Factorisation in Higher-Twist
Single-Spin Amplitudes
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Abstract

We analyse the twist-three amplitudes that can give rise to single-spin asymmetries in hadron-hadron scattering; in so doing we bring to light a novel factorisation property. As already known, the requirement of an imaginary part leads to consideration of twist-three contributions that are also related to transverse spin in deep-inelastic scattering. In particular, when an external line becomes soft in contributions arising from three-parton correlators, the imaginary part of an internal propagator may be exposed. As shown here, it is precisely this kinematical configuration that permits the factorisation. An important feature is the resulting simplification: the calculation of tens of Feynman diagrams normally contributing to such processes is reduced to the evaluation of products of the simple factors derived here and known two-body helicity amplitudes. We thus find clarifying relations between the spin-dependent and spin-averaged cross-sections and formulate a series of selection rules. In addition, the kinematical dependence of such asymmetries, is rendered more transparent.

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1 Introduction: single-spin phenomenology

A large body of information has now been gathered in regard of single-spin asymmetries in semi-inclusive hadronic processes [1], where the striking feature is the magnitude of such effects (up to \( \sim 40\% \)). Such phenomena present a theoretical challenge: to find sizeable interfering spin-flip and non-flip amplitudes with relative imaginary phases, a severe difficulty for a gauge theory with near-massless fermions [2]. At the same time, although subject to some early confusion, there is now a clear understanding of the nature and rôle of three-parton twist-three correlators in the transverse-spin dependence of deep-inelastic scattering (DIS) [3–5]. However, the distribution functions associated with such structures will be difficult to study comprehensively [6], especially if consideration is restricted to DIS. Indeed, although data are steadily becoming available [7], further experimental knowledge will be necessary for a complete description of transverse-spin phenomena. Thus, single-spin asymmetries, which are intimately related to the same twist-three amplitudes, may be an invaluable integration of our knowledge in this area.

The experimental aspects of single-spin asymmetries are well documented [8]: the main point to stress is that the measured effects do not appear at all suppressed, even for values of \( p_T \) where it might be hoped that perturbative QCD (pQCD) should be applicable. On the other hand, it has long been held that they would not be reproducible in pQCD [2], although a satisfactory (but largely incomplete) description of such asymmetries is provided by a number of non-perturbative approaches.

One might question whether or not it even makes sense to apply pQCD to processes that, for the time being, have only been measured at relatively small values of \( p_T \). However, recall that Anselmino et al. [9] have made successful fits to the existing pion data, based on pQCD-inspired models. Moreover, the hyperon data does reach large values of \( p_T \), where there is no hint of the polarisation disappearing. If these transverse-spin effects do have a common origin, then one might hope that a perturbative approach should give a reasonable description down to some typical hadronic scale. In this respect, although Teryaev [10] has recently shown that twist-four effects must become important at large parton \( x \), where twist-three contributions would otherwise induce positivity violation owing to their lower-power dependence on \((1−x)\), this is not an argument against the applicability of pQCD. Rather, it underlines the well-known fact that while higher twist is important for \( x \to 1 \), there is an intermediate region where it is negligible even at very low scales. Indeed, just this type of process, being so-to-speak only slightly higher twist, may well provide clues to the transition between regions.

The basic hurdle lies then in the need for spin-flip amplitudes with relative imaginary phases; in a suitable helicity basis it can be shown that single transverse-spin effects are related to the imaginary part of the interference between spin-flip and non-flip amplitudes. Normally, in a gauge theory, spin-flip can only be generated via fermion masses, and phases by loop corrections. However, some time ago Efremov and Teryaev noted [4] that the loop implicit in diagrams containing an extra partonic leg (arising in higher-twist transverse-spin effects) naturally leads to an unsuppressed imaginary part with spin flip. To understand this, it is helpful to appreciate that the extra loop (naïvely
implying higher order in $\alpha_s$) is accompanied by a large logarithm. Thus, the associated distribution function is to be considered at the level of the usual leading-order densities. In other words, at leading-logarithmic level, the usual infinite sum of terms in $(\alpha_s \log Q^2)^n$ is present; however, just the very first term is missing $^{[1]}$. In practice, the extra power of $\alpha_s$ inherent to these contributions is effectively absorbed into the hadron-parton correlator.

