Beyond pairwise network similarity: exploring mediation and suppression between networks

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Network similarity measures quantify how and when two networks are symmetrically related, including measures of statistical association such as pairwise distance or other correlation measures between networks or between the layers of a multiplex network, but neither can directly unveil whether there are hidden confounding network factors nor can they estimate when such correlation is underpinned by a causal relation. In this work we extend this pairwise conceptual framework to triplets of networks and quantify how and when a network is related to a second network (of the same number of nodes) directly or via the indirect mediation or interaction with a third network. Accordingly, we develop a simple and intuitive set-theoretic approach to quantify mediation and suppression between networks. We validate our theory with synthetic models and further apply it to triplets (multiplex) of real-world networks, unveiling mediation and suppression effects which emerge when considering different modes of interaction in online social networks and different routes of information processing in the nervous system.
networks are usually seen as a parsimonious model to describe the backbone architecture of complex systems. Accordingly, comparing different systems boils down to comparing their architecture, leading to the notion of network similarity measure\(^2\)-\(^7\). In graph theory, two graphs are isomorphic if there exists a vertex permutation that maps one network into the other, naturally leading to a binary (and not very useful in real-world systems) notion of similarity. More useful approaches proceed by projecting networks into a suite of properties summarised in some vector \(p\) (e.g. degree distribution, centrality vectors, eigenspectra, etc.) and, subsequently constructing a similarity metric \(D\) by which two networks \(A\) and \(B\) are close in the space spanned by \(p\) if \(D(A, B) = \|p_A - p_B\|\) is "small". Other ideas include the formalisation of graph kernels\(^8\), comparing networks by comparing the statistics of random walks running over them\(^9,10\), or using statistical approaches such as estimating topological correlations between networks\(^11,12\). While in all these approaches we typically have \(D(A, B) = D(B, A)\), i.e. a symmetric relation, in many cases this undirected relation is hiding an actual direction (whether causal or not). As an example, consider the triadic relation, in many cases this undirected relation is hiding an underlying interaction with a third network. Therefore, when the relation between two networks \(A\) and \(B\) conditioned on a third one \(C\) as the Jaccard index over the edge subsets of \(A\) and \(B\) formed by those edges which are absent in \(C\):

\[
NJ_p(A, B; C) = \frac{|(a \cap b) \setminus c|}{|(a \cup b) \setminus c|}.
\]

Let us see intuitively the effect of conditioning with respect to \(C\) in this way. Suppose initially that \(C\) is totally independent from \(A\) and \(B\). Then we may expect that the Jaccard index, on average, will be the same if evaluated just on the links which are absent in \(C\), so \(NJ_p(A, B; C) \approx NJ(A, B)\). Suppose on the other hand that \(A\) is influencing \(B\) indirectly, with the mediation of \(C\). Then, intuitively, removing the links of \(C\) would effectively push the partial Jaccard index to zero. A similar scenario takes place if \(A\) and \(B\) are indirectly related through direct relation to a confounding factor \(C\). Finally, \(C\) could be suppressing the influence of \(A\) in \(B\). For example, imagine that \(B\) somewhat depends on whether \(A\) and \(C\) interact synergistically, e.g., if links in \(C\) are more likely to occur if they are in one network but not on the other (probabilistic XOR gate); then removing the links of \(C\) will enhance the partial Jaccard index.

To distinguish these three scenarios, we define the Jaccard net difference

\[
\Delta[A, B; C] := \Delta = NJ_p(A, B; C) - NJ(A, B).
\]

Intuitively, if \(C\) is independent of the relation between \(A\) and \(B\) then \(\Delta \approx 0\), if it mediates or confounds such relation then \(\Delta < 0\), and if it acts as a suppressor then \(\Delta > 0\).

In what follows we construct simple generative models of independent, mediated and suppressor interactions, detailed as algorithms, we prove that these correctly generate these three types of trivariate relations, and depict numerical simulations of the outcome for finite networks.

**Independency.** A simple generative model of independency is given by three independent, Erdös–Rényi-type models, where in each of the networks each possible link independently occurs with probability \(p\), see Algorithm 1.

