Exponential estimation of generalized state-space time-delay systems

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Abstract. In this paper, global exponential stability for a class of generalized state-space time-delay systems is considered. Delay-dependent criteria are proposed to guarantee the exponential stability and estimate the convergence rate for the generalized state-space systems with two cases of uncertainties. Finally, some numerical examples are illustrated to show the usefulness of the theory.

1. Introduction

Generalized state-space systems (or singular systems, descriptor systems) are often presented in circuit theory, large-scale systems, power systems, and singular space perturbation theory [1-2]. The time-delay phenomena are also often encountered in various practical systems, such as aircraft stabilization, chemical engineering systems, inferred grinding model, manual control, neural network, rolling mill, ship stabilization, and systems with lossless transmission lines [3]. Hence the stability analysis and controller design of generalized state-space time-delay systems have been studied in the recent years [4-7]. In this paper, global exponential stability for generalized state-space time-delay systems will be considered.

In the recent years, the stability analysis for delayed cellular neural networks (DCNN) [8]-[12] and neutral time-delay system [13]-[14] had been extensively studied. Under some formulations, these two systems will be the special cases of the generalized state-space systems considered in this paper. Cellular neural networks (CNN) is a circuit applications and introduced by [9]. CNN can be applied in many physical systems; such as connected component detection, hole filling, image shadowing, optimization and associative memories, pattern recognition, and signal processing [9]. Delayed cellular neural networks (DCNN) are used to treat the moving images or other purposes [8]. On the other hand, neutral time-delay systems are often encountered in many practical systems; such as distributed networks, population ecology, process including steam, and heat exchanges [3, 13-14]. In the similar derivation and consideration, the generalized state-space systems considered in this paper can be applied to other physical systems.

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2. Problem formulation and main result

Consider the following generalized state-space time-delay systems:

\[ \dot{x}(t) = A_0 x(t) + B_0 x(t-h) + C_0 y(t) + D_0 y(t-\tau), \quad t \geq 0, \]  
(1a)

\[ y(t) = A_1 x(t) + B_1 x(t-h) + D_1 y(t-\tau) + f(x(t), x(t-h), y(t-\tau)), \quad t \geq 0, \]  
(1b)

\[ x(t) = \phi(t), \quad t \in [-h, 0], \]  
(1c)

\[ y(t) = \varphi(t), \quad t \in [-\tau, 0], \]  
(1d)

where \( x(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^m, \) and \( h, \tau \geq 0. \) The matrices \( A_0, B_0, C_0, D_0, A_1, B_1, D_1, \) are known with appropriate dimensions, and the initial vector functions \( \phi, \varphi \) are differentiable and continuous, respectively. Two classes of perturbed function \( f(\cdot) \) are considered in this paper.

(A1) Structured uncertainties:

\[ f(x(t), x(t-h), y(t-\tau)) = M \cdot F(t) \cdot \left[ N_0 x(t) + N_1 x(t-h) + N_2 y(t-\tau) \right], \]  
(2)

where \( M \) and \( N_i, i \in \{0, 1, 2\}, \) are some given constant matrices, \( F(t) \) is an unknown real time-varying continuous function with appropriate dimension and bounded as follows:

\[ F^T(t) \cdot F(t) \leq I, \quad \forall \ t \geq 0. \]  

(A2) Norm bounded uncertainties:

\[ f(x(t), x(t-h), y(t-\tau)) = f_0(x(t)) + f_1(x(t-h)) + f_2(y(t-\tau)), \]  
(3a)

\[ \|f_0(x(t))\| \leq \|F_0 x(t)\|, \quad \|f_1(x(t-h))\| \leq \|F_1 x(t-h)\|, \quad \|f_2(y(t-\tau))\| \leq \|F_2 y(t-\tau)\|, \]  
(3b)

where \( \Gamma_i, \ i \in \{0, 1, 2\}, \) are given matrices.

