EXTENDED ANOMALOUS FOREGROUND EMISSION IN THE WMAP THREE-YEAR DATA

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ABSTRACT

We study the spectral and morphological characteristics of the diffuse Galactic emission in the WMAP temperature data using a template-based multilinear regression, and obtain the following results. (1) We confirm previous observations of a bump in the dust-correlated spectrum, consistent with the Draine & Lazarian spinning dust model. (2) We also confirm the “haze” signal in the inner Galaxy, and argue that it does not follow a free-free spectrum as first thought, but instead is synchrotron emission from a hard electron cosmic-ray population. (3) In a departure from previous work, we allow the spectrum of Hα-correlated emission (which is used to trace the free-free component) to float in the fit, and find that it does not follow the expected free-free spectrum. Instead there is a bump near 50 GHz, modifying the spectrum at the 20% level, which we speculate is caused by spinning dust in the warm ionized medium. (4) The derived cross-correlation spectra are not sensitive to the map zero points, but are sensitive to the choice of CMB estimator. In cases where the CMB estimator is derived by minimizing variance of a linear combination of the WMAP bands, we show that a bias proportional to the cross-correlation of each template and the true CMB is always present. This bias can be larger than any of the foreground signals in some bands. (5) Lastly, we consider the frequency coverage and sensitivity of the Planck mission, and suggest linear combination coefficients for the CMB template that will reduce both the statistical and systematic uncertainty in the synchrotron and haze spectra by more than an order of magnitude.

Subject headings: diffuse radiation — dust, extinction — ISM: clouds — radiation mechanisms: nonthermal — radio continuum: ISM

1. INTRODUCTION

Observations of the cosmic microwave background (CMB) by the Wilkinson Microwave Anisotropy Probe (WMAP) have revolutionized our understanding of cosmology and placed strong constraints on cosmological parameters (Spiegel et al. 2003, 2007; Tegmark et al. 2004). Moreover, the WMAP foreground signal represents the most detailed and sensitive full-sky maps of Galactic microwave emission to date, providing an enormous wealth of information about the physical processes in the interstellar medium (ISM).

1.1. Galactic Emission Mechanisms

There are three well-established types of Galactic foreground signals at WMAP frequencies: free-free (or thermal bremsstrahlung) emission from interaction of free electrons with ions, dust emission from grains heated by the surrounding radiation field, and synchrotron emission from supernova shock-accelerated electrons. In addition, there are two other emission mechanisms which have proven more difficult to characterize: spinning dust and the anomalous “haze” (Finkbeiner 2004). Spinning dust refers to emission from the smallest dust grains which have nonnegligible electric dipole moments and are excited into rotational modes through a variety of mechanisms (see Draine & Lazarian 1998). The physical origin of the haze is uncertain.

Though initially controversial (Bennett et al. 2003), numerous authors have presented evidence for a spinning dust spectrum when combining WMAP data with external data sets (de Oliveira-Costa et al. 2004; Finkbeiner et al. 2004; Boughn & Pober 2007). Using only WMAP data, Bennett et al. (2003) and Hinshaw et al. (2007) point out that it is difficult to spectrally distinguish certain spinning dust models from synchrotron. However, in our companion paper (Dobler & Finkbeiner 2008, hereafter DF08), we show that a spinning dust spectrum is indeed recoverable using exclusively WMAP data, although it is not spatially correlated with the thermal dust emission.

The haze was originally thought (Finkbeiner 2004) to be free-free emission from ionized gas which is too hot to be visible in recombination line maps and too cold to be visible in X-ray maps. However, gas at the required temperature $T \sim 10^5$ K is thermally unstable (Spitzer 1978). Furthermore, we will show in §3 that the spectrum of the haze is inconsistent with free-free emission and is most likely explained as a hard synchrotron component which is morphologically and spectrally distinct from the above mentioned soft synchrotron.

1.2. MEM Analysis

With the first- and third-year data releases, the WMAP team provided a dual foreground analysis: a maximum entropy method (MEM) and a template-fitting algorithm. The former was intended to improve our understanding of the astrophysics of foreground emission while the latter (more statistically stable) algorithm was meant to produce CMB maps with well-characterized noise properties for use in the cosmological analysis (Bennett et al. 2003; Hinshaw et al. 2007).

Although the total observed emission matches the MEM model to better than 1%, “low residual solutions are highly constrained, but not necessarily unique or correct” (Bennett et al. 2003, p. 108). The MEM method is a pixel-by-pixel fit which minimizes the functional $H(\hat{p}) = A(\hat{p}) + \lambda(p)B(\hat{p})$ (Press et al. 1992), where $\lambda$ is a regularizing parameter,

$$A(\hat{p}) = \chi^2(\hat{p}) = \sum_p [T(v, p) - T_m(v, p)]^2 / \sigma^2,$$  

$$B(\hat{p}) = \sum_c T_c(\hat{p}) \ln |T_c(\hat{p})/P_c(\hat{p})|,$$  

$\chi^2(\hat{p})$ is the reduced chi-squared, $T(v, p)$ is the data at a given frequency $v$ and pixel $p$, $T_m(v, p)$ is the model spectrum, $\sigma$ is the noise in the data, $T_c(\hat{p})$ is the contribution of each template to pixel $p$, and $P_c(\hat{p})$ is the prior for each template.

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and the sum is over Galactic emission components. Also, \( P_i(\nu) \) is a prior template for component \( c \), normalized to the same frequency as \( T_e \) (see Bennett et al. 2003; Hinshaw et al. 2007, for details).

The noise properties of a MEM-derived CMB map are complicated, e.g., by the fact that noise is clipped to be nonnegative by the logarithm in equation (2). Because simple noise properties are desirable for a cosmological power spectrum analysis, a template-based map is used instead. In the end, Bennett et al. (2003) and Hinshaw et al. (2007) fit spatial templates for the dust, free-free, and synchrotron ISM emissions to \( WMAP \) Q, V, and W bands. They find that the remaining contamination in the maps is sufficiently subdominant outside the Kp2 mask so that the effect on the CMB power spectrum is negligible.

