Mass Transfer and Stellar Evolution of the White Dwarfs in AM CVn Binaries

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Abstract

We calculate the stellar evolution of both white dwarfs (WDs) in AM CVn binaries with orbital periods of $P_{\text{orb}} \approx 5$–70 minutes. We focus on the cases where the donor starts as a $M_{\text{He}} < 0.2 M_\odot$ helium WD and the accretor is a $M_{\text{WD}} > 0.6 M_\odot$ WD. Using Modules for Experiments in Stellar Astrophysics, we simultaneously evolve both WDs assuming conservative mass transfer and angular momentum loss from gravitational radiation. This self-consistent evolution yields important feedback of the properties of the donor on the mass-transfer rate, $\dot{M}$, as well as the thermal evolution of the accreting WD. Consistent with earlier work, we find that the high $M$’s at early times forces an adiabatic evolution of the donor for $P_{\text{orb}} < 30$ minutes so that its mass–radius relation depends primarily on its initial entropy. As the donor reaches $M_{\text{He}} \approx 0.02$–0.03 $M_\odot$ at $P_{\text{orb}} \approx 30$ minutes, it becomes fully convective and could lose entropy and expand much less than expected under further mass loss. However, we show that the lack of reliable opacities for the donor’s surface inhibit a secure prediction for this possible cooling. Our calculations capture the core heating that occurs during the first $10^7$ yr of accretion and continue the evolution into the phase of WD cooling that follows. When compared to existing data for accreting WDs, as seen by Cheng and collaborators for isolated WDs, we also find that the accreting WDs are not as cool as we would expect given the amount of time they have had to cool.

Unified Astronomy Thesaurus concepts: White dwarf stars (1799); AM Canum Venaticorum stars (31); Close binary stars (254)

1. Introduction

AM Canum Venaticorum (AM CVn) systems are ultra-compact binaries undergoing helium mass transfer with orbital periods, $P_{\text{orb}}$, between 5 and 68 minutes (e.g., Nather et al. 1981; Nelemans et al. 2001, 2004; Ramsay et al. 2018). Their orbital evolution is dominated by loss of angular momentum from gravitational wave radiation (e.g., Tutukov & Yungelson 1979, 1981), and the gravitational wave signal from AM CVn systems should be detectable by missions such as the Laser Interferometer Space Antenna (Nelemans et al. 2004; Amaro-Seoane et al. 2013, 2017; Kremer et al. 2017; Breivik et al. 2018). The study of AM CVn systems is of interest for several reasons. Their orbital evolution can help constrain the efficiency of tides in degenerate stars (e.g., Piro 2019). Unstable helium burning is expected to occur in some AM CVn systems, leading to helium novae (e.g., Ashok & Banerjee 2003), and in the case of dynamical helium-burning, faint thermonuclear “Ia” supernovae (Bildsten et al. 2007; Shen & Bildsten 2009). Accretion is expected to occur via an accretion disk (except at shortest periods where direct-impact accretion occurs; e.g., Marsh et al. 2004), which allows AM CVn systems to be good testing grounds for the theory of nearly pure-helium accretion disks (e.g., Kotko et al. 2012).

There are three possible channels for the formation of AM CVn systems. In the helium white dwarf (He WD) donor scenario, the initial donor is a degenerate, low-mass ($\approx 0.1$–0.2 $M_\odot$) He WD (e.g., Deloye et al. 2007). In the helium star (He star) donor scenario, the donor starts mass transfer as a nondegenerate, helium-burning star with masses ranging from $\approx 0.3$–0.7 $M_\odot$ (e.g., Iben & Tutukov 1991; Yungelson 2008). Note that at long periods $P_{\text{orb}} \gtrsim 40$ minutes, the thermal properties of the He WD and He star donors are predicted to converge (Deloye et al. 2007; Yungelson 2008). The two remain distinguishable by their compositions, with the He star donors expected to contain products of helium burning. In the cataclysmic variable (CV) donor channel (e.g., Podsia-dlowski et al. 2003), the donor starts as a $\approx 1 M_\odot$ star, initiates hydrogen-rich mass transfer around the end of core hydrogen-burning, and eventually loses its hydrogen-rich envelope. In all three channels, the accretor is often assumed to be a $0.6$–1.0 $M_\odot$ carbon-oxygen white dwarf (CO WD).

In this work, we revisit the calculations by Bildsten et al. (2006) on the thermal evolution of the CO WD accretor in the context of the He WD donor channel. In Section 2, we describe our computational setup and construction of initial stellar models for both the donor and the accretor. We evolve donor models of various initial central entropies under mass transfer to a point-mass, and describe their thermal evolution in Section 3. We show that the mass–radius relation of the donor, as well as the time evolution of $P_{\text{orb}}$ depends on their initial central entropy and whether they can cool. In Section 4, we evolve the CO WD accretor along with the donor and the binary orbit. Our results agree with Bildsten et al. (2006) but are more consistent as our models track the time-dependent cooling of the donor. At high mass-transfer rates, $\dot{M}$, near period minimum, the accretor is reheated due to accretion; as the orbit widens and $\dot{M}$ drops, the accretor luminosity eventually becomes just that of a cooling WD. We compute synthetic color–magnitude diagrams for our accretor models for comparisons with observations in Section 5. We show that observed AM CVn systems appear bluer and brighter than expected, which corresponds to a younger WD cooling age. We conclude in Section 6.

