Regenerative braking kinematic analysis and optimisation strategies

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Abstract. Most of electric and hybrid cars are city oriented due to their reduced pollution and small range. In this paper we intend to analyse kinematics and energy distribution during braking in the scope of city use cases. The purpose is to identify optimization criteria based on system limitations. Our proposal is to describe each case from point of view of kinematics (velocity, acceleration), energy (kinematic energy, instantaneous power). Having relations between data and the results helps us to form optimization strategies: a strategy oriented on driver comfort and two resulted from regenerative braking system component limits. Based on results, maximizing acceleration until its comfort value, in city conditions, does not generate unsupported instantaneous power for generator or energy storage system for an average electric car. Power limitation became more important when keeping same strategy for speed higher than 50 kph. In this case electric vehicles with small battery capacity and hybrid vehicles could show lower efficiency, as more kinetic energy will be dissipated by friction brakes. Presented strategies can be used to implement braking system controlling algorithm for regenerative brakes/friction brakes ratio on-the-fly optimization.

1. Introduction
Increased popularity of electrical and hybrid vehicles is determined by their efficiency, which is one of the most attractive characteristics for customers. Manufacturers are encouraged to develop technologies that will improve vehicle’s efficiency. A perspective one can be regenerative braking, as it can offer up to 25% of energy saving depending driving cycle, control strategies and system limitations. This paper intends to present optimization strategies based on deceleration and instantaneous power restrictions that can be applied on several city use cases. By simulating a braking process, we get velocity, acceleration and power variation versus time, an optimization of regenerative ration or distance can be done based on this data.

2. Regenerative brakes city use cases
During city driving cycles regenerative brakes can be used in the following cases:
   1. Speed bump / Speed humps
   2. Approaching intersection
      a. road turn
      b. traffic lights
      c. stop traffic sign
   3. Traffic jam
Each case is characterized by starting speed, final speed, braking distance and, as a result, stored kinetic energy and power distribution over braking process. In the paper, first two use cases will be analysed: velocity reduction approaching road element for speed bumps, speed humps and braking until full stop for second use case.

Kinetic energy of the moving car depends on object mass and velocity:

$$K_E = \frac{1}{2}mv^2 \quad (1)$$

Necessary kinetic energy to be dissipated to reduce car speed from 50 kph to 20 kph can be calculated by difference between car’s kinetic energy at 50 kph minus kinetic energy at 20 kph:

$$K_{E50} - K_{E20} = \frac{m \cdot (50/3.6)^2}{2} - \frac{m \cdot (20/3.6)^2}{2} \approx 85.71 \cdot m \ [J] \quad (2)$$

For the same speed reduction from 25 kph to 5 kph the result will be almost twice smaller, because of squared dependency of car’s velocity.

$$K_{E25} - K_{E5} = \frac{m \cdot (25/3.6)^2}{2} - \frac{m \cdot (5/3.6)^2}{2} \approx 48.98 \cdot m \ [J] \quad (3)$$

As we are interested in power which enters generator, we also need time or distance of braking:

$$P_{G_{in}} = \frac{K_{E\text{ stored}}}{\Delta t}, \text{where } \Delta t = \text{braking time} \quad (4)$$

The power generator will output will be less the one on input, because of energy conversion losses:

$$P_{G_{out}} = P_{G_{in}} \cdot \eta_G = \frac{K_{E\text{ stored}} \cdot \eta_G}{\Delta t} \quad (5)$$

As example calculation in this paper the following data will be considered: mass = 1580 kg, motor/generator power = 110 kW, motor/generator efficiency = 0.95.

2.1. Speed bump / Speed humps / Pedestrian crossing case

Speed bumps and speed humps are among many types of “traffic calming” measures. Speed bumps are applications, usually asphalt, that are 50 – 150 mm high and 0.30 – 0.92 m from front to back. Speed humps are usually less than 100 mm high but 3 – 3.65 m from front to back [1], as is depicted in figure 1. In the source listed, comfortable speeds for speed bumps are considered less than 10 kph and for speed humps, less than 40 kph. Recommended speed for speed humps and bumps crossing vary based on the country.

Figure 1. Speed control traffic elements: speed bump and speed hump.
In this case starting speed of a car is considered 50 kph, final speed 16 kph. The speed curve assumed linear, thus it is possible to calculate distance, velocity, time and power versus time. It is given a dependency of velocity versus distance (figure 2). For further calculation velocity versus time curve is required.

