Multi-field coupling analysis on the film-cooling with transverse and arched trenches

Zhan WANG*, Chao ZHANG**, Wen-jing DU* and Shu-jia LI*

*School of Energy and Power Engineering, Shandong University
17923 Jing-shi Road, Li-xia District, Jinan 250061, China
E-mail: wangzhan@sdu.edu.cn

** Tianjin Key Laboratory for Advanced Mechatronic System Design and Intelligent Control,
School of Mechanical Engineering, Tianjin University of Technology
391 Bin-shui-xi-dao Road, Xi-qing District, Tianjin 300384, China

Received: 8 January 2019; Revised: 30 April 2019; Accepted: 14 June 2019

Abstract
In this study, the properties of the flat-plate film-cooling with transverse and arched trenches are investigated. The conjugate temperature field and thermal stress field were both predicted by the multi-field coupling method. In addition, the transverse trench configuration was investigated and compared as the benchmark case. The results show that the blockage effect formed by the arched trench is helpful to improve the coolant lateral coverage and thus the cooling performance. Meanwhile, the thermal stress concentration may be stronger due to the higher temperature gradient for the arched trench configuration. The distance between the downstream trench edges and the holes is of great importance to the cooling performance. The closer this distance, the more conductive to the blockage effect and the lateral coverage, which is helpful to improve the cooling performance. The distance between the upstream trench edges and the holes shows less impact on the cooling performance. Compared with the non-trench film cooling, the trenched models can form higher temperature gradient, so the thermal expansion near the film hole is more different from that in other region, which tend to form stronger stress concentration near the film hole. Compared with the transverse trench, the arched trench can deliver the coolant jet from the center line to the lateral side, which is helpful to form laterally coverage in the downstream region, especially under the higher blowing ratio. So the arched trench can form lower temperature gradient and relative uniform thermal expansion, which is helpful to decrease the stress concentration around the cooling hole.

Keywords: Film-cooling, Flat-plate, Trench, Multi-field coupling (MFC), Thermal-stress

1. Introduction

Increasing the gas temperature at turbine inlet is one of the important ways to improve the performance of gas-turbines (Han et al., 2000). At the present time, the inlet temperature of most turbines exceeds the acceptable temperature of the vane/blade material, and thus external film-cooling and internal rib-roughed convective and/or impingement cooling have to be applied to assure the turbine safety and durability. Compared with the internal cooling, film-cooling could decrease the temperature on the external surface of blades more effectively. Discrete-hole film-cooling is commonly applied to almost all external surfaces of turbine airfoils.

In the past, round holes were widely used for the external film-cooling, since it is easy to manufacture. However, the round hole may lead to blow-off effects while the jet momentum is higher, which may deteriorate the film-cooling performance. In addition, the well-known “kidney vortices” generated by the interaction between the coolant jet from the round hole and the main flow are disadvantageous to the film-cooling (Sinha et al., 1991) (Thole et al., 1996). In order to improve the lateral coverage and cooling effectiveness, many studies focus on the shape of the cooling holes. Such as the fan-shaped hole (Saumweber et al., 2003), laidback hole (Bohn et al., 2003), convergence slot (Sargison et
The main idea of these holes is to enlarge the area of hole exit and decrease the jet momentum, so that to avoid the lift-off effect and yield the better coolant coverage.

The shaped holes have advantages in cooling performance, but they are not easy to manufacture. Considered the advantage of easy-manufacturing, the transverse trench was put forward to improve the lateral coverage of the round hole. The film cooling characteristics of trench hole was first investigated by Bunker (Bunker, 2002) which based on the experiment of Wang (Wang et al., 2000). Due to the depth under the surface, the coolant jet may be attached to the surface under the higher blowing ratio. Lu (Lu et al., 2005) found that the trench depth of 0.75D was optimum. Dorrington (Dorrington et al., 2007) investigated the trench width and found that the narrow trench performed better film-cooling effectiveness at shallow trench.

