Quasilocal energy-momentum for geometric gravity theories

Chiang-Mei Chen, James M. Nester* and Roh-Suan Tung**

Department of Physics, National Central University, Chung-Li, Taiwan 32054

Abstract

From a covariant Hamiltonian formulation, using symplectic ideas, we obtain covariant quasilocal energy-momentum boundary expressions for general gravity theories. The expressions depend upon which variables are fixed on the boundary, a reference configuration and a displacement vector field. We consider applications to Einstein’s theory, black hole thermodynamics and alternate spinor expressions.

Key words: quasilocal energy. variational principle. Hamiltonian formulation. black hole thermodynamics. spinor formulation. symplectic techniques.

1 Introduction

The source of gravity is the energy-momentum density of all other physical fields. For the gravitational field itself, however, energy-momentum is not so simply described. Although geometries with suitable asymptotic regions have a well defined total energy-momentum, the equivalence principle precludes any proper local density. Hence, the idea of quasilocal quantities for gravity has been advocated and there have been many recent proposals. Here we present a set of quasilocal energy-momentum expressions for quite general geometric gravity theories. Distinguishing features of our Hamiltonian based expressions are that they are covariant and that the different possible expressions are associated with the choice of variables to be held fixed on the boundary and a symplectic boundary variational structure.

* Email: nester@joule.phy.ncu.edu.tw
**Email: m792001@joule.phy.ncu.edu.tw
2 The covariant Hamiltonian

We consider general theories of dynamic geometry. The possible geometric potentials are the coframe $\vartheta^\alpha$ and connection $\omega^\alpha{}_{\beta}$ one-forms and the metric coefficients $g_{\mu\nu}$. The corresponding field strengths are the torsion $\Theta^\alpha := D\vartheta^\alpha = d\vartheta^\alpha + \omega^\alpha{}_{\beta} \wedge \vartheta^\beta$ and curvature $\Omega^\alpha{}_{\beta} := d\omega^\alpha{}_{\beta} + \omega^\alpha{}_{\gamma} \wedge \omega^\gamma{}_{\beta}$ 2-forms and the nonmetricity one-form $G_{\mu\nu} := D g_{\mu\nu} = dg_{\mu\nu} - \omega^\alpha{}_{\mu} g_{\alpha\nu} - \omega^\alpha{}_{\nu} g_{\mu\alpha}$. In place of the usual “second order” Lagrangian 4-form $L = L(g, \vartheta, \omega, G, \Theta, \Omega)$, we introduce covariant canonically conjugate momenta forms to obtain a “first order” Lagrangian 4-form

$$\mathcal{L} := G_{\mu\nu} \wedge \pi^{\mu\nu} + \Theta^\alpha \wedge \tau_\alpha + \Omega^\alpha{}_{\beta} \wedge \rho_\alpha{}^{\beta} - \Lambda(g, \vartheta; \pi, \tau, \rho),$$

which yields “first order” field equations via independent variations of the fields $g, \vartheta, \omega$ and momenta $\pi, \tau, \rho$.

For any fixed slicing of space-time by $t$-constant surfaces $\Sigma$ along with a connecting vector field $N$, the decomposition of the first order Lagrangian 4-form according to the general pattern $\mathcal{L} \equiv dt \wedge i_N \mathcal{L} = dt \wedge (\mathcal{L} \varphi \wedge \pi - \mathcal{H}(N))$ identifies the covariant Hamiltonian 3-form (or density) [1], i.e., the generator of evolution along $N$:

$$\mathcal{H}(N) = i_N \Lambda + G_{\mu\nu} \wedge i_N \pi^{\mu\nu} - i_N \vartheta^\alpha D\tau_\alpha - \Theta^\alpha \wedge i_N \tau_\alpha - \Omega^\alpha{}_{\beta} \wedge i_N \rho_\alpha{}^{\beta} - i_N \omega^\alpha{}_{\beta} (D\rho_\alpha{}^{\beta} - g_{\alpha\nu} \pi^{\beta\nu} - g_{\mu\alpha} \pi^{\mu\beta} + \vartheta^\beta \wedge \tau_\alpha) + d\mathcal{B}(N),$$

where $\mathcal{B}(N) := i_N \vartheta^\alpha \tau_\alpha + i_N \omega^\alpha{}_{\beta} \rho_\alpha{}^{\beta}$.

