Enhanced optical squeezing from quasi-bound states in the continuum and Fano resonances without nonlinearity

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Keywords: optical squeezing, Fano resonances, bound states in the continuum

Abstract

To achieve a high degree of quantum noise squeezing, nonlinear optical interaction process is often employed. Here, we propose to utilize quasi-bound states in the continuum (quasi-BICs) and Fano resonances to enhance optical squeezing without nonlinearity. The theory of quantization for electromagnetic fields in the periodic nanostructure with dispersion and absorption has been developed by means of the Green’s function technique with the plane wave expansion method. The quasi-BICs and Fano resonances of radiation modes are realized by designing the photonic crystal slab structure. Based on these quasi-BICs and Fano resonances, we demonstrate that strong squeezed states can be realized by using the balanced homodyne detection scheme. The squeezing degree can be improved by more than 14 times when a weak squeezed states passes through the structure with quasi-BICs and Fano resonances. The advantage of this method is that it is not only efficient but also easy to implement because the nonlinear optical processes are not employed, which is very beneficial for the quantum information processing and precision metrology.

1. Introduction

Squeezed states of light belong to the most prominent nonclassical resources [1]. They have potential applications in quantum information systems [2–4] and precision metrology [5, 6], including gravitational wave detectors [7–9], which require unprecedented sensitivity. The precondition of these applications is to produce squeezed states with high squeezing degrees. Thus, how to generate strongly squeezed states has become an important topic for decades. So far, there are considerable ways to obtain the squeezed states of light, including the optical parametric process [10–14], four-wave mixing [15–17], cavity-QED [18–20], soliton propagation [21, 22] and so on. The commonality of these methods is that nonlinear optical processes have been employed.

On the other hand, optical bound states in the continuum (BICs) in some nanostructures have attracted extensive attention from both theoretical and experimental sides in recent years [23–41]. The BICs are known as embedded trapped modes, which correspond to discrete eigenvalues coexisting with extended modes of a continuous spectrum [42]. Originally, this concept appeared in quantum mechanics [42], but later it was extended to wave systems. A true BICs is a mathematical object with an infinite value of the Q factor and vanishing resonance width, and it can exist only in ideal lossless structures [28–30]. In practice, BICs can be realized as a quasi-BICs, when both the Q factor and resonance width become finite [38–41]. Recent investigations have shown that the quasi-BICs with high-Q factor can be realized in some optical nanostructures [38–41]. In addition, Fano resonances, typically narrow line shapes that arise as the interference of a narrow dark resonance with a broad bright one [43–45], have also been the focus of attention. The Fano resonances and BICs exhibit a rich phenomenology stemming from, respectively, their asymmetric line shapes and infinite quality factors. Although the quasi-BICs have some similar characteristic compared with Fano resonances, they exhibit some unique behaviors, for example, they have extreme Q factor and almost vanishing line width when the designed structures are close to the case of BICs. The question is whether these resonances are helpful for the generation of strong squeezed states?

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In this work, we combine the concepts of the quasi-BICs and Fano resonances with the modulation of squeezed states, and explore the possibility to enhance the optical squeezing phenomena in the linear nanostructure. Our researches are based on the quantization of the phenomenological Maxwell theory. We first extend the Green’s function theory with the plane wave expansion method to the periodic nanostructure with dispersion and absorption. Then, we design the photonic crystal (PhC) slab structure to realize the quasi-BICs and Fano resonances of radiation modes. Based on such a structure, strong squeezed states of light are realized by using the balanced homodyne detection scheme. The advantage of this method is that it is easy to implement because the nonlinear optical processes are not employed, and it is also more efficient.

The rest of this paper is arranged as follows. In section 2, we present the quantization method of electromagnetic (EM) fields in the periodic nanostructure with dispersion and absorption and the balanced homodyne detection theory. In section 3, we design the PhC slab structure to realize the BICs and provide the calculated results for the generation of strong squeezed states based on the quasi-BICs and Fano resonances. Finally, the summary is given in section 4.

2. Theory and method

We consider a PhC slab structure as shown in figure 1(a), which consists of a square lattice of air holes introduced into a high-index dielectric medium, and the corresponding lattice constant is represented by \( a \). The thickness, relative permittivity, and relative permeability of the PhC slab are denoted by \( d \), \( \varepsilon_r \), and \( \mu_r \), respectively. In this work the PhC slab is taken to be nonmagnetic (\( \mu_r = 1 \)). In the following we provide the theory of quantization for EM fields in the 2D PhC slab structure, then discuss the enhanced optical squeezing for such a structure based on the quasi-BICs and Fano resonances without nonlinearity.

2.1. Theory of quantization of EM fields in 2D PhC slab structure

We first extend the Green’s function theory with the plane wave expansion method [46, 47] to solve the phenomenological Maxwell equation [48] in the 2D periodic nanostructure as shown in figure 1(a). Here the relatively dielectric constant in the structure is periodically distributed in the \( x-y \) plane. We introduce the unit vectors of lattice \( a_1 \) and \( a_2 \), and corresponding reciprocal lattice vectors are represented by \( b_1 \) and \( b_2 \). In this work, the EM fields \( \vec{E}(\mathbf{r}, \omega) \) and \( \vec{H}(\mathbf{r}, \omega) \), the dielectric parameter \( \varepsilon(\mathbf{x}, \mathbf{y}) \) and the noise current \( \vec{J}(\mathbf{r}, \omega) \) are expanded by the plane wave due to the periodicity of the structure

\[
\vec{E}(\mathbf{r}, \omega) = \sum_{mn} \vec{E}_{mn}(z, \omega) e^{i(k_{mn,x}x + k_{mn,y}y)},
\]

\[
\vec{H}(\mathbf{r}, \omega) = \sum_{mn} \vec{H}_{mn}(z, \omega) e^{i(k_{mn,x}x + k_{mn,y}y)},
\]

\[
\varepsilon(\mathbf{x}, \mathbf{y}) = \sum_{mn} \varepsilon_{mn}(\mathbf{x}, \mathbf{y}) e^{i(G_{mn,x}x + G_{mn,y}y)},
\]

\[
\vec{J}(\mathbf{r}, \omega) = \sum_{mn} \vec{J}_{mn}(z, \omega) e^{i(k_{mn,x}x + k_{mn,y}y)},
\]

