Model based simulation of a CVT variator to analyse the slip characteristics of the CVT belt

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Abstract. With the increase in utilization of Continuously Variable Transmission, more research has been done in areas pertaining to improving the efficiency of the Continuously Variable Transmission. While it provides a continuous gear ratio for the vehicle to work in, the overall efficiency is lesser than in a Manual or Automatic Transmission. The decrease in the efficiency is mainly due to the variator and the hydraulic losses. While hydraulic losses can be reduced by using electro-hydraulic control systems, the variator losses are primarily caused by the high clamping forces required for the transfer of the engine torque. Controlling the clamping forces can improve the efficiency of the Continuously Variable Transmission and reduce the slip experienced by the system. Analysis of control strategies to reduce the clamping forces and improve the slip characteristics are the primary objectives of this paper. The study looks into the change in the transmission efficiency with respect to the change in slip percentage experienced by the belt. A mathematical model is designed in SIMULINK based on the equations governing the functioning of a Continuously Variable Transmission. The results of the theoretical belt slip and the belt slip due to change in dynamics is compared and the effects of the change in torque and clamping forces are also discussed. Using this, a control strategy is developed to improve the power transmission efficiency of the model.

1. Introduction

Of the different transmission systems in the market, there has been an increase in the utilization of Continuously Variable Transmissions (CVT) as a gearbox system in new automobiles [1]. While CVTs have been in existence since 1920s, application of CVT in automobiles increased in the 1990s, and has become a conventional system in modern automobiles [4]. The main advantage of a CVT is its capability of running at all the potential gear ratios available within the working range. While the overall transmission efficiency is lesser than Manual (MT) and Automatic Transmissions (AT), CVT can work at greater economic operating points providing better fuel efficiency for the system. Combined with the ability to work in infinite gear ratios, CVTs provide a smooth power transmission from the engine to the drivetrain. A CVT consists of a high-strength belt, and 2 pulleys that are hydraulically or electronically controlled.
Belt mechanics of a CVT has a large effect on the overall efficiency of the CVT system. Controlling the functioning of a CVT, the belt connects the primary and the secondary pulleys of the system [6]. The change in radius of the pulley controls the gear ratio of the CVT and power transfer to the differential unit via a secondary shaft [8]. The primary shaft connects the flywheel to the primary pulley. All these components make up the variator of the CVT system. Variator plays a crucial role in controlling the overall efficiency of the system [10]. Some parameters that control the effectiveness of the CVT are the hydraulic losses experienced by the hydraulic actuation system that controls the CVT functioning, the torque losses experienced by the belt and the slip that is experienced by the belt. Slip plays a crucial role as it can cause damage to the belt and the pulleys.

Slip of a belt depends on two main factors: the clamping forces at the primary and secondary pulley, and the overall torque capacity of the variator [10 and 14]. High clamping forces causes more losses in the CVT [15]. Thus it is essential to have a check on the overall clamping forces acting on the CVT.

Theoretical belt slip can be calculated by determining the angular speed of the primary and the secondary shafts and the change of the gear ratio. This helps in providing a slip output that does not depend on the effect of the torque, the moment and the forces acting on the variator.

Another method to check on the micro slip experienced by the belt is to consider the above parameters along with the losses experienced by the system [10]. Taking the effects into account, a cohesive study can be carried out to understand the effects of the clamping forces on the slip characteristics of the CVT variator. Control techniques can also be looked into based on this.

In this paper, a mathematical approach using SIMULINK [7, 11, and 13] is utilized to understand the relationship between clamping forces of the variator, the frictional forces acting on the variator, the slip experienced by the belt and the overall transmission efficiency of the system. Control strategies to help with controlling the overall slip experienced by the system are also studied upon and the results are recorded and discussed.

2. Belt Mechanics

One crucial component of the variator is the belt. Most CVTs use V-belt pulley [6 and 8] systems to control and run the transmission system. The belt allows for transfer of torque which in turn helps in determining the gear ratio, as the change in radii of the pulleys is dependent on the change in the torque inputs experienced [2]. The length of the belt is as follows

\[ L = Rp(\pi + 2\phi) + Rs(\pi - 2\phi) + 2a(\cos(\phi)) \]  

(1)

Fig. 1: Geometric Belt Configuration. Adopted from Efficiency optimization of the push-belt CVT by variator slip control (p. 13) by B. Bonsen, Copyright 2006 by TU/e.

