What do we learn from the $\rho - \pi$ puzzle

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Abstract

The experimental observation indicates that the branching ratio of $\psi' \to \rho \pi$ is very small while the $\rho - \pi$ channel is a main one in $J/\psi$ decays. To understand the puzzle, various interpretations have been proposed. Meanwhile according to the hadronic helicity selection rule the decay mode $J/\psi \to \rho \pi$ should be suppressed, but definitely, numerical computation is needed to determine how it is suppressed. We calculate the branching ratios corresponding to subprocesses $J/\psi \to ggg \to \rho \pi$ and $J/\psi \to ggg \to \pi \pi$ in the framework of QCD. The results show that the branching ratios are proportional to $\left(\frac{m_u + m_d}{M_{J/\psi}}\right)^2$ for $\rho \pi$ mode and $\left(\frac{m_u - m_d}{M_{J/\psi}}\right)^2$ for $\pi \pi$ mode which is an isospin-violation channel. If only the OZI process is considered, the theoretical prediction on the ratio of $J/\psi \to \rho \pi$ is smaller than data, but not too drastically small. Meanwhile, a possible interpretation for the $\rho \pi$ puzzle is proposed that the suppression is due to interference between OZI and electromagnetic (EM) contributions. Thus based on this observation, we suggest that if the amplitudes of the strong OZI process via an $s$-channel three-gluon intermediate state and electromagnetic one via an $s$-channel virtual photon intermediate state have the same order of magnitude, and constructively interfere for $J/\psi \to \rho \pi$, but destructively interfere for $\psi' \to \rho \pi$, thus simultaneously the $\rho \pi$ puzzle disappears and the sizable width of $J/\psi \to \rho \pi$ is understandable. However, so far, we cannot derive the phase difference from an underlying principle of QCD yet. Alternative interpretations are also discussed in the text.
I. INTRODUCTION

The $\rho - \pi$ puzzle has been standing for years. The puzzle is phrased that $J/\psi \to \rho \pi$ is a main channel in $J/\psi$ decays, while the branching ratio of $\psi' \to \rho \pi$ is very small. It seems to contradict to the general understanding of the charmonia physics.

In the regular theoretical framework, there should be a relation

$$R = \frac{BR(\psi' \to ggg)}{BR(J/\psi \to ggg)} = \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \cdot \frac{\Gamma_t(J/\psi)}{\Gamma_t(\psi')} ,$$

where $\Gamma_t$ is the total width. This estimate on the ratio originates from the fact that if both $J/\psi$ and $\psi'$ are $c - \bar{c}$ bound states, as commonly conjectured, in the hadronic decays, $c$ and $\bar{c}$ annihilate into three gluons which then convert into hadrons, whereas in the leptonic decays, $c$ and $\bar{c}$ annihilate into a virtual photon which turns into a lepton-pair. In this picture, the amplitudes of the hadronic decays which are supposed to occur via a three-gluon intermediate state, and the leptonic decay which occurs via a virtual photon intermediate state are proportional to the wavefunction at origin $\psi(0)$. If everything worked well, the ratio should be close to $12\sim 14\%$, which is called as the 14% rule (now, it is sometimes called as 12% rule, anyhow it is a sizable number.). However the data tell us that this ratio is much smaller than this value[1].

Some theoretical interpretations have been proposed. Rosner et al. [2] suggested that the observed $\psi'$ may be not a pure 2S state which is the first radial excited state of the $c\bar{c}$ system, but a mixture of 2S and 1D states. The amplitudes are instead

$$\langle \rho \pi | \psi' \rangle = \langle \rho \pi | 2^3S_1 \rangle \cos \phi - \langle \rho \pi | 1^3D_1 \rangle \sin \phi \sim 0,$$

$$\langle \rho \pi | \psi'' \rangle = \langle \rho \pi | 2^3S_1 \rangle \sin \phi + \langle \rho \pi | 1^3D_1 \rangle \cos \phi \sim \langle \rho \pi | 2^3S_1 \rangle / \sin \phi,$$

where the mixing angle $\phi$ is fixed as $-27^\circ$ or $12^\circ$ by fitting data. By a destructive interference between the contributions of the two components to the amplitude of $\psi' \to \rho \pi$, the smallness is explained. Suzuki [3] alternatively suggested that the relative phase between the one-photon and gluonic decay amplitudes in $\psi'$ decay may result in the small branching ratio. The final state interactions may also give a reasonable explanation [4]. The first proposal can be tested in the decays of $\psi'' \to \rho \pi$ which has not been well measured yet. In Ref. [3], the author suggested that the one-photon amplitude is sizable and it can be tested in some other modes, for example $\psi' \to \pi \pi$ if the process is dominated by the electromagnetic interaction. The hadronic excess can also receive tests in the decays of other higher excited states of $\psi -$ family and even $\Upsilon -$ family. We will come to this important point at the discussion section below.

