HQET with chiral symmetry on the lattice * 

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We show that by combining the static heavy quark action with the Neuberger action for the light quark, the renormalisation of the heavy-light bilinear and four-quark operators, computed on the lattice, becomes highly simplified: all the heavy-light bilinears get renormalised by a single multiplicative constant, whereas the renormalisation of the complete set of parity even $\Delta B = 2$ four-quark operators involves only four independent constants. The relevant (matching) constants are computed at NLO in perturbation theory and are presented here.

Computation of the $B$-meson decay constants and the corresponding “bag parameters” by using lattice QCD is very demanding and beyond reach to the currently available computing facilities. In such a situation, the results for these quantities obtained in the static limit, $m_Q \to \infty$, are particularly helpful. They can be combined with those obtained by using the propagating heavy quarks with $m_Q < m_b$, to interpolate in the inverse heavy quark mass and thus extracting the physically interesting quantities at $m_Q = m_b$ (see eg. ref. [1]). However, after combining the static HQET action with the standard Wilson light quark Lagrangian on the lattice, the explicit breaking of the chiral symmetry generates many problems in the renormalisation procedure of the local heavy-light operators. In particular, the mixing pattern in the renormalisation procedure of the local heavy-light operators becomes very complicated. The extra mixing is a lattice artifact and turns out to be uncomfortably large for the bare lattice couplings actually used in practice. For that reason, disentangling an operator that we want to match to its continuum counterpart is difficult and is prone to the additional systematic and statistical errors. Furthermore, when working with Wilson light quarks, the frequent appearance of exceptional configurations prohibits studying the quarks lighter than a half of the strange quark mass. The above problems can be overcome if the chiral symmetry of the light quark is exactly preserved on the lattice. In recent years it became evident that the overlap fermion action [2] indeed preserves the chiral symmetry on the lattice without giving up any other symmetry.

In what follows, we will show that the combination of the Neuberger (overlap) light and the static heavy quark action indeed simplifies the renormalisation procedure of the composite operators. We will then present the expressions for the renormalisation constants for the heavy-light bilinears, as well as for the $\Delta B = 2$ four-quark operators, that we derived at one-loop (NLO) in perturbation theory.

1. Action and symmetries

The action that we are interested in, has the following form:

$$S = S_{YM} + S_{\text{light}} + S_{\text{heavy}} .$$ (1)

For the discretised version of the Young-Mills part of the action $S_{YM}$ we take the standard Wilson plaquette action, whereas for the $S_{\text{heavy}}$ part we adopt the static HQET action with the backward derivative prescription of ref. [3], i.e.

$$S_{\text{heavy}} = \sum_n \left\{ \bar{h}^{(+)}(n) \left[ h^{(+)}(n) - U_0(n - \hat{0}) h^{(+)}(n - \hat{0}) \right] 
- \bar{h}^{(-)}(n) \left[ U_0(n) h^{(-)}(n + \hat{0}) - h^{(-)}(n) \right] \right\} ,$$

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where $h(n)$ is the static heavy quark field, and $U_0(n)$ is the link variable in the temporal direction. For the light quark we take the Dirac Lagrangian à la Neuberger\[.\]

$$D_N = \frac{1}{a^2} \left[ 1 + \frac{X}{\sqrt{X^2}} \right], \quad X = D_W - \frac{\rho}{a},$$

where $D_W$ is the Wilson Dirac operator, $2D_W = \gamma_\mu (\nabla_\mu + \nabla_\mu) - a \nabla_\mu \nabla_\mu$.

