Abstract

In this paper, we study the problem of disaster relief inventory prepositioning under uncertainty. Specifically, we aim to determine where to open warehouses and how much relief item inventory to preposition in each, pre-disaster. During the post-disaster phase, prepositioned items are distributed to demand nodes, and additional items are procured and distributed as needed. There is uncertainty in the (1) disaster level, (2) locations of affected areas, (3) demand of relief items, (4) usable fraction of prepositioned items post-disaster, (5) procurement quantity, and (6) arc capacity. We propose and analyze two-stage stochastic programming (SP) and distributionally robust optimization (DRO) models, assuming known and unknown uncertainty distributions, respectively. The first and second stages correspond to pre- and post-disaster phases, respectively. We propose a Monte Carlo Optimization procedure to solve the SP and a decomposition algorithm to solve the DRO model. To illustrate potential applications of our approaches, we conduct extensive experiments using a hurricane season and an earthquake as case studies. Our results demonstrate the (1) the robustness and superior post-disaster operational performance of the DRO decisions under various distributions compared to SP decisions, especially under misspecified distributions and high variability, (2) the trade-off between considering distributional ambiguity and following distributional belief, and (3) computational efficiency of our approaches.

Keywords: Humanitarian logistics, inventory prepositioning, uncertainty, stochastic optimization

1. Introduction

Disasters such as earthquakes, hurricanes, or tornados are hard to predict precisely, and they often strike communities with little warning, leaving devastating impacts on people’s lives and infrastructure behind [Rawls and Turnquist 2010, Sabbaghtorkan et al. 2020]. Within the United States alone, an average of 13.8 disasters occur annually [Office For Coastal Management 2020].
The primary goal of emergency response efforts is to provide needed relief items and assistance to disaster-affected populations as soon as possible. The ability to quickly meet the urgent need for supplies can be the difference between life and death. To achieve this goal, planners often seek to preposition emergency relief inventory at strategic locations to distribute when needed. A major difficulty in creating an effective prepositioning plan is dealing with uncertainty; we do not know when disasters will hit or what their effects will be. In particular, the uncertainty of the post-disaster supply, demand, and road link capacity has substantial impact on planning decisions. There are also the following trade-offs. In the immediate aftermath of a disaster, prepositioned items may be the only resources available for distribution, and procuring additional items post-disaster is not easy and is often very expensive. Road networks may be partially functioning or totally damaged, and the items themselves are often subject to higher costs and limited quantities. However, prepositioning too many supplies or putting them in the wrong place can lead to unacceptably high inventory and transportation costs.

One way to improve the quality of inventory prepositioning decisions is to choose an appropriate way of modeling uncertainty while considering the associated trade-offs. To model uncertainty, most integrated disaster relief response problems are formulated as two-stage stochastic programming models (SP), with the first and second stage corresponding to pre-disaster (e.g., prepositioning of relief inventory) and post-disaster (e.g., distribution of relief items to demand nodes) phases, respectively. The SP approach assumes that the decision maker is risk-neutral and knows the probability distribution of uncertainty or have sufficient data to model it. Accordingly, the objective is to minimize the first-stage (prepositioning) cost plus the expectation of the random second-stage cost (e.g., shortage, transportation, etc.). The expectation is taken with respect to the (assumed) known distributions of random parameters.

In practice, it is unlikely that decision-makers can infer the post-disaster conditions or estimate the actual probability distributions of random parameters accurately, especially with limited data before the disaster and in the immediate aftermath (Velasquez et al., 2020; Zokaee et al., 2016). If the SP uses the true distribution, it will provide an excellent basis for the plan. However, if the model uses misspecified (biased) distributions, then the (optimistically biased) optimal decisions may perform poorly when implemented in practice under the true distribution. This phenomenon is known as the Optimizers’ Curse, i.e., an attempt to optimize based on imperfect estimates of distributions leads to biased decisions (Smith and Winkler, 2006; Esfahani and Kuhn, 2018).

Various decision theory studies show that decision-makers tend to be risk and ambiguity-averse (Halevy, 2007; Hsu et al., 2005), in that they try to avoid the impact of misspecified distributions. In the context of humanitarian logistics, some decision-makers may err on the side of caution and seek robust prepositioning decisions to safeguard the performance in adverse scenarios and mitigate the direct and indirect costs of operations. Distributionally robust optimization (DRO) is a risk and ambiguity-averse approach to model uncertainty. It is particularly useful when the distribution...
of uncertain problem data is hard to estimate and subject to uncertainty (i.e., ambiguous). DRO assumes that the uncertain parameter distribution belongs to an *ambiguity set*. The ambiguity set is a family of all possible distributions of the uncertain parameters that share common properties, e.g., mean value and range. We then optimize based on the distribution within this ambiguity set, i.e., the probability distribution is a decision variable (Rahimian and Mehrotra, 2019).

DRO has recently gained significant attention as an attractive approach for addressing optimization problems with unknown probability distributions for the following reasons. First, by allowing uncertain variables to follow an arbitrary distribution defined in the ambiguity set, DRO alleviates the unrealistic assumption of the decision-maker’s complete knowledge of distributions. Second, various techniques have been developed to derive tractable DRO models of real-world problems (see, e.g., [Basciftci et al. (2019); Jiang et al. (2017); Luo and Mehrotra (2018); Saif and Delage (2020); Shehadeh and Sanci (2021); Wang et al. (2020); Wu et al. (2015)]), and some DRO models are more tractable than their SP counterparts (Delage and Saif, 2018 [Rahimian and Mehrotra, 2019]; Wiesemann et al., 2014). Third, one can use distributional information that is easy to approximate (e.g., mean values and range) to construct the ambiguity sets for the DRO models. For example, one can use the decision-maker’s estimate of intuitive statistics such as average and range of demand (based on their experience in previous disasters) or forecasts from a demand forecasting model to construct a mean-range ambiguity set and DRO model (which often admit tractable reformulations).

Despite the potential advantages, there are no tractable DRO approaches for the specific disaster relief location and inventory prepositioning problem that we study in this paper (see Section 2).

1.1. Contributions of the Paper

We present and analyze both a DRO and an SP model to design a preparedness plan for one or multiple disasters. Specifically, given a set of candidate warehouse locations and different types of relief items, the models determine the number of warehouses to open at strategic locations and each relief item’s quantity to preposition at each selected location. The first stage is the pre-disaster planning phase where locations are selected and inventory is prepositioned. In the second stage, the models optimize the response operations. Uncertainties considered include the (1) disaster level, (2) locations of affected areas, (3) demand of relief items, (4) usable fraction of prepositioned items post-disaster, (5) procurement quantity, and (6) arc capacity between two different nodes. The DRO model seeks to find first-stage prepositioning decisions that minimize the sum of the first-stage cost and the worst-case (maximum) expectation of the second-stage cost. Here, we take the worst-case expectation over an ambiguity set characterized by the known mean and range of the unknown distributions of uncertain parameters (1)–(6). Given a joint probability distribution of uncertain parameters (1)–(6), the SP model minimizes the sum of the planning decisions and the expected cost of the response operations. We will use the SP model as a comparator to evaluate
the potential benefits of the DRO approach. We summarize our main contributions as follows.

- To the best of our knowledge and according to the recent survey of Sabbaghtorkan et al. (2020) and our literature review in Section 2, our paper is the first to compare the value and performance of SP and DRO approaches to address uncertainty and distributional ambiguity in parameters (1)–(6) in the specific location and inventory preposition problem that we study in this paper.

- We propose a computationally efficient decomposition algorithm to solve the DRO model and a Monte Carlo Optimization Procedure to obtain near-optimal solutions to the SP model.

- We also consider a model that minimizes the trade-off between considering distributional ambiguity (DRO pessimism) and following distributional belief (SP optimism). This trade-off has not been studied within the context of the specific location and inventory preposition problem that we study in this paper.

- To illustrate the broad application of our approaches, we conduct extensive numerical experiments using two real-world case studies from the literature, which are based on the Atlantic hurricane season in the US and an earthquake that happened in Yushu County in Qinghai Province, China. Our results demonstrate the (1) computational efficiency of our decomposition and MCO algorithm, (2) superior post-disaster operational performance of the DRO decisions under various distributions compared to SP decisions, and (3) the trade-off between considering distributional ambiguity and following distributional belief. More generally, our results draw attention to the need to model the distributional ambiguity of uncertain problem data in strategic real-world stochastic optimization problems.

1.2. Structure of the paper

The reminder of this paper is structured as follows. In Section 2 we review relevant literature. In Section 3 we define and formulate the location and inventory prepositioning of disaster response operations problem. In Section 4 we present a decomposition algorithm to solve the DRO model and a Monte Carlo Optimization Procedure to obtain near-optimal solutions to the SP model. In Section 5 we conduct extensive numerical experiments using a hurricane season and an earthquake as case studies. Finally, we draw conclusions in Section 6.

2. Literature Review

Disaster operations management consists of four phases: mitigation, preparedness, response, and recovery (Altay and Green, 2006; Ergun et al., 2013; Sabbaghtorkan et al., 2020). Mitigation and preparedness activities occur before a disaster and typically include emergency inventory prepositioning, facility location, and transportation decisions (Aboolian et al., 2013; Altay, 2013; Tucker.
Table 1: Summary of papers modeling inventory prepositioning under uncertainty using stochastic optimization approaches.

| Paper                     | item | Metrics/Objectives | Random Factors | Approach |
|---------------------------|------|---------------------|----------------|----------|
| Chang et al. (2007)       | multi| ✓ ✓ ✓ ✓ ✓            | ✓             | SP       |
| Balcik and Beamon (2008)  | multi| ✓                    | ✓             | SP       |
| Salmeron and Apte (2010)  | multi| ✓ ✓ ✓ ✓             | ✓             | SP       |
| Rawls and Turnquist (2010)| multi| ✓ ✓ ✓ ✓ ✓           | ✓             | SP       |
| Mete and Zabinsky (2010)  | multi| ✓ ✓ ✓ ✓             | ✓             | SP       |
| Novan (2012)              | multi| ✓ ✓ ✓ ✓             | ✓             | SP       |
| Döyen et al. (2012)       | multi| ✓ ✓ ✓ ✓             | ✓             | SP       |
| Bezorgi-Amiri et al. (2013)| multi| ✓ ✓ ✓ ✓ ✓ ✓ ✓      | ✓             | SP       |
| Alem et al. (2010)        | multi| ✓ ✓ ✓ ✓ ✓ ✓         | ✓             | RO       |
| Hu et al. (2017)          | single| ✓ ✓ ✓ ✓             | ✓             | RO       |
| Zokaee et al. (2016) *    | multi| ✓ ✓ ✓ ✓ ✓           | ✓             | RO       |
| Ni et al. (2018)          | single| ✓ ✓ ✓ ✓             | ✓             | RO       |
| Velasquez et al. (2020)   | multi| ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓     | ✓             | DRO      |
| **Our Paper**             | multi| ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ | ✓             | **DRO**  |

Notation: Item is single or multiple (multi) relief items, \( f \) is fixed cost, \( A \) is acquisition/setup/handling cost, \( T \) is transportation cost, \( S \) is shortage cost, \( U \) is unused inventory, \( P \) is procurement cost, \( O \) is other metrics (e.g., response time), \( D \) is demand, \( \rho \) is usable fraction of the prepositioned relief supplies, \( M \) is maximum quantity available to procure post disaster, \( V \) is arc capacity.

Most pioneering inventory prepositioning models assume perfect knowledge about the post-disaster conditions (e.g., demand for relief items, road conditions). In reality, it is unlikely that decision-makers be able to predict the exact post-disaster conditions. Thus, a naive, deterministic approach that uses point estimates for random parameters will likely produce suboptimal prepositioning decisions. To address this, several authors have studied inventory prepositioning under uncertainty. In what follows, we review papers that are most relevant to this work: papers that use stochastic optimization (i.e., SP, robust optimization (RO), and DRO) for prepositioning in disaster response operations. For comprehensive surveys, we refer to (Altay and Green, 2006; Anaya-Arenas et al., 2014; Galindo and Batta, 2013; Gupta et al., 2016) and reference therein. We refer to Sabbaghtorkan et al. (2020) for a review of the literature on prepositioning of supplies in disaster operations management.

Table 1 summarizes relevant and recent studies that use stochastic optimization for inventory prepositioning and delivery of disaster relief supplies. Most of these studies employ two-stage SP to model uncertainty, assuming that random parameters’ probability distributions are fully known. The first stage of these models consists of the location and amount of prepositioned supplies, and the second-stage consists of transportation decisions. Optimization criteria of these studies include...
minimizing the expected total cost (Chang et al., 2007; Döyen et al., 2012; Mete and Zabinsky, 2010; Rawls and Turnquist, 2010), minimizing expected response time (Duran et al., 2011), minimizing expected casualties (Salmerón and Apte, 2010), and maximizing the expected satisfied demand or minimizing expected shortage of relief items (Balcik and Beamon, 2008).

While SP is a powerful modeling approach for modeling uncertainty, it suffers from the following shortcomings. First, to formulate an SP, one needs access to all possible scenarios and their probabilities, which is not realistic for inventory prepositioning due to the unpredictable nature of disasters and effects (Condeixa et al., 2017; Salmerón and Apte, 2010; Velasquez et al., 2020). If we calibrate an SP to a particular training dataset, the resulting decision policy may have disappointing out-of-sample performance under an unseen data set from the same population. This phenomenon is known as Optimizer’s Curse and is reminiscent of overfitting effects in statistics (Smith and Winkler, 2006). Second, SP approaches suffer from the “curse of dimensionality,” and they are often intractable. Form Table 1 we observe that existing SP approaches considered either one random factor or a subset of the random factors that we model in this paper. Additionally, these studies did not consider preparing for multiple disasters of different levels.

