Chiral Shock Waves

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We study the shock waves in relativistic chiral matter. We argue that the conventional Rankine-Hugoniot relations are modified due to the presence of chiral transport phenomena. We show that the entropy discontinuity in a weak shock wave is linearly proportional to the pressure discontinuity when the effect of chiral transport becomes sufficiently large. We also show that rarefaction shock waves, which do not exist in usual nonchiral fluids, can appear in chiral matter. These features are exemplified by shock propagation in dense neutrino matter in the hydrodynamic regime.

I. INTRODUCTION

Recently, relativistic chiral matter has attracted great interest both theoretically and experimentally. Chiral matter is considered to be realized in a wide range of systems from the electroweak plasmas in the early Universe [1, 2], quark-gluon plasmas in heavy ion collisions [3, 4], Weyl (semi)metals [5–8], and electron plasmas in neutron stars [9–11] to neutrino media in core-collapse supernova explosions [12, 13]. A remarkable property of chiral matter is the presence of unusual transport phenomena related to quantum anomalies in field theory [14, 15], called the chiral magnetic effect (CME) in a magnetic field [4, 5, 16, 17] and chiral vortical effect (CVE) in a vorticity [18–21]. These chiral transport phenomena lead to new types of collective modes, such as the chiral magnetic wave [22, 23], chiral vortical wave [24], chiral Alfvén wave [25], chiral heat wave [26], and the chiral plasma instability [1, 2, 27]; see also Refs. [28, 29] for recent related works.

In this paper, we study the shock propagation in relativistic chiral matter. We first argue that the so-called Rankine-Hugoniot relations, or the jump conditions at the shock front [30], must be modified by the presence of chiral transport phenomena. This in turn leads to modifications of the basic properties of shock waves in chiral matter. Our main findings are summarized as follows:

- The entropy discontinuity at the shock front in a weak shock wave is linearly proportional to the pressure discontinuity, $\Delta S \propto \Delta p$ [see Eq. (42)], when the effect of chiral transport becomes sufficiently large. This should be contrasted with the behavior $\Delta S \propto (\Delta p)^3$ in nonchiral matter [30].

- Rarefaction shock waves can appear depending on the direction of the wave propagation. This should be contrasted with the fact that rarefaction shock waves are prohibited in nonchiral matter [30].

We exemplify these features by studying shock waves in dense charge neutral chiral matter in the hydrodynamic regime. Such a situation is realized, e.g., by neutrino media at the core of supernovae [12]. These qualitatively new aspects of shock waves in chiral matter may have possible relevance, e.g., to the dynamics of supernovae. Although our argument does in part depend on this particular case of chiral matter, we expect that the qualitative features of our result is more generic. We discuss the possible applications of our arguments and results to other systems in the conclusion.

This paper is organized as follows. In Sec. II we review chiral hydrodynamics for charge neutral chiral matter. In Sec. III, we summarize the basic properties of shock waves in nonchiral matter. In Sec. IV, we study the properties of shock waves in chiral matter in a vorticity. Section V is devoted to our conclusion and discussion.

II. CHIRAL HYDRODYNAMICS

Our starting point is the relativistic chiral hydrodynamics [20]. With keeping a specific application to the neutrino media in supernovae [12] in mind, we consider chiral hydrodynamics for charge neutral chiral matter. Our argument can be extended to charged chiral matter in an external electromagnetic fields in a straightforward manner. For simplicity, we will here ignore the dissipative effects.

The equations of relativistic chiral hydrodynamics for a single (right- or left-handed) chiral fermion are given by energy-momentum conservation and particle number conservation as

\[ \partial_\mu T^{\mu\nu} = 0, \]
\[ \partial_\mu j^{\mu} = 0. \]

Here the energy-momentum tensor $T^{\mu\nu}$ and particle number current $j^{\mu}$ are given in the Landau-Lifshitz frame by [20]$^1$

\[ T^{\mu\nu} = h u^\mu u^\nu - pg^{\mu\nu}, \]
\[ j^{\mu} = nu^\mu + \xi \omega^\mu, \]

$^1$ We use the metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ in this paper.
where \( h = \epsilon + p \) is the enthalpy density, \( n \) is the particle number density, \( u^\mu = \gamma(1, v) \) is the fluid velocity with \( \gamma = (1 - v^2)^{-1/2} \), and \( \omega^\mu = \frac{1}{2} \epsilon \mu \lambda \rho u_\lambda \partial_\rho u_\rho \) is the fluid vorticity.

