Tunneling current characteristics in bilayer quantum Hall systems

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Abstract

Weakly disordered bilayer quantum Hall systems at filling factor $\nu = 1$ show spontaneous interlayer phase coherence if the layers are sufficiently close together. We study the collective modes in the system, the current-voltage characteristics and their evolution with an in-plane magnetic field in the phase-coherent regime.

Key words: bilayer quantum Hall ferromagnets, collective excitations

Introduction: Weakly disordered bilayer quantum Hall system has been a topic of intense research over the last decade [1–4]. This system consists of two 2D electron layers separated by a distance $d$ comparable to the distance between electrons within one layer. In a strong magnetic field perpendicular to the layers, because of the quenched kinetic energy, physical properties of the system are largely determined by the Coulomb interactions. For sufficiently small layer separation, bilayer quantum Hall system at total filling factor $\nu = 1$ undergoes a phase transition from a compressible state to an incompressible quantum Hall state with spontaneous phase-coherence [3,4]. This phase-coherent state can be described as an easy-plane ferromagnet in pseudospin language where we associate a pseudospin with the layer index. Analogous to the spin-waves in a ferromagnet, the collective modes in this system are pseudospin-waves which transfer charge from one layer to the other. The properties of these collective modes are directly probed by interlayer tunneling experiments [5]. In this paper we present an approximate but fully microscopic theory of interlayer current characteristics which takes into account the contribution of these collective modes.

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**Formalism:** We consider a bilayer system with layer separation \( d \) in a perpendicular magnetic field \( B \) and an in-plane field \( B_{||} \) confined to the region between the layers. In the strong-field limit, the pseudospin and the intra-Landau level index are the only dynamical degrees of freedom. The Hamiltonian for such a system with tunneling amplitude \( \Delta_t \) consists of a one-body tunneling term \( \hat{H}_t \), a one-body disorder potential \( \hat{V}_{\text{dis}} \) and the two-body Coulomb interactions \( \hat{V}_c \) [6]. Due to the in-plane field, the tunneling matrix elements between the layers acquire Aharonov-Bohm phases which vary with wavevector \( Q = dB_{||}/Bl^2 \) where \( l \) is the magnetic length [7]. Since the disorder arises from the impurities far away from the layers, we assume that it does not scatter electrons from one layer to the other and restrict ourselves to potentials with zero interlayer correlations [6].

The tunneling Hamiltonian \( \hat{H}_t \) is the only term in the microscopic Hamiltonian which changes the layer-index of electrons. When the tunneling is zero, the charges in individual layers are conserved separately and there is no interlayer current. Using Fermi’s golden rule to estimate the rate of change of electron-number in each layer, we get

\[
I(V, B_{||}) = \frac{e\Delta_t^2 A}{\hbar} \text{Im} \chi^{TB}(Q, eV)|_{\Delta_t=0} \tag{1}
\]

where \( \chi^{TB}(\tau) = \langle Tc^\dagger_T(\tau)c_B(\tau)c^\dagger_Bc^\dagger_T \rangle \) is the thermal response function, \( T(B) \) stands for the top(bottom) layer index and \( A \) is the area of the sample. Eq.(1) relates the current, an observable, to the two-particle response function \( \chi^{TB} \).

We calculate this response function using the self-consistent Born approximation (SCBA) with vertex corrections and the generalized random phase approximation (GRPA) (Fig. 1). Due to the properties of Landau level wavefunctions, it is possible to evaluate the diagrams in Fig. 1 analytically [6]. In the clean limit, Im \( \chi \) is saturated by \( \delta \)-functions at \( eV = \pm E_{Q_{sw}}^Q \) where \( E_{Q_{sw}}^Q \) is the pseudospin-wave energy at wavevector \( Q \) [2,3]. In the presence of disorder, Eq.(1) predicts that the peak in the \( I-V \) characteristics will be at voltages \( eV = \pm E_{Q_{sw}}^Q \). Thus we find that the collective-mode dispersion can be mapped out by varying the wavevector \( Q \) or equivalently the in-plane field \( B_{||} \) [8].

**Results:** In absence of disorder, the mean-field approximation enhances the symmetric-antisymmetric splitting \( \Delta_{SAS} = \Delta_t + \Delta_{sb} \) [2,3]. The enhancement \( \Delta_{sb} \) survives in the limit \( \Delta_t \to 0 \) giving rise to spontaneous phase-coherence. We characterize the phase-coherent state by a dimensionless order parameter \( M_0 = n_S - n_{AS} \) where \( n_\sigma \) is the integrated \( \sigma \)-state spectral-weight below the Fermi energy (\( \sigma = S, AS \)). We find that disorder-averaging broadens the sharp bands and leads to suppression of the order parameter from its clean-limit value, \( M_0 = 1 \).

We obtain the \( Q \)-dependence of the pseudospin-wave energy \( E_{Q_{sw}}^Q \) and its damp-
ing $\Gamma^Q$ because of disorder from the real and imaginary parts of susceptibility $\chi$ respectively. The inset in Fig 2 shows typical dispersions for various disorder strengths. We find that disorder suppresses the pseudospin-wave velocity from its clean-limit value. We also find that at long wavelengths the damping term grows quadratically with the wavevector, $\Gamma^Q \propto Q^2$. In particular, even in the presence of disorder, the Goldstone mode at $Q = 0$ is undamped. These results agree with those obtained from effective-field-theory models, and the fact that this approximation captures these salient features is a rationale behind choosing this particular set of diagrams. Furthermore, with this approximation we can obtain the energy and the width of the collective-mode at any wavevector. The evolution of a typical $I-V$ curve with increasing in-plane field is shown in Fig. 2. As expected from Eq. (1) the tunneling current has a peak at the pseudospin-wave energy $E_{sw}^Q$. The finite width of the peak is because of the nonzero damping $\Gamma^Q$. We also find that for a fixed wavevector, the peak broadens with increasing disorder.

**Summary:** We have presented a microscopic theory of tunneling current in a disordered $\nu = 1$ bilayer, which includes the effect of collective modes. We find that the current has a maximum at the collective-mode energy $E_{sw}^Q$. This treatment is valid only for nonzero wavevectors. We find that the collective mode is undamped at zero wavevector leading to a breakdown of the perturbation theory in $\Delta_t$. Therefore, in such a case, a non-perturbative calculation in the tunneling amplitude is necessary [6].

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Fig. 1. Diagrammatic summary of SCBA and GRPA. It is necessary to include vertex corrections along with the disorder broadening of quasiparticle bands. The competing Hartree and exchange fluctuations are captured by the direct ($V_x$) and exchange ($\Gamma$) ladder diagrams.

Fig. 2. Typical $I$-$V$ characteristics for various in-plane fields. The disorder strength is characterized by reduction of the order parameter from its clean-limit value, $M_0 = 1$. The position and the width of the peak are related to the pseudospin-wave energy $E_{sw} \propto Q$, and and its damping, $\Gamma_Q \propto Q^2$, respectively. The inset shows typical pseudospin-wave dispersions for various disorder strengths. This linearly dispersing Goldstone mode has been recently observed by Spielman et al. [5].