Remark 1: System (1) can be rewritten in the following form:

\[ EX(t) = Z(X(t), X(t-h), X(t-\tau)), \]  

where the matrix \( E = \text{diag}[I, 0] \) is singular, \( X^T(t) = [x^T(t) \ y^T(t)], \) and

\[ Z(X(t), X(t-h), X(t-\tau)) = \\
\begin{bmatrix}
A_0 x(t) + B_0 x(t-h) + C_0 y(t) + D_0 y(t-\tau) \\
A_1 x(t) + B_1 x(t-h) + D_1 y(t-\tau) + f(x(t), x(t-h), y(t-\tau))
\end{bmatrix}. \]  

Since the matrix \( E \) is singular, the system (1) is said to be a generalized state-space system (or singular system, descriptor system) with time delays [1].

Remark 2: System (1) is a general form of many physical systems; see for examples

(a) Uncertain neutral systems [13-14]:

\[ \dot{x}(t) = [A_0 + \Delta A_0(t)] x(t) + [B_0 + \Delta B_0(t)] x(t-h) + [D_0 + \Delta D_0(t)] y(t-\tau), \quad t \geq 0, \]  
(4a)

where \( \Delta A_0 = M \cdot F(t) \cdot N_0, \Delta B_0 = M \cdot F(t) \cdot N_1, \Delta D_0 = M \cdot F(t) \cdot N_2. \)

System (4a) can be rewritten as

\[ \dot{x}(t) = y(t), \]

\[ y(t) = A_1 x(t) + B_1 x(t-h) + D_1 y(t-\tau) + f(x(t), x(t-h), y(t-\tau)), \]

where

\[ f(x(t), x(t-h), y(t-\tau)) = \Delta A_0 x(t) + \Delta B_0 x(t-h) + \Delta D_0 y(t-\tau), \]

\[ = M \cdot F(t) \cdot \left[ N_0 x(t) + N_1 x(t-h) + N_2 y(t-\tau) \right]. \]

This implies that system (4a) is a special case of system (1) with (A1), and
(b) Delayed cellular neural networks [8-12]:

\[
\dot{z}_i(t) = -c_i z_i(t) + \sum_{j=1}^{n} [g_j g_j(\xi_j(t))] + J_i, \quad t \geq 0.
\]  

System (5a) can be rewritten as follows:

\[
\dot{z}(t) = -Cz(t) + Ag(z(t)) + Bg(z(t - \tau)) + J, \quad t \geq 0,
\]  

where 
\[
z(t) = [z_1(t) \quad z_2(t) \quad \cdots \quad z_n(t)]^T,
\]

\[
J = [J_1 \quad \cdots \quad J_n] \in \mathbb{R}^n
\]
is the external bias vector. The matrices 
\[
C = diag[c_i], \quad c_i > 0, \quad A = [a_{ij}],
\]

\[
B = [b_{ij}] \in \mathbb{R}^{n \times n}
\]
are known. The function 
\[
g(z(t)) = [g_1(z_1(t)) \quad g_2(z_2(t)) \quad \cdots \quad g_n(z_n(t))]^T
\]
is bounded monotonically nondecreasing and satisfies

\[
|g_i(\xi_i) - g_j(\xi_j)| \leq L_{ij} |\xi_i - \xi_j|, \quad \xi_i, \xi_j \in \mathbb{R},
\]

where 
\[
L_{ij}, \quad i, j = 1, 2, \ldots, n
\]
are some given nonnegative constants.

Assume \( \bar{z} = [\bar{z}_1 \quad \bar{z}_2 \quad \cdots \quad \bar{z}_n] \in \mathbb{R}^n \) is an equilibrium point of system (5b), we can obtain the following system:

\[
x(t) = -Cx(t) + Ay(x(t)) + By(x(t - \tau)),
\]  

\[
y(x(t)) = f(x(t)),
\]

where 
\[
x(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T = z(t) - \bar{z},
\]

\[
f(x(t)) = [f_1(x_1(t)) \quad f_2(x_2(t)) \quad \cdots \quad f_n(x_n(t))]^T
\]

\[
f_i(x_i(t)) = g_i(x_i(t) + \bar{z}_i) - g_i(\bar{z}_i), \quad g_i(0) = 0.
\]

By the inequality (5c), we have

\[
|f_i(x_i)| = |g_i(x_i + \bar{z}_i) - g_i(\bar{z}_i)| \leq L_i |x_i|
\]

\[
f^T(x) f(x) = \sum_{i=1}^{n} |f_i(x_i)|^2 \leq \sum_{i=1}^{n} L_i^2 |x_i|^2 = x^T \Gamma x
\]

where 
\[
\Gamma = diag[L_i].
\]

