Tunneling Density of State in Y Junction of Tomonaga-Luttinger Liquid Wires: A Density Matrix Renormalization Group Study

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It is well known that the pristine bulk of an interacting one dimensional (1D) system in Tomonaga-Luttinger liquid (TLL) phase shows power law suppression of quasi-particle tunneling amplitude for all values of TLL parameter \( g \), in the zero energy limit. We perform a density matrix renormalization group (DMRG) study of a fully symmetric Y junction of TLL wires and observe an anomalous enhancement of the tunneling density of states (TDOS) in the vicinity of the junction for both (a) interacting hardcore bosons case and (b) interacting fermions case, when \( g > 1 \). We also observe suppression of TDOS for \( g < 1 \) for both bosonic and fermionic case. We find that the TDOS enhancements follow different power laws for bosonic and fermionic cases which suggests that these represent distinct fixed points owing to statistical correlations which play an important role at the Y junction. Analysis of static conductance for the junction indicates that the fermionic fixed point for \( 3 > g > 1 \) resembles the mysterious M-fixed point of Y junction predicted by Oshikawa, Chamon, and Affleck [J. Stat. Mech. P02008 (2006)].

I. INTRODUCTION

The technological advances at the sub-micron scale have enabled fabrication of one dimensional (1D) wires and their junction with high precision1–5. In a confined quasi-1D geometry, effect of inter-electronic repulsion is omnipresent, and the weakest interaction could drive the system to the Tomonaga-Luttinger Liquid (TLL) phase in the low energy limit.6–8 The TLL phases9,10 of 1D electronic quantum systems have been of sustained interest to condensed matter physicists due to their non-Fermi liquid behavior.9,31–15 The power law decay of the bulk electronic density of states (DOS), \( \rho(\epsilon) \sim |\epsilon - \epsilon_F|^{\alpha} \) is a well known signature of TLL wires, where the value of \( \alpha \) depends on the system parameters. Here \( \alpha > 0 \) indicates the fact that the DOS goes to zero as the energy approaches the Fermi energy which is an effect induced purely due to interaction.

An early study of tunneling into a TLL was reported by Oreg and Finkelstein16 and since then there have been several works reported on the topic.17–23 Amongst these, Jeckelmann in Ref. [20] applied dynamical density-matrix renormalization group (DDMRG) method to a 1D spinless fermion (SF) chain with nearest neighbor interaction and he confirmed that the bulk density of states (DOS) shows a power law suppression as \( \epsilon \rightarrow \epsilon_F \) (\( \epsilon_F \) being the Fermi energy) in the gapless phase, as is expected from the TLL theory. He has also confirmed that the TDOS shows an enhancement (suppression) as \( \epsilon \rightarrow \epsilon_F \) at the boundary of the SF chain for attractive (repulsive) inter-particle density-density interaction, which is consistent with the predictions of TLL theory.24

An interesting variant of the two terminal TLL wire set up is the junction of three or more TLLs. Such multi-wire junction of TLL presents a quantum impurity problem which is distinct from an isolated quantum impurity embedded in the bulk of a pristine TLL owing to its much richer fixed point (FP) structure. In recent times, junctions of TLL wires has gained much interest, especially the three-wire junction (Y junction) which is the simplest non-trivial junction of 1D TLL wires.

This structure can be recognized as a basic constituent of future quantum circuits and has already been explored experimentally.25–31 The first theoretical work on this topic was by Nayak et al. where they used bosonization and boundary conformal field theory techniques to obtain FP conductance of the Y junction hosting a resonant level.32 Since then the studies on the topic has predominantly focused on finding various interesting FPs and analyzing the spectral properties of the system using bosonization, weak interaction renormalization group (WIRG) or functional RG (fRG).18,21,32–50 The ground state properties of Y junctions have also been explored using density matrix renormalization group (DMRG) techniques.51–53 A study of TDOS using bosonization technique for spinless fermion was reported by Agarwal et al. in Ref. [18] and a collection of FPs (stable for \( g > 1 \)) were identified which showed enhancement of TDOS in the zero energy limit and the effect was attributed to an Andreev-like reflection off the junction. This study was recently extended to include spin in Ref. [21]. However it should be noted that a numerical study using DMRG technique focused on evaluation of TDOS of Y junction is presently lacking and is the primary focus of the present work.

