Application of Fuzzy Extension Analytic Hierarchy Process in Location Selection of Logistics Center

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Abstract. This paper use fuzzy extension analytic method to solve the problem of location selection of logistics center. First, establish an evaluation system. There are three first-level indicators at the criterion level, which are social benefits, economic benefits, and infrastructure, and nine second-level indicators at the sub-criteria level. The extension analytic method is used to compare all the factors corresponding to the layer to the first factor and use the number of extension intervals to compare them in pairs, so as to construct a judgment matrix for the extension interval and find the weight vector. Further hierarchical ranking is performed. Secondly, the fuzzy comprehensive evaluation method is used to determine the factor set and evaluation set of influencing factors, and the evaluation matrix is established. The primary element is used to determine the evaluation model. First-level evaluation is performed on each logistics center to obtain a second-level evaluation matrix. The model comprehensively evaluates the two-level evaluation matrix, and then ranks the evaluation levels of each which introduces extension theory, considers human ambiguity, uses interval numbers instead of specific numbers to construct a judgment matrix. The example proves that the ranking is more practical and provides a theoretical basis for site selection.

Keywords: Logistics; Extension analytic hierarchy process; Site selection; Fuzzy comprehensive evaluation method.

1. Introduction

When using the analytic hierarchy process to construct the judgment matrix, the importance of pairwise comparisons is assigned to the scale of 1-9 integer and its reciprocal, without considering the ambiguity of human judgment. In real life, people’s judgments are often within a certain range, For example, when comparing the importance of factor A and factor B, it is generally considered that factor A is closer to the actual situation than factor B if the importance is between 3.5 and 4.5 [1]. Apply the Extended Analytic Hierarchy Process (AHP) to Zhou Yusheng's assessment of the lightning protection reform of the line [2], And obtained the best lightning protection measures for each tower, which has a good lightning protection effect. Wu Tonghan [3] believes that the extension analytic hierarchy process is used to set reasonable thresholds for the types of maintenance equipment to meet the functional requirements of each equipment, and then achieve the rationality of the configurationHan Lihong [4] combined the fuzzy comprehensive evaluation method with the scalable analytic hierarchy process to obtain the evaluation value, and proved the effectiveness of the method through examples. Li Rong [5] and others used the fuzzy analytic hierarchy process in the location problem, and evaluated the evaluation indicators of each logistics center to get the conclusion that the traffic weight is the largest. It provides a basis for site selection. Yang Yu [6] established evaluation indicators for the location of regional logistics, used the extension matter-element analysis method to
evaluate, established a comprehensive evaluation indicator system, and verified the effectiveness of the model through examples.

Extenics is the discipline of laws and methods for solving contradictory problems, and which theoretical pillars is the matter-element theory and the collection of extenics, and its logical cells are primitives \(^{[7-10]}\). This paper combines extension theory and analytic hierarchy process to construct an evaluation index system, applies the judgment theory that meets the requirements of consistency in the theory of extension theory, and then conducts first-level evaluation and second-level evaluation for each logistics center, and then obtains the evaluation value. Select the location of the logistics center with the largest evaluation value.

**2. Extension Analytic Hierarchy Process**

2.1. Establish an Evaluation System

The location of the logistics center is affected by many factors. And the economic development of different alternative regions is different. Establishing an evaluation system needs to consider the actual conditions of each alternative places to choose appropriate indicators. The logistics evaluation indicators constructed are divided into criterion level and sub-criteria level. The criterion level has 3 first-level indicators divided into social benefits, economic benefits, and infrastructure. The sub-

\[ B = (B_1, B_2, B_3) \]

\[ B_1 = \text{Social benefits}, \quad B_2 = \text{Economic benefits}, \quad B_3 = \text{Infrastructure}; \]

criteria level has 9 second-level indicators.

