Growth Index of DGP Model and Current Growth Rate Data

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Abstract

Recently, some efforts focus on differentiating dark energy and modified gravity with the growth function $\delta(z)$. In the literature, it is useful to parameterize the growth rate $f \equiv d\ln \delta/d\ln a = \Omega_\gamma^m$ with the growth index $\gamma$. In this note, we consider the general DGP model with any $\Omega_k$. We confront the growth index of DGP model with currently available growth rate data and find that the DGP model is still consistent with it. This implies that more and better growth rate data are required to distinguish between dark energy and modified gravity.

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1 Introduction

The current accelerated expansion of our universe \cite{1,2,3,4,5,6,7,8,9,49} has been one of the most active fields in modern cosmology. There are very strong model-independent evidences \cite{10} (see also e.g. \cite{11}) for the accelerated expansion. Many cosmological models have been proposed to interpret this mysterious phenomenon, see e.g. \cite{1} for comprehensive reviews.

In the flood of various cosmological models, one of the most important tasks is to discriminate between them. Recently, some efforts have been made. For instance, it is important to determine that the dark energy is cosmological constant or dynamical dark energy \cite{1}. Caldwell and Linder proposed a so-called \(w - w'\) analysis in \cite{12} to discriminate dark energy models, and then was extended in \cite{13,14}. A recent review on \(w - w'\) analysis can be found in \cite{15}. Another tool to discriminate models is the statefinder diagnostic proposed by Starobinsky \textit{et al.} in \cite{16}. For a comprehensive list of relevant works on \(w - w'\) analysis and statefinder diagnostic, one can see e.g. \cite{17} and references therein.

Recently, some efforts to discriminate models focus on differentiating dark energy and modified gravity with the growth function \(\delta(z) \equiv \delta \rho_m/\rho_m\) of the linear matter density contrast as a function of redshift \(z\). By now, most of cosmological observations merely probe the expansion history of our universe \cite{1,2,3,4,5,6,7,8,9,49}. As well-known, it is very easy to build models which share a same cosmic expansion history by means of reconstruction between models. Therefore, to distinguish various models, some independent and complementary probes are required. Recently, it is argued that the measurement of growth function \(\delta(z)\) might be competent, see e.g. \cite{18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38}. If two models, especially dark energy and modified gravity models, share a same cosmic expansion history, they might have different growth history. Thus, they could be distinguished from each other.

One of the leading modified gravity models is the so-called Dvali-Gabadadze-Porrati (DGP) braneworld model \cite{32,33}, which altering the Einstein-Hilbert action by a term arising from large extra dimensions. For a list of references on DGP model, see e.g. \cite{29,34} and references therein. The first approach to study the growth function \(\delta(z)\) of DGP model is numerical solution, see e.g. \cite{25,27,35}. The second approach is to parameterize the growth rate \(f \equiv d \ln \delta / d \ln a\), where \(a = (1 + z)^{-1}\) is the scale factor of our universe.

For many years, it has been known that a good approximation to the growth rate \(f\), within Einstein gravity, is given by

\[
 f \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m^\gamma,
\]

where \(\gamma\) is the growth index, whereas \(\Omega_m\) is the fractional energy density of matter. In the very beginning, Eq. (1) was introduced in \cite{35,37}. There, \(f\) was defined purely in terms of the present value, using the present matter density, not valid for arbitrary redshift. Also, it was not until \cite{21} that it was applied to anything beyond matter, curvature, and a cosmological constant. Finally, not until \cite{28} was it applied to gravity other than general relativity, and then in \cite{19} generalized to modified gravity, varying equation of state, and an integral relation for growth. This parameterized approach has been tested in some works recently, see e.g. \cite{18,19,20,22,28,29,30,31,32,33,34,35}. The theoretical value of \(\gamma\) for \(\Lambda\)CDM model is \(6/11 \simeq 0.545\) \cite{18,19} whereas \(\gamma \simeq 0.55\) for other parameterized dark energy models \cite{19}. The theoretical growth index of flat DGP model (whose \(\Omega_k = 0\) exactly) is \(\gamma = 11/16 = 0.6875\) \cite{18} (In fact, \(\gamma = 11/16\) is a high redshift asymptotic value. \(\gamma = 0.68\) is favored for the fit to the whole growth history to the present, while another approximation is given in Eq. (27) of \cite{18} for any redshift, with \(\gamma = 7/11\) in the asymptotic future). Therefore, it is possible to distinguish the dark energy model (including \(\Lambda\)CDM model) from the flat DGP model.

In this note, we consider the general DGP model with any \(\Omega_k\). We confront the growth index of DGP model with currently available growth rate data and find that the DGP model is still consistent with it. On
the other hand, it is shown that the ΛCDM model is also consistent with current growth rate data [31]. This implies that more and better growth rate data are required to distinguish between dark energy and modified gravity models.