Some studies about the trench design considered the shape of downstream trench edge, in order to force the coolant jet laterally spread to gain better coolant coverage. Bogard (Waye and Bogard, 2007) found the perpendicular trench wall at the trailing edge of the film hole performed better than inclined trench edge in adiabatic effectiveness. Oguntade (Oguntade et al., 2014) applied the shaped trench and found that the overall cooling effectiveness can be increased by 35%. Schreivogel (Schreivogel et al., 2014) (Schreivogel and Pfitzner, 2015) put forward some novel narrow segmented trench geometries, the results showed that the segmented trenches had a broader coolant coverage. Wei (Wei et al., 2016) investigate the sine-wave shaped trench which was obtained by changing the transverse trench, and analyzed the anti-kidney vortices generated, and concluded the sine-wave shaped trench has better cooling coverage and lower aerodynamic loss.

Besides the high temperature, thermal stress is another factor which could threaten the gas turbine. The main reason for the occurrence of the thermal stress is the non-uniformity of the material expansion which should attribute to the non-uniform temperature field. Especially, the cooling holes may lead to serious non-uniformity of the temperature distribution and the material expansion. However, the traditional investigations are only based on the viewpoint of flow and heat-transfer, which cannot solve the problem of thermal stress loaded on the cooling structure. Therefore, it is necessary to study the mechanism of the stress caused by the non-uniform temperature distribution. Since the thermal stress is closely related to the temperature field, the calculation of thermal stress should use multi-field coupling (MFC) method rather than semi-empirical estimation. In MFC method, the pressure and temperature are applied to the solid domain as the loads in the mechanical calculation, and the subsequently calculated stress and displacements of the solid domain are viewed as the final results.

In order to check the stress distribution caused by the film cooling, some literatures studied the thermal stress properties by MFC method. Wang (Wang et al., 2009) analyzed the film-cooling on a laminated-plate with impinging holes and concluded that the thermal stress concentrated near the hole and the constrained boundary surface; and the smaller film cooling injection angle may weaken the stress concentration near the hole. Hu (Hu, 2008) analyzed the internal cooling configuration of a guide vane and indicated that the stress concentration may occur near the high temperature gradient locations. Sierra (Sierra et al., 2009), Nowak (Nowak and Wroblewski, 2007), Amaral (Amaral et al., 2008) predicted the stress distributions in the blade with inner coolant passages, and concluded that the stress concentration occurred at the root, leading edge and trailing edge of the blade. Wang (Wang et al., 2013) analyzed the thermal stress distribution of the flat-plate, and evaluated several geometric parameters of holes based on the temperature and stress distributions.

In summary, many studies focus on the cooling mechanism of the film cooling. Generally speaking, the hole itself may reduce the strength. Especially, the temperature difference between the mainstream and coolant exceeds several hundred degrees, so as a result the thermal stress may exceeds allowable limit and damage the vane/blade. In general, thermal stress calculation is used as the check after cooling design. However, there are many factors that determine the thermal stress in the specific vane/blade. In the flat-plate, the main factors to determine the thermal stress are the hole structure and heat transfer condition, which is beneficial to the study of the mechanism.

The trenched design is a good design to improve the film cooling performance under higher coolant momentum. In the present study, a modified trench design is proposed and investigated by the multi-field coupling numerical method. The arched trench configuration is obtained by changing the downstream trench edge from the straight edge to an arched edge. The aim is to generate the anti-kidney vortices which can improve the cooling performance. The MFC method is utilized to calculate the temperature and stress field, in order to investigate the cooling and thermo-elastic properties.
2. Models and calculating methods

Figure 1 shows the computational domains which consist of the hot main flow, flat-plate with cooling holes and thermal barrier coating (TBC) layer, and the coolant flow. The hole diameter is $D=1\text{mm}$, and the width between two holes is $5D$. The injection angle (the angle between the main flow direction and the hole-axis on the $X$-$Z$ plane) is $30^\circ$. The thickness of TBC is $0.5D$. The origin of the coordinate is at the trailing point of hole’s exit, as shown in Fig. 1(a)(b).

