Study on the stability of waterpower-speed control system for hydropower station with air cushion surge chamber

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Abstract. According to the fact that the effects of penstock, unit and governor on stability of water level fluctuation for hydropower station with air cushion surge chamber are neglected in previous researches, in this paper, the Thoma assumption is broken through, the complete mathematical model of waterpower-speed control system for hydropower station with air cushion surge chamber is established, and the comprehensive transfer function and linear homogeneous differential equation that characterize the dynamic characteristics of system are derived. The stability domain that characterizes the good or bad of stability quantitatively is drawn by using the stability conditions. The effects of the fluid inertia in water diversion system, the air cushion surge chamber parameters, hydraulic turbine characteristics, generator characteristics, and regulation modes of governor on the stability of waterpower-speed control system are analyzed through stability domain. The main conclusions are as follows: The fluid inertia in water diversion system and hydraulic turbine characteristics have unfavorable effects on the system while generator characteristics have favorable effect. The stability keeps getting better with the increase of chamber height and basal area and the decrease of air pressure and air polytropic exponent. The stability of power regulation mode is obviously better than that of frequency regulation mode.

1. Introduction

Air cushion surge chamber is a new-type surge chamber and was proposed by R Svee and L Rathe et al in the 1970s [1, 2]. This type surge chamber can control water hammer pressure and water level fluctuation effectively and are applicable to pumped-storage power station and high head and long pipeline hydropower station. The stability of water level fluctuation is the most important issue in the hydraulic design of air cushion surge chamber. R Svee studied this issue under the premise of the Thoma assumption and proposed the formula of critical stable sectional area [1]. However, the critical stable sectional area calculated from R Svee's formula is usually about several times or even dozens of times as large as that of conventional open-type surge chamber, and overlarge sectional area becomes the limiting factor for the application of air cushion surge chamber. Through the investigations of the throttle of surge chamber and the velocity head at the bottom of the surge chamber, several improved formulae are proposed to reduce critical stable sectional area [3-6]. However, the above researches were all carried out under the premise of the Thoma assumption and the effects of penstock, unit and governor on stability were neglected. Hence, these researches have great limitations.

This paper aims to reveal the influence of all sub systems of diversion power system (i.e. headrace tunnel, surge tank, penstock, turbine, generator and governor) on the stability of air cushion surge
chamber. To achieve this goal, Thoma assumption is broken through and the complete mathematical model of waterpower-speed control system is established. Then overall transfer function is derived and the stability domain that characterizes stability quantificationally is plotted. The effects of influencing factors on stability are analyzed by using stability domain.

The analysis of this paper is based on the following assumptions: (1) rigid water hammer model; (2) neglectation of the head loss and velocity head at the bottom of the surge chamber; (3) neglectation of the nonlinear characteristics of governor: saturation characteristic and speed dead bend.

2. Mathematical Model

The waterpower-speed control system of hydropower station with air cushion surge chamber is illustrated in Figure 1, and its complete mathematical model that includes headrace tunnel, air cushion surge chamber, penstock, hydraulic turbine, generator and governor can be established as follows.

![Figure 1. Waterpower-speed control system of hydropower station with air cushion surge chamber](image)

(a) Diversion power system of hydropower station with air cushion surge chamber

(b) Waterpower-speed control system

(1) Controlled system [7-9]

Momentum equation of headrace tunnel:

\[ h_F = -T_{w} \frac{dQ}{dt} + \frac{2h_0}{Q_f}q_y \]  

Continuity equation of air cushion surge chamber containing state characteristic of gas:

\[ q_y = \frac{F}{(1 + mFp_0/\Delta_0)Q_0} \frac{dh_y}{dt} + q_t \]  

Momentum equation of penstock:

\[ h_y = T_m \frac{dQ}{dt} + \frac{2h_0}{Q_f}q_i + h \]  

Moment equation and discharge equation of hydraulic turbine:

\[ m_i = e_y h + e_x y \]  

\[ q_i = e_y h + e_x y \]  

