Penguin diagrams for the HYP staggered fermions

Keunsu Choi\textsuperscript{a} and Weonjong Lee\textsuperscript{a}

\textsuperscript{a}School of Physics, Seoul National University, Seoul, 151-747, South Korea

We present results of the one-loop corrections originating from the penguin diagrams for the improved staggered fermion operators constructed using various fat links such as Fat7, Fat7+Lepage, Fat7, HYP (I) and HYP (II). The main results include the diagonal/off-diagonal mixing coefficients and the matching formula between the continuum and lattice operators.

1. INTRODUCTION

The low energy effective Hamiltonian of the standard model includes $\Delta S = 1$ four-fermion operators with corresponding Wilson coefficients, which contains all the short-distance physics. The low energy effects of the electroweak and strong interactions can be expressed in terms of matrix elements of the four-fermion operators between hadronic states. Lattice QCD is well-suited to calculate these matrix elements non-perturbatively at low energy. One essential step in using lattice QCD is to find the relationship between the continuum and lattice operators, which is often called “matching formula”. There are two classes of Feynman diagrams at the one-loop level: (1) current-current diagrams and (2) penguin diagrams. At the one loop level, it is possible to treat the penguin contribution and the current-current contribution separately. In the case of the current-current diagrams, the matching formula at the one-loop level is given in [1].

Here, we focus on penguin diagrams in which one of the quarks in the four-fermion operator is contracted with one of the anti-quarks to form a closed loop. Hence, the main goal is to calculate the penguin diagrams for improved staggered operators constructed using various fat links and to provide the corresponding matching formula.

Here, we adopt the same notation and Feynman rules outlined in [1].

2. PENGUIN DIAGRAMS

Here, we study penguin diagrams. On the lattice, the gauge non-invariant four fermion operators such as Landau gauge operators mix with lower dimension operators, which are gauge non-invariant [2]. It is required to subtract these contributions non-perturbatively. However, it is significantly harder to extract the divergent mixing coefficients in a completely non-perturbative way. Therefore, it is impractical to use gauge non-invariant operators for the numerical study of the CP violations. Hence, it is pre requisite to use gauge invariant operators in order to avoid unwanted mixing with lower dimension operators. For this reason, we choose gauge invariant operators in this study.

In the staggered fermion formalism, there are four penguin diagrams at the one loop level as shown in Fig. 1. These diagrams allow mixing with lower dimension operators as well as four fermion operators of the same dimension or higher. The mixing coefficients with lower dimension operators are divergent (i.e. proportional to inverse power of the lattice spacing). The perturbation is, however, not reliable with divergent coefficients. Hence, we must use non-perturbative method to determine them and subtract away the lower dimension operators. In the case of mixing with operators of the same dimension, the perturbation is expected to be reliable as long as the size of one-loop correction is small enough, which can be achieved by using improved staggered fermions.

In Fig. 1 diagrams (a) and (b) have their correspondence in the continuum and diagrams (c)

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and (d) are pure lattice artifacts. However, diagrams (c) and (d) play an essential role to keep the gauge invariance. Basically, the contribution from diagrams (c) and (d) can be re-expressed as a sum of diagrams (e) and (f) as shown in Fig. 2. The first key point is that the sum of diagrams (a) and (e) generates bilinear operators in a gauge invariant form. The main key point is that the contributions from diagrams (b) and (f) leads to four fermion operators of our interests in a gauge invariant form.

The details of bilinear mixing (diagrams (a) and (e)) in Fig. 3 will be presented in [3] and here we skip them. Here we focus on the diagrams (b) and (f) and present the final result. The final result is

$$G_{(b+f)} = \left( -\frac{1}{N_f} \right) \frac{g^2}{(4\pi)^2} \left( \sum_I T_{ab}^I T_{cd}^I \right) I_c$$

Figure 2. Diagram identity.

$$\sum_{\mu} \left( \gamma S' \otimes \xi F' \right) C' D' \left( \gamma \mu \otimes 1 \right)_{CD}$$

$$\delta_{S,\mu} \delta_{F,1} \left[ h_{\mu\mu}(k) \right] \gamma E + F_{0000}$$

where $k = q - p$ is strictly on shell.

$$I_c = \frac{16}{3} \left( -\ln(4m^2a^2) - \gamma_E + F_{0000} \right) - 9.5147$$

$$+ O(m^2a^2)$$

$I_c$ is also given in [2]. The details of deriving Eq. (1) will be presented in [3]. From Eq. (1) we can derive the following theorem:

**Theorem 1 (Equivalence)**

At the one loop level, the diagonal mixing coefficients of penguin diagrams are identical between (a) the unimproved (naive) staggered operators constructed using the thin links and (b) the improved staggered operators constructed using the fat links such as HYP (I), HYP (II), Fat7, Fat7+Lepage, and Fat7.\(^2\) The details on the proof of this theorem will be given in [3].

\(^2\)Note that AsqTad is NOT included on the list. In this
By construction, gluons carrying a momentum close to $k \sim \pi/a$ are physical in staggered fermions and lead to taste changing interactions, which is a pure lattice artifact. In the case of unimproved staggered fermions, it is allowed to mix with wrong taste ($\neq 1$) and the mixing coefficient is substantial. In contrast, in the case of improved staggered fermions using fat links of our interest such as Fat7, Fat7, and HYP (II), the off-diagonal mixing with wrong taste vanishes and is absent. In the case of the improvement using HYP (I) and Fat7 + Lepage, the off-diagonal mixing with wrong taste is significantly suppressed. The details of this off-diagonal mixing will be given in [3].

In summary, the diagonal mixing occurs only when the original operator has the spin and taste structure of $S = \mu$ and $F = 1$ regardless of that of the spectator bilinear. The diagonal mixing coefficient is identical between the unimproved staggered operators and the improved staggered operators constructed using fat links such as Fat7, Fat7 + Lepage, Fat7, HYP (I) and HYP (II). This is a direct consequence of the fact that the contribution from the improvement changes only the mixing with higher dimension operators and off-diagonal mixing, which are unphysical.

The result of this paper, combined with that of [1] provides a complete set of one-loop matching formula.

REFERENCES
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