Double Scattering Effect in Transverse Momentum Distribution of Inclusive $J/\psi$ Production in Photo-Nucleus Collision

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Abstract

In terms of multiple scattering picture, we calculate the double scattering effect in the transverse momentum distribution of $J/\psi$ photoproduction. Applying the generalized factorization theorem, we find that the contributions from double scattering can be expressed in terms of twist-4 nuclear parton correlation functions, which is the same as that used to explain the nuclear dependence in di-jet momentum imbalance and in direct photon production. Using the known information on the twist-4 parton correlation functions, we estimate that the double scattering contributes a small suppression in $J/\psi$ photoproduction. In the analysis we only take into account the leading order in the small velocity expansion for the nonperturbative parts related to the quarkonium.

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1 Introduction

Heavy quarkonium production in high-energy collisions affords us a variety of insights into the underlying dynamics of the strong interaction. Especially in nucleus-nucleus collisions, we may find that a suppression of $J/\psi$ production can serve as a clear signature of the existence of the deconfining phase of QCD \[1\]. This suppression effect was observed by NA38 collaboration later \[2\]. However, successive observation \[3\] pointed out that in proton-nucleus collisions, in which there is no QGP formed, such suppression could also exist. There have been a number of attempts to explain it, such as pre-resonance absorption, gluon shadowing, hadronic co-mover absorption and the energy loss model \[4, 5, 6, 7, 8, 9\], etc. To understand the $J/\psi$ suppression mechanism clearly, its production by scattering in different nuclear facilities must be studied carefully.

In principle, the $J/\psi$ state can be described in a Fock state decomposition

$$|J/\psi\rangle = O(1) |c \bar{c}(3S_1^{(1)})\rangle + O(\nu) |c \bar{c}(3P_J^{(8)})\rangle g + \cdots,$$

where $2S+1L^{(1,8)}$ characterizes the quantum state of the $c \bar{c}$ pair with color-singlet (C1) or color-octet (C8) respectively. This expression is valid for the non-relativistic QCD(NRQCD) framework and the coefficients of each component depend on $\nu$, the small relative velocity between quark and antiquark. The charmonium production can be divided into two steps. The first step is the production of $c \bar{c}$ pair. The $c \bar{c}$ pair can be either $(c \bar{c})_1$ or $(c \bar{c})_8$, and are produced perturbatively and almost instantaneously, with a formation time $\tau_f \approx (2m_c)^{-1} = 0.07 fm$ in the $c \bar{c}$ rest frame. The second step is the formation of a physical state of $J/\psi$ from $c \bar{c}$ pair, that need longer time. The second step is a nonperturbative process, and in different environments such as vacuum, hadron matter and QGP, the original produced $c \bar{c}$ pairs undergo different transition rules to final observable $J/\psi$. All these processes have been studied carefully before. However, besides this nonperturbative effect related to the transitions there is another nonperturbative effect if there are hadrons or nucleus in the initial states. This nonperturbative effect can be analyzed with a twist expansion method \[10, 11\]. Ref. \[12\] gives the effect of the order of next-to-leading twist in inclusive photoproduction of quarkonium. In this paper we will use factorization at higher twist to describe multiple scattering effect for $J/\psi$ production in photo-nucleus collision, and our result can be extended to study the corresponding cases of hardon-nucleus and nucleus-nucleus collisions.

In terms of factorization at higher twist, Luo, Qiu and Sterman (LQS) \[13, 14\] have developed a consistent treatment of multiple scattering at partonic level. They showed that nuclear effect may be brought directly into the scattering formalism of QCD, by treating it as a factorizable nonleading-order correction to hard scattering, and nuclear enhancement appears in this context as a property of multi-parton matrix
elements. In [13] they derive the anomalous nuclear dependence of jet cross sections in elastic scattering and photoproduction in terms of twist-four parton distributions in nuclei. In [15] they expressed the nuclear dependence of di-jet moment imbalance in photo-nucleus collisions and estimated the size of the relevant twist-4 parton distributions by using the Fermi Lab E683 data. Recently Guo [16] applied this method to Drell-Yan process, and calculated its transverse distribution in hadron-nucleus collisions.

All above processes are totally inclusive, and the extension to our case is not quite straightforward. The process we consider is seminclusive, where a $c\bar{c}$ pair is observed indirectly. We neglect the higher order $v^2$ contributions and consider that the formation of $J/\psi$ is totally evolved from the leading order $c\bar{c}(3S_1^{(1)})$ pair. This formation starts with a C1 or C8 $c\bar{c}$ pair produced from initial hard collision between photon and a parton from nucleus, and then on its way out of the nucleus, it undergoes a soft scattering with another parton from the nucleus to form a color singlet $c\bar{c}(3S_1^{(1)})$ pair. The soft scattering for $c\bar{c}$ pair can be done by either its quark or its antiquark picking up the soft gluon from nucleus. Our paper is divided into four parts. In section II, we derived the general formalism and the result of single scattering. In section III, the double scattering contributions are derived. In the last section, we present the numerical results and give a brief discussion.

2 General formalism and the single scattering result

To study the nuclear effect in transverse momentum distribution, we consider the differential cross section for the inclusive process:

$$\gamma(p') + A(p) \rightarrow J/\psi(Q) + X. \quad (2)$$

The momenta of particles are given in brackets, $p'$ is the momentum of incoming photon, $p$ is the momentum per nucleon for nuclear target, and $Q$ is the momentum of outgoing $J/\psi$. In terms of contributions from multiple scattering, we expand the differential cross section as

$$\frac{d\sigma_{\gamma A}}{dQ_\perp^2 dy} = \frac{d\sigma_{\gamma A}^S}{dQ_\perp^2 dy} + \frac{d\sigma_{\gamma A}^D}{dQ_\perp^2 dy} + \cdots \quad (3)$$

The superscript “S” and “D” denote contributions from the “single scattering” and “double scattering”, respectively, and “…” represents contributions from even higher multiple scattering. We choose a frame in which the nucleus $A$ moves in the $z$-direction and the photon in the opposite direction, $Q_\perp$ is the transverse momentum of the $J/\psi$, and $y$ is its rapidity

$$y = \frac{1}{2} \ln \frac{Q_0 + Q_z}{Q_0 - Q_z}. \quad (4)$$
In our approximation the quarkonium mass $m_\psi$ is twice of quark mass $m_c$. We define quantities $s = (p+p')^2$, $t = (p - Q)^2$ and $u = (p' - Q)^2$. For performing the analysis it is convenient to use light-cone coordinate system. In this system the photon carries the momentum $p'^\mu = (0,p'^-,0_\perp)$, and $p^\mu = (p^+,0,0_\perp)$. We introduce in this frame two vectors $n$ and $\bar{n}$ and a tensor $d^\mu\nu_\perp$:

\[
\begin{align*}
n^\mu &= (0,1,0_\perp), & \bar{n}^\mu &= (1,0,0_\perp), \\
d^\mu\nu_\perp &= g^\mu\nu - n^\mu\bar{n}\nu - n^\nu\bar{n}\mu.
\end{align*}
\]

(5)

We will work in the light-cone gauge $n \cdot A(x) = A^+(x) = 0$, where $A^\mu(x) = A^\mu(x) T^a$ is the gluon field.

