The Hadronic Light-by-Light Contribution to Muon $g - 2$: a Short Review

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Abstract

I review the recent calculations and current status of the hadronic light-by-light scattering contribution to muon $g - 2$. In particular, I discuss the main results obtained in a recent work together with Eduardo de Rafael and Arkady Vainshtein where we came to the estimate $\sigma_\mu^{\text{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}$. How the two-photon physics program of low energy facilities can help to reduce the present model dependence is also emphasized.

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1 Introduction

One momenta configuration out of the six possible ones contributing to the hadronic light-by-light to muon g-2 is depicted in Fig. 1 and described by the vertex function

$$\Gamma^\mu(p_2, p_1) = -e^6 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \Pi^{\mu\rho\sigma}(q, k_1, k_2, k_3) \times \gamma_\nu(p_2 + k_2 - m)^{-1} \gamma_\rho(p_1 - k_1 - m)^{-1} \gamma_\sigma$$  \hspace{1cm} (1)

where $q \rightarrow 0$ is the momentum of the photon that couples to the external magnetic source, $q = p_2 - p_1 = -k_1 - k_2 - k_3$ and $m$ is the muon mass.

Figure 1: Hadronic light-by-light scattering contribution.

The dominant contribution to the hadronic four-point function

$$\Pi^{\rho\nu\alpha\beta}(q, k_1, k_3, k_2) = i^3 \int d^4x \int d^4y \int d^4z \ e^{i(-k_1 \cdot x + k_3 \cdot y + k_2 \cdot z)} \langle 0 | T \{ V^\rho(x) V^\mu(y) V^\nu(z) \} | 0 \rangle$$  \hspace{1cm} (2)

comes from the three light quark ($q = u, d, s$) components in the electromagnetic current $V^\mu(x) = [\bar{Q} \gamma^\mu q](x)$ where $\bar{Q} \equiv \text{diag}(2, -1, -1)/3$ denotes the quark electric charge matrix. We are interested in the limit $q \rightarrow 0$ where current conservation implies

$$\Gamma^\mu(p_2, p_1) = -\frac{a_{\text{HLB}}}{4m} [\gamma^\mu, \gamma^\nu] q_\nu.$$  \hspace{1cm} (3)
Therefore, the muon anomaly can then be extracted as

\[
a_{\text{HLbL}} = \frac{e^6}{48m} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{k_1^2k_2^2k_3^2} \left[ \frac{\partial}{\partial q^\mu} \Pi^{\mu\rho\sigma}(q, k_1, k_3, k_2) \right]_{q=0} \times \text{tr}\left\{ (\not{q} + m)\gamma_\mu(\not{q} + m)\gamma_\nu (\not{p} + \not{q} + \not{k}_2 - m)^{-1}\gamma_\rho (\not{p} - \not{k}_1 - m)^{-1}\gamma_\sigma \right\}.
\]

(4)

Here I discuss the results of [1] and [2]. Previous work can be found in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and recent reviews are in [13, 14, 15, 16].

The hadronic four-point function \( \Pi^{\mu\rho\sigma}(q, k_1, k_3, k_2) \) is an extremely difficult object involving many scales and no full first principle calculation of it has been reported yet –even in the simpler large numbers of colors \( N_c \) limit of QCD. Notice that we need it with momenta \( k_1, k_2 \) and \( k_3 \) varying from 0 to \( \infty \). Unfortunately, unlike the hadronic vacuum polarization, there is neither a direct connection of \( a_{\text{HLbL}} \) to a measurable quantity. Two lattice groups have started exploratory calculations [17, 18] but the final uncertainty that these calculations can reach is not clear yet.

Attending to a combined large number of colors of QCD and chiral perturbation theory (CHPT) counting, one can distinguish four types of contributions [19]. Notice that the CHPT counting is only for organization of the contributions and refers to the lowest order term contributing in each case. In fact, Ref. [1] shows that there are chiral enhancement factors that demand more than Nambu-Goldstone bosons in the CHPT expansion in the light-by-light contribution to the muon anomaly. See more comments on this afterwards.

