Charge Radii of the Meson and Baryon Octets in Quenched and Partially Quenched Chiral Perturbation Theory

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(Dated: October 29, 2021)

Abstract

We calculate the electric charge radii of the $SU(3)$ pseudoscalar mesons and the $SU(3)$ octet baryons in quenched and partially quenched chiral perturbation theory. We work in the isospin limit, up to next-to-leading order in the chiral expansion, and to leading order in the heavy baryon expansion. The results are necessary for the extrapolation of future lattice calculations of meson and baryon charge radii. We also derive expressions for the nucleon and pion charge radii in $SU(2)$ flavor away from the isospin limit.

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I. INTRODUCTION

The study of hadronic electromagnetic form factors at low momentum transfer provides important insight into the non-perturbative structure of QCD. A model-independent tool to study QCD at low energies is chiral perturbation theory ($\chi$PT), which is an effective field theory with low-energy degrees of freedom, e.g., the meson octet in $SU(3)$ flavor or the pion triplet in the case of $SU(2)$ flavor. $\chi$PT assumes that these mesons are the pseudo-Goldstone bosons that appear from spontaneous breaking of chiral symmetry from $SU(3)_L \otimes SU(3)_R$ down to $SU(3)_V$. Observables receive contributions from both long-range and short range physics; in $\chi$PT the long-range contribution arises from the (non-analytic) structure of pion loop contributions, while the short-range contribution is encoded in a number of low-energy constants that appear in the chiral Lagrangian and are unconstrained in $\chi$PT. These low-energy constants must be determined from experiment or lattice simulations.

Notable progress toward measuring the proton and neutron form factors has been made in recent years and high precision data are available (see [1, 2] for references). Experimental study of the remaining octet baryons, however, is much harder. The charge radius of the $\Sigma^-$ has only recently been measured [3]. Although more experimental data for the other baryon electromagnetic observables can be expected in the future, progress will be slow as the experimental difficulties are significant. Theory, however, may have a chance to catch up. While quenched lattice calculations have already appeared [1, 5, 6, 7, 8, 9], with the advance of lattice gauge theory, we expect partially quenched calculations for many of these observables in the near future. One problem that currently and foreseeably plagues these lattice calculations is that they cannot be performed with the physical masses of the light quarks. Therefore, to make physical predictions, it is necessary to extrapolate from the heavier quark masses used on the lattice (currently on the order of the strange quark mass) down to the physical light quark masses. For lattice calculations that use the quenched approximation of QCD (QQCD), where the fermion determinant that arises from the path integral is set to one, quenched chiral perturbation theory ($Q\chi$PT) [10, 11, 12, 13, 14, 15, 16] has been developed to aid in the extrapolation. The problem with the quenched approximation is that the Goldstone boson singlet is no longer affected by the $U(1)_A$ anomaly as in QCD. In other words, the QQCD equivalent of the $\eta'$ that is heavy in QCD remains light and must be included in the $Q\chi$PT Lagrangian. This requires the addition of new operators and hence new low-energy constants. In general, the low-energy constants appearing in the $Q\chi$PT Lagrangian are unrelated to those in $\chi$PT and extrapolated quenched lattice data is unrelated to QCD. In fact, several examples show that the behavior of meson loops near the chiral limit is frequently misrepresented in $Q\chi$PT [17, 18, 19, 20]. We find this is additionally true for the meson and baryon charge radii.

These problems can be remedied by using partially quenched lattice QCD (PQQCD). Unlike QQCD, where the masses of quarks not connected to external sources are set to infinity, these “sea quark” masses are kept finite in PQQCD. The masses of the sea quarks can be varied independently of the valence quark masses; usually they are chosen to be heavier. Sea quarks are thereby kept as dynamical degrees of freedom and the fermion determinant is no longer equal to one. By efficaciously giving the sea quarks larger masses it is much less costly to calculate observables than in ordinary QCD. The low-energy effective theory of PQQCD is $PQ\chi$PT [21, 22, 23, 24, 25, 26, 27, 28]. Since PQQCD retains a $U(1)_A$ anomaly, the equivalent to the singlet field in QCD is heavy (on the order of the chiral symmetry breaking scale $\Lambda_\chi$) and can be integrated out [21, 22]. Therefore, the low-energy
constants appearing in PQ\textchiPT are the same as those appearing in \textchiPT. By fitting PQ\textchiPT to partially quenched lattice data one can determine these constants and actually make physical predictions for QCD. PQ\textchiPT has been used recently to study heavy meson \cite{29} and octet baryon observables \cite{30,31,32}.

While there are a number of lattice calculations for observables such as the pion form factor \cite{4,33,34} or the octet baryon magnetic moments \cite{35,36} that use the quenched approximation, there are currently no partially quenched simulations. However, given the recent progress that lattice gauge theory has made in the one-hadron sector and the prospect of simulations in the two-hadron sector \cite{37,38,39,40,41}, we expect to see partially quenched calculations of the electromagnetic form factors in the near future.

The paper is organized as follows. First, in Section II we review PQ\textchiPT including the treatment of the baryon octet and decuplet in the heavy baryon approximation. In Section III we calculate the charge radii of the meson and baryon octets in both Q\textchiPT and PQ\textchiPT up to next-to-leading (NLO) order in the chiral expansion. We use the heavy baryon formalism of Jenkins and Manohar \cite{42,43}, treat the decuplet baryons as dynamical degrees of freedom, and keep contributions to lowest order in the heavy baryon mass, \(M_B\). These calculations are done in the isospin limit of \(SU(3)\) flavor. For completeness we also provide the PQ\textchiPT result for the charge radii for the \(SU(2)\) chiral Lagrangian with non-degenerate quarks in the Appendix. In Section IV we conclude.

II. PQ\textchiPT

In PQQCD the quark part of the Lagrangian is written as \cite{21,22,23,24,25,26,27,28}

\[
\mathcal{L} = \sum_{a,b=u,d,s} \bar{q}_a (i \slashed{D} - m_q)_{ab} q_b + \sum_{\tilde{a},\tilde{b}=\tilde{u},\tilde{d},\tilde{s}} \bar{\tilde{q}}_{\tilde{a}} (i \slashed{D} - m_{\tilde{q}})_{\tilde{a}\tilde{b}} \tilde{q}_{\tilde{b}} + \sum_{a,b=\tilde{j},\tilde{l},\tilde{r}} \bar{q}_{\text{sea},a} (i \slashed{D} - m_{\text{sea}})_{ab} q_{\text{sea},b} + \sum_{j,k=u,d,s,\tilde{u},\tilde{d},\tilde{s},\tilde{j},\tilde{l},\tilde{r}} \bar{Q}_j (i \slashed{D} - m_Q)_{jk} Q_k. \tag{1}
\]

Here, in addition to the fermionic light valence quarks \(u, d,\) and \(s\) and their bosonic counterparts \(\tilde{u}, \tilde{d},\) and \(\tilde{s},\) three light fermionic sea quarks \(j, l,\) and \(r\) have been added. These nine quarks are in the fundamental representation of the graded group \(SU(6|3)\) \cite{44,45,46} and have been accommodated in the nine-component vector

\[
Q = (u, d, s, j, l, r, \tilde{u}, \tilde{d}, \tilde{s}) \tag{2}
\]

that obeys the graded equal-time commutation relation

\[
Q_i^\alpha(x)Q_j^{\beta\dagger}(y) - (-1)^{\eta_j \eta_k} Q_j^{\beta\dagger}(y)Q_i^\alpha(x) = \delta^{\alpha\beta}\delta_{ij}\delta^3(x - y), \tag{3}
\]

where \(\alpha\) and \(\beta\) are spin and \(i\) and \(j\) are flavor indices. The graded equal-time commutation relations for two \(Q\’s\) and two \(Q^{\dagger}\’s\) can be written analogously. The grading factor

\[
\eta_k = \begin{cases} 
1 & \text{for } k = 1, 2, 3, 4, 5, 6 \\
0 & \text{for } k = 7, 8, 9 \end{cases} \tag{4}
\]

takes into account the different statistics for fermionic and bosonic quarks. The quark mass matrix is given by

\[
m_Q = \text{diag}(m_u, m_d, m_s, m_j, m_l, m_r, m_u, m_d, m_s) \tag{5}
\]
so that diagrams with closed ghost quark loops cancel those with valence quarks. Effects of virtual quark loops are, however, present due to the contribution of the finite-mass sea quarks. In the limit \( m_j = m_u, \ m_l = m_d, \) and \( m_r = m_s \) QCD is recovered.

It has been recently realized \([47]\) that the light quark electric charge matrix \( Q \) is not uniquely defined in PQQCD. The only constraint one imposes is for the charge matrix \( Q \) to have vanishing supertrace. Thus as in QCD, no new operators involving the singlet component are subsequently introduced. Following \([30]\) we use

\[
Q = \text{diag} \left( \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, q_j, q_l, q_r, q_j, q_l, q_r \right) \tag{6}
\]

so that QCD is recovered in the limit \( m_j \to m_u, \ m_l \to m_d, \) and \( m_r \to m_s \) independently of the \( q \)’s.

