Exclusive semileptonic B decays to excited D mesons

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Abstract: Exclusive semileptonic B decays to orbitally and radially excited charmed mesons are investigated in the first order of the heavy quark expansion. The merging leading and subleading Isgur-Wise functions are calculated in the framework of the relativistic quark model. It is found that both relativistic and the 1/m_Q corrections play an important role and substantially modify results. An interesting interplay between different corrections is observed.

1. Introduction

The investigation of semileptonic decays of B mesons to excited charmed mesons represents a problem interesting both from the experimental and theoretical point of view. The current experimental data on semileptonic B decays to ground state D mesons indicate that a substantial part (≈ 40%) of the inclusive semileptonic B decays should go to excited D meson states. First experimental data on some exclusive B decay channels to excited charmed mesons are becoming available now [1, 2, 3] and more data are expected in near future. Thus the comprehensive theoretical study of these decays is necessary. The presence of the heavy quark in the initial and final meson states in these decays considerably simplifies their theoretical description. A good starting point for this analysis is the infinitely heavy quark limit, m_Q → ∞ [4]. In this limit the heavy quark symmetry arises, which strongly reduces the number of independent weak form factors [5]. The heavy quark mass and spin then decouple and all meson properties are determined by light-quark degrees of freedom alone. This leads to a considerable reduction of the number of independent form factors which are necessary for the description of heavy-to-heavy semileptonic decays. For example, in this limit only one form factor is necessary for the semileptonic B decay to S-wave D mesons (both for the ground state and its radial excitations), while the decays to P states require two form factors. It is important to note that in the infinitely heavy quark limit matrix elements between a B meson and an excited D meson should vanish at the point of zero recoil of the final excited charmed meson in the rest frame of the B meson. In the case of B decays to radially excited charmed mesons this follows from the orthogonality of radial parts of wave functions, while for the decays to orbital excitations this is the consequence of orthogonality of their angular parts. However, some of the 1/m_Q corrections to these decay matrix elements can give nonzero contributions at zero recoil. As a result the role of these corrections could be considerably enhanced, since the kinematical range for B decays to excited states is a rather small region around zero recoil.

Our relativistic quark model is based on the quasipotential approach in quantum field theory with a specific choice of the quark-antiquark interaction potential. It provides a consistent scheme for the calculation of all relativistic corrections at a given v^2/c^2 order and allows for the heavy quark 1/m_Q expansion. In preceding papers we applied this model to the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons [6], as well as to a description of weak decays of B mesons to ground state heavy and light mesons [7, 8]. The heavy quark expansion for the ground state heavy-to-
heavy semileptonic transitions \( \bar{\psi} q \) was found to be in agreement with model-independent predictions of the heavy quark effective theory (HQET).

2. Decay matrix elements and the heavy quark expansion

In this section we present the heavy quark expansion for weak decay matrix elements between a \( B \) meson and radially excited charmed meson states up to the first order in \( 1/m_Q \) using the HQET. The corresponding formulas for \( B \) decays to orbital excitations can be found in Ref. [10]. Following Ref. [10], we introduce the notation for weak decay matrix elements between a heavy quark and radially excited states destroyed by the fields in \( H_v \) and \( H'_v \), respectively, are given by

\[
\bar{c} \Gamma b \rightarrow \bar{h}^{(c)}_v \Gamma h^{(b)}_v = \xi^{(n)}(w) \text{Tr} \left\{ H'\Gamma H_v \right\},
\]

where \( h^{(Q)}_v \) is the heavy quark field in the effective theory. The leading order Isgur-Wise function \( \xi^{(n)}(w) \) vanishes at the zero recoil \( (w = 1) \) of the final meson for any \( \Gamma \), because of the heavy quark symmetry and the orthogonality of the radially excited state wave function with respect to the ground state one.

