Predicting the Effect of Fly Ash on Concrete’s Mechanical Properties by ANN

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Abstract: Fly ash, as a supplemental pozzolanic material, reduces concrete’s adverse environmental footprint by decreasing the emission of carbon dioxide (CO₂) during the cement manufacturing process. Fly ash, which is a waste material, can enhance both the mechanical characteristics and durability of concrete, and has the capability to play an important role in sustainable design. Considering the widespread interest in applying Fly ash, and despite research studies, the level of replacement is still unclear. In this paper, a novel method using artificial neural networks (ANN) is presented to predict concrete’s mechanical characteristics by adding Fly ash. In this regard, a host of available experimental data, such as the properties of Fly ash, along with concrete additives, was fed into an ANN model. Concrete samples’ tensile and compressive strengths, in addition to their modulus of elasticity, were defined as outputs. It was observed that the predicted outcomes agreed well with the experimental results. To further enhance the research outcomes, simple but practical equations are presented to assess the effect of using Fly ash on concrete’s mechanical characteristics.

Keywords: compressive strength; Fly ash; artificial neural network; prediction

1. Introduction

Three major aspects—Resource Conservation, Life Cycle Costing (LCC), and Human-Friendly Designs (HFD)—fully affect the Sustainability of the Built Environment (SBE). Nowadays, the concept of the 3Rs, i.e., Reduce, Reuse, and Recycle, are commonly utilized in Resource Conservation [1]. Embodies Energy, defined as the energy consumed for raw material extraction, transportation, manufacture, assembly, installation, disassembly, and deconstruction for any product system over the duration of a product’s life, based on [2], is one of the important considerations when measuring the sustainability of a building material. Concrete, as one of the most important and most consumed construction material, which has a negative impact on the environment, needs to be considered in sustainable development. The amount of embodied CO₂ (ECO₂), i.e., the total amount of CO₂ produced in the extraction and transportation of raw materials and their manufacture into the final product, is directly compatible with cement content in concrete mix designs, since in the production process of Portland cement, a huge amount of energy is consumed, and a remarkable amount of CO₂ is produced [3–5]. The ECO₂ is stated as CO₂ produced per unit of volume or mass. For example, the ECO₂ of Portland cement is about 910 ECO₂ kg/tonne [6], which is responsible for 7% of total worldwide CO₂ emissions [7]. The high amount of cement consumption and CO₂ emissions have led to increased global awareness, and inspired researchers to develop sustainable options.

Supplementary Cementitious Materials (SCMs), which are mineral admixtures with pozzolanic activity, can be used to partially replace Portland cement in concrete, to cover the other 3Rs concept in SBE, i.e., reducing. The application of SCMs in the concrete industry has been gaining wide attention as an environmentally-friendly and cost-effective...
approach. Adding SCM to concrete leads to denser calcium silicate hydrate (C–S–H), which reduces permeability and enhances the concrete’s compressive strength. In this regard, further studies are required regarding the pozzolanic reactions, and the proper utilization of these additives to optimize the mechanical properties of the mixtures, as well as reducing the substantial volume of CO$_2$ emissions, to gain a sustainable material [8].

In the past decade, there has been increasing interest in evaluating the cementing efficiency parameter, which is defined as the effect of SCMs on enhancing a specific property, respecting the ratio of water/binder or the cement content, and the recognition of effective parameters of a mineral admixture [9-11]. One of the 3Rs in resource conservation to obtain a sustainable design is reusing material. Fly ash, as the solid waste of coal-firing power stations, predominantly consists of ferric oxide (Fe$_2$O$_3$), aluminum oxide (Al$_2$O$_3$), silicon dioxide (SiO$_2$), and calcium oxide (CaO). Fly ash is a waste material in the power generation industry, and reusing this highly active pozzolan in the construction industry may bring about several advantages. The other benefit of using Fly ash in concrete is the reduction of ECO$_2$ (about 230 times less than Portland cement [6]) which consequently leads to the achievement of sustainable materials.

