Protocol for secure quantum machine learning at a distant place

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The application of machine learning to quantum information processing has recently attracted keen interest, particularly for the optimization of control parameters in quantum tasks without any pre-programmed knowledge. By adapting the machine learning technique, we present a novel protocol in which an arbitrarily initialized device at a learner’s location is taught by a provider located at a distant place. The protocol is designed such that any external learner who attempts to participate in or disrupt the learning process can be prohibited or noticed. We numerically demonstrate that our protocol works faithfully for single-qubit operation devices. A tradeoff between the inaccuracy and the learning time is also analyzed.

I. INTRODUCTION

Advances in quantum information science herald a new era of information technology. Quantum information science has recently penetrated interdisciplinary science and engineering fields. In particular, a current research topic is to adapt the basic idea of machine learning for quantum information processing. Although “learning” is a behavior of humans and other living things, a device or a machine can also learn a task according to the theory of machine learning, which was developed as a subfield of artificial intelligence [1]. In fact, the optimization of control parameters without any pre-programmed knowledge can be referred to as a typical task of machine learning. In this context, the techniques of machine learning have recently been applied to various quantum information protocols [2–6].

Following this trend, here we formulate an intriguing problem. Suppose that one intends to construct an operation to execute a particular quantum task. For this purpose, a quantum machine learning technique can be used to train the operation device for the desired task. However, this device is not necessarily located at the same place as the one who is designing the task to be taught (called a provider hereafter). To realize scalable quantum devices or networks, joint work between different parts of a composite architecture or between separated participants may be necessary. For this purpose, several protocols of distributed quantum information processing have been developed [7,8]. Therefore, a quantum learning protocol performed by a separated learner and provider will also be desirable in some realistic application scenarios.

In this study, we design a protocol to prepare an arbitrary quantum device at a distant place by machine learning. We first assume an arbitrarily initialized device installed at one place where the learner (say Alice) is located. The other, spatially separated, provider (say Bob) determines the target quantum task, which cannot be directly accessed by Alice. Note that the target information does not open to any other people. Alice and Bob use mainly quantum channels to communicate their quantum states. The output state from the device at Alice’s location is sent to Bob so that he can assess the learning process. To obtain feedback from Bob, Alice also sends reference quantum states, and Bob returns them to Alice after performing his task. In designing such a protocol, we employ a specific learning algorithm called single measurement and feedback [9]. When learning is complete, we say that Alice’s operation device has learned to perform the desired quantum task.

We also consider another issue that will be very important in the related field of so-called secure machine learning [10–12]. Alice and Bob do not want any other external learner. Thus, we design the protocol such that any malicious attempts to participate in or disturb the learning can be prohibited or noticed, as long as Alice’s learning elements (i.e., controllable unitary and measurement devices) are not initially correlated [24]. We will demonstrate by Monte Carlo simulations that our protocol works well when learning tasks for qubit states. The learning time and inaccuracy are also analyzed in the demonstration.

II. CONCEPT & METHOD

Here we describe our scenario for developing a remote learning protocol. Suppose that two separated parties, Alice and Bob, intend to teach a device at Alice’s location to perform a quantum task. The target quantum task learned by the device can generally be identified as a unitary transformation from a given initial state \( |\chi_A\rangle \) to a specific final state \( |\tau_B\rangle \) determined by Bob, i.e., the provider. Alice and Bob communicate through quantum and classical channels. The process of our protocol is illustrated in Fig. 1. The tasks performed by Alice and Bob and the channels are described in detail below.

