Flavor in Minimal Conformal Technicolor

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Abstract

We construct a complete, realistic, and natural UV completion of minimal conformal technicolor that explains the origin of quark and lepton masses and mixing angles. As in “bosonic technicolor,” we embed conformal technicolor in a supersymmetric theory, with supersymmetry broken at a high scale. The exchange of heavy scalar doublets generates higher-dimension interactions between technifermions and quarks and leptons that give rise to quark and lepton masses at the TeV scale. Obtaining a sufficiently large top quark mass requires strong dynamics at the supersymmetry breaking scale in both the top and technicolor sectors. This is natural if the theory above the supersymmetry breaking also has strong conformal dynamics. We present two models in which the strong top dynamics is realized in different ways. In both models, constraints from flavor-changing effects can be easily satisfied. The effective theory below the supersymmetry breaking scale is minimal conformal technicolor with an additional light technicolor gaugino. We argue that this light gaugino is a general consequence of conformal technicolor embedded into a supersymmetric theory. If the gaugino has mass below the TeV scale it will give rise to an additional pseudo Nambu-Goldstone boson that is observable at the LHC.
1 Introduction

Conformal technicolor [1] is a solution of the hierarchy problem in which the electroweak scale is determined by the soft breaking of conformal invariance [2]. The minimal theory of this kind is $SU(2)$ technicolor with additional technifermions uncharged under the standard model gauge group, such that the $SU(2)$ gauge coupling has a strong conformal fixed point. The strong conformal symmetry is broken softly by a mass term for the sterile technifermions, which results in the breaking of electroweak symmetry. We will refer to this theory as minimal conformal technicolor (MCTC).

This theory breaks electroweak symmetry via strong dynamics near the TeV scale. Two potentially serious problems with such scenarios are precision electroweak constraints and flavor physics. The precision electroweak constraints in MCTC were discussed in Ref. [3]. This model has a space of vacua given by $SU(4)/Sp(4) \simeq SO(6)/SO(5)$, which contains a pseudo Nambu Goldstone Higgs. The theory can have a standard model-like fit to the precision electroweak data with a light composite Higgs at the price of $\sim 10\%$ fine tuning. Alternatively, the theory may live in the technicolor vacuum provided the $S$ parameter is less than about half the QCD value and there is a positive contribution to the $T$ parameter.

The subject of this paper is a detailed investigation of flavor in MCTC. Electroweak symmetry breaking is communicated to the fermions via higher dimension interactions such as

$$\Delta \mathcal{L}_{\text{eff}} \sim \frac{1}{\Lambda_{d-1}^d}(q_3 \bar{t})\mathcal{H} + \text{h.c.}$$

where $q_3$ and $\bar{t}$ are the third generation quark fields, and $d$ is the UV dimension of the operator $\mathcal{H} = \psi \tilde{\psi}$ that breaks electroweak symmetry. In conformal technicolor (as in “walking” technicolor [4]) $d < 3$, allowing flavor to be generated at a higher scale. We focus on conformal rather than walking dynamics because conformal dynamics is more robust and plausible.\footnote{Walking dynamics is plausible in large-$N_c$ theories at the end of the conformal window [5], but in such theories the $S$ parameter is proportional to $N_c$, and is generically too large.}

Conformal dynamics of non-abelian gauge theories is now being actively investigated on the lattice [6]. In particular, the dimension $d$ can be readily determined in lattice simulations [2] and measurements are starting to appear [7]. Interest in conformal theories in the lattice community is growing and we expect significant progress in the near future. One can also obtain bounds on $d$ using rigorous results of conformal
field theory [8]. The idea is that if $d$ is too close to 1, then the operator $|\mathcal{H}|^2$ has dimension close to 2, and the fixed point is IR unstable. One can use this to obtain rigorous bounds on $d$, but for theories with global symmetries (the case of interest) the bounds are very weak [9].

For what values of $d$ can we get a realistic theory of flavor? The main constraint is that the top quark mass is generated by the interaction in Eq. (1.1). If we choose the coefficient such that $\Lambda_t$ is the scale where the operator becomes strongly coupled in the UV, we have

$$m_t \sim 4\pi v \left( \frac{\Lambda_{\text{CTC}}}{\Lambda_t} \right)^{d-1},$$

(1.2)

where $\Lambda_{\text{CTC}} \sim 4\pi f$ is the $SU(4)$ breaking scale. This gives

$$\Lambda_t \sim 4\pi v \left( \frac{m_t}{4\pi v} \right)^{-1/(d-1)} \sim \begin{cases} 10 \text{ TeV} & d = 3 \\ 40 \text{ TeV} & d = 2 \\ 600 \text{ TeV} & d = 1.5 \end{cases}$$

(1.3)

where we (conservatively) use $f = v$ in the numerical estimates. (Note that $m_t$ renormalized at $4\pi v \sim 2$ TeV in a theory with no light Higgs is 133 GeV.) New physics that generates the operator Eq. (1.1) must appear at or below the scale $\Lambda_t$. This theory has flavor-dependent couplings, but must not generate unacceptably large flavor-changing effects. To know whether this is possible for a given value of $d$ we need a more fundamental theory that generates Eq. (1.1).

In this paper, we address this question by constructing natural and realistic theories of flavor that give a UV completion of MCTC. We recall that MCTC is a theory of electroweak symmetry breaking with gauge group

$$SU(2)_{\text{CTC}} \times SU(2)_L \times SU(2)_R$$

(1.4)

where $Y = T_{3R}$. The theory has fermions

$$\psi \sim (2, 2, 1),$$

$$\tilde{\psi} \sim (2, 1, 2),$$

(1.5)

plus enough additional "sterile technifermions" (charged under $SU(2)_{\text{CTC}}$ but not $SU(2)_L \times SU(2)_R$) so that the gauge coupling has a strongly coupled fixed point. The existence of such a fixed point is expected at the end of the conformal window, and is under active investigation in the lattice gauge theory community. Conformal symmetry is then softly broken by mass terms for the sterile technifermions. At the
scale of the soft breaking the theory confines and spontaneously breaks the $SU(4)$ chiral symmetry acting on $\psi, \bar{\psi}$ via a fermion condensate. Weak interactions that explicitly break $SU(4)$ give rise to a potential can align the condensate to give either a light PNGB Higgs or a technicolor-like theory below the TeV scale.

In this paper, we embed this theory into a supersymmetric theory, with SUSY broken at a scale $M_{SUSY} \gg$ TeV, which plays the role of the flavor scale. It is natural for SUSY to be broken at the same scale in the technicolor and standard model sectors (e.g. in gravity mediation), therefore the superpartners of the standard model particles also have masses of order $M_{SUSY}$. Effective interactions like Eq. (1.1) that generate the quark and lepton masses arise from the exchange of elementary scalars at $M_{SUSY}$. These scalars have the quantum numbers of Higgs bosons, but they do not have VEVs and their role is that of flavor messengers. The quark and lepton masses and mixings are then proportional to ordinary Yukawa couplings. Because $M_{SUSY} \gg$ TeV, the flavor-changing effects from squark and slepton mixing are suppressed or absent.

For completely arbitrary $CP$-violating squark masses, we need $M_{SUSY} \gtrsim 10^3$ TeV, but $M_{SUSY} \gtrsim 10$ TeV may be sufficient if the soft SUSY breaking preserves $CP$ and has some degree of alignment. SUSY and technicolor may therefore solve each others’ flavor problems. This is the basic idea of “bosonic technicolor” [10]. (see also Refs. [11]). SUSY has also been used to UV complete little Higgs theories in Refs. [12], and our models have some features in common with these models.

However, this idea by itself is not sufficient to explain the observed value of the top quark mass. In this scenario, we have

$$m_t \sim 4\pi v \left( \frac{y_t}{4\pi} \right) \left( \frac{y_{TC}}{4\pi} \right) \left( \frac{4\pi f}{M_{SUSY}} \right)^{d-1}.$$  \hspace{1cm} (1.6)

where $y_t$ and $y_{TC}$ are the top and technicolor Yukawa couplings at the scale $M_{SUSY}$. Eq. (1.6) states that the top quark mass is given by its strong-coupling value ($\sim$ TeV) times all suppression factors that make it less than strongly coupled at the scale $4\pi f$ where chiral symmetry breaking occurs. This estimate assumes that the operator $\psi \bar{\psi}$ has dimension $d$ immediately below $M_{SUSY}$. For $d < 3$, this requires that the technicolor coupling is at a non-supersymmetric strong fixed point right below $M_{SUSY}$. If instead the technicolor coupling is weak at the the scale $M_{SUSY}$ and runs to a strong fixed point, the top mass will have additional suppression compared to Eq. (1.6).

The large observed value of the top quark mass implies that the product of all of the suppression factors on the right-hand side of Eq. (1.6) must be $\sim \frac{1}{10}$. This is plausible only if both $y_t$ and $y_{TC}$ are strong ($\sim 4\pi$) at the SUSY breaking scale. We therefore require a theory where the technicolor gauge coupling, $y_{TC}$, and $y_t$ are all strong at the scale $M_{SUSY}$.
This is not a coincidence if all of these couplings are at strong conformal fixed points above \( M_{\text{SUSY}} \). Strong fixed-point Yukawa couplings occur naturally in SUSY conformal theories [13]. SUSY breaking is then a soft breaking of conformal invariance that triggers the transition from a supersymmetric conformal fixed point to a non-supersymmetric conformal fixed point.

In order for \( y_t \) to be at a strong supersymmetric fixed point, the top quark must feel strong gauge interactions above \( M_{\text{SUSY}} \). This can naturally occur in models with an extended color gauge symmetry. We consider two models with color gauge symmetry breaking patterns:

\[
\begin{array}{c}
\text{SU}(3) \times \text{SU}(3) \rightarrow \text{SU}(3), \\
\text{strong} & \text{weak} & \text{weak}
\end{array}
\quad (1.7)
\]

and

\[
\begin{array}{c}
\text{SU}(6) \times \text{SU}(3) \times \text{SU}(3) \rightarrow \text{SU}(3) \times \text{SU}(3), \\
\text{strong} & \text{weak} & \text{weak}
\end{array}
\quad (1.8)
\]

In the first case, the existence of a strong fixed point does not allow all 3 generations of quarks to be charged under the strong \( \text{SU}(3) \). We construct a model where the third generation is charged under the strong \( \text{SU}(3) \) (as in “topcolor” [14]). In the second model, all three quark generations are charged under the strong \( \text{SU}(6) \). Both models have additional flavor-dependent couplings, but we show that they do not give unacceptably large flavor-changing neutral currents for SUSY breaking scales as low as \( \sim 10 \text{ TeV} \).

The overall structure of our models is quite simple. There are only two scales in the model, \( M_{\text{SUSY}} \) and \( \Lambda_{\text{CTC}} \), and both have a natural origin. Above the scale \( M_{\text{SUSY}} \) the theory is supersymmetric, and the technifermions and third generation quarks (and possibly other quarks and leptons) are at a strongly-interacting conformal fixed point. SUSY breaks at the scale \( M_{\text{SUSY}} \), and below this scale all the quarks and leptons become weakly interacting, while the technicolor sector flows to a strongly-interacting non-supersymmetric fixed point. This conformal invariance in the technicolor sector is softly broken by fermion mass terms at the scale \( \Lambda_{\text{CTC}} \), at which scale the technicolor dynamics confines and breaks chiral symmetry.

