Bouncing cosmology in a curved braneworld

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Abstract. We explore the possibility of a non-singular bounce in our universe from a warped braneworld scenario with dynamical branes and a non-zero brane cosmological constant. Such models naturally incorporate a scalar sector known as the radion originating from the modulus of the theory. The presence of brane cosmological constant renders the branes to be non-flat and gives rise to a potential and a non-canonical kinetic term for the radion field in the four dimensional effective action. The kinetic term exhibits a phantom-like behavior within the domain of evolution of the modulus which leads to a violation of the null-energy condition often observed in a bouncing universe. The interplay of the radion potential and kinetic term enables the evolution of the radion field from a normal to a phantom regime where the universe transits from a contracting era to an expanding epoch through a non-singular bounce. Analysis of the scalar and tensor perturbations over such background evolution reveal that the primordial observables e.g., the amplitude of scalar perturbations $A_s$, tensor to scalar ratio $r$ and the scalar spectral index $n_s$ are in agreement with the current constraints reported by the Planck satellite. The implications are discussed.

Keywords: alternatives to inflation, cosmological applications of theories with extra dimensions, cosmological perturbation theory, physics of the early universe

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1 Introduction

One of the major challenges in modern theoretical cosmology is to explain the early stage of the universe, in particular, whether the universe emerged from an initial singularity (also known as the Big-Bang singularity) or the universe underwent a non-singular bounce leading to a possible singularity free expansion of the universe. Some of the early universe scenarios, proposed so far, that can generate an almost scale invariant power spectrum and hence confront the observational constraints are the inflationary scenario [1–8], the bouncing universe [9–68], the emergent universe scenario [69–74] and the string gas cosmology [75–81].

In this work, we study the bouncing scenario in a non-flat warped braneworld model. The bouncing scenario consists of two eras—an era of contraction and an era of expansion of the scale factor, both the eras being connected by a non-singular bounce [9–68]. Beside producing an observationally compatible primordial power spectrum, the bouncing scenario has the merit to give rise to a singularity free evolution of the early universe. Although it has been argued that the Big-Bang singularity may be avoided through a suitable quantum generalization of gravity, the absence of a consistent quantum theory of gravity makes the bouncing description of the universe a promising scenario. In this context it may be mentioned that string theory [82–85] inherently incorporates the quantum nature of gravity and is associated with several extra spatial dimensions. Although originally intended to unify the known forces of nature, it turns out that extra dimensions can also provide plausible resolution to the gauge-hierarchy problem or the finetuning problem in particle physics arising due to large quantum corrections of the Higgs mass [84–95]. In this context warped geometry models are particularly relevant. In particular, the warped geometry models due to Randall-Sundrum (RS) [92] earned a lot of attention since it resolves the gauge hierarchy problem without introducing any intermediate scale (between Planck and TeV scale) in the theory.

The RS scenario consists of an extra spatial dimension (over the usual four dimensional spacetime) with $S^1/\mathbb{Z}_2$ orbifold symmetry, enclosed between two 3-branes which are considered to be flat. The distance $r_c$ between the two branes governs the magnitude of the brane warping and therefore plays the crucial role in resolving the gauge-hierarchy problem. The assumption of flat branes which gives rise to a vanishing brane cosmological constant in the RS scenario can be relaxed in a generalized warped braneworld model [96], which allows the
branes to be non-flat giving rise to de Sitter (dS) or anti-de Sitter (AdS) branes. The interplay of the brane warping (which depends on the interbrane distance \( r_c \)) and the magnitude of the brane cosmological constant leads to the resolution of the gauge-hierarchy problem in such models. The cosmological, astrophysical and phenomenological implications of warped braneworld models (with flat or non-flat branes) have been discussed in [97–119]. Since the resolution of the finetuning problem essentially depends on the interbrane distance \( r_c \), the stabilization of \( r_c \) to the appropriate value becomes crucial. The stabilization is achieved in the RS scenario by introducing a scalar field in the bulk [121, 122], which leads to a potential for \( r_c \) in the four dimensional effective action whose minima can be suitably adjusted to address the finetuning problem. The origin of the bulk scalar however remains unexplored. This problem can evade in the non-flat warped braneworld scenario with dynamical branes such that the interbrane distance attains the status of a field (the so called radion or the modulus). Such a framework enables the radion to generate its own potential along with a non-canonical kinetic term in the four dimensional effective action which in turn can stabilize the modulus to the suitable value [118, 123], without invoking any additional scalar field in the theory.

The non-canonical scalar kinetic term becomes negative for certain values of the modulus which endows the radion a phantom-like behavior where the null energy condition is violated. Such a violation is a generic feature observed in a bouncing universe, which motivates us to explore the prospect of the non-flat warped braneworld model in addressing bouncing cosmology. We investigate the cosmological evolution of the radion field in the FRW background and the subsequent evolution of the primordial fluctuations which allows us to understand the viability of the model in purview of the Planck 2018 constraints.

The paper is organized as follows: in section 2, we briefly describe the non-flat warped braneworld model and its four dimensional effective theory. Having set the stage, section 3 is dedicated for studying the background cosmological evolution while the evolution of the perturbations and confrontation of the theoretical predictions with the latest Planck observations is discussed in section 4. We conclude with a summary of our results and a discussion of our findings in section 5.

2 The non-flat warped braneworld scenario

Randall & Sundrum (RS) [92] proposed the warped braneworld scenario to address the finetuning problem in particle physics. The RS model consists of a 5-dimensional AdS bulk bounded by two 3-branes, namely the visible brane (where our 4-d universe resides) and the hidden brane. The extra dimension denoted by \( \phi \) is associated with a \( S^1/Z_2 \) orbifold symmetry and the hidden brane resides at \( \phi = 0 \) while the visible brane is located at \( \phi = \pi \).

In the RS scenario, the bulk metric is described by

\[
d s^2 = e^{-2A(r_c, \phi)} \eta_{\mu \nu} d x^\mu d x^\nu - r_c^2 d \phi^2
\]  

(2.1)

from which it is evident that the branes are flat which is ensured by the exact cancellation of the brane tension and the cosmological constant induced on the brane [124]. In eq. (2.1) the warp factor is represented by \( e^{-2A} \) with \( A = k_0 r_c |\phi| \) where \( r_c \) is the compactification radius and \( k_0 = \sqrt{-\Lambda/24M^3} \) such that \( \Lambda \) and \( M \) denote the five dimensional cosmological constant and Planck mass respectively. The presence of the exponential warp factor in the metric ensures that \( r_c \sim 12 \) is sufficient to bring down the Higgs mass from the Planck scale to the TeV scale on the visible brane without introducing any new energy scale in the theory.
A generalization of the RS model to incorporate non-flat branes is important since the non-flatness of our universe is often evident from the physical situations, e.g., an expanding universe, the presence of black holes etc. This is achieved by replacing the brane metric $\eta_{\mu\nu}$ with $g_{\mu\nu}$ in the above metric ansatz. This induces a non-zero cosmological constant $\Omega$ on the brane inherited from the bulk which can be both positive or negative. In the situation where the branes are de-Sitter, the warp factor is given by
\[ e^{-A} = \omega \sinh \left( \frac{c_2}{\omega} - k_0 r_c |\phi| \right) \] (2.2)

where, $\omega = (\Omega/3k_0^2)$ is a dimensionless constant directly proportional to the brane cosmological constant $\Omega$ and $c_2 = 1 + \sqrt{1 + \omega^2}$. It can be shown that the above warp factor can give rise to the requisite warping of the Higgs mass on the visible brane (i.e., $k_0 r_c \pi \sim 16 \ln 10$) as in the RS scenario while keeping $\Omega \sim 10^{-124}$ (the magnitude of the present day cosmological constant in Planckian units).

For AdS branes, it can be shown that the warp factor is given by $e^{-A'} = \omega \cosh(\ln \xi + k_0 r_c |\phi|)$, with $c_1 = 1 + \sqrt{1 - \omega^2}$ [96]. We further note that in the event $\omega \to 0$, we retrieve the RS warp factor describing the flat braneworld scenario, for both the dS and AdS branes. Since the observed accelerated expansion of the universe can be explained by a positive brane cosmological constant, we will concentrate mainly on the warped braneworld scenario with de-Sitter branes and investigate its role in bouncing cosmology.

### 2.1 The non-flat warped braneworld with the radion field

In the warped braneworld scenario, the resolution of the gauge hierarchy problem depends crucially on $r_c$, the stable distance between the two branes. This requires a mechanism to stabilize the inter-brane distance to the suitable value. Goldberger & Wise [121] addressed this by invoking a bulk scalar in the five dimensional action which resulted in a potential for $r_c$ in the 4-dimensional effective action. They showed that the stable value of $r_c$ corresponds to the minima of the potential. However, the physical origin of the scalar field in the bulk action is not well understood.

Instead of flat branes if one considers the non-flat warped braneworld scenario, and allows the inter-brane distance to be treated as a 4-dimensional field $T(x)$ (the so called radion or the modulus), then it can be shown that a potential for the modulus is naturally generated in the 4-d effective action which in turn can stabilize the modulus [123]. This modulus potential is completely attributed to the non-flat character of the branes and in the event the branes are flat this potential identically vanishes.

