Low $x$ phenomena

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Abstract

We review recent developments in the application of perturbative QCD to phenomena at small $x$.

Both H1 [1] and ZEUS [2] presented measurements of $F_2(x, Q^2)$ obtained from the 1993 HERA run. A sample of these data is shown in Fig. 1. The dramatic rise of $F_2(x, Q^2)$ with decreasing $x$, discovered in the 1992 data, is now firmly established. The data are compatible with Altarelli-Parisi (or GLAP) evolution from “starting” parton distributions in which the sea quarks have the small $x$ behaviour

$$xS \sim x^{-\lambda} \text{ with } \lambda \sim 0.3, \quad (1)$$

see, for example, the MRS(A) curve [3] in Fig. 1.

Also OPAL [7] presented a measurement of the photon structure function $F_2$ at $x = \text{few} \times 10^{-2}$. No rise with decreasing $x$ is seen at this value of $x$. However both HERA and LEP2 have the potential to measure the photon structure function at much smaller $x$.

1. Deep-inelastic map

To orientate ourselves we show in Fig. 2 a map of the kinematic regime for deep inelastic electron-proton scattering.

(a) GLAP evolution (large $Q^2$)

Starting from a known structure of the proton at $Q^2 = Q_0^2$ we may evolve up to large $\log Q^2$ using the Altarelli-Parisi (or GLAP) equations which are typically of the form

$$\frac{\partial g}{\partial \log Q^2} = P_{gg} \otimes g + ... \quad (2)$$

where the convolution is over the longitudinal momentum fraction. For simplicity, we concentrate on the gluon, the dominant parton at small $x$. Effectively the GLAP equations resum the leading ($\alpha_s \log Q^2$) contributions, which correspond (in a physical gauge) to the $n$-rung gluon ladder of Fig. 3. In fact the leading log arises from the strongly-ordered region of transverse momenta

$$Q^2 \gg k_{Tn}^2 \gg k_{Tn-1}^2 \gg ... \quad (3)$$

When we evolve to high $Q^2$ we probe the proton structure ever more finely, to transverse sizes $\sim 1/\sqrt{Q^2}$, see Fig. 2.

The parton distributions are essential to calculate the cross sections, $\sigma$, for “hard” hadronic processes. First the QCD subprocess, $\hat{\sigma}$, are calculated in the strongly-ordered $k_{Tn}^2 = 0$ approximation and then the factorization theorem gives $\sigma = xg(x, \mu^2) \otimes \hat{\sigma}(\mu^2) + ...$

where the scale $\mu^2 \sim$ hard scattering $p_T^2$.

(b) BFKL equation (small $x$)

Figure 1. A sample of the latest HERA data for $F_2$ [1,2]. The GLAP-based MRS(A) analysis [3] and the BFKL-based AKMS “prediction” [4] give almost indistinguishable descriptions of the HERA data. Also shown are the GRV parton [5] and “hot-spot” shadowing [6] predictions.
On the other hand, when we evolve up to large $1/x$ (i.e. small $x$) we encounter $(\alpha_s \log(1/x))^n$ terms which have to be resummed. Indeed the dramatic rise observed in $F_2$ with decreasing $x$ may be associated with the growth of the gluon density which arises from the resummation of these terms; a growth which, via $g \to q\bar{q}$, is transmitted to the sea quarks probed by the photon. To leading order the summation is accomplished by the BFKL (or Lipatov) equation \[ 3 \], which may be written in the differential form

$$-x \frac{\partial f(x,k_T^2)}{\partial x} =$$

$$\frac{3\alpha_s}{\pi} \int_0^\infty \frac{dk_T^2}{k_T^4} \left[ \frac{f(x,k_T^2) - f(x,k_T^2)}{|k_T^2 - k_T^2|} + \frac{f(x,k_T^2)}{(4k_T^4)^2} \right]$$

$$\equiv K \otimes f. \quad (4)$$

There is now no strong-ordering in $k_nT$ of the emitted gluons and we have to work in terms of the unintegrated gluon distribution $f(x,k_T^2)$ in which the “last” $k_T^2$ integration along the gluon ladder (of Fig. 3a) is unfolded, that is

$$xg(x,\mu^2) = \int \mu^2 \frac{dk_T^2}{k_T^2} f(x,k_T^2). \quad (5)$$

At small $x$ the gluon distribution $f(x,k_T^2,\mu^2)$ becomes independent of the scale $\mu^2$.

