Two-pole structure of the $D_0^*(2400)$

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Abstract. We study the interaction of the $D\pi$, $D\eta$ and $D_sK$ channels in $J^P = 0^+$ in the framework of unitarized chiral perturbation theory in finite volume, predicting the volume dependence of the energy levels. The results successfully describe the energy levels provided in a recent lattice QCD calculation of the same system. We find two poles in the energy region of the $D_0^*(2400)$ resonance, with masses $2105^{+6}_{-8}$, $2451^{+36}_{-26}$ MeV and half-widths $102^{+10}_{-12}$, $134^{+7}_{-8}$ MeV respectively. We study their evolution in the light-flavor $SU(3)$ limit, which allows us to identify the $D_s^0(2317)$ resonance and the pole with lower mass as partners of the same $SU(3)$ multiplet. The mass of the higher pole is obtained between the $D\eta$ and $D_sK$ threshold energies, strongly coupled to the latter channel, thus we expect it to influence the $D_sK$ invariant mass distribution.

1. Introduction
The $D_0^*(2400)$ and $D_{s0}^*(2317)$ resonances are the lowest lying scalar states in the open charm sector. Since their discovery in 2003 [1, 2], they have attracted special attention because they are key pieces in the understanding of the hadron spectroscopy in the charm sector. With an associated $c\bar{u}/d$ and $c\bar{s}$ quark content respectively, their existence was early postulated by Godfrey et al in Ref. [3] as quark model states. Later, they showed up in the experiments with unexpected masses, specially in the case of the $D_{s0}^*(2317)$, whose mass was found around 150 MeV smaller than the theoretical expectations. Concerning the $D_0^*(2400)$ state in which we are interested here, the experimental mass and total width are shown in Figure 1. In that figure we can see how the early measurements provided a mass of $\sim 2.4$ GeV, which is in agreement with, e.g., the value predicted by Godfrey in Ref. [3]. On the other hand, some experiments have found a mass of the order of 2.3 GeV for the neutral $D_0^*$ (2400), which lies close to the $D_{s0}^*$ (2317) mass. This is an interesting point since, according to their constituent quark content, one would expect the $D_{s0}^*$ (2317) to be heavier than the $D_0^*$ (2400), which is the opposite to what the experiments reveal. In the study of these resonances the two-meson thresholds with energies close to the resonance mass play an important role, as it was suggested in Ref. [4] and by lattice QCD (LQCD) studies [5, 6]. In Figure 1 we can find the values of the $D\eta$ and $D_sK$ meson threshold energies, which lie relatively close to the $D_0^*$ (2400) mass. These two channels together
with the $D\pi$ system, when coupled in $S$-wave and isospin $I = 1/2$, will have the same quantum numbers as the $D_0^+(2400)$ resonance. The authors of Ref. [7] performed for the first time a LQCD simulation of the charmed-strangeness sector, $(S, I) = (0, 1/2)$, considering the three two-meson channels already mentioned. We will describe the scattering of the $D\pi$, $D\eta$ and $D_sK$ mesons in coupled channels in a finite volume, in order to compare with the results of Ref. [7]. After that, we will show some of the predictions of the model. The contents of the present manuscript are based on the work of Ref. [8].

### 2. Formalism

#### 2.1. Unitarized $S$-wave scattering amplitudes

We describe the $D\pi$, $D\eta$ and $D_sK$ $S$-wave scattering in coupled channels. Let us label each of the channels by $H_i\phi_j$, ($j = 1, 2, 3$), $H_i$ and $\phi_j$ denote the charmed and light meson inside each channel. The unitarized scattering amplitudes are obtained solving the on-shell Bethe-Salpeter equation in its factorized version,

$$T(s) = \left(V^{-1}(s) - G(s)\right)^{-1}. \quad (1)$$

The unitary matrix $T_{i,j}$ describes the scattering in the coupled channels space $H_i(p_1)\phi_i(p_2) \to H_j(p_3)\phi_j(p_4)$, where $p_1,...,4$ is the four-momentum of the particles and $s$ is the Mandelstam variable. The $V$ matrix is the kernel, its matrix elements being the $S$-wave scattering amplitudes obtained from a next-to-leading order (NLO) chiral Lagrangian, see Ref. [10] for details. The NLO amplitudes depend on six low energy constants (LECs), denoted $h_0,...,s$, that are taken from a previous work [11], where they were fitted to several LQCD scattering lengths in different channels and $(S,I)$ sectors. It is worth mentioning that these sectors did not include the $(S,I) = (0,1/2)$, which is the sector of interest here. $G$ is a diagonal matrix containing the loop functions of the different channels, $G_{jj}(s) = G(s, m_{H_i}, m_{\phi_j})$, see Eq. (16) in Ref. [11]. It is regularized by a substraction constant with its value, together with the already mentioned LECs, constrained in the latter reference. The interaction of the mesons may produce bound states and resonances, as well as virtual states, which will manifest as poles in the complex $s$-plane of $T$ in the different Riemann Sheets (RS). Let us denote the different RS by $(\xi_1, \xi_2, \xi_3)$, where $\xi_j = 0, 1$. The different RS are reached by analytical continuations of the loop function $G_{jj} \to G_{jj} + i\xi_j k_j / 4\pi \sqrt{s}$, where $k_j$ is the modulus of the three-momenta of the mesons in channel $j$, in the center of mass frame. For a pole located at $s = s_p$, the coupling $g_i$ to channel $j$ is given by $g_i g_j = \text{Res} \{T_{i,j}\} |_{s=s_p}$. 

