Magnetic Moment of the $\Lambda_c$, $\Xi^+_{c1}$ and $\Xi^0_{c1}$

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Abstract

The magnetic moment of the $\Lambda_c$, $\Xi^+_{c1}$ and $\Xi^0_{c1}$ vanish when the charm quark mass is taken to infinity because the light degrees of freedom are in a spin zero configuration. The heavy quark spin-symmetry violating contribution from the light degrees of freedom starts at order $1/m_c$, the same order as the contribution from the heavy charm quark. We compute the leading long-distance contribution to the magnetic moments from the spin-symmetry breaking $\Sigma^*_{c} - \Sigma_{c}$ mass splitting in chiral perturbation theory. These are nonanalytic in the pion mass and arise from calculable one-loop graphs. Further, the difference between the magnetic moments of the charged charmed baryons is independent of the charm quark mass and of the subleading local counterterm.
The strong dynamics of a quark are greatly simplified in the limit that its mass becomes much greater than the scale of strong interactions.\textsuperscript{[1]}-\textsuperscript{[3]} Observables involving heavy quarks have a power series expansion in inverse powers of the quark mass and perturbative strong interactions. In the infinite mass limit, baryons containing a single heavy quark can be classified according to the spin of the light degrees of freedom. The \( \mathcal{F} \) of charmed baryons (\( \Lambda_c, \Xi_{c1}^+ \) and \( \Xi_{c1}^0 \)) have \( s_l = 0 \) and the spin of the baryon is carried entirely by the heavy quark. Since the magnetic moment of the heavy quark vanishes as its mass becomes infinite so does the magnetic moment of these baryons. In chiral perturbation theory this translates into there being no magnetic moment counterterm from the electromagnetic current of the light quarks that preserves heavy quark spin symmetry. In this work we point out that the leading contribution to the magnetic moment of the \( \mathcal{F} \) charmed baryons from long-distance physics is calculable in chiral perturbation theory. It arises from the spin-symmetry breaking \( \Sigma_c^* - \Sigma_c \) mass splitting and has nonanalytic dependence on the mass of the pion.

Heavy quark symmetry and chiral symmetry are combined together in order to describe the soft hadronic interactions of hadrons containing a heavy quark\textsuperscript{[4]}-\textsuperscript{[7]}. We are only concerned with dynamics of heavy baryons and so we will not discuss the lagrangian for heavy mesons, we refer the reader to\textsuperscript{[8]} for a review. Light degrees of freedom in the ground state of a baryon containing one heavy quark can have \( s_l = 0 \) corresponding to a member of the flavour \( SU(3) \) \( \mathcal{F} \), \( T_i(v) \) or they can have \( s_l = 1 \) corresponding to a member of the flavour \( SU(3) \) \( 6 \), \( S_{\mu}^{ij}(v) \). In the latter case, the spin of the light degrees of freedom can be combined with the spin of the heavy quark to form both \( J = 3/2 \) and \( J = 1/2 \) baryons, which are degenerate in the \( m_Q \to \infty \) limit. Using the notation of\textsuperscript{[7]} we define the fields,

\[
S_{\mu}^{ij}(v) = \frac{1}{\sqrt{3}} (\gamma_{\mu} + v_{\mu}) \gamma_5 \frac{1}{2} (1 + \hat{\gamma}) B^{ij} + \frac{1}{2} (1 + \hat{\gamma}) B^{*ij}_{\mu} \\
T_{i}(v) = \frac{1}{2} (1 + \hat{\gamma}) B_{i}
\]

(1)

where the \( J = 1/2 \) charmed baryons of the \( 6 \) are assigned to the symmetric tensor \( B^{ij} \)

\[
B^{11} = \Sigma_c^{++}, \quad B^{12} = \frac{1}{\sqrt{2}} \Sigma_c^+, \quad B^{22} = \Sigma_c^0, \quad B^{13} = \frac{1}{\sqrt{2}} \Xi_{c2}^+, \quad B^{23} = \frac{1}{\sqrt{2}} \Xi_{c2}^0, \quad B^{33} = \Omega_c^0.
\]