This paper starts by considering a Y junction of spin-1/2 chains with nearest neighbour anisotropic (XXZ) Heisenberg type interaction. This model can be directly mapped on to a corresponding hard-core boson (HB) model with nearest neighbour interaction. We perform a DMRG study of Y junction for the HB model and the corresponding spinless fermion (SF). We use the correction vector approach to calculate the local contribution to the tunneling density of states (TDOS) of the system.54–57 We first study the Y junction of SF chains and draw a comparison with the existing studies of 1D SF chains and obtain results depicting enhancement of TDOS for \( g > 1 \) limit. Thereafter, we shift our focus to the HB Y junction and verify the existence of enhancement in TDOS near the junction in \( g > 1 \) limit. We further explore the effect of various parameters of the problem on the TDOS spectra, and present quantitative studies on the nature of the spectra. Finally, we
comment on the spatial cut-off scale of the observed TDOS enhancement near the junction.

This paper is organized in four sections. The motivation and existing studies related to our problem has been introduced in Sec. I. Our model and numerical techniques are described in detail, in Sec. II. The calculation of TDOS for the system using the correction vector method has been explained here. Our results are described in Sec. III. We have concluded by summarizing our findings in Sec. IV.

II. MODEL AND NUMERICAL TECHNIQUES

We consider a Y junction of \( N = 3n + 1 \) sites, constituted by three 1D TLL wires of \( n \) sites each, connected at a common central site labeled \( x = 0 \) as shown in Schematic Fig. 1. We study three model Hamiltonians on this geometry. The first model comprises of a of spin-1/2 degrees of freedom placed at site where spin are interacting with their nearest neighbours through an anisotropic (XXZ) Heisenberg like interaction. The model Hamiltonian for the system is given by,

\[
H = \sum_{i=1,k=1}^{n,3} \left[ \frac{J}{2} (S^+_{i,k} S^-_{i+1,k} + h.c.) + J' S^z_{i,k} S^z_{i+1,k} \right] + \sum_{k=1}^{3} \left[ \frac{J}{2} (S^+_0 S^-_{1,k} + h.c.) + J' S^z_0 S^z_{1,k} \right]
\]  

(1)

The first part of the Hamiltonian represents exchange interactions in each of the wires labeled by \( k \). Here \( S^+ \) and \( S^- \) are the raising (lowering) and \( z \)-component of local spin operator, respectively, acting on site \( i \) on leg \( k \) of the system. In the current work, we consider the XXZ model Hamiltonian, therefore \( J' = J_z = J = 1 \) has been kept fixed, and \( J^z \) is considered as the variable parameter. The hard-core boson (HB) is the second model that we considered in this work, and the Hamiltonian can be written as,

\[
H = \sum_{i=1,k=1}^{n,3} \left[ -t(b^+_{i,k} b_{i+1,k} + h.c.) + V n_{i,k} n_{i+1,k} \right] + \mu \left( n_{i,k} + \frac{1}{4} \right)
\]  

(2)

\[
+ \sum_{k=1}^{3} \left[ -t(b^+_0 b^-_{1,k} + h.c.) + V n_0 n_{1,k} \right] + \mu \left( n_0 + \frac{1}{4} \right)
\]

(3)

\( b_{i,k} \) (\( b^+_{i,k} \)) is the boson annihilation (creation) operator at site \( i \) of leg \( k \) and \( n_{i,k} \) is number operator. In the HB limit, the maximum occupation number of the each site is 1, i.e., has two degrees of freedom. The Hamiltonian in Eq. (1) can be exactly mapped to bosonic Hamiltonian\(^{38}\), where \( t = -J/2 \) and \( V = J^2 \) are the transfer integral and density-density interaction strength between nearest neighboring sites, respectively. \( \mu = J^z \) is the chemical potential strength of the system.

