Spin Transfers for Baryon Production in Polarized pp Collisions at RHIC-BNL

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Abstract

We consider the inclusive production of longitudinally polarized baryons in $\bar{p}p$ collisions at RHIC-BNL, with one longitudinally polarized proton. We study the spin transfer between the initial proton and the produced baryon as a function of its rapidity and we elucidate its sensitivity to the quark helicity distributions of the proton and to the polarized fragmentation functions of the quark into the baryon. We make predictions using an SU(6) quark spectator model and a perturbative QCD (pQCD) based model. We discuss these different predictions, and what can be learned from them, in view of the forthcoming experiments at RHIC-BNL.
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1 Introduction

The proton spin structure has attracted a considerable interest in the past few years, but in spite of significant theoretical and experimental progress, a precise understanding is still far from being satisfactory. In particular, the role played by antiquarks and gluons in the nucleon spin remains unsettled, so there is a bad need for more new data. Polarized deep inelastic scattering (DIS) experiments at CERN, DESY, JLab and SLAC will certainly continue helping us to gain some insight into this problem, but we also expect a lot to be achieved by means of the Relativistic Heavy Ion Collider (RHIC) at BNL. This facility, which has been turned on recently, will operate several weeks a year as a polarized $pp$ collider with high luminosity and with a center of mass energy $\sqrt{s} = 500$ GeV, perhaps even higher [1]. A vast spin physics program will be undertaken at RHIC-BNL [1, 2], which will focus not only on the proton spin but also on inclusive production of hadrons in order to study the hadronization mechanism and the quark fragmentation functions $D_q^H(z)$. Here $z$ stands for the momentum fraction of the parent quark $q$ carried by the produced hadron $H$. In the case of baryon production, the measurement of the baryon longitudinal polarization allows to study spin-dependent fragmentation functions $\Delta_L D_q^H(z)$. They contain information on how the spin of the parent polarized quarks is transferred to the baryon, and in turn this gives new insight into the baryon spin structure, which is even more poorly known in the case of the strange baryons. Among these hyperons, $\Lambda$ baryons are specially well suited for polarization studies due to their self-analyzing property in the dominant weak decay channel $\Lambda \rightarrow p\pi^–$. The unpolarized $\Lambda$ fragmentation functions $D_q^\Lambda(z)$ are reasonably well determined from the measurement of the rates in $e^+e^–$ annihilation in the energy range $14 \leq \sqrt{s} \leq 91.2$ GeV. However the poor accuracy of the LEP data on the polarization of the $\Lambda$’s produced at the $Z$ pole does not allow a unique determination of the corresponding fragmentation function $\Delta_L D_q^\Lambda(z)$ [3]. This situation has motivated a recent study of the rapidity distribution of the spin transfer in the reaction $\bar{p}p \rightarrow \bar{\Lambda}X$ at RHIC-BNL [4], which seems to provide a good tool to discriminate between various sets of polarized fragmentation functions compatible with the LEP data. The present work lies along the same lines,
but we also generalize it to all the other octet baryons, and we use different sets of polarized fragmentation functions. We carefully analyze the sensitivity of the various partonic subprocesses which lead to the final baryon, and we try to clarify what can be learned from these spin transfer measurements. The paper is organized as follows: in the next section we recall the basic kinematics and we make a detailed analysis of the subprocesses in order to identify the properties of the dominant ones. In section 3 we give a short review of the theoretical framework which we use to construct the polarized fragmentation functions of the octet baryons. Sec. 4 is devoted to the Λ baryon which deserves special attention, and we compare our results to those from other theoretical works [4, 5]. Sec. 5 contains our predictions at RHIC-BNL for all the other octet baryons. In section 6, based on the dominant subprocess, we try to describe the spin transfers with approximate formulae and elucidate their sensitivity to the helicity distributions of the proton and to the fragmentation functions of the produced baryon. Finally we give our discussion and summary in Section 7.

2 Kinematics and subprocesses analysis

Let us consider the reaction \( \vec{p} p \rightarrow \vec{B} X \), for the single inclusive production of a polarized baryon \( B \) of energy \( E \) (or rapidity \( y \), with \( y > 0 \) if \( B \) is in the direction of the polarized proton) and transverse momentum \( p_T \). Here we assume that both the initial proton and the final baryon are longitudinally polarized. The spin transfer for this reaction is defined as

\[
A^B = \frac{\sigma(s_p, s_B) - \sigma(s_p, -s_B)}{\sigma(s_p, s_B) + \sigma(s_p, -s_B)}
\]

where \( \sigma(s_p, s_B) = E_B d\sigma/d^3p_B \) stands for the invariant cross section, and \( s_p, s_B \) are the proton and baryon spin vectors. \( A^B \) is usually written as \( A^B = \Delta\sigma/\sigma \), and [5]

\[
\Delta\sigma \equiv \frac{E \Delta d^3\sigma}{dp^3} = \sum_{abcd} \int_{x_a}^1 dx_a \int_{x_b}^1 dx_b \Delta f_a^p(x_a, Q^2) f_b^p(x_b, Q^2) \Delta D_c^B(z, Q^2)
\]

\[
\frac{1}{\pi z} \frac{\Delta d\hat{s}}{dt} (ab \rightarrow cd),
\]

