TARGET MASS EFFECTS
IN POLARIZED DEEP-INELASTIC SCATTERING

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Abstract

We present a computation of nucleon mass corrections to nucleon structure functions for polarized deep-inelastic scattering. We perform a fit to existing data including mass corrections at first order in $m^2/Q^2$ and we study the effect of these corrections on physically interesting quantities. We conclude that mass corrections are generally small, and compatible with current estimates of higher twist uncertainties, when available.
1. Introduction

Experimental information on deep-inelastic scattering of polarized leptons off different kinds of polarized nucleon targets has become more and more accurate in the past few years [1]-[9]. This accumulated knowledge, combined with recent theoretical progress in the computation of perturbative QCD quantities relevant to polarized deep-inelastic scattering [10], has allowed a next-to-leading order determination of the polarized parton densities, using data on the structure function $g_1(x,Q^2)$ which determines the cross section asymmetries in the case of longitudinally polarized leptons and nucleons in the Bjorken limit [11]-[14]. Reasonably good determinations of the strong coupling constant $\alpha_s$ and of the axial vector coupling $g_A$ have also been performed [14].

A large part of experimental data in polarized deep-inelastic scattering are taken at relatively low values of $Q^2$. In particular, $Q^2$ is usually around 1 GeV$^2$ for data points in the small-$x$ region, which is particularly interesting because there the effects of $Q^2$ evolution are more important (data at even lower values of $Q^2$ are also available, but they are usually not included in perturbative analyses). In this kinematical region, contributions suppressed by inverse powers of $Q^2$ can become important. These contributions can be of two different origins. There are power-suppressed terms arising from the operator product expansion of the hadronic tensor $W^{\mu\nu}$. These terms originate from matrix elements of operators of non-leading twist (they are usually referred to as dynamical higher twists). Their effect is not controlled by perturbation theory, and it is very difficult to assess their importance. A second class of power-suppressed contributions originates from taking into account the finite value of the nucleon mass $m$ in the kinematics of the leading-twist cross section. These corrections can be computed exactly, and have been studied in detail in the past [15] in the case of unpolarized deep-inelastic scattering. Knowledge of kinematic effects is of course necessary in order to extract information on dynamical higher twists from experimental data.

The problem of calculating kinematic higher twist terms in polarized deep-inelastic scattering has been considered in ref. [16]. There, the reduced matrix elements $a_n, d_n$ of the relevant operators in the OPE were expressed in terms of polarized structure functions, taking mass corrections into account; these expressions reduce to moments

\footnote{The results of ref. [16] have been used in ref. [17] to compute target mass corrections to the Bjorken sum rule.}
of the structure functions in the massless limit, but do not have a simple parton model interpretation in the case \( m \neq 0 \). For this reason, the result of ref. \([16]\) is not directly applicable in a full analysis of polarized deep-inelastic scattering data.

In this paper, we calculate target mass corrections in polarized deep-inelastic scattering extending the analogous work of ref. \([15]\) for the unpolarized case. This has the advantage of yielding the final result in a form which is appropriate for phenomenological applications, that is, we will express moments of polarized structure functions as functions of the reduced operator matrix elements. The details of our calculations are presented in Sect. 2. An important point is of course the interplay between dynamical and kinematical higher twists; in Sect. 2 we discuss this point in some detail. In Sect. 3, we use mass-corrected formulae in an analysis of existing data in the framework of QCD at next-to-leading order, and compare the results with those obtained in the massless limit; in particular, we will compare our results with those of ref. \([14]\), where an estimate is given of the uncertainties coming from higher twist effects on the determination of physically interesting quantities. We will see that the effect of mass corrections is indeed within the uncertainties estimated in ref. \([14]\). Finally, we present our conclusions in Sect. 4.

2. Calculation

Our calculation follows closely the analogous one in the case of unpolarized deep-inelastic scattering, performed by Georgi and Politzer back in 1976 \([15]\). We will adopt the usual notation for the definition of kinematical quantities in deep-inelastic scattering, used for example in ref. \([18]\). In the formalism of the operator product expansion, the antisymmetric part \( T_{\mu\nu}^A \) of the forward amplitude relevant for deep-inelastic scattering,

\[
T_{\mu\nu}(p, q, s) = i \int d^4x e^{iqx} < p, s \mid T[J_{\mu}(x)J_{\nu}(0)] \mid p, s >
\]

is given by \([18]\)

\[
T_{\mu\nu}^A = -i \sum_{n=1}^{\infty} \frac{[1 - (-1)^n]}{2} \left( \frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-2}}
\sum_i \epsilon_{\mu\nu\lambda\sigma} q_{\mu_{n-1}} E_{i,1,1}^n(Q^2, \alpha_s) < p, s \mid R_{1,i,1}^{\sigma\mu_1 \cdots \mu_{n-1}} \mid p, s >
\]
\[ + Q_{\mu \nu \lambda \sigma} \frac{n - 1}{n} E_{2, i}^{m_2} (Q^2, \alpha_s) < p, s \mid R_{2, i}^{\lambda \mu_1 \ldots \mu_{n-2}} \mid p, s > \], \quad (2.2)\]

where \( J_\mu \) is the electromagnetic current, and \( Q_{\mu \nu \lambda \sigma} \) is defined as

\[ Q_{\mu \nu \lambda \sigma} = \epsilon_{\mu \nu \lambda \sigma} q_\mu q_\nu - \epsilon_{\nu \mu \lambda \sigma} q_\mu q_\nu - q^2 \epsilon_{\mu \nu \lambda \sigma}. \quad (2.3)\]

