Spectator theory for three-nucleon electromagnetic currents

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Abstract. In this talk I present some of the more recent developments within the Covariant Spectator Theory. My focus will be on aspects of the derivation of gauge invariant electromagnetic three-nucleon currents which are consistent with the hadronic equations and with the basic assumptions of this framework. Important dynamical ingredients of the three-nucleon currents are also discussed, namely the three-nucleon bound state vertex function and the two-nucleon interaction model they are derived from.

THE COVARIANT SPECTATOR THEORY

The Covariant Spectator Theory (CST) was introduced already several decades ago by Franz Gross [1]. Its purpose is to include relativity in a manifestly covariant way in few-body problems, while keeping their complexity at a level that is still manageable for numerical calculations. It has been already applied successfully to the description of a variety of systems, including the deuteron and nucleon-nucleon (NN) scattering [2, 3], elastic and inelastic electron scattering off the deuteron [4, 5, 6], and the 3N bound state [7, 8].

The CST is based on Relativistic Quantum Field Theory, where an exact and complete scattering amplitude can be expressed in terms of the Bethe-Salpeter equation (BSE) [9]. The Spectator Equation (SE) is a particular re-organization of the complete BSE for a system of heavy particles (nucleons) that interact via the exchange of lighter particles (mesons). Ignoring vertex and self-energy corrections as well as genuine many-body forces, the scattering amplitude is given as an infinite sum of ladder and crossed ladder diagrams of all orders in the coupling constants. It can be generated by an integral equation if the propagators and the irreducible kernel are chosen in a suitable way.

While the BSE includes the full off-shell propagators for all heavy particles, the SE restricts all but one of them to their mass shells. Consequently, contributions to the scattering amplitude which are generated by one equation through iterations may appear as part of the irreducible kernel of the other. When their respective kernels are truncated—which in practical calculations is unavoidable since the complete kernels themselves contain already an infinite number of diagrams—the BSE and the SE are no longer equivalent. It has been shown that the truncated SE, in certain circumstances, converges faster to the full result than the truncated BSE [10]. The simplest and most often used cases are truncations at lowest order, leading to the so-called ladder approximation since crossed-ladder graphs are thereby excluded.
FIGURE 1. Two-nucleon SE (upper panel) with antisymmetrized kernel (lower panel). The symbol “×” on a nucleon line indicates that the particle is restricted to its positive-energy mass shell.

The SE has sometimes been misunderstood as an approximation to the BSE. This point of view is somewhat misleading, since there is really nothing more fundamental about the BSE than the SE (or other so-called quasi-potential equations). It is less confusing to understand the SE as an equation in its own right. It is meant to replace the Schrödinger equation in situations where relativity is important, rather than to approximate a truncated BSE.

Some of the important features of the SE are: it is manifestly covariant although the loop integrations are three-dimensional, all boosts are kinematic (i.e., interaction independent), the off-shell particle has negative-energy components, and cluster separability holds. The latter property guarantees that the solution of the two-body SE is consistent with the one that appears as dynamical input in the three-body SE.

TWO-NUCLEON SCATTERING

The SE for $NN$ scattering is shown in diagrammatic form in Fig. 1. Its antisymmetrized kernel (or “potential”) is truncated to the lowest-order ladder terms, such that it is of one-boson exchange (OBE) form.

The first realistic OBE $NN$ potentials for the SE were constructed by Gross, Van Orden, and Holinde [3]. They are based on the exchange of six different mesons, two pseudoscalar ($\pi$ and $\eta$), two scalar ($\sigma$ and $\delta$), and two vector mesons ($\rho$ and $\omega$).

The potentials have meson-nucleon vertices of the following form:

$$g_s A_s = g_s \left[ 1 + \frac{V_s}{2m} (p' - m + p - m) + \frac{K_s}{4m^2} (p' - m)(p - m) \right],$$

for scalars,

$$g_p A_p = \frac{1}{2} \left[ \frac{V_p}{2m} (p' - m)p - m) \frac{K_p}{4m^2} (p' - m)(p - m) \right]$$

for pseudoscalars, and

$$g_v A_\mu^v = g_v \left[ \gamma^\mu + \frac{K_v}{2m} i\sigma^{\mu\nu} q_v + \frac{K_v (1 - \lambda_v)}{2m} [(p' - m)\gamma^\mu + \gamma^\mu (p - m)] + \cdots \right]$$

for vector mesons. Here, $p$ and $p'$ are the nucleon four-momenta, and $q$ is the meson four-momentum at the meson-$NN$ vertex. Furthermore, $m$ is the nucleon mass, the $g$'s...
are coupling constants, $\kappa_v$ is the usual $f/g$ ratio for vector mesons, $\nu_s$, $\nu_p$, $\kappa_s$, $\kappa_p$, and $\lambda_v$ are off-shell coupling constants. The vertices are regularized by form factors, one for each particle at the vertex, which are parametrized by cut-off masses.

