Rotating and Moving D-Branes in the Presence of Various Background Fields

Farzin Safarzadeh-Maleki and Davoud Kamani

Faculty of Physics, Amirkabir University of Technology (Tehran Polytechnic)
P.O.Box: 15875-4413, Tehran, Iran
e-mails: kamani@aut.ac.ir, f.safarzadeh@aut.ac.ir

Abstract

We calculate the bosonic boundary state associated with a rotating and moving Dp-brane in the presence of the antisymmetric tensor field, a U(1) gauge field and a tachyon field. Rotation and motion are in the brane volume. We reconstruct this boundary state via the group $PSL(2,R)$ to be applicable when the tachyon field is presented. This modified boundary state enables us to calculate the interaction amplitude between two parallel Dp-branes with rotation and motion. The long-range force of this interaction will be obtained. The boundary state also enables us to investigate the tachyon condensation on a rotating and moving Dp-brane.

PACS numbers: 11.25.-w; 11.25.Uv

Keywords: Rotating and moving brane; Boundary state; Interaction; Tachyon condensation.
1 INTRODUCTION

D-branes, as essential objects in string theory [1, 2], have important applications in different aspects such as: braneworld cosmology, stability of time dependent phenomena, solving many timelike singularities of general relativity, describing nonperturbative phenomena in string theory, higher dimensional theories and so on. In addition, D-branes with nonzero background internal fields have shown several interesting properties [3]-[8]. For example, internal fields control the interactions of the branes and/or the background tachyon field specifies instability of the branes.

On the other hand, we have the boundary state formalism for describing the D-branes [9]-[16]. This formalism is not only a useful tool in many complicated situations, even when a clear spacetime is not available, but also it can find the conformal duality which is an important problem in revealing the underlying symmetry of string theory. The overlap of two boundary states, corresponding to two D-branes, through the closed string propagator gives the interaction amplitude of the branes. So far this adequate method has been applied to the stationary mixed branes, moving branes with constant velocities and rotated branes by an angle in the presence of different background fields [16]-[20]. For the branes with the tachyon field see Refs. [21]-[31].

The goal of this paper is to determine a general case including rotating and moving D-branes in the presence of the following background fields: the Kalb-Ramond field, $U(1)$ gauge fields which live in the D-branes worldvolumes and tachyon fields. We shall consider rotation of a brane in its volume, and its velocity is along the brane directions. Because of the various fields inside the branes there are preferred directions and hence these rotations and motions are meaningful. This generalized setup strongly affects interaction of the branes.

In fact, the presence of the tachyon field, especially a tachyon field with the quadratic function of the spacetime coordinates, induces an off-shell theory. In this article we shall use the prescription of Ref. [27] to reconstruct the boundary state associated with a D$p$-brane, in the presence of the quadratic tachyon field. The modified boundary state will be applied for computing the interaction amplitude of two D$p$-branes, which exchange on-shell and off-shell closed strings.

In the last few years significant steps have been made to understand the open string tachyon dynamics and tachyon condensation. These concepts have been studied by the first quantized string theory [32]-[34], by the cubic open string field theory [35], by the RG flow method [36]-[38] and more recently by the boundary string field theory [39]-[41]. In these studies it has been shown that open string tachyon condensation describes the decay
of unstable D-branes into the closed string vacuum (or to stable ones). In this paper, we shall examine the tachyon condensation for a rotating-moving Dp-brane. We observe that tachyon condensation can always make such branes to be unstable and hence reduces the brane dimension.

This paper is organized as follows. In Sec. 2, the boundary state associated with a rotating and moving Dp-brane with background fields will be constructed. In Sec. 3, the interaction of two Dp-branes will be studied. In Sec. 4, the instability of a rotating-moving Dp-brane due to the tachyon condensation phenomena will be examined. Section 5 is devoted to the conclusions.

2 THE BOUNDARY STATE

2.1 Extracting the boundary state from the action

Our starting point is to determine the boundary action corresponding to a rotating and moving Dp-brane. Therefore, we use the following sigma-model action for the closed string:

\[ S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2 \sigma (\sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma (A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta} J^\alpha_{\tau} + T(X^\alpha)), \]  

where \( \Sigma \) is the closed string worldsheet and \( \partial \Sigma \) is its boundary. We shall use \( \{X^\alpha | \alpha = 0, 1, \cdots, p\} \) for the worldvolume directions of the Dp-brane and \( \{X^i | i = p+1, \cdots, d-1\} \) for directions perpendicular to it. This action contains the Kalb-Ramond field \( B_{\mu\nu} \), a \( U(1) \) gauge field \( A_\alpha \) which lives in the worldvolume of the brane, an \( \omega \)-term associated with the rotation and motion of the brane and a tachyonic field. Note that the tachyon and gauge potential are in the open string spectrum, which are attached to the brane.

