Experimental one-step deterministic entanglement purification

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Entanglement purification is to distill high-quality entangled states from low-quality entangled states. It is a key step in quantum repeaters, determines the efficiency and communication rates of quantum communication protocols, and is hence of central importance in long-distance communications and quantum networks. In this work, we report the first experimental demonstration of deterministic entanglement purification using polarization and spatial mode hyperentanglement. After purification, the fidelity of polarization entanglement arises from 0.268 ± 0.002 to 0.989 ± 0.001. Assisted with robust spatial mode entanglement, the total purification efficiency can be estimated as 10\textsuperscript{9} times that of the entanglement purification protocols using two copies of entangled states when one uses the spontaneous parametric down-conversion sources. Our work may have the potential to be implemented as a part of full repeater protocols.

I. INTRODUCTION

Entanglement plays an important role in quantum communications and quantum computations. Quantum teleportation \cite{1}, entanglement-based quantum key distribution \cite{2,3}, entanglement-based quantum secure direct communication \cite{5,6,10}, distributed quantum computation \cite{11} all require to share nonlocal maximally entangled states. However, during the transmission, the maximally entangled states will degrade into mixed states. Such polluted entanglements will decrease the communication efficiency and even make the quantum communication become insecure.

Entanglement purification is a powerful tool to distill high-quality entangled states from the polluted low-quality entangled states \cite{12}, and it is a key step in quantum repeaters \cite{13}. Entanglement purification was first proposed by Bennett et al in 1996 \cite{12} and was well developed in both theory and experiment. Existing entanglement purification protocols can be divided into two groups. The first group is the probabilistic entanglement purification \cite{14,15}. In these protocols, two nonlocal parties require to use two identical noisy pairs of entangled states. After performing the controlled-not (CNOT) operations or other similar operations, they measure the second copy to decide whether the purification is successful or not. If the purification is successful, they will obtain a high-fidelity entangled pair with some probability. Otherwise, both pairs of entangled states should be discarded. This kind of entanglement purification protocols can only purify bit-flip errors in a round. If there exists phase-flip errors, the parties can purify the phase-flip errors in a next round by transforming the phase-flip errors to the bit-flip errors with the Hadamard operations. Therefore, to obtain higher-fidelity entanglements, the nonlocal parties need to repeat these processes by sacrificing a large amount of low-fidelity entangled pairs. Such entanglement purification has been realized in linear optical systems \cite{30,31}, atoms \cite{32}, and spins \cite{33}. Meanwhile, probabilistic entanglement purification protocols can also be realized using only one copy of imperfect hyperentangled states \cite{34,35}. By constructing the CNOT gate between different degrees of freedom (DOFs) of single photons, i.e., the polarization-spatial mode and the polarization-time-bin, the imperfect spatial mode (time-bin) entanglement can be used to purify the polarization
entanglement. The probabilistic entanglement purification was also extended to use two non-identical pairs. It was shown that the discarded pairs still have entanglement and can be reused in the next purification round, if the purification is a failure [27].

Another group is the deterministic entanglement purification. It also uses hyperentanglements [36–41]. In some special noisy environments, if entangled states in spatial mode, time-bin, frequency or other DOFs are more robust than that in the polarization DOF [22–41], the robust entanglement can be used to purify the fragile polarization entanglement deterministically. Deterministic entanglement purification was first used to purify the bit-flip error [57]. After performing the purification operation, the bit-flip error in polarization entanglement can be corrected completely with a success probability of 100%. However, after purification, the robust spatial entanglement is consumed and the hyperentanglement becomes a high-fidelity polarization entanglement. In order to purify the phase-flip error in the next round, the parties still need to exploit the probabilistic entanglement purification protocol. In 2010, the deterministic polarization entanglement purification for both bit-flip error and phase-flip error was proposed [39]. They use the spatial entanglement to purify the bit-flip error and use the frequency entanglement to purify phase-flip error completely. However, this protocol requires the polarization-spatial-frequency hyperentanglement and the cross-Kerr nonlinearity to construct the quantum nondemolition measurement, which makes this protocol hard to realize using existing technologies. In the same year, the deterministic entanglement purification protocol using only polarization-spatial hyperentanglement was proposed [39] [40]. In these protocols, both bit-flip error and phase-flip error in polarization DOF can be corrected deterministically using spatial entanglement in feasible linear optics. Deterministic entanglement purification was also used in double-server blind quantum computation in noisy environments [41].