Here, \( L \) = length of the belt, \( Rp \) = Radius of the primary pulley, \( Rs \) = Radius of the secondary pulley, \( a \) = centre distance and \( \phi \) = belt contact angle. The belt contact angle and the gear ratio can be determined by the below equations
\(R_p - R_s = a(\sin \theta)\) \hspace{1cm} (2)

\[I = \frac{R_p}{R_s}\] \hspace{1cm} (3)

As can be seen, the gear ratio depends on the rate of change of the pulley radius. The change in the pulley radius is dependent on the effect of the torques, forces and the moments experienced by the variator. \(\theta_p\) and \(\theta_s\) are defined as the displacement of the pulley when the pulley experiences change in gear ratio. The speed ratio equation [1] is as follows

\[I_s = \frac{\omega_p}{\omega_s}\] \hspace{1cm} (4)

Where \(\omega_p\) \((d\theta_p/dt)\) and \(\omega_s\) \((d\theta_s/dt)\) are the rotational speed of the input and output shafts connected to the primary and secondary pulleys respectively. This equation helps determine the ratio at which the belt speeds experienced by the primary and secondary pulleys vary.

3. Belt Speed:

Two different methods can be utilized to discuss the effects of slip on the variator. The first method is to solely consider the variator slip based on the change in rotational speed of the primary and the secondary pulley. The second method is to consider the effect of tractive effort on the slip experienced by the variator. The first case can be considered to be the theoretical slip while the second case is considered to be the actual slip of the system.

Calculating the theoretical slip [14] depends on the running velocity of the belt, the rotational velocity experienced by the primary pulley and the rotational velocity of the secondary pulley. The speed of the belt can be determined by using a belt speed sensor [Slip behaviour in variator by measuring the belt speed].

To determine the belt speeds of the primary and secondary pulleys, the equations are as follows:

\[v_p = \omega_p \ast R_p\] \hspace{1cm} (5)

\[v_s = \omega_s \ast R_s\] \hspace{1cm} (6)

\(\omega_p\), and \(R_p\) are the rotational speed and the radius of the primary pulley while \(\omega_s\) and \(R_s\) are the rotational speed and the radius of the secondary pulley respectively. \(v_p\) and \(v_s\) are the belt speeds of the primary and the secondary pulleys.

Using Eqn 5, Eqn 6 and the belt speed input from [14], we can calculate the slip of the variator as follows:

\[V_p = 1 - \left(\frac{v_p}{v_p}\right)\] \hspace{1cm} (7)

\[V_s = 1 - \left(\frac{v_s}{v_s}\right)\] \hspace{1cm} (8)

Where, \(V_p\) is the variator slip in primary pulley and \(V_s\) is the variator slip in the secondary pulley. The belt slip can also be calculated by using Eqn 3 and Eqn 4:

\[V = 1 - \frac{R_s}{R_g}\] \hspace{1cm} (9)
4. Variator Model:

The variator is the most important component of the CVT. Housing the two pulleys, the belt and the input and output shafts, the variator controls the gear ratio and the percentage power transmitted to the drivetrain of the vehicle [10]. Before designing the CVT block, it is imperative to understand the basic mechanical properties of a variator, as the functioning of the same determines the performance of the CVT system as a whole.

In this section, we fixate on the frictional force model used, the clamping forces experienced by the variator and the traction experienced between the belt and the pulley. Based on all these parameters, the torque mechanism of the system is also studied upon. Study of these parameters is necessary as it helps in looking into the direct relation of the belt slip to these characteristics and how it affects the transmission efficiency of the system.

4.1 Viscous Friction Model

Friction is an important parameter when it comes to modelling the characteristics of a CVT. The model developed consists of a Viscous Friction Model [6]. The advantages of this type of friction model is that it combines the effects of both Static Friction, as well as the damping that is experienced by the system.

The frictional force experienced depends on

\[ F_w = |v_s| \ast \min(C_o, |v_s|, \mu_c \ast F_n) \]  

\( F_w \) is the friction force experienced by the system, \( V_s \) is the sliding velocity of the belt, \( C_o \) is defined as the viscous damping constant, \( \mu_c \) is the friction coefficient and \( F_n \) is the normal force acting on the block. Depending on the sliding velocity, \( C_o \) or \( \mu_c \) acts as the viscous damping constant for the system (\( C_o \) for low sliding velocities and \( \mu_c \) for high sliding velocities). The friction block is as shown in the figure.

