Thermodynamic Properties of Effective Chiral Lagrangians with Brown-Rho Scaling

Chaejun Song\textsuperscript{(a,b)}, Dong-Pil Min\textsuperscript{(b)} and Mannque Rho\textsuperscript{(a,c)}

(a) Theory Group, GSI, Planckstr.1, D-64291 Darmstadt, Germany

(b) Department of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea

(c) Service de Physique Théorique, CEA Saclay, F-91191 Gif-sur-Yvette, France

ABSTRACT

We show that effective chiral Lagrangians endowed with Brown-Rho scaling can be mapped to Landau Fermi-liquid fixed point theory in a way consistent with general constraints following from thermodynamics. This provides a unified scheme to treat, starting from normal nuclear matter, hadronic matter under extreme conditions that is encountered in relativistic heavy-ion collisions and in the interior of compact stars.
1 Introduction

In a recent publication [1], we proposed an effective chiral Lagrangian whose parameters (masses and coupling constants) scale in nuclear medium according to the scaling law described by Brown and Rho [2] (referred to in what follows as “BR scaling” in short) that, in the mean field approximation, is to describe the ground state of nuclear matter while when fluctuated in various flavor directions should enable us to extrapolate to a density regime beyond the normal matter as well as to treat excitations above the ground state. The simplest such Lagrangian takes the form

\[
L = \bar{\psi} \left[ i \gamma^\mu (i \partial_\mu - g_v^* (\rho) \omega^\mu) - M^* (\rho) + h \phi \right] \psi \\
+ \frac{1}{2} \left[ (\partial \phi)^2 - m_s^* (\rho) \phi^2 \right] - \frac{1}{4} F_\omega^2 + \frac{1}{2} m_\omega^* (\rho) \omega^2
\]

where \( \psi \) is the nucleon field, \( \omega_\mu \) the isoscalar vector field, \( \phi \) an isoscalar scalar field and the masses with asterisk are BR-scaling as first introduced in [2]. The scaling behavior of the constant \( g_v \) is left arbitrary and the \( h \) is assumed not to scale although it is easy to take into account the density dependence if necessary. It was shown in [1] that in the mean field, this Lagrangian – with the BR scaling suitably implemented – gives a surprisingly good description of the ground state with a compression modulus well within the accepted value \( 200 \sim 300 \) MeV.

As it stands, the Lagrangian (1) does not look chirally invariant. This is because we have dropped pion fields which play no role in the ground state of nuclear matter. In considering fluctuations around the ground state, they (and other pseudo-Goldstone fields such as kaons) should be reinstated. The chiral singlet \( \omega \) field and \( \phi \) field can be considered as auxiliary fields brought in from a Lagrangian consisting of multi-Fermion field operators [3] via a Hubbard-Stratonovich transformation.

The simple Lagrangian (1) embodies the effective field theory of QCD discussed by

\[1^1\text{As suggested in [3, 4], chiral in-medium Lagrangians can be brought to a form equivalent to a Walecka-type model. The scalar field appearing here transforms as a singlet, not as the fourth component of } O(4) \text{ of the linear sigma model.} \]
Furnstahl et al. [5] anchored on general considerations of chiral symmetry. As argued in [1], this Lagrangian should be viewed as an effective Lagrangian that results from two successive renormalization group “decimations”, one leading to a chiral liquid structure [6] at the chiral symmetry scale and the other with respect to the Fermi surface [7]. The advantage of (1) is that it can, on the one hand, be connected to Landau Fermi liquid fixed point theory of nuclear matter as suggested in [1, 8] and, on the other hand, be extrapolated to the regime of hadronic matter produced under extreme conditions as encountered in relativistic heavy ion processes. It would, for instance, allow one, starting from the ground state of nuclear matter, to treat on the same footing the dilepton processes observed in CERES experiments as explained in [9] and kaon production at SIS energy and kaon condensation in dense matter relevant to the formation of compact stars as discussed in [10].

