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Soliton switching and multi-frequency generation in a nonlinear photonic crystal fiber coupler

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Abstract: Soliton switching in nonlinear directional couplers implemented in photonic crystal fibers (PCF) examined here. A vector finite element method (FEM) has been developed to precisely calculate the dispersion along with coupling length of the guided modes. The PCF coupler geometry was carefully designed so that it can support soliton pulses. Soliton switching is demonstrated numerically at 1.55 μm for 100 femto-second (fs) pulses. Our theoretical results explain some of the key spectral features previously observed in the experiment.

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1. Introduction

A silica glass PCF has a central core region of pure silica surrounded by air holes that serves as a cladding. PCFs possess numerous unusual properties, such as bend-loss edge at short wavelengths, a wide spectral range for single-mode behavior, control of the effective-core-area, and engineerable anomalous group velocity dispersion (GVD) at visible and near-infrared wavelengths [1-3]. These properties have recently generated intense research interests. Directional couplers based on dual core fibers are well-known and their linear and nonlinear properties have been investigated theoretically and experimentally [4-6]. This was driven in part by potential applications in nonlinear all-optical control of signals including all-optical switching, all-optical logic gates, multiplexer and demultiplexer (MUX-DEMUX), etc. [1, 6-8]. In standard fibers it was realized that pulse break-up due to GVD was a serious problem, deteriorating significantly the switching characteristics of a nonlinear directional coupler [9-11]. A possible solution to avoid pulse break-up is to use temporal solitons. Yet this approach was never implemented experimentally because of the limited wavelength response of the fiber [9, 12]. PCFs, however, offer more versatility primarily because their group velocity and their dispersion can be tuned dramatically with wavelength using an appropriate PCF design [6-7, 13-14]. In particular, PCFs have been designed and fabricated with very flat GVD response making temporal solitons a solution to pulse break-up because the pulse experiences the same dispersion in a wide range of frequencies [7, 14]. Single core PCF supporting soliton was discussed in [15-18]. It has already been demonstrated theoretically and experimentally that PCFs with two adjacent defect areas which serve as two coupled core regions can be used as an optical directional coupler [4-5]. Previously, nonlinear switching with 100 fs pulses, which was complicated considerably by broadband multi-wavelength generation, has been reported in a dual core PCF by two of the authors [19]. These experimental results indicate that many nonlinear phenomena, including phase-matched interactions, occur in such devices and that a very careful analysis of the linear and nonlinear effects in such couplers is needed not only to explain these results, but also to design couplers to minimize the spurious effects. The coupling length of a dual-core PCF coupler has been evaluated numerically [2]. We evaluate the inter-core coupling along with the dispersion characteristics incorporating the material dispersion which is particularly significant at shorter wavelengths [6]. In the present work we continue using the proposed finite element method to obtain the frequency response of the higher order PCF dispersion parameters and use it to accurately model short duration (<100 fs) pulse (including soliton) propagation at different frequencies. The coupler geometry was carefully designed to achieve appropriate GVD and coupling length so that it can support soliton pulses. Soliton switching and generation of multiple frequencies is explained using this model. Our numerical guided mode analysis of the dual core PCF shows that the cores support two supermodes with different phase velocities which lead to the exchange of power between them [6]. In order to understand in detail the switching and multi frequency generation reported in reference [19], we include self-phase modulation (SPM), stimulated Raman scattering (SRS), the shock term, the effect of GVD and higher order (third and fourth order)
dispersion on femtosecond soliton pulse propagation. A wide range of frequency response of dispersion and coupling characteristics of the PCF coupler has been considered for this purpose [20-22].

2. FEM formulation of guided modes

We have studied a dual core PCF and evaluated the coupling and dispersion characteristics using the vector FEM which provides immunity from spurious modes [23]. In order to analyze the PCF structure shown below in Fig.1, we have discretized the continuous spectrum by enclosing the structure within an electrical wall i.e. we have imposed a zero net field condition along this wall. The vector wave equation for the $E$ field is given by,

$$\nabla \times \left( \frac{1}{\mu_r} \nabla \times E \right) - k_0^2 n^2 E = 0$$

