Notes On The Born-Oppenheimer Approach In A Closed Dynamical System

Dhurjati Prasad Datta
Department of Mathematics
North Eastern Regional Institute of Science and Technology
Itanagar-791109, Arunachal Pradesh, India*
and
International Centre for Theoretical Physics
Post Bag 586, 34100 Trieste, Italy

email:dpd@nerist.ernet.in

Abstract

The various recent studies on the application of the Born-Oppenheimer approach in a closed gravity matter system is examined. It is pointed out that the Born-Oppenheimer approach in the absence of an a priori time is likely to yield potentially new results.

*Permanent address
Recently, there has been a renewed interest in the Born-Oppenheimer (BO) approach in analyzing the quantum evolution of composite systems involving two separate mass (energy) scales [1-8]. It is now well known that the conventional BO approximation method in atomic/molecular physics, for instance, not only offers a natural framework in realizing the Berry adiabatic phase in the quantum state of the lighter system, but the motion of the heavy system also gets influenced by the induced geometric gauge potentials, besides the usual BO potential determined by the energy expectation value of the lighter state[1]. The effects of the geometric electric and magnetic fields on the heavy system have been shown to yield high degree of agreements with the exact results[2]. Moreover, the predictions of the BO analysis in localized quantum mechanical systems are unambiguous and well tested in laboratory (atomic/ molecular physics) experiments[1].

Applications of the BO analysis in closed gravity-matter systems in quantum cosmology, on the other hand, seem still to be rather inconclusive. Although the plausibility of an application of a BO type analysis in a quantized gravity matter system is well accepted (because of the large value of the Plank mass compared to the mass scales of ordinary matter)[3], there seems to be a considerable amount of disagreement in interpreting the results obtained in this method[3-8]. The main reason of these disparities of course relates to the issue of time in quantum gravity. A gravity matter system, for instance, in a quantum cosmological model is a closed dynamical system in the sense that there is no interaction external to those induced by the mutual (and self) interactions between the given gravitational and matter degrees of freedom. The corresponding quantum dynamical equation- the so-called Wheeler -Dewitt (WD) equation- which is obtained by applying the Dirac quantization rule to the classical Hamiltonian constraint in a generally covariant theory of gravity, thus does not involve any external concept of time. This leads to deep conceptual as well as interpretational problems in canonical quantum gravity. A basic motivation in the semiclassical approach [3] in quantum gravity is to understand how a concept of time could emerge intrinsically from an apparently timeless quantum 'evolution' equation in a quasi-classical regime of the (massive) gravitational degrees of freedom. Although a semiclassical approach does not aim at solving the issue of time in quantum gravity, the study is likely to offer new insights into the nature of time and related issues in canonical quantum gravity.

Recent studies in the BO type analysis in quantum cosmology seem to appreciate the importance of incorporating the geometric phases and the associated gauge potentials in the discussions of the effective dynamics of the gravitational degrees of freedom [3-8].
However, as remarked already, the descriptions of the effective dynamics seem to vary because of an arbitrariness in obtaining the back reaction of matter on gravity, which in fact is related to the definition of the intrinsic time. In this paper, we analyze the origin of this ambiguity in realizing an intrinsic time in the BO framework and point out that the BO analysis of a composite system in the absence of an external time would lead to nontrivial predictions[8] in comparison to the conventional treatments.

Let us consider an interacting system described by the Hamiltonian

$H(q, \phi) = \frac{1}{2M} G^{ij} P_i P_j + MV(q) + H_m(\phi, q)$ (1)

Here, $G^{ij}$ denote the metric tensor in the space of heavy configuration variables $q_i (i = 1, 2, 3, \ldots N)$, $P_i$ being the corresponding conjugate momentum. $H_m$ is the Hamiltonian of the lighter system $\phi$, which depends parametrically on the heavy variables $q$. The mass $m$ of the lighter system is much less than that of the heavy system, $m << M$. The composite Hamiltonian (1) can be considered as a model of the minisuperspace quantum cosmological gravity-matter system provided $G^{ij}$ is interpreted as the superspace (Dewitt) metric of compact three geometries $q$ and $\phi$ is the matter fields. The mass $M$ then stands for the Planck mass squared. For a molecular system, on the other hand, $G^{ij} = \delta^{ij}$ and $M$ denotes the molecular mass. The signature of the metric, however, is not very crucial for our present formal discussion.

