Personalized Federated Learning with Exact Stochastic Gradient Descent

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Abstract

In Federated Learning (FL), datasets across clients tend to be heterogeneous or personalized, and this poses challenges to the convergence of standard FL schemes that do not account for personalization. To address this, we present a new approach for personalized FL that achieves exact stochastic gradient descent (SGD) minimization. We start from the FedPer (Arivazhagan et al., 2019) neural network (NN) architecture for personalization, whereby the NN has two types of layers: the first ones are the common layers across clients, while the few final ones are client-specific and are needed for personalization. We propose a novel SGD-type scheme where, at each optimization round, randomly selected clients perform gradient-descent updates over their client-specific weights towards optimizing the loss function on their own datasets, without updating the common weights. At the final update, each client computes the joint gradient over both client-specific and common weights and returns the gradient of common parameters to the server. This allows to perform an exact and unbiased SGD step over the full set of parameters in a distributed manner, i.e. the updates of the personalized parameters are performed by the clients and those of the common ones by the server. Our method is superior to FedAvg and FedPer baselines in multi-class classification benchmarks such as Omniglot, CIFAR-10, MNIST, Fashion-MNIST, and EMNIST and has much lower computational complexity per round.

1. Introduction

Federated learning (FL) aims at training global Machine Learning (ML) models out of distributed datasets in different clients (Shen et al., 2020) so as to overcome privacy concerns. Chronologically, the first FL algorithm is FedAvg (McMahan et al., 2017). In FedAvg, at each round, the server selects a subset of clients and sends them the current model parameters. The clients update model parameters locally by performing a number $E$ of local gradient update steps towards optimizing their loss function and return the locally optimized parameters to the server, which averages them and sends them back to the clients. For $E = 1$, FedAvg is called Federated Stochastic Gradient Descent (Jiang et al., 2019). FedAvg notoriously suffers from data heterogeneity that arises when each client has to solve its own personalized task, i.e. when the clients’ datasets are drawn from different distributions. In such cases, a shared global model and an naive aggregation of its parameters could lead to convergence to a stationary point which is far from optimal (Wang et al., 2020; Li et al., 2020d). Several extensions of FedAvg have been proposed, see e.g. Li et al. (2020c;b); Wang et al. (2020); Reddi et al. (2021); Liu et al. (2019); Yu et al. (2019), that try to improve the behaviour of the algorithm in heterogeneity settings. While such methods can improve convergence still try to learn a fully shared/global ML model across heterogeneous clients. From modelling perspective, this might not be the best choice since it ignores client personalization.

Personalized Federated Learning arises often in real-life applications, such as next word prediction (Hard et al., 2019), emoji prediction (Ramaswamy et al., 2019; Lee et al., 2021), health monitoring (Wu et al., 2020) and personalized healthcare via wearable devices (Chen et al., 2021). Data heterogeneity is a common challenge in (personalized) FL algorithms (Li et al., 2020c;a). If the local datasets among clients are drawn from different data distributions (Konečný et al., 2016), then the prevalent way in which FL is done is not suited to produce a global model that will perform well at the specific ML task of each client. As a result, the training process can be slow or even diverge. Although data heterogeneity exists among clients, we allow clients to share common weights because each client cannot produce a good and robust model on their own, due to insufficient amount of data (Konečný et al., 2016; Arivazhagan et al., 2019).

In this work, in order to address data heterogeneity in personalized FL, we consider the classic feature transfer or multi-task learning model that uses parameter sharing in deep forward neural networks (Caruana, 1997; Ruder, 2017).
This assumes that along with the global or shared model parameters, obtained by a set of backbone neural network layers, for each client there exists an additional set of client-specific output layers with client-specific parameters. Each such parameter set is exclusive to the client, and its purpose is to allow for more flexible modelling of the client-specific data distribution or task. Such a multi-task setting was also used by Arivazhagan et al. (2019); Singhal et al. (2021), from which our work differs significantly in the way we train the FL system; see related work in Section 2.

We propose a new approach for personalized FL, which we call Personalized Federated Learning with Exact Gradient-based Optimization (PFLEGO); see Figure 1. Its main asset is that it achieves exact stochastic gradient descent (SGD) (Robbins & Monro, 1951; Bottou, 2010) minimization of the training loss function, which is the exact equivalent of training with all data concentrated in one place. Our starting point is the FedPer neural network (NN) architecture for personalization (Arivazhagan et al., 2019) (see also Singhal et al. (2021)), whereby the NN has two types of layers: the first ones are common layers, while the few last ones are the client-specific ones used for personalization.

At each optimization round, our algorithm performs the following steps: (a) The server sends to a randomly selected subset of clients the updated weights corresponding to common layers; (b) Given the common weights, each client performs a number of local gradient descent updates of its client-specific weights on its own dataset towards minimizing the loss function, without updating the common weights; (c) Contrary to other methods that we compare against i.e. FedPer, FedAvg, in our approach each client at their final optimization step computes the joint gradient over both the client-specific and the common weights and sends to the server the gradient of the common weights, rather than the raw weights themselves; (d) Finally the server aggregates the gradients of common weights. The sequence of the last two (c)-(d) steps turns out to be the precise equivalent of an SGD update over the full set of parameters that is performed by the clients and the server in a distributed manner. The process continues until convergence.

We validate the superiority of our method over FedAvg and FedPer on benchmarks in multi-class classification such as MNIST, CIFAR-10, Fashion-MNIST, EMNIST and Omniglot. The experimental study shows that our algorithm leads to lower training loss and thus much more effective learning especially in cases where personalized models need to be trained for each client. Our algorithm has much lower, $O(1)$ computational complexity per round compared to $O(\tau)$ of baselines, thus leading to less energy consumption.

The paper is organized as follows. In section 2, we present the related work and compare our proposed approach against prior work; in section 3 we present our approach and present formal evidence to back up the claims about convergence; in section 4 we present experimental evaluation results, and in section 5 we conclude the paper.

2. Related Work

**Personalized FL by fine tuning a global model.** One approach to deal with personalization in FL is to allow clients to fine tune the shared global model using local adaptation (Wang et al., 2019; Yu et al., 2021), or techniques inspired by meta-learning (Chen et al., 2019; Dinh et al., 2022;Fallah et al., 2020; Jiang et al., 2019). These methods differ significantly from ours since they require communicating the full set of parameters of a shared global model (i.e. there is no client-specific model part) between the server and the clients. Note also that fine tuning can be expensive since it requires the individual clients to adapt the full parameter vector of a deep NN.

