Sensitivity of a Ground-based Infrared Interferometer for Aperture Synthesis Imaging

TADASHI NAKAJIMA
National Astronomical Observatory, 2-21-1 Osawa, Mitaka 181-8588, Japan; tadashi.nakajima@nao.ac.jp

Received 2001 April 4; accepted 2001 July 10

ABSTRACT. Sensitivity limits of ground-based infrared interferometers using aperture synthesis are presented. The motivation for this analysis is to compare interferometers composed of multiple large telescopes and a single giant telescope with adaptive optics. In deriving these limits, perfect wave front correction by adaptive optics and perfect cophasing by fringe tracking are assumed. We consider the case in which $n$ beams are pairwise combined at $n(n - 1)/2$ detectors ($C_2$ interferometer) and the case in which all the $n$ beams are combined at a single detector ($C_n$ interferometer). Our analysis covers broadband observations by considering spectral dispersion of interference fringes. In the read-noise limit, the $C_n$ interferometer with one-dimensional baseline configuration is superior to the $C_2$ interferometer, while in the background limit, the advantage of the one-dimensional $C_n$ interferometer is small. As a case study, we compare the point-source sensitivities of interferometers composed of nine 10 m diameter telescopes and a 30 m diameter single telescope with adaptive optics between 1 and 10 $\mu$m for 10 $\sigma$ detection in 1 hr. At $J$ and $H$, the sensitivities of the interferometers are limited to 25–24 mag by read noise and OH airglow background, while that of the single telescope is limited to 28–26 mag by OH airglow background. Longward of 2 $\mu$m, the sensitivities of the interferometers and the single telescope are all limited by instrumental thermal background. At $K$, the sensitivities of the interferometers are around 23 mag, while that of the single telescope is 27 mag. At $N$, the sensitivities of the interferometers are around 10.7 mag, while that of the single telescope is 14.4 mag.

1. INTRODUCTION

Fundamental limitations of optical and infrared aperture synthesis imaging had been an academic subject in which astronomers were not keenly interested. The analyses of sensitivities are mathematically rather involved, and they appeared in journals in optics (Prasad & Kulkarni 1989, hereafter PK89; Nakajima & Matsuhara 2001, hereafter NM01) instead of those in astronomy. The analyses were academic partly because they dealt with ideal conditions in which atmospheric disturbances did not exist. Therefore, their results were applicable only to space interferometers, which were instruments in the distant future. They also were academic partly because optical and infrared interferometers were not considered as routinely used astronomical instruments unlike radio interferometers in radio astronomy.

However, as pointed out by Roddier & Ridgway (1999, hereafter RR99), the situation has changed as a result of the completion of large ground-based telescopes with 8–10 m diameters and of the planning of next-generation giant telescopes with 30–100 m diameters. According to the plans of the giant telescopes, all of them will be equipped with adaptive optics that deliver diffraction-limited images. The giant telescopes are always useful as optical buckets. However, for the purpose of high-resolution imaging, it is not clear whether synthesized images from an interferometric array of large telescopes or diffraction-limited images of giant telescopes are superior. In terms of resolution, the interferometric array is obviously advantageous, but in terms of sensitivity or dynamic range, the quantitative comparison is more complex. Optical/infrared interferometric arrays with small telescopes currently in operation or being built are dedicated to stellar astronomy. With the advent of an interferometric array composed of large telescopes, one essential question is whether interferometry finally has an impact on extragalactic astronomy. This question is directly related to the limiting magnitude.

The development of the fringe-tracking technique has made it possible to cophase a number of telescopes, and a ground-based interferometric array can be regarded as an ideal interferometer without atmospheric disturbances. Therefore, the sensitivity analyses made for space interferometers in the past can now be applied to ground-based interferometers as well. Under these circumstances, it is worthwhile to highlight the results of previous analyses of optical and infrared synthesis imaging in an astronomical journal and apply them to ground-based interferometry composed of large telescopes.

RR99 pioneered the analysis of ground-based interferometry from the point of view of comparison with future giant telescopes. However, their analysis is not of aperture synthesis, and we will show later that their results are not necessarily in agreement with ours. Although this paper mostly utilizes previously obtained formulae for signal-to-noise ratios (S/Ns) in

1289
beams and they are pairwise combined at.

We consider idealized interferometers, but the beam-combination scenario for actual interferometers may be more complicated (Mozurkewich 2000). The schemes are called pairwise and all-on-one combinations in Mozurkewich (2000). In this paper, we present the derivation of expressions for S/Ns of the images obtained by these interferometers in § 2 and § 3. We discuss limiting cases for spectrally broad light in § 4. In § 5, we present the sensitivities of interferometers composed of nine 10 m telescopes in comparison with a single 30 m telescope with adaptive optics. We discuss the sensitivity analysis further in § 6.

