Melting, reentrant ordering and peak effect for Wigner crystals with quenched and thermal disorder

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Keywords: Wigner crystal, order to disorder transition, transport

Abstract

We consider simulations of Wigner crystals in solid state systems interacting with random quenched disorder in the presence of thermal fluctuations. When quenched disorder is absent, there is a well defined melting temperature determined by the proliferation of topological defects, while for zero temperature, there is a critical quenched disorder strength above which topological defects proliferate. When both thermal and quenched disorder are present, these effects compete, and the thermal fluctuations can reduce the effectiveness of the quenched disorder, leading to a reentrant ordered phase in agreement with the predictions of Nelson (1983 Phys. Rev. B 27 2902–14). There are two competing theories for the low temperature behavior, and our simulations show that both capture aspects of the actual response. The critical disorder strength separating ordered from disordered states remains finite as the temperature goes to zero, as predicted by Cha and Fertig (1995 Phys. Rev. Lett. 74 4867–70), instead of dropping to zero as predicted by Nelson. At the same time, the critical disorder strength decreases with decreasing temperature, as predicted by Nelson. The onset of the reentrant phase can be deduced based on changes in the transport response, where the reentrant ordering appears as an increase in the mobility or the occurrence of a depinning transition. We also find that when the system is in the ordered state and thermally melts, there is an increase in the effective damping or pinning. This produces a drop in the electron mobility that is similar to the peak effect phenomenon found in superconducting vortices, where thermal effects soften the lattice or break down its elasticity, allowing the particles to better adjust their positions to take full advantage of the quenched disorder.

1. Introduction

There is a wide class of systems of two-dimensional (2D) particle assemblies that form triangular crystalline phases, including vortices in thin film superconductors [1], colloidal particles [2–4], dusty plasmas [5, 6], magnetic skyrmions [7, 8], active matter [9], and electron solids or Wigner crystals [10, 11]. In the absence of quenched disorder, these systems exhibit a melting transition under increasing temperature that can be characterized by the proliferation of topological defects such as dislocations and disclinations [3, 12, 13]. The melting transition in 2D systems has been intensely studied and can occur as a two step transition with an intermediate hexatic phase or as a single weakly first order phase transition [12, 14, 15]. Even if a hexatic phase is present, the two steps of the transition can occur so close together that it becomes difficult to distinguish whether there are two separate phases in addition to the solid phase [3].

Crystalline systems confined to 2D can also show order to disorder transitions as a function of increasing quenched disorder strength. At $T = 0$ there can be a critical disorder strength above which an amorphous phase appears [17, 18]. When quenched disorder and thermal fluctuations coexist, there can be a competition in which thermal fluctuations soften the interactions between particles but at the same time also reduce the effectiveness of the disordered substrate. Nelson [16] proposed that for a 2D system with
quenched disorder, thermal effects can wash out the quenched disorder, leading to a thermally induced reentrant ordering effect from a low temperature amorphous state to a crystal state as a function of increasing temperature. At still higher temperatures, the system thermally melts. In other studies, it was suggested that the quenched disorder always enhances the appearance of a disordered state, and that introduction of quenched disorder monotonically decreases the temperature at which the transition from a disorder-induced amorphous state to a thermally reordered state occurs [17–19].

