On a gauge-invariant interaction of spin-$\frac{3}{2}$ resonances. *

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We show that the gauge-invariant coupling suggested by Pascalutsa removes non-pole terms from a spin-$\frac{3}{2}$ propagator only for a specific choice of free parameter. For the general case the problem can be solved by including higher order derivatives of spin-$\frac{3}{2}$ fields or by modifying the original coupling. In the latter case the obtained Lagrangian depends on one free parameter pointing to the freedom in choosing an 'off-shell' content of the theory. However, the physical observables must not be affected by the 'off-shell' contributions and should not depend on the free parameter of the Lagrangian.

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I. INTRODUCTION

Since its appearance the theory of higher integer and half-integer spin fields has drawn much attention. The physical motivation was to find an acceptable way of description for higher spin baryon and meson resonances. The interest has mainly been focused on the spin-$\frac{3}{2}$ fields. This is not surprising because of the role which the $\Delta_{33}(1232)$-isobar plays in $\pi N$ interaction above the pion production threshold.

In 1941 Rarita and Schwinger (R-S) have suggested a set of equations which a field function of higher spin should obey [1]. These constraints should directly follow from the corresponding free field Lagrangian of the given spin. Regardless of the procedure used the obtained Lagrangians for free higher-spin fields turn out to be always dependent on arbitrary free parameters. In case of spin-$\frac{3}{2}$ fields this problem is widely discussed in the literature (see e.g. [1, 2, 3, 4, 5]). While the free theory is invariant under so-called 'point-transformations' [2] inclusion of interaction breaks this symmetry. As a result the interaction term depends on a parameter of transformation which is commonly known as an 'off-shell' parameter. It has been shown that this dependence cannot be removed from the full Lagrangian. A critical overview of the problem can be found e.g. in [2, 6].

The possibility to construct consistent higher-spin massless theories has already been pointed out by Weinberg and Witten a long while ago [7]. Recently Pascalutsa has shown [8, 9, 10, 11], that by using a gauge invariant coupling for higher spin fields it is possible to remove extra-degrees of freedom at interaction vertices. As a result, the physical observables do not depend on the off-shell content of the theory.

It is well known, however, that the wave equation for the free spin-$\frac{3}{2}$ field being written in a general form depends on one free parameter $A$ (see e.g. [2]). The commonly used Rarita-Schwinger theory [1] corresponds to the special choice $A = -1$. While the Pascalutsa-coupling removes the unwanted degrees of freedom from the Rarita-Schwinger propagator it leaves the problem unsolved in the more general case $A \neq -1$ resulting in the appearance of 'off-shell' components, for example in the $\pi N$ scattering amplitude. Hence, further investigations of the general properties of the interacting spin-$\frac{3}{2}$ fields become of great importance. In this Letter we discuss the origin of this problem and show how it can be solved. We indicate two alternative approaches. The first recipe consists in constructing a coupling which includes higher derivatives of the spin-$\frac{3}{2}$ field. Another method is based on the generalization of the original gauge invariant interaction to arbitrary values of $A$. In the latter case the obtained Lagrangian depends on one free parameter which also appears in the free field formalism. However, the physical observables should not depend on this parameter. Hence, the matrix element corresponding to the $\pi N$ scattering at tree level does not contain an 'off-shell background'.

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II. SPIN-$\frac{3}{2}$ FIELD.

Rarita and Schwinger suggested a set of constraints which the free spin-$\frac{3}{2}$ field should obey \[1\]

$$
\gamma^\nu \psi_\nu(x) = 0,
\partial^\nu \psi_\nu(x) = 0,
$$

(1)

provided that also the Dirac equation \((\not{\partial} - m)\psi_\nu(p) = 0\) is fulfilled. In the consistent theory the set of equations eq. \[1\] should follow from the equation of motion obtained from the corresponding Lagrangian. The Lagrangian of the free spin-$\frac{3}{2}$ field can be written in a general form as follows (see \[2\] and references therein)

$$
\mathcal{L}_0^2 = \Delta_\mu(x) A^{\mu\nu} \Delta_\nu(x),
$$

(2)

where \(\Delta_\nu(x)\) stands for the spin-$\frac{3}{2}$ field and the \(A^{\mu\nu}\)-operator is

$$
A^{\mu\nu} = (i\not{\partial} - m)g^{\mu\nu} + iA(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu)
+ \frac{1}{2}(3A^2 + 2A + 1)\gamma^\mu \partial^\nu
+ m(3A^2 + 3A + 1)\gamma^\mu \gamma^\nu,
$$

