A Road to the Standard Grand Unified Theory

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Abstract

In this talk\(^1\), we propose a GUT scenario in which doublet-triplet splitting is naturally realized in $SO(10)$ unification using the Dimopoulos-Wilczek mechanism[^4] and the realistic mass matrices of quarks and leptons are obtained in a simple way. For the neutrino sector, bi-maximal neutrino mixing angles are realized. Moreover, the generic interaction is allowed, namely, all the terms which are allowed by the symmetry are included in the scenario. Therefore, once we fix the integer number charges of the anomalous $U(1)_A$ symmetry, which plays an essential role in the scenario, all the scales, GUT breaking scale, mass scales of superheavy particles, are determined. The scenario can be extended into $E_6$ unification, in which a condition for suppression of flavor changing neutral current (FCNC) is automatically satisfied.

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[^1]: This talk is based on the works [1, 2, 3].
1 Introduction

In this talk, we propose a scenario of $SO(10)$ grand unified theory (GUT) with anomalous $U(1)_A$ gauge symmetry, which has the following interesting features\cite{1};

1. The doublet-triplet (DT) splitting is realized using Dimopoulos-Wilczek mechanism \cite{4,5,6,7}.

2. The proton decay via dimension five operator is suppressed.

3. Realistic quark and lepton mass matrices can be obtained in a simple way. Especially in neutrino sector, bi-large neutrino mixing is realized.

4. The symmetry breaking scales are determined by the anomalous $U(1)_A$ charges.

5. The mass spectrum of the super heavy particles is fixed by the anomalous $U(1)_A$ charges.

6. The $\mu$ problem is also naturally solved\cite{2}.

As a consequence of the above features, the fact that the GUT scale is smaller than the Planck scale is strongly connected to the improvement of the undesired GUT relation between the Yukawa couplings $y_\mu = y_s \ (y_e = y_d \ also)$ while keeping $y_\tau = y_b$. Moreover, it is remarkable that the interaction is generic, namely, all the interactions, which are allowed by the symmetry, are taken into account. Therefore, once we fix the field contents with their quantum numbers, all the interactions are determined except the coefficients of order one. Moreover the above scenario can be extended into $E_6$ unification\cite{3}, in which a suppression condition of FCNC is automatically satisfied. In $E_6$ unification, the twisting mechanism\cite{8} is essential.

There the anomalous $U(1)_A$ gauge symmetry\cite{9}, whose anomaly is cancelled by Green-Schwarz mechanism\cite{10}, plays an essential role in the scenario.

2 Relation between VEVs and anomalous $U(1)_A$ charges and neutrino masses

In this section, we explain how the vacua of the Higgs fields are determined by the anomalous $U(1)_A$ quantum numbers\cite{1,2}.

First of all, we show that none of the field with positive anomalous $U(1)_A$ charge gets nonzero VEV if the Froggatt-Nielsen (FN) mechanism works well in the vacuum. Let the gauge singlet fields be $Z_i^{\pm}$ ($i = 1, 2, \cdots n_\pm$) with charges $z_i^{\pm}$
with \( z_i^+ > 0 \) and \( z_i^- < 0 \). From the \( F \) flatness conditions of the superpotential we get \( n = n_+ + n_- \) equations plus one \( D \)-flatness condition,

\[
\frac{\delta W}{\delta Z_i} = 0, \quad D_A = g_A \left( \sum_i z_i |Z_i|^2 + \xi^2 \right) = 0, \quad (2.1)
\]

where \( \xi^2 = \frac{2^{16} Q^A}{192\pi^2} (\equiv \lambda^2 M_P^2) \). At a glance, these look to be over determined. However, the \( F \) flatness conditions are not independent because the gauge invariance of the superpotential \( W \) leads to a relation

\[
\frac{\delta W}{\delta Z_i} z_i Z_i = 0. \quad (2.2)
\]

Therefore, generically SUSY vacuum with \( \langle Z_i \rangle \sim M_P \) exists (Vacuum a), because the coefficients of the above conditions are generally of order 1. However, if \( n_+ \leq n_- \), we can take another vacuum (Vacuum b) with \( \langle Z_i^+ \rangle = 0 \), which automatically satisfy the \( F \)-flatness conditions \( \frac{\delta W}{\delta Z_i} z_i Z_i = 0 \). Then \( \langle Z_i^- \rangle \) are determined by \( F \)-flatness conditions \( \frac{\delta W}{\delta Z_i} z_i Z_i = 0 \) with a constraint (2.2) and \( D \)-flatness condition \( D_A = 0 \). Note that if \( \lambda < 1 \) (i.e., \( \xi < 1 \)), the VEVs of \( Z_i^- \) are less than the Planck scale, that can lead to Froggatt-Nielsen mechanism. If we fix the normalization of \( U(1)_A \) gauge symmetry so that the largest value \( z_i^- \) in the negative charges \( z_i^- \) equals -1 then the VEV of the field \( Z_1^- \) is determined from \( D_A = 0 \) as \( \langle Z_1^- \rangle \sim \lambda \), which breaks \( U(1)_A \) gauge symmetry. (The field \( Z_1^- \) becomes the Froggatt-Nielsen field \( \Theta \).) On the other hand, other VEVs are determined by \( F \)-flatness conditions of \( Z_i^+ \) as \( \langle Z_i^- \rangle \sim \lambda^{-z_i^+} \), which is shown below. Since \( \langle Z_i^+ \rangle = 0 \), it is sufficient to examine the terms linear in \( Z_i^+ \) in the superpotential in order to determine \( \langle Z_i^- \rangle \). Therefore, in general the superpotential can be written

\[
W = \sum_{i} W_{Z_i^+}, \quad (2.3)
\]

\[
W_{Z_i^+} = \lambda^{z_i^+} Z_i^+ (\sum_{j} \lambda^{z_j^-} Z_j^- + \sum_{j,k} \lambda^{z_j^-+z_k^-} Z_j^- Z_k^- + \cdots) \quad (2.4)
\]

\[
= \sum_{i} \tilde{Z}_i^+(\sum_{j} \tilde{Z}_j^- + \sum_{j,k} \tilde{Z}_j^- \tilde{Z}_k^- + \cdots), \quad (2.5)
\]

where \( \tilde{Z}_i \equiv \lambda^{z_i} Z_i \). The \( F \)-flatness conditions of \( Z_i^+ \) fields require