### A. Mesons

For massless quarks, the Lagrangian in Eq. (1) exhibits a graded symmetry \( SU(6|3)_L \otimes SU(6|3)_R \otimes U(1)_V \) that is assumed to be spontaneously broken down to \( SU(6|3)_V \otimes U(1)_V \). The low-energy effective theory of PQQCD that emerges by expanding about the physical vacuum state is PQ\( \chi \)PT. The dynamics of the emerging 80 pseudo-Goldstone mesons can be described at lowest order in the chiral expansion by the \( \mathcal{O}(E^2) \) Lagrangian

\[
\mathcal{L} = \frac{f^2}{8} \text{str} \left( D_\mu \Sigma^\dagger D_\mu \Sigma \right) + \lambda \text{str} \left( m_Q \Sigma + m_Q^\dagger \Sigma^\dagger \right) + \alpha \partial_\mu \Phi_0 \partial^\mu \Phi_0 - \mu_0^2 \Phi_0^2 \tag{7}
\]

where

\[
\Sigma = \exp \left( \frac{2i\Phi}{f} \right) = \xi^2, \tag{8}
\]

\[
\Phi = \begin{pmatrix} M & \chi^\dagger \\ \chi & \bar{M} \end{pmatrix} \tag{9}
\]

\( \Phi \) is defined in the quark basis and normalized such that \( \Phi_{12} = \pi^+ \) (see, for example, \([30]\)). Upon expanding the Lagrangian in (7) one finds that to lowest order the mesons with quark content \( Q\bar{Q}' \) are canonically normalized when their masses are given by

\[
m_{QQ'}^2 = \frac{4\lambda}{f^2} (m_Q + m_{Q'}). \tag{10}
\]

The flavor singlet field given by \( \Phi_0 = \text{str}(\Phi)/\sqrt{6} \) is, in contrast to the \( Q\chi \)PT case, rendered heavy by the \( U(1)_A \) anomaly and can therefore be integrated out in \( \chi \)PT. Analogously its mass \( \mu_0 \) can be taken to be on the order of the chiral symmetry breaking scale, \( \mu_0 \to \Lambda\chi \). In this limit the flavor singlet propagator becomes independent of the coupling \( \alpha \) and deviates from a simple pole form \([21,22]\).
B. Baryons

Just as there are mesons in PQCD with quark content $Q_i Q_j$ that contain valence, sea, and ghost quarks, there are baryons with quark compositions $Q_i Q_j Q_k$ that contain all three types of quarks. Restrictions on the baryon fields $B_{ijk}$ come from the fact that these fields must reproduce the familiar octet and decuplet baryons when $i, j, k = 1-3$. To this end, one decomposes the irreducible representations of $SU(6|3)_V$ into irreducible representations of $SU(3)_{val} \otimes SU(3)_{sea} \otimes SU(3)_{ghost} \otimes U(1)$. The method to construct the octet baryons is to use the interpolating field

$$B^\gamma_{ijk} \sim \left( Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} - Q_i^{\alpha,a} Q_j^{\gamma,c} Q_k^{\beta,b} \right) \epsilon_{abc} (C\gamma_5)_{\alpha\beta},$$

which when restricted to $i, j, k = 1-3$ has non-zero overlap with the octet baryons. Using the commutation relations in Eq. (3) one sees that $B_{ijk}$ satisfies the symmetries

$$B_{ijk} = (-1)^{1+\eta_i \eta_j} B_{skj},$$

$$0 = B_{ijk} + (-1)^{1+\eta_i \eta_j} B_{jik} + (-1)^{1+\eta_i \eta_j + \eta_j \eta_k + \eta_k \eta_i} B_{kji}.$$  (12)

The spin-1/2 baryon octet $B_{ijk} = B^\gamma_{ijk}$, where the indices $i, j, k$ are restricted to 1-3, is contained as a $(8,1,1)$ of $SU(3)_{val} \otimes SU(3)_{sea} \otimes SU(3)_{ghost}$ in the 240 representation. The octet baryons, written in the familiar two-index notation

$$B = \left( \begin{array}{ccc} \frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & \Sigma^- \\ \Sigma^+ & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & \Xi^0 \\ \Sigma^- & \Xi^0 & - \frac{2}{\sqrt{6}} \Lambda \end{array} \right),$$

are embedded in $B_{ijk}$ as

$$B_{ijk} = \frac{1}{\sqrt{6}} \left( \epsilon_{ijl} B_{kl} + \epsilon_{ikl} B_{jl} \right).$$

Besides the conventional octet baryons that contain valence quarks, $qqq$, there are also baryon fields with sea and ghost quarks contained in the 240, e.g., $q_{sea} \tilde{q}$. Since we are only interested in calculating one-loop diagrams that have octet baryons in the external states, we will need only the $B_{ijk}$ of two valence and one sea quark or two valence and one ghost quark. We use the construction in [30].

Similarly, the familiar spin-3/2 decuplet baryons are embedded in the 165. Here, one uses the interpolating field

$$T^{\alpha,\mu}_{ijk} \sim \left( Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} + Q_i^{\beta,b} Q_j^{\gamma,c} Q_k^{\alpha,a} + Q_i^{\gamma,c} Q_j^{\alpha,a} Q_k^{\beta,b} \right) \epsilon_{abc} (C\gamma^\mu)_{\beta\gamma}$$

that describes the 165 dimensional representation of $SU(6|3)_V$ and has non-zero overlap with the decuplet baryons when the indices are restricted to $i, j, k = 1-3$. Due to the commutation relations in Eq. (3), $T_{ijk}$ satisfies the symmetries

$$T_{ijk} = (-1)^{1+\eta_i \eta_j} T_{skj} = (-1)^{1+\eta_i \eta_j + \eta_j \eta_k + \eta_k \eta_i} T_{kji}.$$  (16)

The decuplet baryons are then readily embedded in $T$ by construction: $T_{ijk} = T^\gamma_{ijk}$, where the indices $i, j, k$ are restricted to 1-3. They transform as a $(10,1,1)$ under
SU(3)_{\text{val}} \otimes SU(3)_{\text{sea}} \otimes SU(3)_{\text{ghost}}$. Because of Eq. (16), \(T_{ijk}\) is a totally symmetric tensor. Our normalization convention is such that \(T_{111} = \Delta^{++}\). For the spin-3/2 baryons consisting of two valence and one ghost quark or two valence and one sea quark, we use the states constructed in [30].

At leading order in the heavy baryon expansion, the free Lagrangian for the \(B_{ijk}\) and \(T_{ijk}\) is given by [15]

\[
L = i (\overline{B} v \cdot DB) + 2\alpha_M (\overline{B} B M) + 2\beta_M (\overline{B} M B) + 2\sigma_M (\overline{B} B) \text{str}(M) + i (\overline{T} v \cdot DT) + \Delta (\overline{T} \cdot T) - 2\sigma_M (\overline{T} \cdot T) \text{str}(M), \tag{17}
\]

where \(M = \frac{1}{2} (\xi \xi^\dagger m_Q + \xi^\dagger m_Q \xi)\). The brackets in (17) are shorthands for field bilinear invariants originally employed in [15]. To lowest order in the chiral expansion, Eq. (17) gives the propagators

\[
\frac{i}{v \cdot k}, \quad \frac{i P_{\mu\nu}}{v \cdot k - \Delta} \tag{18}
\]

for the spin-1/2 and spin-3/2 baryons, respectively. Here, \(v\) is the velocity and \(k\) the residual momentum of the heavy baryon which are related to the momentum \(p\) by \(p = M_B v + k\). \(M_B\) denotes the (degenerate) mass of the octet baryons and \(\Delta\) the decuplet–baryon mass splitting. The polarization tensor

\[
P_{\mu\nu} = (v^\mu v^\nu - g_{\mu\nu}) - \frac{4}{3} S^\mu S^\nu \tag{19}
\]

reflects the fact that the Rarita-Schwinger field \((T^\mu)_{ijk}\) contains both spin-1/2 and spin-3/2 pieces; only the latter remain as propagating degrees of freedom (see [42], for example).