RO assumes a complete ignorance about the probability distribution of uncertain parameters. Instead, RO assumes that uncertain parameters reside in a so-called “uncertainty set” of possible outcomes (Bertsimas and Sim, 2004; Ben-Tal et al., 2015; Soyster, 1973), and optimization is based on the worst-case scenario occurring within the uncertainty set. Only a few papers have employed RO for inventory prepositioning (Table 1). Zokaee et al. (2016)’s RO model incorporated the uncertainty associated with demand, supply, and cost parameters, assuming that these uncertain parameters are independent and bounded random variables. They controlled the level of conservativeness using a budget of uncertainty (Bertsimas and Sim, 2004) and reformulated their model as a mixed-integer program.

Two recent papers have considered RO to preposition inventory (Ni et al., 2018; Velasquez et al., 2020). They provide the foundation for our study. Ni et al. (2018) proposed a two-stage RO model to minimize prepositioning (first-stage) cost and (second-stage) shortage costs for one disaster. The uncertainty set included three parameters: demand, usable fraction of prepositioned supplies, and arc capacity. They used an uncertainty budget to control the size of the uncertainty set. Ni et al. (2018) used off-the-shelf solvers for small instances and proposed a Benders decomposition approach that can solve larger instances. The model did not consider the risk of multiple disasters, disaster level, or the possibility of procuring additional relief supplies post-disaster.

Recently, Velasquez et al. (2020) proposed a two-stage RO model that hedged against multiple disasters (e.g., hurricanes) and incorporated the uncertainty in affected areas. In the first stage, the model determines the location and amount of prepositioned relief supplies before any disaster occurs. In the second stage, prepositioned relief items are distributed, and additional items are procured as needed. The objective is to minimize the total cost of prepositioning and distributing
disaster relief supplies. To solve their model, Velasquez et al. (2020) proposed a column-and-constraint generation algorithm. The model assumed that if a disaster occurs at a particular location, the demand for a relief item and the usable fraction of prepositioned supplies at that location are equal to their nominal values (which depends on the disaster type). They assumed that procurement quantity and arc capacities post-disaster are deterministic.

While RO is a helpful framework for dealing with data limitations, there are downsides. By focusing the optimization on the worst-case scenario occurring within the uncertainty set, RO can lead to over-conservatism and suboptimal decisions for more likely scenarios (Chen et al., 2020; Delage and Saif, 2018). Additionally, RO does not fully utilize any distributional information of uncertainty and may yield poor expected performance (Rahimian and Mehrotra, 2019).

An alternative paradigm for modeling uncertainty is DRO. It aims to unify SP and RO while overcoming their drawbacks (Rahimian and Mehrotra, 2019; Saif and Delage, 2020). In DRO, we assume that the distribution of uncertain parameters resides in a so-called “ambiguity set” which is a family of all possible distribution of uncertain parameters characterized through some known properties of uncertain parameters (Esfahani and Kuhn, 2018). The optimization is based on the worst-case distribution within the ambiguity set. There are three primary benefits to using DRO to model uncertainty. First, DRO alleviates the unrealistic assumption of the decision-maker’s complete knowledge of the distribution governing the uncertain problem data. Second, DRO models are often more computationally tractable than their SP and RO counterparts. Finally, DRO avoids the well-known over-conservatism and the poor expected performance of RO and allows for better utilization of the available data. To construct the ambiguity sets, one can use easy to compute information such as mean values and ranges of random parameters and derive tractable DRO models that better mimic reality. We refer to Rahimian and Mehrotra (2019) for a comprehensive survey of DRO. Despite the potential advantages, there are no DRO approaches for the specific location and inventory preposition problem that we study in this paper (Table 1, Sabbaghtorkan et al. (2020)).

From Table 1 and Sabbaghtorkan et al. (2020), we observe that despite the potential advantages, there are no tractable DRO approaches for the specific location and inventory preposition problem that we study in this paper. In this work, we develop a tractable DRO model to consider the realistic lack of distributional information and compare this model to an SP model. We also expand the considered problem by incorporating uncertainty and distributional ambiguity for parameters previously assumed to be deterministic. These include the maximum quantity available to be procured post-disaster, arc capacities, demand, and the usable post-disaster fraction of prepositioned relief items (assumed deterministic in Velasquez et al. (2020)). We also model the possibility of procuring an additional amount of relief supplies post-disaster and include the procurement cost in the second-stage objective, in contrast to Ni et al. (2018)’s recent RO approach (and prior studies) for disaster inventory prepositioning.
3. Formulations and Analysis

In this section, we formally define the stochastic location and emergency inventory prepositioning of disaster relief supplies problem (Section 3.1) and formulate it as a two-stage SP (Section 3.2) and a two-stage DRO model (Section 3.3). In Section 3.4 we derive a solvable reformulation of the DRO model.

3.1. Definitions and assumptions

We consider a directed graph $G(\mathcal{N}, \mathcal{A})$, where nodes $i \in \mathcal{N}$ are candidate locations for warehouses and arcs $a \in \mathcal{A}$ represent roads. We assume the set of candidate locations and the demand nodes are the same, without loss of generality. We let binary decision variable $o_i = 1$ if a facility is open at location $i$, and $o_i = 0$ otherwise. The fixed cost of opening a facility at location $i$ is $f_i$. We consider a set of relief items $\mathcal{T}$. Each location $i \in \mathcal{N}$ has a storage capacity $S_i$, and each unit of relief item $t \in \mathcal{T}$ requires a storage capacity of $s_t$. We can serve the demand for relief item $t \in \mathcal{T}$ at location $i \in \mathcal{N}$ with prepositioned supplies or with supplies procured post-disaster. We assume that there is no limit on the relief supplies available for prepositioning in the first stage, as in prior studies (Sabbaghtorkan et al., 2020; Velasquez et al., 2020). Post-disaster, we assume that relief procurement is limited and more expensive given that supplies are procured with short notice, and price hikes are common (Velasquez et al., 2020). We assume that the maximum order quantity available post-disaster and the capacity of arc $(i, j)$ are random.

There are $|L|$ different types, or levels, of disasters (e.g., major, minor). For all $l \in L$ and $i \in I$, we define $q_{i,l}$ as a 0-1 random variable, which equals to one if a disaster of level $l$ occurs at location $i$, and $q_{i,l} = 0$ otherwise. The level of disaster affects: the demand for relief items at each location, how much of the prepositioned inventory is available (some may be destroyed), how much of each relief item can be ordered after the disaster, and the road capacity connected to the affected location. As in prior literature (Ni et al., 2018; Rawls and Turnquist, 2010; Sabbaghtorkan et al., 2020; Velasquez et al., 2020), we restrict ourselves to a one disaster per disaster-prone node, i.e., $\sum_{l \in L} q_{i,l} \leq 1$, $\forall i \in I$. However, our models consider the possibility that, for example, one hurricane can make a landfall in multiple locations. Moreover, our models and solution methods can handle preparation for multiple disasters, such as an entire hurricane season.

For each $t \in \mathcal{T}$, $i \in \mathcal{N}$, $l \in L$, we let $d_{t,i,l}$ represent the random demand for relief item $t$ if a disaster of type $l$ occurs at node $i$. For all $i \in I$ and $l \in L$, we let $\rho_{i,l}$ represents the fraction of relief supplies prepresents at location $i$ that remains usable after a disaster of type $l$ occurs. For all $t \in \mathcal{T}$, $i \in \mathcal{N}$, and $l \in L$, we let random parameter $M_{t,i,l}$ represent the maximum order quantity available to procure post a disaster of type $l$ at location $i$. For all $(i,j) \in \mathcal{A}$ and $l \in L$, we let random parameter $V_{i,j,l}$ represent the random capacity of arc $(i,j)$ after a disaster of type $l$ occurs at either $i$ or $j$. If a disaster does not occur at either, then the capacity of arc $(i,j)$, equal to the nominal value, $\hat{V}_{i,j}$.
In the first stage, we decide (1) number of facilities to open and their location, and (2) amount of each relief item to store at each open facility. Additional relief items are procured and distributed in the second stage. The quality of these prepositioning decisions is a function of (1) fixed cost of opening facilities and items acquisitions cost (first-stage), (2) cost of unmet demand (second-stage), (3) holding cost (second-stage), (4) procurement cost (second-stage), and (5) shipment cost (second-stage). For all \( t \in \mathcal{T} \) and \( i \in \mathcal{N} \), we let continuous decision variable \( z_{t,i} \) represent the amount of relief item \( t \) prepositioned at location \( i \) and continuous decision variable \( y_{t,i} \) represent the amount of relief item \( t \) procured post-disaster at location \( i \). For all \((i,j) \in \mathcal{A} \) and \( t \in \mathcal{T} \), we let continuous decision variable \( x_{t,i,j} \) represent the amount of item \( t \) shipped through arc \((i,j)\), i.e., flow quantity across \((i,j)\). For all \( t \in \mathcal{T} \) and \( i \in \mathcal{N} \), we let the continuous decision variables \( e_{t,i} \) and \( u_{t,i} \) respectively represent the quantity of unused inventory and unsatisfied demand of item \( t \) at \( i \).

The positive parameters \( c_{t,i} \), \( c_{t,i}^h \), \( c_{t,i}^p \), and \( c_{t,i}^s \) represent the unit acquisition cost of relief item \( t \), unit holding cost of relief item \( t \), unit cost of procuring relief item \( t \) post-disaster, and unit penalty cost of unmet demand for relief item for all \( t \in \mathcal{T} \), respectively. Finally, the positive parameter \( c_{i,j}^f \) represents the unit transportation cost of relief item \( t \) using arc \((i,j)\) in \( \mathcal{A} \), for all \( t \in \mathcal{T} \). For \( a, b \in \mathbb{Z} \), we define \( [a] := \{1, 2, \ldots, a\} \) and \([a,b]_\mathbb{Z} := \{c \in \mathbb{Z} : a \leq c \leq b\} \), i.e., \([a,b]_\mathbb{Z}\) represent the set of running integer indices \([a,a+1,a+2,\ldots,b]\). We summarize the notation in Table 2.

3.2. Two-stage SP Model

In this section, we assume that we know the joint probability distribution \( \mathbb{P} \) of uncertain parameters \( \xi := [q,d,\rho,M,V]^\top \). Accordingly, we formulate the following two-stage SP model

\[
\begin{align*}
\nu &= \min_{o,z} \left\{ \sum_{i \in I} f_i o_i + \sum_{i \in I} \sum_{t \in \mathcal{T}} c_{t,i}^a z_{t,i} + \mathbb{E}[Q(o,z,\xi)] \right\} \\
\text{s.t.} \quad &\sum_{t \in \mathcal{T}} s_t z_{t,i} \leq S_i o_i, \quad \forall i \in I \\
& o_i \in \{0,1\}, \quad z_{i,t} \geq 0 \quad \forall i \in I, t \in \mathcal{T}
\end{align*}
\]

(1a)

(1b)

(1c)

where for a given joint realization of uncertain parameters \( \xi := [q,d,\rho,M,V]^\top \)

\[
Q(o,z,\xi) := \min \left( \sum_{i \in I} \sum_{t \in \mathcal{T}} (c_{t,i}^p y_{t,i} + c_{t,i}^h u_{t,i} + c_{t,i}^s e_{t,i}) + \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{A}} c_{i,j}^f x_{t,i,j}^f \right)
\]

(2a)

\[
\text{s.t.} \quad \sum_{j : (j,i) \in \mathcal{A}} x_{j,i}^f - \sum_{j : (i,j) \in \mathcal{A}} x_{i,j}^f + y_{t,i} - e_{t,i} + u_{t,i} \\
= \sum_{l \in \mathcal{L}} q_{i,l} (d_{t,l,i} - \rho_{i,l} z_{t,i}) - (1 - \sum_{l \in \mathcal{L}} q_{i,l}) z_{t,i}, \quad \forall t \in \mathcal{T}, i \in I \\
y_{t,i} \leq \sum_{l \in \mathcal{L}} q_{i,l} M_{t,i,l}, \quad \forall t \in \mathcal{T}, i \in I \\
\sum_{t \in \mathcal{T}} v_{i,j} x_{t,i,j}^f \leq \sum_{l \in \mathcal{L}} q_{i,l} V_{i,j,l} + (1 - \sum_{l \in \mathcal{L}} q_{i,l}) \hat{V}_{i,j}, \quad \forall (i,j) \in \mathcal{A}
\]

(2b)

(2c)

(2d)
Table 2: Notation.

