The Impact of Non-zero $\theta_{13}$ on Neutrino Mass and Leptogenesis in a SUSY SO(10) Model

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The recent measurement of the reactor angle as $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ come from the Daya Bay collaboration. Evidence of nonzero $\theta_{13}$ was also there at T2K, MINOS and Double Chooz experiments. We study the implication of these recent data on neutrino mass matrix and consequently on leptogenesis in a supersymmetric SO(10) model. To explain the smallness of neutrino mass, in general, we require a heavy Majorana neutrino which is a natural candidate in SO(10) model. In minimal SO(10) model, the symmetry breaking scale or the right-handed neutrino mass scale is close to the GUT scale. It is not only beyond the reach of any present or future collider search but the lepton asymmetry generated from its decay is in conflict with the gravitino constraint as well as unable to fit the neutrino data. We show that addition of an extra fermion singlet can accommodate the observed recent neutrino data in a supersymmetric SO(10) model. This model can generate the desired lepton asymmetry and provide TeV scale doubly-charged Higgs scalars to be detected at LHC.

The standard model (SM) of particle physics is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, where C, L and Y respectively stand for colour, left-handed and hypercharge quantum numbers. In the SM, due to conservation of lepton family number and absence of any right-handed (RH) counterpart, left-handed neutrinos are massless. However, it is well established that neutrino flavour oscillates which require neutrinos to be massive. One can generate tiny neutrino mass either by various type of seesaw mechanisms [1] or via loop corrections [2] going beyond the standard model. Neutrino oscillations can be parametrized using two mass squared differences, three mixing angles and one CP-phase of Pontecorvo-Maki-Nakagawa-Sakata matrix [3]. The third mixing angle, namely the reactor angle, was little known till date.

In neutrino physics, a breakthrough measurement of the third mixing angle $\theta_{13}$ come from Daya Bay experiment [4], and is confirmed by RENO experiment [5]. A more than 5$\sigma$ measurement given by Daya Bay as $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ and the corresponding value from the RENO experiment is $\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst})$. Their central values are very close to what predicted in ref. [6, 7] using the combined data set of T2K [8] and MINOS [9], earlier in the mid of last year, with more than 3$\sigma$ evidence. There was also a similar result from Double Chooz experiment [10] as well. All these new data, thus, make the neutrino mass matrix much more constrained and their consequences to other areas of physics. This new measurement led to prediction and implication of the $\theta_{13}$ angle in different ways [11].

The main ingredient of see-saw mechanism is heavy Majorana neutrino, which is a natural candidate in the left-right symmetric SO(10) grand unified theories (GUTs) [12]. Embedding supersymmetry (SUSY) in such a model have some good features like protecting the Higgs mass from radiative correction that appear due to the huge difference between the weak and unification scales and help to have a good unification of gauge couplings at the GUT scale. We, thus, work here in a scenario of left-right symmetric (LR) supersymmetric SO(10) GUT model. In order to keep the gauge coupling unification intact at the GUT scale, any intermediate scale, here the LR-symmetry breaking scale, has to be very close to the GUT-scale. However, such a heavy symmetry breaking scale or an equally heavy Majorana neutrino is beyond the reach of any present or future collider analysis as well as is unable to produce the low energy light neutrino data.

Observed baryon asymmetry of the universe is another interesting problem. A popular explanation is to generate baryon asymmetry via sphaleron process from lepton asymmetry [13, 14]. The later generally can be produced through the C and CP-violating out-of-equilibrium decay of heavy Majorana neutrinos, which is a member of SO(10)-GUT model and also responsible to explain the tiny neutrino mass.

In the standard thermal leptogenesis, with heavy hierarchical right-handed neutrino spectrum, the CP-asymmetry and the mass of the lightest right-handed Majorana neutrino are correlated. In order to have the correct order of light neutrino mass-squared differences, there is a lower bound on the mass of the right-handed neutrino, $M_N \gtrsim 10^9$ GeV [15], which implies a reheating temperature $\gtrsim 10^9$ GeV. This will lead to an excessive gravitino production and conflicts with the observed data as discussed below.

Gravitino, being the lightest and stable, is a suitable dark matter candidate in a R-parity conserving SUSY. In the post-inflation era, these gravitino are produced in a thermal bath due to annihilation or scattering processes of different standard particles. The relic abundance of gravitino is proportional to the reheating temperature of the thermal bath. One can have the right order of relic
dark matter abundance only if the reheating temperature is bounded to below $10^7$ GeV [16, 17].

