THEORETICAL ASPECTS OF GRAVITATIONAL RADIATION\textsuperscript{a}

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A central problem in gravitational wave research is the generation problem, i.e., the problem of relating the outgoing gravitational wave field to the structure and motion of the material source. This problem has become, in recent years, of increased interest in view of the development of a worldwide network of gravitational wave detectors. We review recent progress in analytical methods of tackling the gravitational wave generation problem. In particular, we describe recent work in an approach which consists of matching a post-Newtonian expansion of the metric near the material source with a multipolar-post-Minkowskian expansion of the external metric. The results of such analytical methods are important notably for providing accurate theoretical predictions for the most promising targets of the LIGO/VIRGO interferometric network: the “chirp” gravitational waveforms emitted during the radiation-reaction-driven inspiral of binary systems of compact objects (neutron stars or black holes).

1 Introduction

I wish to dedicate this talk to Henri Poincaré who introduced (several aspects of) the concept of gravitational wave (“onde gravifique”) ninety years ago. Indeed, in his two seminal papers of June\textsuperscript{a} and July\textsuperscript{b} 1905 (the first one of which preceded Einstein’s paper on special relativity by one week), Poincaré not only introduced what was to become later the basic mathematical structures of special relativity (the Poincaré group and the “Minkowski” metric $(it)^2 + x^2 + y^2 + z^2)$, but also pioneered the idea that one needed, for consistency, a relativistic theory of gravitation. In his two papers written in 1905, he defines a class of “Poincaré invariant” gravity theories and emphasizes that they predict that the gravitational interaction propagates with the velocity of light. In a later work of 1908\textsuperscript{c}, he went as far as speaking of the emission of gravitational waves (“onde d’accélération”) and of the associated loss of energy of the emitting system. He even mentions that the main observable effect of this dissipation of energy into gravitational waves will be a secular acceleration.

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of the mean motions of planets. It is interesting to note that the discovery of binary pulsars, and their subsequent continuous observation, has allowed one precisely to verify the aspects of gravitational waves discussed by Poincaré: propagation of the gravitational interaction with finite velocity, and associated effect on the orbital dynamics. This verification has been observationally possible in two binary pulsar systems: PSR1913+16 and PSR1534+12.

The discovery of binary pulsars is important in three respects for gravitational radiation research: (i) it establishes the reality of gravitational radiation by verifying in a direct manner that gravity propagates with the velocity of light between the companion and the pulsar; (ii) it gives us our first tests of the strong-field regime of gravity thereby confirming the validity of Einstein’s theory in a regime so forth untested; (iii) it establishes the existence of strong sources of gravitational waves, thereby providing fascinating targets for the LIGO/VIRGO network of interferometric detectors.

Indeed, the observation of the secular acceleration of the orbital mean motions of the binary pulsars 1913+16 and 1534+12 proves that, in a few hundred million years, these systems will have shrunk so much that they will constitute an “inspiralling” binary system of neutron stars: i.e. a very close system of two neutron stars, orbiting around each other at a very fast and accelerated pace, the orbital frequency increasing from, say, \( \sim 10 \) Hz to \( \sim 1000 \) Hz in about 20 minutes. Then, when the two stars get near each other they start coalescing together to form only one central object. In the last minutes of the inspiralling motion such systems emit rather strong gravitational waves. The first person to conceive of such systems, and to realize they provided superb targets for gravitational wave detectors, was Dyson, in a prescient paper written years before the discovery of pulsars established the existence of neutron stars. The characteristics of inspiralling and coalescing binaries as gravitational wave sources have then been explored by several authors. The rate of occurrence of such events seem to be high enough to furnish a regular (\( \sim \) monthly) source of signals for the LIGO/VIRGO network. The information content of inspiralling events is of excellent quality: their detection (if done with a suitably high signal to noise ratio) should allow one: (i) to measure directly cosmological distances and thereby to have a clean access to the cosmological parameters \( H_0 \) and \( q_0 \); (ii) to test the nonlinear structure of radiative gravity; (iii) to perform new tests of the existence of a scalar component to gravity; (iv) to probe black hole physics. In view of the importance of inspiralling events, it is crucial to dispose of an accurate theoretical model of the corresponding gravitational wave signals. The aim of the present contribution is to review recent progress in analytical methods of tackling the generation of gravitational waves, and
their application to inspiralling binaries.

