Supersymmetric effects in $B_s \to \ell^+\ell^-\gamma$ decays

S. Rai Choudhury* and Naveen Gaur†

Department of Physics and Astrophysics,
University of Delhi,
Delhi - 110 007, India.

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B-meson decays are very useful probes for testing the Standard Model and its various extensions. Leptonic decays of $B$ have very clean signatures in this respect and hence can be very useful testing grounds. In this work we study the effects of MSSM (Minimal Supersymmetric Extension of Standard Model) on various kinematical distributions in the radiative dileptonic decay ($B_s \to \ell^+\ell^-\gamma$). We study the Forward Backward asymmetry (of the lepton pair), and the various polarization asymmetries of both final state leptons ($\ell^-$ and $\ell^+$). In radiative dileptonic decay of $B$-meson ($B_s \to \ell^+\ell^-\gamma$) the final state photon can also be polarized. So in this channel one can also study the polarization effects of the final state photon.

I. INTRODUCTION

There are many theoretical and experimental reasons for studying flavor-changing neutral current (FCNC) processes. These transitions are forbidden at tree level in Standard Model (SM) and hence provide very stringent tests of SM at loop level. The investigations of various FCNC processes can be used to accurately determine various fundamental parameters of SM like elements of CKM matrix, various decay constants etc. Besides testing SM the FCNC processes can be very useful for discovering indirect effects of possible TeV scale extensions of SM like SUSY (Supersymmetry).

In particular, the processes like $B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$, $B_s \to \ell^+\ell^-\gamma$ etc. are experimentally very clean and possibly are very sensitive to various extensions of SM. Compared to $B \to X_s\gamma$, the flavor-changing semi-leptonic decays (like $B \to X_s\ell^+\ell^-$, $B \to K^*\ell^+\ell^-$, $B_s \to \ell^+\ell^-\gamma$ etc.) can be more sensitive to the actual form of new interactions since here one can measure experimentally various kinematical distributions apart from total decay rate. The various kinematical distributions which can be measured in above mentioned processes can be Forward Backward asymmetry, CP asymmetry, various lepton polarization asymmetries etc.

The flavor changing channel $b \to s\ell^+\ell^-$ decay, which takes place in SM at loop level is very sensitive to the gauge structure of SM. This mode ($b \to s\ell^+\ell^-$) is also very sensitive to the various extensions of SM. New physics (here we are concerned with SUSY) effects manifests in rare B decays in essentially two different ways, either through the new contributions to the existing Wilson coefficients in SM or through the new structures in effective Hamiltonian which were absent in SM. Particularly $b \to s\ell^+\ell^-$ has been extensively studied within SM and in various extensions of it $^1$, $^2$, $^3$, $^4$, $^5$, $^6$, $^7$, $^8$, $^9$, $^{10}$, $^{11}$, $^{12}$, $^{13}$, $^{14}$, $^{15}$, $^{16}$, $^{17}$, $^{18}$, $^{19}$, $^{20}$. Final state lepton polarizations (there can be in general three polarizations namely longitudinal, normal and transverse) provides a very useful probe for establishing new physics $^8$, $^{22}$, $^{23}$.

The simplest and the most favorite extension of the SM has been the Minimal Supersymmetric Standard Model (MSSM). As we know that there are five physical scalars (Higgs) in MSSM as compared to one in SM. The importance of specially Neutral Higgs Bosons (NHBs) $b \to s\ell^+\ell^-$ has been extensively discussed earlier $^8$, $^9$, $^{10}$, $^{11}$, $^{12}$, $^{13}$, $^{14}$, $^{22}$. The importance of lepton polarization in $B_s \to \ell^+\ell^-\gamma$ has already been pointed out in earlier work $^{23}$. In this work we present the complete study of all the lepton polarization asymmetries associated with final state leptons in radiative dileptonic decay ($B_s \to \ell^+\ell^-\gamma$) within the framework of Minimal Supersymmetric Standard Model (MSSM)

*Electronic address: src@ducos.ernet.in
†Electronic address: naveen@physics.du.ac.in
Here we will do the combined analysis the various polarization asymmetries associated with both final state leptons ($\ell^-$ and $\ell^+$) within MSSM. In the radiative dileptonic decay mode ($B_s \rightarrow \ell^+\ell^−\gamma$) apart from lepton polarization we can also have polarized photon (i.e. photon of a particular helicity). We will also study the effects of polarized photon by introducing a variable “photon polarization asymmetry”. This is perhaps a very difficult parameter to be measured experimentally but we include it for completeness. With this variable we now have one more kinematical variable (along with total decay rate, FB asymmetry and three different lepton polarization asymmetries) to test out the exact structure of effective Hamiltonian. In all our analysis we will try to focus on mainly the NHB effects.

This paper is organized as follows: In section II we will first present the QCD corrected effective Hamiltonian of the quark level process $b \rightarrow s\ell^+\ell^-\gamma$, including NHB effects. We will give the corresponding matrix element and then will give the analytic expression of the dilepton invariant mass distribution and forward backward asymmetry. In section III we will give the analytical expressions of the various lepton polarization asymmetries. In section IV we analyse the combinations of the polarization asymmetries. Then in section V we will move to another kinematical variable photon polarization asymmetry. We will finally conclude in section VI with results.

