Generalized $\mu - \tau$ reflection symmetry and leptonic CP violation

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We propose a generalized $\mu - \tau$ reflection symmetry to constrain the lepton flavor mixing parameters. We obtain a new correlation between the atmospheric mixing angle $\theta_{23}$ and the “Dirac” CP violation phase $\delta_{CP}$. Only in a specific limit our proposed CP transformation reduces to standard $\mu - \tau$ reflection, for which $\theta_{23}$ and $\delta_{CP}$ are both maximal. The “Majorana” phases are predicted to lie at their CP-conserving values with important implications for the neutrinoless double beta decay rates. We also study the phenomenological implications of our scheme for present and future neutrino oscillation experiments including T2K, NO$\nu$A and DUNE.

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I. INTRODUCTION

The understanding of flavor mixing and CP violation is a long-standing open question in particle physics. In order to shed light upon the structure of fermion mixing various types of flavor symmetry-based approaches have been invoked [1–5]. Non-Abelian flavor symmetries provide a specially attractive framework. These are typically broken spontaneously down to two distinct residual subgroups in the neutrino and charged lepton sectors, the mismatch between the two leading to specific lepton mixing patterns. A complete classification of lepton mixing matrices from finite residual flavor symmetries has been recently given in [6]. The precise measurement of a non-zero reactor angle [7–10] excludes several flavor symmetry groups and encourages future searches for CP violation in neutrino oscillations. It is interesting to notice that a nearly maximal CP-violating phase $\delta_{CP} \simeq 3\pi/2$ has been reported by the T2K [11], NO$\nu$A [12] and Super-Kamiokande experiments [13], although the statistical significance of all these experimental results is below 3$\sigma$ level. Moreover, such hints of a nonzero $\delta_{CP}$ were already present in global analyses of neutrino oscillation data, such as the one in Ref. [14].

Generic lepton mass matrices may admit both remnant CP symmetries as well as remnant flavor symmetries. Moreover remnant flavor symmetries can be generated by remnant CP transformations [15] [16]. As a result it is an interesting idea to constrain the lepton flavor mixing matrix from CP symmetries rather than flavor symmetries. In particular, the maximal Dirac CP-violating phase can be explained by the so-called $\mu - \tau$ reflection symmetry under

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which a muon (tau) neutrino is transformed into a tau (muon) antineutrino \([17–19]\). Here we obtain a generalized \(\mu - \tau\) reflection symmetry in the context of models based on remnant CP symmetries.

The plan of the paper is as follows. The general form of lepton mixing is reviewed in Sec. II. Based on the residual CP transformation approach we derive in Sec. III a master formula for the lepton mixing matrix. With this we generalize the \(\mu - \tau\) reflection, and show explicitly how the CP phase can be constrained by the experimental measurement of the atmospheric mixing angle. In Sec. IV we investigate the phenomenological implications of our scheme for current and upcoming neutrinoless double beta decay as well as neutrino oscillation experiments.

\section{II. GENERAL FORM OF LEPTON MIXING}

We start with the fully “symmetrical” presentation of the most general unitary lepton mixing matrix, as originally proposed in Refs. [20, 21], given as:

\[
U_{\text{Sym}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13}e^{-i\phi_{12}} & s_{13}e^{-i\phi_{13}} \\
    -s_{12}c_{23}e^{i\phi_{12}} - c_{12}s_{13}s_{23}e^{-i(\phi_{23} - \phi_{13})} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i(\phi_{23} + \phi_{12} - \phi_{13})} & c_{13}s_{23}e^{-i\phi_{23}} \\
    s_{12}s_{23}e^{i(\phi_{23} + \phi_{12})} - c_{12}s_{13}c_{23}e^{i\phi_{13}} & -c_{12}s_{23}e^{i\phi_{23}} - s_{12}s_{13}c_{23}e^{-i(\phi_{12} - \phi_{13})} & c_{13}c_{23}
\end{pmatrix},
\]

(1)

where \(c_{ij} = \cos \theta_{ij}\) and \(s_{ij} = \sin \theta_{ij}\). In this parametrization the relation between flavor mixing angles and the magnitudes of the entries of the leptonic mixing matrix is

\[
\sin^2 \theta_{13} = |U_{e3}|^2, \quad \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \quad \text{and} \quad \sin^2 \theta_{23} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2}.
\]

(2)

The Particle Data Group presents this parametrization of the mixing matrix in a non symmetrical form [22], in which the two “Majorana” phases appear in the diagonal (there are in principle three ways of doing this). The resulting presentation is motivated by the simple description of neutrino oscillation that results, in which the “Majorana” phases manifestly drop out, as they should \(^1\). It is very simple to relate both presentations through a similarity transformation involving a diagonal phase matrix (the reader can verify this by using Eq. (2.5) in [20]).