This implies that (6a)-(6d) is a special case of (1) with (A2) and

\[
A_0 = -C, \quad B_0 = 0, \quad C_0 = A, \quad D_0 = B,
\]

\[
A_1 = B_1 = D_1 = 0, \quad \Gamma_0 = \Gamma, \quad \Gamma_1 = \Gamma_2 = 0.
\]

Definition 1: The system (1) is said to be the globally exponentially stable with convergence rate \( \alpha \), if there are two positive constants \( \alpha \) and \( \rho \) such that

\[
\|x(t)\| \leq \rho \cdot e^{-\alpha t}, \quad t \geq 0.
\]
Lemma 1: For any vectors $x, y \in \mathbb{R}^n$, any matrices $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{p \times n}$, $F(t) \in \mathbb{R}^{m \times p}$ with $F^T(t)F(t) \leq I$, $i \geq 0$, and any constant $\varepsilon > 0$, the following inequality is satisfied:

$$x^TA(t)By + y^TB(t)F(t)A^TBy \leq \varepsilon^{-1}x^TAA^Tx + \varepsilon \cdot y^TB^TBy.$$ 

Lemma 2: Let $P$ and $Q$ be symmetric real matrices, then

$$\xi^TP\xi < 0, \forall \xi \neq 0, \text{s.t.} \xi^TQ\xi \geq 0,$$

holds if and only if there exist $\varepsilon \geq 0$ such that

$$P + \varepsilon \cdot Q < 0.$$ 

Now we present a delay-dependent condition for the global exponential stability of system (1) with (2).

Theorem 1: The system (1) with (2) is globally exponentially stable with convergence rate $\alpha > 0$, if there exist some positive definite symmetric matrices $P, R_i \in \mathbb{R}^{n \times n}$, $i \in \{1, 2, 3\}$, some matrices $Q_j \in \mathbb{R}^{n \times n}$, $j \in \{0, 1, 2, 3, 4\}$, and a positive constant $\varepsilon$, such that the following LMI condition is satisfied:

$$\Xi = \begin{bmatrix} 
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} \\
\Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} \\
\Xi_{33} & \Xi_{34} & \Xi_{35} & 0 & 0 \\
\Xi_{44} & \Xi_{45} & \Xi_{46} & 0 \\
\Xi_{55} & \Xi_{56} & \Xi_{57} \\
\Xi_{66} & 0 \\
\Xi_{77} 
\end{bmatrix} < 0,$$

where

$$\Xi_{11} = PA_0 + A_0^TP + Q_1^TQ_1 + A_1^TQ_1 + 2\alpha \cdot P + R_1 + \varepsilon^{-2\alpha} \cdot R_2,$$

