Form Factor for $B \to Dl\bar{\nu}$ in The Light-Cone Sum Rules With Chiral Current Correlator

Fen Zuo, * Zuo-Hong Li † and Tao Huang *  

1 Institute of High Energy Physics, P.O.Box 918, Beijing 100049, P.R.China§  
2 Department of Physics, Yantai University, Yantai 264005, China  
3 CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China

Abstract

Within the framework of QCD light-cone sum rules (LCSR), we calculate the form factor for $B \to Dl\bar{\nu}$ transitions with chiral current correlator. The resulting form factor depends on the distribution amplitude (DA) of the $D$-meson. We try to use three kinds of DA models of the $D$-meson. In the velocity transfer region $1.14 < y < 1.59$, which renders the Operator Product Expansion (OPE) near light-cone $x^2 = 0$ go effectively, the yielding behavior of form factor is in agreement with that extracted from the data on $B \to Dl\bar{\nu}$, within the error. In the large recoil region $1.35 < y < 1.59$, the results are observed consistent with those of perturbative QCD (pQCD). The presented calculation can play a bridge role connecting those from the lattice QCD, heavy quark symmetry and pQCD to have an all-around understanding of $B \to Dl\bar{\nu}$ transitions.

*Email: zuof@mail.ihep.ac.cn  
†Email: lizh@ytu.edu.cn  
‡Email: huangtao@mail.ihep.ac.cn  
§Mailing address
1 Introduction

Calculation of the form factors for semileptonic transitions of $B$ mesons has been being a subject discussed intensely. Recently, it has been shown that the $B \to \pi$ transition form factor can be consistently analyzed by using the different approaches in the different $q^2$ regions \cite{1,2,3,4}. The perturbative QCD (pQCD) can be applied to the $B \to \pi$ form factor in the large recoil (small $q^2$) region and it is reliable when the involved energy scale is large enough \cite{1}. The QCD light-cone sum rules (LCSR) can involve both the hard and soft contributions to the $B \to \pi$ form factor below $q^2 \simeq 18 \text{ GeV}^2$ \cite{2}. The lattice QCD simulations of the $B \to \pi$ transition form factor \cite{3} are available only for the soft region $q^2 > 15 \text{ GeV}^2$, because of the restriction to the $\pi$ energy smaller than the inverse lattice spacing. Thus the results from these three approaches might be complementary to each other. In Ref.\cite{4} we recalculate the $B \to \pi$ form factor in the pQCD approach, with the transverse momentum dependence included for both the hard scattering part and the nonperturbative wave functions (of $\pi$ and $B$) to get a more reliable pQCD result. By combining the results from these three methods we obtain a full understanding of the $B \to \pi$ transition form factor in its physical region $0 \leq q^2 \leq (M_B - M_\pi)^2 \simeq 25 \text{ GeV}^2$.

Up to now, in comparison with heavy-to-light cases the calculations on heavy-to-heavy transitions can be done only for a certain specific kinematical range, although there have been a lot of discussions in the literature. In the BSW model \cite{5}, the relevant form factors at zero momentum transfer are expressed as an overlap of initial and final meson wave functions for which they take the solutions of the Bethe-Salpeter (BS) equation in a relativistic harmonic oscillator potential. Then one extrapolates the result at $q^2 = 0$ to the whole kinematical region assuming the nearest pole dominance. With the discovering of the heavy quark symmetry, the $B \to D$ form factor have been known better at zero recoil. This is because of the fact that in the heavy quark limit the resulting form factors — Isgur-Wise functions \cite{6} at zero recoil are rigorously normalized. Including the leading symmetry breaking corrections, the deviation from this limit can be estimated at an order of a few percent due to Luke’s theorem and therefore the value of the form factor at this point can be determined within a higher accuracy \cite{7}. However, the dependence of the form factor on the velocity transfer $y = v \cdot v'$ (with $v$ and $v'$ being the velocities of the $B$ and $D$ mesons respectively) is difficult to get even in the leading order, in view of the arbitrary function $\sigma(y)$ \cite{8} which is introduced to simulate higher-resonances in the heavy quark effective theory (HQET). The lattice QCD, despite a rigorous nonperturbative approach, is just adequate to estimate the behavior of the form factors near the zero recoil \cite{9}. Among the other approaches are the QCD sum rules and pQCD. Ref.\cite{10} applies the traditional 3-point sum rule to calculate the form factor at zero momentum transfer. It is concluded in Ref.\cite{11} that pQCD approach is applicable in the large recoil region and can give a consistent result with the experiment.

