Strengthening the bounds on the r-mode amplitude with X-ray observations of millisecond pulsars

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ABSTRACT
r-mode oscillations have been shown to have a significant potential to constrain the composition of fast spinning neutron stars. Due to their high rotation rates, millisecond pulsars (MSPs) provide a unique platform to constrain the properties of such oscillations, if their surface temperatures can be inferred. We present the results of our investigations of archival X-ray data of a number of MSPs, including recent XMM-Newton observations of PSR J1810+1744 and PSR J2241−5236. Using the neutron star atmosphere (NSA) model and taking into account various uncertainties, we present new bounds on the surface temperature of these sources. We were then able to significantly strengthen previous bounds on the amplitude of the r-mode oscillations in millisecond pulsars and find values as low as $\alpha \lesssim 10^{-9}$. This is by now about three orders of magnitude below what standard saturation mechanisms in neutron stars could provide, which requires very strong dissipation in the interior, strongly pointing towards a structurally complex or exotic composition of these sources. At such low temperatures sources could even be outside of the instability region, and taking into account the various uncertainties we obtain for an observed surface temperature a simple frequency bound below which r-modes are excluded in slower spinning pulsars.

Key words: r-mode – neutron stars – gravitational waves – stellar oscillations

1 INTRODUCTION

r-mode asteroseismology provides a unique opportunity to probe the opaque compact star interior. r-modes (Papaloizou & Pringle 1978; Andersson 1998; Lindblom, Owen, & Morsink 1998; Friedman & Morsink 1998; Andersson 2000) are quasi-toroidal oscillations in rotating stars that occur owing to the Coriolis effect. Because r-modes are unstable (Andersson 1998) due to the Chandrasekhar-Friedman-Schutz (CFS) mechanism (Chandrasekhar 1970; Friedman & Schutz 1978), they would spontaneously arise under the emission of gravitational waves and spin the star down, unless the instability can be tamed by dissipation within the star, which depends sensitively on the composition. r-modes are damped in slowly spinning sources, but should be unstable in ordinary neutron stars spinning with millisecond frequencies over a range of typical temperatures, since in this instability region viscous dissipation cannot prevent their instability to gravitational wave emission. In this case the amplitude would be saturated by non-linear, amplitude-dependent enhancement and would strongly heat the star if the amplitude becomes large. Therefore, bounds on the temperatures of these sources conversely set bounds on the amplitude of potential r-modes in observed sources. This has previously been shown for accreting sources in Low Mass X-ray Binaries (LMXBs) (Mahmoodifar & Strohmayer 2013; Alford & Schwenzer 2014; Bhattacharya et al. 2017), as well as millisecond pulsars (MSPs) (Alford 2013; Schwenzer et al. 2017), and led to very low bounds on the r-mode amplitude of $\alpha \lesssim 10^{-8}$.

The dissipation and cooling, that determines the steady state temperature, depends strongly on the matter inside the star. Measurements or bounds on surface temperatures of non-accreting MSPs in turn impose bounds on the r-mode amplitude, since these sources would be hotter if the r-mode amplitude would be higher. However, for non-accreting MSPs detailed X-ray measurements are still limited due to their lower flux. An actual surface temperature measurement is only available for the closest sources such as PSR J0437−4715 (Zavlin et al. 2002). Besides the observed surface temperature, the spin frequency has been shown to

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be the decisive factor for the amplitude bounds (Mahmoodifar & Strohmayer 2013; Schwenzer et al. 2017).

In this paper, we report new results on thermal bounds of fast-spinning MSPs. We also present results of two new XMM-Newton observations of the 602 Hz pulsar PSR J1810+1744 and the 457.31 Hz pulsar PSR J2211−5236 allowing us to perform the first detailed spectral analysis of these sources. Moreover, we report the re-analysis of archival data of several other MSPs, including an update of our earlier analysis for PSR J1231−1411 (Schwenzer et al. 2017) and the fastest spinning pulsar in the Galactic field PSR J0952−0607. We use here an improved version of the method we introduced in (Schwenzer et al. 2017) to take into account dominant microscopic and macroscopic (astrophysical) uncertainties in the analysis, e.g., the distance, to obtain stringent upper limits. Moreover, we now employ a more realistic neutron star atmosphere (NSA) model, appropriate for old systems, instead of assuming a mere blackbody. Based on the new spectral analyses we improve the previous r-mode amplitude bounds for these sources and even within uncertainties we now have several sources with strict upper limits not far above 10−9.

The structure of the paper is as follows. After providing some introductory information on the individual sources we use in this study in Section 2, we provide the details of the observations and data analysis in Section 3. We present the limits on thermal emission obtained from the X-ray data of the sources in Section 4. We sketch the thermal impact of r−modes and derive the resulting r-mode amplitude bounds in Section 5. In Section 6 we derive a general condition if r−modes can be present in observed millisecond sources. Finally, we conclude and discuss our results from X-ray observations in Section 7.

2 MILLISECOND PULSARS INVESTIGATED
We provide below a brief introduction to each neutron star system investigated in our study.

2.1 PSR J1810+1744:
J1810+1744 was discovered in a 350 MHz targeted Green Bank Telescope (GBT) search of unidentified Fermi point sources Hessels (2011), and has been identified as a canonical Black Widow Breton et al. (2013). The dispersion measure (DM) is measured to be 40 pc cm−3 Breton et al. (2013), which corresponds to a distance of 2 kpc. The pulsar has a spin period $P_{\text{spin}} = 1.66$ ms. The pulsar has a very small mass companion $M_{\text{companion}} = 0.045 M_\odot$ orbiting with a period of $P_{\text{orb}} = 3.6$ hr. At low radio frequencies, this source is one of the brightest known millisecond pulsars Polzin (2018). Previous to our new XMM-Newton observation presented in detail below, Gentile et al. (2014) fit the only available X-ray spectrum, with a combination of a thermal and a non-thermal component with fixed temperature and photon index values ($kT = 150$ eV, $\Gamma = 1.5$). The combined flux is measured as $F_{\text{total}} = 2.0^{+0.5}_{-0.6} \times 10^{-14}$ erg cm−2 s−1, where the flux of the individual components are calculated as, $F_{\text{th}} = 0.7^{+0.4}_{-0.3} \times 10^{-14}$ erg cm−2 s−1, $F_{\text{pow}} = 2.0^{+0.6}_{-0.7} \times 10^{-14}$ erg cm−2 s−1.

2.2 PSR J1744−1134:
J1744−1134 was discovered as part of the Parkes 436 MHz survey of the southern sky Bailes (1997). The pulsar has a spin period $P_{\text{spin}} = 4.075$ ms and a period derivative of $\dot{P}_{\text{rot}} = 0.86 \times 10^{-20}$ s s−1 Manchester (2013). For this source the distance has been calculated as $d = 357^{+43}_{-40}$ pc assuming the electron density model of Taylor & Cordes (1993) Toccano (1999), using the dispersion measure of 3.14 pc cm−3 van Haasteren (2011). Using the calculated distance value, the X-ray luminosity of the source has been calculated $L_X = 4 \times 10^{30} d^{-2} \text{erg s}^{-1}$ by Toccano (1999) using ROSAT satellite observations Becker & Trümper (1999).

