J/ψ-Production at Photon-Photon Colliders as a Probe of the
Color Octet Mechanism

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(March 26, 2022)

Abstract

We study $J/\Psi$ production at photon-photon colliders, which can be realised with Compton scattering of laser photons at $e^+e^-$ colliders. We find that the production rate through the color-octet channel is comparable to that through the color-singlet channel. Experimentally the two mechanisms can be studied separately because the processes have different signals.

13.60.Le,13.85.Ni,13.85.Qk,13.60.Hb
It is commonly believed that predictions for quarkonium production can be made with perturbative QCD at a certain level because the mass of the heavy quarks provide a large scale. Before the work of Bodwin et al. [1], such predictions were based on perturbative calculations for the production of a heavy quark pair and on some assumptions of the formation of a quarkonium from this pair. This formation was described by models such as the color singlet model and the color evaporation model. Recently, based on non-relativistic QCD(NRQCD), a factorized form for quarkonium production rates has been proposed [2], where the formation is described systematically by a series of NRQCD matrix elements. Such a formalism is possible because the heavy quark, \( Q \), in the rest frame of the quarkonium moves with a small velocity \( v \), hence an expansion in this small velocity is possible. Predictions based on this factorized form take not only the effects of a heavy quark pair, which have the same quantum numbers as those of the quarkonium, into account, but also those effects from other possible components of the quarkonium with different quantum numbers, such as a heavy quark pair in a color-octet state. Although effects of color-octet states for \( J/\Psi \) production are at higher orders in the small velocity expansion than that of a color-singlet state, they can be significant phenomenologically and can even overwhelm that of the color-singlet states in certain kinematic regions. Actually, after including the effect of color-octet states, the Tevatron data on prompt charmonium production can be fitted well to theoretical predictions [3]. However, it is difficult to identify the detailed significance of color-octet states in a process where one or two hadrons are involved in the initial state. Besides possible inaccuracies introduced by finite orders of perturbation theory, there are many uncertainties in theoretical predictions related to the initial hadrons such as parton distributions and higher twist effects [4], etc. Hence it is important to study the effect of color-octet states at non-hadronic colliders, where theoretical predictions are more certain than those for hadronic colliders. Such studies are carried out for \( e^+e^- \)-colliders in [5].

In this work we study \( J/\Psi \) production at photon-photon colliders. Usually, photon-photon collisions always happen at e.g. \( e^+e^- \)-colliders, and the photon beams can be described in the Weizäcker-Williams approximation. But the luminosity of such beams is
greatly reduced from that of the original collider. It is pointed out in [6] that one can use Compton scattering of laser light to obtain $\gamma\gamma$ collisions at an $e^+e^-$ collider with approximately the same luminosity as that of the $e^+e^-$ beams. Unlike those in Weizäcker-Williams approximation the photons obtained through Compton(back) scattering of laser light are hard, i.e., they have a large probability of carrying a large fraction of the beam energies. With these photon beams the total cross-section due to $\gamma\gamma$ collisions can be calculated by:

$$\sigma_{\text{tot}} = \int dx_1 dx_2 f_\gamma(x_1) f_\gamma(x_2) \sigma_{\gamma\gamma}(x_1, x_2),$$

(1)

where $\sigma_{\gamma\gamma}(x_1, x_2)$ is the cross-section due to photon-photon collisions in which one photon has a fraction $x_1$, another has $x_2$ of the initial beam energy. $f_\gamma(x)$ is the photon distribution, given by:

$$f_\gamma(x) = \frac{1}{N}\{1 - x + \frac{1}{1 - x} - \frac{4x}{x_m(1 - x)} + \frac{4x^2}{x_m^2(1 - x)^2}\},$$

(2)

$$N = (1 - \frac{4}{x_m} - \frac{8}{x_m^2}) \ln(1 + x_m) + \frac{1}{2} + \frac{8}{x_m} - \frac{1}{2(1 + x_m)^2},$$

with

$$x_m = \frac{4E\omega_0}{m_e^2 \cos^2 \theta},$$

(3)

where $E$ is the beam energy, $\omega_0$ is the energy of the laser photon and $\theta$ is the angle between the directions of the beam and the laser photon. The energy fraction $x$, carried by a scattered photon is restricted in the range:

$$0 \leq x \leq \frac{x_m}{1 + x_m}.$$  

(4)

We will consider the photon beams realised at the proposed NLC [9] and take $\omega_0 = 1.26\text{eV}$ and $\alpha = 0$ for our numerical results.