\( \Delta D_c^B(z, Q^2) \) is the fragmentation function of the produced baryon.
with
\[ x_a = \frac{x_T e^y}{2 - x_T e^{-y}}, \quad x_b = \frac{x_a x_T e^{-y}}{2x_a - x_T e^y}, \quad z = \frac{x_T}{2x_b} e^{-y} + \frac{x_T}{2x_a} e^y, \]

where \( x_T = 2p_T/\sqrt{s} \), \( \sqrt{s} \) is the center of mass energy of the \( pp \) collision, and \( \hat{t} = -x_a p_T \sqrt{s} e^{-y}/z \) is the Mandelstam variable at the parton level. The sum is running over all possible leading order subprocesses \( \bar{a}b \to \bar{c}d \) whose spin transfer is defined analogously to \( A^B \), with \( \Delta d\hat{\sigma}/d\hat{t} \) in the numerator and \( d\hat{\sigma}/d\hat{t} \) in the denominator. These quantities are known and their explicit expressions can be found in Refs. [6, 7].

The \( \Delta f^p (f^p) \) are the usual (un)polarized parton distributions of the proton and \( \Delta D_B^{c(\pm)} (z, Q^2) \equiv D_B^{c(\pm)} (z, Q^2) - D_B^{c(\mp)} (z, Q^2) \) describes the fragmentation of a longitudinally polarized parton \( c \) into a longitudinally polarized baryon \( B \). \( D_B^{c(\pm)} (z, Q^2) \) are the probabilities for finding a baryon \( B \) with positive or negative helicity in the parton \( c \) with positive helicity, and \( D_B^{c(\pm)} (z, Q^2) \) is the sum of them. The variable \( Q^2 \) which occurs in the parton distributions and in the fragmentation functions is taken to be \( Q^2 = p_T^2 \). Finally the denominator of \( A^B \), which is the unpolarized cross section \( \sigma \), has a similar expression to (2), with all \( \Delta \)'s removed. In the numerical calculation of the spin transfer, \( p_T \) will be integrated with a minimal cutoff value of \( p_T^{\text{min}} = 13 \text{ GeV} \). In order to study the sensitivity to the \( B \) fragmentation functions, we need to understand the dynamical mechanism at work in this inclusive production. Among the numerous channels which are involved in the summation in Eq.(2), only three subprocesses contribute significantly to the cross section. The dominant subprocess is \( qg \to qg \), which has a gluon \( g \) and a quark \( q \) in the initial and final states, and next we find \( qq \to qq \) and \( qq' \to qq' \), where the quarks carry different flavors. In Fig. 1 and Fig. 2, we show the contributions of these three channels to \( \sigma \) and \( \Delta \sigma \) as a function of \( y \), respectively. In order to stress the role of the fragmentation functions, first the curves in Fig. 1(a) and Fig. 2(a) are produced by setting \( D_B^c = \Delta D_B^c = 1 \) (Here \( B \) is \( \Lambda \)), and then the curves in Fig. 1(b) and Fig. 2(b) are given with the fragmentation functions in the pQCD counting rules analysis, as explained below (see section 3.2). In Fig. 3, the ratios of polarized to unpolarized cross sections are shown for the three most important subprocesses.
Figure 1: Contributions from the three most important channels (solid curve \(qg \rightarrow qg\), dashed curve \(qq' \rightarrow qq'\) and dotted curve \(qq \rightarrow qq\)) to the inclusive \(\Lambda\) production unpolarized cross sections, at \(\sqrt{s} = 500\) GeV. For comparison, the curves in (a) and (b) are produced by setting \(D^A_\Lambda = 1\) and by using the \(D^A_\Lambda\) obtained in the pQCD counting rules analysis, respectively.

We observe that, while the three contributions to the unpolarized cross section are symmetric in \(y\), as expected, this is not the case for the corresponding contributions to the polarized cross section. The negative \(y\) region is strongly suppressed for the channels \(qq \rightarrow qq\) and \(qq' \rightarrow qq'\) because, as shown in Fig. 3, \(\Delta \sigma / \sigma\) is much smaller for \(y < 0\) than for \(y > 0\). In the case of the dominant channel \(qg \rightarrow qg\), \(\Delta \sigma / \sigma = 1\) for all \(y\), but the asymmetry is partly due to the fact that we have assumed \(D^B_g(z) = \Delta D^B_g(z) = 0\) at the initial energy scale as a first approximation (see section 6 for a detailed discussion on the \(y\) dependence of \(A^B\)).
Figure 2: The same labels as in Fig. [1], but for the polarized cross sections.

