Fano profile in a novel double cavity optomechanical system with harmonically bound mirrors

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Abstract

We investigate a double cavity optomechanical system generating single and double Fano resonance (multi-Fano). By altering a single parameter, the tunnelling rate $g$ of the middle mirror, we are able to switch between single and double Fano line shapes. The first spectral line shape is stronger in the case of multi-Fano than in the case of single Fano. Also the behaviour of the steady state value of the displacement of the middle mirror, with respect to $g$, heavily influences the behaviour of double Fano lines in our scheme. This tunability along with using a single pump and signal/probe laser has an added advantage in situations where only low power consumption is available.

Keywords: optomechanics, double Fano resonance, optical switch, single Fano resonance

(Some figures may appear in colour only in the online journal)

1. Introduction

Fano resonance was first understood in the context of Rydberg atoms as an interference effect between discrete and continuum states [1]. Since then it has been observed in various physical processes involving bound states inside a continuum in atom–atom scattering. Besides atomic physics, Fano resonances can be found in nuclear physics, plasmonics and cavity optomechanics [2–4]. A similar effect has been discovered in Mie scattering of small particles with negative dielectric susceptibility and weak dissipation rate [5]. Fano resonances are commonly observed when photons travelling through different paths undergo interference. In cavity optomechanics, they occur due to the constructive and destructive interference between two different pathways the photons travel to build the cavity field [6]. The interaction of the discrete optomechanical ground state with broad continuum state in a $\Lambda$-type system results in an asymmetric line shape. This asymmetric structure of Fano line shape can then be conveniently used to produce fast all optical switches in communication networks [7–10] as opposed to the conventional switching components having long Lorentzian tails. For similar reasons, Fano resonances can be used in optical sensors as it is very sensitive to changes in refractive index [11, 12]. Fano minima has also been used for state transfer and transduction between microwave and optical photons [13]. From a practical point of view it then becomes important to have greater tunability and efficiency to produce such line shapes so that they can be effortlessly incorporated into quantum devices.

On the other hand there has been a recent surge of interest in double cavity optomechanical system (OMS). This is due to the greater tunability of membrane-in-the-middle devices over single cavity Fabry–Perot systems. Double cavity OMSs have been effectively used for ground state cooling [14, 15], entanglement [16–18] and transduction [19] between microwave and optical photons. Hybrid optomechanical technologies comprising of toroidal shaped whispering gallery mode (WGM) micro-resonators have been successfully used to induce non-reciprocity between communication channels [20, 21], an essential step towards quantum communication.

Here we report on the use of a double cavity OMS with two movable mirrors in producing single and double Fano resonances depending on the value of the tunnelling rate $g$ of the middle mirror/membrane. In our system we have used a single pump and probe to build an optical field inside a Fabry–Perot
cavity. The double cavity OMS with two harmonically bound mirrors and single pump and probe lasers seems to be energy efficient from a previously proposed system [6]. Similar results have been achieved in interferometres having mirrors with different mechanical frequencies [22] controlled by mechanical pumps [23]. Another major difference between these models and ours is the dependence on a single parameter, i.e. the photon tunnelling rate $g$, to produce single and double Fano line-shapes.

In our model of double cavity OMS, double Fano line occurs due to the simultaneous coupling of mirrors with the radiation pressure in both the cavities. We assume that this coupling occurs at the natural frequency of the mirrors and is achieved by detuning the pump field with respect to the cavity frequencies of both the cavities, simultaneously. We show that the strength of the second Fano line in the double Fano case depends on the second Fano line in the double Fano case depends on the tunnelling rate $g$, on transition between single and double Fano resonances is also considered. After giving a short description of the system Hamiltonian and establishing the Langevin equations in section 2, we plot single and double Fano resonances in section 3. In section 4 we describe the behaviour of the two Fano line-shapes in double Fano case, w.r.t. $g$. This is followed by a discussion and conclusion in section 5.

2. Basic model

We start with a scheme which we have previously discussed in [24], shown in Figure 1. Cavity A is driven by an intense pump/control laser of frequency $\omega_c$ and has an average photon number $n_\text{p} = \langle a^\dagger a \rangle$, where $a$ ($a^\dagger$) is the annihilation (creation) operator of optical mode confined in cavity A.