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**Figure 1.** Experimental mass $M$ and total width $\Gamma$ for the charged and neutral $D_0^+(2400)$. The values and uncertainties shown are those provided by different experimental facilities see Ref. [9] and references therein. The vertical dashed lines show the thresholds, which coupled in $S$-wave have the same quantum numbers as $D_0^+(2400)$. 

| $D_0^+(2400)^+$ | $D_0^+(2400)^0$ |
|-----------------|-----------------|
| $\Gamma [\text{MeV}]$ | $\Gamma [\text{MeV}]$ |
| 0 | 0 |
| 50 | 50 |
| 100 | 100 |
| 150 | 150 |
| 200 | 200 |
| 250 | 250 |
| 300 | 300 |
| 350 | 350 |
| 400 | 400 |
| 450 | 450 |
| 500 | 500 |

FOCUS (2004)
LHCb (2015)
BELLE (2004)
BABAR (2009)
2.2. Finite volume

The LQCD simulation of Ref. [7] provides the energy levels for the $D\pi$, $D\eta$ and $D_s\bar{K}$ interaction in a finite cubic box of size $L^3$ with periodic boundary conditions, for several values of $L$. When we restrict the interaction to a finite volume imposing periodic boundary conditions on the fields, we get a quantization condition for the three-momentum, $\vec{q} = 2\pi\vec{n}/L$ with $\vec{n} \in \mathbb{Z}^3$. Therefore, the three-momentum integrals in the meson loops of Eq. (1) will become discrete sums over all the possible three-momentum values: $G(s) \rightarrow \tilde{G}_{jj}(s, L)$. Further details on the finite volume loop function $\tilde{G}_{jj}(s, L)$ can be found in Refs. [8, 12, 13]. Besides, the lattice conditions involve the generation of mesons with unphysical masses and, consequently, we adopt the masses obtained in Ref. [7] in order to compare with their results. The energy levels of our meson interaction will be the poles of a finite volume version of Eq. (1) [13],

$$\tilde{T}(s, L) = \left(V^{-1}(s) - \tilde{G}(s, L)\right)^{-1}. \quad (2)$$

In Eq. (2), up to the order we are working here, the kernel $V$ does not receive any volume correction, and the volume dependence of the interaction is introduced through the loop functions. By solving $\tilde{T}^{-1}(E^2, L) = 0$ for different values of the box length $L$, we get the volume dependence of the energy levels $E(L)$. In the non-interacting limit, $V_{i,j} \rightarrow 0$, the energy levels are nothing but the free energies of the mesons in the finite volume, which can be labelled by their three momentum state $E^\text{free}_j(q, L) = \sqrt{m^2_{H_j} + q^2} + \sqrt{m^2_{\phi_j} + q^2}$.

3. Results

3.1. Energy levels

In Figure 2 we show the results of our model in finite volume. The volume dependence of the energy levels obtained solving Eq. (2) are the red solid lines and bands. We find that our predictions are in agreement with the energy levels provided by the LQCD simulation of Moir et al [7], represented as black points in Figure 2. The results in the region above 2.7 GeV are beyond the range of validity of our model, this region is shown as a shaded area. Finally, the free energies are represented with dashed lines.

![Figure 2. Volume dependence of the energy levels of Ref. [7] (black points) compared with our predictions (red coloured bands). The bands are the 1$\sigma$ uncertainties propagated from the free parameters of the model, that were fitted in Ref. [11]. The dashed lines are the free energies. The length $L$ is shown in units of the lattice spacing used in Ref. [7], $a_s = 0.12$ fm.](image-url)

From the deviation of the energy levels with respect to the free energies we can extract information about the interaction. The first energy level, the one below the $D\pi$ threshold energy, is shifted to lower energies, and in Ref. [7] is interpreted as a $D\pi$ bound state. As we shall see in the next section, when we adopt the lattice masses and search for poles in Eq. (1),
Figure 3. Pole positions in the complex energy plane. As can be seen a two-pole pattern is obtained in the case of using the unphysical masses of Ref. [7] (red points) and physical masses (blue points). The black diamond is the PDG average [9] for the $D_0^*$ (2400). We get a two-pole structure in the $D_0^*$ (2400) sector, where both poles are resonances with a similar width and masses above and below the PDG value.

we also get a bound state (see red circle in Figure 3), it must be compared with the vertical lines representing the thresholds, red-dashed type in the case of lattice and blue-solid type for physical masses. The second level in Figure 2, lying between the $D\pi$ and $D\eta$ thresholds, is also very shifted with respect to them, revealing that the interaction is strong. In fact, another pole appears right below the $D\eta$ threshold, see the red square in Figure 3.