Since there is a one-to-one mapping between the HB and XXZ spin-1/2 and the whole energy spectrum is same, hence we solve only HB model, and refer to it as the bosonic Y junction (BY).

The third model is the spinless fermion (SF) model where only a nearest neighbour interaction is included in the hamiltonian. The SF Hamiltonian is given by,

\[
H = \sum_{i=1,k=1}^{n,3} \left[ -t(c^+_{i,k} c_{i+1,k} + h.c.) + V n_{i,k} n_{i+1,k} \right] + \mu \left( n_{i,k} + \frac{1}{4} \right)
\]  

(4)

\[
+ \sum_{k=1}^{2} \left[ -t(c^+_{0} c^-_{1,k} + h.c.) - t'(c^+_{0} c^-_{1,3} + h.c.) \right]
\]

\[
+ \sum_{k=1}^{3} \left[ V n_0 n_{1,k} + \mu \left( n_0 + \frac{1}{4} \right) \right]
\]

\( c_{i,k} \) (\( c^+_{i,k} \)) is the fermion annihilation (creation) operator at site \( i \) of leg \( k \) and \( n_{i,k} \) is number operator. The model Hamiltonian in Eq. (1) can be mapped to this model Hamiltonian using Jordan-Wigner (JW) transformation\(^{39}\). The relation between parameters in these two models are hopping integral \( t = -J/2 \), electron-electron interaction \( V = J^2 \) and the chemical potential \( \mu = J^2 \). The \( t' \) in Eq. (3) can be related to the \( t \) in Eq. (2) as, \( t' = \prod_{i=1}^{n} (-1)^{n_{i,2}+n_{i,3}} \), where\( n_{i,k} \) is the number operator on the site \( i \) of the leg \( k \) and \( n_0 \) is the number operator on the junction site labeled as 0 (as shown in Schematic Fig. 1). It is easily shown that Eq. (3) is essentially same as Eq. (1) and Eq. (2) for a linear 1D chain, however, for the multi-wire junction, the SF system is distinguished due to the non-trivial phase factors associated in the hopping interaction \( t' \) between the junction and the third constituent wire- which accumulates the delocalized JW phase from the other two constituent wires. In our numerical analysis using DMRG for the SF Y junction we have taken \( t' = t \). We refer this SF Y junction system as the fermionic Y junction (FY).

In this paper we study both the BY and FY model system.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Schematic of Y junction formed by three 1D TLL arms of length \( n \) each, joined at an additional central site, \( n_0 = n + 1 \). In our convention, the labeling of the spin site starts from the open end, as illustrated in the figure.}
\end{figure}
To correlate our discrete model parameter with the TLL parameter, we use the results from the 1D bosonic and fermionic systems. The Luttinger parameter \( g_s \) corresponding to the exchange interaction \( J^z \) of spin or bosonic system can be derived using Bethe ansatz (a derivation is presented in Ref. [60]), and is as follows, given by\(^{60} \),

\[
\frac{1}{g_s} = 1 + \frac{2}{\pi} \sin^{-1} \left( \frac{J^z}{J} \right).
\]

And, the Luttinger parameter \( g_f \) corresponding to the inter-particle density-density interaction \( V \) in the half-filled FY junction system is given by Ref. [13],

\[
g_f = \frac{\pi}{2} \frac{1}{\pi - \cos^{-1}(V/2t)}
\]

The limit \( J^z = 0 \) \((V = 0)\) corresponds to the free-particle limit, where \( g_s = 1 \) \((g_f = 1)\). The ferromagnetic (attractive) limit \( J^z < 0 \) \((V < 0)\) corresponds to the Luttinger parameter \( 0 < g_s(g_f) < 1 \) and the AF (repulsive) limit corresponds to \( 1 > g_s(g_f) > \infty \). We study the TDOS in the bosonic and fermionic Y junctions in both the \( 0 > g_s(g_f) > 1 \) and \( 1 > g_s(g_f) > 3 \) limits, to identify the enhancement and suppression regimes.

