Improved Load Balancing in Large Scale Systems using Attained Service Time Reporting

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Our interest lies in load balancing jobs in large scale systems consisting of multiple dispatchers and FCFS servers. In the absence of any information on job sizes, dispatchers typically use queue length information reported by the servers to assign incoming jobs. When job sizes are highly variable, using only queue length information is clearly suboptimal and performance can be improved if some indication can be provided to the dispatcher about the size of an ongoing job. In a FCFS server measuring the attained service time of the ongoing job is easy and servers can therefore report this attained service time together with the queue length when queried by a dispatcher.

In this paper we propose and analyse a variety of load balancing policies that exploit both the queue length and attained service time to assign jobs, as well as policies for which only the attained service time of the job in service is used. We present a unified analysis for all these policies in a large scale system under the usual asymptotic independence assumptions. The accuracy of the proposed analysis is illustrated using simulation.

We present extensive numerical experiments which clearly indicate that a significant improvement in waiting (and thus also in response) time may be achieved by using the attained service time information on top of the queue length of a server. Moreover, the policies which do not make use of the queue length still provide an improved waiting time for moderately loaded systems.

1 INTRODUCTION

Load balancing plays a vital role to achieve a low latency in large scale clusters. It was proven in [17] that Join the Shortest Queue (JSQ) is the optimal policy to distribute jobs in a system with $N$ identical servers and exponential job sizes if jobs must be assigned immediately and no jockeying between servers is allowed. Ever since, the JSQ policy has often been referred to as the golden standard for load balancers. As $N$ grows large, the overhead created by probing all servers at every arrival becomes too large. Therefore the Shortest Queue $d$ (SQ($d$)) policy was introduced and analysed in various papers [1, 10, 15, 16]. A survey of recent advances can be found in [14]. However, the workload in many real systems consists of a mix of many short jobs and some large jobs, where the large jobs contribute a significant part of the total workload (see e.g. [4, 5, 11]). When all servers have the same queue length, the JSQ policy simply assigns the incoming job arbitrarily, while it is better to assign the job to a server which is less likely to be serving a large job.

Moreover, selecting a server with one long job may be worse than selecting a server with multiple small jobs.

Recently the Least Loaded $d$ policy (LL($d$)) was analysed in [2, 7], the LL($d$) policy assigns incoming jobs to a server which has the least amount of work left among $d$ randomly selected servers. This allows one to improve the SQ($d$) policy significantly. The drawback of this policy is however that it only works if job size information is available (which is mostly not the case) or when using a mechanism like late-binding (which brings significant overhead as jobs are not assigned immediately). While a server is typically unaware of its remaining workload, it can measure (up to some accuracy $\Delta$) the time it has spent processing the job(s) in service. This is especially true in FCFS servers. As jobs that have been in service for a substantial amount of time are highly likely to be long jobs that potentially have a long residual service time, using the attained service time to assign jobs to servers may improve performance.

In this paper we propose a collection of load balancing policies that exploit both queue length and attained service time (up to some granularity $\Delta$) information reported by the servers to assign jobs. Note that such policies do not require...
any knowledge of the size of incoming jobs or the job size distribution. We develop a unified analysis which may be used to analyse these load balancing policies in a large scale system under the usual asymptotic independence assumptions. Our main observation is that for small to moderate system loads and \( d \) sufficiently large, the improvement from using the attained service time information is substantial, while this improvement decreases as the system becomes critically loaded. For example, even policies which solely rely on the attained service time of the job in service may outperform the SQ(\( d \)) policy with \( d = 5 \) for a large range of arrival rates (see Figure 6a).

As we may not aspire to approach the performance of the LL(\( d \)) policy (as it is impossible to predict the exact workload using only the attained service time of one job and the queue length), we set ourselves a different goal. We define the Least Expected Workload policy (LEW(\( d \))) as the policy which assigns any incoming job to the server which has the least expected work left at its queue among \( d \) randomly selected servers. Note that the LEW(\( d \)) policy uses knowledge of the job size distribution to estimate the residual service time. We find that many of our policies that do not require such knowledge are able to achieve performance which is similar to that of the LEW(\( d \)) policy.

In our analysis we assume incoming jobs are Phase Type Distributed (further denoted by PH distributed). PH distributions are distributions with a modulating finite state background Markov chain [9] and any general positive-valued distribution can be approximated arbitrary close with a PH distribution. Further, various fitting tools are available online for PH distributions (e.g., [8, 12]).

The main contributions of this paper are as follows:

(1) We propose various load balancing policies that exploit queue length and attained service time information reported by the servers. We demonstrate that all of our policies achieve a significant reduction for the average waiting time of a job compared to SQ(\( d \)) under low to moderate workloads, with a performance that is often close to the LEW(\( d \)) policy. For some policies the waiting time is reduced for all workloads.

(2) We present a unified analysis which is applicable for all policies under consideration (and other variations thereof). This analysis may be of independent interest as it provides a means to assess the performance of an M/PH/1 queue with queue length and attained service time (up to some granularity \( \Delta \)) dependent arrival rates.

(3) We validate the accuracy of the asymptotic independence assumption used in the analysis using simulation experiments.

The paper is structured as follows. In Section 2 we formally define the model of interest. In Section 3 we present seven different policies which we study throughout the paper. In Section 4 we present the method used to analyse our load balancing policies. In Section 5 we verify our analysis by means of simulation of finite systems. Section 6 consists of extensive numerical experimentation investigating the impact of all parameters to our proposed model. Finally, we conclude in Section 7. All code used to generate the table and figures presented in the paper may be found at: https://github.com/THellemans/attainedServiceLB.

2 MODEL DESCRIPTION

The model we consider consists of \( N \) homogeneous servers (with \( N \) large) which all process jobs using FCFS scheduling. We assume jobs arrive according to a Poisson \( \lambda N \) process, these arrivals may occur to multiple dispatchers. We assume job sizes have a Phase Type distribution with representation \((\alpha, A)\). We use the notation \( \mu = -A1 \), where \( 1 \) is a column vector with all its entries equal to one, and denote by \( m \) the number of phases of the job size distribution. Hence, the probability of having a job size smaller than or equal to \( x \) is given by \( 1 - \alpha e^{Ax}1 \). Furthermore, we assume w.l.o.g. that the mean job size is equal to one.
3 LOAD BALANCING POLICIES

In order to define our policies, we first define $\mathcal{R}_n = [0, \infty)^n$ with the lexicographic order. All our policies are based on the same basic idea, from every server, the dispatcher receives the layer and queue length information coded as $(k, \ell) \in \mathbb{N} \times \mathbb{N}$. The dispatcher maps the information $(k, \ell)$ to some value $\xi(k, \ell) \in \mathcal{R}_n$ which is interpreted as a measure for the "version" of the chosen server. The incoming job is then assigned to the server for which the $\xi(k, \ell)$ value is the smallest (amongst the $d$ chosen servers), with ties being broken uniformly at random.

Example 3.1. Some basic examples of policies are random routing, which corresponds to picking $\xi(k, \ell) = 0$ for all $(k, \ell)$ and SQ($d$) for which one sets $\xi(k, \ell) = \ell$.

