Automatic inference of fault tree models via multi-objective evolutionary algorithms

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Abstract—Fault tree analysis is a well-known technique in reliability engineering and risk assessment, which supports decision-making processes and the management of complex systems. Traditionally, fault tree (FT) models are built manually together with domain experts, considered a time-consuming process prone to human errors. With Industry 4.0, there is an increasing availability of inspection and monitoring data, making techniques that enable knowledge extraction from large data sets relevant. Thus, our goal with this work is to propose a data-driven approach to infer efficient FT structures that achieve a complete representation of the failure mechanisms contained in the failure data set without human intervention. Our algorithm, the FT-MOEA, based on multi-objective evolutionary algorithms, enables the simultaneous optimization of different relevant metrics such as the FT size, the error computed based on the failure data set and the Minimal Cut Sets. Our results show that, for six case studies from the literature, our approach successfully achieved automatic, efficient, and consistent inference of the associated FT models. We also present the results of a parametric analysis that tests our algorithm for different relevant conditions that influence its performance, as well as an overview of the data-driven methods used to automatically infer FT models.

Index Terms—Fault tree analysis, evolutionary algorithms, multi-objective optimization, complex systems, model learning, parametric analysis.

LIST OF ABBREVIATIONS

FTA: Fault Tree Analysis
FT: Fault Tree
MCS: Minimal Cut Set
BE: Basic Event
TE: Top Event
EA: Evolutionary Algorithm
MOEA: Multi-Objective Evolutionary Algorithm
m.o.f: Multi objective function

1 INTRODUCTION

Fault Tree Analysis (FTA) is a widely used method in reliability engineering and risk analysis, mainly because it enables modeling complex systems by encoding and displaying logical relationships that can be used, among others, to understand how a system might fail, trace the root cause of the failure, identify critical components, and calculate the system and subsystem failure probabilities.

Fault tree (FT) models exist since the 1960s and have been used in a wide range of domains, including the automotive, aerospace, and nuclear industries [1]. However, a major drawback of FTs is related to their construction, which is traditionally carried out in conjunction with domain expertise and in a hand-crafted manner, resulting in a tedious and time-consuming task. In the case of complex industrial systems, manual development of these models can lead to incompleteness, inconsistencies, and even errors [2].

The above challenge has been discussed since the 1970s, and it is referred to in the literature as construction [3], synthesis [4], or induction [5] of FTs. In this work, we refer to this as automatic inference of FT models, which in general, is the process that automatically (with limited human intervention) produces an FT model given compatible input information.

This problem shares some similarities with System Identification (SI), where the objective is to identify the mathematical model of a given system [6], although one difference we observe between SI and FTs inference is that for SI it is necessary to pre-define a model structure (e.g., based on laws of physics), which is not...
possible in the case of FTs inference as this is a task of the inference process.

We identify FTs inference challenging because there are many possible FTs for a given failure data set, and finding the best match is not trivial. Existing methods fail as (i) they need too much human intervention to add assumptions e.g., to deal with complex dependencies between components; (ii) they do not scale adequately in real-world applications, especially algorithms that perform exhaustive search have exponential time complexity; (iii) they result in complex FT structures, (iv) it is unknown how reliable they are under noisy data.

We are interested in data-driven approaches, whose challenge is illustrated by the following example: Table 1 shows an toy input failure data set (Section 4.1). Suppose the associated system is composed of the components BE1, BE2 and BE3, where 0 and 1 are used as non-faulty and faulty states, respectively. Top corresponds to the system-level failure.

Thus, for this failure data set, we want to find the FT structure that best encodes the logic that describes the failure propagation in the system. Moreover, we are interested in the FT composed of a minimal amount of elements.

The desired output is presented in Fig. 1, where the inferred FT is composed of basic events BE1, BE2, connected to an AND gate (G1), which together with BE3, is connected with an OR gate to the top event Top.

Here the gates, which connect the basic events and the top event, result from the inference process following the logic described by the failure data set.

And although for this example the solution is rather simple, for larger failure data sets (i.e., with more basic events) the solution is not straightforward.

Our contributions with this paper are that (i) we show it is possible to achieve more consistent and efficient FT structures via multi-objective evolutionary algorithms (MOEAs), which simultaneously optimize various criteria in a multi-dimensional space, (ii) we propose a metric to compare FT structures via Minimal Cut Sets using the RV-coefficient, (iii) we carry out a parametric analysis that explains the performance of the algorithm under different assumptions, (iv) we show that maintaining compact FTs benefits scalability by enabling faster convergence. The implementation and all data are available at zenodo.org/record/5536431.

The remaining part of this paper is organized as follows. Section 2 summarizes the related work. Section 3 formally defines FTA and provides the technical background of MOEAs. Section 4 explains our methodology. Section 5 presents how we apply the NSGA-II (an MOEA) to infer FTs. Section 6 presents the results of a thorough parametric analysis. Section 7 discusses our findings and presents our conclusions.

2 RELATED WORK

Different ways of inferring FTs have been discussed in the past. We identify three main groups namely knowledge-based, model-based, and data-driven. The main differences between these categories are that knowledge-based approaches mainly employ different heuristics for knowledge representation and domain expertise [7]; model-based approaches translate existing system models and/or graphs into FTs, and data-driven approaches have structured databases as the primary source of information, where the goal is to identify causal relationships present in a failure data set with minimal domain expertise and human intervention.

Carpignano & Poucet [8] thoroughly review several knowledge-based approaches. An example of a model-based approach can be found in Mheni et al. [9] where the authors used SysML System Models as a base to obtain FT models. However, a major drawback of model-based approaches is the need for a pre-existing model [10].

