Long-distance quantum communication with “polarization” maximally entangled states

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We propose a scheme for long-distance quantum communication where the elementary entanglement is generated through two-photon interference and quantum swapping is performed through one-photon interference. Local “polarization” maximally entangled states of atomic ensembles are generated by absorbing a single photon from on-demand single-photon sources. This scheme is robust against phase fluctuations in the quantum channels, moreover speeds up long-distance high-fidelity entanglement generation rate.

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Keywords: quantum entanglement, quantum repeater, atomic ensemble

Entanglement plays a fundamental role in quantum information science [1] because it is a crucial requisite for quantum metrology [2], quantum computation [3, 4], and quantum communication [3, 5]. Quantum communication opens a way for completely secure transmission of keys with the Ekert protocol [6] and exact transfer of quantum states by quantum teleportation [7]. Because of losses and other noises in quantum channels, the communication fidelity falls exponentially with the channel length. In principle, this problem can be circumvented by applying quantum repeaters [5, 8–11], of which the basic principle is to separate the full distance into shorter elementary links and to entangle the links with quantum swaps [7, 12]. A protocol of special importance for long-distance quantum communication with collective excitations in atomic ensembles has been proposed in a seminal paper of Duan et al. [13]. After that considerable efforts have been devoted along this line [14–20].

In Duan-Lukin-Cirac-Zoller (DLCZ) protocol, entanglement in the elementary links is created by detecting a single photon from one of two ensembles. The probability $p$ of generating one excitation in two ensembles is related to the fidelity of the entanglement, leading to the condition $p \ll 1$ to guaranty an acceptable quality of the entanglement. But when $p \to 0$, some experimental imperfections such as stray light scattering and detector dark counts will contaminate the entangled state increasingly [20], and subsequent processes including quantum swap and quantum communication become more challenging for finite coherent time of quantum memory [16]. To solve this problem, protocols based on single photon source [16, 17] and photon pair source [21] were suggested. However, for the scheme proposed in Ref. [16] the “vacuum” coefficient $c_0$ [13] of the state of the elementary link is near 1, which causes the probability $p_i (i = 1, 2, \ldots, n)$ of successful quantum swap to be very low and thus the capability of the scheme in increasing quantum communication rate to be weak, where $n$ is the nesting level of swap. For the schemes suggested in Refs. [17, 21], the same problem exists owing to the fact that the efficiency of storage of a single photon in a quantum memory is far from ideal. Furthermore, all schemes based on measuring a single-photon via single-photon detectors suffer from the imperfections from the detector dark counts and its incapability of distinguishing one photon from two photons.

Here we present a protocol for long-distance quantum communication using linear optics and atomic ensembles. To overcome the low probability $p$ in DLCZ protocol, we generate the entanglement in every node with on-demand single photon source. To solve the problem of the large “vacuum” coefficient $c_0$ in Refs. [16, 17, 21], the quantum swapping is performed based on “polarization” maximally entangled states [13]. Our scheme can automatically eliminate the imperfection arising from the incapability of the single-photon detectors in distinguishing one photon from two photons and can exclude partially the imperfection due to the detector dark counts, which is the major imperfection on the quality of the entanglement for the previous schemes [17]. With this scheme the quantum communication rate can be significantly increased by several orders of magnitude with higher quantum communication fidelity for a distance 2500 km compared with the DLCZ protocol. To be insensitive to the phase fluctuation in the quantum channel [19,22], our previous propose for quantum communication [23] employs two-photon Hong-Ou-Mandel-type (HOMT) interferences to generate local entanglement, to distribute basic entanglement between distance $L_0$, and to connect entanglement with quantum swap. Because the phase instability in the local quantum channel is easy to control, this scheme uses single-photon Mach-Zehnder-type interferences to generate local entanglement and to connect, and uses two-photon HOMT interferences only to distribute basic entanglement to simplify the physical set-up.

