Numerical simulations of fundamental processes in cavity QED: Atomic decay

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We present results of numerical investigation of a microscopic dynamics of a two-level atom embedded into a "linear crystal" of other two-level atoms. These additional atoms play a rôle of a material media. All atoms interact with a multimode cavity field. We study how the decay of the initially excited atom is affected by the presence of material media and spectral properties of the cavity field.

I. INTRODUCTION

Quantum electrodynamics (QED) lies at the heart of modern quantum theory. QED is a well established and experimentally confirmed theory \cite{1,2} but even fifty years after its foundation many features of the atom-field interaction remain extremely intriguing. In particular, the character of the atom-field interaction can be substantially modified in confined spaces (e.g. the high-Q cavity of a micromaser) due to the fact that local properties of the electromagnetic (EM) field depend on space boundaries. The radiative properties of atoms and the EM field in confined spaces have been investigated for various cavity QED systems \cite{3,4,5,6,7}.

Quantum electrodynamics is a local theory, which means that the dynamics of atoms and electrons depend on local properties of the electromagnetic field with which they interact. But local properties of the electromagnetic field depend also on conditions imposed by the boundaries of the space region in which the field is confined \cite{8}. These conditions are reflected in the quantization of the field. We can either quantize the electromagnetic field in free space or in a "quantization box" of linear dimension $L$. Quantum electrodynamics in a box describes effects associated with processes inside high-quality (perfect) cavities. In addition, quantization in a box can be considered as an approximation to free-space quantization and the two theories must give the same results in the limit $L \to \infty$.

In the first quantum-mechanical description of spontaneous decay of a two-level atom in free space, Weisskopf and Wigner \cite{9} started their calculations with the cavity modes quantized in a box, and then at a certain stage of the calculation, a limit to a continuum of modes was taken. This approach gives in first approximation correct results (exponential decay of the excited level of the atom). The interaction of a two level atom with discrete (cavity) modes has been described systematically by Hamilton \cite{10} who solved the emission and scattering problems exactly for a cubic box by diagonalization of the total atom-field Hamiltonian. Later this approach was utilized by other researchers for a detailed investigation of spontaneous emission of two-level atoms (see for instance papers by Davidson and Kozak \cite{11} and Swain \cite{12}). Essentially, in all these papers on the spontaneous decay of a two-level atom in a cavity (box), the coupling constant between the atom and the cavity modes has been taken to be position independent. This argument is perfectly justifiable in free space, when translational invariance is valid. On the other hand, when the atom interacts with discrete cavity modes in a confined space the position dependence of the coupling can play a significant rôle. The investigation of this problem is not only of theoretical interest. Recent advances in experimental techniques have allowed one to study fundamental processes in cavity quantum electrodynamics (cavity-QED) \cite{3,4,5,6,7} and to verify various effects of the atom-field interaction in confined spaces as predicted by Schelkunoff \cite{13}, Purcell \cite{14}, Barton \cite{15} and others. One of the fundamental processes of cavity QED represents the spontaneous decay of a two–level atom. It is well known that spontaneous emission from an atom positioned very close to the cavity mirror can be significantly suppressed. This effect is called the inhibition of spontaneous emission \cite{13}. The deviation from the exponential Weisskopf–Wigner decay of the atom in free space \cite{9} has been demonstrated in a number of experiments \cite{13}. Many other interesting questions arise for these QED systems. For example, one could ask about the influence of cavity mirrors on the dynamics of the atom and the rôle of the position of the atom on the appearance of Poincaré recurrences (i.e., re-excitations of the atom by radiation reflected by the cavity mirrors) \cite{13}. While the exponential character of the decay is not affected by variations of the position of the atom around the center of the cavity (providing the cavity is large enough), the Poincaré recurrences depend very sensitively on the position of the atom inside the cavity. Namely, variation of the position of the atom within a wavelength of the resonant atomic frequency can result in an almost complete suppression of the first Poincaré recurrence of the excited level of the atom \cite{13}. This means that the atom effectively does not "feel" wave packets reflected from cavity mirrors for times much longer than the time necessary for emitted light to "travel" to the mirrors and back to the atom. Another example concerns the atom which is positioned...
close to one of the mirrors. In general one may expect to see inhibition of the radiation. Nevertheless, taking into account the position dependence of the field–atom interaction it turns out that for some specific distances from the mirror (e.g. one quarter of the resonant wavelength of the radiation field), the atom decays even faster than in free space \[\text{[13]}.\]

A valuable first insight into modification of the spontaneous emission of the atom into vacuum field is offered by the Fermi golden rule \[\text{[1]}\]

\[
\Gamma_a = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho(\omega_a).
\] (1)

The spontaneous emission rate \(\Gamma_a\) is directly proportional to the density of field modes \(\rho(\omega_a)\) in frequency domain at the atomic transition frequency \(\omega_a\); \(V_{fi}\) is the matrix element of the corresponding transition. The presence of boundaries (e.g., in the case when the atom is inserted into a high-\(Q\) microwave cavity) changes the local density of field modes and thereby the spontaneous emission can be suppressed or enhanced. However, it is by no means necessary to change the boundary conditions of the EM field in order to modify the spontaneous emission rate. This goal can be achieved easily by the presence of other atoms which can take part in absorption and re-emission of the radiation field. One important example is that of an atom embedded in a dielectric host.

The main task of our investigation here concerns atom-field interactions in confined geometries. Starting from “first principles” we simulate the dynamics of a system of atoms in a cavity. In particular, we consider a cavity filled with a “crystal” composed of two–level atoms. We investigate the modification of the spontaneous emission and the propagation (scattering) of photon wave packets within this “crystal”. We explore the influence of the position dependence of the atom–field coupling on the dynamics of the system. In this way also the emission and absorption in photonic band gap structures (PBS) \[\text{[20–22]}\] with few atoms can be analyzed. Our microscopic model based on “first principles” provides us with a deeper understanding of the atom–field interaction and offers a framework to study systematically the transition from microscopic to macroscopic (phenomenological) descriptions of the systems under consideration.

In this paper we focus our attention on the modifications of spontaneous emission. We describe the model in Section \[\text{II}\] In Section \[\text{III}\] we study position dependence of the decay of a single two–level atom and in Section \[\text{IV}\] we discuss the effect of inhibition of spontaneous emission. We also analyze how the dynamics of an initially excited atom is modified in the presence of other initially deexcited atoms in the cavity which play the rôle of a dielectric (see Section \[\text{V}\]) and we study in detail the time evolution of atomic populations and quantum-statistical properties of the multimode cavity field. In particular, in Section \[\text{VI}\] we focus our attention on the spectrum of the field. In addition we will also discuss specific technical questions such as the rôle of the frequency cut-off employed here. In Section \[\text{VII}\] we present the convolutionless master equation describing the dynamics of the initially excited atom in dielectrics interacting with multimode cavity field. We summarize our results in Section \[\text{VIII}\].