(3)

where \(A\) is an arbitrary free parameter subject to the restriction \(A \neq -\frac{1}{2}\). The propagator of the free spin-$\frac{3}{2}$ field can be obtained as a solution of the equation, e.g. in momentum space,

$$
A_{\mu\nu}(p) g^{\sigma\rho} G_{\sigma\nu}(p) = g_{\mu\nu}.
$$

(4)

The obtained propagator \(G_{\sigma\nu}(p)\) can be written as an expansion in terms of the spin projection operators \(P^2_{11;\mu\nu}\), \(P^2_{22;\mu\nu}\), \(P^2_{12;\mu\nu}\), and \(P^2_{21;\mu\nu}\) given in \[12\]. The first three operators correspond to different irreducible representations of spin-vector whereas the last two ones stand for a mixing between two spin-$\frac{3}{2}$ representations, see \[12\]. In terms of these projectors the propagator is represented as

$$
G_{\mu\nu}(p, A) = \frac{1}{p^2 - m^2 + i\epsilon} \left( (\not{\partial} + m)P^2_{\mu\nu}(p) + \frac{p^2 - m^2}{m(2A + 1)^2} D^2_{\mu\nu}(p) \right),
$$

(5)

where the 'off-shell' spin-$\frac{1}{2}$ part is given by

$$
D^2_{\mu\nu}(p) = -\frac{(1 + 3A)(2 + 3A)m + (3A + 1)\not{\partial}}{6m}(P^2_{22;\mu\nu}(p)
- \frac{(1 + A)(1 + 3A)}{2\sqrt{3}m}\not{\partial}\left(P^2_{12;\mu\nu}(p) - P^2_{21;\mu\nu}(p)\right)
+ \frac{(3A^2 + 3A + 1)}{\sqrt{3}}\left(P^2_{12;\mu\nu}(p) + P^2_{21;\mu\nu}(p)\right)
- \frac{(1 + A)(2Am - (1 + A)\not{\partial})}{2m}P^2_{11;\mu\nu}(p).
$$

(6)

While the 'off-shell' part does not contain any poles it gives rise to the full propagator in the whole energy-momentum region. The well-known R-S propagator is a special case of eqs. \[5\], \[6\] corresponding to the choice \(A = -1\), i.e.

$$
G^\text{RS}_{\mu\nu}(p) = \frac{1}{p^2 - m^2 + i\epsilon} \left( (\not{\partial} + m)P^2_{\mu\nu}(p) + \frac{p^2 - m^2}{m} \left[ \frac{2(\not{\partial} + m)}{3m} P^2_{22;\mu\nu}(p)
+ \frac{1}{\sqrt{3}}\left(P^2_{12;\mu\nu}(p) + P^2_{21;\mu\nu}(p)\right) \right] \right).
$$

(7)

A commonly used \(\Delta N\pi\)-coupling is written as \(\mathcal{L}_{\text{int}} \sim \bar{\psi}N \theta(z)^{\mu\nu} \Delta_\mu \partial_\nu \pi\) with \(\theta^{\mu\nu}(z) = g^{\mu\nu} + z\gamma^\mu \gamma^\nu\). The free parameter \(z\) is used to scale the 'off-shell' contributions to the interaction vertex but does not affect the pole term. In order to remove the dependence on the off-shell parameters and eliminate the unwanted degrees of freedom Pascalutsa suggested
The corresponding Feynman graph at tree level. The amplitude of interest can be written as
\[ \mathcal{L}_P = \frac{g_N \pi}{m_N m_\pi} \psi_N(x) \gamma_5 \gamma_\mu T_\Delta^{\mu \nu}(x) \partial_\nu \pi(x) + \text{h.c.}, \]
\[ T_\Delta^{\mu \nu}(x) = \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} (\partial_\mu \Delta_\rho(x) - \partial_\rho \Delta_\mu(x)), \tag{8} \]
where \( \epsilon^{\mu \nu \rho \sigma} \) is the fully antisymmetric Levi-Civita tensor. The tensor \( T_\mu(\pi) \) is invariant under the gauge-transformations \( \Delta_\mu(x) \to \Delta_\mu(x) + \partial_\mu \xi(x) \) where \( \xi(x) \) is an arbitrary spinor field. Hence, \( T_\mu(\pi) \) behaves like a 'conserved current' with the constraint \( \partial_\mu T_\mu(\pi) = 0 \). The coupling defined in eq. (8) guarantees that so-called 'off-shell background' does not contribute to the physical observables provided that the free spin-\( \frac{3}{2} \) propagator is chosen in the special form of eq. (7) corresponding to \( A = -1 \). This, however, does not hold in the general case for arbitrary values of \( A \).

