ABSTRACT We present a simulation method for the auralization of the ancient Greek double-reed wind instrument Aulos. The implementation is based on Digital Signal Processing and physical modeling techniques for the instrument’s two parts: the excitation mechanism and the acoustic resonator with toneholes. Single-reeded instruments are in-depth studied firstly because their excitation mechanism is the one used in a great amount of modern wind-reed instruments and secondly because the physics governing the phenomena is less complicated than the double-reeded instruments. We here provide a detailed model of a system comprised of a double-reed linked to an acoustic resonator with toneholes to sonify Aulos. We validate our results by comparing our method’s synthesized signal with recordings from a replica of Aulos of Poseidonia built in our lab. The comparison showed that the fundamental frequencies and the first three odd harmonics of the signals differ 6, 5, 3, and 2 cents on average, respectively, which is below the Just Noticeable Difference threshold.

INDEX TERMS Aulos, digital signal processing, digital waveguides, double-reed, musical acoustics.

I. INTRODUCTION
The realistic sonification of physical musical instruments by digital means constitutes a pole of attraction in several multidisciplinary scientific fields, such as physics, informatics, musicology [1]–[7]. Theoretical acoustics has significantly contributed to a better understanding of the physical phenomena governing the generated sound. On the other hand, computer scientists have developed new cost-effective algorithms running under modern and user-friendly interfaces, which can simulate the phenomena and achieve real-time sonification. These applications are not only valuable for musicians but to the instrument industry as well. Influential pieces of software that, via digital simulation, sonify a virtually customized musical instrument are now commercially available, allowing accurate pre-production testing without physically building the instrument.

State-of-the-art physical modeling (PM) techniques [8]–[10] have been put into practice by commercial companies (Native Instruments,1 Modartt,2 Applied Acoustic Systems,3 etc.) to realistically sonify various popular instruments. Distinct simulation techniques of single-reed wind instruments have been proposed [11]–[15], leading to the development of commercial pieces of software (e.g., Swam Clarinets4). The nonlinear phenomena introduced by the oscillating reed [11], [14], [16], [17] and the linear effect of the oscillating air volume inside the resonator [12], [14], [15],

1https://www.native-instruments.com/
2https://www.modartt.com/
3https://www.applied-acoustics.com/
4https://audiomodeling.com/swam-engine/solo-woodwinds/swam-clarinets/
which defines the resonance and therefore the pitch and timbre of the produced sound, are the fundamental elements governing the physical phenomena of the reed wind instrument. We assumed that these elements are the fundamental aspects of significance in our model and validated the claim with the results.

The resonator (a cavity where air particles that transmit the sound waves are enclosed) of the majority of wind instruments is either a cylindrical pipe or a conical horn. We note here that Aulos (cylindrical resonator) and oboe (conical resonator) have rather different harmonics even if both instruments had the same resonator’s length. The air column oscillates with respect to the tube’s resonant frequency, which is determined by the instrument’s geometry. The pressure wave propagates along the horizontal axis of the acoustic resonator (i.e., direction along x axis in Fig.3) and it is transmitted to the acoustical space (outside the resonator) through the openings (bell and toneholes). The resonator results either in an un-flared or in a flared end (bell). The acoustic attributes of the produced sound depend on the aforementioned air column oscillations. That is the reason why it is the instrument’s geometry, rather than the material that most significantly affects the produced sound [18]–[20].

Wind instruments allow the alteration of the generated pitch through various techniques and mechanisms (e.g., toneholes, slide mechanisms, by means of embouchure). In this work we consider only the use of toneholes. Open toneholes significantly affect the pressure wave propagation by shortening the effective length of the air column as the pressure wave exits the resonator at first open hole closest to the mouthpiece. Closed toneholes do not affect the resonant frequency as much, but their effect cannot be neglected [12], [21]. Fingering pattern, which is a pattern of open and/or closed toneholes, allows variations of the air column’s length, leading to the production of a desired pitch. The potential register holes, which are usually located on the bottom side of the resonator opposite the toneholes, extend the frequency range modifying the air column so as higher resonance frequencies will be produced.

The reed, either single or double, which is located at the tip of the mouthpiece, is the excitation mechanism of the reed wind instruments. The player’s lips abut the reed’s blade(s), and the air coming from the mouth flows inside the instrument’s internal cavity. The pressure difference between the player’s oral cavity and the reed’s channel helps to define the reed movement during excitation. The reed closes as the aforementioned pressure difference increases and opens as it decreases. When the movement mentioned above becomes periodic between two extreme positions (i.e., the reed oscillates), the volume flow entering the resonator stimulates an oscillation to the air particles inside the acoustic resonator. This oscillating flow is obtained by a feedback loop consisting of the nonlinear exciter coupled to the resonator. Researchers, throughout the last decades, have extensively studied the behavior of the nonlinear phenomena governing the operation of single-reed wind instruments [12], [15], [22].

