Stochastic estimation of microactuator buckling

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Abstract. In order to make a robust design of microsystems, it is important to analyze the electrical, thermal and mechanical fields including the actual input parameters. These microdevices, which typically are made of brittle materials such as polysilicon, show wide scatter (stochastic behavior) in properties as well as substantial uncertainty in the shape and geometry because of manufacturing processes. These behaviors necessitate either costly and time-consuming trial-and-error designs or, more efficiently, the development of a probabilistic design methodology for MEMS. Computer aided MEMS simulations regarding performance, power consumption, and reliability is an important design task due to high prototyping costs. Since microbeams have a wide range of applications in MEMS actuation mechanisms, analysis of the thermomechanical behavior of these actuators is very important. In the present work, assessing meaningful uncertainties involved in thermally driven microbeams, the stochastic finite element model (SFEM) is developed and implemented. The analysis shows a large deviation in buckling temperature and thermal stresses for reasonable probability density functions of characteristic parameters. Although computationally significantly more expensive than deterministic electromechanical simulation, the work illustrates the requirement of stochastic modeling for true estimation of microsystems’ performance.

1. Introduction

Some variations in geometrical dimensions, topologies and material properties, which are negligible at macroscopic scales, are more effective and should be taken into account in simulations. These scatters could have strong effects in the design process [1]. The material properties databases of Microsystems have considerable scatter and therefore, probabilistic methods guide the design of these devices to achieve a robust and reliable product in a most efficient procedure. A common actuation mechanism for microelectromechanical systems (MEMS) is joule-heating-induced thermal expansion. [2, 3, 4, 5 and 6] Empirical characterizations of these actuators are abundant. Ease of fabrication, quick and large movement and forces let them [7] have several applications in Microsystems. In previous works [6, 7 and 8] deterministic characterization of this scenario is achieved but they were not satisfactory because they did not contain the stochastic behavior of subsystems and properties. A discussion of modeling factors and process variations is presented. Probabilistic analysis is very essential to improve the reliability of results, moreover using comprehensive probabilistic results (histogram, probabilities, scatter plots, sensitivities ...) [9] the correlation of experimental results and analytical ones become more convenient and clear. This method also plays significant roles for the development of MEMS devices. Eventually the influence of uncertain physical factor uncertainty on thermal buckling of suspended beam is explored. Microstructures fabricated by the MCNC multi-user MEMS process
MUMPs [10] are modeled because of the availability of experimental tests’ results to correlate results from a finite element analysis. This coupled electro, thermal and elastic model has been utilized in the simulation of a laterally driven clamped-clamped heterogeneous microbeam, 2-μm wide, 2-μm thick and 100-μm long. (Fig. 1).

![Image](https://example.com/image.png)

**Figure 1.** Lateral displacement of microbeam under resistive heating [6]

2. Analytical approach for Deterministic thermal buckling

Based on strength of materials [11], critical temperature for a beam, which causes buckling, can be written as:

\[
\Delta T_{cr} = \frac{\pi^2 I}{L^2 \alpha \Delta \varepsilon}
\]  

(1)

where I, L, \( \alpha \) and \( \Delta T \) are cross section moment of inertia, effective length of beam, thermal expansion coefficient and temperature rise, respectively. A passive residual strain of 6.4e-5 has been detected in microbeams via the MCNC MUMPs process [6] The corresponding compressive residual stresses are 9.6 Mpa for polysilicon films with Young’s modulus of 159 Gpa [6, 12, 13]. It is reported [6, 14 and 15] when compressive residual stress exists within a microbeam, the critical temperature to cause the buckling of microbeams decreases. Chiao and Lin [6] mentioned as an effective character in postbuckling behavior of microactuators. In this work the prestressed finite element simulation is used. With this consideration Eq. 1 will be changes to Eq. 2 and a drop of buckling temperature is investigated.

\[
\Delta T'_{cr} = \left( \frac{\pi^2 E L}{L^2 \alpha \Delta \varepsilon} + A \sigma_{res} \right) \left( \Delta E \alpha \right)
\]  

