New holographic dark energy model inspired by the DGP braneworld

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The energy density of the holographic dark energy is based on the area law of entropy, and thus any modification of the area law leads to a modified holographic energy density. Inspired by the entropy expression associated with the apparent horizon of a Friedmann-Robertson-Walker (FRW) Universe in DGP braneworld, we propose a new model for the holographic dark energy in the framework of DGP brane cosmology. We investigate the cosmological consequences of this new model and calculate the equation of state parameter by choosing the Hubble radius, L = H−1, as the system’s IR cutoff. Our study show that, due to the effects of the extra dimension (bulk), the identification of IR-cutoff with Hubble radius, can reproduce the present acceleration of the Universe expansion. This is in contrast to the ordinary holographic dark energy in standard cosmology which leads to the zero equation of state parameter in the case of choosing the Hubble radius as system’s IR cutoff in the absence of interaction between dark matter and dark energy.

I. INTRODUCTION

Nowadays there are many strong evidences, on the observational side, that our Universe is experiencing a phase of accelerated expansion likely driven by some unknown energy component usually dubbed “dark energy” (DE) [1]. Recent astronomical observations indicate that more than 70 percent of the Universe consists of DE with negative pressure [1]. Disclosing the nature and origin of such a DE has been one of the biggest challenges of the modern cosmology. Many theoretical candidates have been proposed as DE. Among them, those which originate from fundamental theory such as quantum gravity has arisen a lot of enthusiasm, recently. For a comprehensive review on DE models see [2].

One of the dramatic candidate for DE, that arose a lot of enthusiasm recently, is the so-called “holographic dark energy” (HDE) proposal (see [3–12] and references therein). This model is based on the holographic principle which states that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume [13] and it should be constrained by an infrared cutoff [14]. It was shown that in quantum field theory, the UV cutoff Λ should be related to the IR cutoff L by the limit set of forming a black hole [14]. If ρD = Λ4 is the vacuum energy density caused by UV cutoff, the total energy of size L should not exceed the mass of the system-size black hole

\[ E_D \leq E_{BH} \rightarrow L^3 \rho_D \leq M_p^2 L. \quad (1) \]

If the largest cutoff L is taken for saturating this inequality, we get the energy density of HDE as [4]

\[ \rho_D = \frac{3c^2 M_p^2}{L^2} = \frac{3c^2}{8\pi G L^2}. \quad (2) \]

The HDE model has been investigated widely in the literature and has also been tested and constrained by various astronomical observation [15–16]. It is fair to claim that simplicity and reasonability of HDE model provides more reliable frame to investigate the problem of DE rather than other models proposed in the literature. It is worth mentioning that in the derivation of HDE density (2), the black hole entropy S plays a crucial role [14]. Indeed, the definition and derivation of holographic energy density depend on the entropy-area relation S ∼ A ∼ L2 of black holes, where A represents the area of the horizon [14]. Any modification of the black holes entropy due to the quantum correction [17] or extra dimension such as in braneworld scenarios [18, 19] will affect directly on the definition of the energy density of the HDE and leads to new cosmological consequences.

