Quark structure of the $\chi_c(3P)$ and $X(4274)$ resonances and their strong and radiative decays

J. Ferretti,1,2 E. Santopinto,3 M. Naeem Anwar,4 and Yu Lu5

1Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120, USA
2Department of Physics, University of Jyväskylä, P.O. Box 35 (YFL), 40014 Jyväskylä, Finland
3INFN, Sezione di Genova, Via Dodecaneso 33, 16146 Genova, Italy
4Institut für Kernphysik, Jülich Center for Hadron Physics and Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany
5Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

We calculate the masses of $\chi_c(3P)$ states with threshold corrections in a coupled-channel model. The model was recently applied to the description of the properties of $\chi_c(2P)$ and $\chi_b(3P)$ multiplets [Phys. Lett. B 789, 550 (2019)]. We also compute the open-charm strong decay widths of the $\chi_c(3P)$ states and their radiative transitions. According to our predictions, the $\chi_c(3P)$ states should be dominated by the charmonium core plus small meson-meson components. The $X(4274)$ is interpreted as a $c\bar{c}\chi_c(3P)$ state. More informations on the other members of the $\chi_c(3P)$ multiplet, as well as a more rigorous analysis of the $X(4274)$’s decay modes, are needed to provide further indications on the quark structure of the previous resonance.

I. INTRODUCTION

Recently, several new meson resonances have been discovered. A fraction of them, the so-called $XYZ$ states, cannot be interpreted in terms of standard quark-antiquark degrees of freedom. Their description needs the introduction of more complicated exotic or multi-quark structures. A well-known example is the $X(3872)$ [now $\chi_c(3872)$] 5,8. A wide range of theoretical descriptions of $XYZ$ states is available. These interpretations include: a) the compact tetraquark (or diquark-antidiquark) model 9–25; b) the meson-meson molecular model 26–35; c) the interpretation in terms of kinematic or threshold effects caused by virtual particles 36–49. Calculations of meson observables (like the spectrum or the decay widths) within the above pictures, when compared with the experimental data, may help to better understand the quark structure of $XYZ$ mesons.

In a previous paper 49, we discussed a novel coupled-channel model approach to the spectroscopy and structure of heavy quarkonium-like mesons based on the Unquenched Quark Model (UQM) formalism 36, 41, 42, 45–48, 53–52. In the UQM, quarkonium-like exotics are interpreted as the superposition of a heavy quarkonium core plus meson-meson molecular-type components. In the approach of Ref. 49, the UQM formalism was used to compute the self-energy corrections to the bare masses of $\chi_c(2P)$ and $\chi_b(3P)$ states due to virtual particle effects. However, from previous UQM calculations, see e.g. Refs. 37, 41, 42, 45, we did not perform a global fit to the whole heavy quarkonium spectrum. We applied the formalism to a single heavy quarkonium multiplet at a time. Moreover, we introduced a “renormalization” prescription for the UQM results. Thanks to this, we could suggest a solution to some long-standing problems of UQM calculations. They include: I) the lack of convergence of UQM results; II) the fact that the self-energy corrections to a state $|A\rangle$, both when $|A\rangle$ is close to meson-meson decay thresholds and when $|A\rangle$ is far away from them, are of the same order of magnitude, which is unphysical.

Here, we make use of the same approach as Ref. 49 to study the quark structure of the $X(4274)$ and $\chi_c(3P)$ states by calculating their masses with threshold corrections. We also compute their open-charm strong decay widths in the $3P_0$ pair-creation model 53–60 and their radiative transitions in the UQM 48 formalism. The $X(4274)$ [also known as $\chi_c(4274)$] was discovered by LHCb in the amplitude analysis of $B^+ \to J/\psi K^+$ decays 61, even though a $3.1\sigma$ evidence for a relatively narrow $J/\psi\phi$ mass peak near $4274 \pm 8$ MeV had been previously presented by CDF 62. Its quantum numbers are $I^G(J^{PC}) = 0^+(1^{++})$ and its total decay width is $49 \pm 12$ MeV 61.

According to our coupled-channel model results, threshold effects should be small to medium-sized in the $\chi_c(3P)$ multiplet. Our $3P_0$ model prediction for the open-charm strong decay width of $X(4274)$ is compatible with the experimental data within the experimental error. Therefore, it is reasonable to treat the $X(4274)$ as a charmonium state. However, due to the total lack of experimental data on the other members of the multiplet, we cannot exclude the presence of small meson-meson components in the $X(4274)$ wave function. Our results for the radiative transitions of the $X(4274)$ and $\chi_c(3P)$s will be an important check and may help to assess the quark structure of the previous resonances.
II. FORMALISM

A. $^3P_0$ pair-creation model

In the $^3P_0$ pair-creation model, the open-flavor strong decay of a hadron $A$ into hadrons $B$ and $C$ takes place in the rest frame of $A$. The decay proceeds via the creation of an additional $q\bar{q}$ pair with $J^{PC} = 0^{++}$ quantum numbers from QCD vacuum [53–55] (see Fig. 1 and the values of the oscillator parameter, $\alpha$[41], Table II]. The values of the oscillator parameter, $\alpha$, and the coefficient $\gamma_0$ were fitted to the open-charm strong decays of higher charmonia [41].

