CKM CP Violation in a Minimal SO(10) Model for Neutrinos and Its Implications

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Abstract

A minimal supersymmetric SO(10) model with one \( 10 \) and one \( 126 \) Higgs superfield has recently been shown to predict all neutrino mixings as well as the solar mass difference squared in agreement with observations. Two assumptions critical to the predictivity and success of the model are that: (i) the superpotential includes only renormalizable terms, thereby limiting the number of free parameters and (ii) the triplet term in the type II seesaw formula for neutrino mass dominates, leading to the sum rule \( M_\nu = c(M_d - M_e) \) that is responsible for large mixings. However, CKM CP phase is constrained to be in the second or third quadrant requiring a significant non-CKM component to CP violation to explain observations. We revisit this issue using type I seesaw formula for neutrino masses and obtain the following results: (i) we show that the above sumrule responsible for large mixing angles can also emerge in type I seesaw models; the detailed predictions are however not compatible with present data for any choice of CP phases. (ii) We then show that addition of a nonrenormalizable term restores compatibility with neutrino data and CKM CP violation both in type I and type II cases. We further find that (iii) the MSSM parameter \( \tan \beta \geq 30 \) in the type I model and (iv) lepton flavor violation and lepton electric dipole moments which are accessible to proposed experiments in both type I and type II models. We also discuss the unification of the gauge couplings in type I model which requires an intermediate scale.
1 Introduction

The observations involving the solar and atmospheric neutrinos together with those using the accelerator and reactor neutrinos have now conclusively established that neutrinos have mass and they mix among themselves. In conjunction with the negative results from CHOOZ and PALO-VERDE reactor experiments, a reasonably clear outline of the mixing pattern among the three generations of neutrinos has emerged. Of the three angles needed to characterize these mixings, $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$, the first two, responsible for solar and atmospheric neutrino deficits respectively, are large, and the third corresponding to reactor neutrinos is small [1, 2]. One of the major experimental issues in neutrino physics now is to make the knowledge of these angles more precise.

Foremost among the theoretical challenges that have already emerged from these discoveries are, first an understanding of the smallness of neutrino masses and second, understanding the vastly different pattern of mixings among neutrinos from the quarks. Specifically, a key question is whether it is possible to reconcile the large neutrino mixings with small quark mixings in grand unified frameworks that unify quarks and leptons.

The first challenge, i.e. the lightness of neutrino masses is elegantly answered by the seesaw mechanism [3] which requires an extension of the standard model that includes heavy right handed neutrinos. The light neutrino mass matrix is obtained by integrating out heavy right handed neutrinos and one gets

$$M^I \nu = -M^D \nu M_R^{-1} (M^D \nu)^T,$$

(1)

where $M^D \nu$ is the Dirac neutrino mass matrix and $M_R$ is the right handed Majorana mass matrix. It is therefore important to explore whether one can answer the second puzzle of large neutrino mixings within the seesaw framework.

The above formula for the neutrino mass matrix is called the type I seesaw formula. The right handed Majorana mass scale, $v_R$, is almost determined by the mass squared difference needed to understand the atmospheric neutrino data to be around $10^{14}$ GeV, if we assume that the Dirac neutrino mass is same as up-type quark mass. Before proceeding to discuss the implications of this large right handed neutrino mass, let us discuss the nature of the seesaw formula.

We will consider a class of models where the right handed neutrino mass is not put in by hand but arises from a renormalizable coupling of the form $f N N \Delta_R$, where $N$ is a right handed
neutrino, \( f \) is a coupling constant and \( \Delta_R \) is a Higgs field whose vacuum expectation value (vev) gives mass to the right handed neutrino. This is a natural feature of models with asymptotic parity conservation, such as those based on \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) or any higher gauge group such as \( SO(10) \), where the \( \Delta_R \) field is part of an \( SU(2)_R \) triplet field. Parity invariance then implies that we also have an \( f \nu \nu \Delta_L \) coupling term as a parity partner of the \( NN \Delta_R \) coupling. In this class of theories, whenever \( \Delta_R \) acquires a vev, so does \( \Delta_L \) and they are related by the formula 

\[
\langle \Delta_L \rangle \equiv v_L = \frac{v_w}{\lambda v_R},
\]

where \( v_w \) is a weak scale and \( v_R \) is the \( \Delta_R \) vev and \( \lambda \) is a coupling constant in the Higgs potential. The \( \Delta_L \) vev contributes a separate seesaw suppressed Majorana mass to the neutrino leading to a modified seesaw formula for neutrino masses given below.

\[
M_{\nu}^{II} = M_L - M^D_{\nu} M^{-1}_R (M^D_{\nu})^T, \tag{2}
\]

where \( M_L = f v_L \) and \( M_R = f v_R \). This formula for the neutrino mass matrix is called the type II seesaw formula [4]. In the case where right handed Majorana masses are heavy enough, the second term in the type II seesaw formula can be negligible, and the first term, \( M_L = f v_L \), is dominant. We will call this pure type II seesaw. When both terms are comparable, we will call this mixed type II seesaw.

Coming to the large scale for right handed neutrino mass (i.e. \( 10^{14} \) GeV or higher), we note that it suggests supersymmetric grand unified theory (GUT) as a natural framework to study neutrino masses, since experimental data suggests the gauge coupling unification scale in the minimal supersymmetric extension of the Standard Model (MSSM) to be, \( M_G \sim 2 \times 10^{16} \) GeV, which is close to the seesaw scale. The minimal grand unification model for neutrinos is the one which is based on the \( SO(10) \) group since all standard model fermions and the right handed neutrino fit into the 16-dimensional representation of \( SO(10) \), resulting not only in a complete unification of the quarks and leptons but also yielding possible relations between the quark and lepton mass matrices. One may therefore hope that the neutrino oscillation parameters might be predictable in an \( SO(10) \) theory.

There are two simple routes to realistic \( SO(10) \) model building. In the first class, one may have smaller representations for the Higgs fields like \( 10 \) and \( 16 \) multiplets. In this case, the nonrenormalizable terms are added to the superpotential to implement the seesaw mechanism. These models have the disadvantage that they break R-parity which then induces rapid proton decay at an unacceptable level.

An alternative is to introduce both \( 10 \) and \( 26 \) Higgs multiplets to give fermion masses.
In this class of models, the R-parity is an automatic symmetry of the model. This naturally prevents the baryon and lepton number violating terms that give rise to rapid proton decay and also guarantees a naturally stable supersymmetric dark matter.

In SO(10) models of this type, both the $\Delta_L$ and $\Delta_R$ fields are part of the $\mathbf{126}$ multiplet. The above mentioned couplings that contribute to seesaw formula arise from the couplings of $\mathbf{16}$ matter spinors to $\mathbf{126}$ multiplet. If the rest of the Higgs sector is appropriately chosen, this leads to type II seesaw formula for neutrinos.