\section{THE MODEL}

We consider a simple one-dimensional model of a cavity in which two-level atoms interact with the cavity modes in the dipole and the rotating-wave approximations. To simplify the model, we neglect all mechanical effects of the cavity field on the atom (i.e., the mass of the atom is assumed to be infinite). Here we note that this 1-D model not only reflects the main features of atom–field interaction but also can be mapped onto an isotropic 3-D model.

Under the assumption of perfectly reflecting mirrors, the operator of the electric field inside the cavity in the Coulomb gauge can be expressed as \[\text{[23,24]}\]

\[
\hat{E}(r) = \sum_{n,\lambda} E_n \hat{e}_\lambda \left( \hat{a}_{n,\lambda} + \hat{a}_{n,\lambda}^\dagger \right) \sin(k_nr),
\] (2)

where \(k_n = \omega_n/\epsilon = n\pi/L\) and \(E_n = \sqrt{\omega_n/\epsilon c}\). The two orthogonal polarization vectors \(\hat{e}_\lambda\) (\(\lambda = 1, 2\)) lie in the plane perpendicular to the cavity axis; \(\hat{a}_{n,\lambda}\) and \(\hat{a}_{n,\lambda}^\dagger\) are annihilation and creation operators of the \(n\)-th mode.

The Hamiltonian describing the free cavity field can be expressed as

\[
\hat{H_F} = \hbar \sum_{\lambda} \sum_{n=1}^{N} \omega_n \hat{a}_{n,\lambda}^\dagger \hat{a}_{n,\lambda},
\] (3)

where we have omitted the zero-point contribution \(\hbar \sum_n \omega_n/2\). Summation over discrete modes in Eq.(3) is performed only up to \(n = N\), which means that in our model we assume a cutoff for the cavity modes.

The Hamiltonian describing a set of \(M\) non-interacting (“free”) two-level atoms with transition frequencies \(\omega_{\alpha}^{(j)}\) can be expressed as

\[
\hat{H_A} = \frac{\hbar}{2} \sum_{j=1}^{M} \omega_{\alpha}^{(j)} \hat{\sigma}_{z}^{(j)},
\] (4)

where \(\hat{\sigma}_{z}^{(j)} = |\epsilon\rangle_{j} \langle \epsilon| - |\sigma\rangle_{j} \langle \sigma|\); \(|\epsilon\rangle_{j}\) and \(|\sigma\rangle_{j}\) denote the upper and lower atomic states, respectively.

When the radius of the atom is much smaller than the wavelength of the resonant electromagnetic radiation then the atom-field interaction can be described within the electric-dipole approximation, i.e., \(\hat{H}_{\text{int}} = -\hat{d} \cdot \hat{E}\). Neglecting for simplicity all polarization effects, the resulting interaction Hamiltonian in the rotating-wave approximation (RWA) reads
\[
\hat{H}_{\text{int}} = -\hbar \sum_{j=1}^{M} \sum_{n=1}^{N} g_n^{(j)} \left| \hat{a}_n \sigma_+^{(j)} + \hat{a}_n^\dagger \sigma_-^{(j)} \right|
\]

where the Pauli spin-flip operators are \( \sigma_+^{(j)} = |e\rangle_j \langle g| \) and \( \sigma_-^{(j)} = |g\rangle_j \langle e| \). The position dependent coupling constants \( g_n^{(j)} \) are given by the expression

\[
g_n^{(j)} = \left( \frac{\omega_n}{\hbar \epsilon_0 L} \right)^{1/2} d_{eg} \sin(k_n r_j)
\]

where \( d_{eg} \) denotes the dipole matrix elements of the atoms. The position dependence of the atom-field coupling constant \( g_n^{(j)} \) given by space–mode functions \( f_n(r) = \sin(k_n r) \) may significantly affect the atomic dynamics.

The total Hamiltonian of the form

\[
\hat{H}_{\text{tot}} = \hat{H}_F + \hat{H}_A + \hat{H}_{\text{int}}
\]

describes the system of \( M \) two-level atoms interacting with \( N \) discrete field modes in 1-D cavity. This model can be solved exactly because the total number of excitations

\[
\hat{R} = \frac{1}{2} \sum_{j=1}^{M} \left( \hat{\sigma}_z^{(j)} + 1 \right) + \sum_{n=1}^{N} \hat{a}_n^\dagger \hat{a}_n
\]

is an integral of motion, i.e. \( [\hat{R}, \hat{H}_{\text{tot}}] = 0 \).

However, it is impossible in general to find a closed analytical solution for the system under consideration. Just a few particular cases can be solved analytically, and among these is the well known Jaynes-Cummings model \([23]\) which describes the dynamics of a two-level atom interacting with a single mode cavity field. Therefore from the very beginning our treatment will be based on a numerical simulation of the cavity QED system. Our numerical treatment allows us to calculate properties of individual atoms and modes of the EM field, i.e., it retains a complete microscopic picture of the problem. The Schrödinger equation for the Hamiltonian \([1]\) can be transformed into a set of coupled linear differential equations for the amplitudes of component states (e.g., basis of eigenstates of bare systems) in a finite-dimensional subspace of the Hilbert space. Numerical solutions can be found using standard methods, e.g., by Runge–Kutta methods. Alternatively one can apply the direct diagonalization of the Hamiltonian \([6]\). Numerical solution allows us to investigate processes with low initial excitation numbers (e.g., initially just one or two atoms are excited). The number of cavity modes can in this case be thousands and the total number of atoms can be up to hundreds.

Using numerical methods, we can analyze the time evolution of the mean values of the following observables:

i) The occupation of the upper level of the \( j \)-th atom

\[
\hat{P}_e^{(j)} = \frac{\sigma_+^{(j)} + 1}{2} = |e\rangle_j \langle e|.
\]

ii) The amplitude of the electric field

\[
\hat{E}(r) = \sum_{n=1}^{N} \left( \frac{\hbar \omega_n}{\epsilon_0 L} \right)^{1/2} \left[ \hat{a}_n + \hat{a}_n^\dagger \right] \sin(k_n r).
\]

iii) The number of excitations of the \( k \)-th mode

\[
\hat{S}(k) = \hat{a}_k^\dagger \hat{a}_k,
\]

which is used to study the spectrum of the radiation.

iv) To analyze the space-time propagation of radiation wave packets we evaluate mean values of the normally-ordered operator for the energy density

\[
\hat{I}(r) = : \epsilon_0 \hat{E}_e^2(r) :.
\]

Here normal ordering (the colons above) is adopted to eliminate the vacuum-state contribution to the energy density of the emitted radiation.

