Closed-loop simulation of decentralized control using RGA for uncertain binary distillation column

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Abstract. This paper presents the results of closed-loop simulation for the decentralized control of uncertain distillation column. The RGA (Relative Gain Array) RGA analysis was used as the basis for selecting the configuration of the decentralized control system. PI controller obtained was then tuned with optimization methods. The simulation results show that the RGA analysis requires accurate range for uncertain systems. In addition, closed-loop simulation results confirm the RGA analysis.

1. Introduction

Decentralized control is famous in chemical process control even though sophisticated methods for designing centralized multivariable control are now available such as Internal Model Control (IMC), Dynamic Matrix Control (DMC), Linear Quadratic Control (LQC), etc. The advantages of decentralized control for multivariable process are as follows \cite{1}:

• Simple algorithm
• Ease of understanding by plant operating personnel (as a result of the simplicity of control structures)
• The availability of standard control designs for the common unit operations
• Flexibility in operation
• Failure tolerance

Theoretically, a centralized control, which handles the whole plant as a single unit, may give the best performance. However, a complete centralized controller has a number of difficulties such as in controller design, tuning, maintenance and modification. In addition, Skogestad \cite{2} explained the main reason of decentralized controller wide acceptance is that it is related to the high cost associated with obtaining good process models, which serve as prerequisite for applying centralized multivariable control. From plant operator’s point of view, a control structure that encompasses a wide range of different units may be beyond understanding. Transparency and intuitiveness of the control structure are important factors for operator acceptance and safe plant operation \cite{3}.

RGA (Relative Gain Array) was first proposed by Bristol \cite{4} as a steady state measure of interactions for decentralized control. Properties of RGA matrix were given by Skogestad and Postlethwaite \cite{5}.
Chen and Seborg [6] presented an analytical expression for RGA uncertainty bounds. Two types of model uncertainty were considered: worst case bounds, where all elements of the steady state process gain matrix are allowed to change simultaneously within their bounds, and statistical uncertainty bounds. A different method by using the structured singular value (μ) analysis framework was introduced for the calculation of the magnitude of the worst-case relative gain [7].

Agustriyanto and Zhang [8] used an optimization method to calculate RGA range under model uncertainties. The model uncertainty type considered is worst case bounds. The lower and upper bounds of an RGA element are calculated as two constrained optimization problems. The method seeks the minimum (for the lower bound) or maximum (for the upper bound) of an RGA element subject to the constraints that allowable model parameters are within their uncertainty bounds. RGA ranges are shown to be important for control pairing analysis. In this paper, closed loop simulation was then performed to evaluate the RGA analysis.

2. Simulation
The system being studied was the binary distillation column used to separate ethanol and water [9]. It is a 19 plate distillation column with 12 inch diameter copper column having variable feed and side stream draw-off locations. This example was also used by Chen and Seborg [6] to test their method for calculating the uncertain ranges of RGA. A schematic diagram of the binary distillation column is shown in Figure 1.

Figure 1. Schematic diagram of the binary distillation column with diagonal control structure

Table 1 shows steady state values of the column and Table 2 shows system constraints. The following Laplace transfer functions have been identified and were used in this study.

\[
G = \begin{bmatrix}
0.66e^{-2.6s} & -0.61e^{-3.5s} & -0.0049e^{-s} \\
6.7s + 1 & 8.64s + 1 & 9.06s + 1 \\
1.11e^{-6.5s} & -2.36e^{-3s} & -0.012e^{-1.2s} \\
3.25s + 1 & 5s + 1 & 7.09s + 1 \\
-33.68e^{-9.2s} & 46.2e^{-9.4s} & 0.87(11.61s + 1)e^{-s} \\
8.15s + 1 & 10.9s + 1 & (3.89s + 1)(18.8s + 1)
\end{bmatrix}
\] (1)
Table 1. Steady state values of binary distillation column

| Variable | Description                                      | Value | Units |
|----------|--------------------------------------------------|-------|-------|
| $y_1$    | Overhead mol fraction of ethanol                 | 0.7   |       |
| $y_2$    | The mol fraction of ethanol in the side stream   | 0.52  |       |
| $y_3$    | Temperature on tray 19                           | 92    | °C    |
| $u_1$    | Overhead reflux flow rate                         | 0.18  | gpm   |
| $u_2$    | Side stream draw-off rate                         | 0.046 | gpm   |
| $u_3$    | Reboiler steam pressure                           | 20    | psig  |

Table 2. System constraints

| Variable | Lower Constraint | Upper Constraint |
|----------|------------------|------------------|
| $u_1$    | 0.068            | 0.245            |
| $u_2$    | 0.00694          | 0.1              |
| $u_3$    | 15.6             | 34.0             |

Generally, the RGA of a non-singular square matrix $K$ is a square matrix and defined as:
\[
RGA = K \otimes (K^{-1})^T
\]  
(2)

Where $\otimes$ denotes element by element multiplication.

