Minimal realization of the Orbital Kondo effect in a Quantum Dot with two Leads

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(Dated: September 1, 2018)

We demonstrate theoretically how the Kondo effect may be observed in the transport of spinless electrons through a quantum dot. The role of conduction electron spin is played by a lead index. The Kondo effect takes place if there are two close levels in the dot populated by a single electron. For temperatures exceeding the Kondo temperature $T \gg T_K$, the conductance is maximal if the levels are exactly degenerate. However, at zero temperature, the conductance is zero at the SU(2) symmetric point but reaches the unitary limit $G = e^2/h$ for some finite value of the level splitting $\Delta \varepsilon \sim T_K$. Introducing the spin-1/2 for electrons and having two degenerate orbital levels in the dot allows to observe the SU(4)-Kondo effect in a single dot coupled to two leads.

PACS numbers: 72.15.Qm, 73.63.Kv, 73.23.-b, 73.23.Hk

I. INTRODUCTION

For several decades the Kondo effect [1] has been one of the most attractive and widely investigated many body phenomena in solid state physics [2]. Experimental observation of the Kondo effect in quantum dots (QD) [3], also predicted almost two decades ago [4], started a new wave of interest in the field. The minimal realization of the Kondo effect requires a single electron channel with spin, coupled to the impurity spin. This may not necessarily be a true spin, but also some internal degree of freedom leading to the orbital Kondo effect. Since in transport experiments the QD is coupled to two leads, the questions why the spin is necessary for the development of Kondo correlations and why the two leads cannot cause an orbital Kondo effect arises. For a single active discrete level in the QD, one lead may be effectively decoupled by a simple unitary transformation [4], eliminating any spinless Kondo behavior. This could cause a belief that there is no Kondo effect in a single dot without spin. However, as was already shown in Ref. [3], having several orbital levels in the dot will cause no decoupled lead to exist and enable the Kondo correlations without spin. As we will show in this paper, the transport through such a QD has a very peculiar form, where both the temperature and gate-voltage dependencies are nonmonotonous. At zero temperature the conductance vanishes for exactly degenerate levels $\Delta \varepsilon = 0$ when the (pseudo)spin is completely screened, while it reaches the unitary limit at finite level splitting $\Delta \varepsilon \approx 2.43 T_K$, where the screening is only partial. (Similar behavior for a related double dot system was found recently in Ref. [3] using the functional renormalization-group approach.) On the other hand, at high temperatures the conductance is maximal at the degeneracy point. Such rich transport properties in the Kondo regime clearly call for an experimental verification.

If the spin degeneracy is present in addition to the two close orbital levels, the SU(4)-Kondo effect develops in a single QD with two single-channel leads. Again the conductance of a QD in the vicinity of SU(4) degenerate point has a peculiar gate-voltage and/or temperature behavior. This behavior may be used for spintronics applications, as suggested in Ref. [2] for double-dot devices.

Even for two degenerate levels in the closed dot, the Kondo correlations do not always develop. Virtual electron jumps from the QD to the leads and back both shift the single electron energies in the dot and $\Delta \varepsilon$ and induce an effective coupling between localized states, which lifts out the degeneracy. Consequently, in order to reach the SU(2)/SU(4) symmetric point, one needs to tune two parameters, the level splitting and the non-diagonal matrix element, which in the following we will parameterize by two components of the effective magnetic field, $B_z$ and $B_x$. Such two-parameter tuning in a single QD may still be easier experimentally than manipulating the double QD. Concurrently, the effective magnetic field dependence of the conductance may serve as an indication of the orbital Kondo effect in a single QD.

We first consider the case of spinless electrons, where analytical results are easily available for both $T \gg T_K$ and $T \ll T_K$. This case may be achieved experimentally, for example, by lifting the spin degeneracy with an external magnetic field (see below). The SU(4)-Kondo effect will be considered in the concluding part of the paper.

Experimentally both orbital and SU(4)-Kondo effects were observed recently in carbon nanotube QDs [14]. The effect in this case originates from the extra valley degeneracy of electronic states in carbon nanotubes, which thus takes place both in the QD and in the leads. Several theoretical papers addressed the possibility of having orbital and SU(4)-Kondo effect in nanotubes, double QDs and QDs with several leads [7, 11, 12]. Reference [7] first predicted the possibility of orbital Kondo effect in the case of QD coupled to two leads. In both carbon nanotubes and double QDs the orbital and/or SU(4)-Kondo effect should be seen in the whole Coulomb blockade valley separating two charging resonances. The emphasis of the current research is on surprisingly rich conductance behavior of the QD with two leads in the Kondo regime, which develops in a narrow region in the parameter space
near the degeneracy point.

