Boson ground state fields in electroweak theory with non-zero charge densities

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ABSTRACT

The "non-linear" self-consistent theory of classical fields in the electroweak model is proposed. Homogeneous boson ground state solutions in the GSW model at the presence of a non-zero extended fermionic charge densities are reviewed and fully reinterpreted to make the theory with non-zero charge densities fruitful. Consequences of charge density fluctuations are proposed.

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1 Introduction

In quantum field theory matter particle is treated as a set of strictly point-like aggregated quanta. Even in classical field theory this creates unpleasant problems such like the infinite self-interaction energy of a point charge, for example. In quantum theory, these zero-point energy divergences do not disappear; on the contrary, they are getting worse, and despite the comparative success of renormalization theory the feeling remains that there ought to be a more elegant way of doing things. Yet it is not only a problem of the aesthetics. Even if in times of Feynman and Dirac more physicists viewed renormalization ideas as mathematically illegitimate (suspect at least) than they used to do today. Even if a certain theory had a fantastic calculational results being in agreement with today’s experiments, one should remember that one is only able to falsify physical theories. There exists also a more practical problem: namely the particular model of quantum field theory is satisfactory only if the renormalization sequence of approximations converges. What we now know is merely that in few of the theories the first few terms give good agreement with experiment. Finally, from the theoretical point of view, particles are not treated in the even way. What do I mean by this? When in quantum mechanics the one-particle probabilistic interpretation of the wave-function $\phi$ for a particle is forced, such interpretation causes a problem for one kind of them (e.g. scalar particles) whereas for others (for example, fermion particles) it causes none. For the Klein-Gordon equation for example, it resulted not only in the rejection of the probabilistic interpretation of the wave-function (in such circumstances this

\footnote{One is only able to falsify physical theories and not to prove them directly. This is in fact a definition of empirical science, recognized since Popper.}
action would sound reasonably) but in the persuasion that the “interpretation
of the Klein-Gordon equation as single-particle equation, with wave-function \( \phi \),
therefore also has to be abandoned” \[3\]. Yet, when quantum field theory enters,
the same functions are "re-interpreted as the charged density” \[3\], and we won-
der why, in quantum mechanics, this interpretation is not commonly allowed
and instead the giving up of the Klein-Gordon equation is forced. For fermion
particles this problem does not appear. Even in the body of quantum mechan-
ics there are statements which are clearly inconsistent. For example when the
scalar wave-function is complex, only probabilistic interpretation is considered
as valid but for the scalar wave-function which is real, ”\( \phi \) corresponds to elec-
trically neutral particles, and \( \rho \) and \( j \) are then the charge and current densities,
rather than the probability and probability current densities.” \[3\].

On the other hand, non-linear field theories possess extended solutions, re-
ferred to as solitons, which represent stable configurations with a well-defined
finite energy. Since non-Abelian gauge theories are non-linear it is natural to
suspect that this new type of solutions may be of relevance to particle physics.
Indeed the last ten years was a period of intensive studies of vortices, magnetic
monopoles and 'instantons', which are all specific types of solitonic solutions.
If gauge theories are taken seriously, so must these solutions be taken too. A
question arises: do they give rise to a new physics \[1\]?

Some years ago one suggested that the electromagnetic field in the presence
of external charge is unstable \[5\]. In effect a new charged non-standard ground
state accompanied by particle-antiparticle pairs might appear. In their paper
\[6\] Müller, Rafelski and Greiner wrote: "The question arises, whether q.e.d.
(quantum electrodynamics) of strong fields is an ensemble of academic problems or whether it can be subject to experimental tests. We believe, that the basic new phenomenon of positron autoionization can be experimentally verified in heavy ion collisions. In the long future even $\gamma$-transitions of the superheavy intermediate molecules - though collision broadened - may be observed and may lead to further tests of the theory. It is encouraging to have recent reports by Armbruster and Mokler and by Saris et al\textsuperscript{3} on first experimental findings in this direction. Both effects, the positron autoionization and especially the $\gamma$-transitions in superheavy quasimolecules can possibly - in the long - be also observed with higher resolution by going through nuclear compound states with life time of the order $10^{-17} - 10^{-18}$ sec. \textsuperscript{3} Similarly in non-Abelian theories a ground state of boson fields induced by the external (non-bosonic) charge may change the physical system. We may expect the appearance of such ground state boson field configurations in very dense microscopic objects created in heavy ion collisions \textsuperscript{3}. Ground state configurations of boson fields are also the subject of interest in the field of astrophysics where the presence of superdense matter is expected in massive compact objects like for example, neutron stars or even more exotic case\textsuperscript{3}.\textsuperscript{3}

The aim of this paper is to examine the phenomenon of homogeneous boson ground state solutions (boson ground fields) in the Glashow-Salam-Weinberg (GSW) theory \textsuperscript{3}. Because the Higgs scalar has not been found and the real symmetry breaking mechanism has not been confirmed, I use in this paper the

\textsuperscript{3} Mokler, P.H., Stein, H.J., Armbruster, P.: Contribution to the Atlanta-Conference on Atomic Spectroscopy, Atlanta (1972). Mokler, P.H.: GSI-Bericht 72-10. See also: Saris, F.W., Mitchell, I.V., Sandy, D.C., Davies, J.A., Laubert, R.: Radiative transitions between transient molecular orbitals in atomic collisions. Contribution to the Atlanta-Conference on Atomic Spectroscopy, Atlanta (1972).
notion of the Higgs scalar in the context of the weakly charged galactical ether\(^4\) in which we are immersed.

2 The General Theory

The mathematics of non-linear Schrödinger and Dirac equations is quite different from that of linear equations. The Hilbert space formulation of quantum theory owes its origin to linearity of the Schrödinger equation. Consequently in non-linear theories the Hilbert space formulation calls for modification. It may work approximately if the non-linear terms are small and treated as perturbations. But it is not always the case.

The "non-linear" self-consistent classical field theory\(^5\), a non-abelian case of which is proposed below, has been previously used with great success in the abelian case by Barut, Kraus, Van Huele, Dowling, Salamin, Ünal \([9], [10], [11], [13], [14]\), (see also \([15], [16]\)).

But here a serious warning has to be given. In Barut coupled equations the wavefunction \(\Psi(x)\) has not the interpretation connected with the full charge density distribution, as in the original linear Schrödinger or Dirac equations, but it is connected with the fluctuations of charge density distribution. Hence that model has its ambition to give the results which till now were attributed to QED, even overcrossing its applicability.

In the model the covariant differentiations, \(D_\mu\) for the Higgs doublet \(H\), and

\(^4\)special thanks to Karol Kołodziej for this notion
\(^5\)This interpretation is connected with the existence of the self field. The self field is small for atomic phenomena, but it is not always small. There is another region where the non-linear term dominates \([4]\). In perturbative QED the self field of the electron is completely absent and it comes back in via a separate quantized radiation field "photon by photon", whereas in the self-consistent classical field concept the whole self field has been put in from the beginning.
\( \nabla_\mu \) for a fermionic field \( R \), are

\[
D_\mu H = \partial_\mu H + igW_\mu H + \frac{1}{2}ig'YB_\mu H ,
\]

\( \nabla_\mu R = \partial_\mu R + \frac{1}{2}ig'YB_\mu R ,
\]

where

\[
W_\mu = W_\mu^a \frac{\sigma^a}{2}
\]

is the gauge field decomposition with respect to the \( su(2) \) algebra generators.

The \( U_Y(1) \) field tensor is defined as

\[
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
\]

and the \( SU_L(2) \) Yang - Mills field tensor as

\[
F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\varepsilon_{abc}W^b_\mu W^c_\nu
\]

where the \( \varepsilon_{abc} \) are the structure constants for \( SU_L(2) \) (\( \varepsilon_{abc} \) is antisymmetric under the interchange of two neighbour indices and \( \varepsilon_{123} = +1 \)). The coupling constant for \( SU_L(2) \) is denoted by \( g \), and by convention the \( U_Y(1) \) coupling is \( g'/2 \). The weak hypercharge operator for the \( U_Y(1) \) group is called \( Y \).

Now, the "Higgs doublet":

\[
H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}
\]

contains the fluctuation \( \varphi \) of the Higgs field.

