Lepton Dipole Moments

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Abstract. From the famous experiments of Stern and Gerlach to the present, measurements of magnetic dipole moments, and searches for electric dipole moments of “elementary” particles have played a major role in our understanding of sub-atomic physics. In this talk I discuss the progress on measurements and theory of the magnetic dipole moments of the electron and muon. I also discuss a new proposal to search for a permanent electric dipole moment (EDM) of the muon and put it into the more general context of other EDM searches.

INTRODUCTION AND THEORY OF THE LEPTON ANOMALIES

Over the past 82 years, the study of dipole moments of elementary particles has provided a wealth of information on subatomic physics. From the pioneering work of Stern[1] through the discovery of the large anomalous magnetic moments of the proton[2] and neutron[3], the ground work was laid for the discovery of spin, of radiative corrections and the renormalizable theory of QED, of the quark structure of baryons and the development of QCD.

A charged particle with spin $\vec{s}$ has a magnetic moment

$$\vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s}; \quad \mu = (1 + a) \frac{e\hbar}{2m}; \quad a \equiv \frac{(g_s - 2)^2}{2};$$  \hspace{1cm} (1)

where $g_s$ is called the gyromagnetic ratio. The expression in the middle is the quantity one finds listed in the Particle Data Tables.[4] The quantity $a$ is the anomalous magnetic dipole moment (or simply the anomaly) which is related to the $g$-factor in the right-hand equation.

The Dirac equation tells us that $g \equiv 2$ for spin angular momentum, and is unity for orbital angular momentum (the latter having been verified experimentally[6]). This can be seen from the non-relativistic reduction for an electron in a weak magnetic field:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ \frac{p^2}{2m} - \frac{e}{2m} (\vec{L} + 2\vec{S}) \cdot \vec{B} \right] \psi,$$ \hspace{1cm} (2)

and the subscript on $g$ is dropped in the following discussion. For point particles, the anomaly arises from radiative corrections, three examples of which are shown in Fig.[1] The “vertex” correction and vacuum polarization are important in the lepton anomaly, while the self-energy term is included in the dressed mass. The situation for baryons is quite different, since their internal quark structure gives them large anomalies.
FIGURE 1. Three examples of radiative corrections. The middle term is absorbed into the dressed mass, but the vertex correction and vacuum polarization play an important role in the anomaly.

The vertex correction in lowest-order gives the famous Schwinger result, $a = \alpha/2\pi$, which was verified experimentally by Foley and Kusch. To lowest order, the S-matrix element for a charged particle in a magnetic field is given by

$$-ie\bar{u}(p') \left( \frac{p+p'}{2m} + \left( 1 + \frac{\alpha}{2\pi} \right) \frac{i\sigma_\lambda q^\lambda}{2m} \right) u(p) A^\lambda(q)$$

In general $a$ (or $g$) is an expansion in $(\alpha/\pi)$,

$$a = C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + \cdots$$

with 1 diagram for the Schwinger (second-order) contribution, 5 for the fourth order, 40 for the sixth order, 891 for the eighth order.

The QED contributions to electron and muon $(g - 2)$ have now been calculated through eighth order, $(\alpha/\pi)^4$ and the tenth-order contribution has been estimated.[8] The first few orders are shown schematically below in Fig. 2.

FIGURE 2. A schematic of the first few terms in the QED expansion for the muon. The vacuum polarization term shown is one of five of order $(\alpha/\pi)^2$.

While magnetic dipole moments (MDMs) are a natural property of charged particles with spin, electric dipole moments (EDMs) are forbidden both by parity and by time reversal symmetry. Interestingly enough, Purcell and Ramsey[9] suggested in 1950 that a measurement of the neutron EDM would be a good way to search for parity violation, well in advance of the paper by Lee and Yang. After the discovery of parity violation, Landau[10] and Ramsey[11] pointed out that an EDM would violate both $P$ and $T$ symmetries. This can be seen by examining the Hamiltonian for a spin one-half particle in the presence of both an electric and magnetic field, $\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$. The transformation properties of $\vec{E}$, $\vec{B}$, $\vec{\mu}$ and $\vec{d}$ are given in the Table[11] and we see that while $\vec{\mu} \cdot \vec{B}$ is even under all three, $\vec{d} \cdot \vec{E}$ is odd under both $P$ and $T$. Thus the existence of an EDM implies that both $P$ and $T$ are violated. In the context of $CPT$ symmetry, an EDM implies $CP$ violation. The standard model value for the electron and muon EDM is $\leq 10^{-35} \, e\text{-cm}$, well beyond the reach of experiments (which are at the $10^{-26} \, e\text{-cm}$ level). Observation of a non-zero $e$ or $\mu$ EDM would be a clear signal for new physics.
TABLE 1. Transformation properties of the magnetic and electric fields and dipole moments.

