Observation of phonon dimensionality effects on electron energy relaxation

J T Karvonen and I J Maasilta
Nanoscience Center, Department of Physics, P. O. Box 35, FIN-40014, University of Jyväskylä, Finland
E-mail: jenni.karvonen@phys.jyu.fi, maasilta@phys.jyu.fi

Abstract. We have investigated electron energy relaxation in copper wires with several film thicknesses (t=14-90 nm) evaporated on (a) 30 nm thick suspended silicon nitride membranes, and (b) bulk nitridized substrates. The experiment was performed at sub-Kelvin temperatures by heating the electron gas by current, and measuring the resulting temperature rise using sensitive normal metal-insulator-superconductor (NIS) tunnel junction thermometers. A clear effect of 2D phonon dimensionality was observed in all wires on top of the membranes, resulting in strengthened electron-phonon interaction below 0.5 K. Thinning the metal film on a bulk substrate also weakens the energy relaxation notably at the lowest temperatures.

1. Introduction
A thin metal wire can be modelled as a thermodynamical system consisting of two subsystems, conduction electrons and phonons, coupled by the thermal resistance $R_{e-p}$ due to electron-phonon (e-p) interaction. If the wire is fabricated on an insulating substrate, the metal phonons see an additional Kapitza boundary resistance $R_K$ for emission into the substrate. At sub-Kelvin temperatures $R_{e-p}$ increases significantly over $R_K$ [1] and becomes the dominant energy relaxation mechanism for electrons in thin metal film. However, the detailed behavior of $R_{e-p}$ depends strongly on the quality of the film, i.e. the level and nature of disorder, and also on the modification of the phonon modes on surfaces and thin membranes [2]-[6]. During the past decades, e-p interaction has been studied experimentally mostly for the case of three dimensional (3D) phonons [1, 7, 8], and no clear observation of the effect of phonon dimensionality [9] was seen until just recently [10]. In this paper, we discuss in more detail how the thickness of the metal film affects the e-p interaction on membrane and bulk substrates.

2. Results and discussion
In thin, suspended membranes, the 3D transversal and longitudinal modes couple to each other, and form a new set of eigenmodes known as the horizontal shear modes ($h$), and symmetric ($s$) and antisymmetric ($a$) Lamb modes [11]. This lowest $a$ mode has quadratic dispersion at low frequencies, changing the e-p interaction in a non-trivial way [3, 5]. In addition, a single free surface can also affect the e-p coupling notably [6]. Hence, even for thin films on bulk substrates, the widely observed result for the heat flow between electrons in a volume $V$ and at a temperature $T_e$ and phonons at $T_p$, $P_{e-p} = \Sigma V(T_e^3 - T_p^3)$ should not strictly hold according to theory.
For the case of a thin (<100 nm) Cu film with a 3D phonon spectrum on a bulk substrate, the effect of boundary resistance is small at sub-Kelvin temperatures [1]. However, for thin metal films on 2D membranes, or even for a thin 2D film and bulk substrate, hardly any theoretical or experimental studies exist. Intuitively it seems reasonable that the phonon modes in the combined system of a metal film and a membrane are strongly coupled and $R_K$ is small, if $\lambda$ is longer than the overall thickness of the system. This means that for thin membranes at low temperatures e-p interaction likely dominates the thermal relaxation.

A schematic of a Cu wire sample on a suspended membrane and the measuring circuit used is shown in figure 1. A more detailed description of the sample fabrication is discussed in Ref. [10]. Table 1 presents the essential dimensions of the samples in this paper. We used the hot-electron technique [7] to measure the e-p interaction by overheating the electrons by Joule heat power $P$ and measuring the resulting electron temperature $T_e$. All the samples had two electrically isolated Cu normal metal wires next to each other. The longer wire ($L = 500 \mu m$) was heated by applying a slowly ramping voltage across the pair of superconducting Nb (or Al) leads in direct metallic contact to Cu, forming SN junctions. These junctions provide excellent electrical, but very poor thermal conductance due to Andreev reflection, as the junctions are biased within the superconducting gap $\Delta$. Thus, input heat is distributed uniformly in the interior of the wire and the electron gas cools dominantly by phonons. In our sample geometry $T_e$ is measured with two additional Al leads forming a NIS tunnel junction pair (SINIS) in the middle of the heated wire, as a function of input Joule power $P = IV$ measured in a four probe configuration. The purpose of the short Cu wire, with additional SINIS thermometer on it, is to give an estimate of the local $T_p$, as the e-p power flow depends on both $T_e$ and $T_p$.