The quantum memory in our scheme can be a cloud of $N_a$ identical atoms with pertinent level structure shown in Fig. 1b. One ground state $|g\rangle$ and two metastable states $|s\rangle$ and $|r\rangle$ may be provided by, for instance, hyperfine or Zeeman sublevels of the electronic ground state of alkali-metal atoms, where long relevant coherent lifetime has been observed [24–26]. The atomic ensemble is optically thick along one direction to enhance the coupling to light [13]. State $|e_1\rangle$ is an excited state. A single photon emitted with a repetition rate

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r from an on-demand single-photon source [13, 27] located halfway between quantum memories L and R in every node is split into an entangled state of optical modes L_m and R_m (Fig. 1a) described by

$$|\psi_{in}(\phi)\rangle = \frac{1}{\sqrt{2}} \left( |0_{L_m}\rangle |1_{R_m}\rangle + e^{i\phi} |1_{L_m}\rangle |0_{R_m}\rangle \right)$$

(1)

where \(\phi\) denotes an unknown difference of the phase shifts in the L and R side channels. This state then is coherently mapped onto the state of atomic ensembles L and R:

$$|\psi(\phi)\rangle_{LR} = \frac{1}{\sqrt{2}} \left( T^{\dagger}_{L} + e^{i\theta} T^{\dagger}_{R} \right) |0_a\rangle_{L} |0_a\rangle_{R}$$

(2)

by applying techniques such as adiabatic passage based on dynamic electromagnetically induced transparency [16], where \(T = 1/\sqrt{\sum_{\alpha=1}^{N_a} |g_{\alpha}\rangle \langle t|\) is the annihilation operator for the symmetric collective atomic mode \(T [13]\) and \(|0_a\rangle \equiv \otimes |g_i\rangle\) is the ensemble ground state. Considering photon loss, which includes the optical absorption in the quantum channel and the inefficiency of the excitation transfer from the optical mode to quantum memory mode, the state of ensembles R and L can be described by an effective maximally entangled (EME) state [13]

$$\rho_{LR}(c_0, \phi) = \frac{1}{c_0 + 1} \left( c_0 |0_a\rangle_{L} |0_a\rangle_{L} + |\psi(\phi)\rangle_{LR} |\psi(\phi)\rangle_{LR} \right)$$

(3)

where \(c_0\) is the vacuum coefficient.

Before proceeding we discuss the conversion of the collective atomic excitation \(T\) into the atomic excitation \(S\) given by \(S \equiv 1/\sqrt{\sum_{\alpha=1}^{N_a} |g_{\alpha}\rangle \langle s|\) (Fig. 2). Consider the atoms have an excited state \(|e_2\rangle\) satisfying the condition that the dipole moments of the atomic transitions \(e r_1 = e(g|r|e_2) = 0, e r_2 = e(s|r|e_2) \neq 0,\) and \(e r_3 = e(t|r|e_2) \neq 0 [28]\). The transition \(|s\rangle \rightarrow |e_2\rangle\) of each of these atoms is coupled to a quantized radiation mode described by an annihilation operator \(a\) with a coupling constant \(g;\) the transitions from \(|e_2\rangle \rightarrow |t\rangle\) are resonantly driven by a classical field-controlled Rabi frequency \(\Omega_{e2}\) (Fig. 2). The interaction Hamiltonian of this systems is in the form [29]

$$H_{in} = \hbar g a \sum_{i=1}^{N} \sigma_{e1}^{i} + \hbar \Omega_{e2}(t) \sum_{i=1}^{N} \sigma_{e2}^{i} + H.c.$$

(4)

where \(\sigma_{\mu\nu} = |\mu\rangle_{i} \langle \nu|\) is the flip operator of the \(i\)th atom between states \(|\mu\rangle\) and \(|\nu\rangle\). This interaction Hamiltonian has the dark state with zero adiabatic eigenvalue [29, 31],

$$|D\rangle = \cos \theta \langle t | S^\dagger | g \rangle |1\rangle - \sin \theta \langle t | T^\dagger | g \rangle |0\rangle,$$

(5)

where \(\tan \theta = g/\Omega_{e2}(t)\) and \(|m\rangle\) denotes the radiation state with \(m\) photon. Thus with this dark state, by applying a retrieval pulse of suitable polarization that is resonant with the atomic transition \(|t\rangle \rightarrow |e_2\rangle\), the atomic excitation \(T\) in an atom ensemble can be converted into the atomic excitation \(S\) while a photon which has polarization and frequency different from the retrieval pulse is emitted [13, 23, 25, 28, 29, 32]. Because this conversion process does not involve the collective enhancement, its efficiency is low.