One may expect that because \( M_{\text{SUSY}} \gg \text{TeV} \) there are no testable consequences of this framework at the TeV scale. Remarkably, there is a robust testable consequence of embedding conformal technicolor in a SUSY theory with a strong conformal fixed point. The strong conformal dynamics above \( M_{\text{SUSY}} \) suppresses the technicolor gaugino mass [15], so there is naturally a \( \text{SU}(2)_{\text{CTC}} \) adjoint field below the TeV scale.
This gives rise to an additional pseudo Nambu-Goldstone boson that can be observed at the LHC.

This paper is organized as follows. In Section 2, we discuss the SUSY completion of the technicolor sector. In Section 3, we discuss a SUSY model in which the strong dynamics in the top sector arises from strong dynamics in the third generation ("topcolor"). In Section 4, we discuss a SUSY model in which all three generations are treated equally. In Section 5, we discuss the phenomenology at the TeV scale, particularly the PNGBs. Section 6 contains some remarks on cosmology, and Section 7 contains our conclusions.

2 Supersymmetric Conformal Technicolor

In this section, we describe the technicolor sector above the SUSY breaking scale $M_{\text{SUSY}}$, and how this matches onto minimal conformal technicolor below $M_{\text{SUSY}}$. This sector is the same for both models of the strong top dynamics that we discuss below.

2.1 Field Content

The gauge group is

$$SU(3)_{\text{SCTC}} \times SU(2)_L \times SU(2)_R$$

(2.1)

with $Y = T_{3R}$. The fields in the technicolor sector are

$$\Psi \sim (3, 2, 1),$$
$$\bar{\Psi} \sim (\bar{3}, 1, 2),$$
$$\Sigma_a \sim (3, 1, 1),$$
$$\bar{\Sigma}_a \sim (\bar{3}, 1, 1),$$

(2.2)

with $a = 1, \ldots, 4$. $SU(3)_{\text{SCTC}}$ has 6 flavors, and therefore has a maximally strongly-coupled (self-dual) conformal fixed point [16].

The technicolor gauge group has been extended from $SU(2)$ to $SU(3)$ for two reasons. The first has to do with the anomalous dimensions of mass-squared terms for technicolored scalars. Mass-squared terms proportional to anomaly-free flavor symmetry generators are not renormalized, while all other mass-squared terms are suppressed [15]. Therefore, some of the technicolored scalars have negative mass-squared
at the scale $M_{\text{SUSY}}$, and hence partial breaking of the technicolor gauge symmetry is inevitable. Actually, because the technicolor gauge group is at a strongly coupled fixed point, we cannot rigorously conclude that the gauge group is spontaneously broken. Here and throughout this paper we make the dynamical assumption that strongly-coupled broken SUSY theories such as this are in the same universality class as a weakly-coupled theory with the same gauge group and matter content. The second reason the gauge group is extended is that the partial breaking of the strong gauge group fixes a mismatch between the number of colors and flavors required for a strong fixed point. For a strong SUSY fixed point, we want $N_c = 2N_f$, while for a non-SUSY fixed point we probably want $N_c \approx 4N_f$. The latter estimate is very uncertain. It is suggested by a number of lattice studies for $N_c = 3$ [17] (but see also Refs. [18]), and agrees with model estimates [19]. We assume VEVs of the form

$$\langle \Sigma \rangle, \langle \tilde{\Sigma} \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * \end{pmatrix}$$

(2.3)

that break $SU(3)_{\text{SCTC}} \to SU(2)_{\text{CTC}}$ at the $M_{\text{SUSY}}$. Below $M_{\text{SUSY}}$ $SU(2)_{\text{CTC}}$ has 7 light fermion flavors (as we will explain below), roughly where we expect a strong conformal fixed point.

There are an odd number of $SU(2)_W$ doublets in Eq. (2.2), so there is a global gauge anomaly. We cancel this by adding the “partner” fields

$$P \sim (1, 2, 1),$$

$$\tilde{P} \sim (1, 1, 2).$$

(2.4)

The fields $\tilde{P}$ do not cancel any gauge anomaly, but they give rise to Dirac fermion masses, as we will see.

### 2.2 Couplings to Flavor Messengers

The higher dimension operators that give masses to quarks and leptons arise from the exchange of heavy electroweak doublet scalars. These have the quantum numbers of the MSSM Higgs fields, namely

$$\Phi \sim (1, 2, 2).$$

(2.5)

These fields have positive mass-squared, and therefore do not get VEVs at the scale $M_{\text{SUSY}}$. We will refer to these fields as “flavor messengers.” Writing out the $SU(2)_R$
doublets explicitly, the relevant fields have $SU(3)_{\text{SCTC}} \times SU(2)_W \times U(1)_Y$ quantum numbers

$$\begin{align*}
\Psi &\sim (3, 2)_0, \\
\tilde\Psi_1 &\sim (3, 1)_{\frac{1}{2}}, \\
\tilde\Psi_2 &\sim (3, 1)_{-\frac{1}{2}}, \\
\Phi_1 &\sim (1, 2)_{\frac{1}{2}}, \\
\Phi_2 &\sim (1, 2)_{-\frac{1}{2}}.
\end{align*}$$

The superpotential couplings are

$$\Delta W = y_{\text{TC1}} \Psi \tilde\Psi_2 \Phi_1 + y_{\text{TC2}} \Psi \tilde\Psi_1 \Phi_2.$$  \hspace{1cm} (2.7)

At the strong fixed point, the operator $\Psi \tilde\Psi$ has dimension $\frac{3}{2}$, so these couplings are relevant. $\Phi_1$ couples to the top quark. As discussed in the introduction, explaining the large value of the top quark mass requires that $y_{\text{TC1}}$ must be strong at the scale $M_{\text{SUSY}}$. Below, we will demonstrate that this is not necessarily a coincidence. In section 2.4, we will show that a mechanism similar to the Giudice-Masiero mechanism for the $\mu$ term [20] can naturally generate relevant interactions of this kind that get strong at $M_{\text{SUSY}}$. Additionally, in section 2.5, we will show that it is very plausible that relevant Yukawa couplings, such as in Eq. (2.7), run to a fixed point when they become strong.

### 2.3 Heavy Fermion Masses

SUSY breaking automatically gives masses to all scalars, but fermion masses must arise from superpotential couplings. The “3rd technicolor” components of the fermions in $\Psi$ and $\tilde\Psi$ get masses with the “partner” fields $P$ and $\tilde P$ via the superpotential interactions

$$\Delta W = y_P \tilde\Sigma \Psi P + y_{\tilde P} \Sigma \tilde\Psi \tilde P.$$  \hspace{1cm} (2.8)

At the strong fixed point, the operators $\tilde\Sigma \Psi$ and $\Sigma \tilde\Psi$ have dimension $\frac{3}{2}$, so the couplings $y_P, y_{\tilde P}$ have dimension $\frac{1}{2}$. These particles carry electroweak quantum numbers, so their mass must be larger than of order 100 GeV. This means that we must require

$$\frac{100 \text{ GeV}}{M_{\text{SUSY}}^{1/2}} \lesssim y_P, y_{\tilde P} \lesssim M_{\text{SUSY}}^{1/2}.$$  \hspace{1cm} (2.9)
The same mechanism that explains why the couplings in Eq. (2.7) are strong near
the scale $M_{\text{SUSY}}$ can work for the couplings $y_P, y_{\tilde{P}}$ as well, leading to masses of these
fields near $M_{\text{SUSY}}$.

The $SU(3)_{\text{SCTC}}$ gaugino has a Majorana mass term from SUSY breaking, but the
Majorana mass operator $\lambda\lambda$ has scaling dimension $d > 3$ from the strong conformal
dynamics [15]. The simplest SUSY breaking scenarios have SUSY broken at an energy
scale much larger than $M_{\text{SUSY}}$, with SUSY breaking communicated to the hidden
sector by weak interactions. In this case, the Majorana mass term will be strongly
suppressed compared to other SUSY breaking terms at $M_{\text{SUSY}}$. The VEVs, Eq. (2.3),
that break $SU(3)_{\text{SCTC}}$ are of order $M_{\text{SUSY}}$, and give a mass to the $SU(3)_{\text{SCTC}}$ gauginos
corresponding to broken gauge generators due to the super-Higgs mechanism. There
is one linear combination of the $SU(2)_{\text{CTC}}$ singlet fermions in $\Sigma_4, \tilde{\Sigma}_4$ that is left
massless. (This can be understood from a counting argument: there are $8 - 3 = 5$
broken generators, but there are 6 fermions in $\Sigma_4, \tilde{\Sigma}_4$.) This fermion can get mass
from superpotential interactions of the form

$$\Delta W \sim (\Sigma \tilde{\Sigma})^2.$$  \hspace{1cm} (2.10)

This operator has dimension 4 at the fixed point. This means that it can naturally
be unsuppressed at the scale $M_{\text{SUSY}}$ and give rise to masses of order $M_{\text{SUSY}}$.

We also need small mass terms for sterile fermions to softly break conformal
symmetry at the TeV scale. These arise from the superpotential couplings

$$\Delta W \sim \Sigma\Sigma\Sigma + \tilde{\Sigma}\tilde{\Sigma}\tilde{\Sigma}$$ \hspace{1cm} (2.11)

where the $SU(3)_{\text{SCTC}}$ indices are contracted with an epsilon symbol. Similarly, we
may also require fermion masses for the technifermions in $\Psi$ and $\tilde{\Psi}$ to control the
vacuum alignment below the electroweak breaking scale [3]. Such masses can arise
from

$$\Delta W \sim \Sigma\Psi\Psi + \tilde{\Sigma}\tilde{\Psi}\tilde{\Psi}.$$ \hspace{1cm} (2.12)

These terms have dimension $\frac{9}{4}$, and are therefore also relevant. The coefficients of the
superpotential terms Eqs. (2.11) and (2.12) must be small at the scale $M_{\text{SUSY}}$, which
is perfectly natural.

### 2.4 Strong Yukawa Couplings from Hidden Sector SUSY breaking

We now discuss whether it is a coincidence for the relevant couplings in Eqs. (2.7)
and (2.8) to be at or near their strong coupling values at $M_{\text{SUSY}}$. This may be viewed
as analogous to the μ problem in the MSSM, which also requires a relevant supersymmetric interaction to be important at the scale of SUSY breaking. In this subsection, we present a simple model of hidden sector SUSY breaking that naturally explains this coincidence using a mechanism similar to the Giudice-Masiero mechanism for the μ problem.

