Simplest Radiative Dirac Neutrino Mass Models

Shaikh Saad

Department of Physics, Oklahoma State University, Stillwater, OK 74078, USA

Abstract

If neutrinos are Dirac particles, their right-handed partners must be present in the theory. Once introduced in the Standard Model (SM), the difference between the baryon number $B$ and the lepton number $L$ can be promoted to a local $U(1)_{B-L}$ symmetry since the corresponding gauge anomalies can be canceled by the right-handed neutrinos. Furthermore, the extremely small neutrino mass can be explained naturally if it is originated from the quantum correction. In this work, we propose simplest models of radiative Dirac neutrino mass using only $U(1)_{B-L}$ symmetry, and without introducing additional fermions and without imposing ad hoc symmetries. In this simple framework, we provide minimal models where Dirac neutrino mass appears at the (i) one-loop, (ii) two-loop and (iii) three-loop. We show that the minimal one-loop model requires three beyond SM scalar multiplets, whereas minimal two-loop and three-loop models require five. The presented two-loop and three-loop Dirac mass models have not appeared in the literature before.

* E-mail: shaikh.saad@okstate.edu
1 Introduction

Neutrinos are massless in the Standard Model (SM), however, experimentally [1–4] it is well established now that this cannot be true. Hence, the SM needs to be extended to incorporate neutrino mass. There exist vast literature on neutrino mass generation, among them, radiative \(^1\) generation of neutrino mass is one of the most widely studied scenario. For models belonging to this class, two most popular mechanisms are: the one-loop Zee-mechanism [6] and the two-loop Zee-Babu mechanism [7,8]. In addition to yet unknown mechanism behind the neutrino mass generation, another greatest mysteries of particle physics is whether the neutrino is Dirac or Majorana like particle. There is no theoretical preference and the issue needs to be settled experimentally. Great amount of experimental effort [9–12] is going on to come to a possible settlement. However, since all the fermions in the SM acquire Dirac mass, it is quite natural to expect that neutrino is also Dirac in nature. Theoretically the nature of the neutrinos may be closely related to the \(U(1)_{B-L}\) symmetry and the way it breaks down, for details see [13,14]. Despite the vast literature on neutrino mass generation of the Majorana type \(^2\), just recently Dirac neutrinos have gained a lot of attention to the particle physics community [16–30].

If neutrinos are Dirac in nature, then the right-handed partners must be present into the theory. The main difficulty faced while constructing a radiative Dirac neutrino mass model is to introduce ad hoc symmetries beyond the SM (BSM) to forbid both the tree-level Dirac mass and the Majorana mass terms. Mechanisms implemented to forbid these terms not only introduces complexity in the theory but also fails to explain the reason behind the presence of these symmetries in the first place. Most of the constructions in the literature use ad hoc discrete symmetries, except only few recent works [24,29,30] that are based on \(U(1)_{B-L}\) symmetry \(^3\). However, in these works, even though \(U(1)_{B-L}\) is the only symmetry used to forbid the unwanted terms, but extra fermion multiplets BSM are introduced to make the theory anomaly free, to generate Dirac neutrino mass, and to have dark matter (DM) candidate. For most of the cases, new fermions come with more than one generation. Furthermore, the presence of few BSM scalar multiplets are also necessary to complete the loop diagrams.

Instead, to reduce the complexities in the theory, we aim to search for the simplest models of Dirac neutrino mass without the presence of the additional fermions. Even though in this work we are not interested in the ultraviolet completion, but the motivation behind this can be justified as follows. The well motivated grand unified theory (GUT) based

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\(^1\)The first radiative neutrino mass model was proposed for Dirac neutrinos [5].

\(^2\) For a recent review on Majorana mass models see [15].

\(^3\) For Dirac neutrino mass models based on \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) symmetry see [5,31–39], on \(U(1)_L \times U(1)_R\) symmetry see [40] and on \(SU(3)_L \times U(1)_{X}\) symmetry see [41–43].
on $SO(10)$ gauge group, in its minimal realization contains only the SM fermions and the right-handed neutrinos. The other widely studied GUT which is based on $SU(5)$ gauge group contains no extra fermions beyond the SM, however, right-handed neutrinos can be trivially added since they are the gauge singlets. In this work we explore the possibilities of building Dirac neutrino mass models using the baryon number minus the lepton number $U(1)_{B-L}$ symmetry offered by the presence of the right-handed neutrinos in the theory to form Dirac pair. Our set-up is the simplest in the sense that many of the complexities can be avoided because, no additional fermions are introduced and no ad hoc symmetries are imposed to generate radiative Dirac neutrino mass. Within this framework, we propose minimal models of Dirac neutrino mass at the (i) one-loop, (ii) two-loop and (iii) three-loop level. We show that within the minimal scenario, one-loop realization requires three BSM scalar multiplets whereas two-loop and three-loop realizations require five multiplets. To the best of the author’s knowledge, the presented two-loop and three-loop Dirac neutrino mass models in this work are not realized in the literature before. Though the presence of DM is not required in this set-up, but the possible DM candidate in this class of models is briefly discussed.

