Non-local gravity and comparison with observational datasets\textsuperscript{1}

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Received February 20, 2015  
Revised March 19, 2015  
Accepted April 5, 2015  
Published April 27, 2015

Abstract. We study the cosmological predictions of two recently proposed non-local modifications of General Relativity. Both models have the same number of parameters as ΛCDM, with a mass parameter $m$ replacing the cosmological constant. We implement the cosmological perturbations of the non-local models into a modification of the CLASS Boltzmann code, and we make a full comparison to CMB, BAO and supernova data. We find that the non-local models fit these datasets very well, at the same level as ΛCDM. Among the vast literature on modified gravity models, this is, to our knowledge, the only example which fits data as well as ΛCDM without requiring any additional parameter. For both non-local models parameter estimation using \textit{Planck} +JLA+BAO data gives a value of $H_0$ slightly higher than in ΛCDM.

Keywords: cosmological parameters from CMBR, dark energy theory

ArXiv ePrint: 1411.7692

\textsuperscript{1}Based on observations obtained with Planck (http://www.esa.int/Planck), an ESA science mission with instruments and contributions directly funded by ESA Member States, NASA, and Canada.
1 Introduction

The observational evidence for the accelerated expansion of the Universe [1, 2] has stimulated renewed interest in modifications of General Relativity (GR) [3]. A possible approach, which has been suggested by different lines of investigations, is to add some non-local terms to GR. Non-locality in this case should not be considered as fundamental. In many physical situations non-local terms emerge from a fundamental local theory, by a classical or a quantum averaging process. For instance, non-local (but causal) effective equations govern the dynamics of the in-in matrix elements of quantum fields, and encode ultraviolet (UV) quantum corrections to the classical dynamics [4, 5]. The cosmological consequences of non-local UV effects have been recently studied e.g. in [6–9]. UV effects are however expected to be relevant only in the large-curvature regime, so for instance for the issue of smoothing the big-bang singularity, but should not be cosmologically relevant in the present epoch. Non-local modifications of GR are however also expected to emerge from infrared (IR) corrections to the effective field equations. These are indeed known to become potentially large in quantum field theory in curved space, most notably in de Sitter, which is the most studied case, see e.g. [10–21], and therefore they can potentially modify the long-distance behavior of GR.

Ultraviolet corrections in quantum field theory in curved space are by now well understood, as summarized in textbooks such as [22]. The situation for IR effects in curved space is much more complicated. Often, they manifest themselves through secularly growing terms in back-reaction computations. Such terms signal the onset of an instability, but it is typically beyond the present technology to follow the fate of the instability when the back-reaction becomes large, and to compute from first principles a corresponding effective (and in general non-local) equation of motion that describes these effects. While a better understanding of infrared effects in curved space would be highly desirable, a simpler phenomenological attitude is to postulate a non-local modification of GR which involves inverse powers of the d’Alembertian, and therefore becomes relevant in the IR, and to study its cosmological consequences. Eventually such a program will only be successful if one will be able to derive such non-local terms from first principles. However, a first step can be to understand what sort of non-local terms can give rise to an interesting cosmology. Identifying a non-local model that works well with respect to the cosmological observations would be of great help in understanding how to derive such an effective theory from fundamental principles (much as understanding the structure of the Fermi theory of weak interactions at low energies was instrumental for building the Standard Model, several decades afterwards).

In this spirit, in recent years there have been many investigations of non-local modifications of GR. For instance, non-local operators appear in the degravitation proposal [23, 24], where the insertion in the Einstein equations of an operator of the form \(1 - m^2/\Box\) was argued to have a screening effect on the cosmological constant (see also [25]). Non-local long-distance modifications of GR have been suggested in [26–29]. Constructing a non-local model that produces a dynamical dark energy and fits well the observations is however quite non-trivial.
For instance, in recent years much attention has been devoted to a non-local cosmological model proposed in [30] (see [31–42]). This model is based on the addition of a term of the form $R f(\square^{-1} R)$ to the Einstein-Hilbert action, where $R$ is the Ricci scalar. The function $f(X)$ was chosen to obtain a viable cosmology for the background evolution. The result turns out to be not very natural, $f(X) = a_1 [\tanh(a_2 Y + a_3 Y^2 + a_4 Y^3) - 1]$, where $Y = X + a_5$, and $a_1, \ldots, a_5$ are coefficients fitted to the observed expansion history. More importantly, once the function $f(X)$ is fixed in this way, one can compute the cosmological perturbations and it turns out that this model is ruled out with great statistical significance, at $7.8\sigma$ from redshift space distortions, and at $5.9\sigma$ from weak lensing [41]. A different non-local approach, which appears to be phenomenologically successful, has been recently developed by our group [43–51] and further discussed in [52–55]. In its simplest form, it is based on the action [48]

$$S_{NL} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - \frac{1}{6} m^2 R \frac{1}{\square} R \right]. \quad (1.1)$$

