Damping rates in the MSSM and electroweak baryogenesis

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Abstract

We present an analysis of the thermalization rate of Higgsinos and winos based on the imaginary part of the two-point Green function in the unbroken phase of the MSSM. We use improved propagators including resummation of hard thermal loops and the thermalization rate is computed at the one-loop level in the high temperature approximation. We find that the damping is typically dominated by scattering with gauge bosons, resulting in a damping rate of about \( \gamma_H \approx 0.025T \), \( \gamma_W \approx 0.065T \). The contribution from scattering with scalars is relatively small. Implications for baryogenesis are also discussed.

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The presence of unsuppressed baryon number violating processes at high temperatures within the Standard Model (SM) of weak interactions makes the generation of the baryon number at the electroweak scale an appealing scenario \cite{1}. The baryon number violating processes also impose severe constraints on models where the baryon asymmetry is created at energy scales much higher than the electroweak scale \cite{2}. Unfortunately, the electroweak phase transition is too weak in the SM \cite{3} (although the existence of a primordial hypermagnetic field could improve the situation \cite{4}). This means that the baryon asymmetry generated during the transition would subsequently be erased by unsuppressed sphaleron transitions in the broken phase. The most promising and well-motivated framework for electroweak baryogenesis beyond the SM seems to be supersymmetry (SUSY). Electroweak baryogenesis in the framework of the Minimal Supersymmetric Standard Model (MSSM) has attracted much attention in the past years, with particular emphasis on the strength of the phase transition \cite{5} and the mechanism of baryon number generation \cite{6,7,8,9,10,11}.

Recent analytical \cite{12,13} and lattice computations \cite{14} have revealed that the phase transition can be sufficiently strongly first order if the ratio of the vacuum expectation values of the two neutral Higgses $\tan \beta$ is smaller than $\sim 4$. Moreover, taking into account all the experimental bounds including those coming from the requirement of avoiding dangerous color breaking minima, the lightest Higgs boson should be lighter than about 105 GeV, while the right-handed stop mass might be close to the present experimental bound and should be smaller than, or of the order of, the top quark mass \cite{13}.

Moreover, the MSSM contains additional sources of CP-violation besides the CKM matrix phase. However, an acceptable baryon asymmetry from the stop sector may only be generated through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter, and their associated CP-violating phases \cite{9}. As a result, the contribution to the final baryon asymmetry from the stop sector turns out to be negligible. On the other hand, charginos and neutralinos may be responsible for the observed baryon asymmetry in the MSSM\cite{9,11,15}. If the strength of the electroweak phase transition is enhanced by the presence of some new degrees of freedom beyond the ones contained in the MSSM, e.g. some extra standard model gauge singlets, light stops (predominantly the right-handed ones) and charginos/neutralinos are expected to give quantitatively the same contribution to the final baryon asymmetry.

CP-violating sources in supersymmetric baryogenesis are more easily built up if the degrees of freedom in the stop and gaugino/neutralino sectors are nearly degenerate in mass \cite{13}. This resonant behaviour is associated with the possibility of absorption (or emission) of Higgs quanta by the propagating supersymmetric particles. For momenta of the order of the critical temperature, this can only take place when, for instance,
the Higgsinos and gauginos do not differ too much in mass. By using the Uncertainty Principle, it is easy to understand that the width of this resonance is expected to be proportional to the thermalization rate of the particles giving rise to the baryon asymmetry [9].

From all these considerations, it is clear that the computation of the thermalization rate of the particles responsible for supersymmetric baryogenesis represents a necessary and crucial step towards the final computation of the baryon number. A detailed computation of the thermalization rate of the right-handed stop from the imaginary part of the two-point Green function has been recently performed in [10] by making use of improved propagators and including resummation of hard thermal loops. The thermalization rate has been computed exactly at the one-loop level in the high temperature approximation as a function of the plasma right-handed stop mass $m_{\tilde{t}_R}(T)$ and an estimate for the magnitude of the two-loop contributions which dominate the rate for small $m_{\tilde{t}_R}(T)$ was also given. If $m_{\tilde{t}_R}(T) \gtrsim T$, the thermalization is dictated by the one-loop thermal decay rate which can be larger than $T$ [10]. For smaller values of $m_{\tilde{t}_R}(T)$, when the thermalization is dominated by two-loop effects (i.e. scattering), $\Gamma_{\tilde{t}_R}$ may be as large as $10^{-3}T$ [10]. With such value, our derivative expansion is perfectly justified since the wall thickness can span the range $(10 - 100)/T$.

