The ’t Hooft determinant resolution of the $\eta'$ puzzle

Alexander A. Osipov$^1$, Alex H. Blin$^2$, Brigitte Hiller$^3$

Centro de Física Teórica, Departamento de Física da Universidade de Coimbra, 3004-516 Coimbra, Portugal

Abstract

The six-quark instanton induced ’t Hooft interaction, which breaks the unwanted $U_A(1)$ symmetry of QCD, is also a source of semi-classical corrections to the low energy effective action. It is argued that there emerges a dimensionless expansion parameter that introduces a new mass scale, $\Lambda_E^2 \simeq 6 \text{ GeV}^2$, in the $0^-, 0^+$ channels. This scale plays a similar role as the large critical mass $M_{\text{crit}}^2 \gtrsim 4.2 \div 6.6 \text{ GeV}^2$ discovered in the framework of QCD sum rules in the $0^-, 0^+$ gluonic channels. In particular, it allows to resolve the $\eta'$ puzzle. To extract $\Lambda_E$ we calculate the masses of the lightest pseudoscalar meson nonet by using the Nambu – Jona-Lasinio (NJL) type $U_L(3) \times U_R(3)$ chiral symmetric Lagrangian together with the ’t Hooft determinant. The mechanism which leads to the large value of $\Lambda_E$ is scrutinized.
1. Introduction

Unfortunately, there is at present no quantitative framework within QCD to deal with its large distance dynamics. The physics of hadrons is approached through phenomenological parametrizations usually based on some simple ansatz with solid symmetry grounds. There are two important experimental facts to support this line of investigation. First, it is known that the chiral symmetry of the massless QCD Lagrangian, which should be a good approximation for light quarks \((u, d, s)\), is not seen in the hadronic spectrum (the \(SU(3)_V\) degenerate multiplets with opposite parity do not exist). It means that the hadronic vacuum is not symmetric under the chiral group. Second, it is seen from the mesonic spectrum that the \(U_A(1)\) symmetry of the QCD Lagrangian is badly broken. The \(SU(3)\) singlet pseudoscalar \(\eta'\) is too heavy to be the ninth Goldstone boson. The \(U_A(1)\) anomaly is responsible for the \(\eta' - \pi, K, \eta\) splitting \[1\]. It has been understood later \[2\] that the \(1/N_c\) expansion can be a relevant approximation to generate hadronic bound states and to find the singlet-octet splitting as a next to the leading \(1/N_c\) order effect.

A qualitatively correct picture of both spontaneous chiral symmetry breaking and \(U_A(1)\) breaking at low energies, which is also compatible with the conclusions coming from the large \(N_c\) expansion, is given by instantons \[3,4\]. The semi-classical theory based on the QCD instanton vacuum provides convincing evidences that \(2N_f\)-quark interactions (\(N_f\) is the number of flavours) actually exist in QCD and that in the leading \(1/N_c\) order they are described by the 't Hooft determinant \[5\],

\[
\mathcal{L}_{2N_f} = \kappa (\det \bar{q} P_L q + \det \bar{q} P_R q)
\]

where the matrices \(P_{L,R} = (1 \mp \gamma_5)/2\) are projectors and the determinant is over flavour indices. We assume here that all interactions between quarks can be taken in the long wavelength limit where they are effectively local\(^4\). At next to the leading \(1/N_c\) order this vertex is modified by the tensor term which we have omitted in eq.\[1\]. Even in this essentially simplified form the determinantal interaction contains all necessary features to describe the dynamical symmetry breaking of the hadronic vacuum and explicitly breaks the axial \(U_A(1)\) symmetry. In the following we will assume that quark fields

\(^4\)To lowest order in \(1/N_c\), hadronic physics can be described as a tree approximation to some local Lagrangian, with local hadron fields and local interaction vertices \[6\].
have colour, $N_c = 3$, and flavour, $N_f = 3$, indices which range over the set $i = 1, 2, 3$. The coupling constant $\kappa$ is a dimensional ($[\kappa] = \text{GeV}^{-5}$) negative parameter with the large $N_c$ asymptotic $\kappa \sim 1/N_c^{N_f}$.