In the following, we denote the accretor and the donor by the subscripts WD and He. Quantities at the center of the stars are denoted by the subscript $c$. Initial and final quantities are
2. Computational Setup and Initial Stellar Model Construction

To obtain realistic, time-dependent mass-transfer histories for AM CVn binaries, we self-consistently evolve a He WD donor and a CO WD accretor using the binary capability of the stellar evolution instrument modules for Experiments in Stellar Astrophysics (MESA; Paxton et al. 2011, 2013, 2015, 2018, 2019). The equation of state (EOS) adopted for the WDs in this study is a blend among Rogers & Nayfonov (2002) and Saumon et al. (1995) at low densities \( \log_10(\rho/g \text{ cm}^{-3}) \lesssim 2.9 \), and Potekhin & Chabrier (2010) at high densities \( \log_10(\rho/g \text{ cm}^{-3}) \approx 2.9 \). Radiative opacities, where allowed, are taken from OPAL (Iglesias & Rogers 1993, 1996) at \( \log_10(T/K) \gtrsim 3.8 \) and Ferguson et al. (2005) at \( \log_10(T/K) \lesssim 3.8 \), and extrapolated otherwise (see Section 3.3). Electron conductive opacities are from Cassisi et al. (2007) with corrections from Blouin et al. (2020) for He under moderate coupling and moderate degeneracy. The latter corrections are implemented by modifying our copy of the MESA source code, but will be included in future releases of MESA. Our nuclear net includes the NCO reaction chain (Bauer et al. 2017), which may impact the helium shell thickness for unstable burning on the accretor. The models presented in this work differ slightly from an earlier version of models presented in van Roestel et al. (2021). The latter adopted the EOS of Timmes & Swesty (2000) during the early phases of mass transfer for the He WD donor, and included no electron conduction opacity correction from Blouin et al. (2020) for both WDs. However, difference in quantities like luminosity is insignificant (of order 10%). Our MESA input files are available on Zenodo at doi:10.5281/zenodo.5532940.

The He WD models are made using a modified version of the make_he_wd test suite, where a 1.5 \( M_\odot \) star with metallicity \( Z = 0.02 \) is evolved from the zero-age main sequence (ZAMS) to the formation of an inert helium core of mass 0.15–0.18\( M_\odot \). We then strip off the envelope, and allow the bare He core to cool to the desired core temperature. Extremely low-mass He WDs are expected to have a hydrogen envelope of mass \( \approx 10^{-3}M_\odot \) (e.g., Istrate et al. 2016). When accreted onto the CO WD the hydrogen may undergo unstable nuclear burning (e.g., Kaplan et al. 2012), which is computationally expensive to follow and has no thermal impact on the accretor; hence, we neglect it in our study. We leave the He WD with almost no hydrogen envelope (\( M_{\text{H}} \lesssim 10^{-20}M_\odot \)), which only impacts the initial mass-transfer phase, but has no impact on the mass transfer at late times (e.g., Kaplan et al. 2012).

The He WD models have initial specific central entropies of \( S_{\text{cen}}^\odot/M_{\text{H}} k_B \approx 2.23, 2.61, 3.07, 3.52 \), where \( N_A \) is Avogadro’s number. In Deloye et al. (2007), the donor models are labeled by their initial central degeneracy parameter, \( \psi_{c,\text{He}} = E_{c,\text{He}}/k_B T_c \approx 1.2 \times 10^{-2} T_c^2/\text{cm}^2 \text{ g}^{-1} \), where \( T_c, \rho_c, \) and \( E_{c,\text{He}} \) are the initial temperature, density, and electron Fermi energy at the center of the He WD. Our models correspond to \( \log_10(\psi_{c,\text{He}}) = 2.89, 2.39, 1.82, 1.34 \), respectively. Our models are chosen to be similar in range of \( \psi_{c,\text{He}} \) to those in Deloye et al. (2007). Deloye et al. (2007) determined the distribution of \( \psi_{c,\text{He}} \) at contact (see their Figure 3) based on post-common-envelope conditions of the surviving double WD systems in the population synthesis study of Nelemans et al. (2001), and modeled the mass transfer of He WDs with \( \log_10(\psi_{c,\text{He}}) \approx 1.1–3.5 \). Our coldest model is less degenerate than theirs due to inadequate EOS coverage in MESA (see below).

The CO WD models are similarly made using a modified version of the make_co_wd test suite. We evolve a ZAMS model of mass 3–6\( M_\odot \) with metallicity \( Z = 0.02 \) up to helium shell burning and the formation of a CO core of the desired mass. We then remove the envelope and allow the CO core to cool to the desired starting core temperature.

The He WD and CO WD models are then taken as initial models in MESA binary, which self-consistently evolves the stellar structures of both stars and the orbital parameters. We assume fully conservative mass transfer at a rate \( \dot{M} \) and model it using the Ritter scheme (Ritter 1988). We assume that orbital angular momentum loss is only due to the emission of gravitational waves. While the accretor may be significantly spun up by the accreted material at the expense of orbital angular momentum (Marsh et al. 2004; Gokhale et al. 2007), we choose to model both stars as nonrotating as a first approximation. Tides may play an important role in synthesizing both components, which helps stabilize the orbit, and in impacting their thermal evolution through tidal dissipation (e.g., Marsh et al. 2004; Fuller & Lai 2012, 2014). Observations show that at long periods \( \dot{P}_{\text{orb}} \gtrsim 60 \) minutes, allowing us to compare our predictions with all known AM CVn systems. Future work with MESA incorporating the EOSs of Chabrier et al. (2019) and Jermyn et al. (2021) can probe the further evolution of the donor at even lower temperatures with better numerical accuracy.

For the accretor, we set \( \tau_{\text{factor}} = 30 \) for numerical convenience, which places the outermost cell at an optical depth of \( \tau = 2/3 \times 30 = 20 \). We also opted to adopt a gray Eddington \( T - \tau \) relation instead of interpolating from the DB tables provided by Odette Toloza and Detlev Koester. The level of discrepant in the accretor effective temperature for these two approaches is \( \lesssim 10\% \). We allow the latent heat of crystallization to be released at a Coulomb coupling parameter \( \Gamma = (Z_{i}^{5/3}) e^{2}/a_{i} k_B T_{i} \) between 225 and 235 (see Equation (1) of Bauer et al. 2020), where \( (Z_{i}^{5/3}) \) is an average of \( Z_{i}^{5/3} \) over all ion species with charge \( Z_i \), and \( a_i = (3/4\pi n_i) \) is the electron separation.

