Inclusive production of a pair of hadrons separated by a large interval of rapidity in proton collisions

D.Yu. Ivanov\(^\dagger\) and A. Papa\(^\ddagger\)

\(^1\) Sobolev Institute of Mathematics and Novosibirsk State University, 630090 Novosibirsk, Russia

\(^2\) Dipartimento di Fisica, Università della Calabria, and Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, I-87036 Arcavacata di Rende, Cosenza, Italy

Abstract

We consider within QCD collinear factorization the inclusive process \(p + p \rightarrow h_1 + h_2 + X\), where the pair of identified hadrons, \(h_1, h_2\), having large transverse momenta is produced in high-energy proton-proton collisions. In particular, we concentrate on the kinematics where the two identified hadrons in the final state are separated by a large interval of rapidity \(\Delta y\). In this case the (calculable) hard part of the reaction receives large higher order corrections \(\sim \alpha_s^n \Delta y^\delta\). We provide a theoretical input for the resummation of such contributions with next-to-leading logarithmic accuracy (NLA) in the BFKL approach. Specifically, we calculate in NLA the vertex (impact-factor) for the inclusive production of the identified hadron. This process has much in common with the widely discussed Mueller-Navelet jets production and can be also used to access the BFKL dynamics at proton colliders. Another application of the obtained identified-hadron vertex could be the NLA BFKL description of inclusive forward hadron production in DIS.

\(^\dagger\) e-mail address: d-ivanov@math.nsc.ru

\(^\ddagger\) e-mail address: papa@cs.infn.it
1 Introduction

The BFKL approach [1] is the most suitable framework for the theoretical description of the
high-energy limit of hard or semi-hard processes, i.e. processes where a hard scale exists
that justifies the application of perturbative QCD.

At high squared center of mass energy $s$ and for squared momentum transferred $t$ not
growing with $s$, large logarithms of the energy compensate the small coupling and must
be resummed at all orders of the perturbative series. The BFKL approach provides a system-
atic way to perform the resummation in the leading logarithmic approximation (LLA),
which means resummation of all terms ($\alpha_s \ln(s))^n$, and in the next-to-leading logarithmic
approximation (NLA), which means resummation of all terms $\alpha_s (\alpha_s \ln(s))^n$.

In the BFKL approach, both in the LLA and in the NLA, the high-energy scattering am-
plitudes are expressed by a suitable factorization of a process-independent part, the Green’s
function for the interaction of two Reggeized gluons, and process-dependent terms, the so-
called impact factors of the colliding particles (see, for instance, [2]).

The Green’s function is determined through the BFKL equation, which is an iterative
integral equation, whose kernel is known at the next-to-leading order (NLO) both for forward
scattering (i.e. for $t = 0$ and color singlet in the $t$-channel) [3, 4] and for any fixed (not
growing with energy) momentum transfer $t$ and any possible two-gluon color state in the
$t$-channel [5].

The impact factors of the colliding particle are a necessary ingredient for the complete
description of a process in the BFKL approach and, therefore, to get a contact with phe-
nomenology. The only impact factors calculated so far with NLO accuracy are those for
colliding quark and gluons [6, 7, 8], for forward jet production [9, 10, 11], for the $\gamma^* \rightarrow \gamma^*$
transition [12] and for the $\gamma^*$ to light vector meson transition [13]. Recently, the NLO
$\gamma^* \rightarrow \gamma^*$ impact factor has been calculated in the coordinate representation [14].

The impact factors for colliding partons [6, 7, 8] played a role in the proof of fulfillment
of bootstrap conditions for the gluon Reggeization in the NLA [2] and are at the basis of the
calculation of the impact factors for forward jets.

The impact factor for the $\gamma^* \rightarrow \gamma^*$ transition is certainly the most important one from the
phenomenological point of view, since it enters the prediction of the $\gamma^*\gamma^*$ total cross section,
which is believed to be the golden channel for the manifestation of the BFKL dynamics [15].
Its calculation turned out to be very complicated and took several year to be completed [12].
A relatively simpler calculation led to the leading-twist NLO impact factor for the transition
from a virtual photon $\gamma^*$ to a light neutral vector meson $V = \rho^0, \omega, \phi$, which was used to
study the $\gamma^*\gamma^* \rightarrow VV$ reaction [16]. However, both for the $\gamma^*\gamma^*$ total cross section and for
the production of two light vector mesons in $\gamma^*\gamma^*$ collisions, accurate experimental data will
possibly come only from future high-energy $e^+e^-$ and $e\gamma$ colliders.

On the other side, the availability of the NLO forward jet impact factors [9] allowed
the NLA calculation [17] of the cross-section for the production of Mueller-Navelet jets [18],
which can be tested at hadron colliders, such as Tevatron and LHC.

In the present work we calculate in the NLO a new impact factor, namely the one for the forward production of an identified hadron, which can be used to determine the cross section for the semi-inclusive process $p+p \to h_1 + h_2 + X$, in the kinematics where the identified hadrons in the final state, $h_1$, $h_2$, have large transverse momenta and are separated by a large interval of rapidity $\Delta y$. This process has much in common with the widely discussed Mueller-Navelet jets production and can be also used to access the BFKL dynamics at proton colliders. Very similarly to the case of forward jet production, the calculation of this impact factor lies at the border between collinear and BFKL factorization: the proton in the initial state emits a parton which, in its turn, produces the hadron in the final state. This calls for the collinear factorization of the hard part of the process with parton distribution functions (PDFs) in the initial-state proton and with the relevant fragmentation functions (FFs) for the production of the final-state hadrons, both functions obeying the standard DGLAP evolution \cite{19}.

The paper is organized as follows. In the next Section we will present the factorization structure of the cross section, recall the definition of BFKL impact factor and discuss the treatment of the divergences arising in the calculation; in Section 3 we give the derivation of LO impact factor; Section 4 is devoted to the calculation of the NLO impact factor; finally, in Section 5 we summarize our results.

## 2 General framework

We consider the process

\begin{equation}
\tag{1}
p(p_1) + p(p_2) \to h_1(k_1) + h_2(k_2) + X
\end{equation}

in the kinematical region where both identified hadrons have large transverse momenta\footnote{See Eq. (3) below for the definition of the transverse part of a 4-vector.}, $k_1^2 \sim k_2^2 \gg \Lambda_{\text{QCD}}^2$. This provides the hard scale, $Q^2 \sim k_{1,2}^2$, which makes perturbative QCD methods applicable. Moreover, the energy of the proton collision is assumed to be much bigger than the hard scale, $s = 2p_1 \cdot p_2 \gg k_{1,2}^2$.

We consider the leading behavior in the $1/Q$-expansion (leading twist approximation). With this accuracy one can neglect the masses of initial protons and identified hadrons. The state of the identified hadrons can be described completely by their (pseudo)rapidities $y_1, y_2$ and transverse momenta $k_{1,2}$.

In QCD collinear factorization the cross section of the process reads

\begin{equation}
\frac{d\sigma}{dy_1 dy_2 d^2 k_1 d^2 k_2} = \sum_{i,j=q,g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1, \mu) f_j(x_2, \mu) \frac{d\tilde{\sigma}(x_1 x_2 s, \mu)}{dy_1 dy_2 d^2 k_1 d^2 k_2},
\end{equation}

\footnote{For massless particle the rapidity coincides with pseudorapidity, $y = \eta$, the latter being related to the particle polar scattering angle by $\eta = -\ln \tan \frac{\theta}{2}$.}
where the $i, j$ indices specify parton types, $i, j = q, \bar{q}, g, f_i(x, \mu)$ denotes the initial protons parton density functions (PDFs), the longitudinal fractions of the partons involved in the hard subprocess are $x_{1,2}$, $\mu$ is the factorization scale and $d\hat{\sigma}(x_1 x_2 s, \mu)$ is the partonic cross section for the production of identified hadrons. The latter is expressed in terms of parton fragmentation functions (FFs), to be specified later.

It is convenient to define the Sudakov decomposition for the identified-hadron momentum,

$$k_h = \alpha_h p_1 + \frac{\vec{k}_h^2}{\alpha_h s} p_2 + k_{h \perp}, \quad k_{h \perp}^2 = -\vec{k}_{h \perp}^2,$$

where the longitudinal fraction $\alpha_h$ is related to the hadron rapidity as $y = \frac{1}{2} \ln \frac{\alpha_h^2 s}{k_h^2}$, $dy = \frac{d\alpha_h}{\alpha_h}$ in the center of mass system.

Another important longitudinal fraction is related with the collinear fragmentation of the parton $i$ into a hadron $h$. The probability for such fragmentation is generically expressed as the convolution of a parton fragmentation function $D_i^h$ and a coefficient function $C_i^h$,

$$d\sigma_i = C_i^h(z) dz \rightarrow d\sigma^h = d\alpha_h \int \frac{dz}{\alpha_h} D_i^h \left( \frac{\alpha_h}{z}, \mu \right) C_i^h(z, \mu).$$

Here $C_i^h$ is the cross section (calculable in QCD perturbation theory) for the production of a parton with momentum fraction $z$; the non-perturbative, large-distance part of the transition to a hadron with momentum fraction $\alpha_h$, is described in terms of the fragmentation function $D_i^h$; $\mu$ stands here for the factorization scale.

Thus, the cross section $d\hat{\sigma}(x_1 x_2 s, \mu)$ in Eq. (2) can be further presented as the convolution of two fragmentation functions with the cross section of the partonic subprocess

$$i(x_1 p_1) + j(x_2 p_2) \rightarrow m(k_1/z_1) + n(k_2/z_2) + X,$$

where $X$ stands for inclusive production of additional, “unidentified” partons. This partonic cross section is free from infrared divergences, but it contains the collinear singularities which are absorbed into the definition of PDFs and FFs, thus leading to a finite result for our process of interest (1) in collinear factorization.

In the case of large interval of rapidity between the two identified hadrons, the energy of the partonic subprocess is much larger than hadron transverse momenta, $x_1 x_2 s \gg \vec{k}_2^2$. In this region the perturbative partonic cross section receives at higher orders large contributions $\sim \alpha_s^n \ln^n \frac{s}{\vec{k}_2^2}$, related with large energy logarithms. It is the aim of this paper to elaborate the resummation of such enhanced contributions with NLA accuracy using the BFKL approach.

Let us remind some generalities of the BFKL method. Due to the optical theorem, the cross section is related to the imaginary part of the forward proton-proton scattering amplitude,

$$\sigma = \frac{\Im m A}{s}.$$
In the BFKL approach the kinematic limit $s \gg \vec{k}^2$ of the forward amplitude may be presented in $D$ dimensions as follows:

$$\mathcal{I}m_s(A) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^2} \Phi_1(\vec{q}_1, s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^2} \Phi_2(-\vec{q}_2, s_0) \left( \frac{s}{s_0} \right)^{\delta+i\infty} \int \frac{d\omega}{2\pi i} G_\omega(\vec{q}_1, \vec{q}_2),$$

where the Green’s function obeys the BFKL equation

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}qK(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_1).$$

The energy scale parameter $s_0$ is arbitrary, the amplitude, indeed, does not depend on its choice within NLA accuracy due to the properties of NLO impact factors $\Phi_{1,2}$ to be discussed below.

What remains to be calculated are the NLO impact factors $\Phi_1$ and $\Phi_2$ which describe the inclusive production of the identified hadrons $h_1$ and $h_2$, with fixed transverse momenta $\vec{k}_1$, $\vec{k}_2$ and rapidities $y_1$, $y_2$, in the fragmentation regions of the colliding protons with momenta $p_1$ and $p_2$, respectively. For definiteness, we will consider the case when the identified hadron belongs to the fragmentation region of the proton with momentum $p_1$, i.e. the hadron is produced in the collision of the proton with momentum $p_1$ off a Reggeon with incoming (transverse) momentum $q$.