This scenario is described by the bulk action,
\[ S = S_{\text{gravity}} + S_{\text{vis}} + S_{\text{hid}} \] (2.3)

such that,
\[ S_{\text{gravity}} = \int_{-\infty}^{\infty} d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G}(2M^3R - \Lambda) \] (2.4)
\[ S_{\text{vis}} = \int_{-\infty}^{\infty} d^4x \sqrt{-g_{\text{vis}}}(L_{\text{vis}} - V_{\text{vis}}) \] (2.5)
\[ S_{\text{hid}} = \int_{-\infty}^{\infty} d^4x \sqrt{-g_{\text{hid}}}(L_{\text{hid}} - V_{\text{hid}}) \] (2.6)
where $\mathcal{R}$ is the bulk Ricci scalar and $G$ the determinant of the bulk metric $G_{\mu\nu}$. In eq. (2.5), $V_{\text{vis}}$ and $L_{\text{vis}}$ refer to the brane tension and matter Lagrangian on the visible brane $\phi = \pi$ while $V_{\text{hid}}$ and $L_{\text{hid}}$ in eq. (2.6) correspond to the brane tension and matter Lagrangian on the hidden brane $\phi = 0$.

The bulk is governed by the Einstein’s equations with the following solution for the metric,

$$ds^2 = e^{-2A(x,\phi)}g_{\mu\nu}dx^\mu dx^\nu - T(x)^2d\phi^2,$$  

which is easily extended from eq. (2.1) with $\eta_{\mu\nu}$ replaced by $g_{\mu\nu}$ and $r_c$ substituted by the radion field $T(x)$. We will concentrate on de-Sitter branes in this work and hence the form of the warp factor is given by

$$e^{-A} = \omega \sinh \left( \ln \frac{c^2}{\omega} - k_0 T(x)|\phi| \right)$$  

which is the same as eq. (2.2) with $r_c$ replaced by $T(x)$.

It is interesting to note that the positivity of the warp factor in eq. (2.8) requires that $\xi = (\Phi/f) = \exp \{-k_0 T(x)\pi\} \geq (\omega/c_2)$ which immediately follows when we write the warp factor in the following way,

$$e^{-A} = \frac{c_2}{2} \exp(-k_0 T(x)|\phi|) - \frac{\omega^2}{2c_2} \exp(k_0 T(x)|\phi|)$$  

and demand $e^{-A} \geq 0$. Further, since the modulus $T(x)$ cannot be negative the maximum value that $\xi$ can attain is unity, when $T(x) \to 0$. Therefore, throughout this work our region of interest in the field space would be $\omega/c_2 \leq \xi \leq 1$. We will use this property of the warp factor when we explore bouncing cosmology with the radion field in section 3.

The effective action $S$ in four dimensions is derived from the bulk action $S$ by integrating over the extra coordinate $\phi$. This can be segregated into three parts, namely,

$$S = S_1 + S_2 + S_3$$  

where,

$$S_1 = \frac{2M^3}{k_0} \int d^4x \sqrt{-\hat{g}} \ h \left( \frac{\Phi}{f} \right) \hat{R}$$  

is the curvature dependent part of the effective action $S$ with $\hat{g}$ the determinant and $\hat{R}$ the Ricci scalar with respect to the brane metric $\hat{g}_{\mu\nu}$. From eq. (2.11) it is evident that the Ricci scalar involves a coupling with the dimensionless radion field $\xi = (\Phi/f) = \exp\{-k_0 T(x)\pi\}$ with $f = \sqrt{6M^3c_2^2/k_0}$ and hence is in the Jordan frame. Here and in the rest of the discussion we shall denote $\xi$ as the radion field. The coupling of the modulus to $\hat{R}$ is denoted by $h(\xi)$ which assumes the form,

$$h(\xi) = \left\{ \frac{c_2^4}{4} + \omega^2 \ln \xi + \frac{\omega^4}{4c_2^2} \left( \frac{1}{\xi^2} \right) - \frac{\omega^4}{4c_2^2} - \frac{c_2^4}{4}\xi^2 \right\}$$  

(2.12)
The second part of the effective action $S_2$ comprises of a potential for the radion field, 

$$S_2 = -2M^3k_0 \int d^4x \sqrt{-\hat{g}} \hat{V}(\xi)$$  \hspace{1cm} (2.13)$$

with

$$\hat{V}(\xi) = 6\omega^4 \ln \xi - \frac{3}{2} \omega^2 c_2^2 \xi^2 + \frac{3}{2} \omega^2 c_2^2 + \frac{3}{2} \omega^6 \left( \frac{1}{\xi^2} \right) - \frac{3}{2} \omega^6 = 6\omega^2 h(\xi)$$  \hspace{1cm} (2.14)$$

It is interesting to note that the potential $\hat{V}(\xi)$ is directly proportional to $h(\xi)$ and vanishes in the event the branes are flat i.e., $\omega \to 0$ [122]. Moreover, it has an inflection point at $\xi_i = \omega/c_2$ which will have important consequences when we explore early universe cosmology in this model.

The third part of the effective action in eq. (2.10) is associated with the kinetic term of the radion given by,

$$S_3 = \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \right) \hat{G}(\xi)$$ \hspace{1cm} (2.15)$$

where,

$$\hat{G}(\xi) = 1 + \frac{4}{3} \frac{\omega^2}{c_2^2} \left( \frac{1}{\xi^2} \right) \ln \xi - \frac{\omega^4}{c_2^4} \left( \frac{1}{\xi^4} \right)$$  \hspace{1cm} (2.16)$$

denotes the non-canonical coupling to the kinetic term which reduces to the canonical form when $\omega \to 0$. Therefore, the non-flatness of the branes generates the brane cosmological constant $\Omega$ which in turn gives rise to the potential for the radion and its non-canonical kinetic term.

Since the observations are generally made in the Einstein frame, we perform a conformal transformation of the Jordan frame metric $\hat{g}_{\mu\nu}$ to remove the coupling of the scalar field to the Ricci scalar. This is achieved by scaling the Jordan frame metric $\hat{g}_{\mu\nu}$ with the conformal field $\zeta(x)$ such that the metric in the Einstein frame is given by $g_{\mu\nu} = \zeta^2(x)\hat{g}_{\mu\nu}$. With this conformal scaling it can be shown that in four dimensions, the Ricci scalar $R$ in the Einstein frame is related to the Ricci scalar $\hat{R}$ in the Jordan frame by,

$$R = \left[ \frac{\hat{R}}{\zeta^2} - \frac{6}{\zeta^4} \hat{g}^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu \zeta \right]$$  \hspace{1cm} (2.17)$$

where $\hat{\nabla}$ denotes covariant derivative with respect to the metric $\hat{g}_{\mu\nu}$. Using eq. (2.17) and the fact that $\sqrt{-\hat{g}} = \zeta^{-4} \sqrt{-g}$ and choosing $\zeta \equiv \sqrt{h(\Phi/f)}$, we arrive at the effective action in the Einstein frame,

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} + \frac{1}{2} G(\xi) \partial^\mu \Phi \partial_\mu \Phi - 8M^3\kappa^2 V(\xi) \right]$$  \hspace{1cm} (2.18)$$

where $2\kappa^2 = 16\pi G_N = \frac{k_0}{2M^2}$ and the potential due to the radion field in the Einstein frame is given by,

$$V(\xi) = \frac{\hat{V}(\xi)}{h(\xi)^2} = \frac{6\omega^2}{h(\xi)}$$  \hspace{1cm} (2.19)$$
while the non-canonical coupling to the kinetic term is given by,

\[ G(\xi) = \frac{\dot{G}(\xi)}{h(\xi)} + \frac{1}{c^2} \left[ \frac{h'(\xi)}{h(\xi)} \right]^2 \]  

(2.20)

where ‘prime’ here implies differentiation with respect to \( \xi \). In figure 1a and figure 1b we plot the variation of \( V \) and \( G \) with the radion field \( \xi \) for \( \omega = 10^{-3} \). It can be shown from eq. (2.19) that the radion potential \( V \) in the Einstein frame continues to have an inflection point at \( \xi_i = \omega/c_2^2 \) which can be confirmed from the vanishing first and second derivatives but a positive third derivative of \( V(\xi) \) with respect to \( \xi \) at \( \omega/c_2 \). Moreover, figure 1b reveals that the non-canonical coupling to the kinetic term \( G(\xi) \) exhibits a transition from a normal to a phantom regime (i.e. from \( G(\xi) > 0 \) to \( G(\xi) < 0 \), where the phantom like behavior remains when \( \xi \) lies in the range \( \xi_i \leq \xi \leq \xi_f \), with \( \xi_f \) denoting the zero crossing of \( G(\xi) \). Also note from figure 1b that for \( \omega = 10^{-3} \), \( \xi_f \approx 0.00148 \).

We thus note that in the non-flat warped braneworld scenario, we have a scalar field, the radion, which is associated with a potential and a non-canonical kinetic term. It is believed that in the early universe the big bang singularity can be avoided in a bouncing scenario which is triggered by a scalar field with a potential. This raises the question whether the radion field can be instrumental in giving rise to a bouncing universe which we address in the next section.

3 Implications in early universe cosmology: background evolution

In this section we explore the role of the radion field in triggering a bouncing universe which can potentially avoid the big bang singularity. The bouncing scenario often invokes a scalar field with a potential and there exist plenty of models in the literature which can give rise to such a scenario (see [11]). In most of the cases the scalar potentials are reconstructed to explain the observations and often their origin remains unexplained. The merit of the non-flat warped braneworld model lies in the fact that the radion field naturally arises from compactification in the effective four-dimensional theory and generates its own potential and
non-canonical kinetic term. Here we consider the implications of the radion field in inducing a bouncing universe.

