Precision Electroweak Measurements and Constraints on the Standard Model

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Abstract

This note presents constraints on Standard Model parameters using published and preliminary precision electroweak results measured at the electron-positron colliders LEP and SLC. The results are compared with precise electroweak measurements from other experiments, notably CDF and DØ at the Tevatron. Constraints on the input parameters of the Standard Model are derived from the results obtained in high-$Q^2$ interactions, and used to predict results in low-$Q^2$ experiments, such as atomic parity violation, Möller scattering, and neutrino-nucleon scattering.
1 Introduction

The experimental results used here consist of the final and published Z-pole results [1] measured by the ALEPH, DELPHI, L3, OPAL and SLC experiments, taking data at the electron-positron colliders LEP and SLC. In addition, published and preliminary results on the mass of the W boson, measured at LEP-II and the Tevatron, and the mass of the top quark, measured at the Tevatron only, are included.

The measurements allow to check the validity of the Standard Model (SM) and, within its framework, to infer valuable information about its fundamental parameters. The accuracy of the W- and Z-boson measurements makes them sensitive to the mass of the top quark \( m_t \), and to the mass of the Higgs boson \( m_H \) through loop corrections. While the leading \( m_t \) dependence is quadratic, the leading \( m_H \) dependence is logarithmic. Therefore, the inferred constraints on \( m_t \) are much stronger than those on \( m_H \).

2 Measurements

The measurement results considered here are reported in Table 1. Also shown are their predictions based on the results of the SM fit to these combined high-\( Q^2 \) measurements, reported in the last column of Table 1. The measurements obtained at the Z pole by the LEP and SLC experiments ALEPH, DELPHI, L3, OPAL and SLD, and their combinations, reported in parts a), b) and c) of Table 1, are final and published [1].

The results on the W-boson mass by UA2 [7], CDF [8] and DØ [9] in Run-I, and the W-boson width by CDF [10] and DØ [11] in Run-I, are combined by the Tevatron Electroweak Working Group based on a detailed treatment of common systematic uncertainties [12]. Including also the recent result on \( m_W \) based on Run-II data by CDF [13], the combined results are:\[ m_W = 80429 \pm 39 \text{ MeV} , \quad \Gamma_W = 2078 \pm 87 \text{ MeV}. \]

Combining these results with the preliminary LEP-II combination [2], the resulting averages used here are:

\[
\begin{align*}
m_W &= 80.398 \pm 0.025 \text{ GeV} \\
\Gamma_W &= 2.140 \pm 0.060 \text{ GeV} .
\end{align*}
\]

For the mass of the top quark, \( m_t \), the published Run-I results from CDF [15] and DØ [16], and recent preliminary and published results based on Run-II data from CDF [17–20] and DØ [21, 22], are combined by the Tevatron Electroweak Working Group with the result: \( m_t = 170.9 \pm 1.8 \text{ GeV} \) [23].

In addition, the following final results obtained in low-\( Q^2 \) interactions and reported in Table 3 are considered: (i) the measurements of atomic parity violation in caesium [24, 25], with the numerical result [26] taken from a recently published revised analysis of QED radiative corrections applied to the raw measurement; (ii) the result of the E-158 collaboration on the electroweak mixing angle\(^3\) measured in Möller scattering [28]; and (iii) the final result of the NuTeV collaboration on neutrino-nucleon neutral to charged current cross section ratios [29].