With two photons as an initial state the charmonium $J/\Psi$ can be produced via the following processes at the leading order of coupling constants $\alpha$ and $\alpha_s$:

$$\gamma + \gamma \rightarrow J/\Psi + G,$$

(5)

$$\gamma + \gamma \rightarrow J/\Psi + \gamma.$$  

(6)
In the first process a $^3S_1$ $c\bar{c}$-pair in a color-octet state is produced and then the pair is transformed into $J/\Psi$, while in the second, a $^3S_1$ $c\bar{c}$-pair in a color-singlet state is produced and then transformed into $J/\Psi$. The transition in the first process happens at order of $v^4$ and in the second at order of $v^0$ in the small velocity expansion. Although the probability for the transition from a color-octet $c\bar{c}$ is smaller than that from a color-singlet $c\bar{c}$, the first process is enhanced by $\alpha_s$ relative to $\alpha$ and it has a cross-section comparable to the second processes. It should be mentioned that at order $v^4$ there are other possible color-octet states which can be transformed into $J/\Psi$, but these states can not be produced at the order of coupling constants which we consider. Further, these two processes can have different experimental signatures. The quarkonium in the final state is accompanied by a jet initiated by the gluon in the first process and by an energetic photon in the second. If these processes are observed, one may easily identify whether the $J/\Psi$ comes from a color-octet $c\bar{c}$ pair or not. Hence studying these processes can help us to understand more about the color-octet mechanism.

The effect of color-octet states in quarkonium production at hadron colliders is studied extensively [7], [8]. Unlike at hadron colliders where many subprocesses can contribute to the quarkonium production we have here only one subprocess. It is straight forward to calculate the cross-sections of the two processes. One of the Feynman diagrams for them is given in Fig.1. For the first process we have:

$$\frac{d\sigma_{\gamma\gamma}}{dt} = \frac{16M_{J/\Psi}}{3} \left(\frac{2}{3}\right)^4 (4\pi)^2 \alpha_s \alpha^2 \langle 0 | O_{J/\Psi}^{I/\Psi}(^3S_1) | 0 \rangle$$

$$\cdot \frac{\hat{s}^2(\hat{s} - M_{J/\Psi}^2)^2 + \hat{t}^2(\hat{t} - M_{J/\Psi}^2)^2 + \hat{u}^2(\hat{u} - M_{J/\Psi}^2)^2}{\hat{s}^2(\hat{s} - M_{J/\Psi}^2)^2(\hat{t} - M_{J/\Psi}^2)^2(\hat{u} - M_{J/\Psi}^2)^2}. \quad (7)$$

The variables $\hat{s}$, $\hat{t}$ and $\hat{u}$ are standard Mandelstam variables, $M_{J/\Psi}$ is twice the $c$ quark mass, $m_c$. The matrix element $\langle 0 | O_{J/\Psi}^{I/\Psi}(^3S_1) | 0 \rangle$ is defined in NRQCD in [2] and represents the probability of transition from a color-octet $^3S_1$ $c\bar{c}$ into the $J/\Psi$. The differential cross-section for the second process can be obtained by the replacement in Eq.(7) via

$$\langle 0 | O_{J/\Psi}^{I/\Psi}(^3S_1) | 0 \rangle \rightarrow 4\alpha \langle 0 | O_{J/\Psi}^{I/\Psi}(^3S_1) | 0 \rangle,$$

$$8\alpha_s,$$
where the matrix element $\langle 0 | O^{J/\Psi}_{1} (3S_{1}) | 0 \rangle$ represents the effect of the transition of a color-singlet $c\bar{c}$ pair, its definition can be found in [2] also.

We take two possible values for the center of mass energy, $\sqrt{s}$, at the proposed NLC, $\sqrt{s} = 500\text{GeV}$ and $1000\text{GeV}$. We obtain the following results for the total cross-section for the process in Eq.(5):

$$
\sigma_{\text{tot}}(J/\Psi + G) = 2.64 \frac{\langle 0 | O^{J/\Psi}_{8} (3S_{1}) | 0 \rangle}{(\text{GeV})^{3}} \text{(pb)}, \quad \text{for } \sqrt{s} = 500\text{GeV}
$$

$$
\sigma_{\text{tot}}(J/\Psi + G) = 0.46 \frac{\langle 0 | O^{J/\Psi}_{8} (3S_{1}) | 0 \rangle}{(\text{GeV})^{3}} \text{(pb)}, \quad \text{for } \sqrt{s} = 1\text{TeV}.
$$

In the above results we have taken $\alpha = 1/128$, $\alpha_{s}(m_{c}) = 0.3$ and $m_{c} = 1.5(\text{GeV})$. The final value of the cross-section depends on the matrix element. This matrix element is extracted from different processes, but its value varies from 0.006(\text{GeV})^{3} to 0.02(\text{GeV})^{3} from the different processes reflecting uncertainties in theoretical predictions. If we take 0.01(\text{GeV})^{3} for the matrix element, the cross-section is 0.0266pb at $\sqrt{s} = 500\text{GeV}$ and 0.0046pb at $\sqrt{s} = 1\text{TeV}$. With a proposed luminosity for the NLC of 100fb$^{-1}$ per year at $\sqrt{s} = 500\text{GeV}$, there will be several hundred events which can be detected through the leptonic decay of $J/\Psi$. Although the event number is not large, once these events are observed, there is direct access to the matrix element.

