Predictions for nonleptonic $\Lambda_b$ and $\Theta_b$ decays

Adam K. Leibovich, Zoltan Ligeti, Iain W. Stewart, and Mark B. Wise

1Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh PA 15260
2Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley CA 94720
3Center for Theoretical Physics, Massachusetts Institute for Technology, Cambridge, MA 02139
4California Institute of Technology, Pasadena, CA 91125

Abstract

We study nonleptonic $\Lambda_b \to \Sigma_c \pi$, $\Sigma_c \pi$, and $\Sigma_c^* \pi$ decays in the limit $m_b, m_c, E_\pi \gg \Lambda_{QCD}$ using the soft-collinear effective theory. Here $\Sigma_c = \Sigma_c(2455)$ and $\Sigma_c^* = \Sigma_c(2520)$. At leading order the $\Lambda_b \to \Sigma_c^{(*)} \pi$ rates vanish, while the $\Lambda_b \to \Lambda_c \pi$ rate is related to $\Lambda_b \to \Lambda_c \ell \nu$, and is expected to be larger than $\Gamma(B \to D^{(*)} \pi)$. The dominant contributions to the $\Lambda_b \to \Sigma_c^{(*)} \pi$ rates are suppressed by $\Lambda_{QCD}^2/E_\pi^2$. We predict $\Gamma(\Lambda_b \to \Sigma_c^{(*)} \pi)/\Gamma(\Lambda_b \to \Sigma_c \pi) = 2 + \mathcal{O}(\Lambda_{QCD}/m_Q, \alpha_s(m_Q))$, and the same ratio for $\Lambda_b \to \Sigma_c^{(*)} \rho$ and for $\Lambda_b \to \Xi_c^{(*)} K$. “Bow tie” diagrams are shown to be suppressed. We comment on possible discovery channels for weakly decaying pentaquarks, $\Theta_{b,c}$ and their nearby heavy quark spin symmetry partners, $\Theta_{b,c}^*$. 

*Electronic address: akl2@pitt.edu
†Electronic address: ligeti@lbl.gov
‡Electronic address: iains@mit.edu
§Electronic address: wise@theory.caltech.edu
Heavy baryon decays are interesting for many reasons. Heavy quark symmetry is more predictive in semileptonic $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ decay than in $B \to D^{(*)} \ell \bar{\nu}$, and may eventually give a determination of $|V_{ub}|$ competitive with meson decays. In this paper we concentrate on the more complicated case of nonleptonic $b \to c \bar{u} d$ baryon transitions, as shown in Table I. These channels provide a testing ground for our understanding of QCD in nonleptonic decays. Our analysis is based on heavy quark symmetry and the soft-collinear effective theory (SCET).

There is considerable experimental interest in these decays. Recently the CDF Collaboration measured

$$\frac{f_{\Lambda_b}}{f_d} \frac{B(\Lambda_b \to \Lambda_c^+ \pi^-)}{B(B^0 \to D^+ \pi^-)} = 0.66 \pm 0.11_{\text{(stat)}} \pm 0.09_{\text{(syst)}} \pm 0.18_{\text{(BR)}},$$

where $f_{\Lambda_b}$ and $f_d$ are the fragmentation fractions of $b$ quarks to $\Lambda_b$ and $B^0$, respectively. Using the input $f_{\text{baryon}}/f_d = 0.304 \pm 0.053$, CDF quoted $B(\Lambda_b \to \Lambda_c^+ \pi^-)/B(B^0 \to D^+ \pi^-) \simeq 2.2$. The measured lifetimes, $\tau(\Lambda_b) = 1.23 \text{ ps}$ and $\tau(B^0) = 1.54 \text{ ps}$, then imply that $\Gamma(\Lambda_b \to \Lambda_c^+ \pi^-)/\Gamma(B^0 \to D^+ \pi^-) \simeq 2.7$. More experimental results on semileptonic and other nonleptonic $\Lambda_b$ decays are expected in the near future.

The part of the weak Hamiltonian relevant for this paper is

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} V^*_{ud} \left[ C_1(m_b) O_1(m_b) + C_2(m_b) O_2(m_b) \right],$$

where $C_i(m_b)$ are the coefficients of the operators $O_i(m_b)$.

| Notation | $s_f$ | $I(J^P)$ | Mass (MeV) | Decays considered |
|----------|------|--------|-----------|------------------|
| $\Lambda_c, \Lambda_b$ | 0 | 0 $(1^+)$ | 2285, 5624 | $\Lambda_b \to \Lambda_c^+ \pi^-$ |
| $\Sigma_c = \Sigma_c(2455)$ | 1 | 1 $(1^+)$ | 2452 | $\Lambda_b \to \Sigma_c^+ \pi^-, \Sigma_c^0 \pi^0, \Sigma_c^0 \rho^0$ |
| $\Sigma_c^* = \Sigma_c(2520)$ | 1 | 1 $(3^+)$ | 2517 | $\Lambda_b \to \Sigma_c^{*+} \pi^-, \Sigma_c^{*0} \pi^0, \Sigma_c^{*0} \rho^0$ |
| $\Xi_c, \Xi_c'$ | 0, 1 | $1/2 (1^+)$ | 2469, 2576 | $\Lambda_b \to \Xi_c^0 K^0$ |
| $\Xi_c^* = \Xi_c(2645)$ | 1 | $1/2 (3^+)$ | 2646 | $\Lambda_b \to \Xi_c^{*0} K^0$ |
| $\Theta_c, \Theta_b$ | 1 | 0 $(1^+)$ | $m_{\Theta_c}, m_{\Theta_b}$ | $\Theta_b^+ \to \Theta_c^0 \pi^+, \Theta_c^+ \to \Theta_c^0 - \Theta_c^- \pi^-$ |
| $\Theta_c^*, \Theta_b^*$ | 1 | 0 $(3^+)$ | $\sim m_{\Theta_c} + 70, \sim m_{\Theta_b} + 22$ | $\Theta_c^* \to \Theta_c \gamma$ or strongly $\Theta_b^* \to \Theta_b \gamma$ |