Since we are interested to study early universe cosmology with the radion field we consider the metric in the Einstein frame to be described by the FRW spacetime in the spatially flat form,

$$ds^2 = dt^2 - a(t)^2 \left[ dx^2 + dy^2 + dz^2 \right]$$ (3.1)

with $a(t)$ is known as the scale factor of the universe. In the generalized RS scenario, the 3-branes can be Minkowskian, de-Sitter or anti de-Sitter depending on the values of the induced brane cosmological constant [96]. Recall, in the present work, we consider the branes to be de-Sitter, i.e. our visible universe is a 3-brane (i.e. having three spatial dimension along with the time coordinate), described by the spatially flat FRW metric. Since the FRW metric is curved irrespective of its spatial curvature, the visible 3-brane is non-flat. At this stage it deserves mentioning that the spatially flat FRW metric is more consistent over the closed or open FRW universe from latest Planck 2018 data through TT, TE, EE + lowE + lensing + BAO data, where TT means temperature cross-correlation of CMB data, TE means cross-correlation between temperature and electric type polarization of CMB data and finally BAO stands for Baryon Acoustic Oscillation [120]. Due to the time dependency of the scale factor, the metric in eq. (3.1) indicates a non-zero curvature on the four dimensional brane geometry and moreover the brane curvature is characterized by the corresponding Ricci scalar given by $R = 6 \ddot{a} + 6 \dot{a}^2$. As we will see from eq. (3.5) that the kinetic as well as the potential energy of radion field contributes to the on-brane Ricci scalar via the effective four dimensional Friedmann equation. Furthermore, as shown in eq. (2.19), the potential energy of the radion field is proportional to the induced brane cosmological constant and thus one may argue that the radion potential energy is generated entirely due to the presence of the non-zero brane cosmological constant. Thereby the brane cosmological constant affects the evolution of the scale factor through the potential energy density of the radion field. Below, we will show that the metric ansatz of eq. (3.1) is consistent with the field equations of motion and moreover it will lead to a non-singular bounce on our visible brane.

It is evident from eq. (2.18) that the energy momentum tensor $T^\mu_\nu$ due to the radion field is given by,

$$T^\mu_\nu = G(\xi) \partial^\mu \Phi \partial_\nu \Phi - \frac{1}{2} \delta^\mu_\nu G(\xi) \partial^\alpha \Phi \partial_\alpha \Phi + 2M^3k_0V(\xi) \delta^\mu_\nu (3.2)$$

such that

$$T^0_0(\xi) = \frac{3M^3c^2}{k_0} G(\xi) \dot{\xi}^2 + 2M^3k_0V(\xi) = \rho \quad (3.3)$$

represents the energy density while

$$-T^i_j(\xi) = \delta^i_j \left[ \frac{3M^3c^2}{k_0} G(\xi) \dot{\xi}^2 - 2M^3k_0V(\xi) \right] = p \quad (3.4)$$

corresponds to the pressure due to the radion field. We note that the radion field $\xi$ depends only on time since the background metric given by eq. (3.1) is only time dependent.
Using eq. (3.3) the Friedmann equation obtained from the temporal component of the Einstein’s equations assume the form,

$$H^2 = \frac{\kappa^2}{3} \rho(t) = \frac{c^2}{4} G(\xi) \dot{\xi}^2 + \frac{k_0}{6} V(\xi)$$  \hspace{1cm} (3.5)$$

while the Friedman equation derived from the spatial component of the Einstein’s equations is given by,

$$\dot{H} = -\frac{\kappa^2}{2} (\rho + p) = -\frac{3}{4} c^2 G(\xi) \dot{\xi}^2$$  \hspace{1cm} (3.6)$$

where $H = \dot{a}/a$ denotes the Hubble parameter. The equation of motion for the radion field is given by,

$$\dot{\rho} + 3H(\rho + p) = 0$$  \hspace{1cm} (3.7)$$

Using eq. (3.3) and eq. (3.4), eq. (3.7) can be written as,

$$\ddot{\xi} + 3H \dot{\xi} + \frac{G'(\xi)}{2G(\xi)} \dot{\xi}^2 + \frac{k_0^2}{3c_2^2} V'(\xi) G(\xi) = 0$$  \hspace{1cm} (3.8)$$

Eq. (3.5), eq. (3.6) and eq. (3.8) are the background equations, although it is important to note that eq. (3.8) is not independent but can be derived from eq. (3.5) and eq. (3.6).

At this stage it deserves mentioning that in a canonical scalar tensor theory, the Friedmann equation becomes $\dot{H} \propto -\dot{\xi}^2$ (eq. (3.6)) and thus the Hubble parameter decreases monotonically with cosmic time. Therefore a bounce phenomena is impossible in a canonical scalar tensor model as it cannot give rise to $\dot{H} > 0$ which is one of the necessary conditions to get a bounce. On the contrary, in a non-canonical scalar tensor theory where the scalar field has non-canonical kinetic term (as $G(\xi)$ in the present context), the Friedmann equations are modified due to the presence of $G(\xi)$ and the modified equations are given by eq. (3.5) and eq. (3.6) respectively. Eq. (3.6) clearly indicates that in a non-canonical scalar tensor model, the sign of $G(\xi)$ actually controls the energy condition, in particular $G(\xi) < 0$ leads to a violation of null energy condition which in turn may ensure a bouncing phase in our visible universe. We have already noted in section 3 that in the present scalar-tensor model the non-canonical kinetic term exhibits a transition from a normal to a phantom regime (i.e. from $G(\xi) > 0$ to $G(\xi) < 0$) where the null energy condition is violated. Therefore it is important to investigate the prospect of bouncing cosmology with the present non-flat warped braneworld model which we explore next. In particular, we first present the background evolution of $H(t)$ and $\xi(t)$ (governed by eq. (3.5) and eq. (3.6)) in the next section and subsequently study the evolution of the perturbations in section 4.

In general, a non-singular bounce is characterized by the conditions $H(t_b) = 0$ and $\dot{H}(t_b) > 0$ where $t_b$ is the cosmic time when the bounce occurs. Keeping these conditions in mind, if we look into eq. (3.5) and eq. (3.6), then it is evident that the model has a possibility to show a bounce phenomena when the non-canonical function $G(\xi)$ becomes negative i.e. when the radion field is in the phantom regime. The analysis in section 3 reveals that $G(\xi)$ is indeed negative in the regime $\xi \sim \omega$. Thus, at first we analytically solve the background equations near $\xi \sim \omega$ to investigate the bounce and then we numerically determine the background evolution for a wide range of $\xi$ (or equivalently for a wide range of cosmic time),
where the boundary conditions of the numerical calculation are provided from the previously found analytic solutions.

In particular we consider,
\[ \xi(t) = \frac{\omega}{c^2} [1 + \delta(t)] \]  
(3.9)

with \( \delta(t) \ll 1 \). Due to the above form of \( \xi(t) \), \( h(\xi) \) in eq. (2.12) simplifies to
\[ h(\xi) = \frac{c_2^2}{4} + O(\omega^2) \approx \frac{c_2^2}{4} \]  
(3.10)
such that \( V(\xi) \) is given by,
\[ V(\xi) \approx \frac{24 \omega^2}{c_2^2} \]  
(3.11)
while \( G(\xi) \) can be approximated as,
\[ G(\xi) \approx -\frac{16}{3c_2} \left[ \ln \left( \frac{c_2}{\omega} \right) - \left\{ 4 + 2 \ln \left( \frac{c_2}{\omega} \right) \right\} \delta \right] \]  
(3.12)
The above simplifications in the form of \( V(\xi) \) and \( G(\xi) \) hold only in the regime where \( \xi(t) \) is given by eq. (3.9) with \( \delta(t) \ll 1 \). With these simplifications the evolution equations for the Hubble parameter \( H(t) \) and the radion field (i.e. eq. (3.5) and eq. (3.6)) turn out to be,
\[ \dot{H} + 3H^2 - \frac{12k_0^2\omega^2}{c_2^2} = 0 \]  
(3.13)
and
\[ \dot{\delta} = \frac{c_2^2}{\omega^2} \frac{6}{4 \ln \left( \frac{c_2}{\omega} \right)} \left[ 1 + \delta \left( \frac{4 + 2 \ln \left( \frac{c_2}{\omega} \right)}{\ln \left( \frac{c_2}{\omega} \right)} \right) \right] \]  
(3.14)
respectively. Eq. (3.13) can be solved to obtain the time evolution of the Hubble parameter which assumes the form,
\[ H(t) = 2k_0 \frac{\omega}{c_2} \tanh \left[ 6 \frac{\omega}{c_2} k_0 t \right] \]  
(3.15)
such that \( \dot{H} \) is given by,
\[ \dot{H} = 12k_0^2 \frac{\omega^2}{c_2^2} \text{sech}^2 \left[ 6 \frac{\omega}{c_2} k_0 t \right] \]  
(3.16)
Using eq. (3.16) in eq. (3.14) we obtain,
\[ \dot{\delta} = -k_0 \sqrt{ \frac{3}{\ln \left( \frac{c_2}{\omega} \right)} } \left( 1 + \frac{A\delta}{2} \right) \text{sech} \left[ 6 \frac{\omega}{c_2} k_0 t \right] \]  
(3.17)
with $A = \frac{4 + 2 \ln(\frac{\omega}{c^2})}{\ln(\frac{\omega}{c^2})}$ and solving the above equation yields the following time evolution for $\delta(t)$,

$$
\delta(t) = -\frac{2}{A} + C_1 \exp \left[ -\frac{A \omega}{6 c^2} \sqrt{\frac{3}{\ln(\frac{\omega}{c^2})}} \tan^{-1} \tanh \left( \frac{3\omega}{c^2} k_0 t \right) \right] \tag{3.18}
$$

where $C_1$ is an integration constant which can be determined by demanding,

$$
\lim_{t \to \infty} \delta(t) \to 0 \quad \text{or equivalently} \quad \lim_{t \to \infty} \xi(t) \to \frac{\omega}{c^2} \tag{3.19}
$$

which implies that $\xi(t)$ (i.e. the radion field) monotonically decreases with time and asymptotically goes to the value $\frac{\omega}{c^2}$ which is the minimum possible value of $\xi$ from the requirement of positive warp factor, as discussed after eq. (2.9). From the condition given by eq. (3.19), the form of $C_1$ turns out to be,

$$
C_1 = \frac{2}{A} \exp \left[ \frac{A \omega}{6 c^2} \sqrt{\frac{3}{\ln(\frac{\omega}{c^2})}} \frac{\pi}{4} \right] \tag{3.20}
$$

which when substituted in eq. (3.18) gives,

$$
\delta(t) = \frac{2}{A} \left[ \exp \left\{ -\frac{A \omega}{6 c^2} \sqrt{\frac{3}{\ln(\frac{\omega}{c^2})}} \left( \tan^{-1} \tanh \left( \frac{3\omega}{c^2} k_0 t \right) - \frac{\pi}{4} \right) \right\} - 1 \right] \tag{3.21}
$$

