One-Photon Transitions between Heavy Baryons in a Relativistic Three-Quark Model

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Abstract
We study one-photon transitions between heavy baryon states in the framework of a relativistic three-quark model. We calculate the one-photon transition rates for ground-state to ground-state transitions and for some specific excited state to ground-state transitions. Our rate predictions for the most important transitions are: \( \Gamma(\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma) = 60.7 \pm 1.5 \) KeV, \( \Gamma(\Xi_c^{*+} \rightarrow \Xi_c^{++} \gamma) = 12.7 \pm 1.5 \) KeV, \( \Gamma(\Lambda_{c1}(2593) \rightarrow \Lambda_c^+ \gamma) = 104.3 \pm 1.3 \) KeV.

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During the last few years there has been significant progress in the experimental study of the spectroscopy of ground state and excited state charm baryons and their strong and electromagnetic decays [1,2]. The one-photon transitions \( \Lambda_c^+ (2593) \rightarrow \Lambda_c \gamma \) and \( \Lambda_c^+ (2625) \rightarrow \Lambda_c \gamma \) were searched for by the CLEO Collaboration but were not seen [3]. The CLEO Collaboration determined upper limits for the branching ratios

\[
B(\Lambda_c^+ (2593) \rightarrow \Lambda_c^+ \gamma) / B(\Lambda_c^+ (2593) \rightarrow \Lambda_c^+ \pi^+ \pi^-) < 0.98, \\
B(\Lambda_c^*+ (2625) \rightarrow \Lambda_c^+ \gamma) / B(\Lambda_c^*+ (2625) \rightarrow \Lambda_c^+ \pi^+ \pi^-) < 0.528.\]

One-photon transitions between charm baryons have been analyzed before in the leading order of the heavy quark mass expansion [4,5], in the nonrelativistic quark model incorporating heavy-quark symmetry [6] and in the bound state picture [7]. When applying heavy quark symmetry to the one-photon transitions one makes no assumptions about the composition of the light-side diquark states mediating the one-photon transitions. A first rough estimate of the unknown coupling parameters entering the effective heavy quark symmetry Lagrangian including electro-magnetism has been attempted using simple dimensional arguments [5]. The light-side diquark transitions have been calculated within the constituent quark model [6] and within a bound state model [7] where the heavy baryon is composed of a heavy meson and a light baryon. In the nonrelativistic constituent quark model one obtains for the ground-state to ground-state transitions [6]

\[
\Gamma(\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma) = 93 \text{KeV}, \quad \Gamma(\Xi_c'^+ \rightarrow \Xi_c^+ \gamma) = 16 \text{KeV}, \quad \Gamma(\Xi_c^0 \rightarrow \Xi_c^0 \gamma) = 0.3 \text{KeV}.
\]

In the bound state picture [7] the \( \Lambda_{c1} \rightarrow \Lambda_c \gamma \) and \( \Lambda_{c1}^* \rightarrow \Lambda_c \gamma \) decays are severely suppressed, whereas the corresponding transitions between bottom baryons are predicted to have significant branching ratios.

In this paper we report on the predictions of the Relativistic Three-Quark Model [8] for the one-photon transitions between heavy baryon states. The Relativistic Three-Quark Model was applied before to a number of different dynamical problems involving the properties of pions [8], light baryons [9] and heavy-light baryons [10]-[12]. In the most recent application the Relativistic Three-Quark Model was used to evaluate the one-pion transition strengths between charm baryons [12].

The Lagrangian describing the couplings of a heavy baryon state to its constituent light and heavy quarks considerably simplifies in the heavy quark limit since the heavy quark field enters as a local field and can be factored from the nonlocal Lagrangian. One has

\footnote{In the following we use the notation \( \Lambda_{c1} \) and \( \Lambda_{c1}^* \) for the excited baryon states \( \Lambda_c^* (2593) \) and \( \Lambda_c^* (2625) \), respectively.}
\[ L_{BQ}^{\text{int}}(x) = g_{BQ} B_Q(x) \Gamma_1 Q^a(x) \int d^4 \xi_1 \int d^4 \xi_2 F_B(\xi_1^2 + \xi_2^2) \]
\[ \times q^b(x + 3\xi_1 - \sqrt{3}\xi_2) CT_2 \lambda_{BQ} q^c(x + 3\xi_1 + \sqrt{3}\xi_2) \varepsilon^{abc} + \text{h.c.} \]
\[ F_B(\xi_1^2 + \xi_2^2) = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \xi_1 + ik_2 \xi_2} \tilde{F}_B \left\{ \frac{k_1^2 + k_2^2}{\Lambda_B^2} \right\} \]

