Lagrangian Relaxation and Fix Heuristic for Integrated Production Planning and Warehouse Storage Allocation Problem under Demand Uncertainty

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We consider an integrated optimization model for production planning and warehouse storage allocation under demand uncertainty. A deterministic problem of the integrated production planning and warehouse storage allocation problem is extended into a stochastic programming problem formulation. A Lagrangian relaxation and fix heuristic is proposed to solve the mixed integer nonlinear programming problem by decomposing the problem into single item capacitated lot sizing and warehouse storage allocation problems. The problem instances based on real data are solved in the numerical experiments. Computational results demonstrate the effectiveness of the proposed method.

1. Introduction

In recent years, rapid development of online shopping such as Amazon has been increasing the importance of warehouse management. Production planning and warehouse storage allocation problems are the key issues of warehousing which involves all stages of supply chain. An effective warehouse storage allocation can distinctly reduce operation costs. Usually, warehouse storage allocation problems and production planning problems are analyzed separately. In the conventional systems, production planning and warehouse storage allocation are determined by the hierarchical scheme where warehouse storage allocation is determined by the production goal that is set by the production planning system. It is expected to reduce total costs by simultaneously optimizing production planning and warehouse storage allocation.

Motivated by real world production warehousing problem, Zhang and Turner developed an integrated strategy to combine production planning and warehouse storage allocation problem such that production process and warehousing can be coordinated effectively [1]. While warehouse storage allocation problem and production planning have been studied extensively [2,3], a few works have been reported that considers warehousing layout and production planning simultaneously by mathematical programming approach. Extensive studies have been conducted on warehouse design and operations with the different operations, components and locations in warehousing. A comprehensive review of research on warehouse operation is provided in [4]. Various researchers have combined the warehouse storage allocation problem with other inventory models such as the replenishment policy model [2]. A Lagrangian decomposition and coordination algorithm has been developed for integrated production planning and warehouse storage allocation problem with deterministic demand [5]. Most of conventional study assumes that demand is deterministic. However, demands are uncertain in real cases. In this paper, we propose an integrated production planning and warehouse storage allocation problem under demand uncertainty. In order to
deal with demand uncertainty, we apply a chance constraint approach in which each uncertain parameter is treated as a random variable with a given probability distribution. The approach is applied to the problem under demand uncertainty with a guaranteed service level to reduce the shortage in the inventory. An equivalent reformulation into deterministic form is presented. The problem is formulated as a mixed integer nonlinear programming problem. The problem based on real data is treated. We propose the Lagrangian relaxation and fix heuristic approach for computing lower and upper bounds. Cube per order index (COI) is used to determine the priority of Lagrangian relaxation and fix heuristic. Computational experiments are executed to investigate the performance of the proposed method and the branch and bound method.

2. Production Planning and Storage Allocation under Demand Uncertainty

2.1 Problem Definition

The deterministic problem formulation is quoted from the model developed by Zhang et al.[1]. Consider a situation that the company aims to organize their warehouse and determine storage locations to meet customer uncertain demand keeping up with current company operations.

![Production area and warehouse model](image)

Fig. 1 The model of production and warehouse area

Fig. 1 shows the model considered in this study. The model consists of production area and warehousing area. The model in this paper focused on uncertainty for simultaneous production planning and warehousing for multi items. The production area produces multiple items with limited production capacity. We assume that the production area is close to its warehousing area in a plant. The production planning decides lot sizes of all items concerned with production costs, setup costs and inventory costs during planning horizon. The warehouse has several storage locations. A dedicated storage assignment policy is used to organize the warehouse space by assigning locations to each item. The challenge faced by the company is that it is difficult to find available space in many periods when the products are transferred from production area to warehouse. The simultaneous production planning and warehouse storage allocation model determines the assignment of items to storage locations to minimize the expected cost to assign/retrieval of travel, reserving space, production quantity, inventory and setup costs. Demand is uncertain and unknown, and shortages are permitted, but the probability distribution of demand is known. The demand is assumed to follow a normal distribution. All items are stored and moved on pallets with the same size. Items are delivered and retrieved using a single command forklift truck. There is one general production area that products come from, and one output point. Costs associated with the placement and retrieval of items are directly proportional to the distance travelled.

