Growth of intermediate mass black holes by tidal disruption events in the first star clusters

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Draft version 5 October 2018

ABSTRACT

We study the stellar dynamics of the first star clusters after intermediate-mass black holes (IMBHs) are formed via runaway stellar collisions. We use the outputs of cosmological simulations of Sakurai et al. (2017) to follow the star cluster evolution in a live dark matter (DM) halo. Mass segregation within a cluster promotes massive stars to be captured by the central IMBH occasionally, causing tidal disruption events (TDEs). We find that the TDE rate scales with the IMBH mass as $N_{\text{TDE}} \sim 0.3 \text{Myr}^{-1} (M_{\text{IMBH}}/1000 M_\odot)^{2}$. The DM component affects the star cluster evolution by stripping stars from the outer part. When the DM density within the cluster increases, the velocity dispersion of the stars increases, and then the TDE rate decreases. By the TDEs, the central IMBHs grow to as massive as $700 - 2500 M_\odot$ in 15 million years. The IMBHs are possible seeds for the formation of supermassive BHs observed at $z \gtrsim 6 - 7$, if a large amount of gas is supplied through galaxy mergers and/or large-scale gas accretion, or they might remain as IMBHs from the early epochs to the present-day Universe.

Key words: intermediate mass black holes – galaxies: star clusters – stellar dynamics

1 INTRODUCTION

A number of quasars have been discovered at redshift $z \gtrsim 6$ (e.g., Mortlock et al. 2011; Wu et al. 2015; Béjar et al. 2018). They are thought to be powered by accreting SMBHs of mass $\gtrsim 10^{9} M_\odot$, which need to be rapidly assembled within 1 Gyr after the Big Bang. The origin of the SMBHs remains largely unknown, but there are a few promising models of BH formation. For example, a very massive Population III star may leave a remnant BH with mass $\sim 100 M_\odot$ (Madau & Rees 2001; Haiman & Loeb 2001; Schneider et al. 2002). Another model considers an even more massive seed BH as a remnant of a supermassive star (SMS) with mass $\gtrsim 10^{5} M_\odot$ (Loeb & Rasio 1994; Oh & Haiman 2002; Bromm & Loeb 2003). The growth of such seed BHs is also under debate. Gas accretion onto an early BH is likely to be suppressed by radiation feedback effects (Alvarez, Wise & Abel 2009; Jeon et al. 2012), whereas it is also possible that hyper-Eddington accretion is achieved under large gas mass accretion (Inayoshi, Haiman & Ostriker 2016; Sakurai, Inayoshi & Haiman 2016).

It has been suggested that an intermediate-mass BH (IMBH) with mass $\sim 1000 M_\odot$ can be formed in a dense star cluster via runaway stellar collisions (Ebisuzaki et al. 2001; Vanbeveren et al. 2009; Devecchi et al. 2010). Dense star clusters can be formed in the early universe by fragmentation of low-metallicity gas clouds (Omukai, Schneider & Haiman 2008). Katz, Sijacki & Haehnelt (2015) perform cosmological simulations and N-body simulations to follow the evolution of early star clusters. They show that the runaway collisions occur within the star clusters of $\sim 10^{3} M_\odot$ in low-mass ($\sim 10^{4} M_\odot$) dark matter haloes. They also show that very massive stars of several hundred solar mass are formed at the cluster center, which leave IMBHs by gravitational collapse. Sakurai et al. (2017) perform cosmological simulations to locate host dark matter halos of the first star clusters, and study the evolution of a number of star clusters in atomic cooling halos at $z \sim 12 - 20$. In all the eight star

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clusters in their simulations, very massive stars with mass \(\sim 400 - 1900 M_\odot\) are formed via stellar collisions.

It is worth studying the later evolution of the first star clusters because tidal disruption events (TDEs) are expected to occur when stars approach the central IMBH. Intriguingly, the TDEs can be bright enough to be observed by SwiftBAT and eROSITA even if a star of \(\lesssim 100 M_\odot\) is disrupted by an IMBH of \(\sim 10^5 M_\odot\) \cite{Kashiyama:2016}, the peak luminosity of the TDEs will be even higher for smaller BHs.

In the present paper, we study the dynamical evolution of the first star clusters after the formation of IMBHs. We run \(N\)-body simulations to follow the cluster evolution for \(\sim 15\) Myr which is comparable to the typical relaxation time of the clusters. The clusters are expected to evolve substantially over a relaxation time. We show that the central IMBHs grow by TDEs and that the TDE rate scales as square of the IMBH mass. The cluster evolution depends on the details of the DM distribution and relative motions between the cluster and the DM halo; stars are stripped from the outer part of the cluster.

We organize the rest of the paper as follows. In Section 2, we describe the initial conditions of our simulations and the details of our \(N\)-body simulations. In Section 3.1, we study mass growth of the IMBHs by TDEs. In Section 3.2, we study how the DM halo affects the TDEs. In Sections 4 and 5, we discuss and summarize our results.

\section{Numerical Methods}

\subsection{Initial conditions of primordial star clusters from Sakurai et al. (2017)}

The initial conditions are generated from the final states of the cluster simulations studied in Sakurai et al. (2017) \cite{Sakurai:2017}. We use the eight models A-H, and three realizations for each model. Hereafter, we refer the initial time \(t = 0\) when the IMBHs are born after 3 million years elapsed since the beginning of our cluster simulations.