**Personalized FL by feature transfer.** Our approach mostly relates to methods that achieve personalization in FL by using a multi-task of feature transfer model, similarly to traditional non-distributed multi-task architectures (Caruana, 1997; Ruder, 2017). This aims to learn a NN whose parameters consist of a common part (also called global or shared) for all clients, and a client-specific part (also called personalized). This approach was followed by Arivazhagan et al. (2019). However, the optimization algorithm in Arivazhagan et al. (2019) differs from our method, as it is based on the standard FedAvg scheme. Specifically, the clients update both the global and the personalized parameters by executing joint gradient descent steps. Then, each client sends back to the server the locally updated global parameters and the server updates the global parameters through averaging. Their method reduces to standard FedAvg when there are no client-specific parameters. In contrast, in our method the clients return gradients over the global parameters.
eters, in a way that our FL algorithm performs exact SGD steps. In another related work FedRecon (Singhal et al., 2021), similarly to FedPer (Arivazhagan et al., 2019), a set of global and client-specific parameters are learned, but at each round their optimization algorithm does not update simultaneously the global and personalized parameters, and therefore their method can be considered as performing block coordinate optimization. In contrast, our distributed optimization algorithm incorporates exact SGD steps where the full set of model parameters are simultaneously updated. This latter property means that our method enjoys theoretical convergence guarantees similar to SGD schemes (Robbins & Monro, 1951; Bottou, 2010).

**FL with gradient return.** Finally, there exist non-personalized FL algorithms optimizing a set of global parameters, where gradients are returned to the server (Yao et al., 2020; Ren et al., 2020). In Yao et al. (2020), each client performs many training steps over local copies of the global parameters and returns to the server the final gradient (after the final iteration) of these parameters, which then aggregates them by performing a gradient step. However, convergence is guaranteed only in the special case that the client performs a single iteration so that their algorithm reduces to SGD optimization. The work by Ren et al. (2020) tries to accelerate training by optimizing batch size. Specifically, clients sample a batch over their private datasets and perform a single gradient step to update their local copies of global parameters, and then they return the gradient to the server. This work differs from our method, since it does not deal with personalized settings. In our scheme, the clients may perform multiple gradient steps over the client-specific parameters before returning the gradient of global parameters to the server.

### 3. Proposed Framework

#### 3.1. Personalized FL Setting

We consider a supervised FL setting in which there is a server and $I$ clients. Each client has a locally stored dataset $D_i = (X_i, Y_i)$, where $X_i = \{x_{i,j}\}_{j=1}^{N_i}$ are the input data samples (e.g. images) and $Y_i = \{y_{i,j}\}_{j=1}^{N_i}$ are the corresponding target outputs (e.g. class labels). The objective of FL is to optimize a shared or global model, together with personalized or client-specific parameters, by utilising all datasets locally stored at clients. As a shared backbone model we assume a deep neural network that consists of a number of common layers with overall parameters $\theta$. The number of outputs of the common layers, i.e. the size of the feature vector, is $M$. More precisely, the shared model receives an input $x$ and constructs in its final output a representation or feature vector $\phi(x; \theta) \in \mathbb{R}^M$. Each client has a copy of the same network architecture corresponding to this shared representation. Each client $i$ has an additional set of personalized layers attached as a head to $\phi(x; \theta)$. For simplicity, we assume that the personalized layers consist of a single linear output layer with weights $W_i$; see Figure 1.

#### 3.1.1. Training Objective for Classification

The learning objective is to train the neural network model by adapting the global parameters $\theta$ and the personalized parameters $\{W_i\}_{i=1}^{I}$. The full set of parameters is denoted by $\psi = \{\theta, W_1, \ldots, W_I\}$. Learning requires the minimization of the following training loss function that aggregates all client datasets:

$$
L(\psi) = \sum_{i=1}^{I} \alpha_i \ell_i(W_i, \theta),
$$

where each scalar $\alpha_i = \frac{N_i}{\sum_{j=1}^{I} N_j}$ quantifies the data proportionality of the different clients, and $\ell_i(W_i, \theta)$ is the client-specific loss,

$$
\ell_i(W_i, \theta) = \frac{1}{N_i} \sum_{j=1}^{N_i} \ell(y_{i,j}, x_{i,j}; W_i, \theta).
$$

The form of each data-individual loss $\ell(y_{i,j}, x_{i,j}; W_i, \theta)$ will depend on the application, e.g. on whether the task is a regression or a classification one. The case of multi-class classification, that we consider in our experiments, is detailed below while other cases can be dealt with similarly.

**Multi-class classification.** A standard task that each client needs to solve is multi-class classification, where the input $x$ is assigned a class label $y \in \{1, \ldots, K_i\}$. In our personalized setting, each client $i$ can tackle a separate classification problem with $K_i$ classes, where classes across clients can be mutually exclusive or partially overlap. The set of personalized weights $W_i$ becomes a $K_i \times M$ matrix that allows to compute the logits in the standard cross entropy loss,

$$
\ell(y_{i,j}, x_{i,j}; W_i, \theta) = -\log \{\Pr(y_{i,j}|x_{i,j}; W_i, \theta)\}.
$$

The class probability is modeled by the Softmax function, i.e. $\Pr(y_{i,j}|x_{i,j}; W_i, \theta) = \frac{e^{\alpha_{y_{i,j}}}}{\sum_{k=1}^{K_i} e^{\alpha_k}}$ where the $K_i$-dimensional vector of logits is $\alpha = W_i \times \phi(x_{i,j}; \theta)$ and $\phi(x_{i,j}; \theta)$ is a $M \times 1$ vector.

#### 3.2. The Proposed Algorithm

The objective of the personalized FL setting above is to minimize the global training loss in (1) over the full set of parameters $\psi$. To this end, we propose a distributed optimization algorithm that incorporates exact Stochastic Gradient Descent (SGD) steps over $\psi$. The stochasticity arises due to
a random client-participation or selection process defined in the sequel, in Section 3.2.1. Without stochasticity, i.e. if all clients participate at every round, these previous steps become exact gradient descent (GD) steps. This means that the proposed algorithm defined in Section 3.2.2 converges similarly to standard GD or SGD methods; see Section 3.3.

