Time-Variant Reliability Analysis Method for Uncertain Motion Mechanisms Based on Stochastic Process Discretization

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\textbf{ABSTRACT} The reliability of a motion mechanism is affected by corrosion, wear, aging and other components’ performance degradations with the extension of service time. This paper tackles this problem by proposing a time-varying reliability analysis method for uncertain motion mechanisms. First, a model of motion mechanism error is constructed by assessing the difference between actual and expected motion. A time-varying reliability analysis method for a motion mechanism is proposed. The time-varying performance function is discretized into several static performance functions, which are further approximated with several normal variables. Then, the correlation coefficient matrix and probability density function of these normal variables are calculated, and the time-varying reliability of a motion mechanism is obtained via high-dimensional Gaussian integration. The study demonstrates that the proposed method successfully transforms the time-varying reliability problem into several time-invariant reliability problems for analysis, and handles the time-varying reliability problem of a nonlinear motion mechanism involving random variables and stochastic processes, and significantly increases the computational efficiency. Finally, the proposed method’s effectiveness is verified by two numerical examples and one practical engineering problem.

\textbf{INDEX TERMS} Process discretization, uncertain mechanism, time-varying reliability, first-order reliability method.

\section{I. INTRODUCTION}

A mechanism is defined as a mechanical device composed of several components used to transfer force and motion and realize specific functions and actions [1]. However, the mechanism’s manufacturing, assembly, boundary conditions, environmental loads, material properties, and artificial assumptions commonly give rise to uncertainties [2]–[4]. These uncertainties may result in undesirable consequences such as reduced motion precision and increased fault rate, which further cause the mechanism’s failure and safety problems [5], [6]. Therefore, it is necessary to explore methods for mechanism reliability analysis with regard to the uncertain factors in the mechanism to obtain analysis results that meet practical engineering requirements.

A motion mechanism’s reliability refers to the mechanism’s ability to complete the specified actions within the specified conditions and motion intervals in the motion process [7]. The past few decades saw progress in reliability analysis methods for motion mechanisms considering uncertainties. Following the “effective rod length theory,” Lee and Gilmore [8] established a reliability analysis model for discontinuous-contact revolute pairs, laying a solid foundation for research on the reliability of mechanisms with kinematic pair clearance. Song et al. [9] used variational inequality to analyze the planar rigid multibody systems with multiple revolute clearance joints motion mechanism, and for this systems provide a nonsmooth strategy. Li et al. [10] used the reliability analysis model established by Lee and Gilmore to conduct a reliability-based virtual experimental study analyzing the mechanism’s motion precision. Taking into account the mechanisms’ parameter uncertainties such as joint clearance, rod length, and mass density, Tuo et al. [11] studied the motion precision of a crank-slider mechanism, establishing a reliability model for its motion precision. Zhang et al. [12] derived a motion precision error model for a
plane mechanism based on the Edgeworth series method and analyzed the reliability of the mechanism’s motion precision. By considering load, friction coefficient, dimension error, and other relevant factors, Lai et al. [13] modeled a plane four-bar mechanism based on the multibody dynamics and gap collision theory and proposed a reliability analysis method for motion mechanisms. Peng et al. [14] proposed a novel uncertainty estimation and optimization strategy for motion planning of overhead cranes with uncertain Li et al. [15] a novel uncertain estimation and optimization strategy for motion planning of overhead cranes with uncertainty. Li et al. [15] proposes a new higher-efficiency interval method for the response bound estimation of nonlinear dynamic systems. This approach uses Chebyshev polynomials to help the interval analysis applied on the estimation, analyzed the offshore boom crane with three degrees of freedom motion mechanisms numerical example.

However, the discussed studies focus on assessing the static reliability of a motion mechanism within the predetermined working scope or at a given time point, without considering the effects of time-varying uncertainties such as wear, corrosion, and aging of mechanism’s components with the extension of service time. In this context, a motion mechanism’s reliability analysis that considers the time factor has gradually attracted the attention of scholars. Zhang and Du [16] considered the correlation between two time-points of a motion mechanism and proposed an analytic solution algorithm for the time-varying reliability of motion mechanisms based on the up-and-down crossing rate method. Further, in their work [17], the authors analyzed the clearance error and connecting rod error of the four-bar mechanism using the dimensionality reduction method. They solved the time-varying reliability of motion mechanisms with the envelope method. Wei et al. [18] combined the envelope method with the first-order reliability method to solve the reliability of the bar system with input variables showing random process features.

Nevertheless, these methods disregarded the performance functions’ autocorrelation, which produces significant calculation errors. Fang et al. [19] proposed a time-varying reliability analysis method based on joint probability for trajectory mechanisms, solving their motion reliability using a multi-dimensional normal distribution function. Linjie et al. [20] tackled the problem of nonlinearity with the limit state function for flexible mechanisms. The authors proposed a modal synthesis method to describe the time-varying laws of system responses under the effects of random parameters and rigid-flexible coupling and analyzed the time-varying reliability of flexible mechanisms.

