Micro-SQUID characteristics

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We report on the dependence on field and temperature of the critical current of micro-SQUIDs: SQUIDs with diameters as small as 1 micron, using Dayem bridges as weak links. We model these SQUIDs by solving the Ginzburg-Landau equations with appropriate boundary conditions to obtain the supercurrent-phase relationships. These solutions show that the phase drops and depression of the order parameter produced by supercurrent flow are often distributed throughout the micro-SQUID structure, rather than being localized in the bridge area, for typical micro-SQUID geometries and coherence lengths. The resultant highly non-sinusoidal current-phase relationships $I_c(\phi)$ lead to reduced modulation depths and triangular dependences of the micro-SQUID critical currents on applied magnetic flux $I_c(\Phi)$. Our modelling agrees well with our measurements on both Al and Nb micro-SQUIDs.

INTRODUCTION

Several different types of specialized Superconducting Quantum Interference Devices (SQUIDs) have been developed for measuring the magnetic response of small samples. One such device is the “micro-SQUID”: a thin film DC-SQUID with Dayem-Bridges as Josephson Junctions. In the micro-SQUID, the entire device is fabricated by electron beam lithography, and the SQUID loop itself serves as the flux-input coil. Micro-SQUIDs have the advantages of very small pickup areas (about 1 $\mu$m$^2$), and relatively small sensitivity to in-plane applied magnetic fields (since they can be made very thin, less than 20 nm). Micro-SQUIDs have been used for the observation of persistent currents in 2 dimensional electron gas rings, for the study of the mechanisms of magnetization reversal in ferromagnetic particles as small as 3 nm in diameter, and have been integrated into a scanning SQUID-AFM with high magnetic field spatial resolution. However, these SQUIDs have two disadvantages: they have hysteretic current-voltage characteristics, and relatively low modulation depths in their critical current - flux characteristics. In this paper we report detailed measurements of micro-SQUID characteristics, and compare them with model calculations. These calculations model the micro-SQUID characteristics well, and could represent a valuable tool for optimizing the properties of this class of SQUIDs.

MEASUREMENT

The basic operating properties of micro-SQUIDs are understood: They have a hysteretic V(I) characteristic, induced by the propagation of a hot spot. As the current is ramped up from zero, the micro-SQUID transits from the superconducting to the normal state at a critical current $I_c$. A voltage step is generated as the normal state resistance of the junction appears and the dissipated energy heats the entire micro-SQUID loop. When the current is lowered the micro-SQUID stays in the resistive state until the current is much smaller than $I_c$. This thermal hysteresis excludes the usual current biasing schemes used for DC-SQUID readout. Therefore a detection technique suitable for hysteretic devices is implemented: A computer controlled circuit simultaneously triggers a current ramp and a 40MHz quartz clock. As soon as a $\partial V/\partial t$ pulse of a preset height is detected at the micro-SQUID, the clock stops and the current is set to zero. The clock reading is transferred to the computer, and the cycle begins again. The critical current is proportional to the duration of the current ramp. The fastest repetition rate is 10kHz, limited by the time needed to settle the current. A single wire is sufficient to connect the micro-SQUID, since the $\partial V/\partial t$ pulse is detected on the current biasing lead. Every time the critical current is measured, the flux state in the micro-SQUID is sampled, and every time the micro-SQUID becomes normally conducting the external field penetrates. As the current is reduced (in 40ns) to zero the micro-SQUID structure becomes progressively superconducting again and screening currents are set up to quantize the total flux through the micro-
SQUID. In the limit of high critical currents different flux configurations can be stabilized in the micro-SQUID ring during the backswitching, leading to multivalued $I_c$ vs $B$ characteristics.

However, the details of the dependence of the critical current of the micro-SQUID on magnetic field and temperature have not been well characterized. We present in this manuscript a detailed description of the underlying physics necessary for the understanding and improvement of this type of SQUID. In this study we made $I_c(T)$ and $I_c$ vs $B$ measurements on micro-SQUID devices made of aluminum or niobium. Each SQUID consists of a 1µm square loop. Bulk aluminum and niobium have very different superconducting properties, such as the coherence length $\xi$, and the normal to superconducting transition temperature $T_c$. The characteristics of micro-SQUIDs (see e.g. Fig.’s 2 and 4) do not resemble those of ideal Josephson SQUIDs, in that they have relatively shallow modulation depths and triangular $I_c$ vs $B$ interference patterns at low temperatures.