![Figure 2. Speed bump/hump use case scheme.](image)

Starting from velocity relation, we can integrate it to get time:

\[ v = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v} \]

\[ T = \int_0^x \frac{dx}{v} \]  \hspace{2cm} (6)

The formula above represents area under curve for 1/v versus distance.

In particular case, for linearly decreasing velocity with respect to distance (figure 3), the following simplifications can be applied:

\[ v = k_1 x + v_0, \text{ where } k_1 \text{ - linear coefficient} \]  \hspace{2cm} (8)

\[ t = \int_0^{x_{final}} \frac{dx}{v_0 - k_1 x} = \frac{1}{k_1} \ln \left( \frac{k_1 x + v_0}{v_0} \right) \]  \hspace{2cm} (9)

![Figure 3. Velocity and time versus distance (speed bump/hump use case).](image)

\[ y = -0.1889x + 13.8889 \]
Having time versus distance, velocity versus time (figure 4), also the acceleration versus time can be plotted (figure 5):

\[ v = v_0 e^{tk_1} \]  
\[ a = \frac{dv}{dt} = \frac{d(v_0 e^{tk_1})}{dt} = k_1 v_0 e^{tk_1} \]  

\[ P = \frac{dK}{dt} = \frac{d}{dt} \left( \frac{1}{2} mv^2 \right) = mv \cdot \frac{dv}{dt} = mv \cdot k_1 v_0 e^{tk_1} \]  

**Figure 4.** Velocity versus time (speed bump/hump use case).

**Figure 5.** Acceleration versus time (speed bump/hump use case).

Total kinetic energy to be dissipated in this case:

\[ K = \frac{m(v_{\text{final}}^2 - v_0^2)}{2} = -136787 \text{ J} \]  

Another relevant element for us is instantaneous power: 

\[ P = \frac{dK}{dt} = \frac{d}{dt} \left( \frac{1}{2} mv^2 \right) = mv \cdot \frac{dv}{dt} = mv \cdot k_1 v_0 e^{tk_1} \]
Instantaneous power result with negative sign due to direction of energy, but we are interested only in the magnitude of power, so absolute value will be plotted (figure 6):

![Figure 6. Power versus time (speed bump/hump use case).](image)

Instantaneous power has maximum value of: 56787.35 W.

For cases when approaching pedestrian crossing (figure 7) (without complete stop) or speed hump, the same steps can be applied for other sets of initial and final velocity.

![Figure 7. Pedestrian crossing use case scheme.](image)

2.2. Stop traffic sign / pedestrian crossing ($v_{\text{final}} = 0 \text{ kph}$)

For this case we assume that car brakes from 50 kph to 0 kph within 80 meters. Starting from velocity versus distance curve, the velocity (figure 9), acceleration (figure 10) and power (figure 11) versus time to be calculated, eqs. (10),(11).

![Figure 8. Stop traffic sign (intersection approaching) use case scheme.](image)
**Figure 9.** Velocity versus time (stop traffic sign use case).

**Figure 10.** Acceleration versus time (stop traffic sign use case).

**Figure 11.** Acceleration versus time (stop traffic sign use case).
Regarding the poor performance of regenerative brakes at low speed and the impossibility to bring car into standstill state, only the braking until 10 kph will be considered.

3. Recuperated energy

In the section I, presented use cases were analysed from point of view of all braking forces acting on the car, from equation for longitudinal motion \[ F_r + F_f = ma + F_b + R_r + R_a + R_g \] (15)

where:

- \( F_r, F_f \) – active (driving) forces on rear and front axles

Taking as example a moving car at 50 kph on horizontal road \( (R_g = 0 \text{ N}) \) with constant speed \( (a = 0) \) and mass of 1580 kg, we get:

\[ R_r = 0.01 \cdot m g \approx 155 \text{ N} \] (16)
\[ R_a = \frac{1}{2} \rho C_D A_f v^2 = 0.3 \cdot v^2 \approx 67.35 \text{ N} \] (17)

where:

- \( \rho \) – air density at normal conditions (at sea level and 15°C) (1.225 kg/m³)
- \( C_D \) – drag coefficient (aerodynamic resistance factor) (0.27 ... 0.35 for modern cars)
- \( A_f \) – front drag surface (in example area of 1.9 m² was considered)

Total active force can be estimated from motion wheel torque. Taking as example 110 kW motor, and 0.9 total efficiency coefficient and angular speed of the wheels for the chosen speed (50 kph).

