Production of energetic dileptons with small invariant masses
from the quark-gluon plasma*

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Abstract

The resummation technique of Braaten and Pisarski is used for a complete
calculation of the $\alpha^2\alpha_s$-contribution to the production rate of dileptons with
energy $E \gg T$ and invariant mass $M \sim T$ from a quark-gluon plasma. In
particular the cancellation of the infrared singularity by medium effects is
discussed in detail.
PACS numbers: 12.38.Mh, 25.30.Rw, 12.38.Cy

*Supported by BMBF, GSI Darmstadt, and DFG

†Heisenberg fellow
I. INTRODUCTION

The thermal radiation of photons and dileptons from a quark-gluon plasma (QGP) is one of the most promising signatures for the formation of a QGP in relativistic heavy ion collisions [1]. Whereas real photons can be produced to lowest order $\alpha \alpha_s$ only via the participation of a gluon, dileptons are produced to lowest order $\alpha^2$ by quark-antiquark annihilation into a virtual photon (Born term) [2]. However, in the case of a small invariant photon mass $M^2 = E^2 - p^2$, where $E$ is the energy and $p = |p|$ the momentum of the photon, radiative corrections of the order $\alpha^2 \alpha_s$ become increasingly important. These corrections have been considered using perturbative QCD at finite temperature [3,4]. In contrast to the production of real photons the dilepton production rate turns out to be infrared finite in the case of a vanishing quark mass. There is a cancellation of the infrared singularities of real and virtual contributions, the latter appearing only for the production of virtual photons [4]. However, also medium effects leading to an effective quark mass $m_q^*$ by the interaction of the quark with the heat bath can screen these infrared divergences. Altherr and Ruuskanen [4] suggested that the effective quark mass simply replaces the invariant photon mass as infrared cutoff if $M < m_q^*$.

Medium effects can be included consistently using the resummation technique of Braaten and Pisarski [5]. This method has been applied to a number of interesting quantities of the QGP leading to gauge invariant and infrared finite results that are complete to leading order in the coupling constant [6]. For example, the production rate of energetic photons has been derived in this way, where a resummed quark propagator containing the effective quark mass $m_q^{*2} = g^2 T^2/6$ has been used [7–9]. Here we want to reconsider the result of Altherr and Ruuskanen [4] performing a complete calculation of the dilepton rate using the Braaten-Pisarski method analogously to the photon case. In particular we will study in detail the role of $m_q^*$ and $M$ as infrared cutoffs. Like Altherr and Ruuskanen, we will restrict ourselves to small photon masses $M \lesssim T$, for which the $\alpha_s$-corrections are of importance, and to large photon energies $E \gg T$.  

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II. DILEPTON PRODUCTION RATE TO ORDER \( \alpha^2 \alpha_s \)

The lowest order matrix elements for the production of virtual photons from the QGP are shown in Fig.1. Besides the real contributions (Compton scattering and annihilation) of Fig.1a,b, appearing also for the production of real photons, there are virtual contributions, namely gluon absorption (Fig.1c) and the interference terms of the Born term with radiative corrections (self energy insertion and vertex correction) in Fig.1d. Due to phase space restrictions and kinematics \( E \gg T \) the gluon absorption as well as the vertex correction can be neglected \[4\].

The matrix elements can be related to the imaginary part of the photon self energy diagrams in Fig.2 \[10\]. Here the processes of Fig.1a,b,c can be obtained by cutting through the internal quark and gluon lines, while the interference term (Fig.1d) corresponds to a cut through the quark lines only. In the latter case the quark self energy becomes on-shell, leading to an infrared singularity which cancels the one from the exchanged massless quark in the real contribution \[4\]. This cancellation can be considered as an example of the KLN-theorem at finite temperature \[11\].