We briefly summarize the physical properties of these clusters (see table 1 of SYFH17). The gas clouds that produce star clusters are located at \(z \sim 12 - 20\) in atomic-cooling halos. The halos have masses of \((1.5 - 4.2) \times 10^7 M_\odot\). In the cluster generation procedures at \(t = -3\) Myr,

- SPH particles are replaced with star particles in a probabilistic manner based on Equation (1) in SYFH17 which is determined to preserve mass conservation and to yield a global star formation efficiency of \(\sim 5 - 10\%\),
- the stellar mass distributions are determined by Salpeter initial mass function (IMF) with the minimum mass \(3 M_\odot\) and the maximum mass \(100 M_\odot\),
- the DM particles are retained as in the original zoom-in simulations with a DM particle mass \(1.87 M_\odot\) and
- the velocities of the star and DM particles are rescale such that the star cluster systems are approximately in virial equilibrium.

The resulting star cluster mass ranges around \((5 - 16) \times 10^4 M_\odot\) with the number of stars being \((6 - 20) \times 10^3\) and the core radius \(\sim 0.2 - 0.7\) pc \cite{Casertano:1985}. The star cluster systems have total DM masses of \(\sim (3 - 7) \times 10^5 M_\odot\) and the number of DM particles of \(\sim (2 - 4) \times 10^7\). To follow the dynamical evolution and runaway stellar collisions within the clusters, a hybrid \(N\)-body simulation code BRIDGE \cite{Fujii:2007} is used. The evolution is followed for 3 Myr, which is approximately equal to the lifetime of massive stars. In most of the simulations of SYFH17, runaway stellar collisions occur and massive stars with masses \(\sim 400 - 1900 M_\odot\) are formed. Such very massive stars are expected to collapse gravitationally to IMBHs with the same masses.

In the present study, we replace the central very massive stars with IMBHs, and follow the dynamical evolution for further 15 million years. Each model cluster has one IMBH at the center. In Table 1, we summarize the initial properties of our star cluster samples. Fig. 1 shows the initial spatial distributions of the stars in our eight models. For the initial conditions, the stars are considered to be ‘bound’ if the stellar energy without DM potential is negative, i.e.,

\begin{equation}
\frac{v_{\text{star}}^2}{2} + \phi_{\text{star}} < 0,
\end{equation}

where \(v_{\text{star}}\) is the velocity of a star and \(\phi_{\text{star}}\) is the gravitational potential generated by only the stars. For comparison, we also generate an initial condition without DM for Model A of Sakurai et al. (2017). The properties of the models are given in Table 1.

\subsection{Hybrid N-body simulations}

We use the parallel hybrid \(N\)-body code BRIDGE \cite{Fujii:2007} to follow the stellar dynamics within the star clusters. The hybrid code computes the gravitational forces of stars by direct calculation using the sixth-order Hermite integrator \cite{Nitadori:2005} self-consistently with the forces of DM calculated by the tree method \cite{Barnes:1986} in which a second-order leapfrog integrator is used. In the Hermite integration scheme, the individual timesteps are adopted to improve the computational efficiency. In order to increase the parallel efficiency, the NINJA scheme is used \cite{Nitadori:2006}. The Phantom-GRAPE library is adopted to accelerate the computation of the forces \cite{Tanikawa:2013}. Parameters of the \(N\)-body simulations are the same as in Table 2 of Sakurai et al. (2017) otherwise noted.

The conditions for merger of two stars is set by the so-called sticky sphere approximation \cite[e.g.,][]{Gaburov:2010}, a pair of stars merge when their separation \(d\) becomes less than the sum of the two stellar radii \(r_1 + r_2\). The values of the stellar radii are derived from a fitting formula of Tout et al. (1996) for zero-age-main-sequence stars. Although the stellar mergers may occur disruptively with non-negligible mass loss, we set the merged stellar mass to the sum of the two stellar mass for simplicity.

TDEs by the IMBHs are also implemented. When the separation between a star and an IMBH is less than the TDE radius \(r_{\text{TDE}} \equiv 1.3 r_\text{s} (M_{\text{BH}}/2m_\text{s})^{1/3}\) \cite{Kochanek:1992} where \(r_\text{s}\) is the stellar mass, the star is assumed to be disrupted. Note that we did not use the Schwarzschild radius of a BH for the merger condition because it is much less than the TDE radius. For example, for an IMBH with mass of \(10^5 M_\odot\) and a star with mass of \(10^2 M_\odot\) and a radius \(\sim 10 R_\odot\), the two
Figure 1. Initial distributions of the stars and DM within 400 pc on a side, projected along z-axis. The green dots represent bound stars whereas the black dots are unbound stars. DM density is shown by colors, and the color-bar indicates the surface density in arbitrary units.
radii are

\[ R_{\text{Sch}} = \frac{2GM_{\text{BH}}}{c^2} = 3.0 \times 10^6 M_{\odot} \text{ cm} \]  
\[ r_1 = 1.5 \times 10^{12} M_{\odot}^{1/3} m_{*}^{1/3} r_{*1} \text{ cm}, \]

where \( M_{\text{BH},x} = M_{\text{BH}}/10^5 M_{\odot}, m_{*,x} = m_{*}/10^x M_{\odot} \) and \( r_{*,x} = r_*/10^x R_{\odot} \). We assume that the stellar mass is all added to the IMBH after the merger. More realistically, a part of the stellar mass can be ejected depending on the stellar orbit and thus may not contribute to the IMBH mass increase (Rees 1988, see Section 4.4.3 for discussion).

