Lectures on D-branes, tachyon condensation, and string field theory

Notes on lectures presented at the CECS School on Quantum Gravity, Valdivia, Chile, January 2002

Washington Taylor

Center for Theoretical Physics
MIT, Bldg. 6-308
Cambridge, MA 02139, U.S.A.
wati@mit.edu

Abstract

These lectures provide an introduction to the subject of tachyon condensation in the open bosonic string. The problem of tachyon condensation is first described in the context of the low-energy Yang-Mills description of a system of multiple D-branes, and then using the language of string field theory. An introduction is given to Witten’s cubic open bosonic string field theory. The Sen conjectures on tachyon condensation in open bosonic string field theory are introduced, and evidence confirming these conjectures is reviewed.

November 2002
1 Introduction

The last seven years have been a very exciting time for string theory. A new understanding of nonperturbative features of string theory, such as D-branes, has led to exciting new developments relating string theory to physically interesting systems such as black holes and supersymmetric gauge field theories, as well as to a new understanding of the relationship between Yang-Mills theories and quantum theories of gravity.

Despite remarkable progress in these directions, however, a consistent nonperturbative background-independent formulation of string theory is still lacking. This situation makes it impossible at this point, even in principle, to directly address cosmological questions using string theory. String field theory is a nonperturbative approach to string theory which holds some promise towards providing a background-independent definition of the theory. These lecture notes give an introduction to string field theory and review some recent work which incorporates D-branes into the framework of string field theory. This work shows that string field theory is a sufficiently robust framework that distinct string backgrounds can arise as disconnected solutions of the theory, at least for open strings. It remains to be seen whether this success can be replicated in the closed string sector.

In this section we review briefly the situation in string theory as a whole, and summarize the goals of this set of lectures. In Section 2 we review some basic aspects of D-branes. In Section 3, we describe a particular D-brane configuration which exhibits a tachyonic instability. This tachyon can be seen in the low-energy super Yang-Mills description of the D-brane geometry. This field theory tachyon provides a simple model which embodies much of the physics of the more complicated string field theory tachyon discussed in the later lectures. In Section 4 we give an introduction to Witten’s cubic bosonic open string field theory and summarize the conjectures made by Sen in 1999, which suggested that the tachyonic instability of the open bosonic string can be interpreted in terms of an unstable space-filling D-brane, and that this system can be analytically described through open string field theory. Section 5 gives a more detailed analytic description of Witten’s cubic string field theory. In Section 6 we summarize evidence from string field theory for Sen’s conjectures. Section 7 contains a brief review of some more recent developments. Section 8 contains concluding remarks and lists some open problems.

Much new work has been done in this area since these lectures were presented at Valdivia in January 2002. Except for a few references to more recent developments in footnotes and in the last two sections, these lecture notes primarily cover work done before January 2002. Previous articles reviewing related work include those of Ohmori [1], de Smet [2], and Aref’eva et al. [3]. An expanded set of lecture notes, based on lectures given by the author and Barton Zwiebach at TASI ’01, will appear in [4]; the larger set of notes will include further details on a number of topics.
1.1 The status of string theory: a brief review

To understand the significance of developments over the last seven years, it is useful to recall the situation of string theory as it was in early 1995. At that time it was clearly understood that there were 5 distinct ways in which a supersymmetric closed string could be quantized to give a microscopic definition of a theory of quantum gravity in ten dimensions. Each of these approaches to quantizing the string gives a set of rules for calculating scattering amplitudes between on-shell string states which describe gravitational quanta as well as an infinite family of massive particles in a ten-dimensional spacetime. These five string theories are known as the type IIA, IIB, I, heterotic \(SO(32)\), and heterotic \(E_8 \times E_8\) superstring theories. While these string theories give perturbative descriptions of quantum gravity, in 1995 there was little understanding of nonperturbative aspects of these theories.

In the years between 1995 and 2000, several new ideas dramatically transformed our understanding of string theory. We now briefly summarize these ideas and mention some aspects of these developments relevant to the main topic of these lectures.

**Dualities:** The five different perturbative formulations of superstring theory are all related to one another through duality symmetries \([5, 6]\), whereby the degrees of freedom in one theory can be described through a duality transformation in terms of the degrees of freedom of another theory. Some of these duality symmetries are nonperturbative, in the sense that the string coupling \(g\) in one theory is related to the inverse string coupling \(1/g\) in a dual theory. The web of dualities relating the different theories gives a picture in which, rather than describing five distinct possibilities for a fundamental theory, each of the perturbative superstring theories appears to be a particular perturbative limit of some other, as yet unknown, underlying theoretical structure.

**M-theory:** In addition to the five perturbative string theories, the web of dualities also seems to include a limit which describes a quantum theory of gravity in eleven dimensions. This new theory has been dubbed “M-theory”. Although no covariant definition for M-theory has been given, this theory can be related to type IIA and heterotic \(E_8 \times E_8\) string theories through compactification on a circle \(S^1\) and the space \(S^1/Z_2\) respectively \([7, 6, 8]\). For example, in relating to the type IIA theory, the compactification radius \(R_{11}\) of M-theory becomes the product \(gl_s\) of the string coupling and string length in the 10D IIA theory. Thus, M-theory in flat space, which arises in the limit \(R_{11} \to \infty\), can be thought of as the strong coupling limit of type IIA string theory. It is also suspected that M-theory may be describable as a quantum theory of membranes in 11 dimensions \([7]\), although a covariant formulation of such a theory is still lacking.

**Branes:** In addition to strings, all five superstring theories, as well as M-theory, contain extended objects of higher dimensionality known as “branes”. M-theory has M2-branes and M5-branes, which have two and five dimensions of spatial extent (whereas a string has one). The different superstring theories each have different complements of D-branes as well as the fundamental string and Neveu-Schwarz 5-brane; in particular, the IIA/IIB
superstring theories contain D-branes of all even/odd dimensions. The branes of one theory can be related to the branes of another through the duality transformations mentioned above. Through an appropriate sequence of dualities, any brane can be mapped to any other brane, including the string itself. This suggests that none of these objects are really any more fundamental than any others; this idea is known as “brane democracy”.

**M(atrix) theory and AdS/CFT:** One of the most remarkable results of the developments just mentioned is the realization that in certain space-time backgrounds, M-theory and string theory can be completely described through simple supersymmetric quantum mechanics and field theory models related to the low-energy description of systems of branes. The M(atrix) model of M-theory is a simple supersymmetric matrix quantum mechanics which is believed to capture all of the physics of M-theory in asymptotically flat spacetime (in light-cone coordinates). A closely related set of higher-dimensional supersymmetric Yang-Mills theories are related to string theory in backgrounds described by the product of anti-de Sitter space and a sphere through the AdS/CFT correspondence. It is believed that these models of M-theory and string theory give true nonperturbative descriptions of quantum gravity in space-time backgrounds which have the asymptotic geometry relevant to each model. For reviews of M(atrix) theory and AdS/CFT, see [9, 10].

The set of ideas just summarized have greatly increased our understanding of nonperturbative aspects of string theory. In particular, through M(atrix) theory and the AdS/CFT correspondences we now have nonperturbative definitions of M-theory and string theory in certain asymptotic space-time backgrounds which could, in principle, be used to calculate any local result in quantum gravity. While these new insights are very powerful, however, we are still lacking a truly background-independent formulation of string theory.

### 1.2 The goal of these lectures

The goal of these lectures is to describe progress towards a nonperturbative background-independent formulation of string theory. Such a formulation is needed to address fundamental questions such as: What is string theory/M-theory? How is the vacuum of string theory selected? (i.e., Why can the observable low-energy universe be accurately described by the standard model of particle physics in four space-time dimensions with an apparently small but nonzero positive cosmological constant?), and other questions of a cosmological nature. Obviously, aspiring to address these questions is an ambitious undertaking, but we believe that attaining a better understanding of string field theory is a useful step in this direction.

More concretely, in these lectures we will describe recent progress on open string field theory. It may be useful here to recall some basic aspects of open and closed strings and the relationship between them.

Closed strings, which are topologically equivalent to a circle $S^1$, give rise upon quantization to a massless set of spacetime fields associated with the graviton $g_{\mu\nu}$, the dilaton $\varphi$, and so on.
and the antisymmetric two-form $B_{\mu\nu}$, as well as an infinite family of massive fields. For the supersymmetric closed string, further massless fields associated with the graviton supermultiplet appear—these are the Ramond-Ramond $p$-form fields $A^{(p)}_{\mu_1\ldots\mu_p}$ and the gravitini $\psi_{\mu \alpha}$. Thus, the quantum theory of closed strings is naturally associated with a theory of gravity in space-time. On the other hand, open strings, which are topologically equivalent to an interval $[0, \pi]$, give rise under quantization to a massless gauge field $A_{\mu}$ in space-time. The supersymmetric open string also has a massless gaugino field $\psi_{\alpha}$. It is now understood that generally open strings should be thought of as ending on a Dirichlet $p$-brane (D$p$-brane), and that the massless open string fields describe the fluctuations of the D-brane and the gauge field living on the world-volume of the D-brane.

It may seem, therefore, that open and closed strings are quite distinct, and describe disjoint aspects of the physics in a fixed background space-time containing some family of D-branes. At tree level, the closed strings indeed describe gravitational physics in the bulk space-time, while the open strings describe the D-brane dynamics. At the quantum level, however, the physics of open and closed strings are deeply connected. Indeed, historically open strings were discovered first through the form of their scattering amplitudes [11]. Looking at one-loop processes for open strings led to the first discovery of closed strings, which appeared as poles in nonplanar one-loop open string diagrams [12, 13]. The fact that open string diagrams naturally contain closed string intermediate states indicates that in some sense all closed string interactions are implicitly defined through the complete set of open string diagrams. This connection underlies many of the important recent developments in string theory. In particular, the M(atrix) theory and AdS/CFT correspondences between gauge theories and quantum gravity are essentially limits in which closed string physics in a fixed space-time background is captured by a simple limiting Yang-Mills description of an open string theory on a family of branes (D0-branes for M(atrix) theory, D3-branes for the CFT describing AdS$_5 \times S^5$, etc.)

The fact that, in certain fixed space-time backgrounds, quantum gravity theories can be encoded in terms of open string degrees of freedom through the M(atrix) and AdS/CFT correspondences leads to the question of how a global change of the space-time background would appear in the quantum field theory describing the appropriate limit of the open string model in question. If such a change of background could be described in the context of M(atrix) theory or AdS/CFT, it would indicate that these models could be generalized to a background-independent framework. Unfortunately, however, such a change in the background involves adding nonrenormalizable interactions to the field theories in question. At this point in time we do not have the technology to understand generically how a sensible quantum field theory can be described when an infinite number of nonrenormalizable interaction terms are added to the Lagrangian. One example of a special case where this can be done is the addition of a constant background $B$ field in space-time. In the associated Yang-Mills theory, such as that on a system of $N$ D3-branes in the case of the simplest AdS/CFT correspondence, this change in the background field corresponds to replacing products of
open string fields with a noncommutative star-product. The resulting theory is a noncommutative Yang-Mills theory. Such noncommutative theories are the only well-understood example of a situation where adding an infinite number of apparently nonrenormalizable terms to a field theory action leads to a sensible modification of quantum field theory (for a review of noncommutative field theory and its connection to string theory, see [14]).

String field theory is a nonperturbative formulation in target space of an interacting string theory, in which the infinite family of fields associated with string excitations are described by a space-time field theory action. For open strings, this field theory is a natural extension of the low-energy Yang-Mills action describing a system of D-branes, where the entire hierarchy of massive string fields is included in addition to the massless gauge field on the D-brane. Integrating out all the massive fields from the string field theory action gives rise to a nonabelian Born-Infeld action for the D-branes, including an infinite set of higher-order terms arising from string theory corrections to the simple Yang-Mills action. Like the case of noncommutative field theory discussed above, the new terms appearing in this action are apparently nonrenormalizable, but the combination of terms must work together to form a sensible theory.

In the 1980’s, a great deal of work was done on formulating string field theory for open and closed, bosonic and supersymmetric string theories. Most of these string field theories are quite complicated. For the open bosonic string, however, Witten [18] constructed an extremely elegant string field theory based on the Chern-Simons action. This cubic bosonic open string field theory (OSFT) is the primary focus of the work described in these lectures. Although Witten’s OSFT can be described in a simple abstract language, practical computations with this theory rapidly become extremely complicated. Despite a substantial amount of work on this theory, little insight was gained in the 1980’s regarding how this theory could be used to go beyond standard perturbative string methods. Work on this subject stalled out in the late 80’s, and little further attention was paid to OSFT until several years ago.

One simple feature of the 26-dimensional bosonic string has been problematic since the early days of string theory: both the open and closed bosonic strings have tachyons in their spectra, indicating that the usual perturbative vacua used for these theories are unstable. In 1999, Ashoke Sen had a remarkable insight into the nature of the open bosonic string tachyon [19]. He observed that the open bosonic string should be thought of as ending on a space-filling D25-brane. He pointed out that this D-brane is unstable in the bosonic theory, as it does not carry any conserved charge, and he suggested that the open bosonic string tachyon should be interpreted as the instability mode of the D25-brane. This led him to conjecture that Witten’s open string field theory could be used to precisely determine a new vacuum for the open string, namely one in which the D25-brane is annihilated through condensation of the tachyonic unstable mode. Sen made several precise conjectures regarding the details of the string field theory description of this new open string vacuum. As we describe in these lectures, there is now overwhelming evidence that Sen’s picture is correct, demonstrating that string field theory accurately describes the nonperturbative physics of D-branes. This
new nonperturbative application of string field theory has sparked a new wave of work on Witten’s cubic open string field theory, revealing many remarkable new structures. In particular, string field theory now provides a concrete framework in which disconnected string backgrounds can emerge from the equations of motion of a single underlying theory. Although so far this can only be shown explicitly in the open string context, this work paves the way for a deeper understanding of background-independence in quantum theories of gravity.

2 D-branes

In this section we briefly review some basic features of D-branes. The concepts developed here will be useful in describing tachyonic D-brane configurations in the following section. For more detailed reviews of D-branes, see [15, 16].

2.1 D-branes and Ramond-Ramond charges

D-branes can be understood in two ways: a) as extended extremal black brane solutions of supergravity carrying conserved charges, and b) as hypersurfaces on which strings have Dirichlet boundary conditions.

a) The ten-dimensional type IIA and IIB supergravity theories each have a set of $(p+1)$-form fields $A^{(p+1)}_{\mu_1...\mu_{p+1}}$ in the supergraviton multiplet, with $p$ even/odd for type IIA/IIB supergravity. These are the Ramond-Ramond fields in the massless superstring spectrum. For each of these $(p+1)$-form fields, there is a solution of the supergravity field equations which has $(p+1)$-dimensional Lorentz invariance, and which has the form of an extremal black hole solution in the orthogonal $9-p$ space directions plus time (for a review see [17]). These “black $p$-brane” solutions carry charge under the R-R fields $A^{(p+1)}$, and are BPS states in the supergravity theory, preserving half the supersymmetries of the theory.

b) In type IIA and IIB string theory, it is possible to consider open strings with Dirichlet boundary conditions on some number $9-p$ of the spatial coordinates $x^\mu(\sigma)$. The locus of points defined by such Dirichlet boundary conditions defines a $(p+1)$-dimensional hypersurface $\Sigma_{p+1}$ in the ten-dimensional spacetime. When $p$ is even/odd in type IIA/IIB string theory, the spectrum of the resulting quantum open string theory contains a massless set of fields $A_\alpha, \alpha = 0, 1, \ldots, p$ and $X^a, a = p + 1, \ldots, 9$. These fields can be associated with a gauge field living on the hypersurface $\Sigma_{p+1}$, and a set of degrees of freedom describing the transverse fluctuations of this hypersurface in spacetime. Thus, the quantum fluctuations of the open string describe a fluctuating $(p+1)$-dimensional hypersurface in spacetime — a Dirichlet-brane, or “D-brane”.

The remarkable insight of Polchinski in 1995 [20] was the observation that Dirichlet-branes carry Ramond-Ramond charges, and therefore should be described in the low-energy supergravity limit of string theory by precisely the black $p$-branes discussed in a). This
connection between the string and supergravity descriptions of these nonperturbative objects paved the way to a dramatic series of new developments in string theory, including connections between string theory and supersymmetric gauge theories, string constructions of black holes, and new approaches to string phenomenology.