The production rate for massless electron and muon pairs derived from the photon self energy is given by \[4\]

\[
\frac{dR}{d^4xd^4p} = \frac{1}{6\pi^4} \frac{\alpha}{M^2} \frac{1}{e^{E/T} - 1} \text{Im} \Pi_{\mu\mu}^\mu(P).
\]  

(1)

As in the case of the photon production rate \[7-9\], we want to include the effect of the medium by using a resummed quark propagator for soft momenta of the exchanged quark. For this purpose we introduce a separation scale \( k_c \) \[12\], restricted by \( gT \ll k_c \ll T \) in the weak coupling limit. For quark momenta smaller than \( k_c \) we will start from the photon self energy shown in Fig.3, where the blob denotes the effective quark propagator, in which the quark self energy in the hard thermal loop approximation has been resummed \[14\]. Owing to the high energy of the photon and energy-momentum conservation we need to take into account only one effective quark propagator and no effective vertices as opposed to the case of soft dileptons \[14\].
For momenta of the exchanged quark larger than $k_c$, the diagrams of Fig.1 or Fig.2, containing only bare propagators, will be considered. After adding up the soft and hard contributions, the separation scale has to drop out, demonstrating the completeness of the calculation [12]. In the case of the photon production a covariant separation scale, i.e. $|K^2| = |\omega^2 - k^2| = k_c^2$ [7], and a non-covariant one ($k = k_c$) [8] have been used, both leading to the same result.

In the limit $M^2 \ll ET$ the hard real contribution (Fig.1a,b) of the dilepton production rate can be taken over from the photon rate [4], yielding in the case of a non-covariant cutoff [8]

\[
\left( \frac{dR}{d^4x d^4p} \right)_{\text{hard}}^{\text{real}} = \frac{10}{27\pi^3} \alpha^2 \alpha_s T^2 e^{-E/T} \left[ \ln \frac{ET}{k_c^2} + \frac{3}{2} + \frac{\ln 2}{3} - \gamma + \frac{\zeta'(2)}{\zeta(2)} \right],
\]

where $\gamma = 0.57722$ and $\zeta'(2)/\zeta(2) = -0.56996$. In the case of a covariant cutoff the constant in the square brackets is changed by adding a term $\ln 4 - 2$ [7,8]. The hard contribution to the photon rate follows from a momentum integration over the square of the matrix elements of Fig.1a,b using distribution functions for the external partons. Applying the Boltzmann approximation for the distributions of the incoming partons and using $k_c \ll T$ the result (4) is found after some tedious manipulations. It can be obtained more easily by calculating the photon absorption rate and applying the principle of detailed balance [13].

The soft contribution to the dilepton rate can be computed by introducing spectral functions for the resummed quark propagator [14] as in the case of the photon rate [7]. Following the arguments in the calculation of the photon rate we arrive at

\[
\left( \frac{dR}{d^4x d^4p} \right)_{\text{soft}}^{\text{real}} = \frac{40}{9\pi^2} \frac{\alpha^2}{M^2} \int \frac{d^3k}{(2\pi)^3} \int d\omega \int d\omega' \delta(E - \omega - \omega') n_F(\omega) n_F(\omega') \left\{ \left( 1 + \hat{q} \cdot \hat{k} \right) [\rho_+(\omega, k) \delta(\omega' + q) + \rho_-(\omega, k) \delta(\omega' - q)] + \left( 1 - \hat{q} \cdot \hat{k} \right) [\rho_+(\omega, k) \delta(\omega' - q) + \rho_-(\omega, k) \delta(\omega' + q)] \right\},
\]

where $n_F$ is the Fermi-Dirac distribution, $q = p - k$ the momentum of the bare quark propagator, and
the spectral functions with

\[
\beta_\pm (\omega, k) = -\frac{m_q^2}{2} \frac{\pm \omega - k}{k (-\omega \pm k) + m_q^2 \left( \pm 1 - \frac{\pm \omega - k}{2k} \ln \frac{k + \omega}{k - \omega} \right)^2 + \left( \frac{\pi}{2} m_q^2 \frac{\pm \omega - k}{k} \right)^2}.
\]