The operators \( R_{\sigma \mu_1 \ldots \mu_{n-1}} \) are given by

\[ R_{1, i}^{\sigma \mu_1 \ldots \mu_{n-1}} = i^{n-1} \left[ \bar{\psi}_5 \gamma_5 \gamma^\sigma D^{\mu_1} \ldots D^{\mu_{n-1}} \frac{\lambda_i}{2} \psi \right]_{S}, \quad i = 1, \ldots, 8; \quad (2.4)\]

\[ R_{1, \psi}^{\sigma \mu_1 \ldots \mu_{n-1}} = i^{n-1} \left[ \bar{\psi}_5 \gamma_5 \gamma^\sigma D^{\mu_1} \ldots D^{\mu_{n-1}} \psi \right]_{S}, \quad (2.5)\]

\[ R_{1, G}^{\sigma \mu_1 \ldots \mu_{n-1}} = i^{n-1} Tr \left[ \epsilon^{\sigma \alpha \beta \gamma} F_\beta \gamma D^{\mu_1} \ldots D^{\mu_{n-2}} F_{\alpha - 1} \right]_{S}. \quad (2.6)\]

Here \( D^\mu \) is the QCD covariant derivative, and the symbol \([ \ldots ]_S\) means complete symmetrization in the indices \( \sigma, \mu_1, \ldots, \mu_{n-1} \); \( F_{\mu \nu} \) is the usual QCD gluon tensor.

The twist-3 operators \( R_{2, i}^{\sigma \mu_1 \ldots \mu_{n-2}} \) are given by

\[ R_{2, i}^{\lambda \mu_1 \ldots \mu_{n-2}} = i^{n-1} \left[ \bar{\psi}_5 \gamma_5 \gamma^\lambda D^{\sigma} D^{\mu_1} \ldots D^{\mu_{n-2}} \frac{\lambda_i}{2} \psi \right]_{S'}, \quad i = 1, \ldots, 8; \quad (2.7)\]

\[ R_{2, \psi}^{\lambda \mu_1 \ldots \mu_{n-2}} = i^{n-1} \left[ \bar{\psi}_5 \gamma_5 \gamma^\lambda D^{\sigma} D^{\mu_1} \ldots D^{\mu_{n-2}} \psi \right]_{S'}, \quad (2.8)\]

\[ R_{2, G}^{\lambda \mu_1 \ldots \mu_{n-2}} = i^{n-1} Tr \left[ \epsilon^{\sigma \alpha \beta \gamma} F_\beta \gamma D^{\mu_1} \ldots D^{\mu_{n-2}} F_{\alpha - 1}^\lambda \right]_{S'}, \quad (2.9)\]

where \([ \ldots ]_{S'}\) indicates antisymmetrization with respect to \( \lambda \) and \( \sigma \) and symmetrization with respect to other indices. The coefficient functions \( E_{1, i}^n (Q^2, \alpha_s) \) and \( E_{2, i}^n (Q^2, \alpha_s) \) have been computed up to order \( \alpha_s^2 \) in perturbative QCD \[19\]. The expansion of the forward scattering amplitude in powers of \( m^2 / Q^2 \) is independent of the perturbative expansion in powers of \( \alpha_s \); we can therefore perform our calculation at leading order in \( \alpha_s \), and then insert perturbative corrections to coefficient functions in the final result.

For \( \alpha_s = 0 \) we have simply \( E_{1, i} = E_{2, i} = 1 \) for \( i = 1, \ldots, 8, \psi \) and \( E_{1, G} = E_{2, G} = 0 \).

The matrix elements of the operators \((2.4-2.5)\) can be written as

\[ < p, s \mid R_{1}^{\sigma \mu_1 \ldots \mu_{n-1}} \mid p, s > = -2mA_n M_1^{\sigma \mu_1 \ldots \mu_{n-1}} \] \quad(2.10)\]

(we have omitted the index \( i \), which is no longer necessary at leading order). The tensor \( M_1^{\sigma \mu_1 \ldots \mu_{n-1}} \) is the most general rank-\( n \) symmetric tensor which can be formed
with one spin four-vector $s$ and $n - 1$ momentum four-vectors $p$; furthermore, it must satisfy the tracelessness conditions
\[ g_{\mu \nu} M_{\mu_1 \cdots \mu_n} = 0 \] (2.11)
for all pairs $i, j$. With these requirements, we find
\[ M_{\mu_1 \cdots \mu_n} = \frac{1}{n \sum_{j=0}^{n-1} \frac{(-1)^j (n-j)!}{2^j n!} g \cdots g [sp \cdots p]_S (m^2)^j } \] (2.12)
up to an overall normalization, which can be absorbed in the definition of the reduced matrix elements $a_n$. The symbol
\[ g \cdots g \] (2.13)
represents a product of $j$ metric tensors $g^{\mu \nu \kappa}$, with indices chosen among $\mu_1, \ldots, \mu_n$ in all possible ways; the remaining $n - 2j$ indices of $M_{\mu_1 \cdots \mu_n}$ are carried by the symmetric product $[sp \cdots p]_S$. When the nucleon mass is neglected, only the first term of the sum, $[s^\sigma p^{\mu_1} \cdots p^{\mu_{n-1}}]_S$, is retained; this is the standard result, used for example in ref. [18].

