Electromagnetic Noise Reduction in GREATEM Signal Using Singular Value Decomposition in a Junkyard

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Abstract. The grounded electrical source airborne transient electromagnetic system (GREATEM) is a popular geophysical method in recent years [1], has advantages such as convenience, high efficiency, large detection range, high signal-to-noise ratio, and good spatial resolution. However, since the GREATEM transient electromagnetic secondary field electromagnetic response belongs to a wide-band signal and a large attenuation amplitude, late-stage data is easily contaminated by multiple noises, which will seriously affect the results of the inversion interpretation of the data. Therefore, it is still meaningful to study the de-noising methods for GREATEM's data. According to the characteristics of the secondary field electromagnetic response, the singular value decomposition method is used to process the theoretical synthesized signal containing white Gaussian noise. The singular value decomposition and reconstruction of the effective singular value and its corresponding vector are performed, and the analog signal is restored for the purpose of denoising. Then process the measured data of a landfill site. Whether it is viewed from the transient electromagnetic secondary field attenuation curve or the electromagnetic induction profile curve, it can be seen that the singular value decomposition is very useful in denoising. Finally, comparing the effect of singular value decomposition method on the inversion results, the study found that this method can effectively improve the quality of inversion imaging.

1. Introduction
GREATEM is a transient electromagnetic measurement system that uses a ground-based, air-receiving mode of operation. The system uses a grounded long electrical dipole source as the transmitter and the airborne coil as the receiving system. With the advancement of the times, people's requirements for geophysical methods have gradually increased. As a method that is convenient, efficient, and not affected by terrain, semi-aerial transient electromagnetic method is widely used in surveying projects at home and abroad [2,3]. In order to study the adaptability of the method in different regions, we conducted experiments in landfills, grassland wetlands, geological landslides, and lakes. We found that semi-aviation data was affected by a variety of noises during the experiment, mainly including atmospheric noise, human noise, motion noise, random noise, and drone noise. Based on the study of noise sources and characteristics, many scholars have studied electromagnetic denoising methods [4]. These methods include finite impulse response low-pass filtering, empirical mode decomposition (EMD), and wavelet threshold filtering [5]. The above methods all have their limitations. Because the effective signal is a wideband signal, low-pass filtering is difficult when the frequency and the frequency of the noise signal overlap. It is difficult to have good results. Empirical modal decomposition has obvious boundary effects when processing data. Wavelet threshold filtering requires strict selection of wavelet bases and threshold values [6].
Signal processing method based on singular value decomposition (SVD) can effectively analyze non-linear and non-stationary signal, has good numerical robustness and unique processing methods, and has been widely used in signal noise reduction, image reconstruction and power system harmonics Source targeting and other areas. Different from the traditional signal analysis idea, it decomposes the matrix containing signal information into a series of signal subspaces corresponding to singular values and singular value vectors. Different subspaces reflect different components and characteristics of the signal, and are generally considered to be larger singular value and its corresponding singular vector represent the ideal signal, and the noise is reflected on the smaller singular value and its corresponding singular vector, and the greater the energy of the singular value of the target signal, the better the denoising effect.

In order to verify the denoising effect of the singular value decomposition method, we first theoretically generate a set of transient electromagnetic secondary field data, then add random noise to synthesize a noisy signal, and then use the singular value decomposition method to filter the noise to verify the method effect. In order to verify the effect of this method on the measured data, we used the measured transient electromagnetic data in a landfill site, and used the singular value decomposition method to separately process the single-point attenuation curve and electromagnetic induction profile curve of the survey line. Both achieved good results. Finally, the inversion imaging was performed with the data before and after processing. The singular value decomposition can effectively improve the inversion imaging quality.

2. The algorithm of SVD
Make x (n) for an observation signal exist a set of uniform sampling sequence x=[x(1),x(2),..., x(n)], each row in turn choose n sampling points data, structure size for m*n(m > n) matrix.

\[
A = \begin{bmatrix}
    x(1) & x(2) & \cdots & x(n) \\
    x(2) & x(3) & \cdots & x(n+1) \\
    \vdots & \vdots & \ddots & \vdots \\
    x(m) & x(m+1) & \cdots & x(m+n+1) \\
\end{bmatrix}
\]  

(1)

Where :n≥2, m≥2 and m+n-1=N. The matrix A ∈ R^{m×n}is called the attractor trajectory matrix, also known as the Hankel matrix. There are orthogonal matrices U ∈ R^{m×m} and V ∈ R^{n×n}, that satisfy:

\[
A = U \begin{bmatrix} 
\Sigma \\
0 \\
0 \\
\end{bmatrix} V'
\]  

(2)

Which is called the singular value decomposition of matrix A. In the formula \(\Sigma = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_r)\), \(\sigma_1, \sigma_2, \cdots, \sigma_r\) are called the nonzero singular value of the matrix A. Nonzero singular values are generally arranged in order from largest to smallest, such as \(\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r\). Formula (2) can be rewritten as a general form:

\[
A = USV' = \sum_{i=1}^{n} u_i \sigma_i v_i'
\]

(3)

Therefore, through Hankel matrix transformation, the singular value decomposition operation can represent the original signal X as a simple linear superposition of a series of sub-signal. When singular value decomposition processes signal, the first p larger singular values reflect the effective signal, the smaller singular values reflect the noisy signal, reset the smaller part of the singular values to zero, and then the noise can be removed in the reconstructed signal.