The first term of the spectral functions (4), containing the dispersion relations $\omega_\pm(k)$ (see Fig.4) of the collective quark modes above the light cone ($\omega > k$), gives rise to the virtual contribution of the dilepton rate. The real soft contribution, on the other hand, follows from the second term below the light cone ($-k < \omega < k$), which comes from the imaginary part of the hard thermal loop quark self energy. Using $E \gg T$ and assuming $\omega$ and $k$ to be soft, the soft real contribution reduces to

\[
\left( \frac{dR}{d^4x d^4p} \right)_{real}^{soft} = \frac{5}{9\pi^4} \frac{\alpha^2}{M^2} e^{-E/T} \int dk d\omega \left[ (k - \omega + E - p) \beta_+ (\omega, k) + (k + \omega - E + p) \beta_- (\omega, k) \right].
\]

The integration range, determined by $\omega = -k + E - p$, $\omega = k$, and the separation scale $k = k_c$ or $k^2 - \omega^2 = k^2_c$, respectively, are shown in Fig.4. Compared to the photon case it is shifted by $E - p > 0$, which renders the integration in (7) more difficult. The logarithmic dependence on the scale $k_c$ can be extracted by adopting the static limit of the effective quark propagator

\[
\beta_\pm (\omega = 0, k) = \frac{m_q^2}{2} \frac{k}{(k^2 + m_q^2)^2 + (\pi m_q^2/2)^2}.
\]

In the case of a non-covariant separation scale $k_c$, where the $\omega$-integration is restricted by the limits $-k + E - p$ and $k$, while the $k$-integration ranges from $(E - p)/2$ to $k_c$, we find to logarithmic approximation

\[
\left( \frac{dR}{d^4x d^4p} \right)_{real}^{soft} = \frac{5}{9\pi^4} \frac{\alpha^2}{M^2} e^{-E/T} m_q^2 \left( \ln \frac{k_c^2}{m_q^2} + A \right),
\]

where we have used $k_c \gg gT$. In order to determine the function $A(E - p)$ beyond the logarithm, we cannot use the static approximation anymore, but have to solve (8) together.
with (5) numerically. We restrict ourselves to $E - p \leq m_q^*$, which holds for realistic values of the coupling constant $g \gtrsim 1$. Then we find that $A$ increases from $A(E - p = 0) = -1.31$, which corresponds to the photon result [8], to $A(E - p = m_q^*) = -1.71$.

Adding the hard contribution (2) to (8) with $m_q^* q^2 = 2 \pi \alpha_s T^2 / 3$, the arbitrary separation scale $k_c$ drops out and we obtain an infrared finite result for the real contribution of the dilepton rate. On the other hand, using a covariant separation scale, (8) is multiplied by a factor $k_c^2 / [(E - p)^2 + k_c^2]$. Note that $E - p = M^2 / (E + p) \ll T$ might be of the same order as $k_c \ll T$ for $M \sim T$. Hence the separation scale does not cancel in this case in contrast to the real photon rate ($E = p$). This shows that we should demand that $\omega$ and $k$ are soft individually as it was also assumed in the derivation of the Braaten-Pisarski method, which is based on the imaginary time formalism in euclidean space-time [5].

Finally, we consider the virtual part coming from the pole contribution to the imaginary part of the photon self energy in (4). Now we have to use a covariant separation scale since the infrared singularity in Fig.2 comes from the on-shell self energy at $K^2 = 0$. The soft part follows from the pole contribution of the diagrams in Fig.3 corresponding to the first term of the spectral functions (4). Since $\omega_\pm(k)$ lies always below $K^2 = k_c^2 \gg m_q^* q^2$ (see Fig.4), there is no hard contribution to the virtual part. Thus the virtual contribution is determined solely by the imaginary part of the self energy in Fig.3 coming from the pole of the effective quark propagator, which gives a finite result when integrated over the entire momentum range.