It is necessary that there is a reliable estimate of $B \to D$ transition in the whole kinematically accessible range $0 \leq q^2 \leq (M_B - M_D)^2 \simeq 11.6 \text{ GeV}^2$, in order to account for the data on $B \to D\ell\bar{\nu}$. For this purpose, it is practical, as shown in $B \to \pi$ case, to combine the result of QCD LCSR with those from the lattice QCD, heavy quark symmetry and
pQCD. The LCSR approach [12], where the non-perturbative dynamics are effectively parameterized in so-called light-cone wave functions, is regarded as an effective tool to deal with heavy-to-light exclusive processes. Although the $B \to D$ transition in question is a heavy-to-heavy one, the $c$-quark is much lighter compared to $b$-quark and so discussing it with LCSR is plausible for the kinematical range where the OPE near light-cone $x^2 = 0$ is valid. The other problem with our practical calculation is that the higher twist DA’s of D meson, which are important but less studied, would enter into the sum rule result. However, an effective approach [13] has been presented to avoid the pollution by some higher-twist DA’s. This improved LCSR method uses a certain chiral current correlator as the starting point so that the relevant twist-3 wave functions make no contributions and the reliability of calculation can be enhanced to a large extend. It’s applicability has been examined by a great deal of studies [2, 14]. In this paper we would like to employ the improved LCSR to discuss the form factor for the $B \to D$ transition and try to give a full understanding of QCD dynamics involved in the $B \to D\bar{\nu}$.

This paper is organized as follows. In the following Section we derive the LCSR for the form factor for $B \to D$. A discussion of the DA models for the $D$-meson is given in section III. Section IV is devoted to the numerical analysis and comparison with other approaches. The last section is reserved for summary.

2 Derivation of LCSR for The $B \to D$ Form Factor

The $B \to D$ weak form factors $f(q^2)$ and $\tilde{f}(q^2)$ are usually defined as:

$$<D(p)|\bar{c}\gamma_\mu b|B(p+q)> = 2f(q^2)p_\mu + \tilde{f}(q^2)q_\mu,$$

with $q$ being the momentum transfer. On the other hand, when applying the heavy quark symmetry to do discussion the following definition is advisable,

$$<D(p)|\bar{c}\gamma_\mu b|B(p+q)> = \sqrt{m_B m_D}[h_+(y)(v + v')_\mu + h_-(y)(v - v')_\mu].$$

If we neglect the masses of leptons in the decay final state of $B \to Dl\bar{\nu}l$, only $f(q^2)$ is relevant and thus we can confine us to the discussion on $f(q^2)$. Obviously, the following relation is observed between $f(q^2)$ and $h_{+(-)}(y)$,

$$f(q^2) = \frac{m_B + m_D}{2\sqrt{m_B m_D}}F_{B \to D}(y)$$

where $F_{B \to D}(y) = h_+(y) - \frac{m_B - m_D}{m_B + m_D}h_-(y)$, $q^2 = m_B^2 + m_D^2 - 2m_B m_D y$.

Using the heavy quark symmetry, the value of form factor $F_{B \to D}(1)$ at zero recoil could be fixed better. Since in heavy quark limit $h_+(1) = 1$ and $h_-(1) = 0$, the form factor $F_{B \to D}(1)$ should be close to 1. A systematic investigation gives $F_{B \to D}(1) = 0.98 \pm 0.07$ [13], with a less model dependence. This result is also confirmed with lattice calculations [9]. PQCD analyses are also made in the large recoil region $y = 1.35 - 1.59$, yielding a result consistent with the data. The LCSR calculation can help to understand
\( \mathcal{F}_{B \to D}(y) \) in the whole kinematical region in complementary to the lattice QCD with the heavy quark symmetry and pQCD approaches.

