Abstract: A novel method of three-dimensional photoelectric stylus micro-displacement measuring system was proposed in this paper. The micro-displacement which was got by cantilever was amplified through optics method, and experiment showed that the max error of measurement was 4um. The experimental prototype of micro-displacement measuring system was built and the working theory of micro-displacement measuring system was analyzed. The design of the main parts such as cantilever and probe of the system was completed and the parameters of the system were calibrated by using gauge blocks. The measurement experiment of thread and cylinder followed. The results showed the system has the capability of guiding experimental teaching task for undergraduates and completing middle or low accuracy measurement in the laboratory.

Keywords: micro-displacement measuring, cantilever, probe, system calibration

1. Introductions

Displacement measurement is very common in the field of measurement. With precision machining technology entering the era of micro-nano technology, there followed a variety of micro and nano level measuring instruments such as Coordinate Measuring Machine (CMM), Scanning Tunnel Microscope (STM), Atomic Force Microscope (AFM) [1-5] and so on which widely used in research are very expensive and not suitable for experiments teaching for undergraduates. To understand the cutting-edge knowledge of micro-nano measurement technology for undergraduates and change the past passive observation way but to actively participate in practices in form of new knowledge learning, it has great significance to explore to build the micro-nano technology experimental teaching platform for undergraduates.

2. Measuring theory and system prototype

2.1. Measuring theory

In figure 1(a), light beam from the laser projects onto the mirror fixed on the surface of the free end of the cantilever and then was reflected to the Position Sensitive Detector (PSD) after a lens. There is a probe on the lower surface of the free end of the cantilever. When the probe had sensed ups and downs of the measured surface, position of the light spot on PSD changes. The distance of spot movement and the distance of probe movement have a one-one correspondence. Measuring the distance of spot movement, the distance of probe movement can be calculated through the geometrical optics method. The topography of the measured surface can be got by measuring a number of sampling points. If the
measurement range of object which fixed on the manual translation stage is beyond the range of measuring system, translation stage helps to complete the measurement.

Experimental device of the measuring system was showed in figure 1(b). Micro-displacement sensing part of the system is the cantilever and probe. So the cantilever should have a low force constant, high resonant frequency and some lateral stiffness. Beryllium-bronze has little elastic hysteresis, high elastic stability and excellent resistance to abrasion, corrosion, high temperature, low temperature and fatigue. So it is selected as the material of cantilever. To avoid the resonance among cantilever, machines and ventilation ducts etc in the lab, the resonance frequency of the cantilever is at least 100HZ. Supposed the length, wideness and thickness of the cantilever is L, w and t, its resonance frequency is:

\[ f_1 = \frac{1.875}{2\pi L^2} \sqrt{\frac{E t^2}{12 \rho}} = \frac{346.3 t}{L^2} \]  
(Elastic modulus E is 133GPa, density is 8.23 g/cm³)

\[ \text{Fig.1 (a) Optical schematic of the system; (b) Experimental system in laboratory; (c) Detailed picture of cantilever and probe} \]

To make \( f_1 \) more than 100, length should be shorter than 41.6 with the thickness selected to 0.5. To avoid the twisting of cantilever and having some lateral stiffness, wideness w is selected to 5. So the design of cantilever is length 40mm, wideness 5mm and thickness is 0.5mm. Probe is the contact part with the measured object in the measuring system, and hence wear resistance of the probe needs to be considered, so carbon steel is a better choice. To ensure the measurement of a variety of surfaces, one end of the probe is designed to hemispherical shape whose diameter is 0.8mm and length of probe is 3mm. As the cantilever is thin and a mirror is fixed on the other side, the union of probe and cantilever is welding shown in figure 1(c).

