Doubly Heavy Baryon $\Xi_{cc}$ Production in $\Upsilon(1S)$ Decay

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$\Upsilon(1S)$ decay to $\Xi_{cc} + \text{anything}$ is studied. It is shown that the corresponding branching ratio can be as significant as that of $\Upsilon(1S)$ decay to $J/\Psi + \text{anything}$. The non-relativistic heavy quark effective theory framework is employed for the calculation on the decay width.

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$\Upsilon(1S)$ decay is a good arena to study QCD and hadron physics. Several instructive results have been obtained. For example, recent searches on the exotic XYZ hadrons via the inclusive channel $\Upsilon \to J/\Psi + \text{anything}$ [1] and on light tetraquark hadrons in several channels of $\Upsilon$ decay [2] have been made. Both reported negative results. As a matter of fact, in the energy region above $J/\Psi$ mass at BEPC and that above $\Upsilon$ mass at B factories, many exotic XYZ hadrons have been observed (for a recent review, see [3]). These exotic particles, except those directly couple to the virtual photon in $e^+e^-$ annihilations, are all produced from the decays of either the exited $c\bar{c}$ bound states or the B hadrons. On the other hand, $\Upsilon$ decay is an environment significantly different from those where the exotic particle production is observed. $\Upsilon$ decays via the OZI-suppressed ways, i.e., the annihilation of the $b\bar{b}$ quarks. The dominant mode ($> 80\%$) is the hadronic one generally refered as '3-gluon' decay [4], and the subsequent hadronization is a special case of multiproduction. The negative results [1] [2] mentioned above can shed light on property of confinement and the unitarity of the hadronization in multiproduction processes as we have pointed out [5–8]. The experimental facts mentioned above confirm that the $c\bar{c}$ pair produced in perturbative

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process prefers to transfer into general hadrons like $J/\Psi$ rather than exotic XYZ’s in this multiproduction process; and that for light hadrons, it is also the similar case, i.e., the above negative experimental results on light exotic hadrons indicate that the dominant decay channels should be $\Upsilon \rightarrow h's$, with $h's$ referring to mesons as well as baryons. In one word, $\Upsilon$ generally decays to mesons and baryons, with exotic ones hardly possible to be observed. But the to-date measured decay channels of $\Upsilon$ are much far from exhausting the total decay width. Especially, almost no baryon channel is measured [4]. So measuring the baryon production is an important task for better understanding the dynamics in $\Upsilon$ decay.

Among all the baryons produced in $\Upsilon$ decay, the doubly heavy baryon $\Xi_{cc}$ is the most heavy. SELEX and LHCb have respectively reported the observations of this kind of baryons with different mass [11, 12]. One of the possibilities can be that different SU(2) multi-states of $\Xi_{cc}$ are observed by these two Collaborations. To measure these multi-states, and further to explore SU(3) multi-states, can surely help to clarify and deepen our knowledge on the property and production mechanism of $\Xi_{cc}$. $\Upsilon$ decay can provide a clean platform for such measurements.

There is a further special reason stands for the observation on $\Xi_{cc}$ in $\Upsilon$ decay. It is noticed that most of the presented data of $\Upsilon$ decay are upper limits [4]. However, the decay channel $\Upsilon \rightarrow J/\Psi + X$ is well measured for several times by several collaborations and has attracted wide interests, which is important on the study of PQCD and NRQCD (for the full literature list, please see a recent review [13]). It was pointed out that, based on the soft $J/\Psi$ spectrum by CLEO measurement which was quite rough at that time, and on the calculation of the partial width [9], the dominant contribution could be $\Upsilon(1S) \rightarrow J/\Psi + c\bar{c}g$. Then the spectrum and branching ratio is confirmed by CLEO II [16, 17] and later by BELLE [1], though detailed calculations show that several competing sub-processes contribute [14, 15]. This fact strongly implies that the perturbative production of $c\bar{c}cc$ in $\Upsilon$ decay is significant. This leads to that the double charm baryon is hence easily produced as argued by the colour connection analysis [18]. For $c_1\bar{c}_2 c_3\bar{c}_4 g$ system from $\Upsilon$ decay, $c_1\bar{c}_2$ and $c_3\bar{c}_4$ respectively come from a virtual gluon. But $c_1\bar{c}_4$ and $c_3\bar{c}_2$ can respectively be in colour singlet, i.e., the colour space can be reduced as

$$ (3_1 \bigotimes 3^*_1) \bigotimes (3_3 \bigotimes 3^*_2) = (1_{14} + 8_{14}) \bigotimes (1_{23} + 8_{23}). $$

