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Updating $\overline{m}_{c,b}(\overline{m}_{c,b})$ from SVZ-moments and their ratios

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Using recent values of $\alpha_s$, the gluon condensates $\langle \alpha_s G^2 \rangle$ and $\langle g^2 f_{abc} G^2 \rangle$ and the new data on the $\psi'/\Upsilon$-families, we update our determinations of the $\overline{\text{MS}}$ running quark masses $\overline{m}_{q,b}(\overline{m}_{c,b})$ from the SVZ-moments $\mathcal{M}_n(Q^2)$ and their ratios $[1,2]$ by including higher order perturbative (PT) corrections, non-perturbative (NPT) terms up to dimension $d = 8$ and using the degree $n$-stability criteria of the (ratios of) moments. Optimal results from different (ratios of) moments converge to the average mean values: $\overline{m}_t(\overline{m}_t) = 1264(6)$ MeV and $\overline{m}_t(\overline{m}_b) = 4188(8)$ MeV in Table 4, which improve and confirm our previous findings $[1,2]$ and the recent ones from Laplace sum rules $[3]$. Comments on some other determinations of $\overline{m}_c(\overline{m}_t)$ and $\langle \alpha_s G^2 \rangle$ from the SVZ-ratios of moments in the vector channel are given in Section 5.

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1. Introduction and SVZ-moments

In Refs. [1,2], we have used different $\mathcal{M}_n(Q^2)$ moments and their ratios $r_n^{\psi'/\Upsilon}/r_n^{\tau'/\mu}$ introduced by SVZ $[4,5]$ for extracting the values of the charm and bottom running quark masses $\overline{m}_{c,b}(\overline{m}_{c,b})$ and the dimension $4: \langle \alpha_s G^2 \rangle$ and $6: \langle g^2 f_{abc} G^2 \rangle$ gluon condensates. Using the recent values of the gluon condensates from Laplace sum rules $[3,15]$ and new data on the $\psi'/\Upsilon$-families masses and leptonic widths $[16]$, we shall improve in this paper our previous results for the quark masses. Here, we shall be concerned with the two-point correlator:

$$-\langle g^{\mu\nu} q^2 - q^\mu q^\nu \rangle \Pi_{\psi}(q^2)$$

$$= i \int d^4 x e^{-iqx} \langle 0 | T J_\psi^{\mu}(x) (J_\psi^{\nu}(0))^\dagger | 0 \rangle,$$

associated to the $J_\psi^{\mu} = \bar{\Psi} \gamma^{\mu} \Psi$ ($\Psi = c, b$) heavy quark neutral vector current. The corresponding moments are $[2]:$

$$\mathcal{M}_n(-q^2 \equiv Q^2) \equiv 4\pi^2 (-1)^n \frac{d^n}{d Q^2} \Pi(-Q^2)$$

$$\equiv \int_{4m_0^2}^{\infty} dt \frac{R(t, m_c^2)}{(t + Q^2)^{n+1}}. \quad (2)$$

Their ratios read:

$$r_n^{\psi'/\Upsilon}(Q^2) = \frac{\mathcal{M}_n(Q^2)}{\mathcal{M}_{n+1}(Q^2)}, \quad r_n^{\tau'/\mu}(Q^2) = \frac{\mathcal{M}_n(Q^2)}{\mathcal{M}_{n+2}(Q^2)}, \quad (3)$$

where the experimental sides are more precise than that of the moments $\mathcal{M}_n(Q^2)$. It has been noticed by $[18,19]$ that the OPE of $\mathcal{M}_n(0)$ breaks down for higher values of $n$, while it has also been mentioned in $[1,2]$ that low moments $n \leq 3$ are sensitive to the way for parametrizing the high-energy part of the spectral function (hereafter called QCD continuum) making the results obtained from low moments model-dependent. Therefore, one should look for compromise values of $n$ (stability in $n$) where both problems are avoided. Another way out is to work with the $Q^2 \neq 0$ moments $[11]$ where the OPE converges faster while the QCD continuum contributions are strongly suppressed.