Consider now the twist-3 operators of eqs. (2.7-2.9). Their matrix elements can be written as
\[ <p, s | R_2^{\lambda \sigma \mu_1 \cdots \mu_{n-2}} | p, s > = m d_n M_2^{\lambda \sigma \mu_1 \cdots \mu_{n-2}} , \] (2.14)
where the tensor $M_2$ must be antisymmetric in $(\lambda, \sigma)$, symmetric in all other indices, and traceless. It is easy to prove that
\[ M_2^{\lambda \sigma \mu_1 \cdots \mu_{n-1}} = \frac{n+1}{n} (s^\sigma \Pi^{\lambda \mu_1 \cdots \mu_{n-2}} - s^\lambda \Pi^{\sigma \mu_1 \cdots \mu_{n-2}}) + \frac{n-1}{n} (p^\sigma M_1^{\lambda \mu_1 \cdots \mu_{n-2}} - p^\lambda M_1^{\sigma \mu_1 \cdots \mu_{n-2}}) \] (2.15)
where
\[ \Pi^{\mu_1 \cdots \mu_n} = \sum_{j=0}^{\frac{n}{2}} \frac{(-1)^j (n-j)!}{2^j n!} g \cdots g p \cdots p (m^2)^j \] (2.16)
is the most general rank-$n$ symmetric, traceless tensor that can be formed with the momentum $p$ alone. For $m^2 = 0$, one recovers the usual result
\[ <p, s | R_2^{\lambda \sigma \mu_1 \cdots \mu_{n-2}} | p, s > = m d_n \left( s^\sigma p^\lambda - s^\lambda p^\sigma \right) p^{\mu_1} \cdots p^{\mu_{n-2}} . \] (2.17)

The reduced matrix elements $a_n, d_n$ contain all the information on the proton spin structure; they are related to moments of polarized structure functions. Our next step
consists in obtaining these relationships in the general case \( m \neq 0 \). To do this, we compute explicitly the amplitude \( T_{\mu \nu}^A \) using our results, eq. (2.10) and eq. (2.14). We decompose \( T_{\mu \nu}^A \) into a twist-2 and a twist-3 component,

\[
T_{\mu \nu}^A = T_{\mu \nu}^{(a)} + T_{\mu \nu}^{(d)}.
\] (2.18)

Let us first consider the contribution of twist-2 operators,

\[
T_{\mu \nu}^{(a)} = 2im\epsilon_{\mu \nu \lambda \sigma} q^\lambda \sum_{n \text{ odd}} \left( \frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-1}} M_1^{\sigma} \cdots M_{n-1}^{\sigma} a_n.
\] (2.19)

Using eq.(2.12) and recalling that \( x = Q^2/(2p \cdot q) \) we find

\[
T_{\mu \nu}^{(a)} = 2im\frac{p \cdot q}{Q^2} \epsilon_{\mu \nu \lambda \sigma} q^\lambda \sum_{n \text{ odd}} x^{-n/2} \sum_{j=0}^{n-1} \left( \frac{x^2 m^2}{Q^2} \right)^j \frac{(n-j)!}{(n-1-2j)!} \left( s^\sigma + (n-2j-1) \frac{s \cdot q}{p \cdot q} p^\sigma \right).
\] (2.20)

Following ref. [15], we change summation index from \( n \) to \( l \), with \( n = 2l + 2j + 1 \), and exchange the summation order of \( l \) and \( j \). This gives

\[
T_{\mu \nu}^{(a)} = 2im\frac{p \cdot q}{Q^2} \epsilon_{\mu \nu \lambda \sigma} q^\lambda \sum_{l=0}^{\infty} x^{-(2l+1)} \left( s^\sigma + 2l \frac{s \cdot q}{p \cdot q} p^\sigma \right) \sum_{j=0}^{\infty} \frac{a_{2l+2j+1}}{(2l+2j+1)^2} \left( \frac{m^2}{Q^2} \right)^j \frac{(2l+j+1)!}{j!(2l)!}.
\] (2.21)

We now define functions \( F_{a,d}(x) \) by

\[
a_n = \int_0^1 dy \ y^n F_{a}(y); \quad d_n = \int_0^1 dy \ y^n F_{d}(y).
\] (2.22)

It is easy to prove that

\[
a_n \frac{n}{n} = \int_0^1 dy \ y^{n-1} G_{a}(y); \quad d_n \frac{n}{n} = \int_0^1 dy \ y^{n-1} H_{a}(y)
\] (2.23)

\[
a_n \frac{n}{n} = \int_0^1 dy \ y^{n-1} G_{d}(y); \quad d_n \frac{n}{n} = \int_0^1 dy \ y^{n-1} H_{d}(y),
\] (2.24)

where

\[
G_{a,d}(x) = \int_x^1 dy F_{a,d}(y); \quad H_{a,d}(x) = \int_x^1 \frac{dy}{y} G_{a,d}(y).
\] (2.25)

These definitions allow us to perform the summation over \( j \) in eq. (2.21); in fact, we can write