To sum up, despite the progress in the time-varying reliability of motion mechanisms, there are still two urgent problems. First, existing studies on motion mechanism reliability primarily focus on the hinge clearance of the motion mechanism, often assuming that the dimensional parameters describing the components in the hinge clearance and the kinematic pair clearance obey normal distribution or uniform distribution. Since these studies paid little attention to the wear, corrosion, or aging of components with the extension of service time or the reliability of the motion mechanism under the combined effects of dynamic external loads and external environments, they cannot reflect practical engineering problems comprehensively. Second, the motion function does not contain time variables and has only one input random process. However, general performance functions have multiple stochastic processes. These functions are not only functions of time but also non-monotonic functions of various input variables (including time, random variables, and stochastic processes), thus endowing the performance function for motion mechanism with strong autocorrelation. The existence of such autocorrelation exerts a significant influence on reliability considering the motion mechanism’s time-variability [21]. If only the correlation between two consecutive time points in the motion mechanism is considered but not the autocorrelation of the function, large reliability errors are produced.

This paper proposes an efficient time-varying reliability analysis method for motion mechanisms. This method’s novelty is reflected in the following:

- First, on the basis of the motion mechanism error model, this study considers manufacturing and assembly errors, as well as other uncertain factors such as time-varying dynamic loads and material performance degradation during motion, thus achieving practical significance.
- Second, with regard to the problem of non-linear time-varying reliability analysis in which the uncertainty parameters in the motion function are arbitrary stochastic process variables or random variables, the problem of motion mechanisms’ time-varying reliability is transformed into a static reliability problem through time-domain discretization. Further, efficient problem solving is realized through high-dimensional Gaussian integration, which improves calculation precision and efficiency and extends the proposed methods’ applicability.

II. TIME-VARYING RELIABILITY MODEL FOR MOTION MECHANISMS

The uncertainties in a motion mechanism result in differences between the actual and expected motion. This difference is usually referred to as motion error. In engineering practice, the input variables such as random material parameter, random size parameter, and random load affect the motion mechanism’s output. For example, wear, corrosion, and load variation of components in the mechanism are all explicit functions of time \( t \). In other words, there are both random parameters and stochastic process parameters in the motion mechanism, and the randomness in input parameters is transferred to the motion error through the output function, which is also a function of time. Therefore, for a motion mechanism involving time-varying problems, an error function at time point \( t \) can be established based on the relationship between actual and expected motion responses [22]:

\[
G(t) = D(X, Y(t), t) - D_d(t)
\]
where \( X_{n \times 1} = (X_1, X_2, \cdots, X_n)^T \) denotes an \( n \)-dimensional random input variable composed of kinematic pair clearance, dimensional variables, and dynamic mechanism errors. Further, \( D(X, Y(t), t) \) and \( D_Q(t) \) represent the actual and expected values of the mechanism’s response during motion, respectively. Finally, \( Y(t) = (Y_1(t), Y_2(t), \cdots, Y_M(t)) \) comprises \( M \) random processes and represents the time-varying load, dimensional attenuation, or material degradation of the components in the motion mechanism. The randomness of \( X \) and \( Y(t) \) is transferred to \( G(t) \) through the function, so \( G(t) \) is also a random process. The time-varying performance function for the mechanism can be established based on the mechanism error:

\[
g_j(X, Y(t), t) = G(t) - \varepsilon
\]

where \( \varepsilon \) denotes the maximum allowable error value. To meet the motion mechanism requirements, the movement error must be controlled within the allowable range. Thus, the motion mechanism failure is defined as the case where the absolute value of its error function \(|G(t)|\) exceeds the allowable value \( \varepsilon \) at any time \( t \in [0, T] \), where \( t \in [0, T] \) is the reference period for the mechanism’s working setting. The reliability \( R_j(t) \) of the mechanism’s \( j \)-th unit at a certain time point is called instantaneous reliability. Now, the instantaneous reliability of the motion mechanism regarding the time-varying point can be defined as:

\[
R_j(t) = \Pr \{ g_j(X, Y(t), t) > 0 \} \tag{3}
\]

### III. TIME-VARYING RELIABILITY ANALYSIS METHOD FOR MOTION MECHANISMS BASED ON PROCESS DISCRETIZATION

The time-variant reliability analysis method based on stochastic process discretization (TRPD) [23] has been applied to reliability analysis of ten-bar truss, automobile engine spindle, and other structures. This method first discretizes the random process in the performance function, thus obtaining the static performance functions at different time points. Next, these static performance functions are approximated with normal variables via the first-order reliability method (FORM) [24]. The normal variables are then assembled into an equivalent static performance function through inputting the random process autocorrelation. Finally, FORM solves the time-varying reliability. However, TRPD has two main disadvantages. First, an additional random variable is introduced in constructing the equivalent static performance function, thus increasing the nonlinearity of the equivalent static performance function and producing calculation errors. Second, the repeated invoking of the equivalent static performance function in FORM requires multiple high-dimensional Gaussian integrations, introducing calculation errors and reducing the computational efficiency.