II. EFFECTIVE INTRADOT HAMILTONIAN

The Hamiltonian of a quantum dot with two close levels \((j = 1, 2)\) coupled to two leads \((L, R)\) has the form

\[
H = \sum_{ik} \varepsilon_k c_{ki}^\dagger c_{ki} + \sum_j \varepsilon_j d_j^\dagger d_j + \frac{U}{2} \sum_{ij} d_i^\dagger d_i d_j^\dagger d_j + \sum_{ij} (t_{ij} c_{ki}^\dagger d_j + H.c.).
\]

Here, \(d_j\) \((c_{ki})\) annihilates the electron at the level \(j\) in the dot \((\text{the one with momentum } k \text{ in the lead } i)\); \(\varepsilon_j\) \((\varepsilon_k)\) are the electron energies in the dot (lead); \(U\) is the charging energy of the dot and \(t_{ij}\) are the dot-lead coupling matrix elements. We take all \(t_{ij}\) to be real. \(\text{(This requires that while the Zeeman splitting of spin-up and spin-down states exceeds the energy difference of two close levels, the magnetic field is parallel to the plane of the dot or small enough to not affect the orbital states in the dot.)}\)

When the Fermi energy \(\varepsilon_F\) in the leads is chosen as \(\varepsilon_j < \varepsilon_F < \varepsilon_k + U\), the dot is charged by one electron.

In the second order of perturbation theory two processes lead to the renormalization of the single-particle interdot Hamiltonian: Electron jumps from the dot to the lead or vice versa, both followed by the return of an electron. The corresponding matrix elements calculated in \([9]\) are (see a similar calculation for the Anderson impurity model in Ref. \([13]\)):

\[
V_{ij} = \frac{\varepsilon_j \delta_{ij}}{2\pi} \frac{\Gamma_{ij} \ln \left| E - \varepsilon_F + U \right|}{\left| E - \varepsilon_F \right|}.
\]

Here \(\Gamma_{ij} = 2\pi \sum_{t=L,R} t_{i|t|j} \delta_{ij}\), \(i, j\) label the level indices 1 or 2 and \(E = (V_{11} + V_{22})/2\). For simplicity we assume the same density of states \(\nu = \frac{dn}{dc}\) in the two leads.

The Hamiltonian of the isolated dot now takes the form

\[
H_d = E \sum_i d_i^\dagger d_i + \frac{U}{2} \sum_{ij} d_i^\dagger d_i d_j^\dagger d_j + \tilde{B} \hat{S},
\]

where \(\hat{S} = \sum_{ij} d_i^\dagger \delta_{ij} d_j^\dagger/2\) and the only nonzero components of the effective magnetic field are \(B_x = 2V_{12}\) and \(B_z = V_{11} - V_{22}\). We assume that \(|\tilde{B}| \ll U\), while \(\varepsilon_F < E < U\).

The QD Hamiltonian, Eq. (1), formally coincides with the Hamiltonian of the double-dot system tuned-coupled to two shared leads, investigated in Ref. [6]. Consequently our result for the conductance in the spinless case found in Sec. V for strongly developed Kondo correlations \((U \gg \Gamma, T = 0)\) closely resembles the correlation-induced resonances found in that paper for intermediate values of \(U/T\).

III. THE KONDO HAMILTONIAN

A convenient way to define the pseudospin states is to perform unitary transformations of both dot and lead states to diagonalize the matrix \(U(T_1)\), where the matrix \(T\) is formed by the tunnelling matrix elements \(t_{ij}\) [Eq. (1)] and the unitary matrices \(U_i(U_j)\) operate in the lead-index(dot-level) space. Fortunately, in the case of interest to us, this procedure is simplified. In order to have strong Kondo correlations, the effective Zeeman splitting in the dot should be small compared to, e.g., the widths of the levels (we require both components \(B_x \sim B_z\) to be small, not the vanishing of one of them).

This, in particular, implies \(|\Gamma_{12}| \ll \max(\Gamma_{11}, \Gamma_{22})\). Since \(\Gamma_{12} \sim \sum t_{1j} t_{2j}\), a simple rotation in the lead-index space (angle \(\alpha\), \(\tan \alpha = t_{1R}/t_{1L} \approx -t_{2L}/t_{2R}\)) allows to define the mixed lead states, each coupled to one of the levels in the dot with the tunnelling matrix elements

\[
t_1 = \sqrt{t^2_{1L} + t^2_{1R}}, \quad t_2 = \sqrt{t^2_{2L} + t^2_{2R}}.
\]

The standard Schrieffer-Wolff transformation leads us now to the Kondo-type Hamiltonian

\[
H_K = \sum_{ki} \varepsilon_k c_{ki}^\dagger c_{ki} + J (s_x S_x + s_y S_y) + J_z s_z S_z + \tilde{B} \hat{S},
\]

where \(\hat{S} = \sum \delta_{ij} c_{ki}^\dagger c_{kj}/2\), and we dropped out irrelevant terms describing electron scattering without spin-spin interaction.

