Intermediate shapes for incremental sheet forming

W J T Daniel
School of Mechanical and Mining Engineering, University of Queensland
billd@uq.edu.au

Abstract. Incremental sheet forming has limitations such as when forming a steep wall angle with a small radius tool. These limits need to be anticipated by using a multistage strategy with intermediate shapes, that avoids excessive strains. In designing methods to smooth a formed shape to achieve an intermediate shape, work on digital image processing is informative. In particular, K. Crane [6] and others have used solution of Poisson’s diffusion equation to obtain a smoothing of a surface. An alternative approach labelled false elasticity smoothing to estimate intermediate shapes is presented here. Two parameters control smoothing of the final shape: one to scale back curvatures and one to scale back membrane strains. Membrane strains are estimated to give nodal forces. The curvature tensor at any element centre is estimated from the final shape to predict nodal moments. Scaling back moments will reduce local curvature, and scaling back forces will reduce the membrane strains. An elastic finite element solution predicts an intermediate shape, typically giving priority to reducing the local peak curvatures. This can be repeated until the sheet is flattened. The application of this approach to trial geometries is presented, and the choice of smoothing parameters discussed.

1. Introduction
Incremental sheet forming (ISF) involves moving a forming tool in a stepped pathway over a clamped sheet to form it gradually. Meridional strain is often estimated with the sine law which assumes a material point moves vertically, at least before contact with the forming tool, implying that deformation does not extend beyond the region formed. Studies of thickness changes show the sine law does predict meridional strain from a single pass of ISF, where this is the dominant strain component, due to a lack of strain in the tangential direction along the toolpath [1]. While ISF can be done beyond conventional forming limits for necking, a steep wall angle will still lead to excessive strain, as the sine law predicts. The only remedy is to involve more material in the region formed by a multipass strategy in which the strain is spread over a greater amount of material, extending the region formed in the intermediate stages. In two point forming with a die, material can be pulled in toward the die on a second pass, no longer moving vertically, such that the accumulated plastic strain is less than that predicted by the sine rule. A lower wall angle created on a previous forming pass does also reduce by itself the strain accumulated in a multipass strategy, as explained in [2], again implying that material does not move vertically.

Small radii are also hard to form in one stage, due to high local stressing. Where more than one pass is needed, appropriate intermediate shapes that control the strain distribution need to be estimated. A limit on thickness strain is often used a first estimate of a fracture forming limit. By volume conservation, this reflects the change in area of an element of a finite element mesh of the sheet being formed. Juanchao Li et al [2] proposed a guide for the number of forming stages \( n \) based on comparing the total thinning, a thickness ratio \( R' \) to that acceptable in each stage \( R_0 \) as \( \varepsilon = \ln(1 - R_0)/\ln(1 - R') \).
Zhang [3] used an estimation of bulging under hydrostatic pressure to predict intermediate shapes for ISF. Cao et al [4] focus on purely conical geometry and propose a method of tracing motion of a material point to estimate deformation when finding intermediate shapes. Zhaobing Liu in [5] set out to create intermediate shapes by prescribing the area change of triangular elements on the initially horizontal sheet formed, then creating the intermediate shape by moving corner nodes in the vertical direction. By local averaging of thickness, the strain distribution implied by the area changes is made to extend beyond the area of sheet finally formed. The creation of the intermediate shape this way has a fundamental problem of being overdetermined because the number triangles with specified area changes is more than the number of nodes that can be shifted, in fact more than twice as many. By working in from the boundary, every second triangle can have a specified area change until the centre of the sheet is approached, but a quadratic equation with two solutions is needed to estimate the vertical movement of a particular node.

Work on digital image processing is informative on alternative approaches to estimating intermediate shapes by starting from the final shape to be formed and smoothing it. Keenan Crane in [6] uses a range of techniques involving solution of Poisson’s equation to transform triangular meshes on arbitrary surfaces, ending with a spherical or a planar mesh.

The present paper explores an alternative way to estimate intermediate shapes starting from the final shape by pretending that the shape was obtained from elastic rather than plastic deformation and performing “false elasticity” spring-back calculations.

2. Analysis method

Meridional strains are estimated from the sine law \( \varepsilon_m = \frac{1}{\cos(\theta)} - 1 \) where \( \theta \) is local wall angle of an element from the horizontal. If \( \varepsilon_m < 2 \) the strain is corrected so that

\[
\varepsilon_m = \varepsilon_m - \frac{\varepsilon_m}{4}
\]

(1)

Else if \( \varepsilon_m \geq 2 \), set \( \varepsilon_m = 1 \) so that the reversal of this strain cannot collapse an element completely. The modified strains are used to estimate forces at nodes of each element elastically:

\[
F = \int B^T D \varepsilon_T dA
\]

(2)

where \( B \) is the strain/nodal-displacement matrix of a constant strain triangle, \( D \) plane stress elastic stress/strain relations and \( \varepsilon_T = [0, -\varepsilon_m, 0] \) with the local element \( y \) axis in the meridional direction. In a similar way nodal moments are estimated from curvature about the element local \( x \) axis, which is in the horizontal tangential direction. The curvature tensor is first estimated from nodal normals by least squares as by Rusinkiewicz in [7], with each nodal normal found by weighting element normals with element areas for elements surrounding a node. Forces and moments are scaled independently to give loading causing a controlled amount of spring-back, aiming at particular meridional strains, while also increasing the radii of small radius bends. A linear elastic solution is then used with particular scalings of the loading to estimate an intermediate shape. The intermediate shape can then be used to re-estimate forces and moments for a subsequent stage of smoothing.