$$\Gamma_{A\rightarrow BC} = \Phi_{A\rightarrow BC}(q_0) \sum_{\ell} \left| \langle BCq0\ell|T^\dagger|A \rangle \right|^2.$$  

(1)

where $l$ is the relative angular momentum between $B$ and $C$ and $J$ represents the total angular momentum of $B$ and $C$. The coefficient

$$\Phi_{A\rightarrow BC}(q_0) = 2\pi q_0 \frac{E_B(q_0)E_C(q_0)}{M_A}$$

(2)

is the phase-space factor for the decay; it depends on the relative momentum $q_0$ between $B$ and $C$, the energies of the two decay products, $E_{B,C}(q_0)$, and the mass of the decaying meson, $M_A$. We assume harmonic oscillator wave functions for the hadrons $A$, $B$ and $C$, depending on a single oscillator parameter $\alpha_{ho}$; see [41, Table II] and [60, Table II]. The values of the oscillator parameter, $\alpha_{ho}$, and of the other pair-creation model parameters, $r_q$ and $\gamma_0$, were fitted to the open-charm strong decays of higher charmonia [41].

![Diagram](image)

FIG. 1: Diagrams contributing to the $A \rightarrow BC$ decay process. $q_i$, with $i = 1, \ldots, 4$, and $q_j$, with $j = 5, \ldots, 8$, are the quarks and antiquarks in the initial and final states, respectively. Picture from Ref. [51]. APS copyright.

Following Refs. [41, 42, 50, 51, 60], we introduce a few changes in the $^3P_0$ pair-creation model operator, $T^\dagger$. These modifications include the substitution of the pair-creation strength, $\gamma_0$, with an effective one, $\gamma_0^{eff}$, to suppress heavy quark pair-creation, see [51, Eq. (12)] and [53], and the introduction of a Gaussian quark form-factor, because the pair of created quarks has an effective size [41, 42, 50, 52].

B. Threshold mass-shifts in a coupled-channel model

We briefly summarize the main features of the coupled-channel model of Ref. [49]. There, higher Fock components, $|BC\rangle$, due to virtual particle effects are superimposed on the $QQ$ bare meson wave functions, $|A\rangle$, of heavy quarkonium states. One has [53, 54, 56, 57, 58, 59, 60]:

$$|\psi_A\rangle = N \left[ |A\rangle + \sum_{BC\ell J} \int dq \frac{BCq\ell J |T^\dagger|A\rangle}{M_A - E_B - E_C} \right].$$

(3)

The sum is extended over a complete set of meson-meson intermediate states $|BC\rangle$, with energies $E_{B,C}(q) = \sqrt{M_{B,C}^2 + q^2}$; $M_A$ is the physical mass of the meson $A$; $q$ is the on-shell momentum between $B$ and $C$, $\ell$ is the relative angular momentum between them, and $J$ is the total angular momentum, with $J = J_B + J_C + \ell$. Finally, the amplitudes $\langle BCq\ell J |T^\dagger|A\rangle$ are computed within the $^3P_0$ pair-creation model [53–56]. See also Sec. II A.

In the coupled-channel approach of Ref. [49], one can study a single multiplet at a time, like $\chi_c(2P)$ or $\chi_b(3P)$. The physical masses of the meson multiplet members are given by

$$M_A = E_A + \Sigma(M_A) + \Delta_{th}.$$  

(4)

In the previous equation,

$$\Sigma(M_A) = \sum_{BC} \int_0^\infty q^2 dq \frac{\left| \langle BCq\ell J |T^\dagger|A\rangle \right|^2}{M_A - E_B(q) - E_C(q)}$$

(5)

is a self-energy correction, $E_A$ is the bare mass of the meson $A$, and $\Delta_{th}$ is a free parameter. Contrary to our previous UQM studies [41, 42], the bare meson masses $E_A$ are not fitted to the whole charmonium spectrum. Their values are directly extracted from the relativized QM predictions of Refs. [57, 58]. The UQM model parameters, which we need in the calculation of the $\langle BCq\ell J |T^\dagger|A\rangle$ vertices, were fitted to the open-flavor strong decays of charmonia; see [41, Table II] and [60, Table II]. Thus, for each multiplet $\Delta_{th}$ is the only free parameter. It is defined as the smallest self-energy correction (in terms of absolute value) among those of the multiplet members; see Sec. II C and [49, Secs. 2.2 and 2.3]. The introduction of $\Delta_{th}$ in Eq. (4) represents our “renormalization” or “subtraction” prescription for the threshold corrections in the UQM.