Technically, this is done using as starting point the definition of inclusive parton impact factor, given in Refs. [6, 7], for the cases of incoming quark(antiquark) and gluon, respectively. This definition involves the integration over all possible intermediate states appearing in the parton-Reggeon collision, see Fig. 1. Up to the next-to-leading order, this means that we can have one or two partons in the intermediate state. Here we review the important steps and give the formulae for the LO parton impact factors.

Note that both the kernel of the equation for the BFKL Green’s function and the parton impact factors can be expressed in terms of the gluon Regge trajectory,

$$j(t) = 1 + \omega(t),$$

and the effective vertices for the Reggeon-parton interaction.

To be more specific, we will give below the formulae for the case of forward quark impact factor considered in $D = 4 + 2\epsilon$ dimensions of dimensional regularization. We start with the LO, where the quark impact factors are given by

$$\Phi_q^{(0)}(\vec{q}) = \sum_{\{a\}} \int \frac{dM_a^2}{2\pi} \Gamma_{aq}^{(0)}(\vec{q}) [\Gamma_{aq}^{(0)}(\vec{q})]^* d\rho_a,$$

where $\vec{q}$ is the Reggeon transverse momentum, and $\Gamma_{aq}^{(0)}$ denotes the Reggeon-quark vertices in the LO or Born approximation. The sum $\{a\}$ is over all intermediate states $a$ which contribute to the $q \to q$ transition. The phase space element $d\rho_a$ of the state $a$, consisting
of particles with momenta $\ell_n$, is $(p_q$ is the initial quark momentum)

$$d\rho_a = (2\pi)^D \delta^D \left( p_q + q - \sum_{n \in a} \ell_n \right) \prod_{n \in a} \frac{d^{D-1}\ell_n}{(2\pi)^{D-1}2E_n}, \quad (11)$$

while the remaining integration in (10) is over the squared invariant mass of the state $a$,

$$M^2_a = (p_q + q)^2.$$

In the LO the only intermediate state which contributes is a one-quark state, $\{a\} = q$. The integration in Eq. (10) with the known Reggeon-quark vertices $\Gamma^{(0)}_{qq}$ is trivial and the quark impact factor reads

$$\Phi^{(0)}_q(\vec{q}) = g^2 \sqrt{N^2 - 1} \frac{2}{2N}, \quad (12)$$

where $g$ is the QCD coupling, $\alpha_s = g^2/(4\pi)$, $N = 3$ is the number of QCD colors.

In the NLO the expression (10) for the quark impact factor has to be changed in two ways. First, one has to take into account the radiative corrections to the vertices, $\Gamma^{(0)}_{qq} \rightarrow \Gamma_{qq} = \Gamma^{(0)}_{qq} + \Gamma^{(1)}_{qq}$.

Second, in the sum over $\{a\}$ in (10), we have to include more complicated states which appear in the next order of perturbative theory. For the quark impact factor this is a state with an additional gluon, $a = qg$. However, the integral over $M^2_a$ becomes divergent when an extra gluon appears in the final state. The divergence arises because the gluon may be emitted not only in the fragmentation region of initial quark, but also in the central rapidity region. The contribution of the central region must be subtracted from the impact factor, since it is to be assigned in the BFKL approach to the Green’s function. Therefore the result for the forward quark impact factor reads

$$\Phi_q(\vec{q}, s_0) = \left( \frac{s_0}{q^2} \right)^{\omega(-\vec{q}^2)} \sum_{\{a\}} \int \frac{dM^2_a}{2\pi} \frac{\Gamma_{aq}(\vec{q}) [\Gamma_{aq}(\vec{q})]^*}{\Gamma_{aq}(\vec{q})} \int d\rho_a \theta(s_{\Lambda} - M^2_a) \int d^D-2k \frac{q^2}{k^2} \Phi^{(0)}_q(\vec{k})K^{(0)}_{r}(\vec{k}, \vec{q}) \ln \left( \frac{s_{\Lambda}^2}{(k - \vec{q})^2s_0} \right). \quad (13)$$

The second term in the r.h.s. of Eq. (13) is the subtraction of the gluon emission in the central rapidity region. Note that, after this subtraction, the intermediate parameter $s_{\Lambda}$ in the r.h.s. of Eq. (13) should be sent to infinity. The dependence on $s_{\Lambda}$ vanishes because of the cancellation between the first and second terms. $K^{(0)}_r$ is the part of LO BFKL kernel related to real gluon production,

$$K^{(0)}_r(\vec{k}, \vec{q}) = \frac{2g^2N}{(2\pi)^D} \frac{1}{(k - \vec{q})^2}. \quad (14)$$

The factor in Eq. (13) which involves the Regge trajectory arises from the change of energy scale ($\vec{q}^2 \rightarrow s_0$) in the vertices $\Gamma$. The trajectory function $\omega(t)$ can be taken here in the
one-loop approximation \((t = -\vec{q}^2)\),

\[
\omega(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k}{k^2(\vec{q} - \vec{k})^2} = -g^2 N \frac{\Gamma(1 - \varepsilon) \Gamma^2(\varepsilon)}{(4\pi)^{D/2} \Gamma(2\varepsilon) (\vec{q}^2)^\varepsilon}.
\] (15)

In the Eqs. (10) and (13) we suppress for shortness the color indices (for the explicit form of the vertices see [6]). The gluon impact factor \(\Phi_g(\vec{q})\) is defined similarly. In the gluon case only the single-gluon intermediate state contributes in the LO, \(a = g\), which results in

\[
\Phi_g^{(0)}(\vec{q}) = \frac{C_A}{C_F} \Phi_q^{(0)}(\vec{q}) ,
\] (16)

here \(C_A = N\) and \(C_F = (N^2 - 1)/(2N)\). In the NLO additional two-gluon, \(a = gg\), and quark-antiquark, \(a = q\bar{q}\), intermediate states have to be taken into account in the calculation of the gluon impact factor.

The identified hadron production vertex we want to calculate is simply related with inclusive parton impact factors. In order to allow the inclusive production of a given hadron, one of these integrations in the definition of parton impact factors is "opened" (see Fig. 2). This means, in practice, that the integration over the momentum of one of the intermediate-state partons is replaced by the convolution with a suitable fragmentation function. We illustrate the procedure starting from the LO impact factor, contributing to the BFKL amplitude in the LLA, then we move on to the NLO impact factor, relevant for the BFKL resummation in the NLA.
3 Impact Factor in the LO

The inclusive LO impact factors of quark (antiquark) and gluon are given in Eqs. (12) and (16). It is important that these expressions are valid also in non-integer dimensions.

The inclusive LO impact factor of proton may be thought of as the convolution of quark and gluon impact factors with the corresponding proton PDFs,

\[ \Phi = C \, dx \left( \frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x) \right), \quad C = g^2 \sqrt{N_c^2 - 1} \frac{1}{2N} = 2\pi\alpha_s \sqrt{\frac{2C_F}{C_A}}. \] (17)

In order to establish the proper normalization for the impact factor in the case of fragmentation with an identified hadron, we insert into the inclusive impact factor, Eq. (17), a delta function and the integration over the “parent parton” transverse momentum, \( d^2 \vec{k} \), and use Eq. (4) to get:

\[ \frac{d\Phi^h}{q^2} = C d\alpha_h \frac{d^2 \vec{k}_h}{k^2} \int \frac{dx}{x} \delta^2 (\vec{k} - \vec{q}) \left( \frac{C_A}{C_F} f_g(x) D_g^h \left( \frac{\alpha_h}{x} \right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h \left( \frac{\alpha_h}{x} \right) \right), \] (18)

an expression for the LO impact factor schematically presented in Fig. 3.

What is left is to express the “parent parton” variables by those of the identified hadron, \( \vec{k} = (x/\alpha_h) \vec{k}_h \) (note that \( d^2 \vec{k}_h/\vec{k}_h^2 = d^2 \vec{k}/\vec{k}^2 \)).

Although the results for the fragmentation impact factors should be expressed in terms of quantities describing the identified hadron, \( \alpha_h, \vec{k}_h \), it is convenient during the calculation to operate with the kinematical variables of the parent parton, \( \alpha, \vec{k} \) (in LO, \( \alpha = x \)). The transition to the hadron variables may be easily done at the end of the calculation.

In what follows we will calculate the projection of the impact factor on the eigenfunctions of LO BFKL kernel, i.e. the impact factor in the so called \((\nu,n)\)-representation,

\[ \Phi(\nu,n) = \int d^2 \vec{q} \frac{\Phi(\vec{q})}{q^2} \frac{1}{\pi \sqrt{2}} (q^2)^{\nu - \frac{1}{2}} e^{in\phi}. \] (19)

Here \( \phi \) is the azimuthal angle of the vector \( \vec{q} \) counted from some fixed direction in the transverse space.

4 The NLO calculation

We will work in \( D = 4 + 2\epsilon \) dimensions and calculate the NLO impact factor directly in the \((\nu,n)\)-representation (19), working out separately virtual corrections and real emissions. To this purpose we introduce the “continuation” of the LO BFKL eigenfunctions to non-integer dimensions,

\[ (q^2)^{\gamma} e^{in\phi} \rightarrow (q^2)^{\gamma - \frac{n}{2}} \left( \vec{q} \cdot \vec{l} \right)^n, \] (20)
where $\gamma = i\nu - \frac{1}{2}$ and $\bar{I}^2 = 0$. It is assumed that the vector $\mathbf{l}$ lies only in the first two of the $2 + 2\epsilon$ transverse space dimensions, i.e. $\mathbf{l} = \mathbf{e}_1 + i \mathbf{e}_2$, with $\mathbf{e}_1^2 = 1$, $\mathbf{e}_1 \cdot \mathbf{e}_2 = 0$. In the limit $\epsilon \to 0$ the r.h.s. of Eq. (20) reduces to the LO BFKL eigenfunction. This technique was used recently in Ref. [20]. An even more general method, based on an expansion in traceless products, was used earlier in Ref. [21] for the calculation of NLO BFKL kernel eigenvalues.

