Influence of Ferrofluid Lubrication on Longitudinally Rough Truncated Conical Plates with Slip Velocity

M M Munshi*, A R Patel† and G M Deheri‡

1 Alpha College of Engineering and Technology, Gujarat Technological University, Kalol, 382 721, Gujarat, India
2 Vishwakarma Government Engineering College, Gujarat Technological University, Ahmedabad, 382 424, Gujarat, India
3 Sardar Patel University, Vallabh Vidhyanagar, Anand, 388 120, Gujarat, India
*Email: raiyan.munshi@gmail.com

1. Introduction

Various industrial applications including aerospace and aeronautical industries, nuclear and civil engineering, modern construction engineering amongst others make use of conical plates as crucial constitutional elements. The dynamic response of these conical plates is significantly impacted by various fluids (stationary or flowing) that they work with. That is why, it is crucial to study the behavior generated by different load types in order to ensure safe functioning in applications. A number of experimental and analytical studies have come forward recently that study the fluid effects on plates and shells. Flat and curved plates and circular cylindrical shells have been the major concern for most of them. While there hasn’t been much work on the fluid effects on conical plates. Thin walled conical plates are used in many different engineering domains. From aircrafts and satellites in aerospace to submarines, waterborne ballistic missiles and torpedoes in ocean engineering and containment vessels in civil, conical shells have a lot of different applications.

Different researchers have come up with a number of ways to study the impact of surface roughness on bearing performance. (Christensen and Tonder 1969a, 1969b, 1970) utilized the concept of stochastic averaging and created a model with lubricated films having longitudinal and transverse roughness. (Burton 1963), (Berthe and Godet 1974) and (Gadelmawla et al. 2002) used this model and came up with a different geometric configurations to understand the impact of surface roughness. When two distinct lubricating surfaces generate positive pressure by approaching each other in normal direction and supports a load, this phenomenon called the squeeze film. A squeeze film has a lot of different applications in automobile and domestic appliances. That is why, researchers like (Bhat and Deheri 1991, Lin et. al. 2013, Prakash and Vij 1973, Ting 1975) worked on studying the impact of a squeeze film bearing. (Neuringer and Rosensweig 1964) designed a basic model of flow to analyze the behavior of Ferrofluid with different external magnetic fields. A lot of papers have been written that study various bearing systems under the Neuringer and Rosensweig model, for example circular plates by (Bhat and Deheri 1992), (Patel et. al. 2012) in journal bearing and (Patel and Deheri 2011) in plane inclined slider bearing.

The study focuses on analyzing the effect of slip velocity in the case of a Ferrofluid squeeze film when the surface of truncated cone-shaped plates has a longitudinal roughness. Each oblique to the bottom plate was utilized by the external magnetic field. The bearing surface has a roughness that is designed with the help of a random stochastic variable having a nonzero mean, skewness and variance. The load carrying ability of a bearing system’s surface is determined by calculating the dispersal of pressure in the system, which is calculated by using the associated stochastically average Reynolds’ equation. The graphs obtained from the study shows that there is a correlation between the longitudinal surface roughness and the bearing system performance. The magnetic fluid lubrication has a positive impact on a system’s bearing capacity. However, the load bearing capacity declines as a result of the effect of slip. A high negative skewness of the surface roughness also has a positive impact on a bearing’s load carrying capacity. One interesting finding shows that contrasting to the results of transverse roughness, standard deviation positively impacts the load bearing capacity. This investigation suggests despite the imp-portance of aspect ratio and semi vertical angle is significant for performance enhancement, it is also essential to maintain the slip at the lowest level.
(Patel and Deheri 2007) focused their study on analyzing the impact of magnetic fluid on bearing performance of porous truncated conical plates. (Deheri et. al. 2007) took their work further and focused on transverse roughness. They concluded that negative variance and negative skewness are crucial with appropriate semi vertical angle. (Andharia and Deheri 2011) further worked on this aspect by taking into account the longitudinal roughness. (Shimpi and Deheri 2014) took into account the slip velocity and deformation impact to extend the study of (Deheri et. al. 2007). (Shimpi and Deheri 2016) also studied truncated conical plates by taking into account longitudinal roughness, slip velocity and deformation. (Vadher et. al. 2011) worked on the study by using hydromagnetic bearing instead of hydrodynamic bearing. The squeeze films with a magnetic fluid base had an impact on conical plates which has been studied by a lot of researchers using a number of different parameters. For example, (Patel and Deheri 2007) used porosity, (Patel and Deheri 2013) studied the model using transverse roughness and porosity, (Andharia and Deheri 2010) worked on longitudinal roughness, (Patel and Deheri 2016) studied it by taking into account longitudinal roughness with slip velocity and (Patel et. al. 2017) worked with longitudinal roughness as well as deformation effect. The design of a structure from porous as well as fluid layer is derived by (Beavers and Joseph 1967). They considered slip boundary condition at the interface. Many researchers have worked with slip velocity; (Munshi et. al. 2017) used circular plates, (Shukla and Deheri 2013) worked on Rayleigh step bearing and (Shah and Bhat 2002) studied inclined slider bearing. From these studies, it can be concluded that slip place a crucial part in changing the bearing capacity of any system. This study aims to change and clearly define the findings (Andharia and Deheri 2011) to find the correlation between slip velocity and Ferrofluid squeeze film in truncated conical plates with roughness pattern of longitudinal.

