Bulk viscosity of a hot QCD/QGP medium in strong magnetic field within relaxation-time approximation

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The bulk viscosity of hot QCD medium has been obtained in the presence of strong magnetic field. The present investigation involves the estimation of the quark damping rate and subsequently the thermal relaxation time for quarks in the presence of magnetic field while realizing the hot QCD medium as an effective Grand-canonical ensemble of effective gluons and quarks-antiquarks. The dominant process in the strong field limit is $1 \rightarrow 2 (q \rightarrow q\bar{q})$ which contributes to the bulk viscosity in a most significant way. Further, setting up the linearized transport equation in the framework of an effective kinetic theory with hot QCD medium effects and employing the relaxation time approximation, the bulk viscosity has been estimated in lowest Landau level (LLL) and beyond. The temperature dependence of the ratio of the bulk viscosity to entropy density indicates towards its rising behavior near the transition temperature.

Keywords: Quark-gluon-plasma, Strong magnetic field, Thermal relaxation time, Bulk viscosity, Effective fugacity.

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I. INTRODUCTION

Relativistic heavy-ion collision experiments (RHIC) set the platform for the creation and study of quark-gluon plasma (QGP) as a near-perfect fluid [1-2]. Recent investigations on the QGP suggests the presence of extremely high magnetic field in the early stages of the collisions, (specially in the non-central asymmetric collisions) [3-6]. In this context, a deeper understating of various aspects of the QGP in the strong magnetic field is the prime focus of the current research on the physics of the RHIC. In particular, Chiral magnetic effect [7-9] and chiral vortex effects [10-12] gained huge attention in the QGP community. More recently, the discovery of global Λ-particle polarization in non-central RHIC [13, 14] opens up a new direction in the study of the QGP in the presence of strong magnetic field.

Recall that the quark-antiquark pair production and fusion processes are kinematically possible in the presence of the strong magnetic field [13-16] via $1 \rightarrow 2$ processes that dominate over $2 \rightarrow 2$ scattering processes while estimating the transport coefficients. This could be understood in terms of the fact that the rate is proportional to coupling constant $\alpha_s$ in the case of the former, whereas that of the binary processes, it is proportional to $\alpha_s^2$ [17]. The magnetic field effects enter in the quark-antiquark degrees of freedom through the Landau levels. The strong magnetic field restricts the calculation to the (1+1)-dimensional ground state i.e., lowest Landau level (LLL) [15-19] (the dimensional reduction). On the other hand, the electrically chargeless gluons are not directly coupled to the magnetic field through dispersion relation. However, the gluonic dynamics in the presence of magnetic field can be affected through the quark loop while defining the gluon vertex through the self-energy where the quark/antiquark loop contributes.

The quantitative study of the transport coefficients in the hot QCD medium is required for the estimation of the experimental observables like transverse momentum spectra and collective flow of the QGP within the dissipative relativistic hydrodynamic framework. In particular, extremely low viscosity to entropy ratio indicate the larger elliptic flow observed in RHIC. Besides providing the basis for understanding the probes of QGP, the transport coefficients give insights to the electromagnetic response of the medium. Recently, a number of ALICE results have shown the relevance of transport processes in the RHIC [20-22]. Since the strong magnetic field is generated in the non-central asymmetric HIC, the dissipative magnetohydrodynamics describes the transport process of the medium. This sets the strong motivation for the estimation of transport coefficients of the QGP in presence of the strong magnetic field.

There have been several attempts to estimate the transport coefficients of the hot QCD medium in the strong magnetic field [23-27]. In a very recent work, Fukushima and Hidaka [28], estimated the longitudinal conductivity in the magnetic field beyond LLL approximation by solving the kinetic equation, considering the scattering amplitude of synchrotron radiation and the pair annihilation processes. The authors have numerically shown that the contribution from LLL is the dominant one.

The goal of the present investigations is to estimate the temperature dependence of the thermal relaxation time and thereby the effective bulk viscosity while encoding the hot QCD medium effects in strong magnetic field background through an effective quasi-particle model. The analysis has been done with relativistic semi-classical...
transport theory, in which, microscopic particle interactions bridges to macroscopic transport phenomena of the thermodynamic system. The kinetic theory approach is followed within the linear response analysis of transport equation in which magnetic field enters through the propagator (matrix element in collision integral) and momentum distribution functions of the quarks and antiquarks. Note that another equivalent approach to investigate the transport coefficients of the hot QCD in the magnetic field background is the hard thermal loop effective theory (HTL) [29, 30]. We are following the former one here.

