Dynamical generation of space-time signature by spontaneous symmetry breaking

F. Darabi

Department of Physics, Azarbaijan University of Tarbiat Moallem, Tabriz, 53714-161 Iran.

June 18, 2018

Abstract

The problem of dynamical generation of 4-D space-time signature at small scales and its stabilization towards Lorentzian signature at large scales is studied in the context of Higgs mechanism in a two-time scenario. It is also shown that Lorentz invariance at small scales can be violated but at large space-time scales is restored.
The initial idea of signature change was due to Hartle, Hawking and Sakharov [1]. This idea would make it possible to have both Euclidean and Lorentzian metrics in the path integral approach to quantum gravity. However, it was later shown that signature change may happen, as well, in classical general relativity [2]. This issue has recently been raised in the Brane-world scenario, as well [3]. There are two different approaches to this problem: continuous and discontinuous. In the continuous approach, in passing from Euclidean to Lorentzian region, the signature of the metric changes continuously, hence the metric becomes degenerate at the border. In the discontinuous approach, however, the metric is non-degenerate everywhere and discontinuous at the border.

Most of the works regarding the signature change dealt with situations where the signature changing metric is defined apriori on the manifold and one looks for the effects of the assumed signature change on the Einstein equations or propagation of particles in such a manifold. However, there are some other viewpoints in which the signature generation of the large scale space-time is studied and considered to be a dynamical phenomenon [4], [5], [6]. On the other hand, it is believed that while the signature of the large scale 4-D space-time is definitely Lorentzian, the phenomena of dynamical topology and signature changes at ultrashort distances can happen as the microscopic fluctuations of the space-time. This is because a more general formulation of gravitation should accommodate geometries with degenerate metrics and nontrivial topologies. It is then interesting to introduce one mechanism which accommodates both small and large scales and addresses the problem of metric signature at these hierarchial scales.

In the present letter, we propose such a mechanism for signature fluctuations at ultrashort
distances and stabilization of Lorentzian signature at large scale 4-D space-time, in the context of Higgs mechanism in a two-time scenario. We introduce a Higgs potential whose minima will define the signature of the 4-D metric. The parameter of the symmetry breaking is so chosen that it leads to a quantum oscillating signature at ultrashort distance and a definite signature at large scale. It is then discussed that the Lorentzian signature of the present large scale universe might have been generated due to a quantum tunneling effect at very early universe and that the immediate inflation could have stabilized this chosen Lorentzian metric and prevented this new baby universe from re-tunneling to an Euclidean phase. The subsequent Big Bang and the observed acceleration of the universe are then considered as different ways by which the universe could have fixed the Lorentzian metric as the preferred one.

Beside this scenario, it is also shown that at the present large scale Lorentzian universe there can be fluctuating signature at small scales which may be accompanied by a constant time-like vector that accounts for a principle violation of the Lorentz invariance at small scales. At large scales, although a same vector can in principle exist but its norm is vanishing and so is non-observable. This leaves the Lorentz invariance as an almost exact symmetry at large space-time scales.

Consider the 5-D two-time metric\(^1\)

\[
dS_{(5)}^2 = \langle \Phi \rangle dt^2 + ds^2 - dT^2, \tag{1}
\]

where \(ds^2\) accounts for 3-space metric, \(T\) is the extra time dimension, and \(\langle \Phi \rangle\) is assumed to be the vacuum expectation value of a dimensionless Higgs field with the following potential

\(^1\)Such two-time metrics are currently the subject of investigations. [7].
\[ V(\Phi) = \frac{1}{2} \alpha \Phi^2 + \frac{1}{4!} \beta \Phi^4 + \frac{1}{6!} \gamma \Phi^6, \]  

where \( \beta < 0 \) and \( \gamma > 0 \) together with \( \alpha \) are the parameters of the potential. It is assumed that the 4-dimensional metric is independent of the extra time, but the Higgs field depends merely on \( T \). The choice of \( \alpha \) is of particular importance in incorporating the notions of large and small scales in the study of signature dynamics. In fact, no absolute line of demarcation can exist between small and large scales without having a positive definite measure of distance. Therefore, we assume a characteristic size in the ultrashort distance regime, described by an absolute scale of length \( l_0 \), which acts as a sort of universal length that determines a lower bound on any scale of length probed in a measurement process. The existence of universal length \( l_0 \) is not compatible with the universal requirement of Lorentz invariance. Such a violation of Lorentz invariance may be a consequence of unification of quantum physics and gravity and is expected to manifest itself at ultrashort distances. We therefore take \( \alpha = l_0/l \) where \( l \) is the characteristic size of the region over which one measures the signature of metric.