As mentioned before, we focus our attention on data-driven approaches, where the applications of machine learning techniques and data analytics fall into this category. In Appendix A, Table 6 (divided into two parts) we summarise and compare relevant literature in data-driven methods for the automatic inference of FT models. Below we briefly discuss these methods.

To the best of our knowledge, the very first attempt to tackle the challenge in a data-driven manner was made by Madden & Nolan [5] with their IFT algorithm, which is based on Quinlan’s ID3 algorithm to induce Decision Trees (DTs) [11]. The authors also continued with this work in the subsequent years [12], [13]. Mukherjee & Chakraborty [14] addressed this challenge via linguistic analysis and domain knowledge to identify the nature of the failure from short plain text descriptions of equipment faults, and from this generate an FT model. Roth et al. [15] propose a method that follows the Structural Complexity Management (StCM) methodology, where their main goal is to derive dependencies that are later on used to infer the Boolean logic operators of the FT models. Inspired by Causal Decision Trees [16], in Nauta et al.

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**TABLE 1: Toy input failure data set.**

| BE1 | BE2 | BE3 | Top |
|-----|-----|-----|-----|
| 1   | 1   | 1   | 1   |
| 1   | 0   | 1   | 1   |
| 0   | 1   | 1   | 1   |
| 1   | 1   | 0   | 1   |
| 0   | 0   | 1   | 1   |
| 0   | 0   | 0   | 0   |

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[17] they proposed an approach based on the Mantel-Haenszel statistical test.

Based on Knowledge Discovery in Database, Waghen & Ouali [18] propose a method for hierarchical causality analysis called Interpretable Logic Tree Analysis (ILTA), that looks for patterns in a data set which are translated into Interpretable Logic Trees. Linard et al. [19] proposed a method that consists of learning a Bayesian Network graph, which is later translated into an FT model. Lazarova-Molnaratal [20] presents DDFTA, an approach based on time series of failure data, binarization techniques, Minimal Cut Sets, and Boolean algebra. This algorithm also manages to infer FT models with VoT gates. In Waghen & Ouali [21] the authors further extended their work to the Multi-level Interpretable Logic Tree Analysis (MLITA), which tackles the problem of multiple cause-and-effect sequences (the latter a limitation of their ILTA algorithm) by incorporating Bayesian probability rules.

The first attempt based on evolutionary algorithms was carried out by Linard et al. [22], here the authors created an algorithm to generate FT models from a labeled binary failure data set using a uni-dimensional cost function based on the accuracy i.e., the proportion of correctly predicted top events by a given FT.

A drawback of the above approach is that it focuses solely on achieving high accuracy without considering the FT structure. The latter has negative implications such as: running the algorithm twice for the same failure data set may yield considerably different FT structures; it is prone to complexity explosion, leading to massive FTs that are difficult to handle; long computational time, and bad convergence of the algorithm.

3 THEORETICAL BACKGROUND

3.1 Fault Tree Analysis

Fault Tree Analysis (FTA) is one of the most famous methods in reliability engineering that supports decisions in the design and maintenance of complex systems. FTA enables qualitative and quantitative analyses. Qualitative analysis is based on the FT structure and aims at finding the critical system components. Here, an important concept refers to the Minimal Cut Sets (MCSs), which are minimal combinations of component failures that lead to a system failure. Small minimal cut sets point to system vulnerabilities.

The quantitative analysis aims at computing various dependability metrics, such as system Reliability; Availability; and the Mean-Time-to-Failure. Estimation of these metrics requires that the FT leaves are equipped with failure probabilities.

A Fault Tree (FT) helps in understanding why a system fails by modeling how low-level failures propagate through the system and lead to the system-level failure. As similarly done by Ruijters and Stoelinga [23], to formally define an FT, we first define GateType = {And, Or} ∪ {VoT(k/N) | k, N ∈ N, k ≤ N}. Then, an FT is a 5-tuple F = ⟨BE, G, T, I, TE⟩ where

- BE is a set of basic events, which may be annotated with a probability of occurrence p_i.
- G is a set of logic gates, with BE ∩ G = ∅.
- E = BE ∪ G for the set of elements.
- T : G → GateTypes is a function that describes the type of each gate.
- I : G → P(E) describes the inputs of each gate. We require I(g) ≠ ∅ and that |I(g)| = N if T(g) = VoT(k/N).

And gate is a tuple ⟨And, I, O⟩ where O outputs true, if every i ∈ I occurs.

Or gate is a tuple ⟨Or, I, O⟩ where O outputs true, if at least one i ∈ I occurs.

VoT(k/N) gate is a tuple ⟨VoT, k, I, O⟩ where O outputs true, if at least k of i ∈ I occur.

TE ∈ E is a unique root called the top event, and it is reachable from all other nodes.

Importantly, the graph formed by (E, I) should be a directed acyclic graph. Fig. 2.(a) and 2.(b) depict respectively the event and gate symbols used to construct the FT model.

Fig. 2.(c) provides an example of an FT. This is the FT of a Container Seal Design adapted from [24]. In this FT, the top event, the sealing function fails, either occurs if a common cause seal failure occurs or if the seals fail independently. For the former, it is necessary that the contamination tape fails, and a basic cause seal failure occurs. For the latter, it is necessary for the metal-to-metal seal, the fused plug, and at least two of the three compression seals to fail.

3.2 Multi-Objective Evolutionary algorithms

Evolutionary algorithms (EAs) are population-based search strategies that use the fundamental principle of natural selection, where the best individuals are more likely to reproduce and, therefore, to pass on to the next generations [25]. When the EA deals with several conflicting objective functions to be simultaneously optimized in a multi-dimensional space [26], we call these Multi-Objective Evolutionary Algorithms (MOEAs). The result of MOEAs is a set of solutions with trade-offs, better known as Pareto-optimal solutions, from which the user can decide on the basis of higher-level qualitative considerations [27].