\[
T_{\mu \nu}^{(a)} = 2im\frac{p \cdot q}{Q^2} \epsilon_{\mu \nu \lambda \sigma} q^\lambda \sum_{l=0}^{\infty} x^{-(2l+1)} (2l+1) \left( s^\sigma + 2l \frac{s \cdot q}{p \cdot q} p^\sigma \right) \int_0^1 dy \ y^{2l} \ H_{a}(y) \sum_{j=0}^{\infty} \left( \frac{y^2 m^2}{Q^2} \right)^j \frac{(2l+j+1)!}{j!(2l+1)!}.
\] (2.26)
The sum over $j$ can now be performed using the identity
\[
\frac{1}{(1 - z)^{n+1}} = \frac{1}{n!} \frac{d^n}{dz^n} \frac{1}{1 - z} = \frac{1}{n!} \sum_{j=0}^{\infty} \frac{(n+j)!}{j!} z^j,
\]
with the result
\[
T^{(a)}_{\mu\nu} = \frac{2im}{p \cdot q} \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} \sum_{n \text{ odd}} x^{-n} n \left( s^\sigma - (1 - n) \frac{s \cdot q}{p \cdot q} p^\sigma \right) \int_0^1 dy \frac{y^{n-1}}{(1 - \frac{y^2 m^2}{Q^2})^{n+1}} H_a(y),
\]
where we have defined $n = 2l + 1$. The same procedure applied to the twist-3 term yields
\[
T^{(d)}_{\mu\nu} = \frac{2im}{p \cdot q} \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} \sum_{n=3,5,\ldots} x^{-n} \int_0^1 dy \frac{y^{n-1}}{(1 - \frac{y^2 m^2}{Q^2})^{n}} \left\{ (n-1) \left[ G_d(y) s^\sigma - n H_d(y) \frac{s \cdot q}{p \cdot q} p^\sigma \right] + 2n \frac{y^2 m^2 G_d(y) + H_d(y)}{1 - \frac{y^2 m^2}{Q^2}} s^\sigma \right\}
\]
It is now possible to obtain the $n^{th}$ moments of the polarized structure functions $g_1$ and $g_2$. The antisymmetric part of the hadronic tensor $W_{\mu\nu}$ is defined by
\[
iW^A_{\mu\nu} = \frac{1}{\pi} \text{Im} T^A_{\mu\nu}.
\]
From the analytic structure of $T^A_{\mu\nu}$ in the complex $\nu \equiv p \cdot q$ plane, one can prove that
\[
T^A_{\mu\nu} = \frac{2}{\pi} \sum_{n=1,3,\ldots} x^{-n} \int_0^1 dy y^{n-1} \text{Im} T^A_{\mu\nu} = \sum_{n=1,3,\ldots} x^{-n} \int_0^1 dy y^{n-1} 2iW^A_{\mu\nu}.
\]
In other words, the coefficient of $x^{-n}$ in $T^A_{\mu\nu}$ gives twice the $n^{th}$ moment of the hadronic tensor $iW^A_{\mu\nu}$.

On the other hand, $iW^A_{\mu\nu}$ is usually parametrized in the following way:
\[
iW^A_{\mu\nu} = \frac{im}{p \cdot q} \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} \left[ g_1(x, Q^2)s^\sigma + g_2(x, Q^2) \left( s^\sigma - \frac{q \cdot s}{p \cdot q} p^\sigma \right) \right].
\]
We can therefore identify the $n^{th}$ moment of $g_1 + g_2$ and $g_2$ as twice the coefficients of $s^\sigma$ and $-p^\sigma(s \cdot q)/(p \cdot q)$ in eqs. 2.28, 2.29 respectively, thus obtaining
\[
g_1^n(Q^2) = \int_0^1 dy \frac{y^{n-1}}{(1 - \frac{y^2 m^2}{Q^2})^{n+1}} \left[ n^2 H_a(y) + \frac{2y^2 m^2}{Q^2} \left[ nG_d(y) + n^2 H_d(y) \right] \right].
\]
Equations (2.33, 2.34) are our main result. They express the moments of the polarized structure functions \( g_1, g_2 \) in terms of matrix elements of the operators appearing in the light-cone expansion of the forward scattering amplitude, at all orders in \( m^2/Q^2 \). Observe that when the twist-3 operator matrix elements \( d_n \) are neglected, eqs. (2.33, 2.34) obey the so-called Wandzura–Wilczek relation [20]

\[
g_2^n(Q^2) = \frac{n - 1}{n} g_1^n(Q^2) \quad (\text{for } d_n = 0).
\] (2.35)

The familiar \( m = 0 \) result [18] is easily recovered by using eqs. (2.22–2.24):

\[
g_1^n(Q^2) = a_n + \mathcal{O} \left( \frac{m^2}{Q^2} \right) \tag{2.36}
g_2^n(Q^2) = \frac{n - 1}{n} (d_n - a_n) + \mathcal{O} \left( \frac{m^2}{Q^2} \right) \tag{2.37}
\]

The inverse Mellin transforms of eqs. (2.33, 2.34) can be computed as in ref. [15]:

\[
g_1(x, Q^2) = \left( x^2 \frac{d^2}{dx^2} + x \frac{d}{dx} \right) \left[ \frac{x}{r\xi} \left( H_a(\xi) + \frac{2\xi^2m^2}{Q^2} H_d(\xi) \right) \right]
\]

\[
- \frac{2xm^2}{Q^2} \frac{d}{dx} \left[ \frac{x\xi}{r} G_d(\xi) \right]
\] (2.38)

\[
g_2(x, Q^2) = x \frac{d^2}{dx^2} \left[ \frac{x^2}{r\xi} \left( \frac{\xi}{x} H_d(\xi) - H_a(\xi) \right) \right],
\] (2.39)

where

\[
\xi = \frac{2x}{1 + r}; \quad r = \sqrt{1 + \frac{4x^2m^2}{Q^2}}.
\] (2.40)

An excellent test of the correctness of our calculation is a comparison with the results of ref. [16]. We have checked that eliminating \( g_1 \) and \( g_2 \) from eqs. (18,19) of ref. [16] using our eqs. (2.38, 2.39) (a non-trivial task) two identities are obtained.

