A RIGIDITY THEOREM FOR HOLOMORPHIC GENERATORS
ON THE HILBERT BALL

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Abstract. We present a rigidity property of holomorphic generators on the
open unit ball $B$ of a Hilbert space $H$. Namely, if $f \in \text{Hol}(B, H)$ is the generator
of a one-parameter continuous semigroup $\{F_t\}_{t \geq 0}$ on $B$ such that for some
boundary point $\tau \in \partial B$, the admissible limit $K\text{-lim}_{z \to \tau} f(z) = 0$, then $f$
vanishes identically on $B$.

Let $H$ be a complex Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and induced norm $\|\cdot\|$. If $H$ is finite dimensional, we will identify $H$ with $\mathbb{C}^n$. We denote by $\text{Hol}(D, E)$ the
set of all holomorphic mappings on a domain $D \subset H$ which map $D$ into a subset
$E$ of $H$, and put $\text{Hol}(D) := \text{Hol}(D, D)$.

We are concerned with the problem of finding conditions for a mapping $F \in \text{Hol}(D, E)$ to coincide identically with a given holomorphic mapping on $D$ when
they behave similarly in a neighborhood of a boundary point $\tau \in \partial D$.

A number of basic results in this direction are due to D. M. Burns and
S. G. Krantz [6]. They establish conditions at a boundary point for a holomor-
phic self-mapping $F$ of the open unit disk $\Delta := \{z \in \mathbb{C} : |z| < 1\}$ to coincide with the identity mapping (see Proposition 1 below). Then they generalize this fact
to the $n$-dimensional case: for holomorphic self-mappings of the open unit ball
(see Proposition 3 below) and of strongly pseudoconvex domains in $\mathbb{C}^n$. Further
developments of this theme are presented by X. J. Huang in [15], where he ob-
tains similar results for weakly pseudoconvex domains. More recently, L. Baracco,
D. Zaitsev and G. Zampieri [3] have proved local boundary rigidity theorems for
mappings defined only on one side as germs at a boundary point, and extended
their results from boundaries of domains to submanifolds of higher codimension.
More higher-dimensional results can be found, for instance, in [2] and [11].

In this paper we present a rigidity theorem for holomorphic generators on the
open unit ball $B$ of a Hilbert space $H$ which generalizes the analogous theorem
for the one-dimensional case [8, 17, 7] and properly contains the above-mentioned
Burns–Krantz theorem for the open unit ball in $\mathbb{C}^n$.

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We begin by recalling the result of D. M. Burns and S. G. Krantz [6] for holomorphic self-mappings of the open unit disk $\Delta$.

**Proposition 1.** Let $F \in \text{Hol}(\Delta)$. If the unrestricted limit

$$\lim_{z \to \tau} \frac{F(z) - z}{(z - \tau)^3} = 0$$

for some $\tau \in \partial \Delta$, then $F \equiv I$ on $\Delta$.

This assertion also holds when the unrestricted limit is replaced with the angular one (see [22] and [5]). Recall that a function $f \in \text{Hol}(\Delta, \mathbb{C})$ has an angular limit $L := \angle \lim_{z \to \tau} f(z)$ at a point $\tau \in \partial \Delta$ if $f(z) \to L$ as $z \to \tau$ in each nontangential approach region

$$\Gamma_k(\tau) := \left\{ z \in \Delta : \frac{|z - \tau|}{1 - |z|} < k \right\}, \quad k > 1.$$  

In this case it is convenient to set $f(\tau) := \angle \lim_{z \to \tau} f(z)$. Moreover, in a similar way, one defines the angular derivative of $f$ at $\tau \in \partial \Delta$ by $f'(\tau) := \angle \lim_{z \to \tau} \frac{f(z) - f(\tau)}{z - \tau}$.