2 Growth index of DGP model

In the DGP model, the Friedmann equation is modified as [33] (see also e.g. [28, 29, 39])

\[ H^2 + \frac{k}{a^2} - \frac{1}{r_c} \sqrt{H^2 + \frac{k}{a^2}} = \frac{8\pi G}{3} \rho_m, \tag{2} \]

where \( H \equiv \dot{a}/a \) is the Hubble parameter; a dot denotes the derivative with respect to cosmic time \( t \); the constant \( r_c \) is the crossover scale; spatial curvature \( k = 0, k > 0 \) and \( k < 0 \) for flat, closed and open universe, respectively. Here, we only consider the self-accelerating branch. It is easy to rewrite Eq. (2) as an equivalent form [33] (see also e.g. [40])

\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \left( \sqrt{\rho_m + \rho_{rc}} + \sqrt{\rho_{rc}} \right)^2, \tag{3} \]

where \( \rho_{rc} \equiv 3/(32\pi G r_c^2) \). One can alternatively recast Eq. (2) as the “standard” form

\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_m + \rho_{de}), \tag{4} \]

where

\[ \rho_{de} = \frac{3}{8\pi G r_c^2} \sqrt{H^2 + \frac{k}{a^2}} = 2\sqrt{\rho_{rc}} \left( \sqrt{\rho_m + \rho_{rc}} + \sqrt{\rho_{rc}} \right), \tag{5} \]

in which we have used Eq. (3) in the last equality. Here, \( \rho_{de} \) can be regarded as an effective “dark energy” component, which compiles the contributions to the Friedmann equation from the extra dimensions [28, 29].

From Eq. (4), we have

\[ 1 - \Omega_k = \Omega_m + \Omega_{de}, \tag{6} \]

where \( \Omega_k \equiv -k/(a^2 H^2) \), and \( \Omega_i \equiv (8\pi G \rho_i)/(3H^2) \) for \( i = m \) and \( de \). From Eqs. (5), (6) and the energy conservation equation \( \dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = 0 \), we find that the effective equation-of-state parameter (EoS) for the effective “dark energy” component \( w_{de} \equiv p_{de}/\rho_{de} \) is given by

\[ w_{de} = \frac{-1 + \Omega_k}{1 + \Omega_m - \Omega_k}. \tag{7} \]

For the flat DGP model whose \( \Omega_k = 0 \) exactly, we get \( w_{de} = -(1 + \Omega_m)^{-1} \), which agrees with the one of [28, 29]. From Eq. (6), it is easy to see that

\[ E^2(z) \equiv \left( \frac{H}{H_0} \right)^2 = \left[ \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{rc}} + \Omega_{k0}(1+z)^2 \right]^2 + \Omega_{b0}(1+z)^2, \tag{8} \]

where we have used \( \rho_m = \rho_{m0}a^{-3} \); the subscript “0” indicates the present value of the corresponding quantity; \( \Omega_{rc} \equiv (8\pi G \rho_{rc})/(3H_0^2) = 1/(4H_0^2 r_c^2) \). From Eq. (8), we have

\[ 1 = \left[ \sqrt{\Omega_{m0} + \Omega_{rc}} + \sqrt{\Omega_{rc}} \right]^2 + \Omega_{b0}. \tag{9} \]

