Finite size Kosterlitz-Thouless transition in 2DXY Fe/W(001) ultrathin films

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Magnetic susceptibility measurements of 3-4 ML Fe/W(001) ferromagnetic films demonstrate that this is a 2DXY system in which a finite size Kosterlitz-Thouless (KT) transition occurs. The films are grown in ultrahigh vacuum and their magnetic response is measured using the magneto-optic Kerr effect (MOKE). The analysis of many independently grown films shows that the paramagnetic tail of the susceptibility is described by $\chi(T) \sim \exp\left(\frac{B}{T/T_{\text{KT}} - 1}\right)^{\alpha}$, where $\alpha = 0.50 \pm 0.03$ and $B = 3.48 \pm 0.16$, in quantitative agreement with KT theory. Below the finite size transition temperature $T_C(L)$, the behaviour is complicated by dissipation (likely due to domain walls in the four-fold magnetic system). A subset of measurements with very small dissipation most closely represents the idealized system treated by theory. In these, the temperature interval between the fitted Kosterlitz-Thouless transition temperature and the finite size transition temperature is $T_C(L)/T_{\text{KT}} - 1 = 0.065 \pm 0.016$. This yeilds an estimate of the finite size affecting the film as $L \sim \mu m$, and gives experimental support to the idea that even a mesoscopic limitation of the vortex-antivortex gas results in a substantial finite size effect at the KT transition. In contrast, fitting the paramagnetic tail to a power law, appropriate to a second order critical transition, does not give reasonable results. The effective critical exponent $\gamma_{\text{eff}} \approx 3.7 \pm 0.7$ does not correspond to a known universality class, and the fitted transition temperature is much further below the peak in the susceptibility than is reasonable.

I. INTRODUCTION

The concept of a topological phase transition occurring in two dimensional (2D) systems was put forward in a series of articles by Berezinskii, Kosterlitz and Thouless more than 45 years ago. These ideas have assumed an important role in condensed matter physics. The original paper considered topological transitions in the melting of a 2D lattice, in neutral superfluids, and in a 2DXY ferromagnetic film, with a quantitative analysis using the 2DXY ferromagnet as a model system following soon thereafter. Subsequent experimental investigations of the Kosterlitz-Thouless (KT) transition have concentrated on superfluids and superconducting Josephson junction arrays, with little work on 2DXY ferromagnetic films. Because each system offers a different window through which to view the KT transition, there is a strong motivation to confirm experimentally that a 2D ultrathin film ferromagnet exhibits the transition. A more detailed understanding of the implications of KT theory in non-ideal, physical realizations of the 2DXY model can then be gained through the experimental study of these accessible, easily prepared systems.

Earlier experimental work on 2DXY spin systems studied planar three dimensional (3D) antiferromagnets and ferromagnets, where the exchange coupling within a 2D plane is much larger than that between planes. These samples are suitable for neutron scattering techniques. A series of theoretical articles by Bramwell, Holdsworth and co-workers pointed out the essential role of finite size effects in the KT transition in these systems, and showed that this results in a temperature range where the magnetization $M(T)$ (or the staggered magnetization in antiferromagnets) scales like an power law with an effective exponent $\beta_{\text{eff}}$. This range ends at a finite size transition temperature $T_C(L) > T_{\text{KT}}$. This power law scaling has been observed in a number of compounds, and a detailed neutron scattering study of the antiferromagnet Rb$_2$CrCl$_4$ showed in addition an internally consistent analysis of the correlation length and susceptibility in terms of a finite size KT transition.

The connection between these ideas and truly 2D ultrathin ferromagnetic films, as envisioned in the original papers by Kosterlitz and Thouless, has been made by compiling published experimental determinations of $\beta_{\text{eff}}$ for ultrathin epitaxial metal films on substrates of different symmetries. For metallic films grown on the faces of cubic substrates, the distribution of exponent values is bimodal, with clusters near the Ising value $\beta = 0.125$ and the 2DXY effective value $\beta_{\text{eff}} = 0.231$ for (001) and (111) substrates. They cluster near the Ising exponent for (110) substrates. This is important, but somewhat ambiguous, evidence of KT behaviour of 2DXY ferromagnetic films. The authors' primary point is that the four-fold anisotropy of the ferromagnetic films grown on (001) substrates is not strong enough to move them into a completely new universality class where $\beta$ is much larger.