2. "C_3" INTERFEROMETER

Let there be n identical principal apertures from which we derive n main beams. Each beam is divided into n - 1 subbeams by the use of beam splitters. The resulting n(n - 1) subbeams are combined pairwise at n_c = n(n - 1)/2 detectors, where n_c is also the number of baselines. The fringe pattern is formed at the focal plane of each detector, which is either a one-dimensional array of identical pixels or a two-dimensional array of identical pixels that range from 1 to P (Fig. 1). For spectrally broad light, the fringe can be dispersed in the cross-fringe direction with the use of a two-dimensional detector. For a focal plane interferometer of this type, the influence of source shot noise on fringe phasor estimation has been fully formulated (Walkup & Goodman 1973; Goodman 1985). Let the interferometer be illuminated by a source and spatially smooth background. The intensity pattern on the rth detector (rth baseline) is given by

\[ I_r(x) = \langle I_r(x) \rangle = \frac{2}{1 + \gamma} \left[ \cos (\mathbf{kx} \cdot \mathbf{B}_r/\lambda_0 + \phi_r) \right] + \langle I_r(x) \rangle, \]

where \( \gamma \) is the signal-to-noise ratio and \( \phi_r \) is the phase of the fringe pattern on the rth detector.

Fig. 1.—Schematic diagram of a focal plane interferometer. There is one detector for each baseline.

PK89 and NM01, we also present new results to cover the full parameter space.

Baseline redundancy is a factor that affects the S/N of a synthesis image for a given number of apertures. Since the number of large telescopes combined to form an interferometric array is likely to be limited, it is important to gain u-v coverage by using nonredundant baselines. Throughout this paper, we consider only interferometers with nonredundant baselines.

There are two types of noise that appear in aperture synthesis, one of which is removable while the other is intrinsic and not removable. Sampling noise is potentially introduced by the incompleteness of u-v coverage. However, deconvolution techniques such as CLEAN and the maximum entropy method (Perley, Schwab, & Bridle 1989), when applied to radio interferometric data, appear to compensate for sampling noise. We thus regard sampling noise as removable noise. On the other hand, noise resulting from the photodetection process is not removable. The S/N of a dirty image to which deconvolution has not been applied is limited by photodetection noise and detector read noise. Here we regard the S/N of a dirty image as the S/N of the synthesized image.

There are two different methods by which a synthesized image can be constructed from the visibility data: inversion without total counts and true inversion. In the former, the zero-frequency phasor is neglected, implying that the total number of photons in the synthesized image is zero. Despite this seemingly unattractive feature, this is the standard method in radio astronomy. In the latter method, the van Cittert–Zernike theorem is strictly applied and the zero-frequency phasor as well as the n(n - 1)/2 complex phasors is used. The image produced by this technique has the desirable property of nonnegativity. In the presence of additive background that overwhelms the signal, the magnitude of the zero-frequency phasor is much greater than those of other phasors.

The zero-frequency phasor determines the DC offset level due to thermal background in the synthesized image. Not only is this background of little astronomical interest, it is also a major source of Poisson fluctuations. In this paper, we present the results obtained for inversion both with and without total counts. However, in practice faint-source detection is limited by OH airglow background at J and H and thermal background in the thermal infrared, so we obtain those detection limits for inversion without total count.

The paper is organized as follows. First we introduce two types of interferometers, the "C_3" interferometer and the "C_n" interferometer with expressions for S/Ns of the images obtained by these interferometers in § 2 and § 3. We discuss limiting cases for spectrally broad light in § 4. In § 5, we present the sensitivities of interferometers composed of nine 10 m telescopes in comparison with a single 30 m telescope with adaptive optics. We discuss the sensitivity analysis further in § 6.
where \( \langle I_0 \rangle \) is the average intensity in each subbeam at the detector, \( B \) is the baseline vector, \( \kappa = 2\pi/\lambda \) is the wavenumber, \( d \) is the distance between the aperture plane and the detector, \( x \) is the spatial vector in the detector plane, and \( \gamma \exp (i\phi) \) is the complex visibility function at the baseline vector \( B \). In deriving equation (1), we have assumed that the incident light is spectrally narrow so that the fringe visibility depends on only the spatial correlation of the field.

Because of the presence of background illumination,

\[
I_0 = I_s^0 + I_b^0,
\]

where \( I_s^0 \) and \( I_b^0 \) are source and background intensities, respectively. Let \( \gamma' \) be the fringe visibility of the source in the absence of the background; then

\[
\gamma = \frac{I_s^0}{I_s^0 + I_b^0} \gamma'.
\]

The complex visibility function \( \gamma' \exp (i\phi) \) is determined by the \( u-v \) coordinates of the \( r \)th baseline \((u, v)\) and the source brightness distribution on the sky \( S(x, y) \) by a Fourier transform relation:

\[
\gamma' \exp (i\phi) = \frac{1}{\int S(x, y) \exp \left[ 2\pi i (ux + vy) \right] dx \, dy} \int S(x, y) dx \, dy.
\]

In an effort to reduce the clutter in the equations, we henceforth drop the vector notation, but bear in mind that spatial frequencies, pixel locations, etc., are really vectors. The photoelectron detection theory (Walkup & Goodman 1973) takes into account the discrete nature of both photons and detector pixels. The average photoelectron count \( \langle k(p) \rangle \) at the pixel location specified by the integer index \( p \) of the detector is proportional to \( \langle f(x) \rangle \):

\[
\langle k(p) \rangle = 2\langle K_0 \rangle [1 + \gamma \cos (\omega_r + \phi)].
\]

Here \( \langle \ldots \rangle \) denotes averaging over the photoelectron-detection process. The product \( \omega_r \) is understood to be the scalar product of the pixel position vector \( p \) and the spatial frequency vector \( \omega_r \), expressed in inverse pixel units.