Since all the model Hamiltonians considered on the Y junction geometry in Eq. (1), (2) and (3) contain many-body interaction terms, hence, the degrees of freedom in the system increases exponentially with the system size \( N \). Therefore, the exact diagonalization (ED) technique are used for smaller system sizes up to \( N \sim 28 \), and Density matrix renormalization group (DMRG) techniques are used for larger system sizes, up to \( N = 610 \). DMRG is a state-of-art numerical techniques based on the systematic truncation of irrelevant degrees of freedom, and renormalization of the system observables with the reduced density matrix wavefunction \(^{61,62} \). For accurate calculations we have used the modified DMRG algorithm especially designed for Y junction, which renders the accuracy of these calculations comparable to that for linear 1D chains.\(^{52} \) To maintain a reliable accuracy in the calculations, eigenvectors corresponding to up to 200 largest eigenvalues are retained in each DMRG sweep. The truncation error of the density matrix eigenvalues is less than \( 10^{-12} \). For better accuracy, we perform finite DMRG upto 10 sweeps, and the total error in gs state is less than 0.01%.

In this paper study of TDOS is our main focus, and the TDOS for a system gives information about the low lying excitations and can be defined as,

\[
\rho_z(\omega) = \int_0^\infty e^{-\omega t}dt \langle \psi_0 | A(t)A(0) | \psi_0 \rangle
\]

\[
= \int_0^\infty \sum_n e^{-\omega t}dt \langle \psi_0 | e^{-iHt}A | \psi_n \rangle \langle \psi_n | e^{+iHt}A \rangle | \psi_0 \rangle
\]

\[
\propto \text{Im} \left[ \sum_n \frac{|\langle \psi_0 | A^+_n \psi_0 | \rangle|^2}{E_n - (E_0 + \omega) + i\eta} \right]
\]

where, \( |\psi_0 \rangle \) and \( E_0 \) are the gs wavefunction and energy, whereas \( |\psi_n \rangle \), and \( E_n \) are \( n^{th} \) eigenstate of the system. \( A^+_n \equiv S^+_n \), \( e^{+iHt}_n \) represents the spin raising and fermionic creation operators, respectively, of particle at \( n^{th} \) site. The spatial numbering in the Y junction system is shown in Fig. 1. The broadening factor \( \eta \) used in the calculation of TDOS in Eq. (6) is generally proportional to the lifetime of quasi-particles, and we keep its value fixed to \( \eta = 0.20 \) throughout all the calculations. We use the TDOS correction vector technique to calculate the TDOS, which is a state-of-art numerical technique for dynamical calculations \(^{54-57} \). TDOS is equivalent to locally injecting a magnon into the gs of system, which can now access all the excited states with a finite transition probability determined by the non-zero transition matrix elements between the gs and the excited state.

## III. Results and Comparisons

In this paper we present TDOS behavior of both the BY and FY junction systems, and our numerical studies show that the TDOS in the proximity of junction (including the junction site) for both the BY and FY systems shows enhancement in the attractive interaction limit, whereas the suppression is observed in the repulsive interaction limit. These results have also been complemented by the static conductance calculations which lead to identification of the FPs responsible for the observed enhancement or suppression of the TDOS. In particular, we show that the FP corresponding to the enhancement in the FY model belongs to the M-FP earlier predicted in literature\(^{44} \). Though we observe similar signatures in the static current conductance for both the BY and FY models, the power law exponents for the TDOS near the junction for both systems are distinct, which can be attributed to the exchange statistics of the particles-bosons in BY, and fermions in FY model. We note here that for a 2–wire junction, the effect of statistics of the particles generally are not reflected in the TDOS spectrum, owing to cancellation of the statistical phase in 1D linear chain. In the last subsection, we present results showing that the observed enhancement has a finite spatial cut-off, and we demonstrate that it is highly localized near the junction. Before explaining the TDOS results, we revisit the gs properties of both models.