where the wave vector is expressed as \( k_{mn} = (k_{mn,x}, k_{mn,y}) = (k_{1x}, k_{2y}) + G_{mn} \) and \( G_{mn} = mb_1 + nb_2 \). Here \( \vec{E}_{mn}(z, \omega) \), \( \vec{H}_{mn}(z, \omega) \) and \( \vec{J}_{mn}(z, \omega) \) are the expansion coefficients of the EM fields and the noise current,
respectively. Using the plane wave expansion method, we substitute equations (1)–(4) into the Maxwell equation \[ \frac{\partial}{\partial z} \mathbf{E}_{mn}(z, \omega) = \sum_{ij;\gamma=x,y} T_{ij;\gamma}^{(mn)} \mathbf{H}_{ij}(z, \omega) + \mathbf{X}_{ij}^{(mn)}(z, \omega), \] (5a)
\[ \frac{\partial}{\partial z} \mathbf{H}_{mn}(z, \omega) = \sum_{ij;\gamma=x,y} T_{ij;\gamma}^{(mn)} \mathbf{E}_{ij}(z, \omega) + \mathbf{X}_{ij}^{(mn)}(z, \omega), \] (5b)
where the subscript \( \gamma \) in the matrices denotes the \( x \) or \( y \) components. If plane waves with \( M = (2N_1 + 1) \times (2N_2 + 1) \) are used to expand the EM fields and run in the range of \(-N_1 \leq i \leq N_1\), \(-N_2 \leq j \leq N_2\), then the dimensions of \( \mathbf{E}_{mn}(z, \omega) \) and \( \mathbf{H}_{mn}(z, \omega) \) are both \( 2M \times 2M \). Here, \( \mathbf{E}_{mn}(z, \omega) = (\cdots, \mathbf{E}_{mN}(z, \omega), \mathbf{E}_{mN_2}(z, \omega), \cdots)^T \) and \( \mathbf{H}_{mn}(z, \omega) = (\cdots, \mathbf{H}_{mN}(z, \omega), \mathbf{H}_{mN_2}(z, \omega), \cdots)^T \) are \( 2M \times 1 \) column matrices and the superscript \( T \) indicates the transpose of the matrices. The matrices \( T_{ij;\gamma}^{(mn)} \) and \( T_{ij;\gamma}^{(mn)} \) are both of dimensions \( 2M \times 2M \). \( T_{ij;\gamma}^{(mn)} \) and \( T_{ij;\gamma}^{(mn)} \) are written as
\[ T_{ij;\gamma}^{(mn)} = \frac{i}{\omega \epsilon_0} \begin{pmatrix} k_{mnx} e^{i m \pi (i-J)} - k_{mnx} e^{-i m \pi (i-J)} & -k_{mnx} e^{-i m \pi (i-J)} k_{bxy} + \omega^2 / \epsilon_0 \delta_{mj} & -k_{mnx} e^{i m \pi (i-J)} k_{bxy} - \omega^2 / \epsilon_0 \delta_{mj} \\ k_{mnx} e^{i m \pi (i-J)} k_{bxy} - \omega^2 / \epsilon_0 \delta_{mj} & -k_{mnx} e^{-i m \pi (i-J)} k_{bxy} + \omega^2 / \epsilon_0 \delta_{mj} & k_{mnx} e^{i m \pi (i-J)} k_{bxy} - \omega^2 / \epsilon_0 \delta_{mj} \\ \end{pmatrix}, \] (6a)
\[ T_{ij;\gamma}^{(mn)} = \frac{i}{\omega \mu_0} \begin{pmatrix} -k_{mnx} \delta_{m} e^{i m \pi (i-J)} & k_{mnx} \delta_{m} e^{i m \pi (i-J)} & -k_{mnx} \delta_{m} e^{-i m \pi (i-J)} \\ \end{pmatrix} \] (6b)
\( \mathbf{X}_{1}^{(mn)}(z, \omega) \) and \( \mathbf{X}_{2}^{(mn)}(z, \omega) \) represent the \( 2M \times 1 \) column matrices of noise current and they are expressed as
\[ \mathbf{X}_{1}^{(mn)}(z, \omega) = \begin{pmatrix} \frac{1}{\epsilon_0 \omega} \sum_{y} \delta_{m-y} e^{i m \pi (i-j)} \mathbf{J}_{by}(z, \omega) k_{mnx} \\ \frac{1}{\epsilon_0 \omega} \sum_{y} \delta_{m-y} e^{i m \pi (i-j)} \mathbf{J}_{by}(z, \omega) k_{mnx} \end{pmatrix}, \quad \text{and} \quad \mathbf{X}_{2}^{(mn)}(z, \omega) = \begin{pmatrix} \mathbf{J}_{mnx}(z, \omega) \\ -\mathbf{J}_{mnx}(z, \omega) \end{pmatrix}. \] (7)
From equations (5a) and (5b), we can obtain the matrix equation of the electric field operator
\[ \left( -\frac{\partial^2}{\partial z^2} + P \right) \mathbf{E}_{mn}(z, \omega) = \omega \mu_0 \mathbf{Y}_{mn}(z, \omega), \] (8)
where \( P = T_{ij;\gamma}^{(mn)} T_{ij;\gamma}^{(mn)} \) and \( \omega \mu_0 \mathbf{Y}_{mn}(z, \omega) = \sum_{ij;\gamma=x,y} T_{ij;\gamma}^{(mn)} \mathbf{X}_{ij}^{(mn)}(z, \omega) = \frac{\partial}{\partial z} \mathbf{X}_{ij}^{(mn)}(z, \omega) \). In the following, we use the Green’s function theory to solve the matrix equation of the electric field. The solution of equation (8) can be given by
\[ \mathbf{E}_{mn}(z, \omega) = \sum_{\alpha\beta} \tilde{\mathbf{G}}_{mn\alpha\beta}(z, \omega) \mathbf{Y}_{\alpha\beta}(z', \omega), \] (9)
where \( \tilde{\mathbf{G}}_{mn\alpha\beta}(z, \omega) \) is the (classical) Green’s function tensor that satisfies the following equation
\[ \left( -\frac{\partial^2}{\partial z^2} + P \right) \tilde{\mathbf{G}}_{mn\alpha\beta}(z, \omega) = \delta(\omega - \omega'), \] (10)
where the Green’s function tensor \( \tilde{\mathbf{G}}_{mn\alpha\beta}(z, \omega') \) is also a \( 2M \times 2M \) matrix, \( I \) is a unit matrix. In order to solve the Green’s function equation, we introduce the Fourier transforms for the \( \mathbf{E}_{mn}(z, \omega) \) and \( \mathbf{Y}_{mn}(z, \omega) \), which are determined by the following formulas
\[ \mathbf{E}_{mn}(k, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \mathbf{E}_{mn}(k, \omega) e^{ikz}, \quad \text{and} \quad \mathbf{Y}_{mn}(k, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \mathbf{Y}_{mn}(k, \omega) e^{ikz}. \] (11)
The explicit form of \( \tilde{\mathbf{G}}_{mn\alpha\beta}(z, \omega') \) is calculated in appendix A and given by
\[ \tilde{\mathbf{G}}_{mn\alpha\beta}(z, \omega') = \sum_{q=1}^{M} \sum_{i,j,y} \frac{S_{mnq\beta} S_{nq\alpha\gamma}^*}{2k_{qy}} e^{i k_{qy} (z-z')}, \] (12)
where \( -k_{qy}^2 \) is the eigenvalue of the \( P \) matrix, here subscripts \( q \) and \( \gamma \) denote the plane wave and the \( x \) or \( y \) component, respectively. The matrix \( S_{mnq\alpha\beta} = (\cdots, S_{mnq\alpha\beta}, S_{mnq\alpha\beta}, \cdots)^T \) represents the corresponding eigenvector. Substituting equations (12) into (9), we obtain
\[ \mathbf{E}_{mn}(z, \omega) = \omega \mu_0 \sum_{q} \sum_{\alpha\beta} S_{mnq\alpha\beta} \left[ e^{i k_{qy}^* (z-z')} \tilde{\mathbf{G}}_{mn\alpha\beta}(z', \omega) + e^{-i k_{qy}^* (z-z')} \tilde{\mathbf{G}}_{mn\alpha\beta}(z', \omega) \right], \] (13a)
where
\[
\hat{\alpha}^+(\mathbf{z}, \omega) = i \int_{-\infty}^{\infty} \sum_{\alpha, \beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} e^{-i \gamma_{q \gamma} r - i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} Y_{\alpha \beta}(z', \omega),
\]
(13b)
\[
\hat{\alpha}^-(\mathbf{z}, \omega) = i \int_{-\infty}^{\infty} \sum_{\alpha, \beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} e^{i \gamma_{q \gamma} r - i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} Y_{\alpha \beta}(z', \omega),
\]
(13c)
here \(k_{q \gamma} = \beta_{q \gamma} + i \gamma_{q \gamma}\), \(\beta_{q \gamma}\), and \(\gamma_{q \gamma}\) are the real and imaginary parts of the wave vector \(k_{q \gamma}\), respectively. The operator \(\hat{\alpha}^\pm(\mathbf{z}, \omega)\) denotes the amplitude operator of the electric field and it obey the following quantum Lagrangian evolutionary equations in the space domain
\[
\frac{\partial \hat{\alpha}^+(\mathbf{z}, \omega)}{\partial z} = -\gamma_{q \gamma} \hat{\alpha}^+(\mathbf{z}, \omega) + \hat{L}^+(\mathbf{z}, \omega),
\]
(14a)
\[
\frac{\partial \hat{\alpha}^-(\mathbf{z}, \omega)}{\partial z} = \gamma_{q \gamma} \hat{\alpha}^-(\mathbf{z}, \omega) + \hat{L}^-(\mathbf{z}, \omega),
\]
(14b)
with
\[
\hat{L}^+(\mathbf{z}, \omega) = \sum_{\alpha} \sum_{\beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} Y_{\alpha \beta}(z, \omega),
\]
(14c)
\[
\hat{L}^-(\mathbf{z}, \omega) = -\sum_{\alpha} \sum_{\beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} Y_{\alpha \beta}(z, \omega).
\]
(14d)
According to the commutation relations between noise current operators [49, 50], the explicit commutation relations between amplitude operators are given in appendix B and they satisfy the following commutation relations
\[
[\hat{\alpha}^+(\mathbf{z}, \omega), \hat{\alpha}^{\dagger}_\alpha q \gamma] = \frac{i}{k_{q \gamma}} \sum_{q, \gamma} \sum_{\alpha, \beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} S_{\alpha \beta q} \rho_{m-\alpha}(\omega) \times \left[ e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} - e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} \right] \delta(\omega - \omega'),
\]
(15a)
\[
[\hat{\alpha}^-(\mathbf{z}, \omega), \hat{\alpha}^{\dagger}_\alpha q \gamma] = \frac{i}{k_{q \gamma}} \sum_{q, \gamma} \sum_{\alpha, \beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} S_{\alpha \beta q} \rho_{m-\alpha}(\omega) \times \left[ e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} - e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} \right] \delta(\omega - \omega'),
\]
(15b)
\[
[\hat{\alpha}^+(\mathbf{z}, \omega), \hat{\alpha}^{\dagger}_\alpha q \gamma] = \frac{i}{k_{q \gamma}} \sum_{q, \gamma} \sum_{\alpha, \beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} S_{\alpha \beta q} \rho_{m-\alpha}(\omega) \times \left[ e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} - e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} \right] \delta(\omega - \omega'),
\]
(15c)
\[
[\hat{\alpha}^-(\mathbf{z}, \omega), \hat{\alpha}^{\dagger}_\alpha q \gamma] = \frac{i}{k_{q \gamma}} \sum_{q, \gamma} \sum_{\alpha, \beta} \frac{S^*_{\alpha \beta q}}{2k_{q \gamma}} S_{\alpha \beta q} \rho_{m-\alpha}(\omega) \times \left[ e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} - e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}'} \right] \delta(\omega - \omega'),
\]
(15d)
where \(\rho_{m-\alpha}(\omega) = \sum_{\alpha} e^{\frac{\omega}{2} \beta_{\alpha \gamma}} \beta_{\gamma m-\alpha}(\omega)\). Because the amplitude operators for different plane wave expansions are not commutative, we use the linear combination of the amplitude operators to construct new generation and annihilation operators of photon. In such a case, the generation and annihilation operators of photon satisfy the Boson commutation relations. In the following, we construct the creation and annihilation operators of photon according to the amplitude operator \(\hat{\alpha}(\mathbf{z}, \omega)\) and have
\[
\hat{\alpha}^{\dagger}(\mathbf{z}, \omega) e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} = \sum_{\alpha, \beta} F_{\alpha \beta q} \hat{\alpha}^{\dagger}_\beta q \gamma(\mathbf{z}, \omega) e^{i \mathbf{k}_{q \gamma} \cdot \mathbf{z}},
\]
(16a)
\[
\hat{\alpha}(\mathbf{z}, \omega) e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}} = \sum_{\alpha, \beta} F_{\alpha \beta q} \hat{\alpha}_\beta q \gamma(\mathbf{z}, \omega) e^{-i \mathbf{k}_{q \gamma} \cdot \mathbf{z}},
\]
(16b)
the linear combination coefficients $F_{\alpha \beta q'}^{\pm}$ and $F_{\alpha \beta q}$ can be determined by the following commutation relations
\[
\begin{align}
[\hat{a}_{mnq}^{\pm}(z, \omega), \hat{a}_{\alpha \beta q'}^{\pm}(z', \omega') &= \delta_{mn} \delta_{\beta \beta'} \delta(\omega - \omega'), \\
[\hat{a}_{mnq}^{-1}(z, \omega), \hat{a}_{\alpha \beta q'}^{-1}(z', \omega') &= \delta_{mn} \delta_{\beta \beta'} \delta(\omega - \omega').
\end{align}
\]
(17a, 17b)