2.2 Problem Formulation

Indices

- $i$: index of an item
- $t$: index of a period within the planning horizon
- $l$: index of a storage location within the warehouse

Sets

- $I$: the set of items
- $L$: the set of storage locations within the warehouse
- $T$: the set of periods within the planning horizon

Parameters

- $a^i_t$: unit loss cost of holding item $i$ in period $t$
- $c^i_t$: variable unit production cost of item $i$ in period $t$
- $\hat{d}^i_t$: realized quantity of demand of item $i$ in period $t$
- $d^i_t$: mean value of demand of item $i$ in period $t$
- $f_i$: resource capacity of production, for example, personnel, budget, production capacity, etc.
- $h^i_t$: unit inventory cost of holding item $i$ in period $t$
- $M$: production capacity per period
- $O^i_l$: unit cost of moving a column of any item from storage location $l$ to the output point
- $P^i_l$: unit cost of moving a column of any item from the production area to storage location $l$
- $R^i_l$: unit cost of reserving storage location $l$
- $s^i_0$: the initial inventory level for item $i$
- $u^i_t$: unit setup cost of item $i$ in period $t$
- $v^i_t$: required resource of item $i$ in period $t$
- $\sigma^i_t$: standard deviation of demand of item $i$ in period $t$
- $\lambda^i_t$: lower bound on the probability that the planned
sales $Q_i^t$ is greater than $d_i$.

**Decision variables**

$l_i^t$: unit opportunity loss cost of item $i$ in period $t$

$n_i^l$: 1, if item $i$ is inventoried in location $l$ during period $t$; 0, otherwise

$q_i^l$: 1, if item $i$ is requested (by demand) from location $l$ during period $t$; 0, otherwise

$Q_i^t$: planned quantity for sale of item $i$ in period $t$

$s_i^t$: inventory level of item $i$ at the end of period $t$

$x_i^t$: 1, if item $i$ is moved from the production area and placed in storage location $l$ during period $t$; 0, otherwise

$y_i^t$: quantity of item $i$ produced during period $t$

$z^l$: 1, if location $l$ is reserved for item $i$ for the planning horizon; 0, otherwise

**Objective function**

$$
\begin{align*}
\sum_{i \in I} \sum_{l \in L} & P_i^l z_i^l + \sum_{i \in I} \sum_{l \in L} \sum_{t \in T} (P_t^l w_i^l + O_t^l q_i^l) \\
+ & \sum_{i \in I} \sum_{t \in T} (c_i^t x_i^t + w_i^t y_i^t + E_t^l q_i^l) + E_t^l q_i^l)
\end{align*}
$$

The objective function is divided into two lines. The first line of the three terms represents warehousing costs. It consists of the cost of reserving locations for items in a dedicated storage policy, the cost associated with the travel of products from the production area to assigned storage locations, and the cost associated with the travel of products from the storage locations to the output point. The second line of the four terms is associated with production planning, the cost of production, setup, expectation of inventory holding costs and expectation of opportunity loss costs. The objective of the problem is to produce and place the products in the best locations to minimize the total expected costs. Constraints are described as follows.

**Constraints on warehouse storage allocation**

$$
\begin{align*}
\sum_{l \in L} z_i^l & \leq 1 \quad (i \in I) \\
\sum_{i \in I} q_i^l & \leq 1 \quad (l \in L, t \in T) \\
\sum_{i \in I} w_i^l & \leq 1 \quad (l \in L, t \in T) \\
\sum_{i \in I} n_i^l & \leq 1 \quad (l \in L, t \in T) \\
\sum_{l \in L} q_i^l & = Q_i^t \quad (i \in I, t \in T) \\
\sum_{l \in L} w_i^l & = x_i^t \quad (i \in I, t \in T)
\end{align*}
$$

