Vacuum alignment with(out) elementary scalars

Tommi Alanne, Helene Gertov, Aurora Meroni and Francesco Sannino

I. INTRODUCTION

Theories in which the gauge group is only a part of the original global symmetry can undergo a vacuum (mis)alignment phenomenon via quantum corrections. In two pioneering papers in the context of technicolor, Peskin [1] and Preskill [2] recognized that quantum corrections stemming from the electroweak (EW) sector can destabilize the original vacuum endangering the EW stability. Later Kaplan and Georgi [3, 4] turned the technicolor misalignment problem into a feature by realising that the Higgs doublet of the Standard Model (SM) can be identified with a doublet of dynamically generated Goldstone bosons (GBs). The final vacuum of the theory depends heavily on the details of the dynamics responsible for generating the SM fermion masses which, for purely fermionic composite extensions, is unknown.

We start with reviewing well-known results in the elementary framework, and then show how the analysis modifies when considering elementary scalars. In the elementary case we start from the general form of the Coleman–Weinberg (CW) effective potential [5] and then deduce general conditions for the stability of the vacuum (mis)alignment phenomenon via quantum corrections. In two pioneering papers in the context of technicolor, Peskin [1] and Preskill [2] recognized that quantum corrections stemming from the gauge group, both \( E \) to the gauge-breaking vacuum, \( \chi \), and unbroken generators, \( \sigma \). Since \( E_0 \) preserves the full gauge group, both \( \sigma \) and \( \chi \) leave the vacuum \( E_0 \) invariant. Furthermore, the unbroken \( \sigma \) also leaves \( E_B \) invariant, i.e.

\[
\sigma^a \cdot E_0 = 0, \quad \sigma^a \cdot E_B = 0, \\
\chi^a \cdot E_0 = 0, \quad \chi^a \cdot E_B \neq 0, 
\]

where the notation \( \cdot \) represents the generators acting on the vacuum. Thus the only non-zero contribution to the gauge boson masses are the \( \chi^a \cdot E_B \)

II. REVIEW OF THE VACUUM ALIGNMENT IN COMPOSITE SCENARIOS

Let us consider underlying composite framework with global chiral-symmetry-breaking pattern \( SO(N) \to SO(N - 1) \). The corresponding coset space is parameterized by

\[
\Sigma = \exp \left( \frac{i}{f} \Pi^a X^a \right) E, 
\]

where the \( \Pi^a \) are the GBs corresponding to the broken generators \( X^a \), \( f \) is the 'pion decay constant', and \( E \) gives the vacuum orientation inside \( SO(N) \).

A. The gauged subsector and fermion embedding

Let \( K \leq SO(N) \) be gauged, and assume that the vacuum alignment \( E_0 \) preserves \( K \), whereas \( E_B \) breaks it maximally to \( K_B \). In the following, we investigate the alignment of the vacuum with respect to the gauged subgroup \( K \).

To this end, we parameterise \( E \) as:

\[
E \equiv E_0 \cos \theta E_0 + \sin \theta E_B, 
\]

where the angle \( \theta \) is \textit{a priori} a free parameter in the range \([0, \pi/2]\).

Let us start by defining the basis with respect to the gauge-breaking vacuum \( E_B \). Let \{\( \tau^a \)\} be the set of generators of the gauge group \( K \), which can be divided into the broken, \{\( \chi^a \)\}, and unbroken generators, \{\( \sigma^a \)\}. Since \( E_0 \) preserves the full gauge group, both \( \sigma^a \) and \( \chi^a \) leave the vacuum \( E_0 \) invariant. Furthermore, the unbroken \( \sigma^a \) also leaves \( E_B \) invariant, i.e.

\[
\sigma^a \cdot E_0 = 0, \quad \sigma^a \cdot E_B = 0, \\
\chi^a \cdot E_0 = 0, \quad \chi^a \cdot E_B \neq 0, 
\]
with masses \( K \) \( \tau \) \( \pi \) \( \Lambda \) \( \theta \) \( \Pi \) \( h \) \( v \) \( f \) \( N \) \( m \) \( L \) \( g \) \( R \) \( S \) \( T \) \( V \) \( W \) \( X \) \( Y \) \( Z \) \( \delta \) \( \theta \) \( \phi \) \( \psi \) \( \chi \) \( \varepsilon \) \( \omega \) \( \alpha \) \( \beta \) \( \gamma \) \( \delta \) \( \epsilon \) \( \zeta \) \( \eta \) \( \theta \) \( \iota \) \( \kappa \) \( \lambda \) \( \mu \) \( \nu \) \( \xi \) \( \omicron \) \( \pi \) \( \rho \) \( \sigma \) \( \tau \) \( \upsilon \) \( \phi \) \( \chi \) \( \psi \) \( \omega \)