Hot QCD medium effects encrypted as the equation of state (EoS) dependence on the transport coefficients within effective linear transport theory are well understood [31–37]. In [38], the authors have recently estimated the EoS/medium dependence on the longitudinal electrical conductivity for the 1 \( \rightarrow \) 2 processes in the strong magnetic field background. In the present work, we followed the effective fugacity quasiparticle model (EQPM), proposed in [39, 40] and extended in the case of the strong magnetic in Ref [38]. The first step towards the evaluation the bulk viscosity is the quark damping rate \( \Gamma_{\text{eff}} \) in the strong field limit that leads to the thermal relaxation time \( \tau_{\text{eff}} \), followed by the estimation of the bulk viscosity \( \zeta_{\text{eff}} \) in the presence of magnetic field by setting up an effective linearized transport equation. This has been done not only in LLL but also with the higher Landau level (HLL) corrections.

The paper is organized as follows. Section II deals with the mathematical formalism for the estimation of the effective thermal relaxation time and the bulk viscosity along with the description of hot QCD effective coupling constant with HLL corrections. Section III constitutes the predictions on the bulk viscosity and the related discussions. Finally, in section IV, the conclusion and outlook of the work are presented.

II. EFFECTIVE DESCRIPTION OF THERMAL RELAXATION AND BULK VISCOSITY IN STRONG MAGNETIC FIELD

Green-Kubo formula is employed to estimate the bulk viscosity of the medium both in the presence and the absence of the strong magnetic field background in the studies [24, 41, 53]. In this work, we are adopting the kinetic theory approach for the analytical calculation of \( \zeta_{\text{eff}} \) in the strong magnetic field, in which we need to start from the relativistic transport equation. The strong magnetic field limit, \( T^2 \ll eB \) has been considered for computing various quantities under consideration in LLL. The contributions from higher Landau levels are negligible (proportional to \( e^{-zqT} \)) in the regime. Now, for the weaker magnetic fields, going beyond LLL might help in understanding the impact of the magnitude of the field on the transport coefficients. A full computation in the weak field domain will also require computation of the quark/antiquark propagators under the same approximation and is beyond the scope of the present work. The formalism for the estimation of effective bulk viscosity includes the quasiparticle modeling of the system followed by the estimation of the thermal relaxation time of the process.

A. EQPM in the strong magnetic field

EQPM describes the hot QCD medium effects with temperature dependent effective fugacities - quasigluon and quasiquark/antiquark fugacities, \( z_q \) and \( z_{\bar{q}} \) respectively [43]. Various quasiparticle models encode the medium effects, viz., effective masses with Polyakov loop [45], NJL and PNJL based quasiparticle models [46] self-consistent and single parameter quasiparticle models [47] and recently proposed quasiparticle models based on the Gribov-Zwanziger (GZ) quantization, leading to a nontrivial IR-improved dispersion relation in terms of the Gribov parameter [48, 50]. EQPM encodes the medium effects as EoS dependence of the distribution functions, enters through the effective fugacities.

Here, we consider the recent (2+1) flavor lattice QCD EoS (LeoS) [51] and 3-loop HTL perturbative (HTLpt) EOS [52, 53]. The 3-loop HTLpt EOS has recently been computed by N. Haque et al., which is very close to the recent lattice results [51, 55]. These EoSs have been carefully embedded in \( z_q \) and \( z_{\bar{q}} \) for both isotropic and to anisotropic hot QCD medium [56, 58]. \( z_q \) and \( z_{\bar{q}} \) have complicated temperature dependence as discussed in Ref. [58].

We have extended the EQPM in the presence of magnetic field \( \vec{B} = B\hat{z} \) [55] in which quasi-quark/antiquark distribution function is given as,

\[
\beta \tilde{f}_q = \frac{z_q \exp \left(-\beta \sqrt{p_t^2 + m^2 + 2l |q_f eB|} \right)}{1 + z_q \exp \left(-\beta \sqrt{p_t^2 + m^2 + 2l |q_f eB|} \right)},
\]

where \( E^l_p = \sqrt{p_t^2 + m^2 + 2l |q_f eB|} \) is the Landau energy eigenvalue and \( q_f e \) is the fractional charge of quarks. \( l = 0, 1, 2, \ldots \) is the order of the energy levels. Since dispersion relation of electrically neutral gluon remain intact in strong magnetic field background, the quasigluon distribution function remains as,

\[
\beta \tilde{f}_{\bar{q}} = \frac{z_{\bar{q}} \exp \left(-\beta |\tilde{p}| \right)}{1 + z_{\bar{q}} \exp \left(-\beta |\tilde{p}| \right)}.
\]

We are working in units where \( k_B = 1, \ e = 1, \ h = 1 \) and hence \( \beta = \frac{1}{T} \). The parton distribution functions leads to the dispersion relations,

\[
\omega_q' = \sqrt{p_t^2 + m^2 + 2l |q_f eB| + T^2 \partial_T \ln(z_q)},
\]

and

\[
\omega_{\bar{q}} = |\tilde{p}| + T^2 \partial_T \ln(z_{\bar{q}}).
\]
The physical significance of the effective fugacity comes in the second term of dispersion relations Eqs. [3] and [4] which corresponds to the collective excitation of quasipartons. Effects of the magnetic field are entering into the system through the dispersion relations and the Debye screening mass [59].

