Beating the channel capacity limit for superdense coding with entangled ququarts

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Quantum superdense coding protocols enhance channel capacity by using shared quantum entanglement between two users. The channel capacity can be as high as 2 when one uses entangled qubits. However, this limit can be surpassed by using high-dimensional entanglement. We report an experiment that exceeds the limit using high-quality entangled ququarts with fidelities up to 0.98, demonstrating a channel capacity of 2.09 ± 0.01. The measured channel capacity is also higher than that obtained when transmitting only one ququart. We use the setup to transmit a five-color image with a fidelity of 0.952. Our experiment shows the great advantage of high-dimensional entanglement and will stimulate research on high-dimensional quantum information processes.

INTRODUCTION
Quantum entanglement (1) reveals the power of the quantum world. An important application of quantum entanglement is quantum superdense coding (SDC) (2), relying on the shared quantum entanglement between Alice (sender) and Bob (receiver). Alice encodes two classical bits of information on her qubit and then sends the qubit to Bob; Bob performs a complete Bell-state measurement on the two qubits and decodes the two classical bits. During the process, Alice only sends one qubit to Bob, while Bob receives two bits of classical information. This protocol has been realized in many systems (3–6). Considering the photon as the natural flying qubit, the performance of SDC in a linear optical system attracts much attention. However, a complete Bell-state measurement with linear optics is forbidden (7, 8), and the theoretical channel capacity limit of 2 is hard to reach. Previous efforts were made to reach this limit (9–11) even in the presence of noise (12).

Compared with two-qubit entanglement, high-dimensional entanglement is more powerful and increases the channel capacity (13, 14). Recently, high-dimensional entanglement has been demonstrated in optical systems (15–20). On the basis of a photon’s orbital angular momentum, Zeilinger’s group also demonstrated the preparation of a complete four-dimensional Bell basis (21) and quantum gates on single high-dimensional photons (22). These techniques pave the way to high-dimensional quantum communication. However, the fidelity of the high-dimensional Bell state in the previous works is slightly lower, that is, 0.78 ± 0.03 (21), and thus, it is hard to carry out the SDC protocol. On the other hand, we have used beam displacers (BDs) to build very stable Mach–Zehnder interferometers (18, 23, 24) and obtained a high-quality three-dimensional path entanglement with a fidelity of 0.975 ± 0.001 (18). Here, we report an experimental demonstration of SDC using path polarization–encoded entanglement. By encoding the spatial and polarization modes of the photon, the system is more compact than the previous one (18), and the fidelity of the four-dimensional entangled state is up to 0.980 ± 0.001. The measured channel capacity is 2.09 ± 0.01, which exceeds the limit of 2 from the qubit system for the first time (2). Moreover, the channel capacity is also higher than the limit of transmitting one ququart (25). We use the setup to transmit a real image and observe a fidelity of 0.952. This makes the efficient SDC using high-dimensional entanglement feasible in the future.

RESULTS
To generate high-quality, high-dimensional entangled states, we use a path-polarization hybrid system, as shown in Fig. 1. We encode horizontally polarized (H) photons in path a1 (a3) as |0⟩, vertically polarized (V) photons in path a1 (a3) as |1⟩, H photons in path a2 (a4) as |2⟩, and V photons in path a2 (a4) as |3⟩ (here, |0⟩, |1⟩, |2⟩, and |3⟩ represent high-dimensional bases, not Fock states). Thus, the two-photon state is \(\Psi_{11} = (|00⟩ + |11⟩ + |22⟩ + |33⟩)/2\). Then, Alice uses four computer-controlled liquid crystal variable retarders (LCs; which introduce birefringence between fast axis and slow axis, depending on the voltage applied on them) to perform local operations on her own photon and sends it to Bob for Bell-state measurement (details can be found in Materials and Methods). Although it is impossible to perform a complete Bell-state measurement on a four-dimensional system with linear optics (26), we can separate the 16 Bell states into seven classes and select only one state in each class for SDC using photon number–resolving detectors (27, 28). In our system, we choose five states for SDC:

\[
\begin{align*}
\Psi_{11} & = \frac{1}{2} (|00⟩ + |11⟩ + |22⟩ + |33⟩), \\
\Psi_{12} & = \frac{1}{2} (|00⟩ − |11⟩ + |22⟩ − |33⟩), \\
\Psi_{13} & = \frac{1}{2} (|00⟩ + |11⟩ − |22⟩ − |33⟩), \\
\Psi_{14} & = \frac{1}{2} (|00⟩ − |11⟩ − |22⟩ + |33⟩), \\
\Psi_{23} & = \frac{1}{2} (|01⟩ + |10⟩ − |23⟩ − |32⟩).
\end{align*}
\]