We now introduce a number of load balancing policies that are all described by defining $\xi : \mathbb{N} \times \mathbb{N} \rightarrow \mathcal{R}_n$. For our policies, we always have $\xi(0, 0) = (0)^n_{i=1}$ and $\xi(k, \ell) \neq (0)^n_{i=1}$ if $\ell \geq 1$. This ensures that we always assign to idle queues if possible. As jobs with larger attained service times are more likely to be large jobs with a potentially large residual service time, $\xi$ is always chosen to be non-decreasing in $k$.

Throughout, we assume that $\xi$ is chosen such that our model remains stable for $\lambda < 1$. More often than not, it suffices to note that the load balancing policy outperforms the random routing policy.

3.1 SQ($d$) with Runtime based Tie Breaking (SQ($d$)-RTB)

This policy mainly relies on the queue length information, but in case multiple chosen servers have the same number of pending jobs, the job is routed to the server for which the job at the head of the queue has currently received the least service, that is, is in the lowest layer $k$. The intuition is that the job in service is more likely to be a short job that will finish soon. For this policy, we set:

$$\xi(k, \ell) = (\ell, k).$$ (1)

We further refer to this policy as the SQ($d$) with Runtime based Tie Breaking policy, denoted by SQ($d$)-RTB. It is similar to SQ($d$) but may improve performance by using the attained service time to resolve ties.

3.2 SQ($d$) with Runtime Exclusion (SQ($d$)-RE($T$))

For this policy, we only rely on the queue length information, as long as the attained service time does not exceed some threshold $T$ (e.g., $T = 2$). When a server is queried it simply replies by stating its queue length and whether or not the attained service time is more than time $T$. This corresponds to setting $\Delta = T$ and $r = 1$ in our model. Whenever there are $1 \leq d' \leq d$ servers for which the attained service time does not exceed the threshold $T$, we assign the job to the server with the least number of jobs amongst the $d'$ chosen queues. If all $d$ servers report an attained service time above $T$ (meaning $k = 2$), the job is routed to the server with the least number of jobs amongst all $d$ chosen servers. The idea
is that we assume a job is large when its runtime significantly exceeds the average runtime of a job. For this policy, we define:

\[ \zeta(k, t) = (k, t), \]  

(2)

We refer to this policy as SQ\(d\) with Runtime Exclusion, denoted by SQ\(d\)-RE\(T\).

### 3.3 SQ\(d\)-RTB with Runtime Exclusion (SQ\(d\)-RTB-RE\(T\))

This policy is a mix of SQ\(d\)-RTB and SQ\(d\)-RE\(T\). We set some threshold \(T > 0\) where we suspect that a job is large once the attained service time exceeds this threshold. When \(d' \geq 1\) of the randomly chosen servers report an attained service time smaller than \(T\), we employ the SQ\(d\)-RTB policy to decide which of these \(d'\) servers receives the incoming job. When the attained service time of the \(d\) selected servers exceeds \(T\), we use SQ\(d\)-RTB to pick a server. This policy can also be described by defining an appropriate \(\xi\):

\[ \xi(k, t) = (\delta\{k > s\}, t, k), \]  

(3)

here \(\delta\{A\}\) is equal to one if \(A\) is true and zero otherwise and \(T = \Delta s = c_s\) for some \(s \leq r\). We refer to this policy as the SQ\(d\)-RTB with Runtime Exclusion policy, denoted by SQ\(d\)-RTB-RE\(T\).

### 3.4 Least Attained Service (LAS\(d\))

For this policy, we assume that the dispatcher assigns incoming jobs to the server for which the job at the head of the queue has attained the least amount of service among \(d\) randomly selected servers. This policy is defined by:

\[ \zeta(k, t) = k. \]  

(4)

We further refer to this policy as the Least Attained Service policy, denoted as LAS\(d\). The success of this policy relies on having highly variable jobs, such that it is more important to know whether the job at the head of a queue is large rather than knowing the number of jobs in the queue. Note that if we pick \(\Delta > 0\) sufficiently small, the probability of having a tie tends to zero.

### 3.5 LAS\(d\) with Queue length based Tie Breaking (LAS\(d\)-QTB)

For this policy, we assign the job to the queue for which the attained service time of the job at the head of the queue is minimal, but in case there is a tie between multiple servers, we assign the incoming job to the server with the fewest number of waiting jobs. To this end, we define:

\[ \zeta(k, t) = (k, t). \]  

(5)

We refer to this policy as LAS\(d\) with Queue length based Tie Breaking, which we denote by LAS\(d\)-QTB.

**Remark.** While for other policies, having a small \(\Delta > 0\) is always beneficial, the performance of LAS\(d\)-QTB may actually improve by have a larger value of \(\Delta\). In particular, letting \(r = 1\) and \(c_1 = T\) this policy reduces to SQ\(d\)-RE\(T\). While setting an arbitrary \(r\) and \(c_1, \ldots, c_r\), this policy may be viewed as an SQ\(d\) policy with multiple thresholds.

### 3.6 Runtime Exclusion (RE\((d, T)\))

For this policy, we set \(r = 1\) and \(\Delta = T\). In this case having \(k = 0\) means that the server is idle, \(k = 1\) means the job in service has an attained service time below \(T\) and \(k = 2\) otherwise. We find:

\[ \zeta(k, t) = k. \]  

(6)

We refer to this policy as the Runtime Exclusion policy, denoted by RE\((d, T)\). It is a special case of the LAS\(d\) policy with \(r = 1\).
3.7 Least Expected Workload (LEW(d))

For this policy we assume that the job size distribution of the incoming jobs is known, such that we can compute the mean residual service time given the attained service time. In this case we can use the more refined information of the expected residual service time $E[X | X > c_k]$ to decide which queue should receive the incoming job. This policy also fits in our framework by defining:

$$\xi(k, \ell) = (\ell - 1) \cdot E[X] + E[X | X \geq c_k] - c_k,$$

we refer to this policy as the Least Expected Workload policy, denoted by LEW(d).

**Remark.** As our main objective is to study load balancers which are not aware of the job size distribution, we only use this policy to see how it compares with the other strategies. This policy may be viewed as an idealized version of what the other policies attempt to do: avoid queues with a large expected workload. As such, we view the performance of LEW(d) as the goal of what the other policies try to achieve. In Section 6 we find that the performance of some policies indeed closely approximates that of LEW(d).

4 MODEL ANALYSIS

In this section we use the cavity approach presented in [3] to study the performance of the load balancing algorithms introduced in the previous section. We refer to the attained service time of a job at a server as its *age* $a$. Note that in our case the age $a$ of a job does not include the waiting time of the job, only the time it has been in service.

4.1 Description of the Cavity Map

The cavity process intends to capture the evolution of a single server assuming asymptotic independence among servers (see below). The state of a single server is captured by the service phase ($j$), the age ($a$) of the job in service and the queue length ($\ell$). The state of a server is thus denoted as a triple ($j, a, \ell$). As arrivals occur according to a Poisson($\lambda N$) process and each arrival has a probability of $d_N$ to select any particular queue, the rate at which a server is selected as one of the $d$ random servers is equal to $\lambda d$, we refer to this rate as the potential arrival rate. At each potential arrival, $d - 1$ independent copies of the queue at the cavity are considered. The state ($j, a, \ell$) for each of these $d - 1$ independent copies at time $t$ has the same distribution as the distribution of the queue at the cavity at time $t$. The potential arrival is assigned to one of the $d$ selected queues based on the ($k, \ell$) values reported by the queue at the cavity and the $d - 1$ independent queues. Thus, the actual arrival rate depends on both the queue length $\ell$ and the layer $k$ containing the age $a$ of the job at the head of the cavity queue at the time of a potential arrival. Let us denote this value by $\lambda_{\text{act}}(k, \ell)$.