1.3. “Anomalous” ISM Emission Mechanisms

Given their dual approach, an important question to address is why have the foreground analyses of the \( WMAP \) team not positively identified the spinning dust emission in DF08 and the haze emission in Finkbeiner (2004)?

With the MEM analysis, there are two pitfalls. First and most importantly, the MEM analysis generates a map of the spectral behavior of the soft synchrotron in each pixel. Since the free-free and dust spectra are kept fixed in the model, deviations away from these assumed prior spectra are absorbed into this “spectral index map” for synchrotron. Second, when minimizing \( H(\nu) \), if the recovered model \( T_m(\nu, p) \) yields negative pixel values, \( \lambda(\nu) \) is increased until the results are greater than zero. These two effects combine so that foreground emission which does not match the prior templates and spectra (i.e., spinning dust and haze emission) are simultaneously absorbed into the synchrotron spectral index map and washed out by adding priors back into each pixel to enforce positivity. Since minimizing \( H(\nu) \) in this way is repeated multiple times (with the synchrotron spectral index map being “updated” with each iteration) the results of the MEM analysis naturally strongly resemble the priors.

With the template fitting algorithm, the difficulty arises in the choice of a synchrotron template. Because the \( WMAP \) signal-to-noise ratio is far superior to previous surveys, Hinshaw et al. (2007) used the difference of the two lowest \( WMAP \) frequency maps (K-Ka) as a template for synchrotron. They acknowledge that this template necessarily contains free-free emission as well, but it also contains the haze and spinning dust emission. Thus, the haze and spinning dust are simultaneously explicitly fit with this template. In addition, the spectra of the dust emission and free-free emission are again kept fixed so deviations are difficult to identify.

Given the numerous challenges involved in a CMB foreground analysis and the serious questions about whether all the relevant foreground emission mechanisms have even been identified yet, we choose an approach that is simple enough to have well characterized noise properties, but flexible enough to allow us to find surprises. In the limit where the spectrum of each component is invariant with position, one still has the choice of assuming a perfect spatial template, or assuming knowledge of the spectrum of each component. Given that the spectrum of each component can vary with position, neither of these approaches is strictly correct. Nevertheless, as we shall see, there is still much to be learned by making a too rigid assumption and then studying the resulting residuals.

\[ T \propto \nu^{\alpha}, \]

where \( \alpha \approx -2.15 \), \( T \) is in antenna temperature, and the proportionality depends only on the electron temperature \( T_e \). Thus, the only free parameter is a scaling factor equivalent to the electron temperature on the sky (but see § 3), although this temperature is expected to vary with position.

**Thermal and spinning dust.**—The emission produced from tiny interstellar dust grains vibrating in equilibrium with the surrounding radiation field has been mapped across the sky by Schlegel et al. (1998). We use their full-sky map evaluated at 94 GHz by Finkbeiner et al. (1999, hereafter the FDS map) as a dust template.\(^2\) The smallest of these dust grains are expected to have a nonnegligible electric dipole moment, and so can also emit radiation at \( WMAP \) frequencies through rotational modes excited by collisions with ions. Thus, our template also traces spinning dust emission. Since the spectral dependence of the spinning dust is not well known, we use our fit to constrain the frequency dependence of the dust-correlated foregrounds.

**Soft synchrotron.**—As relativistic, shock-accelerated electrons travel through the Galactic magnetic field, they emit synchrotron radiation with a characteristic frequency dependence \( \propto \nu^3 \) (in antenna temperature units). At 408 MHz, this emission was measured by Haslam et al. (1982) and we use their full-sky map as a tracer of soft synchrotron. As pointed out by Bennett et al. (2003) and Hinshaw et al. (2007) the spectral index \( \beta \) is expected

\[ \frac{\nu}{\nu_*} \ll 1 \],

where \( \nu_* \) is the characteristic frequency of the synchrotron emission.\(^3\) The H\(\alpha\) and FDS maps can be found online at http://www.skymaps.info.

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2 Bennett et al. (2003) used the synchrotron template described in § 2.1, and an excess south of the Galactic center is indeed visible in their residual maps (their Fig. 11, upper right).

3 The H\(\alpha\) and FDS maps can be found online at http://www.skymaps.info.
to vary across the sky, and in particular, the spectrum may be harder near regions of recent supernova activity. Although we use our fit to evaluate the spectra of the 408 MHz-correlated synchrotron emission, we note that a value of $\beta = -3.05$ removes most of the emission at high latitude (most notably the prominent “north Galactic spur” feature).

$CMB$—As we shall see in § 3, the choice of $CMB$ estimator can dramatically affect the inferred foreground spectra in a given fit. To illustrate this sensitivity, we use six different $CMB$ estimators defined as follows.

$CMB1.$—The published internal linear combination (ILC) map derived by the $WMAP$ team for the three-year data.

$CMB2.$—An ILC with the coefficients that the $WMAP$ team have found best cancel their region 0 foregrounds in the three-year data Hinshaw et al. (2007). This ILC is given by

$$CMB2 = 0.156 T_K - 0.888 T_{Ka} + 0.030 T_Q + 2.045 T_V - 0.342 T_W,$$

where $T_j$ is the observed $WMAP$ temperature data in band $j$, in thermodynamic $\Delta T$ units.

$CMB3.$—An ILC which minimizes the variance over our unmasked pixels. Here,

$$CMB3 = -0.032 T_K - 0.205 T_{Ka} + 0.037 T_Q + 0.441 T_V + 0.760 T_W,$$

where the constant $A$ is determined from the approximate free-free amplitude at 23 GHz and traced to 94 GHz via equation (3).

$CMB4.$—A “high-frequency (HF) estimator” that removes the dominant foregrounds (thermal dust and free-free) from the 94 GHz $WMAP$ data,

$$CMB4 = T_W - FDS - A\alpha,$$

where $\alpha$ is given by equation (3).