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1 Since our last report [2], the following results have been published which are not included in the results used in the following: [3–6].
2 The combined Tevatron result quoted for \( \Gamma_W \) and used here does not take into account the very recently published CDF result based on Run-II data, \( \Gamma_W = 2.032 \pm 0.073 \text{ GeV} \) [14].
3 E-158 quotes in the MSbar scheme, evolved to \( Q^2 = m_Z^2 \). We add 0.00029 to the quoted value in order to obtain the effective electroweak mixing angle [27].
Using neutrino-nucleon data with an average $Q^2 \simeq 20 \text{ GeV}^2$, the NuTeV collaboration has extracted the left- and right-handed couplings combinations $g^2_{\alpha L u d} = 4g^2_{\nu L u} (g^2_{\alpha L u} + g^2_{\alpha L d}) = [1/2 - \sin^2 \theta_{\text{eff}} + (5/9) \sin^4 \theta_{\text{eff}}] \rho_{\nu} \rho_{u d}$ and $g^2_{\alpha R u d} = 4g^2_{\nu R u} (g^2_{\alpha R u} + g^2_{\alpha R d}) = (5/9) \sin^4 \theta_{\text{eff}} \rho_{\nu} \rho_{u d}$: $g^2_{\alpha L u d} = 0.30005 \pm 0.00137$ and $g^2_{\alpha R u d} = 0.03076 \pm 0.00110$, with a correlation of $-0.017$. While the result on $g_{\alpha R u d}$ agrees with the SM expectation, the result on $g_{\alpha L u d}$, measured nearly eight times more precisely, shows a deficit with respect to the expectation at the level of 2.8 standard deviations.

An additional input parameter, not shown in the table, is the Fermi constant $G_F$, determined from the $\mu$ lifetime, $G_F = 1.16637(1) \cdot 10^{-5} \text{ GeV}^{-2}$ [30]. Recent new measurements of $G_F$ yield values which are in good agreement [31, 32]. The relative error of $G_F$ is comparable to that of $m_Z$: both errors have negligible effects on the fit results.

### 3 Theoretical and Parametric Uncertainties

Detailed studies of the theoretical uncertainties in the SM predictions due to missing higher-order electroweak corrections and their interplay with QCD corrections had been carried out by the working group on ‘Precision calculations for the Z resonance’ [33], and later in [34, 35]. Theoretical uncertainties are evaluated by comparing different but, within our present knowledge, equivalent treatments of aspects such as resummation techniques, momentum transfer scales for vertex corrections and factorisation schemes. The effects of these theoretical uncertainties are reduced by the inclusion of higher-order corrections [36, 37] in the electroweak libraries TOPAZ0 [38] and ZFITTER [39].

The use of the higher-order QCD corrections [37] increases the value of $\alpha_S(m_Z^2)$ by 0.001, as expected. The effects of missing higher-order QCD corrections on $\alpha_S(m_Z^2)$ covers missing higher-order electroweak corrections and uncertainties in the interplay of electroweak and QCD corrections. A discussion of theoretical uncertainties in the determination of $\alpha_S$ can be found in References 33 and 40, with a recent analysis in Reference 41 where the theoretical uncertainty is estimated to be about 0.001 for the analyses presented in the following.

Recently, the complete (fermionic and bosonic) two-loop corrections for the calculation of $m_W$ [42], and the complete fermionic two-loop corrections for the calculation of $\sin^2 \theta_{\text{eff}}$ [43] have been calculated. Including three-loop top-quark contributions to the $\rho$ parameter in the limit of large $m_t$ [44], efficient routines for evaluating these corrections have been implemented since version 6.40 in the semi-analytical program ZFITTER. The remaining theoretical uncertainties are estimated to be 4 MeV on $m_W$ and 0.000049 on $\sin^2 \theta_{\text{eff}}$. The latter uncertainty dominates the theoretical uncertainty in SM fits and the extraction of constraints on the mass of the Higgs boson presented below. For a complete picture, the complete two-loop calculation for the partial $Z$ decay widths should be calculated.

The determination of the size of remaining theoretical uncertainties is under continuous study. The theoretical errors discussed above are not included in the results presented in Tables 2 and 3. At present the impact of theoretical uncertainties on the determination of SM parameters from the precise electroweak measurements is small compared to the error due to the uncertainty in the value of $\alpha(m_Z^2)$, which is included in the results.

The uncertainty in $\alpha(m_Z^2)$ arises from the contribution of light quarks to the photon vacuum polarisation ($\Delta \alpha^{(5)}_{\text{had}}(m_Z^2)$):

$$\alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta \alpha(m_Z^2) - \Delta \alpha^{(5)}_{\text{had}}(m_Z^2) - \Delta \alpha_{\text{top}}(m_Z^2)},$$ (3)
where \( \alpha(0) = 1/137.036 \). The top contribution, \(-0.00007(1)\), depends on the mass of the top quark, and is therefore determined inside the electroweak libraries TOPAZ0 and ZFITTER. The leptonic contribution is calculated to third order \([45]\) to be 0.03150, with negligible uncertainty.