In general, we find that the number of jobs present in the queue at the cavity increases by one with a rate equal to $\lambda_{\text{act}}(k, \ell)$, whilst its age $a$ continuously increases at rate one, and the service phase $j$ evolves as dictated by the phase type distribution ($\alpha, A$). When a job completes service, the age $a$ jumps to zero, $\ell$ decreases by one and a new job starts service if present.

In order to formally prove that the results presented in this paper correspond to the limiting behavior as the number of servers $N$ tends to infinity, the modularized program presented in [3] can be followed:

**a. Asymptotic Independence.** Demonstrate $\Pi^N \rightarrow \Pi$ as $N \rightarrow \infty$, where $\Pi^N$ is the stationary distribution for the studied policy with $N$ queues and $\Pi$ is a stationary and ergodic distribution on $(\{1, \ldots, m\} \times [0, \infty) \times \mathbb{N})^\infty$.

Show that the limit $\Pi$ is unique. Show that, for every $k$:

$$\Pi^{(k)} = \bigotimes_{j=1}^{k} \Pi^{(1)}$$
where $\Pi^{(k)}$ is $\Pi$ restricted to its first $k$ coordinates.

b. **The queue at the cavity.** Let $A^N_{act}$ denote the process of actual arrivals to the first queue. Show that $A^N_{act} \to \lambda_{act}$ in distribution as the number of servers $N$ tends to infinity.

c. **Calculations.** Given $\lambda_{act}$, the actual arrival rates, analyse the queue at the cavity in the large $N$ limit using queueing techniques to express $\Pi^{(1)}$ as a function of $\lambda_{act}$:

$$\Pi^{(1)} = T(\lambda_{act}).$$

Moreover, the arrival rate is also determined by the state of a server $\Pi^{(1)}$ we thus have:

$$\lambda_{act} = H(\Pi^{(1)}).$$

We then must determine the fixed point of the cavity map, that is, solve the equation $\Pi^{(1)} = T(H(\Pi^{(1)}))$ to obtain $\Pi^{(1)}$.

In this work, we focus on $c$, the computational step of the program. We present a numerical method to compute $\Pi^{(1)}$ for the load balancing policies in Section 3. For ease of notation we denote $\Pi^{(1)}$ as $\pi$. We validate our results using simulation to show that the obtained solutions indeed correspond to the system under study (as $N \to \infty$).

### 4.2 Obtaining the Steady State

In this section we indicate how to compute $\pi$ given $\lambda_{act}$. Given the discussion in the previous subsection, this step corresponds to determining the steady state of a queueing system with the following characteristics:

1. A single server queue that serves jobs in FCFS manner.
2. Service times of a job follow an order $m$ phase type distribution with parameters $(a, A)$.
3. Poisson arrivals with a rate that depends on the queue length $c$ and the attained service time $a$ (if the queue is busy).
4. No arrivals when the queue length equals some large value $B$.

More specifically, let $0 = c_0 < c_1 < \ldots < c_r < c_{r+1} = \infty$, then the dependence on $a$ is such that the Poisson arrival rate is only influenced by $k$, where $k$ is the unique index such that $a \in (c_{k-1}, c_k]$. In other words, the queueing system under consideration is fully determined by $B, (a, A)$ and a set of arrival rates $\{\lambda_0\} \cup \{\lambda_{k, \ell}|k \in \{1, \ldots, r+1\}, \ell \in \{1, \ldots, B-1\}\}$. The reason why we can assume an arrival rate equal to zero when the queue length equals some $B \ll \infty$ is explained further on.

For the purpose of determining a fixed point of the cavity map, it suffices to develop a method to compute the steady-state probabilities $\pi_{k, \ell, j}^{\text{busy}}$ that the service phase equals $j$, queue length equals $\ell$ and the attained service time $a$ belongs to $(c_{k-1}, c_k]$ given that the queue is busy, for $k = 1, \ldots, r+1, j = 1, \ldots, m$ and $\ell = 1, \ldots, B$. These probabilities are not affected by $\lambda_0$.

We start by defining a finite state discrete-time Markov chain on the state space

$$S = \{(k, \ell, j)|k = 0, \ldots, r; \ell = 1, \ldots, B; j = 1, \ldots, m\},$$

by observing the queue whenever the attained service time equals $c_k$ for some $k = 0, \ldots, r$. Note that this implies that we observe the queueing system exactly $i$ times for a job with length $z \in (c_{i-1}, c_i)$: once when the service starts and $i - 1$ times during its service.

Given that this DTMC is in state $(k, \ell, j)$ we can have two types of transitions:

1. The job in service remains in service for at least $c_{k+1} - c_k$ more time and the chain transitions to a state of the form $(k + 1, \ell', j')$ with $\ell' \geq \ell$.
2. The length of the job in service is below $c_k$ and the chain transitions to a state of the form $(0, \ell', j')$ with $\ell' \geq \ell - 1$. 

Hence, if we order the states in $S$ in lexicographical order, the transition probability matrix $p_{DTMC}$ has the following form

$$p_{DTMC} = \begin{pmatrix}
\Xi^{(1)} & \Lambda^{(1)} \\
\Xi^{(2)} & \Lambda^{(2)} \\
\vdots & \vdots \\
\Xi^{(r)} & \Lambda^{(r)} \\
\Xi^{(r+1)}
\end{pmatrix},$$

where $\Xi^{(k)}$ and $\Lambda^{(k)}$, for $k = 1, \ldots, r + 1$, are square matrices of size $mB$. To express these matrices we define the size $mB$ matrix

$$G = (I_B \otimes \mu) \cdot \begin{pmatrix}
1 & 0 & \ldots & 0 \\
1 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 \\
0 & 0 & \ldots & 1
\end{pmatrix} \otimes \alpha,$$

where $\mu$ was defined as $-A\hat{1}$ and the size $mB$ matrices

$$F^{(k)} = I_B \otimes A + \begin{pmatrix}
-\lambda_{k,1} & \lambda_{k,1} & \lambda_{k,1} & \ldots & \lambda_{k,1} \\
-\lambda_{k,2} & \lambda_{k,2} & \lambda_{k,2} & \ldots & \lambda_{k,2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-\lambda_{k,B-1} & \lambda_{k,B-1} & \lambda_{k,B-1} & \ldots & \lambda_{k,B-1} \\
-\lambda_{k,B} & \lambda_{k,B} & \lambda_{k,B} & \ldots & \lambda_{k,B}
\end{pmatrix} \otimes I_m,$$

for $k = 1, \ldots, r + 1$, with $\lambda_{k,B} = 0$.