Let \( \langle C \rangle \) be the average number of photoelectrons detected by the entire array in one integration period, and let \( 2\langle N \rangle \) be the average number of photoelectrons per detector per integration time. Clearly, then, \( \langle C \rangle = 2\langle N \rangle n_s \), and thus \( \langle N \rangle = \langle C \rangle / [n(n-1)] \). According to equation (5), the average number of photoelectrons per detector is equal to \( 2\langle K_0 \rangle P \), and thus \( \langle K_0 \rangle P = \langle N \rangle \).

### 2.1. Inversion without Total Count

An analysis based on the photodetection theory gives the mean image \( I_1 \) as follows (NM01):

\[
I_1(q) = \langle N \rangle \sum_r \gamma_r \cos (\omega_r + \phi_r),
\]

\[
= \langle N \rangle \sum_r \frac{I_0^r}{I_0^r + I_b^r} \gamma'_r \cos (\omega_r + \phi_r),
\]

(6)

where \( q \) refers to the pixels in the synthesized image; in particular, \( q \) ranges from \(-Q/2\) to \(+Q/2\).

The variance \( V[i_1(q)] \) in the synthesized image \( i_1(q) \) is given by

\[
V[i_1(q)] = n_b \left( \frac{\langle N \rangle}{2} + \frac{P\sigma^2}{2} \right) = \frac{\langle C \rangle}{2} + n_b \frac{P\sigma^2}{2}.
\]

(7)

For a point source (\( \gamma'_r = 1 \)) at the phase center \( (\phi_r = 0) \), the S/N of the synthesized image is

\[
\frac{I_1(0)}{\sqrt{V[i_1(0)]}} = \frac{(\langle C \rangle/2)[I_0^r/(I_0^r + I_b^r)]}{\sqrt{(\langle C \rangle/2 + n_b P\sigma^2/2)}}.
\]

(8)

### 2.2. Inversion with Total Count

An analysis that takes into account shot noise and detector read noise gives the mean image \( I_2 \) as follows:

\[
I_2(q) = \langle N \rangle \sum_r \gamma_r \cos (\omega_r + \phi_r) + 1
\]

\[
= \langle N \rangle \sum_r \left[ \frac{I_0^r}{I_0^r + I_b^r} \gamma'_r \cos (\omega_r + \phi_r) + 1 \right].
\]

(9)

The variance \( V[i_2(q)] \) in the synthesized image \( i_2(q) \) is given by

\[
V[i_2(q)] = n_b \left( \frac{3\langle N \rangle}{2} + \frac{3P\sigma^2}{4} \right) + \langle N \rangle \sum_r \gamma_r \cos (\omega_r + \phi_r)
\]

\[
= \frac{3\langle C \rangle}{4} + n_b \frac{3P\sigma^2}{4} + \langle N \rangle \sum_r \gamma_r \cos (\omega_r + \phi_r).
\]

(10)

This variance is newly derived for this work. For a point source at the phase center, the S/N of the image is

\[
\frac{I_2(0)}{\sqrt{V[i_2(0)]}} = \frac{\langle N \rangle \sqrt{n_b(2I_0^r + I_b^r)(I_0^r + I_b^r)}}{\sqrt{(3\langle N \rangle/2 + 3P\sigma^2/4) + \langle N \rangle I_0^r(I_0^r + I_b^r)}}.
\]

(11)
3. $^{\nu}C_n$ INTERFEROMETER

In the $^{\nu}C_n$ interferometer, all $n$ beams interfere on a single detector and $n_b$ fringes are superposed. Both the baseline configuration and the detector can be either one-dimensional or two-dimensional. As an example, a two-dimensional $^{2}C_3$ interferometer is schematically presented in Figure 2. A special case is the combination of a one-dimensional baseline configuration and a two-dimensional detector for which the superposed fringes are dispersed in the cross-fringe direction so that spectrally broad light can be used without bandwidth smearing (Fig. 3). We call an interferometer of this type an $^{\nu}C_n$ interferometer.

Let the interferometer be composed of $n$ identical apertures, and let it be illuminated by a source and spatially smooth background. The classical intensity distribution of the interference pattern by the $n$ apertures has the average value

$$
\langle I(x) \rangle = \langle I_0 \rangle \left[ n + 2 \sum_{g<h} \gamma_{gh} \cos (\kappa x \cdot B_{g,h}/d + \phi_{g,h}) \right],
$$

where the various symbols have meanings similar to those in equation (1); $gh$ denotes the baseline $gh$ corresponding to the apertures $g$ and $h$. Let $\langle k(p) \rangle$ denote the photoelectron count distribution due to $\langle I(x) \rangle$. As in § 2, we discontinue the vector notation, assume that the total number of pixels is $P$, and note that $\langle k(p) \rangle$ is proportional to $\langle I(x) \rangle$:

$$
\langle k(p) \rangle = \langle Q_o \rangle \left[ n + \sum_{g<h} \gamma_{gh} \cos (p\omega_{g,h} + \phi_{g,h}) \right].
$$

Here $\langle Q_o \rangle$ has approximately the same meaning as $\langle K_o \rangle$ in § 2. However, since there is no beam splitting, $\langle Q_o \rangle = (n-1)\langle K_o \rangle$. We also define $\langle M \rangle = P\langle Q_o \rangle = \langle C \rangle/n$.

