The mean density of the Universe from cluster evolution

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Abstract

The determination of the mean density of the Universe is a long standing problem of modern cosmology. The number density evolution of x-ray clusters at a fixed temperature is a powerful cosmological test, new in nature (Oukbir and Blanchard, 1992), somewhat different from standard analyses based on the dynamical measurement of individual objects. However, the absence of any available sample of x-ray selected clusters with measured temperatures at high redshift has prevented this test from being applied earlier. Recently, temperature measurements of ten EMSS clusters at $0.3 \leq z \leq 0.4$ have allowed the application of this test (Henry, 1997). In this work, we present the first results of a new analysis we have performed of this data set as well as a new estimation of the local temperature distribution function of clusters: a likelihood analysis of the temperature distribution functions gives a preferred value for the mean density of the universe which corresponds to 75% of the critical density. An open model with a density smaller than 30% of the critical density is rejected with a level of significance of 95%.

1 Introduction

The value of the mean density of the Universe, $\rho_0$, is a fundamental constant for Cosmology. According to the theory of general relativity, in the case of a zero cosmological constant, the evolution of the Universe is determined by the ratio of this density to the so-called critical density, $\rho_c$, the latter corresponding to the solution known as the Einstein-de Sitter model. Models with a density greater than this critical density will undergo a finite period of expansion, followed by a collapse sometimes called the “big-crunch”. On the contrary, if the density is smaller than the critical value, the expansion will continue forever. This is all conveniently parameterized by $\Omega_0$, the density parameter, which is equal to the ratio of the actual density to the critical density. The value of $\Omega_0$ also has implications for the local geometry of space, for general relativity provides a link between the two. If $\Omega_0 > 1$, the geometry is spherical; if $\Omega_0 < 1$, the geometry is hyperbolic; while in the special case of $\Omega_0 = 1$, the geometry of space is locally Euclidean. Finally, the value of $\Omega_0$ should be predicted by the theories of the very early Universe. Inflation is certainly the best example of such a theory, whose initial prediction was $\Omega_0 = 1$. It has been recently shown that other values are possible, but at the expense of a lack of simplicity.

The classic method of determining the density of the Universe is known as the Oort method. It consists of evaluating the amount of mass present in a specific object, like a cluster for instance, from knowledge of the velocity dispersion, or the x-ray gas temperature. The mean value of the universal density is then derived under the assumption that the ratio of total to visible mass is constant. Such a method currently leads to low values of $\Omega$, of the order of 0.2 – 0.4 (Adami et al., 1998). The reliability of this method has been a matter of long debate over the last fifteen years, and remains so to this day.

2 Cluster number evolution: a global probe of the mean density of the Universe

It is clear that it is vital to find methods to estimate the mean density of the universe which do not rely on this assumption of a constant ratio between mass and visible material. Geometrical tests, like the luminosity–distance relation, could in principle provide us with such a method. In practice, these tests have always failed to give a reliable answer because they are pervaded by evolutionary effects. The application to distant supernova represents a modern version which has received much attention in recent
years, possibly indicating a non-zero cosmological constant (Riess et al., 1998). Recently, it has been realized that the statistical properties of the cosmological microwave background fluctuations maps could allow one to determine the cosmological parameters with high precision (provided that the theoretical framework of gravitational instability with initial adiabatic fluctuations is correct). Although current observations already lead to interesting conclusions, only satellite-based observations, like those from Planck or MAP (Microwave Anisotropy Probe), will allow to remove the degeneracy between the various parameters entering the problem.

A category of test which is different in nature, is based on the dynamics of perturbations in the expanding universe. Linear perturbations grow at a slower rate in a low density Universe than in $\Omega_0 = 1$ Universe. It follows that knowledge of the cosmic velocity field would lead to the value of $\Omega_0$ (Dekel, 1994). First applications of this test were promising, but it has since been realized that systematic effects could present a stronger limitation to its usefulness than was initially thought (Davis, 1998). Another approach, proposed a couple of years ago, is based on the evolution of the number density of x-ray clusters (Oukbir and Blanchard, 1992). This evolution is directly related to the growth rate of fluctuations and allows, in principle, one to measure the mean density of the Universe. Considerable work has been devoted to the study of this test (Bartlett, 1997, and references therein). These studies show unambiguously that the abundance of x-ray clusters, with temperatures of the order of few keV, is expected to change much more slowly with redshift in a low-density Universe than in an Einstein–de Sitter model, a strong difference that should be easy to measure.