2. Analysis

The system has two plates in the shape of truncated cones. The upper plate is in motion towards the lower plate. The $h(x)$ is considered as

$$ h(x) = ar{h}(x) + h $$

using the works of (Christensen and Tonder 1970, 1969a,b). Also, the study uses $h$ using the probability density function

$$ f(h) = \begin{cases} \frac{35}{32\epsilon^3} \left( 1 - \frac{h^2}{\epsilon^2} \right)^3, & -\epsilon \leq h \leq \epsilon \\ 0, & \text{elsewhere} \end{cases} $$

(2)

Further, the magnetic field’s magnitude is represented by (Andharia and Deheri 2011)

$$ M^2 = k(a \csc \omega - x)(x - b \csc \omega) $$

In the equation, $k$ is the suitable constant. If we assume that the magnetic field existing external has developed as a result of a potential function, $\phi = \phi(x,z)$ making the equation

$$ \cot \phi \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z} = \frac{2x - a \csc \omega - b \csc \omega}{2(a \csc \omega - x)(x - b \csc \omega)} $$

(6)

The general hydrodynamic lubrication assumption modified Reynolds’ (Andharia and Deheri 2011, Shimpi and Deheri 2014) equation as follows:

$$ \frac{1}{x} \frac{d}{dx} \left[ x^3 \frac{d}{dx} \left( \frac{\rho - \frac{1}{2} \mu \bar{M}_0}{\rho - \frac{1}{2} \mu \mu M^2} \right) \right] = \frac{12 \mu \hbar^3}{\sin^2 \omega} $$

(7)

The averaging process of stochastic suggested by (Andharia and Deheri 2011), equation (7) takes the form

$$ \frac{1}{x} \frac{d}{dx} \left[ x \frac{1}{E(h + \bar{h})} \frac{d}{dx} \left( \rho - \frac{1}{2} \mu \mu M^2 \right) \right] = \frac{12 \mu \hbar^3}{\sin^2 \omega} $$

$$ \frac{1}{x} \frac{d}{dx} \left[ \frac{1}{m(h)} \frac{d}{dx} \left( \rho - \frac{1}{2} \mu \mu M^2 \right) \right] = \frac{12 \mu \hbar^3}{\sin^2 \omega} $$

(8)
where
\[ m(h, \alpha, \sigma, \varepsilon) = \frac{b^3}{2} \left[ 1 - 3\alpha b^2 \right] + 6b^{-2} \left( \frac{\sigma^2}{1 + \sigma} \right) - 10b^{-3} \left( \varepsilon + 3\sigma^2 \alpha + \alpha^3 \right) \]

(9)
The following dimensionless quantities are used,
\[ H = \frac{\tilde{b}}{h_0}, \quad X = \frac{x}{a}, \quad M(H, \tilde{S}, \alpha, \sigma, \varepsilon) = h_0^3 m(h, \alpha, \sigma, \varepsilon), \quad \alpha = \frac{\tilde{\alpha}}{h_0}, \quad \sigma = \frac{\tilde{\sigma}}{h_0}, \quad \varepsilon = \frac{\tilde{\varepsilon}}{h_0}, \quad K = \frac{\tilde{b}}{a} \]

(10)
The related pressure limit settings are
\[ P(K \cosec \omega) = 0, \quad P(\cosec \omega) = 0 \quad (11) \]
Solving equation (8) with the aid of equation (11), the non-dimensional type of dispersal of pressure is
\[ P = \frac{1}{2\pi} \left[ \mu^* \left( 1 - X \sin \omega \right) \left( X - K \cosec \omega \right) + 6 M(H) \cosec^3 \omega \left( 1 - X^2 \sin^2 \omega \right) + 2K^2 \ln \left( X \sin \omega \right) \right] \]

(12)
The bearing ability of dimensionless type is obtained
\[ \bar{W} = \frac{-h_0^3}{\mu h_0 \pi (a^2 - b^2)^2 \pi^2 \cosec^2 \omega} \]

(13)
\[ \bar{W} = \frac{2}{(1 - K^2) \cosec \omega} \left[ \frac{\mu^* \cosec^3 \omega (1 - K)^2}{24\pi} + \frac{3M(H) \cosec^3 \omega}{4\pi(1 - K^2)} \left( 1 - K^2 \right) \left( 1 - 3K^2 \right) - 4K^4 \ln(K) \right] \]

