A Unified Accreting Magnetar Model for Long-duration Gamma-Ray Bursts and Some Stripped-envelope Supernovae

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Abstract

Both the long-duration gamma-ray bursts (LGRBs) and the Type I superluminous supernovae (SLSNe I) have been proposed to be primarily powered by central magnetars. A correlation, proposed between the initial spin period ($P_0$) and the surface magnetic field ($B$) of the magnetars powering the X-ray plateaus in LGRB afterglows, indicates a possibility that the magnetars have reached an equilibrium spin period due to the fallback accretion. The corresponding accretion rates are inferred as $M \approx 10^{-4} - 10^{-1} M_\odot \text{yr}^{-1}$, and this result holds for the cases of both isotropic and collimated magnetar wind. For the SLSNe I and a fraction of engine-powered normal Type Ic supernovae (SNe Ic) and the broad-lined subclass (SNe Ic-BL), the magnetars could also reach an accretion-induced spin equilibrium, but the corresponding $B - P_0$ distribution suggests a different accretion rate range, i.e., $M \approx 10^{-7} - 10^{-3} M_\odot \text{yr}^{-1}$. Considering the effect of fallback accretion, magnetars with relatively weak fields are responsible for the SLSNe I, while those with stronger magnetic fields could power SNe Ic-BL. Some SLSNe I in our sample could arise from compact progenitor stars, while others that require longer-term accretion may originate from the progenitor stars with more extended envelopes or circumstellar medium.

Unified Astronomy Thesaurus concepts: Magnetars (992); Supernovae (1668); Gamma-ray bursts (629)

1. Introduction

Rapidly rotating magnetars are promising central engine candidates for some transient astrophysical phenomena. They compete with black holes (BHs) to be the central engines of the gamma-ray bursts (GRBs; e.g., Usov 1992; Dai & Lu 1998; Zhang & Mészáros 2001). They also have been proposed to power the X-ray flares and plateaus (shallow decays) in some GRB afterglows (e.g., Dai et al. 2006; Zhang et al. 2006; Rowlinson et al. 2010, 2013; Dall’Osso et al. 2011; Li & Zhang 2014; Li et al. 2018; Lin et al. 2018; Stratta et al. 2018). In addition, the magnetar-powered model is invoked to interpret the luminosity evolution of different subclasses of supernovae (SNe; e.g., Kasen & Bildsten 2010; Woosley 2010), such as the normal Type Ic SNe (SNe Ic; e.g., Taddia et al. 2018, 2019), broad-lined SNe Ic (SNe Ic-BL; e.g., Wang et al. 2017a, 2017b), and Type I superluminous supernovae (SLSNe I; e.g., Insetra et al. 2013; Liu et al. 2017; Nicholl et al. 2017; Yu et al. 2017; Blanchard et al. 2018, 2019; Villar et al. 2018; Lin et al. 2020).

In those models, magnetars are usually treated as isolated neutron stars (NSs) that spin down due to the magnetic dipole radiation. In the context of core-collapse explosion, however, the stellar debris could circulate into a disk and interact with a nascent magnetar, which has a strong influence on the spin evolution and outflows of the magnetar (magnetar propeller; e.g., Piro & Ott 2011; Metzger et al. 2018). For a given accretion rate, the magnetar could reach an equilibrium spin period, i.e., $P_{\text{eq}} \propto B^{6/7}$ (see also Appendix A), where $B$ is the surface magnetic field of the magnetar. Such a model has been invoked and further developed to study the diverse X-ray light curves of long- and short-duration GRB (LGRB and SGRB) afterglows (Dai & Liu 2012; Gompertz et al. 2014; Gibson et al. 2017, 2018), since the magnetar–disk system could be formed in the cases of both core-collapse explosion and binary compact star mergers. Assuming that the magnetar wind is collimated, Stratta et al. (2018) find a correlation between the surface magnetic field ($B$) and the initial spin period ($P_0$) of isolated magnetar engines for X-ray plateaus in GRB afterglows, in agreement with the $B \propto P_0^{-7/6}$ relation for the accreting magnetars.