First notice that the above expressions in Eq. (2) also hold when using the PDG form. Therefore, the difference between both parameterizations appears only in the way of writing the CP invariants. We start with the usual Jarlskog invariant describing CP violation in conventional neutrino oscillations. This is defined as

\[
J_{\text{CP}} = \Im \left\{ U_{e1}^* U_{\mu3} U_{e3} U_{\mu1} \right\},
\]

and takes the following form in the symmetric parametrization

\[
J_{\text{CP}} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin(\phi_{13} - \phi_{23} - \phi_{12}).
\]

(3)

This invariant is the leptonic analogue of that which characterizes the quark CKM mixing matrix. It is clear that, as expected, in the symmetrical parametrization \(J_{\text{CP}}\) depends, apart from the three mixing angles, on the rephasing

\(^1\) Of course the Majorana phases also drop out when writing in the symmetric form, but in a less obvious way.
invariant phase combination $\phi_{13} - \phi_{23} - \phi_{12}$. This gives a very transparent interpretation of the “Dirac” leptonic CP invariant. On the other hand, concerning the remaining two invariants

$$I_1 = \text{Im} \{U_{e2}^2 U_{e3}^* U_{e1}^* \} \quad \text{and} \quad I_2 = \text{Im} \{U_{e3}^2 U_{e1}^* U_{e2}^* \},$$

associated with the “Majorana” phases [23–25] they take the form

$$I_1 = \frac{1}{4} \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin(-2\phi_{12}) \quad \text{and} \quad I_2 = \frac{1}{4} \sin^2 2\theta_{13} \cos^2 \theta_{12} \sin(-2\phi_{13}).$$

(4)

These invariants appear in lepton number violating processes such as neutrinoless double beta decay which do not depend, as expected, on the “Dirac” invariant $J_{CP}$. Indeed, one can easily check that this is so. In contrast, however, when written in the PDG form, the amplitude for neutrinoless double beta decay involves all three CP phases. Pulling out an overall phase is, of course, possible but would bring in an ambiguity in the extraction of the phases. For all the reasons explained in this section, we prefer the fully symmetric parametrization to the equivalent PDG form.

III. GENERALIZED $\mu - \tau$ REFLECTION

In contrast with flavor symmetry schemes, our generalized CP symmetry approach can constrain not only the mixing angles but also the CP violating phases. It can lead to rather predictive scenarios, where all the mixing parameters depend on a small number of free parameters [26]. We now turn to the method of residual CP symmetry transformations proposed in Ref. [15]. This will allow us to obtain CP-violating extensions systematically. Moreover it will, in principle, allow us to make CP predictions, starting from the general CP-conserving form of the lepton mixing matrix. Without loss of generality, we adopt the charged lepton diagonal basis, i.e. $m_l \equiv \text{diag}(m_e, m_{\mu}, m_\tau)$. Then the neutrino mass matrix $m_\nu$ can be expressed via the mixing matrix $U$ as $m_\nu = U^* \text{diag}(m_1, m_2, m_3) U^\dagger$ under the assumption of Majorana neutrinos. The invariance of the neutrino mass matrix under the action of a CP transformation $X$ implies [15]

$$X^T m_\nu X = m_\nu^*,$$

(5)

where $X$ should be a symmetric unitary matrix to avoid degenerate neutrino masses. As a result we find a master formula for the lepton mixing matrix [15]

$$U = \Sigma O_{3 \times 3} Q_\nu,$$

(6)