Criterion layer:

Sub-criteria layer:

\[ C = (C_{11}, C_{12}, C_{13}) \]

\[ C_{11} = \text{Environmental protection situation}, \quad C_{12} = \text{Market power}, \quad C_{13} = \text{Future development space}; \]

\[ C = (C_{21}, C_{22}, C_{23}) \]

\[ C_{21} = \text{Profitability}, \quad C_{22} = \text{capital turnover}, \quad C_{23} = \text{total assets}; \]

\[ C = (C_{31}, C_{32}, C_{33}) \]

\[ C_{31} = \text{Transportation conditions}, \quad C_{32} = \text{Public facilities}, \quad C_{33} = \text{Information platform facilities}. \]

2.2. Construct Extension Judgment Matrix

In the application of Extension Analytic Hierarchy Process (EAHP) method, after the establishment of the hierarchy structure, for a certain factor of \( k – 1 \), compare all the factors corresponding to layer \( k \) in pairs. Secondly, the extension interval is used to quantitatively express the relative importance of them. Then a judgment matrix \( A \) with extension interval is constructed, where \( a_y = (a_y^{-1}, a_y) \) represents an extension interval number. In order to quantify each element of the judgment matrix, the median of the extension interval number is the integer on the 1-9 scale proposed by T.J.Saaty in AHP.

Extension judgment matrix \( A = (a_{ij})_{nm} \) is positive reciprocal matrix:

\[ a_y = 1, \quad a_y^{-1} = \left( \frac{1}{a_y}, a_y \right), \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n \]

2.3. Calculation Method of Judgment Matrix and Weight Vector

Set \( a_y = (a_j^{-1}, a_j) \) \((i, j = 1, 2, \ldots, n, t = 1, 2, \ldots, T) \) is the extension interval number given by an expert, according to this formula

\[ a_y' = (a_j^{-1}, a_j) \] \((i, j = 1, 2, \ldots, n, t = 1, 2, \ldots, T) \)

The formula (2) is to obtain the comprehensive extension interval of a certain element of the \( k \) layer, and then obtain the extension judgment matrix of a certain evaluation index of \( k – 1 \) and \( k \). And get the extension judgment matrix of a certain element of the pair level,

The steps to determine whether the matrix meets the condition of consistency are:
1) Obtain the normalized vector $x^+, x^-$ corresponding to the judgment matrix according to the judgment matrix $A^+, A^-$. 
2) Find the value of $k, m$ according to the judgment matrix $\left(\begin{array}{c} a_{ij}^+ \\ a_{ij}^- \end{array}\right)$:

$$k = \frac{\sum_{j=1}^{n} a_{ij}^+}{\sum_{j=1}^{n} a_{ij}^-}, \quad m = \frac{\sum_{j=1}^{n} a_{ij}^-}{\sum_{j=1}^{n} a_{ij}^+}$$

3) Find the weight vector

$$S^k = \{S^k_1, S^k_2, \cdots, S^k_m\} = \{kx^-, mx^+\}$$

2.4. Single Hierarchical Arrangement

Let $a = a^+, a^-$, $b = b^+, b^-$ be two extension interval Numbers, $V(a \geq b)$ is obtained by the following formula:

$$V(a \geq b) = \frac{2(a^+ - b^-)}{(b^+ - b^-) + (a^- - a^+)}$$

By the formula of $V\left(S^k_i \geq S^k_j\right)(i = 1, 2, \cdots, n_k; i \neq j)$,

if $\forall i = 1, 2, \cdots, n_k; i \neq j, V\left(S^k_i \geq S^k_j\right) \geq 0$,

$$P^k_{ih} = 1, P^k_{ah} = V(S^k_i \geq S^k_j), (i = 1, 2, \cdots, n_k; i \neq j)$$

Among them $P^k_{ah}$ represents the single ordering of the $i$ factor in the $k$ layer on the $h$ factor in the $k-1$ layer.

2.5. Overall Ranking Levels

When solve for $P^k_a = (P^k_{1a}, P^k_{2a}, \cdots, P^k_{n_ka})$ for $h = i = 1, 2, \cdots, n_{k-1}$, we get the $n_k \times n_{k-1}$ matrix

$$P^k = (P^k_1, P^k_2, \cdots, P^k_{n_k}) = \left[\begin{array}{cccc} p_{11}^k & p_{12}^k & \cdots & p_{1n_k}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2n_k}^k \\ \vdots & \vdots & \ddots & \vdots \\ p_{n_k1}^k & p_{n_k2}^k & \cdots & p_{nn_k}^k \end{array}\right]$$

The ranking vector $W^{k-1} = (W_1^{k-1}, W_2^{k-1}, \cdots, W^{k-1}_{n_{k-1}})$ of the general objective in layer $k-1$ is then the composition order $W^k$ of all elements in $k$ layer to the goal calculated by the following formula:

$$W^k = (W_1^k, W_2^k, \cdots, W_{n_k}^k) = P^k W^{k-1}$$

generally speaking

$$W^k = P^k P^{k-1} \cdots P^2 W^2$$

Where $W^2$ represents a single sort vector[11].

