On the Colour Suppressed
Decay Modes $\bar{B}^0 \to D_s^+ D_s^-$ and $\bar{B}_s^0 \to D^+ D^-$

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We point out that the decay modes $\bar{B}^0 \to D_s^+ D_s^-$ and $\bar{B}_s^0 \to D^+ D^-$ have no factorized contribution. At quark level these decays can only proceed through the annihilation mechanism, which in the factorized limit give zero amplitude due to current conservation. In this paper, we identify the dominating non-factorizable (colour suppressed) contributions in terms of two chiral loop contributions and one soft gluon emission contribution. The latter contribution can be calculated in terms of the (lowest dimension) gluon condensate within a recently developed heavy-light chiral quark model. We find branching ratios $BR(\bar{B}^0 \to D_s^+ D_s^-) \simeq 7 \times 10^{-5}$ and $BR(\bar{B}_s^0 \to D^+ D^-) \simeq 1 \times 10^{-3}$.

I. INTRODUCTION

There is presently great interest in decays of $B$-mesons, due to numerous experimental results coming from BaBar and Belle, and later at LHC.

It has been shown [1] that some classes of $B$-meson decay amplitudes exhibit QCD factorization. This means that, up to $\alpha_s/\pi$ (calculable) and $\Lambda_{QCD}/m_b$ (not calculable), their amplitudes factorize into the product of two matrix elements of weak currents. Typically, the decay amplitudes which factorize in this sense are $B \to \pi\pi$ and $B \to K\pi$ where the energy release is big compared to the light meson masses. However, for various decays of the type $\bar{B} \to DD$ where the energy release is of order 1 GeV, QCD factorization is not expected to hold. (Here $\bar{B}$, $D$, and $\bar{D}$ contain a heavy $b$, $c$, and anti-$c$ quark respectively).

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Such decay modes have been considered in connection with intermediate $D\bar{D}$ states for other $B$-decay modes\[2\].

In a previous paper\[3\], it was pointed out that the decay mode $D^0 \rightarrow K^0\bar{K}^0$ was zero in the factorized limit due to current conservation. However, there are in that case non-factorizable (colour suppressed) contributions in terms of chiral loops and soft gluon emission modelled by a gluon condensate.

In this paper we report on the following observation: The decay modes $\bar{B}^0 \rightarrow D_s^+D_s^-$ and $\bar{B}^0_s \rightarrow D^+D^-$ have no factorized (colour non-suppressed) contributions. At quark level, these decays a priori proceed through the annihilation mechanism $b\bar{s} \rightarrow c\bar{c}$ and $b\bar{d} \rightarrow c\bar{c}$, respectively. However, within the factorized limit the annihilation mechanism will give a zero amplitude due to current conservation, as for $D^0 \rightarrow K^0\bar{K}^0$. But there are non-zero factorized contributions through the axial part of the weak current if at least one of $D$-mesons in the final state is a vector meson $D^*$. Such contributions are, however, proportional to the numerically non-favourable Wilson coefficient $C_1$, which we will neglect in this short paper. In contrast, the typical factorized decay modes which proceed through the spectator mechanism, say $\bar{B}^0 \rightarrow D^+D_s^-$, are proportional to the numerically favourable Wilson coefficient $C_2$. If the mesons in this amplitude are also allowed to be vector mesons, such amplitudes will generate non-factorizable ($\sim 1/N_c$) chiral loop contributions to the process $\bar{B}^0_d \rightarrow D_s^+D_s^-$ due to $K^0$-exchange. These will be considered in the present paper.

There are also non-factorizable ($\sim 1/N_c$) contributions due to soft gluon emission. Such contributions can be calculated in terms of the (lowest dimension) gluon condensate within a recently developed Heavy Light Chiral Quark Model (HL$\chi$QM)\[4\], which is based on Heavy Quark Effective Theory (HQEFT)\[5\]. This model has been applied to processes with $B$-mesons in \[6, 7\]. The gluon condensate contributions is also proportional to the favourable Wilson coefficient $C_2$.