where $\Sigma$ is the Takagi factorization matrix of $X$ fulfilling $X = \Sigma \Sigma^T$, $Q_\nu$ is a diagonal phase matrix whose form is $Q_\nu = \text{diag}(e^{-ik_1 \pi/2}, e^{-ik_2 \pi/2}, e^{-ik_3 \pi/2})$ with the natural numbers $k_i = 0, 1, 2, 3$. Actually, the entries of $Q_\nu$ are $\pm 1$ and $\pm i$ which encode the CP-parity or CP-signs of the neutrino states and it renders the light neutrino mass eigenvalues positive [27]. The matrix $O_{3 \times 3} = O_1 O_2 O_3$ is a generic three dimensional real orthogonal matrix, and it can be parameterized as

$$O_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{pmatrix}, \quad O_2 = \begin{pmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{pmatrix} \quad \text{and} \quad O_3 = \begin{pmatrix} \cos \theta_3 & \sin \theta_3 & 0 \\ -\sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$
A possible overall minus sign of $O_{3\times 3}$ is dropped since it is irrelevant. Therefore the lepton mixing matrix is predicted to depend on three free parameters $\theta_{1,2,3}$ besides the parameters characterizing the residual CP transformation $X$. Notice that if $\Sigma$ is a Takagi factorization matrix of $X$, $\Sigma O_{3\times 3}'$ is also a valid Takagi factorization matrix, where $O_{3\times 3}'$ is an arbitrary real orthogonal matrix which can be absorbed into $O_{3\times 3}$ by parameter redefinition. As a result, the prediction for the lepton mixing matrix in Eq. (4) remains true. Here we focus on a generalization of the widely discussed $\mu - \tau$ reflection \[17\]. This interesting CP transformation takes the following form:

$$X = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\
0 & e^{i\beta} \cos \Theta & i e^{i(\frac{\Theta}{2} + \frac{\delta}{2})} \sin \Theta \\
0 & i e^{i(\frac{\Theta}{2} - \frac{\delta}{2})} \sin \Theta & e^{i\gamma} \cos \Theta \end{pmatrix},$$

where the parameters $\alpha$, $\beta$, $\gamma$, and $\Theta$ are real. The corresponding Takagi factorization matrix is given by

$$\Sigma = \begin{pmatrix} e^{i\frac{\Theta}{2}} & 0 & 0 \\
0 & e^{i\beta} \cos \frac{\Theta}{2} & i e^{i\gamma} \sin \frac{\Theta}{2} \\
0 & i e^{i\gamma} \sin \frac{\Theta}{2} & e^{i\frac{\Theta}{2}} \end{pmatrix}.$$  \tag{9}

As a result the resulting lepton mixing angles are determined as

$$\sin^2 \theta_{13} = \sin^2 \theta_2, \quad \sin^2 \theta_{12} = \sin^2 \theta_3, \quad \sin^2 \theta_{23} = \frac{1}{2} \left( 1 - \cos \Theta \cos 2\theta_1 \right),$$

while the CP violation parameters are predicted as

$$J_{CP} = \frac{1}{2} \sin \Theta \sin \theta_2 \sin 2\theta_3 \cos^2 \theta_2, \quad \sin \delta_{CP} = \frac{\sin \Theta \sin \theta_2 \sin 2\theta_3}{\sqrt{1 - \cos^2 \Theta \cos^2 2\theta_1}},$$

$$\tan \delta_{CP} = \tan \Theta \csc 2\theta_1, \quad \phi_{12} = \frac{k_2 - k_3}{2} \pi, \quad \phi_{13} = \frac{k_3 - k_1}{2} \pi, \quad \delta_{CP} = \frac{k_3 - k_2}{2} \pi - \phi_{23}.$$  \tag{11}

In general, as we saw in the previous section, the lepton mixing matrix is specified by six parameters, three angles and three phases. In our scenario only four free independent parameters appear: $\theta_1$, $\theta_2$, $\theta_3$ and $\Theta$. Notice also that the parameters $\alpha$, $\beta$ and $\gamma$ in Eq. (8) do not appear in the mixing parameters. It follows that the three mixing angles are not correlated with each other. Hence we have no genuine prediction for mixing angles. In contrast, however, an important prediction concerning CP violation is that the “Majorana” phases $\phi_{12}$ and $\phi_{13}$ are restricted to lie at their CP-conserving values, and correspond simply to the CP parities of the neutrino states \[27, 28\]. Moreover, one sees that the atmospheric angle and the Dirac phase $\delta_{CP}$ are given in terms of two parameters $\theta_1$ and $\Theta$, and they are correlated with each other according to \[2\]

