Evidence for strong pairing at arbitrary small $J/t$ is provided in a t-J model on the checkerboard lattice for the sign of hopping leading to frustration in hole motion. Destructive quantum interferences suppress Nagaoka ferromagnetism when $J/t \rightarrow 0$ and reduce drastically coherent hole motion in the fluctuating singlet background. It is shown that, by pairing in various orbital symmetry channels, holes can benefit from a large gain of kinetic energy.

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Although the underlying mechanism that leads to high critical temperature in the cuprates superconductors might be of magnetic origin a detailed scenario is still missing. The inter-layer tunnelling mechanism proposed by Anderson [1] is based on the presence of weakly coupled CuO layers in this material. The original idea is that the pairing mechanism within a given layer (which might be rather weak) could be amplified by the Josephson inter-layer tunnelling of the Cooper pairs. Such a scenario is only possible if the single particle motion is not allowed along the c-axis perpendicular to the layers, i.e. if the coherent single particle tunnelling is blocked by some means. This phenomenon could be observed e.g. in weakly coupled (along the c-axis) two-dimensional (2D) Luttinger liquids [2] (LL) as a result of the orthogonality catastrophe [3].

Despite its beauty and simplicity, this scenario still lacks strong experimental support and, on the theoretical side, a proof that an analog of the one-dimensional LL could be realized in 2D. Nevertheless, I suggest here that a closely related mechanism could be realized in a doped, strictly 2D, antiferromagnetic (AF) frustrated magnet. In the last years, there has been growing experimental and theoretical interests in frustrated magnets which could provide e.g. realizations of new exotic gapped spin liquid phases. Transition metal oxides (like $A_2Ti_2O_7$ titanates [4]) with a pyrochlore type structure, a three dimensional lattice of corner-sharing tetrahedra, exhibit a wide range of interesting physical properties. The AF Heisenberg model defined on the checkerboard lattice [5, 6], a 2D analog of the pyrochlore lattice, is believed to form a gapped Valence Bond Solid (VBS) [7] with an ordered arrangement of plaquette singlets on a subset of the void plaquettes (see Fig. 1(a)) and a rather short magnetic correlation length of only a few lattice spacings. The GS hence breaks translation symmetry and is two-fold degenerate.

The recent discovery of superconductivity in 5d transition metal pyrochlores [8] or in a CoO triangular layer based compound [9] suggests that geometric frustration, which could be magnetic and/or kinetic, might play a key role in the mechanism of unconventional superconductivity. In this Letter, I show that pairing appears at arbitrary small $J/t$ in the hole-doped checkerboard AF magnet for a given sign of the hopping amplitude $t$, leading to frustration in single hole motion. It is shown that hole delocalization i.e. gain in kinetic energy can play a key role in some mechanisms of unconventional...
pairing, as in the inter-layer mechanism discussed above.

To describe the low-energy physics of the weakly doped magnet the standard \( t-J \) model Hamiltonian is used:

\[
H = -t \sum_{i,j,\sigma} P c_{i,\sigma}^\dagger c_{j,\sigma} P + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \delta_{n_i n_j} \tag{1}
\]

where the same couplings \( t \) and \( J \) are used on the vertical, horizontal and diagonal bonds of the checkerboard lattice. Computations are performed by Lanczos exact diagonalisation of a 32-site periodic cluster. Although tilted by 45° w.r.t. the crystallographic axis, the square \( \sqrt{32} \times \sqrt{32} \) cluster (see Fig. 2(a)) exhibits the full \( C_{4v} \) point group symmetries of the infinite lattice (with 2 sites per unit cell) so that all orbital symmetries (\( s, d_{x^2-y^2}, d_{xy}, \) etc...) can be considered [11]. In addition, the most symmetric \( k \)-points are available in the Brillouin zone (BZ) as shown in Fig. 2(b). Note that, since the unit cell contains 2 sites, a folding of the Brillouin zone occurs so that the many-body spectra at \( k \) and \( k + Q_0 \) (\( Q_0 \) is the AF wavevector) are identical (although spectral weights are different). An additional folding is expected in the thermodynamic limit due to the translation symmetry breaking of the VBS GS.

