The anomalous frequency and doping dependence of the Hall constant in the normal state of high-$T_c$ superconductors are investigated within models of strongly correlated electron systems. In Mori theory, the transition of the Hall constant from infinite to zero frequency is described by a memory function. It naturally introduces a second time scale, that, within the $t - J$ model, is identified with the spinon relaxation time of Anderson. This provides us with a phenomenological understanding of the interplay between the frequency and temperature dependence of the Hall constant for frequencies below the Mott-Hubbard gap. Within the single-band Hubbard model in the limit $U \gg t$ ($U$: Coulomb repulsion, $t$: hopping amplitude), the memory function is calculated via its moments and shown to project out the high-energy scale $U$, introduced by doubly occupied sites. This causes the Hall constant to decrease by a factor $(1+\delta)/2$ ($\delta$: doping), when the frequency is lowered from infinity to values within the Mott-Hubbard gap. Finally, it is outlined, how the Hall constant may be calculated in the low frequency regime.

I. INTRODUCTION

Recently, we have proposed a memory function treatment of the Hall effect in strongly correlated electron systems [1]. Its main advantage consists in the fact that we are directly provided with a representation of the Hall resistivity. Therefore, in contrast to ordinary approaches, we may dodge the issue of coping with a quotient of conductivities, which may be precarious due to resonances like the Drude peak.

In our approach, the frequency dependent Hall constant is given as the sum of its infinite frequency limit and a memory function contribution. A perturbative calculation of both terms within the single-band Hubbard model has demonstrated the usefulness of this approach [2]. Our results turned out to be in qualitative agreement with the unusual experimental findings for high-$T_c$ superconductors as, e.g., La$_{2-x}$Sr$_x$CuO$_4$, except for the doping dependence of the Hall constant in the vicinity of half filling, i.e. below $\delta \sim 0.1$. There, a $1/\delta$ law is observed which requires a description in the strong-correlation regime [3]. This parameter regime will be the subject of the present paper, which is organized as follows.

In Sec. II, the main results concerning the representation of the Hall constant within Mori theory are compiled from Ref. [2]. Then, in Sec. III, we study the moments of the memory function in the limit $U \to \infty$. Thereby, the memory function is found to eliminate the high-energy scale set by the Hubbard repulsion $U$. This provides us with an explanation of the frequency dependence of the Hall constant in the cross-over regime from $\omega \gg U$ to $W \ll \omega \ll U$, where $W$ is the bare band width. Furthermore, a simple analytical treatment of the Hall effect within the Hubbard I approximation is presented, the results of which turn out to be in full accord with that obtained by high-temperature expansions. In Sec. IV, we reformulate the frequency dependent Hall constant within the $t - J$ model in order to address the low frequency regime as well. The comparison to a representation of $R_H(\omega)$ in terms of two relaxation rates and effective masses introduced by Anderson [3] provides us with an interpretation of the additional relaxation rate $\tau_H$ in the language of the Mori theory and a phenomenological understanding of the anomalous frequency and temperature dependence of the Hall constant in high-$T_c$ superconductors. Finally, we explain how the emerging picture may be put onto the basis of a microscopic calculation. In this context, we reduce the problem of calculating the memory function in the $t - J$ model to the much easier one of finding the first few moments of the ordinary current-current correlation functions. We conclude this paper in Sec. V with a discussion of our main results.

II. MEMORY FUNCTION APPROACH TO THE HALL CONSTANT

For a more detailed introduction into the following memory function treatment of the Hall effect in strongly correlated electron systems, the reader is referred to Ref. [2]. We choose the simplest model of strongly correlated electrons, namely the single-band Hubbard model on a two-dimensional square lattice:

$$
\hat{H} = -t \sum_{<ij,\sigma>} P_{ij} c_{i\sigma}^+ c_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} . \tag{1}
$$

The second term is a local repulsive interaction. The sum in the hopping term is restricted to nearest neighbors and the Peierls phase factor $P_{ij} \approx \exp\{ie\vec{A}(\vec{R}_j)(\vec{R}_i - \vec{R}_j)\}$ guarantees the gauge invariance [4] (the sign of the charge $e$ is chosen to be negative). The vector potential decomposes into two terms describing the electric and mag-
nentic field, respectively: \( \vec{A}(t, \vec{r}) = \vec{A}^{el}(t) + \vec{A}^{mag}(\vec{r}) \), \( \vec{E}(t) = -\frac{\partial}{\partial t} \vec{A}^{el}(t) \), and \( \vec{H} = \text{rot} \, \vec{A}^{mag}(\vec{r}) \). In linear response theory, the current operators are defined as:

\[
\hat{J}_\nu := -\frac{1}{e} \frac{\delta \hat{H}(t)}{\delta A_\nu^\mu(t)} \bigg|_{A_\nu^\mu = 0}.
\]

(2)

A suitable representation of the Hall constant is obtained within Mori theory. This formalism enables us to separate the dynamics of the current operators \( \hat{J}_x \) and \( \hat{J}_y \) from that of all the other degrees of freedom, the influence of which is then accumulated in memory functions. In the center of this separation stand the superprojectors

\[
P = \left( \frac{\beta}{\chi^0} \right) \sum_{\nu=x,y} |\hat{J}_\nu\rangle \langle \hat{J}_\nu|,
\]

(3)

\[
Q = 1 - P,
\]

(4)

acting in operator space \( \mathcal{L} \) and being defined with respect to Mori’s scalar product

\[
(\hat{A}|\hat{B}) := (1/\beta) \int_0^\beta d\tau < \exp\{\tau L\} \hat{A}^+ \cdot \hat{B} > .
\]

(5)

Here, \( \beta \) is the inverse temperature, the Liouville operator \( L \) maps a given operator onto its commutator with the Hamiltonian: \( L\hat{A} = [\hat{H}, \hat{A}] \), and \( \chi^0 = \beta(\hat{J}_x|\hat{J}_y) \) is the static current susceptibility. This scalar product implies the so-called Kubo identity

\[
\beta(\hat{A}|L|\hat{B}) = < [\hat{A}^+, \hat{B}] > ,
\]

(6)

which will play an important role in the following sections. By representing the current-current correlation function within the Kubo formula for the conductivity tensor in terms of a memory matrix, we obtain the following representation of the Hall constant:

\[
R_H(z) = \frac{N}{e^2 \chi^0} \lim_{\omega \to 0} \frac{\Omega + iM(z)}{\omega_H} .
\]

(7)

The first term is given by the cyclotron frequency \( \Omega = (1/\chi^0) < [\hat{J}_x, \hat{J}_y] > \) while the second is a memory function contribution:

\[
M(z) = \frac{\beta}{\chi^0} (QL\hat{J}_x i z + QLQ\hat{J}_y) .
\]

(8)

Here, \( z \) is a complex frequency that ultimately has to be continued analytically to \( z = \omega + i0^+ \). This function has the structure of a relaxation function for the so-called residual forces

\[
\hat{f}_\nu = iQL\hat{J}_\nu .
\]

(9)

whose dynamics is governed by the projected Liouville operator \( QLQ \) rather than \( L \). Thus, these forces may vary on a time scale that is different from that of the current operators \( \hat{J}_\nu \). In Sec. IV, we shall identify these two time scales with the relaxation rates of the holon and spinon degrees of freedom in Anderson’s tomographic Luttinger liquid theory. The memory function can be represented as a spectral integral

\[
M(z) = \int \frac{d\omega}{\pi} \frac{M''(\omega)}{\omega - z} ,
\]

(10)

where the spectral function \( M''(\omega) \) is given by the discontinuity across the real axis:

\[
M(\omega \pm i0^+) = M'(\omega) \pm iM''(\omega) ,
\]

(11)

and can be shown to be real and even. Since the memory function vanishes as \( 1/\omega^2 \) at high frequencies, the first term on the Rhs. of Eq. (10) represents the Hall constant in the limit \( \omega \to \infty \), that was considered by Shastry et al. It will be denoted as \( R_H^\infty \) from now on.