(2)
The $J = 3/2$ partners of these baryons have the same $SU(3)_V$ assignment in $B_{ij}^*$. The charmed baryons of the $\bar{3}$ representation are assigned to $B_i$ as

$$B_1 = \Xi_{c_1}^0, \quad B_2 = -\Xi_{c_1}^+, \quad B_3 = \Lambda_{c}^+.$$  

The chiral lagrangian describing the soft hadronic interaction of these baryons is given by [7]

$$\mathcal{L}_Q = i T^i_v \cdot D T_i - i S_{ij}^\mu \cdot D S_{ij}^\mu + \Delta_0 S_{ij}^\mu S_{ij} - g_3 \left( \epsilon_{ijk} T^i_j S_{kl}^j + h.c. \right) + i g_2 \epsilon_{\mu\nu\rho\sigma} S_{ik}^\mu v^\nu (A^\rho)_j^i S_{jk}^{\sigma} + \ldots,$$

where the dots denote operators with more insertions of the light quark mass matrix, more derivatives or higher order in the $1/m_Q$ expansion and $D^\alpha$ is the chiral covariant derivative. The axial chiral field $A^\mu = \frac{i}{2} \left( \xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi \right)$ is defined in terms of $\xi = \exp \left( iM/f_\pi \right)$ where $M$ is the octet of pseudogoldstone bosons

$$M = \begin{pmatrix}
\frac{1}{\sqrt{6}} \eta & \frac{1}{\sqrt{2}} \pi^0 \\
\frac{1}{\sqrt{6}} \eta - \frac{1}{\sqrt{2}} \pi^0 & \pi^+ \\
\pi^- & K^0 \\
K^- & -2 \sqrt{\frac{2}{3}} \eta
\end{pmatrix},$$

and $f_\pi = 135$MeV is the pion decay constant. Coupling of the pseudo-Goldstone bosons to the $\bar{3}$ baryons is forbidden at lowest order in $1/m_Q$. Even in the infinite mass limit the $\Sigma_Q^{(*)}$ baryons are not degenerate with the $\Lambda_Q$ baryons as the light degrees of freedom are in a different configuration giving rise to an intrinsic mass difference $\Delta_0$.

The magnetic moment interactions of a heavy quark are described by the lagrange density

$$\mathcal{L}_{\text{mag}} = \frac{e Q}{4m_Q} \sigma_{\mu\nu} h^{(Q)}_\nu \sigma_{\mu\nu} h^{(Q)}_\nu F^{\mu\nu} + \ldots,$$

where $Q$ is the charge of the heavy quark, $F_{\mu\nu}$ is the electromagnetic field tensor and the dots denote terms higher order in $1/m_Q$ [9]. Its inclusion into the heavy baryon chiral lagrangian has been discussed previously [10] [11]. There is no spin symmetry conserving local counterterm describing the magnetic moment interactions of the light degrees of freedom in an $s_l = 0$ configuration. Consequently, at lowest order in the chiral expansion the magnetic moment of all three $\bar{3}$ charmed baryons vanish. However, a magnetic moment for these baryons occurs at order $1/m_c$ from both the light degrees of freedom and from the charm quark itself.
We write the magnetic moment of the $\bar{3}$ charmed baryons as

$$
\mu^i = \mu_c + \mu^i_l \ ,
$$

(7)

where $\mu_c$ is the contribution from the charm quark and $\mu^i_l$ is the contribution from the light degrees of freedom. $\mu_c$ is found from forward matrix elements of (6) between baryon states and is known by heavy quark spin symmetry. It is reproduced in the heavy baryon lagrangian by

$$
\mathcal{L}^{\text{heavy}}_3 = \frac{e Q_c}{4m_c} T_i \sigma^{\mu\nu} T_i F_{\mu\nu} \ ,
$$

(8)

where $Q_c$ is the charge of the charm quark. The spin-symmetry breaking $\Sigma^*_c - \Sigma_c$ mass splitting $\delta$ gives rise to the formally leading contribution to $\mu^i_l$ through the graph shown in fig. 1. The formally subleading counterterm has the form

$$
\mathcal{L}^{\text{c.t.}}_3 = \frac{e\beta}{4m_c} T_i Q^i_j T_j F_{\mu\nu} \ ,
$$

(9)

where $Q$ is the light quark electromagnetic charge matrix

$$
Q = \frac{1}{3} \left( \begin{array}{ccc} 
2 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
\end{array} \right) \ ,
$$

