Spin-Orbit Assisted Variable-Range Hopping in Strong Magnetic Fields

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Abstract

It is shown that in the presence of strong magnetic fields, spin-orbit scattering causes a sharp increase in the effective density of states in the variable-range hopping regime when temperature decreases. This effect leads to an exponential enhancement of the conductance above its value without spin-orbit scattering. Thus an experimental study of the hopping conductivity in a fixed, large magnetic field, is a sensitive tool to explore the spin-orbit scattering parameters in the strongly localized regime.
While the effects of spin-orbit scattering in the weakly localized regime are well understood [1], much less is known on how spin-orbit scattering affects the transport in the strongly localized regime. Several different effects due to spin-orbit scattering have been suggested [2–6], with different, and sometimes contradicting predictions. All of these works rely on the measurement of the magnetoresistance, just as in the weakly localized regime, in order to explore the spin-orbit effects. However, the interplay of the different mechanisms leading to magnetoresistance in the strongly localized regime ambiguates the experimental results, leading to indefinite conclusions.

In this work we propose a different approach which will allow determination of the spin-orbit scattering parameters without relying on magnetoresistance measurements. In particular, we show that in the presence of strong magnetic field, spin-orbit scattering leads to a temperature dependence of the effective density of states, $\rho$, in the Mott variable-range hopping law, $R \sim \exp\left(\left(T_0/T\right)^{1/(d+1)}\right)$, with $T_0 \sim 1/\rho \xi^d$, and $\xi$ the localization length. Thus spin-orbit scattering will lead to an exponential change in the resistance, with a crossover temperature determined by the spin-orbit scattering and by the magnetic field.

The sensitivity of the variable-range hopping resistance to spin-orbit scattering stems from an effect, first pointed out by Kamimura and coworkers [7]. At zero magnetic field each impurity state can be either unoccupied, singly occupied or doubly occupied. Hopping processes can occur from a singly or doubly occupied state to an unoccupied or singly occupied state. However, a strong magnetic field ($g\mu H > kT\left(T_0/T\right)^{1/(d+1)}$) polarizes all singly occupied states, thus blocking all singly occupied to singly occupied hopping processes (and by detailed balance, also all doubly occupied to unoccupied ones), leading to an effectively reduced density of states, and an exponentially enhanced magnetoresistance. Such a strong positive magnetoresistance, which saturates at high fields, has indeed been observed experimentally [10–12].

Here we demonstrate that in the presence of spin-orbit scattering, the high-field effective density of states increases at low temperatures to its zero-field value. The calculation involves two steps: (a) calculation of the probability that the electron can spin-flip during its hop,
due to spin-orbit scattering, and (b) calculation of the contribution of the spin-flip hops to the resistance.

The probability of a spin-flip hop. The hopping probability is proportional to the overlap of the wavefunction the electron hops from with the one the electron hops onto, squared. As mentioned above, in strong magnetic fields, and in the absence of spin-orbit scattering, hopping from a singly occupied state to a singly occupied state involves two wavefunctions with opposite spins and consequently zero overlap. In the presence of spin-orbit scattering, each wavefunction will acquire a component in the opposite spin direction. Assuming a short range spin-orbit scattering, \( V_{so}(r) = U_{so} \delta(r - r_i) \), where \( r_i \) is the position of the scatterer and \( U_{so} \) its strength, the amplitude of the opposite-spin component, say \( A_{\downarrow} \), due to that single scatterer, is given by first order perturbation theory,

\[
A_{\downarrow} = \frac{U_{so}}{g\mu H} \frac{1}{r_i^2} e^{-2r_i/\xi}. \tag{1}
\]

Averaging Eq.(1) over all possible positions of the spin-orbit scatterers, we find

\[
A_{\downarrow} = \frac{U_{so} n_{so}}{g\mu H} \equiv \frac{H_{so}}{H}, \tag{2}
\]

where \( n_{so} \) is the density of spin-orbit scatterers, and where trivial numerical factors have been omitted. Note that the dominating spin-orbit scattering occurs within the localization length, and so we expect \( H_{so} \) to be similar to its weak-localization value \( H \). The spin-flip hopping probability, \( P_{so} \), will thus be proportional to \( (H_{so}/H)^2 \).

The calculation of the resistance. We follow here the approach of Ambegaokar, Halperin and Langer [13]. According to Miller and Abrahams [14] the resistance can be calculated by solving the equivalent random-resistor network, where each pair of impurity states is connected by a classical resistor, \( R = R_0 e^{\Delta \epsilon/kT+2r/\xi} \), where \( \Delta \epsilon \) is the difference in energy between the states, \( r \) is the distance between the impurities, and \( R_0 \) is some microscopic resistance. In the following all resistances will be in units of \( R_0 \). Ambegaokar et al. [13] suggested that due to the exponential spread in the values of the resistors, the resistance of the network will be determined by the lowest resistance, \( R \), such that the network composed
of all resistors with resistances smaller than \( R \), percolates. Since all such resistors have to obey \( |\Delta \epsilon| < kT \log R \), and \( r < (\xi/2) \log R \), the percolation criterion takes the form

\[
z_d = 2\rho_0 kT \log R (\xi \log R/2)^d ,
\]

with \( z_d \) the critical density in \( d \) dimensions (\( z_2 \approx 6.9 \) and \( z_3 \approx 5.3 \) [13]), leading to the Mott hopping law

\[
(\log R)^{d+1} = 2^{d-1}z_d \frac{kT}{\rho_0 \xi^d} = \frac{T_0}{T}.
\]

