Study of Rate-Splitting Techniques with Block Diagonalization for Multiuser MIMO Systems

Andre R. Flores and Rodrigo C. de Lamare

1 Centre for Telecommunications Studies, Pontifical Catholic University of Rio de Janeiro, Brazil
2 Department of Electronic Engineering, University of York, United Kingdom

Emails: andre.flores@cetuc.puc-rio.br, delamare@cetuc.puc-rio.br

Abstract

In this work, we investigate Block Diagonalization (BD) techniques for multiuser multiple-antenna systems using rate-splitting (RS) multiple access. In RS multiple access the messages of the users are split into a common part and a private part in order to mitigate multiuser interference. We present the system model for a RS multiple access system operating in a broadcast channel scenario where the receivers are equipped with multiple antennas. We also develop linear precoders based on BD for the RS multiple access systems along with combining techniques, such as the min-max criterion and the maximum ratio combining criterion, to enhance the common rate. Closed-form expressions to describe the sum rate performance of the proposed scheme are also derived. The performance of the system is evaluated via simulations considering imperfect channel state information at the transmitter. The results show that the proposed schemes outperform conventional linear precoding methods.

Index Terms

Multiple-antenna systems, ergodic sum-rate, rate-splitting, block diagonalization.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) technology enhances the information and error rates performance of wireless communications systems by exploiting multipath propagation through multiple transmit and receive antennas [1]. Modern wireless communications systems deal with multiple users distributed geographically, making multiuser MIMO (MU-MIMO) the focus of many research works over the last decade. Among the key problems of MU-MIMO are the multi-user interference (MUI) and acquisition of channel state information at the transmitter (CSIT), which can decrease dramatically the overall system performance [2], [3]. In order to deal with MUI, many transmit processing techniques have been proposed in the literature [4], [5], [6], [7]. Most of these techniques rely on the quality of CSIT. Nevertheless, obtaining highly accurate CSIT in practice is still an open problem [8].

Rate-splitting (RS) multiple access schemes, proposed in [9], have been adopted in the last years as a promising approach to enhance the sum-rate performance of MIMO systems working under imperfect CSIT. Basically, RS splits the data before transmission into a common stream and private streams. The common stream should be decoded by all users. In contrast, the private streams are decoded only by its corresponding user. The main advantage of RS schemes is that they can adjust the content and the power of the common message in order to partially decode interference and partially treat interference as noise. RS has the ability to control how much interference should be decoded through the common message and how much should be treated as noise [10].

In the literature, RS has been used in conjunction with several linear precoding techniques [8], [11], working under perfect and imperfect CSIT assumption. RS with non-linear precoders has been studied in [12]. Other scenarios of interest, such as massive MIMO and MISO networks with RS have been considered in [13] and [14], respectively. RS has also been studied for robust transmission under bounded CSIT errors [15]. However, most of the works on RS consider only multiple-input single-output (MISO) scenarios and zero-forcing and minimum mean-squared error (MMSE) channel inversion-type precoders. MIMO scenarios have been studied in [16] from a DoF perspective. In [17] a MIMO RS architecture has been proposed for millimetre waves using a ZF precoding. However, the design of linear precoders for RS schemes has not considered Block Diagonalization (BD) type linear precoders, which have the potential to significantly enhance the sum rate performance of ZF and MMSE linear precoders.

In this paper, we generalize linearly precoded RS multiple access schemes to MU-MIMO systems where the users are equipped with multiple antennas. We present BD linear precoder for RS multiple access schemes in MU-MIMO systems. Furthermore, we propose techniques based on the min-max and the maximum ratio combining criteria to enhance the common rate at the receivers with multiple antennas. The performance of the proposed schemes is evaluated using the sum rate figure of merit in a Broadcast Channel (BC) under imperfect CSIT assumption.

The rest of this paper is organized as follows. Section II presents the mathematical model of the system and briefly reviews linear precoding techniques. Section III describes the proposed combining strategies to compute the common rate. Section IV present the analysis of the sum rate performance. Section V shows the simulation results. Finally, Section VI draws the conclusions of this work.

This work is partly funded by the CNPq and FAPERJ Brazilian agencies.
Matrices are denoted by boldface uppercase letters, whereas boldface lower case letters denote column vectors. Standard letters represent scalars. The superscripts $(\cdot)^\top$ and $(\cdot)^H$ are the transpose and Hermitian operators respectively. The cardinality operator is given by $\text{card}(\cdot)$ and $\text{diag}(\cdot)$ represents a diagonal matrix with the entries of vector $\cdot$ in the main diagonal. The trace of a matrix is denoted by $\text{tr}(\cdot)$. $\mathbb{E}[\cdot]$ stands for the expectation operator, $\|\cdot\|$ for the Euclidean norm and $\odot$ for the Hadamard product.