While $\nu_p$ has appeared before as mixing parameter in potentials that allow a mixing of pseudoscalar and pseudovector $\pi NN$ coupling, the scalar off-shell coupling proportional to $\nu_s$ is a new feature not explored previously in $NN$ scattering. It contributes only if at least one of the nucleons at the vertex is off mass shell. In frameworks that do not contain off-shell nucleons, such as in relativistic hamiltonian dynamics or in nonrelativistic quantum mechanics, these couplings vanish.

The terms proportional to $\kappa_s$ and $\kappa_p$ contribute only if both nucleons at the vertex are off mass shell. With their inclusion, Eqs. (1) and (2) represent the most general coupling of nucleons to scalar or pseudoscalar mesons, respectively. However, in the potentials fitted so far we have always set $\kappa_s = \kappa_p = 0$.

THE THREE-NUCLEON BOUND STATE

The SE for the 3$N$ bound state has been derived [8] in complete consistency with the CST of $NN$ scattering. This is achieved by restricting in all intermediate states spectator particles to their positive-energy mass shell (hence the name “spectator theory”). This simple prescription automatically insures that in any intermediate state only one nucleon is off mass shell. As a consequence, loop integrations are again reduced from four to three dimensions, the same as in the nonrelativistic case.

Figure 2 displays the homogeneous integral equation for the 3$N$ vertex function in graphical form. It is given here in its general form, valid both for identical fermions ($\zeta = -1$) and bosons ($\zeta = +1$).

The numerical solution of the Faddeev-type 3$N$ SE was performed [7] in a basis of partial wave helicity states. The propagator of the off-shell nucleon is decomposed into positive and negative energy components, or $\rho$-spin states, which also enter into the classification of the basis states.

The 3$N$ SE was solved for a family of $NN$ potentials, which were constructed by fixing the scalar off-shell coupling parameters to a particular value and fitting the remaining parameters to the energy-dependent Nijmegen $np$ phase shift analysis [11] of 1993. Then the $\chi^2$ to $NN$ databases is calculated from the potentials in a second step, using Arndt’s code SAID [12]. While the off-shell couplings for the $\sigma$ and $\delta$ mesons are in principle independent, initially we related them in terms of a common scaling parameter $\nu$, such that $\nu_{\sigma} = 2.6\nu$ and $\nu_{\delta} = -0.75\nu$ (a result obtained in preliminary independent fits). The
parameters of some of these potentials can be found in Ref. [13].

It turned out that potentials with nonzero values of $\nu$ have a lower $\chi^2$ and are therefore preferred by the fits. The potential named W16 has the minimum value, $\chi^2 = 1.895$, for $\nu = 1.6$, which corresponds to $\nu_\sigma = 4.16$ and $\nu_\delta = -1.2$. The triton binding energy varies rather strongly with $\nu$. The best model in terms of $\chi^2$, W16, yields with $E_t = -8.491$ MeV also the binding energy closest to the experimental value of $-8.48$ MeV.

This family of $NN$ potentials was designed principally to study the scalar off-shell coupling and other relativistic effects in the $3N$ bound state. The number of adjustable parameters was—with only 13—kept comparatively small. We are now in the process of further improving the fit to the $NN$ data by allowing more potential parameters to vary independently. As an example I mention here the (preliminary) model WJL19-1.1 with 19 free parameters. Among other new features, it has different masses for the charged and neutral pions, and its scalar off-shell coupling parameters, $\nu_\sigma = -6.59$ and $\nu_\delta = 2.67$, were varied independently. With a much improved $\chi^2 = 1.254$ it demonstrates that the potentials of the CST are comparable in quality with the best nonrelativistic potentials. They are, however, true meson-exchange potentials in the sense that their parameters are the same in all partial waves or isospin channels. The model WJL19-1.1 yields a somewhat higher triton binding energy of $E_t = -9.116$ MeV. This value is likely to change as the fit is being finalized.

We also calculated the $3N$ bound state in the nonrelativistic limit, using a potential fitted especially for this case. This calculation yields a $3N$ binding energy of $-7.914$ MeV. One can use this case also to estimate the importance of relativistic boost effects, which here turned out to be repulsive but relatively small.