We believe that these non-ideal characteristics occur because the weak links in these SQUIDs are Dayem-Bridges, with dimensions comparable to the coherence length. It is well known that “long” Josephson weak links can have non-sinusoidal Josephson current-phase relationships \[ \alpha \psi + \beta |\psi|^2 \psi = 0 \] \[ \alpha \psi + \beta |\psi|^2 \psi + \frac{\hbar}{2m*} \left( \frac{\partial^2}{\partial x^2} |\psi|^2 - |\psi| \left( \frac{\partial^2}{\partial x^2} \right)^2 \right) - \frac{\hbar^2}{2m*} \left( \frac{\partial^2}{\partial y^2} |\psi|^2 - |\psi| \left( \frac{\partial^2}{\partial y^2} \right)^2 \right) = 0 \] \[ 2 \frac{d\phi}{dx} \frac{d\phi}{dx} + |\psi| \left( \frac{d^2\phi}{dx^2} \right)^2 + 2 \frac{d\phi}{dy} \frac{d\phi}{dy} + |\psi| \left( \frac{d^2\phi}{dy^2} \right)^2 = 0. \]

Setting $|\psi| = \frac{f}{|\psi|_\infty}$, where $f$ is a real function of $x$ and $y$, and $|\psi|_\infty$ is the unperturbed value of the order parameter, $\beta |\psi|^2_\infty = \frac{\hbar^2}{2m*\xi^2(T)}$ \[ 2 \frac{d\phi}{dx} \frac{d\phi}{dx} + \frac{2 \frac{d\phi}{dx}}{\frac{d\phi}{dx^2}} + \frac{2 \frac{d\phi}{dy}}{\frac{d\phi}{dy^2}} + f - f^3 = 0 \]

and Eq. 3 becomes

$$2 \frac{d\phi}{dx} \frac{d\phi}{dx} + \frac{2 \frac{d\phi}{dx}}{\frac{d\phi}{dx^2}} + \frac{2 \frac{d\phi}{dy}}{\frac{d\phi}{dy^2}} + f - f^3 = 0 \quad (5)$$

The supercurrent density $\vec{J}$ becomes

$$\vec{J} = \frac{e^*\hbar}{m^*c} |\psi|_\infty \left[ \frac{\partial}{\partial x} \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \frac{\partial}{\partial y} \right] f^2. \quad (7)$$

It can be shown that Eq. 7 is equivalent to setting the divergence of the supercurrent (Eq. 3) equal to zero.

MODEL

The first Ginzburg-Landau (GL) differential equation is \[ \alpha \psi + \beta |\psi|^2 \psi + \frac{1}{2m*} \left( \frac{\hbar}{i} \nabla - \frac{e^* A}{c} \right) \psi = 0 \] where $m^* = 2m$ and $e^* = 2e$ are the mass and charge of the Cooper pair, $\psi$ is the complex order parameter describing the superconducting state, and $A$ is the vector potential. The supercurrent $\vec{J}$ is given by:

$$\vec{J} = \frac{e^*\hbar}{2m^*i} (\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*) - \frac{e^*g^2}{m^*c} \psi^* \psi \vec{A}. \quad (2)$$

For the purposes of the paper, we include the effects of the vector potential $\vec{A}$ in a lumped circuit element model (see Eq. 12). Therefore in our solution of the GL equations we set $\vec{A} = 0$. Writing the complex order parameter as $\psi = |\psi| e^{i\phi}$, using a coordinate system in which the SQUID is in the $xy$ plane, and neglecting the $z$ dependence of the gradients of $\psi$, the real (Eq. 3) and imaginary (Eq. 4) parts of Eq. 1 become:

$$\alpha |\psi| + \beta |\psi|^3 - \frac{\hbar^2}{2m^*} \left[ \frac{d^2 |\psi|^2}{dx^2} \right] - |\psi| \left( \frac{d^2 |\psi|^2}{dx^2} \right) = 0 \quad (3)$$

and

$$2 \frac{d^2\phi}{dx^2} + |\psi| \left( \frac{d^2\phi}{dx^2} \right)^2 + 2 \frac{d\phi}{dy} \frac{d\phi}{dy} + |\psi| \left( \frac{d^2\phi}{dy^2} \right)^2 = 0. \quad (4)$$

The supercurrent density $\vec{J}$ becomes

$$\vec{J} = \frac{e^*\hbar}{m^*c} |\psi|_\infty \left[ \frac{\partial}{\partial x} \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \frac{\partial}{\partial y} \right] f^2. \quad (7)$$

