Upper Stage Visual Inertial Integrated Navigation Method Based on Factor Graph

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Abstract. In the working process of the upper stage integrated navigation information fusion system, the multi-source navigation information fusion algorithm based on factor graph Bayesian estimation is used to fuse the information of inertial sensors, visual sensors and other sensors. The overall joint probability distribution of the system is described in the form of probability graph model with the dependence of local variables, so as to reduce the complexity of the system, adjust the data structure of information fusion to improve the efficiency of information fusion and smoothly switch the sensor configuration.

1. Introduction
In the design of modern upper stage, the target aircraft needs to use all available information sources as much as possible to provide stable, reliable and accurate navigation information for its own integrated navigation information fusion system, so as to obtain better position and attitude detection results than single sensor. In the development of multi-sensor information fusion, DS evidence theory¹, Bayesian estimation², wavelet theory³, various improved Kalman filtering and other technologies are continuously applied to the field of navigation. In recent years, as a probability graph model, factor graph (FG) has been introduced into the field of navigation because of its characteristics of plug and play, clarifying data dependencies and optimizing data structure. Based on the factor graph method, a highly adaptive framework is proposed to meet the plug and play requirements of various types of sensors. The method can dynamically adjust the fusion architecture according to the needs, which can take advantage of more advantage information. When the sensor is inserted or online, new factor nodes can be directly added to the factor graph.

2. Factor graph model
Factor graph is a kind of bipartite graph and it’s usually defined as:

\[ G = \{ F, V, \varepsilon \} \]

For complex estimation problem, there are two kinds of node sets in factor graph, factor node set \( F \) and variable node set \( V \). Each couple \( f_i \in F \) and \( v_i \in V \) are connected by an edge \( \varepsilon \). For one \( f_i \), several \( v_i \) connected to \( f_i \) comes to be a variable set \( V_i \) called scope of \( f_i \), which can be written as \( \text{Scope}[f_i] \).
Figure 1 is an example of factor graph. A hollow circle represents a variable node and a filled circle represents a factor node. There are four variable nodes and four factor nodes \(x_1, x_2, x_3, x_4\). The scope of the four factor nodes are:

\[
\begin{align*}
Scope[f_A] &= \{x_1\} \\
Scope[f_B] &= \{x_2\} \\
Scope[f_C] &= \{x_1, x_2, x_3\} \\
Scope[f_D] &= \{x_3, x_4\}
\end{align*}
\]

The whole factor graph defines the factorization form of the function \(f(x_1, x_2, x_3, x_4)\):

\[
f(x_1, x_2, x_3, x_4) = f_A(x_1) \cdot f_B(x_2) \cdot f_C(x_1, x_2, x_3) \cdot f_D(x_3, x_4)
\]

For motion estimation, it is usually assumed that the noise of the measured value is Gauss normal distribution model, and the nodes in the factor graph will have clear meaning. The variable node represents a series of unknown random state variables, and the factor node represents the form of probability density function between these variable nodes. This information comes from observation information or a priori information, usually a Gauss density function. The whole factor graph corresponds to a Jacobian matrix.

2.1. Information fusion method design based on factor graph

With the increase of the number and types of sensors, different sensors have different update frequencies and are asynchronous in time\(^{[4,5]}\). The traditional information fusion method is difficult to meet the increasingly complex and changing needs of the system. Simultaneous interpreting principle and data, the model is constructed by combining factor graph data fusion principle. After obtaining different sensor measurement information, the factor map is expanded by simultaneous interpreting of the system variable nodes and factor nodes, and the recursive and update of the system state is achieved for different sensor nodes, and the multi-source information fusion is achieved.

In the motion estimation problem of integrated navigation, when the time \(t\) is set, the state variable of auxiliary information detector is \(x_i\) and the state variable of spatial feature is \(l_j\). For the smoothing problem, it is necessary to estimate the values of all previous state variables at each time. When set to \(t_s\), all state variables are:

\[
V_{t_s} = \{x^T_i, l^T_j\} \\
i \in \kappa_s, j \in J_{\kappa_s}
\]

Where \(\kappa_s\) represents the collection of all state variables obtained from \(t\) to \(t_s\). \(J_{\kappa_s}\) represents a collection of all spatial features contained in all state variables \(\kappa_s\). All measurement sets are \(Z_{t_s} = \{Z_t\}_{t=1}^s\).