Combining (3) and the first term of (4) we find

$$\left( \frac{dR}{d^4x d^4p} \right)_{\text{virt}} = \frac{5 \alpha^2}{9 \pi^2 M^2 m_q^2} \frac{1}{p} \left\{ \int_{k_{\text{min}}^+}^{k_{\text{max}}^+} dk n_F(\omega_+) n_F(E - \omega_+) (\omega_+^2 - k^2) \left[ \frac{M^2}{2} - E (\omega_+ - k) + \frac{(\omega_+ - k)^2}{2} \right] \right. $$

$$\left. - \int_{k_{\text{min}}^-}^{k_{\text{max}}^-} dk n_F(\omega_-) n_F(E - \omega_-) (\omega_-^2 - k^2) \left[ \frac{M^2}{2} - E (\omega_- + k) + \frac{(\omega_- - k)^2}{2} \right] \right\},$$

where the limits $k_{\text{max,min}}^\pm$ are determined from the intersections of the dispersion relations.

$$(9)$$
with \( \omega = \pm k \pm (E - p) \) as shown in Fig.4. Note that the virtual part vanishes for \( M \to 0 \) as \( k_{\min}^\pm \) tends to infinity.

In order to isolate the term proportional to \( \alpha^2 \alpha_s \) and to investigate its dependence on \( M, m^*_q, E, \) and \( T, \) we introduce again a separation scale \( gT \ll k_s \ll T. \) For \( k < k_s \) we may approximate the distribution functions by \( n_F(\omega_{\pm}) \simeq 1/2 \) and \( n_F(E - \omega_{\pm}) \simeq \exp(-E/T), \)
whereas for \( k > k_s \) we may set \( \omega_{\pm} \simeq k + m^*_q/k \) leading to \( \omega_+^2 - k^2 \simeq 2m^*_q \) and \( \omega_- = k \) [16]. Hence the plasmino branch \( \omega_-(k) \) does not contribute for hard momenta as \( \omega_- \) approaches \( k \) exponentially for \( k \gg gT \) [16].

For the part of the integrals in (9) containing the terms with \( M^2/2 \) it is sufficient to restrict to \( k > k_s \) since the hard part is finite for \( k_s \to 0. \) Considering \( E \gg T \) the simplifications \( k_{\max}^+ \simeq (E + p)/2, k_{\min}^- \simeq (E - p)/2, n_F(\omega_+) \simeq n_F(k), \) and \( n_F(E - \omega_+) \simeq n_F(E - k) \) can be assumed. Then this term reduces to the Born term, which reads for \( E \gg T \) and \( E - p \ll T \)

\[
\left( \frac{dR}{d^4xd^4p} \right)^{\text{Born}} = \frac{5}{9\pi^4} \alpha^2 \frac{1}{p} e^{-E/T}.
\]

The Born term is contained in the virtual contribution (9) because the effective quark propagator includes the bare one.

Next we consider the terms under the integral in (9) proportional to \( E(\omega_{\pm} \pm k) \). Using the approximations discussed above, we find for \( k < k_s \)

\[
\left( \frac{dR}{d^4xd^4p} \right)^{\text{virt}} = \frac{5}{18\pi^4} \frac{\alpha^2}{M^2m^*_q} \frac{1}{p} e^{-E/T} \left[ -\int_{k_{\min}}^{k_s} dk (\omega_+^2 - k^2)(\omega_+ - k) + \int_{k_{\min}}^{k_s} dk (\omega_-^2 - k^2)(\omega_- + k) \right].
\]

We proceed analogously to Baier et al. [3] in the case of the photon rate, using \( (\omega_+^2 - k^2)(\omega_+ + k)/m^*_q = \omega_\pm - k(d\omega_\pm/dk) \). Then we obtain with \( k_s \gg m^*_q \)

\[
\left( \frac{dR}{d^4xd^4p} \right)^{\text{virt}} = \frac{5}{9\pi^4} \frac{\alpha^2}{M^2} e^{-E/T} m^*_q \left( \ln \frac{k_s}{m^*_q + k_{\min}^+} + B \right).
\]

