Element segregation in giant galaxies and X-ray clusters

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ABSTRACT
We examine the process of element segregation by gravity in giant elliptical galaxies and X-ray clusters. We solve the full set of flow equations developed by Burgers for a multi-component fluid, under the assumption that magnetic fields slow the diffusion by a constant factor $F_B$. Compared to the previous calculations that neglected the residual heat flow terms, we find that diffusion is faster by $\sim 20\%$. In clusters we find that the diffusion changes the local abundance typically by factors of $1 + 0.3(T/10^8\text{ K})^{1.5}/F_B$ and $1 + 0.15(T/10^8\text{ K})^{1.5}/F_B$ for helium and heavy elements, respectively, where $T$ is the gas temperature. In elliptical galaxies, the corresponding factors are $1 + 0.2(T/10^7\text{ K})^{1.5}/F_B$ and $1 + 0.1(T/10^7\text{ K})^{1.5}/F_B$, respectively. If the suppression factor $F_B$ is modest, diffusion could significantly affect observational properties of hot X-ray clusters and cD galaxies. In particular, diffusion steepens the baryon distribution, increases the total X-ray luminosity, and changes the spectrum and evolution of stars that form out of the helium-rich gas. Detection of these diffusion signatures would allow to gauge the significance of the magnetic fields, which also inhibit thermal heat conduction as a mechanism for eliminating cooling flows in the same environments.

Key words: galaxies: clusters: general – cosmology:theory – dark matter – X-rays: galaxies.

1 INTRODUCTION

In a multicomponent plasma, gravity acts so as to separate different elements. The thermal equilibrium state for the number density of particles of mass $m_i$ and temperature $T$ is proportional to the Boltzmann factor, $e^{-m_i\phi(r)/kT}$, where $\phi(r)$ is the gravitational potential. Thus in self-gravitating objects formed out of a homogeneous mixture of elements, the heavier elements (such as helium, metals) sink towards the center and the lighter elements (such as hydrogen) rise up. Fabian & Pringle (1977) estimated that in high-temperature X-ray clusters significant abundance gradients might arise for metals. However, their calculation did not allow for the suppression of diffusion by magnetic fields, which could play an important role in clusters. Since a charged particle is confined to move along magnetic field lines, diffusion can proceed uninhibited only on the coherence length-scale of the magnetic field. In clusters the scale of magnetic field is likely to be small and the diffusion speed could be suppressed by several orders of magnitude (Chandran & Cowley 1998), making its effect negligible. In recent years, however, it has been suggested that turbulence may suppress transport processes by only a modest factor of a few (Narayan & Medvedev 2001; Malyshevkin 2001). Such models are motivated by the lack of observational evidence for cooling flows in the cores of clusters and elliptical galaxies (Mollendi & Pizzolato 2001; Fabian et al. 2001). Cooling flows would be suppressed if thermal heat conduction by electrons is not strongly inhibited by the magnetic fields.

In this paper we estimate the impact of diffusion on observational properties of galaxies and X-ray clusters for different magnetic suppression factors. Unlike previous analyses (Fabian & Pringle 1977, Rephaeli 1978, Gilfanov & Sunyaev 1984, Qin & Wu 2000, Chuzhoy & Nusser 2003), we solve the full diffusion equations derived by Burgers (1969). We find that if the suppression factor is modest, then diffusion would lead to significant abundance gradients for all elements. In extreme cases, the abundance of helium (which except for deuterium is the fastest diffusing element) can grow above that of hydrogen. Because helium sedimentation changes the mean molecular weight, the gas density profile steepens and the total luminosity of the cluster increases. Since the diffusion speed depends strongly on temperature, this effect may contribute to the luminosity-temperature dependence observed in clusters (Chuzhoy & Nusser 2003). The change in the mean molecular weight would also affect mass estimates of hot clusters and elliptical galaxies under...
the assumption of hydrostatic equilibrium (Qin & Wu 2000, Chuzhoy & Nusser 2003). In addition to its effect on the X-ray emitting gas, the increase in the helium abundance could affect the spectrum and evolution of stars that form out of this gas.

Because the diffusion speed in a plasma scales with temperature as \( T^{3/2} \), all previous work was restricted to X-ray clusters which are the hottest virialized objects. However, we find that because of the lower gas mass fraction, diffusion would proceed faster in hot cD galaxies with \( T \sim 2 \times 10^7 \) K than in clusters with \( T < 10^8 \) K.