II. DILEPTON INVARIANT MASS DISTRIBUTION

The exclusive $B_s \rightarrow \ell^+\ell^-\gamma$ decay is induced by the inclusive $b \rightarrow s\ell^+\ell^-$ one. So, we have to start with QCD corrected effective Hamiltonian for related quark level process $b \rightarrow s\ell^+\ell^-$, which can be obtained by integrating out heavy particles in SM and MSSM.

\begin{equation}
\mathcal{H} = - \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^\ast \sum_{i=1}^{10} \left\{ C_i(\mu) O_i(\mu) + C_Q(\mu) Q_i(\mu) \right\}
\end{equation}

where $O_i$ are current-current ($i = 1, 2$), penguin ($i = 1, \ldots, 6$), magnetic penguin ($i = 7, 8$) and semileptonic ($i = 9, 10$) operators. $Q_i$, ($i = 1, \ldots, 10$) are the operators which results due to NHB exchange diagrams. $C_i(\mu)$ and $C_Q(\mu)$ are Wilson coefficients evaluated at scale $\mu$ and are tabulated in [3, 6, 8, 10].

Neglecting the strange quark mass the effective Hamiltonian (2.1) give the following matrix element for the inclusive $b \rightarrow s\ell^+\ell^-$:

\begin{equation}
\mathcal{M} = \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^\ast \left\{ -2 C_{eff}^{\tau \tau} \frac{m_b}{p^2} \bar{s}i\sigma_{\mu\nu}P^\nu(1 + \gamma_5)b \bar{\ell}\gamma^\mu\ell + C_{eff}^{\tau \tau} \bar{s}\gamma_\mu(1 - \gamma_5)b \bar{\ell}\gamma^\mu\ell \\
+ C_{10} \bar{s}\gamma_\mu(1 - \gamma_5)b \bar{\ell}\gamma^\mu\gamma_5\ell + C_{Q_1} \bar{s}(1 + \gamma_5)b \bar{\ell}\ell + C_{Q_2} \bar{s}(1 + \gamma_5)b \bar{\ell}\gamma_5\ell \right\}
\end{equation}

where $p = p_+ + p_-$ is the sum of momenta of $\ell^-$ and $\ell^+$, $V_{tb}, V_{ts}$ are CKM factors. The coefficients $C_{eff}^{\tau \tau}, C_{Q_1}, C_{Q_2}$ are given in many earlier works [3, 6, 8, 10]. We will also take the long distance effects related to the charm resonances according to the Breit Wigner form [3, 6, 8, 10].

In order to obtain the matrix element for $B_s \rightarrow \ell^+\ell^-\gamma$ decay, a photon line should be hooked to any of the charged internal or external lines. As has been pointed out before [10], contributions coming from hooking a photon line from any charged internal line will be suppressed by a factor of $m_b/M_W^2$, hence we neglect them in our further analysis. When photon is attached to the initial quark lines the corresponding matrix element is the so called structure dependent (SD) part of the amplitude which can be written as:

\begin{equation}
\mathcal{M}_{SD} = \frac{\alpha^{3/2} G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^\ast \left\{ [A \varepsilon_{\mu\alpha\beta\sigma}\varepsilon^\alpha_\rho q^\beta q^\sigma + iB (\varepsilon^\alpha_\mu(pq) - (\varepsilon^\alpha_p)\mu_q) ] \bar{\ell}\gamma^\mu\ell \\
+ [C \varepsilon_{\mu\alpha\beta\sigma}\varepsilon^\alpha_\rho q^\beta q^\sigma + iD (\varepsilon^\alpha_\mu(pq) - (\varepsilon^\alpha_p)\mu_q) ] \bar{\ell}\gamma^\mu_5\ell \right\}
\end{equation}

where

\begin{align*}
A &= \frac{1}{m_{B_s}} \left[ C_{eff}^{\tau \tau} G_1(p^2) - 2C_{eff}^{\tau \tau} \frac{m_b}{p^2} G_2(p^2) \right], \\
B &= \frac{1}{m_{B_s}} \left[ C_{eff}^{\tau \tau} F_1(p^2) - 2C_{eff}^{\tau \tau} \frac{m_b}{p^2} F_2(p^2) \right],
\end{align*}
\[ C = \frac{C_{10}}{m_B^2} G_1(p^2), \]
\[ D = \frac{C_{10}}{m_B^2} F_1(p^2). \] (2.4)

In getting eqn. (2.3) we have used following definitions of the form factors \[21\]
\[ \langle \gamma | \bar{s}\gamma_\mu(1 \pm \gamma_5)b \mid B_s \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu \alpha \beta \sigma} p_\beta q_\sigma G_1(p^2) \mp i[\epsilon_\mu^\ast (pq) - (\epsilon^\ast p)q_\mu]F_1(p^2) \right\} \] (2.5)
\[ \langle \gamma | \bar{s}\sigma_{\mu \nu}(1 \pm \gamma_5)b \mid B_s \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu \alpha \beta \sigma} p_\beta q_\sigma G_2(p^2) \pm i[\epsilon_\mu^\ast (pq) - (\epsilon^\ast p)q_\mu]F_2(p^2) \right\} \] (2.6)

another relation we can get by multiplying \( p_\mu \) on both the sides of eqn. (2.6):
\[ \langle \gamma | \bar{s}(1 \pm \gamma_5)b \mid B_s \rangle = 0 \] (2.7)

Here \( \epsilon_\mu \) and \( q_\mu \) are the four vector polarization and momentum of photon respectively. We can see from eqn. (2.2) that neutral scalar exchange parts doesn’t contribute to the \textbf{structure dependent} part.