The transparency of the middle mirror allows for the tunnelling of photons with rate $g$ into cavity B which has photons with average photon number $n_b = \langle b^\dagger b \rangle$ where $b$ ($b^\dagger$) is the annihilation (creation) operator of optical mode confined in cavity B. This leads to the coupling of the two mirrors $M_1$ and $M_2$ via radiation pressure forces from the cavity fields. In a frame rotating with frequency $\omega_c$, the system Hamiltonian can be written as,

$$H = \sum_{j=1}^{2} \left( \frac{\hat{p}_j^2}{2m_j} + \frac{1}{2} m_j \omega_j \delta_j^2 \right) - \hbar \Delta_1 \hat{a}^\dagger \hat{a} - \hbar \Delta_2 \hat{b}^\dagger \hat{b} - \hbar G_1 (\hat{\rho}_1 - \hat{\rho}_2) + \hbar g (\hat{\rho}_1 \hat{b}^\dagger + \hat{\rho}_2 \hat{b}) + \hbar g (\hat{a}^\dagger \hat{b}^\dagger + \hat{a} \hat{b}) + i \hbar \sqrt{N_c} \epsilon_p \hat{a}^\dagger - \hat{a} \hat{e}^{i\omega_p t} - \hat{a} \hat{e}^{i\omega_p t},$$

(1)

where $m_1$ and $m_2$ are the ‘bare’ masses of mirrors $M_1$ and $M_2$ respectively. Here, $\Omega_j (j = 1, 2)$ is the mechanical frequency of the oscillator $M_j$. In our scheme both the movable mirrors vibrate with the same mechanical frequency $\Omega_1 = \Omega_2 = \Omega_m$. We assume that the field in cavity B couples with equal strength to both mirrors $M_1$ and $M_2$. Also $\delta_1$, $\delta_2$ and $\hat{\rho}_1$, $\hat{\rho}_2$ are the position and momentum operators following the commutation relations, $[\xi_j, \hat{p}_j] = i \hbar (j = 1, 2)$ for mirrors $M_1$ and $M_2$. $G_{1,2}$ denotes the cavity frequency shift per resonator displacement for $M_{1,2}$. It is related to single-photon optomechanical coupling strength by $g_{1,2} = G_{1,2} x_{\text{pr}}$. Here $x_{\text{pr}}$ is the standard deviation of the zero point motion of the oscillator. Thus $G_1 = \frac{\gamma_1}{\eta_1}$ and $G_2 = \frac{\gamma_2}{\eta_2}$, where $\omega_1 (\omega_2)$ are resonant frequencies of cavity A (B) and $L_{1,2}$ are the corresponding cavity lengths. It is to be noted that $G_{1,2}$ can be made very large as the effective length can be made very small [25]. Cavity A is driven by an external input laser field consisting of a strong control field and weak probe field denoted by $a_{\text{in}}(t) = \epsilon_c e^{-i\omega_c t} + \epsilon_p e^{-i\omega_p t}$ with field strengths $\epsilon_c$ and $\epsilon_p$ and frequencies $\omega_c$ and $\omega_p$ respectively. The field strengths are given as $\epsilon_c = \sqrt{P_c / \hbar \omega_c}$ and $\epsilon_p = \sqrt{P_p / \hbar \omega_p}$ where $P_c$ and $P_p$ are the control and probe field powers, respectively. Without loss of generality we assume that $\epsilon_c$ and $\epsilon_p$ are real. $\kappa_{\text{om}}$ is the external decay rate between the i/o system and the optical cavity while $\kappa$ is the total decay rate. $\eta = \kappa_{\text{om}} / \kappa$ is the coupling coefficient and is fixed in the critical coupling regime ($\eta = 0.5$) [26, 27]. Here $\Delta_c = \omega_c - \omega_1 (j = 1, 2)$, is the cavity detuning of cavity A ($j = 1$) and B ($j = 2$) while $\Omega = \omega_p - \omega_c$.
is the cavity resonance frequency and pump laser via Raman scattering. When the pump is detuned by the mechanical frequency \( \Omega_m \), \( \delta \) is the difference between the resonances of the probe field at frequency \( \omega_p \).