For simplifying the calculations the background fields \( G_{\mu\nu} \) and \( B_{\mu\nu} \) are considered to be constant. In addition, for the \( U(1) \) gauge field we apply the gauge \( A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta \) with constant field strength. Besides, we use the following tachyon profile \( T = T_0 + \frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta \), where \( T_0 \) and the symmetric matrix \( U_{\alpha\beta} \) are constant. Finally, the \( \omega \)-term, which is responsible for the rotation and motion of the brane, contains the antisymmetric angular velocity \( \omega_{\alpha\beta} \) and angular momentum density \( J^\alpha_{\tau} \). Its explicit form is given by \( \omega_{\alpha\beta} J^\alpha_{\tau} = 2 \omega_{\alpha\beta} X^\alpha \partial_\tau X^\beta \).

In fact, \( \omega_{\alpha\beta} |_{\alpha \neq \beta} \) denotes the velocity component of the brane along the direction \( X^\alpha \) while \( \omega_{\alpha\beta} \) represents its rotation. Note that in the presence of the Kalb-Ramond field and the local gauge potential there are some preferred alignments in the brane. This implies that the rotation and motion of the brane in its volume are sensible.
Vanishing of the variation of the action with respect to \( X^\mu(\sigma, \tau) \) gives the equation of motion and the following boundary state equations

\[
\left[ (\eta_{\alpha\beta} + 4\omega_{\alpha\beta})\partial_\tau X^\beta + \mathcal{F}_{\alpha\beta}\partial_\sigma X^\beta + U_{\alpha\beta}X^\beta \right]|_{\tau = 0} |B\rangle = 0,
\]

\[
\delta X^\tau|_{\tau = 0} |B\rangle = 0,
\]

(2)

where the total field strength is \( \mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta} \). For simplicity, we assumed that the following mixed elements are zero \( B_{\alpha i} = U_{\alpha i} = 0 \).

Now by using the closed string mode expansion

\[
X^\mu(\sigma, \tau) = x^\mu + \frac{l^2}{2} p^\mu \tau + \frac{1}{2} \sum_{m \neq 0} \frac{1}{m}(\alpha_m^\mu e^{-2im(\tau - \sigma)} + \tilde{\alpha}_m^\mu e^{-2im(\tau + \sigma)}),
\]

(3)

the boundary state equations can be written in terms of the oscillators

\[
\left[ (\eta_{\alpha\beta} + 4\omega_{\alpha\beta})l^2 p^\beta + U_{\alpha\beta}x^\beta \right]|B\rangle^0 = 0,
\]

(4)

\[
(x^i - y^i)|B\rangle^0 = 0,
\]

(5)

\[
\left[ (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta})\alpha_m^\beta + (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta})\tilde{\alpha}_{-m}^\beta \right]|B\rangle^{\text{osc}} = 0,
\]

(6)

\[
(\alpha_m^i - \tilde{\alpha}_{-m}^i)|B\rangle^{\text{osc}} = 0,
\]

(7)

where the set \( \{ y^i \} \) indicates the position of the brane, and \( |B\rangle = |B\rangle^{\text{osc}} \otimes |B\rangle^0 \). According to the eigenvalues in Eq. (4) we deduce the relation

\[
p^\alpha = -\frac{1}{2}\alpha'[\eta + 4\omega]^{-1}U]\alpha_x^\beta.
\]

(8)

This implies that along the worldvolume of the brane momentum of the closed string depends on its center of mass position. Therefore, in the presence of the tachyon field the emitted closed string feels an exotic potential which affects its evolution.

