On the existence of a finite-temperature transition in the two-dimensional gauge glass

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Results from Monte Carlo simulations of the two-dimensional gauge glass supporting a zero-temperature transition are presented. A finite-size scaling analysis of the correlation length shows that the system does not exhibit spin-glass order at finite temperatures. These results are compared to earlier claims of a finite-temperature transition.

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The existence of a finite-temperature spin-glass transition in the two-dimensional gauge glass has been a source of great controversy. While the existence of a transition in three dimensions is undisputed, several different approaches have been made to determine the spin-glass ordering temperature \( T_c \) in two dimensions with different results. As the gauge glass in two dimensions is a model commonly used to describe the vortex-glass transition in dirty high-temperature superconductor thin films, it is important to settle the question of whether the system exhibits spin-glass order at finite temperatures or not. Experimental results on YBCO thin films show no vortex-glass order, whereas bulk materials do exhibit a transition to a glassy phase. A brief summary of the different approaches used to determine \( T_c \) is presented, followed by results from a novel technique that strongly supports the absence of a glassy phase at finite temperatures in the two-dimensional gauge glass.

Early work by Fisher et al. who study the size-dependence of domain-wall energies with a zero-temperature domain-wall renormalization group method, shows evidence of a zero-temperature transition in the two-dimensional gauge glass. Gingras as well as Akino and Kosterlitz who also study the stiffness exponent of the model, find agreeing results.

In the aforementioned work, Fisher et al. also analyze the scaling of Binder ratios of the order parameter and obtain further evidence of a zero-temperature transition. These results are in agreement with Monte Carlo simulations by Reger and Young who conclude that the transition temperature is not finite by studying the scaling of the currents induced by a twist in the boundary conditions. Katzgraber and Young find evidence of a zero-temperature transition for the two-dimensional gauge-glass Hamiltonian with periodic boundary conditions by studying the scaling of the currents and the spin-glass susceptibility. They also conclude that one can scale data for the susceptibility well for any positive \( T_c \), probably due to the extra fitting parameter \( q \), the anomalous dimension of the spin-glass order parameter \( q \), and that studying the scaling of the spin-glass susceptibility alone is not enough to obtain evidence of a finite-temperature transition. This may explain why similar simulations by Choi and Park who study the susceptibility only, find a finite-temperature transition with \( T_c = 0.22 \pm 0.02 \).

Furthermore, Kim has studied the gauge glass with fluctuating twist boundary conditions by means of resistively shunted junction (RSJ) dynamics and finds a finite \( T_c \) in agreement with the work of Choi and Park and with early results by Li (challenged later by Simkin). Surprisingly, Granato who also uses RSJ dynamics but with periodic boundary conditions, obtains \( T_c = 0 \).

More recently Holme et al. have studied the two-dimensional gauge glass by means of Monte Carlo simulations with fluctuating twist boundary conditions. By introducing an extra anomalous exponent \( b \) they are able to scale data for the currents and the helicity modulus and so obtain a finite transition temperature close to 0.20.

As there are diverging opinions on the spin-glass transition temperature of the two-dimensional gauge glass, the issue is reconsidered in the present work using a different approach. The scaling of the finite-system correlation lengths is studied, giving compelling evidence that \( T_c \approx 0 \), at least for the system sizes studied here. These results are supported by data for the spin-glass susceptibility.

The gauge-glass Hamiltonian which describes a disordered granular superconductor, is given by

\[
\mathcal{H} = -J \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j - A_{ij}),
\]

where the sum ranges over nearest neighbors on a square lattice of size \( N = L^2 \) and \( \phi_i \) represent the angles of the \( XY \) spins. The \( A_{ij} \) are quenched random variables uniformly distributed between \([0, 2\pi]\) with the constraint that \( A_{ij} = -A_{ji} \). In this work \( J = 1 \) and periodic boundary conditions are applied.

