Tunnel damage identification method based on relative entropy of wavelet packet energy: An experimental verification

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Abstract. Ineffective methods used to detect damage in subway tunnels can result in severe safety risk. Traditional detection methods use laser scanning, image recognition, and inspection to identify damages in the inner lining of a tunnel after the subway is out of service; these methods have low efficiency and accuracy. In this study, a new method is proposed to analyze the vibration signal of a moving train based on the relative entropy of the wavelet packet energy, which can quickly identify and evaluate the damage of the subway tunnel and its auxiliary structure in real time. The test model includes a three-dimensional printed tunnel and a moving vehicle to simulate the train running in the tunnel. Wireless acceleration sensors are installed inside the vehicle and at the top of the tunnel to record the vibration signals of the car and the tunnel lining. After obtaining the acceleration signal of the vehicle, the relative entropy of energy is calculated using wavelet packet transform, in which sudden changes in entropy reflect damages. The model test results show that the vertical acceleration of the vehicle is sensitive to tunnel damage and the signal energy is mainly concentrated at 30–80 Hz and 200–400 Hz. By comparing the energy of the relative entropy between healthy and damaged signals, the location and degree of damage can be identified.

1. Introduction

With rapid urban development and the construction of numerous tunnels, subways have become the lifeline of urban transportation. Thus, tunnel safety has become an increasingly important issue [1]. The methods currently used to detect tunnel damage rely mainly on monitoring the deformation [2] and structure vibration [3]. However, owing to the cost constraints of these detection methods, wired sensors are usually installed only in sensitive areas of tunnels [4]; most regions rely on repairs conducted via on-site inspection only after the train has stopped or using a machine vision system [5-7]. These methods can identify only the apparent damage; thus, a more efficient detection method is needed to evaluate damages inside and behind the tunnel lining [3].

Numerous studies have discussed damage detection in tracks and bridges using moving vehicle-carried sensors [8-10]. Soheil [11] was the first to propose end-to-end pipelines for comprehensive modal identification of a bridge using moving sensors within vehicles. Yang et al. [12] proposed a new theory for coupled vertical and rotational motions of a two-axle asymmetric vehicle. These methods negate the need for sensor installation in the tunnel, and their indirect application shows great potential.
in terms of economy and efficiency. However, further research is needed on the software and hardware used for these methods to enable robust field applications.

This study proposes a method to identify tunnel structure damage based on the vibration acceleration signal of the service train that will improve the efficiency of the damage detection. First, the train–rail–tunnel coupled model test is designed for which the law of model vibration is studied. Then, the relative entropy of wavelet packet energy is used to identify damages, which verifies the effectiveness of the method using test data.

2. Train–track–tunnel coupled model test

2.1. Train–track–tunnel coupled model

The dynamic coupled model of the metro train–track–tunnel can be simplified according to that given in previous studies [13, 14].

In this model, the car body, bogie, primary- and secondary-spring damping systems, steel rail, track plate, fasteners, and tunnel structure are considered and coupled with each other. The entire system can be divided into train and substructure systems. The balance relationship between the train and the track is established by a force and deformation coordination equation related to the Hertz contact theory. According to the D’Alembert theory, the coupled system vibration equation can be expressed as

\[ M_{sv} \ddot{X}_v + C_{sv} \dot{X}_v + K_{sv} X_v = F_v, \tag{1} \]

where \( M_{sv} \), \( C_{sv} \), and \( K_{sv} \) represent the mass matrix, damping matrix, and stiffness matrix, respectively. The damage in the tunnel structure can be set in the stiffness and mass matrices. The train acceleration curve can be obtained by solving the coupled system equations. Then, the acceleration signal can be analyzed to determine whether damage is present in the tunnel structure, identify the location of the damage, and evaluate the structural damage degree and the service life. This method is described in detail in our previous research.

2.2. Scale model

According to relevant theoretical research, a lab scale model was designed, and the measured signal was analyzed.

2.2.1. Similarity ratio

On the basis of the similarity theory, the main physical parameters of the actual and modeled structures are the same or proportional. The similarity coefficients, considering the laboratory space and model size, are given in Table 1.
Table 1. Similarity parameters

| Name            | Similarity equation | Similarity coefficient |
|-----------------|---------------------|------------------------|
| Length          | $C_L$               | 1/30                   |
| Elastic Modulus | $C_E$               | 1/60                   |
| Density         | $C_\rho$            | 1/2                    |
| Strain          | $C_\varepsilon$     | 1                      |
| Accelerate      | $C_a = \frac{C_E}{C_\rho C_L}$ | 1                      |
| Time            | $C_t = C_L \left(\frac{C_\rho}{C_E}\right)^{-0.5}$ | 0.182                  |
| Frequency       | $C_f = C_t^{-1}$    | 5.477                  |

2.2.2. Model system

The purpose of this test is to verify the method used for identifying the damage in the tunnel structure by measuring and analyzing the acceleration data of the train. Owing to space and material limitations, the primary and secondary springs of the train vehicle as well as the rail fasteners were simplified. The coupled train–track–tunnel system includes a shield tunnel model, train model, traction device, and automatic control system, as shown in Figure 2. The actual model is shown in Figure 3.