The four different types of contributions are:

- Nambu-Goldstone boson exchanges contribution are \( \mathcal{O}(N_c) \) and start at \( \mathcal{O}(p^6) \) in CHPT.
- One-meson irreducible vertex contribution and non-Goldstone boson exchanges contribute also at \( \mathcal{O}(N_c) \) but start contributing at \( \mathcal{O}(p^8) \) in CHPT.
- One-loop of Goldstone bosons contribution are \( \mathcal{O}(1/N_c) \) and start at \( \mathcal{O}(p^4) \) in CHPT.
- One-loop of non-Goldstone boson contributions which are \( \mathcal{O}(1/N_c) \) but start contributing at \( \mathcal{O}(p^8) \) in CHPT.
Based on the counting above there are two full calculations [3, 4, 6] and [5, 7]. There is also a detailed study of the $\pi^0$ exchange contribution [8] putting emphasis in obtaining analytical expressions for this part.

Recently, two new calculations of the pion exchange using also the organization above have been made. In Ref. [10], the pion pole term exchange is evaluated within an effective chiral model. These authors also study the box diagram one-meson irreducible vertex contribution. The results are numerically very similar to the ones found in the literature as can be seen in Table 1. In Ref. [11], the author uses a large $N_c$ model $\pi^0\gamma\gamma^*$ form factor with the pion also off-shell. This has to be considered as a first step and more work has to be done in order to have the full light-by-light within this approach. In particular, it would be very interesting to calculate the contribution of one-meson irreducible vertex contribution within this model.

Using operator product expansion (OPE) in QCD, the authors of [12] pointed out a new short-distance constraint of the reduced full four-point Green function

$$\langle 0 | T [V^\nu(k_1)V^\rho(k_3)V^\sigma(-(k_1 + k_2 + q))] | \gamma(q) \rangle$$

when $q \to 0$ and in the special momenta configuration $-k_1^2 \simeq -k_3^2 >> -(k_1 + k_3)^2$ Euclidean and large. In that kinematical region,

$$T [V^\nu(k_1)V^\rho(k_3)] \sim \frac{1}{k^2} \varepsilon^{\mu\rho\alpha\beta} \hat{k}_\alpha \left[ 7 Q^2 \gamma_\beta \gamma_5 q \right]$$

with $\hat{k} = (k_1 - k_3)/2 \simeq k_1 \simeq -k_3$. See also [20]. This short distance constraint was not explicitly imposed in previous to [12] calculations.

## 2 Leading in $1/N_c$ Results

Using effective field theory techniques, the authors of [9] shown that leading large $N_c$ contribution to $a^{HLbl}_{\pi^0}$ contains an enhanced term at low energy by $\log^2(M_\rho/m_\pi)$ where the rho mass $M_\rho$ acts as an ultraviolet scale and the pion mass $m_\pi$ provides the infrared scale.

$$a^{HLbl}_{\pi^0} = \left( \frac{\alpha}{\pi} \right)^3 N_c \frac{m^2 N_c}{48 \pi^2 f_\pi^2} \left[ \ln^2 \frac{M_\rho}{m_\pi} + \mathcal{O} \left( \ln \frac{M_\rho}{m_\pi} \right) + \mathcal{O}(1) \right]$$

This leading logarithm is generated by the Goldstone boson exchange contributions and is fixed by the Wess–Zumino–Witten (WZW) vertex $\pi^0\gamma\gamma$. In
Figure 2: A generic meson exchange contribution to the hadronic light-by-light part of the muon $g - 2$.

the chiral limit where quark masses are neglected and at large $N_c$, the coefficient of this double logarithm is model independent and has been calculated and shown to be positive in [9]. All the calculations we discuss here agree with these leading behaviour and its coefficient including the sign. A global sign mistake in the $\pi^0$ exchange in [3, 4, 5] was found by [8, 9] and confirmed by [6, 7] and by others [21, 22]. The subleading ultraviolet scale $\mu$-dependent terms [9], namely, $\log(\mu/m_{\pi})$ and a non-logarithmic term $\kappa(\mu)$, are model dependent and calculations of them are implicit in the results presented in [3, 4, 5, 7, 12]. In particular, $\kappa(\mu)$ contains the large $N_c$ contributions from one-meson irreducible vertex and non-Goldstone boson exchanges. In the next section we review the recent model calculations of the full leading in the $1/N_c$ expansion contributions.