The paper is organized as follows. In §2.1 we write the general diffusion equations. In §2.2 we evaluate the diffusion timescales and describe the diffusion in the perturbative regime, where the fractional deviations of the abundances from uniformity are small. In §3 we describe diffusion in the non-linear regime, assuming the NFW mass profile for the dark matter (Navarro, Frenk & White 1997). We summarize our results in §4.

## 2 DIFFUSION EQUATIONS

### 2.1 General equations

Each species of particles \( s \) is described by a distribution function \( F_s(x, v, t) \) normalized to unit integral, a mean number density \( n_s \), an ionic charge \( q_s = Z_s e \) and a particle mass \( m_s \). The cross-section for Coulomb scattering between particles of species \( s \) and of species \( t \) is given by

\[
\sigma_{st} = 2\sqrt{\pi} e^4 Z_s^2 Z_t^2 \left( k_B T \right)^{-2} \ln \Lambda_{st},
\]  
(1)

where \( \ln \Lambda_{st} \) is the Coulomb logarithm. For simplicity, we approximate the Coulomb logarithm for all species by a constant, \( \ln \Lambda_{st} = 40 \), which is characteristic for clusters. The friction coefficient between species \( s \) and \( t \) is

\[
K_{st} = (2/3)n_s n_t \sigma_{st} \left( \frac{2k_B T n_s m_t}{m_s + m_t} \right)^{1/2}.
\]  
(2)

The mean fluid velocity is

\[
u = \frac{\langle \sum_s n_s m_s u_s \rangle}{\langle \sum_s n_s m_s \rangle},
\]  
(3)

where \( u_s \) is the mean fluid velocity of each species. The diffusion velocity of species \( s \) is defined as

\[
w_s = u_s - \nu.
\]  
(4)

and its “residual heat flow vector” is (Burgers 1969)

\[
r_s = \frac{m_s}{2k_B T} \int F_s (v - u) |v - u|^2 dv - \frac{5}{2} w_s.
\]  
(5)

If the gas is in hydrostatic equilibrium (i.e. \( \nabla p/\rho = g \)), the Burgers equations for mass, momentum and energy conservation in spherical symmetry, as formulated by Thoul, Bahcall & Loeb (1994), are

\[
\frac{dn_s}{dt} = -\frac{1}{r^2} \frac{\partial (r^2 n_s u_s)}{\partial r},
\]  
(6)

\[
\frac{d(n_s k_B T)}{dr} + n_s m_s g - n_s Z_s e E = \sum_i K_{si} \left[ (w_s - u_s) + 0.6 (x_{st} r_s - y_{st} r_t) \right],
\]  
(7)

\[
\frac{5}{2} n_s k_B \frac{dT}{dr} = -0.8 K_{st} r_s + \sum_i K_{si} \left( \frac{3}{2} x_{st} (w_s - v_t) - y_{st} \left[ 1.6 z_{st} r_s + r_t \right] + y_{st} - 4.3 x_{st} r_t \right),
\]  
(8)

where \( g \) is the gravitational acceleration, \( E \) is the electric field, \( x_{st} = m_t/(m_s + m_t) \), \( y_{st} = m_s/(m_s + m_t) \) and \( y_{st} = 3h_{st} + 1.3 z_{st} m_t/m_s \). From equations (7) and (8) we can obtain a solution for the diffusion velocity and the heat flow of each species

\[
w_s = \frac{T^{5/2}}{n_p} \left( \sum_i I_{si} \frac{\partial \ln n_i}{\partial r} + J_{st} \frac{\partial \ln T}{\partial r} \right),
\]  
(9)

\[
r_s = \frac{T^{5/2}}{n_p} \left( \sum_i J_{si} \frac{\partial \ln n_i}{\partial r} + J_{st} \frac{\partial \ln T}{\partial r} \right),
\]  
(10)

where the coefficients \( I_{si}, I_{st}, J_{st} \) and \( J_{st} \) depend only on the composition of the gas. In the presence of magnetic fields, we introduce a constant suppression factor, \( F_p^{-1} \), for all these coefficients. The actual suppression factor in reality should depend on the geometry and fluctuations of the magnetic field which may vary with position; but the lack of detailed data on these properties calls for a simple-minded approach of the type we adopt.