\[ P_{motor} = \frac{M_{wheel} \omega_{wheel}}{\eta} \Rightarrow M_{wheel} = \frac{P_{motor} \cdot \eta}{\omega_{wheel}} = \frac{110 \cdot 10^3 \cdot 0.9}{43.27} = 2287.96 \text{ N} \cdot m \] (18)
\[ F_b \geq \frac{M_{wheel}}{r_{wheel}} - R_r + R_a + R_g + ma \Leftrightarrow F_b \geq 7069.72 \text{ N} \] (19)

From the result above we can see that braking force is far bigger that rolling and aerodynamic resistance at low speed, so from this point in paper they will be ignored. Braking force due to car inertia will be ignored in this case study, to be revised latter in context of more complex dynamic model, as it is not the topic for this paper.

![Figure 12. Series braking strategy, one driven axle [3].](image1)

![Figure 13. Parallel braking strategy, one driven axle [4].](image2)
The next thing to consider is the ratio of regenerative brakes and its distribution between axles. Electrical motor could apply braking torque only on driven axle, the second is equipped only with frictional brakes. There are control strategies proposed for both configurations: parallel or series for one driven axle (figure 12) or for both (figure 13) [3-4]. Comparing both strategies, we can consider that the amount of brake force provided by regenerative brakes can roughly vary from 20 % to 50 %, depending on braking power limitation (desired deceleration), factors discussed further.

4. Optimizing strategies
Three optimization strategies will be applied in this paper:
1. Deceleration limit due to comfortable driving (3 – 3.5 m/s²) [5-6]
2. Instantaneous power limitation due to generator
3. Instantaneous power limitation due to energy storage system

In first case by setting maximum comfortable deceleration we can determine minimum braking distance. For greater braking distance, by applying smaller deceleration, total recuperated energy via generator will not be impacted as recovered kinetic energy depends only on initial and final velocity.

\[
\begin{align*}
&\text{Deceleration (}a(t) = \text{const}) \\
&\text{Initial velocity (}v_0\text{)} \\
&\text{Final velocity (}v_{\text{final}}\text{)}
\end{align*}
\]

\[
\begin{align*}
&v(t) \quad D(t) \quad P(t)
\end{align*}
\]

\[
K (\text{kinetic energy})
\]

In this case it is needed to ensure braking distance greater than required braking distance to avoid collision or applying uncomfortable deceleration.

Algorithm description:
- with constant acceleration, velocity will have linear time dependency: \(v = k_1 t + k_2\)
  \[
  a = \frac{dv}{dt} = \frac{d}{dt}(k_1 t + k_2) = k_1 \quad \text{for} \ t = 0, v = v_0 \Rightarrow k_2 = v_0
  \]
  \[
  (20)
  \]

- determine the time when \(v = v_{\text{final}}\)
  \[
  T = \frac{v_{\text{final}} - k_2}{k_1}
  \]
  \[
  (21)
  \]

- calculating distance, by integrating velocity (mention T vs t, and D)
  \[
  v = \frac{dx}{dt} \Rightarrow dx = v dt \Rightarrow D = \int_0^T v \cdot dt = \frac{T(T k_1 + 2k_2)}{2}
  \]
  \[
  (22)
  \]

- calculating instantaneous power and total kinetic energy using formula, eq. (1)
  \[
  P = m v \frac{dv}{dt} = m v a, a = \text{const} \Rightarrow P(t) = m \cdot v(t) \cdot a_{\text{const}}
  \]
  \[
  (23)
  \]

Example: \(a = 3.5 \frac{m}{s^2}\), \(v_0 = 50 \frac{km}{h} \approx 13.889 \frac{m}{s}\), \(v_{\text{final}} = 10 \frac{km}{h} \approx 2,778 \frac{m}{s}\), \(m = 1580 \text{ kg}\)

We plot \(v(t)\) (figure 14) and \(P(t)\) (figure 15) for this example.

Required time for braking \(T = 3.1746 \text{ s}\) and braking distance \(D = 26.45 \text{ m}\), recovered kinetic energy \(K = 146296 \text{ J}\).
Figure 14. Velocity versus time for braking between 50 kph and 10 kph (strategy – limit deceleration).

Figure 15. Power versus time for braking between 50 kph and 10 kph (strategy – limit deceleration)

Discussion: Power in the graph above is the total braking power. If we will consider optimistic case when regenerative braking is done on both axles and generate 50 % of all braking power, resulting instantaneous power has maximum value: 38.4 kW. For selected speed range instantaneous power has relatively small values and further investigations should be done on generator and energy storage system. A remark needs to be added to this case, in real life it is not comfortable to apply constant acceleration like step signal, a ramp in process is required.