Therefore, only two of \( \Omega_{m0}, \Omega_{rc} \) and \( \Omega_{b0} \) are independent model parameters. For the flat DGP model whose \( \Omega_{k0} = 0 \) exactly, Eq. (9) becomes \( \Omega_{m0} = 1 - 2\sqrt{\Omega_{rc}} \).
In Einstein gravity, the growth function \( \delta(z) \) at scales much smaller than the Hubble radius obeys the following differential equation [18, 19, 20, 21, 31, 40]
\[
\ddot{\delta} + 2H \dot{\delta} = 4\pi G \rho_m \delta.
\tag{10}
\]
In modified gravity, Eq. (10) has been modified to [27, 28, 29, 40]
\[
\ddot{\delta} + 2H \dot{\delta} = 4\pi G_{\text{eff}} \rho_m \delta,
\tag{11}
\]
where \( G_{\text{eff}} \) is the effective local gravitational “constant” measured by Cavendish-type experiment, which is time-dependent. In general, \( G_{\text{eff}} \) can be written as
\[
G_{\text{eff}} = G \left( 1 + \frac{1}{3\beta} \right),
\tag{12}
\]
where \( \beta \) is determined once we specify the modified gravity theory. In the DGP gravity, \( \beta \) is given by [27, 29, 39, 40]
\[
\beta = 1 - \frac{2}{r_c} H \left( 1 + \frac{\dot{H}}{3H^2} \right).
\tag{13}
\]
By using Eq. (4), \( \beta \) can be written as [40]
\[
1 + \frac{1}{3\beta} = \frac{4\Omega_m^2 - 4(1-\Omega_k)^2 + \alpha}{3\Omega_m^2 - 3(1-\Omega_k)^2 + \alpha},
\tag{14}
\]
where
\[
\alpha \equiv 2\sqrt{1-\Omega_k} \left( 3 - 4\Omega_k + 2\Omega_m\Omega_k + \Omega_k^2 \right).
\tag{15}
\]
For the flat DGP model whose \( \Omega_k = 0 \) exactly, we get \( \beta = -(1+\Omega_m^2)/(1-\Omega_m^2) \), which agrees with the one of [28, 29]. Following [21, 31], we rewrite Eq. (11) as
\[
(\ln \delta)'' + (\ln \delta)'^2 + \left( 2 + \frac{\dot{H}}{H} \right) (\ln \delta)' = \frac{3}{2} \left( 1 + \frac{1}{3\beta} \right) \Omega_m,
\tag{16}
\]
where a prime denotes the derivative with respect to \( \ln \alpha \). By using Eqs. (4), (6) and the energy conservation equation \( \dot{\rho}_{de} + 3H(1+w_{de})\rho_{de} = 0 \), we find that
\[
\frac{H'}{H} = -\frac{3}{2} + \frac{\Omega_k}{2} - \frac{3}{2} w_{de} (1-\Omega_k-\Omega_m),
\tag{17}
\]
where \( w_{de} \) is given in Eq. (7). Substituting Eq. (17) into Eq. (16), we obtain
\[
(\ln \delta)'' + (\ln \delta)'^2 + (\ln \delta)' \left[ \frac{1}{2} (1 + \Omega_k) - \frac{3}{2} w_{de} (1-\Omega_k-\Omega_m) \right] = \frac{3}{2} \left( 1 + \frac{1}{3\beta} \right) \Omega_m.
\tag{18}
\]
In fact, the growth rate \( f \equiv d\ln \delta/d\ln \alpha = (\ln \delta)' \). By using the definition of \( \Omega_m \), the energy conservation equation \( \dot{\rho}_m + 3H\rho_m = 0 \) and Eq. (17), we have
\[
\Omega_m' = \Omega_m \left[ 3w_{de} (1-\Omega_k-\Omega_m) - \Omega_k \right].
\tag{19}
\]
Therefore, we find that
\[
(\ln \delta)'' = f'' = \Omega_m \left[ 3w_{de} (1-\Omega_k-\Omega_m) - \Omega_k \right] \frac{df}{d\Omega_m}.
\tag{20}
\]
So, Eq. (18) becomes
\[
\Omega_m \left[ 3w_{\text{de}} (1 - \Omega_k - \Omega_m) - \Omega_k \right] \frac{df}{d\Omega_m} + f^2 + f \left[ \frac{1}{2} (1 + \Omega_k) - \frac{3}{2} w_{\text{de}} (1 - \Omega_k - \Omega_m) \right] = \frac{3}{2} \left( 1 + \frac{1}{3\beta} \right) \Omega_m, \tag{21}
\]
where \(w_{\text{de}}\) and \(\beta\) are given in Eqs. (7) and (14), respectively. Substituting Eq. (11) into Eq. (21) and expanding around \(\Omega_m = 1\) (good approximation especially at \(z \gtrsim 1\)), after some tedious algebra, we finally arrive at
\[
10\gamma (\Omega_m - 1) - \Omega_k (1 - 2\gamma) + 3 (1 - 2\gamma) (1 - \Omega_m) = 6 (\Omega_m - 1) + \Omega_m [(\Omega_m - 1) (\Omega_m + 1) + 2\Omega_k], \tag{22}
\]
where we have ignored the higher order terms of small quantities \(1 - \Omega_m\) and \(\Omega_k\). Noting that Eq. (6), namely \(1 - \Omega_m = \Omega_k + \Omega_{\text{de}}\), we consider three cases for Eq. (22).

Case (I) \(\Omega_k = 0\) exactly or \(\Omega_k \ll \Omega_{\text{de}} = 1 - \Omega_m \ll 1\).
In this case, throwing out the terms of \(\Omega_k\) in Eq. (22), then eliminating \((\Omega_m - 1)\) in both sides, and using \(\Omega_m \to 1\) finally, we have \(\gamma = 11/16\). Obviously, it agrees with the known one of the flat DGP model [18].