First derivative differential equation of generator:

\[ T_e \frac{dx}{dt} = m_i - (m_y + e_x y) \]  

where \( z=(Z-Z_0)/Z_0, h_f=(H_F - H_{00})/H_{00}, h=(H - H_0)/H_0, q_i=(Q_i - Q_0)/Q_0, q_f=(Q_f - Q_0)/Q_0, x=(n - n_0)/n_0, Y=(Y - Y_0)/Y_0, m_i=(M_i - M_{00})/M_{00}, m_y=(M_y - M_{00})/M_{00}, \) are the relative deviations of altitude difference between surge chamber water level and upstream reservoir water level \( Z \) (Positive direction is downward), piezometric head at the bottom of surge chamber \( H_F \), working head \( H \), headrace tunnel discharge \( Q_f \), penstock discharge \( Q_0 \), unit frequency \( n \), guide vane opening \( Y \), kinetic moment \( M_i \), resisting moment \( M_y \), respectively. \( Q_0 = Q_{00} = Q_0 \) is initial discharge of pipeline and the subscript '0' refers to the value of initial moment. \( L, f, T_m = L, Q_0/gfH_0 \) and \( h_0 \) are length, sectional area, water inertia time constant and head loss of headrace tunnel, respectively.
length, sectional area, water inertia time constant and head loss of penstock, respectively. Let:

\[ T_F = \frac{FH_0}{(1+mF_0/\Delta_0)Q_0} \]

be air cushion surge chamber time constant. \( F \) is sectional area of surge chamber. \( p_0, \Delta_0, l_0 = \Delta_0/F \) are air absolute pressure and volume in surge chamber. \( m \) is gas polytropic exponent. We can get:

\[ T_F = \frac{FH_0}{Q_0} \quad \text{in the case of } \Delta_0 \to \infty, \]

and it is the expression of open-type surge chamber time constant.

\( e_h, e_x, e_y \) are moment transfer coefficients of turbine. \( e_{qh}, e_{qx}, e_{qy} \) are discharge transfer coefficients of turbine. \( T_a \) is unit inertia time constant. \( e_g \) is load self-regulation coefficient.

(2) Turbine control system [7-9]

Equation of governor:

\[ b_T \frac{dy}{dt} = - (T_d \frac{dx}{dt} + x) \tag{7} \]

where \( b_t \) is temporary droop; \( T_d \) is damping device time constant.

3. Derivation of overall transfer function

For the situation of grid load disturbance, the block diagram of waterpower-speed control system can be obtained from equations (1)-(7) and shown in Figure 2, where \( G(s) = H(s)/Q(s) \) is the transfer function of pipeline system and can be derived from the Laplace transform of equations (1)-(3), \( s \) is complex variable.

**Figure 2.** Block diagram of waterpower-speed control system.

According to Figure 2 and the Laplace transform of equations (1)-(7), the following overall transfer function of waterpower-speed control system is obtained (\( M_g(s) \) and \( X(s) \) are the Laplace transform of resisting moment \( m_g \) and unit frequency response \( x \), respectively, which the former is input signal and the later is output signal):

\[ G(s) = \frac{X(s)}{M_g(s)} = \frac{b_T p_s (b_0 s^3 + b_1 s^2 + b_2 s + b_3)}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5} \tag{8} \]

Equation (8) can be changed into the following form:

\[ (a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5)X(s) = -b_T p_s (b_0 s^3 + b_1 s^2 + b_2 s + b_3)M_g(s) \tag{9} \]

Since the issue of stability is about the performance of reaching a new steady state after disturbance vanishes (i.e. \( M_g(s) = 0 \)), the stability analysis boils down to the study of the following equation:

\[ (a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5)X(s) = 0 \tag{10} \]

Through equation (10), the fifth order linear homogeneous differential equation that characterizes the dynamic characteristics of waterpower-speed control system is obtained:

\[ a_0 \frac{d^5 x}{dt^5} + a_1 \frac{d^4 x}{dt^4} + a_2 \frac{d^3 x}{dt^3} + a_3 \frac{d^2 x}{dt^2} + a_4 \frac{dx}{dt} + a_5 = 0 \tag{11} \]

The expressions of coefficients in equation (11) are presented in Appendix.
4. Stability conditions of the waterpower-speed control system

Using Routh-Hurwitz criterion [10], the stability conditions of waterpower-speed control system is obtained as follows:

1. \( a_i > 0 \) (\( i = 0, 1, 2, 3, 4, 5 \))
2. \( \Delta_2 = a_5 a_2 - a_0 a_3 > 0 \)
3. \( \Delta_4 = \begin{vmatrix} a_1 & a_2 & a_3 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix} = (a_5 a_2 a_3) (a_4 a_4 - a_0 a_3) - (a_5 a_2 a_3)^2 > 0 \)

When the coefficients in equation (11) (i.e. \( a_i (i=0, 1, 2, 3, 4, 5) \)) satisfy the discriminants 1, 2 and 3 at the same time, waterpower-speed control system is stable.