Deceleration limit can have bigger impact on possible recovered kinetic energy by regenerative brakes, as instantaneous power will increase linearly with velocity. In the figure 16 is shown power values for braking between 80 and 50 kph, where it has values between 76.8 kW and 122.88 kW (total braking power). For velocity > 50 kph, higher deceleration rate could be needed to achieve distance requirements, which will increase instantaneous power even more.
The limitation in second case comes from generator, we assume that instantaneous braking power should not exceed generator/motor power.

\[
Power (P(t) = \text{const})
\]

\[
\text{Initial velocity (}v_0\text{)}
\]

\[
\text{Final velocity (}v_{\text{final}}\text{)}
\]

\[
a(t)
\]

\[
v(t)
\]

\[
D(t)
\]

\[
K \text{ (kinetic energy)}
\]

Algorithm description:
- constant power implies constant product between velocity and acceleration:
  \[
P(t) = m \cdot v(t) \cdot a(t) = \text{const} \Rightarrow v(t) \cdot a(t) = \frac{P_{\text{const}}}{m}
\]  
  \[(24)\]
- velocity should decrease over time, at higher speed we have constant power stage, when acceleration higher than comfortable value is not possible to reach due to maximum allowed power. When maximum comfortable acceleration is reached, it is kept constant and power will decrease.
- braking distance and kinetic energy are calculating using relations presented for previous optimization strategy, eq. (22).

Discussion: This optimization strategy can be seen as a complementary to the first one, by adding generator/motor limitation to the model. Analysing the results in the first case, we observe that in city speed range, if no critical deceleration is applied, braking power due to regenerative braking do no exceed 40 kW, which is more relevant for hybrid cars rather than electric only. As algorithm in this case is part of third strategy, more detailed example will be presented in the following section.

The difference of third strategy is that instantaneous power limitation comes from electrical energy storage system, which usually is more strictly than the one specific to motor/generator system. The maximum allowed power depends on charging current, which should be limited due to avoid battery performance and lifetime reducing or its defect.

The most popular type of batteries used in automotive engineering is Li-Ion batteries. One cell can have nominal voltage 3.6 – 3.85 V depending on its type, 4.2 – 4.4 V is the maximum charge voltage and end-of-discharge voltage between 2.8 and 3.0 V. Automotive batteries combine such cells in packs to achieve higher voltage and total required capacity. If we look to the modern city cars, we will...
see that battery capacity vary from 24 kWh (Fiat 500e) to 42 kWh (BMW i3). Battery charging efficiency depends on charging current and cell temperature (in the figure 17 we can see Nissan Leaf Cells characteristics, cell’s capacity of 32.5 Ah).

![Example of discharge profiles (25 °C, BOL)](image)

**Figure 17.** Discharge curves for different charge current (C-rate), for Li-Ion cells (Nissan Leaf Cell).

Main input parameters for our calculation will be:
- battery capacity in kWh
- C-rate in h\(^{-1}\), represents the rate at which the battery is being discharged or charged, calculated by dividing current through the battery by theoretical current draw under which the battery would deliver its nominal rated capacity in one hour (also known as a one-hour discharge).

\[
\begin{align*}
\text{Battery capacity} & [\text{kWh}] \\
\text{State of Charge} & [%] \\
\text{Initial velocity} (v_0) & \\
\text{Final velocity} (v_{\text{final}}) & \\
\text{Max power} (P(t)) & \\
\text{a(t)} & \\
\text{v(t)} & \\
\text{D(t)} & \\
\text{K (kinetic energy)} & \\
\end{align*}
\]

Charging Li-Ion batteries with high currents (C-rate > 1) is not recommended as this stresses the battery which yields in higher temperatures and energy losses, the 0.5C – 1C (0.8C) are the recommended values. Another factor that impacts charge current is State of Charge (SoC). After cells reach their voltage peak (~4.2 V) the charging current starts to decrease until values < 3% of rated current, when charging process is finished. This point represents approximately 85 % SoC, further charging is not possible with rated current (figure 18).