In short, the main novelties of our work is to generate radiative Dirac neutrino mass with only $U(1)_{B-L}$ symmetry, no ad hoc symmetries are imposed and furthermore, other than the right-handed neutrinos that are required to make $U(1)_{B-L}$ anomaly free, no extra fermions are added to the SM.

\section{Framework}

The SM has accidental baryon number $U(1)_B$ and lepton number $U(1)_L$ conservation symmetries that are anomalous and cannot be gauged without the additions of quite a few number of new fermions into the theory. However, the difference between the baryon number and the lepton number symmetry $U(1)_{B-L}$ can be made non-anomalous by adding the right-handed partners, $\nu_{Ri}$ of the SM neutrinos, $\nu_{Li}$. Anomaly cancellation demands one of the two solutions, the lepton number of the right-handed neutrinos to be vector charges: $L_{1,2,3} = \{-1, -1, -1\}$ or chiral charges: $L_{1,2,3} = \{+5, -4, -4\}$ under $U(1)_{B-L}$ \cite{44-46}. Since all the charged fermions in the SM acquire Dirac mass, it is quite natural to expect that $\nu_L$ and $\nu_R$ pair up to form four-component Dirac fermion and as a result the neutrinos also receive Dirac mass.

Since the $U(1)_{B-L}$ symmetry is anomaly free, it is natural to gauge this symmetry. Also unlike discrete and global symmetries, gauge symmetries are known to be respected by gravitational interactions. This is why we prefer to gauge $U(1)_{B-L}$, however, our analysis remains valid for both global and gauge $U(1)_{B-L}$. 


Now, if the solution \( L_{1,2,3} = \{-1, -1, -1\} \) as aforementioned is realized, then just like all the charged fermions, neutrinos receive Dirac mass at the tree-level when the EW symmetry is broken by the SM Higgs VEV from the Yukawa term: \( \mathcal{L}_Y \supset y' \bar{L}_L \nu_R \tilde{H} \) which is gauge invariant, where \( L_L = (\nu_L \ell_L)^T, H = (H^+ H^0)^T \) and \( \tilde{H} = \epsilon H^* \) (\( \epsilon \) is the 2-index Levi-civita tensor). However, this tree-level Dirac mass realization requires the corresponding Yukawa couplings to be of the order of \( y' \sim 10^{-11} \). This may not be a natural solution. However, small Dirac mass for neutrinos can be realized naturally if the other anomaly free solution \( L_{1,2,3} = \{+5, -4, -4\} \) is considered [16], this choice automatically forbids the tree-level Dirac mass term. In our framework, we adopt this chiral charge solution, hence tree-level Dirac mass is forbidden and neutrinos receive Dirac mass from the quantum corrections, which naturally explains the smallness of the observed neutrino mass. Furthermore, we do not require ad hoc symmetries to forbid the Majorana mass terms for \( \nu_L \) and \( \nu_R \), hence our framework is theoretically well motivated. Within this framework, it is worthwhile to search for the minimal models to generate non-zero neutrino mass. The new idea of this paper is to use the chiral anomaly free solution and construct Dirac neutrino mass radiatively without introducing new fermions and without imposing ad hoc symmetries.

In this work we construct simple ways of neutrino mass generation by demanding the following requirements:

- Neutrinos are Dirac type particle.
- Neutrino mass originates from quantum correction.
- No additional fermions are introduced except the three right-handed neutrinos.
- No additional symmetries are imposed except the \( U(1)_{B-L} \) that is automatically offered by the presence of the right-handed neutrinos.

We provide examples of minimal models for one-loop, two-loop and three-loop scenarios. In our framework the lowest order operator to generate radiative Dirac neutrino mass is the following dimension-5 operator:

\[
\mathcal{L}_5 = \frac{h_{ij}}{\Lambda} \bar{L}_i \tilde{H} \nu_R \sigma^0.
\] (2.1)

Here \( \sigma^0 \) is a scalar singlet of the SM and we assume that our theory allows the presence of this operator. Based on this dimension-5 operator we build minimal models of one-loop and two-loop Dirac mass for neutrino. In the three-loop neutrino mass model, dimension-7 operator given in Eq. (2.2) is responsible for non-zero neutrino mass.