The gauge contribution has a minimum at \( \theta = 0 \), meaning that the gauge sector prefers to be unbroken in agreement with Peskin [1] and Preskill [2]. However, the top sector prefers the minimum to be at \( \theta = \pi/2 \). This contribution was not considered in [1] and was analysed only later in recent models of technicolor and composite-Higgs dynamics, see e.g. [6] [7]. Given the large top contribution, it will dominate the potential and try to align the vacuum in the direction where the electroweak symmetry is fully broken. This means that in the composite limit, we need new sources of vacuum misalignment to achieve a pseudo-Goldstone-boson (pGB) Higgs scenario. We will discuss such a source in the next subsection.

C. Explicit symmetry breaking

To achieve the desired vacuum alignment, one can add an \textit{ad hoc} explicit symmetry breaking operator taking the form:

\[
V_B = \pm C_B f^4 \text{Tr}[E_0^2 \Sigma] = \pm C_B f^4 \cos \theta + \ldots ,
\]

where \( C_B \) is just a positive dimensionless constant, and the sign depends on whether the trivial minimum is at \( \theta = 0 \) (positive sign) or \( \theta = \pi/2 \) (negative sign). Since the minimum without the explicit-breaking term is at \( \theta = \pi/2 \), a negative sign is needed to achieve a pGB vacuum.

The method of adding an \textit{ad hoc} operator is well known in the literature [7] [8].

### III. VACUUM ALIGNMENT WITH ELEMENTARY SCALARS

We consider an SO(\(N\)) invariant scalar potential for a scalar field \( \Phi \):

\[
V_0 = \frac{m^2}{2} \Phi \Phi + \frac{\lambda}{4!} (\Phi \Phi)^2 .
\]

If \( m^2 \) is negative SO(\(N\)) breaks spontaneously to SO(\(N-1\)) leaving behind \( N-1 \) GBs. We parameterise the scalar field as

\[
\Phi = (\sigma + i \Pi^a X^a) E ,
\]

where \( \Pi^a \) are the GBs corresponding to the broken generators \( X^a \), and \( E \) gives the vacuum alignment. The gauging of the subgroup \( K \) and adding fermion-Yukawa interactions breaks the global symmetry explicitly, and thus an interesting part of the dynamics arises via quantum corrections. For \( N > 4 \), it is always possible to decompose the \( N \) multiplet into a 4 of the SO(\(4\)) ≅...
SU(2)\(_L\) \(\times\) SU(2)\(_R\) and \(N-4\) singlets. The EW interactions are embedded in such a way that the SO(4) is a subgroup of the unbroken SO(\(N-1\)). In this way the electroweak symmetry is intact at tree-level, and the Higgs doublet is massless.

A. The gauged subsector

Similarly as in the composite framework, Sec. II, we gauge \(K\leq\text{SO}(N)\) by introducing the covariant derivative

\[ D_\mu \Phi = \partial_\mu \Phi - ig A_\mu^a \tau^a \cdot \Phi, \tag{15} \]

and parameterise the vacuum as in Eq. (2), \(E = E_\theta = \cos \theta E_0 + \sin \theta E_B\), where \(E_0\) preserves \(K\) and \(E_B\) breaks it to \(K_B\). Thus, when \(\Phi\) acquires a vev, \(\langle \Phi \rangle = fE\), the gauge bosons obtain masses

\[ (\mu^2)_{ab} = g^2 C(E_B)_{ab} f^2 \sin^2 \theta. \tag{16} \]

The \(C(E_B)_{ab}\) factor encodes the group theoretical embedding of the gauge group as in the composite case.

B. Adding fermions

For simplicity, we consider the usual SM-type fermions, i.e. a Yukawa sector invariant only under the gauge group \(K\) and not the full global symmetry. Further, we assume that the fermions obtain their masses fully via the vev of \(\Phi\). Then, we can write the fermion mass-squared matrix as

\[ \left( m^1 m \right)_{ab} = y^2 D(E_B)_{ab} f^2 \sin^2 \theta. \tag{17} \]

Also in this case the factor \(D(E_B)_{ab}\) contains information about the fermion embedding and is independent of the angle \(\theta\).