It can be shown that Eq. 7 is equivalent to setting the divergence of the supercurrent (Eq. 3) equal to zero.
Using (in MKS units)

difference equation
tions, the differential equation Eq. 5 can be cast as a

\[ x \]

where \( i \) and \( j \) are indices labelling the 2-d matrices in the

directions respectively. Similarly, Eq. 6 becomes

\[ \phi_{i,j+1} + \phi_{i,j-1} + \phi_{i+1,j} + \phi_{i-1,j} - 4(\phi_{i+1,j} - \phi_{i,j})^2 - (\phi_{i,j+1} - \phi_{i,j})^2 + \delta^2 \phi_{i,j} = \delta^2 \phi_{i,j+1} \]  

(8)

where \( i \) and \( j \) are indices labelling the 2-d matrices in the

x and y directions respectively. Similarly, Eq. 8 becomes

\[ \phi_{i+1,j} [f_{i,j} + 2(f_{i+1,j} - f_{i,j})] + \phi_{i-1,j} f_{i,j} + \phi_{i,j+1} [f_{i,j} + 2(f_{i,j+1} - f_{i,j})] + \phi_{i,j-1} f_{i,j} + \phi_{i,j} [-4 f_{i,j} - 2(f_{i+1,j} - f_{i,j}) - 2(f_{i,j+1} - f_{i,j})] = 0. \]  

(9)

Using (in MKS units) \( \psi = \frac{\hbar}{2\mu_0 e^2} \lambda_{eff}^2 \), the total current through the SQUID can be written as:

\[ I = I_0 \sum_j f_{i,j}^2 (\phi_{i+1,j} - \phi_{i,j}), \]  

(10)

where \( I_0 = dh/2\mu_0 e^2 \lambda_{eff}^2 \), and \( d \) is the thickness of the

film from which the micro-SQUID is patterned. Super-
current conservation requires that the sum over \( j \) in Eq. 10 be independent of the value of \( i \) chosen. This was used as a self-consistency check of the solutions presented below. The difference equations Eq. 8, 9 are in the standard form.

\[ a_{i,j} u_{j+1,l} + b_{i,j} u_{j-1,l} + c_{i,j} u_{j,l+1} + d_{i,j} u_{j,l-1} + e_{i,j} u_{j,l} = g_{j,l} \]  

(11)

We solved Eq. 8 and 9 using a general non-linear differential equation solver subroutine (SOR) using the successive over-relaxation method with Chebyshev acceleration. An initial guess was made for the matrices \( f_{i,j} \) and \( \phi_{i,j} \). SOR was used to search for a solution for \( f_{i,j} \) of Eq. 8 for a pre-determined number of iterations, with \( \phi_{i,j} \) fixed. The result for \( f_{i,j} \) was then fixed, and a solution for \( \phi_{i,j} \) in Eq. 9 was sought for the same number of iterations. This procedure was iterated until the sum of the absolute values of the deviations from equality in Eq. 8 and 9 were both less than a fixed error sum value. For the results reported here, the error sum value was chosen to be \( 10^{-3} \). This procedure gave results in agreement with those reported by Likharev and Yakobson for long 1-D microbridges. For modelling of our micro-SQUIDs we chose the boundary conditions: 1) \( \phi \) was fixed at 0 along the entrance to the micro-SQUID structure (at the left of Fig. 1a), and fixed at 0.5 at the exit of the micro-SQUID structure (to the right); 2) \( \phi \) was chosen to be 1 at both the entrance and exits to the micro-SQUID; and 3) the components of the gradients of \( f \) and \( \phi \) were taken to be zero normal to the other boundaries (solid lines in Fig. 1a,b). The boundaries of the model calculations were chosen to match those of electron micrographs of the actual SQUIDs measured. In choosing these boundary conditions we neglect the effects of phase drops and supercurrent depression in the leads to and from the micro-SQUID. We argue that phase drops outside of the SQUID loop should have little effect on the critical current-flux characteristics which we are modelling, and that neglect of lead effects is therefore a good starting approximation.