\[
\mathcal{L}_7 = \frac{h_{ij}}{\Lambda^3} \bar{L}_i \tilde{H} \nu_R \sigma^0 (H^\dagger H).
\] (2.2)
From Eq. (2.1), the topologies that we are interested in can be constructed straightforwardly and are presented in Fig. 1 for one-loop scenario and in Fig. 2 for the case of two-loop. We will use these topologies as a guidance to build minimal models within our set-up. We stress the fact that not all the topologies can satisfy the set of requirements.
| Multiplets | $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ |
|------------|-----------------------------------------|
| Leptons    | $L_L(2, -\frac{1}{2}, -1)$ |
|            | $\ell_R(1, -1, -1)$ |
|            | $\nu_R(1, 0, \{5, -4, -4\})$ |
| Higgs      | $H(2, \frac{1}{2}, 0)$ |

Table I: Quantum numbers of the leptons and SM Higgs doublet.

listed above and we will discuss them case by case. In these figures, the blue colored arrow represents the direction of the iso-spin doublet flow. Each of the different topologies is labeled as: T-x-y-z, where x=I for one-loop (x=II for two-loop), y=F corresponds to the case when the SM Higgs doublet is attached to fermion line (y=H when the SM Higgs doublet is attached to scalar line) and z=i, ii, ... to differentiate among topologies with same x and y labels. The topologies corresponding to the three-loop models will be presented later in the text.

First we would like to determine the BSM scalar multiplets to complete these loop diagrams. To do so, first we look at the structure of the possible fermion bilinears. These will also reveal the multiplets that can give rise to the unwanted tree-level Dirac or Majorana mass terms. We only introduce BSM scalars that are color blind, hence the symmetry that we are interested in is: $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. Under this symmetry, the quantum numbers of the leptons and the SM Higgs doublet are presented in Table I. Since we do not introduce additional fermions, the number of possible fermion bilinears are limited and given by:

- $\mathcal{T}_L \otimes \ell_R \sim (2, -\frac{1}{2}, 0)$
- $\mathcal{T}_L \otimes L_L \sim (1, -1, -2) \oplus (3, -1, -2)$
- $\mathcal{T}_L \otimes \nu_R \sim (2, \frac{1}{2}, +6) \oplus (2, \frac{1}{2}, -3)$
- $\mathcal{T}_R \otimes \ell_R \sim (1, -2, -2)$
- $\mathcal{T}_R \otimes \nu_R \sim (1, -1, +4) \oplus (1, -1, -5)$
- $\mathcal{\bar{T}}_R \otimes \nu_R \sim (1, 0, +10) \oplus (1, 0, +1) \oplus (1, 0, -8)$

From these bilinears, it is clear that to avoid tree-level neutrino Dirac mass term, $(2, -\frac{1}{2}, -6)$ and $(2, -\frac{1}{2}, +3)$ Higgs multiplets cannot be present. If they do, one needs to make sure that the electrically neutral component of the corresponding doublet field does not acquire any VEV, which might be a difficult task to arrange and may require the existence of
some additional discrete symmetries. And similarly, to forbid the Majorana mass terms 
\((1, 0, -10), (1, 0, -1), (1, 0, +8)\) Higgs multiplets are not allowed in our theory.

For our model building purpose, we use the structure of the above bilinears to select the appropriate Higgs multiplets for radiative neutrino mass generation. So the \(U(1)_{B-L}\) symmetry plays the crucial role to forbid all the unwanted terms. As long as this symmetry is preserved, neutrinos are massless at all order of the perturbation theory within our framework. However, due to spontaneously breaking of the \(U(1)_{B-L}\) symmetry, neutrinos receive Dirac mass via quantum correction. We achieve this symmetry breaking by the SM singlet scalar \(\sigma^0\) that carries a non-trivial charge under \(U(1)_{B-L}\). When this field acquires VEV, the dimension-5 operator in Eq. (2.1) generates non-zero neutrino mass at one-loop and two-loop. The lowest dimension for which neutrino mass can be generated is seven of Eq. (2.2) for three-loop scenario. As a result of this symmetry breaking, the imaginary part of \(\sigma^0\) will be eaten up by the corresponding gauge boson \(Z'\). In this framework \(Z'\) can be made sufficiently heavy to evade collider constraints. For low \(Z'\) mass, interesting phenomenology may emerge, however in this work we do not discuss such possibilities rather focus mainly on the ways of neutrino mass generation.

3 Minimal models

In this section we present the models of Dirac neutrino mass at (i) one-loop, (ii) two-loop and (iii) three-loop based on the set-up discussed in the previous section 2.

