Acceleration effect and the possibility of its observation in neutron-optical experiment

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Abstract

The development of ideas about the optical phenomenon called the accelerating matter effect led to the hypothesis of the existence of a very general acceleration effect. Its formulation is that the result of the particle interaction with any object moving with acceleration should be a change in its energy and frequency. The validity of the acceleration effect hypothesis in quantum mechanics has recently been confirmed by numerically solving a number of problems related to the interaction of a wave packet with potential structures moving with acceleration. If these ideas are true, they can be fully attributed to the case of neutron scattering on the atomic nuclei of accelerating matter. Since the time of neutron interaction with the nucleus is very short, the observation of the acceleration effect during scattering by nuclei requires them to move with a very high acceleration. This goal can be achieved if centripetal acceleration is used.

Keywords: acceleration effect, interaction time, neutron scattering, atomic nuclei, centripetal acceleration

1. Introduction

For a long time, research in the field of moving media optics has been reduced only to the case of uniform motion. The situation began to change at the end of the last century, when theoretical works on the interaction of electromagnetic waves \cite{1}, and later neutrons \cite{2,3} with matter moving with acceleration appeared. The theory predicted that in both cases, the wave passage through a refractive sample resulted in a change in its frequency but the connection between these two phenomena was not realized at that time.

Later, in experiments with ultracold neutrons (UCNs), a change in the energy and velocity of the particle was observed after its passing through a refractive sample oscillating in space and moving with alternating acceleration \cite{4,5}. The results were quite satisfactorily consistent with the predictions in the works \cite{2,3}. At the same time, an idea about the generality of the effect predicted for light \cite{1} and for neutron waves \cite{2,3} appeared, since in both cases we mean the passage of a wave through a refractive sample moving with acceleration. Moreover, in \cite{4} it was suggested that any particles should change their energy after passing through a bordered volume of a medium moving with acceleration, since the concept of a refractive index can be introduced for waves of any nature. Later, this idea was developed in \cite{7} presenting an assumption about a change in the frequency and mass of one of the neutrino state components, when a neutrino passes through a layer of matter moving with acceleration.

An important step in understanding the nature of the effect was made in \cite{5}, where it was shown that the conclusion about the change in the frequency of the wave during the passage of a refractive sample moving with acceleration can be made without resorting to detailed calculations, but only based on the idea of the equivalence principle (EP) validity. At the same time, it was demonstrated that the main factor determining the magnitude of the effect is the difference in the times $\delta t$ of the wave propagation through free space and through the region of space, where there is a refractive sample. Knowing this value, it is possible, in a single way, to obtain expressions for the magnitude of the effect for both massive particles and light, which would coincide with the formulas obtained in \cite{1,5}.

Thus, based only on the principle of equivalence, and neglecting both relativistic and non-stationary quantum effects, expressions for the magnitude of the frequency shift during the passage of a sample moving with acce-
eration were obtained. They coincided with the results of a more rigorous theoretical analysis for both light and the slow particle. In both cases, the following formula was valid

$$\Delta \omega = k a \Delta t$$

(1)

Relatively recently, a change in the energy of cold neutrons passing through an accelerating sample was observed under conditions close to the Bragg reflection condition \[8, 9\]. It is natural to believe that the effect observed in these works has the same nature as the change in frequency during refraction, although in this case the time delay of the wave in the sample is due to a slightly different phenomenon.

In a recent paper \[10\], attention was drawn to the important fact that the conclusion about the change in the frequency of the wave passing through the accelerating sample was obtained in \[5\] only based on the assumption of propagation times different for the wave in the sample and vacuum, but in no way on the assumption that this difference is due to refraction in matter. In particular, attention was drawn to the most significant point that in quantum mechanics any interaction is inevitably associated with a time delay described in the first approximation by the so-called group delay time (GDT) \[11, 12\], which originally called as a phase time.

$$\tau = \frac{\hbar d\varphi}{dE},$$

(2)

where $\varphi$ is the phase of the complex amplitude of the interacting wave, for example, in scattering, and $E$ is the energy. Thus, it should be expected that any elementary scatterer moving with acceleration should change the frequency of the wave, and this means an effect that complements the usual Doppler shift but is proportional not to speed, but to acceleration. At the same time, the frequency change in this case should also be described qualitatively by formula \(1\), where the role of the time delay is played by the interaction time.

The work \[13\] was devoted to the verification of this prediction. It presents the calculation results of the evolution of the spectrum of UCNs interacting with a number of potential structures moving with acceleration. In all cases, this interaction resulted in a change in the velocity spectrum corresponding, in the order of magnitude, to the relation $\Delta \langle v \rangle = a \tau$ where $\langle v \rangle$ is the velocity averaged over the wave packet and $\tau$ is the group delay time \[2\]. Thus, the idea of the validity of the acceleration effect in quantum mechanics has been confirmed. If this is indeed the case, the problem of neutron scattering by an atomic nucleus moving with acceleration becomes undoubtedly relevant. Obviously, assuming the possibility of changing the frequency of the neutron wave function in this elementary act, we will be forced to take a new look at the problem of neutron optics of an accelerating medium. This issue is the subject of this work.