C. Radiative Transitions in the QM and UQM formalisms

Radiative transitions of higher charmonia are of considerable interest, since they can shed light on their internal structure and provide one of the few pathways between different $cc$ multiplets. Particularly, for those
states which cannot be directly produced at $e^+e^-$ colliders (such as $P$-wave charmonia), the radiative transitions serve as an elegant probe to explore such systems. In the quark model, the electric dipole ($E1$) transitions can be expressed as

$$\Gamma_{E1,QM} = \frac{4\alpha e^2}{3} C_{AB} |\langle R_B | r | R_A \rangle|^{2} \frac{E^3 \bar{E}_B}{M_A} \delta_{S_A,S_B}. \quad (6)$$

Here, $e_c = \frac{2}{3}$ is the $c$-quark charge, $\alpha$ the fine structure constant, $E_\gamma$ denotes the energy of the emitted photon, and $\bar{E}_B = \sqrt{M_B^2 + E^2}$ is the total energy of the final meson. The spatial matrix elements

$$\langle R_B | r | R_A \rangle = \int_0^{\infty} r^3 dr \ R^*_A(r) \ R(r) \quad (7)$$

involve the initial and final meson radial wave functions and are obtained numerically; for further details, we refer to [14, 33]. From Eq. (7), we know that the value of the decay width depends on the details of the wave functions, which are highly model dependent. The angular matrix elements $C_{AB}$ are given by

$$C_{AB} = \max(L_A, L_B) \left( 2J_B + 1 \right) \left\{ \frac{L_B \ J_B \ S_A}{J_A \ L_A} \right\}^2 \quad (8)$$

where $S_{A,B}$, $L_{A,B}$ and $J_{A,B}$ are the spin, orbital angular momentum and total angular momentum of the initial/final charmonia, respectively.

In the UQM formalism, the wave function of a heavy quarkonium state consists of both a $QQ$ valence configuration and meson-meson higher Fock components, which are the result of the creation of light $q\bar{q}$ pairs from the vacuum; see Eq. [14]. Therefore, the heavy quarkonium bare meson wave function has to be properly renormalized [51, Eq. (9)].

In our specific case, the radial wave functions $R_{A,B}$ of Eq. (7) have to be multiplied by the factors $P_{c\bar{c}}(A,B) \leq 1$, which are the probabilities of finding the wave functions of the $A$ and $B$ states in their valence components. Given this, in the UQM formalism the width of Eq. (9) becomes [48]:

$$\Gamma_{E1,QM} = \Gamma_{E1,QM} P_{c\bar{c}}(A)P_{c\bar{c}}(B). \quad (9)$$

### III. RESULTS

#### A. Open-charm strong decays of $\chi_c(3P)$ states

In this section, we calculate the open-charm strong decay widths of $\chi_c(3P)$ states within the $^3P_0$ pair-creation model. The main features of the model are briefly described in Sec. IIA. When available, we extract the masses of both the initial- and final-state mesons from the PDG [1]; otherwise, we use the relativized QM predictions of Refs. [57–64]. Our theoretical results are given in Table I and can be compared to the $^3P_0$ pair-creation model results of Table XI.

| State  | Channel | Width [MeV] | State  | Channel | Width [MeV] |
|-------|---------|-------------|-------|---------|-------------|
| $\chi_c(3P)$ | $DD^*$ | 6.6 | $\chi_c(3P)$ | $DD$ | 4.0 |
| $D^*D^*$ | 28.0 | $D^*D^*$ | 35.0 |
| $DD_0^*$ | 0.2 | $DD_1(2420)$ | 3.4 |
| $D_sD_s^*$ | 6.3 | $DD_1(2430)$ | 0.8 |
| $D_s^*D_s^*$ | 2.5 | $D_sD_s$ | 2.6 |
| $D_s^*D_s^*$ | 5.1 | $D_s^*D_s^*$ | 4.3 |
| $D_s^*D_s^*$ | 2.5 | $D_s^*D_s^*$ | 4.7 |

TABLE I: Open-charm strong decays of $\chi_c(3P)$ states in the $^3P_0$ pair-creation model. The values of the $\chi_c(3P)$ masses are taken from Refs. [47, 63] (see also Table XVII second column), except for the value of the $\chi_c(3P)$ [or $X(4274)$] mass, which is extracted from the PDG [1]. The values of the charmed and charmed-strange meson masses are taken from the PDG [1], the mixing angle between $D_1(1P_1)$ and $D_1(1P'_1)$ states is taken from [61, Table III].

It is worth noting that: I) our predictions are of the same order of magnitude as those of [57, Table XI]. The discrepancies are in the order of 10–20%, except for the $h_c(3P)$, where they are larger. These differences between our results and those of Ref. [57] arise partly because of different choices of the $^3P_0$ model parameters, and partly because of the values of the masses of the decaying mesons given as inputs in the calculations. In particular, in our case we use for the decaying meson masses either the experimental values [1] or relativized QM predictions [64]. On the contrary, in Ref. [57] the authors extracted the masses from a non-relativistic potential model fit to the charmonium spectrum. Moreover, the use of different masses for the $c\bar{c}$ decaying mesons determines the opening of decay channels, like $\chi_c(3P) \to DD_1^*(2460)$, which were below threshold in [57, Table XII]. Finally, as a check we have also computed the decay widths of $\chi_c(3P)$s by using the same input masses and model parameters as Ref. [57] and we have obtained the same results as Ref. [57]; II) according to our results, the $\chi_c(3P)$s are characterized by relatively large open-charm widths, which are of the order of 40–60 MeV. If our predictions were confirmed by the experiments, we may argue that the $\chi_c(3P)$ mesons should be charmonium-like states, with their wave functions being dominated by a $c\bar{c}$ core; III) of particular interest are our results for the $\chi_c(3P)$
state. Specifically, our theoretical prediction for the total open-charm width of the $\chi_{c1}(3P)$, i.e. 43.6 MeV, is compatible with the total experimental width of the $\chi_{c1}(4274)$ [4], namely 49±12 MeV, under the hypothesis that the open-charm contribution to the total width of the $\chi_{c1}(4274)$ is the dominant one. As discussed in the previous point, this suggests that the wave function of the $\chi_{c1}(4274)$ should be dominated by the charmonium component.