$$\sin^2 \delta_{CP} \sin^2 2\theta_{23} = \sin^2 \Theta.$$  \tag{12}

Taking $\Theta = \pm \frac{\pi}{2}$, both $\theta_{23}$ and $\delta_{CP}$ are maximal, since the residual CP transformation $X$ reduces to the standard $\mu - \tau$ reflection. When $\theta_1 = \pm \frac{\pi}{4}$, the atmospheric mixing angle $\theta_{23}$ is maximal and $\tan \delta_{CP} = \pm \tan \Theta$. On the other hand, we have maximal $\delta_{CP}$ and $\sin^2 \theta_{23} = \sin^2 \frac{\Theta}{2}$ for $\theta_1 = 0, \pi$. Present global fits of neutrino oscillation data indicate the $\theta_{23}$ deviates from the maximal value \[14\]. If non-maximal $\theta_{23}$ was confirmed by forthcoming more

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\[2\] We note that in the $A_4$ flavor-symmetry-based model in Ref. \[29\] we also have a correlation between $\delta_{CP}$ and the atmospheric angle.
FIG. 1: The contour region of $\sin^2 \theta_{23}$ in the plane of $\theta_1$ and $\Theta$ for both normal ordering (NO) and inverted ordering (IO) mass spectrum. The different contours correspond to $1\sigma$, $2\sigma$ and $3\sigma$. The red solid lines represent the best fitting values.

FIG. 2: Contour plot of $|\sin \delta_{\text{CP}}|$ defined in Eq. (11). The thick dashed lines, dotted lines, dot-dashed lines, dashed line and thick solid lines refer to $|\sin \delta_{\text{CP}}| = 0$, $1/2$, $1/\sqrt{2}$, $\sqrt{3}/2$ and 1 respectively.

Sensitive experiments, the standard $\mu - \tau$ reflection would be disfavored, while our present CP transformation would provide a good alternative, with the value of $\Theta$ determined from the measured values of $\theta_{23}$ and $\delta_{\text{CP}}$. We display the contour regions for $\sin^2 \theta_{23}$ and $|\sin \delta_{\text{CP}}|$ in the plane $\theta_1$ versus $\Theta$ in Fig. 1 and Fig. 2 respectively.

Given the $3\sigma$ range of the atmospheric mixing angle $0.393 \leq \sin^2 \theta_{23} \leq 0.643$, the correlation in Eq. (12) allows us to predict the range of the Dirac CP violating phase $|\sin \delta_{\text{CP}}|$ as a function of the parameter $\Theta$ which characterizes the CP transformation $X$. The result is shown in Fig. 3. It is remarkable that $|\sin \delta_{\text{CP}}|$ is predicted to lie in a rather narrow region for a given value of $\Theta$.

On the other hand, as we can see from Eq. (12), the correlation between the atmospheric angle and the CP phase is weighted by the value of the $\Theta$ angle. In Fig. 4 we map the allowed ranges of the $\delta_{\text{CP}}$ phase versus the atmospheric angle for given values of the $\Theta$ parameter determining a given CP scheme. The best fit points (BFP), $1\sigma$ and $3\sigma$
FIG. 3: The regions of $|\sin \delta_{CP}|$ versus $\Theta$, where the atmospheric mixing varies within its experimentally allowed $3\sigma$ range [14].

FIG. 4: Predicted range of $|\delta_{CP}|$ phase, for given illustrative values of the $\Theta$ parameter characterizing our CP scheme, where $\Theta$ is fixed to $\pi/6$, $\pi/4$, $\pi/3$, $3\pi/8$, and $5\pi/12$, $2\pi/5$. The best fits, $1\sigma$ and $3\sigma$ ranges of the atmospheric mixing angle from [14] are indicated. For the benchmark value of $\Theta = 3\pi/8$, $2\pi/5$ and $5\pi/12$, the range of $|\sin \delta_{CP}|$ allowed by the data of $\theta_{23}$ at $3\sigma$ level is given in Table I. One sees that the experimentally observed nearly maximal $\delta_{CP}$ can be reproduced.

| $\Theta$       | $3\pi/8$ | $2\pi/5$ | $5\pi/12$ |
|----------------|----------|----------|-----------|
| $|\sin \delta_{CP}|$ | [0.92, 0.96] | [0.95, 0.99] | [0.97, 1] |