(10)

and $\beta$ is an unknown constant. We find that, to this order

$$
\begin{align*}
\mu^3_l &= -\frac{1}{3} \frac{\beta}{m_c} : \Lambda_c \\
\mu^2_l &= -\frac{1}{3} \frac{\beta}{m_c} - (\Delta m) \frac{g_2^2}{3} \frac{1}{16\pi^2 f_\pi^2} J(m_\pi, \Delta_0) : \Xi_{c1}^+ \ , \\
\mu^1_l &= \frac{2}{3} \frac{\beta}{m_c} + (\Delta m) \frac{g_3^2}{3} \frac{1}{16\pi^2 f_\pi^2} J(m_\pi, \Delta_0) : \Xi_{c1}^0 \ .
\end{align*}
$$

(11)

The function $J(m, \Delta)$ arising from a taylor expansion of the loop integral in powers of $\Delta m$ (the $\Sigma^*_c - \Sigma_c$ mass splitting $\delta$) is given by

$$
J(m, \Delta) = \log \left( \frac{m^2}{\Lambda^2_\chi} \right) - \frac{\Delta}{\sqrt{\Delta^2 - m^2}} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m^2} + i\epsilon}{\Delta + \sqrt{\Delta^2 - m^2} + i\epsilon} \right) \ ,
$$

(12)

$^1$ This mass splitting is responsible for the leading long-distance corrections to $\Lambda_b \to \Lambda_c e^- \nu_e$ at zero-recoil arising from spin-symmetry breaking [12]. The analogous corrections in the meson sector have been computed in [13] [14].
where we have chosen to evaluate the graphs at the chiral symmetry breaking scale $\Lambda_\chi$.

We have not shown the contribution to each magnetic moment from loops involving kaons, as these are suppressed compared to the contribution from pion loops. When these terms are included the scale dependence of the total one-loop contribution is compensated by the scale dependence of $\beta$. We have neglected any SU(3) breaking in the baryon masses and assumed that the $\Sigma^*_c - \Sigma_c$ mass splitting $\Delta m$ is equal to the $\Xi^*_c - \Xi_c$ mass splitting. We can see from (12) that when $\Delta_0 = 0$ the loop contribution has a true infrared divergence regulated by the pion mass. However, when $\Delta_0 \neq 0$ the graph is no longer infrared divergent as $m_\pi \to 0$ and is regulated by the intrinsic mass splitting. The physical spectrum of charmed baryons does not correspond to either regime and so we keep the full functional dependence of (12). This is formally the dominant contribution to the amplitude.

The $\Sigma^*_c$ has not been observed yet \footnote{The SKAT bubble chamber group report a signal for the $\Sigma^{++}_c$ with a mass of $m_{\Sigma^{++}_c} = 2530 \pm 5$MeV \cite{ref1}. This needs independent verification.} and in order to get an estimate of the long-distance contribution we use a nonrelativistic quark model calculation of the $\Sigma^*_c$ mass $m_{\Sigma^*_c} = 2494 \pm 16$MeV \cite{ref2}. This value, combined with the experimental measurements of the other relevant masses \cite{ref3} gives $\Delta m = 41 \pm 16$MeV \footnote{For other estimates of $\Delta m$ see \cite{ref10,ref11} and for the present experimental situation see \cite{ref12}-\cite{ref14}.}. We use this to estimate an intrinsic mass splitting of $\Delta_0 = 194 \pm 11$MeV where the uncertainty depends entirely on that of $m_{\Sigma^*_c}$. The axial coupling constant $g_3$ is, as yet, undetermined. However, in the large-$N_c$ limit of QCD ($N_c$ being the number of colours) it has been shown to be related to the $\pi$-$N$ axial coupling constant $g_3 = \sqrt{3/2}g_A$ where $g_A = 1.25$ \cite{ref15,ref16}. It is this value of $g_3$ we will use in our estimate. We find that for $m_c = 1700$ MeV and $\beta = 0$ the magnetic moments of the 3 charmed baryons are

\begin{align}
\mu(\Lambda_c) & \sim 0.37 \text{ N.M.} \\
\mu(\Xi^+_c) & \sim 0.42 \text{ N.M.} \\
\mu(\Xi^0_c) & \sim 0.32 \text{ N.M.}
\end{align}