The present article investigates the magnetic susceptibility of Fe/W(001) films, and compares it to the theory of a the finite size KT transition. There are a number of advantages to measuring the susceptibility, as compared to the magnetization. Whereas the magnetization signal disappears as the transition is approached, the susceptibility signal exhibits a peak that can be studied in detail. Thus, in addition to the value of exponents predicted by the theory, the peak shape can be compared to the distinct exponential temperature dependence of KT theory. Above $T_{\text{KT}}$, the vortex gas can be probed, and below $T_{\text{KT}}$ the effects of the four-fold symmetry can be studied through the dissipation caused by magnetic domains.

To our knowledge, there is one published measurement of...
the susceptibility of a ferromagnetic ultrathin film 2DXY system with four-fold anisotropy, and this is also for Fe/W(001). This measurement was made as the difference of magnetization curves in which a slightly different d.c. magnetic field is applied. The high temperature tail of the curve can be fit to the functional form predicted by finite size KT theory, but the fitted values of the constants are not consistent with the theory.

The current article reports on the growth, measurement and quantitative analysis of the magnetic susceptibility of many independent Fe/W(001) films. The excellent quantitative agreement with finite size KT theory shows that this is a 2DXY systems that exhibit a KT transition. This accessible ultrathin film system offers new opportunities to study topological phases and transitions by straightforward measurements of the magnetic susceptibility.

II. THEORETICAL DESCRIPTION

The Mermin-Wagner theorem proves that a 2D array of in-plane spins with nearest neighbour exchange coupling \( J \), and no anisotropy, cannot order at finite temperature in the thermodynamic limit. If only spin wave excitations are considered, the susceptibility always diverges and the magnetization is zero. However, because the 2DXY model admits another spin excitation, the vortex, it can undergo a type of transition mediated by the unbinding of vortices and antivortices.

Below the transition temperature \( T_{KT} \), the vortices and antivortices form bound pairs that represent only a small proportion of the spin system, such that the spin waves are dominant. With increasing temperature the pairs are bound more loosely and begin to create an effective medium with reduced exchange coupling \( J_{eff} \). As the thermal energy approaches the binding energy of the pairs, there is a highly non-linear feedback, where reducing the exchange coupling produces more loosely bound pairs, which reduces the coupling, and so on. The exchange coupling is driven to \( J_{eff}/kT = \pi/2 \) at \( T_{KT} \), and then directly to zero above the transition as the vortex-antivortex pairs unbind and proliferate.

Above the KT transition, the susceptibility becomes finite and varies as:

\[
\chi(T) \sim \xi^{2-\eta},
\]

where the exponent \( \eta = 1/4 \) at the transition temperature, and the correlation length \( \xi \) does not diverge like a power law, but as

\[
\xi \sim \exp\left( \frac{b}{\sqrt{T_{KT}/\beta}} - 1 \right).
\]

The best estimates of \( b \) are 1.8 to 1.9.

In the spin wave region of real systems below \( T_{KT} \), the magnetization falls so slowly with the system size that the thermodynamic limit is not reached even for macroscopic samples. For this reason, the magnetization of a 2DXY ferromagnetic film exhibits a substantial magnitude below \( T_{KT} \) due to finite-size effects. This can then be aligned by a four-fold anisotropy to create a net magnetization. Furthermore, the distribution of vortex-antivortex pairs is truncated by the finite size, \( L \), of the system, so that \( J_{eff}/kT \) is renormalized more slowly than in an infinite system. It reaches the value of \( \pi/2 \) at temperature \( T^* \), where

\[
\frac{T^* - T_{KT}}{T_{KT}} = \frac{\beta^2}{(\ln L)^2} = \frac{\beta^2}{(\ln L)^4}.
\]

\( T_{KT} \) continues to denote the transition temperature for the infinite system. Above this temperature, \( J_{eff} \) falls more gradually than in the infinite system, such that there is a finite size transition at \( T_{C}(L) \). Setting \( \xi(T) \sim L \) in eq. (2) yields:

\[
\frac{T_{C}(L) - T_{KT}}{T_{KT}} = \frac{\beta^2}{(\ln L)^2}.
\]

