Nucleon internal degrees of freedom and the uniqueness of the Gamow-Teller state

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Abstract

The Gamow-Teller strength in nuclei can be strongly affected by the internal degrees of freedom of the nucleon. It is demonstrated that this feature is unique to the Gamow-Teller. Excitation modes that involve spatial degrees of freedom are much less influenced by the internal excitations of nucleons. The fact that the observed Gamow-Teller strength is quenched by 30%–40% in all nuclei suggests that indeed this is due to the nucleon excitations in nuclei.
I. INTRODUCTION

Experimentally, for many years it is a well-established fact that the GT or M1 strength is quenched \[1, 2\]. In $\beta$ decay which involves the weak interaction or in charge-exchange reactions such as $(p, n)$, $(^3\text{He}, t)$, etc which are caused by the strong force, one observes strong quenching of the order 30–40%. This is observed in all nuclei studied. The GT and M1 are unique in this respect. No such phenomenon is found for other types of excitations. We should, however, emphasize that this aspect of quenching of strength for other resonances, such as the monopole, quadrupole, etc was not yet studied in great detail. (For the isovector dipole, the strength is well accounted.) As for the observations of other types of strength, they are not detailed enough to determine whether there is significant strength missing.

Basically, two types of theories were introduced to discuss the quenching of GT and M1 strength in nuclei:

A. The quenching of the GT strength in charge-exchange reactions refers to the main peaks where the GT strength is concentrated. So it was suggested that the remaining strength is fragmented and spread out at several tens of MeV above the main peaks \[3–5\]. The nuclear force (and especially the tensor component) causes a mixing of the one particle-one hole (1p-1h) components with 2p-2h excitations, causing the strength to be spread out (see \[1, 3–5\]). There are attempts to locate experimentally this fragmented strength, but it is quite difficult to reach a conclusive result. This mechanism could also affect other giant resonances, but no such effect was clearly observed so far.

B. The other approach taken in the past to explain the missing GT, M1 strength was to consider the influence of internal nucleon degrees of freedom that couple to the nucleus degrees of freedom and remove the strength to very high excitation energies, about 300 MeV above the ground state \[6–8\].

And then there is the possibility that both mechanisms A and B contribute to this puzzle of the missing GT strength. None of these possibilities can be ruled out at present. However, as we will see our considerations in this paper about the role of internal degrees of freedom makes the possibility B more plausible. The mechanisms described in A should be applied to the GT strength as well as to excitations involving the spatial coordinates. The understanding of the source of GT quenching has important consequences for nuclear
experimental and theoretical physics. The quenching of the GT strength should be even more pronounced in double Gamow-Teller transition \[9\] and thus also in double-beta decay \[10\]. The two mechanisms will influence the two-neutrino double-beta decay calculations differently than the neutrinoless double-beta decay calculations \[11\].

II. EXCITATION OF GIANT RESONANCES IN NUCLEI AND THE ROLE OF INTERNAL NUCLEON DEGREES OF FREEDOM

Consider a one-body operator with space, spin, and isospin dependence acting on the nucleons in the nucleus:

\[
\hat{O} = \sum_N \hat{O}_N + \sum_{q,N} \hat{O}_{q,N} \equiv \hat{O}_N + \hat{O}_q
\]

(1)

It is composed of two parts; the first sum referrers to operators acting on the nucleon degrees of freedom in the nucleus, the second sum involves single-particle operators acting on the internal degrees of freedom of each nucleon in the nucleus. We assume that the second sum contains only transition operators from a nucleon to some baryonic resonances, and does not have diagonal parts. Let us now define a coherent nucleon particle-hole state:

\[
|\lambda\rangle = \frac{\hat{O}_N|0\rangle}{\langle 0|\hat{O}_N^+\hat{O}_N|0\rangle^{1/2}}
\]

(2)

as well as a

\[
|\lambda^*\rangle = \frac{\hat{O}_q|0\rangle}{\langle 0|\hat{O}_q^+\hat{O}_q|0\rangle^{1/2}}
\]

(3)

which represents a coherent sum of particle-hole excitations of the type \(|N^{-1}N^*\rangle\) with definite parity, spin, and isospin; \(N^*\) means excited nucleon states (resonances).