3.1 One-loop model

For one-loop Dirac neutrino mass generation, there are two different expected topologies depending on the direction of the flow of the iso-spin doublet within our framework as shown in Fig. 1. However, between these two, only model with topology T-I-F-x can be realized. The absence of the T-I-H-x topology is due to not allowing additional fermions BSM, which is one of our requirements. As a result the internal fermion line cannot be completed for the diagrams of type T-I-H-x. However, by relaxing this requirement, topology of this type can be built using \(U(1)_{B-L}\) symmetry [24,29,30].

A very simple model with T-I-F-i topology can be constructed by extending the SM Higgs sector by only three scalars: a SM singlet scalar to break the \(U(1)_{B-L}\) symmetry and two singly charged scalars. The quantum number of these multiplets are presented in Table II and the corresponding Feynman diagram is shown in Fig. 3. This is the most minimal model of radiative Dirac neutrino mass existing in the literature. It is because, no ad hoc symmetry is imposed, no BSM fermion is introduced and in this case only three BSM Higgs
multiplets are required. Any other model demands more than one BSM chiral fermions along with more than one BSM scalar multiplets, making the model more complicated compared to this simplest realization. By introducing a $\mathbb{Z}_2$ symmetry, this diagram was first realized in [47] and later discussed in [48]. In the context of $U(1)_{B-L}$ symmetry, this model was just recently reappeared in [29].

| Topology | $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ |
|----------|--------------------------------------|
| T-I-F-i  | $\sigma^0(1, 0, 3)$                  |
|          | $S^+ (1, 1, 5)$                      |
|          | $\xi^+ (1, 1, 2)$                    |

Table II: Quantum numbers of the BSM scalars.

Figure 3: One-loop Dirac neutrino mass for the particle content shown in table II. Propagators in red correspond to BSM scalars.

In this model the relevant Yukawa Lagrangian is given by:

$$\mathcal{L}_Y \supset y_{ij}^H \ell_{Li} \ell_{Rj} H + y_{kj}^S \nu_{Rk} \ell_{Rj} S^+ + y_{ij}^\xi \ell_{Li} \ell_{Lj} \xi^+ + h.c. \quad (3.3)$$

Where $i,j = 1-3$ and $k = 2,3$ are the generation indices. Note that $y^\xi$ is $3 \times 3$ anti-symmetric matrix, and since $\nu_{R1}$ carries a different $U(1)_{B-L}$ charge compared to the other two generations as a result one has $y_{11,12,13}^S = 0$. The relevant part of the scalar potential contains the following cubic term to complete the loop in Fig. 3:

$$V \supset \mu S^+ \sigma^0 \xi^- + h.c. \quad (3.4)$$

Neutrino mass will be generated when the neutral singlet $\sigma^0$ that carries $+3$ unit of charge under the $U(1)_{B-L}$ acquires non-zero VEV. Electroweak (EW) symmetry also needs to be

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broken for this diagram to contribute to neutrino mass. This specific charge assignment for \(\sigma^0\) multiplet is required to complete the loop as can be seen from the corresponding Feynman diagram 3. The corresponding neutrino mass matrix has the form:

\[
m_{\nu_{ab}} \sim \frac{1}{16\pi^2} \frac{\mu(\sigma^0)(H^0)}{\Lambda^2} g_{a\bar{a}} g_{ij}^s g_{j\bar{b}}.
\] (3.5)

Here \(\Lambda\) is the mass scale associated with the heaviest BSM scalar multiplet running in the loop. The Yukawa coupling \(y^\xi\) is anti-symmetric, as a result, one of the neutrinos always remain massless to all order since \(\text{det}(y^\xi) = 0\), irrespective of the restrictions on \(y^S\). Since \(\nu_{R1}\) has no interaction due to a different \(B-L\) charge assignment, it is one of the two massless chiral states and contributes to \(N_{\text{eff}}\), the effective number of relativistic degrees of freedom. Since this chiral state does not couple directly to the SM particles, it decouples from the rest of the theory early in the universe, hence does not conflict with any cosmology [28, 49–51].

After the breaking of the \(U(1)_{B-L}\) and EW symmetries, the Dirac neutrino mass term in the Lagrangian has the form \(\mathcal{L}_Y \supset \bar{\nu}_L m_{\nu} \nu_R\). One can go to a basis where the charged lepton mass matrix is diagonal. The phases of the entries \(y^s_{ij}\) can be absorbed in \(L_L\) (and subsequently in \(\ell_R\)). Then \(y^S\) is in general complex, however, recall that \(y^s_{ij} = 0\). Since neutrinos are Dirac, one can also simultaneously work in a basis where \(\nu_R\) are mass eigenstates. Then the Dirac mass matrix can be written as: \(m_{\nu} = U m^\text{diag}_\nu, U = U_{\text{PMNS}}\). Assuming normal mass ordering, this corresponds to:

\[
m_{\nu} = a_0 y^\xi^T m_{E} y^S = \begin{pmatrix} 0 & m_2 U_{e2} & m_3 U_{e3} \\ 0 & m_2 U_{\mu2} & m_3 U_{\mu3} \\ 0 & m_2 U_{\tau2} & m_3 U_{\tau3} \end{pmatrix}, \quad a_0 = \frac{\sin(2\theta)}{16\pi^2} \ln \left( \frac{m^2_{H_2}}{m^2_{H_1}} \right).
\] (3.6)