**Algorithm 1: Uncorrelated (\(U\))**

**Input:** \(p\)

**Output:** Erdös–Rényi adjacency matrices \(A, B, C\) which are uncorrelated (\(\Delta = 0\))

1. \(A \leftarrow 0\)
2. \(B \leftarrow 0\)
3. \(C \leftarrow 0\)
4. For \(i = 1 \text{ to } N - 1\) do
5. 5. For \(j = i + 1 \text{ to } N\) do
6. 6. If \(\text{rand} < p\) then \(A_{ij} = A_{ji} = 1\)
7. 7. If \(\text{rand} < p\) then \(B_{ij} = B_{ji} = 1\)
8. 8. If \(\text{rand} < p\) then \(C_{ij} = C_{ji} = 1\)
9. 9. Return \(A, B, C\)

The following theorem can be proved.

**Theorem 2.1.** Let \(A, B\) and \(C\) be as in Algorithm 1. Then for large networks \(\mathbb{E}(\Delta) = 0\) and the expected values of \(NJ_p(A, B; C)\) and \(NJ(A, B)\) are equal to \(p/(2 - p)\).

The proof of this theorem is given in Supplementary Note 1. We conclude that, on average, \(NJ_p\) and \(NJ\) are nearly equal for uncorrelated networks generated in this way, as partialisation with respect to an independent network \(C\) does not have any

\[
NJ(A, B) = \text{a similarity metric, and a distance can be easily defined as } d(A, B) = 1 - NJ(A, B). \text{ This quantity alone can be used to initially establish if two networks are related. Regardless of the fact that such a relation is effectively undirected or otherwise is causal (influence), in order to explore whether such relation is underpinned by a third network } C \text{ we need to consider the effect of conditioning such relation on } C \text{ (in the sense of causal mediation analysis)}^{15}. \text{ Let us then define the partial network—Jaccard index } NJ_p(A, B; C) \text{ of two networks } A, B \text{ conditioned on a third one } C \text{ as the Jaccard index over the edge subsets of } A \text{ and } B \text{ formed by those edges which are absent in } C:\n\]

\[
NJ_p(A, B; C) = \frac{|(a \cap b) \setminus c|}{|(a \cup b) \setminus c|}.
\]

1. The proof of this theorem is given in Supplementary Note 1.
effect. In Fig. 1, we illustrate this case for finite networks with \( N = 50 \) nodes and \( p = 0.5 \), finding that indeed \( \Delta = 0 \) and that \( \text{NJ}_p(A, B; C) \approx \text{NJ}(A, B) \approx 1/3 \), in good agreement with the theorem.

**Mediation.** Suppose now that \( A \) and \( B \) are both dependent on \( C \), i.e. such that if there is a link in \( C \), then there is a link in \( A \) and \( B \) (see Fig. 2 for an illustration of such case, and Algorithm 2 for a formal recipe of this generative model).

Algorithm 2 (MEDIANATE)

| Input | Output |
|-------|--------|
| \( p \), \( q \), \( N \) | Adjacency matrices \( A, B, C \) where \( C \) mediates the relation between \( A \) and \( B \) \((\Delta > 0)\) |
| \( A \leftarrow 0 \) | \( B \leftarrow 0 \) |
| \( C \leftarrow 0 \) |
| for \( i = 1 \) to \( N - 1 \) do |
| for \( j = i + 1 \) to \( N \) do |
| if \( \text{RAND} < q \) then \( C_{ij} = 1 \) |
| return \( A, B, C \) |

This describes a situation where \( C \) mediates the relation between \( A \) and \( B \) (or, alternatively, \( C \) is confounding that relation). Partializing with respect to \( C \) removes the dependence between \( A \) and \( B \) due to \( C \), which intuitively leads to \( \Delta < 0 \). The following theorem can be proved:

**Theorem 2.2.** Let \( A \) and \( B \) be as in Algorithm 2. If \( A \) and \( B \) share at least one edge besides the common edges shared with \( C \), then \( \Delta < 0 \).

The proof of this theorem can also be found in Supplementary Note 2. In Fig. 1, we show numerical results for finite networks with \( N = 50 \), with \( p = 0.5 \).

**Suppression.** Finally, let us consider the case where \( B \) depends on the interaction of \( A \) and \( C \) such that, an edge occurs in \( B \) with a certain probability if it appears in \( A \) but not in \( C \) or alternatively if it appears in \( C \) but not in \( A \) (see Fig. 2 for an illustration). This is akin to a probabilistic XOR gate. Then on average \( \text{NJ}_p(A, B; C) > \text{NJ}(A, B) \), i.e. partializing with respect to \( C \) in this case evidences suppression effects.