To achieve a LCSR estimate of \( \mathcal{F}_{B \to D}(y) \), we follow [2] and use the following chiral current correlator \( \Pi_{\mu}(p, q) \):

\[
\Pi_{\mu}(p, q) = i \int d^4 x e^{ipx} <D(p)|T\{\bar{c}(x)\gamma_{\mu}(1 + \gamma_5)b(x), \bar{b}(0)i(1 + \gamma_5)d(0)\}|0> \\
= \Pi(q^2, (p + q)^2)p_\mu + \tilde{\Pi}(q^2, (p + q)^2)q_\mu, \tag{4}
\]

In the first place, we discuss the hadronic representation for the correlator. This can be done by inserting the complete intermediate states with the same quantum numbers as the current operator \( \bar{b}i(1 + \gamma_5)d \). Isolating the pole contribution due to the lowest pseudoscalar \( B \) meson, we have the hadronic representation in the following:

\[
\Pi_{\mu}^H(p, q) = \Pi^H(q^2, (p + q)^2)p_\mu + \tilde{\Pi}^H(q^2, (p + q)^2)q_\mu \\
= \frac{<D|\bar{c}\gamma_{\mu}|B><B|\bar{b}\gamma_5d|0>}{m_B^2 - (p + q)^2} \\
+ \sum_H \frac{<D|\bar{c}\gamma_{\mu}(1 + \gamma_5)b|B^H><B^H|\bar{b}(1 + \gamma_5)d|0>}{m_{B^H}^2 - (p + q)^2}. \tag{5}
\]

Note that the intermediate states \( B^H \) contain not only the pseudoscalar resonance of masses greater than \( m_B \), but also the scalar resonances with \( J^P = 0^+ \), corresponding to the operator \( \bar{b}d \). With Eq. (1) and the definition of the decay constant \( f_B \) of the \( B \)-meson

\[
<B|\bar{b}\gamma_5d|0> = \frac{m_B^2}{m_B^2} f_B/m_b, \tag{6}
\]

and expressing the contributions of higher resonances and continuum states in a form of dispersion integration, the invariant amplitudes \( \Pi^H \) and \( \tilde{\Pi}^H \) read,

\[
\Pi^H[q^2, (p + q)^2] = \frac{2f(q^2)m_B^2f_B}{m_b(m_B^2 - (p + q)^2)} + \int_{s_0}^{\infty} \frac{\rho^H(s)}{s - (p + q)^2} ds + \text{subtractions}, \tag{7}
\]

and

\[
\tilde{\Pi}^H[q^2, (p + q)^2] = \frac{\tilde{f}(q^2)m_B^2f_B}{m_b(m_B^2 - (p + q)^2)} + \int_{s_0}^{\infty} \frac{\tilde{\rho}^H(s)}{s - (p + q)^2} ds + \text{subtractions}, \tag{8}
\]

where the threshold parameter \( s_0 \) should be set near the squared mass of the lowest scalar \( B \) meson, the spectral densities \( \rho^H(s) \) and \( \tilde{\rho}^H(s) \) can be approximated by invoking the quark-hadron duality ansatz

\[
\rho^H(s)(\tilde{\rho}^H(s)) = \rho^{QCD}(s)(\tilde{\rho}^{QCD}(s))\theta(s - s_0). \tag{9}
\]

On the other hand, we need to calculate the corrector in QCD theory to obtain the desired sum rule result. In fact, there is an effective kinematical region which makes OPE
applicable: \((p + q)^2 - m_b^2 \ll 0\) for the \(bd\) channel and \(q^2 \leq (m_b - m_c)^2 - 2\Lambda_{QCD}(m_b - m_c)\) for the momentum transfer.

