Unparticle as a particle with arbitrary mass

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Abstract

The unparticle field operator can be expanded in terms of creation and destruction operators corresponding to particles with a continuous mass spectrum. Hence, when the 4-momentum of an unparticle is measured, then the unparticle manifests as an ordinary particle with a definite (but arbitrary) mass.

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1 Introduction

The possibility of the existence of a scale invariant sector of an effective field theory recently proposed in [1] has received a considerable attention. (For the most recent works with extensive lists of references see, e.g., [2, 3, 4].) It has been argued [1] that physics of that sector cannot be described in terms of particles, which is why this sector has been dubbed *unparticle* stuff. In particular, it has been obtained [1] that the unparticle stuff with the scale dimension $d_U$ may appear as a non-integer number $d_U$ of invisible particles.

After the initial work [1], it has been realized that certain aspects of unparticle physics can be viewed as physics of quantum fields with a continuous mass spectrum [5, 6, 7]. This suggests that, at the fundamental level, the unparticle stuff may not be so different from the ordinary notion of particles appearing in quantum field theory. In this paper we further explore such a particle interpretation of the unparticle stuff. The next section deals with particles with a discrete and continuous mass spectra, while the relation with unparticles is discussed in Sec. 3. Additional remarks on canonical quantization of unparticle fields are given in Sec. 4.
2 Particles with various masses

Consider a discrete collection of free hermitian scalar fields $\phi(x,m)$, each having a different mass $m$. These fields satisfy the canonical equal-time commutation relations

$$[\phi(x,m), \pi(x',m')] = i\delta^3(x - x')\delta_{mm'},$$  \hspace{1cm} (1)

where $\pi = \partial_0 \phi$ are the canonical momenta. By expanding the fields in terms of plane waves in the standard way, one obtains the creation and destruction operators $a^\dagger(k,m)$ and $a(k,m)$, respectively, that obey the commutation relations

$$[a(k,m), a^\dagger(k',m')] = f(k,m)\delta^3(k - k')\delta_{mm'},$$  \hspace{1cm} (2)

with other commutators vanishing. The function $f(k,m)$ is a matter of normalization, with a few different normalizations appearing in the literature. As is well known, these commutation relations imply that the quantum fields exhibit a particle interpretation; the representation space of the field algebra is constructed by acting with the creation operators $n = 1, 2, 3, \ldots$ times on the vacuum $|0\rangle$, corresponding to states with an integer $n$ number of particles. In particular, the most general 1-particle state is a state of the form

$$|1\rangle = \sum_m \int d^3k c(k,m)|k,m\rangle,$$  \hspace{1cm} (3)

where $|k,m\rangle = a^\dagger(k,m)|0\rangle$ and $c(k,m)$ is an arbitrary function normalized so that $\sum_m \int d^3k |c(k,m)|^2 = 1$.

The physical meaning of a superposition (3) involving various types of particles specified by different masses $m$ may seem peculiar. Nevertheless, such a peculiarity disappears when the mass $m$ is measured. According to the standard rules of quantum mechanics, when an observable is measured then the value of this observable attains a definite value. Thus, when $m$ is measured, one observes a particle of a definite type specified by the mass $m$. The probability that the measured mass will have the value $m$ is equal to $\int d^3k |c(k,m)|^2$. Furthermore, if the 3-momentum is also measured simultaneously with the mass, one observes a particle with a definite 4-momentum $k = (k,k_0)$, where $k_0$ and $m$ are related as

$$m^2 = k_0^2 - k^2 \equiv k^2.$$  \hspace{1cm} (4)

Now let us generalize it to the case of a continuous mass spectrum. Clearly, the Kronecker $\delta_{mm'}$ gets replaced by the Dirac function $\delta(m - m')$. However, to provide a manifest Lorentz covariance in the momentum space, it is more convenient to replace the independent variables $(k,m)$ by another set of independent variables $(k,k_0)$, where the variables $m$ and $k_0$ are related as in (4). Thus, with an appropriate choice of the normalization $f$, (2) generalizes to

$$[a(k), a^\dagger(k')] = \delta^4(k - k').$$  \hspace{1cm} (5)

Such a commutation relation has also been introduced in [8]. Clearly, just as (2), such a commutation relation also admits a particle interpretation; an $n$-particle state
is obtained by acting $n$ times with the creation operators $a^\dagger(k)$ on the vacuum $|0\rangle$. In particular, (3) generalizes to

$$|1\rangle = \int d^4 k \ c(k)|k\rangle,$$

where $|k\rangle = a^\dagger(k)|0\rangle$ and $c(k)$ is an arbitrary function normalized so that $\int d^4 k \ |c(k)|^2 = 1$. Analogously to the case of a discrete mass spectrum discussed above, when the 4-momentum $k$ is measured, then one observes an ordinary particle with a definite 4-momentum $k$. In this case the mass $m^2 = k^2$ also takes a definite value. The only difference with respect to the discrete-mass case is the fact that now the mass may take any value $m$, with the probability density equal to $\int d^4 k \ \delta(|k| - m)|c(k)|^2$.