2. Group delay time at neutron scattering by the atomic nucleus

Let us calculate the group delay time \(2\) for neutron scattering by the atomic nucleus. The phase of the scattering amplitude of a neutron wave when scattering at the center is

$$\varphi = \arctan \left( \frac{f''}{f'} \right),$$

(3)

where $f'$ and $f''$ are the real and imaginary parts of the scattering amplitude, respectively. In \[3\] it is taken into account that, as a rule, $f'' \ll f'$.

Since we are talking about an isolated nucleus and slow neutrons, the result of the interaction is only elastic scattering and neutron capture. In this case, in accordance with the optical theorem, the total cross-section is defined as

$$\sigma_t = \sigma_s + \sigma_a = \frac{4\pi}{k} f'',$$

(4)

where $\sigma_t$ is the scattering cross-section,

$$\sigma_s = 4\pi |f'|^2 = 4\pi |f'|^2,$$

(5)

and $\sigma_s$ and $\sigma_a$ are the real and imaginary parts of the scattering amplitude, respectively. Assuming the cross-sections $\sigma_s$ and $\sigma_a$ in both \(4\) and \(5\) are known, we calculate the phase of the scattering amplitude

$$\varphi = \frac{k(\sigma_s + \sigma_a)}{\sqrt{4\pi\sigma_s}}.$$  

(6)

Since the capture cross-section of slow neutrons obeys the $1/\nu^2$ law, it can be determined through the cross-section of thermal neutrons $\sigma_a^{th}$

$$\sigma_a = \frac{\sigma_a^{th} k^{th}}{k},$$

(7)

where $k^{th}$ is the wave number of thermal neutrons. Then \(7\) may be rewritten as a sum, the second term of which is a constant

$$\varphi = \frac{k\sigma_s}{\sqrt{4\pi\sigma_s}} + \frac{k^{th}\sigma_a^{th}}{\sqrt{4\pi\sigma_s}}.$$  

(8)
Having differentiated (5) by energy $E = \frac{2\pi}{\hbar} k^2$, where $k$ is the wave number, for the group delay time (3) for neutron scattering by a fixed nucleon we obtain

$$
\tau = \frac{\sqrt{\sigma_s v}}{v \sqrt{4\pi}}.
$$

(9)

Taking into account (5), and meaning $|f''| = b^2$, where $b$ is the scattering length, we obtain, finally

$$
\tau = \frac{|b|}{v}.
$$

(10)

This result seems quite natural, since the scattering length $b$ is, by definition, the magnitude of the spatial shift of the wave function of the scattered wave with respect to the incident one.

Assuming a typical value for the scattering length $b = 5 \times 10^{-13}$ cm, we obtain that for thermal neutrons with a velocity of 2200 m/s, the value of the GDT at scattering on a single nucleus is $\tau_{th} = 2 \times 10^{-18}$ s. For UCNs having a velocity of 5 m/s, this value is equal to $\tau_{UCN} = 10^{-15}$ s.

3. Acceleration effect at neutron scattering by the atomic nucleus and the possibility of its experimental observation

Having come to the conclusion that the elementary process of neutron scattering on the nucleus is characterized by a finite value of the group delay time, we, in accordance with the hypothesis [10], should also assume that neutron scattering by nuclei of matter moving with acceleration is inelastic. In this case, the change in the frequency of the neutron wave function is determined by formula (1), where the role of the time delay is played by the group delay time $\tau$ (10). The energy and velocity of the neutron after scattering will differ from the initial values by

$$
\Delta E = \hbar k \tau, \quad \Delta v = ar.
$$

(11)

It should be emphasized that from the fact of the inelastic nature of neutron scattering by the nuclei of accelerating matter, it inevitably follows that one way or another it is necessary to modify the theory of neutron waves dispersion in an accelerating medium, since the very existence of the refractive index is due to the interference of incident and elastically scattered secondary waves. The possibility of distinguishing the dispersion law in an accelerating medium from the case of a stationary or uniformly moving medium was discussed earlier in [5],[14].

We will discuss the possibility of experimental verification of the validity of this conclusion. It is obvious that the smallness of the GDT leads to the requirements of a very large acceleration with which the matter moves. Using a sample moving with a large linear acceleration in a real experiment is very problematic, since in this case the sample will leave the experimental setup very quickly. Therefore, it seems natural to turn to the case of centripetal acceleration [15–17].

In such a case, the experiment may consist in finding the neutron beam deviation from the original direction when letting it pass through a rotating disk. Schematically, this is shown in the figure.

It should be noted that in such an experimental scheme, the boundaries of matter are stationary, and the velocity and acceleration of elementary scatterers – atomic nuclei – are mutually perpendicular. The geometry of the experiment corresponds to the geometry of the so-called null Fizeau experiment [18,19], the purpose of which was to find the dependence of the phase of the wave function of the transmitted wave on the velocity, which is possible in the case of deviations of the law of neutron dispersion in the medium from the potential law. The immobility of the boundaries should exclude the influence of the accelerated matter effect consisting in a change in the longitudinal velocity of neutrons.