The solutions of the boundary state equations can be found by the coherent state method. Thus, for the oscillating modes we have

\[
|B\rangle^{\text{osc}} = \prod_{n=1}^\infty [\det Q(n)]^{-1}\exp \left[ -\sum_{m=1}^\infty \frac{1}{m}(\alpha_{-m}^\mu S(m)_{\mu\nu} \tilde{\alpha}_{-m}^\nu) \right] |0\rangle_\alpha \otimes |0\rangle_\tilde{\alpha},
\]

(9)

where the matrices are defined by

\[
Q(n)_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta},
\]

\[
S(m)_{\mu\nu} = (\Delta(m)_{\alpha\beta} , -\delta_{ij}),
\]

\[
\Delta(m)_{\alpha\beta} = (Q^{-1}(m) N(m))_{\alpha\beta},
\]

\[
N(m)_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta}.
\]

(10)
The advent of the normalization factor $\prod_{n=1}^{\infty} [\det Q(n)]^{-1}$ is anticipated by the disk partition function. The boundary state associated with the zero modes possesses the following feature

$$|B^{(0)}\rangle = \int_{-\infty}^{\infty} \prod_{\alpha} dp^\alpha \exp \left[ i\alpha' \left( \sum_{a=0}^{p} (U^{-1} A)_{\alpha\alpha} (p^\alpha)^2 + \sum_{\alpha,\beta=0,\alpha\neq\beta}^{p} (U^{-1} A + A^T U^{-1})_{\alpha\beta} p^\alpha p^\beta \right) \right] \times \prod_{i} \delta(x^i - y^i)|p^2 = 0\rangle \otimes \prod_{\alpha} |p^\alpha\rangle,$$

(11)

where $A_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}$. The variety of the parameters in Eqs. (9) and (11) enables us to adjust each part of the boundary state to a desirable state.

The total boundary state is

$$|B\rangle^{(tot)} = \frac{T_p}{2} |B\rangle^{(osc)} \otimes |B\rangle^{(0)} \otimes |B\rangle^{(gh)},$$

(12)

where $|B\rangle^{(gh)}$ is the boundary state corresponding to the conformal ghost fields

$$|B\rangle^{(gh)} = \exp \left[ \sum_{m=1}^{\infty} \left( c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m} \right) \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle \otimes |\tilde{q} = 1\rangle. \right.$$

(13)

In this state the ghost vacuum has been chosen in the picture $(-1, -1)$.

Equations (9) and (12) define an effective tension for the brane, i.e.,

$$T_p(T_p, F, U, \omega) = T_p \prod_{n=1}^{\infty} [\det Q(n)]^{-1}.$$

(14)

### 2.2 Modification of the boundary state

It should be noted that the presence of the tachyon field in the boundary state leads to an off-shell theory. Therefore, we apply the procedure of Ref. [27] to deform it under the $PSL(2, R)$ transformation. Let $z$ be the complex coordinate of the points within a unit disk which is the worldsheet of an emitted closed string by the brane. Thus, the action of the $PSL(2, R)$ on the complex coordinate $z$, is the mapping $z \rightarrow w(z) = (az + b)/(b^*z + a^*)$ where the complex variables $a$ and $b$ satisfy the relation $|a|^2 - |b|^2 = 1$. Since $PSL(2, R)$ is a subgroup of the full conformal group of the disk, one can check that this transformation preserves the shape and location of the boundary of the disk.

Now we study the effect of this transformation on the closed string oscillators which are defined by the following contour integrals

$$\alpha^\mu_m = \sqrt{\frac{2}{\alpha'}} \oint_{C_z} \frac{dz}{2\pi} z^m \partial_z X^\mu(z)$$

\[= \sqrt{\frac{2}{\alpha'}} \oint_{C_w} \frac{dz}{2\pi} z^m \partial_w X^\mu(w) \frac{dw}{dz}. \]

(15)
The contour $C_z$ is a unit circle, i.e. the boundary of the unit worldsheet disk of the emitted closed string. There exists a similar formula for the left-moving oscillators $\tilde{\alpha}_m^\mu$. A mode expansion for $X^\mu$ also exists in terms of $w$ and $\bar{w}$ with the coefficients $\alpha_m^\mu$ and $\tilde{\alpha}_m^\mu$ exactly in the feature of the first line of Eq. (15). Comparing the corresponding coefficients we receive the relations

$\alpha_0^\mu = \alpha_0^\nu$, $\tilde{\alpha}_0^\mu = \tilde{\alpha}_0^\nu$,
$\alpha_m^\mu = M_{mn}(a, b)\alpha_n^\nu$, $\tilde{\alpha}_m^\mu = -M^*_{mn}(a, b)\tilde{\alpha}_n^\nu$, $m, n \in \mathbb{Z} - \{0\}, (16)$

where the matrix $M_{mn}(a, b)$ is defined by

$M_{mn}(a, b) = \oint_{C_z} dz \frac{z^m(b^* z + a^*)^{n-1}}{(az + b)^{n+1}}. (17)$

One can check that a creation (annihilation) oscillator is expressed in terms of the creation (annihilation) oscillators.