The function \( B(E - p) \) decreases from \( B(E - p = 0) = 0, \) where the virtual contribution vanishes, to \( B(E - p = m^*_q) = -0.66, \) where \( k_{\min}^\pm = 0 \) (see Fig.4).
The hard part, \( k > k_s \), using the approximations given above, reads

\[
\left( \frac{dR}{d^4xd^4p} \right)_{\text{virt hard}} \sim -\frac{10}{9\pi^3} \frac{\alpha^2}{M^2} \frac{m_q^*}{k} \int_{k_s}^E \frac{n_F(k)n_F(E-k)}{k} dk.
\] (13)

Considering \( k_s \ll T \ll E \), we find

\[
\left( \frac{dR}{d^4xd^4p} \right)_{\text{virt hard}} \sim -\frac{5}{9\pi^3} \frac{\alpha^2}{M^2} e^{-E/T} \frac{m_q^*}{k_s} \ln \frac{E}{k_s}.
\] (14)

Adding up both the contributions (12) and (14), \( k_s \) cancels and we arrive at

\[
\left( \frac{dR}{d^4xd^4p} \right)_{\text{virt}} \sim -\frac{5}{9\pi^3} \frac{\alpha^2}{M^2} e^{-E/T} \frac{m_q^*}{k_s} \left( \ln \frac{E}{m_q^* + k_{min}^+} + B \right).
\] (15)

We observe that the virtual contribution coming from an interference term is negative, thus reducing the dilepton rate compared to the photon rate. Furthermore, the virtual contribution vanishes for \( m_q^* \to 0 \) as it is also the case for the virtual contribution in naive perturbation theory [4].

The part under the integral in (13) proportional to \((\omega_{\pm} - k)^2\) can be shown to be suppressed by \( m_q^* / E \) relative to (15).

Combining (2), (8), and (15) we end up with our final result for the dilepton production rate

\[
\frac{dR}{d^4xd^4p} = \frac{10}{27\pi^3} \frac{\alpha^2}{\alpha_s} \frac{T^2}{M^2} e^{-E/T} \left( \ln \frac{T(m_q^* + k_{min}^+)}{m_q^*} + C \right),
\] (16)

where the function \( C \) ranges from \( C(E - p = 0) = -0.73 \) to \( C(E - p = m_q^*) = -0.47 \).

For giving an approximate analytic expression for \( k_{min}^+ \) we replace the exact dispersion relation \( \omega_+(k) \) by \( \omega_+^2 = k^2 + m_q^* \), which deviates from the exact one by less than 11% over the entire momentum range. Then we obtain \( k_{min}^+ = |Em_q^2/M^2 - M^2/(4E)| \), restricted by \( k_{min}^+ \leq k_{max}^+ \simeq (E + p)/2 \).

III. DISCUSSION

Now we will discuss our result (16) for various limits of \( M \). For \( M \to 0 \) the virtual contribution vanishes since \( k_{min}^+ \) tends to infinity for \( E - p \to 0 \). Hence the dilepton rate is given by the real contribution which agrees with the photon rate in this limit.
For $M \sim m_q^*$ we get $E - p = M^2/(2E) \ll m_q^*$. Hence $k_{\text{min}}^+$ is given by the intersection of $\omega \simeq k + m_q^{*2}/k$ and $\omega = k + E - p$, i.e. $k_{\text{min}}^+ = m_q^{*2}/(E - p) \sim E$. Thus the result for the photon rate holds approximately, which agrees with the result of Altherr and Ruuskanen [4], if we replace the invariant photon mass in the logarithmic term of their formula (2.18) by the effective quark mass in the case of $M < m_q^*$ as suggested in their paper.