It is possible that the reentrant ordering proposed by Nelson [16] may depend on the size scale and strength of the quenched disorder pinning sites. For systems with long range particle-particle interactions but short range particle-pin interactions, reentrant ordering is possible when the pinning sites are sufficiently small. In figure 1 we show a schematic phase diagram as a function of reduced disorder strength $\sigma/\sigma_c$ versus temperature $T/T_m$ highlighting the ordered and disordered phases. When $\sigma = 0$, a melting transition occurs at $T_m$. The schematic does not distinguish whether there is also a hexatic phase, but simply indicates the point at which topological defects start to proliferate. At $T = 0$, for increasing $\sigma$ there is a transition from a crystal to a disordered state at $\sigma_c$. On the right hand side of the figure, the solid line indicates that the critical value $\sigma_c$ at which the system disorders decreases with increasing temperature. The three lines on the left hand side of the figure indicate different possible low temperature behaviors. The dashed line is the prediction from Nelson [16], where the system is disordered at very low $T$ but thermal effects wash out the effect of the disorder, permitting reordering to occur with increasing temperature. Cha and Fertig [17] predicted the upper blue solid line, where the system remains ordered at low temperatures all the way up to a constant critical $\sigma_c$. The solid red line is what we observe in the present work, where disordering occurs even at $T = 0$ when $\sigma$ is increased, but for $\sigma < \sigma_c$ there is a reentrant ordering with increasing temperature, so that the predictions of both Nelson and of Cha and Fertig occur. We also find that for finite $\sigma$, the thermally induced melting transition at higher temperatures is depressed in temperature. We note that in their simulation work, Cha and Fertig found that at $T = 0$ there is a critical disorder strength $\sigma_c$ for the 2D crystal to disorder; however, they did not consider the effect of finite temperature to see whether there might still be a reentrant thermally reordered phase. Reentrant phases have been observed previously at zero temperature in systems with periodic pinning [20–23], but such reentrance is produced by very different mechanisms from what we consider in this work, where the reentrance arises due to the thermal fluctuations creating what is essentially a floating solid phase.

Another feature of 2D systems with quenched disorder is that the transport of the system should strongly depend on whether the particle arrangement is crystalline, disordered, or fluid [18, 24–28]. If the system is in a crystal state, there can still be a depinning threshold, but the depinning will be an elastic process in which particles keep their same neighbors [18, 24, 27]. If the system is disordered or glassy, the depinning can be plastic where a portion of particles remain immobile while the other particles move, creating local tearing in the assembly [18, 24, 26]. In the fluid phase, strong thermal hopping reduces the effects of the quenched disorder. For $T = 0$, the depinning threshold $F_c$ shows a pronounced change across $\sigma_c$, where in general $F_c$ rapidly increases with increasing $\sigma$ once the system is on the disordered side of the transition where the depinning is plastic [18, 24, 27]. Additionally, both the shape of the velocity-force curves and the fluctuations in the moving state show pronounced changes across the transition from elastic to plastic depinning, going.

Figure 1. Schematic phase diagram as a function of reduced quenched disorder strength $\sigma/\sigma_c$ versus reduced temperature $T/T_m$ for a 2D assembly of repulsive particles in the presence of quenched disorder. Here $\sigma_c$ is the quenched disorder strength at the threshold for dislocation proliferation at $T = 0$, while $T_m$ is the melting temperature at $\sigma = 0$. The green dashed line is the prediction from Nelson [16] showing a reentrant disordered phase. The solid blue line is the prediction from Cha and Fertig [17], and the solid red line is the result we find in this work.
from a single step depinning process on the elastic side to a multiple step process in the plastic phase [18, 24–27]. The jump up in pinning effectiveness upon crossing from elastic to plastic depinning has previously been argued to be the cause of what is called the peak effect phenomenon observed in type-II superconductors [29–41]. In the superconducting vortex system, the driving forces on the vortices arise from an applied current $J$, and the vortices depin above a critical current $J_c$. The peak effect occurs when, as a function of increasing temperature, $J_c$ undergoes a rapid increase at a well defined temperature or magnetic field. If the vortices are already moving when the temperature is increased, there is a drop in their average velocity at the peak effect temperature. The apparent increase in the effectiveness of the pinning with increasing temperature is counterintuitive because it is more natural to expect that increasing the temperature would reduce the effectiveness of the pinning. In the peak effect regime, the thermal fluctuations are argued to reduce the elasticity of the vortex lattice (VL) or induce the formation of dislocations that strongly soften the VL, causing the VL to become amorphous and permitting individual vortices to adjust easily to the pinning landscape in order to become better pinned [29, 32, 33]. As the temperature is increased further, the thermal fluctuations overwhelm the pinning energy and the vortices readily hop out of the pinning sites, causing a decrease in the critical current or equivalently an increase in the vortex velocity at fixed current. It is also possible to observe the peak effect as a function of increasing field, and in this case it is argued that changes in the magnetic penetration depth can reduce the strength of the vortex-vortex interactions and soften the VL. Various measures of the VL structure show that the peak effect is characterized by a transition from an ordered lattice to a disordered or amorphous state [34, 37, 38]. The peak effect occurs in both 2D and three-dimensional (3D) systems, and the 3D peak effect is associated with first order characteristics and history dependence [37, 40]. The peak effect should be a general feature in any type of 2D elastic system coupled to quenched disorder where thermal effects can cause a softening of the lattice, leading to an increase in the depinning threshold or a drop in the velocity across the elastic to plastic transition.