FIG. 1. Solution to the 2-dimensional Ginzburg-Landau differential equations for the quantum mechanical phase \( \phi \) and normalized superfluid density \( f \) for a micro-SQUID. The geometry of the SQUID is that of the Al micro-SQUID for which measurements are presented in Fig. 4. The length of the bridge \( S = 290nm \), bridge width=66 nm, the SQUID length (horizontal, \( x \)) is 1.3\( \mu \)m and width (vertical, \( y \)) is 1.45\( \mu \)m. The calculations for this figure were done for \( \xi = 580nm \), with a total phase drop across the SQUID of \( \phi_s = \pi \). Greyscale images of the superconducting phase \( \phi \) (a) and superfluid density \( f \) (b) in the SQUID are shown. Shown in (d) are cross-sections through these images, along the path shown in (c). For this value of \( S/\xi(T)=0.5 \), the phase drop and superfluid depression are mostly localized in the micro-bridge region.

By taking discrete steps \( \delta \) in both the \( x \) and \( y \) directions, the differential equation Eq. 8 can be cast as a difference equation

\[ f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1} + (-4 - (\phi_{i+1,j} - \phi_{i,j})^2 - (\phi_{i,j+1} - \phi_{i,j})^2 + \delta^2) f_{i,j} = \delta^2 f_{i,j+1} \]  

(8)

where \( i \) and \( j \) are indices labelling the 2-d matrices in the

x and y directions respectively. Similarly, Eq. 8 becomes

\[ \phi_{i+1,j} [f_{i,j} + 2(f_{i+1,j} - f_{i,j})] + \phi_{i-1,j} f_{i,j} + \phi_{i,j+1} [f_{i,j} + 2(f_{i,j+1} - f_{i,j})] + \phi_{i,j-1} f_{i,j} + \phi_{i,j} [-4 f_{i,j} - 2(f_{i+1,j} - f_{i,j}) - 2(f_{i,j+1} - f_{i,j})] = 0. \]  

(9)

Using (in MKS units) \( \psi = \frac{\hbar}{2\mu_0 e^2} \lambda_{eff}^2 \), the total current through the SQUID can be written as:

\[ I = I_0 \sum_j f_{i,j}^2 (\phi_{i+1,j} - \phi_{i,j}), \]  

(10)

where \( I_0 = dh/2\mu_0 e^2 \lambda_{eff}^2 \), and \( d \) is the thickness of the

film from which the micro-SQUID is patterned. Super-
current conservation requires that the sum over \( j \) in Eq. 10 be independent of the value of \( i \) chosen. This was used as a self-consistency check of the solutions presented below. The difference equations Eq. 8, 9 are in the standard form.
Example solutions for $\varphi$ and $f$ using this model are shown in Fig. 3 and 4. Here the geometry is that of the Al micro-SQUID. The $\varphi_{i,j}$ and $f_{i,j}$ matrices both had $102 \times 110$ elements, with $76$ elements/µm. The bridges were $S=289$ nm long and $66$ nm wide. The arms of the SQUID were $237$ nm wide. Figure 3 shows the solution to Eq.'s 8 and 9 for $S/\xi=0.5$, $\varphi_s = \pi$. For a “short” 1-d micro-bridge $(S/\xi << 1)$ with $\varphi_s = \pi$, the solution for $\varphi$ has a step from $0$ to $\pi$, and a depression in $f$ to $0$, with a characteristic width of $\xi$, centered at the center of the bridge. The numerical solution for the full SQUID structure has a step in $\varphi$ and a depression in $f$ near the center of the bridges, but there are significant changes well into the body of the SQUID, and $f$ does not go completely to $0$, meaning that $I_s$ does not go to zero, until $\varphi_s$ is slightly above $\pi$. These effects become more pronounced as $S/\xi$ increases. Figure 3 shows the results for $S/\xi=2.9$, $\varphi_s=4.4$. This is the maximum value of $\varphi_s$ for which a numerical solution to the Ginzburg-Landau equations could be found for $S/\xi=2.9$. In this case almost half of the total phase drop occurs outside of the micro-bridge region.

Supercurrent-phase relationships $I_s(\varphi)$ for this geometry are shown in Figure 3. This procedure is unable to find solutions numerically for the lower branch of the $I_s(\varphi)$ characteristic when this characteristic is doubly valued [3]. This will not affect our results, since we are comparing our modelling with the maximum supercurrent at each value of applied field. This conclusion is supported by our calculations of $I_s(\Phi)$ characteristics using the Likharev-Yakobson [3] $I_s(\varphi)$ characteristics: only the upper branch is important for calculating the $I_s(\Phi)$ characteristics. The coherence lengths $\xi$ in Fig. 3 were chosen to be appropriate for an Al micro-SQUID at the temperatures for which the measurements of Fig. 3 were made. We use the dirty limit expression for the temperature dependent coherence length $\xi(T) = 0.855\sqrt{\xi_0t/(1-T/T_c)}$ [3], estimate the low temperature mean free path $l$ in our films to be about 10nm from transport measurements, and take a value for $\xi_0$ of $100nm$ [6], and $T_c=1.25K$.