In this note, we address an issue which was left unaddressed in [1], namely thermodynamic consistency of the Lagrangian (1) treated in the mean field approximation. For instance, it is not obvious that the presence of the density-dependent parameters in the Lagrangian does not spoil the self-consistency of the model, in particular, energy-momentum conservation in the medium and also certain relations of Fermi-liquid structure of the matter[7]. The purpose of this paper is to show that there is no inconsistency in doing a field theory with BR scaling masses and other parameters.

2 Implementing BR Scaling in the Lagrangian

In [1], we have treated the density-dependent masses and constants as independent of the fields that enter in the Lagrangian. The Euler-Lagrange equations of motion are then the same as for the Lagrangian wherein the masses and constants are not BR-scaling. While this procedure gives correct energy density, pressure and compression modulus, the energy-momentum conservation is not automatically assured. In fact, if one were to compute the pressure from the energy-density $E$, one would find that it does not give $\frac{1}{3} < T_{ii} >$ (where

\[ T_{ii} \]

\[ 2 \]

We thank Bengt Friman for raising this question.
$T_{\mu\nu}$ is the conserved energy-momentum tensor and the bra-ket means the quantity evaluated in the mean-field approximation as defined before) unless one drops certain terms without justification. This suggests that it is incorrect to take the masses and coupling constants independent of fields in deriving, by Noether theorem, the energy-momentum tensor. So the question is: how do we treat the field dependence of the BR scaling masses and constants?

One possible solution to this problem is as follows. In [2], the density dependence of the Lagrangian arose as the “vacuum” expectation value of the scalar field $\chi$ that figures in the QCD trace anomaly. It corresponded to the condensate of a quarkonium component of the scalar $\chi$ with the gluonium component – which lies higher than the chiral scale – integrated out. It was assumed to scale in dense medium in a skyrmion-type Lagrangian subject to chiral symmetry. Now in the language of a chiral Lagrangian consisting of the nucleonic matter field $\psi$ with other massive fields integrated out, this scalar condensate would be some function of the “vacuum” expectation value of $\bar{\psi}\psi$ or $\bar{\psi}\gamma_0\psi$ coming from multi-Fermion field operators mentioned above. How these four-Fermi and higher-Fermi field terms can lead to BR scaling in the framework of chiral perturbation theory was discussed in [4]. We shall follow this strategy in this paper leaving other possibilities (such as dependence on the mean fields of the massive mesons) for later investigation. For this, it is convenient to define

$$\tilde{\rho} u^\mu \equiv \bar{\psi} \gamma^\mu \psi$$

(2)

with unit fluid 4-velocity $u^\mu = \frac{1}{\sqrt{1 - v^2}} (1, \vec{v}) = \frac{1}{\sqrt{\rho^2 - |\vec{j}|^2}} (\rho, \vec{j})$ with the baryon current density $\vec{j} = <\bar{\psi}\gamma_5\psi>$ and the baryon number density $\rho = <\psi^\dagger\psi> = \sum_i \rho_i$. We will take $\rho_i$ to be given by the Fermi distribution function, $\rho_i = \theta(k_F - |\vec{k}_i|)$ at $T = 0$. We should replace $\rho$ in (1) by $\tilde{\rho}$ for consistency of the model. The definition of $\tilde{\rho}$ makes our Lagrangian Lorentz-invariant which will later turn out to be useful in deriving relativistic Landau formulas.

