Three-dimensional inhomogeneity of electron-temperature-gradient turbulence in the edge of tokamak plasmas

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Abstract
Nonlinear multiscale gyrokinetic simulations of a Joint European Torus edge pedestal are used to show that electron-temperature-gradient (ETG) turbulence has a rich three-dimensional structure, varying strongly according to the local magnetic-field configuration. In the plane normal to the magnetic field, the steep pedestal electron temperature gradient gives rise to anisotropic turbulence with a radial (normal) wavelength much shorter than in the binormal direction. In the parallel direction, the location and parallel extent of the turbulence are determined by the variation in the magnetic drifts and finite-Larmor-radius (FLR) effects. The magnetic drift and FLR topographies have a perpendicular-wavelength dependence, which permits turbulence intensity maxima near the flux-surface top and bottom at longer binormal scales, but constrains turbulence to the outboard midplane at shorter electron-gyroradius binormal scales. Our simulations show that long-wavelength ETG turbulence does not transport heat efficiently, and significantly decreases overall ETG transport—in our case by ~40%—through multiscale interactions.

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1. Introduction

In tokamaks, strong magnetic fields and plasma currents generate nested magnetic flux surfaces. On a flux surface, illustrated in figure 1, particles move much faster parallel to the magnetic field than perpendicular to it, causing equilibrium quantities such as temperature and density to be constant within flux surfaces [1]. The radial gradients of equilibrium quantities drive turbulence at scales comparable to ion and electron gyroradii \( \rho_i \) and \( \rho_e \). Such turbulence has a perpendicular eddy length that is very short compared to the perpendicular equilibrium length scale \( \rho_i \) and is radially inhomogeneous from the core to the edge [7–9]. In the tokamak core, turbulence is found to vary slowly along magnetic field lines [10–12]. In these conditions, the turbulence amplitude typically peaks at the outboard midplane—the low magnetic-field side where an interchange-like plasma instability is strongest—and decreases in amplitude smoothly in the parallel direction away from the outboard midplane [10, 13]. Thus, core turbulence typically varies strongly in the plane perpendicular to the magnetic field, but has a predictable profile in the parallel direction.

In contrast, for the turbulence in the edge pedestal of tokamak plasmas—a region of steep pressure gradients in high-performance discharges [14]—we show that both the parallel and perpendicular physics become intriguingly complex, giving turbulence a highly inhomogeneous character. This inhomogeneous turbulence is due to steep pressure gradients and the strong parallel variation in the perpendicular physics of magnetic drifts, generating modes with very different character at different parallel (and, therefore, poloidal) locations. The flux surfaces in the pedestal are highly ‘shaped’ [15–18], unlike the more circular flux surfaces in the plasma core [19, 20]. In figure 1, we show both a highly shaped and circular flux surface. We find that the strong magnetic-field variation in the parallel direction and the steep temperature gradients in the edge pedestal create a non-trivial topography of regions that determines where turbulence can and cannot reside. This topography gives edge pedestal turbulence a novel three-dimensional structure not seen in the core.

Due to the steep gradients in the pedestal, we find that the parallel spatial structure of the turbulence is particularly inhomogeneous and peaked away from the outboard midplane at wavelengths as long as the ion gyroradius, \( k_y \rho_i \sim 1 \), where \( k_y \) is the wavenumber in the binormal direction \( y \) and \( \rho_i \) is the Larmor radius for a species \( s \). Note that the binormal length scale is still determined by electron physics, not intrinsically by \( \rho_i \), but it is quantitatively comparable to \( \rho_i \) at the relevant JET parameters. Thus, the ion–electron scale separation is broken in this system. In contrast, at electron-gyroradius binormal scales, \( k_y \rho_e \sim 1 \), turbulence becomes confined to the tokamak’s low magnetic-field side.

To reveal the importance of the \( k_y \rho_i \sim 1 \) ETG turbulence and its unusual parallel structure, we performed multiscale [21–26] nonlinear gyrokinetic simulations of a JET pedestal using the gyrokinetic code Stella [27]. These multiscale simulations are novel because they resolve the spatiotemporal scales that are needed to observe the complex parallel dynamics and multiscale interactions of \( k_y \rho_i \sim 1 \) ETG turbulence. By means of these numerical experiments, we will show that the electron-temperature-gradient (ETG) turbulence at \( k_y \rho_i \sim 1 \) reduces transport due to ETG turbulence at \( k_y \rho_i \sim 1 \).

ETG turbulence is one of many important transport mechanisms in the edge pedestal. The pedestal, which is a key ingredient in a fusion reactor, appears once external plasma heating crosses a threshold value [28]. This steep-gradient region significantly increases a reactor’s core pressure and hence fusion power [29]. The transport properties of the pedestal are determined by the nature of the turbulence, which is driven by the strong gradients. These turbulent fluxes constrain the pedestal’s magnetohydrodynamic stability [30–34], neoclassical transport [35], and scrape-off-layer processes [36]. Extensive experimental, numerical, and analytic results suggest that ion-temperature-gradient (ITG) [3, 37, 38], ETG [4, 39], microtearing [40], kinetic-ballooning [41, 42], and trapped-electron modes [43] are responsible for anomalous heat losses in the pedestal [26, 44–56]. Pedestal instability and turbulence peaking away from the outboard midplane has been observed for ETG [57–59], ITG [60], microtearing [48, 53, 61], and trapped-electron modes [61].

The rest of this paper is organized as follows. We introduce the gyrokinetic formalism in section 2. In section 3, we
describe the consequences of steep temperature gradients for pedestal ETG physics. Results of linear and nonlinear gyrokinetic simulations are described in sections 4 and 5, respectively. In section 6, we analyze a temperature-gradient scan for nonlinear simulations. Section 7 describes the relation between geometric topography and turbulence. In section 8, we use a numerical experiment to show that \( k_y \rho_e \sim 1 \) ETG turbulence reduces transport at \( k_y \rho_e \sim 1 \). We conclude in section 9.

2. Gyrokinetic turbulence

In the presence of a strong magnetic field, plasma perturbations are anisotropic relative to the mean magnetic field, \( k || / k_\perp \sim \rho_s \ll 1 \), and slow relative to the Larmor frequency, \( \omega_c / \Omega_L \sim \rho_s \). Here \( k || \) and \( k_\perp \) are wavenumbers parallel and perpendicular to the mean magnetic field, \( \rho_s = \rho_i / L_p \) where \( L_p \) is the pedestal width, \( \omega \) is the turbulent frequency, \( \Omega_L = Z_n e B / m_e c \) is the gyrofrequency, \( Z_n \) is the charge number, \( e \) is the proton charge, \( B \) is the magnetic field strength, \( m_e \) is the mass, and \( c \) is the speed of light. Such plasma fluctuations are well-described by gyrokinetics [62-67]. The distribution function of particles of species \( s \) is split into equilibrium and turbulent components, \( f_s = f_{Me} + f_s^b \), where \( f_{Me} \) is a Maxwellian and the turbulent distribution \( f_s^b \) satisfies \( \partial f_s^b / \partial t + v_{s\perp} \cdot \nabla f_s^b = 0 \). We study turbulence governed by the gyrokinetic equation