Debye mass and effective coupling in the strong magnetic field with HLL corrections

EQPM is based on charge renormalization in the hot QCD medium whereas the effective mass model is motivated from the mass renormalization of QCD [60]. Realization of this charge renormalization could be related to the estimation of Debye mass from semi-classical transport theory. There are several investigations on the screening masses of the QGP as a function of the magnetic field [19, 65, 66]. Employing EQPM, we can compute the screening mass as [55, 60],

\[
m_D^2 = -4\pi\alpha_s \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{d}{dp}(2N_c\bar{f}_g + N_f(f^p_g + f^q_g)), \tag{5}
\]

where \(f^p_g\) and \(\bar{f}_g\) is the quasiparton distribution functions as defined in Eqs. [1] and [2], \(\alpha_s(T)\) is the running coupling constant at finite temperature taken from 2-loop QCD gauge coupling constants [61]. Including the effects of HLLs in presence of the strong magnetic field \(\vec{B} = B\hat{z}\), \(m_D\) for quarks and antiquarks becomes,

\[
m_D^2 = \frac{4\alpha_s}{T} \frac{|q_f eB|}{\pi} \int_0^\infty \sum_{l=0}^\infty dp_z(2 - \delta_{l0})\bar{f}_q^l(1 - f^q_{l}), \tag{6}
\]

in which integration phase factor due to dimensional reduction in the strong field [19, 65, 66] can be represented as,

\[
\int \frac{d^3\vec{p}}{(2\pi)^3} \frac{|q_f eB|}{2\pi} \sum_{l=0}^\infty \int \frac{dp_z}{2\pi}(2 - \delta_{l0}). \tag{7}
\]

After performing the momentum integral Eq. [5] using Eq. [1] we obtain,

\[
(m_D^2/\alpha_s) = \frac{24T^2}{\pi}PolyLog[2, z_q] + \frac{12}{\pi} \frac{|q_f eB|}{\pi} \left( \frac{z_q}{1 + z_q} \right) + \frac{8}{T} \frac{|q_f eB|}{\pi} \int_0^\infty \sum_{l=1}^\infty dp_z\bar{f}_q^l(1 - \bar{f}_q^l). \tag{8}
\]

We have plotted the ratio of Debye mass to running coupling constant ratio at \(|eB| = 0.3\) GeV\(^2\) as a function of temperature for different Landau levels in Fig. 1. For the chosen temperature range we are focusing up to \(T = 0.25\) GeV\(^2\). For ideal EoS \(z_q,g = 1\) (ultra-relativistic non-interacting quarks and gluons), the definition of Debye mass can rewrite as,

\[
(m_D^2)_{Ideal} = 4\pi \alpha_s(T)[T^2 + \frac{3}{2\pi^2} \frac{|q_f eB|}{T} + \frac{2}{\pi^2} \frac{|q_f eB|}{T} \int_0^\infty \sum_{l=1}^\infty dp_z\bar{n}_q^l(1 - \bar{n}_q^l)], \tag{9}
\]

with \(\bar{n}_q^l = \frac{1}{\exp(\beta E_q^l) + 1}\). From Eqs. [8] and [9], including HLLs we can define the effective running coupling
constant $\alpha_{eff}^l(T, z_q, z_{\bar{q}}, |eB|)$ so that,

$$m_D^2 = \frac{\alpha_{eff}^l}{\alpha_s} m_{D \text{ideal}}^2. \quad (10)$$

Therefore,

$$m_D^2 = 4\pi \alpha_{eff}^l(T, z_q, z_{\bar{q}}) T^2 + \frac{2 |q_f eB|}{\pi^2} \int_0^{\infty} \sum_{i=1}^{\infty} dp_z n_q^i (1 - \bar{n}_{\bar{q}}^i), \quad (11)$$