On lines suggested by multicolour chromodynamics, however, it can be argued [2] that the $U_A(1)$ anomaly is negligible in the large $N_c$ limit, the deviation of the singlet – octet mixing angle from its ideal value is suppressed, so that mesons come degenerate in mass nonets. Thus, the leading order mesonic Lagrangian must inherit the $U_L(3) \times U_R(3)$ chiral symmetry of massless QCD. To specify the corresponding part of the effective quark Lagrangian we consider the four quark NJL type interactions [7]

$$L_{\text{NJL}} = \frac{G}{2} \left[ (\bar{q} \lambda_a q)^2 + (\bar{q} i \gamma_5 \lambda_a q)^2 \right],$$

where $\lambda_a$, $a = 0, 1, \ldots 8$ are the normalized ($\text{tr} \lambda_a \lambda_b = 2 \delta_{ab}$) Gell-Mann matrices acting in flavour space. The positive four quark coupling $G$, $[G] = \text{GeV}^{-2}$, counts as $G \sim 1/N_c$ and therefore the Lagrangian (2) dominates over $L_6$ at large $N_c$.

There is another approach to the $U_A(1)$ problem which is also based on the method of effective Lagrangians [8]. The way we follow in the present work reflects the quark structure of light pseudoscalar and scalar mesons and has a built-in mechanism for dynamical chiral symmetry breaking. A similar model has been considered for two flavours in [9] and for three flavours in [10, 11] and has been widely explored since that time [12].

We have used the multicolour asymptotics to motivate our choice of many quark interactions (1) and (2). However, we are not going to follow explicitly the idea of $1/N_c$ expansion. As is well known from QCD sum rules, the channels with quantum numbers $J^P = 0^+, 0^-$ are strongly coupled to the gluonic world [13]. It has been argued there that for these channels the pictures emerging at $N_c \to \infty$ and $N_c = 3$ seem to be qualitatively different from each other and the accuracy of the $1/N_c$ expansion becomes worse. From a pure phenomenological point of view it would suffice to mention here the large deviations from the Zweig rule in the pseudoscalar channel, or the well known $\eta'$ puzzle: the mass of this meson, being of order $1/N_c$, is unexpectedly too large. QCD sum rules relate these deviations from the $1/N_c$ counting to the increasing of the mass scale $M_{\text{crit}}$ characterizing the breaking of asymptotic freedom in the corresponding channel, in such a way that at large energies $E \gg M_{\text{crit}}$, the $1/N_c$ counting rehabilitates itself. The mass scale relevant for $\eta'$ physics is $M_{\text{crit}} \gtrsim 2.0 \div 2.6 \text{ GeV}$ [13].

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Since the critical mass is too large, one cannot rely much on the $1/N_c$ expansion. Here we use another approximation, namely the stationary phase method for the semi-classical path integral bosonization of the effective quark Lagrangian \[ \mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - \hat{m})q + \mathcal{L}_{\text{NJL}} + \mathcal{L}_6. \] (3)

In this approximation the interactions $\mathcal{L}_{\text{NJL}}$ and $\mathcal{L}_6$ are considered as contributions of the same order in $\hbar$.

One can also try to improve the lowest order SPA result by taking into account the Gaussian fluctuations of the six-quark 't Hooft determinant around the stationary phase trajectory \[14, 15\]. If $\mathcal{L}_6$ is not small, corrections can be much larger than one could predict starting from $1/N_c$, and be important for the mesonic $(0^+, 0^-)$ mass spectra. The dimensionless parameter ensuring the smallness of semi-classical corrections in the model is \[15\]

\[ \zeta = \frac{\kappa^2 \Omega^{-1}}{32 G^3} \sim \frac{1}{N^3_c} \] (4)

where $\Omega$ is the volume of a small Euclidean spacetime box with a side $2\pi/\Lambda_E$. For certain, we are dealing here with a small effect. However, one cannot neglect such quasi-classical contributions until the mass scale $\Lambda_E$ associated with it is established. The strong $1/N_c$ suppression of \[14\] makes room for a large value of $\Lambda_E$ still leaving $\zeta$ small enough. We argue here that the cut-off $\Lambda_E$ is in a sense similar to the mass scale $M_{\text{crit}}$ advocated in \[13\]. Our assertion is based on a calculation of the mass spectrum of pseudoscalar $(0^-)$ mesons from which one can extract $\Lambda_E$ and show that it is large $\Lambda_E \gtrsim 2$ GeV.