The bare exchange spin-spin scattering amplitudes

\[
J_{s0} = \frac{(t_1^2 + t_2^2)|U|}{|E - \varepsilon_F|} , \quad J_0 = \frac{2t_1 t_2}{t_1^2 + t_2^2} J_{s0}
\]

are renormalized by integrating out the electron states in the leads with \(|\varepsilon_k - \varepsilon_F| > D\). The corresponding renormalization-group equations have the form

\[
J' = -J^2, \quad J' = -J J_z.
\]

Here the derivative \(A' = dA/d(\nu \ln D)\). The two differential equations have an "integral of motion", \(\lambda = \sqrt{J_z^2 - J^2} = \text{constant}\). A simple calculation now gives

\[
J_z = \lambda \coth \left[ \lambda \nu \ln \frac{D}{T_K} \right], \quad T_K = \frac{1}{\lambda \nu} \ln \frac{t_1}{t_2},
\]

where \(T_K\) is the Kondo temperature.

Once the renormalization of the spin-spin interaction becomes sufficient, \(J(D) \gg J_0\), one may neglect the difference between \(J\) and \(J_z\). Then, the scattering on the impurity become \(SU(2)\) invariant and we may again perform a rotation in the pseudospin space leading to

\[
H_K = \sum_{ks} \varepsilon_k c_{ks}^\dagger c_{ks} + J \hat{S} \hat{S} + B S_z,
\]

where \(B = \sqrt{B_x^2 + B_z^2}\). The lead spin states are

\[
c_{k^\uparrow} = c_{kL} + s c_{kR}, \quad c_{k^\downarrow} = -s c_{kL} + c c_{kR},
\]
where \( c = \cos \phi \), \( s = \sin \phi \) and the angle \( \phi \) includes two contributions, the rotation of lead states which diagonalizes the matrix \( T = (t_{ij}) \) and the rotation of both the lead-electron and the dot spins in order to align the z axis with the direction of effective magnetic field

\[
\phi = \frac{1}{2} \arctan \frac{B_x}{B_z} + \arctan \frac{t_{1R}}{t_{1L}}.
\]  

IV. THE LINEAR CONDUCTANCE

First we consider the situation where the Kondo correlations are developed but have not reached the unitary limit, \( \nu J_0 \ll \nu J \ll 1 \). Let also the temperature \( T \) and the voltage \( eV = \varepsilon_{F_L} - \varepsilon_{F_R} \) be small compared to the effective Zeeman splitting in Eq. (10), \( B \gg T, eV \). Then the spin projection of the QD is always \( S_z = -1/2 \) with no fluctuations. The transport is now entirely due to the term \( J_{S_z, S_z} \rightarrow \frac{1}{4} \sin 2\phi \sum_k c_{kR}^\dagger c_{kL} \) in Eq. (9), leading to the conductance

\[
G = \frac{e^2}{h} \left[ \sin 2\phi \frac{\pi \nu J(B)}{2} \right]^2 = \frac{e^2}{h} \left[ \frac{B_z}{B} \frac{\pi \nu J(B)}{2} \right]^2.
\]  

In the last formula we substituted the angle \( \phi \) [Eq. (11)] for symmetric dot-to-lead coupling, \( t_{1R} = t_{1L} \). A different choice of the ratio \( t_{1R}/t_{1L} \) would lead to a simple rotation on the \((B_x, B_z)\) plane. The Kondo correlations enter Eq. (12) through the running coupling constant \( J(B) \).

Slightly more complicated is the case of a finite temperature \( T \gg T_K \). In the spirit of Ref. [14] we may obtain the probability to find the dot in one of the spin states \( n_\uparrow \) and \( n_\downarrow \) via the stationary solution of corresponding master equation with small applied bias \( \varepsilon_{F_L} = \varepsilon_{F_R} + eV \),

\[
\frac{n_\downarrow}{n_\uparrow} = \exp(\frac{eV}{T}) \left[ 1 - \frac{eV}{T} \cos 2\phi \right].
\]  

Then, we obtain the conductance, which includes both non-spin-flip and spin-flip contributions, as follows:

\[
G = \frac{e^2}{h} \left[ \sin 2\phi \frac{\pi \nu J}{2} \right]^2 \left( 1 + \frac{B/T}{\sinh(B/T)} \right).
\]  

Here the running coupling constant \( J \) [Eq. (8)] may be calculated, e.g., at the average \( D = \sqrt{B^2 + T^2} \). Details of the derivation of Eqs. (13) and (14) are given in the Appendix.