\[
\frac{\partial h_s}{\partial t} + (v_s \cdot B + v_{s\perp} + (v_{s\perp}^b / T_s) \cdot \nabla h_s)
= \frac{Z_neF_{Me}}{T_s} \frac{\partial (\phi^b / \psi)}{\partial \psi} - (v_{s\perp}^b / T_s) \cdot \nabla F_{Me},
\]

where \( h_s = (Z_ne^b / T_s)F_{Me} + f_s^b / T_s \), \( t \) is time, \( T_s \) is the equilibrium temperature, \( v_s \) is the parallel velocity, \( \mathbf{b} = \mathbf{B} / B \), \( \mathbf{v}_{Me} \) is the magnetic drift velocity, \( v_{s\perp}^b = \mathbf{v} \times \nabla \psi^b / B \) is the \( \mathbf{E} \times \mathbf{B} \) drift velocity, \( \psi^b \) is the turbulent electrostatic potential, \( (\ldots)_\| \) is an average with respect to the variable \( \Lambda \), and \( \psi \) is the gyrophase angle.

Since the turbulence is anisotropic, behaving differently in the directions perpendicular and parallel to the magnetic field, we can solve equation (1) in a numerically efficient field-following domain called a flux tube [11], which has a narrow perpendicular extent centered on a magnetic field line, but extends far along the field line, typically performing 2\( \pi \) poloidal circuit. To describe the directions perpendicular to the magnetic field, we use the flux coordinates

\[
x = \frac{q_e \psi}{r_i B_i}, \quad y = \frac{1}{B_i} \left( \frac{\partial \psi}{\partial r} (\zeta - q \theta - \Omega c t - \nu) \right),
\]

where \( q_e \) is the safety factor, \( r_i \) is a minor-radial flux coordinate, both evaluated at the flux tube’s center, \( \psi \) is the poloidal flux divided by \( 2\pi r_i B_i \), \( \zeta \) is a reference magnetic field, \( \zeta \) is the toroidal angle, \( \theta \) is the poloidal angle, \( \Omega_c \) is the toroidal flow’s angular frequency, and \( \nu(r, \theta) \) is a function 2\( \pi \)-periodic in \( \theta \) [68] that is nonzero when a magnetic field line’s pitch angle at a poloidal location \( \theta \) differs from the mean pitch angle \( 1/q \) on the flux surface; \( |x| \) is large for highly shaped flux surfaces, and \( \zeta \) is the poloidal location that is for the more circular fluxes in the core. The quantities \( q(r) \) and \( \nu(r, \theta) \) are defined so that \( \mathbf{B} \cdot \nabla y = 0 \). The angle \( \theta \) is defined so that \( \theta = 0 \) is the outboard midplane, \( \theta = \pm \pi \) is the inboard midplane (see figure 1), and \( \theta = \pm \pi / 2 \) is approximately the flux surface’s top/bottom. We Fourier transform locally in the perpendicular plane,

\[
\tilde{\phi}(x, y, \theta, t) = \sum_{k_x, k_y} \hat{\phi}_{k_x, k_y}(\theta, t) \exp(ik_x x + ik_y y),
\]

where the normalized potential is \( \tilde{\phi} = e^{i\theta^b / T_i \rho_i} \) and \( \hat{\phi}_{k_x, k_y}(\theta, t) \) are its Fourier coefficients.

We will frequently use the electron magnetic-curvature drift frequency \( \omega_{tec} \) and the grad-B drift frequency \( \omega \).

\[
\omega_{tec} = \frac{v_e^2}{\Omega_e} k_x \cdot \left[ \mathbf{b} \times \left( \nabla \ln B + \frac{4\pi p}{B^2} \frac{\partial}{\partial r} \nabla r \right) \right],
\]

\[
\omega_{DB} = \frac{v_e^2}{\Omega_e} k_x \cdot \left[ \mathbf{b} \times \nabla \ln B \right],
\]

related to \( \mathbf{v}_{Me} \) through \( \mathbf{v}_{Me} \cdot k_x = \omega_{exc} v_e^2 / v_e^2 + \omega_{DB} v_e^2 / 2v_e^2 \),

where \( v_e \) is the perpendicular velocity, \( v_{\perp e} = \sqrt{2T_e / m_e} \) is the thermal speed, and \( p \) is the equilibrium pressure.

The turbulent wavenumber is

\[
k_x = k_y \nabla x + k_y \nabla y = k_y \left[ \hat{s}(\theta_0 - \theta) - \gamma_{\nu} t + \frac{r}{\nu \nu} \nabla \right] \nabla x
+ \frac{\partial \psi}{\partial r} B_i k_y \nabla \left[ \hat{c} + \frac{\partial \nu}{\partial r} B_i k_y \nabla \theta \right],
\]

which is the electron drift frequency associated with the temperature gradient length scale \( L_{Te} \equiv -\langle B \ln T_e / \partial r \rangle^{-1} \). This frequency appears in the linear-drive term on the right-hand side of equation (1) and is particularly large in the pedestal due to steep temperature gradients. Typically, \( \omega_{DB} \) is comparable in size to the frequency of drift waves [3, 69], as has been shown for the pedestal ETG modes [58] considered in this paper. Crucially, the magnetic drift frequencies \( \omega_{tec} \) and \( \omega_{DB} \) are proportional to \( k_x \), but \( \omega_{DB} \) is proportional only to \( k_y \).

In this paper, we perform linear and nonlinear local, electrostatic, collisionless gyrokinetic simulations for JET-ILW discharge #92174 [70] at \( r/a = 0.974 \). This flux surface was chosen due to its large value of the flow shear \( \gamma_{\nu} \nu_{te} / \nu_{te} = 0.56 \), which is an important parameter for turbulence suppression [71-73]. On this surface, we use the following simulation parameters [58]: \( a / L_{Te} = 42, a / L_t = 11, a / L_e = 10, \rho_i / L_{Te} = 0.12, T_e / T_i = 0.56, T_i = 0.71 \) keV, \( \gamma = 3.36 \), \( q = 5.1, v_{\perp e} / v_e = 0.83, \nu_{\perp e} / v_e = 0.0066 \), where the minor radius at the midplane is \( a = 0.91 \) m, the density gradient is \( \nu_{\perp e} \equiv -\langle (B \ln n / \partial r) \rangle^{-1} \) for equilibrium density \( n \), \( \nu_{\perp e} = \sqrt{2\pi n_e Z^2 e^2} \ln(\Lambda_{\perp e}) / \sqrt{m_e T_e^{3/2}} \), and \( \ln(\Lambda_{\perp e}) \) is the
Coulomb logarithm. For the flux surface shape, shown in figure 1, we use a Miller geometry prescription [74]. For linear simulations, we use the parameters described in [58] with ηE = 0.