However, double-reed wind instruments are not thoroughly studied, firstly because of the complexity of the airflow that such an excitation mechanism introduces, and secondly because there is a limited number of modern instruments of this kind. The most popular double-reed instruments in our times are the oboe, the bassoon, and the bagpipe, whose reed consists of two symmetrical [17] oscillating blades followed by a conical part that works as a conical diffuser. Some alternative approaches for the physical modeling of the double-reed musical instrument of Zournas have also been presented in the literature [23]–[26].

Although not much research on the double-reed instruments has been done, there are some studies [16], [17] [27]–[31] which focus on the double-reed and the complicated, nonlinear phenomena that govern its mechanism. Most of these works study the oboe’s double-reed, but not in combination with the rest of the instrument (i.e., the resonator).

According to archeomusicology, a significant number of ancient instruments were played with a double-reed, e.g., the Greek Aulos [32]–[34] and the renaissance Rauschpfeife [35]. The Aulos of Poseidonia, dated in 5ct BCE, constitutes a typical representative of the classical Greek Aulos in terms of dating, geometry, and function [36]. Therefore, its acoustic study enables us to suggest generalized indications on the sound of Aulos of the classical era. In the current work, we study the double-reed of Aulos, which geometrically differs from the modern ones purely because after the blades, instead of the conical part, there is a cylindrical backbore. Andreopoulou’s work on Aulos [37] focused on the capabilities of scale reproduction but not on the instrument’s simulation techniques. The current work aims to provide a more robust and detailed modeling approach than previous works. Our goal is to develop a complete digital analog of this ancient wind instrument by using physical modeling techniques. In order to do so, we modified the proposed method by Almeida et al. [29], who studied the oboe’s reed, to fit the Aulos reed case and, later, we combined the theory for the nonlinear excitation mechanism with the digital waveguide method [9], [12], [15], [38] to simulate the physics of the resonator with its toneholes. As a result, we created a digital double-reed wind instrument model that allows the customization of the instruments’ geometry, i.e., the mouthpiece’s length and the resonator, the positions, the size, and the number of the toneholes and the fingering pattern. In order to demonstrate a scenario of a simple, user-friendly interface, we also created a Graphical User Interface (GUI) which accepts values for the geometrical variables, virtually demonstrates a simplified geometry of the instrument, and displays the produced signal both in time and frequency domain. This model can export the fundamental frequency and its first harmonics and therefore, we expect it to be a useful tool for archeologists and archeomusicologists to predict the musical scale or a missing part of an instrument. It should be noted here that in this work, the effect of the instrument’s material was not taken into account as, in the family of instruments simulated (i.e., wind instruments with
a cylindrical resonator), it does not significantly affect the produced sound [18]–[20]. Lastly, we compared the audio generated by our model with the recorded samples taken from the replica of Aulos of Poseidonia, physically reconstructed, by following the principles of archeomusicology [39].

The structure of the current work is as follows: first, we describe in detail the modeling for the double-reed and the resonator along with the toneholes. Then, we present the implementation of the read-resonator system and the open and closed toneholes. Finally, we show the validation results based on comparing the replica’s signal vs. the signal generated by the digital modeling.

II. MODEL’S DESCRIPTION

A. THE DOUBLE-REED

As mentioned in the introduction, the sound generated by a wind musical instrument is due to the oscillating air particles that cause the pressure wave propagation along the acoustic tube. In order to study this phenomenon, we first need to simulate the excitation mechanism which triggers these oscillations. In double-reed instruments, the movement of the reed’s two blades which is caused by the alterations of pressure (i.e., the pressure inside the player’s mouth and the pressure inside the reed), is the excitation mechanism. Because of the nonlinear movement of the double-reed (as the reed rapidly closes), the airflow through it is affected by phenomena such as vortices and/or turbulent flow [17], [27], [29] which cannot easily be described by analytical expressions. In this section, we present the calculation of the difference between the mouth pressure and the pressure inside the reed according to the mouth pressure and the pressure exiting the reed (i.e., the pressure right after the reed’s conical part or backbore). This is an essential step towards the simulation of the generated sound of the wind instrument as the pressure at the end of the reed equals the one entering the instrument’s resonator, where the resonance (responsible for the sound production) is taking place.