Theoretically the \( \sim (J/J_0)^2 \) increase of the conductance due to the Kondo effect may become very large. However, since the spin scattering constant \( J \) depends only logarithmically on the cutoff \( D \) [Eq. (8)], the more pronounced in the experiment may be the enhancement due to the last factor in Eq. (14), which accounts for the spin-flip processes near the \( SU(2) \) symmetric point. The non-spin-flip and spin-flip contributions to the conductance may be measured separately in a QD embedded into the Aharonov-Bohm interferometer [12].

Conductance as a function of \( B \) in various regimes as well as the contour plot of the conductance on the \( B_x, B_z \) plane are shown in Figs. 1 and 2.

V. EXACT S-MATRIX

For \( T \ll T_K \), the electron is only elastically scattered on the Kondo impurity and the conductance is determined by a single-particle S-matrix [15, 17]. In the usual case of electron with spin-1/2 and single-level QD the S-matrix connects four incoming and four outgoing waves, corresponding to two "spin orientations" in the two leads. However, only two effective channels acquire a nontrivial scattering phase, while the other two are completely decoupled [4]. In our setup there are only two channels with nontrivial phase behavior, corresponding to the two pseudo-spin directions, and the S-matrix in terms of these effective channels entering the Kondo Hamiltonian Eq. (9) has the form [18]

\[
S = \begin{pmatrix} e^{2i\delta} & 0 \\ 0 & e^{-2i\delta} \end{pmatrix},
\]  

where the phase \( \delta = \delta(B/T_K) \) increases from 0 to \( \pi \) when \( B \) changes from \( +\infty \) to \( -\infty \) and \( \delta(0) = \pi/2 \). In order to
The explicit dependence of the phase $\delta$ increases, reaching the unitary limit ($\vec{B}$ minimal at plane at point, $\vec{B} = 0$) develops along the $B_z$ axis, separating two peaks with $G = e^2/h$.

find the $S$-matrix in the original $L - R$ basis we perform a rotation ($c = \cos \phi, s = \sin \phi$)

$$S^{(LR)} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} e^{2i\delta} & 0 \\ 0 & e^{-2i\delta} \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix}.$$  \hspace{1cm} (16)

The nondiagonal matrix element $S_{12}^{(LR)} = i \sin 2\phi \sin 2\delta$ now determines the conductance

$$G = \frac{e^2}{h} \sin^2 2\phi \sin^2 2\delta. \hspace{1cm} (17)$$

The explicit dependence of the phase $\delta(B/T_K)$ is found from the Bethe ansatz solution [10]. When approaching the $SU(2)$ symmetric point the conductance first increases, reaching the unitary limit ($\delta = \pi/4, 3\pi/4$) at $B = \pm 2.43 T_K$. It vanishes, however, at the symmetric point, $\delta(0) = \pi/2$ [20].

VI. SU(4)-KONDO EFFECT

Taking into account the true electron spin is achieved in Eqs. (1) and (3) by simply adding an extra index to all creation and annihilation operators. The bare Kondo Hamiltonian [analog of Eq. (3)] will now contain a number of new operators describing both spin, isospin, and mixed spin-pseudo-spin scattering [7]. Fortunately, the renormalization-group flow of the corresponding constants at low temperatures (and small level splitting in the dot) results in a simple $SU(4)$ symmetric Hamiltonian, known as a Coqblin-Schrieffer model [21].

$$H_{\text{eff}} = \bar{J} \sum_{\alpha, \beta = 1, \ldots, 4} [\psi_{\alpha}^\dagger \psi_{\beta}] (\alpha) - \frac{1}{4} \psi_{\alpha}^\dagger \psi_{\beta} \psi_{\gamma}^\dagger \psi_{\delta} + BT_z. \hspace{1cm} (18)$$

Here the isomagnetic field $B$ is the same as in eq. (13), but we have replaced $S_z$ by $T_z$ to emphasize that this is an isospin operator, $|\alpha\rangle$'s denote the four states of quantum dot, and $\psi_{\alpha} = \sum_{k} c_{\alpha k}$.