A SQUID with non-sinusoidal current-phase ($I_c(\varphi)$) relationships will have critical current vs. flux interference patterns ($I_c(\Phi)$) that are also non-standard. These can be modelled as follows. Assume that the SQUID has two arms labelled $a$ and $b$, with total phase drops across the two arms $\varphi_a$ and $\varphi_b$, inductances $L_a$ and $L_b$, and micro-bridge supercurrents $I_{sa} = I_a(\varphi_a)$ and $I_{sb} = I_b(\varphi_b)$.

The requirement of a single valued superconducting order parameter leads to

$$\begin{align*}
2\pi n &= \varphi_a - \varphi_b + 2\pi\phi_e + \beta_ag(\varphi_a) - \beta_bg(\varphi_b), \quad (12)
\end{align*}$$

where $\phi_e$ is the externally applied flux $\Phi_e$ divided by the superconducting flux quantum $\Phi_0$; $\beta_a = 2\pi L_a/I_a/\Phi_0$ and $\beta_b = 2\pi L_b/I_b/\Phi_0$. The total supercurrent through the SQUID is

$$I = I_ag(\varphi_a) + I_bg(\varphi_b). \quad (13)$$

The dependence of the critical current on applied field is determined most easily by assuming values for $\phi_e$ and one of the micro-bridge phase differences $\varphi_a$ or $\varphi_b$, then varying the phase of the second micro-bridge until Eq. 12 is satisfied. The values for $\varphi_a$ and $\varphi_b$ are then substituted into Eq. 13 to find the total current. The maximum value for $I$ after repeating this procedure for all initial values of $\varphi_a$ and $\varphi_b$ is the critical current for that value of $\phi_e$.

The effect of the self-induced field is expressed in Eq.
where we assume \( \lambda \) and use the case of Nb, which have critical currents of 2000 \( \mu \)A at 0.2 K. In the determination of \( I_c \) values for the inductances. For this modelling we take the full modulation period (open diamonds), for an Al micro-SQUID, as a function of temperature. The solid triangles are the predictions for a sinusoidal current-phase relationship model as outlined above. The solid squares and inverted triangles are the predictions using the GL current-phase relationship described above.

Figure 4c shows the predictions using the full GL calculations for the current-phase relationship, with the same parameters. In the determination of \( I_0 \) we assume \( \lambda_{c,eff}(T) = \lambda_{c,eff}(0)/\sqrt{1-t^2} \), \( t = T/T_c \), \( d = 38 \text{nm} \) and use \( \lambda_{c,eff}(0) \) as the sole fitting parameter. The best fit between experiment and modelling for \( I_c(T, \Phi = 0) \) is for \( \lambda_{c,eff}(0) = 172 \text{ nm} \). This is to be compared with \( \lambda_L = 44 \text{ nm} \) for bulk aluminum. The effective penetration depth is often longer in thin films than in bulk, and can also be increased by impurity scattering. Further, the effective thickness of the films will be reduced by oxidation. Once this scaling is done, the agreement between theory and experiment for \( I_c(T, \Phi = 0) \) is good. The full model (Figure 4c) fits the \( I_c(T, \Phi) \) experimental data well. This is made clear in the plot of \( I_c(0) \), the critical current at zero applied field, and \( I_c(\Phi_0/2) \), the critical current at the first minimum of the interference pattern, as a function of temperature, in Figure 5. In this figure, the open circles are \( I_c(0) \), and the open diamonds are \( I_c(\Phi_0/2) \). The solid triangles are the predictions for a sinusoidal current-phase relationship model as outlined above. The solid squares and inverted triangles are the predictions using the GL current-phase relationship described above.