When the measured value \(Z_{t_s}\) is given, the goal of motion estimation problem is to estimate the carrier motion state and the value of spatial characteristic state variables. However, because the
measured values have uncertainty and the measurement results contain noise, the essence of motion estimation is to estimate the posterior probability distribution of state variables given a series of measured values:

\[ P(V_{0,x} | Z_{1:t}) \]

According to Bayesian rules, \( P(V_{0,x} | Z_{1:t}) \) can be divided as:

\[
P(V_{0,x} | Z_{1:t}) = \frac{P(V_{0,x}, Z_{1:t})}{P(Z_{1:t})} = \frac{P(Z_{1:t} | V_{0,x})P(V_{0,x})}{P(Z_{1:t})}
\]

According to Markov hypothesis \( (V_s \perp V_{0:s-2} | V_{s-1}) \) and conditional independence:

\[
P(Z_{1:t} | V_{0,s}) = P(Z_1, Z_2, \ldots, Z_s | V_0, V_1, \ldots, V_s)
\]
\[
= \prod_{i=1}^{s} P(Z_i | V_i) = \prod_{i=1}^{s} \prod_{z_j \in Z_i \setminus \{\text{imu}\}} P(z_j | V_i)
\]
\[
P(V_{0,s}) = P(V_0, V_1, \ldots, V_s) = P(V_0)P(V_1 | V_0)P(V_2 | V_1)\cdots P(V_s | V_{s-1})
\]

We can rewrite \( P(V_{0,s} | Z_{1:t}) \) as follows:

\[
P(V_{0,s} | Z_{1:t}) = \frac{P(V_0) \prod_{i=1}^{s} P(V_i | V_{i-1}) \prod_{z_j \in Z_i \setminus \{\text{imu}\}} P(z_j | V_i)}{P(Z_{1:t})}
\]
\[
= \frac{P(V_0) \prod_{i=1}^{s} P(V_i | V_{i-1}, z_{imu}^{i-1}) \prod_{z_j \in Z_i \setminus \{\text{imu}\}} P(z_j | V_i)}{P(Z_{1:t})}
\]
\[
\propto P(V_0) \prod_{i=1}^{s} P(V_i | V_{i-1}, z_{imu}^{i-1}) \prod_{z_j \in Z_i \setminus \{\text{imu}\}} P(z_j | V_i)
\]

The purpose of parameter optimization is to obtain a set of state variable values to maximize a posteriori probability value and obtain the maximum a posteriori estimation:

\[
V_{0,s}^* = \arg \max_{V_{0,s}} P(V_{0,s} | Z_{1:t})
\]
\[
= \arg \max_{V_{0,s}} P(V_0) \prod_{i=1}^{s} P(V_i | V_{i-1}, z_{imu}^{i-1}) \prod_{z_j \in Z_i \setminus \{\text{imu}\}} P(z_j | V_i)
\]

2.2. Visual/Inertia information fusion based on factor graph

Establishment of motion model by the Integrated Data of Visual and INS.

\[ x_i = f(x_{i-1}, u_i) + w_i \]

Visual measure model is:

\[ z_j = h(V_i^j) + v_j = h(x_i, I_j) + v_j \]

The corresponding error is expressed as:

\[ e_{re}^{i,j} (V_i^j, z_j) = x_i - \hat{x}_i = x_i - f(x_{i-1}, u_i) \]
\[ e_{i\mu}^j (V_i, z_{i\mu}^j) = z_j - \tilde{z}_j - h(V_i^j) \]

If the measurement noise is a zero mean normal distribution model, the likelihood function and a priori information will be Gauss density model.

\[
P(V_i | V_{i-1}, z_{i\mu}^{i-1}) \propto \exp\left(-\frac{1}{2}(x_i - f(x_{i-1}, u_i))^T \Omega_{\text{motion}}^{-1} (x_i - f(x_{i-1}, u_i))\right)
\]

\[
P(z_j | V_i^j) \propto \exp\left(-\frac{1}{2}(z_j - h(V_i^j))^T \Omega_{\text{re}}^{-1} (z_j - h(V_i^j))\right)
\]

