Finite element method for modelling of two phase fluid flow in porous media

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Abstract. Petroleum and gas industry is doing enormous research to investigate new process and methods for the extraction of oil and gas from natural reservoirs after primary and secondary recovery, these processes are named as Enhanced Oil Recovery (EOR). A lot of simulation techniques have been adopted to study the behaviour and predict the physical mechanisms of oil recovery. Different characteristic functions including capillary pressures, relative permeabilities and fluid saturations are helpful in predicting the effects of wettability alteration, viscosity, and surface tension on oil recovery. In this work, simulations have been carried out by using finite element method (FEM), for two-phase fluid flow in porous media. A 3-dimensional cylindrical shaped porous media was designed, same as the laboratory-scale setup for EOR. Brooks-Corey model and Ban Genuchten models were used for the calculation of capillary pressure with changing the saturation of water and oil. The proposed model is capable to predict capillary pressure, relative permeability and speed of fluid at different positions in porous media with random wettability conditions.

1. Introduction
Demand of innovative techniques in oil and gas industry is escalating with the passage of time to fulfill the increasing demand of hydrocarbons fuels [1]. Nanotechnology which has become a subject of common interest in almost every field also proved to be a promising technology for Enhanced Oil Recovery (EOR). The small size of nanoparticles makes them important tool for EOR applications as they can penetrate and move easily in the porous rocks without influencing the permeability of reservoir. Nanoparticles can drastically enhance the oil recovery by improving either the injected fluid properties (density, viscosity enhancement, surface tension reduction, thermal conductivity, specific heat improvement and emulsification improvement) or the fluid rock interaction properties (wettability alteration and heat transfer coefficient). [2]

A mathematical model for the flow of immiscible and incompressible fluids through porous media can be developed using finite element method to represent the subsurface oil reservoirs. Anant R. Kukreti et al. [3] solved the nonlinear equations in the time domain and an algorithm was presented for linear and quadratic approximation to demonstrate the two-phase flow through a homogenous square domain porous media. Elsyed et al. [4] studied the qualitative and quantitative understanding of interaction between nanoparticles and porous media. Nanoparticles size, concentration, injection rate and permeability are critical parameters that control the transport of nanoparticles in porous media. The proposed model was used for optimization of nanoparticles application to petroleum reservoirs. [5], [6]
In various oil reservoir stimulation methods, external energy has been transferred from the surface to the subsurface in terms of thermal energy, mechanical energy and electrical energy, for creation of additional forces that can cause instability to the interfaces of oil and the surrounding media [7]. Coupling external energy to the targeted area needs careful consideration, such that insignificant energy losses to the surrounding can be minimized. Traditionally, electrical stimulation method has been widely used to heat up the oil reservoir and improve mobility because energy coupling through electrical charge distributions is more efficient since most of the reservoir bodies are transparent to the magnetic fields [8]. However, it suffers from drawbacks that electrical signals are highly attenuated while propagating through subsurface layers and requires external stimulation to maintain energy level, which will not occur if magnetic signals are taken into consideration [9]. Magnetically coupled system can provide better solutions in terms of selectivity and localized transfer of energy to the targeted areas. In addition, magnetic signals are not affected by other parameters such as surface charges, pH and ion [10, 11].

In enhanced oil recovery (EOR) practical applications, the most commonly used polymer is hydrolysed polyacrylamide (HPAM) [12]. Lab studies and simulations have shown that with the increasing polymer concentration and volume, the recovery rate increase significantly. This finding was supported by the fact that increase in the viscosity and decrease in the permeability of the porous medium makes mobility ratio (M) becomes more favourable compared to water flooding [13, 14].

In another study improvement in the recovery rate on macro scale as a result of increasing volumetric sweep efficiency by injecting HPAM of various concentration into the oil reservoir [15]. They have also concluded that the relationship between capillary number and recover of cores using Newtonian fluids does not apply to fluids with elastic properties i.e. showing Non-Newtonian behaviour. Elastic properties of the fluid caused different velocity distribution in the pores and exert a very strong pulling effect. However, in reservoir environment of high salinity, it is found that the rheological properties of the polymer solution suffer from the salt intolerance, which caused the polymer to lose its viscosity. Similar elastic properties can be achieved, provided higher concentration of polymer or polymer of higher molecular weight is used, which will bring another problem; cost-effectiveness and injectivity [16, 17]. As highlighted by Heemskerk et al (1984), increasing viscoelastic effect by using polymers with higher molecular weight leads to excessive pressure build-up and consequently, low maximum injection rates [16]. Therefore, there is a need to introduce a solution to the existing problem. Tolerance of viscosifying agents towards harsh reservoir conditions i.e. high temperature, high salinity and high shearing rates need to be improve to maximize oil recovery. This early research on ferrofluids have various application in mechanical engineering such as lubrication and sealing of bearings and in brakes [18]. Also, there are a lot of applications in detection and imaging such as magnetic nanoparticles when attached to specific body tissues can be used to enhance MRI [19], and to burn cancer cells [20].