A point $\tau \in \bar{\Delta}$ is a fixed point of $F \in \text{Hol}(\Delta)$ if either $F(\tau) = \tau$, where $\tau \in \Delta$, or $\lim_{r \to 1} F(r\tau) = \tau$, where $\tau \in \partial \Delta = \{ z : |z| = 1 \}$. If $F$ is not an automorphism of $\Delta$ with an interior fixed point, then by the classical Schwarz–Pick lemma and the Julia–Wolff–Carathéodory theorem, there is a unique fixed point $\tau \in \bar{\Delta}$ such that for each $z \in \Delta$, $\lim_{n \to \infty} F_n(z) = \tau$, where the n-th iteration $F_n$ of $F$ is defined by $F_1 = F, F_n = F \circ F_{n-1}, n = 2, 3, \ldots$. This point is called the Denjoy–Wolff point of $F$. Moreover, a boundary fixed point $\tau \in \partial \Delta$ of $F$ is its Denjoy–Wolff point if and only if $F'(\tau) \in (0, 1]$.

A rigidity result for generators of one-parameter continuous semigroups on $\Delta$ (see Proposition 2 below) has been proved in [8] and [17]. To formulate it, we first recall the definitions of these notions.

Let $D \subset H$ be a domain in the Hilbert space $H$. We say that a family $S = \{ F_t \}_{t \geq 0} \subset \text{Hol}(D)$ is a **one-parameter continuous semigroup on $D$** (a semigroup, for short) if

(i) $F_t(F_s(z)) = F_{t+s}(z)$ for all $t, s \geq 0$ and all $z \in D$,

and

(ii) $\lim_{t \to 0^+} F_t(z) = z$ for all $z \in D$.

A semigroup $S = \{ F_t \}_{t \geq 0} \subset \text{Hol}(D)$ is said to be generated if for each $z \in D$, there exists the strong limit

$$\lim_{t \to 0^+} \frac{1}{t} [z - F_t(z)] = f(z).$$

In this case the mapping $f : D \to H$ is called the **(infinitesimal) generator** of $S$.

A well-known representation of generators on $\Delta$ is due to E. Berkson and H. Porta [4], namely:

A function $f \in \text{Hol}(\Delta, \mathbb{C})$ is a generator if and only if there is a point $\tau \in \bar{\Delta}$ and a function $p \in \text{Hol}(\Delta, \mathbb{C})$ with $\text{Re} \, p(z) \geq 0$ for all $z \in \Delta$ such that

$$f(z) = (z - \tau)(1 - \tau z)p(z), \quad z \in \Delta.$$  

This point $\tau$ is the common Denjoy–Wolff point of the semigroup generated by $f$.

The following rigidity result for generators has been proved in [8] and [17].
Proposition 2. Let \( g \in \text{Hol}(\Delta, \mathbb{C}) \) be the generator of a one-parameter continuous semigroup. Suppose that
\[
\angle \lim_{z \to 1} \frac{g(z)}{|z - 1|^3} = 0.
\]
Then \( g \equiv 0 \) in \( \Delta \).

Here we take this opportunity to present a completely different proof of this assertion.

Proof. Suppose that \( g \) does not vanish identically on \( \Delta \). Condition (2) implies that \( \tau = 1 \) is the Denjoy–Wolff point of the semigroup generated by \( g \) (see Lemma 3 in [10]). So, \( g \) has no null point in \( \Delta \) (see Theorem 1 in [10]). Consequently, \( g \) can be represented by the Berkson–Porta formula
\[
g(z) = -(1 - z)^2 p(z), \quad z \in \Delta,
\]
where \( p \) is a holomorphic function of nonnegative real part which does not vanish in \( \Delta \).

Consider the function
\[
g_1(z) := -\frac{z}{(1 - z)^2} \cdot g(z) = z p(z), \quad z \in \Delta.
\]
This function is the holomorphic generator of a semigroup on \( \Delta \) with its Denjoy–Wolff point at zero.

However, the equality
\[
\angle \lim_{z \to 1} \frac{g_1(z)}{z - 1} = \angle \lim_{z \to 1} \frac{-z}{(1 - z)^3} \cdot g(z) = 0
\]
implies that \( g_1(1) = 0 \) and \( g_1'(1) = 0 \). Therefore \( \tau = 1 \), too, is the Denjoy–Wolff point of the semigroup generated by \( g_1 \) (again by Lemma 3 in [10]). The contradiction we have reached proves that \( g \equiv 0 \) on \( \Delta \). \( \square \)

As we have already mentioned above, D. M. Burns and S. G. Krantz generalize their one-dimensional result for holomorphic self-mappings of \( \Delta \) (Proposition 1) to the open unit ball \( B := \{ x \in \mathbb{C}^n : \| x \| < 1 \} \), where \( \| x \| = \sqrt{|x_1|^2 + |x_2|^2 + \ldots + |x_n|^2} \).