**Constraints on production planning**

$$
\begin{align*}
\sum_{i \in I} v_i^l & \leq f_t \quad (i \in I, t \in T) \\
x_i^t & \leq M y_i^t \quad (i \in I, t \in T) \\
s_i^t & = s_i^{t-1} + x_i^t - \min(d_i^t, Q_i^t) \quad (i \in I, t \in T) \\
l_i^t & = \max(0, d_i^t - Q_i^t) \quad (i \in I, t \in T) \\
Pr & \left[ Q_i^t \geq d_i^t \right] \geq \lambda_i \quad (i \in I, t \in T) \\
x_i^t, s_i^t & \geq 0 \quad (i \in I, l \in L, t \in T)
\end{align*}
$$

Constraints (2) limit the number of items that can be assigned to a reserved location. Constraints (3) ensure that only one item can be requested per storage location. Constraints (4) limit the number of items that can be placed into a single storage location. Constraints (5) ensure that only one item can be inventoried per storage location. Constraints (6) state that the number of products requested from storage locations during period $t$ is equal to the demand of that product during period $t$. Constraints (7) ensure that items are placed in storage locations when they are produced, and that only the produced items can be placed into these storage locations. Constraints (8) state that all items remain at the end of time period $t$ is equal to the inventories at the end of time period $t$. Constraints (9) ensure that items can only be retrieved from locations which they have previously been placed or inventoried. Constraints (10) restrict that a location cannot contain an inventoried item and an item placement at the same time. Constraints (11) are the flow of an item from the production area, to a storage location and from the storage location to the output point. Constraints (12) and (13) ensure that items are only retrieved from or placed in locations that have been reserved for them. Constraints (14) represent that items are only inventoried in locations that have been reserved for them. Constraints (15) indicate the domain of the decision variables.
capacity. Constraints (17) state that production quantity of item \(i\) in period \(t\) can be nonnegative if \(y_1^i = 1\). Constraints (18) are the inventory balancing constraints. Because the quantity demand \(d_t^i\) is a random variable, the actual quantity of delivery is \(\min(\overline{d}_t^i, Q_t^i)\). Constraints (19) represent the opportunity loss penalty costs. Constraints (20) are chance constraints indicating that the probability of the planned sales of item \(i\) in period \(t\) is greater than or equal to \(\lambda_t^i\) where \(\lambda_t^i\) is a safety parameter. Constraints (21) indicate the domain of the decision variables.

### 2.3 Equivalent Reformulation

Petkov and Maran (1997) provided an equivalent reformulation of stochastic production planning problems under demand uncertainty into deterministic form [6]. After the reformulation, the problem can be formulated as a mixed integer nonlinear programming problem that can be solved by a commercial solver. Let \(\overline{d}_t^i = \frac{d_t^i - \overline{d}_t^i}{\sigma_t^i}\) and \(K_t^i = \frac{Q_t^i - \overline{d}_t^i}{\sigma_t^i}\). \(\Phi\) is the cumulative probability function of standard normal random variable. \(f\) is the standardized normal distribution function. The inventory holding costs can be replaced by the following equation from (18).

\[
h_t^i s_t^i = h_t^i [s_t^i + \sum_{t'=1}^t x_{t'} - \sum_{t'=1}^t \min(\overline{d}_{t'}^i, Q_{t'}^i)]
\] (22)

The expectation of \(\min(\overline{d}_t^i, Q_t^i)\) can be written by the following equations.

\[
E[\min(\overline{d}_t^i, Q_t^i)] = \Phi(K_t^i)E[\overline{d}_t^i \leq Q_t^i] + (1 - \Phi(K_t^i))E[\overline{d}_t^i > Q_t^i]
\]

\[
= \overline{d}_t^i + \sigma_t^i \Phi(K_t^i)E[\overline{d}_t^i \leq \theta_t^i] + (1 - \Phi(K_t^i))\Phi(K_t^i | K_t^i \leq \theta_t^i]
\]

\[
= \overline{d}_t^i + \sigma_t^i [-f(K_t^i) + (1 - \Phi(K_t^i)K_t^i)]
\] (23)