3 Covariant quasilocal expressions

Noether’s theorem for a translation applied to $\mathcal{L}$ yields the differential identity $d\mathcal{H}(N) \equiv (\text{terms } \propto \text{ field equations})$ and the algebraic identity $\mathcal{H}(N) \equiv (\text{terms } \propto \text{ field equations}) + d\mathcal{B}(N)$. Consequently, the Hamiltonian $\int_\Sigma \mathcal{H}(N)$, the integral of the density over a finite spatial region, has a conserved value on a solution given via Stokes theorem by $\oint_{\partial \Sigma} \mathcal{B}(N)$. This boundary integral gives the quasilocal quantities (energy etc.); the limiting value at infinity should be a total conserved quantity for $N$ asymptotically Killing.

However, the value of the Hamiltonian is not yet firmly fixed. As with other Noether currents we can add any exact differential thereby modifying the expression given above for the Hamiltonian boundary term. Moreover, such
an adjustment only affects the boundary term in the variation of the Hamiltonian and thus does not affect the basic physics: the dynamic equations. The Hamiltonian formulation, however, includes a further principle which fixes the possible forms of \( B \). The proper form is identified by considering the variation of the Hamiltonian density:

\[
\delta H(N) = (\text{field eq. terms}) + di_N(\delta g_{\mu\nu} \pi^{\mu\nu} + \delta \vartheta^\alpha \wedge \tau_\alpha + \delta \omega^{\alpha\beta} \wedge \rho_\alpha \beta).
\]

The total differential term in \( \delta H(N) \) produces a boundary integral which reflects both the choice of variables to be held fixed ("control" variables) and the symplectic structure [2]. The boundary term in \( \delta H(N) \) vanishes (as it should) for fixed control variables on a finite boundary. But for a boundary at infinity the limit of the asymptotic falls generally gives a nonvanishing results (e.g., for Einstein’s theory) then the above \( B(N) \) expression needs adjustment [3].

An acceptable boundary expression for general metric compatible gravity theories was found [1] and soon improved by Hecht [4]. From his work we saw more possibilities. The expressions for the Hamiltonian boundary term which are 4-covariant and yield a 4-covariant symplectic structure for the boundary term in \( \delta H(N) \) were identified [5]. There are two types:

\[
B_\varphi(N) := i_N \varphi \wedge \Delta p - \varsigma \Delta \varphi \wedge i_N \hat{p}, \tag{3}
\]

\[
B_p(N) := i_N \hat{\varphi} \wedge \Delta p - \varsigma \Delta \varphi \wedge i_N p, \tag{4}
\]

depending upon whether the configuration field k-form \( \varphi \) or its conjugate momenta \( p \) is "controlled" on the boundary [2] (i.e., either Dirichlet or Neumann boundary conditions). Here \( \hat{\varphi} \) and \( \hat{p} \) are the values in a reference configuration, \( \Delta \varphi = \varphi - \hat{\varphi}, \Delta p = p - \hat{p}\) and \( \varsigma = (-1)^k \). Thus for the geometric fields

\[
\mathcal{B}(N) = \begin{cases} 
- \Delta g_{\mu\nu} i_N \pi^{\mu\nu} \\
- \Delta g_{\mu\nu} i_N \pi^{\mu\nu}
\end{cases} + \begin{cases} 
i_N \vartheta^\alpha \Delta \tau_\alpha + \Delta \vartheta^\alpha \wedge i_N \hat{\tau}_\alpha \\
i_N \hat{\vartheta}^\alpha \Delta \tau_\alpha + \Delta \vartheta^\alpha \wedge i_N \tau_\alpha
\end{cases}
\]