Now, $\alpha_{eff}^l$ can be expressed as,

\[ \begin{align*}
\alpha_{eff}^l &= \frac{6T^2}{\pi^2} \text{PolyLog}[2, z_q] + \frac{3 |q_f eB|}{\pi^2} \frac{z_q}{(1 + z_q)} \left( T^2 + \frac{3 |q_f eB|}{2\pi^2} + h(T, |eB|) \right) \\
&+ \frac{2 |q_f eB|}{T \pi^2} \int_0^{\infty} \sum_{i=1}^{\infty} dp_z n_q^i (1 - \bar{n}_{\bar{q}}^i) \left( T^2 + \frac{3 |q_f eB|}{2\pi^2} + h(T, |eB|) \right), \quad (12)
\end{align*} \]

where $h(T, |eB|) = \frac{2 |q_f eB|}{T} \int_0^{\infty} \sum_{i=1}^{\infty} dp_z n_q^i (1 - \bar{n}_{\bar{q}}^i)$.

For LLL quarks Eq. (12) reduced to

\[ \begin{align*}
\alpha_{eff}^0 &= \frac{6T^2}{\pi^2} \text{PolyLog}[2, z_q] + \frac{3 |q_f eB|}{\pi^2} \frac{z_q}{(1 + z_q)} \left( T^2 + \frac{3 |q_f eB|}{2\pi^2} \right).
\end{align*} \quad (13)

The temperature behavior of $\alpha_{eff}^l$ with HLLs corrections are depicted in the Fig. 2. As expected, asymptotically the ratio approaches unity. Dominant contribution of $\alpha_{eff}^l$ comes from the LLL, where the HLLs gives the higher order corrections. More interestingly, including HLLs $\alpha_{eff}^l$ are almost identical for $|eB| = 0.3 \text{ GeV}^2$ and $|eB| = 0.6 \text{ GeV}^2$, which implies the weaker dependence of the strength of magnetic field on $\alpha_{eff}^l$. The ratio is showing a small but quantitative change with the HLLs corrections. Hence, these corrections are significant in the estimation of the bulk viscosity in the strong field.

**B. Thermal relaxation in strong magnetic field**

The microscopic interactions, which are the dynamical inputs of the bulk viscosity, are incorporated through the thermal relaxation time ($\tau_{eff}$). The focus of this work is on the dominant $1 \rightarrow 2$ processes (gluon to quark-antiquark pair). The relaxation time, $\tau_{eff}$ can be defined from the relativistic transport equation of quasiparton distribution functions for the process $k \rightarrow p + p'$ in strong magnetic field $\vec{B} = B \hat{z}$ as,

$$\frac{df_q^l}{dt} = C(f_q^l) = -\frac{\delta f_q^l}{\tau_{eff}}, \quad (14)$$

The quantity $\delta f_q^l$ is the non-equilibrium part of the distribution function of quasiquark/antiquark,

$$f_q^l(p_z) = f_q^l + \delta f_q^l, \quad (15)$$

and given by

$$\delta f_q^l = \beta f_q^l(p_z) (1 - f_{\bar{q}}^l(p_z)) \chi_q(p_z), \quad (16)$$

where $\chi(p_z)$ is the response function (primed notation for antiquark). Here, $C(f_q^l)$ is the collision integral which quantifies the rate of change of distribution function. In strong magnetic field background, the collision integral for $1 \rightarrow 2$ processes have the following form \[38\]

$$C(f_q^l) = \alpha_{eff}^l C_2 m^2 \int_{-\infty}^{\infty} \frac{d p_z}{\omega_p} \frac{\beta f_q^l(E_p') f_{\bar{q}}^l(E_p')}{\omega_{p'}^2} \left( \chi_q(p_z') - \chi_{\bar{q}}(p_z') \right), \quad (17)$$

where $C_2$ is the Casimir factor and $\alpha_{eff}^l$ is the effective coupling constant which encoded the EoS dependence. $\omega_p$ is the single quark energy as defined in Eq. (3). The response $\chi$ for quark and antiquark in the strong magnetic field has opposite sign (since their charges are opposite). This implies that $\chi_q(p_z)$ is an odd function as described in \[23\] within LLL approximation. Since the Landau levels enters as $E_l = \sqrt{p_z^2 + m^2 + 2l |eB|}$ in the dispersion relations and distribution functions, the odd nature of $\chi(p_z)$ is completely independent on the order of LL. Hence we have,
\[ C(f_q^i) = -\chi_q(p_z)\alpha_{eff}^i C_2 m^2 \beta \]
\[ \times \int_{-\infty}^{\infty} \frac{dp_z'}{\omega_{p'}^2} f_q^i(E_{p'}^i)(1 + \tilde{f}_g(E_p^i + E_{p'}^i)). \]

(18)

