The Measurement and Uncertainty Analysis of Thermal Resistance in Cryogenic Temperature Sensor Installation

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Abstract. The choice of the appropriate installation method plays an important role for accurate temperature measurement. In the cryogenic and high vacuum environment, due to poor contact between the cryogenic temperature sensor and the surroundings that the sensor is installed and intended to measure, the self-heating from sensor measuring current brings about temperature difference and creates a potential temperature measurement error. The self-heating temperature difference is directly proportional to the thermal resistance for a mounted sensor, which means that lower installation thermal resistance of sensors is advantageous to obtain better measurement results. In this paper, a measurement model for the installation thermal resistance of sensor is built in terms of two currents method which is always used to measure self-heating effect. A cryostat that can provide variable temperature in the accurate temperature measurement and control experiments is designed and manufactured. This cryostat can reach 3K in a few hours and the sample temperature can reach as high as 20 K. Based on the experimental results, the measurement uncertainty of the thermal resistance are also analyzed and calculated. To obtain the best measurement results in our cryostat, the thermal resistances of sensors with two installation methods are measured and compared.
1. Introduction

Operation of resistance thermometer requires dissipation of power in the sensor. The heat flux generated by the measurement current creates the temperature difference between the sensor and the environment intended to measure, which is no doubt to produce temperature measurement error or uncertainty. The self-heating temperature difference relates to the thermal resistance between the sensor and its surroundings. Thermal resistance is a significant factor in temperature measurement uncertainty. How to accurately measure the thermal resistance between the sensor and the environment is an important problem in high precise temperature measurement.

Thermal resistance is found to depend on temperature and details of sensor mounting. Apiezon grease and Varnish are always used to mount thermometer sensors in low temperature, because they have high thermal conductivity in low temperature. Details of construction unknown to a user can strongly affect the thermal resistance. It is difficult to calculate an effective thermal resistance for a mounted temperature sensor.

Many scholars have done research on the thermal resistances of the cryogenic temperature sensors. Rusby [1] studied the thermal resistances of a standards grade rhodium-iron sensor from 0.4 to 280K; Schoepe [2] calculated the thermal resistances of a thick film resistor from 0.4 to 280K. Thermal resistances were measured at cryogenic temperatures from 1 to 300K on several commercially available temperature sensors by Holmes and Courts [3].

In this paper, the thermal resistances of the Cernox temperature sensor that mounted by two different methods (Apiezon N Grease, VGE-7031 varnish) from 4.2 to 14K are calculated. The measurement uncertainty of the thermal resistance is also analyzed. A cryostat cooled by Gifford-McMahon (GM) cryocooler that can provide variable temperature in the accurate temperature measurement and control experiments is designed and manufactured. A thermal damper made of PTFE is inserted between the second cold head and the sample to decrease the temperature oscillation of the sample caused by the second cold head [4].

2. Theoretical background

When the resistance $R$ of the Cernox sensor is measured, the measurement current $I$ dissipates power $P$:

$$P = I^2 R$$  \hspace{1cm} (1)

Heat dissipation in the sensor causes a heat flux that generates a temperature gradient from the sensor to the mounting surface. The Cernox sensor therefore measures a higher temperature $T$ instead of the actual temperature $T_0$, which would be measured without the self-heating effect [6]. The temperature difference $\Delta T$ can therefore be calculated as:

$$\Delta T_0 = T - T_0 = P \cdot R_i = I^2 R \cdot R_i$$  \hspace{1cm} (2)

Where $R_i$ is the thermal resistance between the sensor and its environment, and $I$ is the measurement current, $R$ is the resistance of the temperature sensor at the measurement current. The self-heating value can also be calculated in terms of resistance

$$\Delta T_0 = T_i - T_0 = \frac{1}{s} (X_i - X_0) R$$  \hspace{1cm} (3)

Where $X_0$ and $X_1$ are the resistance ratios of zero-current and current $I_1$ separately.