The improved time-variant reliability analysis method based on stochastic process discretization (ITRPD) [25] leverages TRPD’s advantages. It discretizes the function into several instantaneous performance functions and approximates them with the corresponding normal variables in FORM. The method then calculates the correlation coefficient matrix of these normal variables, obtaining their probability density functions (PDFs) [26]. Now, only one high-dimensional Gaussian integration is sufficient to calculate the time-varying reliability of the unit. Since there is no need to introduce additional random variables in the calculation process, the analysis process is simplified. In addition, the computational efficiency and precision are improved due to performing only one high-dimensional Gaussian integration.

Note that \( R_j(t) \) relates only to the state of the \( j \)-th unit of the mechanism at a fixed time point \( t \) and has nothing to do with the motion state at other time points. It is a static reliability model that changes over time. Thus, the time-varying reliability of the \( j \)-th mechanism unit can be defined as:

\[
R_j(0, T) = \Pr \{ g_j(X, Y(t), t) > 0 \}, \quad \forall t \in [0, T] \tag{4}
\]

The unit’s time-varying reliability \( R_j(0, T) \) refers to the probability that the function is greater than 0 over the entire time interval and is, thus, a function of the time interval. The unit’s instantaneous reliability \( R_j(t) \) is a function of time \( t \), where \( j = 1, 2, \ldots, j \). There is no need to consider the autocorrelation function when calculating instantaneous reliability. However, the calculation of \( R_j(0, T) \) requires considering both the instantaneous distribution of the performance function over the entire interval and the autocorrelation coefficient function of the performance function. This is the essential difference between instantaneous reliability and time-varying reliability.

First, \( m \) time points \( (t_m, m = 1, 2, \ldots, M) \) are uniformly distributed over the time interval \([0, T]\), enabling the time-varying performance function \( g_j(X, Y(t), t) \) to be discretized into \( m \) static performance functions. Now, the time-varying reliability \( R_j(0, T) \) can be expressed as:

\[
R_j(0, T) = \Pr \left\{ \bigcap_{m=1}^{m} (g_j(X, Y(t_m), t_m) > 0) \right\} \tag{5}
\]

where the formula \( Y(t_m) \) is a random vector fixed at a time point \( t_m \), and the variable space \((X, Y(t_m))\) is generally not a standard independent normal space. To perform the first-order Taylor expansion on \( g_j(X, Y(t_m), t_m) \) at the most probable point (MPP) using the first-order reliability method, it is necessary to convert all the components of \( X \) and \( Y(t_m) \) into standard normal variables using the mapping change method. If the components of \( X \) and \( Y(t_m) \) are not independent, Nataf transformation can be used to convert them into independent variables. Then, the time-varying reliability \( R_j(0, T) \) in the independent standard normal space \((U, V_m)\) can be expressed as follows:

\[
R_j(0, T) = \Pr \left\{ \bigcap_{m=1}^{m} (g_j(U, V_m, t_m) > 0) \right\} \tag{6}
\]

The first-order reliability method can be utilized to perform the first-order Taylor expansion on \( g_j(U, V_m, t_m) \).
at MPP as follows:
\[
R_J(0, T) = \Pr \left\{ \bigcap_{m=1}^{M} \left[ \beta_{j,m} + \alpha_{j,U,m} U_j^T + \alpha_{j,V,m} V_m^T > 0 \right] \right\}
\]  
(7)

Equation (7) can be simplified as:
\[
R_J(0, T) = \Pr \left\{ \bigcap_{m=1}^{M} H_{j,m} > 0 \right\}
\]  
(8)

where \( H_{j,m} = \beta_{j,m} + \alpha_{j,U,m} U_j^T + \alpha_{j,V,m} V_m^T \), \( m = 1, 2, \ldots, M \), is the linear approximation of \( g_j(U, V_m, t_m) \) at MPP, and \( H_j = (H_{j,1}, H_{j,2}, \ldots, H_{j,M}) \) is the vector of normal distribution (mean vector \( \beta_j = (\beta_{j,1}, \beta_{j,2}, \ldots, \beta_{j,M}) \); variance = 1). To calculate the time-varying reliability of the motion mechanism, the autocorrelation coefficient matrix \( \rho_j \) for \( H_j \) needs to be obtained. A random \( \beta_{j,m,m'} \) in \( \rho_j \) can be expressed as:
\[
\rho_{j,m,m'} = \frac{\text{Cov}(H_{j,m}, H_{j,m'})}{\sigma_{H_{j,m}} \sigma_{H_{j,m'}}}
\]  
(9)

where \( \sigma_{H_{j,m}} \) and \( \sigma_{H_{j,m'}} \) denote the standard deviations of \( H_{j,m} \) and \( H_{j,m'} \), respectively (both set as 1). Then, the above formula can be converted into:
\[
\rho_{j,m,m'} = \text{Cov}(\beta_{j,m} + \alpha_{j,U,m} U_j^T + \alpha_{j,V,m} V_m^T, \beta_{j,m'}) + \alpha_{j,U,m'} U_j^T + \alpha_{j,V,m'} V_m^T
\]  
(10)