We follow the star cluster evolution for 15 Myr, which is near one half-mass relaxation time (see Table 1)

\[ t_{\text{rh}} = \frac{0.651 \text{ Gyr}}{M_{\odot}} \left( \frac{M_{\odot}}{10^6 M_{\odot}} \right)^{1/2} \left( \frac{\rho_{\odot}}{1 \text{ pc}} \right)^{3/2}, \]

where \( \gamma \sim 0.015 \) (Giersz & Heggie 1996) \( c_{\sigma} \) is the mean stellar mass of the Salpeter IMF, \( m_{*} \) is a cluster mass and \( r_1 \) is a half-mass radius. Although the time 15 Myr is much longer than the lifetimes of massive stars \( \sim 2 \) – 3 Myr, we do not consider gravitational collapses or supernovae in our main simulations. We will discuss the impact of the collapse (compact remnant formation) and supernovae in Section 4.4.1.

3 RESULTS

3.1 IMBH mass growth by TDEs

In this section, we describe the details of IMBH mass growth by TDEs in the first clusters. We confirm that the IMBHs grow only moderately, to have final IMBH masses \( M_{\text{IMBH,f}} \) determined by the properties of the host haloes which are about 1 – 2% of the cluster mass. We also find that the TDE properties are consistent with those found in literatures (e.g., Baumgardt, Makino & Ebisuzaki 2004).

In Fig. 2 we show the mass evolution of the IMBHs which grow by TDEs. In Table 2 we show the final mass of the IMBHs at the end of the simulations. The IMBHs in the star clusters grow to become as massive as 700 – 2500 \( M_{\odot} \) by TDEs. The diversity of the final IMBH masses can be attributed to the diversity of the properties of the host clusters, or equivalently, the host haloes (see SFYH17). Fig. 3 shows that the IMBH mass and a quantity \( M_{\odot} \ln(\rho_{\odot}/\sigma_{*}^2) \) (Sakurai et al. 2017) linearly correlates with each other, where \( \rho_{\odot} \) and \( \sigma_{*} \) are the core density and the core velocity dispersion, respectively. The quantity \( M_{\odot} \ln(\rho_{\odot}/\sigma_{*}^2) \) is derived by integrating the mass growth rate of the IMBH which undergoes runaway collisions (Portegies Zwart & McMillan 2002). In Fig. 4 we also show the cluster mass-IMBH mass relation derived from our simulations (blue circles). The dashed lines are the relations \( M_{\text{IMBH,f}} = 0.02 M_{\odot} \) (upper line) and 0.01\( M_{\odot} \) (lower line). The final masses are roughly proportional to the cluster mass as \( M_{\text{IMBH,f}} \propto M_{\odot} \). We will compare the result with other studies in Section 4.3 to explore the impact of DM on the relation and possible scenario of the observed IMBH formation.

### Table 1. Initial properties of the star cluster models. The model data are taken from the snapshots at \( t \sim 3 \) Myr of the star clusters studied in Sakurai et al. (2017). For each model, all listed quantities are averaged among three realizations.

| Model | \( M_{\odot} (10^4 M_{\odot}) \) | \( N_{\text{star}} (10^3) \) | \( r_{c} (\text{pc}) \) | \( \rho_{c} (10^7 M_{\odot}/\text{pc}^3) \) | \( t_{\text{rh}} \) (Myr) | \( M_{\text{IMBH,f}} (M_{\odot}) \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| A     | 16.4            | 19.9            | 0.151           | 3.18            | 16.8            | 917             |
| B     | 13.0            | 15.7            | 0.132           | 3.35            | 15.4            | 409             |
| C     | 12.1            | 14.7            | 0.137           | 15.2            | 15.4            | 1312            |
| D     | 11.7            | 14.1            | 0.153           | 15.2            | 15.4            | 971             |
| E     | 4.76            | 5.76            | 0.142           | 2.28            | 11.7            | 724             |
| F     | 9.00            | 10.8            | 0.136           | 12.6            | 14.0            | 908             |
| G     | 12.5            | 15.0            | 0.115           | 25.4            | 15.7            | 1628            |
| H     | 7.70            | 9.32            | 0.159           | 5.15            | 12.9            | 964             |
| AnoDM | 1.64            | 19.9            | 0.136           | 9.35            | 16.3            | 874             |

Column 2: total mass of the star cluster, Column 3: total number of stars, Column 4: core radii, Column 5: core density, Column 6: half-mass relaxation time, Column 7: initial IMBH mass. The core radii and the core density are computed using the method described in Casertano & Hut (1985).
Table 2. Summary of the results of the simulations for \( t < 15 \) Myr.

| Model | \( M_{\text{IMBH,f}} \) (M\(_{\odot}\)) | \( N_{\text{TDE}} \) | \( \dot{N}_{\text{TDE}} \) \( (\text{Myr}^{-1}) \) | \( \langle \delta m \rangle \) (M\(_{\odot}\)) | \( \delta m_{\text{min}} \) (M\(_{\odot}\)) | \( \delta m_{\text{max}} \) (M\(_{\odot}\)) |
|-------|-----------------|----------|-----------------|-----------------|-----------------|-----------------|
| A     | 2134            | 20.0     | 1.33            | 62.2            | 3.21            | 145             |
| B     | 1594            | 16.7     | 1.11            | 71.3            | 6.80            | 176             |
| C     | 2255            | 15.3     | 1.02            | 63.1            | 3.52            | 110             |
| D     | 1625            | 10.7     | 0.711           | 61.5            | 4.24            | 168             |
| E     | 927             | 4.00     | 0.267           | 42.7            | 18.4            | 88.1            |
| F     | 1694            | 14.7     | 0.978           | 51.8            | 3.22            | 116             |
| G     | 2266            | 12.0     | 0.800           | 57.4            | 3.62            | 166             |
| H     | 1289            | 4.67     | 0.311           | 67.5            | 44.1            | 97.1            |
| AnoDM | 2087            | 17.7     | 1.18            | 70.2            | 4.46            | 228             |

Column 2: final mass of the IMBHs, Column 3: the number of the TDEs, Column 4: time-averaged TDE rate, Column 5, 6 and 7: the mean mass, the minimum mass and the maximum mass of the disrupted stars respectively. The first four quantities are averaged using three realizations of the simulations. The latter two quantities are derived from the minimum and maximum mass among the three realizations.