In this article we work in left-right symmetric SUSY SO(10) GUT model, rich with an extra SO(10) singlet lepton per generation [18–20]. This extra singlet lepton is a natural member in $E_6$ and many other models. The issue is earlier addressed in different context [21, 22]. However, here, we are to accommodate the recent neutrino data in this model. In addition, we discuss the impact of these new data on other related phenomenology. Our analysis, in a single model, is able to explain various issues like the light neutrino masses and their mixing as measured in recent experiments, have an exact unification of different gauge couplings at the GUT scale, have a low intermediate scale or a lighter right-handed Majorana neutrino as well as to generate right amount of lepton asymmetry to explain the observed baryon asymmetry of the universe without being in conflict with the gravitino constraint.

The spontaneous symmetry breaking prescription of SO(10) group in our model is as follows – At the GUT scale SO(10) is spontaneously broken with a simultaneous vacuum expectation value (vev) to the $\Phi^{54}$ of along the singlet direction under the Pati-Salam group ($G_{PS}) \ SU(2)_L \times SU(2)_R \times SU(4)_C$ [23] and to the singlet direction under the left-right gauge group ($G_{LR}) \ SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ in the $G_{PS}$ multiplet, $(1,1,15)$ contained in a $\Phi^{210}$ of SO(10). At this stage D-parity remains intact and both the gauge couplings of SU(2)$_L$ and SU(2)$_R$ are equal, $g_L = g_R$ [24]. At the next step a vev to the D-Parity odd singlet, also contained in $\Phi^{210}$ of SO(10), breaks the D-parity. To break the LR-symmetry at the next step we assign vev to the RH doublets $\chi_R \oplus \chi_R \subset 16_R \oplus \overline{16}_R$, however the subtlety of this breaking will be discussed later in the sections while at the last step electroweak symmetry is broken by a $10_H$-plet.

The effective Lagrangian at the intermediate, LR-symmetry breaking, scale is

$$L_Y = Y^\psi_L \psi_L^\dagger \phi^{16} + f \psi_R^T \tau_2 \psi_R^\dagger \Delta^{126} R + f \psi_R^T \tau_1 \phi^{16} R + \mu T^1 T + H.c..$$

(1)

The interacting superpotential to the scalar fields at the intermediate scale is given by

$$W = M_{\Delta_R} \Delta_R^{126} \Delta_R^{126} + \lambda \chi_R^{16} \chi_R^{16} \chi_R^{16} + \lambda^* \Delta_R^{126} \Delta_R^{126} \chi_R^{16} \chi_R^{16} \chi_R^{16}$$

(2)

where $\psi_L/R$ are left-/right-handed lepton doublets, while the superscript $16$ stands to represent that they belong to the $16$-plet of SO(10) representation and so on, and $T$, not to be confused with the superscript $T$ for the transposed field, the fermion singlet field, one for each generation. The introduction of the scalar field $\Delta < 126$, we can justify from the scalar interaction terms as follows.

We have assigned a vev to the right-handed doublet component only $\langle \chi_R^{16} \rangle = \langle \chi_R^0 \rangle = v_X$. However, the vanishing F-term conditions give us,

$$\langle \Delta_R^{126} \rangle = v_R = -\lambda \frac{v_X^2}{M_{\Delta_R}}.$$  

(3)

We, thus, have a large induced vev to the neutral component of the triplet scalar $\Delta_0^R$ or $\sum_0^R$, once the neutral doublet component $\chi^0_R$ gets a vev. For example, with a lighter RH-triplet mass $M_\Delta \simeq 100$ GeV–1 TeV, it is possible to have $v_R \simeq 10^3 - 10^7$ GeV for $v_X = 10^6 - 10^7$ GeV, assuming $\lambda \sim O(1)$. Since, $v_R > v_X$, the spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ takes place at a higher scale generating large RH Majorana neutrino masses $M_N \gg M_Y$. This will lead to a small $N_T - T$ mixing which is a crucial point needed to establish the out-of equilibrium conditions for leptogenesis.

