Shrinking fermionic modes, on the lattice and in the continuum

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Abstract

Recent lattice data indicates that volume occupied by topological fermionic modes shrinks to zero in the continuum limit of vanishing lattice spacing. The data apparently cannot be accommodated within, say, conventional instanton model. We review field-theoretic arguments which demonstrate that the topological fermionic modes are to shrink to a vanishing submanifold of the whole four dimensional space provided that measurements are performed with high resolution. Moreover, the data fit well the emerging overall picture of lower-dimensional defects in the Euclidean vacuum of Yang-Mills theories. We also mention results on localization of scalar particles.

1 Introduction

In this talk we will discuss topological defects in the Euclidean Yang-Mills vacuum. By topological defects we understand regions with large absolute value of the density of the topological charge $Q_{\text{top}}(x)$,

$$Q_{\text{top}}(x) = (16\pi^2)^{-1} G_{\mu\nu}^a(x) \tilde{G}_{\mu\nu}^a(x).$$

Usually one thinks about such regions in terms of instantons. For instantons,$^1$

$$\left( \int d^4x Q_{\text{top}}(x) \right)_{\text{instanton}} = 1,$$

which is large compared to the perturbative noise which contains an extra power of $\alpha_s^2$. The instanton picture has been challenged since long because of inconsistencies in the large $N_c$ limit$^{[3, 4]}$. An alternative description could be provided by domain walls$^{[3]}$. The theory of domain walls is not developed in detail, however.

Note that domain walls are, by definition, three-dimensional (3d) defects in the vacuum. Thus, one could argue that in the domain-walls picture topological defects would occupy a vanishing fraction of the whole 4d space. As far as we know, however, this point has never been emphasized. Moreover, the first example of low-dimensional vacuum defects was provided by lattice strings, or vortices$^{[2]}$ which are responsible for the confinement.

The possibility that topological defects could occupy a 3d submanifold of the 4d space was suggested first in Ref$^{[7]}$ in the context of the 3d defects discovered in$^{[8]}$ and closely related to

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$^1$The instanton model has been elaborated in great detail, for review see$^{[1, 2]}$.

$^2$For review of the role of the vortices in the confinement see$^{[5]}$ while identification of the vortices with strings is based on results of papers$^{[6]}$ and introduced in$^{[7]}$. 


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the vortices. Independently, there began to appear data on unusual behavior of fermionic zero modes as function of the lattice spacing. The data does indicate that topological defects shrink to a vanishing subspace of the whole space if measurements are performed with high resolution, that is on scale of the lattice spacing.

2 Low-lying fermionic modes

2.1 Generalities

To uncover topology of the gluonic fields one concentrates on low-lying modes of the Dirac operator. The modes are defined as solutions of the eigenvalue problem

$$D_\mu \gamma_\mu \psi_\lambda = \lambda \psi_\lambda,$$

where the covariant derivative is constructed on the vacuum gluonic fields \(\{A^a_\mu(x)\}\).

For exact zero modes, the difference between the number of modes with positive and negative chirality equals to the total topological charge of the lattice volume:

$$n^+ - n^- = Q_{\text{top}}.$$

Assuming that topological charge fluctuates by order unit independently on pieces of 4d volumes measured in physical units one derives:

$$\langle Q^2_{\text{top}} \rangle \sim \Lambda_{\text{QCD}}^{-4} V_{\text{tot}} \approx (180 \text{ MeV})^4 V_{\text{tot}},$$

where the numerical coefficient here is related to the \(\eta'\)-mass (the Witten-Veneziano relation).

One also considers so called near-zero modes which occupy, roughly speaking the interval

$$0 < \lambda < \frac{\pi}{L_{\text{latt}}},$$

where \(L_{\text{latt}}\) is the linear size of the lattice. Near-zero modes determine the value of the quark condensate via the Banks-Casher relation:

$$\langle \bar{q} q \rangle = -\pi \rho(\lambda \to 0),$$

where \(\lambda \to 0\) with the total volume tending to infinity.