As mentioned above, in strong magnetic field and without spin-orbit scattering, the network separates into two subnetworks (denoted \( A \) and \( B \) in Fig. 1). An electron can hop from a singly occupied site onto an unoccupied one (type \( A \)), which then becomes singly occupied, and then hops onto an unoccupied site, and so forth. Or the electron can hop from doubly occupied sites to singly occupied ones (type \( B \)). As can be seen from Fig.1, the density of the type \( A \) impurities is determined by the density of states around the Fermi energy, while that of type \( B \) impurities is determined by the density of states at energy \( U \) away from the Fermi energy, where \( U \) is the intra-impurity, Hubbard-like repulsion between the electrons. If the density of states is constant on an energy scale \( U \), then the effect of the magnetic field is to replace \( \rho_0 \) in Eq.(4) by \( \rho_0/2 \), leading to the doubling of \( T_0 \) and an exponentially enhanced resistance.

In the presence of spin-orbit scattering impurity states with opposite spin are also connected, with resistance \( R = e^{\Delta\epsilon/kT+2r/\xi}/P_{so} \). This will lead, in the case of constant density of states, to a modified percolation criterion,

\[
(\log R)^{d+1} + [\log(RP_{so})]^{d+1} = \frac{2T_0}{T}.
\]

As long as \( R < 1/P_{so} \), the solution of (1) will yield a smaller resistance than the solution of (3). Thus, at high temperatures spin-orbit scattering will not play any role, as the resistors involving spin-flips will not participate in the percolating cluster. These will start to contribute as the temperature is lowered so that \( R > 1/P_{so} \), and at small enough temperature
Eq. (5) has a solution similar to (4), with $\rho_0/2$ replaced by $\rho_0$, namely all types of hopping processes contribute to the conductance. Thus we expect the effective $T_0$ to be temperature dependent, with an effective density of states $\rho_0/2$ at high temperatures and saturating at a value corresponding to an effective density of states $\rho_0$ at low temperatures. The crossover temperature is determined by $R = 1/P_{so}$, so it depends (see Eq. (2)) on the spin-orbit scattering and on magnetic field. Eqs. (4) and (5) can be solved exactly in 2 and 3 dimensions, and one finds that $T_0^{\text{eff}}(T) \equiv \left[ d(\log R)/(1/T^{1/d+1}) \right]^{d+1}$, is given by

$$
\frac{T_0^{\text{eff}}}{T_0} = \begin{cases} 
\frac{8}{h_2^3(4 + h_2)} \frac{[x^{2/3} + (4 + h_2)^2/3]^3}{2d} & \text{for } 2d \\
\frac{8}{(h_3 - 3\sqrt{x}/2\sqrt{2})^2h_3} \frac{[\log(16 + x^2)]^{3/2}}{3d} & \text{for } 3d
\end{cases}
$$

with $x = |\log P_{so}|^{d+1}T/T_0$, $h_2 = \sqrt{16 + x^2}$ and $h_3 = \sqrt{x + 2}$. These expressions are valid for $\log R > |\log P_{so}|$. For higher temperatures $T_0^{\text{eff}}$ is given by its high temperature value, $2T_0$.

When the density of states is not constant, one has first to calculate the probabilities, $p_A$ and $p_B$, of an impurity of type $A$ or $B$, respectively, to belong to the percolating cluster. To see that these probabilities are not given by the relative densities, $\rho_A$ and $\rho_B$, respectively, one can look at the strong field limit. In this case, since the two subnetworks do not communicate, the percolating network will consist only of one type of impurities, that with the larger density. So if $\rho_A > \rho_B$, then $p_A = 1$ and $p_B = 0$ independent of the values of $\rho_A$ and $\rho_B$.

In order to derive an equation for $p_A$, we realize that in order for an impurity to belong to the infinite cluster, it has to be connected to another impurity on that cluster. This latter impurity can be either of type $A$ or type $B$, and the resistor between these two impurities has to satisfy the condition (3). Thus we find the self-consistent equation,

$$
p_A \rho_B \rho = p_A \rho_A (\log R)^{d+1} + p_B \rho_B (\log (RP_{so}))^{d+1} + p_B (\log R)^{d+1},
$$

(7)
Eq. (5) for the resistance becomes

\[
1 = \frac{T}{T_0} \left\{ (p_A \rho_A + p_B \rho_B)(\log R)^{d+1} + (p_A \rho_B + p_B \rho_A)[\log(RP_{so})]^{2d+2} \right\}. \tag{8}
\]