2 Analytical formalisms for treating the generation of gravitational waves

One can distinguish three basic questions in gravitational radiation theory:

– Question 1 (“asymptotic problem”): What is the asymptotic behaviour, appropriate to isolated systems and consistent with Einstein’s field equations, of radiative gravitational fields far from their sources?

– Question 2 (“generation problem”): What is the link between the preceding asymptotic behaviour and the structure and motion of the sources that generate the gravitational radiation?

– Question 3 (“radiation reaction problem”): What is the back-reaction of the emission of gravitational radiation on the source?

The standard answer to Question 1 is given by the Bondi-Sachs-Penrose description of radiative, asymptotically flat spacetimes, with a sufficiently smooth fall off at $I^+$ and the exclusion of ingoing waves on $I^-$. However, this answer is still a conjecture. Let us note, in this respect, that (i) the estimates used in the global theorem of Christodoulou and Klainerman are not strong enough to establish the standardly assumed peeling at $I^+$; and (ii) some perturbation calculations suggest a violation of peeling in scattering problems (where it is found that $I^+$ cannot be $C^3$).

Questions 2 and 3 have standard answers (discussed in textbooks) only at the lowest approximation. These standard answers go by the (ambiguous) name of “quadrupole formulas”. Actually, one should carefully distinguish: the “far-field quadrupole formula”, the “energy-loss quadrupole formula”, the “radiation-reaction quadrupolar force”, etc... (see, e.g., for a discussion).

In any case, these standard answers are insufficiently accurate to give mathematical models of inspiralling signals adequate for high-precision observations. Indeed, during the final stages of the inspiralling motion the orbital velocities become rather high ($v/c \lesssim 0.3$) and necessitate the consideration of many corrections to the leading “quadrupole” result.

Several methods have been proposed for going beyond the lowest-order results. For instance, some years ago Epstein, Wagoner and Thorne (in an attempt to generalize the Landau-Lifshitz-type derivation of the standard far-field quadrupole formula) introduced a post-Newtonian extension of the quadrupole formalism. Though their formalism is marred by some mathematical difficulties (divergent integrals), it was used to derive $O(v^2/c^2)$ corrections to the quadrupole formula for binary systems, and has been recently used to go to higher orders in $v/c$. A basic problem of the Landau-Lifshitz-
Epstein-Wagoner-Thorne approach is the lack of a clear separation between the near zone and the wave zone. The combined use of an “effective” stress-energy tensor for the gravitational field (with non-compact support) and of formal post-Newtonian expansions quickly leads to the appearance of divergent integrals. By contrast, Blanchet, Damour and Iyer (building on the Fock-type derivation of the quadrupole formula and on the double-expansion method of Bonnor) introduced a new gravitational-wave-generation formalism based on a clean separation between near-zone and wave-zone effects. The Blanchet-Damour-Iyer approach is mathematically well-defined and obtains corrections to the leading quadrupolar formalism in the form of compact-support integrals. [In a recent development of this formalism, Blanchet found it convenient to obtain the corrections in the form of (well-defined) analytically-continued integrals which are (formally) equivalent to compact-support integrals.] The BDI scheme has a “modular structure”: the final results are obtained by combining an “external zone module” (in which the external, vacuum metric is expanded as a multipolar-post-Minkowskian double series) with a “near zone module” (based on a more traditional post-Newtonian-type expansion). When dealing with strongly self-gravitating material sources (such as neutron stars, or black holes) one must also use a “compact body module”. After elimination of the various mathematical intermediaries appearing in the formalism (such as the “algorithmic” multipole moments $M_L$ and $S_L$), the basic structure of the final results of the BDI formalism is the following: the (directly observable) “radiative” multipole moments $U_L$ and $V_L$, parametrizing the angular dependence of the asymptotic gravitational wave amplitude $h^{TT}_{ij}(T, R, \theta, \phi)$, are given in terms of the “source” multipole moments $I_L$ and $J_L$ as a series of terms of increasing nonlinearity:

$$U_{ij}(t) = \frac{d^2 I_{ij}(t)}{dt^2} + \frac{2GM}{c^3} \int_0^\infty d\tau \left( \ln \frac{\tau}{2b} + \frac{11}{12} \right) \frac{d^4 I_{ij}(t-\tau)}{dt^4} + \cdots \quad (1)$$