TABLE I: The decays considered in this paper. Here $s_f$ is the spin of the light degrees of freedom. The mass shown for the $\Sigma_c^{(*)}$ is the average of the charge 0 and +1 states, and for the $\Xi_c$'s the mass is the average in the doublet. The stability of the pentaquark states $\Theta_Q (= \bar{Q}udud)$ and their value of $s_f$ are both conjectures.
where both the Wilson coefficients, \( C_i \), and the four-quark operators

\[
O_1 = (\bar{c} \gamma_\mu P_L b) (\bar{d} \gamma^\mu P_L u), \quad O_2 = (\bar{d} \gamma_\mu P_L b) (\bar{c} \gamma^\mu P_L u),
\]

depend on the renormalization scale which we take to be \( m_b \), and \( P_L = (1 - \gamma_5)/2 \). The combination \( [C_1(m_b) + C_2(m_b)/3] |V_{ud}| \) is very close to unity.

Weak nonleptonic decays are sometimes characterized by diagrams corresponding to different Wick contractions. As shown in Fig. 1, there are more possibilities in baryon than in meson decays. In particular, a “Bow tie” contraction is unique to baryons. The color structure for baryons also differs from mesons: we find that the \( C \) diagram is of the same order in the large \( N_c \) limit as the \( T \) diagram.\(^1\) Nonleptonic meson decay amplitudes are sometimes estimated using naive factorization, which would set \( \langle \Lambda_c \pi | O_1 | \Lambda_b \rangle = \langle \Lambda_c | \bar{c} \gamma_\mu P_L b | \Lambda_b \rangle \langle \pi | \bar{d} \gamma_\mu P_L u | 0 \rangle \). In baryon decays the extra light quark implies that this procedure is ill-defined for all but the tree diagram. In naive factorization the \( \Lambda_b \to \Sigma_c^{(*)} \pi \) decays are very suppressed, since the \( T \) contribution vanishes separately in the isospin and heavy quark limits \(^2\) (just like the semileptonic \( \Lambda_b \to \Sigma_c^{(*)} \ell \nu \) decays), the \( C \) contribution vanishes after doing a Fiertz transformation on the four-quark operator, and the \( E \) and \( B \) amplitudes are identically zero since the \( u \) and \( b \) fields are in different quark bilinears.

In this letter we show that more rigorous techniques can still be applied to make reasonable predictions for all these decays. By expanding in \( m_b, m_c, E_\pi \gg \Lambda_{QCD} \) we show that for \( \Lambda_b \to \Lambda_c^{(*)} \pi^- \) the amplitudes corresponding to the diagrams in Fig. 1 satisfy \( T \gg C \sim E \gg B \), and we find that the experimental result in Eq. 1 is consistent with

\(^1\) If we treated the \( N_c - 3 \) additional quarks in the baryons as flavors that are sterile under the weak interaction then color-commensurate would become color-suppressed.

\(^2\)
theoretical expectations. Next we consider \( \Lambda_b \to \Sigma_c^{(*)} \pi \) decays, and show the leading contributions to these nonleptonic rates are suppressed by \( \Lambda_{QCD}^2/E_{\pi}^2 \), much like in \( B^0 \to D^0\pi^0 \). Using heavy quark symmetry we derive a relation between the decay rate to \( \Sigma_c \) and \( \Sigma_c^* \) and comment on decays to \( \Xi_c \). Finally we consider the detection of possible weakly decaying heavy pentaquarks, \( \Theta_b \) and \( \Theta_c \), with nonleptonic decays.

The proof of factorization at leading order for \( \Lambda_b \to \Lambda_c \pi \) decay follows closely that for \( B^0 \to D^{(*)+}\pi^- \), so we do not review it here. In this case the nonperturbative expansion parameter for SCET is \( \Lambda = \Lambda_{QCD}/E_{\pi} \). Since \( E_{\pi} \) is set by the bottom and charm quark masses, we take this to be of the same order as the expansion parameter for the heavy quark effective theory (HQET), i.e., \( \Lambda \sim \Lambda_{QCD}/m_Q \) (\( Q = c, b \)). Working at leading order in \( \lambda \) and \( \alpha_s(m_b) \) and neglecting the pion mass, the \( \Lambda_b \to \Lambda_c \pi \) matrix element factorizes in the standard way,

