Differential fluid mechanics – basis of the theory of flows with combustion

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Abstract. The basis of invariant models is a system of fundamental equations with an empirical formula for the Gibbs potential and density. This paper presents an analysis of fast direct atomic-molecular, mechanical with a flow or wavy with a group velocity, and slow diffusion transfer of energy. The classification of components, including ligaments, waves, and vortices is given based on the compatibility condition, which determines the rank and the order of the linearized version of equations. The requirements for numerical and laboratory modeling are formulated. The structure of the flow near the strip and the rotating disk is studied. Comparison of calculations with the data of experiments carried out at the stands of the Unique science facilities “GFK IPMech RAS” is given. Expanding developed approaches to flows with combustion is discussed.

1. Introduction

Based on the definition of Mathematics as an axiomatic science concerning the principles of prescribing the content of symbols, rules of operations, accuracy control criteria and condition of implementation in fluid mechanics, as well as the rules of constructing a rational theory of physical phenomena by J.C. Maxwell [1], a fluid is defined as a deformable medium whose properties are characterized by equations of state for potentials (the Gibbs potential $G = G(x, t)$ [2] is chosen as the base) and their derivatives - density $\rho$, pressure $P$, temperature $T$, the concentration of dissolved substances (suspended particles or small drops) $S_i$ as functions of coordinates $x$ and time $t$. Flows are defined as the intrinsic or forced transfer of momentum $p$, a density of a substance $\rho$, a concentration of components $S_i$, and energy $E_i$, which determines the interaction of the fluid with the body. Under natural conditions and technical systems, the thermodynamic potentials and physical parameters of the medium are distributed unevenly. In the field of external mass forces, gravitational or inertial separation of the media occurs and stratification is formed, which is characterized by the frequency $N = \sqrt{g \left[ \ln \rho / \ln \delta \right]}$ and period of buoyancy $T_b = 2\pi / N$. Stratified (strongly $N \sim 1 \text{ c}^{-1}$ or weakly $N \sim 10^{-2} \text{ c}^{-1}$), as well as potentially ($N \sim 10^{-5} \text{ c}^{-1}$) and actually ($N = 0$) homogeneous media.

The greatest changes in potentials and other quantities are observed in the vicinity of the liquid boundaries, where available potential surface energy is concentrated in a layer of the order of the molecular cluster thickness $\delta \sim 10^{-9} \text{ m}$ and in regions with large temperature and concentration gradients. In flows of fluids and gases, energy is changed directly by atomic-molecular processes (during the elimination of a free surface during the merging of fluids, in chemical reactions, combustion), it is transported at the velocity of flows $U$, at the group velocity of waves $c_g$ and most slowly under the action of dissipative processes. Each of the simultaneously operating mechanisms is characterized by its own spatio-temporal scales, which occupy a wide range. The least studied are the mechanisms of energy transfer from atomic-molecular scales $\delta \sim 10^{-10} \text{ m}, t \sim 10^{-10} \text{ s}$, to macroscopic $L_m \sim 1 \text{ m}, t_m \sim 1 \text{ c}$.
2. Mathematical model

The flows are described by systems of fundamental equations for the transport of density, energy (temperature) and the concentration of components, including both molecular and diffusion transport with the corresponding kinetic coefficients [3], which are not given here for brevity. The system of equations is supplemented by physically justified initial and boundary conditions and is analyzed taking into account the compatibility condition, which defines the rank of the complete system, the order of the linearized version, and the degree of the characteristic (dispersion) equation [4] in the approximation of smallness of dissipative parameters.

