Brane Inflation After WMAP Three Year Results

Qing-Guo Huang $^{1,2}$, Miao Li$^{1,2}$ and Jian-Huang She$^{1,3}$

$^1$Institute of Theoretical Physics, Academia Sinica
P. O. Box 2735, Beijing 100080

$^2$The interdisciplinary center of theoretical Studies, Academia Sinica
P. O. Box 2735, Beijing 100080

$^3$Graduate University of the Chinese Academy of Sciences, Beijing 100080, P.R. China

huangqq@itp.ac.cn
mli@itp.ac.cn
jhshe@itp.ac.cn

WMAP three-year data favors a red power spectrum at the level of 2 standard deviations, which provides a stringent constraint on the inflation models. In this note we use this data to constrain brane inflation models and find that KKLMMT model can not fit WMAP+SDSS data at the level of 1 standard deviation and a fine-tuning, eight parts in thousand at least, is needed at the level of 2 standard deviation.

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Inflation dynamically resolves many puzzles concerning the hot big bang cosmology, such as homogeneity, isotropy and flatness of the universe [1], and moreover it generates naturally superhorizon fluctuations without appealing to fine-tuned initial setup. These fluctuations become classical after crossing out the Hubble horizon. During the deceleration phase after inflation they re-enter the horizon, and seed the matter and radiation fluctuations observed in the universe. The microwave anisotropy encodes information from inflation. The ΛCDM model remains an excellent fit to the three-year WMAP result as well as other astronomical data [2].

The results of WMAP three-year data are presented in [2]. For ΛCDM model, WMAP three year data only shows that the index of the power spectrum satisfies

$$n_s = 0.951^{+0.015}_{-0.019},$$

combining WMAP with SDSS, the result is

$$n_s = 0.948^{+0.015}_{-0.018},$$

at a level of 1 standard deviation. A red power spectrum is favored at least at the level of 2 standard deviations. If there is running of the spectral index, the constraints on the spectral index and its running and the tensor-scalar ratio are

$$n_s = 1.21^{+0.13}_{-0.16}, \quad \alpha_s = \frac{dn_s}{d \ln k} = -0.102^{+0.050}_{-0.043}, \quad r \leq 1.5.$$  \hspace{1cm} (3)

What’s important is that WMAP three year data does not favor a scale-invariant spectrum of fluctuations any more. While allowing for a running spectral index slightly improves the fit to the WMAP data, the improvement in the fit is not significant enough to require the running. WMAP group claimed that the simple chaotic inflation model with potential $m^2 \phi^2$ [3] fits the observations very well. The WMAP three year data provides significant constraints on the inflation models and has ruled some inflation models out in [3]. Other discussions are given in [4].

In spite of many phenomenological successes of inflation, there nevertheless exist many serious problems, such as the singularity problem, trans-Plankian problem, as well as questions concerning the origin of the scalar field driving inflation. These problems can only be addressed in a framework broader than effective field theory. And it is expected that quantum theory of gravity should be employed, for which string theory is the only self-consistent scenario till now. So it is important to try to embed the many inflation
models into string theory. However this is not easy. Even the above mentioned simple $m^2 \phi^2$ inflation model has not yet been realized in string theory. In the recent few years, much advance has been achieved in this direction. One possible inflation scenario which has a natural set up in string theory is driven by the potential between the parallel dynamical brane and anti-brane, namely brane inflation [7,8]. However it is generally not easy to get a sufficiently flat inflaton potential in brane inflation models [9]. Kachru et al. in [10] successfully introduced some $D3$-branes in a warped geometry in type IIB superstring theory to break supersymmetry and uplift the AdS vacuum to a metastable de Sitter vacuum with lifetime long enough. Putting an extra pair of brane and anti-brane in this scenario, a more realistic slow roll inflation, named KKLMMT model, is naturally realized [11]. KKLMMT inflation model has also been discussed in [12,13,14] in detail, where a possible conformal-like coupling between the scalar curvature and the inflaton is taken into account and a fine tuning at the level of roughly 1 part in ten is needed. See [15] [16] for more considerations about KKLMMT model.

In this short note, we investigate the constraints on the brane inflation model by using the WMAP three year data. We find that KKLMMT model is not good at fitting the new data. It seems that the better racetrack inflation is more likely to serve such role [17].