3 The relation with unparticle fields

The physics of the particle creation and destruction operators obeying (5) can be viewed as physics of a one-parameter family of massive fields $\phi(x, m)$, where $m$ is a continuous parameter. But there is also a different view of the same creation and destruction operators. Instead of dealing with the family of fields $\phi(x, m)$, one can deal with a single field

$$\Phi(x) = \int \frac{d^4 k}{(2\pi)^4} F(k) \theta(k_0) \theta(k^2) [a(k)e^{-ikx} + a^\dagger(k)e^{ikx}].$$

The function $F(k)$ can be determined by imposing the requirement of scale invariance. Following [1], the requirement of scale invariance implies that the 2-point function must have the form

$$\langle 0| \Phi(x)\Phi(0)|0\rangle = A_{dU} \int \frac{d^4 k}{(2\pi)^4} \frac{F^2(k)}{(2\pi)^4} \theta(k_0) \theta(k^2) e^{-ikx}.$$  

On the other hand, (5) implies

$$\langle 0|a(k)a^\dagger(k')|0\rangle = \delta^4(k - k'),$$

so, using the fact that the step function satisfies $\theta^2 = \theta$, the expansion (7) leads to

$$\langle 0|\Phi(x)\Phi(0)|0\rangle = \int \frac{d^4 k}{(2\pi)^4} \frac{F^2(k)}{(2\pi)^4} \theta(k_0) \theta(k^2) e^{-ikx}.$$  

We see that (10) is compatible with (8), provided that we take

$$F(k) = \sqrt{A_{dU}(2\pi)^4 (k^2)^{(d_U - 2)/2}}.$$  

Thus, we see that the unparticle field operator introduced in [1] can be expanded as

$$\Phi(x) = A_{dU}^{1/2} \int \frac{d^4 k}{\sqrt{(2\pi)^4}} (k^2)^{(d_U - 2)/2} \theta(k_0) \theta(k^2)$$

$$\times [a(k)e^{-ikx} + a^\dagger(k)e^{ikx}].$$
Despite the name “unparticle”, the operators $a(k)$ and $a^\dagger(k)$ in (12) have a well defined particle interpretation, as discussed in Sec. 2.

Now let us resolve an apparent conceptual paradox. On one hand, the operators $a^\dagger(k)$ and $a(k)$ create and destruct states with an integer number of particles, completely independent of the value of the scale dimension $d_U$. On the other hand, it is shown in [1] that the unparticle stuff manifests as a non-integer number $d_U$ of particles. Is there a contradiction between these two results? Not at all! Namely, the result of [1] refers to a non-integer number of invisible particles, that is, to the case in which the unparticle stuff is not observed. On the other hand, as explained in Sec. 2, when the unparticle stuff is observed, then it manifests as ordinary particles with arbitrary mass.

4 Remarks on canonical quantization

As we have explained, the commutation relation (5) is derived from the canonical commutation relation (1), or more precisely, from a version of (1) in which $m$ is a continuous parameter. Is it possible to derive (5) in an alternative way, from an unparticle canonical commutation relation of the form

$$[\Phi(x), \Pi(x')] = i\delta^3(x - x'),$$

(13)

where $\Pi(x)$ is the unparticle canonical momentum with an expansion similar to (12)? Formally, it is possible to construct an operator $\Pi(x)$ that obeys the commutation relation (13). Nevertheless, it is misleading to call such an operator the “canonical momentum”. The very notion of the canonical momentum makes sense only for Lagrangians that are functions of a canonical coordinate and its first time derivative, but not other time derivatives. On the other hand, the effective action describing unparticle fields is of the form [6, 7]

$$S \propto \int d^4x \partial^\mu \Phi(- \partial^\nu \partial_\nu)^{1-d_U} \partial_\mu \Phi,$$

(14)

so the Lagrangian depends on other time derivatives of the unparticle field $\Phi(x)$. Such systems do not admit the usual canonical quantization. Consequently, the commutation relation (5) cannot be derived from a more fundamental canonical commutation relation of the form of (13). This suggests that the massive fields $\phi(x, m)$ from which (5) is derived play a more fundamental role than the effective unparticle field $\Phi(x)$.

As a conclusion, we end with the following remark. Given the fact that the unparticle field admits a particle interpretation just as ordinary fields do, the name “unparticle” seems somewhat misleading. Given our results above, perhaps the expression “uncanonical field” would better describe what this scale invariant field theory really is.

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