The random vectors \( X \) and \( Y(t) \) are independent after conversion. Consequently, \( U \) and \( V_m \) (obtained after conversion) are also independent. Therefore, the above formula can be converted into:
\[
\rho_{m,m'} = \text{Cov}(\alpha_{j,U,m} U_j^T + \alpha_{j,U,m'} U_j^T) + \text{Cov}(\alpha_{j,V,m} V_m^T, \alpha_{j,V,m'} V_m^T)
\]  
(11)

where \( \text{Cov}(\alpha_{j,U,m} U_j^T + \alpha_{j,U,m'} U_j^T) \) equals
\[
\text{Cov}(\alpha_{j,U,m} U_j^T + \alpha_{j,U,m'} U_j^T) = E(\alpha_{j,U,m} U_j^T \cdot \alpha_{j,U,m'} U_j^T) - E(\alpha_{j,U,m} U_j^T)E(\alpha_{j,U,m'} U_j^T)
\]  
(12)

Here, \( E(\bullet) \) denotes the expected calculation. In the variable space, \( U \) is an independent standard normal vector, so \( E(\alpha_{j,U,m} U_j^T) \) and \( E(\alpha_{j,U,m'} U_j^T) \) are both 0. Therefore, Equation (12) can be simplified to:
\[
\text{Cov}(\alpha_{j,U,m} U_j^T + \alpha_{j,U,m'} U_j^T) = E(\alpha_{j,U,m} U_j^T \cdot \alpha_{j,U,m'} U_j^T) = \sum_{i=1}^{I} \sum_{i=1}^{I} E(\alpha_{j,U,m,i} \alpha_{j,U,m,i'} U_j U_j')
\]  
(13)

\( i = 1, 2, \ldots, I \) is the \( i \)-th components of \( U_j \). When \( i \neq i' \), \( E(\alpha_{j,U,m,i} \alpha_{j,U,m,i'} U_j U_j') = 0 \), and \( U_j^2 \) is a standard normal variable with a mean value equal to one. So \( U_j^2 \) is a cardinality variable with the degree of freedom 1. On this basis, the above formula can be written as follows:
\[
\text{Cov}(\alpha_{j,U,m} U_j^T + \alpha_{j,U,m'} U_j^T) = \sum_{i=1}^{I} \sum_{i=1}^{I} \alpha_{j,U,m,i} \alpha_{j,U,m,i'} = \sum_{i=1}^{I} \alpha_{j,U,m,i} \alpha_{j,U,m,i'}
\]  
(14)

Similarly, \( \text{Cov}(\alpha_{j,V,m} V_m^T, \alpha_{j,V,m'} V_m^T) \) in Equation (11) can be written as:
\[
\text{Cov}(\alpha_{j,V,m} V_m^T, \alpha_{j,V,m'} V_m^T) = \sum_{p=1}^{P} \alpha_{j,V,m,p} \alpha_{j,V,m,p'} \rho(V_{j,m,p} V_{j,m,p'})
\]  
(15)

where \( \rho(V_{j,m,p} V_{j,m',p'}) \) denotes the correlation coefficient between \( V_{j,m,p} \) and \( V_{j,m',p'} \), and \( \alpha_{j,V,m,p} \) and \( \alpha_{j,V,m',p'} \) denote the \( p \)-th components of \( \alpha_{j,V,m} \) and \( \alpha_{j,V,m'} \), respectively. Since the input process \( Y(t) \) is a Gaussian process, the correlation coefficients do not change after the respective transformation of \( Y_{j}(t_m) \) and \( Y_{j}(t_{m'}) \). [24] Thus,
\[
\rho(V_{j,m,p} V_{j,m',p'}) = \rho(Y_{j,p}(t_m) Y_{j,p}(t_{m'})) = \rho_{y_{j,p}}(t_m, t_{m'})
\]  
(16)

Substituting it into Equation (15) yields:
\[
\text{Cov}(\alpha_{j,V,m} V_m^T, \alpha_{j,V,m'} V_m^T) = \sum_{p=1}^{P} \alpha_{j,V,m,p} \alpha_{j,V,m'} \rho_{y_{j,p}}(t_m, t_{m'})
\]  
(17)

Every element in \( \rho_j \) can be calculated from Equation (17). Finally, Equation (8) is converted into:
\[
R_J(0, T) = \phi_p(\beta_j; 0; \rho_j)
\]  
(18)

In summary, the described method discretizes the time-varying performance function of a motion mechanism into several instantaneous performance functions and then approximates these instantaneous performance functions with corresponding normal variables using FORM. After that, the method calculates the correlation coefficient matrix of the normal variables, obtaining their PDFs. Finally, it performs one high-dimensional Gaussian integration to calculate the time-varying reliability of the motion mechanism unit. This algorithm has a high calculation precision and does not require the introduction of additional random variables. Besides, since there is no need to perform FORM analysis on the equivalent static problem, the method is characterized by high computational efficiency and good robustness.