An interesting class of renormalizable SO(10) models with $\mathbf{126}$ was proposed in Ref.[5] where it was shown that if in addition to the Higgs multiplets that break SO(10) and do not couple to fermions, one chooses only one $\mathbf{10}$ and one $\mathbf{126}$ Higgs field, then the model provides a completely realistic description of the charged fermion sector of the standard model and is very predictive in the neutrino sector without any extra symmetry assumptions. The reason for this is that all the Yukawa parameters of the model are determined by the quark and charged lepton sector. This model is called the minimal renormalizable SO(10) model, since unlike the SO(10) models with $\mathbf{16}$ Higgs fields, it does not add any nonrenormalizable terms to the superpotential. The presence of the $\mathbf{126}$ representation that allows the neutrino flavor structure to be related to other fermion mass matrices is at the heart of the predictivity of the model.

In the Refs.[6, 7, 8], this model was analyzed using the type I seesaw formula to see whether the neutrino oscillation parameters predicted would agree with observations. They found that the atmospheric and solar mixings can be large, and the phenomenological predictions are studied [9]. However they found that mass squared ratio for the solar and atmospheric data is $\Delta m^2_\odot / \Delta m^2_A \approx 0.2$. This result is incompatible with the recent combined data analysis of the solar mixing angle and mass squared ratio [2].

A new approach to discussing neutrinos in this model was presented during the past year. Considering only the second and third generation sector, the authors of Ref.[10] pointed out that the type II seesaw in the minimal SO(10) model can provide a natural way to understand the maximal atmospheric mixing due to the convergence of bottom quark and tau lepton masses when extrapolated to GUT scale. The reason for this that in the the pure-type II seesaw case, the $\mathbf{126}$ Higgs coupling leads the neutrino mass matrix to be proportional to $M_d - M_e$, where $M_d$ is a down-type quark mass matrix and $M_e$ is a charged-lepton mass matrix. It is then easy to see that $b-\tau$ mass convergence makes the 3-3 entry of the neutrino mass matrix very small, leading to large atmospheric mixing angle. While this observation was interesting, a priori, it was not clear if this would survive once the model is extended to include three generations. It
was however shown in Ref. [11], that the same $b$-$\tau$ mass convergence that led to large atmospheric mixing, also leads to large solar mixing while keeping the 1-3 mixing angle small as required by data. It also predicts the ratio of solar to atmospheric mass squared difference i.e. $\Delta m^2_\odot/\Delta m^2_A$ to be in the right range. This establishes that the minimal renormalizable SO(10) models with 126 Higgs provide a very interesting way to understand neutrino masses and mixings within a complete quark-lepton unified framework.

In Ref. [11], all the CP phases, including the Kobayashi-Maskawa (KM) phase, were set to zero (or 180 degree) and it was assumed that all CP violating effects owe their origin to the SUSY breaking sector. In Ref. [12], the pure type II model including all CP phases was analyzed, and it was found that one can maintain the predictivity of the model despite the appearance of the phases; but the KM phase must be in the second quadrant in the $\rho$-$\eta$ plane to maintain the neutrino predictions (where we have adopted the Wolfenstein parameterization for the CKM matrix). This implies that the model is substantially different in CP violating sector from the standard CKM model [13] where the KM phase must be in the first quadrant. Such non-CKM CP violation, for instance, could be in the squark mixings. Phenomenologically speaking, there is nothing wrong with this possibility although admittedly it will require many random parameters to reproduce the observed data on CP violation.

Since the CKM model is simple and seems consistent with known data on CP violation and it would be interesting to see if the above predictive model for neutrinos can coexist with the simple CKM CP violation. One of the objectives of this paper is to study this using the type I seesaw formula. It was mentioned in Ref. [12] that the main reason for the constraint on CP phase being in the third quadrant in the pure type II model has to do with fitting the electron mass and if a nonrenormalizable term is introduced to fix this problem, then the CP phase could be of CKM type.

In this paper, we focus primarily on the same minimal SO(10) model but with both type I and type II seesaw formula for neutrinos. We obtain the following results: (i) We first show that in a certain limit, one can get the relation $M_\nu \simeq c(M_d - M_e)$, so that the basic mechanism that led to the maximality of solar and atmospheric neutrino mixings in the case of pure type II seesaw formula is preserved in the type I case. As far as we know, this fact was not noticed in any of the earlier papers that analyzed type I seesaw in minimal SO(10). We then show that the model is in conflict with neutrino data with or without CP violating phases. This is different from the pure type II case where, one could get the neutrino predictions correctly within the renormalizable framework without any CP phase in the Yukawa couplings as well.
as the KM phase in second quadrant. (ii) Secondly, we show that we can restore compatibility with KM CP violation for both type I and type II cases by using the SO(10) model as an effective theory at the GUT scale, where we generate a specific nonrenormalizable term. This not only leads to successful predictions for neutrino mixings but also maintains the CKM model for CP violation. Of course, the resulting model is not a minimal model anymore since at the Planck scale, it can emerge from a renormalizable theory with two pairs of $126$ fields (rather than one in the minimal case). (iii) We also find that the type I model works only for values of $\tan \beta$ higher than 30, providing a way to test the model in supersymmetry experiments. (iv) We evaluate the lepton flavor violating processes and electron electric dipole moment (EDM) by using the predicted leptonic mass matrices in both type I and pure type II case and show that they are in a range accessible to the present experiments. (v) We find that for the type I model to work, intermediate scale ($v_R$) must be below the GUT scale. We present a brief discussion of unification of gauge couplings in the model to show that this can indeed happen.

This paper is organized as follows: in section 2, we give a basic framework of the minimal SO(10) model and show that the relation $M_\nu \simeq c(M_d - M_e)$ needed to understand large neutrino mixings follows in a certain limit even with the type I or mixed type II seesaw. We examine its predictions for the case where the type I seesaw formula is used for discussing neutrino masses and KM phase is kept around 60-70 degrees as required in the standard model fit. We find that the predictions for solar neutrino oscillation do not agree with the current combined data analysis of mixing angles and ratio of mass squared differences for this case. We also discuss the restriction on $\tan \beta$ in the mixed type II and type I models. In section 3, to remedy the situation of neutrino masses and mixing angles, we include specific types of non-renormalizable terms in the superpotential and show how they are helpful in reconciling the neutrino predictions with CKM CP violation for both type I and type II models. We also discuss the origins of these terms. In sec. 4, we discuss the predictions of the model giving numerical results for various parameters as well as neutrino masses and mixings. In section 5, we discuss the gauge coupling unification in the model and show that the $B-L$ scale required for obtaining neutrino masses is compatible with gauge coupling unification. In sec. 6, we evaluate the leptonic flavor violation and lepton electric dipole moment for both type I and type II models. Section 7 contains our conclusions.
2 Minimal Renormalizable SO(10) Model and Its Predictions

In the minimal SO(10) model, Yukawa interactions are given as the couplings of the 16-dimensional matter spinor $\psi_i$ with only one 10-dimensional Higgs $H$ and one 126 Higgs $\Delta$. The superpotential for Yukawa interactions is written as

$$W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \Delta.$$  \hspace{1cm} (3)

The Yukawa couplings, $h$ and $f$, are complex symmetric $3 \times 3$ matrices.

The SO(10) gauge symmetry breaks down by the Higgs mechanism. There are several breaking patterns in SO(10) GUT. The SO(10) symmetry is broken to the left-right group, $G_{2231} = SU(2)_L \times SU(2)_R \times SU(3)_c \times U(1)_{B-L}$, using the 54 and 210 Higgses. To break $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$, we will use 126 + 126 Higgs multiplets.

