Practical method for estimating road curvatures using onboard GPS and IMU equipment

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Abstract. This paper describes an experimental method to determine with high accuracy the curvature of a road segment, the turning radius of a car, and the discomfort level perceived by the passengers in the vehicle cabin when passing through a curve. For these experiments we used professional equipment provided with two GPS active antennas with 13 dB gain featuring non-contact 100 Hz speed and distance measurement, and a ten degree Inertial Measurement Unit (IMU) with dynamic orientation outputs. The same experimental measurements also used the low cost GPS equipment available on smartphones, domestic vehicle GPS devices, as well as an Arduino GPS shield in order to compare the results generated by professional equipment. The purpose of these experiments was also to establish if certain road curve sections were correctly executed in order to ensure the safety and comfort of passengers. Another use of the proposed method relates to the road accident reconstruction field, providing experts and forensics with an accurate method of measuring the roadway curvature at accident scenes or traffic events. The research and equipment described in this paper have been acquired and developed under a PhD study and a European funded project won and elaborated by the authors.

1 Introduction

Knowing the accurate roadway curvature at a certain point is highly important to the proper operation of certain trajectory control systems and the lateral dynamics of a vehicle. We mention here some of the well-known systems: Autonomous Cruise Control (ACC), Adaptive Front-lighting System (AFS), automated parking system, lane departure warning system, collision avoidance system, anti-roll system, trajectory control systems for tracking autonomous vehicles, etc. Another use of knowing the accurate roadway curvature is in the field of accident science and road events investigation. Using this knowledge, we can determine the critical slide speed in a curve, the rollover speed, the lateral forces and other important data to the reconstruction of a traffic accident. [2], [7], [9].

From the viewpoint of road design, the curvature radius and the vehicle speed decisively impacts both the dynamic stability and the passengers comfort when passing through a curve. Therefore, curvatures are the critical sections of roads. From the viewpoint of the plane geometry of curves, when the minimum curve radius and/or super elevation are not observed, the lateral acceleration developed by the vehicle may significantly affect the passengers’ safety and comfort. Determining the minimum curvature radius is a significant design criterion, and checking its conformity is a major requirement of any investigation of road accidents occurred on such road sections.
In this paper, the authors define a practical method to calculate and measure the curvature radius using GPS equipment, i.e. an Inertial Measurement Unit (IMU), thus eliminating the deficiencies involved by onsite measurements or the errors generated when using maps.

2 The impact of the road geometry in curves on the passengers’ safety

Figure 1 presents the ideal curvature of a road curve section by unifying two alignments, composed by two clothoid joints and a circular arc.

Several studies showed that most of the vehicle drivers fail to reduce their speed on the curve entry alignment, and they do it only inside the curve, when the uncomfortable sensation becomes predominant. The alignment is generally passed through at a certain speed that is usually higher than those recommended for that specific sector. Subsequently, in the case of a tight curve, the driver tends to suddenly reduce the speed until the discomfort feeling disappears. After the speed reduction, the driver continues the curve by accelerating progressively, fact that allows the vehicle to reach a speed that exceeds the usual one, on the new alignment.

When the alignment is directly continued by an arc of a circle, the perception on acceleration and lateral force is sudden and brutal, fact that sometimes determines the driver to panic and make sudden moves that further unbalance the vehicle. To avoid this sudden increase in the lateral acceleration, the transition curves are used between the alignment and the effective curve, allowing a gradual and moderate increase in the lateral forces and providing the driver the time to accommodate and the perception of the possibility of reaching a hazardous discomfort level, fact that makes him/her consciously initiate the speed reduction maneuvers in due time, and take control of the vehicle.

One of the most familiar models used for the transition curves is the clothoid. This is characterized by the curve $\rho(s)$ (the curve is defined as the reverse curve radius), having a linear variation depending on the length $s$ (the arc length) according to the relation (1).