Thus, for the case of non-integer dimension the LO result for the impact factor reads

$$
\frac{d\Phi^h}{q^2} = C \frac{d\alpha}{\bar{k}^2} \int \frac{dx}{x} \delta^{(2+2\epsilon)} \left( \mathbf{k} - \mathbf{q} \right) \left( \frac{C_A}{C_F} f_q(x) D^h_q \left( \frac{\alpha_h}{x} \right) + \sum_{a=q,\bar{q}} f_a(x) D^h_a \left( \frac{\alpha_h}{x} \right) \right),
$$

which gives in the $(\nu, n)$-representation the result

$$
\pi \sqrt{2} \bar{k}^2 \frac{d\Phi^h(\nu, n)}{d\alpha d^{2+2\epsilon} \bar{k}} = \int \frac{dx}{x} \left( \frac{C_A}{C_F} f_q(x) D^h_q \left( \frac{\alpha_h}{x} \right) + \sum_{a=q,\bar{q}} f_a(x) D^h_a \left( \frac{\alpha_h}{x} \right) \right) \left( 2 \right)^{\nu - \frac{n}{2}} \left( \mathbf{k}, \mathbf{l} \right)^n.
$$

Collinear singularities which appear in the NLO calculation are removed by the renormalization of PDFs and FFs. The relations between the bare and renormalized quantities are

$$
f_q(x) = f_q(x, \mu_F) - \frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu^2} \right) \frac{1}{x} \int \frac{dz}{z} \left[ P_{qq}(z) f_q(\frac{x}{z}, \mu_F) + P_{qg}(z) f_g(\frac{x}{z}, \mu_F) \right],
$$

$$
f_g(x) = f_g(x, \mu_F) - \frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu^2} \right) \frac{1}{x} \int \frac{dz}{z} \left[ P_{qg}(z) f_q(\frac{x}{z}, \mu_F) + P_{gg}(z) f_g(\frac{x}{z}, \mu_F) \right],
$$

where $\frac{1}{\epsilon} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^{\epsilon}}$, and the DGLAP kernels are given by

$$
P_{qq}(z) = C_F \frac{1 + (1 - z)^2}{z},
$$

$$
P_{qg}(z) = T_R \left[ z^2 + (1 - z)^2 \right],
$$

$$
P_{qq}(z) = C_F \left[ \frac{1 + z^2}{1 - z} + \frac{3}{2} \delta(1 - z) \right],
$$

$$
P_{gg}(z) = 2 C_A \left[ \frac{1}{(1 - z)_+} + \frac{1}{z} - 2 + z(1 - z) \right] + \left( \frac{11}{6} C_A - \frac{n_f}{3} \right) \delta(1 - z),
$$

Figure 3: Diagrammatic representation of the LO vertex for the identified hadron production for the case of incoming quark (left) and gluon (right).
with $T_R = 1/2$. Here and below we always adopt the $\overline{\text{MS}}$ scheme. Similarly, for FFs we have

$$D_q^h(x) = D_q^h(x, \mu_F) - \frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} + \ln \frac{\mu_F^2}{\mu^2} \right) \int \frac{dz}{z} \left[ D_q^h(z, \mu_F)P_{qq}(z) + D_g^h(z, \mu_F)P_{qg}(z) \right] ,$$

$$D_g^h(x) = D_g^h(x, \mu_F) - \frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} + \ln \frac{\mu_F^2}{\mu^2} \right) \int \frac{dz}{z} \left[ D_q^h(z, \mu_F)P_{qg}(z) + D_g^h(z, \mu_F)P_{gg}(z) \right] . \tag{28}$$

Here $\mu$ is an arbitrary scale parameter introduced by the dimensional regularization, which cancels out in the results for physical quantities. Note the difference in the non-diagonal terms ($P_{qq} \leftrightarrow P_{qg}$) between the formulas for PDFs’ and FFs’ renormalization, which is due to the fact that in the parton to hadron splitting the hadron is in the first case in the initial state, whereas in the second case it is in the final state.

Owing to $\vec{k} = (x/\alpha_h)\vec{k}_h$, one gets

$$\frac{\pi \sqrt{2} \vec{k}^2}{C} \frac{d\Phi^h(\nu, n)}{d\alpha_h d^2+2\epsilon \vec{k}} = \left( \vec{k}_h^2 \right)^{\gamma - \frac{\epsilon}{2}} \left( \vec{k}_h \cdot \vec{l} \right)^n \int \frac{dx}{x} \left( \frac{C_A}{C_F} f_g(x)D_g^h \left( \frac{\alpha_h}{x} \right) + \sum_{a=q,\bar{q}} f_a(x)D_a^h \left( \frac{\alpha_h}{x} \right) \right) \left( \frac{x}{\alpha_h} \right)^{2\gamma}. \tag{29}$$

Now we can calculate the collinear counterterms which appear due to the renormalization of bare PDFs and FFs. Inserting the expressions given in Eqs. (23), (28) into the LO impact factor (29), we obtain, after some suitable transformations of the integration variables in the terms coming from the PDFs renormalization, the following collinear counterterm $^3$

$$\frac{\pi \sqrt{2} \vec{k}^2}{C} \frac{d\Phi^h(\nu, n)}{d\alpha_h d^2+2\epsilon \vec{k}} \bigg|_{\text{coll. c.t.}} = \frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} + \ln \frac{\mu_F^2}{\mu^2} \right) \int \frac{dx}{x} \int \frac{dz}{z} \left( \vec{k}^2 \right)^{\gamma - \frac{\epsilon}{2}} \left( \vec{k} \cdot \vec{l} \right)^n$$

$$\times \left[ (1 + z^{-2\gamma})P_{qq}(z) \sum_{a=q,\bar{q}} f_a(x)D_a^h \left( \frac{\alpha_h}{x} \right) + \left( \frac{C_A}{C_F} + z^{-2\gamma} \right) P_{qg}(z) \sum_{a=q,\bar{q}} f_a(x)D_a^h \left( \frac{\alpha_h}{xz} \right) \right]$$

$$+ (1 + z^{-2\gamma}) \frac{C_A}{C_F} P_{qg}(z) f_g(x)D_g^h \left( \frac{\alpha_h}{xz} \right) + \frac{C_A}{C_F} \left( \frac{C_A}{C_F} + z^{-2\gamma} \right) P_{gg}(z) f_g(x) \sum_{a=q,\bar{q}} D_a^h \left( \frac{\alpha_h}{xz} \right) \right] . \tag{30}$$

The other counterterm is related with QCD charge renormalization; with $n_f$ active quark flavors we have

$$\alpha_s = \alpha_s(\mu_R) \left[ 1 + \frac{\alpha_s(\mu_R)}{4\pi} \beta_0 \left( \frac{1}{\epsilon} + \ln \frac{\mu_R^2}{\mu^2} \right) \right], \quad \beta_0 = \frac{11C_A}{3} - \frac{2n_f}{3}, \tag{31}$$

$^3$In principle, one can consider different values of the factorization scale for the evolution of PDFs and FFs. Here for simplicity we take these scales equal.
and is given by
\[
\frac{\pi \sqrt{2} \vec{k}^2}{C} \frac{d \Phi(\nu, n)}{d \alpha d^{2+2\epsilon} \vec{k}} |_{\text{charge c.t.}} = \frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} + \ln \frac{\mu_R^2}{\mu^2} \right) \left( \frac{11 C_A}{6} - \frac{n_f}{3} \right) \\
\times \int_{\alpha_h}^{1} \frac{dx}{x} \left( \frac{C_A}{C_F} f_{g}(x) D_{g}^{h} \left( \frac{\alpha_h}{x} \right) + \sum_{a=q,\bar{q}} f_{a}(x) D_{a}^{h} \left( \frac{\alpha_h}{x} \right) \right) \left( \vec{k}^2 \right)^{\gamma - \frac{1}{2}} \left( \vec{k} \cdot \vec{l} \right)^{n}.
\] (32)

To simplify formulae, from now on we put the arbitrary scale of dimensional regularization equal to the unity, \( \mu = 1 \).

In what follows we will present intermediate results always for \( \frac{\pi \sqrt{2} \vec{k}^2}{C} \frac{d \Phi(\nu, n)}{d \alpha d^{2+2\epsilon} \vec{k}} \), which we denote for shortness as
\[
\frac{\pi \sqrt{2} \vec{k}^2}{C} \frac{d \Phi(\nu, n)}{d \alpha d^{2+2\epsilon} \vec{k}} = I.
\] (33)

Note that we always denote by \( \vec{k} \) the transverse momentum of the parton which fragments to the identified hadron. Moreover, \( \alpha_s \) with no argument from now on is to be understood as \( \alpha_s(\mu_R) \).

We will consider separately the subprocesses initiated by a quark and a gluon PDF, and denote
\[
I = I_q + I_g.
\] (34)

We start with the case of incoming quark.

### 4.1 Incoming quark

We distinguish virtual corrections and real emission contributions,
\[
I_q = I_q^V + I_q^R.
\] (35)

Virtual corrections (see Fig. 4) are the same as in the case of the inclusive quark impact factor, therefore we have
\[
I_q^V = -\frac{\alpha_s}{2\pi} \frac{\Gamma[1 - \epsilon]}{(4\pi)^{\epsilon}} \frac{1}{\Gamma(1 + 2\epsilon)} \int_{\alpha_h}^{1} \frac{dx}{x} \sum_{a=q,\bar{q}} f_{a}(x) \left( \frac{\alpha_h}{x} \right) \left( \vec{k}^2 \right)^{\gamma + \epsilon - \frac{1}{2}} \left( \vec{k} \cdot \vec{l} \right)^{n} \\
\times \left\{ C_F \left( \frac{2}{\epsilon} - \frac{4}{1 + 2\epsilon} + 1 \right) - n_f \frac{1 + \epsilon}{(1 + 2\epsilon)(3 + 2\epsilon)} + C_A \left( \ln \frac{s_0}{\vec{k}^2} + \psi(1 - \epsilon) - 2\psi(\epsilon) + \psi(1) \right) \right. \\
\left. + \frac{1}{4(1 + 2\epsilon)(3 + 2\epsilon)} - \frac{2}{\epsilon(1 + 2\epsilon)} - \frac{7}{4(1 + 2\epsilon)} - \frac{1}{2} \right\}.
\] (36)
We expand (36) in $\epsilon$ and present the result as a sum of the singular and the finite parts. The singular contribution reads

$$\left( I^V_q \right)_s = -\frac{\alpha_s}{2\pi} \frac{\Gamma[1 - \epsilon] \Gamma^2(1 + \epsilon)}{\epsilon \Gamma(1 + 2\epsilon)} \int \frac{dx}{x} \sum_{a=q,\bar{q}} \frac{f_a(x)D^h_a}{x} \left( \frac{\alpha_h}{x} \right) \left( \frac{k^2}{x^2} \right)^{\gamma + \epsilon - \frac{\epsilon}{2}} \left( k \cdot \ell \right)^n,$$

whereas for the regular part we obtain

$$\left( I^V_q \right)_r = -\frac{\alpha_s}{2\pi} \int \frac{dx}{x} \sum_{a=q,\bar{q}} \frac{f_a(x)D^h_a}{x} \left( \frac{\alpha_h}{x} \right) \left( \frac{k^2}{x^2} \right)^{\gamma + \frac{\epsilon}{2}} \left( k \cdot \ell \right)^n,$$

where the numerator is $C_F \frac{2}{\epsilon} - 3 - \frac{n_f}{3} + C_A \left( \ln \frac{s_0}{k^2} + \frac{11}{6} \right),$ (37)

Note that $\left( I^V_q \right)_s + \left( I^V_q \right)_r$ differs from $I^V_q$ by terms which are $\mathcal{O}(\epsilon)$.

4.1.1 Quark-gluon intermediate state

The starting point here is the quark-gluon intermediate state contribution to the inclusive quark impact factor,

$$\Phi^{QG} = \Phi_q g^2 q \frac{d^2 + 2 \beta_1 k_1^2}{(2\pi)^3} \frac{d^2 + 2 \beta_2 k_2^2}{(2\pi)^3} \frac{d \beta_1}{\beta_1} \frac{[1 + \beta^2 + \epsilon \beta_1^2]}{\beta_1 k_1^2 k_2^2 (k_2 \beta_1 - k_1 \beta_2)^2} \left\{ C_F \beta_1^2 k_2^2 + C_A \beta_2 \left( k_1^2 - \beta_1 k_2 \cdot \overline{q} \right) \right\}.$$

where $\beta_1$ and $\beta_2$ are the relative longitudinal momenta ($\beta_1 + \beta_2 = 1$) and $k_1$ and $k_2$ are the transverse momenta ($k_1 + k_2 = \overline{q}$) of the produced gluon and quark, respectively.