The profound connection between thermodynamics and gravity has now well established through a numerous and complementary theoretical investigations [20]. It has been shown that the gravitational field equations in a wide range of theories can be recast as the first law of thermodynamics on the boundary of spacetime [21–30]. The studies were also generalized to the brane cosmology, where it was shown that the differential form of the Friedmann equation on the brane can be transformed to the first law of thermodynamics on the apparent horizon [31, 32]. This procedure leads to extract an expression for the apparent horizon entropy in braneworld scenarios which is useful in studying black holes physics on the brane [18, 19]. These results indicate the holographic properties of the gravitational field equations in a wide range of gravity theories. The deep connection between the gravitational equation describing the gravity in the bulk and the first law of thermodynamics on the apparent horizon reflects some deep ideas of holography.
According to the braneworld scenario, all matter fields in standard model of particle physics are confined to a brane embedded in a higher-dimensional bulk, while the gravitational field, in contrast, is usually considered to live in the bulk spacetime. A simple and well studied version of braneworld model was proposed by Dvali-Gabadadze-Porrati (DGP) [33–35]. In this model our four-dimensional Universe is a FRW brane embedded in a five-dimensional Minkowski bulk. It is important to note that the self-accelerating DGP solution has ghost instabilities and it is impossible to realize phantom divide crossing in this branch of solutions without adding extra component(s). To realize phantom divide crossing in the self-accelerating branch, it is necessary to add at least a component as DE on the brane. On the other hand, the normal DGP branch cannot explain late time cosmic speed-up, but it has the potential to realize a phantom-like phase by dynamical screening of the brane cosmological constant. Adding a DE component to the normal branch solution brings new facilities to explain late time acceleration and also better matching with observations. These are the motivations to add DE to this braneworld setup [32]. In this paper we would like to propose a new modified HDE model in the context of the DGP braneworld.

The structure of this paper is as follows. In the next section, we first show that the corresponding Friedmann equation of a flat FRW Universe in DGP braneworld, can be rewritten as the first law of thermodynamics on the apparent horizon. This procedure allows us to derive the entropy expression associated with the apparent horizon in brane cosmology. Then, having the entropy expression at hand, we propose a new modified HDE model in DGP braneworld. In section III, we consider the Hubble radius as IR cutoff and study cosmological implications of the proposed model. In section IV, we extend the study to the case where there is an interaction between the two dark components of the Universe. The last section is devoted to conclusions and discussions.

II. THE MODEL

We consider a homogeneous and isotropic FRW Universe on the brane which is described by the line element

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right),$$

(3)

where $k = 0, 1, -1$ represent a flat, closed and open maximally symmetric space on the brane, respectively. Using the spherical symmetry, we can rewrite the metric as

$$ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega^2,$$

(4)

where $\tilde{r} = a(t)r$, $x^0 = t$, $x^1 = r$ and $h_{ab} = \text{diag} \left( -1, \frac{a^2}{1 - kr^2} \right)$ is the two dimensional sub-manifold. The dynamical apparent horizon is determined by equation $h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0$, which implies that the vector $\nabla \tilde{r}$ is null on the apparent horizon surface [36, 37]. The explicit evaluation of the apparent horizon for the FRW Universe gives the apparent horizon radius

$$\tilde{r}_A = 1/\sqrt{H^2 + k/a^2},$$

(5)

where $H = \dot{a}/a$ is the Hubble parameter. The associated temperature $T = |\kappa|/2\pi$ with the apparent horizon is defined through the surface gravity

$$\kappa = \frac{1}{2\sqrt{-h}} \partial_a \left( \sqrt{-h} h^{ab} \partial_b \tilde{r} \right).$$

(6)

The explicit evolution of the surface gravity at the apparent horizon of FRW Universe reads

$$\kappa = -\frac{1}{\tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right).$$

(7)

Thus the temperature associated with the apparent horizon is given by

$$T = \frac{1}{2\pi \tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right).$$

(8)

where we assume $\dot{\tilde{r}}_A < 2H\tilde{r}_A$. It was shown that there is indeed a Hawking radiation associated with the apparent horizon [38] which gives more solid physical implication of the temperature associated with the apparent horizon.
We are going to rewrite the Friedmann equation in the form of the first law of thermodynamics at apparent horizon. The Friedmann equation in DGP braneworld has the following form \[34\]

\[H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{\rho}{3M_P^2} + \frac{1}{4r_c^2} + \frac{\epsilon}{2r_c}}\right)^2,
\]

where \(\epsilon = \pm 1\) corresponds to the two branches of solutions \[34\], and \(\rho = \rho_m + \rho_D\) is the total energy density of the fluid on the brane where \(\rho_D\) and \(\rho_m\) are energy density of dark matter (DM) and DE on the brane, respectively. In the above equation \(r_c\) is the crossover length scale between the small and large distances given by

\[r_c = \frac{M_P^2}{2M_5^3} = \frac{G_5}{2G_4}.
\]