In order to compare with experimental data, we calculate the ratio of total differential cross section and single scattering contribution,

\[
\rho = \frac{d\sigma_{\gamma A}}{dQ^2_- dy} \left/ \frac{d\sigma_{S A}}{dQ^2_+ dy} \right. \approx 1 + \frac{d\sigma_{D A}^{T}}{dQ^2_- dy} \left/ \frac{d\sigma_{S A}^{S}}{dQ^2_+ dy} \right.
\]

(6)

To analyze the effect of single scattering we need to consider the diagram in Fig.1, in which the up box

Figure 1: A Graphical representation for single scattering process in photon-nucleus collision.
represents the nonperturbative part related to the nucleus A, the down box contains the perturbative part for production of the $c\bar{c}$ pair at the leading order of coupling constant and the transition of the pair into $J/\psi$, they are illustrated in the Feynman diagram given in Fig.2. The contribution from Fig.1 can be written as:

$$2s\sigma^S_{\gamma A} = \int \frac{d^4k}{(2\pi)^4} \hat{S}^{a_1a_2}_{\mu_1\mu_2}(k) \int d^4ze^{ikz} < p_A|A^{a_1\mu_1}(z)A^{a_2\mu_2}(0)|p_A > ,$$

(7)

where $\hat{S}^{a_1a_2}_{\mu_1\mu_2}(k)$ corresponds to the lower part in Fig.1, its leading order part is shown in Fig.2, where the left and right box include all possible tree Feynman diagrams with the external partons shown in the figures.

In the center of mass frame of high energy collision, all partons inside the nucleus are parallel to each other, along the momentum of the nucleus. In order to get the leading order contribution, we make the collinear expansion,

$$\hat{S}^{a_1a_2}_{\mu_1\mu_2}(k) = \hat{S}^{a_1a_2}_{\mu_1\mu_2}(xp) + \frac{1}{2} \frac{\partial^2 S^{a_1a_2}_{\mu_1\mu_2}(k)}{\partial k^\alpha \partial k^\beta} \bigg|_{k=xp} \Delta k^\alpha \Delta k^\beta + \cdots ,$$

(8)

Where $\Delta k = k - xp$. We only keep the first term $\hat{S}^{a_1a_2}_{\mu_1\mu_2}(xp)$, which is independent of $k^-$ and $k_\perp$, so we can perform the integration over $k^-$ and $k_\perp$ directly,

$$2s\sigma^S_{\gamma A} = \int \frac{dx}{x} \hat{S}^{a_1a_2}_{\mu_1\mu_2}(xp) \frac{xp^+}{2\pi} \int dz^- e^{ixp^+z^-} < p_A|A^{a_1\mu_1}(z)A^{a_2\mu_2}(0)|p_A >$$

$$= \int \frac{dx}{x} f_{g/A}(x) \left(-\frac{1}{16} d^{\mu_1\mu_2}_{\perp} \delta^{a_1a_2} S^{a_1a_2}_{\mu_1\mu_2}(xp)\right),$$

(9)

where we have used the matrix element relation[12]

$$< p_A|A^{a_1\mu_1}(z)A^{a_2\mu_2}(0)|p_A > = \frac{1}{16} d^{\mu_1\mu_2}_{\perp} \delta^{a_1a_2} < p_A|A^{a_1}(z)A^{a_2}_{\perp}(0)|p_A > .$$

(10)

The function $f_{g/A}(x)$ is defined as

$$f_{g/A}(x) = -\frac{xp^+}{2\pi} \int dz^- e^{ixp^+z^-} < p_A|A^{a_1}(z)A^{a_2}_{\perp}(0)|p_A > .$$

(11)

We consider the formation of $J/\psi$ from a $c\bar{c}$ pair at leading order in $v^2$. According to the NRQCD factorization formalism[17], the amplitude of $J/\psi$ production from partons can be factorized into
\[ A(gγ \rightarrow ψc) = A(gγ \rightarrow c\bar{c}(3S^1_1)) \bigg|_{\text{pert.}} \otimes A(c\bar{c}(3S^1_1) \rightarrow ψc) \bigg|_{\text{nopert.}}. \] (12)

Here we only keep the leading order contribution comes from color singlet \( c\bar{c}(3S^1_1) \) pair. Contribution from color octet and other higher angular momentum \( c\bar{c} \) pair states are suppressed at least \( v^2 \), we neglect them here. \( c\bar{c}(3S^1_1) \) pair production starts at \( O(\alpha_s^2) \) via process \( γg \rightarrow ψg \), whose Feynman diagram is shown in Fig.3. Now \( \hat{S}^{a_1a_2}_{μ_1μ_2}(xp) \) can be written as

\[ \hat{S}^{a_1a_2}_{μ_1μ_2}(xp) = \frac{1}{2} \Gamma^{(2)} \times L^{a_1}_{μ_1νλ}(k) \otimes R^{a_2}_{λνμ_2}(k) \times M(ψc), \] (13)

where \( \frac{1}{2} \) represents the average over the initial photon spin,

\[ \Gamma^{(2)} = \int \frac{dQ^2 \, dy}{8\pi} \frac{1}{s + t - m^2_φ} \delta(x + \frac{u}{s + t - m^2_φ}) \] (14)

represents the final two body phase space integration for \( J/ψ \) and a gluon. Nonperturbative part \( M(ψc) \) in Eq.(13) cannot be computed from first principles without resorting to lattice QCD, we neglect the low energy dependence of interactions, the nonperturbative amplitude only affect the overall normalization constant,

\[ M(ψc) = \frac{1}{6m_c} \langle 0|O^{(1)}_c(3S_1)|0 \rangle = \text{constant}. \] (15)

In (13) \( L^{a_1}_{μ_1νλ} \) and \( R^{a_2}_{λνμ_2} \) represent the perturbative production amplitude the of \( c\bar{c}(3S^1_1) \) pair shown in

Figure 2: The diagram corresponds to the lower part in Fig.1 for \( J/ψ \) production at lowest order partonic level.
Figure 3: Diagrams represent the $c\bar{c}$ pair production amplitude including all partonic hard subprocess at leading order in perturbative coupling constant.
the left and right parts of Fig.2 respectively. We have

\[
L_{\mu_1 \nu_\lambda}^{a_1}(k) = \frac{1}{\sqrt{3}} \sum_{s_1, s_2} \bar{u}(q_1; s_1) \tilde{L}_{\mu_1 \nu_\lambda}^{a_1}(k, q_1, q_2) v(q_2, s_2) < \frac{1}{2} s_1; \frac{1}{2} s_2 | 1S_z >
\]

\[
= \frac{1}{\sqrt{3}} Tr \left[ L_{\mu_1 \nu_\lambda}^{a_1}(k, q_1, q_2) P_{1S_z}(q_1, q_2) \right],
\]

(16)

where \( \tilde{L}_{\mu_1 \nu_\lambda}^{a_1}(k, q_1, q_2) \) represents the left box of Fig.2 with external quark momentum \( q_1 \) and antiquark momentum \( q_2 \), and its all tree level diagrams are illustrated in the diagrams of Fig.3. The heavy quark and antiquark propagate nearly on -shell with a large combined \( Q = q_1 + q_2 \) and a small relative momentum \( \bar{q} = q_1 - q_2 \). The spin quantum numbers for the \( c\bar{c} \) pair are projected out by the sums over \( SU(2) \) Clebsch-Gordan coefficients \(< \frac{1}{2} s_1; \frac{1}{2} s_2 | 1S_z >\). The \(4 \times 4\) matrix is

\[
P_{1S_z} = \sum_{s_1, s_2} v(q_2; s_2) \bar{u}(q_1; s_1) < \frac{1}{2} s_1; \frac{1}{2} s_2 | SS_z >
\]

\[
= - \frac{1}{2\sqrt{2m_c}} (q_1 - m_c) \psi(Q, S_z)(q_2 + m_c)
\]