We need to consider separately the fragmentation of the quark and of the gluon into the final hadron, therefore we denote the corresponding contributions as $I^{R,q}_{q,q}$ and $I^{R,q}_{q,g}$.

$$I^R_q = I^{R,q}_{q,q} + I^{R,q}_{q,g}.$$

We start with the gluon fragmentation case, $I^{R,q}_{q,g}$ (see Fig. 5).
a) gluon fragmentation

The "parent parton" variables are \( \mathbf{k} = \mathbf{k}_1, \zeta = \beta_1 (\beta_2 = \bar{\zeta} \equiv 1 - \zeta, \bar{k}_2 = \bar{q} - \bar{k}) \), therefore we have

\[
I_{R,q,g}^{\alpha_h} = \frac{\alpha_s}{2\pi} \int \frac{d^2q}{\pi^{1+\epsilon}} \left( \frac{q^2}{\zeta} \right)^{\gamma - \frac{\zeta}{\bar{\zeta}}} \left( \frac{q \cdot \bar{l}}{\zeta} \right)^n \int \frac{dx}{x} \int \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a(x) D^h_{\bar{g}} \left( \frac{\alpha_h}{x\zeta} \right) \times \left[ \frac{1}{(q - \bar{k})^2} + \frac{\zeta}{(q - \bar{k})^2} \left( \frac{\bar{q}^2}{\bar{\zeta}} \right)^{\gamma - \frac{\zeta}{\bar{\zeta}}} \left( \frac{\bar{q} \cdot \bar{l}}{\bar{\zeta}} \right)^n \right].
\] (41)

It is worth stressing the difference between the previous calculations of NLO inclusive parton impact factors and the present case of fragmentation to a hadron with fixed momentum. In parton impact factor case, one keeps fixed the Reggeon transverse momentum \( \mathbf{q} \) and integrates over the allowed phase space of the produced partons, i.e. the integration is of the form \( \int \frac{d\zeta}{\zeta(1-\zeta)} d^2k \ldots \). In the fragmentation case, instead, we keep fixed the momentum of the parent parton \( \zeta, \mathbf{k} \), and allow the Reggeon momentum \( \mathbf{q} \) to vary. Indeed, the expression (41) contains the explicit integration over the momentum \( \mathbf{q} \) with the LO BFKL eigenfunctions, which is needed in order to obtain the impact factor in the \((\nu, n)\)-representation.

The \( \mathbf{q} \)-integration in (41) generates \( 1/\epsilon \) poles due to the integrand singularities at \( \mathbf{q} \to \mathbf{k}/\zeta \) for the contribution proportional to \( C_F \) and at \( \mathbf{q} \to \mathbf{k} \) for the one proportional to \( C_A \). Accordingly we split the result of the \( \mathbf{q} \)-integration into the sum of two terms: "singular" and "non-singular" parts. The non-singular part is defined as

\[
\frac{\alpha_s}{2\pi} \int \frac{dx}{x} \int \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a(x) D^h_{\bar{g}} \left( \frac{\alpha_h}{x\zeta} \right) C_A \frac{\zeta}{(q - \bar{k})^2} \left( \frac{1 + \zeta^2 + \epsilon \zeta^2}{\zeta} \right) \times \left[ \frac{\bar{k}^2}{\zeta} \frac{\bar{q}^2 - \bar{k} \cdot \bar{q}}{(q - \bar{k})^2} \left( \frac{\bar{q} \cdot \bar{l}}{\bar{\zeta}} \right)^n \right.
\]
where $\vec{n}$ is a unit vector, $\vec{n}^2 = 1$. Taking this expression for $\epsilon = 0$ we have

$$
(I^R_{q,g})_s = \frac{\alpha_s}{2\pi} \int \frac{1}{x} \frac{1}{\zeta} \sum_{a=q,g} f_a(x) D_g^h \left( \frac{\alpha_h}{x \zeta} \right) \left( \vec{k}^\perp \right)^{\gamma+\epsilon - \frac{2}{\zeta}} \left( \vec{k} \cdot \vec{l} \right)^n C_{A \vec{C}} \left( \frac{1 + \vec{C}^2 + \epsilon \zeta^2}{\zeta} \right)
$$

Expanding it in $\epsilon$ we get

$$
(I^R_{q,g})_s = \frac{\alpha_s}{2\pi} \int \frac{1}{x} \frac{1}{\zeta} \sum_{a=q,g} f_a(x) D_g^h \left( \frac{\alpha_h}{x \zeta} \right) \left( \vec{k}^\perp \right)^{\gamma+\epsilon - \frac{2}{\zeta}} \left( \vec{k} \cdot \vec{l} \right)^n
$$

$$
\times \left\{ P_{qq}(\zeta) \left[ \frac{C_A}{C_F} + \zeta^{-2\gamma} \right] + \epsilon \left( \frac{1 + \vec{C}^2}{\zeta} \right) \left[ C_F \zeta^{-2\gamma} (\chi(n, \gamma) - 2 \ln \zeta) + 2 C_A \ln \frac{\vec{C}}{\zeta} + \zeta (C_F \zeta^{-2\gamma} + C_A) \right] \right\},
$$

where

$$
\chi(n, \gamma) = 2\psi(1) - \psi \left( \frac{n}{2} - \gamma \right) - \psi \left( \frac{n}{2} + 1 + \gamma \right)
$$

is the eigenvalue of the LO BFKL kernel, up to the factor $N\alpha_s/\pi$.

Note that the divergent part of (45) is canceled by the corresponding term of the collinear counterterm (30).
Figure 6: Diagrammatic representation of the NLO vertex for the identified hadron production for the case of incoming quark: real corrections from quark-gluon intermediate state, case of quark fragmentation.

b) quark fragmentation

Now the “parent parton” variables are $\vec{k} = \vec{k}_2$, $\zeta = \beta_2$ ($\beta_1 = \bar{\zeta}$, $\vec{k}_1 = \vec{q} - \vec{k}$) (see Fig. 6).

The corresponding contribution reads

$$I_{q,q}^R = \frac{\alpha_s}{2\pi(4\pi)^\epsilon} \int \frac{d^2+2\epsilon}{\pi^{1+\epsilon}} \frac{\vec{q}}{(\vec{q}^2)^{\gamma-\frac{\eta}{2}}} (\vec{q} \cdot \vec{l})^n \int \frac{dx}{x} \int \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a(x) D_h^a \left( \frac{\alpha_h}{x\zeta} \right)$$

$$\times \frac{1 + \zeta^2 + \epsilon \zeta^2}{1 - \zeta} \left[ C_F \frac{\zeta^2}{\zeta^2} \frac{\vec{k}^2}{(\vec{q} - \vec{k})^2 (\vec{q} - \vec{k}_2)^2} + C_A \frac{\vec{q}^2 - \vec{k} \cdot \vec{q} + \vec{k}_2^2}{(\vec{q} - \vec{k}_2)^2 (\vec{q} - \vec{k})^2} \right]. \quad (47)$$

We will consider separately the contributions proportional to $C_F$ and $C_A$.

b1) quark fragmentation: $C_F$-term

Note that the integrand of the $C_F$-term is not singular at $\zeta \to 1$. We use the decomposition

$$\frac{\vec{k}^2}{(\vec{q} - \vec{k})^2 (\vec{q} - \vec{k}_2)^2} = \frac{\vec{k}^2}{(\vec{q} - \vec{k}_2)^2 + (\vec{q} - \vec{k})^2} \left( \frac{1}{(\vec{q} - \vec{k}_2)^2} + \frac{1}{(\vec{q} - \vec{k})^2} \right)$$

in order to separate the regular and singular contributions. The regular part is given by

$$\frac{\alpha_s}{2\pi(4\pi)^\epsilon} \int \frac{dx}{x} \int \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a(x) D_h^a \left( \frac{\alpha_h}{x\zeta} \right) (\vec{k}_2^2) \gamma^{\epsilon-\frac{\eta}{2}} (\vec{k} \cdot \vec{l})^n \frac{\zeta^2}{\zeta^2} \left( \frac{1 + \zeta^2 + \epsilon \zeta^2}{1 - \zeta} \right)$$

$$\times C_F \int \frac{d^2+2\epsilon}{\pi^{1+\epsilon}} \frac{1}{(\vec{a} - \vec{n})^2 + (\vec{a} - \vec{n}/\zeta)^2} \left[ (\vec{a}^2)^{\gamma-\frac{\eta}{2}} (\vec{a}/\vec{n})^n - 1 \right] + \frac{(\vec{a}^2)^{\gamma-\frac{\eta}{2}} (\vec{a}/\vec{n})^n - \zeta^{-2\gamma}}{(\vec{a} - \vec{n})^2} \right]. \quad (48)
The next step is to introduce the plus-prescription, which is defined as 

\[
(I_R^{q,q})_F = \frac{\alpha_s}{2\pi} \frac{1}{\alpha_h} \int_0^1 \frac{dx}{x} \int_0^\frac{a}{x} \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a(x) D_a^h \left( \frac{\alpha_h}{x\zeta} \right) \left( \vec{k}^2 \right)^{\gamma-\frac{n}{2}} \left( \vec{k} \cdot \vec{l} \right) \frac{n}{\zeta} (1+\zeta^2) C_F I_2 ,
\] 

(49)

where we define the function 

\[
I_2 = I_2(n, \gamma, \zeta) = \frac{d^2 a}{\pi} \frac{1}{(a-n)^2 + (\bar{a}-\bar{n})^2} \left[ \frac{(\bar{a})^\gamma e^{i\phi} - 1}{(\bar{a} - n)^2} + \frac{(\bar{a})^\gamma e^{i\phi} - \zeta^{-2\gamma}}{(\bar{a} - \bar{n})^2} \right] .
\] 

(50)

The singular part is proportional to the integral 

\[
\int d^{2+2\epsilon} \bar{q} \langle \bar{q} \bar{k} \rangle^{2\epsilon} \left( \frac{\bar{k}^2}{\bar{q} - \bar{k}} \right)^2 \left[ \left( \bar{k}^2 \right)^{\gamma-\frac{n}{2}} \left( \bar{k} \cdot \bar{l} \right)^n \left( \frac{\bar{k}}{\bar{\zeta}} \right)^{2\epsilon-2} \left( 1 + \zeta^{-2\gamma} \right) \right]
\]

therefore, for the singular part of the $C_F$-term we have 

\[
\frac{\alpha_s}{2\pi} \Gamma(1-\epsilon) \frac{\Gamma^2(1+\epsilon)}{(4\pi)^\epsilon} \frac{\Gamma(1+2\epsilon)}{(1+2\epsilon)^\epsilon} \int_0^1 \frac{dx}{x} \int_0^{\frac{\alpha_h}{x}} \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a(x) D_a^h \left( \frac{\alpha_h}{x\zeta} \right) \left( \bar{k}^2 \right)^{\gamma+\epsilon-\frac{n}{2}} \left( \bar{k} \cdot \bar{l} \right)^n
\]

\[
\times C_F \frac{1 + \zeta^2 + \epsilon \zeta^2}{1 - \zeta} \left( \frac{\bar{k}}{\bar{\zeta}} \right)^{2\epsilon} \left( 1 + \zeta^{-2\gamma} \right) .
\] 

(51)

The next step is to introduce the plus-prescription, which is defined as 

\[
\int_a^1 \frac{d\zeta}{(1-\zeta)^+} = \int_a^1 \frac{d\zeta F(\zeta) - F(1)}{(1-\zeta)} - \int_a^0 \frac{d\zeta F(1)}{(1-\zeta)} ,
\] 

(52)

for any function $F(\zeta)$, regular at $\zeta = 1$. Note that 

\[
(1-\zeta)^{2\epsilon-1} = (1-\zeta)^{2\epsilon-1} + \frac{1}{2\epsilon} \delta(1-\zeta) = \frac{1}{2\epsilon} \delta(1-\zeta) + \frac{1}{(1-\zeta)^+} + 2\epsilon \left( \frac{\ln(1-\zeta)}{1-\zeta} \right)^+ + O(\epsilon^2) .
\]