\[
\langle \Lambda_c(v', s') | H_W | \Lambda_b(v, s) \rangle = \sqrt{2} G_F \left( C_1 + \frac{C_2}{3} \right) V_{cb} V_{ud}^* f_\pi E_{\pi} \langle \Lambda_c(v', s') | \bar{c} \gamma_{\nu} P_L b | \Lambda_b(v, s) \rangle,
\]

where \( f_\pi = 131 \text{ MeV} \) is the pion decay constant, \( n \) is a light-like four-vector along the direction of the pion’s four-momentum, \( p_\pi^\mu = E_{\pi} n^\mu \), and the four-velocities of the \( \Lambda_b \) and \( \Lambda_c \) are \( v \) and \( v' \), respectively. Perturbative corrections induce a multiplicative factor in Eq. (4), \( \langle T(x) \rangle_\pi = \int_0^1 dx T(x) \phi_\pi(x) \), where \( T(x) \) is computable and \( \phi_\pi \) is the nonperturbative light-cone pion distribution function \([10, 11]\), and a term proportional to the matrix element of \( \bar{c} \gamma_{\nu} P_R b \). At leading order in \( \alpha_s(m_Q) \), we can set \( \langle T(x) \rangle_\pi = 1 \) and the term involving \( \bar{c} \gamma_{\nu} P_R b \) to 0. This implies that the nonleptonic rate is related to the semileptonic differential decay rate at maximal recoil,

\[
\Gamma(\Lambda_b \to \Lambda_c \pi) = \frac{3\pi^2(C_1 + C_2/3)^2 |V_{ud}|^2 f_\pi^2}{m_{\Lambda_b}^2 r_\Lambda} \left( \frac{d\Gamma(\Lambda_b \to \Lambda_c \ell \bar{\nu})}{dw} \right) |_{w_{\text{max}}},
\]

where \( r_\Lambda = m_{\Lambda_c}/m_{\Lambda_b} \), \( w = v \cdot v' = (m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2)/(2m_{\Lambda_b} m_{\Lambda_c}) \), and \( w_{\text{max}} \) corresponds to \( q^2 = m_{\pi}^2 (\approx 0) \).

The semileptonic \( \Lambda_b \to \Lambda_c \ell \bar{\nu} \) form factors are

\[
\langle \Lambda_c(p', s') | V_{\mu} | \Lambda_b(p, s) \rangle = \bar{u}(p', s') \left[ f_1 \gamma_{\mu} + f_2 v_{\mu} + f_3 v_{\mu}' \right] u(p, s),
\]

\[
\langle \Lambda_c(p', s') | A_{\mu} | \Lambda_b(p, s) \rangle = \bar{u}(p', s') \left[ g_1 \gamma_{\mu} + g_2 v_{\mu} + g_3 v_{\mu}' \right] \gamma_5 u(p, s),
\]

where the \( f_i \) and \( g_i \) are functions of \( w \), and the relevant currents are \( V_\nu = \bar{c} \gamma_{\nu} b \) and \( A_\nu = \bar{c} \gamma_{\nu} \gamma_5 b \). The spinors are normalized to \( \bar{u}(p, s) \gamma^\mu u(p, s) = 2p^\mu \). In the heavy quark limit,

\[
\zeta(w) = f_1(w) = g_1(w),
\]
0 = f_2(w) = f_3(w) = g_2(w) = g_3(w), \quad (7)

where ζ(w) is the Isgur-Wise function for ground state baryons. The differential decay rate is given by

\[
\frac{d\Gamma(\Lambda_b \to \Lambda_c \ell \bar{\nu})}{dw} = \frac{G_F^2 m_{\Lambda_b}^5 |V_{cb}|^2}{24 \pi^3} r_\Lambda^3 \sqrt{w^2 - 1} \left[ 6w + 6wr_\Lambda - 4r_\Lambda - 8r_\Lambda w^2 \right] F_\Lambda^2(w), \quad (8)
\]

where in the \(m_Q \gg \Lambda_{QCD} \) limit \(F_\Lambda(w)\) is equal to the Isgur-Wise function, \(ζ(w)\), and in particular \(F_\Lambda(1) = 1\). In terms of the original form factors

\[
F_\Lambda(w)^2 = \left[ 6w + 6wr_\Lambda - 4r_\Lambda - 8r_\Lambda w^2 \right]^{-1} \left\{ (w - 1) \left[ (1 + r_\Lambda) f_1 + (w + 1)(r_\Lambda f_2 + f_3) \right]^2 
+ (w + 1) \left[ (1 - r_\Lambda) g_1 - (w - 1)(r_\Lambda g_2 + g_3) \right]^2 
+ 2(1 - 2r_\Lambda w + r_\Lambda^2) \left[ (w - 1)f_1^2 + (w + 1)g_1^2 \right] \right\}. \quad (9)
\]