To identify the actual source of ambiguity in the BO method in quantum cosmology and for the sake of generality, we begin our analysis in the framework of the ordinary quantum mechanics with an a priori time. The quantum evolution of the composite system (1) is then governed by the Schrödinger equation

$H(q, \phi) \Psi(q, \phi) = E \Psi(q, \phi) = i \frac{\partial}{\partial t} \Psi(q, \phi)$ (2)

$\Psi$ is thus an energy eigenstate of the total Hamiltonian: $\Psi = e^{-iEt} \Psi_0$. Assuming that $H(q, \phi)$ is not explicitly $t$ dependent and constraining $\Psi$ to the zero energy state (which in fact amounts to a redefinition of the potential $V \rightarrow V - E/M$), the eigenstate equation (2) mimics the WD equation

$H \Psi_0 = 0$ (3)

One can thus proceed to study the effective dynamics of the heavy $q$-system under the influence of the lighter system $\phi$, both with or without an external time, in the present
model. Although the mathematical framework is more or less similar, the two situations are clearly distinct physically; thus necessitating an extra care in interpreting and comparing the relevant results. To keep our discussions sufficiently general we thus choose to work with the more general equation, viz. eq.(2)

\[
\left( \frac{1}{2M} G^{ij} P_i P_j + MV(q) + H_m \right) \Psi_0 = E \Psi_0
\]  

(4)

where \( P_i = -i\hbar \partial/\partial q^i \), in the following. The total energy \( E \) here is a constant (which may even be zero), reflecting the constraint nature of the operator eq.(4).

We make a BO decomposition of \( \Psi \) as

\[
\Psi_0(q, \phi) = \psi(q) \chi(q, \phi)
\]

(5)

where the quantum state of the lighter system \( \chi(q, \phi) \) is not further decomposable. Projecting the total equation (4) on the normalized lighter state \( \langle \chi | \chi \rangle = 1 \) one gets,

\[
\left[ -\frac{\hbar^2}{2M} G^{ij} D_i D_j + MV + \langle H_m \rangle + \frac{1}{2M} \rho \right] \psi(q) = 0
\]

(6)

where \( \langle H_m \rangle = \langle \chi | H_m | \chi \rangle \) and the covariant derivatives \( D_i = \partial/\partial q^i + iA_i \) and \( \bar{D}_i = \partial/\partial q^i - iA_i \) are introduced because of the induced (adiabatic) \( U(1) \) magnetic connection

\[
A_i = -i \langle \chi | \partial/\partial q^i | \chi \rangle
\]

(7)

and \( \rho \) is the electric potential

\[
\rho = -\hbar^2 \langle \chi | G^{ij} \bar{D}_i \bar{D}_j | \chi \rangle = -\hbar^2 \langle \bar{D}^2 \rangle
\]

(8)

Further, on multiplying eq.(6) by \( \chi \) and subtracting it from eq.(4) one obtains following ref.[4-6]

\[
(H_m - \langle H_m \rangle) \chi - \frac{\hbar^2}{2M} \psi^{-1} G^{ij} (D_i \psi) \bar{D}_j \chi = \frac{\hbar^2}{2M} \left( \bar{D}^{-2} - \langle \bar{D}^2 \rangle \right) \chi
\]

(9)

Note that eq.(6) and (9) constitute an exact set of coupled (non linear) equations and should be solved self-consistently. As noted in ref.[5-6] (see also ref.[8]) the electric potential \( \rho \) and the rhs of eq.(9) are related to fluctuations, which are neglected in the conventional BO adiabatic approximation, in the presence of time \( t \). Stated more precisely, the electric potential \( \rho \) corresponds to the first order nonadiabatic correction in
the back reaction on the $q$-modes from level transitions in the time dependent $\phi$ states, which vanishes nevertheless in the pure adiabatic limit. In the intrinsic time formalism (see below), on the other hand, $\rho$ turns out to be the dominant back reaction and cannot be dropped [8]. However, the nonadiabatic corrections in the rhs of eq. (9) is clearly of higher order (in smallness) in a regime when the pure adiabaticity is weakly violated (c.f., ref [5] for an explicit calculation) and can be neglected safely in most of the present discussion (these terms are important however for the energy conservation in the composite system).

