Cross-correlation of 2MASS and WMAP 3: implications for the integrated Sachs–Wolfe effect

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ABSTRACT
We perform a cross-correlation of the cosmic microwave background using the third year Wilkinson Microwave Anisotropy Probe (WMAP) data with the Two Micron All Sky Survey (2MASS) galaxy map (about 828,000 galaxies with median redshift z ≈ 0.07). One motivation is to detect the integrated Sachs–Wolfe (ISW) effect, expected if the cosmic gravitational potential is time dependent; for example, as it is in a flat universe with a dark energy component. The measured spherical harmonic cross-correlation signal favours the ISW signal expected in the concordance Lambda cold dark matter (ΛCDM) model over that of zero correlation, although both are consistent with the data within 2σ. Within a flat ΛCDM model we find a best-fitting value of ΩΛ = 0.85 and ΩΛ < 0.89 (95 per cent CL). The above limits assume a galaxy bias bgal(σ8/0.75) ≈ 1.40 ± 0.03, which we derived directly from the 2MASS autocorrelation. Another goal is to test if previously reported anomalies in the WMAP data are related to the galaxy distribution (the so-called ‘axis of evil’ – AoE). No such anomaly is observed in the 2MASS data nor are there any observed AoE correlations between the 2MASS and WMAP3 data.

Key words: cosmic microwave background – cosmological parameters – cosmology: observations – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION
Our currently favoured Lambda cold dark matter (ΛCDM) cosmological model has received ongoing confirmation and bolstering over recent years, especially from recent observations of the cosmic microwave background (CMB) by the Wilkinson Microwave Anisotropy Probe (WMAP) (Spergel et al. 2003, 2006). This model postulates that more than two thirds of the Universe is composed of ‘dark energy’, a mysterious energy with negative pressure. This dark energy has never been directly observed – only inferred. The case is does require some assumptions about other cosmological parameters. As CMB photons travel through space they pass through the gravitational potential wells of LSS. As they fall into a well, the photons are blueshifted, and as they climb out they are redshifted. In an Einstein–de Sitter universe, these effects cancel and no net shift is observed on large scales. However, if dark energy dominates, large scale gravitational potentials decay and there is an overall net effect. This secondary anisotropy is called the late-time ISW effect.

On large scales, the ISW effect will add power to the CMB anisotropies by

\[
\left( \frac{\Delta T}{T} \right)_{\text{ISW}} = -2 \int_{\eta_0}^{\eta_1} \Phi' (\eta - \eta_L(\mathbf{k}, \eta)) d\eta,
\]

where T is the temperature; η is the conformal time, defined by \(d\eta = \frac{dx}{a R}\), and \(\eta_0\) and \(\eta_1\) are the conformal times today and at the surface of last scattering, respectively; \(\mathbf{k}\) is the unit vector along the line of sight; \(\Phi(\mathbf{x}, \eta)\) is the gravitational potential at position \(\mathbf{x}\) and at conformal time \(\eta\), and \(\Phi' = \partial \Phi / \partial \eta\). Its relative amplitude makes it difficult to distinguish from the primary anisotropies. However, Crittenden & Turok (1996) proposed using the cross-correlation between the LSS and the CMB to detect the ISW effect, independently from the intrinsic CMB fluctuations. In the case where the gravitational potentials decay, a positive correlation is expected. This means that on large scales hotspots in the CMB will correspond to overdense regions in the galaxy distribution. This positive

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correlation is also expected in open universes (Kamionkowski 1996; Kinkhabwala & Kamionkowski 1999), whereas a negative correlation will occur in closed universes. The Sunyaev–Zeldovich effect also produces a negative correlation, but this is expected on smaller scales ($\ell > 20$ for $z \approx 0.07$, see Afshordi, Loh & Strauss 2004).

In our currently favoured cosmological models, we believe the Universe has recently ($z < 1$) become dominated by dark energy, which makes the ISW effect a fitting probe of our cosmological model, though alternative models of gravity predict similar signatures (Carroll et al. 2005; Song, Sawicki & Hu 2007).

The first cross-correlations between the COBE (Cosmic Microwave Background Explorer) CMB map and tracers of the LSS (hard X-ray background and Radio sources) did not reveal any significant detections (Boughn & Crittenden 2002, 2003); however, there have since been a number of reported detections of late-time ISW from cross-correlating the first-year WMAP data (WMAP1) with: radio sources (Boughn & Crittenden 2004; Nolta et al. 2004; Boughn & Crittenden 2005); the hard X-ray background (Boughn & Crittenden 2004, 2005); the Sloan Digital Sky Survey (SDSS) (Fosalba, Gaztanaga & Castander 2003; Scranton et al. 2003; Padmanabhan et al. 2005); the Two Micron All Sky Survey (2MASS) (Afshordi et al. 2004); the APM Galaxy Survey (Fosalba & Gaztanaga 2004); a combination of the above (Gaztanaga, Manera & Multamaki 2006). Recently, the third-year WMAP data (WMAP3) was correlated by Cabre, Gaztanaga, Manera, Fosalba & Castander (2006) with the fourth SDSS data release (DR4), and they detected a significant positive cross-correlation, while Giannantonio et al. (2006) cross-correlated it with high-redshift SDSS quasars and found a $2\sigma$ detection. The above cross-correlations were all performed in angular or harmonic space. McEwen et al. (2007) used a directional spherical wavelet analysis and found a positive detection at the 3.9$\sigma$ level.

