Mathematical methods for determining the guaranteed accuracy of the components content in a mixture made by a conveyor non-mixer

A V Evseev¹, V V Preys¹, V A Lapina¹, G V Kasatkin ²

¹Tula State University, Tula, Russia
²MIREA-Russian Technology University, Moscow, Russia

E-mail: ews1972@mail.ru

Abstract. In work two mathematical techniques of studying of quality of mixes of heterogeneous (tubular) components (length gages) based on the analysis of synthesis of mix in automatic conveyor non-mixer with use of two and more drum dozers are considered. The relevance and novelty of development are shown on examples of receiving the general party of composite tubular product from small similar parties, at the determined forming of its uniformity, with the set qualitative and quantitative characteristics from heterogeneous tubular components. The criteria allowing determining the required guaranteed accuracy of maintenance of components in the mixes received on installation are also offered.

1. Introduction
In works [1, 3, 7, 8] theoretical and pilot researches of processes of the determined formation of uniformity of heterogeneous mixes of different function were conducted. In works [2, 4-6] similar researches at stochastic formation of uniformity of mixes were conducted. In the majority cases researchers drew conclusions on the greatest influence of accuracy of operation of the devices dosing components on quality of the received mixes. However most of scientists consider need of accounting of accuracy of dispensing only at batch processes of mixture at probabilistic formation of uniformity of mixes [5, 6].

Feature of the theory of the determined formation of uniformity of mixes is accounting of these parameters at the continuous process of a nonmixing [9, 10]. In this work similar theoretical developments on the example of use in practice of the linear conveyor non-mixer of the continuous action are carried out [8].

Developments of the mathematical and criteria description and justification of functioning of such inventory which in practice would allow to find communication between qualitative and quantitative operation parameters of the corresponding dosing and other clusters of non-mixer with final characteristics of the mixes received on them are especially important and relevant. Two such developments are provided in this work.

The novelty of techniques consists in binding of probabilistic indicators of accuracy of operation of portioning devices for supply of heterogeneous (tubular) materials to output quality indicators of ready mix of the set level [3, 7, 8].
2. Problem definition
Two methods of studying of quality of mixes, made by batchers of non-mixer are considered and comparison is carried out them.
Mix is formed non-mixer from batchers, which by the piece give out on the conveyor of particle of components of mix. The batcher with number it is ready for delivery particles for unit of time, the general for all batchers. However, in reality it gives out particles, where is the integer random variable concluded in limits
\[ m_i - a_i \leq n_i \leq m_i + b_i, \quad i \in (1, \ldots, k), \]  
where \( a_i, b_i \) - is known integers, \( a_i, b_i < m_i \).
Accuracy \( \varepsilon \) mixes, called further \( \varepsilon \) - is accuracy of maintenance of components in mix – characteristic of proximity of the relation \( n_1:n_2: \ldots :n_k \) to the relation \( m_1:m_2: \ldots :m_k \), is defining quality of mix.
Let us consider two criteria \( \varepsilon \) - accuracy of maintenance of components in mix.
The first – standard criterion
\[ \forall i \in (1, \ldots, k), \quad 1 - \varepsilon \leq \frac{n_i}{m_i} \leq 1 + \varepsilon. \] (2)
The second criterion is applicable at two and more batchers and is formulated so
\[ \forall i, s \in (1, \ldots, k), j \neq s \quad (1 - \varepsilon) \frac{m_s}{m_j} \leq \frac{n_s}{n_j} \leq (1 + \varepsilon) \frac{m_s}{m_j}. \] (3)
Let us set as the purpose to find that accuracy within the specified criteria \( \varepsilon \) the maintenance of components in mix, which is guaranteed when performing conditions (1) irrespective of the fact which random variables are subordinated to the probabilistic law, \( n_i \) and to compare the received estimates.
Note. Mix may contain fillers and insignificant components in the sense that their relative content in mix does not join in mix accuracy assessment. Therefore, all batchers considered further deliver essential components in mix.
This task has arisen when studying quality of the mixes made by non-mixer with batchers of tubular components. The scheme of operation of this device is presented on figure 1.

![Figure 1. Conveyor non-mixer:](image)

1 – the bunker of tubular components, 2 – drum dozers of piece (tubular) components, 
3 – the conveyor of non-mixer, 4 – trenches for forming of single doses of mix, 5 – single dose of mix, 
6 – capacity for ready mix

3. Algorithm of the task solution
These sizes
\[ \tau_i' = \frac{a_i}{m_i}, \quad \tau_i'' = \frac{b_i}{m_i}, \quad i \in (1, \ldots, k), \]

define work accuracy limits \( i \)-th batcher, and size

\[ \tau_i = \max(\tau_i', \tau_i'') \]

is the accuracy of this batcher. With increase \( \tau_i \) accuracy of the batcher decreases and, on the contrary, with reduction \( \tau_i \) accuracy of operation of the batcher increases.

