Computation of Neutron Star Structure
Using Modern Equation of State

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Abstract
Using the modern equations of state derived from microscopic calculations, we have calculated the neutron star structure. For the neutron star, we have obtained a minimum mass about 0.1 M⊙ which is nearly independent of the equation of state, and a maximum mass between 1.47 M⊙ and 1.9! M⊙ which is strongly dependent on the equation of state. It is shown that among the equations of state of neutron star matter which we have used, the stiffest one leads to higher maximum mass and radius and lower central density. It is seen that the given maximum mass for the Reid-93 equation of state shows a good consistency with the accurate observations of radio pulsars. We have indicated that the thickness of neutron star crust is very small compared to the predicted neutron star radius.

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I. INTRODUCTION

For identifying of an astrophysical object as a black hole, it is required to know the maximum gravitational mass of a neutron star for stability against collapse into a black hole. In the other word, it is expected that below a certain maximum mass, degeneracy pressure due to the nucleons is sufficient to prevent an object from becoming a black hole. Therefore, determining the maximum gravitational mass of neutron star is of special importance in astrophysics.

Observationally, the X-ray pulsars and X-ray bursters can offer direct measurement of the neutron star mass, but their accuracy is rather poor and due to large errors, measuring masses of them are not very useful. Fortunately, the masses of neutron stars have been determined with high accuracy using the binary radio pulsars [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

Theoretically, in calculating a maximum mass of neutron star which should be in agreement with the precise observations, the equation of state of neutron star matter plays a crucial role. Theoretical investigations of the neutron star matter indicates that a considerable uncertainties exist in the behavior of the equation of state, especially at high densities. This is due to using the old non phase-shift equivalent nucleon-nucleon potentials [16]. These uncertainties result a significant uncertainty in the maximum mass of neutron star. Therefore having a modern equation of state derived from an accurate many-body calculation using modern nucleon-nucleon potentials is of particular importance in the mass determination for the neutron star [11, 14, 17].

Recently, we have obtained the equation of state of neutron star matter using the microscopic constrained variational calculations based on the cluster expansion of the energy functional [18, 19]. In these calculations, we have employed the modern two-nucleon potentials such as the new Argonne AV$_{18}$ [20] and charged dependent Reid-93 [21]. This is a fully self-consistent technique which does not bring any free parameter into the calculations and its results show a good convergence. In this method, for the asymmetric matter calculations, a microscopic computation of asymmetry energy is done and therefore its results are more accurate with respect to the other methods which use the semi-empirical parabolic approximation. In fact, using the modern nucleon-nucleon potentials which are explicitly depend on the isospin projection ($T_z$), requires a microscopic calculation [22, 23, 24]. In this article,
we investigate some physical properties of neutron star structure using modern equations of state of neutron star matter \[18, 19\].

**II. EQUATION OF STATE AND NEUTRON STAR STRUCTURE**

As it is mentioned in the previous section, the equation of state of neutron star matter has a key role in determining the neutron star structure, especially its maximum mass. In this work, we study the structure of neutron star using the modern microscopic equations of state of neutron star matter employing the AV$_{18}$ and Reid-93 modern two-nucleon potentials as well as the AV$_{14}$ potential \[25\]. In our calculations, we also consider the effect of three-body force using the UV$_{14}$ + TNI potential in which the effect of three-nucleon interaction is included by adding two density dependent terms into the two-body potential \[26\]. The procedure of our calculations is discussed in references \[18, 19\]. Our results for the equation of state of neutron star matter are given in Figure 1. It is seen that by inclusion of three-nucleon interaction, we find a stiffer equation of state. From Figure 2 it is seen that for the Reid-93, AV$_{14}$ and AV$_{18}$ equations of state, the speed of sound (\(C_s\)) is always less than the speed of light, even at high densities. In the case of UV$_{14}$ + TNI equation of state, above \(\rho = 1.28 \text{ fm}^{-3} (\epsilon = 29.61 \times 10^{14} \text{ g/cm}^3)\), the sound speed exceeds the speed of light. However, as we will see, for this equation of state, the neutron star mass reaches a limiting value (maximum mass) below this density.