What is different in the neutral chiral hydrodynamics from the conventional nonchiral hydrodynamics [30] is the presence of the CVE proportional to \( \omega^\mu \) in Eq. (4) [18–21]. The transport coefficient \( \xi \) takes the form of \([20, 21, 31]\)

\[
\xi = C \mu^2 \left( 1 - \frac{2}{3} \frac{n \mu}{h} \right) + DT^2 \left( 1 - \frac{2}{3} \frac{n \mu}{h} \right),
\]

where \( \mu \) is the chemical potential and \( T \) is the temperature. The coefficients \( C \) and \( D \) are related to those of the chiral anomaly and mixed gauge-gravitational anomaly as \([20, 21, 32, 33]\)

\[
C = \pm \frac{1}{4\pi^2}, \quad D = \pm \frac{1}{12},
\]

for right- and left-handed chiral fermions, respectively.

In the following, we will focus on the regime \( \mu \gg T \) for demonstration. This is relevant to, e.g., the dense neutrino matter at the core of supernovae [12]. Note however that our argument itself is not limited to this regime and is applicable to other regimes as well. For a relativistic gas of noninteracting fermions (which is a reasonable assumption for a neutrino gas), the expressions of \( n, p, \epsilon, \) and the entropy \( S \) are given by

\[
n = \frac{\mu^3}{6\pi^2} + \frac{\mu T^2}{6}, \quad p = \frac{\epsilon}{3} = \frac{\mu^4}{24\pi^2} + \frac{\mu^2 T^2}{12}, \quad S = \frac{\pi^2 T}{\mu},
\]

to the leading corrections in \( T/\mu \ll 1 \). From Eqs. (7) and (8), the transport coefficient \( \xi \) in Eq. (5) in the regime \( \mu \gg T \) reduces to

\[
\xi \approx \frac{1}{3} C \mu^2 + \left( \frac{2}{3} C - D \right) T^2.
\]

These relations will be used in Sec. IV.

### III. SHOCK WAVES IN RELATIVISTIC NONCHIRAL MATTER

We first revisit the properties of shock waves in relativistic nonchiral matter [30] before we analyze shock waves in chiral matter. Consider a relativistic gas of particles moving along the \( x \) axis towards the positive \( x \) direction and that there is a surface of discontinuity perpendicular to the direction of propagation of the gas. This surface of discontinuity divides the three dimensional space into two regions, side 1 and side 2. The sides are defined in such a way that the gas moves from side 1 to side 2.

#### A. Rankine-Hugoniot relations

Imposing continuity in particle number flux, energy and momentum flux, \( j^1 = j^2, T_1^\gamma = T_2^\gamma, \) and \( T_1^{\mu \lambda} = T_2^{\mu \lambda} \), we have the following three equations relating the two sides [30],

\[
\frac{v_1 \gamma_1}{V_1} = \frac{v_2 \gamma_2}{V_2}, \quad h_1 v_1 \gamma_1^2 + p_1 = h_2 v_2 \gamma_2^2 + p_2, \quad h_1 v_1 \gamma_1^2 = h_2 v_2 \gamma_2^2,
\]

where the subscripts 1 and 2 stand for the sides 1 and 2 and \( V \) is the volume per particle, \( V \equiv 1/n \). Equations (11)–(13) constitute the Rankine-Hugoniot relations for shock propagation in nonchiral matter.

#### B. Pressure-volume relation

By solving Eqs. (12) and (13) in terms of \( v_1 \) and \( v_2 \), we have [30]

\[
v_1 = \sqrt{\frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)}}, \quad v_2 = \sqrt{\frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}}.
\]

Substituting these expressions into Eq. (11), we obtain the pressure-volume relation [30]:

\[
h_1^2 V_1^2 - h_2^2 V_2^2 + (p_2 - p_1)(h_1 V_1^2 + h_2 V_2^2) = 0.
\]

For given \( p_1 \) and \( V_1 \), it provides the relation between \( p_2 \) and \( V_2 \) under the equation of state \( p = p(\epsilon) \).