The Lagrangian describing the relevant interactions of the \(B_{ijk}\) and \(T_{ijk}\) with the pseudo-Goldstone mesons is

\[
L = 2\alpha (\overline{B} S^\mu B A_\mu) + 2\beta (\overline{B} S^\mu A_\mu B) + \sqrt{\frac{3}{2}} C [\overline{T}^\nu A_\nu B] + \text{h.c.} \tag{20}
\]

The axial-vector and vector meson fields \(A^\mu\) and \(V^\mu\) are defined by analogy to those in QCD:

\[
A^\mu = \frac{i}{2} (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi) \quad \text{and} \quad V^\mu = \frac{1}{2} (\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi). \tag{21}
\]

The latter appears in Eq. (20) in the covariant derivatives of \(B_{ijk}\) and \(T_{ijk}\) that both have the form

\[
(D^\mu B)_{ijk} = \partial^\mu B_{ijk} + (V^\mu)_d B_{ijk} + (-)^{\eta(\eta_j + \eta_m)} (V^\mu)_m B_{ijn} + (-)^{\eta(\eta_j + \eta_n)} (V^\mu)_kn B_{ijn}. \tag{22}
\]

The constants \(\alpha\) and \(\beta\) are easily calculated in terms of the constants \(D\) and \(F\) that are used for the \(SU(3)_{\text{val}}\) analogs of these terms in QCD. Restricting the indices of \(B_{ijk}\) to \(i, j, k = 1, 2, 3\) one easily identifies

\[
\alpha = \frac{2}{3} D + 2F \quad \text{and} \quad \beta = \frac{5}{3} D + F. \tag{23}
\]
FIG. 1: Loop diagrams contributing to the octet meson charge radii in PQ$\chi$PT. Octet mesons are denoted by a dashed line, singlets (hairpins) by a crossed dashed line, and the photon by a wiggly line. Only the third diagram has $q^2$ dependence and therefore contributes to the charge radius.

III. CHARGE RADII

In this Section we calculate the charge radii in PQ$\chi$PT and Q$\chi$PT. The basic conventions and notations for the mesons and baryons in PQ$\chi$PT have been laid forth in the last section; Q$\chi$PT has been extensively reviewed in the literature [10, 11, 12, 13, 14, 15, 16].

A. Octet Meson Charge Radii

The electromagnetic form factor $G_X$ of an octet meson $\phi_X$ is required by Lorentz invariance and gauge invariance to have the form

$$\langle \phi_X(p')|J^\mu_{em}(|\phi_X(p)) = G_X(q^2)(p + p')^\mu \tag{24}$$

where $q^\mu = (p' - p)^\mu$ and $p$ ($p'$) is the momentum of the incoming (outgoing) meson. Conservation of electric charge protects it from renormalization, hence at zero momentum transfer $eG_X(0) = Q_X$, where $Q_X$ is the charge of $\phi_X$. The charge radius $r_X$ is related to the slope of $G_X(q^2)$ at $q^2 = 0$, namely

$$< r_X^2 > = 6 \frac{d}{dq^2} G_X(q^2)|_{q^2=0}. \tag{25}$$

There are three terms in the $O(E^4)$ Lagrangian

$$\mathcal{L} = \alpha_4 \frac{8\lambda}{f^2} \text{str}(D_\mu \Sigma D^\mu \Sigma) \text{str}(m_Q \Sigma + m_Q^\dagger \Sigma^\dagger) + \alpha_5 \frac{8\lambda}{f^2} \text{str}(D_\mu \Sigma D^\mu \Sigma(m_Q \Sigma + m_Q^\dagger \Sigma^\dagger)) + i\alpha_9 \text{str}(L_{\mu\nu} D^\mu \Sigma D^\nu \Sigma^\dagger + R_{\mu\nu} D^\mu \Sigma^\dagger D^\nu \Sigma) + \ldots \tag{26}$$

that contribute to meson form factors at tree level. Here $L_{\mu\nu}$, $R_{\mu\nu}$ are the field-strength tensors of the external sources, which for an electromagnetic source are given by

$$L_{\mu\nu} = R_{\mu\nu} = e \mathcal{Q}(\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu) + ie^2 \mathcal{Q}^2 [\mathcal{A}_\mu, \mathcal{A}_\nu]. \tag{27}$$

2 In short, there are only valence and ghost quarks in the theory and they have the same mass and charge pairwise so that diagrams with disconnected quark loops are zero. The singlet is light and has to be retained in the theory. It is treated perturbatively and resulting observables generally depend upon the parameters $\alpha$ and $\mu_0$. 

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Unlike \( \chi PT \), where the low-energy constants are unique and have no known connection to \( \chi PT \), in PQ\( \chi PT \) the parameters in (26) are the dimensionless Gasser-Leutwyler coefficients of \( \chi PT \) which can be seen by looking at mesons that contain sea quarks only.

To calculate the charge radii to lowest order in the chiral expansion one has to include operators of \( \mathcal{L} \) in (7) to one-loop order  [see Figs. (1) and (2)] and operators of (26) to tree level. Using dimensional regularization, where we have subtracted \( \frac{1}{\epsilon} + 1 - \gamma + \log 4\pi \), we find in PQ\( \chi PT \) for the \( \pi^+ \)

\[
G^{PQ}_{\pi^+}(q^2) = 1 - \frac{1}{16\pi^2 f^2} [ 2F_{u_j} + F_{u_r} ] + \alpha_9 \frac{4}{f^2 q^2}, \tag{28}
\]

which interestingly does not depend on the charges of the sea and ghost quarks, \( q_j, q_l, q_r \).

For the \( K^+ \) we find

\[
G^{PQ}_{K^+}(q^2) = 1 + \frac{1}{16\pi^2 f^2} \left[ (\frac{1}{3} - q_{jl}) F_{uu} - (\frac{4}{3} - q_{jl}) F_{uj} - (\frac{2}{3} - q_r) F_{ur} + \left(\frac{1}{3} + q_r\right) F_{ss} ight. \\
- \left(\frac{2}{3} - q_{jl} + q_r\right) F_{us} - \left(\frac{2}{3} + q_{jl}\right) F_{js} - \left(\frac{1}{3} + q_r\right) F_{rs} \bigg] + \alpha_9 \frac{4}{f^2 q^2} \tag{29}
\]

and for the \( K^0 \) we find

\[
G^{PQ}_{K^0}(q^2) = \frac{1}{16\pi^2 f^2} \left[ (\frac{1}{3} - q_{jl}) F_{uu} + (\frac{2}{3} + q_{jl}) F_{uj} + \left(\frac{1}{3} + q_r\right) F_{ur} + \left(\frac{1}{3} + q_r\right) F_{ss} ight. \\
- \left(\frac{2}{3} - q_{jl} + q_r\right) F_{us} - \left(\frac{2}{3} + q_{jl}\right) F_{js} - \left(\frac{1}{3} + q_r\right) F_{rs} \bigg]. \tag{30}
\]

Here \( q_{jl} = q_j + q_l \) and we have defined

\[
F_{QQ'} = \frac{q^2}{6} \log \frac{m^2_{QQ'}}{\mu^2} - m^2_{QQ'} \mathcal{F} \left( \frac{q^2}{m^2_{QQ'}} \right), \tag{31}
\]

where the function \( \mathcal{F}(a) \) is given by

\[
\mathcal{F}(a) = \left(\frac{a}{6} - \frac{2}{3}\right) \sqrt{1 - \frac{4}{a}} \log \sqrt{1 - \frac{a}{4} + i\epsilon - 1} + \frac{5a}{18} - \frac{4}{3}. \tag{32}
\]

The first derivative of \( F_{QQ'} \) at \( q^2 = 0 \), needed to calculate the charge radii, becomes

\[
6 \frac{d}{dq^2} F_{QQ'}|_{q^2=0} = \log \frac{m^2_{QQ'}}{\mu^2} + 1. \tag{33}
\]
Charge conjugation implies

\[
G^{PQ}_{\pi^-} = -G^{PQ}_{\pi^+}, \quad G^{PQ}_{K^-} = -G^{PQ}_{K^+}, \quad \text{and} \quad G^{PQ}_{K^0} = -G^{PQ}_{\bar{K}^0},
\]  

(34)

which we have also verified at one-loop order. The form factors of the flavor diagonal mesons are zero by charge conjugation invariance. In the limit \(m_j \to \bar{m}, m_r \to m_s\) we recover the QCD result \([18, 44]\) as expected.

It is interesting to note, that duplicating these calculations for \(Q\chi PT\) shows that there is no meson mass dependence at this order. Specifically we find

\[
G^{Q+}_{\pi}(q^2) = -G^{Q-}_{\pi}(q^2) = G^{Q+}_{K}(q^2) = -G^{Q-}_{K}(q^2) = 1 + \frac{4}{f^2} a_9^Q q^2,
\]

(35)

and the form factors of the neutral mesons are zero. Here we annotate the quenched constant \(a_9^Q\) with a “Q” since its numerical value is different from the one in Eq. (26). Eq. (35) reflects that flavor-singlet loops do not contribute to the \(q^2\)-dependence at this order; thus the virtual quark loops are completely removed by their ghostly counterparts. This can readily be seen by considering the quenched limit of Eqs. (28)–(30). The meson mass independence reveals once again the pathologic nature of the quenched approximation and seriously puts into question \(\chi PT\) extrapolations to the physical pion mass.