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**Figure 1.** Transition to a very tight curve using two clothoid arcs before the alignment.
\[ \rho(s) = \rho_0 + \rho_1 \cdot s \]  

(1)

If \( \rho_1 = 0 \), the curvature is constant, and the curve is a circle. The clothoid parameters, \( \rho_0 \) and \( \rho_1 \), should be identified as early as possible, starting from the vehicle entry in that particular curve, or even anticipated, where possible (using a video system, for example). Most of the high speed roads (highways, national roads with two or several lanes in each direction) are built from successive alignments and clothoid segments, with different \( \rho_0i \) and \( \rho_1i \) coefficients for each segment.

Thus, in B point \( s = s_B \), \( \rho \to 0 \) (alignment) and in C point, where \( s = s_C, \rho \to 1/R \) (circle), therefore, if we replace these limit values in the relation (1) it results:

\[ \rho(s) = \frac{s - s_B}{R(s_C - s_B)} \cos s_B \leq s \leq s_C \]  

(2)

Another element that should be considered when designing a curve is the superelevation defined by the angle \( \alpha \) to the horizontal position of the roadway, measured towards the curve exterior. Considering a coordinate system placed in the CG of the vehicle, the forces that act on it are described in figure 2.

Considering the road grip expressed by the side friction factor \( \mu_y \), the balance condition to ensure vehicle stability when passing through a curve is:

\[ F_{cf} \cdot \cos(\alpha) - G \cdot \sin(\alpha) < F_{fl} = \mu_y \cdot \left( G \cdot \cos(\alpha) + F_{cf} \cdot \sin(\alpha) \right) \]  

(3)

The cross slope of the road is defined as \( [%] = 100 \tan(\phi) \). As the superelevation angle has low values, we make the following approximations: \( \cos(\alpha \approx 1; \sin(\alpha \approx \alpha) \)), resulting the relation:

\[ \frac{v^2}{R} \cdot (1 - \alpha \cdot \mu_y) < g \cdot (\mu_y + \alpha) \]  

(4)

The product is \( \alpha \cdot \mu_y \ll 1 \) and it can be neglected. By expressing the vehicle speed \( v \) in km/h the relation (4) can be rewritten as:

\[ R > \frac{v^2}{3.6^2 \cdot 9.81 \cdot (\mu_y + \alpha)} = \frac{v^2}{127 \cdot (\mu_y + \alpha)} \]  

(5)

The value of the curvature radius \( R \) set by the relation (5) represents a minimum value when designing a curve for a certain road passing through speed. If the superelevation is missing, the curve radius should be increased, in order to ensure the same stability and comfort level for the passengers.

From the road accidents reconstruction viewpoint, the curve superelevation and the effects of the lateral acceleration on the vehicle are usually ignored, unfortunately.

We can assume a similar relation to that of the clothoid valid in the case of the variation of superelevation along the clothoid, considering that we start from a null value in B point and we reach the maximum height \( \alpha_{max} \) in C point on the arc of the circle where the relation is applicable (4). The superelevation model adopted by the authors results from the relation:

\[ \alpha(s) = \alpha_0 + \alpha_1 \cdot s \]  

(6)

where \( \alpha_0 \) and \( \alpha_1 \) are the parameters that also have to be estimated for the extreme points of the curve.
G-weight of vehicle with mass $M$, $F_{cf} = \frac{Mv^2}{R}$ – centrifugal force, $F_{fr}$ side friction factor that ensures the trajectory and prevents the lateral sliding, $\alpha$ super elevation angle

Thus, in B points $s_B \to 0$ (alignment) and C point, where $s=s_C$ we have $\alpha \to \alpha_{max}$ (circle)

Therefore, if we replace these values at the ends in relation (6) it results:

$$\alpha(s) = \frac{(s-s_B)\alpha_{max}}{(s_C-s_B)} \cos B \leq s \leq s_C$$

(7)

For the clothoid segment DE, the limit relations defined for the extreme points of the curve are:

In D points $s_D$ we have $\rho \to 1/R$ (arc of circle) and in E point, where $s=s_E$ we adopt $\rho \to 0$ (alignment) so that if we replace these limit values in the relation (1) it results:

$$\rho_1(s) = \frac{s-s_E}{R(s_D-s_E)} \cos D \leq s \leq s_E$$

(8)

As for the variation of super elevation along the clothoid DE, if we write the conditions at the ends, it results: in D point $s=s_D \alpha \to \alpha_{max}$ and in E point, where $s=s_E$ we have $\alpha \to 0$ (alignment). Thus, if we replace these end values in relation (6) it results:

$$\alpha_1(s) = \frac{(s_E-s)\alpha_{max}}{(s_E-s_D)} \cos D \leq s \leq s_E$$

(9)

The estimation accuracy for the parameters and equations of the two transition clothoids depends on the accuracy used to determine the end values, using GPS or IMU equipment, for example.

3. The impact of geometry of the curved road on the passengers’ comfort

Figure 1 presents the linear increase of the lateral acceleration ($y$ axis) during the passing through the clothoid transition curve, from the null value specific to the alignment passing through up to a maximum value, given by the centripetal force specific to the movement inside a perfectly circular curve. If the transition from the alignment to the circular curve is made directly, the passengers shall
face the immediate impact of the centrifugal acceleration at peak value, resulting in a high discomfort level.

To demonstrate the difference between the constant curvature (circle) and a variable linear curvature (clothoid), we run experiments on a road sector with a circular horizontal geometry, i.e. a roundabout. For this experiment, we selected the roundabout in Piața Alba Iulia, Bucharest, with its map presented in figure 3.

The figure above presents the circular trajectories of the vehicle made in 20 tests, where the driver did his best to keep a constant speed. The tests were performed on dry roadway, with a very good condition of the asphalt cover. In several tests we succeeded to keep a constant speed (by using the Cruise-Control function), but most of the times this was not possible, due to the dense traffic and the priority signs in two points of the circular route. Usually, the average speed on that sector is approx. 30-35 km/h.

The results achieved in one of the tests where we succeeded to keep a relatively constant vehicle speed over the whole circular sector were presented in figure 4. We can notice the confirmation of the relation: $\frac{v}{\rho} = \frac{1}{R}$, the radius average value of 58.68 m being very close to the real one, i.e. 60 m. Normally, on a circular curve, the lateral acceleration is constant for a constant speed. In figure 4, the deviations of the vehicle speed from the constant value can be found in the deviations in the lateral acceleration. The minor deviations from the constant value recorded on the R radius graphic come from the adjacent deviations of the vehicle from the perfectly circular trajectory, or from some graphical errors of the road signs between the lanes.

When analyzing mostly the speed graphic (in red) that reflects the constant speed of approx. 10.83 m/sec (39 km/h) ±5% and the lateral acceleration graphic (in blue), we notice the sudden change from an acceleration value of 0 m/sec² to a value of 2 m/sec². Although the entries to a roundabout have a curve designed on a parabolic law, in our test the curve was cut, to simulate an entry from the alignment into the circular arc.

To find a connection between the curvature of a road sector and the discomfort level of the passengers while passing through it, we noticed that the latter is not so much determined by the lateral acceleration value, that remains almost constant throughout the circular sector at 2 m/sec², but mostly by its variation from 0 to 2 m/sec². Therefore, it is natural to associate the discomfort level to the variation in the lateral acceleration (the acceleration derivative).
To calculate this acceleration variation, we considered a vehicle that travels on a circular sector, with a superelevation compliant with the road design guidelines [2] defined as: $e[\%] = 100 \cdot \tan(\alpha)$, $\alpha$ being the superelevation angle according to figure 5.