Combining the above results for \(\Lambda_b \to \Lambda_c^+ \pi^-\) decay with the analogous ones for \(\bar{B}^0 \to D^{(*)+} \pi^-\) we find that

\[
\frac{\Gamma(\Lambda_b \to \Lambda_c \pi^-)}{\Gamma(\bar{B}^0 \to D^{(*)+} \pi^-)} = \frac{8m_{\Lambda_b}^3 (1 - r_\Lambda^2)^3 r_{D^{(*)}}}{m_B^3 (1 - r_{D^{(*)}}^2)^3 (1 + r_{D^{(*)}})^2} \left( \frac{ζ(w_{\text{max}}^\Lambda)}{ξ(w_{\text{max}}^{D^{(*)}})} \right)^2 \quad (10)
\]

where ξ is the Isgur-Wise function for \(B \to D^{(*)}\) semileptonic decay, and \(r_{D^{(*)}} = m_{D^{(*)}}/m_B\). When the \(\Lambda_b \to \Lambda_c^+ \ell^- \bar{\nu}\) rate is measured, one can directly test factorization using Eq. (5) or Eq. (10). In the absence of this data, we have to resort to using model predictions for the baryon Isgur-Wise function. If the ratio of Isgur-Wise functions in Eq. (10) is unity then the prefactor in Eq. (10) implies that \(\Gamma(\Lambda_b \to \Lambda_c \pi^-)/\Gamma(\bar{B}^0 \to D^{(*)+} \pi^-) = 1.6(1.8)\). This enhancement is in rough agreement with the data in Eq. (10). A similar result also follows from the small velocity limit \(m_Q \gg m_b - m_c \gg \Lambda_{QCD}\), in which the nonleptonic rates satisfy \(\Gamma(\Lambda_b \to \Lambda_c \pi) : \Gamma(B \to D^*\pi) : \Gamma(B \to D\pi) = 2 : 1 : 1\), while for the semileptonic rates \(\Gamma(\Lambda_b \to \Lambda_c \ell \bar{\nu}) : \Gamma(B \to D^*\ell \bar{\nu}) : \Gamma(B \to D\ell \bar{\nu}) = 4 : 3 : 1\).

The large \(N_c\) limit provides some support for the ratio of baryon to meson Isgur-Wise functions being close to unity at maximal recoil. In the large \(N_c\) limit the heavy baryons can be treated as bound states of chiral solitons and mesons containing a heavy quark. In this picture, the baryon Isgur-Wise function, \(ζ(w)\), is predicted to be \(ζ(w) = 0.99 e^{-1.3(w-1)} \) \[2\]

\[2\] Updating the parameters by fitting the mass splitting to give \(κ = (0.411 \text{ GeV})^3\), and using \(m_N = \bar{A} = 0.8 \text{ GeV}\) (instead of \(M_N\)) for the mass of the light degrees of freedom leaves the exponent essentially unchanged.
This gives $\zeta(w_{\text{max}}^A = 1.4) = 0.57$, which is indeed close to $\xi(w_{\text{max}}^{B^0} = 1.5) \simeq 0.55$\,\cite{13}. Using this model for $\zeta(w)$, $|V_{cb}| = 0.04$, $\tau_{\Lambda_b} = 1.23$ ps, and Eqs. (5) and (8) yield the prediction that $\mathcal{B}(\Lambda_b \to \Lambda_c^+\pi^-) = 4.6 \times 10^{-3}$. As expected, this is larger than $\mathcal{B}(\mathcal{B} \to D^{(s)}+\pi^-) \simeq 2.7 \times 10^{-3}$.

However, the uncertainty in this prediction is quite large, particularly given that large $N_c$ strictly only applies for $w$ near 1. The same large $N_c$ inputs predict $\mathcal{B}(\Lambda_b \to \Lambda_c^+\ell^-\nu) \approx 6\%$, i.e., this channel is expected to make up a large part of the inclusive $\Lambda_b \to X_c\ell^-\nu$ rate, with the $s_1^P = 1^-$ excited $\Lambda_c$ states making up a significant fraction of the remainder\,\cite{14}.

Order $\Lambda_{QCD}/m_Q$ corrections to these predictions may be significant. The $\Lambda_b \to \Lambda_c^+\pi^-$ amplitude receives contributions from the $T$, $C$, $E$, and $B$ classes of diagrams in Fig.\,\cite{11} In SCET, $|E/T|$ and $|C/T|$ are of order $\Lambda_{QCD}/m_Q$\,\cite{15}, and we will show later that $|B/T|$ is further suppressed. In $B \to D\pi$ decay we know from $\mathcal{B}(B^- \to D^0\pi^-)/\mathcal{B}(\mathcal{B} \to D^+\pi^-) \simeq 1.8$\,\cite{16} that $\Lambda_{QCD}/m_Q$ corrections affect the amplitudes at the $15 - 30\%$ level. In particular $|A(\mathcal{B}^0 \to D^+\pi^-)| = |T + E| = (5.9 \pm 0.3) \times 10^{-7}$ GeV and $|A(B^- \to D^0\pi^-)| = |T + C| = (7.7 \pm 0.3) \times 10^{-7}$ GeV. The ratio of these amplitudes can be reproduced by a power correction involving a hadronic parameter $|s_{\text{eff}}| \simeq 430$ MeV, which is of natural size\,\cite{15}. Since $B_s \to D_s^-\pi^+$ only has a $T$ contribution, accurate measurement of this rate will improve our understanding of the size of $E$ and $C$. CDF recently measured $[f_s \mathcal{B}(B_s \to D_s^-\pi^+)]/[f_d \mathcal{B}(B^0 \to D^-\pi^+)] = 0.35 \pm 0.05^{(\text{stat})} \pm 0.04^{(\text{syst})} \pm 0.09^{(\text{BR})}$\,\cite{16}, and using $f_s/f_d = 0.26 \pm 0.03$ yields $\mathcal{B}(B_s \to D_s^-\pi^+)/\mathcal{B}(B^0 \to D^-\pi^+) \simeq 1.35$. Neglecting $SU(3)$ breaking\,\cite{3} this implies $|A(B_s \to D_s^-\pi^+)| = |T| = (7.3 \pm 1.5) \times 10^{-7}$ GeV and that $|C|$ and $|E|$ may be comparable. The errors are still too large to draw any definite conclusions.