| Variable | Variable type | Mean | Standard deviation |
|----------|---------------|------|--------------------|
| \( X_i \) | Normal | 0.1 | 0.01 |
| \( X_i \) | Normal | 0.2 | 0.02 |
| \( X_i \) | Normal | 0.3 | 0.03 |
| \( X_i \) | Normal | 0.4 | 0.04 |
Generally speaking, a motion mechanism system is often composed of multiple components. The constituent systems include series systems, parallel systems, and hybrid systems. The mechanism units in these systems have different effects on the entire mechanism. Therefore, weight coefficients are introduced to characterize the effect of each mechanism unit on the entire mechanism system. In practical engineering, the failure probability can be expressed as a weighted function of the units. In that case, the reliability of a motion mechanism with a series system is:

$$R_k(0, T) = \prod_{j=1}^{k} \alpha_j R_j(0, T)$$

where \(\alpha\) is the weighting factor that can be determined by the degree of a mechanism unit’s effect on the entire system. Similarly, the reliability of a motion mechanism with a parallel system can be expressed as:

$$R_k(0, T) = 1 - \prod_{j=1}^{k} \alpha_j \{1 - R_j(0, T)\}$$

A hybrid system is a system composed of both series and parallel systems. The reliability of subsystems is calculated using Equations (19) and (20), and then the subsystems are integrated into the corresponding series or parallel systems, thus obtaining the hybrid system’s reliability.

IV. VERIFICATION WITH CALCULATION EXAMPLES

A. NUMERICAL EXAMPLES

Suppose the error function for the motion mechanism with time-varying parameters is:

$$G(t) = X_1X_2 - 2X_2X_3 + 3X_3X_4 F(t) - 4X_4X_1 + 4 \sin(\theta)X_1X_2X_3X_4$$

where the four-dimensional vector input \(X = (X_1, X_2, X_3, X_4)\) contains random vectors that follow the normal distribution, whose parameters are shown in Table 1. Let \(\theta\) denote a random parameter and \(F(t)\) a stationary random process. The mean function \(\mu_F(t, t')\), standard deviation function \(\sigma_F(t, t')\), and autocorrelation coefficient function \(\rho_F(t, t')\) for \(\rho_F(t, t')\) are:

$$\mu_F(t, t') = 1000 + 100 \sin(t)N$$

$$\sigma_F(t, t') = 330 + 33 \sin(t)N$$

$$\rho_F(t, t') = \exp(-0.1 \times (t - t'))$$

In this numerical example, \(T = 30\) years. Monte carlo simulation (MCS) [27] is a general time-varying time-invariant reliability estimation tool with higher computational accuracy. It is often used to verify the accuracy of other methods. MCS and ITRPD are used to calculate the time-varying failure probability \(P_f(0, T)\). The number of discrete points for ITRPD in \([0, T]\) is set to \(P = 20\) to observe the trend of time-varying failure probability over time. To obtain precise results for MCS, the number of discrete points for MCS in \([0, T]\) is set to 100, and the number of samples used is \(n_{MCS} = 1 \times 10^7\).

Figure 1 shows the results obtained by comparing the three methods. One can note that \(P_f(0, t)\) monotonically increases with time because the probability of the mechanism’s failure during its service period generally keeps increasing. This result points to the fundamental difference between the time-varying reliability model and the static reliability model for the mechanism and is consistent with engineering practice.

As is shown by the values in Table 2, the initial failure probabilities \(P_f(0)\) obtained by ITRPD and TRPD are both \(1.23 \times 10^{-3}\), with an error of \(0.08\%\), relative to the calculation result \(1.22 \times 10^{-3}\) obtained by MCS. The failure probabilities \(P_f(0, 10)\) obtained by ITRPD and TRPD are both \(3.52 \times 10^{-3}\), with an error of \(-0.09\%\), whereas MCS yields \(3.52 \times 10^{-3}\). The failure probability \(P_f(0, 20)\) obtained by ITRPD is \(5.77 \times 10^{-3}\), the failure probability \(P_f(0, 20)\) obtained by TRPD is \(5.76 \times 10^{-3}\), with an error of \(-0.42\%\) and \(-0.49\%,\) respectively. And MCS obtains \(5.79 \times 10^{-3}\). The failure probabilities \(P_f(0, 30)\) obtained by ITRPD and TRPD are both \(7.95 \times 10^{-3}\), while the failure probability calculated by MCS is \(8.05 \times 10^{-3}\), with a relative error of \(-1.2\%). These data indicate that the maximum relative error is within 2% when the design life is \(T = 30\), and the results obtained by ITRPD are precise.