In the next section (II), we shortly present the four quark Lagrangian at quark level. In section III we present our analysis of chiral loop contributions within the heavy light chiral perturbation theory. In section IV we give the calculation of non-factorizable matrix elements due to soft gluons expressed through the (model dependent) quark condensate. In section V we give the results and conclusion. Throughout the paper, we will give formulae and figures for the decay mode $\bar{B}^0 \rightarrow D_s^+D_s^-$. The treatment of $\bar{B}^0_s \rightarrow D^+D^-$ will proceed analogously.
II. EFFECTIVE NON-LEPTONIC LAGRANGIAN AT QUARK LEVEL

Based on the electroweak and quantum chromodynamical interactions, one constructs an effective Lagrangian at quark level in the standard way:

$$\mathcal{L}_W = \sum_i C_i(\mu) \, Q_i(\mu) \, ,$$

where all information of the short distance (SD) loop effects above a renormalization scale $\mu$ is contained in the (Wilson) coefficients $C_i$. In our case there are two relevant operators

$$Q_1 = 4 (\bar{q}_L \gamma^\alpha b_L) \, (\bar{c}_L \gamma_\alpha c_L) \, , \quad Q_2 = 4 (\bar{c}_L \gamma^\alpha b_L) \, (\bar{q}_L \gamma_\alpha c_L) \, ,$$

for $q = d, s$. Penguin operators may also contribute, but have small Wilson coefficients. We may write

$$C_i = - \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* a_i$$

where $a_1 \sim 10^{-1}$ and $a_2 \sim 1$ at the scale $\mu = m_b$. Performing perturbative QCD corrections within Heavy Quark Effective Theory (HQEFT) [5], the effective Lagrangian (11) can be evolved down to the scale $\mu \sim \Lambda_\chi \sim 1 \text{ GeV}$ [8, 9], where one finds $|a_1| \simeq 0.4$ and $|a_2| \simeq 1.4$. The $b, c,$ and $\tau$ quarks are then treated within HQEFT.

In order to study non-factorizable contributions at quark level, we may use the following relation between the generators of $SU(3)_c$ ($i, j, l, n$ are colour indices running from 1 to 3):

$$\delta_{ij} \delta_{ln} = \frac{1}{N_c} \delta_{in} \delta_{lj} + 2 t^a_{ln} t^a_{lj} \, ,$$

where $a$ is the color octet index. Then the operators $Q_{1,2}$ may, by means of a Fierz transformation, be written in the following way:

$$Q_1 = \frac{1}{N_c} Q_2 + 2 \tilde{Q}_2 \, , \quad Q_2 = \frac{1}{N_c} Q_1 + 2 \tilde{Q}_1 \, ,$$

where the operators with the “tilde” contain colour matrices:

$$\tilde{Q}_1 = 4 (\bar{q}_L \gamma^\alpha t^a b_L) \, (\bar{c}_L \gamma_\alpha t^a c_L) \, , \quad \tilde{Q}_2 = 4 (\bar{c}_L \gamma^\alpha t^a b_L) \, (\bar{q}_L \gamma_\alpha t^a c_L) \, .$$

To obtain a physical amplitude, one has to calculate the hadronic matrix elements of the quark operators $Q_i$ within some framework describing long distance (LD) effects.

As an example of a typical factorized case we choose the amplitude for $B^0 \to D^+ D_s^-$ obtained from (11) and (2):

$$\langle D_s^- D^+ | \mathcal{L}_W | B^0 \rangle_F = - (C_2 + \frac{1}{N_c} C_1) \langle D_s^- | s \gamma_\mu \gamma_5 c | 0 \rangle \langle D^+ | \bar{c} \gamma_\mu \bar{b} | B^0 \rangle \, ,$$
FIG. 1: Factorized contribution for $\bar{B}^0 \to D^+D_s^-$ through the spectator mechanism, which does not exist for decay mode $\bar{B}^0 \to D_s^+D_s^-$ we consider in this paper. The double dashed lines represent heavy mesons, the double lines represent heavy quarks, and the single lines light quarks.

FIG. 2: Factorized contribution for $\bar{B}^0 \to D_s^+D_s^-$ through the annihilation mechanism, which give zero contributions if both $D_s^+$ and $D_s^-$ are pseudoscalars.

which will in section III be compared with our chiral loop contributions. This term is proportional to the $D$-meson decay constant times the Isgur-Wise function (for $\bar{B} \to D$ transition) and is visualized in figure 1.