3. Pedestal ETG characteristics

Steep temperature gradients in the pedestal drive strong ETG instability far away from the outboard midplane, particularly at k⊥ρe ≪ 1. This is in stark contrast to the tokamak core, where the linear ETG growth rate and nonlinear field amplitudes peak at the outboard midplane at k⊥ρo ≈ 1 [39, 59]. For the JET pedestal region investigated in this paper, two branches of ETG dominate: toroidal and slab ETG modes, which are unstable drift waves mediated by electron magnetic drifts and parallel streaming, respectively [3, 4, 37, 39, 75–77].

For a strong toroidal ETG instability to be present, i.e., for the growth rate to be γ ≈ ωE,T, it has been shown that ωE,T/ωke ≈ A must be satisfied, where A ≈ 3–20 [58]. Note that A is a dimensionless constant for the toroidal ETG instability, and is not specific to the discharge analyzed in this paper. We find

\[ \frac{\omega_{E,T}}{\omega_{ke}} \approx \frac{k_y}{k_i} \frac{R}{L_{Te}} \approx A, \]  \hspace{1cm} (7)

where R is the major radius. Since R/ALTe ≫ 1 in the pedestal, for a strong toroidal ETG instability, we must have

\[ \frac{k_y}{k_i} \approx \frac{R}{AL_{Te}} \gg 1. \]  \hspace{1cm} (8)

In the simple case where γE = ν = 0, for a mode with θ0 = 0, as θ increases away from the outboard midplane, k⊥/k∥ becomes large due to magnetic shear, k⊥ ≈ k∥θ[θ]. For \( \hat{s} \sim 1 \), equation (8) implies

\[ |\hat{s}| \sim \frac{R}{sAL_{Te}} \gg 1, \]  \hspace{1cm} (9)

and so we expect linear toroidal ETG modes to be localized away from the outboard midplane. In section 4, we show numerically that in our JET equilibrium, this is indeed the case for most values of θ0.

For a strong ETG instability, we also require the finite-Larmor-radius (FLR) effects not to be too strong: significant FLR damping occurs for k⊥ρe ≳ 1 as an electron’s gyromotion averages over the smaller-scale perpendicular wavelength, decreasing the linear growth rate. Thus, for a strong instability, we must have

\[ k_y \rho_e \lesssim 1. \]  \hspace{1cm} (10)

Combining equations (7) and (10) we find strong toroidal ETG instability for

\[ k_y \rho_e \lesssim \frac{AL_{Te}}{R}. \]  \hspace{1cm} (11)

Since pedestal parameters often satisfy ALTe/R ≲ ρe/ρo, this implies that toroidal ETG modes can be driven linearly at k⊥ρo ≈ 1. We stress that strong toroidal ETG instability at k⊥ρo ≈ 1 is a quantitative coincidence of ALTe/R ≲ ρe/ρo, and is not a fundamental consequence of kinetic ion physics. However, for notational convenience, we will frequently refer to ‘k⊥ρo ∼ 1 ETG modes.’

Linear slab ETG modes dominate in the JET equilibrium used for this paper at most k⊥ρo values for θ0 = 0. One also finds strong sub-dominant slab ETG instability for θ0 ≠ 0 [58]. In section 7, by examining the topographies of k⊥ and ωE,T/ωke for this JET equilibrium, we show that, for k⊥ρe ≪ 1, both linear toroidal and slab ETG modes are expected to be unstable far away from the outboard midplane.

Nonlinearly, we also expect the ETG turbulence injection scale—the outer scale—at long binormal wavelengths. To estimate the wavenumber kδ associated with this outer scale, we observe that the nonlinear decorrelation rate at the outer scale must be the same as the energy injection rate by the instability, ωke, and then we ‘critically balance’ this rate with the parallel streaming rate v∥/l⊥ [12, 78, 79], to find

\[ k_y^2 \rho_e \sim \frac{L_{Te}}{l_i} \ll 1. \]  \hspace{1cm} (12)

In the pedestal, we expect the parallel correlation length l⊥ to be determined by the characteristic parallel length of the local magnetic drifts and perpendicular wavenumber. In contrast, in the core, the length l⊥ is usually assumed to be of the order of the length of one poloidal turn along a magnetic field line, qR [12], giving k⊥2ρe ≈ (1/q)(L_{Te}/R).

In our equilibrium, we find that at the outer scale, l⊥ ≈ qR/2, giving L_{Te}/l⊥ ∼ 1/300 ∼ ρo/ρ. Therefore, nonlinear pedestal ETG simulations that aim to capture the full ETG turbulence cascade require a wide range of binormal modes from k⊥ρe ∼ (L_{Te}/l_i)(ρ_e/ρ) ∼ 1 to k⊥ρe ∼ 1.

4. Linear simulations

We perform linear gyrokinetic simulations using the code GS2 [39, 80] for 0.7 ≤ k⊥ρo ≤ 150. Due to tokamak toroidal symmetry, the linear system is 2π periodic in θ0 [1, 3, 81]. For this reason, in figures 2 and 3 we plot θ0 between −π and π only. In figure 2(a), we plot the linear growth rate, γE/\( \gamma_{\parallel} \) versus k⊥ρi and θ0. In figure 2(b), we indicate the fastest-growing linear mode, which for this equilibrium is associated with either toroidal or slab ETG instability. In figures 3(a)–(d), we plot some properties of the fastest-growing modes in the k⊥ρo ≲ 20 region, where turbulent amplitudes in the nonlinear simulation described in section 5 are largest. In this region, toroidal ETG modes dominate except for k⊥ρo ∼ 5 where θ0 ∼ 0. In figure 3(a), we show that the maximum linear growth rate, γE, peaks at θ0 ≠ 0. In figure 3(b), we plot \( \theta_{max} \parallel \), the poloidal angle at which the linear modes have maximum amplitude |\( \phi_\parallel \)|. Due to steep gradients, toroidal ETG modes with k⊥ρe ∼ 1 peak away from the outboard midplane [58], as predicted in equation (9). Toroidal ETG modes also satisfy k⊥/k∥ ∼ 1 (see equation (8)), and k⊥ρe ∼ 1 (see equation (10)), shown in figure 3(c). In figure 3(d), we indicate the linear mode type at each (k⊥, θ0), showing dominant toroidal ETG instability.

Since the dominant linear instabilities found by us are ETG modes satisfying k⊥ρe ∼ 1, where \( d_e \) is the electron
electromagnetic instabilities being important nonlinearly. The growth rates of the modes that we find are unaffected by [47, 48, 54, 85–87], we cannot rule out linearly subdominant likely unimportant for these ETG modes. However, since simulations Base150 in section 5.

Figure 2. (a) The maximum linear growth rate $\gamma/\nu_i$ and (b) the dominant mode type, versus $k_i\rho_i$ and $\theta_0 = k_i/(k_i^2 + \rho_i^2)^{1/2}$, for the linear GS2 simulation described in section 4. Green dashed curves in (a) denote the edge of the perpendicular ($\theta_0, k_i$) grid for the nonlinear simulation Base150 in section 5.