As a first step to tackle the challenge of automatically inferring FTs from a failure data set by simultaneously optimizing different metrics, we decided to use one of the most popular algorithms in multi-objective optimization called the Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) (Section 3.2.1) and the Crowding-Distance (Section 3.2.2).
3.2.1 Elitist Non-dominated Sorting Genetic Algorithms (NSGA-II)

The Elitist Non-dominated Sorting Genetic Algorithms (NSGA-II) [28] aim at finding multiple Pareto-optimal solutions. NSGA-II uses the elitist principle which is a diversity preserving mechanism that emphasizes the non-dominated solutions [27]. Moreover, the elitist principle guarantees that the quality of the solution does not decrease, by letting the best individual(s) of the current generation pass to the next one.

Non-dominated MOEAs use the concept of dominance, where two solutions are compared to evaluate whether one dominates the other or not. Non-dominated sorting is a crucial step to uncovering elitist efficient solutions in MOEAs, but it is computationally expensive since it involves numerous comparisons [29]. A set of solutions that do not dominate each other is known as a non-dominated front. In Appendix B.1 we provide a detailed explanation of these concepts and exemplify how the NSGA-II is applied in the automatic inference of FTs.

3.2.2 Crowding-Distance

Since it may be the case that the last non-dominated front obtained through the NSGA-II does not fully accommodate the available slots to complete the new population, the Crowding-Distance is used to decide which individuals of the last front can pass to the next generation, acting as a mechanism that promotes diversity [30]. In Appendix B.2 we provide details on how the Crowding-Distance is applied in the automatic inference of FTs.

4 METHODOLOGY

Fig. 3 depicts the general methodology we followed in this paper. First, we selected some case studies of existing FTs (Section 6.2), these FTs act as ground truth. Then, we selected some parameters of interest (Section 6.4) to be evaluated in the parametric analysis. We used the Monte Carlo method (Section 6.1) to generate failure data sets (Section 4.1) based on selected case studies (Section 6.2). Then we used our FT-MOEA algorithm (Section 5) to infer the FT based on the provided failure data set. Finally, we compared the ground truth with the inferred FTs and evaluate the experiment (Section 6).

4.1 The failure data set

Our methodology needs the following assumptions for the input failure data set:

- **Labeled**: Our data set consists of combinations of BE and their corresponding TE.
- **Binary**: Both BE and TE are binary. This poses an advantage because we can make use of Boolean operations which makes the algorithm faster. In our case, 0 and 1 are used as non-faulty and faulty states, respectively.
- **Monotonic/consistent**: For a given set of BE, if a single BE changes from state 0 to 1, it is possible that TE changes from state 0 to 1, but it will never change from state 1 to 0.
- **Complete**: The total number of unique combinations of BE in the failure data set equals the space complexity $O(2^w)$, where $w$ corresponds to the number of unique BE for a given FT.
- **Noise-free**: There is no corrupted information contained in the failure data set. In other words, the relation $BE \rightarrow TE$ is always true for a given FT.

Table 2 presents an example of the data set associated with the FT in Fig. 2. To generate this data set, we used the Monte Carlo method described in Section 6.1 for $N = 250,000$ data points and using a failure rate for the BE of $p_i = 0.5$. Ob. refers to the observation associated with a unique combination of values of BE,
Fig. 3: General methodology.

and the associated TE. The columns BE1, BE2, ..., BE7 show the states of the set of BE. TE corresponds to the top event. Finally, the last column refers to the count of each of the observations in the failure data set.

TABLE 2: Example of failure data set associated to the example in Fig. 2.

| Ob. | BE1 | BE2 | BE3 | BE4 | BE5 | BE6 | BE7 | TE | Count |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|
| 1   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 1,968 |
| 2   | 0   | 0   | 0   | 0   | 0   | 1   | 0   | 0   | 2,039 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ...   |
| 24  | 0   | 0   | 1   | 0   | 1   | 1   | 1   | 1   | 1,976 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ...   |
| 128 | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1,947 |

$p \approx 0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.50 \ \ N = 250,000$

5 INFERRING FAULT TREES VIA MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS (FT-MOEA)

The input of the algorithm is composed of the failure data set, described in Section 4.1, and the initial parameters of the MOEA, described in Section 5.1. The output of the algorithm is a string that describes the structure of the inferred FT (based solely on And and Or gates), and metrics of interest that tell about the error between the inferred FT and the failure data set. These metrics are explained in detail in Section 5.5.1. Fig. 4 provides an overview of our approach, consisting of the following five steps:

1) Step 1: Initialize the algorithm by loading the failure data set (Section 4.1) as well as the initial parameters (Section 5.1). An optional step is the extraction of the MCSs from the failure data set (Section 5.2), which is executed only when the objective function considers metrics based on MCSs.

2) Step 2: Initialize the population with the parent fault tree(s) (Section 5.3) on which the genetic operators are applied until the desired population size is reached.

3) Step 3: Apply the genetic operators (Section 5.4) to randomly modify the structure of the FTs. These genetic operators are recursively applied to a population of FTs until reaching at least the desired population size.

4) Step 4: For each FT in the offspring population, compute the metrics to be optimized (Section 5.5.1). Then, using the concept of Pareto efficiency through the Elitist Non-Dominated Sorting Algorithm (NSGA-II) and Crowding-Distance (Section 3.2), determine the population of FTs for the next generation.