Algorithm 3 encapsulates the generative model. The following theorem can be proved:

**Theorem 2.3.** Let \( A \), \( B \) and \( C \) be as in Algorithm 3. Then \( E(\Delta) > 0 \).

The proof for this theorem is contained in Supplementary Note 3. In Fig. 1, we show numerical results for finite networks with \( N = 50 \), with \( p = 0.5 \) and \( q = 1 \), which are in full agreement with the theorem.

Coexistence of mediation and suppression effects. When we abandon ideal cases where only suppression or only mediation are present, and we go towards a mixture of the two, it becomes evident that both effects can be hidden and a single \( \Delta \) cannot in principle tell us if the system evidences only one out of the two mechanisms. To investigate the coexistence of both mechanisms, we run a simulation in which Algorithms 2 and 3 above are combined: in each step with probability \( 1 - \mu \) we take Algorithm 2, and with probability \( \mu \) we take Algorithm 3. Since each algorithm is performed independently, the resulting model linearly interpolates between mediation and suppression, such that measurable \( \Delta = (1 - \mu) \Delta_{\text{med}} + \mu \Delta_{\text{syn}} \) where \( \Delta_{\text{med}} \) and \( \Delta_{\text{syn}} \) are...
hidden. The results are depicted in Fig. 3a, for different instances of parameter $p$ and $q = 1$. We can have negative, null or positive values of $\Delta$ underpinned by a balance of both mediation and suppression mechanisms, and actually for the concrete set of parameters, the effect of mediation in $\Delta$ is slightly stronger than the effect of suppression (this unbalance gets more pronounced for $q < 1$). This simple interpolating model thus leads us to conclude that, in real cases, we might for instance be naively measuring $\Delta < 0$ and misleadingly concluding that there is only mediation where in fact both mediation and suppression could be at play. Accordingly, a measure describing the effects of suppression and mediation is not enough to describe and resolve the simultaneous presence of both.

\begin{algorithm}
\caption{Null model}
\begin{algorithmic}[1]
\State $B_2 \leftarrow \text{PullRandom}(B)$
\State $\Delta_S \leftarrow \text{SelectiveRewire}(A, B, C, X)$
\State $\Delta_M \leftarrow \text{SelectiveRewire}(A, B, C, X)$
\State $\Delta_{M,S} \leftarrow \frac{\Delta_S - \Delta_M}{\Delta_{M,S}}$
\State return $\Delta_{M,S}$
\end{algorithmic}
\end{algorithm}

In order to disentangle both effects, we now introduce Algorithm 4, which applies both for constructing null models for mediation (M) and suppression (S). To construct a surrogate where all suppression has been removed, starting from $A, B, C$, we perform a selective rewiring in $B$, where only those links in $B$ which are also present in $A$ but not in $C$ (or that also appear in $C$ but not in $A$) are rewired randomly. Similarly, to construct a surrogate where all mediation has been removed, starting from $A, B, C$, we perform a selective rewiring in $B$, where only those links in $B$ which are also present in $A$ and $C$ are rewired randomly.

We then compute again the net Jaccard difference on the rewired versions, which are being labelled $\Delta_S$ (applied to the case where suppression is removed) and $\Delta_M$ (applied to the case where mediation is removed), respectively. The heuristic is then simple: if there is, e.g., hidden suppression in the data (representative mediation), then $\Delta_S < \Delta$ (respectively $\Delta_M > \Delta$), whereas if such mechanism is absent then $\Delta_S \approx \Delta$ ($\Delta_M \approx \Delta$).

Now, we also need to take into account finite-size effects which irremediably add spurious mediation and suppression effects (i.e. triplets of purely random, uncorrelated networks will show small