$CMB5.$—A model for the thermal dust is subtracted from all of the $WMAP$ bands using $T_{dust} = \langle \nu/94 \text{ GHz} \rangle^{1.7} \times FDS$. A minimum variance ILC is then generated from this thermal dust presubtracted data (denoted by primes). The variance is minimized for,

$$CMB5 = 0.104 T'_K - 0.289 T'_{Ka} + 0.190 T'_Q + 0.317 T'_V + 1.059 T'_W.$$

$CMB6.$—A cleaned map of the three-year data, cleaned with the Tegmark et al. (2003; TOH) method. This method utilizes a linear weighting of the data in which the weights depend on the multipole $\ell$ of the spherical harmonic expansion of each of the five $WMAP$ bands.

In the limit where the noise in each $WMAP$ band is equal to $\sigma_0$, the measurement noise of the ILC is simply

$$\sigma_L = \sigma_0 \sqrt{\sum_b \xi_b^2},$$

so $CMB3$ and $CMB5$ are significantly less noisy than $CMB1$ and $CMB2$. Due to the complicated weighting of the TOH method, the measurement noise properties of $CMB6$ are quite complicated.

Mask.—In addition to masking out all point sources listed in the $WMAP$ team’s three-year catalog, as noted above, we mask all regions of the sky where the $\alpha$ extinction due to dust $A(\alpha) = 2.65E(B - V) \geq 1$ mag. We also mask out the LMC, SMC, M31, Orion-Barnard’s Loop, NGC 5090, and $\zeta$ Oph. This mask covers 21.5% of the sky.

### 2.2. Fitting Procedure

Our model is that the observed $WMAP$ data are a linear combination of the foreground templates plus the $CMB^6$ plus noise. Therefore, we want to solve the matrix equation

$$Pa = w,$$

where $w$ is the $CMB$-subtracted $WMAP$ data and $P$ is a “template matrix” whose columns consist of the foreground templates outlined in § 2.1, for the coefficient vector $a$ whose entries represent the different foregrounds.

The template can be represented schematically by a block-diagonal matrix,

$$P = \begin{pmatrix}
  P_1 & & & \\
  & P_2 & & \\
  & & P_3 & \\
  & & & P_4 \\
  & & & & P_5
\end{pmatrix},$$

where the five blocks correspond to the five $WMAP$ bands, and each block has the form

$$P_b = \begin{pmatrix}
  f_1, b & d_1, b & s_1, b \\
  f_2, b & d_2, b & s_2, b \\
  f_3, b & d_3, b & s_3, b \\
  \vdots & \vdots & \vdots \\
  f_N, b & d_N, b & s_N, b
\end{pmatrix},$$

where $f$, $d$, and $s$ are the templates for free-free, thermal and spinning dust, and soft synchrotron emission (in thermodynamic mK) respectively. This makes $P$ a $5N_p \times 15$ matrix. For each template column, the first index represents the pixel number and the second represents a $WMAP$ frequency band—i.e., $1 = 23$ GHz (K), $2 = 33$ GHz (Ka), etc. The total number of unmasked pixels in each map is $N_p$. For our most general fits, we assume no knowledge of the $f$, $d$, and $s$ spectra, and so those templates do not differ for each band. Rather, we explicitly fit the spectra as discussed below.

Variations in the foreground spectra from place to place on the sky will be explored by fitting smaller regions in upcoming sections.

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4 Available at http://lambda.gsfc.nasa.gov/
5 Available at http://space.mit.edu/home/tegmark/wmap.html
The CMB-subtracted \textit{WMAP} data and the coefficient vector are column vectors,

\[ w = \begin{pmatrix} T_{1,1} - c_1 \\ T_{2,1} - c_2 \\ T_{3,1} - c_3 \\ \vdots \\ T_{Np,5} - c_{Np} \end{pmatrix}, \quad a = \begin{pmatrix} a_{f,1} \\ a_{d,1} \\ a_s,1 \\ a_{f,2} \\ a_{d,2} \\ a_s,2 \\ \vdots \\ a_{f,5} \end{pmatrix}, \]

where \( T_{i,j} \) is the observed \textit{WMAP} temperature data in pixel \( i \) and band \( j \), and \( c \) is one of our CMB estimators. Their lengths are \( 5N_p \) and 15 respectively.

Since \( P \) is not a square matrix with linearly independent rows, it is not invertible. To solve equation (9) we calculate \( P^{+} \), where the plus sign superscript denotes the pseudo-inverse.\footnote{The pseudoinverse is defined as \( P^{+} = B^{*}U^T \), where the singular value decomposition of \( P = USV^T \) and \( S \) is the transpose of \( S \) with all nonzero singular values replaced by their inverse. In the case of a square, nonsingular matrix, \( P^{+} = P^{-1} \).} The solution \( e = P^{+}w \) minimizes the quantity \( e^2 = \| Pa - w \|^2 \), so that if we divide both sides of equation (9) by the uncertainty \( \sigma \) (following Bennett et al. [2003] and Hinshaw et al. [2007], we use the mean measurement noise in each \textit{WMAP} band), the solution\footnote{From the properties of the pseudoinverse, \( PP^+P = P \) and \( [PP^+]^T = PP^+ \), it is easy to show that, if the columns of \( P \) are linearly independent as they are in our case, \( P^{+} = [P^T P]^{-1}P^T \), making our technique equivalent to other \( \chi^2 \) minimization techniques (e.g., Tegmark et al. 2003; de Oliveira-Costa & Tegmark 2006).}  

\[ a = (P/\sigma)^{+}(w/\sigma) \]  

minimizes the quantity

\[ \frac{\| P a - w \|}{\sigma} = \chi^2. \] \label{chi2}

This fitting procedure is flexible in that additional foreground components can be incorporated by simply adding columns to the template matrix. We will exploit this feature in \( \S \) 3 to model the anomalous “haze” excess emission toward the Galactic center.