2.2 Born-Infeld and super Yang-Mills D-brane actions

In this subsection we briefly review the low-energy super Yang-Mills description of the dynamics of one or more D-branes. As discussed in the previous subsection, the massless open string modes on a Dp-brane in type IIA or IIB superstring theory describe a \((p + 1)\)-component gauge field \(A_\alpha\), \(9 - p\) transverse scalar fields \(X^a\), and a set of massless fermionic gaugino fields. The scalar fields \(X^a\) describe small fluctuations of the D-brane around a flat hypersurface. If the D-brane geometry is sufficiently far from flat, it is useful to describe the D-brane configuration by a general embedding \(X^\mu(\xi)\), where \(\xi^\alpha\) are \(p + 1\) coordinates on the Dp-brane world-volume \(\Sigma_{(p+1)}\), and \(X^\mu\) are ten functions giving a map from \(\Sigma_{(p+1)}\) into the space-time manifold \(\mathbb{R}^{9,1}\). Just as the Einstein equations governing the geometry of spacetime arise from the condition that the one-loop contribution to the closed string beta function vanish, a set of equations of motion for a general Dp-brane geometry and associated world-volume gauge field can be derived from a calculation of the one-loop open string beta function \[21\]. These equations of motion arise from the classical Born-Infeld action

\[
S = -T_p \int d^{p+1}\xi \, e^{-\varphi} \sqrt{-\det(G_{\alpha\beta} + B_{\alpha\beta} + 2\pi\alpha'F_{\alpha\beta}) + S_{CS} + \text{fermions}} \tag{1}
\]

where \(G, B\) and \(\varphi\) are the pullbacks of the 10D metric, antisymmetric tensor and dilaton to the D-brane world-volume, while \(F\) is the field strength of the world-volume \(U(1)\) gauge field \(A_\alpha\). \(S_{CS}\) represents a set of Chern-Simons terms which will be discussed in the following subsection. This action can be verified by a perturbative string calculation \[15\], which also gives a precise expression for the brane tension

\[
\tau_p = \frac{T_p}{g} = \frac{1}{g\sqrt{\alpha'} (2\pi\sqrt{\alpha'})^p} \tag{2}
\]

where \(g = e^{\langle \varphi \rangle}\) is the string coupling, equal to the exponential of the dilaton expectation value.

A particular limit of the Born-Infeld action \(1\) is useful for describing many low-energy aspects of D-brane dynamics. Take the background space-time \(G_{\mu\nu} = \eta_{\mu\nu}\) to be flat, and all other supergravity fields \((B_{\mu\nu}, A^{(p+1)}_{\mu_1\cdots\mu_{p+1}})\) to vanish. We then assume that the D-brane is approximately flat, and is close to the hypersurface \(X^a = 0\), \(a > p\), so that we may make the static gauge choice \(X^\alpha = \xi^\alpha\). We furthermore assume that \(\partial_\alpha X^a\) and \(2\pi\alpha'F_{\alpha\beta}\) are small and of the same order. In this limit, the action \(1\) can be expanded as

\[
S = -\tau_p V_p - \frac{1}{4g^2_{YM}} \int d^{p+1}\xi \left( F_{\alpha\beta}F^{\alpha\beta} + \frac{2}{(2\pi\alpha')^2}\partial_\alpha X^a \partial^\alpha X^a \right) + \cdots \tag{3}
\]
where $V_p$ is the $p$-brane world-volume and the coupling $g_{YM}$ is given by

$$g_{YM}^2 = \frac{1}{4\pi^2 \alpha'^2 \tau_p} = \frac{g}{\sqrt{\alpha'}}(2\pi\sqrt{\alpha'})^{p-2}.$$  \hspace{1cm} (4)

Including fermionic terms, the second term in (3) is simply the dimensional reduction to $(p + 1)$ dimensions of the 10D $\mathcal{N} = 1$ super Yang-Mills action

$$S = \frac{1}{g_{YM}^2} \int d^{10}\xi \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\psi} \Gamma^\mu \partial_\mu \psi \right)$$  \hspace{1cm} (5)

where for $\alpha, \beta \leq p$, $F_{\alpha\beta}$ is the world-volume $U(1)$ field strength, and for $a > p, \alpha \leq p$, $F_{aa} \rightarrow \partial_\alpha X^a$ (setting $2\pi\alpha' = 1$).

When multiple D$p$-branes are present, the D-brane action is modified in a fairly simple fashion [22]. Consider a system of $N$ coincident D-branes. For every pair of branes $\{i, j\}$ there is a set of massless fields $(A_\alpha)^i_j$, $(X^a)^i_j$ associated with strings stretching from the $i$th brane to the $j$th brane; the indices $i, j$ are known as Chan-Paton indices. Treating the fields (6) as matrices, the analogue for multiple branes of the Born-Infeld action (1) takes the form

$$S \sim \int \text{Tr} \sqrt{-\det (G + B + F)}.$$  \hspace{1cm} (7)

This action is known as the nonabelian Born-Infeld action (NBI). In order to give a rigorous definition to the nonabelian Born-Infeld action, it is necessary to resolve ordering ambiguities in the expression (7). Since the spacetime coordinates $X^a$ associated with the D-brane positions in space-time become themselves matrix-valued, even evaluating the pullbacks $G_{\alpha\beta}, B_{\alpha\beta}$ involves resolving ordering issues. Much work has been done recently to resolve these ordering ambiguities (see [23] for some recent papers in this direction which contain further references to the literature), but there is still no consistent definition of the nonabelian Born-Infeld theory (7) which is valid to all orders.

The nonabelian Born-Infeld action (7) becomes much simpler in the low-energy limit when the background space-time is flat. In the same limit discussed above for the single D-brane, where we find a low-energy limit giving the $U(1)$ super Yang-Mills theory in $p + 1$ dimensions, the inclusion of multiple D-branes simply leads in the low-energy limit to the nonabelian $U(N)$ super Yang-Mills action in $p + 1$ dimensions. This action is the dimensional reduction of the 10D $U(N)$ super Yang-Mills action (analogous to (5), but with an overall trace) to $p + 1$ dimensions. In this reduction, as before, for $\alpha, \beta \leq p$, $F_{\alpha\beta}$ is the world-volume $U(1)$ field strength, and for $a > p, \alpha \leq p$, $F_{aa} \rightarrow \partial_\alpha X^a$, where now $A_\alpha, X^a$, and $F_{\alpha\beta}$ are $N \times N$ matrices. We furthermore have, for $a, b > p$, $F_{ab} \rightarrow -i[X^a, X^b]$ in the dimensional reduction.
The low-energy description of a system of \( N \) coincident flat D-branes is thus given by \( U(N) \) super Yang-Mills theory in the appropriate dimension. This connection between D-brane actions in string theory and super Yang-Mills theory has led to many new developments, including new insights into supersymmetric field theories, the M(atrix) theory and AdS/CFT correspondences, and brane world scenarios.

2.3 Branes from branes

In this subsection we describe a remarkable feature of D-brane systems, namely a mechanism by which one or more D-branes of a fixed dimension can be used to construct additional D-branes of higher or lower dimension.

In our discussion of the D-brane action \( (\ref{eq:1}) \) above, we mentioned a group of terms \( S_{CS} \) which we did not describe explicitly. For a single D\(_{p}\)-brane, these Chern-Simons terms can be combined into a single expression of the form

\[
S_{CS} \sim \int_{x_{p+1}} \mathcal{A} e^{F+B} \tag{8}
\]

where \( \mathcal{A} = \sum_{k} A^{(k)} \) represents a formal sum over all the Ramond-Ramond fields \( A^{(k)} \) of various dimensions. In this integral, for each term \( A^{(k)} \), the nonvanishing contribution to \( S_{CS} \) is given by expanding the exponential of \( F + B \) to order \((p + 1 - k)/2\), where the dimension of the resulting form saturates the dimension of the brane. For example, on a D\(_{p}\)-brane, there is a coupling of the form

\[
\int_{x_{(p+1)}} A^{(p-1)} \wedge F. \tag{9}
\]

This coupling implies that the U(1) field strength on the D\(_{p}\)-brane couples to the R-R field associated with \((p - 2)\)-branes. Thus, we can associate magnetic fields on a D\(_{p}\)-brane with dissolved \((p - 2)\)-branes living on the D\(_{p}\)-brane. This result generalizes to a system of multiple D\(_{p}\)-branes by simply performing a trace on the RHS of \( S_{CS} \). For example, on \( N \) compact D\(_{p}\)-branes, the charge

\[
\frac{1}{2\pi} \int \text{Tr} F_{\alpha\beta}, \tag{10}
\]

which is the first Chern class of the U(N) bundle described by the gauge field on the \( N \) branes, is quantized and measures the number of units of D\((p - 2)\)-brane charge living on the D\(_{p}\)-branes, which are encoded in the field strength \( F_{\alpha\beta} \). Similarly,

\[
\frac{1}{8\pi^2} \int \text{Tr} F \wedge F \tag{11}
\]

encodes D\((p - 4)\)-brane charge on the D\(_{p}\)-branes.

Just as lower-dimensional branes can be described in terms of the degrees of freedom associated with a system of \( N \) D\(_p\)-branes through the field strength \( F_{\alpha\beta} \), higher-dimensional branes can be described by a system of \( N \) D\(_p\)-branes in terms of the commutators of the matrix-valued scalar fields \( X^a \). Just as \( \frac{1}{2\pi} F \) measures \((p - 2)\)-brane charge, the matrix

\[
2\pi i [X^a, X^b] \tag{12}
\]
measures \((p + 2)\)-brane charge \([16, 21, 25]\). The charge \((12)\) should be interpreted as a form of local charge density. The fact that the trace of \((12)\) vanishes for finite sized matrices corresponds to the fact that the net \(D_p\)-brane charge of a finite-size brane configuration in flat spacetime vanishes.

A simple example of the mechanism by which a system of multiple \(D_p\)-branes form a higher-dimensional brane is given by the matrix sphere. If we take a system of \(D_0\)-branes with scalar matrices \(X^a\) given by

\[
X^a = \frac{2r}{N} J^a, \quad a = 1, 2, 3
\]

where \(J^a\) are the generators of \(SU(2)\) in the \(N\)-dimensional representation, then we have a configuration corresponding to the “matrix sphere”. This is a \(D_2\)-brane of spherical geometry living on the locus of points satisfying \(x^2 + y^2 + z^2 = r^2\). The “local” \(D_2\)-brane charge of this brane is given by \((12)\). The \(D_2\)-brane configuration given by \((13)\) is rotationally invariant (up to a gauge transformation). The restriction of the brane to the desired locus of points can be seen from the relation \((X^1)^2 + (X^2)^2 + (X^3)^2 = r^2 \mathbb{1} + \mathcal{O}(N^{-2})\).

### 2.4 T-duality

We conclude our discussion of D-branes with a brief description of T-duality. T-duality is a perturbative symmetry which relates the type IIA and type IIB string theories. This duality symmetry was in fact crucial in the original discovery of D-branes \([20]\). A more detailed discussion of T-duality can be found in the textbook by Polchinski \([26]\). Using T-duality, we construct an explicit example of a brane within a brane encoded in super Yang-Mills theory, illustrating the ideas of the previous subsection. This example will be used in the following section to construct an analogous configuration with a tachyon.

Consider type IIA string theory on a spacetime of the form \(M^9 \times S^1\) where \(M^9\) is a generic 9-manifold of Lorentz signature, and \(S^1\) is a circle of radius \(R\). T-duality is the statement that this theory is precisely equivalent, at the perturbative level, to type IIB string theory on the spacetime \(M^9 \times (S^1)'\), where \((S^1)'\) is a circle of radius \(R' = \alpha'/R\).

T-duality is most easily understood in terms of closed strings, where it amounts to an exchange of winding and momentum modes of the string. The string winding modes on \(S^1\) have energy \(m = R w / \alpha'\), where \(w\) is the winding number. the T-dual momentum modes on \((S^1)'\) have \(m = n / R'\); it is straightforward to check that the spectrum of closed string states is unchanged under T-duality. T-duality can also be understood in terms of open strings. Under T-duality, an open string with Neumann boundary conditions on \(S^1\) is mapped to an open string with Dirichlet boundary conditions on \((S^1)'\), and vice versa. Thus, a Dirichlet \(p\)-brane which is wrapped around the circle \(S^1\) is mapped under T-duality to a Dirichlet \((p - 1)\)-brane of one lower dimension which is localized to a point on the circle \((S^1)'\). At the level of the low-energy theory on the D-brane, the \((p + 1)\)-dimensional Yang-Mills theory on the \(p\)-brane is replaced under T-duality with the \(p\)-dimensional Yang-Mills theory on the
T-duality takes a diagonal D1-brane on a two-torus (a) to a D2-brane on the dual torus with constant magnetic flux encoding an embedded D0-brane (b).

dual $(p - 1)$-brane. Mathematically, the covariant derivative operator in the direction $S^1$ is replaced under T-duality with an adjoint scalar field $X^a$. Formally, this adjoint scalar field is an infinite size matrix, containing information about the open strings wrapped an arbitrary number of times around the compact direction $(S^1)'$.

We can summarize the relevant mappings under T-duality in the following table

| IIA/$S^1$ | IIB/$(S^1)'$ |
|----------|--------------|
| $R$      | $R' = \alpha'/R$ |
| Neumann/Dirichlet b.c.’s | Dirichlet/Neumann b.c.’s |
| $p$-brane | $(p \pm 1)$-brane |
| $2\pi\alpha'(i\partial_a + A_a)$ | $X^a$ |

The phenomena by which field strengths in one brane describe lower- or higher-dimensional branes can be easily understood using T-duality. The following simple example may help to clarify this connection. (For a more detailed discussion from this point of view see [16].)

Consider a D1-brane wrapped diagonally on a two-torus $T^2$ with sides of length $L_1 = L$ and $L_2 = 2\pi R$. (Figure 1(a)). This configuration is described in terms of the world-volume Yang-Mills theory on a D1-brane stretched in the $L_1$ direction through a transverse scalar field

$$X^2 = 2\pi R \xi_1 / L.$$  \hspace{1cm} (14)

To be technically precise, this scalar field should be treated as an $\infty \times \infty$ matrix whose $(n, m)$ entry is associated with strings connecting the $n$th and $m$th images of the D1-brane on the covering space of $S^1$. The diagonal elements $X^2_{n,n}$ of this infinite matrix are given by $2\pi R(\xi_1 + nL)/L$, while all off-diagonal elements vanish. While the resulting matrix-valued function of $\xi_1$ is not periodic, it is periodic up to a gauge transformation

$$X^2(L) = VX^2(0)V^{-1}$$ \hspace{1cm} (15)

where $V$ is the shift matrix with nonzero elements $V_{n,n+1} = 1$. 

---

**Figure 1**: T-duality takes a diagonal D1-brane on a two-torus (a) to a D2-brane on the dual torus with constant magnetic flux encoding an embedded D0-brane (b).
Under T-duality in the $x^2$ direction the infinite matrix $X^2_{nm}$ becomes the Fourier mode representation of a gauge field on a dual D2-brane

$$A_2 = \frac{1}{R' L'} \xi_1 .$$

(16)

The magnetic flux associated with this gauge field is

$$F_{12} = \frac{1}{R' L}$$

so that

$$\frac{1}{2\pi} \int F_{12} \, d\xi^1 \, d\xi^2 = 1 .$$

(17)

(18)

Note that the boundary condition (15) on the infinite matrix $X^2$ transforms under T-duality to the boundary condition on the gauge field

$$A_2(L, x_2) = e^{2\pi i \xi_2 / L_2'} (A_2(0, x_2) + i \partial_2) e^{-2\pi i \xi_2 / L_2'} $$

$$= e^{2\pi i \xi_2 / L_2'} A_2(0, x_2) e^{-2\pi i \xi_2 / L_2'} + \frac{2\pi}{L_2'},$$

(19)

where the off-diagonal elements of the shift matrix $V$ in (15) describe winding modes which correspond after T-duality to the first Fourier mode $e^{2\pi i \xi_2 / L_2'}$. The boundary condition on the gauge fields in the $\xi_2$ direction is trivial, which simplifies the T-duality map; a similar construction can be done with a nontrivial boundary condition in both directions, although the configuration looks more complicated in the D1-brane picture.

This construction gives a simple Yang-Mills description of the mapping of D-brane charges under T-duality: the initial configuration described above has charges associated with a single D1-brane wrapped around each of the directions of the 2-torus: $D1_1 + D1_2$. Under T-duality, these D1-branes are mapped to a D2-brane and a D0-brane respectively: $D2_{12} + D0$. The flux integral (18) is the representation in the D2-brane world-volume Yang-Mills theory of the charge associated with a D0-brane which has been uniformly distributed over the surface of the D2-brane, just as in (10).

3 Tachyons and D-branes

We now turn to the subject of tachyons. Certain D-brane configurations are unstable, both in supersymmetric and nonsupersymmetric string theories. This instability is manifested as a tachyon with $M^2 < 0$ in the spectrum of open strings ending on the D-brane. We will explicitly describe the tachyonic mode in the case of the open bosonic string in Section 4.1; this open bosonic string tachyon will be the focal point of most of the developments described in these notes. In this section we list some elementary D-brane configurations where tachyons arise, and we describe a particular situation in which the tachyon can be seen in the low-energy Yang-Mills description of the D-branes. This Yang-Mills background with a tachyon provides a simple field-theory model of a system analogous to the more complicated string field theory tachyon we describe in the later part of these notes. This simpler model may be useful to keep in mind in the later analysis.
3.1 D-brane configurations with tachyonic instabilities

Some simple examples of unstable D-brane configurations where the open string contains a tachyon include the following:

**Brane-antibrane:** A pair of parallel Dp-branes with opposite orientation in type IIA or IIB string theory which are separated by a distance \( d < l_s \) give rise to a tachyon in the spectrum of open strings stretched between the branes \([28]\). The difference in orientation of the branes means that the two branes are really a brane and antibrane, carrying equal but opposite R-R charges. Since the net R-R charge is 0, the brane and antibrane can annihilate, leaving an uncharged vacuum configuration.