At first order of the \( 1/m_Q \) expansion there are contributions from the corrections to the HQET Lagrangian

\[
\delta \mathcal{L} = \frac{1}{2m_Q} \mathcal{L}^{(Q)}_{1, v} = \frac{1}{2m_Q} \left[ O^{(Q)}_{\text{kin, } v} + O^{(Q)}_{\text{mag, } v} \right],
\]

and from the tree-level matching of the weak current operator onto effective theory which contains a covariant derivative \( D^\lambda = \partial^\lambda - ig_\lambda A^\lambda \)

\[
\bar{c} \Gamma b \rightarrow \bar{h}^{(c)}_v \left( \Gamma - \frac{i}{2m_c} \not\! \partial \Gamma + \frac{i}{2m_b} \not\! \Gamma \right) h^{(b)}_v. \quad (2.5)
\]

The matrix elements of the latter operators can be parameterized as

\[
\bar{h}^{(c)}_v \not\! \partial \Gamma h^{(b)}_v = \text{Tr} \left\{ \xi^{(c)}_\lambda \bar{H}_v \Gamma H'_v \right\},
\]

\[
\bar{h}^{(c)}_v \not\! \Gamma \Gamma h^{(b)}_v = \text{Tr} \left\{ \xi^{(b)}_\lambda \bar{H}_v \Gamma H'_v \right\}. \quad (2.6)
\]

The most general form for \( \xi^{(Q)}_\lambda \) is [12]

\[
\xi^{(Q)}_\lambda = \xi^{(Q)}_\lambda (v + v') \lambda + \xi^{(Q)}_\lambda (v - v') \lambda - \xi^{(Q)}_\lambda \gamma_\lambda. \quad (2.7)
\]

The equation of motion for the heavy quark, \( i(v \cdot D) h^{(Q)} = 0 \), yields the relations between the form factors \( \xi^{(Q)}_\lambda \)

\[
\xi^{(c)}_+ (1 + w) + \xi^{(c)}_+ (w - 1) + \xi^{(c)}_3 = 0
\]

\[
\xi^{(b)}_+ (1 + w) - \xi^{(b)}_+ (w - 1) + \xi^{(b)}_3 = 0. \quad (2.8)
\]

The additional relations can be obtained from the momentum conservation and the definition of the heavy quark fields \( h^{(Q)}_v \), which lead to the
\[ i\partial_v \bar{\xi}^{(c)}_\nu \Gamma_{\mu}^{(b)} = (\bar{\xi} v_{\nu} - \bar{\Lambda}^{(n)} v_{\nu}) \bar{\xi}^{(c)}_\nu \Gamma_{\mu}^{(b)}, \]
implies that
\[ \xi^{(c)}_+ + \xi^{(b)}_+ + \xi^{(c)}_- + \xi^{(b)}_- = \bar{\Lambda} \xi^{(n)}, \quad \xi^{(c)}_+ + \xi^{(b)}_- - \xi^{(c)}_- - \xi^{(b)}_+ = -\bar{\Lambda}^{(n)} \xi^{(n)}, \quad \xi^{(c)}_+ + \xi^{(b)}_- = 0. \quad (2.10) \]

The functions \( \chi^{(b)}_n \) contribute to the decay form factors \( [2.3] \) only in the linear combination \( \chi_b = 2\chi_1^{(b)} - 4(w-1)\chi_2^{(b)} + 12\chi_3^{(b)} \). Thus five independent functions \( \xi_3, \chi_b \) and \( \chi^{(b)}_1 \), as well as two mass parameters \( \bar{\Lambda} \) and \( \bar{\Lambda}^{(n)} \) are necessary to describe first order \( 1/m_Q \) corrections to matrix elements of \( B \) meson decays to radially excited \( D \) meson states. The resulting structure of the decay form factors is
\[ h_+ = \xi^{(n)} + \varepsilon_c \left[ 2\bar{\chi}_1 - 4(w-1)\bar{\chi}_2 + 12\bar{\chi}_3 \right] + \varepsilon_b \chi_b, \]
\[ h_- = \varepsilon_c \left[ 2\bar{\xi}_3 - (\bar{\Lambda}^{(n)} + \bar{\Lambda}) \bar{\xi}_3 \right] \]
\[ h_V = \varepsilon_c \left[ 2\bar{\chi}_1 + (\bar{\Lambda}^{(n)} + \bar{\Lambda}) \bar{\xi}_3 \right] - 4\bar{\xi}_3 \]
\[ h_{A_1} = \varepsilon_c \left[ 2\bar{\chi}_1 - 4\bar{\xi}_3 \right] + \varepsilon_b \left[ \chi_b + (\bar{\Lambda}^{(n)} + \bar{\Lambda}) \bar{\xi}_3 - 2\bar{\xi}_3 \right], \]
\[ h_{A_2} = \varepsilon_c \left[ 4\bar{\chi}_2 - \frac{2}{w+1} \left( \bar{\Lambda}^{(n)} + \bar{\Lambda} \right) \bar{\xi}_3 \right] \]
\[ + \bar{\xi}_3 \]
\[ h_{A_3} = \varepsilon_c \left[ 2\bar{\chi}_1 - 4\bar{\xi}_3 \right] + \varepsilon_b \left[ \chi_b + \frac{w-1}{w+1} \left( \bar{\Lambda}^{(n)} + \bar{\Lambda} \right) \bar{\xi}_3 - 2\bar{\xi}_3 \right], \quad (2.14) \]
where \( \varepsilon_Q = 1/(2m_Q) \).