\[
\lambda^{z_i^+} (1 + \sum_j \tilde{Z}_j^- + \cdots) = 0, \quad (2.6)
\]

which generally lead to solutions \( \tilde{Z}_j \sim O(1) \) if these \( F \)-flatness conditions determine the VEVs. Thus the \( F \)-flatness condition demands,

\[
\langle Z_j^- \rangle \sim O(\lambda^{-z_j}). \quad (2.7)
\]
Here we have examined the VEVs of singlets fields, but generally the gauge invariant operator $O$ with negative charge $o$ has non-vanishing VEV $\langle O \rangle \sim \lambda^{-o}$ if the $F$-flatness conditions determine the VEV. Note that when $n_+ = n_-$, all the VEVs $\langle Z_i \rangle$ can be determined by the $F$-flatness conditions $\frac{\delta W}{\delta Z_i} = 0$. It means that there is no flat direction, namely no massless field. On the other hand, when $n_+ < n_-$, then there must be some massless fields related with the flat direction.

If the vacuum a is selected, the anomalous $U(1)_A$ gauge symmetry is broken at the Planck scale and the FN mechanism does not work. Therefore, we cannot know the existence of the $U(1)_A$ gauge symmetry from the low energy physics. On the other hand, if the vacuum b is selected, the FN mechanism works well and we can understand the signature of the $U(1)_A$ gauge symmetry from the low energy physics. Therefore, it is natural to assume that the vacuum b is selected in our scenario, in which the $U(1)_A$ gauge symmetry plays an important role for the FN mechanism. Namely, the VEVs of the fields $Z_i^+$ vanish, that guarantee that the SUSY zero mechanism works well.

If an adjoint field $A(45)$ has a VEV by the $F$-flatness condition, the scale of the VEV is determined as $\langle A \rangle \sim \lambda^{-a}$ because $A^2$ can be gauge invariant. Moreover, in addition to the adjoint field $A$, we have to introduce spinor Higgs $C(16)$ and $\bar{C}(\overline{16})$ to break $SO(10)$ into the standard gauge group. The VEV $\langle \bar{C}C \rangle$ are determined by the anomalous $U(1)_A$ charges $c + \bar{c}$ as $\langle \bar{C}C \rangle = \lambda^{-(c+\bar{c})}$. This leads to

$$\langle C \rangle = \langle \bar{C} \rangle = \lambda^{-\frac{1}{2}(c+\bar{c})} \tag{2.8}$$

because of $D$-flatness condition of $SO(10)$ gauge theory. Note that the scale of the VEVs are also determined by the anomalous $U(1)_A$ charges, though the relation is different from the naive expectation $\langle C \rangle = \lambda^{-c}$. This is because the $D$-flatness condition plays a critical role to determine the VEVs. Note that the power is half integer. This fact plays an important role to obtain bi-large mixing angles in neutrino sector, which will be discussed later.

3 Doublet-triplet splitting with anomalous $U(1)_A$ gauge symmetry

In this section, we show that DT splitting is naturally realized in $SO(10)$ GUT with anomalous $U(1)_A$ gauge symmetry.

The minimal Higgs content to break $SO(10)$ into $SU(3)_C \times U(1)_{EM}$ is one adjoint Higgs $A(45)$, a pair of spinor fields $C(16)$ and $\bar{C}(\overline{16})$ and usual Higgs $H(10)$. All of them must have negative anomalous $U(1)_A$ charges because they have non-vanishing VEVs. On the other hand, we have to introduce the same number of the fields with positive anomalous $U(1)_A$ charges in order to make
all fields massive\footnote{Strictly speaking, some component fields are absorbed by the Higgs mechanism, so we do not have to introduce the same number of the fields with positive charges. However, it is not the case in $SO(10)$ unification.}. The content of the Higgs sector with $SO(10) \times U(1)_A$ gauge symmetry is given in Table I, where the symbols $\pm$ denote a $Z_2$ parity quantum numbers.

Table I. The lowercase letters represent the anomalous $U(1)_A$ charges.

\begin{center}
\begin{tabular}{ll}
45 & $A(a = -2, -)$, \quad $A'(a' = 6, -)$ \\
16 & $C(c = -4, +)$, \quad $C'(c' = 4, -)$ \\
16 & $\bar{C}(\bar{c} = -1, +)$, \quad $\bar{C}'(\bar{c}' = 7, -)$ \\
10 & $H(h = -6, +)$, \quad $H'(h' = 8, -)$ \\
1 & $Z(z = -3, -)$, \quad $\bar{Z}(\bar{z} = -3, -)$, \quad $S(s = 5, +)$
\end{tabular}
\end{center}

Here we have listed typical values of the anomalous $U(1)_A$ charges. Among these fields, $A$, $C$, $\bar{C}$, $Z$ and $\bar{Z}$ are expected to obtain non-vanishing VEVs around the GUT scale. As discussed in the previous section, the fields with positive $U(1)_A$ charges have vanishing VEVs. It is surprising that the DT splitting mechanism is naturally embedded into the above minimal model in a sense.

Since the fields with non-vanishing VEVs have negative charges, only the $F$-flatness conditions of fields with positive charge must be counted for determination of their VEVs. Moreover, we have only to take account of the terms in the superpotential which contain only one field with positive charge. This is because the terms with more positive charge fields do not contribute to the $F$-flatness conditions, since the positive fields are assumed to have zero VEV. Therefore, in general, the superpotential required by determination of the VEVs can be written as

$$ W = W_{H'} + W_{A'} + W_S + W_{C'} + W_{\bar{C}'}.$$  \hspace{1cm} (3.1)

Here $W_X$ denotes the terms linear in the $X$ field, which has positive anomalous $U(1)_A$ charge. Note, however, that terms including two fields with positive charge like $\lambda^{2a'}H'H'$ give contributions to the mass terms but not to the VEVs.