Integrating by parts the $\square^{-1}$ operator, this non-local action can be rewritten as

$$S_{NL} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - \frac{1}{6} m^2 f(\square^{-1} R) \right], \quad (1.2)$$

where $f(X) = X^2$. This model works well, compared to cosmological observations, both at the level of background evolution and at the level of cosmological perturbations. The fact that this success is obtained with the simple choice $f(X) = X^2$, rather than with a highly fine-tuned function, certainly makes the model stand out for its simplicity. Nonetheless, it is important to explore also different related non-local models, to see to what extent one can extract general predictions. From this point of view, a first useful observation is that models involving tensor non-localities, e.g. involving terms such as $R_{\mu\nu} \square^{-2} R^{\mu\nu}$ in the action, or terms $\square^{-1} R_{\mu\nu}$ in the equations of motion, do not provide a viable cosmological evolution already at the background level, since they are plagued by fatal run-away instabilities [46, 48, 52] (similar results have been found in [40] for models involving terms such as $R^{\mu\nu} \square^{-1} R_{\rho\sigma}$ in the action). This significantly restricts the class of viable models, and provides potentially useful indications for the construction of the corresponding fundamental theory.\footnote{In this context a common misconception is that, in an effective theory, all terms that are consistent with the symmetries of the problem will necessarily appear. So, in our case, in the effective description also the unwanted terms involved tensor non-localities will necessarily be present. However, this is not correct. An obvious counter-example is given by the Fermi theory of weak interactions. From the point of view of low-energy physics, assuming Lorentz invariance and knowing that parity is not respected, all possible terms quadratic in the fermion bilinears could in principle appear at low energy, e.g. $(\bar{\psi} \gamma_5 \psi)^2$, $(\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma_\mu \psi)$, $(\bar{\psi} \gamma_\mu \gamma_5 \psi)^2$, $(\bar{\psi} \gamma_\mu \gamma_\nu \gamma_5 \psi)^2$, etc. However, given the specific structure of the Standard Model, at low energy the charged currents only involve one very specific structure, namely $[\bar{\psi} \gamma_\mu (1 - \gamma_5) \psi]^2$.}

Another interesting non-local model, introduced in [44], is defined directly at the level of equations of motion, by

$$G_{\mu\nu} - \frac{1}{3} m^2 (g_{\mu\nu} \square^{-1} R)^T = 8\pi G T_{\mu\nu}, \quad (1.3)$$

where the inverse of the d’Alembertian is defined with the retarded Green’s function, and the superscript “$T$” denotes the extraction of the transverse part of a tensor (which is itself a non-local operation). The extraction of the transverse part ensures that the left-hand side of eq. (1.3) has zero divergence, and therefore $T_{\mu\nu}$ is automatically conserved. While the model.
defined by eq. (1.1) corresponds to the simplest possible action in this class, the model (1.3) provides the most compact equation of motion, so in a sense they are both selected by simplicity, and comparing the results obtained with them allows us to have an idea of how much the results depend on the choice of a specific non-local model. These two models are also related by the fact that, when linearizing the equations of motion derived from the action (1.1) around flat space, one finds the same equations of motion as those obtained by linearizing eq. (1.3) [48]. However, beyond the linear level in an expansion over Minkowski space, or for generic backgrounds (such as FRW), the equations of motion of the two theories are different.

