Horizon fluffs: In the context of generalized minimal massive gravity

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Abstract – We consider a metric which describes Bañados geometries and show that the considered metric is a solution of the generalized minimal massive gravity (GMMG) model. We consider the Killing vector field which preserves the form of the considered metric. Using the off-shell quasi-local approach we obtain the asymptotic conserved charges of the given solution. Similar to the Einstein gravity in the presence of negative cosmological constant, for the GMMG model, we also show that the algebra among the asymptotic conserved charges is isomorphic to two copies of the Virasoro algebra. Eventually, we find a relation between the algebra of the near-horizon and the asymptotic conserved charges. This relation shows that the main part of the horizon fluffs proposed by Afshar \textit{et al.}, Sheikh-Jabbari and Yavartanoo appear for generic black holes in the class of Bañados geometries in the context of the GMMG model.

Introduction. – Recently, Donnay \textit{et al.} [1], have shown that the asymptotic symmetries close to the horizon of the non-extremal black hole solution of the three-dimensional Einstein gravity in the presence of a negative cosmological term, are generated by an extension of supertranslations. More recently, we have studied the behaviors and algebras of the symmetries and conserved charges near the horizon of the non-extremal black holes in the context of the so-called generalized minimal massive gravity [2], proposed in ref. [3]. The authors of [4] have studied the spacetime geometry around the non-extremal horizon in the context of the Einstein gravity in the presence of negative cosmological constant. They have proposed a new set of the boundary conditions which leads to a near-horizon symmetry, the Heisenberg algebra. In another paper, we have studied this near-horizon symmetry in the framework of Chern-Simons–like theories of gravity [5]. In other terms, similar to the near-horizon symmetry algebra of the black flower solutions of the Einstein gravity in the presence of negative cosmological constant, we have found an algebra which consists of two $U(1)$ current algebras, but instead of levels $\pm \frac{k}{2}$, the level of our algebra is given by $\pm \frac{k}{2}(\sigma \pm \frac{1}{m} + \frac{\alpha H + F}{2m^2})$. It is worth mentioning that, using an appropriate coordinates transformation, near-horizon fall-off conditions of any locally $AdS_3$ black hole spacetime can be written in the form of a black flower metric. In fact, a black flower metric gives us general near-horizon fall-off conditions for generic stationary non-axially symmetric black holes. Based on the discussions in refs. [6,7], in field theories with local symmetries, there are conserved charges due to the residual gauge symmetries, which make the field configuration distinguishable. In [6] the authors have found a microcanonical description of the black hole microstates as near-horizon soft hairs. Then they have studied the “horizon fluffs” as proposed in [6] for generic black holes in the class of Bañados geometries in [7]. They have done this study in the context of the Einstein gravity in the presence of negative cosmological constant. In the present work, we generalize the analysis in the case of the Einstein gravity to a class of Chern-Simons–like higher curvature theories. We know that the pure Einstein-Hilbert gravity in three dimensions (in the presence of negative cosmological constat also) exhibits no propagating physical degrees of freedom [8,9]. Adding the gravitational Chern-Simons term produces a propagating massive graviton [10]. The resulting theory is called topologically massive gravity (TMG). Unfortunately TMG has