Figure 3: $\Delta \hat{\sigma}/\hat{\sigma}$ corresponding to $x_a = x_b = 0.3$ and $p_T = 13$ GeV for the three most important channels, with the same labels as in Fig. [1].
3 Theoretical framework for $q \rightarrow \Lambda$ fragmentation

There have been many efforts to relate the $\Lambda$ polarization to its spin structure. In most of these analyses, polarized $\Lambda$ fragmentation functions were proposed \cite{3, 5, 8} based on a simple ansatz such as $\Delta D_q^\Lambda(z) = C_q(z) D_q^\Lambda(z)$ with $C_q(z)$ either $z^a$ or constant coefficients, or Monte Carlo event generators without a clear physics motivation. Therefore there is a real need to give more realistic predictions of the $\Lambda$-polarization for future experiments such as those at HERMES and at RHIC-BNL. In order to give more reliable predictions of the spin transfer for the produced $\Lambda$ in $\vec{p}p$ collisions, we employ an SU(6) quark-spectator-diquark model and a perturbative QCD (pQCD) based counting rules analysis since they have clear physics motivations, to describe the polarized quark distributions in the $\Lambda$. In the following subsections, we review these two models for the spin structure of the $\Lambda$ and explain how to relate the quark fragmentation functions to the corresponding quark distribution functions.

3.1 SU(6) quark-diquark spectator model

The model \cite{9, 10} starts from the three quark SU(6) quark model wavefunction of the $\Lambda$,

$$|\Lambda^\uparrow\rangle = \frac{1}{\sqrt{12}}[(u^\dagger d^\dagger + d^\dagger u^\dagger) - (u^\dagger d^\dagger + d^\dagger u^\dagger)]s^\uparrow + (\text{cyclic permutation}). \quad (5)$$

If any of the quarks is probed, the other two quarks can be regarded as a diquark state with spin 0 or 1 (scalar and vector diquarks), i.e., the diquark serves as an effective particle, called the spectator. In terms of quark and diquark states, the wavefunction of the $\Lambda$ can be rewritten as

$$|\Lambda^\uparrow\rangle = \frac{1}{\sqrt{12}}[V_0(ds)u^\dagger - V_0(us)d^\dagger - \sqrt{2}V_+(ds)u^\dagger + \sqrt{2}V_+(us)d^\dagger$$

$$+ S(ds)u^\dagger + S(us)d^\dagger - 2S(ud)s^\uparrow], \quad (6)$$

where $V^s_{q_1q_2}$ stands for a $(q_1q_2)$ vector diquark Fock state with third spin component $s_z$, and $S(q_1q_2)$ stands for a $(q_1q_2)$ scalar diquark Fock state. In this model, some non-perturbative effects between the two spectator quarks or other non-perturbative
gluon effects in the hadronic debris can be effectively taken into account by the mass of the diquark spectator. The model prediction of positive polarizations for the $u$ and $d$ quarks inside the $\Lambda$ at $x \to 1$ has been found to be supported by the available experimental data \cite{10}. According to the wavefunction of the $\Lambda$ in (6), the unpolarized and polarized valence quark distributions $(u_v(x), s_v(x)$ and $\Delta u_v(x), \Delta s_v(x))$ of the $\Lambda$ can be expressed as

$$u_v(x) = \frac{1}{4}a_V(x) + \frac{1}{12}a_S(x);$$

$$s_v(x) = \frac{1}{3}a_S(x);$$

and

$$\Delta u_v(x) = -\frac{1}{12}\tilde{a}_V(x) + \frac{1}{12}\tilde{a}_S(x);$$

$$\Delta s_v(x) = \frac{1}{3}\tilde{a}_S(x),$$

respectively, where $a_D(x)$ ($D = S$ for scalar spectator or $V$ for axial vector spectator) can be expressed in terms of the light-cone momentum space wave function $\varphi(x, \vec{k}_\perp)$ as

$$a_D(x) \propto \int [d^2 \vec{k}_\perp] |\varphi(x, \vec{k}_\perp)|^2, \quad (D = S \text{ or } V)$$

which is normalized such that $\int_0^1 dx a_D(x) = 3$ and denotes the amplitude for quark $q$ to be scattered while the spectator is in the diquark state $D$. The amplitude for the quark spin distributions including the Melosh-Wigner rotation effect \cite{11} reads

$$\tilde{a}_D(x) \propto \int [d^2 \vec{k}_\perp] \frac{(k^+ + m_q)^2 - \vec{k}_\perp^2}{(k^+ + m_q)^2 + \vec{k}_\perp^2} |\varphi(x, \vec{k}_\perp)|^2, \quad (D = S \text{ or } V)$$

with $k^+ = x\mathcal{M}$ and $\mathcal{M}^2 = \frac{m_q^2 + \vec{k}_\perp^2}{x} + \frac{m_D^2 + \vec{k}_\perp^2}{1-x}$, where $m_D$ is the mass of the diquark spectator. In our numerical analysis, we employ the Brodsky-Huang-Lepage (BHL) prescription \cite{12} of the light-cone momentum space wave function of the quark-spectator

