SPIN–ORBIT ANGLES OF KEPLER-13Ab AND HAT-P-7b FROM GRAVITY-DARKENED TRANSIT LIGHT CURVES

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ABSTRACT

Analysis of the transit light curve deformed by the stellar gravity darkening allows us to photometrically measure both components of the spin–orbit angle \( \psi \), its sky projection \( \lambda \) and inclination of the stellar spin axis \( i_s \). In this paper, we apply the method to two transiting hot Jupiter systems monitored with the Kepler spacecraft, Kepler-13A and HAT-P-7. For Kepler-13A, we find \( i_s = 81^\circ \pm 5^\circ \) and \( \psi = 60^\circ \pm 2^\circ \) adopting the spectroscopic constraint \( \lambda = 58.6 \pm 2.0 \) by Johnson et al. In our solution, the discrepancy between the above \( i_s \) and that previously reported by Barnes et al. is solved by fitting both of the parameters in the quadratic limb-darkening law. We also report the temporal variation in the orbital inclination of Kepler-13Ab, \( d\cos i_{\text{orb}}/dt = (-7.0 \pm 0.4) \times 10^{-6} \text{days}^{-1} \), providing further evidence for the spin–orbit precession in this system. By fitting the precession model to the time series of \( i_{\text{orb}} \), \( \lambda \), and \( i_s \) obtained with the gravity-darkened model, we constrain the stellar quadrupole moment \( J_2 = (6.1 \pm 0.3) \times 10^{-5} \) for our new solution, which is several times smaller than \( J_2 = (1.66 \pm 0.08) \times 10^{-4} \) obtained for the previous one. We show that the difference can be observable in the future evolution of \( \lambda \), thus providing a possibility to test our solution with follow-up observations. The second target, HAT-P-7, is the first F-dwarf star analyzed with the gravity-darkening method. Our analysis points to a nearly pole-on configuration with \( \psi = 101^\circ \pm 2^\circ \) or \( 87^\circ \pm 2^\circ \) and the gravity-darkening exponent \( \beta \) consistent with 0.25. Such an observational constraint on \( \beta \) can be useful for testing the theory of gravity darkening.

Key words: planets and satellites: individual (Kepler-13, KOI-13, KIC 9941, HAT-P-7, KOI-2, KIC 10666592) – stars: rotation – techniques: photometric

1. INTRODUCTION

Spin–orbit angle or the stellar obliquity, \( \psi \), the angle between the stellar spin axis and the orbital axis of its planet, serves as a unique probe of the dynamical history of planetary systems. Especially, its connection with the hot Jupiter migration has been extensively studied (e.g., Fabrycky & Winn 2009), but the relationship between the observed samples and the migration process is not straightforward for various reasons. First of all, the initial distribution of the spin–orbit angles is not known. Some studies do suggest that the protoplanetary disk may have already been misaligned with the stellar equator due to the chaotic gas accretion (e.g., Bate et al. 2010; Fielding et al. 2014) or the magnetic star–planet interaction (e.g., Lai et al. 2011). In these cases, the spin–orbit misalignment is primordial, rather than due to the migration. Even after the disk dissipation or the completion of migration, spin–orbit angle can evolve due to the gravitational perturbation from the companion (e.g., Li et al. 2014; Storch et al. 2014). As suggested by the observed correlation between the spin–orbit misalignment and stellar effective temperature (Winn et al. 2010; Albrecht et al. 2012), spin–orbit angle may also be affected by the tidal star–planet interaction (e.g., Xue et al. 2014), whose mechanism is not well understood. To partially resolve these issues, it is beneficial to measure spin–orbit angles for systems with various host-star and orbital properties. For instance, planets on distant orbits or around hot/young stars are valuable targets because we expect that tides have not significantly affected the primordial spin–orbit configuration.

This paper focuses on a relatively new method for the spin–orbit angle determination in transiting systems, which utilizes the gravity darkening of the host star owing to its rapid rotation (Barnes 2009). Stellar rotation makes the effective surface gravity at the stellar equator smaller than that at the pole by a fractional order of \( \gamma \equiv \Omega_r^2 R^2_p/2G M_s \sim (R_*/P_{\text{rot}})^3 \), where \( \Omega_r \), \( R_* \), \( M_* \), \( P_{\text{rot}} \) are angular rotation frequency, radius, mass, break-up rotation period, and rotation period of the star, respectively. According to von Zeipel’s theorem (von Zeipel 1924), this results in the inhomogeneity of the stellar surface brightness through the relation \( \Delta T_{\text{eff}} \propto g_{\text{eff}}^{-2} \). Here, \( T_{\text{eff}} \) and \( g_{\text{eff}} \) are the effective temperature and surface gravity at each point on the stellar surface, and gravity-darkening exponent \( \beta \) characterizes the strength of the gravity darkening, which is theoretically 0.25 for a barotropic star with a radiative envelope. When a planet transits a star with such an inhomogeneous and generally non-axisymmetric brightness distribution, an anomaly of \( \delta \equiv \mathcal{O}(\gamma \beta) \) appears in the light curve, where \( \delta \) is the transit depth. Since the shape of the anomaly depends on the position of the stellar pole relative to the planetary orbit, the obliquity of the stellar spin \( \psi \) can be estimated with the light-curve model taking into account the effect of gravity darkening.

Indeed, this “gravity-darkening method” has many unique aspects. So far, it is the only known method that simultaneously constrains both components of \( \psi \), the sky-projected spin–orbit angle \( \lambda \) and stellar inclination \( i_s \) (see Equation (2) and Figure 1). Moreover, obliquity analysis is possible essentially with the photometric data alone, and its application is not necessarily limited to short-period planets, as far as the transit is observed with sufficient signal-to-noise ratio (Zhou & Huang 2013). It is also interesting to note that the method is (only) applicable to fast-rotating (i.e., young or hot) stars, for
which anomalies of larger amplitudes result. Since rapid rotators are not suitable for the precise spectroscopic velocimetry because of their broad spectral lines, this method is complementary to the conventional spin–orbit angle measurement using the Rossiter–McLaughlin (RM) effect. All these properties make the method suitable for sampling stars for which the tidal effect is not so significant that the primordial information is expected to be well preserved in the current spin–orbit configuration.

Although the gravity-darkening method is valuable in many aspects, the procedure for obtaining $\psi$ may not be fully established. In a representative example of its application, Kepler-13A, the constraint from the gravity-darkening method (Barnes et al. 2011, hereafter B11) is known to be in disagreement with the later spectroscopic measurement of $\lambda$ with the Doppler tomography (Johnson et al. 2014). In addition, inconsistent results arise even within the gravity-darkening analyses, depending on the choice of the limb-darkening coefficients or $\beta$ (Zhou & Huang 2013; Ahlers et al. 2014). For these reasons, it is worth revisiting the reliability and limitation of this method more carefully, in order for this unique method to be applied to more systems in future and provide credible results.