It is known that entanglement purification plays a key role in quantum repeaters, for it determines the efficiency and communication rates of quantum communication protocols, and is hence of central importance in the long-distance communications and quantum networks [25]. The probabilistic entanglement purification requires to perform the protocol for several rounds to obtain a high-fidelity entangled state, by sacrificing a large amount of low-fidelity entangled pairs. Moreover, these protocols require the fidelity of initial low-quality entangled state to be greater than 1/2. In this paper, based on Ref. [39], we report the first experiment of one-step deterministic polarization entanglement purification using hyperentanglement. This paper is organized as follows. In Section II, we briefly introduce the one-step deterministic entanglement purification. In Section III, we explain the experiment setup and results. In Section IV, we make a discussion and conclusion.

![Fig. 1: Protocol of the deterministic entanglement purification using hyperentanglement](image)

FIG. 1: Protocol of the deterministic entanglement purification using hyperentanglement [39]. PBS is the polarizing beam splitter. HWP is the half-wave plate which can convert the horizontally polarized photon to the vertically polarized photon, vice versa. $D_i (i = 1, 2, \cdots, 8)$ is the single photon detector (SD).

II. BASIC PRINCIPLE OF DETERMINISTIC ENTANGLEMENT PURIFICATION

Now we first briefly describe the basic principle of such deterministic entanglement purification [39]. As shown in Fig. 1, the hyperentanglement source emits one pair of hyperentangled state of the form

$$|\phi\rangle = |\phi^+\rangle_P \otimes |\phi^+\rangle_S$$

$$= \frac{1}{2}((|H\rangle|H\rangle + |V\rangle|V\rangle) \otimes (|a_1\rangle|b_2\rangle + |a_2\rangle|b_1\rangle)). \quad (1)$$

After the photon pair transmits through a noisy channel, the polarization part of the hyperentangled state becomes a mixed state

$$\rho_P = F_1|\phi^+\rangle_P \langle \phi^+| + F_2|\phi^-\rangle_P \langle \phi^-|$$

$$+ F_3|\psi^+\rangle_P \langle \psi^+| + F_4|\psi^-\rangle_P \langle \psi^-|. \quad (2)$$

Here

$$|\phi^-\rangle_P = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle - |V\rangle|V\rangle),$$

$$|\psi^\pm\rangle_P = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle \pm |V\rangle|H\rangle). \quad (3)$$

$a_1, a_2, b_1,$ and $b_2$ are the spatial modes as shown in Fig. 1. Suppose that the entanglement in the other DOF, i.e. spatial mode, is robust. Therefore, the whole mixed state can be written as

$$\rho = \rho_P \otimes \rho_S, \quad (4)$$

with $\rho_S = |\phi^+\rangle\langle \phi^+|$. The whole mixed state can be described as follows. It is in the state $|\phi^+\rangle_P \otimes |\phi^+\rangle_S$ with the probability of $F_1$, in the state $|\phi^-\rangle_P \otimes |\phi^+\rangle_S$ with the probability of $F_2$, and so on. As shown in Fig. 1, the purification process will make the state $|\phi^+\rangle_P \otimes |\phi^+\rangle_S$ and $|\phi^-\rangle_P \otimes |\phi^+\rangle_S$ become $|\phi^+\rangle_P$ in the output modes $D_2D_4$ or $D_5D_7$. On the other hand, $|\psi^+\rangle_P \otimes |\phi^+\rangle_S$ and $|\psi^-\rangle_P \otimes |\phi^+\rangle_S$ will become $|\psi^+\rangle_P$ in the output modes $D_2D_7$ or $D_4D_5$. By selecting the output modes $D_2D_1$, $D_5D_7$, $D_2D_7$ or $D_4D_5$, the nonlocal parties can obtain the state $|\phi^+\rangle_P$ or $|\psi^+\rangle_P$. If they obtain $|\psi^+\rangle_P$, by adding a bit-flip operation on one of the photons, they can convert $|\psi^+\rangle_P$ to $|\phi^+\rangle_P$ deterministically.
FIG. 2: Experimental setup. (a) Preparation of hyperentangled states. The pump light beam (775 nm) is separated into two spatial modes by the beam displacer (BD). The two beams are injected into a Sagnac interferometer to pump a type-II (|H⟩_{775nm} → |H⟩_{1550nm} ⊗ |V⟩_{1550nm}) cut periodically poled potassium titanyl phosphate (PPKTP) crystal and generate the two-photon polarization entanglement. It can finally generate the hyperentanglement $|\phi^+⟩_P ⊗ |\phi^+⟩_S$ by tuning the relative phase between the two spatial modes $[34, 45]$. We used 120 mW pumped light to excite 2400 photon pairs per second. The coincidence efficiency of the entangled source is 20%.