III. HALL CONSTANT IN THE CROSS-OVER REGIME FROM \( \omega \gg U \) TO \( \omega \ll U \)

Numerical studies of multi-band Hubbard models for the Cu-O planes of high-\( T_c \) materials indicate that all low-energy excitations are reproducible within a single-band Hubbard model with \( U \sim W \). Unfortunately, this parameter regime is not accessible to reliable analytical calculations. Therefore, we shall exaggerate the impact of \( U \) by considering the range \( U \gg W \) instead.

A. The moments of the memory function in the limit \( U \to \infty \)

The main problem in dealing with the memory function is related to the fact that its dynamics is governed by the projected Liouville operator \( QLQ \) rather than \( L \). We resolve this difficulty by inquiring into the properties of the memory function via its moments. Then, we may get rid of the superprojectors \( Q \) simply by resorting to their definition through Eqs. (3) and (4).

We start by writing the Hall constant in terms of spectral functions:

\[
R_H(z) = \frac{N}{e^2 (\chi^0)^2 H} \int_{-\infty}^\infty d\omega \left\{ S_H(\omega) + K(\omega) \frac{\omega}{\omega - z} \right\} .
\]

(12)

The first spectral function in the integrand is defined as

\[
S_H(\omega) = -i < [\hat{J}_x, \delta(\omega + L)\hat{J}_y] > ,
\]

(13)

and corresponds to the Hall matrix element of the current-current correlation function. The second function is related to the spectral function \( M''(\omega) \), introduced in Eq. (11), by \( K(\omega) = \chi^0 M''(\omega)/(\pi \omega) \), which implies:

\[
K(\omega) = \frac{-i\beta}{\omega} (QL\hat{J}_x i \delta(\omega + QLQ)(QL\hat{J}_y) .
\]

(14)
This function describes the correlation between the residual forces \( f_L \) and \( f_R \). In the following, we shall unravel a simple connection between the moments of the two functions \( S_H(\omega) \) and \( K(\omega) \) in the limit \( U \to \infty \).

First, we note that both functions may be shown to be even and real \( [16] \). Furthermore, we expect both functions to vanish beyond a certain frequency above \( U \), since \( U \) is the highest energy scale in the problem. This assumption will be corroborated below up to mistakes of the order \( t/U \). Hence, for finite but large \( U \), all moments exist and it is sufficient, to consider only the even ones:

\[
S_l = \int_{-\infty}^{\infty} d\omega \, S_H(\omega)\omega^{2l},
\]

\[
K_l = \int_{-\infty}^{\infty} d\omega \, K(\omega)\omega^{2l},
\]

where \( l \geq 0 \). Since we are mainly interested in the dc-Hall constant, we would like to calculate \( K_0 \). Unfortunately, this is not feasible on the basis of Eq. \([14]\), since the inverse of the projected Liouville operator \( QLQ \) does not exist. However, all the other moments \([10]\) can be calculated: Due to Eq. \([4]\), they are given as:

\[
K_l = i\beta(\hat{J}_x) LQL \ldots LQL \hat{J}_y \quad (l \geq 1).
\]

Now, if we insert \( Q = 1 - P \), this expression decomposes into \( 4^l \) terms. Consider a special one consisting of \( p \) superprojectors \( P \). By using the definition of \( P \), this term is seen to decompose further into \( f + 1 \) factors of the form \( \beta(\hat{J}L^{m_j}\hat{J}) \), the orders of which are \( U^{m_j-1} \) due to the Kubo identity \([3]\). Here, \( \sum_{j=1}^{f} m_j = 2l + 1 \). Thus, each factor lowers the relative order in \( U \) by one. Hence, the more superprojectors \( P \) a given term is composed of, the lower its relative order in \( U \) is. Therefore, all superprojectors \( Q \) may be removed from the Rhs. of Eq. \([7]\) to leading order in \( t/U \). And this, in turn, establishes the following relation between the \( l \geq 1 \) moments of Eqs. \([3]\) and \([14]\):

\[
\frac{K_l}{U^{2l}} = - \frac{S_l}{U^{2l}} + o\left(\frac{t}{U}\right).
\]

This equation does not imply that the functions \([3]\) and \([14]\) differ only by a sign in the limit \( U \to \infty \). This conclusion would require positive or negative definite functions and finite moments even in the limit \( U \to \infty \). None of both conditions is satisfied. To proceed anyway, we remind ourselves that in the context of the Hubbard model in the strong correlation limit, any spectral function is believed to separate into individual “peaks” centered around integer multiples of \( U \) \( [10, 11, 12] \) (in this context, any connected structure of a given spectral function, irrespective of its detailed shape, is referred to as a “peak”; for instance, it may vanish at discrete points). This reflects the fact that one-particle excitations may be grouped into two Hubbard bands separated by the so-called charge transfer gap, which is of the order \( U \). Since the current operators produce particle-hole excitations, we expect the functions \([3]\) and \([14]\) to have peak structures centered around \( \omega = 0 \) and \( \pm U \) related to excitations within the two Hubbard bands and across the charge transfer gap, respectively. Therefore, these peaks are expected to have widths of the order of those of the Hubbard bands. In the following, we shall prove this picture at least for the function \([3]\) and derive formulas required to extract more information from the relations \([3]\).

**B. Structure of the functions \( S_H(\omega) \) and \( K(\omega) \)**

The appropriate technique to investigate spectral properties of the Hubbard model in the strong correlation limit was pioneered by Harris and Lange in the special case of single-particle excitations \( [8] \) and generalized by several other authors, see for instance Refs. \([11, 10]\). At the heart of this procedure stands the decomposition of a given operator into terms, which increase the number of doubly occupied sites by integer values \( p \):

\[
\hat{O} = \sum_{p=-\infty}^{\infty} \hat{O}_{pu}.
\]

Together with the Lehmann representation of a given spectral function, one may then address the properties of its individual peaks. The decomposition \([19]\) is accomplished by an iterative procedure based on a canonical transformation of the Hubbard Hamiltonian: \( H \to \exp\{iS\} H \exp\{-iS\} \). The expansion of the operator \( \hat{S} \) up to the order \( l \) in \( t/U \) eliminates those processes from \( \hat{H} \) which change the total number of doubly occupied sites up to the order \( t^l/U^{l-1} \). The corresponding transformed Hamiltonian \( \hat{H}^{(l+1)} \) in turn helps to fix the next order of \( \hat{S} \) in \( t/U \) and so on. Thus, subsequent iterations generate increasing orders in \( t/U \). Once the generator \( \hat{S} \) has been found to a given order, one may decompose any operator to the same order by first decomposing its rotated counterpart \([10]\).