$$\Xi_{12} = PB_0 + \varepsilon^{-2\alpha} \cdot R_2 + Q_1^TB_1 + A_1^TB_1,$$

$$\Xi_{13} = A_1^TB_0,$$

$$\Xi_{14} = PC_0 - Q_0^TQ_1 + A_0^TQ_0,$$

$$\Xi_{15} = PD_0 + Q_0^TD_0 + A_0^TD_0,$$

$$\Xi_{16} = Q_0^TM,$$

$$\Xi_{17} = \varepsilon \cdot N_1^T,$$

$$\Xi_{22} = e^{-2\alpha \cdot [R_1 + R_2]} + B_1^TQ_2 + Q_1^TB_1,$$

$$\Xi_{23} = B_1^TQ_0,$$

$$\Xi_{25} = Q_1^TD_0 + B_1^TQ_1,$$

$$\Xi_{26} = Q_0^TM,$$

$$\Xi_{27} = \varepsilon \cdot N_1^T,$$

$$\Xi_{33} = -Q_0^TQ_0 + h^2 \cdot R_2,$$

$$\Xi_{34} = Q_0^TC_0,$$

$$\Xi_{35} = Q_0^TD_0,$$

$$\Xi_{44} = -Q_0^TR_3 + R_3,$$

$$\Xi_{45} = Q_0^TD_1 + Q_0^TD_4,$$

$$\Xi_{46} = Q_0^TM,$$

$$\Xi_{55} = -e^{-2\alpha \cdot R_3} + Q_0^TD_3 + D_1^TQ_1,$$

$$\Xi_{56} = Q_0^TM,$$

$$\Xi_{57} = \varepsilon \cdot N_2^T,$$

$$\Xi_{66} = -\varepsilon \cdot I,$$

$$\Xi_{77} = -\varepsilon \cdot I.$$

Proof: Define the functional

$$V(x, y) = x^T(t)Px(t) + \int_{t-h}^{t} e^{2\alpha(s-t)} \cdot x^T(s)R_1x(s)ds + h \cdot \int_{t-h}^{t} e^{2\alpha(s-t)} \cdot (s - (t-h))x^T(s)R_2x(s)ds$$

$$+ \int_{t-h}^{t} e^{2\alpha(s-t)} \cdot y^T(s)R_3y(s)ds.$$
\[
\int_{t-h}^{t} \dot{x}(s) ds = x(t) - x(t-h),
\]
the time derivatives of \( \dot{V}(x, y) \), along the trajectories of system (1) with (2) satisfy
\[
\dot{V}(x, y) + 2\alpha \cdot V(x, y) \leq \eta^T_0 \cdot \Omega \cdot \eta_0,
\]
where \( \eta^T_0 = \left[ x^T(t) \ x^T(t-h) \ \dot{x}^T(t) \ y^T(t) \ y^T(t-h) \right] \) and
\[
\Omega = \left[ \begin{array}{cccccc}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\
\ast & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\
\ast & \ast & \Omega_{33} & \Omega_{34} & \Omega_{35} \\
\ast & \ast & \ast & \Omega_{44} & \Omega_{45} \\
\ast & \ast & \ast & \ast & \Omega_{55}
\end{array} \right]
\]
\[
= \left[ \begin{array}{cccccc}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\
\ast & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\
\ast & \ast & \Xi_{33} & \Xi_{34} & \Xi_{35} \\
\ast & \ast & \ast & \Xi_{44} & \Xi_{45} \\
\ast & \ast & \ast & \ast & \Xi_{55}
\end{array} \right]
\]
\[
+ \left[ \begin{array}{c}
Q^T_1 M \\
Q^T_2 M \\
0 \\
Q^T_3 M \\
Q^T_4 M
\end{array} \right] \left[ \begin{array}{c}
F(t) \\
N^T_0 \\
N^T_1 \\
N^T_2 \\
N^T_3
\end{array} \right] + \left[ \begin{array}{c}
N^T_0 \\
N^T_1 \\
0 \\
F^T(t) \\
0
\end{array} \right] \left[ \begin{array}{c}
Q^T_1 M \\
Q^T_2 M \\
0 \\
Q^T_3 M \\
Q^T_4 M
\end{array} \right]^	op.
\]
\[
\Omega_{11} = \Xi_{11} + Q^T_1 M F(t) N_0 + N^T_0 F^T(t) M^T Q_1,
\]
\[
\Omega_{12} = \Xi_{12} + Q^T_1 M F(t) N_1 + N^T_0 F^T(t) M^T Q_2,
\]
\[
\Omega_{13} = \Xi_{13}, \ \Omega_{14} = \Xi_{14} + N^T_0 F^T(t) M^T Q_3,
\]
\[
\Omega_{15} = \Xi_{15} + Q^T_1 M F(t) N_2 + N^T_0 F^T(t) M^T Q_4,
\]
\[
\Omega_{22} = \Xi_{22} + Q^T_2 M F(t) N_1 + N^T_1 F^T(t) M^T Q_2,
\]
\[
\Omega_{23} = \Xi_{23}, \ \Omega_{24} = \Xi_{24} + N^T_0 F^T(t) M^T Q_3,
\]
\[
\Omega_{25} = \Xi_{25} + N^T_1 F^T(t) M^T Q_4 + Q^T_2 M F(t) N_2,
\]
\[
\Omega_{33} = \Xi_{33}, \ \Omega_{34} = \Xi_{34}, \ \Omega_{35} = \Xi_{35},
\]
\[
\Omega_{44} = \Xi_{44}, \ \Omega_{45} = \Xi_{45} + Q^T_3 M F(t) N_2,
\]
\[
\Omega_{55} = \Xi_{55} + Q^T_4 M F(t) N_2 + N^T_2 F^T(t) M^T Q_4.
\]
By Lemma 1 with condition (9b), we have
\[
\Omega \leq \left[ \begin{array}{cccccc}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\
\ast & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\
\ast & \ast & \Xi_{33} & \Xi_{34} & \Xi_{35} \\
\ast & \ast & \ast & \Xi_{44} & \Xi_{45} \\
\ast & \ast & \ast & \ast & \Xi_{55}
\end{array} \right]
\]
From the Schur complement of [17] with (7), we obtain the condition \( \Omega < 0 \) in (9a) and (9c). By the result in (9a) with \( \Omega < 0 \), we conclude the following result