The presence of SiO$_2$ and Al$_2$O$_3$ are the main reasons accounting for the pozzolanic activity of Fly ash. The presence of calcium aluminate hydrate (C-A-H), along with the C-S-H gel, forms a denser matrix, which in respect, improves strength and durability due to reaction with the calcium hydroxide in the hydration process of the cement [12,13]. The spherical-shaped particles of the Fly ash create a ball bearing effect, in that such particles perform as a miniature ball that densifies the paste and improves the particle packing [14]. A host of improvements have been reported regarding the use of Fly ash in concrete [15]. These include increasing concrete characteristics such as durability, mechanical properties, and workability, along with reducing concrete deficiencies; in particular, decreasing the early-age thermal cracking, and reducing heat hydration are deemed as major advantages of the implementation Fly ash in concrete [16-19]. According to chemical compositions, Fly ash is categorized as Class F and Class C. The Class C Fly ash has high-pozzolanic and self-cementing characteristics, while Class F represents the reaction of Fly ash with the excess lime that is incorporated in the Portland cement’s hydration process [20].

The prime objective of the current study is to present a simple but reliable empirical formula based on artificial neural networks (ANN) to predict the mechanical properties of concrete, namely the compressive strength, tensile strength, and modulus of elasticity, in mixtures including Fly ash. The results of this study will lead to more employment of Fly ash as an alternative SCM to replace cement, which will ultimately bring about sustainable development by reusing waste material, and environmental protection by reducing cement consumption. To fulfill this aim, three perceptron neural networks were used, along with a backpropagation algorithm. The number of neurons and hidden layers were specified, and the network was trained using 291 samples containing Fly ash to evaluate the compressive strength, 56 samples to gauge the tensile strength, and 55 samples for predicting the modulus of elasticity. In this matter, the input parameters were selected based on the most influential parameters on the strength of concrete containing Fly ash.

The input parameters considered in the neural network are cement content (C), water content (W), and Fly ash replacement level (F.A.), along with amounts of coarse aggregates (G), fine aggregates (S), and SiO$_2$ (Si) in Fly ash. Accordingly, the output parameters were defined as tensile and compressive strengths, in addition to the concrete’s modulus of elasticity. The inputs were selected in such a way that the effects of physical and chemical factors on the mechanical properties of the concrete could be considered. One of the novelties of this paper is considering the various types of Fly ash, by taking into account the Fly ash reactivity, i.e., SiO$_2$ content. The predicted outcomes were compared to the experimental results, and the error rate was determined. Moreover, simple empirical equations are presented for predicting compressive and tensile strengths, along with the modulus of elasticity of the concrete samples in the presence of Fly ash. In this
practical applications of neural networks in engineering are used to enhance the research outcomes.

2. Artificial Neural Network

Simplifying the models to use and increase their accuracy from complex natural systems with large inputs are the advantages of neural network applications [21]. Neural networks consist of simple operating elements that work in parallel. These systems are information manager models inspired by the human brain. In nature, the performance of neural networks is determined by the way in which the components are interconnected [22]. Therefore, it is possible to construct an artificial structure in accordance with natural networks and determine the relationship between its components by adjusting the values of each connection, as the weight of the connection. After adjusting or training the neural network, researchers apply a specific input leads to a particular result as the output. The most important part of the training is minimizing the error. This is done by changing the weight during the learning step continuously until the error function, like mean square error, is less than a specified limit. The error is determined as follows:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$  (1)

where $N$ stands for the samples number; and $y_i$ and $\hat{y}_i$ are the target and predicted results, respectively. The prediction procedure is iterative, which initializes the $w$ value, estimating $\hat{y}_i$, and calculating the corresponding error. In the initial step, due to the randomly-selected weights, the error is high. The challenge of network learning is to determine the weights that result in less error for all data sets. In most artificial networks, the numbers of weights are high, and so they cannot be found directly. Estimating weights by trial and error also wastes effort and time. The gradient descent approach is an effective way to determine the minimum sets of the error more quickly during the training of the network. Gradient descent, as the name implies, uses the error gradient to descend the error [23]. The error is related to the output of the network; it depends on the weighted output of the hidden neurons, and it depends on the weights. Thus, the chain rule of differentiation can be extended from the error to weight of the first layer, $\frac{dE}{dw_{\text{first}}}$. This method, i.e., backpropagation, which was first proposed by Werbos [24] and then by Rumelhart [25], is a gradient descent algorithm that is based on shifting the network weights to the opposite direction of the performance function slope.