The expectation of the opportunity loss penalty costs can be replaced by the following equations.

\[
a_t^l t_l = a_t^l \max(0, \overline{d}_t^i - Q_t^i)
\]

\[
= -a_t^l [\min(\overline{d}_t^i, Q_t^i) - \overline{d}_t^i]]
\]

\[
E[a_t^l t_l^i] = -a_t^l \sigma_t^i [-f(K_t^i) + (1 - \Phi(K_t^i)K_t^i)]
\] (24)

The probability density function and the cumulative distribution function can be expressed in General Algebraic Modeling System (GAMS). The \(\text{errorf}(.)\) in GAMS implements a variant on \(\text{errorf}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dt\). Therefore, after standardizing the expectation terms in the objective function, we have

\[
z = \sum_{i \in I} \sum_{l \in L} R_l^i z_{l, t}^i + \sum_{i \in I} \sum_{l \in L} P_l w_t^i + \sum_{i \in I} \sum_{l \in L} O_l^i q_t^i
\]

\[
+ \sum_{i \in I} \sum_{l \in L} O_l^i q_t^i + \sum_{i \in I} \sum_{l \in L} (c_t^i x_t^i + u_t^i q_t^i)
\]

\[
+ \sum_{i \in I} \sum_{l \in L} h_t^i (s_t^i + \sum_{t'=1}^t x_{t'} - \sum_{t'=1}^t \overline{d}_{t'}^i)
\]

\[
+ \sum_{i \in I} \sum_{l \in L} \sum_{t'=1}^t h_t^i (\sigma_t^i [f(K_t^i) - (1 - \Phi(K_t^i)K_t^i)])
\]

\[
+ \sum_{i \in I} \sum_{l \in L} (\alpha_t^i x_t^i + \gamma_t^i q_t^i + \pi_t^i w_t^i)
\]

\[
+ \sum_{i \in I} (\alpha_t^i x_t^i + \gamma_t^i q_t^i + \pi_t^i w_t^i)
\]

3. Lagrangian Decomposition and Coordination Algorithm

### 3.1 Lagrangian Decomposition and Coordination Method

The Lagrangian decomposition and coordination algorithm[5] is explained in this section. In order to decompose the original problem, joint constraints (2)-(5), (6), (16) are relaxed by using non-negative Lagrange multipliers \(\alpha_t^i, \beta_t^i, \gamma_t^i, \pi_t^i, \mu_t^i\).

\[
L = z + \sum_{i \in I} \sum_{l \in L} (z_{l, t}^i - 1) + \sum_{i \in I} \sum_{l \in L} \sum_{t \in T} (w_t^i - 1) + \sum_{i \in I} \sum_{l \in L} \sum_{t \in T} (q_t^i - 1)
\]

The subproblem for each product item is formulated as a mixed integer nonlinear programming problem \((SP_l)\).

\[
\min \sum_{i \in I} \sum_{l \in L} R_l^i z_{l, t}^i + \sum_{i \in I} \sum_{l \in L} P_l w_t^i + \sum_{i \in I} \sum_{l \in L} O_l^i q_t^i
\]

\[
+ \sum_{i \in I} \sum_{l \in L} (c_t^i x_t^i + u_t^i q_t^i)
\]

\[
+ \sum_{i \in I} \sum_{l \in L} \sum_{t'=1}^t h_t^i (s_t^i + \sum_{t'=1}^t x_{t'} - \sum_{t'=1}^t \overline{d}_{t'}^i)
\]

\[
+ \sum_{i \in I} \sum_{l \in L} \sum_{t'=1}^t h_t^i (\sigma_t^i [f(K_t^i) - (1 - \Phi(K_t^i)K_t^i)])
\]

\[
+ \sum_{i \in I} \sum_{l \in L} \sum_{t'=1}^t h_t^i (\sigma_t^i [f(K_t^i) - (1 - \Phi(K_t^i)K_t^i)])
\]

\[
+ \sum_{i \in I} \sum_{l \in L} \sum_{t \in T} \sum_{t \in T} (\alpha_t^i x_t^i + \gamma_t^i q_t^i + \pi_t^i w_t^i)
\]

s. t. (6) - (15), (17), (21), (26) - (28)
The subproblem \((SP_i)\) can be regarded as a production planning and warehouse storage allocation problem with a capacitated lot sizing problem which can be solved by a commercial solver. The Lagrangian dual problem is formulated as the following \((LDP)\).

