Research on Wear Theory of L-Shaped Elbow

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Abstract. Starting from general wear theory, the solid particle causes erosion loss in pipeline is studied. The motion trajectory of solid particle and theoretical wear rate equation of material surface when elbow suffers erosion wear are summarized, and the effect of attack angle and impact velocity on wear rate of L-Shaped elbow is qualitatively analyzed, so as to provide theoretical basis for the design of elbow.

1. Introduction

The inner wall of oil and natural gas pipeline suffers various impacts due to existence of solid impurity, and it will cause erosion wear. Wear is the phenomenon that materials of solid friction surface gradually lost due to mechanical action. According to general classification method, wear is divided into abrasive wear, erosion wear, fatigue wear and friction wear. As for metal, the relation between wear and friction is not so closely related. Therefore, abrasive wear and erosion wear generally occur in metal pipe.

The solid impurities in pipeline are always iron filings and sands. The erosive rate of elbow is 50 times higher than at straight section based on field experience and data [1], and the effect of impurity particles on the inner wall of elbow is usually more serious than at straight pipe. Therefore, understanding the mechanism is the key to solve the elbow wear, which will contribute to the development of petroleum industry.

In this paper, the wear theory and the motion trajectory of solid particle in L-Shaped elbow is researched based on elastic-plastic fatigue theory, and the effect of attack angle and impact velocity on wear rate is analyzed, so as to provide theoretical basis for the design of elbow.

2. Research on erosion wear in L-Shaped elbow

2.1. Overview of theory for metal material

Erosion wear includes abrasive wear and impact wear. The former’s impact angle of wear is smaller relative to solid surface, even close to parallel. The distribution of particle velocities is so perpendicular to solid surface that particles are embedded in the impacted objects, but the angular velocity which is tangent on surface makes it slide along the object surface, the synthesizing results of two angular velocity are planning effect; The definition of impact wear is the impact angle is larger or close to vertical relative to solid surface, and the particles impacting the surface with a certain velocity makes it generate tiny plastic deformation and microcrack on the surface. As a result, the whole plastic deformation layers are gradually falling with long-term massive impacts.
The scale of erosion wear can be measured by material loss occurred in per abrasive weight \[2\], weight loss rate of erosion wear is defined as \(\varepsilon\):

\[
\varepsilon = \frac{\text{Material weigh loss}}{\text{Abrasive weight}}
\]

Table-1 gives the theories for the analysis of erosion wear for metal.

| No. | Name                  | Basis              | Applicable conditions       |
|-----|-----------------------|--------------------|-----------------------------|
| 1   | Micro-cutting theory  | Micro-cutting      | Low attack angle            |
| 2   | Deformation-cutting theory | Energy balance | High attack angle           |
| 3   | Fatigue theory        | Friction fatigue   | Elastic-plastic contact     |

2.2. The motion trajectory of solid particle in L-Shaped elbow

Figure-1 gives the trajectory of solid particle in L-Shaped elbow:

Assuming particles move close to lateral wall in elbow and are uniform distribution, only considering the drag force as interaction force between particles and fluid, and all of particles are the same diameter without considering inter-collision. So, the principle of hydrodynamics can be applied to determine the trajectory:

\[ \frac{du}{dt} = G(v - u) \quad (1) \]

To the boundary layer:

\[ G = \frac{3\pi\delta\eta}{m_p} \quad (2) \]

To the outflow:

\[ G = \frac{C_pA_s\rho_f|v - u|}{2m_p} \quad (3) \]

Where

- \(m_p\) = particle mass
- \(t\) = time
- \(u\) = particle velocity vector
- \(v\) = fluid velocity vector
- \(\delta\) = particle size
\[ \eta = \text{fluid viscosity} \]
\[ C_D = \text{resistance coefficient} \]
\[ A_s = \text{the projected area of particles perpendicular to the flow direction} \]
\[ \rho_f = \text{fluid density} \]

Using cylindrical coordinates \((\theta, r, z)\):
\[ u_\theta = r \frac{d\theta}{dt} \]
\[ u_r = \frac{dr}{dt} \]
\[ u_z = \frac{dz}{dt} \]  \hspace{1cm} (4)

Combining with the fluid motion equations, equation (1) can be rewritten:
\[ \frac{du_\theta}{dt} + \frac{u_\theta u_\theta}{r} = G(v_\theta - u_\theta) \]
\[ \frac{du_r}{dt} + \frac{u_r^2}{r} = G(v_r - u_r) \]
\[ \frac{du_z}{dt} = G(v_z - u_z) \]  \hspace{1cm} (5)

Assuming in a very short time \(\Delta t\) that \(u_\theta, u_r, u_z\) change with time alone, integrating equation (6) and equation (7):
\[ \theta = \theta + u_\theta \Delta t / r \]
\[ r = r_0 + u_r \Delta t \]
\[ z = z_0 + u_z \Delta t \]  \hspace{1cm} (6)

And
\[ u_\theta = v_\theta - \frac{u_\theta v_\theta}{G_{\theta 0} + 2u_\theta} + \left( u_{\theta 0} - v_\theta + \frac{u_\theta v_\theta}{G_{\theta 0} + 2u_\theta} \right) \exp\left[ -\left( G + u_{r 0} / r_0 \right) \Delta t \right] \]
\[ u_r = v_r + \frac{u_r^2}{G_{r 0}} + \left( u_{r 0} - v_r - \frac{u_r^2}{G_{r 0}} \right) \exp\left[ -G \Delta t \right] \]
\[ u_z = v_z + \left( u_{z 0} - v_z \right) \exp\left[ -G \Delta t \right] \]  \hspace{1cm} (7)

The trajectory can be determined by taking equation (7) into equation (6) [3].