Sets and Indices

\[ \mathcal{N} \] set of nodes
\[ I \] set of potential facility (warehouse) sites to store the prepositioned emergency supplies, \( I \subseteq \mathcal{N} \)
\[ \mathcal{T} \] set of relief supplies types
\[ \mathcal{A} \] set of arcs
\[ L \] set of disaster levels

Parameters

\( q_{i,l} \) binary random variable that equals 1 if a disaster of level \( l \in L \) occurs at location \( i \) and 0 otherwise
\( c_{a,t} \) unit acquisition cost for prepositioning relief item \( t \)
\( c_{h,t} \) unit holding cost for relief item \( t \)
\( c_{p,t} \) unit cost of procuring relief item \( t \) post-disaster
\( c_{f,i,j} \) unit transportation cost of relief item \( t \) using arc \( (i,j) \)
\( f_{i} \) fixed cost of opening facility
\( S_{i} \) storage capacity at location \( i \)
\( s_{t} \) unit storage space required for relief supply \( t \)
\( v_{t} \) arc capacity required to transport a unit of relief item \( t \)
\( M_{t,i,l} \) random maximum order quantity available to procure post-disaster for relief supply \( t \) at location \( i \)
\( \rho_{i,l} \) random fraction of relief items prepositioned at location \( i \) that remains usable after a disaster of type \( l \)
\( \hat{V}_{i,j} \) nominal capacity of arc \( (i,j) \), i.e., if no disaster occurs
\( V_{i,j,t} \) random capacity of arc \( (i,j) \) if a disaster of type \( l \) occurs
\( d_{t,i} \) random demand for relief item \( t \) if a disaster of type \( l \) occurs at location \( i \)

Decision Variables

\( o_{i} \) binary variable that equals 1 if a facility is open at location \( i \) and 0 otherwise
\( z_{t,i} \) amount of relief item \( t \) prepositioned at location \( i \)
\( x_{t,i,j} \) amount of relief item \( t \) shipped through arc \( (i,j) \), i.e., flow quantity across link \( (i,j) \)
\( y_{t,i} \) amount of relief item \( t \) procured post-disaster at location \( i \)
\( e_{t,i} \) quantity of unused inventory of item \( t \) at location \( i \)
\( u_{t,i} \) quantity of unsatisfied demand of relief item \( t \) at location \( i \)

\[
\sum_{t \in \mathcal{T}} v_{t}x_{t,i,j}^{i} \leq \sum_{t' \in L} q_{j,t'}V_{i,j,t'} + (1 - \sum_{t' \in L} q_{j,t'})\hat{V}_{i,j} \quad \forall (i,j) \in \mathcal{A} \quad (2e)
\]

\[
y_{t,i} \geq 0, \quad e_{t,i} \geq 0, \quad u_{t,i} \geq 0, \quad x_{t,i,j}^{i} \geq 0, \quad \forall t \in \mathcal{T}, i \in (i,j) \in \mathcal{A} \quad (2f)
\]

The objective function (1a) minimizes the sum of the fixed cost of opening facilities (first term), prepositioning cost (second term), and total expected cost of response (third term). The expected cost of response is the sum of the costs of procurement (first term in (2a)), shortage (second term in (2a)), holding (third term in (2a)), and transportation (fourth term in (2a)). Constraints (1b) enforce the storage capacity for prepositioning relief supplies. Constraints (1c) define the domain. In the second-stage, constraints (2b) enforce flow conservation. Constraints (2c) limit the relief supplies procured post-disaster. If no disaster occurs at location \( i \) (i.e., \( \sum_{l \in L} q_{i,l} = 0 \)), then all of the prepositioned items (if any) remain usable. Constraints (2d)–(2e) enforce arc capacities. If there is not a disaster at either end of arc \( (i,j) \), then the arc capacity is not affected, and it is equal to its nominal value, \( \hat{V}_{i,j} \). If a disaster hits node \( i \) or node \( j \), the arc capacity is \( V_{i,j,t} \) or \( V_{i,j,t'} \), respectively. If a disaster hits both nodes, the arc capacity is defined as \( \min\{V_{i,j,t},V_{i,j,t'}\} \). Constraints (2f) specify feasible ranges of the decision variables.
3.3. Two-stage DRO model

The distributions of post-disaster parameter uncertainty (e.g., demand for relief supplies) may be difficult to estimate before a disaster occurs. Historical data on past disasters may be either unavailable or insufficient to model the correct distributions of uncertainty. Future disasters often have different characteristics (e.g., distributions) than previous events. Therefore, in this section, we consider the case when the probability distribution of $P$ of random parameters is not perfectly known or hard to estimate.

We assume that we know or can approximate the mean values and support (i.e., upper and lower bound) of the random parameters. Mathematically, we consider support $\mathcal{R} = \mathcal{R}^q \times \mathcal{R}^d \times \mathcal{R}^\rho \times \mathcal{R}^M \times \mathcal{R}^V$, where $\mathcal{R}^q, \mathcal{R}^d, \mathcal{R}^\rho, \mathcal{R}^M,$ and $\mathcal{R}^V$ are the supports of random parameters $q, d, \rho, M,$ and $V.$ They are defined in (3). This assumption of mean-range support is motivated by a humanitarian logistics expert who noted that the mean and range values of disaster-related uncertainty may be estimated from historical data or approximated by subject matter experts. Advancements in this sector have included developing forecasts for the occurrence and effects of future disasters, and drones and satellite technology are increasingly used for damage assessments. In addition, engineering methods could be used to estimate the mean and range of the potential damage levels ($\rho$) as well as the level of damage for arc capacities.

In addition, we let $\mu^q, \mu^d, \mu^\rho, \mu^M,$ and $\mu^V$ represent the mean values of $q, d, \rho, M,$ and $V$, respectively. For notational brevity, we let $\mu := \mathbb{E}[\xi] = [\mu^q, \mu^d, \mu^\rho, \mu^M, \mu^V]^\top$. We let $\mathcal{P}(\mathcal{R})$ represent the set of probability distributions that are supported on $\mathcal{R}$ such that each distribution matches the mean values of $\xi$. Using $\mathcal{R}$ and $\mu$, we define the following mean-support ambiguity set

$$\mathcal{F}(\mathcal{R}, \mu) := \left\{ P \in \mathcal{P}(\mathcal{R}) : \int_{\mathcal{R}} dP = 1, \mathbb{E}_P[\xi] = \mu \right\}$$

(4)

Note that we do not consider higher moments of random parameters for three primary reasons. First, moment-range ambiguity allows us to derive a tractable reformulation and solution methodology. Various studies have demonstrated that incorporating higher moments of random parameters often undermines the computational tractability of the DRO model, and therefore its applicability in practice. Second, even if we assume that uncertain parameters are dependent, it is not straightforward for decision-makers to approximate the correlation between these uncertain parameters,
especially with limited data during the planning stage. The mean and range are intuitive statistics that a decision-maker may approximate and change in the model. Third, one can argue that the availability of procurement quantity from the supplier after disaster and demand are independent, for example. Finally, DRO models for real-world problems based on mean-range ambiguity sets often allow for tractable reformulations and solution methods.

Using ambiguity set \( \mathcal{F}(\mathcal{R}, \mu) \), we formulate our DR location and inventory prepositioning of disaster response operations problem as the following min-max problem:

\[
\min \left\{ \sum_{i \in I} f_i o_i + \sum_{i \in I} \sum_{t \in T} c_{t,i}^* z_{t,i} + \sup_{P \in \mathcal{F}(\mathcal{R}, \mu)} E_P[Q(o, z, \xi)] \right\}.
\]  (5)

It seeks to find the prepositioning decisions \((o, z)\) that minimize the sum of the fixed facility costs, prepositioning costs, and the worst-case expected cost of response operations.

3.4. Reformulation of the DRO model

In this section, we derive an equivalent solvable reformulation of the min-max model in (5). We first consider the inner maximization problem \( \sup_{P \in \mathcal{F}(\mathcal{R}, \mu)} E_P[Q(o, z, \xi)] \) for a fixed first-stage decision \((o, z)\), where \( P \) is the decision variable, i.e., we are choosing the distribution that maximizes \( E_P[Q(o, z, \xi)] \).

\[
\max_P E_P[Q(o, z, \xi)] \quad (6a)
\]

\[
s.t. \ E_P[\xi] = \mu, \quad (6b)
\]

\[
E_P[\mathbf{1}_R(\xi)] = 1 \quad (6c)
\]

where \( \mathbf{1}_R(\xi) = 1 \) if \( \xi \in \mathcal{R} \) and \( \mathbf{1}_R(\xi) = 0 \) if \( \xi \notin \mathcal{R} \). In Proposition 1, we show that problem (6) is equivalent to the deterministic problem (7) (see Appendix A for a detailed proof).

**Proposition 1.** For any feasible \((o, z)\), problem (6) is equivalent to

\[
\min_{\alpha, \phi, \gamma, \lambda, \tau} \left\{ \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} \left( \mu_{t,i,l}^d \alpha_{t,i,l} + \mu_{t,i,l}^M \phi_{t,i,l} \right) + \sum_{i \in I} \left( \mu_{t,i,l}^d \gamma_{i,l} + \mu_{t,i,l}^\lambda \lambda_{i,l} \right) + \sum_{(i,j) \in A} \mu_{t,j,l}^V \tau_{i,j,l} \right] \right.
\]

\[
+ \max_{\xi \in \mathcal{R}} \left\{ Q(o, z, \xi) + \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} -d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l} \right] + \sum_{i \in I} -\left( \rho_{t,i,l} \gamma_{i,l} + q_{i,l} \lambda_{i,l} \right) \right. 
\]

\[
+ \sum_{(i,j) \in A} -V_{i,j,l} \tau_{i,j,l} \right\} \right\} \quad (7)
\]

Note that \( Q(o, z, \xi) \) is a minimization problem. Thus, in (7) we have a potentially challenging inner max-min problem. We next analyze \( Q(o, z, \xi) \) for the given feasible first-stage solution \((o, z)\) and realized value of \( \xi \) to derive a solvable reformulation of the inner problem in (7). Note that for a fixed \((o, z, \xi)\), \( Q(o, z, \xi) \) is a feasible and bounded linear program (i.e., we have complete recourse).
Let variable $\beta_{t,i}$ represent the dual associated with constraint $(2b)$ for all $i \in I$, $t \in T$, variable $\Gamma_{t,i}$ represents the dual of constraint $(2e)$, variable $\psi_{i,j}$ represents the dual of constraint $(2d)$ for all $(i,j) \in A$, and variable $\varphi_{i,j}$ represents the dual of constraint $(2c)$ for all $(i,j)$. We formulate $Q(o,z,\xi)$ in its dual form as:

$$Q(o,z,\xi) := \max \left\{ \sum \sum \sum (dt_{i,l} - \rho_{i,l}z_{t,i})q_{i,l}\beta_{t,i} - (1 - \sum q_{i,l})z_{t,i}\beta_{t,i} + \sum \sum \sum q_{i,l}M_{t,i,i}\Gamma_{t,i} \right. $$

$$+ \left. \sum (i,j) \in A \sum \sum \sum q_{i,l}V_{i,j,l}\psi_{i,j} + (1 - \sum q_{i,l})\hat{V}_{i,j,l}\varphi_{i,j} \right\} \quad (8a)$$

$$\text{s.t. } -\beta_{t,i} + \beta_{t,j} + v_t\psi_{i,j} + v_t\varphi_{i,j} \leq c_{t,i}, \quad \forall (i,j) \in A, \forall t \in T \quad (8b)$$

$$\beta_{t,i} + \Gamma_{t,i} \leq c_{t,i}, \quad \forall i \in I, \forall t \in T \quad (8c)$$

$$-c_{t,i} \leq \beta_{t,i} \leq c_{t,i}, \quad \forall i \in I, \forall t \in T \quad (8d)$$

$$\Gamma_{t,i} \leq 0, \psi_{i,j} \leq 0, \varphi_{i,j} \leq 0, \quad \forall (i,j) \in A, \forall t \in T \quad (8e)$$

Note that region $(8b)-(8e)$ of $Q(o,z,\xi)$ is feasible and bounded, support $R$ of $\xi$ is finite, and $z \geq 0$. In view of formulation $(8)$, we derive the following reformulation of the inner maximization problem in $(7)$:

$$\max \left\{ \sum \sum \sum (dt_{i,l} - \rho_{i,l}z_{t,i})q_{i,l}\beta_{t,i} - (1 - \sum q_{i,l})z_{t,i}\beta_{t,i} + \sum \sum \sum q_{i,l}M_{t,i,i}\Gamma_{t,i} \right. $$

$$+ \left. \sum (i,j) \in A \sum \sum \sum q_{i,l}V_{i,j,l}\psi_{i,j} + (1 - \sum q_{i,l})\hat{V}_{i,j,l}\varphi_{i,j} \right\} \quad (9a)$$

$$\text{s.t. } -\beta_{t,i} + \beta_{t,j} + v_t\psi_{i,j} + v_t\varphi_{i,j} \leq c_{t,i}, \quad \forall (i,j) \in A, \forall t \in T \quad (9b)$$

$$\beta_{t,i} + \Gamma_{t,i} \leq c_{t,i}, \quad \forall i \in I, \forall t \in T \quad (9c)$$

$$-c_{t,i} \leq \beta_{t,i} \leq c_{t,i}, \quad \forall i \in I, \forall t \in T \quad (9d)$$

$$\Gamma_{t,i} \leq 0, \psi_{i,j} \leq 0, \varphi_{i,j} \leq 0, \quad \forall i \in I, \forall t \in T, (i,j) \in A \quad (9e)$$

$$dt_{i,i} \in [d_{t,i}, \bar{d}_{t,i}], M_{t,i,i} \in [\underline{M}_{t,i,i}, \bar{M}_{t,i,i}], \quad \forall t \in T, i \in I, l \in L \quad (9f)$$

$$\rho_{i,l} \in [\underline{\rho}_{i,l}, \bar{\rho}_{i,l}], \quad q_{i,l} \in \{0,1\}, \quad \forall i \in I, l \in L \quad (9g)$$

$$V_{i,j,l} \in [\underline{V}_{i,j,l}, \bar{V}_{i,j,l}], \quad \forall (i,j) \in A, l \in L \quad (9h)$$