2.1 Model Calculations

The pseudo-scalar exchange is the dominant numerical contribution and was saturated in [3, 4, 5, 6, 7, 8, 10, 11] by Nambu-Goldstone boson’s exchange. This contribution is depicted in Fig. 2 with $M = \pi^0, \eta, \eta'$. The relevant four-point function was obtained in terms of the off-shell $\pi^0\gamma^*(k_1)\gamma^*(k_3)$ form factor $\mathcal{F}(k_1^2, k_3^2)$ and the off-shell $\pi^0\gamma^*(k_2)\gamma(q = 0)$ form factor $\mathcal{F}(k_2^2, 0)$ modulating each one of the two WZW $\pi^0\gamma\gamma$ vertex.

In all cases several short-distance QCD constraints were imposed on these form-factors. In particular, they all have the correct QCD short-distance
Table 1: Results for the $\pi^0$, $\eta$ and $\eta'$ exchange contributions.

| Reference | $10^{10} \times a$ |
|-----------|---------------------|
| $[3, 4, 6]$ | $5.7$ $8.3 \pm 0.6$ |
| $[5, 7]$ | $5.6$ $8.5 \pm 1.3$ |
| $[8]$ with $h_2 = 0$ | $5.8$ $8.3 \pm 1.2$ |
| $[8]$ with $h_2 = -10 \text{ GeV}^2$ | $6.3$ |
| $[10]$ | $6.3 \sim 6.7$ |
| $[11]$ | $7.2$ $9.9 \pm 1.6$ |
| $[12]$ | $7.65$ $11.4 \pm 1.0$ |

The results are in Table 2 where one can see a very nice stability region when $\Lambda$ is in the interval $[0.7, 4.0] \text{ GeV}$. Similar results for the quark loop below $\Lambda$ were obtained in $[3, 4]$ though these authors didn’t discuss the short-distance long-distance matching.

Within the models used in $[3, 4, 5, 6, 7, 8, 10, 11]$, to get the full contribution at leading in $1/N_c$ one needs to add the one-meson irreducible vertex contribution and the non-Goldstone boson exchanges. In particular, below some scale $\Lambda$, the one-meson irreducible vertex contribution was identified in $[5, 7]$ with the ENJL quark box contribution with four dressed photon legs. While to mimic the contribution of short-distance QCD quarks above $\Lambda$, a loop of bare massive heavy quark with mass $\Lambda$ and QCD vertices was used.

The behaviour

$$\mathcal{F}(Q^2, Q^2) \rightarrow \frac{A}{Q^2} \quad \text{and} \quad \mathcal{F}(Q^2, 0) \rightarrow \frac{B}{Q^2}$$

(8)

when $Q^2$ is Euclidean and large and are in agreement with $\pi^0\gamma^*\gamma$ low-energy data. They differ slightly in shape due to the different model assumptions (VMD, ENJL, Large $N_c$, $N\chiQM$) but they produce small numerical differences always compatible within quoted uncertainty $\sim 1.3 \times 10^{-10}$ – see Table 1.

Within the ENJL model, the one-meson irreducible vertex contribution is

\footnote{See however the new measurement of the $\gamma\gamma^* \rightarrow \pi_0$ transition form factor by BaBar $[23]$ at energies between 4 and 40 GeV.$^2$}
Table 2: Sum of the short- and long-distance quark loop contributions \[^5\] as a function of the matching scale $\Lambda$.

| $\Lambda$ [GeV] | $10^{10} \times a_{HLbL}^{\text{HLbL}}$ |
|-----------------|------------------|
| 0.7             | 2.2              |
| 1.0             | 2.0              |
| 2.0             | 1.9              |
| 4.0             | 2.0              |

Table 3: Results for the axial-vector exchange contributions from \[^3\ 4\ 6\] and \[^5\ 7\].

| References | $10^{10} \times a_{HLbL}^{\text{HLbL}}$ |
|------------|------------------|
| \[^3\ 4\ 6\] | 0.17 ± 0.10     |
| \[^5\ 7\]   | 0.25 ± 0.10     |

related through Ward identities to the scalar exchange which we discuss below and both have to be included \[^5\ 7\]. The result of the scalar exchange obtained in \[^5\] is

$$a_{HLbL}^{\text{HLbL}}(\text{Scalar}) = -(0.7 \pm 0.2) \times 10^{-10}. \quad (9)$$

The scalar exchange was not included in \[^3\ 4\ 6\ 8\]. The result of the axial-vector exchanges in \[^3\ 4\ 6\] and \[^5\ 7\] can be found in Table 3.

