Research on finite element analysis of the infrared grating lock-in thermal imaging method

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Abstract

Infrared grating lock-in thermal test is a kind of infrared nondestructive testing methods with the combination of grating incentive and infrared lock-in thermal technology. This method is adopted to realize the vertical crack detecting inside the components. There is no related research up to now by using a grating which changes according to sine rule to the defects detection. In this paper, the numerical simulation of this method is carried out, and the finite element analysis process is introduced in detail. Results show that the choice of appropriate detection frequency and level speed is the key for the infrared grating lock-in method to detect. Our study verifies the feasibility of effective detection for superficial cracks in the film.

Key words: grating, lock-in, numerical simulation, frequency, level speed

Introduction

The non-contact detection of complex construction shows its great significance nowadays[1-3], and the infrared thermal imaging detection technology, as a kind of non-contact detection method characterized by its safety, high precision, large single detection area and being suitable for the site, has become an important mean to assure the product quality and the safe operation of the detection. For the detection of complex artifacts that work under the harsh environment like high temperature oxidation, the traditional nondestructive testing technology has certain limitations. Therefore, it is important to detect the internal defects of metal part accurately and effectively, find the cracks that may cause bad effect timely, especially the cracks hiding inside and being perpendicular to the surface, and improve the work safety.

The vertical cracks are difficult to detect. The main reason is that, in most detection methods, the propagation of the detecting signal is parallel to the vertical cracks and does not interact with them. So, the traditional methods are often invalid or insensitive to detect the them. The eddy current testing method does detect the vertical cracks in conduction. However, it is invalid for insulators.
So we put forward a new detection method called infrared grating lock-in thermal imaging method, which has wider application scope and can detect vertical cracks as well as horizontal cracks. In actual infrared thermal imaging detection, the discretion of the modulation frequency and the horizontal displacement rate of speed are the most important factors that affect the effectiveness of infrared thermal imaging detection method.

In this article, the correlativity between the phase difference of the infrared grating lock-in thermal imaging method and the factors like modulation frequency, horizontal shift rate, crack size, crack depth, and material parameter is uncovered from the aspect of theoretical calculation, and the multi-component quantitative relation curve is obtained as the reliable theoretical guidance for the parameters of the heat source selection in next step of experiment research.

1. The basic principle and detection process

By the traditional infrared phase locking method, infrared grating lock-in thermal imaging detection technology bring in the level of moving heat source excitation component in the horizontal direction. Heat conducts in the vertical and horizontal directions, and the defeat appears through the abnormalities of Specimen surface temperature distribution when heat conduction encountered obstacles at the defeat. The lock-in technique can effectively eliminate the interference of noise signal. In the traditional infrared lock-in thermal imaging test, due to the same direction of the vertical crack and the heat conduction, there is no reflection section, so it is difficult to be detected effectively.

As a supplement to the traditional method[4-7], our new technology imports the horizontal heat fluctuations, and then the problem of vertical cracks detection is solved. In this paper, we studied on the detection of surface crack defects in metal materials and the defeats perpendicular to the surface with the method of infrared optical grating imaging detection technology through theory and test. By using the finite element method, the heat transfer process in the metal is analyzed, and the amplitude and phase diagram of the surface temperature of the specimen is calculated.

We established a two-dimensional heat transfer model, for a rectangular plate with thermal insulation on both sides and the bottom surface, and take the length as 100mm. Then internal crack length is 2 ~ 12mm. In the top heating source excitation, crack depth of 1 ~ 8mm(The depth of the vertical crack up to the point where the depth gauge). Infrared grating lock-in thermal imaging method to detect internal defects mainly includes the process of the heat source heating, heat conduction and the acquisition and processing of thermal images.

(1) Heat conduction differential equation within the object

\[ \frac{\partial^2 T(x, y, t)}{\partial x^2} + \frac{\partial^2 T(x, y, t)}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T(x, y, t)}{\partial t} \]  

(2) Heat conduction over the defect

Horizontal crack : \[ \frac{\partial T}{\partial y} \bigg|_{y=m} = 0 \]

Vertical crack : \[ \frac{\partial T}{\partial x} \bigg|_{x=n} = 0 \]

(3) The initial and boundary conditions:

(4)

(5)

(6)

(7)
\[ T(x, y, 0) = 0 \]
\[ \frac{\partial T}{\partial x} \bigg|_{x=-l} = \frac{\partial T}{\partial x} \bigg|_{x=l} = 0 \]
\[ \frac{\partial T}{\partial y} \bigg|_{y=h} = 0 \]
\[ \frac{\partial T}{\partial y} \bigg|_{y=0} = A \sin(\omega t + \omega x + \phi) \]