5) Step 5: Check whether any of the convergence criteria (Section 5.6) are met. If not, the genetic operators (Step 3) are applied to the new population, and Steps 4 and 5 are applied recursively until at least one of the convergence criteria is met, outputting a Pareto set of inferred FTs where we chose the best individual i.e., the FT with the smallest size, and smallest error(s) within the first Pareto set.

5.1 Step 1 - Initialization

We have the following initial parameters.

- Population size ($ps$): Corresponds to the number of FTs within a generation. Only the best $ps$ FTs can pass to the next generation.

- Selection strategy: For the NSGA-II algorithm we only use the elitist selection strategy.

- Max. generations with unchanged best candidate ($uc$): if after $uc$ number of generations the best individual (i.e., the FT with the smallest size, and smallest error(s) within the best Pareto set) remains unchanged, then we assume the process has converged and is therefore terminated.

- Max. number of generations ($ng$): Terminates the optimization process if the number of generations exceeds $ng$ and none of the other convergence criteria is met.

5.2 Step 1.2 - Extraction of MCSs from the failure data set (optional step)

As mentioned before, MCSs are a minimal combinations of component failures that lead to a system failure. This information is extremely valuable because it encodes the failure modes of the system. By considering this information in our optimization process we are adding additional criteria that can help our
algorithm to find better solutions in a shorter amount of time. However, the user can decide whether to consider MCSs within the optimization process.

One should consider MCSs only if it is guaranteed that the failure data set is noise-free, and if the expected FT is not too complex (see Section 6.4.3 where we address the topic of complexity in FTs). The former because otherwise one cannot be sure whether MCSs are correct, and the latter because computing MCSs is computationally expensive (we will discuss this in Section 5.5.1) and for complex FTs, this could increase the convergence time.

The process of extracting MCSs from a failure data set is done as described by Lazarova-Molnar et al. [20] with the following four steps. Step 1: identify all the combinations of BE that output class 1 (i.e., $\text{TE} = 1$). Step 2: from this sub-set, identify the one with the minimal order (where the order is defined as the number of true BE in an observation) and save this observation as part of the MCSs matrix computed from the failure data set ($M_D$). Step 3: look in the sub-set for other observations that include the previously identified MCS. If any, delete them from the sub-set. Step 4: repeat Steps 2 and 3 until the sub-set is empty.

The obtained $M_D$ can be used as an input argument to compute the accuracy based on the MCSs ($\phi_c$) (see Eq. 3). Table 3 provides an example of $M_D$ for the failure data set described in Table 2.

5.3 Step 2 - parent fault tree(s)

The parent fault tree(s) can be seen as that one (or those ones) from which the offspring population is generated when applying the genetic operators (Section 5.4). Defining the parent FT(s) is important because it determines how far away, from the global optimum, one starts the optimization process. In Linard et al. [22] two parent FTs are used to generate the offspring population, one in which the set of BE are connected to a single Or gate, and the other to a single And gate.

5.4 Step 3 - Genetic operators

The genetic operators are mathematical operations that seek to modify the structure of an FT. We use the seven genetic operators proposed by Linard et al. [22], which also gives the formal definitions of these operators. For completeness, we here provide a short description.

(i) $G$-create: randomly creates an And or Or gate under an existing gate in the set $G$ for a given FT.
(ii) $G$-mutate: randomly selects a gate in the set $G$ and changes its type (i.e., $\text{Or} \rightarrow \text{And}$, or $\text{And} \rightarrow \text{Or}$).
(iii) $G$-delete: from a given FT takes a gate in the set $G$ and deletes it, including its children.
(iv) $BE$-disconnect: from a given FT takes a basic event in the set $BE$ and disconnects it.
(v) $BE$-connect: from a given FT takes a disconnected basic event and randomly places it under a gate in the set $G$.
(vi) $BE$-swap: from a given FT takes a basic event in the set $BE$ and randomly moves it under a different parent gate in the set $G$.
(vii) $Crossover$: randomly chooses two FTs in the offspring population, then an element in the set $E$ of each FT is randomly selected and exchanged.

5.5 Step 4 - Multi-objective function

In Section 5.5.1 we define the metrics to be minimized, and in Section 5.5.2 we describe the different setups of our multi-objective optimization function.

5.5.1 Computation of metrics

We consider three metrics in our multi-objective function, namely the fault tree size ($\phi_s$), the error based on the failure data set ($\phi_d$), and the error based on the MCSs ($\phi_c$).
• **Fault Tree Size (φₕ):** corresponds to the number of elements in the FT i.e., the value of E in the FT (Eq. 1).

\[
φₕ = |E| = |BE| + |G|
\]

Here note that φₕ ≥ 2, as each FT has at least one BE and one G (i.e., the TE).

• **Error based on the failure data set (φₜ):** First we need to compute a one-dimensional vector P with N values, where N is the number of data points. P contains the value of the TE of an FT for a given set BE. For the same set BE, we count with the value of the ground truth top event (TEᵢ), which is in the failure data set (Section 4.1). Thus, φₜ is computed as follows

\[
φₜ = 1 - \frac{\sum_{i=1}^{N} x_i}{N} \left\{ \begin{array}{ll} 
1 & \text{if } P_i = \text{TE}_i \\
0 & \text{otherwise}
\end{array} \right.
\]

(2)

Here φₜ varies in the closed interval [0, 1], where 0 means that an FT manages to perfectly map the given set BE in the failure data set into the corresponding TE in the failure data set.

