Study of the $f_2(1270)$ and $a_2(1320)$ resonances in $\gamma^*(Q^2)\gamma$ collisions

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We discuss studies of the $Q^2$ dependence of the $f_2(1270)$ and $a_2(1320)$ production cross sections in $\gamma^*(Q^2)\gamma$ collisions at current and coming into operation colliders with a high luminosity. Changing the dominant helicity amplitude occurs in the reactions $\gamma^*(Q^2)\gamma \to f_2(1270)$ and $\gamma^*(Q^2)\gamma \to a_2(1320)$ with increasing $Q^2$. This is caused by the coming of the QCD asymptotics. It is shown that the transition to the asymptotic behavior of QCD in the amplitudes $\gamma^*(Q^2)\gamma \to f_2(1270)$, $a_2(1320)$ is provided by the compensation of the contributions of ground vector states $\rho$ and $\omega$ in $Q^2$-channel with the contributions of their radial excitations.

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Physics of two-photons collisions entering into the era of ultra-high statistics gives unique opportunities to study the internal (quark-gluon) structure of hadrons [1–4]. For example, the recent experiments of the Belle Collaboration on the reactions $\gamma\gamma \to \pi^+\pi^-$ [5, 6], $\gamma\gamma \to \pi^0\pi^0$ [7], and $\gamma\gamma \to \pi^0\eta$ [8] established conclusively the smallness of the two-photon widths of the $f_2(980)$ and $a_2(980)$ resonances, which testifies in favor of their four-quark structure [3, 11].

The measurements of the two-photon widths of the light pseudoscalar mesons $P = \pi^0$, $\eta$, $\eta'$ in $\gamma\gamma$ collisions [4] and the transition form factors $F_{\gamma\gamma}\gamma\gamma(pQ^2)$ in $\gamma^*(Q^2)\gamma\gamma$ collisions$^2$ performed by CELLO [12], CLEO [13], BaBar [14, 15] and Belle [16] Collaborations allowed to realize a critical test of QCD calculations of the processes at large $Q^2$.

Production of classical tensor $q\bar{q}$ resonances by two real photons proceeds very intensively: $f_2(1270)$ in the reactions $\gamma\gamma \to \pi^+\pi^-$ [5, 6, 17, 18] and $\gamma\gamma \to \pi^0\pi^0$ [7, 19] and $a_2(1320)$ in the reactions $\gamma\gamma \to \pi^0\eta$ [8, 20] and $\gamma\gamma \to \pi^+\pi^-\pi^0$ [21, 22] (see Fig. 1). This fact is a good reason to start detailed investigations of the $Q^2$ dependence of the $f_2(1270)$ and $a_2(1320)$ production cross sections in $\gamma^*(\gamma^*)\gamma$ collisions at $e^+e^-$ colliders with a high luminosity.$^3$

We now turn to the detailed discussion.

In $\gamma\gamma$ collisions, the $f_2(1270)$ and $a_2(1320)$ resonances can be produced in the states with helicity $\lambda = 0$ and $\pm 2$. Helicity $\lambda$ is defined in the resonance rest frame, in which $\lambda = \lambda_1 - \lambda_2$, where $\lambda_1$ and $\lambda_2$ are the helicities of incoming photons. According to the high-statistics measurements [6, 8, 17, 19, 23, 25] the fraction of the $f_2(1270)$ and $a_2(1320)$ production in states with $\lambda = \pm 2$ in $\gamma\gamma$ collisions is more than 95%.

This remarkable experimental fact of $\lambda = \pm 2$ dominance is naturally reproduced by the effective gauge-invariant Lagrangian, describing the tensor meson production by two photons with opposite helicities only.

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$^1$ In 1982, the prediction $\Gamma(f_0(980) \to \gamma\gamma) \approx \Gamma(a_0(980) \to \gamma\gamma) \approx 0.27$ keV was done in the four-quark MIT bag model [9, 10]. In 2014, the Particle Data Group cited in the Review of Particle Physics the following data [1]: $\Gamma(f_0(980) \to \gamma\gamma) \approx 0.29$ keV and $\Gamma(a_0(980) \to \gamma\gamma) \approx 0.3$ keV, which is an order of magnitude smaller than the $\gamma\gamma$ width of the tensor $q\bar{q}$ meson $\Gamma(f_2(1270) \to \gamma\gamma) \approx 3$ keV. The prediction of the $q\bar{q}$ model $\Gamma(f_2(1270) \to \gamma\gamma)/\Gamma(a_0(980) \to \gamma\gamma) = 25/9$ is excluded experimentally.

$^2$ $\gamma^*(Q^2)$ ($\gamma^*$ below) denotes the photon with virtuality $Q^2$.