where $\mu_r$ and $n$ are the permeability and refractive index of the material in the waveguide respectively, $k_0$ is the free space wave number. The refractive index of the air holes in the PCF is taken to be unity and in the silica glass is wavelength dependent. The dependence can be expressed by a Selmeir expansion [23]. Eq. (1) can be expressed in terms of the transverse (subscript $t$) and longitudinal (subscript $z$) field components and written as

$$\nabla \times \left( \frac{1}{\mu_r} \nabla \times E_t \right) + \frac{1}{\mu_r} (k_z^2 \nabla E_z + k_t^2 E_t) = k_0^2 n^2 E_t$$

$$- \frac{1}{\mu_r} \left[ \nabla \cdot (\nabla E_z + E_z) \right] = k_0^2 n^2 E_z$$

here $k_z$ is the wavevector in the direction of propagation and can be found numerically using vector FEM. The transverse component of electric field $E_t$ and longitudinal component of electric field $E_z$ were also evaluated at the same time [23]. For this optical waveguide we get the effective refractive index $n_{eff}$ by using the relation

$$n_{eff} = \frac{\text{Real}(k_z)}{k_0}$$

![Fig. 1. Dual Core PCF geometry.](image)

Even and odd symmetry modes of the structure are supported by both cores of the dual core PCF and the coupling length $L_c$ is determined by the difference between the wavevector for the even and odd supermodes which is formulated as

$$L_c = \frac{\pi}{k_{ze} - k_{zo}}$$

where $k_{ze}$ and $k_{zo}$ represent the wavevectors of the even and odd supermodes respectively.
3. Soliton pulse propagation

Depending on the excitation wavelength, the dual core PCFs are capable of supporting soliton pulses noting that anomalous dispersion is required to support soliton pulses for the self-focusing nonlinearity of silica glass. The dual core PCFs considered here supports solitons in both the 1.31 μm and 1.55 μm wavelengths windows. Pulse propagation through this coupled structure is described by

\[
i \frac{\partial q_1}{\partial z} - \frac{\beta_1}{2} \frac{\partial^2 q_1}{\partial \tau^2} - i \frac{\beta_1}{6} \frac{\partial^3 q_1}{\partial \tau^3} + \gamma|q_1|^2 q_1 + i \frac{\gamma}{\omega} \frac{\partial}{\partial \tau} (|q_1|^2) q_1 + \frac{\pi}{2 L_c} q_2 = 0
\]

(6.a)

\[
i \frac{\partial q_2}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 q_2}{\partial \tau^2} - i \frac{\beta_2}{6} \frac{\partial^3 q_2}{\partial \tau^3} + \gamma|q_2|^2 q_2 + i \frac{\gamma}{\omega} \frac{\partial}{\partial \tau} (|q_2|^2) q_2 + \frac{\pi}{2 L_c} q_1 = 0
\]

(6.b)

where \( z \) and \( \tau \) are the distance and time respectively; \( q \) refers to the envelope of the electromagnetic wave; \( \eta \) is the small ratio of the cross-phase (XPM) to self-phase (SPM) modulation coefficients; \( \gamma \) is the nonlinear parameter which can be written as

\[
\gamma = \left( \frac{n_2 \omega}{c A_{eff}} \right)
\]

(7)

where \( c \) is the velocity of light. Numerically the effective nonlinear modal area, \( A_{eff} \), can be evaluated by using the following expression involving the modal electric field \( E(x,y) \) evaluated from the finite element solution of Eqs. (2) and (3) [17]

\[
A_{eff} = \frac{\int \int |E(x,y)|^2 dx dy}{\int \int |E(x,y)|^2 dx dy}
\]

(8)

For the PCF coupler shown in Fig. 1 (\( d=2.0 \) μm, \( C=2\Lambda \) and \( d/\Lambda =0.9 \)), \( A_{eff} \) was evaluated as 41 μm² at 1.55 μm. The second and third terms in Eq. (6)correspond to the dispersion, the fourth term corresponds to self and cross phase modulation, the fifth term describes the shock effect and the last term corresponds to the linear coupling [20-22]. These equations were solved numerically using a beam propagation method (BPM) for 100 fs pulses [22]. We found the value of \( L_c \) can be varied significantly by altering the PCF geometry [2, 6]. We increased the coupling length by increasing the \( d/\Lambda \) [6]. \( L_c \) for the PCF discussed in Fig. 1 was 1.8 cm at 1.55 μm. The FEM model provided the wavevector \( k_z \), from which the group velocity dispersion, \( \beta_2 \) - the second derivative of the wavevector \( k \), with respect to angular frequency; \( \omega \), was calculated. We evaluated \( \beta_2 \) for the PCF under discussion in Fig 1 at 1.55 μm to be \(-47\) ps²/km. Subsequently third order dispersion \( \beta_3 \), the third derivative of the wavevector \( k \), was also evaluated as 0.1 ps³/km.