C. The Coleman–Weinberg potential and the renormalisation procedure

To illustrate the different UV structure and to relate to the composite case, let us first write down the one-loop effective potential regulated with hard cut-off, \(\Lambda\):

\[
V_1(\varphi) = \frac{1}{64\pi^2} \text{Str} \left[ \Lambda^4 \left( \log \Lambda^2 - \frac{1}{2} \right) \right] \\
+ 2 M^2(\varphi) \Lambda^2 \\
+ M^4(\varphi) \left( \log \frac{M^2(\varphi)}{\Lambda^2} - \frac{1}{2} \right) + \text{c.t.} , \tag{18}
\]

where \(\varphi\) is the background choice, and \(M(\varphi)\) is the tree-level mass matrix evaluated on the given background. The supertrace is defined as

\[ \text{Str} = \sum_{\text{scalars}} -2 \sum_{\text{Weyl fermions}} +3 \sum_{\text{vectors}} . \tag{19} \]

In the composite framework with a physical cut-off, the leading contributions to the effective potential, below the cut-off, are of the same form as the second term in Eq. (18). In the renormalizable framework, we cancel these divergent contributions via counter terms, and the dominant contribution assumes the form of the last term in squared brackets of Eq. (18). We choose to work in the \(\overline{\text{MS}}\) scheme in which the one-loop CW potential can be written as

\[ V_{\text{eff}} = V_0 + V_1^{\text{scalar}} + V_1^{\text{gauge}} + V_1^{\text{fermion}} , \tag{20} \]

with

\[ V_1^{\text{scalar}} = \frac{1}{64\pi^2} \text{Tr} \left[ M^4(\varphi) \left( \log \frac{M^2(\varphi)}{\mu_0^2} - \frac{3}{2} \right) \right], \]

\[ V_1^{\text{gauge}} = \frac{3}{64\pi^2} \text{Tr} \left[ \mu^4(\varphi) \left( \log \frac{\mu^2(\varphi)}{\mu_0^2} - \frac{5}{6} \right) \right], \tag{21} \]

\[ V_1^{\text{fermion}} = -\frac{4}{64\pi^2} \text{Tr} \left[ \left( m^1(\varphi)m(\varphi) \right)^2 \right. \]

\[ \times \left( \log \frac{m^1(\varphi)m(\varphi)}{\mu_0^2} - \frac{3}{2} \right) \right] , \]

where \(M(\varphi), \mu(\varphi), \text{and } m(\varphi)\) are the background-dependent scalar, gauge boson, and fermion mass matrices, respectively.

We fix the renormalisation scale such that the vev remains at the tree-level value, i.e. the one-loop tadpole contributions in the \(\sigma\) direction vanish,

\[ \frac{\partial (V_1^{\text{scalar}} + V_1^{\text{gauge}} + V_1^{\text{fermion}})}{\partial \sigma} \bigg|_{\sigma=f} = 0. \tag{22} \]

D. The scalar and gauge contributions

The background-dependent masses of \(\sigma\) and the gauge bosons, respectively, read

\[ M_\sigma^2 = \frac{\lambda}{6} (3\sigma^2 - f^2) , \]

\[ (\mu^2)_{ab} = g^2 C(E_B)_{ab} \sigma^2 \sin^2 \theta. \tag{23} \]
Thus, in the $\sigma$ direction

$$
V_{1}^{\text{scalar}} = \frac{1}{64\pi^2} \left( \frac{\lambda}{6} (3\sigma^2 - f^2) \right)^2 \times \left[ \log \frac{\lambda(3\sigma^2 - f^2)}{6\mu_0^2} - \frac{3}{2} \right],
$$

(24)

$$
V_{1}^{\text{gauge}} = \frac{3g^4}{64\pi^2} \sigma^4 \sin^4 \theta \times \left[ A \log \frac{g^2 \sigma^2 \sin^2 \theta}{\mu_0^2} + B \right],
$$

where

$$
A = \text{Tr}[C(E_B)^2] \quad \text{and} \quad B = \text{Tr} \left[ C(E_B)^2 \left( \log C(E_B) - \frac{5}{6} \right) \right].
$$

(25)

Fixing the renormalisation scale using Eq. (22) leads to

$$
\log \mu_0^2 = \frac{1}{18Ag^4 \sin^4 \theta + \lambda^2} \times \left[ 9g^4 \sin^4 \theta \left( 2A \log (g^2 f^2 \sin^2 \theta) + A + 2B \right) + \lambda^2 \log \frac{\lambda f^2}{3e} \right].
$$

(26)

Substituting this in Eq. (20), we get the renormalised effective potential, $V_{\text{eff}}^R$.

E. The vacuum structure

The vacuum energy depends on the angle $\theta$. Therefore, to find the true minimum, we need to further minimize with respect to $\theta$, i.e.

$$
\frac{\partial V_{\text{eff}}^R}{\partial \theta} \bigg|_{\sigma=f} = 0.
$$

(27)

This yields the following condition:

$$
0 = \frac{-g^4 f^4 \sin^3 \theta \cos \theta}{32\pi^2} \left( 18A g^4 \sin^4 \theta + \lambda^2 \right)^2 \times \left[ 972A^3 g^8 \sin^8 \theta + 90A^2 g^4 \lambda^2 \sin^4 \theta - \lambda^4 \left( 2A \log \frac{3g^2 \sin^2 \theta}{\lambda} + A + 2B \right) \right].
$$

(28)

The potential has trivial critical points at $\theta = 0$ and $\theta = \pi/2$. Studying the second derivative, we find that the critical point at $\theta = 0$ is a maximum, and $\theta = \pi/2$ is a minimum.