**B. Octet Baryon Charge Radii**

The electromagnetic form factors at or near zero momentum transfer that enable the extraction of the baryon magnetic moments and charge radii have been frequently investigated in QCD \([43, 50, 52, 53, 54, 55, 56, 57, 58]\). There are also recent quenched and partially quenched calculations of the octet baryon magnetic moments in \(Q\chi PT\) and \(PQ\chi PT\) \([19, 30, 59]\). Here, we extend these calculations to the octet baryon charge radii. We retain spin-3/2 baryons in intermediate states since formally \(\Delta \sim m_\pi\).

Using the heavy baryon formalism \([42, 43]\), the baryon matrix element of the electromagnetic current \(J^\mu\) can be parametrized in terms of the Dirac and Pauli form factors \(F_1\) and \(F_2\), respectively, as

\[
\langle B(p') | J^\mu | B(p) \rangle = \overline{u}(p') \left\{ v^\mu F_1(q^2) + \frac{[S^\mu, S^\nu]}{M_B} q_\nu F_2(q^2) \right\} u(p) \tag{36}
\]

with \(q = p' - p\). The Sachs electric and magnetic form factors defined as

\[
G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_B^2} F_2(q^2) \tag{37}
\]

\[
G_M(q^2) = F_1(q^2) + F_2(q^2) \tag{38}
\]

are particularly useful. The baryon charge \(Q\), electric charge radius \(<r_E^2>\), and magnetic moment \(\mu\) can be defined in terms of these form factors by

\[
Q = G_E(0), \quad <r_E^2> = 6 \frac{d}{dq^2} G_E(q^2)\bigg|_{q^2=0}, \quad \text{and} \quad \mu = G_M(0) - Q. \tag{39}
\]

Here the baryon charge \(Q\) is in units of \(e\).
1. PQχPT

Let us first consider the calculation of the octet baryon charge radii in PQχPT. Here, the leading tree-level correction to the magnetic moments come from the dimension-5 operators

\[ \mathcal{L} = \frac{ie}{2M_B} \left[ \mu_\alpha \left( \mathcal{B}[S_\mu, S_\nu] \mathcal{B}Q \right) + \mu_\beta \left( \mathcal{B}[S_\mu, S_\nu] \mathcal{Q}B \right) \right] F^{\mu\nu} \] (40)

which can be matched on the QCD Lagrangian upon restricting the baryon field indices to 1-3

\[ \mathcal{L} = \frac{ie}{2M_B} \left[ \mu_D \text{tr}(\mathcal{B}[S_\mu, S_\nu]) \{Q, B\} + \mu_F \text{tr}(\mathcal{B}[S_\mu, S_\nu]) [Q, B] \right] F^{\mu\nu} \] (41)

where

\[ \mu_\alpha = \frac{2}{3} \mu_D + 2 \mu_F \quad \text{and} \quad \mu_\beta = -\frac{5}{3} \mu_D + \mu_F \] (42)

at tree level. The magnetic moments contribute the so-called Foldy term to charge radii via \( F_2(0) \) in Eq. (37). Likewise, further leading tree-level corrections to the charge radii come from the dimension-6 operators

\[ \mathcal{L} = \frac{e}{\Lambda_\chi^2} \left[ c_\alpha \left( \mathcal{B}\mathcal{B}Q \right) + c_\beta \left( \mathcal{B}\mathcal{Q}B \right) \right] v_\mu \partial_\nu F^{\mu\nu} \] (43)

and the parameters \( c_+ \) and \( c_- \), defined by

\[ c_\alpha = \frac{2}{3} c_+ + 2 c_- \quad \text{and} \quad c_\beta = -\frac{5}{3} c_+ + c_- \] (44)

are the same as those used in QCD. Here, we take the chiral symmetry breaking scale \( \Lambda_\chi \sim 4\pi f \) for the purpose of power counting. The NLO contributions arise from the one-loop diagrams shown in Figs. (3) and (4). To calculate the charge radii we need the form factors \( F_1 \) to first order in \( q^2 \) and \( F_2(0) \) we find

\[ < r^2_E > = -\frac{6}{\Lambda_\chi^2} (Qc_- + \alpha_D c_+) + \frac{3}{2M_B^2} (Q\mu_F + \alpha_D \mu_D) \]

\[ - \frac{1}{16\pi^2 f^2} \sum_X \left[ A_X \log \frac{m_X^2}{\mu^2} - 5 \beta_X \log \frac{m_X^2}{\mu^2} + 10 \beta_X' \mathcal{G}(m_X, \Delta, \mu) \right]. \] (45)

Here, we have defined the function \( \mathcal{G}(m, \Delta, \mu) \) by

\[ \mathcal{G}(m, \Delta, \mu) = \log \frac{m^2}{\mu^2} - \frac{\Delta}{\sqrt{\Delta^2 - m^2 + i\epsilon}} \log \frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}. \] (46)

Note that in Eq. (45) the only loop contributions we keep are those non-analytic in \( m_X \).

The parameters for the tree-level diagrams are listed in Table I. The computed values for the \( \beta_X, \beta_X', \) and \( A_X \) coefficients that appear in Eq. (45) are listed for the octet baryons in Tables II–IX. The corresponding values for the \( \Lambda \Sigma^0 \) transition are given in Table X. In each table we have listed the values corresponding to the loop meson that has mass \( m_X \). If a particular meson is not listed then the values for \( \beta_X, \beta_X', \) and \( A_X \) are zero.4

---

3 Here we use \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).

4 We have defined the coefficients \( \beta_X \) and \( \beta_X' \) to correspond to those defined in [30] where \( \mu = Q\mu_F + \alpha_D \mu_D + \frac{M_B}{g} \sum_X [\beta_X m_X + \beta_X' \mathcal{F}(m_X, \Delta, \mu)] \) and the function \( \mathcal{F}(m_X, \Delta, \mu) \) is given in [30].
FIG. 3: Loop diagrams contributing to the baryon magnetic moments and charge radii. A thin (thick) solid line denotes an octet (decuplet) baryon. The last two diagrams in the first row contribute to both magnetic moments and charge radii; the magnetic moment part of which has already been calculated in \[30\]. The first diagram in row 1 contributes $q^2$ dependence only to $F_1$ and therefore is relevant for the charge radii. The remaining diagrams have no $q^2$ dependence. These along with the wave function renormalization diagrams in Fig. 4 maintain charge non-renormalization.

FIG. 4: Wave function renormalization diagrams needed to maintain baryon electric charge non-renormalization.

### TABLE I: Tree-level contributions in QCD, QQCD, and PQQCD

|      | $Q$ | $\alpha_D$ |
|------|-----|-------------|
| $p, \Sigma^+$ | 1   | $\frac{1}{3}$ |
| $n, \Xi^0$    | 0   | $\frac{-2}{3}$ |
| $\Sigma^0$    | 0   | $\frac{1}{3}$ |
| $\Sigma^-, \Xi^-$ | $-1$ | $\frac{-1}{3}$ |
| $\Lambda$      | 0   | $\frac{-1}{3}$ |
| $\Sigma^0\Lambda$ | 0   | $\frac{1}{\sqrt{3}}$ |
therefore of higher order in the chiral expansion. We find

As with the meson case, the diagram where the photon couples to the closed meson loop does not contribute as their contribution to the charge radii is of the form \((QCD)\). Therefore we annotate them with a \(Q\). Additional terms involving hairpins \([15, 19]\) Eqs. (41) and (43) contribute, however, their low-energy coefficients cannot be matched onto QCD. Therefore, we list them with a \(Q\). Additional terms involving hairpins \([15, 19]\) do not contribute as their contribution to the charge radii is of the form \((\mu_b^2/\Lambda^4)\) log \(m_q\) and therefore of higher order in the chiral expansion. We find

\[
<r_E^2> = -\frac{6}{\Lambda_X^4} (Qc^Q + \alpha_Dc_\perp^Q) + \frac{3}{2M_B^2} (Q\mu_F^Q + \alpha_D\mu_B^Q) + \frac{1}{16\pi^2 f^2} \sum_X \left[ 5 \beta_X^Q \log \frac{m_X^2}{\mu^2} - 10 \beta_X^Q' G(m_X, \Delta, \mu) \right].
\]

As with the meson case, the diagram where the photon couples to the closed meson loop does not contribute to baryon charge radii in the quenched case, \(c.f., A_X\) is zero. The remaining coefficients appearing in Eq. (47) are listed in Table V and stem from meson loops formed solely from valence quarks.