Based on the magnetic propeller model, we tentatively explore the properties of a portion of LGRBs and SNe that can be explained by considering a magnetar as a dominant power source. Our Letter is organized as follows. In Section 2, we collect a sample of transients that are potentially powered by magnetars, including LGRBs with X-ray plateaus, SLSNe I, SNe Ic, and SNe Ic-BL. In Section 3, we show the $B - P_0$ distribution inferred from isolated magnetars as the central engines for different types of transients, and further discuss its physical implications. A summary is given in Section 4.

2. Data and Sample Selection

Li et al. (2018) systematically studied GRB X-ray plateaus, which are selected from the Neil Gehrels Swift/X-ray Telescope (XRT) data observed during 2004 December–2017 May, and concluded that 19 LGRB X-ray plateaus could be explained by the energy injection from isotropic magnetar wind. Assuming that the magnetar wind is collimated in the plateau phase, however, Li & Zhang (2014) found more potentially magnetar-powered events from the XRT data obtained between 2005 January and 2013 August. Note that four X-ray plateaus in LGRB afterglows (GRB 060526, GRB...
061110A, GRB 070110, and GRB 120422A) can be explained in both scenarios. We include both of the above two magnetar candidate samples in the following analysis, since the wind configuration is still debated.

The sample of SLSNe I are mainly collected from Nicholl et al. (2017) and Villar et al. (2018), which analyzed multiband light curves of a total of 58 spectroscopically identified SLSNe I based on the magnetar-powered model. In addition, three more SLSNe I (PS16aqv, SN 2017dwh, and SN 2018hti; Blanchard et al. 2018, 2019; Lin et al. 2020) are also included in our sample.

Wang et al. (2017a) invoked the magnetar as an alternative energy source to model the light curves and velocity evolution of 11 SNe Ic-BL without detections of companion LGRBs. For SN 2007ru, SN 2010ah, and PTF10qts, we collect $B$ and $P_0$ inferred from the pure-magnetar model, while the parameters for the other eight events are determined based on the fits with the magnetar plus $^{56}$Ni model, which provides better fits with lower $\chi^2$/dof values.

The magnetar model is also proposed to account for the high luminosity of SN 2011kl (Greiner et al. 2015; Wang et al. 2017c), which is associated with ultralong GRB 111209A. Wang et al. (2017b) fitted the bolometric light curve of SN 1998bw (associated with GRB 980425) with the magnetar plus $^{56}$Ni model, and found that the peak and tail of the light curve can be explained by magnetar spin-down. In addition to these two SNe associated with LGRBs (LGRB-SNe), two normal SNe Ic (iPTF15dgt, Taddia et al. 2019; PTF11mbb, Taddia et al. 2018) that are also likely powered by magnetars are included in our sample.

The information of our sample is tabulated in Table 1. From the references listed in Table 1, we collect the parameters ($B$ and $P_0$) inferred from the models that invoke a magnetar as the dominant energy source.

### Table 1

| Transients               | Number | Power Source               | References               |
|--------------------------|--------|----------------------------|--------------------------|
| LGRB X-ray plateaus      | 19     | Magnetar (isotropic wind)  | Li et al. (2018)         |
| LGRB X-ray plateaus      | 36     | Magnetar (collimated wind) | Li & Zhang (2014)        |
| SLSNe I                  | 61     | Magnetar                   | Nicholl et al. (2017), Blanchard et al. (2018), Villar et al. (2018), Blanchard et al. (2019), Lin et al. (2020) |
| SNe Ic-BL without detected LGRBs | 11     | Magnetar/Magnetar-$^{56}$Ni | Wang et al. (2017a)      |
| LGRB-SNe                 | 2      | Magnetar/Magnetar-$^{56}$Ni | Greiner et al. (2015), Wang et al. (2017b) |
| SNe Ic                   | 2      | Magnetar-$^{56}$Ni         | Taddia et al. (2018), Taddia et al. (2019) |