Thermal relaxation time \( \tau_{eff} \), which is the inverse of the quark damping rate \( \Gamma_{eff} \), can be obtained from Eqs. (14) (16) and (18) as, 
\[ \tau_{eff}^{-1} \equiv \Gamma_{eff} \]
\[ = \frac{\alpha_{eff}^i C_2 m^2}{\omega_p(1 - f_q^i)} \int \frac{dp_z'}{\omega_{p'}^2} f_q^i(E_{p'}^i)(1 + \tilde{f}_g(E_p^i + E_{p'}^i)). \]

(19)

Being motivated by the recent work Ref. [17] [23], we could be described by choosing \( z_q/q = 1 \), and in that case, the relaxation time reduces to
\[ \tau_{ideal}^{-1} = \frac{\alpha_q C_2 m^2}{E_p^i(1 - f_q^i)} (1 + \tilde{n}_q(E_p^i)) \ln(T/m), \]
(21)

with \( \tilde{n}_q = \frac{1}{(e^{\beta E_p^i} + 1)} \) and \( \tilde{n}_g = \frac{1}{(e^{\beta E_p^i} - 1)} \) for ideal fermions and bosons respectively.

Since the dominant charge carriers have momenta in the order of \( T \), we are employing \( < p_z >> T \) for the comparison of \( \tau_{eff} \) with \( \tau_{ideal} \) to investigate the EoS dependence. Note that the momentum dependence of the relaxation time is significant in the estimation of bulk viscosity. Therefore, while computing the bulk viscosity, the momentum dependent thermal relaxation time as defined in Eq. (20) is employed. Here, we plotted the temperature variation of \( \frac{\tau_{eff}^{-1}}{\tau_{ideal}^{-1}} \) with \( < p_z >> T \) for the ground state quarks \( (l = 0) \) at \( |eB| = 0.3 \text{ GeV}^2 \) and \( |eB| = 0.9 \text{ GeV}^2 \) in Fig. 3. Hot medium effects are identical for the system under consideration irrespective of the magnitude of the magnetic field. EoS effects in relaxation time are embedded in Eq. (10) through the quasiparton distribution function and the effective coupling defined in Eq. (12). Since \( \alpha_{eff}^i \) is lower than \( \alpha_s \) at the lower temperature, \( \tau_{eff}^{-1} \) to \( \tau_{ideal}^{-1} \) ratio gives lower value in that temperature range.

HLLs corrections are entering through Landau dispersion relation in the quark distribution function. Effect of higher levels in the effective coupling is understood from Eq. (12). The effective thermal relaxation time controls the behavior of bulk viscosity critically.

**C. Bulk viscosity from the relaxation time approximation**

We investigated the bulk viscosity of perturbative QCD in the strong magnetic field \( \vec{B} = B \hat{z} \) by adopting the EQPM, for the dominant \( 1 \rightarrow 2 \) processes. Dynamics of the system is described by the Boltzmann equation for the quasiparton distribution function,
\[ (\partial_t + v_z \partial_z) f_q^i(p_z, t, z) = C(f_q^i) = -\frac{\delta f_q^i}{\tau_{eff}^i}, \]
(22)

where \( C(f_q^i) \) is the collision integral Eq. (17) and the longitudinal velocity \( v_z \equiv \frac{\partial f_q^i}{\partial p_z} = \frac{E_z}{E_p} \). Equilibrium distribution function is defined as,
\[ f_q^i = \frac{1}{(z_q^i - 1) \exp (-\beta (E_p^i - p_z v_z) + 1)}, \]
(23)
in the presence of the flow \( u_z \). For \( u_z = 0 \), \( f_q^i \) reduces to Eq. (1). We consider the linear response regime of the Boltzmann equation in which \( u_z \) and \( \delta f_q^i \) are assumed to be small, with appropriate collision integral to solve \( \delta f_q^i \).
The system in equilibrium is disturbed by an expansion in the direction of magnetic field, which gives the change in pressure ($\delta P_L$). Bulk viscosity is defined as [24],

$$\delta P_L = -3\zeta_{eff} \Theta,$$

with $\Theta(z) \equiv \partial_z u_z$, which defines the magnitude of expansion. We investigated the QCD thermodynamic quantities such as pressure, energy density, entropy density and the speed of sound in the strong magnetic field using the extended EQPM [33]. With LLL approximation, longitudinal pressure (in the direction of $\vec{B}$) is obtained from the fundamental thermodynamic definition,

$$P_L = \sum_f \left| \frac{e q_f B}{2\pi} \right| \frac{1}{2\pi} 2N_c \int_{-\infty}^{\infty} dp_z \ln(1 + z_q \exp(-\beta \sqrt{p_z^2 + m^2})).$$