On the finite lattice, the action $\[1\]$ is invariant with respect to:

1. Chiral symmetry transformations ($\chi S$) $\[4\]$

$$\psi(x) \to i\gamma_5 \psi(x), \quad \bar{\psi}(x) \to \bar{\psi}(x)i(1 - \frac{D_N}{a})\gamma_5;$$

2. Heavy quark spin symmetry transformations (HQS), of which we will need the following ones

$$h^{(\pm)}(x) \to \frac{1}{2} e_{ijk} \gamma_j \gamma_k h^{(\pm)}(x),$$

$$\bar{h}^{(\pm)}(x) \to -\bar{h}^{(\pm)}(x) \frac{1}{2} e_{ijk} \gamma_j \gamma_k, \quad (i = 1, 2, 3);$$

3. Discrete $O(3)$ symmetry, of which we will need just a rotation by $\pi/2$ about the $i$th axis $(x_i \to x_i, x_j \not\to x_j \to e_{ijk} x_k)$:

$$\psi(x) h^{(\pm)}(x) \to \frac{1}{\sqrt{2}} \left[ \psi(x) h^{(\pm)}(x) \right],$$

$$\bar{\psi}(x) \bar{h}^{(\pm)}(x) \to \bar{\psi}(x) \bar{h}^{(\pm)}(x) \frac{1}{\sqrt{2}}.$$

With the above symmetry properties in hands, we now show that all the heavy-light bilinears

$$A^I \to iS,$$

implying that $Z_S = Z_A$ and $Z_V = Z_P$. We thus arrive at the wanted result that

$$Z_A = Z_V = Z_S = Z_P \equiv Z(\alpha \mu). \quad (2)$$

With the notation, $O_{1,2} = (\zeta^{(+)\Gamma_1} g^\alpha) (\zeta^{(-)\Gamma_2} g^\beta)$, we chose the following basis of parity conserving $\Delta B = 2$ operators (in HQET):

$$O_{TT} \in \{O_{VV+AA}, O_{SS+PP}, O_{VV-AA}, O_{SS-PP}\}.$$

Without symmetries discussed above, all entries of the $4 \times 4$ renormalisation matrix are non-zero and independent from one another. HQS, together with $O(3)$, provides relations among the entries, resulting in the following structure $\[5\]$

$$Z = \begin{pmatrix}
Z_{11} & 0 & Z_{13} & 2Z_{13} \\
0 & Z_{22} & Z_{23} & -Z_{13} - 2Z_{23} \\
-Z_{13} & Z_{32} & Z_{33} & Z_{34} \\
2Z_{13} & Z_{23} & Z_{34} & Z_{33}
\end{pmatrix}, \quad (3)$$

as explicitly verified in perturbation theory with Wilson light quarks $\[6\]$. 8 independent entries get reduced to only 4, after applying the $\chi S$ transformations. Indeed, we verify that

$$O_{VV+AA} \leftrightarrow -O_{VV+AA}, \quad O_{SS+PP} \leftrightarrow -O_{SS+PP},$$

$$O_{VV-AA} \leftrightarrow +O_{VV-AA}, \quad O_{SS-PP} \leftrightarrow +O_{SS-PP},$$

which finally brings us to the form,

$$Z = \begin{pmatrix}
Z_{11} & 0 & 0 & 0 \\
0 & Z_{22} & Z_{33} & Z_{34} \\
0 & Z_{33} & Z_{34} & Z_{33}
\end{pmatrix}. \quad (4)$$

2. Perturbative matching

In this section we present results of the procedure in which we match the lattice regularised operators with their continuum counterparts, renormalised in the $\overline{\text{MS}}(\text{NDR})$ renormalisation scheme. Details of the calculation will be presented in ref. $\[2\]$. Here we only spell out the results.

For the renormalisation constant relevant to the bilinear operators [cf. eq. $\[2\]$], we obtain

$$Z_{\overline{\text{MS}}}^{\text{MS}}(\mu a) = 1 + \frac{\alpha_s}{4\pi} \frac{4}{3} \left[ 5 \frac{\alpha_s}{4} - d_S - e - d_H \right] + \frac{3}{2} \ln(\mu^2 a^2) \].$$
where the constant $e = 24.48059730$, comes from the self energy of the static quark leg \[9\], $d_2(\rho)$ comes from the light quark self energy \[9\], and $d_\Sigma$ is the vertex contribution that has not been calculated before. The values for $d_{\Sigma, H}(\rho)$ are listed in tab. 4 for three specific values of parameter $\rho$.