2.2 Lattice data

While the close relation of the low-lying fermionic modes to the topology of the underlying gluon fields is well known since long, it is only recently that these modes have been measured on the original field configurations, without cooling. This recent progress is due to the advent of the overlap operator.

Measurements on original fields confirmed the general relations and . However, they also brought an unexpected result that the volume occupied by low-lying modes apparently tends to zero in the continuum limit of vanishing lattice spacing, \(a \to 0\). Namely,

$$\lim_{a \to 0} V_{\text{mode}} \sim (a \cdot \Lambda_{\text{QCD}})^r \to 0,$$

No suggestion was made, however, how to verify this prediction on the lattice. Moreover, even now, that there is emerging data that topological defects are indeed shrinking to a vanishing, three-dimensional submanifold of the whole space, see below, there is no proof that this submanifold is the same as suggested in Ref. [7]. Nevertheless, it is amusing that such a simple classification scheme of the defects as presented in could determine the dimension of the chiral defects correctly.
where \( r \) is a positive number of order unit and the volume occupied by a mode, \( V_{\text{mode}} \) is defined in terms of the Inverse Participation Ratio (IPR)\(^4\). As for exact numerical values of the critical exponent \( r \), one should address the original papers \([9]\) for details. Roughly speaking, measurements mostly favor \( r \approx 1 \), except for the second paper in Ref. \([9]\) where \( r \) is rather larger.

### 2.3 Chirality and the vortices

A crucial question is then, whether the underlying vacuum structure is the same for the confining fields and fields with non-trivial topology. An attempt to answer this question was undertaken in Ref. \([12]\) through a direct study of correlation between intensities of fermionic modes and of vortices.

In more detail, center vortex is a set of plaquettes \( \{D_i\} \) on the dual lattice, for review see \([5]\). Denote the set of plaquettes dual to \( \{D_i\} \) by \( \{P_i\} \). Then the correlator in point is defined as:

\[
C_\lambda(P) = \frac{\sum_{P_i} \sum_{x \in P_i} \rho_\lambda(x)}{\sum_{P_i} \sum_{x \in P_i} \langle \rho_\lambda(x) \rangle},
\]

where \( \rho_\lambda(x) \) is the density of the fermionic mode with eigenvalue \( \lambda \). Since \( \sum_x \rho_\lambda(x) = 1 \) and \( \langle V_{\text{tot}} \rho_\lambda(x) \rangle = 1 \), definition (7) can be rewritten as

\[
C_\lambda(P) = \frac{\sum_{P_i} \sum_{x \in P_i} (V_{\text{tot}} \rho_\lambda(x) - 1)}{\sum_{P_i} \sum_{x \in P_i} 1}.
\]

Results of the measurements can be found in the original paper \([12]\). Here we briefly summarize the findings.

First of all, there does exist positive correlation between intensities of fermionic modes and density of vortices. Second, the value of the correlator depends on the eigenvalue and the correlation is strong only for the topological fermionic modes. Finally and most remarkably, the correlation grows with diminishing lattice spacing. A simple analysis reveals that, indeed, if the 2d defects are entirely responsible for chiral symmetry breaking or constitute a boundary of 3d defects carrying large topological charge, the correlator (7) grows as an inverse power of the lattice spacing. The data does show that the correlator grows for smaller \( a \) but does not allow yet to uniquely fix the dimensionality of the chiral defects.

### 3 Pieces of theory

#### 3.1 “Subtraction volume”

The result \([6]\) is in striking contradiction with the instanton model and at first sight seems very difficult to appreciate. A more careful analysis demonstrates, however, that the shrinking of topological fermionic modes could have been predicted from field theory\(^5\).

As is explained above the topological fermionic modes just reveal the topological structure of the underlying gluon fields and we can, therefore, concentrate on distribution of \( \tilde{G} \tilde{G}(x) \). Consider correlator of the topological density at two points. From general principles alone, one can show that for any finite \( x \)\(^6\)

\[
\langle \tilde{G} \tilde{G}(x), \tilde{G} \tilde{G}(0) \rangle_{\text{Minkowski}} > 0
\]

\[
\langle \tilde{G} \tilde{G}(x), \tilde{G} \tilde{G}(0) \rangle_{\text{Euclidean}} < 0.
\]

\(^4\)Independent evidence in favor of shrinking of the regions occupied by topologically non-trivial gluon fields was obtained in \([11]\).

