An Efficient Algorithm for Frequency Estimation of Sinusoid Signal Based on Improved Quinn

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Abstract. To balance the performance and complexity for frequency estimation of sinusoid wave using DFT-based methods, an efficient estimation algorithm based on improved Quinn is proposed. The amplitude and phase information are used to interpolate which ensures the accuracy of the frequency shift direction. A method of piecewise frequency shift is performed, and it has the characteristics of higher precision and lower computational costs. Simulation results show that the judgment of direction factor is more correct using Quinn, the Root Mean Square Error (RMSE) of this method is closer to Cramer-Rao Lower Bound (CRLB) under the conditions of different frequencies and SNRs comparing with Quinn and other improved algorithms, and it can attain remarkable decrease of computational cost which facilitates the algorithm implementation.

1. Introduction
The accuracy and efficiency of the carrier frequency estimation of radar signal in the complex electromagnetic environments directly affect the performance and efficiency of radar signal sorting, target recognition, and jamming guidance. The algorithms for frequency estimation of sinusoid signal are mainly divided into autocorrelation function estimation [1-2] and spectrum analysis method based on Discrete Fourier Transform (DFT). The latter is more suitable for real-time signal frequency estimation by the Fast Fourier Transform (FFT) theory and the development of high-speed FPGA devices. Therefore, this paper mainly discusses the frequency estimation method based on DFT.

Rife [3] and Quinn [4] are the two typical spectrum analysis algorithms based on DFT. Rife algorithm uses the two maximum discrete spectrum lines after FFT to estimate the frequency. However, when the estimated frequency is near one of the discrete frequency lines, the estimation accuracy is reduced obviously. An improved Rife algorithm [5-7], based on frequency shift modified is proposed. The main idea is to move the signal frequency to the middle of the adjacent discrete frequencies to improve the accuracy. However, when the Signal-to-Noise Ratio (SNR) is reduced, the probability of misjudgment of the frequency shift direction will increase, which causes the loss of precision. Quinn algorithm uses amplitude and phase information for interpolation operation, which can ensure the accuracy of frequency shift direction when the estimated frequency is near the quantized frequency point and the SNR is low. Quinn algorithm is improved [8,9] and the Root Mean Square Error (RMSE) can approach Cramer-Rao Lower Bound (CRLB) under different SNRs. However, the computation amount is still large, which reduces the use of performance.
This paper analyzes and compares the accuracy of Quinn and Rife algorithms in frequency shift direction judgment with low SNRs, and proposes an efficient sinusoid signal frequency estimation algorithm using piecewise frequency shift based on improved Quinn [9]. Comparing with the improved Quinn algorithm, the RMSE of frequency estimation is closed, and the algorithm complexity is reduced.

2. The Principle of RIFE and QUINN Algorithm

2.1. Rife Algorithm

Assuming that the complex sinusoid signal sequence contaminated by Gaussian white noise can be expressed as

\[ x(n) = A \exp\left(j(2\pi f_0 n \Delta t + \theta) + w(n)\right) \]

\( n = 0, 1, \ldots, N-1 \) \hspace{1cm} (1)

Where, \( A \) is the amplitude of \( x(n) \), \( f_0 \) is the frequency, \( \theta \) is the initial phase, \( \Delta t \) is the sample interval, \( N \) is the number of samples, and \( w(n) \) is a zero-mean Gaussian white noise with the variance of \( \sigma^2 \). Without considering the noise, DFT is carried out by \( x(n) \)

\[ X(k) = \sum_{n=0}^{N-1} x(n) \exp\left(-j \frac{2\pi}{N} kn\right) \]

\( k = 0, 1, \ldots, N-1 \) \hspace{1cm} (2)

