Matching the non-equilibrium initial stage of heavy ion collisions to hydrodynamics with QCD kinetic theory

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A. Kurkela, AM, J.-F. Paquet, S. Schlichting and D. Teaney, arXiv:1805.01604, 1805.00961
A. Kurkela, AM, in preparation
Why is the correct modeling of initial stages important?

- Initial collision geometry ⇒ particle correlations, e.g. flow harmonics. 
  *Pre-equilibrium dynamics smears the geometry, creates flow.*
- Consistency in physics pictures ⇒ independence of the crossover time
  *Need “hydrodynamization“ of initial conditions to justify hydro.*
- Small collision systems (eg. p-p, p-Pb) ⇒ short life-time.
  *Initial stages can be a significant part of the total evolution.*
Equilibration and hydrodynamization in QCD kinetic theory
Initial stages at very high collision energy

Consider a weak coupling limit at $\sqrt{s} \rightarrow \infty$

- Initial state – classical gluonic fields, Yang-Mills evolution, anisotropic
- Final state – viscous hydrodynamics, close to equilibrium

Use QCD kinetic theory to connect early particle production to hydrodynamic simulation.
Thermalization in QCD effective kinetic theory

“Bottom-up” thermalization scenario

QCD effective kinetic theory

Weakly coupled quark and gluon quasi-particles in a soft background.

\[
\partial_\tau f + \frac{\mathbf{P}}{|\mathbf{p}|} \cdot \nabla f - \frac{p_z}{\tau} \partial_{p_z} f = - \left( C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f] \right)
\]

- elastic $2 \leftrightarrow 2$ and inelastic $1 \leftrightarrow 2$ scatterings (with LPM suppression)

*the same QCD processes as in jet quenching*

- only parameter — the coupling constant $\lambda = 4\pi \alpha_s N_c$.

Caveats: in practice need to extrapolate $\lambda$ to realistic values ($\alpha_s \approx 0.3$), isotropic screening (avoiding plasma instabilities).
Classical-statistical lattice simulations of Yang-Mills converge to the first stage of ‘bottom-up’.
Thermalization stages in kinetic theory

Evolution of gluon distribution function

2 ↔ 2 broadening

Collinear cascade

Mini-jet quench

The equilibration time governed by the coupling constant

\[ \alpha_s \quad \Longrightarrow \quad \frac{\eta}{s} \quad \Longrightarrow \quad \tau_R(\tau) = \frac{(4\pi\eta/s)}{T} \]
Thermalization stages in kinetic theory

Evolution of gluon distribution function

large increase of gluon number ($\times 2$)

important for $dN_{ch}/dy$ calculations

The equilibration time governed by the coupling constant

$$\alpha_s \quad \Longrightarrow \quad \eta/s \quad \Longrightarrow \quad \tau_R(\tau) = (4\pi \eta/s)/T$$
From kinetic theory to hydrodynamics

scaling in hydro: \( e(\tau) = \nu g \frac{\pi^2}{30} T_{\text{id.}}^4 \left( 1 - \frac{8}{3} \frac{\eta/s}{\tau T_{\text{id.}}} + C_2 \left( \frac{\eta/s}{\tau T_{\text{id.}}} \right)^2 \right) \)

kinetic theory \( \eta/s = 0.62 \)
kinetic theory \( \eta/s = 0.16 \)

Kinetic equilibration becomes universal for scaled time \( \frac{\tau}{\tau_R(\tau)} = \frac{\tau T(\tau)}{4\pi \eta/s} \)
From kinetic theory to hydrodynamics

scaling in hydro: \[ e(\tau) = \nu g \frac{\pi^2}{30} T_{\text{id.}}^4 \left( 1 - \frac{8}{3} \frac{\eta/s}{\tau T_{\text{id.}}} + C_2 \left( \frac{\eta/s}{\tau T_{\text{id.}}} \right)^2 \right) \]

"ideal" temp.

ideal

viscous

2nd order hydro

hydrodynamics applies

Kinetic equilibration becomes universal for scaled time \[ \frac{\tau}{\tau_R(\tau)} = \frac{\tau T(\tau)}{4\pi \eta/s} \]
**System size and hydrodynamization time**

**Will hydrodynamically flowing QGP be formed for a given system size?**

- Quite universally for longitudinally expanding systems

  also in holography, see e.g. Heller, Kurkela, and Spalinski (2017) [7]

\[ \frac{\tau_{\text{hydro}} T(\tau_{\text{hydro}})}{4\pi \eta/s} \approx 1 \]

\[ \begin{align*}
\tau < R & \quad T \sim \tau^{-1/3} \\
\tau > R & \quad T \sim \tau^{-1}
\end{align*} \]

- Long lived hydrodynamic phase can form if \( \tau_{\text{hydro}} \ll R \)

