Anomalous Quark Chromomagnetic Moment and Single-Spin Asymmetries

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Abstract

We discuss a nonperturbative mechanism for the single-spin asymmetries in the strong interaction. This mechanism is based on the existence of a large anomalous quark chromomagnetic moment induced by the nontrivial topological structure of QCD vacuum. Our estimations within the instanton liquid model for QCD vacuum show that AQCM generates very large SSA on the quark level. Therefore, this mechanism can be responsible for the anomalously large SSA observed in different high energy reactions with hadrons.

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1 Introduction

One of the longstanding problems in strong interaction is the understanding within QCD of the mechanism that is responsible for the large single-spin asymmetries (SSA) observed in numerous high energy reactions with hadrons. Many different approaches were suggested to solve this problem (see recent papers and reviews [1, 2, 3] and references therein). Most of them are based on the assumption of the so-called transverse-momentum-dependent (TMD) factorization [4, 5, 6, 7]. The validity of this assumption is not clear so far [8]. Furthermore, in our paper we will show the existence of the non-perturbative QCD mechanism which violates explicitly the TMD factorization for SSA.

It is well known that SSA arises from interference of different diagrams and should include at least two ingredients. First of all, it should be a helicity-flip in the scattering amplitude and secondly, the amplitude should have a nonzero imaginary part. The small current masses of quarks are only a source in perturbative QCD (pQCD) for helicity-flip. Furthermore, the imaginary part of the scattering amplitude, which comes from loop diagrams, is expected to be suppressed by extra power of the strong coupling constant $\alpha_s$. As a result, pQCD fails to describe large observed SSA. On the other hand, it is known that QCD has a complicated structure of vacuum which leads to the phenomenon of spontaneous chiral symmetry breaking (SCSB) in strong interaction. Therefore, even in the case of a very small current mass of the quarks their dynamical masses arising from SCSB can be large. The instanton liquid model of QCD vacuum [9, 10] is one of the models in which the SCSB phenomenon arises in a very natural way due to quark chirality flip in the field of strong fluctuation of the vacuum gluon field called instanton [12, 11]. The instanton is the well-known solution of QCD equation of motion in the Euclidian space-time which has nonzero topological charge. In many papers (see reviews [9, 10, 13]), it was shown that instantons play a very important role in hadron physics. Furthermore, instantons lead to the anomalous quark-gluon chromomagnetic vertex with a large quark helicity-flip [14, 10]. Therefore, they can give the important contribution to SSA [14, 15, 16, 17, 18, 10, 19].

In this paper, we will present the first consistent calculation of SSA in the quark-quark scattering based on the existence of the anomalous quark chromomagnetic moment (AQCM) induced by instantons [14].

2 Quark-gluon interaction in non-perturbative QCD

In the general case, the interaction vertex of a massive quark with a gluon, Fig.1, can be written in the following form:

$$V_\mu(p_1^2, p'_1^2, q^2)t^a = -g_s t^a [F_1(p_1^2, p_1'^2, q^2)\gamma_\mu + \frac{\sigma_{\mu\nu}q_\nu}{2M_q} F_2(p_1^2, p_1'^2, q^2)],$$

where the first term is the conventional perturbative QCD quark-gluon vertex and the second term comes from the nonperturbative sector of QCD.

In Eq.[1] the form factors $F_{1,2}$ describe nonlocality of the interaction, $p_1, p'_1$ are the momenta of incoming and outgoing quarks, respectively, $q = p'_1 - p_1$, $M_q$ is the quark mass, and $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$.

1 The semi-classical mechanism for SSA based on large AQCM has recently been discussed in papers [20, 21].
The form factor $F_2(p_1^2, p_1'^2, q^2)$ suppresses the AQCM vertex at short distances when the respective virtualities are large. Within the instanton model it is explicitly related to the Fourier-transformed quark zero-mode and instanton fields and reads

$$F_2(p_1^2, p_1'^2, q^2) = \mu_a \Phi_q(|p_1| \rho/2) \Phi_q(|p_1'| \rho/2) F_g(|q| \rho),$$

where

$$\Phi_q(z) = -z \frac{d}{dz}(I_0(z)K_0(z) - I_1(z)K_1(z)), \quad (2)$$

$$F_g(z) = \frac{4}{z^2} - 2K_2(z), \quad (3)$$

$I_\nu(z)$, $K_\nu(z)$ are the modified Bessel functions and $\rho$ is the instanton size.

We assume $F_1 \approx 1$ and $F_2(p_1^2, p_1'^2, q^2) \approx \mu_a F_g(q^2)$ since valence quarks in hadrons have small virtuality.