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\( \text{Non-linear self-interactions of } W^a_\nu \text{ bosons have never been proved either a priori (it's obvious) or in consequences (so, what's the reason for their existence), hence I will count them to be only the curiosity of the model.} \)

\( \text{In the presented model the non-linear hypothetical ground state configurations of the } W \text{ and } Z \text{ fields might have appeared when having being induced by the interaction with the non-zero fermionic charge density fluctuations.} \)
For the sake of simplicity and transparency I specify only the electron and its neutrino. The contributions from other existing fermions can be treated in a similar way. Here we adopt the notation

\[ L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad R = (e_R). \quad (7) \]

In accord with the above statement, connected with the \( \Psi(x) \) function, let us make the following formal replacement of the uncharged (standard model) SM physical configuration (which is in the neighbourhood of \( \varphi = v \)) with fields \( W^a_\mu, B_\mu, \varphi \) by the charged physical configuration (which is in the neighbourhood of \( \varphi = \delta \), see below) with fields \( W^a_\mu, B_\mu, \varphi \) (LHS of Eq.(8)) which we decompose into (RHS of Eq.(8)) the changing part \( W^a_\mu, B_\mu, \varphi \) and the part in the minimum of the effective potential \( U_{ef} \); (\( v \) and \( \delta \) are the different quantities; \( v \) is a background "field", \( \delta \) is the fluctuation):

\[
\begin{align*}
W^a_\mu &=: W^a_\mu + \omega^a_\mu, \\
B_\mu &=: B_\mu + b_\mu, \\
\varphi &=: \varphi + \delta.
\end{align*}
\quad (8)
\]

Now, the "non-linear" Lagrangian density of electroweak model with hidden \( SU_L(2) \times U_Y(1) \) symmetry is:

\[
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H)^+ D^\mu H - \lambda (H^+ H - \frac{1}{2} v^2)^2 + \mathcal{L}_f, \quad (9)
\]

where \( \mathcal{L}_f \) is the fermionic part, \( \lambda \) and \( v \) are constant parameters.

Field equations for the Yang-Mills fields (with still non-shifted field \( \varphi \)) are following\( (\Box = \partial_\mu \partial^\mu) \), for \( B^\mu \)

\[
- \Box B^\mu + \partial^\mu \partial_\nu B^\nu = -\frac{1}{4} g g' \varphi^2 W^{3\mu} + \frac{1}{4} g'^2 \varphi^2 B^\mu - \frac{g'}{2} j^\mu_Y, \quad (10)
\]

\footnote{The formal shape of Eqs.(10)-(12) would be also true for the external boson fields penetrating just discussed configuration of boson ground fields induced by matter sources.}
for $W^{a\mu}(a = 1, 2)$

\[-\Box W^{a\mu} + g\varepsilon_{abc} W^{b\nu} \partial_\nu W^{c\mu} = \]

\[= g^2 \left( \frac{1}{4} \varphi^2 W^{a\mu} - W^b_\nu W^{b\nu} W^{a\mu} + W^{a\nu} W^b_\nu W^{b\mu} \right) - gj^{a\mu}, \]

and for $W^{3\mu}$

\[-\Box W^{3\mu} + g\varepsilon_{abc} W^{b\nu} \partial_\nu W^{c\mu} = \frac{1}{4} g^2 \varphi^2 W^{3\mu} - \]

\[= \frac{1}{4} gg' \varphi^2 B^\mu - g^2 W^b_\nu W^{b\nu} W^{3\mu} + g^2 W^{3\nu} W^b_\nu W^{b\mu} - gj^{3\mu}. \]

Here $j^\mu_Y$ and $j^{a\mu}$ are to be the continuous, extended in space matter current density fluctuations which are given by the equations (hence $L$ and $R$ fields are the wavefunctions which have not the interpretation connected with the full charges of $L$ and $R$ particles, as in the original linear Dirac equations, but they are connected with the charge density distribution fluctuations of $L$ and $R$ particles, exactly as in the Barut case):

\[j^\mu_Y = \bar{L}\gamma^\mu Y L + \bar{R}\gamma^\mu Y R, \]

\[j^{a\mu} = \bar{L}\gamma^\mu \frac{g^a}{2} L, \text{ where } a = 1, 2, 3. \]

Accordingly, the fluctuation $\varphi$ of the Higgs field satisfies\footnote{not operators of quantum field theory with point like charges.} \footnote{Because the fluctuation of the Higgs field is a fluctuation of an ether hence $\Box \varphi = 0$.}

\[-\Box \varphi = \left( -\frac{1}{4} g^2 W^a_\nu W^{a\nu} - \frac{1}{4} g^2 B^\nu B^\nu + \frac{1}{2} gg' W^3_\nu W^3^\nu \right) \varphi - \]

\[= \lambda \varphi^2 + \lambda \varphi^3 + \frac{m}{v} (\bar{e}_L e_R + \text{h.c.}). \]

\[\text{where } m = \frac{\varepsilon}{2} (\bar{e}_L e_R + \text{h.c.}). \]
3 Boson ground state solutions

Now we are interested in such a configuration of fields that the ground state of boson fields \( W_\mu^a, B_\mu \) and \( \varphi \) (LHS of Eq. (8)) are constant and equal just to (RHS of Eq. (8)) \( \omega_\mu^a, b_\mu \) and \( \delta \) respectively:

\[
\begin{align*}
W_\mu^a &= \omega_\mu^a, \\
B_\mu &= b_\mu, \\
\varphi &= \delta.
\end{align*}
\]

(16)

One can parameterize the ground fields \( \omega_\mu^a \) and \( b_\mu \) in the following homogeneous form:

\[
\omega_\mu^a = \left\{ \begin{array}{ll}
\omega_0^a &= \sigma n^a, \\
\omega_i^a &= \vartheta \varepsilon_{aib} n^b \text{ and } n^a n^a = 1,
\end{array} \right.
\]

\[
b_\mu = \left\{ b_0 = \beta, \\
b_i = 0. \right.
\]

(17) (18)

In Eq. (17) \( (n^a) \) plays the role of the unit vector in the adjoint representation of the Lie algebra \( su(2) \). It chooses a direction for the ground field. It is easy to see that (no summation over an index "a")

\[
\omega_\mu^a \omega_\mu^a = \sigma^2 n^a n^a - \vartheta^2 \varepsilon_{aib} \varepsilon_{aib} n^b n^b \text{ and } b_\mu b_\mu = \beta^2 .
\]

(19)

When we define the "electroweak magnetic field" as \( B_i^a = 1/2 \varepsilon_{ijk} F_{jk}^a \) and the "electroweak electric field" as \( E_i^a = F_{i0}^a \) then, in the homogeneous case \( (\sigma = \text{constant}, \vartheta = \text{constant}, \beta = \text{constant}, (n^a) = \text{constant}) \), we obtain for \( \vartheta \neq 0 \) the "electroweak magnetic ground field \( \langle B_i^a \rangle_0 \)" and the "electroweak" \( \varepsilon_{aib} \varepsilon_{aib} n^b n^b \)
electric ground field \(< \mathcal{E}_i^a >_0 \) in the form
\[
\langle B_i^a \rangle_0 = -g\varphi^2 n^i n^a \quad \text{and} \quad \langle \mathcal{E}_i^a \rangle_0 = g\sigma\varphi(\delta_{ai} - n^a n^i) .
\] (20)

The effective potential of our model is given as the ground state expectation value of the Lagrangian density
\[
\mathcal{U}_{ef} = -\langle \mathcal{L} \rangle_0 ,
\] (21)

So the mean matter current density fluctuations \(J_Y^\mu\) and \(J^a^\mu\) are the ground state expectation values of \(j_Y^\mu\) and \(j^a^\mu\) respectively (see Eqs. (13-15)):
\[
J_Y^\mu = (\langle \bar{L}\gamma^\mu YL \rangle_0 + \langle \bar{R}\gamma^\mu YR \rangle_0 ) \quad \text{and} \quad J^a^\mu = \langle \bar{L}\gamma^\mu \sigma^a 2L \rangle_0 .
\] (22)

\(J_Y^\mu\) and \(J^a^\mu\) are the extended in space quantities.