  Kurkela, AM, Paquet, Schlichting and Teaney (2018)[1]

\[ \frac{\tau_{\text{hydro}}}{R} \approx \left( \frac{4\pi(\eta/s)}{2} \right)^{\frac{3}{2}} \left( \frac{dN_{\text{ch}}/d\eta}{63} \right)^{-\frac{1}{2}} \left( \frac{S/N_{\text{ch}}}{7} \right) \left( \frac{\nu_{\text{eff}}}{40} \right)^{\frac{1}{2}} \]

cf. earlier estimates Basar and Teaney (2013) [8], Schlichting and Tribedy (2016)[9]
Conformal scaling and system size

\[
\frac{\tau_{\text{hydro}}}{R} \approx \left( \frac{4\pi(\eta/s)}{2} \right)^{\frac{3}{2}} \left( \frac{dN_{\text{ch}}/d\eta}{63} \right)^{-\frac{1}{2}} \left( \frac{S/N_{\text{ch}}}{7} \right) \left( \nu_{\text{eff}} \right)^{\frac{1}{2}}
\]

-Little time for hydrodynamization in small collision systems.
Transverse dynamics in pre-equilibrium evolution
Event-by-event pre-equilibrium evolution with kinetic theory

\[ T^{\mu\nu}(\tau_{\text{hydro}}, x) = \bar{T}^{\mu\nu}(\tau_{\text{hydro}}) + \delta T^{\mu\nu}(\tau_{\text{hydro}}, x) \]

\( \tau_{\text{hydro}} \sim 1 \text{ fm/c} \)

\( \tau_{\text{EKT}} \sim 0.1 \text{ fm/c} \)

\( 2R \sim 10 \text{ fm} \)
Plane wave perturbations in transverse plane

Equilibration of transverse perturbations using linear response
Linearized gluon distribution function at initial time

\[ f(\tau, p, x_\perp) = \bar{f}_p + \delta f_{k_\perp, p} e^{ik_\perp \cdot x_\perp}. \]

uniform background  transverse perturbations

Some (weak) assumptions about \( \bar{f}_p \) and \( \delta f_{k_\perp, p} \)
Linearize Boltzmann equation in perturbations

\[ (\partial_\tau - \frac{p_z}{\tau} \partial_{p_z}) \bar{f}_p = -C[\bar{f}] \]

background

\[ (\partial_\tau - \frac{p_z}{\tau} \partial_{p_z} + \frac{i p_\perp \cdot k_\perp}{p}) \delta f_{k_\perp, p} = -\delta C[\bar{f}, \delta f] \quad k_\perp \text{ perturbation} \]

Study response functions to energy and momentum perturbations

\[ \delta T^{\mu\nu}_{x}(\tau_{\text{hydro}}, x') = \int d^2 x' \quad G^{\mu\nu}_{\alpha\beta}(x - x', \tau_{\text{hydro}}, \tau_{\text{EKT}}) \quad \delta T^{\alpha\beta}_{x}(\tau_{\text{EKT}}, x'). \]

\( \delta T^{\mu\nu}_{x}(\tau_{\text{hydro}}, x') \) goes into hydro

linear response function

\( \delta T^{\alpha\beta}_{x}(\tau_{\text{EKT}}, x') \) initial
Kinetic theory response functions

Invariant form of non-equilibrium response functions

\[ G^{\mu\nu}(\tau, \tau_0, |\mathbf{x} - \mathbf{x}_0|, e(\tau_0), \lambda) \Rightarrow G^{\mu\nu, \text{univ}} \left( \frac{\tau T_{\text{id.}}}{\eta/s}, \frac{|\mathbf{x} - \mathbf{x}_0|}{(\tau - \tau_0)} \right) \]

All components of energy-momentum tensor from kinetic response

- **energy** $T^{00}$
- **velocity** $T^{0i}$
- **shear-stress** $T^{ij}$

- **long wavelengths & late times** $\Rightarrow$ **hydrodynamic response**
  
  cf. universal velocity response Vredevoogd and Pratt (2008) [10], Shee, Romatschke and Pratt (2013) [11]

- **short wavelengths & early times** $\Rightarrow$ **free streaming response**
  
  Broniowski, Florkowski and Chojnacki, and Kisiel (2008) [12] Liu, Shen and Heinz (2015) [13]
Practical event-by-event kinetic pre-equilibrium: KøMPøST

- Input non-equilibrium $T^{\mu\nu}(\tau_{EKT}, x)$, i.e. $P_L \approx 0$ (e.g. IP-Glasma)
- Equilibrate background $\bar{T}^{\mu\nu}$ according to the scaling curve.
- Propagated linearized $\delta T^{\mu\nu}$ perturbations with Green’s functions
- Pass total $T^{\mu\nu}$ to hydro (sometimes need to regulate response)

```
T^{\mu\nu}(\tau_{EKT}, x)  
^ ^  
X
```

```
T^{\mu\nu}(\tau_{hydro}, x)  
^ ^  
X
```

```
\tau_{hydro} \sim 1.0 \text{ fm/c}  
\tau_{EKT} \sim 0.1 \text{ fm/c}  
2c(\tau_{hydro} - \tau_{EKT}) \sim 10 \text{ fm} 
2R \sim 10 \text{ fm} 
```