Figure 3. Linear and nonlinear mode properties for the $k_i$ region where nonlinear potential amplitudes are highest, $k_i\rho_i \lesssim 20$. (a) Linear growth rate $\gamma$, (b) poloidal location $|\theta_{\text{max}}|/\pi$ of maximum amplitude $|\phi|$ for the linear mode, (c) perpendicular wavenumber evaluated at the linear mode’s maximum and divided by the binormal wavenumber, viz, $k_{i,\text{max}}/k_i$, and (d) the fastest growing mode’s type, all versus $k_i\rho_i$ and $\theta_0$ for the linear simulation described in section 4. The dashed curves in (a)–(c) denote the perpendicular grid boundary for nonlinear simulations. (e) The nonlinear amplitude $\log_{10}(|\phi|^2)_{k_i,\theta}$ and (f) $|\theta_{\text{max}}|/\pi$ versus $k_i\rho_i$ and $\theta_0$ for nonlinear simulations in section 5; these are calculated using $\phi$ averaged over $t_{\nu_i}/a \in [14.8, 15.9]$.

5. Nonlinear simulations

Simulating turbulence away from the outboard midplane imposes demanding radial-resolution requirements, necessitating large numbers of radial grid points [59]. Additionally, capturing the fine structure in the linear spectra in figures 2 and 3 requires narrow perpendicular grid spacing in $\theta_0$ and $k_i$. Given that there is strong linear instability for $k_i\rho_c \gg 1$, we also need a large maximum $k_i\rho_i$.

We perform nonlinear simulations that attempt to satisfy these demanding resolution requirements using the gyrokinetic code stellar. We simulate the non-adiabatic response of both ions and electrons by evolving $h_i$ for both species according to equation (1). Simulations have $\Delta k_y\rho_i = 1.38$, $\Delta k_x\rho_i = 0.71$, 150 $k_x$ modes, 67 $k_y$ modes, 128 parallel grid points, 12 $\mu = m_{i/e}v_i^2/(2B)$ grid points, and 48 $v_{i||}$ grid points. These simulation parameters are referred to as the ‘Base150’ simulation. Simulations are performed with an experimentally relevant flow-shear value $\gamma_E a/\nu_i = 0.65$ [58], without which ETG streamers [39] at $k_i\rho_c \sim 1$ appear at long times (we found them at $t_{\nu_i}/a \approx 12$ in a simulation with $\gamma_E = 0$). At smaller $k_i\rho_i$ values where parallel velocity gradient (PVG) instability [93, 94] could be present, we find that it is likely stabilized by $E \times B$ shear [58]. Hyperviscosity in $k_i$ prevents spectral pile-up at $k_i\rho_c \lesssim 1$, and is discussed further around equation (16).

In figure 2(a), the green dashed curves denote the edge of the $k_i\rho_i$ and $\theta_0$ grids for our nonlinear simulations. The variable $\theta_0$ is not periodic in $2\pi$ in nonlinear simulations, but we can ignore $|\theta_0| > \pi$ because we find very low turbulent amplitudes for these higher values of $k_i$. For $k_i\rho_c \gg 1$, the $|\theta_0|$ gridpoints have small values since $\theta_0 \sim 1/k_i$, limiting the resolution of turbulence at $k_i\rho_c \gg 1$ away from $\theta_0 = 0$. This limitation occurs because the radial grid in nonlinear simulations is evenly spaced in $k_i$, but not in $\theta_0$. We checked this limitation in $\theta_0$ for our nonlinear simulations by doubling the number of radial modes in a cheaper simulation (referred to as Radial100 in table 1) using 100 $k_i$ modes (rather than 150 $k_x$ modes in Base150). This doubles the maximum $|\theta_0|$ value included in the simulation at any given $k_i\rho_i$, allowing us to resolve more structure in $\theta_0$. We found that doubling the number of radial modes did not qualitatively change the nature of the turbulence.

In figure 3(e), we plot the turbulence amplitude $\log_{10}(|\phi|^2)_{k_i,\theta}$ averaged over $t_{\nu_i}/a \in [14.8, 15.9]$ from our nonlinear simulations versus $k_i\rho_i$ and $\theta_0$, zoomed in to skin depth, electromagnetic effects [4, 39, 82–84] are likely unimportant for these ETG modes. However, since electromagnetic modes often dominate in other pedestals [47, 48, 54, 85–87], we cannot rule out linearly subdominant electromagnetic instabilities being important nonlinearly.

We have adopted the collisionless limit because the growth rates of the modes that we find are unaffected by
the $k_y\rho_i \lesssim 20$ region where $\log_{10}(|\langle \hat{\phi}|^2 \rangle)$ is the largest. In figure 3(f), we plot the $\theta$ location where $\log_{10}(|\langle \hat{\phi}|^2 \rangle)$ has a maximum for each $(k_y\rho_i, \theta_0)$ value. For lower $k_y\rho_i$ modes, the mode amplitudes peak far away from $\theta = 0$.

In our simulation, the fastest-growing modes at $k_y\rho_i \approx 1$ and $k_y\rho_i \approx 90$ have linear growth rates $\gamma/\nu_i \approx 1$ and $\gamma/\nu_i \approx 70$, respectively. To resolve these modes and their nonlinear interactions, the simulation must satisfy $t \gg 1/\gamma_{\text{slowest}}$, where $\gamma_{\text{slowest}}/\nu_i \approx 3.5$ is the slowest linear growth rate over all dominant modes with $k_y\rho_i \lesssim 20$ in our simulation domain. This is demonstrated by the simulation time traces in figure 4. For $0 < \nu_i/3 \lesssim 2$, the heat flux is dominated by faster high-$k_y\rho_i$ slab ETG modes similar to ‘conventional’ ETG, with the heat flux peaked at $k_y\rho_i \approx 1$ at $\theta = 0$, shown by the ‘early’ curve of the heat flux in figure 5(a) and the heat flux contours in figure 5(b).

Figures 5(a) and (b) show local contributions to $\nu_i$ at the electron heat flux, $\nu_{\text{ik}}$, and the turbulent electron heat flux through the flux surface is then

$$\nu_{\text{ik}}(t) = \sum_{k_y, k_x} \nu_{\text{ik}, k_x}(k_y, \theta, t).$$

Here, $\nu_{\text{ik}}$ is normalized to ion gyroBohm units, $Q_{\text{GB}} = (\rho_i/\nu_i)^2 \nu_i$, where $\nu_i$ is the equilibrium ion pressure. At the early times $0 < \nu_i/3 \lesssim 2$, while the heat flux appears steady and one might erroneously believe that saturation has been reached, the total electrostatic potential

$$\Phi_e(t) = \sum_{k_y, k_x} \nu_{\text{ik}, k_x}(k_y, \theta, t).$$

is, in fact, still increasing (see figure 4). We plot the parallel structure of $\Phi$ and $\nu_{\text{ik}}$ in figures 6(b) and (d), showing their maximum amplitudes near the outboard midplane at early times.

