Black Holes and Black String-like Solutions in Codimension-2 Braneworlds *

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Abstract

We discuss black hole solutions with a Gauss-Bonnet term in the bulk and an induced gravity term on a thin brane of codimension-2. We show that these black holes can be localized on the brane, and they can be extended further into the bulk by a warp function. These solutions have regular horizons and no other curvature singularities appear apart from the string-like ones. The projection of the Gauss-Bonnet term on the brane imposes a constraint relation which dictates the form of matter on the brane and in the bulk.

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1 Introduction

Recently there has been some interest in codimension-2 braneworlds. The most attractive feature of these models is that the vacuum energy (tension) of the brane instead of curving the brane world-volume, merely induces a deficit angle in the bulk solution around the brane. This observation led several people to utilize this property in order to self-tune the effective cosmological constant to zero and provide a solution to the cosmological constant problem [1]. However, soon it was realized [2] that one can only find nonsingular solutions if the brane energy momentum tensor is proportional to its induced metric. To reproduce an effective four-dimensional Einstein equation on the brane one has to introduce a cut-off (brane thickness) [3, 4, 5] with the price of loosing the predictability of the theory. Alternatively, in the thin brane limit four dimensional gravity is recovered as the dynamics of the induced metric on the brane if the gravitational action is modified by the inclusion of either a Gauss-Bonnet term [6] or an induced gravity term on the brane [7].

We are still lacking an understanding of time dependent cosmological solutions in codimension-2 braneworlds. In the thin brane limit, because the energy momentum tensors on the brane and in the bulk are related, the brane equation of state and energy density are tuned and we cannot get the standard cosmology on the brane [8, 9]. One then has to regularize the codimension-2 branes by introducing some thickness and then consider matter on them [10, 11, 12, 13]. To have a cosmological evolution on the regularized branes the brane world-volume should be expanding and in general the bulk space should also evolve in time. This is a formidable task, so an alternatively approach was followed in [14, 15] by considering a codimension-1 brane moving in the regularized static background. The resulting cosmology, however, was unrealistic having a negative Newton’s constant (for a review on the cosmology in six dimensions see [16]).

We do not either fully understand black hole solutions on codimension-2 braneworlds. Recently a six-dimensional black hole localized on a 3-brane of codimension-2 [17] was proposed. These solutions are generalization of the 4D Aryal, Ford, Vilenkin [18, 19] black hole pierced by a cosmic string adjusted to the codimension-2 branes with a conical structure in the bulk and deformations accommodating the deficit angle. However, it is not clear how to realize these solutions in the thin brane limit where high curvature terms are needed to accommodate matter on the brane. Generalizations to include rotations were presented in [20].

The localization of a black hole on the brane and its extension to the bulk is a difficult task. In codimension-1 braneworlds the first attempt was to consider the Schwarzschild metric and study its black string extension into the bulk [21]. Unfortunately, as suspected by the authors, this string is unstable to classical linear perturbations [22] (for a recent review see [23]). Since then, several authors have attempted to find the full metric using numerical techniques [24]. Analytically, the brane metric equations of motion were considered with the only bulk input coming from the projection of the Weyl tensor [25] onto the brane. Since this system is not closed because it contains an unknown bulk dependent term, assumptions have to be made either in the form of the metric or on the Weyl term [26].
A lower dimensional version of a black hole living on a (2+1)-dimensional braneworld was considered in [27] by Emparan, Horowitz, and Myers. They based their analysis on the so-called C-metric [28] modified by a cosmological constant term. They found a BTZ black hole [29] on the brane which can be extended as a BTZ black string in a four-dimensional AdS bulk. Their thermodynamical stability analysis showed that the black string remains a stable configuration when its transverse size is comparable to the four-dimensional AdS radius, being destabilized by the Gregory-Laflamme instability [22] above that scale, breaking up to a BTZ black hole on a 2-brane.

In this talk we will discuss black holes on a thin conical brane and their extension into a five and six-dimensional bulk with a Gauss-Bonnet term [30, 31]. In the case of a five-dimensional bulk [30] we had found that the BTZ black hole and its short distance corrections [32] solve the junction conditions on a conical 2-brane. These solutions in the bulk are BTZ string-like objects with regular horizons and no pathologies. The warping to five dimensions depends on the length $\sqrt{\alpha}$ where $\alpha$ is the Gauss-Bonnet coupling, and this length scale defines the shape of the horizon. Consistency of the bulk solutions requires a fine-tuned relation between the Gauss-Bonnet coupling and the five-dimensional cosmological constant.