The \(\epsilon = +1\) is the self-accelerating solution in which the Universe may enter an accelerating phase in late time without presence of additional DE component. The \(\epsilon = -1\) branch has been named as the normal branch, where acceleration only appears if the DE component is included on the brane. For \(r_c \gg 1\), the Friedmann equation in standard cosmology is recovered,

\[H^2 + \frac{k}{a^2} = \frac{\rho}{3M_P^2}.
\]

Recent observations indicate that our Universe is spatially flat. For a flat FRW universe on the brane, Eq. (9) reduces to

\[H^2 - \frac{\epsilon}{r_c}H = \frac{\rho}{3M_P^2}.
\]

As we mentioned for \(\epsilon = +1\) and in the absence of any kind of energy or matter field on the brane \(\rho = 0\), there is a de Sitter solution for Eq. (12) with constant Hubble parameter

\[H = \frac{1}{r_c} \Rightarrow a(t) = a_0 e^{Ht},
\]

which leads to an accelerating Universe with constant equation of state parameter, \(w_D = -1\), exactly like cosmological constant. However, this solution suffers some unsatisfactory problems. First of all, it suffers the well-known cosmological constant problems namely, the fine-tuning and the coincidence problems. Besides, it leads to a constant \(w_D\), while many cosmological evidences, especially the analysis of the type Ia supernova data indicates that the time varying DE gives a better fit than a cosmological constant \[39–41\]. In addition, to arrive at Eq. (13) one ignores all parts of energy on the brane including DE, DM and byronic matter, which is not a reasonable assumption in a real universe even at the late time.

We also assume that there is no energy exchange between the brane and the bulk and so the energy conservation equation holds on the brane

\[\dot{\rho} + 3H(1 + w)\rho = 0,
\]

while in general there is an interaction between DM and DE on the brane. Thus both components do not obey the conservation equation, and they obey instead

\[\dot{\rho}_m + 3H\rho_m = Q,
\]

\[\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = -Q,
\]

where \(\omega_D = p_D/\rho_D\) is the equation of state parameter of DE and \(Q = \Gamma\rho_D\) shows the interaction between the DE and DM on the brane. Using the apparent horizon radius \[5\] we can write the Friedmann equation (9) as

\[\rho_m + \rho_D = 3M_P^2\left[\left(\frac{1}{r_A} - \frac{\epsilon}{2r_c}\right)^2 - \frac{1}{4r_c^2}\right],
\]

To obtain a differential form of the Friedmann equation in favor of the first law of thermodynamics, we take the differential of equation (17) and then by using Eqs. (15) and (16) we find

\[H\rho_D(1 + u + \omega_D)dt = 2M_P^2\left(\frac{1}{r_A} - \frac{\epsilon}{2r_c}\right)\frac{dr_A}{r_A^2},
\]
where \( u = \rho_D / \rho_m \) is the ratio of energy densities of the two dark components. Multiplying both sides of equation [15] by a factor of \( 4\pi \tilde{r}_A^3 (1 - \frac{\dot{r}_A}{2H\tilde{r}_A}) \), and using expression (7) for surface gravity, we arrive at

\[ -8\pi M_p^2 \kappa \left( \frac{1}{r_A} - \frac{\epsilon}{2r_c} \right) \tilde{r}_A^2 d\tilde{r}_A = 4\pi H\dot{r}_A^3 \rho_D (1 + u + \omega_D) dt - 2\pi \tilde{r}_A^2 \rho_D (1 + u + \omega_D) d\tilde{r}_A. \]  

(19)

Next, we assume \( E = (\rho_m + \rho_D) \frac{4\pi}{3} \tilde{r}_A^3 \) is the total energy inside the 3-sphere on the brane, where \( V = \frac{4\pi}{3} \tilde{r}_A^3 \) is the volume enveloped by 3-dimensional sphere. Taking differential form of total energy, \( E \), we find

\[ dE = 4\pi \tilde{r}_A^2 (\rho_m + \rho_D) d\tilde{r}_A + \frac{4\pi}{3} \tilde{r}_A (\rho_m + \rho_D) dt. \]  