A neural network is based on the following assumptions:

1- processing of information occurs in simple members called neurons;
2- signals are passed to neurons over connection links;
3- every connection has an associated weight;
4- each neuron transmits inputs from the activation function and determines outputs.

A neural network is known based on its architecture, the learning algorithm, and the activation function. Using neural networks reduces the number of experiments and saves time [26–28].

2.1. Dataset

An in-depth literature review was conducted to develop a neural network that can evaluate the mechanical properties of the concrete containing Fly ash. The dataset includes about 296 samples with six distinguishing features. The collected dataset contained information about the water content (W), cement (C), Fly ash (F.A.), gravel (G), sand (S), and SiO$_2$ content of Fly ash (Si). Besides, the mold of specimens was considered, so the compressive strength was converted to a 150 mm cubic standard mold. The 28-day compressive and tensile strength, along with the modulus of elasticity, were considered as outputs of the network. The datasets are used to train and test the network. The distribution of the parameters considered is presented in Figure 1. Table 1 summarizes statistical
parameters for the concrete containing Fly ash dataset. As can be seen, the amount of cement consumption can be limited by implementing Fly ash as a SCM. This may result in a reduction of ECO₂ in the practical usage of concrete.

Table 1. Statistical parameters for the concrete including Fly ash dataset.

| Attribute                  | Unit    | Min  | Max   | Average | Standard Deviation |
|----------------------------|---------|------|-------|---------|--------------------|
| Water                      | kg/m³   | 100  | 255   | 173.25  | 30.35              |
| Cement                     | kg/m³   | 90   | 675   | 273.9   | 100.44             |
| Fly ash                    | kg/m³   | 10   | 544   | 139.04  | 87.1               |
| Coarse Aggregate           | kg/m³   | 436  | 1278  | 976.83  | 195.03             |
| Fine Aggregate             | kg/m³   | 279  | 1293  | 749.27  | 178.25             |
| SiO₂ content of Fly ash    | %       | 26.61| 79.34 | 52.63   | 9.31               |
| 28-day compressive strength| MPa     | 5    | 121   | 41.35   | 20.64              |
| Tensile strength           | MPa     | 1.5  | 6.2   | 3.62    | 1.09               |
| Modulus of elasticity      | GPa     | 5    | 45.8  | 25.28   | 9.13               |

Figure 1. The histograms of input and the output parameters for mechanical properties of concrete containing Fly ash, (a) Cement (kg/m³), (b) Fly ash (kg/m³), (c) Water (kg/m³), (d) Coarse aggregate (kg/m³), (e) Fine aggregate (kg/m³), (f) SiO₂ content of Fly ash (%), (g) 28-day compressive strength (MPa), (h) Tensile strength (MPa), and (i) Modulus of elasticity (GPa).

2.2. Network Modeling

In general, modeling is the procedure of simulating real-world issues using mathematical functions [21,29]. It is important to optimize the network’s configuration to simultaneously save time and maintain the relativity of the prediction process. Due to the fact that there is no relation to compare the numbers of neurons and hidden layers, such numbers were specified using the Trial and Error procedure. In this regard, the most suitable configuration was selected for the ANN model, following the analysis of various arrangements of hidden layers and assigned neurons in each single layer. It was observed that the network with a single hidden layer including 13, 6, and, 15 neurons for 28-day
compressive strength, tensile strength, and modulus of elasticity had the best performance. The linear normalization of database parameters ranged from 0 to 1. The existing relationship in the initial gleaned database was preserved by the linear transformation [22]. The Levenberg–Marquardt algorithm was used to train the network, considering the algorithms’ suitable convergence, high precision, and time efficient characteristics [30–36]. This algorithm randomly divided the data into three parts, 70% for training, 15% for validation, and the remaining 15% for testing the network’s performance. TANSIG (Equation (2)) and PURELIN (Equation (3)) are respectively considered as the hidden and output layers’ activation functions. The learning process was set to be completed should the desired performance of the network be accomplished.