\[
\begin{align*}
(LDP): & \quad \max q \\
\text{where} & \quad q = \min \sum_{i \in I} L_i - \sum_{i \in L} \alpha^l \\
& \quad - \sum_{i \in L} \sum_{t \in T} (d^l_i + \gamma^l_i + \pi^l_i) - \sum_{t \in T} \mu_t f_t \\
\end{align*}
\]

(31)

3.2 Subgradient Optimization

The subgradient optimization method is used to solve the Lagrangian dual problem. For example, the Lagrange multiplier \(\alpha^{l,(k)}\) is updated by

\[
\alpha^{l,(k+1)} = \alpha^{l,(k)} + \varphi(UB - L^{(k)}(\sum_{i\in L} z^{i,l} - 1)) / (\sum_{i\in L} z^{i,l} - 1)^2
\]

where \(\alpha^{l,(k)}, \beta^{l,(k)}, \gamma^{l,(k)}, \pi^{l,(k)}, \mu^{l,(k)}\) are the Lagrange multipliers at \(k\)th iteration. \(UB\) and \(L^{(k)}\) are the value of upper bound and Lagrange function value at \(k\)th iteration, respectively. \(\varphi\) is the step size which is determined by preliminary experimental results.

3.3 Construction of a Feasible Solution

The solution of the Lagrangian dual problem \((LDC)\) is generally infeasible because \((2)-(5)\) and \((16)\) are relaxed. A feasible solution is constructed by the following heuristic algorithm. The heuristic algorithm successively checks the violations of constraints \((2)-(5)\) and \((16)\) and modifies the solution of the relaxed problem into a feasible one.

**Heuristic algorithm**

Step 1: Initialization
Set the parameters \(I_{plan} \leftarrow I, I_{error} \leftarrow \phi\).

Step 2: Check the items that reserve more than the minimum number of required storage locations.
If there exists item \(i\) that satisfies the condition
\[
\sum_{i \in L} z^{i,l} \leq \max \hat{d}_i, \text{item } i \text{ is canceled from planning.}
\]

\(I_{plan} \leftarrow I_{plan} \setminus \{i\}\).

Step 3: Check the items that do not satisfy the constraints \((2)\). If there is item \(i\) that does not satisfy \((2)\), then \(I_{error} \leftarrow I_{error} \cup \{i\}\).

Step 4: For each item \(i \in I_{error}\), if there exists another location that can reassign the reserved storage location \(l_2\) that satisfies \(\sum_{i \in I} z^{i,l_2} = 0\), the reservation of the violated item is reassigned to the location \(l_2\). If there is no storage location that can reassign the violated location, the planning of item \(i\) is canceled and \(I_{plan} \leftarrow I_{plan} \setminus \{i\}\).

4. Lagrangian Relaxation and Fix

4.1 Upper Bound Computation

We propose a new algorithm for computing an upper bound without using a heuristic algorithm. It is usually difficult to construct a feasible solution with a good upper bound by heuristic algorithms. We use Lagrangian relaxation and fix heuristic method to construct a better feasible solution. In the proposed method, the solution of the subproblem of the item \(i\), is fixed one by one. The following constraints
(33)-(38) are imposed to the subproblems in which the relaxed constraints (2)-(5), (16) are replaced by (33)-(38) such that an item cannot be stored in the already fixed locations. Let \( I_{\text{rest}} \) denote the set of items whose solution is not fixed and the variable whose subscript has fix denote the fixed variables.