#### C. Absence of rarefaction shock waves

Let us consider the case of weak shock waves. For weak shock waves, we have \( \epsilon_2 \to \epsilon_1 \) and \( p_2 \to p_1 \), etc. If we expand the expression for the speed on side 1 given in Eq. (14) in \( \Delta \epsilon/h_1 \ll 1 \) and \( \Delta p/h_1 \ll 1 \), with \( \Delta \epsilon \equiv \epsilon_2 - \epsilon_1 \) and \( \Delta p \equiv p_2 - p_1 \), we find that

\[
(v_1)^2 = \lim_{\Delta \to 0} \frac{\Delta p \epsilon_2 + p_1}{\Delta \epsilon \epsilon_1 + p_2} = \frac{dp}{d\epsilon} \left[ 1 + \frac{\Delta \epsilon}{h_1} \left( 1 - \frac{dp}{d\epsilon} \right) + \cdots \right]
\]

\[
= (c_{s1})^2 \left[ 1 + \frac{\Delta \epsilon}{h_1} \left( 1 - (c_{s1})^2 \right) + \cdots \right],
\]

where \( c_{s1} \) is the speed of sound on side 1, \( (c_{s1})^2 = (dp/d\epsilon)_1 \). Similarly we find that

\[
(v_2)^2 = (c_{s2})^2 \left[ 1 - \frac{\Delta \epsilon}{h_1} \left( 1 - (c_{s2})^2 \right) + \cdots \right],
\]

\[
\Delta \epsilon/h_1 \ll 1 \text{ and } \Delta p/h_1 \ll 1.
\]
where \( c_{2s} \) is the speed of sound on side 2, \((c_{2s})^2 = (dp/ds)_{2s}\). In order to simplify our discussion below, we assume the equations of states on the two sides to be the same, where \( c_s = c_{2s} \).

As can be seen from Eqs. (14) and (15), \((v_1^2) > (v_2^2)\) when \( c_2 > c_1 \), and \((v_1^2) < (v_2^2)\) when \( c_2 < c_1 \). The former is known as a compression shock wave and the latter is known as a rarefaction shock wave. Remember also that, for compression shock waves, we have \( p_2 > p_1 \), and for rarefaction shock waves, we have \( p_1 > p_2 \). This is ensured by the fact that

\[
\frac{p_2 - p_1}{c_2 - c_1} > 0.
\]

Although the equations of relativistic hydrodynamics allow for the existence of both compression shock waves and rarefaction shock waves, only compression shock waves are consistent with the second law of thermodynamics, but rarefaction shock waves are not.

In order to see it, we expand the adiabatic of Eq. (16) using

\[
\Delta H = T \Delta S + V_1 \Delta p + \frac{1}{2} \frac{\partial V}{\partial p_1} |_{1} (\Delta p)^2 + \frac{1}{6} \frac{\partial^2 V}{\partial p^2_1} |_{1} (\Delta p)^3 + \cdots ,
\]

\[
\Delta V = \frac{\partial V}{\partial p_1} |_{1} \Delta p + \frac{1}{2} \frac{\partial^2 V}{\partial p^2_1} |_{1} (\Delta p)^2 + \frac{1}{6} \frac{\partial^3 V}{\partial p^3_1} |_{1} (\Delta p)^3
\]

\[
+ \frac{\partial V}{\partial S_1} |_{1} \Delta S + \cdots ,
\]

the former of which can be derived by using the thermodynamic relation \( V = \partial H/\partial p|_{S} \). Here \( H = hV \) is the enthalpy, \( S = sV \) is the entropy, and we defined \( \Delta H = H_2 - H_1 \), \( \Delta S = S_2 - S_1 \), and \( \Delta V = V_2 - V_1 \). After substituting Eqs. (20) and (21) into Eq. (16), one finds [30]

\[
\Delta S = \frac{1}{12H_1T} \frac{\partial^2 (HV)}{\partial p^2_1} |_{1} (\Delta p)^3 + O((\Delta p)^4) .
\]