We now discuss the determination of the VEVs. If $-3a \leq a' < -5a$, the superpotential $W_{A'}$ is in general written as

$$ W_{A'} = \lambda^{a'+a} \alpha A' A + \lambda^{a'+3a}(\beta(A'A)_1(A^2)_1 + \gamma(A'A)_{54}(A^2)_{54}),$$  \hspace{1cm} (3.2)

where the suffixes 1 and 54 indicate the representation of the composite operators under the $SO(10)$ gauge symmetry, and $\alpha$, $\beta$ and $\gamma$ are parameters of order 1. Here we assume $a + a' + c + \bar{c} < 0$ to forbid the term $CA'AC$, which destabilizes the DW form of the VEV $\langle A \rangle$. If we take $\langle A \rangle = i\tau_2 \times \text{diag}(x_1, x_2, x_3, x_4, x_5)$, the $F$-flatness of the $A'$ field requires $x_i(\alpha \lambda^{-2a} + 2(\beta - \gamma)(\sum_j x_j^2) + \gamma x_i^2) = 0$, which
Once the direction of the VEV \( \langle D \rangle \sim -\lambda^{-2a} \). Here \( N = 1 - 5 \) is the number of \( x_i \neq 0 \) solutions. The DW form is obtained when \( N = 3 \). Note that the higher terms \( A' A^{2L+1} \) \((L > 1)\) are forbidden by the SUSY zero mechanism. If they are allowed, the number of possible VEVs other than the DW form becomes larger, and thus it becomes less natural to obtain the DW form. This is a critical point of this mechanism, and the anomalous \( U(1)_A \) gauge symmetry plays an essential role to forbid the undesired terms. It is also interesting that the scale of the VEV is automatically determined by the anomalous \( U(1)_A \) charge of \( A \), as noted in the previous section.

Next we discuss the \( F \)-flatness condition of \( S \), which determines the scale of the VEV \( \langle CC \rangle \). \( W_S \), which is linear in the \( S \) field, is given by

\[
W_S = \lambda^{s+c+c} S \left( \langle \bar{C}C \rangle + \lambda^{- (c+c)} + \sum_k \lambda^{- (c+c) + 2ka} A^{2k} \right)
\]

if \( s \geq -(c + c) \). Then the \( F \)-flatness condition of \( S \) implies \( \langle \bar{C}C \rangle \sim \lambda^{-(c+c)} \), and the \( D \)-flatness condition requires \( | \langle C \rangle | = | \langle \bar{C} \rangle | \sim \lambda^{-(c+c)/2} \). The scale of the VEV is determined only by the charges of \( C \) and \( \bar{C} \) again. If we take \( c + c = -6 \), then we obtain the VEVs of the fields \( \bar{C} \) and \( C \) as \( \lambda^3 \), which differ from the expected values \( \lambda^{-c} \) and \( \lambda^{-c} \) if \( c \neq -c \).

Finally, we discuss the \( F \)-flatness of \( C' \) and \( \bar{C}' \), which realizes the alignment of the VEVs \( \langle C \rangle \) and \( \langle \bar{C} \rangle \) and imparts masses on the PNG fields. This simple mechanism was proposed by Barr and Raby [5]. We can easily assign anomalous \( U(1)_A \) charges which allow the following superpotential:

\[
\begin{align*}
W_{C'} &= \bar{C}' (\lambda^{a+z} A + \lambda^{a+z} Z) C', \\
W_{\bar{C}'} &= C' (\lambda^{a+z} A + \lambda^{a+z} Z) \bar{C}'.
\end{align*}
\]

The \( F \)-flatness conditions \( F_{C'} = F_{\bar{C}'} = 0 \) give \( (\lambda^{a-z} A + Z) C = (\bar{C} (\lambda^{a-z} A + \bar{Z}) = 0 \). Recall that the VEV of \( A \) is proportional to the \( B - L \) generator \( Q_{B-L} \) as \( \langle A \rangle = \frac{3}{2} v Q_{B-L} \). Also \( C', 16 \), is decomposed into \( (3, 2, 1)_{1/3}, (3, 1, 2)_{-1/3}, (1, 2, 1)_{-1} \) and \( (1, 1, 2) \) under \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). Since \( \langle \bar{C} C \rangle \neq 0 \), not all components in the spinor \( C \) vanish. Then \( Z \) is fixed to be \( Z \sim -\frac{3}{2} \lambda v Q_{B-L}^0 \), where \( Q_{B-L}^0 \) is the \( B - L \) charge of the component field in \( C \), which has non-vanishing VEV. It is interesting that no other component fields can have non-vanishing VEVs because of the \( F \)-flatness conditions. If the \( (1, 1, 2) \) field obtains a non-zero VEV (therefore, \( \langle Z \rangle \sim -\frac{3}{2} \lambda v \)), then the gauge group \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) is broken to the standard gauge group. Once the direction of the VEV \( \langle C \rangle \) is determined, the VEV \( \langle \bar{C} \rangle \) must have the same direction because of the \( D \)-flatness condition. Therefore, \( \langle \bar{Z} \rangle \sim -\frac{3}{2} \lambda v \). Thus, all VEVs have now been fixed.

We do not discuss the detail of the mass spectrum here. But all fields acquire the mass term except one pair of doublet Higgs fields[6]. We will discuss only
the mass matrix of Higgs $H$. Considering the additional mass term $\lambda^2 h' H' H'$, we write the mass matrix of the Higgs fields $H$ and $H'$, which are decomposed from $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$, as

$$
(\mathbf{5}_H, \mathbf{5}_{H'}) \begin{pmatrix}
0 & \lambda^{h+h'+a} \langle A \rangle \\
\lambda^{h+h'+a} \langle A \rangle & \lambda^{2h'} \\
\end{pmatrix} \begin{pmatrix}
\mathbf{5}_H \\
\bar{\mathbf{5}}_{H'} \\
\end{pmatrix}.
$$

(3.6)

The colored Higgs obtain their masses of order $\lambda^{h+h'+a} \langle A \rangle \sim \lambda^{h+h'}$. Since in general $\lambda^{h+h'} > \lambda^{2h'}$, the proton decay is naturally suppressed. The effective colored Higgs mass is estimated as $(\lambda^{h+h'})^2 / \lambda^{2h'} = \lambda^{2h}$, which is larger than the Planck scale, because $h < 0$. One pair of the doublet Higgs is massless, while another pair of doublet Higgs acquires a mass of order $\lambda^{2h'}$. The DW mechanism works well, although we have to examine the effect of the rather light additional Higgs.