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a bulk-boundary unitarity conflict. In other terms either the bulk or the boundary theory is non-unitary, so there is a clash between the positivity of the two Brown-Henneaux boundary \( c \) charges and the bulk energies [11]. But, as mentioned in [3], fortunately GMMG avoids the aforementioned “bulk-boundary unitarity clash”. The calculation of the GMMG action to quadratic order about the aforementioned “bulk-boundary unitarity clash”. The calculation of the GMMG action to quadratic order about the AdS\(_3\) space shows that the theory is free of negative-energy bulk modes. Also the Hamiltonian analysis showed that the GMMG model has no Boulware-Deser ghosts and this model propagate two physical modes. So this model is a viable candidate for the semi-classical limit of a unitary quantum 3D massive gravity. So it is our reason and motivation to generalize the Einstein gravity studies in [6,7] to higher curvature theories in the presence of the Chern-Simons term. But totally, the motivation for the above-mentioned recent studies of near-horizon symmetries is due to the the recent nice work of Hawking et al. [12], where one may be find a solution to the information paradox for black holes by the analysis of near-horizon symmetries. The paper is organised as follows. In the second section, we briefly review the method for obtaining conserved charges of CSLTG by the off-shell quasi-local approach. Then, we introduce the GMMG model as an example of CSLT of gravity. In the third section we consider the Bañados geometries and their symmetries. The conserved charges of asymptotic and near-horizon regions will be considered in the fourth section, where their algebras will be lastly presented. The last section is devoted to conclusions and discussions.

**Quasi-local conserved charges of generalized minimal massive gravity.**

**Conserved charges in the context of CSLTG.** First, let us consider the method of obtaining conserved charges of Chern-Simons-like theories of gravity (CSSLG) by the off-shell quasi-local approach [5]. The Lagrangian 3-form of the Chern-Simons-like theories of gravity is given by [10]

\[
L = \frac{1}{2} \tilde{g}_{rs} a^r \cdot da^s + \frac{1}{6} \tilde{f}_{rst} a^r \cdot a^s \times a^t. \tag{1}
\]

In the above Lagrangian \( a^r = a^r_{\mu} dx^\mu \) are the Lorentz-vector-valued one-forms, where, \( r = 1, \ldots, N \) and \( a \) indices refer to flavour and the Lorentz indices, respectively. We should mention that, here, the wedge products of the Lorentz-vector-valued one-form fields are implicit. Also, \( \tilde{g}_{rs} \) is a symmetric constant metric on the flavour space and \( \tilde{f}_{rst} \) is a totally symmetric “flavour tensor” which are interpreted as the coupling constants. We use a 3D vector algebra notation for the Lorentz vectors in which contractions with \( \eta_{ab} \) and \( \epsilon^{abc} \) are denoted by dots and crosses, respectively.\(^1\) It is worth saying that \( a^r \) is a collection of the dreibein \( e^r \), the dualized spin-connection \( \omega^r \), the auxiliary field \( h^r_{\mu} = \epsilon^r_{\nu} h^\nu_{\mu} \) and so on. Also for all interesting CSLTGs we have \( \tilde{f}_{rst} = \tilde{g}_{rs} \) [13].

The total variation of \( a^r \) due to a diffeomorphism generator \( \xi \) is [14]

\[
\delta_\xi a^r = L_\xi a^r - \delta_\xi^\mu d\chi^a_{\mu}, \tag{2}
\]

where \( \chi^a_{\mu} = \frac{1}{2} \epsilon^{abc} \lambda^b_{\mu c} \) and \( \lambda^b_{\mu c} \) is the generator of the Lorentz gauge transformations \( SO(2,1) \), also \( \delta_\xi^\mu \) denotes the ordinary Kronecker delta. We assume that \( \xi \) may be a function of dynamical fields. In the paper [5], we have shown that the quasi-local conserved charge perturbation associated with a field-dependent vector field \( \xi \) is given by\(^2\)

\[
\hat{\delta}Q(\xi) = \frac{1}{8\pi G} \int_\Sigma (\tilde{g}_{rs} \xi^a a^r - \tilde{g}_{\omega s} \chi^a_s) \cdot \hat{\delta}a^s, \tag{3}
\]

where \( G \) denotes the Newtonian gravitational constant and \( \Sigma \) is a space-like codimension-two surface. We can take an integration of (3) over the one-parameter path on the solution space [15,16] and then we find that

\[
Q(\xi) = \frac{1}{8\pi G} \int_0^1 ds \int_\Sigma (\tilde{g}_{rs} \xi^a a^r - \tilde{g}_{\omega s} \chi^a_s) \cdot \hat{\delta}a^s, \tag{4}
\]

Also, we argued that the quasi-local conserved charge (4) is not only conserved for Killing vectors which are admitted by spacetime everywhere but it is also conserved for the asymptotically Killing vectors.