3. Data processing results to simulate theoretical data containing noise
According to the principle of transient electromagnetics, we know that the signal observed by the GREATEM are generated by the eddy current of the underground medium after the current is turned off, and the signal show an exponential decay trend. The research shows that the GREATEM data has a large attenuation in the early stage and a small attenuation in the middle and late stage, so the noise has a large effect on the signal in the middle and late stage. In order to verify the denoising effect of
the singular value decomposition method on the noisy GREATEM data, this paper first simulated the transient electromagnetic secondary field attenuation data curve that produced a uniform half-space. (Uniform half-space model parameters: Formation resistivity is $\rho_0 = 50 \, \Omega \cdot m$, Offset is 500m, Emission current is 20A, Line source length is 1000m, Height of receiving coil from ground is 50m ), then the theoretical data is combined with random noise to synthesize the analog signal, and then the analog signal is constructed into a Hankel matrix and SVD decomposed and reconstructed. The data curves of the rectangular coordinate system and the logarithmic coordinate system are plotted respectively. In the figure below, the theoretical secondary field attenuation data (black curve) with a time length of 6.4ms, and the theoretical data and Gaussian white noise data are combined to simulate data (green curve) And the denoised data (red curve) after SVD decomposition and reconstruction.

![Theoretical data](image1)

**Figure 1.** Comparison of before and after SVD of secondary field attenuation data

It can be seen from Figure 1 that no matter whether it is a rectangular coordinate system or a logarithmic coordinate system, SVD can well remove random noise. For better quantitative analysis, the detailed description is made by calculating the signal-to-noise ratio and the root mean square error before and after SVD. The signal-to-noise ratio (SNR) and root mean square error (MSE) formulas are as follows:

$$\text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^{L} s(i)^2}{\sum_{i=1}^{L} (f(i) - s(i))^2}$$

(4)

$$\text{MSE} = \frac{\sum_{i=1}^{L} (f(i) - s(i))^2}{L}$$

(5)

**Table 1.** Comparison of signal-to-noise ratio (SNR) and mean square error (MSE) of SVD.

|         | SNR (dB) | MSE   |
|---------|----------|-------|
| Before SVD | 26.1     | 43.9  |
| After SVD | 43.7     | 0.83  |

4. SVD processing results of measured data

In order to further verify the denoising effect of singular value decomposition, we selected a section of GRAM data collected in a landfill for verification. The landfill includes two parts, Phase I and Phase II, as shown in the figure below. Phase I is on the left and Phase II is on the right. Part of the survey line is arranged as follows, the blue curve is the line source, the length is 1100m, the red is the survey line trajectory, the length is about 1000 m (single point distance is about 2.5 m), the offset is 100m, the receiving coil is 50 m from the ground, and the transmitting current is 20 A.
First, we selected a single point on this line for singular value decomposition (SVD) denoising, as shown in Figure 3. The green curve in the figure represents the measured data, and the black curve represents the denoised data. It can be seen that this method can effectively remove random noise in the late stage, but some other noises are not removed.

We extracted all the single points on the entire survey line with equal logarithmic intervals, and extracted 15 data points at each point. Using the same data from different survey points, we plotted the transient electromagnetic induction profile curve and used this data to perform inversion. Then we use the singular value decomposition method to process the transient electromagnetic induction profile curve, and then use the processed data to perform inversion, as shown in Figure 4 and Figure 5.
From the perspective of the electromagnetic induction profile before SVD, the noise is mainly concentrated at a flying point of about 400 meters, and the curve fluctuates sharply at 900-1000, which may be due to the influence of the receiving coil shaking during the drone flying height. After denoising the electromagnetic induction profile, it can be seen that the singular value decomposition can well deal with the effects of flying points and sloshing of the receiving coil. From the inversion imaging, it can be found that the scope and depth of the first and second phase projects of the landfill can be distinguished before and after SVD. However, many banded structures appeared in the inversion imaging before SVD. It is between 700m-1000m. It can be seen from the image of the work area that this section should be a complete mountain body, which is inconsistent with actual geological phenomena. Therefore, these strip structures are mostly caused by random noise, which is not conducive to inversion interpretation. In the inversion results after SVD, the band structure is significantly reduced, and the quality of inversion imaging is significantly improved.

5. Conclusions
In this paper, the singular value decomposition method is first used to process the simulated semi-aeronautical transient electromagnetic signal and the measured signal from a landfill. The experimental results can draw the following conclusions: the singular value decomposition method has
a good effect on the random noise in the semi-aviation transient electromagnetic signal. The de-noising effect can effectively improve the quality of inversion imaging and is conducive to the interpretation of later geological results. Because the method uses singular values to reconstruct the data, the effect of processing random noise is better, and it is more difficult to deal with coherent noise. In the process of SVD decomposition and reconstruction, the selection of the number of singular values is often determined by experience, and it is difficult to achieve adaptive selection.

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