### B. $E1$ radiative transitions of $\chi_c(3P)$ states

Here, we discuss our UQM results for the $E1$ radiative transitions of $\chi_c(3P)$ states. Our predictions, denoted as $\Gamma_{E1,UQM}$ and computed by means of Eq. (10) [57], are given in Table II and Tables V and VI. The QM widths of Eq. (10) [57], $\Gamma_{E1,QM}$, are computed by using Cornell potential model [54, 57, 64] wave functions for both the parent and daughter charmonium states. Our results for the $\Gamma_{E1,QM}$ widths coincide with those reported in Ref. [57]; therefore, they are not shown in the present paper.

The UQM predictions, denoted as $\Gamma_{E1,UQM}$, are calculated by renormalizing the $A$ and $B$ meson wave functions according to the valence probabilities $P_{c}(A)$ and $P_{\bar{c}}(B)$. For simplicity, due to the large amount of work, the calculation of the probabilities $P_{c}(A, B)$ is not performed in the UQM-based coupled-channel formalism of Secs. 11B and 11C but rather in the standard UQM formalism [43, 54], with the model parameter values $\alpha_{0} = 0.5$ GeV and $\gamma_{0} = 0.4$, extracted from Ref. [57], and considering only 1S1S open-charm intermediate states. Moreover, when the initial state is above a $D(\bar{s})D(\bar{s})$ or $D(\bar{s})D(\bar{s})$ threshold, we ignore the contribution of this channel in wave function renormalization, even though the mass shift caused by the previous channel is not zero.

Finally, it is worth noting that: I) the radiative decay widths of $\chi_{c}(3P)$ states span a wide interval, from $O(300$ MeV) to $O(1$ MeV), in the case of $3P \rightarrow 3S + \gamma$ and $3P \rightarrow 1D + \gamma$ transitions, respectively. In particular, the $3P \rightarrow 3S + \gamma$ decay widths are quite large; thus, they might be observed in the next few years; II) our UQM results for $\chi_{c}(3P)$ states are roughly of the same order of magnitude as the QM ones [57]. This is a confirmation of our statement that loop effects can play a relatively important role in determining the properties of $\chi_{c}(3P)$s, though their importance is far from being conclusive. In this respect, it is interesting to estimate the importance of loop effects in the case of other charmonium radiative transitions. See Appendix A, Tables V and VI and the QM results of Ref. [57]. For example, consider the $\chi_{c2}(2P) \rightarrow \psi_{c}(1^{3}D_{1}) + \gamma$ decay, where the ratio between the QM [57] and UQM widths is almost a factor of 2.5; III) finally, we also show that the $E1$ transition widths of $\chi_{c}(3P)$s into $J/\psi + \gamma$ are one order of magnitude suppressed with respect to those into $\psi(3S) + \gamma$. A similar pattern was previously observed in the $\chi_{b}(3P)$ case [48].

In conclusion, these results may provide solid references to search for the other members of the $\chi_{c}(3P)$ multiplet by analyzing the $\chi_{c}(3P) \rightarrow \psi(2S,3S) + \gamma$ radiative transitions. Recently, the CMS Collaboration was able to distinguish for the first time between two candidates of the bottomonium $3P$ multiplet, $\chi_{b1}(3P)$ and $\chi_{b2}(3P)$, through their $Y(nS) + \gamma$ ($n = 1, 2, 3$) decays [68]. We expect the charmonium $3P$ multiplet to be easily searched by means of the same strategy.
those channels denoted by – are suppressed by selection rules.

It is worth noting that in the coupled-channel model calculations, the relative threshold mass shifts between the $\chi_c(3P)$ multiplet members due to a complete set of $1S1P$ meson-meson loops, like $DD_0^*(2300)$, $DD_1(2420)$, and so on. As shown in Ref. [41], charmonium loops, like $\eta_c\chi_c(1P)$, are negligible because of the suppression mechanism of [41, Eq. (12)]. Therefore, these loops are not taken into account in the calculation of the self-energy corrections of $\chi_c(3P)$ states.