Through the above process, we have completed the quantization of EM fields in the periodic nanostructure with dispersion and absorption. In order to further study the optical squeezing phenomenon in the periodic nanostructure, we need to obtain quantized input–output relation for the structure as shown in figure 1(b).

According to equations (5a), (5b) and (13a), the quantized magnetic field can be written as
\[
\hat{H}_{mn}(z, \omega) = \sum_{q'} T_{mnq'} \left[ e^{i \tilde{z} \cdot \nu} \hat{a}_{mnq'}(z, \omega) - e^{-i \tilde{z} \cdot \nu} \hat{a}_{mnq'}^{-1}(z, \omega) \right],
\]
(18)
where the matrix $T_{mnq'} = \frac{i}{\omega} S_{mnq'} \nu_k q'$. Utilizing the Maxwell EM fields boundary condition, we substitute equations (13a) and (18) into the boundary conditions, at the interface $z = z_{i-1}$, the obtained relations are written in matrix form as
\[
\begin{pmatrix}
S_{mnq'}^2 & S_{mnq'}^2 \\
T_{mnq'}^2 & T_{mnq'}^2
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{mnq}(z_{i-1}, \omega) \\
\hat{a}_{mnq}^{-1}(z_{i-1}, \omega)
\end{pmatrix}
= \begin{pmatrix}
S_{mnq'}^2 & S_{mnq'}^2 \\
T_{mnq'}^2 & T_{mnq'}^2
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{mnq}(z_{i}, \omega) \\
\hat{a}_{mnq}^{-1}(z_{i}, \omega)
\end{pmatrix} + \begin{pmatrix}
S_{mnq'}^2 & S_{mnq'}^2 \\
T_{mnq'}^2 & T_{mnq'}^2
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{mnq}(z_{i-1}, \omega) \\
\hat{a}_{mnq}^{-1}(z_{i-1}, \omega)
\end{pmatrix}
\]
(19a)
at the interface $z = z_i$, a similar method can be used to obtain a relation and it also is written in matrix form as
\[
\begin{pmatrix}
S_{mnq'}^3 & S_{mnq'}^3 \\
T_{mnq'}^3 & T_{mnq'}^3
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{mnq}(z_{i}, \omega) \\
\hat{a}_{mnq}^{-1}(z_{i}, \omega)
\end{pmatrix}
= \begin{pmatrix}
S_{mnq'}^3 & S_{mnq'}^3 \\
T_{mnq'}^3 & T_{mnq'}^3
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{mnq}(z_{i-1}, \omega) \\
\hat{a}_{mnq}^{-1}(z_{i-1}, \omega)
\end{pmatrix} + \begin{pmatrix}
S_{mnq'}^3 & S_{mnq'}^3 \\
T_{mnq'}^3 & T_{mnq'}^3
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{mnq}(z_{i-2}, \omega) \\
\hat{a}_{mnq}^{-1}(z_{i-2}, \omega)
\end{pmatrix}
\]
(19b)