\( \Gamma_i \) and \( \lambda_{BQ} \) are spin and flavor matrices, respectively; \( g_{BQ} \) denotes the coupling of the heavy baryon with the constituent quarks; \( \Lambda_B \) is the cutoff parameter defining the distributions of light quarks in the heavy baryon. As we are working to leading order in the heavy quark mass expansion the baryon cutoff parameter \( \Lambda_B \) has to be chosen to be the same for the charm and bottom baryons in order to guarantee the correct normalization of the baryonic Isgur-Wise function in the heavy quark symmetry limit [10]. The quantum numbers of the heavy baryons and the Dirac matrices \( \Gamma_i \) and flavour matrices \( \lambda_{BQ} \) define the structure of the relevant three-quark charm baryon currents. They are listed in TABLE I.

Let us now specify how the electromagnetic interactions are introduced at the quark level. As in a local theory we derive the interaction Lagrangian of the electromagnetic field \( A_\mu \) with the quarks using the standard minimal substitution

\[ L_{\text{em}}(x) = -e A_\mu(x) \bar{q}^a(x) \gamma^\mu Q q^a(x) \]

where \( Q = \text{diag}\{2/3, -1/3, -1/3\} \) is the charge matrix of the light quarks. In the leading order of the heavy quark mass expansion the photon does not couple to the heavy quark. In order to have a gauge-invariant theory one also needs to couple the electromagnetic field into the nonlocal heavy-baryon-quark Lagrangian (1). This can be achieved by the prescription of Mandelstam [13]. Previous applications of the Mandelstam prescription to hadron physics can be found in Refs. [14,9]. Each light quark field \( q(y) \) is multiplied with the exponential factor \( \exp(i eQ \int_x^y dz_\mu A_\mu(z)) \). As a result one obtains a nonlocal gauge invariant interaction Lagrangian for the coupling of heavy baryons to their constituent quarks including electromagnetism. One has

\[ L_{BQ}^{\text{int,em}}(x) = g_{BQ} B_Q(x) \Gamma_1 Q^a(x) \int d^4 \xi_1 \int d^4 \xi_2 F_B(\xi_1^2 + \xi_2^2) \]
\[ \times \exp(i eQ \int_x^{x + 3\xi_1 - \sqrt{3}\xi_2} dz_\mu A_\mu(z)) q^b(x + 3\xi_1 - \sqrt{3}\xi_2) CT_2 \lambda_{BQ} \]
\[ \times \exp(i eQ \int_x^{x + 3\xi_1 + \sqrt{3}\xi_2} dz_\mu A_\mu(z)) q^c(x + 3\xi_1 + \sqrt{3}\xi_2) \varepsilon^{abc} + \text{h.c.} \]
The Lagrangian (3) generates nonlocal vertices which involve the heavy baryons, photons and light and heavy quarks. In general several diagrams contribute to the one-photon transitions of heavy baryons: the standard triangle diagram (FIG.1a) and the contact interaction-type diagrams (FIG.1b). The calculation of the contact interaction-type diagrams was discussed in detail in Ref. [9] where this approach was applied to the study of nucleon electro-magnetic interactions. Only when the contact interaction-type diagrams are included one satisfies the relevant Ward-Takahashi identities for the connected Green functions (see details in Ref. [9]). However, it is not difficult to see that the contact interaction-type contributions are nonleading in the heavy mass expansion, at least when the photon is on its mass shell \( (q^2 = 0) \). Since we are working in the heavy quark limit and with real photons throughout, these contributions can be safely dropped. For transitions involving P-wave states there are, however, leading contact interaction-type contributions resulting from the minimal substitution prescription for the derivatives in the interaction Lagrangian coupling the excited baryon states to the constituent quarks.