The similar analysis \( [10] \) for \( B \) decays to orbitally excited states indicate that it is necessary to introduce two Isgur-Wise functions in leading order of the heavy quark expansion: one function \( \tau(w) \) for decays to \( D_1, D_2^* \) mesons with \( j = 3/2 \) and the other one \( \zeta(w) \) for decays to \( D_1^*, D_1^* \) mesons with \( j = 1/2 \). At subleading order six additional functions \( (\tau_{1,2}, \eta_{ke}, \eta_{1,2,3}) \) arise for the former decays and four functions \( (\zeta_1, \chi_{ke}, \chi_{1,2}) \) for the latter ones.
3. Relativistic quark model

We use the relativistic quark model based on the quasipotential approach for the calculation of corresponding Isgur-Wise functions. Our model has been described in detail at this conference \[\text{[14]}\], so we directly go to the calculation of decay matrix elements of the weak current between meson states. In the quasipotential approach, the matrix element of the weak current \(J^\mu = \bar{c}\gamma^\mu(1 - \gamma^5)b\) between a \(B\) meson and an excited \(D^{**}\) meson takes the form \[\text{[13]}\]

\[
\langle D^{**}|J^\mu_W(0)|B \rangle = \int \frac{d^3p}{(2\pi)^3} \langle p|\bar{c}c\rangle \Gamma_\mu(p,q)\Psi_B(q), \quad (3.1)
\]

where \(\Gamma_\mu(p,q)\) is the two-particle vertex function and \(\Psi_{B,D^{**}}\) are the meson wave functions projected onto the positive-energy states of quarks and boosted to the moving reference frame. The contributions to \(\Gamma\) come from Figs. 1 and 2. In the heavy quark limit \(m_{b,c} \to \infty\) only \(\Gamma^{(1)}\) contributes, while \(\Gamma^{(2)}\) contributes at \(1/m_Q\) order. They look like

\[
\Gamma^{(1)}_\mu(p,q) = \bar{u}_c(p_c)\gamma_\mu(1 - \gamma^5)u_b(q_b)(2\pi)^3\delta(p_q - q_b), \quad (3.2)
\]

and

\[
\Gamma^{(2)}_\mu(p,q) = \bar{u}_c(p_c)\bar{u}_q(p_q)\{\gamma_Q\gamma_\mu(1 - \gamma^5)\}
\]

\[
\times \frac{\Lambda^{(\gamma)}_\mu(k)}{\epsilon_b(k) + \epsilon_c(p_c)}\gamma^0\gamma_Q\gamma_\mu(1 - \gamma^5)\}
\]

\[
\times \frac{\Lambda^{(\gamma)}_\mu(k')}{\epsilon_c(k') + \epsilon_b(q_b)}\gamma^0\gamma_Q\gamma_\mu(1 - \gamma^5)\}
\]

\[
\}
\]

\[
\}
\]

where the superscripts “(1)” and “(2)” correspond to Figs. 1 and 2, \(Q = c \text{ or } b\), \(k = p_c - \Delta\): \(k' = q_b + \Delta\): \(\Delta = p_{D^{**}} - p_B\); \(\epsilon(p) = (m^2 + p^2)^{1/2}\); \(\Lambda^{(\gamma)}_\mu(p) = \frac{\epsilon(p) - (m^2 + \gamma^0(\gamma p))}{2\epsilon(p)}\).