As in the unpolarized case, there is a well-known difficulty in eqs. (2.38, 2.39) at \( x = 1 \): in fact, when \( x = 1 \) the structure functions should vanish for kinematical
reasons, while the RHS of eqs. (2.38,2.39) are clearly nonzero, since \( \xi(x = 1) < 1 \). This problem was discussed in ref. [21] for the unpolarized case; the conclusion reached there is that in the large \( x \) region dynamical higher twist corrections become important and cannot be neglected any more. This is because the twist \( 2 + 2k \) contribution to the \( n^{th} \) moment of a generic structure function has the form

\[
B_{kn}(Q^2) \left( \frac{n \Lambda^2}{Q^2} \right)^k,
\]  

(2.41)

where \( \Lambda \) is a mass scale of the order of a few hundreds MeV, and the coefficients \( B_{kn}(Q^2) \) have no power dependence on \( n, k \) or \( Q^2 \). The crucial feature of eq. (2.41) is the presence of a factor \( n^k \), which arises because there are at least \( n \) twist \( 2 + 2k \) operators of a given dimension for each leading twist operator of the same dimension. One can prove that the behaviour of structure functions in the \( x \sim 1 \) region is governed by moments \( n = \mathcal{O}(Q^2/m^2) \); in fact, when \( x = 1 \) and \( m^2/Q^2 << 1 \) we have

\[
\xi \simeq 1 - \frac{m^2}{Q^2},
\]  

(2.42)

on the other hand, if we assume a \( (1 - \xi)^a \) behaviour for the structure function, with \( a \) of order 1, its \( n^{th} \) moment receives the dominant contribution from the region

\[
\xi \simeq \frac{n}{a+n} \simeq 1 - \frac{a}{n}.
\]  

(2.43)

Comparing eqs. (2.42) and (2.43), we obtain that the relevant moments for the \( x = 1 \) region are of order

\[
n = \frac{aQ^2}{m^2}
\]  

(2.44)

as announced. Inserting this in eq. (2.41), one immediately realizes that the contribution of dynamical higher twists is no longer suppressed by inverse powers of \( Q^2 \) when \( x \) is close to 1, and we cannot expect our result, eqs. (2.33,2.34), to hold in this region. A solution to this problem [22, 23] is that of expanding the result in powers of \( m^2/Q^2 \) up to any finite order. In this way, the dangerous contribution of terms with large powers of \( m^2/Q^2 \) is not included. The expansion remains reliable even when \( Q^2 \) is as low as \( m^2 \), provided \( x \) is not too large; in fact, powers of \( m^2/Q^2 \) always appear multiplied by an equal power of \( x^2 \). The expanded result of course cannot be reliable at \( x \simeq 1 \), but this would not be the case even without expanding in \( m^2/Q^2 \), since we are not including the contributions of eq. (2.41), which are important in
this region. Therefore, we will perform our phenomenological analysis expanding our results, eqs. (2.33,2.34), to first order in $m^2/Q^2$. We have

$$g_1^n(Q^2) = a_n + \frac{m^2 n(n + 1)}{Q^2 (n + 2)^2} (na_{n+2} + 4d_{n+2}) + O\left(\frac{m^4}{Q^4}\right)$$

(2.45)

$$g_2^n(Q^2) = \frac{n - 1}{n} (d_n - a_n) + \frac{m^2 n(n - 1)}{Q^2 (n + 2)^2} [nd_{n+2} - (n + 1)a_{n+2}] + O\left(\frac{m^4}{Q^4}\right).$$

(2.46)

We can now use eqs. (2.36, 2.37) to eliminate the matrix elements $a_n, d_n$ from eqs. (2.45, 2.46) in favour of the moments of $g_1$ and $g_2$ at zero nucleon mass, which we denote by $g_{10}^n$ and $g_{20}^n$, respectively. We obtain

$$g_1^n(Q^2) = g_{10}^n(Q^2) + \frac{m^2 n(n + 1)}{Q^2 (n + 2)^2} \left[ (n + 4) g_{10}^{n+2}(Q^2) + 4 \frac{n + 2}{n + 1} g_{20}^{n+2}(Q^2) \right] + O\left(\frac{m^4}{Q^4}\right)$$

(2.47)

$$g_2^n(Q^2) = g_{20}^n(Q^2) + \frac{m^2 n(n - 1)}{Q^2 (n + 2)^2} \left[ n + 2 \frac{n + 2}{n + 1} g_{20}^{n+2}(Q^2) - g_{10}^{n+2}(Q^2) \right] + O\left(\frac{m^4}{Q^4}\right).$$

(2.48)

3. Phenomenology

Analyses of polarized deep-inelastic scattering data in the context of QCD at next-to-leading order have been performed by different groups [11]-[14] in the zero-mass approximation. In this section we will repeat the same analysis taking mass corrections into account, in order to establish their practical importance. The quantities which are directly measured in polarized deep-inelastic scattering experiments are the asymmetries

$$A_\perp = \frac{\sigma^{\uparrow \rightarrow} - \sigma^{\uparrow \rightarrow}}{\sigma^{\uparrow \rightarrow} + \sigma^{\uparrow \rightarrow}}$$

(3.1)

$$A_\parallel = \frac{\sigma^{\downarrow \uparrow} - \sigma^{\uparrow \uparrow}}{\sigma^{\downarrow \uparrow} + \sigma^{\uparrow \uparrow}},$$