3. B–P Distribution

3.1. X-Ray Plateaus in LGRB Afterglows

The X-ray plateaus in some LGRB afterglows are observed to persist for $\sim 100 - 10^5$ s before the steeper decline. Assuming that the X-ray plateaus are powered by the isotropic wind from the magnetars, the light curves can be used to constrain the surface magnetic field and initial spin period of magnetars. As seen in Figure 1, magnetars with $P_0 \sim 1$ ms usually possess $B \sim 10^{14} - 10^{15}$ G, while those with $P_0 \gtrsim 10$ ms are accompanied by a strong magnetic field of $B \sim 10^{15} - 10^{16}$ G. We perform a linear fit (see Appendix B for the detailed descriptions of the fitting) to the log $B$–log $P_0$ distribution, and find

$$\log B = 14.6^{+0.04}_{-0.05} + (1.13^{+0.11}_{-0.09})\log P_0,$$

where $P_0$ is in units of milliseconds. Such a correlation is consistent with the spin equilibrium state for the accreting magnetars ($B \propto P_0^{3/6}$ for a given accretion rate; e.g., Piro & Ott 2011; see also Equation (A3)). It implies that the initial spin period inferred from observations ($P_0$) could deviate from that of the magnetar at birth, but possibly corresponds to the equilibrium spin period as a result of interaction between the magnetar and surrounding accretion disk. The accretion rates of the disks are inferred as $M \approx 10^{-4} - 0.1 M_{\odot}$ s$^{-1}$. We further estimate the evolutionary timescales for these magnetars to reach the spin equilibrium ($t_{ev} \propto B^{-5/7} M^{-3/7}$; Metzger et al. 2018; see also Equation (A4)), which turn out to be $\sim 0.1 - 1000$ s. We consider $t_{ev}$ as the lower limits for the accretion timescales ($t_{acc}$) and show them in Figure 2. Assuming $M_s \sim M_{acc}$, the total mass of accretion disk can be constrained to be $M_d \gtrsim 10^{-2} - 0.5 M_{\odot}$. We caution that a magnetar could possibly collapse into a BH if it accretes a significant amount of materials and exceeds the maximum mass of a stable NS.\(^7\) Hence, we assume an accretion disk mass of $\lesssim 1 M_{\odot}$. Such a disk mass corresponds to an accretion timescale of $\lesssim 10^{-4}$ s, in agreement with the fallback timescale derived for Wolf-Rayet (WR) stars, i.e., $\sim 10^{-2} - 10^5$ s;\(^8\) for a shorter timescale, the fallback materials could come from the core of the progenitors, which suggests an origin of compact progenitor stars for some LGRBs (e.g., Campana et al. 2006; Woosley & Heger 2006).

Notice that only $\sim 20\%$ of X-ray plateaus out of the Li et al. (2018) sample (including LGRB and SGRB X-ray plateaus) are consistent with the energy budget of magnetars, if the magnetar wind is isotropic. They argue that the BHs could be the central engines for most of the X-ray plateaus. If fallback accretion plays a role in the evolution of magnetars, the accreting magnetars could maintain the spin equilibrium on a longer timescale in the presence of an accretion disk, which may provide a natural explanation for the energy that is beyond the millisecond magnetar budget in some cases. Actually, the configuration of magnetar wind is still debated in the context of

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\(^7\) Some pulsars with a mass of $\sim 2 M_{\odot}$ have been discovered (Demorest et al. 2010; Antoniadis et al. 2013), which sets a lower limit for the maximum mass of NSs ($M_{max}$). Hitherto, there is no consensus on the upper limit of $M_{max}$ (e.g., Lasky et al. 2014; Margalit & Metzger 2017).

\(^8\) The radii ($r_*)$ of the envelopes of WR stars are $\sim 10^{10} - 10^{12}$ cm (Koesterke & Hamann 1995). The freefall timescale of the extended envelopes can be estimated by $t_{ff} \sim (c_s^2/GM)^{1/2}$, i.e., $100 \lesssim t_{ff} \lesssim 10^5$ s.
LGRB afterglow. Magnetar wind could escape via a collimated jet shortly after the SN explosion (Bucciantini et al. 2009), on a longer timescale, wind could still be channeled into the polar region where the preceding jet drill its way out of the stellar envelope. A large number of X-ray plateaus can be explained by the injection of collimated magnetar wind along the GRB jets (Lü & Zhang 2014; Stratta et al. 2018). In Figure 1, we compare the $B$–$P_0$ distributions that are derived from different wind models. Although the broader distributions are suggested in the case of collimated wind, they also follow a similar $B$–$P_0$ correlation (see also Appendix B), i.e.,

$$\log B = 14.44^{+0.03}_{-0.03} + (1.22^{+0.06}_{-0.06}) \log P_0$$

And the inferred mass inflow rates are similar to those obtained by Stratta et al. (2018) and are also consistent with the results based on the isotropic wind model. Therefore, both results suggest that the central magnetars could experience interactions with the surrounding accretion disks and finally reach a spin equilibrium.

