Superconducting proximity effect in graphene nanostructures

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Abstract. The superconducting proximity effect is studied theoretically for graphene(G)-superconductor(S) hybrid structures. Especially S-G-S junction is considered and the critical current is argued focusing on its temperature and junction-length dependences. The free energy of the total system is calculated by use of the tunneling approximation and the critical current is estimated from it. It is reported that the critical current can oscillate as a function of temperature and junction-length in bi-layer junctions. Some detailed comparison with ordinary S-normal metal-S junction is also given and both monolayer and bilayer junctions show some characteristic deviations.

1. Introduction

Graphene has been attracting much interest because of their unique electronic properties as well as potential application[1, 2, 3]. From the very beginning of the research several intriguing phenomena originating from peculiar band structure of monolayer and bilayer graphenes have been reported [4, 5]. Their superconducting properties are also studied theoretically and experimentally. It has been shown that, in spite of the vanishing density of states at the Fermi energy, a finite supercurrent can actually flow through graphene[6, 7, 8]. One of the present authors (AK) has fabricated microstructures consist of ultrathin graphite films and investigated superconducting proximity effect[9, 10, 11] and it has been found that the critical supercurrent depends largely on the gate voltage.

In this paper we calculate the critical current of superconductor(S)-graphene(G)-superconductor(S) Josephson junction using tunneling approximation and path integral formalism [12]. A similar calculation for monolayer graphene has been done by several authors [13, 14, 15]. In this paper we especially pay attention to the difference between monolayer and bilayer junctions focusing on the behaviors at non-zero temperatures. We argue the proximity length characteristic to Josephson junctions and examine the superconducting state within graphene.

The tunneling between superconductors and graphene can be treated in several different methods. In most of the preceding studies, the superconducting lead attached to graphene is treated by introducing non-vanishing superconducting gap to the graphene in the contact area [7, 16, 17]. In this paper we adopt the tunneling approximation. In this method, we
treat perturbatively the tunneling matrix elements between leads and graphene. Usually the junctions look like (a), however we consider the point-contact-like junction as depicted in Fig. 1 (b) for simplicity. We note that, even in junction (a), the critical current is mostly determined by the shortest part of the junction, and therefore the behavior of (a) and (b) may not change significantly. This point will be discussed elsewhere in greater detail.

Figure 1. (a) A conventional S-G-S junction. $S_L$ and $S_R$ indicate superconducting leads, whose separation and width are $d$ and $W$, respectively. (b) Point-contact type S-G-S junction. Here we have shown only one layer of the graphene for simplicity.

2. Model

2.1. Hamiltonian and Band Structure of Graphene

We start from the following tight-binding Hamiltonian which describes the electrons in left and right superconducting leads and monolayer (or bilayer) graphene (for simplicity we hereafter indicate them by $L$, $R$ and $G$, respectively):

\[
H_L = \sum_{k,\sigma} \left( \xi_k c_{k,\sigma}^L c_{-k,\sigma}^L + \Delta^L c_{k,\sigma}^{L\uparrow} c_{-k,\sigma}^{L\downarrow} + \Delta^L c_{-k,\sigma}^{L\downarrow} c_{k,\sigma}^{L\uparrow} \right),
\]
(1)

\[
H_R = H_L \ (L \rightarrow R),
\]
(2)

\[
H_G = -t \sum_{(j,k)\sigma} \left( a_{j,\sigma}^L b_{k,\sigma} + h.c. \right) - t_{\perp} \sum_{(j,k)\sigma} \left( a_{j,\sigma}^{L\uparrow} b_{k,\sigma} + h.c. \right) - t_{\perp} \sum_{(j,k)\sigma} \left( a_{j,\sigma}^{L\downarrow} b_{k,\sigma} + h.c. \right) \]
(3)

\[
\simeq \sum_{k,\sigma} E_k \left( \alpha^{L\uparrow}_{k,\sigma} \alpha^{L\downarrow}_{k,\sigma} - \beta^{L\uparrow}_{k,\sigma} \beta^{L\downarrow}_{k,\sigma} \right).
\]
(4)