The size

\[ \tau_i = \max(\tau_1, \tau_k) \]

is natural to consider by work accuracy (the maximum rejection of the contents of components in mix from set) all non-mixer.

Within standard criterion \( \varepsilon \) – the answer according to the solution of objective is obvious to the accuracy of maintenance of components in mix: the guaranteed accuracy \( \varepsilon \) the maintenance of components in mix is defined by equality

\[ \varepsilon = \varepsilon_1 = \tau. \]  \hfill (4)

Let us pass the decision of task with use of the second criterion of quality of mix. Instead of random variables \( n_i \) let us enter random variables \( x_i \) on formula

\[ x_i = \frac{n_i}{m_i}, \quad i \in (1, \ldots, k). \]

Inequalities (1), (3) in sizes \( x_i, \tau_i', \tau_i'' \) will correspond in the following look

\[ 1 - \tau_i' \leq x_i \leq 1 + \tau_i'', i = 1, \ldots, k, \]  \hfill (5)

\[ (1 - \varepsilon)x_j \leq x_s \leq (1 + \varepsilon)x_j. \]  \hfill (6)

Let's look for the minimum value \( \varepsilon = \varepsilon_2 \), at which inequalities (6) are carried out at all possible values of sizes \( x_i \), meeting conditions (5).

Performance of inequality (6) at all possible values of sizes \( x_j \) means inequality performance

\[ \max \frac{\min x_j}{(1 - \varepsilon)x_j} \leq \frac{\min x_i}{(1 + \varepsilon)x_i} \]

or

\[ (1 - \varepsilon)(1 + \tau_j') \leq x_s \leq (1 + \varepsilon)(1 - \tau_j'). \]

The received inequality is carried out at all possible values of size \( x_s \) only when

\[ \begin{cases} (1 - \varepsilon)(1 + \tau_j'') \leq \min x_s = 1 - \tau_s' \\
1 + \tau_s'' = \max x_s \leq (1 + \varepsilon)(1 - \tau_j'') \end{cases} \]

or

\[ \begin{cases} \varepsilon \geq \frac{\tau_j' + \tau_j''}{1 + \tau_j'} \\
1, s \in (1, \ldots, k), j \neq s, \varepsilon \geq \frac{\tau_s' + \tau_j''}{1 - \tau_j'} \end{cases} \]  \hfill (7)

Search of the specified values of sizes \( j, s \) are the maximum value \( \varepsilon_+ \) right parts of all first inequalities and maximum value \( \varepsilon_* \), the right parts of all second inequalities in system (7)

\[ \varepsilon_* = \max \frac{\tau_j' + \tau_j''}{1 + \tau_j'}, \quad \varepsilon_{*+} = \max \frac{\tau_s' + \tau_j''}{1 - \tau_j'}. \]

After that all inequalities of system (7) will be replaced with one assessment
\[ \varepsilon \geq \max(\varepsilon, \varepsilon_*) \]  
(8)

If to rearrange in places indexes \( j, s \) in the formula determining size \( \varepsilon_** \), we will receive

\[ \varepsilon_** = \max \frac{\tau'_j + \tau'_{s'}}{1 - \tau_{s'}}. \]  
(9)

From here it is visible that \( \varepsilon_** > \varepsilon_* \) and, therefore, assessment (8) is replaced with simpler assessment

\[ \varepsilon \geq \varepsilon_**. \]

Minimum value

\[ \varepsilon = \varepsilon_2 = \varepsilon_**. \]  
(10)

Just also represents the required guaranteed mix accuracy within the second criterion of quality.

4. Discussion of results

The detailed analysis of inequalities (7), (8) has shown that assessment \( \varepsilon = \varepsilon_2 \) qualities of mix by the second criterion more than twice exceeds assessment \( \varepsilon = \varepsilon_1 \) by standard criterion. To see it we will give two examples.

If accuracy limits \( \tau'_i, \tau'_{s'} \) all batchers match and are equal to the same size \( \tau' \) (this size is determined as the accuracy of non-mixer) above, according to formulas (9), (10)

\[ \varepsilon = \varepsilon_2 = \frac{2\tau}{1 - \tau}. \]

This value more than twice exceeds value of size \( \varepsilon = \varepsilon_1 \) from formula (4). From the last expression we will find:

\[ \tau = \frac{2\varepsilon}{1 + \varepsilon}. \]  
(11)

From formula (11) easily there is accuracy of non-mixer, which providing with the second criterion, \( \varepsilon \) - set accuracy of maintenance of components in mix.

Function graph \( \tau = \tau(\varepsilon) \) it is presented on figure 2. It is visible that at approach to zero size \( \tau \) decreases more slowly, than \( \varepsilon \).