\(^5\)The argumentation was worked out by A.I. Vainshtein and the author \([13]\) and outlined in the talk \([14]\).

\(^6\)For references and discussion see \([15]\).
On the other hand, for a pure instanton, or within a zero mode:

$$\langle G\tilde{G}(x), G\tilde{G}(0) \rangle_{\text{instanton}} > 0.$$  \hfill (10)

Since the instanton contribution (10) taken alone violates unitarity, compare to (9), it cannot dominate and the unitarity is restored at any finite $x$ by perturbative contributions. Somewhat schematically, the correlator can be represented at short distances as:

$$\langle G\tilde{G}(x), G\tilde{G}(0) \rangle_{\text{Euclidean}} \sim -c_1\frac{\alpha_s^2}{x^8} + c_2\Lambda_{QCD}^4 \delta(x),$$  \hfill (11)

where $c_{1,2}$ are positive constants and the $1/x^8$ term is perturbative.

The central point is that by measuring topological modes we filter the perturbative noise away and are left with the local term. In the language of dispersion relations, this is a subtraction term, which has no imaginary part [15].

It is only natural then that contributions which are described by subtraction constants in dispersion relations appear as vanishing submanifolds once attempt is made to measure their spatial extension, or volume. Moreover, to see that the volume is small we need measurements with high resolution. Hence, dependence on the lattice spacing exhibited by the data (6). In this sense, the lattice spacing $a$ is to be understood as resolution of the measurements rather than an ultraviolet cut off.

### 3.2 Dimensionality of the chiral defects

Although this type of argument makes observation (6) absolutely natural and predictable, it does not immediately fix the exponent $r$. Further considerations seem to favor $r = 1$, or 3d defects. Here we briefly mention the arguments in favor of 3d topological defects (see also [7]).

First, we already mentioned that domain walls appear naturally in dual formulations of Yang-Mills theories with large $N_c$. It is not a proof of $r = 1$ yet since quantum corrections at $N_c = 2, 3$ could induce non-trivial fractal dimensions so that 3d manifold would percolate through the 4d space and occupy the whole or a finite part of it 7. From experience with the lattice strings, see, e.g. [7], we would still expect zero anomalous fractal dimension and topological defects occupying 3d volume in physical units.

More specifically, one can invoke analogy with quantum mechanics [13]. In case of quantum mechanics, the phenomenon is that if one tries to measure the time spent by a particle under the barrier, this time turns to be zero. A mnemonic rule is that the particle under the barrier lives in imaginary time and when projected to real time the barrier transition has zero duration (for details and references see [14]). Violation of the unitarity in (10) could also be formally removed by changing one coordinate from real to imaginary values. Thus, one expects that only one coordinate collapses to a vanishing interval for instanton transition (if measurements are performed with perfect resolution).

Note also that while the shrinking of topological modes (6) follows from Yang-Mills theory, explaining the observed correlation of the topological modes with the lattice strings is beyond the scope of field theory so far. Probably, clues are provided by theory of the defects in the dual, string formulation but there has been no discussion of the issue in the literature.

### 3.3 Protected and unprotected matrix elements

There is a drastic difference between results of measurements of, say, topological susceptibility [3] and of the instanton size. While the value of [3] does not depend on the resolution, or

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7Such defects were considered in Ref. [16]. Since these topological defects do not depend on the $\Lambda_{QCD}$ scale, they are the same typical for Yang-Mills as, say, for photodynamics. In these notes, we do not consider non-trivial fractal dimensions and by 3d defects always understand 3d volumes in physical units which are a vanishing part of the total volume in the continuum limit.
lattice spacing $a$, the size of topological excitations changes drastically:

$$(\text{size})_{\text{resolution } a} \sim \exp(-\text{const}/g^2(a)) \cdot (\text{size})_{\text{resolution } \Lambda_{\text{QCD}}},$$

where $g^2(a)$ is the bare coupling.