Case (II) \(\Omega_{\text{de}} \ll \Omega_k = 1 - \Omega_m \ll 1\).
In this case, eliminating \((\Omega_m - 1) = -\Omega_k\) in both sides of Eq. (22) and then using \(\Omega_m \to 1\), we have \(\gamma = 4/7\). In fact, it coincides with the curvature solution found firstly in [51] and could be considered as a special case of the results in [21,15] with \(w = -1/3\).

Case (III) \(\Omega_k \sim \Omega_{\text{de}} \sim 1 - \Omega_m \ll 1\).
In this case, \(\Omega_k, \Omega_{\text{de}}\) and \(1 - \Omega_m\) are at the same order. Noting that \(1 - \Omega_m = \Omega_k + \Omega_{\text{de}}\), for convenience, we parameterize \(\Omega_k = m (1 - \Omega_m)\), where \(0 < m < 1\). It is worth noting that generally \(m\) is time-dependent and one considers only an instantaneous value of \(m\). Substituting into Eq. (22), then eliminating \((\Omega_m - 1)\) in both sides, and using \(\Omega_m \to 1\) finally, we have
\[
\gamma = \frac{11 - 3m}{16 - 2m}. \tag{23}
\]
Obviously, when \(m \to 0\) and \(1\), \(\gamma \to 11/16\) and \(4/7\), respectively.

For the flat DGP model whose \(\Omega_k = 0\) always, the only theoretical growth index is given by \(11/16\). For the DGP models whose \(\Omega_k \neq 0\), the situations are different. Noting that \(\Omega_m \propto (1 + z)^3\), \(\Omega_k \propto (1 + z)^2\) and \(\Omega_{\text{de}} \propto (1 + z)^{3(1+w_{\text{de}})}\) with \(w_{\text{de}} < -1/3\) to accelerate the expansion of our universe, \(\Omega_k\) increases faster than \(\Omega_{\text{de}}\) when \(z\) increases. So, for high \(z\), \(\Omega_{\text{de}} \ll \Omega_k = 1 - \Omega_m \ll 1\) eventually. Thus, we have \(\gamma = 4/7\) eventually, regardless of the value of \(\Omega_{k0}\). However, if \(\Omega_{k0}\) deviates from 0 very small, it is still possible that \(\Omega_k \ll \Omega_{\text{de}} = 1 - \Omega_m \ll 1\) at high \(z\), where \(\gamma \approx 11/16\); eventually \(\gamma = 4/7\) at higher \(z\). On the other hand, if \(\Omega_{k0}\) deviates from 0 not so small, the only theoretical growth index is given by 4/7.

Finally, it is worth noting that the above results are obtained as high redshift asymptotic values, rather than values for the whole growth history or the asymptotic future. On the other hand, the general solution of \(\gamma\) is obtained in [18] for the flat DGP model. We refer to [18] for details on this.

3 Confronting with current growth rate data

The most useful currently available growth rate data involve the redshift distortion parameter \(\beta_L\) [41] observed through the anisotropic pattern of galactic redshifts on cluster scales [22] (see also [31]). The parameter \(\beta_L\) is related to the growth rate \(f\) as [31]
\[
\beta_L = \frac{f}{b}, \tag{24}
\]
where $b$ is the bias factor. We present the currently available data for $\beta_L$ and $b$ at various redshifts in Table 1 along with the inferred growth rates. This is an extended version of the dataset used in Ref. [31], and contains a new data point at $z = 0.77$ [45]. Hereafter, we call the five data points except the one at $z = 0.77$ as dataset “Fobs” and call all the six data points as dataset “Fobsxext”.

About the currently available growth rate data, it is worth noting that there is considerable variation in analysis of different references and hence there is no accomplished consensus in fact. On the other hand, one should use these data with caution.

Using the growth rate data in Table 1 we can perform a $\chi^2$ analysis to find the growth index $\gamma$ and check its consistency with the theoretical values. As well-known, the corresponding $\chi^2$ reads

$$\chi^2(p, \gamma) = \sum_i \left( \frac{f_{obs}(z_i) - f_{th}(z_i; p, \gamma)}{\sigma_{f_{obs}}} \right)^2,$$

where $f_{obs}$ and its corresponding 1σ uncertainty $\sigma_{f_{obs}}$, are given in Table 1. $p$ denotes the model parameters; $f_{th}(z_i; p, \gamma)$ can be obtained from Eq. (1), in which $\Omega_m$ can be rewritten as a more convenient form

$$\Omega_m(z) = \frac{\Omega_m(1 + z)^3}{E^2(z)},$$

and $E(z)$ can be found in Eq. (8).