We now assume that we are in the local rest coordinate system in which
\[
J_Y^0 = \varphi_Y , \quad J_Y^i = 0 , \quad J^a^0 = \varphi^a \quad \text{and} \quad J^{ai} = 0 ,
\] (23)

where \(\varphi_Y\) and \(\varphi^a\) are the matter charge density fluctuations related to \(U_Y(1)\) and \(SU_L(2)\), respectively. Using Eq. (21) with Eqs. (10) – (13) we obtain the ground state part of the effective potential \(\mathcal{U}_{ef}\) for the “boson ground fields induced by matter sources” configuration (hereafter, I will call it bgfms configuration):
\[
\mathcal{U}_{ef}(\vartheta, \sigma, \beta, \delta) = -g^2\varphi^2\vartheta^2 + \frac{1}{2}g^2\vartheta^4 - \frac{1}{8}g^2\delta^2(\sigma^2 - 2\vartheta^2) + \frac{1}{4}gg'\delta^2\beta\sigma n^3 - \frac{1}{8}g^2\delta^2\beta^2 + g\varphi^a n^a \sigma + g'\varphi_Y \beta + \frac{1}{4}\lambda(\delta^2 - v^2)^2 .
\] (24)

Now from the field equations, Eqs. (10) – (14), we can obtain four equations:
\[
\partial_0 \mathcal{U}_{ef} = \partial_\sigma \mathcal{U}_{ef} = \partial_\beta \mathcal{U}_{ef} = \partial_\delta \mathcal{U}_{ef} = 0 .
\] (25)

\(^{14}\) paper \(\mathcal{U}_{ef}\) is drastically reinterpreted by the current one
These equations translate into four algebraic equations for the ground fields \( \vartheta, \sigma, \beta \) and \( \delta \):

\[
\left[ \frac{1}{2} \delta^2 - 2\sigma^2 + 2\vartheta^2 \right] \vartheta = 0 , \quad (26)
\]

\[-g(2\vartheta^2 + \frac{1}{4} \delta^2)\sigma + \frac{1}{4} g'\delta^2 \beta n^3 + g^a n^a = 0 , \quad (27)
\]

\[\frac{1}{2} (g\sigma n^3 - g' \beta) \delta^2 + \varphi^r = 0 , \quad (28)
\]

\[
\left[ -\frac{1}{4} g^2 (\sigma^2 - 2\vartheta^2) + \frac{1}{2} gg' \sigma \beta n^3 - \frac{1}{4} g'^2 \beta^2 + \lambda (\delta^2 - \nu^2) \right] \delta = 0 . \quad (29)
\]

These equations are the screening charge analog of the screening current condition in electromagnetism \[17\].

Now let us choose

\[(n^a) = (0, 0, 1) . \quad (30)\]

In this case we have an "electroweak magnetic ground field" different from zero
\(< B^3 >_0 = -g\vartheta^2 \) pointed in the \(x^3\) spatial direction and "electroweak electric
ground fields" different from zero \(< E^1 >_0 = < E^2 >_0 = g\sigma\vartheta \) pointed in the \(x^1\) and \(x^2\) spatial direction respectively.

Using Eqs. \((10) - (15)\) together with Eqs. \((16)-(19)\) and Eq. \((30)\) we obtain
in the ground state of the bgfms configuration the square masses of the boson
fields as follows\[15\]:

\[m^2_{W1,2} = g^2 \left( \frac{1}{4} \delta^2 - \sigma^2 + \vartheta^2 \right) , \quad (31)\]

\[m^2_{W3} = g^2 \left( \frac{1}{4} \delta^2 + 2\vartheta^2 \right) , \quad (32)\]

\[m^2_B = \frac{1}{4} g'^2 \delta^2 , \quad (33)\]
\[ \Delta_{\varphi}^2 = \mathcal{N} \times f(\sigma, \vartheta, \beta, \delta), \quad (34) \]

where \( \Delta_{\varphi} \) (or rather \( \mathcal{N} \)) is a free parameter of the model. Eqs.\((31)\) to \((34)\) together with Eqs.\((26)\) to \((29)\) is the ground state screening current condition in the electroweak analog of the electromagnetic case \(^{17}\).

Let us perform (for \( \delta \neq 0 \)) a "rotation"\(^{17}\) of \( W_3^\mu \) and \( B_\mu \) fields to the physical fields \( Z_\mu \) and \( A_\mu \)

\[
\begin{pmatrix}
Z_\mu \\
A_\mu
\end{pmatrix} =
\begin{pmatrix}
cos \Theta & -sin \Theta \\
-\sin \Theta & \cos \Theta
\end{pmatrix}
\begin{pmatrix}
W_3^\mu \\
B_\mu
\end{pmatrix},
\]

and a "rotation" of \( \sigma \) and \( \beta \) ground fields to their counterparts \( \zeta \) and \( \alpha \) as well as a "rotation" of the charge density fluctuations \( \varrho^3 \) and \( \varrho_Y \) to the corresponding physical quantities \( \varrho_Z \) and \( \varrho_Q \)

\[
\begin{pmatrix}
\zeta \\
\alpha
\end{pmatrix} =
\begin{pmatrix}
cos \Theta & -sin \Theta \\
-\sin \Theta & \cos \Theta
\end{pmatrix}
\begin{pmatrix}
\sigma \\
\beta
\end{pmatrix},
\]

\[
\begin{pmatrix}
(g/cos \Theta) \varrho_Z \\
(g \sin \Theta) \varrho_Q
\end{pmatrix} =
\begin{pmatrix}
cos \Theta & -sin \Theta \\
-\sin \Theta & \cos \Theta
\end{pmatrix}
\begin{pmatrix}
(g) \varrho_a n^a \\
(g'/2) \varrho_Y
\end{pmatrix}.
\]

Now using Eqs.\((31)\) to \((34)\) and defining the \( W^\pm \) fields as \( W^\pm = (W^1 \mp iW^2)/\sqrt{2} \) we can rewrite the square masses of the boson fields\(^{18}\) as follows:

\[ m_{W^\pm}^2 = g^2 \left[ \frac{1}{4} \delta^2 - (\zeta \cos \Theta + \alpha \sin \Theta)^2 + \varrho^2 \right], \quad (39) \]

\(^{16}\) When \( \varphi \equiv \text{constant} \), so \( \varphi = 0 \), as it is in the case of the ether, then the mass of the Higgs field fluctuation is a free parameter, \( \Delta_{\varphi} \), of the model. Hence \( \Delta_{\varphi} \) could not be established in this model; therefore the Higgs field fluctuation is not an elementary particle (? in this model.

\(^{17}\) The standard model relations between the Weinberg angle \( \Theta_W \), \( g \) and \( g' \) are following:

\[
\cos \Theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad \text{and} \quad \sin \Theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}.
\]

Some quantum numbers of the electroweak \( SU_2(2) \times U_Y(1) \) model are given in Table.

\(^{18}\) Because masses (and structures) of bosons change we should denote this fact somehow or other. Hence we reserve the notation \( Z^0 \) (with the upper index 0) for the boson in the Standard Model case whereas \( Z \) for this boson on the ground state of the system with non-zero matter charge density fluctuations.
\[ m_Z^2 = \frac{1}{2} \left[ m_{Z^0\,SM}^2 + 2g^2\delta^2 + \sqrt{(m_{Z^0\,SM}^2 + 2g^2\delta^2)^2 - 2(gg'\delta\dot{\vartheta})^2} \right], \quad (40) \]

\[ m_A^2 = \frac{1}{2} \left[ m_{Z^0\,SM}^2 + 2g^2\delta^2 - \sqrt{(m_{Z^0\,SM}^2 + 2g^2\delta^2)^2 - 2(gg'\delta\dot{\vartheta})^2} \right], \quad (41) \]

\[ \Delta^2 \varphi = N \times f(\zeta, \vartheta, \alpha, \delta), \quad (42) \]

where \( m_{Z^0\,SM}^2 \) is the standard counterpart for the boson \( Z^\mu \) square mass which is equal to

\[ m_{Z^0\,SM}^2 = \frac{1}{4}(g'^2 + g^2)\delta^2. \quad (43) \]

It is illustrative to write the relations between weak isotopic charge density fluctuation \( \varrho^3 \) (see Eq.(23) and Eq.(30)), weak hypercharge density fluctuation \( \varrho_Y \), (below defined, Eq.(46)) standard\(^{19}\) electric charge density fluctuation \( \varrho_{Q\,SM} \), (below defined, Eq.(46)) standard\(^{20}\) weak charge density fluctuation \( \varrho_{Z^0\,SM} \) and their generalizations in our model i.e. the electric charge density fluctuation \( \varrho_Q \) and weak charge density fluctuation \( \varrho_Z \):

\[ \varrho_Q = \varrho_{Q\,SM} + \frac{1}{2}(\frac{g'}{g}ctg\Theta - 1)\varrho_Y, \quad (44) \]

\[ \varrho_Z = \varrho^3 - \varrho_Q \sin^2\Theta, \quad (45) \]

\[ \varrho_{Q\,SM} = \varrho^3 + \frac{1}{2}\varrho_Y \quad \text{and} \quad \varrho_{Z^0\,SM} = \varrho^3 - \varrho_{Q\,SM} \sin^2\Theta_W. \quad (46) \]

Here the \( \Theta \) angle is the modified mixing angle given by the formula

\[ tg\Theta = \left[ \frac{- (1 + 8(\vartheta/\delta)^2)g^2 + g'^2}{2gg'} + \sqrt{\left( \frac{(1 + 8(\vartheta/\delta)^2)g^2 - g'^2}{2gg'} \right)^2 + 1} \right]. \quad (47) \]

It is not difficult to see that electroweak assumptions are formally recovered in the limit \( \vartheta \to 0 \) (and extended \( Q = 0 \)). It is evident from Eq.(47) that

\(^{19}\) unscreened
\(^{20}\) unscreened
transition from zero charge density fluctuations to $g^3 \neq 0$, $\vartheta_Y \neq 0$ is associated with a non-linear response of the system.