2. Expressions of the SVZ-moments $\mathcal{M}_n(Q^2)$

The QCD expressions of the moments can be derived from the ones of $R$. The on-shell expression of the spectral function is transformed into the $\overline{\text{MS}}$-scheme by using the known relation between

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2. For reviews, see e.g. [6-14].
the on-shell and MEA-scheme running quark masses. The sources of different PT contributions up to order $\alpha_s^3$ for $\mathcal{M}_n(Q^2 = 0)$ and up to order $\alpha_s^2$ for $\mathcal{M}_n(Q^2 \neq 0)$ are quoted in [1] and will not be re-quoted here. The same for the different NP contributions up to dimension $d = 8$ where one notice that the $d = 4$ condensate contribution is known to NLO. Some explicit numerical QCD expressions of the moments can be found in Ref. [1]. We shall use the QCD parameters given in Table 1. To the value of $\alpha_s(M_Z)$ quoted there, correspond:

$$\alpha_s(M_t) = 0.397(15) \quad \text{and} \quad \alpha_s(m_t) = 0.227(7),$$

(4)

where we have used the recent determinations from a recent global fit of the (axial-)vector and (pseudo)scalar charmron and bottomonium systems using Laplace sum rules [3]:

$$\bar{m}_t(M_t) = 1264(10) \text{ MeV}, \quad \bar{m}_b(M_t) = 4.184(9) \text{ MeV}. \quad (5)$$

The low-energy part of the spectral function is well described by the sum of different resonances contributions within a narrow width approximation (NWA). For the c-quark channel, it reads:

$$R_c(t) \equiv 4\pi \text{Im} \Pi_c(t + i\epsilon) \equiv \frac{\pi N_c}{Q^2\alpha_s^2} \sum_{J/\psi} M_{J/\psi} \Gamma_{J/\psi \rightarrow \ell^+\ell^-} \delta(t - M_{J/\psi}^2),$$

(6)

where $N_c = 3$; $M_{J/\psi}$ and $\Gamma_{J/\psi \rightarrow \ell^+\ell^-}$ are the mass and leptonic width of the $J/\psi$ mesons; $Q_c = 2/3$ is the charm electric charge in units of $e$; $\alpha = 1/133.6$ is the running electromagnetic coupling evaluated at $M_Z^2$. We shall use the experimental values of the $J/\psi$ parameters compiled in Table 2.

We shall parametrize the contributions from $\sqrt{\mathcal{F}} \geq (4.5 \pm 0.1)$ GeV using either:

- **Model 1**: The approximate PT QCD expression of the spectral function to order $\alpha_s^3$ up to order $(m_t^2/t)^6$ given in [24] and the $\alpha_s^3$ contribution from non-singlet contribution up to order $(m_t^2/t)^2$ given in [25].

- **Model 2**: The asymptotic PT expression of the spectral function known to order $\alpha_s^3$ where the quark mass corrections are neglected.\(^3\)

- **Model 3**: Fits of different data above the $\psi(2S)$ mass: we shall take e.g. the results in [25] where a comparison of results from different fitting procedures can be found in this paper (see e.g. [26]).

3 Original papers are given in Refs. 317 to 321 of the book in Ref. [7].
one $\bar{m}_c(\bar{m}_c) = 1275(23)$ MeV from [25] and the improved recent estimate $1279(10)$ MeV from [26] obtained for $\alpha_s(M_Z) = 0.118$ from the analogous $n = 1$ moment. The sensitivity of the results on the high energy part of the spectral function may question the accuracy of the results quoted in these papers $\mathcal{M}_1(0)$.

- Instead, in the $n$–stability region, the QCD continuum-model dependence of the result disappears (see Fig. 1) and leads to the optimal and more accurate value given in Eq. (7):

$$\bar{m}_c(\bar{m}_c) = 1266(9)\text{ MeV }.$$ (11)

The error due to the parametrization of the spectral function is even reduced when working with the ratio of moments (see Fig. 2) leading to the result in Eq. (8):

$$\frac{\bar{m}_c(\bar{m}_c)}{\bar{m}_c} = 1264(11)\text{ MeV },$$ (12)

but the errors due to the QCD parameters have increased compared to the one of the moment.