(17)

up to \( O(q^2) \) corrections. The project operators’ dependence on \( \bar{q} \) is irrelevant for \( S \)-wave \( c\bar{c} \) production. This indicates \( q_1 = q_2 = q \), and \( Q = 2q \), so

\[
P_{1S_z} = - \frac{1}{2\sqrt{2m_c}} (q - m_c) \psi(Q, S_z)(q + m_c).
\]

(18)

The final perturbative result can be derived as

\[
\frac{d\sigma_S}{dQ^2 dy} = \frac{32\pi \alpha_s e_Q^2 |R(0)|^2}{3s(s + t - m_\psi^2)} f_{g/A}(x) \left\{ \frac{\hat{s}^2(\hat{s} - m_\psi^2)^2 + \hat{t}^2(\hat{t} - m_\psi^2)^2 + (\hat{s} + \hat{t} - m_\psi^2)^2}{(\hat{s} - m_\psi^2)^2(\hat{t} - m_\psi^2)^2(\hat{s} + \hat{t})^2} \right\}
\]

(19)

where

\[
\hat{s} = (xp + p')^2, \quad \hat{t} = (xp - Q)^2,
\]

(20)

and \( x \) is fixed according to the momentum conservation,

\[
x = - \frac{u}{s + t - m_\psi^2}.
\]

(21)
Figure 4: Graphical representation of double scattering contribution in photon-nucleus collisions; a) real diagram, b) and c) interference diagram.
3 The double scattering result

Now we begin to consider the double scattering effect, at lowest order, the cross section corresponding to soft rescattering are shown in Fig.4, its contribution can be written as

\[
2sd\sigma_D^{\gamma A} = \int \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^4} \hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3) \int d^4z_1 d^4z_2 d^4z_3 e^{ik_1 \cdot z_1 + ik_2 \cdot z_2 + ik_3 \cdot z_3} < p_A | A^{\alpha_1 \mu_1}(z_1) A^{\alpha_2 \mu_2}(z_2) A^{\alpha_3 \mu_3}(z_3) A^{\alpha_4 \mu_4}(0) | p_A > .
\]  

(22)

The momenta, color- and Lorentz- indices are marked in Fig.4. In Eq.\[(22)\] $\hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3)$ corresponds to hard production of $c\bar{c}$ pair and its transition to $J/\psi$. To pick up the leading contributions to the nuclear enhancement, we expand the parton momenta at the following according their collinear values:

\[
\begin{align*}
  k_1 &= x_1 p^+ \bar{n}, \\
  k_2 &= x_2 p^+ \bar{n} + k_{2\perp}, \\
  k_3 &= x_3 p^+ \bar{n} + k_{3\perp}.
\end{align*}
\]  

(23)

The minus components of the $k_i$ give even smaller contributions in $\hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3)$ and will be neglected. In addition, $k_{1\perp}$ dependence in $\hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3)$ will not associated with $A$ enhancement and we drop it as well. Later, we will show that $x_2$ and $x_3$ can be fixed by poles as functions of $k_{2\perp}$ (or $k_{3\perp}$) and $Q_\perp$, and that they vanish when $k_{2\perp}$ and $k_{3\perp}$ go to zero.

Once we have dropped the $k_{1\perp}$ and $k_{i\perp}$ (i=1,2,3) dependence in $\hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}$, their integrals give $\delta$ functions and allow us rewrite Eq.\[(22)\] as

\[
2sd\sigma_D^{\gamma A} = \int \frac{p^+ dx_1}{2\pi} \int \frac{p^+ dx_2}{2\pi} \frac{d^2k_{2\perp}}{(2\pi)^2} \int \frac{p^+ dx_3}{2\pi} \frac{d^2k_{3\perp}}{(2\pi)^2} \int dz_1 dz_2 dz_3 \left( e^{ix_1 \cdot \bar{z}_1 + ix_2 \cdot \bar{z}_2 + ix_3 \cdot ar{z}_3} \right) \left( 2\pi \right)^2 \\
\hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3) < p_A | A^{\alpha_1 \mu_1}(z_1) A^{\alpha_2 \mu_2}(z_2) A^{\alpha_3 \mu_3}(z_3) A^{\alpha_4 \mu_4}(0) | p_A > .
\]  

(24)

First of all, we expand the hard part $\hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3)$ in momenta about the collinear direction:

\[
\begin{align*}
  \hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3) &= \hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(x_1 p, x_2 p, x_3 p) \\
  &+ \frac{\partial \hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(x_1 p, x_2 p, x_3 p)}{\partial k_{2\perp}^\rho} \bigg|_{k_{2\perp}=0} k_{2\perp}^\rho + \frac{\partial \hat{C}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(x_1 p, x_2 p, x_3 p)}{\partial k_{3\perp}^\sigma} \bigg|_{k_{3\perp}=0} k_{3\perp}^\sigma + \cdots .
\end{align*}
\]  

(25)
In this discussion we will confine ourselves to experiments with unpolarized beams. Thus odd-twist contributions of the second and third terms in Eq. (25) vanish on taking a spin average. In addition, the leading term $C_{\mu_1 \mu_2 \rho \mu_4}^{a_1 a_2 a_3 a_4}(x_1 p, x_2 p, x_3 p)$ does not contribute to $A$ dependence because it is independent of transverse momenta. We did not list explicitly terms such as $\partial^2 \hat{C}/\partial k_{2\rho}^2$ and $\partial^2 \hat{C}/\partial k_{2\sigma}^3$ because, for reasons that will become clear below, they do not contribute physical double scattering. Therefore, double scattering effect comes entirely from the last single term of Eq. (27)

$$2sd\sigma^D_{\gamma A} = \frac{1}{2} \int \frac{(p^+)^3 dx_1 dx_2 dx}{(2\pi)^3} \int dz_1^- dz_2^- dz_3^- \left( \frac{\partial^2 \hat{C}_{\mu_1 \mu_2 \rho \mu_4}^{a_1 a_2 a_3 a_4}(x_1 p, k_2, k_3)}{\partial k_{2\rho}^2 \partial k_{2\sigma}^3} \right) \bigg|_{k_{2\perp}=k_{3\perp}=0\perp}
$$

$$e^{ix_1 p^+ z_1^- + ix_2 p^+ z_2^- + ix_3 p^+ z_3^-} \int \frac{d^2 k_{2\perp} d^2 k_{3\perp} d^2 p}{(2\pi)^2 (2\pi)^2 (2\pi)^2} e^{ik_{2\perp} \cdot z_1 + ik_{3\perp} \cdot z_3} < p_A \left| A^{a_1}_{\mu_1}(z_1) A^{a_2}_{\mu_2}(z_2) A^{a_3}_{\mu_3}(z_3) A^{a_4}(0) \right| p_A >$$

$$= -\frac{1}{2} \int \frac{(p^+)^3 dx_1 dx_2 dx_3}{(2\pi)^3} \int dz_1^- dz_2^- dz_3^- \left( \frac{\partial^2 \hat{C}_{\mu_1 \mu_2 \rho \mu_4}^{a_1 a_2 a_3 a_4}(x_1 p, k_2, k_3)}{\partial k_{2\perp}^2 \partial k_{3\perp}^3} \right) \bigg|_{k_{2\perp}=k_{3\perp}=0\perp}
$$

$$e^{ix_1 p^+ z_1^- + ix_2 p^+ z_2^- + ix_3 p^+ z_3^-} \left( < p_A \left| A^{a_1}_{\mu_1}(z_1) \frac{\partial A^{a_2}_{\mu_2}(z_2)}{\partial z_2^{\rho}} \frac{\partial A^{a_3}_{\mu_3}(z_3)}{\partial z_3^{\sigma}} A^{a_4}(0) \right| p_A > \right).$$

