A dynamical Origin of Little Higgs
- Hidden Local Symmetry in Large $N_f$ QCD -

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ABSTRACT

In this write-up I summarize the main points of our recent work, in which we showed that the large flavor QCD is regarded as a dynamical origin of little Higgs, and that the mass of such a little Higgs becomes actually very small.

1. Introduction

In little Higgs models [1], the Higgs is introduced as pseudo Nambu-Goldstone (NG) boson, which naturally explains the lightness of Higgs boson.

In Ref. [2], we pointed out that it is easy to formulate little Higgs models using the hidden local symmetry (HLS). We picked up a little Higgs model with two sites and two links, where one site corresponds to the U(1) gauge symmetry and another to the SU($N_f$) gauge symmetry. When the U(1) gauge symmetry is a subgroup of the chiral SU($N_f$)$_L \times$SU($N_f$)$_R$ symmetry, the above little higgs model is equivalent to the HLS with the parameter choice $a = 1$ taken. Based on this, we showed that the large flavor QCD is regarded as a dynamical origin of little Higgs, and that the mass of such a little Higgs becomes actually very small near the chiral symmetry restoration point. In this write-up, I will show main points of our work.

2. Hidden Local Symmetry

The HLS model [3] is based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = \text{SU}(N_f)_L \times \text{SU}(N_f)_R$ is the global chiral symmetry and $H = \text{SU}(N_f)_V$ the HLS whose gauge bosons are identified with the vector mesons (the $\rho$ meson and its flavor partners) denoted as $V_\mu$. Here $N_f$ denotes the number of massless quark flavors in the underlying QCD. The basic dynamical variables in the HLS model are gauge bosons $\rho_\mu = \rho_\mu^a T_a$ of the HLS and two SU($N_f$)-matrix-valued variables $\xi_L$ and $\xi_R$. These can be parameterized as $\xi_{L,R} = e^{i\sigma/F_\sigma} e^{i\pi/F_\pi}$, where $\pi = \pi^a T_a$ denotes the NG bosons (π meson and its flavor partners) associated with the spontaneous breaking of $G$ and $\sigma = \sigma^a T_a$ (with $J^{PC} = 0^{+-}$) the NG bosons absorbed into the (longitudinal) HLS gauge bosons. $F_\pi$ and $F_\sigma$ are the relevant decay constants, with a ratio $a$ defined by $a \equiv F_\sigma^2/F_\pi^2$.

The Lagrangian of the HLS is expressed as

$$\mathcal{L} = \frac{a + 1}{4} F_\pi^2 \text{tr} [\tilde{\alpha}_L \tilde{\alpha}_L^\mu + \tilde{\alpha}_R \tilde{\alpha}_R^\mu] + \frac{a - 1}{2} F_\pi^2 \text{tr} [\tilde{\alpha}_L \tilde{\alpha}_R^\mu] - \frac{1}{2} \text{tr} [V_\mu V^{\mu\nu}] ,$$

(1)

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where the 1-forms $\hat{\alpha}_L^\mu$, $\hat{\alpha}_R^\mu$ are defined as $\hat{\alpha}_L^\mu, \hat{\alpha}_R^\mu = -iD^\mu \xi_{L,R} \cdot \xi_{L,R}^\dagger$ with the covariant derivatives given by $D^\mu \xi_{L,R} = (\partial^\mu - igV_\mu) \xi_{L,R}$. I would like to stress that, when we take $a = 1$ in the above Lagrangian, the second term disappears, and the fields in $\xi_L$ couples to those in $\xi_R$ only through the HLS gauge boson $V_\mu$, which is nothing but the theory space locality. It should be noticed that the parameter choice $a = 1$ together with $g = 0$ corresponds to the Georgi’s “vector limit” [4].

3. Stability of theory space locality

When we consider quantum corrections, it is essential to have a systematic expansion. Fortunately, a systematic chiral perturbation can be done in the HLS including the dynamical effect of the HLS gauge boson [4,5,6,7]. This chiral perturbation is actually justified in the large $N_c$ limit of QCD, where the pion decay constant $F_\pi$ becomes large while the vector meson mass $m_V$ is kept fixed. As a result, the ratio $m_V/4\pi F_\pi$ becomes small in the large $N_c$ QCD. In other words, the HLS gauge coupling becomes small in the large $N_c$ QCD.

One may think that the scalar meson should be included since it is lighter than the vector meson in real-life QCD. However, recently in Ref. [8], we extended the analysis adopted in Refs. [9,10] for studying the $\pi$-$\pi$ scattering in the real-life QCD to the one in the large $N_c$ QCD, and showed that, for $N_c \geq 6$, the unitarity in the scalar channel of the $\pi$-$\pi$ scattering is satisfied without scalar mesons up until the energy scale of $4\pi F_\pi$. This indicates that we do not need the scalar meson in the low-energy region in the large $N_c$ QCD. In the real-life QCD, we know that there is a light scalar meson ($\sigma$ meson), which is actually very broad. I expect that loop corrections from such broad resonances are very small, and that the chiral perturbation in the HLS is still possible, as far as we do not see the scalar channel.

I should note that there is no guarantee for the smallness of $(a - 1)$ in the large $N_c$ QCD. In other words, the theory space locality may not be justified in the large $N_c$ QCD. However, since $a = 1$ is a fixed point of the renormalization group equation for the parameter $a$ [7], there is no divergent correction to $a = 1$ at one-loop level. In Ref. [11], on the other hand, we have shown that there exists a small finite correction when we take non-zero gauge coupling. These results imply that, once the parameter $a$ is tuned to be 1, there is only small violation of the theory space locality even at two-loop level, i.e., the theory space locality is quite stable.