2.3 PSR J1231−1411:
J1231−1411 was discovered by Fermi LAT as one of the brightest gamma-ray MSPs in the sky ($F_{100} = 10.57^{+0.62}_{-0.32} \times 10^{-9}$ erg cm−2 s−1), it has a spin period of $P = 3.684$ ms. This source is in a binary system which has an orbital period of 1.8601 d, with a dispersion measure (DM) of 8.09 pc cm−3 Bassa et al. (2016). The observed period derivative implies a surface magnetic field strength of $(2−3) \times 10^{10}$ G and a spin-down luminosity of $\sim 2 \times 10^{33}$ erg s−1 Ransom et al. (2011). The flux of the source in the 0.5−3.0 keV energy band was found to be $(1.15 \pm 0.05) \times 10^{-15}$ erg cm−2 s−1. In addition, we previously modelled the XMM-Newton data with a blackbody, giving an unabsorbed flux of $F_{\text{bb}} = 15^{+5}_{-4} \times 10^{-14}$ erg cm−2 s−1 in the 0.5−8 keV band, and when taking into account a distance of 0.4 kpc an X-ray luminosity of $L_{\text{bb}} = 2.9^{+1.4}_{-1.0} \times 10^{30}$ erg s−1. Alternatively, the observed spectrum can be modeled with a power-law with exponent $\Gamma = 4.29^{+0.41}_{-0.38}$, and the column density is calculated as $N_{\text{H}} = 1.8^{+0.8}_{-0.5} \times 10^{21}$ cm−2 Schwenzer et al. (2017).

2.4 PSR J2256−1024:
J2256−1024 was discovered during the 350 Mhz GBT pulsar drift scan survey Boyle (2011). This pulsar is a Black-Widow with a degenerate companion of mass 0.1 $M_\odot$, which exhibit radio eclipses. It has a spin period of 2.29 ms and a short orbital period of 5.1 hr Gentile (2018). This source has a DM of 14 pc cm−3, indicating a distance of $\approx 0.65$ kpc, using the NE2001 model Cordes & Lazio (2002). The combined flux was measured as $F_X = 4.6^{+2.5}_{-1.5} \times 10^{-14}$ erg cm−2 s−1, where the flux of the single components were calculated as, $F_{\text{th}} = (5.3 \pm 0.6) \times 10^{-14}$ erg cm−2 s−1, $F_{\text{bb}} = 3.2^{+2.6}_{-1.5} \times 10^{-14}$ erg cm−2 s−1 by Gentile et al. (2014).

2.5 PSR J1723−2837:
J1723−2837 was discovered by Faulkner et al. (2004) during the Parkes Multi-beam survey, it has a spin period of 1.86 ms and the measured spin-down rate is $\sim 7.5 \times 10^{-21}$ s s−1. This pulsar is a Redback millisecond pulsar with a low-mass companion in a 14.8 hr orbit. The pulsar’s DM is 19.69 pc cm−3 indicating a distance of 0.75 kpc Crawford et al. (2013), using the NE2001 model Cordes & Lazio (2002). However, GAIAs second data release Jennings et al. (2018) yielded a very precise distance measurement for this source. Based on GAIA results, Jennings et al. (2018) calculated the distance to be $d = 0.91^{+0.04}_{-0.05}$ kpc. In this paper we will use
this new model independent distance measurement for further analysis. The X-ray spectrum of this source could be modeled with an absorbed power-law model by Kong et al. (2017), based on NuSTAR observations. The best-fit model parameters inferred are $N_H = 3.9^{+2.7}_{-2.0} \times 10^{22}\text{cm}^{-2}$ and $\Gamma = 1.28 \pm 0.04$ and as a consequence the X-ray flux was calculated as $F_X = (9.6 \pm 0.5) \times 10^{-12}\text{erg cm}^{-2}\text{s}^{-1}$ Kong et al. (2017).

### 2.6 PSR J1400–1431:

J1400–1431 was discovered by the Pulsar Search Collaboration in the Green Bank 350 MHz Drift Scan Survey (2013). This pulsar has a dispersion measure of 4.9 pc cm$^{-3}$ and its spin period is 3.08 ms. The binary orbital period is found to be 9.5 d, with a white dwarf companion mass $M_{\text{c,min}} = 0.30 M_\odot$. The X-ray luminosity of the source is reported as $L = 10^{32}\text{erg s}^{-1}$ in the 0.3–10 keV range for a distance of 270 pc, and the spindown luminosity is $< 3 \times 10^{33}\text{erg s}^{-1}$. The inferred spectral parameters of a blackbody model are $K_T = (0.15 \pm 0.2)\text{keV}$, with an apparent radius of $R = 0.06^{+0.05}_{-0.04}\text{km}$, an unabsorbed flux of $F_{\text{abs}} = (1.07 \pm 0.15) \times 10^{-11}\text{erg cm}^{-2}\text{s}^{-1}$, for a fixed value of the atomic hydrogen column density, $N_H = 1.5 \times 10^{20}\text{cm}^{-2}$. For the surface area of the neutron star with a total emitting area of $60\%$ fitting the data, an unabsorbed flux of $F_{\text{unabs}} = (1.15 \pm 0.17) \times 10^{-14}\text{erg cm}^{-2}\text{s}^{-1}$ has been found Swiggum et al. (2017).

### 2.7 PSR J0952–0607:

J0952–0607 was discovered by the Low-Frequency Array (LOFAR) survey at 135 MHz. It is in a 6.42 hr binary with a very low-mass companion and it is the fastest-spinning known pulsar in the galactic field (707 Hz) Bassa et al. (2017). The observed properties of the system show that it is a Black-Widow binary. The DM is reported to be $22.412\text{ pc cm}^{-3}$. This value is used to infer two different distances assuming the YMW16 and the NE2001 models as $d = 1.74\text{kpc}$ and $d = 0.97\text{kpc}$ Bassa et al. (2017), respectively. Ho, Heinke, & Chugunov (2019) recently presented an analysis of the XMM-Newton observation of the system. The spectral fit, using a power-law, results in a photon index of $\Gamma \approx 2.55^{+0.5}_{-0.4}$ and $F_{\text{unabs}} = 9 \times 10^{-15}\text{erg s}^{-1}\text{cm}^{-2}$ in the 0.3–10 keV range. Their result for the luminosity $L_X = 3 \times 10^{30}\text{erg s}^{-1}$, assuming a distance of 1.74 kpc, is ten times lower than the upper limits set using a short Swift exposure by Bassa et al. (2017). Note that the flux result obtained using a single power-law is consistent with the value we find here (see Table 3).

### 2.8 PSR J2241–5236:

J2241–5236 was discovered by Fermi–LAT Keth et al. (2011). It has a spin period 2.19 ms and a period derivative of $6.6 \times 10^{-21}\text{ s}^{-1}$. Using these timing measurements of the pulsar the characteristic age, the surface magnetic field strength and the rate of energy loss has been derived as $\tau = 5 \times 10^9\text{ yr}$, $B = 1.2 \times 10^8\text{ G}$, and $\dot{E} = 2.5 \times 10^{34}\text{erg s}^{-1}$, respectively. The pulsar has a very low mass companion with an orbital period of 3.5 hours. This source has a low dispersion measure, indicating a distance of $\approx 0.5\text{kpc}$. Previous to our new XMM-Newton observation discussed below, the X-ray spectrum of this source was fitted with an absorbed single blackbody model with a temperature, $kT = 0.26\pm0.04\text{keV}$ Keth et al. (2011). Using the Galactic hydrogen column density, $N_H = 1.21 \times 10^{20}\text{cm}^{-2}$, Kalberla et al. (2005) and assuming a distance of 0.5 kpc, results in an X-ray luminosity of $\sim 2 \times 10^{30}\text{ergs}^{-1}$. This is less than 1% of the rotational energy loss rate Keth et al. (2011).

### 3 OBSERVATIONS AND DATA ANALYSIS

We concentrate in this work on the X-ray observations of eight millisecond pulsars which are rapidly rotating, nearby and high quality X-ray observations could be found. These pulsars are introduced in Section 2 and a log of their observations used here, is given in Table 2. For two pulsars (PSR J1810+1744 and PSR J2241–5236), we obtained new XMM-Newton observations and provide here the first spectral results of these data. XMM-Newton observed PSR J1810+1744 for 82 ks on 16th October 2017 and PSR J2241–5236 for 52 ks on 28th November 2018, with ObsIDs 0800880201 and 0824320201 respectively (see Table 1).