---

3By “vacuum” we mean the state of baryon number zero modified from that of true vacuum by the strong influence of the baryons in the system. See later for more on this point.
With this, the Euler-Lagrange equation of motion (EOM) for the nucleon field is

$$\frac{\delta L}{\delta \psi} = \frac{\partial L}{\partial \psi} + \frac{\partial L}{\partial \bar{\rho}} \frac{\partial \bar{\rho}}{\partial \psi} = \left[ i\gamma^\mu (\partial_\mu + ig_\nu^* \omega_\mu - iu_\mu \Sigma) - M^* + h\phi \right] \psi = 0 \quad (3)$$

with

$$\Sigma = \frac{\partial L}{\partial \bar{\rho}} = m^* \bar{\psi} \bar{\omega} \omega^2 \frac{\partial m^*}{\partial \bar{\rho}} - m^* \phi^2 \frac{\partial m^*}{\partial \bar{\rho}} - \bar{\psi} \omega^\mu \gamma_\mu \psi \frac{\partial g^*_\nu}{\partial \bar{\rho}} - \bar{\psi} \psi \frac{\partial M^*}{\partial \bar{\rho}}. \quad (4)$$

This additional term which may be related to what is referred to in many-body theory as “rearrangement terms” plays a crucial role in what follows. The EOM’s for the bosonic fields are

$$(\partial^\mu \partial_\mu + m^*_s^2) \phi = h\bar{\psi} \psi \quad (5)$$

$$\partial_\nu F^\nu_\omega + m^*_\omega \omega^\mu = g^*_\nu \bar{\psi} \gamma^\mu \psi. \quad (6)$$

3 Equation of State (EOS)

We start with the conserved canonical energy-momentum tensor constructed a la Noether from the Lagrangian (1):

$$T^{\mu\nu} = i\bar{\psi} \gamma^\mu \partial^\nu \psi + \partial^\mu \phi \partial^\nu \phi - \partial^\mu \omega^\lambda \partial^\nu \omega^\lambda - \frac{1}{2}((\partial \phi)^2 - m^*_s^2 \phi^2 - (\partial \omega)^2 + m^*_\omega \omega^2 - 2\bar{\Sigma} \bar{\psi} \gamma^\cdot \bar{u} \psi) g^{\mu\nu}. \quad (7)$$

We shall compute thermodynamic quantities from (7) using the mean field approximation which amounts to taking

$$\psi = \frac{1}{\sqrt{V}} \sum_i a_i \sqrt{E_{\xi_i} + m^*_i} \left( \frac{\chi}{\frac{\sigma_{\xi_i}}{E_{\xi_i} + m^*_i}} \right) \exp \left( i\bar{\kappa}_i \cdot \vec{x} - i(g^*_\nu \omega_\nu - u_0 \Sigma + E_i) t \right) \quad (8)$$

$$h\phi = C^2_\hbar < \bar{\psi} \psi > = C^2_\hbar \sum_i \rho_i \frac{m^*_i}{\sqrt{\bar{\kappa}_i^2 + m^2_L}} \quad (9)$$

$$g^*_\nu \omega_0 = C^2_\nu \rho = C^2_\nu \sum_i \rho_i \quad (10)$$

$$g^*_\nu \omega^\cdot = C^2_\nu \sum_i \rho_i \frac{\bar{\kappa}_i}{\sqrt{\bar{\kappa}_i^2 + m^2_L}} \quad (11)$$
where $a_i$ is the annihilation operator of the nucleon $i$, with $\rho_i = \langle a_i^\dagger a_i \rangle$, and $\Sigma = \langle \Sigma \rangle$, $E_{\kappa_i} = \sqrt{\kappa_i^2 + m_L^2}$ with $m_L^* \equiv M^* - h\phi$, $\chi$ is the spinor and $\vec{\sigma}$ is the Pauli matrix. We have defined

\[
C_v(\tilde{\rho}) \equiv \frac{g^*(\tilde{\rho})}{m_v^*(\tilde{\rho})} \quad (12)
\]
\[
C_h(\tilde{\rho}) \equiv \frac{h}{m_v^*(\tilde{\rho})} \quad (13)
\]
\[
\tilde{C}_h(\tilde{\rho}) \equiv \frac{1}{C_h(\tilde{\rho})} \quad (14)
\]
\[
\tilde{\kappa} \equiv \tilde{k} - C_v^2 \tilde{j} + \tilde{u}\Sigma. \quad (15)
\]