Now we turn to $\Lambda_b \to \Sigma_c\pi$ decays. As shown in Table\,\cite{11} there are two $\Sigma_c$ states with different spin which we refer to as $\Sigma_c$ and $\Sigma^*_c$. They form a heavy quark spin symmetry doublet with the spin and parity of the light degrees of freedom, $s_1^{\pi^0} = 1^+$. Under isospin, the $\Lambda_b$ is $I = 0$, the $\Sigma_c^{(*)}$ is $I = 1$, and the Hamiltonian is $I = 1$, so the $\Sigma_c^{(*)}\pi$ final state must be $I = 1$ (it can not be $I = 0$ or 2). Therefore the rates to the two different charge channels are equal,

$$\Gamma(\Lambda_b \to \Sigma_c^{(\ast)0}\pi^0) = \Gamma(\Lambda_b \to \Sigma_c^{(\ast)+}\pi^-).$$

Based on $B$ decay data and the SCET power counting, we expect $\Gamma(\Lambda_b \to \Sigma_c^{(\ast)}\pi)$ to be up to about an order of magnitude smaller than $\Gamma(\Lambda_b \to \Lambda_c\pi)$, since the leading contributions

---

3 In the heavy quark limit of the $T$ amplitude $SU(3)$ will be tested by the measurement of $B_s \to D_s\ell\nu$. 
FIG. 2: Contributions in SCET I to $\Lambda_b \to \Sigma_b^{+}\pi^-$ [(a) and (b)], and to $\Sigma_b^{0}\pi^0$ [(c) and (d)]. Solid dots denote insertions of the suppressed usoft-collinear Lagrangian, $L_{\xi q}^{(1)}$, the double lines are heavy quarks, the dashed lines are collinear quarks, the solid lines are usoft quarks, and the “looped lines” are collinear gluons. The nonleptonic weak vertex is denoted by $\otimes$.

to $\Lambda_b \to \Sigma_b^{(*)}\pi$ are power suppressed.

Again, we use SCET to expand in $\Lambda_{QCD}/m_Q$, $\Lambda_{QCD}/E_\pi$, and $\alpha_s(m_Q)$, keeping only the leading terms that cause the $\Lambda_b \to \Sigma_b^{(*)}\pi$ transitions. These come from the $C$ and $E$ diagrams in Fig. 1 and their contributions can be studied following the analysis of $\bar{B}^0 \to D^{(*)0}\pi^0$ in Ref. [15]. The leading diagrams in SCET I that determine the matching onto power suppressed operators are shown in Fig. 2. To match the $C$ and $E$ diagrams, two insertions of the mixed usoft-collinear Lagrangian, $L_{\xi q}^{(1)}$ [17], is required, each yielding a suppression of $\sqrt{\Lambda_{QCD}/E_\pi}$. This yields the power counting that $|C/T|$ and $|E/T|$ are $\mathcal{O}(\Lambda_{QCD}/E_\pi)$. In contrast, matching the $B$ diagram in Fig. 1 requires four insertions of $L_{\xi q}^{(1)}$ (or other higher dimensional terms in the Lagrangian), and $B$ is therefore power suppressed compared to $C$ and $E$ by at least an additional $\Lambda_{QCD}/E_\pi$.

In addition there is a further matching onto SCET II. The resulting matrix element involves soft and collinear operators which factor [15]. The matrix element of the weak Hamiltonian, $\langle \Sigma_b^{0}(v', s')\pi^0 |H_W| \Lambda_b(v, s) \rangle$, can be written (neglecting $\alpha_s(m_Q)$ corrections) as a convolution integral of a jet function, $J(x, k_1^+, k_2^+)$, with the matrix element involv-

---

4 Since the messenger modes from Ref. [18] do not spoil factorization for cases with a product of color singlet soft and collinear operators, we can neglect them in our analysis.
ing the collinear fields, \( \langle \pi | O_c(x) | 0 \rangle \) which gives \( \phi_\pi(x) \), and that involving the soft fields, \( \langle \Sigma_c(v', s') | O_s(k^+_j) | \Lambda_b(v, s) \rangle \). In what follows we only need the form of the soft operator \[ O_s(k^+_j) = \left( \bar{h}^{(c)}_{v'}(S) \not\! P_L (S^\dagger h^{(b)}_v) \right) \left( \not\! d \right)_{k^+_j} \not\! P_L (S^\dagger u)_{k^+_j}, \] (12)