• **Error based on the MCSs (φₛ):** we propose to compute the error of an FT based on MCSs by means of the RV-coefficient [31]. The RV-coefficient is a generalization of the squared Pearson correlation coefficient and measures the similarity between the MCS matrix computed from the failure data set (Mₗ) (see Section 5.2) and the MCS matrix of a given FT (Mₚ).

\[Mₚ \text{ is computed based on the disjunctive normal form (DNF), which in Boolean logic is also known as an Or of Auds. Thus, we transform a given FT into its DNF and identify from it the MCSs to construct } Mₚ. \]

However, this transformation showed to be computationally expensive for large FT sizes.

In our context, Mₗ is a p × w matrix, Mₚ is a q × w matrix, w corresponds to the number of unique BEs considered in the problem and p and q are the number of MCSs in the failure data set in an FT, respectively. The computation of φₛ is defined as

\[
φₛ = 1 - \frac{\text{tr}(MₗMₚᵀ)}{\sqrt{\text{tr}(MₗᵀMₚ) \text{tr}(MₚᵀMₚ)}}
\]

(3)

Here tr(·) is the trace, and φₛ varies in the closed interval [0, 1], where 0 indicates perfect correlation or similarity between Mₗ and Mₚ. We choose to use the RV-coefficient as a means to compute the error based on MCSs because, for a given problem, the number of unique BEs always remains the same, but the FTs within a population for a given generation often have different numbers of MCSs with respect to the ones found in the failure data set (i.e., p ≠ q).

5.5.2 **Setups of the multi-objective functions**

Since our multi-objective function (m.o.f.) has three arguments, we can play with different setups and assess their influence (see Section 6.4.2 for the results of the parametric analysis). For example, if we want to minimize only the error based on the MCSs (φₛ), we can “turn off” φₜ and φₜ by assigning constant values (i.e., φₛ = φₜ = 1). To differentiate between the different configurations in the m.o.f., we propose the nomenclature presented in Table 4. Here ‘x’ refers to whether a metric is being considered (or active) in the m.o.f.

### TABLE 4: Different setups of the m.o.f.

| m.o.f. | φₛ | φₜ | φₜ |
|--------|----|----|----|
| sdc    | x  | x  | x  |
| dc     | x  | x  |   |
| sc     | x  | x  |   |
| sd     | x  | x  |   |
| c      | x  | x  |   |
| d      | x  |   |   |

5.6 **Step 5 - Convergence criterion**

Our convergence criterion is based on two initial parameters namely the max. number of generations (ng), and the max. generations with unchanged best candidate (see Section 5.1). Additionally, we terminate the convergence process if φₛ = 0 or φₜ = 0 when the minimization of the FT size is “turned off” i.e., for the m.o.f.’s cd, c, or d.

6 **Experimental evaluation**

For our experimental evaluation, we first selected six case studies from the literature (Section 6.2), then we made the implementation of our FT-MOEA algorithm in Python, whose source code and data are available at zenodo.org/record/5536431. We evaluate our algorithm using synthetic failure data sets (Section 6.1). Section 6.3 compares FT-MOEA with FT-EA and shows details on convergence. Our parametric analysis is presented in Section 6.4.

6.1 **The Monte Carlo method**

We use the Monte Carlo method to generate synthetic failure data sets using the case studies presented in Section 6.2 and keeping the same properties of the input data set described in Section 4.1. To generate the synthetic dataset (i) we randomly generate (N) data points, by drawing the BE independently from a binomial distribution (i) with a probability of success equal to pᵢ, where i corresponds to a basic event,
TABLE 5: Case studies and associated relevant information. The number of: unique BEs ($w$), total BEs ($W$), And gates (#And), Or gates (#Or), VoT gates (#VoT), Minimal Cut Sets (#MCS), order of MCSs (O-MCSs), and space complexity ($O(2^w)$).

| Case study | Ref. | $w$ | $W$ | #And | #Or | #VoT | $FT_{size}$ | #MCSs | O-MCSs | $O(2^w)$ |
|------------|------|-----|-----|------|-----|------|-------------|-------|--------|----------|
| CSD$^{(a)}$ | [24] | 6   | 6   | 2    | 2   | 0    | 10          | 3     | {2,3,3} | 64       |
| PT$^{(b)}$  | [24] | 6   | 6   | 1    | 4   | 0    | 11          | 5     | {1,1,2,2,2} | 64     |
| COVID-19$^{(c)}$ | [32] | 9   | 21  | 9    | 3   | 0    | 33          | 6     | {3,4,4,4,4,4} | 512    |
| ddFT$^{(d)}$ | [20] | 8   | 8   | 3    | 1   | 1    | 13          | 6     | {3,4,4,5,6,6} | 256    |
| MPPS$^{(e)}$ | [24] | 8   | 12  | 3    | 8   | 0    | 23          | 7     | {2,2,2,2,2,2,2} | 256    |
| SMS$^{(f)}$ | [33] | 13  | 17  | 0    | 8   | 0    | 25          | 13    | {1,1,1,1,1,1,1,1,1,1,1,1} | 8,192 |

$^{(a)}$ CSD: Container Seal Design; $^{(b)}$ PT: Pressure Tank; $^{(c)}$ COVID-19: COVID-19 infection risk; $^{(d)}$ ddFT: Data-driven Fault Tree; $^{(e)}$ MPPS: Mono-propellant propulsion system; $^{(f)}$ SMS: Spread Monitoring System.

and (ii) computing the corresponding TE by following the logical rules of the given FT (e.g., case studies in Section 6.2). As one of the main requirements for the failure data set is to be complete (see Section 4.1) we satisfied this condition for each case study by drawing enough data points from the Monte Carlo simulation ensuring that the number of unique observations of BE equals the space complexity $O(2^w)$, where $w$ corresponds to the number of unique BE for a given FT.