Now we discuss the generation of local entanglement. Two pairs of ensembles are prepared in the same EME state \(\rho_{LR}(i = 1, 2)\) at every node with the vacuum coefficient \(c_0\) (Fig. 1b). The \(\phi\) parameters in \(\rho_{LR}(i = 1, 2)\) are equal assuming that the two EME states are generated through the same stationary channels. The state of the two pairs of ensembles can be described with \(\rho_{LR}(\phi \otimes \rho_{LR}(\phi)\). By applying retrieval pulses on resonance with the atomic transition \(|t\rangle \rightarrow |e_2\rangle\), the atomic excitations \(T\) are transformed simultaneously into excitations \(S\) while photons are emitted. After the conversion, the stimulated photons overlap at a 50%-50% beam split (BS), and then are recorded by the single-photon detectors \(D_{L_1}, D_{L_2}, (D_{R_1}, D_{R_2})\) which measures the combined radiation from two samples, \(a^{\dagger}_{L_{1+}} a_{L_{1-}}\) or \(a^{\dagger}_{L_{1-}} a_{L_{1+}}, a^{\dagger}_{R_{1+}} a_{R_{1-}}\) or \(a^{\dagger}_{R_{1-}} a_{R_{1+}}\), with \(a_{L_{1+}} = a_{L_{1-}} \pm s_{L_{1+}} a_{L_{1-}}\). In the following discussion, we assume \(\phi_{L_{1+}} = \phi_{R_{1+}}\), which is easy to control for the local transformation [22, 33]. Only the coincidences of the two-side detectors are recorded, so the protocol succeeds with a probability \(p_r\) only if both of the detectors on the left and right sides have a click. Under this circumstance, the vacuum components in the EME states, the state components \(T^{\dagger}_{L_{1+}} T^{\dagger}_{L_{1-}} |\text{vac}\rangle\), and \(T^{\dagger}_{R_{1+}} T^{\dagger}_{R_{1-}} |\text{vac}\rangle\) have no effect on the experimental results, where \(|\text{vac}\rangle\) is the ground state of the ensemble \(|0_a\rangle_{0} |0_a\rangle_{0} |\text{equal}\rangle_{L_{1+}L_{1-}R_{1+}R_{1-}}\). Thus, after the conversion, the state of system of four ensembles can be written as the following polarization maximally entangled (PME) state

$$|\psi\rangle_{\text{PME}}^\phi = (S^{\dagger}_{L_{1+}} S^{\dagger}_{R_{1+}} \pm S^{\dagger}_{L_{1-}} S^{\dagger}_{R_{1-}}) |\text{vac}\rangle / \sqrt{2}.$$  

(6)

Without loss of generality, we assume that the generated PME is \(|\psi\rangle_{\text{PME}}^\phi\) in the following discussion. The success probability for entanglement generation at every node is \(p_r = p^2_r = p_r^2 r^2_{\text{eff}} / 2\), where we denote the probability of emitting one photon by the single-photon source with \(\eta_p\), the efficiency for the atomic ensemble storing a photon by \(\eta_a\), the speed for the atomic ensemble emitting a photon during the process \(T^\dagger |0_a\rangle \rightarrow S^\dagger |0_a\rangle\) by \(\eta_s\), and the single-photon detection efficiency by \(\eta_d\). The average waiting time for successful generating a local entanglement state is \(T_j = 1 / p_r\).

Then we show how to distribute basic entanglement between neighboring nodes at a distance \(L_0\). The atomic ensembles at neighboring nodes A and B are prepared in the state \(|\psi^+\rangle_{\text{PME}}\) then illuminated simultaneously by retrieval laser pulses on resonance of the atomic transition \(|s\rangle \rightarrow |e_3\rangle\), where
$|e_3\rangle$ an excited state, the atomic excitations $S$ are transformed simultaneously into anti-Stokes photons. We assume the anti-Stokes photons are in an orthogonal polarization state $|H\rangle$ from ensemble $A_{R1}, B_{L1}$ and $|V\rangle$ from ensemble $A_{R2}, B_{L2}$, which represent horizontal and vertical linear polarization, respectively.

