Relativistic quark model and scalar diquarks charge radii

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Abstract

In the framework of relativistic quark model the behaviour of electromagnetic form factors of diquarks with $J^P = 0^+$ at small and intermediate momentum transfer are determined. The charge radii of nonstrange and strange scalar diquarks are calculated.

1 Introduction

Diquarks have become now an efficient tool for studying various processes in hadron physics. Diquark model arises naturally if we assume, that the strong two-quark correlations determine the properties of baryons. As a result, baryons can be considered as the connected quark-diquark states [1-3].

As a first step towards describing baryon form factors, one calculate the on-shell electromagnetic form-factors of the constituent diquarks. The diquark form factors are important ingredient in the baryon form factor and contain information about the sizes of the correlated diquark states. Evidence for correlated diquark states in baryons is found in deep-inelastic lepton scattering [4] and in hyperon weak decays [5].

In the framework of the dispersion N/D-method with help of the iteration bootstrap procedure the scattering amplitudes of dressed quarks were constructed [6]. The mass values of the lowest mesons ($J^{PC} = 0^{-+}, 1^{--}, 0^{++}$) and their quark content are obtained. The $qq$-amplitudes in the colour state $\bar{3}_c$ have the diquark levels with $J^P = 0^+$ and the masses $M_{ud} = 0.72$ GeV, $M_{us} = M_{ds} = 0.86$ GeV.

The present paper is devoted to the calculation of electric form factors and charge radii of nonstrange and strange scalar diquarks. The dispersion relation technique allows us to consider the relativistic effects in the composite systems. The double dispersion relations
over the masses of the composite particles are used for the consideration of the electric scalar diquark form factors in the infinite momentum frame (section 2). In the Conclusion the calculation results for the electric form factors, charge radii of scalar diquarks and the status of the considered model are discussed.

2 Electric diquark form factors in the infinite momentum frame

We consider electromagnetic form factor of two-quark system. Let the masses of quarks composing the system be equal \( m_1 \) and \( m_2 \) respectively. The Feynman amplitude for process of scattering of virtual photon on the diquark with \( J^P = 0^+ \) is described by the standard triangle diagram (Fig.1) on which the photon interacts with each of the two quarks. This amplitude is equal:

\[
A(q^2) = \frac{1}{2} \int \frac{d^4 k_2}{i(2\pi)^4} \left[ (P^2 - (m_1 - m_2)^2)k_{1\mu} + \right.
\]

\[
(P^2 - (m_1 - m_2)^2)k_{1\mu}' + q^2 k_{2\mu} \] \times

\[
\left[ (m_1^2 - k_1^2)(m_1^2 - k_1'^2)(m_2^2 - (P - k_1)^2) \right]^{-1} \times
\]

\[
G((k_1 - k_2)^2)G((k_1' - k_2)^2)\epsilon_\mu e_1 f_1(q^2) + [1 \leftrightarrow 2],
\]

(1)

where the latter contribution corresponds to the diagram with particles 1 and 2 rearranged among themselves. \( G \) is the diquark vertex function. Here \( e_{1,2} \) and \( f_{1,2}(q^2) \) are charges and the form factors of quarks \( m_1 \) and \( m_2 \) respectively. Then we obtain:

\[
A(q^2) = \frac{1}{2} \int \frac{d^4 k_2}{i(2\pi)^4} \left[ (P^2 - (m_1 - m_2)^2)k_{1\mu} + \right.
\]

\[
(P^2 - (m_1 - m_2)^2)k_{1\mu}' + q^2 k_{2\mu} \] \times

\[
\left[ (m_1^2 - k_1^2)(m_1^2 - k_1'^2)(m_2^2 - (P - k_1)^2) \right]^{-1} \times
\]

\[
G((k_1 - k_2)^2)G((k_1' - k_2)^2)\epsilon_\mu e_1 f_1(q^2) + [1 \leftrightarrow 2]
\]

(2)

Using the expression [7]:

\[
k_{1\mu} + k_{1'\mu}' = \alpha (P_\mu + P_\mu') + \beta q_\mu + (k_{1\mu} + k_{1'\mu})_\perp,
\]

\[
\alpha = \frac{P^2 + P'^2 + 2m_1^2 - 2m_2^2 - q^2}{2(P^2 + P'^2) - q^2 - (P^2 - P'^2)^2/q^2},
\]

\[
\beta = -\frac{\alpha (P^2 - P'^2)}{q^2},
\]

(3)

we calculate the amplitude \( A(q^2) \):
where the diquark form factor is obtained:

\[
G_D^E(q^2) = \int \frac{d^4k_2}{i(2\pi)^4} \left(\left((1 - \alpha)q^2 + \alpha(P^2 + P'^2 - 2(m_2 - m_1)^2)\right)\right) \times \\
\frac{1}{4(m_1^2 - k_1^2)(m_1^2 - k_2^2)(m_2^2 - k_1^2)} \times \\
G((k_1 - k_2)^2)G((k_1 - k_1')^2)\frac{\varepsilon_1}{\varepsilon_D}f_1(q^2) + [1 \leftrightarrow 2],
\]

(5)

\(\varepsilon_{\mu}q_{\mu} = 0, \varepsilon_D\) is the diquark charge.