Another system that forms a 2D crystal state that can be driven is electron solids or Wigner crystals [42–53]. Solid state systems hosting Wigner crystals usually contain some form of quenched disorder, and a variety of studies have revealed nonlinear transport and possible depinning thresholds [44–46, 48, 54] associated with enhanced noise [51]. Wigner crystals can also undergo melting transitions as a function of increasing temperature [55–59]. There is a growing number of solid state systems in which Wigner crystals could be realized, including dichalcogenide monolayers [60], moiré heterostructures [61, 62], bilayer systems [63], and Wigner crystals at zero field [64], while new advances in materials preparation point to a variety of future experiments that could be done in which the competition between quenched disorder and thermal effects could be studied [11]. Unlike colloidal assemblies or superconducting vortices, imaging experiments for solid state Wigner systems are difficult, so the existence of order to disorder or melting transitions must be determined on the basis of some type of response or transport experiments. An open question is what the Wigner crystal phase diagram is as a function of quenched disorder and temperature, as illustrated schematically in figure 1, and whether the different phases can be deduced from transport measures. Another question is whether Wigner crystals can also exhibit a peak effect or an increase in the effectiveness of the pinning as a function of increasing temperature similar to what is found in superconducting vortex systems across a thermally induced disordering transition.

In this work we perform simulations of a 2D solid state localized electron system in the presence of quenched disorder $\sigma$ and thermal disorder $T$. At $\sigma = 0$, there is a well defined melting temperature $T_m$ characterized by a proliferation of topological defects, while for $T = 0$ there is a well defined critical quenched disorder strength $\sigma_c$ above which topological defects proliferate. We map out the phase diagram as a function of $\sigma$ versus $T/T_m$ and find that when $\sigma < \sigma_c$, an increase in $T$ can cause the system to thermally order as predicted by Nelson [16]. Additionally, when $\sigma$ is finite, thermal disordering occurs at temperatures lower than $T_m$. For sufficiently large $\sigma$, the system is always disordered. We show that the ordered and disordered phases can be detected using transport signatures, where we apply a finite driving force and measure the changes in the average velocity. When $\sigma < \sigma_c$, the system forms a disordered pinned state at low temperatures, but at the reentrant ordering transition, a lattice with elastic behavior emerges and the depinning threshold is strongly reduced, leading to a finite velocity of the Wigner crystal. Under driving, the velocity drops at higher temperatures when the system thermally melts because the electrons can partially adjust their positions in order to maximize their interactions with the quenched disorder, similar to the drop in vortex mobility seen across the peak effect as a function of increasing temperature in superconducting vortex systems [29–40]. The peak effect we observe only occurs for $T/T_m < 1.0$. We show that reentrance in the phase diagram occurs both as a function of increasing disorder strength for fixed disorder density and as a function of increasing disorder density for fixed disorder strength, and that the peak effect remains robust for different values of the drive.
2. Methods

We consider a 2D solid state system with periodic boundary conditions in the x and y directions containing $N_e$ localized electrons at an electron density of $n = N_e/L^2$, where the system is of size $L \times L$. Throughout this work we use $n = 0.44$, $L = 36$, and $N_e = 572$. The system also contains $N_p$ localized immobile pinning sites of density $n_p = N_p/L^2$. Note that the solid state quenched disorder differs from the self-induced dimples observed in liquid helium systems [65]. The initial electron configuration is obtained via simulated annealing, similar to what was done in previous simulations of Wigner crystals in the presence of disorder [66–69]. The equation of motion for electron $i$ in the Wigner crystal is

$$\alpha_d v_i = \sum_j^N \nabla U(r_{ij}) + F_p + F^T_i + F_0.$$  \hspace{1cm} (1)