Proposition 3 (see [6]). Let \( B \subset \mathbb{C}^n \) be the open unit ball. Let \( \Phi : B \to B \) be a holomorphic mapping of the ball to itself such that
\[
\Phi(x) = 1 + (x - 1) + O (\| x - 1 \|^{4})
\]
as \( x \to 1 \). (Here \( 1 \) denotes the distinguished boundary point \( 1 = (1, 0, \ldots, 0) \) of the ball.) Then \( \Phi(x) = x \) on the ball.

At this juncture, a natural question arises: does the rigidity result for generators (Proposition 2) admit an analogous generalization to the open unit balls of either \( \mathbb{C}^n \) or a Hilbert space \( H \)? The following theorem gives an affirmative answer to this question. Moreover, we show that it is sufficient to consider the \( K \)-limit instead of the unrestricted one in the assumption of the theorem.

Let \( B \) be the open unit ball of the Hilbert space \( H \). For \( \alpha > 1 \), we denote by
\[
D_\alpha(\tau) := \left\{ x \in B : |1 - \langle x, \tau \rangle| < \frac{\alpha}{2} (1 - \| x \|^2) \right\}
\]
To this end, we fix a point $y$ where the Korányi approach regions at $M$. ELIN, M. LEVENSHTEIN, S. REICH, AND D. SHOIKHET

$$g(x) = \frac{g(x)}{c}$$

Now we define a holomorphic function

$$f(x) = \frac{g(x)}{c}$$

This function is a holomorphic generator on $\Delta$. To see this, note that by the

**Theorem.** Let $f \in \text{Hol}(\mathbb{B}, H)$ be the generator of a one-parameter continuous semigroup on $\mathbb{B}$. If for some $\tau \in \partial \mathbb{B}$, the $K$-limit

$$\lim_{x \to \tau} \frac{f(x)}{\|x - \tau\|^3} = 0,$$

then $f \equiv 0$ on $\mathbb{B}$.

**Proof.** We prove this assertion by reduction to the one-dimensional case. Namely, we consider the restriction of the orthogonal projection of an appropriate modification of the generator $f$ to a one-dimensional disk touching $\mathbb{B}$ at the point $\tau \in \partial \mathbb{B}$.

To this end, we fix a point $y \in \mathbb{B}$ and define the mapping

$$M_y(x) := \frac{y - P_y x - sQ_y x}{1 - \langle x, y \rangle}, \quad x \in \mathbb{B},$$

where $P_y$ is the orthogonal projection of $H$ onto the subspace generated by $y$ ($P_0 \equiv 0$ and $P_y x = \langle x, y \rangle y$ for $y \neq 0$), $Q_y = I - P_y$ and $s = \sqrt{1 - \|y\|^2}$. This mapping is an automorphism of $\mathbb{B}$ satisfying $M_y^{-1} = M_y$ (cf. p. 98 in [12] and p. 25 in [20]).

Denote by $U_y$ a unitary operator on $\mathbb{B}$ such that $U_y \tau = M_y \tau$. Then the mapping $m := M_y \circ U_y$ is an automorphism of $\mathbb{B}$ which satisfies $m(\tau) = \tau$ and $m(0) = y$.

Obviously, $m$ is a biholomorphism of $\mathbb{B}$ onto $\mathbb{B}$. Therefore, by Lemma 3.7.1 on p. 30 of [9], the mapping

$$f_m(w) = \left[ m'(w) \right]^{-1} f(m(w)), \quad w \in \mathbb{B},$$

is also a holomorphic generator on $\mathbb{B}$.