The matrix $G$ contains the rates at which $(\ell, j)$ changes due to a service completion, which does not depend on the attained service time. The matrix $F^{(k)}$ captures the evolution of the queue length and service phase when there is no service completion and the attained service time is between $c_{k-1}$ and $c_k$. Based on these interpretations we have, for $k = 1, \ldots, r + 1$,

$$\Lambda^{(k)} = e^{F^{(k)}(c_k-c_{k-1})},$$

and $\Xi^{(k)} = \Psi^{(k)}G$, where

$$\Psi^{(k)} = \int_{0}^{c_k-c_{k-1}} e^{F^{(k)} \delta} \, d\delta = (I_{mB} - \Lambda^{(k)}) \cdot (F^{(k)})^{-1}.$$ 

Let $\pi^{(k)}$ be the size $mB$ vector holding the steady state probabilities of the DTMC characterized by $p_{DTMC}$ corresponding to the states of the form $(k, \ell, j)$, then due to the structure of $p_{DTMC}$, we have

$$\pi^{(k)} = \pi^{(k-1)} \Lambda^{(k)} = \pi^{(0)} \prod_{i=1}^{k} \Lambda^{(i)},$$

for $k = 1, \ldots, r + 1$. Using the first balance equation, the vector $\pi^{(0)}$ is therefore found as

$$\pi^{(0)} = \pi^{(0)} \sum_{k=0}^{r-1} \prod_{i=1}^{k} \Lambda^{(i)} \Xi^{(k+1)}.$$

As $\Xi^{(k)} = \Psi^{(k)}G$, the above equation can be restated as $\pi^{(0)} = \pi^{(0)} \Omega^{(1)}$, where $\Omega^{(r+1)} = \Psi^{(r+1)}G$ and $\Omega^{(k)} = \Psi^{(k)}G + \Lambda^{(k)} \Omega^{(k+1)}$.

Hence, the time complexity to compute the steady state probabilities of the DTMC characterized by $p_{DTMC}$ equals $O(mB^3)$.

As stated before, our aim is to compute the steady state probabilities $p_{busy}^{k,\ell,j}$ that the service phase equals $j$, queue length equals $\ell$ and the attained service time $a$ belongs to $(c_{k-1}, c_k]$ given that queue is busy. These probabilities can
now be computed from the steady state probabilities $\hat{\pi}_{k,ℓ,j}$ of the DTMC, by looking at the amount of time that the queue length and service phase equal $(ℓ′, j′)$ in between two points of observation of the DTMC. Note that entry $((ℓ, j), (ℓ′, j′))$ of the matrix $Ψ^{(k)}$ contains the expected amount of time that the queue length equals $ℓ′$ and the server is in phase $j′$ during a single transition of the DTMC from state $(k − 1, ℓ, j)$ to any other state. We therefore have

$$\pi^{busy}_{k,ℓ,j′} = ν(\hat{π}_{(k−1)}^{(k)})_{(ℓ′, j′)}.$$  

for $k = 1, \ldots, r+1, ℓ′ = 1, \ldots, B$ and $j′ = 1, \ldots, m$, with $ν$ being the normalization constant. As each job brings $E[X] = 1$ work on average, the queue at the cavity is empty with probability $1 − ν$. This allows us to express the stationary distribution $π$ by setting $π_0 = λ$ and renormalizing $π^{busy}$ to sum to 1.

### 4.3 Determining the Arrival Rate

In this section we indicate how to determine $λ_{act}$ given $π$. For any $(k, ℓ)$, we define:

$$\mathcal{A}(k, ℓ) = \{(k′, ℓ′) \mid k′ ≥ k\}, \quad \mathcal{B}(k, ℓ) = \{(k′, ℓ′) \mid k′ > k\}.$$  

(13)

We denote $x_{k,ℓ} = \sum_j π_{k,ℓ,j}$ the probability that, in the stationary regime, the queue at the cavity is in layer-$k$ with queue length $ℓ$. Furthermore we let:

$$u_{k,ℓ} = \sum_{(k′, ℓ′) \in \mathcal{A}(k, ℓ)} x_{k′, ℓ′}, \quad v_{k,ℓ} = \sum_{(k′, ℓ′) \in \mathcal{B}(k, ℓ)} x_{k′, ℓ′}, \quad w_{k,ℓ} = u_{k,ℓ} − v_{k,ℓ}. \quad (14)$$

**Remark.** When $ξ$ is injective, we simply have $w_{k,ℓ} = x_{k,ℓ}$.

We obtain the arrival rate in layer-$k$ with queue length $ℓ$ in the following Proposition:

**Proposition 4.1.** The arrival rate to the queue at the cavity, given its queue length $ℓ$ and service layer-$k$ is given by:

$$λ_{act}(k, ℓ) = λ \frac{w_{k,ℓ}}{w_{k,ℓ} + v_{k,ℓ}} \left( u_{k,ℓ}^d - v_{k,ℓ}^d \right). \quad (15)$$

**Proof.** Whenever a job arrives to the system, it samples the queue at the cavity and $d$ i.i.d. servers with distribution $x_{k,ℓ}$. Potential arrivals occur with rate $λ \cdot d$ and each potential arrival joins the queue at the cavity with probability $1/ℓ$ if $j$ of the chosen servers are in some state $(k′, ℓ′)$ with $ξ(k′, ℓ′) = ξ(k, ℓ)$, while all other servers are in state $(k′, ℓ′)$ with $ξ(k′, ℓ′) > ξ(k, ℓ)$. We obtain:

$$λ_{act}(k, ℓ) = λd \sum_{j=0}^{d-1} \frac{1}{j+1} \binom{d-1}{j} w_{k,ℓ}^j v_{k,ℓ}^{d-1-j} . \quad (16)$$

$$= \frac{λ}{w_{k,ℓ}} \left( (w_{k,ℓ} + v_{k,ℓ})^d - v_{k,ℓ}^d \right),$$

which simplifies to (15), finishing the proof.

**Remark.** While (15) is the more elegant formula, the expression in (16) is actually more stable for numerical purposes.

### 4.4 Iterative Procedure

In this section we show how to compute the fixed point $π$ described in Section 4.1 using Sections 4.2 and 4.3. The procedure in Section 4.2 corresponds to the function $T$ such that the stationary distribution of the queue at the cavity $π = T(λ_{act})$. The result in Section 4.3 corresponds to the map $H$ such that $λ_{act} = H(π)$. The fixed point of the cavity map is given by the fixed point $π$ which satisfies $π = T(H(π))$. To this end, we propose the following iterative scheme to compute $π$:

1. Pick some $π^{(0)}$ (e.g. $π^{(0)}_0 = 1$), set some tolerance $tol$ and $n = 0$.
2. Compute $π^{(n+1)} = T(H(π^{(n)}))$. 


We set $d = 5$ and consider all policies in Section 3.

We use the SQ($d$)-RTB for $d = 2, \ldots, 8$.

Fig. 1. Convergence of $\|\pi^{(n)} - \pi^{(n-1)}\|_1$ for $\lambda = 0.8$, $\Lambda = 0.1$ and HEXP(10, 1/2) job sizes.

(3) If $\|\pi^{(n+1)} - \pi^{(n)}\|_1 = \sum_{k,\ell,j} \pi_{k,\ell,j}^{(n+1)} - \pi_{k,\ell,j}^{(n)} < \text{tol}$ we accept $\pi^{(n+1)}$, otherwise increment $n$ by one and return to step 2.