Here \[\text{[13]}\]

\[
p_{c,q} = \epsilon_{c,q}(p)\frac{p_{D^{**}}}{M_{D^{**}}} \pm \sum_{i=1}^{3} n^{(i)}(p_{D^{**}})p^i,
\]

\[\text{[2]}\]

The contribution \(\Gamma^{(2)}\) is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function \(\Gamma^{(2)}\) is explicitly dependent on the Lorentz structure of the \(q\bar{q}\)-interaction.

**Figure 1:** Lowest order vertex function \(\Gamma^{(1)}\) contributing to the current matrix element \((3.1)\).

**Figure 2:** Vertex function \(\Gamma^{(2)}\) taking the quark interaction into account. Dashed lines correspond to the effective potential. Bold lines denote the negative-energy part of the quark propagator.

\[
q_{b,q} = \epsilon_{b,q}(q)\frac{p_B}{M_B} \pm \sum_{i=1}^{3} n^{(i)}(p_B)q^i,
\]

and \(n^{(i)}\) are three four-vectors given by

\[
n^{(i)}(p) = \left\{ \frac{p^i}{M}, \delta_{ij} + \frac{p^{i}p^{j}}{M(E + M)} \right\}.
\]

It is important to note that the wave functions entering the weak current matrix element \[\text{[13]}\] are not in the rest frame in general. For example, in the \(B\) meson rest frame, the \(D^{**}\) meson is moving with the recoil momentum \(\Delta\). The wave function of the moving \(D^{**}\) meson \(\Psi_{D^{**},\Delta}\) is connected with the \(D^{**}\) wave function in the rest frame \(\Psi_{D^{**},0}\) by the transformation \[\text{[13]}\]

\[
\Psi_{D^{**},\Delta}(p) = D^{1/2}_\epsilon(R^W_\Delta)D^{1/2}(R^W_{L_\Delta})\Psi_{D^{**},0}(p),
\]

(3.4)

where \(R^W\) is the Wigner rotation, \(L_\Delta\) is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix \(D^{1/2}(R)\) in spinor representation is given by

\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} D_{c,q}^{1/2}(R^W_{L_\Delta}) S^{-1}(p_{c,q}) S(\Delta) S(p),
\]

(3.5)

where

\[
S(p) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{\alpha p}{\epsilon(p) + m} \right).
\]

is the usual Lorentz transformation matrix of the four-spinor. For electroweak \(B\) meson decays to...
S-wave final mesons such a transformation contributes at first order of the $1/m_Q$ expansion, while for the decays to excited final mesons it gives a contribution already to the leading term due to the orthogonality of the initial and final meson wave functions.

Now we can perform the heavy quark expansion for the matrix elements of $B$ decays to excited $D$ mesons in the framework of our model and determine leading and subleading Isgur–Wise functions. We substitute the vertex functions and determine leading and subleading Isgur–Wise functions. We use the following values for HQET parameters $\bar{\Lambda} = 0.51$ GeV, $\bar{\Lambda}' = 0.80$ GeV, and $\bar{\Lambda}^* = 0.89$ GeV.

4. Semileptonic decays to orbitally excited states

We get the following expressions for leading and subleading Isgur–Wise functions of semileptonic $B$ decays to orbitally excited $D$ mesons [16]:

(i) $B \to D_1^* e\nu$ and $B \to D_2^* e\nu$ decays

$$
\tau(w) = \sqrt{2} \frac{1}{3(w+1)^{3/2}} \times \int \frac{d^3p}{(2\pi)^3} \overline{\psi}_D(p) \frac{2\epsilon_q}{M_D(w+1)} \psi_B(p),
$$

(ii) $B \to D_3^* e\nu$ and $B \to D_1^* e\nu$ decays

$$
\zeta(w) = \sqrt{2} \frac{1}{3(w+1)^{1/2}} \times \int \frac{d^3p}{(2\pi)^3} \overline{\psi}_D(p) \frac{2\epsilon_q}{M_D(w+1)} \psi_B(p),
$$

The contributions of all other subleading form factors, $\eta_i(w)$ and $\chi_i(w)$, to decay matrix elements are suppressed by an additional power of the ratio $(w - 1)/(w + 1)$, which is equal to zero at $w = 1$ and less than 1/6 at $w_{\text{max}} = (1 + \rho^2)/(2 \rho)$. Since the main contribution to the decay rate comes from the values of form factors close to $w = 1$, these form factors turn out to be unimportant. This result is in agreement with the HQET-motivated considerations [16] that the functions parametrizing the time-ordered products of the chromomagnetic term in the HQET Lagrangian with the leading order currents should be small.