In what follows we demonstrate the main features of the atom-field interaction in confined geometries and describe these effects:

a) Modification of spontaneous emission of the atom in the cavity due to the position dependence of the atom-field interaction. A partial re-excitation of the atom caused by the back reflected radiation (Poincaré recurrences).

b) Decay in a “crystal”: modification of spontaneous emission due to the presence of other atoms, which are initially deexcited (the decaying atom can be considered as being embedded in a dielectric “crystal” which is formed by other atoms).

c) A model of quantum measurement: a system of two-level atoms serves as a device to measure the field spectrum of the considered configuration. The transient character of the spectrum can be observed.

### III. DECAY AND RE-EXCITATION OF ATOM

Within the Weisskopf–Wigner theory \([1]\) in free space, an initially excited atom which is coupled to a continuum of vacuum field modes decays exponentially to its ground state. Representing the usual 1D continuum with the discrete model \([4]\) for a large cavity \((L \to \infty)\), the population of the excited atomic level \( P_e \) decays exponentially with the rate \( \Gamma_a \) given by the Fermi golden rule \([4]\), i.e.,

\[
P_e(t) = \exp(-\Gamma_a t), \quad \Gamma_a = \frac{\omega_a |d_{eg}|^2}{\epsilon_0 hc}.
\]

In 1D “free” space \((L \to \infty)\) the decay process is accompanied by the emission of two wave packets (representing the one-photon state) propagating to the left and to the right from the atom. In the case of the “left-right”
symmetry of atomic-wave functions in 1-D (this corresponds to spherical symmetry in 3-D) each of the two wave packets carries half of the initial excitation. This process is irreversible as the energy cannot be reabsorbed by the atom. In confined geometries the situation differs. First, the density of discrete modes is changed due to the boundary conditions. The translational symmetry is lost and the coupling is position-dependent. In particular, when the atom is positioned at the center of the cavity it is coupled only to odd modes of the field (for even modes the coupling constant is equal to zero; for more details see below). Second, the two wave packets are reflected back by the cavity mirrors and can be (partially) reabsorbed by the atom. This partial restoration of the initial state of the atom, the so called Poincaré recurrence, can be viewed as a consequence of constructive quantum interference (see below).

Figure 1 presents the time evolution of the probability of the atomic excitation for four different values of the position of the atom around the center of the cavity, namely \( \Delta r_1 \equiv r_1 - \frac{L}{2} = 0, \pm \frac{\lambda}{16}, \pm \frac{\lambda}{4} \). To the case when the initially excited atom is positioned in the cavity center (dotted line) we will further refer as the “free-space” decay. The central atom interacts only with the odd modes and thus the density of (interacting) modes equals to \( \frac{1}{2\pi c} \). The corresponding “free-space” decay rate \( \Gamma_a \) is given by Eq. (13).

From Fig. 1 we see that the first “exponential” stage of the decay is (almost) position independent. Providing the atom is “far” from the mirrors, i.e., \( \min(r_1, L - r_1) \gg c/\Gamma_a \), the reflected wave packets do not influence the exponential decay. More precisely, the density of modes is doubled when the atom is shifted from the cavity center. Owing to the position dependence of the atom-field coupling (4) the even modes start to interact with the atom and the interaction with the odd modes decreases. Considering the effective squared interaction constant as the average for two neighboring (odd and even) cavity modes it decreases to the half value of the squared coupling constant of the interacting modes for the atom in the cavity center. It means that even though the atom is shifted from the cavity center the Fermi golden rule (9) with the effective squared interaction constant and doubled density of modes leads to the same decay rate (13).

For large enough times, the total excitation energy of the atom is transferred to the field, which in turn is effectively in a one photon (one excitation) state represented by two EM wave packets propagating towards the mirrors. For finite cavities at time approximately \( \frac{L}{c} \) the wave packets are reflected by the mirrors and at \( t_R = \frac{L}{c} \) they approach the atom, which starts to reabsorb the energy from the field. We observe the re-excitation of the atom (i.e. the Poincaré recurrence can be observed). In contrast to the “exponential” stage of the atomic decay, Poincaré recurrences are very sensitive to small position shifts of the atom within a wavelength of the resonant atomic transition. In Fig. 1 we clearly see that if the atom is positioned at the cavity center \( r_1 = L/2 \) then at time \( t_R = L/c \) a very well pronounced Poincaré recurrence of the atomic inversion is seen. One can say that at the moment when the Poincaré recurrence appears the atom “sees” the cavity mirrors (24). On the other hand, with a small shift of the atom from the cavity center to \( \Delta r_1 = \pm \frac{\lambda}{16} \) the first atomic recurrence is almost completely suppressed. For simplicity we consider the two emitted wave packets (one to the left and one to the right) as monochromatic plane waves (at the atomic transition frequency and with the group velocity c). The difference of their geometrical paths is equal to \( \frac{\lambda}{2} \). This path difference results in destructive interference due to the accumulated phase difference of \( \pi \). In other words, the atom does not “see” the wave packets reflected from the cavity mirrors. Obviously, when the two wave packets propagate further, then after the second reflection they accumulate a phase difference of \( 2\pi \) so the corresponding Poincaré recurrence can then be seen (i.e. in this case the atom needs an elapsed time which is twice as long compared with the situation when \( r_1 = \frac{L}{2} \) to “see” the cavity mirrors).

For the case of an atom positioned at \( \Delta r_1 = \pm \frac{\lambda}{2} \) the evolution of the atomic inversion is almost indistinguishable from the case where \( \Delta r_1 = 0 \). At the time of appearance of the first Poincaré recurrence there is a
constructive interference of the wave packets. The trivial phase shift \(2\pi\) results from the difference of the geometrical paths which is then equal to \(\lambda_a\).