Within the instanton liquid model [9, 10], where all instantons have the same size $\rho_c$, AQCM is [22]

$$\mu_a = -\frac{3\pi (M_q \rho_c)^2}{4\alpha_s}. \quad (4)$$

In Eq.(4), $M_q$ is the so-called dynamical quark mass. We would like to point out two specific features of the formula for AQCM. First, the strong coupling constant enters into the denominator showing a clear nonperturbative origin of AQCM. The second feature is the negative sign of AQCM. As we will see below, the sign of AQCM leads to the definite sign of SSA in the quark-quark scattering. The value of AQCM strongly depends on the dynamical quark mass which is $M_q = 170$ MeV in the mean field approximation (MFA) [9] and $M_q = 350$ MeV in the Diakonov-Petrov model (DP) [10]. Therefore, for fixed value of the strong coupling constant in the instanton model, $\alpha_s \approx \pi/3 \approx 0.5$ [10], we get

$$\mu_a^{MFA} = -0.4 \quad \mu_a^{DP} = -1.6 \quad (5)$$

We would like to mention that the Schwinger-type of the pQCD contribution to AQCM

$$\mu_a^{pQCD} = -\frac{\alpha_s}{12\pi} \approx 1.3 \cdot 10^{-2} \quad (6)$$

Figure 1: a) Perturbative helicity non-flip and b) nonperturbative helicity-flip quark-gluon vertices
is by several orders of magnitude smaller in comparison with the nonperturbative con-
tribution induced by instantons, Eq. 5, and, therefore, it can give only a tiny contribution to
spin-dependent cross sections [24].

3 Single-spin asymmetry in high energy quark-quark
scattering induced by AQCM

The SSA for the process of transversely polarized quark scattering off unpolarized quark,
\( q^\uparrow(p_1) + q(p_2) \rightarrow q(p'_1) + q(p'_2) \), is defined as

\[
A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow},
\]

(7)

where \( \uparrow\downarrow \) denote the initial quark spin orientation perpendicular to the scattering plane

\[
d\sigma^\uparrow = \frac{|M(\uparrow\downarrow)|^2}{2I}dPS_2(S, q_t),
\]

(8)

where \( I \) is the initial flux, \( S = (p_1 + p_2)^2 \), \( M(\uparrow\downarrow) \) is the matrix element for the different
initial spin directions, \( dPS_2(S, q_t) \) is the two-particle phase space and \( q_t = p'_1t - p_1t \) is the
transverse momentum transfer. In the high energy limit \( S \gg q_t^2, M_q^2 \), we have \( I \approx S \)
and \( dPS_2(S, q_t) \approx d^2q_t/(8\pi^2S) \).

In terms of the helicity amplitudes [25], [26]

\[
\Phi_1 = M_{++;++}, \quad \Phi_2 = M_{++;-+}, \quad \Phi_3 = M_{+-;+-}, \quad \Phi_4 = M_{+-;+-}, \quad \Phi_5 = M_{++;;+-},
\]

where the symbols + or − denote the helicity of quark in the c.m. frame, SSA is given by

\[
A_N = -\frac{2Im[(\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4)\Phi_5^*]}{|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2}.
\]

(9)

In Fig.2, we present the set of diagrams which give a significant contribution to \( A_N \). Higher
order terms in \( \mu_a \) and \( \alpha_s \) are expected to be suppressed by a small instanton
density in QCD vacuum [9] and by a large power of the small strong coupling constant.

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Figure 2: Contribution to SSA arising from different diagrams.

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Recently, a rather large AQCM has been obtained within the approach based on the Dyson-Schwinger
equations (see review [23] and references therein).
For estimation, we take the simple form for the gluon propagator in the Feynman gauge

\[ P_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 - m_g^2}, \]

where \( m_g \) can be treated as the infrared cut-off related to confinement [29], or as the dynamical gluon mass [30], [31]. Within the instanton model this parameter can be considered as the effect of multiinstanton contribution to the gluon propagator.

By using in the high energy limit the Gribov decomposition for the metric tensor into the transverse and longitudinal parts

\[ g_{\mu\nu} = g_{\mu\nu}^\perp + \frac{2(p_{2\mu}p_{1\nu} + p_{2\nu}p_{1\mu})}{S} \]

and the Sudakov parametrization of the four-momenta of particles [27], [28]

\[ q_i = \alpha_i p_2 + \beta_i p_1 + q_{i,t}, \quad q_{i,t} p_{1,2} = 0, \quad q_{i,t}^2 = -q_i^2 < 0, \]

we finally obtain

\[ A_N = -\frac{5\alpha_s\mu_a q_t (q_t^2 + m_g^2)}{12\pi M_q} \left( \frac{F_2(q_t)}{N(q_t)} \right), \]

where

\[ N(q_t) = \int d^2 k_t \frac{(1 + \mu_a^2 (q_t \cdot k_t + k_t^2) F_2(\rho|q_t|) F_2(\rho|q_t + k_t|))}{(k_t^2 + m_g^2)((k_t + q_t)^2 + m_g^2)} \]

and

\[ D(q_t) = \left( 1 + \frac{\mu_a q_t}{2M_q} F_2(\rho|q_t|) \right)^2 + \frac{\alpha_s^2 (q_t^2 + m_g^2)^2}{12\pi^2} \left( \int \frac{d^2 k_t}{(k_t^2 + m_g^2)((k_t + q_t)^2 + m_g^2)} \right)^2 \]