First we consider some general brane inflation models. We start with a pair of $Dp$ and $\overline{Dp}$-branes ($p \geq 3$) filling the four large dimensions and seperated from each other in the extra six dimensions which are compactified. The D-brane tension provides an effective cosmological constant $V_0$ on the brane, inducing an inflation. The separation of the branes serves as the inflaton with potential

$$V = V_0 \left(1 - \frac{\mu^n}{\phi^n}\right).$$

The second term in (4) comes from the attractive interaction between the branes and $n = d_{\perp} - 2$, where $d_{\perp} = 9 - p$ is the number of the transverse dimensions. Since $d_{\perp} \leq 6$, $n \leq 4$. At the moment with the number of e-folds $N$, the inflaton field $\phi$ has the value

$$\phi_N = \left(n(n+2)NM_p^2 \mu^n\right)^{\frac{1}{n+2}},$$

The authors in [6] proposed an interesting idea to realize an assisted chaotic-like inflation, called N-flation, by using a large number of axion fields in string theory.
where \( M_p \) is the reduced Planck mass in four dimensions. The corresponding slow-roll parameters are

\[
\epsilon_v \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{n^2}{2(n(n+2))^{2(\frac{n+1)}{n+2}}} \frac{\mu^{2n+1}}{M_p^{2n+2}},
\]

\[
\eta_v \equiv M_p^2 \frac{V''}{V} = -\frac{n+1}{n+2} \frac{1}{N},
\]

\[
\xi_v \equiv M_p^4 \frac{V'V'''}{V^2} = \frac{n+1}{n+2} \frac{1}{N^2}.
\]  

In general, both \( V_0 \) and \( \mu^4 \) are roughly the effective brane tension compactified to four dimensions, and thus much smaller than \( M_p^4 \). So the amplitude of the tensor perturbations is negligible \( [9] \). The spectral index and its running become

\[
n_s = 1 - \frac{n+1}{n+2} \frac{2}{N},
\]

\[
\alpha_s = -\frac{n+1}{n+2} \frac{2}{N^2}.
\]  

For \( N \sim 50 \), the running of spectral is roughly \( -\mathcal{O}(10^{-3}) \) which is also negligible. The spectral index for different power \( n \) and number of e-folds \( N \) are showed in fig. 1.

**Figure 1.** The yellow lines correspond to \( n \to \infty \). The region with \( n > 4 \) is not allowed, since the number of the brane transverse dimension is not greater than 6.

We see from the above figure that the smaller the number of the transverse dimensions, the worse the fitting with WMAP data. For \( n = 2, 4 \) (corresponding respectively to the
D5 and D3 brane cases), the fitting results for the number of e-folds for WMAP only are

\[ N = 30.6^{+13.5+48.3}_{-8.50-13.4} \quad \text{for} \quad n = 2, \]
\[ N = 34.0^{+15.0+53.7}_{-9.50-14.9} \quad \text{for} \quad n = 4; \]  

(8)

for WMAP+SDSS

\[ N = 28.8^{+11.7+39.4}_{-7.4-11.8} \quad \text{for} \quad n = 2, \]
\[ N = 32.0^{+13.0+43.8}_{-8.20-13.1} \quad \text{for} \quad n = 4. \]  

(9)

On the other hand, the authors in [3] proposed the relevant e-folding number corresponding to the observation is

\[ N = 54 \pm 7. \]  

(10)

The brane inflation model for \( n = 4 \) is near the boundary of the WMAP three years data only and can not fit the data of WMAP+SDSS at the level of \( 1\sigma \); the case with \( n = 2 \) is outside the allowed range by WMAP only or WMAP+SDSS at the level of \( 1\sigma \). But they still survive at the level of \( 2\sigma \).

We neglect a problem for the brane inflation model in the above discussions, which says that the distance between the brane and anti-brane must be larger than the size of the extra dimensional space if the inflation is slow rolling, or \( \eta \) sufficiently small, in this scenario. Maybe the potential in eq. (4) can emerge in other theories we have not known.

A more realistic embedding of brane inflation into string theory is the so called KKLMMT model [11]. They consider in type IIB string theory highly warped compactifications with all moduli stabilized by the combination of fluxes and nonperturbative effects. With a small number of \( \overline{D}3 \)-branes added, the vacuum is lifted to dS. They consider further in the warped throat a D3-brane moving towards the \( \overline{D}3 \)-brane located at the bottom of the throat, thus realizing the scenario of brane/antibrane inflation. The warping of the geometry also provides a natural mechanism to achieve a sufficiently flat potential and thus a sufficiently small \( \eta \), since the warped geometry gives rise to a redshift which reduces the effective tension of anti-brane and the potential coming from the attractive force between brane and anti-brane is redshifted to be much smaller than the effective tension term [11].

In this note, we consider the potential for KKLMMT model which takes the form, in [12],

\[ V = \frac{1}{2} \beta H^2 \phi^2 + 2T_3 h^4 \left( 1 - \frac{\mu^4}{\phi^4} \right), \]  

(11)

where

\[ T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2} \]  

(12)
is D3-brane tension, $h$ is the warped factor in the throat and

$$
\mu^4 = \frac{27}{32\pi^2} T_3 h^4. 
$$

(13)

The newly added $\phi^2$ term comes from the Kähler potential, D-term and also interactions in the superpotential, and $H$ is the Hubble constant. Generally $\beta$ is of order unity [11], but to achieve slow roll, it has to be fine-tuned to be much less than one [12,13]. In the following, we will proceed to further constrain $\beta$ using the WMAP three year data.