Both the Chandra X-ray Observatory (CXO) and XMM-Newton observatories’ data presented in this paper were analyzed using the standard scientific analysis software for each satellite. For the CXO data analysis, we used CIAO version 4.9 with CALDB version 4.7.3. We used the chandra_repro tool to create calibrated Level 2 event files and the spec_extract tool to extract source and background X-ray spectra and generate appropriate response and ancillary response files. We used circular regions with typical radii of 2″–3″ and 8″–15″, radius, for source and background, respectively.

In a similar way, all the data analysis and calibration regarding the XMM-Newton data has been performed using SAS version 20160201_1833-15.0.0 with the most up to date calibration files as of 1st of June 2018. We used the eprocd and emproc tools to create calibrated event files for the European Photon Imaging Camera (EPIC) pn, MOS1 and MOS2. In general we had to use pn camera data, which resulted in a higher signal. However whenever possible we also used MOS data as well. Using the expfilt tool, we investigated carefully the existence of soft proton contamination. In some cases this resulted in a decrease in the exposure times of the X-ray spectra from the values, these values and other sources total exposure time are in Table 1. For example, in the case of J1810+1744 the most recent observation was obtained by XMM-Newton on October 16, 2017 (ObsID: 0800880201) for a total exposure time of 82 ks, however, unfortunately it was heavily affected by solar particle background and therefore it was only possible to extract 24 and 32 ks of data from the EPIC pn and MOS, respectively. For all the XMM-Newton data, the source and appropriate background spectra were extracted from circular regions of roughly 640 and 1200 pixel radius from the same CCD chip.

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1. http://cxc.cfa.harvard.edu/ciao/
2. https://www.cosmos.esa.int/web/xmm-newton
and the flux of the source measured by pulsar, we also present the corresponding flux measurements if available. Otherwise, it represents the total flux. For each mal component whose temperature is also given in the table, XMM-Newton measure with CXO is power-law does not change and can be fitted with a simple absorbed two observations show that, while the X-ray spectral shape in Figures 1, 2, 3, and 4. Unabsorbed fluxes are given in component, while PSR J1231−1431 and PSR J2241−5236 to the weighted average of the value given by Kalberla et al. (2005) in the direction of the sources in Table 3, using the FTOOL nh. 4 Thereafter, we used a thermal (blackbody) and a non-thermal (power-law) component to model the observed spectra of each source. We fixed the value of the hydrogen column density at the surface of a neutron star for an assumed neutron star mass, radius and distance. In this paper, we make several improvements to our assumptions in the previous paper Schwenzer et al. (2017).

First we improve on our previous assumption that the thermal component from the surface can be represented with a simple BB emission. In addition to a blackbody, we here also employ a fully ionized Hydrogen atmosphere model, namely NSA in Xspec (Zavlin & Pavlov & Shibanov 1996), in radiative and hydrostatic equilibrium. It is well known that even if there is only a small amount of matter collected on the surface of a neutron star, such a layer can act as an atmosphere, and it significantly changes the energy distribution of the photons emanating from the surface (Zavlin & Pavlov & Shibanov 1996; Özel 2013). Typically, such an atmosphere results in broader spectra compared to a pure BB emission, because of the energy dependence of the opacity in the atmosphere. Due to this dependence, more energetic photons at deeper layers effectively become visible to an observer, resulting in more flux at higher energies and hence a broader observed X-ray spectrum. If an observed spectrum from such an atmosphere is modeled with a simple BB function, then to provide a fit to the data with its narrow spectral shape, the best fit BB temperature and apparent radius values become higher and lower, respectively, compared with those obtained with a more realistic spectral model. This is why a temperature obtained by fitting an atmospheric model with a blackbody is called color temperature. Using a more realistic spectral model, like NSA, therefore naturally results in a smaller temperature and a larger emitting radius. The fact that NSA or similar atmosphere model spectra are broader than a BB in particular significantly helps to limit the faint surface emission, since they can better reproduce the high energy tail of a cold source. Therefore, they increase the sensitivity of standard X-ray spectroscopy to such low-energy signals, which likely peak in the UV: As we increase the predicted temperature of the atmosphere to higher values, to see if it affects our fit to the observed spectra, a broader spectrum obviously has a statistically more significant effect on the fit than a narrower (pure blackbody) model Özel (2013). Note that for comparison we also provide limits on the surface temperature of these sources assuming a BB function.

The NSA model provides tabulated X-ray spectra in the 0.05 to 10 keV range. There are three options for the strength of the magnetic field: $B = 0, 10^{12}, 10^{13} \text{G}$. Because the NSA model assumes a fully ionized Hydrogen atmosphere with Thomson scattering, it is only valid within the temperature range $10^6−10^7 \text{K}$ (Zavlin & Pavlov & Shibanov 1996). The normalization of the model is defined as the inverse square of the distance of the object in parsecs. This model takes into account the gravitational redshift and so the best-fit resulting temperatures are unredshifted temperatures $T_\ast$ at the surface of a neutron star for an assumed mass and radius. With such a model the surface (effective)

### Table 1. Exposure times of the observations of MSPs analyzed in this paper.

| Source Name | Chandra | XMM-Newton | SUZAKU |
|-------------|---------|------------|--------|
| PSR J1810+1744 | 20.71   | 82.0       | –      |
| PSR J1744−1134  | 64.14   | –          | –      |
| PSR J1231−1411  | 10.07   | 29.82      | 78.95  |
| PSR J2256−1024  | 20.05   | –          | –      |
| PSR J1723−2837  | 55.07   | 62.08      | –      |
| PSR J1400−1431  | –       | 40.9       | –      |
| PSR J0953−0607  | –       | 71.2       | –      |
| PSR J2241−5236  | 20.19   | 51.99      | –      |

3 https://heasarc.nasa.gov/docs/xanadu/xspec/index.html
4 https://heasarc.gsfc.nasa.gov/lheasoft/ftools/heasarc.html

4 LIMITS ON THERMAL EMISSION FROM X-RAY DATA

Once the modeling of the observed spectra was completed, we moved on to apply the method outlined in Schwenzer et al. (2017), which is to find the highest temperature value of a thermal component emitted from the whole surface that would cause a statistically significant deviation on the existing model, for an assumed neutron star mass, radius and distance. In this paper, we make several improvements to our assumptions in the previous paper Schwenzer et al. (2017).

We grouped the resulting individual spectra to have at least 25 counts per channel. For pulsars PSR J1231−1411 and PSR J1810+1744 for which we have more than one spectrum we fit all the extracted spectra simultaneously. For all the fits we used XSPEC\(^3\) version 12.9.0 Arnaud (1996). We took into account the effect of the interstellar absorption, using the tbabs model Wilms & McCray (2000) assuming interstellar abundances for each source. We fixed the value of the hydrogen column density $n_H$ to $10^{19} \text{cm}^{-2}$, corresponding only to the ther-
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Figure 1. X-ray spectra of PSR J1400−1431 (left panel) and PSR J1723−2837 (right panel) together with the best fit model. Lower panels show the residuals from the model in units of the statistical uncertainty of the data. Colors of the labels in the upper panels denote the corresponding data in Chandra, XMM-Newton and Swift. The Chandra, XMM-Newton and Swift data are shown by red, green, and blue colors, respectively.

Figure 2. As in Figure 1 for PSR J1744−1134 (left panel) and PSR J1231−1411 (right panel). Colors of the labels in the upper panels denote the corresponding data in Chandra, XMM-Newton and Swift. The Chandra, XMM-Newton and Swift data are shown by red, green, and blue colors, respectively.