It can be seen by plotting the adiabatic for any reasonable equation of state that \( \partial^2 (HV)/\partial p^2 > 0 \); for ultrarelativistic gases at low \( T \), see the explicit calculation of the coefficient of the term \((\Delta p)^3\) in Eq. (40) below. The second law of thermodynamics requires that \( S_2 > S_1 \), which then requires that \( p_2 > p_1 \). This corresponds to compression shock waves explained earlier. Hence we see that, in nonchiral relativistic fluids, second law of thermodynamics allows for the existence of compression shock waves alone and that there are no rarefaction shock waves. This is the so-called Zemplén’s theorem.

### IV. SHOCK WAVES IN CHIRAL MATTER

Let us now consider shock waves in chiral matter in the presence of a vorticity \( \omega \). The important point here is that the vorticity induces the CVE in chiral matter, leading to the modifications of the Rankine-Hugoniot relations. As a result, basic properties of shock waves are qualitatively modified compared with those in nonchiral matter in Sec. III.

We again consider a situation where a shock wave is moving along the \( x \) axis towards the positive \( x \) direction. For simplicity, we consider the case with a nonzero constant vorticity in the \( x \) direction,

\[
\omega_x = \frac{1}{2} u_x (\partial_x u_y - \partial_y u_x) \equiv \omega_x ,
\]

with \( \omega_x = \omega_z = 0 \), but the extension of our analysis to more general cases should be straightforward. Note that we are in a system where we cannot go to a frame where \( v_x = v_z = 0 \) at all points, unlike the case of shock waves in nonchiral matter discussed earlier.

In what follows, we consider the regime \( |\omega/\rho| \ll 1 \), where \( \rho = \sqrt{v_x^2 + v_z^2} \) is the distance from the axis of the vorticity. Assuming \( \partial_y v_x = \partial_x v_y = 0 \), we can solve Eq. (23) for \( v_x \equiv \sqrt{v_x^2 + v_z^2} \) to find

\[
v_x = \omega \rho (1 - v_x^2) + O((\omega \rho)^2) .
\]

#### A. Rankine-Hugoniot relations

Now let us look at how the jump conditions at the shock front are modified in the presence of the vorticity. Continuity of particle flux and energy-momentum flux across a surface of discontinuity, \( j^1_x = j^2_x \), \( T^{1xx} = T^{2xx} \), \( T^{1zz} = T^{2zz} \), \( T^{1xy} = T^{2xy} \), and \( T^{1zx} = T^{2zx} \), reads

\[
v_1^1 v_1^1 \gamma_1 + \xi_1 \omega_1 = v_2^2 v_2^2 + \xi_2 \omega_2 \equiv \Omega ,
\]

\[
h_1 (v_1^1)^2 \gamma_1 + p_1 = h_2 (v_2^2)^2 \gamma_2 + p_2 ,
\]

\[
h_1 v_1^1 \gamma_1 = h_2 v_2^2 \gamma_2 ,
\]

\[
h_1 v_1^1 \gamma_1 v_1^1 + h_2 v_2^2 \gamma_2 v_2^2 ,
\]

\[
h_1 v_1^1 \gamma_1 v_1^1 = h_2 v_2^2 \gamma_2 v_2^2 .
\]

The modification, compared with Eqs. (11)-(13) in nonchiral matter, is the presence of the CVE in Eq. (25).

Notice first that Eqs. (28) and (29) require that \( v_1^1 = v_2^3 \). From Eq. (24), we then must have \( \omega_1 [1 - (v_1^1)^2] = \omega_2 [1 - (v_2^3)^2] \). Below we will be interested in weak shock waves, for which \( v_1^1 \) and \( v_2^3 \) are taken to be approximately equal, up to terms that are suppressed by \( (v_1^1)^2 [1 - (v_2^2)^2] \ll 1 \). To analyze the weak shock wave, only Eqs. (25)-(27) are thus relevant, which constitute the modified Rankine-Hugoniot relations for shock propagation in chiral matter.