is the detuning of the probe field with respect to the control field frequency \( \omega_c \). In our semi-classical description the operators are replaced by c-numbers while the noise terms are excluded. The Hamiltonian in equation (1) gives rise to the following Heisenberg Langevin equation,

\[
\dot{a} = i(\Delta_1 + G_1 x_1) a - igb + \sqrt{\eta F} \epsilon_e + \sqrt{\eta F} \epsilon_p e^{-i\delta t} - \frac{\kappa}{2} a, \\
\dot{b} = i(\Delta_2 + G_2 (x_2 - x_1)) b - i g a - \frac{\kappa}{2} b, \\
\dot{x}_j = \frac{p_j}{m_j}, \\
\dot{p}_j = -m_1 \Omega_j^2 x_j - h G_2 b^* b + h G_1 a^* a - \frac{\gamma_j}{2} p_j, \\
\dot{p}_2 = -m_2 \Omega_2^2 x_2 + h G_3 b^* b - \frac{\gamma_2}{2} p_2,
\]

where \( \gamma_{1,2} \) are the decay rates of oscillations of mirror \( M_{1,2} \) respectively. All decay terms in the above equations can be derived using the standard master equation. We do not elaborate on the master equation approach here since it becomes important only if internal system dynamics is of interest.

3. Fano resonances

Fano resonance in a cavity OMS arises due to the interference between photons from to two coherent processes, the direct photon absorption and the indirect Raman process. For simplicity we consider a \( \Lambda \)-type system as shown in figure 2. Here, \( \omega_c \) is the cavity resonance frequency and \( \omega_e \) is the frequency of the strong pump laser. The oscillating mirror with natural frequency \( \Omega_m \) creates sidebands (Stokes and anti-Stokes) on the pump laser via Raman scattering. When the pump is red-detuned from the cavity frequency (\( \Delta = \omega_e - \omega_c \approx -\Omega_m \)) the anti-Stokes sideband becomes resonant with the cavity frequency. This leads to the coherent exchange between photons and phonons. The state \(|f\rangle\) coherently interacts with state \(|e\rangle\) as depicted in figure 2. Further when the weak probe laser is detuned from \( \omega_p \) such that it is near resonance with the cavity frequency \( \omega_c \), photons can be directly injected into the cavity. This is labelled as pathway 1 in figure 2. Some of these photons from probe laser can then be stimulated down to state \(|f\rangle\) to produce phonons with frequency \( \Omega_m \) and then back to the excited state \(|e\rangle\) with the help of the strong pump laser. This is labelled as pathway 2 in figure 2. The cavity field is thus built through these two coherent processes or two different pathways. Just like the two slit experiment, since there is no way of knowing which slit the photons have come out, here there is no way of knowing which pathway the photons have traversed. This leads to the interference between cavity photons. \( \delta \) in figure 2, is the difference between the resonances of the probe field, \( \omega_p \approx \omega \) and \( \omega_p \approx \omega_e + \Omega_m \) [6]. It is a parameter which controls this interference. A complete destructive interference (\( \delta = 0 \)) gives rise to optomechanically induced transparency (OMIT) while in general when \( \delta \neq 0 \), a partial constructive and destructive interference occurs between cavity photons and causes asymmetric Fano profiles.

3.1. Single Fano

In cavity optomechanics, a \( \Lambda \)-type system shown in figure 2 can be formed using either single or multiple cavities separated by dielectric membranes (or partially transmitting mirrors). It has been shown [6] that single Fano resonance can even occur in a single Fabry–Perot cavity provided that the anti-Stokes Raman field is offset from the cavity resonance frequency such that there is a difference \( \Omega_m \) (or \( \delta \) in our case) between the probe field and the anti-Stokes field. This gives rise to partial constructive and destructive interference causing asymmetric Fano line-shape. In a double cavity OMS too we can have single Fano provided that both the end mirrors are fixed. In that case the Hamiltonian in equation (1) will be modified;

\[
\hat{H} = \left( \frac{\hat{p}_1^2}{2m_1} + \frac{1}{2} m_1 \Omega_1^2 \hat{x}_1^2 \right) - \hbar \Delta_1 \hat{a}^\dagger \hat{a} - \hbar \Delta_2 \hat{b}^\dagger \hat{b} - \hbar G_2 \hat{b}^\dagger \hat{b} + \left( \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a} \right) + \hbar \sqrt{\eta F} \epsilon_e (\hat{a}^\dagger - \hat{a}) + \hbar \sqrt{\eta F} \epsilon_p (\hat{a}^\dagger e^{-i\delta t} - e^{i\delta t} \hat{a}).
\]