The channel capacity using five-state SDC is \(\log_2 5 = 2.32\), which is higher than the set’s limit by using entangled qubits and is also higher than the limit of transmitting one ququart. By carefully checking the coincidental events among those single-photon detectors, Bob can determine which state Alice prepared (see the Supplementary Materials for details). For example, coincidence between D1 and D5 means that the state Alice prepared is \(\Psi_{11}\).

First, we test the state preparation and Bell-state measurement. Alice sends each state to Bob, and Bob performs a measurement and checks...
the success probabilities. The results are shown in Fig. 2. For states $\Psi_{11}$, $\Psi_{12}$, $\Psi_{13}$, and $\Psi_{14}$, the average success probability is 0.926 ± 0.002, and for state $\Psi_{23}$, the success probability is 0.972 ± 0.002. This is because the first four states suffer from imperfect Hong-Ou-Mandel interference, while the last is just a polarization projection. In our experiment, the measured visibility of Hong-Ou-Mandel interference is 0.962 ± 0.002 when using a 10-mm-long ppKTP crystal. Hence, the channel capacity is measured to be 2.09 ± 0.01, which is slightly lower than the theoretical prediction of 2.32 but still exceeds the limit when using the entangled qubit system. When one transmits one ququart, the channel capacity is $\log_2 4 = 2$. Our result also exceeds this limit. This means that our experiment proves that high-dimensional SDC can increase the channel capacity; thus, it is possible to perform quantum communication via efficient SDC even in a high-dimensional system.

DISCUSSION

SDC with high-dimensional entanglement is promising in the future. Note that, because the photon pairs are generated from ppKTP, the overall detection efficiency can be as high as 78.6% when using careful alignment and superconductor detectors (29). The computer-controlled LC operation rate is approximately 30 Hz; this part can be replaced by a fast electro-optical modulator with an operation rate of 1 GHz. Together with a pulsed laser source with a repetition rate of tens of megahertz or even faster, the SDC data rate can be as high as 1 MHz. The path-polarization hyperentanglement can also be distributed to distant users by a multicore fiber (30), and our SDC with high-dimensional entanglement is feasible in the future.

Fig. 1. Experimental setup. A continuous-wave violet laser (power, 4 mW; wavelength, 404 nm) is focused by two lenses, and the waist radius is approximately 0.25 mm. Then, the light beam is separated into two paths by a BD. These two beams are injected into a Sagnac interferometer to pump a type II cut periodically poled potassium titanyl phosphate (ppKTP) crystal (1 mm × 7 mm × 10 mm) and generate two-photon polarization entanglement $(|\Phi\rangle + |\Psi\rangle)/\sqrt{2}$ in each path (33). The ppKTP is temperature-controlled by a homemade temperature controller, and the temperature stability is $K$ to ensure the phase stability between the two paths. Then, we encode horizontally polarized photons in path a1 (a3) as $|0\rangle$, vertically polarized photons in path a1 (a3) as $|1\rangle$, $H$ photons in path a2 (a4) as $|2\rangle$, and $V$ photons in path a2 (a4) as $|3\rangle$ and carefully adjust the relative phase between the two paths; the state is prepared in a four-dimensional maximally entangled two-photon state $\Psi_{11} = (|00\rangle + |11\rangle + |22\rangle + |33\rangle)/2$.

Finally, Alice encodes her information using four computer-controlled LCs and sends her photon to Bob. Bob performs a measurement on the two photons and decodes the information that Alice encoded (details can be found in the Supplementary Materials). PBS, polarizing beam splitter; HWP, half-wave plate.
for fast, distant quantum communication. Furthermore, path-polarization hyperentanglement is possible to connect to integrated photonic chips \cite{31, 32}.