Our approach of the double-reed is based on the work of Almeida et al. [16], [29], who studied the oboe’s double-reed excitation mechanism. In order to describe the nonlinear behavior of the double-reed in a simple model, the quasistatic regime is used to define the relationship between two variables of acoustic relevance; the pressure drop ($\Delta p$) across the reed and the volume flow ($q$) at its output. According to quasistatic conditions, since the quantities $p$ and $q$ (Fig.1) vary sufficiently slowly in time, all time derivatives in the nonlinear characteristic relation can be neglected.

The pressure difference ($\Delta p_c$) between the mouth pressure ($p_m$) and the pressure inside the reed ($p_c$) affects the reed’s opening area ($S$). Both double-reed’s, the one that Almeida described [29] and the one used in this study, are inward-striking, i.e., when the mouth pressure is increased, the blades of the reed tend to close. The relation of the pressure difference and the reed’s cross-section area is given by (1)

$$\Delta p_c = p_m - p_c = k_S(S_0 - S)$$

(1)

where, $S_0$ is the reed’s opening at rest, and $k_S$ is the reed’s stiffness coefficient.

Almeida et al. studied the double-reed’s movement by performing experimental measurements in the case of the oboe [28]. For their experiment, they use an artificial mouth which allows them to control and alternate the pressure $p_m$. Initially, the reed is at rest, where the pressure difference ($\Delta p_c$) equals to zero, and the reed’s opening area is at its maximum level ($S_0$). As the pressure $p_m$ increases the blades are progressively closing. When the pressure difference exceeds a specific value, the reed is forced to shut ($S = 0$) rapidly. Consequently, the airflow through the reed stops, the pressure $p_m$ suddenly increases, and the pressure $p_c$ drops to 0. Then the mouth pressure $p_m$ is gradually decreased. The reed remains closed until the value of $p_m$ - and consequently the value of $\Delta p_c$ - becomes small enough to allow for the reed to open again. Then when the reed is open, the air starts flowing again through the reed and, as a result, the mouth pressure quickly decreases and the reed pressure increases. When the pressure difference ($\Delta p_c$) becomes equal to zero, it obliges the reed to stabilize at its initial rest position. It is important to note here, that, in our simulation for simplicity reasons we assume that the mouth pressure $p_m$ is not altered (i.e., its value is constant). This simulated constant mouth pressure (input signal) takes the value of 1 (normalized maximum) which is reached by an initial slope (corresponding to the time needed to reach the constant value). Although this simplification cannot simulate some aspects related to the player’s subtle control of the sound quality (e.g., vibrato and rapid transients), it enables the adequate approximation of the normal playing condition (i.e., production of a self-sustained steady oscillation) [40].

The volume flow ($q$) that enters the instrument is defined by the pressure difference ($\Delta p_c$) and the reed’s opening area, as described by Bernoulli (2).

$$q = S \sqrt{\frac{2(p_m - p_c)}{\rho}} = S \sqrt{\frac{2\Delta p_c}{\rho}}$$

(2)

where, $\rho$ is the density of the air medium.

In the case of the oboe’s excitation mechanism, the part after the blades is conical. This geometrical feature is described by the cross-sections $S_1$ and $S_2$ (Fig.1) and it is considered to be a conical diffuser with a pressure recovery coefficient $c_p$ ($0 \leq c_p \leq 1$) [29], [30]. Therefore, the total
pressure difference between mouth pressure and pressure at the exit of the reed \( (p_r) \) is:

\[
p_m - p_r = (\Delta p)c = c_p \frac{1}{2} \rho \left( \frac{q}{S_c} \right)^2
\]

(3)

where, \( S_c \) is the cross-section at the beginning of the conical diffuser and, assuming an ideal pressure recovery coefficient, \( c_p = 1 - \frac{S_n^2}{S_f^2} \).

Combining (1) and (3), the pressure inside the reed is

\[
p_c = p_r - c_p \frac{1}{2} \rho \left( \frac{q}{S_c} \right)^2
\]

(4)

Therefore, the pressure difference \( (\Delta p)c \) can be expressed according to the mouth pressure and the pressure at the exit of the reed, as it occurs from (1) and (4)

\[
(\Delta p)c = p_m - p_r + c_p \frac{1}{2} \rho \left( \frac{q}{S_c} \right)^2
\]

(5)

Ancient double-reeds (Fig.2), on the other hand, geometrically differ from modern ones. The main difference is that after the blades, instead of a conical diffuser, there is a cylindrical part, called the reed’s backbore, with an inner diameter equal to the inner diameter of the instrument’s resonator.