The trench models are shown in Fig.1(c). There are 7 cases to be analyzed in this paper. Case NT means the non-trench model with TBC layer, case TT1~3 both have the transverse trench, case AT4~6 both have the arched trench in downstream. Case TT1&AT1 both have broader trench in upstream and downstream, case TT2&AT2 both have narrower trench in upstream and downstream, case TT3&AT3 both have narrower trench in downstream but broader trench in upstream. The boundary conditions of the model are set in accordance with a typical real guide vane of the gas turbine. At the main flow inlet, the velocity $U_m=150\text{m/s}$, the temperature $T_m=1500\text{K}$, and the turbulence intensity is $10\%$. At the main flow outlet the pressure $P_m=10.0\text{atm}$. For the coolant flow, the inlet velocity $U_c=75\text{m/s}$, the temperature $T_c=700\text{K}$, turbulence intensity is $5\%$, and the outlet pressure $P_c$ is set depending on different blowing ratios. The blowing ratio $M=\rho_c U_c/\rho_m U_m=1.0$ and 2.0, where $U_c$ is the injection velocity of the hole outlet. The material of the flat-plate is a Ni-based super alloy, the thermal conductivity is given by $\lambda=10^{-5}T^2-0.0006T+11.7\text{W/(mK)}$ which is about $22$~$30\text{ W/(m-K)}$; the heat capacity is $c=0.6\text{kJ/(kg-K)}$, the density is $\rho=8000\text{kg/m}^3$. For TBC layer, the thermal conductivity is $\lambda=1.3\text{ W/(m-K)}$, the heat capacity is $c=0.88\text{kJ/(kg-K)}$, the density is $\rho=5000\text{kg/m}^3$.

![Diagram](image-url)
In this paper, the multi-field coupling method was used to predict the steady state temperature and thermal stress distributions of the flat-plate with the cooling holes. At first, commercial CFD solver CFX, which is a RANS solver based on the finite-volume method, was used for the prediction of the flow and temperature field. The discretization scheme used for the solver is second-order accuracy. The SST k-ω turbulence model (Menter, 1993) was chosen for the simulations. The conjugate heat transfer (CHT) method in which the energy equation (in tensor form) is solved in the fluid and solid domain together, is used to calculate the fluid-solid coupling heat transfer. To verify the solver and the selected turbulence model, validation was conducted. Fig.3 gives the comparisons of the dimensionless temperature distributions on a flat-plate without TBC layer between the present CFD results and the experimental data (Zhang, 2014). The experimental conditions are: $U_m=35m/s$, $T_m=333.15K$, $T_c=298.15K$, $P_c=1.0atm$, $D=4mm$, $\lambda=16.3W/(m\cdot K)$, $c=0.5kJ/(kg\cdot K)$, $\rho=7930kg/m^3$. The good agreements indicate that the solver CFX and SST turbulence model are qualified for the present study. All the grids for the CFD and FEM calculation are hexahedral elements. There are about 2.3 million elements for the CFD calculation, and about 52,000 elements for the FEM calculation, as shown in Fig.2(a). Grid independence validation was also performed, Fig.2(b) shows the dimensionless temperature distribution on solid surface of case NT under different meshes. The dimensionless temperature is defined as $T_d=(T-T_c)/(T_m-T_c)$, where $T$ is the temperature at any location.

To perform the multi-field coupling analysis, the temperature distribution acquired by the CFD simulation should be substituted to the constitutive equations below. Commercial FEM tool ANSYS was used to predict the thermal stress distribution.

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0$$  \hspace{1cm} (1)
\[ \varepsilon_y = \frac{1}{2} \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) \]  
(2)

\[ \sigma_y = \varepsilon_y E \frac{v\alpha \delta T}{(1 + \nu)(1 - 2\nu)} + \frac{\alpha E \Delta T}{1 - 2\nu} \delta_y \]  
(3)

Where \( \sigma \) is the stress, \( \varepsilon \) is the strain, \( u \) is the displacement; \( T \) is the temperature in the flat-plate, which should be transferred from the CFD result, as shown in Fig.4. And other symbols are the coefficients relate to the material property, among which \( \alpha \) is the thermal expansion coefficient \((1.7 \times 10^{-5} \text{K}^{-1})\), \( E \) is the Young's Modulus \((-0.036T^2 + 7.11T + 114700 \text{ MPa})\), \( \nu \) is the Poisson's ratio \((0.3)\). From the equations above, it can be seen that the steady state stress distribution depends on the forces and constraints imposed on the material, and the temperature distribution.