### 3.2. SLSNe Ic, SNe Ic-BL

Core-collapse SNe usually reach their peak luminosities at tens of days after explosion. Around that time, magnetar wind should be near-isotropic and should contribute most of the rotation energy to heat and/or accelerate the ejecta. Based on the model fits to the light curves (Nicholl et al. 2017; Blanchard et al. 2018, 2019; Villar et al. 2018; Lin et al. 2020), the magnetar engines of SLSNe I are characterized by a magnetic field of $10^{12} < B < 10^{14}$ G and a spin with a short period of $I < P_0 < 10$ ms (Figure 1). The magnetars with a longer initial spin period appear to have a stronger magnetic field. This correlation possibly suggests that the engine timescale is roughly comparable to the diffusion timescale of ejecta, which corresponds to $B \propto P_0$ (Nicholl et al. 2017). If the physics of accretion-induced spin equilibrium could apply to the cases of SLSNe I, the inferred accretion rates based on Equation (A3) mainly fall between $10^{-7}$ and $10^{-3} M_{\odot}$ s$^{-1}$.

SNe Ic-BL might be accompanied by the birth of magnetars, since an upper limit of the kinetic energy of SNe Ic-BL (i.e., $\sim$ a few $10^{52}$ erg s$^{-1}$, model dependent though) is comparable to the maximum rotation energy of a millisecond magnetar (Mazzali et al. 2014). Moreover, some SNe Ic-BL (e.g., SN2010ay and PTF10ovq) are unlikely to be explained by the radioactive $^{56}$Ni model and hence the magnetar is invoked as a dominant power source (Wang et al. 2017a). As seen from Figure 1, most SNe Ic-BL without coincident LGRBs in our sample invoke a magnetar with $P_0 \gtrsim 10$ ms and $B \sim 10^{15}$ G, while some events require the magnetic field to be as strong as $10^{16}$ G. Associated with GRB 980425, SN 1998bw requires $B \approx 1.66 \times 10^{15}$ G for the central magnetar, in agreement with most of its non-GRB peers. Compared with the SLSNe I
magnetars, the magnetar candidates for SNe Ic-BL have a stronger magnetic field and longer initial spin period.

Greiner et al. (2015) reported the observations of a luminous SN Ic SN 2011kl (associated with GRB 111209A), which has intermediate luminosity lying between typical SNe Ic-BL and SLSNe I. Following their fitting results, SN 2011kl could be also powered by a magnetar with an initial spin period of \( P_0 \approx 12 \) ms and magnetic field of \( B \approx 7.5 \times 10^{14} \) G. Two normal SNe Ic (iPTF15dtg and PTF11mnb) require a weaker magnetic field than most SNe Ic-BL and LGRB-SNe in our sample (Taddia et al. 2018, 2019).

We note that, although SLSNe I exhibit distinct spectral features at early times (e.g., Quimby et al. 2011, 2018), their post-peak spectra eventually evolve to resemble those of SNe Ic/Ic-BL. Pastorello et al. (2010); Liu et al. (2017); Blanchard et al. (2019); Nicholl et al. (2019). This suggests an underlying connection among these subtypes of SNe. By fitting the \( B-P_0 \) distribution of our SNe samples (including SLSNe I, SNe Ic, and SNe Ic-BL), the following correlation (see also the fitting procedure described in Appendix B) is derived:

\[
\log B = 13.49^{+0.1}_{-0.1} + (1.24^{+0.14}_{-0.14}) \log P_0,
\]

which is consistent with \( B-\dot{P}_{eq} \) correlation expected for the accreting magnetars at the spin equilibrium state. Although SNe Ic/Ic-BL require different properties of magnetars than SLSNe I, similar accretion rates are inferred for most SNe in our sample. Therefore, the nascent magnetar plus the accretion disk system provide a unified picture to explain the production of SLSNe I, SNe Ic, and SNe Ic-BL. Based on the magnetic propeller model, the central magnetar with a low magnetic field will be accelerated to a millisecond spin period responsible for the energy source powering an SLSN I; conversely, a stronger magnetic field leads to a longer equilibrium spin period of a magnetar, and it hence produces an SN Ic/Ic-BL.