Figure 3. As in Figure 1 for PSR J1810+1744 (left panel) and PSR J2256−1024 (right panel). Colors of the labels in the upper panels denote the corresponding data in Chandra, XMM-Newton and Swift. The Chandra, XMM-Newton and Swift data are shown by red, green, and blue colors, respectively.
temperature can be inferred. For the purpose of this paper we used 0 G magnetic field strength, as is the most appropriate case for millisecond pulsars (having fields $\lesssim 10^{12}$ G). For the gravitational redshift, we took into account a fixed neutron star mass of $M_\odot = 1.4$, and three radius values $R = 8, 10, 15$ km.

The distance values we used in this paper and compiled from literature are given in Table 2 together with appropriate uncertainties. Typically these measurements are derived from the inferred dispersion measurements and the Galactic electron density maps (Weisberg 1996). Especially for the distance measurements relying on electron density

Table 2. Observational properties of the millisecond pulsars as compiled from the literature.

| Source Name | Type1 | $f$ (Hz) | $-f$ (10^{-16} s^{-2}) | Distance3 (kpc) | Ref. |
|-------------|-------|----------|-------------------------|-----------------|------|
| J1810+1744  | BW    | 602.41   | 2.0                     | 1               |
| J1744−1134  | BW    | 245.426  | 0.357                   | 2               |
| J1231−1411  | R     | 271.45   | 0.44                    | 3               |
| J2256−1024  | BW    | 436.68   | 0.6                     | 4               |
| J1723−2837  | R     | 538.87   | 0.91 ± 0.04             | 5               |
| J1400−1431  |       | 324.23   | 0.27                    | 6               |
| J0932−0607  | BW    | 707.3    | 1.74                    | 7               |
| J2241−5236  | BW    | 457.31   | 0.52                    | 8               |

1 BW : Black-Widow, R : Redback
2 Assumes the Cordes & Lazio (2002) model of electron density.
3 For the distance based on the dispersion-measure the NE2001 model is used Cordes & Lazio (2002).

References: (1) Gentile et al. (2014); (2) Breton et al. (2013); Polzin (2018) (3) Schwenzer et al. (2017); Bassa et al. (2016) (4) Gentile et al. (2014) (5) Jennings et al. (2018); Kong et al. (2017); Van Staden (2016) (6) Swiggum et al. (2017) (7) Bassa et al. (2017) (8) Keth et al. (2011).

Table 3. Results of the spectral analysis of all sources.

| Source Name | $nH^*$ (10^{22}) cm^{-2} | $kT$ (keV) | $R^1$ (km) | $\Gamma$ | Flux $^2$ (10^{-14} erg/s/cm^2) | $\chi^2 / \text{dof}$ |
|-------------|---------------------------|------------|------------|---------|-------------------------------|---------------------|
| PSR J1810+1744 | 0.087                     | 0.16 ± 0.01 | 1.72 ± 0.09 | 5.55 ± 0.77 | 0.8/15                        |
| PSR J1744−1134 | 0.2                        | 0.159 ± 0.144 | 2.57 ± 0.21 | 7.37 ± 0.85 | 1.09/16                       |
| PSR J1231−1411 | 0.04                      | 0.04 ± 0.01 | 8.35 ± 0.05 | 30.80 ± 1.20 | 0.98/225                      |
| PSR J2256−1024 | 0.03                      | 1.33 ± 0.52 | 2.93 ± 0.20 | 6.46 ± 1.3 | 0.73/6                        |
| PSR J1723−2837 | 0.37                      | 0.133 ± 0.004 | 1.16 ± 0.04 | 108.00 ± 7.30 | 0.82/208                      |
| PSR J1400−1431 | 0.015                     | 0.16 ± 0.01 | 1.16 ± 0.17 | 1.07 ± 0.04 | 0.51/18                       |
| PSR J0932−0607 | 0.04                      | 0.04 ± 0.01 | 2.44 ± 0.23 | 1.07 ± 0.04 | 1.21/12                       |
| PSR J2241−5236 | 0.0121                    | 0.176 ± 0.01 | 2.64 ± 0.15 | 6.23 ± 0.02 | 0.56/25                       |

* $nH$ values have been obtained from Kalberla et al. (2005).
** The value for this source has been taken from Swiggum et al. (2017).
1 $R$ is the apparent emitting radius calculated using the distance and the normalization of the blackbody model.
been able to use one such new measurement. The distance calculated by Jennings et al. (2018). In our analysis we have addition to these, with the second data release of the GAIA mission Gaia Collaboration et al. (2018) astrometric param-

Table 4. Results of the NSA and blackbody temperature for different radius and bounds on the surface temperature in the considered spectral model. The error range for the temperatures stems from the uncertainties of both the radius and distance measurements. The radius is considered to be in the range $8 \text{ km} \leq R \leq 15 \text{ km}$ and the uncertainty in the distance is assumed to be $\pm 15\%$ in cases where it is not provided. The largest temperature corresponds to the smallest radius and source distance, and vice versa.

| Source name       | $f$ (Hz) | Distance (kpc) | $T_{\text{NSA}}^s$ (eV) | $T_{\infty}^{(\text{BB})}$ (eV) | $\sigma_{\text{rigorous}}^{(\text{NSA})}$ (eV) | $\sigma_{\text{fiducial}}^{(\text{NSA})}$ (eV) |
|-------------------|---------|---------------|--------------------------|-------------------------------|----------------------------------|----------------------------------|
| J1810$+$$1744$    | 602.41  | 2.0           | 23.22 $\pm$ 4.5          | 36.0 $\pm$ 3.0                | 3.8 $\times 10^{-9}$             | $1.6 \times 10^{-9}$             |
| J1744$-$$1134$    | 245.426 | 0.357         | 21.69 $\pm$ 4.0          | 43.0 $\pm$ 2.5                | $1.2 \times 10^{-7}$             | $5.1 \times 10^{-8}$             |
| J1231$-$$1411$    | 271.45  | 0.44          | 18.25 $\pm$ 3.7          | 26.0 $\pm$ 1.5                | $5.8 \times 10^{-8}$             | $2.4 \times 10^{-8}$             |
| J2256$-$1024      | 436.68  | 0.6           | 23.45 $\pm$ 4.5          | 40.0 $\pm$ 2.5                | $1.2 \times 10^{-8}$             | $5.6 \times 10^{-9}$             |
| J1723$-$$2837$    | 538.87  | $0.91^{+0.05}_{-0.04}$ | 36.42 $\pm$ 6.3          | 50.0 $\pm$ 3.5                | $1.4 \times 10^{-8}$             | $6.2 \times 10^{-9}$             |
| J1400$-$$1431$    | 324.23  | 0.27          | 10.31 $\pm$ 1.5          | 17.0 $\pm$ 1.0                | $8.3 \times 10^{-9}$             | $3.8 \times 10^{-9}$             |
| J0952$-$$0607^*$  | 707.3   | 1.74          | 19.49 $\pm$ 4.0          | 22.0 $\pm$ 3.5                | $1.4 \times 10^{-9}$             | $6.0 \times 10^{-10}$            |
| J2241$-$$5236$    | 457.31  | 0.5           | 17.04 $\pm$ 3.3          | 29.0 $\pm$ 2.1                | $6.0 \times 10^{-9}$             | $2.6 \times 10^{-9}$             |

* Two different distance values ($d = 1.74 \text{kpc}$ and $d = 0.97 \text{kpc}$, respectively) are assumed.