In this work, we are mainly interested in whether it is possible to determine a non-trivial minimum without adding further ingredients to the model. This can only exist, if

$$
f(\theta) = 0,
$$

(29)

where we defined

$$
f(\theta) \equiv 972A^3 g^8 \sin^8 \theta + 90A^2 g^4 \lambda^2 \sin^4 \theta - \lambda^4 \left( 2A \log \frac{3g^2 \sin^2 \theta}{\lambda} + A + 2B \right).
$$

(30)

We find that $f(\theta)$ has a minimum at $\tilde{\theta}$ for which

$$
\sin^4 \tilde{\theta} = \frac{\lambda^2}{108A g^4},
$$

(31)

and at this minimum

$$
f(\tilde{\theta}) = \frac{1}{12} \lambda^4 \left[ 12A \log(12A) - A - 24B \right].
$$

(32)

In order to have a non-trivial minimum for the model, Eq. (30) has to be zero for some value of $\theta$. Since $f(\theta) \to \infty$ as $\theta \to 0$ this is only possible if $f(\tilde{\theta}) < 0$. Thus, we obtain a condition for the desired minimum depending only on the gauge group structure:

$$
12A \log(12A) - A - 24B < 0.
$$

(33)

We obtain more insight if we write down $A$ and $B$ in the mass eigenbasis of the gauge bosons. To this end, we write

$$
\tilde{C}(E_B)_{ab} = (U^{-1} C(E_B) U)_{ab} \equiv c_a \delta_{ab}.
$$

(34)

Then,

$$
A = \sum_a c_a^2, \quad \text{and} \quad B = \sum_a c_a^2 \left( \log c_a - \frac{5}{6} \right),
$$

(35)

and

$$
12A \log(12A) - A - 24B
$$

$$
= \sum_a \left[ 12c_a^2 \log \left( \sum_b 12c_b^2 \right) - c_a^2 - 24c_a^2 \left( \log c_a - \frac{5}{6} \right) \right] + 19c_a^2.
$$

(36)
All the terms in the sum are positive, so we conclude that the condition of Eq. (33) can never be fulfilled, and thus there is no solution with non-vanishing vacuum alignment.

There is still one possible caveat to be checked: If $\lambda^2 > 108A g^4$, Eq. (31) does not have a solution, and we have to see whether $f(\pi/2)$ can be negative in this case. To do that, let $\lambda = \alpha g^2 \sqrt{108A}$ with $\alpha > 1$. Then

$$f\left(\frac{\pi}{2}\right) = 972A^2 g^8 \left[ \alpha^4 (12A \log(12A) - A - 24B) + A (12\alpha^4 \log \alpha^2 - 11\alpha^4 + 10\alpha^2 + 1) \right],$$

where the first line is positive by Eq. (36) and the second line is always non-negative for $\alpha > 1$ (and zero at $\alpha = 1$). Therefore the previous conclusion holds.

F. The fermion contribution

The same procedure used in the previous section can be applied, without loss of generality, to the Yukawa sector and in particular to the top quark. We define, similarly to the gauge boson sector:

$$A_f = \text{Tr}[D(E_B)^2] \quad \text{and} \quad B_f = \text{Tr}\left[D(E_B)^2 \left( \log D(E_B) - \frac{3}{2} \right) \right],$$

such that we can write the combined gauge and fermion contributions as

$$V_{\text{gauge}} + V_{\text{form}} = \frac{3g^4}{64\pi^2} \sigma^4 \sin^4 \theta \left[ \bar{A} \log \frac{g^2 \sigma^2 \sin^2 \theta}{\mu_0^2} + \bar{B} \right],$$

where we set

$$\bar{A} = A - \frac{4g^4}{3g^4} A_f \quad \text{and} \quad \bar{B} = B - \frac{4g^4}{3g^4} B_f - \frac{4g^4}{3g^4} A_f \log \frac{g^2}{g_0^2}.$$  

This shows that we get the same condition for non-trivial solution as in Eq. (33) for $\bar{A}$ and $\bar{B}$, if $\bar{A} > 0$. However, in this case there is also the possibility to have negative $\bar{A}$. Then, following the procedure in the previous section, we obtain

$$\bar{A} \log(-2\bar{A}) - 3\bar{A} - 2\bar{B} > 0.$$  

The regions where these conditions are fulfilled in the $(\bar{A}, \bar{B})$ plane are shown in Fig. 1. We note that in the case of dominant fermion contributions (or neglecting gauge contributions), as in the SM-like gauge–fermion(top) sector, it is not possible to find a non-trivial solution. Different gauge–fermion realisations are not discussed in this work.