Another puzzle is raised for $J/\psi \to \rho \pi$ decay, it is generally believed that $J/\psi$ hadronic decays are dominated by strong interaction, namely the hadronic decay processes occur via an s-channel three-gluon intermediate state which eventually hadronize into hadrons. Those are the OZI-forbidden processes. More seriously, as Brodsky et al. indicated, there is a so-called helicity selection rule[5], which forbids the process of $J/\psi \to \rho \pi$ as long as the masses of light quarks can be neglected. The helicity selection rule, in fact, would greatly suppress the direct decay of $J/\psi \to \rho \pi$, even though the masses of $u$ and $d$ quarks are not set to zero. However, the data show that it is one of the main channels of $J/\psi$ decays and it demands an explanation along with the $\rho \pi$ puzzle for $\psi'$. 
The final state interaction may play an important role as it does for $D$ and $B$ decays, in decays of $\psi$ charmonia as suggested in our earlier work\cite{6}. The final state interaction process is induced by strong interaction at lower energy region, thus it is governed by the non-perturbative QCD which is not fully understood in the present theoretical framework yet. People need to invoke some phenomenological models to carry out the calculations. In our work\cite{4}, in terms of a simple model, we simultaneously consider the FSI and the direct decay of $J/\psi$ into a vector meson plus a pseudoscalar meson and conclude that both of them contribute to the widths and their interference should be destructive to explain data. The difficulties are how to properly evaluate such effects. A detailed discussion on estimation of final state interaction was presented in our previous works, here we only cite our results to discuss the puzzle.

II. ESTIMATE OF THE CONTRIBUTION FROM THE OZI PROCESS

The first step is properly evaluating the contribution from the OZI process. In a straightforward calculation based on the SM, we estimate the decay width of the OZI forbidden process $J/\psi \rightarrow ggg \rightarrow \rho \pi$ \cite{7}, and find that the width is indeed proportional to $(m_q/m_{J/\psi})^2$ which stands as the hadronic helicity suppression factor. Numerically the branching ratio of $J/\psi \rightarrow \rho \pi$ should be smaller than 0.1%. The same situation appears for $\psi' \rightarrow \rho \pi$. To testify the calculation, we recalculate the subprocess $J/\psi \rightarrow ggg \rightarrow \pi\pi$, which is an isospin violating reaction and usually is supposed to be dominated by the subprocess $J/\psi \rightarrow \gamma^* \rightarrow \pi\pi$ because the EM interaction violates isospin as well known. Our result indicates that in the OZI forbidden subprocess the transition amplitude is proportional to $(m_u - m_d)/m_{J/\psi}$, i.e. the mass difference results in the isospin violation instead. All these consequences are consistent with our physics picture and qualitatively reasonable. Therefore we can trust our calculations for the process $J/\psi \rightarrow ggg \rightarrow \rho\pi$. Our numerical results are listed in Table I.

| $m_u$(MeV) | $m_d$(MeV) | $\Gamma(\phi_1)$(MeV) | $\Gamma(\phi_2)$(MeV) | $\Gamma(\phi_3)$(MeV) | exp(MeV) |
|----------|----------|----------------|----------------|----------------|--------|
| 2        | 2        | $1.04 \times 10^{-4}$ | $7.21 \times 10^{-5}$ | $5.11 \times 10^{-4}$ |        |
| 3        | 3        | $2.36 \times 10^{-4}$ | $1.6 \times 10^{-4}$ | $1.17 \times 10^{-3}$ |        |
| 4        | 4        | $4.12 \times 10^{-4}$ | $2.9 \times 10^{-4}$ | $2.08 \times 10^{-3}$ | $(1.06 \pm 0.08) \times 10^{-3}$ |
| 5        | 5        | $6.69 \times 10^{-4}$ | $4.54 \times 10^{-4}$ | $3.38 \times 10^{-3}$ |        |
| 6        | 6        | $9.75 \times 10^{-4}$ | $6.68 \times 10^{-4}$ | $4.88 \times 10^{-3}$ |        |

Our numerical result for $J/\psi \rightarrow ggg \rightarrow \pi\pi$ is of the same order as the data. On other aspect, the EM sub-process is also responsible for the isospin violating decay mode, i.e. the contribution of $J/\psi \rightarrow \gamma^* \rightarrow \pi\pi$ should also be of the order of the data. This observation implies that the contribution of the OZI and EM sub-processes should have the same order of magnitude, even though the details of the theoretical estimate are somehow model-dependent. This observation would support our proposal for explaining the $\rho\pi$ puzzle and dismissing the suppression from the
helicity conservation (see below discussion).

This is not surprising because the relative ratio between the OZI and EM contribution is roughly proportional to

$$\frac{\alpha^3}{\alpha \kappa} \sim 1.1,$$

where $1/\pi$ comes from the loop integration, the EM coupling $\alpha \sim 1/137$, and at the $J/\psi$ energy scale, $\alpha_s$ could be around 0.3 $\kappa$ represents a numerical factor of order O(1) for a quark-pair production from vacuum. Definitely, this ratio depends on the value of the running constant $\alpha_s$. Since in the energy region of charm mass the non-perturbative QCD effects begin to play roles, the value of $\alpha_s$ should have an uncertainty, that is what we refer above as the details in theoretical calculations.