Longitudinal pressure end up as,

$$P_L = \sum_f \left| \frac{e q_f B}{2\pi} \right| N_c \int_{0}^{\infty} dp_z \frac{p_z^2}{E_p} f_0^q,$$

where $f_0^q$ is the momentum distribution of lowest Landau quarks ($l = 0$). Similarly, energy density of the quarks is defined as

$$\varepsilon_L = \sum_f \left| \frac{e q_f B}{2\pi} \right| N_c \int_{0}^{\infty} dp_z \frac{(\omega_p^0)^2}{\omega_p^0} f_0^q,$$

in which $\omega_p^0$ is the single particle energy for LLL quarks. The integral can be expressed in terms of PolyLog functions. Change in longitudinal pressure leads to the bulk viscosity in the direction of magnetic field as given in Eq. [24] and hence,

$$\zeta_{eff} = \sum_f \frac{1}{3\Theta} \left| \frac{e q_f B}{2\pi} \right| N_c \int_{0}^{\infty} dp_z \frac{p_z^2}{E_p} \delta f_0^q.$$

However, even when $\delta f_0^q = 0$ there will be a change in pressure since the temperature ($\beta \equiv \beta(t)$) decreases in time due to the expansion. This can be directly related to the Landau-Lifshitz condition for the stress-energy tensor in the calculation of the bulk viscosity without magnetic field [67]. We subtract this effect as in Ref. [24] [68], and we have

$$\delta P \rightarrow \delta P_L \equiv \delta(P_L - \Omega \varepsilon_L),$$

with $\Omega \equiv \partial P_L / \partial \varepsilon_L = \partial P_L / \partial T$. To solve this, we have used EQPM definition of pressure and energy density in strong magnetic field as in Eqs. [26] and [27],

$$\Omega = \{- | eB | \frac{2T}{\pi^2} \nu_q PolyLog[2, -z_q]$$

$$+ | eB | \frac{T^2}{\pi^2} \nu_q \ln(1 + z_q) (\partial_T \ln z_q) /$$

$$\{ - \frac{4}{\pi^2} | eB | T PolyLog[2, -z_q]$$

$$+ 5 | eB | (T^2 \partial_T \ln z_q) \frac{1}{\pi^2} \nu_q \ln(1 + z_q)$$

$$+ | eB | T^2 (\partial_T^2 \ln z_q) \frac{T}{\pi^2} \nu_q \ln(1 + z_q) \},$$

where $\nu_q = \sum_f 2N_c | q_f |$ in the presence of magnetic field. Also, we need to evaluate the change in equilibrium distribution function $\delta f_q$ for the calculation of $\delta P_L$. Considering the linear response regime of the Boltzmann equation Eq. [22] with the distribution function as Eq. [25], we have

$$\langle \partial_t + v_z \partial_z \rangle f_0^q(p_z, t, z) = - [(E_p^0 + T^2 \partial_T \ln z_q) \partial_z \beta$$

$$- \beta v_z \partial_z \Theta(z)] f_0^q / f_q^0 - 1).$$

Here, $z_q(T)$ and $\beta(T)$ are functions of time since temperature changes with expansion. Detailed calculations are shown in the Appendix A. In the relaxation time approximation, we can directly connect the relaxation time $\tau_{eff}$ with the collision integral $C(f_q^0)$ as shown in Eq. [22]. Therefore, Eq. [31] becomes

$$\delta f_q^0 = -\tau_{eff} \beta \nu_q f_q^0 (f_q^0 - 1) \Theta(z) (\omega_p^0 \Omega - v_z p_z),$$

where $\partial_t \beta \equiv \beta \Omega \partial \Theta$ as given in [24] [68] and $\tau_{eff}$ is the thermal relaxation time (at $l = 0$ in the LLL approximation) for $1 \rightarrow 2$ processes defined in Eq. [20]. Now we
can estimate $\zeta_{eff}$ by direct substitution of Eqs. (20) (32) and (30) to (29) and end up with,

$$\zeta_{eff} = \frac{1}{3} \frac{| \frac{q e B}{\pi^2} \left[ \frac{\beta}{m^2} \int_0^\infty dp_z \left( p_z^2 - \Omega_{\omega_p} \right)^2 \right] }{ \alpha_{eff} C_2 \ln(T/m)}$$

$$\times \int_0^\infty dp_z \left( p_z^2 - \Omega_{\omega_p} \right)^2 \frac{f_p^2(1 - f_q^2)}{f_q + 1}. \quad (33)$$

Here, $f_q^0$ is the quark distribution function with $l = 0$ level. Bulk viscosity $\zeta_{eff}$ depends on the behavior the term $(p_z^2 - \Omega_{\omega_p} E_p^0)^2$ along with the momentum distribution function and the effective coupling constant.

