Current-induced instability of superfluid Fermi gases in optical lattices

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Abstract. We study the stability of superfluid flow of two-component Fermi gases in one-dimensional optical lattices. We find that the density fluctuation mode, the so-called Anderson-Bogoliubov mode, has the roton-like structure as seen in superfluid ⁴He. With increasing supercurrent, one of the roton-like minima reaches zero before the pair breaking occurs. This means that the instability of the superfluid Fermi gas is due to the spontaneous emission of the roton-like excitations of the Anderson-Bogoliubov mode instead of due to the Cooper pair breaking. We calculate the critical velocity determined by the roton-like structure for one-dimensional optical lattices.

1. Introduction
The realization of the BCS-BEC crossover in Fermi gases with a Feshbach resonance [1] has attracted much attention from both theorists and experimentalists. In particular, one of the most interesting properties of this system is the superfluidity characterized by dissipationless flow. Recently, Miller et al. investigated the superfluidity of Fermi gases in the BCS-BEC crossover region by using a moving optical lattice and measured the superfluid critical velocity, above which the system becomes unstable due to the spontaneous creation of excitations, for different values of the interatomic interaction and the lattice depth [2].

In order to investigate theoretically the stability of superfluid flow, the analysis of the excitation is of essential importance [3]. The instability of superfluid flow is signified by the appearance of excitations with negative energies. Two branches in the excitation spectrum are thought to be responsible for the breakdown of superfluid flow [4, 5]. One is the branch of the single-particle excitations, which corresponds to the energy required to break Cooper pairs. The other is the branch for the density fluctuation mode, the so-called Anderson-Bogoliubov (AB) mode, whose long-wavelength component is the Goldstone mode associated with the spontaneous breaking of U(1) gauge symmetry. In previous work, it was suggested that the instability of a superfluid Fermi gas of the BCS type in a moving optical lattice is due to the anomaly in the single-particle excitations [2, 4].
In this paper, focusing on the BCS region, we investigate the instability of superfluid flow of Fermi gases. Since the experiment at MIT [2] has been done for one-dimensional (1D) optical lattices, we restrict ourselves within 1D problem in the following sections. We apply the generalized random phase approximation (GRPA) [6] to the attractive Hubbard model in order to calculate the excitation spectra in the all-wavelength region. We show that in the absence of supercurrents the AB mode has roton-like minima and that with increasing the supercurrent one of the roton-like minima reaches zero before the single-particle excitation does, i.e., before the pair breaking occurs. This means that the critical velocity of superfluid Fermi gases in one-dimensional optical lattices is determined by the roton-like excitations of the AB mode instead by the single-particle excitations.

2. Formalism

We consider an atomic Fermi gas of two hyperfine states with contact interaction loaded into an optical lattice. The two hyperfine states are described by pseudospins $\sigma = \uparrow, \downarrow$. The population of each hyperfine state is assumed to be equal. We also assume that the filling factor, namely the number of atoms in each hyperfine state per site, is smaller than unity, and that the lattice potential is sufficiently deep such that the tight-binding approximation is valid. The system is well described by the single-band Hubbard model as

$$H = -J \sum_{\sigma} \left( c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) + U \sum_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow}$$

where $c_{j\sigma}$ and $c_{j\sigma}^\dagger$ are creation and annihilation operators of fermions with pseudospin $\sigma$ on the $j$-th site, $J$ is the hopping matrix element between nearest-neighbor sites, and $U$ is the on-site interaction. Here, we assume that the atomic interaction is attractive, i.e., $U$ is negative. We set $\hbar = 1$ from now on.

We calculate the single-particle excitation spectrum in the presence of supercurrents within the BCS mean-field approximation. If the lattice potential is moving with a constant velocity $-v$ or equivalently the uniform supercurrent flows, the center of mass of Cooper pairs has a quasimomentum $2mv$ in the coordinate system where the lattice is static. In this situation, the superfluid gap is given by

$$\Delta_v = -\frac{U}{M} \sum_k (c_{-k+mv\downarrow} c_{k+mv\uparrow})$$

where $M$ is the number of lattice sites. By diagonalizing the mean-field Hamiltonian, we obtain the single-particle excitation spectrum as

$$E_v(q) = 2J \sin(mvd) \sin(qd) + \sqrt{\xi_v(q)^2 + \Delta_v^2}$$

where $\xi_v(q) = 2J(1 - \cos(mvd) \cos(qd)) - \mu$, and $d$ is the lattice spacing. The gap $\Delta_v$ and the chemical potential $\mu$ are determined by solving the system of the gap equation and the number equation self-consistently. The single-particle excitation spectrum reaches zero at certain $q$ when the pair breaking occurs. Thus, the velocity $v_{pb}$ at which the instability due to the pair breaking sets in is determined from the condition $E_v=v_{pb}(q) = 0$ [7].

