Sensitivity of Key Parameters in Aerodynamic Wind Turbine Rotor Design on Power and Energy Performance

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Abstract. In this paper the influence of different key parameters in aerodynamic wind turbine rotor design on the power efficiency, $C_p$, and energy production has been investigated. The work was divided into an analysis of 2D airfoils/blade sections and of entire rotors. In the analysis of the 2D airfoils it was seen that there was a maximum of the local $C_p$ for airfoils with finite maximum $c/l/c_d$ values. The local speed ratio should be between 2.4 and 3.8 for airfoils with maximum $c/l/c_d$ between 50 and 200, respectively, to obtain maximum local $C_p$. Also, the investigation showed that $Re$ had a significant impact on $C_p$ and especially for $Re<2mio$ corresponding to rotors below approximately 400kW this impact was pronounced. The investigation of $C_p$ for rotors was made with three blades and showed that with the assumption of constant maximum $c/l/c_d$ along the entire blade, the design tip speed ratio changed from $X=6$ to $X=12$ for $c/l/c_d=50$ and $c/l/c_d=200$, respectively, with corresponding values of maximum $C_p$ of 0.46 and 0.525. An analysis of existing rotors re-designed with new airfoils but maintaining the absolute thickness distribution to maintain the stiffness showed that big rotors are more aerodynamic efficient than small rotors caused by higher $Re$. It also showed that the design tip speed ratio was very dependent on the rotor size and on the assumptions of the airfoil flow being fully turbulent (contaminated airfoil) or free transitional (clean airfoil). The investigations showed that rotors with diameter $D=1.75m$, should be designed for $X$ around 5.5, whereas rotors with diameter $D=126m$, should be designed for $X$ between 6.5 and 8.5, depending on the airfoil performance.

1. Introduction
With the increasing size of wind turbines and the increasing volume of wind turbines the importance of the exact prediction of power and load performance has become even more important. Several attempts of improving the performance have been made using different airfoil designs, different philosophies in the rotor design and the control. The different rotor design philosophies are, e.g. high design lift versus low design lift, large versus small root chord, winglets versus no winglets and (active) stall regulation with constant rotor speed versus pitch regulation with variable rotor speed.

Several researchers have contributed to the insight into rotor design. Snel [1,2] describes wind turbine aerodynamics in general and gives an overview of the available methods to compute the aerodynamic rotor performance. Fuglsang [3] describes the methods needed in the rotor design process in terms of a guideline and Tangler [4] gives a short historical overview of the rotor design investigations. Also, aerodynamic optimization of rotors are described by Fuglsang and Madsen [5], Giguere and Selig [6]
and Nygaard [7]. Details in rotor design to increase the power efficiency have been investigated by several researchers, e.g. Johansen et al. [8] and Madsen et al. [9] investigated especially the root part of rotors and found that a new root design of the rotors did not increase the power performance significantly. However, the interaction between different parameters in the rotor design, such as Tip-Speed-Ratio, Reynolds number etc. still needs to be investigated simultaneously when addressing design of rotors with maximum power performance.

This paper describes an analysis carried out on wind turbine rotors ranging from 1kW of the size 1.75m in diameter to 5MW of the size 126m in diameter, where the Reynolds number, tip-speed-ratio, the assumption of clean airfoils versus contaminated airfoils and large versus small blade root chords are investigated. Thus, the performance on the small scales, which is in the order of the airfoil chord scale, and the performance on the large scales, which is in the order of the rotor diameter, are all included. The analysis is divided into an investigation of different airfoils and their performance in two dimensions (2D) and an investigation of rotors in three dimensions (3D) based on different design parameters and using the blade element momentum (BEM) method.