In this model the neutral fermions per generation are a left-handed neutrino $\nu$, a right-handed neutrino $N$, both of which are member of $16$-plet of SO(10), and a sterile neutrino, $T$. From the Yukawa interaction, Eq.(1), we see in the $(\nu, N, T)$ basis the $3 \times 3$ mass matrix is given by

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & M_N & M_X \\ 0 & M_X & \mu \end{pmatrix} \begin{pmatrix} \nu^c \\ N \\ T \end{pmatrix}.$$ (4)

Here the $N - T$ mixing matrix arises through the vev of the RH-doublet field with

$$M_N = f v_R,$$ (5)

and the RH-Majorana neutrino mass is generated by the induced vev of the RH-triplet with

$$M_N = f v_R,$$ (6)

where, $v_R$ is given in eq.(3) as described above. The vev of the weak bi-doublet $\Phi(2,2,0,1) \subset 10_H$ of SO(10) yields the Dirac mass matrix for neutrinos,

$$m_D = Y \langle \Phi^0 \rangle.$$ (7)

In our model different mass scales hierarchy is $M_N \gg M_X \gg \mu \gg m_D$. With this hierarchical mass spectrum [20], the eigenvalues of the mass matrix in eq. (4) is given by

$$m_{\nu} \sim -m_D \left[ M_X^{-1} \mu \left( M_X^{-1} \right)^{-1} m_D^T \right],$$ (8)
\[ M_T \sim \mu - \frac{M_X^2}{M_N}, \]  

(9)

Here, we see that the light neutrino masses satisfy a double see-saw structure. It may be noted that the mass matrix structure in eq. (4) ensures that the type-I see-saw contribution is absent and \( M_N \) remains unconstrained by the light neutrino masses. This freedom in \( M_N \) – a hallmark of the model – is vital to ensure adequate leptogenesis.

In order to satisfy both the neutrino data as well as to generate the required amount of lepton asymmetry in this SO(10) model, we note that the mass matrix \( \mu \) can be obtained using eq.(8) once we know the mass matrices \( m_\nu, m_D \) and \( M_X \). Our strategy is described as follows:

To construct the Dirac mass matrix \( m_D \), here we work in a basis in which the down-quark and charged lepton mass matrices are diagonal. The entire mixings in the quark and lepton sectors, thus, can be ascribed to the mass matrices of the up-type quarks and the neutrinos, respectively. On the otherhand, in SO(10) model, the quark-lepton symmetry [23] relates the neutrino Dirac mass matrix \( m_D \) to its counterpart in the up-sector. We, therefore, obtain \( m_D \) using the quark masses and the Cabibbo-Kobayashi-Maskawa mixing angles, upto \( O(1) \) effects due to RG evolution. Using the data enlisted in the Particle Data Group [25] for the CKM matrix elements, its Dirac phase, and the running masses of the three up-type quarks, namely, \( m_u = 2.5 \) MeV, \( m_c = 1.29 \) GeV, \( m_t = 172.9 \) GeV, we have,

\[ m_D \simeq M_U = V_{CKM}^\dagger \text{diag}(m_u, m_c, m_t)V_{CKM}, \]  

(10)

where we have used the CKM phase \( \delta_{CKM} = 1.2 \) radian, and the quark mixing angles \( \sin^2\theta^u_{12} = 0.2253, \sin^2\theta^u_{23} = 0.041, \) and \( \sin^2\theta^u_{13} = 0.0041 \). The Dirac neutrino mass matrix is fixed by the underlying quark-lepton symmetry of SO(10). Neglecting small RG corrections, it is taken to be approximately equal to the up-quark mass matrix.

Next, we see that the matrix \( M_X \) is determined through eq.(5). However, the \( 3 \times 3 \) coupling matrix \( F \) is completely arbitrary. To minimize the number of independent parameters, we take the matrix \( F \) to be real and diagonal. Here, we choose, for example, \( M_X = \text{diag}(0.15, 0.5, 0.8) \times v_\chi \) with \( v_\chi = 10^6 \text{ GeV} \). Once a vev to the RH doublet \( \chi^6_R \) is chosen, we can have a definite induced vev to the RH triplet \( \Delta^{126}_R \) using eq.(3). Not to mention, due to same reason like \( M_X \), we choose \( f \sim \text{diag}(0.1, 0.5, 0.9) \) to obtain, via eq.(6), the \( M_N \) mass matrix of \( O(10^{10}) \) GeV. This will, for a general \( M_X \), lead to a \( N_1 \equiv T_3 \) mixing given by

\[ \sin^2\theta^u_{ij} = \frac{M_{X_{ii}}}{M_{N_{j}}}, \]  