The arrow over $\partial/\partial p$ in (11) and (14) indicates that the derivative acts on the wave function of the $D^{**}$ meson. All the wave functions and meson masses have been obtained in [13] by the numerical solution of the quasipotential equation. We use the following values for HQET parameters $\bar{\Lambda} = 0.51$ GeV, $\bar{\Lambda}' = 0.80$ GeV, and $\bar{\Lambda}^* = 0.89$ GeV.

The last terms in the square brackets of the expressions for the leading order Isgur–Wise functions $\tau(w)$ (4.1) and $\zeta(w)$ (4.4) result from the
wave function transformation \([\psi_0, \psi_1, \ldots]\) associated with the relativistic rotation of the light quark spin (Wigner rotation) in passing to the moving reference frame. These terms are numerically important and lead to the suppression of the \(\zeta\) form factor compared to \(\tau\). Note that if we had applied a simplified non-relativistic quark model \([3, 7]\) these important contributions would be missing. Neglecting further the small difference between the wave functions \(\psi_{D(1/2)}\) and \(\psi_{D(3/2)}\), the following relation between \(\tau\) and \(\zeta\) would have been obtained \([10]\)

\[
\zeta(w) = \frac{w+1}{\sqrt{3}} \tau(w). \quad (4.6)
\]

However, we see that this relation is violated if the relativistic transformation properties of the wave function are taken into account. At the point \(w = 1\), where the initial \(B\) meson and final \(D^{**}\) are at rest, we find instead the relation

\[
\frac{\tau(1)}{\sqrt{3}} - \frac{\zeta(1)}{2} \approx \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \hat{\psi}_{D^{**}}(p) \frac{p}{\varepsilon_q + m_q} \psi_B(p), \quad (4.7)
\]

obtained by assuming \(\psi_{D(3/2)} \approx \psi_{D(1/2)} \approx \psi_{D^{**}}\). The relation \([1, 7]\) coincides with the one found in Ref. \([3]\), where the Wigner rotation was also taken into account.

In Table 1 we present our numerical results for the leading order Isgur–Wise functions \(\tau(1)\) and \(\zeta(1)\) at zero recoil of the final \(D^{**}\) meson, as well as their slopes \(\rho_{2/3}^2\) and \(\rho_{2/3}^0\), in comparison with other model predictions \([10, 18, 19, 21, 22, 23]\). We see that most of the above approaches predict close values for the function \(\tau(1)\) and its slope \(\rho_{2/3}^2\), while the results for \(\zeta(1)\) significantly differ from one another. This difference is a consequence of a specific treatment of the relativistic quark dynamics. Nonrelativistic approaches predict \(\zeta(1) \approx (2/\sqrt{3})\tau(1)\), while the relativistic treatment leads to \((2/\sqrt{3})\tau(1) > \zeta(1)\). The more relativistic the light quark in the heavy–light meson is, the more suppressed \(\zeta\) is with respect to \(\tau\).

We can now calculate the decay branching ratios by integrating double differential decay rates. Our results for decay rates both in the infinitely heavy quark limit and taking account of the first order \(1/m_Q\) corrections as well as their ratio

\[
R = \frac{\text{Br}(B \to D^{**}e\nu)}{\text{Br}(B \to D^{*}e\nu)}_{m_Q \to \infty}
\]

are presented in Table 2. We see that the inclusion of \(1/m_Q\) corrections considerably influences the results and for some decays their contribution is as important as the leading order contribution. This is the consequence of the vanishing of the leading order contribution to the decay matrix elements due to the heavy quark spin-flavour symmetry at zero recoil of the final \(D^{**}\) meson \([10]\), while nothing prevents \(1/m_Q\) corrections to contribute to the decay matrix element at this kinematical point. In fact, matrix elements at zero recoil are determined by the form factors \(f_{V_1}(1)\), \(g_{\pm}(1)\) and \(g_{V_1}(1)\), which receive non-vanishing contributions from first order heavy quark mass corrections:

\[
\sqrt{6} f_{V_1}(1) = -8\varepsilon_c (\vec{\Lambda}' - \vec{\Lambda}) \tau(1) \quad (4.8)
\]

\[
g_{\pm}(1) = -\frac{3}{2}(\varepsilon_c + \varepsilon_b) (\vec{\Lambda}' - \vec{\Lambda}) \zeta(1) \quad (4.9)
\]

\[
g_{V_1}(1) = (\varepsilon_c - 3\varepsilon_b) (\vec{\Lambda}' - \vec{\Lambda}) \zeta(1). \quad (4.10)
\]