**Generalized minimal massive gravity.** The generalized minimal massive gravity (GMMG) is an example of the Chern-Simons-like theories of gravity [3]. In the GMMG, there are four flavours of one-form, \( a^r = \{e, \omega, h, f\} \), and the non-zero components of the flavour metric and the flavour tensor are

\[
\begin{align*}
\tilde{g}_{e\omega} &= -\sigma, & \tilde{g}_{eh} &= 1, \\
\tilde{g}_{ef} &= -\frac{1}{m^2}, & \tilde{g}_{\omega f} &= \frac{1}{\mu}, \\
\tilde{f}_{e\omega} &= -\sigma, & \tilde{f}_{eh} &= 1, \\
\tilde{f}_{ef} &= -\frac{1}{m^2}, & \tilde{f}_{\omega f} &= \frac{1}{\mu}, \\
\tilde{f}_{ee} &= -\frac{1}{m^2}, & \tilde{f}_{\omega \omega} &= \Lambda_0, & \tilde{f}_{ehh} &= \alpha.
\end{align*}
\]

where \( \sigma, \Lambda_0, \mu, m \) and \( \alpha \) are a sign, a cosmological parameter with dimension of mass squared, a mass parameter of the Lorentz-Chern-Simons term, a mass parameter of the new massive gravity term and a dimensionless parameter, respectively.

For all the solutions of the Einstein gravity with negative cosmological constant, we have

\[
R(\Omega) + \frac{1}{2l^2} e \times e = 0, \quad T(\Omega) = 0, \tag{6}
\]

where \( R(\Omega) = d\Omega + \frac{1}{2} \Omega \times \Omega \) is the curvature 2-form, \( T(\Omega) = D(\Omega) e \) is the torsion 2-form and \( \Omega \) is the torsion-free spin-connection. Also, \( D(\Omega) \) denotes the exterior covariant derivative with respect to \( \Omega \) and \( l \) is the AdS\(_3\).

\(^1\)Here we consider the notation used in [10].

\(^2\)We denote the variation with respect to dynamical fields by \( \hat{\delta} \).
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radius. All the solutions of the Einstein gravity with negative cosmological constant will solve the GMMG equations of motion when the following equations are satisfied [2,5]

\[ f^a = F e^a, \quad h^a = H e^a, \]

where \( F \) and \( H \) are constant parameters.

Conserved charges in the context of GMMG. One can simplify the quasi-local charge perturbation (3) in the context of GMMG for all the considered class of solutions which obey eqs. (6)–(11). After some calculations, one finds that \([5]\)

\[
\delta Q(\xi) = \frac{1}{8\pi G} \int_\Sigma \left\{ \left( \frac{\bar{\sigma}}{\mu^2} + \frac{\alpha}{\mu} H + \frac{F}{m^2} \right) - \frac{1}{\mu^2} + 2(1 + \alpha)H + 2\alpha \frac{e}{m^2} F H + \frac{\alpha_2}{\mu^2} H^2 \right\}

\times \left[ (i\xi_\Omega - \chi_\xi) \cdot \delta e + i\xi e \cdot \delta \Omega \right] + \frac{1}{\mu} \left[ (i\xi_\Omega - \chi_\xi) \cdot \dot{\delta} \Omega + \frac{1}{\mu} i\xi e \cdot \dot{\delta} e \right] \right\}, \tag{12}
\]