2. Analysis of 2D airfoils

2.1. Blade element momentum (BEM) method applied on a 2D airfoil

Maximum power performance is desired in aerodynamic rotor design. The power performance is commonly expressed in terms of the power coefficient, \( C_P \), and can for an annular element of the rotor disc be written as:

\[
C_P = \frac{dF_{\text{driving}}}{\frac{1}{2} \rho V_0^2 dA}
\]

Eqn. (1) can be written as:

\[
C_P = [(1-a)^2 + x^2 (1+a')^2] xc_x \sigma
\]

In the relations above the axial and tangential induction, respectively, are computed with the tip loss correction \( F=1 \) in the 2D analysis. Analyzing the momentum theory which is the basis of the BEM method shows that the power coefficient \( C_P \) is maximized if the axial induction \( a = 1/3 \). In this investigation of rotor performance this is assumed to be valid even though advanced computational techniques show that the axial induction should be somewhat higher to obtain maximum power efficiency. With \( a = 1/3 \) equation (2) turns into:

\[
C_P = \left[ \frac{4}{9} + x^2 (1+a')^2 \right] xc_x \sigma
\]

Where \( x \) is chosen and \( a' \) and \( c_x \) is computed and \( \sigma \) is computed through the axial induction, \( a \), assuming that \( a = 1/3 \):

\[
\sigma = \frac{2 \sin^2 \phi F}{c_y}
\]

These equations are solved iteratively because of the dependency between \( a', c_x, C_y, \phi \) and \( \sigma \). In the 2D airfoil analysis the aerodynamic performance of airfoils is expressed in terms of the lift-drag ratio, \( c_l/c_d \), and the corresponding \( c_l' \): \n
\[
c_x = c_l' (\sin \phi - \frac{1}{c_l'/c_d} \cos \phi)
\]

\[
c_y = c_l' (\cos \phi + \frac{1}{c_l'/c_d} \sin \phi)
\]

Investigating the 2D airfoil characteristics for an airfoil, eqn. (5), shows that maximum \( C_P \) is found where \( c_l/c_d \) is maximum. Comparisons of 2D airfoils with different \( c_l/c_d \) ratios are shown in Figure 1.
The plot shows five different airfoils with $c/l/c_d=50, 100, 150, 200$ and $\infty$ (inviscid flow). The different $c/l/c_d$ values are ranging from rather poor airfoil efficiency ($c/l/c_d=50$) to very good airfoil efficiency ($c/l/c_d=200$). The upper limit for the efficiency is shown by the curve corresponding to inviscid flow i.e. no viscous effects in the flow, $c_{\text{visc}}=0$. It is seen that $C_P$ approaches the Betz limit for high $x$ when inviscid flow is assumed. However, for realistic ratios of $c/l/c_d$ there are maxima for $C_P$ at different $x$ depending on the value of $c/l/c_d$. Thus, maximum $C_P$ is found around $x=2.4$ for $c/l/c_d=50$ and around $x=3.8$ for $c/l/c_d=200$. Below the plot three airfoil sections are seen reflecting the solidity and the twist of the airfoil corresponding to $x=2, 4$ and $6$, respectively. These changes in solidity and twist can be seen on wind turbine blades because $x$ changes from the tip speed ratio at the tip to zero at the root. Thus, at the inner part of a blade the chord and twist are large and at the tip they are small.

![Figure 1](image1.jpg)

Figure 1 $C_P$ versus local speed ratio, $x$, for different $c/l/c_d$ ratios. Below the plot the size in terms of solidity, $\sigma$, and the twist of airfoils corresponding to $x=2, 4$ and $6$ are seen.

In Figure 2 the different terms in the calculation of $C_P$, eqn. (3), are seen for $c/l/c_d=150$.

![Figure 2](image2.jpg)

Figure 2 Plot of the different terms in the calculation of $C_P$, eqn. (3) for $c/l/c_d=150$. 

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It is seen that the driving force coefficient ($c_x$) and the solidity ($\sigma$) are decreasing as the local speed ratio ($x$) is increasing, whereas the normalized relative velocity ($4/9 + x^2(1+a')^2$) and the local speed ratio ($x$) are increasing as the local speed ratio ($x$) is increasing. The product of the four curves is $C_p$, which is also seen on the plot and has a maximum at around $x=3.4$ for this $c_l/c_d$ value.

2.2. Reynolds number dependency and roughness sensitivity

Wind turbines of today are manufactured from around 1kW (around 2m rotor diameter) to 5MW (around 125m rotor diameter). Because the size of the rotors and thereby the chord length of each blade section varies with the turbine size also the Reynolds number ($Re$) and thereby the airfoil performance varies with wind turbine size. For small wind turbines $Re$ is 200,000 or below whereas it is 6,000,000 or more for the largest wind turbines.