By starting from a very large parameter \( \alpha \gg 1 \), namely \( l \ll l_0 \), one finds that the potential has one minimum at \( \langle \Phi \rangle = 0 \) as is shown in Fig.1. However, at some (critical) smaller parameter \( l = l_c < l_0 \) ( \( l_c \) is presumably the Planck length \( l_p \) ), the potential will have three minima at points where \( V(\Phi) = 0 \) (see Fig.2). One of these minima is at \( \langle \Phi \rangle = 0 \) and two others are at \( \pm \langle \Phi \rangle_c \). In other words, at \( l = l_c \) there are two phases in equilibrium with each other, one with \( \langle \Phi \rangle = 0 \) and the other with \( \langle \Phi \rangle_c \) (or \( -\langle \Phi \rangle_c \)). The phase with \( \langle \Phi \rangle = 0 \) is stable when \( l \leq l_c \), and meta-stable when \( l \) is a bit larger than \( l_c \). The order parameter, namely \( \langle \Phi \rangle \), is discontinuous at the transition \( l = l_c \). Therefore, we are dealing
with a first order phase transition where two phases can coexist, one with $<\Phi>=0$ and the other with nonzero $<\Phi>$. One may then find meta-stability so that the system can persist in the phase with $<\Phi>=0$, for the parameter $l$ very close to $l_c$. When $l=l_0$, the potential $V(\Phi)$ is unstable at $<\Phi>=0$, but has two negative minima which are stable, as Fig.3. In such case, there can never be coexistent phases at $<\Phi>=0$ and $<\Phi>\neq 0$. Therefore, the unstable phase at $<\Phi>=0$ is removed and a stable phase should be chosen out of two minima at $\pm <\Phi>_0$.

This means, when $l \ll l_0$ we have the degenerate 4-D metric

$$dS^2_{(4)} = ds^2,$$

(3)

everywhere on the manifold, before symmetry breaking. Then, at some smaller parameter $\alpha$ namely $l = l_c < l_0$ the 4-D metric is capable of taking on the following forms

$$\begin{cases}
    dS^2_{(4)} = <\Phi>_c dt^2 + ds^2, & \text{Euclidean} \\
    dS^2_{(4)} = ds^2, & \text{Degenerate} \\
    dS^2_{(4)} = -<\Phi>_c dt^2 + ds^2, & \text{Lorentzian}
\end{cases}$$

(4)
corresponding to three equal minima of the potential where $V(\Phi) = 0$, so that they can be in equilibrium with each other. The manifold at the scale $l_c$ is then capable of being degenerate, Euclidean, and Lorentzian. In other words, the signature of metric oscillates (in $T$) between the above three forms due to quantum tunneling between the minima of the potential. If the parameter $l$ begins to increase above $l_c$ the system will provisionally be meta-stable so that the metric can persist, for the parameter $l$ close to $l_c$, in coexistent phases. Finally, at $l = l_0$ the system is unstable at $<\Phi>=0$ and has two negative minima. This means, the manifold can no longer have a degenerate metric and the signature can just oscillate between Euclidean
and Lorentzian, due to quantum tunneling between the two minima. As \( l \) increases above \( l_0 \) the system becomes more stable so that for \( l \gg l_0 \) the tunneling probability approaches zero and the preferred signature is permanently fixed.

