Towards the MSSM from F-theory

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Abstract

We study the MSSM in F-theory. Its group is the commutant to a structure group SU(5) × U(1)Y of a gauge bundle in E8. The spectrum contains three generations of quarks and leptons plus vectorlike electroweak and colored Higgses. The minimal MSSM Yukawa couplings with matter parity is obtained at the renormalizable level.

1. Introduction

The purpose of this letter is to describe the Minimal Supersymmetric Standard Model (MSSM) in F-theory. String theory provides self-consistent ultraviolet completion of the gauge theory. For example, the constraint of anomaly freedom in the low energy theory originates from the finiteness of string one-loop amplitude.

F-theory is defined by identifying S-duality of IIB theory with the symmetry of a torus, lifting the gauge symmetry to geometry [1]. To have four dimensional theory with N = 1 supersymmetry, we compactify F-theory on Calabi–Yau fourfold that is elliptic fibered. Gauge bosons are localized on a complex surface S, along which the fiber is singular. The structure of the singularities has correspondence to that of the corresponding group, so that the symmetry breaking and enhancement are described by geometric transition [2]. Analogous to the bifundamental representations at D-brane intersections, matter fermions come from the ‘off-diagonal’ components under branching of the gaugino of E8 [3], localized along matter curves [4].

So far there have been constructions of Grand Unified Theories (GUTs) mainly based on the simple group SU(5), and further attention has been paid on the flavor sector [5, 6, 7]. Compared to field theoretic GUT, presumably its best merit is that we do not need adjoint Higgses for breaking down to SM. Instead, we can turn on a flux in the background in E8 [8]. If U(1)Y is constructed via geometry and there is no flux along this part, embeddability to SU(5) GUT singularity guarantees anomaly free spectrum and we do not worry about its breaking by Green–Schwarz mechanism.

Figure 1: The Standard Model group (filled) is obtained as the commutant to SU(5)×SU(2)×U(1)Y background in E8.

2. The Standard Model surface

A gauge group is described by a singular fiber sharing the same name, which is read off from Tate’s table [9]. We claim that the singularity describing the SM group is

\[ y^2 = x^3 + (b_5 + b_4a_1)xy + (b_3 + b_2a_1)(a_1b_5 + z)yz + (b_4 + b_3a_1)x^2z + (b_2 - b_0a_1^2)(a_1b_5 + z)xz^2 + b_0(a_1b_5 + z)^2z^3. \]

For the total space to be Calabi–Yau, the equation should satisfy topological conditions: z, a1, b0 are respectively sections of O(S), O(KB + S), O((n − 6)KB + (n − 5)S) in the base B of elliptic fibration, and KB is its canonical bundle. The surface S is located at z = 0, since the vanishing discriminant of (1)

\[ \Delta = (b_5 + a_1b_4)a_1^4b_3^2P_{\alpha}P_{\beta}z^3 + a_1b_5Qz^4 + O(z^5) \]

signals a singular fiber. We need globally defined sections in B, since the simple group components would be laid away from S.

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Eq. (1) is a deformation of $E_8$ singularity. Surveying the degrees of the coefficients in (1). Tate’s table shows it is generically an $SU(3)$ singularity. However the parameters are specially tuned, so the actual symmetry is larger. Vanishing of each coefficient factor triggers gauge symmetry enhancement, implying matter localization. We see shortly that the parameters $a_1 \equiv P_X$, $b_5 \equiv P_{g_5}$, $P_{g_{55}}$ and $P_{g_{55}}$ are respectively related to $X$-boson and subscribed quarks, which are all the charged fields under $SU(3)$. We can see, neither $Q$ nor $O(z^5)$ is proportional to $a_1$ or $b_5$, hence $P_{g_{55}}$ and $a_1 b_5 \to 0$ respectively enhance the symmetry to $SU(4)$ and $SU(5)$. To see it contains also $SU(2)$, we change the variable $z' = z + a_1 b_5$, in which (2) becomes

$$\Delta = ((b_5 - a_1 b_1)^2 - 4a_1^2 b_1 b_3) a_1^3 b_1^3 P_L z'^2 + a_1^2 b_1^2 P' z'^3 + a_1 b_1 Q' z'^4 + O(z^5).$$ (3)