In our case, \( t/U \)-expansions may be terminated after the zeroth order term since the relation \([18]\) indicates that it is not sensible to go beyond. Then, we do not have to distinguish between original and transformed Fermi operators and the decomposition \([3]\) specialized to the case of the operators \( \hat{D}_{ij}^\sigma \equiv c_{i\sigma}^\dagger c_{j\sigma}^\dagger \), making up the components of the current operator, becomes:

\[
\hat{D}_{ij}^\sigma = \hat{D}_{ij; -U}^\sigma + \hat{D}_{ij; 0}^\sigma + \hat{D}_{ij; U}^\sigma.
\]

In terms of Hubbard operators \( X_{i\sigma}^{0\sigma} \equiv c_{i\sigma}^\dagger (1 - \hat{n}_{i\sigma}) \), \( X_{i\sigma}^{p} \equiv (1 - \hat{n}_{i\sigma}) c_{i\sigma}^\dagger \), and \( X_{i\sigma}^{\delta} \equiv \hat{n}_{i\sigma} c_{i\sigma}^\dagger \), where \( \delta \equiv -\sigma \), the terms of Eq. \([20]\) may be written conveniently as:
\[
\hat{D}_{ij;0} = X_i^{0\sigma} X_j^{0\sigma} + X_i^{2\sigma} X_j^{2\sigma},
\]
\[
\hat{D}_{ij;-U} = X_i^{0\sigma} X_j^{2\sigma},
\]
\[
\hat{D}_{ij;U} = X_i^{2\sigma} X_j^{0\sigma}.
\]

The Lehmann representation of the function (13) is derived straightforwardly:
\[
S_H(\omega) = \frac{1}{Z} \sum_{n,m} M_{nm} (e^{-\beta\epsilon_n} - e^{-\beta\epsilon_m}) \times \delta(\omega - [\epsilon_n - \epsilon_m]),
\]
\[
M_{nm} = \frac{1}{2t} \left\{ < n|\hat{J}_x|m > < m|\hat{J}_y|n > - < n|\hat{J}_y|m > < m|\hat{J}_x|n > \right\},
\]
\[
\text{where states and energies are defined through the eigenvalue equation } (H - \mu N) |n > = \epsilon_n |n >.
\]

Inserting the decomposition of the current operators corresponding to (21), we find that the peak centered around \( \omega = pU \) has a weight given by
\[
W_{pU} = \frac{1}{2t} \left\{ < [\hat{J}_{x;pU}, \hat{J}_{y;-pU}] > + < [\hat{J}_{x;-pU}, \hat{J}_{y;pU}] > \right\}
\]
\[
\text{and that only the peaks } p = 0 \text{ and } p = \pm 1 \text{ survive in leading order in } t/U.
\]

We assume that the function (13) has qualitatively the same triple-peak structure. Although not proven, this assumption is shown to lead to reasonable conclusions.

C. Frequency dependence of the Hall constant in the range \( \omega \gg W \)

Given the peak structure of the functions (13) and (21), only the contributions of the satellite peaks around \( \omega = \pm U \) can be resolved in the \( l \geq 1 \) moments (13) and (14). Thus, the relation (13) implies that the “spectral weights” of the peaks of \( K(\omega) \) and \( S_H(\omega) \) around \( \omega = U \) differ only by a sign. Together with Eq. (12), we may then draw the following conclusions: For \( \omega \gg U \), all peaks of \( S_H(\omega) \) contribute to the Hall constant and none of \( K(\omega) \). In the frequency range \( W \ll \omega \ll U \), the high-frequency peaks cancel each other out while the contribution of the zero-frequency peak of \( K(\omega) \) is negligible. Within this charge transfer gap region, the frequency dependent Hall constant is then lowered by a factor \( p \) in comparison to its infinite frequency limit,
\[
R_H^* = R_H(W \ll \omega \ll U) = p R_H^\infty,
\]
\[
\text{if we define } p \text{ to be the relative spectral weight of the low energy structure of } S_H(\omega):
\]
\[
p = \frac{W_0}{W_0 + W_0 + W_U} = \frac{< [\hat{J}_{x;0}, \hat{J}_{y;0}] >}{< [\hat{J}_x, \hat{J}_y] >},
\]
\[
\text{Eq. (27)} \text{ is valid except for mistakes of the order } t/U. \text{ Therefore, it is sufficient to evaluate it in the limit } U \to \infty.
\]

First, we seek for an interpretation of \( R_H^\infty \). We begin by noting that the infinite frequency Hall constant, i.e., the first term of Eq. (6), may be rewritten as follows:
\[
R_H^\infty = \frac{1}{e} \frac{2a_d}{(2a_n)^2},
\]
\[
\text{where } a_n \text{ and } a_d \text{ is the amplitude of a nearest neighbor hop and a hop diagonally across the unit cell, respectively.}
\]

This holds on account of the following relations, which may be proven by straightforward analysis:
\[
< [\hat{J}_x, \hat{J}_y] > = 8iN e t^2 H a_d,
\]
\[
\chi^0 = 4tN a_n.
\]

In the limit \( U \to \infty \), we have
\[
\lim_{U \to \infty} a_n = < X_i^{0\sigma} X_j^{0\sigma} >_R = \frac{\xi N e t^2 H a_d}{\beta \omega},
\]
\[
\lim_{U \to \infty} a_d = < X_i^{0\sigma} X_j^{0\sigma} >_R = \frac{\xi N e t^2 H a_d}{\beta \omega},
\]
\[
\text{where } \hat{x} \text{ and } \hat{y} \text{ is a primitive lattice vector in the } x \text{- and } y \text{-direction, respectively.}
\]

Here and in the following, expectation values are taken with respect to states without double occupancies. Then, the projected current operators appearing in the nominator of the Rhs. of Eq. (28) take on the following form:
\[
\hat{J}_{ij;0} = it \sum_{<ij>,\sigma} \Delta_{ij}^\sigma P_{ij} X_i^{0\sigma} X_j^{0\sigma},
\]
\[
\text{since the term } X_i^{2\sigma} X_j^{2\sigma} \text{ of Eq. (21)} \text{ may be omitted. Here,}
\]
\[
\Delta_{ij} \equiv \tilde{R}_i - \tilde{R}_j \text{ and } P_{ij} \text{ is the phase factor defined in the text following Eq. (1). From Eqs. (28)-(34), we conclude that}
\]
\[
R_H^\infty \text{ defined in Eq. (27), represents the infinite frequency Hall constant of the } U = \infty \text{-Hubbard model, which is defined as follows:}
\]
\[
\hat{H} = -t \sum_{<ij>,\sigma} P_{ij} X_i^{0\sigma} X_j^{0\sigma}.
\]

In fact, the analysis of Sec. II is straightforwardly carried over to this model, the current operator of which is then found to be given by Eq. (34). Note, that \( p \neq 1 \) expresses the fact that the limits \( \omega \to \infty \) and \( U \to \infty \) do not commute: If we start with the limit \( \omega \to \infty \), the integral over \( S_H(\omega) \) in Eq. (12) extends over all three peaks while when taking the limits in reversed sequence, the high-energy peaks are unattainable from the outset.