The SO(10)-invariant superpotential, Eq. (3), includes MSSM Yukawa couplings plus right handed Majorana mass term which are written by using the MSSM superfields:

$$W_Y \supset Y_u^{ij} Q_i U^c_j H_u + Y_d^{ij} Q_i D^c_j H_d + Y_e^{ij} L_i E^c_j H_d + Y_{\nu}^{ij} L_i N^c_j H_u + \frac{1}{2} f_{ij} N^c_i N^c_j \Delta_R,$$ \hspace{1cm} (4)

where $H_u$ and $H_d$ are MSSM Higgs doublets which are linear combinations of the SM doublets in 10 and 126 Higgs multiplets, and $\Delta_R$ is part of the 126 field $\Delta$. We note that the VEV of $\Delta_R$ gives right handed neutrino masses and breaks $SU(2)_R \times U(1)_{B-L}$ symmetry down to $U(1)_Y$. As noted the SO(10) Higgs fields, $H$ and $\Delta$, contain two pairs of $SU(2)_L$ Higgs doublets, $H^{u,d}_{10}$ and $H^{u,d}_{126}$, and the MSSM Higgs doublets are linear combinations of two pairs,

$$H_u = \alpha_u H^u_{10} + \beta_u H^u_{126},$$ \hspace{1cm} (5)

$$H_d = \alpha_d H^d_{10} + \beta_d H^d_{126},$$ \hspace{1cm} (6)

where $|\alpha_u|^2 + |\beta_u|^2 = 1$. The other linear combinations are assumed to be massive around GUT scale. The MSSM Yukawa couplings $Y^{u,d,e,\nu}$ are given by linear combination of $h$ and $f$, and the fermion mass matrices and Majorana mass matrix for right handed neutrino are given as\(^1\)

\begin{align*}
M_u &= (h^* \alpha_u + f^* \beta_u) v_u, & (7) \\
M_d &= (h^* \alpha_d + f^* \beta_d) v_d, & (8)
\end{align*}

\(^1\)The mass matrices are defined as $-L_m = \bar{\psi}_L M \psi_R + \frac{1}{2} (\nu_R)^T C^{-1} M_R \nu_R + \text{h.c.}$
\[ M_e = (h^* \alpha_d - 3 f^* \beta_d) v_d, \]  
\[ M_\nu^D = (h^* \alpha_u - 3 f^* \beta_u) v_u, \]  
\[ M_R = f^* v_R, \]  
(9) (10) (11)

where \( v_{u,d} \) are VEVs of MSSM Higgs doublets and \( v_R \) is a VEV of \( \Delta_R \). We denote that \( v_u = v \sin \beta \) and \( v_d = v \cos \beta \), where \( v = 174 \text{ GeV} \). Then we have sumrules for leptonic mass matrices:

\[ M_e = c_d M_d + c_u M_u, \]  
\[ M_\nu^D = \frac{c_d + 3}{c_u} (M_d - M_e) + M_u, \]  
\[ M_R = \frac{M_d - M_e}{4 \beta_d v_d} v_R, \]  
(12) (13) (14)

where

\[ c_u = \frac{4 \cot \beta}{\alpha_u/\alpha_d - \beta_u/\beta_d}, \quad c_d + 3 = \frac{-4 \beta_u/\beta_d}{\alpha_u/\alpha_d - \beta_u/\beta_d}. \]  
(15)

So if the quark mass matrices \( M_u \) and \( M_d \) are our input, then leptonic mass matrices are determined by two complex parameters, \( c_u \) and \( c_d \), barring the overall scale and the phase of Majorana neutrino mass matrix, \( M_R \). Thus, using the masses of three charged leptons, we predict the neutrino mass matrices in terms of only one real parameter. This is an interesting feature of the minimal SO(10) model and is responsible for the high predictive power of the model.

Let us count the number of parameters in the fermion mass matrices in the minimal SO(10) model. Rotating the fermion fields by a unitary matrix without violating SO(10) symmetry (fermions in 16 are all rotating simultaneously), up-type quark mass matrix, \( M_u \), can be made real positive diagonal, \( M_u = \text{diag}(m_u, m_c, m_t) \equiv D_u \). Then down-type quark mass matrix, \( M_d \), is a general complex symmetric matrix, which has 6 complex parameters. The matrix can be written as \( M_d = UD_d U^T \) where \( D_d \) is real positive diagonal, \( D_d \equiv \text{diag}(m_d, m_s, m_b) \), and \( U \) is a unitary matrix. The unitary matrix has 9 real parameters (3 angles and 6 phases) and we can parameterize the unitary matrix as \( U = P_u^* \bar{U} P_d \). The matrices \( P_u \) and \( P_d \) are diagonal phase matrices, \( P_u \equiv \text{diag}(e^{i \phi_u/2}, e^{i \phi_c/2}, 1) \) and \( P_d \equiv \text{diag}(e^{i \phi_d/2}, e^{i \phi_s/2}, e^{i \phi_b/2}) \) and \( \bar{U} \) is just same as the Cabibbo-Kobayashi-Maskawa (CKM) matrix, \( V_{CKM} \), in which there are 3 mixing angles and 1 phase (KM phase). The phase matrices, \( P_u \) and \( P_d \), are unphysical in the MSSM quark sector since they can be absorbed in right handed quark fields, but the phases are relevant parameters in the leptonic mass matrices through the sumrules in Eqs.(12,14). One overall
phase of $M_d$ is irrelevant in the sumrules, using the rotation, $M_d \rightarrow e^{i\phi} M_d, M_u \rightarrow M_u$. This rotation is generated by $(\alpha_d, \beta_d) \rightarrow (e^{i\phi} \alpha_d, e^{i\phi} \beta_d)$. So, we can choose the phase $\phi_b$ to be zero.

Finally, we have 19 parameters in the sumrules: 14 real parameters in quark mass matrices $M_u$ and $M_d$ (6 quark masses, 3 mixing angles, 1 KM phase, and 4 unknown phases, $\phi_u,c,d,s$), 2 complex parameters, $c_u$ and $c_d$, and 1 overall scale parameter in the Majorana mass matrix $M_R$ (overall phase of $M_R$ is not counted since it is not physical). On the other hand, the MSSM Yukawa couplings plus the right handed Majorana mass in Eq.(4), give us 32 parameters: 10 parameters in $Y_u$ and $Y_d$ (6 quark masses and 3 mixings and 1 KM phase), 10 parameters in $Y_e$ and $Y_\nu$ in the same way, and then 12 parameters in the Majorana mass matrix which is generally a symmetric matrix. So, in the minimal SO(10) model, we have less parameters than the MSSM.

The quark masses, mixings and the KM phase are our inputs. We can redefine the basis of fermions, and the up- and down-type quark mass matrices are written as $M_u = D_u P^2_u$, $M_d = V_{CKM} D_d P^2_d V_{CKM}^T$. The authors in Ref.[7] assumed that the phase matrices $P^2_{u,d}$ are real and they found a solution to generate the observed quark masses and the CKM matrix and the charged lepton masses in the case where $\phi_{d,s,c} = \pi$ and $\phi_u = 0, \pi$. In our general analysis, we take those phases $\phi_{d,s,u,c}$ to be arbitrary. Those phases are constrained to obtain the bi-maximal neutrino oscillation data.