(20)

Using Eqs. (15) and (16), we get

\[ dE = 4\pi \tilde{r}_A^2 \rho_D (1 + u) d\tilde{r}_A - 4\pi \tilde{r}_A^3 H\rho_D (1 + u + \omega_D) dt. \]  

(21)

Combining Eq. (21) with (19), we obtain

\[ dE - W dV = 16\pi^2 M_p^2 T \left( \frac{1}{r_A} - \frac{\epsilon}{2r_c} \right) \tilde{r}_A^2 d\tilde{r}_A, \]  

(22)

where we have defined the work density as [37]

\[ W = \frac{1}{2}(p - p) = \frac{1}{2} H\rho_D (1 + u - \omega). \]  

(23)

The work term \( W dV \) is defined as the work done by the change of the apparent horizon surface. Expression (22) is just the first law of thermodynamics at the apparent horizon on the brane, namely \( dE = T dS + W dV \). We can define the entropy associated with the apparent horizon on the brane as

\[ S = \int_0^{\tilde{r}_A} dS = 16\pi^2 M_p^2 \int_0^{\tilde{r}_A} \left( \frac{1}{r_A} - \frac{\epsilon}{2r_c} \right) \tilde{r}_A^2 d\tilde{r}_A = \frac{A}{4G_4} \left( 1 - \frac{\epsilon\tilde{r}_A}{3r_c} \right), \]  

(24)

where we have used \( M_p^2 = (8\pi G_4)^{-1} \) and \( A = 4\pi \tilde{r}_A^2 \) is the area of the apparent horizon. Let us note that for \( \epsilon = -1 \), Eq. (24) is similar to the result obtained in [15]. In the limiting case where \( \tilde{r}_A \ll 3r_c \), one recovers the area law for the apparent horizon entropy. Physically, this means that the apparent horizon is not extended in the bulk and located totally on the brane. As a result, the effect of the extra dimension does not appear in the entropy expression.

It is well-known that in each gravity theory, the entropy expression associated with the apparent horizon in cosmology is the same as the entropy associated with the black hole horizon. The only change which is needed to replace the horizon radius \( r_+ \) of black hole with the apparent horizon \( \tilde{r}_A \) [27, 42]. Thus we propose the entropy of the black hole horizon in the DGP braneworld scenario to be given by

\[ S = \frac{A}{4G_4} \left( 1 - \frac{\epsilon r_+}{3r_c} \right), \]  

(25)

where \( A = 4\pi r_+^2 \) is the area of the black hole horizon. It is important to note that the definition and derivation of holographic energy density (\( \rho_D = 3\epsilon^2 M_p^2 / L^2 \)) depends on the entropy-area relationship \( S \sim A \sim L^2 \) of black holes, where \( A \) represents the area of the horizon [14]. Inspired by the entropy relation (25) and following the derivation of HDE [43] and entropy-corrected HDE [44, 45], we can easily obtain the corresponding energy density of the HDE in the DGP braneworld as

\[ \rho_D = \frac{3\epsilon^2 M_p^2}{L^2} \left( 1 - \frac{\epsilon L}{3r_c} \right). \]  

(26)

When \( L \ll 3r_c \), the above equation yields the well-known holographic energy density, as expected. The significant of the corrected term in various regions depends on the fraction \( L/r_c \). We emphasize here that for studying the HDE in the framework of DGP braneworld, it is more reasonable to take the energy density of HDE in the form of (26) instead of (2). This is due to the fact that the well-known area law for the black holes entropy no longer holds on the brane and the entropy associated with the horizon on the brane should be modified as relation (25). This is an important point which was not taken into account in the previous studies on HDE in the DGP braneworld [46]. In the remaining part of this paper, we shall investigate the cosmological implications of the HDE density (26). Since the simple and natural choice for the system’s IR cutoff is the Hubble radius \( L = H^{-1} \), thus in this paper we consider this choice in two different cases, namely in the absence of the interaction and then we consider the interacting case. In both cases we find the equation of state and deceleration parameters of a FRW Universe on the brane. We also plot the related figures to show the evolution of them in each case.
III. NON INTERACTING NEW HDE MODEL