**Wrong-dimension branes:** In type IIA/IIB string theory, a Dp-brane of even/odd spatial dimension \( p \) is a stable BPS state carrying a nonzero R-R charge. On the other hand, a Dp-brane of the *wrong* dimension (i.e., odd/even for IIA/IIB) carries no charges under the classical IIA/IIB supergravity fields, and has a tachyon in the open string spectrum. Such a brane can annihilate to the vacuum without violating charge conservation.

**Bosonic D-branes:** Like the wrong-dimension branes of IIA/IIB string theory, a Dp-brane of any dimension in the bosonic string theory carries no conserved charge and has a tachyon in the open string spectrum. Again, such a brane can annihilate to the vacuum without violating charge conservation.

3.2 Example: tachyon in low-energy field theory of two D-branes

As an example of how tachyonic configurations behave physically, we consider in this subsection a simple example where a brane-antibrane tachyon can be seen in the context of the low-energy Yang-Mills theory. This system was originally considered in \([29, 30]\).

The system we want to consider is a simple generalization of the (D2 + D0)-brane configuration we described using Yang-Mills theory in Section 2.4. Consider a pair of D2-branes wrapped on a two-torus, one of which has a D0-brane embedded in it as a constant positive magnetic flux, and the other of which has an anti-D0-brane within it described by a constant negative magnetic flux. We take the two dimensions of the torus to be \( L_1, L_2 \). Following the discussion of Section 2.4, this configuration is equivalent under T-duality in the \( L_2 \) direction to a pair of crossed D1-branes (see Figure 2). The Born-Infeld energy of this configuration is

\[
E_{\text{BI}} = 2\sqrt{(\tau_2 L_1 L_2)^2 + \tau_0^2} = \frac{1}{g} \left[ \frac{2L_1 L_2}{\sqrt{2\pi}} + \frac{(2\pi)^{3/2}}{L_1 L_2} + \cdots \right] \tag{20}
\]

in units where \( 2\pi \alpha' = 1 \). The second term in the last line corresponds to the Yang-Mills approximation. In this approximation (dropping the D2-brane energy) the energy is

\[
E_{\text{YM}} = \frac{\tau_2}{4} \int \text{Tr} \ F_{\alpha\beta} F^{\alpha\beta} = \frac{1}{4\sqrt{2\pi} g} \int \text{Tr} \ F_{\alpha\beta} F^{\alpha\beta} . \tag{21}
\]
We are interested in studying this configuration in the Yang-Mills approximation, in which we have a $U(2)$ theory on $T^2$ with field strength

$$F_{12} = \begin{pmatrix} \frac{2\pi}{L_1 L_2} & 0 \\ 0 & -\frac{2\pi}{L_1 L_2} \end{pmatrix} = \frac{2\pi}{L_1 L_2} \tau_3. \quad(22)$$

This field strength can be realized as the curvature of a linear gauge field

$$A_1 = 0, \quad A_2 = \frac{2\pi}{L_1 L_2} \xi \tau_3 \quad(23)$$

which satisfies the boundary conditions

$$A_j(L, \xi_2) = \Omega \left( i \partial_j + A_j(0, \xi_2) \right) \Omega^{-1} \quad(24)$$

where

$$\Omega = e^{2\pi i (\xi_1/L_2) \tau_3}. \quad(25)$$

It is easy to check that this configuration indeed satisfies

$$E_{YM} = \frac{1}{2g} \frac{(2\pi)^{3/2}}{L_1 L_2} \Tr \tau_3^2 = \frac{1}{g} \frac{(2\pi)^{3/2}}{L_1 L_2} \quad(26)$$

as desired from $[20]$. Since, however,

$$\Tr F_{\alpha\beta} = 0, \quad(27)$$

the gauge field we are considering is in the same topological equivalence class as $F = 0$. This corresponds to the fact that the D0-brane and anti-D0-brane can annihilate. To understand the appearance of the tachyon, we can consider the spectrum of excitations $\delta A_\alpha$ around the background $[23] [29]$. The eigenvectors of the quadratic mass terms in this background are described by theta functions on the torus satisfying boundary conditions related to $[24]$. There are precisely two elements in the spectrum with the negative eigenvalue $-4\pi/L_1 L_2$. These theta functions, given explicitly in $[29]$, are tachyonic modes of the theory which
Figure 3: The brane-antibrane instability of a D0-D0̅ system embedded in two D2-branes, as seen in the T-dual D1-brane picture.

are associated with the annihilation of the positive and negative fluxes encoding the D0- and anti-D0-brane. These tachyonic modes are perhaps easiest to understand in the dual configuration, where they provide a direction of instability in which the two crossed D1-branes reconnect as in Figure 3. In the T-dual picture it is also interesting to note that the two tachyonic modes of the gauge field have support which is localized near the two brane intersection points. These modes have off-diagonal form

\[ \delta A_t \sim \begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}. \]  

This form of the tachyonic modes naturally encodes our geometric understanding of these modes as reconnecting the two D1-branes near the intersection point.

The full Yang-Mills action around the background (23) can be written as a quartic function of the mass eigenstates around this background. Written in terms of these modes, there are nontrivial cubic and quartic terms which couple the tachyonic modes to all the massive modes in the system. If we integrate out the massive modes, we know from the topological reasoning above that an effective potential arises for the tachyonic mode \( A_t \), with a maximum value of (26) and a minimum value of 0. This system is highly analogous to the bosonic open string tachyon we will discuss in the remainder of these lectures. Our current understanding of the bosonic string through bosonic string field theory is analogous to that of someone who only knows the Yang-Mills theory around the background (23) in terms of a complicated quartic action for an infinite family of modes. Without knowledge of the topological structure of the theory, and given only a list of the coefficients in the quartic action, such an individual would have to systematically calculate the tachyon effective potential by explicitly integrating out all the massive modes one by one. This would give a numerical approximation to the minimum of the effective potential, which could be made arbitrarily good by raising the mass of the cutoff at which the effective action is computed. It may be helpful to keep this example system in mind in the following sections, where an analogous tachyonic system is considered in string field theory. For further discussion of this unstable configuration in Yang-Mills theory, see [29, 30].
4 Open string field theory and the Sen conjectures

The discussion of the previous sections gives us an overview of string theory, and an example of how tachyons appear in a simple gauge theory context, when an unstable brane-antibrane configuration is embedded in a higher-dimensional brane. We now turn our attention back to string theory, where the appearance of a tachyon necessitates a nonperturbative approach to the theory. In subsection 4.1, we review the BRST quantization approach to the bosonic open string. Subsection 4.2 describes Witten’s cubic open string field theory, which gives a nonperturbative off-shell definition to the open bosonic string. In subsection 4.3 we describe Sen’s conjectures on tachyon condensation in the open bosonic string.

4.1 The bosonic open string

In this subsection we review the quantization of the open bosonic string. For further details see the textbooks by Green, Schwarz, and Witten [31] and by Polchinski [26]. The bosonic open string can be quantized using the BRST quantization approach starting from the action

$$ S = -\frac{1}{4\pi\alpha'} \int \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu, \quad (29) $$

where $\gamma$ is an auxiliary dynamical metric on the world-sheet. This action can be gauge-fixed to conformal gauge $\gamma^{ab} \sim \delta^{ab}$. Using the BRST approach to gauge fixing introduces ghost and antighost fields $c^\pm(\sigma), b_{\pm\pm}(\sigma)$. The gauge-fixed action, including ghosts, then becomes

$$ S = -\frac{1}{4\pi\alpha'} \int \partial_a X^\mu \partial^a X_\mu + \frac{1}{\pi} \int \left( b_{++} \partial_+ c^+ + b_{--} \partial_- c^- \right). \quad (30) $$

The matter fields $X^\mu$ can be expanded in modes using

$$ X^\mu(\sigma, \tau) = x_0^\mu + l_s^2 p^\mu \tau + \sum_{n \neq 0} \frac{l_s^2}{n} \alpha_n^\mu \cos(n\sigma)e^{-in\tau}. \quad (31) $$

Throughout the remainder of these notes we will use the convention

$$ \alpha' = \frac{l_s^2}{2} = 1, \quad (32) $$

so that $l_s = \sqrt{2}$. In the quantum theory, $x_0^\mu$ and $p^\mu$ obey the canonical commutation relations

$$ [x_0^\mu, p^\nu] = i\eta^{\mu\nu}. \quad (33) $$

The $\alpha_n^\mu$’s with negative/positive values of $n$ become raising/lowering operators for the oscillator modes on the string, and satisfy the commutation relations

$$ [\alpha_m^\mu, \alpha_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}. \quad (34) $$
We will often use the canonically normalized raising and lowering operators

\[ a^\mu_n = \frac{1}{\sqrt{|n|}} \alpha^\mu_n \]  

which obey the commutation relations

\[ [a^\mu_n, a^\nu_m] = \eta^{\mu\nu} \delta_{m+n,0}. \]  

The raising and lowering operators satisfy \( (\alpha^\mu_n)^\dagger = \alpha^\mu_{-n}, (a^\mu_n)^\dagger = a^\mu_{-n} \). We will also frequently use position modes \( x_n \) for \( n \neq 0 \) and raising and lowering operators \( a_0, a_0^\dagger \) for the zero modes. These are related to the modes in (31) through (dropping space-time indices)

\[ x_n = \frac{i}{\sqrt{n}} (a_n - a_n^\dagger) \]  

\[ x_0 = \frac{i}{\sqrt{2}} (a_0 - a_0^\dagger) \]  

The ghost and antighost fields can be decomposed into modes through

\[ c^\pm (\sigma, \tau) = \sum_n c_n e^{\pm in(\sigma \pm \tau)} \]  

\[ b^\pm (\sigma, \tau) = \sum_n b_n e^{\pm in(\sigma \pm \tau)} . \]  

The ghost and antighost modes satisfy the anticommutation relations

\[ \{c_n, b_m\} = \delta_{n+m,0} \]  

\[ \{c_n, c_m\} = \{b_n, b_m\} = 0 . \]  

A general state in the open string Fock space can be written in the form

\[ \alpha^\mu_{-n_1} \cdots \alpha^\mu_{-n_i} c_{-m_1} \cdots c_{-m_j} b_{-p_1} \cdots b_{-p_l} \mid 0; k \rangle \]  

where \( \mid 0; k \rangle \) is the SL(2,R) invariant vacuum annihilated by

\[ b_n \mid 0; k \rangle = 0, \quad n \geq 1 \]  

\[ c_n \mid 0; k \rangle = 0, \quad n \geq 2 \]  

\[ \alpha^\mu_{-n} \mid 0; k \rangle = 0, \quad n \geq 0 \]  

with momentum

\[ p^\mu \mid 0; k \rangle = k^\mu \mid 0; k \rangle . \]  

We will often write the zero momentum vacuum \( \mid 0; k = 0 \rangle \) simply as \( \mid 0 \rangle \). This vacuum is taken by convention to have ghost number 0, and satisfies

\[ \langle 0; k | c_{-1} c_0 c_1 | 0 \rangle = \delta(k) \]
For string field theory we will also find it convenient to work with the vacua of ghost number 1 and 2

\[ G = 1 : \quad |0_1\rangle = c_1|0\rangle \quad (46) \]
\[ G = 2 : \quad |0_2\rangle = c_0c_1|0\rangle . \quad (47) \]

In the notation of Polchinski [26], these two vacua are written as

\[ |0_1\rangle = |0\rangle_m \otimes |\downarrow\rangle \quad (48) \]
\[ |0_2\rangle = |0\rangle_m \otimes |\uparrow\rangle . \]

where \(|0\rangle_m\) is the matter vacuum and \(|\downarrow\rangle, |\uparrow\rangle\) are the ghost vacua annihilated by \(b_0, c_0\).

The BRST operator of this theory is given by

\[ Q_B = \sum_{n=-\infty}^{\infty} c_n L_{-n}^{(m)} + \sum_{n,m=-\infty}^{\infty} \frac{(m-n)}{2} : c_m c_n b_{-m-n} : -c_0 \quad (49) \]

where the matter Virasoro operators are given by

\[ L_q^{(m)} = \begin{cases} \frac{1}{2} \sum_{n} \alpha_{q-n}^{\mu} \alpha_{\mu n}, & q \neq 0 \\ p^2 + \sum_{n=1}^{\infty} \alpha_{n}^{\mu} \alpha_{\mu n}, & q = 0 \end{cases} \quad (50) \]

Some useful features of the BRST operator \(Q = Q_B\) include:

- \(Q^2 = 0\); i.e., the BRST operator is nilpotent. This identity relies on a cancellation between matter and ghost terms which only works in dimension \(D = 26\) for the bosonic theory.
- \(\{Q, b_0\} = L_0^{(m)} + L_0^{(g)} - 1\).
- \(Q\) has ghost number 1, so acting on a state \(|s\rangle\) of ghost number \(G\) gives a state \(Q|s\rangle\) of ghost number \(G + 1\).
- The physical states of the theory are given by the cohomology of \(Q\) at ghost number 1

\[ \mathcal{H}_{\text{phys}} = \mathcal{H}_{\text{closed}}/\mathcal{H}_{\text{exact}} = \{ |\psi\rangle : Q|\psi\rangle = 0 \}/ \langle |\psi\rangle \sim |\psi\rangle + Q|\chi\rangle \]  

(51)

- Physical states can be chosen as representatives of each cohomology class so that they are all annihilated by \(b_0\).

It is often convenient to separate out the ghost zero-modes, writing \(Q = c_0 L_0 + b_0 M + \tilde{Q}\), where (momentarily reinstating \(\alpha'\))

\[ L_0 = \sum_{n=1}^{\infty} (\alpha_{-n} \alpha_n + nc_{-n} b_n + nb_{-n} c_n) + \alpha' p^2 - 1 \quad (52) \]
In this expression the term in parentheses is simply the oscillator number operator, measuring the level of a given state.

Some simple examples of physical states include the tachyon state

$$|0_1; p\rangle$$  \hspace{1cm} (53)

which is physical when \( p^2 = 1/\alpha' = -M^2 \), and the massless gauge boson

$$\epsilon_\mu \alpha_{-1}^{\mu} |0_1; p\rangle$$  \hspace{1cm} (54)

which is physical when \( p^2 = M^2 = 0 \), for transverse polarizations \( p \cdot \epsilon = 0 \). Note that the transverse polarization condition follows from the appearance of a term proportional to \( c_{-1} p \cdot \alpha_1 \) in \( \tilde{Q} \), which must annihilate the state (54).

### 4.2 Witten’s cubic bosonic SFT

The discussion of the previous subsection leads to a systematic quantization of the open bosonic string in the conformal field theory framework. Using this approach it is possible, in principle, to calculate an arbitrary perturbative on-shell scattering amplitude for physical string states. To study tachyon condensation in string theory, however, we require a nonperturbative, off-shell formalism for the theory—a string field theory.

A very simple form for the off-shell open bosonic string field theory action was proposed by Witten in 1986 \[18\]

$$S = -\frac{1}{2} \int \Psi \star Q \Psi - \frac{g}{3} \int \Psi \star \Psi \star \Psi .$$  \hspace{1cm} (55)

This action has the general form of a Chern-Simons theory on a 3-manifold, although for string field theory there is no explicit interpretation of the integration in terms of a concrete 3-manifold. In Eq. (55), \( g \) is interpreted as the string coupling constant. The field \( \Psi \) is a string field, which takes values in a graded algebra \( \mathcal{A} \). Associated with the algebra \( \mathcal{A} \) there is a star product

$$\star : \mathcal{A} \otimes \mathcal{A} \rightarrow \mathcal{A},$$  \hspace{1cm} (56)

under which the degree \( G \) is additive \((G_{\psi \star \phi} = G_\psi + G_\phi)\). There is also a BRST operator

$$Q : \mathcal{A} \rightarrow \mathcal{A},$$  \hspace{1cm} (57)

of degree one \((G_Q \psi = 1 + G_\psi)\). String fields can be integrated using

$$\int : \mathcal{A} \rightarrow \mathbb{C} .$$  \hspace{1cm} (58)

This integral vanishes for all \( \Psi \) with degree \( G_\psi \neq 3 \).
The elements $Q, \star, \int$ defining the string field theory are assumed to satisfy the following axioms:

(a) Nilpotency of $Q$: $Q^2 \Psi = 0$, $\forall \Psi \in \mathcal{A}$.

(b) $\int Q \Psi = 0$, $\forall \Psi \in \mathcal{A}$.