II. System Model and Linear Precoding

We consider a MIMO BC with $K$ users, where user $k$ is equipped with $N_k$ antennas. The total number of receive antennas is then given by $N_r = \sum_{k=1}^K N_k$. The number of antennas at the transmitter is denoted by $N_t$ and remains in the range $N_t \geq K \geq 2$. The group of data streams intended for user $k$ form a set denoted by $B_k$. Then, the $i$th stream sent to the $j$th user is expressed by $B_{ij}$. The total number of transmitted data streams is $B = \sum_{k=1}^K M_k$ with $M_k = \text{card}(B_k)$. The model satisfies the transmit power constraint $\mathbb{E}[\|x\|^2] \leq E_t$, where the vector $x$ represents the transmitted signal and $E_t$ denotes the total transmit power.

The system employs the RS scheme, which splits the messages into a common part and a private part \cite{13, 9}. Since we focus on the sum rate analysis, it suffices to consider that only one stream is split. The common part is encoded into one common stream and the private parts into $B$ private streams. The receivers share a codebook since the common message has to be decoded by all the users with zero error probability. In contrast, each private stream is decoded only by its corresponding user. This means that each receiver must decode $M_k + 1$ data streams, namely the common stream (decoded by all but intended to only one user) and a set of private streams (decoded by its respective user). This is possible if we apply successive interference cancellation (SIC) techniques \cite{19, 20, 21, 22, 23, 24, 25, 26, 27, 28}. The common stream is first decoded using SIC and all private messages are considered as interference and treated as noise. At the end, the message sent via the private streams is decoded. The strength of RS is its ability to adjust the content and the power of the common message to control how much interference should be decoded by all users (through the common message) and how much interference is treated as noise.

The information sequences in the data streams are modulated and then the splitting process is performed over the message, resulting in a vector of data symbols $s_{\text{RS}} \in \mathbb{C}^{B+1}$. Specifically, the vector of the transmitted symbols is given by $s_{\text{RS}} = [s_c, s_1^T, s_2^T, \ldots, s_K^T]^T$, where $s_c$ is used to designate the symbol of the common stream and $s_k$ contains the $M_k$ private streams of the $k$th user. We assume uncorrelated symbols with zero mean and covariance matrix equal to $R_s = I$. The transmitter processes the symbols using a linear precoder, which maps the symbols to the transmit antennas. A common precoder, $p_c \in \mathbb{C}^{N_t}$, is introduced in order to map the common symbol to the transmit antennas. Then, the precoder is given by $P_{\text{RS}} = [p_c, P_1, P_2, \ldots, P_K]$, where $P_k \in \mathbb{C}^{N_t \times M_k}$ is the precoder of the $k$th user. Moreover, the vector $p_k$ denotes the $k$th column of matrix $P_{\text{RS}}$. The transmitted signal is expressed by

$$x = P_{\text{RS}}A_{\text{RS}}s_{\text{RS}} = a_c s_c + \sum_{i=1}^K P_i \text{diag}(a_i) s_i,$$  \hspace{1cm} (1)

where $A_{\text{RS}} \in \mathbb{R}^{(B+1) \times (B+1)}$ is a general diagonal power loading matrix and the vector $a_{\text{RS}} = [a_c, a_1^T, a_2^T, \ldots, a_K^T]^T \in \mathbb{R}^{(B+1)}$ consists of the coefficients in the main diagonal of the matrix $A_{\text{RS}}$. The vector $a_k \in \mathbb{R}^{M_k}$ contains the power assigned to the $M_k$ symbols intended for the $k$th user. The coefficient $a_c$ denotes the power distributed to the common message. In other words, the total transmit power is allocated partially to the common and private streams. The transmit power constraint is expressed by $\text{tr}(P_{\text{RS}} \text{diag}(a_{\text{RS}}) P_{\text{RS}}^H) \leq E_t$. Assuming normalized precoders, the transmit power constraint is reduced to $|a_c|^2 + \sum_{k=1}^B |a_k|^2 \leq E_t$.

After the precoding operation, the data are sent to the receiver over the channel $H = \tilde{H} + \tilde{H} \in \mathbb{C}^{N_r \times N_t}$, where each coefficient $h_{ij}$ in the channel matrix $H$ represents the link between the $j$th transmit antenna and the $i$th receive antenna. The matrix $\tilde{H}$ represents the estimate of the channel and the matrix $\tilde{H}$ takes into account the quality of the channel estimate by modelling the error produced by the estimation procedure. The channel of the $k$th user is given by $H_k \in \mathbb{C}^{N_r \times N_t}$. It follows that $H = [H_1^H \ldots H_k^H \ldots H_K^H]$. The vector $h_{ij}^T \in \mathbb{C}^{N_t}$ represents the $j$th row of matrix $H$. For simplicity, we consider a flat fading channel which remains fixed during a transmission block.