There are several terms which must be forbidden for the stability of the DW mechanism. For example, $H^2$, $HZH'$ and $HZH'$ induce a large mass of the doublet Higgs, and the term $\bar{C}A'AC$ would destabilize the DW form of $\langle A \rangle$. We can easily forbid these terms using the SUSY zero mechanism. For example, if we choose $h < 0$, then $H^2$ is forbidden, and if we choose $\bar{c} + c + a + a' < 0$, then $\bar{C}A'AC$ is forbidden. (It is interesting that the negative $U(1)_A$ charge $h$, which is required for the DT splitting, enhances the left-handed neutrino masses, as discussed in section 2.) Once these dangerous terms are forbidden by the SUSY zero mechanism, higher-dimensional terms which also become dangerous; for example, $\bar{C}A'\alpha^2 C$ and $\bar{C}A'\bar{C}\alpha C$ are automatically forbidden, since only gauge invariant operators with negative charge can have non-vanishing VEVs. This is also an attractive point of our scenario.

In this section, we have proposed a natural DT splitting mechanism in which the anomalous $U(1)_A$ gauge symmetry plays a critical role, and the VEVs and mass spectrum are automatically determined by the anomalous $U(1)_A$ charges. In the next section, we examine a model with this DT splitting mechanism, which gives realistic mass matrices of quarks and leptons.

### 4 Quark and Lepton sector

In this section, we examine the simplest model to demonstrate how to determine everything from the anomalous $U(1)_A$ charges.

In addition to the Higgs sector in Table I, we introduce only three $\mathbf{16}$ representations $\Psi_i$ with anomalous $U(1)_A$ charges ($\psi_1 = n + 3, \psi_2 = n + 2, \psi_3 = n$) and one $\mathbf{10}$ field $T$ with charge $t$ as the matter contents. These matter fields are assigned odd R-parity, while those of the Higgs sector are assigned even R-parity. Such an assignment of R-parity guarantees that the argument regarding VEVs in the previous section does not change if these matter fields have vanishing VEVs. We can give an argument to determine the allowed region of the anomalous $U(1)_A$
charges to obtain desired terms while forbidding dangerous terms. Though this is a straightforward argument, we do not give it here. Instead, we give a set of anomalous $U(1)_A$ charges with which all conditions are satisfied and a novel neutrino mass matrix is obtained: $n = 3, t = 4, h = -6, h' = 8, c = -4, \bar{c} = -1, c' = 4, \bar{c}' = 7, s = 5$. Then the mass term of $5$ and $\bar{5}$ of $SU(5)$ is written as

$$5_T(\lambda^6 \langle C \rangle, \lambda^5 \langle C \rangle, \lambda^3 \langle C \rangle, \lambda^2 \langle C \rangle).$$  \hspace{1cm} (4.1)$$

Since $\langle C \rangle = \langle C \rangle \sim \lambda^{5/2}$, because $c + \bar{c} = -5$, the massive mode $\bar{5}_M$, the partner of $5_T$, is given by

$$\bar{5}_M \sim \bar{5}_T + \lambda^{5/2} \bar{5}_T.$$  \hspace{1cm} (4.2)$$

Therefore the three massless modes $(\bar{5}_1, \bar{5}_2, \bar{5}_3)$ are written $(\bar{5}_T, \bar{5}_T + \lambda^2 \bar{5}_3, \bar{5}_T)$. The Dirac mass matrices for quarks and leptons can be obtained from the interaction

$$\lambda^2 \bar{\psi}_i \psi_j H.$$  \hspace{1cm} (4.3)$$

The mass matrices for the up quark sector and the down quark sector are

$$M_u = \begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \langle H_u \rangle, \quad M_d = \lambda^2 \begin{pmatrix}
\lambda^4 & \lambda^7/2 & \lambda^3 \\
\lambda^3 & \lambda^{5/2} & \lambda^2 \\
\lambda^1 & \lambda^{1/2} & 1
\end{pmatrix} \langle H_d \rangle.$$  \hspace{1cm} (4.4)$$

Note that the Yukawa couplings for $\bar{5}_2 \sim \bar{5}_T + \lambda^{5/2} \bar{5}_3$ are obtained only through the Yukawa couplings for the component $\bar{5}_T$, because we have no Yukawa couplings for $T$. We can estimate the CKM matrix from these quark matrices as

$$U_{CKM} = \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},$$  \hspace{1cm} (4.5)$$

which is consistent with the experimental value if we choose $\lambda \sim 0.2^3$. Since the ratio of the Yukawa couplings of top and bottom quarks is $\lambda^2$, a small value of $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \sim O(1)$ is predicted by these mass matrices. The Yukawa matrix for the charged lepton sector is the same as the transpose of $M_d$ at this stage, except for an overall factor $\eta$ induced by the renormalization group effect.

The mass matrix for the Dirac mass of neutrinos is given by

$$M_D = \lambda^2 \begin{pmatrix}
\lambda^4 & \lambda^3 & \lambda \\
\lambda^7/2 & \lambda^{5/2} & \lambda^{1/2} \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \langle H_u \rangle \eta.$$  \hspace{1cm} (4.6)$$

\(^{3}\) Strictly speaking, if the Yukawa coupling originated only from the interaction \(^{4}\), the mixing concerning to the first generation becomes smaller than the expected values because of a cancellation. In order to get the expected value of CKM matrix as in \(^{5}\), non-renormalizable terms, for example, $\bar{\psi}_i \psi_j HCC$ must be taken into account.
The right-handed neutrino masses come from the interaction

\[ \lambda \psi_i + \psi_j + 2a \Psi_i \Psi_j \bar{C} \bar{C} \]  

as

\[ M_R = \lambda \psi_i + \psi_j + 2a \langle \bar{C} \rangle_2^2 = \lambda^9 \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \]  

(4.8)

Therefore we can estimate the neutrino mass matrix:

\[ M_\nu = M_D M_R^{-1} M_D^T = \lambda^{-5} \begin{pmatrix} \lambda^2 & \lambda^{3/2} & \lambda \\ \lambda^{3/2} & \lambda & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{pmatrix} \langle H_u \rangle^2 \eta^2. \]  

(4.9)

Note that the overall factor \( \lambda^{-5} \) has negative power, which can be induced by the effects discussed in sections 2 and 3. From these mass matrices in the lepton sector the MNS matrix is obtained as

\[ U_{MNS} = \begin{pmatrix} 1 & \lambda^{1/2} & \lambda \\ \lambda^{1/2} & 1 & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{pmatrix}. \]  

(4.10)