**D. Bulk viscosity beyond LLL approximation**

Effect of HLLs on the effective coupling $\alpha_{eff}^l$ and thermal relaxation $\tau_{eff}$ are defined in the Eq. (12) and Eq. (19) respectively. Higher order Landau level corrections to the QCD thermodynamics (pressure, entropy density etc.) are described in our previous work [38] and utilized in the present work wherever required. The longitudinal pressure and energy density with HLL corrections have the form,

$$P_L = \sum_f \frac{|q e B|}{\pi^2} N_c \int_0^\infty dp_z (2 - \delta_m) \frac{p_z^2}{E_p} f_q^l, \quad (34)$$

and

$$\varepsilon_L = \sum_f \frac{|q e B|}{\pi^2} N_c \int_0^\infty dp_z (2 - \delta_m) \frac{\omega_p^2}{p_z} f_q^l, \quad (35)$$

In which $E_p^l = \sqrt{p_z^2 + m^2 + 2l | \frac{q e B}{\pi} |}$ is the Landau levels of order $l$. The integration phase factor and quasiquark distribution function are defined in Eqs. (7) and (1) respectively. Incorporating these, we can calculate $\Omega \equiv \frac{\partial P}{\partial e}$ with higher order corrections. Finally, the bulk viscosity with higher Landau corrections has the following form,

$$\zeta_{eff} = \frac{1}{3} \frac{| \frac{q e B}{\pi^2} \left[ \frac{\beta}{m^2} \int_0^\infty dp_z \left( p_z^2 - \Omega_{\omega_p} E_p^0 \right)^2 \right] }{ \alpha_{eff} C_2 \ln(T/m)}$$

$$\times \frac{1}{\beta} \sum_{l=0}^\infty \frac{1}{z_q} \int_0^\infty dp_z (2 - \delta_m) \frac{\omega_p^2}{p_z} f_q^l \frac{f_p^2(1 - f_q^2)}{f_q + 1}. \quad (36)$$

In transport theory, the viscosity to entropy ratio $\xi_{eff}/s$ has significant importance. The temperature behavior and the effects of HLLs on $\xi_{eff}/s$ are discussed in the next section.

**III. RESULTS AND DISCUSSIONS**

We initiate our discussions with the hot QCD medium dependence on the thermal relaxation time $\tau_{eff}$ and the effective coupling $\alpha_{eff}^l$. The medium dependence on $\alpha_{eff}^l$ and $\tau_{eff}$ are explicitly shown in Fig. 2 and Fig. 3 respectively. Thermal relaxation time defined in Eq. (20) encoded the microscopic interactions of the system, which are the dynamical inputs for the estimation of bulk viscosity. The hot QCD medium effects embedded through EoS dependence on the bulk viscosity of $P/T$ processes can be inferred from the Eq. (33). The EoS dependence is entering through the quasiparton momentum distribution functions along with the effective coupling. We plotted the variation of $\xi_{eff}/\xi_{ideal}$ with $T/T_c$ for $| eB | = 0.3$ GeV$^2$ with LLL approximation.

FIG. 5: Temperature behavior of ratio of bulk viscosity to entropy (left panel) and $(\varepsilon - P/\Omega)$ (right panel) at $| eB | = 0.3$ GeV$^2$ with LLL approximation.
0.00
0.02
0.04
0.06
0.08
0.10
T/T_c
\(\zeta_{\text{eff}}\)

FIG. 6: Comparison of the temperature behavior of \(\zeta/s\) for the \(1 \rightarrow 2\) processes at \(|eB| = 0.3\) GeV\(^2\) with Lattice data \[69, 70\] and sum rule analysis \[71\] in the absence of magnetic field.

Next, we present the temperature behavior of bulk viscosity to entropy ratio for \(1 \rightarrow 2\) process in strong magnetic field. Explicit dependence of temperature on \(\zeta_{\text{eff}}/s\) is shown in Eqs. \[33\] and \[36\]. The Eq. \[20\] shows that the coupling constant \(\alpha\) entering through the relaxation time (and hence bulk viscosity) of \(1 \rightarrow 2\) processes as \(1/\alpha\) whereas for \(2 \rightarrow 2\) processes as \(1/\alpha^2\). In Fig. \[5\] we have depicted \(\zeta_{\text{eff}}/s\) in the presence of magnetic field as a function of \(T/T_c\), for both the EoS in LLL approximation. The behavior of bulk viscosity depends on the \(\Omega\). The temperature behavior of \((\epsilon - P)/T^4\) is shown in Fig. \[5\]. This term is significantly important in the Eq. \[33\] of \(\zeta_{\text{eff}}/s\). The higher value of \(\zeta_{\text{eff}}/s\) near to the transition temperature \(T_c\) is due the term \((\epsilon - P)/T^4\). At very high temperature \(\zeta_{\text{eff}}/s\) approaches to zero.