Next, we derive an exact analytical expression for \( p \) by taking the additional limit \( T \to \infty \). We may wonder whether this is reasonable. However, since the limit \( U \to \infty \) was already carried out, at least, the condition \( T \ll U \) is satisfied, i.e., the thermal energy cannot excite an electron across the charge transfer gap. Furthermore, we expect neither of the high-frequency objects \( R_H^\infty \) and
\( R_H^\infty \) to depend appreciably on temperature, since they correspond to and generalize the semi-classical expression for the Hall constant \([1]\). In the context of high-temperature expansions, one has to cope with electrons or holes hopping around closed loops, which are defined by a sequence of adjacent lattice sites. Therefore, it is convenient not to expand the phase factors. Then, an electron hopping along a polygon \( ijk\ldots il \) accumulates a phase proportional to the flux \( \phi_{ijk\ldots il} \) enclosed. The procedure to expand expectation values of Hubbard operators like that of Eqs. (32) and (33) or the nominator procedure to expand expectation values of Hubbard operators for the Hall constant \([1]\) in powers of \( \frac{1}{T} \) is explained, e.g., in Ref. [11]. To leading order, we obtain:

\[
\rho = \frac{1 + \delta}{2},
\]

(36)

where here and in the following, the electron density is measured via the average number \( \delta \) of holes per lattice site introduced into the half filled system, i.e. \( \delta = 0 \) corresponds to half filling while \( \delta = 1 \) corresponds to the empty band. Although the high-temperature calculation has introduced another high-energy scale into our system, we expect this result to hold qualitatively for low temperatures as well. For instance, since \( \rho \) is a measure for the difference between the plateau values of \( R_H(\omega) \) on both sides of \( U \), it is expected to increase monotonically as half filling is approached. Further down, we shall derive the same expression for \( \rho \) within a simple approximation valid at \( T = 0 \).

Finally, we calculate \( R_H^\infty \) to leading order in \( \frac{1}{T} \). The leading order of the amplitudes (32) and (33) are found to be \( a_n = (3\beta t/2)\delta(1-\delta) \) and \( a_d = -(3^2\beta t^2/4)\delta(1-\delta)(1-3\delta) \), which results in

\[
R_H^\infty = \frac{1}{|e|} \left( \frac{1 - 3\delta}{2} \right) \left( \frac{1}{\delta + 1 - \delta} \right).
\]

(37)

Together with Eqs. (27) and (30), we recover the result for \( R_H^\infty \) of Ref. [1], that was derived within the \( t - J \) model in leading order in \( \frac{1}{T} \). In this work, it was further shown that, although \( R_H^\infty \) is renormalized as a function of \( T \) and \( J \) when including higher orders in \( \frac{1}{T} \), the doping dependence of the Hall constant retains its most important features: its sign change at \( \delta \approx 1/3 \) and its singular behavior in the vicinity of half filling. What seems to be striking at first sight is the fact that the same properties are encountered for \( R_H^\infty \), i.e. the Hall constant at frequencies well beyond \( U \). At such high frequencies, the dynamics of the electrons is insensitive to the interaction \( U \). However, for nondynamical quantities as matrix elements, the correlations introduced by the Hubbard interaction remain important.

Before we discuss possibilities to extend the moments technique to lower frequencies, we shall rederive the results of this subsection within a simple approximation, valid at \( T = 0 \).

### D. Hubbard I approximation

An expression for the frequency dependent Hall conductivity with vertex corrections having been neglected was derived in Ref. [12]:

\[
\sigma_{xy}(\omega) = \frac{e^3H}{2} \sum_{k0} \left[ \frac{\partial \epsilon_k}{\partial k_x} \right]^2 \frac{\partial^2 \epsilon_k}{\partial k_y^2} \frac{\Pi_H(z,\k)}{z},
\]

(38)

\[
\Pi_H(i\omega_n,\k) \equiv \frac{1}{\beta} \sum_n G_E(\omega_n)^2 \times \left[ G_E(i\omega_n + i\omega_m) - G_E(i\omega_n - i\omega_m) \right].
\]

(39)

Here, the Green’s function is given in terms of its spectral function as

\[
G_E(i\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi \omega} \frac{A_x(\omega)}{\omega_n - \omega},
\]

(40)

and \( c \), in our notation, is negative. Furthermore, it is assumed that the momentum dependence arises solely from the dispersion \( \epsilon_k^c \) of the bare band. In Mori theory, the Hall conductivity is represented as [1]:

\[
\sigma_{xy}(\omega) = \frac{i e^2}{\beta} \frac{L}{Z + L} |\vec{J}_y|.
\]

(41)

Therefore, the function \([13]\) may be connected to the Hall conductivity \( \sigma_{xy}(\omega \pm i0^+) \equiv \sigma_H^\prime(\omega) \pm i \sigma_H^{\prime\prime}(\omega) \) via

\[
S_H(\omega) = \frac{\omega \sigma_H(\omega)}{\pi^2}.
\]

(42)

Since \( \sigma_H^{\prime\prime}(\omega) \) arising from Eqs. (38) and (39) may be shown to be real and odd, the function \([13]\) has indeed the correct analytic properties. Inserting Eqs. (39) and (41) into Eq. (38) and using Eq. (42), we obtain, after a standard calculation [13]:

\[
S_H(\omega) = |e| \sum_{k} \left[ \frac{\partial \epsilon_k}{\partial k_x} \right]^2 \frac{\partial^2 \epsilon_k}{\partial k_y^2} X_k(\omega),
\]

(43)

\[
X_k(\omega) = \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} A_k(\omega_1) A_k(\omega_2) F_k(\omega_1;\omega) - F_k(\omega_2;\omega) + (\omega \to -\omega),
\]

\[
F_k(\epsilon;\omega) \equiv \frac{A_k(\epsilon - \omega)}{2\pi} \left[ f(\epsilon) - f(\epsilon - \omega) \right].
\]

(44)

(45)

By means of a partial integration, the corresponding sum rule is straightforwardly shown to be satisfied (cf. Eqs. [13], [14], and [23]).