(i) Alice’s elements. – Alice prepares a controllable device \( U \) to learn a unitary transformation task from a fiducial state \( |\chi_A\rangle \) (known to both Alice and Bob). Here
U can be expressed as the unitary operator

$$\hat{U}(a) = e^{-i\mathbf{a}^T \mathbf{G}},$$

(1)

where \(a = (a_1, a_2, \ldots, a_{d^2-1})^T\) is a \((d^2 - 1)\)-dimensional (real) vector, and \(\mathbf{G} = (g_1, g_2, \ldots, g_{d^2-1})^T\) is a vector operator whose components are SU(d) group generators \(\mathbf{13, 14}\.\) We assume that \(d\) is the dimension of the Hilbert space of both |\(\tau_B\rangle\) and |\(\chi_A\rangle\). In the process, Alice controls the components \(a_j \in [-\pi, \pi]\) \((j = 1, 2, \ldots, d^2 - 1)\) of the vector \(a\) \(\mathbf{127}\). Measurement devices and a feedback system to update the control parameters according to a learning algorithm are also placed on Alice’s side. Alice also prepares to generate either |0\rangle or |\pm\rangle, which will be used as a reference state in our protocol. Alice sends both her output state obtained by applying \(U\) to the state |\(\chi_A\rangle\) and a reference state to Bob for each trial.

\(\text{(ii) Quantum channels.}\) Alice and Bob are connected by three one-way quantum channels (drawn as gray lines in Fig. 1). Two of the channels are from Alice to Bob \((C_r^{AB} \text{ and } C_o^{AB})\), and the remaining one is from Bob to Alice \((C_i^{BA})\). The channel \(C_r^{AB}\) carries the reference states, either |0\rangle or |\pm\rangle, and \(C_o^{AB}\) transmits Alice’s output states to Bob. The channel \(C_i^{BA}\) is used to deliver the reference state from Bob’s task back to Alice.

\(\text{(iii) Bob’s elements.}\) Bob, the provider, determines the target state |\(\tau_B\rangle\) (known only to Bob) and prepares it for each trial. Note that Bob does not transmit any information on the target state |\(\tau_B\rangle\) directly to Alice. After receiving Alice’s output state and a reference state, Bob operates a full-fledged quantum module, which consists of two Hadamard gates \(\hat{H} = (\hat{S}x + \hat{S}z)/\sqrt{2}\) and a control-swap (C-SWAP) gate, as illustrated in Fig. 1. The C-SWAP gate acts as \(\hat{C}_{\text{swap}} = |0\rangle \langle 0| \otimes \hat{I}_{d^2} + |1\rangle \langle 1| \otimes \hat{S}\), where \(\hat{I}_{d^2}\) is a \(d^2\)-dimensional identity, and \(\hat{S}\) is a swap operator, defined as \(\hat{S}|x\rangle \langle y| = |y\rangle \langle x|\) \(\mathbf{13, 10}\).

We now illustrate how our protocol runs. First, Alice publicly declares the commencement by announcing the fiducial state |\(\chi_A\rangle\) to Bob. The fiducial state is one element of a predetermined set of initial states, which are agreed upon by Alice and Bob in advance. For example, if the set is \{0\} \(\text{ or } \{1\}\) and Alice announces its label, e.g., either 0 or 1, then Bob knows that the input is |\(\chi_A\rangle = |0\rangle\) or |\(\chi_A\rangle = |1\rangle\), respectively. Bob then determines the target state |\(\tau_B\rangle\) according to the input |\(\chi_A\rangle\) and informs Alice that he is also ready. When Alice and Bob identify their signs \(\mathbf{29}\), the process starts:

\(\text{[P.1] For every trial, Alice generates a reference state, either |0\rangle or |\pm\rangle. For the |0\rangle state, Alice applies the learning unitary operator } \hat{U}(a) \text{ to her input state as}\)

$$|\chi_A\rangle \xrightarrow{\hat{U}(a)} |\tilde{\chi}_A(a)\rangle,$$

(2)

where \(a\) is selected on the basis of Alice’s learning algorithm. Note that \(a\) is initially chosen at random. For either |\pm\rangle or |\mp\rangle, Alice applies a random unitary operator \(\hat{U}(r_h)\), such that

$$|\chi_A\rangle \xrightarrow{\hat{U}(r_h)} |\tilde{\chi}_A(r_h)\rangle,$$

(3)

where \(r_h = (r_{h,1}, r_{h,2}, \ldots, r_{h,d^2-1})^T\) is a randomly generated vector (known only to Alice). Thus, the states |\(\tilde{\chi}_A(a)\rangle\) and |\(\tilde{\chi}_A(r_h)\rangle\) are sequentially changed in each trial, depending on the choice of reference states. Alice sends both the reference state and the output state |\(\psi_{A\rightarrow B}\rangle\), prepared as either |0\rangle\rangle |\(\tilde{\chi}_A(a)\rangle\) or |\(\pm\rangle\rangle |\(\tilde{\chi}_A(r_h)\rangle\) to Bob via \(C_r^{AB}\) and \(C_o^{AB}\), respectively. Here, we use the subscripts “r” and “o” to denote the reference and output modes, respectively. Note that Alice does not open the states that are being sent.