TABLE III: Self-energy corrections, $\Sigma(M_A)$ (in MeV), to the bare masses of $\chi_c(3P)$ states, calculated via Eq. (5). The values of the UQM parameters are extracted from [11, Table II]. The first rows show the partial contributions to $\Sigma(M_A)$ from channels $BC$, such as $DD_0^*(2300)$, $DD_1(2420)$, and so on. The last rows provide the total results, obtained by summing the previous partial contributions. The contributions of those channels denoted by – are suppressed by selection rules.

| State | $DD_0^*(2300)$ | $DD_1(2420)$ | $DD_2(2430)$ | $DD_3^*(2460)$ |
|-------|----------------|----------------|----------------|----------------|
| $h_c(3P)$ | -12.3 | - | -0.5 | -30.6 |
| $\chi_{c0}(3P)$ | - | -41.6 | -28.3 | |
| $\chi_{c1}(3P)$ | -0.2 | -13.3 | -12.9 | -18.1 |
| $\chi_{c2}(3P)$ | - | -15.7 | -14.2 | -15.9 |

TABLE IV: Comparison between the experimental masses [1] and our theoretical predictions, $M_A$, for the mass of the $\chi_c(3P)$ states, namely the $h_c(3P)$, $\chi_{c0}(3P)$, and $\chi_{c2}(3P)$ states.

| State | $E_A$ [MeV] | $\Sigma(M_A) - \Delta_{th}$ [MeV] | $M_A^{th}$ [MeV] | $M_A^{exp}$ [MeV] |
|-------|-------------|---------------------------------|-----------------|-----------------|
| $h_c(3P)$ | 4318 | -8 | 4310 | - |
| $\chi_{c0}(3P)$ | 4292 | -25 | 4267 | - |
| $\chi_{c1}(3P)$ | 4317 | 0 | 4317 | 4274±8 |
| $\chi_{c2}(3P)$ | 4337 | -17 | 4320 | - |

It is worth noting that: I) the threshold corrections of Table IV are larger than those of $\chi_h(3P)$ states, but smaller than those of $\chi_c(2P)$ states; see Ref. [41, Table I]. In light of this, we expect the $\chi_c(3P)$ states to be dominated by the $c\bar{c}$ core component; II) at present, the only decay mode of the $X(4274)$ which has been observed experimentally is that into $J/\psi\phi$. This may be compatible with the interpretation of the $X(4274)$ as a multiquark state with non-zero hidden-charm hidden-strange components. However, as discussed in Sec. 4, several properties of the $X(4274)$ (e.g. its total decay width) are compatible with those of a $\chi_{c1}(3P)$ state.

In conclusion, this work indicates that the $X(4274)$’s wave function should be dominated by the $\chi_{c1}(3P)$ component. More information on the other members of the $\chi_c(3P)$ multiplet, as well as a more rig-
orous analysis of the $X(4274)$’s decay modes, are needed to provide further indications on the quark structure of the previous resonance.

IV. $X(4274)$: OTHER INTERPRETATIONS

As pointed out in Ref. [59], molecular states cannot account for the $1^+$ nature of the $X(4274)$. A possible interpretation of the $X(4274)$ is that of a $sar{s}car{c}$ compact tetraquark state. The spectrum of strange and nonstrange hidden-charm compact tetraquark states was computed in Ref. [22] within a relativized diquark-antidiquark model. There, the authors could provide tetraquark assignments to 13 suspected XYZ exotics, including the $Z_c(3900)$, $X(4500)$ and $X(4700)$; however, they could not accommodate the $X(4274)$ within the tetraquark picture. A similar investigation on $sar{s}car{c}$ compact tetraquarks was conducted within the relativized quark model [20]. There, the authors discussed possible assignments to the $X(4140)$, $X(4500)$ and $X(4700)$, but they could not accommodate the $X(4274)$ within a $sar{s}car{c}$ compact tetraquark description [20]. In Ref. [70], the authors made use of QCD sum rules to study the properties of the $X(4140)$ and $X(4274)$. They interpreted the $X(4140)$ as a $1^{++}$ diquark-antidiquark compact tetraquark in the $3{}^−_c$ color configuration, while the $X(4274)$ was described as a diquark-antidiquark bound state with a $6_6^{}c_6^{}$ color wave function. Finally, in Ref. [71] it was suggested that the $X(4140)$, $X(4274)$, $X(4500)$ and $X(4700)$ could be accommodated within two tetraquark multiplets, with the $X(4274)$ characterized by $0^{++}$ or $2^{++}$ quantum numbers.

In Ref. [72], the authors investigated possible assignments for the four $J/ψφ$ structures, reported by LHCb, CMS, D0 and Babar [72,76], in a coupled channel scheme by using a nonrelativistic constituent quark model [77]. In particular, they showed that the $X(4274)$, $X(4500)$ and $X(4700)$ can be described as conventional $3^3P_1$, $4^3P_0$, and $5^3P_0$ charmonium states, respectively. The same interpretation for the $X(4274)$ was proposed in Ref. [78]. In a study of heavy quarkonium hybrids based on the strong coupling regime of pNRQCD [79], the authors found out that the $X(4274)$ is compatible with a $χ_{c1}(3P)$ state, which may be affected by the $D^{*+}_sD^{*-}_s$ threshold.