4 Results

The calculations below are done for the boson fields in extrema of the effective potential $U_{ef}$. It is not difficult to see that the solutions of Eqs. (26)-(29) for boson ground fields in the extrema of the effective potential $U_{ef}$ split into cases discussed just below.

4.1 Ground fields $\vartheta \neq 0$ and $\delta \neq 0$

Eqs. (26)-(29) can be now rewritten as follows:

\[ \sigma = \frac{1}{2g}\vartheta Q_{SM}, \tag{48} \]
\[ \beta = \frac{1}{g^2}(g\sigma n^3 + 2\frac{\vartheta Y}{\delta^2}), \tag{49} \]
\[ \vartheta^6 + \frac{1}{4}\delta^2\vartheta^4 - \frac{1}{4g^2}\vartheta^2 Q_{SM} = 0, \tag{50} \]
\[ \delta^6 + \left(\frac{g}{2\lambda}\vartheta^2 - v^2\right)\delta^4 - \frac{1}{\lambda}\delta^2 = 0. \tag{51} \]

From Eq. (50) we see that the ground field $\vartheta$ is non-zero only if $\vartheta Q_{SM} \neq 0$.

Let us notice that the relation between the weak hypercharge quantum number $Y$ and the electromagnetic charge quantum number $Q$ can be written in the form $Q = pY/2$ (for matter fields), where the corresponding values of $p$ ($p \neq 0$) are given in Table. Then the relation between the weak hypercharge density fluctuation $\vartheta_Y$ and the standard electromagnetic charge density fluctuation $\vartheta Q_{SM}$ can be written in the similar form

\[ \vartheta Q_{SM} = p\frac{\vartheta_Y}{2}. \tag{52} \]
Using Eq.(52) we solved numerically Eqs.(48)-(51) and we obtained the ground fields squared $\vartheta^2$ and $\delta^2$ as the functions of $\varrho_{Q SM}$ with $p$ as a parameter. Different values of $p$ (see Table) represent different matter fields which could be the sources of charge densities. The results of solving Eqs.(48)-(51) for the $\alpha$ and $\zeta$ ground fields (see Eq.(37)) and the $\vartheta$ and $\delta$ ground fields are shown in Figure 1-Figure 4.

Now Eq.(16) has the form:

$$
\begin{align*}
W_{0,3}^\pm &= 0, & W_1^\pm &= \pm i\vartheta/\sqrt{2}, & W_2^\pm &= \vartheta/\sqrt{2}, \\
Z_i &= 0, & Z_0 &= \zeta, & (\zeta = \sigma\cos\Theta - \beta\sin\Theta), \\
A_i &= 0, & A_0 &= \alpha, & (\alpha = \sigma\sin\Theta + \beta\cos\Theta), \\
\varphi &= \delta.
\end{align*}
$$

The masses of the $Z^0$ and $A$ were calculated according to Eqs.(40) and (41) and the appropriate results are shown in Figs.5 - 6. The masses of the $W^\pm$ fields are, according to Eq.(26) and Eq.(31) (for the $\vartheta \neq 0$ configuration of fields), equal to zero.

The results for the ratio $\sin\Theta/\sin\Theta_W$ (see Eq.(17)) and the physical charge density fluctuation $\varrho_Q$ (see Eq.(44)) for boson ground fields given by Eqs.(48)-(51) as functions of $\varrho_{Q SM}$ are presented in Figure 7 and Figure 8, respectively.

In all the figures the curves for different values of $p$ converge for relatively small values of $\varrho_{Q SM}$ (i.e., for values of $\varrho_{Q SM}$ in the range up to values approximately $10^3$ times bigger than those which correspond to matter densities in nucleon matter). In that range of values for $\varrho_{Q SM}$ we have also that $\varrho_Q \approx \varrho_{Q SM}$ (see Figure 8). For these reasons the following calculations in that regime were done for $p = 1$ (for other $p \neq 0$ in Table it would be the same).

The minimal energy density of the bgfms configuration $E_{min}(\varrho_{Q SM}) = U_{ef}(\vartheta \neq 0, \delta \neq 0)$ (see Eq.(24)) for boson ground fields given by Eqs.(48)-(51)
as functions of $\varrho_{Q\,SM}$ is presented in Figure 9. For big value of charge density fluctuation $\varrho_{Q\,SM}$ (i.e., for $\varrho_{Q\,SM}$ which corresponds to matter densities approximately $10^3$ times bigger than characteristic for static nucleon matter) the minimal energy density $E_{\text{min}}(\varrho_{Q\,SM})$ is extremely big and is increasing very rapidly with $\varrho_{Q\,SM}$. For example, $E_{\text{min}} \approx 2.9 \times 10^{178} \, \text{GeV}^4$ for $\varrho_{Q\,SM} \approx 1.3 \times 10^7 \, \text{GeV}^3$.

For this reason a charged star could effectively resist against the gravitational collapse. Even any global (in a star) electric charge density distribution fluctuations in a collapsing uncharged star\footnote{\textit{Few years ago, Marek Biesiada claimed that "irresistible" gravitational collapse should not be realized.}} could resist the gravitational force. It would be also true for an electrically charged elementary particle if it has electroweak structure; hence, it could be the reason that the wavefunction (defined by the charge density distribution) does not collapse.

It is very interesting that when we investigate the function $E_{\text{min}}(\varrho_{Q\,SM})$ more carefully then a subtle structure emerges. It appears a "stable" (bgfms) configuration of charge density fluctuation with $\varrho_{Q\,SM} \neq 0$ (see Figure 9) different from that for the standard "linear" model (with charge density fluctuation $\varrho_{Q\,SM} = 0$ and $E_{\text{min}}(0) = 0$). The numerical calculations for the value of the local minimum of the function $E_{\text{min}}(\varrho_{Q\,SM})$ reveal little dependence on the $\lambda$ parameter of the Higgs potential (see Figure 9) and the results are following:

$$E_{\text{min}}(\varrho_{Q\,SM}) \approx (2.5811 \, \text{GeV})^4 \text{ for } \varrho_{Q\,SM} \approx 0.5539 \, \text{GeV}^3. \quad (54)$$

This bgfms configuration is separated from the static standard model configuration (ground state of the standard model) by a high barrier $\Delta E_{\text{min}}$ which depends on the $\lambda$ parameter (see Figure 9). For example when $\lambda = 1$ then $\Delta E_{\text{min}} \approx (180 \, \text{GeV})^4$. It is not difficult to see that $E_{\text{min}} \to 0$ as $\varrho_{Q\,SM} \to 0$.
for all considered values of $\lambda > 0$ and $p \neq 0$ (see Table).

When we notice that the mass of an electrically charged bgfms configuration with the "radius of the charge fluctuation" $r_q$ is equal to $M_q = 4/3 \pi r_q^3 \times E_{\text{min}}(\varrho_{Q SM})$, and that the matter global electric charge fluctuation $q = 4/3 \pi r_q^3 \varrho_{Q SM}$ then from Eq.(24) and Eqs.(48)-(51) we obtain $M_q \rightarrow \pm qge/2 = \pm q \times 80.13 \text{ GeV}$ (sign "+" for $Q > 0$, sign "-" for $Q < 0$) as $\varrho_{Q SM} \rightarrow 0$ for all considered values of $\lambda > 0$ and $p \neq 0$ (see Table). The function $M_{q=1}(r_q)$ is presented in Figure 10. These configurations of fields lie only on the $M_q - r_q$ curve ($M_q = \pm qM_{q=1}$). For example, a droplet of the new bgfms configuration of fields with charge fluctuation $q$ and described by Eq.(54) will have the "radius of the charge fluctuation" $r_q = q^{1/3} \times 0.149$ fm (in comparison, for proton with full electric charge $Q = 1$ its global electric charge radius $r_Q \approx 0.805$ fm) and the mass $M_q \approx \pm q \times 80.13$ GeV. If one takes into account the mass of a fermion (fermions) playing the role of matter source inducing boson ground fields, then the value of the mass $M_q$ will change of about the order of the mass of this part of a fermion (fermions) which is contained in the region of the fluctuation.

In Figs.3-7 and Figs.9-10 the curves corresponding to $p$ and $-p$ are the same.

Now a few comments are in order. From the Eqs.(39)-(42), Eqs.(48)-(51) and Eq.(53) we can notice that fields $W^+$ and $W^-$ taken together as a pair of massive fields become (in this bgfms configuration of fields) a kind of massless self field and a ground field which is coupled to charged fields with charge density fluctuations $\varrho_{Q SM} \neq 0$ and $\varrho_Y \neq 0$. When these charge density fluctuations go to zero then the $W^+ - W^-$ ground field also goes to zero ($\vartheta \rightarrow 0$). The
ground fields of $Z^\mu$ and $A^\mu$ given by $\zeta$ and $\alpha$ (see Eq.(53)) are nonzero even for 
$\varrho_{Q \, SM} \to 0$ and $\varrho_{Y} \to 0$.