Let us consider, for simplicity, the flat FRW universe. The modified Friedmann equation is given by Eq. (12) where \( \rho = \rho_m + \rho_D \). In the absence of interaction between DE and DM, the continuity equations read

\[
\dot{\rho}_m + 3H\rho_m = 0, \\
\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = 0.
\]

(27)

(28)

Note that since the matter component is mainly contributed by the cold DM we ignore the baryon matter component here for simplicity. From (12), we can write

\[
\Omega_m + \Omega_D = 1 - 2\epsilon\sqrt{\Omega_{rc}},
\]

(29)

where we have used the following definitions

\[
\Omega_m = \frac{\rho_m}{3M_p^2H^2}, \quad \Omega_D = \frac{\rho_D}{3M_p^2H^2}, \quad \Omega_{rc} = \frac{1}{4r_c^2H^2}.
\]

(30)

Considering the Hubble radius \( L = H^{-1} \) as the system’s IR cutoff, we can write (26) as

\[
\rho_D = 3\epsilon^2M_p^2H^2\left(1 - \frac{2\epsilon\sqrt{\Omega_{rc}}}{3}\right).
\]

(31)

Using (30), we have

\[
\Omega_D = \epsilon^2\left(1 - \frac{2\epsilon\sqrt{\Omega_{rc}}}{3}\right).
\]

(32)

This equation implies that for HDE in standard cosmology (\( \Omega_{rc} = 0 \)), with Hubble radius as the IR cutoff, \( \Omega_D = \epsilon^2 \) and thus DE has no evolution during the history of the Universe. As we know this is not a reasonable result. Thus our model may resolve this problem, since in our case \( \Omega_D \) given in (32) is no longer a constant.

Taking the time derivative of Eq. (12) and using Eqs. (27), (28), (29) and (30) we can easily find

\[
\frac{\dot{H}}{H^2} = -\frac{3}{2}\left[\frac{1 - \Omega_D - 2\epsilon\sqrt{\Omega_{rc}}}{1 - \Omega_D - \epsilon\sqrt{\Omega_{rc}} - \frac{\epsilon^2\sqrt{\Omega_{rc}}}{3}}\right].
\]

(33)

Taking the time derivative of Eq. (31), we find

\[
\dot{\Omega}_D = \frac{2\epsilon^2\sqrt{\Omega_{rc}}}{3}\frac{\dot{H}}{H}.
\]

(34)

Using the fact that \( \dot{\Omega}_D = H\Omega_D' \), after substituting Eq. (33) in (34) the evolution of dimensionless ECHDE density as

\[
\Omega_D' = -\epsilon^2\sqrt{\Omega_{rc}}\left[\frac{1 - \Omega_D - 2\epsilon\sqrt{\Omega_{rc}}}{1 - \Omega_D - \epsilon\sqrt{\Omega_{rc}} - \frac{\epsilon^2\sqrt{\Omega_{rc}}}{3}}\right],
\]

(35)

where the prime denotes derivative with respect to \( x = \ln a \). Using (32) we can omit \( \Omega_{rc} \) and rewrite the above relation in the form

\[
\Omega_D' = -3(c^2 - \Omega_D)\left[\frac{2c^2 + \Omega_Dc^2 - 3\Omega_D}{c^2 + \Omega_Dc^2 + c^4 - 3\Omega_D}\right].
\]

(36)

The evolution of the dimensionless HDE density parameter \( \Omega_D \) as a function of \( 1 + z = a^{-1} \) is shown in Fig. (1). From this figure we see that at the early Universe where \( z \to \infty \) we have \( \Omega_D \to 0 \), while at the late time, where \( z \to -1 \), the DE dominated, namely \( \Omega_D \to 1 \).