Throughout our numerical experiments we typically employ the tolerance $\text{tol} = 10^{-10}$. Moreover, as illustrated in Figure 1, $\|\pi^{(n+1)} - \pi^{(n)}\|_1$ decreases exponentially in $n$ and the number of iteration required is typically below 50, where each iteration requires $O(m^3 B^3 r)$ time. If we start with $\pi_0^{(0)} = 1$, then setting $B = n$ during the $n$-th iteration suffices. We can even make use of a smaller $B$ value as the arrival rates $\lambda_{k,\ell}$ decrease very rapidly in $\ell$ for most policies considered.

4.5 Obtaining Performance Measures

Given the stationary distribution $\pi$ obtained in Section 4.4, we show how to obtain the performance measures associated to our model. In particular we are interested in the average queue length, response time and waiting time. The expected queue length can easily be computed as $E[Q] = \sum_{k,\ell,j} \ell \cdot \pi_{k,\ell,j}$, while the expected response time can then be derived using Little’s Law $E[R] = E[Q] / \lambda$. The expected waiting time is then given by $E[W] = E[R] - 1.$

In order to compute the waiting time distribution, we denote by $J_{k,\ell,j}$ the probability that a random job arriving to the system joins a queue with length $\ell$ for which the job at the head of the queue is in phase $j$ and resides in layer-$k$. It is not hard to see (similar to the proof of Proposition 4.1) that:

$$J_{k,\ell,j} = d \cdot \pi_{k,\ell,j} \sum_{s=0}^{d-1} \frac{1}{s+1} \binom{d-1}{s} w_{k,\ell}^s \cdot \nu_{k,\ell}^{d-1-s} = \frac{\pi_{k,\ell,j}}{w_{k,\ell}} \cdot \left( \nu_{k,\ell}^d - \nu_{k,\ell}^0 \right).$$

From this one easily computes the probability of joining a queue with length $\ell$ for which the job currently being served is in phase $j$: $J_{\ell,j} = \sum_k J_{k,\ell,j}$. Let us denote by $X$ a generic job size variable, and by $X_j$ a generic random Phase Type random variable with rate matrix $A$ which starts in phase $j$. Associated with these values we denote $X_{\ell,j}$, the convolution of $X_j$ with $\ell - 1$ i.i.d. copies of $X$. We find that the waiting time distribution is given by:

$$F_W(w) = \sum_{\ell \geq 1} \sum_j J_{\ell,j} F_{X_{\ell,j}}(w).$$
From this it is not hard to obtain the response time distribution:

$$\bar{F}_R(w) = \sum_{\ell \geq 1} \sum_{j} J_{\ell,j} \bar{F}_{X_{\ell+1,j}}(w) + (1 - \lambda^d) \bar{F}_X(w).$$

(19)

4.6 Job Size Distribution

In real systems job sizes are known to be highly variable and a significant part of the total workload is often offered by a small fraction of long jobs, while the remaining workload consists mostly of short jobs (e.g., [4, 5, 11]). A measure for the variability of a distribution is the Squared Coefficient of Variation (SCV), which is defined as $\text{Var}(X) / \mathbb{E}[X]^2$. The SCV of an exponential random variable with mean 1 is exactly equal to one, while measurements in real systems reveal much higher SCVs [6, Chapter 20].

For simplicity we represent these workloads as a hyperexponential (HEXP) distribution (with 2 phases) such that we can vary the SCV in a systematic manner as well as the fraction $f$ of the workload offered by the short jobs. More precisely, with probability $p$ a job is a type-1 job and has an exponential length with parameter $\mu_1 > 1$ and with the remaining probability $1 - p$ a job is a type-2 job and has exponential length with parameter $\mu_2 < 1$. Hence, the type-2 jobs are longer on average and we therefore sometimes refer to the type-2 jobs as the long jobs. The parameters $p, \mu_1$ and $\mu_2$ are set such that the following three values are matched:

- the mean job length (set to one),
- the squared coefficient of variation (SCV) and
- the fraction $f$ of the workload that is offered by the type-1 jobs.

To this end, we use the following equations:

$$\mu_1 = \frac{\text{SCV} + (4f - 1) + \sqrt{(\text{SCV} - 1)(\text{SCV} - 1 + 8f^2)}}{2f(\text{SCV} + 1)},$$

$$\mu_2 = \frac{\text{SCV} + (4\bar{f} - 1) - \sqrt{(\text{SCV} - 1)(\text{SCV} - 1 + 8\bar{f}^2)}}{2f(\text{SCV} + 1)},$$

with $\bar{f} = 1 - f$ and $p = \mu_1 f$. We note that this distribution has a PH representation with $\alpha = (\rho, 1 - \rho)$ and

$$A = \begin{pmatrix} -\mu_1 & 0 \\ 0 & -\mu_2 \end{pmatrix}.$$

Throughout, we denote this job size distribution as HEXP(SCV, f).

Remark. Note that all hyperexponential distributions have a decreasing hazard rate (DHR), which implies that jobs with a higher attained service time have a longer expected residual service time. This is in correspondence with $\xi(k, l)$ being non-decreasing in $k$ for our policies. While real workload distributions may not necessarily always have a DHR, we believe that in general the attained service time is a useful measure to detect whether or not a long job is in progress.

5 Finite System Accuracy

In this section we compare the mean waiting time that corresponds to the fixed point of the cavity map with simulation experiments. The simulation setup is identical to the model, except that the number of servers $N$ is finite. For each policy in Section 3, we selected some arbitrary parameter setting and varied the number of servers from $N = 10$ to $N = 2000$. More specifically, the simulation experiments were performed using the following parameter settings:

- SQ(3)-RTB : $\lambda = 0.7$, $\Delta = 0.01$, SCV = 10 & $f = 1/2$.
- SQ(5)-RE(2) : $\lambda = 0.8$, SCV = 10 & $f = 1/10$. 
which is simply a flattened version of $E$ where waiting time, rendering our models to be accurate for high values of $N$.

Throughout this section we compare the proposed policies relative to the SQ($d$) policy. Therefore, we are interested in the relative improvement of the proposed policies, in particular we focus on the quantity:

$$E_{W, rel, P} = \frac{\mathbb{E}[W^{(SQ(d))}] - \mathbb{E}[W^{(P)}]}{\mathbb{E}[W^{(SQ(d))}]}$$

(20)

where $\mathbb{E}[W^{(P)}]$ denotes the expected waiting time for some policy $P$. This value denotes how much additional waiting time using the SQ($d$) policy yields. Note that if one was mainly interested in the relative improvement in the response time, one would compute:

$$E_{R, rel, P} = \frac{\mathbb{E}[R^{(SQ(d))}] - \mathbb{E}[R^{(P)}]}{\mathbb{E}[R^{(SQ(d))}]}$$

which is simply a flattened version of $E_{W, rel, P}$. As $\mathbb{E}[R] = \mathbb{E}[W] + 1$, and we are mainly interested in the amount delay a job experiences, we focus on $E_{W, rel, P}$ as defined in (20). Throughout our numerical experimentation, we employ one central example as a base case in order to investigate the effect different parameters have on $E_{W, rel, P}$ for the policies $P$ discussed in Section 3. As the base case, we take $d = 5$, $\Delta = 0.1$ and HEXP(10, 1/2) distributed job sizes. In Figure 2 we observe the evolution of $E_{W, rel, P}$ as a function of the arrival rate $\lambda$. From Figure 2, we can already make quite a few observations:

- The relative improvement we obtain from using the attained service time information is significant, with relative improvements in the waiting time close to 75%. However, the improvement decreases as the arrival rate $\lambda$ approaches one. This makes sense as for a higher value of $\lambda$ the queues become longer and queue length information becomes more important than attained service time information.
- All policies which mainly focus on the queue length information improve the performance for all $\lambda$, as such they can be viewed as enhanced SQ($d$) policies. For the policies which mainly use the attained service time to distribute jobs, we observe that there exists some $\lambda_{\text{max}}$ such that these policies outperform SQ($d$) for all $\lambda < \lambda_{\text{max}}$ while they are outperformed by SQ($d$) when $\lambda > \lambda_{\text{max}}$.

### Table 1. Table of the expected waiting time for finite $N$ and the cavity method ($N = \infty$).

| $N$ (finite) | $N = \infty$ |
|-------------|-------------|
| SQ(3)-RTB  | 1.641957177 | 0.926703803 |
| SQ(5)-RE(2) | 3.308384612 | 1.564915051 |
| SQ(10)-RTB-RE(2) | 0.902071168 | 0.933562899 |
| LAS(2)     | 4.868804414 | 3.774355410 |
| LAS(7)-QTB | 7.598382494 | 3.755167910 |
| RE(6, 2)   | 3.435247775 | 0.902071168 |
| LEW(8)     | 6.068363318 | 0.780379145 |

- SQ(10)-RTB-RE(2) : $\lambda = 0.7$, $\Delta = 0.1$, SCV = 20 & $f = 1/3$.
- LAS(2) : $\lambda = 0.6$, $\Delta = 0.5$, SCV = 20 & $f = 1/4$.
- LAS(7)-QTB : $\lambda = 0.9$, $\Delta = 0.01$, SCV = 20 & $f = 2/3$.
- RE(6, 2) : $\lambda = 0.8$, SCV = 10 & $f = 1/4$.
- LEW(8) : $\lambda = 0.9$, $\Delta = 0.5$, SCV = 15 & $f = 1/3$.
Many of our policies perform as good as LEW(d) for loads up to 0.6, while these policies do not make use of the job size distribution. The performance of the SQ(5)-RTB-RE(2) policy is very close to that of LEW(d) for all $\lambda$, where the threshold $T = 2$ was chosen quite arbitrarily (cf. Section 6.6).

The improvement has a plateau at first, then the relative improvement tends to decrease with $\lambda$. For the policies which mainly employ the queue length information to distribute jobs, the curves become somewhat irregular for high loads. This irregularity is discussed when we consider the impact of larger $d$ values (see Section 6.1).

The low traffic limit ($\lambda \to 0^+$) appears to be the same for all policies which solely rely on a threshold and those which rely on the attained service time information. This makes sense as in the low traffic limits, whenever a job has a non-zero waiting time, all $d$ chosen queues have exactly one job waiting, meaning queue length information becomes irrelevant.

In the following sections, we reproduce Figure 2, where we change the value of one parameter. This allows us to investigate the effect this parameter has on our basic example.

**Remark.** As $\lambda$ increases, the number of jobs in each queue increases. This reduces the value of knowing the attained service time of the job at the head of the queue. However, if we were to use a scheduling policy such as Processor Sharing, dispatchers may request information on the attained service time of all jobs in the queue, which could result in a more significant improvement at high loads.

### 6.1 Impact of the number of chosen servers $d$

In Figure 3 we reproduce Figure 2 after setting $d = 20$ rather than $d = 5$ (and thus setting $\Delta = 0.1$, SCV = 10 and $f = 1/2$). We observe that the relative improvement increases significantly by increasing the value of $d$. In particular we make the following observations:

- Increasing the value of $d$ increases the low traffic limit and extends the range of the load at which this value is maintained. This is expected as higher $d$ values result in shorter queues.
- For the same reason, the value of $\lambda_{\text{max}}$ up to which the policies which solely depend on the attained service time outperform SQ(d) increases for larger $d$ values.
(a) Policies which do not make use of any threshold value.

(b) Policies which make use of the threshold value $T = 2$.

Fig. 3. Plots of the improvement in mean waiting time as a function of $\lambda$ for $d=20$, $A = 0.1$ and HEXP(10, 1/2) job sizes. This figure should be compared to Figure 2.

- The gap between the SQ($d$)-RTB-RE(2) and LEW($d$) becomes negligible by increasing the value of $d$ to 20.
- For some policies, the behaviour becomes irregular as $\lambda$ approaches one. To understand the cause of this irregularity, we define $T_d(\lambda)$ as the expected number of jobs which have the least number of jobs pending amongst $d$ randomly selected servers (given that all $d$ chosen servers are busy). Letting $u_k$ denote the probability that a server has $k$ or more pending jobs, we find that:

$$T_d(\lambda) = \sum_{k=1}^{d} k \cdot \sum_{\ell=1}^{\infty} \frac{P_{\ell,k}}{u_{\ell}^d}, \quad (21)$$

with $P_{\ell,k} = \binom{d}{\ell} (u_{\ell+1} - u_{\ell+1})^k \cdot u_{\ell+1}^{d-k}$, the probability that $k$ servers have exactly $\ell$ pending jobs and all other selected servers have at least $\ell + 1$ jobs in their queue. In Figure 4 we plot the evolution of $T_d(\lambda)$ for the SQ($d$) policy with various values of $d$, SCV = 10 and $f = 1/2$ in Figure 4a and $f = 1/10$ in Figure 4b. We observe the same type of irregularity as in Figure 3. For $T_d(\lambda)$ this behaviour can be understood as follows: for low loads the $d$ chosen servers have queue length 1 and the number of chosen servers with queue length one then decreases as $\lambda$ increases. If we look at the mean number of selected servers with queue length 2, then this mean increases with $\lambda$ (except for very high $\lambda$). For larger $\lambda$ values, the expected number of chosen servers with the least number of jobs also depends on the mean number of servers with queue length 2 as all $d$ chosen servers may have a queue length of at least 2. This causes the “waves” close to $\lambda = 1$ which become more pronounced as $d$ grows. These waves also explain the irregularity in Figure 3 (and Figure 2) as a higher value for $T_d(\lambda)$ implies we have more ties in the queue length and the attained service time information is more valuable. In Figure 4b we observe that decreasing the value of $f$ decreases the height of the waves.

To further emphasize the aforementioned observations, we single out two policies which were used to create Figure 3 and show their performance as a function of $\lambda$ for various values of $d =$ 2, 5, 10, 15, 20. In Figure 5a we consider the SQ($d$)-RE(2) policy whilst in Figure 5b we consider the RE($d$, 2) policy. We can clearly see our observations being confirmed. In particular for SQ($d$)-RE(2), we see waves becoming larger as $d$ increases, the plots of $d = 15$ and $d = 20$ even cross at some point.
Here we set $f = 1/2$, this figure is used to motivate the behaviour in Figure 3 and 5 for $\lambda \approx 1$.

