Dynamic analysis and management optimization for maritime supply chains using nonlinear control theory

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ABSTRACT
As world trade becomes ever more globalised and interconnected, the international maritime supply chains are experiencing challenges as well as enjoying greater business opportunities. Chaotic systems are always described by deterministic laws, however they produce completely erratic and divergent behaviors. The study of system dynamics and management strategy has been promoted to improve the efficiency and effectiveness of chaotic maritime supply chains. This paper deals with a nonlinear control theory for how to intensely explore complex dynamical behaviors in multi-echelon supply chain networks. Also, novel sliding mode controllers are designed to regulate the supply chain management to achieve the control and synchronization of chaotic behaviour of the system dynamics. Extensive numerical simulations have been conducted to validate the efficacy and reliability of the presented control algorithms for maritime supply chains under parametric perturbations and external disturbances.

Introduction
Efficient supply chain management issue is a problem faced by all enterprises today. About 90% of the world trade is handled by the international maritime shipping industry. Being the vital link between sea and land, this amount of seaborne trade plays an essential role in the global supply chain networks (Song and Lee 2009). Many researches have explored the possible benefits for different management and optimization strategies in the maritime supply chains. The supply chain management is a chaotic system that represents complex nonlinear behaviours due to uncertain subsystem dynamics such as manufacturers, retailers, distributors, and end customers, which can be transformed into multiple stage network models (Thomsen et al., 1991; Hwarng and Thomsen 2014). In fact, supply chain management optimization is an extremely challenging issue because of the impacts from endogenous as well as exogenous factors which are caused by such as system uncertainties in demand, supply, delivery, and forecasting (Hwarng and Xie 2008). In the context of analysing system dynamics and improving supply chain performance, there exist many models in the literature to investigate the system behaviours, for instance, two-dimensional chaotic analysis (Liu, Xia, and Wang 2018; Hou, Zeng, and Zhao 2009), three-stage system using beer distribution model (Ahmad et al. 2014), hyperchaotic and higher-order chaotic phenomena. (Hwarng and Thomsen 2014; Thomsen, Mosekilde, and Sterman 1991). The cause of the system dynamics and the sources of amplification from the downstream to the upstream of the chain is a disruption known as the bullwhip effect. Many studies have been conducted to identify the causes and mitigate this phenomenon such as the suppression and synchronization of chaotic behaviours in the supply chain to eliminate possible undesirable oscillations and volatility (Göksu, Kocamaz, and Uyaroğlu 2015; Ahmad et al. 2014). Despite the increasing number of publications in logistics and supply chain management, there are only a few papers on control engineering approach to analytically explore the underlying mechanisms and eliminate chaotic behaviours of the maritime supply chain dynamics. This paper presents a dynamical model of the three-stage maritime supply chain system. Chaotic system behaviour has been investigated through the equilibrium points with eigenvalue analysis, 3D phase plane and time history analysis. Then, chaos suppression and synchronization have been realized by utilizing a novel sliding mode control algorithm. Sliding mode control (SMC) is a well-known robust control scheme which has been used in many areas (Young, Utkin, and Ozguner 1999; Fei and Lu 2018). However, less research has been conducted to utilize this control synthesis to improve system performance in multi-echelon supply chain management. The rest of this paper is organized as follows. In Section 2, the dynamical analysis is provided for all echelons of the supply chain system. In Section 3, sliding mode control is designed for chaos suppression and synchronization. Numerical simulations are
carried out to demonstrate the effectiveness of the designed controller to the closed-loop supply chains in Section 4. Finally, conclusions are made in Section 5.

**Chaotic supply chain and dynamical analysis**

**Multi-echelon model**

The goal of supply chain management is to explore decision rules for smooth information and production flows through the chains for maintaining stability and efficiency of supply chain business. In this research, the beer distribution model based on three-echelon product network is proposed for dealing with supply chain dynamics and control synthesis (Göksu, Kocamaz, and Uyaroğlu 2015; Ahmad et al. 2014). The schematic diagram of the model is illustrated in Figure 1. A set of three autonomous differential equations which represent the supply chain system is described as follows:

\[
\begin{align*}
\dot{x} &= ay - (b + 1)x \\
\dot{y} &= cx - xz - y \\
\dot{z} &= xy - (b + 1)z
\end{align*}
\]  

(1)

where \(x, y, z\) are state variables; \(a, b, c,\) and \(m\) are constant parameters \((a, b, c, m \in \mathbb{R})\). In the supply chain system, orders propagate from customers to the factory while products are shipped in the opposite flow. The dynamical model describes the time evolutions of state variables representing for volume demanded, including the shipment sent of retailer \(x\), the inventory level of distributor \(y\), quantity produced by manufacturer \(z\). Moreover, the supply chain performance is affected by some system parameters such as transport risk coefficient \((a)\), safety stock and distortion coefficients \((b,c)\) and customer satisfaction \((m)\).