A cross-correlation between LSS and the CMB can also probe the issue of foreground contamination in the CMB and LSS maps. Recent studies have highlighted anomalous features in CMB data that include alignments of the low-$\ell$ multipoles, and a north–south asymmetry (Eriksen et al. 2004; Land & Magueijo 2005b; Copi et al. 2006). These two qualities of 2MASS make it one of the best available tracers of their mass. Since IR light penetrates more easily through the older stellar populations in galaxies and is therefore a robust tracer of their mass. Since IR light penetrates more easily through the older stellar populations in galaxies and is therefore a robust tracer of their mass. Since IR light penetrates more easily through the older stellar populations in galaxies and is therefore a robust tracer of their mass. Since IR light penetrates more easily through the older stellar populations in galaxies and is therefore a robust tracer of their mass. Since IR light penetrates more easily through the older stellar populations in galaxies and is therefore a robust tracer of their mass. Since IR light penetrates more easily through the older stellar populations in galaxies and is therefore a robust tracer of their mass.

In this section we summarize the two data sets used for the cross-correlation.

2.1 The large-scale structure: 2MASS

We use the publicly available full-sky extended source catalogue (XSC) of the near-IR 2MASS (Jarrett et al. 2000). Following Afshordi et al. (2004), we divide the galaxies into different magnitude bands depending on their $K_{\text{S}}$-band isophotal magnitude $K_{20}$ ($^\prime \langle k_m l 20 c^\prime - 20 \text{ mag arcsec}^{-2}\rangle$ isophotal circular aperture magnitude). We correct these magnitudes for Galactic extinction using the IR reddening maps of Schlegel, Finkbeiner & Davis (1998)\(^1\):

$$K_{20} \rightarrow K_{20} - A_K ,$$

(2)

where $A_K = 0.367 \times E(B - V)$ (Afshordi et al. 2004). We use the same extinction maps to create a mask that excludes regions of sky where the XSC is unreliable. Afshordi et al. (2004) find a limit of $A_K < 0.05$ for which 2MASS is seen to be 98 per cent complete for $K_{20} < 13.85$. We adopt this method herein, and mask areas with $A_K > 0.05$, leaving 69 per cent of the sky with approximately 828 000 galaxies for the analysis.

For our four $K_{20}$ magnitude shells, we adopt the redshift distribution computed by Afshordi et al. (2004) (parametrized by their equations 33 and 35). They fit the redshift distribution from the 2MASS $K_{20}$ luminosity function (Kochanek et al. 2001) for a three-parameter generalized gamma distribution which we use herein. They recall the number counts for the four $K_{20}$ shells and the redshift $z_0$ at which the distributions peak, taken from Afshordi et al. (2004), in Table 1. In Fig. 1 we plot the redshift distribution for each shell as well as for $12 < K_{20} < 14$. The 2MASS overdensity field for galaxies with $K_{20} < 14.0$ is plotted in Fig. 2, where it is convolved with a Gaussian beam of full width at half-maximum (FWHM) 100 arcmin.

\(^1\) Corresponding reddening maps can be found at http://astro.berkeley.edu/davis/dust/index.html.

Table 1. The number of galaxies for our four $K_{20}$ shells and for all four shells combined. We also recall the peak redshift values for the distributions. These values are taken from Afshordi et al. (2004). The parametrizations of these distributions are shown in Fig. 1.

| Magnitude | $N_{\text{tot}}$ | $z_0$ |
|-----------|----------------|-------|
| $12.0 < K_{20} < 12.5$ | 49 902 | 0.043 |
| $12.5 < K_{20} < 13.0$ | 102 947 | 0.054 |
| $13.0 < K_{20} < 13.5$ | 217 831 | 0.067 |
| $13.5 < K_{20} < 14.0$ | 457 267 | 0.084 |
| $12.0 < K_{20} < 14.0$ | 827 947 | 0.073 |

In Section 2 we introduce the 2MASS and WMAP3 data that we use, and in Section 3 we outline our theory. In Section 4 we discuss our choice of fiducial model. In Section 5 we estimate the galaxy bias, $b_g$, from the galaxy autocorrelation function. In Section 6 we discuss our cross-correlation method. In Section 7 we present our results of the cross-correlation and discuss their statistical significance. In Section 8 we investigate AoE type correlations and in Section 9 we present a discussion of our results.
We use the third-year data from NASA’s WMAP satellite (Hinshaw et al. 2007; Spergel et al. 2006). We use the foreground reduced maps of the $Q$ (41 GHz), $V$ (61 GHz) and $W$ (94 GHz) bands. The foreground reduced sky maps were produced by removing a foreground model from the ‘unreduced’ maps. Synchrotron, free–free, and dust emission templates were modelled and then subtracted from the single year ‘unreduced’ maps. Full three-year maps were produced by performing a weighted, pixel-by-pixel, mean of the three single-year maps. This same weighted mean method was then used to combine three-year maps of the same frequency band into a single map for each frequency. Two maps were combined to produce the $Q$- and $V$-band maps; four maps were combined to produce the $W$-band map. We use the Kp2 mask to exclude the Galactic plane and other known foreground sources. We also use the WMAP3 internal linear combination (ILC) map with a Kp2 mask. The WMAP3 ILC map convolved with a Gaussian beam with FWHM 100 arcmin is plotted in the top part of Fig. 2 in Mollweide projection. As with the 2MASS data, we use the HEALPix routine ‘map2alm’ to find the spherical harmonic coefficients.