where \( h^{(Q)}_v \) is an HQET heavy quark field, \( S \) is a soft Wilson line, and the subscripts denote the momentum carried by the fields. For our purposes the most important aspect of the analysis is that \( O_s \) only involves the the combination \( \bar{h}^{(c)}_{v'} \not\! P_L h^{(b)}_v \). Thus, by heavy quark symmetry

\[
\langle \Sigma_c(v', s') | O_s | \Lambda_b(v, s) \rangle = \frac{1}{\sqrt{3}} \bar{u}_{\Sigma_c}(v', s') (\gamma^\mu - v'^\mu) \gamma_5 \not\! P_L u_{\Lambda_b}(v, s) X_\mu, \\
\langle \Sigma_c^*(v', s') | O_s | \Lambda_b(v, s) \rangle = \bar{u}_{\Sigma_c^*}(v', s') \not\! P_L u_{\Lambda_b}(v, s) X_\mu, \tag{13}
\]

where \( v \) and \( v' \) are the four-velocities of the \( \Lambda_b \) and \( \Sigma_c^{(*)} \) respectively. The spinor field normalizations are \( \bar{u}(v, s) u(v, s) = 1 \) for the \( \Lambda_b \) and \( \Sigma_c \), and \( \bar{u}_\alpha(v, s) u^\alpha(v, s) = -1 \) for the \( \Sigma_c^* \). \( X_\mu \) is the most general vector compatible with the symmetries of QCD,

\[
X_\mu = a n_\mu + b v_\mu + c v'_\mu. \tag{14}
\]

Note that in Eq. (12) the part of \( O_s \) involving the light quark fields is parity violating, so we need not worry about the fact that \( \Lambda_b \to \Sigma_c \) is an “unnatural” transition. Using \( m_{\Lambda_b} v = m_{\Sigma_c} v' + E_\pi n \) to eliminate the term proportional to \( v_\mu \) in Eq. (14), it is easy to see that any term in \( X_\mu \) proportional to \( v'_\mu \) does not contribute, so only \( n_\mu \) remains. Hence the ratio of the rates for \( \Lambda_b \to \Sigma_c \pi \) and \( \Lambda_b \to \Sigma_c^* \pi \) are determined model independently at leading order in \( \Lambda_{QCD}/m_Q \) and \( \alpha_s(m_Q) \), similar to the \( B^0 \to D^{(*)0} \pi^0 \) case. We find

\[
\frac{\Gamma(\Lambda_b \to \Sigma_c^* \pi)}{\Gamma(\Lambda_b \to \Sigma_c \pi)} = 2 + \mathcal{O} \left( \frac{\Lambda_{QCD}}{m_Q}, \alpha_s(m_Q) \right). \tag{15}
\]

To evaluate the square of the matrix element in Eq. (13), we used the spin sums from Ref. [14] for the various spin \( \Sigma_c^{(*)} \) states. The explicit calculation shows that the rate to \( \Sigma_c^* \) with \( |s'| = 3/2 \) vanishes, as required by angular momentum conservation.

A practical complication in testing this prediction is that the \( \Sigma_c^{(*)} \) states decay to \( \Lambda_c \pi \), and so both decay channels \( \Lambda_b \to \Sigma_c^{(*)0} \pi^0 \to \Lambda_c \pi^- \pi^0 \) and \( \Lambda_b \to \Sigma_c^{(*)+} \pi^- \to \Lambda_c \pi^0 \pi^- \) contain a \( \pi^0 \) that makes the reconstruction hard at hadron colliders. This can be circumvented by studying \( \Lambda_b \to \Sigma_c^{(*)0} \rho^0 \) decays. In this case the final states are \( \Lambda_c \pi^- \pi^+ \pi^- \). Decays to a vector meson are potentially more complicated due to the fact that “long-distance” contributions
can induce transverse polarizations at the same order in $\Lambda_{QCD}/E_\pi$. However, at leading order in $\alpha_s(m_Q)$ these long-distance contributions vanish for the $\rho^0$ final state \[15\] and we obtain
\[\frac{\Gamma(\Lambda_b \rightarrow \Sigma_c^0\rho^0)}{\Gamma(\Lambda_b \rightarrow \Sigma_c^0\rho^0)} = 2 + O[\Lambda_{QCD}/m_Q, \alpha_s(m_Q)]. \quad (16)\]

It is also worth noting that
\[\frac{\Gamma(B_0 \rightarrow D_0\pi^0)}{\Gamma(B_0 \rightarrow D_0\rho^0)} = \frac{\Gamma(\Lambda_b \rightarrow \Sigma_c^0\pi^0)}{\Gamma(\Lambda_b \rightarrow \Sigma_c^0\rho^0)} + O[\Lambda_{QCD}/m_Q, \alpha_s(\sqrt{m_Q\Lambda_{QCD}})], \quad (17)\]

where in contrast to Eqs. (15) and (16) this prediction requires a perturbative expansion at the intermediate scale $\sqrt{\Lambda_{QCD}/m_Q}$.