In the following section, we will show, using the simplest elementary SO(5) → SO(4) framework, that it is possible to stabilize the vacuum at a non-vanishing value of $\theta$ by extending the scalar sector with a gauge singlet field. In this case no explicit breaking terms are needed. Alternatively one can still add explicit symmetry breaking terms to stabilize the vacuum for a non-vanishing $\theta$.

IV. THE SO(5) → SO(4) ELEMENTARY TEMPLATE

The simplest breaking pattern enabling to embed the entire Higgs doublet of the SM as GB is SO(5) → SO(4). This breaking pattern produces four GBs and it has been extensively used in composite Higgs models, first considered in [9, 10]. Here we are interested in elementary scalar degrees of freedom where the underlying scalar potential is renormalisable. We emphasize that our treatment is different from that of [11], since we consistently calculate the full one-loop potential, and determine the vacuum alignment based on these corrections.

![Figure 1](image-url)
Here and in the following, $X^a$ and $S^a$ are respectively the broken and unbroken generators normalized as $\text{Tr}(S^a S^b) = \text{Tr}(X^a X^b) = 2\delta^{ab}$, $\text{Tr}(S^a X^b) = 0$ (the explicit expressions for the SO(5) generators can be found in Appendix A). We again identify the vacuum of the theory as a superposition of a vacuum preserving the EW group, $E_0$, and a vacuum, $E_B$, which breaks the EW group $\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_Q$ as $E_0 = \cos \theta E_0 + \sin \theta E_B$, where the two vacua can be explicitly written as

$$E_0 = (0, 0, 0, 0, 1)^T, \quad E_B = (0, 0, 1, 0, 0)^T. \quad (42)$$

We parameterise the SO(5) scalar multiplet similarly as in Eq. (14), $\Phi = (\sigma + i \Pi^a X^a) E_0$.

The masses for the $W^\pm$ and $Z$ bosons are:

$$\mu_W^2 = \frac{1}{4} g^2 f^2 \sin^2 \theta,$$

$$\mu_Z^2 = \frac{1}{4} (g^2 + g'^2) f^2 \sin^2 \theta. \quad (43)$$

Therefore, in order to produce the correct EW spectrum, we identify

$$v_0^2 = f^2 \sin^2 \theta. \quad (44)$$

Finally, we couple the top quark to the left doublet of $\Phi$ such that the top quark acquires a mass

$$m_t = \frac{1}{\sqrt{2}} y_t f \sin \theta. \quad (45)$$

Both the gauge bosons and fermions masses are proportional to $f \sin \theta$.

### A. Quantum corrections

The quantum corrections are given in Eqs. (23) and (39) where $\hat{A}$ and $\hat{B}$ now are

$$\hat{A} = \frac{1}{16} \left[ 2 \left( 1 + \frac{g'^2}{g^2} \right)^2 - \frac{y_t^4}{g^4} \right],$$

$$\hat{B} = \frac{1}{16} \left[ \left( 1 + \frac{g'^2}{g^2} \right)^2 \left( \log \frac{g^2 + g'^2}{4 g^2} - \frac{5}{6} \right) \right.$$

$$\left. - \frac{5}{3} - 4 \log 2 \right] + \frac{y_t^4}{g^4} \left( \frac{3}{2} + \log 2 \right) - \frac{y_t^4}{g^4} \log \frac{y_t^2}{g^2}.$$ \quad (46)

Making use of the SM values for the gauge and Yukawa couplings, we obtain:

$$\hat{A} \approx -5.2, \quad \hat{B} \approx 6.9. \quad (47)$$

Note that this coincides with the star in Fig. 1, showing that the EW does not break in this case, and it further illustrates the usefulness of the general results presented in the previous Section.

### B. Vacuum stabilization mechanisms

As shown above, for the SM gauge and fermion embeddings, the vacuum is never stabilized for a $\theta$ away from $0$. In the following, we present an alternative mechanism for stabilising the vacuum in the elementary framework such that a pGB Higgs appears without adding further explicit symmetry breaking operators: We extend the theory with a singlet scalar state. We will see that the vacuum dynamically orients itself in a direction supporting a pGB Higgs with non-vanishing value of $\theta$. Alternatively, we could choose to break explicitly the SO(5) symmetry to SO(4) via a minimal operator that forces the vacuum to align in the desired direction. We will briefly discuss this alternative after discussing the singlet-scalar case.