The cross-section through the color-singlet channel can be estimated with the replacement in Eq.(8). The color-singlet matrix element can be approximately extracted from the leptonic decay of $J/\Psi$. We take its value as 1.1(\text{GeV})^{3} and obtain the cross-section:

$$
\sigma_{\text{tot}}(J/\Psi + \gamma) = 0.031(\text{pb}), \quad \text{for } \sqrt{s} = 500\text{GeV}
$$

$$
\sigma_{\text{tot}}(J/\Psi + \gamma) = 0.005(\text{pb}), \quad \text{for } \sqrt{s} = 1\text{TeV}.
$$

With these estimates one can conclude that the $J/\Psi$ production through the color-octet channel will be comparable to that through the color-singlet channel. The production through these mechanisms can be distinguished experimentally.

Having studied the total cross-section we turn to some distributions for the process in Eq.(5). The distributions studied here are the same for the process in Eq.(6). In Fig.2 we
give the differential cross-section $\frac{d\sigma_{\text{tot}}}{dp_t^2}$ as function of $p_t^2$, where $p_t$ is the transverse momentum of the $J/\Psi$. We define a total cross-section with the $J/\Psi$ having a $p_t$ larger than $p_{\text{min}}$ as:

$$\sigma_{\text{tot}}(p_{\text{min}}) = \int_{p_{\text{min}}}^{p_{\text{max}}} dp_t \frac{d\sigma_{\text{tot}}}{dp_t}.$$  

(13)

$\sigma_{\text{tot}}(p_{\text{min}})$ as function of $p_{\text{min}}$ is drawn in Fig.3. From Fig.2 and Fig.3 one can see that most of $J/\Psi$ will be produced with relatively small $p_t$. The cross section $\sigma_{\text{tot}}(p_{\text{min}})$ decreases rapidly with increasing $p_{\text{min}}$.

For $J/\Psi$ production at hadronic colliders the process of gluon fragmentation plays an important role. The reason is because the contribution from that process is kinematically much less suppressed than that from non-fragmentation processes. However in our case a gluon can be only produced via the diagrams given in Fig.1, hence it can be expected that the contribution from gluon fragmentation will have roughly the same suppression from the kinematics as that for the process in Eq.(5), and the gluon fragmentation begins at the order of $\alpha_s$. Therefore we can expect that the contribution from gluon fragmentation will not be significant. Besides gluon fragmentation, quark fragmentation can also lead to contributions to inclusive $J/\Psi$ production, where the fragmentation is through color-octet mechanisms at the order of $\alpha_s^2$ and is studied in [10]. In our case the production of a quark is less suppressed kinematically than the process in Eq.(6), but the probability for quark fragmentation is very small. The ratio of the probabilities of quark and gluon fragmentation from [10] is:

$$\frac{P(q \rightarrow J/\Psi)}{P(G \rightarrow J/\Psi)} \approx 0.22\alpha_s(2M_c).$$  

(14)

It may be expected that quark fragmentation will not lead to dominant contributions to the inclusive production of $J/\Psi$. However a more detailed study may be needed, which is beyond the scope of this work.

Our results for $J/\Psi$ production through the process in Eq.(5) can also be used for $\chi_{cJ}$, for $J = 0, 1, 2$, by the replacement:

$$\langle 0 | O_{\chi_{cJ}}^{J/\Psi}(3S_1) | 0 \rangle \rightarrow \langle 0 | O_{\chi_{cJ}}^{\chi_{cJ}}(3S_1) | 0 \rangle.$$  

(15)
The matrix element $\langle 0 | O_{S}^{\chi_{c1}}(3S_{1}) | 0 \rangle$ is approximately $0.0073 GeV^3$. With this estimate, the cross section for $\chi_{c1}$ production at the NLC at $\sqrt{s} = 500 GeV$ is 0.078pb. The production of this exited state can be important for overall $J/\Psi$ production because $\chi_{c1}$ decays via $\chi_{c1} \rightarrow J/\Psi + \gamma$ with a branching ratio of 27%. Unlike $J/\Psi$ production, no color singlet contribution exists for $\chi_{cJ}$ which gives a significant contribution.

To summarise, we studied in this work $J/\Psi$ production at a photon-photon collider. It was found that in addition to the color-singlet contribution, a color octet $c\bar{c}$ pair makes a significant contribution to the production rate. Additionally, it is possible to differentiate between the two mechanisms by tagging the photon produced in the color-singlet process. Hence a photon-photon collider will provide an ideal place to study the color-octet mechanism for quarkonium production.

Acknowledgement:

This work is supported by The Australian Research Council and the Australian Postgraduate Award.
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**Figure Captions**

Fig.1: One of six Feynman diagrams for the production of a color-octet $c\bar{c}$ pair. Others are obtained through permutations of photon and gluon lines.

Fig.2: The color octet $p_t^2$-distributions as a function of $p_t^2$(GeV$^2$) in pbGeV$^{-2}$. The solid line is at $\sqrt{s} = 500$GeV while the dotted line is at $\sqrt{s} = 1$TeV.

Fig.3: The color octet cross-section $\sigma(p_{\text{min}})$(pb) as function of $p_{\text{min}}$(GeV) as defined in Eq. (14). The solid and dotted lines are for $\sqrt{s} = 500$GeV and $\sqrt{s} = 1$TeV respectively.
Fig. 1