In order to examine the stability of superfluid Fermi gases, we also calculate the spectrum of density fluctuation mode, which is called the AB mode, by the GRPA [6]. The spectrum of the AB mode is obtained from the poles of density response function $\chi(q, \omega)$ [6]. We numerically calculate the dynamic structure factor $S(q, \omega) = -(1/\pi) \text{Im} \chi(q, \omega)$ to investigate the excitation spectra instead of $\chi(q, \omega)$. $S(q, \omega)$ can be measured in experiments by using, e.g., Bragg spectroscopy [8].
3. Results

In Fig. 1, we show the dynamic structure factors of superfluid Fermi gases in 1D lattices for (a) current-free and (b) current-carrying cases. There the upper continuum and the lower curve (sharp peaks of the dynamic structure factor) correspond to the single-particle excitations and the AB mode, respectively. It is clearly seen that in the long-wavelength limit ($qd \ll 1$) the AB mode is phonon-like, namely gapless and linearly dispersing, reflecting the U(1) gauge symmetry breaking. The dispersion relation for $qd \ll 1$ is analytically given by

$$\omega = 4J N_0^2 Ud^2 \cos(k_F d) \sin(mvd)q$$

$$+ \sqrt{(1 + 2N_0^2 Ud)\{v_F^2 - (4J^2 d^2(1 - 2N_0^2 Ud) - 2N_0^2 Ud v_F^2)\sin^2(mvd)\}|q|},$$

(4)

where $N_0^2 = (2\pi v_F \cos(mvd))^{-1}$. When $v = 0$, Eq. (4) reduces to a well-known formula $\omega = v_F \sqrt{1 + Ud/(\pi v_F^2)}|q|$ obtained by Anderson [9]. As $v$ increases, the slope of the AB phonon dispersion for $q < 0$ gradually decreases. Then, the excitation energy of the AB mode in the long-wavelength limit becomes negative at a certain velocity $v_0$. However, as long as the BCS region is considered, $v_0$ is always larger than $v_{pb}$. Hence, the phonons are not relevant to the critical velocity.

As seen in Fig. 1(a), where the supercurrent is absent, the AB mode has roton-like minima at $|q| \approx 2k_F$. Such roton-like structure has been found also in uniform 1D Fermi gases, and it reflects the tendency towards the formation of density wave order originating from the nesting of the Fermi surface [10]. As $v$ increases, one of the roton-like minima approaches zero; it reaches zero before the single-particle excitation spectrum does, i.e., before the pair breaking occurs (see Fig. 1(b)). The critical velocity $v_c$ is determined as the velocity at which the roton-like minimum reaches zero. Accordingly, when $v_c < v < v_{pb}$, the instability of superfluid flow can be attributed to the spontaneous emission of the roton-like excitations of the AB mode instead of the single-particle excitations. In Fig. 2, the velocity $v_{pb}$ and the critical velocity $v_c$ are shown as functions of $U/J$. One clearly sees that $v_c$ is smaller than $v_{pb}$ and that both $v_c$ and $v_{pb}$ increases monotonically with increasing $|U|/J$. The difference between $v_c$ and $v_{pb}$ also grows with $|U|/J$.

![Figure 1](image_url)

Figure 1. The dynamic structure factor $S(q, \omega)$ for quarter filling and $U = -2.0J$. (a) $S(q, \omega)$ for $v = 0$ is shown, where $|\Delta_v| = 0.4087J$ and $\mu = 0.6241J$. (b) $S(q, \omega)$ for $v = 0.21/(md) \approx v_c$ is shown, where $|\Delta_v| = 0.4196J$ and $\mu = 0.6553J$. 


4. Conclusion
In summary, we have studied the instability of two-component Fermi gases in the BCS superfluid region in moving one-dimensional (1D) optical lattices. We analyzed the attractive Hubbard model by means of the generalized random phase approximation, and calculated the density response function. The excitation spectra have been extracted from the dynamic structure factors. We have shown that the Anderson-Bogoliubov (AB) mode has the roton-like structure at \(|q| \approx 2k_F\). In addition, as the lattice velocity is increased to a certain value, the instability accompanied with the roton-like excitations was found to occur even before the pair-breaking destabilizes the system.

While we have focused on 1D optical lattices in the present paper, Koponen et al. showed that the AB mode in the current-free case is well below the single-particle excitation spectrum in three dimensional (3D) lattices [11]. Hence, the instability due to the AB mode with short wavelength is expected to occur in higher dimensions. Moreover, given that recent experiments regarding Bose gases loaded into 3D optical lattices have measured the superfluid critical velocity [12], the stability of Fermi gases in moving 3D lattices may be also studied in future experiments. All the circumstances indicate that it will be important to extend the analyses in the present paper to higher dimensions.

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