Where, \( |X(k_0)| \) expresses the amplitude of the maximum spectrum line of \( X(k) \), \( |X(k_0 - 1)| \) and \( |X(k_0 + 1)| \) are the adjacent spectrum lines on the left and right sides of \( |X(k_0)| \). \( a \) is introduced as a direction factor. When \( |X(k_0 + 1)| > |X(k_0 - 1)| \), \( a = 1 \), else \( a = -1 \). The expression of frequency estimation of Rife algorithm is expressed as

\[ \hat{f}_a = \Delta f \left( k_0 + a \frac{|X(k_0 + a)|}{|X(k_0 + a)| + |X(k_0)|} \right) \]

\hspace{1cm} (3)

Where, \( \Delta f \) is the frequency resolution and equals \( f_0 / N \). \( f_0 \) is the sample frequency and equals \( 1 / \Delta t \). In the condition of low SNRs, if the real signal frequency is located near a discrete frequency, the influence of noise makes \( a \) mistaken, and the error will be further increased.

To figure out this problem, an improved rife algorithm is proposed in [5], i.e. I-Rife. The main idea is to use the maximum spectrum line and the middle position of the two adjacent spectrum lines, i.e. \( |X(k_0 \pm 0.5)| \) instead of \( |X(k_0 \pm 1)| \), to determine \( a \). This method effectively increases the judgment accuracy of \( a \). Meanwhile, the frequency shift factor \( \delta_k \) is introduced to move the original signal frequency to the middle of the two quantized frequencies, so that the high precision frequency estimation can be obtained in the whole frequency band. The disadvantage is that the algorithm complexity increases.

2.2. Quinn Algorithm

Quinn algorithm fully considers the important influence of the direction factor on the frequency shift, and uses the amplitude and phase information for interpolation calculation. It improves the misjudgment of \( a \) by Rife algorithm under the conditions that the SNR is low and the real signal frequency is near a quantized frequency. The frequency estimation equation of Quinn algorithm is expressed as
\[ \hat{f}_0 = \Delta f (k_0 + \delta) \] (4)

Where, \( \delta \) is the frequency modification factor and expresses as
\[ \delta = \left\{ \begin{array}{ll}
\delta_2, & \delta_2 > 0, \delta_1 > 0 \\
\delta_1, & \text{others}
\end{array} \right. \] (5)

Where, \( \delta_1 \) and \( \delta_2 \) are expressed as
\[ \left\{ \begin{array}{l}
\delta_1 = \beta_1 / (1 - \beta_1) \\
\delta_2 = \beta_2 / (\beta_2 - 1)
\end{array} \right. \] (6)

Where, \( \beta_1 \) and \( \beta_2 \) are calculated by quantized spectral lines and expressed as
\[ \left\{ \begin{array}{l}
\beta_1 = \text{Re} \left\{ X (k_0 - 1) / X (k_0) \right\} \\
\beta_2 = \text{Re} \left\{ X (k_0 + 1) / X (k_0) \right\}
\end{array} \right. \] (7)

Quinn algorithm improves the judgment of direction factor, but the error also increases if \(|\delta|\) is closed to 0. The I-Quinn algorithm [9] also uses the frequency shift method by moving the measured frequency to the middle of quantized frequencies to correct the error, but the computation amount is large. Therefore, this paper proposes an efficient estimation algorithm based on I-Quinn, which can reach the high accuracy of I-Quinn algorithm and reduce algorithm complexity obviously.

3. Improved Efficient Estimation Algorithm

To meet the requirements of high-speed signal processing and reduce the algorithm complexity, an efficient Quinn algorithm based on piecewise frequency shift is proposed in this paper. By setting thresholds and selecting fixed frequency shift values in different ranges, the computing amount of complex multiplication can be effectively reduced. The specific steps of the algorithm are as follows:

1. DFT is carried out by \( x(n) \) to get the \( X (k) \). Calculate the frequency modification factor \( \delta \) by formula (5), (6), (7).