The goal of this paper is to present a computation of the thermalization rate of Higgsinos and winos from the imaginary part of the two-point Green function in the unbroken phase of the MSSM. We use improved propagators including resummation of hard thermal loops and the thermalization rate is computed at the one-loop level in the high temperature approximation.

There are two types of diagram contributing to the damping rate of charginos and neutralinos to one loop. The internal loop has either a scalar or a vector boson, apart from the fermion. We expect the leading processes to involve massless gauge bosons, since these diagrams are IR divergent without plasma resummation.

Let us begin with performing an analysis for the charged $\tilde{H}^\pm$. We first calculate the gauge boson contribution (as accurately as possible with the present understanding of the IR sensitivity) and then compare it with a typical diagram involving a scalar (the stop). We find that scattering with scalars provides typically much less damping. The contribution from unbroken gauge bosons is in some sense the most difficult diagram since it requires subtle resummation schemes. On the other hand, this is also the most studied case and the result can be found in the literature. The relevant diagram is
Depending on the assumptions of the mass and momentum of the external fermion there are a number of different cases:

- **Heavy** $m \gg T$ fermion at rest $p = 0$.
  In this case thermal fermions are not present but only the gauge bosons. The result for the damping rate $\gamma$ is \[\gamma = \frac{g^2 T C_f}{8\pi},\] (1)

  Here $C_f$ denotes the quadratic Casimir invariant of the fermion representation.

- **Massless fermion at rest** $p = 0$.
  This calculation is more involved than the heavy fermion case since one needs to resum all propagators and vertices. In [18] this is done for a few different gauge groups with the result that

  \[\gamma_\equiv -\frac{1}{8} \text{Im} \text{ tr} [\gamma_0 \Sigma(p_0 = M, p = 0)] = a(N, N_f) \frac{g^2 T C_f}{4\pi},\] (2)

  where $a(3, 2) = 1.40$, $a(2, 2) = 1.45$ and $a(2, 4) = 1.57$. The same quantity was calculated in [19] with the result $a(3, 3) = 1.43$. A major problem with this calculation is that the gauge independence of $\gamma$ has not been established. With certain particular IR regularization schemes the result is gauge invariant but depends on how the IR regulator is removed and the external particle put on-shell [20, 21].

- **Moving** $p > 0$ massless fermion.
  There have been several papers concerning this case and most of them conclude that $\gamma$ depends on the magnetic screening [24, 21, 23]. For abelian theories this is not an alternative so another resummation scheme was presented in [25] (apart from the usual HTL resummation) which resulted in a non-exponential but finite decay of correlation functions. It is found that excitation with $p \gg e^2 T$ (which is what we have) decay like

  \[S(t) = e^{-i\omega_p t} \exp \left[-v \frac{e^2 T}{4\pi} t \ln(M v t)\right],\] (3)
where again \( v \) is the velocity and \( M \) is the plasma mass. The expectation value of the damping rate is to leading order

\[
\gamma = v \frac{e^2 T}{4\pi} \ln \left( \frac{4\pi M}{e^2 T} \right),
\]

which has the same leading \( e^2 T \ln \frac{1}{e} \) term as the heavy fermion case. The same leading damping rate has been obtained for the non-abelian plasma in \[26, 23\] by assuming the existence of a magnetic mass of order \( g^2 T \) but the solution in \[25\] is independent of such assumptions. Notice also that in \[23\] the constant in \( \gamma \sim e^2 T (\ln \frac{1}{e} + \text{const}) \) was calculated.

From the results above we conclude that, to the extent that it is calculable, the damping rate of a light fermion due to scattering with gauge bosons is approximately \( \gamma \simeq \frac{g^2 T}{4\pi} C_f \) for both \( p = 0 \) and \( p > 0 \). The correction factor \( \ln \left( \frac{4\pi M}{g^2 T} \right) \) for moving fermions and the variation in the factor \( a(N, N_F) \) is at most of order 2 and does not affect the conclusion significantly.