Figure 5. $\chi^2$/dof as a function of the unredshifted surface temperature of the assumed neutron star atmosphere model for PSR J1810$+$$1744$ (left panel) and PSR J2256$-$1024 (right panel). The limit temperature is determined from the point where the $\chi^2$/dof deviates from the best fit value by $1\sigma$ (Avni et al. (1976)). The thickness of vertical shaded area is due to different distance assumptions.

maps, often the uncertainties are not presented, since the model-dependent systematic uncertainties are often larger than the formal uncertainties of the measurements. In such cases we assumed a $15\%$ uncertainty on the distance and used this to derive our limits on the surface temperature. In addition to these, with the second data release of the GAIA mission Gaia Collaboration et al. (2018) astrometric parameters and distances of several millisecond pulsars have been calculated by Jennings et al. (2018). In our analysis we have been able to use one such new measurement. The distance of pulsar PSR J1723$-$$2837$ (Jennings et al. (2018)) changed from $\approx 0.74 \text{kpc}$ to $0.91^{+0.05}_{-0.04} \text{kpc}$, making it the pulsar with the most precise distance measurement in our sample. Because the distance of an object directly determines the amount of observed flux and correspondingly its temperature, taking into account the uncertainties in the distances when determining the limits on the surface temperature becomes elemental.

Taking into account the above mentioned atmospheric effects and uncertainties in the distance, we followed a very similar method to Schwenzer et al. (2017). We added a thermal component representing the surface emission of the neutron star to the best fit model, presented in Section 3. For a fixed distance and radius (apparent emitting radius in the case where we assumed a BB function for the surface emis-

\[ \text{http://sci.esa.int/gaia/} \]
sion), we increased the temperature of the thermal component from the lower limit of the NSA model (0.001 keV in the case of a BB model) and investigated the resulting $\chi^2$/dof. For each source the resulting change in the $\chi^2$/dof as a function of the temperature of this hot surface component is shown in Figures 5, 6, 7, and 8. Colors in the figures indicate results for different distance and apparent radius assumptions. We report the limiting temperatures in Table 4, which are the values corresponding to a 1σ in $\chi^2$ Avni (1976). The uncertainties in Table 4 reflect the uncertainties in the distance and radius of the neutron star.

One remaining assumption in our calculations is related to the Hydrogen column density. As it is well known, soft X-rays are absorbed in the interstellar medium (ISM) because
of photoelectric absorption and scattering by gas and dust grains. For the analysis of thermal emission from neutron stars, generally, $N_H$ is a fit parameter that shows a correlation with the inferred temperature and emitting radius values. We here utilized the values presented by Kalberla et al. (2005) and used them as a fixed parameter in our analyses (see Table 3). To test the potential effects of this assumption, we allowed the column density to be free while fitting the spectra as well. For this purpose we used four sources with thermal and non-thermal components. These sources are PSR J1810+1744, PSR J1231−1411, PSR J1723−2837 and PSR J2241−5236 respectively. We found that for two pulsars (PSR J1231−1411 and PSR J2241−5236) the inferred $N_H$ values increased by more than 50%. Such a change in the $N_H$ resulted in the limiting temperatures to change to $T_{NSA} = 19.92$ eV and $T_{NSA} = 17.11$ eV, respectively. In one other case, we found the $N_H$ to be lower than the fixed value we used, by more than 80%, which resulted in a decrease in the limiting temperature, as $T_{NSA} = 26.511$ eV, for PSR J1810+1744. Finally, the Hydrogen column density value is found to be in agreement with the fixed value used, which resulted in a decrease in the limiting temperature, $T_{NSA} = 33.33$ eV for PSR J1723−2837. Overall, we found that the variation in our limits to the surface temperature is negligible and at the 7.3% level. Such variations show the complicated correlation between the inferred temperature and Hydrogen column density, which also depend on the signal to noise at lower energies.

5 R-MODE AMPLITUDE BOUNDS

As demonstrated previously Mahmoodifar & Strohmayer (2013); Alford (2013); Schwenzer et al. (2017), the temperature bounds obtained in the last section impose bounds on the amplitude of r-modes Papaloizou & Pringle (1978); Andersson (1998); Lindblom, Owen, & Morsink (1998); Friedman & Morsink (1998); Andersson (2000), which are global toroidal oscillations of rotating stars that are driven unstable by gravitational wave emission via the Friedman-Schutz mechanism Friedman & Schutz (1978). Since r-modes are unstable, the dissipation required to saturate them would strongly heat the star if they are present. The observed rather cold millisecond sources therefore significantly constrain the presence of r-modes within them. The size of r-modes is determined by the dimensionless amplitude parameter $\alpha$ defined in Lindblom, Owen, & Morsink (1998).

Whereas spin-down data was known to set bounds of at most $\alpha \lesssim 10^{-7}$, X-ray data has steadily improved these limits. Initial limits stemmed from sources that were heated by accretion in LMXBs Mahmoodifar & Strohmayer (2013), which allowed to directly measure their surface temperature, leading to bounds $\alpha \lesssim 10^{-8}$. In Schwenzer et al. (2017) it was shown that even though the temperature of cold millisecond pulsars is too low to directly observe a thermal surface component, its absence can set even tighter constraints on the size of the r-mode amplitude, leading to bounds $\alpha \lesssim 10^{-8}$ even when the sizable uncertainties in the analysis are taken into account. Similar results were presented for sources in globular clusters Bhattacharya et al. (2017) and very recently the spectral observation of the 707 Hz pulsar J0952−0607 set the most restrictive bound to date Ho, Heinke, & Chugunov (2019).

Here we use the spectral results for the various millisecond pulsars discussed above, including in particular the novel data on J1810+1744, J2241−5236 and the recent data on J0952−0607, to obtain tighter bounds on the r-mode amplitude. To this end, following Schwenzer et al. (2017), we...
take into account the main uncertainties in the analysis to obtain robust upper bounds. As discussed in the previous section this includes in particular the use of the more realistic neutron star atmosphere (NSA) model.

The bound on the r-mode amplitude that is set by an observed temperature bound stems from simple conservation equations Mahmoodifar & Strohmayer (2013); Schwenzer et al. (2017) and has been given in its general form that exhibits the explicit dependence on the various parameters in Schwenzer et al. (2017). The most important case are sources without fast neutrino cooling (like direct Urca processes), i.e. merely slow modified Urca processes, in which case for surface temperatures roughly below $10^8 K$, as realized for the sources considered here, photon cooling from the surface strongly dominates Schwenzer et al. (2017). In contrast to a spectral blackbody model, the NSA model yields as an output the (unredshifted) surface temperature $T_\infty$ instead of the (redshifted) temperature $T_\infty^*$ as observed far away from the source. In terms of $T_\infty$ the bound on the r-mode amplitude for a star with fast neutrino cooling reads

$$\alpha_{\text{sat}} \leq \sqrt{\frac{3}{2^5 \pi^6} \frac{3^5}{(3 - 2\chi)} \chi^2 G \frac{1}{2} \frac{T_\infty^2}{M R^2 \chi^3}} \quad (1)$$

where $f, M$ and $R$ are spin frequency, mass and radius, while $J$ is a dimensionless constant entering the r-mode gravitational wave emission, that encodes the radial energy density profile $\rho(r)$ of the source

$$J_m \equiv \frac{1}{2M \int_0^R dr \rho(r) > 1/(2\pi)} \quad (2)$$

and the rigorous lower bound is imposed by the stability of the star Alford & Schwenzer (2014). The weakly frequency dependent factor $\chi(\Omega) \approx 1$ describes the deviation of the connection between the rotation frequency $\Omega = (2\pi)$ and the r-mode oscillation frequency $\nu = \omega/(2\pi)$ from their canonical relation $\omega = \Omega$, and is determined by general relativistic and rotation corrections.

