Swarming collapse under limited information flow between individuals

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Abstract. The emergence of collective decision in swarms and their coordinated response to complex environments underscore the central role played by social transmission of information. Here, the different possible origins of information flow bottlenecks are identified, and the associated effects on dynamic collective behaviors revealed using a combination of network-, control- and information-theoretic elements applied to a group of interacting self-propelled particles. We find a sufficient condition on the agents’ bandwidth \( B_n \) that guarantees the effectiveness of swarming while also highlighting the profound connection with the topology of the underlying interaction network. We also show that when decreasing \( B_n \), the swarming behavior invariably vanishes following a second-order phase transition irrespectively of the intrinsic noise level.

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1. Introduction

Information exchange is critical to the execution and effectiveness of natural and artificial collective behaviors: fish schooling, birds flocking, amoebae aggregating or more generally agents swarming [1, 2, 3, 4, 5, 6]. The mechanistic quest initiated with the self-propelled-particles (SPP) model introduced by Vicsek et al [7] has recently focused on gaining insight into exactly how information flows through the swarm, with the ultimate goal of achieving functional predictions about collective animal
behavior [8, 9, 10]. Collective decisions emerge from information exchanges occurring between locally-interacting agents. Hence, a study of swarm dynamics benefits from a description and representation of the true communication that follows those information transfers [11]. Any real communication channel, irrespective of its topology and the nature of the signal, has a finite informational capacity owing to its noisiness and limited bandwidth. Here, we investigate the origins and consequences of information flow bottlenecks, associated with the finite capacity of information channels on dynamic collective behaviors.

Recently, a network-theoretic approach has been proposed [12, 6, 8, 13, 14], which opens new avenues for the study of the unifying concept of swarm information flow representing the propagation of behavioral changes. However, such a high-level structural representation should not hide the complexity of a central part of the real informational channel associated with the agents’ sensory cascade—(i) detection, (ii) processing followed by (iii) response [11]—taking place when information hits a node and is routed through the swarm signaling network (SSN) [12]. This crucial factor can be better fathomed when considering the prototypical swarming behavior of predator avoidance of fish and marine insects [15, 16] or flocking birds [5] in which the detection of an incoming predator triggers a fright response—in the form of a swift directional change—in a limited set of agents. These localized behavioral responses initiated by the informed agents constitute a signal transmitted through the medium—edges of the SSN—which, in turn, is detected by the agents—nodes of the SSN—directly linked to the informed ones [Fig. 1]. The SSN is a self-assembled and temporal adaptive network [17], whose dynamics is tightly coupled to that of the agents in the physical space. It appears therefore necessary to account for the functional details of each agent, and in particular, its sensory cascade, which can conceptually be modeled using the control-theoretic concept of multi-input and multi-output plant [18].

A full description of the information flow through a dynamic swarm would not be complete without another conceptual layer borrowing elements from information theory. Indeed, swarms are dynamic complex adaptive systems and as such they are well known to sense, store and deploy more information than do simple systems, thereby allowing

![Figure 1. (color online) Schematic of a networked flock of birds with the associated communication channel in the form of the swarm signaling network (SSN). Edges represent an interaction between two agents. Nodes are the agents themselves, which are routing behavioral information.](image)

\[ C_e = B_e \log_2(1 + \text{SNR}_e) \]

\[ C_n = B_n \log_2(1 + \text{SNR}_n) \]
the emergence of large-scale order. The Shannon–Hartley theorem provides the capacity $C$ of information transfer through any information channel

$$C = B \log_2(1 + SNR),$$  \hspace{1cm} (1)$$

$B$ being the channel’s bandwidth and $SNR$ the signal-to-noise ratio [19]. In the case of swarms, one has to consider $C_e$ associated with informational signaling through the medium—the network edges “e”—as well as $C_n$ for the sensory cascade internal to each agent—the network nodes “n” [Fig. 1].