We take into account the lateral forces on the $y$ axis of the vehicle, the longitudinal forces on the $x$ axis (moving direction), and the vertical forces on the $z$ axis perpendicular on the $xoy$ plan, oriented upwards. In this situation, the center of gravity of the vehicle, equipped with a 6 degree of freedom inertial measurement unit, there are three active forces: the propulsion longitudinal force ($\vec{F}_x = m \cdot$...
the centrifugal lateral force \( F_{cf} = m \cdot v^2 \cdot \rho \), and the vehicle weight \( G = mg \). We noted with \( \rho \) the roadway curvature. According to transverse section in the image, the center of gravity shows the resultant force \( \mathbf{R} \) generated by the unification of the latter two components listed. If we relate it to the vehicle inclination and the coordinates system of the center of gravity (CG), and of the IMU measuring system, the resultant \( \mathbf{R} \) can be decomposed into a lateral component, \( \mathbf{F}_y \), oriented towards the versor \( \mathbf{y} \) of \( oy \) axis, with a normal trajectory and a speed vector, and a vertical component, perpendicular on the roadway surface that makes \( \beta \) angle with the resultant \( \mathbf{R} \).

\[
\begin{align*}
\mathbf{F}_y &= |\mathbf{F}_y| \cdot \mathbf{y} = R \cdot \sin(\beta) = R \cdot \sin((\alpha + \beta) - \alpha) = R \cdot \sin(\alpha + \beta) \cdot \cos(\alpha) - \\
&= \cos(\alpha) \cdot \left| \left( F_{cf} - G \cdot \tan(\alpha) \right) \right| \cdot \mathbf{y}
\end{align*}
\]

On the longitudinal direction tangent to the trajectory curve, there is a force that can be expressed by the following relation:

\[
\mathbf{F}_x = m \cdot |\mathbf{a}_x| \cdot \mathbf{x} = m \cdot \frac{dv}{dt} \cdot \mathbf{x} \tag{11}
\]

The resultant force can be expressed by the following equation:

\[
\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y = m \cdot \frac{dv}{dt} \cdot \mathbf{x} + \cos(\alpha) \cdot \left( F_{cf} - G \cdot \tan(\alpha) \right) \cdot \mathbf{y} \tag{12}
\]

The resulting acceleration has the following expression:

\[
\mathbf{a} = \frac{dv}{dt} \cdot \mathbf{x} + \cos(\alpha) \cdot \left( \rho \cdot v^2 - g \cdot \tan(\alpha) \right) \cdot \mathbf{y} \tag{13}
\]

The discomfort felt when entering a curve is determined by the lateral acceleration. Its value can be calculated using the expression:

\[
\mathbf{a}_y = \frac{da}{dt} \cdot \mathbf{y} \tag{14}
\]

In the expression (13), the acceleration, speed and curvature vary in time but also depending on the length of the trajectory covered by the vehicle. If we change the deviation variable of the time in the length of the curve, the resulting acceleration and the superelevation in the route length according to the relation:

\[
\frac{d\mathbf{a}}{ds} = \frac{dv}{dt} \cdot \frac{d\mathbf{x}}{ds} + \frac{d}{ds} \left[ \cos(\alpha) \cdot \left( \rho \cdot v^2 - g \cdot \tan(\alpha) \right) \right] \cdot \mathbf{y} \tag{15}
\]

The connection between the derivatives of acceleration and speed depending on time and the length of the curve covered result from the following equations:

\[
\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{ds}{ds} \cdot \frac{dv}{ds} = \frac{1}{v} \cdot \frac{dv}{dt} = \frac{1}{v} \cdot a_x \tag{16}
\]

and
To determine the variation of vector \( \vec{x} \) that represents the longitudinal acceleration derivative along the curve oriented on the tangent direction to the curve, we use Figure 6, where \( \vec{x} \) is the versor of the longitudinal axis of the mobile tridimensional system of coordinates attached to the center of gravity of the vehicle.