3.2.1. CLIENT PARTICIPATION PROCESS

We assume that the optimization of the global loss in (1) is performed in different rounds, where each round involves a communication of the server with some of the clients. More precisely, we assume that at the beginning of each optimization round $t$, a subset of clients $I_t \subset \{1, \ldots, I\}$ is selected uniformly at random to participate. For instance, two sensible options are: (i) the number of clients $r_t := |I_t|$ follows a Binomial distribution $B(I, \rho)$, i.e. each client participates independently with probability $\rho$, or (ii) always a fixed number $0 < r \leq I$ of clients is uniformly selected, i.e. $|I_t| = r$ for any $t$. For both cases an arbitrary client $i \in \{1, \ldots, I\}$ participates in round $t$ with probability

$$\Pr(i \in I_t) = \frac{r}{I},$$

where for case (i) $r = I\rho$ can be a float number while for case (ii), $r$ is strictly an integer.

3.2.2. CLIENT AND SERVER UPDATES

**Client side.** Having first selected the subset of clients $I_t$ to participate in round $t$, the server sends the global model parameters $\theta$ to these clients. Then, each client $i \in I_t$ trains locally the task specific parameters $W_i$ by performing a total number of $\tau$ gradient descent steps. For the first $\tau - 1$ steps, the global parameter $\theta$ is “ignored” (it remains fixed to the value sent by the server), and only the gradient $\nabla_W \ell_i(W_i, \theta)$ over the task parameters $W_i$ is computed. The gradient steps of these $\tau - 1$ steps can have the form

$$W_i \leftarrow W_i - \beta \nabla_W \ell_i(W_i, \theta),$$

where $\beta$ is some learning rate value. In fact, these $\tau - 1$ updates could be replaced by any other optimization procedure, as long as the final loss value $\ell(W_t, \theta)$, i.e. after these $\tau - 1$ steps, is smaller or equal to the corresponding initial value. In contrast, for the final iteration $\tau$, the client simultaneously computes the joint gradient $(\nabla_W \ell_i, \nabla_\theta \ell_i)$ of both $W_i$ and the shared parameters $\theta$, and it performs the final ($\tau$-th) gradient step for $W_i$ using the rule

$$W_i \leftarrow W_i - \rho_t \nabla_W \ell_i(W_i, \theta),$$

where $\rho_t$ is the learning rate at round $t$, and the multiplicative scalar $\frac{1}{\Pr(i \in I_t)} = \frac{1}{r}$ ensures unbiasedness of the full gradient update over all parameters $\psi$; see Section 3.3.

**Algorithm 1 PFLGO**

**Input:** $T$ rounds, $\tau$ local gradient updates, $I$ clients, $r \ll I$ (average) sampled clients per round, $N_i$ data samples at the $i$-th client

**Server:**

- Initialize global parameters $\theta_i$
- for round $t = 1, 2, \ldots, T$
  - $I_t \leftarrow$ (Select a random subset of clients)
  - Receive $g_i$ from each client $i \in I_t$
  - Aggregate: $\theta_{t+1} \leftarrow \theta_t - \rho_t \frac{1}{r} \sum_{i \in I_t} g_i$
- end for

**Client\_Update($\theta_i$):**

- Initialize $W_i$, the first time client $i$ is visited
- for local gradient update $= 1, 2, \ldots, \tau - 1$
  - $W_i \leftarrow W_i - \beta \nabla_W \ell_i(W_i, \theta_i)$
- end for

**Compute joint grad $(\nabla_W \ell_i(W_i, \theta), \nabla_\theta \ell_i(W_i, \theta))$**

**Return $g_i := \nabla_\theta \ell_i(W_i, \theta)$ to server**

**Server side.** The server gathers all gradients from the participating clients, and then it performs a gradient update to the parameters $\theta$ by taking into account also the data proportionality weight of each client. Specifically, the update it performs takes the form

$$\theta \leftarrow \theta - \rho_t \frac{1}{r} \sum_{i \in I_t} \alpha_i \nabla_\theta \ell_i(W_i, \theta),$$

where again the term $\frac{1}{r}$ is included to ensure unbiasedness as detailed next. The whole optimization procedure across rounds is described by Algorithm 1.

3.3. Exact Stochastic Gradient Descent Optimization

An important property of the proposed algorithm is that the final iteration $\tau$ over $W_i$’s at the selected set of clients $I_t$, combined with the update over the global parameter $\theta$ at the server results in an unbiased SGD step over all parameters $\psi = \{\theta, W_1, \ldots, W_f\}$. To prove this rigorously, we introduce the stochastic gradient vector $\nabla^s_\psi \mathcal{L} = \{\nabla_\theta \mathcal{L}, \nabla_W \ell_1, \ldots, \nabla_W \ell_f \}$ where we used the symbol $s$ in $\nabla^s$ to indicate that these gradient vectors are stochastic. For any client $i = 1, \ldots, f$ the vector $\nabla^s_W \mathcal{L}$ is defined as

$$\nabla^s_W \mathcal{L} = 1(i \in I_t) \frac{1}{r} \nabla_W \mathcal{L}(\psi) = 1(i \in I_t) \frac{1}{r} \alpha_i \nabla_W \ell_i(W_i, \theta).$$

Here, $1(i \in I_t)$ is an indicator function that equals one if $i \in I_t$, i.e. if client $i$ was selected in round $t$, and zero otherwise. We also used the fact that $\nabla_W \mathcal{L}(\psi) =$
\( \alpha_i \nabla_W \ell_i(W_i, \theta) \). Note that for selected clients in set \( I_t \), the corresponding vector, \( \nabla^s_W \mathcal{L} = \frac{1}{\tau} \sum_{i=1}^{\tau} \alpha_i \nabla_W \ell_i(W_i, \theta) \) is precisely the gradient used in the client update in (4), while for the remaining clients \( i \notin I_t \), \( \nabla^s_W \mathcal{L} = 0 \). The stochastic gradient \( \nabla^s \mathcal{L} \) is defined as

\[
\nabla^s \mathcal{L} = \frac{1}{\tau} \sum_{i=1}^{\tau} 1(i \in I_t) \alpha_i \nabla_W \ell_i(W_i, \theta). \tag{7}
\]

We can see that in Algorithm 1, the final client update together with the server update can be compactly written as the following gradient update over all parameters \( \psi \):

\[
\psi \leftarrow \psi - \rho_t \nabla^s \mathcal{L}.
\]

This now is a proper SGD step as long as the stochastic gradient \( \nabla^s \mathcal{L} \) is unbiased, as we state next.