In addition to the matter fields, the ghosts fields are also influenced by the $PSL(2, \mathbb{R})$ transformation. The procedure of Eqs. (15) and (16) defines the following transformations for the oscillators of the conformal ghosts

$b_m = N_{mn} b'_n,$
$c_m = P_{mn} c'_n,$
$\tilde{b}_m = -N^*_{mn} \tilde{b'}_n,$
$\tilde{c}_m = -P^*_{mn} \tilde{c'}_n, (18)$

where the matrices are defined by

$N_{mn} = \oint_{C_z} dz \frac{z^{m+1}(b^* z + a^*)^{n-2}}{(az + b)^{n+2}},$
$P_{mn} = \oint_{C_z} dz \frac{z^{m-2}(b^* z + a^*)^{n+1}}{(az + b)^{n-1}}. (19)$

An appropriate redefinition of the boundary state, consistent with the above transformations, is given by

$|\bar{B}\rangle = \int d^2 a \, d^2 b \, \delta(|a|^2 - |b|^2 - 1)|B_{a,b}\rangle^{(mat)} \otimes |B_{a,b}\rangle^{(gh)},$

$|B_{a,b}\rangle^{(mat)} = \frac{T_p}{2} \prod_{m=1}^{\infty} [\det Q_{(m)}]^{-1} \exp \left[ \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \left( -\frac{1}{n} \alpha^-_{m,k}(S_{njk}(a, b))_{\mu\nu} \tilde{\alpha}_{\nu,j} \right) \right]$
\begin{equation}
\times |0\rangle_\alpha \otimes |0\rangle_{\tilde{\alpha}} \otimes |B\rangle^{(0)},
\end{equation}
\begin{equation}
|B_{a,b}\rangle^{(gh)} = \left(|a|^2 + |b|^2\right) \exp \left[\sum_{n=1}^\infty \sum_{j=1}^\infty \sum_{k=1}^\infty \left(\Gamma_{(n)jk}(a, b)c_{-j}\tilde{b}_{-k} - \Gamma^*_{(n)jk}(a, b)b_{-j}\tilde{c}_{-k}\right)\right]
\end{equation}
\begin{equation}
\times \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle \otimes |\tilde{q} = 1\rangle,
\end{equation}
where for simplicity we removed the primes of the new oscillators. The phrases \((S_{(n)jk}(a, b))_{\mu\nu}\) and \(\Gamma_{(n)jk}(a, b)\) are given by
\begin{equation}
(S_{(n)jk}(a, b))_{\mu\nu} = -M_{-n,-k}(a, b)S_{(n)\mu\nu}M^*_{-n,-j}(a, b),
\end{equation}
\begin{equation}
\Gamma_{(n)jk}(a, b) = -P_{-n,-j}(a, b)N^*_{-n,-k}(a, b).
\end{equation}
The boundary state (20) will be used for calculation of the branes interaction.

3 INTERACTION OF TWO D-BRANES

The interaction of two D-branes can be calculated in two different but equivalent approaches: open string one-loop and closed string tree-level diagrams. In the first approach which is a quantum process, an open string is stretched between two D-branes. It forms a cylinder via a loop diagram. In the second approach which is a classical process, a closed string is created by one D-brane. It propagates in the transverse space between the two D-branes, and then the other D-brane absorbs it. Here we use the second approach for finding the interaction amplitude. Therefore, we calculate the overlap of the two boundary states via the closed string propagator
\begin{equation}
A = \langle B_1 | D | B_2\rangle,
\end{equation}
where \(|B_1\rangle\) and \(|B_2\rangle\) are total boundary states corresponding to the D-branes and “D” is the closed string propagator
\begin{equation}
D = 2\alpha' \int_0^\infty dt \ e^{-tH_{\text{closed}}}.\end{equation}
The closed string Hamiltonian is given by
\begin{equation}
H_{\text{closed}} = H_{\text{ghost}} + \alpha' p^2 + 2 \sum_{n=1}^\infty (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n) - \frac{d - 2}{6}.
\end{equation}
The modified boundary state (20) can be applied for obtaining the interaction amplitude of the branes. After a long calculation, for parallel Dp-branes we receive the following
amplitude