Integrating out the lead states with $|E - E_F| > D$, increases the coupling constant $\bar{J}$ like $\bar{J} \sim 1/\ln(D/T_K)$. This renormalization of Eq. (18) continues until $D \approx B$. At $D < B$ the $SU(4)$ symmetry of the Hamiltonian is broken to $SU(2)$ (single level with spin $s = 1/2$). The $SU(2)$ coupling constant is further (logarithmically) renormalized with decreasing $D$, but the corresponding increase of the conductance is smaller than for the $SU(4)$ symmetric case. This effect was interpreted in Ref. [11] as the increase of the Kondo temperature close to the degeneracy point (as done before for the case of singlet to triplet transition [22]). However, these $SU(4)$ vs $SU(2)$ renormalization group effects change $\bar{J}$ only through logarithmic corrections. Thus in a large range of variation of parameters the most pronounced effect will be the conductance increase at $B \approx T$ due to spin-flip processes. A straightforward calculation shows that Eq. (18) for intradot levels occupation remains valid in the presence of the true spin and the conductance is given by

$$G = \frac{2e^2}{h} \left( \pi \nu \bar{J} \sin 2\phi \right)^2 \left( 1 + \frac{B/T}{\sinh(B/T)} \right). \hspace{1cm} (19)$$

At zero temperature, electron scattering is described by a $4 \times 4$ S-matrix, which is diagonal in the basis Eq. (10)

$$S = \text{diag}(e^{2i\delta_1}, e^{2i\delta_1}, e^{2i\delta_2}, e^{2i\delta_2}). \hspace{1cm} (20)$$

When $B$ changes from $+\infty$ to $-\infty$, the first phase decreases from $\delta_1 = \pi/2$ to $\delta_1 = 0$, while the second increases from $\delta_2 = 0$ to $\delta_2 = \pi/2$ [23]. The $SU(4)$ symmetric point $B = 0$ corresponds to $\delta_1 = \delta_2 = \pi/4$ (see Ref. [7] for a NRG calculation of phases). Transformation to the original $L - R$ basis is done via a rotation, similar to that of Eq. (19), separately of the pairs of states $1 - 3$ and $2 - 4$. This gives (compare Ref. [17])

$$G = \frac{2e^2}{h} \sin^2 2\phi \sin^2 (\delta_1 - \delta_2). \hspace{1cm} (21)$$

As in the $SU(2)$ case [Eq. (17)], the conductance vanishes at the $SU(4)$ symmetric point $\delta_1 = \delta_2$. Away from this point the usual spin-1/2 Kondo effect takes place due to transport through the lower of the two levels in the dot. The conductance is maximal here since $\sin^2 (\delta_1 - \delta_2) \approx 1$ (unitary limit).

VII. CONCLUSIONS

In this paper, we discussed how with suitable tuning of the dot levels and the couplings to the two leads an orbital $SU(2)$-Kondo effect can be realized for spinless electrons and a single Quantum Dot. We obtained the nontrivial transport properties of this model. Introducing the real electron spin results in the $SU(4)$-Kondo effect. A nonmonotonic conductance dependence on the
temperature and the effective magnetic field should allow the experimental verification of our findings. One of our important predictions is that the conductance, which is zero at the point of complete screening of the dot pseudospin, $B,T = 0$, increases all the way up to the unitary limit if the ideal screening is destroyed by the effective magnetic field. The question what is the maximal value of the conductance in case of partial screening due to finite temperature remains open. Solving this problem would require a Numerical Renormalization-Group solution at $T$ around $T_K$.

**Acknowledgements.** We thank Yuval Oreg for helpful discussions. This work was supported by the SFB TR 12, by the German Federal Ministry of Education and Research (BMBF) within the framework of the German-Israeli Project Cooperation (DIP) and by the Israel Science Foundation (grant No. 1566/04). PGS’s visit at Weizmann was supported by the EU - Transnational Access program, EU project RITA-CT-2003-506095. When this paper was prepared for publication we became aware of independent calculations on a related problem by V. Kashcheyevs et al [23]. After the submission we also became aware of a preprint [25], whose content partly overlaps with our Sec. [V]