Between \( T^* \) and \( T_{C}(L) \), the magnetization falls to zero like a power law with an effective exponent \( \beta_{eff} \), so that

\[
M(T) \sim \left( 1 - \frac{T}{T_{C}(L)} \right)^{\beta_{eff}}.
\]

The fact that this behaviour has been observed in a number of 2D ferromagnetic films suggests that although the four-fold anisotropy is essential in creating a net magnetization, it does not determine the critical properties leading up to the transition. In this same region, the susceptibility increases with temperature due to softening of the spin waves, until it reaches a maximum at \( T_{C}(L) \). Above this temperature, the distribution of unbound vortices creates a paramagnetic state where the susceptibility once again varies as in eq. (1), with the correlation length in eq. (2) determined by \( T_{KT} \) for the infinite system.

The paramagnetic tail of the experimental susceptibility above \( T_{C}(L) \) is therefore expected to vary as

\[
\chi(T) \sim \chi_0 \exp\left( \frac{B}{(T_{KT}/\beta - 1)^a} \right),
\]

where \( a = 1/2 \) and \( B = (2-\eta)b \). While \( \eta = 1/4 \) at \( T_{KT} \), it may be as low as zero in the paramagnetic state. Thus \( B \) may range from about 3.2 to 3.8. In the paramagnetic region, the gas of free vortices should respond to an applied field without dissipation, so that the imaginary component of the susceptibility will be very small or absent. Eq. (6) is to be compared with the corresponding quantity in a second order critical transition.

\[
\chi(T) \sim \chi_0 \left( \frac{T}{T_{C}} - 1 \right)^{-\gamma},
\]

where \( \gamma \) is the critical exponent at the transition. For a 2D Ising transition, \( \gamma = 7/4 \); for a four state Potts model, \( \gamma = 7/6 \).
Below $T_C(L)$, the measured susceptibility is complicated by the presence of the four-fold crystalline anisotropy. An analysis of experimental magnetization studies of 2DXY system finds no evidence that a four-fold anisotropy affects the behaviour of the magnetization between $T_K$ and $T_C(L)$. However, magnetization experiments are performed in an applied d.c. field that saturates the sample, whereas susceptibility measurements use a small a.c. field that does not remove magnetic domains. The remanent magnetization of Fe/W(001) films is well known to have four-fold symmetry, with the domains along both the (100) and (010) in-plane directions. Domain walls represent an additional low energy magnetic excitation just below the transition that is not included in the theoretical analysis. By analogy with standard critical transitions in ferromagnets, the response and dissipation by domain wall motion is expected to dominate the susceptibility for a temperature interval below $T_C(L)$. The size of this interval, the process by which the domains form or disappear, and the magnitude of the response in a 2DXY ferromagnetic system is not clear.

### III. EXPERIMENTAL METHODS

The growth of Fe/W(001) films has been studied by a number of groups. After some initial uncertainty, it has been established that 2ML of Fe on W(001) are thermally stable up to 700 K, whereas thicker films are stable to lower temperatures. This permits a straightforward thickness calibration of films grown by evaporation in UHV using the Auger Electron spectroscopy (AES) signal from the W substrate during deposition. The AES signal shows a clear break between two linear regions as a function of deposition time, if the films are annealed to 700 K. This thickness calibration is accurate to within 5%. A recent magnetic microscopy study confirmed layer growth for 3 to 4 layers, after which the formation of three dimensional islands begins. The current study uses films of 3 to 4 ML Fe, because the same microscopy study showed the development of large magnetic domains in the films in this thickness range. The films in the present article were grown in two stages – the first 2 ML was grown at room temperature and annealed to 600 K to promote wetting and smoothing, and then an additional 1 or 2 ML was grown at room temperature. The entire film was then annealed to 460 K to ensure stability during $\chi(T)$ measurements that can extend as high as 450 K.