We note in passing that twist is best considered in terms of the power of $Q^2$ with which a given contribution appears in a hadronic cross-section $^{[2]}$: in the single-spin case one expects asymmetries to behave as

$$A \propto \frac{\mu p_T}{\mu^2 + p_T^2} ,$$

(1)

where $\mu$ is some typical hadronic mass scale. Again, Teryaev $^{[10]}$ has discussed how the necessary inclusion of twist four leads to the form of the denominator in eq. (1). Thus, the usual suppression should be observed asymptotically while a roughly linear dependence is expected for low values of $p_T$. The intriguing implication of Teryaev’s analysis is that the point of maximum asymmetry should indicate the onset of the regime dominated by leading twist. If the hyperon data is typical then this already occurs at around 1 GeV for intermediate values of $x$. However, the $p_T$ dependence would suggest that at the point where higher twist is reduced by a factor 10 the asymmetry will still be $\sim 30\%$ of its maximum value.

Much progress has been made in the direction of interrelating the various aspects of polarisation phenomenology $^{[4, 12–17]}$. In particular, in the case of twist-three contributions, the possibility that one of the hard-scattering propagators may generate an imaginary part in the soft limit has already been exploited as a possible mechanism for the large asymmetries mentioned above. Early work concentrated on prompt-photon production $^{[4,12–14]}$; other processes that have been considered are pion production $^{[15]}$ and Drell-Yan $^{[16]}$.

Here we present a systematic analysis to demonstrate how the requirement of an imaginary part (and thus a soft internal propagator) greatly simplifies calculations owing to a novel factorisation property of the Feynman amplitudes involved. After some preliminary definitions in the next section and clarification of the spin-flip requirement at the partonic level, section 3 contains the main derivation and results, illustrating how the factorisation arises and the simple selection rules that follow therefrom. In the concluding section we present the resulting formal expression for the spin-dependent partonic cross-sections, together with some discussion.

While the technique presented opens the way to simpler and more rapid calculation, we do not consider it useful to present yet another evaluation of any particular process for two reasons: firstly, a model input for the unknown parton correlators would, in any case, be required and we have nothing new to add there; and, secondly, many calculations have already been published (as cited above) and this technique should not, of course, produce different results.
2 Preliminaries and definitions

Some relevant twist-three diagrams are displayed in Fig. 1; such diagrams may contribute to single-spin asymmetries owing to the imaginary parts implicit in the internal lines, according to the standard propagator prescription:

$$\frac{1}{k^2 \pm i\varepsilon} = \text{IP} \frac{1}{k^2} \mp i\pi\delta(k^2),$$

(2)

where $\text{IP}$ indicates the principal value. While the imaginary part is never exposed (for kinematical reasons) in the usual two-to-two lowest-order partonic scattering amplitudes, in those containing three-parton correlators it is possible for one internal line to become soft (along a boundary of the three-body phase space). The three boundaries of interest are given by the kinematical limits: $x_i \rightarrow 0$, where $i = q, \bar{q}$ or $g$.

The strong flavour-spin correlation in the measured pion asymmetries prompts initial consideration of the diagrams of the $qqg$ amplitude (fig. 2a). This will certainly demonstrate the full potential of the approach. However, the triple-gluon correlator (fig. 2b) may also contribute \cite{14, 17} and should be taken into account; the technique described here does not depend on the detailed form of the correlators and thus will suffice in this case too. Therefore, we shall concentrate on contributions arising from diagrams of the type shown in fig. 1 and, in particular, on those arising when either a gluon or quark line becomes soft \cite{4, 12}. These may be divided into three classes: gluon insertion into (i) initial external lines, (ii) final external lines and (iii) internal lines. We shall consider these in turn.