Inflation is dominated by the D3-brane tension. With the number of e-folding equals $N$ before the end of inflation, the inflaton field is

$$
\phi_N^6 = 24 N M_P^2 \mu^4 m(\beta),
$$

(14)

where

$$
m(\beta) = \frac{(1 + 2\beta)e^{2\beta N} - (1 + \beta/3)}{2\beta(N + 5/6)(1 + \beta/3)}.
$$

(15)

Now the slow roll parameter can be expressed as

$$
\epsilon_v = \frac{1}{18} \left( \frac{\phi_N}{M_p} \right)^2 \left( \beta + \frac{1}{2 N m(\beta)} \right)^2,
$$

$$
\eta_v = \frac{\beta}{3} - \frac{1}{6 N m(\beta)},
$$

$$
\xi_v = \frac{5}{3 N m(\beta)} \left( \beta + \frac{1}{2 N m(\beta)} \right).
$$

(16)

The amplitude of the the scalar comoving curvature fluctuations has been calculated in detail [12,13]

$$
\Delta_R^2 \approx \frac{V/M_P^4}{24\pi^2 \epsilon_v} = \left( \frac{25}{3\pi^4} \right)^{1/3} \left( \frac{T_3 h^4}{M_P^4} \right)^{\frac{\beta}{2}} N^{\frac{\beta}{6}} f^{-\frac{4}{3}}(\beta),
$$

(17)

where

$$
f(\beta) = m^{-5/4}(\beta)(1 + 2\beta N m(\beta))^{\frac{4}{\beta}}.
$$

(18)

Substituting eq. (13) and (17) into (14), we have

$$
\frac{\phi_N}{M_p} = \left( \frac{27}{8} \right)^{\frac{1}{4}} m^{\frac{1}{6}}(\beta) f^{\frac{4}{3}}(\beta) N^{-\frac{1}{4}} (\Delta_R^2)^{\frac{4}{3}}.
$$

(19)

Now we have

$$
\epsilon_v = \frac{1}{4\sqrt{6N}} (\Delta_R^2)^{\frac{1}{6}} m^{\frac{1}{3}}(\beta) f^{\frac{4}{3}}(\beta) \left( \beta + \frac{1}{2 N m(\beta)} \right)^2.
$$

(20)
Usually the normalization of the primordial scalar power spectrum is $\Delta^2_R \simeq 2 \times 10^{-9}$ for $N \sim 50$. The spectral index and its running are

$$n_s = 1 - 6\epsilon_v + 2\eta_v,$$
$$\alpha_s = -24\epsilon^2_v + 16\epsilon_v\eta_v - 2\xi_v,$$  \hfill (21)

and the tensor-scalar ratio is

$$r = 16\epsilon_v.$$  \hfill (22)

According to eq. (16), the $\beta$ term in the potential makes the power spectrum trending to be blue.

For $\beta \leq 0.1$, the tensor-scalar ratio is roughly $10^{-9} \sim 10^{-5}$ and thus the tensor perturbations are negligible. Now the spectral index and its running are

$$n_s = 1 + \frac{2\beta}{3} - \frac{5}{3m(\beta)} \frac{1}{N},$$
$$\alpha_s = -\frac{10\beta}{3m(\beta)} \frac{1}{N} - \frac{5}{3m^2(\beta)} \frac{1}{N^2}. $$  \hfill (23)

The spectral index is shown in fig. 2. The running of the spectral index is roughly $-\mathcal{O}(10^{-3})$ and is negligible.

**Figure 2.** The tensor perturbations can be neglected. If $\beta < 0$, the $\beta$ term in the potential trends to push D3-brane out of the throat and the inflation does not happen.

For $\beta = 0$, the fitting result is just the case for $n = 4$ in eq. (8) and (9). The KKLMMT model is on the boundary at the level of 1$\sigma$ of the WMAP three years data.
only and does not fit the data of WMAP+SDSS. Larger the value of $\beta$, larger the spectral index. Thus the $\beta$ term in the potential can not improve the fitting results. The constraints on the parameter $\beta$ are $\beta \leq 6 \times 10^{-4}$ at the level of $1\sigma$ and $\beta \leq 8 \times 10^{-3}$ at the level of $2\sigma$ for WMAP only; $\beta \leq 6 \times 10^{-3}$ at the level of $2\sigma$ for WMAP+SDSS. The fine tuning for the parameter $\beta$ is needed!