\[
\lambda_{\max}(P) \| x(t) \|^2 \leq V(x, \varphi)
\]

\[
\leq e^{-2\alpha t} \cdot V(\phi, \varphi) \leq e^{-2\alpha t} \cdot \Lambda(\phi, \varphi),
\]

where

\[
\Lambda(\phi, \varphi) = (\lambda_{\max}(P) + \lambda_{\max}(R_1) \cdot h) \| x \|^2
\]

\[
+ \lambda_{\max}(R_2) \cdot h^2 \cdot \| x \|^2 + \lambda_{\max}(R_3) \cdot \tau \cdot \| x \|^2.
\]

Hence we have

\[
\| x(t) \| \leq \rho \cdot e^{-\alpha t}, t \geq 0,
\]

where \( \rho = \sqrt{\Lambda(\phi, \varphi)/\lambda_{\max}(P)} \). By Definition 1, we conclude that the system (1) with (2) is globally exponentially stable with convergence rate \( \alpha \).

In the next, we propose a delay-dependent condition for global exponential stability of system (1) with (3).

**Theorem 2:** The system (1) with (3) is globally exponentially stable with convergence rate \( \alpha > 0 \), if there exist some positive definite symmetric matrices \( P, R_i \in \Re^{n \times n}, i \in \{1, 2, 3\} \), some matrices \( Q_j \in \Re^{m \times m}, j \in \{0, 1, 2, 3, 4\} \), and some nonnegative constants \( \varepsilon_k, k \in \{0, 1, 2\} \), such that the following LMI condition is satisfied:

\[
\Theta = \begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} & \Theta_{17} & \Theta_{18} \\
* & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & \Theta_{26} & \Theta_{27} & \Theta_{28} \\
* & * & \Theta_{33} & \Theta_{34} & \Theta_{35} & \Theta_{36} & \Theta_{37} & \Theta_{38} \\
* & * & * & \Theta_{44} & \Theta_{45} & \Theta_{46} & \Theta_{47} & \Theta_{48} \\
* & * & * & * & \Theta_{55} & \Theta_{56} & \Theta_{57} & \Theta_{58} \\
* & * & * & * & * & \Theta_{66} & 0 & 0 \\
* & * & * & * & * & * & \Theta_{77} & 0 \\
* & * & * & * & * & * & * & \Theta_{88}
\end{bmatrix} < 0,
\]

where

\[
\Theta_{11} = PA_0 + A_0^T P + Q_i^T A_i + A_i^T Q_i + 2\alpha \cdot P + R_i
\]

\[
- e^{-2\alpha h} \cdot R_2 + \varepsilon_0 \cdot \Gamma_0 \cdot \Gamma_0^T,
\]

\[
\Theta_{12} = PB_0 + e^{-2\alpha h} \cdot R_2 + Q_i^T B_1 + A_i^T Q_2, \quad \Theta_{13} = A_0^T Q_2,
\]

\[
\Theta_{14} = PC_0 - Q_i^T + A_i^T Q_3, \quad \Theta_{15} = PD_0 + Q_i^T D_i + A_i^T Q_4,
\]

\[
\Theta_{16} = Q_i^T, \quad \Theta_{17} = \Theta_{18} = Q_i^T,
\]

\[
\Theta_{22} = -e^{-2\alpha h} \cdot [R_2 + R_3] + B_2^T Q_5 + Q_2^T B_i + \varepsilon_i \cdot \Gamma_i \cdot \Gamma_i^T,
\]