In this paper, we reanalyze a well-known example of the gravity-darkened transit of Kepler-13Ab, with more data than used in the previous analysis by B11. We investigate the systematic effects in the spin–orbit angle determination and propose a joint solution that may solve the discrepancy with the Doppler tomography measurement (Section 3). We will also see that the spin–orbit precession in this system can be used to test the validity of our solution, as well as to determine the stellar quadrupole moment $J_2$ (Section 4).

In addition, we apply the gravity-darkening method for the first time to an F-type dwarf star, HAT-P-7, where the anomaly in the transit light curve has been reported in several studies (e.g., Esteves et al. 2013, 2014; Van Eylen et al. 2013; Benomar et al. 2014). While the RM measurements (Narita et al. 2009; Winn et al. 2009a; Albrecht et al. 2012) have established that $\lambda > 90^\circ$, suggesting a retrograde orbit, the following asteroseismic inferences (Benomar et al. 2014; Lund et al. 2014) have revealed that a pole-on orbit is actually favored. In Section 5, we show that a similar conclusion is also obtained from the gravity-darkening method and discuss the consistency of our result with other constraints on the host-star properties.

2. METHOD

2.1. Model

We basically follow Barnes (2009) in modeling the gravity-darkened transit light curve. The model includes the following 14 parameters, which are listed as “fitting parameters” in Tables 1 and 3:

1. mean stellar density, $\rho_* = 3M_*/4\pi R_*^3$, which corresponds to the semi-major axis scaled by the stellar equatorial radius $^1$, $a/R_*$;
2. limb-darkening coefficient for the quadratic law, $c_1 = u_1 + u_2$;
3. limb-darkening coefficient for the quadratic law, $c_2 = u_1 - u_2$;
4. time of the inferior conjunction, $t_i$;
5. orbital period, $P$;
6. cosine of orbital inclination, $\cos i_{\text{orb}}$;
7. planetary radius normalized to the stellar equatorial radius, $R_p/R_*$;
8. normalization of the out-of-transit flux, $F_0$;
9. stellar mass, $M_*$;
10. stellar rotation frequency, $\Omega$;
11. stellar effective temperature at the pole, $T_{*,\text{pole}}$;
12. gravity-darkening exponent, $\beta$;
13. stellar inclination, $i_*$;
14. sky-projected spin–orbit angle, $\lambda$.

The first eight parameters are common with the light-curve model without gravity darkening. We assume circular orbits for the two targets because the orbital eccentricities are constrained to be very small, if any, from the occultation light curves (Benomar et al. 2014; Shporer et al. 2014).

In the gravity-darkened model by Barnes (2009), the shape of the star is approximated by the spheroid with the oblateness $\gamma = \Omega^2 R_\text{eff}^3/2GM_*=3\pi\eta^2/2G\rho_*$. The surface brightness at each point is modeled as the blackbody emission of the temperature $T_s = T_{*,\text{pole}} (g_{\text{eff}}/g_{\text{eff, pole}})^\beta$, where $g_{\text{eff}}/g_{\text{eff, pole}}$ is the effective surface gravity normalized by its value at the stellar pole. The surface gravity at point $r$ on the stellar surface is calculated by $g_{\text{eff}} = -GM_*/r^2 + 4\pi\beta r \beta_\text{pole}^2 r \beta_\text{pole}$. Here $r$ and $\beta$ are the norm and unit vector of the radius vector $r$, respectively. Similarly, $r_{\lambda}$ and $\beta_{\lambda}$ are those of $r_{\lambda}$, the projection of $r$ onto the stellar equatorial plane. The Planck function $B_\lambda(T_s)$ at each point is convolved with the “high-resolution" Kepler response function$^2$ using the table of the wavelength- and temperature-dependent factor calculated prior to the fitting. The convolved flux is then multiplied by the limb-darkening function

$$I(\mu) = 1 - u_1(1 - \mu) - u_2(1 - \mu)^2,$$

$^1$ In this paper, $R_*$ denotes the equatorial radius of the star.

$^2$ http://keplergo.arc.nasa.gov/CalibrationResponse.shtml
with $\mu$ being the cosine of the angle between $-\mathbf{g}_{\text{eff}}$ and our line of sight (LOS),\(^3\) and integrated over the visible surface of the star to give the total flux. We fix $T_{\text{s,pole}}$ at the observed effective temperature assuming that the difference between $T_{\text{s,pole}}$ and the disk-integrated effective temperature is small. Note that the gravity-darkened transit light curve gives $\rho_2$ alone and cannot constrain $M_\star$ and $R_\star$ separately, as is the case for the transit without gravity darkening.

The configuration of the planetary orbit and stellar spin is specified by three angles, $i_{\text{orb}}$, $i_\star$, and $\lambda$, which are defined in Figure 1 (see also Figure 1 of Benomar et al. 2014). The orbital and stellar inclinations, $i_{\text{orb}}$ and $i_\star$, are measured from the LOS and defined to be in the range $[0, \pi]$. The sky-projected spin–orbit angle, $\lambda$, is the angle between the sky-projected stellar spin and planetary orbital axes. It is measured from the former to the latter counterclockwise in the sky plane, and it is in the range $[0, 2\pi]$. With these definitions, the true spin–orbit angle, or the stellar obliquity, $\psi$, is given by Equation (1) of Benomar et al. (2014):

$$\cos \psi = \cos i_\star \cos i_{\text{orb}} + \sin i_\star \sin i_{\text{orb}} \cos \lambda.$$  (2)

Throughout the paper, we restrict $i_\star$ to be in the range $[0, \pi/2]$, making use of the intrinsic symmetry with respect to the sky plane. We do not lose any physical information of the system with this choice because any of the relative star–planet configurations with $i_\star$ in $[\pi/2, \pi]$ are the same as one of those with $i_\star$ in $[0, \pi/2]$. In other words, the configurations $(i_\star, i_{\text{orb}}, \lambda)$ and $(\pi - i_\star, \pi - i_{\text{orb}}, -\lambda)$ are equivalent. This transformation corresponds to looking at the system from the other side of the plane of the sky.

In the following, we also adopt the constraint on the stellar LOS rotational velocity $v \sin i_\star$ from spectroscopy, which is related to the above model parameters by

$$v \sin i_\star = 2\pi f_{\text{rot}} \left( \frac{3M_\star}{4\pi \rho_2} \right)^{1/3} \sin i_\star.$$  (3)

This, in principle, allows us to break the degeneracy between $M_\star$ and $R_\star$, enabling the determination of the absolute dimension of the system. Nevertheless, the constraint on $M_\star$...
is usually weak as discussed in B11, and so we fix $M_\star$ at the observed value.