It is worth noting that the references in Table 1 have assumed flat ΛCDM (with $\Omega_m = 0.30$ for the five data points except the one at $z = 0.77$, and with $\Omega_m = 0.25$ for the data point at $z = 0.77$) when converting redshifts to distances for the power spectra and therefore their use to test models different from ΛCDM might be unreliable, as stressed in [31]. However, we can get around this problem in a new way. As mentioned above, the key is the redshift-distance relation. If the DGP model and ΛCDM model share the same redshift-distance relation, these growth rate data can also be used in the DGP model. To this end, we should properly select the model parameters of DGP model in order to reproduce the same redshift-distance relation of ΛCDM model. There is a simple and efficient method. As well-known in any textbook, the comoving distance is given by

$$r(z) \equiv \frac{1}{H_0 \sqrt{|\Omega_k|}} F \left( \sqrt{|\Omega_k|} \int_0^z \frac{dz}{E(z)} \right),$$

Table 1: The currently available data for $\beta_L$ and $b$ at various redshifts, along with the inferred growth rates. Notice that Ref. [47] only reports the growth rate and not the $\beta_L$ and $b$ parameters, since the growth rate was obtained directly from the change of power spectrum $L y - \alpha$ forest data in SDSS at various redshift slices. This is an extended version of the dataset used in Ref. [31], and contains a new data point at $z = 0.77$ [45].

| $z$  | $\beta_L$  | $b$  | $f_{obs}$ | Reference |
|------|-------------|------|-----------|-----------|
| 0.15 | 0.49 ± 0.09 | 1.04 ± 0.11 | 0.51 ± 0.11 | [33]      |
| 0.35 | 0.31 ± 0.04 | 2.25 ± 0.08 | 0.70 ± 0.18 | [17]      |
| 0.55 | 0.45 ± 0.05 | 1.66 ± 0.35 | 0.75 ± 0.18 | [44]      |
| 0.77 | 0.70 ± 0.26 | 1.3 ± 0.1  | 0.91 ± 0.36 | [45]      |
| 1.4  | 0.60±0.14   | 1.5 ± 0.20 | 0.90 ± 0.24 | [46]      |
| 3.0  | —           | —       | 1.46 ± 0.29 | [47]      |
where the function \( F(x) = x, \sin x \) and \( \sinh x \) for \( \Omega_k = 0, \Omega_k < 0 \) and \( \Omega_k > 0 \), respectively. For convenience, we use the dimensionless comoving distance \( H_0 r(z) \) instead. It is easy to get the \( H_0 r(z) \) line for the flat \( \Lambda \)CDM model with \( \Omega_{m0,\Lambda} = 0.30 \). Then, we discretize it into many points in a redshift range (for instance, \( 0 \leq z \leq 4 \), which covers the range of current growth data), and manually assign a relative “error” (say, 0.5%) to these discrete points. So, we have many fake “data points” in hand. Then, we fit the DGP model to these fake “data points” and find out the “best fit” parameters. Obviously, the DGP model with these “best fit” parameters will share the same redshift-distance relation with the flat \( \Lambda \)CDM model. Hence, the growth rate data in Table 1 can also be used for the corresponding DGP model. Finally, there is a minor remark on the procedure mentioned above. In fact, it holds only for the low redshift quantities considered here. Comparison of the distances to CMB last scattering (\( z \sim 1100 \)) will show large differences between these matching models. Fortunately, the redshift of current growth data is less than 4 and the procedure mentioned above works well.

Figure 1: The \( H_0 r(z) \) of flat DGP model with the corresponding “best fit” parameters (red dashed line) and flat \( \Lambda \)CDM model with \( \Omega_{m0,\Lambda} = 0.30 \) (black solid line) are shown in left panel. The right panel shows the relative departure between these two \( H_0 r(z) \) lines.

We consider the flat DGP model whose \( \Omega_{k0} = 0 \) exactly at first. Following above method, we find that the “best fit” parameter of flat DGP model is \( \Omega_{r_c} = 0.1519 \) (the corresponding \( \Omega_{m0} = 0.2204 \)). In Fig. 1, we show the \( H_0 r(z) \) of the corresponding flat DGP model along with the one of flat \( \Lambda \)CDM model with \( \Omega_{m0,\Lambda} = 0.30 \). The largest relative departure between these two \( H_0 r(z) \) lines in range \( 0 \leq z \leq 4 \) is less than 1.5%. Similarly, we consider the general DGP model with any \( \Omega_{k0} \) and find that the “best fit” parameters of DGP model are \( \Omega_{m0} = 0.2217 \) and \( \Omega_{r_c} = 0.1734 \) (the corresponding \( \Omega_{k0} = -0.0921 \)). In Fig. 2, we show the \( H_0 r(z) \) of the corresponding DGP model along with the one of flat \( \Lambda \)CDM model with \( \Omega_{m0,\Lambda} = 0.30 \). Obviously, these two \( H_0 r(z) \) lines are degenerate in fact. The largest relative departure between the \( H_0 r(z) \) of DGP model with the corresponding “best fit” parameters and flat \( \Lambda \)CDM model with \( \Omega_{m0,\Lambda} = 0.30 \) in range \( 0 \leq z \leq 4 \) is less than 0.19% surprisingly. The DGP model with corresponding “best fit” parameters
excellently reproduces the same redshift-distance relation of the flat $\Lambda$CDM model with $\Omega_{m,\Lambda} = 0.30$. So, the growth rate data in Table 1 can also be used for the corresponding DGP model. It is worth noting that all the above “best fit” parameters are consistent with the constraints of [29, 48, 39] (see also e.g. [34, 50]) and therefore they are also observationally acceptable.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.pdf}
\caption{The same as in Fig. 1 except for the DGP model with $\Omega_k \neq 0$.}
\end{figure}