$$\varphi(x, \vec{k}_\perp) = A_D \exp\left\{-\frac{1}{8\alpha_D^2}\left[\frac{m_q^2 + \vec{k}_\perp^2}{x} + \frac{m_D^2 + \vec{k}_\perp^2}{1-x}\right]\right\}.$$
with the parameter $\alpha_D = 330$ MeV. We take the quark masses as $m_u = m_d = 330$ MeV and $m_s = 480$ MeV. We choose the diquark masses $m_S = 600$ MeV and $m_V = 800$ MeV for non-strange diquark states, $m_S = 750$ MeV and $m_V = 950$ MeV for diquark states $(qs)$ with $q = u, d$.

3.2 pQCD counting rules analysis

The pQCD counting rules analysis has been successfully used to describe the spin structure of the nucleon [14]. The typical characteristic of the pQCD counting rules analysis lies in that it predicts “helicity retention”, which means that the helicity of a valence quark will match that of the parent nucleon. Explicitly, the quark distributions of a hadron $h$ have been shown to satisfy the counting rule [15] for the large $x$ region,

$$q_h(x) \sim (1 - x)^p, \quad (12)$$

where

$$p = 2n - 1 + 2\Delta S_z. \quad (13)$$

Here $n$ is the minimal number of the spectator quarks, and $\Delta S_z = |S_q - S_h| = 0$ or 1 for parallel or anti-parallel quark and hadron helicities, respectively [14]. We extend the pQCD analysis from the nucleon case to the $\Lambda$. More specifically, we adopt the canonical form for the quark distributions,

$$q_i^\uparrow(x) = \frac{A_i}{B} x^{-\alpha}(1 - x)^3 + \frac{B_i}{B} x^{-\alpha}(1 - x)^4;$$
$$q_i^\downarrow(x) = \frac{C_i}{B} x^{-\alpha}(1 - x)^5 + \frac{D_i}{B} x^{-\alpha}(1 - x)^6. \quad (14)$$

with $q_1 = s$ and $q_2 = u$ or $d$, where $B_\alpha$ is the $\beta$-function defined by $B(1 - \alpha, n + 1) = \int_0^1 x^{-\alpha}(1 - x)^n dx$. Here $\alpha = 1/2$ because the small $x$ behavior is controlled by the Regge exchanges for non-diffractive valence quarks. The helicity retention for the quark distributions in the $\Lambda$ implies that $u(x)/s(x) \rightarrow 1/2$ and $\Delta q(x)/q(x) \rightarrow 1$ (for $q = u, d, s$) for $x \rightarrow 1$, and therefore the flavor structure of the $\Lambda$ near $x = 1$ is a region in which accurate tests of pQCD can be made. There are five constraint
conditions due to the numbers of quarks, the quark contributions to the spin of the Λ, and the helicity retention property,

\[ n_{u}^\uparrow + n_{u}^\downarrow = 1, \]  
\[ n_{s}^\uparrow + n_{s}^\downarrow = 1, \]  
\[ n_{u}^\uparrow - n_{u}^\downarrow = \Delta U, \]  
\[ n_{s}^\uparrow - n_{s}^\downarrow = \Delta S, \]

and

\[ \frac{\bar{A}_{u}}{\bar{A}_{s}} = \frac{1}{2}, \]

where the integrated polarized quark densities \( \Delta U = -0.2 \) and \( \Delta S = 0.6 \) for the Λ can be extracted by using SU(3) symmetry from the deep-inelastic lepton-proton scattering experiment data [16] and the hyperon semileptonic decay constants \( F = 0.459 \) and \( D = 0.798 \) [17]. There might be a large uncertainties in these values of \( \Delta U \) and \( \Delta S \), since SU(3) symmetry breaking may affect the explicit flavor-dependent helicity separation of the octet baryons [18]. Nevertheless, the effect of these uncertainties on the pQCD fragmentation functions in the medium and large \( z \) region, which give the main contributions to the spin transfers, do not change the qualitative features of our results due to the helicity retention property of the pQCD analysis. The five constraints in (15)-(19) leave us with three unknown parameters, which are chosen to be \( \bar{A}_{s}, \bar{C}_{s} \) and \( \bar{C}_{u} \). Following Ref. [13], we let them be the same with the value of 2. The \( d \) quark distributions are the same as those for the \( u \) quark.