**Proposition 1.** The stochastic gradient \( \nabla^s \mathcal{L} \) is unbiased, i.e. it holds \( \mathbb{E}[\nabla^s \mathcal{L}] = \nabla \mathcal{L}(\psi) \) where \( \nabla \mathcal{L}(\psi) \) denotes the exact gradient and the expectation is taken under the client participation process (either case i or ii) defined in Section 3.2.1.

**Proof.** By taking the expectation \( \mathbb{E}[\nabla^s \mathcal{L}] \) for any \( i \), and the expectation \( \mathbb{E}[\nabla^s \mathcal{L}] \), the indicator function \( 1(i \in I_t) \) is replaced by its expected value \( \mathbb{P}(i \in I_t) = \frac{\tau}{T} \) (this value is the same for cases i and ii), which gives the exact gradient. \( \square \)

As a consequence of Proposition 1, when the number of client updates is \( \tau = 1 \), Algorithm 1 is precisely an SGD algorithm with the standard convergence guarantees, as long as \( \sum_{t=1}^{\infty} \rho_t = \infty \) and \( \sum_{t=1}^{\infty} \rho_t^2 < \infty \) (Robbins & Monro, 1951; Bottou, 2010). When the selected clients at each round perform more than one gradient step, i.e. \( \tau > 1 \), convergence can speed up since the first \( \tau - 1 \) GD steps in these clients can result in a systematic/deterministic improvement of the global loss in (1), i.e. \( \mathcal{L}(\psi_{\text{after } \tau - 1 \text{ steps}}) \leq \mathcal{L}(\psi_{\text{init}}) \). Indeed, in practice we observe that convergence gets faster as \( \tau \) increases.

### 3.4. Computational Complexity

Since during the first \( \tau - 1 \) client updates the global parameters \( \theta \) are fixed we can pass the data from the NN only twice for any value of \( \tau \). At the beginning of each round we pass once the data from the NN, store all feature vectors and then carry out \( \tau - 1 \) GD steps to update only the client-specific parameters. For the final \( \tau \) iteration we need to pass the data a second time from the NN to compute the joint gradient. This is a great computational advantage of PFLEGO versus FedAvg and FedPer, since PFLEGO roughly runs \( \frac{\tau}{2} \) times faster. Concretely, given that the complexity per round is dominated by the NN evaluations, then PFLEGO is \( O(1) \) while others are \( O(\tau) \), consequently the clients consume much less energy per round.

### 4. Experiments

To validate the performance benefits of our proposed PFLEGO algorithm, we compare it against FedPer (Ariavazhagan et al., 2019) and FedAvg (McMahan et al., 2017).

#### 4.1. Datasets and Model Architectures

We experiment with the Omniglot, CIFAR-10, MNIST, Fashion-MNIST and EMNIST datasets and corresponding model architectures, detailed below.

##### 4.1.1. Datasets description

**Omniglot.** This dataset was introduced by (Lake et al., 2015) and consists of 1623 105x105 handwritten characters from 50 different alphabets and 20 samples per handwritten character. Omniglot can be a natural choice for personalized learning due to its small number of samples per character and large number of different handwritten characters per alphabet. Each alphabet can be considered as a classification problem with a certain number of classes, e.g. the English alphabet has 24 classes. We use 4 convolutional layers, each layer is followed by one max pooling layer, the architecture we use is the same as the one from (Finn et al., 2017).

**CIFAR-10.** It consists of 32x32 RGB images of 10 different classes of visual categories (Krizhevsky, 2009). We use the same architecture as in (Yao et al., 2020). It includes two convolutional layers of 64 filters each, and a kernel of size 5. Each layer is followed by a max pooling layer of size 3x3 with stride 2. The output of convolutional layers passes through two additional fully connected layers of size 384 and 192. The activation function for the convolutional and fully connected layers is ReLU.

**MNIST.** It consists of 28x28 handwritten grayscale images of single digits from classes 0 to 9. For MNIST, we use an MLP architecture that consists of one fully connected layer of 200 units with a ReLU activation function.

**Fashion-MNIST.** It is similar to MNIST but more challenging and consists of 28x28 grayscale images of 10 different classes of clothing. We use the same MLP architecture as the one in MNIST.

**EMNIST.** It extends the MNIST dataset with grayscale images of single digits from classes 0 to 9. For EMNIST, we use an MLP architecture that consists of one fully connected layer of 200 units with a ReLU activation function.

#### 4.2. Experimental Setup

For Omniglot, we assume that a single alphabet is stored in a different client. Due to the fact that each handwritten system of each alphabet is unique, there is no class label set overlap among the clients, which makes Omniglot the
hardest and highly personalized FL problem in our experiments. At each alphabet, data of each class are split into 75% for training and 25% for testing. Also standard data augmentation is used by including rotated image samples of multiples of 90° (Finn et al., 2017). The setup we follow is based on (Shamsian et al., 2021) that uses $I = 50$ clients, $\tau = 50$ inner steps per client, and $T = 5,000$ communication rounds, and each client has an unique alphabet. For the FedAvg algorithm, the final layer of the common weights is set to 55 outputs, which is equal to the maximum number of classes among all 50 alphabets.

For MNIST, Fashion-MNIST, EMNIST and CIFAR-10, we use $I = 100$ clients, $\tau = 50$ inner steps per client, and $T = 200$ communication rounds. MNIST, Fashion-MNIST and CIFAR-10 are well-balanced datasets and contain 10 classes. EMNIST is more challenging since it has 62 classes and varying number of examples per class. We simulate several FL scenarios by varying the amount of task-personalization among clients. This involves varying the degree of class label set overlap among clients, so that different clients can have different classes in their private datasets.

4.2.1. DEGREE OF PERSONALIZATION

For all datasets except Omniglot which is personalized by design, i.e. for MNIST, CIFAR-10, Fashion-Mnist and EMNIST, we artificially simulate personalized FL problems by varying the degree of personalization. This is quantified by the size $K$ of classes randomly assigned to each client from the total set of classes, so that the smaller the $K$ is, the higher the personalization is, since the probability of class overlap among clients reduces with smaller $K$ values. We consider three degrees of personalization. (i) “high-pers” where each client has $K = 2$ randomly chosen classes from the total set of $C$ classes ($C = 10$ for MNIST, CIFAR-10, Fashion-MNIST and $C = 62$ for EMNIST), (ii) “medium-pers” where $C/2$ classes are randomly assigned to each client, and (iii) “no-pers” where all clients have data points from all $C$ classes, i.e. all clients solve the same task.