From (4) we see that the small $x$ behaviour of $f$ is controlled by the largest eigenvalue $\lambda_L$ of the eigenfunction equation $K \otimes f_n = \lambda_n f_n$, since as $x \to 0$

$$f \sim \exp(\lambda_L \log(1/x)) \sim x^{-\lambda_L}. \quad (6)$$

Indeed for fixed $\alpha_s$ there is an analytic solution for the leading small $x$ behaviour

$$f \sim x^{-\lambda_L}(k_T^2)^2 \exp\left(-\frac{\log^2(k_T^2/k_0^2)}{\log(1/x)}\right) \quad (6)$$

where

$$\lambda_L = (3\alpha_s/\pi)4\log2. \quad (7)$$

This singular $x^{-\lambda_L}$ Lipatov behaviour is in contrast to the naive Regge-type expectations that

$$f \sim x^{1-\alpha_P(0)} \sim x^{-0.08} \quad (8)$$

where $\alpha_P(0)$ is the intercept of the Pomeron.

A second feature of the solution (6) of the BFKL equation is the diffusion in $k_T$ with decreasing $x$, as manifested by the Gaussian form in $\log k_T^2$ with a width which grows as $(\log(1/x))^{1/2}$ as $x$ decreases. The physical origin of the diffusion is clear. Since there is no strong-ordering in $k_T$, there is a “random walk” in $k_T$ as we proceed along the gluon chain and hence evolution to smaller $x$ is accompanied by diffusion in $k_T$. We foresee that the diffusion will be a problem in the applicability of the BFKL equation since, with decreasing $x$, it leads to an increasingly important contribution from the infrared and ultraviolet regions of $k_T^2$ where the equation is not expected to be valid.

For running $\alpha_s$ the singular behaviour and diffusion in $k_T$ are confirmed. In addition it is found that

$$f \sim C(k_T^2)x^{-\lambda} \quad (9)$$

where the value $\lambda \approx 0.5$ is much less sensitive to the treatment of the infrared region in (4) than is the normalization $C$.

(c) Shadowing region

The $x^{-\lambda}$ growth of the gluon cannot go on indefinitely with decreasing $x$. It would violate unitarity. The growth must eventually be suppressed by gluon recombination, which is represented by an additional quadratic term so that (4) has the symbolic form

$$-x \partial f/\partial x = K \otimes f - V \otimes f^2. \quad (10)$$

The additional term contains a factor $\alpha_s^2/k_T^2R^2$ since the gluon-gluon interaction behaves $\sim \alpha_s^2/k_T^2$, whereas $1/R^2$ arises because the smaller the transverse area ($\pi R^2$), in which the gluons are concentrated within the proton, the stronger the effect of recombination. The precise form of this equation, originally proposed by GLR \[ 4 \], is still a matter of debate \[ 10 \]. The region where shadowing should be calculable perturbatively is just below the dashed line in Fig. 2.