The factorized amplitude for $\bar{B}^0 \to D_s^+D_s^-$ obtained from (1) and (2) is visualized in figure 2 and is given by

$$\langle D_s^-D_s^+|\mathcal{L}_W|\bar{B}^0\rangle_F = 4(C_1 + \frac{1}{N_c}C_2)\langle D_s^-D_s^+|\bar{c}L\gamma_\mu c_L|0\rangle\langle 0|d_L\gamma^\mu b_L|\bar{B}^0\rangle.$$ \hspace{1cm} (7)

Unless one or both of the $D$-mesons in the final state are vector mesons, this matrix element is zero due to current conservation:

$$\langle D_s^+D_s^-|\bar{c}\gamma_\mu c|0\rangle\langle 0|d\gamma^\mu\gamma_5 b|\bar{B}^0\rangle \sim f_B(p_D + p_D)^\mu \langle D_s^+D_s^-|\bar{c}\gamma_\mu c|0\rangle = 0.$$ \hspace{1cm} (8)

The genuine non-factorizable part for $\bar{B}^0 \to D_s^+D_s^-$ can be written in terms of colored
currents (see eqs. (4) and (5)):

$$\langle D_s^- D_s^+ | \mathcal{L}_W | B^0 \rangle_{NF} = 8C_2 \langle D_{sL}^- D_{sL}^+ | (\bar{u}_L \gamma^\alpha t^a b_L) (\bar{c}_L \gamma_\alpha t^a c_L) | B^0 \rangle$$

(9)

We observe that the annihilation mechanism amplitude in the non-factorizable case has the numerically favourable Wilson coefficient $C_2$. This amplitude is, within the HλQM visualized later in figure [4].

III. HEAVY LIGHT CHIRAL PERTURBATION THEORY

Our calculations will be based on HQEFT [5], which is a systematic $1/m_Q$ expansion in the heavy quark mass $m_Q$. The heavy quark field $Q(x) = b(x)$ (eventually $c(x)$ or $\bar{c}$) is replaced with a “reduced” field $Q_v^{(\pm)}(x)$ for a heavy quark, and $Q_v^{(-)}(x)$ for a heavy antiquark. These are related to the full field $Q(x)$ in the following way:

$$Q_v^{(\pm)}(x) = P_\pm e^{im_v v \cdot x} Q(x) ,$$

(10)

where $P_\pm$ are projecting operators $P_\pm = (1 \pm \gamma \cdot v)/2$. The Lagrangian for heavy quarks is:

$$\mathcal{L}_{HQEFT} = \pm Q_v^{(\pm)} i v \cdot D Q_v^{(\pm)} + \mathcal{O}(m_Q^{-1}) ,$$

(11)

where $D_\mu$ is the covariant derivative containing the gluon field. In [7] the $1/m_Q$ corrections were calculated for $B - \bar{B}$ -mixing. In this paper these will not be considered.

Integrating out the heavy and light quarks, the effective Lagrangian up to $\mathcal{O}(m_Q^{-1})$ can be written as [4, 10]

$$\mathcal{L} = \mp Tr \left[ H_{a}^{(\pm)} i v \cdot D_{ba} H_b^{(\pm)} \right] - g_A Tr \left[ H_{a}^{(\pm)} H_b^{(\pm)} \gamma_5 A_\mu^{ba} \right] ,$$

(12)

where $H_{a}^{(\pm)}$ is the heavy meson field containing a spin zero and spin one boson:

$$H_{a}^{(\pm)} \equiv P_\pm (P_{a\mu}^{(\pm)} \gamma^\mu - i P_{a5}^{(\pm)} \gamma_5 ) .$$

(13)

The fields $P_{M}^{(\pm)}(P_{M}^{(-)})$ annihilates (creates) a heavy meson (vector for $M = \mu$ and pseudoscalar for $M = 5$) containing a heavy quark (anti-quark) with velocity $v$. Furthermore, $i D_{ba}^\mu = i \delta_{ba} D^\mu - \mathcal{V}_{ba}^\mu$, and $a, b$ are flavour indices. The vector and axial vector fields $\mathcal{V}_\mu$ and $A_\mu$ contain the field $\xi$ (and its first derivative) which is a 3 by 3 matrix containing the (would be) Goldstone octet $(\pi, K, \eta)$:

$$\mathcal{V}_\mu \equiv \frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi \partial_\mu \xi^\dagger) \ ; \ A_\mu \equiv -\frac{i}{2} (\xi \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \ , \ \xi \equiv expi(\Pi/f)$$

(14)
where \( f \) is the bare pion coupling, and \( \Pi \) is a 3 by 3 matrix which contains the Goldstone bosons \( \pi, K, \eta \) in the standard way. The axial chiral coupling is \( g_A \simeq 0.6 \). Eqs. (12), (13), and (14) will be used for the chiral loop contributions.