Here $c_{k,\sigma}^{L(R)}$ is the annihilation operator of electron with wave number $k$ and spin $\sigma$, and $\Delta^L(R)$ the superconducting gap. The gap is assumed to be spatially constant in both leads. $\xi_k = \hbar^2 |k|^2 / (2m)$ with $m$ being electron mass in the leads. We also assume that $\xi_k$ is common in both leads. In Eq. (3) and (4), we have given the graphene part; $a_{j,\sigma}$ and $b_{j,\sigma}$ are the annihilation operator of an electron located at A and B sublattice of the graphene, respectively, and, $t$ is the hopping matrix element. The second line of Eq. (3) is for bilayer graphene. The annihilation operators in the second layer (we also call it lower layer) is given by $\bar{a}_{j,\sigma}$ and $\bar{b}_{j,\sigma}$, and $t_{\perp}$ is the inter-layer hopping matrix element. Since A-B stacking is stable for bilayer system we have introduced only A-B hopping. The summation $\sum'$ is taken over interlayer pairs. Here, we limit the electron hopping in the graphene to the nearest neighbors, which makes the electron dispersion symmetric about the zero energy. The quasiparticle annihilation operators
with positive \((E^+_k)\) and negative \((-E^-_k)\) energy, obtained after diagonalization, are denoted by \(\alpha_{k\sigma}\) and \(\beta_{k\sigma}\), respectively. In case of monolayer, the dispersion is given by

\[
E_k = |\gamma_k| \equiv |te^{i\delta}(1 + e^{i\bar{a}_1} + e^{i\bar{a}_2})|,
\]

where \(\delta\), \(\bar{a}_1\) and \(\bar{a}_2\) are shown in Fig. 2 (a). Near the \(K\) point, \(|\gamma_k|\) is approximated by \(\bar{\hbar}v_F|k - \bar{K}|\) with \(\bar{\hbar}v_F = 3ta/2\) and the same holds for \(K'\). (\(\bar{K}\) shows the position of \(K\)-point.) These structures are usually described as Dirac cones. In case of bilayer, Dirac cones of the two monolayers split into high and low energy bands, and the energy of the latter is given by \(E_k = |\gamma_k|^2/t_\perp\), which has a parabolic form. The higher energy band is usually about 0.2 eV above, and is neglected hereafter.

\[ \text{Figure 2. (a) Honeycomb lattice of graphene and its unit cell (shaded region). Two lattice vectors (}\bar{a}_1\text{ and }\bar{a}_2\text{) are indicated. Open and filled circles, respectively, show the A and B sublattice of graphene. (b) Reciprocal lattice of graphite sheet. The first (hexagon in the center) and the second Brillouin zone (shaded regions) are shown with two reciprocal lattice vectors (}\bar{b}_1\text{ and }\bar{b}_2\text{). }K\text{ and }K'\text{ are the points where two Fermi points exists.} \]

Let us consider the tunneling of electrons between graphene and left leads. We assume the following tunneling Hamiltonian,

\[
H_T^{L(A)} = -t' \sum_{j \in L} (c_{j\sigma}^L a_{j\sigma} + h.c.), \quad H_T^{L(B)} = -t' \sum_{j \in L} (c_{j\sigma}^L b_{j\sigma} + h.c.)
\]

where \(H_T^{L(A)}\) and \(H_T^{L(B)}\) describe the tunneling of electron from left superconductor to A and B site of the graphene, respectively, with \(t'\) being the tunneling matrix element. For right leads, we similarly define \(H_T^{R(A)}\) and \(H_T^{R(B)}\). In case of bilayer system, tunneling occurs only to the upper layer, denoted by \(a_{j\sigma}\) and \(b_{j\sigma}\). In Eq.(6), \(j \in L\) means that the \(j\)-th A (or B) site is in the left junction area. In this paper, the junction area is finally reduced to a lattice point.

2.2. Calculation of Free Energy

We assume \(\Delta^R = \Delta_R e^{i\phi^R}\) and \(\Delta^L = \Delta_L e^{i\phi^L}\), where \(\Delta_0\) and \(\theta^{R(L)}\) are constants, and the free energy \(F(\theta)\), where \(\theta = \theta^R - \theta^L\), can be calculated by imaginary-time path integral formalism
in the following way,

\[ F(\theta) = -\beta^{-1} \ln Z(\beta), \quad Z(\beta) = \int D[\Psi_k^\dagger] e^{-S/\hbar}, \]

\[ S = \int_0^{\beta\hbar} d\tau \left[ \sum_{k\sigma} \Psi_k^\dagger \cdot \hbar \frac{\partial}{\partial \tau} \Psi_k + H - \mu_L N_L - \mu_G N_G - \mu_R N_R \right], \]