3. Examples

In this paper, the fuzzy extension analytic hierarchy process introduced in section 2 is used in the
example of logistics center location. First, an evaluation system is established for the logistics center, the weight coefficients of each layer are obtained through the extension analytic hierarchy process, and then the fuzzy synthesis. The evaluation method conducts the first-level evaluation and the second-level evaluation on the evaluation system, and finally obtains the evaluation results, and then selects the best logistics center.

3.1. Use the Extension Analytic Hierarchy Process to Determine the Index Weight

According to the requirements of the general objective, the extension interval judgment matrix A of the evaluation criteria is obtained by inviting relevant experts to score the indicators by comparing them in pairs. Calculate the extension comprehensive weight vector compared with each evaluation index for the target layer, and construct the extension interval judgment matrix.

(1) This article invites relevant experts to score various evaluation indicators and obtain a judgment matrix, See Table 1:

| $A - B$ | $B_1$ | $B_2$ | $B_3$ |
|-------|------|------|------|
| $B_1$ | <1,1> | <2.8, 3.2> | <3.5, 4.5> |
| $B_2$ | <0.3125, 0.3571> | <1,1> | <1, 3> |
| $B_3$ | <0.2222, 0.2857> | <0.3333, 1> | <1,1> |

Calculated from (4): $P_1 = 3.3679$, $P_2 = 1.736$, $P_3 = 1$
Obtain a single ranking of the overall goal by three evaluation indicators from standardization $P = (0.5518, 0.2844, 0.1638)$

(2) Analyzing the results of the experts’ scoring of the criteria layer for the sub-criteria layer, Use the extension analytic hierarchy process to calculate the weight coefficient of the extension interval judgment matrix, And check its consistency. See Table 2, Table 3, Table 4:

| $B_1$ | $C_{11}$ | $C_{12}$ | $C_{13}$ |
|------|------|------|------|
| $C_{11}$ | <1,1> | <0.4347, 0.5882> | <0.2222,0.285> |
| $C_{12}$ | <1.7,2.3> | <1,1> | <0.2857,0.4> |
| $C_{13}$ | <3.5,4.5> | <2.5,3.5> | <1,1> |

Calculated from (4): $P_1 = 1$, $P_2 = 4.2424$, $P_3 = 8.7994$
Obtain a single ranking of the overall goal by three evaluation indicators from standardization, $P = (0.0712, 0.3021, 0.6267)$.

| $C_{21}$ | $C_{22}$ | $C_{23}$ |
|------|------|------|
| $C_{21}$ | <1,1> | <0.4762, 0.5263> | <1.8, 2.2> |
| $C_{22}$ | <1.9, 2.1> | <1,1> | <2.9, 3.1> |
| $C_{23}$ | <0.4545, 0.5556> | <0.3226, 0.3448> | <1,1> |
Calculated from (4): $P_1 = 6.358$, $P_2 = 15.334$, $P_3 = 1$.

Obtain a single ranking of the overall goal by three evaluation indicators from standardization, $P = (0.2802, 0.6757, 0.0441)$.

### Table 4. Weight of C to B3

| $C_{31}$  | $C_{32}$  | $C_{33}$  |
|-----------|-----------|-----------|
| <1,1>     | <0.2381,0.2631> | <1.9,2.1> |
| $C_{32}$  | <3.8, 4.2>  | <1,1>     |
| <4.8, 5.2> |           |           |
| $C_{33}$  | <0.3448,0.4762> | <0.1923,0.2083> |
|           |           | <1,1>     |

Calculated from (4): $P_1 = 6.845$, $P_2 = 18.174$, $P_3 = 1$.

Obtain a single ranking of the overall goal by three evaluation indicators from standardization, $P = (0.2631, 0.6985, 0.0384)$.