The problem can be demonstrated on the example of the \( s \)- or \( u \)-channel \( \pi N \) scattering amplitude calculated from the corresponding Feynman graph at tree level. The amplitude of interest can be written as
\[ M = \left( \frac{g_N \pi}{m_N} \right)^2 \tilde{\psi}_N(p') \left[ \Gamma_{\mu \rho}(p_\Delta) \frac{1}{m_N^2} G^{\sigma \rho}(p_\Delta) \Gamma^{\mu \nu}_\sigma(p_\Delta) \right] \psi_N(p) (q_\pi')^\nu (q_\pi')^\nu, \tag{9} \]
where \( p'(q_\pi') \) and \( p(q_\pi) \) are momenta of the initial and final nucleon(pion) correspondingly; \( p_\Delta \) stands for the momentum of the \( \Delta \)-isobar and depends on the channel (\( s \)- or \( u \)-) of interest. The 'current conserved' vertex function \( \Gamma_{\mu \rho}(p_\Delta) \) reads
\[ \Gamma_{\mu \rho}(p_\Delta) = \epsilon_{\mu \nu \rho \sigma} \gamma_\nu \gamma_\sigma p_\Delta^\rho. \tag{10} \]
Using the explicit expression for the free spin-\( \frac{3}{2} \) propagator in the form eqs. (5),(6) with \( A = 1 \) it is easy to show that the final result contains an extra contribution coming from the \( \mathcal{P}_{11; \mu \nu}(p) \) projection operator
\[ \left[ \Gamma_{\mu \rho}(p_\Delta) \frac{1}{m_N^2} G^{\rho \sigma}(p_\Delta) \Gamma^{\mu \nu}_\sigma(p_\Delta) \right] = \frac{\hat{p} + m}{p^2 - m^2} \frac{p_\Delta^2}{m_N^2} \mathcal{P}_{11; \mu \nu}(p_\Delta). \tag{11} \]
This result obtained first in \[8, 9, 10, 11\] demonstrates that the interaction chosen in the form of eq. (8) removes the lower spin components which enter to the original Rarita-Schwinger propagator. A closer inspection shows, that this statement, however, is not true in the general case as we discuss in the following. By choosing the free spin-\( \frac{3}{2} \) propagator in the form eqs. (5),(6) with \( A = -1, -\frac{1}{2} \) it is easy to show that the final result contains an extra contribution coming from the \( \mathcal{P}_{11; \mu \nu}(p) \) projection operator
\[ \left[ \Gamma_{\mu \rho}(p_\Delta) \frac{1}{m_N^2} G^{\rho \sigma}(p_\Delta, A) \Gamma^{\mu \nu}_\sigma(p_\Delta) \right] = \frac{\hat{p} + m}{p^2 - m^2} \frac{p_\Delta^2}{m_N^2} \mathcal{P}_{11; \mu \nu}(p_\Delta) + f \left( \mathcal{P}_{11; \mu \nu}(p_\Delta) \right), \tag{12} \]
where \( f \left( \mathcal{P}_{11; \mu \nu}(p_\Delta) \right) \) is a function of parameter \( A \) and operator \( \mathcal{P}_{11; \mu \nu}(p_\Delta) \). The second (non-pole) term on the right side of eq. (12) represents the 'off-shell' spin-\( \frac{3}{2} \) background. By multiplying e.g. the right part of eq. (12) by the operator \( \theta^{\mu \nu} = g^{\nu \tau} + z \gamma_\nu \gamma_\tau \) one can see that the obtained expression will again be sensitive to the 'off-shell' parameter \( z \) - an unwanted feature which we want to eliminate from the theory. Note, that the coupling of eq. (8) is invented under assumption that the interaction must support the local symmetries (gauge invariance) of the free massless R-S formulation, which does not hold for \( A \neq -1 \).