The received signal obtained following the model in (1) is given by

$$y = G(\tilde{H}P_{\text{RS}} \text{diag}(a_{\text{RS}}) s_{\text{RS}} + n),$$  \hspace{1cm} (2)

where $G \in \mathbb{C}^{B \times N_r}$ is a block diagonal receive filter since there is no cooperation between users. Moreover, the matrix $G_k \in \mathbb{C}^{M_k \times N_r}$ denotes the receive filter of the $k$th user. The vector $n \in \mathbb{C}^{N_r \times 1}$ is the additive noise modelled as a circularly symmetric complex Gaussian random vector, i.e., $n \sim \mathcal{CN}(0, R_{nn})$. Without loss of generality, we will consider that the noise is uncorrelated and has the same statistical properties at each antenna, i.e., $\sigma_n^2 = \sigma_{n,i}^2 = \sigma_n^2$, $\forall i, j$, reducing the covariance matrix to $R_{nn} = \sigma_n^2 I$. The SNR is defined as $\text{SNR} = E_t / \sigma_n^2$. 


Given a channel state and considering an imperfect CSIT scenario, the received signal at the $k$th terminal can be written as
\[
y_k = d_k s_k G_k H_k p_c + \sum_{i \in B_k} a_i s_i G_i H_i p_i + \sum_{j=1 \atop j \not\in B_k}^B a_j s_j G_j H_j p_j + G_k n_k. \tag{3}
\]

Let us define the matrix $P^H = GH$ and the matrix $\tilde{F}^H = \tilde{G} H$. The mean power of the $l$th received stream at user $k$ can be expressed as follows:
\[
\mathbb{E} \left[ |y_{lk}|^2 \right] = \alpha^2_l |F^H_l p_c|^2 + \sum_{i \in B_k} \alpha^2_l |F^H_i p_i|^2 \\
+ \sum_{j=1 \atop j \not\in B_k}^B a_j^2 |F^H_j p_j|^2 + \|g_k\|^2 \sigma^2_n. \tag{4}
\]

When working under a perfect CSIT scenario, the term $\tilde{H}$ is reduced to zero and equations (3) and (4) remain the same with $H = \tilde{H}$.

Suppose that we allocate no power to the common stream, i.e., we set $a_c = 0$. In such cases the model represents a conventional MU-MIMO system where no RS is performed. It turns out that the model established is a general framework and conventional MU-MIMO can be seen as a particular case where no power is assigned to the common stream.

In what follows we review the main concepts about the ZF and BD precoding techniques. We consider that the precoder of the private stream is defined as $P = [P_1, P_2, \ldots, P_K]$.

A. Linear ZF Precoding

The ZF precoder \[4\] defined by
\[
P^{(ZF)} = \beta^{(ZF)} F^H (HH^H)^{-1}, \tag{5}
\]
where $\beta^{(ZF)}$ is a scaling factor introduced to satisfy the transmit power constraint that is defined as
\[
\beta^{(ZF)} = \sqrt{\frac{E_{tr}}{\text{tr} \left( (HH^H)^{-1} R_{ss} \right)}}. \tag{6}
\]

B. Linear BD Precoding

BD precoding has been proposed in \[3\] and \[29\] for wireless communications systems, and further studied in \[30\], \[31\], \[32\] due to its potential to increase the sum rate performance. This technique is based on Singular Value Decomposition (SVD).

The precoding matrix for the $k$th user can be written in two parts as follows:
\[
P_k^{(BD)} = P_k^a P_k^b. \tag{7}
\]
The filter $P_k^a$ is used to completely eliminate the MUI, whereas the filter $P_k^b$ allows parallel symbol detection. Let us form the matrix $\hat{H}_k$ by excluding the channel matrix of the $k$th user, i.e. $\hat{H}_k = [H_1^H \ldots H_{k-1}^H H_{k+1}^H \ldots H_K^H]$. By using SVD we get $H_k = \hat{U}_k \hat{\Psi}_k \left[ \hat{V}_k^{(1)} \hat{V}_k^{(0)} \right]^H$. The matrix $\tilde{V}_k = \left[ \hat{V}_k^{(1)} \hat{V}_k^{(0)} \right]^H$ is a unitary matrix with dimensions $N_t \times N_t$. Let us suppose that the rank of $\hat{H}_k$ is given by $\bar{L}_k$. The vector $\tilde{V}_k^{(0)}$ contains the last $N_t - \bar{L}_k$ singular vectors and forms an orthogonal basis for the null space of $\hat{H}_k$. Therefore, we can set the first part of the precoder to
\[
P_k^a = \tilde{V}_k^{(0)}. \tag{8}
\]
The first precoder separates the MU-MIMO channel into $K$ parallel independent channels. Consider the effective channel matrix defined as $H_k = H_k P_k^a$. Performing a second SVD over the effective channel $H_k$, i.e. $H_k = \hat{U}_k \hat{\Psi}_k \left[ \hat{V}_k^{(1)} \hat{V}_k^{(0)} \right]^H$ we obtain the second precoder and the receive filter as given by
\[
P_k^b = \hat{V}_k^{(1)}, \quad G_k^{(BD)} = \hat{U}_k^H. \tag{9}
\]
The matrices $P_k^b$ and $G_k^{(BD)}$ allow us to perform symbol-by-symbol detection.
III. RS COMMON AND PRIVATE RATES