In the case of an atom positioned at \(\Delta r_1 = \pm \frac{\lambda_a}{16}\) the path difference is equal to \(\lambda_a/4\) and the first Poincaré recurrence is intermediate between the extreme cases \((\Delta r_1 = 0, \pm \frac{\lambda_a}{4})\) considered above. Dephasing of the wave packets by \(\frac{\pi}{2}\) results in a partially reduced reabsorption. More rigorous analysis should take into account the multimode structure of the wave packets as an additional source of dephasing due to the different (Rabi) frequencies of the modes. We next note that the second Poincaré recurrence associated with the second reflection from the boundaries starts simultaneously in all cases shown in Fig. 1. At the time \(\simeq 2t_R\) the wave packets are merging in-phase, i.e. their geometrical paths are equal, which results in a partial reexcitation of the atom.

![Graph](image)

FIG. 2. A stroboscopic set of plots describing the space-time propagation of the mean energy density of the cavity field. We assume the same settings as in Fig. 1 with the atom located in the cavity center. We see the two wave packets propagating towards the cavity mirrors. These wave packets have “sharp” fronts. We note that larger the number of modes coupled to the atom the sharper the fronts are. The length of the tails of the wave packets depends on the life-time of the atom.

Summarizing this part of the description of the dynamics of the atom, we can say that while the “exponential” character of the decay of an excited atom inside a large cavity is not influenced by small shifts of the atomic position, the first Poincaré recurrence is a position-dependent interference effect. The basic features can be explained using a simple classical point of view based on two interfering monochromatic waves. This fact may be thought of as rather surprising, as mathematically the Poincaré recurrences can be related to the discrete nature of the cavity modes (with equal frequency spacing). To be more precise, the phase-matching conditions necessary for the appearance of Poincaré recurrences in the atom-field system can be associated with the evolving phase factors \(e^{-ikz}\) of contributing eigenstates \(|\Phi_k\rangle\) of the total Hamiltonian \(\hat{H}\) : a Poincaré recurrence, i.e., a partial restoration of the initial state, can appear at time \(t_R\) such that the relation \(E_k t_R \simeq 2\pi\) is valid for many values of \(k\) (for more details see [27]).

In Fig. 2 we present a stroboscopic set of plots describing the space-time propagation of the energy density of the cavity field for the same physical situation (initial state) as considered in Fig. 1 when the atom is in the center of the cavity. We see two distinct wave packets propagating to the right and to the left. Reflection of the wave packets from the cavity mirrors (at time \(t \simeq \frac{c}{2}\)) is nicely demonstrated and the subsequent re-excitation of the atom is synchronized with the interference of the wave packets in the center of the cavity (compare with Fig. 1).

IV. INHIBITION OF SPONTANEOUS EMISSION

In the previous section we have considered situations when the atom is “far” from the cavity mirrors [i.e. \(\text{min}(r_1, L - r_1) \gg c/\Gamma_a\)] and the wave packets reflected by the mirrors do not directly affect the initial spontaneous decay of the atom. On the other hand, for distances between the atom and one of the cavity mirrors smaller than \(c/\Gamma_a\) (here \(1/\Gamma_a\) is the spontaneous decay time in a free space) deviations from exponential decay should be expected [4][28]. In particular, the decay of a two-level atom which is positioned very close to the cavity mirror can be significantly suppressed. This effect is called the inhibition of spontaneous emission [4]. The inhibition of spontaneous emission is a position-dependent effect which is related to the position dependence of the atom-field coupling constant \(\Phi\). In Fig. 3 we present numerical simulations for the time evolution of the population of the upper level of the atom described by the model interaction Hamiltonian \(\hat{H}\). The atom is assumed to be initially in its excited state and the field in the vacuum. We consider several typical physical configurations. Firstly, for reference, we plot the atomic decay of the atom positioned at \(r_1 = \lambda_a/8\) (solid line) which is indistinguishable from the exponential decay of the atom at the cavity center (i.e., \(P_e \approx e^{-T_a t}\) for \(t \leq t_R\)). For other atomic positions \(r_1 = \lambda_a/16\) and \(r_1 = \lambda_a/32\) (here \(\lambda_a = L/50\)) we clearly see that the closer the atom is to the mirror the slower the spontaneous decay is, that is the inhibition of spontaneous radiation is transparent for the considered positions of the atom. On the other hand for very specific atomic positions close to the mirror the opposite effect - the enhancement of spontaneous
emission - takes place. Namely, for the atomic distance \( r_1 = \lambda_a/4 \) the atom decays as \( P_t = e^{-2t^{*}L/2} \), i.e., it radiates twice as fast compared with the free-space case [see the reference case \( r_1 = \lambda_a/8 \)].

For distances of the atom from the mirror larger than the wavelength of the resonant transition \( \lambda_a \) the interference with the reflected wave packet can either stimulate or suppress the emission of the atom. To be specific we show in Fig. 4 the time evolution of the population of the upper level of the atom which is positioned at three distances \( r_1 = \lambda_a \) (dashed line); \( r_1 = \lambda_a + \lambda_a/4 \) (dotted line); and \( r_1 = \lambda_a + \lambda_a/8 \) (dashed-dotted line) which are compared with the exponential decay of the atom in the cavity center (solid line). Other settings are as in Fig. 3. The suppression and the stimulation of the emission caused by the reflected wave packet are clearly see.

FIG. 3. The time evolution of the population \( P_e(t) \) of the excited atomic level for the atom very close to the cavity mirror. The atom is considered at the following positions: \( r_1 = \lambda_a/2 \) (dotted line), \( r_1 = \lambda_a/4 \) (dashed line), \( r_1 = \lambda_a/8 \) (solid line), \( r_1 = \lambda_a/16 \) (dashed-dotted line) and \( r_1 = \lambda_a/32 \) (dotted line). The “reference” exponential decay of the atom at the cavity center \( r_1 = L/2 \) coincides with the case \( r_1 = \lambda_a/8 \). The initial conditions and other parameters are as in Fig. 3.