The appearance of the non-pole spin-\( \frac{3}{2} \) component in eq. (12) can be easily traced back. The projection operators used in the decomposition of the propagator eq. (5) correspond to various irreducible representations of the spin-vector field. The coupling of eq. (8) leads to the 'current conserved' vertex function eq. (10): \( \mathcal{P}_{11; \mu \nu}(p_\Delta) = \mathcal{P}_{11; \mu \nu}(p_\Delta) = 0 \). Hence, only those terms in eqs. (8),(9) give rise to observables which satisfy the condition
\[ \mathcal{P}_{ij; \mu \nu}(p) = 0, \tag{13} \]
where \( J = \frac{3}{2}, \frac{1}{2} \) and \( i, j = 1, 2 \). The \( \mathcal{P}_{11; \mu \nu}(p) \)-operator fulfills this constraint by definition. The \( \mathcal{P}_{11; \mu \nu}(p) \) projection operator corresponds to the \( \left[ \frac{3}{2} \otimes 1 \right]_1 \) irreducible representation which contains the vector component explicitly.

\[ ^1 \] Throughout the paper we omit isospin indices.
Therefore, it is subject to the condition $p^\mu \mathcal{P}_{11;\mu\nu}^2(p) = 0$ too. As a result both projection operators contribute to the matrix element eq. (12).

The problem reported above can be solved in different ways. The straightforward one is to use a coupling with higher order derivatives of the spin-$\frac{3}{2}$ field which explicitly involves the $\mathcal{P}_{11;\mu\nu}^2(p)$ projection operator:

$$\frac{g_{\Delta N\pi}}{m_\pi m_N} \bar{\psi}_N(x) \left[ \Box \mathcal{P}_{11;\mu\nu}^2(\partial) \Delta^\nu(x) \right] \partial^\mu \pi(x) + \text{h.c.}. \quad (14)$$

The use of $\mathcal{P}_{11;\mu\nu}^2(\partial)$ ensures that only the spin-$\frac{3}{2}$ part of the propagator contributes and the d’Alembert-operator guarantees that no other singularities except the mass pole term $(p^2 - m^2)^{-1}$ appear in the matrix element. Note, that the coupling written in the form of eq. (14) restores the invariance of the full Lagrangian under the 'point-like' transformations $\Delta^\mu \rightarrow \Delta^\mu + z \gamma^\mu \Delta^\nu$.

To keep the interaction term in the full Lagrangian as simple as possible we propose here another coupling

$$\mathcal{L}_I = \frac{g_{\Delta N\pi}}{m_\pi m_N} \bar{\psi}_N(x) \left[ \Gamma_{\nu\eta}(A, \partial) \Delta^\eta(x) \right] \partial^\nu \pi(x) + \text{h.c.}, \quad (15)$$

with the modified vertex operator $\Gamma_{\nu\eta}(A, \partial)$ depending on the parameter $A$:

$$\Gamma_{\nu\eta}(A, \partial) = \gamma_5 \gamma^\mu \epsilon_{\mu\nu\rho\sigma} \theta_\sigma^\eta(A) \partial^\rho, \quad (16)$$

$$\theta_\sigma^\eta(A) = g_{\sigma\eta} - \frac{A + 1}{2} \gamma_\sigma \gamma_\eta. \quad (17)$$

In momentum space at $A = -1$ the vertex function eq. (16) reduces to that suggested by Pascalutsa, eq. (10). The $\theta_{\mu\nu}(A)$-operator has a simple physical meaning: it relates the Rarita-Schwinger theory to the general case which does not contain the Levi-Civita tensor explicitly.

Using the coupling eq. (15) the final result for the matrix element of $\pi N$ scattering is independent on the none-pole spin-$\frac{3}{2}$ terms in the full propagator

$$\left[ \Gamma_{\mu\nu}(A, p_\Delta) G^{\sigma\nu}(p_\Delta, A) \Gamma_{\sigma\nu}^I(A, p_\Delta) \right] = \frac{\theta + m}{p^2 - m^2} \frac{p_\Delta^2}{m_N^2} \mathcal{P}_{11;\mu\nu}^2(p_\Delta) \quad (18)$$

and coincides with that obtained for the case $A = -1$. The coupling eq. (15) can be written in a more compact form which does not contain the Levi-Civita tensor explicitly

$$\mathcal{L}_I = \frac{ig_{\Delta N\pi}}{4m_\pi m_N} \bar{\psi}_N(x) \left[ \gamma^{\sigma\nu} \theta_{\sigma\eta}(A) \partial^\rho \Delta^\eta(x) \right] \partial^\nu \pi(x) + \text{h.c.}, \quad (19)$$

where $\gamma^{\sigma\nu} = \{ \gamma^{\sigma\rho}, \gamma^{\nu\rho} \}$ and $\gamma^{\sigma\rho} = \{ \gamma^\sigma, \gamma^\rho \}$ and $\theta_{\sigma\eta}(A)$ is defined in eq. (17).