where \( \Psi_k = \{ c_{k\uparrow}^\dagger, c_{k\downarrow}^\dagger, \alpha_{k\uparrow}, \alpha_{k\downarrow}, \beta_{k\uparrow}, \beta_{k\downarrow}, \gamma_{k\uparrow}, \gamma_{k\downarrow} \} \) are Grassmann variables, and \( H = H_L + H_R + H_G + H_T \), \( \beta = 1/(k_B T) \) and, \( T \) and \( k_B \) are temperature and Boltzmann constant, respectively. \( \mu_a \) and \( N_a \) \((a = L, R, G)\) are the chemical potential and number operator in \( L, R \) and \( G \) region, respectively. We set \( \mu_L = \mu_R \equiv \mu \) in this paper. \( \mu_G \) is kept in the following calculation, although finally we discuss only \( \mu_G = 0 \) case. The action in the Fourier space is given by

\[ S = \hbar \sum_{\vec{k}, \vec{k}', \omega_n} \Psi_{\vec{k}\omega_n}^\dagger \cdot \Pi(g, \vec{k}', \omega_n) \cdot \Psi_{\vec{k}'\omega_n} \]

\[ \Pi = \begin{pmatrix} \langle G_{\alpha}^R \rangle^{-1} & \Gamma_{\alpha R} & 0 \\ \Gamma_{\alpha L} & \langle G_{\beta}^R \rangle^{-1} & 0 \\ 0 & 0 & \langle G_{\beta}^L \rangle^{-1} \end{pmatrix} \]

\[ G_{\alpha(L)}^{\vec{k} \vec{k}'} = \delta_{\vec{k}, \vec{k}'} \left( \begin{array}{cc} g_{\alpha(L)}^{\vec{k}\omega_n} & f_{\alpha(L)}^{\vec{k}\omega_n} \\ f_{\alpha(L)}^{\vec{k}\omega_n}^* & -g_{\alpha(L)}^{\vec{k}\omega_n}^* \end{array} \right), \quad G_{\alpha(\beta)}^{\vec{k} \vec{k}'} = \delta_{\vec{k}, \vec{k}'} \left( \begin{array}{cc} g_{\alpha(\beta)}^{\vec{k}\omega_n} & 0 \\ 0 & -g_{\alpha(\beta)}^{\vec{k}\omega_n}^* \end{array} \right) \]

where the arguments \((\vec{k}, \vec{k}' \) and \( \omega_n \) are suppressed accordingly. The Green’s functions are given as

\[ g_{\alpha(L)}^{\vec{k}\omega_n} = -\frac{i \omega_n + (\xi_{\vec{k}} - \mu) / \hbar}{\omega_n^2 + \{ (\xi_{\vec{k}} - \mu)^2 + \Delta_0^2 \} / \hbar^2}, \quad f_{\alpha(L)}^{\vec{k}\omega_n} = -\frac{\Delta_{\alpha(L)}^{\vec{k}}}{\omega_n^2 + \{ (\xi_{\vec{k}} - \mu)^2 + \Delta_0^2 \} / \hbar^2} \]

where \( \omega_n = (2n + 1)/(\beta \hbar) \) is the Matsubara frequency. In Eq. 13, \( \pm \) sign should be taken + for \( \alpha \) and - for \( \beta \). The tunneling terms are defined as

\[ \Gamma_{\alpha L} = \Gamma_{A L} + \Gamma_{B L}, \quad \Gamma_{\beta L} = -\Gamma_{A L} + \Gamma_{B L} \]

\[ \Gamma_{A L \vec{k} \vec{k}'} = -\frac{t'}{\sqrt{2} \hbar} \sqrt{n_0^{\alpha(L)}}, \quad \Gamma_{B L \vec{k} \vec{k}'} = -\frac{t'}{\sqrt{2} \hbar} \sqrt{n_0^{\alpha(L)}}, \quad v_{A L}^{\vec{k}(B)}(\vec{k}) = \sum_{j \in L} e^{-i \vec{k} \cdot r_{A L}^{(B)}} \]

where \( \sigma_z \) is the Pauli matrix, dagger (\( \dagger \)) means \( \Gamma_{A L \vec{k} \vec{k}'}^\dagger = \Gamma_{A L \vec{k} \vec{k}'}^\ast \), and \( r_{A L}^{(B)} \) denotes the A (B) site of the \( j \)-th unit cell. The phase factor \( e^{i \phi_{\vec{k}}} \) denotes \( \gamma_{\vec{k}} / |\gamma_{\vec{k}}| \). The right-hand terms, \( \Gamma_{R\vec{k}} \) etc., can be given in the same manner. \( n_a \) \((a = L, R, G)\) are the number of lattice sites in \( L, R \) and \( G \) region, respectively.