These equations may also be viewed as the interface conditions between the two sides of chiral matter in a global rotation \( \Omega = \Omega \mathbf{e} \) under the replacement \( \omega \rightarrow \Omega \).
B. Pressure-volume relation

From Eqs. (25) and (26), we have

\[
(h_1 V_1^2 - h_2 V_2^2)j^2 - 2\omega(h_1 V_1^2 \xi_1 - h_2 V_2^2 \xi_2)j
+ p_1 - p_2 + \omega^2(h_1 \xi_1 V_1^2 - h_2 \xi_2 V_2^2) = 0. \tag{30}
\]

This can be solved for \(j\) as

\[
j = \frac{(h_1 V_1^2 \xi_1 - h_2 V_2^2 \xi_2)\omega \pm \sqrt{D}}{h_1 V_1^2 - h_2 V_2^2}, \tag{31}
\]

with

\[D \equiv (p_2 - p_1)(h_1 V_1^2 - h_2 V_2^2) + h_1 h_2 V_1 V_2(\xi_1 - \xi_2)^2 \omega^2. \tag{32}\]

In order for \(j\) to be real, we must have \(D \geq 0\).

Note that we must have \(|\omega| \ll \mu_1, \mu_2\) for hydrodynamics to make sense, where \(\mu_1, \mu_2\) is the chemical potential of side 1 or 2. Since we are interested in the regime \(\alpha^2 \rho^2 \ll V,\) the limit \(|\omega|/\rho \ll 1\) is within the applicability of hydrodynamics. In this limit, Eqs. (26) and (27) can be solved to obtain the expressions for \(v_1^2\) and \(v_2^2\) which invariably turn out to be identical to the expressions in Eqs. (14) and (15) up to corrections suppressed by powers of \(\omega \rho\). Hence, we can use the leading-order expression for the velocities in the expansion of \(\omega \rho\) in Eq. (25) to obtain the relation on the two sides as

\[
\frac{v_1}{V_1 \sqrt{1 - v_1^2}} = \frac{v_2}{V_2 \sqrt{1 - v_2^2}} = -\omega(\xi_1 - \xi_2). \tag{33}
\]

Here \(v_{1,2}\) are given in Eqs. (14) and (15). Recalling that \(\mu_1, \mu_2\) and \(T_{1,2}\) can be written in terms of \(p_{1,2}\) and \(V_{1,2}\) from Eqs. (7) and (8), both sides of Eq. (33) can be expressed in terms of \(p_{1,2}\) and \(V_{1,2}\) alone, for a given equation of state \(p = p(\rho)\). This provides the pressure-volume relation of the two sides for chiral matter in a constant vorticity.

C. Presence of rarefaction shock waves

Let us now consider how the relation (22) is modified for weak shock waves in chiral matter. The modified relation will be obtained by expanding Eq. (33) in terms of \(\Delta S = S_2 - S_1\) and \(\Delta p = p_2 - p_1\) across the surface of discontinuity. The expansion of the left-hand side of Eq. (33), but ignoring the right-hand side, would lead to the result (22) for nonchiral matter. Here we need to expand the right-hand side in \(\Delta S\) and \(\Delta p\) as well.

For this purpose, we first solve \(p = p(\mu, T)\) and \(S = S(\mu, T)\) in Eqs. (8) and (9) for \(\mu\) and \(T\) in terms of \(p\) and \(S\), by treating \(S \ll 1\) as a perturbation for \(\mu \gg T\). The results to the leading correction in \(S \ll 1\) are given by

\[
\mu(p, S) = \left(1 - \frac{S^2}{2\pi^2}\right)(24\pi^2 p)^{1/4}, \tag{34}
\]

\[
T(p, S) = \frac{S^2}{\pi^2}(24\pi^2 p)^{1/4}. \tag{35}
\]

From these equations, \(\Delta \mu \equiv \mu_2 - \mu_1\) and \(\Delta T \equiv T_2 - T_1\) can be expanded in \(\Delta p\) and \(\Delta S\) as

\[
\Delta \mu = \frac{6\pi^2}{\mu^4} \Delta p - T \Delta S, \tag{36}
\]

\[
\Delta T = \frac{6\pi^2}{\mu^4} \Delta p + \frac{\mu}{\pi^2} \Delta S. \tag{37}
\]