Considering the mixed state:

\[
|\Phi\rangle = |\lambda\rangle + \alpha|\lambda^*\rangle
\]

(4)

with \(\alpha\) being the mixing amplitude, we now evaluate the transition strength between the ground state \(|0\rangle\) and the state \(|\Phi\rangle\) for the operator \(\hat{O}\)

\[
|\langle \Phi|\hat{O}|0\rangle|^2 = |\langle \lambda|\hat{O}_N|0\rangle + \alpha \langle \lambda^*|\hat{O}_q|0\rangle|^2
\]

(5)

We assume that the second term is small enough compared to the first one so we can neglect terms proportional to \(\alpha^2\). Consequently,

\[
|\langle \Phi|\hat{O}|0\rangle|^2 = |\langle \lambda|\hat{O}_N|0\rangle|^2 \left[ 1 + 2\alpha \frac{\langle \lambda^*|\hat{O}_q|0\rangle}{\langle \lambda|\hat{O}_N|0\rangle} \right].
\]

(6)
The expression

\[ \gamma = 2 \alpha \frac{\langle \lambda^* | \hat{O}_q | 0 \rangle}{\langle \lambda | \hat{O}_N | 0 \rangle} \]  

(7)

is the “quenching factor”, although it could also be an enhancement depending on the sign of \( \alpha \). It represents the contribution of the internal excitations to the strength at low energies.

The mixing amplitude is

\[ \alpha = 2 \frac{\langle \lambda | \sum V_{NN^*} | \lambda^* \rangle}{E_{\lambda^*}} \]  

(8)

where \( V_{NN^*} \) is a two-body transition potential for \( NN \rightarrow NN^* \) and \( E_{\lambda^*} \) is the excitation energy of \( |\lambda^*\rangle \) with respect to the g.s. and is approximately equal to the mass of the \( N^* \) relative to the nucleon mass, \( E_{\lambda^*} \approx m_{N^*} - m_N \).

The excitation energy \( \hbar \omega \) in the denominator was neglected because \( \hbar \omega \ll E_{\lambda^*} \). Following several calculations in the past [6] (in particular those involving the \( \Delta_{33} \) resonance) one can write:

\[ \langle \lambda | \sum V_{NN^*} | \lambda^* \rangle \simeq \langle \lambda | \sum V_{NN} | \lambda \rangle \frac{\langle \lambda^* | \hat{O}_q | 0 \rangle}{\langle \lambda | \hat{O}_N | 0 \rangle} G, \]  

(9)

where \( G \) is the ratio of the coupling constant:

\[ G = \frac{g_{mNN^*}}{g_{mNN}}, \]  

(10)

\( m \) stands for the meson exchange, mostly the pion-\( \pi \).

We can write \( \gamma \) in the form:

\[ \gamma = 2 \frac{\langle \lambda | \sum V_{NN} | \lambda \rangle}{E_{\lambda^*}} \frac{\langle \lambda^* | \hat{O}_q | 0 \rangle^2}{\langle \lambda^* | \hat{O}_N | 0 \rangle^2} G. \]  

(11)

The matrix element squared \( \langle \lambda^* | \hat{O}_q | 0 \rangle^2 \) can be estimated in the following way

\[ \langle \lambda^* | \hat{O}_q | 0 \rangle^2 = Z \delta_p + N \delta_n \]  

(12)

where \( Z \) and \( N \) denote the number of protons and neutrons in the nucleus. \( \delta_p \) and \( \delta_n \) are \( \langle p | \hat{O}_q | N_{1/2}^* \rangle \) and \( \langle n | \hat{O}_q | N_{3/2}^* \rangle \) corresponding the transition strengths between the proton (neutron) and the \( N^* \) component with \( T_z = -1/2 \) and the \( T_z = 1/2 \).