The aerodynamic performances for several airfoil families have been investigated using the panel code XFOIL by Drela [10]. In Figure 3 the maximum $c_l/c_d$ as function of $Re$ are seen for the Risø-B1 airfoil family designed for wind turbine rotors. This family is used as an example. Other airfoil families show the same trend where maximum $c_l/c_d$ increases a lot until around $Re=2,000,000$. For $Re$ between 2,000,000 and 9,000,000 maximum $c_l/c_d$ still increases, but at a slower rate. Above $Re=9,000,000$ the increase in maximum $c_l/c_d$ is small.

Two assumptions were made:

1) Fully turbulent flow, which probably under predicts the performance because the predicted $c_d$ is too high. This assumption commonly simulates roughness at the leading edge caused e.g. by bugs or dust. However, in the case of contamination of the airfoils there is a risk that $c_{l,max}$ will reduce more than predicted by this assumption.

2) Free transition, which probably over predicts the performance because $c_d$ is somewhat too low and $c_l$ including $c_{l,max}$ is somewhat too high. However, if the flow has a low turbulence intensity at small scales, i.e. at the size of an airfoil chord, and the airfoils are clean with no roughness at the leading edge, the performance will probably be close to the one predicted with free transition.

The two assumptions as shown in the plot express the uncertainty in the airfoil performance concerning the sensitivity to leading edge roughness.

![Figure 3 Maximum $c_l/c_d$ ratio versus $Re$ for the Risø-B1 family. Lines with dots show XFOIL computations assuming turbulence from the very leading edge. Lines without dots show XFOIL computations assuming free transition with the $e^n$ model with $n=9$.](image-url)
Comparing maximum $c/l_d$ variations with the relative thickness at constant $Re$ shows that maximum $c/l_d$ scales more or less with the relative thickness. For fully turbulent flow at $Re=6,000,000$ the maximum $c/l_d$ is reduced from 106 for the Risø-B1-15 ($t/c=15\%$) to $c/l_d=50$ for the Risø-B1-36 ($t/c=36\%$). This corresponds to a reduction in local $C_P$ for the airfoil from around 0.55 to 0.53, see Figure 1. If the airfoils can obtain the performance according to the assumption of free transition, maximum $c/l_d$ for the thin airfoils will be around 160, so that $C_P$ can be increased from 0.55 to 0.56.

3. Rotor analysis using the blade element momentum (BEM) method

The investigation of the rotor performance is made in three steps:
1. Rotors with blades having constant maximum lift-drag ratio, $c/l_d$, are investigated.
2. Four different existing rotors of different sizes are designed to many different tip speed ratios, $X$, equipped with airfoils and having the same absolute thickness distribution as the existing blades to maintain the required stiffness.
3. The energy production including the regulation of the largest rotor, 5MW, is investigated.

3.1. Rotors with constant maximum lift-drag ratio

Extending the investigation to rotors instead of 2D airfoils requires only minor changes in the set up because the rotor performance is the sum of the performance of the 2D sections along the rotor blade. This is true as long as the BEM method is used because this method assumes that the rotor performance can be treated in 2D in annular elements of the rotor disc. Thus, eqn. (3) to (5) are used, but now with the Prandtl tip correction $F$ defined according to Glauert [11]:

$$F = 2 \frac{\text{arccos}(\exp(-f))}{\pi}$$  \hspace{1cm} (6)

with $f = \frac{B(R-r)}{2r \sin(\phi)}$. In Figure 4 $C_P$ as function of the tip speed ratio, $X$, is seen for different values of the maximum lift-drag ratio, $c/l_d$, which is constant along the blade. Also, it is assumed that that the number of blades $B=3$. It is seen that $C_P$ is around 0.46 for $X_{design}$ around 6 if maximum $c/l_d=50$ and increases to $C_P$ around 0.525 for $X_{design}$ around 12 if maximum $c/l_d=200$. The upper limit for the efficiency is shown by the curve corresponding to inviscid flow. It is seen that $C_P$ approaches the Betz limit for high $X$ when inviscid flow is assumed. If only two blades were assumed the result would be different with the maximum $C_P$ at higher tip speed ratio, $X$. From the 2D analysis the local tip speed, $x$, should be between 2.4 and 3.8 for maximum $c/l_d$ between 50 and 200 to obtain maximum local $C_P$. This analysis for the rotor shows that the tip correction and the fact that the blade operates at all local speed ratios, $x$, between, 0 (zero) and the tip speed ratio ($X$) causes the maximum $C_P$ to appear at $X$ greater than the $x$ corresponding to maximum local $C_P$.