The five components of each coefficient (e.g., \( a_{d,j} \), with \( j = [1,5] \)) represent a fit of both the amplitude and spectrum of the associated emission. By simultaneously fitting all of the spectra for all of the foregrounds, we can completely decouple the bands from each other in our fits. In addition, we also perform less general fits in which various spectra are fixed to follow the spectral behavior making it amenable to presubtraction before forming an ILC.

3. RESULTS

We have performed both full-sky fits as well as fits of smaller regions— the motivation being that both the soft synchrotron and spinning dust spectra vary from place to place across the sky. Our fits are characterized by the \( \chi^2 \) statistic of equation (14) and by the residual map,

\[ r = Pa - w, \] \label{residual_map}

which guides intuition and serves as a visual aid in evaluating the goodness of fit.

The fits can be separated into eight types as outlined in Table 1. The types designate which spectra are fit and whether the fit was over the full sky (FS), the Galactic center (GC: \( (l, b) = (-45^\circ, 45^\circ) \)), or specific regions of interest (RG; see Fig. 3).

| Fit Types and Results | Table 1 |
|-----------------------|---------|
| **Spectra Fit**       | **\( \chi^2/\nu \)** |
| Type | Ho | Dust | Haslam | Haze | FS | GC | RG |
| 1. | x | | | | 3.514 | 6.700 | 4.572 |
| 2. | x | x | | | 2.993 | 5.147 | 4.213 |
| 3. | x | x | | | 3.498 | 6.656 | 4.302 |
| 4. | x | x | x | | 2.977 | 5.126 | 4.168 |
| 5. | x | x | | | 3.506 | 6.550 | 4.241 |
| 6. | x | x | x | | 2.988 | 5.106 | 4.169 |
| 7. | x | x | x | | 3.498 | 6.611 | 4.208 |
| 8. | x | x | x | x | 2.972 | 5.082 | 4.148 |

Notes. — Different types of fits performed with the procedure outlined in \( \S \) 2.2 using CMB5 for a CMB estimator. Fit types are characterized by which spectra are fit, and in which region of the sky: full-sky (FS), Galactic center (GC: \( (l, b) = (-45^\circ, 45^\circ) \)), or specific regions of interest (RG; see Fig. 3).

\( ^a \) The haze template is described in \( \S \) 3.1.

\( ^b \) The \( \chi^2/\nu \) for the RG fits are averaged over all 12 regions.

The haze template is described in \( \S \) 3.1. or the “regions” (RG), which were chosen as regions of interest based on \( r \) for a full-sky fit (see Fig. 3, below). Also shown in Table 1 are the \( \chi^2/\nu \) for each fit type, where \( \nu \) is the number of degrees of freedom and is given by

\[ \nu = N_p - N_a, \] \label{nu}

with \( N_a \) the number of elements in the coefficient vector. Throughout the rest of this paper, we concentrate exclusively on our most general fit types 7 and 8, and unless otherwise noted, use CMB5 for a CMB estimator. This estimator is attractive both because of its uncomplicated noise properties and because the thermal dust emission has arguably the most well-understood amplitude and spectral behavior making it amenable to presubtraction before forming an ILC.

3.1. Full-Sky and Galactic Center Residual Maps

Figure 1 shows \( \Delta T_p \) as well as the unsubtracted maps for K, Ka, and Q bands, stretched to \( \pm 0.25, \pm 0.12, \) and \( \pm 0.08 \) mK respectively. The fit yields \( \chi^2/\nu = 3.49, \) removing 95.8\%, 95.7\%, 96.3\%, 97.5\%, and 99.7\% (for K, Ka, Q, V, and W bands, respectively) of the variance from the \textit{WMAP} data. However, it is clear from the second row of Figure 1 that there is still a remaining emission residual toward the Galactic center (GC). This residual is the “haze” present in the first-year data as shown by Finkbeiner (2004).

Although the average power in the haze is small (with a mean of just 0.59 kJy sr\(^{-1}\) per pixel at 23 GHz within 30\(^{\circ}\) of the GC in the southern sky), our fit may be compensating for its presence in the southern sky.
by adjusting the weights of the other foreground templates. To relax the stress on the fit, we adopt a crude model for the haze emission,

\[
h \propto \begin{cases} 
\frac{1}{r} - \frac{1}{r_0} & \text{for } r < r_0, \\
0 & \text{for } r > r_0,
\end{cases}
\]  

(17)

where \( r \) is the distance to the Galactic center and we arbitrarily set \( r_0 = 45^\circ \). Since the emission mechanism is unknown, we fit the spectrum of the haze as well. The extent to which the other fit parameters change gives an idea of the cross talk between the haze and the other templates.

The third row of Figure 1 shows residual maps for FS8. It is clear from \( r_{FS8} \) alone that the fit is improved, particularly in the southern GC where obscuration from dust and gas is minimal and in the high-latitude north where the north Galactic spur synchrotron feature is no longer over subtracted at 23 GHz (compare rows 2 and 3 of Fig. 1). Although \( \chi^2/\nu_{FS8} = 2.97 \) is only slightly lower than the FS7 fit, we emphasize that the number of degrees of freedom is quite large (\( \nu \sim 155,000 \)) and so the likelihood for the FS8 model is significantly higher. Furthermore, we are including many pixels at large Galactic latitudes where the signal-to-noise ratio is very low and the amplitude of the haze is very small.

Since the ISM may have different properties near the GC (i.e., lower gas temperature due to more efficient cooling because of higher metallicity, increased supernova activity, etc.), it is instructive to consider this region separately from the rest of the sky. Figure 2 shows \( r \) maps for the GC7 and GC8 fits. There is a significant decrease in \( \chi^2/\nu \) from GC7 to GC8 with the inclusion of our haze template—from 6.61 for GC7 to 5.08 for GC8. The substantially oversubtracted regions near the edge of the mask in \( r_{GC7} \) indicate that the fit is indeed attempting to compensate for
the haze by adjusting the amplitudes of other templates. Although there is still some over subtraction in $r_{\text{GC8}}$, the overall quality of the fit is much improved, particularly in the southern sky.