### 2.2. Data Processing

We detrend and normalize the transit light curves of each target along with the consistent determination of the transit times and transit parameters. We first normalize the light curve of each quarter using its median and then iterate the following two steps until the resulting transit times $t_\tau$ and transit parameters converge (typically 10–20 times).

1. The light curve around each transit ($\pm 0.2$ days for Kepler-13A and $\pm 0.15$ days for HAT-P-7) is modeled as the product of a quadratic polynomial:

   $$ a_0 + a_1(t - t_\tau) + a_2(t - t_\tau)^2 $$

   where $t_\tau$ is the time of the transit, and $a_0$, $a_1$, and $a_2$ are the fitted parameters. We use the Levenberg–Markwardt (LM) method to find the best-fit polynomial parameters.

2. Using the set of $t_\tau$ obtained in the first step, we calculate the mean orbital period $P$ and transit epoch $t_0$ by linear fit and use them to phase-fold the normalized and detrended transits. The phase-folded light curve is averaged into one-minute bins and then fitted with the Levenberg–Markwardt model to find the best-fit orbital period $P$ and transit epoch $t_0$.

In the following analysis, we use the one-minute binned, phase-folded light curve obtained in the second step of the final iteration. For each bin, the flux value is given by its mean, and the error is estimated as the standard deviation within the bin divided by the square root of the number of data points.

### 2.3. Fitting Procedure

In fitting the observed light curves, we use the likelihood $\mathcal{L}$ of the model computed by $\mathcal{L} \propto \exp(-\chi^2/2)$, where

$$ \chi^2 = \sum_i \left( \frac{f_i - f_{\text{model},i}}{\sigma_i} \right)^2 + \sum_j \left( \frac{p_j - p_{\text{model},j}}{\delta p_j} \right)^2. $$

In the first term, $f_i$, $f_{\text{model},i}$, and $\sigma_i$ are the observed value, modeled value, and error of the $i$th data point, respectively. The second term is introduced to take into account the constraints from other observations on some (functions of) the model parameters $p_j$. In the following analysis, $p$ is read to be $v \sin i$, and, in some cases, $\lambda$. For each $p_j$, we assume a Gaussian constraint of the form $p_j \pm \delta p_j$, and the value obtained from the model is denoted by $p_{\text{model},j}$.

The maximum likelihood solution is found by minimizing Equation (4) with the LM method using the \texttt{cmpfit} package (Markwardt 2009). Since the complex dependence of $\chi^2$ on $i$, and $\lambda$ is expected, we repeat the fitting procedure from the initial $i$, in $[0, 90^\circ]$ and $\lambda$ in $[-180^\circ, 180^\circ]$ at $10^\circ$ intervals. Initial values of the other parameters are chosen close to the best-fit values obtained from the model without gravity darkening. We also try both positive and negative $\cos i_\text{orb}$ as an initial value to search the whole domain of $i_\text{orb}$, which is now $[0^\circ, 180^\circ]$.

### 3. TRANSIT ANALYSIS OF KEPLER-13Ab

In this section, we report the analysis of the gravity-darkened transit of Kepler-13Ab. We first analyze the whole available short-cadence (SC) data from Q2, 3, and 7–17 using the same stellar parameters as in B11 to test the validity of our method (Section 3.1). Motivated by the recently reported disagreement with $\lambda$ from the Doppler tomography, we also investigate the possible systematics in the spin–orbit determination arising from the choice of stellar parameters. We show that the discrepancy can be absorbed by adjusting the value of $c_2$ and present a joint solution that is compatible with all of the observations made so far.

#### 3.1. Reproducing the Results by B11

In this subsection, we analyze the SC, Pre-search Data Conditioned Simple Aperture Photometry (PDCSAP) fluxes from Q2, 3, and 7–17. Given the clear transit duration variation (TDV) reported by Szabó et al. (2012, 2014), we separately analyze the transits from each quarter, rather than folding all the available data. Since we do not detect significant temporal variations in the parameters other than $\cos i_\text{orb}$ (see Section 4), we report the mean and standard deviation of the best-fit values from the above 13 quarters for each parameter.

First, we use the same stellar parameters as in B11 and obtain the results in the second column of Table 1. Namely, we subtract a constant value $F_2 = 0.45$ from the normalized flux to remove the flux contamination from the companion star, and we impose the constraint $v \sin i = 65 \pm 10$ km s$^{-1}$ based on Szabó et al. (2011). We fix $M_\star = 1.83M_\odot$ and $T_\text{eff} = 8848$ K from Borucki et al. (2011), and $c_2 = 0$. In Figure 2, the best-fit model is overplotted with the data for Q2, which is to be compared with Figure 2 of B11.

Basically, we find a very good agreement with the result by B11 using about 12 times more data. Although the values of $\cos i_\text{orb}$, $i_\star$, and $\lambda$ we report here appear different from those in B11, that is simply because we choose $i_\star$ to be in the range $[0, \pi/2]$. This is physically the same configuration as theirs and corresponds to the top left situation in Figure 3 of B11. That is, $\lambda$ in our solutions with $\cos i_\text{orb} < 0$ should be read as $-\lambda$ in the conventional definition, because $\lambda$ is usually defined for the orbit with $\cos i_\text{orb} > 0$ (see also the discussion after Equation (2)).

In addition to the solution in Table 1, we also find a retrograde solution with $\lambda > 90^\circ$ as noted in B11. Here we do not discuss this solution, however, because the Doppler tomography observation has already excluded the retrograde orbit with high significance (Johnson et al. 2014).
3.2. Systematics due to Stellar Parameters

Although we find consistent values of $\lambda$ and $i_c$ as obtained by B11, those of $\lambda$ significantly differ from $\lambda = 58.6 \pm 2.0^\circ$, the value obtained from the Doppler tomography (Johnson et al. 2014). Motivated by this discrepancy, we investigate the possible origins of systematics in the spin–orbit angle determination with gravity darkening in this subsection.

First, we examine the systematics due to the choice of $M_*$, $v \sin i_c$, $T_{\text{pole}}$, and $F_c$, which are the stellar properties not derived from the light-curve modeling. We perform the same analysis as in Section 3.1, but adopting the following parameters from the most recent photometric and spectroscopic study by Shporer et al. (2014, hereafter S14): $v \sin i_c = 78 \pm 15 \text{ km s}^{-1}$, $M_* = 1.72 \text{ M}_\odot$, $T_{\text{pole}} = 7650 \text{ K}$, and $F_c = 0.47726$. The corresponding results are shown in the third column of Table 1. We find that $i_c$ and $\lambda$ can differ by as large as $10^\circ$ due to the choice of the above parameters, but the difference is not so large as to explain the disagreement with the Doppler tomography. The main difference from the B11 case with this new set of parameters is the different constraint on $f_{\text{rot}} \sin i_c$, which is proportional to the combination $(\rho_*)^{1/2} \sin i_c$ (see Equation (3)). With smaller $M_*$ and larger $v \sin i_c$, the stellar rotation rate slightly higher than the B11 case is favored. We find that the difference in $T_{\text{pole}}$ is less important compared to the above effect. We also find that larger $F_c$ yields larger $R_p/R_*$, which makes the impact parameter or $|\cos i_{\text{orb}}|$ smaller to give the same ingress/egress duration.

