MODELLING AN EXPERIMENT TO MEASURE THE SPEED OF GRAVITY IN SHORT DISTANCES

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Abstract: An experiment to measure the speed of gravitational signals in short distances has been developed with the goal to study its behaviour as traveling through a medium different from air. The experiment is composed of three sapphire devices suspended in vacuum and cooled down to 4.2 Kelvin. The amplitudes of the central device (detector) is monitored by an ultralow phase noise microwave signal using resonance in the whispering gallery modes. The other two sapphire devices are excited by piezoelectric crystals, which make the two devices vibrate at the same frequency and phase. Between the two vibrating devices and the detector, a different medium will be placed, and then the speed is measured and compared with the case where the medium is pure air. The modelling of the experiment is made assuming the detector as a spring-mass system. The results show that the detection is achievable.

1. Introduction

The only group devoted in Brazil to experimental gravity is the Graviton Group which efforts are concentrated in the detection of Gravitational Waves. The first gravitational wave detectors appeared in the beginning of the sixties [1] with the resonant mass gravitational wave detectors [2,3,4,5].

The Mario Schenberg Gravitational Wave detector is the main focus of the Graviton Group. In this detector, six sensors are placed on the surface of a sphere, arranged in a half dodecahedron distribution. These sensors are electromechanical vibration transducers. Each of these transducers mechanically amplifies the vibration on there sphere surface caused by the passing of the gravitational wave. The mechanically amplified vibration is connected to a wall of a resonant cavity. In this resonant cavity microwaves are injected, with the change in size of the cavity, a side band signal appears in the microwave signal that leaves the cavity, these sideband signal is proportional to the intensity of the gravitational wave and its polarization [6,7,8].

The Graviton Group efforts on the field of gravitational wave detection can be summarized in the following papers [9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25]. The Brazilian detector is shown in Figure 1.
2. The experiment
The experiment is based on a system with three sapphire devices cooled to 4K in vacuum, the side devices are excited by PZT to keep the vibration in phase and the central device has is vibration state monitored by microwaves of ultra-low noise, the devices are made of sapphire [26]. The sapphire devices will be suspended in distances of 1 meter between them. The central sapphire device is a block of three connected bars to allow a wider detection bandwidth, but for simplicity it will be shown here as a bar. After measuring the speed in air, the next phase is to insert between detector and emitter a different medium, and measure again the speed and compare speed in these different conditions. The schematics of the experiment can be seen on Figure 2.

3. Sapphire Transducer Device
This sapphire transducer device is based on microwave pumped sapphire piece that works as a detector using parametric amplification and normal amplification on the signal that leaves the sapphire detector device, as can be seen in Figure 3.
The geometry of the sapphire transducer will be in the form of a three cylindrical connected rods as massive as possible, and regarding the manufacture of the sapphire, some care must be taken to achieve the highest possible values for the $Q_e$ and $Q_m$. The shape of the sapphire device is shown as a bar.

**Figure 3:** Complete model of the gravitational wave detector adapted to the design - Low Noise Oscillator connected to the Project receiving Sapphire - Unprecedented

4. Gravitational signal generation and detection
The simplified model of the experiment can be seen in Figure 4.

**Figure 4:** Model of the detector and the emitter of periodic tidal gravitational signals - Carlos Frajuca

5. Calculation of signal intensity
From this Newtonian Theory of Gravity, the calculation of the intensity of the signal is made, taking into account the calculation of force $F_{1x}$ and $F_{2x}$ considered as horizontal forces acting in mass of the detector device. The proposed model, part of a system formed by three sets of masses 'm1' e 'm2', as seen in Figure 4, displaced together by the distance 'b' coupled by a spring.
5.1 Calculations
The analysis of the Newtonian signal generated from the 'm₁e' m₂ masses of two emitting elements, these emitting devices vibrate with an amplitude equal to a.

Therefore, we can obtain tidal interaction forces that have the following expressions:

\[
F_{1x} = \frac{MmG}{(b+X/a. \cos \omega t)} + \frac{MmG}{(2b+X/a. \cos \omega t)} \\
F_{2x} = MmG \left( \frac{1}{(X/a. \cos \omega t)} + \frac{1}{(b+X/a. \cos \omega t)} \right)
\]

(1)

(2)

where: \( G \) is the Newton constant; \( M \) is the mass of the emitting elements; \( m \) is the effective mass of emitters and detectors; \( \omega \) is the angular speed of mass vibration; \( a \) is the length / distance of sapphire piece vibrating; \( X \) is the distance between the emitters and the detector; \( b \) is the effective length of transmitters and receiver and \( t \) is the time.