4 Results and discussion

In Fig.3, the result for \( A_N \) as the function of transfer momentum transfer is presented for different values of the dynamical quark mass \( M_q \) and the parameter infrared cut-off \( m_g \). Our results show that SSA \( A_N \) induced by AQCM is very large and practically independent of particular values of \( M_q \) and \( m_g \). We would like to stress also that \( A_N \) in our approach does not depend on c.m. energy. The energy independence of SSA is in agreement with experimental data and in contradiction with naive expectation that spin effects in strong interaction should vanish at high energy [32]. One can show that this property is directly related to the spin one t-channel gluon exchange. Another remarkable feature of our approach is a flat dependence of SSA on transverse momentum of a final particle, Fig.3. It comes from a rather soft power-like form factor in the gluon-quark vertex, Eq.3 and a small average size of instanton, \( \rho_c \approx 1/3 fm \), in QCD vacuum [9]. Such a flat dependence has recently been observed by the STAR collaboration in the inclusive \( \pi^0 \) production in high energy proton-proton collision [33] and was not expected in the models based on TMD factorization and ad hoc parametrization of Sivers and Collins functions [3]. Finally, the sign of the SSA is defined by the sign of AQCM and should be positive, Eq.10. This sign is very important in explaining of the signs of SSA observed for inclusive production.
of $\pi^+$, $\pi^-$ and $\pi^0$ mesons in proton-proton and proton-antiproton high energy collisions (see discussion and references in [3], [32]).

It is evident that the instanton induced helicity-flip should also give the contribution to SSA in the meson production in semi-inclusive deep inelastic scattering (SIDIS) where large SSA in $\pi^-$ and $K$-meson production was observed by HERMES [34] and by COMPASS Collaborations [35]. In the leading order in the instanton density the nonzero contribution to SSA in SIDIS is expected to come from the interference of diagrams presented in Fig.4. Here, the imaginary part arises from final state perturbative and non-perturbative interactions of the current quark with the spectator system. The real part of the amplitude presented by two first diagrams includes perturbative helicity-conserved photon-quark vertex and the instanton induced helicity-flip vertex. The Pauli form factor corresponding to the last vertex was calculated in [38].

We should emphasize the significant difference between our approach to SSA in SIDIS and perturbative final state interaction model presented in [39]. In particular, one can expect that the main contribution comes from the kinematical region where the virtuality of gluon in Fig.4 is small. Therefore, soft gluon interaction with quarks should be highly nonperturbative. Furthermore, the helicity flip in [39] is related to the wave function of the nucleon. Due to that, SSA coming from this mechanism, might be significant only in the region of small transverse momentum of the final meson $k_t \approx \Lambda_{QCD} \approx 250$ MeV. In our approach, we expect the large SSA at higher transverse momentum because
the averaged instanton size is much smaller than the confinement size $\rho_c \approx R_{\text{conf}}/3$. This qualitative observation corresponds to the experimental data presented by HERMES and COMPASS where large SSA was observed only at rather large $k_t$. Additionally, a significant $Q^2$ dependence of SSA found by COMPASS Collaboration [33] might be related to the strong $Q^2$ dependence of the nonperturbative photon-quark vertex presented by second diagram in Fig.4.

The additional contribution to SSA induced by instantons was suggested in the papers [18] and [19]. It is based on the results from [36], where the effects of instantons in the nonpolarized deep inelastic scattering process were calculated in a careful way [3]. In this case, the effect arises from phase shift in the quark propagator in the instanton field. This contribution might be considered as complementary to the AQCM effect.

In spite of the fact that our estimation is based mainly on single-instanton approximation (SIA) for AQCM [14], the effects of the multiinstantons, which are hidden in the value of dynamical quark mass in Eq.4, are also taken into account in the effective way. The accuracy of such SIA was discussed in various aspects in [40]. By analyzing of several correlation functions the authors claimed that dynamical quark mass can be different from the MFA value $M_q = 170$ MeV. However, as it was discussed above, SSA induced by AQCM has rather a weak dependence on the value of dynamical mass, Fig.3. Therefore, we believe that some effects beyond SIA can not lead to a significant change of our results. Furthermore, we would like to mention that the SSA mechanism based on AQCM is quite general and might happen in any nonperturbative QCD model with the spontaneous chiral symmetry breaking. The attractive feature of the instanton liquid model is that within this model this phenomenon comes from rather small distances $\rho_c \approx 0.3$ fm. As the result, it allows to understand the origin of large observed SSA at large transverse momentum.

In summary, we calculated the SSA in the quark-quark scattering induced by AQCM and found that it was large. This phenomenon is related to the strong helicity-flip quark-gluon interaction induced by the topologically nontrivial configuration of vacuum gluon fields called instantons. Our estimation shows that the suggested mechanism can be responsible for anomalously large SSA observed in different reactions at high energies. We would like to stress that quark-gluon and quark-photon nonperturbative interactions violate the TMD factorization in inclusive meson production in both hadron-hadron and deep inelastic scatterings. Therefore, it cannot be treated as some additional contribution to the Sivers distribution function or to the Collins fragmentation function. It is evident that the nonfactorizable mechanism for SSA based on AQCM can be extended to other spin-dependent observables, including double-spin asymmetries in inclusive and exclusive reactions.

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This approach was applied to the Drell-Yan process [37] as well.
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