\[
\Theta_{23} = B_2^T Q_5, \quad \Theta_{24} = -Q_2^T + B_2^T Q_5,
\]

\[
\Theta_{25} = Q_i^T D_i + B_2^T Q_5, \quad \Theta_{26} = Q_i^T, \quad \Theta_{27} = Q_i^T,
\]

\[
\Theta_{28} = Q_i^T, \quad \Theta_{33} = -Q_i^T - Q_0 + h^2 \cdot R_2, \quad \Theta_{34} = Q_i^T C_0.
\]
\[\Theta_{35} = Q_0^T D_0, \quad \Theta_{44} = -Q_3^T Q_3 + R_3,\]
\[\Theta_{45} = Q_4^T D_1 - Q_4, \quad \Theta_{46} = Q_5^T, \quad \Theta_{47} = Q_5^T, \quad \Theta_{48} = Q_5^T,\]
\[\Theta_{55} = -e^{-\delta_\tau} R_3 + Q_1^T D_1 + D_1^T Q_1 + \varepsilon_2 \cdot \Gamma_2^T \Gamma_2,\]
\[\Theta_{56} = Q_6^T, \quad \Theta_{57} = Q_7^T, \quad \Theta_{58} = Q_7^T, \quad \Theta_{66} = -e_0 \cdot I, \quad \Theta_{77} = -e_1 \cdot I, \quad \Theta_{88} = -e_2 \cdot I.\]

**Proof:** From (3), we have
\[
x^T(t) \Gamma^T \Gamma x(t) - f_0^T f_0 \geq 0,
\]  
\[
x^T(t-h) \Gamma^T \Gamma x(t-h) - f_1^T f_1 \geq 0,
\]  
\[
y^T(t-\tau) \Gamma^T \Gamma y(t-\tau) - f_2^T f_2 \geq 0.
\]

Thus we obtain the following result:
\[
\dot{V}(x_i, y_i) + 2\alpha \cdot V(x_i, y_i) + \varepsilon_0 \cdot \left[ x^T(t) \Gamma^T \Gamma x(t) - f_0^T f_0 \right]
\]
\[
+ \varepsilon_1 \left[ x^T(t-h) \Gamma^T \Gamma x(t-h) - f_1^T f_1 \right]
\]
\[
+ \varepsilon_2 \cdot \left[ y^T(t-\tau) \Gamma^T \Gamma y(t-\tau) - f_2^T f_2 \right] \leq \eta^T \cdot \Theta \cdot \eta,
\]

where
\[
\eta = \begin{bmatrix} x^T(t) & x^T(t-h) & x^T(t) & y^T(t) \\
 y^T(t-\tau) & f_0^T(x(t)) & f_1^T(x(t-h)) & f_2^T(y(t-\tau)) \end{bmatrix},
\]
and the matrix \( \Theta < 0 \) is defined in (11). By Lemma 2 with \( \Theta < 0 \) and (12), we have
\[
\dot{V}(x_i, y_i) + 2\alpha \cdot V(x_i, y_i) < 0, \forall \eta_i 
eq 0.
\]

The result of (10) can be achieved in the same derivation. This proof is completed.

**Remark 3:** By setting \( \alpha = 0 \) in Theorem 1 and Theorem 2, we can obtain the global asymptotic stability results of system (1) with (2) and (3), respectively.

### 3. Numerical examples

**Example 1:** Consider the following neutral time-delay system:
\[
\dot{x}(t) = \left[A + \Delta A(t)\right]x(t) + \left[B + \Delta B(t)\right]x(t-h) + \left[D + \Delta D(t)\right]x(t-\tau),
\]
\[
x(t) = \phi(t), \quad t \in [-H, 0],
\]
where
\[
A = \begin{bmatrix} -4 & 1 \\ 0 & -4 \end{bmatrix}, B = \begin{bmatrix} 0.1 & 0 \\ 4 & 0.1 \end{bmatrix}, D = \begin{bmatrix} \beta & 0 \\ 0 & \beta \end{bmatrix},
\]
\[
\|\Delta A(t)\| \leq \delta, \quad \|\Delta B(t)\| \leq \delta, \quad \|\Delta D(t)\| \leq \delta, \quad H = \max\{h, \tau\}.
\]

By setting \( \beta = \delta = 0 \), system (13) can be reduced to a retarded time-delay system (Example 2 of [15] with \( \delta_0 = \delta_1 = 0.2 \)). From Theorem 2, we obtain some upper bounds of system trajectories for the time...
delay system (13). Some comparisons with the obtained results and [15] are shown in Table 1. From Table 1, the results in this paper are less conservative and have some better exponential estimates.