Next, we allow $c_2 = u_1 - u_2$ to be free and find that the resulting spin–orbit angle is very sensitive to this parameter. When $c_2$ is floated, the constraints on $i_c$ and $\lambda$ become much weaker than the $c_2 = 0$ case, as shown in the fourth and fifth columns of Table 1. The strong dependence on $c_2$ is illustrated in Figure 3, which shows that $\lambda$ and $i_c$ vary by several tens of degrees depending on $c_2$. In fact, the result indicates that the gravity-darkened light curve is actually compatible with the Doppler tomography solution if we choose $c_2 \sim 0.25$; such a solution will be discussed in Section 3.3.

3.3. Joint Solution

In Section 3.2, we found that the gravity-darkened light curve is compatible with the value of $\lambda$ estimated from the Doppler tomography if $c_2 \sim 0.25$. Thus, we repeat the analysis treating $c_2$ as a free parameter for both stellar parameters by B11 and S14, but this time imposing the additional constraint $\lambda = 58.6 \pm 2.0^\circ$ from the Doppler tomography. The results are

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**Figure 2.** Fitting the gravity-darkened model to the Q2 transit of Kepler-13Ab. Middle: black dots are the phase-folded and binned fluxes from Q2. The thick red line shows the best-fit gravity-darkened model, while the thin blue line is the best-fit model without gravity darkening. Bottom: black dots are the residuals of the best-fit gravity-darkened model. Gray open circles are those for the joint solution, where $c_2$ is fitted with the constraint $\lambda = 58.6 \pm 2.0^\circ$ from the Doppler tomography. Top: black dots are the residuals of the best-fit model without gravity darkening. The thick red line is the difference between the best-fit model with gravity darkening and that without gravity darkening. The dashed red line shows the same result for the joint solution. The difference between the two gravity-darkened solutions is only barely visible just after the ingress and before the egress.

**Figure 3.** Constraints on $(\lambda, i_c)$ from the gravity-darkened transit of Kepler-13Ab for the different choices of $c_2$. In this illustration, data from Q2 are used and stellar parameters from B11 are adopted. The solid, dashed, and dotted contours, respectively, show $1\sigma$, $2\sigma$, and $3\sigma$ confidence regions for $(\lambda, i_c)$ obtained from 200,000 Markov chain Monte Carlo (MCMC) samples for three fixed values of $c_2$ (0, 0.12, and 0.25). The shaded areas bounded by the vertical solid, dashed, and dotted lines denote $1\sigma$, $2\sigma$, and $3\sigma$ confidence regions, respectively, for $\lambda$ obtained from the Doppler tomography (Johnson et al. 2014). The sign of $\lambda$ is opposite to their quoted value because we are now dealing with the solution with $\cos i_{\text{orb}} < 0$ (i.e., $\pi/2 < i_{\text{orb}} < \pi$); see also the discussion in the third paragraph of Section 3.1.

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6 We do not examine the dependence on $\beta$ here because B11 have already shown that a different choice of $\beta = 0.19$, suggested by the interferometric observation of Altair (Monnier et al. 2007), does not change the result significantly.
summarized in the last two columns in Table 1. The resulting value of $i_s = 81^\circ \pm 5^\circ$ indicates that the star is close to equator-on, and $\psi = 60^\circ \pm 2^\circ$ is slightly larger than the previous estimate. In terms of $\chi^2_{\text{min}}$, these solutions equally well reproduce the transit anomaly as the solutions discussed so far, and still they are consistent with the Doppler tomography result. Moreover, we obtain a slightly longer $P_{\text{rot}}$, which better agrees with $P_{\text{rot}} = 25.43 \pm 0.05$ hr estimated by Szabó et al. (2012, 2014) than the solution with the gravity darkening alone. For these reasons, the joint solution is most favored from the current observations. We note, however, that the likelihood for the joint solution is not so high as to statistically justify the introduction of the additional free parameter $c_2$. Furthermore, the plausibility of the value of $c_2$ in our joint solution is theoretically unclear. We obtain the theoretical values of $c_{1, \text{th}} \approx 0.6$ and $c_{2, \text{th}} \approx 0.0$ from the table of Sing (2010) if we adopt the effective temperature and surface gravity by S14. Hence, the value of $c_2$ from our joint solution is discrepant from $c_{2, \text{th}}$; they could even have opposite signs depending on the stellar parameters. Nevertheless, it is also true that theoretical values often disagree with the observed ones (e.g., Southworth 2008); in fact, $c_1$ in the light-curve solution with $c_2 = 0$ is also different from $c_{1, \text{th}}$. Therefore, we do not consider the possible deviations from the theoretical values crucial, and we regard it as an open question. An alternative approach to independently assess the validity of our solution is discussed in the next section.

4. SPIN–ORBIT PRECESSION IN THE KEPLER-13A SYSTEM

The shape of Kepler-13Ab’s transit is known to exhibit a long-term variation, which is likely due to the spin–orbit precession induced by the quadrupole moment of the rapidly rotating host star (Szabó et al. 2012, 2014). Indeed, we find the monotonic decrease in $|\cos i_{\text{orb}}|$ from the quarter-by-quarter analysis in Section 3; the constant-value model is rejected at the $p$-value of 0.5% for this parameter using a simple $\chi^2$ test. On the other hand, the other model parameters are found to be consistent with the constant value using the same criterion. Therefore, our analysis confirms that the observed TDVs are actually due to the variation in $\cos i_{\text{orb}}$, further supporting the precession scenario with the more realistic model of the asymmetric transit light curve.

In this section, we further examine this scenario with the gravity-darkened transit model. Unlike the above previous studies (Szabó et al. 2012, 2014) that focused on $i_{\text{orb}}$, the gravity-darkened model allows us to additionally study the (non-)variations in the other two angles, $\lambda$ and $\nu$, which should also be induced if the system is precessing. By fitting the analytic precession model to the time series of $\cos i_{\text{orb}}$, $\lambda$, and $i$, obtained from the light curves, we constrain the stellar quadrupole moment $J_2$ and its moment of inertia coefficient $C$. On the basis of these constraints, we predict the future evolution of the system configuration and argue that the follow-up observations of such a long-term modulation can distinguish the light-curve and joint solutions discussed in Section 3. In the following, we mainly discuss the results obtained with the stellar parameters from S14, though the conclusions remain the same for the B11 parameters.