Eq. (2) can be used for $3 \times 3$ process and all other systems.

The $ij$th element of the RGA [10] is:
\[
\lambda_{ij} = (-1)^{i+j} \frac{K_{ij} \det(K^{ij})}{\det(K)}
\]  
(3)

Here, $K_{ij}$ is the element on the $i$th row and $j$th column of $K$ and $K^{ij}$ is the sub matrix that remains after the $i$th row and $j$th column of $K$ are deleted.

It is obvious that $\lambda_{ij}$ is a function of $K$, that is
\[
\lambda_{ij} = f(K)
\]  
(4)

Now assume that the uncertainty bounds (lower and upper bounds) for steady state gain, $K_{ij}$, $i=1, 2, \ldots, n, j=1,2, \ldots, n$, are given, then there will be $2n^2$ constraints for all elements of steady state gains which can be formulated as follows:
\[
AX \leq b
\]  
(5)

where
- $X$ is a vector of size $n^2 \times 1$ containing all elements of $K$ as its elements:
\[
X = [K_{11}, \ldots, K_{nn}]^T
\]  
(6)
- $b$ is a vector of size $(2n^2) \times 1$ containing the lower and upper bounds of the corresponding elements of $X$.
- $A$ is an appropriate matrix of size $(2n^2) \times (n^2)$ satisfying the inequalities in Eq. (5).

Therefore, the lower bound and upper bound of $\lambda_{ij}$ can be formulated as the following respectively:

Lower Bound: $\min_X \lambda_{ij} = f(X)$
(7)

Upper Bound: $\max_X \lambda_{ij} = f(X)$
(8)

Subject to constraint of Eq. (5).
Note that $\lambda_{ij}$ cannot be determined if $\det(K) = 0$. Therefore, in order to use the above method, the range of $\det(K)$ should not include 0. The range of $\det(K)$ can be calculated by using the same optimization method.

Optimization algorithm that optimized control performance was used to automate trial and error method. Here, $fmincon$ function in Matlab Optimization Toolbox was used to obtain lower and upper bounds.

3. Results and discussion

The nominal steady state gain matrix and RGA are given as:

$$K = \begin{bmatrix} 0.66 & -0.61 & -0.0049 \\ 1.11 & -2.36 & -0.012 \\ -33.68 & 46.2 & 0.87 \end{bmatrix}$$  \hfill (9)

$$RGA = \begin{bmatrix} 1.9454 & -0.6737 & -0.2718 \\ -0.6643 & 1.8991 & -0.2348 \\ -0.2811 & -0.2254 & 1.5065 \end{bmatrix}$$  \hfill (10)

The nominal RGA values indicate that the diagonal control structure, $(y_1-u_1, y_2-u_2, y_3-u_3)$, should be chosen because the sign of the RGA elements for this pairing are all positive and their values are close to 1.

As in Chen and Seborg [6], it is assumed here that the uncertainty for each steady state gain can be expressed as:

$$K_y = K_y \pm \Delta K_y$$  \hfill (11)

where

$$|K_y| < \alpha |K_y|$$  \hfill (12)

Case 1: $\alpha = 0.01$

The uncertainty ranges for RGA elements calculated via optimization method described in the above Section are given below:

$$RGA = \begin{bmatrix} 1.88 \leq \lambda_{11} \leq 2.02 & -0.74 \leq \lambda_{12} \leq -0.61 & -0.31 \leq \lambda_{13} \leq -0.24 \\ -0.73 \leq \lambda_{21} \leq -0.60 & 1.83 \leq \lambda_{22} \leq 1.97 & -0.27 \leq \lambda_{23} \leq -0.20 \\ -0.32 \leq \lambda_{31} \leq -0.25 & -0.26 \leq \lambda_{32} \leq -0.20 & 1.48 \leq \lambda_{33} \leq 1.54 \end{bmatrix}$$  \hfill (13)

The above results indicate that the recommended controller pairing is obvious and unambiguous: $(y_1-u_1, y_2-u_2, y_3-u_3)$.