By demanding that the Lie-Lorentz derivative of \( e^a \) becomes zero explicitly when \( \xi \) is a Killing vector field, we can find the following expression for \( \chi_\xi \) [17,18]:

\[
\chi_\xi^a = i\xi a \omega^a + \frac{1}{2} e^a_{bc} e^{\rho\nu} (i\xi T^c)^{\rho
u} + \frac{1}{2} e^a_{bc} e^{\rho\nu} e_{\nu\nu}^\bullet \nu \mu \xi_\nu, \tag{13}
\]

where \( \nabla \) denotes covariant derivative with respect to the Levi-Civita connection, and one can show that this expression can be rewritten as \([19]\)

\[
i\xi_\Omega - \chi_\xi = -\frac{1}{2} e^a_{bc} e^{\rho\nu} e_{\nu\nu}^\bullet \nu \mu \xi_\nu. \tag{14}
\]

Also, we mention that the torsion-free spin-connection is given by

\[
\Omega_\mu^a = -\frac{1}{2} e^a_{bc} e^\rho_{\nu\nu} \nabla_\nu e_{\rho\sigma}. \tag{15}
\]

Thus, by using eq. (12), we can find that the conserved charge of a given dreibein \( e^a \) (which describes a spacetime) corresponds to a given symmetry generator \( \xi \).

Bañados geometries and their symmetries. – The Bañados geometries can be expressed by the following line-element [20]:

\[
ds^2 = \frac{l^2 dr^2}{r^2} - \left( rdx^++\frac{l^2 L^-}{r}dx^-\right) \times \left( rdx^- - \frac{l^2 L^+}{r} dx^+\right), \tag{16}
\]

where \( x^\pm = t/l \pm \phi \). \( r \) and \( \phi \sim \phi + 2\pi \) are, respectively, the radial, time and angular coordinates. Also, \( L_{\pm} = L_{\pm}(x^\pm) \) are two arbitrary periodic functions. The line-element (16) solves the equations of motion of the Einstein gravity with negative cosmological constant.

The metric under transformations generated by \( \xi \) transforms as \( \delta_{\xi} g_{\mu\nu} = \xi \cdot g_{\mu\nu} \) (see footnote 3). The variation generated by the following Killing vector field preserves the form of the considered metric [21]:

\[
\xi^r = -\frac{T}{2} (\partial_+ T^+ + \partial_- T^-),
\]

\[
\xi^\pm = T^\pm + \frac{2r^2 \partial_x^2 T^\pm + 4 L^\pm \partial_{x^\pm}^2 T^\pm}{2 (r^4 - 4 L_\pm^2 T^\pm)}, \tag{17}
\]

with \( L_{\pm} \) are the dynamical fields appeared in the metric (16). As we know, Bañados geometries obey the standard Brown-Henneaux boundary conditions at spatial infinity [22]. So, by expanding the Killing vector field (17) at spatial infinity we will find the asymptotic Killing vector which is presented in [22], as we expect. If we set \( \delta_\xi L_{\pm} = 0 \), we will get to exact symmetries of Bañados geometries. In this case \( T^\pm \) are not arbitrary functions and this has been considered in [7,23]. In this paper we consider a general case in which \( \delta_\xi L_{\pm} \neq 0 \), generally.

Since \( \xi \) depends on dynamical fields, we need to introduce a modified version of the Lie brackets [24]

\[
[x_1, x_2] = (\xi^1, \xi^2 - \delta_{\xi^1}^g \xi^2 + \delta_{\xi^2}^g \xi^1), \tag{20}
\]

where \( \xi_1 = (T^1_+, T^-_1) \) and \( \xi_2 = (T^1_+, T^-_1) \). In eq. (21), \( \delta_{\xi^1}^g \xi^2 \) denotes the change induced in \( \xi^2 \) due to the variation of metric \( \delta_{\xi^1} g_{\mu\nu} = L_{\xi^1} g_{\mu\nu} \) [24]. By substituting eq. (17) into eq. (20), we can find that

\[
[x(T^1_+, T^-_1), x(T^1_+, T^-_1)] = \delta_{T^1_+} g_{T^1_+ \xi}, \tag{21}
\]

where

\[
T^1_+ = T^1_+ \partial_{x^+} T^1_+ - T^1_+ \partial_{x^-} T^1_+. \tag{22}
\]

Thus the algebra of the Killing vector fields is closed.