The various quantities arising in eq. (1) depend on the particular star configuration, which, for a given equation of state (EoS), can be parametrized by the mass $M$, and are not independent of each other. The radius in particular is approximately monotonously decreasing with increasing mass. In the non-relativistic case, for a neutron star sufficiently away from its mass limit, they scale roughly as $M \sim \frac{1}{R}$ in. In the case of the arising mass and radius dependence mostly cancels out and the expression would be merely linearly dependent on the radius. Only close to the mass limit the mass asymptotes to its maximum value while the radius decreases further. Therefore, there the bound could become slightly weaker compared to the non-relativistic scaling region. Yet, this surely only happens when the bound is already particularly tight since the mass, which can vary over a large range, is at its upper limit. The same holds for $J$, which is likewise maximal for the most massive configuration Alford, Mahmoodifar, & Schwenzer (2012). This generic behavior of the arising factor that incorporates the dependence on the particular star configuration is shown in fig. 9 for an APR EoS Akmal et. al. (1998) and confirms that it is away from the mass limit indeed a monotonously decreasing function of mass. Therefore the uncertainty is in general much weaker than eq. (1) suggests and a strict limit is obtained for the minimum mass corresponding to the minimum $J$ and the maximum radius for a given equation of state. The most strongly constrained and lowest masses have been observed in double neutron star binaries, which very likely present birth mass since no recycling is possible in such systems. These lie around $M \approx 1.3 M_\odot$ Özel & Freire (2016), and this value presents therefore a lower bound for the neutron star masses in millisecond pulsars which have a long accretion history and should therefore have masses significantly larger than their birth mass. Present observations constrain the radius of a $1.4 M_\odot$ neutron star to $10 \text{ km} \lesssim R_{1.4} \lesssim 11.5 \text{ km}$ Özel & Freire (2016). In this low mass regime the radius is very weakly mass-dependent anyway and using for a strict bound nevertheless $10 \text{ km}$, as the lower limit of the above range, as well as the universal lower bound on $J$ eq. (2) in eq. (1) gives therefore a rigorous amplitude bound. Taking further into account that the dominant relativistic corrections increase the r-mode frequency factor $\chi > 1$ Idrisy et al. (2015) gives finally

$$\alpha_{\text{sat}} < 1.0 \times 10^{-9} \left( \frac{T_\infty}{10 \text{ eV}} \right)^2 \left( \frac{500 \text{ Hz}}{f} \right)^4 \left( \frac{1.3 M_\odot}{20 \text{M}_\odot} \right)^{-1} \left( \frac{R}{10 \text{ km}} \right)^{-2} \quad (3)$$

where the general bound is obtained when the bracket containing the dependence on the source properties is one, and a more restrictive bound can be obtained if information on source properties is available.

As discussed previously in Schwenzer et al. (2017), in addition to the temperature dependence of these bounds, the other key parameter is the spin frequency, and fast spinning sources lead to significantly lower limits. If fast cooling processes are absent, eq. (1) is a good approximation for most millisecond pulsars since the power law exponents for photon (θγ = 4) and neutrino emission (θθ > 4) are very different, so that for modified Urca processes (θ = 8) the neutrino emission is suppressed by a factor 16 compared to photon emission already at about $5 \times 10^6 \text{ K}$, i.e. at half the temperature where they are of equal size. Yet, if fast cooling processes are present they can generally compete with photon cooling in observed sources.
The rigorous bounds for the sources with new thermal X-ray data, employing the NSA model, are shown in fig. 10 (coloured solid triangles) and the corresponding numerical values are given in tab. 4. Where timing data is available the data points are compared to their spindown limits (diamonds). As can be seen the spectral bounds are significantly below the spindown limits and those of some millisecond pulsars set very tight bounds on the r-mode amplitude—the most restrictive bound \( \alpha \text{sat} \lesssim 1.4 \times 10^{-9} \) being obtained for the pulsar PSR J0952+0007. However, also the bounds from the two new sources PSR J1810+1744 and PSR J2241–5236, for which we analyzed dedicated new XMM-Newton observations in this work, are not much weaker. Our lowest rigorous bound, obtained for PSR J0952+0007, is comparable to the value given for this source in Ho, Heinke, & Chugunov (2019). In Bhattacharya et al. (2017) a similarly low bound of \( \alpha \lesssim 2.5 \times 10^{-9} \) has been given based on a luminosity bound of 47 Tuc aa. However, this bound does not systematically take into account the uncertainties in the analysis and further involves a particular heating mechanism to lower the bound, which involves its own uncertainties and assumptions. These bounds are by now substantially lower than the best bound stemming from LMXBs Mahmoodifar & Strohmayer (2013).

The assumption entering the rigorous bounds are probably unrealistically conservative and if we use conventional source assumptions, as done in other studies, these bounds would be even tighter. A fiducial 1.4 \( M_\odot \) APR neutron star Alford, Mahmoodifar, & Schwenzer (2012) is e.g. given by the open triangles in fig. 10 and in the last column of tab. 4, which yields values that are nearly a factor two lower. A better understanding of the equation of state of dense matter, e.g. due to advancements owing to the NICER observatory or future gravitational wave measurements, would therefore tighten these bounds. Moreover, as discussed, all these sources are expected to be substantially heavier than the mass limit of \( M \gtrsim 1.3 M_\odot \) assumed here. To reach their high frequencies, the millisecond sources we study need to have been spun up by accretion in a binary. The mass transfer should therefore result in star masses in the upper range of possible mass values, as is clearly observed for the few millisecond sources were mass measurements are available Özel & Freire (2016). Unfortunately, to our knowledge none of the sources presented in section 2 has currently a mass measurement Antoniadis et al. (2016). Such independent mass constraints significantly enhance the bounds, as was e.g. shown for the case of J1023+0038 in Schwenzer et al. (2017).

In fig. 11 our bounds using the more realistic NSA model (triangles) are compared to the previously employed blackbody model (squares), which shows that the new bounds based on the NSA model are significantly stronger, owing to the tighter thermal bounds. As can be seen, this is a generic effect illustrated by the fact that all dotted line segments, connecting the two different bounds for a given source, are roughly parallel and have a similar length. For comparison fig. 11 also shows blackbody results for other sources taken from our previous analysis Schwenzer et al. (2017). Most of these are mere luminosity bounds that did not rely on detailed spectral fitting of a surface component. As can be seen our new results significantly surpass these previous bounds from values around \( 10^{-8} \) to bounds not far above \( 10^{-9} \). However, taking into account that the NSA model generally

![Figure 10](image-url)

**Figure 10.** Bounds on the r-mode amplitude stemming from thermal X-ray data (triangles) using a NSA model compared to those from pulsar timing data (diamonds) for the sources discussed in section 2. Full triangles denote the rigorous upper limits and open triangles the bounds for fiducial standard assumptions on source properties. The small circles show model computations for the special case that fast cooling would be present in the source.

![Figure 11](image-url)

**Figure 11.** Comparison of the amplitude bounds when employing the NSA atmosphere model (large colored triangles, same colors as in fig. 10) vs. a simple blackbody model (small colored squares). The plot also shows our previous blackbody bounds (gray symbols) for the sources that had been studied (Schwenzer et al. (2017)).
strengthens the bounds, there are several sources, for which a proper spectral reanalysis with the NSA model or even the use of additional improved data would be very promising. PSR 1023+0038, which had imposed the tightest bound in Schwenzer et al. (2017), being a particular promising candidate. It has a mass measurement and therefore even the mere blackbody luminosity bound is still competitive with the presently best source PSR J0952+0607.