To limit the number of disasters of type $l \in L$ to $N_l$, we can add constraint $\sum_{i \in A} q_{i,l} = N_l$ to $(9)$. Note that objective function $(9a)$ contains the interaction terms $q_{i,l}dt_{i,i}\beta_{t,i}$, $q_{i,l}\rho_{i,i}\beta_{t,i}$, $q_{i,l}M_{t,i,i}\Gamma_{t,i}$.
To linearize, we define variables \( k_{t,i,l} = q_{t,i} \beta_{t,i}, h_{t,i,l} = k_{t,i,l} d_{t,i,l}, g_{t,i,l} = k_{t,i,l} \rho_{t,i,l}, \Delta_{t,i,l} = q_{t,i} \Gamma_{t,i}, \eta_{t,i,l} = q_{t,i} \psi_{t,i}, \varpi_{t,j,l} = q_{t,j} \varphi_{t,j,l}, \Phi_{t,j,l} = \eta_{t,j} V_{t,j,l}, \Lambda_{t,j,l} = \varpi_{t,j} V_{t,j,l}, \) and \( \sigma_{t,i,l} = \Delta_{t,i,l} M_{t,i,l} \). Also, we introduce McCormick inequalities \((B.1a) - (B.1n)\) in Appendix C for these variables. Note that the McCormick inequalities often rely on big-M coefficients \((\tau \text{ and } \gamma)\) that take large values and thus may undermine computational efficiency. In Appendix C, we derive tight bounds of these big-M coefficients to strengthen the MILP formulation. Accordingly, problem \((9)\) is equivalent to the following mixed-integer linear program (MILP)

\[
\begin{align*}
\text{max} \quad & \sum_{i,l} \sum_{t \in T} \left( \sum_{l \in L} (h_{t,i,l} - g_{t,i,l} z_{t,i}) - (\beta_{t,i} - \sum_{l \in L} k_{t,i,l}) z_{t,i} + \sum_{l \in L} \sigma_{t,i,l} \right) \\
& + \sum_{(i,j) \in A} \left( \sum_{l \in L} \Phi_{i,j,l} + (\psi_{i,j} - \sum_{l \in L} \eta_{i,j,l}) V_{i,j} + \sum_{t' \in L} \Lambda_{i,j,t'} + (\varphi_{i,j} - \sum_{l \in L} \varpi_{i,j,t'}) \hat{V}_{i,j} \right) \\
& + \sum_{l \in L} \sum_{i,l} \left( \sum_{t \in T} - (d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l}) + \sum_{i,l} -(\rho_{i,l} \gamma_{i,l} + q_{i,l} \lambda_{i,l}) + \sum_{(i,j) \in A} -V_{i,j,l} \tau_{i,j,l} \right) \bigg\} \\
\text{s.t.} \quad & (9b) - (9h), \quad (B.1a) - (B.1n) \quad (10a) \\
& (10b)
\end{align*}
\]

Combining the inner problem in the form of \((10)\) with the outer minimization problems in \((7)\) and DRO model in \((5)\), we derive the following equivalent reformulation of our DRO model:

\[
\begin{align*}
\text{min} \quad & \sum_{i \in I} \sum_{t \in T} f_{i} a_{i} + \sum_{i \in I} \sum_{t \in T} c_{i}^{t} z_{t,i} + \sum_{i \in I} \sum_{t \in T} \sum_{l \in L} \left( \mu_{i,t,l}^{V} \alpha_{t,i,l}^{V} + \mu_{i,t,l}^{M} \phi_{t,i,l}^{V} \right) + \sum_{i \in I} \sum_{l \in L} \left( \mu_{i,l}^{V} \gamma_{i,l} + \mu_{i,l}^{M} \lambda_{i,l} \right) \\
& + \sum_{(i,j) \in A} \sum_{l \in L} \mu_{i,j,l}^{V} \tau_{i,j,l} + \delta \bigg\} \quad (11a) \\
\text{s.t.} \quad & (1b) - (1c) \quad (11b) \\
& \delta \geq H(z, \xi) := \max \left\{ \sum_{i \in I} \sum_{t \in T} \sum_{l \in L} \left( h_{t,i,l} - g_{t,i,l} z_{t,i} \right) - (\beta_{t,i} - \sum_{l \in L} k_{t,i,l}) z_{t,i} + \sum_{l \in L} \sigma_{t,i,l} \right\} \\
& \quad + \sum_{(i,j) \in A} \left( \sum_{l \in L} \Phi_{i,j,l} + (\psi_{i,j} - \sum_{l \in L} \eta_{i,j,l}) V_{i,j} + \sum_{t' \in L} \Lambda_{i,j,t'} + (\varphi_{i,j} - \sum_{l \in L} \varpi_{i,j,t'}) \hat{V}_{i,j} \right) \\
& \quad + \sum_{l \in L} \sum_{i \in I} \left( \sum_{t \in T} - (d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l}) + \sum_{i \in I} -(\rho_{i,l} \gamma_{i,l} + q_{i,l} \lambda_{i,l}) \right) \\
& \quad + \sum_{(i,j) \in A} -V_{i,j,l} \tau_{i,j,l} \bigg\} : \quad (9b) - (9h), \quad (B.1a) - (B.1n) \quad (11c)
\end{align*}
\]

**Proposition 2.** For any feasible first-stage decision \( z \), \( H(z, \xi) < \infty \). Furthermore, \( H(z, \xi) \) is a convex piecewise linear function in \( z \) (see Appendix D for a detailed proof.)
4. Solution Approaches

In this section, we present a decomposition algorithm to solve the DRO model (Section 4.1), and a Monte Carlo Optimization procedure to obtain near-optimal solutions to the SP model (Section 4.2). In Section 4.3 we present a model that minimizes the trade-off between considering distributional ambiguity (as in DRO) and following a distributional belief for $\xi$ (as in SP).

4.1. DRO–Decomposition Algorithm

Proposition 2 suggests that constraint (11c) describes the epigraph of a convex and piecewise linear function of decision variables in formulation (11). Therefore, given the two stage characteristic of our problem, it is natural to attempt to solve (11) (equivalently, the DRO model in (5)) via a decomposition algorithm. Algorithm 1 presents our decomposition algorithm. Algorithm 1 is finite because we identify a new piece of the function $H(z, \xi)$ each time the set $\{L(z, \delta) \geq 0\}$ is augmented in step 4, and the function has a finite number of pieces according to Proposition 2.

4.2. Monte Carlo Optimization

Note that it is difficult to obtain an exact optimal solution to the two-stage SP in (1). Indeed, evaluating the values of $E[Q(o, z, \xi)]$ involves taking multi-dimensional integrals (Birge and Louveaux 2011). Therefore, we resort to Monte Carlo approximation approach to obtain near-optimal solutions to (1) in a reasonable time. In the Monte Carlo approach, we replace the distribution of $\xi$ with a (discrete) distribution based on $N$ independent and identically distributed (i.i.d.) samples of $\xi$, and then we solve the sample average approximation (SAA) formulation (14a)–(14b) of the SP in (1).

\[
\nu_N = \min \left\{ \sum_{i \in I} f_i \hat{o}_i + \sum_{i \in I} \sum_{t \in T} c^o_i \hat{z}_{t,i} + \hat{F}_N \right\}
\]

(14a)

\[
\text{subject to (1b), (1c), (2b)–(2f), for all } n = 1, \ldots, N
\]

(14b)

where $\hat{F}_N := \sum_{n=1}^{N} \frac{1}{N} \left[ \sum_{i \in I} \sum_{t \in T} (c^o_i y_{t,i}^n + c^w_i u_{t,i}^n + c^c_i e_{t,i}^n) + \sum_{t \in T} \sum_{(i,j) \in A} c^x_{i,j} x_{i,j}^n \right]$. Note that in the SAA formulation (14a)–(14b), we associate all scenario-dependent parameters, variables, and constraints with a scenario index $n$ for all $n = 1, \ldots, N$. For example, we replace parameters $d_{t,i,l}$ by $d_{t,i,l}^n$ to represent the demand for relief item $t$ if a disaster of type $l$ occurs at location $i$ in scenario $n$. In addition, constraints (2b)–(2f) are incorporated in each scenario.

Algorithm 2 summarizes the Monte Carlo Optimization (MCO) algorithm that determines an appropriate sample size $N$ and obtain near-optimal solutions to the SP model based on SAA formulation within a reasonable time. This algorithm is based on the SAA method in Homem-de Mello and Bayraksan (2014) and Kleywegt et al. (2002) and Shehadeh et al. (2020) with some adaptations to our model. Starting with an initial candidate value of $N$, the MCO algorithm
Algorithm 1: DRO–Decomposition algorithm.

1. **Input.** Feasible region \( \{ [1b] - [1c] \} \); set of cuts \( \{ \mathcal{L}(z, \delta) \geq 0 \} = \emptyset \); \( LB = -\infty \) and \( UB = \infty \).

2. **Master Problem.** Solve the following master problem

\[
Z = \min \left\{ \sum_{i \in I} f_i z_{i,1} + \sum_{i \in I} \sum_{t \in T} c_{t,i} z_{t,i} + \frac{1}{2} \sum_{i \in I} \sum_{t \in T} \sum_{l \in L} \left( \mu_{t,i,l} \alpha_{t,i,l} + \mu_{t,i,l}^{\alpha} \phi_{t,i,l} \right) + \sum_{i \in I} \sum_{t \in T} \left( \mu_{t,i,l} \gamma_{i,l} + \mu_{t,i,l}^{\gamma} \lambda_{i,l} \right) \right. \\
+ \sum_{(i,j) \in A} \sum_{l \in L} \mu_{i,j,l} \tau_{i,j,l} + \delta \\
\left. \right\} \quad \text{s.t.} [1b], [1c] \quad \mathcal{L}(z, \delta) \geq 0 \quad (12a)
\]

and record an optimal solution \( (\alpha^*, \phi^*, \delta^*, \alpha^*, \phi^*, \gamma^*, \lambda^*) \) and set \( LB = Z^* \).

3. **Sub-problem.** With \( (z, \alpha, \phi, \gamma, \lambda, \tau) \) fixed to \( (\alpha^*, \phi^*, \gamma^*, \lambda^*, \tau^*) \), solve the following problem

\[
H(z, \xi) := \max \left\{ \sum_{i \in I} \sum_{t \in T} \left[ \sum_{l \in L} \left( h_{t,i,l} - g_{t,i,l} z_{t,i}^* \right) - \left( \beta_{t,i} - \sum_{l \in L} k_{t,i,l} z_{t,i}^* \right) + \sigma_{t,i}\right] \\
+ \sum_{(i,j) \in A} \sum_{l \in L} \Phi_{i,j,l} + \left( \psi_{i,j} - \sum_{l \in L} \eta_{i,j,l} \right) \hat{V}_{i,j} + \sum_{l \in L} A_{i,j,l'} + \left( \phi_{i,j} - \sum_{l \in L} \omega_{i,j,l'} \right) \hat{V}_{i,j} \right\} \\
+ \sum_{i \in I} \sum_{t \in T} \left[ -V_{i,j,l} \tau_{i,j,l}^* \right] \quad : [0b] - [0l], [B.1a] - [B.1n] \quad (13)
\]

record \( (\beta^*, h^*, g^*, k^*, \sigma^*, \psi^*, \phi^*, \Phi^*, \eta^*, \lambda^*, \omega^*, q^*, d^*, M^*, V^*) \) and \( UB = \min \{ UB, H(z, \xi) + (LB - \delta^*) \} \)

4. **if** \( \delta^* \geq \sum_{i \in I} \sum_{t \in T} \left[ \sum_{l \in L} \left( h_{t,i,l} - g_{t,i,l} z_{t,i}^* \right) - \left( \beta_{t,i} - \sum_{l \in L} k_{t,i,l} z_{t,i}^* \right) + \sigma_{t,i}\right] \\
+ \sum_{(i,j) \in A} \sum_{l \in L} \Phi_{i,j,l} + \left( \psi_{i,j} - \sum_{l \in L} \eta_{i,j,l} \right) \hat{V}_{i,j} + \sum_{l \in L} A_{i,j,l'} + \left( \phi_{i,j} - \sum_{l \in L} \omega_{i,j,l'} \right) \hat{V}_{i,j} \right\} \\
+ \sum_{i \in I} \sum_{t \in T} \left[ -V_{i,j,l} \tau_{i,j,l}^* \right] \quad \text{then} \quad \text{stop and return} \quad \phi^* \quad \text{and} \quad \alpha^* \quad \text{as the optimal solution to formulation} \quad [12] \quad \text{i.e., DRO model} \quad \text{end if} \)
Algorithm 2: Monte Carlo Optimization (MCO) Procedure

Input: $N_0$ is an initial sample size, $K$ is number of replicates, $N'$ is number of scenarios in the Monte Carlo Simulation step, and $\epsilon$ is a termination tolerance.

Output: $N$ is sample size, $\hat{v}_N$ and $\bar{v}_{N'}$ are respectively statistical lower and upper bounds on the optimal value of the SP, and $AOI_N$ is approximate optimality index.

Initialization: $N := N_0$

Step 1. MCO Procedure

for $k = 1, \ldots, K$, do

1. **Scenario Generation**
   - Generate $N$ independent and identical distributed (i.i.d.) scenarios of $(d, \rho, M, V)$

2. **Solving the SAA formulation**
   - Solve the SAA formulation in (14) with the scenarios generated in step 1.1 and record the corresponding optimal objective value $v_N^k$ and optimal solution $(\hat{\delta}, \hat{\zeta})_N^k$.

3. **Cost Evaluation using Monte Carlo Simulation**
   - Generate a new $N'$ i.i.d scenarios of $(d', \rho', M', V')$
   - Use solution solution $(\hat{\delta}, \hat{\zeta})_N^k$ and parameters $(d', \rho', M', V')$ to compute $x', u', \epsilon', y'$, and evaluate the objective function $v_{N'}^k$, as follows:

$$v_{N'}^k = \sum_{i \in I} f_i \hat{a}_i + \sum_{i \in I} \sum_{t \in T} c_{i,t} \hat{z}_{t,i} + \sum_{n=1}^{N'} \frac{1}{N'} \left( \sum_{i \in I} \sum_{t \in T} (c_{i,t} y_{t,i} + c_{i,t} u_{t,i} + \hat{c}_{i,t} e_{t,i}) + \sum_{t \in T} \sum_{(i,j) \in A} \hat{c}_{i,j} x_{i,j} \right)$$

end

Step 2. Compute the average of $\hat{v}_N^k$ and $\bar{v}_{N'}^k$ among the $K$ replications

$$\bar{v}_N = \frac{1}{K} \sum_{k=1}^{K} v_N^k \quad \bar{v}_{N'} = \frac{1}{K} \sum_{k=1}^{K} v_{N'}^k$$

Step 3. Compute the Approximate Optimality Index

$$AOI_N = \frac{\bar{v}_{N'} - \bar{v}_N}{\bar{v}_{N'}}$$

Step 4. If $AOI_N$ satisfies a predetermined termination tolerance (i.e., $|AOI_N| < \epsilon$), terminate and output $N, \bar{v}_N, \bar{v}_{N'}$, and $AOI_N$. Otherwise, update $N \leftarrow 2N$, and go to step 1.

proceeds as follows. First, for $k = 1, \ldots, K$, we repeat the following steps. In step 1.1, we generate a sample of $N$ i.i.d scenarios of $(d, \rho, M, V)$. In step 1.2, we solve the SAA formulation with the scenarios generated in step 1.1 and record the corresponding optimal objective value $v_N^k$ and optimal prepositioning decisions $(\hat{\delta}, \hat{\zeta})_N^k$. In step 1.3, we evaluate the objective function value $v_{N'}^k$ via Monte Carlo simulation of $(\hat{\delta}, \hat{\zeta})_N^k$ with a new sample of $N' >> N$ i.i.d scenarios of $(d, \rho, M, V)$.