In petroleum engineering, there are strong evidences which support that the ferrofluid transport to the target locations is feasible for enhanced oil recovery. Ryoo et al. (2012) studied that the nanoparticles adsorbed at the fluid interfaces under an oscillating field has an acoustic response to determine the oil saturation [21]. Davidson et al. (2012) demonstrated that the heat can be delivered to specific locations of the reservoir with heavy oil by the Neel relaxation effect in ferrofluids [22]. Rahmani et al. (2012) numerically simulated the ferrofluid blobs in porous geometries and suggested that we can improve the micro displacement efficiency of the oil by using magnetic nanoparticles, however he was not able to find out the value of magnetic field within the ferrofluid phase [23].

In this work a mathematical model will be proposed based on finite element method to simulate the fluid flow in porous media to find out the recovery rate and sweep efficiency of oil with water and brine.
2. Mathematical modelling and simulation

2.1. Finite Element Method (FEM)
Finite element analysis of a problem can be carried out using four steps, involving.

1. The solution region is discretized into a finite number of elements or sub-regions.
2. To derive the governing equation for each type of element.
3. To make an assembly of all elements in a solution region.
4. Solving the equations for whole system.

For the system of two phase fluid flow in porous media, the cylindrical shaped porous media was created in three-dimensional space. Each element of cylinder is a tetrahedron to discretize the whole system as shown in figure 1 [24].

![Figure 1. Tetrahedron element for fluid flow discretization.](image)

A mesh consisted of such tetrahedrons and triangles was generated, and the number of triangles increases with the increasing density of fluids. The equations solved for each mesh element to measure the values of different parameters for fluid flow in porous media [25].

2.2 Brine and oil flow in porous media
The parameters which control the capillary forces and affect the hydrocarbon recovery process i.e., the wetting properties of rock - fluid, two phase flow properties (relative permeability and capillary pressure), the matrix permeability, viscosities of phases, the initial water saturation and the boundary conditions. Nanoparticles and nanofluids can accelerate the transfer rate of oil due to the migration of nanoparticles from aqueous phase to oleic phase affecting both the oil properties and rock oil properties to change. For example, oil mobility, may increase owing to the change in viscosity which may enhance the ultimate recovery of oil in fractured reservoirs. Some other mechanisms which can play role in EOR are oil density reduction and alteration of the interfacial tension of oil and water.

For this purpose, we need some solvent (in our case magnetic or dielectric nanoparticles) which can be injected with brine to the reservoir where oil is present. A mathematical model based on Finite element method was developed in COMSOL in which a system of equations will be solved to describe the brine/nanofluid injection in to the petroleum filled core.

2.2.1. Model Description.
A 3D porous scale model is proposed to simulate the displacement of oil by water. The following assumptions were made to simplify the model.
1. The model is three-dimensional, the rock properties are homogenous and gravity term neglected.
2. The process is isothermal.
3. Both the oleic and aqueous phases also the porous media are incompressible.
4. Two phase Darcy's law and Brinkman's model are applicable.
5. The porous medium and fractures are fully saturated with oil in beginning.
6. Adsorption is neglected at the walls of solid rock.
7. The pressure at the outlet is atmospheric pressure.
8. Steady state flow.

The geometrical model is shown in Figure 2. Geometry of porous media for two-phase fluid flow, a cylindrical porous structure of length 1m and radius 0.1m. The fluid enters from the inlet on the inlet and comes out at the outlets on the right face. There is no flow at around the curvature of the cylinder. The input parameters for simulation are shown in Table 1.

![Geometry of porous media for two-phase fluid flow](image)

Figure 2. Geometry of porous media for two-phase fluid flow.