We also return to the WMAP1 data and in Section 6 we compare the cross-correlation results for the $V$ band (which is the least contaminated band with the best resolution). We use the first-year foreground cleaned maps, with the Kp2 mask, and inverse-noise co-add the two available $V$-band maps.

Cross-correlation of 2MASS and WMAP3

3 THEORY

In this section, we present the formalism for the calculation of autocorrelation and cross-correlation functions (ACF and CCF).

In what follows, we use $r$ as the comoving distance and implicitly as a label of redshift epoch $z$. They are related for a given cosmology by $dr = c H(z) d\zeta$, where $c$ is the speed of light in vacuum. Thus, the growth factor $D(z)$, the growth function $f(z)$ (defined below) and the Hubble parameter $H(z)$ all have implicit dependences on $r$.

For the angular CCF:

$$C_{\ell \tau}(\ell) = 4\pi b^2_\ell \int \frac{\Delta^2(k)}{k} \sqrt{W_{\ell}(k)} W_{\tau}(k).$$

(3)

For the angular overdensity ACF:

$$C_{\ell\delta}(\ell) = 4\pi b^2_\ell \int \frac{\Delta^2(k)}{k} |W_{\delta}(k)|^2,$$

where

$$\Delta^2(k) = \frac{4\pi}{(2\pi)^3} k^3 P(k),$$

(5)

$$W_{\delta}(k) = \int dr \Theta(r) j_z(kr) D,$$

(6)

$$W_{\tau}(k) = -\frac{3\Omega_{m,0} H_0^2}{k^2 c^3} \int_0^{\Delta_z} dr j_z(kr) H D(f - 1),$$

(7)

$$\Theta(r) = \frac{r^2 n_c(r)}{\int dr r^2 n_c(r)}.$$

(8)

Integrals are evaluated over the wavenumber $k$ (expressed in $h$ Mpc$^{-1}$), and the comoving distance $r$ (in $h^{-1}$ Mpc). The matter power spectrum, $P(k)$, is related to the galaxy power spectrum through the linear bias $b_\ell$ which we take to be constant on the depth of our survey. This is justified by the Galaxy conserving model of Magliocchetti et al. (2000) (their equation 2) where the bias evolution between redshift zero and the mean redshift of 2MASS was found to be less than 1 per cent. The selection function, $n_c(r)$, for each magnitude shell, is shown in Fig. 1.

The linear growth factor, $D(z)$, is given by

$$D(z) \propto H(a) \int_{a_{\min}}^{a(z)} \frac{da'}{[a'H(a')]^3},$$

(9)

and is normalized such that $D(0) = 1$. In equation (7), the redshift at the surface of last scattering is $z_L \sim 1089$. The growth function $f$ is given by $f(z) \equiv 1 + \delta H(z) / aH(a)$ and can be approximated by $f(z) \simeq \Omega_{m,0}^{1/6} (z)$ (Peebles 1993, their equation 5.120). The dependence on the dark energy density and equation of state is negligible at the present epoch (Lahav et al. 1991; Wang & Steinhardt 1998). The Hubble constant is parametrized by $H_0 = 100 \, h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$ and $j_z(kr)$ is the spherical Bessel function of the first kind of order $\ell$.

These equations are exact in linear theory, and for $\ell > 10$ we replace them with the small-angle approximation or Limber equation (Afshordi et al. 2004), which arise from the Bessel function approximation:

$$\lim_{\ell \to \infty} j_z(x) = \frac{\pi}{2x} e^{-x}.$$  

(10)

Equations (3) and (4) then simplify to

$$C_{\ell \tau}(\ell) = \frac{-3\Omega_{m,0} H_0^2}{c^3 (\ell + 1/2)^2} \int d\zeta D^2 H[f - 1] P\left(\frac{\ell + 1/2}{r}\right),$$

(11)

Footnotes:

1. Cross-correlation of 2MASS and WMAP3

2. Available at http://lambda.gsfc.nasa.gov/.

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Reducing the number of integrals significantly reduces computation time. In Fig. 3 we compare the CCF using the equation exact in linear theory (equation 3) with its small-angle approximation (equation 11). The difference is less than 1 per cent from $\ell = 5$ upwards. However, for lower multipoles the difference is more important; for example, at $\ell = 2$ it is of order 10 per cent. The small-angle approximation makes the assumption that $kr \sim (\ell + 1/2)$. From this we see that for deeper surveys, the small-angle approximation will begin to hold at a higher multipole. One should therefore check at which multipole the Limber equation begins to hold. Throughout this paper we use equations (3) and (4) (11 and 12 when $\ell > 10$) to model our theory.

In Fig. 4 we plot the CCF for different values of the matter density, $\Omega_{m,0}$, for flat cosmologies with dark energy. The correlation increases with increasing $\Omega_{m,0}$. We note that an ISW effect is also observed in open cosmologies without dark energy (Kamionkowski & Spergel 1994; Kamionkowski 1996; Kinkhabwala & Kamionkowski 1999); and for low redshifts ($z < 2$) the signal expected in an open cosmology can be greater than the signal expected in a flat $\Lambda$CDM cosmology (Kamionkowski 1996). Closed cosmologies would produce a negative correlation. In fact, within the family of cosmological models with arbitrary matter density, cosmological constant and curvature, only the Einstein–de Sitter universe ($\Omega_m = 1, \Omega_{\Lambda} = 0$) will give zero correlation. This can be seen from the last term in equation (7); for no correlation, the following condition must be satisfied: $f(z) \simeq \Omega_{m,0}^{0.6} (z) = 1$ for all redshift.