Melnikov and Vainshtein used a model that saturates the hadronic four-point function in \[^2\] at leading order in the $1/N_c$ expansion by the exchange of the Nambu-Goldstone $\pi^0$, $\eta$, $\eta'$ and the lowest axial-vector $f_1$ states. In that model, the new OPE constraint of the reduced four-point function found in \[^12\] mentioned above, forces the $\pi^0\gamma^\ast(q)\gamma(p_3 = 0)$ vertex to be point-like rather than including a $\mathcal{F}(q^2,0)$ form factor. There are also OPE constraints for other momenta regions \[^24\] which are not satisfied by the model in \[^12\] though they argued that this made only a small numerical difference of the order of $0.05 \times 10^{-10}$. In fact, within the large $N_c$ framework, it has been shown \[^25\] that in general for other than two-point functions, to satisfy fully the QCD short-distance properties requires the inclusion of an infinite number of narrow states.

3 Next-to-leading in $1/N_c$ Results

For the next-to-leading in $1/N_c$ contributions to the $a_{HLbL}^{\text{HLbL}}$ there is no model independent result at present and is possibly the most difficult component. Charged pion and kaon loops saturated this contribution in \[^3\ 4\ 5\ 6\ 7\]. To
dress the photon interacting with pions, a particular Hidden Gauge Symmetry (HGS) model was used in [3, 4, 6] while a full VMD was used in [5, 7]. The results obtained are $-(0.45 \pm 0.85) \times 10^{-10}$ in [3] and $-(1.9 \pm 0.5) \times 10^{-10}$ in [5] while using a point-like vertex one gets $-4.6 \times 10^{-10}$. Both models satisfy the known constraints though start differing at $O(p^6)$ in CHPT. Some studies of the cut-off dependence of the pion loop using the full VMD model was done in [5] and showed that their final number comes from fairly low energies where the model dependence should be smaller. The authors of [12] analyzed the model used in [3, 4] and showed that there is a large cancellation between the first three terms of an expansion in powers of $(m_\pi/M_\rho)^2$ and with large higher order corrections when expanded in CHPT orders but the same applies to the $\pi^0$ exchange as can be seen from Table 6 in the first reference in [2] by comparing the WZW column with the others. The authors of [12] took $(0 \pm 1) \times 10^{-10}$ as a guess estimate of the total NLO in $1/N_c$ contribution. This seems too simply and certainly with underestimated uncertainty.

4 Comparing Different Calculations

The comparison of individual contributions in [3, 4, 5, 6, 7, 8, 10, 11, 12] has to be done with care because they come from different model assumptions to construct the full relevant four-point function. In fact, the authors of [10] have shown that their constituent quark loop provides the correct asymptotics and in particular the new OPE found in [12]. It has more sense to

Figure 3: Goldstone boson exchange in the model in [12] contributing to the hadronic light-by-light.
Table 4: Results for the full hadronic light-by-light contribution to $a^{\text{HLBL}}$.

| Full Hadronic Light-by-Light | $10^{10} a_{\mu}$ |
|------------------------------|-------------------|
| $[3, 4, 6]$                  | $8.9 \pm 1.7$     |
| $[5, 7]$                     | $8.9 \pm 3.2$     |
| $[12]$                       | $13.6 \pm 2.5$    |

compare results for $a^{\text{HLBL}}$ either at leading order or at next-to-leading order in the $1/N_c$ expansion.

The results for the final hadronic light-by-light contribution to $a^{\text{HLBL}}$ quoted in $[3, 4, 5, 6, 7, 12]$ are in Table 4. The apparent agreement between $[3, 4, 6]$ and $[5, 7]$ hides non-negligible differences which numerically almost compensate between the quark-loop and charged pion and $[12]$ are in Table 4. Notice also that $[3, 4, 6]$ didn’t include the scalar exchange. Comparing the results of $[5, 7]$ and $[12]$, as discussed above, we have found several differences of order $1.5 \times 10^{-10}$ which are not related to the new short-distance constraint used in $[12]$. The different axial-vector mass mixing accounts for $-1.5 \times 10^{-10}$, the absence of the scalar exchange in $[12]$ accounts for $-0.7 \times 10^{-10}$ and the use of a vanishing NLO in $1/N_c$ contribution in $[12]$ accounts for $-1.9 \times 10^{-10}$. These model dependent differences add up to $-4.1 \times 10^{-10}$ out of the final $-5.3 \times 10^{-10}$ difference between $[5, 7]$ and $[12]$. Clearly, the new OPE constraint used in $[12]$ accounts only for a small part of the large numerical final difference.

