Critical fluctuations in a dynamically expanding heavy-ion collision

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Abstract

For the discovery of the QCD critical point it is crucial to develop dynamical models of the fluctuations of the net-baryon number that can be embedded in simulations of heavy-ion collisions. In this proceeding, we study the dynamical formation of the critical fluctuations of the net-baryon number near the QCD critical point and their survival in the late stages in an expanding system. The stochastic diffusion equation with a non-linear free energy functional is employed for describing the evolution of conserved-charge fluctuations along trajectories in the crossover and first-order transition regions near the QCD critical point.

Keywords: fluctuations, critical phenomena, QCD phase diagram, beam-energy scan

1. Introduction

The phase diagram of QCD in the temperature ($T$) and baryon chemical potential ($\mu_B$) plane is believed to have a QCD critical point and a first-order phase transition line. Experimental search for these structures is one of the most challenging subjects that will be realized by the relativistic heavy-ion collisions [1, 2]. Active experimental analyses for this purpose are ongoing in the beam-energy scan program at RHIC [3, 4]. Future experimental programs, FAIR, NICA, and J-PARC-HI, will also contribute to this project.

Fluctuations are important experimental observables in the search for the phase structure of QCD [1, 2]. In equilibrated media the fluctuations diverge at the second-order phase transition associated with the divergence of the correlation length. It is also known that the higher-order cumulants characterizing non-Gaussianity of fluctuations have a sharper enhancement and characteristic sign changes near the critical point. The experimental search for these behaviors in the fluctuation observables has been very active at RHIC [3].

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In heavy-ion collisions, however, the hot and dense systems created by the collisions are dynamically expanding. Therefore, the non-equilibrium dynamics of fluctuations plays a crucial role when the fluctuations are used for the study of the QCD phase diagram [5, 6]. In the present work, we focus on the fluctuations of conserved charges and study their dynamical evolution within a Bjorken expansion by employing the stochastic diffusion equation (SDE) [6]. By solving this equation numerically, we explore the effects of the non-equilibrium dynamics on the evolution along trajectories in the crossover [7] and first-order transition [8] regions near the critical point.

2. Stochastic diffusion equation

In this study, we focus on the evolution of conserved-charge fluctuations in dynamically expanding systems. The long-wavelength behavior of the fluctuations of a conserved charge $n(z, t)$ is well described by the stochastic diffusion equation (SDE) [6]

$$\partial_t n(z, t) = \Gamma \frac{\delta F[n]}{\delta n} - \partial_z \xi(z, t),$$

with the free energy functional $F[n]$ and the noise term $\xi(z, t)$ satisfying $\langle \xi(z, t) \rangle = 0$ and

$$\langle \xi(z_1, t_1) \xi(z_2, t_2) \rangle = 2\Gamma T \delta(z_1 - z_2) \delta(t_1 - t_2).$$

By substituting a quadratic form of the free energy functional, $F[n] = (1/2) \int dz m^2(n(z, t))^2$, into Eq. (1), we obtain the conventional form of the SDE

$$\partial_t n(z, t) = D \partial^2_z n(z, t) - \partial_z \xi(z, t),$$

with the diffusion coefficient $D = \Gamma m^2$. In Ref. [5], the evolution of the Gaussian fluctuations of $n(z, t)$ has been discussed by solving Eq. (3) analytically. In this model, however, all non-Gaussian cumulants \(\langle n^m \rangle_c\), with \(m \geq 3\) vanish in equilibrium and this property is not suitable for describing their dynamical evolution.

To describe the evolution of non-Gaussian fluctuations, one has to introduce non-linear terms into $F[n]$. In Ref. [6], the form of $F[n]$ obtained by the Taylor expansion around the average density $\langle n \rangle$,

$$F[n] = \int dz \left( \frac{m^2}{2n} (\delta n)^2 + \frac{K}{2n^2} (\partial_z \delta n)^2 + \frac{\lambda_3}{3n^2} (\delta n)^3 + \frac{\lambda_4}{4n^2} (\delta n)^4 + \frac{\lambda_6}{6n^2} (\delta n)^6 \right),$$

with $\delta n = n - \langle n \rangle$ has been employed for describing the non-Gaussianity. As the SDE is no longer solved analytically with the non-linear terms, the SDE with Eq. (4) is solved numerically in the Cartesian coordinate system. The proper description of the evolution of non-Gaussian fluctuations has been confirmed [6, 9].
In the following, we consider the evolution of the conserved charge in Bjorken-expanding systems assuming boost invariance. We employ Milne coordinates, i.e. space-time rapidity $y$ and proper time $\tau$. In this coordinate system, Eq. (1) is rewritten as

$$\partial_\tau n = \frac{\Gamma}{\tau^2} (\partial^2_\mu^2 b) \frac{\partial F[n]}{\partial n} - \frac{1}{\tau} \delta \xi - \frac{n}{\tau}. \quad (5)$$

The last term represents the reduction of the average density due to the expansion. The noise correlation is also modified as $\langle (\delta(y_1, \tau_1) \delta(y_2, \tau_2) \rangle = 2\tau_1 \Gamma T \delta(y_1 - y_2) \delta(\tau_1 - \tau_2)$.