The contribution of the triangle diagram (FIG.1a) to the matrix element of the one-photon transition \( B_Q^i(p) \rightarrow B_Q^f(p) + \gamma(q) \) has the following form in the heavy quark limit

\[
M^i_{\text{inv,}\Delta}(B_Q^i \rightarrow B_Q^f \gamma) = e g_{\text{eff}} g_{\text{eff}} \cdot \bar{u}(v) \Gamma^i_1 \frac{(1+i)}{2} \Gamma^i_1 u(v) \cdot I^{if}_{q_1q_2}(v, q) \tag{4}
\]

\[
g_{\text{eff}} = g_{B_Q} \frac{\Lambda^2_B \sqrt{C_{\text{color}}}}{8\pi^2} \quad \quad C_{\text{color}} = 6
\]

\[
I^{if}_{q_1q_2}(v, q) = \frac{1}{\pi^2} \int \frac{d^4k_1}{d^4k_2} \tilde{F}_B(k_1, k_2, q) \tilde{F}_B(k_1, k_2, 0) \Pi_{q_1q_2}(k_1, k_2, q) \tag{5}
\]

\[
\tilde{F}_B(k_1, k_2, q) \equiv \tilde{F}_B \left\{ -6 \left[ (k_1 + q)^2 + (k_2 - q)^2 + (k_1 + k_2)^2 \right] \right\}
\]

\[
\Pi_{q_1q_2}(k_1, k_2, q) = Q_{q_1q_2} \left[ \Gamma^i_2 S_q(k_1 + k_2) \Gamma^f_2 S_q(k_2 - q) \gamma^\mu S_q(k_2) - Q_{q_1q_1} \left[ \Gamma^f_2 S_q(-k_1 - k_2) \Gamma^i_2 S_q(-k_2) \gamma^\mu S_q(-k_2 + q) \right] \right]
\]

where \( \Gamma^{i(2)}_1 \) and \( \Gamma^{f(2)}_1 \) are the Dirac matrices of the initial and the final baryons, respectively. Here \( S_q(k) = 1/(m_q - k) \) is the light quark propagator \( (q = u, \bar{d}, s) \). The masses of the \( u \) and \( d \) quarks are set equal: \( m_u = m_d = m_q \). The parameter \( \Lambda_{q_1q_2} = M_{Q_{q_1q_2}} - m_Q \) in the denominator of the heavy quark propagator denotes the difference between the heavy baryon mass \( M_{Q_{q_1q_2}} \) and the heavy quark mass \( m_Q \). We use different values for the parameter \( \Lambda_{q_1q_2} \) for baryons containing only nonstrange light quarks and one or two strange
quarks: \( \bar{\Lambda}, \bar{\Lambda}_s \) and \( \bar{\Lambda}_{ss} \), respectively. The appearance of unphysical imaginary parts in the Feynman diagrams is avoided by imposing the condition that the baryon mass is less than the sum of constituent quark masses. In the case of heavy-light baryons this restriction implies that the parameter \( \bar{\Lambda}_{q_1q_2} \) must be less than the sum of light quark masses. Latter constraint serves as an upper limit for our choices of the parameter \( \bar{\Lambda}_{q_1q_2} \). All dimensional parameters are expressed in units of \( \Lambda_B \). The integrals are calculated in the Euclidean region both for internal and external momenta. Finally, the results for the physical region are obtained by analytic continuation of the external momenta after the internal momenta have been integrated out.

In the calculation of (5) we use the \( \alpha \)-parametrization for quark propagators and the Laplace transform for the vertex function

\[
\frac{1}{A} = \int_0^\infty d\alpha e^{-\alpha A}, \quad \tilde{F}_B(6X) = \int_0^\infty ds \tilde{F}_B^L(6s)e^{-sX} \quad (6)
\]

The use of the Laplace transform allows one to perform the calculation of the transition matrix elements for any given function \( \tilde{F}_B \). In the numerical analysis of one-photon transitions of heavy baryons we will use a Gaussian vertex functions for heavy baryons in Eq. (5). As an illustration of our calculational procedure we evaluate a typical matrix element as e.g.

\[
R_{q_1q_2}^{ij}(v, q) = \int \frac{d^4k_1}{\pi^2 i} \int \frac{d^4k_2}{\pi^2 i} \frac{\tilde{F}_B(k_1, k_2, q)\tilde{F}_B(k_1, k_2, 0)}{[-k_1v - \bar{\Lambda}_{q_1q_2}]} \times \text{tr}[\Gamma_2^iS_{q_2}(q_1 + q_2)\Gamma_2^fS_{q_1}(k_2 - q)\gamma^\mu S_{q_1}(k_2)] \quad (7)
\]