This gives bi-maximal mixing angles for the neutrino sector, because \( \lambda^{1/2} \sim 0.5 \). We then obtain the prediction \( m_{\nu_\mu}/m_{\nu_\tau} \sim \lambda \), which is consistent with the experimental data [11, 12]: \( 1.6 \times 10^{-3} \text{(eV)}^2 \leq \Delta m_{\text{atm}}^2 \leq 4 \times 10^{-3} \text{(eV)}^2 \) and \( 2 \times 10^{-5} \text{(eV)}^2 \leq \Delta m_{\text{solar}}^2 \leq 1 \times 10^{-4} \text{(eV)}^2 \). The relation \( V_{e3} \sim \lambda \) is also an interesting prediction from this matrix, though CHOOZ gives a restrictive upper limit \( V_{e3} \leq 0.15 \) [13]. The neutrino mass is given by \( m_{\nu_\tau} \sim \lambda^{-5} \langle H_u \rangle^2 \eta^2/M_P \sim m_{\nu_\mu}/\lambda \sim m_{\nu_e}/\lambda^2 \). If we take \( \langle H_u \rangle \eta = 100 \text{ GeV}, M_P \sim 10^{18} \text{ GeV} \) and \( \lambda = 0.2 \), then we get \( m_{\nu_\tau} \sim 3 \times 10^{-2} \text{ eV}, m_{\nu_\mu} \sim 6 \times 10^{-3} \text{ eV} \) and \( m_{\nu_e} \sim 1 \times 10^{-3} \text{ eV} \). It is surprising that such a rough approximation gives values in good agreement with the experimental values from the atmospheric neutrino and large mixing angle (LMA) MSW solution of the solar neutrino problem. This LMA solution for the solar neutrino problem gives the best fit to the present experimental data [14].

In addition to Eq. (4.3), the interactions

\[ \lambda \psi_i + \psi_j + 2a + h \Psi_i A^2 \Psi_j H \]  

(4.11)

also contribute to the Yukawa couplings. Here \( A \) is squared because it has odd parity. Since \( A \) is proportional to the generator of \( B-L \), the contribution to the lepton Yukawa coupling is nine times larger than that to quark Yukawa coupling, which can change the unrealistic prediction \( m_\mu = m_s \) at the GUT scale. Since the prediction \( m_\mu/m_b \sim \lambda^{5/2} \) at the GUT scale is consistent with experiment, the enhancement factor \( 2 \sim 3 \) of \( m_\mu \) can improve the situation. Note that the
additional terms contribute mainly in the lepton sector. If we set $a = -2$, the additional matrices are

$$\frac{\Delta M_u}{\langle H_u \rangle} = \frac{v^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \frac{\Delta M_d}{\langle H_d \rangle} = \frac{v^2}{4} \begin{pmatrix} \lambda^2 & 0 & \lambda \\ \lambda & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

(4.12)

$$\frac{\Delta M_e}{\langle H_d \rangle} = \frac{9v^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ 0 & 0 & 0 \\ \lambda & 1 & 0 \end{pmatrix}. \quad \text{(4.13)}$$

It is interesting that this modification essentially changes the eigenvalues of only the first and second generation. Therefore it is natural to expect that a realistic mass pattern can be obtained by this modification. This is one of the largest motivations to choose $a = -2$. Note that this charge assignment also determines the scale $\langle A \rangle \sim \lambda^2$. It is suggestive that the fact that the $SO(10)$ breaking scale is slightly smaller than the Planck scale is correlated with the discrepancy between the naive prediction of the ratio $m_\mu/m_s$ from the unification and the experimental value$^4$. It is also interesting that the SUSY zero mechanism plays an essential role again. When $z, \bar{z} \geq -4$, the terms $\lambda^{\psi_i+\psi_j+a+z+h}Z \Psi_i A \Psi_j H + \lambda^{\psi_i+\psi_j+2z+h}Z^2 \Psi_i \Psi_j H$ also contribute to the fermion mass matrices, though only to the first generation.

Proton decay mediated by the colored Higgs is strongly suppressed in this model. As mentioned in the previous section, the effective mass of the colored Higgs is of order $\lambda^{2h} \sim \lambda^{-12}$, which is much larger than the Planck scale. Proton decay is also induced by the non-renormalizable term

$$\lambda^{\psi_i+\psi_j+\psi_k+\psi_l} \Psi_i \Psi_j \Psi_k \Psi_l,$$

(4.14)

which is also strongly suppressed.

Since the spectrum of the superheavy particles is fixed by anomalous $U(1)_A$ charges, we can check whether the three gauge couplings are unified or not. This is a severe constraint to select a realistic model. There is an example in which the three gauge couplings meet at a scale. If we take the anomalous $U(1)_A$ charges as $\psi_1 = 5, \psi_2 = 4, \psi_2 = 2, t = 3, a = -1, a' = 3, h = -4, h' = 5, c = -3, \bar{c} = 0, c' = 2, c'' = 5$, then the three gauge couplings are unified at the unification scale $\Lambda_G \sim \Lambda$. Here we have to take the cutoff scale $\Lambda \sim 2 \times 10^{16}$ GeV. Therefore, in this model, the proton decay due to the dimension 6 operator may be seen in near future.

## 5 A natural solution for the $\mu$ problem

In our scenario, SUSY zero mechanism forbids the SUSY Higgs mass term $\mu H H$. However, once SUSY is broken, the Higgs mass $\mu$ must be induced. The induced mass must be proportional to the SUSY breaking scale.

$^4$ Such an argument has been done also in [15].
We now examine a solution for the $\mu$ problem in a simple example \[2\]. The essential point of this mechanism is that the VEV shift of a heavy singlet field by SUSY breaking. We introduce the superpotential $W = \lambda sS + \lambda sZ$, where $S$ and $Z$ are singlet fields with positive anomalous $U(1)_A$ charge $s$ and with negative charge $z$, respectively ($s + z \geq 0$). Note that the single term of $Z$ is not allowed by SUSY zero mechanism, while usual symmetry cannot forbid this term. This is an essential point of this mechanism. The SUSY vacuum is at $\langle S \rangle = 0$ and $\langle Z \rangle = \lambda^{-z}$. After SUSY is broken, these VEVs are modified. To determine the VEV shift of $S$, which we would like to know because the singlet $S$ with positive charge can couple to the Higgs field with negative charge, the most important SUSY breaking term is the tadpole term of $S$, namely $\lambda sM^2_A S$. Here $A$ is a SUSY breaking parameter of order of the weak scale. By this tadpole term, the VEV of $S$ appears as $\langle S \rangle = \lambda^{-s-2z} A$. If we have $\lambda s+2hS H^2$, the SUSY Higgs mass is obtained as $\mu = \lambda s+2h S M^2_B$, which is proportional to the SUSY breaking parameter $m_S B$ and the proportional coefficient can be of order 1 if $h \sim z$. Note that the $F$-term of $S$ is calculated as $F_S \sim \lambda s S M^2_B$. The Higgs mixing term $B\mu$ can be obtained from the SUSY term $\lambda s A S H^2$ and the SUSY breaking term $\lambda s A S H^2 S H^2$ as $\lambda s A S H^2 S H^2$ and $\lambda s A S H^2 S H^2 S H^2$ and $\lambda s+2h S A S H^2 \sim \mu A$, respectively. Therefore the relation $B \sim m_S B$ is naturally obtained. This is a solution for the $\mu$ problem. Note that the condition $h \sim z$ can be satisfied because both fields $H$ and $Z$ have negative charges.