As shown in [44, 47, 48], these models do not suffer from the vDVZ discontinuity, so they do not need a Vainshtein mechanism, and smoothly recover GR in the limit $m \to 0$. Thus, taking $m = \mathcal{O}(H_0)$, as we will do below for cosmological purposes, the deviations from the predictions of GR at the solar system and lab scales are utterly negligible. The issue of ghosts, which is a typical problem that must be faced in extensions of GR, has also been discussed in detail in [44, 45, 51]. The crucial point is that eq. (1.3), with $\Box^{-1}$ defined in terms of the retarded Green’s function, must be understood as a classical effective equation of motion, derived through some form of classical or quantum averaging from an underlying fundamental local theory. For instance, as we mentioned above, effective non-local (but causal) equations govern the dynamics of the in-in matrix elements of quantum fields [4, 5]. Non-local but causal equations also emerge in a purely classical context when one separates the dynamics of a system into a long-wavelength and a short-wavelength part. One can then obtain an effective non-local equation for the long-wavelength modes by integrating out the short-wavelength modes, see e.g. [56] for a recent example in the context of cosmological perturbation theory. A promising approach from this point of view is the one recently discussed in [57], in which the effective field theory describing cosmology at scales parametrically larger than the horizon is shown to be a special case of the effective field theory of open system, in which the relevant modes are not described by an effective action in the Wilsonian sense, but rather by an effective stochastic equation of motion. In ref. [57] only the limit of scales very large compared to the horizon is considered, since in this limit the stochastic dynamics becomes Markovian and the computation simplifies technically. However, for cosmological purposes one is really interested in the physics near the horizon scale. In this case non-Markovian effects will in general appear, and we expect that these will be described by non-local but causal terms in the effective equations of motion. The issue of the presence of a ghost at the quantum level must therefore be addressed in the full quantum theory from which eq. (1.3) is derived as an effective classical equation.\footnote{The same argument applies to the model (1.1). In this case our non-local model is defined by the non-local equations of motions obtained performing the formal variation of the action and then formally replacing $\Box^{-1}$ with $\Box^{-1}$ret. In this sense, we propose the non-local action (1.1) just as a formal object useful for deriving covariant non-local (but causal) equations of motion, and quantum issues must be addressed in the fundamental theory underlying these classical effective equations of motion.}

In the end, if the comparison with the data should eventually point toward the correctness of a non-local model of this sort, it might also point toward the necessity of refining it. It must certainly be borne in mind that our quantitative results are in any case specific to the models that we use. Ideally, one would like to eventually derive the non-local model from first principles, and this should select the exact non-local structure. The purely phenomenological approach that we rather take here could help in identifying a promising non-local structure, paving the way for a first-principle approach.
The model in a Boltzmann code. For the CMB they only used the Planck CMB shift parameters and did not implement the effect of the Universe. Furthermore, for very general reasons they both predict a phantom DE equation of state. Numerical details of course differ. After fixing the mass $m$ so as to reproduce the observed value of $\Omega_{DE}$, the model (1.1) predicts that, today, $w_{DE} \simeq -1.14$, while eq. (1.3) predicts $w_{DE} \simeq -1.04$. Since cosmological perturbations in the DE sector are mostly proportional to $(1 + w_{DE})$, we also generically find that the predictions of the model (1.3) are intermediate between that of model (1.1) and that of ΛCDM.

The aim of this paper is to perform a detailed comparison of these non-local models with cosmological observations. We will refer to them as the “$R\Box^{-2}R$ model” and the “$g_{\mu\nu}\Box^{-1}R$ model”, respectively. For the $g_{\mu\nu}\Box^{-1}R$ model, cosmological perturbations have already been worked out in [49, 53]. In particular, Nesseris and Tsujikawa [53] showed that the $g_{\mu\nu}\Box^{-1}R$ model is consistent with SNe (Union 2.1), BAO, CMB and growth rate data. However for the CMB they only used the Planck CMB shift parameters and did not implement the model in a Boltzmann code. For the $R\Box^{-2}R$ model, cosmological perturbations have been worked out in [49], where again consistency with SN and structure formation data was found. However, an accurate comparison with CMB data requires to implement the cosmological perturbations of these models in a Boltzmann code. We have now implemented the cosmological perturbations of both non-local models in a Boltzmann code, modifying the CLASS code [58–61]. In this paper we present an accurate comparison of these models with CMB, SNe, BAO and HST data. We perform parameter estimation for these models, and we compare their goodness of fit to that of ΛCDM, using the Markov Chain Monte Carlo (MCMC) code Montepython v2.1.0 [62]. In this paper we present the main results of this analysis. A more extended discussion will be presented elsewhere.