- One can also notice from the Tables in Ref. [26] that the stability of the central values is reached from $\mathcal{M}_{n=2,3}(0)$ which is about 10 MeV below their favoured choice from $\mathcal{M}_1(0)$. A such value is in a better agreement with our previous results quoted in Eq. (7).

- We estimate the errors in the truncation of the PT series by including the $\alpha_s^4$ contribution assumed to be of the same size as the $\alpha_s^3$ one (a geometric growth of the coefficient observed for massless quarks [29] may not be extrapolated for heavy quarks). The induced error is about 5 MeV which is smaller than the one of 19 MeV quoted in Ref. [25] estimated using some iterative or contour improved procedures where the effect of the subtraction scale $\mu$ is also included.

- However, it is not clear that moving the subtraction scale from $\bar{m}_c(\bar{m}_c)$ to higher values, say 3 GeV [25–28] for improving the convergence of the PT series can help due to the ambiguity of the charm quark mass definitions used in the OPE $\langle 1/\bar{m}_c \rangle$ expansion. Indeed apart the Wilson coefficient of $\langle \alpha_s G^2 \rangle$ known to NLO [30], the ones of the high-dimension condensates are only known to LO. Refs. [26–28] choose to work with the pole mass in the OPE which, as emphasized in [25] is ambiguous due to the IR renormalon contribution. Then, the use of the running mass in the OPE can be better justified which is also consistent with the use of the running mass in the PT contributions. However, if one moves the subtraction scale $\mu$ from $\bar{m}_c(\mu) = 1.264$ to 3 GeV, $\bar{m}_c(\mu)$ moves from 1.264 to 0.972 GeV which can induce an enhancement of about $1.3^d$ for the dimension $d$ condensate contributions to the moments. Therefore, a careful analysis including radiative corrections to the Wilson coefficients of each condensate should be done when working at high values of $\mu$. To my knowledge, this point has not yet been carefully studied. In order to circumvent a such large enhancement, which does not arise when working with the Laplace sum rule [3] where an optimal value of $\mu$ has been derived, we limit here to the (usual and natural) choice $\mu = \bar{m}_c$ and do not try to move it arbitrarily around this value.

- Coulombic corrections have been roughly estimated in Ref. [1]. However, it has been also argued in Ref. [17] that this contribution, which is not under a good control, can be safely neglected in the relativistic sum rules. Therefore, we shall not consider such corrections in this paper.

- In [12,17], the set of QCD parameters:

$$\bar{m}_c(\bar{m}_c) = 1275(15)\text{ MeV }, \quad 0.7 \leq \langle \alpha_s G^2 \rangle \times 10^3 \leq 6.3\text{ GeV }.$$ (13)

obtained from the moments used here has been favoured. Examining Figs. 4 and 5 of [17], one can see that the values of $\bar{m}_c(\bar{m}_c)$ from the different moments alone cannot fix accurately the values
Fig. 5. Values of $m_\nu(m_b)$ from the moments $N_{\nu}(m_b^2)$ and their ratios $r_{\nu}(m_b^2)$ for different values of $n$ using the QCD input parameters in Table 1 and Model 1 given previously for the QCD continuum parametrization. In the $n$ axis: 1 = $r_{1/2}$, 2 = $r_{2/3}$, 3 = $r_{3/4}$, 4 = $r_{4/5}$, 5 = $r_{5/6}$, 6 = $r_{6/7}$, 7 = $r_{7/8}$, 8 = $r_{8/9}$.

Fig. 6. Values of $m_\nu(m_b)$ from the moments $N_{\nu}(m_b^2)$ and their ratios $r_{\nu}(m_b^2)$ for different values of $n$ using the QCD input parameters in Table 1 and Model 1 given previously for the QCD continuum parametrization. In the $n$ axis: 7 = $r_{7/8}$, 8 = $r_{8/9}$, 9 = $r_{9/10}$, 10 = $r_{10/11}$, 11 = $r_{11/12}$, 12 = $r_{12/13}$, 13 = $r_{13/14}$.