Now is made the approximation for a small amplitude of the vibrations

\[
\text{where} \quad a = a \cos \omega t
\]

\[
F_{1x} - F_{2x} = \frac{-4(bGmM)}{40b^3GmM(3a^2 - 3ab + 2b^2)} + \frac{12b^2GmM}{x^4} - \frac{8(bGmM(3a^2 - 3ab + 2b^2))}{x^6} + O\left(\frac{1}{x^7}\right)
\]

(Laurent series)

(3)

(4)

using a fourth order approximation around a small device compared to the distance, and considering only the terms with a cos part, the tidal forces is given by:

\[
F_{1x} - F_{2x} = MmG \frac{2ab^2}{X^2},
\]

(5)

and using the expression for a harmonic oscillator, the vibration amplitude in \( b \) is given by:

\[
\Delta b = QGM \left( \frac{2ab^2}{X^2} \right) \cdot \sin \omega t
\]

(6)

\[
\Delta b = 3.10^{-18} m.
\]

(7)

calculated with the characteristics described in the next chapter.

6. Quantum limit, equipment sensitivity and thermal noise
The Standard Quantum Limit (SQL) of a low-loss acoustic oscillator is the minimum vibration amplitude that can be measured with a single phonon. Such amplitude can be measured because sapphire with ultra-low acoustic and dielectric losses are available and a low noise parametric transducer reading, based on a low-noise phase oscillator, which can now reach levels as low as -185 dBc/Hz at 1 kHz off set, are available.
The value to be used in the following calculations are: effective mass $M_{\text{eff}} = 1 \text{ kg}$; Distance between the masses $= 1 \text{ m}$; mechanical quality factor $Q_m = 10^3$; vibration amplitude $a = 10^{-4}$; device length $b = 0.2 \text{ m}$ and Frequency bandwidth BW = 1000 Hz.

6.1 Quantum Limit
Is the vibration amplitude of a single phonon:

$$E = \hbar \nu$$  \hspace{1cm} (8)

That corresponds to the minimum limit, because the smallest number of Phonons is 1; therefore,

$$\hbar \nu = \frac{1}{2} A^2 w^2 m \Rightarrow \hbar \nu = \frac{A^2 w^2 M_{\text{eff}}}{2}$$ \hspace{1cm} (9)

$$\Delta b_{QL} = A = \sqrt{\frac{2\hbar}{M_{\text{eff}} 2\pi \nu}} = \frac{2 \times 10^{-34}}{12 \times 5 \times 10^2} = 2.5 \times 10^{-20} \text{ m}$$ \hspace{1cm} (10)

$$\Delta b_{QL} = 2.5 \times 10^{-20} \text{ m}.$$  \hspace{1cm} (11)

6.2 Equipment Sensitivity Limit
The sensitivity of the displacement of a parametric transducer can be characterized by the frequency shift of the electrical resonance in relation to the displacement $(df/dx)$ in this case this value is $(2 \text{ MHz})/(\mu\text{m})$.

The limit for spectral performance of the transducer can be calculated by:

$$S_x(f) = \left(\frac{df}{dx}\right)^2 S_\phi(f) / 2$$ \hspace{1cm} (12)

$$S_x = \sqrt{10^{39.5} \frac{m}{\sqrt{\text{Hz}}}} = 5.1 \times 10^{-19} \frac{m}{\sqrt{\text{Hz}}}$$ \hspace{1cm} (13)

Using BW = 1000 Hz:

$$S_x = 1.6 \times 10^{-20} \text{ m}.$$  \hspace{1cm} (14)

6.3 Thermal Noise Limit
Variations in detector length due to thermal effects $(\Delta x_{th})$, known as Nyquist noise, should be reduced below quantum limits. Using the detection bandwidth (BW) equal to 1000 Hz.

$$\Delta x_{th} = \sqrt{\frac{K T}{5M_{\text{eff}} Q_{m} \nu_{0} B_{m}}}.$$  \hspace{1cm} (15)

$$\Delta x_{th} = 1.3 \times 10^{-20} \text{ m}.$$  \hspace{1cm} (16)

7. Conclusion
Using the new bandwidth of the detector device the equipment sensitivity, the quantum noise and thermal noise are well below the signal generated by the vibration of the both side sapphire devices, much better than the results of [26]. This demonstrates that the experiment is viable.

Acknowledgments
Carlos Frajuca acknowledges FAPESP for grant #2013/26258-4 and grant #2006/56041-3.

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