When the photon is attached to the lepton lines using the expressions:
\[ \langle 0 \mid \bar{s}b \mid B_s \rangle = 0 \] (2.8)
\[ \langle 0 \mid \bar{s}\sigma_{\mu \nu}(1 + \gamma_5)b \mid B_s \rangle = 0 \] (2.9)
\[ \langle 0 \mid \bar{s}\gamma_\mu \gamma_5 b \mid B_s \rangle = -i f_{B_s} p_{B_s \mu} \] (2.10)

and the conservation of vector current we can get the contribution to the Bremsstrahlung part (called \textbf{internal Bremsstrahlung} IB) part as:
\[ M_{IB} = \frac{\alpha^{3/2} G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^\ast \bar{i} \mathbf{m} f_{B_s} \left\{ (C_{10} + \frac{m_B^2}{2m_b m_b} C_{Q_2}) \frac{\ell}{2p_{-q}} - \frac{P_{B_s} \cdot \ell}{2p_{-q}} \gamma_5 \ell \right\} + \frac{m_B^2}{2m_b m_b} C_{Q_2} \left\{ 2m_{\ell}(\frac{1}{2p_{-q}} + \frac{1}{2p_{+q}}) \bar{\ell} \cdot \ell + \bar{\ell} \left( \frac{P_{B_s} \cdot \ell}{2p_{+q}} - \frac{P_{B_s} \cdot \ell}{2p_{-q}} \right) \right\} \] (2.11)

where \( P_{B_s} \) and \( f_{B_s} \) are the momentum and decay constant of the \( B_s \) meson. \( p_- \) and \( p_+ \) are the four moment of \( \ell^- \) and \( \ell^+ \) respectively.

The total matrix element for \( B_s \to \ell^+ \ell^- \gamma \) is obtained as a sum of \( M_{SD} \) and \( M_{IB} \) terms:
\[ M = M_{SD} + M_{IB} \] (2.12)

From above matrix element we can get the square of the matrix element as,(with photon polarizations summed over)
\[ \sum_{\text{photon pol}} |M|^2 = |M_{SD}|^2 + |M_{IB}|^2 + 2Re(M_{SD}M_{IB}^\ast) \] (2.13)

with
\[ |M_{SD}|^2 = 4 \left( \frac{\alpha^{3/2} G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^\ast \right)^2 \left\{ [ |A|^2 + |B|^2 ] \left[ p^2((p-q)^2) + (p+q)^2 \right] + 2m_B^2(pq)^2 \right\} + [ |C|^2 + |D|^2 ] \]
\[ [p^2((p-q)^2 + (p+q)^2) - 2m_B^2(pq)^2] + 2 \text{Re}(B^*C + A^*D) p^2((p-q)^2 - (p+q)^2) \} \] (2.14)
\[ |M_{IB}|^2 = 4 \left( \frac{\alpha^{3/2} G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^\ast \right)^2 f_B^2 m_B^2 \left\{ (C_{10} + \frac{m_B^2}{2m_b m_b} C_{Q_2}) \left\{ 8 + \frac{1}{(p-q)^2}(-2m_B^2 m_\ell^2 - m_B^2 p^2 + p^4) + 2p^2(p+q) \right\} \right.

\[ + \frac{1}{(p-q)^2}(6p^2 + 4(p+q)) + \frac{1}{(p+q)^2}(2m_B^2 m_\ell^2 - m_B^2 p^2 + p^4 + 2p^2(p-q)) + \frac{1}{(p-q)^2}(-4m_B^2 m_\ell^2 + 2p^4) \right\} \] (2.15)
The differential decay rate of $B^+$ is defined as:

$$
\frac{d\Gamma}{d\Delta} = 16 \frac{\alpha^{3/2} G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^* f_b \ell m_{\ell}^2 \left[ C_{10} + \frac{m_{B_s}^2}{2m_{\ell} m_b} C_{Q_2} \right] \left\{ - \text{Re}(A) \frac{(p-q+p+q)^3}{(p-q)(p+q)} + \text{Re}(D) \frac{(pq)^2(p-q-p+q)}{(p-q)(p+q)} \right\}
$$

The differential decay rate of $B_s \rightarrow \ell^+ \ell^- \gamma$ as a function of invariant mass of lepton pair is given by:

$$
\frac{d\Gamma}{ds} = \frac{\alpha^{3/2} G_F}{2\sqrt{2\pi}} V_{tb} V_{ts}^* f_b m_{\ell}^2 \left[ C_{10} + \frac{m_{B_s}^2}{2m_{\ell} m_b} C_{Q_2} \right] \left\{ - \text{Re}(A) \frac{(p-q+p+q)^3}{(p-q)(p+q)} + \text{Re}(D) \frac{(pq)^2(p-q-p+q)}{(p-q)(p+q)} \right\}
$$