Assuming that magnetars can always reach the spin equilibrium state, the lower limits for accretion timescales in the cases of SLSNe I are estimated as \( t_{acc} \approx 10^{2–10^6} \) s based on Equation (A4), while SNe Ic-BL show a similar \( t_{eq} \) distribution to LGRB X-ray plateaus (Figure 2). Although the limits of accretion timescales for most SLSNe I are consistent with the fallback timescale for the envelopes of the compact progenitor stars, a fraction of SLSNe I require longer accretion timescales. Assuming that the accretion timescale \( (t_{acc}) \) is equivalent to the fallback timescale, the long-term accretion could be due to the fallback of the stellar envelope from large radii or inner ejecta that cannot escape from the central object (Chevalier 1989; Dexter & Kasen 2013). Since early-time bumps observed in some events could be attributed to the cooling of a shocked envelope with a radius of \( \gtrsim 500 \) \( R_\odot \) (e.g., Piro 2015; Nicholl & Smartt 2016; Smith et al. 2016), the immediate progenitors of a portion of SLSNe I could be surrounded by largely extended envelopes, in agreement with the first possible scenario allowing for the late accretion. Alternatively, those bumps might suggest the possible existence of circumstellar medium (CSM; Leloudas et al. 2012). In this scenario, reversed shock could be produced by the ejecta–CSM interaction. Then the inner layer of the ejecta would be decelerated by the reverse shock and finally bound to the gravity of the newborn magnetar, contributing to the late accretion. It remains unknown whether SLSNe I could be associated with LGRBs. But it might be a challenge for an LGRB jet to push through the extended envelope or CSM surrounding the progenitor stars of some SLSNe I. For this subclass of SLSNe I, the magnetic field strength is found to be lower than \( 10^{14} \) G, which is inconsistent with the magnetic field required by LGRBs.

Figure 3 shows some examples of the spin-down luminosity (magnetic dipole radiation luminosity) evolution of isolated magnetars with \( B = 10^{14–10^{16}} \) G and \( P_0 \in [10, 100] \) ms (2) based on the \( B-P \) relation for an accreting magnetar with the accretion rates \( \dot{M} = M/(10^{-5} \text{ M}_\odot \text{ s}^{-1}) = 0.01, 0.05, 0.1, 0.5, 1, 5 \), respectively. The blue horizon lines show the peak luminosity range of the normal SNe Ic (\( 4.5 < \log L_{\text{peak}} < 43 \)), and the purple vertical lines mark the major range of the peak time (\( \approx 10–23 \) days; Prentice et al. 2016).

\[ L_{\text{peak}} = 4 \times 10^{44} \text{ erg s}^{-1} \]

9 Note that three SNe Ic-BL (SN 1997ef, SN 2002ap, and SN 2007bg) deviate significantly from the best-fit power-law relation for all of the SNe sample (Equation (3)) in the \( B-P_0 \) diagram (Figure 1). They might represent a subset of SNe Ic-BL that are powered by magnetars with the magnetic field being as strong as \( \approx 10^{16} \) G. However, we caution that there could be some other reasons for the deviations. Due to the lack of stringent constraints on the magnetar or \( ^{56}\text{Ni} \) contribution in fitting with the magnetar plus \( ^{56}\text{Ni} \) model, it is difficult to obtain accurate parameters of the magnetar. In addition, diverse power sources might also lead to a deviation, since both of the magnetar plus \( ^{56}\text{Ni} \) model and the two-component pure-\( ^{56}\text{Ni} \) model can provide a viable explanation for the emission of SN 1997ef, SN 2002ap, and SN 2007bg (Maeda et al. 2003; Young et al. 2010).
reduced and hence the angular momentum can be sustained to help form a rapidly rotating magnetar. As most SNe Ic prefer higher-metallicity environments (e.g., Modjaz et al. 2020), it is thus expected that only a small subset of SNe Ic are accompanied by the birth of magnetars.