3. Results and analysis

3.1 Film-cooling without trench

First of all, the MFC cooling property without trench should be investigated as the study’s basis. Fig.5 is the dimensionless temperature on the meridian section and top metal surface, and Fig.6 shows the velocity vectors whose background is the dimensionless temperature field. It can be seen that the temperature is lower near the cooling hole according to the strong convection inside the hole, and the well-known kidney vortex can be generated (see in Fig.6(a),(b)). Under the lower blowing ratio \((M=1.0)\), the low temperature region \((T_d<0.55)\) shows extended coverage in the downstream \((X/D<30)\). However, under the higher blowing ratio \((M=2.0)\), the low temperature region \((T_d<0.55)\) is limited to the near downstream \((X/D<15)\). This phenomenon can be explained by Fig.6, the coolant jet can lift-off the top surface when the blowing ratio is higher, thus the kidney vortices may force the high-temperature gas to the top surface.

3.2 Film-cooling with trench

When the trench is installed, the results and analysis are as follows: Fig.7 shows the temperature on meridian section and top metal surface. Under the lower blowing ratio \((M=1.0)\), the low temperature region \((T_d<0.55)\) shows extended coverage in the downstream \((X/D<30)\). However, under the higher blowing ratio \((M=2.0)\), the low temperature region \((T_d<0.55)\) is limited to the near downstream \((X/D<15)\). This phenomenon can be explained by Fig.7, the coolant jet can lift-off the top surface when the blowing ratio is higher, thus the kidney vortices may force the high-temperature gas to the top surface.

Fig.4 Flow chart of the single-way MFC method

Fig.5 Dimensionless temperature field of no-trench case

Fig.6 Flow chart of the single-way MFC method

Fig.7 Dimensionless temperature field of trench case
Fig.6 Velocity vectors of no-trench case

The “Von-Mises Stress”, which is defined as

\[ \sigma = \sqrt{\frac{1}{2} \left( \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 \right)} \]

It can be seen that the stress is generally low (<30MPa) in most region, but is much greater around the hole. According to the principle of elastic mechanics, the non-uniform thermal expansion may lead to internal force in the material, and thereby leads to thermal stress (Chen, 2007). The temperature around holes is lower than other region, so its expansion is also weaker than other region. Thus, the region around the hole would be pulled by the upstream and downstream region, and the thermal stress can be formed around the holes.

The non-uniform stress distribution exists not only in the flat plate, but also around the hole. It can be seen that the stress concentration is in the \( X \)-direction, especially on the leading and trailing, while in the \( Y \)-direction the stress is very small. According to the principle of elastic mechanics, the stress concentration around the hole would appear in the direction perpendicular to the imposed force. So, the direction of internal force imposed on the hole must be along the \( Y \)-direction; otherwise, the stress on the lateral side would be much higher if the internal force is along the \( X \)-direction.

3.2 Properties of transverse trench

From the previous discussion, the pure flat plate cannot prevent the coolant jet from lifting off the surface under higher blowing ratio. The trenched film cooling is a good method to remedy the problem. At first, the MFC properties of transverse trenches (TT1~TT3) were investigated. Fig.8 and Fig.9 present the temperature field on the top metal surface and meridian section, and Fig.10 shows the streamline.

Compared with the non-trench case, the trenched cases show obvious better cooling performance, because the jets bump against the straight edge and lose momentum and are then pushed downwards towards the surface resulting in jet spreading and improved span wise coverage. The region of \( T_d<0.55 \) can generally cover the top surface in most cases with transverse trench. And also, it is clear that the overall cooling performance is worse under higher blowing ratio \( (M=2.0) \) than that under lower blowing ratio \( (M=1.0) \), except the near hole region. It is certain that the jet momentum is much higher under the blowing ration of \( M=2.0 \), that caused the lift-off effect.

Among all the trenched cases, for the case TT1, the trench is far from the jet, so the block-effect and lateral spread are not as obvious as case TT2 and TT3, and the temperature is higher, especially under \( M=2.0 \). For the case TT3, the jet can be mixed by the main flow according to the broader upstream distance. So its jet temperature is certainly higher than the case TT2, and result in relative weaker cooling performance. So, the case TT2 shows the best cooling performance under all blowing ratios.