Table 3 presents \(N_G\) and \(N_{\varphi_F}\) denote the number of times the limit state equation functions are invoked by the
three methods and the times of high-dimensional Gaussian integration, respectively. When the design reference period is 30 years, the \( N_G \) value of ITRPD is 264, while MCS needs to be invoked \( 1 \times 10^9 \) times. While TRPD invokes multiple times on the high-dimensional Gaussian integration \( \Phi_P (\bullet) \), the result requires 12 times \( \Phi_P (\bullet) \), ITRPD is calculated \( \Phi_P (\bullet) \) only once. This result highlights the ITRPD’s computational efficiency in mechanism analysis. When analyzing motion mechanism failure with time-varying problems, a complex “black box function” is usually utilized. In practical engineering, the main computing resources are consumed by the calculation of complex functions, so the computational efficiency of a method is usually measured according to the calculation amount of the functions invoked. According to Table 2, the value of \( P_f (0, 30) \) is 6.6 times the value of \( P_f (0) \). If designers, faced with practical engineering problems, use the static reliability model for design and mistakenly regard the initial failure probability as the failure probability within ten years, the designed mechanism will fail reliability requirements, posing incalculable potential safety risks. This result demonstrates the importance and engineering significance of the time-varying reliability analysis of the mechanism.

### TABLE 3. The amount of calculation in time-varying reliability analysis for numerical examples.

| Method | ITRPD | TRPD | MCS |
|--------|-------|------|-----|
| \( N_G \) | 264 | 264 | \( 1 \times 10^9 \) |
| \( N_{P_N} \) | 1 | 12 | 0 |

### B. CRANK SLIDER MECHANISM

The crank slider mechanism can convert rotary motion into reciprocating linear motion. It is characterized by convenient processing and high manufacturing precision and has been widely applied in all kinds of machinery, such as punching machines, internal combustion engines, automatic feeding mechanisms, and air compression machines. Typically, when the crank slider mechanism is running, it causes additional dynamic pressure in the kinematic pair under strong inertia that changes periodically. Consequently, it increases the friction in the kinematic pair and the internal stress of the components, leading to increased wear and lower efficiency and reducing the mechanism’s reliability. The crank slider mechanism is shown in Figure 2. As seen in Figure 2, the slider is subjected to dynamic load \( P(t) \) in \( x \)-direction. The length of the crank is the length of the connecting bar \( AB \), \( \alpha \) denotes the rotation angle of bar \( AB \), \( \omega \) denotes the crank’s angular velocity, \( b \) and \( h \) stand for the slider’s width and height, and \( E \) is the elasticity modulus. The crank-slider motion mechanism’s reliability is analyzed.

Load \( P \) in Figure 2 is:

\[
P = f(t) \sin(\alpha)\]

\( \alpha \in [2n\pi, (2n + 1)\pi], \quad n = \{0, 1, 2, 3, 4, \cdots \} \] (25)

where \( \alpha \) is the rotation angle of the bar \( AO \) in Figure 3. It can be seen that the moment of inertia for the cross-section of the bar \( AB \) is:

\[
I_Z = \frac{hb^3}{12} \tag{26}
\]

Thus, the maximum bearable load can be expressed as:

\[
F_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 Ehb^3}{12L^2} \tag{27}
\]

The geometric relationship in Figure 2 enables deducing:

\[
\frac{R}{\sin(\beta)} = \frac{L}{\sin(\alpha)} \tag{28}
\]

\[
\sin^2(\beta) + \cos^2(\beta) = 1 \tag{29}
\]

By combining these equations, one obtains:

\[
\cos(\beta) = \sqrt{1 - \left(\frac{R}{L}\right)^2 \sin^2(\alpha)} \tag{30}
\]

Figure 3 shows that:

\[
F_{AB} = \frac{P}{\cos(\beta)} \tag{31}
\]
Substituting Equation (28) into (29) yields:

$$F_{AB} = \frac{f(t) \sin(\alpha)}{\sqrt{1 - \left(\frac{R}{T}\right)^2 \sin^2(\alpha)}}$$  \hspace{1cm} (32)

Thus, the limit state function for the bar $AB$ can be expressed as:

$$G(E, h, b, R, L, f, \alpha) = F_{cr} - F_{AB} = \frac{\pi^2 Ehb^3}{12L^2} - \frac{f(t) \sin(\alpha)}{\sqrt{1 - \left(\frac{R}{T}\right)^2 \sin^2(\alpha)}}$$  \hspace{1cm} (33)

where $b_0$ and $h_0$ denote the initial values of $b(t)$ and $h(t)$, respectively. Further, $k = 3 \times 10^{-5} m/year$, and the material corrosion reduces the effective width and height of the beam according to the following rules:

$$b(t) = b_0 - 2kt$$  \hspace{1cm} (34)

$$h(t) = h_0 - 2kt$$  \hspace{1cm} (35)

where $f(t)$ denotes a non-stationary random process vector. The mean function $\mu_f(t)$, standard deviation function $\sigma_f(t)$, and autocorrelation function $\rho_f(t,t')$ of $f(t)$ are:

$$\mu_f(t) = 30000 + 3000 \sin(t)$$  \hspace{1cm} (36)

$$\sigma_f(t) = 13000 + 1300 \sin(t)$$  \hspace{1cm} (37)

$$\rho_f(t,t') = \exp(-2(t-t')^2)$$  \hspace{1cm} (38)

In this example, the elastic modulus $E(Pa)$, $h$, and $b$ are random variables, with their distribution parameters shown in Table 4.