Where \( \Omega_{\text{motion}}^{-1} = (\Sigma_{\text{motion}})^{-1} \), \( \Omega_{\text{re}}^{-1} = (\Sigma_{\text{re}})^{-1} \), they are information matrix of Visual navigation and INS. For each factor, loss function is:

\[
d(I) = \exp\left(-\frac{1}{2}\|e_q^j(V_s^q, z_q)\|^2\Omega\right)
\]

So, the factor graph of Visual/INS navigation represents a group of Gauss probability density function:

\[
f_q (V_s^q) \propto \exp\left(-\frac{1}{2}\|e_q (V_s^q, z_q)\|^2\Omega\right)
\]

The optimization goal can be written as:

\[
V_{0,s}^* = \arg\max_{V_{0,s}} P(V_{0,s} | Z_{1,s}) = \arg\min_{V_{0,s}} \left( -\left( P(V_{0,s} | Z_{1,s}) \right) \right)
\]

\[
= \arg\min_{V_{0,s}} \left\{ \sum_{i=1}^{s} \|x_i - f(x_{i-1}, u_i)\|^2_{\Omega_{\text{motion}}} + \sum_{j} \|z_j - h(V_i^j)\|^2_{\Omega_{\text{re}}} \right\}
\]

The optimization problem described in the above formula is a nonlinear problem, which can be solved iteratively by Gauss Newton method.

\[ e_{\text{re}}^i (\bar{X}^j + \Delta X^j) = z_j - h(\bar{X}^j) + \left( \frac{\partial h(\bar{X}^j)}{\partial X^j} \right)_{\bar{X}^j} \Delta X^j \]

\[ = \left( \frac{\partial h(\bar{X}^j)}{\partial X^j} \right)_{\bar{X}^j} \Delta X^j = A^i \Delta X^j - b_i^j \]

\[
\Delta X^* = \arg\min_{\Delta X} \|A\Delta X - b\|^2_{\Omega}
\]

\[ A^T \Omega (A\Delta X - b) = 0 \rightarrow A^T \Omega A\Delta = A^T \Omega b \]

QR decomposition is used to process the matrix \( A \):
The solution of the above formula is satisfied
\[ R\Delta X^* - d = 0 \]
Because \( R \) is an upper triangular matrix, the inverse substitution method can be used to obtain the optimal parameters \( \Delta X^* \).

3. Integrated navigation simulation based on factor graph
The track data used in the simulation are as follows:

![Figure 2. Attitude of track data.](image)

![Figure 3. Velocity of track data.](image)

In the simulation experiment, the navigation geographic coordinate system is selected as the "North-East-Earth, NEU" coordinate system, with inertial navigation system as the main reference system and visual navigation as the main auxiliary system.
4. Conclusion

(1) The data fusion method based on factor graph has better immunity, smoother estimation process and higher accuracy;

(2) The factor graph can improve the calculation efficiency on the basis of ensuring the accuracy;

(3) The data fusion method based on Bayesian estimation of factor graph can dynamically adjust the structure of each sensor in the navigation system to achieve the purpose of "plug and play".

The proposed multi-source information fusion autonomous navigation system based on factor graph incremental smoothing information fusion can significantly suppress the error caused by the change of sensor performance. The method can also dynamically adjust the measured noise variance of each sensor.
according to the actual working conditions of intelligent missile, upper stage and other aircraft, realize the dynamic allocation of weights, save calculation time and improve calculation efficiency Improve navigation accuracy.

References
[1] Kang J, Gu Y B, Li Y B . Multi-sensor information fusion algorithm based on DS evidence theory. Journal of Chinese Inertial Technology, 2012, 20(6):670-673.
[2] Su B, Mu R, Long T, et al. Variational Bayesian Adaptive High-Degree Cubature Huber-Based Filter for Vision-Aided Inertial Navigation on Asteroid Missions. IET Radar Sonar Navigation, 2020(6).
[3] Yushan, Sun, Fanyu, et al. Multi-Scale Fusion Algorithm for AUVs Integrated Navigation Systems. Journal of Beijing Institute of Technology, 2019, v.28; No.102(04):54-59.
[4] Peng W.; Mu R.J.; Deng Y.P. Review of intelligent bionic vision navigation. LIDAR Imaging Detection and Target Recognition 2017. 2017, 11, 187.
[5] Multiple estimation channel decoupling and optimization method based on inverse system. Young Scientists Forum. 2018.