At later times ($\nu_i/3 \gtrsim 2$), the slower-growing ETG modes increase in amplitude, causing turbulence to peak away from the outboard midplane. Figure 5(a) shows non-negligible heat transport at lower $k_y\rho_i$ modes near the flux surface’s top/bottom at ‘intermediate’ and ‘late’ times. ‘Intermediate’ times are averaged over $\nu_i/3 \in [7.5, 8.8]$ and ‘late’ times are averaged over the saturated state for $\nu_i/3 \in [14.8, 15.9]$.

![Figure 4. Time traces of the potential $\Phi^2$ and heat flux $\nu_{\text{ik}}$, defined in equations (13) and (14), respectively, for Base150 nonlinear simulation (see row one of table 1).](https://example.com/figure4.png)
do better at resolving this high-\(|\theta_0|\) turbulence near the inboard midplane and determine its importance definitively.

In figure 7, we compare snapshots of \(\hat{x}\) and its correlation functions at the early (left column) and late (right column) times. In panels (a) and (b), we plot \(\hat{x}\) versus \(\theta\) and \(x\) at fixed \(y\). At late times, radially narrow eddies that are extended in \(\theta\) emerge. These are responsible for reducing overall heat transport in the outboard midplane (see figure 6(d)). In the \((y, \theta)\) cross-sections shown in panels 7(c) and (d), the fluctuations that emerge at later times are seen to have \(k_y \rho_i \sim 1\). In panels 7(e)--(h), we plot the (time averaged) correlation functions

\[
\langle C_\phi(x, y)\rangle_t = \left< \frac{\sum |\phi_{k_x, k_y}|^2 e^{ik_x x + ik_y y}}{\sum |\phi_{k_x, k_y}|^2} \right>_t,
\]

at \(\theta = 0\) and \(\theta = 1.57\) at early and later times. At \(\theta = 1.57\), the correlation length in \(y\) increases significantly with time, indicating the importance of the slower growing, low-\(k_y \rho_i\) modes away from \(\theta = 0\), as anticipated in section 3.

To test for convergence, we performed scans in the number of parallel and perpendicular velocity and spatial grid points, and in hyperviscosity. We are unable to resolve turbulence at \(k_y \rho_i \geq 1\) fully, due to computational resource constraints, which currently prevent us from substantially increasing the maximum value of \(k_y \rho_i\) in the simulation at fixed \(\Delta k_x \rho_i < 1\). However, we believe that our results are close to reality thanks to our use of hyperviscosity.

We used dimensionless hyperviscous coefficients \(D_{\theta\theta} = 10^{-6}\) and \(D_{xy} = 3.5 \times 10^{-7}\), with the hyperviscous damping rate \(\gamma_h\) given by [59]

\[
\gamma_h \frac{a}{v_{\theta\theta}} = -D_{xy}(k_x \rho_i)^4 - D_{\theta\theta}(k_y \rho_i)^4.
\]

This was the weakest hyperviscosity possible that still admitted a well-converged simulation at high \(k_y \rho_i\). To determine whether this value of hyperviscous damping is physically acceptable, we performed nonlinear simulations with a 60% smaller perpendicular box size (\(\Delta k_x \rho_i = 1.75\)) at a fixed number of binormal wavenumbers (and so a much larger maximum \(k_y \rho_i\) value), and used much smaller hyperviscous coefficients \(D_{xy} = D_{\theta\theta} = 10^{-3}\). We found that the heat-flux peak for \(\theta = 0\) in figure 5(a) remained at \(k_y \rho_i \approx 50\). We do not show these simulations with a larger maximum \(k_y \rho_i\) and smaller hyperviscosity because they fail to capture low-\(k_y \rho_i\) physics that regulates the heat flux at higher \(k_y \rho_i\) values, discussed in section 8.

We refer the reader to references [52, 96, 97] for thorough investigations of \(k_y \rho_i \gtrsim 1\) pedestal ETG turbulence.

To determine whether the low-\(k_y \rho_i\) heat flux is wholly due to toroidal ETG turbulence, we performed simulations with no magnetic drifts, \(v_{Me} = 0\). We found large heat flux and turbulent amplitudes driven by slab ETG modes at \(k_y \rho_i \sim 1\) away from the outboard midplane and at \(k_y \rho_i \sim 1\) at the outboard midplane. This shows that at \(k_y \rho_i \sim 1\), both toroidal and slab ETG modes are driven strongly away from the outboard midplane. Even so, the toroidal ETG modes are important because, when \(v_{Me} \neq 0\), we observe the poloidally extended radial structures, shown in figure 7(b), which are absent when \(v_{Me} = 0\).
where $T_{\perp}$ and $a_{\perp}$ are the Fourier coefficients of $T$ and $a$, and $\tilde{T}$ and $\tilde{a}$ are the mean values of $T$ and $a$ over the period. Figure 9 shows that the temperature fluctuations are significantly reduced when $a_{\perp}$ is set to zero. The parameter $\tilde{a}$ is the temperature fluctuation amplitude at $\tilde{T}$.

We now show density and temperature fluctuations for ETG turbulence. The heat transport is complicated by the presence of density fluctuations. The density fluctuations are due to the nonlinear coupling between density and temperature fluctuations. The density fluctuations are also coupled to the fluid flow. The density fluctuations are not shown in figure 10. The density fluctuations are shown in figure 10 with $\tilde{\rho}$ fixed and $\tilde{a}$ fixed. We plot $\tilde{\rho}$ versus $\tilde{a}$ for the ETG turbulence. We plot $\tilde{\rho}$ versus $\tilde{a}$ for the ETG turbulence.
Figure 7. (a)–(d) Electrostatic potential \( \tilde{\phi} \) at early \((t_{vt}/a = 1.1, \) left column) and late \((t_{vt}/a = 15.9, \) right column) times. \( \tilde{\phi} \) is plotted in green to indicate a correlation length. Note that the plot ranges in (e)–(h) are much larger than in (a)–(d). In (e), a yellow box shows the plot ranges of (g).

Figure 8. Root-mean-square perturbations of temperature \( \langle T_e \rangle \), density \( \langle N_e \rangle \), and potential \( \langle \Phi \rangle \), averaged over \( t_{vt}/a \in [16.7–17.8] \) for Base150 and normalized so that the maximum value for each is one.

Figure 9. (a) and (b) Temperature fluctuations, (c) and (d) potential plus density fluctuations, showing a near cancellation at many locations, particularly where \( |\tilde{n}_e| \) is largest. Definitions for \( \tilde{T}_e \) and \( \tilde{n}_e \) are given in equations (17) and (18).

With \( L_\alpha \) fixed, the shape of \( \tilde{q}_e \) is similar for \( a/L_{Te} = 42 \) and \( a/L_{Te} = 34 \), but has the maximum value of \( \tilde{q}_e \) away from the outboard midplane for \( a/L_{Te} = 21 \). With \( \eta_\theta \) fixed, the intermediate temperature gradient \( a/L_{Te} = 34 \) has the highest relative off-midplane transport. The reasons behind this dependence of the heat flux’s poloidal profile on the temperature gradient are beyond the scope of this work, but the different sensitivity of the growth rates and stability boundaries of toroidal and slab ETG modes to \( \eta_\theta \) may be playing a role \[58, 101\].

