Mode Confinement in Photonic Quasi-Crystal Point-Defect Cavities for Particle Accelerators

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In this Letter, we present a study of the confinement properties of point-defect resonators in finite-size photonic-bandgap structures composed of aperiodic arrangements of dielectric rods, with special emphasis on their use for the design of cavities for particle accelerators. Specifically, for representative geometries, we study the properties of the fundamental mode (as a function of the filling fraction, structure size, and losses) via 2-D and 3-D full-wave numerical simulations, as well as microwave measurements at room temperature. Results indicate that, for reduced-size structures, aperiodic geometries exhibit superior confinement properties by comparison with periodic ones.

A promising approach to the development of new types of compact, efficient microwave particle accelerators is to realize structures having higher frequencies of operation, since this can increase the accelerating field gradient and reduce the power consumption. Unfortunately, in a particle accelerator, the higher the frequency, the stronger the excitation (by the wakefield effect) of higher order modes (HOMs), with a significant reduction of the beam stability. Nowadays accelerators operate at frequencies \( \lesssim 1 \text{ GHz} \), and they usually rely on waveguide-based HOM dampers in order to efficiently suppress the wakefields. However, at higher (\( \gtrsim 10 \text{ GHz} \)) working frequencies, the standard configurations used for HOM damping become rather cumbersome or even technically unfeasible.

In the past, open metallic photonic-crystal (PC) cavities, based on periodic arrangements of inclusions, have been proposed as candidates for a new generation of accelerating cells since they exhibit electromagnetic (EM) responses that can be highly selective in frequency. This property (and others related) arises from the formation of photonic bandgaps (PBGs), whereby multiple scattering of waves by periodic lattices of inclusions acts to prevent the propagation of EM waves within certain frequency ranges. In these structures, an open resonator can be easily created by introducing a lattice defect, e.g., by removing one inclusion. Unlike conventional closed cavities, such a point-defect PBG cavity can be designed to support only one bound mode (strongly localized within the defect region), and mostly extended HOMs. This suggests the possibility of using PBG-based cavities for effective HOM wakefield suppression, without the need for mode couplers or detuning. Indeed, a prototype of a large-gradient accelerator that relies on a metallic PBG structure has been successfully fabricated and experimentally characterized. Superconducting prototypes have been also tested at 4 K, showing quality factors up to \( \sim 10^5 \), limited by radiation losses only. All-dielectric or hybrid-dielectric PBG structures could also be used, thereby eliminating or reducing the characteristic metallic losses at the frequency of operation, even if at the expense of an increased radiative contribution.

Moreover, it should be observed that spatial periodicity is not an essential ingredient for obtaining PBG and related confinement effects. During the past few years, following up on the discovery of “quasicrystals” in solid-state physics, similar effects have been found in aperiodically-ordered structures too (see, e.g., Ref. 8 for a recent review). Such structures, commonly referred to as “photonic quasicrystals” (PQCs), are typically based on the so-called “aperiodic-tiling” geometries, characterized by weak (local or statistical) rotational symmetries of “noncrystallographic” type (e.g., of order 5, 8, 12). In PQCs, the EM response can be strongly dependent on the lattice short-range configuration and on multiple interactions, and yet almost independent of the incidence angle, and can be engineered/optimized via judicious exploitation of the additional degrees of freedom endowed by aperiodicity (see, e.g., Ref. 8). Moreover, in view of their many non-equivalent sites, PQCs exhibit a number of possible different defects useful for field confinement, as well as intrinsic localized modes. In this connection, PQC-based optical microcavities have already been successfully fabricated, with the ability of strongly confining the light with both high quality factors and small modal volumes.

In this Letter, we present a comparative study of the confinement properties of different point-defected PBG cavities made of cylindrical dielectric rods arranged according to representative aperiodic PQC geometries. The study is aimed at highlighting the potential advantages offered by PQC dielectric structures in the design of PBG-based accelerating resonators, with respect to their periodic PC counterparts. In this framework, we present a body of results from numerical full-wave simulations.
compare the confinement properties of the various configurations (including a reference periodic one) as a function of the structure size, filling fraction, and losses. In addition, for experimental verification, we present the results of measurements at room temperature on cavity prototypes operating in the microwave region (\(\sim 16.5\) GHz).

The structures of interest are composed of sapphire (relative permittivity \(\varepsilon_r = 9.2\), and typical loss-tangent of \(10^{-6}\)) circular rods of radius \(r = 0.15\) cm and height \(h = 0.6\) cm, placed at the vertices of a periodic or aperiodic tiling with lattice constant \(a = 0.75\) cm. The structure is sandwiched between two metallic plates, and presents a bore radius of 0.2 cm for the particle beam transit. Specifically, as shown in Fig. 1, we consider two representative (aperiodic) PQC geometries based on the Penrose (5fold-symmetric, Fig. 1(a)) and on the dodecagonal (12fold-symmetric, Fig. 1(b)) tilings (see, e.g., Refs. 5,14 for details about their generation), and a reference periodic (triangular) PC geometry (Fig. 1(c)).