4.1. Model Parameters from Each Transit

To examine the temporal variations in $\cos i_{\text{orb}}$, $i_s$, and $\lambda$, we fit individual transit light curves, rather than the phase-folded ones, for these parameters. We use the same two models (“light-curve solution” with $c_2 = 0$ and “joint solution” with $c_2$ fitted) as discussed in Section 3. In order not to underestimate the errors in the three angles, we fit all the other model parameters, $\rho_s$, $c_1$, $c_2$ (for the joint model), $t$, $R_g/R_s$, $f_{\text{rot}}$, and $F_0$ as well, which should not vary temporally in our model. Using the best values in Table 1, we impose the constraints on these parameters except for $t$, $F_0$, through the second term of Equation (4). In fitting much noisier individual transits, this prescription assures that the parameters converge to the values consistent with those from the phase-folded light curves, while preserving their differences from transit to transit. We also discard the transits for which the fit does not converge due to the data gaps and/or flare-like brightening features sometimes found in the light curves. The resulting sequences of the transit parameters are plotted in Figure 4.

As mentioned above, we again find the clear linear trend in $\cos i_{\text{orb}}$ from individual transits. We fit the linear model to the time series of $\cos i_{\text{orb}}$ using a Markov chain Monte Carlo (MCMC) algorithm and obtain the rates of change in the upper part of Table 2. Here we only report the slopes for absolute values of $\cos i_{\text{orb}}$ because its actual sign depends on the sign of $\cos i_s$, which can never be determined with the current observations (we arbitrarily choose $\cos i_s > 0$ in this paper, as discussed after Equation (2)). Comparing the light-curve solution and joint solution, we find that the rate of $|\cos i_{\text{orb}}|$ change is insensitive to $\lambda$ or $c_2$ because $|\cos i_{\text{orb}}|$ is mainly determined from the transit duration. With $a/R_s$, calculated from $\rho_s$, our value for $d|\cos i_{\text{orb}}|/dt$ is found to be consistent with $\frac{db}{dt} = (-4.4 \pm 1.2) \times 10^{-5}$ days$^{-1}$ by Szabó et al. (2012), but our constraint is several times better.

Figure 4 also shows the abrupt systematic changes in $R_g/R_s$. These changes occur exactly in phase with the border of different quarters indicated with different colors (black and gray). For this reason, they are unlikely to be of physical origin, but are probably due to the seasonal transit depth variations similar to those reported by Van Eylen et al. (2013) for HAT-P-7. In addition, some of the parameters (most notably $\rho_s$ and $f_{\text{rot}}$) show the long-term modulation of the period ~400 days. Origins of these systematics are beyond the scope of this paper, and they are just treated as the additional scatter in the data.

4.2. Fit to the Observed Angles and Future Prediction

Among the observed time series of transit parameters in Figure 4, those of $\cos i_{\text{orb}}$, $\lambda$, and $i_s$ are fitted using an MCMC algorithm to observationally constrain $J_2$ and $C$. We utilize the
same analytic precession model as in Barnes et al. (2013), which constitutes an analytic solution of the secular equations of motion derived by Boué & Laskar (2009). In this model, the orbital and spin angular momenta precess around the total angular momentum at the same angular rate given by

\[ \dot{\Omega} = \Omega_p \left( \frac{L}{S} + \cos \psi \right) + \sin^2 \psi, \]  

Figure 4. Best-fit model parameters from each transit. The left panels are the results for the light-curve solution with \( c_2 = 0 \), while the right ones are for the joint solution. Errors are from the outputs of the \texttt{cmpfit} package. Parameters from even quarters (2, 8, 10, 12, 14, and 16) are shown in black, while those from odd quarters (3, 7, 9, 11, 13, 15, and 17) are in gray. For the times of inferior conjunctions, \( t_c \), the residuals of the linear fit (i.e., TTVs) are plotted for clarity. Solid lines in \( \cos l_\text{orb} \) panels are the best-fit linear models.
is imposed. The central value is from the result for the moment of inertia coefficient of the host star. With Equation (6), this explains why $M_p/M_*$ is smaller and C is larger for the light-curve solution than for the joint solution.

The constraints from the MCMC fit are summarized in the middle and bottom parts of Table 2, and the best-fit models are plotted with the solid lines in Figure 5. Basically, the precession model is compatible with the observations for both the light-curve solution and the joint solution. The value of $J_2$ and the corresponding precession period, however, are different by a factor of a few, in spite of the similar observed slopes in $\cos i_{\text{orb}}$ and $i_{\text{rot}}$. While $J_2 = (1.66 \pm 0.08) \times 10^{-4}$ for the light-curve solution is consistent with the earlier estimate by Szabó et al. (2012), $J_2 = (2.1 \pm 0.6) \times 10^{-4}$ from observed TDVs and $J_2 = 1.7 \times 10^{-4}$ from the stellar model, the joint solution yields a smaller value, $J_2 = (6.1 \pm 0.3) \times 10^{-5}$.

The difference comes from the different three-dimensional architectures of the system described by the two solutions. Since all of $\cos i_{\text{orb}}$, $\lambda$, and $i_*$ are constrained from the gravity-darkened light curves, the relative configuration of the stellar spin and orbital angular momenta are completely specified in three dimensions. This means that the phase of the precession during the Kepler mission, which corresponds to the left end in the right column of Figure 6, is observationally constrained; from the top panel, we find that $\cos i_{\text{orb}}$ is closer to the bottom of the sine curve for the light-curve solution (blue dashed line), while that for the joint solution (red solid line) resides in the phase of a rapid increase. For this reason, a larger precession rate (i.e., shorter precession period) is required for the light-curve solution to match the observed change in $\cos i_{\text{orb}}$. According to Equations (5) and (6), the larger precession rate can be achieved by increasing either $J_2$ or $L/S$. However, the larger precession rate also induces faster variations in $\lambda$ and $i_*$, contradicting their almost constant observed values (middle and bottom panels in Figure 5). The only way to mitigate this conflict is to make $J_2$ larger (i.e., increase the precession rate) while keeping $L/S$ small, making it more difficult to move stellar spin axis. With Equation (7), this explains why $M_p/M_*$ is smaller and C is larger for the light-curve solution than for the joint solution.
Accordingly, the bottom panel of the right column in Figure 6 exhibits a smaller precession amplitude for $\sum_i$ in the former solution (blue dashed line) than in the latter (red solid line). The approximately three times difference in the precession period would be apparent even on the short timescale (left column in Figure 6). As shown in the middle panel, a change as large as $\sim 10^\circ$ in $\lambda$ is expected within the next $\sim 10$ yr for the light-curve solution, which may well be detectable given the current precision of the spin–orbit angle measurement (nominally down to a few degrees). On the other hand, $\lambda$ for the joint solution is almost constant. From this point of view,
the joint solution may slightly be favored even with the current data, because the nearly constant values observed for \( \lambda \) and \( i_\lambda \) are more natural for the joint solution than for the light-curve one, for the reasons discussed in the previous paragraph. This indication also manifests itself in the fact that the resulting \( M_p/M_\star \) and \( C \) better agree with our prior knowledge in the joint solution.