(15)
where the load bearing capacity is calculated using
\[ w = 2\pi \int_{\cosec \omega}^{\pi} x \tilde{w} \, dx \quad (16) \]

3. Results and Discussion
From equation (15), the load bearing ability increase as per the following equation
\[ \mu^* \cosec^2 \omega \left( \frac{1 - K}{1 + K} \right) \]
more than the regular lubricant-based bearing system. In the nonexistence of slip this investigation diminishes to the study of (Andharia and Deheri 2011). As the equation (15) is linear with regards to \( \mu^* \), a boost in the magnetization would introduce an enhancement in the load bearing ability.
A comparison of overall performance with (Andharia and Deheri 2011) suggests that the impact of slip effect is not all bad. The graphical results are presented below.
Figure 2. Trends of $\bar{W}$ concern to $\tilde{S}$.
Figure 3. Trends of $\overline{W}$ concern to $K$

Figure 4. Trends of $\overline{W}$ concern to $\omega$

Figure 5. Trends of $\overline{W}$ concern to $\sigma$
Following conclusion can be drawn from the above load profiles.

1. There is a substantial increase because of the roughness standard deviation. This performance increases when there is a high negative skewness value for the surface roughness.
2. The slip velocity decreases the system’s bearing capacity.
3. The positive impact of magnetization isn’t strong enough to counter the adverse effect of roughness and slip velocity.
4. However, with an appropriate combination of aspect ratio and semi vertical angle of the cone, the adverse impact created by surface roughness can be decreased to a significant extent, especially in the case of smaller slip parameter values.

A comparison of the graphical results presented in (Deheri et. al. 2007) goes on to show that the longitudinally surface roughness can be more adoptable as compare to transverse surface roughness when no slip is involved.

### 4. Validation

**Table 1. Comparison of \( \overline{W} \) calculated for \( \mu^* \)**

| Quantity | Load bearing Capacity (calculated for \( \alpha = -0.05, \sigma = 0.3, \varepsilon = -0.05, \omega = 55^\circ, K = 0.5, \overline{S} = 0.03 \)) |
|----------|----------------------------------------------------------------------------------|
| \( \mu^* \) | Result of the current study | Result of (Andharia and Deheri 2011) |
| 0.0001 | 0.2575338 | 0.2446277 |
| 0.001 | 0.2575475 | 0.2446395 |
| 0.01 | 0.2576643 | 0.2447581 |
| 0.1 | 0.2588503 | 0.2459441 |
| 1 | 0.2707102 | 0.2578040 |

**Table 2. Comparison of \( \overline{W} \) calculated for \( \alpha \)**

| Quantity | Load bearing Capacity (calculated for \( \mu^* = 0.01, \sigma = 0.3, \varepsilon = -0.05, \omega = 55^\circ, K = 0.5, \overline{S} = 0.03 \)) |
|----------|----------------------------------------------------------------------------------|
| \( \alpha \) | Result of the current study | Result of (Andharia and Deheri 2011) |
| -0.05 | 0.2576640 | 0.2447581 |
| -0.02 | 0.2432010 | 0.2298273 |
| 0 | 0.2340100 | 0.2202664 |
| 0.02 | 0.2251560 | 0.2109807 |
| 0.05 | 0.2124570 | 0.1974948 |

**Table 3. Comparison of \( \overline{W} \) calculated for \( \sigma \)**

| Quantity | Load bearing Capacity (calculated for \( \mu^* = 0.01, \alpha = -0.05, \varepsilon = -0.05, \omega = 55^\circ, K = 0.5, \overline{S} = 0.03 \)) |
|----------|----------------------------------------------------------------------------------|
| \( \sigma \) | Result of the current study | Result of (Andharia and Deheri 2011) |
| 0.1 | 0.2187241 | 0.2088460 |
| 0.2 | 0.2333267 | 0.2223130 |
| 0.3 | 0.2576643 | 0.2447581 |
| 0.4 | 0.2917369 | 0.2761812 |
| 0.5 | 0.3355446 | 0.3165824 |
Table 4. Comparison of $\bar{W}$ calculated for $\varepsilon$

| Quantity | Load bearing Capacity (calculated for $\mu^* = 0.01, \alpha = -0.05, \sigma = 0.3, \omega = 55^\circ, K = 0.5, \bar{S} = 0.03$) | Result of the current study | Result of (Andharia and Deheri 2011) |
|----------|-----------------------------------------------------------------------------------------------------------------|-----------------------------|--------------------------------------|
| $\varepsilon$ |                                                                                                                  |                             |                                      |
| -0.05    | 0.2576643                                                                                                        | 0.2447581                   |                                      |
| -0.02    | 0.2443342                                                                                                        | 0.2237814                   |                                      |
| 0        | 0.2354474                                                                                                        | 0.2097969                   |                                      |
| 0.02     | 0.2265606                                                                                                        | 0.1958125                   |                                      |
| 0.05     | 0.2132305                                                                                                        | 0.1748358                   |                                      |