Here $P_L$ is related to the lepton doublet. Since neither of $P', Q'$ nor $O(z^5)$ is proportional to $a_1$ or $b_5$, vanishing $P_{g_{55}}$ and $a_1 b_5$ respectively enhances the gauge symmetry respectively to $SU(3)$ and $SU(5)$. The position $z' = 0$ is off from the position of $S$, as a ‘back-reaction’ under the symmetry breaking from $SU(5)$, but it is still linearly equivalent to $S$. We can see, to make the embracing $SU(5)$ truly traceless, we need also a backreaction $z \to z - 2a_1 b_5 / 5$ to make the center of mass of the total brane lie on $S$. The factorization structure of $P_{g_{55}} P_{g_{55}}$ and $P_X P_{g_{55}} = a_1 b_5$ hints the existence of an additional $U(1)_Y$, otherwise we have only single factors for colored singlet and colored doublet, respectively. However to guarantee the existence of such $U(1)_Y$ we should check the global factorization structure of (1) [11]. Above all, the most general deformation is by $a_1 \to 0$, implying the embedding of our symmetry to a general $SU(5)$ GUT singularity in the literature [3]. Being its deformation, the factorization structure of $SU(3) \times SU(2) \times U(1)_Y$ is stably preserved against higher order perturbation in $O(z^6)$. Lying along the $E_8$ unification series $E_3 \times U(1)_Y \to E_4 \to E_8$, this SM group is the only possibility.

3. Matter contents

The broken symmetry is $SU(5)_L \times SU(5)_\nu$ ‘structure group’ whose commutant in $E_8$ is the Standard Model group [12]. The spectral cover geometrically describes it (or the dual to gauge bundle) satisfying supersymmetry conditions [13]. Roughly, it generalizes the notion of branes whose symmetry broken by recombination, so sometimes called by flavor brane stack, intersecting $S$ along the matter curves. It is described by factorized spectral cover $C = C_X \cup C_5$:

$$(a_0 s + a_1)(b_0 s^3 - a_0^{-1} a_1 b_0 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5) = 0,$$ (4)

where $s$ transforms as $-c_5 \equiv K_S$, the canonical bundle of $S$. We also define $-t$ as the normal bundle to $S$ in $B$. [14] is embedded in a compact threefold $\mathcal{B}$ whose projection to $S$ we denote $\pi$. The parameters are the ones used in [11] projected on $S$: Using adjunction formula $b_{mn}$ are sections of $(6 - m)c_1 \equiv \eta - mc_1$ and the combination $a_1 b_0 / a_0$ is a section of $5c_1 - t$, with which the full equation for $C$ has no $s^5$ term showing the embeddability to a $SU(6)_l$ structure group [3]. Using this embedding structure, one can be convinced that the parameters in [11] are the only possible combinations. We can recover the conventional $SU(5)$ GUT by $a_1 \to 0$ in [12], making its structure group traceless, reducing the above $C_5$ exactly to the standard one used in $SU(5)$ GUT. The fields charged under two groups satisfy the Green–Schwarz relations [21, 11], fixing $a_0 = 1$ to be trivial section.

We get the matter spectrum from the decomposition of the adjoint $248$ of $E_8$

$$(8, 1, 1) + (1, 3, 1) + (1, 1, 1) + (1, 1, 24) + X(3, 2, 1) - 5/6 + c_0 (3, 1, 10)_1 + d_0 (3, 2, 5)_l / 3 + v_5 (3, 1, 5)_2 / 3 + l_0 (2, 5, 10)_2 / 3 + e_5 (1, 1, 5)_2 / 3,$$

up to Hermitian conjugates. Matter fields are obtained as ‘off-diagonal’ components of the branching [3].

They are further identified by local gauge symmetry enhancement directions. For this, we parameterize their localizing curves using parameters $t_1, t_2, \ldots, t_8$ having one-to-one correspondence with the five weights of $SU(5)_L$ and $t_0$ with $U(1)_Y$. For example, the $X$-boson appears from a local symmetry enhancement to $SU(5)_L$, controlled by $t_0 \to 0$, agreeing with the above. However the physical parameters are only the coefficients in the spectral cover [11]: $b_n / b_0$ are the elementary symmetric polynomials of degree $k$, of $t_1, t_2, t_3, t_4, t_5, t_6, t_0$, and $a_1 \equiv P_X \sim t_0$ [5, 3]. $P_{g_{55}}$ is localized along the curve $b_5 / b_0 \sim t_1 t_2 t_3 t_4 t_5 = 0$ and $P_{g_{55}} \sim \prod_{i<j} (t_i + t_j)$. $P_{g_{55}} \sim \prod_{i<j} (t_i + t_j)$. The counting of $10$ and $\overline{10}$ agrees thanks to the relation

$$t_i + t_j + t_0 = - t_k - t_l - t_m, \quad \epsilon_{ijklm} \neq 0.$$ (5)