The sum rate performance of a system employing an RS architecture and assuming Gaussian signalling consists of a common rate \(R_c\) and a private rate \(R_p = \sum_{k=1}^{K} R_k\), where \(R_k\) denotes the private rate of the \(k\)th user. The common rate represents the contribution of the common stream, whereas the private rate takes into account all private streams. In general for the instantaneous rate, we have \(S_r = R_c + R_p\).

In contrast to conventional RS in MISO systems, the \(k\)th receiver in a MIMO system has a total of \(M_k\) copies of the common symbol available. These copies can be used to enhance the common rate of the system. In this section, we propose combining strategies to improve the common rate performance and also derive an expression for the private rate.

A. Min-Max Criterion

Let us consider (3) from the model described in section II. The \(k\)th user receives \(M_k\) streams each one containing a copy of the common symbol. When decoding the common stream, all private messages are considered additional noise. The common rate of the \(i\)th stream intended for user \(k\) can be computed by

\[
R_{c,k,i} = \log_2 \left(1 + \frac{a_c^2|\mathbf{f}_k^H \mathbf{p}_c|^2}{\sum_{j=1}^{B} a_j^2|\mathbf{f}_k^H \mathbf{p}_j|^2 + \|\mathbf{g}_i\|^2\sigma_n^2} \right). \tag{10}
\]

It is important to note that in equation (10) we consider an imperfect CSIT scenario, i.e., \(\hat{f} = \tilde{f} + \tilde{f}_i\). The error in the channel estimate modelled by \(\tilde{f}_i\) originates MUI, which limits the overall performance of the system. Moreover, (10) be used in a perfect CSIT scenario with \(\mathbf{f}^H\) reduced to \(\mathbf{f}^H\).

When employing the Min-Max criterion, the \(k\)th receiver picks the stream in \(B_k\) that leads to the highest achievable common rate. This is possible because we assume perfect CSI available at the receiver. Mathematically, this can be expressed by

\[
R_{c,k}^{(\text{max})} = \max_{i \in B_k} (R_{c,k,i}). \tag{11}
\]

Let us consider the vector \(\mathbf{r}_{c}^{(\text{max})}\), containing all the maximum rates from all users i.e. containing all the rates \(R_{c,k}^{(\text{max})}\) computed. The common stream should be decoded by all users. In order to satisfy this condition we set the common rate equal to the minimum rate stored in \(\mathbf{r}_{c}^{(\text{max})}\), which leads us to

\[
R_{c}^{(\text{min-max})} = \min R_{c,k}^{(\text{max})}. \tag{12}
\]

Finally, the receiver decodes and subtracts the common stream from the received signal using SIC in order to decode the private stream.

B. Maximum Rate Combining

Another possibility to enhance the common rate by exploiting the multiple streams at the receiver is to use the maximum rate combining (MRC). Let us consider the received vector of (3) and define the combined signal \(\tilde{y}_k = \mathbf{w}^H y_k\), where the vector \(\mathbf{w} = [\omega_1 \omega_2 \ldots \omega_{M_k}]^T\) represents the combining filter used to maximize the SNR. Then, the average power of \(\tilde{y}_k\) is

\[
\mathbb{E} [||\tilde{y}_k||^2] = a_c^2 |\mathbf{\omega}_k^H \mathbf{H}_k \mathbf{p}_c|^2 + \sum_{i \in B_k} a_i^2 |\mathbf{\omega}_k^H \mathbf{H}_k \mathbf{p}_i|^2
\]

\[
+ \sum_{j=1}^{B} a_j^2 |\mathbf{\omega}_k^H \mathbf{H}_k \mathbf{p}_j|^2 + \mathbb{E} [||\mathbf{\omega}_k^H \mathbf{n}_k||^2], \quad (13)
\]

where we introduce the row vector \(\mathbf{\omega}_k^H = \mathbf{w}^H \mathbf{G}_k\) in order to simplify the notation. By evaluating the noise term we obtain

\[
\mathbb{E} [||\mathbf{\omega}_k^H \mathbf{n}_k||^2] = ||\mathbf{\omega}_k||^2\sigma_n^2. \tag{14}
\]