Using the equation of state of the neutron star matter, we can calculate the neutron star mass and radius as a function of central mass density, \(\epsilon_c\), by numerically integrating the general relativistic equation of hydrostatic equilibrium, Tolman-Oppenheimer-Volkoff (TOV) equation \[1\],

\[
\frac{dP}{dr} = -\frac{G}{r^2}[\epsilon(r) + P(r)/c^2]\frac{m(r) + 4\pi r^3 P(r)/c^2}{1 - \frac{2Gm(r)}{rc^2}},
\]

where

\[
\epsilon = \rho[E(\rho) + mc^2]
\]

is the mass density, \(G\) is the gravitational constant, and

\[
m(r) = \int_0^r 4\pi r'^2 \epsilon(r')dr'
\]

has the interpretation of the mass inside radius \(r\). By selecting a central mass density \(\epsilon_c\), under the boundary conditions \(P(0) = P_c, m(0) = 0\), we integrate the TOV equation
outwards to a radius $r = R$, at which $P$ vanishes. This yields the neutron star radius $R$ and mass $M = m(R)$.

For densities greater than $0.05 \text{ fm}^{-3}$, we use our equations of state presented in Figure 1. However, at the lower densities, we use the equation of state calculated by Baym et al. [27], since the details of the equation of state at low densities do not affect our results.

The neutron star gravitational mass (in solar mass $M_\odot$ units) as a function of central mass density, $\epsilon_c$, calculated with the $\text{AV}_{18}$, Reid-93, $\text{AV}_{14}$ and $\text{UV}_{14} + \text{TNI}$ equations of state is shown in Figure 3. Our results show that at low densities, the calculated neutron star masses exhibit a minimum ($\simeq 0.1 \text{ } M_\odot$) which depends weakly on the equation of state of neutron star matter. It can be seen that at high densities, the increasing of mass becomes very slow and finally it approaches a limiting value which strongly depends on the equation of state. This limiting value of mass is the maximum gravitational mass of neutron star and its corresponding central density is the highest possible value for the neutron star central density. A star with the higher central density would be unstable against the gravitational collapse to a black hole. A comparison between Figure 1 and Figure 3 indicates that there is a one-to-one correspondence between the behavior of equation of state of neutron star matter, especially at high densities, and the resulting mass of neutron star. The stiffest equation of state leads to the higher mass and lower central densities. It should be noted that, as mentioned above, the limiting value of neutron star mass with the $\text{UV}_{14} + \text{TNI}$ equation of state is reached below the density in which the speed of sound exceeds the speed of light.

In Figure 4, we have presented the radius of neutron star with the $\text{AV}_{18}$, Reid-93, $\text{AV}_{14}$ and $\text{UV}_{14} + \text{TNI}$ potentials as a function of central mass density. It is seen that as the central density increases, the radius decreases very rapidly and then for $\epsilon_c \geq 7 \times 10^{14} \text{ g/cm}^3$, it reaches a nearly constant value. This shows that the radius of neutron star is nearly independent of the central density. From Figure 4, we also observe that the stiffest equation of state leads to a relatively larger radius for the neutron star.

The gravitational mass versus radius (M-R curve) for the $\text{AV}_{18}$, Reid-93, $\text{AV}_{14}$ and $\text{UV}_{14} + \text{TNI}$ potentials is shown in Figure 5. Our results indicate that for the neutron star, there is a minimum gravitational mass about $0.1 \text{ } M_\odot$ which is nearly identical for different equations of state, and a maximum mass which is different for these equations of state. It is well known that in this mass region, the equilibrium configuration of neutron
stars can exist.

A summary of our results for the properties of maximum mass configuration of neutron star predicted for different equations of state are given in Table III. From Table III, it can be seen that the inclusion of the three-nucleon interaction considerably affects the calculated properties of neutron star maximum mass configuration. Here, we explicitly see that the stiffest equation of state used in our calculations gives the lower central density and higher maximum mass and radius for the neutron star. As it is seen in Table III, our results give the maximum mass of neutron star between 1.47 M⊙ and 1.98 M⊙. This agrees with the measured range of neutron star masses and the results of other theoretical investigations [11, 14, 28, 29]. Furthermore, our predicted maximum mass of neutron star with the Reid-93 potential has a good consistency with the mass determined from the accurate observations of radio pulsars [2, 5, 9]. This is a good confirmation for our equation of state of dense matter with the modern nucleon-nucleon potential, Reid-93. If we use the equation of state of noninteracting beta-stable matter, a maximum mass 0.7 M⊙ is obtained. Comparing this value with those presented in Table III implies that at high densities, the nucleon-nucleon interaction is sufficiently repulsive to produce a considerable shift in the maximum mass of neutron star with respect to the noninteracting case.