![Ancient double-reed with cylindrical backbore.](image)

In the case of the double-reed with a cylindrical backbore, there is a neck located between the blades and the backbore. As shown in Fig.2, \( S_c \) is the section area before the reed’s neck, \( S_n \) is the section area of the neck and \( S_r \) is the section area of the backbore.

Applying Bernoulli’s law between \( S_c \) and \( S_n \) and between \( S_n \) and \( S_r \) respectively, we get:

\[
p_c + \frac{1}{2} \rho \frac{q^2}{S_c^2} = p_n + \frac{1}{2} \rho \frac{q^2}{S_n^2}
\]

(6)

\[
p_n + \frac{1}{2} \rho \frac{q^2}{S_n^2} = p_r + \frac{1}{2} \rho \frac{q^2}{S_r^2}
\]

(7)

The combination of the above relations gives:

\[
p_r = p_c + \left( 1 - \frac{S_n^2}{S_f^2} \right) \frac{1}{2} \rho \frac{q^2}{S_c^2}
\]

(8)

As in oboe’s reed, we can set the constant \( c_p = 1 - \frac{S_n^2}{S_f^2} \).

\[
(\Delta p)c = p_m - p_r
\]

(9)

### B. THE RESONATOR

The airflow coming from the reed enters the instrument’s bore and forces the air particles inside the resonator to vibrate. Assuming a simple closed-open tube without tone-holes, the pressure waves travel along the tube and partially come back every time they reflect at each end, forming a standing wave inside the resonator. When the waves reach the open end, they are reflected back with a phase change of \( \pi \), while, at the closed end, the waves are reflected back without phase change [18]. In the physics theory governing reed instruments, the resonator’s end, where the reed is attached, is considered to be closed because the reed’s opening area is much smaller than the resonator’s diameter. However, the other end is open to the outside air. Therefore, Aulos’ resonator (a cylindrical wind reed instrument) behaves as a closed-open pipe, which produces only the odd harmonics with respect to the fundamental produced frequency.

At the closed end, there is a pressure antinode (i.e., the sound pressure becomes maximum), and the displacement of the air particles equals zero. At the open end, there is a pressure node where the sound pressure equals zero (the pressure is approximately atmospheric), and the displacement of the oscillating air particles becomes maximum. Fig.3 A, B, and C show few examples of pressure waves with wavelengths of...
The resonator is right-going traveling wave (Fig. 2), the total pressure inside the exit (Fig. 5). Considering the resonator’s cross-sectional area equals the one at the reed’s mouth, the pressure entering the instrument’s resonator because the pressure p\text{\text{m}} exceeds the pressure inside the reed, the reed closes, the pressure peeling waves at a point gives the total power at that point in the waveguide. The pressure p_r exiting the reed equals the pressure entering the instrument’s resonator because the resonator’s cross-sectional area equals the one at the reed’s chimney, the tonehole height t is approximated [12] by

\[ t = t_w + \frac{1}{8} \frac{b}{a} \left[ 1 + 0.172 \left( \frac{b}{a} \right)^2 \right] \]  

III. IMPLEMENTATION

A. THE RESONATOR

The resonator is modeled as a one-dimensional digital waveguide by using delay lines [12], [15], [38]. The delay line is an elementary functional unit used to simulate traveling wave propagation and constitutes the fundamental building block of digital waveguide synthesis. The sum of the traveling waves at a point gives the total power at that point in the waveguide. The pressure p_r exiting the reed equals the pressure entering the instrument’s resonator because the resonator’s cross-sectional area equals the one at the reed’s exit (Fig. 5). Considering p^+ as the left-going and p^- as the right-going traveling wave (Fig. 2), the total pressure inside the resonator is:

\[ p_r = p^+ + p^- \]  

The relation between the right-going pressure p^- and the left-going pressure p^+, can be expressed according to a reflection coefficient (r, (12)), which concerns the reflection at the closed end of the bore. For simplicity reasons and because of the symmetrical displacement of the reed’s blades in the X-axis [16], [17], we use the reflection coefficient of a single reed as it is described in [15]. When the mouth pressure exceeds the pressure inside the reed, the reed closes, and the reflection coefficient equals 1. When the reed opening reaches its maximum value, the reflection coefficient equals 0.

\[ p^- = rp^+ + \frac{1 - r}{2} p_m \]  

Smith [15] suggests the reflection coefficient function (13) depicted in Fig. 6, which introduces the nonlinearity, for \( h^\text{c} = \frac{t_w}{L} - p_b^m \) and \( m = \frac{1}{h^\text{c} + 1} \). The point \( h^\text{c} \) is the smallest pressure difference giving reed closure.