The simplest way to calculate the matrix element of four quark operators like \( Q_{1,2} \) in eq. (1) is by inserting vacuum states between the two currents, as indicated in section II. This vacuum insertion approach (VSA) corresponds to bosonizing the two currents in \( Q_{1,2} \) separately and multiply them, i.e. the factorized case. Based on the symmetry of HQEFT, the bosonized current for decay of the \( b\bar{q} \) system is [4, 10]:

\[
\bar{q}_L \gamma^\mu Q^{(+)}_v \longrightarrow \frac{\alpha_H}{2} Tr \left[ \xi^\dagger \gamma^\alpha L H^{(+)}_b \right],
\]

(15)

where \( Q^{(+)}_v \) is a heavy \( b \)-quark field, \( v \) is its velocity, and \( H^{(+)}_b \) is the corresponding heavy meson field. This bosonization has to be compared with the matrix elements defining the meson decay constants \( f_H \) (\( H = B, D \)) are the same when QCD corrections below \( m_Q \) are neglected (see [4, 5]):

\[
\alpha_H = \frac{f_H \sqrt{M_H}}{(C_v + C_\gamma)},
\]

(16)

where \( C_{v,\gamma} \) are Wilson coefficients due to perturbative QCD for scales \( \mu < m_Q \) (\( Q = b, c \) for \( H = B, D \)). We take \( \mu = \Lambda_\chi \), which is the scale where perturbative QCD are matched to our hadronic matrix elements.

For the \( W \)-boson materializing to a \( \bar{D} \) we obtain the bosonized current

\[
\bar{q}_L \gamma^\mu Q^{(-)}_\bar{v} \longrightarrow \frac{\alpha_H}{2} Tr \left[ \xi^\dagger \gamma^\alpha L H^{(-)}_c \right],
\]

(17)

where \( \bar{v} \) is the velocity of the heavy \( \bar{c} \) quark and \( H^{(-)}_c \) is the corresponding field for the \( \bar{D} \) meson.

For the \( b \to c \) transition, we obtain the bosonized current

\[
\bar{Q}^{(+)}_v \gamma^\mu LQ^{(+)}_v \longrightarrow -\zeta(\omega) Tr \left[ H^{(+)}_c \gamma^\alpha L H^{(+)}_b \right],
\]

(18)

where \( \zeta(\omega) \) is the Isgur-Wise function for the \( B \to D \) transition, and \( v' \) is the velocity of the heavy \( c \)-quark. Furthermore, \( \omega \equiv v \cdot v' = v \cdot \bar{v} = M_B/(2M_D) \). Note also that from conservation of momentum we find the relation between the heavy quark velocities:

\[
p_B = p_D + p_D \implies v^\mu = \frac{M_D}{M_B} (v' + \bar{v})^\mu.
\]

(19)
FIG. 3: Non-factorizable chiral loops for $B^0 \to D_s^+ D_s^-$.

For the weak current for $D \bar{D}$ production (corresponding to the factorizable annihilation mechanism) we obtain

$$
Q^{(+)}_{\nu'} \gamma^\mu L Q^{(-)}_{\nu'} \longrightarrow -\zeta(-\lambda) Tr \left[ H^0 \gamma^\alpha L H^0 \right],
$$

(20)

where $\lambda = \bar{v} \cdot v' = [M_B^2/(2M_D^2) - 1]$. The Isgur-Wise function $\zeta(-\lambda)$ in (20) is a complex function, and not so well-known as for the $b \to c$ transition. In the factorized limit, the matrix elements of the four quark operators are obtained by multiplying the bosonized currents above.