\[
\begin{align*}
    z_{i}^{d} + \sum_{i \in I_{\text{rest}}} z_{i}^{l} \leq 1 & \quad i \in I_{\text{rest}}, \forall l \\
    w_{t}^{d} + \sum_{i \in I_{\text{rest}}} w_{i}^{d} \leq 1 & \quad i \in I_{\text{rest}}, \forall l, \forall t \\
    q_{t}^{d} + \sum_{i \in I_{\text{rest}}} q_{i}^{d} \leq 1 & \quad i \in I_{\text{rest}}, \forall l, \forall t \\
    n_{t}^{d} + \sum_{i \in I_{\text{rest}}} n_{i,t}^{d} \leq 1 & \quad i \in I_{\text{rest}}, \forall l, \forall t \\
    v_{t}^{d} + \sum_{i \in I_{\text{rest}}} v_{i,t}^{d} \leq f_{t} & \quad i \in I_{\text{rest}}, \forall l
\end{align*}
\]  

The constraint which limits the available location for the rest items must be considered to obtain a feasible solution when the each subproblem for unfixed items must be considered to obtain a feasible solution. It allocates them in that order to the most accessibility (the number of storage/retrieval requests) for the item’s number of storage locations to its popularity. The COI-based assignment policy ranks the items on the basis of its COI values in an ascending order. The COI-based assignment policy ranks the items on the basis of its COI values in an ascending order. The COI-based assignment policy ranks the items on the basis of its COI values in an ascending order.

\[
\sum_{i \in L} z_{i}^{l} + \sum_{i \in I_{\text{rest}} \setminus \{i\}} (\max_{l} d_{i}^{l}) + \sum_{i \in L} z_{i}^{l} \leq |L| 
\]  

The equations for subgradient optimization method are rewritten with respect to the fixed solution. For example, the Lagrange multiplier \( \alpha_{l}(k) \) is updated by

\[
\alpha_{l}(k+1) = \alpha_{l}(k) + \frac{\varphi(|B - L| - \sum_{i \in I_{\text{rest}}} z_{i}^{d} + \sum_{i \in I_{\text{rest}}} x_{i}^{d} - \sum_{i \in I_{\text{rest}}} x_{i}^{d} \leq L)}{\sum_{i \in I_{\text{rest}}} x_{i}^{d} + \sum_{i \in I_{\text{rest}}} x_{i}^{d} \leq L - 1} \]  

4.2 COI (Cube per-Order Index) Rule

The cube-per-order index is a very widely used rule for allocating storage space to inventories in a warehouse [7]. The cube-per-order index is the ratio of the item’s number of storage locations to its popularity (the number of storage/retrieval requests) for an item. The COI-based assignment policy ranks the items on the basis of its COI values in an ascending order. It allocates them in that order to the most accessible locations closest to the input/output point. We define COI value for item \( i \) as

\[
\text{COI}_{i} = \frac{\sum_{l \in I} z_{i}^{l}}{\sum_{l \in T} Q_{l}^{t}}
\]

It is expected that to allocate based on the priority of COI rule leads to reduce the total costs.

4.3 Overall Optimization Algorithm

The algorithm of the Lagrangian relaxation and fixed heuristic method is described as Algorithm 2. \( I_{\text{rest}} \) indicates the set of the non-fixed items.

Algorithm 2

Step 1: Initialization

Set the parameters, \( \alpha_{l}(0), \beta_{l}(0), \gamma_{l}(0), \phi_{l}(0), \mu_{l} \leftarrow 0 \) and \( I_{\text{rest}} \leftarrow I \).

Step 2: Solving subproblem for each product \( i \)

Solve the subproblem \( (SP)_{i} \) with (33) -(38) for each item \( i \).

Step 3: Construction of a feasible solution

Generate a feasible solution from the solution of subproblems derived at Step 2.

Step 4: Evaluation of convergence

If the difference between lower bound and upper bound are not updated predefined time, go to Step 6.

Step 5: Update of Lagrange multipliers

The Lagrange multipliers are updated by subgradient algorithm, e.g. (39), and return to Step 2.

Step 6: Fixing the solution

Select the item \( i_{\text{min}} \) has the minimum COI of \( I_{\text{rest}} \). Fix the solution of the \( (SP)_{i_{\text{min}}} \) with (33)-(38).