The first two classes can, in principle, both provide an imaginary part: the insertion into an on-shell external line leads to an additional internal propagator, which may reach the soft limit. However, the transversity (see later) of the gluon connected to the hadronic amplitudes in question forces a non-zero transverse momentum in the struck line. Thus, the collinearity of the initial lines forces such a contribution to be of even

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Figure 1: Example contributions to twist-three transverse single-spin effects.
higher twist. On the other hand, the $p_T$ dependence of the final-state parton is just as suggested by the observed phenomena and only final-state external insertions give non-vanishing contributions. The last class leads to an imaginary part only when another external line becomes soft, i.e., when the gluon line carries all the momentum of the polarised hadron ($x_g = \pm 1$). These diagrams may also be written in a factorised form, viewing them in terms of soft fermionic insertions; although the final result is more complicated and both initial- and final-state insertions contribute.

There are two $qqg$ hadronic amplitudes (fig. 2a) for the twist-three contribution \[5\]:

$$D^A(x_1, x_2) \gamma_5 p s_T^\mu, \quad \text{and} \quad D^V(x_1, x_2) \frac{i\epsilon^{\mu p\bar{p}s_T}}{p\bar{p}},$$

where $p$ and $s_T$ are the momentum and (purely transverse) spin vectors of the incoming polarised hadron while $\bar{p}$ belongs to the unpolarised state; typically one takes $p^\mu = E(1, 0, 0, 1)$ and $\bar{p}^\mu = E(1, 0, 0, -1)$ in the partonic centre-of-mass frame. The parton correlators, $D^{A,V}(x_1, x_2)$, have the following symmetry properties under interchange of their arguments:

$$D^A(x_1, x_2) = D^A(x_2, x_1) \quad \text{and} \quad D^V(x_1, x_2) = -D^V(x_2, x_1).$$

(4)

It is instructive to rewrite the hadron-parton amplitudes using a suitable helicity basis, in which the calculation simplifies. To do this we shall adopt a common and convenient notation \[18\] and ignore quark-mass contributions:

$$u_\pm(p) = |p\pm\rangle \quad \text{and} \quad \bar{u}_\pm(p) = \langle p\pm|.$$

(5)

We may thus write

$$\psi = |p+\rangle \langle p+| + |p-\rangle \langle p-|,$$

$$\gamma_5 \psi = |p+\rangle \langle p+| - |p-\rangle \langle p-|.\quad \text{(6)}$$

For the amplitudes \[3\], the gluon is linearly polarised in a plane perpendicular to the beam (parallel and orthogonal to $s_T$ respectively for the axial and vector amplitudes). Thus, the polarisation vectors take the following natural forms:

$$\xi^\mu_A(p) = s_T^\mu \quad \text{and} \quad \xi^\mu_V(p) = -\frac{i\epsilon^{\mu p\bar{p}s_T}}{p\bar{p}}.$$

(7)
A helicity basis may be constructed using these:
\[
\tilde{\xi}^\mu_\pm(p) = \frac{p.\tilde{\eta}^\mu + \tilde{p}.\tilde{\eta}^\mu - p.\tilde{\eta}^\mu \mp i\varepsilon^{\mu\nu\rho\sigma}p_\nu\tilde{p}_\rho}{2\sqrt{p.\tilde{p}p.\eta\tilde{p}.\eta}}
\]
\[
= \frac{1}{\sqrt{2}} \left[ s_T^\mu \mp \frac{i\varepsilon^{\mu\nu\rho\sigma}p_\nu\tilde{p}_\rho}{p.\tilde{p}} \right] = \frac{1}{\sqrt{2}} \left[ \xi^\mu_\pm(p) \pm \xi^\mu_\mp(p) \right],
\]
where the choice of auxiliary vector,
\[
\tilde{\eta}^\mu = s_T^\mu + \frac{p^\mu + \tilde{p}^\mu}{\sqrt{2p.\tilde{p}}} \quad \text{with} \quad \tilde{\eta}^2 = 0,
\]
implicitly fixes the phase convention for circular polarisation. A more conventional choice for the phase is to take \(\tilde{\eta}\) in the scattering plane and perpendicular to the beam axis; in terms of such a set (without the tilde) one has
\[
\tilde{\xi}^\mu_\pm(p) = e^{\pm i\phi_{s\eta}} \xi^\mu_\pm(p),
\]
where \(\phi_{s\eta}\) is the azimuthal angle between \(s_T\) and \(\tilde{\eta}\).