The damping term is $\alpha_d$ and the electron–electron repulsive interaction potential is $U(r_{ij}) = q/r_{ij}$, where $q$ is the electron charge, $r_i$ and $r_j$ are the positions of electrons $i$ and $j$, and $r_{ij} = |r_i - r_j|$. Since the interactions are of long range, we employ a real space version of a modified Ewald summation technique called the Lekner method as in previous work [70, 71]. The pinning force $F_p$ is modeled as arising from randomly placed static short range parabolic traps of radius $r_0 = 0.35$. The thermal fluctuations $F^T_i$ are represented by Langevin kicks with the properties $\langle F^T_i \rangle = 0$ and $\langle F^T_i(t) F^T_j(t') \rangle = 2k_B T \delta_{ij} \delta(t - t')$. We also consider the effect of an applied driving force $F_0 = F_D x$, which could come from an applied voltage. We measure the average velocity per electron, $\langle V \rangle = \sum_i^N v_i \cdot \hat{x}$, allowing us to construct the equivalent of an experimental current–voltage curve. The simulation method we employ for the pinning and dynamics of Wigner crystals was used previously to examine nonlinear velocity-force curves [66], noise [67], and depinning thresholds [69, 72]. If the electrons are subjected to a magnetic field $B$, there can be an additional force term $qB \times v_i$ that can generate a Hall angle for the electron motion [68]; however, in general this term is small and we will not consider it in this work.

3. Results

We first study a sample containing no quenched disorder. To characterize the system we use the fraction of sixfold coordinated electrons, $P_6 = N_e^{-1} \sum_i^N \delta(z_i - 6)$, where the coordination number $z_i$ of electron $i$ is obtained using a Voronoi construction. For a triangular lattice of electrons, $P_6 = 1.0$. In figure 2(a) we plot $P_6$ versus $T/T_m$, where the melting temperature $T_m$ is defined to occur at the point where $P_6$ shows a rapid drop. Figure 2(a) indicates that the melting temperature is well defined. We plot the Voronoi construction for the electron positions at $T/T_m = 0.52$ in figure 3(a), where the lattice is ordered, and at $T/T_m = 1.09$ in figure 3(b), where numerous topological defects have appeared. In figure 2(b) we show $P_6$ versus the maximum disorder strength $\sigma$ in a sample with a pinning density of $n_p = 0.25$ at zero temperature, $T = 0$. There is a well defined disorder strength $\sigma_c \approx 0.105$ above which a proliferation of topological defects occurs.

We next perform a series of simulations at different combinations of $\sigma$ and $T$ for the system from figure 2 with $n_p = 0.25$. In figure 4(a) we plot $P_6$ versus $T/T_m$ at $\sigma = 0$, 0.06, 0.1, 0.12, 0.13, 0.135, 0.14, and 0.16, where $T_m$ is defined to be the temperature at which melting occurs for $\sigma = 0$. For $\sigma = 0$, 0.06, and 0.1, the system starts off ordered at low $T/T_m$ and remains ordered until it melts near $T/T_m \approx 1$. The drop in $P_6$ associated with melting shifts to lower values of $T/T_m$ as $\sigma$ increases. For $\sigma = 0.12, 0.13, 0.135,$ and 0.14, all of which are above the value of $\sigma = 0.105$ for which the system disorders at $T = 0$ in figure 2(b), the system is disordered at lower temperature. As $T/T_m$ increases, there is a critical temperature at which $P_6$ increases back to a value near $P_6 = 1$, indicating that the electrons have ordered under increasing temperature. The onset of this reentrant ordering shifts to higher $T/T_m$ with increasing $\sigma$, while the thermal disordering temperature drops to lower $T/T_m$ as $\sigma$ becomes larger, so for $\sigma = 0.14$ there is only a narrow window, $0.5 < T/T_m < 0.75$, where the system is ordered. For $\sigma = 0.16$, the system is disordered at all temperatures.