(c) Derivation property of $Q$:

$$Q(\Psi \star \Phi) = (Q \Psi) \star \Phi + (-1)^{G_\Psi} \Psi \star (Q \Phi), \quad \forall \Psi, \Phi \in \mathcal{A}.$$ 

(d) Cyclicity: $\int \Psi \star \Phi = (-1)^{G_\Psi} \int \Phi \star \Psi$, $\forall \Psi, \Phi \in \mathcal{A}$.

(e) Associativity: $(\Phi \star \Psi) \star \Xi = \Phi \star (\Psi \star \Xi)$, $\forall \Phi, \Psi, \Xi \in \mathcal{A}$.

When these axioms are satisfied, the action (55) is invariant under the gauge transformations

$$\delta \Psi = Q \Lambda + \Psi \star \Lambda - \Lambda \star \Psi$$

for any gauge parameter $\Lambda \in \mathcal{A}$ with ghost number 0.

When the string coupling $g$ is taken to vanish, the equation of motion for the theory defined by (55) simply becomes $Q \Psi = 0$, and the gauge transformations (59) simply become

$$\delta \Psi = Q \Lambda.$$ 

Thus, when $g = 0$ this string field theory gives precisely the structure needed to describe the free bosonic string. The motivation for introducing the extra structure in (55) was to find a simple interacting extension of the free theory, consistent with the perturbative expansion of open bosonic string theory.

Witten presented this formal structure and argued that all the needed axioms are satisfied when $\mathcal{A}$ is taken to be the space of string fields

$$\mathcal{A} = \{ \Psi[x(\sigma); c(\sigma), b(\sigma)] \}$$

which can be described as functionals of the matter, ghost and antighost fields describing an open string in 26 dimensions with $0 \leq \sigma \leq \pi$. Such a string field can be written as a formal sum over open string Fock space states with coefficients given by an infinite family of space-time fields

$$\Psi = \int d^{26}p \ [\phi(p) \ |0_1; p\rangle + A_\mu(p) \alpha^\mu_1 |0_1; p\rangle + \cdots]$$

Each Fock space state is associated with a given string functional, just as the states of a harmonic oscillator are associated with wavefunctions of a particle in one dimension. For example, the matter ground state $|0\rangle_m$ annihilated by $a_n$ for all $n \geq 1$ is associated (up to a constant $C$) with the functional of matter modes

$$|0\rangle_m \rightarrow C \exp \left(-\frac{1}{4} \sum_{n>0}^\infty n x_n^2 \right).$$
For Witten’s cubic string field theory, the BRST operator $Q$ in (55) is the usual open string BRST operator $Q_B$, given in (49). The star product $\star$ acts on a pair of functionals $\Psi, \Phi$ by gluing the right half of one string to the left half of the other using a delta function interaction

\[
\Psi \mid \begin{array}{c}
\delta
\end{array} \mid \Phi
\]

This star product factorizes into separate matter and ghost parts. In the matter sector, the star product is given by the formal functional integral

\[
(\Psi \star \Phi)[z(\sigma)] \\
\equiv \int \prod_{0 \leq \tilde{\tau} \leq \frac{\pi}{2}} dy(\tilde{\tau}) dx(\pi - \tilde{\tau}) \prod_{\frac{\pi}{2} \leq \tau \leq \pi} \delta[x(\tau) - y(\pi - \tau)] \Psi[x(\tau)]\Phi[y(\tau)],
\]

\[
x(\tau) = z(\tau) \text{ for } 0 \leq \tau \leq \frac{\pi}{2},
\]

\[
y(\tau) = z(\tau) \text{ for } \frac{\pi}{2} \leq \tau \leq \pi.
\]

Similarly, the integral over a string field factorizes into matter and ghost parts, and in the matter sector is given by

\[
\int \Psi = \int \prod_{0 \leq \sigma \leq \pi} dx(\sigma) \prod_{0 \leq \tau \leq \frac{\pi}{2}} \delta[x(\tau) - x(\pi - \tau)] \Psi[x(\tau)].
\]

(65)

This corresponds to gluing the left and right halves of the string together with a delta function interaction

\[
\Psi \mid \begin{array}{c}
\delta
\end{array} \mid \Phi
\]

The ghost sector of the theory is defined in a similar fashion, but has an anomaly due to the curvature of the Riemann surface describing the three-string vertex. The ghost sector can be described either in terms of fermionic ghost fields $c(\sigma), b(\sigma)$ or through bosonization in terms of a single bosonic scalar field $\phi_g(\sigma)$. From the functional point of view of Eqs. (64, 65), it is easiest to describe the ghost sector in the bosonized language. In this language, the ghost fields $b(\sigma)$ and $c(\sigma)$ are replaced by the scalar field $\phi_g(\sigma)$, and the star product in the ghost sector is given by (64) with an extra insertion of $\exp(3i\phi_g(\pi/2)/2)$ inside the integral. Similarly, the integration of a string field in the ghost sector is given by (65) with an insertion of $\exp(-3i\phi_g(\pi/2)/2)$ inside the integral. Witten first described the cubic string
field theory using bosonized ghosts. While this approach is useful for some purposes, we will use fermionic ghost fields in the remainder of these lecture notes.

The expressions (64, 65) may seem rather formal, as they are written in terms of functional integrals. These expressions, however, can be given precise meaning when described in terms of creation and annihilation operators acting on the string Fock space. In the Fock space language, the integral of a star product of two or three fields is described in terms of two- and three-string vertices

$$\langle V_2 \rangle \in \mathcal{H}^* \otimes \mathcal{H}^*, \quad \langle V_3 \rangle \in (\mathcal{H}^*)^3$$

so that

$$\int \Phi * \Psi \rightarrow \langle V_2 \rangle (\langle \Phi \rangle \otimes \Psi)$$

$$\int \Psi_1 * \Psi_2 * \Psi_3 \rightarrow \langle V_3 \rangle (\langle \Psi_1 \rangle \otimes \langle \Psi_2 \rangle \otimes \langle \Psi_3 \rangle)$$

In the next section we will give explicit forms for the two- and three-string vertices (66). In terms of these vertices, the string field theory action becomes

$$S = -\frac{1}{2} \langle V_2 | Q \Psi \rangle - \frac{g}{3} \langle V_3 | \Psi, \Psi \rangle.$$  \hspace{1cm} (68)

This action is often written using the BPZ dual $\langle \Psi |$ of the string field $| \Psi \rangle$, defined by the conformal map $z \rightarrow -1/z$, as

$$S = -\frac{1}{2} \langle \Psi | Q \Psi \rangle - \frac{g}{3} \langle \Psi | \Psi \rangle.$$  \hspace{1cm} (69)

In the remainder of these lectures, however, we will use the form (68). Using explicit formulae for the vertices (66) and the string field expansion (62) leads to the full string field theory action, given by an off-shell action in the target space-time for an infinite family of fields $\phi(p), A_\mu(p), \ldots$. We discuss this action in more detail in Section 5.

### 4.3 The Sen conjectures

The existence of the tachyonic mode in the open bosonic string indicates that the standard choice of perturbative vacuum for this theory is unstable. In the early days of the subject, there was some suggestion that this tachyon could condense, leading to a more stable vacuum (see for example [32]). Kostelecky and Samuel argued early on that the stable vacuum could be identified in string field theory in a systematic way [33], however there was no clear physical picture for the significance of this stable vacuum. In 1999, Ashoke Sen reconsidered the problem of tachyons in string field theory. Sen suggested that the open bosonic string should really be thought of as living on a D25-brane, and hence that the perturbative vacuum for this string theory should have a nonzero vacuum energy associated with the tension of this D25-brane. He suggested that the tachyon is simply the instability mode of the D25-brane,
which carries no conserved charge and hence is not expected to be stable, as discussed in section 3. Sen furthermore suggested that Witten's cubic open string field theory is a natural framework to use to study this tachyon, and that this string field theory should give an analytic description of the true vacuum. More precisely, Sen made the following 3 conjectures [19]:

1. Witten's classical open string field theory should have a locally stable nontrivial vacuum solution. The energy density of this vacuum should be given by the D25-brane tension

\[
\frac{\Delta E}{V} = T_{25} = -\frac{1}{2\pi^2 g^2}.
\]  

2. Lower-dimensional D-branes should exist as solitonic solutions of SFT which break part of the Lorentz symmetry of the perturbative vacuum.

3. Open strings should decouple from the theory in the nontrivial vacuum, since the D25-brane is absent in this vacuum.

In Section 6 of these lectures we discuss the evidence for these conjectures, focusing particularly on the first and third conjectures. First, however, we need to develop the technical tools to do specific calculations in string field theory.

5 Basics of SFT

In this section, we give a more detailed discussion of Witten’s open bosonic string field theory. Subsection 5.1 is a warmup, in which we review some basic features of the simple harmonic oscillator and discuss squeezed states. In Subsection 5.2 we derive the two-string vertex, and in subsection 5.3 we give an explicit formula for the three-string vertex. In subsection 5.4 we put these pieces together and discuss the calculation of the full SFT action. 5.5 contains a brief description of some more general features of Witten’s open bosonic string field theory. For more details about this string field theory, the reader is referred to the reviews [34, 35, 36].

5.1 Squeezed states and the simple harmonic oscillator

Let us consider a simple harmonic oscillator with annihilation operator

\[
a = -i \left( \sqrt{\frac{\alpha}{2}} x + \frac{1}{\sqrt{2\alpha}} \partial_x \right)
\]  

and ground state

\[
|0\rangle = \left( \frac{\alpha}{\pi} \right)^{1/4} e^{-\alpha x^2/2}.
\]
In the harmonic oscillator basis \(|n\rangle\), the Dirac position basis states \(|x\rangle\) have a squeezed state form

\[
|x\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{\alpha}{2} x^2 - i\sqrt{2} \alpha a^\dagger x + \frac{1}{2} (a^\dagger)^2\right) |0\rangle.
\] (73)

A general wavefunction is associated with a state through the correspondence

\[
f(x) \rightarrow \int_{-\infty}^{\infty} dx \ f(x) |x\rangle.
\] (74)

In particular, we have

\[
\delta(x) \rightarrow \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(\frac{1}{2} (a^\dagger)^2\right) |0\rangle
\] (75)

\[
1 \rightarrow \int dx \ |x\rangle = \left(\frac{4\pi}{\alpha}\right)^{1/4} \exp\left(-\frac{1}{2} (a^\dagger)^2\right) |0\rangle
\]

This shows that the delta and constant functions both have squeezed state representations in terms of the harmonic oscillator basis. The norm of a squeezed state

\[
|s\rangle = \exp\left(\frac{1}{2} s(a^\dagger)^2\right) |0\rangle
\] (76)

is given by

\[
\langle s|s\rangle = \frac{1}{\sqrt{1 - s^2}}
\] (77)

Thus, the states (75) are non-normalizable (as we would expect), however they are right on the border of normalizability. As for the Dirac basis states \(|x\rangle\), which are computationally useful although technically not well-defined states in the single-particle Hilbert space, we expect that many calculations using the states (75) will give sensible physical answers.

It will be useful for us to generalize the foregoing considerations in several ways. A particularly simple generalization arises when we consider a pair of degrees of freedom \(x, y\) described by a two-harmonic oscillator Fock space basis. In such a basis, repeating the preceding analysis leads us to a function-state correspondence for the delta functions relating \(x, y\) of the form

\[
\delta(x \pm y) \rightarrow \exp\left(\pm \frac{1}{2} a^\dagger(x) a^\dagger(y)\right) (|0\rangle_x \otimes |0\rangle_y).
\] (78)

we will find these squeezed state expressions very useful in describing the two- and three-string vertices of Witten’s open string field theory.

### 5.2 The two-string vertex \(|V_2\rangle\)

We can immediately apply the oscillator formulae from the preceding section to calculate the two-string vertex. Recall that the matter fields are expanded in modes through

\[
x(\sigma) = x_0 + \sqrt{2} \sum_{n=1}^{\infty} x_n \cos n\sigma.
\] (79)
(We suppress Lorentz indices in most of this section for clarity.) Using this mode decomposition, we associate the string field functional $\Psi[x(\sigma)]$ with a function $\Psi(\{x_n\})$ of the infinite family of string oscillator mode amplitudes. The overlap integral combining (65) and (64) can then be expressed in modes as

$$\int \Psi \Phi = \int \prod_{n=0}^{\infty} dx_n dy_n \delta(x_n - (-1)^n y_n) \Psi(\{x_n\}) \Phi(\{y_n\}). \quad (80)$$

Geometrically this just encodes the overlap condition $x(\sigma) = y(\pi - \sigma)$ described through

$$\Phi \Psi \Psi$$

From (88), it follows that we can write the two-string vertex as a squeezed state

$$\langle V_2 \rangle_{\text{matter}} = (\langle 0 \rangle \otimes \langle 0 \rangle) \exp \left( -\sum_{n,m=0}^{\infty} a_n^{(1)} C_{nm} a_m^{(2)} \right) \quad (81)$$

where $C_{nm} = \delta_{nm} (-1)^n$ is an infinite-size matrix connecting the oscillator modes of the two single-string Fock spaces, and the sum is taken over all oscillator modes including zero. In the expression (81), we have used the formalism in which $\langle 0 \rangle$ is the vacuum annihilated by $a_0$. To translate this expression into a momentum basis, we use only $n, m > 0$, and replace

$$\langle 0 \rangle \otimes \langle 0 \rangle \exp \left( -a_0^{(1)} a_0^{(2)} \right) \rightarrow \int d^{26} p \langle \langle 0; p | \otimes \langle 0; -p | \rangle \exp \left( -\sum_{n=1}^{\infty} (a_n^{(1)} a_n^{(2)} + c_n b_n^{(1)} + c_n^{(2)} b_n^{(1)}) \right). \quad (82)$$

The extension of this analysis to ghosts is straightforward. For the ghost and antighost respectively, the overlap conditions corresponding with $x_1(\sigma) = x_2(\pi - \sigma)$ are $c_1(\sigma) = -c_2(\pi - \sigma)$ and $b_1(\sigma) = b_2(\pi - \sigma)$. This leads to the overall formula for the two-string vertex

$$\langle V_2 \rangle = \int d^{26} p \langle \langle 0; p | \otimes \langle 0; -p | \rangle (c_0^{(1)} + c_0^{(2)}) \exp \left( -\sum_{n=1}^{\infty} (a_n^{(1)} a_n^{(2)} + c_n b_n^{(1)} + c_n^{(2)} b_n^{(1)})) \right). \quad (83)$$

This expression for the two-string vertex can also be derived directly from the conformal field theory approach, computing the two-point function of an arbitrary pair of states on the disk.

5.3 The three-string vertex $|V_3\rangle$

The three-string vertex, which is associated with the three-string overlap diagram

$$\Psi_1 \longrightarrow \Psi_2 \longrightarrow \Psi_3$$
can be computed in a very similar fashion to the two-string vertex above. The details of the calculation, however, are significantly more complicated. There are several different ways to carry out the calculation. One approach is to first rewrite the modes \( \cos n\sigma \) on the full string in terms of modes \( l, r \) on the two halves of the string with \( \sigma < \pi/2, \sigma > \pi/2 \). This rewriting can be accomplished using an infinite orthogonal transformation matrix \( X \). The delta function overlap condition can then be applied to the half-string modes as above, giving a squeezed state expression for \( |V_3\rangle \) with a squeezing matrix which can be expressed in terms of \( X \). The three-string vertex can also be computed using the conformal field theory approach. The three-string vertex was computed using various versions of these approaches in [37, 38, 39, 40, 41].

In these lectures we will not have time to go through a detailed derivation of the three-string vertex using any of these methods. We simply quote the final result from [37, 47]. Like the two-string vertex, the three-string vertex takes the form of a squeezed state

\[
\langle V_3 \rangle = \int d^{26}p_1 d^{26}p_2 d^{26}p_3 \left( \langle 0; p_1 \rangle \otimes \langle 0; p_2 \rangle \otimes \langle 0; p_3 \rangle \right) \delta(p_1 + p_2 + p_3) c_0^{(1)} c_0^{(2)} c_0^{(3)} \kappa \exp \left( -\frac{1}{2} \sum_{r,s=1}^3 [\epsilon_m^{(r)} V_{mn}^{rs} a_n^{(s)} + 2 \alpha_m^{(r)} V_{m0}^{rs} p^{(s)} + p^{(r)} V_{00}^{rs} p^{(s)} + \epsilon_m^{(r)} X_{mn}^{r,s} b_n^{(s)}] \right), \tag{84}
\]

where \( \kappa = 3^{3/2}/2^6 \), and where the Neumann coefficients \( V_{mn}^{rs}, X_{mn}^{r,s} \) are calculable constants given as follows. Define \( A_n, B_n \) for \( n \geq 0 \) through

\[
\begin{align*}
\left( \frac{1 + ix}{1 - ix} \right)^{1/3} &= \sum_{n \text{ even}} A_n x^n + i \sum_{m \text{ odd}} A_m x^m \\
\left( \frac{1 + ix}{1 - ix} \right)^{2/3} &= \sum_{n \text{ even}} B_n x^n + i \sum_{m \text{ odd}} B_m x^m
\end{align*}
\tag{85}
\]

These coefficients can be used to define 6-string Neumann coefficients \( N_{nm}^{r,\pm} \) through

\[
\begin{align*}
N_{nm}^{r,+} &= \begin{cases} 
\frac{1}{3(n+m)}(-1)^n(A_n B_m \pm B_n A_m), & m + n \text{ even}, m \neq n \\
0, & m + n \text{ odd}
\end{cases} \\
N_{nm}^{r,\pm(r+\sigma)} &= \begin{cases} 
\frac{1}{6(n+m\pm\sigma)}(-1)^{n+1}(A_n B_m \pm \sigma B_n A_m), & m + n \text{ even}, m \neq n \\
\frac{\sqrt{3}}{6(n+m\pm\sigma)}(A_n B_m \mp \sigma B_n A_m), & m + n \text{ odd}
\end{cases}
\end{align*}
\tag{86}
\]

Another interesting approach to understanding the cubic vertex has been explored extensively since these lectures were given. By diagonalizing the Neumann matrices, the star product encoded in the 3-string vertex takes the form of a continuous Moyal product. This simplifies the complexity of the cubic vertex, but at the cost of complicating the propagator. For a recent review of this work and further references, see [42].