1. **Adding a singlet**

In the following, we show that it is possible to have a dominantly pGB Higgs with mass 125 GeV by adding a scalar singlet $S$ to the theory. For simplicity, we take this new scalar to be $Z_2$ symmetric and real. The scalar potential then reads:

$$V_0 = \frac{1}{2} m_S^2 \Phi^\dagger \Phi + \frac{1}{2} m^2 S^2$$

$$+ \frac{\lambda}{4!} (\Phi^\dagger \Phi)^2 + \frac{\lambda_S}{4!} (\Phi^\dagger \Phi) S^2 + \frac{\lambda_S}{4!} S^4. \quad (48)$$

The stability of the potential requires:

$$\lambda \geq 0, \quad \lambda_S \geq 0 \quad \text{and} \quad 3\lambda_S + \sqrt{\lambda \lambda_S} \geq 0. \quad (49)$$

Assuming for simplicity that $S$ does not acquire a vev, the background-dependent mass of the new scalar $S$ reads

$$M_S^2(\sigma) = m_S^2 + \frac{\lambda_S}{2} \sigma^2. \quad (50)$$

The singlet contributes to the one-loop effective potential with a term

$$V_1^S = \frac{1}{64\pi^2} M_S^2(\sigma) \left( \log \frac{M_S^2(\sigma)}{\mu_0^2} - \frac{3}{2} \right). \quad (51)$$

We then fix the renormalization scale similarly as in Eq. (22) and minimize again the vacuum energy with respect to the alignment angle, $\theta$. Finally, we search for solutions for $\theta \in (0, \pi/2)$. To this end, it is convenient to express the physical mass of $S$ as $M_S = \beta M_\sigma$. Then the function $f(\theta)$ modifies to:
\[ f(\theta) = 972 \bar{A}^2 g^8 \sin^8 \theta \]
\[ + 18 \bar{A}^2 g^4 \lambda \sin^4 \theta (5\lambda + 2\beta^4 \lambda + 3\beta^2 \lambda S) \]
\[ + 2\lambda^2 (\lambda + \beta^2 \lambda S) (2\beta^4 \lambda - \lambda - 3\beta^2 \lambda S) \]
\[ \times \left( \frac{B + \bar{A} \log \frac{3g^2 \sin^2 \theta}{\lambda}}{\lambda} \right) \]  \hspace{1cm} (52)
\[ + \bar{A} \lambda^2 \left[ (\lambda + \beta^2 \lambda S) (8\beta^4 \lambda - \lambda - 9\beta^2 \lambda S) \right. \]
\[ + 2\beta^2 \lambda S (\lambda - 2\beta^4 \lambda + 3\beta^2 \lambda S) \log \beta^2 \] ,

where \( \lambda \) is proportional to the mass squared of \( \sigma \), \( \lambda = \frac{6 \lambda^2}{\lambda} \). Finding a vacuum requires \( f(\theta) \) to vanish under the constraints that the Higgs mass is properly reproduced. This leaves us with two free parameters besides \( \theta \) (in addition to \( \lambda_S \) which does not affect the vacuum alignment and the Higgs mass constraint) which we choose to be the tree-level masses of \( \sigma \) and \( S \), \( M_\sigma \) and \( M_S \), respectively. We further fix the mass of \( S \) to \( 0.5M_\sigma \) or \( 2M_\sigma \) as benchmark values. The parameter space is further constrained by the requirement of (tree-level) vacuum stability and perturbativity of the quartic couplings (\( \lambda \ll 4\pi \)). We show the results in Fig. 2.

We observe that along the continuous red line in the \( M_\sigma - \theta \) space, there is a ground state. It is useful to note that using Eqs. (23) and (44), one roughly expects \( M_\sigma \propto \sqrt{\lambda} / \sin \theta \) on the ground state. In addition, we require overall stability of the potential expressed in Eq. (49). It is clear that for small or negative \( \lambda_S \), one needs to have a sufficiently large \( \lambda_S \), as it is clear from the top panel of Fig. 2. We are guaranteed that the overall solution occurs for perturbative values of \( \lambda \) since large values of this coupling (the light blue region in the top corner of the plots) do not admit a ground state. The same analysis is repeated for \( M_S = 2M_\sigma \) in the bottom panel of Fig. 2.

The numerical result shows that the introduction of a singlet, dynamically misaligns the vacuum to a value of \( \theta \neq 0 \).