Figure 2 shows the measured calibration data (black circles) for the SINIS thermometer biased with two different values of bias current, and the corresponding curves from BCS theory. In the calibration, the bath temperature of the refrigerator $T_{bath}$ is decreased slowly and the resulting voltage response of the SINIS is measured. This data can then be compared with the

![Figure 1.](image1.png)  
![Figure 2.](image2.png)
Table 1. Parameters for samples. M=suspended SiN membrane of thickness 30nm and B=bulk Si substrate with 30 nm SiN top layer. \( t \) is the thickness of the Cu wire, \( V \) volume and \( l \) electron mean free path.

|     | M1 | B1 | M2 | B2 | M3 | B3 | M4 | B4 |
|-----|----|----|----|----|----|----|----|----|
| Cu  | 14 | 14 | 19 | 19 | 32 | 32 | 90 | 90 |
| \( t \) (nm) | 14 | 14 | 19 | 19 | 32 | 32 | 90 | 90 |
| \( V \) \((\mu m)^3\) | 2.71 | 2.46 | 5.50 | 4.62 | 6.09 | 5.09 | 18.6 | 19.1 |
| \( l \) (nm) | 5.7 | 4.9 | 11.2 | 9.8 | 22 | 19 | 46 | 46 |

BCS theory without any fitting parameters, as the energy gap \( \Delta \) and the tunneling resistance are determined independently from I-V curves. If one calculates a BCS-curve, in which the energy gap \( \Delta \) is opening as the temperature decreases (light gray line), the agreement with the data is excellent except at the very lowest temperatures. The low temperature saturation of the voltage is believed to be caused by external noise heating. However, in the e-p coupling measurement where the electron gas is heated directly, the superconducting leads stay cold at \( T_{\text{bath}} = 60 \) mK so that \( \Delta \) stays fully open. For this reason, the SINIS voltage must be converted to temperature by using a BCS theory curve, in which \( \Delta(0K) \) is maintained (red, dashed line). The calibration data measured with a smaller bias current has a larger slope below 0.8 K, but has less response in the high temperature range, where measurements are performed with the higher \( I_{\text{bias}} \). This way, by using two different bias currents, it is possible to measure the temperature from 50 mK to 1K and even above.

In figure 3 we plot the measured \( T_e \) and \( T_p \) versus heating power density \( P/V \) for membrane M1-M4 [ figure 3(a)] and bulk samples B1-B4 [ figure 3(c)], where the Cu wire thickness ranges from 14 nm to 90 nm. By comparing the graphs we see that both \( T_e \) and \( T_p \) of all membrane samples clearly behave differently from the corresponding bulk samples. At \( P/V < 6 \) pW/(\( \mu m^3 \)), \( T_e \) is lower in membrane samples, which means that electron energy relaxation is stronger. At higher temperatures, the thermal relaxation on membranes weakens significantly, and clearly depends on the Cu thickness. This is, most likely, a signature of the the Kapitza boundary resistance becoming dominant. A reasonable explanation for the difference between bulk and membrane samples is that phonons are 2D in the thin membrane, as the estimated dimensionality cross-over temperature [12] is \( T_{cv} = \hbar c_t/(2k_Bd) \sim 0.5 \) K for the thinnest Cu wire (\( c_t=6200 \) m/s for SiN, \( c_t=2325 \) m/s for Cu and \( d \) is the total thickness). Even sample M4 with the thickest Cu film with \( t = 90 \) nm behaves as 2D. This is still consistent with data from 14nm thick Cu film on 200nm thick SiNx membrane [10], where 3D behavior was observed. For bulk samples [figure 3 (b)] the strength of the e-p scattering seems to depend on the Cu film thickness at lower \( T \). The behavior of sample B4 seems a bit distinct at high powers compared to samples B1-B3.