\[ y = \text{tansig}(x) = \frac{2}{(1 + e^{-2x})} - 1 \]  
\[ y = \text{purlin}(x) = x \]

Network Performance

The mechanical properties of concrete containing Fly ash can be predicted, once the network is appropriately trained. Moreover, the complex relation between the input parameters and their effects on output can be determined. Figure 2 presents the network’s performance regarding the evaluation of the compressive and tensile strengths, and modulus of elasticity. The best validation performance was obtained as 0.0038 at the 28th epoch for compressive strength, 0.0080 at the 3rd epoch for tensile strength, and 0.0027 at the 11th epoch for concrete’s modulus of elasticity in the presence of the Fly ash. The estimation quality is presented in Figure 2 based on the correlation coefficient (R) for all the incorporated data in the network. The coefficient illustrates the correlation between the ANN output and the target, i.e., the experimental data. The R values of almost 1 verify the reliability of outcomes by the network. These values for the compressive strength, tensile strength, and modulus of elasticity are 0.9729, 0.9658, and 0.9955, respectively. This indicates that the outcomes of the proposed network are in good agreement with the experimental results.

![Network Performance](image)

Figure 2. Cont.
3. Results and Discussion

After ensuring the network’s suitable performance and the response accuracy, to estimate the mechanical characteristics of concrete made with Fly ash, three separate networks were trained using 70% of the data, and their performance was evaluated using 15% of the data. The comparison of the 28-day compressive strength of the experimental specimen with the predicted results, based on the ANN is shown in Figure 3a. The results of the network and experimental values regarding the tensile strength and modulus of elasticity are presented in Figure 3b,c, respectively. As can be seen, the networks can predict the real experimental results with suitable accuracy, which is sufficient for practical use. This may encourage the further usage of ANNs to predict the mechanical properties of concrete containing Fly ash. Consequently, a simple method to estimate the behavior of concrete containing Fly ash could result in more employment of this type of concrete by engineers in practical use. The greater the reduction in cement consumption in concrete leads to a greater reduction in ECO₂, and further realization of sustainable development in the construction industry.

The statistical error values for the predicted mechanical properties of Fly ash-based concrete are stated as mean absolute percentage error (MAPE), Nash–Sutcliffe efficiency (NSE) coefficient, correlation coefficient (R), and root mean square error (RMSE). Such metrics can be determined according to Equation (4). Furthermore, Table 2 compares the mentioned metrics (including MSE) following all data sets gleaned from the network. A zero value is the optimal option for the metric (except for NSE and R). On the other hand, one is the desired value regarding the R and NSE parameters. The RMSE represents the existing deviation within the experimental and predicted values. MAPE estimates the error and the ratio of the error regarding the experimental values [37,38]. NSE is used to evaluate the predictive capability of the model. The statistical indicators in Table 2 demonstrate that the estimated compressive strength using the ANN were very close to the experimental values. This further confirms the suitability of the proposed ANN model in predicting the mechanical properties of concrete containing Fly ash.

\[
\text{RMSE} = \sqrt{\frac{\sum(\hat{y} - y)^2}{N}} \quad \text{NSE} = 1 - \frac{\sum(\hat{y} - y)^2}{\sum(y - \bar{y})^2} \\
\text{MAPE} = \frac{100}{N} \sum \left| \frac{\hat{y} - y}{y} \right| \quad \text{R} = \frac{\sum(\hat{y} - \bar{y})(y - \bar{y})}{\sqrt{\sum(\hat{y} - \bar{y})^2 \sum(y - \bar{y})^2}}
\]

where \( y \) and \( \hat{y} \) are the target and predicted values; and the \( \bar{y} \) and \( \bar{\hat{y}} \) parameters are the averages of the target and the predicted values, respectively.
Table 2. MSE, root mean square error (RMSE), Nash–Sutcliffe efficiency (NSE), R coefficient for All data in network.