We also performed two simulations with the circular flux-surface geometry (by setting Miller shaping parameters to ‘circular’ values), one with the experimental gradient for the pedestal \( a/L_{Te} = 42 \) and a second with core-like gradients \( a/L_{Te} = 4.2, \eta_\theta \) fixed, and \( a/L_{ni} \) decreased by a factor of ten to \( a/L_{ni} = 1.1 \). These two simulations are denoted by Scan200f and Scan100e in table 1, respectively. The simulation with \( a/L_{Te} = 4.2 \) had \( \Delta k_y \rho_i = 1.76 \). We chose a relatively small \( \Delta k_y \rho_i \) for this simulation because we wished to determine whether significant \( k_y \rho_i \sim 1 \) ETG turbulence would appear, which it did not. The circular-geometry simulation with \( a/L_{Te} = 42 \) had \( \Delta k_y \rho_i = 0.35 \)—we required this very small \( \Delta k_y \rho_i \) to resolve significant \( k_y \rho_i \sim 1 \) ETG turbulence. To make the \( a/L_{Te} = 42 \) simulation affordable, we used 64 parallel gridpoints.

For the \( a/L_{Te} = 4.2 \) simulation, the heat flux’s profile versus \( \theta \), shown in figure 10(d), resembles that observed in core ETG/ITG turbulence simulations, peaked at the outboard midplane and decaying smoothly in the parallel direction \[10, 11\]. In contrast, for the \( a/L_{Te} = 42 \) simulation, the heat flux had substantial off-midplane contributions, shown in figure 10(d), due to ETG modes away from the outboard midplane. This demonstrates that even in circular flux-surface geometry, steep gradients can produce turbulence with a novel parallel structure.
7. Topography of turbulence

In this section, we show how FLR effects and magnetic-drift profiles determine the parallel distribution of turbulence. These influences act at different scales: while the magnetic-drift profiles $\omega T_e^*/\omega_{ce}$ are independent of $k_\perp$ (at fixed $\theta_0$), the strength of electron FLR damping, measured by the reduction in the linear instability’s growth rate and the resulting turbulence amplitude, is almost always greater at higher $k_\perp \rho_e$ values [58]. The $\omega T_e^*/\omega_{ce}$ topography is mostly relevant for toroidal

Figure 11. Effect of FLR ($k_\perp \rho_e$) topography (a) on the spatial distribution of the turbulent amplitudes $|\hat{\phi}|$ in (b)–(d). Contours of $k_\perp \rho_e = 1$ in (a) are strongly correlated with (c) and (d), indicating the importance of FLR effects for the distribution of slab ETG turbulence at higher $k_\perp \rho_e$ values. The amplitudes are averaged over $n_i/a \in [14.8–16.8]$. The ETG turbulence is driven mainly by toroidal instability in (b), but is driven mainly by slab instability in (c) and (d).
ETG modes, whereas the $k_{\perp} \rho_i$ topography is important for both toroidal and slab ETG modes.

In order for ETG turbulence and transport to be strong, the FLR damping, occurring when $k_{\perp} \rho_i \gtrsim 1$, cannot be too large. Therefore, we expect $|\phi|$ to be higher in regions of the flux surface where $k_{\perp} \rho_i$ is lower. In figure 11(a), we plot the ratio $k_{\perp}/k_i$ for our flux surface as a function of $(\theta, \theta_0)$. It is important to note that $k_{\perp}/k_i$ is independent of $k_i$ at fixed $\theta_0$. Due to strong magnetic shaping in the pedestal, the quantity $k_{\perp}$ varies more strongly in $\theta$ and $\theta_0$ than for flux-surface shapes characteristic of the core. Therefore, we expect turbulence and transport in the pedestal to have a stronger dependence on $\theta$ than in the core. To map out the regions of weaker FLR damping at different $k_{\perp} \rho_i$ values, in figure 11(a), we plot the curves of $k_{\perp} \rho_i = 1$ for different $k_{\parallel} \rho_i$ values. In the areas bounded by these curves, $k_{\perp} \rho_i \lesssim 1$, so, heuristically, we expect weaker FLR damping there, and hence stronger turbulence.

For slab ETG turbulence, amplitudes are inversely correlated with $k_{\perp} \rho_i$. This can be seen by comparing values of $k_{\perp}/k_i$ at given $(\theta, \theta_0)$ in figure 11(a) with $|\phi|^2$ in (c) and (d) at the same $(\theta, \theta_0)$. Figures 11(c) and (d) show $|\phi|^2$ for $k_{\parallel} \rho_i = 12.0$ and $k_{\parallel} \rho_i = 28.9$, respectively. We see that $|\phi|^2$ becomes narrower in $\theta$ and $\theta_0$ at higher $k_{\parallel} \rho_i$. This is because, in figure 11(a), there are fewer regions of $k_{\perp}/k_i$, satisfying the weak FLR-damping constraint, $k_{\perp} \rho_i \lesssim 1$, at higher values of $k_{\parallel} \rho_i$. At lower values of $k_{\parallel} \rho_i$, $k_{\perp} \rho_i \lesssim 1$ is satisfied at more values of $\theta_0$ and $\theta$. Therefore, at lower $k_{\parallel} \rho_i$, we expect slab ETG turbulence to be present over more of the $(\theta, \theta_0)$ plane.

However, slab ETG turbulence does not dominate for all values of $k_{\parallel} \rho_i$. In figure 11(b), we plot $|\phi|^2$ for a relatively small $k_{\parallel} \rho_i = 2.8$: clearly, $|\phi|^2$ does not occupy the lowest $k_{\perp} \rho_i$ values from figure 11(a). This is because the turbulence at $k_{\parallel} \rho_i = 2.8$ is primarily toroidal ETG turbulence. While the parallel extent of slab ETG turbulence is constrained primarily by $k_{\parallel} \rho_i$ increasing along the field line, the parallel extent and location of toroidal ETG turbulence is subject to two constraints. Namely, for strong toroidal ETG turbulence to exist at a given parallel location, not only must FLR damping be relatively weak ($k_{\perp} \rho_i \lesssim 1$), but also the value of $\omega_{pe}^{\perp}/\omega_{ce}$ must allow strong toroidal ETG instability, requiring $\omega_{ce}^{\perp}/\omega_{ce} \approx A \sim 3–20$ (see discussion around equation (7)).

In figure 12(a), we plot $\omega_{pe}^T/\omega_{ce}$ for our flux surface, which shows a topography very different to the FLR constraints in figure 11(a). While the FLR damping in figure 11(a) tends to be weakest around $\theta_0 \simeq 0$ and $\theta \simeq 0$, the magnetic drifts are most favorable to the excitation of turbulence at $\theta_0/\pi \approx \pm 1$ and $\theta \neq 0$. Since the ratio $\omega_{pe}^T/\omega_{ce}$ is independent of $k_i$, but $k_{\perp} \rho_i$ is not, at lower $k_{\parallel} \rho_i$ values where FLR damping is weaker, we expect toroidal ETG modes to be freer to occupy $\theta$ locations where $\omega_{pe}^{\perp}/\omega_{ce}$ is optimal. For example, in figure 12(a), the dashed green line shows the rough FLR damping boundary, $k_{\perp} \rho_i = 1$, for $k_{\parallel} \rho_i = 2.8$. Within this region, $\omega_{ce}^{\perp}/\omega_{ce}$ has optimal values for strong toroidal ETG instability, and hence we expect strong toroidal ETG turbulence at $k_{\parallel} \rho_i = 2.8$.