In Ref. [80], an interpretation of the $X(4274)$ as a $P$-wave $D_sD_{s0}(2317)$ molecular state in a quasi-potential Bethe-Salpeter equation approach was proposed. If the previous state is a hadronic molecule, an $S$-wave $D_sD_{s0}(2317)$ bound state below the $J/ψφ$ threshold should also exist. Finally, in Ref. [81] the authors suggested to assign the $X(4274)$ to a $ψ(2S)φ$ $S$-wave hadrocharmonium configuration.

V. CONCLUSION

We studied the quark structure, the spectrum and the strong open-charm and radiative decay modes of the $X(4274)$ and $χ_c(3P)$ states within an UQM-based coupled-channel model [19] and the quark model formalism [53, 55, 65, 67].

The present coupled-channel model was previously used to study the properties and quark structure of the $χ_c(2P)$ and $χ_b(3P)$ multiplets [19]. There, a prescription to “renormalize” the UQM results for the self-energy/threshold corrections made it possible to distinguish between quarkonia, the $χ_b(3P)$, and quarkonium-like states with significant meson-meson components in their wave functions, the $χ_c(2P)$’s.

According to our new results, the $X(4274)$ can be described as a $χ_{c1}(3P)$ state. The other members of the $χ_c(3P)$ multiplet can be interpreted as $3P$ charmonium cores plus small to medium-sized open-charm meson-meson components.

A comparison between theoretical results for the radiative transitions of $χ_c(3P)$’s (including ours and, for example, those from Ref. [51]) and the forthcoming experimental data may provide exploratory pathways to search for still unobserved $3P$ charmonia. Hence, we suggest the experimentalists to focus on the study of the $χ_c(3P) → ψ(nS) + γ$ decay modes, and especially on the $ψ(2S, 3S) + γ$ transitions.

In conclusion, we hope that this study might be helpful to fulfill a better understanding of higher $P$-wave charmonia. More precise conclusions regarding the quark structure of the $χ_c(3P)$ states will necessarily require more experimental informations on the properties of the still unobserved $h_c(3P)$, $χ_{c0}(3P)$ and $χ_{c2}(3P)$.