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Interpolating model. a Values of Jaccard net difference $\Delta$ (see the text), obtained averaging 50 realisations of an interpolating model giving a blend of mediation and suppression (see the text), where each network has $N = 300$ nodes and varying model parameter $p$, as a function of the interpolation parameter $\mu$ (in each of the $N (N-1)$ steps we apply Algorithm 2 with probability $1 - \mu$ and Algorithm 3 with $\mu$). b Revised version of the left panel, this time selectively removing all suppression and mediation effects according to our theoretical framework. The original curve, for an interpolating model generating a blend of mediation and suppression, is depicted in black. The blue curve (R) is pure randomisation, which generates $\Delta \approx 0$. The dotted green line (S) corresponds to a selective rewiring that removes all hidden suppression: in that case, the curve is kept always below zero (increasing $\mu$ increases the amount of Algorithm 3, but then is selectively rewired, thus effectively randomising the networks and pushing $\Delta$ to zero). The pink curve (M) is the result of a selectively rewiring that removes all hidden mediation: in that case the curve is pushed to the regime $\Delta > 0$. As $\mu$ increases, the amount of Algorithm 3 (generating suppression) is increased, hence pushing $\Delta$ to larger values. The dashed yellow (S,R) and pink (M,R) lines are the result of selectively rewiring on the randomised networks, and only highlight the residual values of suppression or mediation which occur by chance (as a finite-size effect) in randomised networks.}
\end{figure}
but non-zero mediation and suppression due to chance). To counterbalance such effects, we also proceed to selectively remove suppression and mediation from a completely randomly rewired version of $\mathcal{B}$, which we call $\mathcal{B}2$, leading to two new indices: $\Delta_{S}$ and $\Delta_{RM}$. We can finally combine these to produce normalised indices of mediation ($\bar{m}$) and suppression ($\bar{s}$) by normalising them dividing over the maximum possible value of suppression (mediation) attainable by a generative model such as Algorithm 3 (2) to the triplet of networks, i.e.

$$\bar{m} = \frac{\Delta_S - \Delta_{RS}}{M_{\text{max}}}, \quad \bar{s} = \frac{\Delta_M - \Delta_{RM}}{S_{\text{max}}}.$$  \hspace{1cm} (4)

Accordingly, the role that $\mathcal{C}$ plays in the relation of $\mathcal{A}$ and $\mathcal{B}$ is described by the duplet $(\bar{m}, \bar{s})$.

In addition, a significance value for these indices could be defined as:

$$\sigma_{\bar{S}, \bar{M}} = \frac{|\Delta_{[S,M]} - \Delta_{RS,M}|}{\text{std}(\Delta_{RS,M})},$$ \hspace{1cm} (5)

where $\Delta_{[S,M]}$ simply means that the expression equally applies for mediation and suppression, depending on what label we take. It is worth to stress that for large networks the finite-size effects become less common, and $\Delta_{RS,M}$ will tend to zero, for all the realisations, resulting in a small standard deviation, and in any value of suppression and mediation being highly significant.

For illustration, in the left panel of Fig. 4 we show the effects of the sequence of selective rewirings on $\Delta$ applied to a particular example of three independent, Erdős–Rényi networks with $N=300$ nodes and wiring probability $p=0.3$ (i.e., Algorithm 1). The original value of $\Delta$ is very close to zero, as well as the ones obtained from full randomisation of $\mathcal{B}$. Since in this example the networks are independent, any mediation or suppression is only a spurious residual due to finite-size effects, this residual is flagged out in similar terms by a selective rewiring on the actual network $\mathcal{B}$ ($\Delta_{2}$) or on its full randomisation ($\Delta_{22}$), hence the violet and green histograms overlap, and similarly the pink and pale blue ones also overlap. As an additional illustration, we applied the sequence of selective rewirings on the results of the interpolating model. Results are shown in Fig. 3b.

**Empirical networks.** We now turn to real-world networks and consider four types of 3-layer multiplex networks, including (i) different modes of social interaction in Twitter during the 2014’s New York City Climate March (NYC), (ii) different types of social interaction—proximity, phone call/text message, Facebook—as collected in Denmark (Copenhagen), (iii) different interpersonal relations inside a corporation (Lazega law firm) and (iv) different synaptic junctions in a neuronal network (C. elegans), see Table 1 for details.

To begin with, in Fig. 5 we confirm that, with the exception of the pair NYC Retweets vs Replies, all other possible pairs of layers in the four examples are indeed genuinely related—i.e., showing substantially more similarity than a null model. In each case, we plot $N\{\mathcal{A}, \mathcal{B}\}$ (blue bars) and as a reference, as black lines we also plot the average result of $N\{\mathcal{A}, \mathcal{B}\}$ (± standard deviation) after $\mathcal{A}, \mathcal{B}$ have been appropriately randomised. We confirm that the similarity between each pair of networks is not the result of a finite-size effect and thus exploring the role of a third network ($\mathcal{C}$) is justified.

We then turn to analyse the role of $\mathcal{C}$. For illustration, the whole selective rewiring process described in Algorithm 4 is depicted with detail for a specific example (the case of the C. elegans multiplex where we explore the role that the electrical synapses layer play in the relation between the monadic and the polyadic layers) in the right panel of Fig. 4. We provide the original value $\Delta_{0}$ and the distributions of the $\Delta$ values obtained after each of the rewiring procedure, concluding that this network indeed shows non-negligible mediation and suppression effects.