Example 2: Consider the delayed cellular neural networks with the following parameters:

\[ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \]

\[ g_i(z_i) = 0.5|z_i + 1| - |z_i - 1|, \quad i = 1, 2, \tau = 5. \] (14)

| Table 1: Some comparisons for system (13). |
|--------------------------------------------|
| Retarded time-delay system with \( h = 0.5 \) and \( \tau = 0 \) |
| Conditions | Theorem 2 of [15] | Theorem 2 of this paper with \( R_2 = R_3 = 0 \) |
| \( \beta = \delta_2 = 0, \alpha = 0.3 \) | \( \|x(t)\| \leq 3.6331e^{-0.3t} \|x\|_{\|x\|_{L_2}} \) | \( \|x(t)\| \leq 2.4430e^{-0.3t} \|x\|_{\|x\|_{L_2}} \) |
| \( \beta = \delta_2 = 0, \alpha = 0.476 \) | \( \|x(t)\| \leq 3.3052e^{-0.476t} \|x\|_{\|x\|_{L_2}} \) | \( \|x(t)\| \leq 2.6482e^{-0.476t} \|x\|_{\|x\|_{L_2}} \) |
| \( \beta = \delta_2 = 0, \alpha = 0.6 \) | Fail | \( \|x(t)\| \leq 2.8753e^{-0.6t} \|x\|_{\|x\|_{L_2}} \) |
| \( \beta = \delta_2 = 0, \alpha = 0.768 \) | Fail | \( \|x(t)\| \leq 3.5937e^{-0.768t} \|x\|_{\|x\|_{L_2}} \) |
| Neutral time-delay system with \( h = 0.5 \) and \( \tau = 0.5 \) |
| Conditions | Theorem 2 of [15] | Theorem 2 of this paper |
| \( \beta = \delta_2 = 0.2, \alpha = 0.3 \) | Cannot be applied | \( \|x(t)\| \leq \sqrt{6.04 \|x\|_{\|x\|_{L_2}} + 0.31 \|x\|_{\|x\|_{L_2}}} e^{-0.3t} \) |
| \( \beta = \delta_2 = 0.2, \alpha = 0.4 \) | Cannot be applied | \( \|x(t)\| \leq \sqrt{6.92 \|x\|_{\|x\|_{L_2}} + 0.37 \|x\|_{\|x\|_{L_2}}} e^{-0.4t} \) |
| \( \beta = \delta_2 = 0.2, \alpha = 0.48 \) | Cannot be applied | \( \|x(t)\| \leq \sqrt{7.73 \|x\|_{\|x\|_{L_2}} + 0.42 \|x\|_{\|x\|_{L_2}}} e^{-0.48t} \) |

From the inequality in (5c), \( \Gamma \) is the \( 2 \times 2 \) identity matrix. By using the result (6e) in Remark 2, we have

\[ A_0 = -C, \quad B_0 = 0, \quad C_0 = A, \quad D_0 = B, \]

\[ A_1 = B_1 = D_1 = 0, \quad \Gamma_0 = \Gamma, \quad \Gamma_1 = \Gamma_2 = 0. \]

By the Theorem 2 with \( \alpha = 0.1 \), LMI (11) has a feasible solution. We conclude that the delayed cellular neural networks (6) (i.e. (5) with equilibrium point \( \bar{z} \)) with (14) and \( \tau = 5 \) is globally exponentially stable with convergence rate \( \alpha = 0.1 \). Note that \( \lambda_{\text{max}}(A + A') = 0.4 > 0 \) and \( \Gamma = I \), the results of [8] and [11] cannot provide any conclusion.

4. Conclusion

In this paper, the global exponential stability for generalized state-space time-delay systems has been studied. Based on the LMI approach, delay-dependent stability conditions have been proposed to guarantee the stability of systems. Some numerical examples have illustrated the contributions of this paper. From the simulation results, significant improvement over the recent results has been observed.

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