![Figure 4 Rotor $C_P$ as function of tip speed ratio for different lift-drag ratios constant along the blade.](image)
3.2. Four rotors with constraints on the absolute thickness distribution

To reveal the aerodynamic performance for real rotors four different rotors where selected, so both \( R_e \) and the thickness distribution were taken into account. The blades were:

- Wind Dynamic 3WTR designed for 1kW wind turbines with diameter \( D=1.75 \text{m} \)
- LM8.2 designed for 100kW wind turbines with diameter \( D=19.0 \text{m} \)
- LM19.1 designed for 500kW wind turbines with diameter \( D=41.0 \text{m} \)
- NREL Reference Wind Turbine blade (NREL-RWT) made for theoretical investigations for 5MW wind turbines with diameter \( D=126.0 \text{m} \)

Based on these blades and their absolute thickness distributions several blades were designed for different tip speed ratios, \( X \). The designs were based on airfoils of different relative thicknesses and computed for different \( R_e \). Thus, the airfoil characteristics used in the design corresponds to both relative thicknesses and \( R_e \). Three different airfoil series were used:

- The Risø-B1 family with thicknesses 15\% to 36\%
- The DU family with thicknesses 18\% to 35\%
- The NACA636xx with thicknesses 15\% to 21\% and the FFA-w3-xxx with thicknesses 24\% to 36\%

The airfoil characteristics for the airfoil families were computed using XFOIL with its 2D assumption. For each airfoil and \( R_e \) the maximum \( c_l/c_d \) and the corresponding \( c_l \), the design lift, \( c_{l,\text{design}} \), were determined. Based on the design lift the blade planforms were designed for different tip speed ratios using eqn. (3) to (5) for maximum aerodynamic efficiency at each radial station. This means that the chord length at the root part of the blade was not reduced, which is common on some modern wind turbine rotors. In the computations the airfoil characteristics along the blade are described by interpolation of \( c_{l,\text{design}} \) and maximum \( c_l/c_d \).

Figure 5 and Figure 6 show the power coefficient, \( C_P \), versus the tip speed ratio, \( X \), for the four different rotors assuming fully turbulent and free transition airfoil flow, respectively. The maximum \( C_P \) and the corresponding \( X_{\text{design}} \) for each rotor can be seen in Table 1. From the plots and the table it is seen that the bigger rotor the higher maximum \( C_P \). Also, it can be seen that the tip speed ratio, \( X_{\text{design}} \), corresponding to the maximum \( C_P \) moves towards higher values for bigger sizes of rotors. Based on the investigations in 2D it can be concluded that the increase in maximum \( C_P \) is caused by the increase in \( R_e \). Thus, \( R_e \)'s for the different rotors are approximately: Wind Dynamic: \( R_e = 100,000 \), LM8.2: \( R_e = 1.2 \text{mio} \), LM19.1: \( R_e = 2.5 \text{mio} \) and NREL RWT: \( R_e = 7 \text{mio} \). Comparing Figure 5 and Figure 6 show the difference in the assumption of fully turbulent and free transitional airfoil flow. As discussed in the section “Reynolds number dependency and roughness sensitivity” the assumption of fully turbulent flow probably is somewhat conservative whereas the assumption of free transition probably is somewhat optimistic. Therefore, the rotor designs should be somewhere between these extremes.

| Diameter [m] | \( \text{Max. } C_P \) | \( X_{\text{design}} \) |
|--------------|----------------|----------------|
| Wind Dynamic | \( 0.450 \) | \( 5.50 \) |
| LM8.2        | \( 0.465 \) | \( 5.75 \) |
| LM19.1       | \( 0.473 \) | \( 6.25 \) |
| NREL-RWT     | \( 0.480 \) | \( 6.50 \) |

| Fully turbulent airfoil flow | \( \text{Max. } C_P \) | \( X_{\text{design}} \) |
|-----------------------------|----------------|----------------|
| \( \text{Max. } C_P \) | \( 0.465 \) | \( 5.75 \) |
| \( X_{\text{design}} \) | \( 6.25 \) | \( 7.50 \) |