Where, \(H_i\) represents the two singly charged mass eigenstates and \(\theta\) represents the corresponding mixing angle. Solving this system requires \(y^S_{21,31} = 0\) and the rest of the parameters are fixed as:

\[
y^S_{22} = \frac{m_2}{m_\mu} \left( \frac{-c_{23}s_{12}s_{13} - c_{12}s_{23}}{y^S_{23}} \right), \quad y^S_{23} = \frac{m_3}{m_\mu} \left( \frac{c_{13}c_{23}}{y^S_{23}} \right),
\] (3.7)

\[
y^S_{32} = \frac{m_2}{m_\tau} \left( \frac{s_{12}s_{13}s_{23} - c_{12}c_{23}}{y^S_{23}} \right), \quad y^S_{33} = \frac{m_3}{m_\tau} \left( \frac{-c_{13}s_{23}}{y^S_{23}} \right),
\] (3.8)

\[
y^\xi_{12} = y^S_{23}, \quad y^\xi_{13} = y^S_{23} \left( \frac{-c_{23}s_{12} - c_{12}s_{13}s_{23}}{s_{13}} \right).
\] (3.9)

Here \(c_{ij} = \cos(\theta^\text{PMNS}_{ij})\) and \(s_{ij} = \sin(\theta^\text{PMNS}_{ij})\), \(m_2 = \sqrt{\Delta m^2_{21}}\) and \(m_3 = \sqrt{\Delta m^2_{31}}\). For simplicity, here we assumed \(U_{\text{PMNS}}\) is real by setting the Dirac phase \(\delta = 0\). Note that the only free Yukawa coupling is \(y^S_{23}\), the rest of them are fixed in terms of this and by the neutrino oscillation data, hence parameter space of this model is highly restricted. These non-trivial restrictions can have important significance in phenomenology of this model.
Figure 4: $BR(\mu \rightarrow e\gamma)$ as a function of $y_{23}^\xi$, for details see text. The horizontal red line corresponds to the current experimental upper bound $BR(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$ by MEG experiment and the horizontal orange line represents the projected sensitivity $BR(\mu \rightarrow e\gamma) < 6 \times 10^{-14}$ by MEG II experiment [52].

The presence of the Yukawa couplings $y_{ij}^\xi$ give rise to lepton flavor violating (LFV) decays, $\ell_i \rightarrow \ell_j\gamma$, among them the most stringent constraints comes from $\mu \rightarrow e\gamma$ process. Using the above derived relations, the corresponding branching ratio is given by:

$$BR(\mu \rightarrow e\gamma) = \frac{\alpha}{48\pi G_F^2} A^2 \left(y_{23}^\xi\right)^4 \left(\frac{c_{23}s_{12} + c_{12}s_{13}s_{23}}{s_{13}}\right)^2, \quad A = \frac{\cos^2 \theta}{m_{H_1}^2} + \frac{\sin^2 \theta}{m_{H_2}^2}. \quad (3.10)$$

Note that the Yukawa coupling $y^S$ does not contribute to this process in our scenario due to $y_{ij}^S = 0$. Assuming $\theta = 0.1$ and setting $m_{H_{1,2}} = 650,750$ GeV, the $\mu \rightarrow e\gamma$ branching ratio is plotted against $y_{23}^\xi$ and shown in Fig. 4. In this plot all the neutrino observables are varied within their experimental $3\sigma$ range [4]. Other interesting processes of this model are $\mu \rightarrow eee$ and $\mu - e$ conversion in the nuclei and so on, for general analysis of LVF processes and for LHC phenomenology we refer the reader to [47,48].