Figure 2. Diagram of the test system 1: Tunnel model; 2: control system; 3: wheel support; 4: stepper motor; 5: vehicle; 6: wireless acceleration sensor; 7: soft track; 8: wired acceleration sensor; 9: limit sensor

Figure 3. Actual model used in this study
### Table 2. Physical properties of post-cured material

| Appearance       | Density (g·cm⁻³) | Tensile modulus (MPa) | Tensile strength (MPa) | Flexural modulus (MPa) | Poisson ratio |
|------------------|------------------|-----------------------|------------------------|------------------------|---------------|
| Photosensitive resin | White | 1.12–1.18 | 2589–2695 | 38–56 | 2692–2775 | 0.4–0.44 |

The tunnel model includes the tunnel body and track plate constructed via three-dimensional printing using photosensitive resin. The tunnel is simplified into a multi-span continuous beam with lining segments assembled by bolts, which is similar to the actual shield tunnel. The tunnel segments can be replaced to simulate various states of tunnel damage. The track is constructed using two complete metal strips with smooth surfaces to reduce the impact of track irregularities.

The vehicle model is composed of aluminum alloy and is divided into three parts. The wireless sensor and load block are fixed with respective bolts in the vehicle. The vehicle is equipped with grooved wheels that can smoothly run on the track and restrict lateral movement. The vehicle is easily constructed, and single- or multi-car tests can be set according to the requirements.

The traction device is composed of a support, stepper motor, and traction crawler, which can provide stable traction power to the vehicle model to simulate the driving load. The stepper motor can be automatically controlled through computer programming to ensure that the vehicle runs at a uniform speed and that stability is maintained through a large number of repeated tests.

### 2.3. Sensor arrangement

In the test, a three-axis wireless sensor was installed in the middle of the vehicle to collect its acceleration information, which is the vibration response of the train–rail–tunnel coupled system and includes the tunnel damage information. The wired acceleration sensors were placed on the third, fifth, and seventh spans of the tunnel to measure the vibration in the tunnel.

### 3. Wavelet packet energy relative entropy

It is often necessary to analyze several non-stationary signals in engineering. The wavelet packet transform theory has developed rapidly in recent years and has been successfully applied in the fields of image processing, signal denoising, and communication engineering. In this study, the wavelet packet was used to decompose the measured signal and divide the window into several parts. Then, the difference between the signal energy distribution of the healthy structure and that of the damaged structure was measured by the relative entropy of the wavelet packet energy, which reveals the structural damage characteristics.

#### 3.1. Wavelet packet decomposition

Wavelet packet decomposition is a time–frequency analysis method with multi-resolution analysis characteristics that can be used to decompose both low- and high-frequency parts. This decomposition is neither redundant nor omitted. Therefore, the signal containing a large amount of high-frequency information can be analyzed better in time and frequency.
As shown in Figure 4, the original signal is divided into components of different frequency bands after binary decomposition. Each frequency band corresponds to the same frequency width. As the number of decomposition layers increases, the frequency resolution is higher; however, the calculation cost also increases.

3.2. Shannon entropy and relative entropy

Shannon entropy is a statistical indicator used to describe the complexity of random sequences \( \{X_i\} \) and reflects the uniformity of the probability distribution \( P \). As the entropy value increases, the sequence distribution becomes more random. Therefore, the energy entropy value can reflect the energy distribution of the signal at various scales. If \( P = \{p_i\} \) is the probability distribution of the random sequence \( \{X_i\} \), then the information entropy is defined as

\[
W_e = - \sum_{i=1}^{n} p_i \log_2 p_i .
\]

Also known as Kullback–Leibler divergence or information divergence, relative entropy is an asymmetric measure of the difference between two probability distributions. In information theory, the relative entropy is equivalent to the difference between the information entropy of two probability distributions (Shannon entropies)[15]. If \( P = \{p_i\} \) and \( Q = \{q_i\} \) are two probability distributions on discrete random variables \( \{X_i\} \) and \( \{Y_i\} \), then the relative entropy is defined as

\[
W_{re}(P \parallel Q) = \sum_{i=1}^{n} p_i \log_2 \frac{p_i}{q_i} .
\]