With this, we arrive at the background solution for $H(t)$ and $\delta(t)$ in the regime $\xi(t) = \frac{\omega}{c^2}(1 + \delta(t))$ with $\delta(t) \ll 1$,

$$
\xi(t) = \frac{\omega}{c^2}(1 + \delta(t)), \quad \begin{cases} 
H(t) = 2k_0 \frac{\omega}{c^2} \tanh \left[ \frac{6\omega}{c^2} k_0 t \right], \\
\delta(t) = \frac{2}{A} \left[ \exp \left\{ -\frac{A \omega}{6 c^2} \sqrt{\frac{3}{\ln(\frac{\omega}{c^2})}} \left( \tan^{-1} \tanh \left( \frac{3\omega}{c^2} k_0 t \right) - \frac{\pi}{4} \right) \right\} - 1 \right], 
\end{cases} \tag{3.22}
$$

where, $A = \frac{4 + 2 \ln(\frac{\omega}{c^2})}{\ln(\frac{\omega}{c^2})}$. Eq. (3.22) clearly indicates $H(0) = 0$ and $\dot{H} > 0$ at $t = 0$ (corresponding to the bounce time) which are the necessary conditions for a non-singular bounce. Therefore, the present non-flat warped braneworld model predicts a bouncing universe in the visible brane when the radion field lies within the phantom regime, in particular near $\xi \sim \omega$. In the phantom regime, due to the negative kinetic energy of the radion field, the effective null energy condition (NEC) is violated and makes the bounce possible at a certain finite time, in particular at $t = 0$. Here we would like to mention that such NEC violation occurs irrespective of any values of $\frac{\omega}{c^2}$ and $k_0$ (i.e. the model parameters), and moreover the initial conditions of the solution of eq. (3.22) is free from fine tuning of the model parameters. Thereby, we may argue that the bounce solution in the present context is a generic feature and does not require any fine-tuned values of the model parameters.

At this stage, it is important to check whether the radion field, starting from a value in the normal regime, will reach to the phantom regime by its dynamical evolution. For this
Figure 2. The above figure depicts the time evolution of (a) the Hubble parameter $H(t)$ and (b) the radion field magnified 1000 times, i.e. $\xi(t) \times 1000$; while the inset of figure 2b depicts the non-canonical kinetic term $G(\xi)$ (magenta curve) and the zoomed-in version of $\xi(t) \times 1000$ (blue curve) near the zero crossing of $G(\xi)$. Note that bounce occurs at $t = 0$ when the kinetic term of the radion is in the phantom regime. Both the above figures are illustrated for $\omega = 10^{-3}$.

purpose, we solve the coupled equations for $H(t)$ and $\xi(t)$ (i.e. eq. (3.5) and eq. (3.6)) for a wide range of cosmic time numerically. In regard to the numerical calculation, the boundary conditions are provided from the analytic solutions as determined in eq. (3.22), in particular the boundary conditions are given by $H(0) = 0$ and $\xi(0) = 6.0041 \times 10^{-4}$, where we consider $\omega = 10^{-3}$ (later, during the perturbation calculation, we show that such a value of $\omega$ is consistent with the Planck 2018 constraints). The time evolution of the Hubble parameter and the radion field are shown in figure 2a and figure 2b respectively (the radion field plot is magnified 1000 times i.e. $\xi(t) \times 1000$). In the inset of figure 2b, the magenta curve denotes the time evolution of $G(\xi)$ while the blue curve represents the zoomed-in version of $\xi(t) \times 1000$ near the zero crossing of $G(\xi)$. From figure 2b it is evident that $G(\xi)$ exhibits a transition from a normal regime (where $G(\xi) > 0$) to a phantom regime (where $G(\xi) < 0$) with its zero crossing occurs at a finite time before the bounce at $t = 0$. Moreover figure 2b demonstrates that there is no divergence in the dynamical evolution of $\xi(t)$ as $G(\xi)$ transits from normal to the phantom regime. However on the other hand, as evident from eq. (3.6), $\frac{d}{dt} \xi = 0$ diverges at the time when the non-canonical kinetic coupling $G(\xi)$ makes the zero crossing. Here we would like to mention that such divergence of $\xi$ does not lead to any pathology to the radion field equation of motion i.e. to eq. (3.8) and the reason is following: eq. (3.8) can be equivalently expressed as

$$\frac{d}{dt} \left( \frac{3M^4c^2}{k_0} G(\xi) \dot{\xi}^2 + 2M^3k_0V(\xi) \right) + \frac{18M^3c^2}{k_0} H G(\xi) \dot{\xi}^2 = 0$$

which includes $G(\xi)\dot{\xi}^2$ and its derivative with respect to cosmic time. Now eq. (3.6) evidents that $G(\xi)\dot{\xi}^2$ is proportional to $H$ which, along with its derivative, is indeed finite for all possible cosmic time (see figure 2a). Thereby the term $G(\xi)\dot{\xi}^2$ and its derivative with respect to $t$ are finite everywhere even at $G(\xi) \rightarrow 0$, and thus we may argue that the radion field equation of motion does not lead to any inconsistency in the present context.

Figure 2a reveals that the Hubble parameter becomes zero and increases with respect to cosmic time at $t = 0$, which confirms a non-singular bounce at $t = 0$. Before demonstrating the dynamics of the radion field, we recall that for $\omega = 10^{-3}$, the zero crossing of $G(\xi)$ occurs at $\xi = \xi_f \simeq 0.00148$, as shown in figure 1b i.e. $G(\xi)$ exhibits the normal to phantom
transition as $\xi$ crosses $\xi_f = 0.00148$ from higher values. Numerical solution of eq. (3.5) and eq. (3.6) indicates that the radion field starts its journey from the normal regime (i.e. $\xi > \xi_f$) and dynamically moves to the phantom era (i.e. $\xi < \xi_f$) with time by monotonically decreasing in magnitude and asymptotically stabilizes to the value $\xi_i \to \omega/c_2$ which for $\omega = 10^{-3} \sim \xi_i = 5 \times 10^{-4}$. This is in accordance with the analytical results obtained in eq. (3.22) and the time evolution of the background radion field is explicitly illustrated in figure 2b. As the radion field asymptotically tends to $\xi_i$ (i.e., $\xi \to \xi_i + \varepsilon$), the warp factor $e^{-A} \simeq c_2 \varepsilon$ which in turn resolves the gauge-hierarchy problem for $\varepsilon \simeq 10^{-16}$ while the stabilized inter-brane separation $k_0 \pi (T) \to \ln \left( \frac{\omega}{c_2} \right)$ [123]. Therefore, in the non-flat warped braneworld scenario with dynamical branes, the radion generates its own potential which in turn stabilizes the modulus dynamically in the FRW background. Further, the presence of the phantom era enables violation of the null energy condition for the radion field which makes this a promising model to explore the bouncing scenario.

At this stage, it may be mentioned that a holonomy improved non-canonical scalar tensor model may rescue the energy condition in a bouncing scenario. In the holonomy generalized model, the squared Hubble parameter (i.e. $H^2$ in eq. (3.5)) is proportional to the linear as well as quadratic power of energy density, unlike the usual Friedmann equations where $H^2$ is proportional only to the linear power of energy density. Such difference in the field equations may play a significant role to rescue the null energy condition necessary for a non-singular bounce. This investigation is expected to be carried out soon in a future work.

4 Implications in early universe cosmology: evolution of perturbations

In this section, we consider the spacetime perturbations over the background FRW metric and consequently determine the primordial observable quantities like the scalar spectral index ($n_s$), tensor to scalar ratio ($r$) and the amplitude of scalar perturbations ($A_s$). In a bouncing universe, the Hubble parameter becomes zero and consequently the comoving Hubble radius diverges at the bouncing point. However, the asymptotic behaviour of the Hubble radius differentiates various bouncing models which can be broadly classified into two scenarios. In the first case, the comoving Hubble radius decreases and goes to zero asymptotically with time, which corresponds to a late time accelerating universe. In this case, the perturbation modes generate near the bounce, because at that time, the horizon has an infinite size and all the perturbation modes lie within the horizon. In the second situation the Hubble radius diverges asymptotically with time, which indicates a decelerating universe at late time and consequently the primordial perturbation modes relevant for the present era generate at a distant past far away from the bounce. More explicitly, in the latter case, the comoving wave number $k$ begins its journey from the infinite past in the contracting universe, within the sub-Hubble scale, exits the horizon as it contracts, and again re-enters the horizon in the low curvature regime of the expanding phase and becomes relevant for present time observations. Therefore, depending on the asymptotic behavior of the Hubble radius, the perturbation modes in a bounce model generate either near the bounce or far away from the bounce deeply in the contracting regime.