Let us also define the common and private vectors \(\mathbf{r}_{k,c} = \mathbf{H}_k \mathbf{p}_c\) and \(\mathbf{r}_{k,i} = \mathbf{H}_k \mathbf{p}_i\) with \(i \in B_k\). Substituting these terms in (13) we get

\[
\mathbb{E} [||\tilde{y}_k||^2] = a_c^2 |\mathbf{\omega}_k^H \mathbf{r}_{k,c}|^2 + \sum_{i \in B_k} a_i^2 |\mathbf{\omega}_k^H \mathbf{r}_{k,i}|^2
\]

\[
+ \sum_{j=1}^{B} a_j^2 |\mathbf{\omega}_k^H \mathbf{r}_{k,j}|^2 + ||\mathbf{\omega}_k||^2\sigma_n^2. \tag{15}
\]

From the last equation we obtain the SINR for the common message, which is given by
Evaluating the expected value of the received signal power, we can obtain the common rate of the \( k \)-th stream as:

\[
\tilde{T}_k = \sum_{i \in B_k} a_i^2 |\omega_k^H r_{k,i}|^2 + \sum_{j \in B_k} a_j^2 |\omega_k^H r_{k,j}|^2 + \|\omega_k\|^2 \sigma_n^2
\]

Using the property of the dot product and simplifying terms, the SINR can be expressed as follows:

\[
\gamma_{k,c}^{(\text{MRC})} = \frac{a^2 \|r_{k,c}\|^2 \cos \theta}{\sum_{i \in B_k} a_i^2 \|r_{k,i}\|^2 \cos \beta_i + \sum_{j \in B_k} a_j^2 \|r_{k,j}\|^2 \cos \beta_j + \sigma_n^2}
\]

where \( \theta \) is the angle between the vectors \( \omega_k \) and \( r_{k,c} \) and \( \beta_j \) is the angle between \( \omega_k \) and \( r_{k,j} \).

The maximum value of the numerator is achieved when \( \cos \theta = 1 \) and is obtained when the vectors \( \omega_k \) and \( r_{k,c} \) are parallel. By setting \( w = (G_k^H)^{-1} \frac{r_{k,c}}{\|r_{k,c}\|} \) the vectors \( \omega_k \) and \( r_{k,c} \) become parallel which lead us to the following SINR expression:

\[
\gamma_{k,c}^{(\text{MRC})} = \frac{a^2 \|r_{k,c}\|^2}{\sum_{i \in B_k} a_i^2 \|r_{k,i}\|^2 \cos \beta_i + \sum_{j \in B_k} a_j^2 \|r_{k,j}\|^2 \cos \beta_j + \sigma_n^2}
\]

Since all users should decode the common message, the transmitter sets the common rate equal to the minimum rate found across all users i.e., the common rate is given by

\[
R_c^{(\text{MRC})} = \min_{k=1 \cdots K} \log_2 \left( 1 + \gamma_{k,c}^{(\text{MRC})} \right)
\]

C. Private Rate

After decoding the common stream, the system performs SIC to remove the common symbol from the received signal. Considering that the precoder reduces the interference to the noise level we have that the covariance matrix of the effective noise is given by

\[
R_{z_k z_k} = \sum_{i=1}^{K} G_k H_k P_i \text{diag} (a_i \otimes a_i) P_i^H H_k^H G_k^H + R_{nn}
\]

Then, the achievable rate for the \( k \)-th user is

\[
R_k = \log_2 \left( \det \left[ I + P_k G_k \text{diag} (a_k \otimes a_k) P_k^H F_k R_{z_k z_k}^{-1} \right] \right)
\]

IV. Rate Analysis

In this section, we carry out the sum rate analysis of the proposed strategies combined with the BD precoder.

A. RS Min-Max Criterion with the BD precoder

The BD precoder partially removes the MUI interference. However, residual interference remains due to the imperfect CSIT, which lead us to the following vector:

\[
y_k = a_s c_s U_k^H H_k p_c + \Psi_k \text{diag} (a_k) s_k + \tilde{T}^{(k,k)} \text{diag} (a_k) s_k \\
+ \sum_{j=1}^{K} \tilde{T}^{(k,j)} \text{diag} (a_j) s_j + U_k^H n_k,
\]

where the matrix \( \tilde{T}^{(k,j)} = U_k^H \hat{H}_j \tilde{V}_j^{(0)} \tilde{V}_j^{(1)} \) represents the residual interference. Let us also consider the index \( n_k = \sum_{l=1}^{K} M_l \).