Having specified the subset of classes per client, we distribute the data to the clients through the Round Robin (RR) algorithm (Yang et al., 2019), which distributes approximately equal-sized partitions of the data per class to each client; see Appendix A.2 for more details.

Table 1. Test accuracy for MNIST, CIFAR-10, EMNIST and Fashion-MNIST datasets on different degrees of personalization.

| Method / Deg of Pers. | MNIST | CIFAR-10 |
|-----------------------|-------|----------|
|                       | High-Pers | Medium-Pers | No-Pers | High-Pers | Medium-Pers | No-Pers |
| FedPer                | 97.88 ± 0.25 | 92.83 ± 0.46 | 87.23 ± 0.33 | 85.13 ± 1.08 | 61.01 ± 0.84 | 37.88 ± 0.59 |
| FedAvg                | 97.54 ± 0.25 | 92.83 ± 0.52 | 93.12 ± 0.24 | 85.18 ± 0.96 | 64.53 ± 0.75 | 61.33 ± 0.52 |
| PFLEGO                | 98.43 ± 0.21 | 94.22 ± 0.43 | 90.50 ± 0.34 | 87.81 ± 0.94 | 73.23 ± 0.62 | 50.27 ± 0.4 |

Table 2. Test accuracy for Omniglot dataset on high degree of personalization. The average is measured by averaging the last 10 global rounds of the first 1000 global rounds.

| Method | Test Accuracy |
|--------|---------------|
| FedPer | 68.02 ± 1.74  |
| FedAvg | 49.65 ± 1.74  |
| FedRecon | 74.06 ± 1.19 |
| PFLEGO | 74.56 ± 1.23  |

Optimizers and hyperparameter search. Despite the theoretical advantages of plain SGD, in practice when running our PFLEGO algorithm we found out that it is more effective for the server update over $\theta$ to apply the popular
Adam optimizer (Kingma & Ba, 2014), which (unlike simple SGD) uses parameter-wise learning rates and in practice accelerates convergence. Adam is appropriate to use for global parameter $\theta$ as it is straightforward for the server to store moving averages for the first and second moments of the gradients, as required by Adam (Kingma & Ba, 2014).

In contrast, for the client specific parameters, we use SGD. Also, for simplicity the clients apply gradient steps over their entire datasets, instead of sub-sampling subsets or minibatches, so that SGD reduces to gradient descent (GD).

Regarding hyperparameter values, we performed hyperparameter search for the base learning rate $\rho$ of Adam needed...
in server updates, and the learning rate $\beta$ of simple GD in client updates; see details in the Appendix A.3.

4.3. Results

We report training loss (given by (1)) and test accuracy for the compared methods. Training loss allows to visualize how fast each FL algorithm optimizes the model, i.e. how many rounds are required, while test accuracy quantifies predictive classification performance. Results are averages over all clients.

4.3.1. Discussion on the main results

Table 1 reports test accuracy scores for FedPer, FedAvg and our approach PFLEGO for all datasets, except Omniglot, and degrees of personalization (high/medium/no personalization). Table 2 reports the test accuracy for Omniglot. Each reported accuracy value and confidence interval is the mean of the corresponding values at the final 10 global rounds. We observe that our algorithm has significantly higher accuracy from the other baselines for high degree of personalization; see High-Pers columns in Table 1 and Omniglot results in Table 2. In the case of Medium-Pers, our algorithm has better performance than the baselines in all datasets except EMNIST. Finally, as expected, in the case of low personalization, FedAvg outperforms all methods.

To visualize optimization and learning speed, Figure 3 shows training loss and test accuracy with respect to the number of rounds for MNIST and for different degrees of personalization. The corresponding plots for Fashion-MNIST, EMNIST and CIFAR-10 are included in the Supplementary Material. Figure 2 plots the same quantities for the Omniglot dataset. These plots clearly indicate that PFLEGO can require fewer communication rounds to minimize the overall training loss, and it faster achieves high classification accuracy in the high-personalized regime; see e.g. Figure 2 for the Omniglot.

Further, for the most challenging Omniglot benchmark we also compare our approach with FedRecon (Singhal et al., 2021) which follows a stochastic block coordinate descent scheme, as opposed to our SGD approach. In Table 2 we include the test accuracy of FedRecon, from which we observe that our approach outperforms this baseline as well. See Appendix B.3 for more details.

4.3.2. Effect of client learning rate and participation rate $r$

Effect of learning rate. In Figure 4 we conduct an ablation to study the effect of the client learning rate $\beta$ for PFLEGO, and for various values of the server learning rate value $\rho$ (i.e. the base rate in Adam). We observe that as $\beta$ increases we systematically obtain faster convergence. This well aligns with the observations in section 3.3 where we note that overall convergence can improve as long as the first $\tau - 1$ GD steps in the clients result in a deterministic improvement of the global loss in (1), i.e. $\Delta L = L(\psi_{\text{after } \tau - 1 \text{ steps}}) - L(\psi_{\text{init}}) \leq 0$. In fact, the more $\Delta L \ll 0$ the faster the convergence, and essentially this is what we evidence in Figure 4. In practice, we could remove the need to specify $\beta$ in the GD update in (4) since the number of client parameters $W_i$ is rather small (e.g. the final weights in the NN), and therefore we could use an inexpensive second order Newton update. We leave the investigation for such more elaborate updates for future work.

Effect of participation rate $r$. Finally, we examine the effect of clients participation rate $r$ in each optimization round, i.e. the average percentage $r$ of clients participating in each round where a single server update is performed over $\theta$. We consider $r \in \{20, 40, 60, 100\}$%. We consider the CIFAR-10 dataset, with $T = 200$ rounds and $\tau = 50$ inner client GD steps per round. The client learning rate for all algorithms is $\beta = 0.001$ while for PFLEGO the learning rate of the server is $\rho = 0.001$. From Figure 5 we observe that for FedAvg and FedPer, the convergence speed does not vary with $r$, while for PFLEGO the optimization algorithm converges faster as $r$ increases.

5. Conclusion

We propose PFLEGO, an algorithm for personalized FL with the unique, and novel to the best of our knowledge, property that it achieves exact SGD minimization of the total training loss. We rely on a NN architecture that includes shared and client-specific layers. We show rigorously that PFLEGO performs exact SGD-based optimization, where the stochasticity arises due to randomness in client participation, and therefore it achieves faster convergence and lower training loss than state-of-the-art alternatives such as FedAvg and FedPer. Importantly, PFLEGO’s advantage arises in regimes where a high degree of personalization is needed.