2.2. System prototype

In figure 1(a), auxiliary line OB parallels the reflected light beam at the initial position of the cantilever and auxiliary line OA parallels the reflected light beam after a moving distance d of the probe. Supposed that the angle between reflected light beam at the initial position and optical axis of lens is \( \theta \), and that when the probe has a moving distance d, the rotation angle of cantilever free end is \( \alpha \). So

\[ BM = m \tan \theta \quad AM = m \tan(2 \alpha + \theta) \]

The moving distance s of light spot on the PSD is:

\[ s = AB = AM - BM = m[\tan(2 \alpha + \theta) - \tan \theta] \quad (1) \]

\[ \text{Fig.2 Deforming mould of cantilever} \]

One end of the cantilever used in the system is fixed and the other is free, which is equivalent to a
simple beam. In figure 2, when the force \( F \) acts on the free end of the cantilever, displacement \( f(x) \) and rotation angle \( \alpha(x) \) of each point on the cantilever are as \( ^{[6]} \):

\[
f(x) = \frac{F}{EI} \left( \frac{1}{2} L x^2 - \frac{1}{6} x^3 \right) \quad \alpha(x) = \frac{F}{EI} \left( L x - \frac{1}{2} x^2 \right)
\]

(\( E \) is the elastic module of the cantilever. \( I \) is moment inertia of the cantilever.)

When \( x \) is equal to \( L \),

\[
\alpha = \frac{3 f(L)}{2L}
\]

If \( \alpha \) is extreme small,

\[
\tan 2 \alpha \approx 2 \alpha = \frac{3d}{L}
\]  \( (2) \)

By using (2), (1) can be rewritten as:

\[
d = \frac{sL}{3m + 3s \tan \theta + 3m \tan^2 \theta}
\]  \( (3) \)

3. Calibration of system parameters \( ^{[7][8]} \)

In equation (3), \( L, m \) and \( \theta \) are all system parameters which decide the accuracy of the measurement. So they should be calibrated by special method to get the more accurate parameter value. Measured values of the system parameters are approximately: \( L \) is 40mm, \( m \) is 60mm and \( \theta \) is 20 degree.

3.1. Calibration algorithm based on least square and iteration

If the errors of the system parameters mentioned above are small enough, get the total differential of (3) to the error equation as:

\[
\Delta d = \frac{\partial d}{\partial L} \Delta L + \frac{\partial d}{\partial m} \Delta f + \frac{\partial d}{\partial \theta} \Delta \theta
\]  \( (4) \)

To calculate simply, make \( a \) equal to \( \tan \theta \), and (4) can be rewritten as:

\[
\Delta d = \frac{\partial d}{\partial L} \Delta L + \frac{\partial d}{\partial m} \Delta f + \frac{\partial d}{\partial a} \Delta a
\]  \( (5) \)

(5) is expressed by matrix like:

\[
\Delta d = A \Delta C
\]  \( (6) \)

Matrix of partial derivatives coefficient is like:

\[
A = [\frac{\partial d}{\partial L}, \frac{\partial d}{\partial m}, \frac{\partial d}{\partial a}]
\]

Parameter error vector is like:

\[
\Delta C = [\Delta L, \Delta m, \Delta a]^T
\]

According to least square algorithm, parameter error vector is calculated as:

\[
\Delta C = (A^T A)^{-1} A^T \Delta d
\]  \( (7) \)

And the initial value of parameter vector is like:
C = \([40, 60, 0.364]\)^T

By using the height differences among gauge blocks as the actual values of system calibration, nine spot moving distance values s of calibration points are got. Based on the known initial parameter values, measured values \(d_i\) \((i=1, 9)\) of every calibration point and matrix of partial derivatives \(A\) can be calculated out by equation (3). Then calculate \(\Delta C\) using equation (7) and modify vector \(C\) with \(\Delta C\). Using the new parameter vector \(C\), calculate measured values, matrix of partial derivatives and errors between measured values and actual values again then calculate \(\Delta C\) until to the accuracy requirement.

3.2. Experiment data of calibration

The height range of gauge block using in the experiment is 1mm to 1.9mm and the height difference between close gauge blocks is 0.1mm. The maximum reference error declines to 4% after three iterations, as shown in table 1, and reaching to the system requirements.

System parameters after calibration are that: \(L\) is 41.105mm, \(m\) is 59.747mm, \(\theta\) is 6.5 degree.

4. Analysis of system accuracy and measurement

4.1. Analysis of system accuracy

There are two working modes. One is that record the moving distance of the probe to get the topography of the measured surface without moving the translation stage. The other mode is that set a initial position of the cantilever and move Z-aix of the translation stage to make the cantilever to the initial working position after every measurement of the sample points, then record the moving value of the translation stage to get the topography of the measured surface. In the first mode, measuring accuracy is totally decided by the system accuracy, but in the second mode, measuring accuracy is decide by the accuracy of the translation stage.