Using the representation (6) we obtain

\[
R_{q_1q_2}^{ij}(v, q) = \int ds_1 \tilde{F}_B^L(6s_1) \int ds_2 \tilde{F}_B^L(6s_2) e^{2s_2q^2} \int d^4\alpha e^{\alpha_3\bar{\Lambda} - (\alpha_1 + \alpha_4)m_{q_1}^2 - \alpha_2m_{q_2}^2} \times \text{tr}
\]

\[
\Gamma_2^i\left(m_{q_2} - \frac{\vec{\phi}_1 + \vec{\phi}_2}{2}\right)\Gamma_2^f\left(m_{q_1} - \frac{\vec{\phi}_1 + \vec{\phi}_2}{2}\right)\gamma^\mu\left(m_{q_1} - \frac{\vec{\phi}_1 + \vec{\phi}_2}{2}\right)
\]

\[
\int \frac{d^4k_1}{\pi^2 i} \int \frac{d^4k_2}{\pi^2 i} e^{kAk_{2B}}
\]

The integration over \( k_1 \) and \( k_2 \) results in

\[
R_{q_1q_2}^{ij}(v, q) = \int ds_1 \tilde{F}_B^L(6s_1) \int ds_2 \tilde{F}_B^L(6s_2) e^{2s_2q^2} \int d^4\alpha e^{\alpha_3\bar{\Lambda} - (\alpha_1 + \alpha_4)m_{q_1}^2 - \alpha_2m_{q_2}^2} \times \text{tr}
\]

\[
\Gamma_2^i\left(m_{q_2} - \frac{\vec{\phi}_1 + \vec{\phi}_2}{2}\right)\Gamma_2^f\left(m_{q_1} - \frac{\vec{\phi}_1 + \vec{\phi}_2}{2}\right)\gamma^\mu\left(m_{q_1} - \frac{\vec{\phi}_1 + \vec{\phi}_2}{2}\right)\frac{e^{-BA^{-1}B}}{|A|^2}
\]
where

\[ kAk - 2kB = \sum_{i,j=1}^{2} k_i A_{ij} k_j - 2 \sum_{i=1}^{2} k_i B_i, \quad \bar{\phi}_i = \frac{\partial}{\partial B_i} \]

\[ A_{ij} = \begin{pmatrix} 2(s_1 + s_2) + \alpha_2 & s_1 + s_2 + \alpha_2 \\ s_1 + s_2 + \alpha_2 & 2(s_1 + s_2) + \alpha_1 + \alpha_2 + \alpha_4 \end{pmatrix} \]

\[ B_1 = -s_2 q - \alpha_3 v/2 \quad B_2 = (s_2 + \alpha_1) q \]

The kinematics of the one-photon transitions allows one to make use of the approximation: \( qv = (m_i^2 - m_f^2)/(2m_i) \approx 0 \) where \( m_i \) and \( m_f \) are the masses of the initial and the final baryons, respectively, divided by \( \Lambda_B \). Then, by making the variable replacement \( \alpha_i \rightarrow (s_1 + s_2)\alpha_i \) and by using \( \Gamma_2^1 = \gamma_\nu \) and \( \Gamma_2^2 = \gamma_5 \) the overlap integral can be seen to be proportional to \( q^\mu \) such that

\[ R_{\mu\nu}^{\rho}(v, q) = 4i \varepsilon^{\rho\mu\nu\beta} q^\alpha v^\beta J, \quad J = \int_0^\infty \frac{d^3 \alpha_1 \alpha_3}{2|A|^2} \tilde{F}_B^2(6z)\{m_{q_1}(A_{11}^{-1} + A_{12}^{-1}) - m_{q_2}A_{12}^{-1}\} \]

\[ A_{ij} = \begin{pmatrix} 2 + \alpha_2 & 1 + \alpha_2 \\ 1 + \alpha_2 & 2 + \alpha_1 + \alpha_2 \end{pmatrix}, \quad A_{ij}^{-1} = \frac{1}{|A|} \begin{pmatrix} 2 + \alpha_1 + \alpha_2 & -(1 + \alpha_2) \\ -(1 + \alpha_2) & 2 + \alpha_2 \end{pmatrix} \]

The evaluation of the other remaining matrix elements proceeds along similar lines.