Here we set $f = 1/10$, as such this setting can be seen to correspond to that considered in Figure 8.

Fig. 4. Plots of $T_d(\lambda)$ as a function of $\lambda$ for the SQ($d$) policy with various values of $d$ and HEXP($10, f$) job sizes.

(a) The SQ($d$)-RTB-RE($2$) policy.

(b) The RE($d, 2$) policy.

Fig. 5. Plots of the improvement in mean waiting time as a function of $\lambda$ for various values of $d$, $\Delta = 0.1$ and HEXP($10, 1/2$) job sizes. This figure should be viewed as a supplement of Figure 3.

6.2 Impact of the Squared Coefficient of Variation (SCV)

In Figure 6 we use the same parameter settings as in Figure 2, but change the SCV from 10 to 30 (i.e. we set $d = 5$, $\Delta = 0.1$ and $f = 1/2$). As expected, increasing the SCV, increases the relative improvement made by using the attained service time information. However, the improvement is not as significant as one might expect, especially for larger values of $\lambda$. Therefore we investigate this further in Figure 7, where we show the relative improvement as a function of the SCV for $\lambda = 0.7$, we observe that there is a clear improvement if we have a higher SCV, but only up to some point. For most policies we observe a strong improvement until SCV $\approx 15$, after which the improvement seems to flatten, or even decrease. The reason for the slight decrease is probably due to the fact that a higher SCV also results in
Policies which do not make use of any threshold value.

(b) Policies which make use of the threshold value $T = 2$.

Fig. 6. Plots of the improvement in mean waiting time as a function of $\lambda$ for $d = 5$, $\Delta = 0.1$ and HEXP($30, 1/2$) job sizes. This figure should be compared to Figure 2.

Policies which do not make use of any threshold value.

(b) Policies which make use of the threshold value $T = 2$.

Fig. 7. Plots of the improvement in mean waiting time as a function of the SCV for $\lambda = 0.7$, $d = 5$, $\Delta = 0.1$ and HEXP(SCV, 1/2) job sizes.

longer queues and similar to higher $\lambda$ values, this decreases the value of knowing the attained service time somewhat. Additional experiments showed that for smaller $\lambda$ values the decrease as a function of the SCV occurs further on.

6.3 Impact of the Load from Small Jobs $f$

In Figure 8 we repeat the experiment in Figure 2, but we replace the value of $f$ by $f = 1/10$. We observe that, somewhat surprisingly, having a small value for $f$ appears to decrease the gain from using the attained service time information. This may be explained by the fact that, as the value of $f$ decreases, the probability that all chosen servers are working on a large job increases. In particular for our example there is a probability around $(9/10)^5 \approx 59\%$ that all chosen servers are currently working on a large job (given that all chosen servers are currently busy). That is, in 59% of cases,
Policies which do not make use of any threshold value.

(b) Policies which make use of the threshold value \( T = 2 \).

Fig. 8. Plots of the improvement in mean waiting time as a function of \( \lambda \) for \( d = 5 \), \( \Delta = 0.1 \) and HEXP(10, 1/10) job sizes. This figure should be compared to Figure 2.

Policies which do not make use of any threshold value.

(b) Policies which make use of the threshold value \( T = 2 \).

Fig. 9. Plots of the improvement in mean waiting time as a function of \( f \) for \( \lambda = 0.6 \), \( d = 5 \), \( \Delta = 0.1 \) and HEXP(10, f) job sizes. This figure should be viewed as a supplement of Figure 8.

our policies reduce to either random routing or the standard SQ(d) policy. We emphasize this point using Figure 9, where we show the performance of our policies as a function of \( f \), we clearly see how the improvement reaches a peak around \( f \approx 2/3 \). The performance decreases sharply as \( f \approx 0 \) and \( f \approx 1 \), in the first case, (almost) all servers are working on large jobs, turning our policies into random routing resp. SQ(d) while for \( f \approx 1 \), almost all servers are working on short jobs, which again turns our policies into random routing resp. SQ(d).

6.4 Tail of the Waiting Time Distribution

In this section we take a closer look at the tail of the workload distribution \( F_W(w) = P\{W \geq w\} \). In Figure 10 we show \( F_W(w) \) as a function of \( w \) for the same setting as in Figure 2 with \( \lambda = 0.8 \) (i.e. \( d = 5 \), \( T = 2 \) and HEXP(10, 1/2) job sizes).
(a) Policies which do not make use of any threshold value.

(b) Policies which make use of the threshold value $T = 2$.

Fig. 10. Plots of the tails of the waiting time distributions $\bar{F}_W(w) = P\{W \geq w\}$ for $\lambda = 0.8, \Delta = 0.1, d = 5$ and HEXP(10, 1/2) job sizes. This figure should be seen as a supplement to Figure 2.

We observe that the tail behaviour is identical for all policies (and is the same as the tail of SQ(5)). This result is not unexpected as the tail is heavily influenced by the job size distribution of the long jobs. This entails that studying the tail behaviour does not add much value to our discussion, therefore we keep our focus on the mean waiting time.

6.5 The Granularity $\Delta$

We have always used the same granularity $\Delta = 0.1$, one could argue that in a real system the servers have more fine grained information on the attained service time of the job at the head of its queue. In Figure 11a we consider the same setting as in Figure 2 ($d = 5, T = 2$ and HEXP(10, 1/2) job sizes), but with $\Delta = 0.01$. We do not generate the plots associated to SQ(5)-RE(2) and RE(5, 2) as these policies are independent of the granularity (the server simply states whether or not it has exceeded the threshold $T$). With the exception of LAS(5)-QTB, we can hardly spot a difference with the plots in Figure 2. The exception for LAS(5)-QTB can be explained by noting that a smaller granularity $\Delta$ implies fewer ties in the reported $k$ value, meaning the queue length information is neglected more often and as $\Delta$ tends to zero, the performance of LAS(5)-QTB converges to that of LAS(5) (while for a very large granularity its performance resembles SQ($d$)-RE($T$)). We confirm our findings in Figure 11b, where we repeat the plots made in Figure 6, but with $\Delta = 0.01$ rather than $\Delta = 0.1$ (i.e. $d = 5, T = 2$ and HEXP(30, 1/2) job sizes).

6.6 Choice of the threshold $T$

Thus far, we have always used a (somewhat arbitrary) threshold $T = 2 \cdot \mathbb{E}[X]$. With this choice we noticed that we obtain a significant improvement over the standard SQ($d$) policy. However, it would make more sense if we chose a threshold which depends on the mean of the small jobs rather than the mean of all jobs. This way, our threshold is more directly linked to the probability that a job is long given that a server has been working on it for a time $T$. We now look at the impact of $T$ on the performance of the policies which depend on a threshold value. To that end, we use the setting of Figures 2b and 8b with $\lambda = 0.8$ (that is, we take $d = 5$ and consider HEXP(10, $f$) with $f$ equal to 1/2 and 1/10). Let $X_1$ denote the job size distribution for the small jobs, then we have $\mathbb{E}[X_1] \approx 0.53$ in the case of $f = 1/2$. 
For this plot, we set $SCV = 10$, as such it should be compared to Figure 2.