It is noted that in a uniform surrounding medium the matrix $S_{mnq'}^j = \delta_{mn}(j = 1, 3)$. Utilizing the Langvin evolution relation, the electric field operator at $z = z_{i-1}$ can be represented by the electric field operator at $z = z_{i-1}$ and the operator $\Gamma_{mn}(z, \omega)$ as
\[
\begin{pmatrix}
e^{i \tilde{z} \cdot \nu} \hat{a}_{mnq}(z_{i}, \omega) \\
e^{-i \tilde{z} \cdot \nu} \hat{a}_{mnq}^{-1}(z_{i}, \omega)
\end{pmatrix} = \begin{pmatrix} e^{i k_{x \nu} d} & 0 \\
0 & e^{-i k_{x \nu} d}
\end{pmatrix} \begin{pmatrix}
e^{i \tilde{z} \cdot \nu} \hat{a}_{mnq}(z_{i-1}, \omega) \\
e^{-i \tilde{z} \cdot \nu} \hat{a}_{mnq}^{-1}(z_{i-1}, \omega)
\end{pmatrix} + \begin{pmatrix} e^{i k_{x \nu} 1/2d} \sqrt{2 \gamma^2} & 0 \\
0 & e^{-i k_{x \nu} 1/2d} \sqrt{2 \gamma^2}
\end{pmatrix} \begin{pmatrix}
\Gamma_{mnq}^{+}(\omega) \\
\Gamma_{mnq}^{-}(\omega)
\end{pmatrix}
\]
(20)

where the operators $\Gamma_{mnq}^{+}(\omega)$ and $\Gamma_{mnq}^{-}(\omega)$ can be written as
\[
\begin{align}
\Gamma_{mnq}^{+}(\omega) &= i \frac{1}{2 \gamma \nu} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz'' \gamma^{-i k_{x \nu} z''} \sum_{\alpha \beta} \frac{S_{\alpha \beta q}^{*} S_{\alpha \beta q'}}{2 k_{\alpha q'}} \Gamma_{\alpha \beta}(z'', \omega), \\
\Gamma_{mnq}^{-}(\omega) &= -i \frac{1}{2 \gamma \nu} \int_{-\frac{d}{2}}^{\frac{d}{2}} dz'' \gamma^{-i k_{x \nu} z''} \sum_{\alpha \beta} \frac{S_{\alpha \beta q}^{*} S_{\alpha \beta q'}}{2 k_{\alpha q'}} \Gamma_{\alpha \beta}(z'', \omega).
\end{align}
\]
(21a, 21b)

In order to obtain the quantized input–output relation of the amplitude operators, we employ the linear combination of the operator $\gamma_{mnq}^{\pm}(\omega)$ to construct the creation and annihilation operators of photon. Meanwhile, the new constructed operators of photon also satisfy the Boson commutation relations. Firstly, we construct the operators $\Gamma_{mnq}^{+}(\omega)$ and $\Gamma_{mnq}^{-}(\omega)$ according to the operator $\Gamma_{mnq}^{\pm}(\omega)$, they satisfy
\[
\begin{align}
\Gamma_{mnq}^{-}(\omega) &= -\Gamma_{mnq}^{+}(\omega) + \Gamma_{mnq}^{\pm}(\omega), \\
\Gamma_{mnq}^{\pm}(\omega) &= -\Gamma_{mnq}^{-}(\omega) - \Gamma_{mnq}^{\pm}(\omega).
\end{align}
\]
(22a, 22b)

Based on equations (21a) and (21b), the newly constructed operator $\gamma_{mnq}^{\pm}(\omega)$ satisfies the following commutation relations:
\[
[\Gamma_{mnq}^{\pm}(\omega), \Gamma_{\alpha \beta q'}^{\pm}(\omega')] = 2 \frac{1}{2 \gamma \nu} \frac{1}{2 \gamma \nu'} \frac{1}{2 k_{\alpha q'}} \sum_{\nu, \nu'} \frac{S_{\alpha \beta q' \nu}^{*} S_{\alpha \beta q' \nu'}}{2 k_{\alpha q'}} \gamma_{\alpha \beta}(\omega, \omega'),
\]
\[
\times \int_{-\frac{d}{2}}^{\frac{d}{2}} dz' \left[ e^{i k_{x \nu} - k_{x \nu'} z'} + e^{i k_{x \nu} + k_{x \nu'} z'} \right] \left[ \mu_{\alpha}^{2}(\omega) \delta(\omega - \omega'),
\right]
\]
(23a)
\[ \Gamma_{\alpha\beta}(\omega), \Gamma_{\alpha\beta}^+(\omega') = \frac{1}{\sqrt{2\pi\hbar}} \sum_{\nu, \rho, \tau} \frac{S_{\nu\rho\tau}^* S_{\nu\rho\tau}}{2k_{\nu\rho\tau}} \]

\[ \times \int \frac{d^3 \mathbf{z}'}{\sqrt{2\pi\hbar}} \left[ e^{i k_{\nu\rho\tau} \cdot \mathbf{z}'} - e^{i k_{\nu\rho\tau} \cdot (\mathbf{z} - \mathbf{z}')} \right] \rho_{\nu\rho}^0(\mathbf{z} - \mathbf{z}') \delta(\omega - \omega'), \]

\[ \Gamma_{\alpha\beta}(\omega), \Gamma_{\alpha\beta}^+(\omega') = 0, \quad \Gamma_{\alpha\beta}^-(\omega), \Gamma_{\alpha\beta}^+(\omega') = 0. \] 

In order to find the nondependent terms and make them to satisfy the Boson commutation relations, we introduce the operators \( \tilde{\Gamma}_{\alpha\beta}(\omega) \) and \( \tilde{\Gamma}_{\alpha\beta}^+(\omega) \), which are the linearly transformation of the operators \( \Gamma_{\alpha\beta}(\omega) \) and \( \Gamma_{\alpha\beta}^+(\omega) \). And they can be expressed by

\[ \tilde{\Gamma}_{\alpha\beta}^+(\omega) = \sum_{\alpha\beta} \tilde{\epsilon}^+_{\alpha\beta} \tilde{\Gamma}_{\alpha\beta}(\omega), \quad \tilde{\Gamma}_{\alpha\beta}^-(\omega) = \sum_{\alpha\beta} \tilde{\epsilon}^-_{\alpha\beta} \tilde{\Gamma}_{\alpha\beta}(\omega), \]

where the coefficients \( \tilde{\epsilon}^+_{\alpha\beta} \) and \( \tilde{\epsilon}^-_{\alpha\beta} \) can be determined by equations (23a)–(23c). Thus, the new constructed operators satisfy the following relations

\[ \tilde{\Gamma}_{\alpha\beta}^+(\omega), \tilde{\Gamma}_{\alpha\beta}^+(\omega') = \delta_{\alpha\beta} \delta_{\omega\omega'} \]

\[ \tilde{\Gamma}_{\alpha\beta}^-(\omega), \tilde{\Gamma}_{\alpha\beta}^-(\omega') = \delta_{\alpha\beta} \delta_{\omega\omega'}. \]