We show the number of TDEs and the TDE rates in Table 2. The number of the TDEs ranges from \( \sim 4 \) to 20, and the TDE rates are \( \sim 0.3 - 3 \) Myr\(^{-1}\). We show in Fig. 3 that the TDE rates in our simulations correlates with the IMBH mass as

\[
\dot{N}_{\text{TDE}} \sim 0.3 \text{ Myr}^{-1} \left( \frac{M_{\text{IMBH,f}}}{1000 \text{M}_\odot} \right)^2.
\]  

The correlation \( \dot{N}_{\text{TDE}} \propto M_{\text{IMBH,f}}^2 \) is suggested in Baumgardt, Makino & Ebisuzaki (2004), who estimate the TDE rate as

\[
\dot{N}_{\text{TDE}} = 1 \text{ Myr}^{-1} r_{c,0}^{3/5} m_{c,1}^{3/5} M_{\text{BH,3}}^{7/5} \sigma_{c,1}^{-21/5},
\]

where \( r_{c,0} = n_c/10^4 \text{ pc}^{-3} \) and \( \sigma_{c,0} = \sigma_c/10^4 \text{ km s}^{-1} \) which are the core number density and the core velocity dispersion respectively. The estimate is based on a loss cone theory presented in Frank & Rees (1976) with calibrations by the results of the N-body simulations of star clusters in Baumgardt, Makino & Ebisuzaki (2004). We find that our TDE rate can be fitted by the analytical expression (see also Section 4.1).

In Table 2, we show the mean mass, the minimum mass and the maximum mass of the disrupted stars. The minimum mass ranges \( 3 - 44 \) M\(_{\odot}\), which is close to the lowest mass of the IMF of \( \sim m_{\text{min}} = 3 \) M\(_{\odot}\) except for Runs E and H. The maximum mass is around the highest mass of the IMF \( \sim m_{\text{max}} = 100 \) M\(_{\odot}\). The mean mass of the disrupted stars is \( \sim 40 - 70 \) M\(_{\odot}\), which is much higher than the mean mass of the IMF of \( \sim 8 \) M\(_{\odot}\). Overall, stars of any mass between \( m_{\text{min}} \) and \( m_{\text{max}} \) can be disrupted, and massive stars are preferentially disrupted owing to mass segregation that occurs rapidly within the dense star clusters.

We show the mass distribution of the disrupted stars in Fig. 6 for all our 8 cluster models with DM with three realizations. The mass distribution is bimodal with two notable bumps around 8 M\(_{\odot}\) and 80 M\(_{\odot}\). Stars with high masses are
preferentially disrupted as is already discussed. The large number of disrupted low-mass stars is due to the fact that the adopted power-law IMF is weighted toward low masses.

The TDEs are triggered by stellar dynamical interactions around the cluster centers. In Fig. 5, we show two examples of orbits just before TDEs. The orbits are taken from the first and second TDEs in one realization of Run A. In the left panel, the IMBH with mass 1006 M⊙ first forms a binary with a massive star with mass 145 M⊙. The third star with mass 29 M⊙ comes close to the binary and strongly interacts with it. Finally, the latter star collides with the IMBH, leaving the other massive star, which continues to be a binary component with the IMBH. In the right panel, the binary is perturbed by a tertiary star with mass 70.3 M⊙ which comes close to it. The two component stars consequently collide with each other.

3.2 Global star cluster evolution

3.2.1 Model A

Before comparing our 8 models, in order to examine the effect of DM on the star cluster evolution, we first focus on Model A-DM (Model A with DM) and Model AnoDM (Model A without DM). Hereafter in this section, we refer to a specific realization Run X1 from Model X as Run X, where X is A-DM or AnoDM.

In Fig. 5 we show the time evolution of bound stellar mass M_{cl,b} for Model A-DM (red solid lines). We note again that the ‘bound stars’ mean that the stars are bound by their own gravitational potential without including the contribution from the DM component (see Equation 4). The bound mass M_{cl,b} decreases from 1.4 × 10^5 M⊙ to 10^5 M⊙ until 8 Myr and then increases to 1.3 × 10^5 M⊙ until 10 Myr. After this point, the mass decreases again by about 50 percent toward the end of the simulations.

The time variation of M_{cl,b} can be explained by the presence of DM. In Fig. 5 we also show M_{cl,b} for the Model AnoDM by a black solid line for comparison. We see little variation in M_{cl,b} for the Model AnoDM, in good contrast to the Model A-DM (red solid lines).

It is important to identify the cause of variation of M_{cl,b}. In Fig. 6 we show the Lagrangian radii of the clusters for 5, 10, 30, 50, 70 and 90% of the initial bound stellar masses M_{cl,b}. The Lagrangian radii for less than 0.3 M_{cl,b} in the Model A-DM do not significantly differ from those in the Model AnoDM. In contrast, the Lagrangian radii for larger than 0.5 M_{cl,b} in the Model A-DM increase when M_{cl,b} decreases, while those in the Model AnoDM do not significantly vary. The expansion indicates that the DM field strips stars from the outer part of the clusters.