At this point it is convenient to define a standard set of gauge invariant coupling constants for the one-photon transitions discussed in this paper. The general expansion of the transition matrix elements into a minimal set of gauge invariant covariants reads

\[ M_{\text{inv}}^\gamma(\Sigma_c \rightarrow \Lambda_c \gamma) = i \frac{2}{\sqrt{3}} f_{\Sigma_c \Lambda_c \gamma} \bar{u}(v) \not{\partial} \not{\gamma} u(v) \]

\[ M_{\text{inv}}^\gamma(\Sigma_c^* \rightarrow \Lambda_c \gamma) = 2f_{\Sigma_c^* \Lambda_c \gamma} \bar{u}(v) \epsilon(\mu \varphi^* v k) u^\mu(v) \]

\[ M_{\text{inv}}^\gamma(\Lambda_{c1} \rightarrow \Lambda_c \gamma) = \bar{u}(v)[F_{\Lambda_{c1} \Lambda_c \gamma} \cdot g^{\alpha \mu} v q + G_{\Lambda_{c1} \Lambda_c \gamma} \cdot v^\alpha q^\mu] \frac{\gamma_5}{\sqrt{3}} u(v) \varepsilon_\alpha^*(q) \]

\[ M_{\text{inv}}^\gamma(\Lambda_{c1}^* \rightarrow \Lambda_c \gamma) = \bar{u}(v)[F_{\Lambda_{c1} \Lambda_c \gamma}^* \cdot g^{\alpha \mu} v q + G_{\Lambda_{c1} \Lambda_c \gamma}^* \cdot v^\alpha q^\mu] u^\mu(v) \varepsilon_\alpha^*(q) \]

In the heavy quark limit three of the coupling constants become related. The relations read [4,5]

\[ f_{\Sigma_c \Lambda_c \gamma} = f_{\Sigma_c^* \Lambda_c \gamma} = f \]

\[ F_{\Lambda_{c1} \Lambda_c \gamma} = F_{\Lambda_{c1}^* \Lambda_c \gamma} = F \]
\[ G_{\Lambda c \Lambda c \gamma} = G_{\Lambda c^* \Lambda c \gamma} = G \]

Returning to our model calculation the coupling constant \( f \) can be represented as

\[ f = (\mu_1 - \mu_2) \frac{R_{\Sigma Q \Lambda Q \gamma}}{\sqrt{R_{\Lambda Q}} \sqrt{R_{\Sigma Q}}} \]

(8)

\[ R_{\Sigma Q \Lambda Q \gamma} = \frac{1}{4} \int_0^\infty d^3\alpha \alpha_3 (\alpha_1 + \alpha_2) \tilde{F}_B^2(6\alpha) \frac{A_{11}^{-1}}{|A|^2} \]

\[ R_{BQ} = \int_0^\infty d^3\alpha \alpha_3 \tilde{F}_B^2(6\alpha) \left\{ 1 + d_{BQ} \frac{\alpha_3}{m_q^2} \frac{\partial z}{\partial \alpha_3} - \frac{\alpha_3^2}{4m_q^2} A_{12}^{-1} (A_{11}^{-1} + A_{12}^{-1}) \right\} \]

where \( \mu_i = e_i/(2m_q) \) is the magnetic moment of the i-th light quark. Here

\[ z = \frac{\alpha_3^2}{4} A_{11}^{-1} + m_q^2 (\alpha_1 + \alpha_2) - \bar{\Lambda} \alpha_3, \quad d_{BQ} = \begin{cases} 1 & \text{for } B_Q = \Lambda_Q \\ \frac{1}{2} & \text{for } B_Q = \Sigma_Q \end{cases} \]

The calculation of the other two coupling factors \( F \) and \( G \) proceeds along similar lines.

The one-photon decay rates can then be calculated by using the general rate formula

\[ \Gamma = \frac{1}{2J + 1} \frac{|q|}{8\pi M_{BQ}^2} \sum_{\text{spins}} |M_{\text{inv}}|^2 \]

(9)

where \( |q| = qv = (m_i^2 - m_j^2)/(2m_i) \) is the photon momentum in the rest frame of the decaying baryon. In terms of the above coupling constants one obtains

\[ \Gamma (\Sigma_c \rightarrow \Lambda_c \gamma) = \frac{4}{3\pi} f^2 |q|^3 \frac{M_{\Lambda_c}}{M_{\Sigma_c}} \]

\[ \Gamma (\Sigma_c^* \rightarrow \Lambda_c \gamma) = \frac{4}{3\pi} f^2 |q|^3 \frac{M_{\Lambda_c}}{M_{\Sigma_c^*}} \]