Because of the nonlinear terms in the field equations there appeared the 
screening charge problem \[17\] which is very essential in this paper. Now, the 
role of the $|\psi|$ part of the wave function $\psi$ is played not only by fluctuations 
of matter (fermion) fields but by fluctuations of weakly charged scalar $\varphi$ (and 
global gauge ground fields $\zeta$ and $\alpha$). These fluctuation fields together with the 
global gauge ground fields form a system characterized by wave functions $\varphi$, 
$Z^\mu$ and $A^\mu$ which are "macroscopic" in its spatial extension. When, as in this 
case, we have the Higgs fluctuation field $\varphi$, $Z_0$-ground field and $A_0$-ground field 
in the ground state of the system with their ground state expectation values 
$< \varphi >_0 = \delta \neq 0$, $< Z_0 >_0 = \zeta \neq 0$ and $< A_0 >_0 = \alpha \neq 0$ respectively, then in 
the presence of the $W^\mu$ fields the electroweak force field generates "electroweak 
screening charges" connected with the fact that both the basic fermion field and 
Higgs field carry the nonzero charge densities.

4.2 Ground fields $\vartheta = 0$ and $\delta \neq 0$

Using Eqs.(37) – (38) we can rewrite the effective potential $U_{ef}$ (Eq.(24)) in a 
very simple form:

$$U_{ef}(\zeta, \alpha, \delta) = -\frac{1}{8}(g^2 + g'^2)\delta^2\zeta^2 + \varrho_{Z_0 \, SM}\zeta + \varrho_{Q \, SM}\alpha +$$

$$+ \frac{1}{4}\lambda(\delta^2 - v^2)^2.$$  \hspace{1cm} (55)

Now, according to Eqs.(25)-(29) we have $\partial_\alpha U_{ef} = \partial_\zeta U_{ef} = \partial_\delta U_{ef} = 0$ which 
yields

$$\varrho_{Q \, SM} = 0,$$  \hspace{1cm} (56)
\[
\frac{1}{4} \sqrt{g^2 + g'^2} \delta^2 \zeta = \varrho_{Z^0} \text{SM} \tag{57}
\]

and

\[
\lambda (\delta^2 - v^2) - \frac{1}{4} (g^2 + g'^2) \zeta^2 = 0. \tag{58}
\]

Non-zero weak charge density fluctuation \(\varrho_{Z^0} \text{SM}\) leads inevitably to a non-zero \(\zeta\) ground field which implies \(\delta \neq 0\). From Eq. (56) and Eqs. (44)-(47) we can also notice that \(\varrho_Z = \varrho_{Z^0} \text{SM}\) (for \(\vartheta = 0\)).

Now, combining Eqs. (56) and (57) with Eq. (55) we obtain the ground state counterpart of the electroweak effective potential for \(\vartheta = 0\) (see Figure 11)

\[
U_{ef}(\delta, \varrho_{Z^0} \text{SM}; \vartheta = 0, \varrho_Q \text{SM} = 0) = \frac{2 \varrho_{Z^0} \text{SM}}{\delta^2} + \frac{1}{4} \lambda (\delta^2 - v^2)^2. \tag{59}
\]

The solution of Eqs. (57)–(58) leads to

\[
\zeta(\varrho_{Z^0} \text{SM}) = \frac{2 \frac{3\lambda}{\sqrt{g^2 + g'^2}}} \times \left[ \sqrt{\varrho_{Z^0} \text{SM}} + \sqrt{\varrho_{Z^0} \text{SM} + \frac{\lambda v^2}{2\delta^2}} + \sqrt{\varrho_{Z^0} \text{SM} - \sqrt{\varrho_{Z^0} \text{SM} + \frac{\lambda v^2}{2\delta^2}}} \right] \geq 0 \tag{60}
\]

and

\[
\delta^2(\varrho_{Z^0} \text{SM}) = \frac{4 \varrho_{Z^0} \text{SM}}{\sqrt{g^2 + g'^2} \zeta}, \tag{61}
\]

where \(\zeta\) and \(\delta^2\) are the functions of \(\varrho_{Z^0} \text{SM}\) only. It is not difficult to see that in the limit \(\varrho_{Z^0} \text{SM} \rightarrow 0\) (implying \(\zeta \rightarrow 0\) and \(\delta \rightarrow v\)) the well-known electroweak configuration \(\delta = v\) with \(U_Q(1)\) symmetry emerges.

Using Eqs. (17)-(18) and Eqs. (36)-(37) we can rewrite Eq. (10) for the physical field \(A_\mu\) in the form

\[
A_\mu = (\alpha, 0, 0, 0). \tag{62}
\]
Let us notice from Eqs. (56)-(59) that $\alpha$ is not a dynamical parameter so a transformation $0 \rightarrow \alpha$ acquires the interpretation of a gauge transformation. Here the $\alpha$ ground field corresponds to a nonphysical degree of freedom (this is connected with the fact that $\rho_{QSM} = 0$) and it can be removed by an appropriate gauge transformation. So the $U_Q(1)$ group remains untouched and it gives us

$$\alpha = \sigma \sin \Theta_W + \beta \cos \Theta_W = 0 \ . \quad (63)$$

Now we have the result that the ground fields in Eq. (16) can be rewritten as follows:

$$\begin{align*}
W^{1,2}_\mu &= 0 \ , \ W^3_\mu = 0 \\
W^3_\beta &= -\beta \cot \Theta_W \\
B_0 &= \beta \\
B_i &= 0 \\
\varphi &= \delta \ .
\end{align*} \quad (64)$$

or in terms of physical ground fields

$$\begin{align*}
W^{\pm}_\mu &= 0 \ , \ Z_i = 0 \\
Z_0 &= \zeta \quad \text{where} \quad (\zeta = -\frac{1}{\sin \Theta_W} \beta) \\
A_\mu &= 0 \\
\varphi &= \delta \ .
\end{align*} \quad (65)$$

The appearance of the non-zero value of the weak charge density fluctuation $\rho_{Z^0_{SM}}$ and $\zeta$ boson ground state field induced by it (see Eq. (60)) influences the masses of the fields in the model and from Eqs. (39)-(42) ($\vartheta = 0$ and $\alpha = 0$) we obtain (see Figs. 12-13):

$$m^2_{W^\pm} = \frac{1}{4}g^2 \delta^2 - g^2 \zeta^2 \cos^2 \Theta_W \ , \quad (66)$$

$$m^2_{Z^0} = \frac{1}{4}(g^2 + g'^2) \delta^2 \ , \quad (67)$$

$$m^2_A = 0 \ , \quad (68)$$

$$\Delta^2_\varphi = N \times f(\zeta, \delta) \ . \quad (69)$$
Let us notice that the effective mass of the physical field $A_\mu$ is $m_A^2 = 0$.

The minimal energy density of the bgfms configuration $E_{\text{min}}(q_{Z^0 \text{SM}}) = U_{\text{ef}}(\vartheta = 0, \delta \neq 0)$ is (see Figure 14):

$$E_{\text{min}}(q_{Z^0 \text{SM}}) = \frac{1}{2} \frac{\vartheta}{g^2 + g'^2} q_{Z^0 \text{SM}} + \frac{1}{64 \lambda} (g^2 + g'^2) \vartheta^2 \zeta^4. \quad (70)$$

From the Eq. (66) it is clear that the appearance of $q_{Z^0 \text{SM}} > 0$ (so the boson ground field $\zeta > 0$) leads to the instability in the $W^\pm_\mu$ sector if

$$\zeta^3(q_{Z^0 \text{SM}}) > \frac{\sqrt{g^2 + g'^2}}{g^2} q_{Z^0 \text{SM}}. \quad (71)$$

When the equality $\zeta^3(q_{Z^0 \text{SM}}) = q_{Z^0 \text{SM}} \sqrt{g^2 + g'^2}/g^2$ is taken into account we obtain the relationship between $\lambda_{\text{max}}$ and $q_{Z^0 \text{max}}$, where $\lambda_{\text{max}}$ is the value of $\lambda$ and $q_{Z^0 \text{max}}$ is the value of $q_{Z^0 \text{SM}}$ for which we have $m_{W^\pm}^2 = 0$ (see Figure 15). The region of possible bgfms configurations with $\zeta \neq 0$ is on and below the $\lambda_{\text{max}} - q_{Z^0 \text{max}}$ curve.