### 6.2 Case studies

In order to establish a sensible ground truth, we used existing FTs in the literature with different applications. Our selection criteria were the number of cases we used: with different applications. Table 5 presents the case studies we selected together information on their number of cases, with different applications.

In order to establish a sensible ground truth, we used existing FTs in the literature with different applications. Our selection criteria were the number of cases we used: with different applications. Table 5 presents the case studies we selected together information on their number of cases, with different applications.

Fig. 5: Evolution of metrics over generations. In (a) using the m.o.f. $d$, in (b) using the m.o.f. $sdc$, for the MPPS case study ($ps = 400$, $ng = 100$, $uc = 20$). The red dashed circle with the arrow at the bottom of (b) indicates the global optimum.

In Table 5, we include the number of unique BEs ($w$), total BEs ($W$), And gates (#And), Or gates (#Or), VoT gates (#VoT), Minimal Cut Sets (#MCS), order of MCSs (O-MCSs), and space complexity ($O(2^w)$).

6.3 Key findings of the FT-MOEA algorithm

To illustrate our findings and main contributions, we will use the case study Mono-propellant propulsion system (MPPS) from Table 5. We first generate the failure data set as described in Section 6.1, with $N = 250,000$ data points. Then, we use this failure data set as part of the input data of the FT-MOEA algorithm, together with the following initial parameters: $ps = 400$, $ng = 100$, and $uc = 20$.

We first compare the evolutionary process between generations for two m.o.f.’s $d$ and $sdc$. In this way we can respectively compare the approach by Linard et al. [22] (FT-EA, only minimizing $\phi_d$) and our multi-objective optimization process (FT-MOEA, minimizing $\phi_s$, $\phi_d$, and $\phi_o$).

In Fig. 6, we compare both m.o.f.’s across generations using only the metrics of the best FT per generation (i.e., that one in the first Pareto front that has the smallest error $\phi_d$ and $\phi_o$). In Fig. 6.(a) we analyze $\phi_d$ across the generations for both objective functions, we can see that the m.o.f. $d$ more rapidly minimize $\phi_d$ compared to the m.o.f. $sdc$. However, the latter m.o.f. managed to achieve the global optimum in the 20th generation, whereas the former m.o.f. did...
not find the global optimum. Fig. 6(b) compares \( \phi_c \), with pretty similar results.

Fig. 6(c) depicts the variation of \( \phi_s \) over the generations. One can observe that our m.o.f. keeps smaller FT structures, and although the size of the ground truth FT is 23, the FT-MOEAs found one of \( \phi_s = 14 \), i.e., a compressed version of the original one (for more details see Appendix C, Fig. 11).

Fig. 6(d) depicts the cumulative time to convergence (t) for both m.o.f.’s. We can observe that our algorithm manages to find the optimal solution in about 20 minutes. On the other hand, by just minimizing \( \phi_d \) the process takes about 4 hours without finding the global optimum. In Appendix D we provide details on the convergence of metrics over the generations for the whole population.

### 6.4 Parametric analysis

We consider in our parametric analysis the population size, the multi-objective functions, the FT complexity, and the influence of superfluous variables. Additionally, we evaluate the influence of varying the parent FT in the Appendix E. We decided to explore these parameters to understand their impact on the computational time and convergence.

We generate the failure data set as described in Section 6.1. Since the evolutionary algorithm is a stochastic process, we run our algorithm five times per combination of parameters until convergence, and by using *box charts* in Matlab (e.g., Fig. 7) we depict the groups of numerical data through their quartiles.

#### 6.4.1 Population size

Fig. 7 presents the results of the parametric analysis when varying the population size for the m.o.f.’s \( d \) and \( sdc \). Here we use the MPPS case study (Table 5).

From Fig. 7(a) and 7(b), the m.o.f. \( sdc \) is more consistent when the population size is larger, also both errors (\( \phi_c \) and \( \phi_d \)) tend to decrease with larger population sizes. On the other hand, when using the m.o.f. \( d \), it seems that the errors decrease for larger population sizes, but with less consistency.

Fig. 7(c) shows that the m.o.f. \( sdc \) always retrieves smaller FTs compared with the m.o.f. \( d \), even smaller than the ground truth (i.e., \( \phi_s \leq 23 \)) indicated by the horizontal red line.

Fig. 7(d) shows that larger population sizes exponentially increases the computational time for both m.o.f.’s. However, the m.o.f. \( sdc \) is consistently faster.

#### 6.4.2 Multi-objective functions

We evaluate all Setups of our m.o.f. (Table 4), for this, we use the case studies in Table 5, and keep the input parameters \( ps = 400 \), \( ng = 100 \), \( uc = 20 \) fixed. Fig. 8 presents the results for the case studies COVID-19, MPPS, and ddFT. Fig. 15 (Appendix F) presents the results of the case studies CSD, PT, and SMS.

Fig. 8(a) shows the error based on the failure data set (\( \phi_d \)). We observe different behaviors per objective function. The m.o.f. \( dc \) achieves the exact solution for all the cases, whereas the other m.o.f.’s failed in at least one of the cases to find the global optimum.
Fig. 8: Comparing the performance of m.o.f.’s for all the case studies in Table 5, using (ps = 400, ng = 100, uc = 20). In (a) error based on the failure data set (φd), (b) error based on the MCSs (φc), (c) fault tree size (φs), and (d) convergence time.

Similarly, Fig. 8.(b) shows the error based on MCSs (φc). As expected, we observe that the m.o.f. dc achieves φc = 0.0 for all the cases. Nevertheless, notice that a low φd does not imply an optimal FT, as an example, compare φd and φc for the case MPPS for the m.o.f.’s d and sdc.