For the present purpose, it is sufficient to consider the invariant amplitude \(\Pi(q^2, (p + q)^2)\) which contains the desired form factor. The leading contribution is derived easily by contracting the \(b\)–quark operators to a free propagator:

\[
< 0|Tb(x)\bar{b}(0)|0> = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{k_b + m_b}{k^2 - m_b^2}.
\]

(10)

Substituting Eq.\((10)\) into Eq.\((4)\), we have the two-particle contribution to the correlator,

\[
\Pi^{(qq)}_\mu = -2m_b i \int\frac{d^4xd^4k}{(2\pi)^4} e^{i(q-k)x} \frac{1}{k^2 - m_b^2} <D(p)|T\bar{c}(x)\gamma_\mu \gamma_5 d(0)|0>.
\]

(11)

An important observation, as in Ref.\([2]\), is that only the leading non-local matrix element \(<D(p)|\bar{c}(x)\gamma_\mu \gamma_5 d(0)|0>\) contributions to the correlator, while the nonlocal matrix elements \(<D(p)|\bar{c}(x)\gamma_5 d(0)|0>\) and \(<D(p)|\bar{c}(x)\gamma_\mu \gamma_5 d(0)|0>\) whose leading terms are of twist 3, disappear from the sum rule. Proceeding to Eq.\((11)\), we can expand the nonlocal matrix element \(<D(p)|T\bar{c}(x)\gamma_\mu \gamma_5 d(0)|0>\) as

\[
<D(p)|T\bar{c}(x)\gamma_\mu \gamma_5 d(0)|0> = -ip_\mu f_D \int_0^1 du \frac{\varphi_D(u)}{u^2 + m_b^2 - (up + q)^2} + \text{higher twist terms},
\]

(12)

where \(\varphi_D(u)\) is the twist-2 DA of D meson with \(u\) being the longitudinal momentum fraction carried by the c-quark, those DA’s entering the higher-twist terms are of at least twist 4. The use of Eq.\((12)\) yields

\[
\Pi^{(qq)}[q^2, (p + q)^2] = 2f_Dm_b \int_0^1 du \frac{\varphi_D(u)}{m_b^2 - (up + q)^2} + \text{higher twist terms}.
\]

(13)

Invoking a correction term due to the interaction of the \(b\) quark with a background field gluon into \((10)\), the three-particle contribution \(\Pi^{(qg)}_\mu\) is achievable. However, the practical calculation shows that the corresponding matrix element whose leading term is of twist 3 also vanishes. Thus, if we work to the twist-3 accuracy, only the leading twist DA \(\varphi_D\) is needed to yield a LCSR prediction.

Furthermore, we carry out the subtraction procedure of the continuum spectrum, make the Borel transformations with respect to \((p + q)^2\) in the hadronic and the QCD expressions, and then equate them. Finally, from Eq.\((13)\) follows the LCSR for \(F_{B \to D}(y)\), which is applicable to the velocity transfer region \(1.14 < y < 1.59\),

\[
F_{B \to D}(y) = \frac{2m_b^2}{(m_B + m_D)m_B} \sqrt{m_D f_D e^{m_b^2/M^2}} \times \int_\Delta^1 du \frac{\exp \left[ -m_b^2 - (1 - u)(q^2 - um_b^2) \right]}{uM^2} \varphi_D(u),
\]

(14)
where

\[ \Delta = \sqrt{(s_0 - q^2 - m_D^2)^2 + 4m_D^2(m_D^2 - q^2) - (s_0 - q^2 - m_D^2)} \]

\( \frac{2m_D^2}{m_D^2} \), \hspace{1cm} (15)

and \( p^2 = m_D^2 \) has been used.

### 3 D-meson Distribution Amplitude

Now let’s do a brief discussion on an important nonperturbative parameter appearing in the LCSR formula (14), the leading twist DA of D-meson, \( \varphi_D(x) \).

D-meson is composed of the heavy quark \( c \) and the light anti-quark \( \bar{q} \). The longitudinal momentum distribution should be asymmetry and the peak of the distribution should be approximately at \( x \approx 0.7 \). According to the definition in Eq. (12), \( \varphi_D(x) \) satisfies the normalization condition

\[ \int_0^1 dx \varphi_D(x) = 1, \] \hspace{1cm} (16)

which is derived by the leptonic decay \( D \rightarrow \mu \nu \).