In figure 4(b) we plot $\langle V \rangle$ versus $T/T_m$ for the same system in figure 4(a) where we have added a driving force with $F_D = 0.01$. Under this drive, the $\sigma = 0$ system has $\langle V \rangle = 0.01$ for all values of $T/T_m$. When $\sigma = 0.06$ or 0.01, $\langle V \rangle$ starts off at a finite value and increases with increasing $T/T_m$; however, for $\sigma > 0.1$, $\langle V \rangle = 0$ at low temperatures where $P_6$ is low. This indicates that the system is easily pinned in the low temperature disordered phase; however, when the electrons enter the reentrant ordered state, they form an elastic lattice that is less well pinned, leading to an increase in $\langle V \rangle$. Another interesting effect is that at the thermal melting transition, the drop in $P_6$ is also correlated with a drop in $\langle V \rangle$, indicating that the effectiveness of the pinning increases at the thermally induced disordering transition. This behavior is very
Figure 2. (a) The fraction of sixfold coordinated electrons $P_6$ vs reduced temperature $T/T_m$ for a system with no quenched disorder, $\sigma = 0$. There is a well defined melting temperature $T_m$ indicated by the drop in $P_6$. (b) $P_6$ vs disorder strength $\sigma$ in a system with pinning density $n_p = 0.25$ and zero temperature, $T/T_m = 0$. There is a well defined value of $\sigma$, $\sigma_c$, at which topological defects begin to proliferate, leading to a drop in $P_6$.

Figure 3. Voronoi construction of the electron positions for the system in figure 2(a) with no quenched disorder, $\sigma = 0$. White polygons are sixfold coordinated, blue are fivefold coordinated, red are sevenfold coordinated, and gray are fourfold coordinated. (a) At $T/T_m = 0.52$ the lattice is triangular. (b) At $T/T_m = 1.09$, numerous topological defects have appeared and the system is in a liquid state.

Similar to the peak effect phenomenon, where at a finite drive the average superconducting vortex velocity can show a drop with increasing temperature when the system thermally disorders. In the vortex case, it has been argued that the disordered state is softer and can therefore better adapt to the pinning landscape. Figure 4(b) shows that at higher temperatures where the thermal effects start to dominate, the velocity increases with increasing $T/T_m$. For $\sigma = 0.06$ there is only a weak dip in $\langle V \rangle$ at the thermal disordering temperature, and at $\sigma = 0.16$, there is no dip in the velocity since the system is always disordered.

In figure 5 we show a blowup of the $\langle V \rangle$ versus $T/T_m$ plot from figure 4(b) in order to highlight the drop in $\langle V \rangle$ across the thermal melting transition. In general, the drop in velocity only appears at $T/T_m < 1.0$ and shifts to lower values of $T/T_m$ as the pinning strength $\sigma$ increases. This result indicates that both the reentrant ordering transition and the thermal melting can be detected using changes in the transport measures.

Using the features in figure 4, in figure 6 we construct a phase diagram as a function of $\sigma$ versus $T/T_m$ showing the ordered and disordered regimes. Reentrant ordering occurs for $0.1 < \sigma < 0.14$, and the thermally induced disordering transition drops further below $T/T_m = 1.0$ with increasing $\sigma$. The phase
Figure 4. (a) $P_6$ vs $T/T_m$ for the system in figure 2 with $n_p = 0.25$ at $\sigma = 0.0, 0.06, 0.1, 0.12, 0.13, 0.135, 0.14,$ and $0.16$, from top to bottom. For $\sigma = 0.0, 0.06,$ and $0.1$, there is no reentrant ordering, but for $\sigma = 0.12, 0.13, 0.135,$ and $0.14$, there is reentrant ordering with increasing temperature. (b) The corresponding velocity $\langle V \rangle$ vs $T/T_m$ at $F_D = 0.01$. The velocities are low in the disordered regime at lower $T/T_m$, but increase at the reentrant ordering transition. In addition, at the thermal melting transition there is a drop in $\langle V \rangle$ similar to the peak effect phenomenon found for superconducting vortices.

Figure 5. A blowup of the plot of $\langle V \rangle$ vs $T/T_m$ in figure 4(b) for the system in figure 2 with $n_p = 0.25$ at $\sigma = 0.0, 0.06, 0.1, 0.12, 0.13, 0.135, 0.14,$ and $0.16$, from top to bottom, showing that the velocity drops across the thermal disordering transition.

diagram includes the reentrant ordering feature predicted by Nelson [16] as well as a finite value of $\sigma_c$ at $T = 0$ as proposed by Cha and Fertig [17]. For $\sigma > 0.14$, the system is always disordered. The dashed line indicates the upper end of the region in which there is a velocity reduction associated with the peak effect. This line is determined by the point at which $\langle V \rangle$ reaches 75% of $\langle V_d \rangle$, where $\langle V_d \rangle$ is the value of $\langle V \rangle$ just before the velocity dip begins as a function of increasing $T/T_m$. We note that additional lines could be drawn in the disordered regime to differentiate between a low temperature glassy or pinned phase and a higher temperature fluctuating fluid phase.