Let us see how one can obtain a numerical fit of the parameters in the lepton sector at the GUT scale. The charged lepton mass matrix is written as $M_e = c_d M_d + c_u M_u$. So, $\tau$ and $\mu$ masses are approximately (neglecting generation mixing and phases),

$$\pm m_\tau \simeq c_d m_b + c_u m_t, \quad \pm m_\mu \simeq c_d m_s + c_u m_c. \quad (16)$$

Using the relations $m_c/m_t \ll m_s/m_b$ and $m_\tau \simeq m_b$, we obtain $c_d \simeq \pm m_\mu/m_s$ and $c_u \simeq -m_b/m_t (c_d \pm 1)$, and thus $|c_d|$ is about 3, naively. In this naive approximation, however, the electron mass becomes about $c_d m_d$, which is too large, so we cannot neglect the off-diagonal elements, and we need a fine-tuning in $\det(M_d + \kappa M_u)$, where $\kappa \equiv c_u/c_d$, by a suitable choice of $\kappa$. The numerical fit is given in Ref.[7]. The key ingredient to fit the electron mass is the strange quark mass. Actually, the current strange quark mass still has large experimental error and we can make it to be a parameter in the model. We show the values of the strange quark mass (at 1 GeV) and $|c_d|$ by varying the KM phase in Fig.1. We take the KM phase to be in the first and second quadrant without loss of generality, since the sumrule does not change under the conjugation. The three real parameters are consumed to fix the three charged lepton
masses: $\kappa$ and $|c_d|$ are determined and the phase of $c_d$ is still undetermined.

We now determine the neutrino mass matrix. First, in the pure-type II case, the light neutrino mass, $M^\text{II}_\nu$, is given as $f^*v_L$ which is proportional to $M_d - M_e$. The charged-lepton mass matrix is written as $M_e = c_d(M_d + \kappa M_u)$. We define the complex number $\xi$ as

$$\xi \equiv |c_d|(M_d + \kappa M_u)_{33}/m_\tau. \quad (17)$$

By definition, $|\xi| \simeq 1$. The phase of $\xi$ is determined by the charged lepton mass fitting. The $(3,3)$ element of $M_e$ is then $\xi e^{i\sigma} m_\tau$, where $\sigma$ is a phase of $c_d$, $e^{i\sigma} = c_d/|c_d|$. We denote $\hat{\xi} \equiv \xi e^{i\sigma}$.

The pure type II neutrino mass matrix is given as

$$M^\text{II}_\nu \propto (1 - c_d)M_d - c_u M_u$$

$$\ approx \ (1 - c_d) \begin{pmatrix}
  m_d e^{i\phi_d} + V_{us}^2 m_s e^{i\phi_s} & V_{us} m_s e^{i\phi_s} & V_{ub} m_b \\
  V_{us} m_s e^{i\phi_s} & m_s e^{i\phi_s} & V_{cb} m_b \\
  V_{ub} m_b & V_{cb} m_b & (m_b - \hat{\xi} m_\tau)/(1 - c_d)
\end{pmatrix}. \quad (18)$$

In the expression, $c_u m_c$ and $c_u m_u$ terms in the diagonal element are neglected. Choosing the phase $\sigma$ to make $\hat{\xi} \simeq 1$, the $(3,3)$ entry of the matrix is $m_b - m_\tau$. Since $b-\tau$ mass convergence implies that $m_b$ and $m_\tau$ become close to each other as we move to the GUT scale, we can assume that $m_b - m_\tau \approx (1 - c_d)V_{cb} m_b$, which implies that the neutrino mass matrix is given as

$$M^\text{II}_\nu \sim \begin{pmatrix}
  \lambda^2 & \lambda & \lambda \\
  \lambda & 1 & 1 \\
  \lambda & 1 & 1 
\end{pmatrix} m_0, \quad (19)$$
where $\lambda \simeq V_{us} \sim 0.2$. We then obtain bi-large neutrino mixing: both atmospheric and solar angle are large. The third neutrino mass, $m_{\nu_3}$, is about $m_0$. To obtain the large solar mixing, determinant of (2-3) block of the neutrino matrix is required to be less than $\lambda m_0^2$, and then the second neutrino mass, $m_{\nu_2}$, is about $\lambda m_0$. So the mass squared ratio is predicted as $\Delta m^2_\odot / \Delta m^2_A \sim \lambda^2$. The 1-3 neutrino mixing is also predicted as $U_{e3} \sim \lambda$.

Next we consider the case of mixed type II neutrino mass matrix. This case is more complicated than the pure type II case. From Eqs.(13–14), the mixed type II seesaw matrix is given as

$$M_{\nu}^{\text{II mixed}} \propto (M_d - M_e)(1 + \Delta) + 2 \frac{c_u}{c_d + 3} M_u + \left( \frac{c_u}{c_d + 3} \right)^2 M_u(M_d - M_e)^{-1} M_u. \quad (20)$$

The first term is just the same as in the pure type II case. When $\Delta = 0$, we get the type I case. The second and third terms might give a hierarchical structure to the (2-3) block of neutrino mass matrix, and would spoil large atmospheric mixing. The (3,3) element of the Eq.(20) can be written approximately as

$$\left( (1 - \hat{\xi})(1 + \Delta) + 2 \frac{\hat{\xi} - c_d}{c_d + 3} + \zeta \left( \frac{\hat{\xi} - c_d}{c_d + 3} \right)^2 \right) m_\tau, \quad (21)$$

where $\zeta = [(M_d - M_e)^{-1}]_{33} m_\tau$. This element is of the order of $m_\tau$ or larger in general. However, if the (3,3) element can be canceled to the order of $(1 + \Delta)(1 - c_d) V_{cb} m_\tau$, then the bi-large mixing neutrino matrix can be realized. Assuming that $\hat{\xi} \simeq 1$ as in pure type II case, the cancellation condition of the second and third terms is $c_d = -(6 + \zeta)/(2 - \zeta)$. In our numerical analysis, the parameter $\zeta$ is almost real and $\zeta \simeq 1$. Thus, the cancellation condition is $c_d \sim -7$. At that time, the phase of $c_d$ is almost $\pi$, and then the phase of $\xi$ must be almost $\pi$ from the condition $\hat{\xi} \simeq 1$. The condition can be satisfied if the strange quark has a smaller value of mass, since $|c_d| \sim m_\mu/m_s$ naively. According to our numerical analysis, $\hat{\xi}$ is not necessary to be 1 and $c_d$ can be different values from $-7$. However, $|c_d|$ has a lower bound, $|c_d| > 5$ to cancel the (3,3) element of the neutrino mass matrix in our analysis. Consequently, smaller value of strange mass is favored in the mixed type II case.