Using this result, one can write 

\[
C_F \frac{1 + \zeta^2 + \epsilon \zeta^2}{1 - \zeta} \left( \frac{\bar{k}}{\bar{\zeta}} \right)^{2\epsilon} \left( 1 + \zeta^{-2\gamma} \right) = C_F \left[ \frac{2}{\epsilon} \delta(1-\zeta) + \frac{1 + \zeta^2}{(1-\zeta)^+} \left( 1 + \zeta^{-2\gamma} \right) + O(\epsilon) \right]
\]

\[
= C_F \left[ \left( \frac{2}{\epsilon} - 3 \right) \delta(1-\zeta) + \left( \frac{1 + \zeta^2}{(1-\zeta)^+} + 3 \delta(1-\zeta) \right) (1 + \zeta^{-2\gamma}) + O(\epsilon) \right]
\]
\[
= C_F \left( \frac{2}{\epsilon} - 3 \right) \delta(1 - \zeta) + P_{qq}(\zeta) \left( 1 + \zeta^{-2\gamma} \right) + \mathcal{O}(\epsilon) .
\]

Taking this into account and expanding in $\epsilon$ the singular part of the $C_F$-term, one gets the following result for the divergent contribution:

\[
\left( I_{q,q}^R \right)_s^{C_F} = \frac{\alpha_s}{2\pi} \Gamma[1 - \epsilon] \Gamma^2(1 + \epsilon) (1 - \zeta) \sum_{a=q,\bar{q}} \int_0^1 dx \frac{d\zeta}{\zeta} f_a(x) D_a^b \left( \frac{\alpha_s}{x \zeta} \right) \left( k^2 \right)^{\gamma + \epsilon - \frac{2}{\epsilon}} \left( k \cdot l \right)^n \times \left\{ C_F \left( \frac{2}{\epsilon} - 3 \right) \delta(1 - \zeta) + P_{qq}(\zeta) \left( 1 + \zeta^{-2\gamma} \right)
+ \epsilon C_F (1 + \zeta^{-2\gamma}) \left( \zeta + 2(1 + \zeta^2) \left( \frac{\ln(1 - \zeta)}{1 - \zeta} \right) - 2(1 + \zeta^2) \frac{\ln \zeta}{1 - \zeta} \right) \right\}.
\]

Note that the first term of the divergent contribution given in (53) cancels in the sum with the corresponding term of the virtual correction (37), whereas the second one is canceled by the corresponding term of the collinear counterterm (30).

\textbf{b2) quark fragmentation: $C_A$-term}

The $C_A$-contribution needs a special treatment due to the behavior of (47) in the region $\zeta \to 1$.

We use the following decomposition:

\[
\frac{C_A}{(1 - \zeta)} \left( \frac{\bar{q}^2 - \bar{k} \cdot \bar{q}^{1 + \zeta} + \bar{k}^2}{(\bar{q} - \bar{k})^2} \right) = \frac{C_A}{2} \frac{2}{(\bar{q} - \bar{k})^2} \frac{1}{(1 - \zeta)}
+ \frac{C_A}{2(1 - \zeta)} \left[ \frac{1}{(\bar{q} - \bar{k})^2} - \frac{1}{(\bar{q} - \bar{k})^2} - \frac{\bar{\zeta}^2}{(\bar{q} - \bar{k})^2(\bar{q} - \bar{k})^2} \right].
\]

The second term in the r.h.s. is regular for $\zeta \to 1$ and can be treated similarly to what we did above in the case of the $C_F$-contribution. The first term is singular and the integration over $\zeta$ has to be restricted (according to definition of NLO impact factor) by the requirement

\[
M_{QG}^2 \leq s_A , \quad M_{QG}^2 = \frac{k_1^2}{\beta_1} + \frac{k_2^2}{\beta_2} - \bar{q}^2 = \frac{(\bar{q} - \bar{k})^2}{1 - \zeta} + \frac{\bar{k}^2}{\zeta} - \bar{q}^2 ,
\]

and assuming $s_A \to \infty$. Therefore the $\zeta$-integral has the form

\[
\int_0^{1 - \zeta_0} d\zeta \frac{F(\zeta)}{1 - \zeta} , \quad \text{for} \quad \zeta_0 = \frac{(\bar{q} - \bar{k})^2}{s_A} \to 0 .
\]

Using the plus-prescription (52) one can write

\[
\int_0^{1 - \zeta_0} d\zeta \frac{F(\zeta)}{1 - \zeta} = \int_0^1 d\zeta \frac{F(\zeta)}{(1 - \zeta)_+} + F(1) \ln \frac{1}{\zeta_0} , \quad \text{for} \quad \zeta_0 \to 0 ,
\]

(54)
for any function not singular in the limit \( \zeta \to 1 \), and

\[
\frac{C_A}{(1 - \zeta)} \left( \frac{q^2 - k \cdot q + \frac{1 + \zeta}{\zeta} + \frac{k^2}{\zeta}}{(q - k)^2 \left( \frac{q}{\zeta} \right)^2} \right) = \frac{C_A}{2} \delta(1 - \zeta) \frac{2}{(q - k)^2} \ln \frac{s_A}{(q - k)^2}
\]

\[+ \frac{C_A}{2} \frac{2}{(q - k)^2 (1 - \zeta)} + \frac{C_A}{2} \left[ \frac{1}{(q - k)^2} - \frac{1}{(q - k)^2} - \left( \frac{\zeta}{\zeta} \right)^2 \frac{k^2}{(q - k)^2(\bar{q} - \bar{k})^2} \right].
\]

We remind that the definition of NLO impact factor requires the subtraction of the contribution coming from the gluon emission in the central rapidity region, given by the last term in Eq. (13), which we call below “BFKL subtraction term”. After this subtraction the parameter \( s_A \) should be sent to infinity, \( s_A \to \infty \). Our simple treatment of the invariant mass constraint, \( M_{QG}^2 \leq s_A \), anticipates this limit \( s_A \to \infty \), therefore we neglect all contributions which are suppressed by powers of \( 1/s_A \). Moreover, the first term in the r.h.s. of the above equation should be naturally combined with the BFKL subtraction term, giving finally

\[
\frac{C_A}{(1 - \zeta)} \left( \frac{q^2 - k \cdot q + \frac{1 + \zeta}{\zeta} + \frac{k^2}{\zeta}}{(q - k)^2 \left( \frac{q}{\zeta} \right)^2} \right) = \frac{C_A}{2} \delta(1 - \zeta) \frac{1}{(q - k)^2} \ln \frac{s_0}{(q - k)^2}
\]

\[+ \frac{C_A}{2} \frac{2}{(q - k)^2 (1 - \zeta)} + \frac{C_A}{2} \left[ \frac{1}{(q - k)^2} - \frac{1}{(q - k)^2} - \left( \frac{\zeta}{\zeta} \right)^2 \frac{k^2}{(q - k)^2(\bar{q} - \bar{k})^2} \right].
\]

After that, we are ready to perform the \( \bar{q} \)-integration, which naturally introduces the separation into singular and non-singular contributions. The singular contribution reads

\[
\frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{\Gamma(1 + 2\epsilon)} \int \frac{dx}{x} \int \frac{d\zeta}{\zeta} \sum_{a = q, \bar{q}} f_a(x) D_a^h \left( \frac{\alpha_h}{x \zeta} \right) \left( \frac{k^2}{\bar{k}^2} \right)^{\gamma + \epsilon - \frac{n}{2}} \left( \bar{k} \cdot \bar{l} \right)^n
\]

\[\times \frac{C_A}{2} \left( (1 + \zeta^2 + \epsilon \zeta^2) \right) \left\{ \frac{\Gamma(1 + 2\epsilon) \Gamma(\frac{n}{2} - \gamma - \epsilon) \Gamma(\frac{n}{2} + 1 + \gamma + \epsilon)}{\Gamma(1 + \epsilon) \Gamma(1 - \epsilon) \Gamma(\frac{n}{2} - \gamma) \Gamma(\frac{n}{2} + 1 + \gamma + 2\epsilon)} \right\} \left[ \delta(1 - \zeta) \left( \ln \frac{s_0}{k^2} + \psi \left( \frac{n}{2} - \gamma - \epsilon \right) + \psi \left( 1 + \gamma + \frac{n}{2} + 2\epsilon \right) - \psi(\epsilon) - \psi(1) \right) + \frac{2}{(1 - \zeta)_+}
\]

\[+ \frac{\zeta^{-2\epsilon - 2\gamma - 1}}{1 - \zeta} \right] \right) \left( \zeta^{-2\epsilon + \zeta^{-2\epsilon - 2\gamma - 2\epsilon}} \right). \tag{55}
\]

Expanding this expression in \( \epsilon \) and using that

\[
\frac{\zeta^{-2\epsilon - 2\gamma - 1}}{1 - \zeta} = \frac{\zeta^{-2\epsilon - 2\gamma - 1}}{(1 - \zeta)_+},
\]

and

\[
\zeta^{2\epsilon - 1} \left( \zeta^{-2\epsilon} + \zeta^{-2\gamma - 2\epsilon} \right) = \left( \zeta^{-2\epsilon} + \zeta^{-2\gamma - 2\epsilon} \right) \left( \frac{\delta(1 - \zeta)}{2\epsilon} + \frac{1}{(1 - \zeta)_+} + \mathcal{O}(\epsilon) \right),
\]

\[
17
\]
we get a divergent term,
\[
(I_{q,q}^R)^{CA} = \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\epsilon (4\pi)^{\epsilon}} \int_{\alpha_h}^{1} \frac{dx}{x} \int_{1}^{\alpha_h} \frac{d\zeta}{\zeta} \sum_{a=q,q} f_a(x) D_a^h \left( \frac{\alpha_h}{x\zeta} \right) (\vec{k}^2)^{\gamma+\epsilon-\frac{n}{2}} (\vec{k} \cdot \vec{l})^n
\times \left\{ C_A \delta(1 - \zeta) \ln \frac{s_0}{k^2} + \epsilon C_A \left[ \delta(1 - \zeta) \left( \chi(n, \gamma) \ln \frac{s_0}{k^2} + \frac{1}{2} \left( \psi' \left( 1 + \gamma + \frac{n}{2} \right) - \psi' \left( \frac{n}{2} - \gamma \right) - \chi^2(n, \gamma) \right) \right) \\
+ (1 + \zeta^2) \left( 1 + \zeta^{-2}\gamma \right) \left( \frac{\chi(n, \gamma)}{2(1 - \zeta)_+} - \left( \frac{\ln(1 - \zeta)}{1 - \zeta} \right)_+ + \ln \frac{\zeta}{1 - \zeta} \right) \right] \right\}.
\]
(56)

The divergent term given in (56) cancels in the sum with the corresponding term of the virtual correction (37). The remaining singularity in (37) vanishes after the charge renormalization (31).