Fig 2: (a) The moving block of the system. (b)Viscous Friction Model. Adopted from Efficiency optimization of the push-belt CVT by variator slip control (p. 19, 21) by B. Bonsen, Copyright 2006 by TU/e.
4.2 Clamping Forces

The clamping forces determine how much slip is experienced by the belt [10]. The primary and the secondary pulleys are made of two sheaves on either side of the pulley [6]. It is the motion of the sheaves that controls the working of the CVT system as a whole [1] [6]. One sheave is fixed, while the other is a movable sheave. The fixed sheave keeps the CVT system in position while the moveable sheave changes the pitch radius of the CVT belt, which in turn changes the gear ratio of the system [1]. The distance moved by the sheaves depends on the clamping forces it experiences.

To improve the safety and the longevity of the variator, high clamping forces are used. This causes higher losses in the CVT, thus reducing the efficiency of the belt and the CVT. Clamping forces also have a direct impact on the slip experienced by the system [10] [14].

The tension acting on the belt under equilibrium conditions (as shown in Fig 3) is as below

\[ S = 2Rq_t - 2Rq_w \sin \gamma \cos \beta \]  

(11)

Here \( q_t \) is the radial component of the normal force acting on the pulley, while \( q_w \) is the friction force acting on the system as measured using equation (10). Tension on the primary pulley and secondary pulley can be determined by taking the radius value accordingly, while the sum of both determines the tension experienced by the belt. \( \gamma \) is the angle in the radial direction, while \( \beta \) is the groove angle of the pulley.

Taking the rotational components of \( q_t \),

\[ q_t = q_n \sin \beta \]  

(Tangential direction)

(12)

Figure 3: The forces acting on the belt model. 4(a) shows the side view, while 4(b) shows the front view of a single groove of the belt. The belt is a continuous belt model. Adopted from Efficiency optimization of the push-belt CVT by variator slip control (p. 23) by B. Bonsen, Copyright 2006 by TU/e.

Calculating the tension acting on the belt is necessary to determine the torque transmitted by the system from the primary side to the secondary side of the system. As stated in [CVT Clamping forces],

\[ T_p = (S_1 - S_2) \times R_p \]  

(13)
The torque transmitted on the primary side of the pulley is $T_p$. Similarly $T_s$, can be measured. The maximum permissible torque from the system can be calculated by taking the clamping force acting on the pulley, the friction coefficient and the running radius of the pulley. Thus

$$T_m = S_1 \left( \frac{\mu}{e^{\sin \beta}} - 1 \right) * R_p$$  \hspace{1cm} (14)

This is the maximum permissible torque in the belt pulley system that is modelled. The torque ratio is defined as follows

$$\tau = \frac{T_p}{T_m}$$  \hspace{1cm} (15)

The clamping/pulley forces experienced by the primary and the secondary pulleys can be determined by taking the integral of the normal and the frictional forces acting on the pulley. The forces acting on the sheaves of the pulley can be determined using

$$F_{clamp}(\theta) \int_0^\theta (q_n \cos \gamma + q_w \sin \gamma \cos \beta) d\theta$$  \hspace{1cm} (16)

$q_n$ is the normal force experienced by the sheave. The frictional force acting on the sheave is denoted by $q_w$. $\gamma$ and $\beta$ can be taken from Figure 4. The primary and secondary pulley clamping forces can be determined by the above equations separately.

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Figure 4: Forces acting on the pulley sheave. Adopted from *Efficiency optimization of the push-belt CVT by variator slip control* (p. 26) by B. Bonsen, Copyright 2006 by TU/e.

The primary and secondary pulley torques depends on the frictional forces acting on the pulleys. Taking the friction forces into account, the equations change to as below:

$$T_p = \int_0^\theta (q_w \cos \gamma) * R_p d\theta_p$$  \hspace{1cm} (17)
\[ T_s = \int_0^{\theta_s} (q_w \cos \gamma) * R_s d\theta_s \] (18)

From Figure 4, it can be seen that the clamping forces acts directly on the sheaves of the pulley. Higher the clamping forces, higher are the losses, experienced by the variator. While the belt and pulleys are held in position, the efficiency of the CVT reduces. Decreasing the clamping forces causes the belt and the pulleys to move axially from its position. This causes slip in the belt. Hence controlling the clamping forces is a necessary parameter to control the rate of change of ratio with respect to the slip experienced by the system.