For the renormalisation constants of the four fermion operators our results read:

\[
\begin{align*}
Z_{11} &= 1 + \frac{\alpha_s}{4\pi} \left[ \frac{7}{3} c + \frac{10 d_H}{3} + \frac{d_S}{3} - \frac{4 d_\Sigma}{3} \right] \\
&- \frac{4 e}{3} + \frac{2 d_\xi}{3} + 4 \log(a^2 \mu^2) \\
Z_{21} &= \frac{\alpha_s}{4\pi} \left[ -\frac{5}{36} c + \frac{d_H}{2} - \frac{d_S}{36} - \frac{2 d_\xi}{9} \\
&- \frac{2 \log(a^2 \mu^2)}{3} \right] \\
Z_{22} &= 1 + \frac{\alpha_s}{4\pi} \left[ \frac{16}{9} + \frac{2 c}{3} - \frac{4 d_H}{3} + \frac{2 d_S}{9} \\
&- \frac{8 d_\xi}{9} - \frac{4 d_\Sigma}{3} - \frac{4 e}{3} + 4 \log(a^2 \mu^2) \right] \\
Z_{33} &= 1 + \frac{\alpha_s}{4\pi} \left[ \frac{41}{12} c + \frac{7 d_H}{6} - \frac{d_\xi}{6} - \frac{4 d_\Sigma}{3} \\
&- \frac{4 e}{3} + \frac{7 d_\xi}{6} + 7 \log(a^2 \mu^2) \right] \\
Z_{34} &= \frac{\alpha_s}{4\pi} \left[ \frac{1}{2} + \frac{c + 2 d_H - d_\xi}{6} - \frac{3 \log(a^2 \mu^2)}{2} \right] \\
Z_{43} &= \frac{\alpha_s}{4\pi} \left[ \frac{1}{8} + \frac{c}{4} + \frac{d_H}{2} - \frac{d_\xi}{4} - \frac{3 \log(a^2 \mu^2)}{4} \right] \\
Z_{44} &= 1 + \frac{\alpha_s}{4\pi} \left[ \frac{41}{12} c + \frac{7 d_H}{6} - \frac{d_\xi}{6} - \frac{4 d_\Sigma}{3} \\
&- \frac{4 e}{3} + \frac{7 d_\xi}{6} + 7 \log(a^2 \mu^2) \right]
\end{align*}
\]

Table 1

| $\rho$ | 1.0 | 1.2 | 1.4 |
|-------|-----|-----|-----|
| $d_{\Sigma}(\rho)$ | -31.33861723 | -23.20304037 | -17.47396963 |
| $d_H(\rho)$ | 0.55183709 | 0.59728235 | 0.64838696 |
| $d_S(\rho)$ | 1.46989129 | 2.02562759 | 2.55134784 |
| $d_{V}(\rho)$ | 0.03924115 | 0.04665160 | 0.05600630 |

verify the symmetry relation,

\[
Z_{21} = \frac{Z_{22} - Z_{11}}{4}, \quad Z_{33} = Z_{44}, \quad Z_{43} = \frac{Z_{44}}{4}. \quad (5)
\]

3. Concluding remarks

Our proposal to combine the static HQET with the overlap light quark is very rewarding for the renormalisation procedure. It particularly simplifies the renormalisation of the phenomenologically important $\Delta B = 2$ operators. In practice, the use of HQET on the lattice suffers from the poor signal-to-noise ratio. This problem was recently circumvented by replacing $U_0(n) \to U_0^{at}(n)$ (fat-link) in $S_{\text{heavy}}$ \[9\]. Empirically, that replacement results in the statistical accuracy in correlation functions, comparable to what one has in lattice QCD. The implementation of that replacement in the above results will be presented elsewhere.

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