\(\text{[P.2] Then, Bob applies the delivered state } |\psi_{A\rightarrow B}\rangle\text{ and the target state } |\tau_B\rangle\text{ to his module, where the subscript “t” denotes the target mode. It yields the state } |\Psi_{\text{comp}}\rangle\text{ as}\)

$$|\psi_{A\rightarrow B}\rangle |\tau_B\rangle \xrightarrow{\mathbf{25, 26}} |\hat{G} \hat{H} \otimes \mathbf{1}_{d^2} (\hat{C}_{\text{swap}} |\hat{H} \otimes \mathbf{1}_{d^2} \rangle \rangle |\Psi_{\text{comp}}\rangle.$$

(4)

Here, for |\(\psi_{A\rightarrow B}\rangle = |0\rangle\rangle |\(\tilde{\chi}_A(a)\rangle\), the output state |\(\Psi_{\text{comp}}\rangle\) is equal to

$$\sum_{k=0,1} \frac{1}{\sqrt{2}} |k\rangle_r \langle 0\rangle_r \langle \tilde{\chi}_A(a)\rangle_r |\tau_B\rangle_t + \langle -1\rangle_r |\tau_B\rangle_t |\tilde{\chi}_A(a)\rangle_t,$$

(5)

whereas for |\(\psi_{A\rightarrow B}\rangle = |\pm\rangle\rangle |\(\tilde{\chi}_A(r_h)\rangle\), the output state |\(\Psi_{\text{comp}}\rangle\) is given as

$$|+\rangle_r |\tilde{\chi}_A(r_h)\rangle_o |\tau_B\rangle_t \text{ or } |-\rangle_r |\tau_B\rangle_t |\tilde{\chi}_A(r_h)\rangle_t.$$

(6)

Note again that only Alice knows whether the output |\(\Psi_{\text{comp}}\rangle\) is equal to Eq. 5 or Eq. 6. Bob resends the reference state after performing his task, written as \(\hat{\rho}_{\text{ref}} = T_{r,h} |\Psi_{\text{comp}}\rangle \langle \Psi_{\text{comp}}| \rangle \), back to Alice through \(C_r^{BA}\).

\(\text{[P.3] Then, Alice checks the returning state } \hat{\rho}_{\text{ref}} \text{ as follows: First, if the prepared reference state was } |+\rangle\rangle \text{ or } \langle -\rangle \langle -\rangle \langle \mp\rangle \rangle \text{ or } \langle -\rangle \langle -\rangle \langle \pm\rangle \rangle \text{ or } \langle -\rangle \langle -\rangle \langle \mp\rangle \rangle \text{ is not prepared, Alice's protocol is terminated. If the returned state is } |+\rangle\rangle \text{ or } \langle -\rangle \langle -\rangle \langle \pm\rangle \rangle \text{ or } \langle -\rangle \langle -\rangle \langle \mp\rangle \rangle \text{, she considers herself to be a winner. If the returned state is } \langle -\rangle \langle -\rangle \langle \mp\rangle \rangle \text{ or } \langle -\rangle \langle -\rangle \langle \mp\rangle \rangle \text{, she considers herself to be a loser.}\)”