**APPENDIX A: FINITE TEMPERATURE**

In this appendix, we present the details of derivation of Eqs. (13), (14), and (19) for level occupations in the dot and conductance in the case of high temperature $T \gg T_K$. Our calculation may be considered as a generalization of the method of Ref. [14]. Specific for our problem is that the quantum dot is out of resonance and the elementary processes which should be balanced in the stationary solution of the master equation are not adding/removing an electron to the dot but transmission of the electron through the dot accompanied by the switch of the dot internal state. Also, in our case, what is called the effective spin-up and spin-down states of the leads are the quantum superpositions of the true left and right lead states. Applying a finite bias to the QD, one changes the Fermi energy in the left-right leads and not in the spin up-down effective leads. The second and third terms in the Hamiltonian Eq. (9) may symbolically be written as

$$\begin{align*}
BS_z + Js_zS_z + \frac{J}{2}(s_-S_+ + s_+S_-) = \\
= BS_z + \frac{J}{2}[S_z(\langle \uparrow \uparrow \rangle - \langle \downarrow \downarrow \rangle) + S_+(\langle \uparrow \downarrow \rangle + S_-\langle \downarrow \uparrow \rangle),
\end{align*}$$

(A1)

where, for example, $\langle \uparrow \uparrow \rangle = \sum_k c_{k+}^0$. In terms of the original left and right lead operators we may write

$$2s_z = \langle \uparrow \uparrow \rangle - \langle \downarrow \downarrow \rangle = \langle c^2 - s^2 \rangle (L)\langle L \rangle - \langle R \rangle\langle R \rangle + 2c(s_+\langle R \rangle + |R\rangle\langle L \rangle),$$

(A2)

where $c = \cos \phi$, $s = \sin \phi$, and $|L\rangle = \sum_k c_{kL}^\dagger |R\rangle = \sum_k c_{kR}^\dagger$. Similarly

$$s_\mp = |\langle \uparrow \rangle \rangle =$$

$$= -\cos\langle L \rangle\langle L \rangle - |R\rangle\langle R \rangle - s^2\langle L \rangle\langle R \rangle + c^2\langle R \rangle\langle L \rangle.$$ 

(A3)

Now we may write a detailed balance equation for the probabilities to find the dot in states up($\uparrow$) and down($\downarrow$). For example,

$$\dot{n}_+ = -\dot{n}_- \propto n_+\left(\frac{c^2}{2}\langle |\uparrow \rangle\langle \uparrow | + |\downarrow \rangle\langle \downarrow |\rangle + c^4\langle |\downarrow \rangle\langle \uparrow | + |\uparrow \rangle\langle \downarrow |\rangle - n_+\left(\langle \uparrow \rangle\langle \uparrow | + |\downarrow \rangle\langle \downarrow |\rangle\rangle -$$

$$- n_+\left(\langle \uparrow \rangle\langle \downarrow | + |\downarrow \rangle\langle \uparrow |\rangle\rangle + c^4\langle |\uparrow \rangle\langle \uparrow | + |\downarrow \rangle\langle \downarrow |\rangle\rangle\right).$$

(A4)

Here $f$ is a Fermi function for electrons in the lead, $g = 1 - f$, and we also introduced symbolic notations

$$\langle f^{\uparrow\downarrow}g^{\downarrow\uparrow} \rangle = \int \frac{1}{1 + \exp((x - \varepsilon_1)/T)} dx = \frac{\delta e^{-\delta}}{1 - e^{-\delta}},$$

where $\delta = (\varepsilon_2 - \varepsilon_1)/T$.

For zero bias, $eV = 0$, Eq. (A4) transforms into

$$\dot{n}_+ \propto \frac{b}{2\sinh b/2}[n_+e^{-b/2} - n_+e^{b/2}],$$

(A6)

where $b = B/T$. Now let the Fermi energies in the two leads be different

$$E_{FL} = E_{FR} + eV.$$ 

(A7)

In this case, instead of Eq. (A5), we get

$$\dot{n}_+ \propto \frac{b}{2\sinh b/2} \left(1 - (c^2 - s^2)\frac{eV}{T} \frac{d}{db} \right) \frac{b}{e^b - 1}$$

$$- \frac{b}{2} \left(1 - (c^2 - s^2)\frac{eV}{T} \frac{d}{db} \right) \frac{b}{e^b - 1}.$$ 

(A8)

Finally, we end up with the remarkably simple expression

$$\frac{n_-}{n_+} = e^{B/T} \left[1 - \frac{e^B}{T}\right].$$

(A9)

where $c = \cos \phi$, $s = \sin \phi$, and $|L\rangle = \sum_k c_{kL}^\dagger |R\rangle = \sum_k c_{kR}^\dagger$. Similarly

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$$\langle f^{\uparrow\downarrow}g^{\downarrow\uparrow} \rangle = \int \frac{1}{1 + \exp((x - \varepsilon_1)/T)} dx = \frac{\delta e^{-\delta}}{1 - e^{-\delta}},$$

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In this case, instead of Eq. (A5), we get

$$\dot{n}_+ \propto n_+\left(1 - (c^2 - s^2)\frac{eV}{T} \frac{d}{db} \right) \frac{b}{e^b - 1}$$