Indeed, the turbulent amplitude $|\phi|^2$ for $k_{\parallel} \rho_i = 2.8$ and 1.4 in figures 11(b) and 12(b), respectively, has maxima in the optimal regions of $\omega_{pe}^T/\omega_{ce}$ of figure 12(a), demonstrating that at these binormal scales, the turbulence has a strong toroidal ETG character.

At higher values of $k_{\parallel} \rho_i$, FLR damping becomes stronger in regions where $\omega_{pe}^T/\omega_{ce}$ has optimal values for the excitation of toroidal ETG modes and so toroidal ETG turbulence must occupy regions with less favorable, in this case higher, values of $\omega_{pe}^T/\omega_{ce}$. For example, at $k_{\parallel} \rho_i = 12.0$ and 28.9 in figures 11(c) and (d), respectively, the turbulence has a stronger slab ETG character, as suggested by the fact that the amplitudes $|\phi|^2$ are inversely correlated with $k_{\perp}/k_i$. The stronger competition between magnetic drifts and FLR damping at higher values of $k_{\parallel} \rho_i$ causes toroidal ETG turbulence to be less virulent than slab ETG turbulence at these scales. Note that, as discussed earlier, the decrease in the maximum $|\theta_0|$ value with increasing $k_{\parallel} \rho_i$ due to grid and computational resource constraints may also artificially suppress toroidal ETG turbulence at higher $k_{\parallel} \rho_i$.

In the core, the effect of magnetic drifts and FLR damping on toroidal ETG instability is qualitatively different from the one in the pedestal. In the pedestal, the toroidal ETG instability at the outer scale (given by equation (12)) is strongest away from the outboard midplane, whereas in the core, it
occurs at higher \( k_y \rho_i \) due to gentler gradients and is strongest at \( \theta \approx 0 \). In figure 13(a), we plot \( \omega_e^T / \omega_{\text{ce}} \) for the circular flux-surface geometry with \( a/L_{Te} = 4.2 \) (see Scan100e in section 7 and table 1). This confirms that the most favorable values of \( \omega_e^T / \omega_{\text{ce}} \) for toroidal ETG instability in the core are at \( \theta \approx 0 \) and \( \theta_0 = 0 \). The grey regions indicate parallel locations where \( \omega_{\text{ce}} \) is too large (\( 0 < \omega_e^T / \omega_{\text{ce}} \lesssim 2 \)) for instability, even in bad-curvature regions \([58]\). In figure 13(b), we plot \( k_\perp / k_y \) for the tokamak core geometry. As in the pedestal, FLR effects in the core typically favor the outboard midplane as the preferred location for unstable modes with higher \( k_y \rho_i \). Thus, while toroidal ETG instability in the pedestal is favored at low \( k_y \rho_i \) because FLR effects damp the modes at higher \( k_y \rho_i \), toroidal ETG instability in the core can be strong at higher \( k_y \rho_i \). This is because, in the core, unlike in the pedestal, there is an alignment of favorable FLR effects and values of \( \omega_e^T / \omega_{\text{ce}} \) at higher \( k_y \rho_i \). In figure 13(a), we also plot contours of \( k_\perp \rho_i \) for \( k_y \rho_i = 42 \), near the approximate outer scale for the core turbulence, showing that strong toroidal ETG instability is driven at \( \theta \approx 0 \). Recall that unlike the pedestal gradients, core gradients cannot support \( k_y \rho_i \approx 1 \) ETG turbulence because \( a/L_{Te} \) is too small, according to the outer scale estimate in equation (12) for the core, \( k_y^0 \rho_i \approx (\rho_i / \rho_e) (LT_e / qR) \gg 1 \), where we used \( l_b \sim qR \).

If we keep the circular flux-surface geometry but increase the gradient to the pedestal value \( a/L_{Te} = 42 \), strong toroidal ETG turbulence is pushed away from the \( \theta \approx 0 \) (see Scan200f in figure 10(d) and table 1). Increasing \( a/L_{Te} \) from 4.2 to 42 increases \( \omega_e^T / \omega_{\text{ce}} \) in figure 13(a) by a scalar factor of 10, resulting in figure 13(c). Notably, this transformation leaves the \( k_\perp / k_y \) profile unchanged because for the circular flux-surface geometry, we set \( \partial \rho / \partial r \) in equation (5) to zero. Figure 13(c) reveals that regions where toroidal ETG instability is most virulent, viz, \( 3 \lesssim \omega_e^T / \omega_{\text{ce}} \lesssim 20 \), are now located away from the outboard midplane. The emergence of favorable \( \omega_e^T / \omega_{\text{ce}} \) regions away from the outboard midplane, as well as a decrease in the outer scale \( k_y^0 \rho_i \) due to steeper \( a/L_{Te} \), explains why the set-up with circular flux-surface geometry and \( a/L_{Te} = 42 \) in figure 10(d) exhibits significant contributions to the heat flux from off-midplane turbulence, whereas the case with \( a/L_{Te} = 4.2 \) does not: in circular flux-surface geometry with \( a/L_{Te} = 42 \), both slab and toroidal ETG turbulence at lower \( k_y \rho_i \) values are supported, and can be driven away from the outboard midplane.

It is important to recall that our Miller geometry is up-down symmetric \([102–107]\). Accordingly, so are the perpendicular-wavenumber and magnetic-drift topographies in figures 11 to 13, viz, they are invariant under the transformation \( (\theta, \theta_0) \rightarrow (\theta_0, -\theta) \). In contrast, inspection of the poloidal dependence of \( q_T \) and \( \phi_T \) in figures 6(d), 10(c) and (d), 11(b) and (c) and 12(b) reveals an up-down asymmetry in the parallel spatial distribution of turbulence. Thus, in figures 6(d), 10(c) and (d), 11(b) and (c) and 12(b), near \( \theta \approx 0 \), \( q_T \) is larger for \( \theta > 0 \), whereas away from \( \theta \approx 0 \), \( q_T \) is larger for \( \theta < 0 \). Averaging over longer time periods confirms this up-down asymmetry. It is caused by flow shear, with opposite asymmetry for toroidal and slab ETG turbulence. We have verified numerically that the asymmetry is reversed when the sign of \( \gamma_e \) is reversed. The asymmetry occurs because toroidal ETG modes prefer \( \text{sign}(\theta_0) = -\text{sign}(\theta) \), as is seen by inspecting regions of \( \omega_e^T / \omega_{\text{ce}} > 0 \) in figure 12(a). In contrast, examination of figure 11(a) shows that slab ETG modes usually prefer regions of \( \text{sign}(\theta_0) = \text{sign}(\theta) \) where \( k_y \rho_i \) is lower. For \( \gamma_e > 0 \), the effective \( \theta_0 \) decreases with time (see equation (5)) \([108, 109] \) and,
as a result, turbulence amplitudes peak at negative values of \( \theta_0 \).
In turn, toroidal ETG moves to \( \theta > 0 \) and slab ETG to \( \theta < 0 \).
Thus, the effect of flow shear on the relative up-down poloidal
distribution of slab and toroidal ETG transport can be predicted
qualitatively by inspecting the \( k_y \) and \( \omega_{ki}/\omega_{pe} \) topographies.