In the one-step deterministic purification protocol, we can correct both bit-flip and phase-flip errors in polarization entanglement. As shown in Fig. 2(b), we introduce extra noises in the polarization DOF, which include two types of noise — bit-flip (BF) noise ($\rho_P = |\phi^+⟩_P P(\phi^+) \rightarrow F|\phi^+⟩_P P(\phi^+) + (1 - F)|\psi^+⟩_P P(\psi^+)$) and white noise ($\rho_P = |\phi^+⟩_P P(\phi^+) \rightarrow F|\phi^+⟩_P P(\phi^+) +$...
We use two HWPs (setting at $22.5^\circ$ and $-22.5^\circ$) and a liquid crystal (LC2 set at $0^\circ$) phase plate to add the BF noise in polarization DOF. When the voltage of LC2 is $V_{\sigma_x}$, BF ($\sigma_{x} = |H\rangle\langle V| + |V\rangle\langle H|)$ operation is performed. On the other hand, the polarization stays the same when voltage $V_{0}$ is applied to the LC2. For white noise, we need an extra LC1 (setting at $0^\circ$) to perform the phase-flip operation ($\sigma_x = |H\rangle\langle H| - |V\rangle\langle V|)$ by changing the LCs’ voltages periodically, we can tune the ratio of noise from 0 to 1. In our experiment, we loaded 30%, 50% and 70% noise for each type of noise, respectively (details see Appendix). The experimental fidelity of the states before purification are listed in Table I. The average fidelity of the entanglement in spatial mode DOF is about 0.99.

The process of polarization entanglement purification is shown in Fig. 2(c). We use the polarizing beam splitter (PBS) to postselect the photons with different polarization, then combine the different spatial modes by a BD and two HWPs. Here we should mention that the experiment protocol in Fig. 2 is a little different from Fig. 1 and the two HWPs of 45° are set at the same spatial mode. Such a change will make the photons detected by coincidence outputs $\{D_2, D_1\}, \{D_5, D_7\}, \{D_2, D_7\}$ or $\{D_4, D_5\}$ are in the state $|\phi^+\rangle_P$. All the photons after purification are nearly in the pure state $|\phi^+\rangle_P$, and we do not need extra operations on different outputs to get the same maximally entangled state.

After purification, the fidelities of states in polarization DOF will increase. For example, by loading 70% white noise on the polarization part, the fidelity of the polarization entanglement is $F = 0.268 \pm 0.002$, and the fidelity of the spatial mode entanglement is $F = 0.990 \pm 0.001$ before purification. After purification, the fidelity of polarization entanglement ($|\phi^+\rangle_P$) is $F_{D_2, D_4} = 0.989 \pm 0.001$, $F_{D_2, D_7} = 0.985 \pm 0.001$, $F_{D_5, D_7} = 0.980 \pm 0.001$, and $F_{D_4, D_5} = 0.983 \pm 0.001$, respectively. On the other hand, by loading 30% bit-flip noise, the fidelity of the polarization entanglement is $F = 0.693 \pm 0.001$ and the fidelity of spatial mode entanglement is $0.990 \pm 0.001$. After purification, the fidelity of polarization entanglement ($|\phi^+\rangle_P$) is $F_{D_2, D_4} = 0.989 \pm 0.001$, $F_{D_2, D_7} = 0.988 \pm 0.001$, $F_{D_5, D_7} = 0.984 \pm 0.001$, and $F_{D_4, D_5} = 0.988 \pm 0.001$, respectively. We show all the experiment results after purification in Table I (details see Appendix).

IV. DISCUSSION AND CONCLUSION

We demonstrate the first deterministic entanglement purification using spatial mode entanglement. Compared with all two-copy entanglement purification protocols, this experiment use only one pair of hyperentangled state. It will make the purification more efficient. For example, when one loads 20% white noise, we only need to perform one step purification to obtain nearly perfect maximally entangled states. However, by using the purification protocol in Ref. [12], we should perform the purification process for three times to obtain an entangled state with a fidelity of 0.905. Moreover, if we consider the spontaneous parametric down-conversion (SPDC) source, this deterministic entanglement purification becomes more efficient. By using one pair of hyperentangled state, the success probability of obtaining nearly perfect maximally entangled state is about $P_x \approx 0.001$ [34]. By using two-copy entanglement purification [12], in order to perform one bit-flip error purification and one phase-flip error purification, they should at least choose four copies of low-fidelity mixed states, with the probability of $P_x = 10^{-12}$. Therefore, the whole purification efficiency can be estimated as $10^9$ times that of the entanglement purification protocols using two copies of entangled states with SPDC sources [12].