FIG. 1: (Color online) Schematic picture of our protocol. Alice prepares a (fiducial) state |\(\chi_A\rangle\) (which is also known to Bob) and initializes her own (unitary) device \(U\) to be \(\hat{U}\). Then, Bob determines the state |\(\tau_B\rangle\) of target (which is known to only Bob) at a distant place, so that Alice’s device \(U\) learns a desired quantum operation (See the main text for details).
or $|−\rangle$, Alice performs the measurement $M_±$ with the bases $\{|+,−\rangle\}$ on $ψ_{ref}$. Note that Bob’s operation does not alter the reference states $|+\rangle$ and $|−\rangle$ [see Eq. (3)]. Thus, if an unexpected outcome, i.e., “−” (or “+”) for the initially prepared reference state $|+\rangle$ (or $|−\rangle$), appears in $M_±$, Alice can immediately notice that the state transmitted in $C^A_B$ or $C^B_A$ has been altered by an external learner [27]. Second, for the reference state $|0\rangle$, the returned state $ψ_{opt}$ is measured by $M_{0/1}$ with the bases $\{|0\rangle,|1\rangle\}$. In this case, the measurement results are delivered to the feedback system for effective quantum learning.

By iterating steps [P.1]–[P.3], Alice’s device $\hat{U}(a)$ is supposed to learn the desired task,

$$\hat{U}(a_{opt})\langle a_{opt}|\chi_A) \simeq |τ_B⟩,$$

where $a_{opt}$ denotes the optimal vector achieved after learning is complete. To realize this learning process, we can use the following property: If $|τ_A(a_{opt})⟩ = |τ_B⟩$, Bob’s output state $|ψ_{comp}⟩$ is to be $|0⟩$, $|τ_B⟩_o|τ_B⟩_l$ [see Eq. (5)], so Alice cannot obtain the outcome of $|1⟩$ in her measurement $M_{0/1}$. More generally, the probability $Pr(k)$ that Alice measures $|k⟩$ ($k = 0,1$) in $M_{0/1}$ can be calculated as

$$Pr(k) = \frac{1 + (-1)^k f}{2},$$

where $f = |⟨τ_B|τ_A(a_{opt})⟩|^2$. Our learning strategy is thus to update $\hat{U}(a)$ until $|0⟩$ is successively measured, without any single outcome of $|1⟩$, in $M_{0/1}$. This strategy is conceptually equivalent to the maximization of $f$.

III. LEARNING ALGORITHM

To realize the above-mentioned strategy, we employ the quantum learning algorithm based on single measurement and feedback introduced in Ref. [9]. This algorithm requires a finite $N_L$-bit classical first-in-first-out (FIFO) memory in which the measurement results are recorded as “fail” or “not-fail” data. Note that, as the memory size is finite, the newest data have to push the old data out of the memory (see Fig. 2). Thus, the memory retains the latest data for the learning process.

In our case, the learning algorithm is programmed in Alice’s feedback system with the rule for updating the vector $a$ of $U$. The learning algorithm runs as follows: If Alice measures $|0⟩$ in $M_{0/1}$ (that is, “not-fail”), the feedback system reserves judgment regarding whether the current $\hat{U}(a)$ is appropriate and thus leaves the vector $a$ unchanged. Otherwise, if $|1⟩$ is measured (that is, “fail”), $a$ is updated according to

$$a^{(n)} = a^{(n-1)} + \frac{N_F}{N} \tau^{(n)},$$

where $n$ denotes the number of iterations of the effective learning process (or the total number of measurements $M_{0/1}$ performed), $\tau^{(n)}$ is a vector randomly generated at the $n$th iteration step, and $N = \min(N_L, N_F + N_{nf})$. Here, $N_F$ and $N_{nf}$ are the number of “fail” and “not-fail” data recorded in the memory, respectively. Our learning algorithm is intuitively understandable: the greater the number of “fail” events is, the more changes are imposed. Note that the random vector $\tau_l$, rather than any pre-programmed knowledge, is used to develop $a$. This feature is a typical trait of machine learning and is of particular importance in our task, as it implies that any information about the target state $|τ_B⟩$ is not directly referenced to find the optimal vector $a_{opt}$.

The learning process is continued until all the “fail” data are eliminated in the $N_L$ memory blocks. We call this the halting condition. After learning is complete, i.e., the halting condition is satisfied, Alice’s final output state $|τ_A(a_{opt})⟩$ is supposed to be well matched to the target state $|τ_B⟩$, with $f = |⟨τ_B|τ_A(a_{opt})⟩|^2 = 1 - ε_L (ε_L \ll 1)$. Here, we can infer that the learning error $ε_L$ becomes small for large $N_L$, but a large $N_L$ requires a longer learning time, as explicitly shown later.