From the chaos theory of dynamical systems, the chaotic model is described by states of dynamical systems that apparently show random states of disorder and irregularities. Often, this model is governed by deterministic ways that are very sensitive to initial conditions and produces a wide variety of non-linear behaviours depending upon the system parameters. Therefore, the initial values need to be chosen to fit the dynamical system in nature and clearly show the chaotic behaviours for the given initial conditions. From which the system dynamic analysis should be conducted to gain greater insights into underlying mechanisms and then to explore opportunities to mitigate unwanted dynamic behaviours.

To illustrate the dynamical behaviours of the chaotic maritime supply chains, the system parameters are selected as follows. The parameter \(c\) indicates the rate of the information distortion at the distributor and can be estimated as the percentage value in a natural way. As described in Table 1, when \(c = 26\), this means that the order quantity from the retailer is placed to the distributor with 26% distortion rate. The safety stock \(b\) is set to 5\% \((b = 0.05)\) of the actual production volume to avoid market fluctuations. In addition, \(m\) and \(a\) indicate customer satisfaction and transport risk coefficient, respectively. If \(m\) is fixed to have percentage value \(m= 10\%\), then the retailer places orders to the distributor with assuming that least 10\% of the orders are satisfied by the customer. This is of course relatively low, but it is chosen to describe the steady-state behaviors in the maritime supply chain model. Finally, in order to mitigate the risk caused by the distributor and improve the transportation service, the transport risk is specifically set to \(a= 1.1\).

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \(a\)      | 1.1    | \(\Delta c\) | 0.5sin(2 \(t\)) |
| \(b\)      | 0.05   | \(\Delta m\) | 0.3sin(2 \(t\)) |
| \(c\)      | 26     | \(\delta_1\) | 0.1sin(1 \(t\)) |
| \(m\)      | 0.1    | \(\delta_2\) | 0.4cos(7 \(t\)) |
| \(\Delta a\) | 0.3sin(2 \(t\)) | \(\delta_3\) | 0.5sin(3 \(t\)) |
| \(\Delta b\) | 0.2cos(5 \(t\)) | \(x_d, y_d, z_d\) | 0,0 |

Figure 1. Beer distribution model of three-echelon supply chain network.
Dynamical analysis of chaotic supply chain system

In general, system dynamical behaviours can be completely described using three characteristics: eigenvalue analysis, phase portraits and time history of state variables. The dynamical analysis is conducted to gain more insights into nonlinear dynamical phenomena, such as periodicity, stability with bifurcation, and chaotic behaviours. First, the eigenvalue analysis of equilibrium points \( E_i \) is to understand local qualitative behaviours of the nonlinear dynamical system. The system parameters are chosen as shown in Table 1. It can be calculated that the dynamical system (1) has three equilibrium points, in which, \( E_0 = (x_0, y_0, z_0) = (0, 0, 0) \) is the trivial equilibrium point of the system. Other equilibrium points are defined as follows:

\[
E_1 = (x_1, y_1, z_1) = \left( \sqrt{(b + 1)(c - 1)}, \sqrt{(b + 1)(c - 1)}, (c - 1) \right)
\]

(2)

\[
E_2 = (x_2, y_2, z_2) = \left( -\sqrt{(b + 1)(c - 1)}, -\sqrt{(b + 1)(c - 1)}, (c - 1) \right)
\]

(3)

Then, the Jacobian matrix \( (J_i) \) of Eq. (1) evaluated at the equilibrium point \( E_0 \) is given by

\[
J_i = \begin{bmatrix}
-a & a & 0 \\
-c - z_i & -1 & -x_i \\
y_i & x_i & b + 1
\end{bmatrix}
\]

(4)

where \( i = 0, 1, 2 \). Based on eigenvalue analysis, local system behaviours are described as follows. With the equilibrium point \( E_0 \), Jacobian matrix (4) has three eigenvalues: \( \lambda_1 = -22.2407, \lambda_2 = 11.2407 \), and \( \lambda_3 = 1.05 \). Similarly, there are three eigenvalues in case of \( E_1 \) and \( E_2 \): \( \lambda_1 = 6.8312 \), and \( \lambda_2 = -8.3906 \pm 2.5373i \). The equilibrium points \( E_0, E_1, E_2 \) are unstable points based on the Routh-Hurwitz criterion since at least one eigenvalue of each equilibrium point has a positive real part.