Solving heat transfer analysis problems is the theoretical basis of infrared thermal image inspection, and it can help to deepen the recognition of the essence of the infrared thermal image inspection physical process. However, due to the complexity of heat transfer process, usually, we can only get the analytical solution of simple boundary conditions. It is the complexity of solving the unsteady heat transfer analysis that makes the numerical method extremely necessary. Finite difference method starts from the differential equation. After the discrete processing of the area, using difference, difference quotient instead of differential, differential quotient approximately. In this way, the solving of a differential equation and boundary conditions can be summed up in solving a linear algebraic equation, and then we can get a numerical solution. In the actual process of heat transfer, due to the analytical solution is steady-state solution, the numerical solution is transient-state solution, so the results of the numerical solution is closer to the actual situation. In the process of calculating, all the data used in calculating the Amplitude and phase diagram are from the analysis of the transient-state solution of certain heating time, so whether it's in the process of numerical simulation or real experiments, numerical solution is more feasible and has more advantages compared with the analytical solution. However, in the process of the finite difference calculation, the computational accuracy is affected by time step and the the node space, so we need to find a suitable algorithm to ensure that the calculation results are accurate, and the precision is not involved in the analytic solution.

2. The differential processing

2.1 The discrete of heat transfer differential equations

When establishing the difference scheme, we need to discrete type (1), make the discretization of area \(0 \leq x \leq L\) and \(0 \leq y \leq L\) respectively with the node \((I = 1, 2, \ldots, p+1)\) and \((J = 1, 2, \ldots, q+1)\), the distance between two nodes is called distance step. Meanwhile, we discrete the time area \(t \geq 0\) with the note \(n = 1, 2, \ldots\), the interval between two moments is called time step. In each layer of the differential calculation, presupposing a temperature field (usually take the temperature field of the former layer as the initial field of this layer) is necessary, and then until the convergence of iterative calculation.
Make the bulk chemical separation of heat conduction equation (1), and written in the following form:

$$\left( \frac{\partial^{2}u}{\partial x^{2}} \right)_{i,j}^{n+1} + \left( \frac{\partial^{2}u}{\partial y^{2}} \right)_{i,j}^{n+1} = \frac{1}{\alpha} \left( \frac{\partial u}{\partial t} \right)_{i,j}^{n}$$  \hspace{1cm} (8)

The left type expressed in central difference quotient, the right type with the forward difference quotient, we can get:

$$\frac{u_{i-1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i+1,j}^{n+1}}{\Delta x^2} + \frac{u_{i,j-1}^{n+1} - 2u_{i,j+1}^{n+1} + u_{i,j}^{n+1}}{\Delta y^2} + \alpha(\Delta x + \Delta y) = \frac{1}{\alpha} \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} + o(\Delta t)$$  \hspace{1cm} (9)

Ignore: $o(\Delta t + (\Delta x)^2 + (\Delta y)^2)$, and let: $C = \frac{\alpha \Delta t}{(\Delta x)^2 + (\Delta y)^2}$.

According to the principle of von-Neumann analyses, assuming that the boundary value of initial value calculation is accurate and correct, and an error vector is introduced in the calculation at some layer, the error is a small perturbation. If the disturbance intensity is increasing with the passage of time, then this format is not stable. On the other hand, if the amplitude of disturbance attenuation or remained the same over time, the format is stable. Therefore, we put a harmonic component of the error vector into the discrete equations, to draw the amplitude ratio of harmonic component between adjacent two layers. The stability of the format requirements:

$$\left| \frac{\psi(t + \Delta t)}{\psi(t)} \right| = \mu \leq 1$$  \hspace{1cm} (10)

We bring

$$e(t) = \psi(t)e^{i\theta + j\vartheta}$$

Into

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \alpha \left( \frac{u_{i-1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i+1,j}^{n+1}}{\Delta x^2} + \frac{u_{i,j-1}^{n+1} - 2u_{i,j+1}^{n+1} + u_{i,j}^{n+1}}{\Delta y^2} \right)$$  \hspace{1cm} (11)

Meanwhile we let: $\Delta x = \Delta y = h$, \hspace{1cm} $i = j, \theta = \vartheta$, \hspace{1cm} and then:
\[
\frac{\psi(t + \Delta t) - \psi(t)}{\Delta t} e^{\mu_i + \mu_j} = 2\alpha \frac{\psi(t)}{\Delta h^2} \left[ e^{f(t+1)\theta} - 2e^{f\theta} + e^{f(i-1)\theta} \right]
\]

After processing, it can be:

\[
\mu = \frac{\psi(t + \Delta t)}{\psi(t)} = 1 - 4\left(\frac{\alpha \Delta t}{\Delta h^2}\right)(1 - \cos \theta) = 1 - 8C \sin^2 \frac{\theta}{2}
\]

Stable condition is:

\[-1 \leq 1 - 8C \sin^2 \frac{\theta}{2} \leq 1\]

This type automatically set up right end, to make the left side under arbitrary theta value, it should be made: \(C \leq \frac{1}{4}\).