Evaluating the expected value of the received signal power we can obtain the common rate of the \( i \)-th stream intended for user \( k \), which is given by

\[
R_{c,k,i}^{(BD)} = \log_2 \left( 1 + \frac{a^2 |f_k^H p_c|^2}{\beta_{k,i}^{(\text{MM})} + \sigma_n^2} \right)
\]
where

$$
\rho_{k,i}^{(M-M)} = a^2_i |\psi_{i-n_k}^{(k)}|^2 + \sum_{l=1}^{M_k} a^2_{i+n_k} |\psi_{i-n_k,i-n_l}^{(k,k)}|^2 \\
+ \sum_{j=1}^{K} \sum_{q=1}^{M_k} a^2_{n_j+q} |\tilde{\psi}_{i-n_k,q}^{(j,j)}|^2 
$$

(24)

In a perfect CSIT scenario, the precoder and the receiver remove completely the interference and the previous equation is reduced to

$$
R_{c,k,i}^{(BD)} = \log_2 \left( 1 + \frac{a^2_i |\tilde{\Psi}_{i-n_k}^{(k)} p^H_{e} |^2}{a^2_i |\psi_{i-n_k}^{(k)}|^2 + \sigma^2_n} \right) 
$$

(25)

B. RS MRC criterion with the BD precoder

Let us consider the $k$th user and evaluate the vector $r_{k,j}$ with $j \in B_q$. We also define the column index $m = j - \sum_{l=1}^{q-1} M_l$. When $q = k$ the squared module of vector $r_{k,j}$ is reduced to:

$$
||r_{k,j}||^2 = |\psi_{m}^{(k)}|^2 + \tilde{H}_k y_m^{(k)(1)}|^2.
$$

(26)

The BD precoder should reduce the vector $r_{k,j}$ to zero when $q \neq k$ due to the zero inter-user interference restriction imposed. However, the imperfect CSIT assumption originates residual MUI, which leads us to

$$
||r_{k,j}||^2 = \sum_{i=1}^{N_k} \sum_{l=1}^{N_k} \tilde{h}_{i,l}^{(k)} |\tilde{\psi}_{l,n}^{(q)(0)} y_{n,m}^{(q)(1)}|^2,
$$

(27)

Substituting (26) and (27) in (18) we get the SINR expression, which is given by

$$
\gamma_{k,c}^{(MRC)} = \frac{a^2_i ||r_{k,c}||^2}{\rho_{k}^{(MRC)} + \sigma^2_n},
$$

(28)

where

$$
\rho_{k,i}^{(MRC)} = \sum_{i \in B_k} a^2_i |\tilde{\psi}_{i-n_k}^{(k)} u_m^{(k)} + \tilde{H}_k y_m^{(k)(1)}|^2 \cos \beta_i \\
+ \sum_{j=1}^{B} \sum_{j \notin B_k} a^2_j \left[ \sum_{i=1}^{N_k} \sum_{l=1}^{N_k} \tilde{h}_{i,l}^{(k)} |\tilde{\psi}_{l,n}^{(q)(0)} y_{n,m}^{(q)(1)}|^2 \right] \cos \beta_j.
$$

(29)

Under perfect CSIT assumption the MUI interference, which is given by $\tilde{H}_k y_m^{(k)(1)}$ and $||r_{k,j}||^2$ with $j \notin B_k$, is eliminated and the expression in (28) is reduced to

$$
\gamma_{k,c}^{(MRC)} = \frac{a^2_i ||r_{k,c}||^2}{\sum_{i \in B_k} a^2_i |\tilde{\psi}_{i-n_k}^{(k)}|^2 \cos \beta_i + \sigma^2_n}.
$$

(30)

C. RS BD private streams sum rate

Let us consider the matrix $\Phi_k = \Psi_k + \tilde{\Phi}_k$. After the SIC process we get the equations for the private rate

$$
R_p^{(BD)} = \log_2 \det \left[ I + \Phi_k \text{diag} (a_k \odot a_k) \Phi_k H R_{a_k}^{-1} \right].
$$

(31)

Under perfect CSIT assumption we have

$$
R_p^{(BD)} = \log_2 \det \left[ I + \Psi_k \text{diag} (a_k \odot a_k) \Psi_k H R_{a_k}^{-1} \right].
$$

(32)
V. SIMULATIONS

In this section we assess the performance of the proposed MIMO RS schemes employing ZF and BD precoders. A total of 12 transmit antennas was used at the BS for all simulations. We also consider 6 users where each is equipped with 2 receive antennas. The inputs follow a Gaussian distribution with zero mean and variance equal to one. We consider additive white Gaussian noise with the same statistics for all users, such that all users experience the same SNR. The ESR was computed averaging 100 independent channel realizations. For each channel realization we obtained the ASR employing 100 error matrices. The power allocated to the common precoder was set through exhaustive search, where we keep the proportion of power allocated to the private precoders fixed. Conventional RS, which uses the minimum common rate available, was also considered. We termed this strategy as RS in the simulation results.