(3.2)

where the arrows refer to the orientation of the lepton and the proton spin vectors with respect to the beam axis. One can show that

$$A_\perp = d(A_2 - \zeta A_1)$$

(3.3)

$$A_\parallel = D(A_1 + \eta A_2),$$

(3.4)
where $A_1$ and $A_2$ are virtual photon cross section asymmetries, and the coefficients $D, d, \eta, \zeta$ are fixed by the kinematics of the process; in the target rest frame

$$D = \frac{E - \varepsilon E'}{E(1 + \varepsilon R)} \quad \eta = \frac{\varepsilon \sqrt{Q^2}}{E - \varepsilon E'} \quad d = D \sqrt{\frac{2\varepsilon}{1 + \varepsilon}} \quad \zeta = \eta \frac{1 + \varepsilon}{2\varepsilon}$$  \quad (3.5)

where $E$ ($E'$) is the energy of the incoming (outgoing) lepton,

$$\frac{1}{\varepsilon} = 1 + 2 \left( 1 + \frac{Q^2}{4x^2m^2} \right) \tan^2 \frac{\theta}{2}$$ \quad (3.6)

and $\theta$ is the lepton scattering angle. The ratio $R$ is the usual quantity defined in unpolarized deep-inelastic scattering, namely

$$R = \frac{F_2}{2xF_1} \left( 1 + \frac{4x^2m^2}{Q^2} \right) - 1.$$  \quad (3.7)

The asymmetries $A_1$ and $A_2$ are directly related to polarized structure functions through

$$A_1 = \frac{g_1 - \frac{4m^2x^2}{Q^2}g_2}{F_1}$$ \quad \quad (3.8)

$$A_2 = \frac{2mxg_1 + g_2}{\sqrt{Q^2}} F_1.$$ \quad (3.9)

Therefore, both $g_1$ and $g_2$ can be expressed in terms of the measured asymmetries $A_\perp, A_\parallel$ and of unpolarized structure functions, through eqs. (3.3,3.4) and (3.8,3.9).

When the nucleon mass is neglected, one has simply

$$\frac{A_\parallel}{D} = A_1 = \frac{g_1}{F_1} = \frac{g_1}{F_2} 2x(1 + R) \quad (for \ m = 0)$$  \quad (3.10)

while in general this relationship involves both $A_\parallel$ and $A_\perp$.

The usual procedure for analysing data, adopted for example in ref. [14], must be modified in different aspects. First, the term proportional to $m^2/Q^2$ in eq. (3.7), which relates $F_1$ to $R$ and $F_2$, must now be included. Secondly, one must perform a global fit of the measured asymmetries using eq. (2.47) (if the analysis is done in moment space), where the moments $g_n$ are given in terms of moments of the coefficient functions and of the polarized parton distributions, which by definition are proportional to the matrix elements $a_n$. A difficulty immediately arises, because moments of the structure function $g_{20}$ also appear in eq. (2.47); therefore, it is not
possible to treat $g_1$ and $g_2$ independently, as in the $m = 0$ case. One could in principle circumvent this problem using experimental information on $g_2$; unfortunately, $g_2$ data available up to now are restricted to a very limited range of $x$ and $Q^2$, and are affected by large uncertainties [24]. We prefer here to follow a different strategy. We will perform fits to data in two different ways, characterized by two different assumptions on $g_{20}$: either we simply set $g_{20} = 0$, or we relate $g_{20}$ to $g_{10}$ by the Wandzura-Wilczek relation,

$$g_{20}^{n+2}(Q^2) = \frac{n+1}{n+2} g_{10}^{n+2}(Q^2). \quad (3.11)$$

Notice that none of the two assumptions is theoretically justified: there is of course no reason to assume that $g_{20}$ vanishes, nor that twist-3 operators give a negligible contribution. However, both assumptions are consistent with presently available information on $g_2$. We will check that this procedure actually allows to make a reliable estimate of the effect of mass corrections. Notice that some of the experimental collaborations present values of the asymmetry $A_1$, while others give values of the combination of $A_\parallel$ and $A_\perp$ which corresponds to $g_1/F_1$. The two quantities coincide for $m = 0$, as already observed, but they do not when mass corrections are included; so in the first case the asymmetry must be fitted by $[g_1 - (4m^2x^2/Q^2)g_2]/F_1$, in the second case simply by $g_1/F_1$.

The structure function $g_{10}$ is related to the polarized quark and gluon distributions by

$$g_{10}(x, Q^2) = \frac{\langle e^2 \rangle}{2} [C_{NS} \otimes \Delta q_{NS} + C_S \otimes \Delta \Sigma + 2n_f C_g \otimes \Delta g], \quad (3.12)$$

where $\langle e^2 \rangle = n_f^{-1} \sum_{i=1}^{n_f} e_i^2$, $\otimes$ denotes convolution with respect to $x$, and the nonsinglet and singlet quark distributions are defined as

$$\Delta q_{NS} \equiv \sum_{i=1}^{n_f} \left( \frac{e_i^2}{\langle e^2 \rangle} - 1 \right) (\Delta q_i + \Delta \bar{q}_i), \quad \Delta \Sigma \equiv \sum_{i=1}^{n_f} (\Delta q_i + \Delta \bar{q}_i), \quad (3.13)$$

where $\Delta q_i$ and $\Delta \bar{q}_i$ are the quark and antiquark distributions of flavor $i$ and $\Delta g$ is the polarized gluon distribution.