The forward-backward (FB) asymmetry is also very sensitive to the details of the new physics (SUSY here). We can define the FB asymmetry as:

$$
A_{FB} = \left[ - 2 m_{B_s}^2 \ Re(A^*D + B^*C) (1-\hat{s})^2 \hat{s} \sqrt{1-4\hat{m}_{\ell}^2} + 32 f_b m_{\ell}^2 \frac{(-1+\hat{s})}{\sqrt{1-4\hat{m}_{\ell}^2}} \text{Log} \left( \frac{4\hat{m}_{\ell}^2}{\hat{s}} \right) \right]
$$

where $\hat{s} = p^2/m_{B_s}^2$, $\hat{m}_{\ell}^2 = m_{\ell}^2/m_{B_s}^2$, $\hat{s} = \frac{1+\sqrt{1-4\hat{m}_{\ell}^2}}{1-\sqrt{1-4\hat{m}_{\ell}^2}}$ are dimensionless quantities.
FIG. 1: Forward Backward asymmetry of $\mu$ (left) and $\tau$ (right). Parameters are: $m = 200$, $M = 450$, $A = 0$, $\tan\beta = 40$ and $\text{sgn}(\mu)$ is +ve. For SUGRA the pseudoscalar Higgs mass is taken to be 306. All masses are in GeV

$$
\times \left\{ \left( C_{10} + \frac{m_{P}^2}{2m_{\ell}m_{b}}C_{Q_3} \right) \text{Re}(D) + \left( \frac{m_{P}^2}{2m_{\ell}m_{b}}C_{Q_1} \right) \text{Re}(C) \right\} / \triangle (2.20)
$$

III. LEPTON POLARIZATION ASYMMETRIES

We compute the polarization asymmetries from the four fermi interactions defined in eqn. (2.3) and eqn. (2.11). We define the following orthogonal unit vectors $S$ in the rest frame of $\ell^-$ and $W$ in the rest frame of $\ell^+$, for the polarizations of the leptons [18] to the longitudinal direction (L), the normal directions (N) and the transverse direction (T)

$$
\begin{align*}
S_L^\mu &\equiv (0, e_L) = (0, \frac{p_-}{|p_-|}) \\
S_N^\mu &\equiv (0, v^\mu_N) = (0, \frac{q \times p_-}{|q \times p_-|}) \\
S_T^\mu &\equiv (0, e_T) = (0, e_N \times e_L) \\
W_L^\mu &\equiv (0, w_L) = (0, \frac{p_+}{|p_+|}) \\
W_N^\mu &\equiv (0, v^\mu_N) = (0, \frac{q \times p_+}{|q \times p_+|}) \\
W_T^\mu &\equiv (0, w_T) = (0, w_N \times w_L)
\end{align*}
$$

(3.1)

where $p_+$, $p_-$ and $q$ are three momenta of $\ell^+$, $\ell^-$ and photon respectively in the CM frame of $\ell^-\ell^+$ system. Now on boosting above vectors defined by eqns. (3.1, 3.2) to the CM frame of $\ell^-\ell^+$ system. Only the longitudinal vector will get boosted while the other two (transverse and normal) will remain the same. The longitudinal vectors after the boost will become

$$
\begin{align*}
S_L^\mu &\equiv \left( \frac{|p_-|}{m_\ell}, \frac{E_1 p_-}{m_\ell |p_-|} \right) \\
W_L^\mu &\equiv \left( \frac{|p_-|}{m_\ell}, -\frac{E_1 p_-}{m_\ell |p_-|} \right)
\end{align*}
$$

(3.3)
The polarization asymmetries can now be calculated by using the spin projector \( \frac{i}{2}(1 + \gamma_5)S \) for \( \ell^- \) and the spin projector is \( \frac{i}{2}(1 + \gamma_5)W \) for \( \ell^+ \). The lepton polarization asymmetries can be defined as:

\[
P_{-x} = \left( \frac{d^4(S_x, W_x)}{ds} + \frac{d^4(S_x, -W_x)}{ds} \right) - \left( \frac{d^4(-S_x, W_x)}{ds} + \frac{d^4(-S_x, -W_x)}{ds} \right) \tag{3.4}
\]

\[
P_{+x} = \left( \frac{d^4(S_x, W_x)}{ds} + \frac{d^4(S_x, -W_x)}{ds} \right) - \left( \frac{d^4(-S_x, W_x)}{ds} + \frac{d^4(-S_x, -W_x)}{ds} \right) \tag{3.5}
\]

where the sub-index \( x \) is \( L, T \) or \( N \). \( P_L \) denotes the longitudinal polarization asymmetry, \( P_T \) is the polarization asymmetry in the decay plane and \( P_N \) is the normal component to both of them. \( P_L \) and \( P_T \) are P-odd, T-even and hence CP even observable. But \( P_N \) is P-even, T-odd and hence a CP-odd observable \( ^1 \).