The regular contribution differs from (48) only by one factor and reads
\[
(I_{q,q}^R)^{CA} = \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\epsilon (4\pi)^{\epsilon}} \int_{\alpha_h}^{1} \frac{dx}{x} \int_{1}^{\alpha_h} \frac{d\zeta}{\zeta} \sum_{a=q,q} f_a(x) D_a^h \left( \frac{\alpha_h}{x\zeta} \right) (\vec{k}^2)^{\gamma+\epsilon-\frac{n}{2}} (\vec{k} \cdot \vec{l})^n \frac{\zeta}{\zeta^2} (1 + \zeta^2)
\times \left( -\frac{C_A}{2} \right) I_2.
\]
(57)

4.2 Incoming gluon

We distinguish virtual corrections and real emission contributions,
\[
I_g = I_g^V + I_g^R.
\]
(58)

Virtual corrections (see Fig. 7) are the same as in the case of inclusive gluon impact factor,
\[
I_g^V = -\frac{\alpha_s}{2\pi} \frac{\Gamma[1 - \epsilon] \Gamma^2(1 + \epsilon)}{\epsilon \Gamma(1 + 2\epsilon)} \int_{\alpha_h}^{1} \frac{dx}{x} f_g(x) D_g^h \left( \frac{\alpha_h}{x} \right) (\vec{k}^2)^{\gamma+\epsilon-\frac{n}{2}} (\vec{k} \cdot \vec{l})^n \frac{C_A}{C_F}
\]
Figure 8: Diagrammatic representation of the NLO vertex for the identified hadron production for the case of incoming gluon: real corrections from quark-antiquark intermediate state, case of quark fragmentation.

\begin{align*}
\times \left\{ C_A \left( \frac{\ln s_0}{k^2} + \frac{2}{\epsilon} - \frac{11 + 9\epsilon}{2(1 + 2\epsilon)(3 + 2\epsilon)} + \psi(1 - \epsilon) - 2\psi(1 + \epsilon) + \psi(1) \\
+ \frac{\epsilon}{(1 + \epsilon)(1 + 2\epsilon)(3 + 2\epsilon)} \right) + n_f \left( \frac{(1 + \epsilon)(2 + \epsilon) - 1 - \frac{\epsilon}{1 + \epsilon}}{(1 + \epsilon)(1 + 2\epsilon)(3 + 2\epsilon)} \right) \right\}. \tag{59}
\end{align*}

Expanding it in \( \epsilon \) we obtain the following results for the singular,

\begin{align*}
(I^V_g)^s &= -\frac{\alpha_s}{2\pi} \frac{1}{(4\pi)^\epsilon} \frac{\Gamma^2(1 + \epsilon)}{\epsilon \Gamma(1 + 2\epsilon)} \int \frac{dx}{x} f_g(x) D^h_g \left( \frac{\alpha_h}{x} \right) \left( \frac{k_2}{k_1} \right)^{\epsilon - \frac{2}{3}} \left( k \cdot l \right) C_A C_F \\
&= \times \left\{ C_A \left( \frac{\ln s_0}{k^2} + \frac{2}{\epsilon} - \frac{11}{6} \right) + \frac{n_f}{3} \right\}, \tag{60}
\end{align*}

and the finite parts,

\begin{align*}
(I^V_g)^f &= -\frac{\alpha_s}{2\pi} \int \frac{dx}{x} f_g(x) D^h_g \left( \frac{\alpha_h}{x} \right) \left( \frac{k_2}{k_1} \right)^{\frac{\pi^2}{2} - \frac{2}{3}} \left( k \cdot l \right) C_A C_F \\
&= \times \left\{ C_A \left( \frac{67}{18} - \frac{\pi^2}{2} \right) - \frac{5}{9} n_f \right\}. \tag{61}
\end{align*}

For the corrections due to real emissions, one has to consider quark-antiquark and two-gluon intermediate states,

\begin{align*}
I^R_g = I^R_{g,q} + I^R_{g,g}. \tag{62}
\end{align*}

4.2.1 Quark-antiquark intermediate state

The starting point here is the quark-antiquark intermediate state contribution to the inclusive gluon impact factor \( (T_R = 1/2) \),

\begin{align*}
\Phi^{\{QQ\}} = \Phi_g g^2 q^2 \frac{D^{2+2\epsilon} k_1}{(2\pi)^{3+2\epsilon}} d\beta_1 T_R \left( 1 - \frac{2\beta_1 \beta_2}{1 + \epsilon} \right) \left\{ C_F \frac{1}{k_1 k_2} + \beta_1 \beta_2 \frac{k_1 \cdot k_2}{k_1^2 k_2 (k_2 \beta_1 - k_1 \beta_2)^2} \right\}, \tag{63}
\end{align*}
where \( \beta_1 \) and \( \beta_2 \) are the relative longitudinal momenta \((\beta_1 + \beta_2 = 1)\) and \( \vec{k}_1 \) and \( \vec{k}_2 \) are the transverse momenta \((\vec{k}_1 + \vec{k}_2 = \vec{q})\) of the produced quark and antiquark, respectively.

We consider quark fragmentation (see Fig. 8). In this case, the “parent parton” variables are \( \vec{k} = \vec{k}_1, \ \zeta = \beta_1 \) \((\beta_2 = \zeta \equiv 1 - \zeta, \ \vec{k}_2 = \vec{q} - \vec{k})\). The case when antiquark fragments is essentially the same. Therefore, the sum over all possible quarks and antiquarks fragmentation has the form:

\[
I_{R,g,q}^* = \frac{\alpha_s}{2\pi(4\pi)\epsilon} \int \frac{d^2q}{\pi^{1+\epsilon}} \left( \frac{2}{2\epsilon} \right) \frac{d x}{x} \int \frac{d \zeta}{\zeta} f_g(x) \sum_{a = q, \bar{q}} D_a^h \left( \frac{\alpha_h}{x \zeta} \right) \frac{C_A}{C_F} \mathcal{T}_R \left( 1 - \frac{2\zeta \bar{\zeta}}{1 + \epsilon} \right) \left\{ \frac{C_F}{C_A (\vec{q} - \vec{k})^2} + \frac{\bar{\zeta}}{\zeta} \frac{\vec{k} \cdot (\vec{q} - \vec{k})}{(\vec{q} - \vec{k})^2 (\vec{q} - \vec{k}_2)^2} \right\}.
\]

(64)

We can split this integral into the sum of singular and non-singular parts. The non-
singular contribution of (64) reads

\[
\frac{\alpha_s}{2\pi(4\pi)\epsilon} \int \frac{d x}{x} \int \frac{d \zeta}{\zeta} f_g(x) \sum_{a = q, \bar{q}} D_a^h \left( \frac{\alpha_h}{x \zeta} \right) \frac{C_A}{C_F} \mathcal{T}_R \left( 1 - \frac{2\zeta \bar{\zeta}}{1 + \epsilon} \right) \left\{ \frac{C_F}{C_A (\vec{q} - \vec{k})^2} + \frac{\bar{\zeta}}{\zeta} \frac{\vec{k} \cdot (\vec{q} - \vec{k})}{(\vec{q} - \vec{k})^2 (\vec{q} - \vec{k}_2)^2} \right\}.
\]

(65)

Expanding it in \( \epsilon \) we obtain

\[
(I_{R,g,q}^*)_r = \frac{\alpha_s}{2\pi} \int \frac{d x}{x} \int \frac{d \zeta}{\zeta} f_g(x) \sum_{a = q, \bar{q}} D_a^h \left( \frac{\alpha_h}{x \zeta} \right) \frac{C_A}{C_F} \mathcal{T}_R \left( 1 - \frac{2\zeta \bar{\zeta}}{1 + \epsilon} \right) \left\{ \frac{C_F}{C_A (\vec{q} - \vec{k})^2} + \frac{\bar{\zeta}}{\zeta} \frac{\vec{k} \cdot (\vec{q} - \vec{k})}{(\vec{q} - \vec{k})^2 (\vec{q} - \vec{k}_2)^2} \right\} P_{qg}(\zeta) I_3,
\]

(66)

where we define the function

\[
I_3 = I_3(n, \gamma, \zeta) = \int \frac{d^2 \bar{\alpha}}{\pi} \frac{\bar{\alpha} \cdot \bar{n} - 1}{(\bar{\alpha} - \bar{n})^2} \left[ (\bar{\alpha}^2)^{\gamma} e^{in\phi} - \zeta^{-2\gamma} \right].
\]

(67)

For the singular contribution of (64) we have

\[
\frac{\alpha_s}{2\pi} \frac{\Gamma[1 - \epsilon]}{2(4\pi)\epsilon} \frac{1}{\epsilon} \frac{\Gamma^2(1 + \epsilon)}{\Gamma(1 + 2\epsilon)} \int \frac{d x}{x} f_g(x) \int \frac{d \zeta}{\zeta} \sum_{a = q, \bar{q}} D_a^h \left( \frac{\alpha_h}{x \zeta} \right) \frac{C_A}{C_F} \mathcal{T}_R \left( 1 - \frac{2\zeta \bar{\zeta}}{1 + \epsilon} \right) \left\{ \frac{C_F}{C_A (\vec{q} - \vec{k})^2} + \frac{\bar{\zeta}}{\zeta} \frac{\vec{k} \cdot (\vec{q} - \vec{k})}{(\vec{q} - \vec{k})^2 (\vec{q} - \vec{k}_2)^2} \right\}.
\]

(68)
Expanding it in $\epsilon$, we get

$$\left(I^R_{g,g}\right)_s = \frac{\alpha_s}{2\pi} \frac{\Gamma[1 - \epsilon]}{\epsilon(4\pi)^\epsilon} \int_0^1 dx \int_0^1 d\zeta \frac{f_g(x)}{a_{qg}} \sum_{a=q,\bar{q}} D_h^a \left(\alpha_h x\zeta\right) \left(\vec{k}_2^2\right)^{\frac{\epsilon}{2} - \frac{n}{2}} \left(\vec{k} \cdot \vec{l}\right)^n \frac{C_A}{C_F}$$

$$\times \left\{ P_{qg}(\zeta) \left[ \frac{C_F}{C_A} + \zeta^{-2\gamma} \right] + \epsilon \left( 2\zeta \tilde{T}_R \left( \frac{C_F}{C_A} + \zeta^{-2\gamma} \right) + P_{qg}(\zeta) \left( \frac{C_F}{C_A} \chi(n, \gamma) + 2\zeta^{-2\gamma} \ln \zeta \right) \right\}. \quad (69)$$

Note that the divergent part of this expression is canceled by the corresponding term of the collinear counterterm (30).

### 4.2.2 Two-gluon intermediate state

The starting point here is the gluon-gluon intermediate state contribution to the inclusive gluon impact factor,

$$\Phi^{(GG)} = \Phi g^2 g^2 \frac{d^2 q}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \frac{C_A}{2} \left[ \frac{1}{\beta_1} + \frac{1}{\beta_2} - 2 + \beta_1\beta_2 \right]$$

$$\times \left\{ \frac{1}{k_1^2 k_2^2} + \frac{\beta_1^2}{k_1^2 (k_2^2 - k_1^2)\beta_2^2} + \frac{\beta_2^2}{k_2^2 (k_2^2 - k_1^2)\beta_1^2} \right\}, \quad (70)$$

where $\beta_1$ and $\beta_2$ are the relative longitudinal momenta ($\beta_1 + \beta_2 = 1$) and $\vec{k}_1$ and $\vec{k}_2$ are the transverse momenta ($\vec{k}_1 + \vec{k}_2 = \vec{q}$) of the two produced gluons.

