Superluminal propagation of light in gravitational field and non-causal signals: some comments

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Abstract

We examine the recent statement by A. Dolgov and I. Novikov that superluminal propagation of light, which is induced by the quantum corrections to the photon propagation in gravitational field, permits non-causal signal propagation. For this purpose we examine the possibility of the existence of non-causal signals in the model case, when characteristic cone of the signal is outside the usual light cone in Minkowski space-time, as it take place in the case of the photon propagation in gravitation field with quantum corrections taking into account. In such model the signal propagates along invariant intervals. It is shown that there are no non-causal signals in the considered model. The recent statement of A. Dolgov and I. Novikov was obtained by using additional supposition that velocity of the signal with respect to the emitter is independent from its motion. Such supposition does not valid in the case of the photon propagation in gravitational field.

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1 Introduction

The recent studies of photon propagation in gravitational field show that quantum corrections modify the characteristic cone of electromagnetic field equations in such a way that in some cases it may lay outside the light cone \[1, 2, 3, 4, 5, 6\]. This effect follows from the photon effective action in gravitational field in one loop approximation, which has the following form \[2, 4, 5\]

\[
S = \int d^4x \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha}{m_e^2} \left( a R F_{\mu\nu} F^{\mu\nu} + b R_{\mu\nu} F^{\mu\lambda} F^{\nu}_{\lambda} + c R_{\alpha\beta\mu\nu} F^{\mu\nu} F^{\alpha\beta} \right) \right) \tag{1}
\]

Here \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic field tensor, \( \alpha = 1/137 \) is the fine structure constant, \( m_e \) is the mass of electron and \( a, b \) and \( c \) are coefficients, whose exact values are calculated in \[1, 2, 4, 5\]. For our purpose it is essential that these coefficients are nonzero. The terms, which are proportional to the Ricci tensor \( R_{\mu\nu} \) or to the curvature scalar \( R \) vanish in vacuum. It is evident that the action (1) leads to the field equations whose characteristic cone does not coincide with the normal light cone in general case. It means that velocity of the front of the signal would not coincide with the normal light velocity \( c \). Geometrically it means that instead of the normal isotropic interval in normal metric

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{2}
\]

photon propagate along interval whose line element \( ds^2_{ph} \) may be positive, negative or zero depending from direction of propagation, polarization etc. The case \( ds^2_{ph} < 0 \) corresponds to faster than light photon propagation \[\footnote{2} \] (examples see in \[1, 2, 3, 4, 5, 6\]). In some cases such nonisotropic photon propagation in normal metric may be described as propagation along isotropic direction in modified metric

\[
ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu \tag{3}
\]

where \( \tilde{g}_{\mu\nu} \) is effective metric tensor.

The most important that photon propagation is determined by general covariant equations and hence it propagate along invariant curve, which may be space-like in normal metric \[4\] and does not depend on the emitter motion. The possibility to describe photon propagation by means of isotropic curves in modified metric \[3\] is not essential.

In the standard Special Relativity it is usually expected (see, for instance \[7, 8\]), that ”time travel and faster than light space travel are closely connected” and ”if you can do one you can do another”. In few words such connection is explained as follows \[8\]: ”You just have to travel from \( A \) to \( B \) faster than light would normally take. You then travel back, again faster than light, but in a different Lorentz frame. You can arrive back before you left” (in more details this discussed in \[7\]). Therefore, the faster than light photon propagation ”may imply serious problems for the theory” \[\footnote{4} \].

In this note we examine the possibility of the appearing of non-causal signals, which declared \[7\], in the model case than the superluminal propagation of signal occurs in flat
space-time. As in the case of the superluminal photon propagation in gravitational field, it will be assumed that the superluminal signal propagate along isotropic line in some effective metric, which is not coincide with normal Lorentzian metric. It will be shown that the non-causal signal propagation in this case is impossible.

2 Superluminal signals in Special Relativity

Consider the superlight signal propagation in Special Relativity. Namely, consider the usual Minkowski \((t, x)\)-plane with interval

\[
 ds^2 = dt^2 - dx^2
\]

and suppose that it is possible to send superluminal signals, whose interval has the form

\[
 ds^2 = u^2 dt^2 - dx^2 = 0
\]

where \(u > 1\). Such supposition may be considered as a simplest model for superluminal photon propagation due to the quantum corrections.

Let such superluminal signal is emitted at the point \(x_0 = 0\) at the moment \(t_0 = 0\) (the event \(p_0\)), then it is detected at the point \(x_1\) at the moment \(t_1\) and immediately re-emitted in reverse direction (the event \(p_1\)) and finally is detected at the initial point \(x_0\) at some moment \(t_2\) (the event \(p_2\)) as it is shown at figure 1, where the thin lines \(AA'\) and \(BB'\) represent the normal light cone for the Minkowski metric (4), the thick lines represent the tachyonic cone (isotropic cone in modified metric (5)), the arrows define the directions of the superluminal signal propagation and \(u = 2\). It is clear, that intervals \(p_0p_1\), \(p_1p_2\) and \(p_0p_2\) are invariants.