\[ r = \begin{cases} 1 - m(h^\text{c} - h^\text{c}_\lambda), & -1 \leq h^\text{c}_\lambda \leq h^\text{c} \leq 1 \\ 1, & h^\text{c}_\lambda \leq h^\text{c} \leq 1 \end{cases} \]  

B. THE REED-RESONATOR SYSTEM

The combination of (5), (11), and (12) gives the relation between the pressure difference (\( \Delta p \)), and the mouth pressure, the reflection coefficient and the left-going resonator pressure p^+.

\[ (\Delta p)_c = \frac{(1 + r) \left( \frac{1}{2} p_m - p^+ \right)}{1 - c p \left( \frac{q}{S_c} \right)^2} \]  

for Aulos (c_p=0)

Therefore, the total pressure p_r inside the resonator now occurs from (3).

\[ p_r = p_m - (\Delta p)_c + c \left( \frac{q}{S_c} \right)^2 \]  

for Aulos (c_p=0)
C. TONEHOLES

The effect of the tonehole, either open or closed, can be represented as a lumped circuit, symmetric $T$ transmission line model (Fig.7) with shunt impedance ($Z_s$) and series impedences ($Z_a$) [21]. The shunt impedance corresponds to the standing wave inside the resonator, and the predicted value is given by $Z_s = \frac{P_1}{U_1}$.

The series impedance for an open-hole represents a negative length correction to the resonator, and the predicted value is given by $Z_a = \frac{P_2}{U_2}$.

![FIGURE 7. T-transmission line model of the tonehole.](image)

In the case of a pressure node at the tonehole junction, the volume flow ($U$) is symmetric across the junction. If there is a pressure anti-node at the tonehole junction, the symmetry across the junction is related to the pressure ($P$).

The characteristic impedance of the instrument’s resonator is

$$R_0 = \frac{\rho c}{\pi a^2}$$  \hspace{1cm} (16)

where $\rho$ is the air density, and $c$ is the speed of sound in the propagation medium (i.e., air).

According to Keefe [21], the lumped circuit, assuming that $|Z_a/Z_s| \ll 1$, is represented in (17), where the values of $Z_a$ and $Z_s$ depend on the status of the hole (open or closed).

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_a \\ Z_s^{-1} & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix}$$  \hspace{1cm} (17)

The reflectance and the transmittance [12] of the tonehole are given by

$$r = \frac{Z_a Z_s - R_0^2}{Z_a Z_s + 2R_0 Z_s + R_0^2}$$  \hspace{1cm} (18)

$$t = \frac{2R_0 Z_s}{Z_a Z_s + 2R_0 Z_s + R_0^2}$$  \hspace{1cm} (19)

1) OPEN TONEHOLES

The series impedance for an open-hole represents a negative length correction to the resonator, and the predicted value has a similar order of magnitude as the predicted closed-hole series impedance. The shunt impedance of an open-hole in the absence of dissipation and with negligible radiation is an inerance that involves both inner and outer length corrections as well as the tonehole’s chimney height.

An open tonehole’s specific resistance $\xi_e$ accounts for visco-thermal losses at the walls of the tonehole. Considering $r_c$ as the radius of the resonator’s curvature, the specific resistance is given by

$$\xi_e = 0.25(kb)^2 + \alpha r + \frac{1}{4} k d_c \ln \left(\frac{2b}{r_c}\right)$$  \hspace{1cm} (20)

where $k = \omega/c$ is the wavenumber, $\alpha$ can be calculated by (21), and it is described in the literature as either the attenuation coefficient [41] or the real part of the propagation wavenumber [42], and $d_v = \sqrt{2\eta/(\rho \omega)}$ is the viscous boundary layer thickness in relation with the air’s shear viscosity $\eta$.

$$\alpha = 3 \cdot 10^{-5} \sqrt{\frac{\omega}{b}}$$  \hspace{1cm} (21)

The effective length of an open tonehole is given by (22).

$$l_e = \frac{k^{-1} \tan(k r) + b \left[1.4 - 0.58 \left(\frac{b}{a}\right)^2\right]}{1 - 0.61 k b \tan(k r)}$$  \hspace{1cm} (22)

The open tonehole ($a$) series equivalent length [21] is given by

$$l_a^{(o)} = \frac{0.47 b \left(\frac{b}{a}\right)^4 \tanh \left(1.84 \frac{b}{a}\right) + 0.62 \left(\frac{b}{a}\right)^3 + 0.64 \left(\frac{b}{a}\right)}{\coth \left(1.84 \frac{b}{a}\right) + 0.62 \left(\frac{b}{a}\right)^3 + 0.64 \left(\frac{b}{a}\right)^2}$$  \hspace{1cm} (23)

2) CLOSED TONEHOLES

The shunt impedance for a closed tonehole in the low-frequency limit is an acoustic compliance, whose volume equals the closed-hole volume. The series impedance for a closed tonehole may be written as a negative length correction; that is, the presence of a closed-tonehole at a pressure node is equivalent to a reduction in the main resonator length in the vicinity of the tonehole.