3.2 Two-loop models

In this subsection, we present the simplest two-loop Dirac neutrino mass models within our framework. As aforementioned, we are interested in topologies presented in Fig. 2 as a realization of the dimension-5 operator of Eq. (2.1). First we point out that, out of the four topologies, the diagram of type T-II-F-x cannot be generated with only the SM fermions. Similar to that of T-I-H-x topology as discussed before, the internal fermion line cannot be completed and demands chiral fermions BSM, however, we do not pursue in this direction here. Below we present the models with topology of type T-II-H-x. Realization of this topology in our set-up demands five BSM scalar multiplets. By comparison with the existing models in the literature, we will show that our models require less number of BSM.
Table III: Quantum numbers of the BSM scalars.

| Topology   | SU(2)_L × U(1)_Y × U(1)_{B−L} |
|------------|---------------------------------|
| T-II-H-i   | \(\sigma^0(1,0,3)\)           |
|            | \(S^+(1,1,5)\)                 |
|            | \(\chi^{++}(1,2,2)\)          |
|            | \(\eta(2\frac{1}{2},0)\)      |
|            | \(\Omega^+(1,1,-3)\)          |

Figure 5: Two-loop Dirac neutrino mass for the particle content shown in table III.

For topology T-II-H-i the particle content with quantum numbers is presented in Table III and the Feynman diagram is shown in Fig. 5. A second Higgs doublet neutral under \(U(1)_{B−L}\) is required in this model. To complete the loop, in addition to two singly charged scalars, a doubly charged scalar is introduced, these three multiplets along with the neutral scalar that breaks the \(U(1)_{B−L}\) symmetry are all iso-singlets and carry non-trivial \(B−L\) charge. The relevant Yukawa couplings are given by:

\[
\mathcal{L}_Y \supset y_{H}^{ij} \bar{L}_{Li} \ell_{Rj} H + y_{\eta}^{ij} \bar{L}_{Li} \ell_{Rj} \eta + y_{S}^{ij} \bar{L}_{Li} \ell_{Rj} S^+ + y_{\chi}^{ij} \bar{\ell}_{Li} \ell_{Rj} \chi^{++} + h.c. \tag{3.11}
\]

Here \(y^\chi\) is a \(3 \times 3\) symmetric matrix in the generation space. And the relevant part of the scalar potential that contribute to complete the loop diagram is:

\[
V \supset \mu \, S^+ \Omega^+ \chi^{--} + \lambda \, H \eta \Omega^- \sigma^0 \, s + h.c. \tag{3.12}
\]

The neutrino mass matrix in this case has the following form:

\[
m_{\nu_{ab}} \sim \frac{1}{(16\pi^2)^2} \frac{\lambda \mu (\sigma^0) \langle H^0 \rangle}{\Lambda^2} \, y_{ai} y_{j}^{\chi} y_{jb}. \tag{3.13}
\]

For topology T-II-H-iii, the model is presented in Table IV, and the corresponding Feynman diagram in Fig. 6. Model with T-II-H-iii topology can be found by replacing the singly charged scalar \(\Omega^+\) of the previous model (Table III) by an iso-spin doublet \(\zeta = \ldots\)
| Topology   | $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ |
|------------|------------------------------------------|
| T-II-H-iii | $\sigma^0(1,0,3)$                        |
|            | $S^+(1,1,5)$                             |
|            | $\chi^{++}(1,2,2)$                       |
|            | $\eta(2,\frac{1}{2},0)$                 |
|            | $\zeta(2,-\frac{3}{2},-2)$              |

Table IV: Quantum numbers of the BSM scalars.

Figure 6: Two-loop Dirac neutrino mass for the particle content shown in table IV.

$(\zeta^- \zeta^{--})^T$ having hypercharge of $-3/2$. This doublet that can not acquire VEV and also cannot couple directly to the fermions, carries non-trivial $B - L$ charge. For this model, Yukawa Lagrangian is identical to that of Eq. (3.11). Then the relevant part of the scalar potential responsible for completing the loop diagram is given by:

$$V \supset \mu \zeta^\dagger \eta \chi^{--} + \lambda H \epsilon S^+ \sigma^0 + h.c. \quad (3.14)$$

In this case, the Dirac neutrino mass matrix also has the same structure as T-II-H-i given in Eq. (3.13).

And finally for the topology of type T-II-H-ii, the needed particle representations are presented in Table. $V$ and the corresponding Feynman diagram is presented in Fig. 7. However, note that for the realization of this topology, singly charged scalar $\xi^+$ which carries +2 unit of $U(1)_{B-L}$ charge is needed to complete the loop. Due to the presence of this multiplet that allows the additional Yukawa coupling $y_{ij}^S L_{Li} \epsilon L_{Lj} \xi^+$ on the top of the terms in Eq. (3.11), this model automatically accommodates Dirac neutrino mass at the one-loop level given in Fig. 3.

As already discussed, the different charge assignment of $\nu_{R1}$ leads to $y_{ij}^S = 0$. Since this results in $det(y^S) = 0$, one of the neutrinos is massless. In all these models two chiral states remain massless, one of them is $\nu_{R1}$ that contributes to $N_{\text{eff}}$ as discussed earlier. Particle content can be extended trivially for these two-loop models to give all three generations Dirac mass, which is unlike the case for the one-loop model presented earlier where one of
the neutrinos is always massless due to anti-symmetric nature of the Dirac mass matrix. Of course such extension \(^4\) is not required, since one zero neutrino mass is fully consistent with experimental data.