### 3.2. Fuzzy Comprehensive Evaluation

(1) Judgment set $V = \{v_1, v_2, v_3, v_4, v_5\}$, $v_1 = (0.9~1)$, $v_2 = (0.7~0.9)$, $v_3 = (0.5~0.7)$, $v_4 = (0.3~0.5)$, $v_5 = (0.1~0.3)$, $V = \{$ Very good, good, fair, bad, very bad $\}$. Calculate the membership degree of each evaluation scale from the score of each expert, Obtain the membership degrees of the three alternative places A, B, and C. The calculation results are shown in the table below:

### Table 5. The evaluation result of A region

|   | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|-------|-------|-------|-------|-------|
| $C_{11}$ | 0.15  | 0.35  | 0.15  | 0.2   | 0.15  |
| $C_{12}$ | 0.2   | 0     | 0.5   | 0.2   | 0.1   |
| $C_{13}$ | 0.4   | 0.4   | 0.15  | 0.05  | 0     |
| $C_{21}$ | 0.55  | 0.25  | 0.2   | 0     | 0     |
| $C_{22}$ | 0.25  | 0.15  | 0.4   | 0.1   | 0.1   |
| $C_{23}$ | 0.15  | 0.25  | 0.4   | 0.1   | 0.1   |
| $C_{31}$ | 0.4   | 0.3   | 0.15  | 0.05  | 0.1   |
| $C_{32}$ | 0.15  | 0.45  | 0.2   | 0.1   | 0.1   |
| $C_{33}$ | 0.25  | 0.3   | 0.15  | 0.2   | 0.1   |

### Table 6. The evaluation result of B region

|   | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|-------|-------|-------|-------|-------|
| $C_{11}$ | 0.25  | 0.45  | 0.2   | 0     | 0.1   |
| $C_{12}$ | 0.25  | 0     | 0.5   | 0.25  | 0     |
| $C_{13}$ | 0.45  | 0.45  | 0.05  | 0.05  | 0     |
| $C_{21}$ | 0.5   | 0.25  | 0.2   | 0     | 0.05  |
| $C_{22}$ | 0.1   | 0.25  | 0.4   | 0.15  | 0.1   |
3.3. First-level Evaluation

This paper uses the main element deterministic comprehensive evaluation model to make a first-level evaluation of the alternative addresses A, B, and C.

Single factor weight:

The weight of $B$ to $A$ is $w = [0.5518, 0.2844, 0.1638]$

The weight of $C$ to $B_1$ is $w_1 = [0.0712, 0.3021, 0.6267]$

The weight of $C$ to $B_2$ is $w_2 = [0.2802, 0.6757, 0.0441]$

The weight of $C$ to $B_3$ is $w_3 = [0.2631, 0.6985, 0.0384]$

1) First-level evaluation of Alternative $A$

The calculation of the comprehensive evaluation vector is as follows:

$B_{12} = w_2 \circ R_{12} = \begin{bmatrix} 0.2802 & 0.6757 & 0.0441 \end{bmatrix} \times \begin{bmatrix} 0.55 & 0.25 & 0.2 & 0 & 0 \\ 0.25 & 0.15 & 0.4 & 0.1 & 0.1 \\ 0.15 & 0.25 & 0.4 & 0.1 & 0.1 \end{bmatrix}$

$= \begin{bmatrix} 0.2802 & 0.25 & 0.4 & 0.1 & 0.1 \end{bmatrix}$

$B_{13} = w_3 \circ R_{13} = \begin{bmatrix} 0.2631 & 0.6985 & 0.0384 & 0.2631 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.3 & 0.15 & 0.05 & 0.1 \\ 0.15 & 0.45 & 0.2 & 0.1 & 0.1 \\ 0.25 & 0.3 & 0.15 & 0.2 & 0.1 \end{bmatrix}$

$= \begin{bmatrix} 0.2631 & 0.45 & 0.2 & 0.1 & 0.1 \end{bmatrix}$

In summary, the evaluation matrix of Alternative $A$ is obtained

$B_A = \begin{bmatrix} B_{11} \\ B_{12} \\ B_{13} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.4 & 0.3021 & 0.2 & 0.1 \\ 0.2802 & 0.25 & 0.4 & 0.1 & 0.1 \\ 0.2631 & 0.45 & 0.2 & 0.1 & 0.1 \end{bmatrix}$