(11)

In neutrino physics, breakthrough measurement of the third mixing angle \( \theta^u_{13} \) come from different experiments. Its evidence come in the mid last year from T2K and MINOS. Their combined data predicted non-zero \( \theta^u_{13} [6, 7] \) with more than \( 3\sigma \) evidence. Recently, Daya Bay as well as RENO experiment come up with a more than \( 5\sigma \) measurement. However, their central value is very close to what predicted in [6, 7]. To construct the neutrino mass matrix we use the combined neutrino mixing data, including the recent T2K and MINOS results, given in Table-I from Ref.[6] with the new reactor flux estimate. In addition, here, we assume all other CP-phases in the lepton sector, except the one through the CKM matrix in the quark sector, to be zero. Assuming the lightest neutrino mass eigenvalue \( m_1 \) for normal hierarchy and \( m_3 \) for the inverted hierarchy) to be zero, we obtained two other mass eigenvalues using different values of \( \delta m^2(= m_2^2 - m_1^2) \) and \( \Delta m^2(= m_3^2 - (m_2^2 + m_1^2)/2) \) from Table-I. We construct \( m_\nu \) from these mass eigenvalues via the PMNS matrix, \( U_{PMNS} \) as

\[ m_\nu = U_{PMNS}^T \text{diag}(m_1, m_2, m_3)U_{PMNS}. \]  

(12)

With the knowledge of \( m_\nu, m_D \) and \( M_X \) we then use the inverse see-saw mass formula, from eq.(8), to obtain elements of the matrix \( \mu \) for both normal and inverted hierarchical light neutrino masses. The \( \mu, M_X \) and \( M_N \)
matrices are used in eq. (9) to compute the mass eigenvalues of the singlet fermions and their mixings. Thus the two input matrices $M_N$ and $M_X$ (chosen diagonal) – eq. (4) – completely determine the singlet neutrino, $T_1$, masses and mixings consistent with the recent data on the light neutrino mass spectrum and their mixing. Out of three eigenvalues only one, denoting it as $T_1$ mass varies between (0, 0.1624) $\times 10^6$ GeV and is denoted by the cross (cyan) in Fig. 1. Here, we see that $T_1$ mass equals to 0.6124 $\times 10^4$ GeV and is denoted by the cross (cyan) in Fig. 1. Here, we see that $T_1$ mass varies between (0.4709 – 0.8526) $\times 10^4$ GeV (brown-central dark) for 1$\sigma$, 0.2874 $\times 10^4 – 0.1152$ $\times 10^5$ GeV (yellow - whitish) for 2$\sigma$ and 0.1388 $\times 10^4 – 0.1533$ $\times 10^5$ GeV (red - outer dark) for 3$\sigma$ allowed experimental neutrino data. A similar analysis is shown for the inverted hierarchy is given in Fig. 2.

Below we discuss various solutions of the Boltzmann equations. These determine the number densities in a co-moving volume $Y_T = n_T/n_S$ and $Y_L = n_L/n_S$, where $n_T$ and $n_S$ are respectively the number densities of leptons and the entropy. We can read the equations as -

\[
\frac{dY_T}{dz} = - (Y_T - Y^{eq}_T) \left[ \frac{\Gamma_D^T}{zH(z)} + \frac{\Gamma_s^T}{zH(z)} \right],
\]

\[
\frac{dY_L}{dz} = \epsilon_T \frac{\Gamma_D^T}{zH(z)} (Y_T - Y^{eq}_T) - \frac{\Gamma_W^T}{zH(z)} Y_L.
\]

where $\Gamma_D^T$, $\Gamma_s^T$ and $\Gamma_W^T$ represent the decay, scattering, and wash out rates, respectively, that take part in establishing a net lepton asymmetry. We refrain from presenting their detailed expressions here and due to negligible contribution from supersymmetric processes [26], we have not included them. The Hubble expansion rate $H(z)$, where $z = M_T/T$, and the CP-violation parameter $\epsilon_T$ are compared with the Hubble expansion rate, $H$, as a function of $z$, for the best fit values of neutrino data only are shown for the Normal Hierarchy (Left) and Inverted Hierarchy (Right).
are given by

\[ H(z) = \frac{H(M_T)}{z^2}, \quad H(M_T) = 1.67 g_\ast^{1/2} \frac{M_T^2}{M_{pl}}, \]

\[ \epsilon_T = \frac{\Gamma(T \rightarrow l\phi) - \Gamma(T \rightarrow l\phi^*)}{\Gamma(T \rightarrow l\phi) + \Gamma(T \rightarrow l\phi^*)}. \]  