| Target Response                  | MSE   | RMSE  | MAE   | MAPE  | NSE   | R       |
|----------------------------------|-------|-------|-------|-------|-------|---------|
| 28-day compressive strength      | 0.0016| 0.0401| 0.028 | 14.89 | 0.9639| 0.9729  |
| Tensile strength                 | 0.0036| 0.0602| 0.0436| 11.87 | 0.9324| 0.9658  |
| Modulus of elasticity            | 0.0004| 0.0221| 0.0119| 3.78  | 0.9902| 0.9955  |

3.1. Stability Analysis

The ratio of predicted to experimental result, versus the affecting parameters, which indicate the stability in estimating the mechanical properties of concrete made with Fly ash, are presented in Figure 4. The affecting parameters, which are normalized in [0, 1] domain, due to large range of variety, are listed in Table 1. The variation of data about the horizontal line started from the ratio of 1 shows the stability of parameters, and the closer the data are to this line, the more stability in the parameters. As can be seen in Figure 4, all the data indicate a suitable stability and capability of presenting accurate results.
3.2. Sensitivity Analysis

As discussed earlier in Section 2, the weight of each neuron indicates its importance. The Garson’s factor [39] is used to assess the relative importance of the parameters in the network. The equation for a network with a single hidden layer can be written as follows:

\[
Q_{xz} = \frac{\sum_{y=1}^{L} \left( \frac{w_{xy}}{\sum_{r=1}^{N} w_{ry}} v_{yz} \right)}{\sum_{x=1}^{N} \left( \sum_{y=1}^{L} \frac{w_{xy}}{\sum_{r=1}^{N} w_{ry}} v_{yz} \right)}
\]

where \(\sum_{r=1}^{N} w_{ry}\) is the weights summation within the neurons of N input and the y hidden; \(v_{yz}\) stands for weight of each connection of y and the output neuron z [40]. The sensitivity analysis result is plotted in Figure 5.

It was observed that almost all of the parameters have comparable participation in determining the mechanical properties of concrete made with Fly ash. Besides, as no parameters have relatively low effectiveness, it can be concluded that no irrelevant or excess parameters have been incorporated. By excluding the excess parameters in the prediction process, the degradation of learning algorithm is prevented, which in respect results in further accuracy of the prediction [41–43]. The results of the sensitivity analysis indicate that the presence of coarse aggregate and water have a significant effect on enhancing the 28-day compressive strength of a concrete with Fly ash. On the other hand, the SiO\(_2\) content of Fly ash has a negligible effect on 28-day compressive strength. This may be attributed to the time-consuming effects of SCM on improving the compressive strength of concrete. The mechanical characteristics of concrete with Fly ash are considerably influenced by Fly ash’s physical properties and chemical composition. These findings are in accordance with the results of [44–46], which further confirm the accuracy of the network in estimating the
mechanical properties of concrete containing Fly ash. In addition, this further determines the efficiency of the proposed network in estimating the mechanical properties of concrete containing Fly ash, taking into account its chemical characteristics.

Regarding the tensile strength, the effective parameter is the SiO$_2$ content, which is responsible for the pozzolanic reaction. In general, the presence of silica in cement enhances its mechanical characteristics, e.g., increases in the tensile strength by forming Si–O–Si bonds, which are stronger compared to the bonds of Al–O–Al and Si–O–Al [47]. Therefore, it is essential to consider the chemical characteristic of SCM in predicting the mechanical properties of concrete. Furthermore, all parameters affect the modulus of elasticity of concrete containing Fly ash equally. By comparing the results of the sensitivity analysis with experimental evidence and facts, it can be said that the proposed network is able to accurately anticipate the mechanical properties of concrete with less cement than ordinary concrete. The network can be used as a tool for technicians to further utilize this type of concrete due to its benefits in terms of sustainable development and environmental protection.