In both pure and mixed type II (and of course in the type I case), the neutrino mass matrix structure in Eq.(19) which gives bi-large mixing can be obtained. Then we have a good prediction of the 1-3 mixing angle, $|U_{e3}| \simeq \lambda$. Actually, in the numerical studies in Ref.[12] (type II) and Ref.[8] (type I), the prediction of $|U_{e3}|$ is about 0.17-0.18. Another important prediction of the minimal SO(10) model is the solar mixing angle and the ratio of mass squared differences, $\Delta m^2_\odot / \Delta m^2_A$. The predictions are almost determined by the (2-3) block of the matrix. The (2-3)
block is approximately written as

\[ M_{\nu}^{(2-3)} \propto \begin{pmatrix} m_s e^{i\phi_s} & V_{cb} m_b \\ V_{cb} m_b & \epsilon m_{\tau} \end{pmatrix}, \tag{22} \]

where \( \epsilon \) is a cancellation factor, \( \epsilon m_{\tau} \simeq (M_d - M_e)_{33} / (1 - c_d) \) in the pure type II case, and \( \epsilon m_{\tau} \) is approximately Eq.(21) divided by \( 1 - c_d \) in the mixed type II case. The \( |\epsilon| \) should be less than \( V_{cb} \sim \lambda^2 \) to give rise to a large atmospheric mixing. The condition to obtain a large maximal mixing angle is that the determinant of the (2-3) block is canceled,

\[ |\epsilon e^{i\phi_s} m_s m_{\tau} - V_{cb}^2 m_b^2| \lesssim O(\lambda)V_{cb}^2 m_b^2. \tag{23} \]

This condition also provides a small mass differences squared ratio, \( \Delta m^2_\odot / \Delta m^2_A \sim \lambda^2 \). To satisfy the condition, one needs \( \epsilon e^{i\phi_s} \simeq +|\epsilon| \), and \( |\epsilon| \) can not be very small. Then, maximal atmospheric mixing requires a relation \( m_s - |\epsilon| m_{\tau} \ll 4 V_{cb} m_b \). Assuming \( m_s \simeq |\epsilon| m_{\tau} \), we obtain the condition for the bi-maximal neutrino mixing and the small mass squared ratio as

\[ m_s \simeq V_{cb} m_b. \tag{24} \]

From the experimental data, the strange/bottom mass ratio at the GUT scale is always smaller than \( V_{cb} \), thus a larger strange mass gives rise to larger mixing angles in the minimal SO(10) model. Furthermore, in the minimal model, the strange quark mass is constrained due to electron mass fitting. As we can see in Fig.1, the maximal values of the strange mass depend on the KM phase, and the second quadrant KM phase gives larger strange quark mass. As a result, the bi-maximal neutrino mixings and the small mass squared ratio favor the second quadrant KM phase. This result is in agreement with the numerical studies in Ref.\[12\]. In the case of mixed type II or type I, smaller values of strange quark mass are favored to obtain the cancellation of the (3,3) component of the neutrino mass matrix as we have noted. So in type I and mixed type II case, there is a tension between getting large neutrino mixings in general and getting their precise values as well as the desired mass square ratio. As a result, the minimal SO(10) model with type I or mixed type II predicts that the solar mixing angle is not very large and the mass squared ratio is not very small as long as the atmospheric neutrino mixing is maximal. We give our numerical data plotting for the solar mixing angle and mass squared ratio in the Fig.2. In the mixed type II or type I case, the mixing angle and mass squared ratio are bounded for any KM phase, and recent data fitting excludes such bounds more than 2\( \sigma \) level \[2\].
Next we will see the $\tan \beta$ bound in the minimal SO(10) model. From Eq. (15), we obtain

$$\frac{\alpha_u}{\alpha_d} = \frac{1 - c_d}{c_u} \cot \beta, \quad \frac{\beta_u}{\beta_d} = -\frac{c_d + 3}{c_u} \cot \beta.$$  \hspace{1cm} (25)

Since $\alpha_{u,d}$ and $\beta_{u,d}$ are Higgs mixings, we have unitarity constraint $|\alpha_{u,d}|^2 + |\beta_{u,d}|^2 = 1$. Thus we have $(|\alpha_u/\alpha_d| - 1)(|\beta_u/\beta_d| - 1) \leq 0$, and

$$\min \left( \left| \frac{c_d + 3}{c_u} \right|, \left| \frac{1 - c_d}{c_u} \right| \right) \leq \tan \beta \leq \max \left( \left| \frac{c_d + 3}{c_u} \right|, \left| \frac{1 - c_d}{c_u} \right| \right).$$  \hspace{1cm} (26)

In the case where $\hat{\xi} \simeq 1$ which is favored to obtain bi-large structure, $(1 - c_d)/c_u \simeq m_t/m_b$, and then we have lower bounds of $\tan \beta$, such as $\tan \beta \gtrsim m_t/m_b |c_d + 3|/(1 - c_d)|$. In the case where $c_d \sim -7$ which is favored in the mixed type II (and type I) case, the lower bound of $\tan \beta$ is roughly 30. To obtain small $\tan \beta$ such as $\tan \beta \sim 10$, $c_d \sim -3.5$ is needed. Such small $|c_d|$ can happen only in the case where the KM phase is close to 180 degree in the minimal SO(10) model.

In the type I seesaw case, the scale of the right handed Majorana mass is predictable. Using the Eqs. (13-14), the light neutrino mass scale, $m_0$ in Eq. (19), can be written as

$$m_0 \simeq (1 - c_d) \left( \frac{c_d + 3}{c_u} \right)^2 V_{cb} m_b \frac{4\beta_d v_d}{v_R}. \hspace{1cm} (27)$$

So using the mass squared magnitude from the atmospheric neutrino data, we can fix the mass parameter, $m_0$, and obtain the right handed Majorana mass matrix, Eq. (14). The lightest
Majorana mass, which is an important scale for leptogenesis, is about $10^{11}$ GeV. Also we obtain the upper bound of $v_R$ since $|\beta| \leq 1$. Now let us summarize the predictions of the minimal SO(10) model. In both pure type II and mixed type II, we can obtain the bi-large mixing structure, Eq.(19), for the solar and the atmospheric oscillation. In the bi-large neutrino matrix, the 1-3 mixing is well predicted as $|U_{e3}| \simeq 0.17 - 0.18$. In the mixed type II (and type I) case, larger $|c_d|$ is favored to obtain bi-large structure in the neutrino mass matrix. In order to obtain both bi-maximal neutrino mixing and small mass squared ratio, a larger strange quark mass is favored, and this requires a smaller $|c_d|$. So, in the mixed type II (and type I) case, the bi-maximal neutrino and the small mass squared ratio are not favored, and it is hard to achieve a best fit value of the recent combined data analysis. In the pure type II case, the condition of bi-large mixing is just $\xi \simeq 1$, and $|c_d|$ is not constrained. However, due to the electron mass fitting, the strange mass is related to the KM phase, and if the KM phase is 60-80 degree, the strange mass can not be large and thus it is hard to achieve the bi-maximal mixing and the small mass squared ratio in the pure type II case as well. If the KM phase is in the second quadrant and especially for the larger value of KM phase, the strange quark mass can be large and the best fit values for neutrino oscillation can be achieved. The $\tan \beta$ in the mixed type II and type I model is greater than 30 roughly and the $B - L$ breaking scale has an upper-bound in the type I model.