We assume that SUSY is broken in the hidden sector by a chiral superfield $X$ with $F_X \neq 0$. We assume the usual couplings between $X$ and the visible sector fields

$$\Delta K \sim \frac{1}{M_P^2} X^\dagger X q^\dagger q + \cdots$$  \hspace{1cm} (2.13)

where $M_P$ is the Planck scale. These give rise to visible sector SUSY breaking at the scale

$$M_{\text{SUSY}} \sim \frac{\langle F_X \rangle}{M_P}.$$  \hspace{1cm} (2.14)

Suppose that in addition the hidden sector has a field $Y$ with

$$\langle Y \rangle \sim \langle F_X \rangle^{1/2}, \quad \langle F_Y \rangle \lesssim \langle F_X \rangle^{1/2} M_{\text{SUSY}}.$$  \hspace{1cm} (2.15)

Since $\langle F_X \rangle$ corresponds to the scale of SUSY breaking dynamics, this is very natural as we will see below. A coupling between the hidden and visible sectors of the form

$$\Delta W \sim \frac{1}{M_P^{1/2}} Y \bar{\Psi} \Phi_2 \Phi_1$$  \hspace{1cm} (2.16)

then generates an effective Yukawa coupling

$$y_{\text{TC1}} \sim \frac{\langle Y \rangle}{M_P^{1/2}} \sim M_{\text{SUSY}}^{1/2}. $$  \hspace{1cm} (2.17)

This is just what we require in order for the Yukawa coupling to be strong at the scale $M_{\text{SUSY}}$. The second condition in Eq. (2.15) ensures that visible sector SUSY breaking from $\langle F_Y \rangle$ is no larger than $M_{\text{SUSY}}$.

A simple hidden sector that accomplishes this has a superpotential

$$W = \kappa X + \frac{\lambda}{M_P} Y^4.$$  \hspace{1cm} (2.18)

Note that this preserves a $Z_4$ symmetry under which $Y \mapsto iY$. We are looking for VEVs of the form Eq. (2.15) with $\langle F_X \rangle \sim \kappa$, so we can expand around $X,Y = 0$ in inverse powers of $M_P$. In this case, a theorem due to Weinberg [21] guarantees that if there is a vacuum with the desired properties in the global SUSY limit, turning on
SUGRA corrections will only perturb the VEVs. We therefore analyze the potential neglecting SUGRA corrections. The most general \( Z_4 \) invariant Kähler potential is therefore
\[
K = X^\dagger X + Y^\dagger Y + \frac{1}{4M^2} \left[ c_{XX}(X^\dagger X)^2 + 4c_{XY}X^\dagger XY + c_{YY}(Y^\dagger Y)^2 \right] + \mathcal{O}(M^{-3}).
\]
where we have omitted holomorphic terms. The potential is then
\[
V = \frac{1}{M^2} \left[ -c_{XX}\kappa^2|X|^2 - c_{XY}\kappa^2|Y|^2 + \lambda|Y|^4 \right] + \mathcal{O}(M^{-3}).
\]
Assuming
\[
c_{XX} < 0, \quad c_{XY} > 0,
\]
the potential is minimized for
\[
\langle X \rangle = 0, \quad |\langle Y \rangle|^2 = -\frac{c_{XY}\kappa^2}{2\lambda},
\]
which implies
\[
\langle F_X \rangle = \kappa, \quad \langle F_Y \rangle = \frac{\lambda Y^3}{M} \sim \frac{\langle F_X \rangle^{3/2}}{M} \sim \langle F_X \rangle^{1/2} M_{\text{SUSY}}
\]
for \( \lambda, c_{XY} \sim 1 \). If we ignore SUGRA corrections, this theory has a massless "R-axion" from the spontaneous breaking of a \( U(1)_R \) symmetry under which \( R(X) = 2 \), \( R(Y) = \frac{1}{2} \). When we include SUGRA corrections, a crucial ingredient is the addition of a constant term in the superpotential to cancel the cosmological constant. This explicitly breaks \( U(1)_R \) and gives rise to a mass for the R-axion [22]. Of course there can be other sources of explicit \( U(1)_R \) breaking that also contribute to the R-axion mass.

2.5 Strong Yukawa Couplings from New Fixed Points

The conformal fixed point is no longer exact when a relevant coupling like \( y_{TC1} \) is nonzero. The strength of such couplings grows in the IR, and causes the theory to flow away from the original fixed point. A natural possibility is that the theory flows to a new fixed point where the coupling \( y_{TC1} \) is itself at a strong fixed point value.
There is a highly nontrivial consistency check on this hypothesis from “a maximization” [23]. Provided that the superconformal $U(1)_R$ symmetry can be identified with an anomaly-free $U(1)_R$ symmetry acting on fundamental fields, the quantity

$$a = \frac{3}{32} \text{tr} (3R^3 - R)$$

(2.24)

is maximized over all allowed values of the $R$ charges. This allows one to find the dimensions of all chiral operators for a claimed fixed point. These dimensions must be physical, for example the dimension of scalar operators must be $\geq 1$.

The complete theory has strong dynamics in both the technicolor and the strong sector, and the strong Yukawa coupling for the top quark means that these sectors are also strongly coupled to each other. Nonetheless, we will discuss the possible new fixed point in the technicolor sector to illustrate the main points in a simpler setting. We consider the case where only the coupling $y_{TC1}$ is strong. Here, we must maximize $a$ subject to the condition $R(\Psi\bar{\Psi}\Phi_1) = 2$. This gives a new fixed point with dimensions ($d = \frac{3}{2}R$ for chiral operators)

$$d(\Psi) = 0.78, \quad d(\bar{\Psi}_1) = 0.73, \quad d(\bar{\Psi}_2) = 0.84, \quad d(\Phi_1) = 1.37.$$  

(2.25)

This is to be compared with the fixed point for small $y_{TC1}$, in which $d(\Psi) = d(\bar{\Psi}_{1,2}) = \frac{3}{4}$ and $d(\Phi_1) = 1$. We see that the dimensions of the strongly interacting fields have shifted only slightly, while the field $\Phi_1$ has a large anomalous dimension in the new fixed point. All gauge-invariant operators have physical dimensions, and no scalar field has a dimension near 1, which is consistent with a strongly interacting fixed point.

Because the field $\Phi_1$ has dimension larger than 1 at the new fixed point, weakly coupled Yukawa interactions involving $\Phi_1$ are irrelevant interactions at the new fixed point. The implications of this will be considered later when we discuss complete models including quarks.

### 2.6 Low Energy Effective Theory

For clarity and completeness, we summarize the effective theory below the scale $M_{\text{SUSY}}$. The gauge group is

$$SU(2)_{\text{CTC}} \times SU(2)_W \times U(1)_Y$$

(2.26)

where the $SU(2)_{\text{CTC}}$ is assumed to be at a strong fixed point. Supersymmetry is completely broken, and only gauge fields and fermion fields survive below $M_{\text{SUSY}}$. 

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The fermion fields are
\[
\begin{align*}
\psi & \sim (2,1)_0, \\
\tilde{\psi}_1 & \sim (2,1)_{\frac{1}{2}}, \\
\tilde{\psi}_2 & \sim (2,1)_{-\frac{1}{2}}, \\
\chi_a & \sim (2,1)_0, \\
\lambda & \sim (3,1)_0,
\end{align*}
\]
where \(a = 1, \ldots, 6\). The \(\psi\) and \(\tilde{\psi}\) fields are the fermion components of the superfields \(\Psi\) and \(\tilde{\Psi}\); the \(\chi_a\) are the fermion components of \(\Sigma_{1,2,3}, \tilde{\Sigma}_{1,2,3}\) (the components without VEVs); and \(\lambda\) arises from the \(SU(3)_{\text{SCTC}}\) gaugino. The theory has mass terms for the fields \(\chi, \psi, \tilde{\psi}\) arising from the superpotential terms in Eqs. (2.11) and (2.12):
\[
\Delta L_{\text{mass}} = -\chi^T K \chi - \kappa \psi \tilde{\psi} - \tilde{\kappa} \tilde{\psi}_1 \tilde{\psi}_2, \tag{2.28}
\]
where \(K\) is an antisymmetric \(6 \times 6\) matrix. There is also a mass term \(\lambda \lambda\) arising from the gaugino mass for the \(SU(3)_{\text{SCTC}}\) gauge multiplet, but this is highly suppressed by the strong conformal dynamics, as discussed in Section 2.3. We assume that \(K > \kappa, \tilde{\kappa}\), so that the conformal invariance is softly broken by the \(K\) terms. In fact, the \(K\) mass terms arise from an operator Eq. (2.11) that is more relevant than the operator Eq. (2.12) that generates the \(\kappa, \tilde{\kappa}\) terms, so this is natural. At the scale \(\Lambda_{\text{CTC}}\) where the \(K\) terms get strong, the theory is assumed to confine and spontaneously break the approximate \(SU(4)\) chiral symmetry acting on \(\psi, \tilde{\psi}\). The low-energy effective theory below the scale \(\Lambda_{\text{CTC}}\) is discussed in detail in Ref. [3] for a model without the gaugino field \(\lambda\). The implications of the additional light \(\lambda\) will be discussed in Section 5 below.

3 Supersymmetric Topcolor

We now turn to the top sector above the SUSY breaking scale. In this section, we describe the top sector in a model where only the third generation has strong dynamics above \(M_{\text{SUSY}}\). This is similar to “topcolor” models [14], so we refer to these models as “supersymmetric topcolor.”

3.1 Field Content

The gauge group is
\[
SU(3)_{tC} \times SU(3)_{C'} \times SU(2)_W \times U(1)_Y. \tag{3.1}
\]
The quark fields have quantum numbers

\[ q_3 \sim (3, 1, 2)_{1/6}, \]
\[ \tilde{t} \sim (\bar{3}, 1, 1)_{-2/3}, \]
\[ \tilde{b} \sim (3, 1, 1)_{1/3}, \]
\[ q_i \sim (1, 3, 2)_{1/6}, \]
\[ \tilde{u}_i \sim (1, \bar{3}, 1)_{-2/3}, \]
\[ \tilde{d}_i \sim (1, \bar{3}, 1)_{1/3}, \]

where \( i = 1, 2 \) runs over the first two generations. The breaking of the strong color group requires Higgs fields

\[ \Delta \sim (3, 3, 1)_{0}, \]
\[ \tilde{\Delta} \sim (\bar{3}, 3, 1)_{0}. \]

Mixing between the third generation and the first two is mediated by an additional pair of vector-like quarks:

\[ U \sim (1, 3, 1)_{2/3}, \]
\[ \tilde{U} \sim (1, \bar{3}, 1)_{-2/3}, \]
\[ D \sim (1, 3, 1)_{-1/3}, \]
\[ \tilde{D} \sim (1, \bar{3}, 1)_{1/3}, \]

From the fields listed so far, \( SU(3)_{tC} \) has 5 flavors, a theory that is dual to a \( SU(2) \) gauge theory with 5 flavors. This dual theory is likely weakly coupled, so we add another “junk” flavor:

\[ J \sim (3, 1, 1)_{-1/3}, \]
\[ \tilde{J} \sim (3, 1, 1)_{1/3}. \]

These fields have the hypercharges of a vectorlike down quark to avoid fractional electric charges.
3.2 Color Symmetry Breaking

The mass-squared term for the scalars charged under the strong $SU(3)_{tC}$ group must have both positive and negative eigenvalues, as explained in Section 2.1. We assume that the negative eigenvalues result in VEVs of order $M_{\text{SUSY}}$ of the form

$$\langle \Delta \rangle, \langle \tilde{\Delta} \rangle \propto 1_3,$$

resulting in the gauge symmetry breaking pattern

$$SU(3)_{tC} \times SU(3)_{C'} \rightarrow SU(3)_C.$$  (3.7)

3.3 Heavy Fermion Masses

As noted previously, scalar masses of order $M_{\text{SUSY}}$ are automatically generated for all fields. We assume that all scalar mass-squared terms are positive for all fields except for $\Delta$ and $\tilde{\Delta}$. These fields get VEVs, but the excitations at the minimum of the potential all have positive mass-squared by definition. On the other hand, fermion masses can only arise from superpotential terms. In this subsection we describe the superpotential terms required to give all unwanted fermions masses.