For weak charge density fluctuation $q_{Z^0 \text{SM}} \leq gv^3/(8 \cos^2 \Theta_W) \approx 1.655 \times 10^6 \text{ GeV}^3$ this configuration of fields is stable for an arbitrary $\lambda$ (see Figure 15). For values of $q_{Z^0 \text{SM}}$ bigger than $1.655 \times 10^6 \text{ GeV}^3$, this configuration of fields for given $\lambda$ will be destabilized at certain value of $q_{Z^0 \text{SM}} = q_{Z^0 \text{max}}$ and the system could reach the charged (with $q_{Q \text{SM}} \neq 0$) stable configuration of fields with $\vartheta \neq 0$. For $\lambda < g^2/(16 \cos^4 \Theta_W) \approx 0.0422$ the configuration of fields is stable for all values of weak charge density fluctuations $q_{Z^0 \text{SM}}$ (see Figure 15).

We can also examine the mass $M_{i3} = 4/3 \pi r_{i3}^3 \varrho_{\text{min}}(q_{Z^0 \text{SM}})$ (see Figure 16) of a bgfms configuration with non-zero weak charge density fluctuation. Here $r_{i3}$ is the ”radius of the charge fluctuation” of this configuration which has the matter global weak isotopic charge fluctuation $i^3 = 4/3 \pi r_{i3}^3 q_{Z^0 \text{SM}}$. 

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Taking into account the mass of a fermion (fermions), playing the role of matter source inducing boson ground fields, will change the value of the mass \( M_{I^3} \) of about the order of the mass of this part of a fermion (fermions) which is contained in the region of the fluctuation. These configurations of fields lie only on the \( M_{i^3} - r_{i^3} \) curve, where \( M_{i^3} = \pm i^3 \times M_{I^3=1} \) (sign "+" for \( I^3 > 0 \), sign "-" for \( I^3 < 0 \)). The function \( M_{i^3=1}(r_{i^3}) \) is presented in Figure 16. In the case of neutron (or neutrino) its mass \( m_{n_{22}} \) (or \( m_{\nu} \)) corrects the mass \( M_{i^3} \) of the droplet by the value of \( \sim m_n \) (or \( m_{\nu} \)). Hence according to Figure 16 the mass of the droplet lies on the curve which gives its mass which is slightly above the mass \( m_n \). The mass of the droplet (\( \sim m_n + \sim 1 \text{ keV} \)) lies almost in the region of uncertainty of the neutron mass \( m_n \approx 0.9396 \text{ GeV} \). In the case of a neutrino the mass of a connected droplet might be even bigger depending on its radius (which is presumed to be very small). But the most important fact is that the physical ground fields \( \delta \) and \( \zeta \) (see Eq. (65)) are present in the droplet region - the droplet is the configuration of fields. Because of this the droplet cannot decay to neutron and photon unless the energy equal to the sum of the mass of the \( Z \) particle and the value of \( |\Delta \varphi| \) outside the droplet is supplied to the droplet.

We can also obtain the upper (according to the stability of this configuration of fields within the \( W^\pm \) sector) limit \( M_{i^3,\text{max}} \) for the value of the mass \( M_{i^3} \) with the region of possible bgfms configurations which lie on and below the \( M_{i^3,\text{max}} - \lambda_{\text{max}} \) curve (see Figure 17).

\[^{22}\] It is energetically more favorable for the fluctuation to appear in the whole region of neutron \( (r_{i^3} = r_Z) \) than in its part (our model is too simple to prove it).
With the experimental knowledge of the mass $M_{i,\text{max}} \sim \text{mass of a host fermion}$, and using this curve (see Figure 17) the value of $\lambda$ can be calculated. The function $M_{i=1}(r_i)$ is also presented in Figure 18.

In conclusion, when extended $I^3 \neq 0$ and the weak charge density fluctuation is non-zero $\varrho_{Z^0_{SM}} \neq 0$ (but $\varrho_{Q_{SM}} = 0$) then the field configuration with $Z_\mu$ gauge ground field exists. The asymptotic case $\varrho_{Z^0_{SM}} \to 0$ produces the electroweak assumptions (extended $I^3 = 0$) with and $\delta = v$.

### 4.3 The droplet and the process of pair production $\gamma + \gamma \to e^+ + e^-$

Now very important fact should be noticed. It is clear that in droplets of bosonic ground state fields induced by $\varrho_{Z^0_{SM}} \neq 0$ (but $\varrho_{Q_{SM}} = 0$) the conversion $\gamma + \gamma \to e^+ + e^-$ is not allowed. The reason is very simple. Let us imagine that an electrically charged particle (such as $e^+$ or $e^-$) appears inside the droplet of this configuration of fields. Because it has non-zero value of $Q$ (in the extended form), it should give rise to the appearance (see Section 4.1, Figure 10) of a droplet with the electric charge $Q = (1 + \text{charge fluctuation } q)$ for $e^+$ (or $Q = (-1 + \text{charge fluctuation } q)$ for $e^-$) and the mass equal at least to $M_q \approx q \times 80.13 \text{ GeV}$. Now we recall that inside the initial droplet with $\varrho_{Z^0_{SM}} \neq 0$ (but $\varrho_{Q_{SM}} = 0$) a photon has the effective mass equal to zero (see Eq.(18)). Hence the droplet with $\varrho_{Z^0_{SM}} \neq 0$ (but $\varrho_{Q_{SM}} = 0$) is transparent for photons observed in gamma-ray bursts [18].
5 Conclusions

In the present paper the "non-linear" self-consistent theory of classical fields in the electroweak model has been proposed. Homogeneous boson ground state solutions in the GSW model at the presence of a non-zero extended fermionic charge densities have been reviewed and fully reinterpreted to make the theory with non-zero charge densities [8] sound. Consequences of charge density fluctuations are proposed [21].

But in order to understand the model in the broader context let us for a moment simplify our considerations taking into account a real scalar field theory model defined by the following Lagrangian density:

\[ \mathcal{L} = \frac{1}{2} \dot{\phi}^2(x, t) - \frac{1}{2} (\nabla \phi(x, t))^2 - V(\phi) \]  \( (72) \)

where \( \dot{\phi} = \frac{\partial \phi}{\partial t} \), \( \nabla \phi = \sum_i \vec{i} \frac{\partial \phi}{\partial x^i} \) (\( \vec{i} \) is the versor).

\( V(\phi) \) is a function of \( \phi \) and the dependence on the coupling constant \( g \) is given by

\[ V(\phi) = \frac{1}{g^2} \tilde{V}(\chi), \quad \chi = g\phi \]  \( (73) \)

where \( \tilde{V} \) is an even function which is independent of \( g \). Depending on the choice of the functional form of \( \tilde{V} \) one can consider various models. One of its realization is presented on Figure 19.

The Hamiltonian density derived from the Lagrangian of Eq.(72) is given by

\[ \mathcal{H} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \]  \( (74) \)

Let \( \phi_0 \) be the scalar timeless field solution of the equation of motion for the field \( \phi \) in the ground state of the system given by this Hamiltonian. I use the name of a scalar ground field for the solution \( \phi_0 \).
When we are interested in the Lagrangian density

\[ \mathcal{L} = \bar{\Psi} (\gamma^\mu i \partial_\mu - m) \Psi + J^\mu A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]  

(75)

where \( J^\mu = -e \bar{\Psi} \gamma^\mu \Psi \) is the electron current density fluctuation and \( A_\mu \) is the total electromagnetic field, four-potential \( A_\mu = A_\mu^e + A_\mu^s \), with the superscripts \( e \) and \( s \) standing for external field and self field (which is adjusted by the radiative reaction to suit the electron current and its fluctuations, see [4]), respectively, then, in the minimum of the corresponding total Hamiltonian, the solution of the equation of motion for \( A^s_\mu \) is called electromagnetic ground field.

More generally we have used the name of a boson ground field for a solution of an equation of motion for a boson field in the ground state of a whole system of fields (fermion, gauge boson, scalar) under consideration. This boson field is the self field (or can be treated like this) when it is coupled to a source—"basic" field. By "basic" field we have meant a wave (field) function which is proper for a fermion, a scalar or a heavy boson. This concept of a wave function and the Schrödinger wave equation is dominant in the nonrelativistic physics of atoms, molecules and condensed matter. In the relativistic quantum theory this notion had been largely abandoned in favour of the second quantized perturbative Feynman graph approach, although the Dirac wave equation is used approximatively in some problems. What was done by Barut and others was an extension of the Schrödinger’s "charge density interpretation" of a wave function\[23\] to a "fully-fledged" relativistic theory. They implemented successfully this "natural (fields theory) interpretation" of a wave function in many specific problems with coupled Dirac and Maxwell equations (for characteristic boundary conditions).

\[23\] Electron is a classical distribution of charge.
But the "natural interpretation" of the wave function could be extended to the Klein-Gordon equation \[22\] coupled to Einstein field equations, thus being a rival for quantum gravity in its second quantization form. In both cases the second quantization approach is connected with the probabilistic interpretation of quantum theory, whereas the "natural interpretation" together with the self field concept goes in tune with the deterministic interpretation composing a relativistic, self-consistent field theory.