3 Inclusion of Nonrenormalizable terms

As we have seen in the previous section, the fitting of three charged-lepton masses (especially, electron mass) gives relatively small strange quark mass in the case where the KM phase is 60-80 degree. Then the $(2,2)$ element of the neutrino mass matrix is relatively smaller than $(2,3)$ element, and as a result, we get bounds for the solar mixing angle and the mass squared ratio. In this section, in order to reproduce the CKM model of CP violation at low energies, we employ corrections to the SO(10) model which originate from non-renormalizable terms in the superpotential.

The most simple non-renormalizable term is written including an extra singlet $S$

$$W_Y^{nr} = \frac{1}{2M_P} S \left( h'_{ij} \psi_i \psi_j H_{10} + f'_{ij} \psi_i \psi_j H_{126} \right) .$$

(28)

In this case, there is no change from the results of the renormalizable case since the effect of these terms is simply to redefine the couplings $h$ and $f$ of Eq.(3). We therefore seek other renormalizable terms.
We will employ $210$ Higgs multiplet to remedy the situation. Note that $210$ Higgs (to be denoted $\Sigma$) can be used to break the SO(10) symmetry down to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ if we give the $(1, 1, 1)$ and $(1, 1, 15)$ (under $G_{224}$) submultiplets of $210$ vev. One can then include the term
\[
\frac{1}{2} M_P h'_{ij} \psi_i \psi_j H \Sigma
\] (29)
Since $\Sigma$ has four SO(10) indices, the combination $H \Sigma$ that can couple to spinors is either a $120$ or $126$ representation. We restrict the high scale theory (ultraviolet completion) from which the nonrenormalizable term originates in such a way that no $120$ term in the effective $H \Sigma$ term couples to matter. For instance, we could add a second $126 + \overline{126}$ pair of field with a high mass $M \gg M_U$ which does not develop a VEV with a superpotential given as follows\(^2\):
\[
W' = h' \psi \psi \Delta' + M \Delta' \Delta' + \Delta' \Sigma H
\] (30)
The effective theory below the scale $M$ has a nonrenormalizable term of the type in Eq. (30) where only $126$ part of $\Sigma H$ field product contributes. If only $(1, 1, 1)$ and $(1, 1, 15)$ submultiplets of $210$ acquire vev, the effective operator that contributes to fermion masses then has $(2, 2, 15)$ quantum numbers under $G_{224}$. This leads to mass formulae for the quarks and leptons of the following type.
\[
M_u = (h^* \alpha_u + f^* \beta_u) v_u + h'^* \alpha_u v_u <\Sigma>/M_P,
\] (31)
\[
M_d = (h^* \alpha_d + f^* \beta_d) v_d + h'^* \alpha_d v_d <\Sigma>/M_P,
\] (32)
\[
M_e = (h^* \alpha_d - 3 f^* \beta_d) v_d - 3 h'^* \alpha_d v_d <\Sigma>/M_P,
\] (33)
\[
M^D_{\nu} = (h^* \alpha_u - 3 f^* \beta_u) v_u - 3 h'^* \alpha_u v_u <\Sigma>/M_P,
\] (34)
\[
M_R = f^* v_R,
\] (35)
where $v_{u,d}$ are VEVs of MSSM Higgs doublets and $v_R$ is VEV of $\Delta_R$. Note also that $h'$ is not a symmetric matrix in general, but we assume that $h'$ is a symmetric matrix like $h$ and $f$ for simplicity.

We can now rewrite the above equations as follows
\[
M_e = M^0_e + \Delta M e,
\] (36)
\[
M^D_{\nu} = \frac{c_d + 3}{c_u} (M_d - M^0_e) + M_u + \frac{1 - c_d}{c_u} \Delta M e,
\] (37)
\[
M_R = \frac{M_d - M^0_e}{4 \beta_d v_d} v_R,
\] (38)
\(^2\)For another nonminimal version with multiple $126$ at the GUT scale, see Ref.[15].
where $M_0^e = c_d M_d + c_u M_u$ and $\Delta M_e = -4h^* \alpha_d v_d \langle \Sigma \rangle / M_P$. In this sumrules, we can break the relation of the (2,2) element between charged-lepton and neutrino mass matrices, and thus we can increase (2,2) element of seesaw neutrino mass matrix. In the mixed type II (and type I) case, we have to increase the (2,2) element while keeping $|c_d|$ to be larger than 5. In fact, we find a large solar mixing solution in type I when we switch on the (2,2) element of $\Delta M_e$.

In the pure type II case, we do not need the (2,2) element of $\Delta M_e$ since a small $|c_d|$ can give rise to maximal mixings. Instead, we need the (1,1) element to cancel and to give the desired electron mass and we then find a large solar mixing solution in the case where KM phase is in the first quadrant.

4 Numerical Results

As already noted, the minimal SO(10) model is predictive because fermions have only two Yukawa couplings, one with $10$ and one $\overline{126}$ Higgses. This leads to a sumrule for the leptonic mass matrices which determine the the Dirac neutrino and right handed Majorana mass matrices from the observed neutrino data. In a generic model for neutrino masses, the neutrino oscillation data gives us information about the neutrino masses and mixing angles of light neutrino mass matrix, but no information about the Dirac neutrino and the right handed Majorana mass matrices separately. On the other hand, a separate information of the Dirac neutrino and the right handed Majorana mass matrices are important to extract the predictions of the models for lepton flavor violation and leptonic CP violation. In the minimal SO(10) model however, these matrices are completely determined.

In type I case that we are considering in this paper, we get:

$$Y_\nu = \begin{pmatrix}
-0.00159 - 0.00014i & 0.00067 - 0.0036i & 0.017 + 0.015i \\
0.00369 - 0.00026i & 0.0182 + 0.0017i & -0.046 - 0.0228i \\
-0.022 + 0.0085i & -0.02 - 0.0477i & 0.58 + 0.126i
\end{pmatrix}, \quad (39)$$

and

$$M_R = \begin{pmatrix}
0.00059 + 0.00048i & -0.00022 + 0.0012i & -0.0058 - 0.0051i \\
-0.00022 + 0.0012i & -0.0014 - 0.0063i & 0.011 - 0.013i \\
-0.0058 - 0.0051i & 0.011 - 0.013i & -0.037 - 0.0086i
\end{pmatrix} \times 10^{14.08} \text{ GeV.} \quad (40)$$

The above matrices are calculated for $\tan \beta = 40$ and are in the basis where the charged lepton masses are diagonal. The scale $10^{14.08} \text{ GeV}$ is the VEV of $\overline{126}$ Higgs which couples to fermions and this magnitude of the scale is the maximum possible value given the inputs of the quark
masses and the CKM mixings. The $h$ and $f$ in the basis where $M_u$ diagonal are given as follows:

$$h^* = \begin{pmatrix} 0.0012 - 0.0002i & 0.00067 - 0.00014i & 0.0011 + 0.0035i \\ 0.00067 - 0.00014i & -0.0029 - 0.00122i & -0.037 + 0.0043i \\ 0.0011 + 0.0035i & -0.037 + 0.0043i & 1.15 + 0.129i \end{pmatrix},$$

$$f^* = \begin{pmatrix} -0.0012 - 0.00006i & -0.0025 + 0.000062i & 0.00088 - 0.0021i \\ -0.0025 + 0.000062i & -0.0097 + 0.00061i & 0.021 - 0.0011i \\ 0.00088 - 0.0021i & 0.021 - 0.0011i & -0.0012 - 0.0345i \end{pmatrix}.$$  \hfill (41)