Figure 2 shows a soliton pulse propagating through the PCF coupler. The energy is transferred from the bar channel (the waveguide which received the input pulses) to the other waveguide (named the cross channel) after traveling \( L_c \) which is 1.8 cm. For a 100 fs pulse propagating through this waveguide at 1.55 um wavelength the soliton period \( Z_0=1.57(\frac{T_0^2}{\beta_2}) \)

\( =33 \) cm [22]. Note that there is no pulse break-up in the time domain, at least until \( Z_0 \). At least 80 nJ is required for soliton pulse to propagate. The result shows similar coupling characteristic as that of the nonlinear optical fiber directional coupler reported in ref [9-11].
Fig. 2. Soliton pulse propagation at 1.55 μm through the PCF coupler shown in Fig. 1 (d=2.0 μm, C=2λ and d/λ =0.9) (a) bar channel and (b) cross channel.

A similar the 2D view of the 100 fs pulse depicts the soliton pulse propagation in Fig.3, for comparison with non-soliton propagation. At a lower peak pulse power (20W) the pulse experiences only dispersion and causes pulse broadening as shown in Fig 3(a). When we increase the peak power (to around 800W) the nonlinearity starts taking over and cancels out the dispersion. Fig. 3(b) shows the undistorted soliton pulse.

Fig. 3. 2D view of 100 fs pulse propagation through the PCF bar channel: (a) Dispersed pulse at 20W peak power (b) Soliton pulse at 800W peak power.

4. Pulse switching in dual core PCF

At low peak intensity of the pulse, the dual core PCF behaves like a linear coupler. The two cores completely exchange power after traveling a distance of Lc. As the intensity increases, an increased fraction of the input power remains in the same waveguide where the pulse was originally launched. Fig. 4 shows this clearly. Here we see that at low intensity the normalized power P1 in the bar channel and P2 in the cross channel are completely exchanged after propagating 1.8 cm which is the coupling length of the PCF. This linear characteristic is shown in Fig. 4(a). On the contrary at higher intensities shown in Fig. 4(b), most of the power tends to remain in the launching core. Transmission curve in Fig. 5 shows the switching characteristic prominently. We observe switching around 0.9 TW/cm².
5. Multi-frequency generation

At a higher input power, it is evident that additional frequencies are generated due to several nonlinear processes. Multi frequency generation was experimentally demonstrated in [19]. It was quickly realized from that experiment that the switching characteristics deteriorated due to this energy transfer from one frequency to another. Multiple nonlinear processes such as SPM, SRS (Stimulated Raman Scattering), THG (Third Harmonic Generation) and wave-mixing were responsible for this high frequency generation. Since the frequencies generated covered a broad spectrum, it was essential to incorporate wide spectra of dispersion and coupling characteristic in our model. We include all these and modify the propagation equation as follows

\begin{align}
&i \frac{\partial \tilde{q}_1}{\partial z} - \beta_2 \frac{\partial^2 \tilde{q}_1}{\partial \tau^2} - i \beta_4 \frac{\partial^4 \tilde{q}_1}{4 \partial \tau^4} + \frac{\beta_4}{24} \frac{\partial^6 \tilde{q}_1}{\partial \tau^6} + \frac{\gamma |q_1|^2 |q_1|^2 + \eta |q_2|^2 |q_2|^2}{0} + \frac{\gamma |q_1|^2 |q_1|^2 + \eta |q_2|^2 |q_2|^2}{\partial z} = 0 \tag{9.a}
\end{align}

\[ \text{Fig. 4. Normalized power in bar channel } P_1 \text{ and cross channel } P_2 \]
(a) at 4.16 GW/ cm² (b) at 583 GW/ cm²

\[ \text{Fig. 5. Transmission of the dual core PCF coupler (d=2.0 μm, C=2Λ and d/Λ=0.9) versus input intensity.} \]
The $\beta_4$ shown in the fourth term is the fourth derivative of the wavevector $k_z$ with respect to $\omega$ (fourth order dispersion). $T_R$ is the nonlinear response function associated with the delayed Raman response. Experimentally $T_R$ was estimated to be 3 fs at 1.55 $\mu$m [22]. $\kappa_0$ is the constant part of the series expansion of frequency dependent coupling coefficient $\kappa$, which is related to the even and odd mode wave vector by

$$\kappa = \frac{k_{\text{even}} - k_{\text{odd}}}{2} = \frac{\pi}{2L_c}$$

(10)