After the conversion, the stokes photons from site $A$ and $B$ at every node are directed to the polarization beam splitter (PBS) and experience two-photon Bell-state measurement (BSM) (shown in Fig. 3) at the middle point to generate an entanglement between the atomic ensembles $A_L$ and $B_R$ ($i=1,2$). Only the coincidences of the two single-photon detectors $D_1$ and $D_4$ ($D_1$ and $D_2$) or $D_2$ and $D_3$ ($D_2$ and $D_4$) are recorded, so the protocol is successful only if each of the paired detectors have a click. Under this circumstance, the vacuum components in the EME states, one-excitation components like $S^\dagger_{A_L} |\text{vac}\rangle$, and the two-excitation components $S^\dagger_{A_L} S^\dagger_{B_R} |\text{vac}\rangle$ and $S^\dagger_{A_L} S^\dagger_{B_R} |\text{vac}\rangle$ have no effect on the experimental results [35]. A coincidence click between single-photon detectors, for example, $D_1$ and $D_4$ will project the four atomic ensembles into EME state [22, 34, 35]

$$|\Psi_{AB}^+\rangle = \frac{1}{\sqrt{2}} (S^\dagger_{A_L} S^\dagger_{B_R} + S^\dagger_{A_L} S^\dagger_{B_R}) |\text{vac}\rangle.$$  

(7)

The success probability for entanglement generation within the attenuation length is $p_b = \eta_{\text{eff}}^2 (\eta_{\text{eff}}^2)/2$, where $\eta_{\text{eff}}$ denotes the efficiency for the atomic ensemble emitting a photon during the process $S^\dagger |0_0\rangle \rightarrow |0_0\rangle$ and $\eta_i = \exp[-L_0/(2L_{\text{att}})]$ is the fiber transmission efficiency with the attenuation length $L_{\text{att}}$.

After successful generation of EME states within the basic link, we can extend the quantum communication distance through entanglement swapping with the configuration shown in Fig. 4. We have two pairs of ensembles—$A_1, A_2, B_{L1}$ and $B_{L2}$, and $B_{R1}, B_{R2}, C_1$ and $C_2$, — located at three sites A, B, and C. Each pair of ensembles is prepared in the EME state (Eq. (7)). The stored atomic excitations of four ensembles $B_{L1}$ and $B_{L2}$, and $B_{R1}, B_{R2}$ are transferred into light at the same time with near unity efficiency. The stimulated optical excitations interfere at a 50%-50% beam splitter, and then are detected by single-photon detectors $D_1, D_2, D_3$, and $D_4$. Only if each pair of detectors ($D_1, D_2$) and ($D_3, D_4$), has a click, the protocol is successful with a probability $p_1 = \eta_{\text{eff}}^2 (\eta_{\text{eff}}^2)/2$ and a

PME state in the form of equation (7) is established among the ensembles $A_1, A_2, C_1$, and $C_2$ with a doubled communication distance. Otherwise, we need to repeat the previous processes.

The scheme for entanglement swapping can be applied to arbitrarily extend the communication distance. For the $i$th ($i = 1, 2, ..., n$) entanglement swapping, we first prepare simultaneously two pairs of ensembles in the EME states (Eq. (7)) with the same communication length $L_{i-1}$, and then make entanglement swapping as shown by Fig. 4 with a success probability $p_i = \eta_{\text{eff}}^2 (\eta_{\text{eff}})/2$. After a successful entanglement swapping, a new EME state is established and the communication length is extended to $L_i = 2L_{i-1}$. Since the $i$th entanglement swapping needs to be repeated on average $1/p_i$ times, the average total time needed to generating a EME state over the distance $L_n = 2^n L_0$ is given by [22]

$$T_{\text{tot}} = \left( \frac{L_0}{c} \right) \left( 1 + \frac{1}{r_{\text{pr}}} \right) \left( \frac{1}{\prod_{i=1}^n p_i} \right) \left( \frac{3}{2} \right)^n$$

with $c$ being the light speed in the optical fiber.