The fermion components of the Higgs fields $\Delta, \tilde{\Delta}$ and the “junk” flavors $J, \tilde{J}$ can get mass from superpotential terms

$$\Delta W \sim (\Delta \tilde{\Delta})^2 + (\Delta \tilde{\Delta})(J \tilde{J}).$$  (3.8)

These couplings are marginal at the strong fixed point, and therefore naturally generate fermion masses of order $M_{\text{SUSY}}$. Alternatively, they can get masses from coupling to a singlet field $S$ via

$$\Delta W \sim S(\Delta \tilde{\Delta} + J \tilde{J}).$$  (3.9)

The coupling of these has dimension $\frac{1}{2}$, and can be naturally strong at $M_{\text{SUSY}}$ via the mechanisms discussed in Sections 2.4 and 2.5. The singlet $S$ can naturally get a VEV of order $M_{\text{SUSY}}$, so the mass of the fermionic components of $\Delta$ is naturally of order $M_{\text{SUSY}}$.

There are also a number of weakly-coupled vectorlike fields. These can get masses from ordinary $\mu$ terms

$$\Delta W \sim \Phi_1 \Phi_2 + J \tilde{J} + U \tilde{U} + D \tilde{D}$$  (3.10)

or from couplings to the singlet field $S$ in Eq. (3.9).
3.4 Flavor

We now study the flavor structure of this model. The superpotential interactions that generate the quark masses are

$$\Delta W = (y_u)_{ij} q_i \Phi_1 \tilde{u}_j + y_t q_3 \Phi_1 \tilde{t} + (z_u)_{ij} q_i \Phi_1 \tilde{U} + z_t U \tilde{U} + \mu_U U \tilde{U}$$

$$+ (y_d)_{ij} q_i \Phi_2 d_j + y_b q_3 \Phi_2 \tilde{b} + (z_d)_{ij} q_i \Phi_2 \tilde{D} + z_b D \tilde{D}.$$  \hfill (3.11)

The usual $R$ parity assignments can be extended to the fields in this sector to forbid relevant and marginal $B$ and $L$ violating interactions.

The mass terms $\mu_{U,D}$ are assumed to be of order $M_{\text{SUSY}}$, and can be either explicit mass terms of order $M_{\text{SUSY}}$ (e.g. from the Giudice-Masiero mechanism) or the VEV of the singlet field $S$ in Eq. (3.9). We integrate out these fields to obtain the effective flavor-dependent couplings below the scale $M_{\text{SUSY}}$. We define

$$\Delta_t \equiv z_t \langle \tilde{\Delta} \rangle, \quad \Delta_b \equiv z_b \langle \Delta \rangle,$$

and assume $\Delta_t, b \ll M_{\text{SUSY}}$. We obtain

$$\mathcal{L}_{\text{eff}} = \left[ (y_u)_{ij} q_i \tilde{u}_j + y_t q_3 \tilde{t} - \frac{(z_u)_{ij} \Delta_t}{\mu_U} q_i \tilde{t} \right] \mathcal{H}_2^\dagger$$

$$+ \left[ (y_d)_{ij} q_i \tilde{d}_j + y_b q_3 \tilde{b} - \frac{(z_d)_{ij} \Delta_b}{\mu_D} q_i \tilde{b} \right] \mathcal{H}_1^\dagger,$$

where we have defined

$$\mathcal{H}_1 = \frac{y_{TC1}}{M_{\text{SUSY}}} \frac{\psi_1}{\psi_{\tilde{1}}}, \quad \mathcal{H}_2 = \frac{y_{TC2}}{M_{\text{SUSY}}} \frac{\psi_{\tilde{2}}}{\psi_2}.$$  \hfill (3.15)

We see that this contains mixing between the third generation and the first two generations.

This theory has flavor violation beyond minimal flavor violation. To see this, we use a language where the heavy fermions mix with the light fermions (rather than an effective field theory language where the heavy fermions are integrated out and their effects are parameterized by higher-dimension operators). The fermion mass terms are

$$\Delta \mathcal{L} = (m_u)_{ij} u_i \tilde{u}_j + m_t t \tilde{t} + \mu_U U \tilde{U} + \Delta_t U \tilde{t} + (\delta_u)_{ij} u_i \tilde{U}$$

$$+ (m_d)_{ij} d_i \tilde{d}_j + m_b b \tilde{b} + \mu_D D \tilde{D} + \Delta_b D \tilde{D} + (\delta_d)_{ij} d_i \tilde{D}.$$  \hfill (3.16)

where

$$(m_u)_{ij} = (y_u)_{ij} \langle \mathcal{H}_2 \rangle, \quad m_t = y_t \langle \mathcal{H}_2 \rangle, \quad (\delta_u)_{ij} = (z_u)_{ij} \langle \mathcal{H}_2 \rangle,$$

$$(m_d)_{ij} = (y_d)_{ij} \langle \mathcal{H}_1 \rangle, \quad m_b = y_b \langle \mathcal{H}_1 \rangle, \quad (\delta_d)_{ij} = (z_d)_{ij} \langle \mathcal{H}_1 \rangle.$$  \hfill (3.17)
We naturally have

\[ m_{u,d}, \delta_{u,d} \ll \Delta_{u,d}, \mu_{U,D} \quad (3.19) \]

since the terms on the left are proportional to the electroweak symmetry breaking scale, while those on the right are proportional to \( M_{\text{SUSY}} \). In this language, the new flavor-violating effects come from mixing with the “fourth generation” fields \( U, \tilde{U}, D, \tilde{D} \). Non-minimal flavor violation for the light generations is therefore suppressed by the fact that the flavor violation arises from mixing through a heavy fourth generation messenger.

We first discuss the constraints from \( \Delta F = 2 \) processes. We focus on the down sector, which is the most sensitive. The largest effect in this model comes from exchange of the massive color octet gauge bosons that arise from \( SU(3)_{tC} \times SU(3)_{C'} \to SU(3)_C \). These gives flavor-violating effects because the 3rd generation fields \( d_i, \tilde{d}_i \) have different \( SU(3)_{tC} \times SU(3)_{C'} \) quantum numbers from \( b, \tilde{b} \). The heavy gauge eigenstates are approximately the \( SU(3)_{tC} \) gauge bosons because these are strongly coupled. The effective Lagrangian below the scale \( M_{\text{SUSY}} \) therefore contains the flavor-violating terms

\[ \Delta L_{\text{eff}} \sim \frac{1}{(\Delta)^2} \text{tr} (J_{tC}^\mu)^2 \quad (3.20) \]

where

\[ (J_{tC}^\mu)_A = \sum_{I,J=1}^3 \left[ (V_d)_{3I}^* (V_d)_{3J} d_I'^\dagger \sigma^\mu T_A d'_J - (V_{\tilde{d}})_{3I}^* (V_{\tilde{d}})_{3J} \tilde{d}_I'^\dagger \sigma^\mu T_A \tilde{d}_J \right], \quad (3.21) \]

where \( T_A \) are \( SU(3) \) generators, and the primed fields are the mass eigenstates:

\[ d'_I = (V_d)_{I,J} d_J, \quad \tilde{d}'_I = (V_{\tilde{d}})_{I,J} \tilde{d}_J, \quad (3.22) \]

with \( I, J = 1, \ldots, 4 \). The flavor-violating effects are suppressed by the scale \( M_{\text{SUSY}} \), and require mixing into and out of the third generation. This cannot be too strongly suppressed if we hope to reproduce the observed third generation mixing.

To estimate the mixing angles, we have found mass parameters that give an approximate fit to the observed masses and CKM matrix elements. A sample point of our viable space is given in Table 1. The resulting CKM matrix is found to be

\[ V_{\text{CKM}} \simeq \begin{pmatrix} 0.97 & 0.24 & 0.003 \\ 0.24 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix}. \quad (3.23) \]
| Up Sector | Down Sector |
|-----------|-------------|
| \( (m_u)_{11} \) | \( (m_d)_{11} \) | \( 4.6 \times 10^{-3} \) | \( 8 \times 10^{-3} \) |
| \( (m_u)_{12} \) | \( (m_d)_{12} \) | \( 7 \times 10^{-2} \) | \( 3.8 \times 10^{-2} \) |
| \( (m_u)_{21} \) | \( (m_d)_{21} \) | \( 6.1 \times 10^{-2} \) | \( 3.6 \times 10^{-2} \) |
| \( (m_u)_{22} \) | \( (m_d)_{22} \) | \( 1.5 \) | \( 0.13 \) |
| \( m_t \) | \( m_b \) | \( 160 \) | \( 5.7 \) |
| \( (\delta_u)_1 \) | \( (\delta_d)_1 \) | \( 4.4 \) | \( 1.1 \) |
| \( (\delta_u)_2 \) | \( (\delta_d)_2 \) | \( 3.2 \) | \( 2.8 \) |
| \( \mu_U \) | \( \mu_D \) | \( 14.2 \times 10^3 \) | \( 11.7 \times 10^3 \) |
| \( \Delta_t \) | \( \Delta_b \) | \( 9.5 \times 10^3 \) | \( 1.5 \times 10^3 \) |

Table 1. Values for quark masses (in GeV) that give an approximate fit to the CKM matrix. The corresponding CKM matrix is given in Eq. (3.23).

All values are within 10\% or less of their experimental values, which is sufficient for the estimates we make here.

We then find

\[
(V_d)_{31} \sim \frac{(\delta_d)_1 \Delta_b}{\mu_D m_b} \simeq 2 \times 10^{-2},
\]

(3.24)

\[
(V_d)_{32} \sim \frac{(\delta_d)_2 \Delta_b}{\mu_D m_b} \simeq 6 \times 10^{-2},
\]

(3.25)

\[
(V_d)_{31} \sim \frac{(\delta_d)_1 \Delta_b m_d}{\mu_D m_b} \simeq 4 \times 10^{-4},
\]

(3.26)

\[
(V_d)_{32} \sim \frac{(\delta_d)_2 \Delta_b m_s}{\mu_D m_b} \simeq 2 \times 10^{-3}.
\]

(3.27)

Note that right-handed mixing is suppressed relative to that for left-handed fields. The reason for this is simply that the fields \( \bar{d}_i \) are \( SU(2)_W \) singlets, and so mixing into the singlet fourth generation through their coupling to \( \Phi \) first requires a mass insertion for the light field. This is in contrast to the left-handed case, where the mass insertion must be included for the external \textit{heavy} field.