Clearly, one cannot think of deriving (12) in perturbation theory. Rather, one should think in terms of theory of measurements [13, 14]. As a matter of fact, the chiral condensate does not depend on the resolution and can be measured either without or with cooling while the size of topologically non-trivial regions of gluon field depends power-like on the resolution. One can talk, therefore, about ‘protected’ and ‘unprotected’ matrix elements.

The question is then whether we can judge theoretically which matrix element is protected. Without trying to formulate here a general recipe, turn to examples.

The local matrix element (5) is given by

$$\langle \bar{q} q \rangle \sim \lim_{m_q \to 0} m_q \int \frac{d\lambda}{(\lambda^2 + m_q^2)},$$

where $\lambda$ is the eigenvalue, see (1), and $\rho(\lambda)$ is the density of states. With lattice spacing $a \to 0$ the number of eigenfunctions grows power-like since $\lambda_{\text{max}} \sim 1/a$. However, it is clear that all the modes with $\lambda \gg m_q$ are canceled from (13) in the limit of the vanishing quark mass, $m_q \to 0$. Thus, quark condensate gives an example of a protected matrix element.

To define instanton size in terms of a matrix element one can try a non-local generalization of (13). Namely, consider the correlator

$$\langle \bar{q}^\alpha(x), q^\beta(0) \rangle \equiv f_1(x^2),$$

where the spinor indices are contracted the same way as in case of local condensate (13) and $\alpha, \beta$ are color indices. Because of chirality conservation, the nonlocal correlator is not contributed by perturbation theory and determined by instantons.

True, our correlator is not acceptable yet because it is not gauge invariant. To amend this, one can insert the phase factor:

$$\langle \bar{q}^\alpha(x)\Phi_{\alpha,\beta}(x,0)q^\beta(0) \rangle \equiv f_2(x^2).$$

An advantage of this definition is that one does not need to fix the path of integration in the P-exponent (14).

The question is now, whether the matrix elements (14), (15) are protected or not. It is obvious that the matrix element (14) changes greatly if measurements are performed with resolution of order $a$. Indeed the P-exponent entering (14) has ultraviolet divergent action, the same as, say, Wilson line $^9$. Therefore,

$$f_1(x) \sim \exp \left( -\text{const} \frac{|x|}{a\alpha_s} \right),$$

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$^8$This definition, in its gauge invariant versions, see below, was suggested to me by V.A. Rubakov.

$^9$I have learned the argument from Ph. de Forcrand, in another context.
and we conclude that the matrix element (14) is not protected against huge resolution-dependent corrections.

Turn now to another possibility, that is correlator (15). It is again very sensitive to the resolution (provided by the lattice spacing $a$). Indeed, the scalar particle acquires a quadratically divergent mass,

$$M_\phi^2 \sim \alpha_s/a^2,$$

and the correlator (15) vanishes at distances of order $a/\alpha_s$. Of course, one could try to renormalize mass but then many orders of perturbation theory should be taken into account and correlator (15) is no simpler than (14) discussed above.

3.4 Unitarity

Thus, the correlators (14), (15) are very sensitive to the resolution and subtractions. To get rid of the problem of divergences consider first the quenched approximation when both the spinor and scalar fields are non-dynamical external probes. To learn more on the resolution dependence, turn again to the unitarity constraints. Inserting a complete set of intermediate states:

$$f_2(x^2) \sim \Sigma_n \exp(-m_n \sqrt{x^2}),$$

where $m_n$ are masses of spin-1/2 ‘hadrons’ in case of pure gluodynamics. Assuming that there are no such bound states of gluons, we come to conclusion that

$$f_2(x^2) \sim \delta(x^2),$$

which is equivalent to derivation of shrinking of the fermionic topological modes to a vanishing 4d volume in the continuum limit.

Note, however, that in principle one cannot rule out that there exist spin-1/2 states in pure gluodynamics (Skyrme states). Then the proof of the shrinking is lacking. More realistically, if the fermions are treated dynamically there are spin-1/2 colorless states. The width of the $f_2(x^2)$ distribution is controlled then by the corresponding masses.