In this work, the degree of personalization i.e. the overlap between the subsets of classes available to each client was assumed to be known a priori. If this were not known, a learning algorithm would need to be devised, which gradually learns the degree of personalization needed e.g through estimating similarity of tasks of different clients, so as to decide whether a personalization algorithm (e.g. PFLEGO) would be needed or not. In this work, we aimed at optimizing total training loss over clients. Another issue would be to elaborate on fairness aspects by ensuring similar loss across clients through a carefully selected objective function and subsequent decentralized FL process.
References

Arivazhagan, M. G., Aggarwal, V., Singh, A. K., and Choudhary, S. Federated learning with personalization layers, 2019.

Bottou, L. Large-scale machine learning with stochastic gradient descent. In *Proceedings of COMPSTAT*’2010, pp. 177–186. Springer, 2010.

Caruana, R. Multitask learning. *Mach. Learn.*, 28(1):41–75, July 1997. ISSN 0885-6125.

Chen, F., Luo, M., Dong, Z., Li, Z., and He, X. Federated meta-learning with fast convergence and efficient communication, 2019.

Chen, Y., Wang, J., Yu, C., Gao, W., and Qin, X. Fedhealth: A federated transfer learning framework for wearable healthcare, 2021.

Dinh, C. T., Tran, N. H., and Nguyen, T. D. Personalized federated learning with moreau envelopes, 2022.

Fallah, A., Mokhtari, A., and Ozdaglar, A. Personalized federated learning: A meta-learning approach, 2020.

Finn, C., Abbeel, P., and Levine, S. Model-agnostic meta-learning for fast adaptation of deep networks, 2017.

Hard, A., Rao, K., Mathews, R., Ramaswamy, S., Beaufays, F., Augenstein, S., Eichner, H., Kiddon, C., and Ramage, D. Federated learning for mobile keyboard prediction, 2019.

Jiang, Y., Konečný, J., Rush, K., and Kannan, S. Improving federated learning personalization via model agnostic meta learning, 2019.

Kingma, D. P. and Ba, J. Adam: A method for stochastic optimization, 2014. cite arxiv:1412.6980.

Konečný, J., McMahan, H. B., Ramage, D., and Richtárik, P. Federated optimization: Distributed machine learning for on-device intelligence, 2016.

Krizhevsky, A. Learning multiple layers of features from tiny images, technical report. 2009.

Lake, B. M., Salakhutdinov, R., and Tenenbaum, J. B. Human-level concept learning through probabilistic program induction. *Science*, 350:1332 – 1338, 2015.

Lee, S., Zheng, X., Hua, J., Vikalo, H., and Julien, C. Opportunistic federated learning: An exploration of egocentric collaboration for pervasive computing applications, 2021.

Li, T., Sahu, A. K., Talwalkar, A., and Smith, V. Federated learning: Challenges, methods, and future directions. *IEEE Signal Processing Magazine*, 37(3):50–60, May 2020a. ISSN 1558-0792.
Yang, H. H., Liu, Z., Quek, T. Q. S., and Poor, H. V. Scheduling policies for federated learning in wireless networks, 2019.

Yao, X., Huang, T., Zhang, R.-X., Li, R., and Sun, L. Federated learning with unbiased gradient aggregation and controllable meta updating, 2020.

Yu, H., Jin, R., and Yang, S. On the linear speedup analysis of communication efficient momentum sgd for distributed non-convex optimization, 2019.

Yu, T., Bagdasaryan, E., and Shmatikov, V. Salvaging federated learning by local adaptation, 2021.
A. Experimental settings

A.1. Neural network architectures

In Table 3 we present the architectures for MNIST, Fashion-MNIST, EMNIST, CIFAR-10 and Omniglot. We also provide an equivalent pictorial visualization of the used NN architectures; see Figure 6.

Table 3. Architecture details for all datasets. B denotes the number of data inputs to the neural network, A denotes the activation function

| Layer                      | Kernel details | Stride | Output shape |
|----------------------------|----------------|--------|--------------|
| Input                      | None           | None   | B × 28 × 28 × 1 |
| Flatten                    | 784            | None   | B × 784      |
| Dense                      | 200, A=ReLU    | None   | B × 200      |
| Dense(classifier)          | 10             | None   | B × 10       |

| Layer                      | Kernel details | Stride | Output shape |
|----------------------------|----------------|--------|--------------|
| Input                      | None           | None   | B × 32 × 32 × 3 |
| Conv1                      | F=64, K=5 × 5, A=ReLU | 1     | B × 32 × 32 × 20 |
| Max Pool 1                 | K=3 × 3        | 2      | B × 16 × 16 × 20 |
| Conv2                      | F=64, K=5 × 5, A=ReLU | 1     | B × 16 × 16 × 20 |
| Max Pool 2                 | K=3 × 3        | 2      | B × 8 × 8 × 20  |
| Flatten                    | 4096           | None   | B × 1280     |
| Dense                      | 384, A=ReLU    | None   | B × 800      |
| Dense                      | 192, A=ReLU    | None   | B × 500      |
| Dense(classifier)          | 10             | None   | B × 10       |

| Layer                      | Kernel details | Stride | Output shape |
|----------------------------|----------------|--------|--------------|
| Input                      | None           | None   | B × 28 × 28 × 1 |
| Conv1                      | F=64, K=3 × 3, A=ReLU | 1     | B × 28 × 28 × 64 |
| MaxPool1                   | K=2 × 2        | 2      | B × 14 × 14 × 64 |
| Conv2                      | F=64, K=3 × 3, A=ReLU | 1     | B × 14 × 14 × 64 |
| MaxPool2                   | K=2 × 2        | 2      | B × 7 × 7 × 64 |
| Conv3                      | F=64, K=3 × 3, A=ReLU | 1     | B × 3 × 3 × 64 |
| MaxPool3                   | K=2 × 2        | 2      | B × 1 × 1 × 64 |
| Conv4                      | F=64, K=3 × 3, A=ReLU | 1     | B × 3 × 3 × 64, A=ReLU |
| MaxPool4                   | K=2 × 2        | 2      | B × 1 × 1 × 64 |
| Flatten                    | 64             | None   | B × 64      |
| Dense(classifier)          | 55             | None   | B × 55      |

In Table 4 we summarize the details of the different datasets.