For the hadronic contribution, we no longer use the value 0.02804 \(\pm 0.00065\) \([46, 47]\), but rather the new evaluation 0.02758 \(\pm 0.00035\) \([48]\) which takes into account published results on electron-positron annihilations into hadrons at low centre-of-mass energies by the BES collaboration \([49]\), as well as the revised published results from CMD-2 \([50]\) and new results from KLOE \([51]\). The reduced uncertainty still causes an error of 0.00013 on the SM prediction of \(\sin^2 \theta_{\text{lept}}\), and errors of 0.2 GeV and 0.1 on the fitted values of \(m_t\) and \(\log(m_H)\), included in the results presented below. The effect on the SM prediction for \(\Gamma_{\ell\ell}\) is negligible. The \(\alpha_S\) values from the SM fits presented here are stable against a variation of \(\alpha(\mu^2)\) in the interval quoted.

There are also several evaluations of \(\Delta \alpha^{(5)}_{\text{had}}(m_Z^2)\) \([52–62]\) which are more theory-driven. The most recent of these (Reference 62) also includes the new results from BES, yielding 0.02749 \(\pm 0.00012\).

To show the effects of the uncertainty of \(\alpha(\mu^2)\), we also use this evaluation of the hadronic vacuum polarisation. Note that all these evaluations obtain values for \(\Delta \alpha^{(5)}_{\text{had}}(m_Z^2)\) consistently lower than - but in agreement with - the old value of 0.02804 \(\pm 0.00065\).

4 Selected Results

Figure 1 shows a comparison of the leptonic partial width from LEP-I, \(\Gamma_{\ell\ell} = 83.985 \pm 0.086\) MeV \([1]\), and the effective electroweak mixing angle from asymmetries measured at LEP-I and SLD, \(\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23153 \pm 0.00016\) \([1]\), with the SM shown as a function of \(m_t\) and \(m_H\). Good agreement with the SM prediction using the most recent measurements of \(m_t\) and \(m_W\) is observed. The point with the arrow indicates the prediction if among the electroweak radiative corrections only the photon vacuum polarisation is included, which shows that the precision electroweak Z-pole data are sensitive to non-trivial electroweak corrections. Note that the error due to the uncertainty on \(\alpha(m_Z^2)\) (shown as the length of the arrow) is not much smaller than the experimental error on \(\sin^2 \theta_{\text{eff}}^{\text{lept}}\) from LEP-I and SLD. This underlines the continued importance of a precise measurement of \(\sigma(e^+e^- \to \text{hadrons})\) at low centre-of-mass energies.

Of the measurements given in Table 1, \(R^{0}_{\ell}\) is one of the most sensitive to QCD corrections. For \(m_Z = 91.1875\) GeV, and imposing \(m_t = 170.9 \pm 1.8\) GeV as a constraint, \(\alpha_S = 0.1221 \pm 0.0037\) is obtained. Alternatively, \(\sigma^{0}_{\text{lep}} \equiv \sigma^{0}_{\text{had}}/R^{0}_{\ell} = 2.0003 \pm 0.027\) nb \([1]\) which has higher sensitivity to QCD corrections and less dependence on \(m_H\) yields: \(\alpha_S = 0.1177 \pm 0.0030\). Typical errors arising from the variation of \(m_H\) between 100 GeV and 200 GeV are of the order of 0.001, somewhat smaller for \(\sigma^{0}_{\text{lep}}\).

These results on \(\alpha_S\), as well as those reported in the next section, are in very good agreement with world averages \((\alpha_S(m_Z^2) = 0.118 \pm 0.002\) \([63]\), or \(\alpha_S(m_Z^2) = 0.1178 \pm 0.0033\) based solely on NNLO QCD results excluding the LEP-I lineshape results and accounting for correlated errors \([64]\)).
### Table 1: Summary of high-$Q^2$ measurements included in the combined analysis of SM parameters.