Acknowledgments

We are grateful to Ulf-G. Meißner for a careful reading of the manuscript, and to Bing-Song Zou for mentoring on the formalism of this manuscript. J. Ferretti acknowledges financial support from the US Department of Energy, Grant No. DE-FG-02-91ER-40608, and the Academy of Finland, Project no. 320062. Y. Lu and M. N. Anwar are supported by the DFG (Grant No. TRR110) and the NSF (Grant No. 1162131001) through the funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD”. M. N. Anwar acknowledges partial support from the Munich Institute for Astro- and Particle Physics (MIAPP) which is funded by the DFG under Germany’s Excellence Strategy—EXC-2094-390783311.
| Process | $P_{cc}(A)$ | $P_{cc}(B)$ | $\Gamma_{\text{UQM}}$ [keV] |
|---------|-------------|-------------|-----------------|
| $\psi(2S) \rightarrow \chi_{c2}(1P) + \gamma$ | 0.660 | 0.660 | 16 |
| $\psi(2S) \rightarrow \chi_{c0}(1P) + \gamma$ | 0.660 | 0.700 | 29 |
| $\psi(3S) \rightarrow \chi_{c2}(2P) + \gamma$ | 0.940 | 0.610 | 8 |
| $\psi(3S) \rightarrow \chi_{c2}(2P) + \gamma$ | 0.940 | 0.770 | 39 |
| $\psi(3S) \rightarrow \chi_{c2}(1P) + \gamma$ | 0.940 | 0.660 | 8 |
| $\psi(3S) \rightarrow \chi_{c2}(1P) + \gamma$ | 0.940 | 0.700 | 0 |
| $\psi(3S) \rightarrow \chi_{c2}(1P) + \gamma$ | 1.000 | 1.000 | 68 |
| $\psi(4S) \rightarrow \chi_{c2}(3P) + \gamma$ | 1.000 | 0.960 | 125 |
| $\psi(4S) \rightarrow \chi_{c2}(3P) + \gamma$ | 1.000 | 0.770 | 0 |
| $\psi(4S) \rightarrow \chi_{c2}(3P) + \gamma$ | 1.000 | 0.660 | 0 |
| $\psi(4S) \rightarrow \chi_{c2}(3P) + \gamma$ | 1.000 | 0.700 | 0 |
| $\chi_{c2}(1P) \rightarrow J/\psi + \gamma$ | 0.660 | 0.770 | 216 |
| $\chi_{c2}(1P) \rightarrow J/\psi + \gamma$ | 0.700 | 0.770 | 84 |
| $\chi_{c2}(2P) \rightarrow \psi(2S) + \gamma$ | 0.610 | 0.660 | 124 |
| $\chi_{c2}(2P) \rightarrow \psi(2S) + \gamma$ | 0.770 | 0.590 | 33 |
| $\chi_{c2}(2P) \rightarrow \psi(2S) + \gamma$ | 0.770 | 0.660 | 37 |
| $\chi_{c2}(2P) \rightarrow \psi(2S) + \gamma$ | 0.770 | 0.770 | 32 |
| $\chi_{c2}(2P) \rightarrow \psi(1^3D_2) + \gamma$ | 0.610 | 0.660 | 35 |
| $\chi_{c2}(2P) \rightarrow \psi(1^3D_1) + \gamma$ | 0.610 | 0.680 | 1 |
| $\chi_{c2}(2P) \rightarrow \psi(1^3D_1) + \gamma$ | 0.790 | 0.680 | 12 |
| $h_c(2P) \rightarrow \eta_c(1^3D_2) + \gamma$ | 0.760 | 0.590 | 27 |
| $\psi_2(1^3D_2) \rightarrow \chi_{c1}(1P) + \gamma$ | 0.570 | 0.660 | 24 |
| $\psi_2(1^3D_1) \rightarrow \chi_{c1}(1P) + \gamma$ | 0.680 | 0.660 | 2 |
| $\psi_2(1^3D_1) \rightarrow \chi_{c2}(1P) + \gamma$ | 0.680 | 0.700 | 193 |
| $\psi_2(2^3D_2) \rightarrow \chi_{c2}(2P) + \gamma$ | 0.960 | 0.610 | 140 |
| $\psi_2(2^3D_1) \rightarrow \chi_{c2}(2P) + \gamma$ | 0.970 | 0.790 | 230 |
| $\psi_2(2^3D_1) \rightarrow \chi_{c2}(2P) + \gamma$ | 0.970 | 0.760 | 249 |
| $\psi_2(2^3D_2) \rightarrow \chi_{c2}(1P) + \gamma$ | 0.970 | 0.660 | 4 |
| $\psi_2(2^3D_1) \rightarrow \chi_{c1}(1P) + \gamma$ | 0.960 | 0.700 | 18 |
| $\psi_2(2^3D_1) \rightarrow \chi_{c1}(1P) + \gamma$ | 0.960 | 0.950 | 60 |
| $\psi_2(2^3D_1) \rightarrow \chi_{c1}(1P) + \gamma$ | 0.960 | 0.940 | 0 |
| $\psi_2(2^3D_2) \rightarrow \chi_{c1}(1P) + \gamma$ | 0.970 | 0.940 | 5 |
| $\eta_c(2^3D_2) \rightarrow \eta_c(1^3F_3) + \gamma$ | 0.970 | 0.930 | 49 |
| $\chi_{c2}(1^3F_3) \rightarrow \psi_2(1^3D_3) + \gamma$ | 0.930 | 0.660 | 25 |
| $\chi_{c2}(1^3F_2) \rightarrow \psi_2(1^3D_2) + \gamma$ | 0.940 | 0.660 | 1 |
| $\chi_{c2}(1^3F_2) \rightarrow \psi_2(1^3D_2) + \gamma$ | 0.940 | 0.680 | 304 |
| $\chi_{c2}(2^3F_4) \rightarrow \psi_2(2^3D_4) + \gamma$ | 1.000 | 0.960 | 296 |
| $\chi_{c2}(2^3F_4) \rightarrow \psi_2(2^3D_4) + \gamma$ | 1.000 | 0.970 | 324 |
| $\chi_{c2}(2^3F_4) \rightarrow \psi_2(2^3D_4) + \gamma$ | 1.000 | 0.970 | 56 |
| $h_c(2^3F_3) \rightarrow \eta_c(2^3D_2) + \gamma$ | 1.000 | 0.970 | 351 |
| $h_c(2^3F_3) \rightarrow \eta_c(2^3D_2) + \gamma$ | 1.000 | 0.660 | 1 |
| $h_c(2^3F_3) \rightarrow \eta_c(2^3D_2) + \gamma$ | 1.000 | 0.660 | 0 |
| $h_c(2^3F_3) \rightarrow \eta_c(2^3D_2) + \gamma$ | 1.000 | 0.680 | 14 |
| $h_c(2^3F_3) \rightarrow \eta_c(2^3D_2) + \gamma$ | 1.000 | 0.950 | 27 |
| $\psi_2(1^3G_3) \rightarrow \chi_{c2}(1^3F_2) + \gamma$ | 1.000 | 0.950 | 1 |
| $\eta_c(1^3G_4) \rightarrow h_c(1^3F_3) + \gamma$ | 1.000 | 0.940 | 401 |