Next, we evaluate Eqs. (43) and (44) in the so-called Hubbard I approximation [14]:

\[
\frac{A_k(\omega)}{2\pi} = \frac{1 + \delta}{2} \delta(\omega - \frac{1 + \delta}{2} \epsilon_k) + \frac{1 - \delta}{2} \delta(\omega - U - \frac{1 - \delta}{2} \epsilon_k).
\]

(46)
At $U = \infty$, only the first term contributes. Then, the quantity \((44)\) becomes
\[
S_H(\omega; U = \infty) = 1 + \frac{\delta}{2} \left( -i < [\hat{J}_x, \hat{J}_y] > \right) \delta(\omega).
\]  
(48)

and by means of an integration by parts, we obtain
\[
S_H(\omega; U = \infty) = 1 + \frac{\delta}{2} \left( -i < [\hat{J}_x, \hat{J}_y] > \right) \delta(\omega) .
\]  
(49)

This may be proven by calculating the moments \((17)\) in the limit $U \to \infty$.

Finally, we calculate $R_H^\infty$, given by Eq. \((29)\), analytically within the approximation \((46)\) and at $U = \infty$. Although this has been done numerically some time ago \([13]\), our simple analytical treatment allows for a direct comparison with the exact high-temperature result \((37)\) and demonstrates that the resulting doping dependence does not rely on the location of the Fermi surface. This last point is blurred in the Boltzmann equation based approach of Ref. \([16]\).

The amplitudes on the Rhs. of Eq. \((29)\) may be written as follows:
\[
a_n = \frac{1}{N} \sum_k \cos k_x n_{k\sigma} ,
\]  
(50)
\[
a_d = \frac{1}{N} \sum_k \cos k_x \cos k_y n_{k\sigma} .
\]  
(51)

Here, the density is given as
\[
n_{k\sigma} = \frac{1 + \delta}{2} f\left( \frac{1 + \delta}{2} \epsilon_k \right),
\]  
(52)

since in Hubbard I approximation at $U = \infty$, only the first term on the Rhs. of Eq. \((46)\) is to be kept. Momentum dependences arise solely from the dispersion $\epsilon_k$ of the bare band, why next, we are looking for expressions for the following functions:
\[
A(\epsilon) \equiv \frac{1}{N} \sum_k \cos k_x \delta(\epsilon - \epsilon_k) ,
\]  
(53)
\[
B(\epsilon) \equiv \frac{1}{N} \sum_k \cos k_x \cos k_y \delta(\epsilon - \epsilon_k) .
\]  
(54)

A convenient way to find smooth approximations for these functions is to calculate them in the limit of infinite dimensions $d \equiv 4$:
\[
A(\epsilon) = -\frac{e}{\sqrt{d}} D^0(\epsilon) ,
\]  
(55)
\[
B(\epsilon) = \frac{1}{d} (\epsilon^2 - 1) D^0(\epsilon) ,
\]  
(56)

where $D^0(\epsilon) = (1/\sqrt{\pi}) \exp(-\epsilon^2)$ is the density of states of the bare system. Except for the prefactors $1/\sqrt{d}$ and $1/d$, which ultimately cancel each other out, the functions \((55)\) and \((56)\) may be compared to those calculated numerically on a two dimensional lattice. This reveals that the main effect of the limit $d \to \infty$ is to smooth out the logarithmic singularities at zero energy encountered in the case of the functions $D^0(\epsilon)$ and \((54)\) in $d = 2$. Therefore, this limiting procedure does certainly not affect the validity of our present analysis in any serious manner. At $T = 0$, Eqs. \((60)-(66)\) imply:
\[
2a_n = \frac{1 + \delta}{2} d D^0(2\epsilon_F / 1 + \delta) ,
\]  
(57)
\[
2a_d = -\frac{\epsilon_F}{d} D^0(2\epsilon_F / 1 + \delta) .
\]  
(58)

A relation between the Fermi energy and the doping parameter is established straightforwardly, which, in terms of the function
\[
H(\delta) \equiv \frac{2\epsilon_F(\delta)}{1 + \delta} ,
\]  
(59)

may be written as:
\[
\frac{1 - 3\delta}{1 + \delta} = \text{erf}(H(\delta)) .
\]  
(60)

In summary, the function \((28)\) at $U = \infty$ and $T = 0$ may be written as follows:
\[
|e| R_H^\infty = \frac{2}{1 + \delta} \frac{H(\delta)}{D^0(\epsilon_F)} .
\]  
(61)

The most important features are: Firstly, at $\delta = 1/3$, $R_H^\infty$ vanishes. Secondly, at $\delta \rightarrow 1$, we recover the exact result $R_H^\infty \approx -1/(|e|(1 - \delta))$. And thirdly, at $\delta \rightarrow 0$, we find $R_H^\infty \approx 1/(|e|2\delta)$. The last two statements are proven with the asymptotic relation $D^0(\epsilon)/\epsilon \approx \pm 1 - \text{erf}(\epsilon)$, valid in the limit $\epsilon \rightarrow \pm \infty$. All these points are in exact agreement with \((57)\). We take this as an indication that, on the one hand, the high-temperature result \((57)\) remains qualitatively valid even at low temperatures, and, on the other, that the Hubbard I approximation is remarkably good in the case of the quantity \((28)\). In Ref. \([16]\), the
doping dependence of the Hall constant in Hubbard I approximation was discussed in terms of the Fermi surface. This is misleading for two reasons.

For one thing, the Hubbard I approximation misplaces the Fermi surface: The Luttinger theorem, which relates the volume enclosed by the Fermi surface to the electron density \( n \), is violated in this approximation. In contrast to this, angle resolved photoemission experiments (ARPES) on cuprates like \( \text{Nd}_{2-\delta}\text{Ce}_{\delta}\text{CuO}_{4} \) appear to be consistent with LDA bandstructure calculations which, in turn, imply the validity of this theorem. Despite this flaw, the approximation (13) yields a doping dependence for the Hall constant which is in good agreement with experiments on \( \text{La}_{2-\delta}\text{Sr}_{\delta}\text{CuO}_{4} \).

For another, it was pointed out in Refs. [6] and [20], that the high-frequency object \( R_{H}^\infty \) is not directly related to the location and topology of the Fermi surface. Instead, in a strongly correlated system, the entire Brillouin zone tends to get populated. In consequence, the weighted density average (14) receives contributions from the entire Brillouin zone rather than just from the vicinity of the Fermi surface.

We can demonstrate this more explicitly by slightly changing the form of the lower Hubbard band in Eq. (13): We broaden the delta function a little bit and shift some relatively small amount \( Z \) of spectral weight to a new delta function contribution \( Z \delta (\omega - L(\epsilon_{k})) \), with the function \( L(\epsilon_{k}) \) being undetermined yet. This amounts to replacing Eq. (52) by \( n_{k} = h(\epsilon_{k}) \) with the function \( h(\epsilon_{k}) \) differing from \( \frac{1}{2} f(\frac{1}{2} \epsilon_{k}) \) only in the following respects: The step at \( \epsilon_{k} = \frac{2 \epsilon_{F}}{1+}\omega \) and of height \( \frac{1}{1+}\delta \) is smoothed out while a new, much smaller one of height \( Z \) occurs at \( \epsilon_{k} = L^{-1}(\epsilon_{F}) \). This last condition fixes the new Fermi surface. By choosing the function \( L(\epsilon) \) appropriately, we may place the Fermi surface wherever we want. As long as \( Z \ll \frac{1}{1+}\delta \), the crucial average \( a_{d} = \int_{-\infty}^{\infty} d\epsilon B(\epsilon)h(\epsilon) \) does not differ very much from the result in Eq. (58).