In the northern sky, there is a large structure just northwest of the GC in and around the region of Rho-Oph. Typically, extended regions of over- or undersubtraction in our fits are indicative of variations in the physical conditions of the emission media. However, in this circumstance, it is unclear if the undersubtraction is due to an oversimplification of the radial haze profile given our template for this component (eq. [17]). The next step is to subdivide the sky into certain regions of interest where our fit is the least successful and fit those regions explicitly.

### 3.2 Regional Fits and Composite Maps

Our regions of interest were identified as regions of particularly notable over- or undersubtraction in $r_{\text{FS8}}$. Figure 3 shows the boundaries of these regions superimposed on the raw WMAP data at 23 GHz, $r_{\text{FS8}}$, as well as a composite map for $r_{\text{RG8}}$ (a fit which includes our haze template). The residual map (which is actually the residual maps of each region stitched together with no smoothing) is shown for both RG8 and RG7 fits in Figure 4 (the maps stretched to the same units as Fig. 1).

At all frequencies, the RG8 fit more effectively removes the foregrounds than the RG7 fit. There are more regions of oversubtraction in $r_{\text{RG7}}$ (both at high latitudes and around the mask edges) compared to $r_{\text{RG8}}$, the region boundaries are somewhat less continuous in $r_{\text{RG7}}$, and most importantly, the haze is still present in the $r_{\text{RG7}}$ maps. These composite $r$ maps are completely unsmoothed and it is a testament to the quality of our RG8 fit that there are no discernible large-scale brightness gradients between adjacent regions.

Lastly, we define the residual haze map

$$r_H = r_{\text{RG8}} + a_h,$$

where the appropriate amount of haze is added back in to each region (i.e., the same amount as was subtracted by the fit). The bottom row of Figure 4 shows $r_H$. Despite the fact that the haze fit coefficients are not constrained to be continuous across the region boundaries, $r_H$ has no clear discontinuities.

### 3.3 Foreground Spectra

Figure 5 shows the FS8 spectra (coefficient vectors) in kJy sr$^{-1}$ per template unit: Rayleighs for Hα, mK for FDS and Haslam, and arbitrary units for the haze. The most striking feature of these spectra are that they are very sensitive to the estimator used for the CMB. For example, the synchrotron spectrum actually appears to turn up at high frequencies for CMB1, CMB2, and CMB3, while CMB4 and CMB5 give the more physically motivated power-law–type spectra. This dependence is entirely due to the contamination of the CMB by foregrounds, which can never fully be removed for any CMB estimator. Thus, when we remove the estimator from the WMAP data to perform the foreground fit, we have inadvertently added (or subtracted) some foregrounds from the data with essentially the spectrum of the CMB ($I_\nu \propto \nu^2$). We emphasize that the contamination is small, so that it has minimal effect on the variance of the CMB estimator, but it is large compared to the relative amplitudes of the foregrounds. This is especially true for estimators which minimize the variance of an ILC; in this case the contamination is proportional to the cross-correlation of the true CMB with the true foregrounds (see § 4).

Despite the large uncertainties, there are concrete conclusions that can be drawn from Figure 5. First, the Hα-correlated emission does not follow the $I_\nu \propto \nu^{0.15}$ frequency dependence as expected. Instead there is a bump in the spectrum around 50 GHz. In our companion paper (DF08) we argue that the Hα-correlated emission has a spectrum that is consistent with a classical $\nu^{0.15}$ spectrum plus a WIM spinning dust component. Second, although the soft synchrotron and haze spectra vary substantially with CMB estimator type, for a given type, the haze is always harder than the normal soft synchrotron, a point which we explore in more detail below.

Finally, like Bennett et al. (2003), Finkbeiner (2004), Davies et al. (2006), and Hinshaw et al. (2007), we find that the dust-correlated emission falls from 94 to 61 GHz but then rises to 23 GHz consistent with emission from both thermal and spinning dust. Since these spectra are the result of fits over large areas of the sky and the spinning dust spectrum is expected to vary with position, it is not surprising that we do not see a peak in the dust-correlated emission in the range 20–40 GHz as in the Draine & Lazarian (1998) models. Rather, we are seeing a superposition of many spinning dust spectra with varying peak frequencies. This has led to the misidentification of the this dust-correlated emission as synchrotron in the past (Bennett et al. 2003; Hinshaw et al. 2007).

It is tempting to conclude that the RG fits for the individual regions can be used to construct a map of the variation in the
spinning dust spectra across the sky. Such a map would represent an “excitation map” or “irradiation map” for the dust grains. However, this is simply not possible given the contamination of the CMB estimator by the foregrounds.10

3.4. Comparison of the Haze and Soft Synchrotron

From Figure 5 it seems that the haze spectrum is inconsistent with a $\nu^{-0.15}$ free-free spectrum. While some of the CMB estimators do yield such hard spectra at low frequencies, those fits turn up at high frequencies. This behavior is much less likely than simply a contamination of the CMB estimator by the haze emission. Furthermore, the haze is most clearly visible in the southern GC where dust obscuration is minimal so that the haze should either show up in the H$\alpha$ map (if it is $T < 10^5$ K gas) or in the ROSAT X-ray data (if it is $T > 10^5$ K gas). Since neither of these is the case, and since gas at intermediate temperatures is thermally unstable (see Spitzer 1978), we concluded that the most likely source of the haze is synchrotron emission from relativistic electrons. Figure 5 also shows that this synchrotron emission is harder than the soft synchrotron traced by the Haslam 408 MHz map.11

Figure 6 shows density contours for a scatter plot of the unmasked pixel values in $r_H$ and the total synchrotron residual map, $r_{H+S} = r_H + a_s$, in antenna temperature for various frequency combinations. The pixels shown have $I = (\pm 25^\circ, 25^\circ)$ and $b = (-45^\circ, 0)$. Despite the large scatter, the $r_H$ emission appears to be a distinctly separate component of synchrotron emission with a spectral index that is significantly harder than the synchrotron (as shown in Table 2 this behavior persists for all CMB estimator types, although the precise spectral indices are very uncertain due to the CMB estimator bias describe in § 4). This point is underscored in an RGB color coded map of the 23, 33, and 41 GHz $r_H$ and $r_S$ maps shown in Figure 7. The haze emission is distinctly bluer (harder spectral index) than the total, redder (softer) synchrotron emission in the Galactic center.