For the sake of an estimate, it is reasonable to assume [7] that \( \delta_p = \delta_n = \delta \). We see that the expression of \( \gamma \) will be enhanced by the factor \( A \), the number of nucleons in the nucleus.

The ratio of the coupling constant \( G \) is of the order of 1 [6] and the energy \( E_{\lambda^*} \) is several hundred MeV (about 300 MeV for the \( \Delta_{33} \) resonance, 600 MeV for \( D'_{13} \) resonance, 500 MeV for the Roper resonance, etc).
The matrix element $\langle \lambda | \sum V_{NN} | \lambda \rangle$ can be estimated by realizing that this is a shift of the excitation due to the particle-hole interaction [8, 12]. Such matrix element is typically of the order of $\hbar \omega = x 41 A^{-1/3}$ MeV, where $|x|$ is about 1–2 depending on the type of nuclear excitation and the nucleus. For isoscalar excitations, $x$ is negative because the particle-hole interaction is attractive but for isovector excitation it is repulsive and $x$ is positive.

Let us now estimate the ratio

$$f = \frac{\delta}{\langle \lambda | \hat{O}_N | 0 \rangle^2} \quad (13)$$

Here we must distinguish two types of nuclear excitations. The ones that depend on the spatial coordinate $r$ and those that do not. Many of the collective nuclear excitations are $r$-dependent. The best example is the isovector dipole resonance. The corresponding operator exciting the dipole resonance is $\hat{O}(\text{dip}) = \sum_i r_i Y_1(\theta_i) t_i$. But there are more; isovector monopole, quadrupole with the operators $\sum_i r_i^2 t_i, \sum_i r_i^2 Y_2(\theta_i) t_i$, etc. One can also introduce the spin into these operators. As for example the spin-isovector dipole $\sum_i \sigma_i r_i Y_1(\theta_i) t_i$, spin-isovector monopole $\sum \sigma_i r_i^2 t_i$, etc.

There are also the excitations that do not involve spatial coordinates, only spin-isospin operators, the Gamow-Teller (GT) and M1 with operators

$$\hat{O}_N = \sum_i \sigma_i t_i. \quad (14)$$

III. TRANSITION OPERATORS WITH SPATIAL DEPENDENCE

When we estimate the ratio $f$ (in Eq. (13)) for excitations involving the radial coordinate mentioned above we understand that the $\hat{O}_q$ operator will involve also the same type of spatial coordinate but for quarks inside the nucleon, thus $r_N$. Therefore, the ratio $f$ will be proportional to $(r_N/R)^n$, that is the ratio of the radius of the nucleon (or the $N^*$ resonance) and the radius of the nucleus-$R$ to some power $n \geq 2$, depending on the type of excitation one considers. For example for a dipole $n = 2$, for a monopole $n = 4$, etc. Clearly, this ratio is small. It means that the ratio $f$ will be reduced by a large factor for a dipole, quadrupole, monopole, etc.

In the past, we estimated this factor [7] for the dipole taking into account the coupling to the $D'_{13}$ resonance and using the experimental $\sigma_{-1} \approx 30 \mu b$ cross sections for the $N \to D'_{13}$ transition [13] and for the nuclear dipole from Ref. [14]. We found the reduction factor $f$
to be about 1/2000 and the quenching factor $\gamma$ to be $4 \times 10^{-3}$ for $^{90}$Zr and $1.5 \times 10^{-3}$ for $^{208}$Pb.