5. Analysis of stability for waterpower-speed control system

By substituting the characteristic parameters of the control system in different conditions into the stability conditions given in Section 4, the domain in which the stability conditions are satisfied can be drawn in coordinate system which the abscissa and ordinate are usually \( b_t \) and \( T_d \), respectively. This domain is called stability domain [7] and its boundary is called stability boundary. The opposite side of stability boundary is unsteady domain. Stability domain, stability boundary and unsteady domain are illustrated in Figure 3 (a). Three working points A, B and C located in stability domain, stability boundary and unsteady domain, respectively, are selected and their frequency responses under load disturbance are plotted in Figure 3 (b). It is can be seen from Figure 3 (b) that the fluctuation types of these three frequency responses are damped fluctuation, persistent fluctuation and divergent fluctuation, respectively.

![Diagram](image)

(a) Stability domain (b) Fluctuation types of frequency responses

**Figure 3.** Schematic diagrams of stability domain and fluctuation types of frequency responses.

Only stable system (i.e. the characteristic parameters such as frequency response are all damped) is of practical value [10-12]. The stability of system can be represented by the size of stability domain (i.e. the position of stability boundary). Then a hydropower station with upstream air cushion surge chamber is selected as example for the analyses of influencing factors on the stability for waterpower-speed control system. The basic information of this hydropower station are as follows: two units with a headrace tunnel; rated output is 610MW; rated speed is 166.7r/min; rated head is 288.0m; rated discharge is 228.6m³/s.

The following analyses are carried out by controlling variate method and the default values of variables are as follows: \( T_{wy} = 23.84 \text{s}; \) \( T_{wy} = 1.26 \text{s}; \) \( p_0 = 200 \text{m}; \) \( l_0 = 20 \text{m}; \) \( m = 1.4; \) \( F = 5928 \text{m}^2; \) \( T_a = 9.46 \text{s}; \) \( e_g = 0; \) \( e_h = 1.5, e_x = -1, e_y = 1, e_{gh} = 0.5, e_{gt} = 0, e_{ht} = 1; \) in which \( F = n_f F_{th}; \) \( F_{th} \) is the critical stable sectional area of air cushion surge chamber and its value is 6240m² which is computed from R Svee formula [1], i.e.

\[
F_{th} = \frac{L z f_i}{2 \alpha_g (H_0 - 2 h_0)} (1 + \frac{m p_0}{l_0}) \]

\( n_f \) is amplification coefficient of sectional area of air cushion surge chamber with the default value of 0.95.
5.1. Effect of water inertia of pipeline

Water inertia of pipeline consists of water inertia of headrace tunnel $T_{wy}$ and water inertia of penstock $T_{wt}$. Their effects on stability are shown in Figure 4.

![Figure 4](image)

Figure 4. Effects of water inertia of pipeline on stability.

Figure 4 shows that:

1) $T_{wy}$ has a great effect on stability. The stability domain becomes very small when the value of $T_{wy}$ is very large. With the homogeneous decrease of $T_{wy}$, the stability domain increases and the range of increase becomes larger at the same time. However, if $T_{wy}$ continues to decrease after it reduce to a certain value, the stability domain will no longer change, and moreover, there is a domain in which $b_t$ is lesser while $T_d$ is biggish and the system becomes to unstable from stable (i.e. the shadow domain in Figure 4 (a)). Hence, there is a critical value of $T_{wy}$ about the stability which is corresponding to the condition that the critical stable sectional area of air cushion surge chamber is equal to the actual area. When $T_{wy}$ is greater than the critical value, the decrease of $T_{wy}$ will cause the stability of system to become worse. However, when $T_{wy}$ is less than the critical value, the stability of system will remain unchanged with the variation of $T_{wy}$.

2) $T_{wt}$ has a biggish effect on stability. With the homogeneous decrease of $T_{wt}$, the stability domain increases homogeneously, i.e. the stability of system becomes better. But the range of increase is very limited. When $T_{wt}$ is equal to zero, the stability domain has an evident increase which indicates that the effect of water inertia in penstock on the stability of system is unfavorable.

5.2. Effect of air cushion surge chamber parameters

The selection of air cushion surge chamber parameters, i.e. $p_0$, $l_0$, $m$ and $n_f$, is the key to the design of an surge chamber. The effects of $p_0$, $l_0$, $m$ and $n_f$ on stability are shown in Figure 5.