In the pQCD calculations [11], a simple model (we call model I) is adopted as

\[ \varphi_D^{(I)}(x) = 6x(1-x)(1-C_d(1-2x)) \] \hspace{1cm} (17)

which is based on the expansion of the Gegenbauer polynomials. Eq. (17) has a free parameter \( C_d \) which ranges from 0 to 1. We will take \( C_d = 0.7 \) as input.

On the other hand, it was suggested in [16] that the light-cone wave function of the D-meson be taken as:

\[ \psi_D(x, k_{\perp}) = A_D \exp\left[-b_D^2\left(\frac{k_{\perp}^2 + m_c^2}{x} + \frac{k_{\perp}^2 + m_d^2}{1-x}\right)\right] \] \hspace{1cm} (18)

which is derived from the Brosky-Huang-Lepage (BHL) prescription [17]. \( \psi_D(x, k_{\perp}) \) can be related to the DA by the definition

\[ \varphi_D(x) = \frac{2\sqrt{3}}{f_D} \int \frac{d^2k_\perp}{16\pi^3} \psi_D(x, k_{\perp}). \] \hspace{1cm} (19)

Substituting Eq. (18) into Eq. (19), we have a model of the DA (model II)

\[ \varphi_D^{(II)}(x) = \frac{\sqrt{3}A_D}{8\pi^2 f_D b_D^2} x(1-x) \exp\left[-b_D^2 \frac{xm_d^2 + (1-x)m_c^2}{x(1-x)}\right], \] \hspace{1cm} (20)

where the parameters \( A_D \) and \( b_D \) can be fixed by the normalization [16] and the probability of finding the \(|q\bar{q}>\) Fock state in the D- meson, \( P_D \)

\[ P_D = \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} |\psi_D(x, k_{\perp})|^2. \] \hspace{1cm} (21)
As discussed in Ref. [16], \( P_D \approx 0.8 \) is a good approximation for the \( D \)-meson (As we have checked, change of \( P_D \) makes a numerical effect less than 2%). Then, taking \( P_D \approx 0.8 \), \( f_D = 240 \text{MeV} \), \( m_c = 1.3 \text{GeV} \) and \( m_d = 0.35 \text{GeV} \), we have \( A_D = 63.6 \text{GeV}^{-1} \), \( b_D^2 = 0.292 \text{GeV}^{-2} \).

Furthermore, as argued in Ref. [18], a more complete form of the light-cone wave function should include the Melosh rotation effect in spin space:

\[
\psi^f_D(x, k_\perp) = \chi_D(x, k_\perp) \exp \left[ -b_D^2 \left( \frac{k_\perp^2 + m_c^2}{x} + \frac{k_\perp^2 + m_d^2}{1-x} \right) \right]
\]

(22)

with the Melosh factor,

\[
\chi_D(x, k_\perp) = \frac{(1-x)m_c + x m_d}{\sqrt{k_\perp^2 + ((1-x)m_c + x m_d)^2}}.
\]

(23)

It can be seen from Eq. (23) that \( \chi_D(x, k_\perp) \to 1 \) as \( m_c \to \infty \), since there is no spin interaction between the two quarks in the heavy-flavor meson, i.e., the spin of the heavy constituent decouples from the gluon field, in the heavy quark limit [6]. However the \( c \)-quark is not heavy enough to neglect the Melosh factor.

After integration over \( k_\perp \) the full form of \( D \) meson DA can be achieved (model III):

\[
\varphi_D^{(III)}(x) = \frac{A_D \sqrt{3x(1-x)}}{8\pi^{3/2} f_D b_D} y \left[ 1 - Erf \left( \frac{b_D y}{\sqrt{x(1-x)}} \right) \right] \exp \left[ -b_D^2 \left( \frac{x m_d^2 + (1-x)m_c^2 - y^2}{x(1-x)} \right) \right],
\]

(24)

where \( y = x m_d + (1-x)m_c \) and the error function \( \text{Erf}(x) \) is defined as \( \text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt \). Using the same constraints as in Eq. (16) and (21), the parameters \( A_D \) and \( b_D \) are fixed as \( A_D = 62.8 \text{GeV}^{-1} \) and \( b_D^2 = 0.265 \text{GeV}^{-2} \).