Section a) summarises LEP-I averages, Section b) SLD results ($A_\ell$ includes $A_{LR}$ and the polarised lepton asymmetries), Section c) the LEP-I and SLD heavy flavour results, and Section d) electroweak measurements from LEP-II and the Tevatron. The total errors in column 2 include the systematic errors listed in column 3. Although the systematic errors include both correlated and uncorrelated sources, the determination of the systematic part of each error is approximate. The SM results in column 4 and the pulls (difference between measurement and fit in units of the total measurement error) in column 5 are derived from the SM fit including all high-$Q^2$ data (Table 2, column 4).

(a) The systematic errors on $m_Z$ and $\Gamma_Z$ contain the errors arising from the uncertainties in the LEP-I beam energy only.

(b) Only common systematic errors are indicated.

|                     | Measurement with Total Error | Systematic Error | Standard Model fit | Pull  |
|---------------------|-----------------------------|------------------|--------------------|-------|
| $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$ [48] | $0.02758 \pm 0.00035$ | $0.00034$ | $0.02768$ | $-0.3$ |
| a) **LEP-I**        |                             |                  |                    |       |
| line-shape and      |                             |                  |                    |       |
| lepton asymmetries: |                             |                  |                    |       |
| $m_Z$ [GeV]         | $91.1875 \pm 0.0021$        | $^{(a)}0.0017$  | $91.1875$          | $0.0$ |
| $\Gamma_Z$ [GeV]    | $2.4952 \pm 0.0023$         | $^{(a)}0.0012$  | $2.4957$          | $-0.2$ |
| $\sigma_0^{0,\text{had}}$ [nb] | $41.540 \pm 0.037$ | $^{(b)}0.028$ | $41.477$ | $1.7$ |
| $R_\ell^0$          | $20.767 \pm 0.025$          | $^{(b)}0.007$  | $20.744$          | $0.9$ |
| $A_0^{0,\ell}$      | $0.0171 \pm 0.0010$         | $^{(b)}0.0003$ | $0.0165$          | $0.7$ |
| + correlation matrix [1] |                         |                  |                    |       |
| $\tau$ polarisation:|                             |                  |                    |       |
| $A_\ell (P_\tau)$  | $0.1465 \pm 0.0033$         | $0.0016$        | $0.1481$          | $-0.5$ |
| q̅q charge asymmetry:|                             |                  |                    |       |
| $\sin^2 \theta^{\text{eff}}_{Q_\text{had}}$ | $0.2324 \pm 0.0012$ | $0.0010$ | $0.23138$ | $0.8$ |
| b) **SLD**          |                             |                  |                    |       |
| $A_\ell$ (SLD)      | $0.1513 \pm 0.0021$         | $0.0010$        | $0.1481$          | $1.5$ |
| c) **LEP-I/SLD Heavy Flavour** |                         |                  |                    |       |
| $R_0^0$             | $0.21629 \pm 0.00066$       | $0.00050$       | $0.21586$         | $0.7$ |
| $R_0^c$             | $0.1721 \pm 0.0030$         | $0.0019$        | $0.1722$          | $0.0$ |
| $A_{0,b}$           | $0.0992 \pm 0.0016$         | $0.0007$        | $0.1038$          | $-2.9$ |
| $A_{0,c}$           | $0.0707 \pm 0.0035$         | $0.0017$        | $0.0743$          | $-1.0$ |
| $A_b$               | $0.923 \pm 0.020$           | $0.013$         | $0.935$           | $-0.6$ |
| $A_c$               | $0.670 \pm 0.027$           | $0.015$         | $0.668$           | $0.1$ |
| + correlation matrix [1] |                         |                  |                    |       |
| d) **LEP-II and Tevatron** |                         |                  |                    |       |
| $m_W$ [GeV] (LEP-II, Tevatron) | $80.398 \pm 0.025$ | $80.374$        | $1.0$ |
| $\Gamma_W$ [GeV] (LEP-II, Tevatron) | $2.140 \pm 0.060$ | $2.092$     | $0.8$ |
| $m_t$ [GeV] (Tevatron [23])  | $170.9 \pm 1.8$           | $1.5$          | $171.3$           | $-0.2$ |
Figure 1: LEP-I+SLD measurements [1] of $\sin^2 \theta_{\text{eff}}$ and $\Gamma_{\ell\ell}$ and the SM prediction. The point shows the predictions if among the electroweak radiative corrections only the photon vacuum polarisation is included. The corresponding arrow shows variation of this prediction if $\alpha(m_Z^2)$ is changed by one standard deviation. This variation gives an additional uncertainty to the SM prediction shown in the figure.
5 Standard Model Analyses