To summarize: Depending on the model, the role of a self field can be played by electromagnetic field \[9, 10, 11, 12, 13, 14, 15, 16\], boson \(W^+ - W^-\) ground-field (this paper, for example), or by the gravitational field (metric tensor) \(g_{\mu\nu}\). The main law for arising of these self fields would be taking the lead existence of "basic" fields.

Now, in view of this language, let us conclude what have been done in the present paper. In the presence of the external matter sources boson Higgs ground field and gauge ground fields were examined. In general, we notice two physically different configurations of fields. When the charge density fluctuation \(\varrho_{Q\,SM}\) is not equal to zero, then bgfms (boson ground fields induced by matter sources) classes \[24\] of configurations of fields with \(\vartheta \neq 0\) and \(\delta \neq 0\) exist. In this configuration of fields the \(W^{\pm}_{\mu}\) bosons are massless while the electromagnetic fields \(A_{\mu}\) and bosons \(Z_{\mu}\) are massive. The appearance of the mass of the \(A_{\mu}\) field is the result of the screening effect \[17\]. We observe very deep energy

\[24\] Crossing from one bgfms class (they are characterized by values of ground fields) of configuration of fields to another class which has different quantum number is somehow forbidden. So, also, one bgfms droplet with certain values of quantum numbers does not convert to another configuration of fields with different quantum numbers. Hence, for example, an electrically neutral bgfms droplet might not convert itself to one of its predecessors (to neutron in neutron star - for example).
density minimum (see Figure 9) with $E_{min} \approx 44.382 \text{ GeV}^4$ and the charge density fluctuation $\rho_{Q\ SM} \approx 0.5539 \text{ GeV}^3$ for which we obtained a droplet of this configuration of field with the "radius of the charge fluctuation" $r_q \approx q^{1/3} \times 0.149 \text{ fm}$ and the mass (in the thin wall approximation) $M_q = \pm q \times 80.13 \text{ GeV}$ (for matter global electric charge fluctuation equal to $q$). Hence droplets of this configuration of fields could be experimentally observed by their very small ratio $|q|/M_q \leq 1/80.13 \text{ GeV}^{-1}$. The mass of a droplet of this configuration of fields $M_q \rightarrow \pm q g v/2 = \pm q \times 80.13 \text{ GeV}$ as $\rho_{Q\ SM} \rightarrow 0$ for all values of $\lambda > 0$. These bgfms configurations lie on the $M_q - r_q$ curve (see Figure 10) or equivalently on the $E_{min} - \rho_{Q\ SM}$ curve (see Figure 9) only.

When $\rho_{Q\ SM} = 0$ and $\rho_{Z^0\ SM} \neq 0$ then the second configuration of fields ($\vartheta = 0$ and $\zeta \neq 0$), with the $Z_\mu$ gauge ground field, exists. When $\rho_{Z^0\ SM} \rightarrow 0$ then this case gives the electroweak assumptions of the GSW model with $\delta = v$. Now the region of possible bgfms configurations lies on and below the $\lambda_{max} - \rho_{Z^0\ max}$ curve (see Figure 15). For $\lambda < g^2/(16 \cos^4 \Theta_W) \approx 0.0422$, the configuration of fields is stable for all values of the weak charge density fluctuation $\rho_{Z^0\ SM}$. However, it is stable for an arbitrary $\lambda$ only if $\rho_{Z^0\ SM} \leq g v^3/(8 \cos^2 \Theta_W) \approx 1.655 \times 10^6 \text{ GeV}^3$. For $\rho_{Z^0\ SM} > \rho_{Z^0\ max} \approx 1.655 \times 10^6 \text{ GeV}^3$ this configuration of fields, for given $\lambda$, will be destabilized at certain value of $\rho_{Z^0\ SM} = \rho_{Z^0\ max}$ and the system could reach the charged (with $\rho_{Q\ SM} \neq 0$) stable configuration of fields with $\vartheta \neq 0$.

The further evolution of the system for $\rho_{Q} \neq 0$ and $\vartheta \neq 0$ seems to be very interesting. It may lead, in the electromagnetic field outside of the droplet of this bgfms configuration, to the particle-antiparticle pair production. In effect,
the bgfms droplet will gain lower charge by catching one of them and the other will be sent. The obtained bgfms droplet will have lower energy, reaching a metastable point for another bgfms configuration with lower $g_Q$ charge and so on until both $g_Q = 0$ and $g_{Z^0} < g_{Z^0_{\text{max}}}$.

Field theories with point-like charges do not have such solutions and hence do not give new possibilities to describe more complicated structures (wavefunctions) of matter.

Are there new observations behind this fact? We can answer this question in affirmative. The difference between the inward structure of the neutron and the inward structure of a droplet consisting of bosonic ground state fields appearing in the GSW electroweak model may be a supporting impulse to start off the relativistic fireball in the collapsing object (neutron star mergers) commonly believed to be responsible for gamma-ray bursts.

Here, on earth, we perceived the appearance of such ground state boson field configurations in very dense microscopic objects created in heavy ion collisions. There is no necessity to repeat these experiments but to reinterpret the results.

The discussed model is homogeneous on the level of one droplet. Next calculations should incorporate more realistic shapes of the charge densities of extended matter sources. These shapes would follow from the coupled Klein-Gordon-Maxwell (Yang-Mills) or Dirac-Maxwell (Yang-Mills) equations for charge density fluctuations as it is required in the self-consistent models. Sometimes even Einstein equations should be entangled. Finally, presented model is a

\footnote{On this base an alternative source of energy which may fertilize the gamma-ray bursts was proposed.}

\footnote{Hence, I see a matter particle to be, from the mathematical point of view, a self-consistent}
step towards (the self field formalism of) a classical theory of a one elementary particle which is a material, extended entity, having electroweak, gravitational, etc. self fields of its own, coupled to it. So, next models shall be concerned with a structure of one particle.

solution of field equations involved in the description of this particle.
6 Table: Quantum numbers

Some quantum numbers in the $SU_L(2) \times U_Y(1)$ electroweak theory.

|                | Weak Isotopic Charge $I^3$ | Weak Hypercharge $Y$ | Electric Charge $Q = I^3 + Y/2$ | $p = 2Q/Y$ |
|----------------|---------------------------|---------------------|---------------------------------|------------|
| **Leptons**    |                           |                     |                                 |            |
| $\nu_L$        | 1/2                       | -1                  | 0                               | 0          |
| $\ell_L$       | -1/2                      | -1                  | -1                              | 2          |
| $\ell_R$       | 0                         | -2                  | -1                              | 1          |
| **Gauge Bosons**|                           |                     |                                 |            |
| $W^+$          | 1                         | 0                   | 1                               |            |
| $W^0$          | 0                         | 0                   | 0                               |            |
| $W^-$          | -1                        | 0                   | -1                              |            |
| $B$            | 0                         | 0                   | 0                               |            |
| **Higgs Boson**|                           |                     |                                 |            |
| $H^+$          | 1/2                       | 1                   | 1                               | 2          |
| $H^0$          | -1/2                      | 1                   | 0                               | 0          |
| **Some matter**| -1/2                      | 1                   | 0                               | 0          |
| **source**     | 1/2                       | 1                   | 1                               | 2          |
| **configurations** | 0                         | 2                   | 1                               | 1          |

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Figure captions

Figure 1
The α ground field of the $A_0$ gauge boson fields as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0, \delta \neq 0)$.

Figure 2
The ζ ground field of the $Z_0$ gauge boson fields as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0, \delta \neq 0)$.

Figure 3
The $\vartheta$ ground field of the $W_{i=2}^a=1$ and $W_{i=1}^a=2$ gauge boson fields as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0, \delta \neq 0)$.

Figure 4
The $\delta$ ground field as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0, \delta \neq 0)$.

Figure 5
The mass $m_Z$ of the $Z^\alpha$ gauge boson fields as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0, \delta \neq 0)$.

Figure 6
The mass $m_A$ of the $A^\alpha$ gauge boson fields as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0, \delta \neq 0)$. The region of $\varrho_{Q SM}$ which is in the range up to values approximately $10^3$ times bigger than these for the matter densities in nucleon matter is indicated by the arrow.