### 2 Results

We use as datasets the CMB data from the Planck 2013 data release [63], type-Ia supernovae from JLA [64], and BAO data from BOSS [65] and 6dF [66]. When comparing with BAO we note that, as for other modified gravity models, one would have to take into account any additional scale dependence in the growth. However, for the non-local models discussed here, the power spectrum shows no scale dependence for $k > 0.02 h_0$/Mpc (see figure11 of [49]).

| Param  | ΛCDM          | $g_{\mu\nu}\Box^{-1}R$ | $R\Box^{-2}R$ |
|--------|---------------|-------------------------|---------------|
| $100 \omega_b$ | 2.204$^{+0.028}_{-0.029}$ | 2.204$^{+0.028}_{-0.03}$ | 2.204$^{+0.029}_{-0.029}$ |
| $\omega_c$     | 0.1195$^{+0.0026}_{-0.0027}$ | 0.1195$^{+0.0026}_{-0.0026}$ | 0.1191$^{+0.0027}_{-0.0028}$ |
| $H_0$           | 67.56$^{+1.3}_{-1.3}$          | 68.95$^{+1.3}_{-1.3}$          | 71.67$^{+1.5}_{-1.5}$          |
| $10^9 A_s$      | 2.193$^{+0.052}_{-0.06}$       | 2.194$^{+0.048}_{-0.062}$       | 2.198$^{+0.053}_{-0.059}$       |
| $n_s$           | 0.9629$^{+0.0072}_{-0.0074}$    | 0.9629$^{+0.0077}_{-0.0081}$    | 0.9628$^{+0.0074}_{-0.0073}$    |
| $z_{re}$        | 11.1$^{+1.1}_{-1.1}$           | 11.1$^{+1.1}_{-1.2}$           | 11.16$^{+1.2}_{-1.1}$           |

Table 1. 68% limits for the cosmological parameters of ΛCDM and of the two non-local models, using the Planck CMB data only. The last row gives the minimum chi-square of the best fit. $H_0$ is in units of km s$^{-1}$ Mpc$^{-1}$.
which includes the regime of interest for BAO peaks. We can therefore safely compare the models with BAO data sets, in the linear regime. As a set of cosmological parameters we vary in our analysis the baryon density today \( \omega_b \), the cold dark matter density \( \omega_c = \Omega_c h_0^2 \), the Hubble parameter today \( H_0 = h_0 (100 \text{ km s}^{-1} \text{Mpc}^{-1}) \), the amplitude of scalar perturbation \( A_s \), the scalar spectrum index \( n_s \) and the redshift at which the Universe is half-reionized \( z_{\text{re}} \). In \( \Lambda \)CDM the dark energy density \( \Omega_\Lambda \) is a derived parameter, fixed by the flatness condition. Similarly, in our model the mass parameter \( m^2 \) is a derived parameter, fixed again from the condition \( \Omega_{\text{tot}} = 1 \). The non-local models and \( \Lambda \)CDM therefore have the same set of free parameters, which facilitates their comparison.

Table 1 shows the mean values with the 1\( \sigma \) errors for these parameters, obtained from our MCMC using the \textit{Planck} data only. In table 2 we show the results obtained combining \textit{Planck}, JLA and BAO. Our values for \( \Lambda \)CDM are in agreement, within the statistical error, with those reported in [63]. We see from the tables that the \( g_{\mu \nu} \square^{-1} R \) case gives intermediate predictions between \( \Lambda \)CDM and the \( R \square^{-2} R \) model. Figure 1 shows the separate 1\( \sigma \) and 2\( \sigma \) contours for CMB, BAO and SNe in the plane \(( H_0, \Omega_c )\). We notice that for the \( g_{\mu \nu} \square^{-1} R \) model these dataset are fully consistent, while in the \( R \square^{-2} R \) model there is a slight tension between CMB and SN data, which explains the higher \( \chi^2 \) for this model in table 2 (although even for this model the datasets are in agreement within 2\( \sigma \)).