As discussed, the present bounds were obtained for the standard case that photon emission from the atmosphere dominates the cooling of the star. This is the case when merely standard neutrino cooling mechanisms, like modified Urca emission, are present. However, fast cooling mechanisms, like pair breaking emission or even direct Urca processes, could be present in these sources. In this case, even at the observed low temperatures, neutrino emission from the bulk would still dominate. As noted in Mahmoodifar & Strohmayer (2013); Schwenzer et al. (2017) this could in principle lead to slightly weaker bounds. These depend on the particular neutrino cooling mechanism involving additional microscopic physics and it is therefore harder to give a rigorous expression. However, as done previously Mahmoodifar & Strohmayer (2013); Schwenzer et al. (2017) one can get an idea of the maximum impact of fast cooling by considering the extreme case of hadronic direct Urca cooling, presenting the fastest know cooling mechanism. In this case the bound reads Schwenzer et al. (2017)

$$\alpha_{\text{sat}} \leq \sqrt{\frac{3 \alpha^2 L_{\text{QCD}}}{2 F J^2 L_{\text{damp}} GM^2 R^3}} T^4$$

(4)

where the dimensionless quantity $\tilde{L}$ characterizes the neutrino emission in the star. We use a maximum mass neutron star model with an APR equation of state ($\approx 2.2 M_\odot$), that features a sizable core where direct Urca emission is allowed Alford, Mahmoodifar, & Schwenzer (2010). In the low temperature region such massive and compact source would according to eq. (4) impose a stronger bound and therefore we correct for the deviation from our fiducial parameter set from a $1.4 M_\odot$ star discussed above. This way we take into account that for another equation of state direct Urca processes might already be possible in a $1.4 M_\odot$ source. The results are given by the small circles in fig. 10 and as can be seen even with these extreme assumptions the obtained bounds are roughly comparable to those stemming from pure photon emission.

The obtained amplitudes are not too far from the regime $\lesssim 10^{-11} - 10^{-10}$ where r-modes could be completely ruled out in these sources, since even the fastest millisecond pulsars could cool out of the instability region Alford (2013). What is more is that these amplitudes are many orders magnitude below those that well established saturation mechanisms, like for instance mode-coupling Bondarescu & Wasserman (2013),can provide. Therefore, neutron stars with minimal damping require another strong non-linear dissipation mechanism that can completely damp or saturate r-modes at such low amplitudes. Such an additional mechanism is not established at this point in ordinary neutron matter and therefore the minimal picture of neutron stars is currently incompatible with the astrophysical data. This points to fascinating new physics and there are several interesting proposals Madsen (1999); Alford (2013); Alford & Schwenzer (2014).

The bounds on the r-mode amplitude impose finally corresponding bounds on the gravitational wave signal from these sources. As shown in Alford & Schwenzer (2014); Schwenzer et al. (2017), despite the large spin frequencies of these sources, even the weaker previous amplitude bounds led to a gravitational wave strain for theses sources that would be too low to be detectable with current gravitational wave detectors. The strengthened bounds underline this conclusion and show that a potential r-mode emission of known millisecond pulsars is unfortunately out of reach at present. Young sources present therefore a far more promising target to detect gravitational wave emission due to r-modes Alford & Schwenzer (2014).

6 PRESENCE OF R-MODES

It had been shown, that, if millisecond sources are ordinary neutron stars with known damping mechanisms, they would be trapped within the r-mode instability region. This holds since we on one side observe their tiny spindown rates and on the other side the r-mode amplitudes obtained from well constrained saturation mechanisms would strongly heat these sources so that they cannot cool out of the instability region. However, the increasingly restrictive temperature bounds, imposing corresponding bounds on the r-mode amplitude in millisecond sources, also open the possibility that r-modes can be completely absent in many sources even in case a minimal neutron star scenario is realized. As shown in Alford (2013) for amplitudes as low as $\alpha \lesssim 10^{-11} - 10^{-10}$ the r-mode heating would be small enough that all of these very old sources should have by now cooled out of the instability region. R-modes and the accompanying gravitational wave emission are therefore absent and no saturation mechanism is required at present. Yet, as argued in Alford (2013) this would nevertheless have required a humongous dissipation in their interior during their evolution since it is known from observed LMXBs that they start their cooling evolution after recycling right within the instability region of a minimal neutron star. In this section we discuss this possibility in detail and take into account the uncertainties in the analysis to derive a simple but rigorous condition based on X-ray observations for when r-modes will be absent in a millisecond pulsar. This information should be useful for continuous gravitational wave searches. Millisecond pulsars undergoing r-mode oscillations would be ideal sources for gravitational wave astronomy due to their enormous stability. They would allow us to perform precision multi-messenger gravitational wave observations over long time intervals that could determine various source properties ranging from bulk observables, like mass, radius and moment of inertia, to thermal properties and even independent distance measurements that are not affected by interstellar absorption Aasi et al. (2013); Alford & Schwenzer (2014); Kokkotas (2016).

General analytic estimates for the boundary segments of the r-mode instability region had been obtained in Alford, Mahmoodifar & Schwenzer (2010). In the low temperature region, relevant for millisecond pulsars, shear viscosity is the relevant mechanism, which in neutron matter (as well as in most other forms of matter) dominantly stems from the scattering of relativistic particles due to long-ranged interactions mediated by Landau-damped transverse gauge bosons.
Strengthening the bounds on the r-mode amplitude with X-ray observations of millisecond pulsars

Shternin & Yakovlev (2008). Both in neutron matter and non-CFL quark matter electromagnetic electron scattering is the relevant mechanism, whereas in the quark case there are analogous additional contributions from quark scattering, reducing the mean free path and increasing the instability region. Generalizing the result given in Alford, Mahmoofifar & Schwenzer (2010) by taking into account a general r-modes dispersion relation $\omega = -\beta/3\chi \Omega$ via the correction factor $\chi \approx 1$, the low-temperature segment of the instability boundary is in terms of the core temperature $T$

$$\Omega_{ib} = 1.12 \frac{S^{1/6} \Lambda^{1/9}_{QCD}}{(3 - 2\chi)^{1/6} \chi^{5/6} J^{1/3} G^{1/6} M^{1/3} R^{1/2}} T^{12/18}$$

where $\tilde{J}$ and $\tilde{S}$ are dimensionless constants describing weighted averages of the energy density and of the shear viscosity over the entire source and $\Lambda_{QCD} = 1$ GeV is a generic normalization scale. This requires to connect the core temperature to the observed surface temperature. Taking into account that the thermal conductivity in the core of a neutron star is very large, it has been shown Gudmundsson et al. (1983) that the core temperature $T$ is to a very good approximation a function of the single quantity $T_s^4/g_s$, where $g_s \equiv GM/(R\sqrt{1 - 2GM/R})$ is the surface gravitational acceleration. This function is determined by the detailed heat transport in the crust and the connection stemming from numerical simulations can generally up to corrections at the 10%-level be approximated by a simple power law. Since the millisecond sources considered here are currently not perceivably accreting and likely have not done so for a long time, a model of a catalyzed iron crust without light element admixtures is appropriate. The power law dependence takes in this case the form Gudmundsson et al. (1983)

$$\frac{T}{10^8 \text{K}} \approx 1.288 \frac{10^{14} \text{g cm}^{-2}}{g_s} \left(\frac{T_s}{10^9 \text{K}}\right)^{1.455}$$

and a quantitatively very similar power law (prefactor 1.429, exponent 0.444) is obtained for the leading component in the independent analysis Potekhin (1997) (where the accuracy of the fit was improved to the 1%-level by adding a second, subleading power law component that contributes less than 3% for $T \geq 10^8$ K). Quantitatively comparing the two results shows that they deviate by less than 10% for $T_s \geq 10^8$ K so that the core-surface temperature relation should be accurate at this level. Inserting this above the non-power-law correction factor $\sqrt{1 - R/2R_s}$, where $R_s = 2GM$, enters via a very low power $\approx 0.06$ and therefore amounts for the realistic regime of neutron star radii $R > 2R_s$ to less than 3 percent. Similarly in the relevant regime $1 - \chi < 1$ the dependence on the r-mode dispersion relation can be simplified, and the result for the boundary of the r-mode instability boundary takes the form

$$f_{ib} = (403 \pm 40) \times \frac{\tilde{S}_{1/4}}{\chi^{1/2} \tilde{J}_{1/4}} \times \left(\frac{M}{1.4 M_\odot}\right)^{-0.207} \left(\frac{R}{10 \text{ km}}\right)^{-0.753} \left(\frac{T_s}{10^9 \text{K}}\right)^{-0.506}$$