According to equations (19a), (19b), (20) and the Boson commutation relation inside the PhC slab in equations (25a) and (25b), we can obtain the quantized input–output relation

\[ \begin{align*}
O_{\alpha\beta}(z_1, \omega) & = (T_{11} \frac{1}{2} \frac{1}{2} T_{21}) O_{\alpha\beta}(z_1, \omega) + (A_{11} \frac{1}{2} A_{12}) \tilde{\Gamma}_{\alpha\beta}(\omega) \\
O_{\alpha\beta}(z_1, \omega) & = (T_{11} \frac{1}{2} \frac{1}{2} T_{21}) O_{\alpha\beta}(z_1, \omega) + (A_{11} \frac{1}{2} A_{12}) \tilde{\Gamma}_{\alpha\beta}(\omega)
\end{align*} \]

For convenience, we introduce operators \( \tilde{O}_{\alpha\beta}(z_1, \omega) = e^{i/\hbar} T_{11} O_{\alpha\beta}(z_1, \omega) \) and \( \tilde{O}_{\alpha\beta}(z_1, \omega) = e^{i/\hbar} T_{21} O_{\alpha\beta}(z_1, \omega) \), where the scattering coefficients can be expressed as

\[ T_{11} = (a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1}, \]

\[ T_{21} = -(a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1}, \]

\[ T_{12} = -(a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1}, \]

\[ T_{22} = (a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1}, \]

and the absorption coefficients can be written as

\[ A_{11} = (a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} e^{i/\hbar} a_{12} a_{11}^{-1} + 1) e^{i/\hbar} e^{2/\hbar} \frac{2}{\sqrt{3/2}}, \]

\[ A_{12} = (a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} e^{i/\hbar} a_{12} a_{11}^{-1} - 1) e^{i/\hbar} e^{2/\hbar} \frac{2}{\sqrt{3/2}}, \]

\[ A_{21} = (a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} (1 + e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1}) e^{i/\hbar} e^{2/\hbar} \frac{2}{\sqrt{3/2}}, \]

\[ A_{22} = (a_{11} - e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1})^{-1} (1 + e^{i/\hbar} a_{12} a_{11}^{-1} e^{i/\hbar} a_{21} a_{12}^{-1}) e^{i/\hbar} e^{2/\hbar} \frac{2}{\sqrt{3/2}}, \]

with

\[ \begin{align*}
\frac{1}{2} S_{\alpha\beta}^{(2)} S_{\alpha\beta}^{(0)} & = -T_{\alpha\beta}^{(2)} T_{\alpha\beta}^{(0)}, \quad \frac{1}{2} S_{\alpha\beta}^{(2)} S_{\alpha\beta}^{(0)} = -T_{\alpha\beta}^{(2)} T_{\alpha\beta}^{(0)} \\
\frac{1}{2} T_{\alpha\beta}^{(2)} T_{\alpha\beta}^{(0)} & = -S_{\alpha\beta}^{(2)} S_{\alpha\beta}^{(0)}, \quad \frac{1}{2} T_{\alpha\beta}^{(2)} T_{\alpha\beta}^{(0)} = -S_{\alpha\beta}^{(2)} S_{\alpha\beta}^{(0)}
\end{align*} \]

\[ \frac{1}{2} S_{\alpha\beta}^{(2)} S_{\alpha\beta}^{(0)} = -T_{\alpha\beta}^{(2)} T_{\alpha\beta}^{(0)} \]

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2.2. The method of the balanced homodyne detection to measure the squeezed states of light in the PhC slab structure

In the following, we give the method utilizing the balanced homodyne detection scheme to enhance the optical squeezing [51]. The positive frequency component (\( F_{\alpha\beta}(z, t) \)) of electric field operator at a detector in the region \( z > d \) is expressed as

\[ F_{\alpha\beta}(z, t) = \frac{1}{\sqrt{2\pi\hbar}} \sum_{\nu, \rho, \tau} \frac{S_{\nu\rho\tau}^* S_{\nu\rho\tau}}{2k_{\nu\rho\tau}} \]

\[ \times \int \frac{d^3 \mathbf{z}'}{\sqrt{2\pi\hbar}} \left[ e^{i k_{\nu\rho\tau} \cdot \mathbf{z}'} - e^{i k_{\nu\rho\tau} \cdot (\mathbf{z} - \mathbf{z}') \right] \rho_{\nu\rho}^0(\mathbf{z} - \mathbf{z}') \delta(\omega - \omega'), \]

\[ \Gamma_{\alpha\beta}(\omega), \Gamma_{\alpha\beta}^+(\omega') = 0, \quad \Gamma_{\alpha\beta}^-(\omega), \Gamma_{\alpha\beta}^+(\omega') = 0. \]
where $\hat{b}_L^+(\omega)$ and $\hat{b}_L^-(\omega)$ denote the upward-propagating input field operator and the downward-propagating outgoing field operator, respectively. The schematic arrangement of the propagation geometry for the operators of the EM waves is given in figure 1(b). According to the above derived quantized input-output relation, the operator for the down-wards propagation outgoing field is given in terms of the up-wards and down-wards input fields by

$$\hat{b}_L^+(\omega) = R \hat{b}_L^-(\omega) + T \hat{b}_R^-(\omega) + \hat{F}(\omega),$$  \hspace{1cm} (31)$$

where $R$ and $T$ denote the transmission and reflection coefficients and they are determined by equations (27a) and (27b), respectively. The operator $\hat{F}(\omega)$ denotes the noise associated with the dissipation in the PhC slab, according to the input-output relation its form is given by

$$\hat{F}(\omega) = i \frac{1}{\sqrt{2\gamma_{eq}}} \sum_{mn} \sum_{z} \hat{b}_L^{+}(\omega) \frac{\hat{E}_eq\omega^2}{\pi} \int [-\frac{1}{2d} \int dz^{*} [\xi e^{-i\omega z^2} + \zeta e^{i\omega z^2}] f(z', \omega),$$  \hspace{1cm} (32)$$

where $\xi = (A_1 - A_{12})$ and $\zeta = (A_1 + A_{12})$, the absorption coefficients $A_1$ and $A_{12}$ are determined by equations (28a) and (28b), respectively. $f(z', \omega)$ is spatially distributed Boson vector field operator with the following commutation relation

$$[\hat{f}(z, \omega), \hat{f}(z', \omega')] = \delta(z - z') \delta(\omega - \omega').$$  \hspace{1cm} (33)$$

