Spectral Distribution of Fano Interferences in Classical Damped Oscillation

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Abstract. We present the classical analogy of Fano interferences in helpful view to understand Fano behavior of phonons excitation as the interaction of two harmonic oscillators in damped oscillation system. In order to obtain more informative explanation, we demonstrate the coupled oscillator motions in javascript using numerical integration method of 4th Order Runge Kutta. A detailed discussion of Fano spectral distribution is shown by considering some varied oscillation parameters including natural oscillation frequency, damping factor, coupling constant, as well as applied external force. It is further shown that the oscillation phase-shift differs in those varied oscillation parameters. The range of allowed values of parameters to get the appropriate results will be listed in this article.

1. Introduction

Fano resonance has become a subject of interest topic today especially in plasmonic field showing the distinctive spectral distribution of scattering resonance. Invented by Ugo Fano in 1935 [1], the ensuing study of Fano resonance was then investigated in Wood optical anomaly by Sarrazin et. al., Bragg resonance interaction with Fabry-Perot band in photonic crystal [2] and transmission-reflection process in photonic crystal plates [3]. Furthermore Fano resonance has contributed to the many applied study areas such as optical filtering [4], polarization filtering [5], sensing, laser, modulator, optical resonator [6, 7] and nonlinear optical.

The typical characteristic of Fano resonance can be described in the role of the optical interference effects between transmission and scattered wave that gives rise the asymmetrical and narrowing profile [8, 9]. Despite of the quantum and optical phenomena, Fano resonance can be simply explained in classical mechanics system as a coupled mass-spring oscillator with a presence of damping response and coupling effect [10, 11]. In this paper, we study and visualize Fano resonance by means of classical analogy employing some various parameters, such as spring constant, coefficient of friction (b), natural frequency ($\omega$), and external force amplitude. The effects of each parameter variation will be revealed in detailed explanation to determine their contribution in Fano resonance phenomena.
2. Formulation

The light-matter interaction has been widely understood to underline an asymmetric Fano resonance phenomena as the response of resonant amplitude to the given frequency representing the interference effects between the incoming and scattered field [12, 13, 14]. In the simple treatment, classical analogy of Fano resonance has been formulated for coupled mass spring oscillation system [9] as illustrated in the figure 1.

![Figure 1. Fano resonant classical model of the coupled-mass-spring oscillation system](image)

We start to analyse a simple coupled-mass spring system illustrated in figure 1 with different oscillation parameters under the given external force, coupling effect between the adjacent spring, and linear damping response. It follows that the particle mass 1 explicitly satisfies the following kinematics equation

\[
\ddot{x}_1 + b_1 \dot{x}_1 + \omega_1^2 x_1 + v_{12} x_2 = a_1 e^{i\omega t},
\]

where \(b_1\) denotes the linear damping coefficient of mass 1 while \(v_{12}\) corresponds to the coupling factor associated with the interaction strength between the adjacent springs. The corresponding spring constant and particle mass is given on its own natural frequency \(\omega = \sqrt{k_1/m_1}\) while the external force coefficient of \(a_1\) are directly correlated to the oscillation mode representing the bright mode or dark mode. In the similar expression, the motion of mass 2 can be governed by

\[
\ddot{x}_2 + b_2 \dot{x}_2 + \omega_2^2 x_2 + v_{12} x_1 = a_2 e^{i\omega t},
\]

For steady state solution of harmonic oscillation, the expression of displacement of masses are given respectively by \(x_1 = c_1 e^{i\omega t}\) and \(x_2 = c_2 e^{i\omega t}\) where \(c_1\) and \(c_2\) represent the oscillation amplitude of mass 1 and mass 2 respectively. Inserting the two into equation (1) and (2) leads to the following matrix

\[
\begin{pmatrix}
\omega_1^2 - \omega^2 \\
v_{12}
\end{pmatrix}
\begin{pmatrix}
\omega_2^2 - \omega^2
\end{pmatrix}
\begin{pmatrix}
c_1 \\
c_2
\end{pmatrix}
= 
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix}
\]

Considering matrix inverse method, we obtain the following expression of Fano resonance amplitude

\[
c_1 = \frac{a_1(\omega_2^2 - \omega^2 + i\omega b_2)v_{12}}{v_{12}^2 - (\omega_2^2 - \omega^2 + i\omega b_2)(\omega_1^2 - \omega^2)} - a_2v_{12}
\]

\[
c_2 = \frac{a_1v_{12} - (\omega_1^2 - \omega^2 + i\omega \gamma_2)v_{12}}{v_{12}^2 - (\omega_2^2 - \omega^2 + i\omega \gamma_2)(\omega_1^2 - \omega^2 + i\omega \gamma_2)}a_2
\]

Further, the kinematics motion by expressed by equation (1) and (2) will be calculation using numerical integration of 4th order Runge Kutta giving the adequate accurate results depending on the iteration interval length. As a result, we can determine the displacement of the two masses to describe the oscillation process with varied different oscillation parameter.
3. Result and Discussion

The natural frequency of bright mode and dark mode are selected at different values in order to achieve the narrow spectrum of Fano resonance characteristic [13, 15].