In step 2, we compute the average of $v_N^k$ and $v_{N'}^k$, among the $K$ replications as $\bar{v}_N = (1/K) \sum_{k=1}^{K} v_N^k$ and $\bar{v}_{N'} = (1/K) \sum_{k=1}^{K} v_{N'}^k$, respectively. The statistical results in Mak et al. (1999) and Linderoth et al. (2006) infer that $\bar{v}_N$ and $\bar{v}_{N'}$ are respectively statistical lower and upper bounds of the optimal value of the SP model. In step 3, we compute the approximate optimality index $|AOI_N = (\bar{v}_{N'} - \bar{v}_N)/\bar{v}_{N'}|$ as a point estimate of the relative optimality gap between $\bar{v}_N$ and $\bar{v}_{N'}$. Finally, if $AOI_N$ satisfies a predetermined termination tolerance, the algorithm terminates and
outputs $N, \overline{v}_N, \overline{v}_N', \text{ and } AOI_N$. Otherwise, we increase the sample size (i.e., $N \leftarrow 2N$), and go to step 1.

4.3. Trade-off Model

As pointed out by Chen et al. (2020), Hurwicz (1951) is arguably the first to present a decision criterion that model the trade-off between pessimistic and optimistic objectives, which has the following formulation under distributional ambiguity.

\[
(1 - \theta) \sup_{P \in \mathcal{F}} \mathbb{E}_P[Q(x, \xi)] + \theta \inf_{P \in \mathcal{F}} \mathbb{E}_P[Q(x, \xi)]
\]  \hspace{1cm} (15)

Where the cost function depends on the here-and-now first-stage decision $x$, and it is typically convex in $\xi$ a given $x$ (our recourse function in the second-stage formulation is also convex). Parameter $\theta \in [0, 1]$ represents the level of optimism. If $\theta = 0$, then problem (15) recovers the DRO criterion that solves the DRO model. On the other hand, if $\theta = 1$, then problem (15) recovers the optimistic criterion which solves the SP model under prefect distributional belief. $0 < \theta < 1$ represent a trade-off between the optimistic and pessimistic perception of the objective value. As in Chen et al. (2020), we formulate the trade-off model as follows

\[
\min_{(o, z)} \left\{ \text{first-stage objective} + (1 - \theta) \sup_{P' \in \mathcal{F}} \mathbb{E}_{P'}[Q(o, z, \xi)] + \theta \mathbb{E}_P[Q(o, z, \xi)] \right\}
\]  \hspace{1cm} (16)

Problem (16) finds first-stage planning decisions that minimize the trade-off between considering distributional ambiguity and following a distributional belief for $\xi$.

5. Computational Experiments

In this section, we present computational experiments comparing the proposed DRO and SP approaches. We also consider a model that minimizes the trade-off between pessimistic (DRO) and optimistic objectives (SP) (see 4.3 for the formulation). We compare these models, both computationally and operationally, using two case studies. In the first, we consider prepositioning relief items for hurricane season (Section 5.1). In the second, we consider prepositioning relief items for earthquakes (Section 5.2). For each, we compare the DRO, SP, and trade-off models’ optimal decisions and their in-sample and out-of-sample simulation performance. In Section 5.3 we study the computational performance of the DRO decomposition algorithm on larger, randomly generated networks. We implemented the models and algorithms in AMPL 2016 with CPLEX V12.6.2 and ran the experiments on a laptop with an Intel Core i7 processor, 2.6 GHz CPU, and 16 GB (2667 MHz DDR4) of memory.
5.1. Hurricane case study

In this section, we consider the planning process for the Atlantic hurricane season in the US. Our data is based on a case study presented in Rawls and Turnquist (2010) and Velasquez et al. (2020). The Atlantic hurricane season runs from June through November every year and affects the North Atlantic Ocean, the Caribbean Sea, and the Gulf of Mexico. One month before the season, experts release predictions on how many hurricanes and major storms to expect (Atlantic Oceanographic and Meteorological Laboratory). However, they do not release specific landfall locations; these are only available immediately before a storm hits land. Based on the expected number of storms, planners seek to determine where to locate water, food, and medical kits and how much of each to keep in selected warehouses.

We present the network of 30 nodes and 112 transportation arcs in Figure E.8 in Appendix E. Each node represents a potential warehouse or demand locations. For illustrative purposes, we consider two types of disasters minor \((l = 1)\) and major \((l = 2)\), and three relief supplies: water, medical kits, and food. Water is assumed to be in 1000 gallons units, medical kits are in single units, and food is stored in the form of ready-to-eat meals (MREs), and is measured in units of 1000 MREs. We summarize the costs of procurement and transportation as well as storage volumes in Table E.7 in Appendix E. We set the fixed cost and capacity for potential warehouses to $188,400 and 408,200\(\text{ft}^3\), respectively, as in Rawls and Turnquist (2010) and Velasquez et al. (2020).

We make the following assumptions: (1) demand is zero at locations that are not potential landfall nodes and uncertain at coastal locations susceptible to hurricanes, (2) only one hurricane can impact each node (it is very unlikely that two hurricanes hit the same location in the same hurricane season Velasquez et al. (2020)), (3) post-disaster procurement is twice as expensive as pre-disaster acquisition (i.e., \(c^p = 2c^a\), as in Velasquez et al. (2020)), (4) shortage cost is four times of the acquisition cost (i.e., \(c^u = 4c^a\)), and the holding cost equal to the acquisition cost (i.e., \(c^h = c^a\)). These are consistent with prior literature Sabbaghtorkan et al. (2020).

Our uncertainty is defined as follows. We present the mean demand after a disaster at each landfall node in Table E.8 in Appendix E. For notation convenience, we let \(t = 1, t = 2, \) and \(t = 3\) respectively represent water, food, and medical kits. We set the standard deviation \(\sigma_i^a\) of the demand for each relief item \(t\) to 0.5\(\mu_i^a\). For the fraction of usable prepositioned items available after the disaster, we let \(\mu_i^{p,1} = 0.5\) and \(\{\bar{\rho}_{i,2}, \overline{\rho}_{i,1}\} = [0.4, 0.6]\) (i.e., 10% below and above \(\mu_i^p = 50\%\)), and \(\mu_i^{p,2} = 0.05\) and \(\{\underline{\rho}_{i,2}, \overline{\rho}_{i,1}\} = [0, 0.10]\). The maximum procurement quantity of each relief item \(t\) is \(M_{t,i,l} = 2\mu_i^{a,1}\). We let \(\mu_i^{M,1} = 2\mu_i^{a,1}\) and \([M_{t,i,l}, \overline{M}_{t,i,l}] = [0.9\mu_i^{M,1}, 1.10\mu_i^{M,1}]\). In the text, we designate the number of minor and major disasters to be nMinor and nMajor, respectively. For illustrative purposes, we consider two combinations, where (nMinor, nMajor) is (2, 1) and (4, 2), respectively. To limit the number of minor and major disasters to nMinor and nMajor, respectively,
Table 3: The amount and locations of prepositioned relief supplies. Medium Facility

| nMinor | nMajor | Model   | # Open | Location   | Water    | Food    | Medical Kits |
|--------|--------|---------|--------|------------|----------|----------|--------------|
| 2      | 1      | SP      | 2      | 26, 27     | 3,550    | 3,507    | 9,283        |
| 7      |        | DRO     | 5      | 5, 8, 10, 11, 12, 24, 30 | 15,273  | 18,800  | 133,243      |
|        |        |         |        | Trade (0.3) | 12, 19, 24, 25, 26 | 7,768   | 9,679   | 48,701       |
|        | 5      | Trade (0.5) | 10, 12, 14, 24, 30 | 7,309 | 6,257 | 34,902 |
|        |        | Trade (0.7) | 2, 12, 21, 23, 30 | 4,572 | 5,131 | 35,520 |
| 4      | 2      | SP      | 5      | 10, 12, 23, 24, 27 | 8,753 | 8,849 | 32,742 |
| 10     |        | DRO     | 9      | 3, 6, 9, 10, 12, 14, 24, 27, 28, 30 | 11,818 | 18,701 | 126,496 |
|        |        |         |        | Trade (0.3) | 3, 8, 10, 12, 19, 26, 27, 28, 30 | 13,587 | 19,245 | 90,875 |
|        | 9      | Trade (0.5) | 2, 12, 14, 21, 25, 26, 27, 28, 30 | 12,684 | 13,017 | 84,550 |
|        |        | Trade (0.7) | 8      | 10, 12, 14, 21, 26, 27, 30 | 10871 | 10226 | 80713 |

We add constraints $\sum_{i\in I} q_{i,1} = n_{\text{Minor}}$ and $\sum_{i\in I} q_{i,1} = n_{\text{Major}}$ to the sub-problem in (13).

To approximate the lower and upper bound of $(d, V)$, we follow the same procedure as in prior DRO studies (see, e.g., Jiang et al. (2017); Shehadeh (2020)) as follows. We first generate $N = 1000$ in-sample data of $d$ and $V$ by following a lognormal (LogN) and a truncated normal distribution (as in Ni et al. (2018)), respectively, with the generated mean values and standard deviations of these random parameters. Second, we respectively use the 20%-quantile and 80%-quantile values of the $N$ in-sample data as approximations of lower and upper bound of each random parameter.

5.1.1. Optimal solutions

In this section, we compare the optimal preposition decisions for the hurricane season yielded by the DRO, SP, and trade-off models. The trade-off (denoted as Trade henceforth) considers both the DRO and SP recourse objectives that are weighted by a parameter $\theta$ (see Section 4.3). We present results for $\theta = 0.3, 0.5, 0.7$ (low weight on the DRO objective; equally weighted objectives; and high weight on the DRO objective, respectively). We run the experiments twice; first with medium-sized facilities ($S_i = 408,200$, $f_i = $188, 400) and second with large facilities ($S_i = 780,000$, $f_i = $300,000). For the SP model, we first used the MCO Algorithm in Section 4.2 to determine an appropriate sample size to use for SAA. Based on the results presented in Tables F.9 in Appendix F we used a sample size of $N = 100$.

We present the results for the amount and location of prepositioned relief supplies in Table 3 (medium-sized facilities) and Table 4 (large facilities). We make the following observations. First, to mitigate random parameters’ distributional ambiguity, the DRO model opens a larger number of facilities and prepositions a larger quantity of each relief item than the SP model (for either medium or large facilities). By doing so, the DRO model satisfies a larger amount of demand (reflected by significantly smaller shortage cost in Table 5 and Figures 1-3 presented later in section 5.1.2) and procures fewer relief items post-disaster (see Section 5.1.2).

Second, the trade-off model opens less (more) facilities and prepositions smaller (larger) amounts of relief items than the DRO (SP) model. This results in satisfying a smaller (larger) demand and procuring a larger (smaller) amount of relief items than the DRO (SP) model (see Section 5.1.2).
Table 4: The amount and locations of prepositioned relief supplies. Large Warehouses

| nMinor | nMajor | Model   | # Open | Location | Water  | Food  | Medical Kits |
|--------|--------|---------|--------|----------|--------|--------|--------------|
| 2      | 1      | SP      | 2      | 12, 27   | 4,170  | 4,017  | 13,146       |
|        |        | DRO     | 4      | 10, 12, 27, 28 | 5,875  | 10,152 | 39,510       |
|        |        | Trade (0.3) | 4      | 10, 18, 24, 25 | 7,685  | 9,677  | 48,701       |
|        |        | Trade (0.5) | 4      | 12, 14, 18, 30 | 7,411  | 6,495  | 40,946       |
|        |        | Trade (0.7) | 3      | 2, 12, 30   | 4,669  | 5,117  | 35,304       |
| 4      | 2      | SP      | 3      | 10, 12, 27   | 9,081  | 8,879  | 32,766       |
|        |        | DRO     | 6      | 3, 9, 12, 27, 28, 30 | 13,864 | 18,364 | 121,251      |
|        |        | Trade (0.3) | 7      | 10, 12, 14, 19, 27, 28, 30 | 13,235 | 19,606 | 91,808       |
|        |        | Trade (0.5) | 6      | 8, 12, 20, 26, 27, 30 | 13,683 | 13,017 | 82,241       |
|        |        | Trade (0.7) | 5      | 10, 12, 19, 27, 30  | 10,501 | 10,313 | 77,522       |

Fourth, we observe that the trade-off model with $\theta = 0.3$ and $\theta = 0.5$ opens the same or larger number of facilities than when $\theta = 0.7$. For example, when the facility size is medium and (nMinor, nMajor)=(4,2), Trade (0.3), Trade (0.5), Trade (0.7) opens 9, 9, and 8 facilities, respectively. Fifth, we observe that using all models, the decision-maker opens fewer large facilities than medium-sized facilities. This makes sense as facilities have a larger capacity and are more expensive in the former case.