At finite temperature $T$, the Boson vector field operator has the expectation values

$$\langle f \hat{f}(z, \omega) \hat{f}(z', \omega') \rangle_f = \langle f \hat{f}(z, \omega) \hat{f}(z', \omega') \rangle_f = 0,$$  \hspace{1cm} (34a)$$

$$\langle f \hat{f}(z, \omega) \hat{f}(z, \omega) \rangle_f = \langle f \hat{f}(z, \omega) \hat{f}(z, \omega) \rangle_f = 0,$$  \hspace{1cm} (34b)$$

$$\langle f \hat{f}(z, \omega) \hat{f}(z', \omega') \rangle_f = \theta(\omega, T) \delta(z - z') \delta(\omega - \omega'),$$  \hspace{1cm} (34c)$$

where $\theta(\omega, T) = [e^{\omega/kT} - 1]^{-1}$ denotes the mean number of thermal photons at frequency $\omega$ and temperature $T$. The $f$) indicates the states of a dissipative reservoir inside the slab, which are more formally represented by a statistical mixture, and the noise operator commutation relation satisfies the consistency

$$\langle \hat{F}(\omega), \hat{F}^+(\omega') \rangle = [1 - |R|^2 - |T|^2] \delta(\omega - \omega').$$  \hspace{1cm} (35)$$

The balanced homodyne detection scheme [51] has been used to measure the squeezed states of light in such a structure. For the field on the downwards side of the slab, the difference between the integrated photocurrents in the two arms of a balanced detector can be indicated by the following operator [52]

$$\hat{O} = i \int_{t_i}^{t_{i} + T} dt \{ \hat{b}_L^+(t) \hat{a}_{LO}(t) - \hat{b}_L(t) \hat{a}_{LO}^+(t) \},$$  \hspace{1cm} (36)$$

where $\hat{a}_{LO}(t)$ denotes the field of the local oscillator, which is assumed to be in a coherent state $|\langle \alpha_{LO}\rangle|$ with a single-mode amplitude of the form

$$\alpha_{LO}(t) = F_{LO}^{1/2} e^{\phi_{LO} + i \omega_{LO} t}.$$  \hspace{1cm} (37)$$

Here $F_{LO}$, $\phi_{LO}$ and $\omega_{LO}$ show the mean photon flux, phase and frequency of the local oscillator, respectively. For simplicity, we consider a special case, when the local oscillator is much stronger than the signal, the measurement operator in equation (36) can be denoted as a dimensionless homodyne electric field operator that depicts the property of the signal measured at the detector

$$\hat{O} = (F_{LO} T_{LO})^{1/2} \hat{E} (\phi_{LO}, \omega_{LO}),$$  \hspace{1cm} (38)$$

where the electric field operator $\hat{E} (\phi_{LO}, \omega_{LO})$ can be expressed as

$$\hat{E} (\phi_{LO}, \omega_{LO}) = \frac{e^{i\phi_{LO}}}{\sqrt{2\pi T_{LO}}} \int d\omega [\hat{b}_{L}^+(\omega) e^{i(\omega - \omega_{LO})t_0} \times \xi(\omega - \omega_{LO}) + \text{h.c.}],$$  \hspace{1cm} (39)$$

with

$$\xi(\omega - \omega_{LO}) = e^{i(\omega - \omega_{LO})t_0} - 1 \omega - \omega_{LO}.$$  \hspace{1cm} (40)$$

We focus on the quantum noise properties of the output field and the variance of the field transmitted through to the down of the PhC slab can then be determined by
\[
\langle (\Delta E(\phi_{LO}, \omega_{LO}))^2 \rangle_{\text{out}} = \frac{1}{2\pi T_0} \int_0^{\infty} \omega \left| \xi(\omega - \omega_{LO}) \right|^2 + \frac{1}{\pi T_0} \int_0^{\infty} \omega' \int_0^{\infty} \omega \left\{ \langle \hat{b}_L^+(\omega), \hat{b}_L(\omega') \rangle \right\} \times \xi(\omega - \omega_{LO}) \xi^*(\omega') e^{-i(\omega - \omega_{LO})T_0} - \text{Re}[\langle \hat{b}_L^+(\omega), \hat{b}_L^+(\omega') \rangle] e^{2i\phi_{LO} \xi(\omega - \omega_{LO})} \\
\times \xi(\omega' - \omega_{LO}) e^{i(\omega' - \omega_{LO})T_0} - 2i(\omega - \omega_{LO})T_0 \right\].
\]

(41)

With the standard notation \(\{A, B\} \equiv \langle AB \rangle - \langle A \rangle \langle B \rangle\) for a correlator. The expectation values inside the integrals in equation (41) are over a product state that comprises the incident fields of the state traveling upwards \(|L\rangle\) and downwards \(|R\rangle\) to the slab. We take \(|L\rangle\) as a continuous-mode squeezed vacuum state, such as that produced by a degenerate parametric amplifier pumped at frequency \(2\Omega\) \[33, 54\],

\[
|L\rangle = \hat{S}(\{\rho(\omega), \varphi(\omega)\})|0\rangle,
\]

where \(\varphi(\omega)\) and \(\rho(\omega)\) are the phase and strength, respectively. At the same time, the signal field traveling downward to the PhC is chosen as a squeezed coherent state with the same pump frequency \(2\Omega\)

\[
|R\rangle = \hat{D}(\{\alpha(\omega)\})\hat{S}(\{\sigma(\omega), \varphi(\omega)\})|0\rangle,
\]

where the coherent operator can be denoted as

\[
\hat{D}(\{\alpha(\omega)\}) = \exp \left\{ \int d\omega \alpha(\omega) \hat{b}_R^+(\omega) - \text{h.c.} \right\},
\]

(45)

where \(\alpha(\omega) = \langle \alpha(\omega) \rangle \exp[i\varphi(\omega)]\) represents the complex amplitude of the coherent component of the state \(|R\rangle\), and the parameters \(\varphi(\omega)\) and \(\sigma(\omega)\) illustrate the phase and the amplitude, respectively. The expectation values in equation (41) are now evaluated with the use of equation (31) as

\[
\langle \hat{b}_L^+(\omega), \hat{b}_L(\omega') \rangle = \langle [R(\omega)]^2 \sinh^2 \rho(\omega) + |T(\omega)|^2 \sinh^2 \sigma(\omega) \rangle \delta(\omega - \omega'),
\]

\[
\langle \hat{b}_L^+(\omega), \hat{b}_L^+(\omega') \rangle = \frac{1}{2} \delta(\omega + \omega' - 2\Omega) \times \langle [R^2(\omega)] \sinh 2\rho(\omega) e^{i\varphi(\omega)} + T^2(\omega) \sinh 2\sigma(\omega) e^{i\varphi(\omega)} \rangle.
\]

(46a)

(46b)

When the local oscillator frequency is equal to the central frequency \(\Omega\) of the squeezing, for sufficiently long \(T_0\) in which case the products of the \(\xi(\omega - \omega_{LO})\) in equation (40) tend to the delta function \(\delta(\omega - \omega_{LO})\), then the measured field variance in equation (40) simplifies to

\[
\chi_{VA}(\omega_{LO}, \phi_{LO}) = \langle (\Delta E(\phi_{LO}, \omega_{LO}))^2 \rangle_{\text{out}} = 1 + 2 \left[ |R|^2 \sinh^2 \rho + |T|^2 \sinh^2 \sigma \right] - \text{Re}[e^{2i\phi_{LO}} R^2 e^{-i\varphi} \sinh 2\rho + T^2 e^{-i\varphi} \sinh 2\sigma].
\]

(47)