We note that both errors φd and φc (especially for the ddFT case study) appear to increase when φs is minimized, (i.e., m.o.f. ssc, sd, and sdc), this is because FT-MOEA may converge to a FT with a slightly larger error, but smaller size.

Fig. 8.(c) shows the sizes of the inferred FTs (φs). Here we can clearly observe the influence of minimizing φs. When accounted, φs for most of the cases is equal or less than the ground truth, indicated with the horizontal lines for the different case studies (see Appendix C for examples and details). When not accounted for, in some cases, φs may be significantly larger than the ground truth.

6.4.3 Fault tree complexity

Fig. 8.(d) depicts the convergence time. We observe that in general, for all the m.o.f.’s, sorting the case studies based on the convergence time from the longest to the shortest, we have ddFT, MPPS, COVID-19, CSD, PT, and SMS. This indicates that there is a relationship between the complexity of the underlying FT model of a failure data set and the time that takes the algorithm to find it.

We believe this complexity is ruled by the amount of MCSs and their orders (see O-MCSs in Table 5). For example, the case study ddFT has six MCSs and with orders between 3 and 6, the FT-MOEA generally took the longest time to converge. In contrast, the SMS case study has 13 MCSs but all have order 1, here the FT-MOEA converged almost immediately. Thus, the greater the number of MCSs and their orders, the longer will take the algorithm to retrieve the global optimum. We believe that further research is needed to better quantify this type of complexity.

6.4.4 Influence of superfluous variables

Real-world data sets may consider a different number of BEs where possibly not all of them contribute to the failure of the system. In other words, regardless of the state of superfluous BEs, they will not have any effect on the TE. We call these superfluous variables (ρ). We assess ρ ranging from 0 to 6 using the MPPS case study with ps = 400, ng = 100, and uc = 20, and the m.o.f.’s sdc and d. The results are presented in Fig. 9.

Fig. 9: Influence of superfluous variables (ρ) on (a) size of the resulting FT (φs), (b) additional or missing number of BEs (± BEs). Using the m.o.f.’s sdc and d, and the MPPS case study (ps = 400, ng = 100, uc = 20).

Fig. 9.(a) presents φs for different values of ρ. Here φs is smaller than the ground truth (indicated with the dashed horizontal red line) when using the m.o.f. sdc. On the other hand, when using the m.o.f. d, φs seems to increase for a larger number of ρ.

Fig. 9.(b) shows for different values of ρ the additional or missing number of BEs (± BEs). Recall that the case study MPPS has 7 unique BEs. Thus, we subtract this number from the unique number of BEs per each inferred FT. As we can observe, the m.o.f. sdc, despite ρ, it always outputted an FT with 7 BEs (i.e., ± BEs = 0), in other words, the superfluous variables were removed in the optimization process. On the other hand, the m.o.f. d shows inconsistency for different values of ρ, and it seems to perform poorly for larger values of ρ.
7 Discussion and Conclusions

We demonstrate that it is possible to infer efficient and interpretable FT structures by implementing multi-objective evolutionary algorithms. However, there are several aspects that still need to be addressed before thinking about real-world applications.

Even though our algorithm fares better than alternative approaches, its scalability will become an issue for FTs with many BEs. A plausible way to overcome this drawback is by means of “guided” multi-objective evolutionary algorithms. This concept consists of helping the optimization process by injecting additional knowledge. One way to address this is by searching out for patterns (e.g., Waghen & Ouali [21]) in the failure data set that enables identifying parts of the FT and then assembling them.

Another way is by guiding the application of the genetic operators. In our current procedure, these are randomly applied, whereas one may be able to increase the chances of finding the global optimum by targeting parts of the FT that most likely need to be modified. The latter might be possible to achieve by implementing Bayesian optimization. A more challenging path is exploring deep learning-based approaches like the one by Cranmer et al. [34] to derive symbolic rules from a Graph Neural Networks.

We found that the inclusion of MCSs in the multi-objective optimization function greatly improves the optimization process. However, the FT-MOEA computes the MCSs based on the disjunctive normal form, which is computationally expensive for large FTs. Moreover, MCSs cannot be computed in noisy data (see some preliminary results regarding the effects of noise in the Appendix G). Thus, alternatives to overcome these issues are worth exploring. Finally, some sparse but equally interesting challenges are enlisted next.

- Having noise-free, balanced and complete failure data sets for complex engineering systems is virtually impossible. Thus, a more thorough evaluation of the performance of our algorithm under incomplete, noisy, and unbalanced failure data sets is necessary.
- Real-world problems often contain symmetries, e.g., when two basic events are known to be fully exchangeable. This property could be used as an advantage, because we expect to reduce the solution space, leading to faster convergence. Thus, research in this direction is needed.
- In order to obtain more compact and efficient FT structures, exploring alternatives that enable the inference of more sophisticated gates (e.g., VoT gates) is necessary.
- We believe that a methodology similar to the one used in this paper could be applied for the inference of other reliability models, such as reliability block diagrams, as well as Boolean circuits.
- System identification techniques may be applicable to the inference of FTs, and research in this direction is needed.
- Further research is needed to better understand and quantify the complexity in the inference of FT models. In addition, guidelines and metrics to properly identify the practical capabilities of FT inference algorithms remain an open problem.

Our novel algorithm, the FT-MOEA, has in general a better performance than its predecessor, the FT-EA, by converging faster, inferring more compact FT structures, achieving lower error levels, better removing superfluous variables, and being consistent.