Table 4. Summary of dataset characteristics

| Dataset        | Input Dimension | Labels | Train set | Test set | Feat. vector units M |
|----------------|-----------------|--------|-----------|----------|----------------------|
| MNIST          | 28x28x1         | 10     | 60,000    | 10,000   | 200                  |
| CIFAR-10       | 32x32x3         | 10     | 50,000    | 10,000   | 192                  |
| Fashion-MNIST  | 28x28x1         | 10     | 60,000    | 10,000   | 200                  |
| EMNIST         | 28x28x1         | 62     | 697,932   | 116,323  | 200                  |
| Omniglot       | 105x10x1        | 1623   | 19,280    | 13,180   | 64                   |
**A.2. Dataset splitting across clients**

In Figure 7, we see an example assignment of 7 data points to 3 clients. We demonstrate a split in which all data points belong to the same class. In our work, each class is processed and distributed in the same manner. For instance, if we were to distribute the data points belonging in one class e.g. class with labels of 0, we would follow the procedure below: (a) shuffle all the data points of class 0, (b) filter all clients that were predetermined to contain the labels of class 0, (c) iterate through all filtered clients from step (b) and assign 1 data point to them. Step (c) is repeated in a cyclic manner until all data points are exhausted.

In our example, first, the 7 data points are randomly permuted; then we traverse the permuted sequence from left to right and assign one data point to each client in a cyclic manner until all examples are exhausted. In contrast with random partition, RR does not require to shuffle the data samples first, however we do not omit the shuffling step of the data samples at the RR partition during our experiments.

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**Figure 6.** Model architectures. **Left:** MLP architecture for MNIST, Fashion-MNIST and EMNIST; **Middle:** 4-convolutional layer architecture for Omniglot; **Right:** 2-convolutional layer architecture for CIFAR-10.

**Figure 7.** Assignment of 7 data points to 3 clients with Round Robin partition.
A.3. Hyperparameter selection

For MNIST, CIFAR-10, EMNIST, and Fashion-MNIST we perform an hyperparameter search for $\beta$ in the set of values $\{0.001, 0.002, \ldots, 0.007\}$, and for $\rho$ is in the set $\{0.001, 0.002, 0.003\}$. For Omniglot we further examine two additional values for $\beta$ of $\{0.008, 0.009\}$. In Table 5 we give the values of the best hyperparameters for various degrees of personalization. In all experiments, the server samples 20% of the clients in each round.

| Dataset       | PFLEGO Hyperparameters | Personalization | FedPer Hyperparameters | Personalization | FedAvg Hyperparameters | Personalization |
|---------------|------------------------|-----------------|------------------------|-----------------|------------------------|-----------------|
| MNIST         | $\beta = 0.006, \rho = 0.002$ | High-pers       | $\beta = 0.007$       | High-pers       | $\beta = 0.007$       | High-pers       |
|               | $\beta = 0.006, \rho = 0.003$ | Medium-pers     | $\beta = 0.007$       | Medium-pers     | $\beta = 0.007$       | Medium-pers     |
|               | $\beta = 0.007, \rho = 0.003$ | No-pers         | $\beta = 0.007$       | No-pers         | $\beta = 0.007$       | No-pers         |
| CIFAR-10      | $\beta = 0.002, \rho = 0.001$ | High-pers       | $\beta = 0.007$       | High-pers       | $\beta = 0.007$       | High-pers       |
|               | $\beta = 0.007, \rho = 0.001$ | Medium-pers     | $\beta = 0.005$       | Medium-pers     | $\beta = 0.007$       | Medium-pers     |
|               | $\beta = 0.007, \rho = 0.001$ | No-pers         | $\beta = 0.005$       | No-pers         | $\beta = 0.006$       | No-pers         |
| Fashion-MNIST | $\beta = 0.007, \rho = 0.001$ | High-pers       | $\beta = 0.007$       | High-pers       | $\beta = 0.007$       | High-pers       |
|               | $\beta = 0.006, \rho = 0.002$ | Medium-pers     | $\beta = 0.007$       | Medium-pers     | $\beta = 0.007$       | Medium-pers     |
|               | $\beta = 0.007, \rho = 0.003$ | No-pers         | $\beta = 0.007$       | No-pers         | $\beta = 0.007$       | No-pers         |
| EMNIST        | $\beta = 0.007, \rho = 0.001$ | High-pers       | $\beta = 0.007$       | High-pers       | $\beta = 0.007$       | High-pers       |
|               | $\beta = 0.006, \rho = 0.002$ | Medium-pers     | $\beta = 0.007$       | Medium-pers     | $\beta = 0.007$       | Medium-pers     |
|               | $\beta = 0.007, \rho = 0.003$ | No-pers         | $\beta = 0.007$       | No-pers         | $\beta = 0.007$       | No-pers         |
| Omniglot      | $\beta = 0.009, \rho = 0.001$ | High-pers       | $\beta = 0.009$       | High-pers       | $\beta = 0.009$       | High-pers       |

Table 5. Best hyperparameters on various personalized settings.

B. Additional Experiments

B.1. Personalization study for MNIST, Fashion-MNIST and CIFAR-10 datasets

We provide the experiments of MNIST, Fashion-MNIST, EMNIST and CIFAR-10 on all three degrees of personalization. Our algorithm outperforms the other baselines in the high personalization cases, and FedAvg outperforms the other algorithms when personalization is not needed.

Figure 8. Train loss (top row) and test accuracy (bottom row) over 200 rounds of PFLEGO, FedAvg and FedPer on Fashion-MNIST. Each of the three columns corresponds to a certain degree of personalization. Settings: 100 clients, 200 rounds 50 inner steps, r=20.
Figure 9. Train loss (top row) and test accuracy (bottom row) over 200 rounds of PFLEGO, FedAvg and FedPer with CNN model on CIFAR-10 dataset. Each of the three columns corresponds to a certain degree of personalization. Settings: 100 clients, 200 rounds 50 inner steps, r=20.

Figure 10. Train loss (top row) and test accuracy (bottom row) over 200 rounds of PFLEGO, FedAvg and FedPer with MLP model on Emnist dataset. Each of the three columns corresponds to a certain degree of personalization. Settings: 100 clients, 200 rounds 50 inner steps, r=20.

B.2. Accuracy on effect of participation

We provide the test accuracy on the study conducted for the effect of participation, see Figure 11. The more we increase the number of participation, the more gradients will be sent back to the server for aggregation. Then the server can perform an SGD step to the common weights that minimizes the total loss. (1).
Figure 11. Test accuracy of the ablation study of client participation using the CIFAR-10 for the High-pers case. PFLEGO is compared against FedAvg and FedPer in terms of the ability to minimize the train loss across rounds. The panel in each column corresponds to a different participation percentage $r$.