4 CHOICE OF FIDUCIAL MODEL
AND PRIORS

In this paper we focus on how to constrain dark energy from the ISW effect alone. This requires of course assumptions about the other cosmological parameters. To have the ISW result independent of the CMB, we prefer to assume ‘round’ cosmological parameters.
For the baryon density we assume $\Omega_b = 0.05$, based on big bang nucleosynthesis (Copi, Schramm & Turner 1995).

The normalization of the power spectrum, parametrized as $\sigma_8$, is still highly uncertain, with reported values in the range $\sigma_8 \approx 0.75$ (e.g. the recent WMAP3 result) to $\sigma_8 \approx 1.0$ (Massey et al. 2005). We fix $\sigma_8 = 0.75$, and we solve for the galaxy biasing $b_8$. We note that in linear theory we actually constrain the product $b_8 \sigma_8$, so we can easily scale the result for any preferred value of $\sigma_8$.

In principle one should marginalize over the prior associated with each parameter. However, as we show in Section 7, the cross-correlation signal is very weak and is actually compatible with the null hypothesis of no correlation. By widening the priors on $b_8$ and $\sigma_8$, we find this makes the signal even less significant. We are aware that widening priors for each parameter will reduce the significance of our result, but we chose to opt for a more optimistic view, and acknowledge that the significance of our result is tentative.

### 5 Galaxy Biasing from the Galaxy ACF

We compute the ACF of 2MASS, $C_{gg}(\ell)$, and use it to constrain the galaxy bias, $b_8$, in equations (4) and (12). Since we assume a constant bias across the magnitude shells we make the fit using all the galaxies at once, $12 < K_{20} < 14$. The redshift distribution for all of 2MASS is plotted in Fig. 1, and in Fig. 5 we plot the results of the bias fitting.

Rigorously, equations (3) and (4) (and equations 11 and 12) only hold for a linear matter power spectrum (calculated using CAMB (Lewis, Challinor & Lasenby 2000)), as we have assumed the redshift dependent power spectrum was separable: $P(k, z) = D^2(k)P_{lin}(k)$. We observe that using a linear power spectrum, one can fit the ACF well for linear scales, that is, for $\ell < 30$. However, we find that using a non-linear power spectrum (from Smith et al. 2003, as implemented in CAMB) provides a better fit to the data, and to higher $\ell$, suggesting that using $P(k, z) = D^2(k)P_{non-lin}(k)$ can be considered a valid approximation to about $\ell \sim 50$; since the bias does not change on the depth of our survey, we do not expect its scale-dependence to change much. This issue does not arise when considering the CCF, as the ISW effect should only arise on linear scales.

We therefore decide to fit the ACF using a non-linear power spectrum for $\ell \leq 50$ (to avoid highly non-linear scales), and we bin the data into six logarithmically spaced bins. This reduces correlation between different multipoles so that we can assume the bins are

which are in accord with other cosmological measurements, rather than adopting exact values from another analysis, for example, the WMAP3 TT correlation function.

Based on inflation, we assume the Universe is flat, therefore $\Omega_m = 1 - \Omega_b$. We also assume the spectral index to have the Harrison–Zeldovich value $n = 1$ (although some inflationary models and the recent WMAP3 data suggest $n \approx 0.95$). Based on the HST key project, we take for the Hubble parameter $h = 0.7$ (the 1σ error bar is about 10 per cent). For the matter density we take $\Omega_m = 0.30$, in accord with supernova 1a data combined with the flatness of the universe (we note however that the recent 2dFGRS data (Cole et al. 2005) and WMAP3 (Spergel et al. 2006) favour $\Omega_m \approx 0.25$).
where the diagonal terms of the covariance matrix $\mathbf{M}$, our observed overdensity $\text{ACF}$ after shot noise subtraction, given by $\sigma_d(\ell = 1 - 25)$ the best-fitting bias changes slightly to $b_g, \text{lin} = 1.38 \pm 0.05$ which does not affect the $\chi^2$ values reported in Table 2. The autocorrelation $C_{gg}$ is shown in Fig. 5 with the fit $b_g = 1.40$.

independent and have Gaussian error bars. Hence, scatter about the expected signal is just due to cosmic variance, and the likelihood can be written:

$$-2 \ln \mathcal{L} = \det(\mathbf{M}) + d^T \mathbf{M}^{-1} d + \text{constant},$$

where the diagonal terms of $\mathbf{M}$ contain the variance of $C_{gg}$ (e.g. Dodelson 2003),

$$\sigma_d^2(C_{gg}) \approx \frac{1}{f_{\text{sky}}} \frac{2}{2\ell + 1} C_{gg}^2$$

and $d = (\hat{C}_{gg} - C_{gg})$, $f_{\text{sky}}$ is the fraction of sky observed; $\hat{C}_{gg}$ is our observed overdensity $\text{ACF}$ after shot noise subtraction, given by $1/\hat{N}$ where $\hat{N}$ is the mean number of galaxies per steradian; $C_{gg}$ is the theory (equations 4 and 12). Note that the covariance matrix $\mathbf{M}$, and its determinant, depend on $b_g$.