Last, it is instructive to investigate the decoupling limit for the singlet \( S \), i.e. \( M_S \gg v \). In this limit we obtain:
\[ \sin^2 \theta = \lambda_S \frac{v^2}{M_S^2} \left( 3A + 2B + 2A \log \frac{g^2 v^2}{M_S^2} \right) \]
\[ 4M_S^2 \left( 2A + B + A \log \frac{g^2 v^2}{M_S^2} \right) \] \hspace{1cm} (53)
\[ + \mathcal{O} \left( (f/M_S)^4 \right) . \]

As expected in the exact decoupled limit the EW symmetry remains intact. For finite values of the \( S \) mass the portal coupling is directly responsible for a non-vanishing vacuum value of \( \theta \). In this sense, this mechanism has a dynamical origin.

FIG. 2. The red curve gives the minimum with respect to \( \theta \), and yellow (blue) area is excluded by tree-level vacuum stability (perturbativity). We fix respectively \( M_S = 0.5M_\sigma \) (\( \beta = 0.5 \)) and \( M_S = 2M_\sigma \) (\( \beta = 2 \)) in the upper and lower panels.

2. Explicit symmetry breaking

Adding an ad hoc explicit symmetry-breaking term can be done similarly as in the composite case by adding the operator of the form:
\[ V_B = \pm C_B f^3 E_0^1 \Phi, \] \hspace{1cm} (54)
where \( C_B \) is again a positive dimensionless constant. Contrary to the composite case, we are now interested in moving the minimum away from zero, so the ad hoc operator has to be positive. In the \( \sigma \) background, \( V_B \) reads
\[ V_B = C_B f^3 \sigma \cos \theta. \] \hspace{1cm} (55)
SU(2)R embedded in the SU(4) are identified with
\[ T_L^i = \frac{1}{2} \begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad T_R^i = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & -\sigma_i^T \end{pmatrix}, \] (56)
where \( \sigma_i \) are the Pauli matrices. The generator of the hypercharge is then identified with the third generator of the SU(2)R group, \( T_Y = T_R^3 \). Further, the vacuum that leaves EW intact is given by
\[ E_0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \] (57)
while the alignment that breaks the EW symmetry to U(1)Q is given by
\[ E_B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \] (58)

Following the discussion in the previous sections, we define the vacuum of the theory following Eq. 2 such that \( E_0^T E_B = 1 \). The EW group is gauged by introducing the covariant derivative
\[ D_\mu M = \partial_\mu M - i(G_\mu M + MG_\mu^T), \] (59)
where
\[ G_\mu = gW_\mu^T \hat{T}_L^i + g'B_\mu T_R^3, \] (60)
and the generators \( T_L^i \) and \( T_R^3 \) are given by Eq. (56). When \( \sigma \) acquires a vacuum expectation value, \( f \), the weak gauge bosons acquire masses
\[ m_W^2 = \frac{1}{4} g'^2 f^2 \sin^2 \theta, \quad \text{and} \]
\[ m_Z^2 = \frac{1}{4} (g' + g)^2 f^2 \sin^2 \theta. \] (61)

Further, we couple the SM fermions, in particular the top quark, to the EW doublet within \( M \). The Yukawa term is then given by
\[ \mathcal{L}_{\text{Yuk}} = y_t (Qt^c)_{\alpha}^\dagger \text{Tr}[P_\alpha M] + \text{h.c.}, \] (62)
where \( P_{1,2} \) pick the components of the SU(2)R doublet. The top quark then acquires the following mass:
\[ m_t = \frac{y_t}{\sqrt{2}} f \sin \theta. \] (63)

In the simplest case, where all the massive scalars have equal tree-level masses, the vacuum

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1 As discussed in [6], there are actually two inequivalent vacua of this kind, but with either choice, the physical properties of the pGBs are the same.
alignment depends on one effective quartic coupling, \( \lambda \), and we find phenomenologically viable solutions with \( \theta \ll 1 \), i.e. \( f \) being in the multi-TeV range [12, 13]. Furthermore, the DM candidate can account for the full observed DM abundance, and be consistent with the experimental constraints in the region \( m_{\text{DM}} > 125 \text{ GeV} \). A benchmark scenario for the vacuum alignment with \( \lambda = 0.1 \) is shown in Fig. 4.

In practice the dynamics, for this example, is similar to the case in which we added a singlet state. In other words the potential of the theory is sufficiently rich to allow for a ground state with a non-vanishing \( \theta \).

**V. CONCLUSIONS**

We have investigated the vacuum alignment problem in renormalizable extensions of the SM that can feature a pGB Higgs. We have shown that the structure of the calculable radiative corrections differs from the composite-Goldstone-Higgs paradigm yielding different ways in which the vacuum stabilization mechanisms work. We provided sufficiently general conditions showing, for example, that renormalizable theories with a single massive scalar singlet cannot radiatively stabilize the vacuum. However when generalizing the theory by including at least one more massive scalar state, the vacuum can be made stable without the aid of other mechanisms such as the introduction of new global-symmetry-breaking operators often invoked in composite extensions. We then applied our results to phenomenologically relevant examples.