Next, to investigate the dynamical behaviours over time, the set of initial conditions are chosen as \( (x_0, y_0, z_0) = (0.040, 0.020, 0.035) \). These initial conditions are described as follows: \( x_0 \) is shipment sent of the retailer at the end of the previous period, \( y_0 \) is the inventory level of the distributor at the beginning of the current period, and \( z_0 \) is the remaining goods produced by the manufacturer from the end of the previous period. Those are chosen at a relatively small level to fit a supply chain system in practice. It is noted that depending on initial conditions, a small change in one state of a nonlinear dynamical system can result in large differences in a later state. The time evolutions of state variables and 3D phase plane of the strange attractor of the chaotic system under initial conditions are shown in Figures 2 and 3, respectively. All characteristics show the uniformity of chaos across all levels of the supply chain system. For the time-series simulation, the units of time \( t \) can be selected arbitrarily depending on specific applications, i.e., daily, weekly, or monthly basis. In particular, the amplitude levels of retailer, distributor and manufacturer in time evolution are shown in Figure 2. It is clearly observed that chaotic motion with small amplitude at the downstream; it fluctuates gradually across upstream and reaches the highest level at the manufacturer echelon. This phenomenon is known as bullwhip effects which have a negative influence on the smoothness of the maritime supply chain networks. Figure 3 illustrates 3D phase portraits for the supply chains, in which the dynamic interaction between echelons in the network is nonlinear and unstable. That dynamical behaviour can lead businesses to have either an excess or lack of inventory and shipment sent to the customer. This adverse effect should be effectively managed by the decision-makers in the supply chains based on the business operation strategy. In what follows, the active controller is designed to mitigate the negative impact of bullwhip effect as well as optimize overall business operations.

Active control synthesis for chaotic supply chains

Chaos suppression

Chaos behaviour under parametric uncertainty and disturbances in the supply chain system can be eliminated or synchronized by using nonlinear control synthesis (Göksu, Kocamaz, and Uyaroğlu 2015; Ahmad et al. 2014). In this paper, the management optimization for chaotic supply chains is achieved by a novel sliding mode control synthesis. First, the model in Eq. (1) is rewritten with perturbed form and active control input as follows:

\[
\dot{x} = (a + \Delta a)y - (m + \Delta m + 1)x + \delta_1 + u_1
\]

\[
\dot{y} = (c + \Delta c)x - xz - y + \delta_2 + u_2
\]

\[
\dot{z} = xy - (b + 1 + \Delta b)z + \delta_3 + u_3
\]

(5)

In turn, the state-space representation (5) in the vector-matrix form can be described by

\[
\dot{v} = Av + f(v) + \Delta v + \delta + u
\]

(6)

where \( v = [x, y, z]^T \in \mathbb{R}^3 \) are measurable states vector of the system; \( u = [u_1, u_2, u_3]^T \in \mathbb{R}^3 \) is the control input vector; \( \delta = [\delta_1, \delta_2, \delta_3]^T \) are external perturbations vector. The system matrix \( A \) with fixed nominals parameter of the system is given by

\[
A = \begin{bmatrix}
-m - 1 & a & 0 \\
c & -1 & 0 \\
0 & 0 & -b - 1
\end{bmatrix}
\]

(7)
Besides, $f(v) = [0, -xz, xy]^T$ is a vector of nonlinear terms in the supply chain system. The perturbed term $\Delta v$ represents the parametric uncertainty applied to the four states of the system,

$$
\Delta = \begin{bmatrix}
-\Delta m & \Delta a & 0 \\
\Delta c & 0 & 0 \\
0 & 0 & -\Delta b 
\end{bmatrix}
$$

(8)

Figure 2. Time histories of state variables of chaotic supply chain system: (a) $x(t)$, (b) $y(t)$ and (c) $z(t)$.