6 SUSY breaking and FCNC

We discuss SUSY breaking in this section. Since we should assign the anomalous $U(1)_A$ charges dependent on the flavor to produce the hierarchy of Yukawa couplings, generically the non-degenerate scalar fermion masses are induced through the anomalous $U(1)_A D$-term. Various experiments on the FCNC processes give strong constraints to the off-diagonal terms $\Delta$ in the sfermion mass matrices on the basis on which the flavor changing terms appear only in the non-diagonality of the sfermion propagators as in Ref.\[17\]. The sfermion propagators can be expanded in terms of $\delta = \Delta/\bar{m}^2$ where $\bar{m}$ is an average sfermion mass. As long as $\Delta$ is sufficiently smaller than $\bar{m}^2$, it is enough to take the first term of this expansion and, then, the experimental information concerning FCNC and CP violating phenomena is translated into upper bounds on these $(\delta F)_{ij}^{XY}$’s, where $F = U, D, N, E$.

\[5\] If doublet-triplet splitting is realized by fine-tuning or some accidental cancellation, the Higgs mixing $B\mu$ can become intermediated scale $m_S B M_X$ as discussed in Ref.\[16\], where $M_X$ is the GUT scale. However, once the doublet-triplet splitting is naturally solved as in Ref.\[1\], such a problem disappears.

\[6\] The large SUSY breaking scale can avoid the flavor changing neutral current (FCNC) problem, but in our scenario it does not work because the anomalous $U(1)_A$ charge of the Higgs $H$ is inevitably negative to forbid the Higgs mass term in tree level.
the chirality index $X,Y = L,R$ and the generation index $i,j = 1,2,3$. For example, the experimental value of $K^0 - \bar{K}^0$ mixing gives

\[
\sqrt{|\text{Re}(\delta_{12}^D)_{11}^L(\delta_{12}^D)_{22}^R|} \leq 2.8 \times 10^{-3} \left( \frac{\tilde{m}_q(\text{GeV})}{500} \right),
\]

\[
|\text{Re}(\delta_{12}^D)_{11}^L|, |\text{Re}(\delta_{12}^D)_{22}^R| \leq 4.0 \times 10^{-2} \left( \frac{\tilde{m}_q(\text{GeV})}{500} \right),
\]

with $\tilde{m}_q$, an average value of squark masses\footnote{The CP violation parameter $\epsilon_K$ gives about one order severer constraints on the imaginary part of $(\delta_{12}^D)_{XY}$ than the real part. We here concentrate ourselves only on the constraints from the real part of $K^0 - \bar{K}^0$ mixing, since under the other experimental constraints to the CP phase originated from SUSY breaking sector, which are mainly given by electric dipole moment, we may expect that the CP phases are small enough to satisfy the constraints from the imaginary part of the $K^0 - \bar{K}^0$ mixing.}. The $\mu \rightarrow e\gamma$ process gives

\[
|\text{Re}(\delta_{12}^E)_{11}^L|, |\text{Re}(\delta_{12}^E)_{22}^R| \leq 3.8 \times 10^{-3} \left( \frac{\tilde{m}_l(\text{GeV})}{100} \right)^2,
\]

where $\tilde{m}_l$ is an average mass of scalar leptons. In the usual anomalous $U(1)_A$ scenario, $\Delta$ can be estimated as

\[
(\Delta_{ij}^F)_{XX} \sim \lambda^{f_i-f_j}(|f_i-f_j|) \langle D_A \rangle,
\]

since the mass difference is given by $(f_i-f_j) \langle D_A \rangle$, where $f_i$ is the anomalous $U(1)_A$ charge of $F_i$. Here the reason for appearing the coefficient $\lambda^{f_i-f_j}$ is that the unitary diagonalizing matrices are given by

\[
\begin{pmatrix}
1 & \lambda^{f_i-f_j} \\
-\lambda^{f_i-f_j} & 1
\end{pmatrix}.
\]

Therefore if the condition $\psi_1 = t$ is satisfied, namely the sfermion masses of $\bar{5}_1$ and $\bar{5}_2$ are almost degenerate, the constraints from these FCNC processes become weaker. This is because the constraints from the $K^0 - \bar{K}^0$ mixing and the CP violation to the product $(\delta_{12}^D)_{11}^L \times (\delta_{12}^D)_{22}^R$ are much stronger than those to $(\delta_{12}^D)^2_{11}^L$ or $(\delta_{12}^D)^2_{22}^R$ as shown in eq. (6.1) and (6.2). Therefore suppression of $(\Delta_{12}^D)^2_{RR}$ makes the constraints much weaker.

In the next section, we show that in $E_6$ unification, the above condition is automatically satisfied.

\section{$E_6$ unification}

In the case of $E_6$, 16 and 10 of $SO(10)$ are naturally included in a single multiplet 27 of $E_6$. The fundamental representation of $E_6$ contains 16 and 10 of $SO(10)$
automatically: Under $E_6 \supset SO(10) \supset SU(5)$,
\[
27 \rightarrow \left[\frac{(16, 10) + (16, \bar{5}) + (1, 1)}{16}\right] + \left[\frac{(10, \bar{5}) + (10, 5)}{10}\right] + \left[\frac{(1, 1)}{1}\right] \quad (7.1)
\]
where the representation of $SO(10), SU(5)$ are explicitly denoted in the above. Therefore the $E_6$ model naturally has the freedom for replacing matter fields $(16, \bar{5})$ by $(10, \bar{5})$. In order to see how the replacement happens, we introduce the following Higgs fields which are relevant to determine the mass matrices of matter multiplets $\Psi_i(27)$, whose $U(1)_A$ charges are denoted as $\psi_i (i=1,2,3)$:

1. $\Phi(27)$ and $\Phi(\bar{27})$: $\langle \Phi \rangle = \langle \Phi \rangle^\dagger = \lambda^{-(\phi+\bar{\phi})}$ break $E_6$ into $SO(10)$,

2. $C(27)$ and $\bar{C}(\bar{27})$: $\langle C \rangle = \langle C \rangle^\dagger = \lambda^{-(c+c)}$ break $SO(10)$ into $SU(5)$,

3. $H(27)$: Higgs field which includes the Higgs doublets.