**TABLE V**: The coefficients \(\beta_X, \beta_X', \) and \(A_X\) in SU(3) flavor PQ\(\chi PT\) for the proton.

| \(X\) | \(\beta_X\) | \(\beta_X'\) | \(A_X\) |
|------|-------------|-------------|--------|
| \(\pi\) | \(-\frac{1}{9} (7D^2 + 6DF - 9F^2) - \frac{1}{3} (5D^2 - 6DF + 9F^2) q_{jl}\) | \(-\frac{1}{9} + \frac{18}{9} q_{jl}\) | \(-\frac{1}{9} + \frac{18}{9} q_{jl}\) |
| \(K\) | \(-\frac{1}{9} (5D^2 - 6DF + 9F^2) (1 + 3q_r)\) | \(-\frac{1}{9} + \frac{18}{9} q_r\) | \(-\frac{1}{9} + \frac{18}{9} q_r\) |
| \(uj\) | \(-\frac{2}{9} (D + 3F)^2 + \frac{1}{9} (5D^2 - 6DF + 9F^2) q_{jl}\) | \(-\frac{1}{9} q_r q_j^2\) | \(-\frac{2}{9} - \frac{8}{9} q_r q_j\) |
| \(ur\) | \(-\frac{1}{9} (D + 3F)^2 + \frac{1}{9} (5D^2 - 6DF + 9F^2) q_r\) | \(-\frac{1}{9} q_r q_j^2\) | \(-\frac{2}{9} - \frac{8}{9} q_r q_j\) |

2. **Q\(\chi PT\)**

The calculation of the charge radii can be easily executed for Q\(\chi PT\). The operators in Eqs. (41) and (43) contribute, however, their low-energy coefficients cannot be matched onto QCD. Therefore we annotate them with a \(Q\). Additional terms involving hairpins \([15, 19]\) do not contribute as their contribution to the charge radii is of the form \((\mu_b^2/\Lambda^4)\) log \(m_q\) and therefore of higher order in the chiral expansion. We find
TABLE V: The coefficients $\beta_X$, $\beta'_X$, and $A_X$ in $SU(3)$ flavor PQ$\chi$PT for the $\Sigma^0$.

| $X$ | $\beta_X$ | $\beta'_X$ | $A_X$ |
|-----|------------|-------------|-------|
| $\pi$ | $\frac{2}{9} (D^2 + 3F^2) (1 - 3q_{jl})$ | $(-\frac{1}{5} + \frac{1}{18} q_{jl}) C^2$ | $-\frac{2}{3} + 2q_{jl}$ |
| $K$ | $-\frac{1}{3} (5D^2 + 6DF + 3F^2) - (D - F)^2 q_{jl} - \frac{2}{3} (D^2 + 3F^2) q_r$ | $(-\frac{1}{10} + \frac{2}{9} q_{jl} + \frac{1}{18} q_r) C^2$ | $\frac{1}{3} + q_{jl} + 2q_r$ |
| $\eta_s$ | $-\frac{1}{3} (D - F)^2 (1 + 3q_r)$ | $\left(\frac{1}{27} + \frac{1}{9} q_r\right) C^2$ | $\frac{2}{3} + q_r$ |
| $uj$ | $-\frac{2}{9} (D^2 + 3F^2) (1 - 3q_{jl})$ | $\left(\frac{1}{18} - \frac{1}{18} q_{jl}\right) C^2$ | $\frac{2}{3} - 2q_{jl}$ |
| $ur$ | $-\frac{1}{9} (D^2 + 3F^2) (1 - 6q_r)$ | $\left(\frac{1}{108} - \frac{1}{18} q_r\right) C^2$ | $\frac{1}{3} - 2q_r$ |
| $sj$ | $\frac{1}{3} (D - F)^2 (2 + 3q_{jl})$ | $\left(-\frac{2}{27} - \frac{1}{9} q_{jl}\right) C^2$ | $-\frac{2}{3} - q_{jl}$ |
| $sr$ | $\frac{1}{3} (D - F)^2 (1 + 3q_r)$ | $\left(-\frac{1}{27} - \frac{1}{9} q_r\right) C^2$ | $-\frac{1}{3} - q_r$ |

TABLE VI: The coefficients $\beta_X$, $\beta'_X$, and $A_X$ in $SU(3)$ flavor PQ$\chi$PT for the $\Sigma^-$.  

| $X$ | $\beta_X$ | $\beta'_X$ | $A_X$ |
|-----|------------|-------------|-------|
| $\pi$ | $\frac{2}{9} (D^2 + 3F^2) (1 - 3q_{jl})$ | $(-\frac{1}{5} + \frac{1}{18} q_{jl}) C^2$ | $-\frac{2}{3} + 2q_{jl}$ |
| $K$ | $\frac{1}{9} (D^2 - 6DF - 3F^2) - (D - F)^2 q_{jl} - \frac{2}{3} (D^2 + 3F^2) q_r$ | $(-\frac{1}{10} + \frac{1}{9} q_{jl} + \frac{1}{18} q_r) C^2$ | $\frac{1}{3} + q_{jl} + 2q_r$ |
| $\eta_s$ | $-\frac{1}{3} (D - F)^2 (1 + 3q_r)$ | $\left(\frac{1}{27} + \frac{1}{9} q_r\right) C^2$ | $\frac{2}{3} + q_r$ |
| $uj$ | $\frac{2}{9} (D^2 + 3F^2) (2 + 3q_{jl})$ | $\left(-\frac{2}{27} - \frac{1}{9} q_{jl}\right) C^2$ | $\frac{2}{3} - 2q_{jl}$ |
| $ur$ | $\frac{2}{9} (D^2 + 3F^2) (1 + 3q_r)$ | $\left(-\frac{2}{27} - \frac{1}{9} q_r\right) C^2$ | $\frac{2}{3} - 2q_r$ |
| $sj$ | $\frac{1}{3} (D - F)^2 (2 + 3q_{jl})$ | $\left(-\frac{2}{27} - \frac{1}{9} q_r\right) C^2$ | $\frac{2}{3} - q_{jl}$ |
| $sr$ | $\frac{1}{3} (D - F)^2 (1 + 3q_r)$ | $\left(-\frac{1}{27} - \frac{1}{9} q_r\right) C^2$ | $-\frac{1}{3} - q_r$ |

IV. CONCLUSIONS

We have calculated the charge radii for the octet mesons and baryons in the isospin limit of PQ$\chi$PT and also derive the result for the nucleon doublet and pion triplet away from the isospin limit for the $SU(2)$ chiral Lagrangian. For the octet mesons and baryons we have also calculated the Q$\chi$PT results.

Knowledge of the low-energy behavior of QQCD and PQQCD is crucial to properly extrapolate lattice calculations from the quark masses used on the lattice to those in nature. For the quenched approximation, where the quark determinant is not evaluated, one uses

TABLE VII: The coefficients $\beta_X$, $\beta'_X$, and $A_X$ in $SU(3)$ flavor PQ$\chi$PT for the $\Xi^0$.

| $X$ | $\beta_X$ | $\beta'_X$ | $A_X$ |
|-----|------------|-------------|-------|
| $\pi$ | $\frac{1}{9} (D - F)^2 (1 - 3q_{jl})$ | $\left(-\frac{1}{27} + \frac{1}{9} q_{jl}\right) C^2$ | $-\frac{1}{3} + q_{jl}$ |
| $K$ | $\frac{1}{9} (11D^2 + 6DF + 3F^2) - \frac{2}{9} (D^2 + 3F^2) q_{jl} - (D - F)^2 q_r$ | $\left(\frac{5}{27} + \frac{1}{18} q_{jl} + \frac{1}{18} q_r\right) C^2$ | $-\frac{1}{3} + q_r + 2q_{jl}$ |
| $\eta_s$ | $-\frac{2}{9} (D^2 + 3F^2) (1 + 3q_r)$ | $\left(\frac{1}{27} + \frac{1}{9} q_r\right) C^2$ | $\frac{2}{3} + 2q_r$ |
| $uj$ | $-\frac{1}{9} (D - F)^2 (1 - 3q_{jl})$ | $\left(\frac{4}{27} - \frac{1}{9} q_{jl}\right) C^2$ | $\frac{4}{3} - q_{jl}$ |
| $ur$ | $-\frac{1}{3} (D - F)^2 (2 + 3q_{jl})$ | $\left(\frac{2}{27} - \frac{1}{9} q_{jl}\right) C^2$ | $\frac{2}{3} + 2q_{jl}$ |
| $sj$ | $\frac{2}{9} (D^2 + 3F^2) (2 + 3q_{jl})$ | $\left(-\frac{2}{27} - \frac{1}{9} q_{jl}\right) C^2$ | $\frac{2}{3} - q_{jl}$ |
| $sr$ | $\frac{2}{9} (D^2 + 3F^2) (1 + 3q_r)$ | $\left(-\frac{1}{27} - \frac{1}{9} q_r\right) C^2$ | $-\frac{1}{3} + 2q_r$ |
that our result is free of quenching artifacts. While the expansions about the chiral limit
$NLO$ result is not more divergent than its QCD counterpart. This, however, does not mean
$Q = 0$. In other words, diagrams that have bosonic or fermionic mesons running in loops com-
pletely cancel so that