After a EME state has been generated between two remote sites, quantum communication protocols, such as cryptography and Bell inequality detection, can be performed with that EME state like the DLCZ scheme [13]. The established long-distance EME state can be used to faithfully transfer unknown state through quantum teleportation with the configuration shown in Fig. 5. Two pairs of atomic ensembles $L_1, R_1$ and $L_2, R_2$ are prepared in the EME state. The unknown state which is to be transferred is described by $(\alpha S^\dagger_1 + \beta S^\dagger_2) |0_0\rangle_{L_1 L_2}$, with unknown coefficient $\alpha$ and $\beta$, where $S^\dagger_1$ and $S^\dagger_2$ are the collective atomic operators for the two ensembles $L_1$ and $L_2$. The collective atomic excitations in the ensembles $L_1, L_2$ and $L_2, L_2$ are transferred into optical excitations simultaneously.
FIG. 4: (Color online) Configuration for entanglement swapping.

50%-50% beam splitter, the optical excitations are measured by detectors \(D_1, D_{1l}\) and \(D_2, D_{2l}\). Only if there is one click in \(D_1, D_{1l}\) and one click in \(D_2, D_{2l}\), the state transfer is successful, and the unknown state \((\alpha S^+_1 + \beta S^{-}_1)|0\rangle_1|0\rangle_2\) appears in the ensembles \(R_1\) and \(R_2\) up to a local \(\pi\)-phase rotation. Unlike the DLCZ protocol, this scheme does not need posterior confirmation of the presence of the excitation to teleportation unknown state.

Now we evaluate the perform of the scheme numerically. The conversion efficiency \(\eta\) may be low, assuming to be 0.01. If we assume that \(r = 50\) MHz, \(\eta_p = 1\), \(\eta_s = \eta_{e_2} = 0.9\), \(\eta_d = 0.9\), \(L_a = 2500\) km, \(L_{att} = 22\) km for photons with wavelength of 1.5\,\mu m [17], \(c = 2.0 \times 10^8\) km/s, and \(n = 4\), equation (8) gives the average total time \(T_{tot} = 2251\) s, in contrast to the average total time \(T_{tot} = 650000\) s for the DLCZ protocol and \(T_{tot} = 15300\) s for single-photon source (SPS) protocol [17] with the above parameters. Thus, compared with the SPS protocol, this scheme can significantly reduce the average total time for successful quantum communication. Note that \(e_2\) can be enhanced by putting the atomic ensembles in a low-finesse ring cavity [13] and one can exploited many kinds of on-demand single-photon sources, such as molecule-based sources with max rate 100 MHz and quantum-dot-based sources with max rate 1 GHz [36].

To enhance the conversion efficiency \(\eta_{e_2}\), we can use a cavity with a quality factor \(Q\). According to the the literature [13], in the free-space limit the signal-to-noise ratio \(R_{sn}\) between the coherent interaction rate and the decay rate can be estimated as

\[
R_{sn} \sim \frac{4Nddl^2}{\kappa} \sim \frac{3\rho_dL_a}{k_t^2} \sim d_o, \tag{9}
\]

where \(\rho_d\) and \(d_o\) denote the density and the on-resonance optical depth of the atomic ensemble, respectively, \(L_a\) is the length of the pencil-shape atomic ensemble, \(k_t = \omega_t/c = 2\pi/\lambda_t\) is the wave vector of the cavity mode, and \(\kappa\) is the cavity decay rate. \(\kappa\) is relate to the quality factor \(Q\) of the cavity \(\kappa = \omega_t/Q\) [37]. Thus for the case of the cavity with a quality factor \(Q\), we have the signal-to-noise ratio

\[
R_{sn} \sim \frac{4Nddl^2}{\kappa} \sim 3\rho_dL_a/k_t^2 \sim Q \sim 3\frac{\rho_dL_a^2}{4\pi^2} Q \sim d_o, \tag{10}
\]

which shows that the cavity quality factor \(Q\) and the atom number of the ensemble \(N\) play a similar role in enhancing the atom-photon interaction. To estimate the magnitude of the signal-to-noise ratio \(R_{sn}\), we assume \(3\rho_dL_a^2/4\pi^2 \sim 10^{-2}\) for the case of a single atom. Then we have \(R_{sn} \sim 10 \sim d_o\) for \(Q = 1000\). According to the research [38], the maximum total efficiency for a single photon storage in an atomic ensemble followed by retrieval can be larger than 0.5 for \(d_o = 10\). Thus, the conversion efficiency \(\eta_{e_2}\) larger than 0.01 is feasible if the atomic ensemble is placed in the cavity with a quality factor \(Q = 1000\).