For $M \sim T$, i.e. $E - p \sim T^2/E$, we have to distinguish two cases. First $E - p \sim m_q^*$, which leads to $k_{\text{min}}^+ \sim m_q^*$ (see Fig.4). Then the logarithm in (16) is given by $\ln(T/m_q^*)$. Secondly, in the case $E - p \ll m_q$ we find from Fig.4 $k_{\text{min}}^+ \gg m_q^*$. Now if $k_{\text{min}}^+ \sim E$ we recover the photon result, while for $k_{\text{min}}^+ \sim T$ the logarithmic term is given by $\ln(T^2/m_q^{*2})$.

Finally we will comment on the extrapolation of our result to realistic values of the coupling constant, $\alpha_s = 0.2-0.5$, i.e. $g = 1.5-2.5$. In any case the lowest order result in $\alpha_s$ fails if the rate (14) becomes negative. (This unphysical behaviour can originate from extracting the leading order contribution and extrapolating to large values of the coupling constant. A similar problem has been encountered in the case of transport rates determining thermalization times and the viscosity of the QGP [3,17].) Thus one might expect that for $M \sim T$ the result gets unphysical, since $m_q^* = T$ for $g = \sqrt{6}$. However, for $E \gg T$ we find $E - p \ll T \sim m_q^*$ and $k_{\text{min}}^+ \gg T$. Hence we end up with a logarithmic term which is always positive. However, the constant $C$ behind the logarithm is of the same order (typically half of the value of the logarithm), indicating the necessity to go beyond the logarithmic approximation. (This is more important for dileptons than for photons due to the negative virtual contribution.)

As an example we have chosen $\alpha_s = 0.3$ and $T = 300$ MeV. In Fig.5 the dilepton production rate is shown as a function of $E$ for $M = 300$ MeV and in Fig.6 as a function of $M$ for $E = 3$ GeV. Also shown is the result of Altherr and Ruuskanen [4]. (Here we replaced $M$ in their formula (2.18) by $m_q^*$ for $M < m_q^*$.) For the chosen set of parameters both approaches give similar results, since for $M < m_q^* = 238$ MeV both rates are given by the photon result approximately (see Fig.6). However, the dilepton rate found by Altherr and Ruuskanen breaks down at somewhat larger values of $E$ as shown in Fig.5. In Fig.5
also the Born term (10) is depicted for comparison.

Summarizing, we conclude that the complete calculation using a resummed quark propagator according to the Braaten-Pisarski method leads to a finite result for the dilepton rate to order $\alpha^2 \alpha_s$, where the effective quark mass $m_q^*$ cuts off the infrared divergence. There is no need for using the invariant photon mass $M$ as an infrared cutoff even for $M$ of the order of the temperature.

ACKNOWLEDGMENTS

We would like to thank E. Braaten and V. Ruuskanen for helpful discussions.
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FIG. 1. Matrix elements for dilepton production to order $\alpha^2 \alpha_s$.

FIG. 2. Photon self energy diagrams related to the matrix elements of Fig.1 by cutting rules.

FIG. 3. Photon self energy diagrams with an effective quark propagator.
FIG. 4. Energy $\omega$ and momentum $k$ of the exchanged quark restricted by kinematics and the separation scale $k_c$. The real part of the dilepton rate arises from the region below the light cone, whereas the virtual part follows from the quark dispersion relations $\omega^\pm(k)$.
FIG. 5. Contribution of order $\alpha^2\alpha_s$ to the dilepton production rate versus the energy of the virtual photon $E$ for $T = 300$ MeV, $\alpha_s = 0.3$, and $M = 300$ MeV.
FIG. 6. Contribution of order $\alpha^2\alpha_s$ to the dilepton production rate versus the invariant photon mass $M$ for $T = 300$ MeV, $\alpha_s = 0.3$, and $E = 3$ GeV.