Since the kinematically allowed range for these decays is not broad \((1 \leq w \leq w_{\text{max}} \approx 1.32)\), the contribution to the decay rate of the rather small \(1/m_Q\) corrections is substantially increased \([10]\). This is confirmed by numerical calculations. From Table 2 we see that the decay rate \(B \to D^*_2e\nu\), for which all contributions vanish at zero recoil, is only slightly increased by subleading \(1/m_Q\) corrections. On the other hand, \(B \to D^*_1e\nu\) and \(B \to D^*_0e\nu\) decay rates receive large \(1/m_Q\) contributions. The situation is different for the \(B \to D^*_1e\nu\) decay. Here the \(1/m_Q\) contribution at zero recoil is not equal to zero, but it is suppressed by a very small factor \((\varepsilon_c - 3\varepsilon_b)\) (see Eq. \((4.11)\)), which is only \(\approx 0.015\ \text{GeV}^{-1}\) for our model parameters. As a result, the \(B \to D^*_1e\nu\) decay rate receives \(1/m_Q\) contributions comparable to those for the \(B \to D^*_2e\nu\) rate. The above discussion shows that the sharp increase of \(B \to D^*_1e\nu\) and \(B \to D^*_0e\nu\) decay rates by first order \(1/m_Q\) corrections does not signal the breakdown of the heavy quark expansion, but is rather a result of the interplay of kinematical and dynamical effects. Thus we have good reasons to expect that higher order \(1/m_Q\) corrections will
agrees with the ALEPH one. However, there are

cay is only within the CLEO upper limit and dis-

\[ \zeta(1) \leq 0.70 \pm 0.16 \quad 0.44 \quad 0.42 \]

\[ \rho_{1/2}^3 \leq 1.37 \quad 1.0 \quad 1.0 \quad 1.4 \quad 2.5 \pm 1.0 \quad 0.83 \quad 0.73 \]

Table 1: The comparison of our results for the values of the leading Isgur–Wise functions \( \tau \) and \( \zeta \) at zero recoil of the final \( D^{*+} \) meson and their slopes \( \rho_{j} \) with other predictions.

| Decay         | \( m_Q \to \infty \) | With \( 1/m_Q \) | Experiment               |
|---------------|----------------------|------------------|--------------------------|
| \( B \to D_1 e^+ \nu \) | 1.4 0.32 2.7 0.63 1.97 | 0.56 ± 0.13 ± 0.08 ± 0.04 | \( 0.74 \pm 0.16 \) |
| \( B \to D_2 e^+ \nu \) | 2.1 0.51 2.5 0.59 1.16 | < 0.8            | < 0.2                    |
| \( B \to D_{1*} e^+ \nu \) | 0.31 0.073 0.39 0.09 1.23 |                |                          |
| \( B \to D_{2*} e^+ \nu \) | 0.25 0.061 0.59 0.14 2.3  |                |                          |

Table 2: Decay rates \( \Gamma \) (in units of \( |V_{cb}/0.04|^2 \times 10^{-15} \) GeV) and branching ratios \( BR \) (in %) for \( B \to D^{*+} e^+ \nu \) decays in the infinitely heavy quark limit with the account of first order \( 1/m_Q \) corrections. \( R \) is a ratio of branching ratios with the account of \( 1/m_Q \) corrections to branching ratios in the infinitely heavy quark limit.

influence these decay rates at the level of 10–20%.