Fig. 7. Values of $m_\nu(m_b)$ from the moments $N_{\nu}(m_b^2)$ and their ratios $r_{\nu}(m_b^2)$ for different values of $n$ using the QCD input parameters in Table 1 and Model 1 given previously for the QCD continuum parametrization. In the $n$ axis: 10 = $r_{10/11}$, 11 = $r_{11/12}$, 12 = $r_{12/13}$, 13 = $r_{13/14}$.

Table 3. Masses and electronic widths of the $\Upsilon$ family from PDG 16 [16].

| Name | Mass [MeV] | $\Gamma_{\gamma^{\pm\pm}e^{-}}$ [keV] |
|------|------------|---------------------------------|
| $\Upsilon(1S)$ | 9460.3(26) | 1.340(18) |
| $\Upsilon(2S)$ | 10023.2(31) | 0.612(11) |
| $\Upsilon(3S)$ | 10355.2(5) | 0.443(8) |
| $\Upsilon(4S)$ | 10579.4(12) | 0.272(29) |
| $\Upsilon(5S)$ | 10891(4) | 0.317(7) |
| $\Upsilon(6S)$ | 10987(10) | 0.133(3) |

$m_c = 0$ (Model 2) from the threshold $\sqrt{s} = 4.5$ GeV. We use the coefficients:

$$\lambda_{\nu} = 0, \quad \delta_{\nu} \approx 3.6, \quad \gamma_{\nu} \approx 0.6, \quad \alpha_{\nu} \approx -2.3, \quad \beta_{\nu} \approx 4.3,$$

fixed from $\tau$-decay data by assuming that they can be applied here. We found that, in the example $n = 1$ and $Q^2 = 0$, this effect is completely negligible even allowing a low value of $\sqrt{s} = 1.65$ GeV at which the fit of the coefficients has been performed.

6. Running $m_b(m_b)$ bottom quark mass from $M_{\Lambda}(Q^2)$

The previous analysis is extended to the $b$-quark mass. We shall use the data input in Table 3. Behaviours of the ($\alpha_{\nu}$) of moments versus the degree of the moments are given in Figs. 5 to 7. We deduce as optimal values the overlapping regions of the one from the moments and the ratios of moments. We obtain to order $\alpha_{\nu}^2$ (in units of MeV):

$$m_b(m_b) = 4189.9(8.2)_{\chi^2}(1.7)_{\alpha_{\nu}^2}(0.8)_{C^2}(0.2)_{C^3}(0.2)_{C^4},$$

and to order $\alpha_{\nu}^2$ (in units of MeV):

$$m_b(m_b) = 4189.2(6.4)_{\chi^2}(1.6)_{\alpha_{\nu}^2}(3.6)_{\alpha_{\nu}^2}(0.5)_{C^2}(0.0)_{C^3}(0.0)_{C^4},$$

$$m_b(m_b) = 4187.7(4.3)_{\chi^2}(1.5)_{\alpha_{\nu}^2}(5.0)_{\alpha_{\nu}^2}(0.3)_{C^2}(0.3)_{C^3}(0.3)_{C^4}.$$  

These results are quoted in Table 4.

7. Conclusions

We have updated our previous results in Refs. [1,2] from SVZ- ($\alpha_{\nu}$) of moments. These results are confirmed and improved by the new ones summarized in Table 4. The simultaneous
use of the higher moments and their ratios reduce notably the errors in the mass determinations. Though it is difficult to estimate the systematic errors of the approach, we can expect that they are at most equal to the ones quoted in this paper. These new results are also in perfect agreement with the ones quoted in Eq. (5) from a recent global fit of the (axial-)vector and (pseudo)scalar charm and bottomonium systems using Laplace sum rules [3]. Some comments on the existing estimates of the quark masses and gluon condensates from SVZ-(ratios of) moments are given in Section 5. Our results are comparable with recent results from non-relativistic approaches [33] but more accurate.

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