\( I_{\text{rest}} \leftarrow I_{\text{rest}} \setminus \{i_{\text{min}}\} \).

Step 7: Evaluation of fixed solution

If \( I_{\text{rest}} \neq \phi \), go to Step 2.

Step 8: Evaluation of upper bound

The upper bound derived Step 6 is compared with the one derived the heuristic algorithm introduced in 3.3 in Step 3 and obtain an upper bound.

5. Computational Results

Computational experiments were conducted to investigate the effectiveness of the proposed method. The program code for the proposed method was coded by the General Algebraic Modeling System (GAMS).

An Intel(R) Core(TM) i7-4700K 3.5GHz Processor with 8GB memory was used for computations. A general purpose mixed integer nonlinear programming solver which is called discrete and continuous optimizer (DICOPT) was used to solve the original problem and \( (SP)_{i} \) for each item \( i \) for the proposed method. Three cases of problem instances are solved. Case 1 is \(|I| = 2, |L| = 5, |T| = 5\), Case 2 is \(|I| = 15, |L| = 45, |T| = 6\) and Case 3 is \(|I| = 25, |L| = 65, |T| = 5\). Parameters for problem instances except the oppos-
tunity loss costs are based on real data [1]. The value of opportunity loss costs is set from 0 to 250. The detailed computational results of Case 1 for deterministic and stochastic cases by the proposed method (LF) and DICOPT are shown in Table 1. For the deterministic case, inventory costs are zero because the production quantity is almost the same as the quantity of demands at each time period and there are no stocks. However, for stochastic cases, the stocks are kept in order to reduce the lost opportunity costs. The solution derived by LF is the same with that of DICOPT for stochastic case. An optimal solution can be found by LF. The computation time for LF is larger than that of DICOPT. This is because the problem size is sufficiently small. There is no necessary to use LF when the original problem can be solved by DICOPT within a reasonable CPU time.

In order to investigate the effects of the performance under different stochastic environment, two cases of demand situations are compared. Case 1-2, 2-2 and Case 3-2 are the situation when the deviations of demand are larger than those of Case 1-1, Case 2-1 and Case 3-1. These parameters are shown in Table 2. Table 3 shows the computational results of the cases when the deviations are changed. The increase of the deviation leads to the rise of inventories in order to reduce the opportunity costs. Therefore, the validity of the formulations is confirmed from the results of the influence of stochastic parameters.

In order to compare the performance of the integrated approach, the performance of the proposed method (LF) is compared with the separated approach where the production planning and storage allocation problems are solved separately (SEPARATED). Table 4 shows the computational results. DICOPT is the results of solving the original problem, LDC is the conventional Lagrangian decomposition and coordination method realized by Algorithm 1, and LF is the proposed method realized by Algorithm 2. LB is the lower bound and UB is the upper bound derived by each method. DGAP is the duality gap calculated by (UB-LB)/UB×100. In the separated approach, the feasibility constraints that the production quantity and inventories are less than |L|×0.7 are added to the production planning problem in order to derive a feasible solution for the warehouse storage allocation. If the constraints are eliminated in the production planning problem, the feasible solution cannot be derived because the overproduction by production planning leads to the lack of the storage spaces. The performance of UB for integrated approach is better than that for the separated approach in all cases. The computation time for LF is shorter than that of DICOPT for stochastic case. An optimal solution can be found by LF. The computation time for LF is shorter than that of DICOPT for Case 2 and Case 3. The performance of UB for LF is better than that of LDC in all cases. This is because the performance of the heuristic algorithm to construct a feasible solution explained in 3.3 is not better than the proposed method. LF is effective for large scale problems because the computational efforts can be reduced by decomposing the original problem into several subproblems.