Let me first briefly review the properties of a single hole doped into the VBS GS of the checkerboard lattice (see Fig. 3). Numerical computations based on the analysis of the hole Green function [12] (see also Figs. 3a-b)) show the existence of quasiparticle (QP) poles [13] whose Z-factor increases with \( J/t \) as shown in Fig. 3(c). The mass of the single hole can be estimated e.g. from the dispersion of its QP pole along the \( k_x = k_y \) direction of the Brillouin zone (for which the 32-site cluster contains 5 points). As seen in Ref. [12] and shown in Fig. 3(d) the single hole bandwidth is much smaller than in the (gapless) square lattice where \( W \sim 2.2J \) [14]. In the latter case, the strong renormalization of the coherent motion can be explained e.g. by long-wavelength spinwaves scattering [14]. In contrast, in the checkerboard lattice with \( t > 0 \) the QP is extremely massive, with a very narrow bandwidth, although the VBS host has no low energy excitations. This results from a subtle interplay between the intrinsic frustrated nature of the hopping and the long-ranged plaquette order induced by magnetic frustration. Indeed, one expects destructive interferences [15] between the paths available for the hole to hop coherently from one plaquette to the next. However, incoherent motion still occurs. In that respect, the behavior when \( J/t \to 0 \) (for which exact statements can be given) is very instructive: while for \( t < 0 \) the GS is a Nagaoka ferromagnet (of energy \(-6t\)), for \( t > 0 \) the \( S = 1/2 \) GS remains stable down to \( J = 0 \) where its energy becomes \(-4t\) (much lower than the fully polarized state of energy \(-2t\) where the hole is localized on a plaquette). Interestingly enough, the triangular lattice of Fig. 1(b) exhibits the very same behavior at \( J/t \ll 1 \).

As shown below, I argue that holes doped into the checkerboard AF magnet pair up very strongly when \( t > 0 \). The pairing is studied here by considering the same 32-site cluster doped with two holes. The two hole GS energies are computed for all possible orbital symmetries. The hole-hole binding energies \( E_B \) obtained by subtracting twice the single hole GS energy are plotted in Fig. 4. The results for \( t > 0 \) and \( t < 0 \) reveal striking differences which point toward two completely different physical behaviors. The case \( t < 0 \) shows a behavior qualitatively (even quantitatively) similar to the case of the

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**FIG. 3:** Single hole dynamics on the checkerboard lattice; Spectral functions for \( t < 0 \) (a) and \( t > 0 \) (b). The total weight is a constant (sum-rule) and set to 1; (c) Z-factor at the X-point vs \( J/t \); (d) Dispersal width along the \( k_x = k_y \) direction in the BZ. For comparison the case of the square lattice is shown.

**FIG. 4:** Energies of two holes doped on a square 32-site cluster (\( \sqrt{32} \times \sqrt{32} \)) in various symmetry channels (as indicated). The energy reference \( (E_B = 0) \) is chosen as the bottom of the two-hole continuum. Boundstates are characterized by \( E_B < 0 \). The thick red line corresponds the local (negative) slope at \( J/t \to 0 \). Thin red arrows show the large (small) kinetic (magnetic) energy gain (lost) obtained from the estimated slope of \( E_B \) vs \( J \) at \( J \approx 0.35 \) (see text). Data obtained on a \( 4 \times 4 \) cluster are also shown using smaller symbols.
well-known square lattice \[16\] (for which the sign of \(t\) is irrelevant): (i) the orbital symmetry of the BS is \(d_{x^2-y^2}\)-wave and (ii) the binding energy scales like \(J\) (above a size dependent critical value) as expected for a purely magnetic mechanism. In contrast, the \(t > 0\) case shows a completely new behavior: (i) the binding energy appears immediately at infinitesimal \(J\); (ii) finite size effects in this regime are well controlled: binding is vanishing at \(J = 0\) (for all sizes) and the magnitude of the (negative) slope characterizing the linear behavior \(E_B \propto J/t\) increases with system size; (iii) the lowest BS has s-wave symmetry; (iv) other exotic stable BS appear with non-conventional orbital symmetries like \(d_{x^2-y^2}, d_{xy}\) or even \(g\)-wave symmetries. It should be noted that, although the s-wave BS wavefunction has the full symmetry of the crystal by definition, it could be very anisotropic like the superconducting gap in the inter-layer scenario \[1\]. Importantly enough, while the BS stability in the thermodynamic limit at small \(J/t\) values (the physical range of this parameter) remains controversial in the case \(t < 0\) (as for the square lattice \[16\]), the data (and their behavior with size) for \(t > 0\) provide a very strong evidence for the robustness of the BS down to \(J = 0\). Such a remarkable phenomenon is due to the nature of the pairing enhancement mechanism which always benefit from a gain in kinetic energy. Indeed, using Feynman-Hellmann theorem, the change of magnetic energy in the BS is given by \(E_B^{\text{mag}} = J \frac{\partial E_B}{\partial B}\) and hence can be directly estimated from the local slope of the function \(E_B(J)\) using the simple construction shown in Fig. 4. Hence, the difference \(E_B^{\text{kin}} = E_B - E_B^{\text{mag}} < 0\) (\(\propto -J^2/t\) when \(J/t \rightarrow 0\)) should be of kinetic origin. It is remarkable that, at small intermediate \(J/t\) couplings (typically \(\sim 0.2 - 0.6\) for 32 sites), the stability of the BS benefits only from a large gain of kinetic energy while an overall magnetic energy is lost in the pairing process. In that regime, the frustrating AF magnetic coupling is essential to stabilize the plaquette VBS structure. Therefore, one expects that such a behavior appears for even smaller \(J/t\) as singlet plaquette correlations build up for larger clusters. Then, one cannot completely exclude the fact that the \(J = 0\) limit becomes singular.