Based on the above arguments, before moving to the perturbation calculations, we would like to investigate the asymptotic behaviour of the comoving Hubble radius (defined by $1/aH$) in the context of present model. Using the background solution of the Hubble parameter from figure 2a, we give the evolution of $\frac{1}{aH}$ with respect to cosmic time in figure 3a which clearly demonstrates that the comoving Hubble radius monotonically decreases with time.
Figure 3. The above figure depicts the time evolution of (a) the comoving Hubble radius $\frac{1}{aH}$ and (b) the inverse Hubble parameter $H^{-1}$. Both the above figures are illustrated for $\omega = 10^{-3}$.

and goes to zero asymptotically on both sides of the bounce. Here it may be mentioned that unlike to the comoving Hubble radius, the inverse Hubble parameter does not go to zero asymptotically but reaches to a constant value at late stage of the universe, which is depicted in figure 3b showing the behaviour of $H^{-1}$ vs. $t$. This corresponds to a late time accelerating universe. The asymptotic evolution of the comoving Hubble radius leads to the perturbation modes generate near the bouncing regime where the Hubble radius has an infinite size such that all the perturbation modes are contained inside the horizon. In this regard the present scenario is different from the usual matter bounce models where the Hubble radius diverges asymptotically and the perturbation modes generate far away from the bounce. Therefore in the next section we solve the perturbation equations near the bouncing point $t = 0$.

4.1 Scalar perturbation

The scalar metric perturbation over FRW metric can be written in the longitudinal gauge as,

$$ds^2 = a^2(\eta) \left( (1 + 2\Psi) d\eta^2 - (1 - 2\Psi) \delta_{ij} dx^i dx^j \right)$$

(4.1)

where the line element is expressed in ($\eta, \vec{x}$) coordinates with $\eta$ being the conformal time defined by $d\eta = \frac{dt}{a(t)}$ and the variable $\Psi(\eta, \vec{x})$ symbolizes the scalar metric fluctuation. Here it may be mentioned that the spacelike and the timelike components of scalar perturbation are considered to be same, this is because the background evolution has no anisotropic stress in the present context. Moreover, we expand the radion field as,

$$\Phi(\eta, \vec{x}) = \Phi_0(\eta) + \delta\Phi(\eta, \vec{x})$$

(4.2)

in terms of the background radion $\Phi_0(\eta)$ and the fluctuation $\delta\Phi(\eta, \vec{x})$. As a result, the scalar perturbation, in the longitudinal gauge, follows the following equations (upto first order perturbations) [125],

$$\nabla^2 \Psi - 3\mathcal{H}\Psi' - 3\mathcal{H}\Psi = \frac{\kappa^2}{2} a^2 \delta T_0^0$$

$$(\Psi' + \mathcal{H}\Psi)_{,i} = \frac{\kappa^2}{2} a^2 \delta T_i^0$$

$$\left[ \Psi'' + 3\mathcal{H}\Psi' + (2\mathcal{H}' + \mathcal{H}^2) \Psi \right] \delta_j^i = -\frac{\kappa^2}{2} a^2 \delta T_j^i$$

(4.3)
which explicitly depends on the non-canonical term $G$. The variation of matter energy-momentum tensor, i.e., $\delta T_{\mu\nu}$ (recall, the radion field is the only matter field in the present context) present in the right hand side of the above equations, can be obtained from eq. (3.2) and are given by,

$$
\delta T_0^0 = \frac{1}{a^2} \left[ G(\Phi_0) \Phi_0' \delta \Phi' + \frac{1}{2} G'(\Phi_0)(\Phi_0')^2 \delta \Phi + 2a^2 M^3 k_0 V'(\Phi_0) \delta \Phi \right]
$$

$$
\delta T_1^1 = \frac{1}{a^2} \partial_1 \left[ G(\Phi_0) \Phi_0' \delta \Phi \right]
$$

$$
\delta T_i^j = -\frac{1}{a^2} \delta^i_j \left[ G(\Phi_0) \Phi_0' \delta \Phi' + \frac{1}{2} G'(\Phi_0)(\Phi_0')^2 \delta \Phi - 2a^2 M^3 k_0 V'(\Phi_0) \delta \Phi \right]
$$

where we explicitly used the fluctuation of the radion field shown in eq. (4.2) and recall, $V(\Phi)$ and $G(\Phi)$ are the radion potential and the non-canonical kinetic term of the radion field respectively. In eq. (4.4) and in the rest of the discussion the primes in $V(\Phi_0)$ and $G(\Phi_0)$ are with respect to the background radion field $\Phi_0$ while the primes in $H$ and $\Phi_0$ are with respect to the conformal time $\eta$. Substituting the expressions of $\delta T_{\mu\nu}$ into the set of equations eq. (4.3), we get

$$
\nabla^2 \Psi - 3H\Psi' - 3\dot{H} \Psi = \frac{\kappa^2}{2} \left[ G(\Phi_0) \Phi_0' \delta \Phi' + \frac{1}{2} G'(\Phi_0)(\Phi_0')^2 \delta \Phi + 2a^2 M^3 k_0 V'(\Phi_0) \delta \Phi \right]
$$

$$
\Psi' + H\Psi = \frac{\kappa^2}{2} \Phi_0' \delta \Phi
$$

$$
\Psi'' + 3H \Psi' + (2H' + H^2) \Psi = \frac{\kappa^2}{2} \left[ G(\Phi_0) \Phi_0' \delta \Phi' + \frac{1}{2} G'(\Phi_0)(\Phi_0')^2 \delta \Phi - 2a^2 M^3 k_0 V'(\Phi_0) \delta \Phi \right]
$$

(4.5)

respectively. The second of eq. (4.5) can be used to obtain $\delta \Phi$ in terms of $\Psi$ and $\Psi'$, substituting which into the other two equations leads to the evolution of $\Psi(\eta, \vec{x})$ as,

$$
\Psi'' - \nabla^2 \Psi + 6H \Psi' + (2H' + 4H^2) \Psi = -4a^2 M^3 k_0 \left( \frac{V'(\Phi_0)(\Psi' + H\Psi)}{G(\Phi_0)\Phi_0'} \right)
$$

(4.6)

which explicitly depends on the non-canonical term $G(\Phi_0)$ and for $G(\Phi_0) = 1$, eq. (4.6) reduces to that of the canonical scalar field case [125]. To solve the above perturbation equation, we will use the background evolution of the Hubble parameter and the radion field, which are obtained in the cosmic time coordinate. Thus we first transform eq. (4.6) in terms of the cosmic time and for this purpose, we need the following relations,

$$
\Psi' = a \dot{\Psi} \quad \text{and} \quad \Psi'' = a^2 \ddot{\Psi} + a^2 H\dot{\Psi}
$$

with overdot and prime representing $\frac{d}{dt}$ and $\frac{d}{d\eta}$ respectively. As a result, eq. (4.6) turns out to be,

$$
\ddot{\Psi} - \frac{1}{a^2} \nabla^2 \Psi + \left[ 7H + \frac{2k_0^2}{3c_s^2} \frac{V'(\xi_0)}{G(\xi_0)\xi_0} \right] \dot{\Psi} + \left[ 2\dot{H} + 6H^2 + \frac{2k_0^2}{3c_s^2} \frac{H V'(\xi_0)}{G(\xi_0)\xi_0} \right] \Psi = 0
$$

(4.7)

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter in cosmic time and recall, $\xi_0 = \frac{\Phi_0}{f}$ (with $f = \sqrt{\frac{6M^3 c_s^2}{k_0}}$) is the dimensionless radion field. Eq. (4.7) clearly reveals how the dynamics of the scalar
perturbation (i.e. the acceleration term \(\ddot{\Psi}\)) depends on the background evolution of \(H(t)\) and \(\xi_0(t)\). In particular, the second term in the left hand side leads to an oscillation of \(\Psi\), the third term denotes a friction term and the fourth term indicates a restoring force. As mentioned earlier, the perturbation modes generate near the bounce and thus we are interested to solve the perturbation equations near \(t = 0\), in which case, the background Hubble parameter and the radion field evolution follow eq. (3.22). Using such background evolution of \(H(t)\) and \(\xi(t)\) along with the near-bounce expression of \(G(\xi)\) (see eq. (3.12)), we determine \(\frac{V'(\xi_0)}{G(\xi_0)^2}\) (present in the above equation) as,

\[
\frac{V'(\xi_0)}{G(\xi_0)^2} = \frac{36\omega^2 \delta(t)}{\xi_0^2} = \frac{36\omega^2 \delta(t)}{\xi_0^2} + 48B\omega c_2 \sinh^2(B\pi/8) + 72\omega^2 \left(2 - 2 \cosh(B\pi/4) + \sinh(B\pi/4)\right) \frac{t}{(3 - 2eB\pi/4)^2(2 - \ln \frac{c_2}{\omega})}
\]

(4.8)

where \(B = \frac{\Delta \omega}{c_2} \sqrt{\frac{\delta}{\ln \left(\frac{c_2}{\delta}\right)}}\) and we retain the expression of \(\frac{V'(\xi_0)}{G(\xi_0)^2}\) up to the leading order in \(t\).