3.2. Poles
As we have commented above, when we use the lattice meson masses of Ref. [7] and search for poles in Eq. (1), we find two poles which are the red points in Figure 3. The pole with lower mass is a bound state, mostly coupled to $D\pi$. On the other hand, when we change to physical masses, due to the heavy pion mass obtained in Ref. [7] the $D\pi$ threshold energy changes dramatically, as can be seen in the vertical lines of Figure 3. The lower pole is no longer a bound state and it is shifted to a lower mass becoming a resonance changing of RS, and being still mostly coupled to $D\pi$. The higher pole also shows a similar pattern of couplings using lattice and physical masses, it does not change of RS but it moves slightly to lower masses staying between the $D\eta$ and $D_s K$ thresholds, and thus, becoming also a resonance. We end up with two resonance candidates in the $D_0^*$ (2400) sector, and we succesfully describe the LQCD information from [7] as a prediction of our model. In the following we will study the evolution of the poles in the light $SU(3)$ flavour limit.

3.3. Light-flavour $SU(3)$ limit
The two-pole structure has been previously reported in Refs. [14, 15, 16] and is partially rooted in the $SU(3)$ structure of the interaction of Ref. [10]. To explore it, we change the masses of the mesons recovering the light-flavor $SU(3)$ limit: $m_{H_j} \rightarrow M$, $m_{\phi_j} \rightarrow m$ [14]. In practice, this is achieved by introducing an $SU(3)$ breaking parameter, denoted $x$, $m_{\phi_j} (x) = m_{\phi_j} + x (m - m_{\phi_j})$ (similar for $m_{H_j}$). In the $SU(3)$ limit the interaction decomposes into irreducible representations (irreps), $\mathbf{3} \otimes \mathbf{8} = \mathbf{15} \oplus \mathbf{6} \oplus \mathbf{3}$, and it can be diagonalized $V \rightarrow V_d = \text{diag}(V_{15}, V_{6}, V_{3})$. This fact, together with the choice of a common substraction constant for all the channels, also makes the $T$-matrix diagonalizable. Thus, for a particular choice of $m$ and $M$, we are able to look for poles in each $SU(3)$ irrep as three independent one channel problems, see Ref. [8] for further details. It turns out that the $V_{15}$ is repulsive, and therefore there are no poles associated to this irrep. The rest of the irreps are attractive, being the $\mathbf{3}$ the most. This representation shows a bound state and the $\mathbf{6}$, whose interaction is weaker, a virtual state. With the model of Ref. [10] one can
describe all the possible \((S,I)\) sectors, not only the \((0,1/2)\) studied here. We have also looked for poles in the \((1,0)\) sector, the corresponding to the \(D_{s0}^*\) (2317) resonance. The model predicts a bound state located at \(2315^{+28}_{-28}\) MeV, not far from the experimental value quoted in Ref. [9]. As it happens in the \((0,1/2)\) sector, the \((1,0)\) sector was not considered to constrain the LECs in Ref. [11] so this bound state is another prediction. Besides, the authors have also explored the pole spectroscopy in all the \((S,I)\) \(J^P = 0^+,1^+\) combinations, in the charm and bottom sector. A similar \((0,1/2)\), \((1,0)\) pole pattern is found, while for \((-1,0)\) we find a virtual state, see Ref. [8]. We have followed the pole positions in the complex energy plane when varying \(x\) from 0 to 1, tracing the trajectories in Figure 4. The \((0,1/2)\) pole with a \(\sim 2.1\) GeV mass, denoted as lower pole, and the \((1,0)\) bound state, identified with \(D_{s0}^*\) (2317), have the same \(SU(3)\) origin in the \(\mathbf{3}\) representation. The apparent puzzle mentioned in the introduction (namely, \(D_s^*\) (2400) heavier that \(D_{s0}^*\) (2317)) is solved in this way, the lower pole of the two \(D_{s0}^*\) (2400) states we find is actually lighter than \(D_{s0}^*\) (2317). On the other hand, the higher pole is originated from the virtual state in the \(\mathbf{6}\) irrep, which is less attractive than the \(\mathbf{3}\). Given its strong coupling to the \(D_s K\) channel, we expect it to influence the invariant mass distribution of that channel in \(B\) decays, something that could be tested in future experiments.

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