4. Little Higgs in Large $N_f$ QCD

It is well-known that, in the large flavor QCD, the chiral symmetry restoration occurs at some critical number of flavors $N_f^{\text{crit}}$ [12]. In Ref. [13], we showed that, due to the vector manifestation, the parameters of the HLS $g$ and $a$ approach the fixed point values of the RGEs near the critical point, i.e., $(g, a) \to (0, 1)$ for $N_f \to N_f^{\text{crit}}$.

Since the parameter $a$ approaches 1 near the critical point, the theory space locality is actually guaranteed near the restoration point. This implies that the large flavor QCD can be regarded as an ultraviolet (UV) completion of a little Higgs model. In Ref. [2],
we calculated the mass of the little Higgs in the large flavor QCD by gauging the U(1) subgroup of the chiral symmetry. The generator of the U(1) gauge symmetry and the generator corresponding to the little Higgs (pseudo NG boson) are given by

\[
Q = \begin{pmatrix} 2/3 & -1/3 & \cdots \\
\cdots & \cdots & \cdots \\
\end{pmatrix}, \quad T_{PN} = \begin{pmatrix} 0 & 1 & 0 & \cdots \\
0 & 0 & 0 & \cdots \\
0 & 0 & \cdots & \cdots \\
\vdots & \vdots & \cdots & \cdots \\
\end{pmatrix}.
\]

(2)

It should be noticed that, when we set the parameter \(a\) to 1, the HLS theory becomes a little Higgs model with two sites and two links.

The loop correction to the mass of the little Higgs in the HLS is given by

\[
m_{H}^{2}\bigg|_{HLS} = \frac{\alpha_{U(1)}}{4\pi} \left[ (1 - a)\Lambda^{2} + 3aM_{\rho}^{2}\ln\Lambda^{2} + \cdots \right].
\]

(3)

Here, I would like to stress that the quadratic divergence disappears when we take \(a = 1\), due to the theory space locality, and the mass is expressed as

\[
m_{H}^{2}\bigg|_{HLS} = \alpha_{U(1)}\alpha_{HLS} 3F_{\pi}^{2}\ln\Lambda^{2}.
\]

(4)

In several little Higgs models, this is the end of story. In the present case, however, I consider that the little Higgs model is a low energy effective field theory derived from the large flavor QCD. As a result, there is a remnant of the underlying theory, which gives a contribution to the mass of the little Higgs. In our work, we calculated the UV contribution by using the current algebra formula [14,15]:

\[
m_{H}^{2}\bigg|_{UV} = \frac{3\alpha_{U(1)}}{4\pi F_{\pi}} \int_{\Lambda}^{\infty} dQ^{2} \left[ \Pi_{V}(Q^{2}) - \Pi_{A}(Q^{2}) \right] = \alpha_{U(1)}\alpha_{QCD} \frac{3(N_{c}^{2} - 1)}{N_{c}^{2}} \frac{\langle \bar{q}q \rangle_{A}^{2}}{F_{\pi}^{2}\Lambda^{2}},
\]

where \(\Pi_{V}\) and \(\Pi_{A}\) are the vector and axial vector current correlators. The second expression is obtained by substituting the current correlators derived in the operator product expansion. The mass of the little Higgs is given by the sum of the UV contribution in Eq. (5) and the low-energy contribution in Eq. (3).

The mass of the little Higgs (pseudo NG boson) in the setup given in Eq. (2) becomes the \(\pi^{+}-\pi^{0}\) mass difference in the real-life QCD, which is estimated as

\[
\Delta m_{\pi}^{2} = m_{H}^{2}\bigg|_{HLS} + m_{H}^{2}\bigg|_{UV} \simeq 1000 \text{ MeV}^{2} + 200 \text{ MeV}^{2} = 1200 \text{ MeV}^{2}.
\]

(6)

The predicted value is very close to the experimental value, \(\Delta m_{\pi}^{2}\big|_{\text{exp}} = 1261 \text{ MeV}^{2}\), which shows the validity of our calculation.

For making the estimation of the mass of the little Higgs in the large flavor QCD, we used the following scaling properties of the parameters in the large flavor QCD:

\[
F_{\pi} \sim m_{\text{dyn}}, \quad M_{\rho} \sim gF_{\pi} \sim \langle \bar{q}q \rangle F_{\pi} \sim m_{\text{dyn}}^{4-\gamma_{m}}, \quad (a - 1) \sim \langle \bar{q}q \rangle \sim m_{\text{dyn}}^{6-2\gamma_{m}},
\]

(7)

where \(m_{\text{dyn}}\) is the dynamical fermion mass. \(\gamma_{m}\) is the anomalous dimension of the fermion mass operator, which satisfies \(\gamma_{m} \lesssim 1\) in the large flavor QCD. Substituting these scaling
properties into the sum of the contributions in Eqs. (5) and (3), one can easily show that
the UV contribution becomes dominant near the critical point. Then, the mass of little
Higgs scales as $m_H^2 \sim m_{\text{dyn}}^{4-2\gamma m}$. As a result, the scaling property of the ratio of the little
Higgs mass to the new physics scale is given by

$$\frac{m_H}{4\pi F_\pi} \sim m_{\text{dyn}}^{1-\gamma m} \rightarrow \begin{cases} 0 & \text{for } \gamma m < 1, \\ 0.01 & \text{for } \gamma m = 1 \end{cases}$$

(8)

where the estimation for $\gamma m = 1$ is done numerically. This implies that the large hierarchy
between the mass of little Higgs and that the scale of new physics can be easily achieved
in the large flavor QCD.

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6. References

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