A more detailed discussion of the derivation of the Neumann coefficients using CFT and oscillator methods will appear in [4].

Note that in some references, signs and various factors in \( \kappa \) and the Neumann coefficients may be slightly different. In some papers, the cubic term in the action is taken to have an overall factor of \( g/6 \) instead of \( g/3 \); this choice of normalization gives a 3-tachyon amplitude of \( g \) instead of \( 2g \), and gives a different value for \( \kappa \). Often, the sign in the exponential of \( 84 \) is taken to be positive, which changes the signs of the coefficients \( V_{nm}^{rs}, X_{nm}^{r,s} \). When the matter Neumann coefficients are defined with respect to the oscillator modes \( \alpha_n \) rather than \( \alpha_n \), the matter Neumann coefficients \( V_{nm}^{rs}, V_{nd}^{rs} \) must be divided by \( \sqrt{nm} \) and \( \sqrt{n} \). Finally, when \( \alpha' \) is taken to be \( 1/2 \), an extra factor of \( 1/\sqrt{2} \) appears for each 0 subscript in the matter Neumann coefficients.
where in $N^r, \pm (r+\sigma)$, $\sigma = \pm 1$, and $r+\sigma$ is taken modulo 3 to be between 1 and 3. The 3-string matter Neumann coefficients $V_{nm}^{rs}$ are then given by

$$V_{nm}^{rs} = -\sqrt{mn}(N_{nm}^{rs} + N_{nm}^{r,-s}), \quad m \neq n, \text{ and } m, n \neq 0$$

$$V_{nn}^{rr} = \frac{1}{3} \left[ 2 \sum_{k=0}^{n} (-1)^{n-k}A_k^2 - (-1)^n - A_n^2 \right], \quad n \neq 0$$

$$V_{nm}^{r,r+\sigma} = \frac{1}{2} \left[ (-1)^n - V_{nm}^{rr} \right], \quad n \neq 0$$

$$V_{0n}^{rs} = -\sqrt{2n}(N_{0n}^{rs} + N_{0n}^{r,-s}), \quad n \neq 0$$

$$V_{00}^{rr} = \ln(27/16)$$

The ghost Neumann coefficients $X_{nm}^{rs}, m \geq 0, n > 0$ are given by

$$X_{nm}^{rr} = \left( -N_{nm}^{r,r} + N_{nm}^{r,-r} \right), \quad n \neq m$$

$$X_{nm}^{(r \pm 1)} = m \left( \pm N_{nm}^{r,r+1} + N_{nm}^{r,-(r+1)} \right), \quad n \neq m$$

$$X_{nn}^{rr} = \frac{1}{3} \left[ (-1)^n - A_n^2 + 2 \sum_{k=0}^{n} (-1)^{n-k}A_k^2 - 2(-1)^nA_nB_n \right]$$

$$X_{mn}^{(r \pm 1)} = -\frac{1}{2}(-1)^n - \frac{1}{2}X_{nm}^{rr}$$

The Neumann coefficients have a number of simple symmetries. There is a cyclic symmetry under $r \rightarrow r + 1, s \rightarrow s + 1$, which corresponds to the obvious geometric symmetry of rotating the vertex. The coefficients are also symmetric under the exchange $r \leftrightarrow s, n \leftrightarrow m$. Finally, there is a “twist” symmetry,

$$V_{nm}^{rs} = (-1)^{n+m}V_{nm}^{sr} \quad (89)$$

$$X_{nm}^{rs} = (-1)^{n+m}X_{nm}^{sr}.$$ 

This symmetry follows from the invariance of the 3-vertex under reflection.

### 5.4 Calculating the SFT action

Given the action (68) and the explicit formulae (83, 84) for the two- and three-string vertices, we can in principle calculate the string field action term by term for each of the fields in the string field expansion

$$\Psi = \int d^{26}p \left[ \phi(p) |01; p\rangle + A_\mu(p) \alpha_{-1}^\mu |01; p\rangle + \chi(p) b_{-1} c_0 |01; p\rangle + B_{\mu\nu}(p) \alpha_{-1}^\mu \alpha_{-1}^\nu |01; p\rangle + \cdots \right].$$

(90)

Since the resulting action has an enormous gauge invariance given by (59), it is often helpful to fix the gauge before computing the action. A particularly useful gauge choice is the Feynman-Siegel gauge

$$b_0 |\Psi\rangle = 0.$$

(91)
This is a good gauge choice locally, fixing the linear gauge transformations \( \delta |\Psi\rangle = Q |\Lambda\rangle \). This gauge choice is not, however, globally valid; we will return to this point later. In this gauge, all fields in the string field expansion which are associated with states having an antighost zero-mode \( c_0 \) are taken to vanish. For example, the field \( \chi(p) \) in (90) vanishes. In Feynman-Siegel gauge, the BRST operator takes the simple form

\[
Q = c_0 L_0 = c_0 (N + p^2 - 1)
\]  

(92)

where \( N \) is the total (matter + ghost) oscillator number.

Using (92), it is straightforward to write the quadratic terms in the string field action. They are

\[
\frac{1}{2} \langle V_2 |\Psi, Q\Psi \rangle = \int d^{26}p \left\{ \phi(-p) \left[ \frac{p^2 - 1}{2} \right] \phi(p) + A_\mu(-p) \left[ \frac{p^2}{2} \right] A^\mu(p) + \cdots \right\}.
\]  

(93)

The cubic part of the action can also be computed term by term, although the terms are somewhat more complicated. The leading terms in the cubic action are given by

\[
\frac{1}{3} \langle V_3 |\Psi, \Psi, \Psi \rangle = \int d^{26}p d^{26}q \frac{k g}{3} e^{(\ln 16/27) (p^2 + q^2 + p \cdot q)} \left\{ \phi(-p)\phi(-q)\phi(p + q) + \frac{16}{9} A_\mu(-p) A_\mu(-q) \phi(p + q) \right.
\]

\[
- \frac{8}{9} (p^\mu + 2 q^\mu)(2 p^\nu + q^\nu) A_\mu(-p) A_\nu(-q) \phi(p + q) + \cdots \right\}
\]  

\[
\int d^{26}p d^{26}q \frac{k g}{3} e^{(\ln 16/27) (p^2 + q^2 + p \cdot q)} \left\{ \phi(-p)\phi(-q)\phi(p + q) + \frac{16}{9} A_\mu(-p) A_\mu(-q) \phi(p + q) \right.
\]

(94)

In computing the \( \phi^3 \) term we have used

\[
V^{rs}_{00} = \delta^{rs} \ln \left( \frac{27}{16} \right)
\]  

(95)

The \( A^2 \phi \) term uses

\[
V^{rs}_{11} = -\frac{16}{27}, \quad r \neq s,
\]  

(96)

while the \( (A \cdot p)^2 \phi \) term uses

\[
V^{12}_{10} = -V^{13}_{10} = -\frac{2\sqrt{2}}{3\sqrt{3}}
\]  

(97)

The most striking feature of this action is that for a generic set of three fields, there is a nonlocal cubic interaction term, containing an exponential of a quadratic form in the momenta. This means that the target space formulation of string theory has a dramatically different character from a standard quantum field theory. From the point of view of quantum field theory, string field theory seems to contain an infinite number of nonrenormalizable interactions. Just like the simpler case of noncommutative field theories, however, the magic of string theory seems to combine this infinite set of interactions into a sensible model.

[Note, though, that we are working here with the bosonic theory, which becomes problematic quantum mechanically due to the closed string tachyon; the superstring should be better...]

28
behaved, although a complete understanding of superstring field theory is still lacking despite recent progress \cite{13,14}. For the purposes of the remainder of these lectures, however, it will be sufficient for us to restrict attention to the classical action at zero momentum, where the action is quite well-behaved.

5.5 General features of Witten’s open bosonic SFT

There are several important aspects of Witten’s open bosonic string field theory which are worth reviewing here, although they will not be central to the remainder of these lectures.

The first important aspect of this string field theory is that the perturbative on-shell amplitudes computed using this SFT are in precise agreement with the results of standard perturbative string theory (CFT). This result was shown by Giddings, Martinec, Witten, and Zwiebach in \cite{45,46,47}; the basic idea underlying this result is that in Feynman-Siegel gauge, the Feynman diagrams of SFT precisely cover the appropriate moduli space of open string diagrams of an arbitrary genus Riemann surface with boundaries, with the ghost factors contributing the correct measure. The essential feature of this construction is the replacement of the Feynman-Siegel gauge propagator $L_0^{-1}$ with a Schwinger parameter $t$.

$$\frac{1}{L_0} = \int_0^\infty dt \ e^{-tL_0}.$$  \hspace{1cm} (98)

The Schwinger parameter $t$ plays the role of a modular parameter measuring the length of the strip, for each propagator. This sews the string field theory diagram together into a Riemann surface for each choice of Schwinger parameters; the result of \cite{45,46,47} was to show that this parameterization always precisely covers the moduli space correctly. Thus, we know that to arbitrary orders in the string coupling the SFT perturbative expansion agrees with standard string perturbation theory, although string field theory goes beyond the conformal field theory approach since it is a nonperturbative, off-shell formulation of the theory.

A consequence of the perturbative agreement between SFT and standard perturbative string theory is that loop diagrams in open string field theory must include closed string poles at appropriate values of the external momenta. It is well-known that while closed string theory in a fixed space-time background (without D-branes) can be considered as a complete and self-contained theory without including open strings, the same is not true of open string theory. Open strings can always close up in virtual processes to form intermediate closed string states. The closed string poles were found explicitly in the one-loop 2-point function of open string field theory in \cite{48}. The appearance of these poles raises a very important question for open string field theory, namely: Can closed strings appear as asymptotic states in open string field theory? Indeed, standard arguments of unitarity would seem to imply that open string field theory cannot be consistent at the quantum level unless open strings can scatter into outgoing closed string states. This question becomes particularly significant in the context of Sen’s tachyon condensation conjectures, where we expect that all open
string degrees of freedom disappear from the theory in the nonperturbative locally stable vacuum. We will discuss this issue further in Section 8.

6 Evidence for the Sen conjectures

Now that we have a more concrete understanding of how to carry out calculations in open string field theory, we can address the conjectures made by Sen regarding tachyon condensation. In subsection 6.1, we discuss evidence for Sen’s first conjecture, which states that there exists a stable vacuum with energy density $-T_{25}$. In Subsection 6.2, we discuss physics in the stable vacuum and Sen’s third conjecture, which states that open strings decouple completely from the theory in this vacuum. There is also a large body of evidence by now for Sen’s second conjecture (see [49, 50, 51] for some of the early papers in this direction), but due to time and space constraints we will not cover this work here.4

6.1 Level truncation and the stable vacuum

Sen’s first conjecture states that the string field theory action should lead to a nontrivial vacuum solution, with energy density

$$-T_{25} = -\frac{1}{2\pi^2 g^2}. \quad (99)$$

In this subsection we discuss evidence for the validity of this conjecture.

The string field theory equation of motion is

$$Q\Psi + g\Psi \star \Psi = 0. \quad (100)$$

Despite much work over the last few years, there is still no analytic solution of this equation of motion. There is, however, a systematic approximation scheme, known as level truncation, which can be used to solve this equation numerically. The level $(L, I)$ truncation of the full string field theory involves dropping all fields at level $N > L$, and disregarding any cubic interaction terms between fields whose total level is greater than $I$. For example, the simplest truncation of the theory is the level $(0, 0)$ truncation. Including only $p = 0$ components of the tachyon field, with the justification that we are looking for a Lorentz-invariant vacuum, the theory in this truncation is simply described by a potential for the tachyon zero-mode

$$V(\phi) = -\frac{1}{2} \phi^2 + g\tilde{\kappa}\phi^3. \quad (101)$$

where $\tilde{\kappa} = \kappa/3 = 3^{7/2}/2^6$. This cubic function is graphed in Figure 4. Clearly, this potential has a local minimum at

$$\phi_0 = \frac{1}{3g\tilde{\kappa}}. \quad (102)$$

4A more extensive summary of this work will appear in 4
5as of January, 2003
At this point the potential is

\[ V(\phi_0) = -\frac{1}{54} \frac{1}{g^2 \kappa^2} = -\frac{2^{11}}{3^{10}} \frac{1}{g^2} \approx (0.68) \left( -\frac{1}{2 \pi^2 g^2} \right) \]  

(103)

Thus, we see that simply including the tachyon zero-mode gives a nontrivial vacuum with 68% of the vacuum energy density predicted by Sen. This vacuum is denoted by an open circle in Figure 4.

At higher levels of truncation, there are a multitude of fields with various tensor structures. However, again assuming that we are looking for a vacuum which preserves Lorentz symmetry, we can restrict attention to the interactions between scalar fields at \( p = 0 \). We will work in Feynman-Siegel gauge to simplify calculations. The situation is further simplified by the existence of the “twist” symmetry mentioned in Section 5.3, which guarantees that no cubic vertex between \( p = 0 \) scalar fields can connect three fields with a total level which is odd. This means that odd fields are not relevant to diagrams with only external tachyons at tree level. Thus, we need only consider even-level scalar fields in looking for Lorentz-preserving solutions to the SFT equations of motion. With these simplifications, in a general level truncation the string field is simply expressed as a sum of a finite number of terms

\[ \Psi_s = \sum_i \phi_i |s_i\rangle \]  

(104)

where \( \phi_i \) are the zero-modes of the scalar fields associated with even-level states \( |s_i\rangle \). For example, including fields up to level 2, we have

\[ \Psi_s = \phi |0_1\rangle + B (\alpha_{-1} \cdot \alpha_{-1}) |0_1\rangle + \beta b_{-1} c_{-1} |0_1\rangle . \]  

(105)
The potential for all the scalars appearing in the level-truncated expansion \((104)\) can be simply expressed as a cubic polynomial in the zero-modes of the scalar fields

\[
V = \sum_{i,j} d_{ij}\phi_i\phi_j + g\bar{\kappa} \sum_{i,j,k} t_{ijk}\phi_i\phi_j\phi_k .
\]  

Using the expressions for the Neumann coefficients given in Section 5.3, the potential for all the scalar fields up to level \(L\) can be computed in a level \((L, I)\) truncation. For example, the potential in the level \((2, 6)\) truncation is given by

\[
V = -\frac{1}{2}\phi^2 + 26B^2 - \frac{1}{2}\beta^2 + \bar{\kappa}g\left[\phi^3 - \frac{130}{9}\phi^2\beta - \frac{11}{9}\phi^2\beta + \frac{30212}{243}\phi B^2 + \frac{2860}{243}\phi B\beta + \frac{19}{81}\phi^2\beta\right. \\
\left. - \frac{2178904}{6561}B^3 - \frac{332332}{6561}B^2\beta - \frac{2470}{2187}B\beta^2 - \frac{1}{81}\beta^3\right] (107)
\]

As an example of how these terms arise, consider the \(\phi^2B\) term. The coefficient in this term is given by

\[
g \langle V_3|(|0_1\rangle \otimes |0_1\rangle \otimes \alpha_{-1} \cdot \alpha_{-1}|0_1\rangle) = -g\bar{\kappa}(3 \cdot 26) V_{11}^{11} = -g\bar{\kappa}\frac{130}{9}
\]

where we have used \(V_{11}^{11} = 5/27\).

In the level \((2, 6)\) truncation of the theory, with potential \((107)\), the nontrivial vacuum is found by simultaneously solving the three quadratic equations found by setting to zero the derivatives of \(107\) with respect to \(\phi, B,\) and \(\beta\). There are a number of different solutions to these equations, but only one is in the vicinity of \(\phi = 1/3g\bar{\kappa}\). The solution of interest is

\[
\phi \approx 0.39766 \frac{1}{g\bar{\kappa}} \\
B \approx 0.02045 \frac{1}{g\bar{\kappa}} \\
\beta \approx -0.13897 \frac{1}{g\bar{\kappa}} 
\]

Plugging these values into the potential gives

\[
E_{(2,6)} = -0.95938 T_{25} ,
\]

or 95.9% of the result predicted by Sen. This vacuum is denoted by an open circle in Figure 4.