For Higgsinos, the gauge boson contributions reflect weak and hypercharge symmetries. For the contribution from the weak gauge bosons we obtain \( \gamma \propto 3g_2^2 T/(16\pi) \simeq 0.0244 T \) \((g_2 = 0.640)\). The hypercharge contribution reads \( g_1^2 T/(16\pi) \simeq 0.00122 T \) \((g_1 = 0.247)\) which is negligible compared to the weak contribution. Thus we conclude that the gauge boson contribution to the damping rate of \( \tilde{H}^\pm \) is

\[
\gamma_{gb} \simeq 0.025 T
\]

up to a factor 2 depending on non-perturbative effects through the magnetic mass and variations in the factor \( a(N, N_F) \).

Let us now turn to scattering with scalars. There are a number of diagrams where the external fermion scatter with various scalars such as Higgs or squarks. They differ in structure depending on whether the fermion is Dirac or Majorana and whether the external Higgsino has a diagonal or off-diagonal mass term. However, the kinematics and coupling constants are all similar. Moreover, they do not suffer from the IR sensitivity of gauge boson scattering. Therefore, we shall only calculate explicitly the simplest diagram which is the stop-bottom loop with an external \( \tilde{H}_2^\pm \) where the internal chiral fermion has no Majorana mass and the external particle has only a diagonal Dirac mass. Taking the notation from \[27\], the imaginary part is given by

\[
\text{Im } \Sigma(P) = -\frac{h^2 \text{sign}(p_0)}{\sin 2\phi_P} \int d^4 K \frac{1}{(2\pi)^4} \frac{1}{2} \sin 2\phi_K \frac{1}{2} \sinh 2\phi_{K-P} \cdot 2\pi \delta \left( (K - P)^2 - M_\xi^2 \right) \frac{1}{2} (1 - \gamma_5) 2\pi A_6(k_0, k),
\]

\[6\]
\[
A_b(K) = \frac{1}{2\pi i} \left( S_b(k_0 - i\epsilon, k) - S_b(k_0 + i\epsilon, k) \right), \\
S_b(K) = \frac{s(K)k_0\gamma_0 - r(K)\mathbf{k} \cdot \gamma}{s(K)^2k_0^2 - r(K)^2k^2}, \\
s(K) = 1 - \frac{\mathcal{M}_b^2}{2k_0k} \ln \left( \frac{k_0 + k}{k_0 - k} \right), \\
r(K) = 1 + \frac{\mathcal{M}_b^2}{k^2} \left[ 1 - \frac{k_0}{2k} \ln \left( \frac{k_0 + k}{k_0 - k} \right) \right].
\] (7)

The notation is that \(\mathcal{M}_b, \mathcal{M}_t, \mathcal{M}_{\tilde{H}_\pm}\) and \(\mu\) stands for the thermal bottom, stop, Higgsino mass and the vacuum Higgsino mass, respectively. Since \(s(K)\) and \(r(K)\) have imaginary parts for \(k_0^2 < k^2\) there is a continuous part of the spectral function \(A_b\) apart from the \(\delta\)-functions. We shall use the notation \(s(k_0 - i\epsilon, k) = s_R(K) + i s_I(K)\) and similarly for \(r(K)\).

We want to take the expectation value of \(\text{Im } \Sigma\) in the external state of Higgsinos with thermally corrected masses. Since they have several branches and a continuous spectral weight below the light cone it is not possible to use a simple particle picture for the external states. One straightforward way to include collective effects is to multiply \(\text{Im } \Sigma\) with the spectral function for \(\tilde{H}_\pm\), i.e. \(A_{\tilde{H}_\pm}(P)\), and take the trace and integrate over \(p_0\). This gives the damping for a Higgsino averaged over all states with a given momentum. When the external states have several branches we have to be careful not to overcount the degrees of freedom, but the spectral function conveniently satisfies an exact sum rule which counts the number of degrees of freedom. Assuming that the external Higgsino mass is dominated by thermal effects we can use Eq. (2) for the damping rate:

\[
\gamma_{b/\tilde{t}} = \frac{1}{2} \langle \text{Im } \Sigma \rangle(p) = -\frac{1}{4} \int_0^\infty dp_0 tr [A_{\tilde{H}_\pm}(P)\text{Im } \Sigma(P)],
\] (8)

where we have divided by a factor 2 to average over the spins for incoming positive energy solutions. Finally we should do the \(K\) integral. For simplicity we choose \(p = 0\) so that there is no continuous contribution from the external states. Then the angular part is easy so we rewrite it as

\[
\int d^4K \delta((K - P)^2 - \mathcal{M}_t^2) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} dk_0 k(k_0) \theta((k_0 - p_0)^2 - \mathcal{M}_t^2).
\] (9)
Finally, we end up with

$$\frac{1}{4} \int_{0}^{\infty} dp_{0} \text{tr} [A_{H^{\pm}} \text{Im} \Sigma] = - \frac{\bar{h}_{i}^{2}}{32\pi^{2}} \sum_{p_{0}^{2} > 0} p_{0}^{2} \int dk_{0} k \theta((k_{0} - p_{0})^{2} - \mathcal{M}_{i}^{2})$$

$$\times \text{sign}(k_{0} - p_{0}) \left[ \frac{1}{e^{\beta k_{0}} + 1} + \frac{1}{e^{\beta(k_{0} - p_{0})} - 1} \right]$$

$$\times \left\{ \theta(k_{0}^{2} - k^{2}) \frac{(k_{0} - \eta k)(k_{0}^{2} - k^{2})}{4\mathcal{M}_{b}^{2}(k_{0}^{2} - k^{2} - \mathcal{M}_{b}^{2})} s(K) k_{0} 2\pi \sum_{i} \delta(k_{0} - k_{0}(i)) \right\}$$

$$+ \theta(k^{2} - k_{0}^{2}) s_{1} k_{0} \left[ \frac{s_{1} R}{(s_{1} R - s_{1}^{2}) k_{0}^{2}} - \frac{(r_{R}^{2} - r_{f}^{2}) k^{2}}{2 s_{R} k_{0} [s_{R} s_{f} k_{0}^{2} - r_{R} r_{f} k^{2}]} \right] \right\}$$

where \( k = \sqrt{(k_{0} - p_{0})^{2} + \mathcal{M}_{i}} \). This should be summed over the two solutions \( p_{0}^{(\pm)} = (\sqrt{\mu^{2} + 4\mathcal{M}_{H^{\pm}}^{2}} \pm \mu^{2})/2 \). In first term of the curly bracket the \( \delta \)-function is used for the \( k_{0} \) integral so in practice one has to find all simultaneous solutions to

$$\left( k_{0} - p_{0} \right)^{2} - k^{2} - \mathcal{M}_{i}^{2} = 0 , \quad \text{and} \quad D_{b}(K) = 0 ,$$

and use the values of \( k_{0} \) and \( k \) in the integrand. For the continuous part it is in principle necessary to integrate over all \( k_{0} \) and for each value of \( k_{0} \) and solve \( k \) from \( k = \sqrt{(k_{0} - p_{0})^{2} - \mathcal{M}_{i}^{2}} \). At the same time there are the factors \( \theta((k_{0} - p_{0})^{2} - \mathcal{M}_{i}^{2}) \theta(k^{2} - k_{0}^{2}) \) that restrict the integration domain.

We have evaluated the damping rate for a Higgsino at rest numerically using \( \bar{h}_{i}^{2} = 1 \) for the Higgsino Yukawa coupling (see Fig. (I)). The scalar contribution to the thermalization rate (in the units of temperature) is presented for two different values of the effective thermal Higgsino mass: \( p_{0} = 1.0T \) (solid line) and \( p_{0} = 0.75T \) (dotted line), and for range of the right-handed stop mass form 0 to \( T \). It shows that for large enough stop mass (\( \mathcal{M}_{i} \gtrsim 0.5T \) and \( \mathcal{M}_{i} \gtrsim 0.25T \) for \( p_{0} = 0.75T \)) the main contribution to the damping rate comes from the (weak) gauge bosons. For the lower end of the stop mass range, however, the scalar contribution is considerably larger (\( \gamma_{b/i} \approx 0.01T \)) but still smaller than \( \gamma_{g_{b}} \).