Combining Eqs. (28) and (31) yields

\[
\omega_D = -1 - \frac{2}{3} \left(1 + \frac{\epsilon^2\sqrt{\Omega_{rc}}}{3\Omega_D}\right)\frac{\dot{H}}{H^2},
\]

(37)
and thus the equation of state parameter is obtained as

$$
\omega_D = -1 + \left(1 + \frac{c^2 \sqrt{\Omega_{r_c}}}{3\Omega_D}\right) \left[\frac{1 - \Omega_D - 2\epsilon \sqrt{\Omega_{r_c}}}{1 - \Omega_D - \epsilon \sqrt{\Omega_{r_c}} - \frac{c^2}{3} \sqrt{\Omega_{r_c}}}\right].
$$

In the limiting case where $\Omega_{r_c} \to 0$, we obtain $\omega_D = 0$ which is a wrong equation of state for DE and cannot derive the acceleration of the Universe expansion [3]. This is an expected result, since in this regime the effects of the extra dimension disappear and the standard cosmology is recovered. It was already shown that in the absence of interaction, choosing $L = H^{-1}$ cannot produce the cosmic acceleration [3]. However, as one can see from Eq. (38) in the framework of DGP braneworld with modified HDE density [24], the identification of IR-cutoff with Hubble radius, $L = H^{-1}$, can lead to accelerated expansion. If we substitute $\Omega_{r_c}$ from Eq. (32), we can further rewrite,

$$
\omega_D = \frac{2c^2(c^2 - \Omega_D)}{\Omega_D(c^2 + \Omega_D c^2 + c^4 - 3\Omega_D)}.
$$

From Eq. (32), we see that the limiting case $\Omega_{r_c} = 0$ can be translated to $\Omega_D = c^2$, and thus from (39) we have $\omega_D = 0$. Therefore, the result of standard cosmology is recovered; no acceleration with Hubble radius as IR cutoff.
The deceleration parameter is given by
\[ q = -1 - \frac{\dot{H}}{H^2}. \] (40)

Substituting Eq. (33) into (40) one can obtain the deceleration parameter as
\[ q = -1 + \frac{3}{2} \left[ \frac{1 - \Omega_D - 2\epsilon \sqrt{\Omega_r}}{1 - \Omega_D - \epsilon \sqrt{\Omega_r} - \frac{c^2}{3} \sqrt{\Omega_r}} \right]. \] (41)

Using relation (32) one can rewrite Eq. (11) in terms of \( c^2 \) and \( \Omega_D \),
\[ q = -\frac{c^4 + 6\Omega_D - 2c^2\Omega_D + 5c^2}{c^2 + \Omega_D c^2 + c^4 - 3\Omega_D}. \] (42)

Again for \( \Omega_D = c^2 \) we have \( q = \frac{1}{3} \) which implies a decelerated phase for the Universe, corresponding to HDE with the Hubble radius as IR cut-off in standard cosmology \([3]\). The evolution of equation of state and deceleration parameters are shown in figure 2 for different value of the parameter \( c^2 \). From these figures we see that the Universe has a transition from deceleration to the acceleration phase \( \omega_D < -\frac{1}{3} \). In this model the EoS parameter cannot cross the phantom line \( \omega_D = -1 \) in future. In addition the transition from deceleration to the acceleration phase occurs around \( z \simeq 0.6 \) compatible with the recent observations \([47,49]\).

It is worth noting that although the equation of state and the deceleration parameters do not depend explicitly on the crossover length scale \( r_c \), which is the characterization of the DGP braneworld, they depend on \( r_c \) via the relation between the dimensionless density parameters in Eq. (32). For \( r_c > 1(\Omega_r \rightarrow 0) \), the effects of the extra dimension vanish and the results of the standard cosmology are restored \([3]\).