![Figure 5](image)
Figure 5 shows that:

1) $p_0$, $l_0$, $m$ and $n_f$ have obvious influence on the stability of the system. Like $T_{wy}$, there are critical values of $p_0$, $l_0$, $m$ and $n_f$ about the stability and these critical values correspond to the condition that the critical stable sectional area of air cushion surge chamber is equal to the actual area.

2) For $p_0$ and $m$, if they are more than their own critical values, the stability will become better when $p_0$ and $m$ decrease. And with the homogeneous decrease of $p_0$, the range of increase of stability domain becomes larger. However, if they are less than their own critical values, the stability domain will not change any more when $p_0$ and $m$ decrease.

3) For $l_0$ and $n_f$, if they are less than their own critical values, the stability will become better when $l_0$ and $n_f$ increase. And with the homogeneous increase of $l_0$ and $n_f$, the range of increase of stability domain becomes larger. If they are less than their own critical values, the stability domain will not change any more when $l_0$ and $n_f$ increase.

5.3. Effect of hydraulic turbine characteristics

The six transmission coefficients reflect the moment characteristic and discharge characteristic of hydraulic turbine. In order to analyse the effects of hydraulic turbine characteristics on the stability, the stability domains of ideal hydraulic turbine and actual hydraulic turbine are compared in Figure 6. The six transmission coefficients of actual hydraulic turbine, which is corresponding to the same operating point of ideal hydraulic turbine, are as follows: $e_h=1.493$, $e_x=0.985$, $e_y=0.753$, $e_{qh}=0.681$, $e_{qx}=0.308$, $e_{qy}=0.869$.

Figure 6 shows that: The stability domain of actual hydraulic turbine is smaller than that of ideal hydraulic turbine. This result indicates that the stability of the former is worse than that of the latter. It is well known that the essence of the effect of hydraulic turbine characteristics on stability is the effect of hydraulic turbine efficiency [13]. Therefore, the hydraulic turbine efficiency has unfavorable effect on stability.

5.4. Effect of generator characteristics

According to equation (6), $T_e$ and $e_g$ are the parameters that represent the generator characteristics, and their effects on stability are shown in Figure 7.
Figure 7 shows that:
1. $T_a$ has little effect on stability. With the homogeneous increase of $T_a$, the stability domain increases homogeneously, i.e. the stability of system becomes better.
2. $e_g$ has great effect on stability. $e_g=0$ stands for the omission of function of load self-regulation. When $e_g$ takes positive value and increases, the stability domain enlarges by a large margin. This result illustrates that $e_g$ has a significant improvement on stability. In comparison with $T_a$, the range of increase of stability boundary becomes smaller with the homogeneous increase of $e_g$.

5.5. Effect of regulation modes of governor

There are three regulation modes of governor, i.e. frequency regulation, power regulation and servomotor stroke regulation [11]. Frequency regulation mode and power regulation mode are all closed-loop control modes and are conditionally stable, however, servomotor stroke regulation mode is open-loop control mode and is unconditionally stable [14].

In this section, the effects of frequency regulation mode and power regulation mode on stability are compared. The equation of governor for frequency regulation mode is equation (7). The control pattern of power regulation mode is selected as "integration and feedforward" and its regulation block diagram and governor equation are shown in Figure 8 and equation (12), respectively.

$$\frac{d}{dt} \Delta P_g = b_p T_{y,s} \left( p_s - p_t \right)$$

in which $p = (P_i - P_{i0})/P_{i0}$ and $p_g = (P_g - P_{g0})/P_{g0}$ are the relative deviations of actual output and given output of generator; $b_p$ is permanent droop; $T_y$ is servomotor response time constant.

By taking the derivative of equation (12) and then substituting equation (6) and $p = m_t + x$, we can obtain the following form of equation (12) in the condition of $T_y=0$:

$$b_p T_{y,s} \frac{d}{dt} T_a = -b_p T_a \frac{d}{dt} - b_p (e_g - 1) x$$

Figure 7. Effects of generator characteristics on stability.

Figure 8. Regulation block diagram of "integration and feedforward" control pattern.
Similarly, according to the Laplace transform of equations (1)-(6) and (13), we can obtain the overall transfer function and linear homogeneous differential equation of waterpower-speed control system for power regulation mode. This linear homogeneous differential equation is fifth order as well and its expression is omitted.

The stability domains of frequency regulation mode and power regulation mode are plotted and the effect of \( b_p \) on stability is analysed. The results are shown in Figure 9.