FIG. 2: Longitudinal Polarization asymmetry of \( \ell^+ \mu \) (left) and \( \tau \) (right). Parameters are : \( m = 200 \), \( M = 450 \), \( A = 0 \), \( \tan \beta = 40 \) and \( \text{sgn}(\mu) = + \text{ve} \). For SUGRA the pseudoscalar Higgs mass is taken to be 306. All masses are in GeV

The longitudinal polarization asymmetries for the leptons are:

\[
P_L^- = \left[ \frac{8m_B^2}{3} \Re(A^*C + B^*D) L_1(\hat{s}) + 128 \frac{\Re(B) L_3(\hat{s}) + \Re(C) L_4(\hat{s})}{m_B^2} \left( C_{10} + \frac{m_B^2}{2m_\ell m_b} C_{Q_2} \right) \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_1} \right) L_2(\hat{s}) \right]
\]

\[
+ 32 \left( C_{10} + \frac{m_B^2}{2m_\ell m_b} C_{Q_2} \right) f_\ell m_b^2 \left( \Re(A) L_5(\hat{s}) + \Re(D) L_6(\hat{s}) \right) \right] / \Delta \tag{3.6}
\]

\[
P_L^+ = \left[ - \frac{8m_B^2}{3} \Re(A^*C + B^*D) L_1(\hat{s}) + 128 \frac{\Re(B) L_3(\hat{s}) - \Re(C) L_4(\hat{s})}{m_B^2} \left( C_{10} + \frac{m_B^2}{2m_\ell m_b} C_{Q_2} \right) \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_1} \right) L_2(\hat{s}) \right]
\]

\[
+ 32 \left( C_{10} + \frac{m_B^2}{2m_\ell m_b} C_{Q_2} \right) f_\ell m_b^2 \left( \Re(B) L_5(\hat{s}) - \Re(D) L_6(\hat{s}) \right) \right] / \Delta \tag{3.7}
\]

\(^1\) because time reversal operation changes the sign of the momentum and spin, and parity transformation changes only the sign of momentum
with

\[ L_1(\hat{s}) = \hat{s}(1 - \hat{s})^2 \sqrt{1 - \frac{4m_f^2}{\hat{s}}} \]

\[ L_2(\hat{s}) = \frac{\left\{ (\hat{s} - 4m_f^2\hat{s} - 2\hat{s}^2 - 4m_f^2\hat{s}^2 + 3\hat{s}^3) \sqrt{1 - \frac{4m_f^2}{\hat{s}}} + (2m_f^2 - 8m_f^2\hat{s} + 8m_f^2\hat{s} - 8m_f^2\hat{s}^2 - 3\hat{s}^3) \ln(\hat{s}) \right\}}{(1 - \hat{s})^2(\hat{s} - 4m_f^2)} \]

\[ L_3(\hat{s}) = \frac{4m_f^2 - \hat{s} - 12m_f^2\hat{s} + 3\hat{s}^2 + (2m_f^2 + 2m_f^2\hat{s} - 2\hat{s}^2) \ln(\hat{s}) \sqrt{1 - \frac{4m_f^2}{\hat{s}}}}{(4m_f^2 - \hat{s})} \]

\[ L_4(\hat{s}) = \frac{(-1 + \hat{s}) \left\{ \hat{s} \sqrt{1 - \frac{4m_f^2}{\hat{s}}} + (2m_f^2 - \hat{s}) \ln(\hat{s}) \right\}}{(4m_f^2 - \hat{s})} \]

\[ L_5(\hat{s}) = \frac{(-1 + \hat{s})(-\hat{s} \sqrt{1 - \frac{4m_f^2}{\hat{s}}} + 2m_f^2 \ln(\hat{s}))}{(\hat{s} - 4m_f^2)} \]

\[ L_6(\hat{s}) = \frac{(-1 + \hat{s})(\hat{s} \sqrt{1 - \frac{4m_f^2}{\hat{s}}} + (2m_f^2 - \hat{s}) \ln(\hat{s})}{4m_f^2 - \hat{s}} \]

(3.8)

FIG. 3: Transverse Polarization asymmetry of $\ell^+\mu$ (left) and $\tau$ (right). Parameters are: \( m = 200, \ M = 450, \ A = 0, \ tan\beta = 40 \) and $sgn(\mu)$ is +ve. For SUGRA the pseudoscalar Higgs mass is taken to be 306. All masses are in GeV

The transverse polarization asymmetries $P_T^-$ and $P_T^+$ are:

\[ P_T^- = \pi m_t \left[ -2m_B^2 Re(A^*B)T_1(\hat{s}) + \frac{64f_0^2m_f}{m_B^2} \left( C_{10} + \frac{m_B^2}{2m_f m_b} C_{Q_2} \right) \left( \frac{m_B^2}{2m_f m_b} C_{Q_1} \right) T_2(\hat{s}) \right. \]

\[ + 8 \left( C_{10} + \frac{m_B^2}{2m_f m_b} C_{Q_2} \right) f_5(Re(B)T_3(\hat{s}) + Re(C)T_4(\hat{s})) \]

\[ + 8 \left( \frac{m_B^2}{2m_f m_b} C_{Q_1} \right) f_5(Re(A)T_5(\hat{s}) + Re(D)T_6(\hat{s})) / \Delta \]  