Now we discuss imperfections in our architecture for quantum communication. In the basic entanglement generation, the contamination of entanglement from processes containing two excitations can be arbitrarily suppressed with unending advances in single-photon sources [27, 30]. In the whole process of basic entanglement generation, connection, and entanglement application, the photon loss includes contributions from channel absorption, spontaneous emissions in atomic ensembles, conversion inefficiency of single-photon into and out of atomic ensembles, and inefficiency of single-photon detectors. This loss decreases the success probability but has no effect on the fidelity of the quantum communication performed. Decoherence from dark counts in the basic entanglement generation and the entanglement connection can be excluded, for example, if a dark count occurs on the up side (\(D_1\) and \(D_2\)) (Fig. 4), because in this case there are two clicks in the down side detectors (\(D_2\) and \(D_4\)), thus the protocol fails and the previous steps need to be repeated. Considering that the probability for a detector to give a dark count denoted by \(p_d\) smaller than \(5 \times 10^{-6}\) is within the reach of the current techniques [17], we can estimate the fidelity imperfection \(\Delta F \equiv 1 - F\) for the case of long-distance PME states by

\[
\Delta F = 2^{n^2}p_d < 3.2 \times 10^{-4} \tag{11}
\]

for \(n = 4\).

The imperfection that the detectors cannot distinguish between one and two photons only reduces the probability of successful entanglement generation and connection, but has no influence on both of the fidelity of the PME state generated and the quality of quantum communication. For instance, if two photons have been miscounted as one click in detectors \(D_{1l}\) and \(D_{1l1}\) in Fig 5 then there is no click in the detectors \(D_{2l}\) and \(D_{2l2}\), thus the protocol says that the state transfer fails. Like DLCZ protocol, the phase shifts arising from the stationary quantum channels and the small asymmetry of the stationary set-up can be eliminated spontaneously when we generate the PME state from the EME state, and thus have no effect on the communication fidelity. Because the basic entanglement between distance \(L_0\) is generated through two-photon interference, this scheme is robust against the phase fluctuation in the quantum channels [22].
In conclusion, we have proposed a robust scheme for long-distance quantum communication based on “polarization” maximally entangled state. Through this scheme, the rate of long-distance quantum communication may increase compared with the SPS protocol. At the same time, higher fidelity of long-distance quantum communication can be expected. Considering the simplicity of the physical set-ups used, this scheme may open the probability of efficient long-distance quantum communication.

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[1] P. Zoller et al., Eur. Phys. J. D 36, 203 (2005).
[2] V. Giovannetti, S. Lloyd, and L. Maccone, Science 306, 1330 (2004).
[3] J.I. Cirac, P. Zoller, H.J. Kimble, and H. Mabuchi, Phys. Rev. Lett. 78, 3221 (1997).
[4] L.-M. Duan and H.J. Kimble, Phys. Rev. Lett. 92, 127902 (2004).
[5] H.-J. Briegel, W. Dür, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. 81, 5932 (1998).
[6] A. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[7] C.H. Bennett et al., Phys. Rev. Lett. 73, 3081 (1993).
[8] L. Childress et al., Phys. Rev. A 72, 052330 (2005); Phys. Rev. Lett. 96, 070504 (2006); J.I. Cirac et al., Phys. Rev. Lett. 78, 3221 (1997); W. Yao et al., Phys. Rev. Lett. 95, 030504 (2005); E. Waks et al., Phys. Rev. Lett. 96, 153601 (2006); C.H. Bennett et al., Phys. Rev. Lett. 76, 722 (1996); D. Deutsch et al., Phys. Rev. Lett. 77, 2818 (1996); W. Dür et al., Phys. Rev. A 59, 169 (1999);
[9] P. van Loock et al., Phys. Rev. Lett. 96, 240501 (2006).