The aim of this paper has been to establish a reliable value for the damping rate of Higgsinos in MSSM above the electroweak phase transition. Our claim is that the damping is typically dominated by scattering with gauge bosons and that the contribution from scattering with scalars is relatively small. We found that the gauge boson contribution to the damping is given by \( \gamma_{gb} \sim 0.025T \). Scattering with stop/bottom gives a largest contribution for small stop mass (see Fig. (I)) \( \gamma_{b/i} \lesssim 0.01T \), but is much smaller for higher stop mass. Other scattering processes (involving gauginos and scalars Higgses) can be expected to give similar phase space integrals as the stop/bottom scattering but with gauge coupling constants instead of \( \bar{h}_{i} \). We can therefore conclude that they are relatively insignificant. In a similar way we can estimate the damping rate of \( \tilde{H}^{0} \) and \( \tilde{W} \). Actually, neutral Higgsino damping rate coincides
with the charged Higgsino rates, because we are working in the unbroken phase. They also scatter with gauge bosons and we expect this to be the dominant process. In that case the damping rates would be $\gamma_{\tilde{H}^0} \simeq \gamma_{\tilde{b}} \simeq 0.025T$ and $\gamma_{\tilde{W}} \simeq g_2^2 T/2 \pi = 0.065T$.

We would like to reiterate why we only considered one single diagram including scattering with scalars. Even though all of them can be calculated with high accuracy using resummed propagators there is no reason why they should differ significantly numerically, apart from have different overall coupling constants. On the other hand, scattering of moving fermions with gauge bosons is highly non-trivial problem that can only be estimated using assumption about the value of the non-perturbative magnetic mass. Different estimates give qualitatively the same result but the accuracy can of course not be guaranteed. (The situation is different in abelian theories where there exists a resummation procedure controlling the IR sensitivity [25].) It is, therefore, for the moment, no point in trying to achieve higher accuracy in the scalar scattering diagrams which anyway are subdominant. The case is different for $\tilde{B}^0$ which is gauge singlet and thus does not have (one-loop) gauge boson contribution. However, the form of its scalar contribution differs from the scalar contribution of the Higgsinos because of the Majorana nature of $\tilde{B}^0$ [25].

**Figure 1:** Scalar contribution to the Higgsino damping rate as a function of right-handed stop mass in units of $T$. The different curves are for $p_0 = 1.0T$ (solid line) and $p_0 = 0.75T$ (dotted line).
Let us briefly discuss the implications of our findings. As we already mentioned, the precise knowledge of the thermalization rate of the supersymmetric particles is a key ingredient for the computation of the final baryon asymmetry. Sizeable decay rates of the particles propagating in the plasma destroy the quantum interference out of which the the CP-violating sources are built up and therefore reduce the baryon asymmetry. Small decay rates, on the other side, are relevant when the particles reflecting off the advancing bubble wall have comparable masses and resonance effects show up \[9\]. In such a case, the thermalization rates provide the natural width of these resonances and as the present calculation demonstrates, in supersymmetric theories these depend in a complicated way on the particles involved and their plasma masses. Moreover, as shown in \[23, 30\], memory effects proportional to the damping rate of the Higgsinos play a crucial role in determining the lowest value of the CP-violating phases necessary to account for the observed baryon asymmetry. In particular, memory effects introduce an enhancement factor in the final baryon asymmetry of the order of

$$\left( \frac{\gamma_{\tilde{H}}}{5 \times 10^{-2} T} \right) \left( \frac{v_\omega}{0.1} \right)^{0.1} \times 10^{-2}$$

with respect to the results found in \[9\]. Here $v_\omega$ is the velocity of the bubble wall propagating in the plasma. Combining our findings with the analysis of Ref. \[30\] indicate that the phase $\phi_\mu$ of the parameter $\mu$ satisfy the inequality

$$|\sin(\phi_\mu)| \gtrsim 5 \times 10^{-2} \left( \frac{v_\omega}{0.1} \right).$$

These small values of the phase $\phi_\mu$ are consistent with the constraints from the electric dipole moment of the neutron even if the squarks of the first and second generation have masses of the order of 300 GeV.

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