In the case of six-dimensional bulk, we found solutions of four-dimensional Schwarzschild-AdS black holes on a 3-brane which in the six-dimensional spacetime look like black string-like objects with regular horizons [31]. In the case of constant deficit angle the localization of the four-dimensional black hole requires matter in the two extra dimensions. The energy-momentum tensor corresponding to this matter scales as $1/r^6$. This fact defines a length scale in the six-dimensional spacetime above which we recover the standard four-dimensional General Relativity (GR), while at small distances GR is strongly modified.

## 2 BTZ String-Like Solutions in Five-Dimensional Braneworlds of Codimension-2

We consider the following gravitational action in five dimensions with a Gauss-Bonnet term in the bulk and an induced three-dimensional curvature term on the brane

$$S_{\text{grav}} = \frac{M_5^3}{2} \left\{ \int d^5x \sqrt{-g} \left[ R + \alpha \left( R^2 - 4R_{MN}R^{MN} + R_{MNL}R^{MNL} \right) \right] \\
+ r_c^2 \int d^3x \sqrt{-g^{(3)}} R^{(3)} \right\} + \int d^5x L_{\text{bulk}} + \int d^3x L_{\text{brane}},$$

where $\alpha \geq 0$ is the GB coupling constant and $r_c = M_3/M_5^3$ is the induced gravity “cross-over” scale. The bulk metric is

$$ds_5^2 = g_{\mu\nu}(x, \rho)dx^\mu dx^\nu + a^2(x, \rho)d\rho^2 + L^2(x, \rho)d\theta^2,$$

where $g_{\mu\nu}(x, 0)$ is the braneworld metric and $x^\mu$ denote three dimensions, $\mu, \nu = 0, 1, 2$ whereas $\rho, \theta$ denote the radial and angular coordinates of the two extra dimensions.
The Einstein equations resulting from the variation of the action (1) are
\[ G_{MN}^{(5)} + r_c^2 G_{(3)\mu}^{(3)\nu} g_M^{\mu} g_N^{\nu} \frac{\delta(\rho)}{2\pi L} - \alpha H_M^{(5)} = \frac{1}{M_5^3} \left[ T_M^{(B)N} + T_M^{(br)\mu} g_M^{\mu} g_N^{\nu} \frac{\delta(\rho)}{2\pi L} \right] , \] (3)

where \( H_M^{(5)} \) is coming from the Gauss-Bonnet term. To obtain the braneworld equations we expand the metric around the brane as \( L(x, \rho) = \beta(x) \rho + O(\rho^2) \). We demand that the space in the vicinity of the conical singularity is regular which imposes the supplementary conditions that \( \partial_\mu \beta = 0 \) and \( \partial_\rho g_{\mu\nu}(x, 0) = 0 \) [6].

The extrinsic curvature is given by \( K_{\mu\nu} = g'_{\mu\nu} \). The second derivatives of the metric functions contain \( \delta \)-function singularities
\[ \frac{L''}{L} = -(1 - L') \frac{\delta(\rho)}{L} + \text{non - singular terms} , \] (4)
\[ \frac{K'_{\mu\nu}}{L} = K_{\mu\nu} \frac{\delta(\rho)}{L} + \text{non - singular terms} . \] (5)

From the above singularity expressions and using the Gauss-Codazzi equations, we can match the singular parts of the Einstein equations (3) and get the following “boundary” Einstein equations
\[ G_{\mu\nu}^{(3)} = \frac{1}{M_5^3 (r_c^2 + 8\pi(1 - \beta)\alpha)} T_{\mu\nu}^{(br)} + \frac{2\pi(1 - \beta)}{r_c^2 + 8\pi(1 - \beta)\alpha} g_{\mu\nu} . \] (6)

We assume that there is a localized (2+1) black hole on the brane. The brane metric is
\[ ds_3^2 = \left( -n(r)^2 dt^2 + n(r)^{-2} dr^2 + r^2 d\phi^2 \right) . \] (7)