Figure 8 shows the total intensity of the haze as a function of distance south of the Galactic center. The radial bins are 20° wide and separated by 1° in latitude. Our simple $1/r$ profile for the haze emission is not adequately describing the structure of the haze, which is also clear from the residual maps in Figures 1 and 2. Furthermore, the systematic error bars due to chance correlation between the haze and the CMB are quite large, particularly at high frequencies.

4. CMB ESTIMATOR BIAS

While the statistical uncertainty in our fits comes from measurement noise in the WMAP data, systematic uncertainties in our fits are dominated by contamination of the CMB estimator by the foreground components. We have used six different CMB estimators, but they can be separated into two categories: ILC-type estimators in which multiple WMAP bands are combined to approximately cancel the foregrounds (CMB1, CMB2, CMB3, and CMB5), and HF estimators in which a model for the thermal dust and free-free emission is removed from the highest WMAP band to approximately leave the CMB (CMB4).

4.1. ILC Estimators

Hinshaw et al. (2007) point out that the WMAP data consist of the “true” CMB, $T_C$, plus some additional contamination by foreground components $T_f$ which have some spectral dependence.
Fig. 5.—Foreground spectra for the FS8 fits with our six CMB estimators: solid line, CMB1; dotted line, CMB2; short-dashed line, CMB3; dot-dashed line, CMB4; double-dot-dashed line, CMB5; and long-dashed line, CMB6. The error bars on the coefficients are the formal error bars on the fit (see text).

Fig. 6.—Number density contours of unmasked pixel [with $l = (-25^\circ, 25^\circ)$ and $b = (-45^\circ, 0^\circ)$] temperatures in the $r_{10}$ (blue contours) and $r_{45,5}$ (dashed contours) maps for the RG8 fits at 23, 33, and 41 GHz for CMB5. Although the spectral slope is somewhat uncertain between each band and can vary significantly depending on which CMB estimator is used (see Table 2), the best-fit spectral slope for the haze emission is distinctly harder than for the total synchrotron.
where \( i \) and \( j \) represent the observing band and foreground component respectively. That is,

\[
W_i = T_c + \sum_j N_{i,j} T_j
\]

so that an internal linear combination of the WMAP bands is

\[
L = \sum_i \zeta_i W_i = \sum_i \zeta_i T_c + \sum_j \zeta_N_{i,j} T_j = T_c + \sum_j \Gamma_j T_j, \quad (19)
\]

where \( \zeta_i \) are constrained to sum to one in order to preserve unity response to the CMB, and \( \Gamma_j \) parameterizes the contamination of \( L \) by foreground \( j \).

Let us consider that there are four types of foreground emission: free-free (\( F \)), dust (\( D \)), soft synchrotron (\( S \)), and the haze (\( H \)). Each of these has an associated \( \zeta_i \), and so the ILC is given by

\[
L = T_c + \Gamma_F T_F + \Gamma_D T_D + \Gamma_S T_S + \Gamma_H T_H. \quad (20)
\]

Since \( \zeta_i \) are chosen to minimize the variance in \( L \),

\[
\frac{\partial \langle L^2 \rangle}{\partial T_j} = 0 \quad (22)
\]

Table 2

| CMB Estimator | \( \beta_S \) | \( \beta_H \) | \( \beta_S \) | \( \beta_H \) | \( \beta_S \) | \( \beta_H \) |
|---------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1.............. | -2.86        | -2.58        | -2.81        | -2.14        | -2.13        | -2.23        |
| 2.............. | -2.78        | -2.47        | -2.71        | -1.98        | -1.98        | -2.01        |
| 3.............. | -2.75        | -2.53        | -2.66        | -2.24        | -2.56        | -2.31        |
| 4.............. | -3.15        | -3.33        | -3.22        | -2.57        | -3.14        | -2.76        |
| 5.............. | -3.10        | -3.13        | -3.15        | -2.39        | -2.67        | -2.52        |
| 6.............. | -3.00        | -3.05        | -3.01        | -2.37        | -2.92        | -2.51        |

Notes.—Best-fit spectral slopes for \( r_{H+S} (\beta_S) \) and \( r_H (\beta_H) \) maps for the RG8 fits at 23, 33, and 41 GHz over unmasked pixels (with \( \ell = (-25^\circ, 25^\circ) \) and \( b = (-45^\circ, 0^\circ) \); see Fig. 6). Although there is significant scatter in the inferred spectral index, the haze residual is always harder than the total synchrotron signal.