$^3$ Currently, the maximum luminosity $\approx 2 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ is reached at the KEKB $e^+e^-$ collider [4, 3]. The luminosity of $8 \times 10^{35}$ cm$^{-2}$ s$^{-1}$ is planned to have at the SuperKEKB factory [1].

Figure 1: Cross sections of the reactions (a) $\gamma\gamma \to \pi^+\pi^-$ [5, 6, 17, 18], (b) $\gamma\gamma \to \pi^0\pi^0$ [7, 12], (c) $\gamma\gamma \to \pi^0\eta$ [8], and (d) $\gamma\gamma \to \pi^+\pi^-\pi^0$ [21, 22] as functions of the invariant mass, $\sqrt{s}$, of the final meson system. In plots (a)–(c) $\theta$ denotes the polar angle of one of the outgoing mesons with respect to the incident photon direction in the $\gamma\gamma$ center-of-mass system. The reactions $\gamma\gamma \to \pi^0\pi^0$, plot (b), and $\gamma\gamma \to \pi^+\pi^-\pi^0$, plot (d), seem more preferable in the sense of the smallness of the physical background under the $f_2(1270)$ and $a_2(1320)$ peaks, respectively.
\[ L = g_{T\gamma\gamma} T_{\mu
u} F_{\mu\nu} F_{\nu\sigma}, \]  
(1)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the tensor of the electromagnetic field \( A_\mu \), \( T_{\mu\nu} \) is the field of the tensor meson \( T \) (\( T = f_2(1270), a_2(1230) \)); \( T_{\mu\nu} = T_{\nu\mu}, T_{\mu\nu} = 0 \), \( \partial_\mu T_{\mu\nu} = 0 \); \( g_{T\gamma\gamma} \) is the coupling constants of the \( T \) meson to the energy-momentum tensor of the electromagnetic field.

Using Lagrangian \([11]\) one can unambiguously predict the hierarchy of the \( Q^2 \) dependences of the helicity amplitudes \( V_{\lambda_1\lambda_2}^{(s)}(T; s, Q^2) = V_{\lambda_1\lambda_2}(T; s, Q^2) \) describing the \( \gamma^*\gamma \rightarrow T \) vertices \([26, 28]\):

\[
V_{1,-1}^{(2)}(T; s, Q^2) = V_T(s, Q^2) \left(1 + \frac{Q^2}{s}\right),
\]
(2)

\[
V_{1,0}^{(1)}(T; s, Q^2) = V_T(s, Q^2) \sqrt{\frac{Q^2}{2s}} \left(1 + \frac{Q^2}{s}\right),
\]
(3)

\[
V_{1,1}^{(0)}(T; s, Q^2) = -V_T(s, Q^2) \frac{Q^2}{\sqrt{6s}} \left(1 + \frac{Q^2}{s}\right).
\]
(4)

Here \( s = (q_1 + q_2)^2 \); \( q_1 \) and \( q_2 \) are the four-momenta of the incident photons, \( q_1^2 = 0, q_2^2 = -Q^2 \);

\[
V_T(s, Q^2) = g_{T\gamma\gamma} s F_T(Q^2)/2, \quad F_T(0) = 1,
\]
(5)

\[ g_{T\gamma\gamma} s = 2V_{1,-1}^{(2)}(T; s, 0) = \sqrt{320} \sqrt{s} \Gamma_{T\gamma\gamma}(s), \]
(6)

and \( F_T(Q^2) \) is the transition form factor which is common for all vertices.

The vertex \( V_{1,0}^{(1)}(T; s, Q^2) \) vanishes for \( Q^2 \rightarrow 0 \) as \( \sqrt{Q^2} \). This is a consequence of gauge invariance. The vertex \( V_{1,1}^{(0)}(T; s, Q^2) \) is proportional to \( Q^2 \) for \( Q^2 \rightarrow 0 \) owing to a specific selection of the \( \gamma^*\gamma T \) interaction which consists with the experimental fact of \( \lambda = \pm 2 \) dominance in \( \gamma \gamma \rightarrow T \) transitions (see \( V_{1,-1}^{(2)}(T; s, 0) \) in Eq. (6)).

For small \( Q^2 \), the dominance of \( V_{1,-1}^{(2)}(T; s, Q^2) \) over \( V_{1,0}^{(1)}(T; s, Q^2) \) and \( V_{1,1}^{(0)}(T; s, Q^2) \) is certainly maintained. However, for large \( Q^2 \) the situation changes radically. Asymptotically

\[
V_{1,-1}^{(2)}(T; s, Q^2) \sim F_T(Q^2)^2 Q^2, \quad \lambda = 0\]
(7)

\[
V_{1,0}^{(1)}(T; s, Q^2) \sim F_T(Q^2)^3 Q^3, \quad \lambda = 1\]
(8)

\[
V_{1,1}^{(0)}(T; s, Q^2) \sim F_T(Q^2)^4 Q^4, \quad \lambda = 2\]
(9)

and the \( \gamma^*\gamma \rightarrow T \) vertex with \( \lambda = 0 \) becomes dominant.