5 Conclusions and Prospects

To give a result at present for the hadronic light-by-light contribution to the muon anomalous magnetic moment, the authors of $[1]$, from the above considerations, concluded that it is fair to proceed as follows:

**Contribution to $a^{\text{HLBL}}$ from $\pi^0$, $\eta$ and $\eta'$ exchanges**

Because of the effect of the OPE constraint discussed above, we suggested to take as central value the result of Ref. $[12]$ with, however, the largest error quoted in Refs. $[5, 7]$:  

$$a^{\text{HLBL}}(\pi, \eta, \eta') = (11.4 \pm 1.3) \times 10^{-10}. \quad (10)$$

Recall that this central value is quite close to the one in the ENJL model when the short-distance quark loop contribution is added there.
Contribution to $a^{\text{HLbL}}$ from pseudo-vector exchanges

The analysis made in Ref. [12] suggests that the errors in the first and second entries of Table 2 are likely to be underestimates. Raising their $\pm 0.10$ errors to $\pm 1$ puts the three numbers in agreement within one sigma. We suggested then as the best estimate at present

$$a^{\text{HLbL}}\text{(pseudo vectors)} = (1.5 \pm 1) \times 10^{-10}.$$  \hfill (11)

Contribution to $a^{\text{HLbL}}$ from scalar exchanges

The ENJL–model should give a good estimate for these contributions. We kept, therefore, the result of Ref. [5, 7] with, however, a larger error which covers the effect of other unaccounted meson exchanges,

$$a^{\text{HLbL}}\text{(scalars)} = -(0.7 \pm 0.7) \times 10^{-10}.$$  \hfill (12)

Contribution to $a^{\text{HLbL}}$ from a dressed pion loop

Because of the instability of the results for the charged pion loop and unaccounted loops of other mesons, we suggested using the central value of the ENJL result but with a larger error:

$$a^{\text{HLbL}}(\pi\text{−dressed loop}) = -(1.9 \pm 1.9) \times 10^{-10}.$$  \hfill (13)

From these considerations, adding the errors in quadrature, as well as the small charm contribution $0.23 \times 10^{-10}$, we get

$$a^{\text{HLbL}} = (10.5 \pm 2.6) \times 10^{-10},$$  \hfill (14)

as our final estimate.

The proposed new $g_\mu - 2$ experiment accuracy goal of $1.6 \times 10^{-10}$ calls for a considerable improvement in the present calculations. The use of further theoretical and experimental constraints could result in reaching such accuracy soon enough. In particular, imposing as many as possible short-distance QCD constraints [3, 4, 5, 6, 7, 8, 11] has result in a better understanding of the numerically dominant $\pi^0$ exchange. At present, none of the light-by-light hadronic parametrization satisfy fully all short distance QCD constraints. In particular, this requires the inclusion of infinite number of narrow states for other than two-point functions and two-point functions with soft insertions [25]. A numerical dominance of certain momenta configuration can help to minimize the effects of short distance QCD constraints not satisfied, as in the model in [12].
More experimental information on the decays $\pi^0 \rightarrow \gamma\gamma^*$, $\pi^0 \rightarrow \gamma^*\gamma^*$ and $\pi^0 \rightarrow e^+e^-$ (with radiative corrections included [22, 26, 27]) can also help to confirm some of the neutral pion exchange results. A better understanding of other smaller contributions but with comparable uncertainties needs both more theoretical work and experimental information. This refers in particular to pseudo-vector exchanges. Experimental data on radiative decays and two-photon production of these and other C-even resonances can be useful in that respect.

New approaches to the pion dressed loop contribution, together with experimental information on the vertex $\pi^+\pi^-\gamma^*\gamma^*$ in the intermediate energy region (0.5 – 1.5 GeV) would also be very welcome. Measurements of two-photon processes like $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$ can be useful to give information on that vertex and again could reduce the model dependence. The two-gamma physics program low energy facilities like the experiment KLOE-2 at DAΦNE will be very useful and well suited in the processes mentioned above which information can help to decrease the present model dependence of $a_{\mu}^{HLbL}$.

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