In this approximation, the energy density is

\[
\mathcal{E} = < T^{00} > = < i\tilde{\psi}\gamma^0 \partial^0 \psi + \frac{1}{2} m_v^2 \phi^2 - \frac{1}{2} m_w^2 \omega^2 + \Sigma \tilde{\psi}\gamma^\mu u_\mu \psi >
\]
\[
= \frac{1}{2} C_v^2 (\rho^2 + \tilde{j}^2) + \frac{1}{2} \tilde{C}_h^2 (m_L^* - M^*)^2 + \sum_l \rho_l \sqrt{\kappa_l^2 + m_L^2} - \Sigma \tilde{u} \cdot \tilde{j}. \quad (16)
\]

Note that the $\Sigma$-dependent terms cancel out in the comoving frame ($\vec{v} = 0$), so that the resulting energy-density is identical to what one would obtain from the Lagrangian in the mean field with the density-dependent parameters taken as field-independent quantities as in $[1]$. Given the energy density (16), the pressure can be calculated by (at $T = 0$)

\[
p = -\frac{\partial E}{\partial V} = \rho^2 \frac{\partial \mathcal{E}/\rho}{\partial \rho} = \mu\rho - \mathcal{E}
\]
\[
= \frac{1}{2} C_v^2 (\rho^2) - \Sigma \rho - \frac{1}{2} \tilde{C}_h^2 (m_L^* - M^*(\rho))^2 - \frac{\gamma}{2\pi^2} \left( E_F \left( \frac{m_L^2}{8} k_F - \frac{1}{12} k_F^3 \right) - \frac{m_L^4}{8} \ln[(k_F + E_F)/m_L] \right) \quad (17)
\]

where $\mu$ is the chemical potential – the first derivative of the energy density with respect to $\rho$ in the comoving frame ($\vec{v} = 0$):

\[
\mu \equiv \frac{\partial}{\partial \rho} \mathcal{E}|_{\vec{v}=0} = C_v^2 \rho + E_F - \Sigma_0 \quad (18)
\]
with $E_F = \sqrt{k_F^2 + m_L^2}$ and $\Sigma_0 = \langle \Sigma \rangle_{\bar{v}=0}$. To check that this is consistent, we calculate the pressure from the energy-momentum tensor \textbf{(7)} in the mean field at $T = 0$:

$$p_i \equiv \frac{1}{3} <T_{ii}>_{\bar{v}=0}$$

$$= \frac{1}{3} \left(i \bar{\psi} \gamma^i \partial^i \psi - \frac{1}{2} (m^*_u \omega^2 - m^*_s \phi^2 - 2 \bar{\Sigma} \psi^\dagger \psi) g^{ii} \right)_{\bar{v}=0}$$

$$= \frac{1}{2} C_v^2 (\rho) \rho^2 - \frac{1}{2} \tilde{C}_h (\rho) (m^*_L - M^*(\rho))^2 - \Sigma_0 \rho$$

$$- \frac{\gamma}{2\pi^2} \left( E_F (\frac{m^*_L}{8} k_F - \frac{1}{12} k_F^3) - \frac{m^*_L^4}{8} \ln\left((k_F + E_F)/m_L^*\right) \right).$$

This agrees with (\textbf{17}). Thus our EOS conserves energy and momentum.

4 Landau Fermi-Liquid Parameters

The next issue we address is the connection between the mean-field theory of the chiral Lagrangian \textbf{(1)} and Landau’s Fermi-liquid fixed point theory as formulated in \textbf{[1, 8]}. As far as we know, this connection is the only means available to implement chiral symmetry of QCD in dense matter based on effective field theory. For this, we shall follow closely Matsui’s analysis of Walecka mean field model \textbf{[11]} exploiting the similarity of our model to the latter.