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**Appendix A: SO(5)/SO(4) generators**

We first identify the SU(2)\(_L\) \( \times \) SU(2)\(_R\) subgroup of SO(5) by fixing the left and right generators as

\[
(T_{LR})_{ij}^a = -\frac{i}{2} \left[ \frac{1}{2} c_{abc} \left( \delta_{i}^a \delta_{j}^b - \delta_{j}^a \delta_{i}^b \right) \right.
\]

\[
\left. \pm \left( \delta_{i}^a \delta_{j}^b - \delta_{j}^a \delta_{i}^b \right) \right],
\]

where \( a = 1, 2, 3 \) and \( i, j = 1, \ldots, 5 \). The generator \( T_{LR}^5 \) is then identified as the generator of hypercharge. We list here the SO(5) generators adopted in this work.

\[
X_{ij}^1 = -i \left[ \sin \theta \left( \delta_{i}^1 \delta_{j}^3 - \delta_{j}^1 \delta_{i}^3 \right) \right.
\]

\[
\left. + \cos \theta \left( \delta_{i}^3 \delta_{j}^3 - \delta_{j}^3 \delta_{i}^3 \right) \right],
\]

\[
X_{ij}^2 = -i \left[ \sin \theta \left( \delta_{i}^2 \delta_{j}^3 - \delta_{j}^2 \delta_{i}^3 \right) \right.
\]

\[
\left. + \cos \theta \left( \delta_{i}^3 \delta_{j}^3 - \delta_{j}^3 \delta_{i}^3 \right) \right],
\]

\[
X_{ij}^3 = -i \left[ -\sin \theta \left( \delta_{i}^3 \delta_{j}^3 - \delta_{j}^3 \delta_{i}^3 \right) \right.
\]

\[
\left. + \cos \theta \left( \delta_{i}^3 \delta_{j}^3 - \delta_{j}^3 \delta_{i}^3 \right) \right],
\]

\[
X_{ij}^4 = -i \left( \delta_{i}^1 \delta_{j}^3 - \delta_{j}^1 \delta_{i}^3 \right).
\]

The generators are normalised such that \( \text{Tr}[X_{ij}^a X_{ij}^a] = 2 \delta^{ab} \).

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[1] M. E. Peskin, Nucl. Phys. B 175, 197 (1980). doi:10.1016/0550-3213(80)90051-6
[2] J. Presskill, Nucl. Phys. B 177, 21 (1981). doi:10.1016/0550-3213(81)90265-0
[3] D. B. Kaplan and H. Georgi, Phys. Lett. B 136 (1984) 183. doi:10.1016/0370-2693(84)91177-8
[4] D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B 136 (1984) 187. doi:10.1016/0370-2693(84)91178-X
[5] S. R. Coleman and E. J. Weinberg, Phys. Rev. D 7 (1973) 1888. doi:10.1103/PhysRevD.7.1888
[6] J. Galloway, J. A. Evans, M. A. Luty and R. A. Tacchi, JHEP 1010, 086 (2010) doi:10.1007/JHEP10(2010)086 [arXiv:1001.1361 [hep-ph]].
[7] G. Cacciapaglia and F. Sannino, JHEP 1404 (2014) 111 doi:10.1007/JHEP04(2014)111 [arXiv:1402.0233 [hep-ph]].
[8] E. Katz, A. E. Nelson and D. G. E. Walker, JHEP 0508 (2005) 074 doi:10.1088/1126-6708/2005/08/074 [hep-ph/0504252].
[9] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719 (2005) 165 doi:10.1016/j.nuclphysb.2005.04.035 [hep-ph/0412089].
[10] R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D 75 (2007) 055014 doi:10.1103/PhysRevD.75.055014 [hep-ph/0612048].
[11] F. Feruglio, B. Gavela, K. Kanshin, P. A. N. Machado, S. Rigolin and S. Saa, JHEP 1606 (2016) 038 doi:10.1007/JHEP06(2016)038 [arXiv:1603.05668 [hep-ph]].
[12] T. Alanne, H. Gertov, F. Sannino and K. Tuominen, Phys. Rev. D 91, no. 9, 095021 (2015) doi:10.1103/PhysRevD.91.095021 [arXiv:1411.6132 [hep-ph]].
[13] H. Gertov, A. Meroni, E. Molinaro and F. Sannino, Phys. Rev. D 92, no. 9, 095003 (2015) doi:10.1103/PhysRevD.92.095003 [arXiv:1507.06666 [hep-ph]].