![Fig. 1: (color online) Point-defect PBG cavity geometries for Penrose (a), dodecagonal (b), periodic (triangular) (c) lattices, with structure radius \(R = 7a\) and filling fraction \(r/a = 0.2\).](image)

The structures are obtained by cutting, from suitably large lattices with same rod properties and lattice constant, a circular area of radius \(R\), and finally removing the central rod, as shown in Fig. 1 in coincidence with the beam transit aperture. We assume time-harmonic (\(\exp(-i2\pi ft)\)) excitation and transverse magnetic (TM) polarization (electric field parallel to the rods axis).

Our full-wave numerical study of the EM response of the structures is based on the combined use of a commercial 3-D simulator (CST Microwave Studio) based on finite integration in the time domain and an in-house 2-D code based on a Bessel-Fourier multipole expansion. The two main loss mechanisms are related to radiation (in view of the open character of the structure) and Ohmic dissipation in the metallic plates, the sapphire dielectric losses being actually negligible. Assuming the metallic plate separation smaller than half a wavelength, so that the field distribution is rather uniform in the direction along the rods axis, the radiation losses can be efficiently modeled using the 2-D code (which assumes the dielectric rods infinitely long). This renders a parametric study computationally affordable.

Figure 2 shows, for the three geometries of interest, the radiative quality factor \((Q_r = f_r^{(r)} / 2f_r^{(i)}\), with \(f_r^{(r)} + i f_r^{(i)}\)

![Fig. 2: (color online) Radiative quality factor of the fundamental TM mode, for Penrose (triangles), dodecagonal (squares), and periodic (circles) geometries, as a function of the filling fraction \(r/a\), and for different values of the structure radius \(R\).](image)

denoting the complex eigenfrequency obtained by solving a source-free problem\(^{15}\) dependence on the filling fraction \(r/a\) and the structure size \(R\). We observe that, for all geometries, a maximum \(Q_r\) is achieved for \(r/a\) ranging between 0.18 and 0.20. Differences among the geometries emerge when considering the dependence on the structure size \(R\). Specifically, dodecagonal PQC cavities outperform the periodic ones for small to moderate structure sizes, and become eventually comparable for \(R > 8a\). Particularly intriguing is the behavior of the Penrose PQC cavities, which exhibit the lowest \(Q_r\) for smaller sizes \((R \lesssim 4a)\), and the highest for moderate cavity sizes. Similar indications emerged from the study of the electric field spatial distribution of the resonant modes, shown in Fig. 3 for a moderate structure size \(R = 5a\), so as to better visualize the power radiated outside the cavity.

![Fig. 3: (color online) Normalized electric field intensity maps (in dB) of the fundamental resonant TM mode for Penrose (a), dodecagonal (b), and periodic (c) configurations, with \(R = 5a\) and \(r/a = 0.2\).](image)

As compared with the periodic case, radiation in PQC structures appears to be much better confined. In the case of the dodecagonal structure, the field configuration is also more isotropic than in the periodic case, due to its higher rotational symmetry. Results tend to become much more similar for larger structures.
We also studied other tiling geometries (e.g., octagonal\textsuperscript{17}), whose responses, however, turned out not to be particularly interesting (being always intermediate between the dodecagonal and periodic ones), and are accordingly not shown.

For modeling the conducting losses of the metal plates, which become dominant for larger-size structures, we use the commercial (CST Microwave Studio) 3-D simulator, whose eigenmode solver option also allows the computation of all the main parameters (shunt impedance, conducting and dielectric loss contributions) of interest for accelerating cavities, with the exception of the radiation losses.

Table I summarizes, for the cases under test and for the fundamental TM mode of point-defect mid-sized cavities ($R = 4a, 5a$) having Penrose (PEN), dodecagonal (DOD), and periodic triangular (PER) geometries, and filling fraction $r/a = 0.2$.

| TABLE I: Simulated characteristic parameters for the fundamental TM mode of point-defect mid-sized cavities ($R = 4a, 5a$) having Penrose (PEN), dodecagonal (DOD), and periodic triangular (PER) geometries, and filling fraction $r/a = 0.2$ |
|---|---|---|---|---|
| $f_c$ [GHz] | $Q_c$ | $Q_r$ | $r_s$ [MΩ/m] |
| PEN | DOD | PER | PEN | DOD | PER |
| 16.788 | 16.424 | 16.556 | 16.770 | 16.429 | 16.561 |
| 11400 | 11800 | 11900 | 12200 | 11900 | 11900 |
| 5000 | 17200 | 5700 | 147000 | 93000 | 40800 |
| 60 | 180 | 100 | 280 | 280 | 240 |

The $r_s$ values well compare with data previously reported on PBG metallic cavities\textsuperscript{2}, and suggest that hybrid dielectric structures can be successfully exploited for the design of high-gradient accelerators.