In the first mode, different combination values shown in table 2 of gauge blocks are measured. The error curve between measured values and actual values is shown in figure 3(a) which shows that the max measuring error is 4um.

In the second mode, system accuracy is calibrated by using the translation stage whose smallest scale is 1um. Within the range 1mm, get a sample point every 10um with moving distance of translation stage as the actual value and system measuring data as measured value. The error curve between measured values and actual values is shown in figure 3(b) which shows that the max measuring error is 6um.

So in order to improve accuracy, when the measured range is within the measuring range of the system, it is better to not use the translation stage to complete the measurement.

Fig.3 (a) Error between actual data and measurement of mode one; (b) Error between actual data and measurement of mode two

4.2. Measurement experiment

Screw threads on a M2 bolt is measured by using the experimental device shown in figure 4(a). As shown in the thread waveform, three wave troughs’ position is 0.18, 0.58 and 0.98, so the measured
thread pitch is 0.4 that verifies the accuracy in mode one. Measurement of a cylinder followed as shown in figure 4(b). The radius was calculated to 1.7m through Matlab simulation.

Tab.1 Data of system calibration

| Iteration number | parameters | Reference error |
|------------------|------------|-----------------|
| 1                | C          | 40  60  0.364   | 12.5%           |
|                  | d          | 0.088  0.172  0.258  0.348  0.427  0.515  0.599  0.686  0.775 |
|                  | ΔC         | -1.794  1.564 -0.349 |
| 2                | C          | 38.206  61.564  0.015 |
|                  | d          | 0.093  0.182  0.274  0.370  0.455  0.550  0.641  0.737  0.834 |
|                  | ΔC         | 2.697 -1.675  0.110 |
| 3                | C          | 40.903  59.89  0.125 |
|                  | d          | 0.102  0.199  0.299  0.404  0.496  0.599  0.698  0.802  0.903 |

Tab.2 Accuracy calibration data of mode one

| Actual value | Measured value |
|--------------|----------------|
| 0.007        | 0.007          |
| 0.033        | 0.031          |
| 0.077        | 0.079          |
| 0.133        | 0.130          |
| 0.177        | 0.175          |
| 0.233        | 0.233          |
| 0.277        | 0.273          |
| 0.333        | 0.332          |
| 0.377        | 0.374          |
| 0.433        | 0.433          |
| 0.477        | 0.478          |
| 0.533        | 0.536          |

5. Conclusions

This paper presents a designing method of micro-displacement measuring system used in experimental teaching for undergraduate. Working theory of the system is introduced and system parameters are calibrated, then measuring experiment follows. Experiments show that the max measuring error is 4μm meeting the design requirements. The system has the capability of guiding experimental teaching task for undergraduates and completing middle or low accuracy measurement in the laboratory.

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References

[1] Wang Bo-xiong, Chen Fei-fan, Dong Ying. Metrology for micro-and nanotechnology[M]. Beijing:Tsinghua University Press,2006.8.
[2] Bai ChunliL. Nano Science and Technology [M]. Kunming: Yunnan Science and Technology Press,1995. p.1-87.
[3] Hu Xiaotang. Micro-nano Detection Technology [M]. Tianjin: Tianjin University Press,2009, p.1-120.
[4] Bai Chunli. Scanning Tunneling Microscopy and Application [M].Shanghai: Shanghai Science and Technology Press,1992.10, p.92-105.
[5] J. Tersoff, D. R. Humann, Theory and Application for the Scanning Tunneling Microscope [J].Phys. Rev. Lett, 1983,50(25)
[6] Long K K J. Basic structural mechanics - calculation of internal force and displacement of the beam. Beijing: Science Press, 2009, p.38-87.
[7] Zou Xuna, Li Dehua. Coordinate model and algorithm of multi-joint arm. Optics and Precision Engineering, 2001, 9(3):252-257.
[8] Liu Zhenyu, Chen Yyinglin, Qu Daokui,etal... Research on Robot Calibration [J].Robor,2002,24.