With the above expression, eq. (4.7) turns out to be,

\[
\dot{\Psi} - \nabla^2 \Psi + \left[-\sqrt{\alpha p + (q + 14)\alpha t}\right] \ddot{\Psi} + \left[4\alpha - 2\alpha \sqrt{\alpha p t}\right] \Psi(\vec{x}, t) = 0
\]

(4.9)

near the bounce (i.e. in the leading order of \(t\)), where \(\alpha = \frac{6k_0\omega^2}{c_2^2}\) and \(p\) and \(q\) have the following expressions,

\[
p = 16\sqrt{\frac{2}{3}} \left(\frac{B \sinh^2(B\pi/8)}{3 - 2eB\pi/4(2 - \ln \frac{c_2}{\omega})}\right) \quad \text{and} \quad q = 8\left(2 - 2 \cosh(B\pi/4) + \sinh(B\pi/4)\right) \frac{t}{(3 - 2eB\pi/4)^2(2 - \ln \frac{c_2}{\omega})}
\]

respectively. In terms of the Fourier transformed scalar perturbation variable \(\Psi_k(t) = \int d\vec{x} e^{-i\vec{k} \cdot \vec{x}} \Psi(\vec{x}, t)\), eq. (4.9) can be written as,

\[
\dot{\Psi}_k + \left[-\sqrt{\alpha p + (q + 14)\alpha t}\right] \ddot{\Psi}_k + \left[k^2 + 4\alpha - 2\alpha \sqrt{\alpha p t}\right] \Psi_k(t) = 0
\]

(4.10)

Solving eq. (4.10) for \(\Psi_k(t)\), we get

\[
\Psi_k(t) = b_1(k) \exp \left[\sqrt{\alpha p t} - 7\alpha t^2 - \frac{q}{2} \alpha t^2\right] H \left[-1 + \frac{k^2 + 4\alpha}{\alpha(q + 14)}, -p + (q + 14)\sqrt{\alpha t}, \frac{\sqrt{2}(q + 14)}{\sqrt{2}(q + 14)}\right]
\]

(4.11)

with \(H[n, x]\) is the \(n\)-th order Hermite polynomial. \(b_1(k)\) is the integration constant which can be determined from the initial Bunch-Davies vacuum condition given by \(\lim_{t \to 0} v_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}\), where \(v_k(\eta)\) is the canonical Mukhanov-Sasaki variable. The Bunch-Davies vacuum choice is justified since the primordial modes at \(t = 0\) (or equivalently \(\eta = 0\)) are well inside the Hubble horizon. The Bunch-Davies vacuum condition on the Mukhanov-Sasaki variable immediately leads the corresponding condition on \(\Psi_k(t)\) from the following relation [125],

\[
\lim_{t \to 0} \Psi_k(t) = \frac{\kappa^2 f}{2k^2} \lim_{t \to 0} \left[\sqrt{G(\xi)} \xi v_k(\eta)\right] = \frac{i\kappa^2 f}{2\sqrt{2k}^{3/2}} \lim_{t \to 0} \left[\sqrt{G(\xi)} \xi\right]
\]

(4.12)

and by using the background evolution of \(\xi(t)\) along with the expression of \(G(\xi)\), we determine the initial condition of \(\Psi_k(t)\) as follows,

\[
\lim_{t \to 0} \Psi_k(t) = \frac{\sqrt{3}}{2k^{3/2}} \left(\frac{\omega}{c_2}\right)^{3/2} \left(\frac{k_0}{M}\right)^{3/2} e^{B\pi/4} (3 - 2eB\pi/4)^{1/2}
\]

(4.13)
This makes the integration constant $b_1(k)$ have the following form,

$$b_1(k) = \frac{\sqrt{3}}{2k^{3/2}} \left( \frac{\omega}{c^2} \right) \left( \frac{k_0}{M} \right)^{3/2} \left\{ \frac{e^{B\pi/4}(3 - 2e^{B\pi/4})^{1/2}}{H \left[ -1 + \frac{k^2 + 4\alpha}{\alpha(q+14)} \frac{-p}{\sqrt{2(q+14)}} \right]} \right\}$$

Substituting the above expression of $b_1(k)$ into eq. (4.11) yields the following solution for the scalar perturbation variable,

$$\Psi_k(t) = \frac{\sqrt{3}}{2k^{3/2}} \left( \frac{\omega}{c^2} \right) \left( \frac{k_0}{M} \right)^{3/2} e^{B\pi/4}(3 - 2e^{B\pi/4})^{1/2} e^{[p\sqrt{\alpha} t - 7\alpha t^2 - 4\alpha t^2]} \times \left\{ \frac{H \left[ -1 + \frac{k^2 + 4\alpha}{\alpha(q+14)} \frac{-p}{\sqrt{2(q+14)}} \right]}{H \left[ -1 + \frac{k^2 + 4\alpha}{\alpha(q+14)} \frac{-p}{\sqrt{2(q+14)}} \right]} \right\} \quad (4.14)$$

where $p$ and $q$ are given below eq. (4.9). Consequently the solution of $\Psi_k(t)$ immediately leads to the scalar power spectrum for $k$-th modes as,

$$P_{\Psi}(k, t) = \frac{k^3}{2\pi^2} |\Psi_k(t)|^2 = \frac{3}{8\pi^2} \left( \frac{\omega}{c^2} \right)^2 \left( \frac{k_0}{M} \right)^3 e^{B\pi/2}(3 - 2e^{B\pi/4}) e^{[2p\sqrt{\alpha} t - 14\alpha t^2 - q\alpha t^2]} \times \left\{ \frac{H \left[ -1 + \frac{k^2 + 4\alpha}{\alpha(q+14)} \frac{-p}{\sqrt{2(q+14)}} \right]}{H \left[ -1 + \frac{k^2 + 4\alpha}{\alpha(q+14)} \frac{-p}{\sqrt{2(q+14)}} \right]} \right\}^2 \quad (4.15)$$

Here we would like to mention that our main aim in this section is to investigate whether the theoretical predictions of $n_s$, $A_s$ and $r$ match with the Planck 2018 results which put a constraint on these observable quantities around the CMB scale. Therefore the scale of interest in the present context is around the CMB scale given by $k_{\text{CMB}} \approx 0.02\text{Mpc}^{-1} \approx 10^{-40}\text{GeV}$. With the background solution of Hubble parameter from eq. (3.22), we determine the expression of the time when $k_{\text{CMB}}$ crosses the horizon by using the horizon crossing relation $k = aH$, and is given by,

$$t_h = \frac{k_{\text{CMB}}}{12k_0^2 \left( \frac{\omega^2}{c^2} \right)} \quad (4.16)$$

where, $t_h$ is the horizon crossing time of the CMB scale and recall, $k_0$ being the bulk curvature scale. As we will show later that the model stands to be a viable one in regard to the Planck constraints for the parameter ranges: $\omega = 10^{-3}$ and $\frac{k_0}{M} = [0.601, 0.607]$ respectively. Such parametric ranges make the horizon crossing instance of $k_{\text{CMB}}$ as $t_h \sim 10^{-68}\text{GeV}^{-1} \approx 10^{-93}\text{sec}$ (the conversion $1\text{GeV}^{-1} = 10^{-25}\text{sec}$ may be useful). This estimation of $t_h$ along with eq. (3.22) indicate that the scale factor, around the horizon crossing instance of $k_{\text{CMB}}$, practically behaves as $a(t \simeq t_h) = 1 + 6k_0^2 \left( \frac{\omega^2}{c^2} \right) t^2$; which in turn confirms the fact that the CMB scale crosses the horizon near the bouncing regime. Correspondingly, the scalar power
The spectrum at horizon crossing can be expressed as,

\[
P_{\Psi}(k, t)_{|_{h.c}} = \frac{3}{8\pi^2} \left(\frac{\omega}{c^2}\right)^2 \left(\frac{k_0}{\mathcal{M}}\right)^3 e^{B\pi/2} (3 - 2eB\pi/4) e^{[2p\sqrt{\kappa} t_0 - 14\alpha t_0^2 - \alpha t_0^2]} \]

\[
\times \left\{ \frac{H\left[-1 + \frac{k^2 + 4\alpha}{a(q+14)} \frac{-p+(q+14)\sqrt{\kappa} t_h}{\sqrt{2(q+14)}}}{H\left[-1 + \frac{k^2 + 4\alpha}{a(q+14)} \frac{-p}{\sqrt{2(q+14)}}\right]} \right\}^2. \tag{4.17}
\]

With eq. (4.17), we can determine the observable quantities like the scalar spectral index of the primordial curvature perturbations \(n_s\), the scalar perturbation amplitude \(A_s\) etc. However, before proceeding to calculate \(n_s\) and \(A_s\), we will perform first the tensor perturbation, which is necessary for evaluating the tensor-to-scalar ratio \(r\).