The results obtained via optimization method had wider range for each element of RGA compared to the results from the analytical method. This was because no approximation had been made in optimization method, while Taylor series expansion was used in analytical method. There exists a combination of $K$ which is still within the constraint Eq. (20) and will give the lower and upper bound of relative gains in Eq. (22). This indicated that the proposed method could find more accurate RGA uncertain bounds than the method proposed previously.

Case 2: $\alpha = 0.1$

The uncertainty ranges for RGA calculated via optimization is given below:

$$RGA = \begin{bmatrix} 1.48 \leq \lambda_{11} \leq 3.65 & -1.89 \leq \lambda_{12} \leq -0.24 & -0.76 \leq \lambda_{13} \leq -0.02 \\ -1.86 \leq \lambda_{21} \leq -0.23 & 1.46 \leq \lambda_{22} \leq 3.42 & -0.66 \leq \lambda_{23} \leq -0.01 \\ -0.79 \leq \lambda_{31} \leq -0.03 & -0.66 \leq \lambda_{32} \leq -0.04 & 1.29 \leq \lambda_{33} \leq 2.01 \end{bmatrix}$$  \hfill (14)
Case 3: $\alpha = 0.25$

The uncertainty range for RGA cannot be determined as for $\alpha = 0.25$, the value of $\text{det}(K)$ range will include 0 ($K$ can become singular). While using analytical method\(^6\), the value of $\alpha = 0.5$ can still be tolerated.

Closed loop simulation was then performed for the recommended controller pairing ($y_1 - u_1, y_2 - u_2, y_3 - u_3$). Table 3 shows the tuning parameters which were obtained via optimization where the sum of absolute errors for setpoint tracking was minimized. The following setpoint changes are used for tuning purpose:

• $y_1$ set-point was changed from 0 to 0.1 at $t = 10$ min
• $y_2$ set-point was changed from 0 to -0.1 at $t = 300$ min
• $y_3$ set-point was changed from 0 to -5 at $t = 500$ min

The output values were recorded for 1000 min simulation time with 1 min sampling time. Therefore, the objective function in the optimization routine is to find a set of controller parameters which will minimize sum of the absolute errors for the specified simulation time.

| Controllers | $K_c$  | $\tau_i$ |
|--------------|--------|----------|
| $y_1-u_1$    | 0.0871 | 0.6053   |
| $y_2-u_2$    | -0.3829| 5.1813   |
| $y_3-u_3$    | 6.1938 | 3.4343   |

For the nominal values (i.e. no uncertainties), simulation results for set-point changes are shown in Figure 2. In Figure 2, the solid lines represent the controlled variables and the dashed lines represent the set-points. In all the simulations for this example, $y_1$ set-point was changed from 0 to 0.1 at $t = 10$ min, $y_2$ set-point was changed from 0 to -0.1 at $t = 300$ min, and $y_3$ set-point was changed from 0 to -5 at $t = 500$ min. It can be seen in Figure 2 that the control performance was satisfactory.

Simulations are then performed for the arbitrarily altered process gains (which reflect gain uncertainties) as follows:

\[
\bar{G} = \begin{bmatrix}
G_{11}(1-\alpha) & G_{12}(1+\alpha) & G_{13}(1-\alpha) \\
G_{21}(1+\alpha) & G_{22}(1-\alpha) & G_{23}(1+\alpha) \\
G_{31}(1-\alpha) & G_{32}(1+\alpha) & G_{33}(1-\alpha)
\end{bmatrix}
\]  

(15)

Figures 3 to 5 show the results for three different values of $\alpha$. Once again, the solid lines represent the controlled variables and the dashed lines represent the set-points. As shown in Figure 5, the controller settings based on the nominal model cannot guarantee the closed loop stability for $\alpha = 0.25$.

4. Conclusion

A method for calculating RGA ranges for uncertain process models is presented in this paper. Constrained optimization was used to find the uncertain RGA ranges. The proposed method was then applied to the binary distillation column. Closed loop simulation results confirm the analysis based on the proposed method.
Figure 2. Simulation results for $\alpha = 0$

Figure 3. Simulation results for $\alpha = 0.01$

Figure 4. Simulation results for $\alpha = 0.1$

Figure 5. Simulation results for $\alpha = 0.25$

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