It is worth noting that the model under consideration contains a second dimensionless parameter \[15\],

\[ \epsilon = \frac{\kappa |\Delta|}{4G^2} \sim \frac{1}{N^3_c} \] (5)

with $\Delta = m - \hat{m}$, where $m$ stands for the constituent and $\hat{m}$ for the current quark mass. The series expansion in $\epsilon$ closely corresponds to the $1/N_c$ expansion of the model. However, the fit to the meson mass spectrum shows that $\epsilon \simeq 0.7$, being in contradiction even with the $N_c = 3$ estimate from \[5\].

This relatively large (in comparison with 1) value implies large $1/N_c$ corrections which convert the $1/N_c$ series into a badly convergent one. Sharing ideas of paper \[13\], we explain this behaviour of the series by the existence of a large critical mass in the channel. In this sense, the model perfectly
reflects the known resolution of the $\eta'$ puzzle by generating quasi-classically a large mass scale parameter $\Lambda_E$ which makes $\eta'$ light on its natural scale $m_{\eta'}/\Lambda_E^2 \sim \zeta \sim 10^{-1}$.

2. Characteristic scale of semi-classical corrections

Let us briefly recall some results of bosonization of the quark Lagrangian \cite{14}. Important details can be found in \cite{15}. On the first stage one should linearize the many fermion vertices by introducing auxiliary bosonic fields. The pure quark Lagrangian, $\mathcal{L}$, is transformed to a mixed meson-quark one

$$\mathcal{L}_{\text{mix}}(q, \phi, \sigma) = \mathcal{L}_q + \mathcal{L}_r + \Delta \mathcal{L}_r .$$  \hfill (6)

The first term describes the tree level interactions of constituent quarks with pseudoscalar, $\phi_a(x)$, and scalar, $\sigma_a(x)$, $U(3)$ flavour nonets

$$\mathcal{L}_q = \bar{q} (i\gamma^\mu \partial_\mu - m - \sigma - i\gamma_5 \phi) q .$$  \hfill (7)

The second term is the leading order stationary phase result

$$\mathcal{L}_r = \frac{G}{12} \text{tr} (U_{st} U^\dagger_{st}) + \frac{1}{6} \text{tr} (W U^\dagger_{st} + W^\dagger U_{st})$$

$$= h_a \sigma_a + \frac{1}{2} h^{(1)}_{ab} \sigma_a \sigma_b + \frac{1}{2} h^{(2)}_{ab} \phi_a \phi_b + \ldots .$$  \hfill (8)

Here we used the stationary phase condition

$$G U_a + W_a + \frac{3\kappa}{32} A_{abc} U_b^\dagger U_c^\dagger = 0$$  \hfill (9)

where the totally symmetric constants $A_{abc}$ are defined through the flavour determinant $\det W = A_{abc} W_a W_b W_c$. Our notations are the following. The trace is taken over flavour indices, any flavour matrix written without open index is understood as summed with the Gell-Mann $\lambda_a$ matrices ($a = 0, 1\ldots8$), for instance, $W = W_a \lambda_a$. The mesonic fields are grouped in the covariant combinations $W_a = \sigma_a + \Delta_a - i\phi_a$. The field $U_{st}$ represents the exact solution of the stationary phase condition \cite{9}, which we seek in the form $U_a = s_a - ip_a$ expanding $s_a, p_a$ in increasing powers of bosonic fields $\phi_a, \sigma_a$.

$$s^\text{st}_a = h_a + h^{(1)}_{ab} \sigma_b + h^{(1)}_{abc} \sigma_b \sigma_c + h^{(2)}_{abc} \phi_b \phi_c + \ldots$$  \hfill (10)

$$p^\text{st}_a = h^{(2)}_{ab} \phi_b + h^{(3)}_{abc} \phi_b \phi_c + \ldots$$  \hfill (11)
with coefficients \( h_{ab...}^{(k)} \) explicitly depending on the quark masses and coupling constants \( G, \kappa \). The coefficients are fixed by the series of coupled equations following from (9) and obtained by equating to zero the factors before independent combinations of mesonic fields. Due to recurrence of the considered equations all coefficients are determined once the first one, \( h_a \), has been obtained [15].