The ground state of BY and FY system with even \( n \) (odd \( N \)) contains \( \rho = N/2 + 1 \) bosons and \( \rho = N/2 + 1 \) fermions, respectively, for an isotropic interaction \( t = V \); whereas for the spin–1/2 model, the gs lies in \( S^z = 1/2 \) manifold at \( J = J^z \). For odd \( n \) (even \( N \)) system and isotropic interaction, the gs of the spin–1/2 system has three spin–1/2 ups delocalized at the edge of each leg and a down spin delocalized near junction sites, however, overall the gs of the system is a triplet state. Whereas, for even \( n \) (odd \( N \)), gs is always a doublet state having one spin–1/2 delocalized at the junction site\(^{52} \). In the anisotropic attractive limit, gs of the spin–1/2 (BY and FY) system lies in \( S^z = 0 \) \((\rho = N/2)\) sector for \( J_z/J < 1(V/t < 0) \).
A. Tunneling density of state (TDOS)

The TDOS spectrum for 1D TLL wires has been extensively studied in literature, where the BY and FY model spectrum are indistinguishable, however for quasi-1D or multiwire junctions, difference in TDOS spectrum is expected between the BY and FY systems because of non-trivial phase factors involved in the FY model junction, any well defined analytic study of which is lacking in literature. As the Y junction system are well known for their unique behavior of density of states near the junction, here we study the TDOS of this system near the junction for both the BY and the FY models. Since the TDOS of the 1D SF model has been extensively studied \cite{Liu2010}, therefore, let us first recapitulate the TDOS results of the 1D SF system, and then compare it with the Y junction system. The power law exponent $\alpha$ corresponding to the TDOS of the bulk or mid-chain TDOS $\alpha_b$, and TDOS of the boundary or open end $\alpha_E$ of the interacting 1D SF chain is given by,

$$\alpha_b = \frac{(g_f - 1)^2}{2g_f}$$  \hspace{1cm} (7) \\
$$\alpha_E = \frac{1}{g_f - 1}$$  \hspace{1cm} (8)

where $g_f$ is the Luttinger parameter as defined in Eq. (5).

To compare the power law exponent $\alpha_b$ and $\alpha_E$ obtained for the 1D SF chain with that obtained for the FY system ($\alpha_{FY}$), we begin by calculating TDOS $\rho_0(\omega)$ at the junction site $x = 0$, for $V/t = 1,0$ and $-1$, as a function of frequency $\omega$, as shown in Fig. 2. We notice that the TDOS of junction site near the Fermi-energy for $V = -1$ shows a peak at $\omega \to 0$ which is a signature of enhancement, whereas it shows a flat region near Fermi energy $\omega \approx 0$ for the non-interacting limit $V/t = 0$. The peak near $\omega = 0$ for $V = 0$ is consistent with the system size and owes its origin in the broadening factor $\eta$ and the degeneracy at the Fermi-point of the half-filled FY system. For $V = +1$, the TDOS shows a peak at a large $\omega$ which though similar to 1D SF model, differs in terms of the power law exponent. For the 1D SF chain, $\alpha_b$ and $\alpha_E$ are 0.04 and 1/3 for $V = 1$, whereas we observe $\alpha_{FY} = 0.125 \pm 0.05$ for our FY junction. For $V = -1$, $\alpha_b$ and $\alpha_E$ are 0.08 and $-1/3$, and we observe $\alpha_{FY} = -1.65 \pm 0.2$ for the FY junction.