IV. NUMERICAL ANALYSIS

We perform numerical simulations to analyze our learning protocol. Here, we consider the single-qubit target states (i.e., $d = 2$) for a numerical proof-of-principle demonstration. In the simulations, we investigate mainly the learning and survival probabilities. The learning probability $P_L(n)$ is defined as the probability that learning is completed before or at a certain number $n$ of effective iteration steps. The survival probability $P_S(n)$ is defined as $P_S(n) = 1 - P_L(n)$; thus, it is the probability that learning is not completed until $n$. In Fig. 3 we draw $P_L(n)$ and $P_S(n)$ for $N_L = 100$ by averaging over 1000 simulation data. In each simulation, the target state $|τ_B⟩$ is randomly chosen. We find that $P_S(n)$ is well fitted to the exponential decay function

$$e^{−(n+1−N_L)/n_c},$$

where $n_c$ is a characteristic constant, and $n \geq N_L$ because of the definition of the halting condition. As $P_L(n)$ is an accumulate distribution function (by definition), the average number $\bar{n}$ of iterations to complete the (effective)}
The survival probability \( P(t) \) is well fitted to the exponential decay function \( e^{-(n+1-N_L)/n_c} \) (green dashed line), where \( n_c \) is a characteristic constant that characterizes the average number of effective iterations \( n \) required to complete the learning process; \( \pi = n_c + N_L \). We obtain \( n_c \approx 352 \) and thus \( \pi \approx 452 \). The actual average iteration number in the simulations is \( \approx 478 \).

For further analysis, simulations are also performed by increasing \( N_L \) from 50 to 500 at intervals of 50. In Fig. 3(a), we plot \( \pi \) with respect to \( N_L \). Each point in the graph is obtained by averaging 1000 simulation data. The data points are very well fitted to \( \pi = c_1 N_L^{α} \) with \( c_1 \approx 0.72 \) and \( α \approx 1.39 \). Note that the obtained value of \( α \) is smaller than \( d^2 - 1 \) (in our case, \( d = 2 \)), which means that learning is efficient compared to that obtained by any classical strategy [3]. We also plot the learning error \( \tau_L \) (averaged over 1000 data) in Fig. 3(b). The data points are also well fitted to \( \tau_L = c_2 N_L^{-β} \), and we find \( c_2 \approx 1.12 \) and \( β \approx 0.81 \). From these results, we can see the tradeoff relation between the inaccuracy (i.e., \( \tau_L \)) and the learning time (i.e., \( \pi \)) depending on \( N_L \). To see this more clearly, we draw the graph of \( \tau_L \) versus \( \pi \) in Fig. 5 (see Appendix A). By data fitting, we obtain \( \tau_L \approx 1.10 \times \pi^{0.59} \) (green dashed line in Fig. 5).

V. DISCUSSIONS

Before closing, we discuss the reason that it is impractical for any other external learner (say Eve) to furtively intrude in the learning process. First, note that the target state \( |τ_B⟩ \) is neither directly moved to Alice nor removed from Bob’s side. Note further that the optimized vector \( a_{opt} \) cannot be viewed on Alice’s side after learning is complete. Thus, what Eve can do is to intercept the transmitted particles in the channels \( C_{AB}^r, C_{AB}^o, \) and \( C_{BA}^r \) and to learn \( |τ_B⟩ \) or \( |τ_A(a_{opt})⟩ \) from the intercepted particles. However, this is quite formidable owing to the following complications:

[C.1] If the qubit states transmitted through \( C_{AB}^r \) or \( C_{BA}^r \) are altered, Alice immediately perceives the alterations by the measurement \( M_h \), as described above. This method of using a “cheat-sensitive” (sub)system is often used in quantum cryptographic tasks.