3.3. Algorithm

The main steps of the algorithm used to identify the tunnel damage are shown in Figure 5. The algorithm is divided into the three steps:
Figure 5. Main steps of the algorithm used to identify tunnel damage

**Step 1**: Decompose the signal $x(t)$ into N-layer using the wavelet packet, where the wavelet function is sym8 wavelets. Then, the multi-resolution analysis wavelet packet transform coefficients can be expressed as

$$D = \{d_{j,k}(t), k = 1, \ldots, 2^j, j = 1, 2, \ldots, N\} \quad (4)$$

where $d_{j,k}(t)$ is a series of signal slices decomposed by the wavelet packet, and the subscript $j$ is decomposition order; $k$ is the order of different frequency ranges.

**Step 2**: Divide all of the wavelet coefficients into sub-intervals according to time. The interval length is $l$, and the translation length is $l_{inc}$. Then, the wavelet coefficients of each sub-interval are expressed as

$$D_i = \{d_{i,j,k}(t), k = 1, \ldots, 2^j, j = 1, 2, \ldots N, i = 1, \ldots, M\} \quad (5)$$

Define the wavelet energy of the sub-interval as the sum of the squares of the wavelet coefficients in the interval:

$$e_{i,j,k} = \sum_{l=0}^{(i-1)l_{inc}+l} |d_{i,j,k}(t)|^2 \quad (6)$$

Then, the sum of the wavelet energy of all frequency bands in each sub-interval is:

$$E_{i,j} = \sum_{k=1}^{2^j} e_{i,j,k} = \sum_{k=1}^{2^j} \sum_{l=0}^{(i-1)l_{inc}+l} |d_{i,j,k}(t)|^2 \quad (7)$$

**Step 3**: Calculate the relative entropy of wavelet energy in the sub-interval:

$$Wre_{i,j} = \sum_{k=1}^{2^j} p_{i,j,k} \ln \left( \frac{p_{i,j,k}^D}{p_{i,j,k}^H} \right) \quad (8)$$

where superscript D is the damage signal, and H is the healthy signal; $p_{i,j,k} = \frac{e_{i,j,k}}{E_{i,j}}, \sum_{k=1}^{2^j} p_{i,j,k} = 1$.

The closer the energy distribution in each frequency period of each time period, the more the entropy value tends to be zero. Therefore, the difference in energy distribution between healthy and damage signals can be identified to facilitate locating the damage.
4. Analysis of test results

4.1. Test conditions

The average speed of actual subway trains is ~35 km/h, which was converted to 1.76 m/s in the model, according to the similarity coefficient. When the train is under the full load condition, the total mass of passengers and train is ~300 tons, which was ~11.1 kg in the experiment. However, owing to the limitations of the experimental equipment, the speed and mass of the vehicle were not very high. Therefore, in the standard conditions of the test, the train speed was 0.88 m/s, and the total mass of the vehicle including the load and sensor was 5.08 kg.

To simulate two main types of damage in tunnel structures, two procedures were applied as follows. 1) Masses were placed in the third, fifth, and seventh spans of the tunnel to simulate the local load changes in the tunnel. 2) The lining segments of the fifth span were removed to simulate the local stiffness damage in the tunnel lining. Different degrees of damage were applied for each damage type. The test conditions are shown in Table 3.

| No. | Total mass (kg) | Velocity / m·s⁻¹ | Additional mass (kg) | Stiffness loss | Damage location |
|-----|----------------|-----------------|---------------------|----------------|----------------|
| 0   | 5.08           | 0.88            | 0                   | 0              | -              |
| 1   | 6.55           | 0.88            | 0                   | 0              | -              |
| 2   | 8.01           | 0.88            | 0                   | 0              | -              |
| 3   | 9.48           | 0.88            | 0                   | 0              | -              |
| 4   | 10.94          | 0.88            | 0                   | 0              | -              |
| 5   | 5.08           | 1.17            | 0                   | 0              | -              |
| 6   | 5.08           | 1.46            | 0                   | 0              | -              |
| 7   | 5.08           | 1.755           | 0                   | 0              | -              |
| 8   | 5.08           | 0.88            | 2.93                | 0              | 5th span       |
| 9   | 5.08           | 0.88            | 4.39                | 0              | 5th span       |
| 10  | 5.08           | 0.88            | 5.86                | 0              | 5th span       |
| 11  | 5.08           | 0.88            | 5.86                | 0              | 3rd span       |
| 12  | 5.08           | 0.88            | 5.86                | 0              | 7th span       |
| 13  | 5.08           | 0.88            | 0                   | Remove top lining | 5th span     |
| 14  | 5.08           | 0.88            | 0                   | Remove side lining | 5th span    |
| 15  | 5.08           | 0.88            | 0                   | Remove half ring lining | 5th span |