To investigate further the structure of the two hole BS wavefunctions it is of interest to compute the static hole-hole correlation \(C_{hh}(r)\), i.e. the probability to find the two holes at a given separation \(r\). Results are shown in Fig. 5a as a function of distance. The data for \(t < 0\) show a short-distance “depression” in the hole-hole correlations and a higher probability for the holes to sit at distances \(\simeq 3\) or 4. As expected, the size of the pair wavefunction tends to grow as \(J/|t|\) is reduced (similarly to the case of the square lattice \[14\]) as seen also from the behavior of the average hole-hole separation \(R_{hh} = \sqrt{\langle r^2 \rangle}\) plotted w.r.t \(J/|t|\) in Fig. 5b. Below a critical \(J/|t|\) value, the d-wave BS might then be unstable. In contrast, for \(t > 0\), the hole-hole correlations in the s-wave BS are rather constant with distance showing no tendency for the two holes to repel each other (even for small \(J/|t|\) values). The fact that a strong signature of an effective “attraction” in the quantities plotted in Figs. 5a & (b) is missing is probably directly connected to the origin of the pairing which is of kinetic nature and, hence, should naturally lead to larger pair sizes. Incidentally, one can also expect that such pairing states are likely to be less sensitive to extended Coulomb repulsion.

From the numerical data presented above a simple scenario might be drawn for \(t > 0\). Although its coherent motion is very much suppressed, a single hole bears a large incoherent motion (see e.g. the large low-energy incoherent weight in Figs. 5a-b)) and, hence, can melt the plaquette VBS in its vicinity. This region, which might be fairly extended in space, becomes more favorable for a second hole to gain kinetic energy leading to correlated (or assisted) hopping. It is interesting to point out similarities with the case of some frustrated tight binding lattices showing both (i) single particle states localised in so-called Aharonov-Bohm cages \[17\] and (ii) interaction-induced delocalized two-particle BS \[18\].

In this Letter, I give evidence for two very different behaviors regarding hole pairing in the t-J model depending whether hole motion is frustrated or not, i.e. depending on the sign of the hole hopping on a frustrated lattice such as the checkerboard lattice. For \(t < 0\) and sufficiently large \(J/|t|\) values pairing originates from a local magnetic effective attraction analogous to the case of the non-frustrated square lattice. On the contrary, for \(t > 0\), pairing occurs at arbitrary small \(J/t\). The stability of the pair BS always benefits from a large gain of kinetic energy which might even overcome lost of magnetic energy in some regime. In this case, the binding energy...
FIG. 6: Schematic correlated pair hopping process in the checkerboard AF magnet as indicated by red arrows. The plaquette units of the VBS are shown in green. Note that the pair wavefunction extends on several units (see text).

reaches a few tenths of $t$ so that high superconducting critical temperature would be expected. Interestingly enough, pairing occurs in several pairing channels with different orbital symmetries. This mechanism bears similarities with the inter-layer mechanism proposed for the high temperature cuprate superconductors \[1\]. Note also that, at finite hole density, a paired state with an enhanced kinetic energy gain is very likely to be lower in energy than a phase separated state stabilized only by a magnetic energy gain. Lastly, I notice that triangular and checkerboard lattices behave in a very similar way for $J/t \ll 1$.

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