**Figure 6.** Determination of the \( \vec{x} \) versor derivative relative to the curve length.

The derivative of \( \vec{x} \) versor in relation with the distance crossed in the curve results from the following relation:

\[
\left| \frac{d\vec{x}}{ds} \right| = \lim_{\Delta s \to 0} \frac{\Delta \vec{x}}{\Delta s} = \lim_{\Delta s \to 0} \frac{\vec{x}(s + \Delta s) - \vec{x}(s)}{\Delta s} = \lim_{\Delta s \to 0} \frac{\Delta \vec{y}}{\Delta s} = \lim_{\Delta s \to 0} \frac{\vec{x} \cdot \sin(\Delta \theta)}{\Delta s} = \frac{1}{\Delta s} \Rightarrow \Delta \theta = 1 = \frac{\Delta \theta}{\Delta s} \approx \lim_{\Delta s \to 0} \frac{\Delta \theta}{\Delta s} = \frac{1}{r} = \rho
\]

In the previous relation, we used the fact that the versor’s norm is 1, and for very small angles, \( \sin(\Delta \theta) \approx \Delta \theta \). As orientation, \( \frac{\Delta \vec{y}}{\Delta s} \) is perpendicular on the vector tangent to the curve, thus being oriented towards the normal to the arc followed, i.e. the versor \( \vec{y} \). Therefore, we can write:

\[
\frac{d\vec{x}}{ds} = \frac{\vec{y}}{r} = \rho \cdot \vec{y}
\]

Thus, the first item in the equation (16) that refers to the normal component to the trajectory resulting from the longitudinal acceleration deviation may be noted by taking into account (20) as follows:

\[
\frac{dv}{dt} \cdot \frac{d\vec{x}}{ds} \cdot \vec{y} = \frac{dv}{dt} \cdot \rho \cdot \cos(\alpha) \cdot \vec{y}
\]

By replacing (20) in the relation (15), we obtain:

\[
\frac{d\vec{a}}{ds} = \frac{dv}{dt} \cdot \rho \cdot \cos(\alpha) \cdot \vec{y} + \frac{d}{ds} \left[ \cos(\alpha) \cdot \left( \rho \cdot v^2 - g \cdot \tan(\alpha) \right) \right] \cdot \vec{y}
\]

We also considered the fact that in the formula (20) and Figure 6, the curvature \( \rho \cdot \vec{y} \) is defined in the perfectly horizontal surface, while the general relation (15) refers to a supraelevated road that makes an angle \( \alpha \) to the horizontal surface, generating a versor \( \cos(\alpha) \cdot \vec{y} \). Because \( \vec{y} \cdot \vec{y} = 1 \), we have:

\[
\frac{d\vec{a}}{ds} \cdot \vec{y} = \frac{dv}{dt} \cdot \rho \cdot \cos(\alpha) + \frac{d}{ds} \left[ \cos(\alpha) \cdot \left( \rho \cdot v^2 - g \cdot \tan(\alpha) \right) \right]
\]

To conclude, the derivative \( \frac{d}{ds} \left[ \cos(\alpha) \cdot \left( \rho \cdot v^2 - g \cdot \tan(\alpha) \right) \right] \) can be rewritten as:
\[
\frac{da}{ds} \cdot \vec{y} = \cos(\alpha) \cdot \left[ \rho \cdot \frac{dv}{dt} + v^2 \cdot \frac{d\rho}{ds} + 2 \cdot \rho \cdot v \cdot \frac{dv}{ds} - g \cdot \frac{d\alpha}{ds} \right] - \sin(\alpha) \cdot \rho \cdot v^2 \cdot \frac{d\alpha}{ds} = (23)
\]

By taking into account the equations (16) and (17) that can be replaced in the relation (23), we generate the following expression for the acceleration derivative in relation with the lateral axis:

\[
\frac{da}{dt} \cdot \vec{y} = v \cdot \cos(\alpha) \cdot \left[ 3 \cdot \rho \cdot a_x + v^2 \cdot \frac{d\rho}{ds} - \left( g + \rho \cdot v^2 \cdot \tan(\alpha) \right) \cdot \frac{d\alpha}{ds} \right] (24)
\]