For each point inside the grid area, calculation is according to the following general formula:

\[
u_{i,j}^{n+1} = C(u_{i,j-1}^n + u_{i,j+1}^n + u_{i+1,j}^n + u_{i-1,j}^n) + (1 - 4C)u_{i,j}^n \quad (15)
\]

2.2 The discrete of boundary conditions

For boundary conditions, the heating surface is the second kind of boundary, the rest of the boundaries which have convection with the outside air are the third kind boundary condition. Three kinds of simple boundary conditions are defined as follows: the first kind of boundary conditions for a given object boundary temperature of each point; the second category of each point on the boundary conditions for a given object boundary heat flux values; the third type for a given boundary surface between each point and the surrounding fluid convection heat transfer coefficient and the surrounding temperature of the fluid. To remove the four various points on the boundary of the endpoint, here in the top border, for example, the point at the top of the T value is unknown, so need according to the boundary conditions, the unknown T point value used to calculate the equivalent point B value of replacement:

\[
\frac{u_{i,2}^n - u_{i,0}^n}{2\Delta y} = f_i
\]

The same procedure may be easily adapted to obtain the unknown points on the other three boundaries:

The left boundary:

\[
\frac{u_{2,j}^n - u_{0,j}^n}{2\Delta x} = f_i \quad (17)
\]

The right boundary:

\[
\frac{u_{p+1}^n - u_{p-1}^n}{2\Delta x} = f_r \quad (18)
\]

The bottom boundary:

\[
\frac{u_{i,q+1}^n - u_{i,q-1}^n}{2\Delta y} = f_b
\]

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The initial boundary conditions are given as: 
\[ f_l = f_r = f_b = 0, \quad f_t = q \]
And then we bring the boundary conditions into the general \( \text{type()} \), the difference equation of the boundary conditions can be achieved:

For \( x=1, 2 \leq y \leq q \)
\[ u_{1,j}^{n+1} = C(u_{2,j}^n - 2hf_j + u_{2,j}^n + u_{1,j+1}^n + u_{1,j-1}^n) + (1 - 4C)u_{1,j}^n \quad (20) \]

For \( x=p+1, 2 \leq y \leq q \)
\[ u_{p+1,j}^{n+1} = C(u_{p+1,j}^n + 2hf_j + u_{p+1,j}^n + u_{p+1,j+1}^n + u_{p+1,j-1}^n) + (1 - 4C)u_{p+1,j}^n \quad (21) \]

For \( y=1, 2 \leq x \leq p \)
\[ u_{i,1}^{n+1} = C(u_{i,2}^n - 2hf_i + u_{i,2}^n + u_{i+1,1}^n + u_{i-1,1}^n) + (1 - 4C)u_{i,1}^n \quad (22) \]

For \( y=q+1, 2 \leq x \leq p \)
\[ u_{i,q+1}^{n+1} = C(u_{i,q+1}^n + 2hf_i + u_{i,q+1}^n + u_{i,q+1+1}^n + u_{i,q+1-1}^n) + (1 - 4C)u_{i,q+1}^n \quad (23) \]

As for the four endpoints, as shown in the red dotted box: if the point O is the upper left vertex of the boundary, then the value of upper point T and left point L is unknown and nonexistent. In the last step, we convert the value of the point T into the value of internal point B according to the top boundary condition \( f_t \).

Besides, we need to convert the value of L into the value R and the left boundary \( f_l \) in similar way. Similarly, we use the same methods dealing with three other endpoints, from which expression for each endpoint:

\[ x=1, y=1 \]
\[ \frac{u_{1,0}^n - u_{1,0}^{n-1}}{2\Delta y} = f_i \quad (24) \]
\[ u_{1,1}^{n+1} = C(u_{2,1}^n - 2hf_i + u_{2,1}^n + u_{1,2}^n + u_{1,2}^n - 2hf_i) + (1 - 4C)u_{1,1}^n \quad (25) \]