This means that such combination of the pair can be colour singlet and easy to translate to
$J/\psi$ for proper invariant mass. One can recognize that the colour space can also be reduced as

$$(3_1 \boxtimes 3_3) \boxtimes (3_2^* \boxtimes 3_1^*) = (3_{13}^* + 6_{13}) \boxtimes (3_{24} + 6_{24}).$$

In such colour states, the two-charm pair can combine with a light quark to become $\Xi_{cc}$ for proper invariant mass. This simple analysis implies that the production rate of $\Xi_{cc} + \bar{c}\bar{c}g$ is expected not small once the $J/\Psi + \bar{c}\bar{c}g$ production rate is not small.

In this paper, we devote to study the production of $\Xi_{cc}$ in $\Upsilon$ decay. We calculate the corresponding partial width and the momentum distribution of $\Xi_{cc}$. Multi-states like $\Xi_{cc}^+$ or $\Xi_{cc}^{++}$ could have different width and lead to quite different feasibility or difficulty in observing them, but their production mechanism is completely the same in $\Upsilon$ decay. Therefore we do not make any distinction for the investigation on the production. In the super B factory, once the center of mass energy is tuned on the $\Upsilon$ resonance, a large sample of $\Upsilon$ decay data can be obtained and could be employed for the measurement. The following calculations show that the branching ratio of $\Xi_{cc}$ production can be order of $10^{-4}$. For the $\Upsilon$ decay, the process with two charm pairs production is easy to be triggered by 3-jet like event shape and strangeness enhancement (e.g., the $K^+$ value) [17, 20], of which some of the the charm meson production events can be vetoed by lepton pair or hadron pair mass around $J/\Psi$ mass. In this way, one can get a clean and large sample of events to study the doubly charm baryon multi-states.

In the process $\Upsilon \to \Xi_{cc} + \bar{c}\bar{c}g$, both bottom and the charm quarks are heavy. For the initial bound state, the colour singlet $b\bar{b}$ pair with $C=-1$, it directly leads to the non-relativistic wave function formulations [21–24], where the relative momentum between $b$ and $\bar{b}$ is vanishing, namely same as the case of positronium. For the final bound state, a factorization formulation within the heavy quark effective theory framework [25, 26] is employed. One subtle point is that, the non-relativistic formulations are investigated in the rest frame of each bound state, respectively; and then a corresponding covariant form of description is obtained, which can be employed in any frame. Here we start from the initial state: The differential width of the process $\Upsilon \to \Xi_{cc} + \bar{c}\bar{c}g$ can be formulated as [9]

$$\frac{d\Gamma}{dR} = \frac{|B_T \langle \Xi_{cc} | \bar{c}\bar{c}g | S(\frac{3}{2} S_1, 1) >|^2}{T},$$

where $dR$ is the phase space volume element for $\Xi_{cc}$ and $\bar{c}, \bar{c}, g$ without the constrain of energy momentum conservation; $S$ is the S-Matrix; $B_T$ is related to the wave function of $\Upsilon$
at origin as

\[ B_\Upsilon = \frac{\Psi_\Upsilon(0)}{\sqrt{V_2m_b}}. \]  

(2)

For convenience, we normalize all final state particle states to be \(2EV\) (where \(E\) is the particle’s energy and \(V\) is the volume of the total space). This normalization is also used for all free quarks in bound states. For the initial state, \(B_\Upsilon\) normalizes the state of \(\Upsilon\) to be 1, so that the width can be directly written as above. In Eq. (1) the sum over all spin states for final particles and average of the 3 spin states for \(\Upsilon\) are not explicitly shown and the ‘time’ \(T\) is \(2\pi\delta(0)\).

For the factorization of the initial bound state, the width is written, based on the above Equation, as

\[ d\Gamma = dR' \frac{1}{3M_\Upsilon^2} |\Psi_\Upsilon(0)|^2 < \Xi_{cc}\bar{c}\bar{c}g|T|b\bar{b}^{(3)S_1, 1}> |^2. \]  

(3)

Here \(dR' = dR(2\pi)^4\delta^{(4)}(P_i - P_f)\), the factors time \(T\) and volume \(V\) are cancelled by the \(\delta^{(4)}(0)\). \(T\) is the \(T\) matrix with \(S_{fi} = \delta_{fi} + (2\pi)^4\delta^{(4)}(P_i - P_f)T_{fi}\). Sum over all spin states is inexplicitly indicated.