To make estimates, we choose the disk parameters to be the same as in the experiment [20]. It used a disk made of single crystal silicon, 3 mm thick and 15 cm in diameter, rotating at a frequency of 335 Hz. To be certain, let us assume that the experiment will use UCNs with an energy of 100 neV. The transmission of such a disk for the UCNs is not too small and is about 10%. Let us assume that the beam passes at a distance $R = 6$ cm from the axis. Therefore, the centripetal acceleration experienced by silicon nuclei will be $a = 2.7 \times 10^7$ cm/s². Taking into account that the relative velocity of the neutron and atomic nuclei is approximately 13 m/s, we ob-
tain a value for the GDT $\tau = 3.8 \times 10^{-16}$ s and for the change in the neutron velocity with a single scattering [11] $\Delta v = 10^{-8}$ cm/s.

In order to estimate the total change in the neutron velocity as it passes through the disk, it is necessary to somehow estimate the effective number of scatterings. Strictly speaking, the question of the number of scatterings during wave propagation in the medium is not entirely correct due to the essentially coherent nature of the interaction, but some estimates of the number of scatterings per unit path can probably be made. We will proceed from the well-known pattern of summation of waves scattered in a thin layer of matter proposed by E. Fermi to derive an expression for the refractive index of neutron waves (see [21] as an example). The result of such summation at some point of observation $x$ is the wave function

$$\psi_1 = 2\pi \rho b e^{ik x} \xi = -i \frac{k^2}{2k} e^{ik x},$$

where $\rho$ is number of nuclei per unit of volume, $k_b = \sqrt{4\pi \rho b}$ is the so-called boundary wave number, and $\xi \ll k$ is the layer thickness. As a very rough estimate of the path length at which the primary wave is completely replaced by the scattered ones, we take the thickness of the layer, at which $|\psi|^2$ formally turns out to be equal to one.

$$\xi_m = \frac{2k}{k_b}.$$  \tag{13}

It can be assumed that after passing through the path of matter $\xi_m$, the neutron will experience at least one scattering. Then, to estimate the total amount of scattering when passing through a disk of thickness $d$, we must take the value $N = d/\xi_m$. In the considered case of ultra-cold neutrons and a silicon disk with a thickness of 3 mm $N = 6 \times 10^4$. Consequently, the total increment of velocity during the passage of the rotating disk will be of the order $\Delta V \approx N \Delta v = 6 \times 10^{-4}$ cm/s. Taking into account that the velocity of the UCNs with the above energy is 4.4 m/s, we obtain that, as a result of multiple scattering on the nuclei of accelerating matter, the velocity vector will change by an angle $\alpha = 1.5 \times 10^{-6}$.

At a distance of 1 m from the disk to the detector, the beam displacement when the disk is rotated will be on the order of 1.5 microns. Such a shift is comparable to the sensitivity of an optical installation designed to search for the electric charge of a neutron [22].

It can be expected that under the action of centrifugal force, the disk will experience stretching directed along the radius. In addition, the density will also change with the radius.

Thus, the disk will act as a prism, which will lead to a deviation in the radial direction of the beam passing through it. That means that deformation of the disk under the action of centrifugal force will lead to the appearance of a systematic effect. Calculations show that the angle of refraction of the beam resulting from this effect is an order of magnitude smaller than the above estimate.

4. Conclusion

This work is a development of two previous works. In [10], a hypothesis was put forward about the universality of the acceleration effect. They consisted in changing the wave number of a wave or particle when interacting with an object moving with acceleration, and the magnitude of the effect is determined by the acceleration of the object and the interaction time. The validity of this hypothesis in relation to essentially quantum phenomena was verified in [13]. It presents the results of numerical calculations of the evolution of the UCNs spectrum in interaction with a number of potential structures moving with acceleration. The result of the interaction was a change in the velocity spectrum corresponding in order of magnitude to the relation $\Delta v \approx a \tau$, where $a$ is the acceleration, and $\tau$ is the group delay time. Thus, having received confirmation of the validity of the acceleration effect in quantum mechanics, we moved on to the analysis of an important special case of its manifestation in neutron physics.

It was shown that the GDT during neutron scattering on an atomic nucleus is equal, in the order of magnitude, to the time of neutron flight through a region of space equal to the scattering length. Based on this, we came to the conclusion about the inelastic nature of the interaction during neutron scattering on the nuclei of matter moving with acceleration. At the same time, in the elementary act of scattering, the amount of energy transfer $\Delta E = \hbar k \tau$ should be determined by the acceleration of the medium and the GDT at scattering. The last section of the article discusses the possibility of experimental verification of these predictions.

Acknowledgement

This work was done within the framework of the Joint Institute for Nuclear Research topic plan theme 03-4-1128-2017-2022. Authors are very grateful for the V. G. Baryshevskii, V. A. Bushuev and F. S. Dzeparov for the fruitful discussions.
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