|   | E | B | \(\vec{\mu}\) or \(\vec{d}\) |
|---|---|---|----------------|
| P | - | + | + |
| C | - | - | - |
| T | + | - | - |

The connection between the magnetic and electric dipole moments can be seen by writing the interaction Lagrangian as

\[
L_{dm} = \frac{1}{2} \left[ D\vec{\mu} \sigma^{\alpha\beta} \frac{1 + \gamma_5}{2} + D^* \vec{\mu} \sigma^{\alpha\beta} \frac{1 - \gamma_5}{2} \right] \mu F_{\alpha\beta} \tag{5}
\]

where the dipole operator \(D\) has \(\text{Re } D = a\mu \frac{e}{2m_p}\) and \(\text{Im } D = d\mu\).

The standard model value of \(a\) has three contributions from radiative processes: QED loops containing leptons (\(e, \mu, \tau\)) and photons; hadronic loops containing hadrons in vacuum polarization loops; and weak loops involving the weak gauge bosons \(W, Z\), and Higgs. Thus \(a_{e,\mu}(\text{SM}) = a_{e,\mu}(\text{QED}) + a_{e,\mu}(\text{hadronic}) + a_{e,\mu}(\text{weak})\). A difference between the experimental value and the standard model prediction would signify the presence of new physics beyond the standard model. Examples of such potential contributions are lepton substructure, extra gauge bosons, anomalous \(W - \gamma\) couplings, or the existence of supersymmetric partners of the leptons and gauge bosons.

The electron anomaly is now measured to a relative precision of about four parts in a billion (ppb),\(^{[14]}\) and the muon is measured to 0.7 parts per million (ppm).\(^{[15]}\) The relative contributions of heavier particles to \(a\) scales as \((m_e/m_m)^2\), and the muon has a sensitivity factor of about 40,000 over the electron to higher mass scale radiative corrections. This gives the muon an overall advantage of 230 in measurable sensitivity to larger mass scales, including new physics. At a precision of 0.7 ppm, the muon anomaly is sensitive to \(\geq 100\) GeV scale physics.

In fact, the contribution of anything heavier than an electron to the electron anomaly is at the level of about 3 ppb. So while the the electron \((g - 2)\) experiments are triumphs of experimental and theoretical physics, they are purely a test of QED. Since the independent measurements of \(\alpha\) are less precise (7.4 ppb) than the present accuracy on the electron anomaly, the measurement of \(a_e\) has been used to determine the best measurement of the fine-structure constant.\(^{[16]}\) The uncertainty in \(\alpha\) is not an issue for \(a\mu\).

The CERN experiment\(^{[12]}\) observed the contribution of hadronic vacuum polarization shown in Fig. 3(a) at the 10 standard deviation level. Unfortunately, the hadronic contribution cannot be calculated directly from QCD, since the energy scale is very low \((m_m e^2)\), although Blum\(^{[13]}\) has performed a proof of principle calculation on the lattice. Fortunately dispersion theory gives a relationship between the vacuum polarization loop and the cross section for \(e^+e^- \rightarrow\) hadrons,

\[
a_{\mu}(\text{Had}; 1) = \left(\frac{\alpha a_{\mu}}{3\pi}\right)^2 \int_4^{\infty} ds \frac{K(s) R(s)}{s^2}, \quad \text{where } R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)} \tag{6}
\]
and experimental data are used as input. The factor $s^{-2}$ in the dispersion relation means that values of $R(s)$ at low energies (the $\rho$ resonance) dominate the determination of $a_\mu(\text{Had}; 1)$. This information can also be obtained from hadronic $\tau^-$ decays such as $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$, which can be related to $e^+ e^-$ annihilation through the CVC hypothesis and isospin conservation.[17, 18]

The quantity $R$ is not directly measured. The cross-section for $e^+ e^-$ is determined using some other normalization, followed by careful subtractions for initial state radiation, vacuum polarization etc., and the denominator is calculated from QED. Most of these effects would cancel in the ratio if $R(s)$ were measured directly.