A first fit (called fit A in ref. [14]) is performed with the initial parton distributions parametrized at $Q_0^2 = 1$ GeV$^2$ according to the conventional form

$$\Delta f(x, Q_0^2) = \mathcal{N}_f \eta_f x^{\alpha_f} (1-x)^{\beta_f} (1 + \gamma_f x^{\delta_f}) \quad (3.14)$$

where $\Delta f$ denotes $\Delta q_{NS}$, $\Delta \Sigma$ or $\Delta g$; the factor $\mathcal{N}_f$ is chosen so that the first moment of $\Delta f$ is equal to $\eta_f$. We have fixed

$$\delta_\Sigma = \delta_g = 1, \quad \delta_{NS} = 0.75, \quad \beta_g = 4, \quad \gamma_\Sigma = \gamma_g \quad \text{(fit A)}. \quad (3.15)$$
In a second class of fits (fit B) the rise at small $x$ is at most logarithmic:

$$
\Delta \Sigma = N\Sigma \eta_\Sigma x^{\alpha_\Sigma} (\log 1/x)^{\beta_\Sigma}
$$

$$
\Delta q_{NS} = N_{NS} \eta_{NS} \left[ (\log 1/x)^{\alpha_{NS}} + \gamma_{NS} x (\log 1/x)^{\beta_{NS}} \right] \quad \text{(fit B)}, \quad (3.16)
$$

$$
\Delta g = N_g \eta_g \left[ (\log 1/x)^{\alpha_g} + \gamma_g x (\log 1/x)^{\beta_g} \right]
$$

The motivations for the choice of these particular parametrizations are discussed in ref. [14].

We have first performed fits A and B with fixed $\alpha_S(m_Z) = 0.118$ for $m = 0$, and with the obtained values of the fit parameters we have recomputed the structure function $g_1$ using the mass-corrected formula, eq. (2.47). The effect of target mass corrections turns out to be very small. As an example, in fig. [1] we show $g_1(x, Q^2)$ for the proton at $Q^2 = 1$ GeV$^2$ and $Q^2 = 10$ GeV$^2$ and $m = 0$ for both fits A and B; in the same plot, we also show $g_1$ obtained including mass corrections, using the same values of the fit parameters found for $m = 0$, for both assumptions $g_{20} = 0$ and $g_{20} = g_{20}^{WW}$. Differences among the three curves for $Q^2 = 1$ GeV$^2$ are sizeable only in the large-$x$ region, while at $Q^2 = 10$ GeV$^2$ the three curves are practically indistinguishable on this scale.

We have then repeated the same fits for $m = 0.938$ GeV with both assumptions on $g_{20}$, $g_{20} = 0$ and $g_{20} = g_{20}^{WW}$. The results for $g_1$ (proton) are shown in fig. [2]. Observe that also in this case the difference between the solid curves, which correspond to the
Figure 2: The structure function $g_1$ for the proton at $Q^2 = 1, 10 \text{ GeV}^2$ in fits A and B, with and without target mass corrections (fitted parameters).

$m = 0$ case, and the dashed and dotted curves, corresponding to $m \neq 0$, are quite large at $Q^2 = 1 \text{ GeV}^2$ for $x > 0.2$; when $x$ is very large ($x > 0.7$), however, the effect is not physical because in this region the approximation under which mass corrections have been computed is not reliable. However, these differences become negligible at higher values of $Q^2$ because of the $m^2/Q^2$ suppression factor, as one can see by looking at the $Q^2 = 10 \text{ GeV}^2$ curves; this guarantees that the uncertainty of our calculation connected with the expansion in $m^2/Q^2$ at first order has little effect on the results of our fits to data, since data at large $x$ are taken at high values of $Q^2$. The effect at small $x$, $x < 0.01$, is also rather large; however in this region there are few or no data and thus $g_{10}$ itself is affected by a substantial uncertainty.

The results of fits A and B for $m \neq 0$ are shown in the second and third columns of table I. The errors quoted in the table are statistical errors originating from experimental uncertainties only.

In ref. [14] the uncertainty on $g_A$ from higher twist corrections was estimated to be $\pm 0.03$. We can see from table I that target mass corrections actually induce deviations of this order from the $m = 0$ value for $g_A$. Deviations are always in the direction of making $g_A$ larger than in the $m = 0$ case. We observe also that the values of $g_A$ for some of the fitted parameters are

\[ g_{10} \]

The results in the first column of table I are slightly different from those of ref. [14] for two reasons: first, we are fixing $\beta_g = 4$ in fit A, second, we are using more recent data sets for proton SMC data and neutron E154 data.
Table 1: Results of fits A and B with fixed $\alpha_s(m_Z) = 0.118$. 

|       | FIT A |       |       | FIT B |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|
|       | $m = 0$ | $g_{20} = 0$ | $g_{20} = g_{20}^{WW}$ |       | $m = 0$ | $g_{20} = 0$ | $g_{20} = g_{20}^{WW}$ |
| $g_A$ | 1.167 ± 0.045 | 1.192 ± 0.040 | 1.200 ± 0.043 |       | 1.253 ± 0.057 | 1.292 ± 0.056 | 1.277 ± 0.058 |
| $\eta_{\Sigma}$ | 0.426 ± 0.037 | 0.408 ± 0.027 | 0.416 ± 0.031 |       | 0.455 ± 0.038 | 0.423 ± 0.034 | 0.428 ± 0.037 |
| $\eta_g$ | 0.98 ± 0.25 | 0.81 ± 0.35 | 0.83 ± 0.32 |       | 1.40 ± 0.32 | 1.00 ± 0.33 | 0.99 ± 0.35 |
| $a_0$(10 GeV$^2$) | 0.19 ± 0.04 | 0.21 ± 0.08 | 0.21 ± 0.07 |       | 0.13 ± 0.05 | 0.18 ± 0.06 | 0.19 ± 0.06 |
| d.o.f. | 114-10 | 114-10 | 114-10 |       | 114-11 | 114-11 | 114-11 |
| $\chi^2$ | 91.2 | 86.9 | 89.5 |       | 86.8 | 86.3 | 90.1 |
| $\chi^2$/d.o.f. | 0.88 | 0.84 | 0.86 |       | 0.84 | 0.84 | 0.87 |
Table 2: Results of fits A and B with fixed $g_A = 1.2573$.

