in \text{even edges of } C} \hat{c}_{dj} = 0$

$\|C\|_2$ where $C$ is viewed as a vector of $\hat{c}_{dj}$ values associated with the edges in the cycle. The number of dimensions of the vector is equal to the length of the cycle. So $\text{ratio}(C) = \sum(c(d,j)) / \|C\|_2$ is simply normalizing $\sum(c(d,j))$ by the length of the vector $C$. It can be easily shown theoretically that for a random vector $C$ of length $\|C\|_2$ in a high dimensional Euclidean space the root mean squared value of $|\text{ratio}(C)| = |\sum(c(d,j))| / \|C\|_2$ is equal to 1. Thus, a value of $|\text{ratio}(C)|$ much smaller than 1 indicates that $|\sum(c(d,j))|$ is biased towards smaller values. This gives us a method to test the validity of the Document Independence Hypothesis by measuring $|\sum(c(d,j))|$ and $|\text{ratio}(C)|$ for different cycles $C$.

We measured the quantities $|\sum(c(d,j))|$ and $|\text{ratio}(C)|$ computed over different cycles $C$ in the bipartite graphs of documents and positions over different queries. We found a total of 218, 143 cycles of lengths ranging from 4 to 20. Note that since this is the bipartite graph the cycle of the smallest length is 4 and all cycles must be of even length. Figure 11 shows the distribution of the length of the different cycles.

For each cycle $C = (d_1, j_1, d_2, j_2, d_3, \ldots, d_k, j_k, d_1)$, we compute the quantity $|\sum(c(d,j))|$ as described in Claim 5.1. Figure 12 shows the distribution of $|\sum(c(d,j))|$. We also plot $|\text{ratio}(C)|$ in Figure 13.

As can be seen from Figure 12, the median value of $|\sum(c(d,j))|$ is bounded by about 1 and from Figure 13 the median value of $|\text{ratio}(C)|$ is less than 0.1 for all cycle lengths. While the median value $|\sum(c(d,j))|$ leaves the validity of the Document Independence Hypothesis inconclusive, the small value of $|\text{ratio}(C)|$ can be viewed as mild evidence in favor of the hypothesis.

6. USING RELATED QUERIES TO INCREASE COVERAGE

In addition to finding their use in predicting CTRs, the goodness values obtained from our model can be employed in designing effective search quality metrics that are very well aligned with user satisfaction. In this section, we will present a method to infer the goodness values of documents that are not directly associated with a given query (via clicks) and the illustrate the use of these inferred values in computing a click-based feature for ranking search results.

One of the primary drawbacks of any click-based approach is the sparsity of the underlying data as a large number of documents are never clicked for a query. We present a method to extend the goodness scores for a query to a larger set of documents. We may be able to infer the goodness of more documents for a query by looking at similar queries. Assuming we have access to a query similarity matrix $S$, we may infer new goodness values $L_{dq}$ as

$$L_{dq} = \sum_{q'} S_{qq'} G_{dq'},$$

where, $S_{qq'}$ denote the similarity between queries $q$ and $q'$. This is essentially accumulating goodness values from similar queries by weighting them with their similarity values. Writing this in matrix form gives $L = SG$. The question then is how to obtain the similarity matrix $S$.

One method to compute $S$ is to consider two queries to be similar if they share a lot of good documents. This can be obtained by taking the dot product of the goodness vectors spanning the documents for the two queries. This operation can be represented in matrix form as $S = GG'$. Another way to visualize this is to look at a complete bipartite graph with queries on the left and documents on the right with the goodness values on the edges of the graph. $GG'$ is obtained by first looking at all paths of length 2 between two queries and then adding up the product of the goodness values on the edges over all the 2-length paths between the queries.

A generalization of this similarity matrix is obtained by looking at paths of longer length, say $l$ and adding up the product of the goodness values along such paths between
two queries. This corresponds to the similarity matrix $S = (GG')^l$. The new goodness values based on this similarity matrix is given by $L = (GG')^l G$. We only use non-zero entries in $L$ as valid ratings.