4.1 Quasiparticle interactions

The quasiparticle energy $\varepsilon_i$ and quasiparticle interaction $f_{ij}$ are, respectively, given by first and second derivatives with respect to $\rho_i$:

$$\varepsilon_i = \frac{\partial \varepsilon}{\partial \rho_i}, \quad f_{ij} = \frac{\partial f}{\partial \rho_j}. \quad (20)$$

A straightforward calculation gives

$$\varepsilon_i = C_v^2 \rho + \sqrt{k_i^2 + m_L^2} + C_v^2 \frac{\partial C_v}{\partial \rho_i} - C_v^2 \tilde{C}_h \frac{\partial C_v}{\partial \rho_i}$$

$$+ \tilde{C}_h (m^*_L - M^*)^2 \frac{\partial \tilde{C}_h}{\partial \rho_i} - \tilde{C}_h (m^*_L - M^*) \frac{\partial M^*}{\partial \rho_i} - \Sigma \cdot \frac{\partial \vec{j}}{\partial \rho_i} \quad (21)$$
and

\[ f_{ij} = \frac{\partial \epsilon_i}{\partial \rho_j} \bigg|_{\vec{j} = \vec{v} = 0} \]

\[ = C_v^2 + 4C_v \rho \frac{\partial C_v}{\partial \rho} + \frac{m^*_i \partial m^*_j}{E_i} + \rho^2 \left( \frac{\partial C_v}{\partial \rho} \right)^2 + C_v \frac{\partial^2 C_v}{\partial \rho^2} + \rho \left( \frac{\partial C_h}{\partial \rho} \right)^2 \]

\[ + (m^*_L - M^*)^2 \left( \frac{\partial C_h}{\partial \rho} \right)^2 + \frac{\partial C_h}{\partial \rho} (m^*_L - M^*) \frac{\partial (m^*_L - M^*)}{\partial \rho_j} \]

\[ - 2 \frac{\partial C_h}{\partial \rho} (m^*_L - M^*) \frac{\partial (m^*_L - M^*)}{\partial \rho_j} - \frac{\partial C_h}{\partial \rho} (m^*_L - M^*) \frac{\partial^2 (m^*_L - M^*)}{\partial \rho^2} \]

\[ - (C_v^2 - \frac{\Sigma_0}{\rho}) \frac{\vec{k}_i}{E_i} \cdot \frac{\vec{j}}{\partial \rho} \cdot \frac{\vec{j}}{\partial \rho}. \]

(22)

with \( E_i = \sqrt{k_i^2 + m_i^2} \). Note that \( C_v, C_h, \) and \( M^* \) are functions of \( \langle \tilde{\rho} \rangle = u_0 \rho - \vec{u} \cdot \vec{j} \) in the mean field approximation. In arriving at (22), we have used the observation that in the limit \( \vec{j} \to 0 \), we have

\[ \frac{\partial u_0}{\partial \rho_i} \to 0, \]

\[ \frac{\partial^2 u_0}{\partial \rho_i \partial \rho_j} \to \frac{1}{\rho^2} \frac{\partial^2 \tilde{j}}{\partial \rho_i \partial \rho_j}, \]

\[ \frac{\partial \tilde{u}}{\partial \rho_i} \to \frac{1}{\rho} \frac{\partial \tilde{j}}{\partial \rho_i}, \]

\[ \frac{\partial \langle \tilde{\rho} \rangle}{\partial \rho_i} \to 1, \]

\[ \frac{\partial^2 \langle \tilde{\rho} \rangle}{\partial \rho_i \partial \rho_j} \to - \frac{1}{\rho} \frac{\partial \tilde{j}}{\partial \rho_i} \cdot \frac{\partial \tilde{j}}{\partial \rho_j}. \]

and that if \( f \) is taken to be a function of the expectation value of \( \tilde{\rho} \), then as \( \vec{j} \to 0 \), we have

\[ \frac{\partial f}{\partial \rho_i} = \frac{\partial f}{\partial \tilde{\rho}} \frac{\partial \langle \tilde{\rho} \rangle}{\partial \rho_i} \to \frac{\partial f}{\partial \tilde{\rho}} \]

\[ + \frac{\partial f}{\partial \langle \tilde{\rho} \rangle} \frac{\partial \langle \tilde{\rho} \rangle}{\partial \rho_j} \to \frac{\partial^2 f}{\partial \rho^2} \]

\[ - \frac{1}{\rho} \frac{\partial \tilde{j}}{\partial \rho_i} \cdot \frac{\partial \tilde{j}}{\partial \rho_j}. \]