The origin of inhibition or enhancement of spontaneous emission in the context of the model used in this paper relies on the position dependence of the atom-field coupling (3). In particular, for \( r_1 = \lambda_a/4 \) the spatial–mode function \( \sin(k_a r_1) \approx 1 \) for all the modes close to the resonant frequency \( \omega_a \) irrespective of whether \( n \) is even or odd. This means that the density of modes is increased by a factor of two compared with the case of the atom at the cavity center \( r_1 = L/2 \) when the modes with even \( n \) are decoupled from the atom \( \sin(k_a L/2) = 0 \) for even \( n \) and \( \sin(k_a L/2) = 1 \) for odd \( n \). The increased density of modes implies an enhancement of the spontaneous emission. In a similar way, when \( r_1 = \lambda_a/8 \) the spatial–mode function \( \sin(k_a r_1) \approx 1/\sqrt{2} \) for all \( n \) around the atomic transition frequency. However, the decrease of the squared interaction constants is compensated for by an increase in the density of interacting modes (compared with the atom at the cavity center) and thus the spontaneous emission rate retains the value \( \Gamma_a \). For the other extreme case \( r_1 = \lambda_a/2 \) all modes around \( \omega_a \) are significantly decoupled from the atom (now \( \sin(k_a r_1) \approx 0 \)) which results in the dramatic inhibition of the spontaneous emission as seen in Fig. 3. For other positions shown in Fig. 4 \((r_1 = \lambda_a/16, \lambda_a/32)\) the slowing of the spontaneous emission rate is given by the factor \( \Gamma(r_1)/\Gamma_a \approx 1 - \cos(2k_a r_1) \). We note that in the case \( r_1 = \lambda_a/16 \) the atom decays completely while for \( r_1 = \lambda_a/32 \) the exponential decay law is interrupted by the Poincaré recurrence at \( 2t_R \).

FIG. 4. The time evolution of the population \( P_e(t) \) of the excited atomic level for the atom at the following positions: \( r_1 = \lambda_a \) (dashed line); \( r_1 = \lambda_a + \lambda_a/4 \) (dotted line); and \( r_1 = \lambda_a + \lambda_a/8 \) (dashed-dotted line) which are compared with the exponential decay of the atom in the cavity center (solid line). Other settings are as in Fig. 3. The suppression and the stimulation of the emission caused by the reflected wave packet are clearly see.

For distances of the atom from the mirror larger than the wavelength of the resonant transition \( \lambda_a \) the interference with the reflected wave packet can either stimulate or suppress the emission of the atom. To be specific we show in Fig. 4 the time evolution of the population of the upper level of the atom which is positioned at three distances \( r_1 = \lambda_a \) (dashed line); \( r_1 = \lambda_a + \lambda_a/4 \) (dotted line); and \( r_1 = \lambda_a + \lambda_a/8 \) (dashed-dotted line) which are compared with the exponential decay of the atom in the cavity center (solid line). The phase accumulated by the wave packet during the round trip from the atom to the neighboring mirror and back is in the case \( r_1 = \lambda_a \) equal approximately to 5\( \pi \) (here the additional contribution of \( \pi \) is due to the reflection from the mirror), i.e., there is a destructive interference between the wave packet and the atom which results in the suppression of the radiation. On the other hand, when \( r_1 = \lambda_a + \lambda_a/4 \) the accumulated phase is approximately 6\( \pi \), which leads to constructive interference. It means that the reflected wave packet, when it arrives at the position of the atom, starts to stimulate the atomic emission. In the units used in this simulation, the arrival time of the reflected wave packet is at approximately \( t \approx 0.16 \) which coincides with the change of the initial exponential decay of the atom. When the atom is at the position \( r_1 = \lambda_a + \lambda_a/8 \), the accumulated phase of the reflected wave packet is 11\( \pi/2 \) which gives rise to a partial suppression of radiation.
V. SPONTANEOUS EMISSION IN A “CRYSTAL”

Atomic radiation can be crucially modified by the presence of other atoms in the cavity. Obviously, if the distance between the atoms is large enough then the exponential decay of the originally excited atom is not affected significantly. On the other hand when the atoms are placed close together the situation is different (one of the consequences is a collective behavior of the atoms which might result in superradiance, see for instance [28]).

In this section we consider a specific initial condition when the initially excited atom is surrounded by a collection of two-level atoms in the ground state. These additional atoms can be considered as a linear “crystal”. By changing the density of the atoms we can model systems such as atomic structures embedded in optical lattices (for interatomic distances comparable with the wavelength of the atomic transition) or dielectrics (for much smaller interatomic distances). The cavity QED system with trapped atoms [29] represents a quite new experimental system. The transfer of excitations between particular atoms which are captured in an optical potential is mediated by the cavity field [30].

FIG. 5. The modification of the spontaneous emission of the atom at the cavity center being surrounded by identical atoms which form a linear “crystal”. The central atom is initially excited and the others de-excited. The vacuum field is in the vacuum field. The modification of the spontaneous emission depends dramatically on the interatomic distance \( a \). In the case of the “lattice” constant being \( a = \lambda_a / 2 \) (long dashed line) we can observe strong suppression of the spontaneous emission while for \( a = \lambda_a / 4 \) (short dashed line) an enhancement of radiation compared with the single atom system (dotted line) takes place. The origin of this behavior is related to either destructive or constructive interference effects, respectively. From other examples, for \( a = \lambda_a / 8 \) (solid line) and \( a = \lambda_a / 16 \) (dot–dashed line) it is seen that by increasing the density of the linear “crystal” the atom radiates more slowly. Moreover, the de-excitation is incomplete, as an increasing part of the excitation is captured by the initial state. This subradiant behavior has been already analyzed for the extreme case when all the atoms are located at the same position (e.g., the cavity center) [31]. The initial excitation is captured in the asymmetric atomic state and only a small part \( \sim 1 / M \) is radiated in the cavity field.

The modification of the spontaneous emission of the atom embedded in a linear “crystal” of two–level atoms is shown in Fig. 5. The regular crystal built of \( M = 101 \) atoms fills the central part of the cavity. Initially the only excited atom is located at the cavity center and decays

The regular “crystal” represents a rather specific and idealized case. Positions of atoms can fluctuate due to various reasons (for example in the case of optical lattices with shallow wells formed from optical potentials). To simulate the situation when the atoms are not regularly distributed in the cavity we next consider random configurations of the atoms. Specifically, the atoms are placed randomly such that within each lattice constant there is just one atom. Depending on the particular posi-
tions of the atoms, the dynamics of the originally excited atom can change dramatically. The atomic radiation can be either enhanced or suppressed. To obtain some effective “macroscopic” picture from our simulations, we have averaged over many random configurations.

We present the results in Fig. 6. The dashed (dotted) line in this figure shows the time evolution of the atomic population of the initially excited atom when the atoms are regularly positioned with the lattice constant \(a = \lambda_a/4\) (\(a = \lambda_a/8\)), representing enhancement (suppression) of radiation with respect to the “free-space” decay (solid line). The results of numerical simulations corresponding to averaging over many (100) random configurations of atoms are presented for the average distance between atoms \(\langle a \rangle = \lambda_a/4\) (○) and \(\langle a \rangle = \lambda_a/8\) (△). In both cases the radiation of the atom is suppressed compared with the “free-space” decay. Another common feature of the dynamics in this case is that in both cases the atom does not radiate away completely the initial excitation energy. Both effects (suppression of radiation and “excitation trapping”) are caused by the collective influence of the crystal atoms.

