Structures of vortices in a superconductor under a tilted magnetic field

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Abstract. We study three-dimensional structures of vortices in a type II superconductor under a magnetic field by solving the Ginzburg-Landau equations. First, we show under a perpendicular magnetic field, two vortices are almost parallel but bend toward the center of the superconductor due to the Meissner current. Second, we show under a tilted magnetic field, two vortices enter the superconductor from edge to edge and they are parallel to the magnetic field.

1. Introduction

Since theoretical prediction by Abrikosov [1], vortices in a type II superconductor under an external field have been studied experimentally and theoretically for decades [2,3,4]. Vortices form a triangular lattice as a static structure under a uniform magnetic field, [1]. Also pinning of vortices have been studied until now [5,6]. Vortices show non-equilibrium phenomena, as well [7,8]. In these studies, the magnetic field is always uniform and vortices are parallel to the field. Recently, Fukui et al. studied magnetic flux structures under a helical magnetic field. They considered a bilayer system with a superconductor and a chiral helimagnet [8,9,10,11]. Solving the Ginzburg-Landau (GL) equation, they obtained a peculiar vortex structure. Two vortices are not parallel to the local field. But two vortices tilt toward axis of helical magnetic field. They considered two origins of this peculiar vortex structure.

Between parallel or anti-parallel vortices, there is a repulsive or attractive interaction, respectively. This is because the circular current around a vortex causes the Lorentz force to another vortex and this force is repulsive for parallel vortex and attractive for anti-parallel vortex. But for non-parallel two vortices case, it is clear that the interaction is neither purely repulsive nor attractive. If two vortices are perpendicular, the Lorentz force becomes zero. So, the current around a vortex may cause a torque to another vortex.

Another explanation is an edge effect. Under an external magnetic field, the Meissner current flows around the surface. Flowing pattern of the Meissner current is well studied for uniform field. But for helical magnetic field, it is not clear how the Meissner current flows. Also, at the edge, the Meissner current density becomes large even for the uniform field. Therefore, it may be difficult that vortices enter from the edge and exit from the edge. So, the Meissner current may cause vortices avoid the edge, then the peculiar vortex structure may occur.

In order to clarify these origins are important or not, we study the edge effect on the vortex structure. For this purpose, we investigate vortex structures in a superconductor under uniform perpendicular and tilted magnetic fields in this study. Although there are several studies on effects of the tilted magnetic field for anisotropic film and bulk superconductors [12,13] and for multiply connected superconductors [14,15], there are few studies on edge effects for finite superconductors under the tilted magnetic field.
2. Model

In order to treat spatial variation of the order parameter, we solve the three-dimensional GL equations.

\[ \alpha \psi + \frac{1}{2} \beta |\psi|^2 \psi + \frac{1}{2m^*} \left( \frac{\hbar}{i} \nabla - \frac{e^*}{c} A \right)^2 \psi = 0 \]  

(1)

\[ J = \frac{e^* h}{2m^* i} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^2}{m^* c} \psi^* \psi A \]  

(2)

Here, \( \alpha \) and \( \beta \) are expansion coefficients, \( \psi \) is an order parameter, \( m^* \) is an effective mass of superconducting electron pair, \( A \) is a vector potential, \( J \) is a density of superconducting current, \( e^* \) is an effective charge of superconducting electron pair and \( c \) is the speed of light. In numerical simulation, we solve two equations self-consistently.

We set a boundary condition on the order parameter as follows,

\[ \left. \left( \frac{\hbar}{i} \nabla - \frac{e^*}{c} A \right) \psi \right|_n = 0 \]  

(3)

where \( n \) means a normal component at a boundary. Also, we use the London gauge,

\[ \text{div} A = 0 \]  

(4)

\[ A \cdot n = 0 \]  

(5)

at boundaries. Here \( n \) is a normal vector at the boundary. In numerical simulation, we use the finite element method. We treat three-dimensional system, therefore we divide system into tetrahedrons. Then, we expand \( \psi \) and \( A \) using volume coordinate \( N_i \) as follows.

\[ \psi^e(x, y, z) = \sum_i \psi_i^e N_i^e(x, y, z) \quad (i = 1, 2, 3, 4) \]  

(6)

\[ A(x, y, z) \sum_i A_i^e N_i^e(x, y, z) \quad (i = 1, 2, 3, 4) \]  

(7)

Inserting these equations into equations (1) and (2), we get

\[ \sum_i \left[ P_{ij}((A)) + P_{ij}^R((\psi)) \right] Re \psi_j^e + \sum_i \left[ Q_{ij}((A)) + Q_{ij}^R((\psi)) \right] Im \psi_j^e = V_i^R((\psi)) \]  