Heisenberg Langevin equations (2a)-(2e) will also be modified according to equation (3);

\[
\dot{a} = i(\Delta_1 + G_1 x_1) a - igb + \sqrt{\eta F} \epsilon_e + \sqrt{\eta F} \epsilon_p e^{-i\delta t} - \frac{\kappa}{2} a, \\
\dot{b} = i(\Delta_2 - G_2 x_1) b - i g a - \frac{\kappa}{2} b, \\
\dot{x}_1 = \frac{p_1}{m_1}, \\
\dot{p}_1 = -m_1 \Omega_1^2 x_1 - h G_2 b^* b + h G_1 a^* a - \frac{\gamma_1}{2} p_1,
\]

where \( x_1 \) is the displacement of the middle mirror/membrane. The above equations are almost similar to the ones in [26], where a toroidal WGM microresonator was studied instead of a

Figure 2. Schematic diagram of photon pathways in a \( \Lambda \)-type system. Pathway 1 is the direct photon absorption at cavity frequency \( \omega \) and pathway 2 is the indirect or Raman induced photon absorption due to detuning of pump laser by the mechanical frequency \( \Omega_m \).
Fabry–Perot cavity. Using standard mean field and linearization approach [28] and breaking the above \( c \)-numbers (equations \((4a)–(4d))\) into their respective Fourier components (equations \((4a)–(4d))\) (appendix), we arrive at the anti-Stokes field in cavity A:

\[
A_i^- = -\frac{(gG_2|g|q_1 + iD_1G_1|g|q_3 + D_3\sqrt{\gamma\kappa}\epsilon_p)}{D_1D_3 + g^2},
\]

where \(D_1 = \Theta_1 + i\Omega, \ D_2 = \Theta_1 - i\Omega, \ D_3 = \Theta_2 + i\Omega, \ D_4 = \Theta_2 - i\Omega\) and \(\Theta_j = i\Delta_j - \kappa/2, \ (j = 1, 2)\). Since we are using a single input–output port, we can measure only the backward reflection coefficient of the input probe field. The standard input–output relation leads to;

\[
a_{\text{out}} = C_{\text{ch}}e^{-i\omega_1't} + C_{\text{ph}}e^{-i\omega_3't} - \sqrt{\gamma\kappa}A_1^+e^{-i(2\omega_2 - \omega_3)t},
\]

where \(C_{\text{ch}} = \epsilon_c - \sqrt{\gamma\kappa} \sigma, \ C_{\text{ph}} = \epsilon_p - \sqrt{\gamma\kappa}A_1^-\), are the complex coefficients for steady state and backward reflection, respectively. Using \(C_{\text{ph}}\) we calculate the normalised backward reflection coefficient, \(T_b = |C_{\text{ph}}/\epsilon_p|^2\) (appendix, equation \((A2))\).

Since the pump laser is red-detuned by the mechanical frequency \(\Omega_m\) from equation \((6),\) the Stokes field, with coefficient \(A_1^+\), will be off-resonant by \(2\omega_1 - \omega_2\) from the cavity. Hence we do not show it here but it is straightforward to calculate. Figure 3 gives the normalised backward reflection coefficient \(T_b\) for different values of tunnelling rate \(g\) and \(\Omega/\Omega_m\in[0.98, 1.02]\). This is done to emphasise changes in the complete destructive interference (leading to OMIT) of the cavity field to a constructive and destructive interference (leading to asymmetric Fano line shapes). OMIT occurs at \(g/\Omega_m = 0\) (figure 3(a)) since both the probe and the anti-Stokes Raman field completely destructively interfere at cavity resonance. For \(0.0 < g/\Omega_m \leq 0.4\) (figures 3(b)–(e)) \(T_b\) intensity changes gradually with peak around 0.9. For \(g/\Omega_m \sim 0.6\) (figure 3(f)) the peak drastically increases to around 1.0.