3.1. Inversion without Total Count

We find the mean synthesized image $I_s$ to be

$$
I_s(q) = \langle M \rangle \sum_{i<j} \gamma_{ij} \cos (q\omega_{ij} + \phi_{ij}).
$$

The variance $V[i_s(q)]$ in the synthesized image $i_s(q)$ is given by

$$
V[i_s(q)] = \frac{n\langle M \rangle + Pa^2}{2n_b} + \langle M \rangle(n-2) \sum_{i<j} \frac{I_0^r}{I_0^r + I_0^i} \gamma_{ij} \cos (q\omega_{ij} + \phi_{ij}).
$$

This variance is derived in NM01. For a point source at the
phase center, the S/N of the image is

\[
\frac{I_s(0)}{\sqrt{V[i_s(0)]}} = \frac{\langle M \rangle [I_s'(I_0 + I_0')] \sqrt{n_p}}{\sqrt{(n(M) + P\sigma^2)}/2 + \langle M \rangle (n-2)[I_s'(I_0 + I_0')]}
\]

\[
= \frac{\langle C \rangle [I_s'(I_0 + I_0')] \sqrt{(n-1)/n}}{\sqrt{C + P\sigma^2 + \langle C \rangle [I_s'(I_0 + I_0')] [2(n-2)/n]}
\]

(16)

3.2. Inversion with Total Count
We find the mean synthesized image \(I_s(q)\) to be

\[
I_s(q) = \langle M \rangle \left[ \frac{n}{2} + \sum_{\gamma=0}^n \gamma \cos (q\omega_0 + \phi_0) \right]
\]

= \frac{\langle C \rangle}{n} + \sum_{\gamma=0}^n \gamma \cos (q\omega_0 + \phi_0).
\]

(17)

(18)

The variance \(V[i_s(q)]\) in the synthesized image \(i_s(q)\) is given by

\[
V[i_s(q)] = \frac{\langle M \rangle}{2} \left[ n \left( n_p + \frac{1}{2} \right) + (n-1) \right]
\]

\[
\times \sum_{\gamma=0}^n 2(j-1)\gamma \cos (q\omega_0 + \phi_0) + P\sigma^2 \left[ \frac{n_p}{2} + \frac{1}{4} \right].
\]

(19)

This variance is newly derived for this work. For a point source at the phase center, the S/N of the image is

\[
\frac{I_s(0)}{\sqrt{V[i_s(0)]}} = \frac{\langle C \rangle n}{\sqrt{(C)(3n^2 - 5n + 3) + P\sigma^2(2n^2 - n + 1)}}.
\]

(20)

4. LIMITING CASES FOR SPECTRALLY BROAD LIGHT
In this section, we consider the source shot-noise limit, background shot-noise limit, and detector read-noise limit for a point source and an extended source. For each case, the results of inversion with and without total count are shown.

Formulae for means, variances, and S/Ns were obtained for spectrally narrow light in the previous section. For a two-dimensional \(^{\prime}C_s^\prime\) interferometer, the maximum spectral bandwidth is limited to about \(\Delta \nu B\), where \(B\) is the maximum baseline length. If we use a bandwidth broader than this, fringes will be smeared out. In order to disperse fringes for a general two-dimensional baseline configuration, pupil remapping that transforms the exit pupil to a \(^{\prime}C_s^\prime\) interferometer is necessary. The effect of pupil remapping on the signal-to-noise ratio analysis is beyond the scope of this paper. However, we can imagine that the S/N of the one-dimensional \(^{\prime}C_s^\prime\) interferometer is a good measure of the S/N of an interferometer with pupil remapping. For \(^{\prime}C_s^\prime\) interferometers, fringes can be dispersed in the cross-fringe direction. The formulae for S/Ns obtained for spectrally narrow light hold also for dispersed fringes as can be easily verified as follows.

We here consider the case with the \(^{\prime}C_s^\prime\) interferometer and inversion without total count. Let \(I_s\) be the number of pixels in the fringe direction and \(I_p\) be that in the cross-fringe direction. Therefore, \(P = I_s P_s\). We assume that the photon spectrum within the bandwidth is not steep and the number of photons in one detector row is given by \(\langle C \rangle/I_s\). Then the S/N of an image synthesized from a set of fringes in detector rows corresponding to \(1/P_s\) of the spectral bandwidth is given by

\[
S/N = \frac{\langle C \rangle/I_s \sum \gamma \cos (\omega q + \phi)}{\sqrt{n_s (\langle C \rangle/I_s + P_s \sigma^2/2)}}.
\]

(21)

If we further assume that the total bandwidth is narrow enough that the source structure does not change within it, \(P_s\) images can be co-added to form a final image. The S/N of the final image is given by

\[
S/N = \sqrt{P_s} (s/n)
\]

\[
= \frac{\langle N \rangle \sum \gamma \cos (\omega q + \phi)}{\sqrt{n_s (\langle N \rangle + P_s \sigma^2/2)}}.
\]

(22)

This happens because the photon count \(\langle N \rangle\) and \(\langle C \rangle\) and the pixel number \(P_s\) are linear in the expressions of the mean and variance. The situation is the same for the other S/N expressions.