A few words on the length scales \( l_0 \) and \( l \) are in order. In one hand, the existence of a universal length \( l_0 \) in the small scale regime is in sharp contrast with the universal requirement of Lorentz invariance. On the other hand, one can not distinguish between large and small scales without having a positive definite metric. According to Blokhintsev [9], accompanying the notion of a universal length (Lorentz non-invariance) with a constant time-like vector \( N_\mu = (1,0,0,0) \), it is possible to distinguish between small and large scales in Minkowski space-time by taking the positive definite interval

\[
S^2_{(4)} = \bar{\eta}_{\mu\nu} x^\mu x^\nu, \quad \bar{\eta}_{\mu\nu} = \eta_{\mu\nu} + 2N_{\mu}N_{\nu}.
\]

Therefore, the existence of a universal length \( l_0 \) at ultrashort distance regime is inevitably related to the existence of an internal time-like vector \( N_\mu \) over the manifold. Given the Euclidean metric \( \bar{\eta}_{\mu\nu} \) one may determine the absolute size of a distance by comparing \( S_{(4)} \) with the universal length \( l_0 \). In the same way, one may determine the absolute size of the distance \( l \) and determine the meaningful value of the parameter \( \alpha \).

With no loss of generality one may take the distance \( l \) to be the radius of universe, namely the scale factor \( R \). Therefore, one may expect different phases in the potential according to the evolution of \( R \). At very early universe \( R \ll l_0 \) (or \( R < l_c \)) we have not a meaningful notion of the metric because it is degenerate. At \( R = l_c < l_0 \), the metric can fluctuate between Euclidean, degenerate and Lorentzian forms. This is plausible in the quantum gravity regime \( l_c = l_p \) and is consistent with quantum tunneling in cosmology in which the universe
tunnels from nothing \((R = 0)\), through a Euclidean region, to the planck length \(l_p\) at which the universe may fluctuate between Euclidean \((R \leq l_p)\), degenerate \((R = l_p)\) and Lorentzian \((R \geq l_p)\) forms. As long as the scale factor is close to the planck length these quantum oscillations \((\text{in } T)\) persist with no preference for Euclidean or Lorentzian metric to be the permanent one. This means the true vacuum of the potential is not yet selected. However, once some suitable initial conditions are provided for one of the quantum tunnelings to the Lorentzian region, the universe can start time \((t)\) evolution in the scale factor towards \(l_0\) and this stabilizes the Lorentzian metric because \(\alpha\) decreases. Therefore, at the scale \(R = l_0 > l_p\) the metric is definitely Lorentzian which means \(- < \Phi >_0\) is selected as the preferred minimum of the potential. However, the probability for quantum tunneling to the Euclidean region is not yet excluded, because the barrier between the two minima is not so high and wide. The new born Lorentzian universe can get rid of death \((\text{through quantum re-tunneling to the Euclidean region})\) by undergoing an inflation in the scale factor. This inflation launches the small scale factor \(R = l_0\) \((\text{presumably the Grand unification scale})\) to a distance tens of order greater than \(l_0\) in a very short period of time, namely \(10^{-35} - 10^{-33}\) seconds. Hence \(\alpha\) becomes very small and leads to a very high barrier so that the probability for quantum tunneling becomes very small, as well. In this regard, the inflation helps the Lorentzian metric to be stabilized in a tiny fraction of a second. After inflation, the big bang also causes the universe to be expanded rapidly which leads to more stabilization of the Lorentzian metric. This stabilization is continued as long as the universe expands. One may then interpret the expansion of the universe as a way to avoid re-tunneling to the Euclidean metric. The recent observed acceleration of the universe may also be addressed and justified according to this
scenario.

Now suppose there is a constant internal vector on the 4-D manifold as

\[ N_\mu = (\sqrt{\alpha} \langle \Phi \rangle |, 0, 0, 0), \]  

(6)

associated with the properties of vacuum through \( \langle \Phi \rangle \). Then, for \( R \ll l_0, \alpha \gg 1 \) we have \( N_\mu = (0, 0, 0, 0) \) corresponding to the absolute minimum at \( \langle \Phi \rangle = 0 \), before symmetry breaking. When \( R = l_c \), the parameter \( \alpha \) decreases and the vector \( N_\mu \) is found in a non-vanishing form, namely \( N_\mu = (\sqrt{\alpha} \pm <\Phi> c \mid , 0, 0, 0) \). This means that before symmetry breaking, when \( R \ll l_0 \), there is no \( N_\mu \) at all, and one is appeared at the symmetry breaking at \( R = l_c \). At \( R > l_c \) where only Euclidean and Lorentzian metrics are available through the quantum tunneling, one may use this non-vanishing \( N_\mu \) to relate the Euclidean metric \( \bar{g}_{\mu\nu} \) and Lorentzian one \( g_{\mu\nu} \)

\[
\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{2}{\alpha} N_\mu N_\nu, 
\]  