Because the values of $x_1, x_2$ and $x_3$ will be fixed by $\delta$ functions and poles of the hard part $\hat{C}_{\mu_1 \mu_2 \rho \mu_4}^{a_1 a_2 a_3 a_4}$, we move the exponentials $e^{ix_1 p^+ z_1^- + ix_2 p^+ z_2^- + ix_3 p^+ z_3^-}$ and integrals inside the derivative of $\partial^2/\partial k_{2\rho}^2 \partial k_{3\sigma}^3$.

$$2sd\sigma^D_{\gamma A} = -\frac{1}{2} \int \frac{(p^+)^3 dz_1^- dz_2^- dz_3^-}{(2\pi)^3} \left( < p_A \left| A^{a_1}_{\mu_1}(z_1) \frac{\partial A^{a_2}_{\mu_2}(z_2)}{\partial z_2^{\rho}} \frac{\partial A^{a_3}_{\mu_3}(z_3)}{\partial z_3^{\sigma}} A^{a_4}(0) \right| p_A > \right)$$

$$\left( \frac{\partial^2}{\partial k_{2\perp}^2 \partial k_{3\perp}^3} \right) \int dx_1 dx_2 dx_3 e^{ix_1 p^+ z_1^- + ix_2 p^+ z_2^- + ix_3 p^+ z_3^-} \hat{C}_{\mu_1 \mu_2 \rho \mu_4}^{a_1 a_2 a_3 a_4}(x_1 p, k_2, k_3) \bigg|_{k_{2\perp}=k_{3\perp}=0\perp}$$

According to the discussion in \[12, 13\], the dominant contribution of the four-gluon matrix element can be written as

$$< p_A \left| A^{a_1}_{\mu_1}(z_1) G^{a_2 \rho}(z_2) G^{a_3 \sigma}(z_3) A^{a_4}(0) \right| p_A >$$

$$= \frac{1}{4 \times 8 \times 8} \int \frac{d^2 k_{2\perp} d^2 k_{3\perp} d^2 p}{(2\pi)^2 (2\pi)^2 (2\pi)^2} < p_A \left| A^{a_1 \perp}(z_1) G^{a_2 \perp}(z_2) G^{a_3 \perp}(z_3) A^{a_4}(0) \right| p_A >$$

Therefore, the cross section for double scattering turns out to be

$$ds\sigma^D_{\gamma A} = \frac{1}{2s} \left( \frac{1}{8} \right) \int \frac{dz_1^- dz_2^- dz_3^-}{(2\pi)^3} < p_A \left| A^{a_1 \perp}(z_1) G^{a_2 \perp}(z_2) G^{a_3 \perp}(z_3) A^{a_4}(0) \right| p_A > F(z_1^{-}, z_2^{-}, z_3^{-})$$

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where

\[
F(z_1, z_2, z_3) = \frac{\partial^2}{\partial k_{2\perp} \partial k_{3\perp}} \left( \int dx_1 dx_2 dx_3 e^{ix_1 p^+ z_1 - x_2 p^+ z_2 + ix_3 p^+ z_3} \times \hat{C}_{a_1 a_2 a_3 a_4}(p_1, k_2, k_3) \delta_{a_2 a_3} \delta_{a_1 a_4} \right) \bigg|_{k_{2\perp} = k_{3\perp} = 0}. \tag{31}
\]

Equation (31) has been derived at lowest order in the hard scattering. As it stands, it is not gauge invariant. Gauge invariant is incorporated at higher orders and leading power by taking into account diagrams involving more fields \( n \cdot A \), which will form ordered exponentials between the physical fields shown in (30), as discussed, for instance, in [11, 13]. Such fields, however, can not correspond to physical rescattering, since they can be eliminated by a change of gauge. This is the reason why we neglected two derivatives with respect to \( k_2 \) and none with respect to \( k_3 \), or vice versa. Such a term would involve only three physical fields instead of four. As we shall see below, four physical fields are required to produce nuclear effect.

From Eq. (30) and (31) all integrals of \( x_1, x_2, x_3 \) can now be done explicitly without knowing the detail of the multi-parton matrix elements. Now we are ready to discuss the calculation of the hard part \( \hat{C}_{a_1 a_2 a_3 a_4}(p_1, k_2, k_3) \). As is shown in the diagram of Fig.4, during the collision, firstly the photon strongly interacts with a gluon from one nucleon in nucleus. After a hard scattering, the outgoing \( c\bar{c} \) pair interacts with a soft gluon from another nucleon in the nucleus to form a color-singlet \( c\bar{c}(3S_1) \) pair. So there are two partons from nucleus participate in the scattering process. The kinematics can only fix one parton momentum, thus we need to integrate over the other parton’s momentum. In \( \hat{C}_{a_1 a_2 a_3 a_4}(p_1, k_2, k_3) \) we will encounter one phase space \( \delta \) function and two virtual propagator poles involving \( x_i's \), which correspond to the zero momenta fraction partons. The leading contribution of the integration over the extra parton momentum is given by the residues at these poles, which will fix \( x_2 \) and \( x_3 \) to be functions of the transverse momentum \( k_{i\perp} \). Then in the limit \( k_{i\perp} \to 0 \) the \( x_2 \) and \( x_3 \) will vanish. Therefore \( e^{ix_2 p^+ z_2} \) and \( e^{ix_3 p^+ z_3} \) will reduce to unity eventually. We may expect a significant contribution from the free integrals \( \int dz_2^\perp dz_3^\perp \). This will be the origin of the \( A \) dependence.

Consider subprocess shown in Fig.4, there are three four-momentum linking the partonic part and corresponding four gluon matrix element. In inclusive DIS [13] and Drell-Yan [16] processes, the final results depend only on the real diagram with one extra gluon in each side of the cut, and the role of interference diagrams with both gluons in one side of the cut is to take care of the infrared sensitivities of the short distance hard parts. The process we are considering is seminclusive and the subsequent \( c\bar{c} \) pair is in color-singlet state. This demands that for the real scattering subprocess shown in Fig.4(a), the original \( c\bar{c} \) pair produced from hard scattering of photon and one parton should be in color-octet state, while for the virtual
subprocesses shown in Fig.4(b) and Fig.4(c), the corresponding c\bar{c} pair should be in color-singlet state. Thus we must consider the two different types of physical rescattering separately, and as we shall see below, each of their contributions remains infrared finite in itself. This is quite different from the inclusive situations, in which divergences are canceled only after summing over all sorts of cut in a Feynman diagram.