Relevance Metrics

We measure the effectiveness of our algorithm by comparing the ranking produced when ordering documents for query based on their relevance values to human judgments. We quantify the effectiveness of our ranking algorithm using three well known measures: NDCG, MRR, and MAP. We refer the reader to [19] for an exposition on these measures. Each of these measures can be computed at different rank thresholds $T$ and are specified by NDCG@T, MAP@T, and MRR@T. In this study we set $T = 1, 3, 10$.

The normalized discounted cumulative gains (NDCG) measure discredits the contribution of a document to the overall score as the document’s rank increases (assuming that the most relevant document has the lowest rank). Higher NDCG values correspond to better correlation with human judgments. Given a ranked result set $Q$, the NDCG at a particular rank threshold $k$ is defined as

$$NDCG(Q, k) = \frac{1}{|Q|} \sum_{j=1}^{|Q|} Z_k \sum_{m=1}^k g^j(i) - 1 \frac{1}{\log(1+j)},$$

where $g^j(i)$ is the (human judged) rating ($0=bad, 2=fair, 3=good, 4=excellent, and 5=definitive$) at rank $j$ and $Z_k$ is normalization factor calculated to make the perfect ranking at $k$ have an NDCG value of 1.

The reciprocal rank (RR) is the inverse of the position of the first relevant document in the ordering. In the presence of a rank-threshold $T$, this value is 0 if there is no relevant document in positions below this threshold. The mean reciprocal rank (MRR) of a query set is the average reciprocal rank of all queries in the query set.

Isolated Ranking Metrics

One way to test the efficacy of a feature is to measure the effectiveness of the ordering produced by using the feature as a ranking function. This is done by computing the resulting NDCG of the ordering and comparing with the NDCG values of other ranking features. Two commonly used ranking features in search engines are BM25F [18] and PageRank. BM25F is a content-based feature while PageRank is a link-based ranking feature. BM25F is a variant of BM25 that combines the different textual fields of a document, namely title, body and anchor text. This model has been shown to be one of the best-performing web search scoring functions over the last few years [19, 4]. To get a control run, we also include a random ordering of the result set as a ranking and compare the performance of the three ranking features with the control run.

We compute the NDCG scores for this algorithm. We start with a goodness matrix $G$ with $\gamma = 0.6$ containing 936606 non-zero entries. Figure 14 shows the NDCG scores parameter $l$ set to 1 and 2 respectively. The number of non-zero entries increase to over 7.1 million for $l = 1$ and over 42 million for $l = 2$. However, the number of judged <query, document> pairs only increase from 74781 for $l = 2$ to 87235 for $l = 1$. This implies that most of the documents added by extending to paths of length 2 are not judged results in the high value of NDCG scores for the Random ordering. If we were to judge all these ‘holes’ in the ratings, we think that we will see a lower NDCG score for the Random ordering.

7. CONCLUSIONS

In this paper, we presented a model based on a generalization of the Examination Hypothesis that states that for a given query, the user click probability on a document in a given position is proportional to the relevance of the document and a query specific position bias. Based on this model we learn the relevance and position bias parameters for different queries and documents. We do this by translating the
model into a system of linear equations that can be solved to obtain the best fit relevance and position bias values. We use the obtained bias curves and the relevance values to predict the CTRs given a query, url, and a position. We measure the performance of our algorithm using well-used metrics like log-likelihood and perplexity and compare the performance with other techniques like the plain examination hypothesis and the user browsing model.

Further, we performed a cumulative analysis of the position bias curves for different queries to understand the nature of these curves for navigational and informational queries. In particular, we computed the position bias parameter values for a large number of queries and found that the magnitude of the position bias parameter value indicates whether the query is informational or navigational. We also propose a method to solve the problem of dealing with sparse click data by inferring the goodness of unclicked documents for a given query from the clicks associated with similar queries.