### 3.3. Validation of ANN Model

Soft computing tools, as powerful computational methods, have been utilized to solve complex problems, especially in the engineering field, and also can be utilized for simulating, assessing, and approximating with high accuracy. The accuracy of the model to produce new results based on new input parameters, i.e., generalization, should be determined according to proven facts or theoretical results. A generalization of an ANN is its ability to handle unseen data. In other words, engineers can use the results of a network to predict the outcomes of an assumed mix design.

Following the objective of this research to estimate the properties of concrete with less ECO$_2$, the dosages of Fly ash, which replaced the cement content, and SiO$_2$ as a chemical characteristic of Fly ash, were considered as variables. Changes in the concrete’s tensile and 28-day compressive strength, along with variations of the modulus of elasticity were determined. This evaluation is necessary, as the mechanical properties of concrete containing Fly ash are influenced by the physical and chemical characteristics of Fly ash [44,46]. The other independent variables in the input layer were chosen to be around their median, in order to gain a various compressive strengths. The assumed mix design is listed in Table 3. The w/(c + F.A.) ratio was presumed to be 0.5, and the Fly ash replaced the cement in the mixture up to 50% of cement weight. It should be mentioned that all of these outcomes have been determined in the absence of experimental evaluation, which is only possible using neural networks. However, the accuracy of the network in predicting the results were evaluated in Sections 3.1 and 3.2.
Table 3. The assumed mixture of concrete (All units in (kg/m$^3$)).

| Mixture | Cement | Water | Coarse Aggregate | Fine Aggregate | Fly Ash |
|---------|--------|-------|-----------------|----------------|---------|
| C30     | 370    | 185   | 1232            | 698            | 18.5–185|

For many years, the utilization of Fly ash as a partial replacement of cement in concrete has been a common practice. However, the quantity of Fly ash to replace the cement for typical application to obtain optimum results has not yet been determined, despite the many advantages of this type of concrete in sustainable development. Figure 6a shows the variations of concrete’s compressive strength after 28 days based on Fly ash and SiO$_2$ contents. The replacement level of Fly ash varies from 5 to 50%. This amount of replacement leads to a reduction in cement use. As is shown, an increase in Fly ash dosage in the C30 mixture decreases the 28-day compressive strength if the SiO$_2$ content is more than 40%. Moreover, the optimum Fly ash replacement level can be considered between 10 to 20%, which is in accordance with the results of previous tests [48–51]. In a concrete with a low Fly ash replacement level, Fly ash has the characteristics of the pozzolanic material, however; by using higher volumes of Fly ash, i.e., a replacement level more than 50%, merely a portion of Fly ash contributes in the pozzolanic reaction, and the remaining portion has no contribution during either short or long curing times, and acts as a packing material [52–54]. American and British standards limit replacing the cement with Fly ash to up 35% in the preparation of concrete for the structural applications [55,56]. The tensile strength and compressive strength show comparable trends for concrete made with Fly ash, see Figure 6b.

Figure 6c shows the variation of modulus of elasticity versus Fly ash replacement level and its SiO$_2$ content. As can be seen, an increase in Fly ash replacement level for low SiO$_2$ content leads to a decrease in the modulus of elasticity, and a contrasting trend can be seen in SiO$_2$ content higher than 60%. An increase in Fly ash replacement level leads to a decrease in modulus of elasticity for lower SiO$_2$ content, and an increase in higher content. This can be attributed to the packing effect of a high replacement level of Fly ash, which cannot participate in the hydration process.