(42)

The neutrino mixing angles and mass squared ratio are given as:

$$\sin^2 \theta_{12} = 0.87, \; \sin^2 \theta_{23} = 0.92, \; |U_{e3}| = 0.22, \; \Delta m^2_{\odot}/\Delta m^2_A = 0.051. \hfill (43)$$

At $\tan \beta = 50$, the same matrices are given by:

$$Y_\nu = \begin{pmatrix} -0.0017 - 0.0004i & 0.0012 - 0.0039i & 0.016 + 0.019i \\ 0.004 - 0.0056i & 0.0194 + 0.0111i & -0.053 - 0.023i \\ -0.023 + 0.01i & -0.025 - 0.052i & 0.660 + 0.13i \end{pmatrix},$$

$$M_R = \begin{pmatrix} 0.00117 + 0.00026i & -0.00076 + 0.0024i & -0.010 - 0.012i \\ -0.00076 + 0.0024i & -0.00999 - 0.0118i & 0.0275 - 0.0234i \\ -0.010 - 0.012i & 0.0275 - 0.0234i & -0.066 - 0.025i \end{pmatrix} \times 10^{13.82} \text{ GeV}. \hfill (44)$$

The neutrino mixing angles and mass squared ratio are given as:

$$\sin^2 \theta_{12} = 0.81, \; \sin^2 \theta_{23} = 0.90, \; |U_{e3}| = 0.22, \; \Delta m^2_{\odot}/\Delta m^2_A = 0.06. \hfill (45)$$

In the pure type II case, we get:

$$Y_\nu = \begin{pmatrix} 0.0008 - 0.001i & -0.001 - 0.0057i & 0.004 - 0.026i \\ 0.003 + 0.0013i & -0.0056 + 0.00015i & -0.053 - 0.008i \\ -0.000039 + 0.0014i & -0.012 + 0.00096i & 0.627 + 0.1019i \end{pmatrix},$$

$$M_R = \begin{pmatrix} -0.0016 - 0.000047i & -0.0036 - 0.000029i & 0.00077 + 0.0028i \\ -0.0036 - 0.000029i & -0.014 - 0.00044i & 0.026 + 0.0008i \\ 0.00077 + 0.0028i & 0.026 + 0.0008i & -0.0158 - 0.0078i \end{pmatrix} \times 10^{16.24} \text{ GeV}. \hfill (46)$$

The above matrices are calculated for $\tan \beta = 50$ and are in the basis where the charged lepton masses are diagonal. The scale $10^{16.24} \text{ GeV}$ is the scale where SO(10) gets broken to $G_{2231}$ which
subsequently breaks down to the SM. The neutrino mixing angles and mass squared ratio are given as:

\[
\sin^2 \theta_{12} = 0.85, \quad \sin^2 \theta_{23} = 0.91, \quad |U_{e3}| = 0.22, \quad \Delta m^2_{\odot}/\Delta m^2_A = 0.084. \quad (49)
\]

At \( \tan \beta = 40 \), the same matrices are:

\[
Y_\nu = \begin{pmatrix} -0.00027 - 0.00066i & -0.0046 + 0.00089i & -0.023 - 0.0023i \\
-0.00073 + 0.0028i & 0.00074 - 0.0037i & 0.007 - 0.052i \\
-0.0008 - 0.0001i & 0.0013 - 0.0079i & -0.103 + 0.57i \end{pmatrix}, \quad (50)
\]

and

\[
M_R = \begin{pmatrix} -0.00078 + 1.5 \cdot 10^{-6}i & -0.0018 + 0.00040i & 0.00039 + 0.0012i \\
-0.0018 + 0.000040i & -0.007 - 8.7 \cdot 10^{-6}i & 0.012 + 7.8 \cdot 10^{-6}i \\
0.00039 + 0.0012i & 0.012 + 7.8 \cdot 10^{-6}i & -0.0081 - 0.00013i \end{pmatrix} \times 10^{16.24} \text{ GeV}. \quad (51)
\]

The neutrino mixing angles and mass squared ratio are given as:

\[
\sin^2 \theta_{12} = 0.85, \quad \sin^2 \theta_{23} = 0.92, \quad |U_{e3}| = 0.20, \quad \Delta m^2_{\odot}/\Delta m^2_A = 0.058. \quad (52)
\]

Using these fits, we will calculate the lepton flavor violating processes and the amount of leptonic CP violation in type I and pure type II scenarios in the mSUGRA model in sec. 6.

Even though all the examples we have given have large \( U_{e3} \), its value can be much smaller due to the presence of new parameters in the higher dimensional term.

## 5 Gauge Coupling Unification

From the results of the previous section (Eq. (40) and (45)), we see that for the type I model to be successful in predicting neutrino masses, the intermediate scale \( v_R \) must be below the GUT scale and around \( 10^{14} \text{ GeV} \) for our examples. We assume the resulting symmetry breaking chain is of type, \( SO(10) \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \). This is very different from the gauge coupling unification scenario in MSSM. It is therefore important check if the type I models are compatible with gauge coupling unification.

In our model, in the scale region \( M_{SUSY} \leq \mu \leq v_R \), the spectrum is that of familiar MSSM. Above the \( v_R \) scale, the symmetry group expands to \( SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). In addition to the new gauge bosons associated with this new symmetry, the new matter and Higgs that contribute are the following: three RH neutrinos as part of the \( SU(2)_R \) lepton doublet; one bidoublet from which the MSSM doublets emerged. In addition we need either the \( B - L = 2 \)
Figure 3: The couplings $1/\alpha_i$ are plotted as a function of scale.

triplet pair ($\Delta_L + \bar{\Delta}_L \oplus \Delta_R + \bar{\Delta}_R$) and $x[(3,3,1, -2/3) + (\bar{3}, 3, 1, 2/3)] \oplus y[(3,1,3, -2/3) + (\bar{3}, 3, 1, 2/3)$ to have successful gauge coupling unification. If we use the Higgs spectrum from Ref. [16] in the context of our model, we find that $x \oplus y$ (from 210) and $\Delta_L + \bar{\Delta}_L \oplus \Delta_R + \bar{\Delta}_R$ are at about the $v_R$ scale. Using these fields, the Fig. 3 shows the coupling unification in this model and the gauge unification happens at about $10^{15.5}$ GeV. We have chosen the $x, y$ and $\Delta$ Higgs masses to be 6 times the $v_R$ scale to get this value.

First of all it is gratifying that the type I model is compatible with gauge coupling unification with the desired value of $v_R$ scale. Furthermore the lowering of the ultimate unification scale implies that the gauge exchange contributions to the proton decay in this model lead to proton lifetimes much lower than the MSSM unifying without an intermediate scale and will be around the current lower limit $5 \times 10^{33}$ yrs when we include the threshold corrections [17]. The profile of the proton decay modes in the type I case will be substantially different from the pure type II case recently discussed [18] and is presently under investigation.