The decays $\Lambda_b \rightarrow \Xi_cK$ decays are also Cabibbo-allowed. (These decays involve “$\bar{s}s$ popping” so only $\Xi_c^0K^0$ is allowed, not $\Xi_c^+K^-$). They are similar to $\Lambda_b \rightarrow \Sigma_c^{(*)}\pi$ in the sense that the leading contribution in the heavy quark limit vanishes. As shown in Table II there are three $\Xi_c$ “ground states”, $\Xi_c$, $\Xi_c'$, and $\Xi_c^*$. The $\Xi_c$ and $\Xi_c'$ can mix, but the former is expected to be mostly the state that transforms as $\overline{3}$ under flavor $SU(3)$, while the latter is mostly a $6$. The $\Xi_c^*$ also transforms as a $6$, and forms a heavy quark spin symmetry doublet with the $\Xi_c'$. Thus, a relation similar to Eq. (15) also holds in this case, i.e.,
\[\frac{\Gamma(\Lambda_b \rightarrow \Xi_c^*K)}{\Gamma(\Lambda_b \rightarrow \Xi_c'K)} = 2 + O[\Lambda_{QCD}/m_Q, \alpha_s(m_Q)]. \quad (18)\]

Since the spin of the light degrees of freedom is $s_l = 1$, we expect from heavy quark symmetry that $\Theta_Q$ come with a doublet partner of similar mass, $\Theta_Q^*$, as shown in Table II, with a mass splitting of order $\Lambda_{QCD}^2/m_Q$. From the mass splittings for the $\Sigma_c$ and $\Xi_c$ we expect $m_{\Theta_c} - m_{\Theta_c} \sim 70$ MeV and $m_{\Theta_b} - m_{\Theta_b} \sim 22$ MeV. In this case the $\Theta_Q^*$ may

\[\text{(15)} \quad \text{(16)}\]

\[\text{(17)}\]

\[\text{(18)}\]

5 It is possible that the $\Theta_Q$ are above the strong decay thresholds \[20\]. The assumptions in our analysis are that (i) $\Theta_Q$ decay weakly; and (ii) the spin of the light degrees of freedom is $s_l = 1$, as suggested by \[19\]. If (i) is correct but (ii) is not, it would be easy to modify our predictions, including Eq. \[19\].
FIG. 3: Comparison of the weak nonleptonic decays of the Λ_b and Θ_b.

also be stable with respect to the strong interactions and decay to Θ_Qγ. Since the splitting for Θ_c^{(*)} is larger, it is possible that the Θ_c^{(*)} is just above the strong decay threshold, making the spectroscopy even more interesting (like in D^{*} decays).

It may be possible to discover the Θ_{b,c} via the decay chains

\[ \Theta_b^+ \to \Theta_c^0 \pi^+ \quad \text{and} \quad \Theta_c^0 \to \Theta^+ \pi^- \to K_S p \pi^- \to \pi^+ \pi^- p \pi^- \]  \tag{18}

that are Cabibbo-allowed and lead to all charged final states. The most interesting aspect of the Θ_b^+ → Θ_c^0 decay is that in the diquark picture the correlation is maintained, as shown in Fig. 3 and so no additional suppression factor is expected. In weak Θ_b decays to ordinary baryons this would not be the case. While we do not know the Θ_Q production rates, we can estimate the branching ratios in Eq. (18). The lifetime of a weakly decaying Θ_{b,c} is expected to be comparable with other weakly decaying hadrons that contain a charm or a bottom quark. The Θ_b^+ → Θ_c^0π^+ amplitude factorizes, and is related to Θ_b^+ → Θ_c^0ℓ\bar{ν} via a formula identical to Eq. (5). For the nonleptonic rate we obtain in the heavy quark limit,

\[
\frac{\Gamma(\Theta_b^+ \to \Theta_c^0 \pi^+)}{\Gamma(\Lambda_b \to \Lambda_c \pi^-)} = \frac{m_{\Theta_b}^3(1 - r_{\Theta}^2)^3}{m_{\Lambda_b}^3(1 - r_{\Lambda}^2)^3} \frac{1}{\zeta(w_{\max}^\Lambda) \zeta(w_{\max}^\Theta)} \frac{1}{144r_{\Theta}^4} \left\{ 4[\eta_1(w_{\max}^\Theta)]^2 r_{\Theta}^2(1 + 18r_{\Theta}^2 + r_{\Theta}^4) - 4\eta_1(w_{\max}^\Theta)\eta_2(w_{\max}^\Theta) r_{\Theta}(1 - r_{\Theta}^2)^2(1 + r_{\Theta}^2) + \left[(\eta_2(w_{\max}^\Theta))^2 (1 - r_{\Theta}^2)^4 \right] \right\} \tag{19}
\]

where \( r_{\Theta} = m_{\Theta_c}/m_{\Theta_b} \), and \( \eta_{1,2} \) are the two Isgur-Wise functions that parameterize the weak \( \Theta_b \to \Theta_c^{(*)} \) matrix elements where \( \eta_1(1) = 1 \). In particular

\[
\langle \bar{\Theta}_c(v', s') | \bar{c} \Gamma_b | \bar{\Theta}_b(v, s) \rangle = \frac{1}{3} \left[ g^{\alpha\beta} \eta_1(w) - v^\alpha v^\beta \eta_2(w) \right] \bar{u}(v', s') \gamma_5 (\gamma_\alpha + v'_\beta) \Gamma(\gamma_\beta + v_\beta) \gamma_5 u(v, s). \tag{20}
\]

Thus, \( B(\Theta_b^+ \to \Theta_c^0 \pi^+) \) is expected to be similar to \( B(\Lambda_b \to \Lambda_c \pi) \). If the Θ_Q states exist then an analysis of the \( \Lambda_{\text{QCD}}/m_Q \) corrections would be warranted, as the mass of the light
degrees of freedom is sizable. We expect $\mathcal{B}(\Theta_c^0 \to \Theta^+\pi^-)$ to be at the few percent level, while the other branching ratios in Eq. $[13]$ may be of order unity.