We notice transition in the nature of TDOS from enhancement to suppression on increasing $V/t$ at $V/t = 0$. The repulsive interaction FY model ($V/t > 0$) shows suppression, whereas in the attractive regime ($V/t < 0$) it shows enhancement.

Similar to the FY model, the BY junction also shows a qualitatively similar TDOS pattern. The TDOS $\rho_x(\omega)$ of the junction site for the BY model for various values of $J^x/J$ are shown as a function of frequency $\omega$ are shown in Fig. 3. We notice that TDOS of junction sites $\rho_x(\omega)$ for $J^x = -1$ shows enhancement and the maxima of $\rho_x(\omega)$ decreases with $J^x$. The $\rho_x(\omega)$ follows a power law with $\alpha_{BY} < 0$ and $|\alpha_{BY}|$ decreases with increasing $J^x$ (in the $J^x/J < 0$ regime), as shown in Fig. 3. The behavior of the $\rho_x(\omega)$ with $\omega$ changes at $J^x = 0$, and for $J^x > 0$ TDOS of junction sites shows enhancement to suppression (E-S) transition. The inset of Fig. 3 shows the exponent $\alpha_{BY}$ vs. $J^x$ plot. The $\alpha$ increases with $J^x$, and for $J^x = +1$, $\alpha > 0$.

Whereas the regime of enhancement and suppression is qualitatively similar for the BY and FY junctions, the quan-
titative details differ—e.g., in terms of the power law exponent α. The power law exponent are $\alpha_{BY} = -0.90$ for the BY junction, and $\alpha_{FY} = -1.65$ for the FY junction at $J^z/J = -1$ and $V/t = -1$, respectively. As discussed before, this difference could be attributed to the difference in the exchange statistics of the particles of the respective models.

### B. Conductance, $G_{\beta,\gamma}$ and M-Fixed Point

Since we observed qualitatively similar TDOS enhancements at the junction in both the BY and FY systems, though the TDOS power law exponents differed for the two models, it becomes important to identify the stable FP the Y junction flows into, to correctly characterize the system. Rahmani et al. have investigated an FY model junction with periodic boundary conditions (pbc) at half-filling where they developed a boundary conformal field theory based approach to find the conductivity in these systems\textsuperscript{44}. In absence of any external field, the Y junction preserves the time reversal symmetry and can be described by the elusive M-FP reported in literature. At the M-FP of this Y junction - for attractive interactions ($1 < g < 3$), the following relation is expected to be followed away from the boundary, for $\ell \to \infty$ and $x \to \infty$\textsuperscript{44}:

$$G_{\beta,\gamma} = \lim_{x \to \infty} \langle J_{R}^{\beta}(x)J_{L}^{\gamma}(x) \rangle_{GS} \left[ 4\ell \sin \left( \frac{\pi x}{\ell} \right) \right]^{2} \frac{e^{2}}{h} \quad (9)$$

where, $J_{R}^{\beta}(x)$ and $J_{L}^{\gamma}(x)$ are the chiral currents defined on any two constituent wires $\beta$ and $\gamma$ (of the Y junction). In the FY model, the current is simply given by: $J(x) = \imath \left( c_{i}^{\dagger}c_{i+1} - c_{i}c_{i+1}^{\dagger} \right)$, where $c_{i}^{\dagger}(c_{i})$ represents the creation (annihilation) operator at the site $i$. Similarly for spin system, $J(x) = \imath \left( S_{i}^{z}S_{i+1}^{z} - S_{i+1}^{z}S_{i}^{z} \right)$, where $S_{i}^{\pm}$ are the spin raising (lowering) operators. In the large $\ell$ and $x$ limit, $G_{\beta,\gamma}$ should have a constant value, and the following relation is expected hold:

$$\langle J^{\beta}(x)J^{\gamma}(x) \rangle_{GS} \propto \left[ \frac{1}{\pi \ell} \sin \left( \frac{\pi x}{\ell} \right) \right]^{-2} \quad (10)$$