For $\beta > 0.1$, a large amplitude of the tensor perturbations emerges and a large running is possible. If the running of the spectral index is large enough, a blue power spectrum is allowed (3). We show the spectral index and its running and the tensor-scalar ratio for different $\beta$ in fig. 3.

\[ \text{Figure 3. The top three figures are the spectral index and its running and the tensor-scalar ratio respectively vary with the number of e-folds for different $\beta$. The bottom figure shows the correlation between the spectral index and its running.} \]

Unfortunately, the bottom figure in fig 3 shows that the KKLMMT model can not provide a reasonable running of the spectral index.

To summarize, WMAP three year data provides a stringent constraint on the brane inflation. KKLMMT model can not fit the new data well. In order to stabilize the moduli as in [10], the mass of the inflaton is roughly the same as the Hubble parameter during the period of inflation and thus the inflation is impossible. We need to fine-tune the parameter
to reduce the mass of the inflaton. Our fitting results show that a fine-tuning, eight parts in thousand at least, is needed at the level of 2σ. It seems that the better racetrack inflation proposed in [17] is favored by WMAP three-year data.

One may go on to ask whether such stringent fine-tuning is possible in KKLMMT-type models. Thus we need to look further into the origin of β. In KKLMMT’s original model, with superpotential

\[ W = W_0 + g(\rho)f(\rho), \]  

(24)

where \( W_0 \) comes from the fluxes, \( f(\rho) = 1 + \delta \phi^2 \), \( \delta \) a constant, and \( g(\rho) \) an arbitrary function of \( \rho \), the parameter \( \beta \) has the form \[ \beta = 1 - \left| \frac{V_{\text{AdS}}}{V_{\text{dS}}} \right|(\gamma - 2\gamma^2), \]  

(25)

with \( \gamma = \frac{4}{g} \). Lack of precise knowledge of the superpotential \( W(\rho, \phi) \), when \[ |V_{\text{AdS}}| \gg V_{\text{dS}}, \]  

(26)

domestic tuning can be performed to get any value for \( \beta \). For \( T^2 \times T^4/Z_2 \) and \( T^6/Z_N \) models, the α’ collection to the superpotential is calculated [18], and correction to the inflaton mass is thus determined with the result

\[ \beta = 1 - \left| \frac{V_{\text{AdS}}}{V_{\text{dS}}} \right|\Delta, \]  

(27)

where \( \Delta \simeq 0.1 \). There is still enough room for fine-tuning. Moreover, inflaton mass also receives contributions from the Kähler potential [19] and D-terms [20]. For further discussion of inflaton mass problem see [21] and references therein.

To solve the fine tuning problem in the KKLMMT scenario, Cline and Stoica proposed a novel dynamical mechanism in [16]. They introduced an additional parameter \( \psi_0 \), which is approximated as zero in KKLMMT’s original paper, the position of the antibrane relative to the equilibrium position of the brane in the absence of the antibrane. They further considered the presence of multiple mobile branes. With sufficiently large number of branes, the inflaton potential has a metastable minimum, where the branes are initially confined. These branes can tunnel out of the minimum, making the minimum shallower and shallower, and the potential more and more flat. Finally the remaining branes will roll together into the throat, driving the inflation. In this way, a sufficiently flat inflaton potential is achieved through a dynamical mechanism, without the need of fine-tuning.
In their scenario, the inflaton potential is modified to be

\[ V = \frac{TN}{A^3} \left( 1 - \frac{1}{6} \phi^2 \right)^{-2} \left( 1 - \frac{3b(N/A)^3}{(\phi - \phi_0)^4} \right)^{-1}, \]  

with \( \phi \) the canonically normalized inflaton field, \( \phi_0 \) the canonically normalized value of \( \psi_0 \) and \( N \) the number of branes. While \( T = 2\tau/g_s^4 \), with 3-brane tension \( \tau \) and string coupling \( g_s \). The properly combined parameter \( A = (2\sigma)^{2/3}\epsilon^{-1/3} \), with warp factor \( \epsilon \) and Kähler modulus \( \sigma \).

The scalar spectral index at 55 e-foldings before the end of inflation is calculated in [16] to be 0.93 < \( n_s \) < 1.15, while correspondingly the running of the spectral index is -0.012 < \( dn/d\ln k \) < -0.001. In the range permitted by WMAP 3-year data without running, where 0.93 < \( n_s \) < 0.963, their prediction of running is -0.003 < \( dn/d\ln k \) < -0.001, small enough to be negligible.

In conclusion, Cline and Stoica’s scenario [16] removes the fine-tuning problem of the brane inflation models, and fits well with the WMAP 3-year result without running. But their scenario can not produce a large enough running to fit the WMAP 3-year result with running.

Recently, there is also discussion of KKLMMT scenario in light of WMAP 3-year data in [22], where they seek string effects from CMB B polarization. They expect their predictions to be general and do not depend on the exact form of the inflaton potential. So they do not consider the constraints of \( \beta \) as we do above, and simply choose a fixed value \( \beta = 0.02 \).

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