The magnetic susceptibility was measured using the magneto-optic Kerr effect (MOKE) in the longitudinal geometry. Details of the apparatus and procedures are described in ref. HeNe laser light passes through a polarizing crystal, enters the chamber through a UHV window, scatters from the sample at 45°, exits through a second UHV window and a second polarizing crystal that is almost crossed with the first. The transmitted light then falls on a photodiode. An optical compensation technique is used to ensure that the light falling on the second polarizer is linearly polarized, so that the sensitivity of the method is optimized. An air coil attached to the sample holder is used to create a small in-plane a.c. field within the scattering plane. The output of the photodiode is connected to a lock-in amplifier that detects the Kerr rotation in phase ($\operatorname{Re} \chi$) and out
of phase (\(\text{Im}\chi\)) with the a.c. magnetic field. The sample can be rotated about its normal, so that any in-plane component of the magnetization can be probed.

The magnetic susceptibility measurements of the Fe/W(001) films fall into three qualitative groups. These are shown in fig.(1), using a solid line for \(\text{Re}\chi(T)\) and a dashed line for \(\text{Im}\chi(T)\). Roughly 1/3 of the measurements are like the curve labelled I, and shown in the inset to fig.(1). These have a relatively small magnitude and a very small imaginary component. These will be referred to as Type I signals. In another 1/3 of the measurements, the peak in \(\text{Re}\chi(T)\) is an order of magnitude stronger, the peak is wider, and it is accompanied by strong peak in \(\text{Im}\chi(T)\). These are termed Type II signals. Finally, the remaining 1/3 of the measurements (Type III) have a complex form that is different from film to film. All of these measurements are qualitatively different than the susceptibility of the 2D Ising system Fe/W(110)\(^{27,28}\). Even the narrower peaks in Type I signals have a normalized full width at half maximum \(\Delta T/T_{\text{peak}} \approx 0.50\), which is more than twice the value of 0.018 observed in the second order critical transitions of Fe/W(110). This is despite the fact that the susceptibility of the latter has a substantial imaginary component.

Fig.(1b) shows the peak width and height as a function of the a.c. field strength. These data, which were collected for a Type II film, suggest that a purely linear response is measured only when the a.c. field strength is about 0.3 Oe or less, and that the signal strength is more than 0.15 Oe. While it would be possible to use this very small field for Type II signals, it presents a problem for Type I signals, where the susceptibility is an order of magnitude smaller. In order to maintain a consistent field for all measurements, a compromise of 0.55 Oe has been used. This maintains a good signal-to-noise in the field for all measurements, a compromise of 0.55 Oe has been used. This maintains a good signal-to-noise ratio.

The effect of the heating rate on the measured susceptibility is illustrated in fig.(1b). The particular concern is the possible deformation of the shape of the paramagnetic tail by relaxation from a non-equilibrium state if the heating rate is too high. To quantify this effect, the location of the point of inflection, \(T_{\text{inf},T}\), of the high temperature tail has been found for measurements at different heating rates on the same film. This is then used to form the measure \(\chi(T_{\text{inf},T})/\chi(T_{\text{peak}})\), or “how far down the curve does the point of inflection occur”. This gives an indication of the range of data to which it will be possible to fit eq.(6). It can be seen that the heating rate of 0.1 K/s used in this study presents no difficulty.

IV. RESULTS AND ANALYSIS

Because the Type I measurements have a very small imaginary component, they conform most closely to the idealized theoretical model. Fig.(2a) shows a representative example of a Type I signal from a Fe/W(001) film. This is a different data set than that shown in fig.(1). To determine if this data is described by KT theory, the paramagnetic tail is fit to eq.(6). This requires four parameters to be determined: \(\chi_0\), \(B\), \(T_{\text{KT}}\), and the exponent \(a\), in addition to a lower and upper limit, \(T_{\text{min}}\) and \(T_{\text{max}}\), respectively, to the temperature range where the data is fitted.