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(A8)

Finally, we end up with the remarkably simple expression

$$\frac{n_-}{n_+} = e^{B/T} \left[1 - \frac{e^B}{T}\cos 2\phi\right].$$

(A9)

There is a nontrivial nonequilibrium distribution even for $B = 0$. This is because of the electrons are injected through the leads, which are themselves the superpositions of pseudospin-up and -down states. Conductance through the QD described by the Hamiltonian Eq. (9) may be split into several contributions. First, the non-spin-flip transitions through the ground state $n_- \rightarrow n_-$ give

$$I_{n_- \rightarrow n_-} = \frac{e}{h} eV n_- \left(\frac{J}{2}\frac{2c}{s}\right)^2.$$ 

(A10)
The overall coefficient may always be recovered by comparison with Eq. (12). Transitions (non-spin-flip) through the excited state differ only by the factor \( n_+ \) instead of \( n_\downarrow \). Similarly the transitions with flipping the impurity from the ground state \( n_\downarrow \to n_+ \) give

\[
I_{n_\downarrow \to n_+} = \frac{e}{\hbar} (B - eV) n_\downarrow [J_\uparrow^2]^2 \frac{e^{-b+v}}{1 - e^{-b-v}} \tag{A11}
\]

\[
- \frac{e}{\hbar} (B + eV) n_\downarrow [J_\uparrow^2]^2 \frac{e^{-b-v}}{1 - e^{-b+v}}.
\]

Here \( v = eV/T \). The first term in the r.h.s. of Eq. (A11) describes the charge transfer from the left to the right lead, and the second term accounts for the charge transfer from the right to the left lead. Similarly the \( n_+ \to n_\downarrow \) contribution reads

\[
I_{n_+ \to n_\downarrow} = \frac{e}{\hbar} (B + eV) n_+ [J_\uparrow^2]^2 \frac{1}{1 - e^{-b-v}} \tag{A12}
\]

\[
- \frac{e}{\hbar} (B - eV) n_+ [J_\uparrow^2]^2 \frac{1}{1 - e^{-b-v}}.
\]

Straightforward calculation now gives Eq. (13).

To make explicit the analogy between the \( SU(4) \) and \( SU(2) \) cases we rewrite the term in the Hamiltonian Eq. (9) describing the spin-dependent scattering in the notations used in Eq. (10).

\[
J \hat{S} \hat{s} = \frac{J}{2} \sum_{\alpha \beta} \langle \alpha | \psi_\alpha^\dagger \psi_\beta \rangle - \frac{J}{4} \sum_{\alpha} \langle \alpha | \sum_{\beta} \psi_\beta^\dagger \psi_\beta \rangle.
\]

(A13)

The only difference between the spin scattering in the \( SU(4) \) and \( SU(2) \) forms is now in the relative amplitude of potential scattering \( \sum |\alpha\rangle \langle \alpha| \sum \psi_\alpha^\dagger \psi_\beta \). The potential scattering, however, does not lead to any spin-flip processes and, in our geometry, even does not contribute to any current at all. Indeed, in the potential scattering term, one may always perform individual unitary rotations of lead and dot operators so that each lead will be connected to its own level. The electron in this case may oscillate between lead and connected level, but will never be transferred from one lead to another.

Equation (A1) is still valid in the case of \( SU(4) \) Hamiltonian Eq. (10). Simply now \( n_+ \) and \( n_\downarrow \) are the probabilities to find the dot in the pseudo-spin up or down state with any orientation of the usual spin. Consequently, the occupation ratio \( n_\downarrow/n_+ \) is still given by Eq. (A9), only now \( n_+ + n_\downarrow = 1/2 \).

In the calculation of the conductance one should take into account that since potential scattering does not contribute to the conductance and since the flip of usual spin does not cost any energy, the contributions to conductance due to processes with and without flip of spin coincide. Thus the calculation become almost trivial. First, one replaces \( J \) by \( 2J \) in Eqs. (12), and (14) due to a different definition of the strength of Kondo interaction in Eqs. (10) and (11). Second, the conductance is doubled due to the processes with flipping of the usual spin. Thus, we obtain Eq. (19).