4. Discussions and Conclusions

By involving the fallback accretion effect in the magnetar-powered scenario, we study the correlation between the surface magnetic field ($B$) and initial spin period ($P_0$) of magnetar candidates that can account for the emissions of transients, such as LGRB X-ray plateaus, SLSNe candidates that can account for the emissions of transients, such as the late-time decline of the accretion rate can also in

The Astrophysical Journal Letters, 903:L24 (6pp), 2020 November 10

Lin et al.

future will provide further clues for the nature of these GRBs and stripped-envelope SNe.

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Appendix A

Magnetic Propeller Model

The interaction between a magnetar with its surrounding disk has an effect on its spin evolution and hence the outflows (e.g., Piro & Ott 2011; Gompertz et al. 2014; Gibson et al. 2017, 2018; Metzger et al. 2018). The interaction can be modeled depending on the relative locations of Alfvén radius ($r_A$), corotation radius ($r_c$), and light cylinder radius ($r_L$). Alfvén radius is usually considered as the inner radius of the disk, where the ram pressure of the inflowing materials balances with the magnetic pressure of the magnetar. It can be given by

$$r_m = \frac{(GM)}{\Omega^2} \frac{1}{3}$$ (A1)

where $G$ is the gravitational constant, $M/R/B$ denotes the mass/radius/magnetic field strength of the central magnetar, and $M$ is the mass inflow rate at the inner edge of the disk. Given that inflowing materials rotate at the local Keplerian angular velocity, i.e., $\Omega_K = (GM/p^3)^{1/2}$, their corotation with the magnetar occurs at a radius of

$$r_c = \frac{(GM)}{\Omega^2} \frac{1}{3}$$ (A2)

where $\Omega = 2\pi/p$ and $P$ are the angular velocity and spin period of the magnetar, respectively. The radius of the light cylinder is defined as $r_L = c/\Omega$, inside which the magnetic field lines are usually considered to rotate rigidly with the magnetar.

If $r_m < r_c < r_L$, materials at the inner edge of the disk revolve faster than the local magnetic field lines and tend to be funneled before fall onto the surface of the magnetar. Thus, the magnetar gains its angular momentum and subsequently the corotation radius decreases until $r_c \sim r_m$. Conversely, if $r_c < r_m < r_L$, the slow-rotating inner disk is sped up to a super-Keplerian velocity by the magnetar, which results in a mass ejection from disk and sharp spin-down of the magnetar (propeller regime). The spin-down of the magnetar, in return, leads to an increase of $r_c$. Consequently, such a magnetar–disk system tends to evolve toward $r_c = r_m$ if the spin evolution of the magnetar is dominated by the interaction with the accretion disk. When $r_c$ equals $r_m$ the accreting magnetar would reach an equilibrium spin period (e.g., Piro & Ott 2011)

$$P_{eq} = 2\pi(GM)^{2/5}R^{8/5}B^6/M^{3/5}$$ (A3)

The equilibrium spin period is independent of the initial spin period of magnetar but correlates with the magnetic field strength and mass inflow rate. With $M = 1.4 M_\odot$ and $R = 12$
parameterized as \( \sigma \) are the corresponding errors. where \( x_i \) and \( y_i \) are the observational quantities, and \( \sigma_x \) and \( \sigma_y \) are the corresponding errors.

If there is extra variability of data, which can be parameterized as \( \sigma_y \), the likelihood function is modified as (D’Agostini 2005)

\[
f \propto \prod_i \frac{1}{\sqrt{\sigma_y^2 + m^2\sigma_x^2}} \exp \left[ -\frac{(y_i - mx_i - c)^2}{2(\sigma_y^2 + m^2\sigma_x^2)} \right].
\] (B2)

We fit the data of LGRB X-ray plateaus with Equation (B1) as the likelihood function, and obtain Equation (1) for the isotropic wind model and Equation (1) for the collimated wind model. However, the effect of the extra variability of data (\( \sigma_y \)) should be considered for our SN sample, given the difference in the models (see references listed in Table 1 for details) and the lack of stringent constraint on the magnetar or \(^{56}\)Ni contribution. In fitting, the mean uncertainties of data are adopted as errors for the likelihood function. For the two SNe Ic, the uncertainties of \( B \) and \( P_0 \) are not given in the literature, so we set the half of the values as the corresponding uncertainties. The fitting result of the SNe is given in Equation (3) with \( \sigma_f = 0.52^{+0.05}_{-0.04} \).

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