\[
\mathcal{A} = \frac{T_p^2 \alpha' V_{p+1}}{4(2\pi)^{d-p-1}} \prod_{n=1}^{\infty} \det \left[ Q_{(n)1}^\dagger Q_{(n)2} \right]^{-1} \\
\times \int d^2a \ d^2b \ d^2a' \ d^2b' \delta(|a|^2 - |b|^2 - 1) \delta(|a'|^2 - |b'|^2 - 1) \\
\times \left( |a|^2 + |b|^2 \right) \left( |a'|^2 + |b'|^2 \right) \int_0^\infty dt \{ e^{(d-2)t/6} \\
\times \prod_{n=1}^{\infty} \prod_{j=1}^{\infty} \prod_{k=1}^{\infty} \prod_{j'=1}^{\infty} \prod_{k'=1}^{\infty} \left[ 1 - \Gamma^{*}_{(n)jk}(a, b)\Gamma_{(n)j'k'}(a', b')e^{-4nt} \right]^2 \\
\times \left( \det(1 - S^{(1)\dagger}_{(n)jk}(a, b)S^{(2)}_{(n)j'k'}(a', b')e^{-4nt}) \right)^{-1} \\
\times \frac{1}{\sqrt{\det(R_1 R_2)}} \left( \sqrt{\alpha'^t} \right)^{d-p-1} \exp \left( -\frac{1}{4\alpha'^t} \sum_{i=p+1}^{d-1} (y_i^1 - y_i^2)^2 \right) \}
\]

(24)

where \( V_{p+1} \) is the common worldvolume of the branes, and the symmetric matrices \( R_1 \) and \( R_2 \) possess nonzero elements only along the branes worldvolume

\[
(R_t)_{\alpha\beta} = 2\alpha' (-i\mathcal{M}_t - iU_t^{-1} A_t - iA_t^T U_t^{-1} + t1)_{\alpha\beta}, \quad l = 1, 2,
\]

\[
\mathcal{M}_t = \begin{pmatrix}
(U_t^{-1} A_t)_{00} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & (U_t^{-1} A_t)_{pp}
\end{pmatrix},
\]

\[
(A_t)_{\alpha\beta} = \eta_{\alpha\beta} + 4(\omega_t)_{\alpha\beta}.
\]

(25)

This amplitude includes contribution of all possible on-shell and off-shell closed string states that the Dp-branes can emit them. The constant overall factor of the amplitude, behind the integrals, indicates a hue of the strength of the interaction, which depends on the parameters of the theory. The second determinant is the contribution of the matter part oscillators, the factor including \( \Gamma^* \Gamma \) comes from the ghosts and the other factors in the time integral are related to the zero modes of the string coordinates.

### 3.1 The long-range force

For distant D-branes only the massless states of closed string, i.e. gravitation, dilaton and Kalb-Ramond states, exhibit a considerable contribution to the interaction amplitude. In other words, after a long enough time these massless states become dominant. Therefore, we shall concentrate on the long-range force.

In the 26-dimensional spacetime we acquire the following limit

\[
\lim_{t \to \infty} e^{\Omega t} \prod_{n=1}^{\infty} \prod_{j=1}^{\infty} \prod_{k=1}^{\infty} \prod_{j'=1}^{\infty} \prod_{k'=1}^{\infty} \left[ 1 - \Gamma^*_{(n)jk}(a, b)\Gamma_{(n)j'k'}(a', b')e^{-4nt} \right]^2
\]

8
\[ \times \left( \det(1 - S^{(1)\dagger}_{n;jk}(a, b)S^{(2)}_{n;j'k'}(a', b')e^{-4nt}) \right)^{-1} \]

\[ = \lim_{t \to \infty} e^{4t} + \sum_{j} \sum_{k} \sum_{j'} \sum_{k'} \left[ \text{Tr} \left( S^{(1)\dagger}_{n;jk}(a, b)S^{(2)}_{n;j'k'}(a', b') \right) \right. \]

\[ - 2 \Gamma^*_n(a, b) \Gamma_n(a', b') \].

(26)

The application of this limit in the interaction amplitude specifies the long-range force amplitude

\[ A_{(\text{massless})} = \frac{T^2_p \alpha'}{4(2\pi)^{25-p}} \prod_{n=1}^{\infty} \det [Q_{(n)1}Q_{(n)2}]^{-1} \]

\[ \times \int d^2a \ d^2b \ d^2a' \ d^2b' \ \delta(|a|^2 - |b|^2 - 1)\delta(|a'|^2 - |b'|^2 - 1) \]

\[ \times \left[ \lim_{t \to \infty} e^{4t} + \sum_{j} \sum_{k} \sum_{j'} \sum_{k'} \left( \text{Tr} \left( S^{(1)\dagger}_{n;jk}(a, b)S^{(2)}_{n;j'k'}(a', b') \right) \right) \right. \]

\[ - 2 \Gamma^*_n(a, b) \Gamma_n(a', b') \] \]

\[ \times \frac{1}{\sqrt{\det(R_1^\dagger R_2)}} \left( \sqrt{\frac{\pi}{\alpha' t}} \right)^{25-p} \exp \left( -\frac{1}{4\alpha' t} \sum_i \left( y_{2,i} - y_{1,i} \right)^2 \right) \].