\[
+ \begin{cases} 
i_N \omega^{\alpha\beta} \Delta \rho_\alpha \beta + \Delta \omega^{\alpha\beta} \wedge i_N \hat{\rho}_\alpha \beta \\
i_N \hat{\omega}^{\alpha\beta} \Delta \rho_\alpha \beta + \Delta \omega^{\alpha\beta} \wedge i_N \rho_\alpha \beta
\end{cases}, \tag{5}
\]

where for each bracket the upper (lower) line is to be used if the field (momentum) is controlled. Hence, as in thermodynamics, there are several kinds of “energy”, each corresponds to the work done in a different (ideal) physical process [2]. Our quasilocal boundary expressions are covariant—aside from the manifestly non-covariant explicit connection terms in \( \mathcal{B} \). These terms include a real physical effect plus an unphysical dynamical reference frame effect. These effects can be separated using the identity

\[
(i_N \omega^{\alpha\beta}) \vartheta^\beta \equiv i_N \Theta^\alpha + DN^\alpha - L_N \vartheta^\alpha. \tag{6}
\]
Via this identity the boundary term contains a time derivative of certain frame components. Such terms have been noted previously in Einstein’s theory [6].

The variation of the Hamiltonian, in addition to the field equation terms, now includes for each variable one of the boundary terms

\[ di_N(\delta \varphi \wedge \Delta p), \quad \text{or} \quad di_N(-\Delta \varphi \wedge \delta p), \]  

which reflect the symplectic structure and the control mode. Specifically, for the geometric variables, the total differential term in \( \delta \mathcal{H}(N) \) is of the form \( di_NC \) where

\[
C = \left\{ \begin{array}{l}
\delta g_{\mu\nu} \Delta \pi^{\mu\nu} \\
-\Delta g_{\mu\nu} \delta \pi^{\mu\nu}
\end{array} \right\} + \left\{ \begin{array}{l}
\delta \vartheta^\alpha \wedge \Delta \tau_\alpha \\
-\Delta \vartheta^\alpha \wedge \delta \tau_\alpha
\end{array} \right\} + \left\{ \begin{array}{l}
\delta \omega^\alpha_\beta \wedge \Delta \rho^\beta_\alpha \\
-\Delta \omega^\alpha_\beta \wedge \delta \rho^\beta_\alpha
\end{array} \right\},
\]

here again the upper (lower) line in each bracket corresponds to controlling the field (momentum). Our quasilocal expressions are uniquely determined by the Hamiltonian variation (8) and the requirement that all of the quasilocal quantities vanish when the fields have the reference configuration values.

This general formalism readily specializes to coordinate or orthonormal frames and Riemannian, Riemann-Cartan or teleparallel geometry. For General Relativity and the Poincaré Gauge Theory (asymptotically flat or constant curvature) our expressions reduce to known ones [7,8] and give the total quantities at spatial and future null infinity [9]. Non-vanishing reference configurations (e.g., Minkowski or de Sitter metric, frames, connections) play an essential role in obtaining these total values. Our quasilocal expressions likewise depend on a reference configuration. A reasonable choice for the reference configuration is to embed the spatial surface and its boundary into a Minkowski space [10]; more generally one could use (anti) de Sitter space, a homogeneous cosmology, a Schwarzschild solution, etc. The evolution vector field can be selected to correspond to a Killing field of the reference configuration to obtain the quasilocal energy-momentum (and angular momentum). As in Ref. [10], alternate choices of the evolution vector field can be used to distinguish between quasilocal quantities and conserved charges, mass and energy, etc.