(2)

Likely if any tensile residual stress exists in the microbeam it make the beam buckle later. A microbeam with properties listed in Table 1 is examined to estimate its critical temperature with residual stress and without it. When \( \sigma_{res} \) equals 10Mpa. The result shows a 24°C decrement in the buckling temperature from 505°C to 481°C. When 505°C temperature rise adds to initial temperature of 27°C yields almost the same result with previous works [6, 8].

| \( \alpha \) | E     | b    | L     | \( \sigma_{res} \) | T rise without residual stress | T with residual stress |
|------------|-------|------|-------|------------------|-------------------------------|----------------------|
| 2.6e-6     | 159e9 Gpa | 2um  | 100um | 10 Mpa           | 505°C                        | 481°C                |
b and h are width and height of the beam respectively. To this point all the variables are assumed to be deterministic but as it would be stated in this paper there are considerable scatter in the input variables. There are two methods to tackle the probabilistic analysis, analytical and numerical methods.

3. Analytical approach for Probabilistic thermal buckling and its restrictions

The equation 1 and 2 show deterministic temperature rise for buckling initiation with and without residual stress existence respectively. Input parameters and result of a numerical example of microbeam buckling temperature are summarized in Tables (2) and (3). Analytical approach is restricted to Gaussian distributions for input variables. In this method T, critical temperature, has mean and standard deviation. Using Eq. (2) and input variables in Table (2), the stochastic buckling temperature is achieved and summarized in Table (3).

| Input Parameters | Distribution Type | Mean value | Deviation |
|------------------|-------------------|------------|-----------|
| $\alpha$         | Normal            | 2.6e-6     | 0.1e-6    |
| $E$              | Normal            | 150e3 Mpa  | 10 Mpa    |

Table (3).Output parameter and its stochastic data

| Output Parameter | Distribution Type | Mean value | Deviation |
|------------------|-------------------|------------|-----------|
| $\Delta T_{cr}$  | Normal            | 510°C      | 10        |

Results display the importance of including probabilistic behavior of material parameters in buckling analyses. Gaussian distributed inputs and analytically solved problems are two main limitation of this procedure, therefore a finite element based probabilistic simulation is implemented to include general distributed inputs and general domain containing complicated features.

4. Probabilistic Methods through Numerical Approach

The probabilistic methods execute the deterministic problem several times, each time with a different set of values for the random input variables. One execution run with a given set of values for the random input variables is called a sampling point. The common feature for the Probabilistic Method is Monte Carlo Simulation Method. A fundamental characteristic of the Monte Carlo Simulation method is the fact that the sampling points are located at random locations in the space of the random input variables. There are various techniques available in literature that can be used to evaluate the random locations of the sampling points [17, 18]. The direct Monte Carlo simulation and Latin Hypercube Sampling are two approaches. The direct Monte Carlo Simulation method is not used because its random sampling has no memory. Instead of that the Latin Hypercube Sampling technique is implemented and the range of all random input variables is divided into n intervals with equal probability. For each random variable each interval is “hit” only once with a sampling point.

5. Probabilistic modelling approach

The presented model is based on a surface micro machined clamped-clamped micro beam, as shown in Figures 1. A one-dimensional model of the beam is developed using ANSYS code using 60 beam element. In this probabilistic analysis uncertain parameters are described by statistical distribution functions such as the truncated gaussian. Three groups of random variables are effective in thermal buckling analysis: geometry, material properties and loading conditions. In this work scatter of material properties and fabrication induced residual stresses are included.