5.1.2. Out-of-sample simulation performance

In this section, we compare how the DRO, SP, and Trade models perform when the underlying uncertainty distributions are perfectly specified and misspecified. For brevity, we present results for (nMinor, nMajor)=(4,2) disasters. We evaluate the distributional information as follows. First, we fix the optimal first-stage decisions $(o, z)$ yielded by each model in the SP model. Then, we solve the second-stage recourse problem in (2) using the following two sets of $N' = 10,000$ out-of-sample data $(q^n, d^n, M^n)$, for all $n \in [N']$, to compute the corresponding second-stage procurement, shortage, holding, and transportation costs. To generate the $N'$ out of sample data points:

1. **Perfect distributional information:** we use the same parameter settings and distributions that we use for generating the $N$ in-sample data points to generate the $N'$ data points.

2. **Misspecified distributional information:** we vary the distribution type of random parameters to generate the $N'$ data (Wang et al., 2020). That is, we perturb the distribution of the demand by a parameter $\Delta$ and obtain a parameterized uniform distribution $[(1-\Delta)d, (1+\Delta)d]$, for which a higher value of $\Delta$ corresponds to a higher variation level (we similarly perturb the distribution of the other parameters). We apply $\Delta \in \{0, 0.25, 0.5\}$. A zero value of $\Delta$ indicates that we do not perturb the demand’s distribution, but rather we simulate the optimal solutions under a uniform distribution defined on the range of the demand (i.e., we vary the in-sample distribution). We generate 10,000 samples from these uniform distribution to test the performance of the optimal solutions obtained from the DRO, SP, and Trade models. This is to simulate the performance of the models’ optimal decisions when the in-sample data is biased (i.e., true distributions are different).
Table 5: Out-of-sample performance of optimal solutions under perfect distributional information (LogN).

| Model | Metric | Total | 2nd-stage | Procurement | Shortage | Holding | Shipping | TC% | 2nd% |
|-------|--------|-------|-----------|-------------|----------|---------|----------|-----|------|
| SP    | Mean   | 105,253,797 | 46,097,297 | 38,601,400 | 2,028,850 | 4,541,360 | 925,687 | 925,687 |
|       | Median | 86,748,227 | 27,591,727 | 25,011,200 | 0        | 1,695,540 | 88,4987 | 88,4987 |
|       | 0.75q  | 208,476,580 | 149,320,080 | 116,009,000 | 13,025,600 | 18,806,900 | 1,478,580 | 1,478,580 |
|       | 0.95q  | 131,500,250 | 72,343,750 | 64,561,900 | 0        | 6,699,120 | 1,082,730 | 1,082,730 |
| DRO   | Mean   | 182,855,732 | 54,250,870 | 4,463,470 | 78,920 | 48,577,500 | 1,130,980 | 15%  |
|       | Median | 182,503,295 | 53,898,433 | 712,803 | 0        | 51,754,000 | 1,151,800 | 49%  |
|       | 0.75q  | 202,630,482 | 74,025,620 | 15,354,700 | 0        | 66,763,800 | 1,436,730 | 2%   |
|       | 0.95q  | 225,013,272 | 96,408,410 | 15,354,700 | 0        | 79,187,500 | 1,866,210 | -55% |
| Trade (0.3) | Mean | 180,091,614 | 52,564,014 | 3,017,060 | 36,284 | 48,356,700 | 1,153,970 | 12%  |
|       | Median | 180,433,400 | 52,905,800 | 0 | 0 | 51,754,000 | 1,151,800 | 48%  |
|       | 0.75q  | 198,290,080 | 70,762,480 | 3,202,880 | 0        | 66,061,900 | 1,497,700 | 2%   |
|       | 0.95q  | 222,007,540 | 94,569,940 | 12,902,200 | 0        | 79,801,300 | 1,866,440 | -58% |
| Trade (0.5) | Mean | 133,651,749 | 35,698,849 | 15,256,300 | 254,479 | 19,033,700 | 1,154,370 | -29% |
|       | Median | 122,670,790 | 24,717,890 | 6,198,860 | 0        | 17,410,300 | 1,108,730 | -12% |
|       | 0.75q  | 149,469,100 | 51,516,260 | 17,495,900 | 0        | 32,596,100 | 1,424,260 | -40% |
|       | 0.95q  | 210,935,320 | 112,982,420 | 66,457,100 | 0        | 44,675,600 | 1,849,720 | -32% |
| Trade (0.7) | Mean | 115,928,789 | 40,655,689 | 29,351,400 | 547,039 | 9,753,760 | 1,003,490 | -13% |
|       | Median | 96,168,583 | 20,895,483 | 13,690,400 | 0        | 6,252,450 | 952,633 | -32% |
|       | 0.75q  | 141,537,780 | 66,264,680 | 48,064,900 | 0 | 16,916,300 | 1,283,480 | 7%   |
|       | 0.95q  | 205,841,650 | 130,568,550 | 97,149,600 | 0        | 79,801,300 | 1,669,440 | -14% |

The out-of-sample results with perfect distributional information are presented in Table 5. We make the following observations. The SP model clearly has the lowest total cost among all models. However, the DRO and Trade models satisfy more post-disaster demand by opening more facilities and allocating more relief items (resulting in a higher fixed and total costs). This is reflected by a significantly smaller shortage cost on average and at all quantiles, a lower post-disaster procurement cost, and a significantly lower second-stage cost at the higher quantiles (i.e., 0.75-q and 0.95-q). Trade (0.7) has a similar total cost to that of SP. However, Trade (0.7) has a significantly lower second-stage cost, i.e., a better post-disaster operational performance. Note that it is not surprising that the total cost of the SP is lower than that that in DRO and trade-off models since we assume perfect information of the exact demand distribution in this simulation.

We present the results from misspecified distributions in Figures 1-3. These show the normalized histogram of out-of-sample objective value (i.e., total cost) and second-stage cost for each of the three levels of variation, $\Delta$. It is quite evident that the DRO and Trade (0.3) solutions consistently outperform the solutions from the SP, Trade (0.5), and Trade (0.7) models. This relationship holds for all levels of variation, $\Delta \in \{0, 0.25, 0.5\}$, and across the criteria of mean and quantiles of total and second stage costs. In particular, the DRO model has substantially lower shortage costs than the other considered models. For example, when $\Delta = 0.25$, the average shortage costs of the DRO is 2,533. In contrast, the average shortage costs for the Trade (0.3), Trade (0.5), Trade (0.7), and SP are 1,021,290, 3,252,750, 10,478,500, and 34,592,400, respectively. In addition, the DRO and Trade (0.3) seem to be more stable in terms of attaining the lowest standard deviations (i.e., variation) in the total and second-stage costs. The superior performance of DRO and Trade (0.3) models, which focus on hedging against distributional ambiguity, reflects the value of modeling the distributional ambiguity of random parameters.
5.2. Earthquake Case Study

To further validate our approach, we consider a second case study for earthquake response. In this context, the capacities of the transportation arcs are affected. We use data from the 2010 earthquake that hit Yushu County in Qinghai Province, China (presented in Ni et al. (2018)). This 7.1 magnitude earthquake caused large-scale social and economic destruction. Figure G.9 in Appendix G shows a diagram of the affected area, which consists of 13 nodes and 15 road links.

When an earthquake hits, the nodes closest to the epicenter may receive the worst damage, whereas others may have fewer effects. This characteristic is reflected in how we estimate the
Table 6: The amount and locations of prepositioned relief supplies. Earthquake case study

| Model     | # Open | Location | Amount | Fixed | Acquisition |
|-----------|--------|----------|--------|-------|-------------|
| SP        | 4      | 7        | 136    | 659   | 3843        |
|           | 9      | 6        | 462    | 566   |             |
|           | 10     | 6        | 268    | 681   |             |
| DRO       | 4      | 8        | 800    | 735   | 6099        |
|           | 8      | 8        | 406    | 407   |             |
|           | 11     | 7        | 402    | 402   |             |
|           | 13     | 7        | 154    | 154   |             |
| Trade (0.3) | 4   | 6        | 641    | 735   | 5117        |
|           | 8      | 6        | 308    | 308   |             |
|           | 11     | 6        | 407    | 402   |             |
|           | 13     | 7        | 154    | 154   |             |
| Trade (0.5) | 4   | 7        | 344    | 691   | 4140        |
|           | 8      | 7        | 333    | 333   |             |
|           | 11     | 7        | 386    | 386   |             |
|           | 13     | 7        | 154    | 154   |             |
| Trade (0.7) | 4   | 7        | 351    | 691   | 3694        |
|           | 8      | 8        | 219    | 219   |             |
|           | 11     | 8        | 362    | 362   |             |
|           | 13     | 7        | 154    | 154   |             |

post-disaster proportion of usable inventory $\mu^\rho$ in Table G.10 in Appendix G. We follow the seismic intensity categories presented in Ni et al. (2018). Table G.10 also presents data on the fixed, acquisition, shortage, and holding costs. As in Ni et al. (2018), we set $\mu^d_i = 100$, $\sigma^d_i = 10$, $\sigma^\rho = 0.1$, $\mu^V = 300$, and $\sigma^V = 30$. We let $\mu^M = \mu^d$, $\sigma^M = 0.5\mu^d$, $[\underline{M} , \overline{M}] = [0.9\mu^M , 1.10\mu^M]$], and $[\underline{\rho} , \overline{\rho}] = [0.9\mu^\rho , 1.10\mu^\rho]$. To approximate the lower and upper bounds on $(d, V)$, we follow the same procedure described earlier in Section 5.1.

5.2.1. Analysis of optimal solutions

In this section, we compare the optimal prepositioning decisions yielded by the DRO, SP, and trade-off models and their out-of-sample simulation (i.e., operational) performance. The optimal prepositioning decisions are shown in Table 6. Each of the models open four facilities. They all select the facility at node 11, though this is the only facility shared between the DRO and SP solutions. The DRO and Trade (0.3) models allocate more relief items than the others. As such, the DRO and Trade (0.3) models result in the higher acquisition costs. Next, we analyze how these optimal solutions perform via in-sample and out-of-sample simulation. We follow the same procedure described in Section 5.1.2, and the results are presented in Figures 4-7.

When the decision-maker has perfect distributional information, the total cost and second stage costs are reflected in Figure 4. We first observe that the DRO and Trade (0.3) models yield the highest total cost. This makes sense because these models have the highest prepositioning cost. However, the DRO model has the lowest second-stage cost, followed by the Trade (0.3) model (Figures 4a). These models better hedge against uncertainty, which is realized in the second-stage.

The results for the out-of-sample performance are shown in Figures 5-7. These reflect the context where the decision-maker has misspecified the distributional information. We apply variation levels of $\Delta = \{0, 0.25, 0.5\}$. It is clear from these figures that the DRO and Trade (0.3) models
maintain a robust performance. They have a substantially lower second-stage cost, under mis-
specified distributions and increased variability. Consistent with the results in Section 5.1.2, the
DRO and Trade (0.3) show better stability attaining the lowest variations in the second-stage and
total costs across all scenarios. When the level of variation is low, $\Delta = 0$, the DRO and Trade
(0.3) models result in the highest total costs. Yet, as $\Delta$ increases, the total costs yielded become
comparable.

The DRO and Trade (0.3) models focus on hedging against uncertainty and distributional ambi-
guity. Their out-of-sample performance underscore the value of incorporating both the uncertainty
and ambiguity into inventory prepositioning models.

5.3. Computational Performance of DRO-Decomposition Approach

In this section, we study the performance of the DRO-decomposition algorithm on larger, randomly
generated networks. We follow the procedure in [Ni et al.] (2018) to generate networks with $|I| \in$
\{40, 60, 100\} nodes. Specifically, for each instance with $|I|$ nodes, we first generate the nodes on
10$\times$10 square, then we label them from 1 to $|I|$. Second, we construct a spanning tree by connecting
nodes $i$ and $j$ for any $i \in I \setminus \{1\}$ and some $j$ randomly selected from the set \{1, $\ldots$, $i - 1$\}. Third,
we randomly generate $0.2 |I| + 1$ pairs of nodes and add the corresponding undirected arcs to the
network. Each network contains $1.2|I|$ undirected arcs. We use the Euclidean metric to compute
the distance between each pair of nodes $i$ and $j$ of arc $(i,j)$.

We use the set of relief items from Case Study 1 (in Table E.7). Similar to Section 5.1-5.2, the uncertain parameters $(q, d, \rho, M, V)$ are characterized by their mean values and ranges. For these experiments, we set $\mu_d^{\text{water}}, \mu_d^{\text{food}}$, and $\mu_d^{\text{kits}}$ to their average values across all nodes in Table E.8. We uniformly generate $\mu_{ij}^{V}$ from $U(20|I|, 25|I|)$ and $\sigma_{ij}^{V}$ from $U(2|I|, 2.5|I|)$, as in Ni et al. (2018). We generate $\mu_{\rho}$ and the range of each random parameters as described in Section 5.1. For each network $|I| \in \{40, 60, 100\}$, we conduct experiments with $(n_{\text{Minor}}, n_{\text{Major}}) \in \{(2,1), (4,2), (6,3)\}$ and disaster-prone nodes $\in \{10, 15, 20\}$ for a total of 27 instances.

The details for the experiments are presented in Tables H.11-H.13 in Appendix H for $(n_{\text{Minor}}, n_{\text{Major}}) = (2,1), (4,2), \text{and} (6,3)$, respectively. In each, we report the number of iterations of Algorithm 1 before it converges to the optimum (# of Iter), total CPU seconds taken by the master and the subproblems, the total number of branching nodes (# of B&B), and the total number of MIP simplex iterations (# of MIPiter). From these results, we first observe that solution times and computational effort increase as the potential number of disasters increases. For example, the ranges of solution times increases from $(2-44)$ to $(4.1-2748)$ to $(3.3-3618)$ seconds as $(n_{\text{Minor}}, n_{\text{Major}})$ increases from $(2,1)$ to $(4,2)$ to $(6,3)$, respectively. This makes sense because an increase in the total number of disasters implies an increase in random parameters. For example, under $(4,2)$, at most 6 nodes will generate random demands vs. at most 9 nodes with $(6,3)$. 

