Waveguiding in supported phononic crystal plates

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Abstract. We investigate, with the help of the finite element method, the existence of absolute band gaps in the band structure of a free-standing phononic crystal plate and of a phononic crystal slab deposited on a substrate. The two-dimensional phononic crystal is constituted by a square array of holes drilled in an active piezoelectric (PZT5A or AlN) matrix. For both matrix materials, an absolute band gap occurs in the band structure of the free-standing plate provided the thickness of the plate is on the order of magnitude of the lattice parameter. When the plate is deposited on a Si substrate, the absolute band gap still remains when the matrix of the phononic crystal is made of PZT5A. The AlN phononic crystal plate loses its gap when supported by the Si substrate. In the case of the PZT5A matrix, we also study the possibility of localized modes associated with a linear defect created by removing one row of air holes in the deposited phononic crystal plate.

1. Introduction
Phononic crystals i.e. periodic arrays of inclusions inserted in a matrix \cite{1,2} are receiving a great deal of attention for their potential applications as components of telecommunication devices such as filters and demultiplexers of acoustic waves \cite{3}. Consequently we report here a theoretical investigation using the finite element method, of the band structures of 2D phononic crystal plates made of arrays of air holes drilled in an active piezoelectric matrix. We consider two types of piezoelectric materials namely PZT5A and AlN. The large contrast in physical properties of the air holes and the piezoelectric matrices, may lead to the formation of absolute band gaps in the band structure of unsupported phononic crystal plates. We also study the impact a silicon substrate may have on the existence of the absolute band gap in a supported phononic crystal plate. Finally we look at the waveguiding properties of supported phononic crystal plates containing a linear defect.

2. Absolute band gaps in free-standing phononic crystal plate
We first consider the case of a free standing plate cut inside an infinite 2D phononic crystal (see figure 1(a)). The 2D phononic crystal is composed of a square array of parallel cylindrical holes in a
solid matrix and the plate is cut perpendicular to the cylinders axis. It has been proven [4-6] that the existence of an absolute band gap in the band structure of a free standing plate strongly depends on the physical parameters (density and elastic properties of the constituents) and on the geometrical parameters such as the period \( a \) of the array of inclusions, the shape of the inclusions (cylinders, square rods), the filling factor \( f \) of the inclusions and the thickness \( h \) of the plate. The finite element computation of the band structure is conducted on a unit cell of dimension \( a \times a \times h \) containing a single hole [7]. In particular, an absolute band gap occurs in the band structure provided \( h \) is of the order of magnitude of \( a \) [4,5]. This comes from the confinement of the vibration modes in the thickness of the slab and from the constraints imposed by the free surface boundary conditions on the wave vector perpendicular to the surfaces. Two examples of band structures for free standing plates of 2D piezoelectric phononic crystals are presented in figure (1). Both band structures exhibit an absolute forbidden band in the domain of frequency of the GHz when the geometrical parameters \( a \) and \( h \) are of the order of magnitude of the micrometer. These frequencies are particularly relevant to applications in the field of radiofrequency telecommunications. A wider band gap is observed in the case of AlN compared to that of PZT5A.

![Figure 1](image_url)

**Figure 1.** (a) Free-standing phononic crystal plate of thickness \( h \) (the basic phononic crystal is composed of a square array of parallel cylindrical air inclusions (holes) of radius \( R \) drilled in a PZT5A (figure 1(b)) or AlN (figure1(c)) piezoelectric matrix) and the first Brillouin zone \( \Gamma XM \) of the square array. (b) and (c) Elastic band structures calculated with the finite element method for the freestanding phononic crystal plate with a filling factor \( f = 0.7 \). The geometrical parameters are \( h = a = 0.77 \mu m \) in figure 1(b) and \( h = a = 2.31 \mu m \) in figure 1(c). The components of the wave vector at the \( \Gamma, X \) and \( M \) points are \((0,0), (\pi/a,0)\) and \((\pi/a,\pi/a)\), respectively.