The strongest bound from these interactions comes from the \( B_s - \bar{B}_s \) mass splitting, specifically the effective operator \( (b^\dagger \sigma^\mu \sigma^\nu s)^2/\Lambda^2 \), which requires \( \Lambda \gtrsim 130 \text{ TeV} \) [24]. To accomplish this with the parameters given in Table 1, we need \( \langle \Delta \rangle \gtrsim 8 \text{ TeV} \).

Another potentially large contribution comes from tree-level \( Z \) exchange. As we will show, this gives very weak bounds because it requires mixing into and out of the
fourth generation. The $Z$ coupling is

$$\mathcal{L}_{\text{int}} = g Z_\mu J_\mu^Z$$  \hspace{1cm} (3.28)$$

with

$$J_\mu^Z = \frac{1}{2}(\cos\theta_W - \sin\theta_W)(V_d)_i^* (V_d)_4 J d_i^\dagger \sigma^\mu d_j^\dagger + \text{diagonal terms},$$  \hspace{1cm} (3.29)$$

where we have used the unitarity of the $4 \times 4$ mixing matrix $V_d$. The flavor-changing effective Lagrangian is therefore

$$\Delta \mathcal{L}_{\text{eff}} = -\frac{g^2}{8 m_Z^2} (\cos\theta_W - \sin\theta_W)^2 (V_d)_i^* (V_d)_4 J (V_d)_4^* (V_d)_4 L \times (d_i^\dagger \sigma^\mu d_j^\dagger)(d_K^\dagger \sigma^\mu d_L).$$  \hspace{1cm} (3.30)$$

The mixing angles to the fourth generation are

$$\langle (V_d)_4 \rangle \sim \frac{(\delta_d)_i}{\mu_D} \sim 10^{-4},$$

$$\langle (V_d)_3 \rangle \sim \frac{m_b \Delta_b}{\mu_D^2} \sim 5 \times 10^{-5}.$$  \hspace{1cm} (3.31)$$

The strongest bounds for these operators come from the $K$ system. In particular, the imaginary part of the operator $(s^\dagger \sigma^\mu d)^2/\Lambda^2$ must satisfy $\Lambda \gtrsim 1.7 \times 10^4$ GeV. In the current case we see that such a limit provides no real constraint, as the relevant mixing angles give an amplitude proportional to $(\delta_d/\mu_D)^4 \sim 10^{-16}$. We conclude that there is no strong bound from these effects.

We have found that the strongest flavor bound on this model is $\langle \Delta \rangle \gtrsim 8$ TeV. We would like to translate this to a bound on $M_{\text{SUSY}}$. The field $\Delta$ has a quartic interaction of order $g t_C^2 \sim 16\pi^2$, so we have

$$\langle \Delta \rangle \sim \frac{M_{\text{SUSY}}}{4\pi},$$  \hspace{1cm} (3.32)$$

since $M_{\text{SUSY}}$ gives the scale of the SUSY breaking mass-squared terms. We conclude that the bound is roughly $M_{\text{SUSY}} \gtrsim 100$ TeV.

We close this section by noting that there are flavor-dependent quartic superpotential interactions

$$\Delta W \sim (q_3 \tilde{t})(q_3 \tilde{b})$$  \hspace{1cm} (3.33)$$

that are marginal. If these are unsuppressed at the scale $M_{\text{SUSY}}$ they can lead to dangerous flavor-changing effects. However, these terms can be forbidden by symmetries
that also forbid the “μ term” $\Phi_1 \Phi_2$. We expect such symmetries to be approximate symmetries of the fundamental theory to explain why the μ term is of order $M_{\text{SUSY}}$. We therefore expect that these operators will be strongly suppressed in the fundamental theory. In any case, it is technically natural for the terms Eq. (4.21) to be suppressed since they are superpotential interactions.

3.5 New Yukawa Fixed Points

Now that we have specified the top sector, we revisit the question of why the top quark Yukawa coupling is strong at the scale $M_{\text{SUSY}}$. The most natural explanation is that the top quark Yukawa coupling is at a strongly-coupled fixed point. Assuming that both $y_t$ and $y_{TC1}$ flow to fixed points, $a$-maximization (discussed in Section 2.5) gives the dimensions of the chiral fields:

$$
\begin{align*}
  d(\Psi) &= 0.77, & d(\Psi_2) &= 0.80, & d(\tilde{\Psi}_1) &= 0.74, & d(\Sigma) &= 0.74, \\
  d(q_3) &= 0.77, & d(\tilde{t}) &= 0.80, & d(\Phi_1) &= 1.44, & d(\Phi_2) &= 1, \\
  d(\tilde{b}) &= d(J) = d(\Delta) = 0.74.
\end{align*}
$$

(3.34)

Note that the dimensions of the strongly coupled fields are close to $\frac{3}{4}$, their value in the fixed point with vanishing Yukawa couplings, while $\Phi_1$ has a large anomalous dimension. This is consistent with the assumption that the fixed point is strongly coupled.

Since $d(\Phi_1) = 1.44$, up-type Yukawa couplings of the first two generations are irrelevant while the down-type Yukawa couplings are marginal. This fixed point cannot persist up to arbitrarily high scales, otherwise the charm quark mass is too small. We denote the scale where $\Phi_1$ gets a large anomalous dimension by $\Lambda_1$. Since the charm Yukawa is an ordinary dimensionless coupling at scale above $\Lambda_1$, we require $y_c(\Lambda_1) \lesssim 1$. Scaling down to $M_{\text{SUSY}}$ then gives

$$
  y_c(\Lambda_1) \lesssim \left( \frac{M_{\text{SUSY}}}{\Lambda_1} \right)^{0.42},
$$

(3.35)

and the charm mass is given by

$$
  m_c \sim m_t \frac{y_c(M_{\text{SUSY}})}{y_t(M_{\text{SUSY}})}.
$$

(3.36)

Assuming $y_t(M_{\text{SUSY}}) \sim 4\pi$, we find the bound

$$
  \frac{\Lambda_1}{M_{\text{SUSY}}} \lesssim \left( \frac{m_t}{4\pi m_c} \right)^{1/0.42} \sim 10^3.
$$

(3.37)
We see that the scale $\Lambda_1$ cannot be too far from the SUSY breaking scale, so this new fixed point does not completely eliminate the need for an explanation why relevant couplings get strong near the SUSY breaking scale (e.g. the hidden sector discussed in Section 2.4). However, these fixed points mean that the coincidence is only within a few orders of magnitude in the scale.

If we assume that $y_{TC2}$ also flows to a strong fixed point, we find

$$\begin{align}
d(\Psi) &= 0.80, \quad d(\tilde{\Psi}_2) = 0.77, \quad d(\tilde{\Psi}_1) = 0.83, \quad d(\Sigma) = 0.72, \\
d(q_3) &= 0.77, \quad d(\tilde{t}) = 0.80, \quad d(\Phi_1) = 1.42, \quad d(\Phi_2) = 1.37, \\
d(\tilde{b}) &= d(J) = d(\Delta) = 0.73.
\end{align}$$ (3.38)

Now the Yukawa couplings for the leptons and the first two generations of up- and down-type quarks are irrelevant with roughly the same dimension ($d(y) \simeq -0.4$).

It is an appealing feature of this model that the masses of the light fermions are suppressed because they arise from irrelevant couplings. This mechanism for flavor in conformal field theories was explored in Ref. [25] and also in the context of warped extra dimensions (dual to conformal field theories) in Refs. [26]. As with the previous fixed point, this fixed point cannot be valid up to arbitrarily high scales.

We might ask whether the second fixed point requires $y_t$ and $y_{TC1}$ to become strong at precisely the same scale. For example, if $y_{TC1}$ becomes strong at a higher scale, we have the fixed point discussed in Section 2.5. In that fixed point $d(q_3\tilde{t}\Phi_1) = 2.87$, so the top coupling is still (barely) relevant, and therefore continues to grow in the IR. When it gets strong, we reach the second fixed point discussed above.

Another issue that arises in these new fixed points is the size of the “$\mu$ term” $\Delta W \sim \Phi_1\Phi_2$ that gives rise to the mass of the fermion components of $\Phi_{1,2}$. As above, we assume that $\Phi_1$ gets a large anomalous dimension at the scale $\Lambda_1$, while $\Phi_2$ gets a large anomalous dimension at the scale $\Lambda_2 < \Lambda_1$. The size of the fermion mass term at the SUSY breaking scale is then

$$\mu \sim \frac{M_{SUSY}^2}{\Lambda_1} \left( \frac{\Lambda_1}{\Lambda_2} \right)^{0.56} \left( \frac{\Lambda_2}{M_{SUSY}} \right)^{0.21}.$$ (3.39)

We must have $\mu \gtrsim 100$ GeV to avoid conflict with experiment. To get a feeling for the bounds, we consider the case $\Lambda_1 = \Lambda_2$ for simplicity. We then obtain

$$\frac{\Lambda_{1,2}}{M_{SUSY}} \lesssim \left( \frac{M_{SUSY}}{100 \text{ GeV}} \right)^{1/0.79} \sim 5 \times 10^3 \left( \frac{M_{SUSY}}{100 \text{ TeV}} \right)^{1.26}.$$ (3.40)

We see that these bounds are similar to the ones that come from requiring that the charm quark mass be sufficiently large.

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Finally, note that the superpotential couplings Eq. (3.8) that give masses to the fermion components of $\Delta, \tilde{\Delta}, J, \tilde{J}$ are no longer exactly marginal in the new fixed points discussed here. However, they are only slightly relevant (e.g. the coupling has mass dimension $+0.08$ for the second fixed point), so it is still natural for these couplings to generate fermion masses of order $M_{\text{SUSY}}$.

4 Supersymmetric Extended Color

We now describe a different model in which all three generations of SM quarks are charged under the same color group above $M_{\text{SUSY}}$. These models involve introducing additional colors of quarks, so we refer to them as “extended color” models.

4.1 Field content

The gauge group of the color sector is

$$SU(6)_{\text{EC}} \times SU(3)_{\text{C1}} \times SU(3)_{\text{C2}} \times SU(2)_W \times U(1)_Y$$

where only $SU(6)_{\text{EC}}$ is strong at the scale $M_{\text{SUSY}}$. The quark fields are contained in the fields

$$Q_i \sim (6, 1, 1, 2)_{\frac{1}{6}},$$
$$\bar{U}_i \sim (6, 1, 1, 1)_{-\frac{2}{3}},$$
$$\bar{D}_i \sim (6, 1, 1, 1)_{\frac{1}{3}},$$

where $i = 1, 2, 3$ runs over all three generations. The breaking of the color group is accomplished by the Higgs fields

$$\Delta_1 \sim (6, 3, 1, 1)_0,$$
$$\tilde{\Delta}_1 \sim (6, 3, 1, 1)_0,$$
$$\Delta_2 \sim (6, 1, 3, 1)_0,$$
$$\tilde{\Delta}_2 \sim (6, 1, 3, 1)_0.$$  

To give masses to the extra color components of the quarks in Eq. (4.2) we introduce three generations of “partner quarks” charged under $SU(3)_{\text{C2}}$:

$$\tilde{Q}_i \sim (1, 1, 3, 2)_{-\frac{1}{6}},$$
$$U_i \sim (1, 1, 3, 1)_{\frac{2}{3}},$$
$$D_i \sim (1, 1, 3, 1)_{-\frac{1}{3}},$$
where \( i = 1, 2, 3 \).