(8)

\[ \sum_i \left[ -Q_{ij}((A)) + Q_{ij}^R((\psi)) \right] Re \psi_j^e + \sum_i \left[ P_{ij}((A)) + P_{ij}^R((\psi)) \right] Im \psi_j^e = V_i^I((\psi)) \]  

(9)

\[ \sum_i R_{ij}((\psi)) A_{ix} + \sum_j S_{ij}^{xy} A_{iy} + \sum_j S_{ij}^{xz} A_{iz} = T_i^x - U_i^y + U_i^z \]  

(10)

\[ \sum_i R_{ij}((\psi)) A_{iy} + \sum_j S_{ij}^{yx} A_{ix} + \sum_j S_{ij}^{yz} A_{iz} = T_i^y + U_i^x - U_i^z \]  

(11)
\[
\sum_{j} R_{ij}(\{\psi\}) A_{jx} + \sum_{j} S_{ij}^{xx} A_{jx} + \sum_{j} S_{ij}^{yy} A_{jy} = T_i^x + U_{i}^y - U_{i}^x
\]

(12)

Here, integrals and coefficients are given in Appendix.

3. Result

Our system size is \(26\xi_0 \times 16\xi_0 \times 16\xi_0\) and consider a vacuum layer surrounding the superconductor (figure 1). Here, \(\xi_0 = \frac{\hbar}{\sqrt{2m^*}|\alpha(T = 0)|}\) is a coherence length at zero temperature. We set GL parameter \(\kappa=2.0\). First, we consider a uniform field along \(z\)-direction, to compare with a tilted field case. We set \(T = 0.1T_c\), where \(T_c\) is the transition temperature and \(H = 3.7\frac{\Phi_0}{\xi_0}\). Here \(\Phi_0 = \frac{hc}{2e}\) is a flux quantum. In figure 2, we show order parameter distributions. Here, \(\psi_0\) is an order parameter at zero temperature.

We can see that superconducting region and length between vortices became smaller around \(z = 8.0\xi_0\). This is because, vortices are pushed to the center of the superconductor, around \(z = 8.0\xi_0\), due to the Meissner current surrounding the superconductor. This situation is schematically shown in figure 3, where two blue regions are penetrating fields and two white regions at the center are vortices. Therefore, vortices are not straight and not parallel to the applied field.

Second, we consider a constant field \((H = 3.0\frac{\Phi_0}{\xi_0})\) tilted \(\pi/4\) from \(z\)-axis to investigate the effect of screening currents at edges of the superconductor. Then, the magnetic field is

\[
H = \begin{pmatrix}
0, H \sin \frac{\pi}{4}, H \cos \frac{\pi}{4}
\end{pmatrix}
\]

We set \(T = 0.3T_c\). From figure 4, we can see vortices enter from bottom edge to upper edge. In this case, vortices are not repelled by the Meissner current around the edge of the superconductor. Therefore, we can say the Meissner current around the edge does not always repel vortices. Although we cannot exclude the possibility of effect of the Meissner current, we think previous result may come from peculiar vortex-vortex interaction. So, we will investigate interaction between vortices under spatial varying fields.

4. Summary

We study the origin of previous result which is that directions of vortices are not parallel to the field in the chiral helimagnet/superconductor bilayer structure. Using the GL equations with the three-dimensional finite element method, we investigated vortex structures under perpendicular and tilted magnetic field. Under the perpendicular field, two vortices in the superconductor are almost parallel to the field and apart from each other. But around the center of the superconductor, distance between them becomes short, because of repulsive force from the Meissner current surrounding the superconductor.

Under the tilted magnetic field, two vortices enter the superconductor from the edge. So, the Meissner current around the edges do not prevent entrance of vortices in present case. We think previous peculiar vortex structure may come from the interaction between non-parallel vortices.

Our system is rather small comparing with usual experimental systems. But, we think results of the edge effect may applicable to finite superconductors with edges.

In the future, we will study vortex structures under spatial varying fields to investigate the interaction between non-parallel vortices. Also, we will obtain most stable vortex structure by comparing free energy.
Figure 2. Two-dimensional order parameter distributions in cross section at \( z = 0 \sim 16\xi_0 \). Applied field is \( H = 3.7\frac{\Phi_0}{\xi_0^2} \).
Figure 3. Schema of relation between vortices and penetrating fields.
Figure 4. Two-dimensional order parameter distributions in cross section at $z = 0 \sim 16 \xi_0$. Applied field is $H = 3.0 \frac{\Phi_0}{\xi_0^2}$. 