For instance, the first two corrections (of order $v^2/c^2$ and $v^3/c^3$) to the radiative quadrupole moment are given by
with

\[
I_L(t) = \int d^3\hat{x}\sigma(t, \hat{x}) + \frac{1}{2(2\ell + 3)c^2} \frac{d^2}{dt^2} \int d^3\hat{x}\hat{x}^2 \sigma(t, \hat{x})
\]

\[
- \frac{4(2\ell + 1)}{(\ell + 1)(2\ell + 3)c^2} \frac{d}{dt} \int d^3\hat{x}\hat{x}^k L^{\ell k} \sigma_k(t, \hat{x}) + \ldots
\]

(2)

where \(\sigma \equiv (T^{00} + T^{kk})/c^2\), \(\sigma_i \equiv T^{0i}/c\). The integral appearing on the right of equation 1 represents the effect of the backscattering of the gravitational waves on the Schwarzschild-like curvature associated to the total mass \(M\) of the source (“tails”)44,38. Beyond the terms written in equation 1 there are many other nonlinear contributions (of formal higher order in \(v/c\)). For instance at the quadratically nonlinear order one has a contribution to \(U_L(t)\) depending upon the gravitational wave flux emitted in the past,

\[
\frac{8\pi c^{\ell - 2\ell!}}{(\ell + 1)(\ell + 2)} \int_{-\infty}^{t} dt' \left( \frac{dE_{GW}^{\ell}(t')}{dt'\,d\Omega} \right)_L,
\]

(3)

which has been discussed (in different guises, and under different names) by several authors45,46,47,48,49. For explicit applications of the presently discussed scheme, see50,51,52,53,23,54,55,56,57. Besides “hereditary” effects in the wave zone (such as the integral contributions in equations 1 and 3), it has also been possible to investigate the leading hereditary effects appearing in the near-zone field: they enter at the fourth post-Newtonian level, i.e. \((v/c)^8\) beyond the Newtonian approximation, and correspond to “tail” modifications of the \(O(v^5/c^5)\) Burke-Thorne radiation reaction potential35.

### 3 Inspiralling compact binaries

The accurate mathematical modelling of the gravitational wave signals emitted by inspiralling compact binaries needs two (related) inputs: (i) a solution of the “generation problem”; and (ii) a solution of the “radiation-reaction problem” adequate for treating compact objects. The schemes discussed in the previous section were primarily aimed at solving the generation problem, i.e. at giving \(h^{TT}_{ij}\) as a retarded functional of the structure and motion of the source. However, to have an explicit representation of \(h^{TT}_{ij}\) as a function of time, one needs to know the time evolution of the source, i.e. to solve the problem of motion, including radiation-reaction effects. Actually, it has been recently emphasized20 that the radiation-reaction part of the problem was the most crucial one in that it determined the time evolution of the phase of the gravitational
wave signal (which follows, modulo a factor two for circular orbits, the orbital phase). [We work here within the “restricted waveform” approximation, i.e. we focus on the main Fourier component of the signal.] Indeed, an accurate modelling of the (radiation-reaction-driven) phase $\phi_{GW}(t)$ of the gravitational wave is essential for a successful detection based on correlating the observed, noisy $h^{\text{obs}}(t)$ with some theoretical template $h^{\text{theory}}(t) = a_{GW}(t) \cos \phi_{GW}(t)$ (more so than the modelling of the evolution of the amplitude $a_{GW}(t)$). The results discussed in the previous section are certainly accurate enough for predicting $a_{GW}(t)$. The situation for what concerns the radiation-reaction driven phase $\phi_{GW}(t)$ is less satisfactory. Indeed, the only complete results available for the equations of motion of a compact binary are the $O((v/c)^5)$-accurate equations of motion. Beyond this level, only partial results are known: namely the $O((v/c)^7)$ radiation-reaction terms, and the $O((v/c)^8)$ hereditary contribution to the radiation-reaction. The (expected) “conservative” contributions (of order $(v/c)^6 + (v/c)^8 + \cdots$) to the equations of motion are unknown, as well as the higher-order contributions to radiation reaction. The only way one can presently deal with this problem is to heuristically rely on a (naive) energy-balance argument to relate the loss of mechanical energy of the binary system to the asymptotic flux of gravitational waves. In technical terms, one writes