Fig. 1 summarizes the sum rate performance of the ZF precoder and the BD precoder, both operating in a MU-MIMO system. For this simulation we consider a fixed error variance in the channel equal to 0.1. The proposed techniques achieve a better results because they exploit the multiple antennas at the receiver, enhancing the common stream. The BD precoder allows not only the enhancement of the common stream but also of the private stream obtaining a better performance in terms of sum rate. The best performance is achieved by the BD-RS-MRC due to the use and combination of all available signals at the receive antennas. The curves obtained exhibit saturation because of the imperfect CSIT assumption, which originates MUI that scales with the SNR.

For the second scenario we evaluate the performance of the proposed schemes operating at different noise levels as depicted in Fig. 2. The SNR was set to 25 dB. The results show that the RS strategies increase the robustness of the system across all error variances. The proposed MIMO BD-RS-MRC strategy achieves the highest sum-rate, which is up to 35% higher when compared to the conventional BD precoder.

For the last example, we consider that the quality of the channel estimate improves with the SNR, i.e. \( \sigma_e^2 = \beta \left( \frac{E_{tr}}{\sigma_n^2} \right)^{-\alpha} \). The parameters \( \beta \) and \( \alpha \) were set to 0.94 and 0.2 respectively. Fig. 3 shows that the proposed schemes are more robust than conventional precoding schemes. MIMO BD-RS-MRC shows a sum rate improvement of 33.33% when compared to
conventional BD precoding, whereas the MIMO ZF-RS-MRC achieves a sum rate 35% higher than the conventional ZF-strategy.

Fig. 3. Sum-rate performance with imperfect CSIT and varying channel estimate errors.

VI. CONCLUSION

In this paper, we have proposed MIMO RS strategies combined with the BD precoder and two criteria to enhance the common rate by taking advantage of the multiple antennas at the receivers. In general, all BD precoder schemes outperform their ZF precoder counterpart in terms of sum rate. Moreover, the BD-RS-MRC scheme achieves the best performance among all the proposed techniques, attaining an improvement of more than 30% when compared to conventional techniques. Simulation results have also shown that the BD-RS scheme is more robust when compared to ZF techniques under imperfect CSIT scenarios.