In Table 2 we present the experimental data from CLEO \([1]\) and ALEPH \([2]\), which are available only for the \( B \to D_1 e^+ \nu \) decay. For \( B \to D_{2*} e^+ \nu \), these experimental groups present only upper limits, which require the use of some additional assumptions about the hadronic branching ratios of the \( D_{2*} \) meson. Our result for the branching ratio of the \( B \to D_{2*} e^+ \nu \) decay with the inclusion of \( 1/m_Q \) corrections is in good agreement with both measurements. On the other hand, our branching ratio for the \( B \to D_{2*} e^+ \nu \) decay is only within the CLEO upper limit and disagrees with the ALEPH one. However, there are some reasons to expect that the ALEPH bound is too strong \([10]\).

Finally we test the fulfilment of the Bjorken sum rule \([21]\) in our model. This sum rule states

\[ \rho^2 = \frac{1}{4} \sum_m \frac{|\zeta(m)(1)|^2}{4} + 2 \sum_m \frac{|\tau(m)(1)|^2}{3} + \ldots, \]

(4.11)

where \( \rho^2 \) is the slope of the \( B \to D^{(*)+} e^+ \) Isgur–Wise function, \( \zeta(m) \) and \( \tau(m) \) are the form factors describing the orbitally excited states discussed above and their radial excitations, and ellipses denote contributions from non-resonant channels. We see that the contribution of the lowest lying \( P \)-wave states implies the bound

\[ \rho^2 > \frac{1}{4} + \frac{|\zeta(1)|^2}{4} + 2 \frac{|\tau(1)|^2}{3} = 0.81, \]

(4.12)

which is in agreement with the slope \( \rho^2 = 1.02 \) in our model \([11]\) and with experimental values \([25]\).

5. Semileptonic decays to radially excited states

In the case of semileptonic \( B \) decays to radially excited \( D \) mesons we get the following expressions for leading and subleading Isgur-Wise functions \([26]\):

\[ \zeta^{(1)}(w) = \left( \frac{2}{w+1} \right)^{1/2} \times \int \frac{d^3 p}{(2\pi)^3} \overline{\psi}_{D^{(*)+}}(p) \frac{2q}{M_{D^{(*)+}}(w+1)} \Delta \psi_B(p), \]

(5.1)

\[ \tilde{\zeta}_3(w) = \frac{\Lambda^{(1)} + \Lambda}{2} - m_q + \frac{1}{6} \frac{\Lambda^{(1)} - \Lambda}{w-1} \]

(5.2)

\[ \tilde{\chi}_1(w) \approx \frac{1}{20} \frac{w-1}{w+1} \frac{\Lambda^{(1)} - \Lambda}{w-1} \zeta^{(1)}(w) \]

\[ + \frac{\Lambda^{(1)}}{2} \left( \frac{2}{w+1} \right)^{1/2} \times \int \frac{d^3 p}{(2\pi)^3} \overline{\psi}_{D^{(*)+}}(p) \frac{2q}{M_{D^{(*)+}}(w+1)} \Delta \psi_B(p), \]

(5.3)
factors contributing to the decay matrix elements at zero recoil

\[ h_+(1) = \varepsilon_c [2\tilde{\chi}_1(1) + 12\tilde{\chi}_3(1)] + \varepsilon_b\chi_b(1), \]

\[ h_A(1) = \varepsilon_c [2\tilde{\chi}_1(1) - 4\tilde{\chi}_3(1)] + \varepsilon_b\chi_A(1). \]

Such nonvanishing contributions at zero recoil result from the first order $1/m_Q$ corrections to the wave functions (see Eq. (5.4) and the last terms in Eqs. (5.3), (5.3)). Since the kinematically allowed range for these decays is not broad ($1 \leq w \leq w_{\text{max}} \approx 1.27$) the relative contribution to the decay rate of such small $1/m_Q$ corrections is substantially increased. Note that the terms $\varepsilon_Q(\tilde{\Lambda} - \Lambda)\xi^{(a)}(w)/(w - 1)$ have the same behaviour near $w = 1$ as the leading order contribution, in contrast to decays to the ground state $D^{(*)}$ mesons, where $1/m_Q$ corrections are suppressed with respect to the leading order contribution by the factor $(w - 1)$ near this point (this result is known as Luke’s theorem [12]). Since inclusion of first order heavy quark corrections to $B$ decays to the ground state $D^{(*)}$ mesons results in approximately a 10-20% increase of decay rates [13], one could expect that the influence of these corrections on decay rates to radially excited $D^{(*)'}$ mesons will be more essential. Our numerical analysis supports these observations.