From the parton model considerations \([25, 29, 30]\) and the QCD analysis of hard exclusive processes \([30, 31]\) it follows that for large \( Q^2 \) the tensor meson production amplitude with zero helicity (in the \( \gamma^*\gamma \) center-of-mass system) should tend to the constant value (with logarithmic accuracy), and other amplitudes should be suppressed by powers of \( Q^2 \). This implies that \( F_T(Q^2) \sim 1/Q^4 \) for large \( Q^2 \). In the generalized vector meson dominance model (GVDM) such an asymptotic behavior is provided by the compensation in \( Q^2 \)-channel of the contributions of ground and excited states of vector mesons \( V = \rho, \omega, \phi, V' = \rho', \omega', \phi', \omega'' \), etc. \([28]\).

It is interesting to find out, at least roughly, how fast the angular distributions can vary with \( Q^2 \) in the reactions \( \gamma^*\gamma \rightarrow f_2(1270) \rightarrow \pi\pi, \gamma^*\gamma \rightarrow a_2(1230) \rightarrow \pi^0\eta, \) and \( \gamma^*\gamma \rightarrow a_2(1320) \rightarrow \rho^\pm\pi^\mp + \pi^-\pi^0 \) for \( 0 < Q^2 < 40 \text{ GeV}^2 \) (in the case of the processes \( \gamma^*\gamma \rightarrow \pi^0, \eta, \eta' \) the asymptotic regime apparently occurs near \( 40 \text{ GeV}^2 \)).

We put \( m_\rho = m_\omega \), \( m_{\rho'} = m_{\omega'} \), etc., and will consider that the resonance \( f_2(1270) \) does not contain strange valent quarks (as \( \omega, \omega' \), etc.). Then, in GVDM, a simplest expression for \( F_T(Q^2) \) with the required asymptotic behavior has the form

\[
F_T(Q^2) = \frac{1}{(1 + Q^2/m_{\rho'}^2)(1 + Q^2/m_{\rho'}^2)}.
\]
(10)

Figure 2 shows the \( Q^2 \) dependences of the normalized vertex functions \( 2V_{\lambda_1\lambda_2}^{(s)}(T; m_T^2, Q^2)/(g_{T\gamma\gamma}m_T^2) \) calculated according Eqs. \([26, 28]\) and \([31]\) at \( m_\rho = 0.775 \text{ GeV}, m_{\rho'} = 1.465 \text{ GeV}, \) and \( s = m_T^2 \).

As is seen from Fig. 2, the main at \( Q^2 = 0 \) vertex function with helicity \( \lambda = 2 \) decreases very rapidly with increasing \( Q^2 \). For \( Q^2 \gtrsim 10 \text{ GeV}^2 \) the vertex function with helicity \( \lambda = 0 \) be-
comes main and close to its asymptotic value.

The angular distributions in the reactions $\gamma^*\gamma \rightarrow f_2(1270) \rightarrow \pi\pi$ [26], $\gamma^*\gamma \rightarrow a_2(1320) \rightarrow \pi^0\eta$, and $\gamma^*\gamma \rightarrow a_2(1320) \rightarrow \rho^+\pi^- \rightarrow \pi^+\pi^-\pi^0$ reshape as $Q^2$ increases with the same rate.

The differential cross sections for $\gamma^*\gamma \rightarrow f_2(1270) \rightarrow \pi\pi$ and $\gamma^*\gamma \rightarrow a_2(1320) \rightarrow \pi^0\eta$ (integrated over the azimuth angle of one of the outgoing mesons in the $\gamma^*\gamma$ center-of-mass system) have the following form: $\sin^2\theta$ for the tensor meson decays from the helicity $\lambda = \pm 2$ states, $4\cos^2\theta\sin^2\theta$ for $\lambda = \pm 1$, and $4(3\cos^2\theta - 1)^2$ for $\lambda = 0$, where $\theta$ is the polar angle of one of the outgoing mesons. These angular distributions are equally normalized. Thus, the $\sin^2\theta$ distribution dominating at $Q^2 = 0$ should be replaced by the $\frac{4}{3}(3\cos^2\theta - 1)^2$ distribution with increasing $Q^2$.