4.1. Numbers of Pixels
Let the central frequency be \(\nu\) and the bandwidth be \(\Delta \nu\). Then there are \(2\nu/\Delta \nu\) fringe cycles in each interferogram. For the Nyquist sampling theorem, \(4\nu/\Delta \nu\) pixels are needed in the fringe direction.

For the \(^{\prime}C_s^\prime\) and \(^{\prime}C_s^\prime\) interferometers, we consider the number of pixels in the cross-fringe direction (dispersion direction) in each dispersed fringe. The fractional bandwidth \(\Delta \nu/\nu\) for which bandwidth smearing is small is given by \(\Delta \nu/\nu < D/B\). If we require that \(\Delta \nu/\nu = D/(2B)\), bandwidth smearing will be negligible. For a spectral bandwidth of \(\Delta \nu\), \(\Delta \nu/\nu = 2(\Delta \nu)/(D/B)\)
pixels are needed as the number of pixels in the cross-fringe direction. Thus, the number of pixels of the two-dimensional detector is \( P = 4\nu/\Delta\nu \times 2(\Delta\nu)/(D/B) = 8(B/D)\).

For the "C"\(_2\) interferometer, the longest baseline can be aligned to one of the two sides of a two-dimensional detector and the minimum number of pixels is \( P = (4\nu/\Delta\nu)^2 = 16(B/D)^2\) and the maximum allowed fractional bandwidth is \( D/B\).

### 4.2. S/N Tables

S/N expressions of "C"\(_2\), "C"\(_n\), and "C"\(_i\) interferometers are given for a point source and an extended source, for source shot-noise limit, background shot-noise limit, and detector read-noise limit, and for inversion with and without total count in Tables 1–5. In some cases, the S/N expressions are not well defined and are omitted from the tables.

We define a quantity \( F_{\nu} \) that is proportional to the photon spectrum of the source, collecting area, and integration time. We express the total photon count for the "C"\(_2\) and "C"\(_i\) interferometers as \( \langle C \rangle = F_{\nu}\Delta\nu \). Then the total photon count for the two-dimensional "C"\(_n\) interferometer is \( \langle C \rangle = F_{\nu}(D/B) \). The S/Ns are expressed in terms of \( F_{\nu}\Delta\nu \) and \( D/B\).

### 4.3. Point Source/Source Shot-Noise Limit

The case with a point source in the shot-noise limit is given in Table 1.

By taking ratios of the S/N expressions, we find

\[
\frac{S/N_1}{S/N_3} = \sqrt{\frac{3n-4}{2n-2}} \sqrt{\frac{n}{D/B}},
\]

\[
\frac{S/N_2}{S/N_4} = \sqrt{\frac{4(3n^2-5n+3)}{5n^2}} \sqrt{\frac{n}{D/B}},
\]

\[
\frac{S/N_3}{S/N_5} = \sqrt{\frac{4(3n^2-5n+3)}{5n^2}} \geq 1,
\]

\[
\frac{S/N_1}{S/N_1} = \sqrt{\frac{8}{5}}.
\]

In a typical situation, \( \Delta\nu/\nu > D/B \) and the "C"\(_2\) and "C"\(_i\) interferometers are superior to the two-dimensional "C"\(_n\) interferometer. As expression (26) shows, the "C"\(_2\) interferometer is superior to the "C"\(_i\) interferometer by a factor of \( \sqrt{12/5} \) in S/N for a large \( n \) for inversion with total count. In the source shot-noise limit, the total count information is genuinely of the source. The highest S/N is obtained for the "C"\(_2\) interferometer for inversion with total count (eq. [27]).

### 4.4. Background Limit

In the background limit, the only valid method of analysis is inversion with total count since the total count is dominated by background photon counts. The S/N expressions are given in Table 2.

By taking the ratios, we find

\[
\frac{S/N_1}{S/N_3} = \sqrt{\frac{n}{2(n-1)}} \sqrt{\frac{\Delta\nu/\nu}{D/B}}
\]

and

\[
\frac{S/N_2}{S/N_4} = \sqrt{\frac{n}{2(n-1)}} \leq 1.
\]

As in the case with the source shot-noise limit, the two-dimensional "C"\(_n\) interferometer has the disadvantage of a narrow bandwidth. For a large \( n \), the "C"\(_i\) interferometer is slightly superior to the "C"\(_2\) interferometer by a factor of \( \sqrt{2} \) in S/N.

### 4.5. Read-Noise Limit

In the read-noise limit, the numbers of pixels for the "C"\(_2\) and "C"\(_n\) interferometers are \( P = 8(B/D) \) and that for the two-dimensional "C"\(_n\) interferometer is \( P = 16(B/D)^2 \). The S/N expressions are given in Table 3.