First of all, we consider contribution of real double scattering diagram illustrated in Fig.4(a). By the similar approach which we have taken in deriving Eq.(13), we can write its hard part $\hat{C}_{a_1a_2a_3a_4}(1)$ as

$$\hat{C}_{a_1a_2a_3a_4}(1)(x_1p,k_2,k_3)p^{\mu_1}p^{\mu_2} = \frac{1}{2} \Gamma^{(2)} \times L_{a_1a_2}(x_1p,k_2,k_3) \otimes R_{a_3a_4}(x_1p,k_2,k_3) \times M(\psi_c) \tag{32}$$

Nonperturbative part $M(\psi_c)$ has been defined in Eq.(15). The final state gluon quarkonium two particle phase space can be written as

$$\Gamma^{(2)} = \frac{dQ^2}{8\pi} \frac{1}{s+t-m_c^2} \delta(x_1+x_2 + \frac{u-2Q \cdot k_{2\perp} + k_{2\perp}^2}{s+t-m_c^2}). \tag{33}$$

The represents of amplitudes $L_{a_1a_2}$ and $R_{a_3a_4}$ are shown in Fig.5 and Fig.6 respectively. The c\bar{c} pair can interacts with nucleon through one of its constituents quark or antiquark scattering with soft gluon. In case of the quark participating the interaction, as shown in Fig.5(a), we have

$$L_{a_1a_2}(x_1p,k_2,k_3) = g_s Tr \left\{ (\frac{q-m_c}{2\sqrt{2m_c}}(q+S_z)(q+m_c)b(\frac{q-x_2p-k_{2\perp}+m_c}{2})T^{a_3}L_{a_1a_2}(x_1p,q',q) \right\}$$

Figure 5: Graphical representation of the left amplitude $L_{a_1a_2}$ in Eq.(32) for c\bar{c} interacting with soft gluon, (a) shows the quark picks up the gluon, (b) shows the antiquark picks up the gluon.
where \( q' = q - x_2p - k_{2\perp} \) is the momentum of outgoing quark after hard scattering, \( \tilde{L}_{\mu_1\nu_\lambda}^{a_1a_2}(x_1p, q', q) \) has been defined in Eq.(16) and shown in Fig.3. In Eq.(34) we have taken \( k_{2\perp} = -k_{1\perp} = k_{\perp} \) by momentum conservation. In case of the antiquark participating the interaction, as shown in Fig.5(b), we have

\[
L_{\mu_1\nu_\lambda}^{a_1a_2(b)}(x_1p, k_2, k_3) = g_s Tr \left\{ \frac{(-\hat{q} + x_2\hat{p} + \hat{k}_{2\perp} + m_c)\hat{p}(\hat{q} + m_c)\hat{q}(Q, S_z)(\hat{q} + m_c)T^{a_2} \tilde{L}_{\mu_1\nu_\lambda}^{a_1}(x_1p, q', q)}{2\sqrt{2m_c((-q + x_2p + k_{2\perp})^2 - m_c^2 + i\varepsilon})} \right\}.
\]

The total contribution of Fig.5 is

\[
L_{\mu_1\nu_\lambda}^{a_1a_2}(x_1p, k_2, k_3) = L_{\mu_1\nu_\lambda}^{a_1a_2(a)}(x_1p, k_2, k_3) + L_{\mu_1\nu_\lambda}^{a_1a_2(b)}(x_1p, k_2, k_3)
\]

\[
= \frac{-g_s}{2\sqrt{2m_c} x_2 + \frac{2p_{k_\perp} - k_{\perp}^2}{2p_{q}} - i\varepsilon} \tilde{L}_{\mu_1\nu_\lambda}^{a_1a_2}(x_1p, k_2, k_3),
\]

where

\[
\tilde{L}_{\mu_1\nu_\lambda}^{a_1a_2}(x_1p, k_2, k_3) = Tr \left[ (\hat{q} - m_c)\hat{q}(Q, S_z)(\hat{q} + m_c)(1 - \frac{\hat{p}\hat{k}_{\perp}}{2p_{\perp}})T^{a_2} \tilde{L}_{\mu_1\nu_\lambda}^{a_1}(x_1p, q', q) - \tilde{L}_{\mu_1\nu_\lambda}^{a_1}(x_1p, q, q')T^{a_2}(1 - \frac{\hat{k}_{\perp}\hat{p}}{2p_{\perp}})(\hat{q} - m_c)\hat{q}(Q, S_z)(\hat{q} + m_c) \right].
\]

In above Eq.(36) we have explicitly extracted the pole contribution from the quark or antiquark propagator in amplitude \( L_{\mu_1\nu_\lambda}^{a_1a_2(a)}(x_1p, k_2, k_3) \).

Similarly, we can derive the corresponding result for amplitude \( R_{\lambda\nu\mu_4}^{a_2a_4} \) from the diagrams shown in Fig.6.

Fig.6(a) represents the case of soft gluon attaching to heavy quark line, which gives

\[
R_{\lambda\nu\mu_4}^{a_2a_4(a)}(x_1p, k_2, k_3) = g_s Tr \left\{ \frac{(\hat{q} + x_3\hat{p} + \hat{k}_{3\perp} + m_c)\hat{p}(\hat{q} + m_c)\hat{q}(Q, S_z)(\hat{q} - m_c)T^{a_2} \tilde{R}_{\lambda\nu\mu_4}^{a_4}(x_1p, q'', q)}{2\sqrt{2m_c}|q + x_3p + k_{3\perp}|^2 - m_c^2 - i\varepsilon} \right\}.
\]

\[
= g_s Tr \left\{ \frac{(2p_{\perp} - q + k_{\perp})\hat{p}(\hat{q} + m_c)\hat{q}(Q, S_z)(\hat{q} - m_c)T^{a_2} \tilde{R}_{\lambda\nu\mu_4}^{a_4}(q'', q)}{2\sqrt{2m_c}(k_{\perp}^2 + 2x_3p_{\perp}q - 2q_{\perp}k_{\perp} - i\varepsilon})} \right\},
\]

where \( q'' = q + x_3p + k_{3\perp} \). Fig.6(b) shows the case of soft gluon attaching to heavy antiquark, it gives

\[
R_{\lambda\nu\mu_4}^{a_2a_4(b)}(x_1p, k_2, k_3) = g_s Tr \left\{ \frac{(\hat{q} + m_c)\hat{q}(Q, S_z)(\hat{q} - m_c)\hat{p}(-\hat{q} - x_3\hat{p} - \hat{k}_{3\perp} + m_c)T^{a_2} \tilde{R}_{\lambda\nu\mu_4}^{a_4}(x_1p, q, q'')}{2\sqrt{2m_c}|(-q - x_3p - k_{3\perp})|^2 - m_c^2 - i\varepsilon} \right\}
\]

\[
= g_s Tr \left\{ \frac{(2p_{\perp} - q - \hat{k}_{\perp})\hat{p}(\hat{q} + m_c)\hat{q}(Q, S_z)(\hat{q} - m_c)T^{a_2} \tilde{R}_{\lambda\nu\mu_4}^{a_4}(q, q'')}{2\sqrt{2m_c}(k_{\perp}^2 + 2x_3p_{\perp}q + 2q_{\perp}k_{\perp} - i\varepsilon})} \right\},
\]

where \( q'' = q + x_3p + k_{3\perp} \).
The total result of the amplitude $R^{a_3a_4}_{\lambda\nu\mu_4}$ is