3.2. Multi-Fano

Fano profiles depend on the number of dressed states and resonances of the probe laser. For example a single dressed state with single resonance gives rise to a single OMIT. On the other hand the occurrence of a double OMIT has been
recently shown [29] in a piezo-mechanical system coupled with an OMS. The piezo-mechanical coupling, forms two
dressed states with two resonances, changing a single OMIT
into a double OMIT. Now a shift in the detuning of the
coupling laser by an amount \( \delta \) will give rise to more number
of resonances than dressed states leading to asymmetric Fano
profiles. Here we specifically show the existence of double
asymmetric Fano-line shapes in a double cavity OMS. The
system dynamics are defined by equations (2a)–(2e) and
following the same procedure given in section 3.1, the anti-
stokes field component \( A_i \) is given as:

\[
A_i = \frac{g G_2 (q_2 - q_1) \overline{b} - i D_2 G_1 q_1 \overline{d} - D_3 \sqrt{\eta \kappa \epsilon}}{D_1 D_3 + g^2}.
\]  

The normalised backward reflection coefficient \( T_b \) is \( |C_{pb}/
\epsilon_p|^2 \) as defined below equation (6), is given in appendix
equation (A4)). We plot \( T_b \) in figures 4(a)–(f). The x-axis
range here is again \( \Omega/\Omega_m \in [0.98, 1.02] \) so as to emphasise
changes in Fano profiles. As in figure 3(a), in figure 4(a) we
have a single OMIT structure at \( g/\Omega_m = 0.0 \). This system
does not show double OMIT since at \( g/\Omega_m = 0.0 \) the system
turns into a single cavity OMS with complete destructive
interference of the Raman anti-stokes and the probe field at
cavity resonance with response function \( \eta T = \sqrt{\eta \kappa \epsilon} A_i /
\epsilon_p \) of the OMS being,

\[
\eta T = \frac{\eta \kappa \epsilon}{\frac{\alpha}{2} - i \gamma + \frac{\beta}{\alpha - i \gamma}}.
\]  

We note that instead of having a large parameter space, we
have fixed all the parameters except for the tunnelling rate \( g \).
This makes the system experimentally easy to access. Comparing
figures 3 and 4 we see that the Fano line-shapes are
roughly similar up to around \( g/\Omega_m = 0.2 \). At around
\( g/\Omega_m = 0.3 \) we observe the first signs of single Fano going
into double Fano. At \( g/\Omega_m = 0.4 \) we clearly see the separa-
tion of the two distinct asymmetric Fano line-shapes. The first
Fano line-shape for double Fano case is more sharp than that
for the single Fano case (figure 3). This means easier

Figure 4. Backward reflection coefficient with middle and one end mirror not fixed. \( m_1 = m_2 = 20 \) ng, \( P_c = 1 \) mW,
\( G_1 = 2 \pi \times 13 \) GHz nm\(^{-1} \), \( G_2 = 2 \pi \times 13 \) GHz nm\(^{-1} \), \( \gamma_1 = \gamma_2 = 2 \pi \times 41 \) kHz, \( \kappa = 2 \pi \times 15 \) MHz, \( \Delta_\alpha = \Delta_1 = -\Omega_m \) and
\( \Omega_1 = \Omega_2 = \Omega_m = 2 \pi \times 51.8 \) MHz.
switching when implemented in all-optical switches. We also note that the difference between strength of dips (y-axis) for the two Fano lines remain more or less constant with increasing $g$. Hence a sharper first Fano line also means a sharper second Fano line.

4. Fano profile behaviour

Next we try to model the behaviour of the double Fano profile. Using analytical tools (Mathematica), we measure the distance (in terms of $\Omega/\Omega_m$) between the two Fano line-shapes in the backward reflection ($T_b$) plot. The dependency of the measured separation values between the two line-shapes is plotted with respect to $g$ for $0.4 \leq g/\Omega_m \leq 1$. We choose this interval due to prominence of the weaker line-shape. For comparison, in figure 5(a), we also plot scaled up steady state values $s_1$ and $s_2$ for displacements of mirrors $M_1$ and $M_2$ respectively:

$$s_1 = \frac{\hbar (G_1|E|^2 - G_2|E|^2)}{m_1 \Omega_1^2},$$

$$s_2 = \frac{\hbar G_2|E|^2}{m_2 \Omega_2^2}.$$  

It can be seen in figure 5(a) that the behaviour of measured values closely mimics that of $s_1$. It should be noted that large separation value amounts to well-resolved Fano lines. From figure 5(a) it can be seen that the transmittivity (or tunnelling rate) $g$ of the middle mirror $M_f$ should be relatively high to guarantee well-resolved Fano lines. There is a rapid increase for $0.6 \leq g/\Omega_m \leq 0.9$ saturating at $g/\Omega_m = 0.95$ before decreasing till $g/\Omega_m = 1$. We use two functions to fit the data as shown in figure 5(b), a generalised logistic function and the Moffat function [30].