Employing the project operator formulation (e.g., [21]), and the radial wave function \(R_\Upsilon\) to describe the initial bound state, we get the decay amplitude as,

\[ \mathcal{M}_{fi} = \frac{1}{2} \frac{1}{\sqrt{4\pi}} \frac{1}{\sqrt{M_\Upsilon}} R_\Upsilon(0)Tr[O_0(P + M_\Upsilon)(-\not{\epsilon})]. \]  

(4)

\(O_0\) is the amplitude for \(b\bar{b} \rightarrow \Xi_{cc}\bar{c}\bar{c}g\), with relative momentum of \(b\bar{b}\) vanishing. \(P\) and \(\epsilon\) are 4-momentum and polarization vector of \(\Upsilon\), respectively.

In the final state of the \(\Upsilon\) decay, the unobserved part \(X\) can be divided into a perturbative part \(X_P\) and a non-perturbative part \(X_N\). To the lowest-order (tree level) in PQCD,

\[ \mathcal{M}_{fi} = \int \frac{d^4q_1}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1) \int d^4x_1 e^{-iq_1x_1} \times <\Xi_{cc}(k) + X_N|Q_i(x_1)Q_j(0)> \]  

(5)

We assign \(k_1, k_2, P_1, P_2, P_3, k\) as the momenta of the corresponding particles, \(b, \bar{b}, \bar{c}, c, g, \Xi_{cc}\), respectively, \(k_1 = k_2 = P/2\). \(A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1)\), which includes the initial wave function, can be directly read from FIG.1. Both \(i\) and \(j\) are Dirac and color indices. In the matrix element, \(X_N\) represents the non perturbative effects. \(Q(x)\) is the Dirac field for charm quark.
Taking the absolute square of the above amplitude, one gets

\[
d\Gamma = \frac{1}{2M_Y} \sum_{X_N} \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 P_1}{(2\pi)^3} \frac{d^3 P_2}{2E_1} \frac{d^3 P_3}{2E_2} \frac{d^3 P_3}{2E_3} \times (2\pi)^4 \delta^4(Q - P_1 - P_2 - P_3 - k) \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2} \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_3}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1) \times [\gamma^0 A^\dagger(k_1, k_2, P_1, P_2, P_3; q_3) \gamma^0]_{kl} \int d^4 x_1 d^4 x_3 e^{-iq_1 x_1 + iq_3 x_3} \times <0|Q_k(0)Q_l(x_3)|\Xi_{cc} + X_N>| <\Xi_{cc} + X_N|Q_j(x_1)\overline{Q}_j(0)|0>,
\]

where the spin summation of the baryon \(\Xi_{cc}\), and the polarization and color summation of two anti-charm quarks are implied. Here we take nonrelativistic normalization for the baryon \(\Xi_{cc}\). We can eliminate the sum over \(X_N\) by using translational covariance. Defining
the creation operator $a^\dagger(k)$ for $\Xi_{cc}$ with the three momentum $k$, we obtain

$$d \Gamma = \frac{1}{2M_{\Xi_{cc}}^2} \frac{d^3 k}{(2\pi)^3} \frac{d^3 P_1}{(2\pi)^3} \frac{d^3 P_2}{(2\pi)^3} \frac{d^3 P_3}{(2\pi)^3} \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_3}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1) \times [\gamma^0 A^\dagger(k_1, k_2, P_1, P_2, P_3; q_3) \gamma^0]_{k_1}$$

$$\times \int d^4 x_1 d^4 x_2 d^4 x_3 e^{-iq_1 x_1 + iq_3 x_3 - iq_2 x_2} \times < 0|Q_k(0)Q_l(x_3)a^\dagger(k)\bar{Q}_i(x_1)\bar{Q}_j(x_2)|0 > ,$$

(7)

with $q_2 = k - q_1$.