An alternative form to the exact solution of eq.(9) is provided by the \( 1/N_c \) expansion which gives the stationary phase solution, \( U^a_{st} \), in form of the series

\[
U^a_{st} = -\frac{1}{G} \left( W_a + \frac{3\kappa}{32G^2} A_{abc} W^b W^c + \mathcal{O}(1/N_c^2) \right),
\]

yielding for \( \mathcal{L}_r \)

\[
\mathcal{L}_r = -\frac{1}{4G} \text{tr} (WW^\dagger) - \frac{\kappa}{(4G)^3} \left( \det W + \det W^\dagger \right) + \mathcal{O}(1/N_c).
\]

In fact, the large-\( N_c \) limit forces a series expansion for the coefficients of Lagrangian (8) and this is how the dimensionless parameter (5) reveals itself. To see this, let us assume for a moment that \( SU_f(3) \) flavour symmetry is preserved. In this case the only coefficient among \( h_a \) which is different from zero is \( h_0 \) and it is given by

\[
h_0 = -\frac{8G}{\kappa} \sqrt{\frac{3}{2}} \left( 1 - \sqrt{1 - \frac{\kappa \Delta}{4G^2}} \right).
\]

If the ratio \( \epsilon = \kappa \Delta/(4G^2) \) is small, one can expand the square root inside \( h_0 \) in powers of \( \epsilon \). This automatically leads to the same expansion for all other coefficients in (8) and, finally, one obtains Lagrangian (13) at leading order in \( \epsilon \). This limit is not affected by the semi-classical corrections \( \Delta \mathcal{L}_r \), because these are at most of order \( \sim 1/N_c \). One can argue, however, that numerically \( \epsilon \approx 0.7 \) (see our discussion in Sec.3) what is relatively large, to support the fast convergence of the series.

The third term in (6) is the next to the leading order correction in the semi-classical expansion of the bosonized Lagrangian [15].

\[
\Delta \mathcal{L}_r = -\Omega^{-1} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \text{tr} [F_{\alpha\beta}(\phi, \sigma)]^n
\]

where

\[
F_{\alpha\beta}(\phi, \sigma) = \frac{3\kappa}{16} A_{\epsilon\beta} \left( \begin{array}{cc} -h_{ac}^{(1)} p_{st} & h_{ac}^{(1)} p_{st} \\ h_{ac}^{(2)} p_{st} & h_{ac}^{(2)} p_{st} \end{array} \right)_{\alpha\beta},
\]
with $s_a^\text{st} = s_a^\text{st} - h_a$. The factor $\Omega^{-1}$ may be written as an ultraviolet divergent integral regularized by introducing a cut-off $\Lambda_E$\footnote{One should not confuse this parameter with the standard ultraviolet NJL cut-off $\Lambda \simeq 1$ GeV for the quark loops, which represents the mass scale of spontaneous chiral symmetry breaking and determines the value of the quark condensate to leading order.}

$$\Omega^{-1} = \delta_E^4(0) \sim \int_{-\Lambda_E/2}^{\Lambda_E/2} \frac{d^4k_E}{(2\pi)^4} = \frac{\Lambda_E^4}{(2\pi)^4}. \quad (17)$$

One can speculate about the value of the cut-off. By definition $\Lambda_E$ belongs to the “quark territory” $\Lambda_E \gtrsim 1$ GeV. On the other side, only effects that go beyond standard perturbation theory are included into the Lagrangian $L_6$. Therefore, the characteristic volume of quantum fluctuations, $\Omega$, can not deviate much from the size determined by non-perturbative fluctuations corresponding to classical solutions of the non-linear Yang-Mills equations, i.e. instantons. The relevant mass scale is generated by the gluon vacuum condensate. Thus, we have as a crude estimate

$$\Omega \simeq \langle 0| \frac{\alpha_s}{\pi} C^{a}_{\mu\nu} C^{a}_{\mu\nu}|0 \rangle^{-1} \simeq (330 \text{ MeV})^{-4}, \quad \Lambda_E \simeq 2.1 \text{ GeV}. \quad (18)$$

The crucial question is, however, whether this large value is in agreement with the general idea of a semi-classical expansion: corrections must be small in comparison with the leading order result. At first sight, it seems that we have just the opposite case. The factor $\Omega^{-1} \sim \Lambda^4_E$ and one can naively expect that large values of $\Lambda_E$ will severely break the convergence of the quasi-classical series.