III. DISCUSSION

As indicated in [5], to understand the sizable rate of $J/\psi \to \rho \pi$ which should be suppressed by the hadronic selection rule, the structure of $J/\psi$ may be not a pure $c\bar{c}$ charmonium, but consists of other components, such as hybrids $c\bar{c}g$, $c\bar{c}q\bar{q}$ and etc.

For another aspect, to understand the smallness of the ratio $R$, it is suggested that there is a possible mechanism which would suppress $\psi' \to \rho \pi$, but does not much influence on $J/\psi$, or a completely different alternative picture is that there may be something obscure in $J/\psi$ structure while the smallness of $\psi' \to \rho \pi$ is normal. Our above numerical results show that even though the hadronic selection rule works in the cases of $J/\psi \to \rho \pi$ and $\psi' \to \rho \pi$, the suppression is not too serious and the theoretical prediction is only smaller than the data by less than one order.

A theory should be raised to compromise the both anomalies.

The proposed mixing structures of $\psi'$ and $\psi''$ may still be a possibility for the $\rho \pi$ puzzle, but it needs further experimental test. However, it cannot explain the enhancement of $J/\psi \to \rho \pi$ which should be suppressed by the hadronic helicity conservation and more complicated mechanisms may be needed.

The final state interaction (FSI) might cause a suppression which induces the small R-value, and moreover it also enhances the branching ratio of $J/\psi \to \rho \pi$, but because the theoretical estimate depends on several phenomenological input parameters, it is not very accurate. Thus one cannot be fully convinced that FSI is the final answer yet.

Coming to the anomaly caused by the helicity selection rule, if the $J/\psi$ is not a pure $c-\bar{c}$ bound state, this would be a great challenge to our understanding because the $c\bar{c}$ structure of $J/\psi$ has been recognized almost from very beginning of its discovery. If it is really true, all the previous works based on the potential models where many parameters are fixed by fitting data should be re-considered.

To alleviate the constraint from the helicity conservation, there may be some other mechanisms which were not taken into account, or may exist contributions from new physics beyond the standard model (SM). However, the later seems not very promising because the concerned energy range is rather low and SM works perfectly well to explain the data for all processes and almost no room remains for new physics.
Pretty interesting, there is also an alternative opinion towards the subject, Suzuki [3], Zhao [8] deny the small R-value as a "puzzle", because they consider that the electromagnetic interaction may play an important role in $\psi'$ decays where $c\bar{c}$ annihilate into a virtual photon which later fragment into hadrons. In the picture, it is supposed that a destructive interference between the contribution of three-gluon and single-photon processes would suppress $\psi' \rightarrow \rho\pi$.

Following this idea, we further propose an alternative possibility, namely interferences between the OZI and EM contributions which have the same order of magnitude, result in not only small R, but also the measured rate of $J/\psi \rightarrow \rho\pi$.

As indicated above by our calculation, the process $J/\psi \rightarrow \pi\pi$ is suppressed by isospin violation, two main contributions to the process, i.e. the OZI and EM contributions should have the same order of magnitude. And by our aforementioned simple estimate of order of magnitude, one can be convinced that the OZI and EM contributions to $J/\psi \rightarrow \rho\pi$ and $\psi' \rightarrow \rho\pi$ may also have the same order of magnitude. Thus even though the contribution from the OZI process to $J/\psi \rightarrow ggg \rightarrow \rho\pi$ is smaller than the data, a constructive interference with the EM contribution may double the branching ratio and the final result is close to the data. Meanwhile, a destructive interference greatly diminishes the branching ratio of $\psi' \rightarrow \rho\pi$. Since the two types of contributions are close to each other in magnitude, the destruction may explain data.

There have been some other theoretical explanations besides that we discussed above, in the work by Mo, Yuan and Wang [9], the authors described the recent status of theoretical research as well as the experimental measurements on the interesting subject.

As a brief summary, we can list several interpretations for the rho\pi puzzle and hadronic helicity constraint in the framework of the standard model.

First one is that $\psi'$ is a mixing of 2s and 1d states and meanwhile $J/\psi$ may be a high Fock state, such as a hybrid or contains a sizable component of hybrid.

Secondly, the final state interaction may explain the sizable branching ratio of $J/\psi \rightarrow \rho\pi$ and suppression of $\psi' \rightarrow \rho\pi$.

Third one is that a constructive interference between the OZI and EM contributions enhances the branching ratio of $J/\psi \rightarrow \rho\pi$ and their destructive interference remarkably diminishes the branching ratio of $\psi' \rightarrow \rho\pi$. This interpretation may be more reasonable, but it still suffers from the argument that a fine tuning is required to result in a very small (by orders) branching ratio of $\psi' \rightarrow \rho\pi$ and so far we do not have a convincing calculation to show why it is constructive for $J/\psi \rightarrow \rho\pi$, but destructive for $\psi' \rightarrow \rho\pi$.

Of course, the three mechanisms may all exist and contribute to the double "puzzles" altogether. We need to design new experiments to testify all the possibilities. Thanks to the newly operating BEPC II and BES III, the high luminosity and detection precision provide such opportunities to carry out very accurate measurements by which we may draw more definite conclusion about the above interpretations or suggest new ones.

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