Here, we fit the growth index $\gamma$ of these “reproduced” DGP models to the growth rate data in Table 1. For the case of flat DGP model whose $\Omega_k = 0$ exactly, setting $\Omega_m$ and $\Omega_r$ as the corresponding “best fit” values and minimizing $\chi^2$ with respect to $\gamma$, we find that for dataset $F_{\text{obs}}$, $\gamma = 0.438^{+0.126}_{-0.111}$ with 1σ uncertainty, or $\gamma = 0.438^{+0.272}_{-0.205}$ with 2σ uncertainty; for dataset $F_{\text{obs}ext}$, $\gamma = 0.429^{+0.123}_{-0.108}$ with 1σ uncertainty, or $\gamma = 0.429^{+0.264}_{-0.205}$ with 2σ uncertainty. Obviously, the only theoretical growth index $\gamma = 11/16 = 0.6875$ is consistent with these results at 2σ uncertainty.

For the case of DGP model with $\Omega_k \neq 0$, setting $\Omega_m$, $\Omega_r$, and $\Omega_k$ as the corresponding “best fit” values and minimizing $\chi^2$ with respect to $\gamma$, we find that for dataset $F_{\text{obs}}$, $\gamma = 0.465^{+0.134}_{-0.117}$ with 1σ uncertainty, or $\gamma = 0.465^{+0.290}_{-0.221}$ with 2σ uncertainty; for dataset $F_{\text{obs}ext}$, $\gamma = 0.457^{+0.131}_{-0.115}$ with 1σ uncertainty, or $\gamma = 0.457^{+0.282}_{-0.217}$ with 2σ uncertainty. Obviously, the theoretical growth index $\gamma = 11/16 = 0.6875$ is consistent with these results at 2σ uncertainty, whereas the other theoretical growth index $\gamma = 4/7 \approx 0.5714$ is consistent with these results at 1σ uncertainty.

4 Concluding remarks

In summary, we consider the growth index of general DGP model with any $\Omega_k$ in this note. We confront the growth index of DGP model with current growth rate data and find that the DGP model is still consistent with it. On the other hand, it is shown that the $\Lambda$CDM model is also consistent with current growth rate data [31]. This implies that more and better growth rate data are required to distinguish between dark energy and modified gravity models.
Some remarks are in order. Firstly, it is worth noting that the references in Table 1 have assumed flat ΛCDM (with $\Omega_m=0.30$ for the five data points except the one at $z=0.77$, and with $\Omega_m=0.25$ for the data point at $z=0.77$) when converting redshifts to distances for the power spectra and therefore their use to test models different from ΛCDM might be unreliable, as stressed in [31]. Although we can get around this problem by means of reproducing the same redshift-distance relation between the DGP model and ΛCDM model, the really complete solution to this problem lies on the reanalyzing the power spectra with the proper redshift-distance relation in the corresponding modified gravity. Secondly, in fact, we fixed $\Omega_m$ while fitting for $\gamma$ in section 3. This is required to ensure the validity of matching procedure of finding an equivalent DGP model. We admit that this weakens our conclusion. It is desirable to find a new method to improve the validity of matching procedure in the future works. Thirdly, one may use other parameterizations to the growth rate $f$, such as $f = \Omega_m^2 (1 + \eta)$ [30], or $f = \Omega_m^\gamma$ with $\gamma = \gamma_0 + \gamma_0' z$ [38]. However, the additional parameters $\eta$ and $\gamma_0'$ are found to be negligible in fact [30, 31, 38]. On the other hand, since currently available growth rate data points are so few and have large errors, it is difficult to tightly constrain these additional parameters. Fourthly, although as shown in [23] that non-trivial dark energy clustering or interaction between dark energy and dark matter might bring some troubles, we consider that combining the probes of expansion history and growth history is still promising to distinguish the dark energy and modified gravity. Of course, new idea to distinguish dark energy and modified gravity is still desirable.