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References

[1] S. Kabir, “An overview of fault tree analysis and its application in model based dependability analysis,” Expert Systems with Applications, vol. 77, pp. 114–135, 2017.
[2] J.-P. Signoret, A. Leroy et al., “Automated fault tree building,” Springer Series in Reliability Engineering, pp. 423–426, 2021.
[3] S. L. Salem, G. Apostolakis, and D. Okrent, “Computer-oriented approach to fault-tree construction,” California Univ., Tech. Rep., 1976.
[4] A. Hunt, B. Kelly, J. Mullh, F. Lees, and A. Rushton, “The propagation of faults in process plants: 6, overview of, and modelling for, fault tree synthesis,” Reliability Engineering & System Safety, vol. 39, no. 2, pp. 173–194, 1993.
[5] M. G. Madden and P. J. Nolan, “Generation of fault trees from simulated incipient fault case data,” WIT Transactions on Information and Communication Technologies, vol. 6, 1994.
[6] T. Johnson and P. Husbands, “System identification using genetic algorithms,” in International Conference on Parallel Problem Solving from Nature. Springer, 1990, pp. 85–89.
[7] G. Latif-Shabgahi, “Comparing selected knowledge-based fault tree construction tools,” in Proc. IASTED Int. Conf. Intell. Syst. Control, 2002.
[8] A. Carpinigno and A. Pouget, “Computer assisted fault tree construction: a review of methods and concerns,” Reliability Engineering & System Safety, vol. 44, no. 3, pp. 265–278, 1994.
[9] F. Mheni, N. Nguyen, and J.-Y. Cholley, “Automatic fault tree generation from sysml system models,” in 2014 IEEE/ASME International Conference on Advanced Intelligent Mechatronics. IEEE, 2014, pp. 715–720.
[10] C. E. Dickerson, R. Roslan, and S. Li, “A formal transformation method for automated fault tree generation from a uml activity model,” IEEE Transactions on Reliability, vol. 67, no. 3, pp. 1219–1236, 2018.
[11] J. R. Quinlan, “Induction of decision trees,” Machine learning, vol. 1, no. 1, pp. 81–106, 1986.
[12] M. G. Madden, “Hierarchically structured inductive learning for fault diagnosis,” WIT Transactions on Information and Communication Technologies, vol. 20, 1998.
[13] M. G. Madden and P. J. Nolan, “Monitoring and diagnosis of multiple incipient faults using fault tree induction,” IEEE Proceedings-Control Theory and Applications, vol. 146, no. 2, pp. 204–212, 1999.
[14] S. Mukherjee and A. Chakraborty, “Automated fault tree generation: bridging reliability with text mining,” in 2007 Annual Reliability and Maintainability Symposium. IEEE, 2007, pp. 83–88.

[15] T. Ravi, M. Wolf, and U. Lindemann, “Integrated matrix-based fault tree generation and evaluation,” Procedia Computer Science, vol. 44, pp. 599–608, 2015.

[16] J. Li, S. Ma, T. Le, L. Liu, and J. Liu, “Causal decision trees,” IEEE Transactions on Knowledge and Data Engineering, vol. 29, no. 2, pp. 257–271, 2016.

[17] K. Waghen and M.-S. Ouali, “Multi-level interpretable logic tree analysis: A data-driven fault tree methodology for causality analysis,” Expert Systems with Applications, vol. 136, pp. 376–391, 2019.

[18] A. Linard, D. Bucur, and M. Stoelinga, “Induction of fault trees through bayesian networks,” 09 2019.

[19] S. Lazarova-Molnar, P. Nilofar, and G. K. Barta, “Data-driven fault tree modeling for reliability assessment of cyber-physical systems,” in Proceedings of the 2020 Winter Simulation Conference, 2020.

[20] K. Waghen and M.-S. Ouali, “Multi-level interpretable logic tree analysis: A data-driven approach for hierarchical causality analysis,” Expert Systems with Applications, vol. 178, p. 115035, 2021.

[21] A. Linard, D. Bucur, and M. Stoelinga, “Fault trees from data: Efficient learning with an evolutionary algorithm,” in International Symposium on Dependable Software Engineering: Theories, Tools, and Applications. Springer, 2019, pp. 19–37.

[22] E. Ruijters and M. Stoelinga, “Fault tree analysis: A survey of the state-of-the-art in modeling, analysis and tools,” Computer science review, vol. 15, pp. 29–62, 2015.

[23] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: Nsga-ii,” IEEE transactions on evolutionary computation, vol. 6, no. 2, pp. 182–197, 2002.

[24] Q. Long, X. Wu, and C. Wu, “Non-dominated sorting methods for multi-objective optimization: Review and numerical comparison,” Journal of Industrial & Management Optimization, vol. 17, no. 2, p. 1001, 2021.

[25] S. Roth, A. A. Hafidi et al., “Impact of selection methods on the diversity of many-objective pareto set approximations,” Procedia computer science, vol. 112, pp. 844–853, 2017.

[26] P. Robert and Y. Escoufier, “A unifying tool for linear multivariate statistical methods: The rq-coefficient,” Journal of the Royal Statistical Society: Series C (Applied Statistics), vol. 25, no. 3, pp. 257–265, 1976.

[27] T. Bakeli, A. A. Hafidi et al., “Covid-19 infection risk management during construction activities: An approach based on fault tree analysis (fta),” Journal of Emergency Management, vol. 18, no. 7, pp. 161–176, 2020.

[28] A. Mentes and I. H. Helvacioglu, “An application of fuzzy fault tree analysis for spread mooring systems,” Ocean Engineering, vol. 38, no. 2-3, pp. 285–294, 2011.

[29] M. Cramer, A. Sanchez-Gonzalez, P. Battaglia, R. Xu, K. Cramer, D. Spergel, and S. Ho, “Discovering symbolic models from deep learning with inductive biases,” arXiv preprint arXiv:2006.11287, 2020.

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