An increase of the density of the linear “crystal” (e.g. for \(a = \lambda_a/16\)) rapidly diminishes differences between the regular crystal and the corresponding “random” crystal with one atom randomly positioned within the lattice constant. For completeness we included in Fig. 6 also the case of the regular “crystal” with \(a = \lambda_a/2\) (□) and the average over random configurations with \(\langle a \rangle = \lambda_a/2\). Here the destructive interference effect which leads to the strong inhibition of the radiation for the regular crystal is deteriorated for random atomic configurations shown in Fig. 4.

![Figure 7](image)

**FIG. 7.** The total excitation of the atoms \(R_{\text{atoms}} = \sum P_e^{(j)}\) which form a regular linear “crystal” as in Fig. 3. For the interatomic distance \(a = \lambda_a/4\) also small “crystals” made of \(M = 11\) (dotted line) and \(M = 21\) (dot-dashed line) are considered.

We note that the modification of the spontaneous emission is a local effect, i.e., the atomic decay is influenced only by neighboring atoms. To check this we have performed simulations with only 10 neighboring atoms \((M = 11)\). We have found that the “exponential” stage of the spontaneous emission is unchanged comparing with the case \(M = 101\), and differences occur only on a much longer time scale.

It is instructive to note that part of the initial excitation energy is captured in the atomic system. In particular, the sum of atomic excitations \(R_{\text{atoms}} = \sum P_e^{(j)}(t)\) depends mainly on the number of atoms \(M\). This tendency is confirmed in Fig. 6 which shows the cases \(M = 11, 21, 101\) for \(a = \lambda_a/4\). In general, the oscillation patterns reflect complex interference effects. Nevertheless one can trace a very general tendency in the picture - the “crystal” atoms which surround the initially excited atom play the role of semi-transparent mirrors placed very close to the atom. Therefore the results partially resemble the case of the single atom in the vicinity of a mirror (compare Fig. 6 with Fig. 3).

### VI. Spectrum of the Cavity Field

Within the framework of cavity QED when the field interacting with the atoms is confined within ideal mirrors, there is nothing like a stationary regime which is necessary for the derivation of a time-independent spectrum of the field. The spectrum is intrinsically time dependent. In this case an operational definition of time-dependent spectrum can be given by excitation probabilities of the cavity modes [see Eq. (6)]

The spectrum of the cavity field is affected by the position of the atom. In particular, if the atom is located in the cavity center even modes are completely decoupled from the atom and only odd modes can become excited [see Eq. (6)] establishing in this way oscillations in the spectrum of modes. However, at the point when the total excitation energy of the atom is transferred to the field, the envelope of the spectrum is “Lorentzian” irrespective of the position of the atom (providing that the decay is exponential).

On the other hand, it should be stressed that the spectrum of the interacting modes is highly transient even during the exponential decay period. It undergoes a gradual narrowing from a broad flat spectrum (initially all modes are in the vacuum state with the same probability) towards a Lorentzian-like line of width \(\Gamma_a\). The narrowing is accompanied by transient oscillations of the spectral envelope. This transient behavior is illustrated in Fig. 8 which shows the spectral envelopes at different time moments during the exponential decay of the atomic excitation. At the time \(t \approx 2\) the envelope of the cavity-field spectrum reaches its quasi-stationary shape, being very close to the corresponding (Lorentzian) emission spectrum usually associated with the free-space emission.
FIG. 8. Populations of cavity modes (i.e. the spectrum) at times $t = 0.3, 0.7, 1.3$ (in units of $1/\Gamma_a$) for the atom located at the cavity center. The atom is initially prepared in its excited state and the cavity field is in the vacuum (for other conditions see Fig. 1). For comparison purposes we show the overlaps of the eigenstates of the total Hamiltonian $\mathcal{H}$ with the initial state (these overlaps are denoted by the symbol $\Delta$). The ordering of eigenstates is given by their eigenvalues on frequency axis. The even modes which do not interact with the atom have no overlap with eigenstates of the total Hamiltonian (see oscillations in the function $|\langle \psi_0 | \Phi_k \rangle|^2$).

It is worth noticing that there is a close relation between the emission spectrum and the “spectrum” of squared scalar products (overlaps) between eigenvectors $|\Phi_k\rangle$ of the total Hamiltonian $\mathcal{H}$ and the given initial state $|\Psi(t_0)\rangle$, i.e.,

$$S_e(k) = |\langle \Psi(t_0) | \Phi_k \rangle|^2.$$  

(14)

From Fig. 8 it is evident that the “spectrum” of overlaps (shown as •) resembles the emission spectrum of the completely deexcited atom. In other words, the “spectrum” of overlaps offers an important time-independent characterization of the system under consideration. If there exists a quasi-stationary spectrum of the cavity modes, it should mimic the “spectrum” of overlaps. In addition, a shift of the atomic transition frequency in the spectrum of eigenvalues can be associated with the energy shift. In our calculations we use a broadband approximation, in which the frequency dependence of the atom-field coupling can be neglected. Thus for the symmetrical upper frequency cutoff $\omega_{\text{cut}} = 2\omega_a$ the energy shift equals zero.

We next turn our attention to the fact that an additional system of two–level atoms with different transition frequencies $\omega_a^{(j)}$ inside the cavity can serve as a device to measure the spectrum of the cavity modes. Within this model of measurement the initially deexcited (analyzer) atoms are placed far enough from the decaying atom. The coupling of the analyzer atoms to the cavity modes is so weak that the dynamics of the cavity field is essentially unaffected on the relevant time scale. It means that the linewidths of the analyzer atoms are much smaller than the linewidth $\Gamma_a$ of the decaying atom. The analyzer atoms with different transition frequencies $\omega_a^{(j)}$ thus function as narrow frequency filters interacting effectively only with the cavity modes on exact resonance with particular $\omega_a^{(j)}$. The upper level excitation probabilities of the analyzer atoms are proportional to the intensity of the cavity field at the position of the analyzer atoms $|\Omega|^2$. In order to map the excitations of analyzer atoms to the spectrum of the cavity modes it is necessary to demand that the coupling constants (linewidths) of the analyzer atoms are equal. In general, the analyzer atoms give the local and time dependent frequency spectrum which is not necessarily the same as excitations of cavity modes (11) which give the “instantaneous” spectrum in the whole cavity. In the case of the atomic decay the two spectra agree if the times they are read out from the analyzer atoms and the modes are chosen appropriately.