As depicted in figure 2, the resonant peak is located around its natural frequency which represent the material characteristics. However, when both bright and dark mode are calculated in those natural frequency, it exhibits the destructive amplitude. In this nature, the correlation between positions of peak of spectrum and the natural frequency was unclear, thus we could not characterize materials by the destructive resonance.

In this work, we set dark mode external force $a_2$, mass $m_1$ and $m_2$, initial position $x_A$ and $x_B$, and initial amplitude shift $x'_A$ and $x'_B$. The adjustable parameters were bright mode external force $a_1$, bright mode friction coefficient $b_1$, and coupling constant $v_{12}$, which are given by table 1.

**Table 1.** Oscillation parameters with constant parameters $m_1=m_2=1$, $a_2=0$, $\omega=2$, $\omega_1=1$, $\omega_2=1.21$, $x_A=4$, $x_B=8$, $x'_A=4.5$, $x'_B=8.5$.

| Parameter | I    | II   | III  | IV   | V    | VI   | VII  | VIII |
|-----------|------|------|------|------|------|------|------|------|
| $a_1$     | 1    | 1    | 1    | 0.1  | 0.5  | 1.2  | 1    | 2    |
| $b_1$     | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.4  | 2    |
| $b_2$     | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| $v_{12}$  | 0.1  | 0.5  | 1    | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  |

It is shown that increasing $v_{12}$ will narrowed the peaks. For $v_{12} = 0.1$, two adjacent peaks present showing contrastly difference as the increasing value of coupling constant. The bright mode resonance occurs at around its natural frequency in which phase value will be zero as explained in equation (4). It is known that the bright mode amplitude sensitively depend on frequency, whereas dark mode amplitude is varied due to the coupling constant. In dark mode, the resonance peak is redshifted as increased coupling constant. The amplitude of bright mode is proportionally related to the external force, on the contrary in dark mode, the external force will affect inversely but not significant to the amplitude. The amplitude response to the frequency for bright and dark mode are respectively shown in figure 4 and 5.
Figure 3. The effect of coupling constant and amplitude response to frequency (a) bright mode motion, (b) dark mode motion ( – \( v_{12} = 0.1 \) – \( v_{12} = 0.5 \) – \( v_{12} = 1 \)), (c) amplitude response to frequency in bright mode, (d) amplitude response to frequency in dark mode, (e) phase response to frequency in bright mode, and (f) phase response to frequency in dark mode, with constant parameters \( m_1=m_2=1, \ a_2=0, \omega=2, \omega_1=1, \omega_2=1.21, x_A = 4, x_B = 8, x_{A'} = 4.5, x_{B'} = 8.5 \).

Figure 4. The effect of external force and amplitude response as function of frequency in bright mode (a) \( a_1=0.1 \) (b) \( a_1=0.5 \) (c) \( a_1=1.2 \).

Figure 5. The effect of external force and amplitude response as function of frequency in dark mode (a) \( a_1=0.1 \) (b) \( a_1=0.5 \) (c) \( a_1=1.2 \).
It can be seen in figure 6 that the increasing friction coefficient will reduce the amplitude. Similar to the previous case, the dark mode was relatively more stable when this parameter was involved. It is also interesting to note that the dark mode was relatively more stable when this friction parameter however unstable when the friction coefficient is small enough.

Figure 6. The effect of frictional coefficient (a) bright mode motion, (b) dark mode motion (\( b_f = 0.02 \), \( b_f = 0.4 \), \( b_f = 2 \)), (c) amplitude response to frequency in bright mode, (d) amplitude response to frequency in dark mode, (e) phase response to frequency in bright mode, and (f) phase response to frequency in dark mode, with constant parameters \( m_1 = m_2 = 1 \), \( a_1 = 1 \), \( a_2 = 0 \), \( \omega = 2 \), \( \omega_f = 1 \), \( \omega_2 = 1.21 \), \( x_A = 4 \), \( x_B = 8 \), \( x_A' = 7 \), \( x_B' = 8.5 \), \( v_{12} = 0.1 \).