2. Small $x$ behaviour of $F_2$

The small $x$ behaviour of $xg$ (and $x\bar{q}$) arising from GLAP evolution depends on the form of the starting distributions. For singular starting distributions, $xg \sim x^{-\lambda}$ with $\lambda > 0$, the small $x$ behaviour is stable to evolution in $Q^2$. The larger the value of $\lambda$ the sooner the stability sets in with decreasing $x$. MRS(A) partons \[ 3 \], with $xg \sim x^{-0.3}$, are an example of this behaviour. On the other hand, for non-singular starting distributions,
\( xg \sim x^{-\lambda} \) with \( \lambda \leq 0 \), we find the double leading logarithm (DLL) form

\[
xg \sim \exp(2[\xi(Q_0^2, Q^2)\log(1/x)]^\frac{1}{2}).
\]

That is \( xg \) grows as \( x \to 0 \) faster than any power of \( \log(1/x) \) but slower than any power of \( x \). The larger the “evolution length”

\[
\xi = \int_{Q_0^2}^{Q^2} dq^2 \frac{3\alpha_s(q^2)}{q^2},
\]

the faster the growth. An example is the “dynamical” GRV partons \([1]\) which evolve from valence-like forms at a low scale \( Q_0^2 = 0.3 \text{ GeV}^2 \), and for which (11) mimics a behaviour \( xg \sim x^{-0.4} \) in the HERA regime.

Given the solution \( f(x, k_T^2) \) of the BFKL equation we can use the \( k_T \)-factorization theorem to predict \( F_2 \), see Fig. 3b:

\[
F_2 = f \otimes F^{\text{box}} + F_2^{\text{bg}} \simeq C'(k_T^2)x^{-\lambda} + F_2^{\text{bg}}
\]

where \( \lambda \simeq 0.5 \), and \( F_2^{\text{bg}} \simeq 0.4 \) is determined from the large \( x \) behaviour of \( F_2 \). Once the overall normalisation of the BFKL term is adjusted by a suitable choice of the infrared parameters in (4), then an excellent description of all the \( F_2(x, Q^2) \) HERA data is obtained. Indeed the BFKL-based “prediction” \([1]\) gives an equally good, and almost indistinguishable, description as the GLAP-based order fit \([1]\), see Fig. 1. With GLAP, the steepness is either incorporated (as a factor \( x^{-\lambda} \)) in the starting distributions or generated by evolution from a low scale \( Q_0^2 \). The steepness can be adjusted to agree with the data by varying \( \lambda \) or \( Q_0^2 \). On the other hand the leading \( \log(1/x) \) BFKL prediction for the shape \( F_2 - F_2^{\text{bg}} \sim x^{-\lambda} \), with \( \lambda \simeq 0.5 \), is prescribed. It remains to see how well it survives a full treatment of sub-leading effects.

Conventional shadowing with gluons spread uniformly across the proton \((R = 5 \text{ GeV}^{-1})\) leads to only a small suppression in \( F_2 \) in the HERA regime. If the gluons were concentrated in “hot spots” of area \( \pi R^2 \) with, say, \( R = 2 \text{ GeV}^{-1} \) the effect would be much stronger, see Fig. 1. But could shadowing be identified since we do not know the partons at small \( x \)? Simulated \( F_2 \) data (of accuracy and \( x \) range which may eventually be accessible at HERA) have been used \([1]\) to see how well \( R \) could be determined. The conclusion is that the interplay between the linear and non-linear terms in (10) leads to a considerable ambiguity between the size of the parton distributions and the amount of shadowing.

Is GLAP evolution adequate in the HERA regime? For sufficiently small \( x \) the \((\alpha_s \log 1/x)^n\) terms must be resummed with the full \( Q^2 \) dependence (and not just the leading and next-to-leading \( \log Q^2 \) terms). Ellis et al. \([2]\) have made a theoretical study of the applicability of GLAP evolution and find that it is adequate in

the HERA small \( x \) regime, provided that the evolution occurs from a sufficiently singular starting distribution, \( xg \sim x^{-\lambda} \) with \( \lambda \gtrsim 0.35 \).

3. Identification of BFKL behaviour

The inclusive nature of \( F_2 \), and the necessity to provide “non-perturbative” input distributions of parton densities for its description, prevents its observed small \( x \) behaviour being a sensitive discriminator between BFKL and conventional dynamics. For this purpose it is necessary to look into the properties of the final state.