It is noted that for convenience we have omitted the various functional dependencies from the coefficients on the right-hand side in equation (47), and the frequency-dependent function are all evaluated at \(\omega_{LO}\). For the input fields \(|L\rangle\) and \(|R\rangle\), those for traveling upwards and downwards have

\[
\chi_L(\omega_{LO}, \phi_{LO}) = \langle (\Delta E(\phi_{LO}, \omega_{LO}))^2 \rangle_L = 1 + 2 \left| R \right|^2 \sinh^2 \rho - \text{Re}[e^{2i\phi_{LO}} R^2 e^{-i\varphi} \sinh 2\rho],
\]

\[
\chi_R(\omega_{LO}, \phi_{LO}) = \langle (\Delta E(\phi_{LO}, \omega_{LO}))^2 \rangle_R = 1 + 2 \sinh^2 \sigma - \text{Re}[e^{2i\phi_{LO}} e^{-i\varphi} \sinh 2\sigma].
\]

(48a)

(48b)

Meanwhile, for presenting our results clearly, we use a logarithmic scale for the noise relative to the ordinary vacuum level, it is expressed as \[12\]

\[
S = 10 \log_{10} \left[ \frac{\chi_{VA}(\omega_{LO}, \phi_{LO})}{\chi_{SN}(\omega_{LO})} \right],
\]

(49)

where \(\chi_{SN}(\omega_{LO}) = 1\) denotes the vacuum noise level.

3. Results and discussions

In this section, we present numerical results for the enhancement of the squeezed states of light from the 2D PhC slab structure. In the calculations, we take silicon nitride \(Si_3N_4\) as a component of the PhC slab and its relative permittivity is taken as \(\varepsilon_s = 4.07\) \[55\], the thickness and the radii of the air of the slab are chosen as \(d = 1.20a\) and \(r = 0.235a\), respectively, and the lattice constant \(a = 700\) nm. For such a slab structure, BICs and Fano resonances can be observed at a certain wavelength range by choosing appropriate parameters. Because the absorption of silicon nitride material is very weak in the wavelength range of our investigation and it can be neglected, thus in our calculation we only consider a nonabsorption structure.
In the following, we test the enhancement of squeezed states of light by using the quasi-BICs and Fano resonances. Thus, we let a weak squeezed state of light with $S_0 = -0.50$ dB be incident on the designed PhC slab as shown in figure 1. Figure 3(a) shows the calculated results for the quantum quadrature squeezing spectrum as a function of the wavelength at the incidence angle $\theta = 0^\circ$. The solid line and dashed line correspond to the cases with phases of incident squeezed states $\phi = 0^\circ$ and $45^\circ$, respectively. The parameters of PhC slab are taken identical with those in figure 2. From figure 3(a), it is observed clearly that the strong squeezed state is located at the wavelength $\lambda = 751.657$ nm, which corresponds to the wavelength of the quasi-BICs. The degree of squeezing is enhanced by about 14 times of magnitude at $\phi = 45^\circ$ compared with that of the incident squeezed state of light.

For comparison, the corresponding squeezing spectra around the Fano resonance, where the resonance peak is located at $\lambda = 801.313$ nm, are also plotted in the figure 3(b). It is found that the squeezing degree is enhanced by about 11 times of magnitude compared with that of the input squeezed state of light. Meanwhile, in addition to high degree of squeezing, it is interesting that the resonance peaks of the squeezing spectra in the case of quasi-BICs have narrowed relative line width. The relative line width can be depicted by $\Delta \lambda / \lambda$ in the spectrum;
here $\lambda_c$ denotes the center wavelength of the resonance peak and $\Delta \lambda$ indicates the full width at half-maximum of the peak. Comparing figure 3(a) with (b), we find that the relative line width $\frac{\Delta \lambda}{\lambda_c}$ at the quasi-BICs is several times narrower than that at the Fano resonance.

In order to further explore the influences of geometric parameters of PhC slab on the phenomena, in figure 4 we provide the calculated results of the quantum quadrature squeezing spectra as a function of the wavelength $\lambda$ for different tunable variables of the structure. Figures 4(a) and (b) correspond to the cases with different radii of air holes around the quasi-BICs and Fano resonance, respectively. It can be seen that the quantum quadrature squeezing spectra show blue shift with the increase of $r$ for all cases of the quasi-BICs and Fano resonance. In contrast, the quantum quadrature squeezing spectra show red shift with the increase of $d$, which the effects of the PhC slab thickness on the phenomena are given in figures 4(c) and (d) for the quasi-BICs and Fano resonance, respectively. This means that we can always obtain strong squeezing states of light in any required wavelength by designing the structure parameters of the PhC slab, which is very convenient for precision metrology and quantum communication.

The above discussions show that the degrees of squeezing using quasi-BICs are always bigger than that using Fano resonances. In fact, if we change the structure parameters of the PhC slab, the best squeezing can also be obtained by using Fano resonances. Figure 5(a) displays the calculated transmission coefficient $t$ as a function of the wavelength $\lambda$. The parameters are chosen to be the following: $d = 1.20a$, $r = 0.20a$ and $a = 700$ nm, and the incidence angle of the EM beams is taken as $0^\circ$. It is clearly observed that many sharp resonant peaks appear, which correspond to various Fano resonances. In the following, we utilize the Fano resonance marked by the arrow in figure 5(a) to enhance the optical squeezing. Figure 5(b) shows the corresponding quantum quadrature squeezing spectra as a function of the wavelength around such a Fano resonance. The degree of optical squeezing in such a case is enhanced by about 15 times of magnitude compared with that of the input squeezed states of light. In addition, the influences of the structure parameters on the squeezed states of light are also considered, the calculated results are shown in the figures 5(c) and (d). Similar blue shift and red shift with the change of $r$ and $d$ are observed again. Finally, we would like to point out that the squeezed states can not be directly generated by using our designed structures because we utilize linear materials, only modulations for the squeezed state can be realized. However, such modulations are important. This is because the weak squeezed states are easy to be generated experimentally and strong squeezed states are difficult to be realized.
4. Summary

In summary, we have designed a PhC slab structure to enhance the optical squeezing based on the quasi-BICs and Fano resonances without nonlinearity. In order to calculate the optical squeezing in such a slab structure, the rigorous Green's function theory with the plane wave expansion methods has been developed. Meanwhile, the universal balanced homodyne detection scheme has been introduced to measure the squeezed states of light. We have found that the degree of optical squeezing can be enhanced about by 14 times of magnitude compared with that of the input weak squeezed states of light using the quasi-BICs and Fano resonances. This is different from the previous investigations on the enhancement of squeezing states of light utilizing the nonlinear optical process. The influences of the structure parameters on the optical squeezing have also been discussed. The realized squeezing spectra are sensitive to the phase and structure parameters of the incident squeezed states, and exhibit narrowed relative line width. We believe these phenomena are very beneficial for the precision metrology and quantum communication processing.

Acknowledgments

This work was supported by the National Natural Science Foundation of China through Grant Nos. 91850205 and 11574031.

