Application of improved fiber model in the nonlinear static analysis of RC shear wall

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Abstract. The RC shear wall under monotonic loading was analyzed with a two–dimensional model of flexibility method based on fiber, considering the axial, bending and shear effects. The calculation program to simulate a shear wall’s nonlinear performance under monotonic load was programmed. The model is validated by comparing calculated values with experimental data on laboratory RC shear wall. After verifying the feasibility and the validity of the model, the factors affecting the nonlinear performance of shear wall are analyzed, such as aspect ratio, axial compression ratio, reinforcement ratio, concrete strength.

1. Introduction
Fiber model is the analytical model that is the closest to real mechanical behavior of structures. The unit of fiber model is mainly applied to simulate beam and column, which ignores the effect of shear and simulates the bending of component better. In recent years, many researchers are trying to consider shear effect in fiber model to better simulate component like shear wall which cannot ignore shear effect. Although some results were achieved, there are not models that being widely accepted. Future research is required to get a relative accurate and easy model to analyze nonlinear shear wall.

2. Fiber Wall Model
2.1 Description of Mechanical state
To derivate the Stiffness matrix of the shear wall, the mechanical behavior of shear wall units, cross section of shear wall and fiber need to be considered. When the rigid body displacement is not considered, the unit load vector

\[ Q = (M_a, M_v, N)^T \]

The element of the vector is the bending moment and axial force of each end of beam and column. The unit mechanical state is

\[ Q = K_{ele} \cdot q \]

When the rigid body displacement is considered, the mechanical properties under rigid body displacement can be derived by geometric transformation matrix \( L_{ele} \). The mechanical behavior of shear wall which is described by section force vector, section displacement vector and section stiffness matrix, namely

\[ D(x) = k_{sec} \cdot d(x) \]

Section force matrix is

\[ D(x) = (M_s(x), N(x), Q(x))^T \]

where the element is bending moment, axial force and shear force. Section displacement matrix is

\[ d(x) = (\varphi(x), \varepsilon(x), \gamma(x))^T \]

where the element is the curvature, axial strain and shear strain. Considering axial force, bending moment and shear force separately when the force is acting on the fiber. When calculating axial force and bending moment, the geometry meshing and formula derivation is the same with unit fiber model of beam and column [1]. When considering shear force, the
section shear strain is assumed as \( \gamma_i(x) = \frac{3}{2} \left( 1 - \frac{4x^2}{h^2} \right) \gamma(x) \). The reason of the assumption is that the shear stress on the shear wall is parabolic distribution in the traverse section. Where \( \gamma_i(x) \) is the shear strain of \( i \)th fiber strip of the section; \( y_i \) is the horizontal ordinate of \( i \)th center point of fiber strip; \( h \) is the transverse length of the section; \( \gamma(x) \) is the shear strain of the section. The shape function \( b(x) \) of internal force was established by the relation of section force and unit force[2]. The shape function is derived by energy method and the matrix is

\[
\frac{l-x}{l} \begin{bmatrix} x & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 7 & 0 \end{bmatrix},
\]

where \( l \) is the length of the unit.

2.2 Stiffness matrix and Constitutive relation
As the description of unit, section and the mechanics condition, the tangent line stiffness matrix of the section is

\[
k_{sec} = k(x) = \begin{bmatrix}
\sum_{i=1}^{n} E_i \cdot A_i \cdot y_i^2 - \sum_{i=1}^{n} E_i \cdot A_i \cdot y_i & 0 & 0 \\
- \sum_{i=1}^{n} E_i \cdot A_i \cdot y_i & \sum_{i=1}^{n} E_i \cdot A_i & 0 \\
0 & 0 & \frac{3}{2} \sum_{i=1}^{n} \left( 1 - \frac{4y_i^2}{h^2} \right) \frac{dV(y)}{dy} \left( \gamma(y, x) \right) \\
\end{bmatrix},
\]

where \( E_i \) is the \( i \)th fiber’s tangential modulus under the current strain; \( A_i \) is the \( i \)th fiber’s section area; \( y_i \) is the \( i \)th fiber’s horizontal ordinate; \( n(x) \) is the quantity of fiber which is considered the axle bending. From the relation between section force and fiber force, the resistance under the current situation is

\[
D_k(x) = \begin{bmatrix}
\sum_{i=1}^{n} \sigma_i \cdot A_i \cdot y_i & \sum_{i=1}^{n} \sigma_i \cdot A_i & \sum_{i=1}^{n} V(y_i) \\
\end{bmatrix}^T,
\]

where \( \sigma_i \) is the axial stress of the \( i \)th axle bending fiber.