2. Calculate the direction factor \( a = \text{sign}(\delta) \).

3. Select four frequency shift ranges A, B, C, and D according to the value range of \(|\delta|\), and make frequency shift factor \( \delta_k \) different values in different ranges that show as follows:
   - A: \( 0.35 \leq |\delta| \leq 0.5 \), \( \delta_k = 0 \);
   - B: \( 0.25 \leq |\delta| \leq 0.35 \), \( \delta_k = 0.2 \);
   - C: \( 0.15 \leq |\delta| \leq 0.25 \), \( \delta_k = 0.3 \);
   - D: \( 0 \leq |\delta| \leq 0.15 \), \( \delta_k = 0.5 \);

4. Calculate \( x_1(n) \) by frequency shift of \( x(n) \)
\[ x_1(n) = x(n) \exp \left( j \frac{2\pi n}{N} a \delta_k \right) \] (8)

5. DFT is carried out by \( x_1(n) \) to get the position of maximum amplitude of spectrum line \( k \), and frequency modification factor \( \delta_k \). Thus, the final frequency estimation expression is as follows:
\[ \hat{f}_o = \Delta f (k_i + \delta_i - a\delta_i) \] (9)

The core idea of this algorithm is the piecewise frequency shift processing. In range A, the frequency of the measured signal located in the middle of two quantized frequencies. In this case, Quinn algorithm is directly used to estimate the frequency; In range D, the signal frequency is near the maximum quantized frequency. When the SNR is low, to prevent the quadric error caused by the wrong frequency shift direction, \( \delta_i \) is directly set as 0.5. No matter whether the frequency shift direction is correct or not, the frequency modification factor \( \delta_i \) of \( x_i(n) \) always falls in the range A; In ranges B and C, the correct rate of direction judgment is close to 100\%, Thus, make \( \delta_i \) equal 0.2 and 0.3 respectively to ensure that the frequency points in these ranges move to the middle of quantized frequencies to the greatest extent. After setting the four ranges, the frequency of \( x_i(n) \) always locates in region A, and it reduces the interference of the second largest spectrum line caused by noise and improves the accuracy.

Considering the algorithm complexity, firstly, when the value of \( \delta_i \) is fixed, \( \exp(j2\pi n a\delta_i / N) \) can be stored in the RAM in advance and used by a lookup table, that reduces complex multiplication. Secondly, because of \( \delta_i \leq 0.5 \), The position of the maximum spectrum line of \( x_i(n) \) is still the one of \( k_n, k_o, k_o + 1 \). Thus, DFT is no need for \( x_i(n) \) to calculate \( k_n \).

4. Performance Simulation

4.1. Direction Factor Judgment Analysis

The accuracy of the direction factor directly affects the accuracy of the algorithm, which is more serious than the error of interpolation. In this section, we simulate and analyze the direction judgment accuracy of three algorithms: Rife, Quinn, and I-Rife.

Assuming that the signal frequency is between 400Hz and 402Hz, and 10000 frequency points are selected at the same interval for direction judgment and analysis. The samples \( N \) equals 1000 at each frequency point, \( f_s \) is 2048Hz, SNR is -5dB. According to the calculation, the two quantized frequency points after FFT are 400Hz and 402Hz. The simulation results of \( \delta \) by the three algorithms are shown in Figure 1, and the quantitative analysis results of direction accuracy and RMSE are shown in Table I.

![Figure 1. Simulation results of modification factors of three algorithms](image)
Table 1. Quantitative analysis results

| quantitative analysis | Algorithm       |             |             |
|----------------------|-----------------|-------------|-------------|
|                      | Rife            | I-Rife      | Quinn       |
| Accuracy(%)          | 80.48           | 94.12       | 93.58       |
| RMSE                 | 0.1907          | 0.1613      | 0.1610      |

4.2. Estimation Error in Different Frequencies
To verify the performance of this algorithm in different frequencies, we choose Rife, Quinn, I-Rife and I-Quinn algorithms for comparison and analysis with the algorithm proposed in this paper. In a frequency resolution of 401Hz ~ 403Hz, select 50 frequency points at the same interval, and carry out 1000 times of Monte Carlo simulations for each frequency point. The simulation results with the SNRs from -5dB to 0dB are respectively shown in Figure 2 and Figure 3.