As we show in Section 3, the initial thermal properties and subsequent thermal evolution of the donor are important in setting the binary orbital evolution and \( \dot{M} \) history. Therefore, for each \( M_{\text{He}} = 0.15M_\odot \) model of fixed \( S_{\text{cen}}^\odot/M_{\text{H}} k_B \), we generate an adiabatic mass–radius relation, by stripping the 0.15\( M_\odot \) He WD model to the desired mass via the relax_mass control in MESA. The rate of mass change for this construction is chosen
such that the core of the model evolves adiabatically. We obtain the cold WD (fully degenerate) limit similarly by stripping the $0.15 M_\odot$ He WD models to the desired mass, and subsequently allowing them to cool to a central temperature of $T_{c,\text{He}} = 10^3$ K. To avoid the log $Q \geq 5$ stopping condition, we irradiate the surface layer of the He WD model. This is done in part for numerical convenience, but is also expected for realistic AM CVn systems where, on geometrical grounds, the accreting WD should strongly irradiate the donor.

For comparison, we also evolved a $M_{\text{He}} = 0.35 M_\odot$ He star model taken from Brooks et al. (2015). We chose $M_{\text{WD}} = 0.5 M_\odot$ and $P_{\text{orb}} = 20$ minutes, which corresponds to the first model shown in Table 1 of their paper and of Yungelson (2008). We also obtained an adiabatic mass-radius relation for the He star using the relax_mass control described above. The initial model for this construction is taken from the binary simulation when core He burning is quenched in the He star and its mass decreases to $0.2 M_\odot$.

3. Donor Evolution

The mass-transfer rate, $\dot{M}$, determines whether the accretor is heating or cooling, and sets the evolution of binary parameters (e.g., orbital period, $P_{\text{orb}}$; Bildsten et al. 2006). It depends on the donor’s mass-radius relation, which in turn is set by its thermal evolution (Deloye et al. 2007). Thus, we start by exploring the effect of varying the donor’s initial central specific entropy, $S_{c,\text{He}}$, on $\dot{M}$ and the age-period relation. We run MESA models for initial donor masses of $M_{\text{He}} = 0.15 M_\odot$ or $0.18 M_\odot$ with $S_{c,\text{He}}/N_\text{A}k_b = 2.23, 2.61, 3.07, 3.52$. As we are first exploring the $\dot{M}(t)$ and $P_{\text{orb}}(t)$ evolution, we model the accretor as a point mass with an initial $M_{\text{WD}} = 0.75 M_\odot$. We show the effect of an initially more massive accretor by another model with $S_{c,\text{He}}/N_\text{A}k_b = 3.07$ and $M_{\text{WD}} = 1.05 M_\odot$.

3.1. Donor Mass–Radius Relation

The resulting donor’s mass–radius relations are shown in the top panel of Figure 1, and the associated power-law index $n = d \ln R_{\text{He}}/d \ln M_{\text{He}}$ in the bottom panel. Initially, the donor contracts $(n > 0)$ as the outermost radiative layer is stripped off (see Deloye et al. 2007 and Kaplan et al. 2012 for in-depth discussions). Eventually, the underlying layers drive an expansion in radius $(n < 0)$, with the lowest entropy model closest to $n = -1/3$ as expected for a fully degenerate object. In general, the less degenerate, higher entropy models show a more positive n. As $M_{\text{He}}$ decreases, the power-law slope becomes more positive, in part because Coulomb effects become more important (Deloye & Bildsten 2003).

For less degenerate donors (especially the $S_{c,\text{He}}/N_\text{A}k_b = 3.52$ donor), $n$ becomes more positive when they start to cool. Initially, the donors evolve adiabatically as the mass change timescale, $\tau_m = M_{\text{He}}/\dot{M}$, is much shorter than the thermal timescale, $\tau_{\text{th}} = \int c_p T \, dm/L_{\text{He}}$. This is confirmed by the agreement of the $R(M)$ relations from the binary calculations (solid lines) and the adiabatic models (dotted–dashed lines) in Figure 1, below $P_{\text{orb}} \approx 30$ minutes when $\tau_m \ll \tau_{\text{th}}$. As $\dot{M}$ drops with time, eventually $\tau_{\text{th}} \lesssim \tau_m$ ($\tau_{\text{th}} = \tau_m$ is labeled by square symbols). Then the donor becomes fully convective (labeled by circle symbols) and starts to cool (Deloye et al. 2007), as we discuss in the end of this section. Therefore, less degenerate donors start to converge to the mass–radius relation of a fully degenerate He WD, as seen in Figure 1 (see also Figure 8 of Deloye et al. 2007).

The mass–radius relation depends only slightly on $M_{\text{He}}$, as seen in Figure 1. At a given $S_{c,\text{He}}$, the difference between the $M_{\text{He}} = 0.15 M_\odot$ (solid lines) and $0.18 M_\odot$ (dashed lines) tracks is only evident at larger $M_{\text{He}}$ ($P_{\text{orb}} \lesssim 20$ minutes), and is negligible at small $M_{\text{He}}$ especially when donor cooling happens.