B.3. Experiments for FedRecon

We provide an additional experiment for the Omniglot dataset, and we compare the stochastic block coordinate FedRecon algorithm with our approach PFLEGO that uses a SGD scheme. In Figure 12 we include the visual comparison between FedRecon and PFLEGO, in which we can see the performance gains of PFLEGO regarding convergence speed, e.g., PFLEGO has lower training loss. The best hyperparameters for PFLEGO and FedRecon are $\rho = 0.001$ and $\beta = 0.009$

Figure 12. Training loss (left), training accuracy (middle) and test accuracy (right) for the Omniglot dataset for PFLEGO and FedRecon. Settings: 50 clients, 5000 rounds 50 inner steps, $r = 20$, $\rho = 0.001$, $\beta = 0.009$

Table 6. Test accuracy for Omniglot dataset on high degree of personalization for PFLEGO and FedRecon. The average is measured by averaging the last 10 global rounds of the first 1000 global round

|           | FedRecon     | PFLEGO       |
|-----------|--------------|--------------|
|           | 74.06 ± 1.19 | 74.56 ± 1.23 |

C. FedPer, FedAvg and FedRecon algorithms

In this section we provide the pseudocodes for the Federated Averaging (FedAvg) and Federated Personalized (FedPer) algorithms that we used in our experiments for comparison against our algorithm PFLEGO.

In Algorithm 2 we provide the FedPer pseudocode. In this version the algorithm uses one personalized layer. At each round, the server selects a subset of clients and distributes the global parameters. Given the global parameters, these clients copy the global parameters to their global weights. Then, each client performs a number of gradient updates to its client-specific weights $W_i$ and to its global weights $\theta$. Each client uses the same learning rate $\beta$ on the gradient updates of $W_i$ and of $\theta$. After the last gradient update step, each client returns its updated common weights $\theta$ to the server. The server collects the common weights from clients and averages them. Then the server sets the global model parameters to the average value, and proceeds to the next round. The personalized weights $W_i$ for FedPer and PFLEGO algorithm are initialized with uniform values in $[0, 1)$.
Algorithm 2 FedPer

**Input:** $T$ number of rounds, $\tau$ local gradient updates, $I$ clients, $r \ll I$ sampled clients per round, $N_i$ number of data samples at the $i$-th client

**Server:**
- Initialize global parameters $\theta_1$
- for round $t = 1, 2, ..., T$
  - $I_t \leftarrow$ (Select a random subset of $r$ clients)
  - Receive $\theta'_i$ from each client $i \in I_t$
  - Aggregate: $\theta_{t+1} \leftarrow \sum_{i \in I_t} a_i \theta'_i$
- end for

**Client$_i$.Update($\theta_t$):#runs on client $i$
- Initialize $W_i$, the first time the client is visited
- Copy $\theta_t$ to $\theta_i$
- for local gradient update $= 1, 2, ..., \tau$
  - $W_i \leftarrow W_i - \beta \nabla_{W_i} \ell_i(D_i; W_i, \theta_i)$
  - $\theta_i \leftarrow \theta_i - \beta \nabla_{\theta_i} \ell_i(D_i; W_i, \theta_i)$
- end for
- Return $\theta_i$ as $\theta'_i$ to server

In Algorithm 3 we provide the Federated Averaging algorithm pseudocode. At each round, a random subset of clients copy the model parameters of the server to their local weights. These clients perform a number of gradient updates to their local weights $\theta$. After the last gradient step, each of these clients return their updated local parameters to the server. The server collects all updated parameters and averages them. Then, the server updates its model parameter to the average value, and proceeds to the next round.

In Algorithm 4 we provide the FedRecon algorithm pseudocode, in which the clients perform only 1 gradient update to their global parameters; therefore this is equivalent to sending the gradient back to the server. In this version, the algorithm uses one personalized layer for its client-specific parameters, and clients do not drop the client-specific parameters at the end of each round. At each round, the server selects a subset of clients and distributes the global parameters. Given the global parameters, these clients copy the global parameters to their global weights. Then each client performs a number of gradient updates to their client-specific weights $W_i$. After the last gradient update step on $W_i$, each client computes the gradient w.r.t. the local copies of the global parameters to the server. The server collects all gradients from the clients, and then the server aggregates them to the global parameters $\theta$, and proceeds to the next round. The personalized weights $W_i$ for FedPer and PFLEGO algorithm are initialized with uniform values in $[0, 1)$.

Algorithm 3 FedAvg

**Input:** $T$ number of rounds, $\tau$ local gradient updates, $I$ clients, $r \ll I$ sampled clients per round, $N_i$ number of data samples at the $i$-th client

**Server:**
- Initialize global parameters $\theta_1$
- for round $t = 1, 2, ..., T$
  - $I_t \leftarrow$ (Select a random subset of $r$ clients)
  - Receive $\theta'_i$ from each client $i \in I_t$
  - Aggregate: $\theta_{t+1} \leftarrow \sum_{i \in I_t} a_i \theta'_i$
- end for

**Client$_i$.Update($\theta_t$):#runs on client $i$
- Copy $\theta_t$ to $\theta_i$
- for local gradient update $= 1, 2, ..., \tau$
  - $\theta_i \leftarrow \theta_i - \beta \nabla_{\theta_i} \ell_i(D_i; \theta_i)$
- end for
- Return $\theta_i$ as $\theta'_i$ to server
Algorithm 4 FedRecon

**Input:** $T$ rounds, $\tau$ local gradient updates, $I$ clients, $r \ll I$ (average) sampled clients per round, $N_i$ data samples at the $i$-th client

**Server:**
  - Initialize global parameters $\theta_1$
  - for round $t = 1, 2, ..., T$ do
    - $I_t \leftarrow$ (Select a random subset of clients)
    - Receive $g_i$ from each client $i \in I_t$
    - Aggregate: $\theta_{t+1} \leftarrow \theta_t - \rho_t \frac{1}{r} \sum_{i \in I_t} a_i g_i$
  - end for

**ClientUpdate($\theta_t$):** #runs on client $i$
  - Initialize $W_i$, the first time client $i$ is visited
  - for local gradient update $= 1, 2, ..., \tau$ do
    - $W_i \leftarrow W_i - \beta \nabla W_i \ell_i(W_i, \theta_t)$
  - end for
  - Return $g_i := \nabla \theta \ell_i(W_i, \theta_t)$ to server