REFERENCES

[1] E. Telatar, “Capacity of multi-antenna Gaussian channels,” European Transactions on Telecommunications, vol. 10, no. 6, pp. 585–596, Nov. 1999.
[2] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, “An overview of massive MIMO: Benefits and challenges,” IEEE Journal of Selected Topics in Signal Processing, 2014.
[3] R. C. de Lamare, “Massive mimo systems: Signal processing challenges and future trends,” EURASIP Journal on Wireless Communications and Networking, in press.
[4] M. Joham, W. Utschick, and J. A. Nossek, “Linear transmit processing in MIMO communications systems,” IEEE Transactions on Signal Processing, vol. 53, no. 8, pp. 2700–2712, Aug 2005.
[5] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, “Zero-forcing methods for downlink spatial multiplexing in multiuser mimo channels,” IEEE Transactions on Signal Processing, vol. 52, no. 2, pp. 461–471, Jan. 2004.
[6] K. Zu and R. C. de Lamare, “Low-complexity lattice reduction-aided regularized block diagonalization for mu-mimo systems,” IEEE Communications Letters, vol. 16, no. 6, pp. 925–928, June 2012.
[7] W. Zhang, H. Ren, C. Pan, M. Chen, R. C. de Lamare, B. Du, and J. Dai, “Large-scale antenna systems with ul/dl hardware mismatch: Achievable rates analysis and calibration,” IEEE Transactions on Communications, vol. 63, no. 4, pp. 1216–1229, April 2015.
[8] H. Joudeh and B. Clerckx, “Sum-rate maximization for linearly precoded downlink multiuser MISO systems with partial CSIT: A rate-splitting approach,” IEEE Transactions on Communications, vol. 64, no. 11, pp. 4847–4861, 2016.
[9] T. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” IEEE Transactions on Information Theory, vol. 27, no. 1, pp. 49–60, Jan. 1981.
[10] Y. Mao, B. Clerckx, and V. Li, “Rate-splitting multiple access for downlink communication systems: Bridging, generalizing and outperforming SDMA and NOMA,” EURASIP Journal on Wireless Communications and Networking, in press.
[11] C. Hao, Y. Wu, and B. Clerckx, “Rate analysis of two-receiver MISO broadcast channel with finite rate feedback: A rate-splitting approach,” IEEE Transactions on Communications, vol. 63, no. 9, pp. 3232–3246, July 2015.
[12] A. Flores, B. Clerckx, and R. C. de Lamare, “Tomlinson-harashima precoded rate-splitting for multiuser multiple-antenna systems,” 15th International Symposium on Wireless Communication Systems, 2018.
[13] M. Dai, B. Clerckx, D. Gesber, and G. Caire, “A rate splitting strategy for massive MIMO with imperfect CSIT,” IEEE Transactions on Wireless Communications, vol. 15, no. 7, pp. 1611–1624, July 2016.
[14] C. Hao and B. Clerckx, “MISO networks with imperfect CSIT: A topological rate-splitting approach,” IEEE Transactions on Communications, vol. 65, no. 5, pp. 2164–2179, 2017.
[15] H. Joudeh and B. Clerckx, “Robust transmission in downlink multiuser MISO systems: A rate-splitting approach,” IEEE Transactions on Signal Processing, vol. 64, no. 23, pp. 6227–6242, 2016.
[16] C. Hao, R. B., and B. Clerckx, “Achievable DoF regions of MIMO networks with imperfect CSIT,” IEEE Transactions on Information Theory, vol. 63, no. 10, pp. 6587 – 6606, October 2017.
[17] O. Kolawole, A. Panazafironoulos, and T. Ratarajah, “A rate-splitting strategy for multi-user millimeter-wave systems with imperfect CSI,” IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 2018.
[18] B. Clerckx, H. Joudeh, C. Hao, M. Dai, and R. B., “Rate splitting for MIMO wireless networks: a promising PHY-layer strategy for LTE evolution,” IEEE Communications Magazine, vol. 54, no. 5, pp. 98–105, 2016.
[19] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, “Simplified processing for high spectral efficiency wireless communication employing multi-element arrays,” IEEE Journal on Selected Areas in Communications, vol. 17, no. 11, pp. 1841–1852, Nov 1999.
[20] Xiaodong Wang and H. V. Poor, “Iterative (turbo) soft interference cancellation and decoding for coded cdma,” IEEE Transactions on Communications, vol. 47, no. 7, pp. 1046–1061, July 1999.
[21] R. C. De Lamare, R. Sampaio-Neto, and A. Hjorungnes, “Joint iterative interference cancellation and parameter estimation for cdma systems,” IEEE Communications Letters, vol. 11, no. 12, pp. 916–918, December 2007.
[22] R. C. De Lamare and R. Sampaio-Neto, “Minimum mean-squared error iterative successive parallel arbitrated decision feedback detectors for ds-cdma systems,” IEEE Transactions on Communications, vol. 56, no. 5, pp. 778–789, May 2008.
[23] R. Fa and R. C. D. Lamare, “Multi-branch successive interference cancellation for mimo spatial multiplexing systems: Design, analysis and adaptive implementation,” IET Communications, vol. 5, no. 4, pp. 484–494, March 2011.
[24] P. Li, R. C. de Lamare, and R. Fa, “Multiple feedback successive interference cancellation detection for multiuser mimo systems,” IEEE Transactions on Wireless Communications, vol. 10, no. 8, pp. 2434–2439, August 2011.
[25] R. C. de Lamare, “Adaptive and iterative multi-branch mmse decision feedback detection algorithms for multi-antenna systems,” IEEE Transactions on Wireless Communications, vol. 12, no. 10, pp. 5294–5308, October 2013.
[26] P. Li and R. C. de Lamare, “Distributed iterative detection with reduced message passing for networked mimo cellular systems,” IEEE Transactions on Vehicular Technology, vol. 63, no. 6, pp. 2947–2954, July 2014.
[27] A. G. D. Uchoa, C. T. Healy, and R. C. de Lamare, “Iterative detection and decoding algorithms for mimo systems in block-fading channels using ldpc codes,” IEEE Transactions on Vehicular Technology, vol. 65, no. 4, pp. 2735–2741, April 2016.
[28] Z. Shao, R. C. de Lamare, and L. T. N. Landau, “Iterative detection and decoding for large-scale multiple-antenna systems with 1-bit adcs,” IEEE Wireless Communications Letters, vol. 7, no. 3, pp. 476–479, June 2018.
[29] L. Choi and R. D. Murch, “A transmit preprocessing technique for multiuser mimo systems using a decomposition approach,” IEEE Transactions on Wireless Communications, vol. 3, no. 2, pp. 20–24, Jan. 2004.
[30] H. Sung, S.-R. Lee, and I. Lee, “Generalized channel inversion methods for multiuser mimo systems,” IEEE Transactions on Communications, vol. 57, no. 11, pp. 3489–3499, November 2009.
[31] K. Zu, R. C. de Lamare, and M. Haardt, “Generalized design of Low-Complexity block diagonalization type precoding algorithms for multiuser MIMO systems,” IEEE Transactions on Communications, vol. 61, no. 10, pp. 4232–4242, October 2013.
[32] W. Zhang et al., “Widely linear precoding for large-scale mimo with iq: Algorithms and performance analysis,” IEEE Transactions on Wireless Communications, vol. 16, no. 5, pp. 3298–3312, May 2017.
[33] T. M. Cover and J. A. Thomas, Elements of information theory. New York: Wiley, 2006.