Spring back of infinite honeycomb sheets beyond plastic deformation

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Abstract. Cellular structures are promising for applications where high stiffness and strength are required with the minimal use of material. They are often used in applications where the plastic deformation plays an important role, such as those involving crashworthiness, energy absorption, and stents. The elastic analysis of a honeycomb sheet has been carried out in the past [1]. The present analysis extends this classical work in the elasto-plastic regime. Recoil analysis due to elastic recovery is absent from the published literature. This work aims to develop an analytical model to calculate the spring back for a simplified case, that of an infinite honeycomb sheet. An elastic-perfectly plastic material model is assumed. The recoil for a clamped beam with a load and moment applied at the free edge is analytically calculated first. This is carried out by relating the stress distribution of the cross section to the final deformed shape. The part corresponding to the elastic contribution is subsequently subtracted in order to obtain the final configuration after the external load is removed. This simple elasto-plastic analysis is then incorporated into the analysis of an infinite sheet made of uniform hexagonal cells. The translational symmetry of the lattice is exploited along with the analysis of a beam under tip loading through to plastic stage and recoil. The final shape of the struts upon the removal of the remote stress is completely determined by the plastic deformation which cannot be recovered. The expression for the beam thus obtained is then used to build an analytical model for an infinite honeycomb sheet loaded in both directions.

1. Introduction
Periodic cellular architectures are frequently found in several natural material and engineered structures. Synthetic structured materials can be found in sandwich panels, heat exchangers, cushioning foams and biological implants. In the last twenty years, they have become an important class of material since they afford light weight structures with high stiffness and strength. By minimizing the weight of an engineering system, its performance can be greatly improved such as in aircraft construction where the core of sandwich panels is made of honeycomb sheets [2]. On the other hand, by minimizing the material employed in biological implants, side effects can be drastically decreased, such as the restenosis in stenting procedure used to treat cardiovascular arterial obstructions [3].

The mechanical properties of cellular solids depend on the constituent material and the cellular architecture (relative density and cell shape). Based on the cell shape, several classifications have been made. The first is the distinction between two-dimensional and three-dimensional cells (often called “foam”). The analysis of two-dimensional cellular solids is simpler, therefore the mechanical properties can be analytically related to the parameters that describe
the unit cell structure by using Euler-Bernoulli beam analysis [2]. Further, the cellular geometry can be classified on the basis of the presence of translational symmetry in case of lattice material vs. disordered cellular solids that lack this attribute (see figure 1). The hexagonal lattice is perhaps the most frequently studied ordered structure – the regularity coupled with the apparent isotropy of this lattice being the important reasons for the great interest amongst physicists, engineers and mathematicians [1].

![Figure 1](image.png)

**Figure 1.** An example of (a) aluminium honeycomb sheet and (b) a disordered polyurethane cellular material [1].

Many engineered products and material such as cores of sandwich panels for doors or aerospace components, heat exchangers and crash absorbers possess hexagonal internal structure. Secondly, several generic properties that characterise two-dimensional cellular solids can be extended to three-dimensional geometries. Due to their geometrical regularity, the apparent mechanical properties can be easily obtained in a closed form [1]. Most work in the past has been limited to linear-elastic analysis of such structures with the exception of the collapse using a plastic hinge analysis [1]. On the other hand, many applications require detailed elasto-plastic analysis, including the calculation of elastic response followed by plastic deformation. When a plastically deformed material is unloaded, the elastic deformation is recovered leading to recoil or spring back. The work of Ashby and Gibson provided the apparent Young’s modulus, Poisson’s ratio and collapse pressure for the cellular material by using defined equations. However, plastic analysis and recoil calculations for such lattices are missing in the literature.

The purpose of this study is to extend the work by Gibson and Ashby [1] by including an analytical model that can capture the recoil for two dimensional infinite honeycomb elastic lattices. Given that the load is applied at infinity, the apparent constitutive model must provide the spring back of such structured sheets.