The clamping force ratio is defined as follows:

\[ \varphi = \frac{F_p}{F_s} \] (19)

This condition is only applicable when \( I > 0 \). The above ratio can help in controlling the rate of change of ratio change, thus controlling the change in the radii of the pulleys and in turn the CVT ratio, the slip experienced and the torque measured in the variator.

5. Tractive Effort of the variator

One outcome of the system that is directly related to change in slip is the traction efficiency of the variator. The tractive curve helps us to find the relation between the torque transmitted and the slip generated in the belt. As the maximum input torque (\( T_p \)) is related to the clamping forces (section 4), the tractive coefficient equation changes to the below

\[ \mu_t = \frac{T_p \cos \beta}{2F_s R_s} \] (20)

The traction coefficient is directly proportional to the change in the belt slip as measured from section 2. Traction in the belt is defined as the friction coefficient along the wrapped arc of the belt on the pulley. The traction increases with an increase in the slip until the cut-off value for the same is reached. Hence the relative study between the two factors is also looked at.

6. Experimentation

The below set of assumptions were made while developing the CVT model.

- Friction force model used was the Viscous Friction Model. It provided the right balance between the static and the rolling friction acting on the CVT block.
- V-belt is the most commonly used belt in CVTs. The model is developed on the same principle.
- The belt model has very little bending stiffness. Thus the bending moments are neglected.
- The pulley is made of blocks and bands. The tension force is assumed to be distributed evenly among all bands.
- The belt is assumed to run in a circular path. The force model changes accordingly.
- Inertial and Centrifugal forces are not taken into consideration.
- Friction coefficient is kept constant.

6.1 Model Configuration

The SIMULINK model developed is as in figure. The model consists of subsystems that mimic the functioning of the CVT, along with providing outputs to determine the transmission efficiency, the belt
slip percentages, the torque loss ratio, the clamping force ratio and the frictional forces acting on the system. The inputs are taken from [4, 10, and 12]. The torque map is taken from [10], the CVT ratio values is taken from [4] as most CVTs working range while the angular speeds of the primary and secondary pulley is taken from [12]. The inputs chosen helped record accurate and stable results in terms of the slip experienced by the CVT system. The angular velocity of the primary pulley is a constant of 104.72 rad/s. The secondary pulley velocity varies between 74.35 rad/s to 234.54 rad/s. The secondary pulley angular speed is kept varying because the clamping forces experienced at the secondary pulley is always higher than the primary pulley [12]. The maximum input torque provided to the system is 174.10 Nm [10]. This torque helps in determining the maximum friction force experienced by the model. The CVT ratios are maintained between 2.33 and 0.41 [4]. Using this, the maximum and minimum vehicle velocities are also determined. This data is fed to the model in the form of 2D look up tables [11] which helps in finding the correlation between the vehicle speed, the engine torque and the radii of the primary and secondary pulleys.

Figure 5: CVT Model as developed using Simulink

The radii varies between 0.7m and 0.3m. The model is run for 81 seconds and a fixed step solver configuration is used. As the time step if fixed, the Runge-Kutta method is used to solve the ODEs in the system model. Once the model is run, the results are recorded.

A control strategy [16] is used to control the clamping forces experienced by the CVT variator. Two methods can be utilized to control the clamping forces and the frictional forces acting on the system. The first method is by controlling the torque inputs [10] to the force inducing system by adding a safety factor. The other method is by controlling the hydraulic forces [1] acting on the vehicle. Only the first method is used in this model. Both the theoretical slip, the actual slip and transmission efficiency of the system varies accordingly in each scenario. The pulley speeds are varying in the time scale when the actual belt slip is calculated. All the results will be discussed in the coming sections.
7. Results and Discussions:

7.1 Theoretical Belt Slip

The first result to be discussed is the theoretical belt slip or the macro slip. As discussed in section 3, the theoretical slip experienced by the belt at the primary and the secondary positions is dependent on the belt speed, and the angular velocity of the pulley. Two cases are taken for the results. Figure shows the two results obtained for the different belt speeds and slip experienced by the variator pulleys.