This reasoning illustrates that, in the presence of strong correlations, the doping dependence of \( R_{H}^\infty \) is in fact uncorrelated to the Fermi surface location. Also note, that it does not matter whether the function \( h(\epsilon) \) arises from coherent or incoherent excitations.

**IV. HALL CONSTANT IN THE LOW FREQUENCY REGIME**

In this section, we discuss the frequency dependence of the Hall constant for frequencies below the Mott-Hubbard gap. Therefore, the appropriate model to start with is the \( t-J \) model. It is straightforward to show that Eqs. (13), (14) are still valid, however, with all quantities being redefined within the \( t-J \) model [21]. Apart from the redefinition of the Liouville operator, this amounts to replacing the canonical Fermi operators through projected ones in all quantities that appear, i.e. \( c_{\alpha} \rightarrow X_{\alpha}^{\text{th}} \), and \( c_{\alpha}^{+} \rightarrow X_{\alpha}^{0} \). In particular, the current operator is then given by Eq. (14). If we renormalize the functions (13) and (14) according to

\[
S_{H}(\omega; t - J) = -i < [\hat{J}_{x,0}, \hat{J}_{y,0}> s(\omega) \quad (62)
\]

\[
K(\omega; t - J) = -i < [\hat{J}_{x,0}, \hat{J}_{y,0}> k(\omega) \quad (63)
\]

the analog of Eq. (12) reads:

\[
R_{H}(z) = R_{H}^{*} \left( 1 + \int_{-\infty}^{\infty} d\omega k(\omega) \frac{\omega}{\omega - z} \right) \quad (64)
\]

Here, \( R_{H}^{*} \) is the infinite frequency Hall constant of the \( t-J \) model that was already investigated in Ref. [3], and which has been introduced in Eq. (7) in the special case \( J = 0 \). Furthermore, we have taken into account that the function \( s(\omega) \) is normalized to unity, while \( k(\omega) \) represents the unknown memory function contribution. From the discussion in Sec. III B, we expect the function \( k(\omega) \) to have only one peak centered around zero frequency, because, in the \( t-J \) model, doubly occupied sites can occur only virtually.

Before we set about discussing possibilities to calculate this function via its moments, we try to gain some phenomenological insight.

**A. Phenomenological discussion**

Very recently, the normal state ac-Hall constant was measured in \( \text{YBa}_{2}\text{Cu}_{3}\text{O}_{7} \) thin films for frequencies up to 200 cm\(^{-1} \) [22]. In this work, the experimental data have been fitted successfully in terms of parameters introduced by Anderson [3] to account for the observed \( T^{2} \) dependence of the inverse Hall angle in high-\( T_{c} \) materials. Anderson’s theory is based on spin charge separation with two different relaxation times and effective masses associated with the spinon and holon degrees of freedom: \( \tau_{tr} \) is the decay time of the holons with effective mass \( m_{tr} \), scattering off thermally excited spinons. On the other hand, a transverse relaxation rate \( 1/\tau_{H} \) is determined by the scattering between the spinons. Apart from this, \( \sigma_{xx} \) and \( \sigma_{xy} \) have the ordinary Drude form, i.e. \( \sigma_{xx} \propto \tau_{tr}/m_{tr} \) and \( \sigma_{xy} = \sigma_{xx} \omega_{c} \tau_{H} \). Here, the cyclotron motion is characterized by a mass \( m_{H} \), i.e. \( \omega_{c} \propto 1/m_{H} \). In Ref. [22], Anderson’s theory was extended to finite frequencies via the replacements \( \tau_{tr} \rightarrow \tau_{tr}/(1 - i\omega \tau_{tr}) \) and \( \tau_{H} \rightarrow \tau_{H}/(1 - i\omega \tau_{H}) \). This led to the following representation of the frequency dependent Hall constant:

\[
R_{H}(\omega) = \frac{m_{tr}}{m_{H} ne} \left( 1 + \frac{\tau_{H} - \tau_{tr}}{\tau_{tr}} \frac{1}{1 - i\omega \tau_{H}} \right) \quad (65)
\]

This result is equivalent to the exact expression (64), provided the following identifications are made:

\[
k(\omega) = \frac{\tau_{H} - \tau_{tr}}{\tau_{tr}} L_{1/\tau_{H}}(\omega) \quad (66)
\]

\[
R_{H}^{*} = \frac{m_{tr}}{m_{H} ne} \quad (67)
\]
Here, \( L_I(\omega) \) denotes the Lorentzian of width \( \Gamma \) normalized to unity. Therefore, the unusual relaxation time \( \tau_H \) is a measure for the width of the function \((66)\) and thus determines the decay rate of the correlation between the residual forces \( f_{x,0} \) and \( f_{y,0} \) of the \( t - J \) model (cf. Eqs. \[(3)\) and \[(4)\]). Furthermore, the integrated weight of the function \((66)\) measures the deviation of the relaxation time \( \tau_H \) from the ordinary transport time \( \tau_{tr} \). This deviation is related directly to that of the Hall constant at zero frequency from its value at high frequencies:

\[
\frac{\tau_H - \tau_{tr}}{\tau_{tr}} = \frac{R_H(\omega = 0) - R_H^*}{R_H^*} . \tag{68}
\]

In the phenomenological expression \((66)\), the temperature dependence is entirely contained in the two relaxation rates. They are expected to vary as \( \tau_{tr} \propto 1/T \) and \( \tau_H \propto 1/T^2 \). For dimensional reasons, we take \( \tau_H \propto J/T^2 \), since \( J \) is the only energy scale characteristic of our model. From these properties, we may infer the following: For one thing, the memory function does not only describe the unusual frequency dependence of the Hall constant in high-\( T \) superconductors. Also, the observed anomalous temperature dependence is mainly due to this memory function contribution. For another, we expect \( \tau_{tr} \) to be relatively smaller than \( \tau_H \) at low temperatures, since \( \tau_H/\tau_{tr} \propto J/T \). Thus, Eq. \((68)\) suggests that the Hall constant \( \text{increases} \) when zero frequency is approached. This enhancement was actually observed in the above mentioned measurements on YBa\(_2\)Cu\(_3\)O\(_{6+x}\)\cite{22}. In any case, Eq. \((68)\) indicates that the sign of the Hall constant is solely described by the high-frequency object \( R_H^* \), as claimed in Ref. \cite{3}.

Does the discussion so far point towards spin-charge separation as advocated by Anderson? Obviously, the current operators \( \hat{J}_\nu \) are related to the charge degrees of freedom only. Consequently, the spin physics must be accounted for by the residual forces \( \hat{f}_\nu \). This is also reflected by the proportionality \( \tau_H \propto J/T^2 \). However, up to now, we do not have a compulsory argument why these residual forces should describe exclusively spin degrees of freedom.