It is a straightforward, although computationally intensive, project to generalize this calculation to higher levels of truncation. This calculation was carried out to level \((4, 8)\) by Kostelecky and Samuel \([33]\) many years ago. They noted that the vacuum seemed to be converging, but they lacked any physical picture giving meaning to this vacuum. Following Sen’s conjectures, the level \((4, 8)\) calculation was done again using somewhat different
methods by Sen and Zwiebach \cite{52}, who showed that the energy at this level is \(-0.986 \, T_{25}\). The calculation was automated by Moeller and Taylor \cite{53}, who calculated up to level \((10, 20)\), where there are 252 scalar fields. Up to this level, the vacuum energy converges monotonically, as shown in Table 1. These numerical calculations indicate that level truncation of string field theory leads to a good systematic approximation scheme for computing the nonperturbative tachyon vacuum \(^6\).

It is interesting to consider the tachyon condensation problem from the point of view of the effective tachyon potential. If instead of trying to solve the quadratic equations for all \(N\) of the fields appearing in \((106)\), we instead fix the tachyon field \(\phi\) and solve the quadratic equations for the remaining \(N - 1\) fields, we can determine an effective potential \(V(\phi)\) for the tachyon field. This was done numerically up to level \((10, 20)\) in \cite{53} \(^6\). At each level, the tachyon effective potential smoothly interpolates between the perturbative vacuum and the nonperturbative vacuum near \(\phi = 0.4/g\kappa\). For example, the tachyon effective potential at level \((2, 6)\) is graphed in Figure 4. In all level truncations other than \((0, 0)\) and \((2, 4)\), the tachyon effective potential has two branch point singularities at which the continuous solution for the other fields breaks down; for the level \((2, 6)\) truncation, these branch points occur at \(\phi \approx -0.127/g\kappa\) and \(\phi \approx 2.293/g\kappa\); the lower branch point is denoted by a solid circle in Figure 4. As a result of these branch points, the tachyon effective potential is only

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
level & \(g\kappa(\phi)\) & \(V/T_{25}\) \\
\hline
\(0, 0\) & 0.3333 & -0.68462 \\
\(2, 4\) & 0.3957 & -0.94855 \\
\(2, 6\) & 0.3977 & -0.95938 \\
\(4, 8\) & 0.4005 & -0.9840 \\
\(4, 12\) & 0.4007 & -0.98782 \\
\(6, 12\) & 0.4004 & -0.99514 \\
\(6, 18\) & 0.4004 & -0.99518 \\
\(8, 16\) & 0.3999 & -0.99777 \\
\(8, 20\) & 0.3997 & -0.99793 \\
\(10, 20\) & 0.3992 & -0.99912 \\
\hline
\end{tabular}
\caption{Tachyon VEV and vacuum energy in stable vacua of level-truncated theory}
\end{table}

\(^6\)These were the best values for the vacuum energy and effective potential at the time of the lectures. At strings 2002, Gaiotto and Rastelli reported results up to level \((18, 54)\) \cite{54}. They found the surprising result that while the energy monotonically approaches \(-T_{25}\) up to level 12, at level \((14, 42)\) the energy becomes \(-1.0002 \, T_{25}\), and that the energy continues to decrease, reaching \(-1.0005 \, T_{25}\) at level \((18, 54)\). In \cite{55}, it was shown that this calculation could be theoretically extrapolated to higher levels using the result found in \cite{56} that perturbative amplitudes converge in level truncation with errors described by a power series in \(1/L\). This extrapolation suggests that the energy turns around again near \(L = 28\), and then increases again, asymptotically approaching \(-T_{25}\) as \(L \to \infty\). Further analysis supporting this conclusion was given in \cite{57}, where the effective tachyon potential was extrapolated to higher order using results calculated up to level 18.
valid for a finite range of $\phi$, ranging between approximately $-0.1/g\bar{\kappa}$ and $0.6/g\bar{\kappa}$. In \[58\] it was demonstrated numerically that the branch points in the tachyon effective potential arise because the trajectory in field space associated with this potential encounters the boundary of the region of Feynman-Siegel gauge validity. As mentioned earlier, Feynman-Siegel gauge is only valid in a finite-size region around the perturbative vacuum. It seems almost to be a fortunate accident that the nonperturbative vacuum lies within the region of validity of this gauge choice. It is also worth mentioning here that in the “background-independent” formulation of SFT, the tachyon potential can be computed exactly \[59\]. In this formulation, there is no branch point in the effective potential, which is unbounded below for negative values of the tachyon. On the other hand, the nontrivial vacuum in the background-independent approach arises only as the tachyon field goes to infinity, so it is harder to study the physics of the stable vacuum from this point of view.

Another interesting perspective on the tachyon effective potential is found by performing a perturbative computation of the coefficients in this effective potential in the level-truncated theory. This gives a power series expansion of the effective tachyon potential

$$V(\phi) = \sum_{n=2}^{\infty} c_n (\bar{\kappa} g)^{n-2} \phi^n$$

$$= -\frac{1}{2} \phi^2 + (\bar{\kappa} g) \phi^3 + c_4 (\bar{\kappa} g)^2 \phi^4 + c_5 (\bar{\kappa} g)^3 \phi^5 + \cdots$$

In \[53\], the coefficients up to $c_{60}$ were computed in the level truncations up to (10, 20). Because of the branch point singularity near $\phi = -0.1/g\bar{\kappa}$, this series has a radius of convergence much smaller than the value of $\phi$ at the nonperturbative vacuum. Thus, the energy at the stable vacuum lies outside the naive range of perturbation theory\[7\].

### 6.2 Physics in the stable vacuum

We have seen that numerical results from level-truncated string field theory strongly suggest the existence of a classically stable vacuum solution $\Psi_0$ to the string field theory equation of motion \[100\]. The state $\Psi_0$, while still unknown analytically, has been determined numerically to a high degree of precision. This state seems like a very well-behaved string field configuration. While there is no positive-definite inner product on the string field Fock space, the state $\Psi_0$ certainly has finite norm under the natural inner product $\langle V_2 | \Psi_0, c_0 L_0 \Psi_0 \rangle$, and is even better behaved under the product $\langle V_2 | \Psi_0, c_0 \Psi_0 \rangle$. Thus, it is natural to assume that $\Psi_0$ defines a classically stable vacuum for the theory, around which we can expand the action to find a new “vacuum string field theory”. Expanding

$$\Psi = \Psi_0 + \tilde{\Psi},$$

\[112\]  

\[7\]In \[55\], however, it was shown that the method of Padé approximants enables us to compute the vacuum energy to excellent precision given a reasonably small number of the coefficients $c_n$. Thus, the stable vacuum is in some sense accessible from purely perturbative calculations.
we get the action

$$S(\Psi) = S(\Psi_0 + \tilde{\Psi}) = S_0 - \frac{1}{2} \int \tilde{\Psi} \ast \tilde{Q} \tilde{\Psi} - \frac{g}{3} \int \tilde{\Psi} \ast \tilde{\Psi} \ast \tilde{\Psi}. \quad (113)$$

where

$$\tilde{Q} \Phi = Q \Phi + g(\Psi_0 \ast \Phi + \Phi \ast \Psi_0). \quad (114)$$

This string field theory around the stable vacuum has precisely the same form as Witten’s original cubic string field theory, only with a different BRST operator $\tilde{Q}$, which so far is only determined numerically. Note that this formulation of vacuum string field theory is distinct from the VSFT model of Rastelli, Sen, and Zwiebach (RSZ) [60]. These authors make an Ansatz that the BRST operator takes a pure ghost form, along the lines of $Q \to c_0$, and they conjecture that the theory with such a BRST operator is equivalent to the VSFT model given by the BRST operator (114). We discuss the RSZ model again briefly in the next section.

Sen’s third conjecture states that there should be no open string excitations of the theory around $\Psi = \Psi_0$. This implies that there should be no solutions of the linearized equation $\tilde{Q} \tilde{\Psi}$ in the VSFT (113) other than pure gauge states of the form $\tilde{\Psi} = \tilde{Q} \Lambda$. In this subsection we discuss evidence for this conjecture.

It may seem surprising to imagine that all the perturbative open string degrees of freedom will vanish at a particular point in field space, since these are all the degrees of freedom available in the theory. This is not a familiar phenomenon from quantum field theory. To understand how the open strings can decouple, it may be helpful to begin by considering the simple example of the $(0, 0)$ level-truncated theory. In this theory, the quadratic terms in the action become

$$- \int d^{26} p \phi(-p) \left[ \frac{p^2 - 1}{2} + g \bar{\kappa} \left( \frac{16}{27} \right) p^2 \cdot 3 \langle \phi \rangle \right] \phi(p). \quad (115)$$

Taking $\langle \phi \rangle = 1/3\bar{\kappa}g$, we find that the quadratic term is a transcendental expression which does not vanish for any real value of $p^2$. Thus, this theory has no poles, and the tachyon has decoupled from the theory. Of course, this is not the full story, as there are still finite complex poles. It does, however suggest a mechanism by which the nonlocal parts of the action (encoded in the exponential of $p^2$) can remove physical poles.

To get the full story, it is necessary to continue the analysis to higher level. At level 2, there are 7 scalar fields, the tachyon and the 6 fields associated with the Fock space states

$$(\alpha_{-1} \cdot \alpha_{-1}) |0_1, p\rangle \quad b_{-1} \cdot c_{-1} |0_1, p\rangle$$
$$c_0 \cdot b_{-1} |0_1, p\rangle \quad (p \cdot \alpha_{-2}) |0_1, p\rangle$$
$$(p \cdot \alpha_{-1})^2 |0_1, p\rangle \quad (p \cdot \alpha_{-1}) c_0 b_1 |0_1, p\rangle \quad (116)$$

Note that in this analysis we cannot fix Feynman-Siegel gauge, as we only believe that this gauge is valid for the zero-modes of the scalar fields in the vacuum $\Psi_0$. An attempt at
analyzing the spectrum of the theory in Feynman-Siegel gauge using level truncation was made in [33], with no sensible results. Diagonalizing the quadratic term in the action on the full set of 7 fields of level $\leq 2$, we find that poles develop at $M^2 = 0.9$ and $M^2 = 2.0$ (in string units, where the tachyon has $M^2 = -1$) [61]. These poles correspond to states satisfying $\tilde{Q}\tilde{\Psi} = 0$. The question now is, are these states physical? If they are exact states, of the form $\tilde{\Psi} = \tilde{Q}\tilde{\Lambda}$, then they are simply gauge degrees of freedom. If not, however, then they are states in the cohomology of $\tilde{Q}$ and should be associated with physical degrees of freedom. Unfortunately, we cannot precisely determine whether the poles we find in level truncation are due to exact states, as the level-truncation procedure breaks the condition $\tilde{Q}^2 = 0$. Thus, we can only measure approximately whether a state is exact. A detailed analysis of this question was carried out in [61]. In this paper, all terms in the SFT action of the form $\phi_i \psi_j(p) \psi_k(-p)$ were determined, where $\phi_i$ is a scalar zero-mode, and $\psi_{j,k}$ are nonzero-momentum scalars. In addition, all gauge transformations involving at least one zero-momentum field were computed up to level $(6, 12)$. At each level up to $L = 6$, the ghost number 1 states in the kernel $\text{Ker} \tilde{Q}^{(1)}_{(L,2L)}$ were computed. The extent to which each of these states lies in the exact subspace was measured using the formula

$$\% \text{ exactness} = \sum_i \frac{(s \cdot e_i)^2}{(s \cdot s)}$$

(117)

where $\{e_i\}$ are an orthonormal basis for $\text{Im} \tilde{Q}^{(0)}_{(L,2L)}$, the image of $\tilde{Q}$ acting on the space of ghost number 0 states in the appropriate level truncation. (Note that this measure involves a choice of inner product on the Fock space; several natural inner products were tried, giving roughly equivalent results). The result of this analysis was that up to the mass scale of the level truncation, $M^2 \leq L - 1$, all the states in the kernel of $\tilde{Q}^{(1)}$ were $\geq 99.9\%$ within the exact subspace, for $L \geq 4$. This result seems to give very strong evidence for Sen’s third conjecture that there are no perturbative open string excitations around the stable classical vacuum $\Psi_0$. This analysis was only carried out for even level scalar fields; it would be nice to check that a similar result holds for odd-level fields and for tensor fields of arbitrary rank.

Another more abstract argument that there are no open string states in the stable vacuum was given by Ellwood, Feng, He and Moeller [62]. These authors argued that in the stable vacuum, the identity state $|I\rangle$ in the SFT star algebra, which satisfies $I \star A = A$ for a very general class of string fields $A$, seems to be an exact state,

$$|I\rangle = \tilde{Q}|\Lambda\rangle.$$  

(118)

If indeed the identity is exact, then it follows immediately that the cohomology of $\tilde{Q}$ is empty, since $\tilde{Q}A = 0$ then implies that

$$A = (\tilde{Q}\Lambda) \star A$$

$$= \tilde{Q}(\Lambda \star A) - \Lambda \star \tilde{Q}A$$

$$= \tilde{Q}(\Lambda \star A).$$

(119)
So to prove that the cohomology of $\tilde{Q}$ is trivial, it suffices to show that $\tilde{Q}|\Lambda\rangle = |I\rangle$. While there are some subtleties involved with the identity string field, Ellwood et al. found a very elegant expression for this field,

$$|I\rangle = (\cdots e^{\frac{1}{8}L-16}e^{\frac{1}{4}L-8}e^{\frac{1}{2}L-4})e^{L-2}|0\rangle. \quad (120)$$

(Recall that $|0\rangle = b_{-1}|0_{1}\rangle$. ) They then looked numerically for a state $|\Lambda\rangle$ satisfying (118). For example, truncating at level $L = 3$,

$$|I\rangle = |0\rangle + L_{-2}|0\rangle + \cdots \quad (121)$$

$$= |0\rangle - b_{-3}c_{1}|0\rangle - 2b_{-2}c_{0}|0\rangle + \frac{1}{2}(\alpha_{-1} \cdot \alpha_{-1})|0\rangle + \cdots$$

while the only candidate for $|\Lambda\rangle$ is

$$|\Lambda\rangle = \alpha b_{-2}|0\rangle, \quad (122)$$

for some constant $\alpha$. The authors of [62] showed that the state (121) is best approximated as exact when $\alpha \sim 1.12$; for this value, their measure of exactness becomes

$$\frac{|\tilde{Q}|\Lambda\rangle - |I\rangle}{|I\rangle} \rightarrow 0.17, \quad (123)$$

which the authors interpreted as a 17% deviation from exactness. Generalizing this analysis to higher levels, they found at levels 5, 7, and 9, a deviation from exactness of 11%, 4.5% and 3.5% respectively. At level 9, for example, the identity field has 118 components, and there are only 43 gauge parameters, so this is a highly nontrivial check on the exactness of the identity. Like the results of [61], these results strongly support the conclusion that the cohomology of the theory is trivial in the stable vacuum. In this case, the result applies to fields of all spins and all ghost numbers.

Given that the Witten string field theory seems to have a classical solution with no perturbative open string excitations, in accordance with Sen’s conjectures, it is quite interesting to ask what the physics of the vacuum string field theory (113) should describe. One natural assumption might be that this theory should include closed string states in its quantum spectrum. Unfortunately, addressing this question requires performing calculations in the quantum theory around the stable vacuum. Such calculations are quite difficult (although progress in this direction has been made in the $p$-adic version of the theory [63]). Even in the perturbative vacuum, it is difficult to systematically study closed strings in the quantum string from theory. We discuss the question again briefly in the final section.

### 7 Further developments

In this section we review briefly some further developments which we do not have time to explore in great detail in these lectures. In Subsection 7.1 we discuss the pure ghost BRST
operator Ansatz of RSZ (Rastelli, Sen, and Zwiebach) for vacuum string field theory. In Subsection 7.2 we discuss “sliver” states and related states; these states are projectors in the SFT star algebra, and are closely related to D-branes in the RSZ VSFT model. These topics will be discussed in further detail in [4].

7.1 The vacuum string field theory model of RSZ

In [60], Rastelli, Sen, and Zwiebach made an intriguing proposal regarding the form of Witten’s string field theory around the stable tachyon vacuum. Since the exact form of the BRST operator \( \tilde{Q} \) given by (114) is not known analytically, and is difficult to work with numerically, these authors suggested that it might be possible to “guess” an appropriate form for this operator (after suitable field redefinition), using the properties expected of the BRST operator in any vacuum. They suggested a simple class of BRST operators \( \hat{Q} \) which satisfy the properties (a-c) described in Section 4.2 (actually, they impose the slightly weaker but still sufficient condition \( \int (\hat{Q}\Psi \star \Phi + (-1)^{G\Psi}\Psi \star \hat{Q}\Phi) \) instead of condition (b)). In particular, they propose that after a field redefinition, the BRST operator of the string field theory in the classically stable vacuum should be an operator \( \hat{Q} \) expressable purely in terms of ghost operators. For example, the simplest operator in the class they suggest is \( \hat{Q} = c_0 \), which clearly satisfies \( \hat{Q}^2 \), and which also satisfies condition (c) and the weaker form of condition (b) mentioned above.