3.2. Mass Accretion Rate Histories

We show the time evolution of $\dot{M}$ in the top panel of Figure 2. As the He WD donor fills its Roche lobe, the mass-transfer rate rapidly rises to $\sim 10^{-7} - 10^{-8} M_\odot$ yr$^{-1}$. All $\dot{M}$’s peak within $10^6$ yr after mass transfer initiates, with more degenerate donors having higher $\dot{M}$ due to their smaller radii (e.g., Kaplan et al. 2012). Afterwards, $\dot{M} \propto t^{-1.3}$ as analytically expected (Bildsten et al. 2006). At a fixed age, $\dot{M}$ is lower for more degenerate donors. The ordering of $\dot{M}$ with donor degeneracy both at peak $\dot{M}$ and in the power-law phase agrees with Deloye et al. (2007) (their Figure 6). The top panel of
Figure 2. Time evolution of the mass-transfer rate, $M$, as a function of orbital period, $P_{\text{orb}}$, (top panel), orbital period, $P_{\text{orb}}$, (middle panel), and donor mass, $M_{\text{He}}$, (bottom panel), for various initial donor central specific entropies, $S_{\text{He}}^{c}$. The colored solid (gray dotted-dashed) lines are for $M_{\text{WD}} = 0.75 (1.05) M_{\odot}$ and $M_{\text{He}} = 0.15 M_{\odot}$, the colored dashed lines are for $M_{\text{WD}} = 0.75 M_{\odot}$ and $M_{\text{He}} = 0.18 M_{\odot}$, and the solid gray line is for the He star model. For comparison, we show $M \propto t^{-1.3}$ (dashed gray line; Bildsten et al. 2006) as analytically expected.

Figure 2 also shows that $M$ does not vary significantly with $M_{\text{WD}}$, as shown by the models with $M_{\text{WD}} = 0.75 M_{\odot}$ and $1.05 M_{\odot}$ at $S_{\text{He}}^{c}/N_{\text{K}} = B_{3.07}$.

The time evolution of $P_{\text{orb}}$ is tied intimately to the donor’s mass—radius relation and its thermal evolution, as shown in the middle panel of Figure 2. Less degenerate donors have a larger $P_{\text{orb}}$ at period minimum, but $P_{\text{orb}}$ evolution with time slows down after an age of $\approx 10^{8}$ yr, so that at the end, more degenerate donors catch up and have larger $P_{\text{orb}}$ at a fixed age. For the less degenerate donors, the reason for their slower time evolution is that they eventually cool when $\tau_{\text{He}} \lesssim \tau_{m}$.

Due to increased orbital angular momentum loss from gravitational wave emission, a higher total mass also speeds up the $P_{\text{orb}}$ evolution with time. This is more clearly shown by the pair of $S_{\text{He}}^{c}/N_{\text{K}} = 3.07$ models with different $M_{\text{WD}}$, and barely observable in the pairs of models with different $M_{\text{He}}$. However, a higher total mass increases the likelihood of a helium flash being triggered on the accretor, since less accumulated mass on the accretor is required given a higher

$M$ (a higher $M_{\text{He}}$ leads to a higher peak $M$; see top panel of Figure 2) and a higher $M_{\text{WD}}$ (e.g., Bauer et al. 2017). Indeed, our calculations that simultaneously evolve the stellar structure of the accretor show that the $M_{\text{WD}} = 0.75 M_{\odot}$ accretor undergoes a helium flash for the $M_{\text{He}} = 0.18 M_{\odot}$, $S_{\text{He}}^{c}/N_{\text{K}} = B_{3.07}$ model at $P_{\text{orb}} \approx 6$ minutes, while the $S_{\text{He}}^{c}/N_{\text{K}} = 3.52$ model avoids a helium flash due to the counterintuitive effect of a higher central entropy.

We show the evolution of $M$ with $P_{\text{orb}}$ in Figure 3. For $P_{\text{orb}} < 30$ minutes, an initially hotter donor gives a higher $M$ at fixed $P_{\text{orb}}$. But starting at $P_{\text{orb}} \approx 30$ minutes, the $M$ for the initially hotter donors start to converge to that for cold donors, reflecting the eventual cooling of the donors to a fully degenerate configuration. During this cooling, $M$ drops more sharply with $P_{\text{orb}}$. As analytically derived by Cannizzo & Nelemans (2015), $M \propto P_{\text{orb}}^{-\xi}$, where $\xi = (45 - 6n)/(3n - 1)$. Initially as $n \approx -1/3$, $\xi \approx -4.67$, but during cooling, $n \approx 0$ and $\xi \approx -6.67$. Thus, the sharp drop in $M$ with $P_{\text{orb}}$ is consistent with analytical expectations. Even though hotter donors are at longer $P_{\text{orb}}$ at an age of $10^{8}$ yr (due to a larger minimum $P_{\text{orb}}$), this ordering reverses before an age of $10^{9}$ yr.

For comparison, we semi-analytically derive $M - P_{\text{orb}}$ relations for an adiabatic donor evolution (dotted-dashed lines in Figure 3). Given an adiabatic mass—radius relation from MESA, we numerically integrate the binary orbital parameters ($M_{\text{He}}, M_{\text{WD}}$, and binary separation $a$) assuming conservative mass transfer and orbital angular momentum loss solely due to gravitational waves (see Equations (1) and (2) in Brooks et al. 2015). The initial values are taken to be when $M_{\text{He}}$ decreases to 0.14 $M_{\odot}$ in the MESA binary run with $M_{\text{He}} = 0.15 M_{\odot}$ and $M_{\text{WD}} = 0.75 M_{\odot}$, which correspond roughly to the moment of peak $M$ in Figure 3. The excellent overlap between the solid and dotted—dashed lines in Figure 3 at $P_{\text{orb}} \lesssim 30$ minutes illustrates the initially adiabatic donor evolution. The resulting $S_{\text{He}}^{c}/N_{\text{K}} = 3.52$ adiabatic line corresponds approximately to the hot donor lines in Figure 2 of Bildsten et al. (2006) who assumed adiabatic donor evolution as well. Since our models...
eventually cool, they track closely cool models regardless of $S_{\text{He}}^{i}$ once $P_{\text{orb}} \gtrsim 40$–50 minutes.