Expressions (3) can thus be rewritten as
\[
D^A(x_1, x_2) \left[ |p_+\rangle \langle p_+| - |p_-\rangle \langle p_-| \right] \frac{1}{\sqrt{2}} \left[ e^{i\phi} \xi_+(p) + e^{-i\phi} \xi_-(p) \right],
\]
\[
D^V(x_1, x_2) \left[ |p_+\rangle \langle p_+| + |p_-\rangle \langle p_-| \right] \frac{1}{\sqrt{2}} \left[ e^{i\phi} \xi_+(p) - e^{-i\phi} \xi_-(p) \right].
\]

Note that, since \(\xi_- = \xi_+^\ast\), the last factors in the two expressions above are respectively purely real and purely imaginary. One also clearly sees how the axial (vector) contributions are related to amplitudes involving quark (gluon) helicity differences. The necessary phases are generated by combinations of the propagator imaginary parts and the gluon polarisation-vector phases.

The triple-gluon amplitudes have been considered by Ji \[17\] and lead to more complex expressions involving a number of correlation functions. However, the common simplifying characteristic is that the associated gluon polarisation projectors are restricted to the transverse plane and so can be represented by physical polarisation vectors.

### 3 Factorisation in single-spin \(\tau = 3\) amplitudes

Let us consider first of all the case of soft-gluon insertions into external quark lines, as in the left-hand diagram of fig. 2a. Extracting the imaginary part of the quark line (marked • in the figure) to the left of the gluon vertex forces \(x_g = 0\); taking this into account, the vertex may be written as
\[
\xi^\mu_X(p) \langle k, h_k | \gamma_\mu \bar{k} \ldots = \langle k, h_k | \xi^\mu_X \sum_h |k, h\rangle \langle k, h| \ldots \quad (X = A, V),
\]
where the ellipsis indicates the rest of the amplitude to the left of the vertex, and colour factors have been suppressed. Including the remnant factors from the imaginary
propagator part and factoring the $\langle k, h |$ projector above into the rest of the amplitude, eq. (12) reduces to a simple factor:

$$-i\pi \frac{k \cdot \xi_X(p)}{k \cdot p} \delta(x_g),$$

(13)
multiplying the now pure two-to-two amplitudes (see the right-hand diagram of fig. 3a). The complex-conjugate diagrams acquires a minus sign, arising from the opposite sign of the $i\epsilon$ in the propagator.

Soft-gluon insertions into external gluon lines lead to expressions of the type:

$$\sum_{\chi} V_{\mu\sigma\nu} \xi^\mu_X(p) \xi^\sigma_X(k) \xi^\nu_X(k) \xi^\rho_X(k) \ldots,$$

(14)

where the rightmost circular gluon polarisation vector will be factored into the remaining amplitude (represented by the ellipsis), and $V_{\mu\sigma\nu}$ is just the three-gluon vertex here:

$$V_{\mu\sigma\nu} = g_{\mu\sigma}(p - k)_\nu + g_{\nu\mu}(-k - p)_\sigma + g_{\sigma\nu}2k_\mu.$$

(15)

Only the last term survives (owing to the gauge choice) and we obtain

$$-i\pi \frac{k \cdot \xi_X(p)}{k \cdot p} \delta(x_g) \delta(\lambda_\nu, -\lambda_h),$$

(16)

![Figure 3: Graphical representation of the amplitude factorisation in the case of soft external (a) gluon and (b) quark lines. The solid circle indicates the line from which the imaginary piece is extracted, and $\xi$ refers to the gluon entering the factorised vertex.](image-url)
which has the same structure as the previous case, except that the gluon helicity is flipped ($\lambda = -\lambda_k$). And with the phase conventions adopted one has

$$k.\xi_\pm(p) = \frac{1}{\sqrt{2}} |k_T| e^{\pm i\phi_{k\eta}},$$

where $\phi_{k\eta}$ is the azimuthal angle between $k_T$ and $\vec{\eta}$. The particular phase dependence on $\phi_{k\eta}$ is just what is needed: in combination with that coming from the initial state gluon ($\phi_{s\eta}$, see above), it leads to the expected $\sin\phi_{k\eta}$ dependence of the final cross-section.