Acknowledgements

The authors would like to thank Anitha Kannan for providing the code to compute the relevance and CTRs using the UBM model.

8. REFERENCES

[1] Eugene Agichtein, Eric Brill, Susan T. Dumais, and Robert Ragno. Learning user interaction models for predicting web search result preferences. In SIGIR, pages 3–10, 2006.

[2] Andrei Z. Broder. A taxonomy of web search. SIGIR Forum, 36(2):3–10, 2002.

[3] Mark Claypool, David Brown, Phong Le, and Makoto Waseda. Inferring user interest. IEEE Internet Computing, 5(6):32–39, 2001.

[4] Nick Craswell, Stephen E. Robertson, Hugo Zaragoza, and Michael J. Taylor. Relevance weighting for query independent evidence. In SIGIR, pages 416–423, 2005.

[5] Nick Craswell, Onno Zoeter, Michael Taylor, and Bill Ramsey. An experimental comparison of click position-bias models. In WSDM, pages 87–94, 2008.

[6] Georges Dupret and Benjamin Piwowarski. A user browsing model to predict search engine click data from past observations. In SIGIR, pages 331–338, 2008.

[7] Steve Fox, Kuldeep Karnawat, Mark Mydland, Susan T. Dumais, and Thomas White. Evaluating implicit measures to improve web search. ACM Trans. Inf. Syst., 23(2):147–168, 2005.

[8] Laura A. Granka, Thorsten Joachims, and Geri Gay. Eye-tracking analysis of user behavior in www search. In SIGIR, pages 478–479, 2004.

[9] Fan Guo, Chao Liu, Anitha Kannan, Tom Minka, Michael J. Taylor, Yi Min Wang, and Christos Faloutsos. Click chain model in web search. In WWW, pages 11–20, 2009.

[10] Fan Guo, Chao Liu, and Yi Min Wang. Efficient multiple-click models in web search. In WSDM, pages 124–131, 2009.

[11] Kalervo Järvelin and Jaana Kekäläinen. Cumulated gain-based evaluation of ir techniques. ACM Trans. Inf. Syst., 20(4):422–446, 2002.

[12] Thorsten Joachims. Optimizing search engines using clickthrough data. In KDD, pages 133–142, 2002.

[13] Thorsten Joachims, Laura A. Granka, Bing Pan, Helene Hembrooke, and Geri Gay. Accurately interpreting clickthrough data as implicit feedback. In SIGIR, pages 154–161, 2005.

[14] T. Minka, J.M. Winn, J.P. Guiver, and A. Kannan. Infer.NET 2.2. 2009. Microsoft Research Cambridge. http://research.microsoft.com/infernet.

[15] Filip Radlinski and Thorsten Joachims. Minimally invasive randomization for collecting unbiased preferences from clickthrough logs. In AAAI, pages 1406–1412, 2006.

[16] Matthew Richardson, Ewa Dominowska, and Robert Ragno. Predicting clicks: estimating the click-through rate for new ads. In WWW, pages 521–530, 2007.

[17] Matthew Richardson, Ewa Dominowska, and Robert Ragno. Predicting clicks: estimating the click-through rate for new ads. In WWW, pages 521–530, 2007.

[18] Stephen E. Robertson, Steve Walker, Susan Jones, Micheline Hancock-Beaulieu, and Mike Gatford. Okapi at trec-2. In TREC, pages 21–34, 1993.

[19] H. Zaragoza, N. Craswell, M. Taylor, S. Saria, and S. Robertson. Microsoft cambridge at trec-13: Web and hard tracks. In TREC, pages 418–425, 2004.
Click through rate

average over all queries
median over all queries

position
The graph shows the percentage of queries as a function of relative error. The curves represent different methods: Query Specific EH, EH, and UBM. The x-axis represents the relative error, while the y-axis shows the percentage of queries.
Discount factor vs. NDCG@10 for Goodness
Normalized Position Bias over different query categories