4 Einstein’s theory and a spherically symmetric example

For Einstein’s theory our quasilocal expressions reduce to only two possibilities depending upon the choice of boundary conditions—the frame (i.e., the metric or intrinsic geometry) or the connection (the extrinsic geometry) could be held
fixed on the boundary. These Dirichlet and Neumann quasilocal expressions are

\[
B_\vartheta = i_N \beta_\alpha^\alpha \Delta \epsilon^\alpha_\beta + \Delta \omega^\alpha_\beta \land i_N \epsilon^\alpha_\beta, \\
B_\omega = i_N \beta_\alpha^\alpha \Delta \epsilon^\alpha_\beta + \Delta \omega^\alpha_\beta \land i_N \rho^\alpha_\beta, 
\]

with the corresponding boundary terms in the Hamiltonian variation

\[
\delta H_\vartheta(N) \approx d_i N (\Delta \omega^\alpha_\beta \land \delta \epsilon^\alpha_\beta), \\
\delta H_\omega(N) \approx d_i N (\delta \epsilon^\alpha_\beta \land \Delta \omega^\alpha_\beta), 
\]

respectively, where \( \epsilon^\alpha_\beta = * (\partial^\alpha \land \partial^\beta) \). The expression (9) differs slightly from one due to Katz [11]. It is also similar to the famous expression of Brown and York [10]. Significant differences from the latter are that (i) our expression is covariant, (ii) our expression is not specialized to the timelike boundary being orthogonal to the spacelike hypersurface, (iii) our expression includes interaction terms between the physical system and the reference configuration, and (iv) we have relaxed the seemingly natural choice \( \text{lapse} = 1 \) as it prevents the quasilocal energy from attaining to the total energy in the limit for asymptotically anti-de Sitter solutions. (There is now a new work [12] which also considers quasilocal quantities in asymptotically non-flat spaces.)

Consider the static spherically symmetric metric \( ds^2 = -e^{2\Phi} dt^2 + e^{-2\Phi} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \) where \( e^{2\Phi} = 1 - 2m/r + \lambda r^2 \) and the reference configuration is given by \( m = 0 \), i.e., Minkowski or (anti) de Sitter. For the quasilocal energy within a centered sphere we find \( E_\vartheta = r \alpha (e^{\Phi_0} - e^\Phi) \) and \( E_\omega = r \alpha e^{\Phi - \Phi_0} (e^{\Phi_0} - e^\Phi) \). An appropriate choice of lapse is \( \alpha = e^{\Phi_0} = \sqrt{1 + \lambda r^2} \) which corresponds to the reference configuration timelike Killing vector. For this choice both expressions give \( m \) for the total energy, while the energy within the horizon is \( 2m \) for \( E_\vartheta \) and \( \infty \) for \( E_\omega \). The quasilocal energy exterior to the horizon is negative in both cases. Note that there is no simple relationship between the quasilocal energies of Schwarzschild anti-de Sitter space referenced to Minkowski space, Schwarzschild anti-de Sitter space referenced to anti-de Sitter space and de Sitter space referenced to Minkowski space.

5 Application to black hole thermodynamics

One application of our expressions is to black hole thermodynamics [10,12,13]. For this purpose we want to control the quasilocal energy-momentum so we must allow \( N \) to vary. Hence we Legendre transform to the “microcanonical” Hamiltonian \( H_{\text{micro}}(N) := H(N) - f_{\partial \Sigma} B(N) \). We choose the connection as
one of our control variables, and use (6) but drop the unphysical dynamic reference frame contribution due to $\mathcal{L}_N \bar{\psi}^\alpha$. We take our boundaries at $\infty$ and on the bifurcate Killing horizon. For the evolution vector field $N$ we use the Killing field $\chi := \partial_t + \Omega_H \partial_\phi$ which is normal to and vanishes on the horizon. We obtain the “first law” for a general gravity theory by evaluating “on shell”

$$0 = \delta H_{\text{micro}}(\chi) = \oint_{\partial \Sigma} \delta B(\chi) = \oint_\infty \delta B(\chi) - \oint_H \delta B(\chi) = (\delta E + \Omega_H \delta J) - \oint_H D_\alpha \chi^\beta \delta \rho_\beta^\alpha. \quad (13)$$

The last integral, which for Einstein’s theory has the form $\simeq \kappa \delta A$, identifies the entropy for these general geometric gravity theories.

