Goos-Hänchen and Imbert-Fedorov shifts for Gaussian beams
impinging on graphene-coated surfaces

Simon Grosche, Marco Ornigotti and Alexander Szameit

Institute of Applied Physics, Friedrich-Schiller-Universität Jena,
Max-Wien-Platz 1, D-07743 Jena, Germany

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We present a theoretical study of the Goos-Hänchen and Imbert-Fedorov shifts for a fundamental Gaussian beam impinging on a surface coated with a single layer of graphene. We show that the graphene surface conductivity $\sigma(\omega)$ is responsible for the appearance of a giant and negative spatial Goos-Hänchen shift.

When an optical beam impinges upon a surface, non-specular reflection phenomena may occur, such as the Goos-Hänchen (GH) [1–3] and Imbert-Fedorov (IF) [4,5] shifts, resulting in an effective beam shift at the interface.

A comprehensive review on beam shift phenomena can be found in Ref. [6]. Although Goos and Hänchen published their work more than 60 years ago [1], this field of research is still very active, and in the last decades a vast amount of literature has been produced on the subject, resulting not only in a better understanding of the underlying physical principles [7–14], but also in the careful investigation of the effects of various field configurations [7,15,17–19] and reflecting surfaces [20–23] on the GH and IF shifts.

In recent years, on a parallel trail, graphene attracted very rapidly a lot of interest, thanks to its intriguing properties [24,25]. Its peculiar band structure and the existence of the so-called Dirac cones [26], for example, give the possibility to use graphene as a model to observe QED-like effects such as Klein tunneling [27], Zitterbewegung [28], the anomalous quantum Hall effect [29] and the appearance of a minimal conductivity that approaches the quantum limit $e^2/h$ for vanishing charge density [30]. In addition, the reflectance and transmittance of graphene are determined by the fine structure constant [31], and a single layer of graphene shows universal absorbance in the spectral range from near-infrared to the visible part of the spectrum [32].

Among the vast plethora of applications, graphene also proved to be a very interesting system where to observe beam shifts. Very recently, in fact, the occurrence of GH shift in graphene-based structures has been reported, both for light beams (where giant GH shift has been observed [33]) and for Dirac fermions [34]. Despite all this, a full theoretical analysis of GH and IF shifts in a graphene-based structure has not been yet carried out.

In this Letter, we therefore present a theoretical analysis of the GH and IF shifts occurring for a monochromatic Gaussian beam impinging onto a glass surface coated with a single layer of graphene. The results of our investigations show on one hand, that the appearance of a giant GH shift is ultimately due to the graphene’s surface conductivity $\sigma(\omega)$, and on the other hand, that the presence of the single layer of graphene introduces a dependence of the phases of the reflection coefficients on the incidence angle, thus resulting in a nonzero spatial GH shift also when total internal reflection does not occur.

We start our analysis by considering a monochromatic Gaussian beam with frequency $\omega = ck$ (with $k$ being the vacuum wave number), impinging on a dielectric surface characterized by the refractive index $n$ and coated with a single layer of graphene [Fig. 1 (a)]. The graphene layer is characterized by the optical conductivity $\sigma(\omega)$, whose expression can be given in the following dimensionless form [26]

$$\sigma(\Omega) = \frac{4\alpha}{\Omega} + \pi\alpha \left[ \Theta(\Omega - 2) + \frac{i}{2\pi} \ln \left(\frac{(\Omega - 2)^2}{(\Omega + 2)^2}\right) \right],$$

where $\Omega = \hbar\omega/\mu$ is the dimensionless frequency, $\mu$ is the chemical potential, $\alpha \approx 1/137$ is the fine structure constant [28] and $\Theta(x)$ is the Heaviside step function [35].

According to Fig. 1(b), we define three Cartesian reference frames: the laboratory frame $K = (O, x, y, z)$ attached to the reflecting surface, the (local) incident frame $K_i = (O, x_i, y_i, z_i)$ attached to the incident beam, and the (local) reflected frame $K_r = (O, x_r, y_r, z_r)$ attached to the reflected beam. These three reference frames are connected via a rotation of an angle $\theta$ around the $y$ direction [37]. The reflecting surface is located at $z = 0$, with the $z$-axis pointing towards the interface. With this choice of geometry, the incident beam comes from the region $z < 0$ and propagates in the $x$-$z$ plane.