From the Yuakwa interactions of the two-loop models, Eq. (3.11) one sees that compared to the one-loop model, more parameters are involved in the neutrino mass generation. Even though the parameter space is somewhat relaxed compared to the one-loop scenario, there exist non-trivial restrictions on the parameter space from experimental bounds on various processes. As already pointed out, phenomenologies associated with the Yukawa couplings \(y_{ij}^S\) with the singly charged scalar \(S^+\) are studied in \([47, 48]\). The Yukawa couplings \(y^\chi\) of the doubly charged scalar \(\chi^{++}\) with the fermions contribute to interesting flavor violating processes as well. Furthermore, due to the presence of the second Higgs doublet, the off-diagonal couplings to the iso-doublets with the SM fermions cannot be rotated away, hence lead to LVF processes. Few of the most interesting LFV processes are: radiative rare decays \(\ell_a \rightarrow \ell_b \gamma\), tree-level tri-lepton decays \(\ell_a \rightarrow \ell_b \ell_c \ell_d\) and \(\mu - e\) conversion in the nuclei. Some of these processes put stringent constraints on the off-diagonal couplings. Whereas, flavor conserving processes give rise to electron and muon electric dipole moment and also

\(^4\) To give Dirac mass to all the neutrinos, one needs to introduce another singly charged scalar \(S^+(1, 1, -4)\) and a charged neutral guy \(\sigma^0(1, 0, -6)\). This neutral member needs to acquire VEV as well, which will give rise to a physical Goldstone bosons in the theory.
anomalous magnetic moments via loop diagrams. The phenomenology of this class of models are very similar to the Zee model and Zee-Babu model and detail phenomenological studies of these processes including LHC signals have been carried out in many different works. Instead of listing all, we refer the reader to the most recent analysis [53,54] and the references therein.

3.3 Three-loop models

Following the same spirit of the one-loop and two-loop models, one can also build models for three-loop Dirac neutrino mass within our framework. Here we show that minimal model can be constructed with just five BSM scalars, this number is the same as that of the two-loop models presented above. Three-loop models can be realized using the dimension-7 operator given in Eq. (2.2). For each of the models presented above, the required BSM multiplets are uniquely fixed for each of the different topologies. Which is also true for models with three-loop. Note that, in building the minimal models as discussed earlier, we have used only the fermion bilinears. However, minimal model with Dirac mass appearing at the three-loop can be realized once the gauge interaction of the SM fermions are also used as one of the vertices. Hence the type of the topology we are interested in has the general form shown in Fig. 8, and we are going to use this diagram as a guidance to build models. The direction of the flow of the iso-spin doublet is unique in this type of topologies (T-III-F-x) as can be seen from Fig. 8. No models with topology of type T-III-H-x is found.

![Figure 8: Three-loop topologies T-III-F-x within our framework.](image)

In our set-up, the minimal model for three-loop realization is presented in Table VI with the corresponding Feynman diagram shown in Fig. 9. Completion of this diagram
requires two singly charged scalars, one of them with zero $B - L$. Furthermore, a doubly charged scalar and an iso-doublet with hypercharge $-3/2$ neutral under $B - L$. Unlike the two-loop models, a second SM like Higgs doublet is not required here. We also present two next-to-minimal models that share the same diagram (corresponding topologies are denoted by T-III-F-ii-a and T-III-F-ii-b) but with different set of scalar multiplets. This second set of topologies are variant of the minimal three-loop scenario and require six BSM scalar multiplets. The particle content of these models are presented in Table VII and the associated Feynman diagram is presented in Fig. 10.
Table VII: Quantum numbers of the BSM scalars. A simple variation of this model can be constructed with same topology (denoted by T-III-F-ii-b) by replacing \( \zeta(2, -\frac{3}{2}, 0) \to \zeta(2, -\frac{3}{2}, 3) \) and \( \Omega^+(1, 1, 3) \to \Omega^+(1, 1, 0) \).

Figure 10: Three-loop Dirac neutrino mass for the particle content shown in table VII. By a trivial \( B - L \) charge swapping: \( \zeta(2, -\frac{3}{2}, 0) \to \zeta(2, -\frac{3}{2}, 3) \) and \( \Omega^+(1, 1, 3) \to \Omega^+(1, 1, 0) \) another model (with topology T-III-F-ii-b) can be constructed.