Knowledge of the hadronic contribution has steadily improved over the past 15 years. When the muon $(g - 2)$ experiment E821 began at Brookhaven in the early 1980s, $a_\mu(\text{Had}; 1)$ was known to about 5 ppm. Now the uncertainty is about 0.5 ppm. This progress has not come without some pain. The hadronic light-by-light contribution has changed sign twice, with the positive sign now having been confirmed by a number of authors.[19] New high-precision data from Novosibirsk on $e^+ e^- \rightarrow$ hadrons lowered both the value and the error on $a_\mu(\text{Had}; 1)$. The value obtained from $\tau$-decay differed from that using $e^+ e^-$. Recently a normalization error was uncovered, which moves the value of $a_\mu(\text{Had}; 1)$ obtained from $e^+ e^-$ closer to that obtained from $\tau$-decay.[21] and a revised value[22] of $a_\mu(\text{Had}; 1)$ shows a difference between the standard model value and the experimental result from E821 to be: $(22.1 \pm 7.2 \pm 3.5 \pm 8.0) \times 10^{-10}$ (1.9$\sigma$) and $(7.4 \pm 5.8 \pm 3.5 \pm 8.0) \times 10^{-10}$ (0.7$\sigma$) for the $e^+ e^-$ and $\tau$-based estimates respectively. The second error is from the hadronic light-by-light contribution and the third one from E821. Further measurements of the hadronic cross section are underway at Frascati and BaBar, using initial state radiation to lower $\sqrt{s}$ below the beam energy (sometimes called radiative return).

One of the very useful roles measurements of $a_\mu$ have played in the past is placing serious restrictions on physics beyond the standard model. With the development of supersymmetric theories as a favored scheme of physics beyond the standard model, interest
in the experimental and theoretical value of $a_\mu$ has grown substantially. SUSY contributions to $a_\mu$ could be at a measurable level in a broad range of models. Furthermore, there is a complementarity between the SUSY contributions to the MDM, EDM and transition moment for $\mu \to e$. The MDM and EDM are related to the real and imaginary parts of the diagonal element of the slepton mixing matrix, and the transition moment is related to the off diagonal one, as shown in Fig. 4.

**MEASUREMENT OF THE MUON ANOMALY**

The method used in the third CERN experiment and the BNL experiment are very similar, save the use of direct muon injection\[^{23}\] into the storage ring,\[^{24, 25}\] which was developed by the E821 collaboration. These experiments are based on the fact that for $a_\mu > 0$ the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field. The spin precession frequency $\omega_S$ consists of the Larmor and Thomas spin-precession terms. The spin frequency $\omega_S$, the momentum precession (cyclotron) frequency $\omega_C$, and the difference frequency $\omega_a$ are given by

$$\omega_S = \frac{geB}{2mc} + \frac{(1 - \gamma) eB}{\gamma mc}; \quad \omega_C = \frac{eB}{mc\gamma}; \quad \omega_a = \omega_S - \omega_C = \left(\frac{g - 2}{2}\right) \frac{eB}{mc}. \quad (7)$$

The difference frequency is the frequency with which the spin precesses relative to the momentum, which is proportional to the anomaly, rather than to $g$. A precision measurement of $a_\mu$ requires precision measurements of the precession frequency $\omega_a$ and the magnetic field, which is expressed as the free-proton precession frequency $\omega_p$ in the storage ring magnetic field.

The muon frequency can be measured as accurately as the counting statistics and detector apparatus permit. The design goal for the NMR magnetometer and calibration system was a field accuracy of 0.1 ppm. The $B$ which enters in Eq. 7 is the average field seen by the ensemble of muons in the storage ring. The need for vertical focusing implies that a gradient field is needed, but the usual magnetic gradient used in storage rings is ruled out, since a sufficient magnetic gradient for vertical focusing would spoil the ability to use NMR to measure the magnetic field to the necessary accuracy. An electric quadrupole is used for vertical focusing, taking advantage of the “magic” $\gamma = 29.3$ at which an electric field does not contribute to the spin motion relative to the momentum. With both an electric and a magnetic field, the spin difference frequency is given by

$$\omega_a = -\frac{e}{mc} \left[ a_\mu \bar{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right) \vec{B} \times \vec{E} \right], \quad (8)$$

which reduces to Eq. 7 in the absence of an electric field. For muons with $\gamma = 29.3$ in an electric field alone, the spin would follow the momentum vector.