The $U(1)_A$ invariant superpotential for low energy Yukawa terms is,
\[
W_Y = \left(\frac{\Theta}{M_P}\right)^{\psi_i \psi_j + h} \Psi_i \Psi_j H, \quad (7.2)
\]
and that for the replacement is
\[
W = \lambda^{\psi_i + \psi_j + h} \Psi_i \Psi_j \Phi + \lambda^{\psi_i + \psi_j + c} \Psi_i \Psi_j C, \quad (7.3)
\]
where we suppress the coefficients of order one and for the above we assume that $\psi_i + \psi_j + h \geq 0$ for each $i, j$ pair so that there appears no SUSY zero.

The VEVs $\langle \Phi \rangle = \langle \Phi \rangle^\dagger = \lambda^{-(\phi+\bar{\phi})}$ and $\langle C \rangle = \langle C \rangle^\dagger = \lambda^{-(c+c)}$ induce the masses between $5$ and $\bar{5}$ as
\[
\begin{pmatrix}
\Psi_1(10, 5) \\
\Psi_2(10, 5) \\
\Psi_3(10, 5)
\end{pmatrix} =
\begin{pmatrix}
\lambda^{2\psi_1+r} & \lambda^{\psi_1+\psi_2+r} & \lambda^{\psi_1+\psi_3+r} \\
\lambda^{\psi_1+\psi_2+r} & \lambda^{2\psi_2+r} & \lambda^{\psi_2+\psi_3+r} \\
\lambda^{\psi_1+\psi_3+r} & \lambda^{\psi_2+\psi_3+r} & \lambda^{2\psi_3+r}
\end{pmatrix}
\begin{pmatrix}
\Psi_1(16, 5) \\
\Psi_2(16, 5) \\
\Psi_3(16, 5)
\end{pmatrix} \chi^{\frac{1}{2}(\phi-\bar{\phi})}, \quad (7.4)
\]
where we define a parameter $r$ as
\[
\lambda^r \equiv \lambda^{\frac{1}{2}(c-\varepsilon-\phi+\bar{\phi})}. \quad (7.5)
\]
Since $\psi_3 < \psi_1, \psi_2$, $\Psi_3$ has larger masses than $\Psi_1$ and $\Psi_2$. Therefore $3$ massless modes tends to consist of $\Psi_1$ and $\Psi_2$. Actually under some conditions, the $3$ massless modes become
\[
\begin{array}{l}
\bar{5}_1 = \Psi_1(16, \bar{5}) + \lambda^{\psi_1-\psi_3} \Psi_3(16, \bar{5}) + \lambda^{\psi_1-\psi_2+r} \Psi_2(10, \bar{5}) + \lambda^{\psi_1-\psi_3+r} \Psi_3(10, \bar{5}) \\
\bar{5}_2 = \Psi_1(10, \bar{5}) + \lambda^{\psi_1-\psi_3+r} \Psi_3(16, \bar{5}) + \lambda^{\psi_1-\psi_2} \Psi_2(10, \bar{5}) + \lambda^{\psi_1-\psi_3} \Psi_3(10, \bar{5}) \\
\bar{5}_3 = \Psi_2(16, \bar{5}) + \lambda^{\psi_2-\psi_3} \Psi_3(16, \bar{5}) + \lambda^r \Psi_2(10, \bar{5}) + \lambda^{\psi_2-\psi_3+r} \Psi_3(10, \bar{5}),
\end{array} \quad (7.7)
\]
\[8 \text{ We assume that } \psi_1 > \psi_2 > \psi_3 \]
where the first terms of the right hand side are the main components of these massless modes and the other terms are mixing terms with heavy states, \( \Psi_3(16, \bar{5}) \), \( \Psi_2(10, 5) \) and \( \Psi_3(10, \bar{5}) \). This is almost the same situation as discussed in the previous section. Actually if we take \( r = 1/2 \), namely,

\[
1 = c - \bar{c} - \phi + \bar{\phi},
\]

(7.9)

the massless modes discussed in the previous section are obtained. Namely, all the quark and lepton mass matrices are obtained even in this \( E_6 \) unification. Only the difference is that in \( E_6 \) unification the charge of the main part of second generation \( \bar{5}_2 \) is fixed as \( \psi_1 \). Therefore the condition for suppression of \( K^0\bar{K}^0 \) mixing, which was discussed in the previous section, is automatically satisfied.

Now that the constraints from the \( K^0\bar{K}^0 \) mixing (and the CP violation) become weaker as discussed above, we have larger region in the parameter space, where the lepton flavor violating processes like \( \mu \rightarrow e\gamma \) are appreciable. Actually, if the ratio of the VEV of \( D_A \) to the gaugino mass squared at the GUT scale is given by

\[
R \equiv \frac{\langle D_A \rangle}{M_{1/2}^2},
\]

(7.10)

the scalar fermion mass square at the low energy scale is estimated as

\[
m_{F_i}^2 \sim f_i R M_{1/2}^2 + \eta_F M_{1/2}^2,
\]

(7.11)

where \( \eta_F \) is a renormalization group factor. Therefore in our scenario, the eq. (6.2) for \( (\delta_{12}^D)_{LL} \) becomes

\[
(\delta_{12}^D)_{LL} \sim \lambda \frac{\langle \psi_2 \rangle R M_{1/2}^2}{\eta_M + \frac{\psi_1 + \psi_2}{2} R} = \lambda \frac{(\psi_1 - \psi_2) R}{\eta_M + \frac{\psi_1 + \psi_2}{2} R} \leq 4.0 \times 10^{-2} \left( \frac{(\eta_M + \frac{\psi_1 + \psi_2}{2} R)^{1/2} M_{1/2} (\text{GeV})}{500} \right),
\]

(7.12)

which is rewritten

\[
M_{1/2} \geq 1.25 \times 10^4 \lambda \frac{(\psi_1 - \psi_2) R}{\eta_M + \frac{\psi_1 + \psi_2}{2} R} (\text{GeV}).
\]

(7.13)

Though the main contribution to \( (\delta_{12}^D)_{RR} \) vanishes, through the mixing in eq. (7.6) and (7.7), \( (\delta_{12}^D)_{RR} \) is estimated as

\[
(\delta_{12}^D)_{RR} \sim \lambda \frac{\lambda^2}{\eta_M + \psi_1 R} R^1 \sqrt{\psi_2 (\psi_1 - \psi_2)},
\]

(7.14)

where the mixing \( \lambda^2 \) is different from the naively expected value \( 1 = \lambda^{\psi_1 - \psi_1} \).