2. Informational bottlenecks in collective behaviors

Returning to the problem of global information flow through the SSN, the max-flow min-cut theorem [20] informs us about informational bottlenecks, whereby the maximum information flow rate is given by the minimum capacity of the network edges or nodes. In other words, the spread of information through the swarm is either limited by the signaling through the medium (extrinsic limit) or by the agent’s sensory cascade (intrinsic limit). As can be seen from (1), the informational capacity is either bandwidth- or $SNR$-limited, therefore suggesting 4 possible distinct informational bottlenecks due to a low value of: (i) $SNR_e$ in the medium, (ii) intra-agent $SNR_n$, (iii) $B_e$ of the medium, or (iv) $B_n$ of agent. Note that option (iii) is not of physical interest since in most media, $B_e$ is typically very high [11]. The influence of low $SNR_e$ in the medium can therefore be used to explain empirical evidences of some specific swarming breakdowns: e.g. schools of fish disperse at dusk [21, 22, 23]. Essentially, the phase transitions uncovered using Vicsek’s model can be traced back to information flow breakdowns through noisy channels: either with low $SNR_e$ [24] as in option (i), or with low $SNR_n$ [7] as in option (ii). Other empirical evidences are stressing the importance of a sufficiently-high bandwidth $\dagger$ in complex adaptive systems: e.g. information transmission through signaling relay during the collective migration of social amoebae [2], or the induction of differential anesthesia when chemically reducing the firing frequency of neurons in dorsal root ganglions [25].

3. Swarm modeling framework

Here, we focus on the overlooked option (iv) discussed in the previous section and study the effects of information flow breakdowns in a swarm stemming from the finiteness of the agents’ bandwidth $B_n$. Using a group of SPPs, we establish a sufficient condition on $B_n$ guaranteeing the emergence of a collective response. We also show that when reducing $B_n$, in the presence of noise, the group undergoes a continuous phase transition from a globally ordered state to a disordered one.

We consider a minimalistic model consisting of $N$ topologically interacting SPPs [7, 8, 12, 13, 4, 6], moving at constant speed $v_0$ through a $\ell \times \ell$ domain having periodic

$\dagger$ Equivalent to information update since the Nyquist rate $2B = f_s$ relates bandwidth and frequency of update
boundaries. Each individual $i$ is characterized by its direction of travel $\theta_i$, and a canonical swarming behavior of the consensus type is examined. To account for the finiteness of the bandwidth, we consider synchronous information exchanges occurring every $T_n = 1/2B_n$, where the unit interval $T_n$ is the minimum time interval between condition changes of data transmission signal, a.k.a. the symbol duration time [19]. The agents move synchronously at discrete time steps $T_n$ by a fixed distance $\delta = v_0 T_n$ upon receiving informational signals from their neighbors as per the linear update rule

$$\theta_i(t + T_n) = \theta_i(t) + \frac{T_n}{k_i} \sum_{j \sim i} \{ \theta_j(t) - \theta_i(t) \} + \eta_n \xi_i(t),$$

where $k_i = k$ is the fixed number of individuals in the topological neighborhood $j \sim i$ of $i$ and $\eta_n \xi_i(t)$ is a Gaussian white noise ($\xi_i(t) \in [-\pi, \pi]$). Even though directions are intrinsically nonlinear quantities, such linear consensus models are known [26, 27, 13, 12, 6] to yield phase transitions similar to those obtained with nonlinear models, such as the one in Ref. [7]. In addition, Eq. (2) is a discrete-time version of the minimal model consistent with experimental correlations in natural flocks of birds, while also predicting the propagation of order throughout entire flocks [4, 13].

At each instant, the dynamical swarm behavior is governed by:

$$\Theta(t + T_n) = P_n(t) \Theta(t) + \eta_n \Xi(t),$$

with $(\Theta(t), \Xi(t)) = (\{\theta_i(t)\}_{1 \leq i \leq N}, \{\xi_i(t)\}_{1 \leq i \leq N})$ and $P_n(t) = (I - T_n \tilde{L}(t))$ are Perron matrices dependent on the unit interval $T_n$ [26], with $\tilde{L}(t) = L(t)/k$, $L(t)$ being the SSN graph Laplacian characterizing the instantaneous communication topology between individuals. The system (3) fully embodies the time-dependent relationship between the information flow and communication structure at the core of our problem. To allow for an analytical study of this system, we first neglect the effects of noise, and given any initial state $\Theta_0$, at any point in time the swarm’s state is $\Theta(t = mT_n) = [\Pi_{j=1,\ldots,m} P_n((j - 1)T_n)] \Theta_0$. In the presence of a fixed communication topology, the stability of the dynamical system would be governed and controlled by the spectral properties of the constant Perron matrix [26]. In the present case, however, the constantly reconfigurable and switching network requires a generalization of such stability analysis. We prove that a necessary and sufficient condition for the system (3) to be stable is that it is stable at every point in time $t_j = j T_n$. This key result is obtained by studying the convergence of infinite products of matrices $P_n(t_j)$ by means of the joint spectral radius $\hat{\rho}$ defined as [29]

$$\hat{\rho} := \limsup_{j \to \infty} \left( \max_{t_1, \ldots, t_j \in [1, m]} \|P_n(t_1) \cdots P_n(t_j)\| \right)^{1/j}.$$

Note that the growing body of evidence in support of a topological model of interaction between flocking birds [8, 12, 13, 4, 6] guided our choice. However, qualitatively similar results were obtained with the exact same model with metric interactions, which is consistent with the recent evidence of a unique universality class in the noise-induced critical behavior of SPPs, regardless of the metric or topological nature of interactions [28].
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By taking \( t_1 = \cdots = t_j \) in Eq. (4) and invoking Gelfand’s spectral radius formula, we have

\[
\hat{\rho} \geq \lim_{j \to \infty} \| P_n(t)^j \|^{1/j} = \rho(P_n(t)),
\]

for any \( t \in [1, m] \). Therefore, if \( \rho(P_n(t)) > 1 \), for any \( t \), then \( \hat{\rho} > 1 \). On the other hand, for any \( \varepsilon > 0 \), there exists a matrix norm \( \| \cdot \| \) such that (e.g. \([30, \text{Lemma 5.6.10}]\))

\[
\hat{\rho} \leq \lim_{j \to \infty} \left( \max_{t_1, \ldots, t_j \in [1, m]} \| P_n(t_1) \|^{1/j} \right) \cdots \cdot \left( \max_{t_1, \ldots, t_j \in [1, m]} \| P_n(t_j) \|^{1/j} \right)
\]

\[
= \max_{t \in [1, m]} \| P_n(t) \|
\]

\[
\leq \max_{t \in [1, m]} \{ \rho(P_n(t)) + \varepsilon \}.
\]

Therefore, if \( \rho(P_n(t)) < 1 \) for all \( t \), we can choose \( \varepsilon \) small enough so that \( \hat{\rho} < 1 \). In that case, the consensus reaching of the swarm is therefore guaranteed \([29, 27]\). Furthermore, it can easily be shown that if the bandwidth satisfies the sufficient condition

\[
B_n > B^0_n = \frac{1}{4} \max_{1 \leq i \leq N} | \lambda_i(\tilde{L}(t)) | \quad \text{for all} \quad t,
\]

then the consensus reaching of the swarm is guaranteed. Condition (7) reveals the profound connection between, on the one hand the switching communication topology—through the maximum eigenvalue of the normalized directed graph Laplacian of the signaling network, and on the other hand, the necessary information flow for effective swarming.

4. Phase transition with bandwidth as control parameter

As a next step, we seek evidences of such required minimum information flow by simulating the dynamics of \( N \) SPPs governed by (2), with decreasing bandwidth \( B_n \)—the control parameter—in the presence of different levels of intrinsic noise \( \eta_n \). The effectiveness in swarming is classically measured by the order parameter \( \phi(t) \equiv \frac{1}{N} \left| \sum_{j=1}^{N} e^{i \theta_j(t)} \right| \). For large bandwidths, \( B_n \gg B^0_n \), swarms of vastly different sizes systematically produce large-scale order, even in the presence of relatively high noise levels [Fig. 2]. Continued reduction in \( B_n \) below \( B^0_n \) consistently yields a swarming collapse—corresponding to a disordered state of the system lacking large-scale self-organization—irrespective of the swarm size \( N \) or noise level \( \eta_n \) [Fig. 2]. These phase transitions are of second order since the Binder cumulant \( U \equiv 1 - \langle \phi^4 \rangle / 3 \langle \phi^2 \rangle^2 \) remains positive for all values of the control parameter \( B_n \) [Fig. 3(a)]. However, it is very likely that similarly to noise-induced phase transitions, the observation of continuous phase transitions is only apparent owing to strong small-size effects \([24, 31, 32]\). Indeed, in our particular framework, we are dealing with real-life finite-size swarms. For such swarms, the population \( N \) is relatively small, especially compared to the thermodynamical limit,
which is classically invoked to fully characterize the very nature of a phase transition from the statistical physics standpoint.