Though the outer part of the stars are stripped by DM, we find that most of the stripped stars are still bound by the DM halo in the Model A-DM, i.e., v_{eq}^2 + \phi_{star} + \phi_{DM} < 0, where \phi_{DM} is a DM potential. Furthermore, the DM halo can prevent stars to be completely ejected from the parent halo even after the stars gain high velocities after strong binary interactions near the cluster center.

The stripping of stars from the outer regions by DM and the suppression of ejection can be studied by examining the velocity distributions of the stars. In Fig. 7 we show the stellar velocity distribution for the Run A-DM at t = 0 (blue) and 15 Myr (red). The top panel shows the distributions for r > 1 pc, where r is the radius from the cluster center. The distribution of the outer stars shifts with time toward high velocities, and the number of high velocity stars with v > 10 km s^{-1} increases while that of low velocity stars with v < 10 km s^{-1} decreases. The velocity increase is more directly seen from the velocity color map shown in Fig. 8. The number of high velocity stars increases because of the increase of DM mass in r < 100 pc. According to the virial theorem, stronger gravitational force increases the stars’ velocities. The peak at ~ 20 km s^{-1} in the distribution at t = 15 Myr is consistent with the orbital velocity 10 km s^{-1} < v < 30 km s^{-1} for r_{eq} \approx 10 pc < r < 100 pc, where r_{eq} \approx 10 pc is a radius at which the enclosed DM mass is equal to the enclosed stellar mass. The velocity enhancement by the increase of the DM mass allows the escape or stripping of the stars.

The velocity enhancement is not seen in the Run An-
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Figure 7. We plot the stellar orbits just before TDEs for two characteristic cases. The orbits are taken from the first and second TDEs in Run A. The points represent the positions of the stars. In the right panel, the tertiary star (blue) passes from left to right on a nearly straight orbit before the TDE.

Figure 8. We plot the evolution of the bound stellar mass $M_{cl,b}$ for each star cluster model. We also show the results with a supernova model for Models A-DM and AnoDM (see Section 4.4.1).

oDM when comparing the blue and black histograms in the top panel of Fig. 10 (see also Fig. 11). Interestingly, the distribution spreads with time so that the numbers of both low and high velocity stars increase. The increase of the low velocity stars is caused by expansion of the cluster due to two-body relaxation (see black lines in Fig. 9), whereas the increase of the high velocity stars with $\gtrsim 50 \text{ km s}^{-1}$ is due to ejections caused by binary interactions in the inner region of the cluster.

Comparing the velocity distributions for the Runs A-DM (red) and AnoDM (black) at $t = 15 \text{ Myr}$, we find that high velocity stars with $\gtrsim 50 \text{ km s}^{-1}$ are less common in the former model. This suggests that DM gravity can prevent the ejections of the stars.

In contrast to the outer stars, the presence of DM has little impact on the inner stars. In the bottom panel of Fig. 10, we show the velocity distributions for $r < 1 \text{ pc}$. The distribution in the Run A-DM does not significantly change with time; indeed, the distributions are similar between the Runs A-DM and AnoDM at $t = 15 \text{ Myr}$.

We find that the DM motion relative to the star cluster causes the oscillatory behavior of the bound mass evolution for the Model A-DM in Fig. 8. In Fig. 12, we show snapshots of DM density (color) and stellar distributions (dots) at four epochs. A high density DM clump which is located at about 100 pc lower-left from the center at $t = 0 \text{ Myr}$ moves towards the upper right direction. At $t = 7.45 \text{ Myr}$, the clump temporarily merges with the high density region where the star

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1 Since the velocity distributions are almost the same in the Runs A-DM and AnoDM at the beginning of the simulations, we do not show the distribution at $t = 0 \text{ Myr}$ for the Run AnoDM.
cluster resides, enhancing the DM density at \( r \lesssim 50 \, \text{pc} \). Then the bound stellar mass \( M_{\text{cl,b}} \) (Fig. 8) decreases with the increase of the stellar velocity. The DM clump then passes through the central region at \( t = 11.18 \, \text{Myr} \), and \( M_{\text{cl,b}} \) increases again. Finally, the clump merges with the central region, and begins stripping the stars from the cluster.

The first and the second drops of \( M_{\text{cl,b}} \) in Fig. 8 are caused by a series of processes associated with DM motions. To see this, in Fig. 13 we show the evolution of normalized DM mass \( M_{\text{DM}}/M_{\text{DM}}(t=0) \) at several regions \( r < 10 \, \text{pc}, 10 - 30 \, \text{pc}, 30 - 100 \, \text{pc} \) and \( 100 - 300 \, \text{pc} \). For \( \lesssim 8 \, \text{Myr} \) during which the first drop of \( M_{\text{cl,b}} \) occurs, the DM mass in the inner regions \( \lesssim 30 \, \text{pc} \) increases by a factor of 2. The mass increase indicates that the DM falls toward the central region and deepens the potential well, causing the first drop. At \( \gtrsim 11 \, \text{Myr} \) during which the second drop occurs, the inner DM mass is roughly constant and instead the outer mass in \( 30 - 100 \, \text{pc} \) continues to increase. As the outer DM mass increases, stars in the outer regions are accelerated and get unbound, causing the second drop.