If the diffusion is “switched-on” when the gas is in hydrostatic equilibrium, then initially there would be no mass flow (\( u = 0 \)). For each sinking helium ion there must be approximately four protons and two electrons that rise up. This means that there is a net outflow of particles, which leads to a decrease in the total pressure \( (p = \Sigma n_s k_B T) \). For a fixed mass density, the drop in gas pressure means that the assumption of hydrostatic equilibrium \( (\nabla p/\rho = g) \) can not hold once diffusion starts to operate. In practice the sound speed of the gas is much larger than the diffusion speed, i.e. the dynamical timescale of the system is much shorter than the diffusion timescale. Thus, the gas will always be close to hydrostatic equilibrium and the above equations for the relative velocities of all species remain valid. To find a small mean fluid velocity, we solve the equation of motion,

\[
\frac{du}{dt} = -\frac{\nabla p}{\rho} + g,
\]  
(11)

where \( g = g(r) \) is the gravitational field dictated by the total mass distribution.

If we wish to evaluate the diffusion velocity of hydrogen \( (s \equiv p) \) and helium \( (s \equiv \alpha) \), which are the dominant components of the cosmic plasma, we may neglect the contribution of all other species. Thus, the drift velocity of helium ions is

\[
w_\alpha = \frac{T^{5/2}}{n_p} \left[ I_p \frac{\partial \ln n_p}{\partial r} + I_\alpha \frac{\partial \ln n_\alpha}{\partial r} + J_{st} \frac{\partial \ln T}{\partial r} \right],
\]  
(12)
The velocity of the protons can be obtained simply from the relation $w_p = w_a (X - 1)/X$.

The diffusion velocity of heavy elements is determined primarily by the drag forces from hydrogen and helium (Chuzhoy & Nusser 2003). Helium provides an inward radial friction force for the metals, while protons push them radially outwards. The resulting velocity can be determined by balancing these friction forces. Together with the pressure gradient force, the gravitational and electric forces for the metals increase their diffusion velocity relative to hydrogen by a factor $\sim (1 + 4/Z)$ (Chuzhoy & Nusser 2003). For heavy elements such as iron ($Z = 26$) this correction can be ignored. Using equation (7) we obtain the approximation for the diffusion velocity of all heavy elements

$$w_a = \frac{(1 - X)w_a - 1.2(1 - X)r_a - 0.6Xr_p}{(2 - X)}$$  \hspace{1cm} (13)

The total heat flow and the temperature evolution are given by

$$Q = \sum \left( n_s r_s kT \right),$$  \hspace{1cm} (14)

$$\sum \frac{\partial (s n_s kT)}{\partial t} = \frac{1}{r^2} \frac{\partial (s^2 Q)}{\partial r}$$  \hspace{1cm} (15)

Somewhat surprisingly it turns out that $Q$ and $dT/dt$ are generally nonzero even in the absence of a temperature gradient. As implied by equation (10), the density gradients can alone be responsible for the heat transport and eventually cause temperature gradients. However, because the coefficients $J_s$ are several orders of magnitude smaller than $J_T$, the effect of the heat flow on the temperature profile is negligible in absence of an initial temperature gradient. We also find from Burgers equations that the density gradients can not produce the heat transport and temperature gradients after all elements reach equilibrium distribution.

## 2.2 Diffusion in the perturbative regime

The helium to hydrogen ratio starts from the uniform distribution set by the big bang. In this section, we derive simple analytic results for small deviations from this uniform initial state. In the interiors of X-ray clusters and elliptical galaxies, the density changes by a much larger factor than the temperature and so $|\partial \ln n_s/\partial r| \gg |\partial \ln T/\partial r|$. Since $I_s$ and $I_T$ are of similar magnitude, equation (12) simplifies to

$$w_{a,0} = \frac{T_{\alpha}^{5/2}}{n_p} (I_p^a + I_T^a) \frac{\partial \ln n_p}{\partial r}$$  \hspace{1cm} (16)

Initially the hydrogen fraction $X \approx 3/4$ everywhere and $I = I_p^a + I_T^a = 2.6 \times 10^7 F_B \text{ cm}^{-1} \text{s}^{-1} \text{K}^{-5/2}$, where $F_B$ is the suppression factor due to the magnetic field. Using the assumption of approximate hydrostatic equilibrium, we can rewrite equation (16) as

$$w_{a,0} = 60 \text{ km s}^{-1} \times \left( \frac{g}{10^{-7.5} \text{ cm s}^{-2}} \right) \left( \frac{T}{10^8 \text{K}} \right)^{3/2} \left( \frac{n_p}{10^{-3} \text{ cm}^{-3}} \right)^{-1} F_B^{-1}$$  \hspace{1cm} (17)

The velocity obtained above is $\sim 20\%$ higher than the estimate by Chuzhoy & Nusser (2003) that neglected the heat flow terms and $\sim 30\%$ lower than the estimate by Qin & Wu (2000) that also neglected electric fields.