In figure 7(a) we plot $P_6$ versus $T/T_m$ for the system in figure 4 at a fixed $\sigma = 0.12$ for varied pinning densities of $n_p = 0, 0.15, 0.25, 0.35, 0.45$ and $0.55$. For $n_p = 0$ and 0.15 there is no disordered phase at low $T/T_m$, however, for $0.25 \leq n_p \leq 0.45$, there is a regime of reentrant ordering. For $n_p = 0.55$, the system is disordered for all values of $T/T_m$. Figure 7(b) shows the corresponding $\langle V \rangle$ versus $T/T_m$ curves. The drop in $\langle V \rangle$ at the thermal ordering transition indicates that the same peak effect appears that was found for varied disorder strengths. For $n_p = 0.55$, $\langle V \rangle$ monotonically increases with increasing $T/T_m$. 
Figure 6. Phase diagram as a function of pinning strength $\sigma$ vs reduced temperature $T/T_m$ constructed from the features in figure 4 for a system with $n_p = 0.25$. Green indicates the ordered regime and red indicates the disordered regime. Reentrant ordering occurs for $0.1 < \sigma < 0.14$. The peak effect appears between the disordering transition and the thermal melting regime, and the dashed line indicates the end of the peak effect window.

Figure 7. (a) $P_o$ vs $T/T_m$ for the system in figure 4 at fixed $\sigma = 0.12$ for $n_p = 0, 0.15, 0.25, 0.35, 0.45, \text{ and } 0.55$, from top to bottom. (b) The corresponding $\langle V \rangle$ vs $T/T_m$ under a drive of $F_D = 0.01$ showing a drop in velocity across the thermal melting transition.

In figure 8 we show a phase diagram of the ordered and disordered states as a function of $T/T_m$ versus pinning density $n_p$ constructed using the features in figure 7. The same reentrant ordering phase appears that was found in figure 6 at constant pinning density for changing pinning strength. The dashed line indicates where the drop in velocity associated with peak effect is lost.

We have also tested how robust the peak effect phenomenon is for varied driving force $F_D$. In figure 9 we plot $\langle V \rangle$ versus $T/T_m$ for a system with $n_p = 0.25$ and $\sigma = 0.12$ at $F_D = 0.01, 0.0075, 0.005, \text{ and } 0.0025$. As $F_D$ decreases, at low temperatures there is an extended window where the system is pinned, while at higher temperatures the peak effect region remains robust. The thermal melting transition shifts to slightly lower values of $T/T_m$ with decreasing driving force. This result suggests that if $F_D$ is low enough, the system could show a reentrant pinning near the thermal disordering transition.
4. Discussion

Our simulation results indicate that the two competing theories from Nelson [16] and Cha and Fertig [17] describing the low temperature behavior of 2D systems with quenched disorder both capture aspects of the actual response. The critical disorder strength $\sigma_c$ separating ordered from disordered states remains finite as $T \to 0$, as predicted by Cha and Fertig, instead of dropping to zero as predicted by Nelson. At the same time, $\sigma_c$ decreases with decreasing temperature, as predicted by Nelson, instead of remaining constant, as predicted by Cha and Fertig. This behavior has not been observed in previous simulations and should be general for other 2D systems with quenched disorder. We also demonstrate that a peak effect should be observable in Wigner crystal systems. Interestingly, the peak effect we find differs from the superconducting vortex peak effect that can arise due to changes in the interaction potential with temperature [73–77] or a change in the effective dimensionality of the vortices [78, 79]. The peak that arises in our Wigner crystal simulations occurs when the changing amount of thermal fluctuations modifies the elasticity of the electron lattice, which is much closer to the original Pippard picture of the peak effect [29]. We argue that part of the reason why earlier simulations did not produce a clear peak effect as a function of temperature alone is that when finite temperature is present, long simulations spanning 20–50 million simulation time steps are required in order
to obtain proper time averaging and avoid transient signatures [80]. The earlier studies generally treated situations with $T = 0$ where transient behavior is not important.