### Table 1. Rock and fluid properties.

| Parameter               | Value | Unit |
|-------------------------|-------|------|
| Oil density             | 820   | Kg/m$^3$ |
| Brine density           | 1203  | Kg/m$^3$ |
| Oil viscosity           | 0.001 | Kg/m.s |
| Brine viscosity         | 0.000 | Kg/m.s |
| Porosity                | 0.26  | -    |
| Permeability            | 200m  | mD   |
| Initial Oil saturation  | 0.9   | -    |
| Initial Brine saturation| 0.1   | -    |
| Initial velocity of Brine| 0.04 | m/s |

2.2.2. Governing equations

By keeping in view, the above assumptions, the following equations will be used for the simulation of fluid flow in porous media. Mass conservation equations for aqueous and oleic phase are given by
\[
\frac{\partial (\phi S_a)}{\partial x} = \nabla \cdot \left( \frac{k_a}{\mu_a} \nabla p_a \right) + q_a
\] \hspace{1cm} (1)

\[
\frac{\partial (\phi S_o)}{\partial x} = \nabla \cdot \left( \frac{k_o}{\mu_o} \nabla p_o \right) + q_o
\] \hspace{1cm} (2)

Where \( k, \mu, S, p, \phi \) are the permeability, viscosity, saturation, pressure and porosity of porous media. The subscripts \( o \) and \( a \) represent the oleic and aqueous phases. In this system, the space of the porous media is completely filled with both the phases, so the following relation occurs.

\[ S_a + S_o = 1 \] \hspace{1cm} (3)

Darcy’s law for single phase flow and two-phase flow in porous media are given in equations (4) and (5).

\[ q = \frac{k}{\mu} \frac{\Delta p}{L} \] \hspace{1cm} (4)

\[ q_a = -\frac{k_{ra}}{\mu_a} (\nabla p_a - S_{uA}) \] \hspace{1cm} (5)

The extension of Darcy’s Law in which an effective viscosity term \( \beta \) is added. The correction term accounts for the flow through medium where the grains of the media are porous themselves.

\[ \beta \nabla^2 q + q = -\frac{k}{\mu} \nabla p \] \hspace{1cm} (6)

The modified equation is known as the Brinkman’s equation and is used to find out the velocity of the fluid in single phase flow. The term \( q \) is the fluid velocity.

2.3 Injection of Nanoparticles

The nanoparticles are normally injected into the porous medium as an aqueous slurry, which has to come into contact with the hydrocarbons in the reservoir. The main issue related to the field application of this technology is the reduced mobility of nanoparticles. In order to enhance the mobility of nanoparticles on the subsurface, it is important to (i) prevent the formation of large aggregates that tend to be easily filtered and (ii) reduce the attachment of nanoparticles to the soil grains. To date, the best strategy to achieve these goals has been to modify the surface of the nanoparticles using polymers or surfactants. A number of other studies have investigated the nanoparticles mobility using transport tests in order to develop a powerful tool for the design and the implementation of full scale nanoparticle applications. The deposition of particles on the porous media is an important mechanism controlling the mobility of colloids in aquifer systems and the effectiveness of the technology. In porous media, the dynamic of the particles inside of a pore is governed mainly by drag, gravity and Brownian forces, but close to the sand surface also the electric double layer (EDL) and the Van der Waals forces (VdW) come into play and become prevailing.

2.3.1 Governing equations for Nanoparticles injection

The velocity field \( u \) was solved using 2D steady state Navier-Stokes equation and the continuity equation:

\[ \rho(u \cdot \nabla)u = \nabla \left[ -pI + \mu (\nabla u + (\nabla u)^T) \right] \] \hspace{1cm} (7)
Here, $p$ is a pressure. The inlet velocities of the both outer channels were given as a velocity magnitude of 0.0001 m/s, respectively. The outlet conditions of those channels were given as the same as the atmospheric pressure. No slip boundary condition was imposed on all the solid boundaries, including the inlet and the outlet of the inner channels. Nanoparticle concentration $c$ was solved using the time-dependent convection-diffusion equation: \[
\frac{\partial c}{\partial t} + u \nabla c = D \nabla^2 c \tag{9}
\]

Here, $D$ is a diffusion coefficient of a nanoparticle with its radius $r$. Equation (10) is the Stokes-Einstein equation, where $k_B=1.38 \times 10^{-23} \text{ m}^2\text{kg/(s}^2\cdot\text{K})$ is the Boltzmann constant and $T$ is the absolute temperature, assumed as 293.15 K. The inflow condition was given as $c=1 \text{ mol/m}^3$ for the outer channel inlets.