Similar conclusions can be drawn from the data and modelling for a Nb micro-SQUID (Fig. 7). In this case the \( f_{i,j} \) and \( \varphi_{i,j} \) matrices were 100x100, with 67 elements/micron. The micro-bridges were 184 nm long and 100 nm wide, with the arms of the SQUID 285 nm wide. \( L_c(\varphi) \) characteristics for the values of \( S/\xi(T) \) appropriate for the data of Fig. 7 are shown in Fig. 6. In this case \( S/\xi(T) \) is large, the variations of \( \varphi \) and \( f \) are spread throughout the SQUID structure, and \( I_c(\varphi) \) is nearly linear, at all temperatures. For the calculations of the \( I_c(\Phi) \) characteristics in Fig. 7, the total inductance of the SQUID was taken to be 1.4 pH, the dirty limit expression \( \xi(T) = 0.85(\xi_0d)^{1/2}(1-t)^{1/2} \) was again used for the coherence length, with \( l = 6.5 \text{nm} \), \( \xi_0 = 39 \text{nm} \), \( \lambda_{c,eff}(T) = \lambda_{c,eff}(0)/\sqrt{1-t^2} \), \( d = 30 \text{nm} \), and \( T_c = 8.23 \text{K} \). Figure 6a shows experimental measurements of the depen-

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**Figure 5.** Plot of the experimentally measured critical currents at zero field (open circles) and at an applied flux of half the full modulation period (open diamonds), for an Al micro-SQUID, as a function of temperature. The solid triangles are the predictions as described in the text for the conventional sinusoidal current-phase relationship. The solid inverted triangles and squares are modelling using the Ginzburg-Landau calculations described in the text.

**Figure 6.** Calculated supercurrent-phase relationship for the Nb SQUID, for various values of \( S/\xi \) appropriate for the data at selected temperatures shown in Fig. 7.
dence of the SQUID critical current on applied magnetic field at selected temperatures. A multivalued critical current appears for currents higher than 0.4 and 0.5 mA for our given inductance. The sections of the \( I_c(\Phi) \) characteristics are nearly linear, with sharp discontinuities in the slopes at \( \Phi = (n + 1/2)\Phi_0 \), \( n \) an integer.

As can be seen from Figures 7c and 8, the full modelling does poorly in describing the \( I_c(\Phi) \) characteristics. Figure 7b shows modelling for an assumed sinusoidal current-phase relationship, using symmetric experimentally determined values for the micro-bridge critical currents and symmetric inductances of 0.7pH for each arm of the SQUID. As for the Al micro-SQUID case, this modelling does poorly in describing the \( I_c(\Phi) \) characteristics. Figure 7c shows modelling using the GL calculations expressions for the micro-bridge current-phase relationship, with symmetric inductances of 0.7pH in each arm, and a best fit value of \( \lambda_{eff}(0) = 173 \) nm. This is to be compared with \( \lambda_L(0) = 44 \) nm for bulk Nb with a \( T_c \) of 9.26K [18]. As can be seen from Figures 7c and 8, the full modelling predicts the temperature dependence of the SQUID critical current well, shows a triangular \( I_c(\Phi) \) dependence, and does significantly better than that using a sinusoidal current-phase relationship for the modulation depth.

**DISCUSSION**

Our data shows that the modulation depth is reduced compared to the short bridge model as soon as the coherence length becomes shorter than the bridge length. This effect is much more pronounced in the case of Nb as the intrinsic short coherence length of Nb leads rapidly to values of \( S/\xi \) larger than 1. The model describes very well the overall lineshape in the case of both the Al micro-SQUIDs, as well as the Nb micro-SQUIDs, which show pronounced triangular \( I_c(\Phi) \) characteristics. It is remarkable that the Nb micro-SQUIDs show quantum interference at all temperatures, even when the micro-bridges are much longer than the coherence length, and the phase gradients and supercurrent density depressions extend throughout the body of the SQUID. This implies that the heat dissipation that takes place when the SQUID enters the voltage state occurs well outside of the micro-bridge regions.

How can the sensitivity of these micro-SQUIDs be improved? The Nb SQUIDs are limited in their sensitivity by the short coherence length of Nb, as the modulation depth is maximal if the coherence length is equal to or longer than the bridge length. The coherence length could be increased by increasing the mean free path (epitaxial films or thicker layers), but an increase in critical current would lead to a diminution of the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length. A further way to increase sensitivity is by increasing the coherence length. A further way to increase sensitivity of the mean free path and thus to a diminution of the coherence length.
and allow for a standard DC-SQUID detection scheme.

**CONCLUSION**

In summary, the temperature and field dependence of the critical currents in micro-SQUIDs can be understood by means of numerical calculations based on the phase dependence of the critical current predicted by a 2-d Ginzburg-Landau numerical calculation. It is important to do this calculation for the full micro-SQUID structure in many cases, because of the spreading of the variation in the superconducting phase and depression in the supercurrent density beyond the micro-bridge region in this type of device.

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