It is easy to see that the parameters $P_{m,n}, m = X, g, u, d, e, l$, here calculated from group theory, agree with those in [12] and [3]. Again $t_0 \to 0$ reduces the matter curves to those of $SU(5)$, so the $SU(5)$ GUT structure is preserved.

In the perturbative picture, parallel D-branes do not intersect. Here, the base of elliptic fibration has a similar structure generalizing Hirzebruch surface, where the zero section of the fiber has nonzero ‘self-intersection’. Thus $SU(3)$ and $SU(2)$ components intersect yielding chiral fermions $X$ and $g$. In the sense of regarding the intersecting branes as connected cycles [12], this is not so different.

1. The compact (non-Calabi–Yau) threefold, a projectivized fiber $\pi : \mathcal{O}_S \oplus K_S \to S$, with the trivial bundle $\mathcal{O}_S$ and the canonical bundle $K_S$.

2. Since the trace part of $SU(5)_L$ is the $U(1)_Y$ degree of freedom, it is sometimes called $SU(5) \times U(1)_Y$. 


By dimensional reduction, we obtain Yukawa coupling from covariant derivatives of gaugino of an enhanced group, as usual [3]. So the gauge invariance guarantees the presence of Yukawa couplings. They are

\[ lh_d d^c: (t_i + t_j + t_6) + (t_k + t_l + t_6) + (t_m - t_6) = 0, \]

\[ q h_u u^c: (t_i) + (-t_i - t_j - t_6) + (t_k + t_6) = 0, \]

\[ q h_d d^c: \begin{cases} (t_m) + (t_k + t_l + t_6) + (t_i + t_j) = 0, \\ (t_m) + (-t_m - t_i - t_j) + (t_i + t_j) = 0, \end{cases} \]

(6)

where all the indices run from 1 to 5, and all different.

Due to the relation [5], LHS’s of (6) vanish, and in each relation, we have two or more ways of writing the same coupling. For example in the last line, the first relation reduces the same relation to that of SU(5) GUT, in the limit \( t_6 \to 0 \). The second relates the matter curves for all \( q, h_d, d^c, h_u \), and \( u^c \). We may track this from the left-right symmetric models and their extension (Pati–Salam and trinification) which relates up and down sectors by SU(2)R. We can switch between up-type and down-type fermions, like doing between Georgi–Glashow SU(5) and flipped SU(5).

4. Matter curves and monodromy

For a general spectral cover, we have monodromy condition for connecting different matter curves [10]. In our case, the SU(5) has \( S_3 \) monodromy as reflected in the symmetric polynomial relations of the coefficients: The connected ones form an orbit and are treated as the identical surface. It leads to just one kind of lepton doublet, without distinguishing \( l, h_d \) or \( h_u^c \).

To remedy this, we further mod out the others by \( Z_4 \) monodromy generated by the cyclic permutation of \( (t_1, t_2, t_3, t_4) \). That is, for instance, \( q \) is distinguished by different orbits \( \{ t_1, t_2, t_3, t_4 \} \) and \( \{ t_5 \} \), whereas \( X \) is still \( t_6 \). There are distinct candidates for the matters. For example lepton or Higgs belongs to one of the following \( Z_4 \) orbit

\[ l, h_d, h_u^c: \{ t_1 + t_4 + t_6 \}, \{ t_1 + t_3 + t_4 + t_6 \}, \{ t_4 + t_5 + t_6 \}, \]

with \( i = 1, \ldots, 4 \). To have lepton Yukawa coupling [6], \( h_d \) and \( l \) must not share \( t_5 \), therefore essentially we have two allowed cases for choosing SU(2) doublets. We choose the fields as in Table 1. We named the colored exotics as colored Higgses \( h_{c1} \) and \( h_{c2} \). Note also that, by SU(5) unification structure the fields belonging to a single multiplet has holomorphic curves.