We then expand \(\Delta \xi \equiv \xi_2 - \xi_1\) in \(\Delta S\) and \(\Delta p\). When \(\mu \gg T,\) using Eqs. (6), (10), (36), and (37), we have

\[
\Delta \xi = \lambda \frac{\Delta p}{\mu^2}, \tag{38}
\]

where

\[
\lambda = 4\pi^2 C = \pm 1, \tag{39}
\]

for right- and left-handed chiral matter, respectively. Note here that the coefficient of \(\Delta S\) in Eq. (38) is proportional to \(2C/3 - 2D/\pi^2\) and vanishes identically regardless of the chirality.

Collecting the leading terms in the expansion of Eq. (33) in \(\Delta S\) and \(\Delta p\), we arrive at

\[
\Delta S = \frac{216\pi^6}{\mu^4 T}(\Delta p)^3 + \lambda \frac{6\sqrt{2}\pi^2}{\mu^4 T} \omega \Delta p + \cdots, \tag{40}
\]

where “\(\cdots\)” includes terms that are higher order in \(\omega \Delta p\) and also terms that are higher order in \(\Delta p\) as compared to \((\Delta p)^3\). Note that the first term on the right-hand side of Eq. (40) corresponds to the result (22) for nonchiral matter. The correction in the second term due to the vorticity \(\omega\) becomes negligible and reduces to the result (22) when \(|\omega| \ll \omega_c(\Delta p)\) for a given \(\Delta p\), where

\[
\omega_c(\Delta p) = 18\sqrt{2}\pi^4 (\Delta p)^2 / \mu^4. \tag{41}
\]

Let us now consider the case with \(|\omega| \gg \omega_c\). However, \(\omega\) still needs to be small enough such that \(|\omega|/\rho \ll 1\) remains valid. In this case, Eq. (40) is approximately

\[
\Delta S \approx \lambda \frac{6\sqrt{2}\pi^2}{\mu^4 T} \omega \Delta p. \tag{42}
\]

This is our main result.

Recall that the second law of thermodynamics requires that \(\Delta S > 0\). From Eq. (42), this allows for only one particular sign of \(\omega \Delta p\) for chiral matter with \(\lambda = +1\) or \(\lambda = -1\). For a fixed sign of \(\omega \Delta p\), we have \(p_2 > p_1\) or \(p_1 > p_2\) depending on the sign of \(\omega\). (Here recall that the sign of \(\omega\) indicates whether the vorticity of the fluid is pointing in the same or opposite direction of the shock wave propagation.)

In particular, for neutrino matter (where \(\lambda = -1\)), we must have \(\omega \Delta p < 0\) to satisfy \(\Delta S > 0\). For \(\omega < 0\) we have a compression shock wave (\(p_2 > p_1\)), while for \(\omega > 0\) we have a rarefaction shock wave (\(p_1 > p_2\)). We thus find that rarefaction shock waves can exist in chiral matter due to the CVE. This should be contrasted with the nonchiral matter where the rarefaction shock wave is prohibited as we have seen in Sec. III.
V. CONCLUSION AND DISCUSSION

In this paper, we have explored the shock propagation in relativistic chiral matter. We have seen that the conventional Rankine-Hugoniot relations are modified due to the presence of chiral transport phenomena. In particular, we have shown that rarefaction shock waves can appear in chiral matter, depending on the relative direction of the wave propagation and the vorticity; in this sense, the shock wave found in this paper is chiral, similarly to the other chiral waves [22–26]. It would be interesting to study possible phenomenological consequences of the chiral shock wave, e.g., in the dynamics of supernovae.

Although we limit ourselves to dense and cold charge neutral chiral matter in this paper, our argument should be applicable to hot and/or dense charged chiral matter in an external magnetic field as well. In this case, the chiral magnetic effect modifies the Rankine-Hugoniot relations in a way qualitatively similar to what we found in this paper. Such a new type of shock wave may be relevant to the electroweak plasmas in the early Universe.

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