In summary, Anderson’s notion of two distinct relaxation rates is naturally backed up within the Mori theory. They may be interpreted as the time scales set by the current operators and their associated residual forces.

\section*{B. Moments approach to the memory function}

Of course, it would be interesting to calculate the function \( k(\omega) \) of Eq. \((74)\) quantitatively within the \( t - J \) model in order to relate its width and its integrated weight to the parameters \( t, J, T \) and doping \( \delta \). From the above discussion, we expect that the relevant information about its overall form may be put into only few parameters. As already mentioned, even two parameters as in Eq. \((66)\) have been sufficient to obtain an excellent fit of experimental data \cite{22}. In this subsection, we suggest the following procedure to construct \( k(\omega) \):

\[
k_l = \int_{-\infty}^{\infty} d\omega \, k(\omega) \omega^{2l} . \tag{69}
\]

exist, why we have to replace the Lorentzian \((66)\) by a “short-range” function, e.g. a Gaussian multiplied by a polynomial. We proceed by relating the moments of \( k(\omega) \) to that of the function \( s(\omega) \) and the optical conductivity. Finally, we discuss a possibility to calculate these moments.

The ordinary conductivity is given in Mori theory by the following expression \cite{3}:

\[
\sigma_{xx}(\omega+i0^+) = i e^2 \beta(\hat{J}_{x,0}) \frac{1}{\omega + L + i0^+} \hat{J}_{x,0} , \tag{70}
\]

the real part of which may be shown to be an even function of \( \omega \)\cite{3}, and, due to a sum rule, can be written in terms of a function \( c(\omega) \), that is normalized to unity:

\[
\Re\{\sigma_{xx}(\omega+i0^+)\} = \pi e^2 \chi^0 c(\omega) . \tag{71}
\]

In the appendix, we show that all the moments \((69)\) may be traced back recursively to that of the functions \((62)\) and \((73)\):

\[
s_l = \int_{-\infty}^{\infty} d\omega \, s(\omega) \omega^{2l} , \tag{72}
\]

\[
c_l = \int_{-\infty}^{\infty} d\omega \, c(\omega) \omega^{2l} , \tag{73}
\]

provided \( l \geq 1 \). Thereby, only the definition of the superprojectors \( Q \) has to be used in Eq. \((77)\). The result may be written as follows:

\[
k_l = -s_l + \sum_{j=1}^{l} a_j s_{l-j} \quad \text{for} \ l \geq 1 , \tag{74}
\]

where the coefficients \( a_j \) are polynomials in the moments \((23)\) and are listed in the appendix up to \( j = 6 \). If we had a good method for calculating the moments \((22)\) and \((23)\), we could construct the unknown function \( k(\omega) \) via its first \( n \) moments as already explained. This approximation is reliable, if the zeroth moment \( k_0 \), calculated from Eq. \((69)\), converges fast enough with increasing \( n \). Since in the \( t - J \) model, \( k(\omega) \) has only one peak around \( \omega = 0 \), the first few moments are expected to be sufficient for this to happen. An advantage of this approach is that moments are \textit{global} properties of a spectral function and as such are less sensitive to its detailed resonance structure and to approximations involved. Also, some approximation schemes are better suited for the calculation of moments rather than the underlying spectral function.
For instance, within a high-temperature expansion, moments are accessible, at least in principle, while the corresponding spectral function is not. However, in the case of the moments (72), only very few moments, and to only low orders in 1/T, are within reach. This is all the worse, since now, we are interested in the Hall constant at low frequencies, i.e., no other high-energy scale is present as it was in the context of $R_{\text{H}}^\ast$.

Another example is the exact diagonalization technique [23]. In this method, spectral functions are calculated numerically via the exact eigenstates and eigenenergies on the basis of their Lehmann representation. While being exact, the intrinsic problem of this method is the constraint of working on relatively small clusters. Therefore, the delta functions have to be broadened in order to obtain smooth functions. In contrast to this, no additional approximation is required when calculating moments. Of course, the smallness of the clusters remains the major restriction of this method. Nevertheless, when combined with the moments approach as suggested above, it is a means of extracting reliable information about the frequency dependent Hall constant within the $t-J$ model and should therefore be the subject of a future work.

V. DISCUSSION AND CONCLUSIONS

In this paper, we have derived first results for the Hall effect in correlated electron systems in the strong correlation limit within the recently developed memory function approach [1]. We have focused our attention mainly on the memory function term, which is neglected in Boltzmann type approaches. The important new physics to be expected from this contribution comprises the unusual frequency and temperature dependence of the Hall constant, observed in the normal state of high-$T_c$ superconductors.

In the single-band Hubbard model, a finite amount of spectral weight for particle-hole excitations, caused by the Hall current, is always pinned at the energy $U$. This is valid no matter how large the correlation strength $U$ is. We have shown that the memory function removes these high-energy excitations in the limit $U \to \infty$, thus accounting for the frequency dependence of the Hall constant down to frequencies within the charge transfer gap. The corresponding decrease of the Hall constant by a factor $(1+\delta)/2$ was calculated exactly to leading order in $1/T$ and corroborated within an approximate treatment, valid at $T = 0$. However, our analysis did not provide us with information about the frequency dependence of the Hall constant at lower frequencies. The reason is that it was based on moments, which are dominated by the high-frequency contributions. We have also calculated the infinite frequency Hall constant analytically within the so-called Hubbard I approximation. In essence, we recovered the exact result for $U = \infty$ and $T \to \infty$ and explained, why this result does not rely on the location of the Fermi surface.

Finally, the Hall constant at low frequencies was investigated within the $t-J$ model, an effective model acting in the reduced Hilbert space without doubly occupied sites. We observed that our memory function formalism distinguishes inherently between two time scales: Firstly, the dynamics of the current operators is characterized by the ordinary transport relaxation time $\tau_{tr}$. And secondly, the impact of all the other degrees of freedom on this charge transport is taken into account by fluctuating forces, that introduce an unusual time scale $\tau_H$. On the other hand, it was pointed out by Anderson, that temperature dependences of transport and Hall effect measurements can best be understood in terms of two relaxation times, following a $1/T$ and $1/T^2$ law [3]. We have shown that the time scales encountered within the Mori theory are identical to those introduced by Anderson. Furthermore, we have shown that the deviation of the unusual decay time $\tau_H$ from the ordinary transport time $\tau_{tr}$ is intimately connected to the frequency dependence of the Hall constant at low frequencies. Thus, the temperature and frequency dependence of the Hall constant result from each other and are both due to the memory function contribution. It would be very interesting to investigate this interplay further, both theoretically and experimentally. As for the theoretical side, we have proposed an approach based on moments. It allows the exact treatment of the superprojector that reflects the distinction between the two time scales, leaving us with the problem of finding the first few moments of the ordinary current-current correlation functions. Except for a well studied prefactor, the memory function term is mainly determined by two parameters. Therefore, we expect its first few moments to provide us with enough information to fix these parameters.