(3.9)

\[ P_T^+ = \pi m_t \left[ -2m_B^2 Re(A^*B)T_1(\hat{s}) + \frac{64f_0^2m_f}{m_B^2} \left( C_{10} + \frac{m_B^2}{2m_f m_b} C_{Q_2} \right) \left( \frac{m_B^2}{2m_f m_b} C_{Q_1} \right) T_2(\hat{s}) \right. \]

\[ + 8 \left( C_{10} + \frac{m_B^2}{2m_f m_b} C_{Q_2} \right) f_5(Re(B)T_3(\hat{s}) - Re(C)T_4(\hat{s})) \]
Finally the normal polarization asymmetries $P_N^-$ and $P_N^+$ are:

$$P_N^- = m_\ell \pi \left[ -m_B^2 \text{Im}(A^* D + B^* C) N_1(\hat{s}) ight.$$ \[+ 8 \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_1} \right) f_b \left\{ \text{Im}(A) N_2(\hat{s}) + \text{Im}(D) N_3(\hat{s}) \right\} \right] \] \[+ 8 \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_2} \right) f_b \left\{ \text{Im}(B) N_4(\hat{s}) + \text{Im}(C) N_5(\hat{s}) \right\} \] \[+ 8 \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_1} \right) f_b \left\{ \text{Im}(B) N_4(\hat{s}) - \text{Im}(C) N_5(\hat{s}) \right\} \] \[+ 8 \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_2} \right) f_b \left\{ \text{Im}(A) N_2(\hat{s}) - \text{Im}(D) N_3(\hat{s}) \right\} \] \[+ 8 \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_1} \right) f_b \left\{ \text{Im}(B) N_4(\hat{s}) - \text{Im}(C) N_5(\hat{s}) \right\} \] \]

$$P_N^+ = m_\ell \pi \left[ m_B^2 \text{Im}(A^* D + B^* C) N_1(\hat{s}) \right.$$ \[+ 8 \left( \frac{m_B^2}{2m_\ell m_b} C_{Q_1} \right) f_b \left\{ \text{Im}(A) N_2(\hat{s}) - \text{Im}(D) N_3(\hat{s}) \right\} \]

$$(3.12)$$

$$(3.13)$$
with

\[ N_1(\hat{s}) = (-1 + \hat{s})\sqrt{\hat{s}} \sqrt{1 - \frac{4m_{\ell}^2}{s}} \]
\[ N_2(\hat{s}) = \frac{\hat{s}(1 + \hat{s})\sqrt{1 - \frac{4m_{\ell}^2}{s}}}{(2m_{\ell} + \sqrt{s})} \]
\[ N_3(\hat{s}) = \frac{(1 - \hat{s})\hat{s}\sqrt{1 - \frac{4m_{\ell}^2}{s}}}{(2m_{\ell} + \sqrt{s})} \]
\[ N_4(\hat{s}) = \frac{(1 - \hat{s})\hat{s}\sqrt{1 - \frac{4m_{\ell}^2}{s}}}{(2m_{\ell} + \sqrt{s})} \]
\[ N_5(\hat{s}) = \frac{\hat{s}(1 - 8m_{\ell}^2 + \hat{s})\sqrt{1 - \frac{4m_{\ell}^2}{s}}}{(2m_{\ell} + \sqrt{s})} \]  

(3.14)

IV. COMBINED ANALYSIS OF THE POLARIZATION ASYMMETRIES

One can get useful information about new physics by observing the asymmetries of \( \ell^- \) and \( \ell^+ \) and the combination of these asymmetries. As has been shown in many other earlier works (in model independent ways) regarding these asymmetries in decay modes \( B \rightarrow X_s\ell^-\ell^+ \) and \( B \rightarrow K^*\ell^-\ell^+ \) that within SM some linear combination of these asymmetries vanish. The major results of those investigations were (within Standard Model):

- \( P_L^- + P_L^+ = 0 \).
- \( P_T^- - P_T^+ \approx 0 \).
- \( P_N^- + P_N^+ = 0 \).

It was argued that any deviations from above result will give a definite signal for new physics.

We will analyse the same combination of the various polarization asymmetries within the radiative dileptonic decay \( (B_s \rightarrow \ell^+\ell^-\gamma) \) and will try to figure out what happens to these combinations within SM and if SUSY effects are included.

FIG. 5: Sum of Longitudinal Polarization asymmetries of \( \ell^- \) and \( \ell^+ \), \( \mu \) (left) and \( \tau \) (right). Parameters are : \( m = 200 \), \( M = 450 \), \( A = 0 \), \( tan\beta = 40 \) and \( sgn(\mu) \) is +ve. For SUGRA the pseudoscalar Higgs mass is taken to be 306. All masses are in GeV.
(A) For $P_L^− + P_L^+$ the result is:

$$P_L^− + P_L^+ = 64 f_b \, m_ℓ^2 \left[ 4 \frac{f_b}{m_{B_s}} \left( C_{10} + \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_2} \right) \left( \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_1} \right) L_2(\hat{s}) \right. + \left. \left( C_{10} + \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_2} \right) Re(B) L_3(\hat{s}) + \left( \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_1} \right) Re(A) L_5(\hat{s}) \right] / \Delta \quad (4.1)$$

we can very easily see that within SM (when $C_{Q_1}$ and $C_{Q_2}$ are zero) the sum of longitudinal polarization asymmetries of $\ell^−$ and $\ell^+$ doesn’t vanish.