In the following we will consider explicitly the decay mode $B^0 \to D_s^+ D_s^-$. The analysis of $B_s^0 \to D^+ D^-$ proceed the same way. To calculate the chiral loop amplitudes we need the (factorized) amplitudes for $B_s^0 \to D^+ D^-$ and $B^0 \to D^{*+} D^{*-}$, which proceed through the spectator mechanism as in figure 1. The point is that the leading chiral coupling obtained from (12) is a coupling between a pseudoscalar meson $H$, vector meson $H^*$ a light pseudoscalar $M (= \pi, K, \eta)$. Using the bosonized currents in eqs. (17) and (18), we obtain the following chiral loop amplitude for the process $B^0 \to D_s^+ D_s^-$ from the figure 3:

$$
A(B^0 \to D_s^+ D_s^-)_{\chi} = (V_{cd}^*/V_{cs}^*) A(B_d^0 \to D_d^+ D_d^-)_{F} \cdot R^x,
$$

(21)

where the factorized amplitude for the process $B^0 \to D^+ D^-$ is

$$
A(B^0 \to D^+ D^-)_{F} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 \zeta(\omega) f_D M_D \sqrt{M_B M_D} (\lambda + \omega).
$$

(22)

The quantity $R^x$ is a sum of contributions $R_{1,2}^x$ from the left and right part of figure 3 respectively. In the $\overline{MS}$ scheme the results for $R_{1,2}^x$ are

$$
R_1^x = \frac{m_K^2}{2(4\pi f)^2} g_A^2 \left\{ \frac{2(\omega + 1)}{\omega + \lambda} r(-\omega) - 1 \right\} \ln \left( \frac{m_K^2}{\Lambda^2} \right) - 1,
$$

(23)

$$
R_2^x = \frac{m_K^2}{2(4\pi f)^2} g_A^2 \left\{ \frac{2(\omega + 1)}{\lambda + \omega} r(-\lambda) - 1 \right\} \ln \left( \frac{m_K^2}{\Lambda^2} \right) - 1.
$$

(24)
FIG. 4: Non-factorizable contribution for $B^0 \to D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emission. The wavy lines represent soft gluons ending in vacuum to make gluon condensates.

Adding these two contributions we find:

$$R^x = \frac{m_K^2}{(4\pi f)^2} g_A^2 \left[ \left\{ \frac{(\omega + 1)}{(\omega + \lambda)} [r(-\omega) + r(-\lambda)] - 1 \right\} \ln \left( \frac{m_K^2}{\Lambda^2} \right) - 1 \right]$$

(25)

As usual, the $1/N_c$ suppression is due to $f^2 \sim N_c$. The function $r(x)$ is also appearing in loop calculations [8, 9] of the anomalous dimension in HQEFT (for $x > 1$ and $x < -1$ respectively):

$$r(x) \equiv \frac{1}{\sqrt{x^2 - 1}} \ln \left( x + \sqrt{x^2 - 1} \right), \quad r(-x) = -r(x) + i\pi \frac{i}{\sqrt{x^2 - 1}},$$

(26)

which means that the amplitude gets an imaginary part. Numerically, we find

$$R^x \simeq 0.12 - 0.26i.$$  

(27)

IV. NON-FACTORIZABLE SOFT GLUON EMISSION

The genuine non-factorizable part (see eqs. [4], [5] and [9]) can, within the framework presented in this section, be written in a quasi-factorized way in terms of matrix elements of colored currents:

$$\langle D_s^+ D_s^- |\mathcal{L}_W |\overline{B^0} \rangle_{NF}^G = 8 C_2 \langle D_s^+ D_s^- |\overline{c_L} \gamma_\mu \gamma_5 t^a c_L |G\rangle \langle G| \overline{d_L} \gamma_\mu \gamma_5 t^a b_L |\overline{B^0} \rangle,$$

(28)

where a $G$ in the bra-kets symbolizes emission of one gluon (from each current) as visualized in figure 4. We observe that the annihilation mechanism amplitude in the non-factorizable case has the numerically favourable Wilson coefficient $C_2$. 
In order to calculate the matrix elements (28), we will use a model which incorporates emission of soft gluons modelled by a gluon condensate. This will be the Heavy Light Chiral Quark Model (HLχQM) recently developed in [4]. This model belongs to a class of models extensively studied in the literature [11, 12, 13, 14, 15, 16]. For details we refer to ref. [4].