### 3.3 Fragmentation functions via Gribov-Lipatov relation

Unfortunately, we cannot directly measure the above described quark distributions of the Λ, since it is not possible to use the Λ as a target due to its short life time.
Also one obviously cannot produce a beam of charge-neutral Λs. What one can observe with experiments is the quark to Λ fragmentation, and therefore one needs a relation between the quark fragmentation and distribution functions. In order to connect the fragmentation functions with the distribution functions, we use [10] the Gribov-Lipatov (GL) relation [19]

\[ D_h^q(z) \sim z q_h(z), \tag{20} \]

where \( D_h^q(z) \) is the fragmentation function for a quark \( q \) splitting into a hadron \( h \) with longitudinal momentum fraction \( z \), and \( q_h(z) \) is the quark distribution of finding the quark \( q \) inside the hadron \( h \) carrying a momentum fraction \( x = z \). The GL relation is only known to be valid near \( z \to 1 \) in an energy scale \( Q_0^2 \) in leading order approximation [20]. However, with this relation, predictions of Λ polarizations [10] based on quark distributions of the Λ in the SU(6) quark diquark spectator model and in the pQCD based counting rules analysis, have been found to be supported by all available data from longitudinally polarized Λ fragmentation in \( e^+e^- \)-annihilation [21, 22, 23], polarized charged lepton DIS process [24, 25], and most recently, neutrino (antineutrino) DIS process [26]. Thus we still use (20) as an ansatz to relate the quark fragmentation functions for the Λ to the corresponding quark distributions.

4 Spin transfer for Λ production in \( \vec{p}p \) collisions

The spin transfer for the produced Λ in \( \vec{p}p \) collisions is mainly determined by three subprocesses. The cross section of the most important subprocess \( qg \to qg \) strongly depends on the quark \( q \) distribution in the colliding protons. The strange quark contribution to the spin transfer to the Λ is suppressed due to the fact that the strange quark is not a valence quark of the proton. As opposed to the \( e^+e^- \) annihilation process where the Λ polarization is dominated by the strange quark fragmentation, \( \vec{p}p \) collisions should be a suitable place to check the \( u \) and \( d \) quark fragmentation functions by measuring the large rapidity dependence of the spin transfer to the Λ. In order to show the dominant quark contributions, the \( u \) quark to Λ fragmentation
functions in the pQCD analysis and the SU(6) diquark model are shown in the left-upper part of Fig. 4.

By inserting the fragmentation functions obtained in the pQCD analysis and the SU(6) quark diquark model into (1), and taking the minimal cutoff of the transverse momentum \( p_T = 13 \) GeV, we obtain the spin transfers for \( \Lambda \) production in polarized \( pp \) collisions at \( \sqrt{s} = 500 \) GeV. The results are shown in the left-upper part of Fig. 5. We also show in Fig. 5 the results at \( \sqrt{s} = 200 \) GeV, an energy at which RHIC-BNL will be operating. As expected, the spin transfer is a bit larger than at \( \sqrt{s} = 500 \) GeV, since the cross section is smaller.

In our numerical calculations, we adopt the LO set of unpolarized parton distributions of Ref. [27] and polarized parton distributions of LO GRSV standard scenario [28]. The spin transfers as a function of the transverse momentum of the produced \( \Lambda \) in \( \bar{p}p \) collisions at \( \sqrt{s} = 500 \) GeV, with the specified values of its rapidity \( y = 0 \) and \( y = 2 \), are given in Fig. 6 (a) and (b) respectively. We find that one should measure the spin transfers in the large \( p_T \) region for \( y = 0 \) and select the events with \( p_T \sim 30 \) GeV for \( y = 2 \) in order to distinguish between different sets of fragmentation functions in the two models.

Now let us make a comparison between our results and those in Refs. [4, 5]. In Ref. [4], Florian, Stratmann and Vogelsang made their predictions within three different scenarios of the polarized fragmentation functions. Scenario 1 corresponds to the SU(6) symmetric non-relativistic quark model, according to which the whole \( \Lambda \) spin is carried by the \( s \) quark. Scenario 2 is based on an SU(3) flavor symmetry analysis and on the first moment of \( g_1 \), and leads to the prediction that the \( u \) and \( d \) quarks of the \( \Lambda \) are negatively polarized. Scenario 3 is built on the assumption that all light quarks contribute equally to the \( \Lambda \) polarization. It is very interesting that the best agreement with available LEP data was obtained within scenario 3 [3], i.e. the \( u \) and \( d \) quark fragmentation functions are positively polarized. Our predictions in the SU(6) quark diquark model and the pQCD analysis are similar to those with scenario 3 polarized fragmentation function in Ref. [4]. In addition, our predictions are also close to those predicted in Ref. [3], where positive polarized \( u \) and \( d \) quark fragmentation functions were used. After all, our analysis shows that the \( u \) and \( d \)
Figure 4: The ratios of polarized to unpolarized fragmentation functions for non-strange quarks, both in the pQCD counting rules analysis (solid curves) and the SU(6) quark-diquark spectator model (dashed curves).
Figure 5: The spin transfers as functions of rapidity of the produced Λ, Σ, Ξ of octet baryon members, in \( \bar{p}p \) collisions at \( \sqrt{s} = 500 \text{ GeV} \), with the spin-dependent fragmentation functions in the pQCD counting rules analysis (solid curves) and the SU(6) quark-diquark spectator model (dashed curves). Note that the dashed and solid curves in (b) almost overlap.
Figure 6: The same as Fig. 5, but for $\bar{p}p$ collisions at $\sqrt{s} = 200$ GeV.
quarks to $\Lambda$ fragmentation functions are positively polarized at least at large $z$, which is consistent with the results we found in other processes [10].

