Numerical Calculation of the Process Removal of Localized Convective Structures in a Layer of a Porous Medium

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Abstract. Thermal convection in a flat horizontal layer of a porous medium with solid impermeable boundaries on which the heat flow is given is considered. The porous medium is saturated with a viscous incompressible fluid pumped along the layer. In the system under discussion, with a vertical heat flow inhomogeneous along the layer, localized convective structures may occur in the region where the heat flow exceeds the critical value corresponding to homogeneous heating from below and corresponding to the beginning of convection in the layer. With an increase in the rate of longitudinal pumping of the liquid through the layer, a transition from a state in which the localized convective structures are stable to a state in which the localized convective flow is completely washed out of the region of its excitation occurs. Calculations were performed in the framework of Darcy-Boussinesq. Results of the numerical calculation of the process removal of localized convective structures from the zone of its excitation with an increase in the rate of longitudinal pumping of liquid through the layer are presented. The map of the system state modes is obtained.

1. Introduction

Thermal convection in porous media is of interest both for applied problems related to technical and natural processes, and from the point of view of mathematical physics. In connection with many practical manifestations, special attention is drawn to the development of convective flows from localized heat sources. Thus, for a homogeneous liquid in different versions of the problem statement, a convective torch from a point source in an infinite medium and from a horizontal linear source was taken into account [1]. The stability of the convective flow of a homogeneous liquid caused by heating inhomogeneity was studied in [2]. For the first time, the problem of thermal convection in a horizontal layer of a porous medium bounded by solid impermeable isothermal boundaries was theoretically considered in [3], and experimentally in [4]. The study of a convective torch from a point source of heat in an unlimited array of porous media was first carried out in [5], from a horizontal linear source — in [6]. Free and forced convection in the vicinity of a linear heat source or a heated cylinder was discussed in [7]. Large-scale convection in a liquid-saturated horizontal layer of a porous medium bounded by solid boundaries, with a given heat flow and fluid pumping through the layer was the object of study in [8, 9].

In this paper, we study the process of leaching localized convective structures in a horizontal layer of a porous medium from the region of their excitation.

2. Problem statement

The paper considers a horizontal layer of porous medium saturated with viscous incompressible liquid. The layer is limited to solid surfaces, which is specified inhomogeneous horizontal vertical heat flux. The coordinate system is chosen so that the plane (x,y) is horizontal, and the lower and upper boundaries of the layer correspond to the coordinates z = 0 and z = h. Temperature differences are considered small, so we can assume that the density of the liquid depends on them linearly. Figure 1 shows the scheme of the problem.
Convective filtration in the layer was described in terms of the current function $\psi$ and the temperature $T$ by the equations

$$
\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{Ra}{Pe} \frac{\partial T}{\partial X},
$$

(1)

$$
\frac{\partial T}{\partial t} = \left[ \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right] + Pe \left[ \frac{\partial \psi}{\partial X} \frac{\partial T}{\partial X} - \frac{\partial \psi}{\partial Y} \frac{\partial T}{\partial Y} \right].
$$

(2)

The equations are written in dimensionless form and contain the following dimensionless parameters: the filtration Rayleigh number and the Péclet number

$$
Ra = Kg\beta(T_h - T_c)H/\alpha\nu.
$$

(3)

$$
Pe = U_o H/\alpha.
$$

(4)

where $K$ - the permeability of the medium, $g$ - acceleration due to gravity, $\beta$ - coefficient of thermal expansion, $T_h$ - the temperature of the heater, $T_c$ - the temperature of the upper boundary, $H$ - the layer thickness, $\alpha$ - the effective thermal conductivity of the saturated medium, $\nu$ - the kinematic viscosity of the fluid, $U_o$ - the speed of longitudinal pumping of the liquid through the layer.

The initial conditions for the fields of the current function and the temperature were set in the form:

$$
T(X,Y,0) = 0.
$$

(5)

$$
\psi(X,Y,0) = Y.
$$

(6)

The boundary conditions on the upper and lower boundaries of the layer:

$$
\psi(X,1,t) = 1.
$$

(7)

$$
T(X,1,t) = 0.
$$

(8)

$$
\psi(X,0,t) = 0.
$$

(9)

$$
T(X,0,t) = 1, \text{ for } -L/2 \leq X \leq L/2.
$$

(10)

$$
\frac{\partial T}{\partial Y}(X,0,t) = 0, \text{ for } X < -L/2 \text{ и } X > L/2.
$$

(11)
The problem was solved numerically using the finite difference method. The calculations were carried out in a horizontal layer cell with a length of 10 (sufficient to exclude the influence of lateral boundaries at the considered lengths of the excitation region $L$). On the vertical boundaries periodicity condition were set.