It is easy to check that with the above field content added to three generations of leptons in the MSSM, all gauge anomalies cancel. The \( SU(6)_\text{EC} \) gauge group has 12 flavors, just what we need for a strong conformal fixed point. The gauge groups \( SU(3)_{C1} \times SU(3)_{C2} \) are assumed to be weakly coupled at the SUSY breaking scale.

### 4.2 Color Symmetry Breaking

As explained in Section 2.1, the mass-squared terms for scalars charged under the strong \( SU(6)_\text{EC} \) must have both positive and negative eigenvalues. We assume that this gives rise to VEVs of order \( M_{\text{SUSY}} \) for the Higgs fields,

\[
\langle \Delta_1 \rangle, \langle \tilde{\Delta}_1 \rangle \propto \begin{pmatrix} 1_3 \\ 0_3 \end{pmatrix}, \quad \langle \Delta_2 \rangle, \langle \tilde{\Delta}_2 \rangle \propto \begin{pmatrix} 0_3 \\ 1_3 \end{pmatrix}. \tag{4.5}
\]

This breaks

\[
SU(6)_\text{EC} \times SU(3)_{C1} \times SU(3)_{C2} \rightarrow SU(3)_C \times SU(3)_{C'} \tag{4.6}
\]

The unbroken gauge groups \( SU(3)_C \times SU(3)_{C'} \) are weakly coupled below \( M_{\text{SUSY}} \). The ordinary quarks carry \( SU(3)_C \), which is therefore identified with color.

There are no light particles charged under the unbroken \( SU(3)_{C'} \). This gauge group confines at a scale below \( M_{\text{SUSY}} \), and the heavy particles charged under \( SU(3)_{C'} \) are “quirks” [27] with masses of order \( M_{\text{SUSY}} \). The weak gauging of \( SU(3)_{C'} \) does not play any important role in this model, and we can turn off this gauging and break the global \( SU(3)_{C'} \) symmetry without any important consequences for physics below \( M_{\text{SUSY}} \).

### 4.3 Heavy Fermion Masses

As discussed in Section 3.3, SUSY breaking gives masses of order \( M_{\text{SUSY}} \) to all scalars, but we need superpotential interactions to give masses to unwanted fermion fields.

The “extra” \( SU(6)_\text{EC} \) colors of the quark fields \( \tilde{Q}_i, \tilde{U}_i, \tilde{D}_i \) get masses from

\[
\Delta W = (z_Q)_{ij} \tilde{Q}_i \tilde{\Delta}_2 \tilde{Q}_j + (z_u)_{ij} \tilde{U}_i \Delta_2 U_j + (z_d)_{ij} \tilde{D}_i \Delta_2 D_j. \tag{4.7}
\]

The VEVs of \( \Delta_2, \tilde{\Delta}_2 \) then give masses to these particles proportional to \( M_{\text{SUSY}} \). These couplings have dimension \(+\frac{1}{2}\) at the strong fixed point, so they can naturally become strong near \( M_{\text{SUSY}} \) via the mechanism described in Section 2.4. These new couplings violate flavor, and their impact on flavor physics will be discussed below.
The fermion components of the Higgs fields $\Delta_1, \tilde{\Delta}_1, \Delta_2, \tilde{\Delta}_2$ can get masses from the superpotential terms

$$\Delta W \sim (\Delta_1 \tilde{\Delta}_1)^2 + (\Delta_2 \tilde{\Delta}_2)^2 + (\Delta_1 \tilde{\Delta}_1)(\Delta_2 \tilde{\Delta}_2).$$

These terms are marginal at the fixed point we are considering. Alternatively, they can arise from coupling to a singlet field $S$:

$$\Delta W \sim S \Delta_1 \tilde{\Delta}_1 + S \Delta_2 \tilde{\Delta}_2.$$  \hfill (4.9)

The coupling of these terms has dimension $+\frac{1}{2}$, so they can also naturally become strong near the scale $M_{\text{SUSY}}$. Alternatively, these couplings can themselves be at a strong fixed point, as discussed in Section 2.5, and discussed further below.

4.4 Flavor

The quark masses arise from the Yukawa interactions

$$\Delta W = (y_u)_{ij} Q_i \Phi_1 \tilde{U}_j + (y_d)_{ij} Q_i \Phi_1 \tilde{D}_j.$$  \hfill (4.10)

In addition, the superpotential couplings Eq. (4.7) mix the quarks with heavy fermions at the scale $M_{\text{SUSY}}$. This generally gives rise to new flavor violation, which we discuss in this section.

The fields $Q_i, \tilde{U}_i, \tilde{D}_i$ decompose as

$$Q_i = q_i + Q_i, \quad \tilde{U}_i = \tilde{u}_i + \tilde{U}_i, \quad \tilde{D}_i = \tilde{d}_i + \tilde{D}_i,$$  \hfill (4.11)

where $q_i, \tilde{u}_i, \tilde{d}_i$ are the ordinary quark fields with $SU(6)_{\text{EC}}$ indices 1, 2, 3, and $Q_i, \tilde{U}_i, \tilde{D}_i$ are heavy fields with $SU(6)_{\text{EC}}$ indices 4, 5, 6. All of these fields are triplets under the unbroken $SU(3)_C$. We further write the $SU(2)_W$ doublets as

$$q_i = \left( \begin{array}{c} u_i \\ d_i \end{array} \right), \quad Q_i = \left( \begin{array}{c} T_i \\ B_i \end{array} \right), \quad \tilde{Q}_i = \left( \begin{array}{c} \tilde{T}_i \\ \tilde{B}_i \end{array} \right).$$  \hfill (4.12)

As in Section 3.4, we find it convenient to use a language where we do not integrate out the heavy fermions. The mass matrix for the colored fields is then

$$W_{\text{mass}} = (\Delta_Q)_{ij} \left[ T_i T_j + B_i \tilde{B}_j \right] + (\Delta_u)_{ij} \tilde{U}_i U_j + (\Delta_d)_{ij} \tilde{D}_i D_j + (m_u)_{ij} \left[ u_i \tilde{u}_j + T_i \tilde{T}_j \right] + (m_d)_{ij} \left[ d_i \tilde{d}_j + B_i \tilde{B}_j \right],$$  \hfill (4.13)

where

$$(\Delta_Q)_{ij} = (z_Q)_{ij} \langle \tilde{\Delta}_2 \rangle, \quad (\Delta_u)_{ij} = (z_u)_{ij} \langle \Delta_2 \rangle, \quad (\Delta_d)_{ij} = (z_d)_{ij} \langle \Delta_2 \rangle.$$  \hfill (4.14)
and $m_{u,d}$ are given by Eq. (3.17).

There are $SU(6)_{EC}$ gauge bosons that give rise to transitions between ordinary quarks and heavy fermions. The mass terms $\Delta_{Q,u,d}$ for the heavy fermions are not generally diagonal in the same basis as the Yukawa couplings, so this leads to flavor violation. The most sensitive effects come from box diagrams with two $SU(6)_{EC}$ gauge boson exchange, which give e.g.

$$\Delta L_{\text{eff}} \sim 16\pi^2 \frac{[(\Delta_Q)_{12}]^2}{M_{\text{SUSY}}^4} \left( s^\dagger \sigma^\mu d \right)^2,$$

where $\Delta_Q$ is evaluated in the mass basis. We have used $g_6 \sim 4\pi$ and used $M_{\text{SUSY}}$ for the mass of the heavy gauge bosons. This requires a high degree of alignment. Assuming the effective operator $(s^\dagger \sigma^\mu d)^2 / \Lambda^2$ has an unsuppressed imaginary part, we require $\Lambda \gtrsim 10^4 \text{ TeV}$ [24], giving

$$\frac{(\Delta_Q)_{12}}{M_{\text{SUSY}}} \lesssim \frac{M_{\text{SUSY}}}{10^5 \text{ TeV}}.$$  

For example, for $M_{\text{SUSY}} \sim 100 \text{ TeV}$ we require alignment $\sim 10^{-3}$. This bound may be somewhat weakened if the new physics phase is aligned with the standard model phase. If we only use the bound for the $CP$-conserving part of the operator, the bound is weaker by an order of magnitude.

We now argue that this high degree of alignment can arise very naturally. Suppose that at some high scale there is a (gauged or approximate global) flavor symmetry

$$SU(3)_Q \times SU(3)_{\tilde{U}} \times SU(3)_{\tilde{D}},$$  

(4.17)

with

$$Q \sim (3,1,1),$$  

$$\tilde{U} \sim (1,\bar{3},1),$$  

$$\tilde{D} \sim (1,1,\bar{3}),$$  

$$\tilde{Q} \sim (\bar{3},1,1),$$  

$$U \sim (1,3,1),$$  

$$D \sim (1,1,3).$$  

(4.18)

Then the couplings $z_{Q,u,d}$ (and hence the masses $\Delta_{Q,u,d}$) do not violate the flavor symmetry Eq. (4.17) as long as they are proportional to the identity matrix in flavor
space. The ordinary Yukawa couplings violate the flavor symmetry; they can be viewed as spurions transforming under the flavor symmetry as

\[ y_u \sim (\bar{3}, 3, 1), \quad y_d \sim (\bar{3}, 1, 3). \]  \hfill (4.19)

Even if these are the only spurions that break the flavor symmetries, there can be contributions to the couplings \( z_{Q,u,d} \) that are not proportional to the identity matrix, e.g.

\[ z_Q \sim (\bar{3} \times 3, 1, 1) \sim 1 + y_u y_u^\dagger + y_d y_d^\dagger + \cdots \]  \hfill (4.20)

However, these contributions are diagonal in the basis that diagonalizes the Yukawa couplings. This is precisely the condition for minimal flavor violation. Constructing detailed models of flavor is beyond the scope of the present work, but we believe that this flavor structure is quite reasonable.

There are also flavor-dependent quartic superpotential interactions of the form

\[ \Delta W \sim (\bar{Q} \bar{U})(\bar{Q} \bar{D}) \]  \hfill (4.21)

that are marginal. If the flavor symmetry Eq. (4.17) is broken only by the Yukawa coupling spurions Eq. (4.19) then these will also be diagonal in the basis where the Yukawa interactions are diagonal, and therefore pose no problems for flavor. In any case, it is technically natural for the terms Eq. (4.21) to be suppressed since they are superpotential interactions.