5 Extension of the analysis to other octet baryons

In addition to the measurement of polarization of the produced $\Lambda$, the detection technique of $\Sigma$ and $\Xi$ hyperons is also getting more and more mature in order to measure the various quark to hyperon fragmentation functions [29, 30, 31]. Except for the $\Sigma^0$, which decays electromagnetically, all other hyperons in the octet baryons have their major decay modes mediated by weak interactions. Because these weak decays do not conserve parity, information from their decay products can be used to determine their polarization [29, 30]. The polarization of $\Sigma^0$ can be also re-constructed from the dominant decay chain $\Sigma^0 \rightarrow \Lambda \gamma$ and $\Lambda \rightarrow p\pi^-$ [31]. Therefore we can use the measurable fragmentation functions in order to extract information on the spin and flavor content of hyperons, using the available experimental facilities. Hence, it is important to extend the analysis for the $\Lambda$ to other octet baryons. This can be done directly by adopting the same parameters for the SU(6) quark-diquark model and pQCD analysis as those in Ref. [13]. In Fig. 4, we show the ratios of non-dominant quark polarized to unpolarized fragmentations into octet baryons $\Sigma^0$, $\Sigma^+$, $\Sigma^-$, $\Xi^0$ and
Figure 8: The same as Fig. 5, but for the produced proton (a) and neutron (b).

Ξ⁻. The spin transfers as functions of the rapidity of the produced octet baryons in \( \vec{p}p \) collisions, at \( \sqrt{s} = 500 \text{ GeV} \), and for the spin-dependent fragmentation functions, both in the pQCD counting rules analysis (solid lines) and the SU(6) quark-diquark spectator model (dashed lines), are presented in Fig. 5. For completeness the results at \( \sqrt{s} = 200 \text{ GeV} \) are shown in Fig. 6. By comparing the spin transfers in Fig. 5 and Fig. 6 with the corresponding spin structure of the fragmentation functions in Fig. 4, we find that the spin transfers at large \( y \) are mainly related to the non-dominant \( u \) and \( d \) quark fragmentation ratios of polarized to unpolarized fragmentation functions at medium \( z \) values. This can explain qualitatively the different predictions of the spin transfers in different models. The predictions for the spin transfers in the two models are qualitatively similar for \( \Lambda \) and \( \Sigma \), as can be seen from Fig. 5(a)-(d). However, we find that the spin transfers for the produced \( \Xi \) can provide more clear information to distinguish between the SU(6) quark diquark model and pQCD based analysis. Hence, the \( \Xi \) polarizations in \( \vec{p}p \) collisions deserve experimental attention.

In order to complete our analysis, we include the spin transfers as functions of the rapidity of the produced proton and neutron in \( \vec{p}p \) collisions at \( \sqrt{s} = 500 \text{ GeV} \). The results are given in Fig. 8. As shown in Fig. 8(b), the spin transfer for the produced neutron is also a suitable quantity for distinguishing the two sets of fragmentation functions in the two different models, but experimentally it is difficult to measure the polarization of a fast neutron or that of a fast proton.
6 An approximate estimate of spin transfers

Usually it is hard to extract exact information on the inclusive production of longitudinally polarized baryons in $pp$ collisions because of the following three complex aspects of the spin transfer: (1) Many subprocesses are involved; (2) The contribution of every subprocess includes four factors, i.e. the quark helicity distributions of the proton, the polarization of the produced baryon fragmentation functions, and the subprocess cross sections $\Delta \hat{\sigma}$ and $\hat{\sigma}$; (3) The kinematic variables are integrated over. In order to extract some useful information from the above complex situation, we focus our attention on the dominant subprocess, i.e. $qg \rightarrow qg$. We can use the mean value theorem and take the cross sections out of the corresponding integrals in both the numerator and denominator in the expression for the spin transfer. Fortunately, $\Delta \hat{\sigma}/\hat{\sigma}$ for this subprocess is equal to one for all $y$ (see Fig. 3). So the contribution of this subprocess to the spin transfer only comes from two factors, i.e. the quark helicity distributions and the polarization of the fragmentation functions. In order to pin down the roles played by the above two factors in the spin transfer, we go back to review the results shown in Fig. 1 and Fig. 2. From Fig. 1, one can see that there is a symmetry for $y \leftrightarrow -y$ in the unpolarized cross sections. However, there is a strong asymmetry in the polarized cross sections, as shown in Fig. 4. This asymmetry may arise from the corresponding asymmetry between $\Delta f^p_a(x_a)$ and $f^p_b(x_b)$ when $a \leftrightarrow b$, and another possible source for this asymmetry is the asymmetry of $z$ when $y \leftrightarrow -y$. By comparing Fig. 2(a) and (b), one can see that the asymmetry in the polarized cross section is mainly due to the asymmetry between $\Delta f^p_a(x_a)$ and $f^p_b(x_b)$ when $a \leftrightarrow b$ since the polarized cross section asymmetry still remains when $\Delta D^B_q$ is removed. In addition, we find that the magnitude of the polarized cross section in Fig. 2(b) can be approximately obtained from that in Fig. 2(a) by multiplying it with a factor of $\Delta D^A_q$ at $z \simeq 0.65$. It turns out that the asymmetry in the polarized cross section mainly comes from the helicity distributions of the proton and the magnitude of the cross section is related to the fragmentation functions at $z \simeq 0.65$.