\[ x=1, y=q+1 \]
\[ \frac{u_{1,q+2}^n - u_{1,q+1}^n}{2\Delta y} = f_b \quad (26) \]
\[ u_{1,q+1}^{n+1} = C(u_{2,q+1}^n - 2hf_i + u_{2,q+1}^n + u_{1,q+1}^n + u_{1,q+1}^n + 2hf_i) + (1 - 4C)u_{1,q+1}^n \]

\[ x=p+1, y=1 \]
\[ \frac{u_{p+1,2}^n - u_{p+1,0}^n}{2\Delta y} = f_i \quad (27) \]
\[ u_{p+1,1}^{n+1} = C(u_{p+1,1}^n + 2hf_i + u_{p+1,1}^n + u_{p+1,2}^n + u_{p+1,2}^n - 2hf_i) + (1 - 4C)u_{p+1,1}^n \quad (28) \]

\[ x=p+1, y=q+1 \]
\[ \frac{u_{p+2,q+1}^n - u_{p+1,q+1}^n}{2\Delta x} = f_r \quad (29) \]
\[ u_{p+1,q+1}^{n+1} = C(u_{p+1,q}^n + 2hf_x + u_{p+q+1}^n + u_{p,q+1}^n + u_{p,q+1}^n + 2hf_x) + (1 - 4C)u_{p+1,q+1}^n \]

For the convenience of calculation, we let \( p = q \).

**2.3 The solution of the defect area**

Defective area can be regarded as three thin layers consisted of the ontology material layer, defects, ontology material layer after the defect, so that for the points in the defect layer, we regard the left and right sides of the points corresponding to two numerical solutions.

For two endpoints of crack, due to the points T and B are not on the crack, but in the nondestructive area, so we need to calculated separately.

![Figure 2. Mesh generation of vertical crack](image)

For each point on the crack of removing the endpoint:

\[ x = l, y = A \]

\[ U_{i,j}^{n+1} = C(U_{i-1,j}^n + 2\Delta h_x + U_{i,j+1}^n + U_{i,j-1}^n + U_{i,j+1}^n) + (1 - 4C)U_{i,j}^n \]

\[ U_{i,j}^{n+1} = C(U_{i+1,j}^n - 2\Delta h_x + U_{i,j+1}^n + U_{i,j-1}^n + U_{i,j+1}^n) + (1 - 4C)U_{i,j}^n \]

For each point on the crack of removing the endpoint:

\[ x = l - 1, y \in [A, B] \]

\[ U_{i,j}^{n+1} = C(U_{i-1,j}^n + U_{i+1,j}^n + U_{i,j+1}^n + U_{i,j-1}^n) + (1 - 4C)U_{i,j}^n \]

\[ x = l + 1, y \in [A, B] \]

\[ U_{i,j}^{n+1} = C(U_{i-1,j}^n + U_{i+1,j}^n + U_{i,j+1}^n + U_{i,j-1}^n) + (1 - 4C)U_{i,j}^n \]

Due to the influence of the defect, the area in the non-defect area which is closed to the defect around both sides needs separate treatment, as shown in type():

\[ x = l - 1, y \in [A, B] \]

\[ U_{i,j}^{n+1} = C(U_{i-1,j}^n + U_{i+1,j}^n + U_{i,j+1}^n + U_{i,j-1}^n) + (1 - 4C)U_{i,j}^n \]

\[ x = l + 1, y \in [A, B] \]

\[ U_{i,j}^{n+1} = C(U_{i-1,j}^n + U_{i+1,j}^n + U_{i,j+1}^n + U_{i,j-1}^n) + (1 - 4C)U_{i,j}^n \]

According to the differential equation for accurate analytic solutions of the given boundary
conditions, although the theory is complete, due to the complexity of the actual structure shape and the variable temperature conditions, it is not possible to get the temperature field accurately rely on the traditional analytic method. Especially under the condition of varieties of boundary conditions coexistence, mathematical analysis is helpless, so we can only adopt the method of numerical calculation. The finite difference method has been extensively applied in the field of heat conduction. It is a numerical simulation of physical phenomena, a real situation approximation. By classifying the objects into finite element, it solves the quantity of a finite number of unknown nodes to approximately simulate the real environment of the infinite unknown quantity. Therefore the finite element method is the most convenient and effective tool to solve these problems.