In the semiclassical WKB regime of the $q$-modes one substitutes for the effective wave function $\psi(q)$ the ansatz:

$$\psi = \exp \left( i \int A_i dq^i \right) \psi_{eff} = \sigma \exp \left( i \int A_i dq^i + \frac{1}{\hbar} S \right)$$

in eq. (6), so that the effective quasi-classical motion of the $q$-modes is given by the Hamilton-Jacobi equation

$$\frac{1}{2M} G^{ij} \frac{\partial S}{\partial q^i} \frac{\partial S}{\partial q^j} + MV + \left< H_m \right> + \frac{1}{2M} \rho = E$$

Here, $\sigma$ (in eq (10)) is the WKB prefactor (the Van Vleck determinant) and the intrinsic (WKB) time $\tau$ is introduced via the vector field [3]

$$\frac{d}{Nd\tau} = G^{ij} \frac{\partial S}{\partial q^i} \frac{\partial S}{\partial q^j} \rightarrow \frac{dq^i}{Nd\tau} = G^{ij} P_j, \quad P_i = \frac{\partial S}{\partial q^i}$$

The intrinsic WKB time $\tau$ parametrizes, up to a possible reparametrization, the classical trajectories of the $q$-modes as integral curves normal to the level surfaces of the Hamilton-Jacobi function $S$. This is made explicit by introducing the lapse $N$ in eq. (12). The time $\tau$ thus corresponds, as it should, to the Leibniz (Mach-Einstein) time[9], in contrast to the absolute Newtonian time $t$ in eq. (2).

The quantum evolution of the lighter state is thus described by (neglecting higher order fluctuations in eq. (9))

$$(H_m - \left< H_m \right>) \chi = \frac{i\hbar}{N} \left( \frac{d}{d\tau} - \left< \chi \right| \frac{d}{d\tau} \chi \right) \chi$$

Note that eq. (13) (in fact eq. (9)) when contracted with $\chi^*$ is satisfied identically. Each term of eq. (13) (eq. (9)) thus corresponds to a pure fluctuation with zero mean. So far our discussion was perfectly general and is valid for either of the two formalisms, with or without an external time. The point of bifurcation occurs as one proceeds to interpret eq. (13). Recall that our main goal in this paper is to emphasize a subtle difference between
the possible predictions of the two formalisms, which as it will turn out, can be amply illustrated even limiting our discussion in the regime when the pure adiabaticity is only weakly violated. Dropping of the RHS of eq.(9) is thus justified. In general, the neglected term represents a higher order quantum gravitational correction to the quantized matter evolution. The effects of this correction term on the matter evolution will be considered separately. We however emphasize that although the term involving $\rho$ in eq.(11) encodes the effects of the higher order back reaction from the level transitions in the matter state, it nevertheless becomes important in the intrinsic formalism[8].

In ordinary quantum mechanics with an a priori time $t$, eq.(13) yields a unique, unambiguous interpretation (of course, guided by the inputs from laboratory experiments). In fact, $\chi$ can be considered as the unique horizontal lift[10] of the ray $\tilde{\chi}$ satisfying the parallel transport law $<\chi | d/d\tau | \chi > = 0$. Stated otherwise, eq.(13) corresponds to the parallel transport law for the Schrödinger equation

$$i\hbar \frac{d}{dt}\chi_s = H_m\chi_s$$

(14)

where the Schrödinger state function is given by $\chi_s = e^{-i\gamma}\tilde{\chi}$, the total phase $\gamma$ being the sum of the dynamical phase and the adiabatic phase: $\gamma = \gamma_d + \gamma_g = \int <H_m> dt - \int A_i dq^i$. Further, the time derivative in the lhs of eq.(14) can actually be thought of as a total derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dq^i}{Ndt} \frac{\partial}{\partial q^i}$$

(15)

the second intrinsic derivative takes care of the (adiabatic) fluctuation of the state $\chi_s$ over its mean dynamical evolution (cf.,the rhs of eq.(13)). Note that the BO decomposition (5) realizes eq.(13) at a level when the evolution of lighter system is only horizontal. The vertical (dynamical) component[1,10] of the actual evolution in the Schrödinger state $\chi_s$ is recovered through the explicit time derivative from the external time $t$. Note also that the phases $\gamma_d$ and $\gamma_g$ are distinct, both in magnitude and, evidently, in their origin. $\gamma_d$ is purely dynamical and related to the external time $t$ (and to the Hamiltonian $H_m$), whereas the latter one $\gamma_g$ is a geometric contribution of the adiabatic $U(1)$ gauge group and independent of $t$. In fact, $\gamma_g$ is both gauge and reparametrization invariant[10]. It should however be emphasized that a nonzero $\gamma_g$ always indicates the presence of a small scale independent evolution in the quantum state over the dominant dynamical evolution associated with the mean energy $<H_m>$[8]. Note that the above treatment is strictly (adiabatic) gauge invariant. Moreover, the total wave function $\Psi_0$ is also independent of
the dynamical phase $\gamma_d$, because, as a consequence of the energy conservation in the total system, the effective wave function $\psi$ also picks an equal dynamical phase $\gamma_d$, but for an opposite sign.