TABLE I: Predicted range of $|\sin \delta_{CP}|$ for the benchmark values $\Theta = 3\pi/8$, $2\pi/5$ and $5\pi/12$, allowed by the current $3\sigma$ range $38.8^\circ \leq \theta_{23} \leq 53.3^\circ$ given in [14].
IV. PHENOMENOLOGICAL IMPLICATIONS

We have seen that our generalized $\mu - \tau$ reflection symmetry schemes make well-defined predictions for CP violation. In the following, we shall investigate the phenomenological implications of these predictions for lepton number violating processes such as neutrinoless double beta decay ($0\nu\beta\beta$), as well as conventional neutrino oscillations.

A. Neutrinoless double beta decay

The rare decay $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$ is the lepton number violating process “par excellence”. Its observation would establish the Majorana nature of neutrinos irrespective of their underlying mass generation mechanism [30, 31]. Within the simplest light neutrino exchange mechanism its amplitude is sensitive to the “Majorana phases”. Up to nuclear matrix elements [32] and experimental factors [33, 34] the amplitude for the decay is proportional to the effective mass parameter

$$|m_{ee}| = |m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{-i2\phi_{12}} + m_3 \sin^2 \theta_{13} e^{-i2\phi_{13}}|,$$

where we used the symmetric parametrization of the lepton mixing matrix. It is clear that only the two “Majorana phases” appear but not the “Dirac phase” [21].

The crucial prediction of our CP scheme concerns CP violation, in particular, the absence of Majorana CP violation, as seen in Eq. (11). Within our scheme the Majorana phases are predicted as $\phi_{12} = k_2 - k_1 2\pi$ and $\phi_{13} = k_3 - k_1 2\pi$. In other words, these phase factors are predicted to lie at their CP conserving values, which correspond to the CP signs of neutrino states [27, 28]. This implies that the two Majorana phases ($\phi_{12}, \phi_{13}$) can only take the following nine values ($0, 0$), ($0, \pm \pi/2$), ($\pm \pi/2, 0$) and ($\pm \pi/2, \pm \pi/2$).

The effective mass $m_{ee}$ is an even function of the phases $\phi_{12}$ and $\phi_{13}$. Hence, the difference of signs between Majorana phase values is irrelevant, hence the only relevant values for Majorana phases are ($0, 0$), ($0, \pi/2$), ($\pi/2, 0$) and ($\pi/2, \pi/2$). This means that for each possible neutrino mass ordering, there are only four independent regions for the effective mass. Now, inputting the experimentally allowed $3\sigma$ ranges of neutrino oscillation parameters [14], the resulting regions of the effective mass $|m_{ee}|$ correlate with the lightest neutrino mass as shown in Fig. 5.

The first comment is that, compared with the generic case, the predictions of our scheme for the neutrino-mass-induced neutrinoless double beta decay amplitude are in some cases rather powerful. Consider, for example, the case of inverted ordering (IO), when the lightest neutrino mass is $m_3$. In this case the predicted effective mass for $\phi_{13} = 0$ and $\phi_{13} = \pi/2$ almost coincide, as shown in Fig 5. However, the predictions for $\phi_{12} = 0$ and $\phi_{12} = \pi/2$ can be probably be distinguished from each other in the next generation of experiments.

Turning to the case of normal neutrino mass ordering (NO) it is remarkable that one can place a lower bound for the effective mass despite the possibility of destructive interference amongst the three light neutrinos. Indeed no such interference can take place for ($0, 0$) and ($0, \pi/2$). This situation is analogous to what occurs in a number of flavour symmetry models [35–42].

For completeness we now summarize the above results as tables [I] and [III] for the cases of normal and inverted ordering, respectively. In these tables, the first column gives possible forms of the $Q_{\nu}$ matrix, while the second and third columns show the corresponding (CP conserving) values of the Majorana phases, and the resulting allowed
FIG. 5: Effective mass $|m_{ee}|$ describing neutrinoless double beta decay in our scenario where the Majorana phases are predicted at their CP conserving values 0 and $\pm \pi/2$. The red and blue dashed lines indicate the regions currently allowed at 3$\sigma$ by neutrino oscillation data [14] for inverted and normal neutrino mass ordering, respectively. The allowed values of $|m_{ee}|$ for different values of $\phi_{12}$ and $\phi_{13}$ are displayed. For comparison we show the most stringent upper bound $|m_{ee}| < 0.120$eV from EXO-200 [43, 44] in combination with KamLAND-ZEN [45]. The upper limit on the mass of the lightest neutrino is derived from the lastest Planck result $\sum m_i < 0.230$eV at 95% level [46].