4.5 New Yukawa Fixed Points

We now address the question of whether Yukawa couplings involving strongly coupled fields (such as the top quark) can run to strong fixed point values above the scale \( M_{\text{SUSY}} \). Assuming that such Yukawa couplings do run to a fixed point, we can use \( a \)-maximization (discussed in Section 2.5) to find the dimensions of the chiral fields in the theory. Assuming that \( y_t \) and \( y_{TC1} \) are strong, we have

\[
\begin{align*}
d(\bar{\Psi}) &= 0.76, \quad d(\bar{\Psi}_2) = 0.78, \quad d(\bar{\Psi}_1) = 0.74, \quad d(\Sigma_u) = d(\bar{\Sigma}_u) = 0.74, \\
d(Q_3) &= 0.76, \quad d(\bar{U}_3) = 0.78, \quad d(\Phi_1) = 1.45, \quad d(\Phi_2) = 1, \\
d(\Delta) &= d(Q_{1,2}) = d(\bar{D}_{1,2,3}) = d(\bar{U}_{1,2}) = 0.75.
\end{align*}
\hfill (4.22)
\]

In this fixed point the lepton Yukawa couplings are marginal, so there is no problem getting lepton masses. The Yukawa couplings of the light up-type quarks are just
barely relevant \(d(y_{u,c}) = 0.06\), while the Yukawa couplings of all three generations of down-type quarks are very relevant \(d(y_{d,s,b}) \approx 0.5\). It is consistent for this fixed point to exist all the way up to the Planck scale, but then it would be a complete coincidence that the up-type and down-type quark masses have similar sizes (within an order of magnitude or two). Also, the dimension of the Higgsino mass term \(\Phi_1\Phi_2\) is 1.45, so there is no reason for the Higgsino mass to be near \(M_{\text{SUSY}}\). It is more natural to assume that this fixed point is reached at a scale \(\Lambda_1\) not too far above \(M_{\text{SUSY}}\). If we assume that above \(\Lambda_1\) there is an ordinary Higgsino mass term \(\Phi_1\Phi_2\) of order \(M_{\text{SUSY}}\), then there is a bound on the scale \(\Lambda_1\) from requiring that the Higgsino be heavier than 100 GeV, but this bound is very weak:

\[
\frac{\Lambda_1}{M_{\text{SUSY}}} \lesssim \left( \frac{M_{\text{SUSY}}}{100 \text{ GeV}} \right)^{1/0.45} \approx 10^6 \left( \frac{M_{\text{SUSY}}}{100 \text{ TeV}} \right)^{2/2}.
\]

(4.23)

Requiring that the quark masses arise naturally is probably more restrictive, but it is difficult to quantify.

We can also consider a fixed point where \(y_t, y_{TC1}\) and \(y_{TC2}\) are all at strong fixed points. In this case we find

\[
d(\Psi) = 0.80, \quad d(\bar{\Psi}_2) = 0.76, \quad d(\bar{\Psi}_1) = 0.84, \quad d(\Sigma_a) = d(\bar{\Sigma}_a) = 0.73,
\]

\[
d(Q_3) = 0.77, \quad d(\bar{U}_3) = 0.79, \quad d(\Phi_1) = 1.45, \quad d(\Phi_2) = 1.37,
\]

\[
d(\Delta) = d(Q_{1,2}) = d(\bar{D}_{1,2,3}) = d(\bar{U}_{1,2}) = 0.75.
\]

(4.24)

Now both the up- and down-type Yukawa interactions are barely relevant, but the lepton Yukawa couplings are irrelevant. Therefore, this fixed point cannot persist up to arbitrarily high scales. As in Section 3.5 this gives a bound on the scale \(\Lambda_2\) where this new fixed point is reached:

\[
\frac{\Lambda_2}{M_{\text{SUSY}}} \lesssim \left( \frac{m_t}{4\pi m_r} \right)^{1/0.37} \approx 150.
\]

(4.25)

We conclude that neither of these fixed points can be valid to arbitrarily high scales (although for the first it is a matter of naturalness). We still need an explanation of why relevant couplings get strong near the SUSY breaking scale (e.g. the hidden sector discussed in Section 2.4). But now there is much less of a coincidence to explain.

5 PNGB Phenomenology

All but one of the fields in the SUSY extensions to MCTC discussed in Sections 2–4 can get masses of order \(M_{\text{SUSY}}\), and therefore do not affect the phenomenology
at the TeV scale. The one important exception is the $SU(2)_{CTC}$ component of the technicolor gaugino, which we call $\lambda$. (See Section 2.6 for a discussion of the effective theory below the SUSY breaking scale.) The gaugino mass term $\lambda\lambda$ has a large positive anomalous dimension in the SUSY conformal theory above the scale $M_{\text{SUSY}}$ [15]. If SUSY is broken in a hidden sector at a scale far above $M_{\text{SUSY}}$ and communicated to the visible sector by weak interactions, the coefficient of $\lambda\lambda$ will be very suppressed compared to soft SUSY breaking terms with canonical dimensions. Below the scale $M_{\text{SUSY}}$, the operator $\lambda\lambda$ matches onto an operator in the non-SUSY conformal technicolor theory, and we do not know the scaling dimension of this operator. However, it is very plausible that the $\lambda\lambda$ operator is still suppressed at the scale $\Lambda_{CTC}$ where conformal symmetry is broken. For example in gravity-mediated SUSY breaking, SUSY is broken at the intermediate scale $(M_{\text{SUSY}}M_P)^{1/2}$, while flavor bounds and the top quark mass suggest $M_{\text{SUSY}} \sim 100$ TeV. In this case, the suppression of $\lambda\lambda$ above $M_{\text{SUSY}}$ occurs over 7 orders of magnitude, while the scaling between $M_{\text{SUSY}}$ and $\Lambda_{CTC}$ is two orders of magnitude. While we cannot claim that a light gaugino at the scale $\Lambda_{CTC}$ is an absolute prediction of this framework, it appears to be rather generic in this framework. We stress that this is not just a consequence of the specific models presented here, but is expected in any model where conformal technicolor is completed by a strong SUSY conformal theory. As we have already argued, this is strongly motivated by the top quark mass. In this section, we therefore explore the phenomenological consequences if the operator $\lambda\lambda$ is suppressed at the scale $\Lambda_{CTC}$ where conformal symmetry is broken.

In this case, the strong dynamics at the scale $\Lambda_{CTC}$ have an additional, approximate, anomaly-free $U(1)_\lambda$ global symmetry:

$$\lambda \mapsto e^{-i\alpha}\lambda, \quad \psi \mapsto e^{-i\alpha}\psi, \quad \bar{\psi} \mapsto e^{-i\alpha}\bar{\psi}. \quad (5.1)$$

We expect that $U(1)_\lambda$ will be spontaneously broken by a gaugino condensate $\langle \lambda\lambda \rangle \neq 0$, so there is an associated PNGB that we call $\eta$. In addition, there is an approximate global $SU(4)$ chiral symmetry acting on the fields

$$\Psi^a = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}, \quad (5.2)$$

that is spontaneously broken to $Sp(4)$ by a condensate $\langle \Psi^a\Psi^b \rangle = -\langle \Psi^b\Psi^a \rangle$. The scale of electroweak symmetry breaking depends on a vacuum alignment angle $\theta$, defined by

$$\langle \Psi^a\Psi^b \rangle \propto \Phi^{ab} = \begin{pmatrix} \cos \theta \epsilon & \sin \theta \, 1_2 \\ -\sin \theta \, 1_2 & -\cos \theta \epsilon \end{pmatrix}, \quad (5.3)$$
where
\[
\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\] (5.4)

Electroweak symmetry is broken at a scale
\[
v = f \sin \theta = 246 \text{ GeV}.
\] (5.5)

An important point is that the theory has an unbroken \( SU(2) \) custodial symmetry for all values of \( \theta \) [3]. For \( v \ll f \) we have a composite Higgs model [28]. The scale \( f \) is related to the scale of conformal symmetry breaking via \( \Lambda_{\text{CTC}} \sim 4\pi f \).

There are 2 physical PNGB fields in \( SU(4)/Sp(4) \) that can be parameterized by
\[
\xi = e^{i\Pi/f}, \quad \Pi = hX_h + AX_A
\] (5.6)
where
\[
X_h = -\frac{i}{2} \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}, \quad X_A = \frac{1}{2} \begin{pmatrix} \cos \theta & 1_2 & \sin \theta \epsilon \\ \sin \theta \epsilon & -\cos \theta & 1_2 \end{pmatrix}.
\] (5.7)

The field \( h \) is \( CP \)-even, while \( A \) and \( \eta \) are \( CP \)-odd. For \( v \ll f \), \( h \) is a composite Higgs. We denote the additional PNGB arising from the spontaneous breaking of \( U(1)_{\lambda} \) by \( \eta \). It transforms as
\[
\eta \mapsto \eta + 2\alpha f_\eta
\] (5.8)
where \( f_\eta \sim f \) is the decay constant of the PNGB. The field \( \eta \) is \( CP \)-odd, so \( \eta \) and \( A \) will mix in general.

The potential for the PNGBs arises from interactions that explicitly break the \( SU(4) \times U(1)_{\lambda} \) symmetry. We follow Ref. [3] and consider the case where the PNGB potential is dominated by top quark loops and technifermion mass terms. Justification and more detailed discussion of the potential can be found in that paper. We focus on the \( CP \)-odd PNGBs, since the potential for \( h \) is unchanged from Ref. [3]. The technifermion mass terms Eq. (2.28) break \( U(1)_{\lambda} \), and therefore give mass to the \( \eta \). Defining
\[
\mathcal{K} = \begin{pmatrix} \kappa \epsilon & 0 \\ 0 & \bar{\kappa} \epsilon \end{pmatrix},
\] (5.9)
the leading term in the potential is
\[
V_{\text{mass}} = \frac{1}{4} \hat{C}_\kappa \text{tr}(\mathcal{K} \Phi \Phi^T) e^{i\eta/f_\eta} + \text{h.c.}
\] (5.10)
\[
= \hat{C}_\kappa \left[ (\kappa - \bar{\kappa}) \cos \theta \left( \frac{A^2}{f^2} + \frac{\eta^2}{f_\eta^2} \right) + (\kappa + \bar{\kappa}) \frac{A\eta}{ff_\eta} + \cdots \right].
\] (5.11)
Here we assume that $\kappa, \tilde{\kappa}$ are real, and

$$\hat{C}_\kappa \sim \frac{\Lambda_{CTC}^d}{16\pi^2}. \tag{5.12}$$

The potential due to top quark loops does not break $U(1)_\lambda$, and is therefore the same as in Ref. [3]. Minimizing the total effective potential with respect to $h$ gives

$$\cos \theta = \frac{\hat{C}_\kappa (\kappa - \tilde{\kappa})}{C_t}, \tag{5.13}$$

where $C_t$ is an effective Lagrangian coefficient given by

$$C_t \sim \frac{3m_t^2\Lambda_{CTC}^2}{16\pi^2 \sin^2 \theta}. \tag{5.14}$$

Combining this with the estimate of the top quark mass, we find

$$m_h^2 = c_t N_c m_t^2 \tag{5.15}$$

with $c_t \sim 1$. The $A-\eta$ mass matrix is then

$$\Delta V_{eff} = \frac{1}{2} \frac{m_h^2}{\sin^2 \theta} \begin{pmatrix} A \\ \eta \end{pmatrix}^T \begin{pmatrix} 1 & r_\kappa r_\eta \cos \theta \\ r_\kappa r_\eta \cos \theta & r_\eta^2 \cos^2 \theta \end{pmatrix} \begin{pmatrix} A \\ \eta \end{pmatrix}, \tag{5.16}$$

where

$$r_\kappa = \frac{\kappa + \tilde{\kappa}}{\kappa - \tilde{\kappa}}, \quad r_\eta = \frac{f}{f_\eta}. \tag{5.17}$$

The potential has a minimum at $A, \eta = 0$ provided that $|r_\kappa| < 1$. The mass eigenstates are

$$\begin{pmatrix} A_2 \\ A_1 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} A \\ \eta \end{pmatrix}, \tag{5.18}$$

where

$$\tan \alpha = \frac{1}{2} \left(-x + \sqrt{x^2 + 4}\right), \quad x = \frac{1 - r_\eta^2 \cos^2 \theta}{r_\kappa r_\eta \cos \theta}. \tag{5.19}$$

The mass eigenvalues are

$$m_{A_2,A_1}^2 = \frac{m_h^2}{2 \sin^2 \theta} \left[1 + r_\eta^2 \cos^2 \theta \pm \sqrt{(1 + r_\eta^2 \cos^2 \theta)^2 - 4(1 - r_\kappa^2)r_\eta^2 \cos^2 \theta}\right]. \tag{5.20}$$

Note that

$$m_{A_1}^2 \leq (1 - r_\kappa^2) \frac{m_h^2}{\sin^2 \theta}. \tag{5.21}$$
so there is always one mass eigenstate lighter than $m_h / \sin \theta$, the value of the $CP$-odd PNGB mass in the model without the light $\lambda$.