As advocated already in ref.[8], the small scale geometric evolution could indeed be exploited to introduce a meaningful concept of intrinsic time in a (closed) dynamical system without an external time. Below we show that this is, in fact, the only reasonable choice, contrary to the recent claims[4-7].

To begin with, let us first note that one could perhaps follow formally the steps similar to the above external time formalism even in the closed (gravity-matter) system[4-7]. This is because of the explicit gauge invariance of the total wave function $\Psi_0 = \psi \chi$ and the almost identity like character of eq.(13). One could thus write formally $\Psi_0 = \tilde{\psi} \tilde{\chi}$, where $\psi = e^{i\gamma} \tilde{\psi}$ and $\chi = e^{-i\gamma} \tilde{\chi}$, $\gamma = \gamma_d + \gamma_g = \int N < H_m > d\tau + \gamma_g$, and then identify $\chi$, as above, as the Schrödinger state in intrinsic time:

$$i\hbar \frac{d}{N d\tau} \chi = H_m \chi$$

(16)

This apparently fulfills the aim of obtaining the semiclassical Einstein equations with mean energy as back reaction[3-7]. However, the above arguments cannot be justified rigorously. Note that the split of the total phase $\gamma$ into a dynamical phase and a geometric phase is purely formal and could be misleading. In the absence of an external time, the expectation value $< H_m >$ looses its distinguished dynamical character and instead gets linked with the adiabatic gauge connection $A$. For, the total phase $\gamma$ in this case is geometric and must be treated as a single unit. Explicitly, in the absence of time both $< H_m >$ and $A$ gets related to each other by a suitable choice of the adiabatic gauge. Moreover, the reparametrization invariance of $\gamma$, rather than only of $\gamma_g$, is also evident.

The validity of the arguments leading to eq.(16) can thus be ascertained at best for a particular gauge choice: $< \chi | d/d\tau | \chi > = 0$. There are however other equivalent gauge choices. One particularly interesting gauge is $A = A_i dq^i = N < H_m > d\tau$ which together with eq.(13) also turns $\chi$ a Schrödinger state satisfying eq.(16) (total phase in $\chi$ in this case is, however, zero in contrast to the former gauge with total phase $\gamma$). However, this gauge yields a different quasi classical equation for the $q$-modes, viz.:

$$\frac{1}{2M} G^{ij} P_i P_j + MV + \frac{1}{2M} \rho = E$$

(17)

which misses the zeroth order back reaction from the mean energy $< H_m >$. The predictions of the theory thus become gauge dependent which is physically unacceptable.
Further, there is as such no reason to prefer one theory over another, at least at this level of our analysis. An extra input is necessary to restrict the theory. Some aspects of this ambiguity has already been discussed at length in the literature[11]. The conclusions drawn in refs.[4-7] are definitely motivated by the demand of obtaining the semiclassical Einstein equations even in a quantum cosmological model.(However, the semiclassical Einstein equations with mean energy (-momentum tensor) as back reaction may very well be in suspect(in the absence of any experimental clue) in the context of quantum cosmology. In any case, the standard semiclassical theory is expected to be unambiguous in the black hole back ground with an asymptotic Minkowsky time). Our approach[8] differs from the conventional ones in that we try to interpret eq.(13) minimally using only the demand of adiabatic gauge invariance of the intrinsic formalism.

To this end, one demands the theory to be gauge invariant, and aims at expressing the quasi-classical equation (11) in terms of gauge invariant quantities eg., the adiabatic field strength \( F = dA, d \) being the exterior derivative in the \( q \) space. To this effect we first compute \( F \) in the gauge \( A = N < H_m > d\tau = d\int_0^{(r)} N < M_m > d\tau \). It follows that \( F = 0 \) and hence by gauge invariance, \( F \) vanishes globally. Consequently, the flat adiabatic connection \( A_T \) corresponding to the total phase \( \gamma \) of the state \( \chi \) itself vanishes for any simply connected region in the \( q \)-space.(The gravitational sector of the superspace is expected to be simply connected away from the big bang singularity). The triviality of the adiabatic gauge bundle on the \( q \)-space tells us that the total adiabatic phase \( \gamma \) (split formally as \( \gamma_d + \gamma_g \) in eq.(16)) picked by the factored wave function \( \chi \) (or \( \psi \)) is necessarily zero and consequently the associated connection must not contribute in the gauge invariant equations of motion. Consequently, the mean adiabatic energy \( < H_m > \) (which appears now only as a part of the total connection \( A_T \) ) must be unobservable in the intrinsic description. The quasi-classical dynamics of the heavy system thus remain unaffected by the back reaction from the mean adiabatic energy of the lighter system in the intrinsic description. The back reaction is then determined only by the (gauge invariant) electric potential \( \rho \) which happens to be the first nonadiabatic correction on the adiabatic phase (zero in the present situation). The quasi-classical equation of motion of the heavy system is thus given by eq.(17)[8]:

\[
\frac{1}{2M}G^{ij}P_iP_j + MV + \frac{1}{2M}\rho = E
\]  

(18)

The physical reason for the nonobservability of the adiabatic energy \( < H_m > \) in the intrinsic description may be stated thus[8]. The measurement of energy in any (localized)
dynamical system presupposes the existence of time and involves a definition of the zero point. In the present model however the intrinsic WKB time emerges only at a level when the quantized lighter system follows the quasi-classical q-modes adiabatically. The mutual interactions between the heavy and the lighter systems induce, on the other hand, a gauge freedom in the composite system, which in turn reflects a possible arbitrariness in the choice of the zero point in the measurement of energy of the (internal) lighter system. In the case with an external time, the gauge freedom is harmless as the zero point in energy is well defined from the outset and amounts to a suitable readjustment in the scale of time \( t \). More importantly, the mean energy gets delinked from the adiabatic gauge in the presence of time \( t \). In the present case, however, the definition of intrinsic time is equivalent to a choice of gauge in the Hilbert space of the lighter system, which in turn helps fixing the zero point at the mean adiabatic energy (the only available value of energy at this level), thus leading to a renormalization in the matter Hamiltonian (\( H_m \rightarrow (H_m - < H_m >) \)). Another consequence of the definition of time through a gauge choice is that it makes the induced gauge interaction in the composite system trivial at the adiabatic level (of course, this is a consequence of the reparametrized nature of the intrinsic time). The zero point in energy is thus realized globally so far as a single intrinsic time variable suffices the description of the dynamics of the composite system. Only the (gauge invariant) energy differences over the mean adiabatic energy (i.e., the energy fluctuations) in the state \( \chi \) thus have physical meaning in the intrinsic formalism.

The Schrödinger equation (13) is now interpreted so as to describe the time dependent nonstationary evolution of the lighter state \( \chi \), over the (nonobservable) adiabatic evolution described by eq. (16). This is achieved by the method of dynamical renormalization [12,13]. Explicitly, at the adiabatic level the state \( \chi \) is realized as a stationary state \( \chi_0 \)

\[
H_m \chi_0 = < H_m > \chi_0
\]

To capture the residual (fluctuating) motion (in connection with the nonadiabatic geometric phase [8]) in \( \chi \) in eq. (13), one makes a unitary transformation by writing \( \tilde{\chi} = U\chi \), where \( U \) is the interaction picture evolution operator: \( U = \exp(-ih^{-1} \int N \tilde{H}_m d\tilde{\tau}) \). Here, \( \tilde{H}_m = H_m - < H_m > = (dH_m/d\tilde{\tau})d\tilde{\tau} \), denotes the renormalized (interaction) Hamiltonian and the time variable \( \tilde{\tau} \) is introduced via \( id/d\tilde{\tau} = id/d\tau - h^{-1}N < H_m > \), so that \( d\chi/d\tilde{\tau} = 0 \). The actual time dependent evolution of the (transformed) fluctuating state \( \tilde{\chi} \) is thus given by the (intrinsic) Schrödinger equation
\[ i\hbar \frac{d}{d\tau} \bar{\chi} = N \bar{H}_m(\tau) \bar{\chi}(\tau) \]  
(19)

with \( \bar{\chi}(0) = \chi_0 \). The (nonadiabatic) Pancharatnam connection [1,8] \( A_P \) for the matter state \( \chi \) (eq.(13)) is now realized as the dynamical phase of the intrinsic equation (19): 
\[ A_P = i \langle \phi | d | \phi > = N\hbar^{-1} < \bar{\chi} | \bar{H}_m | \bar{\chi} > d\tau = N\Delta \epsilon d\tau, \]
where \( \bar{\chi} = e^{-i \int A_P(\phi)} \) and \( \phi \in \mathcal{P} \), the projective ray space of the matter Hilbert space. Moreover, \( d \) stands for the corresponding ray space Fubini-Study exterior derivative and \( \Delta \epsilon \) is identified with the uncertainty in the original instantaneous stationary state \( \chi_0 \): 
\[ \Delta \epsilon = \sqrt{\langle \chi_0 | (\bar{H}_m - \langle \bar{H}_m >)^2 | \chi_0 >} \] [8,13].