Table 1 Maximum \( C_P \) and design \( X \) for the four different rotors using Risø-B1 airfoil family.
3.3. Aerodynamic performance for the NREL RWT 5MW with different rotors

In the former sections only maximum $C_P$ at different tip speed ratios, $X$, have been shown. However, the focus in wind turbine design should be to minimize price per energy. This means that the annual energy production for a given design should be maximized considering also the corresponding loads. This requires the inclusion of the control, because the rotor should operate between e.g. 5 and 25m/s. Therefore, the NREL RWT rotor was investigated to find the tip speed ratio that was optimal in terms of energy production. Blades were designed at four different $X$ and equipped with the Risø-B1 airfoil family with the assumption of free transitional airfoil flow. The variation in $Re$ was included. HAWTOPT [12] was used for the design of the rotors and in the prediction of the performance. The objective in the design process was to maximize $C_P$ maintaining the absolute thickness distribution to maintain the blade stiffness. Along the blade span each airfoil section did not necessarily operate at the design lift unlike the designs in the former section. One type of design was to limit the maximum chord to 4.65m like the existing NREL RWT blade. The other type of design was to set no limits on the maximum chord. The annual energy production ($AEP$) was predicted using a Weibull distribution.
where the mean wind speed was \( V_{\text{mean}} = 8 \text{m/s} \), the roughness parameter was \( z_0 = 0.01 \) (smooth onshore surface), the hub height was \( h_A = 89.56 \) and the constant \( C = 1.9 \).

Figure 7 shows \( C_p \) and \( AEP \) versus \( X \) with and without constraints on the maximum chord. For the blade with unconstrained maximum chord it is seen that the \( AEP \) follows \( C_p \), i.e. \( AEP \) has its maximum at the same \( X \) as \( C_p \). Also, it can be seen that 2% change in \( C_p \) (e.g. from \( C_p = 0.50 \) to 0.49) results in around 1% change in \( AEP \) (e.g. from 19.4GWh to 19.2GWh). Furthermore, constraining the chord to maximum 4.65m reduced \( C_p \) with between 2% and 3%, which is between 1% and 1.5% on \( AEP \). The plot indicates that if the blade has a limited maximum chord the design tip speed ratio, \( X_{\text{design}} \), should be decreased to obtain maximum \( C_p \). Finally, this investigation indicates that optimizing blades for \( C_p \) is a simple way to optimize \( AEP \). It should be noted that \( X_{\text{design}} \) is found to be lower in the investigation using HAWTOPT compared to the results shown in Figure 6. This is due to the uncertainty of the performance of the thickest airfoil sections as described in section “Four rotors with constraints on the absolute thickness distribution”. Thus, more knowledge of the performance of thick airfoils could change the shown results. Finally, consideration of the loads should be taken into account and this could change the conclusion concerning the optimal tip speed ratio.

![Figure 7 Cp and AEP versus X for rotors with and without constraints on the maximum chord.](image)

**4. Conclusion**

In this work the influence of different key parameters on the power efficiency, \( C_p \), and loads was investigated. The work was divided into an analysis of 2D airfoils/blade sections and of entire rotors.

In the analysis of the 2D airfoils it was seen that there was a maximum of the local \( C_p \) for airfoils with finite \( c_l/c_d \) values. The local speed ratio should be between 2.4 and 3.8 for airfoils with \( c_l/c_d \) between 50 and 200, respectively, to obtain maximum local \( C_p \). Investigating the Reynolds number (\( Re \)) the Risø-B1 airfoil family showed that at low \( Re = 200,000 \) the maximum \( c_l/c_d \) was around 50 corresponding to a local \( C_p \) of around 0.53 and at high \( Re = 9 \text{mio} \) \( c_l/c_d \) could be up to 180 corresponding to a local \( C_p \) of over 0.56. Thus, \( Re \) has a significant impact on \( C_p \) and especially for \( Re < 2 \text{mio} \) corresponding to rotors below approximately 400kW this impact is pronounced.