![Figure 2. Function graph \( \tau = \tau(\varepsilon) \)](image)

According to formulas (4) and (11), we can conclude that:

1) with the set accuracy is (10%)\((\varepsilon = 0,1)\) the maintenance of components in mix the accuracy of non-mixer is required \((\tau = 0,1)\)(10%) by standard criterion of quality and \( \tau = 0,04762(4,762\%) \) by the second criterion,
2) with the set accuracy (5%) (\( \varepsilon = 0.05 \)) we receive (\( \tau = 0.05 \)) (5%) by standard criterion of quality and (\( \tau = 0.0244(2.44\%) \)) by the second criterion.

Now we will give the following specific example.

Let mix be formed from three components with number of particles \( n_1, n_2, n_3 \), having restrictions

\[ 32 \leq n_1 \leq 45, \quad 88 \leq n_2 \leq 115, \quad 150 \leq n_3 \leq 171. \]  
(12)

Let us find out with what accuracies \( \varepsilon_1, \varepsilon_2 \) the received mix is close to the relation: 40:100:160 (2:5:8).

In this example

\[ m_1 = 40, \quad m_2 = 100, \quad m_3 = 160. \]

We bring out of restrictions (12)

\[
\begin{align*}
40 - 8 & \leq n_1 \leq 40 + 5 \Rightarrow a_1 = 8, \quad b_1 = 5 \Rightarrow \tau_1' = \frac{8}{40} = 0.2, \quad \tau_1^\ast = \frac{5}{40} = 0.125, \quad \tau_1 = 0.2; \\
100 - 12 & \leq n_2 \leq 100 + 15 \Rightarrow a_2 = 12, \quad b_2 = 15 \Rightarrow \tau_2' = \frac{12}{100} = 0.12, \quad \tau_2^\ast = \frac{15}{100} = 0.15, \quad \tau_2 = 0.15; \\
160 - 10 & \leq n_3 \leq 160 + 12 \Rightarrow a_3 = 10, \quad b_3 = 11 \Rightarrow \tau_3' = \frac{10}{160} = 0.0625, \quad \tau_3^\ast = \frac{12}{160} = 0.075, \quad \tau_3 = 0.075.
\end{align*}
\]

On the found values of accuracies of operation of batchers on formulas (4), (11) we receive

\[
\begin{align*}
\varepsilon_1 &= \tau = max(\tau_1, \tau_2, \tau_3) = 0.2 \ (20\%), \\
\varepsilon_2 &= \varepsilon^\ast = max(\frac{\tau_1' + \tau_2^\ast}{1 - \tau_1}, \frac{\tau_1 + \tau_2^\ast}{1 - \tau_1}) = 0.4375 \ (43.75\%).
\end{align*}
\]

Have received that value \( \varepsilon_2 \) more than twice exceeds value \( \varepsilon_1 \). Bad indicators of the guaranteed accuracy of mix relate to bad indicators of accuracy of operation of the first and second batchers. Percentage accuracy of operation of the first batcher is concluded ranging from 100\( \tau_1^\ast = 12.5\% \) to 100\( \tau_1' = 20\% \). Similar indicators of operation of the second and third batchers are respectively equal \( 12 - 15\% \) and \( 6,25 - 7.5\% \).

5. Conclusions

The techniques of determination of the guaranteed mix accuracy and also determination of accuracy of work of non-mixer stated above at present value \( \varepsilon \) - accuracy of maintenance of components in mix, are applicable without any restrictions and to mixes from bulks. Only for them number \( n_i \) represent the masses or volumes and are continuous random variables with estimates (1) in whom sizes \( a_i, b_i \) - not necessarily integers.

Besides, formulas (4), (10) can use when random variables \( n_i \) are independent, and restrictions (1) are executed with probabilities \( p_i \). In this case, the formula (4) and formula (10) give values \( \varepsilon_1 \) and \( \varepsilon_2 \) with probability

\[ p = p_1 \cdot p_2 \cdot \ldots \cdot p_n. \]

For example, if in the example reviewed above there is reservation that conditions (12) are satisfied with probabilities \( p_1 = 0.91, \ p_2 = 0.95, \ p_3 = 0.89 \), that the found assessment \( \tau = 0.4375 \) accuracy of maintenance of components in mix it is executed with probability \( p = p_1 \cdot p_2 \cdot p_3 \approx 0.77 \).

The considered techniques of definition of guaranteed \( \varepsilon \) - accuracy of maintenance of components in mixes and its comparison with standard technique supplements research [3] on studying of quality of multicomponent mixtures. Undoubted advantage of the specified technique is its simplicity in application and the fact that she does not demand knowledge of probabilistic laws of distribution of components of mix. It is only enough to know ranges of dispersion of all components of mix which are defined in the course of observations.
6. References

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