It has been proposed, that unconventional density waves (UDW) are possible candidates for systems with hidden order parameter. Unlike in conventional density waves, no periodic modulation of either the charge-, or the spin-density is present in UDW, in spite of a clear thermodynamic signal. Although the unconventional spin density wave (USDW) has been suggested for the "antiferromagnetic" phase of URu$_2$Si$_2$, the micromagnetism seen by neutron scattering has not been understood. We present here the calculation of the local spin density due to impurities in USDW, which describes quantitatively the neutron scattering data by Amitsuka et.al. Further, we propose that the pseudogap phase in high temperature superconductors (HTSC) should also be USDW. Strong evidence for this are the micromagnetism seen by Sidis et.al., and the optical dichroism seen by Campuzano et.al.

1. Introduction

Many unusual features of the high temperature superconductors (HTSC) are understood in terms of a momentum dependent order parameter $\Delta(k)$. It follows from the success of these developments naturally, that investigating the properties of unconventional condensates in the density wave sector may as well turn out to be a fruitful enterprise. Indeed, in the last few years researchers encountered mysterious low temperature phases in a number of materials, where a clear and robust thermodynamic phase transition is not accompanied by an order parameter detectable by conventional means. This situation is often referred to as hidden order.

Prime example is the 17.5 K transition in URu$_2$Si$_2$ with very small magnetic moment in the low temperature phase. It has been suggested, that an unconventional spin density wave (USDW) may be responsible for this behavior, since in the
clean system USDW does not exhibit periodic spin density modulation due to the vanishing momentum average of the order parameter. The origin of the small but finite magnetic moment, termed micromagnetism, however remains elusive. The magnetic phase diagram of some $\alpha$-(ET)$_2$ salts also offers a low temperature phase (LTP) with resemblance to charge density waves (CDW), but without the characteristic X-ray satellites. In this situation a quasi-one dimensional UCDW scenario is called for, and it’s case is strengthened by the evaluation of the corresponding threshold electric field for sliding conductivity. Finally, the pseudogap phase of high temperature superconductors was proposed to be a kind of UCDW as well. However, micromagnetism was also observed in YBCO by neutron scattering.

In this paper we propose a mechanism by which the USDW exhibits weak periodic spin density modulation in the presence of random magnetic impurities. Comparing the temperature dependence of neutron scattering intensity due to this modulation with experimental data on URu$_2$Si$_2$, we argue that this mechanism is responsible for the micromagnetism in this material, and possibly also in the pseudogap phase of HTSC.

2. General features of unconventional density waves

The unconventional spin density wave (USDW) with spin polarization in the $\alpha$ direction ($\alpha = x, y, z$) is described in the Nambu formalism by the Green’s function

$$G^{-1}(k, i\omega_n) = i\omega_n - \rho_3\xi(k) + \rho_1\sigma_\alpha\Delta(k),$$

where the $\rho$ and $\sigma$ Pauli matrices operate on the electron-hole, and spin space respectively, $\xi(k)$ is the electron spectrum measured from the Fermi energy, and $\Delta(k)$ is the momentum dependent order parameter. In the quasi-one dimensional case it can for example $\Delta(k) = \Delta \cos(bk_y)$, while for URu$_2$Si$_2$ $\Delta(k) = \Delta[\cos(ak_x) - \cos(ak_y)]$ has been proposed. Clearly, due to the vanishing momentum average of $\Delta(k)$, the expectation value of any component of the spin density is zero. It turns out, that the quantity which does have a periodic modulation with the nesting vector $Q$, is the spin current density $\mathbf{J}$. Nevertheless, in the ideal system there is no spin density modulation, therefore the micromagnetism remains unexplained, although the Green’s function definitely prefers a unique direction ($\alpha$) in spin space. There is no such spin direction for UCDW, therefore the mechanism for micromagnetism proposed below will not work in that case.

3. Local spin density and micromagnetism

We assume that we have magnetic impurities in our sample at positions $\mathbf{R}_j$ with spin $\mathbf{S}_j$ interacting with the electron spin density through a Heisenberg exchange coupling $U(\mathbf{r}) = J(\mathbf{r} - \mathbf{R}_j)\mathbf{S}_j\mathbf{\sigma}$. The first diagram on Fig. shows the change in the thermodynamic potential due to impurities:

$$\Delta \Omega = T \sum_{\omega_n} \sum_k \text{Tr}[U(k, k)G(k, i\omega)],$$

(1)
where $U_{s,s'}(k,k') = \int d^3r \varphi^*_k + sQ r U(r) \varphi_{k'+s'Q} r$ is the matrix element of the impurity potential between Bloch states $(s, s' = \pm 1$ spanning the electron hole space). $\Delta \Omega$ vanishes for any kind of CDW, but for USDW each impurity feels a potential energy $\Delta \Omega = \Delta \Omega_j (J_\Delta / P) \cos(QR_j)$, trying to align the impurity spin parallel to the spin direction favoured by the USDW, including its periodicity given by $Q$. $P$ is the component of the interaction driving the USDW transition, while $J$ is the coefficient of the term originating from $U(k,k)$ (a matrix element of the exchange integral $J(r)$ between Bloch states), having the same momentum dependence as the order parameter does (for example $\cos(ak_x) - \cos(ak_y)$).

![Fig. 1](image.png)

Fig. 1. Diagrams contributing to the thermodynamic potential and to the spin density respectively, in first order in the impurity potential.

Once the impurity spins are coherently aligned much like in a conventional SDW, the electron spin polarization around them will also develop coherently, facilitating the observation of a magnetic order by neutron scattering. The electron spin density induced by impurities can be calculated from the second diagram on Fig. 1 as

$$\langle \sigma(r) \rangle = T \sum_{\omega_n} \sum_{k,k'} \text{Tr} [\sigma_{k',k}(r) G(k,i\omega) U(k,k') G(k',i\omega)] ,$$

(2)

where $[\sigma_{k',k}(r)]_{s,s'} = \varphi^*_{k'+sQ} r \varphi_{k+sQ} r \sigma_{s'} r$. A similar expression has already been evaluated for the impurity pinning potential in UDW, which allows us to express the electron spin density at the impurity site $R_j$. The excess spin polarization due to the USDW turns out to be parallel to the USDW’s preferred spin direction ($\alpha$) as well, and the temperature dependence of its magnitude normalized to the $T = 0$ value is given by

$$\frac{\sigma(T)}{\sigma(0)} = \frac{\Delta(T)}{\Delta(0)} \frac{1}{0.5925} \int_0^1 \tanh \frac{\beta \Delta(T) x}{2} E(\sqrt{1 - x^2})(K(x) - E(x)) dx .$$

(3)

The neutron scattering intensity is proportional to the square of this amplitude, and its normalized temperature dependence is plotted on Fig. 2, along with the experimental data on URu$_2$Si$_2$.

4. Concluding remarks

We have proposed a mechanism for the development of micromagnetism in USDW, and shown that the temperature dependence of the magnetic moment is consistent
Fig. 2. Normalized neutron scattering intensity as a function of reduced temperature. Crosses denote URu$_2$Si$_2$ data (Amitsuka et.al.), the solid line is the theoretical prediction from Eq. 3.

with measurements on URu$_2$Si$_2$, lending further support for the suggestion that the low temperature phase of this material is a USDW. We also believe that the same mechanism may apply for the pseudogap phase in HTSC, suggesting that it is not UCDW, but USDW instead. This is further corroborated by the optical dichroism observed in this phase.

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