(14)

In Fig.-4, we have shown the variation of the decay and inverse decay rate of \( T_1 \) for both normal (left) and inverted (right) hierarchical light neutrino cases. Corresponding to the best fit neutrino data, \( Y_L \), \( Y_T \) and \( Y_T^{EQ} \) are shown in the central (cyan) lines and the dot-dashed (red), dashed (maroon) and solid (blue) boundary lines correspond to 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) ranges.

\[ Y_2 \approx Y_3 \]

It is seen from the figure that although it is perturbed at \( \sim \) the right order (\( Y_2^{EQ} \)) the best fit values of neutrino data, \( Y_L \), \( Y_T \) and \( Y_T^{EQ} \) are satisfied to generate the lepton asymmetry. We have shown that the lepton asymmetry production results in Fig.-5 and in Fig.-6. We assume that in the very initial stages the number densities \( T_i \), \( i = 1, 2, 3 \), are zero. \( T_1 \) decay through the channel \( l\phi \) to produce the lepton asymmetry. One important point to note here is that in this process of leptogenesis, reheating temperature is constant with the gravitino constraint. In the figure, corresponding to the best fit values of neutrino data, \( Y_L \), \( Y_{T1} \) and \( Y_{T1}^{EQ} \) are plotted as the central (cyan) lines. The effect of 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) ranges of neutrino data on the evolution of \( Y_L \) and \( Y_{T1} \) are shown respectively with the dot-dashed (red), dashed (maroon) and solid (blue) boundary lines. It is seen from the figure that although it is perturbed at a lower value of \( z \) but as the universe expands \( Y_L \) achieves the right order (\( \sim 10^{-10} \)) starting off from a vanishing initial value while that for \( Y_{T1} \) are well separated. However, for the inverted hierarchy case, effect for 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) are overlapping due to a relatively monochro-

\[ \mathcal{L}_{NRO} = -\frac{1}{2M_U} Tr[F^\mu_\nu(\eta_1 \Phi_{210} + \eta_2 \Phi_{54})F^\mu_\nu]. \] 

(15)

The reason behind this is that above interaction lead to finite corrections to the gauge couplings at the GUT-scale so that the gauge couplings of left-right gauge group emerge from one effective GUT-gauge coupling. The upshot of this is that with these additional contributions it is possible to lower \( M_R \) to as low as \( 10^9 \rightarrow 10^{11} \) GeV as required in this model. The grand unification scale is as large as \( M_V \sim 10^{17-18} \) GeV and the model predicts a stable proton for all practical purposes. Another way to achieve this gauge coupling unification is to introduce some additional scalar multiplet at the intermediate scale [22].
Finally, we comment on the experimental prospect of doubly charged scalar of this model at LHC or ILC [22]. Using the D-parity mechanism in this model we make RH-triplets in $126_H \oplus \overline{126}_H$ carrying $B - L = \pm 2$. Their masses are from 100 GeV to a few TeV. This light triplet scalar comes out as a necessary condition to enhance the induced $v_{ve}v_{\nu_R}$ or to break the LR gauge symmetry at high scale. Consequently, we have heavy RH Majorana neutrinos as well as the corresponding gauge bosons. This forbids $\Delta R^{\pm}$ to decay into right-handed gauge bosons. However, after being produced via Drell-Yann process at LHC, these doubly-charged scalars will decay to fermions to be detected at LHC.

In conclusion, in view of the recent neutrino data we have presented a left-right symmetric SUSY SO(10) model. This model is capable to solve a multi-dimensional problems. This model has the following features

- By virtue of its construction, it is consistent with the most recent neutrino masses and mixing angles obtained at MINOS, T2K, Daya Bay, RENO experiments.
- We have discussed the variation of the $T_1$ mass due to $1\sigma$, $2\sigma$, $3\sigma$ variation for both the normal and inverted hierarchy light neutrino cases.
- It generates a correct lepton asymmetry via the decays of SO(10) singlet neutrino with a mass scale to be consistent with the gravitino constraint.
- It can also have a good unification of different gauge couplings at the GUT scale.
- The model is also a source of light doubly-charged scalar. It's mass range is within the reach of the LHC.

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