Figure 3. 3D phase plane of chaotic supply chain system.
Let $e = [e_1, e_2, e_3]^T \in \mathbb{R}^3$ be the error vector between the desired values $v_d = [x_d, y_d, z_d]^T \in \mathbb{R}^3$ and the actual output $v$. Then, the error vector is rewritten as follows:

$$e = v - v_d = [(x - x_d), (y - y_d), (z - z_d)]^T \quad (9)$$

In the sliding mode control theory, two design steps are needed in order to realize an effective controller. The first step is to define the proper sliding surface, while the second one is to determine the suitable control law. The control law drives the state trajectory converges to sliding surface within a finite time and keep its motion along the sliding surface in the state space. The sliding surface is the geometrical locus consisting of the boundaries of the control structures that the system will move along. The control performance will be confirmed as effectively robust enough with a chaotic system under parametric uncertainties and external perturbations if the system trajectories remain within sliding surfaces (Shahi and Mazinan 2015). In this study, the sliding surface is proposed by

$$\sigma = \dot{e} + k e \quad (10)$$

where $k$ is positive constant ($\in \mathbb{R}^+$) which determines the convergence rate. In practice, the main drawback of sliding mode control (SMC) is numerical chattering which could lead to negative impacts on decision-making and management costs of the supply chain system. Hence, an approximate implementation of sliding mode control techniques is introduced where the discontinuous term in the controller is replaced by a continuous smooth approximation. Specifically, the controller is designed as follows:

$$u(t) = -U \frac{\sigma}{|\sigma| + \epsilon} \quad (11)$$

where $U$ is a sufficiently large positive constant, and $\epsilon > 0$. The schematic diagram of the supply chain system with SMC scheme is illustrated in Figure 4.

**Synchronization scheme**

Similarly, the SMC can be realized to synchronize the perturbed supply chain system with a master system. The reference supply chain model by fixing key parameters is described as the master system. To observe the switching synchronization for chaotic supply chain system, the master-slaver system is given by

$$\begin{align*}
\dot{v}_m &= A v_m + f(v_m) : \text{Master system} \\
\dot{v} &= A v + f(v) + \Delta v + \delta + u : \text{Slave system} 
\end{align*} \quad (12)$$

where the state vector for the master system is denoted by $v_m = [x_m, y_m, z_m]^T \in \mathbb{R}^3$; the vector of nonlinear terms in the master system is defined as $f(v_m) = [0, -x_m z_m, x_m y_m]^T$. The synchronization strategy facilitates today’s maritime supply chains to achieve quickly to market changes in demand, planning, and product design. This management algorithm is particularly suitable for the just-in-time decision-making for maritime supply chain management.

**Numerical simulation**

Extensive numerical simulation tests have been performed to illustrate chaotic suppression and synchronization of the multi-stage supply chains. The system parameters are selected as shown in Table 1.

For chaotic suppression, numerical tests are carried out with the following initial conditions: $(x_0, y_0, z_0) = (0.040, 0.020, 0.035)$. As stated before, it is noted that depending on initial conditions, a small change in one state of a nonlinear dynamical system can result in large differences in a later state. There are no optimal values for initial conditions but they can be chosen to be typical for a particular system. The control actions have been activated at $t = 15$ (time periods) in all simulations. The observed results for chaos suppression with the sliding mode controllers are shown in Figure 5. As expected, system tracking can be converged asymptotically to desire value after the SMC is activated. Hence, despite some disruptor factors such as the bullwhip effect, the SMC can eliminate the periodic trajectories of all state variables in an appropriate time period with ensuring robust stability.

The time responses of state variables for synchronization scheme of supply chain systems are illustrated in Figure 6. To introduce the chaotic behaviour for the supply chains, the same parameters with initial conditions used in chaos suppression are selected to test the synchronization characteristics. For the master system, new initial conditions are given as follows: $(x_{m0}, y_{m0}, z_{m0}) = (10, -3, 20)$. The values of $x_{m0}, y_{m0}$

![Figure 4. Block diagram of supply chain management using active controller.](image-url)
and $z_{m0}$ are chosen such that the dynamical behaviors of the master system are far different from the state variables for the original (slave) system. It makes clearer to observe how state variables of the slave system converge to the master system. As depicted in Figure 6, the designed controllers are successfully implemented for synchronizing chaotic supply chains. All state variables of the slave can converge to the master system after a short period time since the controller has activated at $t = 15$ (time periods). Based on these results, the SMC algorithm can be utilized for efficient synchronization of chaotic supply chains. In addition, the synchronized management system has made the adaptation possible to enterprises on accomplishing goals for the same shipment sent, inventory and produced quantity at all echelons and has significantly mitigated the management risk for market changes.