**TABLE 4. Random variable distribution parameters of the crank slider mechanism.**

| Variable | Variable type | Mean  | Standard deviation |
|----------|---------------|-------|--------------------|
| $E(Pa)$  | Lognormal     | $2 \times 10^4$ (Pa) | $9 \times 10^6$ (Pa) |
| $b_0$    | Lognormal     | 0.02m | 0.002m             |
| $h_0$    | Lognormal     | 0.01m | 0.001m             |

Similar to the previous example, MCS, TRPD and ITRPD are used to calculate the time-varying failure probability $P_f(0, T)$ of the crank slider mechanism. The number of discrete points for $[0, T]$ in ITRPD is $P = 20$. To obtain precise results from MCS, the number of discrete points for $[0, T]$ in MCS was set to 100, and the number of samples used was $n_{MCS} = 1 \times 10^7$. Figure 4 shows the results calculated by the three methods. Since dynamic load $f(t)$ is a non-stationary random process and the mean value function causes significant fluctuations in values, the $P_f(0, t)$ curve shows a non-linear growth. However, the failure probability again keeps increasing over time, and both the ITRPD and MCS curves show a high fitting degree. According to Table 5, the $P_f(0, 10)$ results calculated by ITRPD, TRPD and MCS are $7.26 \times 10^{-3}$, $6.84 \times 10^{-3}$ and $7.07 \times 10^{-3}$, respectively. The calculation errors of ITRPD and TRPD under a design reference period of 10 years are 2.7% and −3.1%, respectively.

When the design reference period is ten years, the $N_G$ values of ITRPD and TRPD are 520, whereas MCS needs to be invoked $5 \times 10^8$ times. Next, TRPD is invoked 30 times, while ITRPD is invoked $\Phi_P(\bullet)$ only once. The numerical example of the crank slider mechanism suggests that ITRPD has more calculation accuracy than that of TRPD, and the efficiency is also higher than that of TRPD and MCS methods. This represents a significant improvement of the ITRPD method over TRPD for mechanism motion reliability calculations.

**TABLE 5. Calculation results of time-varying failure probability for the crank slider.**

| Method | ITRPD | TRPD | MCS |
|--------|-------|------|-----|
| $P_f(0, 10)$ | $7.26 \times 10^{-3}$ | $6.84 \times 10^{-3}$ | $7.07 \times 10^{-3}$ |
| relative error | 2.7% | -3.1% | - |

**TABLE 6. Consumption calculation of time-varying reliability analysis for the crank slider.**

| Method | ITRPD | TRPD | MCS |
|--------|-------|------|-----|
| $N_G$  | 520   | 520  | $5 \times 10^6$ |
| $N_{\Phi}$ | 1    | 30   | 0   |

### C. RELIABILITY ANALYSIS OF A FOUR-BAR MECHANISM WITH FOLDING WINGS

As a mechanical device that transfers motion laws and loads, the motion mechanism has been widely applied in aircraft. Thus, its reliability directly affects the aircrafts’ safety and the completion of missions. While developing various new aircraft types in China, many serious problems...
related to the motion mechanism reliability have emerged, posing bottlenecks to aircraft development. As discussed, the traditional method for the reliability analysis of static motion mechanisms based on given time points has numerous limitations. In this context, it is necessary to conduct the time-varying reliability analysis based on the actual aircraft working conditions.

Linkage folding mechanisms are widely used in aircraft, including folding wings, landing gear mechanisms, and hatch-door opening and closing mechanisms. Linkage mechanisms must be transmitted through intermediate components, leading to long transmission routes and serious error accumulation. The motion precision of the linkage mechanisms seriously affects the functions’ completion. In this example, the folding wing mechanism is subjected to severe wear and corrosion problems during the working process, resulting in large motion errors compared to actual situations. As shown in Figure 5, the folding wing is composed of several expandable units. For the convenience of the analysis, only one of the expandable units on the airfoil is selected, while other units can be inferred on this basis. With the extension of time, component parameters change due to size attenuation and material degradation, thus affecting the mechanism’s reliability. As shown in Figure 6, $R_1$, $R_2$, $R_3$, and $R_4$ of the four-bar mechanism with expandable units denote the lengths of bar AB, bar BC, bar CD, and bar AD, respectively. The angle parameter of the motion mechanism, and $\psi$ and $\delta$ denote output response values. The reliability of the four-bar mechanism motion is analyzed.