“Asymptotic” and “near-horizon” conserved charges and their algebras. –

Asymptotic conserved charges. In this subsection we are going to obtain the conserved charges corresponding \( L_\xi \) denotes the ordinary Lie derivative along the vector field \( \xi \).
to the asymptotic symmetries of generic black holes in the class of Bañados geometries in the context of the GMMG model. Then we will obtain the algebra satisfied by these conserved charges.

The dreibein which corresponds to the line-element (16) is given as
\[ e^0 = \frac{1}{2} \left( r^2 - \frac{L^+}{r} \right) dx^+ + \frac{1}{2} \left( r^2 - \frac{L^-}{r} \right) dx^- , \]
\[ e^1 = \frac{1}{2} \left( r^2 + \frac{L^+}{r} \right) dx^+ - \frac{1}{2} \left( r^2 + \frac{L^-}{r} \right) dx^- , \]
\[ e^2 = \frac{l}{r} dx . \]

The torsion-free spin-connection which corresponds to dreibein (23) is given by
\[ \Omega^0 = \frac{1}{2l} \left( r^2 - \frac{L^+}{r} \right) dx^+ + \frac{1}{2l} \left( r^2 - \frac{L^-}{r} \right) dx^- , \]
\[ \Omega^1 = \frac{1}{2l} \left( r^2 + \frac{L^+}{r} \right) dx^+ - \frac{1}{2l} \left( r^2 + \frac{L^-}{r} \right) dx^- , \]
\[ \Omega^2 = 0 , \]
where we used eq. (15). One can use eq. (14), eq. (17), eq. (23) and eq. (24) to find that
\[ (i_\xi \Delta \chi) \cdot \hat{e} + i_\xi e \cdot \hat{\Omega} = \]
\[ l \left( T^+ \hat{\Omega} d \xi^+ - T^- \hat{\Omega} d \xi^- \right) , \]
\[ (i_\xi \Delta \chi) \cdot \hat{\Omega} + \frac{1}{2} i_\xi e \cdot \hat{\Delta} e = T^+ \hat{\Omega} d \xi^+ + T^- \hat{\Omega} d \xi^- . \]

By substituting eq. (25) into eq. (12) and taking an integration over one-parameter path on the solution space, we can obtain the conserved charge corresponding to the Killing vector field (17) as
\[ Q(\xi) = Q^+(T^+) + Q^-(T^-) , \]
where
\[ Q^\pm(T^+) = \frac{l}{8\pi G} \left( \sigma + \frac{\alpha H}{\mu} + \frac{F}{m^2} + \frac{1}{\mu l} \right) \int_{\Sigma} T^\pm \hat{\Omega} d\Sigma . \]

Now, it is clear that the space-like codimension-two surface \( \Sigma \) can be taken as a circle of arbitrary radius, and this is a consequence of the quasi-local formalism.

The algebra of conserved charges can be written as [25]
\[ \{ Q(\xi_1) , Q(\xi_2) \} = C (\chi_1 , \chi_2) + \mathcal{C} (\xi_1 , \xi_2) , \]
where \( \mathcal{C} (\xi_1 , \xi_2) \) is the central extension term. Also, the left-hand side of eq. (28) can be defined as
\[ \{ Q(\xi_1) , Q(\xi_2) \} = \delta_{\xi_2} Q(\xi_1) . \]

Therefore one can find the central extension term by using the following equationat:
\[ C (\xi_1 , \xi_2) = \delta_{\xi_2} Q(\xi_1) - Q (\{ \xi_1 , \xi_2 \}) . \]