$$\frac{dE_{\text{mechanical}}}{dt} = -\frac{dE_{GW \text{ flux}}}{dt} = \sum_{\ell=2}^{\infty} \frac{G}{c^{2\ell+1}} \times \left[ \frac{(\ell + 1)(\ell + 2)}{(\ell - 1)\ell!(2\ell + 1)!!} \left( \frac{dU_L}{dt} \right)^2 + \frac{4\ell(\ell + 2)}{c^2(\ell - 1)(\ell + 1)!(2\ell + 1)!!} \left( \frac{dV_L}{dt} \right)^2 \right],$$

in the right-hand side of which one inserts the best available results (from solving the generation problem) on the radiative multipole moments generated by a compact binary.

The highest-accuracy results obtained along these lines have been derived in the limiting case of a very small test mass orbiting a heavy central mass modeled as a Schwarzschild (or Kerr) black hole. Indeed, in such a case one can treat the generation problem as a linear perturbation of Schwarzschild or Kerr. The latter perturbation problem can, thanks to the work of many people (Regge, Wheeler, Zerilli, . . ., Teukolsky, . . ., Sasaki, Nakamura, . . ., Chandrasekhar, . . .), be reduced to integrating some linear ordinary differential equations for the radial dependence. This integration can be done numerically. Moreover, the post-Newtonian expansion (in powers of $v/c$) of the
generated gravitational wave amplitudes can be derived analytically[3,4,5]. The results of this approach are important testbeds for the full problem and can help us in posing the important questions (such as: “how fast does the post-Newtonian expansion converge?”[3]), but they fall short of providing us with the answers we really care about. Indeed, if we introduce the dimensionless parameter measuring the deviation from the test mass limit,

$$\nu \equiv \eta \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$$  

(5)

(where \(m_1\) and \(m_2\) are the masses of the members of an inspiralling binary), we expect that many of the systems that LIGO/VIRGO will detect will be made of two nearly equal neutron star masses: \(m_1 \approx m_2 \approx 1.4M_\odot\). This corresponds to the maximum possible deviation from the test-mass limit (\(\nu = \frac{1}{4}\) instead of \(\nu \ll 1\)) for which the black-hole perturbation results become unreliable. This is why we badly need the general-purpose analytical methods discussed in the previous section. Only such methods can, at present, deal with inspiralling binaries having comparable masses \(m_1 \sim m_2\). [Note, however, that black-hole perturbation methods are directly relevant for dealing with some of the target sources of low-frequency space interferometers such as LISA: e.g. the fall of a neutron star into a very massive black hole.]

The main result one is interested in is an analytical expression giving the time evolution of the gravitational wave phase, i.e. something we can call the “phasing formula” of inspiralling binaries:

$$\phi_{GW} = 2\phi_{\text{ORBITAL}} = F[t_\oplus; p_i]$$  

(6)

where \(t_\oplus\) is the (proper) time at the Earth laboratory recording the continuous arrival of the (main) gravitational wave signal

$$h_{GW}(t_\oplus) = a_{GW}(t_\oplus) \cos \phi_{GW}(t_\oplus),$$  

(7)

and where \(\{p_i\}\) is a set of parameters carrying information about the emitting binary system. It is interesting to note that the “phasing formula” is nothing but a continuous analog of the discrete “timing formula” which is basic to relativistic pulsar timing. Indeed, the timing formula of binary pulsars[6,7] can be written as

$$\phi_{PSR}^N = F[t_N^{\oplus}; p_i],$$  

(8)

where \(t_N^{\oplus}\) is the (proper) time of arrival at the Earth observatory of the \(N\)th (\(N \in \mathbb{N}\)) pulse emitted when the rotational phase of the spinning pulsar was \(\phi_{PSR}^N \simeq 2\pi N + \text{const.} \) Here also \(\{p_i\}\) is a set of parameters carrying information about the binary system, and the basic aim of pulsar timing is to measure as
many \( p_i \)'s as possible. The analogy between equations 6 and 8 is clear: in one case one is timing from Earth a continuous orbital phase, in the other case one is timing a stroboscopic rotational phase.