### 4.2 Tensor perturbation

In this section, we consider the tensor perturbation on the FRW metric background which is defined as follows,

\[
ds^2 = -dt^2 + a(t)^2 (\delta_{ij} + h_{ij}) dx^i dx^j, \tag{4.18}
\]

where \(h_{ij}(t, \vec{x})\) is the tensor perturbation. The variable \(h_{ij}(t, \vec{x})\) is itself a gauge invariant quantity, and the tensor perturbed action up to quadratic order is given by [126–128],

\[
\delta S_h = \int dtd^3\vec{x}a(t)^2 \left[ h_{ij} \dot{h}^{ij} - \frac{1}{a^2} (\partial_i h_{ij})^2 \right], \tag{4.19}
\]

where \(z_T(t)\), in the non-canonical scalar-tensor theory i.e. the case of the present context, has the following form [126],

\[
z_T(t) = \frac{a(t)}{\kappa}. \tag{4.20}
\]

Eq. (4.19) indicates that the speed of the tensor perturbation is \(c_T^2 = 1\) i.e. the gravitational waves propagate with the speed of light which is unity in the natural units. This is in agreement with the event GW170817 according to which, the gravitational wave and the electromagnetic wave have the same propagation speed. At this stage, it deserves mentioning that the speed of the gravitational wave depends on the background model, as for example, the \(c_T^2\) is not unity in scalar-Einstein-Gauss-Bonnet (GB) gravity theory and the deviation of \(c_T^2\) from unity is proportional to the GB coupling function considered in the model. However, there exists a certain class of GB coupling function for which the gravitational wave propagates with \(c_T^2 = 1\) leading to the compatibility of the GB model with GW170817 (the bouncing phenomenology in such a class of Gauss-Bonnet theory which is compatible with GW170817 has been recently discussed in [67]). On other hand, the non-canonical scalar-tensor theory always leads to \(c_T^2 = 1\) irrespective of the form of the non-canonical coupling function. Coming back to eq. (4.20), the tensor perturbation is ensured to be stable in the present context as the condition \(z_T(t)^2 = \frac{a(t)^2}{\kappa^2} > 0\) holds. The action eq. (4.19) leads to the following equation for the tensor perturbed variable \(h_{ij}\),

\[
\frac{1}{a(t)z_T^2(t)} \frac{d}{dt} \left[ a(t) z_T^2(t) \dot{h}_{ij} \right] - \frac{1}{a^2} \partial_i \partial_j h_{ij} = 0 \tag{4.21}
\]
The Fourier transformed tensor perturbation variable is defined as $h_{ij}(t, \vec{x}) = \int d\vec{k} \sum_{\gamma} \epsilon_b^{(\gamma)}_{ij} \hat{h}_{(\gamma)}(\vec{k}, t) e^{i \vec{k} \cdot \vec{x}}$, where $\gamma = \pm$ and $\gamma = \mp$ represent two polarization modes. Moreover, $\epsilon^{(\gamma)}_{ij}$ are the polarization tensors satisfying $\epsilon_{ii}^{(\gamma)} = k^{2} \epsilon_{ij}^{(\gamma)} = 0$. In terms of the Fourier transformed tensor variable $h_k(t)$, eq. (4.21) can be expressed as,

$$\frac{1}{a(t) z_k^2(t)} \frac{d}{dt} \left[ a(t) z_k^2(t) \hat{h}_k \right] + \frac{k^2}{a^2} h_k(t) = 0 \quad (4.22)$$

The two polarization modes obey the same eq. (4.22) and thus we omit the polarization index. Moreover, both the polarization modes even follow the same initial condition and hence have the same solution. Therefore, in the expression of the tensor power spectrum, we will introduce a multiplicative factor 2 due to the contribution from both the polarization modes. As mentioned earlier, the perturbation modes generates near the bouncing regime (because at that time all the perturbation modes lie within the Hubble horizon) where the background Hubble parameter ($\mathcal{H}(t)$) follow the evolution presented in eq. (3.22). From the solution of $\mathcal{H}(t)$ the form of the scale factor turns out to be $a(t) = (\cosh \left[ \frac{6k_0 t}{c^2} \right])^{1/3}$ which can be expanded in a Taylor series about $t = 0$ (i.e. about the bounce point) as,

$$a(t) \simeq 1 + \frac{6 \omega^2}{c^2} k_0^2 t^2 + \mathcal{O}(t^3)$$

We are interested to solve the perturbation near the bounce (i.e., $t = 0$) where the scale factor can be approximated to be $a(t) \simeq 1 + \frac{6 \omega^2}{c^2} k_0^2 t^2$. Using this expression of the near-bounce scale factor, we determine $a(t) z_k^2(t)$ as,

$$a(t) z_k^2(t) = \frac{a^3(t)}{k^2} \simeq \frac{1}{k^2} (1 + 3 \alpha t^2) \quad (4.23)$$

with $\alpha = 6 k_0^2 \omega^2 / c^2$. Substituting this expression of $a(t) z_k^2(t)$ into eq. (4.22) and after some algebra, we get the following equation for the Fourier transformed tensor perturbation variable,

$$\ddot{h}_k + 6 \alpha h_k + 2 \mathcal{H} h_k = 0 \quad (4.24)$$

at leading order in $t$ (since the perturbation modes generate near the bouncing phase i.e., near $t = 0$). Solving eq. (4.24) for $h_k(t)$, we get,

$$h_k(t) = b_2(k) e^{-3 \alpha t^2} \mathcal{H} \left[ -1 + \frac{k^2}{6 \alpha}, \sqrt{3 \alpha} t \right] \quad (4.25)$$

where $b_2(k)$ is an integration constant and can be determined from an initial condition. As an initial condition, we consider that the tensor perturbation field starts from the adiabatic vacuum, more precisely the initial configuration is given by, $\lim_{t \to 0} \left[ z_T(t) h_k(t) \right] = \frac{1}{\sqrt{2k}}$. This immediately leads to the expression of $b_2(k)$ as,

$$b_2(k) = \frac{1}{z_T(t \to 0)} \left[ 2 \Gamma \left( 1 - \frac{k^2}{12 \omega^2} \right) \right] = \kappa \left[ 2 \Gamma \left( 1 - \frac{k^2}{12 \omega^2} \right) \right]. \quad (4.26)$$

In the second equality of the above equation, we use $z_T(t \to 0) = 1/\kappa$ from eq. (4.20). Putting this expression of $b_2(k)$ into eq. (4.25) yields the final solution of $h_k(t)$ as follows,

$$h_k(t) = \left( \frac{2 \kappa \Gamma \left( 1 - \frac{k^2}{12 \omega^2} \right)}{\sqrt{2 \pi k^3} \frac{k^2}{2 \omega^2}} \right) e^{-3 \alpha t^2} \mathcal{H} \left[ -1 + \frac{k^2}{6 \alpha}, \sqrt{3 \alpha} t \right] \quad (4.27)$$
Eq. (4.27) represents the solution of the tensor perturbation for both the polarization modes. The solution of $h_k(t)$ immediately leads to the tensor power spectrum as,

$$P_h(k, t) = \frac{k^3}{2\pi^2} \sum_{\gamma} |h_k^{(\gamma)}(t)|^2$$

$$= \frac{2k^2}{\pi^3} \left( \frac{k \Gamma(1 - \frac{a^2}{\alpha^2})}{2^\frac{4\alpha^2}{a^2}} \right)^2 e^{-6\alpha^2} \left\{ H \left[ -1 + \frac{k^2}{2\alpha}, \sqrt{3\alpha} t \right] \right\}^2$$

(4.28)

It may be noticed that $\gamma = +$ and $\gamma = \times$ modes contribute equally to the power spectrum, as expected because their solutions behave similarly. At the horizon crossing $k = aH \simeq 2\alpha t_h$, the tensor power spectrum turns out to be,

$$P_h(k, t) \bigg|_{h.c} = \frac{12k^2\alpha^2}{\pi^3 M^2 c^2} \alpha_h^2 \left( \frac{k \Gamma(1 - \frac{a^2}{\alpha^2})}{2^\frac{4\alpha^2}{a^2}} \right)^2 \frac{1}{e^{6\alpha^2} \left\{ H \left[ -1 + \frac{2\alpha t_h^2}{3\alpha}, \sqrt{3\alpha} t_h \right] \right\}^2}$$

(4.29)

with $t_h$ being the horizon crossing instance.

Now we can explicitly confront the model at hand with the latest Planck observational data [129], so we shall calculate the spectral index of the primordial curvature perturbations $n_s$ and the tensor-to-scalar ratio $r$, which are defined as follows,

$$n_s - 1 = \frac{\partial \ln P_{\Psi}}{\partial \ln k} \bigg|_{h.c}, \quad r = \frac{P_h(k, t)}{P_{\Psi}(k, t)} \bigg|_{h.c}$$

(4.30)

As evident from these expressions, $n_s$ and $r$ are evaluated at the time of the horizon exit near the bouncing point (symbolized by ‘H.C’ in the above equations), for positive times when $k = aH$ i.e. when the mode $k$ crosses the Hubble horizon. Using eq. (4.15), we determine $\frac{\partial \ln P_{\Psi}}{\partial \ln k}$ as follows,

$$\frac{\partial \ln P_{\Psi}}{\partial \ln k} \bigg|_{\alpha(q+14)} = \frac{4k^2}{\alpha(q+14)} \begin{cases} H(1,0) \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] & - \frac{H(1,0)}{H} \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] \\ H \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] & \left\{ H \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] \right\} \end{cases}$$

(4.31)

where $H^{(1,0)}[z_1, z_2]$ is the derivative of $H[z_1, z_2]$ with respect to its first argument. Therefore eq. (4.31) immediately leads to the spectral index as,

$$n_s = 1 - \frac{4k^2}{\alpha(q+14)} \begin{cases} H(1,0) \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] & - \frac{H(1,0)}{H} \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] \\ H \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] & \left\{ H \left[ -1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \sqrt{-p(q+14)} \right] \right\} \end{cases} \bigg|_{h.c}$$

(4.32)