We use heavy quark effective field theory to deal with the $\Xi_{cc}$ state. In $\Xi_{cc}$ rest frame, the heavy quarks move with a small velocity $v_c$. Hence, the Fourier transformed matrix element can be expanded in $v_Q$ with fields of NRQCD. The relation between NRQCD fields and Dirac fields $Q(x)$ in the rest frame is

$$Q(x) = e^{-im_c t} \begin{pmatrix} \psi(x) \\ 0 \end{pmatrix} + O(v_c) + ..., \quad (8)$$

where $\psi(x)$ is NRQCD field. We will work at the leading order of $v_c$. We denote $v$ as the velocity of $\Xi_{cc}$ with $v^\mu = k^\mu / M_{\Xi_{cc}}$ to express our result of Fourier transformed matrix element in a covariance way. Hence, the Fourier transformed matrix element in the rest frame is

$$v^0 \int d^4 q_1 d^4 q_2 d^4 q_3 e^{-iq_1 x_1 - iq_2 x_2 + iq_3 x_3} \times < 0|Q_k(0)Q_l(x_3)a^\dagger(k)\bar{Q}_i(x_1)\bar{Q}_j(x_2)|0 >$$

$$= \int d^4 q_1 d^4 q_2 d^4 q_3 e^{-iq_1 x_1 - iq_2 x_2 + iq_3 x_3} \times < 0|Q_k(0)Q_l(x_3)a^\dagger(k = 0)\bar{Q}_i(x_1)\bar{Q}_j(x_2)|0 > .$$

(9)

Using Eq.(8), the matrix element in Eq.(9) can be expanded with $\psi(x)$ and $\psi^\dagger(x)$. The spacetime dependence of the matrix element with NRQCD field is controlled by the scale $m_c v_c$. At the leading order of $v_c$ one can neglect the spacetime dependence and the mass of the baryon $M_{\Xi_{cc}}$ is approximated by $2m_c$. With the approximation the matrix element in Eq.(9) is

$$< 0|\psi_{\lambda_3}^{a_3}(0)\psi_{\lambda_4}^{a_4}(0)a^\dagger(0)\psi_{\lambda_1}^{a_1}(0)\psi_{\lambda_2}^{a_2}(0)|0 >$$

(10)
where we suppress the notation $k = 0$ and it is always implied that NRQCD matrix elements are defined in the rest frame of $\Xi^{cc}$. The superscripts $a_i (i = 1, 2, 3, 4)$ are used to label the color of quark fields, while the subscripts $\lambda_i (i = 1, 2, 3, 4)$ for the quark spin indices. We obtain the matrix element by two parameters, $h_1, h_3$ as following:

$$< 0 | \psi^{a_3}_{\lambda_3}(0) \psi^{a_4}_{\lambda_4}(0) a^+ a \psi^{a_1}_{\lambda_1}(0) \psi^{a_2}_{\lambda_2}(0) | 0 >$$

$$= (\varepsilon)_{\lambda_4 \lambda_3} (\varepsilon)_{\lambda_2 \lambda_1} \cdot (\delta_{a_1 a_4} \delta_{a_2 a_3} + \delta_{a_1 a_3} \delta_{a_2 a_4}) \cdot h_1$$

$$+ (\sigma^i \varepsilon)_{\lambda_4 \lambda_3} (\varepsilon \sigma^i)_{\lambda_2 \lambda_1} \cdot (\delta_{a_1 a_4} \delta_{a_2 a_3} \delta_{a_1 a_3} - \delta_{a_2 a_4}) \cdot h_3,$$

(11)

where $\sigma^i (i = 1, 2, 3)$ are Pauli matrices. $\varepsilon = i \sigma^2$ is totally anti-symmetric. The parameters $h_1$ and $h_3$ are defined as:

$$h_1 = \frac{1}{48} < 0 | [\psi^{a_1} \varepsilon \psi^{a_2} + \psi^{a_2} \varepsilon \psi^{a_1}] a^+ a \psi^{a_2 \dagger} \varepsilon \psi^{a_1 \dagger} | 0 >$$

$$h_3 = \frac{1}{72} < 0 | [\psi^{a_1} \varepsilon \sigma^n \psi^{a_2} - \psi^{a_2} \varepsilon \sigma^n \psi^{a_1}] a^+ a \psi^{a_2 \dagger} \sigma^n \varepsilon \psi^{a_1 \dagger} | 0 > .$$

(12)