The experimental signal is the $e^\pm$ from $\mu^\pm$ decay, which were detected by lead-scintillating fiber calorimeters.\[^{26}\] The time and energy of each event was stored for analysis offline. Muon decay is a three-body decay, so the 3.1 GeV muons produce a continuum of positrons (electrons) from the end-point energy down. Since the highest
energy $e^\pm$ are correlated with the muon spin, if one counts high-energy $e^\pm$ as a function of time, one gets an exponential from muon decay modulated by the $(g - 2)$ precession. The expected form for the positron time spectrum is $f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_d t + \phi)]$, however in analyzing the data it is necessary to take a number of small effects into account in order to obtain a satisfactory $\chi^2$ for the fit.\cite{15}

![Figure 5](image-url)  

**Figure 5.** The time spectrum of positrons with energy greater than 2.0 GeV from the Y2000 run. The endpoint energy is 3.1 GeV. The time interval for each of the diagonal “wiggles” is given on the right.

The experimental results thus far are shown in Fig. 6 with the average

$$a_\mu(\text{exp}) = 11659203(8) \times 10^{-10} \pm 0.7 \, \text{ppm} \tag{9}$$

being dominated by results from E821. The theory value does not reflect the re-analysis just made available,\cite{22} but rather $a_\mu(\text{Had};1)$ determined from older data\cite{17} is shown. One additional data set from E821 is being analyzed; the only data set obtained with negative muons. The result should be finalized in Fall ’03 with an expected uncertainty between 0.7 and 0.8 ppm. We are exploring the possibility of an upgraded experiment at Brookhaven or at J-PARC\cite{27} which could reach 0.1 ppm (BNL) or 0.06 ppm (J-PARC) precision.

![Figure 6](image-url)  

**Figure 6.** Measurements of $a_\mu$. The theory comes from Davier and Höcker\cite{17} with the sign of the hadronic light by light corrected.

**MEASUREMENT OF THE ELECTRON ANOMALY**

While measurements of the electron anomaly have a history stretching back to Kusch,\cite{6} a major breakthrough in precision came with the trap experiments of Dehmelt,\cite{14}
Single electrons were captured in a Penning trap, and the difference (beat) frequency $\omega_s$ was measured (see Eq. 7). Conventional resonance techniques were employed to measure $\omega_c$, and this work was carried out using a hyperbolic trap where the cavity shifts of the measured magnetic moments placed serious limitations on the precision available.\[28\]

FIGURE 7. The energy levels of the quantum cyclotron. The energy difference between the spin-flip transition and the cyclotron excitation arises because $g \neq 2 (a \neq 0)$.

Recently Gabrielse has developed a cooled cylindrical trap which he calls a quantum cyclotron.\[29\] The trap is cooled to mK temperatures, so that thermal excitation of the quantum cyclotron levels is very improbable, and the axial motion is reduced. The quantum levels of this system are shown in Fig. 7 where the three frequencies now become quantum transitions. The spin flip transition is slightly different in energy from the cyclotron energy since $a \neq 0$. The anomaly is determined by the ratio of two frequencies

$$\frac{\omega_a}{\omega_c} = \frac{g - 2}{2} = a; \quad \text{where} \quad \omega_a \simeq 170 \text{ MHz}; \quad \omega_{s,c} \simeq 160 \text{ GHz}$$

(10)

without the need for fundamental constants. The goal of is an order of magnitude improvement on the precision of $a$, or $\sim \pm 0.3$ ppb.

In the cylindrical trap, spontaneous emission is suppressed by two orders of magnitude. Thus one can stimulate these quantum transitions and measure the two frequencies. The benefits of cooling the trap can be seen in Fig. 8 where transitions to the first and second energy states are observed until the temperature is lowered to 0.08 K. The trap is built and working, and the first results can be expected in late 2003.

FIGURE 8. Thermal excitation of the levels of the quantum cyclotron. As the trap is cooled, the electron spends more time in the ground state. At 0.08 K the electron can spend on the order of two hours without a thermal excitation (courtesy of G. Gabrielse).
EDM SEARCHES, ESPECIALLY FOR THE MUON

While the MDM has a substantial standard model value, the predicted EDMs for the leptons are unmeasurably small and lie orders of magnitude below the present experimental limits given in Table 2. Thus the presence of an EDM at a measurable level would signify physics beyond the standard model. Since the presently known CP violation is inadequate to describe the baryon asymmetry in the universe, additional sources of CP violation should be present. SUSY models do predict an EDM.[33] For a new physics contribution to $a_\mu$ of $3 \times 10^{-9}$ (of the order which might have been seen in E821 before the CDM2 normalization error was found),

$$d_{\mu}^{\text{NP}} \simeq 3 \times 10^{-22} \left( \frac{a_\mu^{\text{NP}}}{3 \times 10^{-9}} \right) \tan \phi_\text{CP} \ e^-\text{cm} \quad (11)$$

where $\phi_\text{CP}$ is a CP violating phase.