\begin{equation}
\chi \end{equation}

\begin{equation}
TABLE VIII: The coefficients $\beta_X$, $\beta_X'$, and $A_X$ in $SU(3)$ flavor PQ\chi PT for the \( \Xi^- \).
\begin{array}{|c|c|c|c|}
\hline
X & \beta_X & \beta_X' & A_X \\
\hline
\pi & \frac{1}{3}(D - F)^2 (1 + 3q_{jl}) & \left(-\frac{1}{15} + \frac{q_{jl}}{6q_{jr}}\right) C^2 & -\frac{1}{3} + q_{jl} \\
K & \left(-\frac{1}{3} (D^2 - 6DF - 3F^2) - \frac{2}{3} (D^2 + 3F^2) q_{jl} - (D - F)^2 q_{jr} \right) & \left(\frac{1}{15} + \frac{1}{3} q_{jl} \right) C^2 & -\frac{1}{3} + 2q_{jl} + q_{jr} \\
\eta_s & \left(-\frac{1}{2} (D^2 + 3F^2) (1 + 3q_{jr}) \right) & \left(\frac{1}{15} + \frac{1}{3} q_{jl} \right) C^2 & \frac{2}{3} + 2q_{jr} \\
u_j & \left(\frac{1}{3} (D - F)^2 (2 + 3q_{jl}) \right) & \left(-\frac{2}{3} - \frac{q_{jl}}{6q_{jr}}\right) C^2 & -\frac{2}{3} - q_{jr} \\
u_r & \left(\frac{1}{3} (D - F)^2 (1 + 3q_{jr}) \right) & \left(-\frac{2}{3} - \frac{1}{6} q_{jl} \right) C^2 & -\frac{2}{3} - q_{jr} \\
s_j & \left(-\frac{2}{3} (D^2 + 3F^2) (2 + 3q_{jl}) \right) & \left(-\frac{2}{3} - \frac{1}{6} q_{jl} \right) C^2 & -\frac{2}{3} - 2q_{jr} \\
s_r & \left(-\frac{2}{3} (D^2 + 3F^2) (1 + 3q_{jr}) \right) & \left(-\frac{2}{3} - \frac{1}{6} q_{jl} \right) C^2 & -\frac{2}{3} - 2q_{jr} \\
\hline
\end{array}

\begin{equation}
TABLE IX: The coefficients $\beta_X$, $\beta_X'$, and $A_X$ in $SU(3)$ flavor PQ\chi PT for the $\Lambda$.
\begin{array}{|c|c|c|c|}
\hline
X & \beta_X & \beta_X' & A_X \\
\hline
\pi & \left(\frac{2}{27} (7D^2 - 12DF + 9F^2) (1 - 3q_{jl}) \right) & \left(-\frac{1}{18} + \frac{q_{jl}}{6q_{jr}}\right) C^2 & -\frac{1}{3} + 2q_{jl} \\
K & \left(-\frac{1}{27} (5D^2 + 30DF - 9F^2) - \frac{1}{9} (D + 3F)^2 q_{jl} - \frac{2}{3} (7D^2 - 12DF + 9F^2) q_{jr} \right) & \left(\frac{5}{36} + \frac{1}{6} q_{jl} \right) C^2 & \frac{1}{3} + q_{jl} + 2q_{jr} \\
\eta_s & \left(-\frac{1}{27} (D + 3F)^2 (1 + 3q_{jr}) \right) & 0 & \frac{1}{3} + q_{jr} \\
u_j & \left(-\frac{2}{27} (7D^2 - 12DF + 9F^2) (1 - 3q_{jl}) \right) & \left(\frac{1}{18} - \frac{1}{6} q_{jl} \right) C^2 & \frac{2}{3} - 2q_{jr} \\
u_r & \left(-\frac{2}{27} (7D^2 - 12DF + 9F^2) (1 - 6q_{jr}) \right) & \left(\frac{1}{18} - \frac{1}{6} q_{jl} \right) C^2 & \frac{1}{3} - 2q_{jr} \\
s_j & \left(\frac{1}{27} (D + 3F)^2 (2 + 3q_{jl}) \right) & 0 & -\frac{2}{3} - q_{jr} \\
s_r & \left(\frac{1}{27} (D + 3F)^2 (1 + 3q_{jr}) \right) & 0 & -\frac{1}{3} - q_{jr} \\
\hline
\end{array}

Table X: The coefficients $\beta_X$, $\beta_X'$, and $A_X$ in $SU(3)$ flavor PQ\chi PT for the $\Lambda\Sigma^0$ transition.
\begin{array}{|c|c|c|c|}
\hline
X & \beta_X & \beta_X' & A_X \\
\hline
\pi & \left(-\frac{1}{3\sqrt{3}} D^2 \right) & \left(-\frac{1}{3\sqrt{3}} C^2 \right) & 0 \\
K & \left(-\frac{2}{3\sqrt{3}} D^2 \right) & \left(-\frac{2}{3\sqrt{3}} C^2 \right) & 0 \\
u_j & \left(\frac{4}{3\sqrt{3}} D(D - 3F) \right) & \left(-\frac{4}{3\sqrt{3}} C^2 \right) & 0 \\
u_r & \left(\frac{2}{3\sqrt{3}} D(D - 3F) \right) & \left(-\frac{2}{3\sqrt{3}} C^2 \right) & 0 \\
\hline
\end{array}

Q\chi PT to do this extrapolation. QQCD, however, has no known connection to QCD and is
of merely academic interest. Observables calculated in QQCD are often found to be more
divergent in the chiral limit than those in QCD. This behavior is due to new operators
included in the QQCD Lagrangian, which are non-existent in QCD. For the octet meson
and baryon charge radii we find that such operators enter at NNLO. Hence, formally our
NLO result is not more divergent than its QCD counterpart. This, however, does not mean
that our result is free of quenching artifacts. While the expansions about the chiral limit
for QCD and QQCD are formally similar, $< r^2 > \sim \alpha + \beta \log m_Q + \ldots$, the QQCD result
is anything but free of quenched oddities: for certain baryons, $\Sigma^-$ and $\Xi^-$ in particular,
diagrams that have bosonic or fermionic mesons running in loops completely cancel so that
$\beta = 0$. In other words, $< r^2 > \sim \alpha + \ldots$ and the result is actually independent of $m_Q$!
The same behavior is found for the charge radii of all mesons in QQCD as the meson loop

\begin{equation}
TABLE X: The coefficients $\beta_X$, $\beta_X'$, and $A_X$ in $SU(3)$ flavor PQ\chi PT for the $\Lambda\Sigma^0$ transition.
\begin{array}{|c|c|c|c|}
\hline
X & \beta_X & \beta_X' & A_X \\
\hline
\pi & \left(-\frac{1}{3\sqrt{3}} D^2 \right) & \left(-\frac{1}{3\sqrt{3}} C^2 \right) & 0 \\
K & \left(-\frac{2}{3\sqrt{3}} D^2 \right) & \left(-\frac{2}{3\sqrt{3}} C^2 \right) & 0 \\
u_j & \left(\frac{4}{3\sqrt{3}} D(D - 3F) \right) & \left(-\frac{4}{3\sqrt{3}} C^2 \right) & 0 \\
u_r & \left(\frac{2}{3\sqrt{3}} D(D - 3F) \right) & \left(-\frac{2}{3\sqrt{3}} C^2 \right) & 0 \\
\hline
\end{array}

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TABLE XI: The coefficients $\beta_X^Q$ and $\beta_X^{Q'}$ in $SU(3)$ flavor $Q\chi$PT for the octet baryons.

|           | $\beta_X^Q$ | $\beta_X^{Q'}$ |
|-----------|-------------|----------------|
| $\pi$     | $-\frac{4}{3}D_Q^2$ | $0$           |
| $K$       | $0$         | $0$           |
| $\Sigma^+$| $0$         | $-\frac{1}{3}C_Q^2$ |
| $\Sigma^0$| $0$         | $0$           |
| $\Sigma^-$| $0$         | $0$           |
| $\Lambda$ | $0$         | $0$           |
| $\Xi^0$   | $0$         | $0$           |
| $\Sigma\Lambda$ | $-\frac{4}{3\sqrt{3}}D_Q^2$ | $-\frac{1}{6\sqrt{3}}C_Q^2$ |

Contributions entirely cancel.

PQCD, on the other hand, is free of such eccentric behavior. The formal behavior of the charge radius in the chiral limit has the same form as in QCD. Moreover, there is a well-defined connection to QCD and one can reliably extrapolate lattice results down to the quark masses of reality. The low-energy constants appearing in PQCD are the same as those in QCD and by fitting them in $PQ\chi$PT one can make predictions for QCD. Our $PQ\chi$PT result will enable the proper extrapolation of PQCD lattice simulations of the charge radii and we hope it encourages such simulations in the future.

Acknowledgments

We would like to thank Martin Savage for very helpful discussions and for useful comments on the manuscript. This work is supported in part by the U.S. Department of Energy under Grant No. DE-FG03-97ER4014.