We will look for black string solutions of the Einstein equations (3) using the five-dimensional metric (2) in the form
\[ ds_5^2 = f^2(\rho) \left( -n(r)^2 dt^2 + n(r)^{-2} dr^2 + r^2 d\phi^2 \right) + a^2(r, \rho) d\rho^2 + L^2(r, \rho) d\theta^2 . \] (8)

The space outside the conical singularity is regular, therefore, we demand that the warp function \( f(\rho) \) is also regular everywhere. We assume that there is only a cosmological constant \( \Lambda_5 \) in the bulk and we take \( a(r, \rho) = 1 \). Then, from the bulk Einstein equations
\[ G_{MN}^{(5)} - \alpha H_{MN} = - \frac{\Lambda_5}{M_5^3} g_{MN} , \] (9)
we find the solutions which are summarized in Table 1 [30].

In the above table \( L_3 \) is the length scale of \( AdS_3 \) space. Note that in all solutions there is a fine-tuned relation between the Gauss-Bonnet coupling \( \alpha \) and the five-dimensional cosmological constant \( \Lambda_5 \), except for the solution in the fourth row.

To introduce a brane we must solve the corresponding junction conditions given by the Einstein equations on the brane (6) using the induced metric on the brane given by (7). We found that the BTZ black hole is localized on the brane in vacuum.
**Table 1: BTZ String-Like Solutions in Five-Dimensional Braneworlds of Codimension-2**

| $n(r)$     | $f(\rho)$                  | $L(\rho)$                  | $-\Lambda_5$ | Constraints |
|------------|-----------------------------|----------------------------|--------------|-------------|
| BTZ        | $\text{cosh} \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $\forall L(\rho)$         | $\frac{3}{4a}$ | $L_3^2 = 4 \alpha$ |
| BTZ        | $\text{cosh} \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $2 \beta \sqrt{\alpha} \sinh \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $\frac{3}{4a}$ |             |
| BTZ        | $\text{cosh} \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $2 \beta \sqrt{\alpha} \sinh \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $\frac{3}{4a}$ | $L_3^2 = 4 \alpha$ |
| BTZ        | $\pm 1$                     | $\frac{1}{\gamma} \sinh (\gamma \rho)$ | $\frac{3}{4\beta^2}$ |             |
| $\forall n(r)$ | $\text{cosh} \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $2 \beta \sqrt{\alpha} \sinh \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $\frac{3}{4a}$ |             |
| $\sqrt{-M + \frac{r^2}{L_5^2} - \frac{\zeta}{r}}$ | $\gamma = \sqrt{-2\Lambda_5}$ | $L_3^2 = 4 \alpha$ |             |
| $\sqrt{-M + \frac{r^2}{L_5^2} - \frac{\zeta}{r}}$ | $\pm 1$                     | $2 \beta \sqrt{\alpha} \sinh \left( \frac{\rho}{2 \sqrt{\alpha}} \right)$ | $\frac{1}{4a}$ | $\Lambda_5 = -\frac{1}{4a} = -\frac{3}{L_5^2}$ |

Table 1: BTZ String-Like Solutions in Five-Dimensional Braneworlds of Codimension-2

When $n(r)$ is of the form $n(r)^2 = -M + r^2/L_5^2 - \zeta/r$, which is the BTZ black hole solution with a short distance correction term and it corresponds to the BTZ conformally coupled to a scalar field [32], the energy momentum tensor necessary to sustain such a solution on the brane is given by $T^\beta_\alpha = \text{diag} \left( \frac{\zeta}{2r_3}, \frac{\zeta}{2r_3}, -\frac{\zeta}{r_3} \right)$.

These solutions extend the brane BTZ black hole into the bulk. Calculating the square of the Riemann tensor we find that at the AdS horizon ($\rho \to \infty$) all solutions give finite result and hence the only singularity is the BTZ black hole singularity extended into the bulk. The warp function $f^2(\rho)$ gives the shape of a 'throat' to the horizon of the BTZ string-like solution. The size of the horizon is defined by the scale $\sqrt{\alpha}$ and this scale is fine-tuned to the length scale of the five-dimensional AdS space.