Table 5. Comparison of $\bar{W}$ calculated for $\omega$

| Quantity | Load bearing Capacity (calculated for $\mu^* = 0.01, \alpha = -0.05, \sigma = 0.3, \varepsilon = -0.05, K = 0.7, \bar{S} = 0.03$) | Result of the current study | Result of (Andharia and Deheri 2011) |
|----------|-----------------------------------------------------------------------------------------------------------------|-----------------------------|--------------------------------------|
| $\omega$ |                                                                                                                  |                             |                                      |
| 40$^\circ$ | 0.6794927                                                                                                        | 0.6454508                   |                                      |
| 45$^\circ$ | 0.4641394                                                                                                        | 0.4408880                   |                                      |
| 50$^\circ$ | 0.3369186                                                                                                        | 0.3200416                   |                                      |
| 55$^\circ$ | 0.2576643                                                                                                        | 0.2447581                   |                                      |
| 60$^\circ$ | 0.2062889                                                                                                        | 0.1959567                   |                                      |

Table 6. Comparison of $\bar{W}$ calculated for $K$

| Quantity | Load bearing Capacity (calculated for $\mu^* = 0.01, \alpha = -0.05, \sigma = 0.3, \varepsilon = -0.05, \omega = 55^\circ, \bar{S} = 0.03$) | Result of the current study | Result of (Andharia and Deheri 2011) |
|----------|-----------------------------------------------------------------------------------------------------------------|-----------------------------|--------------------------------------|
| $K$      |                                                                                                                  |                             |                                      |
| 0.1      | 0.3941064                                                                                                        | 0.3743720                   |                                      |
| 0.2      | 0.3728085                                                                                                        | 0.3541385                   |                                      |
| 0.3      | 0.3412229                                                                                                        | 0.3241333                   |                                      |
| 0.4      | 0.3021199                                                                                                        | 0.2869877                   |                                      |
| 0.5      | 0.2576643                                                                                                        | 0.2447581                   |                                      |

Undoubtedly, Tables 1-6 underline that at least 4% enhancement in the load bearing capacity is registered here. Further the effect of variance is more sharp in comparison with the investigation of (Andharia and Deheri 2011). The consolidated effect of $\varepsilon$ and $\alpha$ are akin to that of (Andharia and Deheri 2011). However the effect of semi vertical angle $\omega$ surges ahead. However, the negative impact of roughness displays more variation in this situation despite the fact that standard deviation raises the load bearing capacity.

Conclusions

From this study, it can be concluded that appropriate magnetic strength can counter the slip effect in the case of small order of roughness. Thus, for industrial applications using such a system is more appropriate, especially if the slip is at the lowest level. Considering the life period perspective, this study is beneficial since it aids the process of choosing the ideal aspect ratio, angle. Such an angle can, in turn, reduce the negative effects of roughness slip combine, even for moderate magnetic field.

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### Nomenclature

| Symbol | Description |
|--------|-------------|
| $a, b$ | dimensions of the bearing |
| $b$ | film thickness (mm) |
| $\overline{h}$ | mean film thickness (mm) |
| $h$ | deviation from mean level |
| $h_0$ | central film thickness (mm) |
| $h_i$ | initial film thickness (mm) |
| $h_2$ | film thickness after time $\Delta t$ |
| $K$ | aspect ratio $b/a$ (width/height) |
| $M$ | magnetic field (Gauss) |
| $M$ | magnitude of magnetic field (N/A.m) |
| $p$ | pressure found in the area covered by the film (N/mm$^2$) |
| $\bar{p}$ | anticipated degree of pressure |
| $\tilde{p}$ | non-dimensional pressure generated due to the film |
| $\omega$ | load capacity (N) |
| $\overline{\omega}$ | non-dimensional load capacity |
| $\alpha$ | variance (mm) |
| $\varepsilon$ | skewness (mm$^3$) |
| $\sigma$ | standard deviation (mm) |
| $\overline{\alpha}$ | non-dimensional variance |
| $\overline{\varepsilon}$ | skewness in dimensionless form |
| $\overline{\sigma}$ | dimensionless standard deviation |
| $S$ | slip parameter |
| $\omega$ | semi vertical angle of the cone |
| $\mu$ | fluid viscosity (N.s/m$^2$) |
| $\mu_0$ | permeability of the free space (N/A$^2$) |
| $\mu$ | magnetic susceptibility of particles |
| $\mu^*$ | dimensionless magnetization parameter |
| $\dot{h}_0$ | normal velocity (m/s) |