The normalised indices of mediation and suppression for the rest of permutations in all the real-world multiplex networks are reported in Fig. 6. The first thing we can observe is that overall there is substantially more mediation than suppression, although we also observe the latter mechanism. All effects are statistically significant ($\sigma \gg 1$ in every case) except for the suppression in the Lazega advice layer and the NYC Retweet layer, where $\sigma = 2$ in both cases.

In the case of the Copenhagen multiplex, the only layer which evidences a significant role in the relation of the other two is the phone/sms layer, which we show displays both mediation and suppression effects, although the former is notably stronger. For the proximity network, we considered averaged values over the
that all pair of layers are genuinely related with the possible exception of the one that displays a larger mediation role. The increased stronger suppressing effect, whereas the monadic chemical layer displays a suppressor effect (the one played by the advice layer is the closest proximity range). Also, the phone network was built comparable to the one of Facebook links, corresponding to the whole 4 weeks, and used an adjacency matrix of a density of them accounting for a different spatial scale) and at the same time keeping the edge density constant, we build non-overlapping ranges are non-uniform but the number of edges in each of which is indeed the smallest multiplex). suppression role of the layer of electrical synapses reflects the evidence that chemical and electrical synapses closely interact and serve related functions\cite{13}, so that when either of the chemical layers is taken as $C$ the presence of the other chemical layer accounts for a reduced suppression/mediation.

As a final analysis, and in order to show how suppression and mediation can be functionally modulated within a particular real-world example, we examine the role played by the proximity layer in the relation between the Facebook and phone calls/SMS layers when such layer is systematically varied. In this multiplex, the proximity network is originally reconstructed using Bluetooth signal strength between participants, by assigning a link between each pair of nodes whose relative Bluetooth strength, averaged over the whole period of the recording (4 weeks) belongs to a given range. In order to build different proximity networks (each of them accounting for a different spatial scale) and at the same time keeping the edge density constant, we build non-overlapping Bluetooth intensity ranges by taking into account the original Bluetooth intensity distribution (see the inset of Fig. 7). In this way, ranges are non-uniform but the number of edges in each of this range is the same, hence the resulting proximity networks have all the same edge density while describing different scales of physical proximity. The intuition is that only the smaller scale is a meaningful proximity network, and for larger spatial scales the whole 4 weeks, and used an adjacency matrix of a density comparable to the one of Facebook links, corresponding to the closest proximity range. Also, the phone network was built irrespective of the timing of the interaction.

In the case of the Lazega law firm, all three layers display very high mediation, but such effect is notably stronger for the co-working network, i.e. within this firm dyadic friendships are related to the dyadic advisory relations, and this is mediated by the fact that these are co-working. Only the friendship layer displays a suppressor effect (the one played by the advice layer is non-significant), i.e. pairs of individuals that are not co-working can have an advisory relationship (or otherwise) because they are friends, or pairs of co-working will also have an advisory relationship without the needs of them being friends.

In the case of the Twitter triplet (NYC), only the Replies layer has all the same edge density while describing different scales of physical proximity. The intuition is that only the smaller scale is a meaningful proximity network, and for larger spatial scales the

### Table 1 Summary of network specificities.

| Triplet         | Type                               | $N$  | Network #1                  | Network #2                  | Network #3                  |
|-----------------|------------------------------------|------|-----------------------------|-----------------------------|-----------------------------|
| C. elegans      | Neuronal; directed; unweighted     | 279  | Monadic (1639 edges)        | Polyadic (1393 edges)       | Electrical (1031 edges)     |
| NYC             | Twitter; directed; weighted        | 102439 | Retweet (213754 edges)      | Mentions (131679 edges)     | Replies (8062 edges)        |
| Lazega law firm | Social (offline); directed; unweighted | 71  | Co-work (892 edges)         | Friendship (575 edges)      | Advice (1104 edges)         |
| Copenhagen      | Social (offline/online); undirected; weighted | 751  | proximity (13020 edges)     | Facebook (12847 edges)      | Calls/SMS (1760 edges)      |

The first three examples are multiplex networks collected from comunelab.fbk.eu/data.php, whereas the fourth one was found through icon.colorado.edu. The C. elegans multiplex describes the C. elegans connectome, where layers correspond to different synaptic junctions: chemical monadic (“MonoSyn”), polyadic (“PolySyn”) and electrical (“Electr”)\cite{20,21,22,23,24}. The NYC multiplex describes Twitter activity during an exceptional event, the NYC Climate March in 2014, and layers correspond to retweet, mentions and replies\cite{22}. The Lazega law firm depicts three kinds of social relationships (Co-work, Friendship and advice) between partners and associates of a corporate law partnership\cite{25,26,27}. Finally, the Copenhagen multiplex describes social interaction in three layers corresponding to phone calls and text messages (merged), Facebook friendships and proximity as measured with the strength of Bluetooth signal\cite{28}.