Fig. 7.—RGB representation of \( r_H \) and \( r_{H+S} \) for RG8 with CMB5. The color coding indicates the spectral index, in antenna temperature, of a given pixel. In particular, the bluer haze region (box) indicates a harder spectrum than the redder synchrotron emission.
for all \( j \), and we have explicitly assumed mean subtracted maps so that \( \langle L \rangle = \langle T_c \rangle = \langle T_f \rangle = 0 \). Now,

\[
L^2 = T_c^2 + 2 \sum_j \Gamma_j T_c T_j + 2 \sum_{k \neq j} \Gamma_k T_k T_j + \sum_j \Gamma_j^2 T_j^2,
\]

so that equation (22) reads

\[
0 = 2 \langle T_s T_f \rangle + 2 \sum_{k \neq j} \Gamma_k \langle T_k T_j \rangle + 2 \Gamma_j \langle T_j^2 \rangle.
\]

(Note that the notation in eq. [24] can be compacted into \( 0 = \sum_\psi \Gamma_\psi \langle T_s T_\psi \rangle \), where \( \psi = \{c,F,D,S,H\} \) and \( \Gamma_c \equiv 1 \).) Thus, we can solve explicitly for

\[
\Gamma_j = -\frac{\langle T_s T_j \rangle + \sum_{k \neq j} \Gamma_k \langle T_k T_j \rangle}{\langle T_j^2 \rangle}.
\]

In the limit of only one foreground \( T_f \), the second term (which represents cross-correlation between the different foreground morphologies) disappears, and this reduces to \( \Gamma = \Gamma(T_c, T_f) = -\sigma_{eff}/\sigma_f^2 \), as derived with a similar method by Hinshaw et al. (2007).

According to equation (25), the ILC map is biased toward anti-correlation between different “true” foreground emission morphologies and the “true” CMB. However, since we do not know the true \( T_c \) or \( T_f \) a priori, we have no information about how much of each foreground to add back into \( L \) in order to correct for this factor.

We can use Monte Carlo techniques to estimate the amplitude (but not the sign) of the error in \( L \) due to foreground contamination as follows. We construct 100 realizations of the CMB by generating random phases for the measured power, \( C_i \), in each Fourier mode of the binned three-year WMAP power spectrum \( \langle \ell_{\text{max}} = 986 \rangle \) from Spergel et al. (2007). For each realization \( T' \), we can estimate \( \Gamma \) with a reasonable foreground template \( T_j' \), which we suppose traces the morphology of the emission (see § 2.1).

Figure 9 shows histograms of

\[
\Gamma' \sigma_j = \Gamma(T'_c, T'_j) \sigma_f,
\]

Fig. 8.—Integrated haze (from the bottom of Fig. 4) in kJy sr\(^{-1}\) as a function of radial distance south of the Galactic center. The radial bins are 20\(^{\circ}\) wide and separated by 1\(^{\circ}\) in longitude. The inner error bars are due to the formal error on the fit coefficients, the outer error bars are the 1 \( \sigma \) standard deviation of the temperature fluctuations in a given radial bin, and the dotted lines represent the bias due to chance correlation between the CMB and the haze.
where $\sigma_j$ is the standard deviation over unmasked pixels of the foreground template $T_0^j$ ($T_0^j = H\alpha$, FDS, Haslam, and haze), for all 100 realizations of the CMB sky. We multiply by $\sigma_j$ to obtain the contamination in a “typical” pixel in the ILC map. Each histogram has a mean consistent with zero, $\langle \Gamma' \rangle \approx 0$ for each $T_0^j$, as expected since the CMB realization $T_0^j$ is just as likely as $-T_0^j$. The implication is that we can only estimate the uncertainty in $T_{\text{ILC}}$, we cannot explicitly correct for the bias.

It is important to note that, although $L$ cannot be corrected for the cross-correlation bias, the net effect is to decrease the variance relative to the true CMB, $T_c$. Since the bias of the variance is not mean zero, the effect on the power spectrum of the CMB can be estimated (see Hinshaw et al. 2007).

4.2. HF-Type Estimators

The bias in a HF estimator is easy to understand and due entirely to the residual foreground emission after subtraction of thermal dust and free-free. Assuming perfect subtraction of these two foregrounds, whatever other foregrounds are left in the HF estimator are subtracted off all lower frequency bands, leading to a systematic bias toward softer spectra for the soft synchrotron and haze components.

4.3. Forecasts for Planck

The principal advantage of the Planck mission over WMAP is the large range of frequency coverage. In particular, the multiple channels at very high frequency reduce the foreground problem to (mostly) a single emission mechanism, thermal dust. In order to exploit this feature, we suggest a new approach based on an ILC-type CMB estimator. However, instead of minimizing the variance (which we have seen leads to large cross-correlation uncertainties), we choose the ILC coefficients $\xi_i$ to cancel as many power-law foreground components as possible. This estimator can then be subtracted off of lower frequency bands with very minimal bias contamination of the inferred foreground spectra.

Planck’s frequency coverage is 30, 44, 70, 100, 143, 217, 353, 545, and 857 GHz. If we estimate that spinning dust emission is negligible above 100 GHz, then these bands contain mostly thermal dust ($T \propto \nu^{\beta_\perp}$, with $1.6 \leq \beta_\perp \leq 2.3$) and also small amounts of free-free, synchrotron, and haze ($T \propto \nu^{\beta_\parallel}$ with $-3.1 \leq \beta_\parallel \leq -2.1$).
In order to find an ILC that optimally cancels out this range of power-law indices, we formulate the error function,

$$\delta_{\beta_4}(\beta) = \frac{T_{94}}{\sum \zeta_i T_i}$$

for a foreground spectrum, normalized to 94 GHz, $T_i = T_{94}(\nu_i/94 \text{ GHz})^\beta$, where $\nu_i$ are the Planck bands above 100 GHz and the sum is over bands. The physical interpretation of $\delta_{\beta_4}(\beta)$ is that it represents the fractional bias in the inferred 94 GHz amplitude of a foreground with power-law index $\beta$. We minimize $\int \delta_{\beta_4} d\beta$ with respect to $\zeta_i$ over the range $\beta_i$ and $\beta_{i+1}$ defined above. Figure 10 shows the resultant $\delta_{\beta_4}$. With these $\zeta_i$ we can form the Planck ILC $L_P = \sum \zeta_i P_i$, where $P_i$ are the Planck data in thermodynamic mK at band $i$. We find that

$$L_P = -1.49 P_{545} + 3.21 P_{217} - 0.74 P_{353} + 0.02 P_{545} - 3.21 \times 10^{-5} P_{857}$$

minimizes the area under the $\delta_{\beta_4}$ curve over the $\beta$ range of interest. Using only the high-frequency channels in this way avoids ambiguities due to an uncertain spinning dust spectrum.