We need to consider the case of a gluon which fragments, the case when the other gluon fragments being taken into account by a factor 2 (see Fig. 9). Thus, we obtain the following integral:

$$I^R_{g,g} = \frac{\alpha_s}{2\pi(4\pi)^\epsilon} \int d^2 q \left(\vec{q}^2\right)^{\frac{\epsilon}{2} - \frac{n}{2}} \left(\vec{q} \cdot \vec{l}\right)^n \int_0^1 dx \int_0^1 d\zeta f_g(x) D_h^g \left(\alpha_h x\zeta\right) \frac{C_A}{C_F}$$
The calculation goes along the same lines as in the Section 4.1.1 (case b₂). First, we separate the $\zeta \to 1$ singularity, then we add the BFKL subtraction term. Using (54) one obtains

$$
\begin{align*}
\times C_A \left[ \frac{1}{\zeta} + \frac{1}{1-\zeta} - 2 + \zeta \tilde{\zeta} \right] & \left\{ \frac{1}{(\bar{q} - \bar{k})^2} + \frac{1}{(\bar{q} - \bar{k} - \bar{q})^2} + \frac{\tilde{\zeta}^2}{\tilde{\zeta}^2 (\bar{q} - \bar{k})^2 (\bar{q} - \bar{k} - \bar{q})^2} \right\} . \quad (71)
\end{align*}
$$

Again, we can split the result into the sum of singular and non-singular parts. The non-singular contribution differs from (148) only by a factor, it reads

$$
\frac{\alpha_s}{2\pi(4\pi)^\epsilon} \frac{1}{x} \int d\zeta f_g(x) D_g^h \left( \frac{\alpha_h}{x\zeta} \right) \left( \bar{k}^2 \right)^{\gamma - \frac{n}{2}} \left( \bar{k} \cdot \bar{l} \right)^n \tilde{\zeta}^2 \left[ \frac{1}{\zeta} + \frac{1}{1-\zeta} - 2 + \zeta \tilde{\zeta} \right] \frac{C_A}{C_F}.
$$

Expanding it in $\epsilon$ we obtain

$$
\begin{align*}
(I_{g,g}^R)_{r} = \frac{\alpha_s}{2\pi} \frac{1}{x} \int d\zeta f_g(x) D_g^h \left( \frac{\alpha_h}{x\zeta} \right) \left( \bar{k}^2 \right)^{\gamma - \frac{n}{2}} \left( \bar{k} \cdot \bar{l} \right)^n.
\end{align*}
$$
\[
\times \frac{\zeta^2}{\zeta^2} \left[ \frac{1}{\zeta} + \frac{1}{(1 - \zeta)} - 2 + \zeta \frac{C_A}{C_F} \right] C_A \frac{C_A}{C_F} I_2 .
\] (73)

For the singular contribution we obtain

\[
\frac{\alpha_s \Gamma[1 - \epsilon]}{2\pi} \int \frac{1}{(4\pi)^{\epsilon}} \frac{1}{\epsilon (1 + 2\epsilon)} \frac{1}{x} \int \frac{d\zeta}{\zeta} f_g(x) D_g \left( \frac{\alpha_h}{x^{\epsilon}} \right) \left( \frac{1}{k^2} \right)^{\gamma + \epsilon - \frac{n}{2}} \left( \frac{1}{k \cdot l^2} \right)^n C_A \frac{C_A}{C_F}
\]

\[
\times \left\{ \left[ \frac{1}{\zeta} + \frac{1}{(1 - \zeta)} - 2 + \zeta \frac{C_A}{C_F} \right] \left( \frac{1}{\zeta} \right)^{2 - 2\gamma} (1 + \zeta^{-2\gamma}) + \frac{\Gamma(1 + 2\epsilon) \Gamma(\gamma - \epsilon) \Gamma(\gamma + 1 + \gamma + \epsilon)}{\Gamma(1 + \epsilon) \Gamma(1 - \epsilon) \Gamma(\epsilon - \gamma) \Gamma(\epsilon + 1 + \gamma + 2\epsilon)} \right\}
\]

Expanding this result in \( \epsilon \), we get for the divergent contribution

\[
\frac{\alpha_s \Gamma[1 - \epsilon]}{2\pi} \int \frac{1}{(4\pi)^{\epsilon}} \frac{1}{\epsilon (1 + 2\epsilon)} \frac{1}{x} \int \frac{d\zeta}{\zeta} f_g(x) D_g \left( \frac{\alpha_h}{x^{\epsilon}} \right) \left( \frac{1}{k^2} \right)^{\gamma + \epsilon - \frac{n}{2}} \left( \frac{1}{k \cdot l^2} \right)^n C_A \frac{C_A}{C_F}
\]

\[
\times C_A \left\{ 2 \left[ \frac{1}{\zeta} + \frac{1}{(1 - \zeta)} - 2 + \zeta \frac{C_A}{C_F} \right] (1 + \zeta^{-2\gamma}) + \delta(1 - \zeta) \left( \ln \frac{s_0}{k^2} + \frac{2}{\epsilon} \right) \right\}
\]

\[
= \frac{\alpha_s \Gamma[1 - \epsilon]}{2\pi} \int \frac{1}{(4\pi)^{\epsilon}} \frac{1}{\epsilon (1 + 2\epsilon)} \frac{1}{x} \int \frac{d\zeta}{\zeta} f_g(x) D_g \left( \frac{\alpha_h}{x^{\epsilon}} \right) \left( \frac{1}{k^2} \right)^{\gamma + \epsilon - \frac{n}{2}} \left( \frac{1}{k \cdot l^2} \right)^n C_A \frac{C_A}{C_F}
\]

\[
\times \left\{ P_{gg}(\zeta) (1 + \zeta^{-2\gamma}) + \delta(1 - \zeta) \left[ C_A \left( \ln \frac{s_0}{k^2} + \frac{2}{\epsilon} - \frac{11}{3} \right) + \frac{2n_f}{3} \right] \right\} .
\] (75)

Finally, the \( \epsilon \) expansion of the divergent part has the form

\[
(I_{g g}) = \frac{\alpha_s \Gamma[1 - \epsilon]}{2\pi} \int \frac{1}{(4\pi)^{\epsilon}} \frac{1}{\epsilon (1 + 2\epsilon)} \frac{1}{x} \int \frac{d\zeta}{\zeta} f_g(x) D_g \left( \frac{\alpha_h}{x^{\epsilon}} \right) \left( \frac{1}{k^2} \right)^{\gamma + \epsilon - \frac{n}{2}} \left( \frac{1}{k \cdot l^2} \right)^n C_A \frac{C_A}{C_F}
\]

\[
\times \left\{ P_{gg}(\zeta) (1 + \zeta^{-2\gamma}) + \delta(1 - \zeta) \left[ C_A \left( \ln \frac{s_0}{k^2} + \frac{2}{\epsilon} - \frac{11}{3} \right) + \frac{2n_f}{3} \right] \right\}
\]

\[
+ \epsilon C_A \left[ \delta(1 - \zeta) \left( \chi(n, \gamma) \ln \frac{s_0}{k^2} + \frac{1}{2} \left( \psi' \left( 1 + \gamma + \frac{n}{2} \right) - \psi' \left( \frac{n}{2} - \gamma \right) - \chi^2(n, \gamma) \right) \right) 
\]

\[
+ \left( \frac{1}{\zeta} + \frac{1}{(1 - \zeta)} - 2 + \zeta \frac{C_A}{C_F} \right) (\chi(n, \gamma)(1 + \zeta^{-2\gamma}) - 2(1 + 2\zeta^{-2\gamma}) \ln \zeta)
\]

23
The term in the divergent contribution (76) proportional to \( P_{gg} \) cancels with the corresponding term of the collinear counterterm (30), while the term in (76) proportional to \( \delta(1 - \zeta) \) cancels in the sum with the singular part of the virtual corrections (60). The uncanceled divergence in the virtual corrections (60) vanishes after the QCD charge renormalization (31).

Summarizing, all the infrared and ultraviolet divergences arisen in the calculation have disappeared after taking into account PDFs and FFs renormalization and QCD charge renormalization. Collecting all intermediate contributions we obtain the final result for the identified hadron NLO impact factor, which reads

\[
\begin{align*}
+2(1 + \zeta^{-2\gamma}) & \left(\left(\frac{1}{\zeta} - 2 + \zeta \bar{\zeta}\right) \ln \bar{\zeta} + \left(\frac{\ln(1 - \zeta)}{1 - z}\right)_+\right) \right] \}. 
\end{align*}
\tag{76}
\]

The results for the NLO coefficient functions read

\[
\begin{align*}
C_{gg}(x, \zeta) &= P_{gg}(\zeta) \left(1 + \zeta^{-2\gamma}\right) \ln \left(\frac{\vec{k}_h^2}{\mu_F^2 \alpha_h^2}\right) - \beta_0 \frac{2}{2} \ln \left(\frac{\vec{k}_h^2 x^2 \zeta^2}{\mu_R^2 \alpha_h^2}\right) \\
&+ \delta(1 - \zeta) \left[C_A \ln \left(\frac{s_0 \alpha_h^2}{\vec{k}_h^2 x^2}\right) \chi(n, \gamma) - C_A \left(\frac{67}{18} - \frac{\pi^2}{2}\right) + \frac{5}{9} n_f\right] \\
&+ \frac{C_A}{2} \left(\psi' \left(1 + \gamma + \frac{n}{2}\right) - \psi' \left(\frac{n}{2} - \gamma\right) - \chi^2(n, \gamma)\right) \\
&+ C_A \left(\frac{1}{\zeta} + \frac{1}{(1 - \zeta)_+} - 2 + \zeta \bar{\zeta}\right) \chi(n, \gamma)(1 + \zeta^{-2\gamma}) - 2(1 + 2\zeta^{-2\gamma}) \ln \zeta + \frac{\bar{\zeta}^2}{\zeta^2} I_2\right) \\
&+ 2 C_A (1 + \zeta^{-2\gamma}) \left(\left(\frac{1}{\zeta} - 2 + \zeta \bar{\zeta}\right) \ln \bar{\zeta} + \left(\frac{\ln(1 - \zeta)}{1 - \zeta}\right)_+\right),
\end{align*}
\]

\[
\begin{align*}
C_{gq}(x, \zeta) &= P_{gq}(\zeta) \left(\frac{C_F}{C_A} + \zeta^{-2\gamma}\right) \ln \left(\frac{\vec{k}_h^2 x^2 \zeta^2}{\mu_F^2 \alpha_h^2}\right) + 2 \zeta \bar{\zeta} T_R \left(\frac{C_f}{C_A} + \zeta^{-2\gamma}\right) \\
&+ P_{gq}(\zeta) \left(\frac{C_F}{C_A} \chi(n, \gamma) + 2 \zeta^{-2\gamma} \ln \bar{\zeta} + \frac{\bar{\zeta}}{\zeta} I_3\right),
\end{align*}
\tag{79}
\]
\[ C_{qg}(x, \zeta) = P_{qg}(\zeta) \left( \frac{C_A}{C_F} + \zeta^{-2\gamma} \right) \ln \left( \frac{\vec{k}_h^2 x^2 \zeta^2}{\mu_F^2 \alpha_h^2} \right) + \zeta \left( C_F \zeta^{-2\gamma} + C_A \right) \] (80)

\[ + \frac{1 + \bar{\zeta}^2}{\zeta} \left[ C_F \zeta^{-2\gamma} (\chi(n, \gamma) - 2 \ln \zeta) + 2 C_A \ln \frac{\zeta}{\bar{\zeta}} + \bar{\zeta} I_1 \right] , \]

\[ C_{qg}(x, \zeta) = P_{qg}(\zeta) \left( 1 + \zeta^{-2\gamma} \right) \ln \left( \frac{\vec{k}_h^2 x^2 \zeta^2}{\mu_F^2 \alpha_h^2} \right) - \frac{\beta_0}{2} \ln \left( \frac{\vec{k}_h^2 x^2 \zeta^2}{\mu_R^2 \alpha_h^2} \right) \] (81)

\[ + \delta(1 - \zeta) \left[ C_A \ln \left( \frac{s_0 \alpha_h^2}{\vec{k}_h^2 x^2} \right) \chi(n, \gamma) + C_A \left( \frac{85}{18} + \frac{\pi^2}{2} \right) - \frac{5}{9} n_f - 8 C_F \right. \]

\[ + \frac{C_A}{2} \left( \psi'(1 + \gamma + \frac{n}{2}) - \psi'(\frac{n}{2} - \gamma) - \chi^2(n, \gamma) \right)) \] \[ + C_F \bar{\zeta} (1 + \zeta^{-2\gamma}) \]

\[ + (1 + \zeta^2) \left[ C_A(1 + \zeta^{-2\gamma}) \frac{\chi(n, \gamma)}{2(1 - \zeta)_+} + (C_A - 2 C_F(1 + \zeta^{-2\gamma})) \frac{\ln \zeta}{1 - \zeta} \right] \]

\[ + \left( C_F - \frac{C_A}{2} \right)(1 + \zeta^2) \left[ 2(1 + \zeta^{-2\gamma}) \left( \frac{\ln(1 - \zeta)}{1 - \zeta} \right)_+ + \bar{\zeta} I_2 \right] . \]