3. Results and Discussion for Simulation

In case of homogeneous porous media, if the injected fluid (water) is less viscous as compared to kerosene oil it will show front instabilities. This process is known as fingering, a particular case of Rayleigh-Taylor instability for fluid dynamics [26]. Water fingers propagate through the porous media and leave the clusters of oil behind, it occurs when the displacing fluid is less viscous (more mobile) than the displaced phase. This phenomenon occurs when the injection velocity and viscosity ratio are very small [27].

3.1. Saturation of fluids

In our case for inhomogeneous porous media there is a high permeable layer in the medium, the injected fluid enters this area first as shown in figure (3b). Because higher permeability zone possesses lower flow resistivity and the fluids have tendency to flow through lower resistive area. This layer creates a path toward the outlet, which results in an early breakthrough of the injected fluid (figure 3c). After breakthrough, most of the injected fluids flow directly through this path and some other parts of the medium remain in swept (figure 3d). To increase the sweep efficiency, a first order solution is to add magnetic nanoparticles to the injected water. Nanoparticles have the tendency to reach the inner pores of reservoir. As they can be activated by EM radiation, the viscosity of the nanoparticles solution increases, causing high resistivity zone in front of the water which has been flooded after the polymer. Hence the water can be diverted to the low permeability zone. This process will be simulated for core scale fluid flow during my research work.
The iso-surface flow pattern of fluids in porous media is shown in Figure 4. This flow patterns can help us understand the porosity of the medium and the flow behaviour of fluids in porous structure.

3.2. Pressure and Velocity measurement in porous media

Darcy’s law can be used to calculate the permeability at each point of the porous media. For this purpose, we need to know about the velocity and pressure at each point which is calculated from simulation. Figure 5(a) shows the surface graph of the porous media at the time t=0s, the pressure is maximum at the inlet and decreases as we move towards the outlet. Red arrows show the direction of fluid flow. Pressure curves in Figure 5(b) shows that the difference at the two ends of porous medium is maximum when the fluid begins to flow but this difference decreases as the water continuously flows and sweep out the oil from porous medium.
Figure 5. (a) Pressure at the surface of porous media (b) Pressure curves at different time durations.

Figure 6 shows the different curves for velocity of brine entering from and inlet and moving towards outlet along the length of porous media. We find that at the beginning of time the velocity of fluid was static and linear, but after travelling a certain distance the velocity changes due to the changing pore size. If the porous media is coarser the fluid will have a steady velocity, but if the pore size are very small the velocity will increase. We can see at time 50s there is a decrease in velocity, which is due to the increase in pore size. At 150s the velocity increases sharply, and this happens die to the smaller pore area.

Figure 6. Velocity measurement along the arc length at different duration of time.

3.3. Capillary Pressure

The relation of capillary pressure and saturation is very vital for the characterization of two-phase fluid flow phenomenon in porous media. The difference in the fluid viscosities are responsible for rise in the capillary pressure. We have mathematically studied the capillary pressure inside the porous media at different durations of times. Capillary pressure increases when the two phases come in contact with each other or with the walls of container. The general expression to find out the capillary pressure at the boundary of the pore is given by this formula,

\[ P_c = \sigma \left( \frac{1}{R_1} \pm \frac{1}{R_2} \right) \]  

Where \( R_1 \) and \( R_2 \) are the radii of curvature of the two sides of pore.
Figure 7. Capillary Pressure vs saturation curves at different time durations for (a) increasing saturation of water (b) decreasing saturation of oil.

Capillary pressure curves shown in figure 7 tells us about the wettability of the porous medium. As the porous medium become more water wet the saturation of oil will decrease and hence the capillary pressure of the system will increase with the passage of time. Capillary pressure reaches zero at lower saturation of water but as the saturation of water increases the capillary pressure reaches to a steady state and does not increase with increasing saturation of water. The bumps in the graphs shows the heterogeneity of porous medium due to the presence of fractures, bugs, laminations and others. Y. Liu et al. also recommended the Brooks-Corey model to study the capillary pressure and saturation curves [28].

4. Conclusion

In this work, a mathematical model based on Finite element method was used to simulate two phase flows in porous media. The effect of permeability contrast in lowering the water sweep efficiency in the porous media was studied. The results show that the early breakthrough of water depends on the viscosity of the fluids and hence the mobility ratio. The velocity of the fluid and capillary pressure studies shows that the wettability of rock increases with the increasing saturation of water and hence it lowers the recovery of oil. With the injection of nanoparticles, the capillary pressure will decrease and hence the wettability of rock will decrease to increase the oil recovery.

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