For a neutron star, the binding energy is defined as the energy needed to transform it into a dispersed configuration. Formally, the binding energy can be obtained from the difference between amu mass $M_A$ and gravitational mass, where $M_A$ is given by [28]

$$M_A = m_A \int_0^R \frac{4\pi r^2 \epsilon(r)}{[1 - 2m(r)G/rc^2]^{1/2}} dr.$$  

Here, we take $m_A$ as one amu ($1.66 \times 10^{-24}$ g). For all values of gravitational masses (greater than the predicted minimum mass), our equations of state give positive binding energies for neutron stars. This indicates that the neutron stars are always stable against the dispersed configuration.

We have calculated the mass density versus radial coordinate for a neutron star with the mass equal to 1.47 M⊙ for the Reid-93 equation of state. This is presented in Figure 6 which shows that up to $r \simeq 7.8$ km (within the core), the mass density is nearly uniform, and then it drops very rapidly (close to the crust). Our results indicate that the crust thickness is about 0.43 km which is only 5.2% of our predicted neutron star radius. This implies that the core contains the most fraction of neutron star matter.
By solving the TOV equation, we have also calculated the properties of neutron star with the gravitational mass equal to $1.4 \, M_\odot$. Our results for different equations of state are presented in Table II. Comparing Tables III and II indicates that for a $1.4 \, M_\odot$ neutron star, our used equations of state give higher radius and lower central density with respect to those predicted for maximum mass configuration. The differences between these two cases become more significant for the stiffest equation of state.

III. SUMMARY AND CONCLUSION

In recent years, a wide range of information on neutron stars are available from X-ray pulsars, X-ray bursters and radio pulsars. Therefore, the study of neutron stars structure on the basis of the equation of state of dense matter is of special interests in astrophysics. Theoretical investigation of neutron star structure is also important, since the observational results leads to the constraints on the equation of state of dense matter.

In present paper, we have computed some properties of neutron star structure using the modern equations of state of neutron star matter obtained by microscopic constrained variational calculations based on the modern potentials. A minimum value for the gravitational mass of neutron star is predicted near $0.1 \, M_\odot$ which is nearly identical for different equations of state. It is seen that a higher maximum mass is obtained, when we consider the effect of three-nucleon interaction in our calculations. We have shown that the radius of neutron star becomes nearly constant for all central densities approximately greater than $7 \times 10^{14} \, \text{g/cm}^3$. The properties of maximum mass configuration of neutron star are predicted. These properties change considerably by considering the three-nucleon interaction. Our results show that the stiffest equation of state gives the higher maximum mass and radius and lower central density for the neutron star. The obtained maximum mass of neutron star is between $1.47 \, M_\odot$ and $1.98 \, M_\odot$ which agrees with the masses determined from observations. In addition, the predicted value $1.47 \, M_\odot$ by our used Reid-93 equation of state is in a good agreement with the accurately determined mass of radio pulsars. We have seen that in our calculated mass region, the binding energy of neutron star is positive and therefore it is stable with respect to the dispersed configuration. We have computed the mass density profile for a neutron star with the mass equal to $1.47 \, M_\odot$ for the Reid-93 equation of state. It is shown that the neutron star crust has a thickness about $0.43 \, \text{km}$ which is very small.
with respect to the obtained neutron star radius. We have also computed the properties of a 1.4 $M_\odot$ neutron star with the different equations of state. A higher radius and lower central density are predicted for a 1.4 $M_\odot$ neutron star compared to the values calculated for the maximum mass configuration. Finally, a good agreement between our results for the neutron star structure and those of other theoretical calculations is observed.