**Conclusions**

The maritime supply chains have been recognized that the system behaviors are chaotic with parametric uncertainties and external disturbances. However, few supply chain models were investigated using nonlinear dynamics and control theory. The purpose of this paper is to explore the dynamic system properties including the stability of equilibrium points, phase portrait and time-series responses of the realistic supply chain models. By using the nonlinear system theory, the dynamical behaviors show that the distortion of the demand is amplified along with information flow and chaotic oscillations are present at all echelons in the maritime supply chain networks. Furthermore, the novel sliding mode control is designed to deal with chaos suppression and synchronization under uncertainties and disturbances. Extensive numerical simulations have been carried out to validate the proposed control synthesis for decision-making. As a result, synchronization and suppression schemes in the chaos supply chain model are successfully realized in an appropriate time period, confirming the effectiveness and reliability of the proposed control method. Finally, the maritime supply chain optimization algorithms can help the enterprise mitigate the volatility of information flows under a tumultuous market and to match manufacturing output to actual demand with high customer satisfaction.

![Figure 5. Time histories of state variables for chaos suppression with controllers activated at $t = 25$ (time periods): (a) $x(t)$, (b) $y(t)$ and (c) $z(t)$.](image-url)
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No potential conflict of interest was reported by the authors.

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References
Ahmad, I., A. B. Saaban, A. B. Ibrahim, and M. Shahzad. 2014. "Chaos Control and Synchronization of a Novel Chaotic System Based upon Adaptive Control Algorithm." International Journal of Advances in Telecommunications, Electrotechnics, Signals and Systems 3 (2): 53–62. doi:10.11601/ijates.v3i2.90.

Fei, J., and C. Lu. 2018. "Adaptive Fractional Order Sliding Mode Controller with Neural Estimator." Journal of the Franklin Institute 355 (5): 2369–2391. doi:10.1016/j.jfranklin.2018.01.006.

Gökşu, A., U. E. Kocamaz, and Y. Uyaroğlu. 2015. "Synchronization and Control of Chaos in Supply Chain Management." Computers & Industrial Engineering 86: 107–115. doi:10.1016/j.cie.2014.09.025.

Hou, J., A. Z. Zeng, and L. Zhao. 2009. "Achieving Better Coordination through Revenue Sharing and Bargaining in a Two-stage Supply Chain." Computers & Industrial Engineering 57 (1): 383–394. doi:10.1016/j.cie.2008.12.004.

Hwarng, H. B., and X. Thomsen. 2014. "Interpreting Supply Chain Dynamics: A Quasi-chaos Perspective." European Journal of Operational Research 233 (3): 566–579. doi:10.1016/j.ejor.2013.09.025.

Hwarng, H. B., and N. Xie. 2008. "Understanding Supply Chain Dynamics: A Chaos Perspective." European Journal of

Figure 6. Time histories for chaos synchronization with controllers activated at $t = 15$ (time periods): (a) $x(t)$, (b) $y(t)$ and (c) $z(t)$.
Liu, Z., T. Xia, and J. Wang. 2018. “Fractional Two-dimensional Discrete Chaotic Map and Its Applications to the Information Security with Elliptic-curve Public Key Cryptography.” *Journal of Vibration and Control* 24 (20): 4797–4824. doi:10.1177/1077546317734712.

Shahi, M., and A. H. Mazinan. 2015. “Automated Adaptive Sliding Mode Control Scheme for a Class of Real Complicated Systems.” *Sadhana* 40 (1): 51–74. doi:10.1007/s12046-014-0314-x.

Song, D. W., and P. T. Lee. 2009. “Maritime Logistics in the Global Supply Chain.” *International Journal of Logistics Research and Applications: A Leading Journal of Supply Chain Management* 12 (2): 83–84. doi:10.1080/13675560902749258.

Thomsen, J. S., E. Mosekilde, and J. D. Sterman. 1991. “Hyperchaotic Phenomena in Dynamic Decision Making.” In Mosekilde E., Mosekilde L. (eds), *Complexity, Chaos, and Biological Evolution*, 397–420. New York, NY: Springer.

Young, K. D., V. I. Utkin, and U. Ozguner. 1999. “A Control Engineer’s Guide to Sliding Mode Control.” *IEEE Transactions on Control Systems Technology* 7 (3): 328–342. doi:10.1109/87.761053.