For experimental verification, we fabricated prototypes of the simulated structures, by suitably placing single crystal sapphire rods between two Oxygen-Free High-Conductivity (OFHC) copper plates. In the experimental setup, input and output tiny dipole antennas are placed on the top plate; the input antenna feeds the cavity near (but non in correspondence of) the defect region, so as to avoid extreme reflection from that port, whereas the output antenna is sufficiently far from the dielectric rods. Both input and output antennas are then connected to a HP8720C Vector Network Analyzer. Careful attention is paid to ensure that measurements are carried out in a weak coupling configuration (insertion losses at the resonance $\sim -40$dB). From the frequency response of the transmission scattering parameters, we evaluate the resonance frequency $f_c$ and the unloaded overall quality factor $Q_T$ for mid-size structures ($R/a = 4, 5$) featuring each of the geometries under study. Concerning the spectral response, results confirm the intrinsically monomodal behavior of PQC cavities, with the defect mode being the only one transmitted well within the corresponding bandgap. The measured resonance frequencies agree pretty well (within 2%) with the numerical predictions in Table I. There is also a good agreement between the experimentally measured $Q_T$ and the values extracted from simulations ($Q_T = (Q_c^{-1} + Q_r^{-1})^{-1}$), as shown in Fig. 4.

For $R = 4a$, the performance is limited by radiation losses only, and therefore the $Q_T$ factor is strongly dependent on the geometry, as expected from the results shown in Fig. 2. Surprisingly, the measured quality factors are always higher than those predicted by the simulations. Conversely, for $R = 5a$, $Q_T$ is almost independent of the dielectric rods arrangement, since it is strongly dominated by the conductive losses. The slight discrepancy between measurements and simulations in this latter case should be ascribed to the inherent uncertainty in the copper conductivity values considered in the modeling (see, e.g., Ref. [16]).

It is worth mentioning that the quality factors of our hybrid-dielectric aperiodic structures are higher than those found for metallic periodic cavities ($\sim 4 \cdot 10^8$) of similar size and resonance frequency ($\sim 17.1$ GHz).\textsuperscript{2} This is due to the combined reduction of both Ohmic and radiative losses, by using dielectric inclusions and exploiting aperiodic geometries. Additional advantages of hybrid-dielectric resonators include a reduced complexity in the fabrication, and the possibility of operating at higher frequencies using HOMs without mode competition. Moreover, it is worth noting that, in principle, breakdown and charging can be strongly reduced or avoided in a dielectric accelerating structure\textsuperscript{17,18}.

To sum up, our comparative study indicates that, although in the limit of very large structure size the confinement properties of periodic and aperiodic geometries tend to become comparable, for small to moderately-sized structures specific aperiodic configurations (in our
case, Penrose and dodecagonal) turn out to outperform the reference periodic one. In particular, for the structure size $R = 5a$, we found the Penrose-type geometry to provide a considerable improvement (of nearly a factor 4) in the radiative quality factor, as compared with the periodic (triangular) counterpart (see Table I). The physical interpretation of such observation can be addressed recalling the mechanisms underlying the bandgap formation in PQCs, which, at variance with the periodic PC case, are not necessarily related to long-range interactions. With specific reference to Penrose-type PQCs, it was shown in Ref. 7 that the bandgap pertaining to the point-defect mode of interest here is actually attributable to short-range interactions. Accordingly, for reduced-size structures, we intuitively expect such a geometry to provide a more effective field confinement than a periodic counterpart.

An improvement in the quality factor, achievable via a simple spatial rearrangement of the inclusions, constitutes indeed an attractive perspective toward the design of cavities for compact, efficient microwave particle accelerators. However, as shown by our numerical and experimental results, such improvement can actually be washed out by conducting losses in the metallic plates. In this framework, current and future studies are aimed at the fabrication and testing of cryogenic hybrid-dielectric prototypes. Preliminary results show an improvement between 2 and 3 in the overall quality factor, with decreasing the operating temperature to $\sim 100$ K, mainly due to the reduction of the conducting losses. The use of superconducting plates could be also foreseen, along the lines of the results presented in Ref. 4. In this last case, however, a deeper study should be undertaken in order to understand to what extent the improvement in the cavity performance would justify the higher complexity introduced by the necessity of operating at 4 K (or even less).

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