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References

1] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003) [astro-ph/0207347]; T. Padmanabhan, Phys. Rept. 380, 235 (2003) [hep-th/0212290]; S. M. Carroll, astro-ph/0310342; R. Bean, S. Carroll and M. Trodden., astro-ph/0510059; V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000) [astro-ph/9904398]; S. M. Carroll, Living Rev. Rel. 4, 1 (2001) [astro-ph/0004075]; T. Padmanabhan, Curr. Sci. 88, 1057 (2005) [astro-ph/0411044]; S. Weinberg, Rev. Mod. Phys. 61, 1 (1989); S. Nobbenhuis, Found. Phys. 36, 613 (2006) [gr-qc/0411093]; E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) [hep-th/0603057]; A. Albrecht et al., astro-ph/0609591; R. Trotta and R. Bower, astro-ph/0607066;
M. Kamionkowski, arXiv:0706.2986 [astro-ph];
B. Ratra and M. S. Vogely, arXiv:0706.1565 [astro-ph];
E. V. Linder, arXiv:0705.4102 [astro-ph];
M. S. Turner and D. Huterer, arXiv:0706.2186 [astro-ph];
J. Frieman, M. Turner and D. Huterer, arXiv:0803.0982 [astro-ph].

[2] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) astro-ph/9805201;
S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) astro-ph/9812133;
J. L. Tonry et al. [Supernova Search Team Collaboration], Astrophys. J. 594, 1 (2003) astro-ph/0305008;
R. A. Knop et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 598, 102 (2003) astro-ph/0309368;
A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004) astro-ph/0402512.

[3] A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 659, 98 (2007) astro-ph/0611572.

[4] P. Astier et al. [SNLS Collaboration], Astron. Astrophys. 447, 31 (2006) astro-ph/0510447;
J. D. Neill et al., Astron. J. 132, 1126 (2006) astro-ph/0605148.

[5] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007) astro-ph/0603449;
L. Page et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 335 (2007) astro-ph/0603450;
G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 288 (2007) astro-ph/0603451;
N. Jarosik et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 263 (2007) astro-ph/0603452.

[6] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) astro-ph/0310723;
M. Tegmark et al. [SDSS Collaboration], Astrophys. J. 606, 702 (2004) astro-ph/0310725;
U. Seljak et al., Phys. Rev. D 71, 103515 (2005) astro-ph/0407372;
J. K. Adelman-McCarthy et al. [SDSS Collaboration], Astrophys. J. Suppl. 162, 38 (2006) astro-ph/0507711;
K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239, astro-ph/0403256, astro-ph/0305492.

[7] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 74, 123507 (2006) astro-ph/0608632.

[8] S. W. Allen et al., Mon. Not. Roy. Astron. Soc. 353, 457 (2004) astro-ph/0405340;
S. W. Allen et al., arXiv:0706.0033 [astro-ph];
A. Mantz, S. W. Allen, H. Ebeling and D. Rapetti, arXiv:0709.4294 [astro-ph].

[9] W. M. Wood-Vasey et al. [ESSENCE Collaboration], Astrophys. J. 666, 694 (2007) astro-ph/0701041;
G. Miknaitis et al. [ESSENCE Collaboration], Astrophys. J. 666, 674 (2007) astro-ph/0701043.

[10] C. Shapiro and M. S. Turner, Astrophys. J. 649, 563 (2006) astro-ph/0512586.

[11] M. Seikel and D. J. Schwarz, arXiv:0711.3180 [astro-ph];
Y. Gong, A. Wang, Q. Wu and Y. Z. Zhang, JCAP 0708, 018 (2007) astro-ph/0703583;
Y. Gong and A. Wang, Phys. Rev. D 73, 083506 (2006) astro-ph/0601453.
11

[12] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. 95, 141301 (2005) [astro-ph/0505494];
E. V. Linder, Phys. Rev. D 73, 063010 (2006) [astro-ph/0601052].

[13] R. J. Scherrer, Phys. Rev. D 73, 043502 (2006) [astro-ph/0509890].

[14] T. Chiba, Phys. Rev. D 73, 063501 (2006) [astro-ph/0510598].

[15] E. V. Linder, Gen. Rel. Grav. 40, 329 (2008) [arXiv:0704.2064].

[16] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, JETP Lett. 77, 201 (2003) [astro-ph/0201498];
U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. 344, 1057 (2003) [astro-ph/0303009].

[17] H. Wei and R. G. Cai, Phys. Lett. B 655, 1 (2007) [arXiv:0707.4526].

[18] E. V. Linder and R. N. Cahn, Astropart. Phys. 28, 481 (2007) [astro-ph/0701317].

[19] E. V. Linder, Phys. Rev. D 72, 043529 (2005) [astro-ph/0507263].