**SPECTATOR THEORY OF ELECTROMAGNETIC THREE-NUCLEON CURRENTS**

For the calculation of elastic and inelastic electron scattering from the $3N$ bound state a conserved electromagnetic $3N$ current has to be derived within the CST. As we will see, special care is needed to avoid double counting and to arrive at expressions consistent with the basic assumptions of the spectator formalism.

In the one-photon approximation, a gauge invariant current is obtained if all Feynman diagrams are summed in which the photon is attached to all propagators and to all vertices with momentum-dependent couplings. This is a clear prescription for any given Feynman diagram. However, if we are dealing with an infinite sum of diagrams, defined through an integral equation, a more general procedure is needed that systematically attaches the photon in all the right places.

Unfortunately, one cannot simply use results from the nonrelativistic theory as guidance. It is instructive to consider the elastic current in impulz approximation for the BSE [14]. Figure 3 shows that diagram 3(B), which one might naively identify with the impulse approximation in analogy with the nonrelativistic case, includes some contributions twice, for instance diagram 3(d).
FIGURE 3. An example for the overcounting problem in the BSE. The upper part displays the decomposition of the full vertex function into components with a given particle, indicated by a dot, as spectator during the last $NN$ interaction in the final state. The lower part shows how certain contributions to the impulse approximation current appear twice if one simply takes the “obvious” diagram (B) from nonrelativistic theory.

The origin of this double-counting problem can be traced back to the fact that the two-body amplitude in diagram 3(d) can be “pulled out”, by applying the 3$N$ equation, from the vertex function in the initial as well as in the final state. In contrast to nonrelativistic diagrams, in a Feynman diagram it makes no difference if this two-body amplitude is extracted from the right or left side of the diagram, since it can “slide freely” past the point were the photon is attached to the third nucleon. The double-counting problem in covariant scattering theories has been known for a long time, especially in the context of simple systems of nucleons and pions [15, 16].

A method to generate systematically the infinite series of diagrams that represents the current, and avoids double counting, was introduced by Kvinikhidze and Blankleider [17]. They call it the “gauging of integral equations method.” It is based on the observation that coupling a photon to an amplitude given by an integral equation satisfies the same distributive rule as the differentiation of a product. For instance, if

$$\Gamma = 2MGP\Gamma$$  \hspace{1cm} (4)

is the Faddeev equation for the vertex function $\Gamma$, with $M$ being the $NN$ amplitude, $G$
Kvinikhidze and Blankleider applied their gauging method also to the 3N SE [18]. However, their result contained diagrams in which the spectator particle appeared off mass shell, clearly inconsistent with the assumptions of the CST. The situation was even more confusing since Adam and Van Orden [19] worked out an alternative derivation of the 3N current, using a purely algebraic technique, and obtained a result in which all spectators are on mass shell. This apparent contradiction was resolved when it was shown that the two results are in fact equivalent [20].

This equivalence is illustrated in Fig. 4. Diagram (A) is part of diagram (B), which has one spectator off mass shell. A closer inspection shows that (A) is actually identical to (A’) which in turn contributes to (B’) where all spectators are on mass shell. Applying this idea one can replace all diagrams of type (B), such as the ones appearing in the current of Kvinikhidze and Blankleider, by others satisfying the spectator-on-mass-shell constraint.

We derived the gauge invariant electromagnetic 3N current for the cases of elastic and inelastic scattering from the 3N bound state within the spectator theory in a diagrammatic approach [20]. The “core diagrams” of Fig. 5 play a central role since they contribute in both cases. In fact, elastic scattering (Fig. 6) is completely determined through the core diagrams. The first 6 diagrams of Fig. 6 comprise the Complete Impulse Approximation (CIA) which is gauge invariant by itself. The remaining 4 diagrams involve the photon coupling to the interacting NN pair and belong therefore to the interaction current.

In the case of electrodisintegration a number of diagrams need to be added in which the photon couples to the outgoing nucleons. In Fig. 7, diagrams (a1) and (a2) represent
FIGURE 7. The electromagnetic 3N breakup spectator current.

the relativistic impuls approximation, (b) contains the final state interactions, and (c) and (d) are interaction currents.

The derived currents will be used to calculate the elastic form factors of the 3N bound state as well as its electrodisintegration. These calculations are currently in progress.

ACKNOWLEDGMENTS

This work was performed in collaboration with Franz Gross (Jefferson Laboratory) and Teresa Peña (IST Lisbon). It was supported by FEDER and the Fundação para a Ciência e a Tecnologia under grant POCTI/FNU/40834/2001.

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