All these three-loop models share the same Yukawa sector:

\[
\mathcal{L}_Y \supset y_{ij}^H \ell_{L_i} R_j H + y_{kj}^S \ell_{R_k} \ell_{R_j} S^+ + y_{ij}^\chi \ell_{R_i} \ell_{R_j} \chi^{++} + h.c. \quad (3.15)
\]

The relevant part of the scalar potential for the model with topology T-III-F-i is:

\[
V \supset \mu \zeta \epsilon H x^+ + \lambda \chi^{--} x^+ S^{0*} + \lambda' \zeta^\dagger H x^- x^- + h.c. \quad (3.16)
\]

whereas for topology of type T-III-F-ii-a(b) it is:

\[
V \supset \lambda \zeta \epsilon H \Omega^+ \sigma^{0*} + \lambda' \zeta^\dagger H \Omega^- \Omega^- + \mu \chi^{--} S^+ \Omega^+ + h.c. \quad (3.17)
\]
All these three-loop models have the same form for the neutrino mass matrix:

\[ m_{\nu ab} \sim \frac{1}{(16\pi^2)^3} \frac{g^2\lambda\lambda'\mu(\sigma^0)(H^0)^3}{\Lambda^4} y_{ai}^H y_{ij}^X y_{jb}^S. \]  

(3.18)

Here \( g \) represents the \( SU(2) \) gauge coupling constant. Two of the neutrinos receive non-zero mass after the \( U(1)_{B-L} \) and the EW symmetry are broken. The reason for the masslessness of the two chiral states is the same as the two-loop case as discussed above.

The phenomenology of the three-loop models are not very different from the one-loop and two-loop models since the Yukawa couplings given in Eq. (3.16) does not contain any new flavor violating interactions that are already discussed above. So we do not repeat the discussion here. Rather we briefly discuss the possibility of a DM candidate in the class of models presented in this work. Even though we do not demand the presence of a DM in our set-up, one can readily gets a DM candidate if a Majorana mass term is allowed only for the \( \nu_{R1} \) state that carries a different charge under \( U(1)_{B-L} \). From the structure of the bilinears listed earlier in the text, one can see that introducing a multiplet with quantum number \( \sigma^0(1,0,+10) \) can allow a Majorana mass of the form: \( \nu_{R1}^c \nu_{R1} \sigma^0 \). Once this scalar gets a VEV, \( \nu_{R1} \) receives a Majorana mass and serve as a Majorana DM candidate [51]. In that case, the imaginary part of \( \sigma^0 \) will become a physical Goldstone bosons due to an accidental global \( U(1) \) symmetry respected by the scalar potential, hence contributes to \( N_{\text{eff}} \). However, this does not cause any serious problem since, this scalar does not couple directly to the SM fermions except the SM Higgs doublet. It can decouple from the thermal bath early in the universe if its coupling with the SM Higgs is somewhat suppressed \( \lesssim 10^{-3} \), for details see for example [49,51,55,56].

Finally, we compare the simplest models presented here with the existing models in the literature that use only \( U(1)_{B-L} \) symmetry, which is the case in our set-up as well but allow extra fermions in the theory. In [24,29,30], the one-loop realization requires two BSM chiral fermions and three scalar multiplets. Similarly, two-loop realizations require more BSM states compared to our scenario here, in [24], four chiral fermions and four scalars are introduced for two-loop implementation. On the other hand, three-loop model is not realization in the literature before. We remind the readers that within our framework, implementation of minimal one-loop model requires three, and minimal two-loop and three-loop models require five BSM scalar multiplets.

4 Conclusions

The baryon number minus the lepton number is a global symmetry of the SM that can be made anomaly free by introducing only three right-handed neutrinos \( \nu_R \). Due to the presence
of the right-handed partners $\nu_R$ of the SM left-handed neutrinos $\nu_L$, it is natural to expect that like rest of the fermions of the SM, neutrinos are also Dirac in nature. Building models of radiative Dirac neutrino mass usually require the presence of ad hoc symmetries as well as additional fermions beyond the SM. These two requirements introduce extra complexities into the theory. By embracing the point of view that the imposition of ad hoc symmetries is least desirable, and well motivated ultraviolet completion of the SM may not contain additional fermions, in this work, we have proposed new simple models of radiative Dirac neutrino mass. In our framework, no tree-level Dirac and Majorana mass terms appear due to symmetry reasons. Within this set-up, we construct minimal models of Dirac neutrino mass at the (i) one-loop, (ii) two-loop and (iii) three-loop level. It is shown that the minimal one-loop model requires three scalar multiplets beyond the SM, whereas minimal two-loop and three-loop models require five. The presented two-loop and three-loop models of Dirac neutrino mass have not appeared in the literature before. Possible dark matter candidate in this class of models is briefly discussed.

Acknowledgments

The author would like to thank Dr. Ernest Ma for discussion.

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