### IV. INTERACTING NEW HDE MODEL

In the presence of the interaction between DE and DM, the conservation equation do not hold separately, they instead obey \([15] \) and \([16] \). Recent observational evidences provided by the galaxy cluster Abell A586 supports the interaction between DE and DM \([50]\). The dynamics of interacting DE models with different \( Q \)-classes have been studied in ample detail in \([51]\). Here we choose \( Q = 3b^2 H(\rho_D + \rho_m) \) as the interaction term, where \( b^2 \) is a coupling constant.

Taking the time derivative of Eq. (12) and using Eqs. (29), (30), (15) and (16), we find
\[ \frac{\dot{H}}{H^2} = \frac{-3(1 - \Omega_D - 2\epsilon \sqrt{\Omega_r} + 3b^2(1 - 2\epsilon \sqrt{\Omega_r})}{2(1 - \Omega_D - \epsilon \sqrt{\Omega_r} - \frac{c^2}{3} \sqrt{\Omega_r})}. \] (43)

Using Eqs. (41) and (13) we can obtain the evolution of the dimensionless density \( \Omega_D \) as
\[ \Omega_D' = 3(c^2 - \Omega_D)\frac{(-2c^2 - c^2\Omega_D + 3\Omega_D) + b^2(-2c^2 + 3\Omega_D)}{(c^2 - 3\Omega_D + c^2\Omega_D + c^4)}. \] (44)

where we have used Eq. (32) for omitting \( \Omega_r \). The evolution of the dimensionless HDE density parameter \( \Omega_D \) as a function of \( 1 + z = a^{-1} \) is shown in Fig. 3. Again we see that at the early time \( \Omega_D \rightarrow 0 \), while at the late time, \( \Omega_D \rightarrow 1 \).

Combining Eqs. (16), (29) and (30) we arrive at
\[ \omega_D = -1 - \frac{2}{3H} \left( \frac{\dot{H}}{H^2} \right) - \frac{2c^2 \epsilon \sqrt{\Omega_r}}{9\Omega_D} \frac{\dot{H}}{H^2} - \frac{b^2(1 - 2c^2 \epsilon \sqrt{\Omega_r})}{\Omega_D}. \] (45)

Substituting Eq. (13) into Eq. (45), we find the equation of state parameter of interacting new HDE as
\[ \omega_D = \frac{\sqrt{\Omega_r}(c^2 \Omega_D + c^2 \Omega_r - 2c^2 \Omega_D - 2c^2 \Omega_r) - b^2(1 - 2c^2 \epsilon \sqrt{\Omega_r})(1 - \epsilon \sqrt{\Omega_r})}{\Omega_D(1 - \epsilon \sqrt{\Omega_r} - \Omega_D - \frac{c^2}{3} \sqrt{\Omega_r})}. \] (46)
Substituting Eq. (43) into (40) we obtain the deceleration parameter as

\[ q = \frac{1 - 4\epsilon \sqrt{\Omega_r} - \Omega_D + \frac{2\epsilon^2}{3} \sqrt{\Omega_r} + 3b^2(1 - 2\epsilon \sqrt{\Omega_r})}{2(1 - \epsilon \sqrt{\Omega_r} - \Omega_D - \frac{\epsilon^2}{3} \sqrt{\Omega_r})}. \]  

Using relation (32), one can rewrite Eqs. (46) and (47) in the following form

\[ \omega_D = \frac{2c^2(c^2 - \Omega_D) + \frac{b^2}{3}(3\Omega_D - 2c^2)(3\Omega_D - c^2)}{\Omega_D \left( c^2 + \Omega_D c^2 + c^4 - 3\Omega_D \right)}, \]  

\[ q = -\frac{c^4 + 6\Omega_D - 2c^2 \Omega_D - 5c^2 + 3b^2(2c^2 - 3\Omega_D)}{\left( c^2 + \Omega_D c^2 + c^4 - 3\Omega_D \right)}. \]  