We should point out that while the charm quark mass is not well known \footnote{The SKAT bubble chamber group report a signal for the $\Sigma^{++}_c$ with a mass of $m_{\Sigma^{++}_c} = 2530 \pm 5$MeV \cite{ref1}. This needs independent verification.} and this leads to a large uncertainty in the magnetic moments, the difference between any two of the three magnetic moments does not depend explicitly on $m_c$ at this order. Assuming that $\beta$ can be neglected, these differences depend only on quantities that can be determined...
experimentally, the $\Sigma^*_c - \Sigma_c$ mass splitting and axial coupling constant $g_3$. However, it is possible that $\beta$ should not be neglected as the $\Sigma^*_c - \Sigma_c$ mass splitting is small, much smaller than the corresponding splitting in the meson sector and because the intrinsic $\Sigma^{(*)}_c - \Lambda_c$ mass splitting suppresses the infrared divergence arising in the chiral limit. We see from (7) and (11) that the difference between the magnetic moment of the $\Lambda_c$ and the $\Xi^+_c$ depends only on $\Delta m$ and $g_3$,

$$\mu(\Xi^+_{c1}) - \mu(\Lambda_c) \sim 0.05 \left( \frac{\Delta m}{41 \text{MeV}} \right) \sqrt{2} \left( \frac{g_3}{g_A} \right) \text{N.M.} \quad (14)$$

Further, the contribution from both the local counterterm and loop graph cancel in the sum of the baryon magnetic moments giving the charm quark magnetic moment as the average over the baryon moments,

$$\mu_c = \frac{1}{3} \left( \mu(\Lambda_c) + \mu(\Xi^+_{c1}) + \mu(\Xi^0_{c1}) \right) \quad (15)$$

In conclusion, we have shown that the leading contribution from the light degrees of freedom to the magnetic moments of the $\Lambda_c, \Xi^+_{c1}$ and $\Xi^0_{c1}$ baryons results from the spin symmetry breaking $\Sigma^*_c - \Sigma_c$ mass splitting and is calculable in chiral perturbation theory. It is found to depend on the third component of isospin, vanishing for the $\Lambda_c$ but giving an equal and opposite contribution to the $\Xi^+_{c1}$ and $\Xi^0_{c1}$. While suppressed by about a factor of two by the intrinsic $\Sigma_c - \Lambda_c$ mass splitting, the contributions are at the 10% level, depending on the $\Sigma^*_c - \Sigma_c$ mass splitting and axial coupling constant $g_3$. There is also a contribution from a formally subleading local counterterm which may be important due to the small $\Sigma^*_c - \Sigma_c$ splitting and finite intrinsic $\Sigma_c - \Lambda_c$ splitting. Mixing between the baryons of the $6$ and $\overline{3}$ is not only SU(3) violating but also vanishes in the heavy quark limit. Therefore we estimate that the effect of such mixing is small compared to the terms found in this work.

We have shown that the difference between the charged charmed baryon magnetic moments is independent of the subleading counterterm and does not depend explicitly on the charm quark mass. It is given in terms of experimentally observable quantities, the $\Sigma^*_c - \Sigma_c$ mass splitting and an axial coupling constant $g_3$. It is possible that the magnetic moment of the charged charmed baryons will be measured by the spin precession that occurs during channeling in bent crystals \[26\]. This would allow the difference between the charged charmed baryon magnetic moments to be measured and compared with this work. If, in addition, the magnetic moment of the neutral charmed baryon were to be
measured the magnetic moment of the charm quark itself could be extracted as it is equal to the average of the baryon moments $\mu_c$.

The same analysis applies to the magnetic moments of $b$-baryons in the $\mathbf{3}$ of SU(3). In this case it is the $\Sigma_b^* - \Sigma_b$ mass splitting that gives rise to the leading long-distance corrections and the contribution of the $b$ quark is $\mu_b = -\frac{1}{3} \frac{1}{m_b}$.

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4 This is also true for the radiative $D^* \to D\gamma$ transitions $^{27}$
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Figure Captions

Fig. 1. Graphs generating the leading long-distance contribution to the magnetic moment of $\bar{3}$ baryons. The wiggly line denotes a photon, the dashed line a $\pi$, and $\Sigma_c^{(*)}$ denotes both $\Sigma_c$ and $\Sigma_c^*$ baryons that can be in the intermediate state.
Figure 1