The normalized absorption spectrum read out from the analyzer atoms is show in Fig. 9. Here we have considered that the decay rates (i.e. linewidths) $\Gamma$ of the analyzer atoms are mutually equal and $\Gamma = 10^{-4}\Gamma_a$. A bunch of one hundred of analyzer atoms is positioned $\Delta r = 0.5$
apart of the decaying atom in the cavity center. The atomic frequencies \( \omega_a^{(j)} \) are equally spaced around the transition frequency \( \omega_a \) of the central atom. We see very good agreement with the envelope of the spectrum of the cavity modes. More precisely, in Fig. 9 we compare the spectrum of the cavity modes at times \( t = 0.3 \) and \( t = 2 \) with the absorption spectrum of the analyzer atoms at delayed times \( t + t_f \) where \( t_f \) is the time of flight of the light from the decaying (central) atom to the analyzer atoms.

![Spectrum of the decaying atom](image)

**FIG. 9.** The spectrum of the cavity modes at times \( t = 0.3 \) (•) and \( t = 2 \) (○) is compared with the normalized absorption spectrum of the analyzer atoms at times \( t = 0.3 + t_f \) (dashed line) and \( t = 2 + t_f \) (solid line). Here \( t_f \) is time of flight of the light from the decaying atom at the cavity center with \( \Gamma_a \) as in Fig. 3 to the analyzer atoms with mutually equal decay rates \( \Gamma = 10^{-4} \Gamma_a \). The distance of the analyzer atoms from decaying atoms is \( \Delta r = 0.5 \) and the time of flight \( t_f = 0.5 \) in chosen units (\( c = 1 \)).

### A. Numerics and cutoff dependence

Our numerical calculations have been performed in the broadband approximation for the interaction constants given by Eq. (6), i.e., we have neglected the frequency dependence of the coupling constants replacing \( \omega_n \) by \( \omega_a \). This approximation is valid only for large enough cavities with \( L \gg \lambda_a \) and “weak” interaction regimes with \( \Gamma_a \ll \omega_a \).

A rather subtle point is the choice of the frequency cutoff. Strictly speaking, the model interaction Hamiltonian (8) with the interaction constants (4) within the broadband approximation leads in second-order perturbation theory to logarithmically divergent energy shifts for \( \omega_{cut} \rightarrow \infty \) \([8]\). (Note that in our numerical calculations we have eliminated the shift of the excited atomic level by choosing a symmetrical frequency cutoff \( \omega_{cut} = 2\omega_a \).) Without the broadband approximation, when the frequency dependence of the interaction constants (4) is taken into account, the energy shifts diverge linearly. It is well known \([1\, 2]\) that if instead of the dipole approximation \( \hat{H}_{int} = -d \cdot \vec{E} \) we start with \( \hat{H}_{int} = -\vec{p} \cdot \vec{A} \), then after the RWA is applied one obtains the interaction Hamiltonian (8) but with a different frequency dependence of the interaction constant, i.e.

\[
g_n^{(j)} = \sqrt{\frac{\omega_a}{\omega_n}} \left( \frac{\omega_a}{\hbar c_0} L \right)^{1/2} d_{eg}^{(j)} \sin(k_a r_j).
\tag{15}
\]

In the broadband approximation the interaction constants given by Eq. (4) and Eq. (15) are identical and the results do not depend on the choice of the interaction Hamiltonian. On the other hand, without the broadband approximation the results are biased by the choice of the frequency dependence of the atom–field coupling. From the mathematical point of view, the coupling given by the expression (15) does not lead in second-order perturbation theory to divergent energy shifts for \( \omega_{cut} \rightarrow \infty \). Obviously at the point when the two effective Hamiltonians considered above lead to different results one has to be careful whether the model is physically relevant (for more details see Ref. [2]).

### VII. MASTER EQUATION FOR THE ATOM IN DIELECTRICS

The system of atoms and field modes under consideration in an ideal cavity represents a closed system with unitary dynamics governed by Schrödinger equation. In this Section we will consider the decaying atom as an open system in the environment represented by field modes and other initially unexcited atoms. This analysis can give us a deeper insight into the problem of dynamical evolution of the atom in dielectrics interacting with many cavity modes. In general, an open system \( S \) (in our case the atom which is initially excited) interacts with an environment \( E \) (the other atoms surrounding the originally excited atom and cavity modes) \([33]\). In this section we consider the archetypal system + environment model which is specified as follows: Let \( \mathcal{H}_S \) denotes a Hilbert space of the system \( S \), and \( \mathcal{H}_E \) the Hilbert space associated with the environment \( E \). The Hamiltonian \( \hat{H}_{SE} = \hat{H}_S \otimes 1_E + 1_S \otimes \hat{H}_E \) of the composite system \( S \otimes E \) acts on \( \mathcal{H}_S \otimes \mathcal{H}_E \). It is assumed that \( S \otimes E \) is a closed finite-dimensional system which evolves unitarily. The density operator \( \hat{\rho}_{SE}(t) \) of this composite system is governed by the von Neumann equation with the formal solution \( \hat{\rho}_{SE}(t) = \exp[-i(t-t_0)\hat{H}_{SE}]\hat{\rho}_{SE}(t_0)\exp[i(t-t_0)\hat{H}_{SE}] \), where the initial state is \( \hat{\rho}_{SE}(t_0) = \hat{\rho}_{S}(t_0) \otimes \hat{\rho}_{E}(t_0) \) and \( \hbar = 1 \). The reduced dynamics of the system \( S \) is then defined as

\[
\hat{\rho}_{S}(t) := \mathcal{T}(t, t_0)\hat{\rho}_{S}(t_0) = \text{Tr}_E \left[ \hat{\rho}_{SE}(t) \right]. \tag{16}
\]

By definition, \( \mathcal{T}(t, t_0) \) is a linear map which transforms the input state \( \hat{\rho}_{S}(t_0) \) onto the output state \( \hat{\rho}_{S}(t) \). In our
recent paper [34] we have addressed the question how to determine (reconstruct) the master equation which governs the time evolution of the reduced density operator $\hat{\rho}_e(t)$. It has been shown that this master equation can be written in the convolutionless form (we omit the subscript $S$)

$$\frac{d}{dt} \hat{\rho}(t) = \hat{L}(t, t_0) \hat{\rho}(t).$$  \hspace{1cm} (17)

which is possible due to the fact that in the finite-dimensional Hilbert spaces matrix elements of density operators are analytic functions. Consequently, $\hat{T}(t, t_0)$ are non-singular operators (except perhaps for a set of isolated values of $t$) in which case the inverse operators $\hat{T}^{-1}(t, t_0)$ exist and the Liouvillian superoperator can be expressed as

$$\hat{L}(t, t_0) := \left[ \frac{d}{dt} \hat{T}(t, t_0) \right] \hat{T}^{-1}(t, t_0).$$  \hspace{1cm} (18)

We note that $\hat{T}(t, t_0)$ is uniquely specified by $\hat{H}_{se}$ and by the initial state $\hat{\rho}_e(t_0)$ of the environment.