Controlling the damping parameter with varying order of velocity (\( v^n \)), the amplitude fluctuations are plotted in terms to the oscillation frequencies in following figure which present the similar phenomena as Doppler effect.

Figure 7. Amplitude modulation period as function of frequency of oscillation with various damping order (– second order, – third order, – fourth order).
4. Conclusion

Fano classic resonance modeling is conducted with variations of parameters, such as frequency, coupling constants, damping factors, and external forces. The position of resonance mainly depends on the natural frequency, as well as damping factor, and coupling constant of the spring whereas the external force determine magnitude of resonance. The phase difference between the resulted light and dark modes is obtained due to the varied oscillation frequency. According to the time dependent deviation graph, increasing damping factor reduces the deviation of amplitude. This condition also indicates that amplitude modulation period depends on damping order. For the further analysis about Fano resonance, the greater damping order of Fano resonance should be studied.

Reference

[1] Hayashi, S., Nesterenko, D.V., Rahmouni, A., Ishitobi, H., Inouye, Y., Kawata, S., and Sekkat, Z. 2016. Light-tunable Fano resonance in metal-dielectric multilayer structures. *Scientific Reports*, 6, 33144.

[2] Sarrazin, M., Vigneron, J.-P., and Vigoureux, J.-M. 2003. Role of Wood anomalies in optical properties of thin metallic films with a bidimensional array of subwavelength holes. *Physical Review B*, 67(8), 085415.

[3] Rybin, M.V., Khanikaev, A.B., Inoue, M., Samusev, K.B., Steel, M.J., Yushin, G., and Limonov, M.F. 2009. Fano resonance between Mie and Bragg scattering in photonic crystals. *Physical Review Letters*, 103(2), 023901.

[4] Fan, P., Yu, Z., Fan, S., and Brongersma, M.L. 2014 *Nature Materials*, 13(5), 471-475.

[5] Cai, D.-J., Huang, Y.-H., Wang, W.-J., Ji, W.-B., Chen, J.-D., Chen, Z., and Liu, S.-D. 2015. Fano resonances generated in a single dielectric homogeneous nanoparticle with high structural symmetry. *The Journal of Physical Chemistry C*, 119(8), 4252-4260.

[6] Fan, S. and Suh, W. 2003. Temporal coupled-mode theory for the Fano resonance in optical resonators. *Journal of the Optical Society of America A*, 20(3), 569-572.

[7] Orta, R., Tibaldi, A., and Debernardi, P. 2016. Bimodal resonance phenomena—part II: high/low-contrast grating resonators. *IEEE Journal of Quantum Electronics*, 52 (12), 6600408.

[8] Hao, F., Sonnefraud, Y., Dorpe, P.V., Maier, S.A., Halas, N.J., and Nordlander, P. 2008 Symmetry breaking in plasmonic nanocavities: subradiant LSPR sensing and a tunable Fano resonance. *Nano letters*, 8(11), 3983-3988.

[9] Miroshnichenko, A.E., Flach, S., and Kivshar, Y.S. 2010. Fano resonance in nanoscale structures. *Reviews of Modern Physics*, 82(3), 2257-2298.

[10] Joe, Y. S., Satamin, A. M., and Kim, C. S. 2006. Classical analogy of Fano resonances. *Physica Scripta*, 74(2), 259-266.

[11] Riffe, D. M. 2011. Classical Fano oscillator. *Physical Review B*, 84(6), 064308-1–064308-8.

[12] Wang, Q., Huang, Y., and Yao, Z. 2016. Analysis of transition from Lorentz resonance to Fano resonance in plasmonic and metamaterial system. *Optical and Quantum Electronics*, 48(2): 83.

[13] Kato, H., Naito, Y., and Ishii, T. 2017. The control of resonance curve using the shape modulation of the scattering region in elastic waveguide. *Proceedings of Meetings on Acoustics*, 29, 065002.

[14] Ott, C., Kaldun, A., Raith, P., Meyer, K., Laux, M., Evers, J., Keitel, C.H., Greene, C.H., and Pfeifer, T. 2013. Lorentz meets Fano in spectral line shapes: a universal phase and its laser control. *Science*, 340 (6133), 716–720.

[15] He, J., and Yang, S. 2016. Line shapes in a plasmonic waveguide system based on plasmon-induced transparency and its application in nanosensor. *Optics Communications*, 381, 163-168.