We have thus completed a full circle in interpreting eq.(13) self consistently in an intrinsic description. In fact, eq.(19) is obtained as a verticle realization of the parallel transport law (13). This means in turn that the intrinsic geometric motion, in connection with the irreducible quantal fluctuations, in the state \( \chi \) is realized as the dominant dynamical evolution for the state \( \bar{\chi} \). Further, the relevant Hamiltonian is now \( \bar{H}_m \), instead of the original one \( H_m \).

The present derivation of the intrinsic time Schrodinger eq.(19), being exact, not only extends our previous discussions[8] but also reflects clearly the role played by the irreducible quantum fluctuations in connection with the nonadiabatic phase, in obtaining it. Note also the close similarity of the two equations: the external time eq.(14) and the intrinsic time eq.(19)[13].

To sum up, eqs.(18) and (19) constitute the two main equations in a self consistent treatment of the BO analysis without an external time. This set of equations are obtained minimally from the exact set, eqs.(6) and (9) using the demand of adiabatic gauge invariance. This seems not only the correct set of equations for studying the semiclassical limit of a closed gravity-matter system in quantum cosmology, but applications of this formalism even in ordinary quantum mechanics can not be ruled out. Some of the possible approaches have already been discussed[8,13]. A more elaborate investigation is under way and will be reported elsewhere. In the framework of quantum cosmology (and semiclassical gravity) this offers an insight into the problem why the present value of the cosmological constant in the universe is negligibly small; the reason being, as envisaged above, the nongravitating (nonobservable) nature of the mean vacuum energy in the universe[8]. The intrinsic dynamical renormalization, as presented above, seems to set the renormalized vacuum energy unambiguously to zero in a cosmological background. The present discussion, however, is restricted to the minisuperspace of a homogeneous cosmology. The generalization of this study to a more general superspace is an interesting problem for the future.
We close with two remarks.

1. Note the lapse dependence of the intrinsic Schrödinger equation (19). Though the freedom in the adiabatic gauge is utilized to define the zero point in energy in the lighter system thereby fixing the characteristic time scale for the quantum evolution, the lapse in the rhs of eq.(19) indicates a still existing residual freedom in its choice. As it is well known the lapse dependence, in the case of a gravitational background is a consequence of the nonuniqueness and foliation dependence of quantum mechanics (quantum field theory) in a curved space-time. In the present formalism there is however an interesting choice which maps eq.(19) to the extrinsic equation (14). Note that in the latter equation the external time \( t \) scales as \( \epsilon^{-1}, \epsilon = \langle H_m \rangle \) thus fixing \( N \) uniquely to \( N = 1 \). In the intrinsic time equation one however has \( N \tau \simeq (\Delta \epsilon)^{-1} = (\nu \epsilon)^{-1}, \nu \ll 1 \). Thus with the choice \( N = \nu^{-1}; \tau \simeq t \simeq \epsilon^{-1} \). In this fluctuation gauge, so to speak, the residual reparametrization gauge freedom is removed, thus allowing one to (approximately) identify the scaled eq.(19) with the external time eq.(14). One thus reproduces a replica of the original external time Schrödinger equation even in a smaller (intrinsic) time scale of the irreducible quantal fluctuations[13]. The dynamics of the heavy system in the two context however remain manifestly different.

2. Both the gauge invariant electric potential \( \rho \) and the Pancharatnam connection \( A_P \) have the same origin, viz: the uncertainty \( \Delta \epsilon \) in the state \( \chi \). Further, there is no violation of the energy conservation (unitarity)[6,8] in the total system even in the present intrinsic time formalism. These follow from the exact coupled eqs.(6) and (9) and the observation that the geometric gauge connection consists, in general, of two parts: adiabatic and nonadiabatic (the later being of lower order). The adiabatic component however vanishes in the intrinsic formalism. The total wave function \( \Psi_0 \) thus remains strictly gauge invariant in both the formalisms: with or without time \( t \). The non-unitarity reported in refs.[3,7] is an artifact of the approximate description of the composite system.

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