2. The elasto-plastic bending and spring back of an infinite honeycomb sheet subjected to uniaxial remote force

Consider a regular two-dimensional cellular hexagonal honeycomb as the one shown in figure 2(a). The honeycomb geometry is kinematically mobile, therefore the cell walls deform primarily in flexure. It is common to refer to this type of structure as bending-dominated [4]. The constitutive relationship used here is the elastic-perfectly plastic idealisation of the material behaviour.

By using thin beam theory, expressions for the deflection of the cell wall under elastic deformation have been developed in the past [1]. Even though flexure is the dominant mechanism, the stretching influences the structural response. Therefore, this model has been extended by introducing the effects of the axial force [1]. Since this model is valid only for the elastic deformation of the honeycomb cell, it is not valid when the stress exceeds the yield stress. The aim of this study is to provide an analysis valid also when the honeycomb sheet is subjected
to plastic deformation. The calculation of the beam deflection has been carried out by relating it to the stress distribution of the cross section by using the fundamental relationship between curvature and load of a cantilever beam provided by [5].

Consider the honeycomb sheet shown in figure 2(a). When a remote uniaxial stress is applied along one of the principal directions, the force experienced at the cell wall end is shown in figure 2 (c).

The resulting moment distribution of the cell wall is shown in figure 3. The mechanics of elasto-plastic deformation of beams has been studied by Yu and Johnson [5] who considered a cantilevered beam under tip load in the context of predicting spring back in the die-punch problem. We have used this analysis by exploiting the symmetry and using the outcome of [5]. As a result, the deformation of the cell wall is obtained. As expected, the deflected shape is cubic, which is obtained by integrating a fourth order equilibrium equation. By repeating this solution symmetrically for the four inclined cell walls, the final cell shape under remote stress can be obtained for the plastic phase first and then the recoil.

\[ [M] : F \sin(\theta) l/2 \]

Figure 3. Moment distribution along the cell wall under horizontal remote stress.

Now the elastic spring back after removing the horizontal stress applied at infinity is considered. According to [5], the unloading is obtained by superimposing the elastic effects caused by the application of a reverse force at the free end of the beam. The resulting qualitative shape of the cell wall is shown in figure 4.
3. Results

By using the analytical model described in the previous section, an elasto-plastic analysis followed by the elastic spring back analysis of an infinite honeycomb sheet was carried out. Consider the infinite honeycomb sheet in figure 2 (a) subjected to a horizontal stress. Figure 5 (a) shows the final shape obtained after the application of the load. The same analysis has been performed by applying the remote stress along direction perpendicular to the first. These results are plotted in figure 5 (b). As can be noticed, the sheet shrinks in the perpendicular direction of the force application and elongates in the other due to the Poisson’s ratio effect of the porous structure. The amount of this lateral response has been exactly calculated here using the analytical model described in the previous section.
Consider the same infinite honeycomb sheet subjected to remote stress. Assume that this sheet is now unloaded. When the elastic deformation is recovered, the only deformation left on the sample is the one related to the permanent plasticity. Such mechanical response is often known as spring back or recoil. Figure 6 shows the final deformed shape of the sheet. To the best of our knowledge, it is the first time that spring back of infinite lattice structures has been reported.

This model can analytically predict the recoil of an infinite honeycomb sheet. Even though hexagonal shape is the most common, several other shape geometries can be found in real life. Therefore, the next step will involve the extension of this approach to more general geometries.

![Figure 6. Springback of an infinite honycomb sheet subjected to (a) horizontal remote stress and (b) perpendicular remote stress after the unloading.](image)

4. Conclusions
Cellular materials are nowadays applied in several applications which require plastic deformation. An inevitable effect is the spring back or recoil. This needs to be accounted for in the analysis so that the shape of the structure upon removing external loading is what intended by the designer. Using a simple mechanical approach which relates the curvature to the cross section stress distribution, we have carried out the final deformation under elasto-plastic conditions. By subtracting the elastic effects from the section stress distribution, the final curvature after the unload has been calculated. One of the advantages of the proposed approach is the analytical expressions for various response quantities as opposed to computational methods such as the Finite Element method which does not provide an explicit parametric dependence of the results. Further details will be published elsewhere [6].

References
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