For the \( I_{1,2,3} \) functions we obtain the following results:

\[ I_2 = \frac{\zeta^2}{\bar{\zeta}^2} \left[ \zeta \left( \frac{2 F_1(1, 1 + \gamma - \frac{n}{2}, 2 + \gamma - \frac{n}{2}, \zeta)}{\frac{n}{2} - \gamma - 1} - \frac{2 F_1(1, 1 + \gamma + \frac{n}{2}, 2 + \gamma + \frac{n}{2}, \zeta)}{\frac{n}{2} + \gamma + 1} \right) \right. \] (82)

\[ + \zeta^{-2\gamma} \left( \frac{2 F_1(1, -\gamma - \frac{n}{2}, 1 - \gamma - \frac{n}{2}, \zeta)}{\frac{n}{2} + \gamma} - \frac{2 F_1(1, -\gamma + \frac{n}{2}, 1 - \gamma + \frac{n}{2}, \zeta)}{\frac{n}{2} - \gamma} \right) \]

\[ + (1 + \zeta^{-2\gamma}) (\chi(n, \gamma) - 2 \ln \bar{\zeta}) + 2 \ln \zeta \] ,

\[ I_1 = \frac{\bar{\zeta}}{2 \zeta} I_2 + \frac{\zeta}{\bar{\zeta}} \left[ \ln \zeta + \frac{1 - \zeta^{-2\gamma}}{2} (\chi(n, \gamma) - 2 \ln \bar{\zeta}) \right] , \] (83)

\[ I_3 = \frac{\bar{\zeta}}{2 \zeta} I_2 - \frac{\zeta}{\bar{\zeta}} \left[ \ln \zeta + \frac{1 - \zeta^{-2\gamma}}{2} (\chi(n, \gamma) - 2 \ln \bar{\zeta}) \right] . \] (84)

Using the following property of the hypergeometric function,

\[ 2 F_1(1, a, a + 1, \zeta) = a (\psi(1) - \psi(a) - \ln \bar{\zeta}) + \mathcal{O}(\bar{\zeta} \ln \bar{\zeta}) , \]

one can easily see that

\[ I_2 = \mathcal{O}(\ln \bar{\zeta}) , \quad I_1 = \mathcal{O}(\ln \bar{\zeta}) , \quad I_3 = \mathcal{O}(\ln \bar{\zeta}) , \]

which implies that the integral over \( \zeta \) in (77) is convergent in the upper limit.
5 Summary

In this paper we have calculated the NLO vertex (impact factor) for the forward production of an identified hadron from an incoming quark or gluon, emitted by a proton. This is a necessary ingredient for the calculation of the hard inclusive production of a pair of rapidity-separated identified hadrons in proton collisions \([1]\). This process, similarly to the production of Mueller-Navelet jets, can be studied at the LHC hadron collider.

Another natural application of the obtained identified hadron production vertex could be the NLA BFKL description of inclusive forward hadron production process in DIS,

\[
e(p_1) + p(p_2) \rightarrow h(k) + X ,
\]

where in the low-\(x\) event the hadron \(h(k)\) with high transverse momentum is detected in the fragmentation region of incoming proton \(p(p_2)\). Data for such reaction in the case of forward \(\pi^0\)-production were published by the H1 collaboration at HERA \([22]\).

At the basis of our calculation of the hard part of the vertex was the definition of NLO BFKL parton impact factors; then the collinear factorization with the PDFs of the incoming partons and with the FF for the production of the identified hadron (in the \(\overline{\text{MS}}\) scheme) was suitably considered.

We have presented our result for the vertex in the so called \((\nu, n)\)-representation, which is the most convenient one in view of the numerical determination of the cross section for the production of a pair of rapidity-separated identified hadrons along the same lines as in Ref. \([16]\).

We have explicitly verified that soft and virtual infrared divergences cancel each other, whereas the infrared collinear ones are compensated by the PDFs’ and FFs’ renormalization counterterms, the remaining ultraviolet divergences being taken care of by the renormalization of the QCD coupling.

In our approach the energy scale \(s_0\) is an arbitrary parameter, that need not be fixed at any definite scale. The dependence on \(s_0\) will disappear in the next-to-leading logarithmic approximation in any physical cross section in which the identified hadron production vertices are used. Indeed, our result for the NLO vertex, given by Eqs. \((77)-(81)\), contains contributions \(\sim \ln(s_0)\) and these terms are proportional to the LO quark and gluon vertices multiplied by the BFKL kernel eigenvalue \(\chi(n, \nu)\). This fact guarantees the independence of the identified hadrons \([1]\) or single hadron \([85]\) cross section on \(s_0\) within the next-to-leading logarithmic approximation. However, the dependence on this energy scale will survive in terms beyond this approximation and will provide a parameter to be optimized with the method adopted in Refs. \([16]\).
Acknowledgements

D.I. thanks the Dipartimento di Fisica dell’Università della Calabria and the Istituto Nazionale di Fisica Nucleare (INFN), Gruppo collegato di Cosenza, for the warm hospitality and the financial support. This work was also supported in part by the grants and RFBR-11-02-00242 and NSh-3810.2010.2.
A Useful integrals

We give here some useful integrals:

\[ \int \frac{d^{2 + 2} \vec{k}' \left( \frac{1}{\vec{k}'^2 + (\vec{k}' - \vec{k})^2} \right)}{\vec{k}'^2} = \frac{1}{2} \int \frac{d^{2 + 2} \vec{k}' \left( \frac{1}{\vec{k}'^2 + (\vec{k}' - \vec{k})^2} \right)}{\vec{k}'(\vec{k}' - \vec{k})^2} = \pi^{1 + \epsilon} \left( \vec{k}'^2 \right)^{\epsilon - 1} \frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{\epsilon \Gamma(1 + 2 \epsilon)}, \]

(A.1)

\[ \int \frac{d^{2 + 2} \vec{k}' \left( \vec{k}'^2 \right)^\alpha}{(\vec{k}' - \vec{k})^2} = \pi^{1 + \epsilon} \left( \vec{k}'^2 \right)^{\alpha + \epsilon} \frac{\Gamma(-\epsilon - \alpha) \Gamma(\epsilon) \Gamma(1 + \epsilon + \alpha)}{\Gamma(-\alpha) \Gamma(1 + \alpha + 2 \epsilon)}. \]

(A.2)

\[ \int_0^{2\pi} d\phi \frac{\cos n\phi}{a^2 - 2ab \cos \phi + b^2} = \frac{2\pi}{b^2 - a^2} \left( \frac{a}{b} \right)^n, \quad a < b \]

(A.3)

In the integrals below, \( \vec{l}^2 = 0 \) is assumed:

\[ \int \frac{d^{2 + 2} \vec{k}' \left( \vec{k}'^2 \vec{k}' \cdot \vec{l} \right)^\alpha}{(\vec{k} - \vec{k}')^2} = \pi^{1 + \epsilon} \left( \vec{k}' \cdot \vec{l} \right)^\beta \left( \vec{k}'^2 \right)^{\alpha + \epsilon} \frac{\Gamma(-\alpha - \epsilon) \Gamma(\epsilon) \Gamma(1 + \epsilon + \alpha + \beta)}{\Gamma(-\alpha) \Gamma(1 + \alpha + \beta + 2 \epsilon)}, \]

(A.4)

\[ \int \frac{d^{2 + 2} \vec{k}' \ln(\vec{k} - \vec{k}')^2 \left( \vec{k}'^2 \vec{k}' \cdot \vec{l} \right)^\beta}{(\vec{k} - \vec{k}')^2} \times \frac{\Gamma(-\alpha - \epsilon) \Gamma(\epsilon) \Gamma(1 + \epsilon + \alpha + \beta)}{\Gamma(-\alpha) \Gamma(1 + \alpha + \beta + 2 \epsilon)} \]

\[ \times \left\{ \ln \vec{k}'^2 + \psi(\epsilon) + \psi(1) - \psi(-\alpha - \epsilon) - \psi(1 + \alpha + \beta + 2 \epsilon) \right\}. \]

(A.5)

References

[1] V.S. Fadin, E.A. Kuraev, L.N. Lipatov, Phys. Lett. B 60 (1975) 50; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Zh. Eksp. Teor. Fiz. 71 (1976) 840 [Sov. Phys. JETP 44 (1976) 443]; 72 (1977) 377 [45 (1977) 199]; Ya.Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.

[2] V.S. Fadin and R. Fiore, Phys. Lett. B 440 (1998) 359.

[3] V.S. Fadin, L.N. Lipatov, Phys. Lett. B 429 (1998) 127.

[4] G. Camici and M. Ciafaloni, Phys. Lett. B 430 (1998) 349.

[5] V.S. Fadin and R. Fiore, Phys. Lett. B 610 (2005) 61 [Erratum-ibid. 621 (2005) 61]; Phys. Rev. D 72 (2005) 014018.
[6] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Rev. D 61 (2000) 094005.
[7] V.S. Fadin, R. Fiore, M.I. Kotsky and A. Papa, Phys. Rev. D 61 (2000) 094006.
[8] M. Ciafaloni and D. Colferai, Nucl. Phys. B 538 (1999) 187.
[9] J. Bartels, D. Colferai and G.P. Vacca, Eur. Phys. J. C 24 (2002) 83; Eur. Phys. J. C 29 (2003) 235.
[10] F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa and A. Perri, JHEP 1202 (2012) 101.
[11] D.Yu. Ivanov and A. Papa, JHEP 1205 (2012) 086.
[12] J. Bartels, S. Gieseke and C. F. Qiao, Phys. Rev. D 63 (2001) 056014 [Erratum-ibid. 65 (2002) 079902]; J. Bartels, S. Gieseke and A. Kyrieleis, Phys. Rev. D 65 (2002) 014006; J. Bartels, D. Colferai, S. Gieseke and A. Kyrieleis, Phys. Rev. D 66 (2002) 094017; J. Bartels and A. Kyrieleis, Phys. Rev. D 70 (2004) 114003; V.S. Fadin, D.Yu. Ivanov and M.I. Kotsky, Phys. Atom. Nucl. 65 (2002) 1513 [Yad. Fiz. 65 (2002) 1551]; Nucl. Phys. B 658 (2003) 156.
[13] D.Yu. Ivanov, M.I. Kotsky and A. Papa, Eur. Phys. J. C 38 (2004) 195.
[14] I. Balitsky and G.A. Chirilli, Phys. Rev. D 83 (2011) 031502.
[15] J. Bartels, A. De Roeck and H. Lotter, Phys. Lett. B 389 (1996) 742; S.J. Brodsky, F. Hautmann and D.E. Soper, Phys. Rev. D 56 (1997) 6957; Phys. Rev. Lett. 78 (1997) 803 [Erratum-ibid. 79 (1997) 3544].
[16] D.Yu. Ivanov and A. Papa, Nucl. Phys. B 732 (2006) 183; Eur. Phys. J. C 49 (2007) 947; F. Caporale, A. Papa and A. Sabio Vera, Eur. Phys. J. C 53 (2008) 525.
[17] D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon, JHEP 1012 (2010) 026.
[18] A.H. Mueller, H. Navelet, Nucl. Phys. B 282 (1987) 727.
[19] V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438; G. Altarelli, G. Parisi, Nucl. Phys. B 126 (1977) 298; Y.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.
[20] R. Kirschner, M. Segond, Eur. Phys. J. C 68 (2010) 425.
[21] A.V. Kotikov and L.N. Lipatov, Nucl. Phys. B 582 (2000) 19.
[22] A. Aktas et al. [H1 Collaboration], Eur. Phys. J. C 36 (2004) 441.