The RSZ model of vacuum string field theory has a number of attractive features.

- This model satisfies all the axioms of string field theory, and has a BRST operator with vanishing cohomology.

- In the RSZ model, the equation of motion factorizes into the usual equation of motion

\[
\hat{Q}\Psi_{\text{ghost}} + g\Psi_{\text{ghost}} \star \Psi_{\text{ghost}} = 0
\]  

(124)

for the ghost part of the field, and a projection equation

\[
\Psi_{\text{matter}} = \Psi_{\text{matter}} \star \Psi_{\text{matter}}
\]  

(125)

for the matter part of the field, where the full string field is given by

\[
\Psi = \Psi_{\text{ghost}} \otimes \Psi_{\text{matter}}.
\]  

(126)

Thus, finding a solution of the equation of motion reduces to the problem of solving the equation of motion in the ghost sector and identifying projection operators in the string field star algebra. It was also recently shown [64, 65, 66] that by taking the BRST operator \( \hat{Q} \) to be given by a ghost insertion localized at the string midpoint, the ghost equation also has essentially the form of the projection equation. Thus, this seems to be a very natural choice for the BRST operator of the RSZ model.
A number of projection operators have been identified in the string field star algebra. These projection operators have many of the properties desired of D-branes. We will briefly review some aspects of these projection operators in the next subsection.

Given the projection operators just mentioned, the ratio of tensions between D-branes of different dimensionality can be computed and has the correct value \[ 67^8. \]

Despite the successes of the RSZ model, there are some difficult technical aspects of this picture. First, it seems very difficult to actually prove that this model is related to the VSFT around the stable vacuum in the Witten model, not least because we lack any analytic control over the Witten theory. Second, the RSZ model seems to have a somewhat singular structure in several respects. Formally, the action on any well-behaved Fock space state satisfying the equation of motion will vanish \[ 70. \] Further, the natural solutions of the projection equation corresponding to the matter sector of the equation of motion have rather singular properties \[ 71. \] Some of these singular properties are related to the fact that some of the physics in the RSZ model seems to have been “pushed” into the midpoint of the string. In the Witten model, the condition that, for example, \( Q^2 = 0 \) involves a fairly subtle anomaly cancellation between the matter and ghost sectors at the midpoint. In the RSZ model, the matter and ghost sectors are essentially decoupled, so that the theory seems to have separate singularities in each sector, which cancel when the sectors are combined. These are all indications of a theory with problematic singularities. While the Witten theory seems to be free of singularities of this type, it remains to be seen whether resolving the singularities of the RSZ model or finding an analytic approach to the Witten theory will be a more difficult problem to solve.

### 7.2 Projection operators in SFT

From the point of view of the RSZ model of VSFT just discussed, projection operators in the matter sector of the star algebra play a crucial role in constructing solutions of the equations of motion. Such projection operators may also be useful in understanding solutions in the original Witten theory. Quite a bit of work has been done on constructing and analyzing projectors in the star algebra since the RSZ model was originally proposed. Without going into the technical details, we now briefly review some of the important features of matter projectors.

The first matter projector which was explicitly constructed is the “sliver” state. This state was identified in conformal field theory in \[ 72, \] and then constructed explicitly using matter oscillators in \[ 73. \] The sliver state takes the form of a squeezed state

\[
\exp \left[ \frac{1}{2} a^\dagger \cdot S \cdot a^\dagger \right] |0\rangle.
\]

This result was known at the time of the lectures. There was quite a bit of recent work on the problem of computing the exact D-brane tension \[ 68. \] A very nice recent paper by Okawa \[ 69 \] resolved the question and demonstrated that not only the ratio of tensions, but also the tension of an individual brane, is correctly reproduced in the RSZ VSFT theory when singularities are correctly controlled.
By requiring that such a state satisfy the projection equation $\Psi \star \Psi = \Psi$, and by making some further assumptions about the nature of the state, an explicit formula for the matrix $S$ was found in [73].

Projectors like the sliver have many properties which are reminiscent of D-branes. This relationship between projection operators and D-branes is familiar from noncommutative field theory, where projectors also play the role of D-brane solitons [74] (for a review of noncommutative field theory, see [14]). In the RSZ model, by tensoring an arbitrary matter projector with a fixed ghost state satisfying the ghost equation of motion (124), states corresponding to an arbitrary configuration of D-branes can be constructed. Particular projectors like the sliver can be constructed which are localized in any number of space-time dimensions, corresponding to the codimension of a D-brane. Under gauge transformations, a rank 1 projector can be rotated into an orthogonal rank 1 projector, so that configurations containing multiple branes can be constructed as higher rank projectors formed from the sum of orthogonal rank one projectors [75, 77]. This gives a very suggestive picture of how arbitrary D-brane configurations can be constructed in string field theory. While this picture is quite compelling, however, there are a number of technical obstacles which make this still a somewhat incomplete story. As mentioned above, in the RSZ model, many singularities appear due to the separation of the matter and ghost sectors. In the context of the matter projectors, these singularities manifest as singular properties of the projectors. For example, the sliver state described above has a matrix $S$ which has eigenvalues of $\pm 1$ for any dimension of D-brane [71, 78]. Such eigenvalues cause the state to be nonnormalizable elements of the matter Fock space. In the Dirichlet directions, this lack of normalizability occurs because the state is essentially localized to a point and is analogous to a delta function. In the Neumann directions, the singularity manifests as a “breaking” of the strings composing the D-brane, so that the functional describing the projector state is a product of a function of the string configurations on the left and right halves of the string, with no connection mediated through the midpoint. These geometric singularities seem to be generic features of matter projectors, not just of the sliver state [79, 78]. These singular geometric features are one of the things which makes direct calculation in the RSZ model somewhat complicated, as all these singularities must be sensibly regulated. These singularities do not seem to appear in the Witten theory, where the BRST operator and numerically calculated solutions seem to behave smoothly at the string midpoint. On the other hand, it may be that further study of the matter projection operators and their cousins in the ghost sector which satisfy (124) will lead to analytic progress on the Witten theory.

8 Conclusions and open problems

The work described in these lectures has brought our understanding of string field theory to a new level. We now have fairly conclusive evidence that open string field theory can successfully describe distinct vacua with very different geometrical properties, which are not
related to one another through a marginal deformation. The resulting picture, in which a complicated set of degrees of freedom defined primarily through an algebraic structure, can produce different geometrical backgrounds at different solutions of the equations of motion, represents an important step beyond perturbative string theory. Such an approach, where different backgrounds with different low-energy degrees of freedom arise from a single underlying formalism, is clearly necessary to discuss questions of a cosmological nature in string theory. It is clearly essential, however, to generalize from the work described here in which the theory describes distinct open string backgrounds, to a formalism where different closed string backgrounds appear as solutions to an equation of motion for a single set of degrees of freedom.

Clearly, it is an important goal to have a formulation of string/M-theory in which all the currently understood vacua can arise in terms of a single well-defined set of degrees of freedom. It is not yet clear, however, how far it is possible go towards this goal using the current formulations of string field theory. It may be that the correct lesson to take from the work described here is simply that there are nonperturbative formulations in which distinct vacua can be brought together as solutions of a single classical theory, and that one should search for some deeper fundamental algebraic formulation where geometry, and even the dimension of space-time emerge from the fundamental degrees of freedom in the same way that D-brane geometry emerges from the degrees of freedom of Witten’s open string field theory. A more conservative scenario, however, might be that we could perhaps use the current framework of string field theory, or some limited refinement thereof, to achieve this goal of providing a universal nonperturbative definition of string theory and M-theory. Following this latter scenario, we propose here a series of questions aimed at continuing the recent developments in open string field theory as far as possible towards this ultimate goal. It is not certain that this research program can be carried to its conclusion, but it will be very interesting to see how far open string field theory can go in reproducing important nonperturbative aspects of string theory.

Some open problems:

1) The first important unsolved problem in this area is to find an analytic description of the tachyonic vacuum. Despite several years of work on this problem, great success with numerical approximations, and some insight from the RSZ vacuum string field theory model, we still have no good analytic understanding of the difference between the D-brane vacuum and the empty vacuum in Witten’s open cubic string field theory. It seems almost unbelievable that there is not some elegant analytic solution to this problem. Finding such an analytic solution would almost certainly greatly enhance our understanding of this theory, and would probably lead to other significant advances.

2) Another interesting and important unsolved problem is to find, either analytically or numerically, a solution of the Witten theory describing two D25-branes. If open string field theory is truly a background-independent theory, at least in the open string sense,
it should be just as feasible to go from a vacuum with one D-brane to a vacuum with two D-branes as it is to go from a vacuum with one D-brane to the empty vacuum (or from the vacuum with two D-branes to the vacuum with one D-brane, which is essentially the same problem as going from one to none). Despite some work on this problem [80], there is as yet no evidence that a double D-brane solution exists for the Witten theory on a single D-brane. Several approaches which have been tried (and will be described in more detail in [80]) include: i) following a positive mass field upward, looking for a stable point; this method seems to fail because of gauge-fixing problems—the effective potential often develops a singularity before reaching the energy $+T_{25}$, ii) following the intuition of the RSZ model and constructing a gauge transform of the original D-brane solution which is $\ast$−orthogonal to the original D-brane vacuum. It can be shown formally that such a state, when added to the original D-brane vacuum gives a new solution with the correct energy for a double D-brane; unfortunately, however, we have been unable to identify such a state numerically in level truncation.

There are several other problems closely related to the double D-brane problem. One related problem is the problem of studying a D0-brane lump solution from the tachyon field on a D1-brane wrapped on a small circle. When the circle is sufficiently small, the mass of the D0-brane is larger than that of the wrapped D1-brane. In this case, it seems much more difficult to construct the D0-brane lump solution than it is when the D0-brane has mass smaller than the D1-brane [81]. Another possibly related problem is the problem of translating a single D-brane of less than maximal dimension in a transverse direction. It was shown by Sen and Zwiebach [82] (in a T-dual picture) that after moving a D-brane a finite distance of order of the string length in a transverse direction, the level-truncated string field theory equations develop a singularity. Thus, in level truncation it does not seem possible to move a D-brane a macroscopic distance in a transverse direction. In this case, a toy model [84] suggests that the problem is that the infinitesimal marginal parameter for the brane translation ceases to parameterize the marginal trajectory in field space after a finite distance, just as the coordinate $x$ ceases to parameterize the circle $x^2 + y^2 = 1$ near $x = 1$. This is similar in spirit to the breakdown of Feynman-Siegel gauge along the tachyon potential discussed in section 6.1.

To show that open string field theory is sufficiently general to address arbitrary questions involving different vacua, it is clearly necessary to show that the formalism is powerful enough to describe multiple brane vacua, the D0-brane lump on an arbitrary radius circle, and translated brane vacua. It is currently unclear whether the obstacles to finding these vacua are conceptual or technical. It may be that the level-truncation approach is not well-suited to finding these vacua. If this is true, however, we may need a clearer mathematical formalism for describing the theory. There is currently

\[^9\text{although this can be done formally [83], it is unclear how the formal solution relates to an explicit expression in the oscillator language}\]
some ambiguity in the definition of the theory, in terms of precisely which states are allowed in the string field. Level-truncation in some sense gives a regularization of, and a concrete definition to, the theory. Without level truncation, we would need some more definitive mathematical tools for analyzing various features of the theory, such as the other vacua mentioned here.

3) Another open question involves the role that closed strings play in open string field theory. As has been known since the earliest days of the subject, closed strings appear as poles in perturbative open string scattering amplitudes. This was shown explicitly for Witten’s SFT in [48], where it was shown that closed string poles arise in the one-loop 2-point function. If Witten’s theory is well-defined as a quantum theory, it would follow from unitarity that the closed string states should also arise in some natural sense as asymptotic states of the quantum open string field theory. It is currently rather unclear, however, whether, and if so how, this might be realized. There are subtleties in the quantum formulation of the theory which have never completely been resolved [35]. Both older SFT literature [85, 86] and recent work [87, 71, 65, 88, 89] have suggested ways in which closed strings might be incorporated into the open string field theory formalism, but a definitive resolution of this question is still not available. If it is indeed possible to encode closed string degrees of freedom in some way in the quantum open string field theory, it suggests that one could use the Witten formalism in principle to not only compute general closed string scattering amplitudes, but perhaps even to address questions of closed string vacua. This is clearly an optimistic scenario, but one can imagine that the open string theory might really contain all of closed string physics as well as open string physics. This scenario is perhaps not so farfetched, as it really represents simply a lifting to the level of string field theory of the AdS/CFT story, where the massive as well as the massless modes are included. Furthermore, the fact that, as discussed in Section 5.5, the open string diagrams precisely cover the moduli space of Riemann surfaces with an arbitrary number of handles (and at least one boundary), suggests that by shrinking the boundaries to closed strings, one might neatly describe all perturbative closed string amplitudes in the open string language. On the other hand, it seems quite possible that the closed string sector of the theory is encoded in a singular fashion (like the encoding of the D-brane in the RSZ VSFT model), so that extracting the closed string physics from the open string field theory may involve such complicated manipulations that one is better off directly working with a closed string formalism. It would certainly be nice to have a clearer picture of how far one can go in this direction purely from the open string point of view.

4) Another obvious, but crucial, question is how this whole story can be generalized to superstrings. The naive Witten cubic superstring field theory has technical problems arising from contact terms between picture-changing operators [90, 91]. It has been suggested that these problems can be resolved directly in the cubic theory [43].
Berkovits has also suggested a new non-polynomial string field theory framework which seems to deal successfully with the contact term problem, at least in the NS-NS sector \[44\]. Some preliminary work indicates that numerical calculations on the tachyon condensation problem for the open superstring can be carried out in the Berkovits model with analogous results to those described here for the bosonic open string, although the results to date for the superstring are much more limited \[92\]. It would be nice to have a more complete picture for the superstring, and some sense of how issues like the closed string question would work in the supersymmetric framework.

5) Perhaps the most important lesson we have learned from the body of work discussed in these lectures is that open string field theory is a consistent framework in which geometrically distinct open string backgrounds can arise as classical solutions of a single theory. A fundamental outstanding problem in string theory is to find a framework in which different closed string backgrounds arise in a similar fashion from some fixed set of degrees of freedom within a single well-defined theory. In principle, we would hope that all the different closed string backgrounds would arise as solutions of the equations of motion for the fundamental underlying degrees of freedom of string field theory, either by incorporating closed strings into the open string field theory framework as described above, or by working directly in some formulation of closed string field theory. It is quite challenging to imagine a single set of degrees of freedom which would encode, in different phases, all the possible string backgrounds we are familiar with. A particularly pressing case is that of M-theory. In principle, a nonperturbative background-independent formulation of type II string theory should allow one to take the string coupling to infinity in such a way that the fundamental degrees of freedom of the theory are still actually at some finite point in their configuration space in the limit. This would lead to the vacuum associated with M-theory in flat space-time. It would be quite remarkable if this can be achieved in the framework of string field theory. Given the nontrivial relationship between string fields and low-energy effective degrees of freedom, however, such a result cannot be ruled out. If this picture could be successfully implemented, it would give a very satisfying understanding of how the complicated network of dualities of string and M-theory could be represented in terms of a single underlying set of degrees of freedom.

**Acknowledgments**

I would like to thank CECS and the organizers of the School on Quantum Gravity for their support and hospitality, and for an extremely enjoyable summer school experience. Thanks to Erasmo Coletti, Ian Ellwood, David Gross, Nicolas Moeller, Joe Minahan, Greg Moore, Leonardo Rastelli, Martin Schnabl, Ashoke Sen, Jessie Shelton, Ilya Sigalov, and Barton Zwiebach, for many discussions and collaborations which provided the material for these lectures. Thanks also to TASI ’01, where some of this material was presented prior to this
School. Particular thanks to Barton Zwiebach for suggestions and contributions to these lecture notes, which have substantial overlap with a more extensive set of lecture notes based on lectures by Zwiebach and myself at TASI ’01, which will appear presently. This work was supported by the DOE through contract #DE-FC02-94ER40818.

References

[1] K. Ohmori, “A review on tachyon condensation in open string field theories,” hep-th/0102085.

[2] P. J. De Smet, “Tachyon condensation: Calculations in string field theory,” hep-th/0109182.

[3] I. Y. Aref’eva, D. M. Belov, A. A. Giryavets, A. S. Koshelev and P. B. Medvedev, “Noncommutative field theories and (super)string field theories,” hep-th/0111208.

[4] W. Taylor and B. Zwiebach, “TASI ’01 lectures on D-branes, tachyons, and string field theory,” to appear.

[5] Hull, C. M., and P. K. Townsend, “Unity of superstring dualities,” Nucl. Phys. B 438, 109, 1995; hep-th/9410167.