To estimate the contamination of $L_P$ by synchrotron and haze emission we must make a couple of assumptions. First, let us assume that these foregrounds follow a power law with amplitudes of roughly our fit results at 23 GHz from Figure 5 and indices $-3.1 \leq \beta_s \leq -2.7$ and $-2.7 \leq \beta_h \leq -2.4$, respectively. Second, we make the implicit approximation that the morphologies of the foreground emission mechanisms do not change with frequency and that they are still well represented by the templates in §2.1.

Figure 10 shows our estimates for the error bars on the “mock-true” soft synchrotron and haze spectra at WMAP frequencies given three CMB estimators: a WMAP-only ILC (leading to cross-correlation bias errors), our WMAP HF estimator CMB4, and the $L_P$ estimator. As we have already seen, the WMAP-only ILC clearly yields very large uncertainties, although they have the advantage of being mean zero (averaged over an ensemble of CMB realizations). CMB4 does significantly better, although now the errors are systematically biased toward softer spectra. With $L_P$, the bias errors are almost completely eliminated. For a foreground with indices $\beta = -3.1$, $-2.7$, and $-2.4$, the bias errors at 23 (94) GHz are $-0.11\%$, $0.06\%$, and $+0.13\%$ ($-7.6\%$, $-2.2\%$, and $+3.3\%$), respectively.

We emphasize that we have by no means attempted to formulate the “optimal” foreground removal algorithm for Planck. There have been numerous discussions on the topic describing and comparing different cleaning methods. Rather, we have demonstrated that, by exploiting the many bands and high-frequency coverage, this simple Planck ILC will substantially reduce the systematic biases in the inferred foreground spectra compared to WMAP.

5. DISCUSSION

We have carried out a multilinear regression fit to the WMAP data using foreground templates for free-free, soft synchrotron, and dust emission. We perform fits over both the (nearly) full sky, as well as smaller regions of interest. Our method simultaneously fits the amplitude and spectrum of each emission component and is immune to priors or initial “guesses” for the template amplitudes.

Importantly, we find that the spectra of the foreground emission mechanisms cannot be determined to high accuracy with the WMAP data. The root cause is that any estimator of the CMB will necessarily be contaminated by the foregrounds to some degree, so that subtracting the CMB estimator from the WMAP data systematically biases the inferred foreground spectra. Thus, the fit spectrum for each foreground is contaminated by some amount of CMB spectrum. The degree of systematic bias in the foreground spectra can be quite large and varies dramatically among our different CMB estimators. It is worth noting that the degree of contamination of a given CMB estimator does not significantly affect the variance of the estimator.

We find that, on removing the free-free, dust, and soft synchrotron emission, the three-year WMAP data still contain the
“haze” seen by Finkbeiner (2004). Since this haze emission is present in all residual maps regardless of which CMB estimator is used, we conclude that it is not an artifact of imperfect CMB subtraction. We have included a simple \( 1/r \) template for the haze to relax the stress on the other foreground components in our fit. This template imperfectly matches the morphology of the haze, but allows us to simultaneously fit an approximate amplitude and spectra of the haze emission as well.

Despite the above issue of bias errors, we can make the following concrete statements about our fit foreground spectra:

1. We find that the H\( \alpha \) correlated emission does not follow a simple \( I_\nu \propto \nu^{-0.15} \) free-free power law as expected from theory. Rather we find a bump in the H\( \alpha \) -correlated emission which we argue in our companion paper (DF08) can be explained by a mixture of free-free gas and spinning dust. We emphasize that this bump is not due to the contamination of the CMB estimator by the foregrounds.

2. The dust-correlated emission has the now familiar fall off from 23 to 41 GHz consistent with a superposition of Draine & Lazarian (1998) spinning dust spectra and then a rise at 94 GHz from thermal dust emission. We cannot use our multiregion fits to map out variations in the spinning dust spectrum from place to place across the sky since the uncertainty from the systematic bias dominates.

3. Our soft synchrotron spectra are very sensitively dependent on which CMB estimator is used. For example, with the WMAP team’s published ILC, the soft synchrotron spectrum actually turns up at high frequencies, while other estimators show a steadily falling spectrum (with \( T_\nu \propto \nu^\beta \), where \( \beta \approx -3.0 \)). This ambiguity is entirely due to the contamination of the CMB estimator by soft synchrotron-type emission.

4. The haze spectrum is similarly very uncertain due to CMB estimator bias; however, for a given estimator, the haze spectrum is always harder than the soft synchrotron. For reasons outlined in § 3.4, we suspect that the haze emission is due to hard synchrotron from a separate component of very energetic electrons near the Galactic center. Possible sources include products of dark matter annihilation (Hooper et al. 2007) or a single energetic event in the last million years, e.g., a gamma-ray burst (A. E. Broderick et al. 2008, in preparation).

Upcoming experiments such as Planck will significantly reduce the problem of CMB estimator bias in the inferred foreground spectra due to the improved high-frequency coverage. In particular, for frequencies \( \nu > 100 \) GHz the Planck data will be virtually free of all foregrounds except for thermal dust. Removing this one bright component with five bands is significantly less ambiguous than removing three or four bright components with five bands. Furthermore, what little free-free, soft synchrotron, and haze emission there is at these high frequencies can be roughly eliminated in a Planck CMB estimator through the appropriate choice of ILC coefficients. We estimate that the systematic uncertainties in soft synchrotron and haze spectra will be reduced by more than an order of magnitude compared to WMAP.

Galactic foreground emission represents the most significant contaminant in determining cosmological parameters. Ironically, the reverse statement is also true and in order to make progress toward determining the “true” spectra of the foreground emission, a more reliable estimate of the CMB sky is required.

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