IV. TRANSITION OPERATORS WITH NO SPATIAL DEPENDENCE

As opposed to the transition operators which depend on the spatial coordinate we will now deal with simple operators that do not depend on $r$. Among the simple ones there is one outstanding operator, the Gamow-Teller operator (see for example [1])

$$\hat{O}_{GT} = \sum_i \sigma(i) t_{\mu}(i); \quad \mu = 0, \pm 1.$$  (15)

We will concentrate on the $\mu = -1$ component of this operator. The GT excitation of the nucleon is the $\Delta_{33}$ resonance with a mass of 1232 MeV. Eq. (1) will be written now as

$$\hat{O}_{GT} = \hat{O}_N + \hat{O}_\Delta,$$  (16)

where

$$\hat{O}_N = \sum_i \sigma(i) t_{-}(i),$$  (17)

and

$$\hat{O}_\Delta = \sum_i S(i) T_{-}(i)$$  (18)

is the nucleon to $\Delta$ part of the GT operator [6, 8].

Let us now use Eq. (11) to estimate the quenching factor $\gamma$, for $|\lambda\rangle = |GT\rangle$ (For simplicity we will deal with $N > Z$ nuclei and with cases when the $GT_+$ excitations are blocked by the Pauli principle.). The matrix element $\langle GT| \sum V_{NN} |GT\rangle$ is the shift of the GT state from the unperturbed position. In the nucleus $^{208}$Pb this shift is about 10 MeV. The energy $E_{\lambda^*}$ in Eq. (11) is the position of the $\Delta$ resonance above the nucleon, that is about 300 MeV. The transition matrix element squared $|\langle0|\hat{O}|GT\rangle|^2$ of the particle-hole state in the nucleus with a nucleon converted into a $\Delta_{33}$ is according to a quark model [6] equal to $\frac{32}{25} A$, the matrix element squared $\langle GT| \hat{O}_N |0\rangle^2$ for the GT is $N - Z$ (the strength of the GT excitation) and the coupling constant $G = 4/3$. Putting all these numbers into Eq. (11), we obtain $\gamma = 0.53$ for $^{208}$Pb, and $\gamma = 0.51$ for $^{90}$Zr. Thus for the GT strength, the quenching factor is of the order of one. Our estimates are not precise and the numbers we present for $\gamma$ are approximate, however one sees a qualitative change when one compares the
quenching factor for the excitations involving only spin-isospin operators to the ones that involve spatial coordinates.

V. CONCLUSION

Our analysis shows that the considerable quenching of strength due to internal excitation of the nucleon occurs uniquely for the GT or M1. Experimentally the clear and systematic quenching of strength is found only for the GT or M1 and not for the multipole \( r \)-dependent resonances. This suggests that the GT quenching is to a large extent due to the mixing with the \( \Delta \) particle-N hole configurations. Often a question is asked if the GT or M1 strength is quenched, does this apply to other types of excitations involving the spin. For example, are the so-called “first forbidden” \( \beta \) decay transitions also reduced? The operators relevant to such transition are of the type \( r \sigma t \) or \( r^2 \sigma t \) [15, 16], isovector dipole resonance, and isovector spin monopole, respectively. Some of these were observed, the spin dipole [17] and spin isovector monopole [18]. These observations are not detailed enough to determine whether there is significant strength missing. If the quenching of strength in the GT resonance is mostly due to the mixing with \( \Delta_{33} \), then in the view of our discussion, the above spin transitions which contain the coordinate \( r \) will not be considerably quenched. Obviously, there is still a lot of experimental work to be done to determine the total strength of \( r \)-dependent excitations.

In summary, the GT strength is unique when considering the influence of internal degrees of freedom of the nucleon. The reason is that the GT and M1 excitation do not involve a spatial coordinate. Many of the nucleons in the nucleus occupy both the spin up and down states and the Pauli principle limits therefore the nuclear GT or M1 transitions. The second main reason is that the internal excitations of the quarks are spatially confined. This confinement limits the amplitudes of the spatial vibrations when compared to the analogous nuclear vibrations.

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