To determine the fitted temperature range, the value \(a = 1/2\) is chosen. (It was verified that the range does not depend upon this choice.) A least squares fit to eq.(6) is made for \(\chi_0\), \(B\), and \(T_{\text{KT}}\) as a function of \(T_{\text{min}}\) and \(T_{\text{max}}\). Fig.(2b) and (c) show the fitted value of \(B\) and the value of the reduced \(\chi^2\) statistic as a function of the bounds. It can be seen that within the range 406.5 K < \(T_{\text{min}}\) < 408.7 K, both the fitted value of \(B\) and the \(\chi^2\) statistic change very little and are essentially independent of the bounds within the fitted uncertainty. Close inspection of the data indicates that if \(T_{\text{min}}\) is moved closer to the susceptibility peak than 406.5 K, it approaches the point of inflection in the data curve (likely due to finite size effects) where no diverging function will fit well. If \(T_{\text{min}}\) is moved further into the tail than 408.7K, it starts to exclude a large proportion of the curve with a high signal-to-noise ratio. This independence of the fit inside a temperature range that is clearly identified by extrinsic factors is precisely what is expected for a fit that properly represents the data. \(T_{\text{min}}\) is therefore chosen on the lower edge of this range, on the principle that maximizing the range will reduce the error in the fitted parameters. Similar criteria are applied to the choice of the upper temperature bound, as shown in fig.(2c). In this case, moving \(T_{\text{max}} > 438\) K starts to include very noisy data in the fit. There are about 710 data points in the selected fitting region.

Having determined the fitting range, least squares fits are made for \(\chi_0\), \(B\), and \(T_{\text{KT}}\) for a selection of values of \(a\) from 0.10 to 1.50. The values of \(B\) are plotted in fig.(2d) as a function of \(a\), with the solid line interpolating the values as a guide to the eye for the data set in fig.(2a). The value of the reduced statistical \(\chi^2\) for the best fit is essentially independent of the value of \(a\), and provides no statistical basis for determining \(a\). However, because KT theory gives independent predictions of \(a\) and \(B\), both must be met simultaneously. The fact that the interpolated curve passes through the theoretically predicted range 0 < \(a < 1/2\), 3.2 < \(B < 3.8\), shows that the data is consistent with KT theory. The fitted line is shown in fig.(2d); it is mostly obscured by the data itself. The fitted \(T_{\text{KT}}\) indicated by a vertical dashed line.

Fig.(2d) includes the results of fitting \(B\) as a function of \(a\) for 7 further measurements of Type I signals as interpolated dotted lines. The inset gives more detail near \(a = 1/2\). Six of the eight curves pass through the small
FIG. 2. a) A magnetic susceptibility of Type I. The temperature range of the data that are fit to eq. (5) is indicated by $T_{\text{min}}$ and $T_{\text{max}}$. The fitted curve is shown with a solid line, and the fitted temperature $T_{\text{KT}}$ by a dashed line. The peak of the susceptibility is indicated by a second dashed line, and the fitted temperature range of the data that are fit to eq. (6) is indicated by $T_{\text{C}(L)}$. b) The fitted value of $B$ (left hand scale, solid symbols) and the $\chi^2$ statistic of the fit (right hand scale, open symbols) as a function of $T_{\text{min}}$. The dashed line shows the value of $T_{\text{min}}$ selected for the fit in part a). c) As in part b), but for the selection of $T_{\text{max}}$. d) The parameter $B$ found in least squares fits to the data in part a), as a function of the parameter $a$, is shown by the points. The solid line interpolates the points. A similar fitting procedure yields the interpolated dotted lines for Type I measurements from seven additional films. The inset expands the region near $a = 1/2$, and shows the fitted values of $B$, with error bars, for $a = 1/2$. box that represents the theoretical range of $B$, and give an average value $B = 3.49$ with a standard deviation of 0.22 when $a = 1/2$. Two curves are not in quantitative agreement with KT theory. Given that the fitted values of $B$ lie many standard deviations from the other six curves, this is not likely to be a statistical variation. However, as we cannot identify a systematic experimental explanation for these outliers, we have no reason to exclude them from the figure. Including these in the average yields $B = 3.48 \pm 0.74$. The width of the box along the horizontal axis establishes a conservative estimate of the uncertainty in $a$. The top left and bottom right corners of the box are situated so that the average values of $B(a)$ for the six measurements pass through them. This determines $a = 0.50 \pm 0.03$.

Because the Type I measurements have a very small dissipative, imaginary component of susceptibility, the position of the peak of the curve should fairly represent the finite size transition temperature $T_{\text{C}(L)}$. A striking feature in fig. (2a) is that the fitted value of $T_{\text{KT}}$ is more than 20 K below the peak maximum. This is precisely the behaviour expected for a finite size KT transition, and is expressed quantitatively by eq. (1). The average value of $T_{\text{C}(L)}/T_{\text{KT}} - 1$ for the eight data sets is $0.065 \pm 0.016$, which gives an estimate of the finite size $L \sim \mu m$. A physical parameter of this order of magnitude is the dimension of the magnetic domains. These observations give experimental support to the idea that this 2DXY system has very significant finite size effects even in mesoscopic samples, and that Fe/W(001) ferromagnetic films support a finite size KT transition.