The closed tonehole ($c$) series equivalent length [21] is given by

$$l_a^{(c)} = \frac{0.47 b \left(\frac{b}{a}\right)^4 \cot \left(\frac{b}{a}\right) + 0.62 \left(\frac{b}{a}\right)^3 + 0.64 \left(\frac{b}{a}\right)}{\cot \left(1.84 \frac{b}{a}\right) + 0.62 \left(\frac{b}{a}\right)^3 + 0.64 \left(\frac{b}{a}\right)^2}$$  \hspace{1cm} (26)

The series impedance $Z_a^{(c)}$ and the shunt impedance $Z_s^{(c)}$ of the T-transmission line model for a closed tonehole [21] are given by

$$Z_a^{(c)} = -j R_0 \left(\frac{a}{b}\right)^2 k t_a^{(c)}$$  \hspace{1cm} (27)

$$Z_s^{(c)} = R_0 \left(\frac{a}{b}\right)^2 \cot(k r)$$  \hspace{1cm} (28)

D. IMPLEMENTATION OF THE PHYSICAL MODELING

Fig.8 presents the Graphical User Interface of the application we have developed in MATLAB 2020b to implement the physical modeling calculations described above. For the example shown in Fig.8, the parameters are set according to the ancient Greek Aulos of Poseidon (36). The user sets Aulos main physical parameters, then the fingering (open/closed toneholes) and the geometrical features of the
FIGURE 8. Graphical user interface for implementing the physical modeling of Aulos showing the model’s input parameters and the results as playable audio and its plots in the time and frequency domain. The parameters are set according to the geometrical features of the Aulos of Poseidonia.

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IV. VALIDATION

In order to evaluate our physical modeling approach described above, we coded the model using MATLAB 2020b and compared its audio output signals (Fig.9b) with the recorded signals (Fig.9a) from the replica instrument of Aulos of Poseidonia. Since the double reed instrument Aulos can be described as a closed-open pipe and therefore it is producing only the odd harmonics [18], our comparison is focusing, in the frequency domain, on the fundamental frequencies and their odd harmonics. In some of the recorded signals, even harmonics were present, as can be seen in Fig.9a2. This could be due to the factors, for simplification reasons, not considered in our model such as the reed’s saliva and the shape of the main resonator not being perfectly cylindrical. Nevertheless, the study of this phenomenon is beyond the scope of this work.

We recorded the low pipe replica of the Aulos of Poseidonia with the musician (Georgios Barbarekos) performing the seven fingerings to reproduce a scale. The microphones were placed approximately 1m away from the instrument off axis from the instrument’s bell and the musician played in piano level every note once. Fig.9 shows the case of one open tonehole for both the recording (Fig.9a) and the physical model (Fig.9b). Their signals are shown in the time and frequency (using Fourier transformation of 1 second recorded signal) domain. The recordings took place at the studio of the Laboratory of Music Acoustics and Technology, Department of Music Studies, National and Kapodistrian University of Athens, on March 4th, 2021. The equipment used: Microphone Preamplifier: Millennia HV-3D, A/D Converter:

5 This particular replica of the Poseidonia aulos was physically reconstructed at the premises of the Speech and Accessibility Laboratory of the National and Kapodistrian University of Athens, Department of Informatics and Telecommunications, on the course of the HERMES: “Towards a training music archaeology project on the reconstruction and use of ancient Hellenic Musical Instruments” project implementation [4]. This specific reconstruction is based on detailed images, measurements, and designs of the original find provided by Reichlin-Moser, Paul J. & Barbara [2013] in Der Paestum Aulós aus der Tomba del Prete. (Illustration by Verena Pavoni, Visuelle Gestaltung: Ulrich Schuwey)
We first compare the time domain characteristics of the recorded (Fig. 9a₁) and the simulated (Fig. 9b₁) signal by studying their dynamic temporal envelope. We observe that our model simulates adequately the temporal characteristics of the recording. The recorded and generated signals both need the same amount of time to reach their steady-state oscillation (i.e., attack of approx. 100msec followed by a prolonged sustain), marked with a yellow dashed line in Fig. 9a₁ and Fig. 9b₁. This is expected due to the inherent properties of the digital waveguides which adequately transform the signal propagation in the continuous space of the physical instrument into the discrete space of the virtual instrument.