The amplitude of the reaction $\gamma^*\gamma \rightarrow a_2(1320) \rightarrow \rho^+\pi^- \rightarrow \pi^+\pi^-\pi^0$ is described by two diagrams and therefore the corresponding angular distributions for $\lambda = \pm 2, \pm 1, 0$ have a rather cumbersome form. Nevertheless these distributions are sensitive to the $a_2(1320)$ helicity $\lambda$. They are exhaustively represented in Refs. [21, 23]. Here we consider as an example the contribution of one diagram $\gamma^*\gamma \rightarrow a_2(1320) \rightarrow \rho^+\pi^- \rightarrow \pi^+\pi^-\pi^0$ only. Then the angular distributions (integrated over the azimuth angle of the outgoing $\pi^-$ meson) corresponding to the $\lambda = \pm 2, \pm 1, 0$ helicity contributions are

$$\sin^2\theta_{\rho^+}\sin^2\theta_{\pi^+}(\cos^2\theta_{\rho^+}\sin^2\varphi_{\pi^+} + \cos^2\varphi_{\pi^+}), \quad (11)$$

$$\sin^2\theta_{\pi^+}[\sin^2\varphi_{\pi^+}(2\cos^2\theta_{\rho^+} - 1)^2 + \cos^2\varphi_{\pi^+}\cos^2\theta_{\rho^+}], \quad (12)$$

$$6\sin^2\theta_{\rho^+}\sin^2\theta_{\pi^+}\cos^2\theta_{\rho^+}\sin^2\varphi_{\pi^+}, \quad (13)$$

respectively, where $\theta_{\rho^+}$ is the polar angle of the $\rho^+$ in the $\gamma^*\gamma$ center-of-mass system, with the $z$-axis along the incident photon direction; the angles $\theta_{\pi^+}$ and $\varphi_{\pi^+}$ describe the decay of the $\rho^+$ in its helicity system; $\varphi_{\pi^+}$ is measured from the plane defined by the momenta of the $\rho^+$ and photons. As $Q^2$ increases, the distribution from Eq. (11) should be replaced by that from Eq. (13).

Note that the form factors of a more general form than that in Eq. (10) may be required for the treatment of real data, for example,

$$F_T(Q^2) = \frac{1 + \xi Q^2}{(1 + Q^2/m_{\rho^+}^2)(1 + Q^2/m_{\rho^+}^2)(1 + Q^2/m_{\rho^-}^2)}, \quad (14)$$

with varying masses $m_{\rho^+}$ and $m_{\rho^-}$ and an additional free parameter $\xi$.

Deviations from the above picture are possible in principle since the tensor meson production in $\gamma^*\gamma$ collisions can be described in the general case by three independent invariant amplitudes. However, our scenario is based on the well-established dominance of the $\lambda = \pm 2$ helicity states in the tensor meson production by two real photons. This allows us to hope that possible deviations will be small.

Thus, the experiments on the reactions $\gamma^*\gamma \rightarrow f_2(1270)$ and $\gamma^*\gamma \rightarrow a_2(1320)$ will allow to check the theoretical predictions about the changing of the dominant helicity amplitude with increasing $Q^2$. The dynamics of this change can be tracked by analyzing the angular distributions of the final mesons in the reactions $\gamma^*\gamma \rightarrow f_2(1270) \rightarrow \pi\pi$, $\gamma^*\gamma \rightarrow a_2(1320) \rightarrow \pi^0\eta$, and $\gamma^*(Q^2)\gamma \rightarrow a_2(1320) \rightarrow \rho^+\pi^- \rightarrow \pi^+\pi^-\pi^0$. The information obtained on three transition form factors, corresponding to the $\lambda = 2, 1$, and 0 $\gamma^*\gamma$ helicity amplitudes, would be crucial for the selection of dynamical models of the $f_2(1270)$ and $a_2(1320)$ resonance production.

We have shown that the transition to the asymptotic behavior of QCD in the amplitudes $\gamma^*(Q^2)\gamma \rightarrow f_2(1270), a_2(1320)$ is provided by the compensation of the contributions of ground vector states $\rho$ and $\omega$ in $Q^2$-channel with those of their radial excitations.

More recently, the Belle Collaboration represented the first data on the processes $\gamma^*(Q^2)\gamma \rightarrow f_2(1270)$ extracted from the measured differential cross section of the reaction $\gamma^*\gamma \rightarrow f_2(1270) \rightarrow \pi^0\pi^0$ for $Q^2$ up to 30 GeV$^2$ [32]. In Fig. 3, the curves transferred from Fig. 2 are compared with the Belle data which we multiplied by a factor $(1 + Q^2/m_{\rho^+}^2)$ in order to match the definition of the transition form factors with $\lambda = 2, 1$, and 0 used in Ref. [32] with our definition of the normalized vertex functions. The theoretical curves are in satisfactory agreement with the data.

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Figure 3: Comparison of the $Q^2$ dependencies of the normalized vertex functions $2\Gamma_{\lambda_{1},\lambda_{2}}(f_{j}(1270); m^2_{j}, Q^2)/(g_{\gamma\gamma} m^2_{j})$, calculated according to Eqs. (2)–(6) and (10), with the Belle data. [32]. The curves are the same as in Fig. 2. The Belle data are reduced to our normalization.