As mentioned earlier, the perturbation modes are generated and also cross the horizon near the bounce. Thus we can safely use the near-bounce scale factor in the horizon crossing condition to determine $k = aH = 2\alpha t_h$ (where $t_h$ is the horizon crossing time). Using this
relation, eq. (4.32) turns out to be,
\[
n_s = 1 - \frac{16\alpha t_h^2}{(q+14)} \left\{ \frac{H^{(1,0)}}{H} \right\} \left[ \begin{array}{c}
-1 + \frac{4(\alpha t_h^2 + 1)}{(q+14)}; \quad -p \\
-1 + \frac{4(\alpha t_h^2 + 1)}{(q+14)}; \quad -p + \frac{\sqrt{2(q+14)}}{2(q+14)} \end{array} \right] \right\}_{h,c}
\]
Furthermore, the tensor-to-scalar ratio is given by,
\[
r = \left. \frac{P_h(k,t)}{P_P(k,t)} \right|_{k=a(t_h)H(t_h)}
\]
where the solutions of $P_h$ and $P_P$ are shown in eq. (4.29) and eq. (4.17) respectively. Eq. (4.33)
and eq. (4.34) clearly indicate that $n_s$ and $r$ depend on the dimensionless parameters $\omega$ and $\alpha t_h^2$ which is further connected to the Ricci scalar at horizon crossing by $\alpha t_h^2 = (R_h/\sqrt{2\omega} - 1)$. Therefore, we can argue that the observable quantities $n_s$ and $r$ depend on $\omega$ and $R_h/\alpha$. With this information, we now directly confront the theoretical expressions of scalar spectral index eq. (4.33) and tensor-to-scalar ratio eq. (4.34) derived from the present model with the Planck 2018 constraints [129]. In particular, we estimate the allowed values of $R_h/\alpha$ and $\omega$ which in turn can give rise to $n_s$ and $r$ in agreement with the Planck data. This is presented in figure 4 where we compute $n_s$ and $r$ for three choices of $R_h/\alpha$ (viz, $R_h/\alpha = 14$ (blue point), $R_h/\alpha = 16$ (black point) and $R_h/\alpha = 19$ (red point) with $\omega = 10^{-3}$. The allowed values of $r$ and $n_s$ from Planck data within $1 - \sigma$ and $2 - \sigma$ constraints are illustrated by the yellow and the blue regions respectively in figure 4. We note that with $\omega = 10^{-3}$ and all the three aforesaid values of $R_h/\alpha$ the model estimated $n_s$ and $r$ are within the $1 - \sigma$ constraints reported by Planck 2018 data.

At this stage it may be mentioned that scalar-tensor models (with single scalar field) which exhibit a matter bounce scenario asymptotically, such that the perturbations are generated far away from the bouncing point deeply in the contracting regime, are generally not consistent with the Planck results since it gives rise to an exactly scale invariant power spectrum [14]. Such inconsistency with Planck observation was also confirmed in [66] from a slightly different viewpoint, namely from an $F(R)$ gravity theory. It turns out that $F(R)$ models can be equivalently mapped to scalar-tensor ones via conformal transformation of the metric and, thus, the inconsistencies of the spectral index in the two different models are well justified. However there exists counter example of this argument in the context of two field matter bounce in [22] where the authors proposed a cosmological evolution which undergoes the phases like matter contraction, then a period of ekpyrotic contraction, followed by a non-singular bounce, and then a phase of fast roll expansion. In such scenario, it has been showed that the primordial curvature perturbation dominated by a scale invariant component while there are other terms which can lead to a scale dependence at small length scales, in particular there is a subdominant $k^{3/2}$ dependence in the expression of scalar power spectrum. Unlike to such scenarios, here we demonstrate that a scalar-tensor gravity model indeed leads to a viable bouncing model when the primordial perturbations are generated near the bounce.

Furthermore the scalar perturbation amplitude $(A_s)$ is constrained to $\ln [10^{10}A_s] = 3.044 \pm 0.014$ from the Planck results [129]. From eq. (4.15) we note that the amplitude of scalar perturbations $A_s$ not only depends on $\omega$ and $R_h/\alpha$ but also on the ratio of the 5D bulk
Figure 4. 1σ (yellow) and 2σ (light blue) contours for Planck 2018 results [129], on $n_s - r$ plane. Additionally, we present the predictions of the present bounce scenario with $\frac{R_h}{\alpha} = 14$ (blue point), $\frac{R_h}{\alpha} = 16$ (black point) and $\frac{R_h}{\alpha} = 19$ (red point).

Curvature ($k_0$) and the 5D Planck mass ($M$) i.e. $\frac{k_0}{M}$. In particular, the scalar perturbation amplitude becomes $A_s = 9.5 \times 10^{-9} \left( \frac{k_0}{M} \right)^3$ when we take $\omega = 10^{-3}$ and $\frac{R_h}{\alpha} = 16$. This is in accordance with the Planck constraints mentioned above provided $\frac{k_0}{M}$ lies within $\frac{k_0}{M} = [0.601, 0.607]$ such that the bulk curvature is constrained to be less than the 5D Planck mass, which in turn confirms the validity of the background classical solution. However, it may be mentioned that the allowed range of $\frac{k_0}{M}$ is sensitive to the choice of $\omega$, i.e. a different $\omega$ will lead to a different allowed range for the parameter $\frac{k_0}{M}$. As an example, $\omega = 10^{-4}$ leads to the scalar perturbation amplitude as $A_s = 9.5 \times 10^{-11} \left( \frac{k_0}{M} \right)^3$ which becomes consistent with the Planck results for $\frac{k_0}{M} > 1$. However with the condition $\frac{k_0}{M} > 1$, the assumption of the background classical solution ceases to hold true, which is not desirable. Therefore, as a whole, the observable quantities $n_s$, $r$ and $A_s$ are simultaneously compatible with the Planck constraints for the parameter ranges: $\omega = 10^{-3}$, $14 \leq \frac{R_h}{\alpha} \leq 19$, $\frac{k_0}{M} = [0.601, 0.607]$ respectively. Such parametric ranges make the horizon crossing Ricci scalar of the order $R_h \sim k_0^2 \left( \frac{\omega}{c^2} \right)^2 \sim 10^{-8} M^2 = 10^{28} \text{GeV}^2$.

Before concluding, we would like to mention that the present paper studies a non-singular bounce from a warped braneworld scenario with dynamical branes, which is found to yield a nearly scale-invariant power spectra of primordial perturbations. However, in the background of the contracting era, the anisotropy grows with the scale factor as $a^{-6}$ and thus the contracting stage becomes unstable to the growth of anisotropies, which is known as the BKL instability [130]. Thus similar to many other bounce models, except the ekpyrotic bounce scenario [22, 24, 131, 132], the present model also suffers from the BKL instability. Thereby it may be an interesting study to explore the possible effects of radion dynamics in an ekpyrotic bounce scenario to avoid the BKL instability. This however may be considered in a future work.

5 Conclusion

We consider a five dimensional warped braneworld scenario with two 3-branes embedded within the 5D spacetime, where the branes have a non-zero cosmological constant $\omega$ leading to a non-flat brane geometry. With dynamical branes the interbrane distance is treated as
a $4-d$ scalar field, the so-called radion or the modulus, which generates its own potential when the $4-d$ effective action is obtained as a consequence of compactification of the extra coordinate. Such a radion field is also associated with a non-canonical kinetic term at the level of the four dimensional effective action which exhibits a transition from a normal to a phantom regime (i.e. from $G(\xi) > 0$ to $G(\xi) < 0$) as the radion field goes from higher to lower values. With the vanishing of the brane cosmological constant $\omega$, the branes become flat such that the radion potential ceases to exist while the radion kinetic term becomes canonical. Such a non-flat warped braneworld scenario is important as it can simultaneously address the gauge-hierarchy problem and the stabilization of the modulus without the necessity of any additional scalar field of unknown origin.

The presence of the phantom regime is further interesting as the cosmological evolution of the radion field in the FRW background leads to a violation of the null energy condition, necessary to ensure a non-singular bounce in our visible universe. This motivates us to explore the prospect of the radion field in triggering a bouncing universe which in turn can potentially avoid the Big-Bang singularity. Note that the radion field by which the bounce is driven arises naturally from compactification in the effective four-dimensional theory and generates its own potential due to the presence of the brane cosmological constant, unlike most of the scalar-tensor bounce models where the scalar potentials are constructed by hand to explain the observations and often their origin remains unexplained.

An analysis of the background cosmological evolution of the Hubble parameter and the radion field reveals that the radion field starts its journey from the normal regime (i.e. $G(\xi) > 0$ regime) and decreases monotonically in magnitude with cosmic time until it transits to the phantom era where the bounce occurs. With further time evolution the radion asymptotically stabilizes to the value $\frac{\omega}{c_s^2}$ which also represents the inflection point of the modulus potential. Such an asymptotic magnitude of the radion field can stabilize the modulus to the appropriate value where the gauge-hierarchy issue can also be adequately addressed.

With the background evolution, we further investigate the cosmological evolution of the scalar and tensor perturbations to the FRW metric from the present model. The primordial perturbation modes in the present context generate near the bounce because at that time the relevant perturbation modes are within the horizon, unlike the usual matter bounce scenario where the perturbation modes generate deeply in the contracting regime far away from the bouncing point. As a result the tensor perturbation is found to be suppressed in comparison to the scalar perturbation and the ratio of tensor to scalar perturbation amplitude becomes less than unity in accordance with the Planck results. Moreover, the speed of propagation of the tensor perturbation $c_T$ turns out to be the same as the speed of light, in agreement with the event GW170817. We compute the scalar spectral index $n_s$, the tensor to scalar ratio $r$ and the amplitude of the scalar perturbations $A_s$ from the present model which turns out to be pleasantly in agreement with the latest Planck 2018 observations, well within the $1-\sigma$ regime.

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