Figure 2. Error estimation of different algorithms(SNR = -5dB)

Figure 3. Error estimation of different algorithms(SNR = 0dB)

It can be noticed from Figure 2 and Figure 3 that the performance of the algorithm in this paper is better than Rife, Quinn, I-Rife, and close to I-Quinn in the whole frequency resolution. It can still maintain high estimation accuracy near the quantized frequency points.
4.3. Estimation Error in Different SNR

The estimation error in different SNRs can reflect the adaptability of an algorithm. In this section, the above five algorithms are simulated under different SNRs. Select SNRs from 10dB to 5dB with the step of 1dB. In each SNR, the signal frequency is randomly selected in a frequency resolution of 401Hz ~ 403Hz with uniform distribution. 2000 times of Monte Carlo simulations are carried out, and the results are shown in Figure 4.

It can be noticed from Figure 4 that the algorithm in this paper and I-Quinn are closest to CRLB with the whole SNR range, and with the increase of SNR, the distance from CRLB is smaller. The performance of this algorithm is better than I-Rife because of the high accuracy of direction factor judgment under the condition of low SNRs.

![Figure 4. Error estimation in different SNR of different algorithms](image)

4.4. Algorithm Complexity Analysis

In the aspect of algorithm complexity, the five algorithms are analyzed and compared. Considering that complex multiplication has more calculation amount than complex addition, only complex multiplication is counted for analysis and comparison.

| Algorithm Complexity Analysis | Complex multiplication amount | Quantitative analysis \((N=128, \mu=0.375)\) | Increasing proportion |
|-----------------------------|-----------------------------|---------------------------------|---------------------|
| Rife                        | \((N/2)ibN\)                 | 448                             | 0%                  |
| Quinn                       | \((N/2)ibN + 2\)             | 450                             | About 0%            |
| I-Rife                      | \((N/2)ibN + 4i\)            | 960                             | 114.3%              |
| I-Quinn                     | \((N/2)ibN + 6\mu iN + 2\)   | 738                             | 64.7%               |
| This Paper                  | \((N/2)ibN + 4\mu iN + 2\)   | 629.2                           | 40.4%               |

Assuming that the number of samples \(N\) is an integer power of 2, the \(N\)-point FFT need \((N/2)ibN\) complex multiplication. Calculating a discrete frequency point need \(N\) complex multiplication. TABLE II compares the computation amount of all algorithms in detail. Quinn has twice complex multiplication more than Rife, and it can be ignored when \(N\) is large; I-Rife needs to increase \(4N\) complex multiplication due to four discrete spectrum lines; I-Quinn needs to adds \(6\mu iN[9]\) complex multiplication to calculate \(x_i(n)\) and its maximum spectrum line; The algorithm in this paper adopts piecewise frequency shift...
processing. The probability of signal falling in the range of B, C and D is 0.7. Because the values of fixed frequency shift are stored in RAM beforehand, the complex multiplication added by the algorithm is $0.7 \times 2N = 1.4N$. When $N$ is 128, the algorithm in this paper only increases by 40.4% compared with Rife, which is lower than 64.7% of I-Quinn. With the approximate estimation accuracy, the hardware resource requirements and operation efficiency of this algorithm are relatively better.

5. Conclusion
After analyzing the accuracy of direction factor judgment, this paper proposes an efficient sinusoid signal frequency estimation algorithm based on improved Quinn. Firstly, it can use the amplitude and phase information to judge the direction of interpolation and improve the accuracy. Secondly, A method of piecewise frequency shift is proposed, which can reduce the algorithm complexity and reach the approximate precision compared with I-Quinn. Simulation results show that the estimation RMSE of this algorithm is close to CRLB in different frequency points or SNRs, and the computation amount is less, which is easy to be realized in engineering application.

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