The electric field in the incident frame can be then written, using its angular spectrum representation [36], as follows:

$$E_i(r) = \sum_{\lambda=1}^{2} \int d^2K \hat{e}_\lambda(U,V,\theta) A_\lambda(U,V,\theta) e^{ik_r \cdot r},$$

where $d^2K = dUdV$, $\hat{e}_\lambda(U,V,\theta)$ is the local reference frame attached to the incident field [38], $A_\lambda(U,V,\theta) =$
characterized by its surface conductivity \( \sigma(\Omega) \), whose explicit expression is given by Eq. (1). The dielectric substrate (green) is characterized by the refractive index \( n \). The different Cartesian coordinate systems \( K, K', K_r \) are shown.

\[
\alpha_\lambda(U, V, \theta) A(U, V) \text{ and } \mathbf{k}_i \cdot \mathbf{r}_i = UX_i + VY_i + WZ_i, \quad \text{being } X_i = k_0 x_i \text{ the normalized coordinate in the incident frame.} \quad \text{being } Y_i \text{ and } Z_i \text{ are defined in a similar manner.} \quad \alpha_\lambda(U, V, \theta) = \mathbf{e}_\lambda(U, V, \theta) \cdot \mathbf{f} \text{ accounts for the projection of the beam's polarization } \mathbf{f} = f_p \mathbf{\hat{x}} + f_s \mathbf{\hat{y}} \text{ (normalized according to } |f_p|^2 + |f_s|^2 = 1) \text{ onto the local basis, and} \quad A(U, V) \text{ is the beam's spectral amplitude, which here is assumed to be Gaussian, i.e.,} \]

\[
A(U, V) = e^{-w_0^2(t^2 + V^2)},
\]

being \( w_0^2 \) the spot size of the beam. In the remaining of the manuscript, we will consider only well collimated beams, namely the paraxial assumption \( U, V \ll 1 \) is implicitly understood.

Upon reflection, the electric field can be then written as follows:

\[
\mathbf{E}_r(\mathbf{r}_r) = \frac{\sum}{\lambda = 1} \int d^2 \mathbf{k}_\lambda (-U, V, \pi - \theta) \mathbf{\hat{A}}_\lambda(U, V, \theta) e^{i \mathbf{k}_\lambda \cdot \mathbf{r}_r},
\]

where \( \mathbf{k}_r \cdot \mathbf{r}_r = -UX_r + VY_r + WZ_r \) and \( \mathbf{A}_\lambda(U, V, \theta) = r_\lambda(U, V, \theta) A_\lambda(U, V, \theta) \), with \( r_\lambda(U, V, \theta) \) being the Fresnel reflection coefficients associated to the single plane wave component of the field [39]. The minus sign in front of \( U \) in \( \mathbf{e}_\lambda \), as well as in \( \mathbf{k}_r \cdot \mathbf{r}_r \), accounts for the specular reflection of the single plane wave component [38].

The presence of a single layer of graphene deposited on the dielectric surface modifies its reflection coefficients as follows [40]:

\[
r_s(\theta) = \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta - \sigma(\Omega)}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta + \sigma(\Omega)}},
\]

\[
r_p(\theta) = \frac{n^2 \cos \theta - \sqrt{n^2 - \sin^2 \theta} [1 - \sigma(\Omega) \cos \theta]}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta} [1 + \sigma(\Omega) \cos \theta]},
\]

where \( \theta \) is the incident angle, \( n \) is the refractive index of the dielectric medium and \( \sigma(\Omega) \) is the graphene's surface conductivity, as defined by Eq. (1). The modulus \( R_\lambda \) and phase \( \phi_\lambda \) of the reflection coefficients \( r_\lambda = R_\lambda e^{i \phi_\lambda} \) (with \( \lambda \in \{p, s\} \)) are shown in Fig. 2, together with the correspondent quantities for the case of a single dielectric surface without the graphene coating. While the presence of the graphene layer does not modify significantly \( R_\lambda \) for neither \( p \)- or \( s \)-polarization (as it appears clear from Figs. 2(a) and (c), respectively), the change induced in the phases \( \phi_\lambda \) of the reflection coefficients is considerable. For a normal air-glass interface, in fact, we have \( \partial \phi_\lambda / \partial \theta = 0 \) being \( \theta \) the angle of incidence. Here, instead, we have \( \partial \phi_\lambda / \partial \theta \neq 0 \). A closer inspection of Eqs. (5), moreover, reveals that such a novel \( \theta \)-dependence of the phases \( \phi_\lambda \) is entirely due to the graphene conductivity \( \sigma(\Omega) \).