Algorithm description:
- depending on battery nominal capacity and its state of charge, a maximum supported instantaneous power for battery can be found. Also, should be considered ratio of regenerative brake into total brake force:

\[
P_{\text{max@battery}} = \begin{cases} 
\text{battery capacity (C – rate = 1), for SoC} \leq 85\% \\
< \text{battery capacity (C – rate < 1), for SoC} > 85\% 
\end{cases}
\] (25)
- from figure 18 we can see that current at SoC ≈ 100% drops to about 3% of its rated value, so for our model a linear dependency can be applied for SoC between 85 and 100%.

\[
\text{SoC} = 85\% \Rightarrow \text{ChargeCurrent} = 100\% \times \text{RatedCurrent}
\]
\[
\text{SoC} = 100\% \Rightarrow \text{ChargeCurrent} = 3\% \times \text{RatedCurrent}
\]  

(26)

- total time brake time and brake distance is the sum of their terms for each stage. Second stage have velocity linearly decreasing with time, so for distance the relation (eq. 22) can be used. In the first state, velocity do not vary linearly with the time, but entire graph can be approximated with trend line (figure 19).

\[
D_{\text{total}} = \frac{T(Tk_{1 \text{overall}} + 2k_{2 \text{overall}})}{2}
\]  

(27)

where:

- \(D_{\text{total}}\) – total braking distance, sum of both stages
- \(T\) – total braking time, sum of both stage
- \(k_{1 \text{overall}}, k_{2 \text{overall}}\) – coefficient of linear equation of the trend line for both stages

Example: We will consider regenerative brakes an both axles and its ratio in total braking force of 50%. Initial speed of 60 kph and final speed of 10 kph. For a battery with capacity of 32.5 kWh and rated charge current corresponding to C-rate of 1C, SoC ≤ 85%, maximum allowed braking power is 75 kW. Vehicle mass, as previous is 1580 kg. Additionally, deceleration constrain is added, as a result we will have 2 stages:

- constant braking power, linearly decreasing velocity until reaching maximum comfortable deceleration

\[
\text{stage 1: } P(t) = \text{const}, \quad v(t) = k_1 t + k_2, \quad a(t) = \frac{P(t)}{m \cdot v(t)} \quad \text{and} \quad a(t) \leq a_{\text{comfort}}
\]  

(28)
- constant deceleration (maximum comfortable deceleration), decreasing velocity according to deceleration and resulting braking power.

\[ \text{stage 2: } a(t) = \text{const}, \quad v(t) = \int a(t) \, dt = a_{\text{const}} \, t \quad \text{and} \quad v(t) \leq v_{\text{final}}, \]

\[ P(t) = m \cdot v(t) \cdot a(t) \quad (29) \]

Figure 19. Velocity versus time for braking from 60 kph to 10 kph (strategy – battery limitation).

Figure 20. Acceleration versus time for braking from 60 kph to 10 kph (strategy – battery limitation).

Discussion: At higher values of the velocity (in our example it can be observed for \( v(t) > 49 \text{ kph} \)), power limitation is dominant and it determines deceleration. Deceleration reaches its maximum comfortable value at around 48 kph and by continuing reducing velocity, smaller instantaneous power results. In context of city speed range and scenarios a battery with capacity of 40 kWh can be sufficient, and regarding the facts that regenerative brakes shows higher efficiency in
cities and most of electric cars are city oriented the more important is to increase ratio of regenerative braking over mechanical braking. Resulted total braking distance in this example is 38.73 meters and total braking time of 3.97 seconds.

Figure 21. Power versus time for braking from 60 kph to 10 kph (strategy – battery limitation).

5. Conclusion
In this paper we tried to apply optimization strategies for regenerative braking, based on system limitations; passenger’s comfort, motor/generator and energy storage system. As context, city uses case were chosen: velocity reduction (speed bumps, speed humps) and vehicle stop (stop sign, intersection, pedestrian crossing). Braking distance can be optimized by setting maximum deceleration, same time ensure driver’s and passenger’s comfort, it does not impact total amount of kinetic energy recovered. Factors that are important for regeneration brakes efficiency are instantaneous power limitation, based on its value the ration between friction brakes and braking with generator can be optimized. Based on obtained results for city speed ranges (until 50 kph), we can conclude that most of current city cars can handle a ratio of 50%, even up to 70%. A remark must be added, the calculation does not include braking due to aerodynamic, rolling and inertia resistance, so it can be possible to go beyond 70% ratio. If look to the result for higher velocity values, we included 2 examples: until 60 kph and until 80 kph, then power limit does not permit to reach high deceleration rates, thus cannot satisfy braking distance requirements. In these cases, a lower deceleration should be adopted to recover as much energy as possible. A more comprehensive information about environment and braking control strategies can permit more precise situation classification and braking process planning.

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