### 3.3. Donor Thermal Evolution Uncertainties under Mass Transfer

The impact of the thermal evolution of the donor is substantial for the observed $M(P_{\text{orb}})$ and $P_{\text{orb}}(t)$ relations and so deserves some additional scrutiny. This evolution is set by both the initial entropy and whether entropy can be lost as mass is transferred, which we now discuss.

The donor can only cool when its thermal timescale, $\tau_{\text{th}}$, is comparable to or shorter than the mass-transfer timescale $\tau_{\text{m}}$. The former is inversely proportional to the donor luminosity, $L_{\text{He}}$, so lowering $L_{\text{He}}$ may delay the cooling of the donor, and speed up the evolution of $P_{\text{orb}}$ with age.

The thermal evolution of the donor with $M_{\text{WD}} = 0.75 M_{\odot}$ and $S_{\text{He}}^{i}/N_{k}\sigma_{B} = 3.07$ is shown in Figure 4. The left panel shows a sequence of temperature-density profiles color coded by $P_{\text{orb}}$, and the right panel shows the entropy profiles of the same models. The black dashed line in the left panel shows the evolution of the core in $T - \rho$ space. At $P_{\text{orb}} \lesssim 30$ minutes, the core evolves adiabatically as $\tau_{\text{m}} \ll \tau_{\text{th}}$, and starts to cool for $P_{\text{orb}} \gtrsim 30$ minutes. This is corroborated by the entropy profiles, which show that the central entropy is roughly constant for $P_{\text{orb}} \lesssim 30$ minutes and only drops due to cooling at longer periods. This decrease of central entropy even before the donor becomes fully convective. Due to radiative diffusion, heat is lost from the center, the occurrence of which we now describe.

Before the donor becomes fully convective and cools, it has an outer convective zone (blue thick lines in Figure 4), and a radiative core. The outgoing luminosity in the thin radiative surface is set by $L/M_{\text{He}} = (64\pi G/3)(\sigma_{g}\sigma_{F}/\kappa P)^{3/2}N_{\text{ad}}/\kappa \propto 1/\kappa$, at the top of the convective boundary (Arras & Bildsten 2006). Thus, the cooling luminosity of the donor is inversely proportional to the opacity at the top radiative-convective boundary for a fully convective star. At $P_{\text{orb}} \gtrsim 20$ minutes, this is at $\log_{10}(\rho/g \text{ cm}^{-3}) \approx 0$ and $\log_{10}(T/K) \lesssim 4$. However, there are no radiative opacity tables currently at this range, and instead MESA extrapolates from the Ferguson et al. (2005) low-temperature opacity tables at the same $\log_{10}(T)$. We defer investigation of the opacity to future studies, and note that a higher opacity at the top radiative-convective boundary can delay cooling of the donor and affect the age-period relation. We also note that irradiation can play an important role in keeping the top radiative-convective boundary at a higher $T$ (and potentially $\kappa$), but this is subject to the same opacity uncertainties described above.

The same uncertainty in the donor cooling physics exists for the He star donors at late times. From Table 1 of Yungelson (2008), the typical age and $P_{\text{orb}}$ for $M_{\text{He}} = 0.35 - 0.4 M_{\odot}$, a few hundred megayears and $\approx 40$ minutes at the end of their calculations. If we take the corresponding binary parameters, and integrate forward taking $n = -0.16$, we get an age since mass transfer between $\approx 1.5$ and 3 Gyr at $P_{\text{orb}} = 65$ minutes. These are lower limits nonetheless, since $n = -0.16$ is fitted for $P_{\text{orb}} \approx 10$–35 minutes. Eventually the He star donors should start to cool (Deloye et al. 2007), as hinted by $n \gtrsim 0$ at low masses for the mass–radius relation in Figure 4 of Yungelson (2008). Indeed, this is explicitly shown by our He star model in the middle panel of Figure 2, where the age increases significantly around $P_{\text{orb}} \approx 40$ minutes as the He star starts to cool (see also Figure 1).

To summarize, we have quantified the scatter in the age-$P_{\text{orb}}$ relation considering all He WD models with donor cooling. In the middle panel of Figure 2, the range of ages at which $P_{\text{orb}} \approx 65$ minutes is 4–5 Gyr. The importance of the period-age relation, itself tied closely to the thermal properties of the donor, will become clear once we discuss the heating and cooling of the accreting WD in Section 4.
4. Heating and Cooling of the Accreting WD

We now focus on the effects of mass transfer on the accretor properties, particularly \( T_{c,WD} \). Given the small scatter in period-age relation over a range of \( T_{c,He}^i \) and \( M_{WD}^i \), we fix \( M_{He}^i = 0.15 \, M_\odot \) and the helium-burning rate is balanced by thermal neutrino losses. The initially high \( \Gamma = 225 \) for the core composition of the MESA model, as shown by the gray dashed line in the right panel. For each profile, we indicate the transition from the He envelope to the core by an open circle, and convection zone by overplotting a thick line. Core evolution is shown by the black dashed line and black arrow. We indicate the transition to degeneracy by where the electron pressure is twice the ideal gas electron pressure.

Figure 5, color coded by orbital period. As the WD further cools, the surface convection zone deepens and eventually crystallization starts when \( \Gamma = 250 \), at \( P_{orb} \approx 50 \) minutes.

The time evolution of \( T_{c,WD} \) and \( L_{WD} \) is shown in Figure 6. Consistent with Figure 5, the core first evolves adiabatically, then becomes significantly heated once the conduction front reaches the center at \( 10^7 \) yr, and cools thereafter. An initially cooler WD is heated to a lower peak \( T_{c,WD} \) and hence shows a lower \( L_{WD} \) around peak \( T_{c,WD} \). We note that, if a steady accreting disk is present, its bolometric accretion luminosity would always dominate \( L_{WD} \). We discuss the challenges of differentiating between luminosity from accretion and from the cooling WD in the Appendix.