Substituting equation (17) into the continuity equation gives

$$\frac{\partial n_{s \alpha}}{\partial r} = \frac{n_{s \alpha}}{12 \text{ Gyr}} \left( \frac{f_g}{0.1} \right) \left( \frac{T}{10^8 \text{K}} \right)^{3/2} F_B^{-1}$$  \hspace{1cm} (18)

where $f_g$ is the local gas fraction. Equation (18) implies that at the initial stages of diffusion, the local helium density grows as $e^{\tau/\tau} \sim (1 + t/\tau)$, where

$$\tau = 12 \text{ Gyr} \left( \frac{f_g}{0.1} \right) \left( \frac{T}{10^8 \text{K}} \right)^{-3/2} F_B$$  \hspace{1cm} (19)

Similarly the density of hydrogen declines as $e^{-t/3\tau}$ for $X = 3/4$, so that the relative abundance of helium grows as $e^{t/3\tau}$. The initial diffusion velocity of metals is $\sim \frac{1}{3}$ of helium velocity, and so the metallicity grows as $\sim e^{2t/3\tau}$.

## 3 DIFFUSION IN GALAXIES AND CLUSTERS

For the radial distribution of the mass density of the dark matter, we assume the NFW profile (Navarro, Frenk & White 1997)

$$\rho_d(r) = \frac{\rho_s}{(c r/r_{vir})(1 + c r/r_{vir})^2},$$  \hspace{1cm} (20)

where $\rho_s = \text{ const}$, $c$ is the concentration parameter and $r_{vir}$ is the virial radius. We make separate calculations for $c = 4$ and $c = 15$, which are characteristic values for clusters and elliptical galaxies, respectively (Navarro, Frenk & White 1997; Wechsler et al. 2002). We assume that the gas is initially isothermal and in hydrostatic equilibrium. Neglecting the contribution of the baryons to the gravitational potential, one obtains the following radial mass profile for the gas (Makino, Sasaki & Suto 1998)

$$\rho_b(r) = \rho_0 e^{-\eta (1 + c r/r_{vir})^{3/2}/c},$$  \hspace{1cm} (21)

where $\eta = 4\pi G m_p \rho_{b, \text{vir}}^2 / c^2 kT$. Assuming that the gas is at the virial temperature, $\eta = 10$ and $\eta = 16.5$ correspond to $c = 4$ and $c = 15$ respectively. For clusters, we chose the ratio $\rho_0/\rho_s$ so that the gas mass fraction within the virial radius is equal to the cosmic value of $(\Omega_b/\Omega_m) = 0.17$ inferred by WMAP (Bennett et al. 2003). In elliptical galaxies, typically $\sim 90\%$ of the baryonic mass is in stars (Brigenti & Mathews 1997), and so we take the gas fraction ten times lower. We assume that the initial abundances are uniform and that the temperature remains constant in time as well as in space.

We solved equations (6)–(8) and (11) for the above initial conditions. Because the gas was assumed to be isothermal, the evolution of metallicity and helium abundance (fig. 1–4) depends mainly on the local gas mass fraction (fig. 5...
and 6). In clusters, the gas mass fraction declines monotonically with radius initially, accounting for the rise in metallicity and helium abundance towards the center. In ellipticals, the gas fraction profile is more complex, which in turn results in non-monotonic abundance profiles.