Thus, there is a hint that in the unquenched approximation the topological modes might not shrink any longer to a vanishing 4d volume. Moreover, our derivation of the shrinking given above and based on consideration of the correlator of topological charge densities also utilizes the quenched approximation. Indeed, we assumed a local correspondence between integrals over topological modes and bumps of topological charge density. Which is true only in the quenched approximation.

4 Localization of scalars

4.1 Quasiclassical picture

After experience with fermions, it is only natural to study localization properties of test color scalar particles in Yang-Mills vacuum [17]. To this end, one considers solutions of the equation

$$D^2 \phi_\lambda = \lambda^2 \phi_\lambda,$$

where the covariant derivative is constructed on the vacuum-field configurations $\{A^a_{\mu}\}$. The analysis has been performed for SU(2) case only but for various values of the color spin, $T=1/2, 1, 3/2$.

For the purpose of orientation let us begin with the instanton case where the equation (20) can be solved analytically [18]. Moreover, let us assume that the momentum of the scalar is much larger than the inverse instanton size, $p^2 \gg \rho^{-2}$. Then the effect of interaction with the external instanton field is a mass shift of the scalar:

$$p^2 \to p^2 - T(T+1)\rho^{-2},$$

which is equivalent to derivation of shrinking of the fermionic topological modes to a vanishing 4d volume in the continuum limit.
where $p^2$ is the momentum squared of a free particle. Eq. (21) looks like introduction of a tachyonic mass. But this is true only as an expansion at large $p^2$. There are no actual tachyonic states since all the eigenvalues of the Eq. (20) are positive definite.

4.2 Hard fields, tachyonic mass

If hard, or original vacuum fields, $A_\mu \sim 1/a$, are used to evaluate the eigenvalues $\lambda^2$ for scalar particles, then there is an ultraviolet divergent radiative mass correction,

$$\delta M^2 \sim \alpha_s a^{-2}.$$ (22)

On the lattice, indeed, the minimum eigenvalue $\lambda_{\text{min}}^2 \sim a^{-2}$. According to the standard perturbative prescription, one is free to renormalize the mass to any physical value by subtractions. For example, subtraction constant $M^2 = -\lambda_{\text{min}}^2$ would correspond to zero renormalized mass. After fixing the renormalized mass, the theory is fully determined perturbatively.

Non-perturbative treatment of the problem on the lattice [17] brings results different from the text-book pattern. Namely, there turn to be two candidates for identification with the radiative mass correction. Apart from $\lambda_{\text{min}}^2$ there is another remarkable eigenvalue, $\lambda_{\text{mob}}^2$ such that for

$$\lambda_{\text{min}}^2 < \lambda^2 < \lambda_{\text{mob}}^2$$ (23)

the corresponding eigenfunctions are localized on finite volumes. If we identify the mobility edge with the radiative mass and introduce subtraction term $M^2 = -\lambda_{\text{mob}}^2$ then there is an advantage that higher eigenvalues $\lambda_n^2 > \lambda_{\text{mob}}^2$ might be associated with standard plane waves. However, the renormalized eigenvalue

$$\tilde{\lambda}^2 \equiv \lambda^2 - \lambda_{\text{mob}}^2$$

is then negative in the interval (23). Tachyonic states are becoming reality.

More generally, existence of the two scalars, $\lambda_{\text{mob}}^2$ and $\lambda_{\text{min}}^2$ associated with a single particle defies the standard classification scheme of states with respect to the Poincare group.

4.3 Further critical exponents

There are new type of states, localized states, and new critical exponents can be introduced [17]:

$$\lambda_{\text{mob}}^2 - \lambda_{\text{min}}^2 \sim \Lambda_{QCD}^2 (a \cdot \Lambda_{QCD})^{-\alpha} ; V_{\text{loc}}(\lambda_{\text{min}}) \sim \Lambda_{QCD}^{-4} (a \cdot \Lambda_{QCD})^\beta,$$ (24)

where $V_{\text{loc}}(\lambda_{\text{min}})$ is the localization volume.