### 3 Black String-Like solutions in Six-Dimensional Braneworlds of Codimension-2

The metric as in the five-dimensional case is

$$ds^2 = g_{\mu\nu}(r, \chi)dx^\mu dx^\nu + a^2(r, \chi)d\chi^2 + L^2(r, \chi)d\xi^2,$$

now with $\mu, \nu = 0, 1, 2, 3$ whereas $\chi, \xi$ denote the radial and angular coordinates of the two extra dimensions.

The corresponding Einstein equations are

$$G^{(6)}_{MN} + r^2 \frac{2}{M_6} G^{(4)}_{\mu\nu} g^\mu_N g^\nu_M \frac{\delta(\chi)}{2\pi L} - \alpha H^N_M = \frac{1}{M_6^4} \left[ -\Lambda_6 + T^{(B)N}_M + T^{(br)\nu}_\mu g^\mu_N \frac{\delta(\chi)}{2\pi L} \right],$$

where $H^N_M$ is the corresponding six-dimensional term. To obtain the braneworld equations we expand the metric around the 3-brane as $L(r, \chi) = \beta(r)\chi + O(\chi^2)$, and as in the five-dimensional case the function $L$ behaves as $L'(r, 0) = \beta(r)$, where a prime now denotes
derivative with respect to $\chi$. The “boundary” Einstein equations are
\[
G^{(4)}_{\mu\nu} \left( r^2 + 8\pi(1 - \beta)\alpha \right) = \frac{1}{M_6} T_{(br)}^{(\mu\nu)} + 2\pi(1 - \beta)g_{\mu\nu} \\
+ \pi L(r, \chi) E_{\mu\nu} - 2\pi\beta\alpha W_{\mu\nu},
\]
(12)
where the term
\[
E_{\mu\nu} = (K_{\mu\nu} - g_{\mu\nu} K),
\]
(13)
appears because of the presence of the induced gravity term in the gravitational action, while the term
\[
W_{\mu\nu} = g^{\lambda\sigma} \partial_{\lambda}g_{\mu\lambda}\partial_{\chi}g_{\nu\sigma} - g^{\lambda\sigma} \partial_{\lambda}g_{\chi\lambda}\partial_{\chi}g_{\mu\nu} \\
+ \frac{1}{2}g_{\mu\nu} \left[ (g^{\lambda\sigma} \partial_{\lambda}g_{\chi\lambda})^2 - g^{\lambda\sigma} g^{\delta\rho}\partial_{\chi}g_{\lambda\delta}\partial_{\chi}g_{\sigma\rho} \right],
\]
(14)
is the Weyl term due to the presence of the Gauss-Bonnet term in the bulk.

We will look for black string solutions of the Einstein equations (11) using the six-dimensional metric (10) in the form
\[
ds_6^2 = F^2(\chi) \left( -A(r)^2dt^2 + A(r)^{-2}dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \\
+ a^2(r, \chi)d\chi^2 + L^2(r, \chi)d\xi^2.
\]
(15)
The solutions are summarized in Table 2 [31].

| $A^2(r)$ | $F(\chi)$ | $L(\chi)$ | $-\Lambda_6$ | Constraints & $T^{(B)}$ |
|-----------|------------|------------|--------------|------------------------|
| $1 + \frac{r^2}{L_4^2} - \frac{\zeta}{r}$ | $\cosh \left( \frac{\chi}{2\sqrt{3\alpha}} \right)$ | $\forall L(\chi)$ | $\frac{5}{12\alpha}$ | $\alpha = \frac{L_4^2}{12}$, $T_\chi = T_\xi = \frac{-6\alpha\zeta^2}{r^6 F(\chi)^3}$ |
| $1 + \frac{r^2}{L_4^2} - \frac{\zeta}{r}$ | $\cosh \left( \frac{\chi}{2\sqrt{3\alpha}} \right)$ | $\frac{2\sqrt{3\alpha}\beta}{\gamma}$ | $\frac{5}{12\alpha}$ | $\alpha = \frac{L_4^2}{12}$, $T_\chi = T_\xi = \frac{-6\alpha\zeta^2}{r^6 F(\chi)^3}$ |
| $1 + \frac{r^2}{L_4^2} - \frac{\zeta}{r}$ | $\pm 1$ | $\frac{\beta}{\gamma} \sinh \left( \frac{\chi}{\sqrt{3\alpha}} \right)$ | $\frac{6}{L_4^2} \left( 1 - \frac{2\alpha}{L_4^2} \right)$ | $\gamma = \frac{1}{L_4^2} \left( \sqrt{1 - \frac{L_4^2}{12\alpha}} \right)$, $T_\chi = T_\xi = \frac{-6\alpha\zeta^2}{r^6 F(\chi)^3}$ |
| $1 + \frac{r^2}{L_4^2} - \frac{\zeta}{r}$ | $\pm 1$ | $\frac{\beta}{\gamma} \chi \sinh \gamma$ | $\frac{6}{L_4^2} \left( 1 - \frac{2\alpha}{L_4^2} \right)$ | $\gamma = \frac{1}{L_4^2} \left( \sqrt{1 - \frac{L_4^2}{12\alpha}} \right)$, $T_\chi = T_\xi = \frac{-6\alpha\zeta^2}{r^6 F(\chi)^3}$ |