|       | FIT A                     |FIT B                     |
|-------|---------------------------|---------------------------|
|       | $m = 0$ | $g_{20} = 0$ | $g_{20} = g_{20}^{WW}$ |
|       | $\alpha_s(m_Z)$ | 0.118 ± 0.005 | 0.117 ± 0.004 | 0.120 ± 0.003 |
|       | $\eta_\Sigma$ | 0.433 ± 0.039 | 0.415 ± 0.027 | 0.423 ± 0.028 |
|       | $\eta_g$ | 1.04 ± 0.45 | 0.91 ± 0.29 | 0.85 ± 0.40 |
|       | $a_0(10 \text{ GeV}^2)$ | 0.18 ± 0.08 | 0.20 ± 0.09 | 0.20 ± 0.09 |
| d.o.f. | 114-10 | 114-10 | 114-10 |
| $\chi^2$ | 94.4 | 89.1 | 90.9 |
| $\chi^2$/d.o.f. | 0.91 | 0.86 | 0.87 |
|       |FIT A                     |FIT B                     |
|       | $m = 0$ | $g_{20} = 0$ | $g_{20} = g_{20}^{WW}$ |
|       | $\alpha_s(m_Z)$ | 0.123 ± 0.003 | 0.118 ± 0.005 | 0.121 ± 0.004 |
|       | $\eta_\Sigma$ | 0.448 ± 0.036 | 0.407 ± 0.036 | 0.418 ± 0.033 |
|       | $\eta_g$ | 1.01 ± 0.32 | 0.73 ± 0.33 | 0.72 ± 0.31 |
|       | $a_0(10 \text{ GeV}^2)$ | 0.14 ± 0.07 | 0.22 ± 0.07 | 0.21 ± 0.06 |
| d.o.f. | 114-11 | 114-11 | 114-11 |
| $\chi^2$ | 84.8 | 86.3 | 89.2 |
| $\chi^2$/d.o.f. | 0.82 | 0.84 | 0.87 |

obtained with the two different assumptions for $g_{20}$ are quite close to each other, thus suggesting that the assumed form of $g_{20}$ has a small impact on the final results.

We have also performed fits A and B using $\alpha_s$ as a free parameter, with $g_A$ fixed to its measured value $g_A = 1.2573$. The results are shown in table 4. Also in this case, mass corrections induce variations on $\alpha_s(m_Z)$ which are compatible with the higher twist uncertainty of ±0.004 estimated in ref. [14].

The values of the first moment of the quark singlet distribution, $\eta_\Sigma$, and of the first moment of the gluon distribution at $Q^2 = 1 \text{ GeV}^2$, $\eta_g$, are also shown in tables 4-4. Also for these quantities, the assumption on $g_{20}$ has little effect. The introduction of mass terms tend to give smaller values for $\eta_\Sigma$ and $\eta_g$, but deviations from the values
of the $m = 0$ case are all within statistical errors.

In tables [1][2] we also present values of the singlet axial charge

$$a_0(Q^2) = \int_0^1 dx \left[ \Delta \Sigma(x, Q^2) - n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(x, Q^2) \right]$$

(3.17)

for $Q^2 = 10$ GeV$^2$. Values of $a_0$ for $m \neq 0$ are slightly larger than in the massless fits. The only case in which a sizeable deviation from the $m = 0$ result is observed is fit B with $g_A$ fixed. Also in this case, however, the difference is compatible with the statistical error.

Finally, we compare the polarized parton distribution functions for quark singlet, quark non-singlet and gluon obtained with and without mass corrections. They are shown, for $Q^2 = 10$ GeV$^2$ in figs. [3][4] and [5] for fits A and B with fixed $\alpha_s(m_Z) = 0.118$.

As expected from the above discussion, curves for $g_{20} = 0$ and $g_{20} = g_{20}^{WW}$ are quite close to each other. Quark distributions are hardly affected by mass corrections. The polarized gluon density $\Delta g$ is determined, in this procedure, only through the effect of scaling violations. It is therefore less constrained than quark distributions. However, the modifications of the gluon distribution induced by mass corrections, although sizeable, are considerably less important than the uncertainty originated by the choice of different functional forms for the input distribution (see ref. [14]).
Figure 4: The quark nonsinglet distribution for the proton at $Q^2 = 10 \text{ GeV}^2$ in fits A and B, with and without target mass corrections.

Figure 5: The gluon distribution at $Q^2 = 10 \text{ GeV}^2$ in fits A and B, with and without target mass corrections.
4. Conclusions

We have computed target mass corrections to nucleon structure functions in polarized deep-inelastic scattering. Our results are consistent with those obtained in ref. [16] using a different technique.

Target mass corrections can in principle be important in the context of polarized deep-inelastic scattering, because part of the data are taken at relatively low values of $Q^2$. For this reason, we have performed an analysis of all available polarized deep-inelastic scattering data in the framework of perturbative QCD, including the contribution of target mass corrections up to terms of order $m^2/Q^2$. We have proved that this approximation is reliable in the kinematical range of presently available data. We have found that the effect of mass corrections is generally small; for example the value of the axial coupling $g_A$ is enlarged by approximately 0.03 when target mass corrections are included. The strong coupling constant $\alpha_s(m_Z)$ receives corrections of approximately 0.004. Both deviations are compatible with higher twist uncertainties estimated in previous works.

Quark distributions are practically unchanged by the introduction of mass terms. The polarized gluon distributions, which is only determined through scaling violations, is comparatively more affected by mass corrections, but also in this case the values of the fitted parameters do not deviate from those obtained for $m = 0$ by more than one standard deviation.

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