The results for $\alpha, \beta$ turn once again absolutely unexpected. Namely, the values of $\alpha, \beta$ depend crucially on the color spin:

$$\alpha(T = 1/2) \approx 0 , \quad \beta(T = 1/2) \approx 0 ; \quad \alpha(T = 1) \approx 1 , \quad \beta(T = 1) \approx 2 .$$ (25)

In other words, the interval (23) appears to be divergent in the continuum limit for the adjoint case. Then the renormalization program for the scalar particles in the adjoint representation cannot actually be performed [17].

10 There exist various detailed mechanisms of localization. The observed pattern of localization of the scalars does not correspond to the Anderson localization.
4.4 Preludes to interpretations

There has been no detailed theoretical discussion of the data \[17\] on localization of scalar particles in the literature and we can add only straightforward remarks.

The measurements brought results which could not be foreseen perturbatively. What is the basic difference between the two approaches, perturbative and non-perturbative? Non-perturbatively, or on the lattice equations of motion are not valid for any particular vacuum-fields configuration \(\{A^a_{\mu}\}\):

\[
D_{\mu}G_{\mu\nu} = \langle j^{' \nu} \rangle, \quad |\langle j^{' \nu} \rangle| \sim a^{-3},
\]

(26)

with a non-vanishing ‘current’ \(j^{' \nu}\). The Feynman graphs, on the other hand, are equivalent to evaluating matrix elements and equations of motion are (imposed to be) true.

Of course, one is tempted to disregard the spurious currents (26). However, it does not seem wise to throw away these currents in the Yang-Mills case. Indeed, confinement is nothing else but a manifestation of instability of the perturbative vacuum and the currents (26) provide original fluctuations which allow the system to learn about the non-perturbative instability. Killing all the fluctuations (26) on the lattice-spacing scale would extinguish confinement on the \(\Lambda_{QCD}\) scale.

Mostly, the fluctuations on the lattice-spacing scale (26) are dying without producing any effect but sometimes they develop into long-range structures. It is like in the Universe, original fluctuations of the density of matter at short distances develop into huge voids and clusters at later stages.

The instability of the perturbative vacuum is quantum-mechanical in nature and is revealed through barrier transtions. This implies that regions of strong instabilities are allowed to occupy only a vanishing part of the whole volume,

\[
V_{\text{instab}} \sim \exp(-\text{const}/g^2(a))V_{\text{tot}} \sim (a \cdot \Lambda_{QCD})^pV_{\text{tot}}.
\]

(27)

Lattice strings and chiral defects considered above are example of such regions of instability corresponding to

\[
\rho_{\text{strings}} = 2, \quad \rho_{\text{chiral}} = 3,
\]

respectively.

Fluctuations off the perturbative vacuum provide a random component of the non-Abelian fields. Stochastic fields, in turn, provide localization. Scalar particles are better tools to study localization since for fermions the gyroscope effect resists the localization strongly. Thus, studies of localization and confinement are closely related to each other.

Measurements [17] reveal a variety of stochastic fields on different scales:

- spin \(T = 1/2\) is sensitive to the \(\Lambda_{QCD}\) scale;  
- spin \(T = 1\) is sensitive to the \(\sqrt{\Lambda_{QCD} \cdot a}\) scale;  
- spin \(T = 3/2\) is sensitive to the \(a\) scale.  

(28)  
(29)  
(30)

Note that stochastic fields on the scale of \(\Lambda_{QCD}\) are commonly discussed. The appearance of the other scales is new. The data reveal also a kind of threshold behaviour of the dependence on the value of the coupling of scalars to gluons. For example, stochastic fields on the mixed scale of \(\sqrt{\Lambda_{QCD} \cdot a}\) are strong enough for binding spin \(T=1\) scalars but the spin \(T=1/2\) escapes from that scale and get bound only on the scale of \(\Lambda_{QCD}\). In a way, we are lucky to have such simple means to probe various scales, by varying the color spin. There is no simple explanation of the phenomenon, however.