Table 2: Black String-Like Solutions in Six-Dimensional Brane Worlds of Codimension-2
4 The rôle of the Gauss-Bonnet Term

In codimension-2 braneworlds there is a relation connecting the Gauss-Bonnet term projected on the brane with the components of the bulk energy-momentum tensor corresponding to the extra dimensions [7]. In six dimensions it reads

$$-\frac{1}{2} R^{(4)} - \frac{1}{2} \alpha \left( R^{(4)} R^{(4)} - 4 R_{\mu \nu} R_{\mu \nu} + R_{\mu \nu \kappa \lambda} R_{\mu \nu \kappa \lambda} \right) = \frac{1}{M_6^4} T^{(B)} - \frac{\Lambda_6}{M_6^2}. \quad (16)$$

All bulk solutions have to satisfy this relation which acts as a consistency relation. For the Schwarzschild-AdS solution of the form $A(r)^2$ appearing in the above table the square of the Riemann tensor reads

$$R^{(4)}_{\mu \nu \kappa \lambda} = \frac{192 \zeta^2 e^{\frac{4 \chi}{L_4}}}{(1 + e^{\frac{4 \chi}{L_4}})^4 r^6} + \frac{60}{L_4^4}, \quad (17)$$

while the Ricci scalar and Ricci tensor are constants. Therefore, for the relation (16) to be satisfied the bulk energy-momentum tensor $T^{(B)} |_{\chi} = 0$ has to scale as $1/r^6$ with the right coefficients. This is actually what happens considering the result appearing in the table. Thus, the presence of the Gauss-Bonnet term in the bulk, which acts as a source term because of its divergenceless nature, dictates the form of matter that must be introduced in the bulk in order to sustain a black hole on the brane. It is interesting to observe that this "holographic matter" does not depend explicitly on the extra dimension but only through the warp function $F(\chi)$ which at large $\chi$ goes to zero. On the other hand on the brane, in the infrared limit we recover the conventional four-dimensional gravity, while in the ultraviolet limit we have strong gravity effects modifying the four-dimensional gravity in a non-trivial way \(^2\).

5 Conclusions

We discussed black holes localization on a thin brane of codimension-2 and their extension into an AdS bulk. To reproduce gravity on the brane, we introduced a Gauss-Bonnet term in the bulk and an induced gravity term on the brane. We showed that black holes can be localized on the conical brane, in five dimensions a BTZ black hole while in six dimensions a Schwarzschild-AdS, and these black hole solutions can be extended into the bulk with a warp function. Consistency of the bulk equations requires a fine-tuned relation between the Gauss-Bonnet coupling constant and the length of the AdS space. The use of this fine-tuning gives to the non-singular horizon the shape of a throat up to the horizon of the AdS space with no other curvature singularities except the brane string-like singularity.

The presence of the Gauss-Bonnet term is important in our considerations. It allows the existence of black string solutions in five dimensions and in six dimensions it specifies

\(^1\)A similar relation involving the Gauss-Bonnet term was presented in [33] in a different context.

\(^2\)Black hole solutions in codimension-2 braneworlds were also recently discussed in [34].
the form of matter which is needed in the bulk in order to sustain a black hole on the 
brane.

We have not discussed the issue of stability of the solutions we found. The presence of 
the GB term in the bulk renders the problem difficult to tackle.

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