In this paper we will employ the above three kinds of models to do numerical calculation. All these DA’s of the \( D \)-meson are plotted in Fig. (1) for a comparison. It is shown that they are of similar shape and all of them exhibit a maximum at \( x \simeq 0.6 - 0.7 \) as expected.

4 Numerical Result and Discussion

Apart from the DA of \( D \)-meson, the decay constant of \( B \)-meson \( f_B \) is among the important nonperturbative inputs. For consistency, we use the following corrector

\[
K(q^2) = i \int d^4x e^{iqx} \left< 0 | \bar{q}(x) (1 + \gamma_5) b(x), \bar{b}(0)(1 - \gamma_5) q(0) | 0 \right>,
\]

(25)

to recalculate it in the two-point sum rules. The calculation should be limited to leading order in QCD, since the QCD radiative corrections to the sum rule for \( \mathcal{F}_{B \to D}(y) \) are not taken into account. The value of the threshold parameter \( s_0 \) is determined by a best fit requirement in the region \( 10 \text{GeV}^2 \leq M^2 \leq 20 \text{GeV}^2 \), where \( M^2 \) is the corresponding Borel
Table 1: Parameter sets for $f_B$ and $\mathcal{F}_{B \rightarrow D}(y)$, $s'_0$ and $s_0$ for $f_B$ and $\mathcal{F}_{B \rightarrow D}(y)$ respectively; $m_b$ and $f_B$ are given in GeV, $s_0$ and $s'_0$ in GeV^2.

| set | $m_b$ | $s'_0$ | $f_B$  | $s_0$  |
|-----|-------|--------|--------|--------|
| 1   | 4.85  | 29.5   | 0.076  | 30.3   |
| 2   | 4.80  | 29.8   | 0.090  | 30.8   |
| 3   | 4.75  | 30.0   | 0.103  | 31.3   |

The same procedure is performed for $\mathcal{F}_{B \rightarrow D}(y)$, resulting in different values of the threshold parameter $s_0$. The result is listed in Tab.1. Choosing $s_0$ and $s'_0$ in this way, the dependence of $f_B$ and $\mathcal{F}_{B \rightarrow D}(y)$ on the Borel parameter is very weak and thus we can simply evaluate them at $M^2 = \bar{M}^2 = 15$GeV^2. The other input parameters are taken as $m_B = 5.279$GeV, $m_D = 1.869$GeV. As we have ignored all the radiation corrections, we don’t expect our values of $f_B$ to be good predictions of that quantity.

With the parameters chosen, it is straightforward to calculate the form factor $\mathcal{F}_{B \rightarrow D}(y)$ in the region $1.14 < y < 1.59$. The results with different sets of parameters are plotted in Fig.(2), where only model II has been used for simplicity. It is shown that the change of parameters can induce a uncertainty of about $10 - 15\%$ if we let $m_b$ vary between $4.75 - 4.85$ GeV. By fitting the data, the behavior of $\mathcal{F}_{B \rightarrow D}(y)$ has been known using the parametrization

$$\mathcal{F}_{B \rightarrow D}(y) = \mathcal{F}_{B \rightarrow D}(1)[1 - \hat{\rho}^2_D(y - 1) + \hat{c}_D(y - 1)^2 + O((y - 1)^3)],$$

with

$$\hat{\rho}^2_D = 0.69 \pm 0.14, \quad \hat{c}_D = 0,$$

$$\hat{\rho}^2_D = 0.69^{+0.12}_{-0.15}, \quad \hat{c}_D = 0.00^{+0.59}_{-0.00},$$