Considering the above, similar inputs (in our case a constant mouth pressure) for both the physical and the virtual instrument result to a correspondingly similar attack. Taking into account that the time introduced into the model is related to the adopted excitation method [12] is the first validation of our double-reed implementation. The prolonged sustain is due to the continuous existence of the excitation force (i.e., the constant air supply from the player).

Secondly, we compare the two signals in the frequency domain. In order to study the timbre characteristics, we consider the fundamental and the three first harmonics (Table 1).

The difference in cents was calculated using the formula $1200 \log_{2} \left( \frac{a}{b} \right)$ where $a$ and $b$ are the frequencies (Hz) being compared. The musician during the recordings reproduced the scale that the instrument naturally generates according to its geometry and to the available reeds. This resulted in a diatonic scale the tonic center of which is defined by the third fingering pattern, forming with the 6th and 7th fingering patterns the approximately pure intervals of the fourth and fifth, respectively. In the case of four open tone-holes, the fundamental frequency of the recorded signal is 320Hz, and the simulated one is 323Hz deviating by 3Hz, which is translated to -16 cents. This mismatch is the most remarkable frequency difference of all the fingerings we compared, and it is still slightly above Just Noticeable Difference (JND) [43], [44]. In more detail, the absolute deviation in cents of the fundamental frequencies shows an average deviation of 6 cents and a standard deviation of $\pm 8$ cents. The first harmonics show an average deviation of 5 cents and a standard deviation of $\pm 3$ cents; the second harmonics show an average deviation of 8 cents and a standard deviation of $\pm 3$ cents. All the above results can be considered perceptually insignificant as they are well below the JND. Inharmonicity (i.e., the deviation of the overtone frequencies from the perfect integer multiples of the fundamental frequency), which is a factor that defines the

| Fingering | Timbre Characteristics |
|-----------|------------------------|
|           | Fundam. 1st OT 2nd OT 3rd OT |
| CCCCCC    | PM (Hz) 206 617 1028 1439 |
| Rec (Hz)  | 205 615 1026 1439 |
| Dev. (c)  | 8 6 3 0 |
| CCCCCO    | PM (Hz) 220 660 1099 1539 |
| Rec (Hz)  | 221 662 1103 1543 |
| Dev. (c)  | -8 -5 -6 -4 |
| CCCCCO    | PM (Hz) 241 722 1203 1684 |
| Rec (Hz)  | 241 723 1202 1684 |
| Dev. (c)  | 0 -2 1 0 |
| CCOOOO    | PM (Hz) 264 792 1321 1849 |
| Rec (Hz)  | 264 793 1320 1849 |
| Dev. (c)  | 0 -2 1 0 |
| OOOOOO    | PM (Hz) 323 968 1613 2258 |
| Rec (Hz)  | 320 963 1610 2248 |
| Dev. (c)  | 16 9 3 8 |
|          | Rounded average of absolute Dev. (c) 6 5 3 2 |

TABLE 1. Comparison of the signals from our physical model (PM) vs. the corresponding Aulos of Poseidonia’s low pipe replica recordings (Rec) (C: Closed tonehole, O: Open tonehole, Dev: Deviation in cents, Fundam.: Fundamental, OT: Overtone).
TABLE 2. Inharmonicity of the signals from our physical model (PM) vs. the corresponding Aulos of Poseidonia’s low pipe replica recordings (Rec) (C: Closed tonehole, O: Open tonehole, IH: Inharmonicity in cents, Diff.: Difference between the inharmonicity of PM and the inharmonicity of Rec in cents, OT: Overtone, abs: Absolute, Avg: Average).

| Fingering | IH (c) | 1st OT | 2nd OT | 3rd OT | Rounded Avg of abs IH per Fingering |
|-----------|--------|--------|--------|--------|-----------------------------------|
| CCCCCC    | PM     | -3     | -4     | -4     | 3                                 |
|           | Rec    | 0      | 2      | 5      |                                    |
|           | Diff   | -3     | -5     | -9     |                                    |
| CCCCCC    | PM     | 0      | -2     | -4     | 1                                 |
|           | Rec    | -3     | -3     | -4     | 3                                 |
|           | Diff   | 3      | 1      | 3      |                                    |
| CCCCCO    | PM     | -2     | -3     | -3     | 3                                 |
|           | Rec    | 0      | -4     | -3     | 2                                 |
|           | Diff   | -2     | 1      | 0      |                                    |
| CCCOOO    | PM     | 0      | 1      | 1      | 1                                 |
|           | Rec    | 2      | 0      | 1      |                                    |
|           | Diff   | -2     | 1      | 0      |                                    |
| CCOOOO    | PM     | 4      | 5      | -7     | 5                                 |
|           | Rec    | 2      | 7      | -2     | 4                                 |
|           | Diff   | 2      | -2     | -5     |                                    |
| COOOOO    | PM     | -2     | -2     | -2     | 2                                 |
|           | Rec    | 5      | 11     | 6      | 7                                 |
|           | Diff   | -7     | -13    | -8     |                                    |
| OOOOOO    | PM     | 2      | 2      | 2      | 2                                 |
|           | Rec    | 0      | 2      | 3      | 2                                 |
|           | Diff   | 2      | 0      | 3      |                                    |