This is not entirely true. To clarify the point and to learn one important feature of Lagrangian (15), let us use again the $1/N_c$ expansion. The first terms of (15) in this framework are

$$\Delta L = \frac{-\kappa^2 \Omega^{-1}}{8(2G)^2} \left[ \text{tr} \left( W W^\dagger \right) + \frac{3\kappa}{(4G)^2} \left( \text{det} W + \text{det} W^\dagger \right) + \mathcal{O}(1/N_c^2) \right]. \quad (19)$$

One can make here a few interesting observations. First, the correction to the result (13) starts from a term of $1/N_c^2$ order. Due to fine cancellations in (15) it is two orders less than one would naively expect. Second, every term in eq.(19) is suppressed by the factor $\zeta$ (for its definition see eq.(14)) as compared with corresponding terms in (13). This dimensionless parameter measures the size of the semi-classical contribution. Its value must be small, but not
necessarily so much suppressed as it follows from the above estimations. Actually, nothing forbids us to suppose that such a suppression is partly compensated due to the increase of the mass scale $\Lambda_E$, in such a way that $\zeta$ still remains small, $\zeta \sim \mathcal{10}^{-1}$. 

3. A model estimation of $\Lambda_E$

Let us see how our expectations look numerically. To check our guess one should turn to the calculation of the pseudoscalar mass spectrum and extract $\Lambda_E$ by confronting model results with experimental data. To make our consideration not too overfilled with details of numerical calculations we give here only final results. We postpone the details until a future publication \cite{[16]}

In the following we shall consider the case of $SU(2)_I \times U(1)_Y$ symmetry, i.e. we take $\hat{m}_u = \hat{m}_d \neq \hat{m}_s$. Accordingly, there are altogether six parameters, $\hat{m}_u$, $\hat{m}_s$, $G$, $\kappa$, $\Lambda$ and $\Lambda_E$, shown in Table 1. The last two columns present the dimensionless parameters of the model, $\zeta$ and $\epsilon$, related to the different types of power expansions. In Table 2 are the results of our calculations of the pseudoscalar spectrum, together with the weak decay constants $f_\pi$, $f_K$ and mixing angle $\theta_p$ in the singlet – octet basis ($\phi_0$, $\phi_8$). Inputs are indicated by (*). The Latin letter labels on the left hand side identify the sets in the tables.

TABLE 1. Main parameters of the model given in the following units: $[m] = \text{MeV}$, $[G] = \text{GeV}^{-2}$, $[\kappa] = \text{GeV}^{-5}$, $[\Lambda] = \text{GeV}$. Sets $(a, b)$ correspond to the leading order SPA. Sets $(c, d)$ include semi-classical corrections. Constituent quark masses $m_i$ and corresponding leading order results $\hat{m}_i$ are also given.

| Set | $\hat{m}_u$ | $\hat{m}_d$ | $m_u$ | $m_d$ | $m_s$ | $\hat{m}_s$ | $m_s$ | $G$ | $-\kappa$ | $\Lambda$ | $\Lambda_E$ | $\zeta$ | $\epsilon$ |
|-----|-------------|-------------|-------|-------|-------|-------------|-------|-----|-----------|----------|-----------|--------|--------|
| a   | 5.3         | 315         | -     | 170   | 513   | -           | 8.89  | 687 | 0.92      | -        | -         | 0.67   |
| b   | 6.1         | 380         | -     | 185   | 576   | -           | 12.6  | 1116| 0.83      | -        | -         | 0.66   |
| c   | 4.1         | 446         | 381   | 123   | 598   | 559         | 7.14  | 201 | 1.1       | 2.4      | 0.076     | 0.44   |
| d   | 4.5         | 471         | 412   | 132   | 620   | 585         | 8.81  | 298 | 1.03      | 2.3      | 0.076     | 0.45   |

The two first sets $(a, b)$ are obtained in the framework of leading order SPA, along lines suggested in our recent work \cite{[17]}. The quantum fluctuations
given by the Lagrangian $\Delta L_r$ are not taken into account. One can see that reasonable fits to the pseudoscalar spectrum at leading order correspond to a large value of $\epsilon \simeq 0.7$ and clearly show the slow convergence of the $1/N_c$ series.