The Lagrangian for the HLχQM is

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQEFT} + \mathcal{L}_{\chi QM} + \mathcal{L}_{\text{Int}}. \quad (29)$$

The first term is given in equation (11). The light quark sector is described by the Chiral Quark Model ($\chi QM$), having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons:

$$\mathcal{L}_{\chi QM} = \bar{\chi} \left[ \gamma^\mu (iD_\mu + V_\mu + \gamma_5 A_\mu) - m \right] \chi. \quad (30)$$

Here $m$ is the SU(3) invariant constituent light quark mass, and $\chi$ is the flavour rotated quark fields given by $\chi_L = \xi^T q_L$, $\chi_R = \xi q_R$, where $q^T = (u, d, s)$ are the light quark fields. The left- and right-handed projections $q_L$ and $q_R$ are transforming after $SU(3)_L$ and $SU(3)_R$ respectively. In (30) we have discarded terms involving the light current quark mass which is irrelevant in the present paper. The covariant derivative $D_\mu$ in (30) contains the soft gluon field forming the gluon condensates. The gluon condensate contributions are calculated by Feynman diagram techniques as in [4, 6, 7, 17, 18]. They may also be calculated by means of heat kernel techniques as in [15, 16, 19].

The interaction between heavy meson fields and heavy quarks are described by the following Lagrangian [4]:

$$\mathcal{L}_{\text{Int}} = -G_H \left[ \bar{\chi}_a \overline{H_a^{(\pm)}} Q_v^{(\pm)} + \overline{Q_v^{(\pm)}} H_a^{(\pm)} \chi_a \right], \quad (31)$$

where $G_H \sim \sqrt{2m/f}$ is a coupling constant. In [4] it was shown how (12) could be obtained from the HLχQM. Performing this bosonization of the HLχQM, one encounters divergent loop integrals which will in general be quadratic-, linear- and logarithmic divergent [4]. Also, as in the light sector [18] the quadratic and logarithmic integrals are related to the quark condensate and the gluon condensate respectively.

Within the model, one finds the following expression for the Isgur-Wise function [4]

$$\zeta(\omega) = \frac{2}{1 + \omega} (1 - \rho) + \rho r(\omega), \quad (32)$$
where $\rho$ is a hadronic parameter giving the deviation from the leading value $^4$:

$$G_H^2 = \frac{2m}{f^2} \rho \quad \rho \equiv \frac{(1 + 3g_A) + \frac{\mu^2}{m_f^2}}{4(1 + \frac{\eta m_f^2}{8\pi f^2})}, \quad \mu_2^2(H) = \frac{3}{2}m_Q(M_{H^*} - M_H),$$

(33)

where $\eta = (1 + 2/\pi)$. Numerically, the deviation of $\rho$ one is of order 10%. The simple expression in (32) is modified by perturbative QCD corrections down to $\mu = \Lambda$ (analogous to eq. (16)) and chiral loop corrections. Our model dependent expression $\zeta(\omega)$ in (32) give a good description of the Isgur-Wise function.

The left part in figure 2 with gluon emission gives us the bosonized coloured current:

$$(q^a_L t^a \gamma^\alpha Q^{(+)}_{\bar{v}B})_{1G} \rightarrow -\frac{G_H g_s}{64\pi} G^a_{\mu\nu} \text{Tr} \left[ \xi^\dagger \gamma^\alpha L H_b^{(+)} \left( \sigma^{\mu\nu} - \frac{2\pi f^2}{m^2 N_c} \{\sigma^{\mu\nu}, \gamma \cdot v\} \right) \right],$$

(34)

where $G^a_{\mu\nu}$ is the octet gluon tensor, and $H_b^{(+)}$ represents the heavy $\bar{B}$-meson fields. Similarly the (heavy) $D$- and $\bar{D}$-mesons are represented by $H_c^{(+)}$ and $H_\bar{c}^{(+)}$ corresponding to a heavy quark field $Q^{(+)}_{\nu'}$ and heavy anti-quark field $Q^{(-)}_{\bar{v}}$ respectively. The symbol $\{ , \}$ denotes the anti-commutator.