We fit for the bias using our fiducial model (flat universe with $\Omega_m = 0.30, \Omega_b = 0.05, h = 0.7, \sigma_8 = 0.75$) and we plot the resulting likelihood curve for Fig. 6. The best-fitting value for the multipole range $\ell = 1 - 50$ is $b_g = 1.40 \pm 0.03$ (to 2 degree precision (d.p.)) at $1\sigma$ (fitting for a Gaussian). If we remove non-linear scales to the bias fitting, and consider only the multipole range $\ell = 1 - 25$, the error on the bias increases slightly while its value remains roughly the same ($b_g, \text{lin} = 1.38 \pm 0.05$). We find our results in Section 7 do not differ significantly for these two values, and herein we take $b_g = 1.40 \pm 0.03$.

The determination of the bias is particularly sensitive to $\sigma_8$ because they both act as overall normalization factors, and in linear theory $C_{gg} \propto (b_g \sigma_8)^2$.

6 CROSS-CORRELATION METHOD

We perform the cross-correlation in harmonic space. We have used masks on all the maps, and thus we cannot obtain true values of the multipole coefficients $a_m$, as the power of the harmonic estimator will be reduced and correlations will be induced between multipoles. However, we scale for the loss of sky cover and use a full covariance matrix to account for the correlation between bins. We have obtained the spherical harmonic coefficients of our four 2MASS $K_{\text{gb}}$ shells, and our four WMAP3 maps, as outlined in Section 2. We perform the cross-correlation:

$$C_{gT}(\ell) = \frac{1}{(2\ell + 1)} \sum_m R(e^{\nu m}a_m^{*}).$$

We further bin the data, using logarithmically spaced bins and $\ell \geq 3$. We avoid $\ell = 2$ due to its anomalously low power in the CMB. For the analysis we use five bins $3 \leq \ell \leq 30$, as this is where the ISW signal is expected to dominate. In Figs 7 and 8 we plot the correlation using six bins with $3 \leq \ell \leq 200$.

In Fig. 7 we compare the $C_{gT}$ results of the WMAP1 and WMAP3 V-band data. Surprisingly we see a slight change, especially in the first point which corresponds to $\ell = 3 - 5$. The power in the WMAP3 maps has changed very little (see e.g. fig. 19 in Hinshaw et al. 2007), yet we see a slight change in the cross-correlations, especially in the first point which corresponds to $\ell = 3 - 5$. The difference may be due to a slight change in the structure of these multipoles, perhaps due to the improved gain model (Jarosik et al. 2007).

We will be comparing two hypotheses: a null hypothesis of no cross-correlation, and that returned by equation (3) (and 11) for our fiducial $\Lambda$CDM cosmology. To assess the fit we will compare the $\chi^2$ values returned by the two hypotheses. Similarly we can compare the un-normalized evidence $E$, where $-2 \ln E = \chi^2$ (if all parameters are fixed), or given by equation (18) (if we marginalize over a parameter, where $b_g$ in the equation can be replaced by any parameter). For this we use a covariance matrix, estimated from simulations, and thus we account for the correlation between $\ell$ bins and those between $K$ shells.

We make 500 simulations of a Gaussian CMB using the best-fitting theoretical $C_{TT}$ from WMAP3. We apply the Kp2 mask, and correlate them with the four $K$-shell 2MASS maps. By not varying the 2MASS maps we have slightly underestimated the errors,
The covariance matrix is defined as

\[ C_{ij} = \langle (d_i - \mu_i)(d_j - \mu_j) \rangle \]

and is calculated from simulations in which the galaxies and the CMB are uncorrelated. We use five logarithmically spaced bins between \( \ell = 3 - 30 \) at which our simulations will be achromatic, which is what we observe, but the null hypothesis is not ruled out.

because we have not accounted for the cosmic variance of the 2MASS data. We also note that the ISW signal in not inbuilt in the simulations; this should be inconsequential since this signal is negligible compared to the cosmic variance of the CMB.

We compute the \( \chi^2 \) to find the favoured model and the improvement \( \Delta(-2 \ln \mathcal{E}) = \Delta(\chi^2) \). As usual we have \( \chi^2 = \mathbf{d}^T \mathbf{M}^{-1} \mathbf{d} \) where \( \mathbf{C}_{\ell T} \) is calculated for the two models, one with no correlation, that is, \( C_{\ell T} = 0 \) and another with correlation due to ISW effect, given by equation (3) (and 11) for our fiducial cosmology. The covariance matrix is defined as

\[ M_{ij} = \sigma^2(C_{\ell T}) = \frac{1}{f_{\delta x}(2\ell + 1)}(C_{\ell T}^2 + C_{gg}C_{TT}) \]

which is the general form of equation (14). We find that error bars calculated using equation (17) are larger but of the same order of magnitude than those estimated from simulations.

7 CROSS-CORRELATION RESULTS

In this section we discuss the significance of the cross-correlation results and determine an upper limit on \( \Omega_\Lambda \).

7.1 Null hypothesis

In Fig. 8 we plot the results of the cross-correlation, for six logarithmically separated bins between \( \ell = 3 - 200 \), with 1σ error bars.