(27)

Since the tachyon state has negative mass squared the divergent term is corresponding to the exchange of the closed string tachyon, while the other terms represent the contribution of the gravitation, dilaton and Kalb-Ramond states in the interaction. Note that closed string emission is independent of the locations of the branes, hence the position factors do not change.

### 4 INSTABILITY OF A ROTATING-MOVING D-BRANE

D-branes in the presence of the tachyonic mode of the string spectrum are under the experience of instability [32]. This is due to the rolling of the tachyon from an unstable maximum to an incorrect vacuum (IR fixed point). This implies that adding the tachyon as a perturbation (deformation) to the theory leads to instability. In other words, tachyon condensation can make lower dimensional unstable branes as intermediate states.

From technical point of view for constructing the tachyon condensation we should take at least one of the tachyon’s elements to infinity. In the boundary states (9) and (11) [or equivalently Eq. (20)] there are four matrices \( Q_{(n)} \), \( \Delta_{(m)} \), \( L = U^{-1}A + ATU^{-1} \) and the diagonal matrix \( (U^{-1}A)_{\alpha\alpha} \), in which the tachyon has been inserted. So it is sufficient to
take the limit of these matrices to evaluate the behavior of the brane under the experience of tachyon condensation.

Now we apply the limit $U_{pp} \to \infty$. According to the following limit

$$\lim_{U_{pp} \to \infty} (U^{-1})_{\alpha\alpha} = \lim_{U_{pp} \to \infty} (U^{-1})_{\alpha p} = 0, \quad \alpha = 0, 1, \cdots, p,$$

the elements of the diagonal matrix $(U^{-1}A)_{\alpha\alpha} = (U^{-1})_{\alpha\gamma} A_{\gamma\alpha} = (U^{-1})_{\alpha\gamma} A_{\gamma'} A_{\alpha\gamma'} + (U^{-1})_{\alpha p} A_{p\alpha}$ with $\gamma' \neq p$ reduce to $(U^{-1})_{\alpha\gamma} A_{\gamma'} A_{\alpha\gamma'}$. For $\alpha = p$ this element also vanishes and hence we receive

$$\sum_{\alpha=0}^{p} (U^{-1}A)_{\alpha\alpha}(p^\alpha)^2 = \sum_{\alpha'=0}^{p-1} (U^{-1}A)_{\alpha'\alpha'}(p'^\alpha)^2.$$

In the same way, the variables $\{L_{\alpha\beta}|\alpha, \beta = 0, 1, \cdots, p; \alpha \neq \beta\}$ reduce to $\{L_{\alpha'\beta'}|\alpha', \beta' = 0, 1, \cdots, p-1; \alpha' \neq \beta'\}$.

The effect of the tachyon condensation on the factor $\prod_{n=1}^{\infty} [\det(Q_{(n)})]^{-1}$ is given by the limit

$$\lim_{U_{pp} \to \infty} \prod_{n=1}^{\infty} \left[ \det \left( \eta + 4\omega - F + \frac{iU}{2n} (p+1) \times (p+1) \right) \right]^{-1} = \prod_{n=1}^{\infty} \frac{2n}{iU_{pp}} \left[ \det \left( \eta + 4\omega - F + \frac{iU}{2n} p \times p \right) \right]^{-1}.$$

(29)

The $p \times p$ matrix in the right-hand side is equal to the $(p+1) \times (p+1)$ matrix in the left-hand side without its last row and last column.

Now look at the matrix $\Delta_{(m)}$. In the limit $U_{pp} \to \infty$ its last row vanishes except $\Delta_{(m)pp}$ which goes to $-1$. However, the elements of its last column, i.e. $(\Delta_{(m)})_{\alpha'p}|\alpha' \neq p$ do not vanish. In fact, the resultant matrix possesses an eigenvalue “$-1$”. Therefore, in the diagonal form, it elucidates that the direction $x^p$ has been omitted from the Neumann directions and has been added to the Dirichlet directions.