(B) For $P_T^− - P_T^+$ the result is:

$$P_T^− - P_T^+ = 16\pi m_ℓ f_b \left[ \left( C_{10} + \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_2} \right) Re(C) T_4(\hat{s}) + \left( \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_2} \right) Re(D) T_6(\hat{s}) \right] / \Delta \quad (4.2)$$

we can again see the same pattern that within SM the difference of the transverse polarization asymmetries of $\ell^−$ and $\ell^+$ doesn’t vanish.

(C) For $P_N^− + P_N^+$ the result is:

$$P_N^− + P_N^+ = 16\pi m_ℓ \left[ \left( C_{10} + \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_2} \right) Im(A) N_2(\hat{s}) + \left( \frac{m_{B_s}^2}{2m_ℓ m_b} C_{Q_1} \right) Im(B) N_4(\hat{s}) \right] \quad (4.3)$$

we have the repetition of the same pattern that the sum of the polarization asymmetries of $\ell^−$ and $\ell^+$ doesn’t vanish.

V. PHOTON POLARIZATION ASYMMETRY

In radiative dileptonic decay mode of B-meson ($B_s \to \ell^+ \ell^− \gamma$) even the final state photon can emerge with a definite polarization. So one can study the effects of polarized photon also in this particular decay mode. Here we introduce such a variable which we call photon polarization asymmetry. Defined as:

$$H = \frac{dΓ(ε^*=ε_1)}{d\hat{s}} - \frac{dΓ(ε^*=ε_2)}{d\hat{s}} \quad \frac{dΓ(ε^*=ε_1)}{d\hat{s}} + \frac{dΓ(ε^*=ε_2)}{d\hat{s}}$$

(5.1)
where \( \epsilon_1 \) and \( \epsilon_2 \) are the two polarization states which a photon can have\(^2\).

Now in order to evaluate the variable \( H \) we have to consider polarized photon in decay rate calculation. In CM (center of mass frame of dileptons) the various four vectors (of \( B_s \) meson, photon, leptons and polarizations of photons) can be taken as\(^3\):

\[
\begin{align*}
P_B &= (E_B, 0, 0, p_B) \\
q &= (p_B, 0, 0, p_B) \\
p_- &= (\frac{\sqrt{s}}{2}, 0, p \sin \theta, -p \cos \theta) \\
p_+ &= (\frac{\sqrt{s}}{2}, 0, -p \sin \theta, p \cos \theta) \\
\epsilon_1 &= \frac{1}{\sqrt{2}} (0, 1, i, 0) \\
\epsilon_2 &= \frac{1}{\sqrt{2}} (0, 1, -i, 0)
\end{align*}
\]

where \( p_B = \frac{(m_{B_s}^2 - s)}{2 \sqrt{s}} \) and \( p = \frac{\sqrt{s}}{2} \sqrt{1 - \frac{4m_{\ell}^2}{s}} \).

Using the above identities one can easily calculate \( H \) as:

\[
H = \left[ \frac{8m_{B_s}^2}{3} Re(A^* B)(1 - \hat{s})^2(2m_{\ell}^2 + \hat{s}) + \frac{8m_{B_s}^2}{3} Re(C^* D)(1 - \hat{s})^2(-4m_{\ell}^2 + \hat{s}) \\
+ 32f_{\beta}m_{\ell}^2 \left( C_{10} + \frac{m_{B_s}^2}{2m_{\ell}m_b} C_{Q_2} \right) Re(B) \frac{(1 - \hat{s})ln(\hat{z})}{\sqrt{1 - \frac{4m_{\ell}^2}{s}}} \\
+ 32f_{\beta}m_{\ell}^2 \left( \frac{m_{B_s}^2}{2m_{\ell}m_b} C_{Q_1} \right) Re(A) \frac{\hat{s}(8m_{\ell}^2 - 2\hat{s} - \sqrt{1 - \frac{4m_{\ell}^2}{s}})ln(\hat{z})(-1 + 4m_{\ell}^2 + \hat{s}))}{4m_{\ell}^2 - \hat{s}} \right] / \Delta
\]

\(^2\) photon being mass-less particle can only have two polarization states which we have called \( \epsilon_1 \) and \( \epsilon_2 \) which actually are conjugate to each other. These states can equally be called positive and negative helicity states respectively.

\(^3\) here we are choosing leptons to be lying in YZ-plane and B-meson is moving along Z-direction.
VI. RESULTS AND DISCUSSION

We have performed the numerical analysis of the forward-backward asymmetry and various polarization asymmetries whose analytical expressions are given in earlier sections.

We are working under MSSM, which is the simplest having the least number of parameters among the various extensions of the SM. But even the MSSM has a very large number of parameters making it very difficult to do any reasonable phenomenology in such a large parameter space. We therefore focus on some unified models which reduce this large number of parameters to a manageable number. One of the most favorite such model is the Supergravity (SUGRA) model. In SUGRA model in addition to the SM parameters we have: $m_0$ (unified mass of all the scalars), $M$ (unified mass of all the gauginos), $A$ (unified trilinear couplings), $\tan\beta$ (ratio of the VEV of the two Higgs) and $\text{sgn}(\mu)$ as parameters $^4$. This sort of model is called Minimal Sugra (mSUGRA) model.