2.3. Theoretical equation of erosion wear rate

The particles are also influenced by centrifugal force in the elbow which changes motion direction of particles. Because of these changes, the particles impact the inner wall of its lateral side with high speed when move through elbow. Small part of the particles slide through surface of the elbow’s inner wall, while the others rebound when colliding with it. These rebounded particles impact the medial side of elbow’s inner wall, and then rebound to the lateral side. Their radial velocity tends to zero after many times’ impacting so that the particles slip and roll close to the wall.
θk

Figure 2. Particles’ motion trajectory in the flow field.

The angle (θk in the figure above), particle whose initial positions is in the channel’s internal diameter begins to slip and roll close to the wall, is called boundary angle or critical angle. Once exceeding the angle, the particles would just slip and roll close to the wall without impacting. As a result, the flowing of particles in the annular pipes can be divided into two levels. The first level (within the θk’s range), the particles’ relative motion to the wall is mainly impacting; the second level (beyond the θk’s range), the particles’ relative motion is mainly slipping or rolling.

The main factors affecting the boundary angle θk are particle size, particles’ relative density, and density of fluid and channel’s geometric characteristics. Data indicate that θk is usually beyond L-Shaped, therefore, the particles’ influence on the wall of the L-Shaped Elbow is mainly impacting, which is called erosion wear.

As to steel pipes, the fatigue theory of elastic contact is applied to calculate wear rate. Set t=2, the theoretical equation of erosion wear rate is:

\[ \varepsilon = K_\alpha \rho^{\frac{3}{2}} \left( \frac{k f}{\sigma_0} \right)^{\frac{4}{3}} \left( \cot \alpha - f \right) \left( v \sin \alpha \right)^{3.2} \]  

### 3. Effects of attack angle and impact velocity on wear rate

The formula above can be used to verify the real wear condition and qualitatively analyze the influences of attack angle and impact velocity on wear rate. Take the natural gas pipelines with high strength low alloy steel as an example, set \( K_\alpha \) (relate to \( \Gamma \) function) of formula (8) = 1.0; \( \rho = 7850 \text{ kg/m}^3 \); \( \rho_p \) is abrasive’s density, set \( \rho_p = 2700 \text{ kg/m}^3 \); steel’s yield strength \( \sigma_0 \) is 360×10⁶ Pa, set hardness=180×10⁶ Pa, scale factor \( k \) (2×hardness/\( \sigma_0 \)) is 1; friction coefficient of particles and inner wall is 0.3; elastic constant \( \Theta = (1-\mu)/E \), about 3.5×10⁻¹² Pa.

Set attack angle \( \alpha \) respectively 15°, 30°, 45°, 60°, 70°, and set particles’ impact velocity \( v \) respectively 5 m/s, 10 m/s, 20 m/s, 30 m/s. Calculation results are as follows:

| \( \alpha \) | \( v \) (5m/s) | 10m/s | 20m/s | 30m/s |
|---|---|---|---|---|
| 15° | 8.75×10⁻⁶ | 80.39×10⁻⁶ | 738.74×10⁻⁶ | 2703.85×10⁻⁶ |
| 30° | 40.02×10⁻⁶ | 275.88×10⁻⁶ | 2535.20×10⁻⁶ | 9279.05×10⁻⁶ |
| 45° | 44.48×10⁻⁶ | 408.79×10⁻⁶ | 3756.64×10⁻⁶ | 13749.64×10⁻⁶ |
| 60° | 33.72×10⁻⁶ | 309.87×10⁻⁶ | 2847.58×10⁻⁶ | 10422.40×10⁻⁶ |
| 70° | 10.10×10⁻⁶ | 92.81×10⁻⁶ | 852.87×10⁻⁶ | 3121.57×10⁻⁶ |
Drawing from the table:

(1) Gas velocity has a great influence on the wearing of square elbow. With the increasing of gas velocity (particles’ impact velocity $v$ is in direct proportion to gas velocity), wear rate of pipe’s inner wall increases quickly at a certain attack angle. The reason is drag force which particle impurities bear enhances with the gas velocity’s growth, then kinetic energy increases and wear loss rises when impact happen. And the faster gas velocity grows, the quicker wear rate increases.

(2) With the augment of attack angle at some impact velocity, wear rate first increases and then decreases. Material surface will form shearing when particles contact with it. This kind of shearing is weak when attack angle is small, and not enough to cause large damage, continuous shear stress will lead to great tearing when attack angle increases to some degree, and then cause large damage, attack angle further increases, and particles’ pressure to the inner wall grows while shearing drops. Experience shows that the most serious wear of plastic material happens when attack angle is between 15° and 45° [4]. The conclusion above has been compared with the one of Kliafas & Holt’s experiment and demonstrated to be accord with it on the whole [5], which proves the validity of selecting theory.

4. Conclusion
This text concludes three theories about erosion wear in metal material, and also reckoned motion trajectory equation of particles in the L-Shaped Elbow and material surface’s theoretical equation. The result is quite accurate when compared with relative experimental ones.

By using the equation above, further study on the effects of attack angle $\alpha$ and impact velocity $v$ on pipe’s ware rate is carried out. The results show that gas velocity affects wear greatly in square elbow and wear loss increases with velocity increasing; under the same particles carrier rate and size, wear rate firstly increases and then decreases while attack angle enhances. The most severe wear happens to plastic materials when attack angle is between 15° and 45°.

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