Figure 9 shows that:

1. Under the same conditions, the stability domain of power regulation mode is far larger than that of frequency regulation mode. This result indicates that the stability of power regulation mode is better than that of frequency regulation mode when they take the same governor parameters.

2. With the homogeneous decrease of \( b_p \), the stability domain of power regulation mode becomes larger and the range of increase of stability domain becomes larger at the same time. This result indicates that the decrease of \( b_p \) will cause the stability of system to become better.

6. Conclusions
This paper studies the stability of waterpower-speed control system for hydropower station with air cushion surge chamber. The major conclusions of this study are summarized as follows:

1. The dynamic characteristics of waterpower-speed control system is represented by a fifth order linear homogeneous differential equation. The stability conditions of this system is that the coefficients of this fifth order equation (i.e. \( a_i (i=0, 1, 2, 3, 4, 5) \)) and their second and fourth order determinants (\( \Delta_2, \Delta_4 \)) are all positive.

2. The stability of waterpower-speed control system can be quantificationally characterized by stability domain in coordinate system which the abscissa and ordinate are usually \( b_t \) and \( T_d \), respectively. The larger the stability domain, the better the stability.

3. The parameters of pipeline (including \( T_{wy}, T_{wt}, p_0, l_0, m \) and \( n_f \)) have considerable effect on stability. There are critical values of \( T_{wy}, p_0, l_0, m \) and \( n_f \). In one side of the critical values, the stability will become better with the increase of \( l_0 \) and \( n_f \) and the decrease of \( T_{wy} \) and \( m \); in the other side of the critical values, the stability remains unchanged when these parameters change. \( T_{we} \) has unfavorable effect on stability.

4. Hydraulic turbine characteristics, generator characteristics and regulation modes of governor have great effects on stability. Hydraulic turbine characteristics have unfavorable effect on stability. The increase of \( T_a \) and \( e_g \) can make the stability better and the improvement of \( e_g \) is more effective. The stability of power regulation mode is obviously better than that of frequency regulation mode. With the decrease of \( b_p \), the stability of power regulation mode becomes better.

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Appendix
The expressions of coefficients in equation (11) are as follows:

\[
\begin{align*}
a_0 &= f_1 f_9, \quad a_1 = f_1 f_{10} + f_2 f_9 + f_3 f_{12}, \quad a_2 = f_1 f_{11} + f_2 f_{10} + f_3 f_9 + f_5 f_{13} + f_6 f_{12}, \\
a_3 &= f_2 f_{11} + f_3 f_{10} + f_4 f_9 + f_5 f_{13} + f_7 f_{12}, \quad a_4 = f_3 f_{11} + f_4 f_{10} + f_5 f_{13} + f_6 f_{12}, \quad a_5 = f_4 f_{11} + f_5 f_{13},
\end{align*}
\]
\[ f_1 = e_{\phi}f_e T_e T_{\omega_0} T_{\kappa_0}, \quad f_2 = T_e \left[ T_{\omega_0}(1 + e_{\phi} \frac{2h_{\omega_0}}{H_0}) + T_{\kappa_0} e_{\phi} \frac{2h_{\kappa_0}}{H_0} \right], \quad f_3 = e_{\phi}(T_{\omega_0} + T_{\kappa_0}) + T_e \frac{2h_{\omega_0}}{H_0}(1 + e_{\phi} \frac{2h_{\kappa_0}}{H_0}), \]

\[ f_4 = 1 + e_{\phi} \frac{2(h_{\omega_0} + h_{\kappa_0})}{H_0}, \quad f_5 = T_e T_{\omega_0} T_{\kappa_0}, \quad f_6 = T_e( T_{\omega_0} \frac{2h_{\omega_0}}{H_0} + T_{\kappa_0} \frac{2h_{\kappa_0}}{H_0}), \quad f_7 = T_{\omega_0} + T_{\kappa_0} + T_e \frac{2h_{\omega_0}}{H_0} \frac{2h_{\kappa_0}}{H_0}, \]

\[ f_8 = \frac{2(h_{\omega_0} + h_{\kappa_0})}{H_0}, \quad f_9 = b_i T_e T_{\omega_0} \quad f_{10} = b_i T_e (e - e_2) + T_e e_4, \quad f_{11} = e_7, \quad f_{12} = b_i T_e e_3 e_2 - T_e e_4 e_2, \quad f_{13} = -e_4 e_2. \]

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