8. Multiscale ETG–ETG interactions

We now demonstrate that \( k_y\rho_i \sim 1 \) ETG turbulence decreases
overall ETG transport substantially, in our case by \( \sim 40\% \).
We show this by introducing artificial damping for low \( k_y\rho_i \) modes
from an initial condition corresponding to the saturated state
of our Base150 calculation.

We damp modes with \( k_y\rho_i \leq k_y\text{cutoff}\rho_i = 4.3 \) to test whether
\( k_y\rho_i \sim 1 \) ETG turbulence affects \( k_y\rho_e \sim 1 \) ETG turbulence and
transport. To damp \( k_y\rho_i \sim 1 \) modes, we multiply the perturbed
distribution function \( f_i^{\rho} \) for these modes by \( 10^{-4} \) at each
timestep. At \( t = t_0 \), just before these modes are damped, there
is a significant heat flux contribution from each low \( k_y\rho_i \) value.
In figure 14(a), we show how \( \tilde{q}_{e,k_y} \) evolves after time \( t_0 \) when
we begin damping them.

At the time immediately after these modes are damped, the
heat flux drops by roughly \( 5\% \). This instantaneous decrease
in the heat flux represents the loss of heat flux carried by the
now-damped \( k_y\rho_i \leq k_y\text{cutoff}\rho_i = 4.3 \) modes. At this time,
modes with \( k_y\rho_i > k_y\text{cutoff}\rho_i \) still carry information about multисcale interactions with the \( k_y\rho_i \leq k_y\text{cutoff}\rho_i \) modes. Therefore,
we will call the heat flux at this time \( \tilde{Q}_{e,\text{before}} \) and use it as the
point of comparison with the heat flux in the new saturated
state at later times, \( \tilde{Q}_{e,\text{after}} \).

As \( t \) increases, there is negligible heat flux from \( k_y\rho_i \leq k_y\text{cutoff}\rho_i \) modes and there is a significant increase in \( \tilde{q}_{e,k_y} \) at
larger values of \( k_y\rho_i \). Figure 14(b) shows how, over a period
of several linear times of the slowest undamped linear modes,
the total electron heat flux \( \tilde{Q}_e \) increases from \( \tilde{Q}_{e,\text{before}} \approx 5.2 \) to
\( \tilde{Q}_{e,\text{after}} \approx 8.4 \) in the new steady state. Thus, there is a \( \sim 40\% \)
reduction in \( \tilde{Q}_e \) when the lower-\( k_y\rho_i \) modes are allowed to play
a role. We have demonstrated that \( k_y\rho_i \sim 1 \) ETG turbulence suppresses higher-\( k_y\rho_i \) ETG transport.

We have shown that retaining \( k_y\rho_i \sim 1 \) ETG modes is crucial
to capture correctly the \( k_y\rho_i \sim 1 \) electron heat flux. This is relevant
for tokamak turbulence modeling [52, 110–115] that aims to predict experimental fluxes accurately. While this result
is not the first to show \( k_y\rho_i \sim 1 \) turbulence suppressing \( k_y\rho_e \sim 1 \) turbulence [21, 23, 24, 116], it is the first to show ETG turbulence at \( k_y\rho_i \sim 1 \) suppressing ETG turbulence and transport at \( k_y\rho_e \sim 1 \). It is important to re-emphasize
that the ETG turbulence at \( k_y\rho_i \sim 1 \) is strongly-driven not
because of kinetic-ion physics, but because the pedestal
temperature gradients are so steep (see discussion surrounding equation (12)).

The multiscale mechanism for the suppression of electron-scale
transport in the pedestal remains to be investigated in
future work. In the core, cross-scale interactions between
electron-scale turbulence (driven by ETG instability) and
ion-scale turbulence (driven by ITG and other instabilities)
can suppress electron-scale and enhance ion-scale transport
[22–24, 116]. In contrast, because steep temperature gradients in the pedestal break electron–ion scale separation, interactions between electron-scale turbulence and ion-scale turbulence, where turbulence at both scales is driven by ETG instability, is possible.

9. Discussion

The main result of this paper is that ETG turbulence in a typical
JET pedestal has a rich three-dimensional spatial structure in
directions both parallel and perpendicular to the magnetic
field. This structure arises due to the steep temperature gradient
and the highly shaped magnetic geometry. Steep temperature
gradients enable strong ETG turbulence to be driven at much
longer binormal wavelengths than in core tokamak plasmas,
often at wavelengths numerically comparable to the ion gyro-
radius, \( k_y\rho_i \sim 1 \). The \( k_y\rho_i \sim 1 \) ETG turbulence has the highest
fluctuation amplitudes but produces modest heat transport due
its short radial correlation length, and also reduces the overall turbulent heat transport through multiscale interactions.

Experimental measurements of off-midplane potential fluctuations are needed to test our predictions, but could prove challenging due to turbulence diagnostics conventionally being located at the outboard midplane, with some exceptions [2, 117]. Our results might be consistent with Beam-Emission-Spectroscopy measurements of ion-gyroradius scale turbulence in MAST, showing correlation lengths that are longer in the binormal direction than in the radial direction [79, 118], hinting at experimental signatures of $k_\perp \gg k_\parallel$ anisotropic turbulence of a nature described in this paper.

The parallel spatial distribution of toroidal and slab ETG turbulence at all scales can be qualitatively predicted from the perpendicular-wavenumber and magnetic-drift profiles (see figures 11(a) and 12(a)). Both have complex topography due to strong magnetic shaping in the pedestal. Due to FLR damping, turbulence and transport are highest in the outboard midplane for $k_\perp \rho_s \sim 1$, but for $k_\parallel \rho_s \sim 1$, electrostatic-potential fluctuations are largest near the flux surface’s top and bottom (see figures 11 and 12). The adiabatic ion nature of toroidal ETG turbulence prevents large heat transport arising from large density fluctuations away from the outboard midplane.

The results of sections 7 and 8 suggest using magnetic shaping to optimize transport in the pedestal and internal transport barriers. This could be achieved by modifying parallel correlation lengths and hence the outer scale of the turbulence (see equation (12)) using FLR effects and magnetic-drift profiles, and by maneuvering toroidal and slab ETG turbulence into similar poloidal locations, so that their multiscale interactions could suppress $k_\perp \rho_s \sim 1$ transport.

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Code and data availability

The data used for the material in this paper are available at the following dataset archive [119].

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