We further observe the existence of a transition line for which the critical bandwidth varies with the noise, i.e. $B_n^c = B_n^c(\eta_n)$. As expected, the variations of the susceptibility $\chi \equiv \ell^2(\langle \varphi^2 \rangle - \langle \varphi \rangle^2)$ with $B_n$ reveal the occurrence of large fluctuations of the order parameter near the phase transition [Fig. 4]. In addition, along the transition line $B_n^c(\eta_n)$, we have that $B_n^c$ decreases with decreasing $\eta_n$. This important observation is consistent with our intuition that a higher noise level would require a higher volume of information to be exchanged for the swarm to self-organize. From the information-theoretic viewpoint, this trend can readily be explained if we assume the existence of a minimum “critical” rate of information $R^c$ below which a collapse of swarming occurs. At the critical point, the max-flow min-cut theorem [20] gives us $C_n = R^c$ and given expression (1), we have that $B_n^c \downarrow$ with $\eta_n \downarrow$. Despite the singularity in Shannon’s capacity at the zero-noise limit, our approach allows us to determine the critical bandwidth in this limit through the intersection of the $U(B_n)$ for several nonzero values of $\eta_n$ [Fig. 3(b)]. It is worth adding that all the above observations remain unchanged for larger values of $N$, other values of the density $\rho = N/\ell^2$, for a wide range of $v_0$, and for other values of $k > 7$. As mentioned earlier, in the thermodynamic limit,
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**Figure 3.** (color online) Stationary values of $U$: (a) wide $B_n$ range; (b) At the critical bandwidth $B_n^c=0.286 \pm 0.001$, $U$ has the same value for not too high $\eta_n$. ($v_0=0.3$, $k=7$, $\rho=N/\ell^2=100$, and equivalent statistics for all data points.)

**Figure 4.** (color online) Stationary values of $\chi$: (a) $N = 1024$; (b) $N = 128$ and $N = 1024$. ($v_0=0.3$, $k=7$, $\rho=N/\ell^2=100$, and equivalent statistics for all data points.)

we still expect to obtain a phase transition albeit possibly of the first order kind as is the case with noise-induced phase transitions [24, 31, 32]. However, our focus here was on swarms numbering in the thousand—as is typical with topologically-interacting flocks of starlings [4], and was not on determining the fine nature of the phase transition taking place when reducing the bandwidth.

5. Conclusions

Most SPP simulations heretofore reported in the literature (e.g. [10, 12, 4, 24, 6]) have been generated using arbitrary, yet sufficiently high, bandwidth levels. Therefore, in those past works, the collapse of swarming is rooted in the noisiness of the signaling channel. Owing to their evolutionary-optimized character, biological swarming agents such as fish and birds do not naturally exhibit the bandwidth-induced phase transition uncovered here. We speculate that this effect can readily be tested experimentally in a simple setup consisting of fish schooling in a tank with a stroboscopic light shining onto them. By reducing the frequency $f$ of the flash, we artificially force the decrease
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in $B_n$ and we expect that at a given critical frequency $f^c = 2B_n^c$, the coordinated schooling behavior will disappear. Beyond the consequences for self-organized biological systems, our work also highlights the importance of having sufficient information signaling capacity when designing artificial swarms so as to ensure their effectiveness. At a more qualitative level and to the best of our knowledge, our analysis provides the first physical explanation for the required minimum firing frequency of neurons in dorsal root ganglia to maintain consciousness and thereby avoid inducing differential anesthesia [25].

Acknowledgments

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