Substituting

$$[m'(w)]^{-1} = [m^{-1}(x)]'_x = U_y^* M_y'(m(w))$$

in (4), we have

$$f_m(w) = U_y^* M_y'(m(w)) f(m(w)), \quad w \in \mathbb{B}.$$
We claim that under our assumptions, \( g \equiv 0 \) on \( \Delta \). Indeed,

\[
g(z) = (U_y^* M_y'(m(z\tau)) f(m(z\tau)), \tau) = \langle M_y'(m(z\tau)) f(m(z\tau)), U_y \tau \rangle
\]
and, consequently,

\[
g(z) = \frac{1}{|z - 1|^3} \left\langle f(m(z\tau)), [M_y'(m(z\tau))]^* U_y \tau \right\rangle
\]

Note that each automorphism \( h \) of \( \mathbb{B} \) is the restriction to \( \mathbb{B} \) of a holomorphic mapping defined either on the larger ball \( B(0, R) \) centered at zero of radius \( R = \frac{1}{|y|} \) if \( h(0) \neq 0 \) or on all of \( H \) if \( h \) fixes the origin. So, \( M_y \) and \( m \) are, in fact, holomorphic mappings defined either on the open ball \( B(0, R) \) of radius \( R = \frac{1}{|y|} > 1 \) if \( y \neq 0 \) or on \( H \) if \( y = 0 \). Hence the first factor on the right-hand side of equality (8) has a finite limit as \( z \to 1 \), and so has the second factor of the inner product.

Now we show that the first factor of the last inner product in (8) tends to zero as \( z \to 1 \) nontangentially in \( \Delta \).

For \( z \) close enough to 1 in the nontangential approach region

\[
\Gamma_k = \left\{ z \in \Delta : \frac{|z - 1|}{1 - |z|} < k \right\}, \quad k > 1,
\]

\( m(z\tau) \) belongs to the Korányi region \( D_\alpha(\tau) \) whenever \( \alpha > k \). Indeed, it can be shown by direct calculations that the function \( m \) satisfies the equality

\[
\frac{1 - \langle m(z\tau), \tau \rangle}{1 - |m(z\tau)|^2} = L \frac{|1 - z|^2}{1 - |z|^2}, \quad z \in \Delta,
\]

where

\[
L := \left. \frac{d}{dz} \langle m(z\tau), \tau \rangle \right|_{z=1} = \frac{1 - \langle y, \tau \rangle}{1 - \langle U_y \tau, y \rangle} = \frac{|1 - \langle y, \tau \rangle|^2}{1 - \|y\|^2} > 0.
\]

Consequently, we have for \( z \in \Gamma_k \),

\[
\frac{1 - \langle m(z\tau), \tau \rangle}{1 - |m(z\tau)|^2} = L \frac{|1 - z|^2}{1 - |z|^2} \left. \frac{1}{1 - \langle m(z\tau), \tau \rangle} \right| < L k \frac{|1 - z|}{1 - \langle m(z\tau), \tau \rangle}.
\]

Since

\[
\lim_{z \to 1^-} \left. \frac{1 - \langle m(z\tau), \tau \rangle}{1 - |m(z\tau)|^2} \right| = L,
\]

it follows that if \( z \in \Gamma_k \) is close enough to 1, then \( m(z\tau) \) is in \( D_\alpha(\tau) \) \( (\alpha > k) \). Hence, by hypothesis (8) of the theorem,

\[
\angle \lim_{z \to 1^-} \frac{f(m(z\tau))}{\|m(z\tau) - \tau\|^3} = 0.
\]

Therefore equality (8) implies that

\[
\angle \lim_{z \to 1^-} \frac{g(z)}{|z - 1|^3} = 0,
\]
and by Proposition 2, \( g \equiv 0 \) on \( \Delta \). So, by (7),

\[
(f(m(z\tau)), [M_y'(m(z\tau))]^* U_y \tau) = 0 \quad \text{for all} \quad z \in \Delta.
\]