$$Y_{gen}(x) = a + \frac{c}{1 + Te^{-b(x - \mu)^2}},$$

$$Y_{moff}(x) = A \left(1 + \left(\frac{x - \mu}{\sigma}\right)^2\right)^{-\beta}.$$  

with appropriate fit parameters. A generalised logistic function is a cumulative function depicting, among many things, diffusion. On the other hand Moffat function is a point spread function used in astrophysics to fit ‘seeing-limited’ images of point sources [31], a blurring of light caused by the Earth’s atmosphere. It is a measure of diffusion of light emanating from a point source registering on a photographic plate after passing through the Earth’s atmosphere. Both the fits give low $\chi^2$/d.o.f (degree of freedom) values, of the order of $10^{-4}$, but the Moffat function gives a better fit as seen from figure 5(b).

From the fits we can see that in our scheme the separation between the two Fano lines in double a Fano profile is related to the transmission of photons through the middle mirror and mimics a diffusion process. We do not suggest that photon diffusion occurs between cavity A and B but nonetheless it might be useful in future studies to understand the effects of changing $g$ on Fano lines in terms of diffusion.

5. Discussion and conclusion

Here we do not delve into quantitatively understanding the results given in figures 3 and 4 following equations for the normalised backward reflection coefficients, but try to intuitively understand them. We have assumed that the strong pump laser is red-detuned from resonant frequencies of cavity A and B by the same amount $\Omega_m$ i.e. $\Delta_1 = \Delta_2 = -\Omega_m$. Since mirror $M_2$ has the same natural frequency $\Omega_m$ as $M_1$, we get another state $|f_2\rangle$ degenerate with $|f_1\rangle$ as shown in Figure 6. For non-zero value of the tunnelling rate $g$, the middle mirror $M_f$ allows photons into cavity B. Just as in the case of single Fano explained in section 3, the simultaneous coupling of photons in cavity A and B with mirrors $M_1$ and $M_2$ gives rise to the shifts $\delta_1$ and $\delta_2$, respectively. Single OMIT occurs when $\delta_1 = \delta_2 = 0$ i.e. when field in cavity A couples to mirror $M_1$ ($g/\Omega_m = 0$). Figure 7 gives the strength of fields in both the cavities. At lower values of the tunnelling rate $g$ (i.e. $g/\Omega_m \sim 0.2$), the field strength in cavity B is much smaller than that in cavity A. This means that it cannot substantially affect the motion of mirror $M_2$ via radiation pressure. Thus the
coupling of state $|f_2\rangle$ with state $|e\rangle$ is practically non-existent. $\delta_1 \neq 0$ but $\delta_2 = 0$, implying only single Fano. Hence Fano profiles in figures 3 and 4 look pretty similar up to around $g/\Omega_m = 0.2$. Once the value of $g/\Omega_m \geq 0.3$, the optical field in cavity B becomes strong enough to influence motion of $M_2$ and hence $\delta_2 = 0$. This might be the reason we start observing a shift from single to double Fano around $g/\Omega_m = 0.3$ in figure 4. The line-shapes become prominent for $g/\Omega_m \geq 0.4$ since the field strengths in cavity A and B increase with g.

Although ideally cavity A and B should contain the same number of photons at $g/\Omega_m = 1.0$, the discrepancies in field strengths at $g/\Omega_m = 1.0$ as shown in figure 7 might be due to approximations (for e.g. truncation of higher order optomechanical coupling, mean field approach and rotating wave approximation) used in calculating these values. These are good approximations for large cavity field strength but they do have their short comings. For instance, in single-photon optomechanics nonlinear photon–phonon interaction becomes important and one can no longer assume linear response of oscillating mirrors to solve the equations of motion [32]. It is to be noted that in our study the single photon optomechanical coupling $\kappa_{0,2} < \kappa_{1,2}$ and hence still in the weak optomechanical regime. In the single photon strong coupling regime one has to include higher order expansion of cavity frequency $\omega(x)$, leading to higher order optomechanical coupling rate.