![Figure 8. Cross-correlation \( C_{\ell T} \)(uK) results for the ILC (small triangle), Q (open triangle), V (open square) and W (open pentagon) CMB maps with four different magnitude bins of the 2MASS galaxy surveys. The dashed lines are 1σ error bars about the null hypothesis, as evaluated from simulations. An ISW effect is expected to be achromatic, which is what we observe, but the null hypothesis is not ruled out.](https://academic.oup.com/mnras/article-abstract/377/3/1085/1079144)

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Null hypothesis | \( b_\phi = 1.40 \) | \( \Omega_\Lambda = 0.85 \) | \( \Omega_\Lambda = 0.85 \) | \( \Omega_\Lambda = 0.85 \) |
| ILC | 11.3 | 9.7 | 9.7 | 7.3 | 10.1 |
| Q | 12.1 | 10.4 | 10.4 | 8.1 | 10.9 |
| V | 11.0 | 9.5 | 9.5 | 7.4 | 10.0 |
| W | 10.8 | 9.1 | 9.1 | 6.9 | 9.6 |

Log evidence \((-2 \ln \mathcal{E})\) values for cross-correlation of each WMAP3 maps (V, W, Q and ILC) with four 2MASS magnitude shells, using different model assumptions and priors. In all cases \( \sigma_b \) is taken to be 0.75. Model 1 is the null hypothesis of no correlation. In model 2, we have considered a flat universe with \( \Omega_m = 0.30, \Omega_\Lambda = 0.05, h = 0.7 \) and \( b_\phi = 1.40 \) (for \( b_\phi = 1.38 \) only the W-map result changes from 9.1 to 9.2). In model 3, we have widened the prior used on \( b_\phi \) to that which we obtained by fitting the 2MASS ACF on scales \( \ell = 1-50 \), that is, \( b_\phi = 1.40 \pm 0.03 \) (the results are unchanged when using \( b_\phi \) in the range \([0.95, 1.4] \). The evidence ratio for model 1 (null hypothesis) and 5 (marginalized \( \Lambda \)CDM) is \( \Delta(-2 \ln \mathcal{E}) = 1.1 \) which means the data prefer a \( \Lambda \)CDM cosmology to the null hypothesis, but only marginally. For models 1, 2 and 4 the evidence is related to the \( \chi^2 \) by

\[ \chi^2 = -2 \ln \mathcal{E} \]

The cross-correlation is achromatic, indicative of an ISW type cross-correlation. However, the results appear completely compatible with the null hypothesis, and in fact scatter much less than expected.

We use five logarithmically separated bins between \( \ell = 3 - 30 \) to compute the \(-2 \ln \mathcal{E}\) values for the (ILC, Q, V, W) maps. These values can be found in Table 2 (model 1). By comparing to simulations, we find these values are low at the \(~95\) per cent level.

7.2 \( \Lambda \)CDM fiducial model

To rule out the null hypothesis, at face value without considering a competing theory, we would actually need a high \( \chi^2 \) value. However, we are comparing two theories and thus can ask the more subtle question of which the data prefer. To do this we can consider the ratio of the evidences, \( \Delta(-2 \ln \mathcal{E}) \).

In Fig. 9 we plot the results again for the two furthest redshift shells, which contain the most galaxies, for five bins at low \( \ell \) where the expected signal dominates. We also show the theoretical signal expected from our fiducial \( \Lambda \)CDM.

Using these bins and the four \( K_{20} \) shells, the \(-2 \ln \mathcal{E}\) values found for the ISW theory, using \( b_\phi = 1.40 \) and our fixed fiducial cosmology are reported in Table 2 (model 2). There is an improvement of \( \Delta(-2 \ln \mathcal{E}) = 1.5 \) and thus we confirm that the data prefer the ISW theory to the null hypothesis (of no correlation, or equivalently Einstein–de Sitter). However, rule of thumb has it that 'strong' evidence is \( \Delta(-\ln \mathcal{E}) \geq 3 \), and thus this improvement is not compelling.

We widen the uncertainty around the bias value, using the error bars from Section 5. There we found \( P(b_\phi) \) was approximately a Gaussian with \((\mu, \sigma) = (1.40, 0.03)\), although we use the curve from Fig. 6 for our marginalization:

\[ \mathcal{E} = \int P(\text{data} | \text{theory}, b_\phi) P(b_\phi) db_\phi. \]
Figure 9. The observed $C_{TT}$ (μK) (squares) from the V-band CMB map and the furthest magnitude shells of the 2MASS Survey ($15 < K_{20} < 13.5$ and $13.5 < K_{20} < 14$), with 1σ cosmic variance. These two shells contain the most galaxies. The solid line represents the theory for our fiducial model (flat universe with $\Omega_m = 0.30, \Omega_\Lambda = 0.70, h = 0.7, \sigma_8 = 0.75$) and $b_g = 1.40$. We have used five bins between $\ell = 3–30$ which corresponds to scales for which the ISW signal dominates.

where we have assumed uniform priors on the theory. In Table 2, we record our $-2\ln \mathcal{L}$ results for a $\Lambda$CDM with a prior on the bias (models 3), which are identical at 2 d.p. with those obtained from fixing $b_g = 1.40$. We get very similar results if we use the 2D probability distribution function $P(b_g, \sigma_8)$ and marginalize over both $b_g$ and $\sigma_8$ (using a Gaussian with $\mu_{b_g} = 0.56$ and $\sigma_{b_g} = 0.08$). The results are also unchanged if we use the strictly linear bias $b_{\ell, \text{lin}} = 1.38 \pm 0.05$.