As a basic theory of infrared thermal image inspection, solving the analytic solution of heat transfer is helpful to strengthen the essence of the physical process in the infrared detection. But we can only get a few number of analytic solution with some simple boundary conditions, which is limited to the complexity of the heat transfer process. Starting from the differential equation, finite difference method dealing the area with the discrete, and then using difference, difference quotient instead of differential, differential quotient approximately. So that the solving of the differential equation and boundary conditions can be summed up in solving a linear algebraic equation, and numerical solution is approved. Finite difference method is limited to the rule of difference grid, only see a node, but ignore the features of units which link the nodes together, while the units are basic cells of the whole, in the process of calculating the temperature of each node, the unit will play their due contributions. This limitation reflects the difference between the numerical solution and analytical solution. Also, the finite difference calculation, the calculation precision is affected by time step and the node spacing; we need to find the right algorithm to ensure the accuracy of calculation results, while the analytical solution has nothing to do with precision.

3. The simulation results

The form of the heat source with the grating as below:

\[ y = A \sin(\omega t + \varphi) \]

\[ T = \frac{2\pi}{\omega} \]

\[ \lambda = vT = \frac{2\pi}{k} \]

\[ \omega_z = k = \frac{\omega}{v} \]

\[ y = A \sin(\omega t + \omega_z x + \varphi) \]  

(40)

Based on the traditional infrared lock-in thermal imaging method, the infrared grating lock-in thermal imaging method introduces the grating of horizontal direction to create horizontal heat conduction. Since that it is difficult to detect vertical crack by the traditional methods, and due to the lack of quantitative research current for lock-in thermography method, we are trying to detect vertical crack effectively through the infrared grating lock-in thermal imaging method, meanwhile realizing quantitative research[7-9].

3.1 The effects of heating time on test results
Results are visible from figure 1, the change law under the influence of time about the same in horizontal and vertical crack detection. In the initial stage, the longer the heating the more sufficient of heat conduction to internal, this lead to a bigger phase difference, which is equal to better detecting result. When heated to a certain time, the increase of heating time for testing result is no longer effective due to the heat transfer process going gradually stabilized inside the object; When the heating time continue increasing, the detection result is poorer, because the internal temperature distribution gradually converge everywhere after a long time of heating.

3.2 Contrast of amplitude figure in horizontal crack detection with the new and traditional method

From amplitude figure in detecting the horizontal crack, the test results of amplitude with the infrared grating lock-in thermal imaging method is less than the traditional method, this is the same as the conclusion from the phase diagram because the added horizontal heat transmission disturbance plays a hinder role in the overall heat transfer inside the object. The introduction of the horizontal heat transfer component is not affected by the same level of crack with its conduction direction, thus the crackle of amplitude is slightly larger than the traditional method whose heat conduction is affected bigger by the existence of horizontal crack.
3.3 Contrast of amplitude figure in vertical crack detection

As depicted in the picture, the traditional method is invalid for vertical crack detection. When using infrared grating lock-in thermal imaging method to detect, due to the disturbance affected by the horizontal heat transfer component, its amplitude is less than the traditional way proceeds in most area.

![Figure 5. Amplitude figure in vertical crack detection with the new and traditional method](image)

3.4 The test result of detecting horizontal crack using the new method with the change of level speed $v(\text{equal to } 1/\omega x)$

Because $\omega x$ is inversely proportional to the horizontal moving velocity $v$, so when the greater the $\omega x$, horizontal displacement rate is smaller. As the chart shows, in the initial stage, due to the decrease of the horizontal displacement rate, a greater degree of heating heat source of objects lead to better heat transfer effect, and phase difference increased gradually. At the maximum peak, the blocking effect of heat conduction caused by the decline of horizontal displacement rate leads the temperature distribution within the object gradually to converge, and phase difference present decreasing trend which is equal to poorer detection result.

![Figure 6. The phase of detecting horizontal crack using the new method with the change of level speed $v$](image)
3.5 The test result of detecting vertical crack using the new method with the change of level speed $v$ (equal to $1/\omega x$)

In the initial stage, it is too fast to detecting effectively. Due to the blocking of vertical crack for horizontal heat transfer, as mobile rate drops, phase difference quickly achieve maximum peak, detection results show a trend of decline in the following as the horizontal displacement rate falling, the reason is the smaller horizontal speed, the closer to the traditional method, and the worse efficiency got.

![Figure 7. The phase of detecting vertical crack using the new method with the change of level speed $v$](image)

4. Conclusion

In guarantee of the horizontal crack detection by almost the same with traditional methods, the infrared grating lock-in thermal imaging method can also achieve the effectiveness of vertical crack detection. Through the finite element simulation model, the process of heat conduction and the influence of heat source excitation parameters on the test results were studied, the results show that in horizontal crack detection with the proposed method, the test results have been affected more by modulation frequency. When testing the vertical crack, test results were more easily affected by the horizontal moving speed. Our work is helpful to determine appropriate incentive parameters in the actual detection and find as many cracks as possible accurately and quickly.

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