$$R^{a_3a_4}_{\lambda\nu\mu_4}(x_1p,k_2,k_3) = R^{a_4a_3(a)}_{\lambda\nu\mu_4}(x_1p,k_2,k_3) + R^{a_4a_3(b)}_{\lambda\nu\mu_4}(x_1p,k_2,k_3)$$

$$= \frac{g_s}{2\sqrt{2m_c}} \frac{1}{x_3 - \frac{2q_{\perp}k_{\perp} - k_{\perp}^2}{2p\cdot q} - i\varepsilon} \tilde{R}^{a_3a_4}_{\lambda\nu\mu_4}(x_1p,k_2,k_3),$$

where

$$\tilde{R}^{a_3a_4}_{\lambda\nu\mu_4}(x_1p,k_2,k_3) = Tr \left[ R^{a_4}_{\lambda\nu\mu_4}(x_1p,q'',q) T^{a_3}(1 - \frac{k_{\perp}q_{\perp}}{2p\cdot q})(\bar{q} + m_c)\bar{q}(Q,S_z)(\bar{q} - m_c) \right.$$  

$$\left. - (\bar{q} + m_c)\bar{q}(Q,S_z)(\bar{q} - m_c)(1 - \frac{k_{\perp}q_{\perp}}{2p\cdot q}) T^{a_3} \tilde{R}^{a_4}_{\lambda\nu\mu_4}(x_1p,q,q'') \right].$$

Now we can perform the integration through $x_i (i = 1, 2, 3)$ in function $F(z_1^-, z_2^-, z_3^-)$. From Eq. (36) and (37), we see that both variables $x_2$ and $x_3$ have poles in the upper half complex plane. So after fixing $x_1$ from $\delta$ function, we can carry out the other integrals $\int dx_2 dx_3$ by using contour integrations in $x_2$ and $x_3$, closing at infinity, and circling the poles, we get

$$F(z_1^-, z_2^-, z_3^-) = \frac{dQ^2 dy^2}{16\pi} \frac{g_5^2}{s + t - m_v^2} \frac{M(\psi_c)}{8m_\psi^2} \frac{\partial^2}{\partial k_{\perp}^2 \partial k_{\perp}^2} \left\{ \int dx_1 dx_2 dx_3 e^{ix_1p^+z_1^- + ix_2p^+z_2^- + ix_3p^+z_3^-} \right\} \int dx_1 dx_2 dx_3 e^{ix_1p^+z_1^- + ix_2p^+z_2^- + ix_3p^+z_3^-}.$$
\begin{align*}
\times& \frac{1}{(x_2 + \frac{2q_k - k^2}{2p_q} - i\varepsilon)} \frac{1}{(x_3 - \frac{2q_k - k^2}{2p_q} - i\varepsilon)} \delta(x_1 + x_2 + \frac{u - 2Q \cdot k - k^2}{s + t - m_\psi^2}) \\
\times& \tilde{L}_{\mu_1,\nu_2}^{a_1,a_2}(x_1p, k_2, k_3) \otimes \tilde{R}_{\lambda_3,\mu_4}^{a_1,a_2}(x_1p, k_2, k_3)d_{\perp}^{\mu_1,\mu_4} \delta^{a_1,a_2} \delta^{a_2,a_3} \right)_{k_\perp = 0} \\
= & \frac{dQ_1^2 dy}{16\pi} \frac{g_2^2}{s + t - m_\psi^2} \frac{\mathcal{M}(\psi_c)}{8m_\psi^2} \partial^2 \left\{ \int dx_2 dx_3 e^{ix_1 p^+ z_1^+ + ix_2 p^+(z_2^- - z_1^-) + ix_3 p^+ z_3^-} \\
\times& \tilde{L}_{\mu_1,\nu_2}^{a_1,a_2}(x_1p, k_2, k_3) \otimes \tilde{R}_{\lambda_3,\mu_4}^{a_1,a_2}(x_1p, k_2, k_3) \right\}_{k_\perp = 0} \\
= & \frac{dQ_1^2 dy}{16\pi} \frac{g_2^2}{s + t - m_\psi^2} \frac{\mathcal{M}(\psi_c)}{8m_\psi^2} \partial^2 \left\{ e^{ix_1 p^+ z_1^+ + ix_2 p^+(z_2^- - z_1^-) + ix_3 p^+ z_3^-} \\
\times& ((2\pi \theta(z_2^- - z_1^-))((2\pi \theta(z_3^-)))d_{\perp}^{\mu_1,\mu_4} \delta^{a_1,a_2} \delta^{a_2,a_3} \\
\times& \tilde{L}_{\mu_1,\nu_2}^{a_1,a_2}(x_1p, k_2, k_3) \otimes \tilde{R}_{\lambda_3,\mu_4}^{a_1,a_2}(x_1p, k_2, k_3) \right\}_{x_2 = -x_3 = -\frac{2q_k - k^2}{2p_q}}_{k_\perp = 0} ,
\end{align*}

where

\begin{equation}
x_1 = \frac{u - 2Q \cdot k - k^2}{s + t - m_\psi^2} .
\end{equation}

By substituting Eq. (42) into Eq. (40), we can obtain the lowest order real double scattering cross section

\begin{align*}
\frac{d\sigma^{(1)}_{2A}}{dQ_1^2 dy} &= -\frac{g_2^2}{2s \times 8^3} \frac{1}{16\pi(s + t - m_\psi^2)} \frac{\mathcal{M}(\psi_c)}{8m_\psi^2} \int dz_1 dz_2 dz_3 \frac{p^+}{2\pi} \theta(z_2^- - z_1^-) \theta(z_3^-) \\
&< p_A \left| A_{\alpha}(z_1^-) G_{\perp}^{b_1}(z_2^-) G_{\perp}^{b_2}(z_3^-) A_{\alpha}(0) \right| p_A = d_{\perp}^{\mu_1,\mu_4} \delta^{a_1,a_2} \delta^{a_2,a_3} \\
\frac{\partial^2}{\partial k^\mu_1 \partial k^\mu_2} \left\{ e^{ix_1 p^+ z_1^+ + ix_2 p^+(z_2^- - z_1^-) + ix_3 p^+ z_3^-} \tilde{L}_{\mu_1,\nu_2}^{a_1,a_2}(x_1p, k_2, k_3) \\
\otimes \tilde{R}_{\lambda_3,\mu_4}^{a_1,a_2}(x_1p, k_2, k_3) \right\}_{x_2 = -x_3 = -\frac{2q_k - k^2}{2p_q}}_{k_\perp = 0} .
\end{align*}

One important step in getting the final result is taking the derivative with respect to \( k_\perp \) as define in Eq. (27).

From Eq. (15) and (11), it is obviously that when \( k_\perp = 0 \), \( x_2 = x_3 = 0 \), and \( x_1 = x \), therefore \( q' = q'' = q \), we found

\begin{equation}
\tilde{L}_{\mu_1,\nu_2}^{a_1,a_2}(x_1p, k_2, k_3)|_{k_\perp = 0} = \tilde{R}_{\lambda_3,\mu_4}^{a_1,a_2}(x_1p, k_2, k_3)|_{k_\perp = 0} = 0. \tag{45}
\end{equation}

This indicates that there are cancellations in both the left and right amplitudes \( \tilde{L}_{\mu_1,\nu_2}^{a_1,a_2} \) and \( \tilde{R}_{\lambda_3,\mu_4}^{a_1,a_2} \) of eq. (42). These are the cancellations between the amplitudes of gluon interacting with heavy quark shown in
Fig. 5(a), 6(a) and corresponding ones of antiquark cases in Fig. 5(b), 6(b). These eliminate the twist-two and even some twist-four terms contained in Eq. (22). Nevertheless certain twist four terms will survive and we study them in the following.