Where $s$ is the sensitivity of the Cernox thermometer at the measurement temperature:

$$s = \frac{dR}{dT}$$  \hspace{1cm} (4)
The resistance ratio of zero-current is unable to be measured. For the highest accuracy measurements, corrections should be applied by measuring at two currents, $I_1$ and $I_2$, and extrapolating to zero current. In addition

$$X_1 - X_0 \cong \frac{I_1^2 (X_2 - X_1)}{I_2^2 - I_1^2}$$  \hspace{1cm} (5)$$

Substituting Eq.(5) to Eq.(3) yields:

$$\Delta T_0 = T_1 - T_0 = \frac{1}{s} \frac{I_1^2 (X_2 - X_1)}{I_2^2 - I_1^2} R = I_1^2 R_i \cdot R_t$$  \hspace{1cm} (6)$$

Therefore, the thermal resistance $R_t$ of the temperature sensor and its mounting environment is:

$$R_t = \frac{1}{sX_i} \frac{X_2 - X_1}{I_2^2 - I_1^2}$$  \hspace{1cm} (7)$$

Differentiation of Eq.(7) leads to the propagation-of-error equation:

$$dR_t = \frac{1}{sX_i} \left( \frac{1}{I_2^2 - I_1^2} dX_2 - \frac{1}{I_2^2 - I_1^2} dX_1 - \frac{2(I_2^2 - X_1)I_2^2 dI_2}{(I_2^2 - I_1^2)^2} \frac{dI_1}{I_2} + \frac{2(X_2 - X_1)I_1^2 dI_1}{(I_2^2 - I_1^2)^2} \frac{dI_2}{I_1} \right)$$

As a consequence, the uncertainty due to the measured effective thermal resistance can be expressed as:

$$u^2(R_t) = \frac{1}{(sX_i)^2 (I_2^2 - I_1^2)^2} \left( u^2(X_2) + \frac{X_2^2}{X_1^2} u^2(X_1) \right) + \frac{4R_t^2}{(I_2^2 - I_1^2)^2} \left( I_2^2 u^2(I_2) + I_1^2 u^2(I_1) \right) + R_t^2 \frac{u^2(s)}{s^2}$$  \hspace{1cm} (9)$$

Where $u(X)$ and $u(I)$ are the uncertainties due to the measurement of resistance ratios and excitation current respectively, while $u(s)$ is the uncertainty due to the fitting of temperature sensitivity. The uncertainties $u(I_1)$ and $u(I_2)$ can be obtained from calibration specification of Fluke 1594A, as seen in Table 1, while $u(s)$ is the uncertainty due to the fitting of temperature sensitivity. It can be seen clearly that choose the $1: \sqrt{2}$ current ratios to calculate the thermal resistance and its uncertainty is most convenient.

The thermal resistance of the Cernox sensor we calculated above consists of two parts: one is the internal thermal resistances, which is related to the structure and materials of the thermometer itself, and the other is the interfacial boundary thermal resistance of which we are concerned. The finite element calculations (FEM) have been performed to calculate the internal thermal resistance by C. Gaiser and B. Fellmuth. Their results show that the internal thermal resistance of the Cernox sensor decreased from 100K/W to 1K/W when the temperature raised from 1K to 20K. The resistance
Table 1. Accuracy of current measurements with Fluke 1594A

| Current range (mA) | Current uncertainty |
|-------------------|---------------------|
| 0.001 to 0.005    | 0.00005             |
| 0.005 to 0.02     | 1%                  |
| 0.02 to 0.2       | 0.5%                |
| 0.2 to 2          | 0.2%                |
| 2 to 20           | 0.5%                |