where $\tilde{J}_{1/4}$ and $\tilde{S}_{1/4}$ are the corresponding values for a standard 1.4 $M_\odot$ neutron star Alford, Mahmoofifar, & Schwenzer (2012) with an APR equation of state Akmal et. al (1998). In order to determine if r-modes could be completely absent in observed sources, we are interested in the maximal size of the instability region, that standard neutron stars without enhanced damping could have, i.e. the minimal possible frequency value eq. (7) can take. As seen in fig. 9 the product of factors encoding the source dependence is basically independent of the mass, with a shallow minimum around 2$M_\odot$. To get a bound we use here correspondingly a 2$M_\odot$ APR Akmal et. al (1998) model Alford, Mahmoofifar, & Schwenzer (2012) and estimate the residual uncertainty due to the equation of state as 10%. The shear viscosity of dense neutron matter stems from sufficiently constrained leptonic processes Shternin & Yakovlev (2008) and the dimensionless factor $\theta$ should therefore be uncertain by at most a factor two. The uncertainty due to the r-mode dispersion is around 10% and the small deviation of the temperature exponent from 1/2 imposes over the relevant temperature range merely a 1% effect. This yields for a given temperature a minimal frequency to which the instability region can extend. A source spinning with a smaller frequency $f$ will even within the sizable uncertainties be outside of the instability region, so that it cannot emit gravitational waves due to r-modes. We obtain correspondingly finally the simple bound, that a source is stable if

$$f \leq (389^{+165}_{-116}) \text{Hz} \sqrt{\frac{T_s}{10^9 \text{K}}}$$

The comparison of the sources discussed in this work with the boundary of the instability region is shown in fig. 12. As can be seen, within the sizable uncertainties none of the considered sources is clearly outside of the instability region of a neutron star with standard viscous damping mechanisms, yet. Therefore r-mode gravitational wave emission cannot be excluded at present. However some of them could be excluded in the future if tighter temperature limits can be obtained or the theoretical uncertainties on the instability region can be reduced. The figure also shows the two sources J0437–4715 Durant et al. (2012) and J2124–3358 Rangelov et al. (2017) for which an actual surface temperature estimate could be obtained using additional UV observations. As seen, even though within the sizable uncertainties the presence of r-modes cannot be completely excluded, they are very likely outside of the instability region.

At very low temperatures the shear viscosity of dense hadronic matter can be dominated by screened, short-ranged interactions Shternin & Yakovlev (2008) which reduces the instability region and correspondingly makes it even easier that fast spinning sources can be outside. This effect will have to be included once even lower temperature bounds or measurements are obtained.

7 DISCUSSION AND CONCLUSION

Using new dedicated XMM-Newton observations of PSR J1810+1744 and PSR J2241–5236 and refined X-ray spectral analyses of several other millisecond pulsars, we set new bounds on the surface emission of these sources. We obtained our lowest limit for PSR J1400–1431, which equals the lower limit of the temperature limit where the NSA model is valid, as defined in Xspec. This is a remarkable limit, as it hints at a surface temperature lower than, 10^7 K.
At such lower temperature values the fully ionized Hydrogen atmosphere assumption may not be valid anymore. Apart from PSR J1400–1431, our results are in general found to be around 20 eV. The exact value for a neutron star, obviously depends on the continuum emission, the distance, hydrogen column density in the line of sight and the instrument being used. On the latter, it is obvious that instruments sensitive to far ultraviolet are necessary. The proposed LUVOIR mission may eventually contribute a lot to constraining the surface emission of these old neutron stars (The LUVOIR Team 2018). The problem at such low wavelengths though is that at these regions the companion object may be so dominant in the total emission of the system that it may be impossible to constrain the surface temperature of the neutron star (see however, Rangelov et al. (2017)). Therefore one other option would be to discover more nearby millisecond pulsars, and observe them in the soft X-ray band. The recently launched eROSITA may provide critical contributions (Merloni, et al. 2012).

Since r-modes would strongly heat such sources, our results impose tighter bounds on the r-mode amplitude than obtained in previous analyses, as well as obtained from accreting sources in LMXBs or mere spin down limits. The key finding of r-mode astero-seismology has been that at temperatures present in LMXBs, standard dissipative mechanisms in neutron stars cannot damp r-modes in sources spinning faster than $f \gtrsim 100\,\text{Hz}$ Lindblom, Owen, & Morsink (1998), Alford & Schwenzer (2014) (see in Table 2). For a faster-spinning star made of neutron matter, this therefore requires a mechanism that eventually saturates the mode due to a nonlinear, amplitude-dependent enhancement of the dissipation. Several such mechanisms have been proposed, but our new frequency bounds $\alpha \lesssim 10^{-9}$ are by now so far below the saturation amplitudes $\alpha \lesssim 10^{-6} - 10^{-5}$ that standard mechanisms in neutron stars Bondarescu & Wasserman (2013) might provide, that it becomes more and more unlikely that the saturated r-mode scenario is realized Haskell et al. (2015). An exception is a hybrid star with a sharp interface, where phase conversion dissipation, presenting the strongest known dissipation mechanism Alford et al. (2015), can provide even lower saturation amplitudes.

Correspondingly our results show that there must be significant additional damping in these sources and further enlarge the discrepancy between standard, well-constrained damping mechanisms and the astrophysical data. Therefore, a minimal neutron star composition is by now basically ruled out by the astrophysical data, and it is likely that there is actually a mechanism, that does not merely saturate r-modes at very low amplitudes, but that completely damps them away. This requires very strong dissipation that could either stem from exotic phases of matter with inherently strong dissipation Alford, Han & Schwenzer (2019) or from the structural complexity of the star.

The first case is simpler since it is described by the local hydrodynamic dissipation coefficients, which are generally determined by the dynamics of the low energy degrees of freedom of the corresponding phase. An important example is ungapped quark matter, where the bulk viscosity due to non-leptonic, flavor-changing interactions is resonantly enhanced under the conditions in cold neutron stars, since the scale width of the weak processes matches the time scale of r-mode oscillations Alford & Schwenzer (2014). Color-superconductivity in general significantly suppresses the dissipation Alford, Han & Schwenzer (2019). Goldstone bosons could become relevant if their mean free path becomes large. However, at compact star temperatures it even largely exceeds the size of the star so that Goldstone bosons behave ballistically which complicates the analysis preventing so far precise quantitative predictions.

The simplest example for a dissipative mechanism due to the structural complexity of the star is the Ekman-layer rubbing of the fluid in the core along a solid crust Lindblom, Owen, & Ushomirsky (1998). Yet, even under most favorable assumptions this cannot provide the required damping Alford & Schwenzer (2014). Further examples for dissipation in structurally complex sources are given by a hadronic-quark interface Alford et al. (2015), non-uniform (“pasta”) phases Horowitz & Berry (2008), extended field configurations like fluxtubes and/or vortices Haskell et al. (2014), or the interactions of the r-mode with more localized oscillation modes, e.g. in the superfluid core Gusakov et al. (2014); Ho, Heinke, & Chugunov (2019). All these are more complicated to describe since they require detailed model assumptions about the poorly constrained structural composition of neutron stars and therefore only allow very rough estimates for the possible dissipation.

As had been shown in Alford (2013), at very low saturation amplitude, slower spinning MSPs—those that would be trapped inside the instability region if the r-mode amplitude would be larger—can actually cool out of the r-mode instability region on times much shorter than their billion year ages, and therefore r-modes are not expected in these sources. We derived an analytic expression for the limiting spin-frequency, below which r-modes can be ruled out in a
given source. This also allows us to estimate the involved uncertainties and we find that within the sizable uncertainties r-modes cannot be ruled out at present in the considered sources.

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