Verification of Vickers indenter geometry by means of three-dimensional coordinate measurement

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Abstract. The paper describes an accurate but practical method of the verification of a Vickers hardness indenter geometry. Three-dimensional coordinates of selected points on the surface of four faces of a Vickers indenter are measured with the laser probe 3D profile measurement instrument. The measurement is made with respect to the datum of the indenter so that the misalignment of the axis of pyramid will not interfere to the verification results of geometry. The directions of pyramid faces are analyzed with equations of the best-fit planes to data points estimated by the least-squares algorithm. The flatness of faces is also evaluated as the range of orthogonal distances of data points from the fitted plane. The results show that the geometry of a Vickers indenter is successfully verified with proposed method and the resolution and repeatability of the method are good enough to verify calibration-machine-grade indenters.

1. Introduction

The Vickers hardness, expressed as the testing force divided by the contact area between the indenter and the specimen, is the most widely applicable method of hardness test from macro- to micro-range of testing force and wide range of hardness number. This versatile feature comes from its self-similarity of indenter shape, i.e., no matter how large or small the indentation is, its geometry is similar and its surface area can be directly calculated from the diagonal length. So far, verifying the indenter geometry is one of the most important items in the hardness machine verification. The definition and permissible errors of the indenter geometry are defined relevant standards [1, 2] and machine manufacturers and calibration laboratories are required to ensure their machines fulfil these requirements. However, any instructions or guidelines have not been given how to verify the indenter geometry. Calibration laboratories are now using specially designed instrument only for the verification of indenter geometry, which consists of adjustable sample holder and the optical device to detect normals of the pyramid faces. It enables us to evaluate angular coordinates of the indenter [3, 4].

Recently, microscopes that have capability to reveal three-dimensional structure of small objects have been commercially available and there were attempts to verify Vickers indenters with the scanning white light interferometry [5] or the confocal laser scanning microscopy [6], already. However, it is not easy to obtain consistent results with analyzing software prepared for these kinds of microscopes. Usually the software is operated with a pointing device and graphical user interface and cannot avoid to include an error due to the operator. According to our experience, the repeatability of angle measurement with this operation was 0.16° of standard deviation. It shows that the method doesn’t have an accuracy enough to verify the angle between opposite faces of Vickers indenters.

Another problem in the verification of Vickers indenter geometry is that the tilt of the axis of Vickers pyramid significantly affects on the evaluation of direction of four faces in the plane perpendicular to the axis (in this paper, this parameter is expressed as in-plane direction). Our simulation showed that 0.1° of tilt of the axis results the maximum of 0.35° of error in the in-plane direction. Therefore, it is quite important to consider the datum of a series of measurement.
In this paper, we propose an accurate but practical technique to measure the geometry of Vickers indenter by means of three-dimensional coordinate measurement with a commercial microscope and to analyze the data to evaluate its geometry. The validity of the method is verified with the repeatability of measurement.

2. Method

The definition of the Vickers indenter geometry and its permissible error for hardness calibration machines [2] are illustrated in Fig. 1. Besides those items, it is required to verify the point of pyramid but it will not be mentioned in this paper. Taking Rayleigh’s criterion, the limit of resolution of microscopes is derived from the wavelength and the numerical aperture of an objective lens [7]. When we consider the wavelength of the visible light, the limit of resolution is comparable to the permissible length of the line of conjunction at the indenter tip, i.e., 0.25 µm. It requires us to use other equipment which has higher magnification and resolution like an atomic force microscope. This procedure might be reported in a separate paper.

Three-dimensional coordinates of selected points on four faces of a Vickers indenter are measured with the microscope and the data are analyzed to evaluate angles and flatness. The calculation was carried out with the code written in Scilab according to the procedure in 2.3.