It has been well emphasized in many works $^5$ that it is not necessary that all the scalars have a unified mass at GUT scale. To suppress $K^0 - \bar{K}^0$ mixing its sufficient to have the universal mass to all the squarks. We will hence explore this type of SUGRA model also where the squarks and Higgs sector has different universal mass. For the results shown in Figures all the MSSM parameters were evolved from GUT scale to electroweak scale using SuSpect $^2$. For MSSM parameter space analysis we have taken a 95% CL bound of $^2$:

$$2 \times 10^{-4} < Br(B \to X_s\gamma) < 4.5 \times 10^{-4}$$

which is in agreement with CLEO and ALEPH results.

In the SUGRA model where the condition of universality of the scalar masses has been relaxed we take the mass of pseudo-scalar Higgs to be a parameter. In Figure (4) we have plotted the forward-backward asymmetry of the lepton in SM, mSUGRA and SUGRA model. As we can see that both SUGRA and mSUGRA gives fairly large deviation from the SM values specially when we have $\tau$ in the final states. In Figures (2, 3, 4) we have plotted the longitudinal, transverse and normal polarization asymmetries respectively of $\ell^+$. As we can see that all the three asymmetries shows fairly large deviation from their respective SM values. The deviation from the SM values is largely because of the new Wilson coefficients $C_{Q1}$ and $C_{Q2}$. These coefficients depends crucially on the two MSSM parameters the Higgs mass and $\tan\beta$. So for relatively large $\tan\beta$ (we have taken this to be 40 here in our calculations) we can have very

$^4$ convention about $\text{sgn}(\mu)$ which we are following is such that $\mu$ occurs in chargino mass matrix with positive sign
large variations in the polarization asymmetries.

Another important observation here is that in earlier model independent analysis of the B-meson decay modes like $B \to X_s \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$ [2], the sum of longitudinal and normal polarization asymmetries (of $\ell^-$ and $\ell^+$) vanishes separately in SM. Also the difference of the transverse polarization asymmetries of $\ell^-$ and $\ell^+$ vanishes. But here in the radiative dilepton decay mode ($b \to s \ell^+ \ell^- \gamma$) this hasn’t happened. So that means that the results quoted in Fukawe et. al. and Aliev et. al. [22] (that these quantities identically vanish in SM) was a process dependent statement. Here in the process we are considering the $C_{10}$ (which is non-zero in SM) is behaving exactly like $C_Q$. As we can see from figures (5,7) that the sum of polarization asymmetries, of $\ell^+$ and $\ell^-$ (for longitudinal and normal) doesn’t vanish even within SM. From figure(3) we can conclude that the difference of the transverse asymmetries also doesn’t vanish within SM. Figure(5) shows the photon polarization asymmetry for completeness. This is much more challenging experimentally to measure as compared to the $\tau^\pm$ polarization asymmetries. So the lepton polarization asymmetries (as compared to photon one) are the best places to look for physics beyond SM in FCNC semi-leptonic B-decays.

From our theoretical and numerical analysis we can thus conclude:

1. From the analysis of the FB-asymmetry, all the three polarization asymmetries associated with the final state leptons we can say that the deviation in these quantities from their respective SM values is fairly substantial almost over the whole region of dileptons invariant mass.

2. As noted in earlier papers regarding general polarization asymmetries in various semi-leptonic decay modes of B-meson [2] that the sum of the longitudinal and normal polarization asymmetries (independently) for lepton ($\ell^-$) and anti-lepton ($\ell^+$) vanishes in SM. Similarly the difference of transverse polarization asymmetries of lepton ($\ell^-$) and anti-lepton ($\ell^+$) also vanishes in SM. But here in radiative dileptonic decay mode ($B_s \to \ell^+ \ell^- \gamma$) this is no-longer true. Here the sum of the longitudinal polarization asymmetries of $\ell^-$ and $\ell^+$ in SM is very small (not exactly zero) for leptons to be muon (see fig.(3)) and the deviation is fairly large in mSUGRA and SUGRA model.

3. The photon polarization asymmetry also shows a large variation from the respective SM value both for muons and tau but this measurement would be much more difficult experimentally. Nonetheless this is still another kinematical variable which could be used to study radiative decays.

4. As we can see that the variation of all the kinematical variables which we have analysed in our work shows large deviation from the respective SM values. But the deviation from the SM is much more enhanced in SUGRA model then in mSUGRA model. The reason for this is the structure of the new Wilson coefficients $C_{Q_2}$. These coefficients depends on Higgs mass (in fact the dependence of these coefficients to Higgs mass is inverse). So as the Higgs mass increases the value of these coefficients decreases. In mSUGRA framework where all the scalars have a common unified mass the Higgs mass turn out to be very high ( and hence value of these coefficients small). Whereas in SUGRA framework as the Higgs sector has a different unification so the masses could be low ( and hence fairly large values of the new coefficients).

So in brief one can say that even the radiative decay mode can be a useful probe in finding out the SUSY signatures and can also probe into the structure of effective Hamiltonian.

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