$\kappa_1$ is the first derivative of $\kappa$ with respect to $\omega$. This $\kappa_1$ term is added in the propagation equation to incorporate the frequency response of the coupling. The frequency dependent dispersion and coupling characteristics for the PCF specimen were evaluated using the FEM model and those agree well with the experimental results. Experimentally $L_c$ can be measured by using a cut back method and at 1.55 $\mu$m it was measured as about 5.6 mm for the PCF coupler used in [19]. The value of $L_c$ from our model confirms this result (Fig. 6.). Fig. 7 shows the frequency response of $\kappa_0$ and $\kappa_1$.

Fig. 6. Coupling length of the dual core PCF sample (Five rings, $d=1.0$ $\mu$m, $\Lambda=2.5\mu$m and $C=10\mu$m): (a) PCF geometry (b) $L_c$.

Fig. 7. Coupling coefficient of the dual core PCF sample: (a) $\kappa_0$ (b) $\kappa_1$.

Figure 8 shows the frequency response of $\beta_2$, $\beta_3$ and $\beta_4$ of the dual core PCF sample used in the experiment. From our evaluated dispersion results it is found that the GVD changes
from normal to anomalous in around 1.1 \( \mu \text{m} \) (equivalent to 272 THz) which also known as the zero dispersion wavelength \( \lambda_d \) and that can be shifted by changing the \( \text{d}/\Lambda \) [6-7, 12].

Fig. 8. Frequency dependent dispersion characteristics of the PCF sample (a) \( \beta_2 \) (b) \( \beta_3 \) and (c) \( \beta_4 \).
By numerically solving the propagation Eq. (9), we observed pulse breaking primarily due to strong SPM as the input peak pulse power increases beyond the switching threshold power. Fig. 9 shows the intensity of the output pulses (for both bar and cross channels) for peak powers of 4, 40 and 114 kW respectively. At a relatively lower input power (up to 4 kW), distortion free pulse propagation is observed. Pulses could not maintain their shape as the power increased. As can be observed from the figures, the intensity modulation becomes more intense as the power is increased.

It is obvious from the time domain coupler response shown in Fig. 9 that higher input power causes increased frequency generation due to multiple nonlinear processes. Fig. 10 shows the wavelength response of the cross channel output for a 1.55 μm input pulse at power levels of 4, 40 and 114 kW respectively. The pulse energy spread into a wide wavelength range (from 0.5 μm to 3.0 μm). The primary reason for the generation of higher frequencies is due to SPM. For generation of lower frequencies (longer wavelengths), process such as SRS and soliton self frequency shift become important.

Note that no sharp spectral features such as those observed experimentally were found. This allows us to identify the origin of these features as phase-matchable processes such as multi-wave (three and four wave) mixing and third harmonic generation which were not considered here. Inclusion in the analysis requires searching for detailed phase-matching over broad spectral regions and this problem will be addressed in the future.
Fig. 9. Input pulse shape and the pulse shapes output from a 9mm PCF coupler (both bar and cross channel) are at a peak pulse power of: (a) 4 kW (b) 40 kW and (c) 114 kW.
6. Conclusion

We have demonstrated soliton switching implemented in nonlinear PCF couplers. A numerical model has been developed to evaluate coupling coefficients, dispersion and nonlinear parameters over wide spectral regions. It is evident that these parameters can be tuned by altering the PCF geometry. We also numerically demonstrated all-optical switching in the nonlinear directional couplers implemented in the dual core PCF. At 1.55 μm we have observed that the switching occurs around 0.9 TW/cm². At higher input power the coupler
generates multiple higher order frequencies due to multiple nonlinear processes such as SPM, SRS, and wave mixing etc. Among these SPM is the most dominant since it spreads the pulse energy over broad spectral regions.