5. Summary

We have investigated the quenched disorder versus temperature phase diagram for a solid state 2D Wigner crystal system. For zero quenched disorder, there is a well defined melting temperature that is characterized by the proliferation of topological defects. For zero temperature there is also a well defined quenched disorder strength at which the system becomes disordered. The phase diagram shows that when the disorder strength is larger than the value at which the zero temperature system becomes disordered, an increase in temperature can produce a thermally induced or reentrant ordering transition when the thermal fluctuations become strong enough to overwhelm the effect of the quenched disorder without melting the lattice. At higher temperatures, introduction of quenched disorder reduces the thermal disordering temperature of the system, while for strong enough quenched disorder the system is always disordered. The phase diagrams we obtain show both the reentrant ordering feature predicted by Nelson for 2D systems with quenched and thermal disorder as well as a well defined quenched disorder strength where the electrons disorder at zero temperature, as predicted by Cha and Fertig. We also show that these phases can be observed through features in the transport curves, where the effectiveness of the pinning strongly drops at the reentrant ordering transition as the system goes from plastic behavior to elastic behavior. In the presence of quenched disorder, the thermal disordering transition is associated with a drop in the average electron velocity, similar to the peak effect phenomenon found in superconducting vortex systems. The drop occurs when thermal fluctuations break down the elasticity of the crystal and allow the electrons to adjust more easily to the pinning substrate. At higher temperatures, the velocity goes back up again when the electrons begin to hop readily out of the pinning barriers. We show that these effects are robust for a range of pinning strengths, densities, and drives. Our predictions could be tested by examining transport signatures in solid state Wigner crystals where both thermal and quenched disorder effects arise. Our results should be general to the broader class of 2D systems with quenched disorder.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

We gratefully acknowledge the support of the U.S. Department of Energy through the LANL/LDRD program for this work. This work was supported by the US Department of Energy through the Los Alamos National Laboratory. Los Alamos National Laboratory is operated by Triad National Security, LLC, for the National Nuclear Security Administration of the U. S. Department of Energy (Contract No. 89233218NCA000001).

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References

[1] Guillamón I, Suderow H, Fernández-Pacheco A, Sese J, Cordoba R, De Teresa J M, Ibarra M R and Vieira S 2009 Nat. Phys. 5 651–5
[2] Murray C A and Van Winkle D H 1987 Phys. Rev. Lett. 58 1200–3
[3] Zahn K, Lenke R and Maret G 1999 Phys. Rev. Lett. 82 2721–4
[4] Reichhardt C and Olson Reichhardt C J 2003 Phys. Rev. Lett. 90 095504
[5] Thomas H, Morfill G E, Demmel V, Goree J, Feuerbacher B and Möhlmann D 1994 Phys. Rev. Lett. 73 652–5
[6] Chiang C H and Lin I 1996 Phys. Rev. Lett. 77 647–50
[7] Huang P, Schonenberger T, Cantoni M, Heinen L, Magrez A, Rosch A, Carbone F and Ronnow H M 2020 Nat. Nanotechnol. 15 761
[8] Zázvorka J, Dittrich F, Ge Y, Kerber N, Raab K, Winkler T, Lützies K, Veis M,Virnau P and Kläui M 2020 Adv. Funct. Mater. 30 2004037
[9] Digregorio P, Levis D, Cugliandolo L F, Gonnella G and Pagonabarraga I 2022 Soft Matter 18 566–91
[10] Monceau P 2012 Adv. Phys. 61 325–581
[11] Shayegan M 2022 Nat. Rev. Phys. 4 212–3
[12] Strandburg K J 1988 Rev. Mod. Phys. 60 161–207
[13] von Grünberg H H, Keim P, Zahn K and Maret G 2004 Phys. Rev. Lett. 93 255703
[14] Marcus A H and Rice S A 1996 Phys. Rev. Lett. 77 2577–80
[15] Du D, Dostakakis M, Hilou E and Biswal S L 2017 Soft Matter 13 1548–53
[16] Nelson D R 1983 Phys. Rev. B 27 2902–14