7.3 Assessing the goodness of fit

By comparing our $\chi^2$ values to those obtained from simulations, we find that they are low to ~95% per cent, which is also evident from the lack of scatter in Fig. 8. However, we are interested in maximizing the evidence, or equivalently minimizing the $\chi^2$, and thus we find that moving from the null hypothesis to the $\Lambda$CDM model, the fit is improved (raising the interesting question: is minimizing the $\chi^2$ always appropriate?).

In the above analysis, we consider 20 correlated data points (five angular points in each of the four radial shells), and calculate the exact $\chi^2$ using a full covariance matrix. We can also consider each magnitude shell separately and calculate the $\chi^2$ for each one, as a consistency check. The $\chi^2$ values obtained for the null hypothesis are $$(1.70, 0.67, 0.94, 2.36)$$ going from the nearest to the furthest $K_{20}$ shell, and for our fiducial model we find $$(1.72, 0.58, 0.83, 2.16).$$ When the data are thus considered, our fiducial model is not always a better fit than the null hypothesis, and data in the closest shell prefer the null hypothesis.

7.4 Upper limit on $\Omega_\Lambda$ and marginalization

Above we have compared our fiducial cosmology to the null hypothesis. Alternatively, we can chose to consider only our fiducial $\Lambda$CDM model and use the cross-correlation to constrain its parameters. We vary $\Omega_\Lambda$, and $\Omega_\Lambda$ keeping all other parameters fixed ($\Omega_m = 1 - \Omega_\Lambda$), and fit it to the measured correlation. In Fig. 10 we plot the resulting likelihood. We find a best fit of $\Omega_\Lambda = 0.85$, and upper limits of 0.87, 0.89, 0.90 at 1, 2, 3σ, respectively. Corresponding $-2\ln \mathcal{L}$ values can be found in Table 2 (model 4).

This relatively high value for $\Omega_\Lambda$ is in good agreement with other studies of the ISW effect; Cabre & Castander (2006) find $\Omega_\Lambda = 0.8–0.85$ at the 1σ level. As can be seen, the null hypothesis ($\Omega_\Lambda = 0$) is less than 2σ away from the best-fitting result, confirming that we cannot confidently rule it out.

In the last column of Table 2, we present the marginalized evidence for our $\Lambda$CDM model (model 5), where the prior on the theory is flat over $\Omega_\Lambda = 0–0.95$, and the likelihood of our model is taken from Fig. 10. For this model $\Delta(-2\ln \mathcal{L}) = 1.1$ on average, so the data prefers a $\Lambda$CDM model to the null hypothesis, but only marginally.

Bearing in mind this issue, and the fact that the $\chi^2$ values are low for all models, any claim for an ISW detection using 2MASS and WMAP3 remains tentative, and indeed our results are also consistent with the null hypothesis of no correlation within the 2σ level.

8 AXIS OF EVIL

We briefly consider other possible sources and statistics of a positive cross-correlation between the CMB and LSS. As discussed in Section 1, interesting anomalies dubbed the AoE have been observed in the WMAP data of the CMB that indicate a possible departure from statistical isotropy (Land & Magueijo 2005a). Here we give a brief description of the AoE anomaly, but the reader should refer to Land & Magueijo (2005a) for a detailed description.

In harmonic space, a statistically isotropic map is expected to have independent $\ell$ modes. For a given multipole $\ell$, the power should also...
be distributed randomly amongst each multipole mode, but it is possible to rotate any map so that the power in a given multipole is mostly in a given multipole mode. However, if for a given frame, several multipoles have power mostly distributed in one mode, then the map can no longer be considered statistically isotropic. Land & Magetso (2005a) found that there existed a set of nearly identical frames (with axis dubbed the ‘AoE’) in which the multipoles of the CMB are dominated by one mode in these frames (shaded region). We do not observe a similar pattern in the 2MASS results (solid line). Negative values of $m$ are not shown as $a_{l-m}^2 = a_{l+m}^2$.

Figure 11. The fraction of power in each $m$ mode for multipoles $l = 2, 3, 4, 5$. The $a_{l,m}$'s are computed in the AoE frames returned by the CMB. By definition the multipoles of the CMB are dominated by one $m$ mode in these frames (shaded region). We do not observe a similar pattern in the 2MASS results (solid line). Negative values of $m$ are not shown as $a_{l-m}^2 = a_{l+m}^2$.

Possible sources of these anomalies are: foreground contamination; astrophysical effects (e.g. lensing and moving cluster effect); alternative cosmological paradigms. If the AoE was in some way due to local inhomogeneities (e.g. Vale 2005) then one might also have power mostly distributed in one $m$ mode, then the map can no longer be considered statistically isotropic. Land & Magueijo (2005a) found that there existed a set of nearly identical frames (with axis dubbed the ‘AoE’) in which the multipoles of the CMB maps showed phase correlations.

For the first test, we examine if the AoE signal observed in the CMB is also present in the 2MASS data. In Fig. 11 we plot the power observed in each $m$ mode, for the AoE frames returned by the CMB. For each multipole, this is the frame where one $m$ mode dominates. We plot the power ratio $2|a_{l,m}|^2/C_l$ (without the 2 factor for $m = 0$) evaluated in the AoE frames, for the 2MASS ($12 < K < 14$) and as well as the CMB (using the cleaned ILC map of Tegmark, de Oliveira-Costa & Hamilton 2003). By definition, in these CMB frames the AoE pattern of one $m$ significantly dominating each multipole. We do not observe a similar pattern in the 2MASS data, and thus conclude that if there exists a source responsible for the AoE features, then is unlikely to be the same source of the $C_l^T$ correlation observed above.