Here we divide \( \Pi \) into two parts as \( \Pi = -\mathbf{\tilde{G}}^{-1} + \mathbf{\tilde{G}}, \) where \( \mathbf{\tilde{G}} \propto t' \), and then perform the perturbative expansion with respect to \( t' \). The lowest order terms that contribute to the
Josephson coupling energy $F_J$ of two superconductors are the fourth order terms. These terms are further classified by the tunneling sites: 1) the tunneling to left and right superconductors occurs within the same sublattice (A or B), which we denote by $F_J^{AA}$, 2) the tunneling to left and right superconductors occur at the different sublattice, which we denote by $F_J^{AB}$. Then we obtain

$$
\beta F_J^{AA} = -2 \sum_{k, k', \omega_n} \text{Re}\{\chi_{A k k'}^{L} \chi_{A k k'}^{R*}\} g_{\omega_n}^{\alpha \beta} g_{\omega_n}^{\alpha \beta} , \quad \beta F_J^{AB} = -2 \sum_{k, k', \omega_n} \text{Re}\{\chi_{A k k'}^{L} \chi_{B k k'}^{R*}\} g_{\omega_n}^{\alpha \beta} g_{\omega_n}^{\alpha \beta}
$$

(17)

where $g_{\omega_n}^{\alpha \beta} = g^\alpha + g^\beta$ and $\bar{g}_{\omega_n}^{\alpha \beta} = g^\alpha - g^\beta$. Here $\chi_{A k k'}^{R(L)}$ can be further calculated as,

$$
\chi_{A k k'}^{R(L)} = \left( \frac{t'}{2\hbar} \right)^2 \frac{1}{n_G n_{L(R)}} \sum_{l} v_{A}^{L(R)}(k - \bar{l}) v_{A}^{L(R)}(\bar{l} - \bar{k}) e^{i(\theta_{l,k} - \theta_{l,k'})} f_{l}^{L(R)},
$$

(18)

$$
\chi_{A k k'}^{R(L)} = \left( \frac{t'}{2\hbar} \right)^2 \frac{1}{n_G n_{L(R)}} \sum_{l} v_{B}^{L(R)}(k - \bar{l}) v_{B}^{L(R)}(\bar{l} - \bar{k}) e^{i(\theta_{l,k} - \theta_{l,k'})} f_{l}^{L(R)}.
$$

(19)

After some calculation, we obtain $F_J^{AA}$ as

$$
\beta F_J^{AA} = -2 \left( \frac{t'}{2\hbar} \right)^4 \left( \frac{s_1 k_F^*}{2\pi} \right)^2 \sum_{\omega_n} \sum_{\bar{j} \in \bar{R}, \bar{j}' \in \bar{L}} \frac{\Delta_{0}^2 \cos(\theta_{\bar{l}, \bar{l}'})}{(\hbar \omega_n)^2 + \Delta_{0}^2} |g_{\omega_n}^{\alpha \beta}(\bar{j} - \bar{j}', \omega_n)|^2,
$$

(20)

where $s_1$ and $k_F^*$ are the unit cell area and the Fermi wave number of the lead superconductors. We have introduced the real space representation of $g_{\omega_n}^{\alpha \beta}$. The A-B term $F_J^{AB}$ is obtained by changing $g_{\omega_n}^{\alpha \beta}(\bar{j} - \bar{j}', \omega_n)$ to $\bar{g}_{\omega_n}^{\alpha \beta}(\bar{j} - \bar{j}', \omega_n)$. The supercurrent flowing through the junction can be calculated from the Josephson energy $F_J$ by

$$
I(\theta) = \frac{2e}{h} \frac{dF_J}{d\theta}.
$$

(21)

Usually contact between superconductor and graphene is macroscopic, including many lattice points. Therefore $F_J$ is given by the average of $F_J^{AA}$ and $F_J^{AB}$. However it is also possible to consider point contacts, such as STM using superconducting tips, and we also discuss later the behavior of $F_J^{AA}$ and $F_J^{AB}$ separately in this paper.