In the following, several different SM analyses as reported in Table 2 are discussed. The $\chi^2$ minimisation is performed with the program MINUIT [65], and the predictions are calculated with ZFITTER as a function of the five SM input parameters $\Delta \alpha^{(5)}(m_Z^2)$, $\alpha_S(m_Z^2)$, $m_Z$, $m_t$ and $\log_{10}(m_H/{\text{GeV}})$ which are varied simultaneously in the fits; see [1] for details on the fit procedure. The somewhat large $\chi^2$/d.o.f. for all of these fits is caused by the large dispersion in the values of the leptonic effective electroweak mixing angle measured through the various asymmetries at LEP-I and SLD [1]. Following [1] for the analyses presented here, this dispersion is interpreted as a fluctuation in one or more of the input measurements, and thus we neither modify nor exclude any of them. A further significant increase in $\chi^2$/d.o.f. is observed when the NuTeV results are included in the analysis.

To test the agreement between the Z-pole data [1] (LEP-I and SLD) and the SM, a fit to these data is performed. The result is shown in Table 2 column 1. The indirect constraints on $m_W$ and $m_t$ from this data sample are shown in Figure 2 compared with the direct measurements. Also shown are the SM predictions for Higgs masses between 114 and 1000 GeV. As can be seen in the figure, the indirect and direct measurements of $m_W$ and $m_t$ are in good agreement, and both sets prefer a low value of the Higgs mass.

For the fit shown in column 2 of Table 2, the direct $m_t$ measurement is included to obtain the best indirect determination of $m_W$. The result is also shown in Figure 3. Also in this case, the indirect determination of W boson mass, $80.360 \pm 0.020$ GeV, is in good agreement with the direct measurements from LEP-II and the Tevatron, $m_W = 80.398 \pm 0.025$ GeV. For the fit shown in column 3 of Table 2 and Figure 4, the direct $m_W$ and $\Gamma_W$ measurements from LEP-II and the Tevatron are included instead of the direct $m_t$ measurement in order to obtain the constraint $m_t = 179^{+12}_{-9}$ GeV, in good agreement with the direct measurement of $m_t = 170.9 \pm 1.8$ GeV.

Finally, the best constraints on $m_H$ are obtained when all high-$Q^2$ measurements are used in the fit. The results of this fit are shown in column 4 of Table 2. The predictions of this fit for observables measured in high-$Q^2$ and low-$Q^2$ reactions are listed in Tables 1 and 3 respectively. In Figure 5 the observed value of $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$ as a function of $m_H$ is plotted for this fit including all high-$Q^2$ results. The solid curve is the result using ZFITTER, and corresponds to the last column of Table 2. The shaded band represents the uncertainty due to uncalculated higher-order corrections, as estimated by ZFITTER.

The 95% confidence level upper limit on $m_H$ (taking the band into account) is 144 GeV. The 95% C.L. lower limit on $m_H$ of 114.4 GeV obtained from direct searches [66] is not used in the determination of this limit. Including it increases the limit to 182 GeV. Also shown is the result (dashed curve) obtained when using $\Delta \alpha^{(5)}(m_Z^2)$ of Reference 62.