3. Trial geometries tested

The trial smoothing software is first tried on the hemisphere of Figure 1, with a sheet thickness of 1.016 mm. Elastic properties of aluminium were used. ABAQUS was used as a trial elastic solver. The intermediate shape of Figure 2 is obtained with force and moment scaling factors of 0.4 and 4. The higher scaling of the moments is discussed below. It causes flat material around the edge to be drawn up in the intermediate shape. The finest mesh corresponds to the former position of the 5mm radius.
Figure 1. Dimensioned hemispherical surface

Figure 2. Hemisphere smoothed with $f_F = 0.4, f_M = 4$. Contours: minimum membrane principal strain.

Figure 3. (a) A second stage of smoothing the hemisphere with $f_F = 0.2, f_M = 0.4$ and (b) a third stage of smoothing with $f_F = 0.51, f_M = 0.34$.

Two subsequent steps of smoothing where then done (opposite in order to the steps of forming), re-estimating the loading from the new mesh geometry. The first notably reduces the height, as in Figure 3(a). The second both reduces the height and further increases the edge radius, the force and moment factors being adjusted to go close to flattening the sheet, as in Figure 3(b).

The jellybean shape of Figure 4(a) is tested next. It is hard to form due to its steep walls and small radii. It is also created with two edges excessively close to the clamped boundary, and is unsymmetrically positioned on the sheet formed. This increases the stiffness variation of the sheet to better test the present approach. If we seek an intermediate shape, the appropriate scaling of the moments estimated is higher if the force scaling is not too high, as we wish to draw more material into the region formed without causing unwanted local bowing of the sheet near where we are unbending a curve. In Figure 4(b) the forces are scaled by 0.5 and the moments by 5. This not only reduces the depth of forming the sheet and unbends the small radii, but generates larger curves top and bottom, as desired to smooth the shape. On the other hand, if the forces are scaled by 0.9 on a first stage of smoothing, which almost flattens the sheet, as in Figure 4(c), then a high moment scaling cannot be
used, as it will cause unwanted local excess deformation. The case in Figure 4(c) uses a 0.9 factor on the moments as well as the forces. The previous plots involve fixing nodes vertically outside the region planned to be formed – that is, in the region where no thickness change is due to forming has been estimated by local averaging of predicted thickness changes to smooth them. This prevents the sheet from bowing down, as in Figure 4(d) where these restraints are removed.

(a) Jellybean shape with finite element mesh.     (b) Smoothing of the jellybean with \( f_F = 0.5, f_M = 5 \).  
Contours: displacement (blue down, red up)

(c) Smoothing the jellybean with \( f_F = 0.9, f_M = 0.9 \).  
(d) Smoothing the jellybean as in (b) without restraints in the unformed region of the panel.

Figure 4. Smoothing the jellybean shape.

Physical insight into the reason for scaling the moments differently can be gained as follows. If material moves vertically, as in the sine law used to estimate forces (Figure 5(b)), then applying moments to unbend an arc does not affect material to the right outside the region formed. The change in curvature can be related to the change in arc length, as in Figure 6. This is of interest as the forces applied to cause springback relate to the arc length reduction but the moments applied cause a curvature reduction. If however the material does not move vertically, as we expect in a second stage of forming, then a much larger change in curvature can occur for the same change in arc length.
Figure 5. Unbending an arc
(a) motion of a material point is inclined, bisecting the arc
(b) motion of a material point is vertical, sine law

Figure 6. Curvature change compared to arc length change, when unbending an arc.

The case in Figure 5(a) is examined by maintaining the chord length constant while unbending the arc. Material beyond the bend is deformed on either side. The geometry of this situation leads to little change in arc length, but a large reduction in curvature, as in Figure 6. Hence it is not unexpected that a much larger scaling of moments than forces can be needed where we wish to not only unbend an arc but also bend material on either side to obtain the intermediate shape like that of Figure 2 or Figure 4(b).

4. Conclusions

A false elasticity method of estimating intermediate shapes for incremental sheet forming is proposed. The forces applied to oppose meridional membrane strain are effective in shrinking the object formed, and reducing the wall angle to one appropriate for an intermediate shape. Application of moments to flatten the part by opposing curvature is less simple, as there is a need to avoid an excessive springback to a shape that crosses the original plane of the sheet, as in Figure 4(d). Moments need reacting to prevent this. In general this could imply the need for a nonlinear elastic analysis with the undeformed
plane of the sheet acting as a contact surface. A further reason for applying constraints in a simulation is the need to avoid interference with the die when forming over a die. Simulations involving this and other asymmetrical or stepped geometries are being undertaken.

Moments also can need to be scaled by a factor significantly greater than one, as the material beyond the region formed has significant bending stiffness and is also being subjected to bending. The higher scaling has the desired effect of including extra material in the region deformed, thereby implying a strain history in the real forming process in which the history of deformation is reversed, which has the effect of reducing the accumulation of strain locally at a bend by involving more material in the stretching process.

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