### 3. Band structures of supported phononic crystal plate

Since micrometer thick free-standing phononic crystal plates are not easily realizable technologically, we calculate the band structure of the phononic crystal plate of section 2 supported on a commonly used substrate made of silicon (see figure 2(a)). The substrate thickness is five times the lattice parameter. The dimensions of the finite element unit cell are \( axax(5a+h) \). We have verified that there is no significant difference between the band gaps of the structure with thicker substrates. A thickness \( d=5a \) is a good compromise between accuracy and computational load. figures 2(b) and 2(c) show the calculated band structures of the supported air/PZT5A and air/AlN phononic crystal plates, respectively. The thick solid straight lines represent the dispersion curves of the slower elastic waves propagating in Silicon i.e. transverse waves. Only the plate modes that lie below the Silicon transverse modes are confined inside the phononic crystal plate. In the case of the air/PZT supported plate (see figure 2(b)) and focusing on frequencies in the vicinity of 1.5 GHz, there are no plate modes over a very wide range of wave vectors. This frequency domain behaves like an absolute forbidden band.
figure 2(c), the air/AlN supported plate does not show such a behavior. This result arises from the differences in speeds of sound of the piezoelectric materials compared to that of Silicon. Indeed the longitudinal $v_l$ and transverse $v_t$ speeds of sound in PZT, AlN and Si are ordered in the following manner $v_{l\text{AlN}}>v_{l\text{Si}}>v_{l\text{PZT}}$ and $v_{t\text{AlN}}>v_{t\text{Si}}>v_{t\text{PZT}}$. Since the speeds of sound of AlN exceed that of Si, this structure cannot supports localized modes in the phononic crystal plate. The existence of a band gap for confined elastic waves in the air/PZT5A/Silicon system suggests the possibility of waveguiding in a defected supported phononic crystal plate.

**Figure 2.** (a) Phononic crystal plate of thickness $h$ deposited onto a homogeneous substrate of thickness $d$. (b) Elastic band structures for the air/PZT phononic crystal plate of thickness $h = a = 0.77\mu$m deposited onto a silicon substrate of thickness $d = 5a$. (c) Same as (b) but for a air/AlN plate of thickness $h = a = 2.31 \mu$m. The thick solid lines represent the dispersion curves of the slower elastic waves propagating in Silicon i.e. transverse waves.

4. Waveguiding in supported phononic crystal plate
We consider a supported air/PZT5A phononic crystal plate with a linear defect created by filling a row of holes with PZT5A. This structure becomes non-periodic and the band structure is now calculated with the help of a super-cell constituted of 1x7 unit cells in the plane of the plate, which means a super-cell of dimensions $ax7ax(5a+h)$. The filled hole is located on the fourth unit-cell. Figures 3(a) and 3(b) illustrate the band structure along the ΓX direction of propagation, of a perfect supported phononic crystal plate and the defected one, respectively. Figure 3(a) differs from figure 2(b) in that the bands are folded in a smaller Brillouin zone. In figure 3(b) we see additional modes in the frequency range of the band gap of figure 3(a). The displacement field in the supported plate corresponding to the mode localized within the band gap at the X point and a frequency of 1.469 GHz is illustrated in figures 3(c) and (d). The displacement field is localized within the defect. The linear defect can therefore act as a waveguide.

5. Conclusion
We investigated using the finite element method, the band structures of 2D phononic crystal plates made of arrays of air holes drilled in an active piezoelectric matrix. We show that free standing air/PZT5A and air/AlN phononic crystal plates exhibit absolute band gaps. We calculated the band
structures of the same phononic crystal plates supported on a Silicon substrate. The supported air/PZT plate is the only one to retain a band gap due to its physical properties that differ significantly from those of the substrate. We demonstrate localization of modes in a linear defect. Although the conditions necessary for the existence of a gap and the localization of modes in the supported phononic crystal plate are quite narrow, the defect modes could be used to realize functional devices such as specific frequency filters or demultiplexers.

References
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