We compared the bulk viscosity to entropy ratio of \(1 \rightarrow 2\) processes with that from sum rule analysis \[71\] and lattice data results \[69\] as in Fig. \[6\]. In \[71\], the universal properties of bulk viscosity in the absence of magnetic field are studied from the sum rule analysis. We observe that the magnetic field enhances the \(\zeta/s\). HLLs corrections are significant for the higher temperature ranges. We plotted the HLLs corrections to the bulk viscosity in the strong magnetic field background in the chosen temperature range in Fig. \[7\]. Corrections up to \(l = 3\) Landau level are shown in the figure. Higher order corrections beyond third Landau level seems to be negligible in the chosen temperature range. Since the HLLs thermal occupation depends on \(\exp(-\sqrt{eB/T})\), higher order corrections are significant at very high temperature. The dominant contributions of the higher order corrections are entering through the effective coupling and the momentum distribution function. Evaluation of the higher order corrections to the matrix element of the processes are beyond the scope of this work.

IV. Conclusion and Outlook

In conclusion, the bulk viscosity of the hot magnetized QCD medium gets significant contributions from both the magnetic field and the EoS. The most significant contributions in the strong magnetic field limit to the bulk viscosity come from the \(1 \rightarrow 2\) processes in the medium (as these are not possible in the absence of the field). The bulk viscosity has been computed from semi-classical transport theory approach within the relaxation time approximation. The thermal relaxation time for the quarks is obtained from their respective damping rates in the medium considering the same process. The effects of magnetic fields are encoded in the effective quark/antiquark momentum distribution functions in the form of the Landau levels and also in their energy dispersion relations. On the other hand, the gluon dynamics is affected through the effective coupling that has been obtained in our analysis, again following the transport theory approach. The hot QCD medium effects in the thermal relaxation time of the quarks are found to be negligible at very high temperature. Furthermore, the leading order term in the bulk viscosity of hot perturbative QCD in strong field limit has been estimated from the EQPM using relaxation time approximation and compared against the es-
timations with and without the magnetic field in other approaches. The results in the present work turned out to be consistent with other recent works. All the analysis is done in LLL approximation first, and then the effects from the HLLs have been included. The HLLs corrections of the bulk viscosity are found to be quite significant at the higher temperatures.

We intend to calculate other transport coefficients such as shear viscosity and charge diffusion coefficient in the near future. Looking at the non-linear aspects of the electromagnetic response of the QGP would be another direction to work.

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Appendix A: Boltzmann equation in the linear response regime

We need to solve the Boltzmann equation with appropriate collision integral for $1 \to 2$ processes. We have,

$$\left(\partial_t + v_z \partial_z\right) f_q^0(p_z, t, z) = -\frac{\delta f_q^0}{\tau_{eff}}, \quad \text{(A1)}$$

where $\tau_{eff}$ is the thermal relaxation time for $1 \to 2$ process. We consider $u_z$ and $\delta f_q^0$ to be small since the prime focus is on the linear response regime. Using extended EQPM quasi quark momentum distribution defined in Eq. (4) the Eq. (A1) becomes,

$$\delta f_q^0 = -\tau_{eff} f_q^0 (f_q^0 - 1) \times \left[ E_p^0 \partial_t \beta + z_q \partial_z \left( \beta v_z \Theta(z) \right) \right], \quad \text{(A2)}$$

with $\Theta(z) \equiv (\partial_z u_z)$. Since temperature is time dependent, Eq. (A2) becomes,

$$\delta f_q^0 = -\tau_{eff} f_q^0 (f_q^0 - 1) \times \left[ (E_p^0 - \partial_q \ln z_q) (\partial_t \beta - \beta v_z \Theta(z)) \right]. \quad \text{(A3)}$$

Finally, we have used $\partial_t \beta = \beta \Omega \Theta(z)$ as defined in the Ref [24]. Thus we end up with

$$\delta f_q^0 = -\tau_{eff} f_q^0 (f_q^0 - 1) \Theta(z) \times \left[ (E_p^0 + T^2 \partial_q \ln z_q) \Omega - v_z p_z \right], \quad \text{(A4)}$$

where $(E_p^0 + T^2 \partial_q \ln z_q) \equiv \omega_p^0$ is the single particle energy in EQPM.

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