There is little doubt that we are only at the very beginning of learning the stochastic fields in the vacuum.
5 Conclusions

Lattice measurements on original, hard fields $A_\mu^a \sim 1/a$ brought unexpected, power-like dependences on the lattice spacing \[6, 8, 9, 17\]. Generically, one can talk about low-dimensional defects in vacuum \[7\]. Theory of such defects is in its infancy. In this talk we presented a few observations on power-like dependences on the lattice spacing, which are not necessarily closely related to each other.

In particular, we argued that the lattice spacing plays the role of resolution \[13\]. A simple analogy from quantum mechanics is measurements of instantaneous velocity of a particle: it tends to infinity with space resolution tending to zero.

More specifically, in case of topological fermionic modes (in the quenched approximation) one can utilize dispersion relations to argue that the volume occupied by the modes vanishes indeed in the continuum limit. The physics behind is that large fluctuations of the topological density are due to under-the-barrier transitions in the Euclidean space. Therefore, they do not correspond to any physical intermediate states and are described by local, or subtraction terms in field theory \[13\].

Thus, chirality-related defects seem to be the first case when shrinking to a vanishing (in the continuum limit) submanifold of the whole 4d space can be understood and predicted within standard formulation of field theory. Moreover, lattice measurements are always restricted to some finite values of the lattice spacing and one could suspect that 'lower-dimensional defects' are in fact pre-asymptotic. The field-theoretic derivation of existence of a low-dimensional defect has an advantage of holding directly in the continuum limit.

Another, and probably most interesting aspect is that dual formulations of YM theories with large $N_c$ predict existence of low-dimensional vacuum defects. One of the earliest of such predictions is existence of domain walls, or 3d defects related to topology \[3\]. Although the argumentation does not apply literally if $N_c$ is not large, the basic geometrical constructions could survive in the $N_c = 2, 3$ cases as well. Then, to uncover low-dimensional defects one needs measurements with high resolution since only explicit lattice dependence distinguishes, say, 3d defects from 4d excitations.

As for the scalars, let me mention in conclusion only a single point. At this conference a lot of attention is devoted to hypothetical new forms of matter, due to scalar fields, see, e.g., \[19\]. The lattice simulations indicate strongly that one can, indeed, expect unusual states for scalar fields which look like localized states when continued to the Euclidean space. I am not sure that specifically such states have ever been discussed in the literature on new forms of matter.

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References

[1] Th. Schafer, E.V. Shuryak, “Instantons in QCD”, Rev. Mod. Phys. 70 (1998) 323, arXiv:hep-ph/9610451.

[2] M. Teper, “Topology in QCD”, Nucl. Phys. Proc. Suppl. 83 (2000) 146, arXiv:hep-lat/9909124.
[3] E. Witten, “Instantons, The Quark Model, And The 1/N Expansion”, Nucl. Phys., B145 (1978) 110; “Theta dependence in the large N limit of four-dimensional gauge theories”, Phys. Rev. Lett. 81 (1998) 2862, [arXiv:hep-th/9807109].

[4] I. Horvath, et al., “On the local structure of topological charge fluctuations in QCD.”, Phys. Rev. D67 (2003), 011501. [arXiv:hep-lat/0203027].

[5] J. Greensite, “The Confinement problem in lattice gauge theory”, Progr. Part. Nucl. Phys. 51 (2003) 1, [arXiv:hep-lat/0301023].

[6] F. V. Gubarev et al., “Fine tuned vortices in lattice SU(2) gluodynamics”, Phys. Lett. B574 (2003) 136, [arXiv:hep-lat/0212003]; V.G. Bornyakov, et al., “Anatomy of the lattice magnetic monopoles”, Phys. Lett. B537 (2002) 291, [arXiv:hep-lat/0103032].

[7] V.I. Zakharov, “Lower-dimensional vacuum defects in lattice Yang-Mills theory”, Yad. Fiz. 68 (2005) 603, [arXiv:hep-ph/0410034]; “Dual string from lattice Yang-Mills theory”, AIP Conf. Proc. 756 (2005) 182, [arXiv:hep-ph/0510111].