From the eq. (6.1) for \( \sqrt{(\delta_{12}^D)_{LL} (\delta_{12}^D)_{RR}} \), the constraint to the gaugino mass \( M_{1/2} \) is given by

\[
M_{1/2} \geq 1.8 \times 10^5 \lambda^{1.75} R^{1.5} \frac{\sqrt{\psi_2 (\psi_1 - \psi_2)}}{(\eta_M + \psi_1 R)^{1.5}}.
\]

(7.15)
On the other hand, the eq. (6.3) for $(\delta E_{12})_{RR}$ leads to

\[ M_{1/2} \geq 1.6 \times 10^3 \left( \frac{\lambda(\psi_1 - \psi_2)R^{1/2}}{\eta_{ER} + \frac{\psi_1^2 + \psi_2^2}{2} R} \right) \text{ (GeV)}. \] (7.17)

Taking probable values, $\psi_1 = 5$, $\psi_2 = 4$, $\eta_{DL} \sim \eta_{DR} \sim 6$ and $\eta_{ER} \sim 0.15$, the lower limits of the gaugino mass are roughly estimated as in Table.1.

| $R$  | 0.1 | 0.3 | 0.5 | 1   | 2   |
|------|-----|-----|-----|-----|-----|
| $(\delta_{D_{12}}^{P})_{LL}$ | 15  | 38  | 53  | 73  | 86  |
| $\sqrt{(\delta_{D_{12}}^{P})_{LL}(\delta_{D_{12}}^{P})_{RR}}$ | 120 | 300 | 420 | 560 | 690 |
| $|{(\delta_{E_{12}}^{P})_{RR}}|$ | 370 | 260 | 210 | 150 | 110 |

Table 1. Lower bound of gaugino mass $M_{1/2}$ at GUT scale (GeV).

Note that when $R = 0.1$, the $\mu \rightarrow e\gamma$ process gives the severest constraint in these FCNC processes. Therefore the lepton flavor violating processes might be seen in future, though the prediction is strongly dependent on the detail of the SUSY breaking sector.

The reason for suppression of $(\Delta_{D_{12}}^{P})_{RR}$ is that the anomalous $U(1)_{A}$ charge of $\mathbf{5}_2$ becomes the same as that of $\mathbf{5}_1$ because the fields $\mathbf{5}_1$ and $\mathbf{5}_2$ are originated from a single field $\Psi_1$. This is a non-trivial situation. The massless mode of the second generation $\mathbf{5}_2 = \Psi_1(\mathbf{10}, \bar{\mathbf{5}}) + \lambda^{5/2}\psi_3(\mathbf{16}, \bar{\mathbf{5}})$ has Yukawa couplings through the second term $\lambda^{5/2}\psi_3(\mathbf{16}, \bar{\mathbf{5}})$. However, for SUSY breaking term which is proportional to the anomalous $U(1)_{A}$ charge, the contribution from the first term dominates the one from the second term, which realizes the degenerate SUSY breaking terms between the first and the second generation. It is suggestive that the requirement to reproduce the bi-large mixing angle in neutrino sector leads to this non-trivial structure, which suppresses the FCNC processes. In this way, such a non-trivial structure is automatically obtained in $E_6$ model, which is much different from the $SO(10)$ model in which the condition can be satisfied only by hand.

We should comment on $D$-term contribution to the scalar fermion masses. Generically such $D$-term has non-vanishing VEV when the rank of the gauge group is reduced by the symmetry breaking and SUSY breaking terms are non-universal. In our scenario, when $E_6$ gauge group is broken to $SO(10)$ gauge group, the $D$-term contribution gives different values to the sfermion masses of $\mathbf{16}$ and $\mathbf{10}$ of $SO(10)$, which destroys the natural suppression of FCNC in the $E_6$ unification. However, if SUSY breaking parameters become universal by some reason, the VEV of $D$ can become negligible. Actually, the condition $m_{\phi}^2 = m_{\chi}^2$ makes the VEV of the $D$ much suppressed. Therefore in principle, we can control the $D$-term contribution, though it is dependent on the SUSY breaking mechanism.

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14
8 Conclusion

In this talk, we proposed a GUT scenario of $SO(10)$ unified model in which DT splitting is naturally realized by the DW mechanism. The anomalous $U(1)_A$ gauge symmetry plays an essential role in the DT splitting. Using this mechanism, we examined the simplest model in which realistic mass matrices of quarks and leptons, including the neutrino, can be determined by the anomalous $U(1)_A$ charges. This model predicts bi-maximal mixing angles in the neutrino sector, a small value of $\tan \beta$, and the relation $V_{e3} \sim \lambda$. Proton stability is naturally realized. It is interesting that once we fix the anomalous $U(1)_A$ charges for all fields, the order of each parameter and scale is determined, except that of the SUSY breaking.

Extension into $E_6$ unification\[3\] is also interesting that the mass matrices with bi-maximal mixing discussed in this paper appear again in the $E_6$ unified model. Moreover, the condition $\psi_1 = t$, which makes the constraints from the FCNC process weaker, is automatically satisfied. In subsequent paper\[20\], it is shown that the DT splitting mechanism can be non-trivially incorporated into $E_6$ unification.

It is very suggestive that the anomalous $U(1)_A$ gauge symmetry motivated by superstring theory plays a critical role in solving the two biggest problems in GUT, the fermion mass hierarchy problem and the doublet-triplet splitting problem. This may be the first evidence for the validity of string theory from the phenomenological point of view.

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