For this plot, we set $SCV = 30$, as such it should be compared to Figure 6.

Fig. 11. Plots of the improvement in mean waiting time as a function of $\lambda$ for $d = 5$, $\Delta = 0.01$ and HEXP($SCV, 1/2$) job sizes.

(a) For this plot, we set $f = 1/2$, as such it should be compared to Figure 2b.
(b) For this plot, we set $f = 1/10$, as such it should be compared to Figure 8b.

Fig. 12. Plots of the improvement in mean waiting time as a function of the threshold $T$ for $\lambda = 0.8$, $d = 5$, $\Delta = 0.1$ and HEXP($10, f$) job sizes. We denote the job size distribution of the small jobs by $X_1$. While $E[X_1] \approx 0.12$ for $f = 1/10$. In Figure 12 we notice that the plots attain a maximum and this maximum appears to be close to the value $T \approx 2E[X_1]$.

In Figure 13 we use this improved threshold value of $T = 2E[X_1]$ and show the improvement in mean waiting time as a function of the arrival rate $\lambda$. We observe that (comparing with Figures 2b and 8b) there is a noticeable improvement in performance. This is especially true for the case of $f = 1/10$, which is quite natural as for this case we now take $T = 0.24$ instead of $T = 2$ while for $f = 1/2$ the threshold value of 2 is replaced by the value $T = 1.06 > 0.24$. However, we note that in both cases, the improvement was still significant for all considered policies even when we were using a suboptimal threshold value.
(a) Plot for $f = 1/2$, this figure should be compared to Figure 2b.

(b) Plot for $f = 1/10$, this figure should be compared to Figure 8b.

Fig. 13. Plots of the improvement in mean waiting time as a function of $\lambda$ for $d = 5$, $A = 0.1$, HEXP(10, $f$) job sizes and using the improved threshold value $T = 2 \cdot E[X_1]$, with $X_1$ the job size distribution of the small jobs.

7 CONCLUSIONS AND FUTURE WORK

In this paper we showed that the mean waiting time of the SQ($d$) policy can be significantly reduced using simple policies if the servers report the attained service time in addition to their queue length when the workload consists of a mixture of short and long jobs. The attained service time is a quantity that is easy to measure in FCFS servers. It is even possible to achieve a sizeable improvement over SQ($d$) by simply setting a single threshold value $T$ such that a server can flag a job as large when this threshold is exceeded. While choosing a good value for $T$ may further improve performance, most policies are not too sensitive to $T$ and there is a wide range of $T$ values which achieve a notable performance improvement.

We found that the improvement coming from the attained service time information diminishes as the system load approaches one, as the attained service time information becomes less valuable in the presence of long queues. However, if one were to use another scheduling policy which is aware of the attained service time of multiple jobs in its queue, larger gains may also be feasible at high load (e.g. using Processor Sharing).

In our experiments we have focused solely on order 2 hyperexponential job sizes, which represent a system in which both large and small jobs arrive. We believe however the insights obtained in our numerical experiments extend far beyond this class of distributions. Further, the approach developed in the paper to assess the system performance can be used for any phase-type distribution (including fittings of heavy tailed distributions).

The main idea presented in this paper can also be used in systems where there is some form of memory at the dispatcher. For instance, the approach of [13] could be adapted such that the dispatcher maintains both an upper bound on the number of jobs in each queue and a lower bound on the attained service time of the job at the head of each queue.

REFERENCES

[1] Reza Aghajani, Xingjie Li, and Kavita Ramanan. 2018. The PDE Method for the Analysis of Randomized Load Balancing Networks. ACM SIGMETRICS Performance Evaluation Review 46, 1 (2018), 132–134.

[2] Urtzi Ayesta, Tejas Bodas, and Ina Verloop. 2018. On a unifying product form framework for redundancy models. (2018).

[3] M. Bramson, Y. Lu, and B. Prabhakar. 2010. Randomized load balancing with general service time distributions. In ACM SIGMETRICS 2010. 275–286. https://doi.org/10.1145/1811039.1811071
[4] Pamela Delgado, Diego Didona, Florin Dinu, and Willy Zwaenepoel. 2016. Job-aware scheduling in eagle: Divide and stick to your probes. In Proceedings of the Seventh ACM Symposium on Cloud Computing. 497–509.

[5] Pamela Delgado, Florin Dinu, Anne-Marie Kermarrec, and Willy Zwaenepoel. 2015. Hawk: Hybrid datacenter scheduling. In 2015 {USENIX} Annual Technical Conference ({USENIX} {ATC} 15). 499–510.

[6] Mor Harchol-Balter. 2013. Performance modeling and design of computer systems: queueing theory in action. Cambridge University Press.

[7] T. Hellemans and B. Van Houdt. 2018. On the Power-of-d-choices with Least Loaded Server Selection. arXiv preprint arXiv:1802.05420 (2018).

[8] J. Kriege and P. Buchholz. 2014. PH and MAP Fitting with Aggregated Traffic Traces. Springer International Publishing, Cham, 1–15. https://doi.org/10.1007/978-3-319-05359-2_1

[9] G. Latouche and V. Ramaswami. 1999. Introduction to Matrix Analytic Methods and stochastic modeling. SIAM, Philadelphia.

[10] M. Mitzenmacher. 2001. The power of two choices in randomized load balancing. IEEE Transactions on Parallel and Distributed Systems 12, 10 (2001), 1094–1104.

[11] K. Ousterhout, P. Wendell, M. Zaharia, and I. Stoica. 2013. Sparrow: Distributed, Low Latency Scheduling. In Proceedings of the Twenty-Fourth ACM Symposium on Operating Systems Principles (SOSP ’13). ACM, New York, NY, USA, 69–84. https://doi.org/10.1145/2517349.2522716

[12] A. Panchenko and A. Thümmler. 2007. Efficient Phase-type Fitting with Aggregated Traffic Traces. Perform. Eval. 64, 7-8 (Aug. 2007), 629–645. https://doi.org/10.1016/j.peva.2006.09.002

[13] Mark van der Boor, Sem Borst, and Johan van Leeuwaarden. 2019. Hyper-scalable JSQ with sparse feedback. Proceedings of the ACM on Measurement and Analysis of Computing Systems 3, 1 (2019), 1–37.

[14] Mark van der Boor, Sem C Borst, Johan SH van Leeuwaarden, and Debankur Mukherjee. 2018. Scalable load balancing in networked systems: A survey of recent advances. arXiv preprint arXiv:1806.05444 (2018).

[15] Thirupathaiah Vasantam, Arpan Mukhopadhyay, and Ravi R Mazumdar. 2018. The mean-field behavior of processor sharing systems with general job lengths under the sq (d) policy. Performance Evaluation 127 (2018), 120–153.

[16] N.D. Vvedenskaya, R.L. Dobrushin, and F.I. Karpelevich. 1996. Queueing System with Selection of the Shortest of Two Queues: an Asymptotic Approach. Problemy Peredachi Informatsii 32 (1996), 15–27.

[17] W. Winston. 1977. Optimality of the Shortest Line Discipline. Journal of Applied Probability 14, 1 (1977), 181–189.