In conclusion, we have successfully shown the presence of double Fano profiles in our scheme. We have also shown that the same scheme can be used to generate single and double Fano lines-shapes by simply controlling the tunnelling rate $g$ of the middle mirror $M_1$. The behaviour of the separation between the line-shapes closely follows the behaviour of the steady state displacement of $M_1$ and mimics a diffusion process. The first Fano line has a larger dip in the multi-Fano case than in the single Fano for the same value of $g/\Omega_m$. The double Fano line-shapes also become resolved for $0.6 \leq g/\Omega_m \leq 0.95$. Hence our scheme can be implemented for sensitive devices in need of sharp spectral lines like in all-optical switches and quantum sensors.

**Appendix**

The Fourier components for cavity and mechanical c-numbers given in equations (2) and (4) are,

\begin{align}
\delta a(t) &= A_i e^{-i\Omega t} + A_i^* e^{i\Omega t}, \quad (A1a) \\
\delta b(t) &= B_i e^{-i\Omega t} + B_i^* e^{i\Omega t}, \quad (A1b) \\
\delta \chi_1(t) &= q_i e^{-i\Omega t} + q_i^* e^{i\Omega t}, \quad (A1c) \\
\delta \chi_2(t) &= q_2 e^{-i\Omega t} + q_2^* e^{i\Omega t}. \quad (A1d)
\end{align}

Single Fano (assuming $D_1 = D_3, D_2 = D_4$ and $G_1 = G_2 = G$):

\begin{equation}
T_b = \left|1 - \eta \kappa \left[ \frac{ig^2 G|\bar{E}|^2 - iGD_1|\bar{b}|^2 - gD_2 G(\bar{a}^* \bar{b} + \bar{a} \bar{b}^*)}{(D_1 D_3 + g^2)(C_1' + C_2' + C_3')}ight] \right|^2, \quad (A2)
\end{equation}

where,

\begin{align}
C_1' &= \frac{-gG(\bar{a}^* \bar{b} + \bar{a} \bar{b}^*) + iG(D_1|\bar{a}|^2 + D_1^*|\bar{b}|^2)}{(D_1 D_3 + g^2)}, \quad (A3a) \\
C_2' &= \frac{-gG(\bar{a}^* \bar{b} + \bar{a} \bar{b}^*) - iG(D_1^*|\bar{a}|^2 + D_1|\bar{b}|^2)}{(D_1^* D_3^* + g^2)}, \quad (A3b) \\
C_3' &= \frac{1}{\hbar G\chi_1(\Omega)}. \quad (A3c)
\end{align}

$\chi_1(\Omega)$ is the mechanical susceptibility of mirror $M_1$.

Double Fano (assuming $D_1 = D_3, D_2 = D_4$ and $G_1 = G_2$):
where,
\[ A = -\frac{g_G \sigma^+ B^* + ig_D |D|^2}{D_0^2 + g^2} - \frac{g_G \sigma^+ B - ig_D |D|^2}{(D_0^2 + g^2)^2}, \] (A5a)
\[ B = \frac{i g_G |D| |D|^2}{(D_0^2 + g^2)^2} - \frac{i g_D |D|^2}{D_0^2 + g^2} - \frac{1}{\hbar g_1 \chi_2(\Omega)}, \] (A5b)
\[ C_1 = -\frac{g_G \frac{1}{2} \left(\frac{1}{2} \right) \sigma^+ B^* + \left(\frac{1}{2} \right) \sigma^+ B}{(D_0^2 + g^2)^2} + \frac{1}{\hbar g_1 \chi_1(\Omega)}, \] (A5c)
\[ C_2 = g_G \frac{1}{2} \left(\frac{1}{2} \right) \sigma^+ B^* - \frac{1}{\hbar g_1 \chi_1(\Omega)}, \] (A5d)
\[ \chi_2(\Omega) \text{ is the mechanical susceptibility of mirror } M_2. \]

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