Figure 6: Comparison of out-of-sample simulation results under misspecified distribution, $\Delta = 0.25$

Figure 7: Comparison of out-of-sample simulation results under misspecified distribution, $\Delta = 0.50$
Second, we observe that for a fixed network size ($|I|$), the computational effort increases as the number of nodes vulnerable to disaster increases. From 10 vulnerable nodes to 20, the (solution time; # of B&B; # of MIPiter) increase from (4.1; 13,668; 185,751) to (1,282; 3,668,080; 43,998,783), given (nMinor, nMajor)=(4,2) and $I = 40$. Third, we observe that the solution times of the subproblem are significantly larger than the master problem. This makes sense because our subproblem is a MILP, and the size of this MILP increases as $|I|$, the number of disasters, and the number of nodes prone to disaster increases. As pointed out by Artigues et al. (2015); Keha et al. (2009); Klotz and Newman (2013), an increase in MILP size suggests an increase in solution time for the linear programming (LP) relaxation of the MILP and, thus, the solution time via commercial solvers.

These results show that our algorithm can solve large instances of the problem within a reasonable time as all solution times are within one hour. In fact, it can solve all nine instances corresponding to (2,1) within a few seconds. Except when the number of disaster nodes is 20, the range of solution times is 4 seconds to 5 minutes under (nMinor, nMajor)=(4,2) and 4 seconds to 22 minutes under (6,3).

6. Conclusion

In this paper, we study the location and inventory prepositioning of disaster relief supplies under uncertainty. Specifically, given a set of warehouse locations, a set of demand nodes, and a set of relief items, we want to determine the number and locations of warehouses to open and each relief item’s quantity to preposition at each open location (preparedness phase). In the aftermath post-disaster, we consider distributing prepositioned relief items to demand locations and procuring and distributing additional supplies as needed. We consider the following random factors (1) type of disaster, (2) locations of affected areas, (3) demand of relief items, (4) usable fraction of prepositioned items post-disaster, (5) procurement quantity, and (6) arc capacity between two different nodes. To model this uncertainty, we propose and analyze two stochastic optimization models—a two-stage SP and a two-stage DRO, assuming known and unknown distributions of uncertainty, respectively. The first and second stages of these models correspond to pre- and post-disaster phases, respectively. We propose a decomposition algorithm to solve the DRO model and a Monte Carlo Optimization procedure to obtain near-optimal solutions to the SP.

We conduct extensive experiments using a hurricane season and earthquake case studies, and it is clear that the DRO approach outperforms the SP approach when the distribution of the uncertain parameters is misspecified. It is only when the distribution is known with certainty (perfect information) that the SP approach performs better than DRO. Uncertainty is a common theme in the humanitarian relief context, and there is often uncertainty about the uncertainty itself (distributional ambiguity). Our results suggest that incorrectly assuming that a distribution is
known with certainty could lead to poor results in practice. Yet, for decision-makers hesitant to apply the inherently pessimistic DRO approach, we also present a trade-off model that considers both the (pessimistic) DRO and (optimistic) SP models. For misspecified distributions, using a trade-off model can mitigate the pessimism of a pure DRO approach while maintaining better performance than a pure SP with a misspecified distribution.

Taken together, our results illustrate the (1) the applicability of our approach to multiple types of humanitarian logistics problems, (2) the computational efficiency of our approach, (3) the robustness of DRO prepositioning decisions compared to SP decisions, especially under misspecified distributions and high variability, and (4) the trade-off between considering distributional ambiguity (DRO) and following distributional belief (SP). More broadly, our results draw attention to the need to model the distributional ambiguity of uncertain problem data in strategic real-world stochastic optimization problems such as planning for disasters. We encourage researchers, especially in uncertain context of humanitarian logistics, to consider the quality of the information they have when parameterizing their models. This work indicates that DRO may provide a helpful intermediary between the perfect information required for stochastic programs and the lack of information used in robust optimization approaches.

We suggest the following areas for future research. First, we assumed that the random parameter distributions are unimodal. We would like to extend our approach by incorporating multi-modal probability distributions and higher moments of random parameters in a data-driven DRO approach. Partnerships with disaster relief agencies could provide data for new contexts and further improve the realism of the models. Second, we aim to include the possibility of restoring arcs functionality provided and the movement on restored arcs or during the restoration process. In particular, we would like to study how both the availability and restoration of damaged transportation networks impact preparedness and response decisions. Third, incorporating the cost of human suffering incurred by the shortage of relief supplies in the post-disaster relief operations would be another relevant and useful extension of our approach. We believe DRO is a natural fit for modeling disaster relief and have aimed to lay the groundwork for continued work in this area.

Acknowledgment
We want to thank all of our colleagues who have contributed significantly to the related literature as well as the many disaster relief practitioners who tirelessly plan to reduce the impact of these events. Dr. Karmel S. Shehadeh dedicates her effort in this paper to every little dreamer in the whole world who has a dream so big and so exciting. Believe in your dreams and do whatever it takes to achieve them—the best is yet to come for you.
Appendix A. Proof of Proposition 1

Proof. For feasible first-stage decisions \((o,z)\), we can formulate problem (6) as the following linear functional optimization problem.

\[
\max_{P \geq 0} \int R Q(o, z, \xi) \, dP \\
\text{s.t.} \int R d_{t,i,l} \, dP = \mu_{t,i,l}^d \\
\int R M_{t,i,l} \, dP = \mu_{t,i,l}^M \\
\int R \rho_{t,l} \, dP = \mu_{t,l}^\rho \\
\int R V_{i,j,l} \, dP = \mu_{i,j,l}^V \\
\int R q_{i,l} \, dP = \mu_{i,l}^q \\
\int R \, dP = 1
\]

(A.1a) \hspace{1cm} (A.1b) \hspace{1cm} (A.1c) \hspace{1cm} (A.1d) \hspace{1cm} (A.1e) \hspace{1cm} (A.1f) \hspace{1cm} (A.1g)

Letting \(\alpha_{t,i,l}, \phi_{t,i,l}, \gamma_{i,l}, \tau_{i,j,l}, \lambda_{i,l}, \) and \(\theta\) be the dual variables associated with constraints (A.1b)–(A.1g), we present problem (A.1) in its dual form:

\[
\min \left\{ \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} \left( \mu_{t,i,l}^d \alpha_{t,i,l} + \mu_{t,i,l}^M \phi_{t,i,l} \right) + \sum_{i \in I} \left( \mu_{t,i,l}^\rho \gamma_{i,l} + \mu_{t,i,l}^q \lambda_{i,l} \right) + \sum_{(i,j) \in A} \mu_{i,j,l}^V \tau_{i,j,l} \right] + \theta \right\} \\
\text{s.t.} \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l} + \sum_{i \in I} \rho_{t,l} \gamma_{i,l} + q_{t,l} \lambda_{i,l} + \sum_{(i,j) \in A} V_{i,j,l} \tau_{i,j,l} \right] \\
+ \theta \geq Q(o, z, \xi), \ \forall \xi \in R
\]

(A.2a) \hspace{1cm} (A.2b)

where \(\alpha, \phi, \gamma, \tau, \lambda, \) and \(\theta\) are unrestricted in sign, and constraint (A.2b) is associated with the primal variable \(P\). Note that strong duality hold between (A.1) and (A.2) under the standard assumptions that (1) \(\mu_{t,i,l}^d\) lies in the interior of the set \(\{\int_R d_{t,i,l} \, dQ : Q \text{ is a probability distribution over } R\}\), (2) \(\mu_{t,i,l}^\rho\) lies in the interior of the set \(\{\int_R \rho_{t,l} dQ : Q \text{ is a probability distribution over } R\}\), (3) \(\mu_{i,j,l}^V\) lies in the interior of the set \(\{\int_R V_{i,j,l} dQ : Q \text{ is a probability distribution over } R\}\), (4) \(\mu_{t,i,l}^M\) lies in the interior of the set \(\{\int_R M_{t,i,l} dQ : Q \text{ is a probability distribution over } R\}\) (Bertsimas and Popescu 2005; Jiang et al. 2017; Shehadeh 2020). Note that constraint (A.2b) is equivalent to \(\theta \geq \max_{\xi \in R} \{Q(o, z, \xi) + \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} -d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l} \right] + \sum_{i \in I} -\left( \rho_{t,l} \gamma_{i,l} + q_{t,l} \lambda_{i,l} \right) + \sum_{(i,j) \in A} -V_{i,j,l} \tau_{i,j,l} \}\).

Since we are minimizing \(\theta\) in (A.2), the dual formulation of (A.1) is equivalent to (7).
Appendix B. McCormick Inequalities

\[ k_{t,i,l} \geq g_{i,l} \beta_{t,i}, \quad k_{t,i,l} \geq \beta_{t,i} + \beta_{t,i}(q_{i,l} - 1), \quad k_{t,i,l} \leq q_{i,l} \beta_{t,i}, \quad k_{t,i,l} \leq \beta_{t,i} + \beta_{t,i}(q_{i,l} - 1) \]  
\[ (B.1a) \]

\[ h_{t,i,l} \geq k_{t,i,l} d_{t,i,l} + d_{t,i,l}(k_{t,i,l} - k_{t,i,l}), \quad h_{t,i,l} \geq k_{t,i,l} d_{t,i,l} + d_{t,i,l}(k_{t,i,l} - k_{t,i,l}) \]  
\[ (B.1b) \]

\[ h_{t,i,l} \leq k_{t,i,l} d_{t,i,l} + d_{t,i,l}(k_{t,i,l} - k_{t,i,l}), \quad h_{t,i,l} \leq k_{t,i,l} d_{t,i,l} + d_{t,i,l}(k_{t,i,l} - k_{t,i,l}) \]  
\[ (B.1c) \]

\[ g_{t,i,l} \geq k_{t,i,l} \rho_{t,i} + \rho_{t,i}(k_{t,i,l} - k_{t,i,l}), \quad g_{t,i,l} \geq k_{t,i,l} \rho_{t,i} + \rho_{t,i}(k_{t,i,l} - k_{t,i,l}) \]  
\[ (B.1d) \]

\[ g_{t,i,l} \leq k_{t,i,l} \rho_{t,i} + \rho_{t,i}(k_{t,i,l} - k_{t,i,l}), \quad g_{t,i,l} \leq k_{t,i,l} \rho_{t,i} + \rho_{t,i}(k_{t,i,l} - k_{t,i,l}) \]  
\[ (B.1e) \]

\[ \Delta_{t,i,l} \geq q_{i,l} \Delta_{t,i,l}, \quad \Delta_{t,i,l} \geq \Gamma_{t,i}, \quad \Delta_{t,i,l} \leq 0, \quad \Delta_{t,i,l} \leq \Gamma_{t,i} + \Gamma_{t,i}(q_{i,l} - 1) \]  
\[ (B.1f) \]

\[ \eta_{i,j} \geq q_{i,j} \psi_{i,j}, \quad \eta_{i,j} \geq \psi_{i,j}, \quad \eta_{i,j} \leq 0, \quad \eta_{i,j} \leq \psi_{i,j} + \psi_{i,j}(q_{i,j} - 1) \]  
\[ (B.1g) \]

\[ \varpi_{i,j} \geq q_{i,j} \varpi_{i,j}, \quad \varpi_{i,j} \geq \varpi_{i,j} - \varpi_{i,j}, \quad \varpi_{i,j} \leq 0, \quad \varpi_{i,j} \leq \varpi_{i,j} + \varpi_{i,j}(q_{i,j} - 1) \]  
\[ (B.1h) \]

\[ \Phi_{i,j,l} \geq \eta_{i,j,l} V_{i,j,l} + V_{i,j,l}(\eta_{i,j,l} - \eta_{i,j,l}), \quad \Phi_{i,j,l} \geq V_{i,j,l} \eta_{i,j,l} \]  
\[ (B.1i) \]

\[ \Phi_{i,j,l} \leq \eta_{i,j,l} V_{i,j,l} + V_{i,j,l}(\eta_{i,j,l} - \eta_{i,j,l}), \quad \Phi_{i,j,l} \leq V_{i,j,l} \eta_{i,j,l} \]  
\[ (B.1j) \]

\[ \Lambda_{i,j,k} \geq \varpi_{i,j,k} V_{i,j,k} + V_{i,j,k}(\varpi_{i,j,k} - \varpi_{i,j,k}), \quad \Lambda_{i,j,k} \geq V_{i,j,k} \varpi_{i,j,k} \]  
\[ (B.1k) \]

\[ \Lambda_{i,j,k} \leq \varpi_{i,j,k} V_{i,j,k} + V_{i,j,k}(\varpi_{i,j,k} - \varpi_{i,j,k}), \quad \Lambda_{i,j,k} \leq V_{i,j,k} \varpi_{i,j,k} \]  
\[ (B.1l) \]

\[ \sigma_{t,i,l} \geq \Delta_{t,i,l} M_{t,i,l} + M_{t,i,l}(\Delta_{t,i,l} - \Delta_{t,i,l}), \quad \sigma_{t,i,l} \geq M_{t,i,l} \Delta_{t,i,l} \]  
\[ (B.1m) \]

\[ \sigma_{t,i,l} \leq \Delta_{t,i,l} M_{t,i,l} + M_{t,i,l}(\Delta_{t,i,l} - \Delta_{t,i,l}), \quad \sigma_{t,i,l} \leq M_{t,i,l} \Delta_{t,i,l} \]  
\[ (B.1n) \]

Appendix C. Strengthening the MILP Formulation

First, observe from constraint (B.1d) that \(-c_{t,i}^0 \leq \beta_{t,i} \leq c_{t,i}^0\). Thus, w.l.o.g, we can assume that \(\beta_{t,i} = -c_{t,i}^0\) and \(\beta_{t,i} = c_{t,i}^0\). Second, given that \(k_{t,i,l} = q_{i,l} \beta_{t,i}\), then w.l.o.g., \(k_{t,i,l} \in [\beta_{t,i}, \beta_{t,i}]\). Third, observe from constraints (B.1b) and (B.1c) that \(\Gamma_{t,i} \leq c_{t,i}^0 - \beta_{t,i} \) and \(\Gamma_{t,i} \leq 0\). Note that in the optimal solution \(\Gamma_{t,i} = \min\{c_{t,i}^0 - \beta_{t,i}, 0\}\). Therefore, \(\Gamma_{t,i} = 0\). And \(\Gamma_{t,i} = c_{t,i}^0 - c_{t,i}^0\) if \(c_{t,i}^0 \leq c_{t,i}^0\), and \(\Gamma_{t,i} = 0\) otherwise. Fourth, given that \(\Delta_{t,i,l} = q_{i,l} \Gamma_{t,i,l}\), then w.l.o.g., \(\Delta_{t,i,l} = \Gamma_{t,i,l}\) and \(\Delta_{t,i,l} = 0\).