![Figure 6a](image1)
![Figure 6b](image2)
![Figure 6c](image3)

Figure 6. Variation of mechanical properties versus Fly ash dosage and SiO$_2$ content for (a) $F_c$, (b) $F_t$, (c) $E$. 
3.4. Development of Empirical Approach to Determine Concrete Compressive Strength

The suitable accuracy of results, as well as the comparable contribution of the parameters in the network not only enhance the reliability of using ANN simulation, but also can be used to develop an estimating equation to make it easier for technicians to use. Such an equation considers the assigned weights and assumed biases of the ANN model to predict the mechanical properties of eco-friendly concrete, i.e., concrete containing Fly ash. The generation of empirical design charts and/or equations can reduce the limitation of neural networks for use in design. The pattern formula used here for estimating the mechanical properties of concrete with Fly ash was based on the research conducted by Leung et al. [57]. To come up with an empirical formula, the most effective parameter on the output, among the other parameters described in Table 1, needs to be determined. Figure 5 indicates that the cement of the mixture has a more sustainable response than the other parameters. The variations of 28-day compressive strength, tensile strength, and modulus of elasticity versus cement value, while the other affecting parameters are set in their reference values, is plotted in Figure 7. Table 4 lists the curve fitting equation, along with MSE error of the equation to estimate each mechanical property. The same procedure is performed for other inputs, while the cement is kept constant at its reference value. It is assumed that the variation of mechanical properties with each parameter is independent of the other parameters and can be expressed as Equation (6). In Equation (6), \((MechanicalProperties)\_chart\) is equal to the value of each mechanical properties that can be directly read from Figure 7. A correction function has to be derived in order to account for the effect of other input parameters on the mechanical property. Using the curve fitting tools in MATLAB, a line that fits to the presented curve and has the lowest values of least square error is determined.

\[
MechanicalProperties = [C_w \times C_{FA} \times C_G \times C_S \times C_{si}] \times (MechanicalProperties)\_chart
\]  

(6)

The equations are obtained based on each desired mechanical characteristic after the curve fitting process ended. These equations are listed in Table 5. A simple empirical approach to predict the mechanical properties of Fly ash concrete is provided using the summarized equations in Table 5, along with the v chart presented in Figure 7. Additionally, the results of the empirical approach are compared to experimental ones in Figure 8. Table 6 indicates the error distribution as percentage difference within the experimental and predicted outcomes. The MSE and correlation coefficient of the network in estimation of the compressive and tensile strength of concrete, along with its modulus of elasticity, are 0.0016 and 0.9729, 0.0036 and 0.9658, and 0.0004 and 0.9955, respectively. The ANN model approached 83.16%, 83.92%, and 98.18% of samples, respectively, in the error range of ±20% for various mechanical properties. The empirical approach exhibits a reasonable precision. This approach can estimate 34.36% of compressive strength with ±20% range of error, 64.94% with ±40% range of error, and more than 84% of samples with ±60% range of error. The empirical approach can predict 7.14% and 14.54% with the range of errors equal to ±20%, 32.14%, 21.1% with ±40% range of error, and the 64.28% and 80% of samples with ±60% range of error, for the tensile strength and modulus of elasticity, respectively. The empirical approaches proposed in this study are not only practical and convenient to use, but are also developed with good correlation and low MSE error.
Figure 7. Variation of mechanical properties versus cement (kg/m³) while other inputs set in their reference value for (a) Fₛ, (b) Fₜ, (c) E.

Table 4. Equation of curve fitting with its MSE for each output.

| Target Response                  | Curve Fitting Equation | MSE     |
|----------------------------------|------------------------|---------|
| 28-day compressive strength      | 1.76 \times 10^{-6}x^2 + 1.89 \times 10^{-5}x^2 - 0.7362x - 51.58 | 0.0002  |
| Tensile strength                 | -8.49 \times 10^{-4}x^{0.035} + 2.33x^{0.0023} | 0.0055  |
| Modulus of elasticity            | -1.385 \times 10^{-3}x^3 + 1.156 \times 10^{-3}x^2 - 0.2594x + 44.3 | 0.088   |

Table 5. Correction function for each output.