6 Lepton Flavor Violating Processes and Lepton Edm

6.1 Lepton Flavor Violation

We now discuss the lepton flavor violating processes e.g. $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ etc. The operator for $l_i \rightarrow l_j + \gamma$ is:

$$\mathcal{L}_{l_i \rightarrow l_j \gamma} = \frac{ie}{2m_l} l_j \sigma^{\mu\nu} q_\nu (a_t P_L + a_r P_R) l_i \cdot A_\mu + h.c. \quad (53)$$
where $P_{L,R} \equiv (1 \mp \gamma_5)/2$ and $\sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. The decay width for $l_i \rightarrow l_j + \gamma$ can be written as:

$$\Gamma(l_i \rightarrow l_j + \gamma) = \frac{m_{\mu e}^2}{64\pi} (|a_l|^2 + |a_r|^2)$$  (54)

Then the branching ratio is obtained by multiplying this decay width with the life time of the $l_i$ lepton. The supersymmetric contributions include the neutralino and chargino diagrams [19].

We work in the basis where the charged lepton masses are diagonal at the highest scale of the theory. We first start with pure type II seesaw model. We assume that $v_R$ (the scale where $G_{2231}$ breaks down to SM) is at the GUT scale. So we have just MSSM and right handed neutrinos below the GUT scale. The right handed masses have hierarchies and therefore get decoupled at different scales. The flavor-violating pieces present in $Y_\nu$ induces flavor violations into the charged lepton couplings and into the soft SUSY breaking masses e.g. $m^2$ terms etc. through the following RGEs:

$$\frac{dY_\nu}{dt} = \frac{1}{16\pi^2} (Y_\nu Y_\nu^\dagger + \cdots) Y_\nu$$  (55)

$$\frac{dm_{LL}^2}{dt} = \frac{1}{16\pi^2} (Y_\nu Y_\nu^\dagger m_{LL}^2 + m_{LL}^2 Y_\nu Y_\nu^\dagger + \cdots)$$

We use the mSUGRA universal boundary conditions at the GUT scale to draw the figures. In Figs. [16] we show the BR[$\mu \rightarrow e\gamma$] and BR[$\tau \rightarrow \mu\gamma$] as a function of $m_{1/2}$ for different values of $A_0$. The lightest neutralino is the dark matter candidate in this model and we satisfy the $2\sigma$ range of the recent relic density constraint $\Omega_{CDM} = 0.1126^{+0.008}_{-0.009}$ [20] in the parameter space. When we satisfy the relic density constraint, the $m_0$ gets determined. For example, $m_0$ varies between 60-100 GeV for $A_0 = 0$ GeV line in the graph. The Figs also show that the larger $\tan \beta$ has larger lepton flavor violation.

In type I model, the upper bound of $v_R$, VEV of 126 which couples to fermions, is determined and this value is not the GUT scale. For example, $v_R$ can be $\leq 10^{14}$ GeV. Then, $G_{2231}$ symmetry can be maintained between the GUT and the $v_R$ scale if other Higgs fields do not break $B-L$ symmetry. This feature induces larger lepton violation since the right handed neutrino is a part of the doublet which has right handed electrons. The right slepton masses get new flavor violating contributions through the flavor violating pieces present in $Y_\nu$. The new contributions to the RGEs:

$$\frac{dY_\nu}{dt} = \frac{1}{16\pi^2} (4Y_\nu Y_\nu^\dagger + \cdots) Y_\nu$$  (56)

$$\frac{dm_{LL}^2}{dt} = \frac{1}{16\pi^2} (Y_\nu Y_\nu^\dagger m_{LL}^2 + m_{LL}^2 Y_\nu Y_\nu^\dagger + 2Y_\nu m_{RR}^2 Y_\nu^\dagger + \cdots)$$
Figure 4: The $\text{BR} [\mu \to e\gamma]$ is plotted as a function of $m_{1/2}$ for different values $A_0$ and $\tan \beta = 10$, 40 and 50 in pure type II.

The new effects arising from the RGEs make the lepton flavor violation to be larger in this case and is depicted in Fig. 5. In Fig. 6, we plot $\tau \to \mu \gamma$ and find that the Br can be as large as $10^{-8}$ which can be explored in the near future.

6.2 Electric Dipole Moment of Electron

The EDM, $d_f$ for fermion $f_i$ appears in the effective Lagrangian as:

$$L_f = \frac{i}{2} \bar{d}_f \vec{\sigma}_{\mu\nu} \gamma^5 \gamma_f F_{\mu\nu} \quad \text{(57)}$$

We have contributions from the chargino and neutralino diagrams to $d_f$ [21]. The electron EDM is plotted in Fig. 7 for the type II and in Fig. 8 for the type I. We again use the same SUSY parameter space as in the case of lepton flavor violation. We find that the maximum value of EDM is $\sim 10^{-31}$ ecm for the type II. For the type I, the EDM is large and is around $10^{-26}$ ecm and a large region of parameter space can be explored very soon. The muon EDM is shown.
Figure 5: The BR[\mu \rightarrow e\gamma] is plotted as a function of \(m_{1/2}\) for different values \(A_0\) and \(\tan \beta = 40\) and 50 in type I.

Figure 6: The BR[\tau \rightarrow \mu\gamma] is plotted as a function of \(m_{1/2}\) for \(\tan \beta = 40\) in pure type II and type I.
Figure 7: The electron EDM is plotted as a function of $m_{1/2}$ for different values $A_0$ and $\tan \beta = 40$ and 50 in pure type II.

in Fig. 9 and the maximum value shown is about $10^{-27}$ ecm and the scaling is broken in this model.

The $\sin 2\beta$ calculated from the $B^0 \to \phi K_s$ mode in this model is 0.67, which is not different from the SM prediction.

7 Conclusion

In summary, we have revisited the minimal renormalizable SO(10) models with a 126 Higgs multiplet that has recently been shown to predict neutrino mixings in agreement with observations, with the primary goal of reconciling CKM CP violation with successful neutrino predictions. We consider the most general type II and type I seesaw formula for neutrino masses that includes the effect of the right handed neutrino mass matrix. We find that in these models the basic ingredients that went into understanding maximal neutrino mixings i.e. the relation $M_\nu = c(M_d - M_e)$, can be recovered in certain limits. However they do not help to keep the CKM phase in the first quadrant and in the type I case are incompatible with detailed neutrino data. We remedy this by including a specific class of nonrenormalizable terms in the Yukawa superpotential, that follows from a simple high scale theory. This specific set of non-
Figure 8: The electron EDM is plotted as a function of $m_{1/2}$ for different values $A_0$ and $\tan \beta = 40$ and 50 in type I.

Figure 9: The muon EDM is plotted as a function of $m_{1/2}$ is plotted as a function of $m_{1/2}$ for 40 in pure type II and type I.
renormalizable terms also reconcile pure type II models with the CKM model of CP violation. We find that the type I model requires an intermediate $v_R$ scale (i.e. $v_R \ll M_U$) which is lower than the standard GUT scale, however it is compatible with gauge coupling unification which happens around $10^{15.5}$ GeV. The proton life time is around the current experimental limit. We also find that the mixed type II and type I models require a value of the supersymmetry parameter $\tan \beta$ larger than 30 to be compatible with present neutrino data. We then study the phenomenological implications of the type I, the mixed type II and the pure type II models for neutrino mixings, lepton flavor violation and lepton edms. We find that these predictions provide a new way to test these models.

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