Finally, these results yield value for $\eta$ in the temperature region 20-30 K above $T_{\text{KT}}$ where eq. (5) is fit to the data. Using the best value of $B$ (without outliers) yields $\eta = 0.12 \pm 0.09$. There is a large uncertainty, but it seems that $\eta$ is reduced from the value of 0.25 expected at $T_{\text{KT}}$.

In contrast, when the data is fit to eq. (7), as would be appropriate for a second order critical transition, the average value of the fitted effective exponent $\gamma_{\text{eff}} = 3.5 \pm 0.8$. This is not close to any known universality class. In addition, the fitted Curie temperature is, on average, 9 K below the peak in the susceptibility curve. This is very different from the second order phase transition of Fe/W(110), where the fitted Curie temperature is below, but within a degree K of the susceptibility peak.$^{27,28}$ For these reasons, the fits to a power law are unphysical.

The Type II and Type III measurement have large imaginary components of the susceptibility. In a finite size KT system with four-fold anisotropy, the region between $T_{\text{KT}}$ and $T_{\text{C}(L)}$ is complicated by the presence of three types of magnetic excitations. There are spin waves, a population of vortex-antivortex pairs, and domain walls between the magnetic domains. While the presence of domain walls is clear in microscopy measurements,$^{13}$ this magnetic excitation is not included in the more idealized theoretical treatments of KT theory. We speculate that the primary difference in the susceptibility measurements of Type I, Type II and the more
magnetic susceptibility measurements of 3-4 ML Fe/W(001) films have demonstrated that a finite size KT transition to a gas of vortices and antivortices.

V. CONCLUSIONS

Magnetic susceptibility measurements of 3-4 ML Fe/W(001) films have demonstrated that a finite size KT transition occurs in this system. This is, to our kno
edge, the first time that a truly two dimensional 2DXY ferromagnetic system has been shown to vary as is predicted for a gas of vortices and antivortices in the paramagnetic state. The measurements on many independently grown films follow the predicted exponential form with quantitative constants \( a = 0.50 \pm 0.03 \) and \( B = 3.46 \pm 0.16 \) in agreement with KT theory. Stated more exactly, the measurements yield a tight correlation between the parameters \( a \) and \( B \) in eq.\( T \), but there is no purely statistical basis for choosing any point on the correlation curve. However, KT theory predicts values of \( a \) and \( B \) independently, such that two tests of the applicability of the theory may be made. Given the predicted value of \( a \), \( B \) is determined independently to be in agreement with KT theory with a small range of uncertainty. Given the predicted range of \( B \), \( a \) is determined independently to be in agreement with KT theory with a small range of uncertainty. We know of no theoretical description that would support different values of \( a \) and \( B \).

Below the finite size transition temperature \( T_C(L) \), the susceptibility in many samples has a large imaginary component due to dissipation. Comparing to magnetic microscopy results, the dissipation is likely due to domain wall excitations in the magnetic system with four-fold symmetry. Within the subset of measurements where this dissipation is small, the fitted value of \( T_{KT} \) is substantially below the peak in the susceptibility at \( T_C(L) \), as predicted. These samples have a characteristic separation of the these two temperatures, with an average value of \( T_C(L)/T_{KT} - 1 = 0.065 \pm 0.016 \). This gives an estimate of the finite size affecting the transition as \( L \sim \mu \). This is the order of magnitude of the dimension of magnetic domains, and is consistent with the idea that a mesoscopic limitation of the vortex anti-vortex gas leads to the finite size transition.

Finally, these results indicate that KT theory is applicable to a 2DXY system with four-fold in-plane anisotropy, and with residual anisotropies due to sample imperfections. This opens an additional window into the fascinating properties of topological states and transitions. The ability to experimentally realize and study a KT transition in a 2DXY system using a simple and accessible ultrathin ferromagnetic film will allow the detailed investigation of many interesting questions in 2D magnetism.

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