5.1.1. Material Variables
All of the material random variables were assumed to be independent for computational simplicity. Truncated Gaussian distributions of random variables $E$ and $\alpha$ have four distribution parameters, namely a mean value $\mu_G$ and a standard deviation $\sigma_G$ of the non-truncated Gaussian distribution, and the lower limit and the upper limit. With these assumptions the parameters for truncated gaussian distribution functions of $E$ and $\alpha$ are listed in Table 4 and graphically illustrated in figures 2 and 3.

| Parameter | Mean     | Standard Deviation | Lower limit | Upper limit |
|-----------|----------|--------------------|-------------|-------------|
| $\alpha$  | 2.6E-06  | 1.0E-07            | 2.4E-06     | 2.8E-06     |
| $E$       | 1.59E+11 | 10E+09             | 1.40E+11    | 1.60E+11    |

![Figure 2. distribution functions of $E$](image2)

![Figure 3. distribution functions of $\alpha$](image3)

5.1.2. Process induced variables (Residual stress)

As mentioned in section 2 the compressive residual stresses exists in microbeams [6]. Residual stresses is a complicated function of several factors like elasticity modulus, process temperature and etc. And these factors have large scatters therefore the residual stress should be considered as a random variable in the model. In this paper In order to show the effect of residual stress two models are used. In the first one residual stress is ignored and in the other one the residual stress is considered as a truncated Gaussian distribution with the listed parameters in the Table 5.

| Parameter | Mean     | Standard Deviation | Lower limit | Upper limit |
|-----------|----------|--------------------|-------------|-------------|
| $\sigma_{res}$ | 10 Mpa   | 2 Mpa              | 0 Mpa       | 20 Mpa      |

6. Post-processing results

For the probabilistic analysis 2000 Latin Hypercube samples have been run. The simulation is run two times in the first run residual stress is not considered but the second simulation includes residual stress and its scatter. The resulting statistics of the buckling temperature, is given in Tables 11 and 12 and shows the importance of considering residual stress and its scatter in the simulations. Therefore the forthcoming results are all related to the residual stressed model. In Fig. 4 X axis is the number of runs in Monte Carlo simulation (2000 run) and Y axis represents buckling temperature.
Table 11. Critical temperature statistics without residual stress effect

| Name | Mean | Standard Deviation | Minimum | Maximum |
|------|------|--------------------|---------|---------|
| T1   | 506  | 19.6               | 446     | 581     |

Table 11. Critical temperature statistics with residual stress effect

| Name | Mean | Standard Deviation | Minimum | Maximum |
|------|------|--------------------|---------|---------|
| T1   | 493.3| 19.3               | 419     | 552560.8|

Figure 4. Buckling temperature calculated in each run

The histogram of the critical temperature is illustrated in Fig. 5. The relative frequency shown in the histograms is equal to the number of samples within a certain interval divided by the total number of samples (2000 in this case). Fig. 6 shows the sensitivities of input variables for the buckling temperature. The sensitivities are given as absolute values (bar chart) and relative to each other (pie chart). Here, the critical temperature is sensitive more to thermal expansion coefficient and residual stress because the impact of elasticity modulus on the result is not significant enough to be worth considering. This is a reduction of the complexity of the problem from three input variables down to only two.

Figure 5. The DISTRIBUTED FUNCTION OF T  
Figure 6. sensitivity diagram

7. CONCLUSIONS

As illustrated in the present paper, probabilistic methods can be used to quantify the reliability of devices [13] specially MEMS devices and to achieve a more robust design and improved quality. Probabilistic sensitivities can be used to reduce the complexity of the problem and derive measures for improving the product quality. This provides guidance for necessary design changes in a most efficient
way. In addition, probabilistic methods are capable of identify where reductions of the manufacturing costs are possible. This work will be extended to include the temperature dependency of properties, where the effect of both geometric tolerance and material property variations are factored in. Buckling behavior of micro machined beam is analyzed with a developed stochastic MEMS model using finite element simulation. This modeling technique can be applied to predict the functionality of microstructures that operate based on joule-heating effects. Simulation results of a typical microbeam with $2 \mu m$ in width, $2 \mu m$ in thickness and 100 $\mu m$ in length obtained and the critical temperature mean prediction, modified from 506 $^C$ to 493 $^C$ by characterizing more realistic criteria which decrease the distance from analysis to reality. In addition to that, importance of probabilistic procedure is confirmed with its special outputs, correlation sensitivity charts and scatter plots. In this paper, the results are used to put the random input variables in a proper order with reference to their effectiveness.

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