The two characteristic features of BFKL dynamics are the absence of strong-ordering of the gluon \( k_T \)’s along the chain (the diffusion in \( k_T \)) and the consequent \((x/x')^{-\lambda} \) or \( \exp(\lambda \Delta y) \) growth of the cross section, where \( x \) and \( x' \) are the longitudinal momentum fractions of the gluons at the ends of the chain, which spans the rapidity interval \( \Delta y = \log(x'/x) \). Recall \( \lambda \simeq 0.5 \). Some processes which exploit these characteristic features are shown in Fig. 4.

The idea \([3]\) in Fig. 4a is to detect deep-inelastic \((x, Q^2)\) events which contain a measured jet \((x_j, k_T^2)\) in the kinematic regime where (i) the jet longitudinal momentum, \( x_j \), is as large as is experimentally feasible \((x_j \simeq 0.1)\), (ii) \( z \equiv x/x_j \) is small, and (iii) \( k_T^2 \simeq Q^2 \) is sufficient to suppress diffusion into the infrared region. The beauty of this measurement is that attention is focussed directly on the BFKL \( z^{-\lambda} \) behaviour at small \( z \). The difficulty is to cleanly separate the forward going jet from the proton remnants. The preliminary results from the H1 collaboration \([4]\) are encouraging and favour the BFKL over the conventional description.

Inspection of Fig. 4b suggests, that due to the relaxation of strong-ordering of the gluon \( k_T \)’s at small \( x \), more transverse energy \( E_T \) should be emitted in the central region (between the current jet and the proton remnants) than would result from conventional evolution. Indeed Monte Carlo predictions based on QCD cascade models \([5]\) fall well below the observed central plateau of height \( E_T \approx 2.1 \text{ GeV per unit of rapidity} \([6]\). A BFKL-based calculation \([7]\), at the parton level, yields about 1.7 GeV per unit of rapidity, but much less if conventional dynamics is used. No hadronization effects have been allowed for.

† A good description has been obtained by a Monte Carlo based on the colour dipole model. This model contains the essence of BFKL dynamics provided the full integration over the emitted gluon \( k_T \) is performed.
Recently there has been renewed interest in the original proposal of Mueller and Navelet \cite{17} that the cross section for the production of a pair of minijets should, according to BFKL dynamics, rise as \( \exp(\lambda \Delta y) \) as the rapidity interval \( \Delta y \) becomes large. The studies \cite{18} show that the effect is masked by the fall-off of the parton densities at large \( x \), but that instead the rate of weakening of the azimuthal (back-to-back) correlation between the jets, could possibly be an indicator of BFKL effects.

BFKL dynamics may be also identified via the weakening of the azimuthal correlation between a pair of jets produced in deep-inelastic scattering at HERA, see Fig. 4d. At sufficiently large values of \( \Delta \phi \equiv \phi - \pi \), BFKL dynamics dominates over the fixed-order QCD contribution from 3+1 jet production, leading to a distinctive tail in the azimuthal distribution which directly probes the \( k_T \) dependence of the gluon distribution \cite{19}.

4. Conclusions

In the HERA small \( x \) regime GLAP evolution (from appropriately parametrized starting distributions) is able to mimic BFKL dynamics as far as the description of \( F_2(x, Q^2) \) is concerned. Moreover it will be difficult to isolate shadowing contributions even with improved measurements of \( F_2 \). Measurements which are less inclusive than \( F_2 \), offer more chance to identify the characteristic BFKL \( x^{-\lambda} \) behaviour and diffusion in \( k_T \). However opening up the final state brings the problems of hadronization and jet identification, and loses some of the small \( x \) “reach” (e.g. \( x \to x/x_j \) where \( x_j \sim 0.1 \)). On the theoretical side, the sub-leading corrections to the BFKL leading \( \log(1/x) \) formalism are urgently needed for future quantitative studies of small \( x \) phenomena.

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