| Correction Function | Compressive Strength | Tensile Strength | Modulus of Elasticity |
|---------------------|----------------------|-----------------|-----------------------|
| Cₜ₀                 | -1.132 \left( \frac{W}{116} \right) + 2.132 | -0.6669 \left( \frac{W}{1127} \right) + 1.667 | -1.416 \left( \frac{W}{1178} \right) + 2.236 |
| Cₚₐₐ              | -0.2379 \left( \frac{G}{115} \right) + 1.238 | 0.1069 \left( \frac{A}{1227} \right) + 0.7876 | -0.1752 \left( \frac{A}{1172} \right) + 1.181 |
| C₉                 | -2.182 \left( \frac{E}{11742} \right) + 3.182 | -1.154 \left( \frac{G}{130} \right) + 2.13 | -2.562 \left( \frac{G}{137} \right) + 3.463 |
| Cₛ                 | -0.5528 \left( \frac{F}{1176} \right) + 1.84 | 0.004462 \left( \frac{G}{130} \right) + 1.244 | -1.215 \left( \frac{G}{137} \right) + 2.215 |
| Cₓᵧ                | 0.1894 \left( \frac{S}{115} \right) + 0.8106 | -0.2364 \left( \frac{S}{130} \right) + 1.313 | 0.5123 \left( \frac{S}{137} \right) + 0.4564 |

Table 6. Precision of proposed methods and distribution of data in the error range.

| Mechanical Properties | Approach   | MSE     | R   | No. of Data in Error Range and Percentage to Total Data |
|-----------------------|------------|---------|-----|-------------------------------------------------------|
|                       |            |         |     | ±20% | ±40% | ±60% |
| Compressive strength  | ANN Analysis | 0.0016  | 0.9729 | 242 (83.16%) | 275 (94.5%) | 283 (97.25%) |
|                       | Empirical  | 0.025   | 0.6358 | 100 (34.36%) | 189 (64.94%) | 245 (84.19%) |
| Tensile strength      | ANN Analysis | 0.0036  | 0.9658 | 47 (83.92%)  | 52 (92.85%)  | 56 (100%)    |
|                       | Empirical  | 0.224   | 0.585 | 4 (71.4%)    | 18 (32.14%)  | 36 (64.28%)  |
| Modulus of elasticity | ANN Analysis | 0.0004  | 0.9955 | 54 (98.18%)  | 55 (100%)    | 55 (100%)    |
|                       | Empirical  | 0.083   | 0.6196 | 8 (14.54%)   | 16 (29.1%)   | 44 (80%)     |
Figure 8. Comparison between experimental and predicted results using artificial neural networks (ANN) and empirical equation for (a) $F_c$, (b) $F_t$, (c) $E$.

4. Conclusions

A considerable amount of experimental data regarding the concrete containing Fly ash was collected. The mechanical characteristics of eco-friendly concrete made with Fly ash, which is a waste material in the power generation industry, were correlated with six input parameters including cement, water, Fly ash, and coarse and fine aggregates, along with the SiO$_2$ content of Fly ash, through the development of artificial neural networks. The network resulted in accurate prediction of concrete’s compressive and tensile strengths, in addition to the modulus of elasticity, in the presence of Fly ash with MSEs of 0.0016, 0.0036, and 0.0004, respectively. The novelty of the paper is to consider the characteristics of Fly ash in predicting the mechanical properties of concrete. With this in mind, any type of Fly ash with any characteristics can be used to produce sustainable and eco-friendly concrete. Almost 83% of the reported results based on the simulation were between ±20% of the experimental compressive strength for the ANN model, which indicated that the proposed ANN was successfully trained to generalize the provided data. The ANN prediction for tensile strength, along with the modulus of elasticity, show more than 83% and 98% within ±20% of the experimental values, respectively. Moreover, the ANN’s simulation results were dispersed around the bisector, which indicates neither over-estimation nor under-estimation. Simple but practical equations were derived based on network results to use the achieved outcomes of the ANN simulation for predicting the mechanical properties of concrete containing Fly ash to address the lack of such a study in this area by ANN. The available experimental data was used to verify the precision of the proposed equation and a suitable agreement was derived.
The mechanical properties of concrete with less ECO₂ can be anticipated using the proposed ANN. The results of the generalization of the ANN indicated that the mechanical properties of concrete with Fly ash are under influenced by the Fly ash replacement level, as well as the SiO₂ content. The results of the proposed network can be implemented to obtain the optimum replacement level of Fly ash considering its characteristics.

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