To further examine the dynamical reaction of the stars when the DM clump falls toward the central region, we compare the evolution of DM mass and stellar mass within 30 pc in Fig. 14. At \( t \lesssim 8 \, \text{Myr} \), the DM mass increases since the DM streams from the outer region to within 30 pc. Later, the DM mass decreases until \( t \sim 12 \, \text{Myr} \) and then remains roughly constant with \( \sim 1.4 \times 10^6 M_\odot \). The evolution of the stellar mass including both bound and unbound stars (dashed line) traces that of the DM: the stellar mass also
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Figure 13. Evolution of DM masses ($M_{\text{DM}}$) normalized at the initial masses in regions of $r < 10$ pc, $10 - 30$ pc, $30 - 100$ pc and $100 - 300$ pc. The initial masses in the regions are $M_{\text{DM}}(t = 0) = 6.3 \times 10^4, 8.4 \times 10^5, 1.1 \times 10^7$ and $2.8 \times 10^7 M_{\odot}$, respectively. We also plot the corresponding bound stellar mass evolution $2M_{\text{cl, b}}/M_{\text{cl, b0}}$, where a factor 2 is introduced to clearly show the oscillation (see Fig. 8).

Figure 14. Evolutions of DM mass (red solid), stellar mass including both bound and unbound stars (black dashed) and bound stellar mass (black dotted) within 30 pc from the center. The DM motions can also affect the TDE rate. To show this, we follow a longer time evolution of Models A-DM and AnoDM to $\sim 22.5$ Myr. In Table 3 we show the TDE rates before and after the DM falls in the central region, $t < 15$ Myr and $t > 15$ Myr. In Table 3, we show the TDE rates before and after the DM falls in the central region, $t < 15$ Myr and $t > 15$ Myr. Before the DM density increases, the TDE rate is higher in Model A-DM. Afterwards, however, the rate becomes lower than in Model AnoDM. The faster decrease of the TDE rate is attributed to the increase of DM density that deepens the total gravitational potential, and to stripping of massive stars that weakens mass segregation within the star cluster. Fig. 15 shows clearly an extended stellar distribution of massive stars in Run A-DM.

3.2.2 Comparison among the models

There are notable differences in the time evolution of $M_{\text{cl, b}}$ between our models (Fig. 8). In Model A and C, $M_{\text{b}}$ oscillate with time, whereas $M_{\text{b}}$ decreases monotonically from $\sim 10^5 M_{\odot}$ to $5 \times 10^4 M_{\odot}$ in Model G. The other models do not show significant variation of $M_{\text{b}}$ and exhibit little decrease or no change.

The different behaviors are attributed to the details of the DM motions. For example, in our Model D, a DM clump gradually falls in the central region, leading to the decrease of $M_{\text{cl, b}}$ (Fig. 8) as is also seen in Model A. Contrastingly, in Model G, the central DM density does not vary significantly and $M_{\text{cl, b}}$ changes little. Overall, we conclude that the degree of stellar stripping and the behavior of star clusters sensitively depend on the details of the DM dynamics.

4 DISCUSSION

4.1 Comparison of TDE rates

We estimate the typical TDE rate to be $\sim 0.1 - 1 M_{\odot}^{-1}$ for our clusters with mass $M_{\text{cl}} \sim 10^5 M_{\odot}$ (Fig. 5). Using the box size of the cosmological simulations $10 h^{-1}$ Mpc and the number of the clusters in the box, the “cosmic” TDE rate is inferred, very roughly, to be $\sim 1 - 10 h^3$ Gpc$^{-3} M_{\odot}^{-1}$. Although there is no TDE observation at $z \sim 10 - 20$ which can be directly compared to our estimate, it is intriguing to compare it with a local observation in order to gain an insight of difference in properties between the primordial and local clusters. Recently, Lin et al. (2018) find a
X-ray outburst 3XMM J215022.4-055108 which might originate from a tidal disruption of a star by an IMBH of a few $10^4 M_\odot$ in a massive star cluster with mass $\sim 10^7 M_\odot$. They estimate that a TDE rate is $\sim 10$ Gpc$^{-3}$ yr$^{-1}$. If we scale the TDE rate as in Equation (5) to BH mass with a few $10^6 M_\odot$ which is comparable to our IMBH masses, the TDE rate would be $\sim 0.1$ Gpc$^{-3}$ yr$^{-1}$. This value is close to our estimate, implying that the primordial and local clusters have some similar properties. For example, cluster core density and cluster velocity dispersion would be similar, otherwise TDE rates would be significantly different considering that the rates have strong dependencies on those properties (see Equation [6]). Although there are a number of differences in, e.g., the number of stars within a cluster, mass function of stars and stellar populations, the comparison of the TDE rates can be a probe to explore differences in the distant and local cluster properties. Previous theoretical studies also estimate the rate of stellar TDEs by IMBHs (Baumgardt, Makino & Ebisuzaki 2004; Chen & Shen 2018; Fragione et al. 2018). As we show in Fig. 3, a TDE rate of Equation (6) derived by the loss cone theory is consistent with ours.

Because we do not consider the processes like tidal captures (e.g., Baumgardt et al. 2006), which could enhance the TDE rate, our derived rate in the above could be conservative. We also note that the deviation of the initial conditions from an isotropic distribution could cause suppression of stellar flux to the central IMBH (Lezhnin & Vasiliev 2015).