3. Experiment setup

In order to obtain the cryogenic temperature, a new cryostat was designed [5], as shown in Fig. 1. This cryostat had a simple structure that consists of a two-stage GM cryocooler (Sumitomo (SHI) Cryogenics of America, Inc. RDK-415D), cryostat wall, radiation shield, thermal damper, sample holder, Cernox temperature sensors, temperature controller (Lakeshore, 340) and other measuring instruments (Fluke 1594A). All the measurement devices used for acquiring data were connected by IEEE-488 cables and controlled by a personal computer using a program written by LabView software. The radiation shield was made of copper and the thickness is 1 mm. It was connected to the first stage of the cryocooler by a flange. The sample holder and heat sink are made of oxygen-free high-conductivity copper (OFHC) in order to get a good heat conduction. The thermal damper with 1.2 mm of thickness and 45 mm of diameter is made of PTFE and located between the sample holder and the cold head to reduce the temperature fluctuations. A constantan resistance of 101.3 Ω is used as the heater which is installed on the end of the sample holder nearby the thermal damper. Two temperature sensors (Cernox-1050-SD SN 70210 and Cernox-1050-SD SN 67685) are attached to the bottom of oxygen-free copper block and their temperatures are automatically controlled by the temperature controller (Lakeshore 340). Two Rhodium-Iron RTDs are used to monitor the temperature of the cold head and the sample holder during the test because the Cernox RTDs we used is calibrated from 4K to 40K. The wires from the measurement instruments (Fluke 1594A) to temperature sensors are twisted to reduce the inductive effect.

![Figure 1. Schematic of the cryostat](image-url)
All measurements were performed with a DC bridge Fluke 1594A (uncertainty 0.8ppm) in combination with a 10000ohm standard resistor (uncertainty 1.5ppm) placed in a temperature-controlled bath. Thermal resistances of two different mounting methods (VGE-7031 Varnish, Apiezon N Grease) were measured at cryogenic temperatures (4.2K, 6K, 8K, 10K, 14K) on Cernox temperature sensor (Cernox-1050-SD SN 70210) by two-current method. The measurement uncertainty of the thermal resistance is also analyzed and calculated by the present theory. It is worth mentioning that the increase in the sensor excitation current will result in a decreasing temperature measurement standard deviation. A larger measurement current will also lead to more obvious temperature difference between the two measurement currents $I_1$ and $I_2$. Nevertheless, a larger excitation current will dissipate more power in the temperature sensor, raising its temperature above the mounted environment. It is a significant problem to choose an appropriate measurement current to balance the standard deviation and the self-heating effect. In our experiment, we choose $35\mu A$ as the measurement current for 4.2K and 6K, and $65\mu A$ for 8K, 10K and 14K.

4. Results and discussion

The temperature measurement results of the Cernox thermometer (X70210) measured by two-current method was shown in Figure 2. The temperature $T_1$ is first measured with $35\mu A$ current at 4.2K and 6K, then temperature $T_2$ is measured with $35\sqrt{2}\mu A$ current and the measurement with the first current is usually repeated to check the stability. The measurement method is the same in other temperature points, only difference is the measurement currents changed to $65\mu A$ and $65\sqrt{2}\mu A$. It can be seen clearly that the temperature fluctuation is less than 2mK at all temperatures. The thermal resistances can be calculated by the measurement results of the Figure 2. The thermal resistance is shown as a function of temperature for the Cernox sensors in Table 2 and Figure 3. The thermal resistances of the Cernox thermometer (X70210) installed by VGE-7031 Varnish and Apiezon N-Grease were measured. The relative difference of the thermal resistance mounted by these two methods is less than 2%. In other words, the temperature uncertainty of the two mounting methods caused by self-heating is roughly equivalent. The thermal resistances of the Cernox thermometer (X70208) mounted by N-Grease were also measured. The thermal resistances of these two thermometers have the same order of magnitude.
Table 2. Thermal resistance of the Cernox temperature sensors mounted by two methods

| Temperature(K) | Rt-Var\(^a\)(K/W) | Rt-Gre\(^b\)(K/W) | Rt-Var\(^c\)(K/W) |
|---------------|-------------------|-------------------|-------------------|
| 4.2           | 4063.31           | 3963.25           | 3500.755          |
| 6             | 1662.43           | 1629.64           | 1302.534          |
| 8             | 942.52            | 886.25            | 667.295           |
| 10            | 678.00            | 600.49            | 460.249           |
| 14            | 441.58            | 378.25            | 233.1             |