(27)

corresponding to the linear and quadratic fits [19], respectively. With the three DA models, the resulting dependence of $\mathcal{F}_{B \rightarrow D}(y)$ on the velocity transfer $y$, along with that extracted experimentally is illustrated in Fig.(3). In what follows, we denote the LCSR results for the form factor by $\mathcal{F}^{LC}_{B \rightarrow D}(y)$ and those extracted experimentally by $\mathcal{F}^{exp}_{B \rightarrow D}(y)$. For comparison, a figure-copy which expresses the pQCD results in [11] is given in Fig.(4). In the region to which the LCSR method is applicable, the central values of $\mathcal{F}^{LC}_{B \rightarrow D}(y)$ turn out to be a bit smaller than the corresponding those of $\mathcal{F}^{exp}_{B \rightarrow D}(y)$, using the DA models II and III as inputs; however, both of them are in accordance with each other within the error. The situation with model I DA is about the same. The central value of the form factor at the largest recoil is $\mathcal{F}^{exp}_{B \rightarrow D}(1.59) = 0.58$ versus $\mathcal{F}^{LC}_{B \rightarrow D}(1.59) = 0.40 - 0.50$, depending on the DA models. We note that the behavior of $\mathcal{F}^{LC}_{B \rightarrow D}(y)$ is essentially unchanged when the three different DA’s are used. From the present calculations, therefore it is too early to draw a conclusion which DA model is more suitable to reflect the characteristics of QCD dynamics inside the $D$ meson. When a comparison is made between the pQCD and LCSR predictions, the consistent results
can also be observed at the larger recoil. Of course, the two approaches describe the
different dynamics in $B \to D$ transitions. Whereas the use of LCSR approach is to
assume that the soft exchanges dominate in the weak decay in question, applying pQCD
method to do calculation corresponds to the viewpoint that the hard exchanges do.

In fact, the kinematical region we give, which makes LCSR results valid, is a con-
servative estimate. It is possible to extrapolate the present LCSR calculation to the
small recoil region. If it is true, we find that in the whole kinematically accessible
range $1.0 \leq y \leq 1.59$, the yielding LCSR estimates are compatible with the data.
For instance, at zero recoil it follows that $F_{B \to D}^{LC}(1) = 1.02$ (using model III), which
is in a good agreement with the evaluation obtained using the heavy quark symmetry:
$F_{B \to D}(1) = 0.98 \pm 0.07$. Nevertheless, we have to emphasize that a full understanding
of the dynamics involved in $B \to D$ transition should be obtained by combining the
three different approaches — the lattice QCD calculations with the heavy quark symme-
try considered, LCSR results and pQCD predictions, which are complementary to each
other. The LCSR results with chiral current correlator may act as a bridge connecting
those of other approaches.

5 Summary

We have discussed the form factor for $B \to D$ transitions $F_{B \to D}(y)$, using the improved
QCD LCSR approach where with the chiral current correlator chosen only the leading
twist DA of the $D$-meson is relevant at twist-3 accuracy. The resulting LCSR’s for
$F_{B \to D}(y)$ are available in the velocity transfer region $1.14 < y < 1.59$. Calculation is
done using three different twist-2 DA models for D meson. It is shown the numerical
results are less sensitive to the choice of DA, and are of a central value slight smaller
than but within the error in a agreement with those obtained by fitting the data on
$B \to Dl\bar{\nu}$. In the larger recoil region $1.35 < y < 1.59$ where pQCD is applicable, the
results presented here are consistent with ones of pQCD. From the practical calculations,
we find that the present results might be extrapolated to the smaller recoil region so that
the $B \to D$ transitions are calculable in the whole kinematically accessible range, using
the improved LCSR approach.

Also, we argue that for understanding the form factor for $B \to Dl\bar{\nu}$ in the whole kine-
matical range a combined use is necessary of three different methods: the lattice QCD
(with the heavy quark symmetry considered), improved LCSR and pQCD approaches,
which are adequate to do calculation in different kinematical regions and so could be
complementary to each other. The LCSR approach plays a bridge role in doing such
calculation.

The present findings can be improved once the QCD radiative correction to the LCSR
is taken into account and a more reliable twist-2 DA of $D$ meson becomes available. From
the previous discussion in [13], however, it is expected that the QCD radiative correction
can not change the present results too much.
ACKNOWLEDGEMENTS

This work was supported in part by the Natural Science Foundation of China (NSFC). We would like to thank Dr X. G. Wu for helpful discussions.