3. Results

Figure 2 shows the dependence of the maximum value of the current function on time, obtained in the calculations, illustrating the temporal evolution of the initial perturbations of the ground state corresponding to the regime of homogeneous pumping, for the Rayleigh number equal to 100, and different values of the Péclet number (for clarity, the maximum values of the function $\tilde{\psi} = \psi - \psi_0$, are presented, where $\psi_0 = 1$ corresponds to the regime of homogeneous pumping). The inserts show the structure of the steady flow for each of these values of the Péclet number. As can be seen from the figure, with a small Péclet number ($Pe=1$), the intensity of the convective flow increases with time and after the transition process reaches a constant value, i.e. a stationary convection regime in the layer is observed. At a higher Péclet number ($Pe=8$) at the initial stage, the perturbations also increase, but after the transition process, a mode of stationary oscillations is established. With an even greater Péclet number ($Pe=15$), the initial perturbations of the ground state fade with time and after the transition process, the maximum value of the current function goes to a value equal to 1, which corresponds to the regime of homogeneous pumping (the value $\tilde{\psi} = \psi - \psi_0$ decreases to zero), i.e. it can be said that the convective structures are completely washed out of the layer. Note that in this situation, for any number of Rayleigh, there is a weak convective flow near the boundaries of the heater caused by the heterogeneity of heating (the presence of a horizontal temperature gradient).

Figure 3 shows a map of modes on the $Pe(L)$ plane obtained by processing numerical data for different values of the Rayleigh number (for the construction of curves, data on the time evolution of initial disturbances similar to those shown in figure 2, in different ranges of parameters). Crosses in the figure marked the points corresponding to the parameters for which the figure 2 shows the dependence of the maximum value of the current function on time. For $Ra=100$ above the curve with points marked by circles, a uniform pumping mode is implemented - the excited convective flow is completely washed out of the region of its excitation by longitudinal pumping. Below the curve with the points shown by the squares, after the transition process a stationary regime is established. At values of parameters in an interval between the specified curves the stationary oscillation mode is observed. For $Ra=80$ and $Ra=60$, the lower curves are absent, at these values, the pumping of the liquid through the layer at any speed leads to the appearance of an oscillating convection mode. Above the curves with the points shown by the circles, the behavior is similar to the case $Ra=100$. 
Figure 2. The dependence of the maximum value of the current function on time at $L=1$, $Ra=100$, $Pe=1$ (dashed), $Pe=8$ (solid), $Pe=15$ (dotted).

Figure 3. Convection mode map at $Ra=60$ (dashed), $Ra=80$ (dotted), $Ra=100$ (solid).

Figure 4 shows the dependence of the wave number on the Péclet number for the oscillatory modes of convection. The inserts show the contours of the current function, corresponding to the extreme points of the range of values of the Péclet number, shown in the figure. As can be seen, the wave number of structures monotonically increases with the increase in the number of Péclet by a law close to linear.

Figure 5 presents the dependence of the oscillation frequency on the Péclet number. As can be seen, with the increase in the Péclet number, a monotonic increase in the frequency of oscillations is observed by a law close to linear.

Figure 6 shows the dependence of the maximum of stream function on the Péclet number. The asymptotic approximation of the curve to the value $\varphi = 1$, corresponding to the regime of homogeneous
pumping, indicates the attenuation of convection with an increase in the pumping rate, associated with the leaching of convective structures from the region of their excitation.

Figure 7 shows the typical structure of the convective flow occurring in the layer. The vortices washed out by the main current from the field of excitation (moving to the right) in the direction of fluid pumping are visible.

Figure 8 shows the temperature field corresponding to the same parameter values.

**Figure 4.** The dependence of the wave number from the Péclet number at $L=4$, $Ra=100$.

**Figure 5.** The dependence of the frequency from the Péclet number at $L=4$, $Ra=100$.

**Figure 6.** The dependence of the maximum value of the current function from the Péclet number at: $L=4$, $Ra=100$.

**Figure 7.** Contours of stream function for a set of parameters: $L=4$, $Ra=100$, $Pe=5$, $t=2.5$. 
4. Conclusion

Convection in a horizontal layer of a porous medium saturated with a liquid in the presence of an inhomogeneous heat flow at the boundaries and liquid pumping along the layer is studied. The map of convection regimes in the parameter plane of the length of the region of excitation - the Péclet number, for several Rayleigh numbers, is obtained. It is found that at low Péclet numbers a stable localized stationary convective flow is observed. At high Péclet numbers a localized oscillating current is implemented. At even higher Péclet numbers localized convective structures are completely washed out of the region of their excitation and there is a regime of homogeneous fluid pumping.

Thus, with the increase in the rate of liquid pumping through the layer (at a given length of the excitation region), the system from the mode of stationary convection passes to the vibrational mode, and then to the mode of homogeneous pumping, similar to what was observed in the scenarios of "leaching" of localized structures arising from random spatial inhomogeneity of the parameters in [10].

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