In figure 2 we show the marginalized likelihood for \( \omega_b, \omega_c, n_s \) and \( H_0 \), while in figure 3 we show the 1\( \sigma \) and 2\( \sigma \) contours of the likelihood function in the \(( H_0, \omega_c )\) plane, for \( \Lambda \)CDM and for the two non-local models. Among the various parameters, the most significant difference is in \( H_0 \), which in both non-local models is higher than in \( \Lambda \)CDM.
Local measurements seem to give higher values of $H_0$ compared to the values obtained from CMB assuming $\Lambda$CDM even if, in the case of local measurements, it is not clear to what extent the systematic effects are under control. In particular, HST gave $H_0 = 73.8 \pm 2.4$ (in units of km s$^{-1}$Mpc$^{-1}$) [67], which has been subsequently reduced to $H_0 = 72.7 \pm 2.4$ after the recalibration to NGC 4258 in [68]. Similarly, the HST Key Project and the Carnegie Hubble Project obtained a high value $H_0 = 74.3 \pm 1.5$(stat) $\pm 2.1$(sys) [69]. At present there is no consensus on whether these high values are due to unaccounted systematics in the SN data [70], statistical fluctuations [71], or could be an indication of deviations from $\Lambda$CDM. The re-analysis in [70] in particular gives $H_0 = 70.6 \pm 3.3$. In any case, it is interesting to observe that in the non-local models $H_0$ automatically comes out slightly higher than in $\Lambda$CDM. In table 3 we show the results of adding to the $Planck + JLA + BAO$ dataset also the value $H_0 = 70.6 \pm 3.3$ from [70]. Increasing $H_0$ further, the fit tends to favor the non-local models over $\Lambda$CDM.

Furthermore, a recent re-analysis in [72] identifies a bias due to a correlation between peak brightness of Type Ia SNe and the local star-formation rate. Correcting for this bias, they find $H_0 = 70.6 \pm 2.6$ when using LMC distance, Milky Way parallaxes and NGC 4258.
Figure 2. The likelihood for $\omega_b$, $\omega_c$, $n_s$ and $H_0$, for the Planck+BAO+JLA datasets, for $\Lambda$CDM (solid black line), $g_{\mu\nu}\Box^{-1}R$ (dotted red) and $R\Box^{-2}R$ (dot-dashed, blue).

Figure 3. The 1$\sigma$ and 2$\sigma$ contours in the $(H_0,\omega_c)$ plane for the three models, using Planck+JLA+BAO.
Figure 4. A plot of the Planck data for $l(l + 1)C_l/(2\pi)$, together with the curves obtained with the best-fit parameters determined fitting to the Planck+JLA+BAO dataset, for ΛCDM (solid black line), the $g_{\mu\nu}\Box^{-1}R$ model (red dashed line, almost indistinguishable from the black line) and the $R\Box^{-2}R$ model (blue, dot-dashed).

Finally, in figure 4 we show the best-fit prediction for the CMB multipoles. It is interesting to observe that, at low multipoles, the $R\Box^{-2}R$ model has a smaller amplitude, which goes in the direction indicated by the data, although of course in this region cosmic variance dominates. A similar result has been shown in [55] (although without performing parameter estimation for the model).

The conclusion of this analysis is that these non-local models fit the present cosmological data as well as ΛCDM, and particularly the $g_{\mu\nu}\Box^{-1}R$ model fits the data even slightly better. Both models have the same number of parameters as ΛCDM, and in this sense they are quite unique, among the vast literature on modified gravity models. To the best of our knowledge, there is no other model that is competitive with ΛCDM from the point of view of fitting current observations (at a level of accuracy which tests not only the background evolution but also the cosmological perturbations of the model), without being an extension of ΛCDM with extra free parameters. This non-local approach therefore seems to provide an interesting new line of attack to the problem of finding a dynamical explanation for dark energy. From the point of view of the analysis of cosmological data, these non-local models also provide a welcome competitors, against which we can test the validity of ΛCDM.

Acknowledgments

We thank Benjamin Audren for providing us a preliminary version of Montepython including JLA data, Alex Barreira for comparison of our codes for the $C_l$ for the $R\Box^{-2}R$ model, and Julien Lesgourgues for useful discussions. The work of YD, SF, MK and MM is supported by the Fonds National Suisse. The work of MM is supported by the SwissMap NCCR. VP acknowledges the DFG TransRegio TRR33 grant on “The Dark Universe”.

The development of Planck has been supported by: ESA; CNES and CNRS/INSU-IN2P3-INP (France); ASI, CNR, and INAF (Italy); NASA and DoE (U.S.A.); STFC and UKSA (U.K.); CSIC, MICINN and JA (Spain); Tekes, AoF and CSC (Finland); DLR and MPG (Germany); CSA (Canada); DTU Space (Denmark); SER/SSO (Switzerland); RCN (Norway); SFI (Ireland); FCT/MCTES (Portugal). A description of the Planck Collaboration and a list of its members, including the technical or scientific activities in which they have been involved, can be found at [http://www.cosmos.esa.int/web/planck/planck-collaboration](http://www.cosmos.esa.int/web/planck/planck-collaboration).
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