In the case of road sectors with very wide curvature radius, the superelevation can be neglected, and the lateral acceleration variation can be determined by using the relation (25) –see[3]and [4].

\[
\frac{da}{dt} \cdot \vec{y} = 3 \cdot \rho \cdot v \cdot a_x + v^3 \cdot \frac{d\rho}{ds} (25)
\]

and (26), in the case of traveling at constant speed.

\[
\frac{d\ddot{a}}{dt} \cdot \vec{y} = v^3 \cdot \frac{d\rho}{ds} (26)
\]

The formula (24) may take specific types when applied to road sectors with particular geometry mainly used in road construction, such as a circular sector or a clothoid arc. For example, for a circular sector with the radius \(r\) (curvature \(\rho = \frac{1}{r}\)) and a constant superelevation \(\alpha\), it results:

\[
\frac{da}{dt} \cdot \vec{y} = \frac{3 \cdot v \cdot a_x \cdot \cos(\alpha)}{r} (27)
\]

From the abovementioned relation we can also determine the relation between the minimum radius of a curve and the lateral acceleration variation, i.e. the discomfort level produced. This level is at maximum value in the absence of superelevation and of a progressive transition curve, resulting in the relation (28) we can also find in [5].

\[
r_{\text{min}} = \frac{3 \cdot v \cdot a_x \cdot \cos(\alpha)}{\frac{da}{dt} \cdot \vec{y}} (28)
\]

In the case of crossing a transition sector in the shape of a clothoid arc whose curvature has a linear variation according to formula (1), the lateral acceleration variation is calculated using the relation:

\[
\frac{da}{dt} \cdot \vec{y} = 3 \cdot \rho \cdot a_x \cdot \cos(\alpha) \cdot \rho(s) + \rho_1 \cdot v^3 \cdot \cos(\alpha) (29)
\]

and if the passing over speed has no variation, the acceleration variation becomes:

\[
\frac{da}{dt} \cdot \vec{y} = \rho_1 \cdot v^3 \cdot \cos(\alpha) (30)
\]

The road building practice accepts 0.9 m/sec³ as the maximum variation values of the lateral acceleration, and regarding the superelevation, the upper limit in practice is 6% -8%. The latter should be applied in relation with the curve length, without requiring speed reductions, resulting thus in the passengers’ safety level.
4. Experimental calculations

The calculation accuracy of the curve depends on the way the driver succeeds keeping his vehicle on the half of the lane or on the white marking that separate the traffic lane from the emergency lane. Any lane "cut" will bias the results of calculating the real curve. Another condition that increases the accuracy of measurements and achieved results is related to the way in which the driver succeeds to keep his speed constant during the test. Therefore, it is advisable to use the "Cruise Control" mode, if the vehicle has this option. The accomplishment of the two requirements depends on the road traffic in the subject sector.

Figures 7 and 8 present the variation of the dynamic parameters (speed, lateral acceleration, turning speed) on a curved sector on DN2 (E85), near Buzau Municipality.

The curvature radius $R$ is calculated using the measured parameters.

Figure 7 and figure 8 present only three results. Figure 9 illustrates the road sector by using Google Earth digital map. We performed 9 crossings, with the related measurements, out of which 5 in one direction and 4 in the opposite direction.

After determining the curvature, we identified the alignments, the transition curves (sectors I-II and III-IV) and the central arc II-III. In figure 7, we marked the deviation for each point (I, II, III and IV) in the 9 measurements.