To compute the GH and IF shifts, we calculate the center of mass of the intensity distribution in the reflected frame, namely [38]

\[
\langle \mathbf{R} \rangle = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |\mathbf{E}_r|^2 dX_r dY_r}{\int_{-\infty}^{+\infty} |\mathbf{E}_r|^2 dX_r dY_r} = \langle X_r \rangle \mathbf{\hat{x}} + \langle Y_r \rangle \mathbf{\hat{y}},
\]

where \( \mathbf{R} = (X_r, Y_r)^T \). Spatial (\( \Delta \)) and angular (\( \Theta \)) GH and IF shifts are then defined as follows:

\[
\Delta_{GH} = \langle X_r \rangle \bigg|_{z=0}, \quad \Theta_{GH} = \frac{\partial \langle X_r \rangle}{\partial z},
\]

\[
\Delta_{IF} = \langle Y_r \rangle \bigg|_{z=0}, \quad \Theta_{IF} = \frac{\partial \langle Y_r \rangle}{\partial z}.
\]

The explicit expressions of the GH and IF shifts for a fundamental Gaussian beam read, according to [41], as follows:

\[
\Delta_{GH} = w_p \frac{\partial \phi_p}{\partial \theta} + w_s \frac{\partial \phi_s}{\partial \theta},
\]

\[
\Delta_{IF} = -\cot \theta \left[ \frac{w_p a^2_s + w_s a^2_p}{a_p a_s} \sin \eta 
+ 2 \sqrt{w_p w_s} \sin(\eta - \phi_p + \phi_s) \right],
\]

\[
\Theta_{GH} = -\left( w_p \frac{\partial \ln R_p}{\partial \theta} + w_s \frac{\partial \ln R_s}{\partial \theta} \right),
\]

\[
\Theta_{IF} = \frac{w_p a^2_s - w_s a^2_p}{a_p a_s} \cos \eta \cot \theta,
\]

where \( f_p = a_p, f_s = a_s \exp(i\eta) \) and \( w_3 = a^2_p R^2_p / (a^2_p R^2_p + a^2_s R^2_s) \) (where \( \lambda \in \{p, s\} \)) is the fractional energy contained in each polarization state.
As suggested by Figs. 2(a) and (c), the changes in $R_\lambda$ introduced by the graphene layer are negligible. We therefore expect to observe no changes in the angular shifts $\Theta_{GH}$ and $\Theta_{IF}$, as they are functions of $R_\lambda$ solely. The spatial shifts $\Delta_{GH}$ and $\Delta_{IF}$, on the other hand, contain a dependence on the phases $\phi_\lambda$, and they are therefore affected by the presence of the graphene coating. Let us first discuss the IF shift. In this case $\phi_p - \phi_s$ is very close to $\pi$ [Figs. 2(b) and (d)], and the resulting spatial shift $\Delta_{IF}$ will be nonzero (but very small) even for linear polarization, in contrast to the case without graphene.

More interesting is the case of the spatial GH shift. For a normal air-dielectric interface, one has $\partial \phi_\lambda / \partial \theta = 0$ and therefore, according to Eq. (8a), $\Delta_{GH} = 0$. It is in fact well known since the pioneering work of Goos and Hänchen [1], that $\Delta_{GH} \neq 0$ occurs only in total internal reflection, where $R_\lambda = 1$ and $\partial \phi_\lambda / \partial \theta \neq 0$. For the case of a graphene-coated surface, on the other hand, the phase $\phi_\lambda$ varies with $\theta$ for both $s$- and $p$-polarizations, as Figs. 2 (b) and (d), respectively, show. In this case, then, we observe a nonzero spatial GH shift even without total internal reflection.

The spatial GH shift $\Delta_{GH}$ occurring at a graphene-coated dielectric surface is depicted in Fig. 3(a) and (b) for $p$- and $s$-polarization, respectively. As can be seen, for both polarizations we have $\Delta_{GH} \neq 0$ although no total internal reflection takes place. In particular, $\phi_p$ varies very rapidly from 0 to $-\pi$ in the vicinity of the Brewster angle $\theta_B$. This corresponds to a giant and negative spatial GH shift. On the other hand, $\phi_s$ varies very smoothly with $\theta$, thus resulting in a nonzero (but very small) spatial GH shift for $s$-polarization.

In conclusion, we have presented a detailed theoretical analysis of GH and IF shifts of a Gaussian beam impinging onto a graphene-coated dielectric surface. Our analysis revealed that the main effect of the graphene layer is to introduce, through its surface conductivity $\sigma(\omega)$, a dependence of the phases $\phi_\lambda$ of the reflection coefficients on the incident angle $\theta$. This, ultimately, reflects in the appearance of a nonzero spatial GH and IF shifts. In particular a giant and negative spatial GH shift in the vicinity of the Brewster’s angle for $p$-polarization has been predicted, in agreement with the recently published experimental results [33]. The authors thank the German Ministry of Education and Science (ZIK 03Z1HN31) for financial support.
Fig. 3. Spatial GH shift $\Delta_{GH}$ for (a) $p$-polarization and (b) $s$-polarization for a graphene-coated surface. Since $\partial \phi / \partial \theta \neq 0$, in both cases $\Delta_{GH} \neq 0$. In particular, since $\phi_p$ varies very rapidly with $\theta$ in the vicinity of the Brewster angle, the corresponding spatial shift for $p$-polarization [Panel (a)] is giant in modulus, and negative due to the fact that $\phi_p$ varies from 0 to $-\pi$ [See Fig. 2(b)].

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