Three selection rules emerge:

1. The transverse nature of the gluon kills all contributions of initial-state insertions ($k = p$ or $\bar{p}$). Note that, for insertions into the incoming lines from the other (unpolarised) hadron, this depends on the choice of $p$ as the gauge-fixing vector for the gluons from the other hadron.

2. Unless the second hadron is also polarised, the $qqg$ axial contribution vanishes owing to parity conservation, as it is proportional to a helicity difference for the incoming quark from the first hadron.

3. Although proportional to a quark helicity sum, the $qqg$ vector contribution does not survive as it is multiplied by $D^V(x, x)$, which vanishes according to eq. (18).

Note also that the axial contribution, were it non-vanishing, would lead to a $\cos \phi$ dependence, i.e., to an up-down asymmetry.

It is possible to treat the case of soft external quark lines similarly, as in the left-hand diagram of fig. 3b. For want of better terminology, we shall call these soft-quark insertions; although a description in terms of insertion would be more pertinent to the case of a supersymmetric theory. The only subtlety is the change in nature of the resulting external particle: a fermionic insertion changes a fermion to a boson and vice versa. The imaginary piece of the gluon line to the left of the vertex forces $x_q = 0$; taking this into account and explicitly including the effective soft-quark spinor, the vertex may be written as

$$\sum_\lambda \langle k, h_k | \gamma_\mu | p, h \rangle \xi_\mu^\lambda(k) \xi^{\nu*}_\lambda(k) \ldots,$$

where again the rightmost term will be factored into the remaining amplitude. Including the various factors from the denominator etc., eq. (18) reduces to:

$$-\frac{i\pi}{k.p} \delta(x_q) \cdot i\hbar \sqrt{2k.p} e^{ih\phi} \delta_{\lambda,-h},$$

where the factored gluon polarisation vector carries helicity $-h$ (see the right-hand diagram of fig. 3b). Here the selection rule excluding initial-state insertions applies only to the partons from the same hadron.

\footnote{We shall comment later on the possible contribution of higher-order poles.}
We also see that both the axial and vector structures may contribute here, as they are proportional to $D^{A,V}(0,x)$. Moreover, the well-known helicity-conservation rules (forbidding the so-called maximally violating amplitudes \cite{18,19}) force the non-zero contributions to come only from the terms in eq. (11) with $(h_q,\lambda_g) = (\pm, \mp)$. Thus, the axial and vector contributions arise in simple linear combinations:

$$D^A(0,x) \pm D^V(0,x) = D^\pm(0,x) = \mp D^\mp(x,0),$$

see ref. \cite{5} for the relevant definitions. There only remains to calculate the case of insertions where the gluon is the external line and the quark, internal. This is, however, simply the complex conjugate of factor (19).

It is worth making a few further observations. Factorisation of the amplitudes immediately clarifies the possibility of large asymmetries, where once they were believed to be suppressed. First of all, the colour overlap is only slightly modified while the phase-space is unaltered, and thus little is lost for reasons of mismatch; the (supersymmetric \cite{18,19}) Ward identities guarantee the close similarity between amplitudes where a fermion line is replaced by a gluon. Indeed, the interference is not between differing kinematical configurations (as often found in early analyses) but simply between spin-flip and non-flip amplitudes; the quark-insertion factor shown in eq. (19) explicitly displays the spin-flip nature (between quark and gluon).