The analysis of the measured and calculated data allow us to make important observations on the roadway that cannot be emphasized by just looking at the route on a map, such as Google Map.
Figure 8. Dynamic parameters (speed $v$, turning speed $\psi$, lateral acceleration $a_y$).
The curvature radius $R$ is calculated using the measured parameters.
Observations resulted:
- the transition areas are of radial type - most probable clothoids with a curvature that have a linear variation on the crossed length;
- the curvature slope of the I-II joint is more abrupt than the III-IV joint, resulting in a faster increase of the lateral acceleration;
- the speed where the sliding occurs (of course, also depending on the road slipperiness) has a lower value if entering the sector from Bucharest towards Focșani than the opposite side. In other words, in certain conditions, it is easier to drift on the I→IV direction than in the I←IV direction;
- the road designer should have considered that when passing through the curve in the I→IV direction, the vehicles run on the lane that has a smaller curvature radius, fact that somehow facilitates the sliding (a slightly higher centripetal acceleration). The difference between the centers of gravity of two vehicles that run on the outside and inner curve is about 6m, fact that means a plus of 0.16 m/sec² in the lateral acceleration at a speed of 120 km/h for the vehicle on the minimal curve. Logically, the I-II joint should have been designed with a lower variation than the III-IV joint;
- the discomfort level, directly proportional to the variation in time of the lateral acceleration will also be more accentuated when running in the I→IV direction compared to the I←IV opposite side;
- the central curve II-III should have been an arc of circle (constant curve or radius), nut it has a curve discontinuity noticed in all 9 measurements – as a road construction error and not a design one;
- the central sector II-III is not an arc of circle, as provided for in the design guidelines. Therefore, any approximation to a circular curve used in reconstruction generate errors;
the sector in question (II – III) does not have a superelevation, as provided for in the guidelines in
force. Therefore, the sliding is more pronounced;

- when reconstructing road accidents, the experts use digital or printed maps to determine the
curvature radius. Triangulation is very difficult to apply, especially on the roads with dense. Even
in the case of using extremely precise digital maps such as Google Earth or Google Map, together
with image processing programs or CAD, there is still a big margin of error due to the impossibility
of detecting the curvature variation (constant or linear variable curvature) or of detecting the point
where a sector of linear curvature meets one of a constant curvature (circle) or an alignment
(tangent). Figure 10 presents four possible approximation options for the curvature of the sector in
figure9 using arcs of circles with radiuses between 213 and 300 m.

![Figure 10. Curvature radiuses of arcs of circles (in red) that can express a portion
from the curve in figure 9.](image)

At a first sight, all of them provide an extremely precise approximation of the subject sector,
although the errors are significant. We remind you the fact that the curvature radius is included in one
of the formula preferred by most experts, i.e. the critical speed, $v_{cr} = \sqrt{\mu \cdot g \cdot R}$.

Also, the calculation of the chord $c$ and of the sagitta $h$ by direct measurement fails to determine the
exact curvature radius. We calculated the arc radius $R$ according to the known relation $R = \frac{c^2}{2h} + \frac{h}{2} = 200.64$ m for $c=63.4$ m and $h=2.52$ m, using a Bosch laser equipment to determine the distances.
We positioned the distances and the selected arc on the white line that separates the two traffic
directions, for an increased accuracy and also to prevent disturbing the dense traffic in the target area.

Figures 7 and figure8 present in green the variation of the curvature radius on the target road sector,
giving a minimum value of 230 m. The accuracy of the GPS equipment we used is 0.027 m/sec for
speed, 1.2 m 95% CEP (Circle of Error Probable) for the absolute coordinates, and under 50 cm/km
for distance. To perform the measurements, we used a GPS base station, RTK (Real Time Kinematic)
technique and DGPS (Differential GPS), that allowed to reach a GPS coordinates measurement
accuracy of 28 cm.

The experimental test results are validated by the curvature calculation with CAD programs like
CATIA or Autocad. Such calculation requires the input of the most accurate trajectory data, i.e. not
only the accurate calculation of coordinates, but also the actual vehicle moving on the curvature
section, even on the roadway median, if possible. Any curve "cutting", even without leaving the lane or direction of moving, changed the calculated curvature. The red line in figure 7 represents the curvature calculated for the GPS track points determined at a 2Hz update rate equivalent to that of a smartphone or a standard GPS device (50 track points determined on the 323 m sector), and the yellow line represents the curvature variation for a curve generated using a 5th degree linear interpolation, starting from the coordinates of the determined track points.