Fig.11 presents the corresponding thermal stress the cases above. Generally, the cases with transverse trench show higher stress around the holes than the pure plate. Particularly, under \( M=2.0 \) the case TT2 and TT3 show much larger stress (>200MPa) in the up-downstream of the hole. This phenomenon is caused by the large temperature gradient in
case TT2 and TT3. This is the side effect by the excellent cooling ability. Besides, the upstream cooling mainly lie on the convection in the hole, while the downstream cooling has the contribution of film coverage, so the upstream stress concentration is more stronger than the downstream stress under the higher blowing ratio of $M=2.0$.

$$a) \ M = 1.0 \ \\ b) \ M = 2.0$$

Fig.8 Temperature on top metal surface of flat-plate (TT1~TT3)

$$\text{Fig.9 Temperature on meridian section (TT1~TT3)}$$

$$\text{Fig.10 Streamline upon top surface (TT1~TT3)}$$

$$a) \ M = 1.0, \text{ top metal surface} \ \\ b) \ M = 2.0, \text{ top metal surface}$$
3.3 Properties of Arch Trench

From the previous narration, the transverse can spread the coolant jet and then enhance the coolant coverage. However, under higher blowing ratio of $M=2.0$, the jet remains lift off so that low temperature area is limited around the hole, and then causes stronger stress concentration. The case AT1~AT3 is the modification for case TT1~TT3, in which the downstream trench edge is an arch bent downstream. Fig.12 shows the temperature contours on the top surface in case AT1~AT3, Fig.13 shows the corresponding streamlines.

Compared with the transvers trenches, the arched trenches show similar cooling properties. In case AT1, the downstream trench edge is also far from the jet, so the block-effect is weak and then the temperature is higher, especially under $M=2.0$. In case AT3, the jet can also be mixed by the main flow in the hole’s upstream, so that result in relative weaker cooling performance. The case AT2 shows the best cooling performance among all three arched trenches. From the comparison of lateral-averaged temperature distribution, it is clear that the arched trenches show better cooling performance than the corresponding transverse trenches, as shown in Fig.15. As mentioned above, the blockage of trench edge can force the jet spread laterally. In transverse trenches, the blockage is more abruptly, so smaller amount of jet can be pushed laterally. While in the arched trenches, the blockage is more smooth, so larger amount of jet can be pushed laterally and then form better coolant coverage, as shown in Fig.13.

The stress distribution in arched trenches is also similar with the transverse trenches. The case AT2 and AT3 also show much larger stress (>200MPa) around the hole, especially under the blowing ratio of $M=2.0$. But it should also be recognized that the stress concentration is relative weaker in the arched trenches compared with the corresponding transverse trenches, as shown in Fig.16. This phenomenon mainly lies in the temperature distribution, as shown in Fig.8 and Fig.12. Although the arched trenches show lower temperature, they have relative uniform temperature gradient around the hole, and the thermal expansion around the hole may also be relative uniform, so that the inner force can be weaker.
Fig. 13  Streamline upon top surface (TT1~TT3)

Fig. 14  Stress in flat-plate (AT1~AT3)

Fig. 15  Temperature distribution on top metal surface
4. Conclusions

In this study, the properties of the flat plate film-cooling with transverse and arched trenches are investigated, and the film-cooling with non-trench model was also analyzed as a comparison. Both conjugate temperature and thermal stress field were predicted by the multi-field coupling (MFC) method. The main conclusions are as follows,

1. Due to the blockage effect of the trench, the coolant jets bump against the straight edge and lose momentum and are then pushed downwards towards the surface resulting in jet spreading and improved cooling performance. Meanwhile, the thermal stress concentration may be stronger according to the higher temperature gradient.

2. The thermal stress tends to concentrate around the cooling hole. Compared with the non-trench film cooling, the trenched models can form higher temperature gradient, so the thermal expansion near the film hole is more different from that in other region, which tend to form stronger stress concentration near the film hole.

3. Compared with the transverse trench, the arched trench can deliver the coolant jet from the center line to the lateral side, which is helpful to form laterally coverage in the downstream region, especially under the higher blowing ratio. So the arched trench can form lower temperature gradient and relative uniform thermal expansion, which is helpful to decrease the stress concentration around the cooling hole.

Acknowledgement

The authors wish to acknowledge the financial supports from the National Science Foundation of Tianjin through Grant No. 18JCQNJC07200 and the National Natural Science Foundation of China through Grant No. 51506150.