Adding all these together we observe that under the tachyon condensation the D$p$-brane loses its $x^p$-direction and reduces to a D$(p-1)$-brane. This implies that the rotation and motion of the brane do not induce a resistance against the instability, and hence collapsing of the brane takes place.

**An example**

Now for our system consider the following special case, i.e. a rotating-moving D2-brane. Thus, under the limit $U_{22} \to \infty$ we obtain

$$\lim_{U_{pp} \to \infty} \Delta_{(m)} = \begin{pmatrix}
\Delta_{(m)00} & \Delta_{(m)01} & \Delta_{(m)02} \\
\Delta_{(m)10} & \Delta_{(m)11} & \Delta_{(m)12} \\
0 & 0 & -1
\end{pmatrix}.$$

(30)
The eigenvalues of this matrix are as in the following

$$\lambda_\pm = \frac{1}{2} \left( \Delta_{(m)00} + \Delta_{(m)11} \pm \sqrt{(\Delta_{(m)00} - \Delta_{(m)11})^2 + 4\Delta_{(m)01}\Delta_{(m)10}} \right),$$

$$\lambda_0 = -1.$$  \hspace{1cm} (31)

According to the eigenvalue $-1$, after the tachyon condensation we obtain a new Dirichlet direction, i.e. $x^2$, while $x^0$ and $x^1$ remain Neumann directions. Similarly, the determinant of the matrix $(Q_{(n)})_{3 \times 3}$ reduces to

$$\lim_{U_{22} \to \infty} \det Q_{(n)} = \frac{i}{2n} U_{22} \det \begin{pmatrix} Q_{(n)00} & Q_{(n)01} \\ Q_{(n)10} & Q_{(n)11} \end{pmatrix}.$$  \hspace{1cm} (32)

The reduction also takes place for other tachyon-dependent variables. Therefore, the D2-brane reduces to a D-string along the $x^1$-direction.

5 CONCLUSIONS

In this article we studied the boundary state of a closed string, emitted (absorbed) by a rotating-moving $D_p$-brane, in the presence of the Kalb-Ramond field, a $U(1)$ gauge potential and a tachyon field. The boundary state equations reveal that in the worldvolume subspace the closed string momentum depends on its center of mass position. This fact demonstrates that the tachyon field exhibits a potential which acts on the closed string.

We deformed the boundary state by the $PSL(2, R)$ group. The modified boundary state is a useful tool when the tachyon field is presented in the system, and it is applicable for both on-shell and off-shell closed strings.

We obtained the interaction amplitude of two parallel rotating-moving $D_p$-branes. The variety of the adjustable parameters controls the treatment of the interaction. From the interaction amplitude the long-range force was also extracted. The constant overall factor of the interaction amplitude indicates the strength of the interaction. For mediating all closed string states or only the massless states, which specify the texture of the interaction, we received two different strengths, as expected. For demonstrating the importance of branes interaction, for example, we can say that branes interaction in the braneworld has been proposed as the origin of inflation [42]-[44], and an epoch in the early universe that brings into begin the radiation-dominated big-bang.

Finally, we studied the tachyon condensation on a rotating-moving $D_p$-brane via its corresponding boundary state. We observed that rotation and motion of the brane cannot
prevent it from instability. Thus, the tachyon condensation is terminated by the collapsing of the brane through reducing the brane dimension.

References

[1] J. Polchinski, “String Theory”, in two volumes. Cambridge: Cambridge University Press. (1998).

[2] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.

[3] M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Phys. Lett. B 400 (1997) 52.

[4] C.G. Callan and I.R. Klebanov, Nucl. Phys. B 465 (1996) 473.

[5] S. Gukov, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B 423 (1998) 64.

[6] M. Li, Nucl. Phys. B 460 (1996) 351.

[7] T. Kitao, N. Ohta and J.G. Zhou, Phys. Lett. B 428 (1998) 68.

[8] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys. B 308 (1988) 221.

[9] M.B. Green and P. Wai, Nucl. Phys. B 431 (1994) 131; M. Li, Nucl. Phys. B 460 (1996) 351; C. Schmidhuber, Nucl. Phys. B 467 (1996) 146.

[10] M.B. Green and M. Gutperle, Nucl. Phys. B 476 (1996) 484.

[11] M. Billo, P. Di Vecchia and D. Cangemi, Phys. Lett. B 400 (1997) 63.

[12] F. Hussain, R. Iengo and C. Nunez, Nucl. Phys. B 497 (1997) 205.

[13] O. Bergman, M. Gaberdiel and G. Lifschytz, Nucl. Phys. B 509 (1998) 194.