We can now calculate the decay branching ratios by integrating double differential decay rates. Our results for decay rates both in the infinitely heavy quark limit and taking account of the first order $1/m_Q$ corrections as well as their ratio

\[ R' = \frac{\text{Br}(B \to D^{(*)'}e\nu)_{\text{with } 1/m_Q}}{\text{Br}(B \to D^{(*)'e\nu})_{m_Q \to \infty}} \]

are presented in Table 3. We find that both $1/m_Q$ corrections to decay rates arising from corrections to HQET Lagrangian [5.3]–[5.6], which do not vanish at zero recoil, and corrections to the current [5.3], (2.11), vanishing at zero recoil, give significant contributions. In the case of $B \to D^{(*)'e\nu}$ decay both types of these corrections tend to increase the decay rate leading to approximately a 75% increase of the $B \to D^{(*)'e\nu}$ decay rate. On the other hand, these corrections give opposite contributions to the $B \to D^{(*)'e\nu}$ decay rate: the corrections to the current give a neg-
Inclusion of first order 1/m corrections. Σ(B \to D^{(*)}e^\nu) represent the sum over the channels. R' is a ratio of branching ratios taking account of 1/mQ corrections to branching ratios in the infinitely heavy quark limit.

### Table 3: Decay rates Γ (in units of [V_{ts}/0.04]^2 \times 10^{-15} \text{ GeV}) and branching ratios BR (in %) for B decays to radially excited \( D^{(*)} \) mesons in the infinitely heavy quark limit and taking account of first order 1/mQ corrections. Σ(B \to D^{(*)}e^\nu) represent the sum over the channels. R' is a ratio of branching ratios taking account of 1/mQ corrections to branching ratios in the infinitely heavy quark limit.

| Decay       | mQ \to \infty | With 1/mQ |
|-------------|---------------|-----------|
| B \to D'e^\nu | 0.53          | 0.12      |
| B \to D*'e^\nu | 0.70          | 0.17      |
| Σ(B \to D^{(*)}e^\nu) | 1.23          | 0.29      |

6. Conclusions

In this paper we have applied the relativistic quark model to the consideration of semileptonic B decays to orbitally and radially excited charmed mesons, in the leading and subleading orders of the heavy quark expansion. We have found an interesting interplay of relativistic and finite heavy quark mass contributions. In particular, it has been found that the Lorentz transformation properties of meson wave functions play an important role in the theoretical description of these decays. Thus, the Wigner rotation of the light quark spin gives a significant contribution already at the leading order of the heavy quark expansion for decays to orbitally excited mesons. This contribution considerably reduces the leading order Isgur–Wise function ζ with respect to τ. As a result, in this limit the decay rates of \( B \to D_2^*e^\nu \) and \( B \to D_1^*e^\nu \) are approximately an order of magnitude smaller than the decay rates of \( B \to D_1e^\nu \) and \( B \to D_2e^\nu \). On the other hand, inclusion of the first order 1/mQ corrections also substantially influences the decay rates. This large effect of subleading heavy quark corrections is a consequence of vanishing the leading order contributions to the decay matrix elements due to heavy quark spin-flavour symmetry at the point of zero recoil of the final charmed meson. However, the subleading order contributions do not vanish at this kinematical point. Since the kinematical range for these decays is rather small, the role of these corrections is considerably increased. Their account results in an approximately twofold enhancement of the \( B \to D_1e^\nu, \ B \to D_2^*e^\nu \) and \( B \to D'e^\nu \) decay rates, while the \( B \to D_2e^\nu, \ B \to D_1^*e^\nu \) and \( B \to D'^*e^\nu \) rates are increased only slightly. The small influence of 1/mQ corrections on the latter decay rate is the consequence of the additional interplay of 1/mQ corrections. We thus see that these subleading heavy quark corrections turn out to be very important and considerably change results in the infinitely heavy quark limit. For example, the ratio of branching ratios \( \text{Br}(B \to D_2^*e^\nu)/\text{Br}(B \to D_1e^\nu) \) changes from the value of about 1.6 in the heavy quark limit, \( m_Q \to \infty \), to the value of about 1 after subleading corrections are included. Finally we find that the semileptonic B decays to first orbital and radial excitations of D mesons amount in total to approximately 2% of the B decay rate.
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