**TABLE V:** As Table [II] but for the radiative transitions of different charmonia. Our UQM predictions, $\Gamma_{E1\text{-UQM}}$, are computed according to Eq. [4].
| Process                                                                 | $P_{c2}(A)$ | $P_{c2}(B)$ | $\Gamma_{\text{UQM}}$ [keV] | $\Gamma_{\text{exp}}$ [keV] |
|------------------------------------------------------------------------|------------|------------|-----------------------------|-----------------------------|
| $\psi(2S) \rightarrow \chi c(1P) + \gamma$                           | 0.660      | 0.660      | 16                          | 27.9 ± 0.6                  |
| $\psi(2S) \rightarrow \chi c(1P) + \gamma$                           | 0.660      | 0.670      | 24                          | 28.7 ± 0.7                  |
| $\psi(2S) \rightarrow \chi c(1P) + \gamma$                           | 0.660      | 0.700      | 29                          | 28.8 ± 0.6                  |
| $\chi c(1P) \rightarrow J/\psi + \gamma$                             | 0.660      | 0.770      | 216                         | 374.3 ± 10                  |
| $\chi c(1P) \rightarrow J/\psi + \gamma$                             | 0.670      | 0.770      | 166                         | 288.8 ± 8.4                 |
| $\chi c(1P) \rightarrow J/\psi + \gamma$                             | 0.700      | 0.770      | 84                          | 151.2 ± 5.4                 |
| $h_c(1P) \rightarrow \eta c(1S) + \gamma$                            | 0.670      | 0.800      | 267                         | 357 ± 42                    |
| $\psi_1(1^3D_1) \rightarrow \chi c(1P) + \gamma$                      | 0.680      | 0.660      | 2                           | < 17.4                      |
| $\psi_1(1^3D_1) \rightarrow \chi c(1P) + \gamma$                      | 0.680      | 0.670      | 57                          | 67.7 ± 6                    |
| $\psi_1(1^3D_1) \rightarrow \chi c(1P) + \gamma$                      | 0.680      | 0.700      | 193                         | 187.7 ± 16                  |

TABLE VI: Our UQM predictions for E1 radiative decay widths of lower charmonia are compared to the available experimental results. 

Appendix A: E1 radiative transitions of charmonium states

In Table VI we enlist our UQM results for the E1 radiative transition widths of higher-lying charmonia, including 2S, 3S, 1P and 2P resonances. The widths are calculated as explained in Sec. [111B In Table VI we compare our UQM predictions to the available experimental data [1]. Our results can also be compared to the QM predictions of Ref. [57]. It is worth noting that our predictions are in good accordance with the existing experimental results [1]. This is a further indication of the importance of the radiative transitions in the study of the properties of both the well-established and still unobserved heavy quarkonium resonances.

[1] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).
[2] A. Esposito, A. Pilloni and A. D. Polosa, Phys. Rept. 668, 1 (2016).
[3] S. L. Olsen, T. Skwarnicki and D. Zieminska, Rev. Mod. Phys. 90, 015003 (2018).
[4] F. K. Guo, C. Hanhart, Ulf-G. Meiβner, Q. Wang, Q. Zhao and B. S. Zou, Rev. Mod. Phys. 90, 015004 (2018).
[5] Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019).
[6] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003).
[7] D. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. 93, 072001 (2004).
[8] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 162002 (2004).
[9] R. L. Jaffe, Phys. Rev. D 15, 281 (1977).
[10] I. M. Barbour and D. K. Ponting, Z. Phys. C 5, 221 (1980); I. M. Barbour and J. P. Gilchrist, Z. Phys. C 7, 225 (1981) Erratum: [Z. Phys. C 8, 282 (1981)].
[11] J. D. Weinstein and N. Isgur, Phys. Rev. D 27, 588 (1983).
[12] B. Silvestre-Brac and C. Semay, Z. Phys. C 57, 273 (1993).
[13] D. M. Brink and F. Stancu, Phys. Rev. D 57, 6778 (1998).
[14] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71, 014028 (2005).
[15] N. Barnea, J. Vijande and A. Valcarce, Phys. Rev. D 73, 054004 (2006).
[16] E. Santopinto and G. Galatà, Phys. Rev. C 75, 045206 (2007).
[17] D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha, Phys. Rev. D 76, 114015 (2007); D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Atom. Nucl. 72, 184 (2009).
[18] C. Deng, J. Ping and F. Wang, Phys. Rev. D 90, 054009 (2014).
[19] L. Zhao, W. Z. Deng and S. L. Zhu, Phys. Rev. D 90, 094031 (2014).
[20] Q. F. Lü and Y. B. Dong, Phys. Rev. D 94, 074007 (2016).
[21] M. N. Anwar, J. Ferretti, F. K. Guo, E. Santopinto and B. S. Zou, Eur. Phys. J. C 78, 647 (2018).
[22] M. N. Anwar, J. Ferretti and E. Santopinto, Phys. Rev. D 98, 094015 (2018).
[23] M. A. Bedolla, J. Ferretti, C. D. Roberts and E. Santopinto, Spectrum of fully-heavy tetraquarks from a diquark-antidiquark perspective, arXiv:1911.00060.
[24] G. Yang, J. Ping and J. Segovia, Doubly-heavy tetraquarks, arXiv:1911.00215.
[25] J. Ferretti and E. Santopinto, Hidden-charm and bottom tetra- and pentaquarks with strangeness in the hadro-quarkonium and compact tetraquark models, arXiv:2001.01067.
[26] J. D. Weinstein and N. Isgur, Phys. Rev. D 41, 2236 (1990).
[27] A. V. Manohar and M. B. Wise, Nucl. Phys. B 399, 17 (1993).
[28] N. A. Törnqvist, Z. Phys. C 61, 525 (1994); Phys. Lett. B 590, 209 (2004).
[29] K. Martins, D. Blaschke and E. Quack, Phys. Rev. C 51, ...