The concrete constitutive model in this paper is first raised by Kent and Park in 1973[3] and modified by Scott et al[4]. The concrete unload and reload model is combined with linear model and curvilinear model. Zhu bolong unloading curve is applied when the concrete is unloading. Linear model that points to unloading point is applied when the concrete is reloaded. Please see the bibliography for details[5]. The skeleton curve of rebar is two – line model[5] under repetitive loading. The equivalent elasticity modulus is \( E' = 0.01E \) when the stress in the rebar is beyond the elastic limit.

The modified model by Takeda is applied in this paper as the shear subunit. Shear hysteretic model and hysteretic curve are the same models mentioned in the reference[6].

2.3 Model and Algorithm
When considering shear effect, fiber is meshed alone traverse section because the shear strain alone traverse section is distributed like a parabola. The model in this paper is based on flexibility method[7]. A FORTRAN programming based on flexibility method[7] and computer program flow diagram[5] is established.

3. Factors that affects nonlinear properties of shear wall

3.1 The influence of depth – width ratio on the nonlinear properties of shear wall
The depth – width ratio of shear wall is 0.8, 1, 1.5, 2, 2.5 respectively. The height of the shear wall is 1.6m, 2m, 3m, 4m and 5m respectively. The load – displacement curve with different depth – width ratio and the stiffness degradation curve with different depth – width ratio is shown below.

![Load displacement curve](image1)

![Stiffness degradation curve](image2)

From the displacement diagram, with the more depth – width ratio, the yield load and the maximum bearing capacity became smaller. While the yield displacement, ultimate displacement became larger. Especially when the depth – width ratio is 2 and 2.5, they increased over 50%. The ductility factor does not fluctuate significantly.

From the stiffness degradation diagram, with the more depth – width ratio, the stiffness degradation of shear wall is slower and vice versa. At the beginning of loading, especially the low depth – width ratio, the stiffness degradation of shear wall is quick.

3.2 The influence of axial compression ratio on the nonlinear properties of shear wall

The axial compression ratio is 0.1, 0.2, 0.3 respectively. The load – displacement curve and stiffness degradation are shown below.

![Load displacement curve](image3)

![Stiffness degradation curve](image4)

From the load – displacement curve diagram, when the axial compression ratio increases, the stiffness, yield load, ultimate capacity and yield displacement increases greatly. However, the ultimate displacement decreases a lot. The ductility of shear wall also decreases. When the axial compression ratio is low, the load – displacement curve appears clear yield plateau. In the other side, the load – displacement curve does not appear clear yield plateau, and the stiffness degradation is fast.

From the stiffness degradation diagram, there is not clear relation between stiffness degradation and axial compression. With the displacement increases, the axial compression increases, and the stiffness of shear wall increases, while the stiffness degradation does not change much when it comes to yield plateau.

3.3 The influence of reinforcement ratio on the nonlinear properties of shear wall

The parameter of RC shear wall is as the statement before. The depth – width ratio is 2.5 and the axial compression ratio is 0.1. Reinforcement ratio is 0.2%, 0.4%, 0.6%, 0.8%. Load – displacement curve with different reinforcement ratio and stiffness degradation curve with different ratio is shown below.
From the load – displacement diagram, with the reinforcement ratio increases, the peak load and ultimate displacement increases generally. Although they decrease a bit with ductility when the reinforcement ratio is 0.4% and 0.6%, most load – displacement curve coincides. The only difference reflects on the yield plateau.

From the stiffness degradation, reinforcement ratio varied in small scale barely affects the stiffness degradation. Thus, it is not useful enough to increase the stiffness of shear wall by increasing reinforcement ratio.

3.4 The influence of concrete strength on the nonlinear properties of shear wall
From the load – displacement curve diagram, with the concrete strength increases, yield load, ultimate load, ductility factor and ultimate displacement increases slightly. However, yield displacement decreases. Increasing concrete strength does not help a lot improve the nonlinear properties of RC shear wall.

From the stiffness degradation diagram, with the concrete strength increases, the stiffness of shear wall increases at the beginning. When the shear wall begins cracking and yielding, stiffness decreases rapidly and is irrelevant to the concrete strength. The phenomenon shows that concrete strength just affects the stiffness of shear wall without cracking and yielding.

4. Summary
Traditional fiber model is modified in this paper. Shear effect is added into the model to simulate the mechanical behavior of the shear wall. Comparing the simulation and experiment, the modified fiber model can better simulate the nonlinear properties of RC shear wall under bending moment, pressure and shear force. After checking the practicability of model, the factors which affects the nonlinear properties of shear wall such as depth -width ratio, axial compression ratio, reinforcement ratio and concrete strength are analyzed.

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