By substituting eq. (19), eq. (21) and eq. (26) into eq. (30) we will obtain the central extension term, then by substituting the obtained result into eq. (28) we have
\[ \{ Q^+(T^+) , Q^+(T^+) \} = \frac{l}{4\pi G} \left( \sigma + \frac{\alpha H}{\mu} + \frac{F}{m^2} + \frac{1}{\mu l} \right) \int_{\Sigma} T^+ \hat{\Omega} d\Sigma , \]
\[ \frac{1}{8} \int_{\Sigma} \left( T^+ \hat{\Omega} d \xi^+ - T^- \hat{\Omega} d \xi^- \right) , \]
\[ \{ Q^+(T^+) , Q^-(T^-) \} = 0 . \]

By introducing the Fourier modes \( Q^\pm = Q^\pm (e^{imx^+}) \), we find that
\[ i \{ Q^m_1 , Q^m_2 \} = (m - n) Q^m_{n+n} \]
\[ + \frac{l}{8G} \left( \sigma + \frac{\alpha H}{\mu} + \frac{F}{m^2} + \frac{1}{\mu l} \right) \times \left[ n^3 \delta_{m+n,0} + \frac{2}{\pi} (m - n) \hat{\bar{L}}(m+n) \right] , \]
\[ i \{ Q^m_1 , Q^m_2 \} = 0 , \]
where
\[ \hat{\bar{L}}(m+n) = \frac{1}{\pi} \int_{\Sigma} e^{imx^+} \hat{\bar{\Omega}} d\Sigma . \]

Now we set \( \hat{\bar{L}} \equiv Q^\pm \) and replace the Dirac brackets by commutators \( i\{,\} \rightarrow [\cdot,\cdot] \). After making a constant shift on the spectrum of \( \hat{\bar{L}} \) [26], we find that
\[ [\hat{\bar{L}}^m_1 , \hat{\bar{L}}^n_1 ] = (m - n) \hat{\bar{L}}^m_1 + \frac{c_+}{12} n^2 (n-1) \delta_{m+n,0} , \]
\[ [\hat{\bar{L}}^m_1 , \hat{\bar{L}}^n_1 ] = 0 , \]
where \( c_\pm \) are the central charges and they are given by
\[ c_\pm = \frac{3}{2G} \left( \sigma + \frac{\alpha H}{\mu} + \frac{F}{m^2} + \frac{1}{\mu l} \right) . \]

It is obviously seen that \( \hat{\bar{L}} \) are the generators of the Virasoro algebra and then the algebra among the asymptotic conserved charges is isomorphic to two copies of the Virasoro algebra. The Virasoro algebra has been derived in 3D gravity with higher curvature [28] and in TMG [29]. Recently, it has been done in GMMG for asymptotically AdS3 spacetimes. In fact, the authors in [30] have shown that the algebra among the asymptotic conserved charges is isomorphic to two copies of the Virasoro algebra, central charges [31] for asymptotically AdS3 spacetimes. Therefore the results given in this subsection respect the generic results.

**Near-horizon conserved charges.** One can use the method presented in the second section to find conserved charges of near-horizon geometries as those presented in [4], which obey the near-horizon fall-off conditions of non-extremal black holes in three dimension. We have done this work in paper [5] and we found the near-horizon
conserved charges in the context of GMMG. Also, we showed that the obtained near-horizon conserved charges obey the following algebra:

\[
\begin{bmatrix}
\hat{j}_m^+ \quad \hat{j}_n^-
\end{bmatrix} = \pm \frac{c_+}{12} m \delta_{m+n,0}, \quad (39)
\]
\[
\begin{bmatrix}
\hat{j}_m^- \quad \hat{j}_n^+
\end{bmatrix} = 0. \quad (40)
\]

Similar to the near-horizon Virasoro algebra in the Einstein gravity with negative cosmological constant [4], the algebra (39) and (40) consists of two \(U(1)\) current algebras, but instead of levels \(\pm \frac{1}{2}\), the level of algebra is given by \(\pm \frac{1}{2}\) here.