[20] D. Huterer and E. V. Linder, Phys. Rev. D 75, 023519 (2007) [astro-ph/0608681].

[21] L. M. Wang and P. J. Steinhardt, Astrophys. J. 508, 483 (1998) [astro-ph/9804015].

[22] Y. Wang, [arXiv:0710.3885 [astro-ph];
Y. Wang, [arXiv:0712.0041 [astro-ph]].

[23] M. Kunz and D. Sapone, Phys. Rev. Lett. 98, 121301 (2007) [astro-ph/0612452];
Bertschinger and P. Zukin, [arXiv:0801.2431 [astro-ph];
H. Wei and S. N. Zhang, [arXiv:0803.3292 [astro-ph], accepted for publication in Phys. Rev. D.

[24] S. Wang, L. Hui, M. May and Z. Haiman, Phys. Rev. D 76, 063503 (2007) [arXiv:0705.0165].

[25] A. Cardoso, K. Koyama, S. S. Seahra and F. P. Silva, [arXiv:0711.2563 [astro-ph]].

[26] K. Koyama, Gen. Rel. Grav. 40, 421 (2008) [arXiv:0706.1557].

[27] K. Koyama and R. Maartens, JCAP 0601, 016 (2006) [astro-ph/0511634].

[28] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D 69, 124015 (2004) [astro-ph/0401515].

[29] A. Lue, Phys. Rept. 423, 1 (2006) [astro-ph/0510068].

[30] C. Di Porto and L. Amendola, [arXiv:0707.2686 [astro-ph];
L. Amendola, M. Kunz and D. Sapone, [arXiv:0704.2421 [astro-ph];
D. Sapone and L. Amendola, [arXiv:0709.2792 [astro-ph]].

[31] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 77, 023504 (2008) [arXiv:0710.1092].

[32] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485, 208 (2000) [hep-th/0005016].

[33] C. Deffayet, Phys. Lett. B 502, 199 (2001) [hep-th/0010186];
C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D 65, 044023 (2002) [astro-ph/0105068].

[34] Z. K. Guo, Z. H. Zhu, J. S. Alcaniz and Y. Z. Zhang, Astrophys. J. 646, 1 (2006) [astro-ph/0603632].
I. Sawicki, Y. S. Song and W. Hu, Phys. Rev. D 75, 064002 (2007) [astro-ph/0606285].

P. J. E. Peebles, Large-Scale Structure of the Universe, Princeton University Press (1980);
P. J. E. Peebles, Astrophys. J. 284, 439 (1984).

O. Lahav, P. B. Lilje, J. R. Primack and M. J. Rees, Mon. Not. Roy. Astron. Soc. 251, 128 (1991).

D. Polarski and R. Gannouji, Phys. Lett. B 660, 439 (2008) [arXiv:0710.1510];
R. Gannouji and D. Polarski, arXiv:0802.4196 [astro-ph].

M. S. Movahed, M. Farhang and S. Rahvar, astro-ph/0701339.

T. Chiba and R. Takahashi, Phys. Rev. D 75, 101301 (2007) [astro-ph/0703347].

A. J. S. Hamilton, astro-ph/9708102.

S. Nesseris and L. Perivolaropoulos, JCAP 0701, 018 (2007) [astro-ph/0610092].

E. Hawkins et al., Mon. Not. Roy. Astron. Soc. 346, 78 (2003) [astro-ph/0212373];
L. Verde et al., Mon. Not. Roy. Astron. Soc. 335, 432 (2002) [astro-ph/0112161];
E. V. Linder, arXiv:0709.1113 [astro-ph].

N. P. Ross et al., astro-ph/0612400.

L. Guzzo et al., Nature 451, 541 (2008) [arXiv:0802.1944].

J. da Angela et al., astro-ph/0612401.

P. McDonald et al. [SDSS Collaboration], Astrophys. J. 635, 761 (2005) [astro-ph/0407377].

J. S. Alcaniz and N. Pires, Phys. Rev. D 70, 047303 (2004) [astro-ph/0404146].

T. M. Davis et al., Astrophys. J. 666, 716 (2007) [astro-ph/0701510].
It compiled the joint 192 SNIa data from ESSENCE [9] and new Gold [3].
The numerical data of the full sample are available at http://www.ctio.noao.edu/essence or http://braeburn.pha.jhu.edu/~ariess/R06

V. Barger, Y. Gao and D. Marfatia, Phys. Lett. B 648, 127 (2007) [astro-ph/0611775];
B. Wang, Y. G. Gong and R. K. Su, Phys. Lett. B 605, 9 (2005) [hep-th/0408032].

J. N. Fry, Phys. Lett. B 158, 211 (1985).