APPENDIX: CHARGE RADII IN $SU(2)$ FLAVOR WITH NON-DEGENERATE QUARKS

In this section, we consider the case of $SU(2)$ flavor and calculate charge radii for the pions and nucleons. We keep the up and down quark masses non-degenerate and similarly for the sea-quarks. Thus the quark mass matrix reads $m_Q^{SU(2)} = \text{diag}(m_u, m_d, m_j, m_l, m_u, m_d)$. Defining ghost and sea quark charges is constrained only by the restriction that QCD be recovered in the limit of appropriately degenerate quark masses. Thus the most general form of the charge matrix is

$$Q^{SU(2)} = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, q_j, q_l, q_j, q_l\right).$$  \hspace{1cm} (A.1)

The symmetry breaking pattern is assumed to be $SU(4|2)_L \otimes SU(4|2)_R \otimes U(1)_V \rightarrow SU(4|2)_V \otimes U(1)_V$.  

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For the $\pi^+$, $\pi^-$, and $\pi^0$ we find

$$G_{\pi^+}^{PQ}(q^2) = 1 + \frac{1}{16\pi^2 f^2} \left[ \left(\frac{1}{3} + q_l\right) F_{dd} + \left(\frac{2}{3} - q_j\right) F_{uu} - (1 - q_j + q_l) F_{uu} - \left(\frac{1}{3} + q_j\right) F_{jd} \right. $$

$$\left. - \left(\frac{1}{3} + q_l\right) F_{id} - \left(\frac{2}{3} - q_j\right) F_{ju} - \left(\frac{2}{3} - q_l\right) F_{iu} \right] + \alpha_0 \frac{4}{f^2} q^2,$$

(A.2)

$$G_{\pi^-}^{PQ} = -G_{\pi^+}^{PQ}, \text{ and } G_{\pi^0}^{PQ} = 0, \text{ respectively.}$$

The baryon field assignments are analogous to the case of $SU(3)$ flavor. The nucleons are embedded as

$$B_{ijk} = \frac{1}{\sqrt{6}} (\epsilon_{ij} N_k + \epsilon_{ik} N_j),$$

(A.3)

where the indices $i, j$ and $k$ are restricted to 1 or 2 and the $SU(2)$ nucleon doublet is defined as

$$N = \left( \begin{array}{c} p \\ n \end{array} \right).$$

(A.4)

The decuplet field $T_{ijk}$, which is totally symmetric, is normalized to contain the $\Delta$ resonances $T_{ijk} = T_{ijk}$ with $i, j, k$ restricted to 1 or 2. The spin-3/2 baryon quartet is then contained as

$$T_{111} = \Delta^{++}, \quad T_{112} = \frac{1}{\sqrt{3}} \Delta^+, \quad T_{122} = \frac{1}{\sqrt{3}} \Delta^0, \quad \text{and } T_{222} = \Delta^-.$$  

(A.5)

The construction of the octet and decuplet baryons containing one sea or one ghost quark is analogous to the $SU(3)$ flavor case and we will not repeat it here.

The free Lagrangian for $B$ and $T$ is the one in Eq. (20) (with the parameters having different numerical values than in the $SU(3)$ case). The connection to QCD is detailed in [31]. Similarly, the Lagrangian describing the interaction of the $B$ and $T$ with the pseudo-Goldstone bosons is the one in Eq. (20). Matching it to the familiar one in QCD (by restricting the $B_{ijk}$ and $T_{ijk}$ to the $qqq$ sector)

$$\mathcal{L} = 2 g_A \nabla^\mu A_\mu N + g_1 \nabla^\mu S^\mu N \text{tr}(A_\mu) + g_{\Delta N} \left( T_{\nu}^{kij} A_{\mu}^\nu N_j \epsilon_{kl} + h.c. \right),$$

(A.6)

one finds at tree-level

$$\alpha = \frac{4}{3} g_A + \frac{1}{3} g_1, \quad \beta = \frac{2}{3} g_1 - \frac{1}{3} g_A, \quad \text{and } \mathcal{C} = -g_{\Delta N}.$$  

(A.7)

The contribution at leading order to the charge radii from the Pauli form factor $F_2(q^2)$, involves only the magnetic moments which arise from the PQQCD Lagrangian [31]

$$\mathcal{L} = \frac{ie}{2 M_N} \left[ \mu_\alpha \left( \mathcal{B}[S_\mu, S_\nu] \mathcal{Q}^{SU(2)} + \mathcal{B}[S_\mu, S_\nu] \mathcal{B} \right) \right. $$

$$\left. + \mu_\gamma \text{str}(\mathcal{Q}^{SU(2)}) \left( \mathcal{B}[S_\mu, S_\nu] \mathcal{B} \right) \right] F^{\mu\nu}.$$  

(A.8)

Note that in the case of $SU(2)$ flavor the charge matrix $\mathcal{Q}$ is not supertraceless and hence there appears a third operator. In QCD, the corresponding Lagrange density is conventionally written in terms of isoscalar and isovector couplings

$$\mathcal{L} = \frac{ie}{2 M_N} \left( \mu_0 \nabla [S_\mu, S_\nu] N + \mu_1 \nabla [S_\mu, S_\nu] \tau^3 N \right) F^{\mu\nu}.$$  

(A.9)
TABLE XII: The coefficients $\beta_X$, $\beta'_X$, and $A_X$ in $SU(2)$ flavor PQChPT for the proton.

| $X$ | $\beta_X$ | $\beta'_X$ | $A_X$ |
|-----|------------|-------------|-------|
| $uu$ | $\frac{2}{9}(4g_A^2 + 2g_1 g_A + g_1^2)(2 - 3q_j)$ | $(-\frac{1}{27} + \frac{1}{18} q_j)g_{\Delta N}$ | $-\frac{4}{3} + 2q_j$ |
| $ud$ | $-\frac{4}{9}g_A^2(5 + 6q_j) + \frac{4}{9}g_1 g_A(2 - 3q_j) + \frac{1}{9}g_1^2(1 - 9q_j - 6q_l)$ | $(-\frac{2}{9} g_1 + \frac{1}{18} q_l)g_{\Delta N}$ | $q_j + 2q_l$ |
| $dd$ | $-\frac{2}{9}g_1^2(1 + 3q_l)$ | $(\frac{1}{27} + \frac{1}{18} q_l)g_{\Delta N}$ | $\frac{1}{3} + q_l$ |
| $uj$ | $-\frac{2}{9}(4g_A^2 + 2g_1 g_A + g_1^2)(2 - 3q_j)$ | $(-\frac{2}{27} - \frac{1}{18} q_j)g_{\Delta N}^2$ | $\frac{4}{3} - 2q_j$ |
| $ul$ | $-\frac{2}{9}(4g_A^2 + 2g_1 g_A + g_1^2)(2 - 3q_l)$ | $\frac{2}{27} - \frac{1}{18} q_l)g_{\Delta N}^2$ | $\frac{3}{3} - 2q_l$ |
| $dj$ | $\frac{1}{3}g_1^2(1 + 3q_j)$ | $(-\frac{1}{27} - \frac{1}{9} q_j)g_{\Delta N}^2$ | $\frac{1}{3} - q_j$ |
| $dl$ | $\frac{1}{3}g_1^2(1 + 3q_l)$ | $(-\frac{1}{27} - \frac{1}{9} q_l)g_{\Delta N}^2$ | $\frac{1}{3} - q_l$ |

and one finds that the QCD and PQQCD coefficients are related by [31]

$$\mu_0 = \frac{1}{6}(\mu_{\alpha} + \mu_{\beta} + 2\mu_{\gamma}), \quad \text{and} \quad \mu_1 = \frac{1}{6}(2\mu_{\alpha} - \mu_{\beta}). \quad (A.10)$$

Likewise, the leading tree-level corrections to the charge-radii come from the Lagrangian

$$\mathcal{L} = \frac{e}{\Lambda_X^2} \left[ c_\alpha (\overline{B} B Q^{SU(2)}) + c_\beta (\overline{B} Q^{SU(2)} B) + c_\gamma \text{str}(Q^{SU(2)}) (\overline{B} B) \right] v_\mu \partial_\nu F^{\mu\nu} \quad (A.11)$$

that matches onto the QCD Lagrangian

$$\mathcal{L} = \frac{e}{\Lambda_X^2} \left( c_0 \overline{N} N + c_1 \overline{N} \tau^3 N \right) v_\mu \partial_\nu F^{\mu\nu} \quad (A.12)$$

with

$$c_0 = \frac{1}{6}(c_\alpha + c_\beta + 2c_\gamma), \quad \text{and} \quad c_1 = \frac{1}{6}(2c_{\alpha} - c_{\beta}). \quad (A.13)$$

Evaluating the charge radii at NLO order in the chiral expansion yields

$$< r_E^2 > = -\frac{6c}{\Lambda_X^2} + \frac{3\alpha}{2M_N^2} - \frac{1}{16\pi^2 f^2} \sum_X \left[ A_X \log \frac{m_X^2}{\mu^2} - 5 \beta_X \log \frac{m_X^2}{\mu^2} + 10 \beta_X \mathcal{G}(m_X, \Delta, \mu) \right]. \quad (A.14)$$

The coefficients $c$ are given by $c_p = c_0 + c_1$ and $c_n = c_0 - c_1$ while $\alpha_p = \mu_0 + \mu_1$ and $\alpha_n = \mu_0 - \mu_1$. The remaining coefficients are listed in Table XII for the proton and Table XIII for the neutron.