Finally, from constraints (B.1d), \(v_i \psi_{i,j} + v_i \varphi_{i,j} \leq c_{i,j}^0 + \beta_{t,j} - \beta_{t,j}\). Given that \(\psi_{i,j} \leq 0\) and \(\varphi_{i,j} \leq 0\), then w.l.o.g., \(\psi_{i,j} \leq \min\{1/v_i (c_{i,j}^0 - c_{i,j}^0)\} \) if \(c_{i,j}^0 \leq c_{i,j}^0\) and \(\psi_{i,j} = 0\) otherwise. It is easy to verify that \(\eta \in [\psi, 0]\) and \(\varpi \in [\varphi, 0]\).
Appendix D. Proof of Proposition 2

Proposition 2. For any feasible first-stage decision $z$, $H(z, \xi) < \infty$. Furthermore, $H(z, \xi)$ is a convex piecewise linear function in $z$.

Proof. First, note that the feasible regions $\Omega := \{(9b) - (9h), (B.1a) - (B.1n)\}$ and $\mathbb{R}$ are independent of $z$ and bounded. Therefore,

$$\max \left\{ \sum_{i \in I} \sum_{t \in T} \left[ \sum_{l \in L} (h_{t,i,l} - g_{t,i,l}z_{t,i}) - (\beta_{t,i} - \sum_{l \in L} k_{t,i,l})z_{t,i} + \sum_{l \in L} \sigma_{t,i,l} \right] \right.$$  

$$+ \sum_{(i,j) \in A} \left[ \sum_{l \in L} \Phi_{i,j,l} + (\psi_{i,j} - \sum_{l \in L} \eta_{i,j,l})\hat{V}_{i,j} + \sum_{l' \in L} A_{i,j,l'} + (\varphi_{i,j} - \sum_{l' \in L} \omega_{i,j,l'})\hat{V}_{i,j} \right]$$  

$$+ \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} \left( d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l} \right) + \sum_{i \in I} - (\rho_{i,l} \gamma_{i,l} + q_{i,l} \lambda_{i,l}) + \sum_{(i,j) \in A} - V_{i,j,l} \tau_{i,j,l} \right] \right\} < \infty$$

Second, for any feasible $(\beta, \Gamma, \psi, \varphi, d, \rho, k, h, g, \Phi, \Lambda, \sigma, \Delta, \eta)$

$$\max \left\{ \sum_{i \in I} \sum_{t \in T} \left[ \sum_{l \in L} (h_{t,i,l} - g_{t,i,l}z_{t,i}) - (\beta_{t,i} - \sum_{l \in L} k_{t,i,l})z_{t,i} + \sum_{l \in L} \sigma_{t,i,l} \right] \right.$$  

$$+ \sum_{(i,j) \in A} \left[ \sum_{l \in L} \Phi_{i,j,l} + (\psi_{i,j} - \sum_{l \in L} \eta_{i,j,l})\hat{V}_{i,j} + \sum_{l' \in L} A_{i,j,l'} + (\varphi_{i,j} - \sum_{l' \in L} \omega_{i,j,l'})\hat{V}_{i,j} \right]$$  

$$+ \sum_{l \in L} \left[ \sum_{i \in I} \sum_{t \in T} \left( d_{t,i,l} \alpha_{t,i,l} + M_{t,i,l} \phi_{t,i,l} \right) + \sum_{i \in I} - (\rho_{i,l} \gamma_{i,l} + q_{i,l} \lambda_{i,l}) + \sum_{(i,j) \in A} - V_{i,j,l} \tau_{i,j,l} \right] \right\}$$

is the maximum of a linear functions of $z$, and hence convex and piecewise linear. Finally, it is easy to see that each linear piece of this function is associated with one distinct extreme point of polyhedra $\Omega$ and $\mathbb{R}$. Given that each has a finite number of extreme points, the number of pieces of this function is finite. This complete the proof.
Appendix E. Data related to Hurricane Case Study

Figure E.8: Map of the facility and transportation network. Nodes vulnerable to disruption are shown in red.

Table E.7: Acquisition cost, transportation cost, and storage volume of relief supplies (Rawls and Turnquist 2010; Velasquez et al. 2020).

| Relief item               | Acquisition cost $c_i^t$ ($/unit) | Transportation cost $c_{i,j}^t$ ($/unit-mile) | Storage volume $s_t$ (ft$^3$/unit) |
|---------------------------|-----------------------------------|-----------------------------------------------|-----------------------------------|
| Water (1000 gallons)      | 647.7                             | 0.3                                           | 144.6                             |
| Food (1000 meals)         | 5420                              | 0.04                                          | 83.33                             |
| Medical kits              | 140                               | 0.00058                                       | 1.16                              |

Table E.8: Mean demand for water, food, and medical kits generated by a minor and major disaster at each potential landfall node.

| Landfall node | Water | Food | Medical kits |
|---------------|-------|------|--------------|
|               | minor | major| minor        | major |
| 2             | 500   | 2000 | 1000         | 2000  |
| 5             | 1500  | 7500 | 1800         | 7500  |
| 11            | 1000  | 1500 | 500          | 9000  |
| 13            | 1000  | 2200 | 500          | 1500  |
| 14            | 1000  | 12000| 1800         | 4000  |
| 15            | 600   | 4000 | 500          | 1800  |
| 21            | 1500  | 9000 | 1500         | 4000  |
| 22            | 1500  | 7500 | 1800         | 9000  |
| 29            | 1000  | 2200 | 500          | 1500  |
| 30            | 1500  | 9000 | 1500         | 10500 |
| Total         | 9600  | 47900| 9400         | 39800 |
Appendix F. MCO Convergence Results

Table F.9: The Approximate Optimality Index (AOI$_N$) between the statistical lower bound $v_N$ and upper bound $v_{N'}$ on the objective values of SP and their 95% Confidence Interval (95%CI) for each instance and each sample size, $N$

| (nMinor, nMajor) | $N$ | $95\% CI^{v_N}$ | $95\% CI^{v_{N'}}$ | AOI$_N$ | Time |
|------------------|-----|----------------|----------------|---------|------|
| (2, 1) 5        | 5   | 48569558, 63216812 | 69516537, 71465964 | 0.2     | 0.12 |
| 10              | 5   | 53514528, 60874322 | 61316102, 62978580 | 0.08    | 0.40 |
| 30              | 30  | 57846768, 63731832 | 61152383, 62578997 | 0.017   | 0.70 |
| 40              | 40  | 56095622, 61356578 | 60982111, 61576539 | 0.030   | 1.95 |
| 50              | 50  | 60338000, 61565560 | 6028447, 6342163 | 0.013   | 1.00 |
| 100             | 100 | 59389165, 62447305 | 60424825, 61764305 | 0.003   | 2.30 |
| (4, 2) 5        | 5   | 97907218, 109527052 | 106418968, 102606732 | 0.057   | 0.45 |
| 10              | 10  | 100012921, 107700430 | 106799402, 107313598 | 0.030   | 1.46 |
| 30              | 30  | 101765522, 107163628 | 105463970, 106932931 | 0.016   | 1.74 |
| 40              | 40  | 103218419, 108630914 | 106395116, 107607984 | 0.010   | 2.15 |
| 50              | 50  | 103334694, 107666076 | 10664255, 107628645 | 0.015   | 1.97 |
| 100             | 100 | 104643866, 108416344 | 106037279, 107158322 | 0.003   | 2.30 |
| (6, 3) 5        | 5   | 139350470, 151548430 | 150611748, 152643052 | 0.040   | 0.30 |
| 10              | 10  | 142104626, 149846274 | 149599196, 149725708 | 0.020   | 1.15 |
| 30              | 30  | 145259753, 152943848 | 148963381, 149573001 | 0.010   | 2.15 |
| 40              | 40  | 146396483, 152199997 | 14922838, 150526362 | 0.010   | 2.37 |
| 50              | 50  | 147858180, 151750721 | 148808189, 149054111 | 0.010   | 2.50 |
| 100             | 100 | 146965469, 149626531 | 148128318, 149288282 | 0.003   | 5.00 |

For each instance, our process was as follows. For the SP model, we first optimized the sample size. We ran the MCO algorithm (Algorithm 2) with $N_o = 5$, $N' = 10000$, $K = 20$, and $\epsilon = 0.1$. These results (approximate optimality index; confidence intervals; solution times) are presented in Tables F.9. Based on these results, we find that $N = 100$ is an appropriate sample size to obtain near-optimal solutions and tight estimates on the SP model’s objective value via its SAA within a reasonable time. Then, we used this value for the Case Study 1 SP experiments.
Appendix G. Data related to Earthquake Case Study

Figure G.9: Map of the facility and transportation network.

Table G.10: Input Parameters for Each Node (Ni et al., 2018).

| Node | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $f_i$ | 203 | 193 | 130 | 117 | 292 | 174 | 130 | 157 | 134 | 161 | 234 | 220 | 176 |
| $c_i^a$ | 3.4 | 2.33 | 2.32 | 2.69 | 2.63 | 3.44 | 3.43 | 3.53 | 2.33 | 2.5 | 3.37 | 2.84 | 3.76 |
| $c_i^b$ | 2.81 | 2.58 | 2.86 | 2.42 | 3.28 | 3.05 | 2.77 | 2.68 | 2.52 | 3.14 | 2.93 | 2.85 | 2.87 |
| $c_i^c$ | 11.48 | 14.32 | 12.14 | 16.19 | 12.01 | 14.9 | 9.42 | 11.91 | 10.68 | 11.24 | 13.1 | 11.09 | 10.1 |
| $\mu_i$ | 0.05 | 0.05 | 0.2 | 0.18 | 0.18 | 0.72 | 0.76 | 0.7 | 0.7 | 0.6 | 0.78 | 0.62 | 0.68 |
Appendix H. Computational Performance of DRO–Decomposition Algorithm

Table H.11: Computational details of solving larger networks. (nMinor, nMajor)=(2,1).

| I   | Nodes | # of Iter | Sol. time (s) | # of B&B | # of MIPiter |
|-----|-------|-----------|---------------|----------|--------------|
|     |       |           | Master       | Sub      |              |
| 40  | 10    | 8         | 0.3          | 2        | 491          | 16823        |
| 15  | 7     | 0.3       | 5            | 12018    | 179831       |
| 20  | 13    | 1         | 43           | 109902   | 1940607      |
| 60  | 10    | 8         | 0.5          | 7        | 7285         | 119705       |
| 15  | 10    | 1         | 11           | 20156    | 231152       |
| 20  | 9     | 1         | 28           | 70275    | 1297574      |
| 80  | 10    | 9         | 1            | 4        | 3785         | 93141        |
| 15  | 5     | 0.5       | 5            | 8920     | 171488       |
| 20  | 7     | 1         | 23           | 5667     | 1316733      |

Table H.12: Computational details of solving larger networks. (nMinor, nMajor)=(4,2).

| I   | Nodes | # of Iter | Avg. time (s) | # of B&B | # of MIPiter |
|-----|-------|-----------|---------------|----------|--------------|
|     |       |           | Master       | Sub      |              |
| 40  | 10    | 9         | 0.3          | 3.8      | 13668        | 185751       |
| 15  | 10    | 0.64      | 351          | 1139550  | 20103690     |
| 20  | 9     | 1         | 1281         | 3668080  | 43998783     |
| 60  | 10    | 9         | 0.5          | 7        | 34050        | 421495       |
| 15  | 10    | 0.6       | 211          | 1126600  | 21931900     |
| 20  | 6     | 0.4       | 2748         | 2540100  | 30927200     |
| 100 | 10    | 9         | 1.1          | 9        | 35088        | 564888       |
| 15  | 10    | 1.1       | 230          | 1002580  | 15916600     |
| 20  | 6     | 1         | 1453         | 5171070  | 76742300     |

Table H.13: Computational details of solving larger networks. (nMinor, nMajor)=(3,6).

| I   | Nodes | # of Iter | Avg. time (s) | # of B&B | # of MIPiter |
|-----|-------|-----------|---------------|----------|--------------|
|     |       |           | Master       | Sub      |              |
| 40  | 10    | 9         | 0.3          | 3        | 3213         | 88511        |
| 15  | 9     | 1         | 1300         | 5117360  | 80771385     |
| 20  | 7     | 1         | 3600         | 3032800  | 49453472     |
| 60  | 10    | 9         | 1            | 70       | 453942       | 5445302      |
| 15  | 10    | 1         | 811          | 4353180  | 76544457     |
| 20  | 5     | 1         | 3602         | 16580700 | 339927442    |
| 100 | 10    | 9         | 1            | 7        | 22265        | 338269       |
| 15  | 9     | 2         | 1220         | 5607820  | 91696167     |
| 20  | 3     | 1         | 3618         | 12094500 | 95327781     |

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