\(^a\)Thermal resistance of the Cernox temperature sensor (X70210) mounted by varnish
\(^b\)Thermal resistance of the Cernox temperature sensor (X70210) mounted by N-Grease
\(^c\)Thermal resistance of the Cernox temperature sensor (X70208) mounted by varnish

Figure 2. Temperature measured by two-current method

The uncertainty of the thermal resistances (X70210) at different temperatures installed by Varnish was analyzed in Table 3. It is remarkable that the uncertainty of the thermal resistance decreasing with the
increasing of the temperature, while the relative uncertainty of the thermal resistance for different temperatures are almost equal. The uncertainty of the thermal resistance at all temperature points are less than 3%, it means that the calculation results of the thermal resistance is credible.

| Temperature(K) | $\delta u(X)$ (ppm) | $\delta u(s)$ | $\delta u(R_t)$ | $\delta R_t$ | $\delta \Delta u(R_t)$ (%) |
|---------------|---------------------|---------------|----------------|-------------|---------------------|
| 4.2           | 0.32                | -0.0441       | 90.86          | 4063.31     | 2.236               |
| 6             | 0.40                | 0.0450        | 37.20          | 1662.43     | 2.238               |
| 8             | 0.10                | 0.0521        | 21.08          | 942.52      | 2.236               |
| 10            | 0.10                | 0.0290        | 15.17          | 678.00      | 2.238               |
| 14            | 0.10                | 0.0089        | 10.06          | 441.58      | 2.277               |

- $\delta u(X)$ Uncertainty of resistance ratio
- $\delta u(s)$ Uncertainty of the sensitivity of the Cernox thermometer
- $\delta u(R_t)$ Uncertainty of the sensitivity of the thermal resistance
- $\delta R_t$ Thermal resistance
- $\delta \Delta u(R_t)$ Relative uncertainty of the thermal resistance

5. Conclusions
Cernox thermometer self-heating is a significant factor in high accuracy cryogenic temperature measurement that cannot be eliminated. The temperature difference caused by self-heating is depended on the thermal resistance of the sensor and its environment. In this paper, the thermal resistance of the Cernox temperature sensor and its surroundings from 4.2K to 14K is calculated by two-current method. The results show that the thermal resistances of the two mounting methods (VGE-7031 Varnish and Apiezon N Grease) are roughly equivalent. It can be observed that with the increasing of the temperature, the effective thermal resistances are gradually decreased. The uncertainty of the thermal resistance is also analyzed in this paper. The uncertainty of the thermal resistance decreased with the increasing of the temperature, while the relative uncertainty for different temperature is equal and less than 3%.

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References
[1] R.L.Rusby, The rhodium-iron resistance thermometer: ten years on: Temperature: Its Measurement and Control in Science and Industry, Vol. 5, part 2, J.F. Schooley, ed. American Inst. Phys. New York (1982) 829-833.
[2] R. Dozer and W. Schoepe, Thermal impedance between a thick-film resistor and liquid helium below 1K. Cryogenics 33 (1993) 936-937.
[3] S. Scott Courts, Thermal resistance of cryogenic temperature sensor from 1-300K, CEC/ICMC 1999.
[4] B Dong, L. Q. Liu, L.Y. Xiong, G. Zhou, X. Zhang, L. Zhang, Temperature control and stabilization in cryostat equipped with cryocooler. Proceedings of ICEC24-ICMC2012: 31-34.
[5] Jack W. Ekin, Experimental Techniques for Low-Temperature Measurements, New York: Oxford University Press, 2006, pp. 87-149.
[6] Batagelj V, Bojkovski J, Methods of reducing the uncertainty of the self-heating correction of a standard platinum resistance thermometer in temperature measurement of the highest accuracy Meas. Sci. Technol. 14 2151-2158