We identify two Abelian symmetries generated by

\[ U(1)_Y: \text{diag}(1, 1, 1, 1, 1, -5), \]

\[ U(1)_M: \text{diag}(1, 1, 1, 1, -4, 0), \]

in the basis \( \{ t_1, t_2, t_3, t_4, t_5, t_6 \} \). The first is \( U(1)_Y \) and the second is the famous ‘B – L’ symmetry, commutant to SU(5) inside SO(10), providing continuous version of matter parity. Since both \( h_u \) and \( h_d \) have even \( U(1)_M \) charges, we can forbid lepton or baryon number violating operators

\[ lh_u, ll e^c, l q d^c, u^c d^c d^c. \]

Table 1: Matter curves with \( Z_4 \) monodromy. All the indices take value in \( Z_4 = \{ 1, 2, 3, 4 \} \) and are different. The primed ones, charged exotics, have always odd \( M + 2s \) charges.

It turns out that neither \( X \) nor \( q' \) exists in four dimensions, for later choice of the flux (see [13] below), so there will be no problem.

The resulting spectral cover is further factorized \( C_5 \to C_{q'} \cup C_q \), thus [11]

\[ (s + a_1)(d_0 s + d_4)(e_0 s^4 + e_1 s^3 + (e_2 + e_7) s^2 + e_3 s + e_4) = 0 \]

(10)

with ‘traceless condition’ \( a_0 d_0 e_0 + d_1 e_0 + d_0 e_1 = 0 \). The new covers are for \( t_5 \sim d_1 / d_0 \), and \( e_1 \) will be again related to elementary symmetry polynomials of degree \( i \) out of \( t_1, t_2, t_3, t_4 \). Only \( e_2 / e_0 \sim (t_1 + t_4) (t_2 + t_4) \) and \( e_3 / e_0 \sim t_1 t_3 + t_2 t_4 \) are nontrivially \( Z_4 \) closed. We have additional degree of freedom to choose \( d_0 \), a section \( x \in H_4(S, Z) \). It follows \( e_k \) are sections \( e_k \sim \eta - k c_1 - x \).

As discussed around [6], there are fields related by non-vanishing Yukawa couplings. They originate from the same spectral cover. Since the defining conditions \( t_i = 0 \) are stronger than \( t_i + t_j = 0 \), we can derive all associated matter curves from the ‘fundamental’ spectral covers \( t_i = 0 \) for \( C_X, C_{q'}, C_q \) [3, 17]. For example the matter curve for \( d^c \) is the common intersection between \( C_q(t_i) \) and \( C_{q'}(-t_i) \), extracting the redundant component.

From the symmetry enhancement directions, as shown in Table 1 we can find the following classes of the matter
curves:

\[ X : C_X \cap \sigma = -c_1 \cap \sigma \]

\[ q, u^c, e^c : C_q \cap \sigma = C_q \cap C_X = (\eta - 4c_1 - x) \cap \sigma \]

\[ q', u', e' : C_{q'} \cap \sigma = C_{q'} \cap C_X = (-c_1 + x) \cap \sigma \]

\[ d^c, l : (C_q - (\sigma + c_1)) \cap C_{q'} \]

\[ = \sigma \cap (\eta - 4c_1 + 2x) + (\eta - c_1 - x) \cap x \]

\[ h_{e_1}, h_{d_1} : C_q \cap (2\sigma + c_1) \]

\[ = 2\sigma \cap (\eta - 2c_1 - x) + (\eta - x) \cap c_1 \]

\[ h_{e_2}', h_{d_2}' : (C_q - 2\sigma) \cap (C_q - 4(\sigma + c_1)) \]

\[ = 2\sigma \cap (\eta - 4c_1 - x) + (\eta - x) \cap (\eta - 4c_1 - x) \]

where we omitted pullback. The associated matter curves have \( \sigma \)-independent components that lie outside \( S \) but on the spectral cover. Since \( \delta_0 \) is trivial section, thus \( C_X \sim \sigma \) and the matter curve with and without \( t_0 \) are homologous, for \( q, u^c, e^c \) for example. Again this exhibits the unification relations to \( SU(5) \) GUT. More detailed calculation is to be found elsewhere \[18\].