The investigation of \( C_p \) for rotors was made with three blades and showed that with the assumption of constant maximum \( c_l/c_d \) along the entire blade, the design tip speed ratio changed from \( X_{\text{design}} = 6 \) to \( X_{\text{design}} = 12 \) for \( c_l/c_d = 50 \) and \( c_l/c_d = 200 \), respectively, with corresponding values of \( C_p = 0.46 \) and \( C_p = 0.525 \). Thus the design tip speed ratio and \( C_p \) are very dependent on the airfoil efficiency.
An analysis of existing rotors re-designed with new airfoils but maintaining the absolute thickness distribution to maintain the stiffness showed that big rotors are more aerodynamic efficient than small rotors. It also showed that the design tip speed ratio was dependent on the rotor size and on the assumptions of the airfoil flow being fully turbulent (contaminated airfoil) or free transitional (clean airfoil). The investigations showed that rotors with diameter $D = 1.75m$, should be designed for $X_{design}$ around 5.5, whereas rotors with diameter $D = 126m$, should be designed for $X_{design}$ between 6.5 and 8.5, depending on the airfoil performance.

To investigate the annual energy production, $AEP$, four blades were designed for a rotor with $D = 126m$ corresponding to 5MW. With no constraints on the maximum chord it was seen that $AEP$ had its maximum at the same tip speed ratio as $C_P$ had its maximum. If the maximum chord was constrained to 4.65m according to the existing 126m rotor it was seen that $AEP$ and $C_P$ had its maximum at a slightly lower tip speed ratio compared to the blades with no constraints on the maximum chord. Also, it was observed that an increase of 2% in $C_P$ corresponded to an increase in $AEP$ of around 1%. Finally, to find the optimal tip speed ratio, also the loads computed using aeroelastic simulations including rotor control should be considered to minimize the price pr. kWh.

5. Nomenclature

\begin{align*}
a &\quad \text{Axial velocity induction [-], } \quad a = \frac{1}{1 - (\frac{4\sin^2 \phi}{\alpha_y} + 1)} \\
a' &\quad \text{Tangential velocity induction [-], } \quad a' = \frac{1}{1 - (\frac{4\sin \phi \cos \phi}{\alpha_y} - 1)} \\
A &\quad \text{Rotor area } [m^2], \quad dA = 2\pi dr \\
B &\quad \text{Number of blades [-]} \\
c &\quad \text{Chord length } [m] \\
c_d &\quad \text{Drag coefficient [-]} \\
c_l &\quad \text{Lift coefficient [-]} \\
c_{ldesign} &\quad \text{Design lift coefficient } (c_l \text{ at maximum lift-drag ratio }) [-] \\
c/c_d &\quad \text{Lift-drag ratio (measure of the airfoil efficiency) [-]} \\
c_x &\quad \text{Driving force coefficient [-], } c_x = c_l \sin \phi - c_d \cos \phi \\
c_y &\quad \text{Axial force coefficient [-], } c_y = c_l \cos \phi + c_d \sin \phi \\
C_P &\quad \text{Power coefficient [-]} \\
D &\quad \text{Rotor diameter } [m] \\
F &\quad \text{Tip loss correction [-]} \\
F_{drive} &\quad \text{Driving force } [N] \\
r &\quad \text{Actual radius } [m] \\
R &\quad \text{Rotor radius } [m] \\
V_0 &\quad \text{Wind speed } [m/s] \\
W &\quad \text{Relative velocity } [m/s], \quad W^2 = (V_0(1 - a))^2 + (\omega r(1 + a'))^2 \\
x &\quad \text{Local speed ratio [-], } x = \frac{\omega r}{V_0} \\
X &\quad \text{Tip speed ratio [-], } \quad X = \frac{\omega r}{V_0} \\
X_{design} &\quad \text{Tip speed ratio at which } C_P \text{ is maximum [-]} \\
\alpha &\quad \text{Local inflow angle to rotor plane } [^\circ], \quad \alpha = \arctan(\frac{1 - a}{x(1 + a')}) \\
\rho &\quad \text{Air density } [kg/m^3] \\
\sigma &\quad \text{Rotor solidity [-], } \sigma = \frac{cB}{2\pi r} \\
\omega &\quad \text{Rotor angular speed } [rad/s] \\
\end{align*}
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