In an earlier paper [34] we have propose a general algorithm how to reconstruct the Liouvillian superoperator $\hat{L}(t, t_0)$ from the knowledge of the unitary evolution of the composite $S \otimes E$ system [see Eq.(14)]. From this knowledge the master equation (17) can be uniquely determined. The dynamics of the open system (in our case the atom) is given exclusively in terms of the system operators. Environmental degrees of freedom are completely eliminated from the reduced dynamics. Nevertheless, the state of the environment may change during the time evolution due to the interaction with the system. That is, there is no need to employ the assumption that the environment is a “big” reservoir which does not change under the action of the system.

To apply the formalism presented in Ref. [34] we remind ourselves that the initial state of the atom-field system considered in the present paper reads

$$|\Psi(t_0)\rangle = |e_1\rangle |g\rangle^0 \otimes |\bar{E}\rangle,$$  \hspace{1cm} (19)

where $|e_1\rangle$ describes the excited state of the initially excited atom, while $|g\rangle^0 := |g\rangle_2 \otimes \ldots \otimes |g\rangle_M$ describes the rest of the $M$ atoms which are initially in the ground state. The vector $|0\rangle_{\bar{E}}$ denotes the vacuum of the cavity field. Because the model Hamiltonian $\hat{H}_{tot}$ [7] is chosen so that the number of excitations in the system is an integral of motion we can express the state vector of the atom field system at time $t$ as

$$|\Psi(t)\rangle = c_1(t) |e_1\rangle |g\rangle^0 \otimes |\bar{E}\rangle$$

$$+ \sum_{j=2}^M c_j(t) |g\rangle_1 |e_j\rangle^0 \otimes |\bar{E}\rangle$$

$$+ \sum_{k} d_k(t) |g\rangle_1 |g\rangle_k |1k\rangle_{\bar{E}},$$  \hspace{1cm} (20)

where $|e_j\rangle^0$ describes the state vector of a set of $M-1$ atoms out of which the $j$-th atom is excited, while $|1k\rangle_{\bar{E}}$ describes the state of the cavity field with the $k$-th mode in the Fock state $|1\rangle$ and all other modes in the vacuum state. Using this notation we can express the master equation for the originally excited atom as [34]

$$\frac{\partial}{\partial t} \hat{\rho} = \frac{i}{2} [\hat{\rho}, \hat{\sigma}_+ \hat{\sigma}_-]$$

$$+ \frac{\Gamma(t)}{2} \left[ 2\hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ - \hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} - \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_- \right],$$  \hspace{1cm} (21)

where the time-dependent decay rate $\Gamma(t)$ and the time-dependent dynamical energy shift $\delta(t)$ can be expressed through the probability amplitude $c_1(t)$ as

$$\Gamma(t) = \text{Re} [\eta(t)]; \quad \delta(t) = \text{Im} [\eta(t)].$$  \hspace{1cm} (22)

where

$$\eta(t) = -2 \left[ \frac{1}{c_1(t)} \frac{dc_1(t)}{dt} \right].$$  \hspace{1cm} (23)

In general the parameter $\eta$ cannot be derived in an analytical form. In Fig. 11 we present results of numerical evaluation. We assume the initially excited atom to be in the center of the cavity. In the chosen units we obtain from the Fermi golden rule [see Eq.(13)] the decay rate $\Gamma_a = \pi$ (see dashed line). We consider two cases: firstly when there is just a single excited atom in the cavity

![Graph](image-url)
(dotted line) and secondly, when the excited atom is surrounded by 100 atoms (solid line) which create a “linear” crystal with regular spacing between atoms \((a = \lambda_{0}/8)\). In the case of the single atom \(\Gamma(t)\) oscillates around the value \(\Gamma_{a}\). The amplitudes of these oscillations are relatively small till the recurrence time when it eventually takes negative values (i.e. the atom starts to absorb energy from the field - compare with Fig. 3). In the second case which mimics the decays of the atom in dielectrics the time evolution of \(\Gamma(t)\) is more complex. At the initial instants \(\Gamma(t)\) oscillates around the value \(\Gamma_{a}\), but then it rapidly decreases and takes negative values - this is correlated with the absorption of energy from the wave packets reflected by surrounding atoms (see Fig. 3).

VIII. CONCLUSIONS

In this paper we have numerically studied a microscopic model of the cavity QED describing an atom interacting with multimode electromagnetic field. The initially excited atom is surrounded by a set of other atoms which represent a dielectric “crystal” on a microscopic level. We have shown how the decay of the atom is modified due to the mode structure of the cavity field and the presence of neighboring atoms.

In the cavity QED model considered here we have neglected all mechanical effects of the EM field on atoms. To account for effects of quantized center–of–mass motion one can generalize the model to a system of trapped atoms interacting with EM field in the cavity. We can assume a situation when due to laser cooling the atoms embedded in a cavity (or PBS) are effectively trapped by harmonic (optical) potential. There is a close analogy with the model of trapped ions [29].

In our next paper we focus on the propagation of one-photon wave packets in a cavity. These wave packets are represented as superpositions of many cavity modes. For example, we will study in detail scattering of a photon wave packet on a single two-level atom. We will show that the atom within this framework effectively behaves as quantum beam splitter or semi-transparent mirror. We will also discuss dynamics of the wave packet incident on a “crystal” composed of two–level atoms which fill some part of the resonator and thereby the transmission of the energy of EM field depends on the density of the atoms. This model will help us to understand on the microscopic level how photon wave packets propagate through dielectric media and to estimate (at least qualitatively) the speed of their propagation from first principles. In addition the model will allow us to formulate the quantum version of the extinction theorem [28] which on the classical level [28] explains how electromagnetic waves packets propagate in material media.

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