Rounded Avg of abs IH per OT

| PM | 2 | 3 | 3 |
| Rec | 2 | 4 | 3 |

timbre, is similar for both the recorded and the synthesized signal (approx. 3 cents in average), see Table 2. Moreover, the total inharmonicity (calculated as the average absolute deviation for the first three overtones, demonstrated in the last column of Table 2) differentiates less than 2 cents for both the recorded and the synthesized signal in all fingerings, except for the CCOOOO which differs by 5 cents.

The microphones in the recordings where placed 1m away from the instrument and the output signal from our model was in the beginning of the mouthpiece (see Fig.8). The effect of the bell is to radiate out of the instrument the high frequencies and to reflect back the low frequencies. This reflectance can be modeled as a low-pass filter [12]. Thus, the signal generated by our model (where the output is positioned inside the resonator) compared with the relative recording (microphone as well as a potential listener positioned outside the resonator) is missing energy in the high frequencies.

V. CONCLUSION

In this investigation, we presented a digital model of Aulos, a double-reed wind instrument, implemented with physical modeling techniques. We considered two parts of the instrument in order to describe and simulate the physical phenomena (the excitation mechanism and the resonator with toneholes). Aulos excitation mechanism is the double-reed, consisting of two symmetrical blades followed by a cylindrical backbore in contrast with the conical backbore of the recently used double-reeds such as the oboe’s one. The nonlinear behavior of the double-reed is described by using quasistatic regimes defining the relation between the pressure difference and the volume flow across the reed. We used a one-dimensional digital waveguide to simulate the instrument’s resonator simulating the pressure traveling wave propagation and the air particles’ oscillation inside the resonator. The effect of the open and/or closed toneholes is modeled with respect to their impedances as a lumped T-transmission line circuit.

The comparison of the signal recorded by the replica of Aulos of Poseidonia, built in our lab, with the signal generated by the relevant physical modeling digital implementation of the instrument showed good agreement, especially in the frequency domain. In the time domain, our model simulates temporal characteristics of the recording, but further improvements could be made by applying an Attack, Decay, Sustain, Release (ADSR) envelope. In the frequency domain for a scale (seven notes), the fundamental frequencies and the three first harmonics of the signals differ only 6, 5, 3, and 2 cents on average, respectively, which is below the Just Noticeable Difference threshold. Further, the resonant frequencies in both signals only slightly differentiate from the odd multiples of the corresponding fundamental, as expected by the theory, and their total average absolute inharmonicity is approximately 3 cents for both of them.

Moreover, by clearly describing the physical phenomena, we expect to inspire researchers to: a) improve the current model by resolving the constrain of the calculated pressure point been inside the resonator and the absence of the even harmonics (as per the theory of a closed-open pipe), b) describe musical instruments as acoustic metamaterials like various apparatus are used in other fields of acoustics [45], [46] and, c) by knowing the acoustics of ancient theatres [47], [48], making them virtually sound again as they use to in their natural auditory space.

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REFERENCES

[1] A. Allen and N. Raghuvanshi, “Aerophones in flatland: Interactive wave simulation of wind instruments,” ACM Trans. Graph., vol. 34, no. 4, p. 134, 2015, doi: 10.1145/2767001.
[2] K. Bakogiannis, S. Polychronopoulos, D. Marini, C. Terzes, and G. T. Kouroupetroglou, “ENTROTUNER: A computational method adopting the Musician’s interaction with the instrument to estimate its tuning,” IEEE Access, vol. 8, pp. 53185–53195, 2020, doi: 10.1109/ACCESS.2020.2981007.
[3] S. Bilbao, “Direct simulation for wind instrument synthesis,” in Proc. 11th Int. Digit. Audio Effects Conf., Espoo, Finland, Sep. 2008, pp. 145–152. [Online]. Available: https://bit.ly/3vIumuP
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