![Figure 6](image.png)

Figure 6: (a) When $W_s = 74.35 \text{ rad/s}$ to $104.23 \text{ rad/s}$, $R_g = 1.98$ to $0.87$, Frictional force = 8kN, static friction = 0.37, rolling friction = 0.29 (b) Slip experienced by the primary and secondary pulley.

From the figure 6(a), it can be seen that the belt speed at the primary and the secondary end of the pulley is between 0.65 m/s to 0.95 m/s. Figure 6(b) shows the slip experienced by the variator. The red line indicates the speed and the slip at the primary end of the variator, while the blue line indicates the same at the secondary end of the variator. The slip is seen to vary between 5% and 9%. The slip varies and reduces at the primary end, as the belt speed decrease. This is because the belt doesn’t experience very high axial forces, hence the slip decreases. Towards the end of the cycle, the slip starts to increase again, as the speed and the axial forces at the input end of the variator increases. The slip is constantly increasing at the output end of the pulley. This is because, with increasing angular speeds of rotation, the slip increases, hence the clamping forces increases and thus the secondary belt speeds of the pulley also increases.

7.2 Torque Ratio vs Clamping Force Ratio

The second parameter that is taken into effect when studying the effect of micro slip is the correlation between the torque ratio and the clamping force ratio. This in turn can help in understanding the effect of belt torque in the model. The results are generated for a cycle of change in the gear ratios and the vehicle speeds. While an experimental case was run, this system can be furthered to run drive cycles. From the vehicle data provided to the model, the following output was obtained with respect to time.
As can be seen from the graph, the clamping force ratio and the torque ratios are directly proportional to each other. The clamping force ratio has a maximum value of 1.23 while the torque ratio varies from 0.3 to -0.15. At regions where the torque ratio is negative, the gear ratio is seen to be less than 1, which is the overdrive conditions. This in turn affects the slip experienced by the variator.

7.3: Effect of slip due to change in gear ratios

Case 1: For constantly decreasing gear ratios:

It can be seen that the slip and transmission efficiency of the system is inversely proportional to each other. At 15% slip, the power transmission that occurs is at 38%. As the slip decreases, the transmission efficiency increases. Towards the end of the run cycle, the transmission efficiency is at 86%, while the belt slip is around 3% of the original value.

Case 2: For a varying gear ratio with and without safety factor
Fig 9: (a) when the varying gear ratios are used, (b) after the safety factor is added to the model

It is again proved that the transmission efficiency of the model is inversely proportional to the slip experienced by the variator belt. The slip recorded in Fig 9(a) is between 2% to 10%. While 88% efficiency is obtained at many instances when the slip is around 2%, the overall efficiency of the system varies drastically. This is because the clamping forces become higher to keep the variator in position at stages when the slip is 8% and higher. Thus the efficiency falls to below 70% in these cases.

In Fig 9(b), after adding the safety factor into consideration, it can be seen that the slip is controlled between 3% and 5%. This is done by controlling the input torque entering the system, and thus reducing the effect of the clamping forces. So, when the slip is at this range, it can be seen that the transmission efficiency lies between 80% and 90%. Even though, it doesn’t reach the maximum value as in case (a), when adding this factor to the system, a consistent transfer of power is obtained. Thus a smoother transition in speed and torque can be obtained when the slip is low and is controlled.

8. Conclusions:

- The theoretical slip depends on the belt speed, the pulley position and the gear ratios experienced by the CVT. The slip increases with increase in the belt speeds, as the forces experienced by the system increases.
- Slip can be considered in two factors, the macro slip and micro slip. While macro slip is largely dependent on the speed of the belt, the micro slip is dependent on the clamping forces of the pulleys and the effect of torque on the variator.
- The clamping force ratio and the torque ratio are directly proportional to each other.
- The micro slip is inversely proportional to the rate of power transmitted to the drive shaft, greater the slip, lesser is the efficiency.
- The slip can be controlled by controlling the input torque provided to the variator. This is done by adding a safety/cut off factor to the system.
- Controlling the slip can control the power transmission occurring. A more uniform power transmission is possible.
- Further improvements can be done to the control technique. While not part of the paper work, controlling the hydraulic forces acting on the variator can help control the clamping forces and the slip of the system.
- Power transmission efficiency can also be improved by looking into controlling the macro slip experienced by the belt.
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