The basic classification of the structural components of flows constructed on an analysis of the solutions of complete systems of fundamental equations includes ligaments, waves, vortices, wakes and jets. Basic ligaments are described by a group of singularly perturbed solutions of a linearized system of fundamental equations. The transverse scales of the ligaments $\delta$ are determined by the dissipative properties of the medium (coefficients of kinematic viscosity $\nu$, temperature diffusivity $\kappa_T$ or dissolves substance diffusion $\kappa_S$) and the characteristic values of the temporal variability - the time it took to establish the process $\Delta t$, the wave frequency $\omega$, or the momentum and energy transfer velocity $U$:

$$\delta_{T}^{\nu} = \sqrt{\nu \Delta t}, \quad \delta_{\omega}^{\nu} = \sqrt{\nu / \omega}, \quad \delta_{U}^{\nu} = \nu / U.$$

The number of ligaments is determined by the form of the equation of state and is equal to four in the case of taking into account the density variability only, as well as six and eight if thermal conductivity and diffusion (Fig. 1) are included. The density variability introduction gives room to calculate 3D flows over the entire range of parameters as the well-posed problem. The homogeneous density approximation leads to the degeneration of the fundamental system with respect to singular components. Ligaments exist in both fast and slow currents. A large number of coexisting components with incommensurable parameters explains the unsteadiness of the flows. In the experiment, ligaments correspond to thin shells, interfaces and fibers that draw the structure of flows.

![Fig. 1. Classification of flow components](image_url)

The spatial and temporal analysis of the structural components was used in the formulations of technical conditions for stands of USF “HPC IPMech RAS” [5] and in the development of codes for numerical calculation of flows, taking into account the resolution conditions for thin structures of unsteady flows, supplementing the well-known Courant criterion [6].

At this stage, attention was paid to the numerical and experimental study of the flow around the plate, both with sharp and smooth edges. A reduced system of equations was used, in which the
density $\rho$ depends only on salinity $S$ and $\rho_0(z) = \rho_{oo} S(z) = \rho_{oo} (-z/\Lambda)$ (in the experiments, table salt was used)

$$
\rho = \rho_0 + \rho_{oo} \cdot s, \quad \text{div} \, v = 0,
$$

$$
\frac{\partial v}{\partial t} + \nabla \cdot (v \, v) = -\frac{1}{\rho_{oo}} \nabla P + \nabla \cdot (v \nabla v) - s \cdot g, \quad (1)
$$

with physically justified initial and boundary conditions of no-slip for velocity and no-flux for matter [6]. Here $N = \sqrt{R/\Lambda}$, $T_b = 2\pi / N$, $\Lambda = [d \ln \rho / dz]^{-1}$ are the frequency, period and scale of buoyancy, $v$ is velocity, $P$ is pressure, $g$ is the acceleration of gravity, $\nu$ and $\kappa$ are the kinematic viscosity and diffusion coefficients, $s$ is perturbations of salinity. The system of equations (1) contains a number of eigen-scales, as linear $\Lambda$, $h$, $L$, $W$ (scale of stratification and dimensions of a body), $\lambda = U T_b$ is length of the attached internal wave, $U$ is the body velocity, Stokes $\delta^v_U = \sqrt{v/N}$ and $\delta^s_U = \sqrt{\kappa_s / N}$ or Prandtl’s $\delta^v_U = v / N$ $\delta^s_U = \kappa_s / U$ length scales. Temporal scales include the buoyancy period $T_b$, parameters of the body motion $T_k = L / U$, $T_g = U / g$, and of the fine structure that is $\tau^v_U = \nu / U^2$ viscous and diffusive $\tau^s_U = \kappa / U^2$ length scales. System (1) allows to calculate all flow parameters, forces, and moments without involving additional hypotheses or simplifications. Calculation methods and experiments are developed taking into account intrinsic scales of studied flows. Of practical interest are the calculations of the fields of the observed quantities, in particular, the density gradients visualized by schlieren instruments.

As can be seen in the calculated flow pattern shown in Fig. 2, in the vicinity of the leading edge of the uniformly moving plate, all structural components, both large and small, with incommensurable parameters, are simultaneously formed. The interaction of many components (eight in a linear description) is manifested in the unsteadiness and permanent evolution of the flow, in which systems of vortices run along a streamlined surface and form a clear trace structure.