[1] P. Mergell, U. G. Meissner, and D. Drechsel, Nucl. Phys. A596, 367 (1996), hep-ph/9506375.
[2] H. W. Hammer, U.-G. Meissner, and D. Drechsel, Phys. Lett. B385, 343 (1996), hep-ph/9604294.
[3] I. Eschrich (SELEX) (1998), hep-ex/9811003.
[4] J. van der Heide, M. Lutterot, J. H. Koch, and E. Laermann (2003), hep-lat/0303006.
[5] A. Tang, W. Wilcox, and R. Lewis (2003), hep-lat/0307006.
[6] M. Gockeler et al. (QCDSF) (2003), hep-lat/0303019.
TABLE XIII: The coefficients $\beta_X$, $\beta_X'$, and $A_X$ in $SU(2)$ flavor PQ\(\chi\)PT for the neutron.

| $X$  | $\beta_X$                                           | $\beta_X'$                                    | $A_X$                  |
|------|------------------------------------------------------|------------------------------------------------|------------------------|
| uu   | $\frac{\beta}{2} g^2_3 (2 - 3q_l)$                  | $(-\frac{2}{27} + \frac{2}{3} q_l) g^2_{3N}$ | $-\frac{2}{3} + q_l$   |
| ud   | $\frac{4}{9} g^2_3 (7 - 6q_l) - \frac{4}{9} g_3 A(1 + 3q_l) + \frac{4}{9} g^2_3 (4 - 6q_l - 9q_l)$ | $(-\frac{1}{9} + \frac{1}{3} q_l) g^2_{3N}$ | $-1 + 2q_l + q_l$     |
| dd   | $-\frac{2}{9} (4 g^2_3 A + 2 g_1 A + g^2_1) (1 + 3q_l)$ | $(\frac{2}{27} - \frac{1}{4} q_l) g^2_{N}$   | $\frac{2}{3} - q_l$   |
| vj   |                                                            | $-\frac{2}{9} (\frac{2}{3} - q_l)$           |                         |
| ul   |                                                            | $-\frac{2}{3} (\frac{2}{3} - q_l)$           |                         |
| dj   | $\frac{2}{3} (4 g^2_3 A + 2 g_1 A + g^2_1) (1 + 3q_l)$ |                                                            |                         |
| dl   | $\frac{2}{3} (4 g^2_3 A + 2 g_1 A + g^2_1) (1 + 3q_l)$ | $(-\frac{1}{3} - \frac{1}{3} q_l) g^2_{3N}$ | $-\frac{2}{3} - 2q_l$  |

[7] W. Wilcox, T. Draper, and K.-F. Liu, Phys. Rev. D46, 1109 (1992), hep-lat/9205015.
[8] T. Draper, R. M. Woloshyn, and K.-F. Liu, Phys. Lett. B234, 121 (1990).
[9] D. B. Leinweber, R. M. Woloshyn, and T. Draper, Phys. Rev. D43, 1659 (1991).
[10] A. Morel, J. Phys. (France) 48, 1111 (1987).
[11] S. R. Sharpe, Phys. Rev. D46, 3146 (1992), hep-lat/9205020.
[12] C. W. Bernard and M. F. L. Golterman, Phys. Rev. D46, 853 (1992), hep-lat/9204007.
[13] C. W. Bernard and M. Golterman, Nucl. Phys. Proc. Suppl. 26, 360 (1992).
[14] M. F. L. Golterman, Acta Phys. Polon. B25, 1731 (1994), hep-lat/9411005.
[15] J. N. Labrenz and S. R. Sharpe, Phys. Rev. D54, 4595 (1996), hep-lat/9605034.
[16] S. R. Sharpe and Y. Zhang, Phys. Rev. D53, 5125 (1996), hep-lat/9510037.
[17] D. Arndt, Phys. Rev. D67, 074501 (2003), hep-lat/0210019.
[18] M. J. Booth (1994), hep-ph/9412228.
[19] M. J. Savage, Nucl. Phys. A700, 359 (2002), nucl-th/0107038.
[20] D. Arndt and B. C. Tiburzi (2003), hep-lat/0308001.
[21] S. R. Sharpe and N. Shoresh, Int. J. Mod. Phys. A16S1C, 1219 (2001), hep-lat/0011089.
[22] S. R. Sharpe and N. Shoresh, Phys. Rev. D64, 114510 (2001), hep-lat/0108003.
[23] S. R. Sharpe and N. Shoresh, Phys. Rev. D62, 094503 (2000), hep-lat/0006017.
[24] S. R. Sharpe and N. Shoresh, Nucl. Phys. Proc. Suppl. 83, 968 (2000), hep-lat/9909090.
[25] M. F. L. Golterman and K.-C. Leung, Phys. Rev. D57, 5703 (1998), hep-lat/9711033.
[26] S. R. Sharpe, Phys. Rev. D56, 7052 (1997), hep-lat/9707018.
[27] C. W. Bernard and M. F. L. Golterman, Phys. Rev. D49, 486 (1994), hep-lat/9306005.
[28] N. Shores (2001), Ph.D. thesis, UMI-30-36529.
[29] M. J. Savage, Phys. Rev. D65, 034014 (2002), hep-ph/0109190.
[30] J.-W. Chen and M. J. Savage, Phys. Rev. D65, 094001 (2002), hep-lat/0111050.
[31] S. R. Beane and M. J. Savage, Nucl. Phys. A709, 319 (2002), hep-lat/0203003.
[32] M. J. Savage (2002), hep-lat/0208022.
[33] G. Martinelli and C. T. Sachrajda, Nucl. Phys. B306, 865 (1988).
[34] T. Draper, R. M. Woloshyn, W. Wilcox, and K.-F. Liu, Nucl. Phys. B318, 319 (1989).
[35] D. B. Leinweber, D. H. Lu, and A. W. Thomas, Phys. Rev. D60, 034014 (1999), hep-lat/9810005.
[36] E. J. Hackett-Jones, D. B. Leinweber, and A. W. Thomas, Phys. Lett. B489, 143 (2000), hep-lat/0004006.
[37] D. Arndt, S. R. Beane, and M. J. Savage (2003), nucl-th/0304004.
[38] S. R. Beane and M. J. Savage, Phys. Rev. D67, 054502 (2003), hep-lat/0210046.
[39] S. R. Beane and M. J. Savage, Phys. Lett. B535, 177 (2002), hep-lat/0202013.
[40] URL http://www.jlab.org/~dgr/lhpc/march00.pdf
[41] URL http://www.jlab.org/~dgr/lhpc/sdac_proposal_final.pdf
[42] E. Jenkins and A. V. Manohar (1991), talk presented at the Workshop on Effective Field Theories of the Standard Model, Dobogoko, Hungary, Aug 1991.
[43] E. Jenkins and A. V. Manohar, Phys. Lett. B255, 558 (1991).
[44] A. B. Balantekin, I. Bars, and F. Iachello, Phys. Rev. Lett. 47, 19 (1981).
[45] A. B. Balantekin and I. Bars, J. Math. Phys. 22, 1149 (1981).
[46] A. B. Balantekin and I. Bars, J. Math. Phys. 23, 1239 (1982).
[47] M. Golterman and E. Pallante, Nucl. Phys. Proc. Suppl. 106, 335 (2002), hep-lat/0110183.
[48] J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985).
[49] J. Gasser and H. Leutwyler, Nucl. Phys. B250, 517 (1985).
[50] E. Jenkins, M. E. Luke, A. V. Manohar, and M. J. Savage, Phys. Lett. B302, 482 (1993), hep-ph/9212226.
[51] U.-G. Meissner and S. Steininger, Nucl. Phys. B499, 349 (1997), hep-ph/9701260.
[52] V. Bernard, H. W. Fearing, T. R. Hemmert, and U. G. Meissner, Nucl. Phys. A635, 121 (1998), hep-ph/9801297.
[53] B. Kubis, T. R. Hemmert, and U.-G. Meissner, Phys. Lett. B456, 240 (1999), hep-ph/9903285.
[54] B. Kubis and U. G. Meissner, Eur. Phys. J. C18, 747 (2001), hep-ph/0010283.
[55] B. Kubis and U.-G. Meissner, Nucl. Phys. A679, 698 (2001), hep-ph/0007056.
[56] S. J. Puglia and M. J. Ramsey-Musolf, Phys. Rev. D62, 034010 (2000), hep-ph/9911542.
[57] S. J. Puglia, M. J. Ramsey-Musolf, and S.-L. Zhu, Phys. Rev. D63, 034014 (2001), hep-ph/0008140.
[58] L. Durand and P. Ha, Phys. Rev. D58, 013010 (1998), hep-ph/9712492.
[59] D. B. Leinweber (2002), hep-lat/0211017.