ranges for the effective mass parameter $|m_{ee}|$.

| Normal Ordering | CP signs $Q_\nu$ | $|m_{ee}|$ ($10^{-2}$ eV) |
|-----------------|-----------------|--------------------------|
| diag(1, 1, 1)   | (0, 0)          | [0.32, 7.22]             |
| diag(1, 1, −i)  | $\left(0, \frac{\pi}{2}\right)$ | $[9.50 \times 10^{-2}, 6.89]$ |
| diag(1, −i, 1)  | $\left(0, \frac{\pi}{2}\right)$ | [0, 3.31]               |
| diag(1, −i, −i) | $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ | [0, 2.94]               |

TABLE II: The allowed ranges for the effective mass in neutrinoless double beta decay for the case of normal ordering. Notice that in our generalized $\mu - \tau$ reflection scenario the Majorana phases can only be 0 and $\pm \pi/2$.

B. CP violation in conventional neutrino oscillations

The existence of leptonic CP violation would show up as the difference of oscillation probabilities between neutrino and anti-neutrinos in the vacuum [47]:

$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = -16 J_{\alpha\beta} \sin \Delta_{21} \sin \Delta_{23} \sin \Delta_{31},$$
In the left panel we show the transition probability \( \nu_\mu \rightarrow \nu_e \) in vacuum has the form \( P(\nu_\mu \rightarrow \nu_e) \approx P_{\text{atm}} + 2\sqrt{P_{\text{atm}}P_{\text{sol}}} \cos(\Delta_{32} + \delta_{\text{CP}}) + P_{\text{sol}} \), where \( \sqrt{P_{\text{atm}}} = \sin \theta_{23} \sin 2\theta_{13} \sin \Delta_{31} \) and \( \sqrt{P_{\text{sol}}} = \cos \theta_{23} \cos \theta_{13} \sin 2\theta_{12} \sin \Delta_{21} \) [47]. Hence, the neutrino anti-neutrino asymmetry in the vacuum is

\[
A_{\mu e} = \frac{P(\nu_\mu \rightarrow \bar{\nu}_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)} = \frac{2\sqrt{P_{\text{atm}}P_{\text{sol}}} \sin \Delta_{32} \sin \delta_{\text{CP}}}{P_{\text{atm}} + 2\sqrt{P_{\text{atm}}P_{\text{sol}}} \cos \Delta_{32} \cos \delta_{\text{CP}} + P_{\text{sol}}}. \quad (14)
\]

In order to describe long baseline neutrino oscillations it is important to include the effect of matter associated to neutrino propagation in the Earth, as it can induce a fake CP violating effect. In this case the expressions for \( \sqrt{P_{\text{atm}}} \) and \( \sqrt{P_{\text{sol}}} \) in matter have the form:

\[
\sqrt{P_{\text{atm}}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)} \Delta_{31}, \quad \sqrt{P_{\text{sol}}} = \cos \theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{aL} \Delta_{21}, \quad (15)
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{CP signs } Q_\nu & (\phi_{12}, \phi_{13}) & |m_{ee}| (10^{-2} \text{ eV}) \\
\hline
\text{diag}(1,1,1) & (0,0) & [4.59, 8.20] \\
\text{diag}(1,1,-i) & (0, \frac{\pi}{2}) & \\
\text{diag}(1,-i,1) & (\frac{\pi}{2}, 0) & [1.10, 3.45] \\
\text{diag}(1,-i,-i) & (\frac{\pi}{2}, \frac{\pi}{2}) & \\
\hline
\end{array}
\]

**TABLE III:** Same as above for the case of inverted ordering.
FIG. 7: The transition probability \( P(\nu_\mu \to \nu_e) \) at a baseline of 295km which corresponds to the T2K experiment. The mixing angle \( \theta_{23} \) is taken within its currently allowed 3\( \sigma \) regions \( 0.393 \leq \sin^2 \theta_{23} \leq 0.643 \) \cite{14}. Remaining oscillation parameters as in Fig. 6.