4.2 Implications to ‘halo stars’

We have shown that stars in the outer part of a star cluster can be stripped by the presence of DM. The stripped stars are still bound within the DM halo, and then the stars would become ‘halo stars’ which spread out beyond the cluster tidal radius, similarly to the observed stellar populations in local star clusters (Marino et al. 2014). Peñarrubia et al. (2017) use analytic models and numerical experiments to show that the presence of DM helps forming a stellar halo. With the DM, the density profile of stars becomes shallower at large distances and the velocity dispersion of the stars is increased.

4.3 Evaporation of star clusters and the fates of IMBHs

An isolated star cluster can be evaporated by the two-body relaxation process in a typical timescale of $10 - 10^6$ yr (Spitzer & Hearn 1958; Johnstone 1993), with the exact timescale depending on the stellar mass distribution. If there are tidal shocks or tidal force from a galaxy, the evaporation timescale can be as short as or shorter than $10^6$ yr (Guedin & Ostriker 1997; Portegies Zwart et al. 2001). The cluster evaporation can be also accelerated by the motion of the DM: DM strips stars from a cluster when it increases the effective mass density within the cluster and according increases the stellar velocity dispersion. The evaporation would eventually cause destruction of the star cluster. Although we have shown that the inner regions of the star clusters are not significantly affected by the DM, it is likely that the inner regions would be also dissolved if the evolution is followed over a much longer time, since the inner region $r \lesssim 1$ pc actually expands gradually (see Fig. 9).

When a star cluster evaporates, the central IMBHs can no longer grow by collisions of stars due to scarcity of intruding stars. For the IMBHs to grow and become supermassive, a further external supply of mass by gas accretion or stellar/BH mergers is necessary. For example, if the host halo merges with other halo(s) or galaxies, gas will fuel the central IMBHs, and finally BH mergers will increase the mass (e.g., Kauffmann & Haehnelt 2000). Large-scale cosmic gas flows can also fuel the BH (e.g., Dekel & Birnboim 2006; Belovary et al. 2013).

IMBHs formed at the cluster centers would also continue floating in the sea of stars which are not bound by themselves but are still loosely bound by the DM halo. Even in this case the stars and IMBHs could be incorporated in other galaxies during the cosmic evolutions. The origin of some observed IMBH candidates (Maillard et al. 2004; Schödel et al. 2005) may be explained by our IMBH formation scenario.

It is interesting to study the ratio of the BH mass to the cluster/bulge mass, which is determined when the BH mass growth ceases. Fig. 4 shows that the IMBH mass is $1 - 2\%$ of the star cluster mass. Our result is consistent with those obtained by a theoretical work (Portegies Zwart & McMillan 2002), meaning that DM does not significantly affect the shape of the relation. Our result is also consistent with those in local observations (Lutzgendorf et al. 2013) see also figure 6 of Sakurai et al. (2015), indicating that the observed candidate IMBHs could have been formed by the runaway stellar collision scenario. When we extrapolate our results to SMBH regimes and compare them with the well-known SMBH mass-bulge mass relation (Magorrian et al. 1998; Merritt & Ferrarese 2001), we overestimate the BH mass by an order of magnitude. During evolution from IMBHs to SMBHs, substantial mergers and accretion could have occurred to shape the Magorrian relation.

4.4 Caveats of the simulations

4.4.1 Stellar evolution

We follow the cluster evolution for a longer time of 15 Myr than the lifetimes of massive stars. For example, a star with mass $\sim 15 M_\odot$ with solar metallicity has a main-sequence (MS) lifetime of $\sim 10$ Myr (Bузzon 2002). The lifetimes are shorter for stars with lower metallicity (Schaerer 2002).

At the end of their lives, stars with mass between 8 and 40 $M_\odot$ are thought to explode as supernovae and lose a substantial amount of mass (Heger et al. 2003). If the stars in our cluster simulation actually explode and lose mass, the cluster would expand accordingly by a factor of $(M_0 - \Delta M)/(M_0 - 2\Delta M)$, where $M_0$ is original mass of the cluster and $\Delta M$ is the lost mass (Hills 1980). In this case, the star cluster loses at most $\int_{4 M_\odot}^{100 M_\odot} m \frac{dN}{dm} / \int_{4 M_\odot}^{100 M_\odot} m \frac{dN}{dm} dm \sim 43\%$ with our adopted Salpeter IMF $dN/dm \propto m^{-2.35}$, and hence it would expand by a factor of $\sim 4$. The star cluster density and velocity dispersion would decrease by a factor of $\sim 4^3$ and $\sim 2$, respectively. From Equation (6), the TDE rates would then drop by a factor of $\sim 3.5$. Therefore the IMBH growth by TDEs can be suppressed accordingly.

In order to test if and how the mass loss by the super-
supernovae affects the global evolution of the clusters, we perform additional simulations with a model of supernovae (Hurley, Pols & Tout 2000) for the model A with and without DM. In the simulations, we convert stars to compact remnants at the end of their lifetimes. We also model mass loss for stars \( \lesssim 25 \, M_\odot \) whereas more massive stars are assumed to collapse to leave remnant BHs. We suppress close encounters of stars/compact objects by increasing the softening length to leave IMBHs. We suppress close encounters of stars/compact objects by increasing the softening length to prevent formation of extremely hard binaries. Also in the test runs, we do not consider possible gravitational wave events by BH mergers. In Fig. [8], we show the evolution of bound stellar mass \( M_{\text{cl,b}} \) for our test simulations with dotted lines. The supernovae accelerate expansion of the clusters and evaporation of stars from the clusters. However, the characteristic evolution of the clusters such as the oscillatory behavior of \( M_{\text{cl,b}} \) (Fig. [5]) remains essentially unaffected. We confirm the dominant role of the DM halo on the cluster evolution similarly to our main simulations.