![Fig. 2. The field of the horizontal component of the density gradient near the leading edge of the inclined plate in a highly stratified fluid: $L = 0.1 \text{ m}$, $h = 0.05 \text{ m}$, $\alpha = 5^\circ$, $U = 1 \text{ m/s}$](image)

A physically justified mathematical formulation of the problem allows one to calculate flows in a unique formulation without involving additional hypotheses in the entire range of the problem parameters while the Prandtl’s microscale exceeds the size of the molecular cluster $\delta^v_U >> \delta_c \sim 10^{-9}$ m. At high body velocities, it is necessary to consider the processes of energy exchange between the kinetic and internal components in more detail. Calculations carried out in a two-dimensional formulation of flow around a strip of various shapes moving in a homogeneous and stratified fluid showed good agreement with experiments.
3. Comparison of calculations with the experiment

As an example Fig. 3 shows the calculation of the field of the horizontal component of the density gradient and schlieren visualization of the flow pattern (the values of the density and index of refraction are related by a linear relation here).

In the calculated and observed patterns of flows behind the vertical plate, all structural components are represented – upstream disturbance, unsteady and attached internal waves, wake, vortices and ligaments that are high gradient interfaces that outline the wake and visualize vortex geometry. The calculation and observation parameters are in good agreement [7].

The multiscale character is preserved in a growing vortex flow, which is formed around a disk that starts uniform rotation with a constant angular velocity in a continuously stratified fluid shown in Fig. 4 at \( t = 2.4 \) s after the start of the motion [8].

![Fig. 3. The schlieren image of the horizontal component of the density gradient (left) and the calculated flow pattern around the vertical plate](image1)

\[ h = 2.5 \text{ cm}, \ T_h = 12.5 \text{ c}, U = 0.75 \text{ cm/c}, \ Re = 187.5,\]
\[ Fr = U / ND = 0.6 \]

Here, the toroidal annular flow breaks up into a system of transverse vortex systems, all of whose elements are contoured by ligaments — thin interfaces and filaments, in which the density gradient substantially exceeds that existing in the undisturbed stratified medium.

![Fig. 4. Schlieren visualization of the vortex flow beyond the edge of a rotating vertical disk in the stratified fluid and its individual vortex components](image2)

\[ T_h = 8 \text{ s}, \ R = 6 \text{ cm}, \ \Omega = 3 \text{ s}^{-1}, \ Re = 10000, \]
\[ Fr = 3.8, \ t = 2.4 \text{ s} \]

The temperature inhomogeneity effects significantly complicate the mathematical description of flows and increase the number of co-existing ligaments [4], which are experimentally visualized in the form of thin filaments and interfaces that regularize the flow pattern in Fig. 5.
Fig. 5. Ligaments, cells and internal waves during side-wall heating of a uniformly stratified fluid at $t = 2, 5, 13, 30$ min

Calculations of the convective flow during lateral heating of a stratified fluid based on a system of fundamental equations [9] are in good agreement with the schlieren imaging of the side heated convective flow shown (overheat temperature $\Delta T = 1.3^\circ$C of a fluid with a buoyancy scale $\Lambda = 13$ m). The height of the cells is equal to $h = \alpha \Delta T \Lambda$ ($\alpha$ - coefficient of thermal expansion.

In energy-loaded flows, the nature of the processes substantially depends on the rate of conversion of potential surface or chemical energy in thin ligaments in the contact areas of dissimilar media.

4. Conclusion

A scaled invariant system of fundamental equations of fluid mechanics, including equations of state for the Gibbs potential and density of the medium, is the natural basis of a coordinated experimental and theoretical study of flows.

Ligaments, with scales ranging from atomic-molecular to macroscopic sizes, are the most universal components of flows.

All energy transfer processes, both fast - atomic-molecular -, and slow - translational or dissipative, forming thin ligaments, vortices, jets, wakes, and all types of waves, play an important role in the formation of flows.

The current state of techniques for computational and experimental mechanics allows us to develop consistent models of combustion processes based on the complete solution of a system of fundamental equations, including ligaments, on which primary chemical reactions proceed.

5. Acknowledgments

This work was partially supported by the RFBR grant No. 18-05-00870. The experiments were performed at the stands of USF “HPC IPMech RAS”. The calculations were carried out jointly with Ya.V. Zagumenny (IHM NAS of Ukraine) using high-performance computing resources of the Scientific and Research Center of M.V. Lomonosov Moscow State University.

6. References

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