Eq. (43) denotes that the derivatives on the exponential $e^{ix_1 p^+ z_1^- + k_2 p^+ (z_2^- - z_1^-) + i x_3 p^+ z_3^-}$ do not contribute, and that we can therefore set $e^{ix_1 p^+ z_1^- + k_2 p^+ (z_2^- - z_1^-) + i x_3 p^+ z_3^-} = e^{i x_1 p^+ z_1^-}$. Now we defined four gluon matrix element

$$\frac{M_g(x)}{x} = \int \frac{p^+ d\zeta^+}{2\pi} \frac{d\zeta^-}{2\pi} e^{ix_1 p^+ z_1^-} \theta(z_2^- - z_1^-) \theta(z_3^-) \times < p_A | A^{a_1} (z_1^-) G^{b_1} (z_2^-) G^{b_2} (z_3^-) A^{a_2} (0) | p_A > . \quad (46)$$

Substituting Eq. (45) into Eq. (44), we obtain the contribution from real double scattering as

$$\frac{d\sigma_{DA}^{(1)}}{dQ^2 dy} = - \frac{g_s^2}{2s \times 8^3} \frac{1}{8(s + t - m_{\psi}^2)} \frac{M_g(x) M(\psi_c)}{8 m_{\psi}^2} \frac{\partial^2}{\partial k_{\perp}^2 \partial k_{\perp,\rho}} \left\{ \tilde{L}_{\mu_1 \nu_\lambda}^{a_1 a_2} (x_1 p, k_2, k_3) \otimes \tilde{R}_{\lambda \mu_4}^{a_3} (x_1 p, k_2, k_3) \right\} \bigg|_{x_2 = -x_3 = -\frac{2q_{\perp} - k_2}{2p_{\perp}, q}} \bigg|_{k_{\perp} = 0} . \quad (47)$$

Now we begin to consider the contribution from the virtual diagrams shown in Fig. 4(b) and Fig. 4(c). The graphical represents of the amplitudes corresponding to the left and right parts of diagram in Fig. 4(b) are given in Fig. 7 and Fig. 8 respectively, and the corresponding represents for Fig. 4(c) can also be given similarly. Following the same methods as that we use to derive Eq. (17), we obtain the differential cross section coming from virtual double scattering as

$$\frac{d\sigma_{VA}^{(2)}}{dQ^2 dy} = - \frac{g_s^2}{2s \times 8^3} \frac{1}{8(s + t - m_{\psi}^2)} \frac{M_g(x) M(\psi_c)}{8 m_{\psi}^2} \frac{\partial^2}{\partial k_{\perp}^2 \partial k_{\perp,\rho}} \left\{ \tilde{L}_{\mu_1 \nu_\lambda}^{a_1 a_2} (x_1 p, k_2, k_3) \otimes \tilde{R}_{\lambda \mu_4}^{a_3} (x_1 p, k_2, k_3) \right\} \bigg|_{x_2 = -x_3 = -\frac{2q_{\perp} - k_2}{2p_{\perp}, q}} + \tilde{L}_{\mu_1 \nu_\lambda}^{a_1} (x_1 p, k_2, k_3) \otimes \tilde{R}_{\lambda \mu_4}^{a_3 a_2} (x_1 p, k_2, k_3) \bigg|_{x_2 = x_3 = -\frac{2q_{\perp} + k_2}{2p_{\perp}, q}} \bigg|_{k_{\perp} = 0} , \quad (48)$$

where

$$\tilde{L}_{\mu_1 \nu_\lambda}^{a_1 a_2} (x_1 p, k_2, k_3) = Tr \left\{ T^{a_2} T^{a_3} (1 - \frac{\not k_{\perp}}{2p_{\perp}, q}) (\not q - m_c) \phi(Q,S_z) \not q + m_c \right\}$$

$$\left(1 + \frac{\not k_{\perp}}{2p_{\perp}, q} \right) \tilde{L}_{\mu_1 \nu_\lambda}^{a_1} (x_1 p, q_1, q_2), \quad (49)$$
Figure 7: Graphical representation of the amplitude corresponding to the left part of diagram in Fig.4(b); a) two soft gluons attach to quark line, b) two soft gluons attach to antiquark line, c) and d) one gluon attaches to quark line and another attaches to antiquark line
\[ \tilde{R}_{\lambda \nu \mu_4}^{a_2 a_3 a_4}(x_1 p, k_2, k_3) = \text{Tr} \left\{ T^{a_3} T^{a_2} \left( 1 + \frac{k_{\perp} \cdot \hat{p}}{2 p \cdot q} \right) (\hat{q} + m_c) \psi(Q, S_z)(\hat{q} - m_c) \right\} \cdot \left( 1 - \frac{\hat{p} \cdot k_{\perp}}{2 p \cdot q} \right) \tilde{R}_{\lambda \mu_1 \mu_4}^{a_4}(x_1 p, \bar{q}_1, \bar{q}_2) \right\}, \tag{50} \]

\[ \tilde{L}_{\mu_1 \nu \lambda}^{a_1}(x_1 p, k_2, k_3) = \text{Tr} \left[ (\hat{q} - m_c) \psi(Q, S_z)(\hat{q} + m_c) \tilde{L}_{\mu_1 \nu \lambda}^{a_1}(x p, q, q) \right] \tag{51} \]

\[ \tilde{R}_{\lambda \nu \mu_4}^{a_4}(x_1 p, k_2, k_3) = \text{Tr} \left[ (\hat{q} + m_c) \psi(Q, S_z)(\hat{q} - m_c) \tilde{R}_{\lambda \mu_1 \mu_4}^{a_4}(x p, q, q) \right], \tag{52} \]

where

\[ x_1 = x - \frac{k_{\perp}^2}{q \cdot p} \]
\[ \bar{q}_1 = q - x_2 p - k_{\perp}, \]
\[ \bar{q}_2 = q - x_3 p + k_{\perp}. \]