**Relation between asymptotic and near-horizon algebras.**

To relate the asymptotic Virasoro algebra (36) and the near-horizon algebra (39), we need a twisted Sugawara construction [27] as follows:

\[
\hat{\Delta}_m = \frac{im}{\sqrt{12}} \hat{j}_m^+ + \frac{6}{c_+} \sum_{p \in \mathbb{Z}} \hat{j}_{m-p}^+ \hat{j}_p^+. \quad (41)
\]

It is straightforward to show that

\[
\begin{bmatrix}
\hat{\Delta}_m^+ \quad \hat{\Delta}_n^-
\end{bmatrix} = (m-n) \hat{\Delta}_{m+n}^+ + \frac{c_+}{12} n^2 \delta_{m+n,0}, \quad (42)
\]

Now, we can get to the asymptotic Virasoro algebra by making a shift on the spectrum of \(\hat{\Delta}_m^+\) by a constant,

\[
\hat{L}_m^\pm = \hat{\Delta}_m^\pm + \frac{c_+}{24} \delta_{m,0}. \quad (43)
\]

In this way, we could relate the near-horizon symmetry algebra to the asymptotic one in the context of the GMMG model.

In fact, the algebra established by \(\hat{\Delta}_m^\pm\) is the near-horizon Virasoro algebra. In other words, eq. (41) relates the near-horizon algebra to the near-horizon Virasoro algebra with central charges \(c_\pm\). Now we have two Virasoro algebras, the asymptotic one with central charges given in (36) and the near-horizon one with unity central charges given in (39). Then we have used eq. (43) to relate the near-horizon Virasoro algebra to the asymptotic Virasoro algebra.

Usually, we identify residual symmetries and the conjugate conserved charges through imposing certain boundary conditions in the asymptotic region of spacetime. We have shown that Bañados geometries are solutions of GMMG. Then they form the solution phase space with a given symplectic two-form [31–34]. We can promote the asymptotic symmetries to symplectic symmetries (by virtue of eq. (18)). Symplectic symmetries are transformations on the solution phase space which do not change the symplectic two-form. The concept of symplectic symmetry extends the notion of asymptotic symmetry inside the bulk [21]. It is clear from eq. (26) that the value of conserved charges and algebra among them are independent of choosing a space-like codimension-2 surface \(\Sigma\). Therefore we expect that the Virasoro algebra will be held at any place in spacetime (either near the horizon or asymptotically). In fact, the near-horizon algebra and the asymptotic Virasoro algebra are both symplectic symmetries of the phase space of locally \(AdS_3\) black hole solutions in the corresponding coordinate systems. Because we have used two different conventions in the near-horizon and asymptotic Virasoro algebra then the extra term in the right-hand side of eq. (43) has appeared.

**Conclusion.** – We have considered the GMMG model as an example of CSLT. We argued that Bañados geometries are solutions of GMMG. In the subsection “Asymptotic conserved charges”, we found the asymptotic conserved charges of the Bañados geometries, conjugate to asymptotic Killing vector fields (17) which preserve the form of the metric, using the off-shell quasi-local approach. Thus, it does not matter that the integration surface is located at spatial infinity or it is to be a circle of finite radius. Also, in that subsection, we showed that the algebra among the asymptotic conserved charges is isomorphic to two copies of the Virasoro algebra (see eq. (36) and eq. (37)) with central charges \(c_\pm\). As we have mentioned in subsection “Near-horizon conserved charges”, the algebra of near-horizon conserved charges is given by eq. (39) and eq. (40). Eventually, later on, we have related the algebra of near-horizon conserved charges to the asymptotic one. So, in this paper we have shown that the main part of the horizon fluffs proposed in refs. [6,7] appears for generic black holes in the class of Bañados geometries in the context of the GMMG model. In other words, we showed that asymptotic conserved charges satisfy two copies of the Virasoro algebra on the one hand, and by using a twisted Sugawara construction on the other hand, we related the near-horizon symmetry algebra to the asymptotic one.

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