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[1] S. Shapiro and S. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars (Wiley, New York, 1983).
[2] J.M. Weisberg and J.H. Taylor, Phys. Rev. Lett. 52, 1348 (1984).
[3] E.P. Liang, Astrophys. J. 304, 682 (1986).
[4] M.Y. Fujimoto and R.E. Taam, Astrophys. J. 305, 246 (1986).
[5] J.H. Taylor and J.M. Weisberg, Astrophys. J. 345, 434 (1989).
[6] S.R. Heap and M.F. Corcoran, Astrophys. J. 387, 340 (1992).
[7] G.E. Brown, J.C. Weingartner and R.A.M.J. Wijers, Astrophys. J. 463, 297 (1996).
[8] M.C. Miller, F.K. Lamb and P. Psaltis, Astrophys. J. 508, 791 (1998).
[9] S.E. Thorsett and D. Chakrabarty, Astrophys. J. 512, 288 (1999).
[10] J.A. Orosz and E. Kuulkers, Mon. Not. R. Astron. Soc. 305, 132 (1999).
[11] J.M. Lattimer and M. Prakash, Astrophys. J. 550, 426 (2001).
[12] H. Heiselberg, preprint [astro-ph/0201465].
[13] P.G. Jonker, M. van der Klis and P.J. Groot, Mon. Not. R. Astron. Soc. 339, 663 (2003).
[14] P. Haensel, preprint [astro-ph/0301073].
[15] H. Quaintrell, Astron. Astrophys. 401, 313 (2003).
[16] L. Engvik, M. Hjorth-Jensen, R. Machleidt, H. Muther and A. Polls, Nucl. Phys. A627, 85 (1997).
[17] J.M. Lattimer and M. Prakash, Phys. Rep. 333, 121 (2000).
[18] G.H. Bordbar and N. Riazi, Astrophys. Space Sci. 282, 563 (2002).
[19] G.H. Bordbar, Int. J. Theor. Phys. 43, 399 (2004).
[20] R.B. Wiringa, V. Stoks and R. Schiavilla, Phys. Rev. C51, 38 (1995).
[21] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen and J. J. de Swart, Phys. Rev C49, 2950 (1994).
[22] G.H. Bordbar and M. Modarres, J. Phys. G: Nucl. Part. Phys., 23, 1631 (1997).
[23] G.H. Bordbar and M. Modarres, Phys. Rev. C57, 714 (1998).
[24] G.H. Bordbar, Int. J. Mod. Phys. A18, 2629 (2003).
[25] R.B. Wiringa, R.A. Smith and T.L. Ainsworth, Phys. Rev. C29, 1207 (1984).
[26] I.E. Lagaris and V.R. Pandharipande, Nucl. Phys. A359, 349 (1981).
[27] G. Baym, C. Pethick and P. Sutherland, Astrophys. J. 170, 299 (1971).

[28] R.B. Wiringa, V. Fiks and A. Fabrocini, Phys. Rev. 38, 1010 (1988).

[29] M. Baldo, I. Bombaci and G.F. Burgio, Astron. Astrophys. 328, 274 (1997).
TABLE I: Maximum gravitational mass ($M_{\text{max}}$), corresponding radius ($R$), central mass density ($\epsilon_c$), and central number density ($\rho_c$) obtained with the AV$_{18}$, Reid-93, AV$_{14}$ and UV$_{14}$ + TNI potentials.

| Potential     | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) | $\epsilon_c$ ($10^{14}$ g/cm$^3$) | $\rho_c$ (fm$^{-3}$) |
|---------------|-------------------------------|----------|-----------------------------------|----------------------|
| Reid-93      | 1.47                          | 8.23     | 35.39                            | 1.7                  |
| AV$_{14}$    | 1.56                          | 8.28     | 33.93                            | 1.6                  |
| AV$_{18}$    | 1.65                          | 8.79     | 31.16                            | 1.45                 |
| UV$_{14}$ + TNI | 1.98                        | 9.81     | 27.17                            | 1.2                  |

TABLE II: Properties of neutron star with the gravitational mass equal to 1.4 $M_\odot$ for different equations of state.

| Potential     | $R$ (km) | $\epsilon_c$ ($10^{14}$ g/cm$^3$) | $\rho_c$ (fm$^{-3}$) |
|---------------|----------|-----------------------------------|----------------------|
| Reid-93      | 8.51     | 27.71                            | 1.4                  |
| AV$_{14}$    | 8.81     | 22.83                            | 1.2                  |
| AV$_{18}$    | 9.58     | 18.25                            | 0.95                 |
| UV$_{14}$ + TNI | 11.16  | 11.32                            | 0.6                  |
FIG. 1: Pressure ($P$) versus mass density ($\epsilon$) for the neutron star matter with the Reid-93 (full curve), $AV_{14}$ (dashed curve), $AV_{18}$ (dotted curve) and $UV_{14} + TNI$ (heavy dotted curve) potentials.
FIG. 2: As Figure 1, but for the sound speed \( C_s/c \) versus number density \( \rho \).
FIG. 3: As Figure 1 but for the gravitational mass of neutron star (in units of the solar mass $M_\odot$) versus central mass density ($\epsilon_c$).
FIG. 4: As Figure 1, but for the radius versus central mass density ($\epsilon_c$).
FIG. 5: As Figure [11] but for the mass-radius relation.
FIG. 6: Mass density as a function of radial coordinate for a $1.47 \, M_\odot$ neutron star with the Reid-93 potential.