Appendix A. Derivation of Green’s function in the quantization process

In the appendix, we give the detailed derivation of Green’s function in the quantization process. $\hat{G}_{m,n;\alpha \beta}(z, z', \omega)$ denotes the (classical) Green’s function tensor and satisfies the following equation

$$\left(-\frac{\partial^2}{\partial z^2} + \mathcal{P}\right) \hat{G}_{m,n;\alpha \beta}(z, z', \omega) = i\delta(z - z'),$$

(A1)
where \( G_{mnc,\beta}(z, z', \omega) \) is a \( 2M \times 2M \) matrix and \( I \) is a unit matrix. In order to solve the Green’s function, we introduce the Fourier transforms for the \( \tilde{E}_{mn}(z, \omega) \) and \( \tilde{Y}_{mn}(z, \omega) \) in equation (8), which are determined by the following formulas

\[
\tilde{E}_{mn}(z, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \tilde{E}_{mn}(k, \omega) e^{ikz}, \quad \text{and} \quad \tilde{Y}_{mn}(z, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \tilde{Y}_{mn}(k, \omega) e^{ikz}. \tag{A2}
\]

Then we substitute equation (A2) into the (8), we obtain

\[
(k^2I + P) \tilde{E}_{mn}(k, \omega) = i\omega \mu_0 \tilde{Y}_{mn}(k, \omega), \tag{A3}
\]

the corresponding Green’s function tensor satisfies the following equation

\[
(k^2I + P) G_{mnc,\beta}(k, k', \omega) = i\delta(k - k'), \tag{A4}
\]

where Green’s function \( G_{mnc,\beta}(k, k', \omega) \) is also a \( 2M \times 2M \) matrix, \( \tilde{E}_{mn}(k, \omega) \) and \( \tilde{Y}_{mn}(k, \omega) \) denote column vectors. The eigenvalue of the matrix \( P = -k^2 \tau \), where subscripts \( q \) and \( \gamma \) denote the plane wave and the \( x \) or \( y \) component, respectively. The matrix \( S_{mnc,q\gamma} = (\cdots, S_{mnc,q\gamma}, S_{mnc,q'\gamma}, \cdots)^T \) represents the corresponding eigenvector and the matrix \( S_{mnc,q\gamma} \) satisfy the following relation

\[
\sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma} S_{mnc,q'\gamma}^* \delta(k - k') = \delta_{mnc} \delta_{\alpha\beta} \delta(k - k'). \tag{A5}
\]

From equation (5), the Green’s function \( G_{wmnc,\beta}(k, k', \omega) \) can be obtained as

\[
G_{wmnc,\beta}(k, k', \omega) = \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{wmnc,q\gamma}^* S_{wmnc,q'\gamma} \delta(k - k') - \frac{1}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{wmnc,q\gamma}^* S_{wmnc,q'\gamma} \left( \frac{1}{k - k_{\gamma} + i\delta} + \frac{1}{k + k_{\gamma} + i\delta} \right) e^{i(k(z-z'))}, \tag{A6}
\]

where \( \delta \) is a infinitesimal. Then the Green’s function \( G_{mnc,\beta}(z, z', \omega) \) can be calculated according to the Green’s function \( G_{mnc,\beta}(k, k', \omega) \), we have

\[
G_{mnc,\beta}(z, z', \omega) = \frac{2\pi i}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma}^* S_{mnc,q'\gamma} e^{ik_{\gamma} (z-z')} = \frac{2\pi i}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma}^* S_{mnc,q'\gamma} e^{i(k_{\gamma} + i\delta)(z-z')}, \tag{A7}
\]

if \( z > z' \), then

\[
G_{mnc,\beta}(z, z', \omega) = \frac{2\pi i}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma}^* S_{mnc,q'\gamma} e^{i(k_{\gamma} + i\delta)(z-z')} = \frac{2\pi i}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma}^* S_{mnc,q'\gamma} e^{i(k_{\gamma} + i\delta)(z-z')}, \tag{A8}
\]

if \( z < z' \), we also assume \( k' = -k \), then

\[
G_{mnc,\beta}(z, z', \omega) = \frac{2\pi i}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma}^* S_{mnc,q'\gamma} \int_{-\infty}^{\infty} (-1) dk \left( \frac{1}{k - k_{\gamma} - i\delta} - \frac{1}{k + k_{\gamma} + i\delta} \right) e^{ik(z-z')}, \tag{A9}
\]

Combined with equations (A8) and (A9), we obtain

\[
G_{mnc,\beta}(z, z', \omega) = \frac{i}{2} \sum_{q=1}^{M} \sum_{\gamma=x,y} S_{mnc,q\gamma}^* S_{mnc,q'\gamma} e^{i(k_{\gamma} + i\delta)(z-z')}, \tag{A10}
\]

**Appendix B. Derivation of the commutation relations between amplitude operators in the periodic nanostructure**

In this section, we give the commutation relations between amplitude operators. In order to obtain the commutation relations of the amplitude operators, we firstly calculate the commutation relations between the
noise current operators. The noise current operator \( \hat{J}(\mathbf{r}, \omega) \) and the Boson vector field operator \( \hat{J}(\mathbf{r}, \omega) \) satisfy the following relation
\[
\hat{J}(\mathbf{r}, \omega) = \frac{\hbar g \omega^2}{\pi} \hat{\varepsilon}(\mathbf{r}, \omega) \hat{J}(\mathbf{r}, \omega),
\]
where \( \hat{\varepsilon}(\mathbf{r}, \omega) \) illustrates the imaginary part of the relative permittivity and the Boson vector field operator \( \hat{J}(\mathbf{r}, \omega) \) satisfies the following commutation relations
\[
[\hat{J}_i(\mathbf{r}, \omega), \hat{J}_j^+(\mathbf{r}', \omega')] = \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega'),
\]
\[
[\hat{J}_i^+(\mathbf{r}, \omega), \hat{J}_j^+(\mathbf{r}', \omega')] = [\hat{J}_i(\mathbf{r}, \omega), \hat{J}_j(\mathbf{r}', \omega')] = 0.
\]

Inserting the expressions of plane wave expansions for the noise current operator and the Boson vector field operator into the commutation relations of the noise current operator, we obtain
\[
\hat{J}_{mn}(z, \omega) = \frac{\hbar g \omega^2}{\pi} \sum_{\alpha} \sqrt{\lambda} \hat{f}_{m-n, \alpha}(z, \omega).
\]

where \( \lambda_{m-n, \alpha} = \hbar g \omega^2 \delta_{m-n, \alpha}(\omega) \). In the following, we give the commutation relations between amplitude operators. Based on the equations (13b), (13c), (B3) and (B6), we have
\[
[\hat{a}_{mn\alpha q}^+(z, \omega), \hat{a}_{13q}'^+(z', \omega')] = \delta_{\alpha q} \hat{\rho}_{m-n, \alpha}(\omega) \delta(z - z') \delta(\omega - \omega').
\]

Then, we can obtain
\[
[\hat{a}_{mnq}^-(z, \omega), \hat{a}_{13q}'^+(z', \omega')] = \int_{-\infty}^{\infty} dz' \sum_{q'} \sqrt{\lambda} \hat{f}_{m-n, \alpha}(z, \omega) e^{i k_{mnq}^-(z-z')} e^{i k_{13q}^+(z'-z')} \hat{f}_{mn}(z', \omega).
\]

Integrating equation (B8), we get
\[
[\hat{e}_{mnq}^-(z, \omega), \hat{e}_{13q}'^+(z', \omega')] = \frac{i}{\hbar} \sum_{k_{mnq}} \sum_{k_{13q}} \hat{S}_{mnq} \hat{S}_{13q}^* \rho_{m-n, \alpha}(\omega) \delta(\omega - \omega') e^{i k_{mnq}^-(z-z')} e^{i k_{13q}^+(z'-z')} \theta(z - z').
\]

where \( \theta(z - z') = e^{i k_{mnq}^-(z-z')}; \) and \( \theta(z' - z) = e^{i k_{13q}^+(z'-z')} \). Similarly, we can derive other commutation relations between amplitude operators.

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