Given the constraints on the other four SM input parameters, each observable is equivalent to a constraint on the mass of the SM Higgs boson. The constraints on the mass of the SM Higgs boson resulting from each observable are compared in Figure 6. For very low Higgs-masses, these constraints are qualitative only as the effects of real Higgs-strahlung, neither included in the experimental analyses nor in the SM calculations of expectations, may then become sizeable [67]. Besides the measurement of the W mass, the most sensitive measurements are the asymmetries, i.e., $\sin^2 \theta_{\text{eff}}^{\text{lept}}$. A reduced uncertainty for the value of $\alpha(m_Z^2)$ would therefore result in an improved constraint on $\log m_H$ and thus $m_H$, as already shown in Figures 1 and 5.
Table 2: Results of the fits to: (1) all Z-pole data (LEP-I and SLD), (2) all Z-pole data plus direct \(m_t\) determination, (3) all Z-pole data plus direct \(m_t, m_W, \Gamma_W\) determinations, (4) all Z-pole data plus \(m_t, m_W, \Gamma_W\) determinations (i.e., all high-\(Q^2\) results). As the sensitivity to \(m_H\) is logarithmic, both \(m_H\) as well as \(\log_{10}(m_H/\text{GeV})\) are quoted. The bottom part of the table lists derived results for \(\sin^2 \theta_{\text{eff}}^\text{lept}, \sin^2 \theta_W\) and \(m_W\). See text for a discussion of theoretical errors not included in the errors above.

|                  | - 1 - all Z-pole data | - 2 - all Z-pole data plus \(m_t\) | - 3 - all Z-pole data plus \(m_W, \Gamma_W\) | - 4 - all Z-pole data plus \(m_t, m_W, \Gamma_W\) |
|------------------|-----------------------|-----------------------------------|-----------------------------------------------|-----------------------------------------------|
| \(m_t\) [GeV]    | \(173^{+13}_{-10}\)  | \(170.9^{+1.8}_{-1.8}\)           | \(179^{+12}_{-9}\)                            | \(171.3^{+1.7}_{-1.7}\)                       |
| \(m_H\) [GeV]    | \(111^{+190}_{-60}\) | \(99^{+52}_{-35}\)               | \(145^{+240}_{-81}\)                         | \(76^{+33}_{-24}\)                           |
| \(\log_{10}(m_H/\text{GeV})\) | \(2.05^{+0.43}_{-0.34}\) | \(2.00^{+0.18}_{-0.19}\)         | \(2.16^{+0.42}_{-0.35}\)                     | \(1.88^{+0.16}_{-0.17}\)                     |
| \(\alpha_S(m_Z^2)\) | \(0.1190 \pm 0.0027\) | \(0.1189 \pm 0.0027\)           | \(0.1190 \pm 0.0028\)                         | \(0.1185 \pm 0.0026\)                       |
| \(\chi^2/\text{d.o.f. (P)}\) | \(16.0/10\) (9.9\%) | \(16.0/11\) (14\%)             | \(17.4/12\) (14\%)                           | \(18.2/13\) (15\%)                         |

|                  | \(\sin^2 \theta_{\text{eff}}^\text{lept}\) | \(\sin^2 \theta_W\) | \(m_W\) [GeV] |
|------------------|-----------------------------------------------|----------------------|--------------|
|                  | \(0.23149\) \(\pm 0.0016\)                  | \(0.22331\) \(\pm 0.0062\) | \(80.363 \pm 0.032\) |
|                  | \(0.23143\) \(\pm 0.00016\)                | \(0.22289\) \(\pm 0.00038\) | \(80.360 \pm 0.020\) |
|                  | \(0.23138\) \(\pm 0.00014\)                | \(0.22311\) \(\pm 0.00029\) | \(80.385 \pm 0.020\) |
|                  | \(0.23138\) \(\pm 0.00013\)                | \(0.22311\) \(\pm 0.00029\) | \(80.374 \pm 0.015\) |

Table 3: Summary of measurements performed in low-\(Q^2\) reactions, namely atomic parity violation, \(e^-e^-\) Møller scattering and neutrino-nucleon scattering. The SM results and the pulls (difference between measurement and fit in units of the total measurement error) are derived from the SM fit including all high-\(Q^2\) data (Table 2 column 4) with the Higgs mass treated as a free parameter.