The more decisive conclusion will be obtained with the future follow-up observations of \( \lambda \) using Doppler tomography, as well as the transit duration observations to better constrain \( \cos i_\text{orb} \) and hence the precession rate. If our joint solution is correct, variations in \( \lambda \) will not be detected in the near future. On the other hand, if the light-curve solution is actually correct and \( \lambda \) from the Doppler tomography is somehow systematically biased, then \( \lambda \) should change; this temporal variation would be observable with the Doppler tomography even if it were biased. Or, it may even turn out that the precession scenario is wrong. In any case, tracking the future evolution of the system configuration can be used for an independent test of our solution, not to mention for better constraining the stellar internal structure via \( J_2 \) and \( C \), for which few observational constraints have been obtained.

5. ANOMALY IN THE TRANSIT LIGHT CURVE OF HAT-P-7

Armed with the methodology established using the distinct anomaly in Kepler-13A (Section 3), we discuss another, more subtle anomaly in this section. Here the methodology is further extended to include the information from asteroseismology, as well as from the RM effect, and applied to an F-type star.

It has been pointed out in several studies that the transit light curve of HAT-P-7 exhibits a small anomaly of \( \Theta (10^{-5}) \). Morris et al. (2013), who reported this anomaly first, attributed it to the local spot-like gravity darkening induced by the gravity of the Jupiter-mass companion HAT-P-7b. They ruled out the gravity darkening of stellar rotational origin on the basis of the inspection that the anomaly is localized in a part of the transit. Later analyses with more data (e.g., Estelles et al. 2013, 2014; Van Eylen et al. 2013; Benomar et al. 2014), however, have shown that the anomaly is seemingly correlated over the whole transit duration, as in the top panel of Figure 8. Moreover, the amplitude of the observed anomaly may be too large to be explained by the spot scenario. According to Jackson et al. (2012), the planet’s gravity induces the surface temperature variation of “a few 0.1 K,” which leads to the surface brightness variation of \( \Delta F \approx 100 \text{ppm} \). If a planet crosses over a spot fainter by \( \Delta F \) than the other part of the stellar disk, the amplitude of the expected anomaly in the relative flux is about \( \Delta F \propto (R_p/R_\star)^2 \approx \Theta (\text{ppm}) \), which is an order-of-magnitude smaller than the observed one. We therefore analyze this anomaly assuming that it originated from the gravity darkening induced by stellar rotation, whose effect should not be localized but manifest during the whole transit duration.

Unlike the case of Kepler-13A, the anomaly in the transit light curve is not clear on a quarter-by-quarter basis for HAT-P-7. In addition, no TTVs/TDVs have been detected for this planet. For these reasons, we deal with the light curve obtained by folding all the available SC, PDCSAP fluxes (Q0–17) processed as described in Section 2.2. We use the spectroscopic constraint \( v \sin i_\lambda = 3.8 \pm 1.5 \text{ km s}^{-1} \) throughout this section. This value is based on Pál et al. (2008), though its error bar is enlarged to take into account other estimates for this quantity that give slightly different values (e.g., Winn et al. 2009a).

5.1. Robustness of the Observed Anomaly

If the observed anomaly is really due to gravity darkening, it should be persistent over the entire observation span. It is important to confirm the property because Morris et al. (2013) only reported the bump before the mid-transit time. Thus, we divide the transits into four consecutive groups (Q0–4, 5–9, 10–13, 14–17), phase-fold and fit each of them with the model without gravity darkening separately, and examine the shapes of the residuals. Although fewer numbers of transits lead to noisier phase-folded light curves, 10-minute binned residuals in the left column of Figure 7 exhibit a similar feature (brightening before mid-transit and dimming after it) in every span of data.

Besides, Van Eylen et al. (2013) reported seasonal variation in the transit depth depending on the quarter, which is reproduced in our analysis with Q0–17 data. To confirm that the anomaly is not an artifact related to this seasonal variation, we also perform a similar analysis as above but this time grouping the transits that have similar depths. As shown in the right column of Figure 7, we find that the same feature is apparent regardless of the season, and the anomaly is not affected by the systematic depth variation. For this reason, along with its unconstrained origin, we do not try to make corrections for this systematic in the following analyses.

5.2. Results

As in Section 3, we consider both the light-curve solution and the joint solution that takes into account the constraints from other observations. First, the light-curve solution is obtained with \( c_2 \) fixed to be zero (Figure 8, second and third columns in Table 3). In this case, we find two solutions with different signs of \( \cos i_\text{orb} \), which are indistinguishable in terms of the minimum \( \chi^2 \). The values quoted in Table 3 are the median, 15.87 percentile, and 84.13 percentile of the MCMC posteriors sampled with emcee (Foreman-Mackey et al. 2013). Our model reasonably reproduces the global feature of the anomaly (positive before the mid-transit and negative after it), yielding \( \Delta \chi^2 \approx 166 \) for \( \approx 420 \) degrees of freedom. We compute the Bayesian information criterion (BIC) for the best-fit models with and without gravity darkening and find \( \Delta \text{BIC} = 129 \), which formally indicates that the gravity-darkened model is strongly favored.

Our solution points to a nearly pole-on configuration with \( i_\lambda \approx 0^\circ \). This conclusion is consistent with the recent asteroseismic analyses by Benomar et al. (2014) and Lund et al. (2014), but the nominal constraint on \( i_\lambda \) from the gravity-darkened model is much tighter. On the other hand, \( \lambda \) is not
constrained very well with the light-curve asymmetry alone. The difficulty is inevitable in the pole-on configuration, where the brightness distribution on the stellar disk is almost axisymmetric even in the presence of gravity darkening. In such a case, \( \psi \) is always close to 90° regardless of \( \lambda \).