For the second test, we looked at 2MASS independently over $l = 2–20$ and found no evidence for any AoE type structure (a correlation of multipole frames as defined above). Clearly the 2MASS catalogue is highly anisotropic, with structures such as the supergalactic plane visible by eye as well as the Galactic mask. Is it not strange that a statistic that measures evidence for deviations from statistical isotropy in the CMB does not find any such feature in 2MASS? The AoE statistic highlights a very particular type of phase correlation, in harmonic space, and is by no means a conclusive test for general deviations from statistical isotropy. The fact that a clearly non-Gaussian and anisotropic map, such as 2MASS, does not return a anomalous AoE signal for $l = 2–20$ perhaps highlights the weakness of this statistic, and throws caution at how one defines and selects appropriate statistics. Perhaps the non-Gaussianity is ‘washed out’ by the depth of the survey – anomalies might be more significant in the shallowest part of the survey. However, there is currently no conclusive way to test a map for deviations from statistical isotropy.

9 DISCUSSION

We calculate the cross-correlation between the 2MASS galaxy survey and the WMAP3 data. This updates the work of Afshordi et al. (2004) who cross-correlated 2MASS with WMAP1.

The cross-correlation signal expected in a $\Lambda$CDM universe scales with the linear galaxy bias, $b_g$, and in linear theory with the product $(b_g \sigma_8)$. We fix $\sigma_8 = 0.75$, and use a flat fiducial cosmology (based on inflation). We use $Q_m = 0.30$ (based on supernovae Ia data). $Q_m = 0.05$ (based on big bang nucleosynthesis), $h = 0.7$ (based on the HST key project), $n = 1$ (i.e. the Harrison–Zeldovich spectral index).

Fitting the fiducial cosmology to the angular autocorrelation function of the 2MASS galaxy survey for $12 < K < 14$, yields a linear bias value of $b_g = 1.40 \pm 0.03$, assessed from multipole scales $l = 1–50$ ($b_{g,50} = 1.38 \pm 0.05$ assessed from multipole scales $l = 1–25$). As the 2MASS galaxy survey is shallow, we assume $b_g$ is constant with redshift over the depth of our survey.

The measured cross-correlations obtained from four different WMAP maps (V, W, Q and ILC) and four different magnitude shells of 2MASS ($12 < K < 15$, $12.5 < K < 13$, $13 < K < 13.5$ and $13.5 < K < 14$) show an achromatic signal, as expected from an ISW effect. However, the observed signal is also within the $1\sigma$ error bars obtained from cross-correlating random simulations of the CMB with 2MASS data. This means the data are consistent with the null hypothesis of no correlation.

We compare our observation with the ISW signal expected in our fiducial $\Lambda$CDM model, with a fixed bias and an uncertainty around the bias value which we found when fitting the autocorrelation function. The $\Lambda$CDM model finds a lower $\chi^2$, and thus is a better fit compared to the null hypothesis, but in both cases of treating the bias the evidence change is only $\Delta(-2\ln E) \approx 1.5$. Whichever model is considered – the null hypothesis or a $\Lambda$CDM universe – the $\chi^2$ values are low when compared to simulations.

Varying the dark energy density component of the $\Lambda$CDM model and assuming flatness, we find that the data prefer a high value of $\Omega_L \approx 0.85$, with $\Omega_{\Lambda} < 0.89$ (95 per cent CL).

Thus, there is a higher correlation between WMAP3 and 2MASS than that expected by a $\Lambda$CDM universe with $\Omega_{\Lambda} = 0.7$. However, the observed cross-correlation may not only be due to an ISW effect. Other signals might contribute to the correlation: positive curvature (even if very small) or cosmic magnification (higher redshift galaxies can contribute more than expected if they are lensed by the lower redshift galaxies and their luminosity function is such that the effect leads to a positive correlation).

When we marginalize over $\Omega_{\Lambda}$, the evidence ratio between the null hypothesis and $\Lambda$CDM becomes $\Delta(-2\ln E) \approx 1.1$ so we can say that the data prefer a $\Lambda$CDM universe, but only marginally. In any case, the correlation observed is consistent with both hypotheses within $2\sigma$.

We also investigate AoE type anomalies, which detect phase correlations between different multipoles in harmonic space. These phase correlations are not expected in a statistically isotropic map.
(Land & Magueijo 2005a). We do not observe an AoE type of structure in the 2MASS catalogue. As non-Gaussian features are expected in the LSS we feel this result raises issues about the use of the AoE statistic as a general test for statistic anisotropy.

We do not observe correlations between the CMB fractional power distribution as measured in its AoE frame and that of 2MASS, constraining the possible explanations of the low-\ell anomalies in the CMB.

Future spectroscopic and photometric galaxy redshift surveys (e.g. the Dark Energy Survey, the Wide Field Multi-Object Spectrograph) will yield more galaxies out to higher redshifts. Afshordi (2004) showed that an all sky survey with 10 million galaxies and uniform sky coverage between 0 < z < 1 would lead to a detection of the ISW effect at the 5\sigma level. It remains to be assessed what combination of depth and sky coverage is optimal for detecting the ISW effect.

We highlight that a claim for an ISW detection could be greatly weakened if one considers some of the uncertainty around the cosmological parameters. For our 2MASS–WMAP3 correlation the signal-to-noise ratio is poor, however with a stronger data set a full Markov Chain Monte Carlo exploration of parameter space should be done.

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