2.1. Equipment

Four faces of a Vickers indenter are measured with modified version of the laser probe 3D profile measurement instrument, Mitaka Koki model NH-3SP [8]. The instrument can detect the position in z-axis with an auto-focus mechanism with probe laser beam induced through an objective lens and scan a sample surface on a motorized stage in x- and y-direction to reconstruct three dimensional coordinates of data points. The resolution is 0.001 µm in x-, y- and z-directions. The scales of this instrument in all three directions are calibrated with certified length standards in prior to the use. Remarkable feature of this instrument is that its measurable range is not limited in the field of view of the microscope. It enables us to evaluate any geometry element with respect to the datum in the distance (the author already reported this feature can be used to verify Rockwell diamond indenter geometry [9]). An indenter is firmly fixed on a flat surface on the X-Y stage of the equipment so that the seating face of the indenter will be identical to the flat surface.

![Figure 1. Definition and permissible errors of a Vickers indenter.](image1)

![Figure 2. Distribution of data points on the indenter surface and the datum.](image2)
2.2. Procedure of measurement

At the beginning of verification, positions of eight points on the flat surface are measured and used to set the datum for successive measurement in the equipment’s software. Then positions of nine (3×3) points on each face of a Vickers indenter are measured since the minimum of nine data points is recommended to define a plane element [10] (Fig. 2). Considering the maximum size of indentation for our standard Vickers hardness machine, the locations of measurement on the indenter surface are distributed in the area corresponding to 680.9 µm of diagonal length (the size for the indentation at 200 HV 50).

2.3. Evaluation of indenter geometry

The best-fit plane is obtained for each face of Vickers pyramid through the least-squares method. The algorithm employed here is based on the orthogonal distances from geometrical element [11]. To fit a plane \( a(x-x_o)+b(y-y_o)+c(z-z_o)=0 \) to data points \((x_i,y_i,z_i) (i=1,2,\cdots,9)\), the signed orthogonal distance from a data point to the plane is \( d_i = a(x_i-x_o)+b(y_i-y_o)+c(z_i-z_o) \) where \((x_o,y_o,z_o)\) is a point on a plane (the centroid of all data points can be used) and \((a,b,c)\) is the unit normal vector of the plane. The plane can be determined where the objective function

\[
 f(a,b,c)=\sum_{i=1}^{9}d_i^2=\sum_{i=1}^{9}\left\{ a(x_i-x_o)+b(y_i-y_o)+c(z_i-z_o) \right\}^2
\]

is to be minimized under the constraint \( g(a,b,c)=a^2+b^2+c^2-1=0 \). This can be solved with a Lagrange multiplier \( \lambda \), i.e. \( \nabla f=\lambda \nabla g \). In vector-matrix notation, these equations (known as the normal equations) can be written as \( (\mathbf{A}^T\mathbf{A})\mathbf{u}=\lambda \mathbf{u} \) where

\[
 \mathbf{u}=\begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \text{and} \quad \mathbf{A}=\begin{bmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \\ \vdots & \vdots & \vdots \\ x_9-x_0 & y_9-y_0 & z_9-z_0 \end{bmatrix}.
\]

One of the most commonly used technique to solve the normal equations is utilizing the singular value decomposition. With this technique, the matrix \( \mathbf{A} \) can be decomposed as \( \mathbf{A}=\mathbf{U}\Sigma\mathbf{V}^T \) where \( \mathbf{U} \) and \( \mathbf{V} \) are square matrices and \( \lambda \) is obtained as the first element of the diagonal matrix \( \Sigma \).

Once equations for all four faces are determined, we can calculate the angles between opposite faces and in-plane directions of faces [12]. If the normals of opposite faces \( i \) and \( j \) are obtained as \( \mathbf{u}_i=(a_i,b_i,c_i) \) and \( \mathbf{u}_j=(a_j,b_j,c_j) \), respectively, the angle between opposite faces can be calculated using the inner product of the normals, i.e., \( \alpha_i^j=\cos^{-1}\mathbf{u}_i \cdot \mathbf{u}_j \). Meanwhile, the in-plane angle can be calculated with the in-plane components of the normal. Assuming \( \mathbf{u}_i'=(a_i/s,b_i/s) \) is the unit component vectors of \( \mathbf{u}_i \) in \( xy \) plane, the in-plane direction of face \( i \) can be calculated using the inner product of \( \mathbf{u}_i' \) and the unit vector in \( x \)-direction \((1,0)\), i.e., \( \theta_i=\cos^{-1}a_i/s \) where \( s=(a_i^2+b_i^2)^{1/2} \) is the length of vector \( \mathbf{u}_i' \).