One remaining issue regarding our solution is that the resulting rotation frequency may be too large. Given the age \((\approx 2 \text{ Gyr})\) and \( B - V = 0.495 \pm 0.022; \text{Lund et al. 2014} \) of the host star, the rotation frequency from the light-curve solution, \( f_{\text{rot}} = 7.7 \pm 0.2 \mu \text{Hz} \) (equivalent to \( f_{\text{rot}} \approx 1.5 \text{ days} \)), is consistent with the gyrochronology relation by Meibom et al. (2009); see Section 6 of Lund et al. (2014). However, our value of \( f_{\text{rot}} \) is much larger than those from asteroseismology, \( 0.70^{+1.02}_{-0.43} \mu \text{Hz} \) (68% credible interval by Benomar et al. 2014) and \(<0.8748 \mu \text{Hz} \) (1\( \sigma \) upper limit by Lund et al. 2014). In fact, the prior used in these analyses, \( |f_{\text{rot}}| < 8 \mu \text{Hz} \), does not fully cover the range we investigate here with the gravity-darkened light curve. Still, the discrepancy is only weakly reduced even with the new analysis adopting the prior range extended up to 17 \( \mu \text{Hz} \), which yields \( f_{\text{rot}} = 0.82^{+2.02}_{-0.50} \mu \text{Hz} \) as the 68% credible interval (by courtesy of Othman Benomar; see also Benomar et al. 2014).

To examine if the gravity-darkened model is compatible with the seismic analysis, we then search for a joint solution including the constraints from both the RM measurement and asteroseismology. From the RM effect, we incorporate the constraint \( \lambda = 172^\circ \pm 32^\circ \), which comes from the average and standard deviation of the analyses for the three different radial velocity data (Benomar et al. 2014). From asteroseismology, we adopt the above updated posterior for \( f_{\text{rot}} \) as the prior, and we performed an MCMC sampling with emcee. To properly take into account the uncertainty from the limb-darkening profile, \( c_2 \) is also floated. The resulting credible intervals are summarized in the fourth and fifth columns in Table 3, and the model that maximizes the likelihood multiplied by the prior on \( f_{\text{rot}} \) is plotted with a dashed line in Figure 8. We again find two equally good solutions, both of which indicate nearly pole-on configurations with slightly prograde and retrograde orbits, \( \psi = 101^\circ \pm 2^\circ \) and \( \psi = 87^\circ \pm 2^\circ \). Although the resulting \( f_{\text{rot}} \) still prefers a higher rotation rate than that from asteroseismology, their difference is now mitigated to the 2\( \sigma \) level; we construct the probability distribution for \( \Delta f_{\text{rot}} \), \( f_{\text{rot}} \) from our joint analysis minus \( f_{\text{rot}} \) from asteroseismology, using their posteriors, and find its 2\( \sigma \) credible region as \( \Delta f_{\text{rot}} = 4.9^{+4.0}_{-3.0} \mu \text{Hz} \). We argue that the level of discrepancy is acceptable, considering that the rotational mode splitting is not clearly detected in the power spectrum of HAT-P-7’s light curves.

Finally, it is also worth considering the case where \( \beta = 0.25 \), given the unconstrained nature of the gravity darkening in F dwarfs. Smaller values of \( \beta \sim 0.08 \) are usually expected for solar-like stars with convective envelopes (e.g., Lucy 1967; Claret 1998), while Espinosa Lara & Rieutord (2011, 2012) argue that \( \beta \) is close to 0.25 in the limit of slow rotation under several assumptions. We repeat the above joint analysis floating \( \beta \) with the prior uniform between 0 and 0.3, and we obtain \( \beta = 0.26^{+0.03}_{-0.03} \) for both solutions in Table 3. On the one hand, this fact may support the claims by Espinosa Lara &
may explain the discrepancy between the solution by B11 and the Doppler tomography result by Johnson et al. (2014). Our new “joint solution” includes $i_s = 81^\circ \pm 5^\circ$, $\lambda = -59^\circ \pm 2^\circ$, $\psi = 60^\circ \pm 2^\circ$, and $P_{\text{rot}} = 24 \pm 2$ hr. Although the joint solution is compatible with all of the observations made so far, introducing the additional free parameter $c_2$ is not statistically justified, nor is it clear whether the best-fit value for $c_2$ is physically plausible.

To examine the above issues from a dynamical point of view, we also analyze the spin–orbit precession in this system. By analyzing the light curves from each quarter separately, we confirm that the variation in $|\cos i_{\text{ort}}|$ causes the TDVs first reported by Szabó et al. (2012), with the more elaborate model taking into account the gravity darkening. This variation is consistent with the precession of the stellar spin and orbital angular momenta around the total angular momentum of the system, induced by the oblateness of the rapidly rotating host star. We thus fit each transit with the gravity-darkened model to determine $\cos i_{\text{ort}}$, $\lambda$, and $i_s$ as a function of time, and then we fit them with the precession model to constrain the stellar quadrupole moment $J_2$. For the light-curve solution and the joint solution, we respectively find $J_2 = (1.66 \pm 0.08) \times 10^{-4}$ and $J_2 = (6.1 \pm 0.3) \times 10^{-5}$, which are different by a factor of a few. Our results predict detectable variations in $\lambda$ on a 10 yr timescale for the light-curve solution, while it should be almost constant for the joint solution. The difference suggests that the future follow-up observations can be used to confirm or refute the joint solution we proposed, as well as to improve the constraint on $J_2$.

6.2. HAT-P-7

Although the anomaly in the transit light curve is much more subtle compared to Kepler-13Ab, we confirm that the asymmetric residual (not only the bump reported by Morris et al. 2013 but also the dip) exists continuously in the transits of HAT-P-7b. Thus, we perform the analysis assuming that the gravity darkening is a viable explanation for the anomaly. The gravity-darkened transit model favors a nearly pole-on orbit ($\psi = 101^\circ \pm 2^\circ$ or $\psi = 87^\circ \pm 2^\circ$) and a gravity-darkening exponent $\beta$ close to 0.25. The constraint on $\psi$ is insensitive to the choice of the limb-darkening parameters or the gravity-darkening exponent.

On the other hand, the stellar rotation rate from the gravity-darkening analysis is about 2σ higher than the value from asteroseismology. In addition, the value of $\beta \approx 0.25$ we obtained may be too large for a star with a convective envelope. These facts, as well as the subtlness of the detected anomaly, may suggest some incompleteness in the current modeling or other origins for the anomaly and should be addressed in future studies.

7. CONCLUSION

Our present analysis reproduces the results by B11 with more data and thus strengthens the reliability of the gravity-darkening method for the spin–orbit angle determination. In contrast, we also find that the spin–orbit angle obtained from the gravity-darkened transit light curve strongly depends on the assumed limb-darkening profile. Depending on its choice, the resulting spin–orbit angle can vary by several tens of degrees. Thus, the reliable modeling of the limb-darkening effect is crucial for this method.

