Gamma-ray Bursts in Wavelet Space

Zsolt Bagoly*, István Horváth†, Attila Mészáros**‡ and Lajos G. Balázs§

*Laboratory for Information Technology, Eötvös University, H-1117 Budapest, Pázmány P. s. 1/A, Hungary
†Department of Physics, Bolyai Military University, H-1456 Budapest, POB 12, Hungary
**Astronomical Institute of the Charles University, V Holešovičkách 2, CZ-180 00 Prague 8, Czech Republic
‡Stockholm Observatory, AlbaNova, SE-106 91 Stockholm, Sweden
§Konkoly Observatory, H-1525 Budapest, POB 67, Hungary

Abstract. The gamma-ray burst's lightcurves have been analyzed using a special wavelet transformation. The applied wavelet base is based on a typical Fast Rise-Exponential Decay (FRED) pulse. The shape of the wavelet coefficients' total distribution is determined on the observational frequency grid. Our analysis indicates that the pulses in the long bursts' high energy channel lightcurves are more FRED-like than the lower ones, independently from the actual physical time-scale.

INTRODUCTION

The shape of the gamma-ray burst's (GRB's) 64 ms resolution lightcurves in the BATSE Gamma-Ray Burst Catalog [4] carry an immense amount of information. However, the changing S/N ratio complicates the detailed comparative analysis of the lightcurves. During the morphological analysis of the GRB's [3, 5, 6] a subclass with Fast Rise-Exponential Decay (FRED) pulse shape were observed. This shape is quite attractive because its phenomenological simplicity. Here we use a special wavelet transformation with a kernel function based on a FRED-like pulse. Similar approach have been used by [8], but their base functions were constructed differently.

THE FRED WAVELET TRANSFORM

We have used the Discrete Wavelet Transform (DWT) matrix formalism (e.g. [7]): here for an input data vector \(v\), the one step of the wavelet transform is a multiplication with a special matrix \(F\):

\[
F = \begin{bmatrix}
    c_0 & c_1 & c_2 & c_3 \\
    c_3 & -c_2 & c_1 & -c_0 \\
    c_0 & c_1 & c_2 & c_3 \\
    c_3 & -c_2 & c_1 & -c_0 \\
    \vdots & \vdots & \ddots & \vdots \\
    c_2 & c_3 & \vdots & c_0 & c_1 \\
    c_1 & -c_0 & \vdots & c_3 & -c_2
\end{bmatrix}
\] (1)
where the \( c_0, \ldots, c_3 \) are the 4-stage FIR filter parameters defining the wavelet. To obtain these values we require the matrix \( F \) to be orthogonal (e.g. no information loss), and the output of the even (derivating-like) rows should disappear for a constant and for a FRED-like \( e^{-t/\tau} \) input signal. These requirements give two different solutions for \( c_0, \ldots, c_3 \): a rapidly oscillating one and a smooth one. In the following we’ll use the later one.

Our filter process with the FRED wavelet transform consists of the usual \textit{transform} \( \rightarrow \) \textit{filter/cut} \( \rightarrow \) \textit{inverse transform} digital filtering steps. During the filtering we’ll loose some information, however this could be quite small. To demonstrate the efficiency of the algorithm on Fig. 1. we reconstructed the 100-320 keV 64ms lightcurve of BATSE trigger 0143 from the biggest 5% of the total wavelet coefficients. The excellent reconstruction of each individual pulse is obvious.

The wavelet transformation algorithm divides the phase-space into equal area regions. On Fig. 2. the wavelet transform are shown. Here the dark segments are the really important coefficients - however they cover only a small portion of the total area which explains the high efficiency of the reconstruction.
**WAVELET SCALE ANALYSIS**

For a frequency-like wavelet scale analysis we would like to create a power-spectrum like distribution along the frequency axis. However, one should be careful. In the classical signal processing one uses the power spectrum from the Fourier-transform, because the signals are electromagnetic-like usually, e.g. the power (or energy) is proportional to the square of the signal. Here the lightcurves measure photon counts — so the signal’s energy is simply the sum of the counts. For this reason we approximate the signal’s strength as a sum the magnitude of the coefficients along the given frequency rows.