Thus, when the curvilinear trajectory is approximated by polynomials using the formula:

\[ s(x) = \sum_{i=0}^{n} A_i \cdot x^i, \]

the curvature \( \rho(x) = 1/R(x) \) can be determined using the ratio:

\[ \rho(x) = \frac{1 + \left(\frac{ds}{dx}\right)^2}{\frac{d^2s}{dx^2}}, \]

where the critical speed is given by the ratio:

\[ v_{cr} = \sqrt{\mu \cdot g \cdot \left(\frac{1}{\rho(x)}\right)}. \]

Both cases show the correspondence between the on site evaluation of the curvature radius pursuant to the measurements (see figure 7 and figure 8) and the graphical one presented in figure 11.

Figure 11. Graphical evaluation of the curvature variation on the specific route.

The permanent change of the curvature radius can be easily noticed, and the critical speed formula can be applied when the trajectory point used for calculating the speed and its exact curvature are being known.

Figure 12 shows how some little deviations of 50-60 cm between curvilinear coordinates may significantly change the curvature radius. In other words, the critical speed when "cutting" a curve is
different from the critical speed that allows keeping the lane, but it is impossible to estimate these small differences when performing a technical investigation in the absence of brake traces.

![Figure 12](image)

**Figure 12.** Little variation in the followed trajectory generates major changes to the curvature radius.

### 5 Conclusions

This paper aims at presenting some experimental methods and the practice results achieved by the authors, that enable the on-board calculation of roadway curvatures, and the identification of the points of discontinuity that fail to meet one or several technical design or construction conditions, thus becoming potential sources of traffic events or passenger discomfort. In addition, the paper describes a number of experiments that improve the accuracy and performances of expert motor vehicle technical investigations compared to the conventional ones. The benefits of the methods and experiments herein mainly address the following aspects: Improving the accuracy of roadway curvature calculation, i.e. curves and critical speed calculation, implicitly. Curves are usually approximated to circular arcs, whose radius are determined using graphical methods from programs like Google Maps, and sometimes by measuring the subtended chord and the circle sagitta; The possibility of analyzing the execution accuracy of some road curve segments (e.g. how accurate was a joint approximated to a clothoid) and measuring the real longitudinal and lateral speed and acceleration developed over the road segment of interest. This enables the analysis of accident blackspots; Possibility of estimating the actual forces acting on a vehicle passing through curves at various speeds. Presently, the accident reconstructions consider only the centrifugal force input, without including other factors in the calculation of the actual lateral speed and acceleration; Measuring the drift angle and turning speed at various speeds and road holding conditions at the event scene enables the calculation of a specific vehicle’s conditions of entering a sideslip; The actual determination of a vehicle’s braking parameters by measuring its deceleration and braking distance in relation to the brake pad trigger, compared to their determination based on brake traces; The actual determination of a vehicle’s acceleration parameters and behavior, particularly at high speeds; The possibility of finding the real braking characteristics of vehicles equipped with ABS and ESP (with the system’s response time depending on the pad pressing time, the actual braking distance and deceleration characteristics, the distances covered at different response times and speed, etc.); Calculation of the vehicle dynamic parameters on
upward and downward slopes. Presently, expert technical investigations fail to consider the changes to distances, kinematic and dynamic parameters of a vehicle moving inclined on a horizontal and longitudinal axis; The possibility of determining the road holding on a specific sector based on real time measurements of various dynamic and kinematic parameters. Both the longitudinal and lateral friction coefficients (sideslip angles) can be calculated. Expert technical investigations ignore the lateral grip parameters, while the longitudinal ones are being approximated based on some standard tables.

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