![Figure 1: Suprlight photon propagation in Minkowski space-time](image-url)
with respect to arbitrary coordinate transformations. In particular, let us make Lorentz transformation

\[ x' = \beta (x - Vt), \quad t' = \beta (t - Vx) \]  

(6)

where \( \beta = 1/\sqrt{1-V^2} \). The interval (4) is invariant under this transformations, while the interval (5) takes the form

\[ ds^2 = \beta^2 (adt'^2 - 2bdt'dx' - cdx'^2) \]  

(7)

where \( a = (u^2 - V^2) \), \( b = (1 - u^2)V \) and \( c = (1 - u^2V^2) \). In this moving reference frame superluminal signal propagates with velocities

\[ u'_+ = \frac{u - V}{1 - uV} \]  

(8)

in positive direction of the \( x' \)-axis (the possibility \( u'_+ < 0 \) means that \( t'_1 < t'_0 \)) and

\[ u'_- = \frac{-u + V}{1 + uV} \]  

(9)

in negative direction of \( x' \)-axis. Thus, in the moving reference frame the velocity of the signal propagation not only changes its value, but become asymmetric with respect to space reflection.

It is clear, that equations (8) and (9) are the direct consequence of the velocity-addition formula, which is well known from special relativity. So, our description of superluminal signal propagation by means of isotropic curves in modified metric (5) is equivalent to the description of superluminal signal propagation by means of invariant space-like curves in normal metric (6).

In particular, for \( V = 1/u \) equations (8), (9) give \( u'_+ = \infty \) and \( u'_- = - (1 + u^2)/2u \), the events \( p_0 \) and \( p_1 \) become simultaneous, i.e. \( t'_1 = t'_0 \) but in both reference systems \((x, t)\) and \((x', t')\) signal propagates from \( p_0 \) to \( p_1 \) and from \( p_1 \) to \( p_2 \) but not reverse (see figure 2). In particular, if reference system \((x'', t'')\) moves with respect to the system \((x, t)\) with velocity \( V = -1/u \) than in the reference system \((x'', t'')\) the simultaneous events are \( p_1 \) and \( p_2 \) and the signal propagates from \( p_0 \) to \( p_1 \) and from \( p_1 \) to \( p_2 \) but not reverse. It is easy to see that there is no reference frame where the signal propagates from \( p_1 \) to \( p_0 \) or from \( p_2 \) to \( p_1 \). So, the appearance of non-causal signals in the considered model is impossible.

The generalization of the considered model to the superluminal photon propagation in gravitational field, is straightforward and will not considered here.

### 3 Conclusion

Thus it is shown that if the superluminal signal propagate along invariant space-like interval (or along isotropic interval in modified metric), then the non-causal signal propagation is impossible. So, the superluminal propagation of light in gravitation field, which was studied by several authors \[1, 2, 3, 4, 5, 6\], does not lead to the appearance of non-causal signals.

The ”standard” conclusions (see, for instance, \[7, 8\]) about connection between faster than light space travel and time travel, are made in supposition that the superluminal signal
Figure 2: Suprlight photon propagation in Minkowski space-time in moving reference frame

has the same velocity $u$ independent from the source motion (in particular, in \cite{7} we find: "Let us suppose for simplicity that the picture is symmetric so that the second source/detector emits a tachyon with the same velocity $u$ with respect to itself ..."). It is easy to see that this supposition, which is natural in Newtonian mechanics, can not be applied to the generally relativistic case of superluminal photon propagation, which induced by quantum corrections in gravitational field. In particular, as it was shown in the simplest model of superluminal signal propagation in Minkowski space-time, if the equations of the signal propagation are generally covariant, as in the case of superluminal photon propagation in gravitational field, than the space velocity of the signal satisfies to the well known velocity-addition formula. As a result, in the moving reference system velocity of signal changes by such a way that non-causal signals are impossible.

Some remarks about the above model of superluminal photon propagation are necessary. First, this model demonstrates the principle possibility of coexistence of several non-equivalent metric (more exact causal) structures in the same space-time. In another context such possibility was supposed in \cite{9}. Some cosmological implications of such possibility were discussed in \cite{10} and the possible geometric description of such situation and some its implications, in particular, the Lorentz invariance of ”standard” fields and appearance of ”dark” matter, were considered in \cite{11, 12}. Second, the conclusions about superluminal photon propagation in gravitational field \cite{1, 2, 3, 4, 5, 6} was obtained in the test field approximation when the gravitational field is given and independent from the electromagnetic one. It is not strange that characteristic cones of electromagnetic and gravitational field equations are not coincide. If the self-consistent system of gravitation and electromagnetic fields would be considered with quantum correction to the photon propagation taking into account than both gravitational and electromagnetic fields would have the same characteristic cone and by this reason the electromagnetic signal will propagate with the same velocity as gravitational
Nevertheless, it is not necessary that in such self-consistent mode the gravitational and electromagnetic signals would propagate along isotropic cone in the metric which solves modified Einstein-Maxwell equations. The detailed investigation of this problem requires rather cumbersome calculations and separate consideration.

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