[6] Witten, E. “String Theory Dynamics in Various Dimensions,” Nucl. Phys. B 443; 85 1995; hep-th/9503124.

[7] Townsend, P. K., “The eleven-dimensional supermembrane revisited,” Phys. Lett. B 350, 184, 1995; hep-th/9501068.

[8] Hořava, P. and E. Witten, “Heterotic and type I string dynamics from eleven dimensions,” Nucl. Phys. B 460, 506, 1996; hep-th/9510209.

[9] W. Taylor, “M(atrix) theory: matrix quantum mechanics as a fundamental theory,” Rev. Mod. Phys. 73, 419, 2001; hep-th/0101126.

[10] Aharony, O., S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323, 183, 2000; hep-th/9905111.

[11] G. Veneziano, “Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories,” Nuovo Cim. 57A 190 (1968).

[12] D. J. Gross, A. Neveu, J. Scherk and J. H. Schwarz, “Renormalization and unitarity in the dual resonance model,” Phys. Rev. D2 (1970) 697.

[13] C. Lovelace, “Pomeron form factors and dual Regge cuts,” Phys. Lett. B34 (1971) 500.
14. M. R. Douglas and N. A. Nekrasov, “Noncommutative field theory,” Rev. Mod. Phys. 73, 977 (2001), hep-th/0106048.

15. J. Polchinski, “TASI Lectures on D-branes,” hep-th/9611050.

16. W. Taylor, “Trieste lectures on D-branes, gauge theory and M(atrices),” hep-th/9801182.

17. M. J. Duff, R. R. Khuri and J. X. Lu, “String solitons,” Phys. Rept. 259, 213 (1995), hep-th/9412184.

18. E. Witten, “Non-commutative geometry and string field theory,” Nucl. Phys. B268, 253, (1986).

19. A. Sen, “Universality of the tachyon potential,” JHEP 9912, 027 (1999), hep-th/9911116.

20. J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges,” Phys. Rev. Lett. 75 (1995), 4724, hep-th/9510017.

21. R. G. Leigh, “Dirac-Born-Infeld action from Dirichlet sigma model,” Mod. Phys. Lett. A4 (1989), 2767.

22. E. Witten, “Bound States of Strings and p-Branes,” Nucl. Phys. B460 (1996), 335, hep-th/9510135.

23. A. A. Tseytlin, “Born-Infeld action, supersymmetry and string theory,” hep-th/9908105.

P. Koerber and A. Sevrin, “The non-abelian D-brane effective action through order alpha’**4,” JHEP 0210, 046 (2002), hep-th/0208044.

S. Stieberger and T. R. Taylor, “Non-Abelian Born-Infeld action and type I - heterotic duality. II: Nonrenormalization theorems,” Nucl. Phys. B 648, 3 (2003), hep-th/0209064.

D. T. Grasso, “Higher order contributions to the effective action of N = 4 super Yang-Mills,” JHEP 0211, 012 (2002), hep-th/0210146.

24. W. Taylor and M. Van Raamsdonk, “Multiple D0-branes in weakly curved backgrounds,” Phys. Lett. B558 (1999), 63, hep-th/9904095.

“Multiple Dp-branes in weak background fields,” Nucl. Phys. B573, 703 (2000), hep-th/9910052.

25. R. C. Myers, “Dielectric-branes,” JHEP 9912 (1999), 022, hep-th/9910053.

26. Polchinski, J. String theory (Cambridge University Press, Cambridge, England, 1998).
[27] W. Taylor, “D-brane Field Theory on Compact Spaces,” Phys. Lett. B394 (1997), 283; hep-th/9611042

[28] T. Banks and L. Susskind, “Brane - Antibrane Forces,” hep-th/9511194

[29] A. Hashimoto and W. Taylor, “Fluctuation Spectra of Tilted and Intersecting D-branes from the Born-Infeld Action,” Nucl. Phys. B503 (1997), 193-219; hep-th/9703217

[30] E. Gava, K. S. Narain and M. H. Sarmadi, “On the bound states of p- and (p+2)-branes,” Nucl. Phys. B 504, 214 (1997), hep-th/9704006.

[31] Green, M. B., J. H. Schwarz, and E. Witten, Superstring theory (Cambridge University Press, Cambridge, England, 1987).

[32] K. Bardakci, “Spontaneous symmetry breaking in the standard dual string model,” Nucl. Phys. B133 (1978), 297.

[33] V. A. Kostelecky and S. Samuel, “On a nonperturbative vacuum for the open bosonic string,” Nucl. Phys. B336 (1990), 263-296.

[34] A. Leclair, M. E. Peskin and C. R. Preitschopf, “String field theory on the conformal plane (I),” Nucl. Phys. B317 (1989), 411-463.

[35] C. Thorn, “String field theory,” Phys. Rep. 175 (1989), 1.

[36] M. R. Gaberdiel and B. Zwiebach, “Tensor constructions of open string theories 1., 2.,” Nucl. Phys. B505 (1997), 569, hep-th/9705038; Phys. Lett. B410 (1997), 151, hep-th/9707051

[37] D. J. Gross and A. Jevicki, “Operator formulation of interacting string field theory (I), (II),” Nucl. Phys. B283 (1987), 1; Nucl. Phys. B287 (1987), 225.

[38] E. Cremmer, A. Schwimmer and C. Thorn, “The vertex function in Witten’s formulation of string field theory” Phys. Lett. B179 57 (1986).

[39] S. Samuel, “The physical and ghost vertices in Witten’s string field theory,” Phys. Lett. B181 255 (1986).

[40] N. Ohta, “Covariant interacting string field theory in the Fock space representation,” Phys. Rev. D34 (1986), 3785; Phys. Rev. D35 (1987), 2627 (E).

[41] J. Shelton, to appear.

[42] I. Bars, “MSFT: Moyal star formulation of string field theory,” hep-th/0211238
[43] I. Y. Aref’eva, A. S. Koshelev, D. M. Belov and P. B. Medvedev, “Tachyon condensation in cubic superstring field theory,” Nucl. Phys. B 638, 3 (2002), hep-th/0011117.

G. Bandelloni and S. Lazzarini, “The geometry of W3 algebra: A twofold way for the rebirth of a symmetry,” Nucl. Phys. B 594, 477 (2001), hep-th/0011208.

[44] N. Berkovits, “Super-Poincare invariant superstring field theory,” Nucl. Phys. B450 (1995), 90, hep-th/9503099.

“Review of open superstring field theory,” hep-th/0105230.

“The Ramond sector of open superstring field theory,” JHEP 0111, 047 (2001), hep-th/0109100.

[45] S. B. Giddings and E. J. Martinec, “Conformal Geometry and String Field Theory,” Nucl. Phys. B278, 91 (1986).

[46] S. B. Giddings, E. J. Martinec and E. Witten, “Modular Invariance In String Field Theory,” Phys. Lett. B 176, 362 (1986).

[47] B. Zwiebach, “A Proof That Witten’s Open String Theory Gives A Single Cover Of Moduli Space,” Commun. Math. Phys. 142, 193 (1991).

[48] D. Z. Freedman, S. B. Giddings, J. A. Shapiro and C. B. Thorn, “The Nonplanar One Loop Amplitude In Witten’s String Field Theory,” Nucl. Phys. B 298, 253 (1988).

[49] J. A. Harvey and P. Kraus, “D-branes as unstable lumps in bosonic open string field theory,” JHEP 0004, 012 (2000), hep-th/0002117.

[50] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, “Lumps and p-branes in open string field theory,” Phys. Lett. B 482, 249 (2000), hep-th/0003031.

[51] N. Moeller, A. Sen and B. Zwiebach, “D-branes as tachyon lumps in string field theory,” JHEP 0008, 039 (2000), hep-th/0005036.

[52] A. Sen and B. Zwiebach, “Tachyon condensation in string field theory,” JHEP 0003, 002 (2000), hep-th/9912249.

[53] N. Moeller and W. Taylor, “Level truncation and the tachyon in open bosonic string field theory,” Nucl. Phys. B 583, 105 (2000), hep-th/0002237.

[54] D. Gaiotto and L. Rastelli, “Progress in open string field theory,” Presentation by L. Rastelli at Strings 2002, Cambridge, England; http://www.damtp.cam.ac.uk/strings02/avt/rastelli/.

[55] W. Taylor, “A Perturbative Analysis of Tachyon Condensation,” hep-th/0208149.

[56] W. Taylor, “Perturbative diagrams in string field theory,” hep-th/0207132.

[57] D. Gaiotto and L. Rastelli, “Experimental string field theory,” hep-th/0211012.
[58] I. Ellwood and W. Taylor, “Gauge invariance and tachyon condensation in open string field theory,” hep-th/0105156

[59] A. A. Gerasimov and S. L. Shatashvili, “On exact tachyon potential in open string field theory,” JHEP 0010, 034 (2000), hep-th/0009103
D. Kutasov, M. Marino and G. Moore, “Some exact results on tachyon condensation in string field theory,” JHEP 0010, 045 (2000), hep-th/0009148
D. Ghoshal and A. Sen, “Normalisation of the background independent open string field theory JHEP 0011, 021 (2000), hep-th/0009191
D. Kutasov, M. Marino and G. Moore, D. Kutasov, M. Marino and G. W. Moore, “Remarks on tachyon condensation in superstring field theory,” hep-th/0010108
S. Moriyama and S. Nakamura, “Descent relation of tachyon condensation from boundary string Phys. Lett. B 506, 161 (2001), hep-th/0011002
A. A. Gerasimov and S. L. Shatashvili, “Stringy Higgs mechanism and the fate of open strings,” JHEP 0101, 019 (2001), hep-th/0011009
I. Y. Aref’eva, A. S. Koshelev, D. M. Belov and P. B. Medvedev, “Tachyon condensation in cubic superstring field theory,” Nucl. Phys. B 638, 3 (2002), hep-th/0011117
P. Kraus and F. Larsen, “Boundary string field theory of the D D-bar system,” Phys. Rev. D 63, 106004 (2001), hep-th/0012198
M. Alishahiha, H. Ita and Y. Oz, “On superconnections and the tachyon effective action,” Phys. Lett. B 503, 181 (2001), hep-th/0012222
G. Chalmers, “Open string decoupling and tachyon condensation,” JHEP 0106, 012 (2001), hep-th/0103056
M. Marino, “On the BV formulation of boundary superstring field theory,” JHEP 0106, 059 (2001), hep-th/0103089
V. Niarchos and N. Prezas, “Boundary superstring field theory,” Nucl. Phys. B 619, 51 (2001), hep-th/0103102
K. S. Viswanathan and Y. Yang, “Tachyon condensation and background independent superstring field theory,” Phys. Rev. D 64, 106007 (2001), hep-th/0104099
M. Alishahiha, “One-loop correction of the tachyon action in boundary superstring field theory,” Phys. Lett. B 510, 285 (2001), hep-th/0104164

[60] L. Rastelli, A. Sen and B. Zwiebach, “String field theory around the tachyon vacuum,” Adv. Theor. Math. Phys. 5, 353 (2002), hep-th/0012251

[61] I. Ellwood and W. Taylor, “Open string field theory without open strings,” Phys. Lett. B 512, 181 (2001), hep-th/0103085

[62] I. Ellwood, B. Feng, Y. H. He and N. Moeller, “The identity string field and the tachyon vacuum,” JHEP 0107, 016 (2001), hep-th/0105024

[63] J. A. Minahan, “Quantum corrections in p-adic string theory,” hep-th/0105312

[64] H. Hata and T. Kawano, “Open string states around a classical solution in vacuum string field theory,” JHEP 0111, 038 (2001), hep-th/0108150

[65] D. Gaiotto, L. Rastelli, A. Sen and B. Zwiebach, “Ghost structure and closed strings in vacuum string field theory,” hep-th/0111129
[66] K. Okuyama, “Ghost kinetic operator of vacuum string field theory,” JHEP 0201, 027 (2002), hep-th/0201015

[67] L. Rastelli, A. Sen and B. Zwiebach, “Classical solutions in string field theory around the tachyon vacuum,” Adv. Theor. Math. Phys. 5, 393 (2002), hep-th/0102112

[68] H. Hata and S. Moriyama, “Observables as twist anomaly in vacuum string field theory,” JHEP 0201, 042 (2002), hep-th/0111034

L. Rastelli, A. Sen and B. Zwiebach, “A note on a proposal for the tachyon state in vacuum string field theory,” JHEP 0202, 034 (2002), hep-th/0111153

H. Hata, S. Moriyama and S. Teraguchi, “Exact results on twist anomaly,” JHEP 0202, 036 (2002), hep-th/0201177

[69] Y. Okawa, “Open string states and D-brane tension from vacuum string field theory,” JHEP 0207, 003 (2002), hep-th/0204012

[70] D. J. Gross and W. Taylor, “Split string field theory II,” JHEP 0108, 010 (2001), hep-th/0106036

[71] G. Moore and W. Taylor, “The singular geometry of the sliver,” JHEP 0201, 004 (2002), hep-th/0111069

[72] L. Rastelli and B. Zwiebach, “Tachyon potentials, star products and universality,” JHEP 0109, 038 (2001), hep-th/0006240

[73] V. A. Kostelecky and R. Potting, “Analytical construction of a nonperturbative vacuum for the open bosonic string,” Phys. Rev. D 63, 046007 (2001), hep-th/0008252

[74] R. Gopakumar, S. Minwalla and A. Strominger, “Noncommutative solitons,” JHEP 0005, 020 (2000), hep-th/0003160

[75] M. R. Douglas and N. A. Nekrasov, “Noncommutative field theory,” Rev. Mod. Phys. 73, 977 (2001), hep-th/0106048

[76] L. Rastelli, A. Sen and B. Zwiebach, “Half strings, projectors, and multiple D-branes in vacuum string field theory,” JHEP 0111, 035 (2001), hep-th/0105058

[77] D. J. Gross and W. Taylor, “Split string field theory I,” JHEP 0108, 009 (2001), hep-th/0105059

[78] D. Gaiotto, L. Rastelli, A. Sen and B. Zwiebach, “Star algebra projectors,” JHEP 0204, 060 (2002), hep-th/0202151

[79] M. Schnabl, “Wedge states in string field theory,” JHEP 0301, 004 (2003), hep-th/0201095

—“Anomalous reparametrizations and butterfly states in string field theory,” Nucl. Phys. B 649, 101 (2003), ep-th/0202139.
[80] I. Ellwood and W. Taylor, to appear.

[81] N. Moeller, A. Sen and B. Zwiebach, “D-branes as tachyon lumps in string field theory,” JHEP 0008, 039 (2000), ep-th/0005036.

[82] A. Sen and B. Zwiebach, “Large marginal deformations in string field theory,” JHEP 0010, 009 (2000), hep-th/0007153.

[83] J. Kluson, “Marginal deformations in the open bosonic string field theory for N D0-branes,” hep-th/0203089. “Exact solutions in open bosonic string field theory and marginal deformation in CFT,” hep-th/0209255.

[84] B. Zwiebach, “A solvable toy model for tachyon condensation in string field theory,” JHEP 0009, 028 (2000), hep-th/0008227.

[85] A. Strominger, “Closed strings in open string field theory,” Phys. Rev. Lett. 58 629 (1987).

[86] J. A. Shapiro and C. B. Thorn, “BRST invariant transitions between open and closed strings,” Phys. Rev. D36 432 (1987); “closed string-open string transitions in Witten’s string field theory,” Phys. Lett. B 194, 43 (1987).

[87] A. A. Gerasimov and S. L. Shatashvili, “Stringy Higgs mechanism and the fate of open strings,” JHEP 0101, 019 (2001), hep-th/0011009.

S. L. Shatashvili, “On field theory of open strings, tachyon condensation and closed strings,” hep-th/0105076.

[88] A. Hashimoto and N. Itzhaki, “Observables of string field theory,” JHEP 0201, 028 (2002), hep-th/0111092.

[89] M. Alishahiha and M. R. Garousi, “Gauge invariant operators and closed string scattering in open string field theory,” Phys. Lett. B 536, 129 (2002), hep-th/0201249.

[90] C. Wendt, “Scattering amplitudes and contact interactions in Witten’s superstring field theory,” Nucl. Phys. B314 (1989) 209.

[91] J. Greensite and F. R. Klinkhamer, “Superstring amplitudes and contact interactions,” Phys. Lett. B304 (1988) 108.

[92] N. Berkovits, A. Sen and B. Zwiebach, “Tachyon condensation in superstring field theory,” Nucl. Phys. B 587, 147 (2000), hep-th/0002211.

P. De Smet and J. Raeymaekers, “Level four approximation to the tachyon potential in superstring JHEP 0005, 051 (2000), hep-th/0003220.

A. Iqbal and A. Naqvi, “Tachyon condensation on a non-BPS D-brane,”, hep-th/0004015.
P. De Smet and J. Raeymaekers, “The tachyon potential in Witten’s superstring field theory,” JHEP 0008, 020 (2000). [hep-th/0004112]