6. SUMMARY

6.1. Kepler-13A

First, we analyze the gravity-darkened transit light curve of Kepler-13A adopting the same model and stellar parameters as in the previous study by B11. We reproduce the spin–orbit angles obtained by B11 with more data (called the “light-curve solution” in this paper) and also find that the choice of the stellar mass, stellar effective temperature, $v\sin i_s$, or contaminated flux affects $\lambda$ or $i_s$ by less than about 10°. If we fit $c_2 = u_1 - u_2$ as well as $c_1 = u_1 + 2u_2$ in the quadratic limb-darkening law, on the other hand, a broader range of the spin–orbit angle is allowed. In fact, this additional degree of freedom
Nevertheless, if $\lambda$ is constrained from other observations, $i_\psi$ is well determined along with the limb-darkening parameters. Hence, the gravity-darkening method still provides valuable information on the true spin–orbit angle $\psi$, which is complementary to $\lambda$ from the RM effect or Doppler tomography. Indeed, such an example is already seen in an eclipsing binary system DI Her (Philippov & Rafikov 2013). In addition, synergy with asteroseismology is also promising because it constrains $f_{\text{rot}}$ and $i_\psi$, which are both essential in the modeling of gravity darkening. The joint analyses of these kinds may in turn help us to better understand the mechanisms of gravity darkening itself, since they enable the measurements of $\beta$ for stars not in close binary systems and hence free from the strong tidal distortion.

If combined with continuous, high-precision photometry as achievable with space-borne instruments, the gravity-darkening method also provides a way to monitor the angular momentum evolution in the system. Modeling of the spin–orbit precession allows us to access the internal structure of the rotating star through its quadrupole moment or moment of inertia. It is also possible to determine the three-dimensional configuration of the system from a dynamical point of view (see Barnes et al. 2013; Philippov & Rafikov 2013). Such information will be valuable in simulating the dynamical histories of individual systems to decipher the origin of the spin–orbit misalignment.

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Table 3
Results for the Transit of HAT-P-7 b

| Constraints | Light-curve Solution ($c_2 = 0$) | Joint Solution ($c_2$ Fitted) |
|-------------|----------------------------------|-------------------------------|
|             | Solution 1                       | Solution 2                    | Solution 1                       | Solution 2 |
| $v \sin i_\psi$ (km s$^{-1}$) | 3.8 ± 1.5                       | 3.8 ± 1.5                     | 3.8 ± 1.5                       | 3.8 ± 1.5 |
| $\lambda$    | ...                              | ...                           | 172 ± 32                        | 172 ± 32  |
| Fitting Parameters |                               |                               |                               |            |
| $M_\ast$ ($M_\odot$)   | 1.59 (fixed)                     | 1.59 (fixed)                  | 1.59 (fixed)                    | 1.59 (fixed)|
| $T_{\text{pole}}$ (K)   | 6310 (fixed)                     | 6310 (fixed)                  | 6310 (fixed)                    | 6310 (fixed)|
| $\rho_\ast$ (g cm$^{-3}$) | 0.2789 ± 0.0006                  | 0.2789 ± 0.0006               | 0.2790 ± 0.0005                 | 0.2784 ± 0.0005 |
| $c_1$          | 0.498 ± 0.003                    | 0.498 ± 0.003                 | 0.507± 0.008                    | 0.508± 0.007 |
| $c_2$          | 0 (fixed)                        | 0 (fixed)                     | 0.07± 0.06                      | 0.06± 0.05  |
| $t_c$ (10$^{-5}$ days)$^a$ | -1.5 ± 0.4                      | -1.5 ± 0.4                    | -1.6 ± 0.4                      | -1.1 ± 0.4  |
| $P$ (day)      | 2.04735471 (fixed)               |                               |                               |            |
| $\cos i_{\text{lab}}$ | -0.1195 ± 0.0004                 | 0.1195 ± 0.0004               | -0.1194 ± 0.0003                | 0.1198 ± 0.0003 |
| $R_{\text{eff}}/R_\ast$ | 0.07757$^{+0.00005}_{-0.00009}$ | 0.07757$^{+0.00005}_{-0.00009}$ | 0.07759 ± 0.00003               | 0.07749$^{+0.00003}_{-0.00004}$ |
| $F_0$          | 0.9999998 ± 0.000005             |                               |                               |            |
| $f_{\text{rot}}$ (µHz)  | 7.7 ± 0.2                        | 7.7 ± 0.2                     | 6.1$^{+2.62}_{-1.7}$            | 5.6$^{+2.48}_{-1.7}$ |
| $i_\psi$ (deg)$^b$   | 3.3$^{+1.3}_{-1.0}$              | 3.3$^{+3.3}_{-1.0}$           | 5.3$^{+3.3}_{-2.0}$             | 5.3$^{+3.7}_{-2.1}$ |
| $\lambda$         | 133$^{+11}_{-88}$                | 142$^{+12}_{-16}$             | 136$^{+11}_{-16}$               |            |
| $\beta$          | 0.25 (fixed)                     | 0.25 (fixed)                  | 0.25 (fixed)                    | 0.25 (fixed)|
| Derived Parameters |                               |                               |                               |            |
| $R_{\text{rot}}$ (day) | 1.51 ± 0.03                     | 1.51 ± 0.03                   | 1.9$^{+0.7}_{-0.6}$             | 2.1$^{+0.9}_{-0.6}$ |
| $\psi$ (deg)      | 99$^{+1}_{-2}$                   | 81$^{+1}_{-2}$                | 101 ± 2                        | 87 ± 2     |
| Impact Parameter   | 0.496 ± 0.001                    | 0.496 ± 0.001                 | 0.496 ± 0.001                   | 0.497 ± 0.001 |
| Stellar Oblateness | 0.0149 ± 0.0006                  | 0.0149 ± 0.0007               | 0.009± 0.003                    | 0.008± 0.004 |
| $\chi^2_{\text{min}}$/dof | 453/424                        | 455/424                       | 450/424                        | 451/424    |

Note. The quoted values and uncertainties are 50, 15.87, and 84.13 percentiles of the marginalized MCMC posteriors. For the light-curve solution, $\chi^2_{\text{min}}$ is the value of $\chi^2$ computed from Equation (4) for the maximum likelihood model. Equation (4) is also used for the joint solution, but $\chi^2_{\text{min}}$ in this case is computed for the model that maximizes the likelihood function multiplied by the prior on $f_{\text{rot}}$.

$^a$ Measured from the transit epoch $t_0$(BJD − 2,454,833) = 120.358522 ± 0.000005 obtained with the transit model without gravity darkening.

$^b$ Posterior from the seismic analysis is used as the prior.

$^c$ We impose the prior uniform in $\cos i_\psi$, rather than in $i_\psi$, which corresponds to the isotropic distribution for the spin direction.