### 6 Spinor expressions

Some of the proposed quasilocal expressions for Einstein’s theory are formulated in terms of auxiliary spinor fields. Here we indicate the relationship between such expressions and our expressions and briefly consider the application of our formalism to spinor formulations.

Via certain new spinor-curvature identities [14] several new quadratic spinor Lagrangians for Einstein’s theory have been found [15]. The different versions depend on whether the connection is varied independently and how the vanishing torsion constraint is imposed. One of the simplest is

$$\mathcal{L}_{qs} := 2 \{ \overline{D(\psi \vartheta)} \gamma_5 D(\vartheta \psi) \} \equiv - \overline{\psi} \psi R * 1 + d\{ \overline{D(\psi \vartheta)} \gamma_5 \vartheta \psi + \overline{\psi} \vartheta \gamma_5 D(\vartheta \psi) \}. \quad (14)$$

It differs from the usual Einstein-Hilbert action by just a total differential. The variables are a Dirac matrix valued orthonormal frame one-form $\vartheta := \gamma_\alpha \vartheta^\alpha$ and a “normalized” spinor field $\psi$ (i.e., $\overline{\psi} \psi = 1$, $\overline{\psi} \gamma_5 \psi = 0$). Asymptotically $\psi \sim \text{const} + O(1/r)$ so the Lagrangian is $O(1/r^4)$ which guarantees finite action.

The corresponding covariant (we have dropped a term proportional to $i_N \omega$ which generates the frame gauge transformations and vanishes on shell) Hamiltonian 3-form has the form

$$\mathcal{H}_{qs}(N) := 2 \{ D(\overline{\psi} \sigma) \gamma_5 D(\vartheta \psi) + D(\overline{\psi} \vartheta) \gamma_5 D(\sigma \psi) \} \equiv -2 \overline{\psi} \psi N^\mu G_{\mu \nu} * \vartheta^\nu + 2d\{ \overline{\psi} \sigma \gamma_5 D(\vartheta \psi) + D(\overline{\psi} \vartheta) \gamma_5 \sigma \psi \}, \quad (15)$$

$$\equiv -2 \overline{\psi} \psi N^\mu G_{\mu \nu} * \vartheta^\nu + 2d\{ \overline{\psi} \sigma \gamma_5 D(\vartheta \psi) + D(\overline{\psi} \vartheta) \gamma_5 \sigma \psi \}, \quad (16)$$
which is just the ADM Hamiltonian up to a total differential. It is asymptotically \( O(1/r^4) \) and its variation has an \( O(1/r^3) \) boundary term which vanishes asymptotically so there is no need for a further adjustment [3] by an additional boundary term. Expression (15) is similar to the Hamiltonian 3-form associated with the Witten positive energy proof [16]:

\[
H_w(\psi) := 2(D\overline{\psi}\gamma_5 D(\vartheta \psi) + D(\overline{\vartheta \psi})\gamma_5 D\psi)
\]

\[
\equiv -2N^\mu G_{\mu\nu} \vartheta^\nu + 2d(\overline{\vartheta \psi}\gamma_5 D\psi - D\overline{\psi}\gamma_5 \vartheta \psi),
\]

wherein \( N^\mu = \overline{\psi}\gamma^\mu \psi \). Again this is the ADM Hamiltonian up to a total differential but \( H_w \) (unlike \( H_{qs} \)) is not related to a Lagrangian. For these spinor Hamiltonians once again on a solution only the boundary terms contribute to the value.