On the other hand, Refs. [34, 35] also demonstrated the polarization entanglement purification using spatial mode entanglement and time-bin entanglement. In their protocols, the spatial mode entanglement [34] and time-bin entanglement [35] can be used to purify the bit-flip error. After performing the purification successfully, such spatial mode entanglement and time-bin entanglement is consumed. In order to purify the phase-flip error, they should first perform the Hadamard operation to transform the phase-flip error to bit-flip error, and then select two copies of polarization states and perform the purification operation in a next round. In a practical transmission, the environment noise will cause the entanglement encoded in different DOFs result in different effect. In the entanglement purification protocols [34, 35], they require that the fidelity of polarization entanglement, spatial mode entanglement or time-bin entanglement are all larger than $1/2$. If the noise makes the fidelity of polarization entanglement degrade to less than $1/2$, while the spatial mode entanglement or time-bin entanglement is still robust, using the approaches in Refs. [34, 35] may not obtain a higher-fidelity polarization entanglement. In this way, it is better to use the existing deterministic entanglement purification protocol.

In conclusion, we have reported the first experimental demonstration of the deterministic entanglement purification assisting with polarization and spatial mode hyperentanglement. We show the robust entanglement encoded in the other DOFs have the ability to purify the polarization entanglement deterministically. This work is general and can be effectively extended to use the robust entanglement in other DOFs, such as the time-bin, frequency, and so on. It also has the potential to be implemented as a part of full repeater protocols.

We note that a similar work was reported recently [46].

V. APPENDIX

A. Noise loading setup

The noise loading setup is shown in Fig. 2(b). LC1 is used to load PF noise ($\sigma_z$ operation) in polarization

$\frac{1}{\sqrt{2}} (|\phi^+\rangle_P |\psi^+\rangle_P + |\phi^-\rangle_P |\psi^-\rangle_P + |\phi^-\rangle_P |\psi^+\rangle_P + |\phi^+\rangle_P |\psi^-\rangle_P)$.
With the help of these operations, we can transform the state to any other three Bell state. If the four Bell states occur alternately with the same probability, white noise is loaded. The details of the time sequence in our experiment is shown in Table II. We take 10 s as a cycle. Various voltages are applied at different times in each cycle. In our experiment, we load 30%, 50%, and 70% noise, respectively.

For loading white noise, we control the proportion of loading $\sigma_z$ operation and $\sigma_x$ operation. By loading $\sigma_z$ and $\sigma_x$ operations, we can transform the state $|\phi^+\rangle_P$ to arbitrary Bell state as

$$|\phi^+\rangle_P \xrightarrow{\sigma_z} |\phi^-\rangle_P, |\phi^+\rangle_P \xrightarrow{\sigma_x} |\psi^+\rangle_P, |\phi^+\rangle_P \xrightarrow{\sigma_z \sigma_x} |\psi^-\rangle_P.$$  

By adjusting the duty cycle between different voltages, any proportion of BF noise and white noise can be loaded. As shown in Fig. 3 and Table II, we take 10 s as a cycle. Various voltages are applied at different times in each cycle. In our experiment, we load 30%, 50%, and 70% noise, respectively.

We have designed an experimental setup to measure the entanglement in spatial mode DOF using polarizers (Fig. 4). We use BDs, HWPs, a QWP and a PBS to realize the state tomography of spatial mode entanglement. Notice that the first two BDs and HWPs realize the conversion between polarization and spatial qubits. At this time, the spatial mode DOF quantum state is converted to the polarization DOF quantum state, and then we use the standard polarization tomography setup for measurement.

C. Purification results

In Fig. 5 and Fig. 6, we show the density matrices of the polarization state before (left) and after (right) the purification in the case of BF noise and white noise, respectively. It is obvious that the fidelity of the purified quantum state can be improved significantly. The exactly values of the fidelity of $|\phi^+\rangle_P$ under different noise ratios are shown in Table I.

FIG. 4: Measurement of spatial qubit. BD1 and BD2 are used to convert the polarization and spatial qubit. The compensator (Comp) is used to compensate for the optical path difference. Thus the standard polarization tomography setup is used to measure the spatial qubit.

FIG. 3: Voltage timing diagram of two LC phase plates. For bit-flip noise, only the operations $I$ and $\sigma_x$ are needed. The voltage of LC1 is set at $V_0$, and by changing the ratio of $t_1$ and $t_2$, we can load any proportion of bit-flip noise. For white noise, both LCs are needed to realize the $\{ I, \sigma_x, \sigma_y, \sigma_z \}$ operations (corresponding to time $t_1, t_2, t_3$ and $t_4$, respectively). With the help of these operations, we can transform $|\phi^+\rangle_P$ to any other three Bell state. If the four Bell states occur alternately with the same probability, white noise is loaded. The details of the time sequence in our experiment is listed in Table III.

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FIG. 5: The density matrices before and after purification in the case of BF noise.

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FIG. 6: The density matrices before and after purification in the case of white noise.
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