Note that in the limit $\tilde{\kappa} \to 0$ ($\kappa \to 0$) where $r_\kappa \to \pm 1$ one linear combination of $A$ and $\eta$ is massless, as can be easily seen from the determinant of the mass matrix Eq. (5.16). This arises because the mass terms preserve an anomaly-free $U(1)$ symmetry in this limit, namely (for $\kappa = 0$)

$$
\lambda \mapsto e^{-i\alpha} \lambda, \quad \psi \mapsto e^{-2i\alpha} \psi, \quad \bar{\psi} \mapsto \bar{\psi}.
$$

(5.22)

This limit is therefore completely natural. In this limit, the mass eigenvalues are

$$
m_{A_2}^2 \simeq \frac{m_h^2}{\sin^2 \theta} \left( 1 + r_\eta^2 \cos^2 \theta \right),
$$

(5.23)

$$
m_{A_1}^2 \simeq \frac{m_h^2}{\sin^2 \theta} \frac{r_\eta^2 \cos^2 \theta}{1 + r_\eta^2 \cos^2 \theta} (1 - r_\kappa^2),
$$

(5.24)

where

$$
1 - r_\kappa^2 = \begin{cases} 
-4\tilde{\kappa}/\kappa & \text{if } |\tilde{\kappa}| \ll |\kappa|, \\
-4\kappa/\tilde{\kappa} & \text{if } |\tilde{\kappa}| \gg |\kappa|. 
\end{cases}
$$

(5.25)

We now consider the couplings of the $CP$-odd PNGBs. The $A$ field has suppressed couplings to fermions due to the $Sp(4)$ custodial symmetry. Specifically, we have [3]

$$
g_{A\bar{f}f} \sim \frac{m_f}{v} \sin \theta \times N_c \left( \frac{m_t}{4\pi v} \right)^2 r_\kappa \cos \theta.
$$

(5.26)

The first factor is the “going rate” for a PNGB with decay constant $f = v / \sin \theta$, while the second factor is typically $\sim 10^{-2}$. On the other hand, the $\eta$ field has unsuppressed couplings to fermions via the operator that generates fermion masses:

$$
\mathcal{L}_{\text{eff}} = m_f \bar{f} \tilde{f} \eta \gamma_5 \eta + \text{h.c.}
$$

(5.27)

$$
= m_f \bar{f} \tilde{f} + \frac{m_f}{f_\eta} \bar{f} \gamma_5 \eta + \cdots,
$$

(5.28)

where the $\eta$ dependence is dictated by the fact that the mass operator breaks $U(1)_\lambda$ by 2 units. Neglecting the suppressed coupling of $A$ to fermions we obtain couplings

$$
g_{A\bar{f}f} = -\frac{m_f}{v} r_\eta \sin \theta \sin \alpha,
$$

(5.29)

$$
g_{A_1\bar{f}f} = \frac{m_f}{v} r_\eta \sin \theta \cos \alpha,
$$

(5.29)
It does not appear natural to have either \( \sin \alpha \) or \( \cos \alpha \) very small, and small \( \sin \theta \) is the fine-tuned standard model limit. Therefore, these couplings are generally comparable to the corresponding standard model Yukawa couplings.

The dominant production mechanism for single \( CP \)-odd PNGBs at a hadron collider is therefore gluon fusion with a top loop or associated production with \( \bar{t}t \). The dominant decay is to the heaviest kinematically allowed fermion pair, just like the \( CP \)-odd Higgs fields in multi-Higgs models. This is challenging at the LHC, associated production with \( \bar{t}t \) and a boosted \( A_{1,2} \) may be possible, since this is similar to the analogous Higgs search discussed in Ref. [29]. This has the possibility to measure the coupling of the scalar to the top quark, and can therefore distinguish between the \( A_{1,2} \) and the Higgs. Another possibility is pair production of \( CP \)-odd PNGBs via heavy resonances associated with the strong dynamics at the scale \( \Lambda_{CTC} \). This is similar to strong double Higgs production in composite Higgs scenarios, which may require very high integrated luminosity [30]. We intend to investigate these signals in future work.

6 Cosmology

In this section, we make some brief remarks about the cosmology of these models. As written, the models have serious cosmological problems if the reheat temperature of the universe is larger than \( \Lambda_{CTC} \sim \text{few TeV} \). The reason is the theory at the scale \( 4\pi f \) contains resonances with half-integer charge, for example resonances with the quantum numbers \( \psi \chi \) or \( \tilde{\psi} \chi \) (see Eq. (2.27)). Such particles are necessarily absolutely stable, since there is no final state that they can decay to. These particles are strongly interacting at the scale \( \Lambda_{CTC} \), and therefore have large annihilation cross sections. If their present cosmic abundance is determined by thermal freeze-out, their abundance is somewhat less than that of dark matter. Nonetheless, it is almost certain that such particles are ruled out by the many constraints on charged stable matter. For example, direct searches for the flux of fractionally charged particles \( X \) at the earth give a limit \( \Phi_X \lesssim 10^{-14} \text{ cm}^{-2} \text{ s}^{-1} \) [31], many orders of magnitude less than the expected flux for dark matter

\[
\Phi_X \sim 10^8 \text{ cm}^{-2} \text{ s}^{-1} \left( \frac{\Omega_X}{\Omega_{DM}} \right) \left( \frac{m_X}{\text{TeV}} \right)^{-1}.
\]

See Refs. [32] for additional constraints.

We can change the hypercharge assignments to eliminate half-integer charged particles at the scale \( \Lambda_{CTC} \). For example, we can assign the particles \( \chi \) in Eq. (2.27) hypercharge \( Y = \pm \frac{1}{2} \). In this case, the particles that previously had half-integer

\
charges are still stable if we add no additional interactions, but higher-dimension interactions can be added to allow them to decay. However, we then find that the SUSY model discussed in Section 2 has half-integer charges that cannot be eliminated by changing the hypercharge assignments. We suspect that SUSY completions can be found without fractionally charged particles, but we will not pursue this here.

A simple solution to these difficulties is to assume that the universe has a reheat temperature low enough that the troublesome particles discussed above are never in thermal equilibrium. This means a temperature below the temperature where they would normally freeze out, of order 100 GeV for particles at the scale \( \Lambda_{\text{CTC}} \). This preserves the successes of big-bang nucleosynthesis, the highest temperature direct experimental test of cosmology. A cosmology with a low reheat temperature does not allow standard mechanisms for dark matter and baryogenesis, but there are simple and plausible mechanisms in the literature that can be used in the present context \[33\]. We leave a full investigation of the cosmology of these models to future work.

7 Conclusions

We have constructed complete models of flavor that reduce to minimal conformal technicolor at low energies. The models are based on supersymmetry broken at a high scale \( M_{\text{SUSY}} \gg \text{TeV} \), and generate the required higher-dimension operators via heavy scalar exchange. We have argued that such a theory requires strong (super)conformal dynamics at the scale \( M_{\text{SUSY}} \) in both the technicolor and the top sector. In such a model, the top quark mass is given by

\[
m_t \sim 4\pi v \left( \frac{4\pi f}{M_{\text{SUSY}}} \right)^{d-1}
\]

where \( d \) is the dimension of the “Higgs” operator in the theory above the scale \( 4\pi v \), and \( f \gg v \) for a composite Higgs model. We argued that the models are safe from flavor-changing neutral currents for \( M_{\text{SUSY}} \gtrsim 100 \text{ TeV} \). Saturating this bound, the observed value of the top quark mass can be obtained with \( d \simeq 1.9 \) for \( f = v \). Vacuum misalignment \( (f > v) \) can relax these bounds even more.

The models have a number of desirable features. Most importantly, there are only two mass scales in the theory, the SUSY breaking scale \( M_{\text{SUSY}} \) and the scale \( \Lambda_{\text{CTC}} \sim 4\pi f \ll M_{\text{SUSY}} \) where conformal symmetry is broken and chiral symmetry breaking takes place. Both of these scales arise naturally, with no fine-tuning. Nontrivial SUSY fixed points or generalizations of the Giudice-Masiero mechanism can explain why supersymmetric mass terms are near the SUSY breaking scale, so that all unwanted fermions naturally get a mass of order \( M_{\text{SUSY}} \).
Another interesting result of this work is that this class of models makes a robust prediction for physics below the TeV scale, namely the existence of an additional $U(1)$ PNGB. This has couplings similar to a pseudoscalar Higgs, and may be observable using jet substructure techniques.

A less attractive feature of the specific models we present is that they require additional nontrivial flavor structure. This arises from the sector responsible for making the top quark strongly interacting at the SUSY breaking scale. For the “topcolor models” of Section 3, additional flavor-dependent couplings are required to mix third generation quarks with the first two generation. For the “extended color” models of Section 4 additional flavor dependent couplings are required to give mass to the extra quark colors. The need for this additional flavor structure arises for technical reasons, and is not obviously inherent in our approach. Specifically we require that the strong color group $SU(N_c)$ gauge group have $N_f = 2N_c$ flavors in order to be at a strong (self-dual) conformal fixed point. In the “topcolor” model of Section 3 the strong color group is $SU(3)$, but this is not strong if all 3 generations feel the strong color force. In the “extended color” model of Section 4 we extend the strong color group to $SU(6)$, but then we need additional flavor-dependent couplings to eliminate the extra colors of quarks. All this depends on a “Higgs” picture of the dynamics where we identify the fundamental degrees of freedom with the composite quarks that emerge below the SUSY breaking scale. This picture allows us to construct specific models, but it is quite possible that there are strongly coupled conformal theories with 3 generations of composite fermions that do not require flavor-dependent interactions beyond the usual Yukawa couplings. Finding such models would be very interesting.

However, the details of the models should not distract from the fact that this work gives the first concrete proposal for a realistic and UV complete dynamics of flavor in a theory of strong electroweak symmetry breaking. We believe that this removes a significant barrier to taking this class of theories seriously as a framework for new physics at the TeV scale.

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