In conclusion, we demonstrate the SDC protocol using four-dimensional entanglement for the first time and achieve a channel capacity of 2.09 ± 0.01, which exceeds the limit of SDC using two-dimensional entanglement and the limit from transmitting one ququart. This experiment will stimulate the research of applications based on high-dimensional entanglement.

**MATERIALS AND METHODS**

**Alice’s encoding process**

The initial state shared by Alice and Bob is

$$\Psi_{11} = \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle),$$

and Alice performs single-photon operations on her own photon and realizes the state transformation from $\Psi_{11}$ to the other four Bell states. These operations are realized by four computer-controlled LCs, as shown in Fig. 4. Then, Alice can easily control the state by applying different voltages on the LCs. If all the LCs are set to introduce the 0 phase, then the two-photon state is

$$\Psi_{23} = \frac{1}{2}(|01\rangle + |10\rangle - |23\rangle - |32\rangle)$$

If LC2 is set to introduce the $\pi$ phase, while the others remain at 0, then the two-photon state is

$$\Psi_{12} = \frac{1}{2}(|00\rangle - |11\rangle + |22\rangle - |33\rangle)$$

If LC2, LC3, and LC4 are set to introduce the $\pi$ phase, while LC1 remains at 0, then the two-photon state is

$$\Psi_{13} = \frac{1}{2}(|00\rangle + |11\rangle - |22\rangle - |33\rangle)$$

If LC3 and LC4 are set to introduce the $\pi$ phase, while LC1 and LC2 remain at 0, then the two-photon state is

$$\Psi_{14} = \frac{1}{2}(|00\rangle - |11\rangle - |22\rangle + |33\rangle)$$

If all the LCs are set to introduce the $\pi$ phase, then the two-photon state is

$$\Psi_{11} = \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle)$$

Bell-state measurement

A complete four-dimensional Bell-state measurement is impossible with linear elements \cite{26}. However, one can separate the 16 four-dimensional Bell states into seven classes and select only one state from each class for SDC \cite{27, 28}. In our experiment, we only choose five states given in Eq. 1. Then, Bob can determine which state Alice prepared by the measurement setup shown in Fig. 1. When the two photons inject into PBS1, they are separated into two classes
by photon’s polarization. For states $\Psi_{11}$, $\Psi_{12}$, $\Psi_{13}$, and $\Psi_{14}$, one photon is reflected and the other is transmitted, while for state $\Psi_{23}$, both photons are reflected or transmitted. After HWP H1 (set at 22.5°) and PBS2, states $\Psi_{11}$, $\Psi_{12}$, $\Psi_{13}$, and $\Psi_{14}$ are separated into two classes by the relative phase between $|00\rangle$ and $|11\rangle$, for example, if the relative phase is $0$ ($\Psi_{11}$), then the two photons will both transmit or reflect from PBS2. Here, this separation depends on the two-photon Hong-Ou-Mandel interference (12). Finally, the last part of the measurement setup (consisting of HWPs, BDs, and PBSs) will separate $\Psi_{11}$ and $\Psi_{13}$ ($\Psi_{12}$ and $\Psi_{14}$) to two classes by the relative phase between $|00\rangle$ and $|22\rangle$. By carefully checking the coincidental events among those single-photon detectors, Bob can determine which state Alice prepared. For example, coincidence between D1 and D5 means that the state Alice prepared is $\Psi_{11}$. Thus, Bob can determine which state Alice prepared and decode the information that Alice encoded.

**Channel capacity**

The channel capacity (10, 11) is the maximal mutual information $H(x; y)$ in our experiment, the mutual information can be calculated as

$$H(x; y) = \sum_{x} \sum_{y} p(x)p(y|x)\log \left( \frac{p(y|x)}{\sum_y p(y)p(y|x)} \right)$$

(7)

where $p(x) = \{ p(\Psi_{11}), p(\Psi_{12}), p(\Psi_{13}), p(\Psi_{14}), p(\Psi_{23}) \}$. The channel capacity in our experiment is 2.09 ± 0.01, which is slightly smaller than the limit of 2.32 owing to the imperfect four-dimensional Bell-state measurement.

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/4/7/eaat9304/DC1

Fig. S1. Measurement on four-dimensional two-photon Bell state.

Fig. S2. Realization of single-photon four-dimensional identical operation and $U$ gate.

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