(10)

\[ \Gamma (\Lambda_{c1} \rightarrow \Lambda_c \gamma) = \frac{1}{3\pi} \frac{3F^2 - G^2}{2} |q|^3 \frac{M_{\Lambda_c}}{M_{\Lambda_{c1}}} \]

\[ \Gamma (\Lambda_{c1}^* \rightarrow \Lambda_c \gamma) = \frac{1}{3\pi} \frac{3F^2 - G^2}{2} |q|^3 \frac{M_{\Lambda_c}}{M_{\Lambda_{c1}^*}} \]
Let us now specify our model parameters. In our numerical evaluation of the one-photon transition rates we make use of the same set of model parameters are were used to study the properties of light and heavy baryons [9,10] and one-pion transitions between charmed baryons [12]. In particular, the coupling constants $g_{BQ}$ in Eqs. (3) are calculated from the compositeness condition (see, ref. [10]), which means that the renormalization constant of the hadron wave function is set equal to zero $Z_{BQ} = 1 - g^2_{BQ}(M_{BQ}) = 0$ where $\Sigma_{BQ}$ is the heavy baryon mass operator. The masses of the light non-strange $u$ and the $d$ quarks ($m_u = m_d = m_q$) were determined from an analysis of nucleon data: $m_q = 420$ MeV [9]. The parameters $\Lambda_B$, $m_s$, $\bar{\Lambda}$ are taken from the analysis of the $\Lambda^+ \rightarrow \Lambda^0 + e^+ + \nu_e$ decay data [12]. To reproduce the present average value of $B(\Lambda^+ \rightarrow \Lambda^0 + e^+ + \nu_e) = 2.2 \%$ we used the following values for our parameters: $\Lambda_B = 1.8$ GeV, $m_s = 570$ MeV and $\bar{\Lambda} = 600$ MeV. The values of the unknown parameters $\bar{\Lambda}_s$ and $\bar{\Lambda}_{ss}$ were determined [12] from the relations $\bar{\Lambda}_s = \bar{\Lambda} + (m_s - m)$ and $\bar{\Lambda}_{ss} = \bar{\Lambda} + 2(m_s - m)$, which give $\bar{\Lambda}_s = 750$ MeV and $\bar{\Lambda}_{ss} = 900$ MeV. Using the values of $\Lambda_B = 1.8$ GeV and $\bar{\Lambda} = 600$ MeV one obtains a satisfactory fit to the decay $\Lambda_b^0 \rightarrow \Lambda^+_c e^- \bar{\nu}_e$ decay: the width $\Gamma(\Lambda_b^0 \rightarrow \Lambda^+_c e^- \bar{\nu}_e) = 5.4 \times 10^{10}$ s$^{-1}$ and the slope of the $\Lambda_b$ Isgur-Wise function $\rho^2 = 1.4$. Hence, in this paper the model parameters are set to $m_q = 420$ MeV, $m_s = 570$ MeV, $\Lambda_B = 1.8$ GeV, $\bar{\Lambda} = 600$ MeV, $\bar{\Lambda}_s = 750$ MeV, $\bar{\Lambda}_{ss} = 900$ MeV. Finally, the mass values of the charm baryon states including current experimental errors are taken from [1,2] (see TABLE I). The masses of the excited bottom baryons $\Lambda_{b1}$ and $\Lambda^*_{b1}$ are estimated from the heuristic relation: $m_{\Lambda_{b1}} = m_{\Lambda^*_{b1}} + (m_{\Lambda_b^0} - m_{\Lambda^+_c})$.

We now present our numerical results for the one-photon decay rates of heavy baryons. Our results are listed in TABLE II. The errors in our rate values reflect the experimental errors in the charm baryon masses [1,2] (see TABLE I). For the sake of comparison we also list the results of the model calculations [5]-[7] mentioned earlier on. Our results are quite close to the results of the nonrelativistic quark model [6]. In [5] the coupling strengths were parametrized in terms of unknown effective coupling parameters $c_{RT}$. A first rough estimate of the unknown coupling parameters can be obtained by setting them equal to 1 on dimensional grounds [5]. As is evident from TABLE II such an estimate is basically supported by our dynamical calculation. We do not agree with the predictions on the charm and bottom p-wave decay rates of [7] except for the $\Lambda^+_{b1} \rightarrow \Lambda_b^0 \gamma$ rate where we are closer to the rate calculated in [7].

Recently the radiative decays of bottom baryons were studied with the use of the light-cone QCD sum rules [15] in the leading order of heavy quark effective theory. For the decay rates of the $\Sigma_b$ and $\Sigma^*_b$ baryons to $\Lambda_b^0 \gamma$ the authors of [15] obtained

$$\Gamma(\Sigma_b \rightarrow \Lambda_b \gamma) = \alpha_{eff} |q|^3 \quad \text{and} \quad \Gamma(\Sigma^*_b \rightarrow \Lambda_b \gamma) = \alpha_{eff}^* |q|^3$$
where the couplings $\alpha_{\text{eff}}$ and $\alpha_{\text{eff}}^*$ are approximately equal to each other. The authors of [15] quote $\alpha_{\text{eff}} \approx \alpha_{\text{eff}}^* \approx 0.03 \text{ GeV}^{-2}$. In order to compare our model results with the results in [15] we set $M_{\Lambda_Q} = M_{\Sigma_Q}$ in Eq. (10). We then obtain $\alpha_{\text{eff}} = 4f^2/(3\pi) \approx 0.015 \text{ GeV}^{-2}$ which is one-half the prediction of Ref. [15].