Finally, the full Lagrangian for the interacting $\Delta N\pi$ fields can be written in the form

$$\mathcal{L}^\frac{3}{2} = \mathcal{L}^\frac{3}{2}_0 + \mathcal{L}_I + \mathcal{L}^N_0 + \mathcal{L}^N_0, \quad (20)$$

where $\mathcal{L}^\sigma_0 = \pi(\Box + m^2)\pi$ and $\mathcal{L}^N_0 = \bar{\psi}_N(i\partial - m)\psi_N$ stand for the free Lagrangians of pion and nucleon fields, respectively. The free spin-$\frac{3}{2}$ Lagrangian $\mathcal{L}^\sigma_0$ and $\Delta N\pi$ coupling $\mathcal{L}_I$ are given by expressions eq. (2) and eq. (15). The Lagrangian eq. (20) depends on one arbitrary parameter $A$ which points to the freedom in choosing the 'off-shell' content of the theory. Although $\mathcal{L}^\sigma_0$ contains one free parameter the physical observables should not depend on it.

A similar conclusion can be made for the electromagnetic coupling. In the notation of (18) the generalized coupling can be written as

$$\mathcal{L}_{\gamma\Delta} = \frac{3ie}{2M_N(M_N + M_\Delta)} \bar{\psi}_N(x) \left[ g_M \theta_{\rho\sigma}(A) \partial_{\mu} \Delta^\sigma F_{\mu\rho} + i g_E \gamma_5 \theta_{\rho\sigma}(A) \partial_{\mu} F_{\rho\sigma} \right] + \text{h.c.}, \quad (21)$$

where $F_{\mu\rho}$ is the electromagnetic tensor and $\tilde{F}_{\mu\rho}$ is its dual. At $A = -1$ the coupling eq. (21) reduces to the one used in (13).
III. SUMMARY.

We have shown that the gauge-invariant $\Delta N\pi$ coupling suggested by Pascalutsa for spin-$\frac{3}{2}$ fields removes the 'off-shell' degrees of freedom only for a specific choice of the spin-$\frac{3}{2}$ propagator but not in the general case. This can be understood by the observation that the vertex function behaves like a 'conserved current'. Hence, only those terms in a propagator are eliminated which do not have vector components. In the general case the spin-$\frac{3}{2}$ propagator contains a term associated with the $\left[\frac{1}{2} \otimes 1\right]_{\frac{1}{2}}$ irreducible representation. The corresponding projection operator $P_{\frac{1}{2} \otimes 1\mu \nu}(p)$ does not contribute to the well-known Rarita-Schwinger propagator therefore no 'off-shell background' appears. However, this is not true in the general case. We have shown that the problem can be solved by introducing higher order derivatives to the interaction Lagrangian or by generalizing the original $\Delta N\pi$ coupling suggested by Pascalutsa. In the latter case the full Lagrangian of the interacting $\Delta N\pi$ fields depends on one free parameter which reflects the freedom in choosing an 'off-shell' content of the theory. We have shown on the example of $\pi N$ scattering amplitude that physical observables should not depend on this free parameter.

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[1] W. Rarita and J. S. Schwinger, Phys. Rev. 60, 61 (1941).
[2] M. Benmerrouche, R. M. Davidson, and N. C. Mukhopadhyay, Phys. Rev. C39, 2339 (1989).
[3] L. M. Nath, B. Etemadi, and J. D. Kimel, Phys. Rev. D3, 2153 (1971).
[4] L. P. S. Singh and C. R. Hagen, Phys. Rev. D9, 910 (1974).
[5] C. Fronsdal, Nuovo Cimento, Suppl. vol. IX, ser. X, 416 (1958).
[6] H. Haberzettl, (1998), nucl-th/9812043.
[7] S. Weinberg and E. Witten, Phys. Lett. B96, 59 (1980).
[8] V. Pascalutsa, Phys. Lett. B503, 85 (2001), hep-ph/0008026.
[9] S. Deser, V. Pascalutsa, and A. Waldron, Phys. Rev. D62, 105031 (2000), hep-th/0003011.
[10] V. Pascalutsa, Phys. Rev. D58, 096002 (1998), hep-ph/9802288.
[11] V. Pascalutsa and R. Timmermans, Phys. Rev. C60, 042201 (1999), nucl-th/9905065.
[12] P. Van Nieuwenhuizen, Phys. Rept. 68, 189 (1981).
[13] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. D76, 111501 (2007), 0711.0147.