$h_1(h_3)$ represents the probability for a cc pair in a $^1S_0(^3S_1)$ state and in the color state of 6(3*) to transform into the baryon. It is the Pauli exclusion principle determines that only these two kinds of combination of colour and spin states, which are asymmetric, are possible [26]. With these results the Fourier transformed matrix element in Eq.(9) can be expressed as:

$$\nu^0 \int d^4 x_1 d^4 x_2 d^4 x_3 e^{-i q_1 x_1 - i q_2 x_2 + i q_3 x_3}$$

$$< 0 | Q_k^{a_3}(0) Q_l^{a_4}(x_3) a^i(k) a(k) \overline{Q}_i^{a_1}(x_1) \overline{Q}_j^{a_2}(x_2) | 0 >$$

$$= (2\pi)^4 \delta^4(q_1 - m_c \nu)(2\pi)^4 \delta^4(q_2 - m_c \nu)(2\pi)^4$$

$$\delta^4(q_3 - m_c \nu) \times [ - (\delta_{a_1 a_4} \delta_{a_2 a_3} + \delta_{a_1 a_3} \delta_{a_2 a_4})$$

$$(\bar{P}_v C \gamma_5 P_v)_{ji} (P_v \gamma_5 C \bar{P}_v)_{ik} h_1$$

$$+ (\delta_{a_1 a_4} \delta_{a_2 a_3} - \delta_{a_1 a_3} \delta_{a_2 a_4}) (\bar{P}_v C \gamma^\mu P_v)_{ji}$$

$$(P_v \gamma^\mu C \bar{P}_v)_{ik} (\nu \mu \nu - g_{\mu \nu}) h_3] + ...$$

(13)

where $P_v = \frac{1 + \gamma^\mu}{2}, \bar{P}_v = \frac{1 + \gamma^\mu}{2}; C = i \gamma^5 \gamma^0$, the charge conjugation operator.
With the above formula, we obtain the decay width as following:

\[
\Gamma = \frac{16\pi^4\alpha_s^2|\mathcal{R}_\Upsilon(0)|^2M_{\Xi cc}}{9M_T^2\left[(P_1 + k/2)(P_2 + k/2)\right]^2}
\]

\[
\sum_{c=1}^{8}\sum_{\xi=1}^{6}\mathcal{A}_{\xi}^{abc}(\sum_{\zeta=1}^{6}\mathcal{A}_{\xi}^{a'b'c})H^{aba'b'}
\]

\[
d^3k \prod_{i=1}^{3} \frac{d^3P_i}{(2\pi)^3E_k} (2\pi)^4\delta^4(Q - P_1 - P_2 - P_3 - k),
\]

where

\[
H^{aba'b'} = -(Tr[T^aT^a'T^bT^{b'}] + Tr[T^aT^a']Tr[T^bT^{b'}]) \times h_1 \times B_1
\]

\[
+ (Tr[T^aT^a'T^bT^{b'}] - Tr[T^aT^a']Tr[T^bT^{b'}]) \times h_3 \times B_2,
\]

\[
B_1 = Tr[\gamma^\alpha(P^\alpha_2 - m_c)\gamma^\alpha'P_v\gamma^\beta\vec{P}_v\gamma^\beta(-P^\beta_1 - m_c)\gamma^\beta\vec{P}_v\gamma^\beta P_v],
\]

\[
B_2 = Tr[\gamma^\alpha(P^\alpha_2 - m_c)\gamma^\alpha'P_v\gamma^\beta\vec{P}_v\gamma^\beta(-P^\beta_1 - m_c)\gamma^\beta\vec{P}_v\gamma^\beta P_v](v_\rho v_\nu - g_{\mu\nu}).
\]

The function \(\mathcal{A}_{\xi}^{abc}(\xi = 1, 2, 3, 4, 5, 6)\) are given in Appendix A.

The radial wave function for \(\Upsilon\) at origin can be obtained, e.g., by fitting its leptonic decay width. On the other hand, the value of \(h_1\) and \(h_3\), is difficult to be obtained. There are no experiment results now. Here we employ a potential model with the radial wave function \(R_{cc}(r)\) at origin \[10\] to get the numerical value of \(h_3\)

\[
h_3 = \frac{|R_{cc}(0)|^2}{4\pi},
\]

with its value to be 0.0287\(GeV^3\). There is no practical model for \(h_1\), which can be taken as a free parameter, the reason is explained later. In the numerical calculations, we take \(\Psi_\Upsilon(0) = 2.194\, GeV^{3/2}\), \(M_\Upsilon = 9.46\, GeV\), \(M_\Xi = 3.621\, GeV\), \(m_b = 4.73\, GeV\), \(\alpha_s(m_c) = 0.253\).