Table 2 Setting of different demand deviations $\sigma_t$

| Case | Method | LB [-] | UB [-] | DGAP [%] | Total time [s] |
|------|--------|--------|--------|----------|----------------|
| 1-1  | DICOPT | -      | 446.83 | 2.54     | 2.54           |
| 1-1  | LF     | 415.82 | 417.82 | 0.30     | 59.79          |
| 1-1  | LDC    | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | LF     | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | LDC    | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | LF     | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | LDC    | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | DICOPT | -      | 446.83 | 2.54     | 2.54           |

Table 3 Influence of demand deviation on the performance for each case

| Case | Method | LB [-] | UB [-] | DGAP [%] | Total time [s] |
|------|--------|--------|--------|----------|----------------|
| 1-1  | SEPARATED | -      | 446.83 | 2.54     | 2.54           |
| 1-1  | DICOPT  | -      | 417.82 | 0.54     | 0.54           |
| 1-1  | LDC     | 415.82 | 417.82 | 0.30     | 59.79          |
| 1-1  | LF      | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | LF      | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | LDC     | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | DICOPT  | -      | 446.83 | 2.54     | 2.54           |
| 3-1  | LF      | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | LDC     | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | DICOPT  | -      | 446.83 | 2.54     | 2.54           |

Table 4 Comparison of performance for LF, LDC and DICOPT

| Case | Method | LB [-] | UB [-] | DGAP [%] | Total time [s] |
|------|--------|--------|--------|----------|----------------|
| 1-1  | SEPARATED | -      | 446.83 | 2.54     | 2.54           |
| 1-1  | DICOPT  | -      | 417.82 | 0.54     | 0.54           |
| 1-1  | LDC     | 415.82 | 417.82 | 0.30     | 59.79          |
| 1-1  | LF      | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | LF      | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | LDC     | 415.82 | 417.82 | 0.30     | 59.79          |
| 2-1  | DICOPT  | -      | 446.83 | 2.54     | 2.54           |
| 3-1  | LF      | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | LDC     | 415.82 | 417.82 | 0.30     | 59.79          |
| 3-1  | DICOPT  | -      | 446.83 | 2.54     | 2.54           |
Table 5  Effects of the COI rule to the performance of the proposed method

| Case | Method | LB [\%] | UB [\%] | DGAP [%] | Total time [s] |
|------|--------|---------|---------|----------|----------------|
| 1-1  | LFWC   | 415.82  | 417.07  | 0.30     | 60.71          |
| 1-1  | LF     | 415.82  | 417.07  | 0.30     | 59.79          |
| 2-1  | LFWC   | 4044.94 | 4148.97 | 2.51     | 11287.43       |
| 2-1  | LF     | 4044.94 | 4131.79 | 2.10     | 6198.44        |
| 3-1  | LFWC   | 5651.48 | 5732.21 | 1.41     | 49548.73       |
| 3-1  | LF     | 5651.48 | 5712.67 | 1.08     | 42163.00       |

The proposed method LF and the LF without COI rule (LFWC) in order to demonstrate the effectiveness of COI rule. The fixing priority for LF without COI rule (LFWC) is according to the minimum objective value of \( SP_i \) with (33)–(38) of \( I_{rest} \) for \( i_{min} \). From the computational results, the performance of UB for LF is better than that of LFWC in all cases. Also, the computation time for LF is shorter than that of LFWC. This is because the fixing priority of COI rule contributes the performance of the proposed method. It indicates that the item with the more planned quantity for sales has higher priority to be located at available spaces. Also, the item with less number of reserved location has higher priority in order to utilize the limited storage space efficiently. The item with less the number of the reserved location has little effect on storage spaces of other non-fixed items. The effectiveness of COI rule for fixing the subproblem is confirmed for the proposed method.

6. Summary and Conclusions

In this paper, a Lagrangian relaxation and fix heuristic approach is proposed for the integrated optimization of production planning and warehouse storage allocation problem under demand uncertainty. The stochastic formulation of the problem of mixed integer nonlinear formulation has been developed. Computational results show that the performance of the Lagrangian relaxation and fix heuristic with COI rule outperforms that of the conventional Lagrangian relaxation method. Future work is to reduce the convergence time of the proposed method and to solve the practical sized instance for \(|I| = 153, |L| = 813, |T| = 12\) with a reasonable computation time.

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