Previously, we estimated the critical temperature of the two-dimensional gauge glass by studying the scaling of the currents induced by a twist in the boundary conditions. This has an advantage over studying the crossing of Binder ratios because the latter will not splay enough for \( XY \) systems at \( T < T_c \), making it almost impossible to accurately determine the crossing point. Studying the currents requires some care as well, as these have different finite-size scaling forms depending on a finite or zero transition temperature. Finally, the currents show strong statistical fluctuations and so require a large amount of
disorder realizations. Therefore in this work the scaling of the correlation lengths is studied as these show little statistical fluctuations, have a simple finite-size scaling form and splay out well for statistical fluctuations, have a simple finite-size scaling region represents the area around $T_c$ is finite, it must be less than 0.13, the lowest temperature simulated. The shaded region shows a scaling plot of the data for $L^{1/ν}T$ for $T_c = 0$ and $1/ν = 0.39$. One can see that the data scale very well, especially for low temperatures.

The spin-glass order parameter for the gauge glass is given by

$$ q = \frac{1}{N} \sum_j^{N} \exp[i(ϕ_j^α - ϕ_j^β)] , \quad (2) $$

where “α” and “β” represent two replicas of the system with the same disorder. Equation (2) is generalized to finite wave vectors $k$:

$$ q(k) = \frac{1}{N} \sum_j^{N} \exp[i(ϕ_j^α - ϕ_j^β) + ik \cdot R_j] . \quad (3) $$

The wave vector dependent spin-glass susceptibility $χ_{SG}(k)$ is then given by

$$ χ_{SG}(k) = N[⟨q(k)⟩^2]_{av} . \quad (4) $$

Here, $⟨⋯⟩$ denotes a thermal and [⋯]_{av} a disorder average. The finite-system correlation length $ξ_L$ is determined from an expansion of the spin-glass susceptibility for small $kξ_L$ in the framework of an Ornstein-Zernicke approximation:

$$ [χ_{SG}(k)/χ_{SG}(0)]^{-1} = 1 + (kξ_L)^2 + O[(kξ_L)^4] . \quad (5) $$

Keeping only the leading term in the expansion and taking into account the lattice periodicity as well as corrections to scaling, we obtain

$$ ξ_L = \frac{1}{2 \sin(|k_{min}/2|)} \left[ \frac{χ_{SG}(0)}{χ_{SG}(k_{min})} - 1 \right]^{1/2} , \quad (6) $$

where $k_{min} = (2π/L, 0, 0)$ is the smallest nonzero wave vector. In Eq. (6), $χ_{SG}(0)$ is the standard spin-glass susceptibility.

The finite-size scaling of $ξ_L$ can be understood by studying the finite-size scaling of the spin-glass susceptibility extended to finite wave vectors $k$

$$ χ_{SG}(T, L, k) = L^{2-ηC}[L^{1/ν}(T - T_c), κL] . \quad (7) $$

From Eq. (6) $k_{min}^2ξ_L^2 \sim χ_{SG}(0)/χ_{SG}(k_{min})$ up to an additive constant, and so the $L$-dependent prefactor in Eq. (7) cancels. The dependence of $χ_{SG}$ on $kL$ drops out as well, as in Eq. (6) $k = k_{min} = 2π/L$. We therefore obtain the simple finite-size scaling form

$$ ξ_L/L = \tilde{X}[L^{1/ν}(T - T_c)] , \quad (8) $$

and so, the introduction of an anomalous exponent is not plausible. This has also the advantage that the finite-size scaling has few parameters. From Eq. (6) we see that data for different system sizes $L$ will cross at $T = T_c$. If $T_c = 0$, one expects that data for different $L$ will decrease with increasing $L$ and not cross for any finite temperature.

For the simulations the parallel tempering Monte Carlo method is used. Because the equilibration test for short-range spin glasses introduced by Katzgraber et al. does not work for the gauge glass as the disorder is not Gaussian, equilibration is tested by the traditional technique of requiring that different observables are independent of the number of Monte Carlo steps. By doubling the number of Monte Carlo sweeps between each measurement until the last three agree within error bars, equilibration is ensured. The equilibration times used for the different system sizes are listed in Ref. 9. The temperature replicas are used with the highest temperature being $T = 1.058$.