[1] J. Kondo, Progr. Theor. Phys. 32, 37 (1964).
[2] A.S. Hewson The Kondo Problem to Heavy Fermions, Cambridge University Press (1997).
[3] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav, and M. A. Kastner, Nature (London) 391, 156 (1998); S.M. Cronenwett, T.H. Oosterkamp, and L.P. Kouwenhoven, Science 281, 540 (1998); J. Schmid, J. Weis, K. Eberl, and K. von Klitzing, Physica (Amsterdam) 256-258B, 182 (1998); Y. Ji, M. Heiblum, D. Sprinzak, D. Mahalu, and H. Shtrikman, Science 290, 779 (2000).
[4] L.I. Glazman and M.E. Raikh, JETP Lett. 47, 452 (1988); T.K. Ng and P.A. Lee, Phys. Rev. Lett. 61, 1768 (1988).
[5] D. Boese, W. Hofstetter, and H. Schoeller, Phys. Rev. B 64, 125309 (2001).
[6] V. Meden and F. Marquardt, Phys. Rev. Lett. 96, 146801 (2006).
[7] L. Borda, G. Zarrad, W. Hofstetter, B.I. Halperin, and J. von Delft, Phys. Rev. Lett. 90, 026602 (2003).
[8] P.G. Silvestrov and Y. Imry, Phys. Rev. Lett. 85, 2565 (2000).
[9] P.G. Silvestrov and Y. Imry, Phys. Rev. B 65, 035309 (2002).
[10] P. Jarillo-Herrero, J. Kong, H.S.J. van der Zant, C. Dekker, L.P. Kouwenhoven, and S. De Franceschi, Nature (London) 434, 484 (2005).
[11] M. Eto, J. Phys. Soc. Jpn. 74, 95 (2005).
[12] G. Zarrad, A. Brataas, and D. Goldhaber-Gordon, Solid State Commun. 126, 463 (2003); K. Le Hur and P. Simon, Phys. Rev. B 67, 201308(R) (2003); K. Le Hur, P. Simon, and L. Borda, ibid. 69, 045326 (2004); R. Lopez, D. Sanchez, M. Lee, M.-S. Choi, P. Simon, and K. Le Hur, ibid. 71, 115312 (2005); K. Kikoin, Y. Avishai, M.N. Kiselev, cond-mat/0407063 (to be published); M.R. Galpin, D.E. Logan, and H.R. Krishnamurthy, Phys. Rev. Lett. 94, 184606 (2005); A.L. Chudnovskiy, Europhys. Lett. 71, 673 (2005); M.-S. Choi, R. Lopez and R. Aguado, Phys. Rev. Lett. 95, 067204 (2005); M.R. Galpin, D.E. Logan, and H.R. Krishnamurthy, J. Phys.: Condens. Matter 18, 6545 (2006), G. Zarrad, Philos. Mag. 86, 2043 (2006), T. Kuzmenko, K. Kikoin, and Y. Avishai, Phys. Rev. B 73, 235310 (2006).
[13] F.D.M. Haldane, Phys. Rev. Lett. 40, 416 (1978).
[14] C.W.J. Beenakker, Phys. Rev. B. 44, 1646 (1991).
[15] R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umanovsky, and H. Shtrikman, Nature (London) 385, 417 (1997).
[16] P. Nozières, J. Phys. (Paris) 39, 1117 (1978); J. Low
Temp. Phys. **17**, 31 (1974).

[17] M. Pustilnik and L.I. Glazman, Phys. Rev. Lett. **87**, 216601 (2001).

[18] In the case of a single level in the dot and an electron with spin one should write a separate $S$-matrix for each spin orientation $\text{diag}(S) = (e^{\pm 2i\delta}, 1)$. A calculation similar to Eq. (16) now gives a conductance different from Eq. (17) by an overall factor of 2 and a replacement $2\delta \rightarrow \delta$ (Ref. [3]).

[19] P.B. Wiegmann and A.M. Tsvelick, J. Phys. C **16**, 2281 (1983); N. Andrei, K. Furuya, and J.H. Lowenstein, Rev. Mod. Phys. **55**, 331 (1983).

[20] The nonmonotonous conductance behavior considered in Ref. [17] occurs in a quantum dot with $S > 1/2$ and requires a Kondo effect developing in two stages.

[21] B. Coqblin and J.R. Schrieffer, Phys. Rev. **185**, 847 (1969).

[22] M. Eto and Y.V. Nazarov, Phys. Rev. Lett. **85**, 1306 (2000).

[23] The phases $\pm \pi/2$ are physically indistinguishable in eqs. (20) and (14). This makes these two equations consistent at the $B \rightarrow \pm \infty$ limit.

[24] V. Kashcheyevs, A. Schiller, A. Aharony, and O. Entin-Wohlman, Phys. Rev. B **75**, 115313 (2007).

[25] H.-W. Lee and S. Kim, [cond-mat/0610496](http://arxiv.org/abs/cond-mat/0610496) (to be published).