2) First-level evaluation of Alternative $B$

In the same way, the first-level evaluation matrix of alternative address $B$ is obtained:
3) First-level evaluation of Alternative C
In the same way, the first-level evaluation matrix of alternative address C is obtained:

\[
B_2 = \begin{bmatrix}
B_{21} & B_{22} & B_{23}
\end{bmatrix} = \begin{bmatrix}
0.45 & 0.45 & 0.3021 \ 0.2802 & 0.25 & 0.4 \ 0.2631 & 0.5 & 0.15
\end{bmatrix}.
\]

3.4. Second-level Comprehensive Evaluation
1) Secondary comprehensive evaluation of Alternative A
Use the main element deterministic comprehensive evaluation model \( M(\land, \lor) \) to comprehensively evaluate the alternative site of \( A \):

\[
C_i = w \circ B_i = \begin{bmatrix} 0.5518 & 0.2844 & 0.1638 \end{bmatrix} \cdot \begin{bmatrix} 0.4 & 0.4 & 0.3021 & 0.2 & 0.1 \ 0.2802 & 0.25 & 0.4 & 0.1 & 0.1 \ 0.2631 & 0.45 & 0.2 & 0.1 & 0.1 \end{bmatrix}.
\]

\[
= \begin{bmatrix} 0.4 & 0.4 & 0.3021 & 0.2 & 0.1 \end{bmatrix}
\]

If the principle of maximum membership degree is adopted, the comprehensive evaluation of Alternative A is "very good" or "good". If the weighted average principle is adopted, firstly normalize \( C_i \) to get:

\[
C_i' = \begin{bmatrix} 0.2853 & 0.2853 & 0.2155 & 0.1426 \end{bmatrix}
\]

Secondly, suppose the value of "very good" is (0.9-1), the value of "good" is (0.7-0.9), the value of "good" is (0.5-0.7), and the value of "general" is (0.3-0.5), the value of "poor" is (0.1-0.3), so the comprehensive evaluation value of alternative site A is possible.

\[
P_i = C_i' \times V^T = \begin{bmatrix} 0.2853 & 0.2853 & 0.2155 & 0.1426 & 0.0713 \end{bmatrix} \times \begin{bmatrix} 0.95 \\
0.80 \\
0.60 \end{bmatrix} = 0.7001
\]

The evaluation result \( P = 0.7001 > 0.7 \) is "good", so the alternative address conditions are at a "good" level.

2) Secondary comprehensive evaluation of Alternative B
The same method, secondary comprehensive evaluation of alternative address B:

\[
P_2 = C_2' \times V^T = \begin{bmatrix} 0.2899 & 0.2899 & 0.1946 & 0.1612 & 0.0644 \end{bmatrix} \times \begin{bmatrix} 0.95 \\
0.80 \\
0.60 \end{bmatrix} = 0.7079
\]

The evaluation result \( P = 0.7079 > 0.7 \) is "good", so the alternative address conditions B is at a "good" level.
3) Secondary comprehensive evaluation of Alternative $C$

The same method, secondary comprehensive evaluation of alternative address $C$

$$P_1 = C'_1 \times V = \begin{bmatrix} 0.2889 & 0.3662 & 0.2045 & 0.0722 & 0.0722 & 0.7339 \\ 0.60 & 0.40 & 0.20 & \end{bmatrix}$$

The evaluation result $P = 0.7339 > 0.7$ is "good", so the alternative address conditions $C$ is at a "good" level.

From the perspective of the three comprehensive scores of A, B, and C, the score of C is higher than other values, so C is selected as the logistics center.

4. Conclusion

This article applies the fuzzy extension analytic hierarchy process to the location of the logistics center. The extension analytic hierarchy process uses interval numbers instead of concrete numbers to construct the judgment matrix. This method not only considers the fuzzy judgment of people, but also achieves the consistency of the judgment matrix and the weight vector at the same time. The feasibility of fuzzy extension analytic hierarchy process is verified by examples of logistics centers A, B, and C, and the best logistics center is selected. This method provides a theoretical basis for logistics location selection.

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