Combining Eqs. (7) and (8), we arrive at the final equation for the resistance,

\[
1 = \frac{T}{2T_0} \left\{ (\log R)^{d+1} + \sqrt{(\log R)^{2d+2}(\rho_A - \rho_B)^2 + 4\rho_A \rho_B[\log(RP_{so})]^{2d+2}} \right\}. \tag{9}
\]

In Fig. 2 we plot $T_0^{\text{eff}}/T_0$, for $\rho_A = \rho_B$, and for $\rho_A = 2\rho_B$. Indeed, we see that the effective $T_0$ reduces from its high-temperature value ($T_0/\max\{\rho_A, \rho_B\}$) to $T_0$, due to the increase in the effective density of states. The transition starts to occur when $\log R = 1/\log P_{so}$, and approaches smoothly its low temperature limit.

To conclude, we have made detailed predictions how to determine the spin-orbit scattering parameters in the strongly localized regime. The suggested measurements are performed in a constant magnetic field, and avoid the complications arising from the various contributions to the resistance as the magnetic field changes. We predict that the effective density of states, at large magnetic fields, will start to deviate from its high temperature value at temperature given by $T = T_0/|\log P_{so}|^{d+1}$, where $P_{so}$ can be controlled both by the strength of spin-orbit scatterers and their density, and by the magnetic field. We predict that the effective density of states will approach at low temperatures a value, about twice its high temperature value, depending on the uniformity of the density of states. These changes in the effective density of states can be easily probed by the changes in the exponent in the Mott variable-range resistance.
Figure Captions

1. Schematic picture of the impurity states in strong magnetic fields. Due to the polarization of the singly occupied states, only hopping processes (denoted by arrows) between impurities of type $A$, or between impurities of type $B$, are allowed in the absence of spin-flip process, leading to a reduction in the effective density of states.

2. The effective $T_0$, appearing in the Mott variable-range hopping formula. $T_0^{\text{eff}}$ changes from its high temperature value ($T_0 / \max\{\rho_A, \rho_B\}$) to its low temperature value, $T_0$, due to the increasing relevance of spin-flip processes at low temperature, leading to an increase of the effective density of states.
REFERENCES

[1] For a review, see G. Bergmann, Phys. Rep. 107, 1 (1984); B. L. Altshuler, A. G. Aronov, M. E. Gershenzon, and Yu. V. Sharvin in Physics Reviews, ed. by I. M. Khala’tnikov (Harwood Academic Publishers, Switzerland, 1987), p. 225.

[2] Y. Meir, N. S. Wingreen, O. Entin-Wohlman and B. L. Altshuler, Phys. Rev. Lett. 63, 798 (1989); Y. Meir and O. Entin-Wohlman, Phys. Rev. Lett. 70, 1988 (1993).

[3] Y. Shapir and Z. Ovadyahu, Phys. Rev. B 40, 12441 (1989).

[4] J.-L. Pichard, M. Sanquer, K. Slevin and P. Debray, Phys. Rev. Lett. 65, 1812 (1990).

[5] E. Medina and M. Kardar, Phys. Rev. Lett. 66, 3187 (1991); E. Medina and M. Kardar, Phys. Rev. B 46, 9984 (1992).

[6] M. Eto, Phys. Rev. B 51, 13066 (1995).

[7] A. Kurobe and H. Kamimura, J. Phys. Soc. Jpn 51, 1904 (1982); H. Kamimura, Prog. Theor. Phys. 72, 206 (1982); H. Kamimura, A. Kurobe and T. Takemori, Physica 117 & 118 B+C, 652 (1983); A. Kurobe, J. Phys. C 19, 2201 (1986).

[8] K. A. Matveev et al., Phys. Rev. B 52, 5289 (1995).

[9] Y. Meir, Europhys. Lett. 33, 471 (1996).

[10] W. Jiang, J. L. Peng, J. J. Hamilton and R. L. Greene, Phys. Rev. B 49, 690 (1994).

[11] A. Frydman and Z. Ovadyahu, Sol. State Somm. 94, 745 (1995); and to be published.

[12] L. Essaleh et al., Phys. Rev. B 52, 7798 (1995).

[13] V. Ambegaokar, B. I. Halperin and and J. S. Langer, Phys. Rev. B 4, 2612 (1971); See also M. Pollak, J. Non-Crystal. Solids 11, 1 (1972); B. I. Shklovskii, Sov. Phys. JETP 34, 1084 (1972).

[14] A. Miller and E. Abrahams, Phys. Rev. 120, 745 (1960); for more details, see B. I.
Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, Berlin, 1984).

[15] A. S. Skal and B. I. Shklovskii, *Sov. Phys. JETP* **16**, 1190 (1971).
$T_0^{\text{eff}}$ vs $(T_0/T)^{1/d+1}$

- Solid line: $2d$, $\rho_A = \rho_B$
- Dashed line: $2d$, $\rho_A = 2\rho_B$
- Dash-dotted line: $3d$, $\rho_A = \rho_B$
- Dotted line: $3d$, $\rho_A = 2\rho_B$