If we take the ratios of the S/Ns, we find

\[
\frac{S/N_1}{S/N_3} = \frac{\sqrt{2}}{n-1} \sqrt{\frac{\Delta\nu}{\nu}} \left( \frac{B}{D} \right)^{1/2},
\]

\[
\frac{S/N_2}{S/N_4} = \frac{1}{n-1} \leq 1,
\]

\[
\frac{S/N_3}{S/N_5} = \frac{4}{n} \sqrt{\frac{n^2-n+1}{3(n^2-n)}} \sqrt{\frac{\Delta\nu}{\nu}} \left( \frac{B}{D} \right)^{3/2},
\]

\[
\frac{S/N_4}{S/N_6} = \frac{2}{n} \sqrt{\frac{2(n^2-n+1)}{3(n^2-n)}} \leq 1.
\]

The "C"\(_n\) interferometer is always superior to the "C"\(_2\) interferometer because of the smaller number of detector pixels.
The two-dimensional \( ^{C_n} \) interferometer has a limited use for a small \( B/D \).

### 4.6. Extended Source/Source Shot-Noise Limit

For a fully extended source, there is no signal other than the total count. The only valid method of analysis for the fully extended source is inversion with total count. The S/N expressions are given in Table 4.

By taking the ratios, we find

\[
\frac{S/N_2}{S/N_4} = \sqrt{\frac{n^2-n+1}{3}} \sqrt{\frac{\Delta v}{\nu D}} \quad (34)
\]

\[
\frac{S/N_2}{S/N_4'} = \sqrt{\frac{n^2-n+1}{3}} \geq 1. \quad (35)
\]

The \( ^{C_n} \) interferometer is superior to the \( ^{C_n} \) and \( ^{C_n'} \) interferometers.

### 4.7. Extended Source/Background Limit

For an extended source, there is no signal other than the source total count. However, the total count is dominated by background photons. Therefore, neither inversion without total count nor inversion with total count is the valid method of analysis.

### 4.8. Extended Source/Read-Noise Limit

As in the case with extended source/shot-noise limit, the only valid method of analysis is inversion with total count. The S/N expressions are given in Table 5.

By taking ratios, we find

\[
\frac{S/N_2}{S/N_4} = \sqrt{\frac{4(n^2-n+1)}{3(n^2-n)}} \left( \frac{\Delta v}{\nu D} \right)^{3/2}, \quad (36)
\]

and

\[
\frac{S/N_2}{S/N_4'} = \sqrt{\frac{2(n^2-n+1)}{3(n^2-n)}} \leq 1. \quad (37)
\]

The two-dimensional \( ^{C_n} \) interferometer is useful only for a small \( B/D \). For large \( n \), \( (S/N_2)/(S/N_4) \) attains an asymptotic value of \( \sqrt{23} \).

### 5. Sensitivities of Interferometers Composed of Nine 10 Meter Telescopes

In this section, we calculate the sensitivities of \( ^{C_n} \) and \( ^{C_n'} \) interferometers composed of nine 10 m telescopes and compare them with the sensitivity of a 30 m single telescope with adaptive optics; 10 \( \sigma \) detection limits for a point source are obtained for 1 hr integration and plotted in Figure 4.

#### 5.1. \( J \) and \( H \)

The throughput of the interferometers is assumed to be 10\%, while that of the single telescope with adaptive optics is assumed to be 60\%. The read-noise level is assumed to be 10 electrons pixel\(^{-1}\), and the maximum baseline length, which determines the number of detector pixels, is assumed to be 200 m. The coherent integration time is assumed to be 1 s,
while a larger value will decrease the effect of detector read noise.

It turned out that both read noise and sky background due to OH airglow affect the performance of the interferometers. We evaluate the S/Ns for inversion without total count because sky background is much greater than the signal near the detection limit. The sensitivity of the telescope is limited by sky background. At $J$, limiting magnitudes are 24.7, 25.7, and 28.0, respectively, for the $C_i$ interferometer, the $C_i$ interferometer, and the single telescope. At $H$, they are 23.6, 24.0, and 26.2, respectively.

### 5.2. Thermal Infrared Wavelengths

For the interferometers, we evaluate the S/Ns for inversion without total count because the instrument background is much stronger than the source signal near the detection limits. The temperature of the interferometer optics is assumed to be 273 K, and the optical throughput is assumed to be 10%. Therefore, the background emission is estimated from a 273 K blackbody with 90% emissivity. For the single telescope with adaptive optics, the throughput of the optics is assumed to be 60%. The temperature of the optics is assumed to be 273 K, and its emissivity is assumed to be 40%.

Longward of 2 $\mu$m, the sensitivities do not depend on beam combination geometry very much. This implies that detector read noise is negligible. At 2.2 $\mu$m, the sensitivities of the interferometers are around 330 nJy, or 22.9 mag, while that of the single telescope is 10 nJy, or 26.9 mag. Longward of 5 $\mu$m, the single telescope is 5.8 mag more sensitive than the interferometers. At 10.5 $\mu$m, the sensitivities of the interferometers are about 2 mJy, or 10.7 mag, while that of the single telescope is 57 $\mu$Jy, or 14.4 mag. At wavelengths shorter than 2 $\mu$m, the baseline length affects the sensitivities of the interferometers because of the change in the number of pixels, while longward of 2 $\mu$m, it does not matter as long as the source is unresolved. Here we do not consider the case for fully extended objects. As we have seen from the previous section, the interferometers are not sensitive to fully extended objects under background-limited conditions.