Note that for these spinor expressions there is no explicit need for a reference configuration, the spinor field \textit{implicitly} plays this role [17]. In order to compare these spinor expressions with the quasilocal expressions discussed earlier we introduce an explicit reference configuration. We then find that both spinor expressions are related to our expression (9):

\[
B_{qs}(N) := 2(\overline{\psi}N\gamma_5 D(\vartheta \psi) + D(\overline{\vartheta \psi})\gamma_5 N\psi)
\]

\[
= \Delta \omega^\alpha_{\beta} i_N \epsilon_\alpha^\beta - 2(\overline{\psi}N\gamma_5 \vartheta \overrightarrow{D}\psi + \vartheta \overrightarrow{D}\overline{\psi}\gamma_5 N\psi),
\]

\[
B_w(\psi) := -2(\psi\gamma_5 \vartheta D\psi + D\overline{\psi}\gamma_5 \vartheta \psi)
\]

\[
= \Delta \omega^\alpha_{\beta} i_N \epsilon_\alpha^\beta - 2(\overline{\psi}\gamma_5 \vartheta \overrightarrow{D}\psi + \overrightarrow{D}\overline{\psi}\gamma_5 \vartheta \psi).
\]

In the limit \( r \to \infty \) these spinor boundary expressions also give the correct total energy-momentum. Expressions like Eq. (21) have been used in several quasilocal energy investigations [18].

Having introduced a reference configuration, instead of just comparing the Hamiltonian boundary expression we can apply our general formalism. The momenta conjugate to the spinor fields can be introduced; then, in addition to the frame and connection type terms we had earlier, the quasilocal expression acquires spinor terms of the sort (3, 4) and the variation of the Hamiltonian contains extra spinor field contributions of the type (7).

7 Discussion

Which quasilocal expression gives the correct physics? The physical role of the spinor field especially still seems mysterious. One way to investigate these various quasilocal expressions is to do more direct calculations for exact solutions,
e.g., [19]. However, a deeper theoretical investigation could be more revealing. Our formulation provides a good starting point for such an investigation. Note that all of the expressions presented here correspond to the work done in some (ideal) physical process. The situation is similar to thermodynamics with its different energies (enthalpy, Gibbs, Helmholtz, etc.) An even better analogy is the electrostatic work required while controlling the potential on the boundary of a region vs. that required while controlling the charge density [2]. Thus for the spinor expressions we simply have a different boundary symplectic structure corresponding to different control variables. What must be held fixed in each case is found by calculating the boundary term in the variation of the Hamiltonian. Mathematically this is straightforward. But no matter which technical procedure is used for the relation between the frame, the connection and the vanishing torsion condition the complete results for the variational symplectic structure turns out to be rather complicated. Briefly, for the spinor expressions the main conclusion is, not surprisingly, that we must hold the orthonormal frame and the spinor field fixed on the boundary. Hence to understand the physics of the quasilocal spinor expressions one must understand the physical meaning of controlling the spinor field on the boundary. For the Witten Hamiltonian the relation $N^\mu = \bar{\psi} \gamma^\mu \psi$ already gives part of the answer.

Thus, for general geometric gravity theories, from a covariant Hamiltonian formulation using differential forms, by always working with four dimensionally covariant variables and by using symplectic ideas, we have found several manifestly covariant Hamiltonian boundary term expressions for the quasilocal quantities: energy-momentum and angular momentum. Our quasilocal expressions depend only on observer independent geometric quantities. They are differential 2-form expressions which can be evaluated on any closed 2-surface. These quasilocal expressions depend on (i) a field configuration, (ii) a reference configuration (or spinor field), and (iii) an evolution vector field $N$ on the boundary. Our formulation should be a good basis for further investigations aimed at understanding both the physical meaning of the quasilocal quantities for the different control modes and the physical interpretation of the contributions from auxiliary spinor fields.

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