(23)

(24)

In the absence of the baryon current, \( \vec{j} = 0 \), the quantities \( \frac{\partial m^*_i}{\partial \rho_j} \) and \( \frac{\partial \tilde{j}}{\partial \rho_j} \) simplify to

\[ \frac{\partial m^*_i}{\partial \rho_j} = \frac{\partial m^*_i}{\partial \rho} - 2C_h \frac{\partial C_h}{\partial \rho} \sum_l \rho_l \frac{m^*_l}{E_l} - C_h^2 \frac{m^*_i}{E_i}, \]

\[ \frac{\partial \tilde{j}}{\partial \rho_j} \]

\[ \to \frac{\partial^2 m^*_i}{\partial \rho^2} \]

\[ + 2C_h \frac{\partial C_h}{\partial \rho} \sum_l \rho_l \frac{m^*_l}{E_l} \]

\[ + \frac{1}{1 + C_h^2 \sum_l \rho_l \frac{m^*_l}{E_l^2}}. \]

(25)
and

\[ \frac{\partial \tilde{j}_j}{\partial \rho_j} = \frac{\tilde{k}_j}{1 + (C_v^2 - \frac{\Sigma}{\rho}) \sum_i \rho_i \tilde{k}_i^2 \tilde{m}^* \tilde{m}^*} \]  

Writing in the standard way

\[ f_1 = (2l + 1) \int \frac{d\Omega}{4\pi} P_l(\frac{\tilde{k}_i \cdot \tilde{k}_j}{k_F^2}) f_{ij}(|\tilde{k}_i| = |\tilde{k}_j| = k_F) \]

we see that the last term in \([22]\) contributes to \(f_1\) and the sum of the rest at the Fermi surface (i.e. \(|\tilde{k}_j| = k_F\)) to \(f_0\). So

\[ F_0 \equiv \frac{\gamma k_F E_F}{2\pi^2} f_0 = \frac{3E_F}{k_F} \rho f_0 \]

\[ = \frac{3E_F}{k_F} \rho \left[ C_v^2 + 4C_v \rho \frac{\partial C_v}{\partial \rho} + \frac{m_L^* \partial m^*_L}{E_F \partial \rho} \right] + \rho^2 \left[ \frac{\partial C_v}{\partial \rho} \right]^2 + C_v \frac{\partial^2 C_v}{\partial \rho^2} \]  

\[ + (m_L^* - M^*)^2 \left\{ \left( \frac{\partial \tilde{C}_h}{\partial \rho} \right)^2 + \tilde{C}_h \frac{\partial^2 \tilde{C}_h}{\partial \rho^2} \right\} + 2\tilde{C}_h \frac{\partial \tilde{C}_h}{\partial \rho} (m_L^* - M^*) \frac{\partial}{\partial \rho} (m_L^* - M^*) \]  

\[ - 2\tilde{C}_h \frac{\partial \tilde{C}_h}{\partial \rho} (m_L^* - M^*) \frac{\partial M^*}{\partial \rho} - \tilde{C}_h \frac{\partial M^*}{\partial \rho} \frac{\partial}{\partial \rho} (m_L^* - M^*) \]  

and

\[ F_1 \equiv \frac{\gamma k_F E_F}{2\pi^2} f_1 = -\frac{3(C_v^2 - \frac{\Sigma}{\rho})}{E_F + (C_v^2 - \frac{\Sigma}{\rho})} \rho \]