According to Figure 6, the following formula is derived:

$$
\begin{align*}
R_1 \cos \theta + R_2 \cos \delta - R_3 \cos \psi - R_4 &= 0 \\
R_1 \sin \theta + R_2 \sin \delta - R_3 \sin \psi &= 0 \\
R_3 \sin \psi &= 2 \arctan \frac{D \pm \sqrt{D^2 + E^2 - F^2}}{E + F}
\end{align*}
$$

where $\psi$ and $\delta$ denote output response values. Now:

$$
R_0 = R_{01} - 2kt
$$

In the four-bar mechanism, $R_1$, $R_2$, $R_3$, and $R_4$ denote the lengths of bars AB, BC, CD, and AD, respectively. The corrosion strength on the entire bar is isotropic. Its change laws over time are as follows:

$$
\begin{align*}
R_1 &= R_{01} - 2kt \\
R_2 &= R_{02} - 2kt \\
R_3 &= R_{03} - 2kt
\end{align*}
$$

where $R_{01}$, $R_{02}$, and $R_{03}$ denote the initial values of $R_1(t)$, $R_2(t)$, and $R_3(t)$, respectively, $k = 3 \times 10^{-8}m$. In this numerical example, $R_{01}$, $R_{02}$, and $R_{03}$ are processed as random vectors, and $R_4(t)$ is processed as a stationary random process vector.

In the folding wing four-bar unit mechanism example, due to the material degradation and geometric dimension, the motion mechanism reliability is no longer a fixed value but shows a downward trend with the extension of service life. Again, MCS, TRPD and ITRPD are used to calculate the time-varying failure probability $P_f(0, T)$. The number of discrete points for $[0, T]$ in MCS is $P = 20$. To obtain precise results for MCS, the number of discrete points for $[0, T]$ in MCS is set to 100, and the number of samples used is $n_{MCS} = 1 \times 10^9$. Figure 7 shows the results obtained by the three methods. The corresponding curves again show a high fitting...
According to the data in Table 8, the $P_f(0, 10)$ results calculated by ITRP, TRPD and MCS are $9.09 \times 10^{-3}$, $9.08 \times 10^{-3}$ and $9.28 \times 10^{-3}$, respectively. The calculation errors of ITRPD and TRPD under a design reference period of 10 years are $-2.08$ and $-2.21$, respectively.

As shown in Table 9, when the design reference period is ten years, the $N_G$ values of ITRP and TRPD are both 315, and MCS needs to be invoked $1 \times 10^9$ times. Next, TRPD is invoked 15 times $\Phi_P(\bullet)$, while ITRPD is invoked $\Phi_P(\bullet)$ only once. The specific numerical examples demonstrate that ITRPD has a higher calculation efficiency than that of MC and TRPD in analyzing the motion mechanism. For the folding wing with large deformation in the motion process as time passes, the dimensional change caused by the components’ wear should be taken into account. According to Figure 7, the $P_f(0, 10)$ value is 9 times that of $P_f(0)$. The results show that the time-varying failure probability of folding wings presents a rising trend due to the influence of time-varying parameters. Therefore, in designing the configuration of the folding wing mechanism, the time-varying effect of reliability needs to be considered, and a reliability margin should be given at the beginning of the design to ensure normal operation of the folding wing mechanism during the whole life cycle.

The folding wing mechanism is composed of several series systems. The series system selected for the research is composed of two four-bar units. Since the two units are equally important, it is assumed that $\alpha_1 = \alpha_2 = 0.5$. According to the presented time-varying reliability method of the folding wing four-bar unit motion mechanism, the time-varying failure probability of the other four-bar unit in the series system can be calculated, as detailed in Figure 8. The $P_f(0, 10)$ value is $9.09 \times 10^{-3}$. According to Equation (19), the result of the time-varying failure probability of the folding wing series
system is obtained (see Figure 9). The \( P_f(0, 10) \) value is \( 9.8 \times 10^{-3} \). These data indicate that with the increase in the set reference period, the failure probability continuously increases, which is consistent with engineering practice.

V. CONCLUSION AND DISCUSSION
This paper deals with the problem of the overall degradation of the motion mechanism’s component performance with the extension of service time. A time-varying reliability analysis method for motion mechanisms is proposed. First, a motion mechanism error model is established according to the difference between the actual and expected motion of a motion mechanism. By relying on time discretization, the time-varying problem is transformed into several time-invariant reliability problems. The reliability of each unit is analyzed using FORM, and a correlation method is used to calculate the correlation coefficient matrix. In addition, high-dimensional Gaussian integration is performed to obtain the time-varying reliability of the motion mechanism unit based on the equivalent correlation coefficient matrix. Finally, the weighting coefficient is introduced to calculate the time-varying reliability of the motion mechanisms system.

The proposed method handles the time-varying reliability problem of nonlinear motion mechanisms involving both random variables and random processes. For example, in the crank slider mechanism example, the external load parameter is a random process variable, and the material’s elastic modulus is processed as a random vector. By employing the proposed method, the maximum calculation error does not exceed 3%, and the number of times the functions are invoked is 520, which is much lower than that in the case of MCS. And ITRPD only needs to be invoked once for the high-dimensional Gaussian integral. These results demonstrate the efficiency and precision of the proposed method.

Nevertheless, while the method proposed in this paper advances practical problem solving, it may fail to converge in scenarios characterized by high reliability and strong nonlinearity. Therefore, the time-varying reliability analysis method for motion mechanisms will be further developed and utilized in research on mechanism dynamics.

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