|                  | Measurement with Total Error | Standard Model High-\(Q^2\) Fit | Pull |
|------------------|-----------------------------|----------------------------------|------|
| \(APV\) [26]    |                             |                                  |      |
| \(Q_W(Cs)\)     | \(-72.74 \pm 0.46\)        | \(-72.899 \pm 0.032\)           | 0.3  |
| \(Møller\) [28] |                             |                                  |      |
| \(\sin^2 \theta_{\text{MS}}(m_Z)\) | \(0.2330 \pm 0.0015\)        | \(0.23109 \pm 0.00013\)            | 1.3  |
| \(\nu N\) [29]  |                             |                                  |      |
| \(g_{\nu \text{Lud}}^2\) | \(0.30005 \pm 0.00137\)    | \(0.30391 \pm 0.00016\)            | 2.8  |
| \(g_{\nu \text{Rud}}^2\) | \(0.03076 \pm 0.00110\)     | \(0.03011 \pm 0.00003\)                 | 0.6  |
Figure 2: The comparison of the indirect measurements of $m_W$ and $m_t$ (LEP-I+ SLD data) (solid contour) and the direct measurements ($p\bar{p}$ colliders and LEP-II data) (dashed contour). In both cases the 68% CL contours are plotted. Also shown is the SM relationship for the masses as a function of the Higgs mass. The arrow labelled $\Delta \alpha$ shows the variation of this relation if $\alpha(m_Z^2)$ is changed by one standard deviation. This variation gives an additional uncertainty to the SM band shown in the figure.
Figure 3: The 68% confidence level contour in $m_W$ and $m_H$ for the fit to all data except the direct measurement of $m_W$, indicated by the shaded horizontal band of ±1 sigma width. The vertical band shows the 95% CL exclusion limit on $m_H$ from the direct search.
Figure 4: The 68% confidence level contour in $m_t$ and $m_H$ for the fit to all data except the direct measurement of $m_t$, indicated by the shaded horizontal band of ±1 sigma width. The vertical band shows the 95% CL exclusion limit on $m_H$ from the direct search.
$D_{c2}^{had} = D_{c2}(5)$

Excluded Preliminary

The dashed curve is the result obtained using the evaluation of $\Delta \alpha_{\text{had}}^{(5)}(m_Z^2)$ from Reference 62. The dotted curve is the result obtained including also the low-$Q^2$ data.

Figure 5: $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ vs. $m_H$ curve. The line is the result of the fit using all high-$Q^2$ data (last column of Table 2); the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on $m_H$ from the direct search. The dashed curve is the result obtained using the evaluation of $\Delta \alpha_{\text{had}}^{(5)}(m_Z^2)$ from Reference 62. The dotted curve is the result obtained including also the low-$Q^2$ data.
Figure 6: Constraints on the mass of the Higgs boson from each pseudo-observable. The Higgs-boson mass and its 68% CL uncertainty is obtained from a five-parameter SM fit to the observable, constraining $\Delta \alpha^{(5)}_{\text{had}}(m_Z^2) = 0.02758 \pm 0.00035$, $\alpha_S(m_Z^2) = 0.118 \pm 0.003$, $m_Z = 91.1875 \pm 0.0021$ GeV and $m_t = 170.9 \pm 1.8$ GeV. Because of these four common constraints the resulting Higgs-boson mass values are highly correlated. The shaded band denotes the overall constraint on the mass of the Higgs boson derived from all pseudo-observables including the above four SM parameters as reported in the last column of Table 2. The vertical line denotes the 95% CL lower limit from the direct search for the Higgs boson. Results are only shown for constraints falling in the range of values shown.
6 Conclusions

The preliminary and published results from the LEP, SLD and Tevatron experiments, and their combinations, test the Standard Model (SM) successfully at the highest interaction energies. The combination of the many precise electroweak results yields stringent constraints on the SM and its free parameters. Most measurements agree well with the predictions. The spread in values of the various determinations of the effective electroweak mixing angle in asymmetry measurements at the Z pole is somewhat larger than expected [1].

Prospects for the Future

The measurements from data taken at or near the Z resonance, both at LEP as well as at SLC, are final and published [1]. Improvements in accuracy will therefore take place in the high energy data (LEP-II), where each experiment has accumulated about 700 pb$^{-1}$ of data, and of course at the Tevatron. The measurements of $m_W$ are likely to reach a precision not too far from the uncertainty on the prediction obtained via the radiative corrections of the Z-pole data, providing an important test of the Standard Model.

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