5. Flux and spectrum

To obtain four dimensional chiral spectrum, we should turn on a magnetic flux \( \gamma \) on the spectral cover \( \gamma \epsilon H^2(C, \mathbb{Z}) \) \[13\] \[14\]. With above factorization we turn on the universal flux only on \( C_q \) \[13\].

\[ \gamma_q = C_q \cap (4\sigma - \pi^*(\eta - 4c_1)), \quad \gamma_{q'} = \gamma_X = 0. \]  \( \text{(12)} \)

We can show it has integral cohomology and traceless \( p_{C, \gamma} = 0 \) inside \( SU(5)_L \times U(1)_Y \), where \( p_{C} : C \rightarrow S \). It follows that \( q', X \) and the associated matters are all zero with no multiplicity

\[ n_X = n_{q'} = n_{u^c} = n_{e^c} = 0. \]  \( \text{(13)} \)

We calculate the net number of fields, or the differences \( n_I \) between the numbers of fermions \( f \) and antifermions \( f^c \) by Riemann–Roch–Hirzebruch index theorem. It assumes the form for associated matter curves, namely the product of \( 4\sigma - p_\eta^*(\eta - 4c_1) \) with the curves in \( \text{(11)} \), letting \( p_\eta \) the projection \( C_q \rightarrow S \) \[2d\] \[10\]. For example, we obtain

\[ n_q = \sigma \cap (\eta - 4c_1 - x) \cap (4\sigma - p_\eta^*(\eta - 4c_1)) \mid_S \]

\[ = -\eta \cdot (\eta - 4c_1 - x) \]

\[ n_{q'} = (\sigma \cap (\eta - 4c_1 + 2x) + (\eta - c_1 - x) \cap x) \cap (4\sigma - p_\eta^*(\eta - 4c_1)) \mid_S \]

\[ = -\eta \cdot (\eta - 4c_1 + 2x) + 4(\eta - c_1 - x) \cdot x, \]

where the dot product \( \cdot \) means the intersection between the divisors on \( S \) and we used the Poincaré dual flux. In fact, Kodaira vanishing theorem states that either chiral or antichiral fermion has exclusively nonzero in most cases \[3\]. The anomaly cancellation condition for \( SU(3) \), \( 2n_q - n_{d_1} - n_{e^c} = 0 \), requires also \( x = 0 \) or \( 4x = 5\eta - c_1 \). We choose \( x = 0 \) to have the spectrum

\[ n_q = n_{d_1} = n_{u^c} = n_{c_1} = -\eta \cdot (\eta - 4c_1), \]

\[ n_{h_4} = n_{h_{c_1}} = n_{h_{c_2}} = n_{h_{c_3}} = -2\eta \cdot (\eta - 4c_1). \]  \( \text{(14)} \)

Given the base \( B \) of elliptic fibration, \( c_1, -t \) are determined purely by the property of \( S \). Choosing \( S \) such that \(-\eta \cdot (\eta - 4c_1) = 3 \), we get three generations of SM fermions, plus six pairs of doublet Higgses and six pairs of colored Higgses. Since we make use of the section \(-c_1 \) for \( U(1)_Y \) we need \( h^0(S, K_S) > 0 \). However it might imply the SM adjoint Higgses unless \( S \) is torsion-free.

We obtained the SM gauge group and three generations of quarks and leptons \textit{without} resorting to an intermediate unification. For the vectorlike electroweak and colored Higgses, we have the standard doublet-triplet splitting problem, also tightly related to the \( \mu \)-problem. We can further elaborate the model employing different factorization and/or flux. Already this model has desirable symmetries to shed light on a dynamical resolution. Because of gauge invariance \( \text{(6)} \) and the vanishing theorem, bare mass terms of F-theory scale, close to the Planck scale, are forbidden. However below some intermediate scale \( M_f \) where the global symmetries and \( U(1)_M \) are broken, mass terms would be \textit{dynamically} generated

\[ W = W_{\text{MSSM}}(\mu = 0) + m_c h_{c_1} h_{c_2} + m_h h_{h_4}. \]  \( \text{(15)} \)

We expect some ‘standard solution’ would generate the mass matrix \( m_c \) and \( m_h \) at the scale \( M_f \). We again emphasize the corresponding fields are distinguished by \( U(1)_M \) charges in Table \( \text{[I]} \). We expect it would be broken down to \( Z_2 \) symmetry, becoming \textit{matter parity} \[22\]. Then still the terms \( \text{[S]} \) are forbidden in the low energy. Also this structure can be related to dynamical supersymmetry breaking \[23\].

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