We pass to the infinite momentum frame and use the dispersion integration over the masses of composite particles [7]. The momentum of the composite particle (diquark) along the \(z\)-axis is large, \(P_z \to \infty\). Hereafter we introduce the notation \(P = k_1 + k_2, P' = P + q\) for the initial and final state momenta \((P^2 = s, P'^2 = s')\). \(s\) and \(s'\) are the initial and final energy of the composite system. The double discontinuity defines the form factor of the two-quark system (diquark):

\[
G_D^E(q^2) = \frac{1}{4}GG' \left[ e_1f_1(q^2)D_1(s, s', q^2)\Delta_1(s, s', q^2) + \\
e_2f_2(q^2)D_2(s, s', q^2)\Delta_2(s, s', q^2) \right]/\varepsilon_D,
\]

(7)

Further we calculate the following terms:

\[
D_1(s_1, s'_1, q^2) = \frac{1}{4}[1 - \alpha_1(s'_1 + s_1 - 2(m_1 - m_2)^2)] \\
\alpha_1 = \frac{b_1 + q^2a_1/s_1}{2(1 - q^2a_1^2/s_1)}, \Delta_1(s_1, s'_1, q^2) = \frac{b_1a_1 + 1}{b_1 + c_1a_1} \\
b_1 = 1 + \frac{m_1^2 - m_2^2}{s_1}, c_1 = b_1^2 - \frac{4k_1^2\cos^2\phi}{s_1} \\
a_1 = \frac{-b_1 + \sqrt{(b_1^2 - c_1)(1 - s_1c_1/q^2)}}{c_1} \\
s_1 = \frac{m_1^2 + x(m_2^2 - m_1^2)}{x(1 - x)}, s'_1 = s_1 + q^2(1 + 2a_1)
\]

(8)
\[ m_{i\perp}^2 = m_i^2 + k_{\perp}^2, \ i = 1, 2 \]

and

\[ D_2 = D_1(1 \leftrightarrow 2), \Delta_2 = \Delta_1(1 \leftrightarrow 2) \]

Finally we obtain:

\[
G_E^D(q^2) = \frac{1}{(4\pi)^3} \int_0^{\Lambda_{k\perp}} dk_{\perp}^2 \int_0^{2\pi} d\phi \int_0^1 dx \frac{1}{x(1-x)} \times \\
\sum_{i=1,2} \frac{GG'}{(s_i - M_D^2)(s_i' - M_D^2)} e_i f_i(q^2) \times \\
D_i(s_i, s_i', q^2) \Delta_i(s_i, s_i', q^2)
\]

The eq.(10) was used in the calculation of the diquark form factors provided the normalization \( G_D^E(0) = 1 \).

### 3 Conclusion

In the present paper in the framework of dispersion integration technique we investigate the behaviour of electric diquark form factors with \( J^P = 0^+ \) at small and intermediate momentum transfer \( Q^2 \leq 0.5 \text{ GeV}^2 \). The charge radii values of nonstrange and strange scalar diquarks are calculated. The scalar diquark masses were calculated [6]: \( M(ud) = 0.72 \text{ GeV}, M(us) = M(ds) = 0.86 \text{ GeV} \). The quark masses are equal: \( m_u = m_d = 0.385 \text{ GeV}, m_s = 0.510 \text{ GeV} \). Analogously [6] we use the dimensionless pair energy cut-off parameter: \( \lambda = 12.2 \), that allows us to define the momentum cut-offs: \( \Lambda_{k\perp}(qq) = 0.3 \text{ GeV}^2, \Lambda_{k\perp}(qs) = 0.41 \text{ GeV}^2 \), where \( q = u, d \). We consider the interaction of constituent quark with electromagnetic field and take into account the nonstrange and strange quark form factors: 

\( f_q(q^2) = \exp(\gamma_q q^2), \gamma_q = 0.33 \text{ GeV}^{-2} \) and \( f_s(q^2) = \exp(\gamma_s q^2), \gamma_s = 0.2 \text{ GeV}^{-2} \) [6]. The behaviour of the scalar diquark electric form factors are shown in Fig.2. The calculated charge radii are equal:

\[ < r_{ud}^2 >^\frac{1}{2} = 0.55 \text{ fm}, < r_{us}^2 >^\frac{1}{2} = 0.65 \text{ fm}, < r_{ds}^2 >^\frac{1}{2} = 0.5 \text{ fm}. \]

In the present paper electromagnetic properties of diquarks are investigated in the framework of the relativistic description. These values of scalar diquark charge radii are compatible with other results for the diquark effective radii [8-11] and experimental data [1]. In the papers [8,9] assuming soft symmetry breaking in the diquark sector, the bosonisation of a quasi-Goldstone ud-diquark is performed. In the chiral limit the ud-diquark mass and diquark charge radius are defined by the gluon condensate \( M_{ud} = 300 \text{ MeV}, < r_{ud}^2 >^\frac{1}{2} \simeq 0.5 \text{ fm}. \)

This model allows to explain the relatively low mass of the scalar diquark.

A approach is based on a local effective quark model, a Nambu-Jona-Lasinio model with a colour-octet current-current interaction [10,11]. One calculated the electromagnetic form
factors of scalar and axial vector diquark bound states using the gauge-invariant proper-time regularization. In the paper [11] the scalar diquark masses $M_{ud}$ and scalar diquark charge radii $<r_{ud}^2>$ for different values of the effective diquark coupling constants are calculated. The scalar diquark charge radius $<r_{ud}^2>$ is equal $(0.5 - 0.55)$ $fm$.

But the non-relativistic, QCD-based, potential quark model for the proton and neutron inevitably predicts a spin-0 diquark structure with a charge radius of the $0.35$ $fm$ or smaller [12,13]. Such conflict between model and experimental data might possibly as the influence of relativistic effects.

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Figure 1: Triangle diagram which defines the form factor of diquark.

Figure 2: The scalar diquarks electric form factor at small and intermediate momentum transfer $Q^2 \leq 0.5\ \text{GeV}^2 (Q^2 \equiv -q^2)$. 