As shown in Figure 6, we placed a mass block directly on the top of the tunnel to simulate the additional mass. Moreover, the tunnel lining was directly dismantled to increase the local tunnel damage effect, as shown in Figure 7. Because the lining segments are assembled, the damage position can be altered by the changing segments.
4.2. Tunnel damage identification
The sampling frequency of the on-board acceleration sensor was 4000 Hz. After the test data were measured, data analysis was performed using MATLAB calculation software. Then, the relative entropy of the wavelet packet energy was calculated, where the interval length \( l \) was 300 and the translation length \( l_{\text{inc}} \) was 100.

4.2.1. Different vehicle loads
As shown in Figure 8, the Fast Fourier Transform algorithm was used to obtain a spectrogram of the vehicle’s vertical acceleration. Under standard conditions, the peak amplitude of the car acceleration was \( \approx 0.2 \, \text{g} \), which was concentrated in two main frequency bands of 20–80 Hz and 200–400 Hz. As the mass of the vehicle increased, the amplitude of the acceleration of the vehicle decreased slightly; however, the bandwidth of the low frequency bands decreased, and the energy was more concentrated. Notably, the huge amount of inertia made car braking difficult, which can harm the motor.
Figure 8. Acceleration signals and spectra diagrams of vehicle with different loads

4.2.2. Different vehicle velocities
Figure 9 shows that when the speed of the vehicle increased, the excitation by the car and the amplitude of the vertical acceleration both increased. However, the energy of each frequency band became larger and the noise interference strengthened, which increased the difficulty in identifying the damage. Therefore, with a large number of tests, it can be convenient to identify the damage using medium vehicle loads and low speed conditions.
Figure 9. Acceleration signals and spectra diagrams of vehicle with different velocities
4.2.3. Additional masses on tunnel
To avoid interference during the experiments, no sensor was placed outside the tunnel. Additional masses weighing 2.93 kg, 4.39 kg, and 5.86 kg were placed on the fifth span of the tunnel. The relative entropy of the vertical acceleration signal of the standard condition and the signal with additional mass were calculated. As shown in Figure 10, the relative entropy value experienced a large mutation to 1.6, 2.4, and 3.4 on the fourth, sixth, and eighth spans, respectively, at ~0.9 s, which demonstrated an abnormality in that part of the tunnel. Then, 4.39 kg masses were placed on the fourth, sixth, and eighth spans. As shown in Figure 10, a sudden change occurred in the relative entropy curve at the relevant time, which proves that the algorithm used for tunnel damage identification is effective.

4.2.4. Local stiffness loss of tunnel
The loss of stiffness of the tunnel lining can be simulated by removing the tunnel segments. In the seventh span of the tunnel, the top block, quarter ring, and half ring were removed to simulate three types of stiffness loss. The relative entropy value changed suddenly to 2.5, 3, and 4 on the fourth, sixth, and eighth spans, respectively, at 1.28 s, where the tunnel stiffness was lost. However, the relative entropy after removing the top block was not sensitive.

![Figure 10. Relative entropy of tunnel with different additional mass values](image)

![Figure 11. Relative entropy of tunnel with different mass positions](image)
Figure 11. Relative entropy in tunnel with different stiffness loss values

5. Conclusion
This study proposes a method for identifying tunnel damage based on the wavelet packet energy relative entropy algorithm, which is verified and analyzed through model tests. The main results of the study are summarized in the following points.
(1) The vehicle–rail–tunnel coupled model was designed to measure the vertical acceleration of the car. Under test standard conditions, the peak acceleration of the vehicle was ~0.2 g, and the energy was mainly concentrated in two frequency bands of 20–80 Hz and 200–400 Hz.
(2) The use of wavelet packet energy relative entropy to identify tunnel damage can effectively identify the location of additional mass and stiffness loss and can distinguish the different degrees of damage.
(3) Track irregularities and accidental error led to significant noise in the signal, which adversely affected the detection of slight damage.

This method is suitable for model experiments with smooth track and linear structures; however, field tests still require verification. In practical applications, this method can be used for quick identification. Other methods can be used for a more detailed detection and evaluation to determine if repairs are necessary. In subsequent research, the signal should be properly filtered to improve the signal-to-noise ratio. Such studies will incorporate machine learning to improve the identification efficiency.

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