Bearing in mind that there are two Fermi points in graphene dispersion, the real-space Green’s function can be calculated as follows,

$$
\begin{align*}
\bar{g}_{\omega_n}^{\alpha (\beta)}(\bar{r}, \omega_n) &= s_0 \int_{BZ} \frac{d^2 \bar{k}}{(2\pi)^2} g_{\omega_n}^{\alpha (\beta) \bar{r}} e^{i \bar{k} \cdot \bar{r}} = s_0 \int_{BZ} \frac{d^2 \bar{l}}{(2\pi)^2} \left\{ g_{\bar{k} + \bar{l}, \omega_n}^{\alpha (\beta) \bar{r}} + g_{\bar{k} - \bar{l}, \omega_n}^{\alpha (\beta) \bar{r}} e^{i \bar{l} \cdot \bar{r}} \right\} e^{i \bar{l} \cdot \bar{r}} \\
&= s_0 \left( e^{i \bar{K} \cdot \bar{r}} + e^{i \bar{R} \cdot \bar{r}} \right) \int_{R^2} \frac{d^2 l}{(2\pi)^2} \frac{1}{i \omega_n - (\pm E_{\bar{l}} - \mu_G)/\hbar} e^{i \bar{r} \cdot \bar{l}},
\end{align*}
$$

(22)

where $s_0 = 3\sqrt{3} a^2 / 2$ is the unit cell area of the graphene with $a$ being the smallest in-plane carbon atom spacing, and BZ denotes the first Brillouin zone. In the last line, the integral domain is expanded to infinity for simplicity.
3. Critical Current of Superconductor-Graphene-Superconductor junction

3.1. Monolayer case

In monolayer case, we obtain

$$\left| g^{\alpha \beta}(r_{jj}', \omega_n) \right|^2 = s_0^2 \Omega_{\phi A}^j \left| \frac{\omega_n - i \mu_G / \hbar}{\pi v_F^2} \right|^2 K_0 \left( \frac{|r_{jj}'| \omega_n - i \mu_G / \hbar}{v_F} \text{sign}(\omega_n) \right)$$

(23)

$$\Omega_{\phi A}^j = |e^{i \vec{K} \cdot \vec{r}_{jj}'} + e^{i \vec{K}' \cdot \vec{r}_{jj}'}|^2 = 2 \left\{ 1 + \cos(\vec{K} - \vec{K}') \cdot \vec{r}_{jj}' \right\}$$

(24)

where $r_{jj}' = r_{j} - r_{j}'$. Here we used the formula

$$\int_0^\infty \frac{tdJ_0(ta)}{t^2 + z^2} dt = K_0(za), \quad (a > 0, \text{Re}(z) > 0),$$

(25)

where $J_n(z)$ and $K_n(z)$ are the $n$-th order Bessel function and modified Bessel function of the second kind, respectively. $\Omega_{\phi A}^j$ gives a atomic scale modulation of the Josephson energy as a function of the tunnel site, whose average over lattice sites is 2. By substituting this to Eq. (20), we obtain $F_{AA}^j$. In a similar way, $F_{AB}^j$ can be obtained in a slightly different functional form. However numerical difference between these two is almost negligible and we discuss only $F_{AA}^j$ in this section.

In Fig. 3, we have plotted the junction length $d$ and temperature dependence of the critical current in monolayer system. We set $\mu_G = 0$ in the following plots. Fig. 3 (a) is the logarithmic plot of $I_c$ as a function of $d$. It can be seen that $I_c$ decay exponentially as a function of $d$, when $d > l_0$, where $l_0 = (\hbar v_F)/\Delta_0(0)$, and $\Delta_0(0)$ is the gap at $T = 0$. (Note that $l_0$ is not exactly the coherence length of superconducting leads, since $v_F$ here is that of graphene.) Therefore we can define the proximity length $\xi_g(T)$ by $I_c \sim \exp(-d/\xi_g(T))$. Fig. 3 (b) shows the logarithmic plot of $I_c$ vs. $\sqrt{T}$, $T$ and $T^2$. Remarkably, the curve is most linear for $T^2$-plot, which means that $\xi_g(T)$ behaves like $\propto 1/T^2$. This is somewhat unusual, since in the S-conventional metal-S junction, we usually obtain $\xi_g(T) \propto 1/T$ for clean-metal case, and $\xi_g(T) \propto 1/\sqrt{T}$ for dirty-metal case. A similar behavior is experimentally observed for junction using gated thin graphite films [18], although its relation to present result is not conclusive at this moment.