4 DISCUSSION

Our calculations indicate that in the absence of magnetic suppression, the diffusion timescale in hot clusters and cD galaxies is comparable to the age of these systems. For a suppression factor $F_B \gtrsim 5$, the diffusion can be well described in the perturbative regime (§2.2) for most clusters and galaxies, except in the central regions of very hot clusters. The abundance change depends on the gas mass fraction, and characteristic numbers were derived by adopting $f_g = 0.17$.
and $f_B = 0.017$ for clusters and cD galaxies, respectively. Interior to the virial radius of clusters with an age $t \sim 5$ Gyr, the helium and metal abundance would increase by a factor of $1 + 0.3(T/10^8 \text{ K})^{1.5}/F_B$ and $1 + 0.15(T/10^8 \text{ K})^{1.5}/F_B$, respectively. For elliptical galaxies with an age $t \sim 10$ Gyr, the corresponding factors are $1 + 0.2(T/10^7 \text{ K})^{1.5}/F_B$ and $1 + 0.1(T/10^7 \text{ K})^{1.5}/F_B$, respectively. Although the effect of diffusion on the distribution of metals is rather modest, its effect on the distribution of helium is more substantial. As indicated by Figures 1 and 2, some regions inside the hottest X-ray clusters or cD galaxies, show an increase of $\sim 50\%$ of helium abundance for $F_B = 3$.

Detection of the diffusion signatures can be used to gauge the effect of magnetic fields which also limit thermal heat conduction from suppressing cooling flows in the same environments. Although the increased helium abundance can not be observed directly, it leads to a number of indirect observable signatures.

As shown by Fig. 5, diffusion results in a substantial steepening of the gas distribution. This would in turn lead to an increase in the total X-ray luminosity. Since diffusion is fastest in high temperature clusters, this process may contribute to the discrepancy between the observed luminosity-temperature relation $L \propto T^3$ (Mushotzky 1984; Edge & Stewart 1991; David et al. 1993) and the form $L \propto T^2$ expected from self-similar arguments (Kaiser 1986).

The mass profile of clusters and elliptical galaxies is usually inferred from the observed distribution of hot X-ray gas under the assumption of hydrostatic equilibrium. Increasing the helium fraction by $\sim 50\%$ is equivalent to increasing the mean molecular weight by $\sim 10\%$, which in turn results in overestimating the mass also by $\sim 10\%$. Thus, in cases where alternative methods for mass determination (such as gravitational lensing or the Sunyaev-Zel’dovich effect) reach better precision, these methods may be used to constrain the diffusion velocity and the magnetic suppression coefficient.

The change in the helium fraction would also affect the evolution of stars formed from the enriched gas, D’Antona et al. (2002), who analyzed globular clusters, found that increasing the initial helium abundance by 20% results in the formation of a very blue horizontal branch. It would be particularly interesting to search observationally for the distinct spectral and evolutionary characteristics of helium-rich stars within X-ray clusters and cD galaxies.

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REFERENCES

Bennett, C. et al. 2003, ApJS, 148, 1
Brigenti, F., Mathews W.G. 1997, ApJ, 486, L83

Burgers, J.M. 1969, Flow Equations for Composite Gases (Academic Press, New York)
Chandran, P.D.G., Cowley, S.C. 1998, Phys. Rev. Lett., 80, 3077
Chuzhoy, L., Nusser, A. 2003, MNRAS, 342, L5
D’Antona, F., Caloi, V., Montalban, J., Ventura, P., Gratton, R. 2002, A&A, 395, 69
David, L. P., Slyz, A., Jones, C., Forman, W., Vrtilek, S.D., & Arnaud, K.A. 1993, ApJ, 412, 479
Edge, A.C. & Stewart, G.C. 1991, MNRAS, 252, 414
Fabian, A.C., Mushotzky, R.F., Nulsen, P.E.J., Peterson J.R. 2001, MNRAS, 321, L20
Fabian, A.C., Pringle, J.E. 1977, MNRAS, 181, 5P
Gilfanov, M.R., Sunyaev, R.A. 1984, Soviet Astron. Lett., 10, 137
Kaiser, N. 1986, MNRAS, 222, 323
Makino, N., Sasaki, S., Suto, Y. 1998, ApJ, 497, 555
Malyshkin, L. 2001, ApJ, 554, 561
Molendi, S., Pizzolato, F. 2001, ApJ, 560, 194
Mushotzky, F.F. 1984, Phys. Scr., T7, 157
Narayan, R., Medvedev, M.V. 2001, ApJ, 562, L129
Navarro, J.F., Frenk, C.S., White, S.D.M. 1997, ApJ, 490, 493
Qin, B., Wu, X. 2000, ApJ, 529, L1
Rephaeli, Y. 1978, ApJ, 225, 335
Thoul, A., Bahcall, J.N., Loeb, A. 1994, ApJ, 421, 828
Wechsler, R.H., Bullock J.S., Primack J.R., Kravtsov A.V., Dekel A. 2002, ApJ, 568, 52

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