Recent calculations by Blouin et al. (2020) show a lower conductive opacity in WD envelopes, than the Cassisi et al. (2007) conductive opacities that are used in typical WD cooling models (e.g., Bédard et al. 2020). We incorporated this correction in all AM CVn models presented in this work. Compared to a model without this correction, the reheated accretor reaches a slightly lower peak central temperature, due to faster transport of heat out of the envelope. In the cooling phase, the accretor has a higher luminosity at the same central temperature (by \( \approx 30\% \)) and cools faster. Equivalently, in well agreement with Blouin et al. (2020), this leads to a lower luminosity at a fixed cooling age. However, the difference in inferred cooling ages is much less than 1 Gyr, for a 0.9 \( M_\odot \) DB WD with \( T_{eff} \approx 10^4 \) K (their Figure 4), so its effect on our calculations is small.

We compare our results with Bildsten et al. (2006), who modeled the thermal state of the accretor using quasistatic envelope methods. For early times, before the core gets heated, they found that the WD luminosity varies only with \( M \) and \( M_{WD} \) as \( L_{WD} \propto M^{1.4} M_{WD}^{0.3} \) for \( M > 10^{-9} M_\odot \) yr\(^{-1} \). We
Figure 6. Time evolution of the accretor core temperature, $T_{c,WD}$ (top panel), and accretor luminosity (bottom panel), since the start of mass transfer. The solid (dashed) lines show models with an initial accretor mass $M_{WD} = 0.75 \, (0.85) M_\odot$, each set with a range of initial core temperatures for the accretor, $T_{c,WD}$ (5×10$^6$ for deep blue to 2×10$^7$ K for light green). We show where the models start to crystallize ($T_c = 225$) by a circle (square) symbol for the $M_{WD} = 0.75 \, (0.85) M_\odot$ models. We also show the accretion luminosity, $L_{\text{acc}} \equiv GM_{WD}M/2R_{WD}$ (dotted-dashed line).

Figure 7. Luminosity of the accretor, $L_{WD}$, as a function of the mass-transfer rate, $M$, during the rapid accretion phase for the same set of models as in Figure 6. We show $L_{WD} \propto T_c^{1.4}$ (dotted-dashed gray line) as expected from Bildsten et al. (2006) for comparison.

Figure 8. Luminosity of the accretor, $L_{WD}$, as a function of the core temperature of the accretor, $T_{c,WD}$, during the core-cooling phase. The models are the same as in Figure 6. At late times, $L_{WD} \propto T_c^{2.5}$ as expected for cooling WDs, and is independent of $T_{c,WD}$ (Bildsten et al. 2006).

However, inferring the cooling ages of AM CVn accretors (see Section 5) is subject to the caveat that, compared to normal CO WDs, AM CVn accretors may have a significant fraction of their total mass in a thick He shell, if unstable He burning can be avoided (e.g., our $M_{He} = 0.15 M_\odot$, $M_{WD} = 0.75 M_\odot$ model would have $q_{He} = M_{He}/M_{WD} = 0.15/0.9 \approx 0.17$ in the end). Since He has a different specific heat capacity than a C/O mixture, the cooling luminosity of an AM CVn accretor would be different than that of a CO WD of the same total mass, at a given cooling age.

If we combine $L_{WD} = -C_V(dT_c/dt)$, where $C_V$ is the total heat capacity, with $L_{WD} \propto T_c^{5/2}$ (e.g., Townsley & Bildsten 2004) and solve for the cooling time $\tau_{\text{cool}}$, then $\tau_{\text{cool}} \propto C_V L_{WD}^{-3/5}$. As seen in Figure 5, at the ages 1–5 Gyr that we are interested in, the $C/O$ is near the crystallization limit and the He is approximately in the liquid state. Then we can approximate, for an AM CVn accretor that has a C/O core mass $M_{CO}$ and He shell mass $M_{He}$, $C_V \approx 3(\mu_{He} M_{He}/\mu_{CO} m_p) + 2(\mu_{He}/\mu_{CO} m_p)$, where $\mu_{He} = 4$, $\mu_{CO} \approx 14$, and the specific heat capacities for C/O and He are
motivated by our MESA models (see also, Baiko & Yakovlev 2019). Similarly, for a CO WD of the same total mass $M_{\text{tot}} = M_{\text{CO}} + M_{\text{He}}$, $C_V \approx 3(k_B M_{\text{CO}}/\mu_{\text{CO}}\eta_p)$. Hence, for a CO WD and an AM CVn accretor of the same total mass $M_{\text{tot}}$ and cooling luminosity $L_{\text{WD}}$, the ratio of their cooling ages is $\tau_{\text{cool},\text{AM CVn}}/\tau_{\text{cool},\text{C/O}} \approx (M_{\text{CO}}(M_{\text{WD}}) + (2/3)(\mu_{\text{CO}}/\mu_{\text{He}}))(M_{\text{He}}/M_{\text{tot}})$. For our fiducial case, $M_{\text{He}} = 0.15 M_\odot$ and $M_{\text{WD}} = 0.75 M_\odot$, this ratio is about 1.22. Therefore, when comparing the contours of constant age for CO WDs with thin He envelopes ($q_{\text{He}} = 10^{-2}$) in Figure 10 to observed data points, the AM CVn accretors may have older actual cooling ages than inferred, by tens of percent. Nonetheless, as we will show in Section 5, there exists a cooling age discrepancy of $\approx 3-4$ Gyr at $P_{\text{orb}} \approx 65$ minutes between theory and observations, so the corrections to the cooling age presented here are insignificant.