We plot the $\langle J^{\beta}(x)J^{\gamma}(x) \rangle_{GS}$ as a function of $\left[ \frac{1}{\pi \ell} \sin \left( \frac{\pi x}{\ell} \right) \right]$ in log-log scale in Fig. 4 to confirm the validity of this relation. Fig. 4a and 4b correspond to the FY and BY model junctions, respectively. And we observe that in both the attractive $V < 0(J_{z}/J < 0)$ and repulsive limits $V/t > 0(J_{z}/J > 0)$, the slope is found to be in the vicinity of $-2$ (represented by solid lines in 4a and 4b), which is consistent previous works. The oscillatory nature of the static current correlations is clearly visible in the repulsive ($V > 0$) limit, again consistent with Ref. [44]. This strongly suggests that our Y junction systems could be in the vicinity of the M-FP.

### C. Nature of TDOS and cut-off scale for enhancement

So far, we have illustrated that the BY and FY junctions are connected to a stable M-FP in the parameter regime $1 < g < 3$ or the attractive limit ($V/t = J^z/J < 0$). Now existing studies in literature regarding the effect of impurities in quantum wires point to a finite spatial cut-off on the enhancement caused by the impurities. In similar spirit, we wish to study the spatial extent of the enhancement observed in the attractive limit of the Y junction. Since the TDOS spectra of the bosonic and fermionic model on Y junctions are similar, here we present the results of only BY model. To estimate the spatial extent of the enhancement in TDOS, we plot the maximum intensity of TDOS $\rho_{x'}(\omega_p)$ at peak frequency $\omega_p$, as a
function of (scaled) distance from the junction $x' = x/\ell$ in Fig. 5 a for the BY junction. The $\rho_x(x')$ is inversely proportional to the $\eta$ in case of resonance condition and proportional to the sum of the squares of all the transition matrix elements $\frac{1}{\eta} \sum_n |\langle \psi_n | S^+_x | \psi_0 \rangle|^2$. Therefore, keeping the $\eta$ same, we can extrapolate the sum of the matrix elements for different $N$.

The spatial dependence of the $\rho_x(x')$ as function of scaled spatial unit $x'$ at $J^z = -1/2$ (enhancement regime) for four system sizes $N = 106, 202, 406$ and 610 are shown in Fig. 5. The finite size dependence of $\rho_x(x')$ for sites near the junction is weak as shown in the inset of Fig. 5, but it is strong for sites away from the junction, as clear from Fig. 5 (main). We also note that the extent of enhancement of $\rho_x(x')$ is limited to neighborhood of junction. A detailed study of the TDOS $\rho_x(x')$ as a function of scaled $x'$ would reveal that the TDOS enhancement survives over a few sites away from the junction, e.g., over $r \lesssim 5$ sites for $N = 406$ in the limit $J^z = -1/2$. Therefore, we can conclude that the enhancement of TDOS is highly localized near the junction of the system in the thermodynamic limit.

IV. SUMMARY

In summary, we have considered the simplest possible Y junction comprising of three equi-length 1D TLL wires which are symmetrically coupled to the central site. We have shown that the spin $-1/2$ XXZ or the bosonic Y junction are distinct from the fermionic Y junction as they show distinct power law dependences in TDOS for $g > 1$. This difference can be attributed to the non-trivial phase factors associated in the hopping between the junction site and the constituent arms, and stems from the quantum exchange statistics of constituent particles. Next, we have tried to identify the FPs associated with the Y junction by studying the static current-current correlations which indicated that our Y junction might be in the neighbourhood of an exotic FP (the M-FP) in the $1 < g < 3$ limit. Finally, we investigated the spatial extent of TDOS enhancement through a finite size scaling study and observed that the TDOS peak amplitude near the junction is weakly dependent on the system size $N$, and the enhancement of the TDOS is highly localized near the junction site.

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