In the absence of the interaction, i.e., \( b^2 = 0 \), Eqs. (48) and (49) reduce to (39) and (42), respectively. Again from relation (32) we see that the limit of standard cosmology where \( \Omega_{rc} \) corresponds to \( \Omega_D = c^2 \), and in this limit the
FIG. 5: The evolution of the deceleration parameter $q$ versus redshift parameter $z$ for interacting new HDE in DGP braneworld.

results of standard cosmology for interacting HDE with Hubble radius as systems’s IR cutoff is restored \[8\],

$$\omega_D = -\frac{b^2}{c^2(1-c^2)}.$$ \hspace{1cm} (50)

$$q = \frac{1}{2} - \frac{3b^2}{2(1-c^2)}.$$ \hspace{1cm} (51)

Again, we see that for the choice of $L = H^{-1}$, in standard cosmology, an interaction is the only way to have an EoS different from that for dust \[8, 9\]. However, for the model presented in this paper, and in the context of DGP braneworld, even in the absence of interaction, the the natural and simple choice of $L = H^{-1}$, can leads to the accelerated Universe. This is one of the main result we have addressed in the present paper.

The evolution of the $\omega_D$ and $q$ are shown in figures 4 and 5 for different model parameters. In one figure we fix $b^2$ and set different values for $c$, while in another figure we fix $c$ and set different values for $b^2$. From these figures we clearly see that the transition from the deceleration into acceleration phase can occur around $z \sim 0.6$, which is consist with recent observations.

V. CONCLUSIONS AND DISCUSSIONS

The energy density expression of the HDE is based on the area law of the black hole entropy, and thus any modification of the area law leads to a modified holographic energy density. Since in the DGP braneworld the area law of the entropy can be modified as given in (25), thus it is more reasonable to propose a new energy density for the HDE which is based on this modified entropy expression.

In this paper, we first showed that the Friedmann equation describing the evolution of the FRW Universe in DGP can be recast as the first law of thermodynamics on the apparent horizon. This procedure leads to extract an entropy expression associated with the apparent horizon. We expect the entropy of the black hole horizon to have the same expression but replacing the apparent horizon radius $\tilde{r}_A$ with the black hole horizon radius $r_+$. Then, inspired by the entropy expression associated with the apparent horizon of FRW universe in DGP braneworld, we proposed a new model for the HDE in the framework of the DGP braneworld. We believe that if one is interested in studying the HDE in the framework of DGP braneworld, one should take the energy density in the form of (20) instead of the usual form $\rho_D = 3c^2M_p^2/L^2$. Compared to the energy density of the HDE in standard cosmology, the new HDE describing by expression (20) consists an additional term which incorporates the effects of un-compact extra dimension onto the brane. Clearly in the limiting case where the effects of the extra dimension vanishes,($L \ll 3r_c$), the energy density of usual HDE is recovered.

Then, we studied the cosmological implications of this new model for flat FRW Universe on the brane. For this purpose, we chose the Hubble radius $L = H^{-1}$, as system IR cutoff. The Hubble radius is not only the most natural and obvious but also the simplest choice for IR cutoff in flat universe. It was already shown that the Hubble radius
in flat Universe can result an accelerated Universe provided the interaction between the two dark components of the universe is taken into account [8]. In other words, in the absence of interaction it leads to the EoS of dust, $w_{\omega D} = 0$ [3]. Interestingly enough, we found that, for the new HDE in DGP braneworld, the identification of IR cutoff with Hubble horizon, $L = H^{-1}$, can lead to an acceleration of the universe expansion, even in the absence of the interaction between two dark components. This is contrast to the HDE in standard cosmology, where $w_{\omega D} = 0$ if one choose $L = H^{-1}$ in the absence of interaction between the two dark components. We also examined our model by taking into account the interaction term and derived the cosmological parameters. In order to see the behavior of the EoS and deceleration parameter, we plotted these parameters versus redshift in both cases. Our studies show that in both cases, our Universe has a transition from deceleration to the acceleration phase around $z \approx 0.6$, which is compatible with observational evidences [47,49].

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