[14] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Nucl. Phys. B 507 (1997) 259.

[15] M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys. B 526 (1998) 199.

[16] H. Arfaei and D. Kamani, Phys. Lett. B 452 (1999) 54, arXiv: hep-th/9909167; Nucl. Phys. B 561 (1999) 57, arXiv: hep-th/9911146.
[17] B. Jensen, “$D(p)$-Branes Moving at the Speed of Light”, arXiv: hep-th/9804196.

[18] P. Di Vecchia and A. Liccardo, “$D$ branes in string theories, II”, arXiv: hep-th/9912275.

[19] H. Arfaei and D. Kamani, Phys. Lett. B 475 (2000) 39, arXiv: hep-th/9909079; D. Kamani, Mod. Phys. Lett. A 15 (2000) 1655, arXiv: hep-th/9910043.

[20] C. Bachas, Phys. Lett. B 374 (1996) 37.

[21] S.P. de Alwis, Phys. Lett. B 505 (2001) 215.

[22] P. Kraus, F. Larsen, Phys. Rev. D 63 (2001) 106004.

[23] K. Okuyama, Phys. Lett. B 499 (2001) 305.

[24] A. Sen, JHEP 0204: 048, 2002; JHEP 0405 (2004) 076.

[25] M. Naka, T. Takayanagi and T. Uesugi, JHEP 0006 (2000) 007.

[26] T. Lee, K.S. Viswanathan and Y. Yang, J. Korean Phys. Soc. 42 (2003) 34.

[27] E.T. Akhmedov, M. Laidlaw and G.W. Semenoff, JETP Lett. 77 (2003) 1-6; PismaZh. Eksp. Teor. Fiz. 77 (2003) 3-8; M. Laidlaw and G. W. Semenoff, JHEP JHEP 0311: 021, 2003; M. Laidlaw, “The Off-Shell Boundary State and Cross-Caps in the Genus Expansion of String Theory”, arXiv: hep-th/0210270; M. Laidlaw, “Tachyons, Boundary Interactions, and the Genus Expansion in String Theory”, PhD Thesis at University of British Columbia, 2003, arXiv: hep-th/0309055.

[28] S.J. Rey and S. Sugimoto, Phys. Rev. D 67 (2003) 086008.

[29] T. Okuda and S. Sugimoto, Nucl. Phys. B 647 (2002) 101.

[30] Z. Rezaei and D. Kamani, Braz. J. Phys. 41 (2011) 177, arXiv:1107.0380; Zh. Eksp. Teor. Phys. 140: 1096-1102, 2011 / [J. Exp. Theor. Phys. 113: 956-962, 2011], arXiv:1106.2097; Zh. Eksp. Teor. Phys. 141: 267-275, 2012 / [J. Exp. Theor. Phys. 114: 234-242, 2012], arXiv:1107.1183.

[31] P. Mukhopadhyay and A. Sen, JHEP 0211 (2002) 047.

[32] A. Sen, Int. J. Mod. Phys. A 14 (1999) 4061; Int. J. Mod. Phys. A 20 (2005) 5513; JHEP 9808 (1998) 010; JHEP 9808 (1998) 012; JHEP 9910 (1999) 008; JHEP 9912 (1999) 027; M. Frau, L. Gallot, A. Lerda and P. Strigazzi, Nucl. Phys. B 564 (2000) 60.
[33] E. Witten, JHEP 9812 (1998) 019.

[34] O. Bergman and M.R. Gaberdiel, Phys. Lett. B 441, (1998) 133.

[35] E. Witten, Nucl. Phys. B 268 (1986) 253.

[36] J.A. Harvey, D. Kutasov and E.J. Martinec, arXiv: hep-th/0003101.

[37] S. Dasgupta and T. Dasgupta, arXiv: hep-th/0010247.

[38] A.A. Gerasimov and S.L. Shatashvili, JHEP 0010 (2000) 034.

[39] D. Kutasov, M. Marino and G. Moore, JHEP 0010 (2000) 045.

[40] P. Kraus and F. Larsen, Phys. Rev. D 63 (2001) 106004; T. Takayanagi, S. Terashima and T. Uesugi, JHEP 0103 (2001) 019.

[41] E. Witten, Phys. Rev. D 46 (1992) 5467; Phys. Rev. D 47 (1993) 3405.

[42] A. H. Guth, Phys. Rev. D 23, 347 (1981).

[43] A. D. Linde, Phys. Lett. B 108, 389 (1982).

[44] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220(1982).