TABLE 2. The light pseudoscalar nonet characteristics (in units of MeV, except for the angle $\theta_p$, which is given in degrees) are presented in a full correspondence with the parameter sets of Table 1.

|   | $\langle \bar{u}u \rangle^{1/3}$ | $\langle \bar{s}s \rangle^{1/3}$ | $m_\pi$ | $m_K$ | $f_\pi$ | $f_K$ | $m_\eta$ | $m_\eta'$ | $\theta_p$ |
|---|---|---|---|---|---|---|---|---|---|
| a | 244 | 204 | 138* | 494* | 92* | 121* | 487 | 958* | -12.0 |
| b | 233 | 182 | 138* | 499* | 92* | 115.8* | 477 | 958* | -15.0 |
| c | 287* | 263 | 138* | 499* | 92* | 115.8 | 503* | 958* | -12.5 |
| d | 274* | 244 | 138* | 494* | 92* | 113* | 494 | 958* | -13.3 |

In the sets $(c, d)$ we include the semi-classical correction to the leading order result. In these cases a new mass scale parameter $\Lambda_E$ enters the fitting process. Consequently, we have included the value of the light quark condensate as an additional input. One can see that quantum fluctuations lead to a small effect, slightly improving the fit. Nevertheless, our expectations seem to be realized. The cut-off $\Lambda_E$ has a large value $\Lambda_E \simeq 2.4$ GeV, with $\zeta \simeq 0.08$ being small enough.

Let us try to understand why $\Lambda_E \simeq 2.4$ GeV and not much larger. The reason for this is very simple and is contained in the value for $\epsilon \simeq 0.44$. This value indicates that cancellations in $\Delta L_r$ are not so strong as one would expect by using the large-$N_c$ arguments to obtain (19). Being 35% less, in comparison with sets $(a, b)$, the model parameter $\epsilon$ is still too large to vouch for suppression in Lagrangian $\Delta L_r$ in full measure.

On the other hand, the calculations would become unreliable if the value of $\Lambda_E$ would lead to such large corrections that they could compete with the leading term. We have found that at $\Lambda_E \simeq 2.4$ GeV the correction amounts at most to $\sim 20\%$ of the leading term (in the case of weak decay constants $f_\pi$ and $f_K$). This is a strong signal that we are already quite close to exhaust the reserves of the quasi-classical expansion and that a further increase of $\Lambda_E$ would start to destroy the fast convergence of the series, giving, in addition, a worse fit.
4. Discussion of the result

We have pointed out a new interesting feature related with the ’t Hooft determinantal resolution of the $U_A(1)$ problem, which provides one more argument in favour of this six-quark interaction: it also unravels the obviously non-trivial $\eta'$ puzzle. Indeed, the phenomenological consequences of the ’t Hooft interaction are well-known. For instance, it leads to spontaneous chiral symmetry breaking with the characteristic mass scale $\Lambda \sim 1$ GeV extracted from the quark loops. It also explains the deviations from Zweig’s rule. In accordance with standard $N_c$ counting rules, the six-quark interaction yields a flavour singlet $\eta'$ meson mass of order $m_{\eta'}^2 \sim 1/N_c$, as it is commonly expected. However, it has not been clear why this $1/N_c$ suppressed mass is not much smaller than its actual value of almost one GeV. The mass scale for chiral symmetry breaking, $\Lambda$, is too low to explain this phenomenological fact.

To find the answer we have suggested to bosonize the quark determinantal interaction and to take into account the next to the lowest order term in the semi-classical expansion of the bosonized 't Hooft Lagrangian. This formal expansion in powers of the dimensional parameter $\hbar$ actually contains a small dimensionless parameter $\zeta$, which hides the large characteristic scale $\Lambda_E \approx 2.4$ GeV.

Our solution is obtained in the framework of a simple model, although it is strongly based on the instanton picture of the QCD vacuum. Accumulating the most essential features of the instanton physics, it is not an accident at all, that it resolves the $\eta'$ puzzle in a very similar way to the known solution in the framework of QCD sum rules. There are, however, important differences: in the latter approach the $\eta'$ gets its mass through mixing with glue states and these are expected to be much heavier, giving a new large mass scale parameter for the pseudoscalar channel.

The ’t Hooft determinantal interaction is a remnant of gluodynamics at large distances, i.e. at scales where quarks interact with each other through their zero modes in the instanton background. By means of the semi-classical expansion of the bosonized ’t Hooft Lagrangian we are able to “touch” the border of the non-perturbative region from its low energy side, as opposed to the QCD sum rules method. Amusingly, the numerical result is in a perfect agreement with our expectations and with the value obtained on the basis of QCD sum rules.
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