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APPENDIX A: REDUCTION OF THE MOMENTS OF THE MEMORY FUNCTION

First of all, we recall the definition of the moments (13) and (14), however with all operators and superoperators now being redefined within the $t-J$ model. They are related to the moments (19) and (72) via the equations $K_l = -i<[\hat{J}_{x,0},\hat{J}_{y,0}]>l_1$ and $S_l = -i<[\hat{J}_{x,0},\hat{J}_{y,0}]>l_1$. In addition, we require the moments (73), renormalized as $C_l = \chi^0c_l$. Next, we define the quantities
To prove these relations, we only have to use the following to the moments
\( K \)
for
\( n \geq 0 \). By using the definition (3) for each of the first superprojector on the right hand sides, we obtain the following recursion relations:

\[
X_l^n = X_{l-1}^{n+1} - \frac{Y_l^n}{\chi^0} X_{l-1}^0 \quad \text{for odd}, \quad (A3)
\]

\[
X_l^n = X_{l-1}^{n+1} - \frac{X_l^n}{\chi^0} Y_{l-1}^0 \quad \text{for even}, \quad (A4)
\]

\[
Y_l^n = Y_{l-1}^{n+1} - \frac{Y_l^n}{\chi^0} Y_{l-1}^0 \quad \text{for odd}, \quad (A5)
\]

\[
Y_l^n = Y_{l-1}^{n+1} \quad \text{for even}. \quad (A6)
\]

To prove these relations, we only have to use the following facts: \( X_l^n \) and \( Y_l^n \) vanishes for all odd and even integers \( n \), respectively. This is due to the fact that the functions (3) and (4) are even, hence their odd moments vanish. Moreover, the quantities (A1) are of first order in the magnetic field. To relate the unknown moments \( K_l \) to the moments \( S_l \) and \( C_l \), we have to supplement the recursion formulas (A3)-(A6) by the following equations:

\[
K_l = X_{2l+1}^0, \quad (A7)
\]

\[
S_l = -X_{1}^{2l}, \quad (A8)
\]

\[
C_l = Y_{2l-1}^0. \quad (A9)
\]

However, only even numbers of iterations occur. Therefore, we may combine two successive iteration steps into one. This leads to the following effective recursion relations:

\[
H_l^n = H_{l-1}^{n+1} - \frac{C_{n+1}}{C_0} H_{l-1}^0 + \frac{S_n}{C_0} N_{l-1}^0, \quad (A10)
\]

\[
N_l^n = N_{l-1}^{n+1} - \frac{C_0}{C_0} N_{l-1}^0, \quad (A11)
\]

where we have defined

\[
H_l^0 := X_{2l+1}^0 \quad \text{for } n \geq 0, l \geq 0, \quad (A12)
\]

\[
N_l^0 := Y_{2l-1}^{2l+1} \quad \text{for } n \geq 1, l \geq 0, \quad (A13)
\]

and where the contact to the moments \( K_l, S_l \) and \( C_l \) is established by means of the equations

\[
K_l = H_l^0, \quad (A14)
\]

\[
S_l = -H_l^l, \quad (A15)
\]

\[
C_l = N_l^0. \quad (A16)
\]

This recursive procedure results in Eq. (7) with the first six coefficients being given as follows:

\[
a_1 = 2c_1, \quad (A17)
\]

\[
a_2 = 2c_2 - 3c_1^2, \quad (A18)
\]

\[
a_3 = 2c_3 - 6c_1c_2 + 4c_1^3, \quad (A19)
\]

\[
a_4 = 2c_4 - 6c_1c_3 - 3c_2^2 + 12c_1^2c_2 - 5c_1^4, \quad (A20)
\]

\[
a_5 = 2c_5 - 6c_1c_4 - 6c_2c_3 + 12c_1^2c_3 + 12c_1c_2^2 - 20c_1^3c_2 + 6c_1^5, \quad (A21)
\]

\[
a_6 = 2c_6 - 6c_1c_5 - 6c_2c_4 + 12c_2^2c_4 - 3c_3^2 + 24c_1c_2c_3 - 20c_1^2c_3 + 4c_2^3 - 30c_1^2c_2^2 + 30c_1c_2 - 7c_1^6. \quad (A22)
\]

[1] E. Lange, cond-mat/9606176, to appear in Phys. Rev. B together with the present work.
[2] N. P. Ong, Physical Properties of High Temperature Superconductors, edited by D. M. Ginsberg, World Scientific, Singapore (1990), Vol. 2.
[3] P. W. Anderson, Phys. Rev. Lett. 67, 2092 (1991).
[4] L. Friedman, T. Holstein, Ann. Phys. (N. Y.) 21, 494 (1963); Phys. Rev. 165, 1019 (1968); K. G. Wilson, Phys. Rev. D 10, 2445 (1974).
[5] E. Fick, G. Sauer, G. M. W. Springer, The Quantum Statistics of Dynamic Processes, Springer (1990).
[6] B. S. Shastry, B. I. Shraiman, R. P. Singh, Phys. Rev. Lett. 70, 2004 (1993).
[7] S. Bacci, E. Gagliano, R. Martin, J. Annett, Phys. Rev. B 44, 7504 (1991).
[8] B. Harris, R. Lange, Phys. Rev. 157, 295 (1967).
[9] A. H. MacDonald, S. M. Girvin, D. Yoshioka, Phys. Rev. B 37, 9753 (1988).
[10] H. Eskes, A. Oles, M. Meinders, W. Stephan, Phys. Rev. B 50, 17980 (1994).
[11] C. J. Thompson et al., J. Phys. A: Math. Ge. 24, 1261 (1991).
[12] P. Voruganti, A. Golubnitschew, S. John, Phys. Rev. B 45, 13945 (1992).
[13] A. L. Fetter, J. D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill (1971).
[14] J. Hubbard, Proc. R. Soc. London, Ser. A 276, 238 (1963).
[15] In Ref. [1], the Hall effect was treated numerically within the Hubbard I approximation on the basis of the Boltzmann equation. This amounts to the same thing as considering \( K_{l+1}^0 \).
[16] H. Fukuyama, Y. Hasegawa, Physica B 148, 204 (1987).
[17] A. Abrikosov, L. P. Gorkov, I. E. Dzyaloshinskii, Methods of Quantum Field Theory in Statistical Physics, Pergamon Press (1965).
[18] D. M. King et al., Phys. Rev. Lett. 70, 3159 (1993).
[19] R. O. Anderson et al., Phys. Rev. Lett. 70, 3163 (1993).
[20] B. S. Shastry, B. I. Shraiman, R. P. Singh, Phys. Rev. Lett. 71, 2838 (1993).
[21] P. Fulde: Electron Correlations in Molecules and Solids. Springer, 1991.
[22] S. G. Kaplan et al., Phys. Rev. Lett. 76, 696 (1996).
[23] For a review, see E. Dagotto, Rev. Mod. Phys., 66, 763 (1994).