[C.2] Even though Eve can intercept the states moving through \( C_{AB}^o, C_{BA}^o, \) and \( C_{BA}^o \) without being discovered, it is still impossible to learn \( |τ_B⟩ \) or \( |τ_A(a_{opt})⟩ \) because the intercepted particles, \( |τ_A(a)⟩\langle τ_A(a)| \) and \( |χ(r_b)⟩\langle χ(r_b)| \), are highly mixed and indistinguishable. Actually, the state of \( N_{int} \) intercepted particles is close to the random mixture \( \frac{1}{2} I_d \) when \( N_{int} \gg 1 \) because \( a \) and \( r_b \) are continuously changed in each trial of the learning process.

[C.3] We finally note that learning is very sensitive to any external alteration of Alice’s estimation states \( |τ_A(a)⟩ \) transmitted in \( C_{AB}^o \) (see Appendix B). Thus, even for any super-Eve who can sort out \( |τ_A(a)⟩ \) in \( C_{AB}^o \), Alice...
can be aware of any ill-intentioned attempts by monitoring the learning time; any alteration is indicated by learning that is too late or cannot be completed, even though unexpected outcomes do not appear in $M_{\perp}$.

In summary, we presented a protocol for quantum machine learning at a distant place. In particular, our protocol was designed such that an external learner cannot participate in or harm the learning process. We demonstrated by Monte Carlo simulations that learning can be faithfully completed for single-qubit target states. We also analyzed the tradeoff between the inaccuracy and the learning time. We expect that our protocol will be developed for realistic applications in quantum information and quantum cryptography tasks.

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Appendix A: Detailed data in Figs. 4 and 5

Here we provide the detailed data in Figs. 4 and 5. By performing numerical simulations while increasing $N_{L}$ from 50 to 500 at intervals of 50, we characterize the learning probabilities $P_{L}(n)$ and survival probabilities $P_{S}(n)$. The simulations are performed 1000 times for each $N_{L}$. For all the cases of $N_{L}$, the survival probabilities $P_{S}(n)$ are well fitted to the fitting function $e^{-\left(n+1-N_{L}\right)/n_{c}}$ [as in Eq. (10)] with the characteristic constant $n_{c}$. The parameters $n_{c}$ and the (estimated) average number of iterations $\bar{n} = N_{L} + n_{c}$ are listed in Tab. I. Here, $\bar{n}_{\text{sim}}$ denotes the average number of iterations actually counted in the simulations. We also find the learning error $\epsilon_{L}$ (averaged over 1000 simulations) for each $N_{L}$. The identified values of $\epsilon_{L}$ are also given in Tab. I. We note again that the fitting parameters $n_{c}$ have finite values for all cases. We thus expect that learning can be completed faithfully for the given $N_{L}$.

Appendix B: Effect on learning of any alterations

Here we consider a situation in which particles in the state $|\tilde{\eta}_{A}(a)\rangle$ moving through $C^{AB}_{o}$ are altered with a certain probability $p_{\text{int}}$ by some malicious Eve. Here, we assume a super-Eve who can sort out Alice’s estimation state $|\tilde{\eta}_{A}(a)\rangle$, discarding the blinded state $|\tilde{\chi}_{A}(r_{h})\rangle$, in $C^{AB}_{o}$ for his/her own effective learning. Eve’s aim is to learn Alice’s vector $a$ and thus to obtain the optimal vector as close to $a_{\text{opt}}$ as possible when Alice’s learning is complete. Eve can thus adopt the strategy of learning Alice’s vector $a$ using a stolen particle for each trial and resend the newly generated particle of his/her estimated state $|\tilde{\eta}_{E}(\text{e})\rangle$ to Bob, where $\text{e}$ is a vector of Eve’s own device.