Nomenclature

| Symbol | Description | Unit |
|--------|-------------|------|
| c      | Heat capacity of flat-plate [kJ·kg⁻¹·K⁻¹] |      |
| D      | Diameter of the cooling hole [mm] |      |
| E      | Young's modulus [MPa] |      |
| M      | Blowing ratio |      |
| P      | Pressure [MPa] |      |
| T      | Temperature [K] |      |
| U      | Velocity [m·s⁻¹] |      |
| α      | Thermal expansion coefficient [K⁻¹] |      |
| ε      | Strain [mm/mm] |      |
| ρ      | Density of flat-plate [kg·m⁻³] |      |
| ν      | Poisson's ratio |      |
| σ      | Stress [MPa] |      |
| δ      | Kronecker delta of Tensor; δ=1, when i=j; δ=0, i≠j; |      |
| u      | Displacement in the strain equation |      |
| i,j,k  | Tensor subscription |      |
| m      | Mainflow |      |

Subscript

- c: Coolant
- d: Dimensionless
- m: Mainflow

Fig.16 Stress distribution inside cooling hole (TT and AT cases)
References

Amaral, S., Verstraete, T., Braemmbussche, R. V., and Arts, T., Design and optimization of the internal cooling channels of a HP turbine blade: part I-methodology, Proceeding of ASME Turbo Expo (2008), Paper No. GT2008-51077.

Bohn, D., Ren, J., and Kusterer, K., Conjugate heat transfer analysis for film cooling configurations with different hole geometries, Proceeding of ASME Turbo Expo (2003), Paper No. GT2003-38369.

Bunker, R. S., Film cooling effectiveness due to discrete holes within a transverse surface slot, Proceeding of ASME Turbo Expo (2002), Paper No. GT2002-30178.

Chen, M. X., Mechanics of elastic and plastic (2007), Science Press (in Chinese).

Dorrington, J. R., Bogard, D. G., and Bunker, R. S., Film effectiveness performance for cool ant holes embedded in various shallow trench and crater depressions, Proceeding of ASME Turbo Expo (2007), Paper No. GT2007-27992.

Han, J. C., Dutta, S., and Ekkad, S., Gas turbine heat transfer and cooling technology (2000), Taylor & Francis.

Hu, J., Investigation on closed-loop steam cooling schemes of a gas turbine guide vane (2008), Master Degree Thesis of Chinese Academy of Sciences (in Chinese).

Lu, Y., Hasan, N., and Ekkad, S. V., Film cooling from a row of holes embedded in transverse slots, Proceeding of ASME Turbo Expo (2005), Paper No. GT2005-68598.

Menter, F. R., Zonal Two Equation k-ω Turbulence Models For Aerodynamic Flows, 24th fluid dynamic conference of AIAA (1993), Paper No. AIAA93-2906.

Nowak, G. and Wroblewski, W., Thermal mechanical optimization of cooled turbine blade, Proceeding of ASME Turbo Expo (2007), Paper No. GT2007-28196.

Oguntade, H. I., Andrews, G. E., Burns, A., and Ingham, D. B., Conjugate heat transfer predictions of effusion cooling with shaped trench outlet, Proceeding of ASME Turbo Expo (2014), Paper No. GT2014-25257.

Sargison, J. E., Oldfield, M. L. G., and Guo, S. M., Flow visualization of the external flow from a converging slot-hole film-cooling geometry, Experiments in Fluids, Vol.38 (2005), pp.304-318.

Saumweber, C., Schulz, A., and Wittig, S., Free-stream turbulence effects on film-cooling with shaped holes, ASME Journal of Turbomachinery, Vol.125 (2003), pp.65-73.

Schreivogel, P. and Pfiztnner, M., Heat transfer measurements downstream of trenched film cooling holes using a novel optical two-layer measurement technique, Proceeding of ASME Turbo Expo (2015), Paper No. GT2015-42385.

Thole, K., Gritsch, M., and Schulz, A., Flowfield measurement for film-cooling holes with expanded exits, Proceeding of ASME Turbo Expo (1996), Paper No. 96-GT-174.

Sierra, F. Z., Bolaina, C., Kubiac, J., Han, J. C., Narzary, D., and Nebradt, J., Heat transfer and thermal mechanical stress distributions in gas turbine blades, Proceeding of ASME Turbo Expo (2009), Paper No. GT2009-59194.

Sinha, A.K., Bogard, D. G., and Crawford, M. E., Film-cooling effectiveness downstream of a single row of holes with variable density ratio ASME Journal of Transaction, Vol.113 (1991), pp.442-456.

Thole, K., Gritsch, M., and Schulz, A., Flowfield measurement for film-cooling holes with expanded exits, Proceeding of ASME Turbo Expo (1996), Paper No. 96-GT-174.