**TABLE 2.** Measured limits on electric dipole moments, and their standard model values

| Particle | Present EDM Limit (e-cm) | Standard Model Value (e-cm) |
|----------|--------------------------|-----------------------------|
| $n[30]$ | $6.3 \times 10^{-26}$ | $10^{-31}$ |
| $e^-[31]$ | $\sim 1.6 \times 10^{-27}$ | $10^{-38}$ |
| $\mu[12]$ | $< 10^{-18}$ (CERN) | $10^{-35}$ |
| | $\sim 10^{-19}$ (E821)$^*$ | |
| | $\sim 10^{-24}$ † | |

$^*$ Estimated limit, work in progress.
† Proposed new dedicated experiment.[33]

A new experiment to search for a permanent EDM of the muon with a design sensitivity of $10^{-24}$ e-cm is now being planned for J-PARC.[32] This sensitivity lies well within values predicted by SUSY models.[33] Feng, et al.,[34] have calculated the range of $\phi_\text{CP}$ available to such an experiment, which is shown in Fig. 9.

**FIGURE 9.** The range of $\phi_\text{CP}$ available to a dedicated muon EDM experiment.[34] The two bands show the one and two standard-deviation ranges if $a_\mu$ differs from the standard model value by $(3 \pm 1) \times 10^{-9}$.
With an EDM present, the spin precession relative to the momentum is given by

\[
\vec{\omega} = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] + \frac{e}{2m} \left[ \eta \left( \frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right) \right]
\]  

(12)

where \( d_\mu = \frac{\eta}{2} \left( \frac{e\hbar}{2mc} \right) \simeq \eta \times 4.7 \times 10^{-14} \ e-\text{cm} \) and \( a_\mu = \frac{\tilde{g} - 2}{2} \). For reasonable values of \( \beta \), the motional electric field \( \vec{\beta} \times \vec{B} \) is much larger than electric fields that can be obtained in the laboratory, and the two vector frequencies are orthogonal to each other. The EDM has two effects on the precession: the magnitude of the observed frequency is increased, and the precession plane is tipped relative to the magnetic field.

E821 was operated at the magic \( \gamma \) so that the focusing electric field did not cause a spin precession. The EDM signal in E821 is very difficult to observe, since the tipping of the precession plane is very small (\( \leq 5 \) mrad). The dedicated experiment will be operated at 500 MeV/c, off of the magic \( \gamma \), and will use a radial electric field to stop the \( (g - 2) \) precession. Thus the EDM would cause a steady build-up of the spin out of the plane with time. Detectors would be placed above and below the storage region, and a time-dependent up-down asymmetry would be the signal of an EDM.

**SUMMARY AND CONCLUSIONS**

Measurements of the muon and electron anomalies played an important role in our understanding of sub-atomic physics in the 20th century. The electron anomaly was tied closely to the development of QED. The subsequent measurement of the muon anomaly showed that the muon was indeed a “heavy electron” which obeyed QED.\[12\] With the sub-ppm accuracy now available for the muon anomaly,\[15\] there may be indications that new physics is beginning to appear. Marciano\[35\] has pointed out that using a few very well measured standard model parameters, rather than a global fit to all electroweak measurements, predicts a Higgs mass which is much smaller than the present experimental limit from LEP. If one argues that any discrepancy between the standard model value of \( a_\mu \) and the experimental one is an indication that the hadronic contribution has been underestimated, and uses this discrepancy to “determine the hadronic contribution”, the Higgs mass limit gets even smaller. Marciano concludes that “hints of ‘New Physics’ may be starting to appear in quantum loop effects.”

The non-observation of an electron EDM is becoming an issue for supersymmetry, just as the non-observation of a neutron EDM implies such a mysteriously (some would say un-naturally) small \( \theta \)-parameter for QCD. The search for EDMs will continue, and if one is observed, the motivation for further searches in other systems will be even stronger. The muon presents a unique opportunity to observe an EDM in a second-generation particle, where the \( CP \) phase might be different from the first generation, or the scaling with mass might be quadratic rather than linear.

It is clear that the study of lepton moments (and neutron EDM searches) will continue to be a topic of great importance in the first part of the 21st century. Both the theoretical and experimental situations are evolving. Stay tuned for further developments.
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