### Fig. 5 Similarity between pairs of real-world networks.

Values of the Jaccard index $NJ(A,B)$ computed on the four empirical multiplex networks considered, see Table 1 (for each multiplex, we consider the three pair permutations). As a reference, black horizontal lines display the proximity (13020 edges) Facebook (12847 edges) Calls/SMS (1760 edges)

### Fig. 6 Normalised suppression-thingation plane for real-world networks.

We display the normalised indices of mediation and suppression for four real-world multiplex networks. For each multiplex we permute the role of network $C$ across the three layers, and for each case the specific layer which plays the role of $C$ is highlighted in parenthesis. Dot clouds are the result of repeating the rewireing procedures 500 times (for multiplexes with a large number of nodes, the figure displays very little dispersion, and the cloud is only perceptible for the Lazega triplet which is indeed the smallest multiplex).
resulting proximity networks do not really imply any real interaction between the nodes. Then, for each resulting proximity network, we estimate the role it plays in the relation between the other two networks and plot it in the \((m, s)\)-plane. Results are shown in the outer panel (a) of Fig. 7. For proximity networks describing large spatial scales, the network is essentially independent of the other two, and it is only when the proximity network captures a smaller spatial scale (i.e., when links describe real physical proximity) that an indirect effect (notably, mediation) gets amplified (see panel b of the same figure). This evidence would speak to the fact that taking Facebook links as a proxy of a wider and less compromising community-based relationship, one tends to communicate by phone/SMS with people who are closer in daily life (same classroom/dorm/sports club).

## Conclusion

In this paper, we have proposed a simple strategy to assess the role that a given network might play in shaping the relation between two other networks, thus enlarging the paradigm of network similarity beyond the classical pairwise comparison. This approach is aligned to a recent endeavour that aims at going beyond dyadic interactions in the characterisation of complex systems\(^{18}\) and takes inspiration from the causal mediation literature\(^{15,19}\). We make use of a set-theoretic approach to define a similarity metric between a pair of networks and to further explore if such a relation is independent of, mediated by, or suppressed by a third network that might be hidden. We introduce simple generative models that, we prove, produce pure mediation and suppression. We then explore the coexistence of mediation and suppression and develop a procedure to disentangle both indirect effects. The whole methodology is subsequently applied to a range of real-world, three-layer multiplex networks, and we unveil previously unnoticed mediation and suppression effects in social and neuronal networks which we also briefly interpreted.

Altogether, this work sparks further research in several areas. First, the simplicity and tractability of our selected approach make it easily applicable across the disciplines (in that sense, note that user-friendly versions of all codes are freely available). Second, our approach can be readily generalised to consider not just isolated triplets of networks. Indeed, one can sequentially apply this protocol to a multiplex network of an arbitrary number of layers, or to a temporal network, and accordingly derive concepts of causality and directionality in those contexts.

As it currently stands, our method is applicable to both directed and undirected networks, but the current version does not take into account weights, something which could be taken into account in further generalisations. At the same time, observe that this work focuses on the special case where two networks are considered to be related if they are close according to a distance based on Jaccard’s index. This is a concrete choice that was selected for simplicity, however, the conceptual framework is much more general and flexible, and mediation and suppression between networks can be explored in a variety of ways. For instance, further work could explore (i) different distance functions—including those that do not require the networks to have the same number of nodes, thus extending the formalism to deal with multilayer networks—and (ii) other types of relations beyond graph distances.

## Data availability

All relevant data are in the public domain and have been appropriately referenced.

## Code availability

All codes are available at https://github.com/danielemarinuzzo/partialjaccard.

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AUTHOR CONTRIBUTIONS

L.L., S.S. and D.M. participated in the conceptualisation of the study and discussed and interpreted results. L.L. and D.M. built the theory and its numerical implementation, developed the analyses and wrote the paper.

COMPETING INTERESTS

The authors declare no competing interests.

ADDITIONAL INFORMATION

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