In conclusion, we have investigated electromagnetic decays of heavy baryons. We have obtained predictions for the rates of the two-body transitions $B_i^Q(p) \rightarrow B_f^Q(p') + \gamma(q)$. We have compared our results with the results of other model calculations [5]-[7]. Unfortunately there is no data yet to compare our results with. For the one-photon decays from the $p$-wave states $\Lambda_{c1} \rightarrow \Lambda_c + \gamma$ and $\Lambda_{c1}^* \rightarrow \Lambda_c + \gamma$ our predicted rates are one order of magnitude below the upper limits given by the experiments calling for an one-order of magnitude improvement of the experimental upper limits. Although the $\Xi_c^+' \rightarrow \Xi_c + \gamma$ one-photon decays have now been seen [2] it will be close to impossible to obtain rate values for these decays because the $\Xi_c^+$-states are far too narrow. The total widths of the $\Sigma_c$, $\Sigma_c^*$ and $\Xi_c^*$ states are larger because they also decay via one-pion emission. In fact the widths of the $\Sigma_c^{++}$ and $\Sigma_c^{*0}$ have been determined [1]. One can hope that one-pion branching ratios can be experimentally determined for the $\Sigma_c$, $\Sigma_c^*$ and $\Xi_c^*$ one-photon decay modes in the near future. We are looking forward to compare the predictions of the Relativistic Three-Quark Model for the one-photon rates with future experimental data.

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FIG. I: Diagrams contributing to one-photon heavy baryon transition $B_Q^i \to B_Q^f \gamma$. 

(a) Triangle diagram

(b) Contact interaction-type diagrams
List of Tables

**TABLE I** Masses and spin and flavour quantum numbers of charm and bottom baryons. Column 4 gives the structure of the coupling of the quark constituents where $\gamma^5 = \tilde{\gamma} + \gamma \gamma$. The $\lambda_i$ in column 5 are the usual Gell-Mann matrices and $\lambda_u = \text{diag}\{1,0,0\}$, $\lambda_d = \text{diag}\{0,1,0\}$.