Publicly available at github.com/KMPST/KoMPoST

Kurkela, AM, Paquet, Schlichting and Teaney (2018)[1, 2]
Results: event-by-event initial stage matching

\[ \tau_{EKT} = 0.2 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.4 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.6 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.8 \text{ fm} \]

\[ \tau_{\text{hydro}} = 1.0 \text{ fm} \]

\[ \tau_{\text{hydro}} = 1.2 \text{ fm} \]

\[ \tau_{\text{out}} = 2.00 \text{ fm} \]

Smooth matching between kinetic phase and hydrodynamics \(\Rightarrow\) independence of \(\tau_{\text{hydro}}\).
Shear-stress tensor from KøMPøST evolution

Kinetic evolution automatically satisfies constitutive equations.

Note: MC-Glauber initial conditions

Expect hydro to apply for $\tau T_{id.}/(4\pi\eta/s) > 1$
Shear-stress tensor from KøMPøST evolution

Kinetic evolution automatically satisfies constitutive equations. **Note:** MC-Glauber initial conditions

Expect hydro to apply for $\tau T_{id.}/(4\pi\eta/s) > 1$
Final hadronic observables

Thermal pions at freeze-out $T_{fo} = 145 \text{ MeV}$

Approximate independence of $\tau_{\text{hydro}}$ for kinetic pre-equilibrium evolution!
Overlapping descriptions of initial stages at heavy ion collisions

\[ \frac{\langle P_T \rangle}{\langle e \rangle} \quad \frac{\langle P_L \rangle}{\langle e \rangle} \]

τ_{EKT} \quad τ_{hydro}

2+1D Yang-Mills

Kinetic theory

2+1D hydro.

No longer a cartoon picture!
Equilibration with fermions
Hydrodynamization with $N_f = 3$ quarks

Chemically equilibrating QCD plasma

Quarks produced from gluon fusion $g + g \rightarrow q + \bar{q}$ and splitting $g \rightarrow q + \bar{q}$

Chemical equilibration does not significantly delay hydrodynamization.
Hydrodynamization with $N_f = 3$ quarks

QCD equilibration—small, but systematic difference from pure glue

Kurkela, AM, *in preparation*

\[
\frac{(e_g + e_q)}{e_{id}} - \frac{\tau}{\tau_R} = 10, \quad \lambda = 1.0, \quad \lambda = 0.1
\]

Yang-Mills fit

\[
\eta/s \approx 1 - 1.9 \times 10^3
\]

Initial conditions

$\tau/\tau_R$
Summary

**KøMPøST**—linearized kinetic theory propagator of heavy ion initial conditions.  

- Background evolution described by a scaling solution in $\tau T_{\text{Id.}}/(\eta/s)$
- 10 kinetic Green’s functions $G^{\mu\nu}(\tau T_{\text{Id.}}/(\eta/s), r/(\tau - \tau_0))$
- Demonstrated small dependence on crossover time $\tau_{\text{hydro}}$

*Smooth and consistent matching between classical field hydrodynamic descriptions using kinetic theory.*

Outlook

- Going beyond linear response (important in small collision systems).
- Hydrodynamization with full QCD degrees of freedom

*Chemical equilibration does not appear to delay hydrodynamization.*
Backup
Energy momentum tensor evolution in kinetic theory

\[ \frac{T_{\mu\nu}}{T_{\mu\nu}^{id}} \]

\[ \frac{\tau_{T_{id.}}}{(4\pi\eta/s)} \]

\( \lambda = 10 (\eta/s \approx 0.62) \)
\( \lambda = 15 (\eta/s \approx 0.34) \)
\( \lambda = 20 (\eta/s \approx 0.22) \)
\( \lambda = 25 (\eta/s \approx 0.16) \)

scaling curve fit
Free streaming and low-|\textbf{k}| limits of kinetic theory

Extreme limits of kinetic response:
- small-|\textbf{k}| expansion à la Pratt, cf. [10, 11]
- free streaming response functions, cf.[12, 13]

![Energy density and shear-stress tensor graphs](image)

**Note:** MC-Glauber initial conditions
Free streaming and low-\(|k|\) limits of kinetic theory

Extreme limits of kinetic response:
- small-\(|k|\) expansion à la Pratt, cf. [10, 11]
- free streaming response functions, cf.[12, 13]

**Note:** MC-Glauber initial conditions
Hydrodynamization with $N_f = 3$ quarks

QCD plasma energy equilibration (no initial fermions)  

$\eta/s \approx 1 - 1.9 \times 10^3$

Chemical equilibration does not significantly delay hydrodynamization, but early time QCD behavior systematically different than Yang-Mills.
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