Figure 7
The ratio $\sin \Theta \sin \Theta_W$ (the $\Theta$ angle is the modified mixing angle, see Eq.(47)) as the function of the standard electric charge density fluctuation $\varrho_{Q SM}(\vartheta \neq 0$,}
The physical electric charge density fluctuation $\varrho_Q$ (see Eq.(44)) as the function of the standard electric charge density fluctuation $\varrho_{Q\,SM}(\vartheta \neq 0, \delta \neq 0)$. The region of $\varrho_{Q\,SM}$ which is in the range up to values approximately $10^3$ times bigger than those for the matter densities in nucleon matter is indicated by the arrow.

The minimal energy density of the bgfms configuration $\mathcal{E}_{\text{min}}(\varrho_{Q\,SM}) = \mathcal{U}_{\text{ef}}(\vartheta \neq 0, \delta \neq 0)$ (see Eq.(24)) for boson ground fields given by Eqs.(48)-(51) (for all values of $p \neq 0$ from Section 6) as the function of the standard electric charge density fluctuation $\varrho_{Q\,SM}$ which is in the range up to values approximately $10^3$ times bigger than charge densities for nucleon matter. The region in the vicinity of the "stable" bgfms configuration (see Eq.(54)) is indicated by the arrow.

The mass $M_{q=1}$ of the bgfms configuration for boson ground fields given by Eqs.(48)-(51) as the function of the "radius of the charge fluctuation" $r_q (\vartheta \neq 0, \delta \neq 0$ and for all values of $p \neq 0$ from Section 6). The region in the vicinity of the "stable" bgfms configuration is indicated by the arrow.

The self-consistent classical effective potential $\mathcal{U}_{\text{ef}}(\delta; \vartheta = 0, \varrho_{Q\,SM} = 0)$ as the function of the $\delta$ Higgs ground field ($\lambda = 1$).
The square mass $m_{W^\pm}^2$ of the $W^\pm_\mu$ gauge boson fields (see Eq.(66)) as the function of the standard weak charge density fluctuation $\varrho_{Z^0}_{SM}(\vartheta = 0, \delta \neq 0, \varrho_{Q_{SM}} = 0)$.

**Figure 13**

The mass $m_Z$ of the $Z$ gauge boson fields (Eq.(67)) as the function of the standard weak charge density fluctuation $\varrho_{Z^0}_{SM}(\delta \neq 0, \varrho_{Q_{SM}} = \vartheta = 0)$.

**Figure 14**

The minimal energy density of the bgfms configuration $E_{\text{min}}(\varrho_{Z^0}_{SM}) = \mathcal{U}_{ef}(\vartheta = 0, \delta \neq 0, \varrho_{Q_{SM}} = 0)$ (see Eq.(70)).

**Figure 15**

The partition of the $(\lambda, \varrho_{Z^0}_{SM})$ plane into two regions of stability and instability of the bgfms configuration of fields with $\vartheta = 0$ and $\delta \neq 0$. The region of possible bgfms configurations lies on and below the $\lambda_{\text{max}} - \varrho_{Z^0}_{\text{max}}$ curve where $\lambda_{\text{max}}$ is the value of $\lambda$ and $\varrho_{Z^0}_{\text{max}}$ is the value of $\varrho_{Z^0}_{SM}$ for which we have $m_{W^\pm}^2 = 0$.

**Figure 16**

The mass $M_{i_3=-1/2}$ of the bgfms configuration as the function of the "radius of the charge fluctuation" $r_{i_3}$. Taking for granted that the radius $r_{i_3}$ of the fluctuation is of $1\text{ fm}$ order we see that its mass is of $\sim m_n + 1\text{ keV}$ order (if the extended charge density is carried by neutron with mass $m_n$).

**Figure 17**

The upper mass $M_{i_3=1,\text{max}}$ of a bgfms configuration of fields with $\vartheta = 0$ and $\delta \neq 0$ as the function of the $\lambda_{\text{max}}$ non-linear Higgs parameter where $\lambda_{\text{max}}$ and $M_{i_3=1,\text{max}}$ are the values of $\lambda$ and $M_{i_3=1}$ respectively for which $m_{W^\pm}^2 = 0$ (on
the curve). Here the region of possible bgfms configurations of fields is on and below the $\lambda_{max} - M_{i\beta, \text{max}}$ curve.

**Figure 18**

The upper (according to the stability of the bgfms configuration with $\vartheta = 0$ and $\delta \neq 0$ in the $W^\pm$ sector) mass $M_{i\beta = 1, \text{max}}(r_{i\beta})$ of a bgfms configuration (with the weak isotopic charge $i^3 = 1$) as the function of the "radius of the charge fluctuation" $r_{i\beta}$ of this configuration.

**Figure 19**

The potential $\tilde{V}(\chi)$ for a toy model in scalar field theory.
References

[1] Rüger A., "Attitudes Towards Infinities: Responses to Anomalies in Quantum Electrodynamics, 1927-1947.", Studies in the History and Philosophy of Science 22, 309-37 (1992).

[2] Popper K.R., "The Logic of Scientific Discovery", (1959). There are other publications on the topic (e.g., Syska J., "What is science", http://www.biblest.com.pl/instytut/SCIENCE/science1.html).

[3] Ryder L.H., "Quantum field theory", pp.31,139,31 (Cambridge University Press 1985).

[4] For more details on these foundational questions and further references, see e.g. Barut A.O., Annals N.Y.Acad.Sci. 480, 393 (1987), and in "Quantum Mechanics versus Local Realism", ch.18, (Edited by Selleri F.) (Plenum Press 1988).

Barut A.O., "The Schrödinger and the Dirac Equation-Linear Nonlinear and Integrodifferential" in "Geometrical and Algebraic Aspects of Nonlinear Fields Theory", (Edited by De Filippo S., Marinaro M., Marmo G., Vilasi G.) (Elsevier Science Publishers B.V. North-Holland 1989).

[5] Gerschtein S.S. and Zeldovich Ya., J. Exp. Theor. Fiz. 57, 654 (1969).
Pieper W. and Greiner W., Z. Phys. 218, 327 (1969).
Zeldovich Ya. and Popov V., Uspekhi Fiz. Nauk 105, 403 (1971).
Müller B., Rafelski J. and Greiner W., Nucl. Phys. B68, 585 (1974).
Klein A. and Rafelski J., Phys. Rev. D11, 300 (1975); Z. Phys.A284, 71 (1977).
Rafelski J., Fulcher L.P. and Klein A., Phys.Rep. 38C, 228 (1978).
Gärtner P., Heinz U., Müller B. and Greiner W., Z. Phys. A300, 143 (1981).

[6] Müller B., Rafelski J. and Greiner W., Z. Phys. 257, 62, 183 (1972).
Greiner C., Rischke D.H., Stöcker H. and Koch P., Phys. Rev. D 38, 2797 (1988).

[7] Rafelski J., Fulcher L.P. and Klein A., Phys. Rep. 38C, 228 (1978).
Jetzer P. and van der Bij J.J., Phys. Lett. B227, 341 (1989).
Lee T.D., Phys. Rep. 221, 251 (1992).

[8] Maňka R. and Syska J., Phys.Rev.D 49, Nu.3, 1468 (1994).

[9] Barut A.O., Kraus J., Found.Phys. 13, 189 (1983).

[10] Barut A.O., Van Huele J.F., Phys.Rev.A 32, 3187 (1985).

[11] Barut A.O., Dowling J.P., Phys.Rev.A 36, 649 (1987).

[12] Barut A.O., Found. Phys. 18, 95 (1988); Ann. Phys. (Leipzig) 45, 31 (1988); Found. Phys. Lett 1, 47 (1988).

[13] Barut A.O., Phys.Scr. T21, 18 (1988).
Barut A.O., Salamin Y.I., Phys.Rev.A 37, 2284 (1988).

[14] Barut A.O. and Ünal N., J.Math.Physics27, 3055 (1986).
Barut A.O. and Ünal N., Physica142 A, 467,488 (1987).

[15] Jaynes E.T., "Coherence and Quantum Optics", 495-509, (Edited by by Mandel L. and Wolf E.) (Plenum, New York 1978).
[16] Milonni P.W., "Foundations of Radiation Theory and Quantum Electrodynamics", 15, (Edited by Barut A.O.) (Plenum, New York 1980).

[17] Aitchison I.J.R. and Hay A.J.G., "Gauge Theories in Particle Physics", second ed., (Adam Hilger, Bristol and Philadelphia 1989).

[18] Janka H.-Th. and Ruffert M., A&A 307, L33 (1996).

[19] Ghosh S.K. and Deb B.M., Phys.Reports 92, No.1, (1982).

[20] Monroe C., Meekhof D.M., King B.E., Wineland D.J., Science Vol.272, 1131 (1996).

[21] Biesiada M. and Syska J., Physica Scripta Vol.59, 95 (1999).

[22] J. Syska, "Self-consistent classical fields in field theories", PhD thesis, (University of Silesia, unpublished, 1995/99).
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