### 3. Crossover region

Let us investigate the evolution of the net-baryon number density $n_B(y, \tau)$ along trajectories in the crossover region by solving Eq. (5) numerically \[7\]. We consider the four trajectories in the QCD phase diagram shown in the left panel of Fig. 1. We employ Eq. (4) for the free energy functional, where the $T$ and $\mu_B$ dependence of the parameters $\Gamma, m^2, K$ and $\lambda$ has been fixed from the static universality class of the 3D Ising model and a mapping onto the QCD phase diagram in line with Ref. [6]. The QCD critical point is located at $T = 150$ MeV and $\mu_B = 390$ MeV. We set the initial proper time and temperature as $\tau_0 = 1$ fm and $T_0 = 200$ MeV. The relation between $\tau$ and $T$ is assumed to be $T(\tau) = T_0 (\tau_0/\tau)^{1/3}$.

We first performed numerical simulations without the non-linear terms and compared the numerical results with the analytic solution \[5\] to check the correct implementation of our code. We verified that the numerical and analytic solutions show an accurate agreement. In the right panel of Fig. 1, we show the evolution of the second-order cumulant $\langle n_B^2 \rangle_c$ as a function of $\tau$ for the four different trajectories. In our parametrization of $T(\tau)$, the medium passes through the transition line at $\tau = \tau_0 \approx 1.37$ fm, while $\langle n_B^2 \rangle_c$ has a peak at $\tau = \tau_0 \approx 1.5$ fm for the trajectory closest to the critical point. This difference highlights a retardation effect.

In Fig. 2 we show the cumulants $\langle n_B^m \rangle_c$ for $m = 2, 3$, and $4$ at $T = 149$ MeV just below the transition for the four considered trajectories taking the non-linear terms into account. The figure shows that the absolute values of $\langle n_B^m \rangle_c$ are nonzero and become large as the trajectory approaches the critical point. These results show that our numerical simulations of Eq. (5) with Eq. (4) reproduce the expected critical enhancement of the cumulants.

### 4. First-order transition

Let us now study the evolution along a trajectory across the first-order phase transition \[8\]. Near the first-order transition $F[n]$ has two local minima and the global minimum flips at the phase boundary. To model this behavior of $F[n]$, we employ the following functional form

$$F[n] = \int dz \left( \frac{a}{2} (n - n_s)^2 + \frac{b}{4} (n - n_s)^4 - cn + K (\partial_\tau n)^2 \right), \quad (6)$$
where $a < 0$, $b$, $n_0$, $c$, and $K$ are parameters. For $c = 0$, Eq. (6) has two degenerate local minima around the local maximum at $n = n_0$, while the left (right) local minimum becomes the global minimum for $c > 0$ ($c < 0$). The coefficient $\Gamma$ is chosen in such a way that the $T$ dependence of the diffusion coefficient is consistent with the behavior employed in Ref. [2].

In the left panel of Fig. 3, we show an example of the time evolution of the density profile obtained with $\Delta y_{\text{eff}}$, $\Delta y_{\text{eff}}$, $\Delta y_{\text{eff}}$, $\Delta y_{\text{eff}}$, and $\Delta y_{\text{eff}}$. The two black horizontal dashed lines show the proper times at which the average density is in a local maximum for $c = 0$: the medium is in the mixed phase between these two lines. The panel shows that the domain formation due to the phase separation manifests itself with the first-order transition, and the density inhomogeneity generated by the transition survives even at very large proper time.

To see effects of the domain formation on observables, we show the equal-time correlation functions $C(\Delta y, \tau) = \langle \delta n(\Delta y, \tau) \delta n(0, \tau) \rangle$ for different $\tau$ in the right panel of Fig. 3. The panel shows that $C(\Delta y, \tau)$ has a local maximum around $\Delta y = 2.0$ corresponding to the typical size of a domain, and this structure survives even at the late time $\tau = 20$ fm. This result suggests that such a structure in $C(\Delta y, \tau)$ can be used as a signal for the existence of the first-order transition.

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