In summary, we studied nonleptonic $\Lambda_b$ decays to $\Lambda_c\pi$, $\Sigma_c\pi$ and $\Sigma_c^*\pi$. Eqs. $[10]$, $[15]$, $[16]$, and $[18]$ are our main results. In the $m_Q \gg \Lambda_{QCD}$ limit the $\Lambda_b \to \Lambda_c\pi$ rate is related to $\Lambda_b \to \Lambda_c \ell \bar{\nu}$, and we found that $\Gamma(\Lambda_b \to \Lambda_c\pi)$ is expected to be larger than $\Gamma(B \to D^{(*)}\pi)$, as observed by CDF. At leading order in $\Lambda_{QCD}/m_Q$ the $\Lambda_b \to \Sigma_c^{(*)}\pi$ rates vanish, but an analysis of the leading contributions suppressed by $\Lambda_{QCD}/m_Q$ was still possible. We predict $\Gamma(\Lambda_b \to \Sigma_c\pi) / \Gamma(\Lambda_b \to \Sigma_c\pi) = \Gamma(\Lambda_b \to \Sigma_c^*\rho) / \Gamma(\Lambda_b \to \Sigma_c\rho) = 2 + \mathcal{O}[\Lambda_{QCD}/m_Q, \alpha_s(m_Q)]$. We also discussed properties of pentaquarks with a $\bar{b}$ or $\bar{c}$, including a possible discovery channel if they decay weakly.

**Acknowledgments**

We thank Shin-Shan Yu for asking questions that raised our interest in some of these topics, and Marjorie Shapiro and Dan Pirjol for helpful discussions. This work was supported in part by the National Science Foundation under Grant No. PHY-0244599 (A.K.L.); by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098 (Z.L.); by the Department of Energy under cooperative research agreement DF-FC02-94ER40818 (I.W.S); and by the Department of Energy under Grant No. DE-FG03-92-ER40701 (M.B.W). Z.L. and I.W.S. were also supported by DOE Outstanding Junior Investigator awards.

[1] N. Isgur and M. B. Wise, Phys. Lett. B 237 (1990) 527; Phys. Lett. B 232 (1989) 113.

[2] N. Isgur and M. B. Wise, Nucl. Phys. B 348 (1991) 276; H. Georgi, Nucl. Phys. B 348 (1991) 293; H. Georgi, B. Grinstein and M. B. Wise, Phys. Lett. B 252 (1990) 456.

[3] N. Isgur and M. B. Wise, Phys. Rev. Lett. 66, 1130 (1991).

[4] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D 63, 014006 (2001) [hep-ph/0005275]; C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D 63, 114020 (2001) [hep-ph/0011336]; C. W. Bauer and I. W. Stewart, Phys. Lett. B 516, 134 (2001)
[5] CDF Collaboration, CDF note 6396, available at:

http://www-cdf.fnal.gov/physics/new/bottom/030702.blessed-lblcpi-ratio/

[6] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66 (2002) 010001.

[7] H. D. Politzer, Phys. Lett. B 250 (1990) 128.

[8] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. 87 (2001) 201806 [hep-ph/0107002].

[9] M. J. Dugan and B. Grinstein, Phys. Lett. B 255, 583 (1991).

[10] H. D. Politzer and M. B. Wise, Phys. Lett. B 257, 399 (1991).

[11] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591 (2000) 313 [hep-ph/0006124].

[12] E. Jenkins, A. V. Manohar and M. B. Wise, Nucl. Phys. B 396 (1993) 38 [hep-ph/9208248].

[13] In the absence of a “World average”, we use the two measurements with the smallest errors:

K. Abe et al. [BELLE Collaboration], Phys. Lett. B 526, 247 (2002) [hep-ex/0111060];

N. E. Adam et al. [CLEO Collaboration], Phys. Rev. D 67, 032001 (2003) [hep-ex/0210040].

[14] A. K. Leibovich and I. W. Stewart, Phys. Rev. D 57 (1998) 5620 [hep-ph/9711257].

[15] S. Mantry, D. Pirjol and I. W. Stewart, [hep-ph/0306254]

[16] CDF Collaboration, CDF note 6708, available at:

http://www-cdf.fnal.gov/physics/new/bottom/031002.blessed-bs-br/

[17] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B 643 (2002) 431 [hep-ph/0206152].

[18] T. Becher, R. J. Hill and M. Neubert, [hep-ph/0308122] T. Becher, R. J. Hill, B. O. Lange and M. Neubert, [hep-ph/0309227]

[19] R. L. Jaffe and F. Wilczek, [hep-ph/0307341]

[20] M. Karliner and H. J. Lipkin, [hep-ph/0307343] S. Sasaki, [hep-lat/0310014]