4.2 Compression modulus and \(F_0\)

The compression modulus \(K\) defined by

\[ K \equiv 9\rho \frac{\partial^2 \mathcal{E}(\tilde{j} = 0)}{\partial \rho^2} \]  

comes out to be

\[ K = \frac{3k_F^2}{E_F} + 9\rho \left[ C_v^2 + 4C_v \rho \frac{\partial C_v}{\partial \rho} + \frac{m_L^* \partial m^*_L}{E_F \partial \rho} \right] + \rho^2 \left[ \frac{\partial C_v}{\partial \rho} \right]^2 + C_v \frac{\partial^2 C_v}{\partial \rho^2} \]  

\[ + (m_L^* - M^*)^2 \left\{ \left( \frac{\partial \tilde{C}_h}{\partial \rho} \right)^2 + \tilde{C}_h \frac{\partial^2 \tilde{C}_h}{\partial \rho^2} \right\} + 2\tilde{C}_h \frac{\partial \tilde{C}_h}{\partial \rho} (m_L^* - M^*) \frac{\partial}{\partial \rho} (m_L^* - M^*) \]  

\[ - 2\tilde{C}_h \frac{\partial \tilde{C}_h}{\partial \rho} (m_L^* - M^*) \frac{\partial M^*}{\partial \rho} - \tilde{C}_h \frac{\partial M^*}{\partial \rho} \frac{\partial}{\partial \rho} (m_L^* - M^*) - \tilde{C}_h^2 (m_L^* - M^*) \frac{\partial^2 M^*}{\partial \rho^2} \]  

8
Comparing (28) and (31), we verify that our model satisfies the relativistic Landau Fermi-liquid formula for the compression modulus [12];

\[ K = \frac{3k_F^2}{E_F}(1 + F_0). \]  

(32)

4.3 First sound velocity

The first sound velocity \( c_1 \) in the relativistic case is defined by

\[
    c_1^2 = \frac{\partial p}{\partial \mathcal{E}} \equiv \frac{\partial}{\partial \mathcal{E}} \left( \rho \frac{\partial \mathcal{E}}{\partial \rho} \right) = \frac{\partial \rho}{\partial \mathcal{E}} \frac{\partial}{\partial \rho} (\mu \rho - \mathcal{E}) = \frac{K}{9\mu}.
\]

(33)

From (32), we have that [12]

\[ c_1 = v_F \sqrt{\frac{E_F}{3\mu}} (1 + F_0). \]

(34)

This is of course satisfied in our model.

4.4 Relativistic Landau effective mass

Baym and Chin have shown [12] that the relativistic Landau liquid satisfies the mass relation

\[ k_F \left( \frac{\partial k_i}{\partial \varepsilon_1} \right)_{k = k_F, \vec{v} = 0} = \mu(1 + F_1/3). \]

(35)

In our model \( k_F \left( \frac{\partial k_i}{\partial \varepsilon_1} \right)_{k = k_F, \vec{v} = 0} = E_F \). One can see from equations (18) and (29) that (35) is satisfied exactly in our model.

5 Discussions

We showed that a simple effective chiral Lagrangian with BR scaling parameters is thermodynamically consistent, a point which is important for studying nuclear matter under extreme conditions. It is clear however that this does not require that the masses appearing
in the Lagrangian scale according to BR scaling only. What is shown in this paper is that masses and coupling constants could depend on density without getting into inconsistency with general constraints of chiral Lagrangian field theory. This point is important for applying (1) to the density regime $\rho \sim 3\rho_0$ appropriate for the CERES dilepton experiments and also kaon production at GSI where deviation from the simple BR scaling of (2) might occur.