The flatness of each face can be obtained as the range of signed distance of data points from the best-fit plane. Now we have the data of signed distance of data points from the plane \( \mathbf{d}=(d_1,d_2,\cdots,d_n) \), the flatness of a face can be calculated as \( \ell=\max(\mathbf{d})-\min(\mathbf{d}) \) according to the definition [13].

3. Results and discussion

The geometry of a Vickers indenter manufactured by Tokyo Diamond Mfg. for Mitutoyo standard Vickers hardness machine is measured to demonstrate the validity of proposed method. The results are summarized in Table 1. The values in the table are the average of three runs in the
### Table 1. Results of the verification of geometry of a Vickers indenter

| Item                              | Location | Value   |
|-----------------------------------|----------|---------|
| Angle between opposite faces      | 1 – 3    | 136.04° |
|                                   | 2 – 4    | 136.03° |
| Flatness of face                  | 1        | 0.08 µm |
|                                   | 2        | 0.04 µm |
|                                   | 3        | 0.04 µm |
|                                   | 4        | 0.04 µm |
| In-plane direction (errors from 90°) | 1 – 2    | -0.28° |
|                                   | 2 – 3    | 0.10°   |
|                                   | 3 – 4    | 0.17°   |
|                                   | 4 – 1    | 0.02°   |
| Misalignment of the axis of indenter |         | 0.07°   |

Same setting so that the repeatability of the instrument is taken into account. It is apparent that the indenter under verification satisfies the requirement for a hardness calibration machine.

Nine locations on a single face of Vickers indenter are distributed in a small square of 120 µm × 120 µm. In general, it is getting harder to make accurate angle measurement when the horizontal dimension gets smaller. Nevertheless, the repeatability of measurement is evaluated in ranges of 0.03° in angles between opposite faces, 0.02 µm in flatness, 0.07° in in-plane angles and 0.0006° in the misalignment of axes of the pyramid and the datum. It shows proposed verification method with the instrument has the measurement capability good enough to prove the compliance to the requirement of the indenter geometry for a hardness calibration machine.

### 4. Conclusions

It has been considered that some special instrument is necessary to verify the geometry of Vickers indenter but our attempt revealed the commercial optical microscope with three-dimensional coordinate measurement capability is available for this purpose. The resolution and repeatability of the results are good enough for the verification of calibration-machine-grade indenters. Since the coordinate measurement technique is not limited for such a microscope we used, proposed measurement procedure and data analysis can be applied to the data obtained with other kinds of equipment. It may also be useful for the indenter verification in nano-range where no other means can be used than the atomic force microscopy.

### References

[1] ISO 6507-2:2018, *Metallic materials – Vickers hardness test – Part 2: Verification and calibration of testing machines*
[2] ISO 6507-3:2018, *Metallic materials – Vickers hardness test – Part 3: Calibration of reference blocks*
[3] Yamamoto K 1960 *Report of the Central Inspection Institute of Weights and Measures, Tokyo* 8 56-66
[4] Liguori A et al 2002 *VDI Berichte* 1685 365-371
[5] Takagi S et al 2006 Proc. XVIII IMEKO World Congress 17 – 22
[6] Germak A and Origlia C 2009 Proc. XIX IMEKO World Congress 981-984
[7] Lipson SG and Lipson H 1969 *Optical Physics* (London, Cambridge University Press)
[8] Miura K and Okada M 2000 *Advances in Abrasive Technology III* (Society of Grinding Engineers, Tokyo) 303-308
[9] Takagi S 2014 *Acta IMEKO* 31 15-21
[10] BS 7172: 1989, *Guide to assessment of position, size and departure from nominal form of geometric features*
[11] Shakarji C M 1998 *J. Res. Natl. Inst. Stand. Technol.* 103, 633-641
[12] Takagi S 2010 Proc. IMEKO 2010 TC3, TC5 and TC22 Conference 133-136
[13] ISO 1101:2017, *Geometrical product specifications (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out*