**Figure 3.** (a) Logarithmic plot of the critical current in monolayer system as a function of the junction length $d$. (b) Three different plots of the temperature dependence of $I_c$ where $d = 2l_0$. The parameter $s$ denotes $\sqrt{T/T_c}$, $T/T_c$ and $(T/T_c)^2$ for three curves.
3.2. Bilayer case

In bilayer case, we obtain

\[
|\tilde{g}^{\alpha\beta}(\vec{r}_{j\prime j}, \omega_n)|^2 = s_0^2 \Omega_{\alpha\beta} \left( \frac{m_b}{\pi \hbar} \right)^2 4 \text{Im} \left[ K_0 \left( \left| \vec{r}_{j\prime j} \right| \sqrt{\frac{2m_b(i\hbar \omega_n + \mu_G)}{\hbar}} \right) \right]^2
\]

\[
|\tilde{g}^{\alpha\beta}(\vec{r}_{j\prime j}, \omega_n)|^2 = s_0^2 \Omega_{\alpha\beta} \left( \frac{m_b}{h\pi} \right)^2 4 \text{Re} \left[ K_1 \left( \left| \vec{r}_{j\prime j} \right| \sqrt{\frac{2m_b(i\hbar \omega_n + \mu_G)}{\hbar}} \right) \right]^2
\]

\[
\Omega_{\alpha\beta} = \left| e^{i\bar{\vec{r}}_{jj'} - i\theta_j} - e^{i\bar{\vec{r}}_{jj'} + i\theta_j} \right|^2
\]

where \(m_b = 2\nu_f^2/t_\perp\), \(\tilde{r}_{j\prime j} = \tilde{r}_A - \tilde{r}_{j\prime j}, \tilde{r}_{j\prime j} = \tilde{r}_A - \tilde{r}_{j\prime j}\), and \(\theta_j\) is the angle between \(\tilde{r}_{j\prime j}\) and the \(x\)-axis. (The sign of the square root in the arguments of \(K_0\) and \(K_1\) should be determined so that the real part is positive.) By use of Eq. (20), we obtain the Josephson energy of bilayer system. In Fig. 4 (a), we have plotted the contribution of \(J_{AA}\) and \(J_{AB}\) to \(I_c\) separately. One can see, in contrast to monolayer case, \(J_{AA}\) and \(J_{AB}\) contributions show oscillation as a function of \(d\). However, as seen from Fig. 4 (b), this oscillation is washed out when we average the two contributions. The \(d\) dependence of \(I_c\) is then simple exponential decay as in the case of monolayers. In Fig. 4 (c), the temperature dependence of \(I_c\) is examined. In this case, \(\xi_g(T) \propto 1/\sqrt{T}\) is a good approximation, which probably originates from the square root in the argument of \(K_0\) and \(K_1\).

4. Discussion

In this paper we have studied the critical current of S-G-S josephson junction. It was found that the behavior of critical current as a function of junction length \(d\) and temperature is largely affected by the band structure of graphene.

In monolayer case, we found an anomalous temperature dependence of the proximity length \(\xi_g(T) \propto 1/T^2\). The origin of this behavior is not clear at this stage, however contributions from \(\omega_n\)-sum is important, since taking account of only the most dominant terms gives ordinary \(\xi_g(T) \propto 1/T\) behavior as one can see from Eq. (24). Therefore \(\xi_g(T) \propto 1/T^2\) comes from the mixed contributions from many modes and is probably difficult to be reduced to a simple picture.

In bilayer case, we found a peculiar oscillation of the critical current as a function of junction length. (The oscillation can also be seen in temperature domain.) The oscillation disappears if we average over contributions of tunneling at A- and B-sublattice of graphene. From Eq. (28) the origin of oscillation can be attributed to the interference between electrons in upper and lower band at the Dirac points. However the curvature of the band is also important, since we do not see any corresponding phenomena in monolayer case. If we understand detailed origin of this oscillation, it may shed a new light on the subtle quantum interference effect in bilayer graphene, which however is a future problem.

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Figure 4. (a) Logarithmic plot of the contributions of $F_{J_{AA}}$ and $F_{J_{AB}}$ to $I_c$ as a function of the junction length $d$. ($T = 0.3T_c$) (b) Three plots with different temperatures of the $d$-dependence of $I_c$, where both contribution from $F_{J_{AA}}$ and $F_{J_{AB}}$ are included. (c) Three different plots of the temperature dependence of $I_c$. The parameter $s$ denotes $\sqrt{T/T_c}$, $T/T_c$ and $(T/T_c)^2$ for three curves. ($d = 2l_0$)

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