$\Delta/\Delta_0: 0 \rightarrow 0.8$
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Appendix
We show integrals and coefficients in Eq. (8) to (12). Integrals are shown in Eq. (13) to (19),

\[ I_{ij} = \int_{V_e} N^e_i N^f_j dV \]  (13)
\[ I_{i_1 i_2 i_3} = \int_{V_e} N^e_{i_1} N^e_{i_2} N^e_{i_3} dV \]  (14)
\[ I_{i_1 i_2 i_3 i_4} = \int_{V_e} N^e_{i_1} N^e_{i_2} N^e_{i_3} N^e_{i_4} dV \]  (15)
\[ f^x_{i_1 i_2 i_3} = \int_{V_e} \frac{\partial N^e_i}{\partial x_i} N^e_{i_2} N^e_{i_3} dV \]  (16)
\[ K^{x_i x_j}_{i_1 i_2} = \int_{V_e} \frac{\partial N^e_i}{\partial x_i} \frac{\partial N^e_j}{\partial x_j} dV \]  (17)
\[ f^x_{i_1 i_2} = \int_{V_e} \frac{\partial N^e_i}{\partial x_i} dV \]  (18)
\[ f^x_{i_1 i_2} = \int_{V_e} \frac{\partial N^e_i}{\partial x_i} N^e_{i_2} dV \]  (19)

where \( \alpha, \beta = x, y, z \). Then, coefficients are shown in Eq. (20) to (30),

\[ P_{ij} (\{ A \}) = \sum_{\alpha} K^{\alpha \alpha}_{ij} + \sum_{i_1 i_2} L_{i_1 i_2 i_3 i_4} \sum_{\alpha} A^e_{i_1 \alpha} A^e_{i_2 \alpha} - \frac{1}{\xi^2 (T)} I_{ij} \]  (20)
\[ p^{2R} (\{ \psi \}) = \frac{1}{\xi^2} \sum_{i_1 i_2} L_{i_1 i_2 i_3 i_4} (3 Re \psi^e_{i_1} Re \psi^e_{i_2} + Im \psi^e_{i_1} Im \psi^e_{i_2}) \]  (21)
\[ p^{2I} (\{ \psi \}) = \frac{1}{\xi (T)^2} \sum_{i_1 i_2} L_{i_1 i_2 i_3 i_4} (Re \psi^e_{i_1} Re \psi^e_{i_2} + 3 Im \psi^e_{i_1} Im \psi^e_{i_2}) \]  (22)
\[ Q_{ij} (\{ A \}) = \sum_{\alpha} \sum_{i_1 i_2} (J^\alpha_{i_1 i_2} - J^\alpha_{i_2 i_1}) A^\alpha_{i_1} \]  (23)
\[ Q^2_i (\{ \psi \}) = \frac{2}{\xi (T)^2} \sum_{i_1 i_2} L_{i_1 i_2 i_3 i_4} Re \psi^e_{i_1} Im \psi^e_{i_2} \]  (24)
\[ R_{ij} (\{ \psi \}) = \kappa^2 \xi (T)^2 \sum_{\alpha} K^{\alpha \alpha}_{ij} + \sum_{i_1 i_2} L_{i_1 i_2 i_3 i_4} \psi^e_{i_1 i_2} \psi^e_{i_1 i_2} \]  (25)
\[ S^{\alpha \beta}_{ij} = \kappa^2 \xi (T)^2 \left( K^{\alpha \beta}_{ij} - K^{\beta \alpha}_{ij} \right) \]  (26)
\[ T^\alpha_i = \sum_{i_1 i_2} J^\alpha_{i_1 i_2} Im \left( \psi^e_{i_1 i_2} \psi^e_{i_1 i_2} \right) \]  (27)
\[ U^\alpha_i = \kappa^2 \xi (T)^2 \frac{2 \pi}{\Phi_0} H J^\alpha_i \]  (28)
\[ V^R_i (\{ \psi \}) = \frac{2}{\xi (T)^2} \sum_{i_1 i_2 i_3} L_{i_1 i_2 i_3 i_4} Re \left( \psi^e_{i_1 i_2} \psi^e_{i_1 i_2} \right) Re \psi^e_{i_1 i_2} \]  (29)
\[ V^I_i (\{ \psi \}) = \frac{2}{\xi (T)^2} \sum_{i_1 i_2 i_3} L_{i_1 i_2 i_3 i_4} Re \left( \psi^e_{i_1 i_2} \psi^e_{i_1 i_2} \right) Im \psi^e_{i_1 i_2} \]  (30)

where \( x_i, x_j = x, y, z \).
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