However, in this case, it takes much longer to complete the learning process because some particles of $|\tilde{\eta}_{A}(a)\rangle$ are altered as $|\tilde{\eta}_{A}(a)\rangle \rightarrow |\tilde{\eta}_{E}(\text{e})\rangle$. To corroborate this, we perform numerical simulations of single-qubit target states ($d = 2$). Here, we set $N_{L} = 100$ and consider three cases: $p_{\text{int}} = 0.1$ (red), 0.2 (green), and 0.3 (blue). We perform 1000 simulations to draw the graphs. In each simulation, the target state $|\tau\rangle$ is randomly chosen. The survival probabilities $P_{S}(n)$ are also well fitted to Eq. (10) (black solid lines).

| $N_{L}$ | $n_{c}$ | $\bar{n} = N_{L} + n_{c}$ | $\epsilon_{L}$ |
|--------|--------|-----------------|-----------|
| 50     | $\pm 143$ | $\pm 193 (\pm 195)$ | $\pm 0.04727$ |
| 100    | $\pm 352$ | $\pm 452 (\pm 478)$ | $\pm 0.02690$ |
| 150    | $\pm 718$ | $\pm 868 (\pm 872)$ | $\pm 0.01964$ |
| 200    | $\pm 996$ | $\pm 1196 (\pm 1257)$ | $\pm 0.01505$ |
| 250    | $\pm 1365$ | $\pm 1615 (\pm 1658)$ | $\pm 0.01268$ |
| 300    | $\pm 1711$ | $\pm 2011 (\pm 2111)$ | $\pm 0.01089$ |
| 350    | $\pm 2176$ | $\pm 2526 (\pm 2754)$ | $\pm 0.00981$ |
| 400    | $\pm 2478$ | $\pm 2878 (\pm 3125)$ | $\pm 0.00882$ |
| 450    | $\pm 3207$ | $\pm 3657 (\pm 3806)$ | $\pm 0.00836$ |
| 500    | $\pm 3758$ | $\pm 4258 (\pm 4532)$ | $\pm 0.00760$ |

TABLE I: Values of $n_{c}$, $\bar{n}$ ($\bar{n}_{\text{sim}}$), and $\epsilon_{L}$ in Figs. 4 and 5

FIG. 6: (Color online) (Color online) (a) Learning probability $P_{L}(n)$ and (b) survival probability $P_{S}(n)$ on a log scale, assuming some Eve who can steal particles moving in $C^{AB}_{o}$ with a certain probability $p_{\text{int}}$. We assume that Eve can adopt the best learning strategy for her learning (see the main text). Here, we set $N_{L} = 100$ and consider the qubit target states, i.e., $d = 2$. We consider three cases: $p_{\text{int}} = 0.1$ (red), 0.2 (green), and 0.3 (blue). We perform 1000 simulations to draw the graphs. In each simulation, the target state $|\tau\rangle$ is randomly chosen. The survival probabilities $P_{S}(n)$ are also well fitted to Eq. (10) (black solid lines).
cannot be completed, Alice stops the learning process so that Eve cannot complete the process $e \rightarrow a_{\text{opt}}$.

| $p_{\text{int}}$ | $\bar{\epsilon} = N_L + n_c \left( \bar{\epsilon}_{\text{sim}} \right)$ | $\epsilon_L$ |
|-----------------|--------------------------------|-------------|
| 0.1             | $\simeq 1.736 \times 10^4 \ (\simeq 1.747 \times 10^4)$ | $\simeq 0.019$ |
| 0.2             | $\simeq 1.808 \times 10^4 \ (\simeq 1.956 \times 10^4)$ | $\simeq 0.021$ |
| 0.3             | $\simeq 2.473 \times 10^5 \ (\simeq 2.767 \times 10^5)$ | $\simeq 0.022$ |

TABLE II: Values of $n_c$, $\bar{\epsilon}$ ($\bar{\epsilon}_{\text{sim}}$), and $\epsilon_L$ in Fig. 6.

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[24] Such an assumption could be strong in a device-independent quantum-cryptographic scenario [18]. However, this condition is essential in machine learning because one should trust his/her machine to identify, evaluate, and control the data in the learning process.
[25] The components $a_j$ can be matched to some real control parameters in experiments, e.g., beam-splitter and phase-shifter alignments in a linear optical system [19] or radio frequency pulse sequences in a nuclear magnetic resonance system [21].
[26] Here, Alice and Bob can use the user authentication scheme to identify their signs (or any malicious external learner pretending to be Alice or Bob) [21, 22].
[27] We assumed that there are no noise effects in the channels $C^{AB}_i$ and $C^{BA}_i$. 