Finally, the accretor effective temperature, $T_{\text{eff WD}}$, is shown as a function of $P_{\text{orb}}$ in Figure 9. Models with $M_{\text{WD}} = 0.65$, 0.75 and 0.85 $M_\odot$, all with $T_{\text{eff WD}} = 2 \times 10^4$ K and identical initial donor models, are shown. The WD starts cooling at $\approx 20$ minutes, so subsequent evolution of $T_{\text{eff WD}}$ with $P_{\text{orb}}$ is set by the period-age relation of the binary. For comparison, we show the $M_{\text{WD}} = 0.65$ models with a hot/cold donor from Bildsten et al. (2006). Our $M_{\text{WD}} = 0.65$ accretor model initially tracks closely the hot donor model and later at long periods ($P_{\text{orb}} \gtrsim 60$ minutes) the cold donor model. This is because our donor model starts with high entropy and we allow for its subsequent cooling. The period-age relation then starts to deviate from the hot donor model, which cools adiabatically, and to converge toward the cool donor model. We also show the DBV strip (22,400 $\leq T_{\text{eff}}/K \leq 32,000$; e.g., Córsico et al. 2019), though no AM CVn accretors are presently known to be pulsating. Finding one would certainly enable a new probe of both the WD thermal state and thickness of the He layer.

5. Comparison to Observations

We showed that at $M \lesssim 3 \times 10^{-9} M_\odot$ yr$^{-1}$, the accretor luminosity is just that of a cooling WD. If in addition we assume that, active accretion is not contaminating the data (Bildsten et al. 2006), then combining a cooling WD model with the age-$P_{\text{orb}}$ relation yields a theoretical prediction for the luminosity of an AM CVn system at a given $P_{\text{orb}}$. We compare this prediction for the accretor to observed systems, and show that the observed systems appear $\approx 3-4$ Gyr younger than predicted at $P_{\text{orb}} \approx 65$ minutes.

To compare with observations, we compute absolute magnitudes from our MESA models. Given an effective temperature, $T_{\text{eff}}$, and surface gravity, $\log g$, from MESA, we interpolate from the pure-helium model atmosphere table from the Montreal group (Bergeron et al. 2011; Blouin et al. 2018; Bédard et al. 2020) and obtain a bolometric correction for various photometric systems.

Our $V$-band absolute magnitude, $M_V$, when plotted against $P_{\text{orb}}$, follows closely the cool donor track of Bildsten et al. (2006). However, we discovered that their $M_V$ is always lower by $\approx 0.7$ mag (i.e., brighter) than the $M_V$ we obtain using the Montreal bolometric corrections with the corresponding $T_{\text{eff}}$ and an assumed $\log g = 8.5$. For example, at $P_{\text{orb}} \approx 63$ minutes, their cool donor model has $T_{\text{eff WD}} \approx 6000$ K, which should give $M_V \approx 14.949$, but their $M_V$ reads $\approx 14.3$ instead. This explains why our $M_V - P_{\text{orb}}$ track agrees better with their cool donor track than with their hot donor track, despite better agreement in $T_{\text{eff}} - P_{\text{orb}}$ with the latter.

Regardless, as shown by Ramsay et al. (2018) (their Figure 2), our $V$ or $g$-band absolute magnitudes, $M_V$ or $M_g$, are lower than observed by 2 magnitudes at $P_{\text{orb}} \approx 60$ minutes. We show this discrepancy instead by comparing our MESA tracks on a color–magnitude diagram ($M_V$ versus $g - r$ for Sloan Digital Sky Survey (SDSS) and Panoramic Survey Telescope and Rapid Response System (Pan-STARRS-1) filters; Figure 10), to observed AM CVn systems with $P_{\text{orb}} > 50$ minutes, both color coded by $P_{\text{orb}}$. By $P_{\text{orb}} \approx 20$ minutes, our $M_{\text{WD}} = 0.75 M_\odot$ (or 0.85 $M_\odot$) accretor has accumulated most of the donor’s mass, and starts to cool as a $M_{\text{WD}} = 0.9 M_\odot$ (or 1.0 $M_\odot$) WD. This explains the good agreement of our MESA tracks even before $P_{\text{orb}} \approx 40$ minutes with the corresponding 0.9 $M_\odot$ (or 1.0 $M_\odot$) cooling tracks from Bédard et al. (2020).

Nevertheless, even though the observed long-period systems lie close to the WD cooling tracks, they appear brighter and bluer than expected from our MESA tracks for the corresponding $P_{\text{orb}}$. If we interpret the observed source as dominated by the luminosity of the cooling WD, then their photometric cooling ages, as inferred from the color–magnitude diagram, are between 0.5 and 2 Gyr. In contrast, the MESA age-period relation (Section 3) would give a cooling age of 4–5 Gyr at $P_{\text{orb}} \approx 65$ minutes.

6. Conclusion

In this work, we perform binary calculations with MESA for AM CVn binaries with a He WD donor. In Section 3, we evolved He WD donor models of various initial central specific entropies, $S^*_{\text{He}}$, and showed that the initial entropy and the subsequent thermal evolution of the donor dictates the mass-transfer history and hence the age-period evolution of the binary. We find that at $P_{\text{orb}} \approx 40$ minutes, the donor starts to cool as its thermal timescale, $\tau_{\text{th}}$, is comparable to or shorter than its mass-transfer timescale, $\tau_{\text{ms}}$, although this is subject to uncertainties in the surface opacity as described in Section 3.3. We then evolve the accretor along with the donor and the orbit

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Footnote:

https://www.astro.umontreal.ca/~bergeron/CoolingModels/
in Section 4. At the initially high \( M \) near period minimum, the accretor is heated due to compressional heating. Later as \( M \) drops while the orbit widens, the accretor behaves simply as a cooling WD (at \( P_{\text{orb}} \gtrsim 30 \) minutes). Our calculations show well agreement with the semi-analytic predictions of Bildsten et al. (2006), but are more accurate since we self-consistently consistent both binary components.