The results from two x-ray satellites, ASCA and ROSAT, have considerably improved the quality of the data: the luminosity function of x-ray clusters now seems well established (Ebeling et al., 1997), and the number of x-ray clusters at high redshift with measured temperatures has substantially increased. This has allowed a first indirect application of this test to a sample of high-$z$ x-ray clusters, leading to a rather high density for the Universe, close to the critical value (Sadat et al., 1998). Finally, Henry (1997) has for the first time provided the temperature of a set of clusters at significantly high redshifts ($0.3 < z < 0.4$), selected in a well-defined way, thereby allowing an estimation of the temperature distribution function. In the following, we present the estimation of the mean density of the Universe on the basis of this set of clusters and of a new set of local clusters ($z \sim 0.05$).

### 3 Data Analysis

We use a compilation of ROSAT observations of clusters and select those above a flux limit of $2.210^{-11}$ erg/s/cm$^2$, leading to a sample of fifty clusters. Temperatures for these fifty clusters were taken from the literature. Although it is believed that this sample is complete, it remains possible that some clusters have been missed. In such a case, the value of the mean density of the universe we infer would be underestimated. The temperature distribution function can be estimated following the standard method:

$$N(> T) = \sum 1/V_m$$

where $V_m$ is the volume of the sample out to the maximum depth at which the cluster would have been detected, given its intrinsic luminosity and the flux limit of the sample.

An important source of possible bias in the estimation of the temperature distribution function comes from the temperature measurement errors (Eke et al., 1998; Viana, Liddle, 1998): because there are considerably more low temperature than high temperature clusters, this can produce an apparent cluster abundance which is higher than the actual value. This effect was early pointed by Evrard (1989) for the velocity dispersion distribution function. In the present case, the errors differ significantly from one measurement to another and are correlated with the apparent luminosity, which determines the volume $V_m$. A correction to individual clusters is therefore preferable to a simple mean correction over the whole sample. We have therefore applied a Bayesian correction to individual temperature measurements. The main difference with previous estimation lies in the fact that we obtain a number density for clusters with $T \sim 4$ keV significantly higher. We have inferred the temperature distribution function at high redshift in a similar way by using the data as provided by Henry (1997).
4 Estimation of the mean density

The method we chose to estimate $\Omega_0$ is the maximum likelihood estimate on the number density of x-ray clusters in independent bins of the temperature distribution function at $z = 0.05$ and $z = 0.33$. Such an anaylsis requires knowledge of the distribution function of the estimator of the mean number density of clusters. This distribution $p$ was found by using the bootstrap re-sampling technique. The likelihood function is then computed as:

$$\mathcal{L} = \prod_i p(N_i | N(\sigma_c, n, z_i, \Omega_0))$$

where $\sigma_c$ is the amplitude of the fluctuations on cluster scales, $n$ is the power spectrum index of the primordial fluctuations, $z_i$ is the redshift of the i-th bin considered, $N_i$ is the actual observed number density of clusters in this bin, while $N$ is number of clusters predicted by the model using the Press and Schechter mass function (Press and Schechter, 1974). The best estimate parameters correspond to those for which $\mathcal{L}$ is maximum. A 68%, 95%, confidence intervals on one parameter can be obtained by considering the region enclosed by $\Delta\mathcal{L} = 0.5$ and $\Delta\mathcal{L} = 2$, respectively (this is only indicative, as it only holds for a normal distribution, which is not valid in our case). The maximum likelihood value we obtain is $\Omega_0 = 0.74$. The 95% range, according to normal statistics, is [0.3 - 1.2] (symmetrized). A preliminary analysis of the various sources of systematic uncertainties indicates that this number is not likely to change by a large amount.

5 Conclusion

The method we have applied to determine the mean density of the Universe has the considerable advantage of being global, relying on the dynamics of the Universe as a whole. Furthermore, numerous studies have confirmed the power and robustness of this method. The conclusion of a high density universe we have obtained, consistently with Sadat et al. (1998), could represent a major and fundamental advance in the understanding of our Universe; and, consequently, it calls for considerable prudence. The temperature distribution function we obtain at $z = 0.05$ is based on ROSAT fluxes which are believed to be accurate as well as on recent temperature measurements for the largest set available (fifty clusters). Great caution should be taken with the sample of high redshift clusters: any unidentified systematic effect in the selection function could undermine our estimate. In the near future, two new spectro-imaging, x-ray satellites, AXAF and XMM, will very likely bring much more light on the nature of distant clusters, allowing a definitive answer to these questions. It is therefore tempting to believe that the mean density of the Universe will be robustly determined before the end of the century.

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Figure 1: The likelihood function normalized to one obtained from the analysis of the relative abundance of clusters between $z = 0.05$ and $z = 0.33$. This function clearly favors a high density universe.