Thanks to recent works which explicitly developed the analytical methods discussed in the previous section to the \((v/c)^4\) and \((v/c)^5\) orders one knows the phasing formula of inspiralling binaries (of arbitrary masses) to the following accuracy: introducing the dimensionless time variable

\[
\hat{t} = \frac{c^3 \nu}{5G(m_1 + m_2)}(t_c - t)
\]  

(9)

(where \( t_c \) denotes the coalescence time) we can write

\[
\phi_c - \phi_{\text{ORBITAL}} = \frac{1}{\nu} \hat{t} \left\{ 1 + A_2(\nu)\hat{t}^2 + A_3(\nu)\hat{t}^3 + A_4(\nu)\hat{t}^4 + B_5(\nu)\hat{t}^5 \ln \hat{t} + \mathcal{O}(\hat{t}^6) \right\}
\]

(10)

where

\[
A_2(\nu) = \frac{5}{24} \left( \frac{743}{336} + \frac{11}{4} \nu \right)
\]

(11)

\((v^2/c^2)\) corrections: see \[32\] \[33\]

\[
A_3(\nu) = -\frac{3}{4} \pi
\]

(12)

\((v^3/c^3)\) or “tail” corrections see \[32\] for the \( \nu = 0 \) limit, and \[35\], \[36\], \[38\] for \( \nu \neq 0 \)

\[
A_4(\nu) = \frac{5}{64} \left( \frac{1855099}{225792} + \frac{56975}{4032} \nu + \frac{371}{32} \nu^2 \right)
\]

(13)

\((v^4/c^4)\) corrections: see \[35\] for the \( \nu = 0 \) limit and \[37\], \[38\], \[39\] for \( \nu \neq 0 \), and

\[
B_5(\nu) = -\pi \left( \frac{38645}{172032} + \frac{15}{2048} \nu \right)
\]

(14)

\((v^5/c^5)\) corrections, after the absorption of any \( A_5(\nu) \) in the definition of \( \phi_c \): see \[38\] for the \( \nu = 0 \) limit and \[42\] for \( \nu \neq 0 \). The numerical importance of finite-mass-ratio effects \((\nu \neq 0)\) is to be noted. In particular, the equal mass case \((\nu = \frac{1}{4})\) represents (with respect to the test-mass limit) an increase of \( A_2 \) by 31\% and of \( A_4 \) by 52\%! (By contrast the corresponding change in \( B_5 \) is only 0.8\%).
4 Conclusions

- A general comment on the present brief review of analytical approaches to gravitational radiation is that it confirms the perennial validity of a remark by Poincaré: to the effect that real problems are never definitively solved, but only more or less solved (“il y a seulement des problèmes plus ou moins résolus”).
- Though I have insisted on the need of computing the (hard to get) higher-order contributions to the phasing formula of inspiralling binaries because of their importance for extracting the maximum possible information from gravitational wave signals, it should be stated that these corrections are (probably) not needed for searching and discovering gravitational wave signals in the noise.
- The ultimate post-Newtonian accuracy which is really needed in the phasing formula for an acceptably accurate determination of the information-carrying parameters \( \{ p_i \} = \{ t_c, \nu \tau (m_1 + m_2), \nu \} \) is still unclear at present (see \[20, 63, 69, 70\]).
- I have considered above only the simplest case where the members of an inspiralling system are slowly spinning. This is the situation one can plausibly expect in most neutron star-neutron star systems. However, systems containing black holes (if they exist in appreciable number) might contain fast spinning objects. See e.g. \[52, 57\] for spin-dependent effects.
- If very high post-Newtonian contributions to the phasing formula are really needed, one might need to reconsider the presently developed analytical approaches. It might, for instance, become necessary to define them in a fully algorithmic manner allowing the use of computer-based algebraic programmes, or it might become necessary to match them to numerical relativity results (see e.g. \[71, 72\]). A better understanding of the mathematical nature of the post-Newtonian expansion might also help.
- I anticipate that a serious obstacle to improving the present \((v/c)^5\) accuracy of the phasing formula will come from the need to extend the accuracy of the equations of motion of compact binaries beyond the \(G^5\) level treated in \[42\].
- The increasingly slow convergence of the post-Newtonian series toward the end of the inspiralling stage points out the importance of improving the sensitivity of the detectors on the low-frequency side (say a few tens of Hz, corresponding to gravitational waves emitted early on). I note, in this respect, that the VIRGO detector puts a particular emphasis on improving its low-frequency sensitivity.

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9
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