In particular, this equality holds for \( z = 0 \); i.e.,

\[
\langle f(y), [M_y'(y)]^* U_y \tau \rangle = 0 \quad \text{for each} \quad y \in \mathbb{B}.
\]
By direct calculations, one obtains that
\[ M'_y(x)h = \frac{1}{(1 - \langle x, y \rangle)^2} \left[-(1 - \langle x, y \rangle)(P_y + sQ_y)h + \langle h, y \rangle(y - P_yx - sQ_yx) \right]. \]
Hence,
\[ M'_y(y)h = -\frac{1}{1 - \|y\|^2}(P_y + sQ_y)h, \]
and equality (9) is equivalent to
\[ \langle f(y), (P_y + sQ_y)U_y\tau \rangle = 0. \]
Substituting
\[ U_y\tau = M_y\tau = \frac{y - P_y\tau - sQ_y\tau}{1 - \langle \tau, y \rangle} \]
in this equality, we obtain
\[ \langle f(y), y - \tau + \|y\|^2\tau - \langle \tau, y \rangle y \rangle = 0 \text{ for all } y \in \mathbb{B}. \]
Let \( y = y_1\tau + \tilde{y} \), where \( y_1 = \langle y, \tau \rangle \) and \( \langle \tilde{y}, \tau \rangle = 0 \).
Similarly, \( f(y) = f_1(y)\tau + \tilde{f}(y) \) with \( f_1(y) = \langle f(y), \tau \rangle \) and \( \langle \tilde{f}(y), \tau \rangle = 0 \) for all \( y \in \mathbb{B} \).
Using this notation, we have
\[ \langle f_1(y)\tau, y_1\tau - \tau + \|y\|^2\tau - |y_1|^2\tau \rangle = -\langle \tilde{f}(y), \tilde{y} - \tau_1\tilde{y} \rangle \]
and
\[ (1 - \tau_1 - \|\tilde{y}\|^2)f_1(y) = (1 - y_1)(\tilde{f}(y), \tilde{y}). \]
Differentiating this equality with respect to \( \tau_1 \), we conclude that it can hold only if \( f_1(y) = 0 \) and
\[ \langle \tilde{f}(y), \tilde{y} \rangle = 0 \text{ for all } y \in \mathbb{B}. \]
Now let \( \sigma \) be an arbitrary unit vector orthogonal to \( \tau \), i.e., \( \langle \sigma, \tau \rangle = 0 \). Suppose that \( \tilde{y} = y_2\sigma + u \), where \( y_2 = \langle \tilde{y}, \sigma \rangle \) and \( \langle u, \sigma \rangle = 0 \).
Similarly, \( \tilde{f}(y) = f_2(y)\sigma + v(y) \) with \( f_2(y) = \langle \tilde{f}(y), \sigma \rangle \) and \( \langle v(y), \sigma \rangle = 0 \) for all \( y \in \mathbb{B} \). Then by (10),
\[ f_2(y)\tau_2 = -\langle v(y), u \rangle. \]
Differentiating this equality with respect to \( \tau_2 \), we obtain \( f_2(y) = 0 \). Hence, \( f \equiv 0 \) on \( \mathbb{B} \).

Following L. A. Harris [8], we define the numerical range of each \( h \in \text{Hol}(\mathbb{B}, H) \) which has a norm continuous extension to \( \overline{\mathbb{B}} \) by
\[ V(h) := \{ \langle h(x), x \rangle : \|x\| = 1 \}. \]

For an arbitrary holomorphic mapping \( h \in \text{Hol}(\mathbb{B}, H) \) and for each \( s \in (0, 1) \), we define the mapping \( h_s : \frac{1}{s}\mathbb{B} \to H \) by
\[ h_s := h(sx), \quad \|x\| < \frac{1}{s}, \]
and set
\[ L(h) := \lim_{s \to 1^-} \sup \text{Re}(V(h_s)). \]
It is known (Theorem 1 in [14]) that the mapping \( I - h \) is a generator if and only if \( L(h) \leq 1 \). So the following corollary is an immediate consequence of our theorem.

**Corollary.** Let \( h \in \text{Hol}(B, H) \) with \( L(h) \leq 1 \). If for some \( \tau \in \partial B \), the K-limit

\[
K \lim_{x \to \tau} \frac{h(x) - x}{|x - \tau|^3} = 0,
\]

then \( h \equiv I \) on \( B \).

Since obviously \( L(h) \leq 1 \) for all self-mappings of \( B \), this corollary properly contains Proposition 3.

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