We can study the temperature dependence in more detail by plotting the logarithmic derivatives \( d(\log P/V)/d(\log T_e) \) vs. \( P/V \). This is useful, since for all the data \( T_e >> T_p \), so that \( P_{e-p} \approx T_e^n \) and \( d(\log P/V)/d(\log T_e) = n \). Figure 3 (b) shows \( n \) for membrane samples and (d) for bulk samples. Membrane samples M1-M2 have a plateau corresponding to \( n \approx 4.5 \) between \( P/V = 0.1-6 \) pW/(\( \mu m^3 \)), while \( n \) for samples M3-M4 decreases continuously indicating the emerging importance of boundary resistance. In contrast, \( n \) for the thinnest bulk samples B1-B3 decreases from much higher values and, surprisingly, sample B4 also plateaus at \( n \approx 4.5 \).

Let us discuss the effect of Cu thickness on e-p interaction on bulk substrate further [Figures 3 (c) and (d)]. At the lowest temperatures, electron energy relaxation is weakened when Cu is thinner. Note that samples B1-B3 do not give the typical \( n = 5 \) results, although the Cu
Figure 3. (color online) (a) $T_e$ and $T_p$ versus $P/V$ for membrane samples. Only one $T_p$ curve is shown for clarity (all samples giving similar results) (b) Numerical logarithmic derivatives of the data in (a) for samples M1-M4. Symbols are given in (a). (c) $T_e$ and $T_p$ versus $P/V$ for bulk samples. Only one $T_p$ curve is shown for clarity. (d) Numerical logarithmic derivatives of the data in (c) for samples B1-B4. From top to bottom, Green line: B1, Magenta line: B2, Blue line: B3 and Black line: B4.

thickness of sample B3 is almost the same as in Refs. [1] (CuAu $t=36$ nm) and [8] (Cu $t=32$ nm), where samples are made on oxidized Si. This can be explained possibly by a difference in the surface phonon modes [6] or in the disorder of the Cu film [4]. Exponents higher than 6 can be explained by the combined effect of strong disorder and surface modes. The clear plateau at $n \approx 4.5$ for sample B4 may be caused by the fact that Kapitza resistance becomes dominant, as it will have a stronger effect for thicker films, or that the sample is in the clean limit ($q\ell > 1$, where $q$ is the thermal phonon wavevector) and both transverse and longitudinal phonons interact with electrons [4].

3. Conclusion
In conclusion, our measurements show clear evidence that the behaviour of e-p coupling drastically changes in wires on thin (30 nm) membranes, even if the metal film is as thick as 90 nm. At the lowest temperatures $T_e < 0.5$ K, e-p coupling is strengthened by a factor 2-3, and at higher temperatures the thermal relaxation is weakened compared to bulk samples. Thinning the metal film on bulk substrates also changes the e-p interaction.
4. Acknowledgments
Discussions with T. Kühn and A. Sergeev are acknowledged. This work was supported by the Academy of Finland project Nos. 118665 and 118231 and Vilho, Yrjö ja Kalle Väisälä rahasto.

References
[1] Wellstood F C, Urbina C and Clarke J 1994 Phys. Rev. B 49 5942
[2] Belitz D and Das Sarma S 1987 Phys. Rev. B 36 7701
[3] Johnson K, Wybourne M N and Perrin N 1994 Phys. Rev. B 50 2035
[4] Sergeev A V and Mitin V 2000 Phys. Rev. B 61 6041
[5] Glavin B A, Pipa V I, Mitin V V and Stroscio M A 2002 Phys. Rev. B 65 205315
[6] Qu S X, Cleland A N and Geller M R 2005 Phys. Rev. B 72 224301
[7] Roukes M L, Freeman M R, Germain R S, Richardson R C and Ketchen M B 1985 Phys. Rev. Lett. 55 422
[8] Karvonen J T, Taskinen I J and Maasilta I J 2007 J. Low Temp. Phys. 146 213
[9] DiTusa J F, Lin K, Park M, Issacson M S and Parpia J M 1992 Phys. Rev. Lett. 68 1156
[10] Karvonen J T and Maasilta I J 2007 Phys. Rev. Lett. 99 145503
[11] Auld B A 1990 Acoustic Fields and Waves in Solids vol II 2nd ed (Malabar: Robert E. Krieger)
[12] Kühn T, Anghel D V, Pekola J P, Manninen M and Galperin Y M 2004 Phys. Rev. B 70 125425