\(m_c/m_b\) is taken to be parameter, and the dependence of branching ratio on \(m_c/m_b\) is studied as shown in FIG.2.

With \(m_c/m_b = 0.25\) the partial width is \(\Gamma = (0.0126h_1 + 0.240h_3)\, KeV\). Here we see that the perturbative part corresponding to \(h_1\) is much smaller than that of \(h_3\). So if there is
no special enhancement on $h_1$, this part of contribution can not be significant. Here for simplicity we take $h_1 = h_3$, and the decay width is 7.256eV, leading to the branching ratio as $1.34 \times 10^{-4}$. The $\Xi_{cc}$ momentum distributions are shown in FIG.3 and FIG. 4. The momentum distributions of $\bar{c}$ are shown in FIG.5 and FIG 6.

![FIG. 2: Dependence of branching ratio on $m_c/m_b$.](image)

![FIG. 3: The momentum distribution of $\Xi_{cc}, h_3 = 0$.](image)

![FIG. 4: The momentum distribution of $\Xi_{cc}, h_1 = 0$.](image)

The experiment of BELLE in 2016 has collected $102 \times 10^6 \ U\ events [1, 13]$. So it is possible to make a scan on the $\Xi_{cc}$ production. In the future, further precise measurement
on the production of $\Xi_{cc}$ can even be made with more large luminosity at BELLE2. Similar productions characteristic of the partonic state with four charm (anti)quarks can also be studied in $\Upsilon$ decay.

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Appendix A

The functions $\bar{A}_\xi (\xi = 1, \ldots, 6)$ in the decay width are:

$$
\bar{A}_1 = Tr^s[T^c T^{b^a} T^a] \frac{1}{[(q - P_3)^2 - m^2][(q - P_1 - k/2)^2 - m^2]}
\times Tr[\xi^* (P_3)(m + P_3 - q)\gamma_\alpha (q - P_1 - \frac{k}{2} + m)\gamma_\beta (M + P) ]
\delta]$$

$$
\bar{A}_2 = Tr^s[T^b T^{a^c} T^a] \frac{1}{[(P_2 + k/2 - q)^2 - m^2][(q - P_1 - k/2)^2 - m^2]}
\times Tr[\gamma_\alpha (m + P_2 + \frac{k}{2} - q)\xi^*(P_3)(q - P_1 - \frac{k}{2} + m)\gamma_\beta (M + P) ]
\delta]$$

$$
\bar{A}_3 = Tr^s[T^c T^{a^b} T^b] \frac{1}{[(P_3 - q)^2 - m^2][(q - P_2 - k/2)^2 - m^2]}
\times Tr[\xi^* (P_3)(m + P_2 - q)\gamma_\beta (q - P_2 - \frac{k}{2} + m)\gamma_\alpha (M + P) ]
\delta]$$

$$
\bar{A}_4 = Tr^s[T^a T^{c^b} T^b] \frac{1}{[(P_1 + k/2 - q)^2 - m^2][(q - P_2 - k/2)^2 - m^2]}
\times Tr[\gamma_\beta (m + P_1 + \frac{k}{2} - q)\xi^*(P_3)(q - P_2 - \frac{k}{2} + m)\gamma_\alpha (M + P) ]
\delta]$$

$$
\bar{A}_5 = Tr^s[T^b T^{a^c} T^c] \frac{1}{[(P_2 + k/2 - q)^2 - m^2][(q - P_3)^2 - m^2]}
\times Tr[\gamma_\alpha (m + P_2 + \frac{k}{2} - q)\gamma_\beta (q - P_3 + m)\xi^*(P_3)(M + P) ]
\delta]$$

$$
\bar{A}_6 = Tr^s[T^a T^{b^c} T^c] \frac{1}{[(P_1 + k/2 - q)^2 - m^2][(q - P_3)^2 - m^2]}
\times Tr[\gamma_\beta (m + P_1 + \frac{k}{2} - q)\gamma_\alpha (q - P_3 + m)\xi^*(P_3)(M + P) ]
\delta]$$

(A1)

Here $Tr^s[...]$ means only keeping the symmetric part; $m = m_b, M = M_T, q = P/2$. $\varepsilon(P_3)$ is the polarization vector of the gluon with momentum $P_3$.

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