### 5.3. Loss-free Interferometers

In the above evaluation of sensitivities, it is assumed that the throughput of the interferometers is 10% while that of the single telescope is 60%. It is natural to question whether the difference in sensitivities is due to that in throughput or something more fundamental.

As discussed by PK89, there is a basic difference between an interferometer and a single telescope in that the former takes the correlation of signal from multiple apertures. However, the loss of S/N by this process is only a factor of $\sqrt{2}$ in the case of an ideal $C_i$ interferometer with inversion without total count in the source shot-noise limit. A similar situation is expected in the case of background shot-noise limit, where the number of detector pixels does not affect the S/N. Then the only situation in which the difference is conspicuous is the read-noise limit, where the interferometer requires many more detector pixels to sample fringes. It is of theoretical interest to assume ideal interferometers and compare their performance with that of the single telescope.

Again we compare the interferometers with nine 10 m telescopes and a 30 m single telescope. Since each of the 10 m telescopes and the 30 m single telescope require adaptive optics, the throughputs are assumed to be 60%. The sensitivity limits are plotted in Figure 5, where the difference is about a factor of 5. Therefore, the fundamental difference between the interferometers and the single telescope is rather small, and the large difference seen in the previous analysis is due to that in throughput.

### 5.4. Baseline Dependence

The S/N expressions of the interferometers are not explicitly baseline dependent. Since the astronomical source is assumed to be unresolved, only the total number of detector pixels is dependent on the lengths of baselines. Detector read noise is more significant as the baselines become longer. In order to
TABLE 5

S/N (Extended Source/Read-Noise Limit)

| Case                        | S/N         | Expression                                                                 |
|-----------------------------|-------------|-----------------------------------------------------------------------------|
| C/ inversion with total count | S/N_C       | \[
\frac{1}{\sqrt{12(n^2 - n)(B/D)}} \left( \frac{F(D)}{J} \right) \] |
| C/ inversion with total count | S/N_N       | \[
\frac{1}{\sqrt{16(n^2 - n + 1)(B/D^2)}} \left( \frac{F(B/D)}{J} \right) \] |
| C/ inversion with total count | S/N_0       | \[
\frac{1}{\sqrt{8(n^2 - n)}(B/D)} \left( \frac{F(D)}{J} \right) \] |

use long baselines, it is essential to keep detector read noise low so that the S/N is not limited by read noise.

For a source of finite extent, long baselines start to resolve it, and the visibilities \( \gamma \) of those baselines decrease from unity, and the phases \( \phi \) become less coherent depending on the symmetry of the source. For the extended source, the S/Ns of the interferometers are calculated as follows. For a particular baseline configuration, the complex visibilities are obtained by the Fourier transform relation (4) and the number of detector pixels are calculated as above. By inserting those numbers and taking the summation over different baselines, one can obtain the S/Ns of the dirty images.

6. DISCUSSION

6.1. Interferometry for Extragalactic Astronomy

One of the motivations of this analysis is to evaluate the capability of interferometry for extragalactic astronomy. We inevitably have to consider extended objects for this purpose and take into account the guide star availability. Fringe tracking requires the presence of a natural guide star in the infrared. If the adaptive optics also needs the same natural guide star, the limiting magnitude is around \( K = 25 \) indicates S/N = \( 10^5 \) for a \( K = 15 \) quasar. In other words, jets can be searched for with a dynamic range of \( 10^5:1 \).

Now we consider resolving broad-line regions (BLRs). According to the photoionization model (Davidson & Netzer 1979), the size of a BLR scales as \( \sqrt{L} \), where \( L \) is the luminosity of the AGN. For an AGN or a nearby quasar toward which the geometry is Euclidean, the observed flux \( f \) is given by

\[
\frac{f}{L} = \frac{d}{d_L^2},
\]

where \( d \) is the distance to the object. Then the angular size \( \theta \propto \sqrt{f} \). Therefore, the brightest quasar has the largest BLR in angular size; 3C 273 is the brightest quasar (\( K = 10 \)) and easiest to observe because Pa_c is redshifted to the K band. The necessary spatial resolution is of order 1 mas, so the necessary baseline length is 400 m. The FWHM of the line is 3400 km s\(^{-1}\), and the line/continuum ratio below 8000 km s\(^{-1}\) is 0.08 (Sellgren et al. 1983). In order to obtain the spatial structure of the BLR, we wish to spectrally resolve the broad

![Fig. 4.—Detection limits of the \(^3C_2\) and one-dimensional \(^3C_n\) interferometers and that of the single giant telescope as functions of wavelength. Symbols are sensitivities at J and H of \(^3C_2\) (asterisks), one-dimensional \(^3C_n\) (squares), and telescope (circles). Assumed parameters are described in the text.](image1)

![Fig. 5.—Loss-free interferometers. Throughputs of the interferometers and the single telescope are assumed to be the same. Meanings of the symbols are the same as in Fig. 4.](image2)
line or at least separate the blue and red components. If we require a spectral resolution of 3000 km s\(^{-1}\), the continuum flux per spectral resolution is equivalent to \(K = 14\). A spectral image will be composed of an unresolved source of \(K = 14\) and a resolved source with 8% of the flux of the unresolved source. Again, the complex visibilities are close to those of a point source, and the S/N argument made above for the jet applies to this case as well. We expect that the BLR of 3C 273 will be resolved with a dynamic range of more than \(10^{5}:1\). At least for AGNs and quasars, infrared interferometry with large telescopes appears effective.