In the above we have ignored the possibility, discussed in the literature \cite{15}, that the correlator $D^V(x_1,x_2)$ might be accompanied by an extra pole in $(x_1 - x_2)$. Should this prove to be the case, then the requirement of an imaginary part would still force the $\delta$-function from the propagator. A Taylor expansion of $D^V(x_1,x_2)$ about the point $(x_1 - x_2) = 0$ would pick out the first derivative of the correlator but leave all other algebraic manipulations as before. Thus, the selection rule excluding terms in $D^V$ would be avoided while the factorisation property would remain unaltered.

Finally, the apparent higher order in $\alpha_s$ of the diagrams is removed by the absorption of the gluon propagator and vertices into the hadronic blob itself (as dictated by gauge invariance), leaving an effective tree-level leading-order graph. Moreover, the expressions may now be written in compact form and require little effort to calculate; all two-to-two pQCD amplitudes are already well known. Only the slightly modified colour factors remain to be evaluated, a task easily performed with the aid of a symbolic manipulation programme.

4 Conclusions

The resulting forms of the amplitudes given above greatly simplify the calculation of the asymmetries: the calculation of the tens of Feynman diagrams normally contributing is reduced to the evaluation of products of the simple factors derived here and known two-body helicity amplitudes. Since all two-body helicity amplitudes have indeed already been calculated in pQCD we shall merely present formal expressions for

\footnote{The author is particularly indebted to Oleg Teryaev for clarifying discussions on this point.}
the asymmetries, as sums over a very limited number of amplitudes for fixed helicities. The soft-insertion factorisation thus allows the partonic cross-section to be expressed in the following compact form:

$$\Delta \hat{\sigma} = \sum_{i,j} C_{ij} \mathcal{M}_i(x, \bar{x}, k_T) \mathcal{M}_j^\dagger(x, \bar{x}, k_T),$$

(21)

where $C_{ij}$ represents both the insertion factors given above and modified colour factors, and the $\mathcal{M}_i$, the individual two-body amplitudes. This much simplified form is ideal for the development of a computer programme (e.g., MadGraph [20]) based on helicity-amplitude subroutines (e.g., Helas [21]) for the automatic generation of cross-sections for any twist-three single-spin asymmetry.

In concluding, let us first of all highlight a difference in the interpretation of the origin of the $x_F$ dependence with respect to ref. [12], where the presence of the derivative of a $qgq$ correlator was claimed responsible for the rise in polarisation effects towards the edges of parton phase-space. Here, in contrast, the remnant factors of $(-t)^{-1/2}$, $(-u)^{-1/2}$ are seen to lie at the origin of this behaviour. It should be stressed that this transparency is due to the factorisation procedure presented.

It is also worth pointing out that the triple-gluon contributions, being insensitive to flavour, are also suggested by the experimentally observed approximately equal magnitudes and opposite signs of the $\pi^+$ and $\pi^-$ asymmetries, where one might have expected a ratio of the order of three to one (with opposite signs), according to SU(6). The (flavour-blind) triple-gluon contribution could lead to just the required net shift of both asymmetries in the same direction.

With the above formulation in terms of four-body amplitudes, it will not be difficult to set up an analysis of the existing data, from which a general parametrisation of the partonic correlators may be determined in a manner similar to that of Anselmino et al. [9]. On the other hand, the procedure adopted here is purely pQCD based and, in particular, requires no assumptions as to the nature of intrinsic $s_T-k_T$ correlations. Indeed, the factorisation property presented should help in clarifying the physical significance of the trade-off between the operator-product expansion description in terms of fields with only “good” components [8] and the $k_T$ dependence augmenting the parton picture [9].

As an example process, we have considered left-right asymmetries for final-state hadrons produced in hadron-hadron collisions with a single initial state polarised. However, it is clear that the proposed factorisation may be extended to many other processes in straightforward manner, including those involving polarised and unpolarised twist-three fragmentation functions. As remarked above, one could also consider measuring the up-down asymmetry predicted to exist for scattering involving one transverse polarisation and one longitudinal. While this asymmetry also contains twist-2 contributions, it would allow for a cross-check measurement of some of the distributions invoked here. The obvious advantage of the single-spin measurements (apart from their experimental accessibility) lies in their automatic and complete filtering of all twist-2 effects.
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