Now let us show why there is a very strong asymmetry in the $y$-dependence of the spin transfer. As an approximation, we only consider the dominant subprocess
Figure 9: $x_a$ as a function of $y$ for $p_T = 17$ GeV, $z = 0.65$ and $\sqrt{s} = 500$ GeV. The solid and dashed curves correspond to $x_b = 0.2$ and $x_b = 0.7$, respectively.

Figure 10: The quark helicity distributions of the proton. (a) $(\Delta u + \Delta d)/(u + d)$; (b) $\Delta u/u$; (c) $\Delta d/d$; for the LO set of unpolarized parton distributions of Ref. [27] and the polarized parton distributions of the LO GRSV standard scenario [28] at $Q^2 = 10$ GeV$^2$. 
$gg \rightarrow gg$, the spin transfer can be expressed in the simple form

$$A^B = \frac{\Delta q(x_a) g(x_b) \Delta D^B_q(z) + \Delta g(x_a) q(x_b) \Delta D^B_g(z)}{q(x_a) g(x_b) D^B_q(z) + g(x_a) q(x_b) D^B_g(z)}.$$  \hspace{1cm} (21)$$

The quark distributions and fragmentation functions in this expression should be understood in an average sense. The $g \rightarrow B$ fragmentation functions are much less known than the $q \rightarrow B$ fragmentation functions, and $\Delta D^B_g$ is customarily set to zero at an initial energy scale. So for the moment let us neglect the $g \rightarrow B$ fragmentation functions. Actually, this is what we have done in the above exact calculation of $A^B$. Therefore, for the $\Lambda$, we find an approximate formula

$$A^B = \left[ \frac{\Delta u + \Delta d}{u + d} \right] (x_a) \left[ \frac{\Delta D^A_u}{D^A_u} \right] (z \simeq 0.65). \hspace{1cm} (22)$$

In this expression there is a $y$-dependence in $x_a$ via

$$x_a = \frac{x_b p_T e^y}{x_b z \sqrt{s - p_T e^{-y}}}$$  \hspace{1cm} (23)$$

by setting $p_T = 17$ GeV, $z = 0.65$, $\sqrt{s} = 500$ GeV and $x_b$ in the range $[0.2, 0.7]$. In Fig. 9, we show $x_a$ as a function of $y$ and the two curves indicate that the $y$-dependence of $x_a$ is stable when $x_b$ varies in the range $[0.2, 0.7]$. The important feature we notice in Fig. 9 is that there is a strong asymmetry in $x_a$ when $y \leftrightarrow -y$, i.e. for a given $|y|$, the value of $x_a$ for $y < 0$ is much lower than that for $y > 0$. In order to see how this asymmetry is reflected in the asymmetry in the spin transfer, we show the helicity distributions of the proton in Fig. 10. For the $\Lambda$ case, we only need Fig. 10(a). Figs. 10(b)-(c) will be used for the other octet baryons. From Fig. 10(a), we find that the ratio $(\Delta u + \Delta d)/(u + d)$ increases with $x_a$. By combining information from Fig. 8, Fig. 10(a), and Eq. (22), the asymmetry for the approximate formula, as shown in Fig. 11(a), can be easily understood.

On the other hand, by looking at the results in Fig. 3, we observe that Fig. 3(a) and (b) are similar, (c) and (d) are mirror symmetric, (e) and (f) are almost mirror symmetric, which motivates us to extend the approximate estimate from the $\Lambda$ to
the other octet baryons. For Fig. 5(b) we use the same formula as for (a). Similarly, we approximate Fig. 5(c) and (e) with \( \frac{[\Delta u/u(x_a)][\Delta D^B_u/D^B_d]}{u(x_a)D^d_u(z) + d(x_a)D^d_d(z)} \) and for Fig. 5(d) and (f), we replace the \( u \) quark by the \( d \) quark. It means that the asymmetry allows only to test the ratio of polarized to unpolarized fragmentation functions of the dominant quark in a region where \( z \approx 0.65 \). The asymmetry is mainly driven by the corresponding quark helicity asymmetries of the \( u \) and \( d \) quark distributions in the proton. Our approximate formulae for all octet baryons are shown in Table 1, and the approximate \( y \)-dependence of the spin transfers is shown in Fig. 11. We find that the spin transfers are well described by our approximate formulae in the given region of \( y \).