**TABLE II** Decay rates $\Gamma$ for heavy baryon states.

| Baryon | $J^P$ | Quark Content | $\Gamma_1 \otimes C \Gamma_2$ | $\lambda_{BQ}$ | Mass (MeV) [1] |
|--------|------|---------------|----------------|-------------|---------------|
| $\Lambda_c^+$ | $1^-_2$ | c[ud] | $I \otimes C \gamma^5$ | $i \lambda_2/2$ | 2284.9 ± 0.6 |
| $\Xi_c^+$ | $1^-_2$ | c[us] | $I \otimes C \gamma^5$ | $i \lambda_5/2$ | 2465.6 ± 1.4 |
| $\Xi_c^0$ | $1^-_2$ | c[ds] | $I \otimes C \gamma^5$ | $i \lambda_7/2$ | 2470.3 ± 1.8 |
| $\Xi_c^{*+}$ | $1^-_2$ | c{us} | $\gamma^5 \otimes C \gamma^\mu$ | $\lambda_4/(2\sqrt{3})$ | 2.5734 ± 3.1 |
| $\Xi_c^{0*}$ | $1^-_2$ | c{ds} | $\gamma^5 \otimes C \gamma^\mu$ | $\lambda_6/(2\sqrt{3})$ | 2.5773 ± 3.2 |
| $\Sigma_c^{++}$ | $1^-_2$ | c{uu} | $\gamma^5 \otimes C \gamma^\mu$ | $\lambda_u/\sqrt{6}$ | 2458.2 ± 0.6 |
| $\Sigma_c^+$ | $1^-_2$ | c{ud} | $\gamma^5 \otimes C \gamma^\mu$ | $\lambda_1/(2\sqrt{3})$ | 2453.6 ± 0.9 |
| $\Sigma_c^0$ | $1^-_2$ | c{dd} | $\gamma^5 \otimes C \gamma^\mu$ | $\lambda_d/\sqrt{6}$ | 2452.2 ± 0.6 |
| $\Xi_c^{++}$ | $3^+2$ | c{us} | $I \otimes C \gamma^\mu$ | $\lambda_4/2$ | 2644.6 ± 2.1 |
| $\Xi_c^{0*}$ | $3^+2$ | c{ds} | $I \otimes C \gamma^\mu$ | $\lambda_6/2$ | 2643.8 ± 1.8 |
| $\Sigma_c^{*++}$ | $3^+2$ | c{uu} | $I \otimes C \gamma^\mu$ | $\lambda_u/\sqrt{2}$ | 2519.4 ± 1.5 |
| $\Sigma_c^{*0}$ | $3^+2$ | c{dd} | $I \otimes C \gamma^\mu$ | $\lambda_d/\sqrt{2}$ | 2517.5 ± 1.4 |
| $\Lambda_{c1}$ | $1^-_2$ | c[ud] | $\gamma^5 \otimes C \gamma^5 \tilde{\gamma}^\mu$ | $i \lambda_2/(2\sqrt{3})$ | 2593.9 ± 0.8 |
| $\Lambda_{c1}^*$ | $3^+2$ | c[us] | $I \otimes C \gamma^5 \tilde{\gamma}^\mu$ | $i \lambda_5/2$ | 2626.6 ± 0.8 |
| $\Xi_{c1}$ | $3^-_2$ | c{us} | $I \otimes C \gamma^5 \tilde{\gamma}^\mu$ | $i \lambda_5/2$ | 2815.0 ± 2.1 |
| $\Lambda_b^0$ | $1^-_2$ | b[ud] | $I \otimes C \gamma^5$ | $i \lambda_2/2$ | 5624 ± 9 |
| $\Lambda_{b1}$ | $1^-_2$ | b[ud] | $\gamma^5 \otimes C \gamma^5 \tilde{\gamma}^\mu$ | $i \lambda_2/(2\sqrt{3})$ | 5933 ± 10 |
| $\Lambda_{b1}^*$ | $3^-_2$ | b[ud] | $I \otimes C \gamma^5 \tilde{\gamma}^\mu$ | $i \lambda_2/2$ | 5966 ± 10 |
| $B_Q \rightarrow B_Q' \gamma$ | This approach | Other approaches | Experiment [1] |
|-------------------------------|----------------|------------------|----------------|
| $\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma$ | $60.7 \pm 1.5$ KeV | 93 KeV [6] | |
| $\Sigma_c^{*+} \rightarrow \Lambda_c^+ \gamma$ | $151 \pm 4$ KeV | | |
| $\Xi_c^{++} \rightarrow \Xi_c^{+} \gamma$ | $12.7 \pm 1.5$ KeV | 16 KeV [6] | | |
| $\Xi_c^0 \rightarrow \Xi_c^{0} \gamma$ | $0.17 \pm 0.02$ KeV | 0.3 KeV [6] | | |
| $\Xi_c^{*+} \rightarrow \Xi_c^{+} \gamma$ | $54 \pm 3$ KeV | | |
| $\Xi_c^{*0} \rightarrow \Xi_c^{0} \gamma$ | $0.68 \pm 0.04$ KeV | | |
| $\Lambda_{c1}(2593) \rightarrow \Lambda_c^+ \gamma$ | $0.104 \pm 0.001$ MeV | $0.191c_{RT}^2$ MeV [5] | $< 2.36_{-0.86}^{+1.31}$ MeV |
| | | $0.016$ MeV [7] | |
| $\Lambda_{c1}^*(2625) \rightarrow \Lambda_c^+ \gamma$ | $0.137 \pm 0.002$ MeV | $0.253c_{RT}^2$ MeV [5] | $< 1$ MeV |
| | | $0.021$ MeV [7] | |
| $\Xi_{c1}^{*+}(2815) \rightarrow \Xi_c^{+} \gamma$ | $0.177 \pm 0.005$ MeV | | |
| $\Xi_{c1}^{*0}(2815) \rightarrow \Xi_c^{0} \gamma$ | $0.463 \pm 0.014$ MeV | | |
| $\Lambda_{b1} \rightarrow \Lambda_b^{0} \gamma$ | $0.126 \pm 0.022$ MeV | 0.09 MeV [7] | | |
| $\Lambda_{b1}^* \rightarrow \Lambda_b^{0} \gamma$ | $0.168 \pm 0.026$ MeV | 0.119 MeV [7] | |