Given that we theoretically expect that accretor is simply a cooling WD at \( P_{\text{orb}} \gtrsim 30 \) minutes, we compute synthetic color–magnitude diagrams for our accretor models and compare with observations (Ramsay et al. 2018) in Section 5. We show that the observed systems, if interpreted as the cooling WD, appear much younger (in terms of cooling age) than expected at the corresponding \( P_{\text{orb}} \) (by \( \approx 3-4 \) Gyr at \( P_{\text{orb}} \approx 65 \) minutes). This may be attributed in part to uncertainties in the initial entropy distribution and subsequent cooling of the donor, which affects the age-period relation of our models. Better understanding of the donor cooling can be obtained by incorporating opacities of warm dense helium in MESA. Another possibility is that the observed luminosities are contaminated by the accretion disk or boundary layer (discussed in the Appendix), although this appears unlikely in the case of Gaia14aae as revealed by eclipse modeling (Green et al. 2018). Finally, isolated WDs show evidence of a cooling delay (e.g., Cheng et al. 2019; Kilic et al. 2020), and we speculate that a similar phenomenon in the accretor may well explain the observed discrepancy. The recent discovery of five new eclipsing AM CVn systems with \( P_{\text{orb}} = 35-62 \) minutes by van Roestel et al. (2021), which constrains the luminosity contributions from different components and the degree of cooling of the donor, will help disentangle these various scenarios.

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**Appendix**

**Accretion Luminosity**

We now address the possibility that observed AM CVn systems have contributions from the accretion disk or boundary layer. As the bottom panel of Figure 6 shows, if accretion is steady, the accretion luminosity, \( L_{\text{acc}} = GM_{\text{orb}}M/2R_{\text{WD}} \), would be more than 5 times greater than the WD luminosity, \( L_{\text{WD}} \), at all times.

If we assume steady-state accretion, and model the disk as a multicolor blackbody, we would expect the disk to dominate over the cooling WD in the \( V \) band. We note that Bildsten et al. (2006) proposed the opposite because they assumed a uniform, averaged disk temperature defined by \( L_{\text{acc}} = 5\sigma T^4 \), where \( S \) is the disk surface area. Their approach underestimated the temperature in the inner disk and hence the disk contribution to the \( V \) band (again, assuming steady state). This implies we cannot expect a priori that the cooling WD dominates over the disk contribution in the \( V \) band.

However, the disk may not be in a steady state between 20 and 60 minutes (Ramsay et al. 2018; Rivera Sandoval et al. 2020, 2021), as it is expected to be thermally and viscously unstable if the mass-transfer rate, \( \dot{M} \), is between the limits \( \dot{M}_{\text{cr}} \), and \( \dot{M}_{\text{cr}} \) (Kotko et al. 2012, their Equation A2 for \( Z = 0.02 \)), shown in Figure 3. The upper limit, \( \dot{M}_{\text{cr}} \), is evaluated at the tidal radius \( R_{\text{t}} = 0.6a/(1 + q) \), where \( a \) is the binary separation and \( q = M_{\text{wd}}/M_{\text{wd}} \) is the mass ratio. The lower limit, \( \dot{M}_{\text{cr}} \) is evaluated at \( R_{\text{WD}} \), the accretor radius at the end of the simulation (essentially the radius of a 0.75 + 0.15 = 0.9 \( M_{\odot} \) WD).
As noted by Cannizzo & Nelemans (2015), Kotko et al. (2012) did not account for the zero-torque inner boundary condition in converting from \( T_{\text{eff}}(R) \) to \( M(R) \) locally in the disk. This affects the lower stability condition \( M_{\text{cr}} \), since the global condition for the disk to be cold and stable is \( T_{\text{eff}} < T_{\text{eff,cr}}(R) \) for every radius \( R \) within the disk. Because the disk effective temperature peaks at \((49/36) R_{\text{WD}}, \) with peak value given by 0.4887 \( \sigma_b \) where \( T_b = 3 GM_{\text{WD}}/8 \pi R_{\text{WD}} \sigma_b \), the lower stability condition should actually be \( 0.488 T_b < T_{\text{eff,cr}}(R = (49/36) R_{\text{WD}}). \)\(^4\) Assuming \( T_{\text{eff,cr}} \propto R^1 \), where \( \beta \approx -0.09 \), and solving for \( M \), the actual \( M_{\text{cr}} \) should be a factor \((49/36)^{0.08} \approx 15.8 \) times \( M_{\text{cr}}(R = R_{\text{WD}}) \). Thus, the green dashed line in the top panel of Figure 3 should be 15.8 times higher if we account for the zero-torque inner boundary condition, and we would expect no disk outbursts for \( P_{\text{orb}} \gtrsim 50 \) minutes. However, this is to be taken with caution since AM CVn disk outbursts are yet to be fully understood (e.g., Kotko et al. 2012; Rivera Sandoval et al. 2020).

Moreover, observations suggest an insignificant disk contribution for long-period AM CVn systems. Eclipse modeling of Gaia14aae suggests that the WD contribution is \( \approx 80\% \) (Green et al. 2018, though a He WD donor is unlikely for this system). In addition, optical spectra and photometry of long-period systems can be fitted with a single blackbody of temperature \( T_{\text{eff}} \approx 10^3 \) K (e.g., SDSS J1137+4054, \( P_{\text{orb}} = 59.6 \) minutes, and SDSS J1505+0659, \( P_{\text{orb}} = 67.8 \) minutes; Carter et al. 2014), whereas an accretion disk spectrum is expected to be flatter (e.g., Nagel et al. 2009).

These observational constraints can be met by an optically thick boundary layer. A boundary layer can be consistent with our point of view because the boundary layer may be optically thin for \( M \lesssim 10^{-10} M_\odot \) yr\(^{-1} \), and it may not spread over the entire WD surface, which implies an effective temperature much higher than \( T_{\text{acc}} \) (e.g., Piro & Bildsten 2004).

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\(^4\) The Kotko et al. (2012) approach essentially takes \( T_b < T_{\text{eff,cr}}(R = R_{\text{WD}}) \) and solves for \( M \).