For the creation of a $D\bar{D}$ pair in the right part of figure 2 the analogue of (34) is

$$(Q^{(+)}_{\nu'} t^a \gamma^\alpha L Q^{(-)}_{\bar{v}})_{1G} \rightarrow \frac{G_H^2 g_s}{32\pi} G^a_{\mu\nu} \text{Tr} \left[ H_c^{(+)} \gamma^\alpha L H_\bar{c}^{(-)} \right] \times \left( \frac{\tilde{r}}{\pi} \sigma^{\mu\nu} + \frac{1}{4m(\lambda - 1)} \{\sigma^{\mu\nu}, \gamma \cdot t\} \right),$$

(35)

where $t = v' - \bar{v}$, and $\tilde{r} \equiv r(-\lambda)$. Multiplying the currents in eqs. (34) and (35), and using the replacement:

$$g_s^2 G^a_{\mu\nu} G^a_{\alpha\beta} \rightarrow 4\pi^2 \frac{\alpha_s}{\pi} G^2 \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}),$$

(36)

we obtain the bosonized version for the operator $\tilde{Q}_1$ in eq. (5) (see also eq. (28)) as the product of two traces. (The expression may be simplified by using the Dirac algebra, but we do not enter these details here).

Taking the pseudoscalar parts of (34) and (35), we find the gluon condensate contribution for $\overline{B}^0 \rightarrow D_s^+ D_s^-$ within our model:

$$A(\overline{B}^0 \rightarrow D_s^+ D_s^-)_{G} = C_2 \alpha_s G^2 \frac{(G_H \sqrt{M_B})^3}{384m} \left( 1 + \frac{3\tilde{r}}{\pi} \right).$$

(37)

(For our algebraic manipulations, the program FORM [20] was useful). The ratio between this amplitude and the factorized one in (22) scales as $M_D/(N_c M_B)$ times hadronic parameters calculated within HL$\chi$QM. We define a quantity $R^G$ for the gluon condensate
amplitude analogously to $R^x$ in (21) and (23) for chiral loops. Numerically, we find that the ratio between the two amplitudes in (37) and (22) is

$$R_G \simeq 0.055 + 0.16i,$$

which is of order one third of the chiral loop contribution in eq. (25).

V. DISCUSSION AND RESULTS

Our amplitude is complex as expected. In the chiral loop amplitude these are due to physical cuts (exchanges of physical particles) to the one-loop order we consider in this paper. The Wilson coefficients turn complex when the c-quark is treated within HQEFT. This is also the case for the matrix elements that these Wilson coefficients should be matched to. There is a potential problem with a quark model without confinement that the amplitude may get an imaginary part due to production of free quarks. Still, within HQEFT one can hardly distinguish $m_c$ from $M_D$ because of the reparametrization invariance. Thus, at the present stage, it is not clear how well our model describes imaginary matrix elements, and we will not go into such details here, as the numerical consequences turn out to be minor.

Adding the amplitudes $R_x$ and $R_G$ and multiplying with the Wilson coefficient $a_2 \simeq 1.33 + 0.2i$, we obtain the quantity:

$$\tilde{R}_T \equiv a_2 (R_x + R_G) \simeq 0.26 - 0.11i.$$  (39)

Dropping the imaginary parts of the three quantities would give instead the value $\simeq 0.25$. Anyway, we have found that the amplitude for $B^0 \to D^+_s D^-_s$ is of order $15 - 20\%$ of the factorizable amplitude for $B^0 \to D^+ D^-_s$, before the different KM-factors are taken into account. We obtain the branching ratios

$$BR(B^0 \to D^+_s D^-_s) = 6.5 \times 10^{-5} \times |\frac{V_{cb}}{0.041} \frac{V_{cs}^*}{0.974} \frac{\tilde{R}_T}{0.25} \frac{\zeta(\omega)}{0.9}|^2$$  (40)

and

$$BR(B^0_s \to D^+ D^-) = 8.9 \times 10^{-4} \times |\frac{V_{cb}}{0.041} \frac{V_{cs}^*}{0.974} \frac{\tilde{R}_T}{0.25} \frac{\zeta(\omega)}{0.9}|^2$$  (41)

The difference between the branching ratios is mainly due to the difference in KM factor. Taking into account the comments above, we end up with the conclusion that

$$BR(B^0 \to D^+_s D^-_s) \simeq 7 \times 10^{-5} \text{, and } BR(B^0_s \to D^+_s D^-_s) \simeq 1 \times 10^{-3}.$$  (42)
The ongoing searches at Belle might soon give the limit on the rate $B^0 \to D^+_s D^-_s$, while the detection of the $B^0_s$ mode might be presently more difficult due to troubles with $B^0_s$ identification.

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