| Baryon | Approximate Formula for \( A^B \) |
|--------|----------------------------------|
| \( p \) | \( \frac{[\Delta u(x_a)\Delta D^p_u(z) + \Delta d(x_a)\Delta D^p_d(z)]}{u(x_a)D^d_u(z) + d(x_a)D^d_d(z)} \) |
| \( n \) | \( \frac{[\Delta u(x_a)\Delta D^n_u(z) + \Delta d(x_a)\Delta D^n_d(z)]}{u(x_a)D^d_u(z) + d(x_a)D^d_d(z)} \) |
| \( \Sigma^+ \) | \( \frac{\Delta u(x_a)}{u(x_a)} \frac{\Delta D^{\Sigma^+}_u(z)}{D^{\Sigma^+}_u(z)} \) |
| \( \Sigma^0 \) | \( \frac{(\Delta u(x_a) + \Delta d(x_a))}{(u(x_a) + d(x_a))} \frac{\Delta D^{\Sigma^0}_u(z)}{D^{\Sigma^0}_u(z)} \) |
| \( \Sigma^- \) | \( \frac{\Delta d(x_a)}{d(x_a)} \frac{\Delta D^{\Sigma^-}_d(z)}{D^{\Sigma^-}_d(z)} \) |
| \( \Lambda^0 \) | \( \frac{(\Delta u(x_a) + \Delta d(x_a))}{(u(x_a) + d(x_a))} \frac{\Delta D^{\Lambda}_u(z)}{D^{\Lambda}_u(z)} \) |
| \( \Xi^- \) | \( \frac{\Delta d(x_a)}{d(x_a)} \frac{\Delta D^{\Xi^-}_d(z)}{D^{\Xi^-}_d(z)} \) |
| \( \Xi^0 \) | \( \frac{\Delta u(x_a)}{u(x_a)} \frac{\Delta D^{\Xi^0}_u(z)}{D^{\Xi^0}_u(z)} \) |

The spin transfer in the positive \( y \) region depends mainly on the helicity distributions in the proton. This is because our present knowledge about the gluon to a octet baryon fragmentation function is very poor and usually the polarized gluon
Figure 11: The spin transfers as functions of the rapidity of the produced octet baryons in $\bar{p}p$ collisions at $\sqrt{s} = 500$ GeV. The thin curves are obtained using the approximate formulae in Table 1 and the thick curves are the exact calculations. The solid and dashed curves correspond to the results with the fragmentation functions in the pQCD counting rules analysis and the SU(6) quark-diquark spectator model, respectively. Note that the dashed and solid curves in (b) almost overlap.
fragmentation functions are set to zero at an initial energy scale and they are only produced via QCD evolution. There is a strong suppression of the spin transfer in the negative $y$ region, due to the smallness of the quark helicity distribution for small $x$. If the gluon fragmentation functions have a significant polarization, then the spin transfer in the negative $y$ region would not be zero although it would be much smaller than in the positive $y$ region.

By means of the approximate formulae for the $\Lambda$, we can immediately check whether the $u$ quark fragmentation is positively or negatively polarized, according to the measured results of the spin transfer. If the spin transfers are positive, then the $u$ and $d$ fragmentation functions should be positively polarized at large $z$ ($\Delta D^\Lambda_u(z) > 0$), and \textit{vice versa}. With our approximate formula the results of Refs. \cite{4} and \cite{5} can also be easily understood.

## 7 Summary

In summary, we have considered the inclusive production of longitudinally polarized baryons in $\vec{p}p$ collisions at RHIC-BNL, with one longitudinally polarized proton. We predicted the spin transfer between the initial proton and the produced baryon as a function of its rapidity by means of an SU(6) quark diquark spectator model and a pQCD analysis. The same analysis was extended from the $\Lambda$ case to other octet baryons. We found that three subprocesses including $qg \rightarrow qg$, $qq \rightarrow qq$ and $qq' \rightarrow qq'$ have dominant contributions to the spin transfers. We pointed out some sensitive kinematics regions where one can distinguish between different sets of fragmentation functions. We tried some approximate formulae to describe the spin transfer and found that the asymmetry allows only to test the ratio of polarized to unpolarized fragmentation functions of the dominant quark in a region where $z = 0.65$ and it is mainly driven by the corresponding quark helicity asymmetries of the $u$ and $d$ quarks in the proton. Our predictions for the spin transfers with positively polarized $u$ and $d$ quark fragmentations to $\Lambda$ should be checked soon by the RHIC-BNL experimental data, and if the measurements for other produced octet baryons in $\vec{p}p$ collisions can be realized, they will enrich our knowledge of the hadronization mechanism.
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