This signal’s strength indicate on Fig 3. (BATSE trigger 0143) the maximum power to be around \( f \approx 500 \text{Hz} \). In each energy channel the signals are similar (observe the logarithmic scale), because the signal is strong even at high energies (channel 4). For BATSE trigger 7343 (with optical redshift \( z = 1.6006 \)) one can observe a strong high frequency cutoff: some of the signal’s high frequency part is missing. However all the 4 channels are visible, while the maximum power is around \( f \approx 62.5 \text{Hz} \). It is interesting to remark that the signal’s shape is quite similar to trigger 0143 if that is scaled down by a factor of \( \approx 2.4 - 2.8 \) in frequency.

**WAVELET FILTERING AND SIMILARITY**

The FRED wavelet transform measures the similarity between the different wavelet kernel functions (here all are FRED-based) and the actual signal. To quantify the similarity we define a magnitude cutoff in the wavelet space so, that the reconstructed \( T_{50} \) value from the filtered data should be similar to the original values. The \( T_{50} \) value and its \( \sigma_P \) error from the photon count statistics could be easily determined from the original lightcurve. To keep only the important features we define the \( T_{50\text{break}} \) breakpoint where

\[
| T_{50\text{break}} - T_{50} | = 10.0 \sigma_P
\]

Using a cut-off point it is possible to define a Compressed Size (CS) for a burst: it is the number of bins (in the wavelet space) needed to restore the curve at the break.
FIGURE 4. The relative value of the CS’s against the total lightcurves’ CS for channels 2 and 4.

The CS value is a robust measure quantifying the similarity between the FRED kernel and the different channels’ lightcurves. Our analysis suggest that all the low energy channels #1, #2 and #3 behaves similarly, while the high energy (> 320 keV) channel is different (which is not very surprising, e.g. [1]). Fig. 4. shows the ratio of the CS’s against the total count lightcurves’ CS for channels 2 and 4. This distributions indicate that the pulse-shapes in the long bursts’ high energy channel are more FRED-like than the lower ones - and this is independent from the actual FRED time-scale!

ACKNOWLEDGMENTS

This research was supported in part through OTKA grants T024027 (L.G.B.), and T034549, Czech Research Grant J13/98: 113200004 and by a grant from the Wenner-Gren Foundations (A.M.).

REFERENCES

1. Bagoly, Z., Mészáros, A., Horváth, I., Balázs, L. G., & Mészáros, P. 1998, ApJ, 498, 342.
2. Kouveliotou, C., Meegan, C.A. & Fishman, G.J. 1993, ApJ, 413, L101
3. Kouveliotou, C., Paciesas, W. S., Fishman, G. J., Meegan, C. A., & Wilson, R. B. 1992, The Compton Observatory Science Workshop, 61
4. Meegan, C., Malozzi, R.S., Six, F. & Connaughton, V. 2001, Current BATSE Gamma-Ray Burst Catalog, http://gammaray.msfc.nasa.gov/batse/grb/catalog
5. Norris, J. P., Nemiroff, R. J., Bonnell, J. T., Paciesas, W. S., Kouveliotou, C., Fishman, G. J., & Meegan, C. A. 1994, American Astronomical Society Meeting, 26, 1333
6. Norris, J. P., Scargle, J. D., Bonnell, J. T., & Nemiroff, R. J. 1998, Gamma-Ray Bursts, 4th Hunstville Symposium, 171
7. Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P. 1992, Numerical Recipes in Fortran, Second Edition, Cambridge University Press, Cambridge
8. Quilligan, F., McBreen, B., Hanlon, L., McBreen, S., Hurley, K. J. & Watson, D., 2002, A&A, 385, 377