[8] A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn, V.I. Zakharov, “Three dimensional vacuum domains in four dimensional SU(2) gluodynamics”, Phys.Lett. B613 (2005) 52, [arXiv:hep-lat/0408014]; M.I. Polikarpov, S.N. Syritsyn, V.I. Zakharov, “A novel probe of the vacuum of the lattice gluodynamics”, JETP Lett. 81 (2005) 143, [arXiv:hep-lat/0402018].

[9] C. Aubin, et al. “The Scaling Dimension of Low Lying Dirac Eigenmodes And Of The Topological Charge Density”, Nucl. Phys. Proc. Suppl. 140 (2005) 626, [arXiv:hep-lat/0410024]; F.V. Gubarev, S.M. Morozov, M.I. Polikarpov, V.I. Zakharov, “Localization of low lying Eigenmodes for chirally symmetric Dirac operator”, JETP Lett. 82 343(2005), [arXiv:hep-lat/0508016]; Y. Koma et al., “Localization properties of the topological charge density and the low lying eigenmodes of overlap fermions”, PoS LAT2005:300,2005 [arXiv:hep-lat/0509164]; C. Bernard et al., “More evidence of localization in low-lying Dirac spectrum” PoS LAT2005:299,2005 [arXiv:hep-lat/0510025]; V. Weinberg et al., “The QCD vacuum probed by overlap fermions”, [arXiv:hep-lat/0610087].

[10] R. Narayanan, H. Neuberger, “A Simulation of the Schwinger model in the overlap formalism”, Nucl. Phys. B 443, (1995) 305, [arXiv:hep-th/9411108]; H. Neuberger, “Exactly massless quarks on the lattice”, Phys. Lett. B417 (1998) 141, [arXiv:hep-lat/9707022]; “More about exactly massless quarks on the lattice”, Phys. Lett. B427 (1998) 353, [arXiv:hep-lat/9801031].

[11] P.Yu. Boyko, F.V. Gubarev, S.M. Morozov, “SU(2) gluodynamics and HP1 sigma-model embedding: Scaling, topology and confinement”, Phys. Rev. D73 (2006) 014512, [arXiv:hep-lat/0511050]; P.Yu. Boyko, F.V. Gubarev, S.M. Morozov, “SU(2) gluodynamics and HP1 sigma-model embedding: Scaling, topology and confinement”, Phys. Rev. D73 (2006) 014512 [arXiv:hep-lat/0511050].

[12] A.V. Kovalenko, S.M. Morozov, M.I. Polikarpov, V.I. Zakharov, “On topological properties of vacuum defects in lattice Yang-Mills theories”, [arXiv:hep-lat/0512036].

[13] A.I. Vainshtein, V.I. Zakharov, in preparation.
[14] V.I. Zakharov, “Matter of resolution: From quasiclassics to fine tuning”, in “Sense of Beauty in Physics. A volume in honour of Adriano Di Giacomo”, Pisa University Press (2006), p 61, arXiv:hep-ph/0602141.

[15] M. Aguado, E. Seiler, “The Clash of positivities in topological density correlators”, Phys. Rev. D72 (2005) 094502, arXiv:hep-lat/0503015.

[16] H.B. Thacker, “D-branes and topological charge in QCD”, PoS LAT2005 (2006) 324, arXiv:hep-lat/0509057.

[17] J. Greensite et al., “Localized eigenmodes of covariant Laplacians in the Yang-Mills vacuum”, Phys. Rev. D71 (2005) 114507, arXiv:hep-lat/0504008;
J. Greensite et al., “Peculiarities in the spectrum of the adjoint scalar kinetic operator in Yang-Mills theory.”, Phys. Rev. D74 (2006) 094507, arXiv:hep-lat/0606008.

[18] G. ’t Hooft “Computation Of The Quantum Effects Due To A Four-Dimensional Pseudoparticle”, Phys. Rev. D14 (1976) 3432.

[19] R. Gannouji, D. Polarski, A. Ranquet, A.A. Starobinsky, “Scalar-Tensor Models of Normal and Phantom Dark Energy”, JCAP 016 (2006) 0609, arXiv:astro-ph/0606287;
V.A. Rubakov, “Phantom without UV pathology”, arXiv:hep-th/0604153.