Figure III shows data for $ξ_L/L$ for different system sizes $L$. The data decrease with increasing $L$ for all temperatures. In particular, there is no crossing of the data around $T \approx 0.20$ (shaded area) where the spin-glass transition is supposed to take place. The inset of Fig. III shows a finite-size scaling plot of the data according to Eq. (6). One can see that the data scale very well for $T_c = 0$ and $1/ν = 0.39 ± 0.02$, values found in Ref. 9. The error bar in $1/ν$ is determined by varying its value until the data do not scale well. An attempt to scale the
data in Fig. 2 to any value of a finite $T_c$, in particular $0.20$, fails for any choice of $\nu$.

Standard finite-size scaling predicts that the spin-glass susceptibility scales as

$$\chi_{SG} \sim L^{2 - \eta}$$

which means that at $T = T_c$ it should scale as a power law $\sim L^{2 - \eta}$. In Fig. 2 data for $\ln(\chi_{SG}/L^2)$ vs $\ln(L)$ is shown. If the data were at $T_c$, one would expect a straight line. In particular, $T = 0.20$ shows a strong downward curvature indicating that the system is above the critical transition temperature. A fit of the data in Fig. 2 to a straight line for the different temperatures yields quality of fit probabilities between $10^{-9}$ for the lowest, and $10^{-20}$ for the highest temperature, respectively. A second-order polynomial fit to the data, instead, has quality of fit probabilities $\sim 0.9$. The curvature of the second-order fits is determined and shown in the inset of Fig. 2. Because at $T < T_c$ [$T > T_c$] one expects $\chi_{SG}(L)$ to bend upward [downward], the curvature has to go through zero at $T_c$. The data in the inset of Fig. 2 do not cross the horizontal axis for the temperature range studied and extrapolate within error bars to $T_c \approx 0$. This is in agreement with results by Katzgraber and Young who obtain good scaling of the susceptibility at low temperatures for $T_c = 0$, $\eta = 0$, and $1/\nu = 0.39$.

To conclude, evidence of a zero-temperature transition in the two-dimensional gauge glass with periodic boundary conditions is presented by studying the finite-size scaling of the correlation lengths. This approach is better than previous methods used because the correlation lengths show small statistical fluctuations and have a simple finite-size scaling form. In addition, the curvature of the spin-glass susceptibility as a function of system size indicates that there is no spin-glass transition at $T \approx 0.20$, at least for the system sizes studied. Moreover the curvature extrapolates within error bars to $T_c \approx 0$. These results are in agreement with experimental data on YBCO thin films where a finite-$T$ is absent, and different numerical approaches. In particular, it is shown that the system does not exhibit a spin-glass transition at $T \approx 0.20$, as claimed in Refs. 7, 8, and 10. It is noteworthy that Refs. 8 and 10 find the aforementioned finite temperature transition by studying the gauge glass with fluctuating twist boundary conditions instead of the commonly used periodic boundary conditions. Could this be one source of the discrepancies?

Finite-size effects can influence the data, and so an analysis with larger system sizes is desirable. Nevertheless, the data presented here strongly support the absence of a transition at $T \approx 0.20$, although a transition at $T < 0.13$, the lowest temperature studied, while unlikely the case, cannot be ruled out completely.

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In Ref. 3 a finite-size scaling plot of the spin-glass susceptibility is performed with $1/\nu = 0.39$, $T_c = 0$, and $\eta = 0$. The data scale well at the lowest temperatures studied. For higher temperatures, strong corrections to scaling are visible. In fact, the best fit “by eye” yields $1/\nu = 0.50$. This slight discrepancy can be explained by recent work (Ref. 4) of Carter et al. who show that for a zero-temperature transition the scaling can have considerable corrections due to large corrections to scaling in the expression for the correlation length.

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