There is another complication, actually a simplification in our model, related to the evolution timescales. Star formation likely lasts for a finite period of time. For example, the radiation hydrodynamics simulations of Kimm et al. (2016) show that star cluster formation within an atomic-cooling halo lasts for \( \sim 10 \, \text{Myr} \). To fully examine the effect of the mass loss by supernovae on the star cluster evolutions, it is necessary to consider self-consistent processes from star formation to the cluster dynamical evolution.

### 4.4.2 Effects of gas

Our simulations do not incorporate a diffuse gas component that may remain after the star cluster formation. The diffuse gas can alter the stellar dynamics. Lupi et al. (2014) use a semi-analytic model and show that the gas inflow can help stellar BHs to avoid ejections from the cluster and can also help runaway collisions by deepening the potential well. Silva et al. (2018) argue that the diffuse gas remaining in star clusters can decrease the central density of the star clusters if the gas mass is 10 times larger than the stellar mass.

In our star cluster models, the gas inflow from the outer part of the parent halos may actually promote TDEs. However, since the mass of the gas that we removed when generating the initial conditions is \( 10 - 20 \, M_\odot \) (table 1 of Sakurai et al. 2017), it could decrease the density of the cluster and TDE rates. Examining either of the two effects has larger effect is an intriguing future work.

### 4.4.3 Mass increase rates of the IMBHs

We may overestimate the mass increase of the IMBHs by assuming that a tidally disrupted star is fully swallowed. If we assume the mass of the disrupted star \( m_* = 100 \, M_\odot \) and \( M_{\text{BH}} = 1000 \, M_\odot \) with \( \beta = 1 \), the star can be fully bound after the disruption only when its orbit has a small eccentricity \( e \lesssim 0.07 \) (see equation 20 of Hayasaki, Stone & Loeb 2013). In our simulations, the eccentricities of the disrupted stars are often \( e \sim 1 \). In such parabolic-orbit cases, about a half of the disrupted “debris” is actually ejected outward, and only another half is accreted on to the IMBH (Rees 1988). Considering the possible partial ejection, the IMBH growth derived in our simulations may be the upper limits. Despite the overestimation, our result that the IMBH grows moderately by the TDEs does not change.

There are some other physical mechanisms that can affect our results. For example, an accretion disk can be formed around an IMBH after TDE(s) (Shen & Matzner 2014, e.g.). The disk, if survives for a long time, can decelerate the motions of intruding stars by dissipative forces (Ostriker 1983) and enhance the growth of the BH (Just et al. 2012; Kennedy et al. 2016; Panamarev et al. 2018). Primordial binaries, detailed stellar evolution and stellar wind and partial tidal disruptions can also modify our results. Including these mechanisms in the N-body simulations would help understand more detailed evolution of the IMBHs in star clusters.

### 5 SUMMARY

We study the stellar dynamics around central IMBHs within early star clusters hosted by DM halos. The IMBHs grow by TDEs of intruding stars, to become as massive as \( 700 \, M_\odot \) to \( 2500 \, M_\odot \) at the end of the simulations. The diversity can be attributed, partly, to the variety of physical properties of the parent clusters. Specifically, we find the final IMBH mass \( M_{\text{IMBH,f}} \) is approximately linearly proportional to the cluster mass \( M_\text{cl} \), yielding a relation \( M_{\text{IMBH,f}} \sim 0.01 - 0.02 \, M_\text{cl} \). The TDE rates by the IMBHs are \( N_{\text{TDE}} \sim 0.3 - 1.3 \, \text{Myr}^{-1} \). We show the rates follow the relations \( N_{\text{TDE}} \sim 0.3 \, \text{Myr}^{-1} \left( M_{\text{IMBH}} / 1000 \, M_\odot \right) \). The disrupted stars have masses in the range from \( 3 \, M_\odot \) to \( \gtrsim 100 \, M_\odot \). These are close to the minimum and maximum mass of the IMF \( m_{\text{min}} = 3 \, M_\odot \) and \( m_{\text{max}} = 100 \, M_\odot \), respectively. The high mass stars are disrupted typically after migration toward the cluster center through mass segregation.

The DM halo affects the evolution of \( M_{\text{cl,b}} \) in a complicated manner (Fig. [5]). Comparing our model A simulations with and without DM, we find that \( M_{\text{cl,b}} \) decreases when the DM density increases and effectively “heats” the star cluster. By increasing the stellar velocity dispersion, the DM halo strips stars from the outer part, but stars in the dense inner region are not significantly affected by the DM. We also find that the DM can prevent ejections of stars from the parent halo. TDE rates are also affected by the DM motion. Massive stars are stripped from the outer part of the cluster, and mass segregation, which effectively promotes TDEs, is suppressed.

Though the IMBHs formed in the star clusters would not grow solely by the TDEs, they are still promising candidates for seeding SMBHs at high redshift \( z \gtrsim 6 \). Studying further IMBH evolution with halo mergers in a cosmological context will help understanding the fates of the early IMBHs.
ACKNOWLEDGEMENTS

We are grateful to Kazumi Kashiyama and Kojiro Kawana for discussions on our estimate of TDE rates. The computations in this work are carried out on Cray XC30 and XC50 at CICA, National Astronomical Observatory of Japan. This study is financially supported by Grant-in-Aid for JSPS Overseas Research Fellowships (YS) and JSPS KAKENHI Grant number 2680108, 17H6360 (MF).

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