### 6.2. Comments on RR99

In this subsection, we compare our results with that of RR99 on Fizeau-type interferometers or two-dimensional \(C_n\) interferometers. First of all, for spectrally broad light, we have seen that two-dimensional \(C_n\) interferometers or Fizeau-type interferometers have limitations in the ratio of the telescope aperture diameter and the baseline length to avoid bandwidth smearing. This is not discussed in RR99. Second, RR99 states that the \(C_n\) interferometer is superior to the \(C_2\) interferometer. However, we have seen from the analysis of limiting cases that the situation is not so simple. As PK89 noted, there is no major difference in performance between both interferometers in the source shot-noise limit. This is also true with the case for the background limit. In the read-noise limit, the \(C_n\) interferometer is superior to the \(C_2\) interferometer because of the difference in the number of detector pixels.

These differences originate from that in the definitions of the S/Ns. RR99 defines the S/N as that of the central peak in the interferogram directly formed by beam combination. On the other hand, we define the S/N as that of the dirty image formed by aperture synthesis. In the latter, the interferogram formed by beam combination is Fourier transformed to measure complex visibilities that fill the \(\nu-\nu\) plane and then they are Fourier transformed back to the image plane to form a dirty image. In RR99, the detector pixels of concern at the focal plane are only a few pixels at which the peak is located, while in the latter many pixels that sample the entire interferogram are considered.

### 6.3. Remaining Issues

We have obtained the sensitivity limits of the interferometers for perfect wave front correction by adaptive optics and perfect fringe tracking. In reality, both adaptive optics and fringe tracking require a guide star and there is a limitation in the precision of wave front and delay compensations. Imperfection in phase directly influences the fringe visibility \(\gamma\), which is proportional to the S/Ns. An analysis that takes into account this imperfection in phase is difficult because the introduction of atmospheric disturbances opens up a vast parameter space. Our analysis corresponds only to the best case designing.

In this paper, we considered only nonredundant baselines. However, if we consider a closely packed array of telescopes, we have to take into account the redundancy of the baselines. Although the behavior of S/Ns is expected to be the same as in the nonredundant case, deriving formulae of S/Ns for the redundant case is nontrivial or analytically impossible.

Finally, one essential and difficult question is whether the definition of the S/N as that of the dirty image is valid. In other words, we have left out the issue of deconvolution by assuming that it is perfect. We believe that this is practically of no consequence because the S/N of the dirty beam is usually much higher than that of the dirty image. Nevertheless, deconvolution is a nonlinear process and the error propagation through it is not fully understood.

### 7. CONCLUSIONS

We present the formulae of S/Ns of interferometers for the purpose of comparing ground-based interferometers composed of multiple large telescopes with a single giant telescope with adaptive optics. In deriving sensitivity limits, perfect wave front correction by adaptive optics and perfect cophasing by fringe tracking are assumed. We consider the case in which \(n\) beams are pairwise combined at \(n(n - 1)/2\) detectors (\(C_n\) interferometer) and the case in which all \(n\) beams are combined at a single detector (\(C_n\) interferometer).

1. In the read-noise limit, the \(C_n\) interferometer with one-dimensional baseline configuration is superior to the \(C_2\) interferometer, while in the background limit, the advantage of the \(C_n\) interferometer is small.

2. We compare the point-source sensitivities of interferometers composed of nine 10 m diameter telescopes and a 30 m diameter single telescope with adaptive optics between 1 and 10 \(\mu m\) for 10 \(\sigma\) detection in 1 hr. Between 1 and 2 \(\mu m\), the sensitivities of the interferometers are limited by detector read noise and sky background, while longward of 2 \(\mu m\), they are limited by the instrumental thermal background. At the \(K\) band, the sensitivities of the interferometers are around 23 mag while that of the single telescope is 27 mag.

I thank H. Matsuhara for allowing me to use his figures and the referee P. Lawson for useful comments.
REFERENCES

Davidson, K., & Netzer, H. 1979, Rev. Mod. Phys., 51, 715
Goodman, J. W. 1985, Statistical Optics (New York: Wiley)
Mozurkewich, D. 2000, in Principles of Long Baseline Interferometry: Course Notes from 1999 Michelson Summer School, ed. P. Lawson (JPL Publ. 00-009; Pasadena: JPL), 231
Nakajima, T., & Matsuhara, H. 2001, Appl. Opt., 40, 514
Perley, R. A., Schwab, F. R., & Bridle, A. H., eds. 1989, ASP Conf. Ser. 6, Synthesis Imaging in Radio Astronomy, NRAO Workshop 21 (San Francisco: ASP)

Prasad, S., & Kulkarni, S. R. 1989, J. Opt. Soc. Am., A6, 1702
Roddier, F., & Ridgway, S. T. 1999, PASP, 111, 990
Sellgren, K., Soifer, B. T., Neugebauer, G., & Matthews, K. 1983, PASP, 95, 289
Walkup, J. F., & Goodman, J. W. 1973, J. Opt. Soc. Am., 63, 399