Now we can work out the derivative, the calculation is tedious but straightforward. The final result for lowest order double scattering cross section is

\[ \frac{d\sigma_D^{(2)}_{gA}}{dQ_{\perp}^2 dy} = \frac{d\sigma_D^{(1)}_{gA}}{dQ_{\perp}^2 dy} + \frac{d\sigma_D^{(2)}_{gA}}{dQ_{\perp}^2 dy} \]

\[ = \frac{16\pi^3 \alpha_s^3 e_Q^2 |R(0)|^2 M_{g/A}(x)}{s(s + t - m_{\psi}^2)m_{\psi}^3} \left( \hat{s} \hat{t} m_{\psi}^2 \right) \frac{f(\hat{s}, \hat{t}, m_{\psi}^2)}{(\hat{s} - m_{\psi}^2)^4(\hat{t} - m_{\psi}^2)^4(\hat{s} + \hat{t})^4}, \tag{53} \]
where the function $f$ is

$$
f(s, t, m^2_{\psi}) = \left\{ -72Q^2_{\perp}m^2_{\psi}s^2 + 16Q^2_{\perp}m^4_{\psi}s^2(82s + 73t) + 8Q^2_{\perp}m^2_{\psi}s^2(-179s^2 - 550st - 88t^2) \\
+ 8Q^2_{\perp}m^2_{\psi}s^2(-89s^3 + 518s^2t - 135st^2 - 158t^3) \\
+ 8Q^2_{\perp}m^6_{\psi}s^2(387s^4 + 290s^3t + 1327s^2t^2 + 1204st^3 + 337t^4) \\
+ 8Q^2_{\perp}m^6_{\psi}s^2(-4025s^5 - 870s^4t - 2263s^3t^2 - 2560s^2t^3 - 1811st^4 - 516t^5) \\
+ 8Q^2_{\perp}m^8_{\psi}s^2(128s^6 + 498s^5t + 1529s^4t^2 + 2222s^3t^3 + 2182s^2t^4 + 1160st^5 + 306t^6) \\
+ 256Q^2_{\perp}m^2_{\psi}s^3t(-s^5 - 3s^4t - 5s^3t^2 - 5s^2t^3 - 3st^4 - t^5) \\
+ 9m^2_{\psi}(s^2 + 2st + t^2) + 4m^4_{\psi}(-53s^3 - 23s^2t + 17st^2 - 13t^3) \\
+ m^4_{\psi}(-181s^4 - 788s^3t - 1552s^2t^2 - 1208st^3 - 263t^4) \\
+ m^4_{\psi}(1081s^5 + 3875s^4t + 6582s^3t^2 + 7030s^2t^3 + 4537st^4 + 1295t^5) \\
+ m^8_{\psi}(-1296s^6 - 5283s^5t - 10856s^4t^2 - 14170s^3t^3 - 12382s^2t^4 - 7039st^5 - 1958t^6) \\
+ m^8_{\psi}(823s^7 + 2921s^6t + 6357s^5t^2 + 11051s^4t^3 + 13065s^3t^4 + 9643s^2t^5 + 4659st^6 + 1289t^7) \\
+ m^6_{\psi}(-224s^8 - 615s^7t - 525s^6t^2 - 1609s^5t^3 - 4268s^4t^4 - 4537s^3t^5 - 2267s^2t^6 - 843st^7 - 320t^8) \\
+ m^4_{\psi}(36s^9 - 727s^8t - 1561s^7t^2 - 894s^6t^3 - 290s^5t^4 + 117s^4t^5 - 389st^6 - 192t^7) \\
+ 3m^2_{\psi}s^3t^2(44s^5 + 173s^4t + 239s^3t^2 + 116s^2t^3 - 13st^4 - 19t^5) \right\}/9. \quad (54)
$$

In deriving (53) we have used the following relation for the long distance matrix element

$$
<0|O^\psi_1(q^3 S_1)|0> = \frac{3N_c}{2\pi} |R(0)|^2. \quad (55)
$$

4 Numerical results and discussion

For the purposes of numerical evaluation, we evaluate the ratio of transverse momentum double scattering differential cross section to corresponding single scattering contribution

$$\rho^{(1)} = \frac{d\sigma^{DA}}{dQ^2_{\perp}} / \frac{d\sigma^{SA}}{dQ^2_{\perp}}$$

$$= \int dy \frac{d\sigma^{DA}}{dQ^2_{\perp} dy} / \int dy \frac{d\sigma^{SA}}{dQ^2_{\perp} dy} \quad (56)$$

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We perform the integration through rapidity $y$ in Eq.(13) and Eq.(53) and the integration range are taken $|y| < 0.4$. Before embarking on the numerical calculation, we have to know the non-perturbative parton distribution $f_{g/A}(x)$ and multi-parton correlation function $M_g(x)$.

In Eq.(14), the effective nuclear gluon distribution function $f_{g/A}(x)$ should have the same operator definition of the normal parton distribution with free nucleon states replaced by the nuclear states. For a nucleus with atomic number $A$, we define

$$f_{g/A}(x) = Af_{g/N}(x), \quad (57)$$

where $f_{g/N}(x)$ is normal gluon distributions in a free nucleon. In this effective nuclear parton distribution, we have neglected the EMC effect and the difference between the normal parton distribution in a free neutron and in a free proton. We adopt $f_{g/N}(x)$ from [18].

The authors of [13, 16] proposed following phenomenological expressions for the twist-four matrix elements that occur in soft rescattering:

$$M_i(x) = A^{1/3} \lambda^2 f_{i/A}(x), \quad (58)$$

where $i = q, \bar{q},$ and $g$. The $f_{i/A}(x)$ are the effective twist-2 parton distributions in nucleus defined in Eq.(57). The constant $\lambda^2$ has units of mass squared due to the dimension difference between twist-4 and twist-2 matrix elements. The $\lambda^2$ should be determined by experimental measurement. It was estimated in [13] by using the measured nuclear enhancement of the momentum imbalance of two jets in photon-nucleus collisions, and was found to be order of

$$\lambda^2 \approx 0.05 - 0.1 GeV^2.$$

In our following calculation we use $\lambda^2 = 0.05 GeV^2$.

From the definition of the correlation functions in Eq.(46), the lack of oscillation factors for both $z_2^-$ and $z_3^-$ integrals can in principle give nuclear dependence proportional to $A^{2/3}$. The $A^{1/3}$ dependence is a result of the assumption that the positions of two field strengths (at $z_2^-$ and $z_3^-$, respectively) are confined within one nucleon.

In Fig.9 we show the ratio of double scattering and single scattering versus $Q_\perp$. We plot $\rho^{(1)}$ as a function of $Q_\perp$ for $s = (20 GeV)^2$ and $s = (40 GeV)^2$ respectively. The ratio is normalized by $A^{1/3}$. We find that the overall contribution of double scattering give negative sign, while it is positive in the processes of Drell-Yan and DIS. From Fig.9, it is obviously that the absolute value of the ratio increase with $Q_\perp$, this indicates that in high $Q_\perp$ $J/\psi$ is more suppressed by the double scattering effect than that in low $Q_\perp$. We find that the two curves for $s = (20 GeV)^2$ and $s = (40 GeV)^2$ are very close, this indicates that at this range the ratio is
almost independent of the invariant $s$. Although our quantity result is not large, about 17% at $Q_\perp = 8\,\text{GeV}$ for $s = 40\,\text{GeV}$, but if we include the extra $A^{1/3}$ factor, it can give observable effect in large $A$ case. Moreover our result lies in reducing some theoretical uncertainties.

In summary, we have applied LQS method, which was developed in inclusive DIS process, to study the multi-scattering effect on $J/\psi$ photoproduction. We consider the interaction between nuclear matter and $c\bar{c}$ pair produced from the hard collision of a photon and a parton in nucleus. This interaction undergoes through either quark or antiquark absorbing a soft gluon. We find the lowest order double scattering has not a great influence and only contributes a slight suppression comparing with the single scattering cross section. Our analysis can be extended to study the multi-scattering effects in $J/\psi$ production from hard-nucleus or nucleus-nucleus collisions, which will involve initial state strong interactions that are absent in the case of photoproduction. These two hadron production processes will be more complicated because for initial state we must consider two types of double scattering, soft-hard scattering and double hard scattering.

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