(7)

with no restriction on the norm of \( N_\mu \). This is because the permanent metric is not fixed. However, far from the critical point, \( R \gg l_c \) where the true vacuum of the system is singled out as the Lorentzian metric \( g_{\mu\nu} \), we find, by contracting Eq.(7) with \( g^{\mu\nu} \), that the internal vector has negative norm

\[ N_\mu N^\mu = -\alpha, \]

which accounts for the time-like nature of \( N_\mu \), as is required according to Blokhintsev point of view. Notice that the norm of \( N_\mu \) is almost vanishing for \( \alpha \ll 1 \), namely \( R \gg l_0 \), and becomes important at small scales \( R \simeq l_0 \). Therefore, the internal vector \( N_\mu \) is non-observable at large scales which means the Lorentz invariance is an almost exact symmetry at large distances compared with \( l_0 \).
Concluding remarks

We have discussed that as long as suitable initial conditions for time evolution of the scale factor were not satisfied at very early universe the 4-D metric could have been oscillating quantum mechanically between Euclidean and Lorentzian phases as degenerate vacuums of a Higgs potential. However, once these conditions were satisfied in one of the tunnelings into a Lorentzian vacuum (early universe), the immediate inflation and the subsequent Big Bang could have stabilized this chosen metric. In this regard, inflation, Big Bang and even the present observed acceleration of the universe can be considered as different behaviors of the universe to escape the death through re-tunneling to the Euclidean metric. We have also shown that Lorentz invariance at small scales can be violated but at large space-time scales it can be restored exactly.

Acknowledgment

This work has been supported financially by the Research Department of Azarbaijan University of Tarbiat Moallem, Tabriz, Iran.
Figure captions

Figure 1. The potential at $l \ll l_0$.

Figure 2. The potential at $l = l_c < l_0$

Figure 3. The potential at $l = l_0$
References

[1] J. B. Hartle, S. W. Hawking, Phys. Rev. D. 28 (1983), 2960.

[2] T. Dereli, R. W. Tucker, Class. Quantum Grav. 10 (1993), 365; G. F. R. Ellis, A. Sumruk, D. Coule and C. Hellaby, Class. Quantum Grav. 9 (1992), 1535; S. A. Hayward, Class. Quantum Grav. 9 (1992), 1851; M. Kossowski, M. Kriele Proc. R. Soc. Lond. A. 446 (1995), 115; C. Hellaby, T. Dray, Phys. Rev. D. 49 (1994), 5096; F. Darabi, H. R. Sepangi, Class. Quantum Grav. 16 (1999), 1565; F. Darabi, Phys. Lett. A 259 (1999), 97; M. Mohseni, Phys. Lett. A 267 (2000), 240.

[3] M. Mars, J. M. M. Senovilla, R. Vera, Phys. Rev. Lett. 86 (2001), 4219; G. W. Gibbons, A. Ishibashi, Topology and Signature Changes in Braneworlds, hep-th/0402024.

[4] R. Precacci, Nucl. Phys. B 353 (1991), 271.

[5] J. Greensite, Phys. Lett. B 300 (1993), 34; A. Carlini, J. Greensite, Phy. Rev. D 49 (1994), 866.

[6] S. D. Odintsov, A. Romeo and R. Tucker, Class. Quantum Grav. 11 (1994), 2951; E. Elizalde, S. D. Odintsov and A. Romeo, Class. Quantum Grav. 11 (1994), L61.

[7] P. Wesson, Phys. Lett. B 538 (2002), 159.

[8] M. Le Bellac, Quantum and Statistical Field Theory, (Clarendon Press, Oxford) (1991).

[9] D. I. Blokhintsev, Physics Letters 12 (1964), 272.