The crucial question is really how to understand the scaling masses and constants as one varies temperature and density as considered in (2). If one takes the basic assumption of (1), (8) that the chiral Lagrangian in the mean field with BR scaling parameters corresponds to Landau’s Fermi-liquid fixed point theory, then one should consider first fixing the Fermi momentum $k_F$ and let renormalization group flow come to the fixed points of the effective mass $M^*$ for the nucleon and Landau parameters $F$ (7). In this case, the scaling quantities would seem to be dependent upon $\Lambda/k_F$, not on the fields entering into the effective Lagrangian. This paper however shows that if one wants to approach the Fermi-liquid fixed point theory starting from an effective chiral Lagrangian of QCD, it is necessary to take into account the fact that the scaling arises from the effect of multi-Fermi interactions figuring in chiral Lagrangians as implied by chiral perturbation theory described in (4). This is probably due to the fact that we are dealing with two-stage “decimations” in the present problem – with the Fermi surface formed from a chiral Lagrangian as a nontopological soliton (i.e., “chiral liquid” (6)) – in contrast to condensed matter situations where one starts ab initio with the Fermi surface without worrying about how the Fermi surface is formed. Our result suggests that there will be a duality in describing processes manifesting the scaling behavior. In other words, the change of “vacuum” by density exploited in (3) could equally be represented by a certain (possibly infinite) set of interactions among hadrons – e.g., four-Fermi and higher-Fermi terms in chiral Lagrangians – canonically taken into account in many-body theories starting from the usual matter-free vacuum. A notable evidence may be found in the two plausible explanations of the low-mass enhancement in
CERES dilepton yields in terms of scaling vector-meson masses \cite{3} and in terms of hadronic interactions giving rise to increased widths \cite{3}.

How to go from one decimation to the next in hadronic physics remains an open problem as stressed in \cite{1}.

Acknowledgments

We have benefited from valuable discussions with Gerry Brown and Bengt Friman. Part of this work was done while two of us (M.R. and C.S.) were visiting the Theory Group of GSI whose hospitality is gratefully acknowledged. The work of M.R. at GSI was supported by a Franco-German Humboldt Research Prize and the work of C.S. and D.P.M. in part by KOSEF through the Center for Theoretical Physics of Seoul National University and in part by Korea Ministry of Education (Grant No. BSRI97-2418).
References

[1] C. Song, G. E. Brown, D.-P. Min, and M. Rho, Phys. Rev. C 56 (1997) 2244

[2] G. E. Brown and M. Rho, Phys. Rev. Lett. 66 (1991) 2720

[3] G.E. Brown and M. Rho, Nucl. Phys. A596 (1996) 503

[4] T.-S. Park, D.-P. Min and M. Rho, Nucl. Phys. A596 (1996) 515; G. Gelmini and R. Ritzi, Phys. Lett. B357 (1995) 431

[5] R.J. Furnstahl, H.-B. Tang and B.D. Serot, Phys. Rev. C 52 (1995) 1368

[6] B.W. Lynn, Nucl. Phys. B402 (1993) 281

[7] J. Polchinski, Recent Directions in Particle Theory: From Superstrings and Black Holes to the Standard Model (TASI-92), ed. J. Harvey and J. Polchinski (World Scientific, Singapore, 1994) 235; R. Shankar, Rev. Mod. Phys. 66 (1994) 129; T. Chen, J. Frölich and M. Seifert, cond-mat/9508063

[8] B. Friman and M. Rho, Nucl. Phys. A606 (1996) 303

[9] G.Q. Li, C.M. Ko, and G.E. Brown, Phys. Rev. Lett. 75(1995) 4007; Nucl. Phys. A606(1996) 568

[10] G.Q. Li, C.-H. Lee and G.E. Brown, Phys. Rev. Lett.(in press) and nucl-th/9711002

[11] T. Matsui, Nucl. Phys. A370 (1981) 365

[12] G. Baym and S. Chin, Nucl. Phys. A262(1976) 527

[13] W. Cassing, E.L. Bratkovskaya, R. Rapp and J. Wambach, nucl-th/9708020