References

[1] T. Kurimoto, H. N. Li, and A. I. Sanda, Phys. Rev. D65, 014007 (2002); C. D. Lü, M. Z. Yang, Euro. Phys. J. C28, 515 (2003).

[2] T. Huang, Z. H. Li, and X. Y. Wu, Phys. Rev. D63, 094991 (2001).

[3] K. C. Bowler, et al., Phys. Lett. B486, 111 (2000).

[4] T. Huang, X. G. Wu, Phys. Rev. D71, 034018 (2005).

[5] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C29, 637 (1985).

[6] N. Isgur and M. B. Wise, Phys. Lett. B232, 113 (1989); ibid. B237, 527 (1989).

[7] Z. Ligeti, Y. Nir and M. Neubert, Phys. Rev. D49, 1302 (1994).

[8] M. Neubert, Phys. Rev. D45, 2451 (1992).

[9] S. Hashimoto, et al., Phys. Rev. D61, 014502 (2000).

[10] A. A. Ovchinnikov and V. A. Slobodynyuk, Z. Phys. C44, 433 (1989); V. N. Baier and A. G. Grozin, Z. Phys. C47, 669 (1990).

[11] T. Kurimoto, H. N. Li and A. I. Sanda, Phys. Rev. D67, 054028 (2003).

[12] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B345, 137 (1990); I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, ibid. 312, 509 (1989).

[13] T. Huang, Z. H. Li, Phys. Rev. D57, 1993 (1998); Z. G. Wang, M. Z. Zhou, and T. Huang, Phys. Rev. D67, 094006 (2003)

[14] T. Huang, Z. H. Li, and H. D. Zhang, J. Phys. G25, 1179 (1999); T. M. Aliev, et al., Phys. Rev. D67, 094009 (2003); T. M. Aliev, et al., Eur. Phys. J. C38, 85 (2004).

[15] I. Caprini, L. Lellouch, and M. Neubert, Nucl. Phys. B530, 153 (1998).

[16] X. H. Guo and T. Huang, Phys. Rev. D43, 2931 (1991).
[17] S. J. Brodsky, T. Huang and G. P. Lepage, in *Particles and Fields-2*, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983), P143; G. P. Lepage, S. J. Brodsky, T. Huang, and P. B. Mackenzie, *ibid.*, p83; T. Huang, in *Proceedings of XXth International Conference on High Energy Physics*, Madison, Wisconsin, 1980, edited by L. Durand and L. G. Pondrom, AIP Conf. Proc. No. 69 (AIP, New York, 1981), p1000.

[18] T. Huang, B. Q. Ma, and Q. X. Shen, *Phys. Rev.* D49, 1490 (1994).

[19] BELLE Collaboration, K. Abe et al., *Phys. Lett.* B526, 258 (2002).
Figure 1: Different kinds of $D$-meson DAs, solid and dashed curves correspond to model III and II, while the dotted line expresses model I.

Figure 2: Dependence of $F_{B\rightarrow D}$ on the different sets of parameters $m_b, f_B, s_0$. The three curves correspond to the parameter set 1–3 from bottom to top. Here we use model II for the $D$-meson DA for simplicity.
Figure 3: $F_{B \rightarrow D}$ as a function of the velocity transfer (with the parameters in the set 2). The thin lines expresses the experiment fits results, the solid line represents the central values, the dashed(dash-dotted) lines give the bounds from the linear(quadradic) fits. The thick lines correspond to our results, with the solid, dashed and dash-dotted lines for model III, II and I respectively.

Figure 4: pQCD results for $F_{B \rightarrow D}(y)$ copied from Ref.[11]. As in Fig.(3), the solid line represents the central values, the dashed(dash-dotted) lines give the bounds from the linear(quadratic) fits. The circles corresponds to pQCD results using model I for the $D$-meson DA with $C_D = 0.5, 0.7, 0.9$ from bottom to top.