FIG. 8: The transition probability \( P(\nu_\mu \to \nu_e) \) at a baseline of 810km which corresponds to the NO\( \nu \)A experiment. The mixing angle \( \theta_{23} \) is considering into the currently allowed 3\( \sigma \) regions \( 0.393 \leq \sin^2 \theta_{23} \leq 0.643 \) \cite{14}. Remaining oscillation parameters as in Fig. 6.

where \( a = G_F N_e/\sqrt{2}, \) \( G_F \) is the Fermi constant and \( N_e \) is the density of electrons. The approximate value of \( a \) is \((3500\,\text{km})^{-1}\) for \( \rho Y_e = 3.0\,\text{g cm}^{-3}\), where \( Y_e \) is the electron fraction \cite{47}. The relative phase \((\Delta_{32} + \delta_{CP})\) between \( \sqrt{P_{\text{atm}}} \) and \( \sqrt{P_{\text{sol}}} \) remains unchanged.

Within the framework of our generalized of \( \mu - \tau \) reflection scenario, the transition probability \( P(\nu_\mu \to \nu_e) \) in matter has the form

\[
P(\nu_\mu \to \nu_e) \approx P_{\text{atm}} + P_{\text{sol}} \pm 2\sqrt{P_{\text{atm}}} \sqrt{P_{\text{sol}}} \cos \left( \Delta_{32} \pm \arcsin \left( \frac{\sin \Theta}{\sin 2\theta_{23}} \right) \right).
\] (16)
The neutrino anti-neutrino asymmetry in matter is given by

\[ A_{\mu e} = \pm \frac{2\sqrt{P_{\text{atm}}} \sqrt{P_{\text{sol}}} \sin \Delta_{23} \sin \Theta}{(P_{\text{atm}} + P_{\text{sol}}) \sin 2\theta_{23} \pm 2\sqrt{P_{\text{atm}}} \sqrt{P_{\text{sol}}} \sqrt{\sin^2 2\theta_{23} - \sin^2 \Theta \cos \Delta_{23}}} \],

where \( \sqrt{P_{\text{atm}}} \) and \( \sqrt{P_{\text{sol}}} \) are given in Eq. (16).

In Fig. 6 we show the \( \nu_\mu \to \nu_e \) transition probability and the neutrino anti-neutrino asymmetry in matter. In this figure we take the atmospheric mixing angle within its currently allowed 3\( \sigma \) region, while for the remaining neutrino oscillation parameters are taken at their best fit values [14]. In Figs. 7, 8 we show the behavior of the transition probability \( P(\nu_\mu \to \nu_e) \) in terms of neutrino energy \( E \) and the CP parameters \( \Theta \) describing our approach, for baseline values 295 and 810 km, which correspond to the current T2K and NO\( \nu \)A experiments, respectively.

Note that so far we have discussed the predictions of our scenario for neutrino oscillations at the T2K and NO\( \nu \)A experiments, for a fixed sign combination in Eq. (16), which is \((+ , +)\). We now consider the variation of our prediction with respect to the choice of sign combination. For definiteness we now consider the future DUNE experiment. First we display in the left panel of Fig. 9 the behaviour of the \( \nu_\mu \to \nu_e \) transition probability with respect to energy for the \((+ , +)\) case and two fixed values of the model parameter \( \Theta \). In the right panel of Fig. 9 we display the model-dependence of the \( \nu_\mu \to \nu_e \) transition probability for different sign combinations.

### V. CONCLUSION

CP violation is the least studied aspect of the lepton mixing matrix. Other unknown features in the neutrino sector include the neutrino mass ordering and the octant of the atmospheric mixing parameter \( \theta_{23} \), not yet reliably determined by current global oscillation fits. In this letter we have proposed a generalized \( \mu - \tau \) reflection scenario for leptonic CP violation and derived the corresponding restrictions on lepton flavor mixing parameters. We found that the “Majorana” phases are predicted to lie at their CP-conserving values with important implications for the
neutrinoless double beta decay amplitudes, which we work out in detail. In addition to this prediction concerning the vanishing of the “Majorana-type” CP violation, we have obtained a new correlation between the atmospheric mixing angle $\theta_{23}$ and the “Dirac” CP phase $\delta_{\text{CP}}$. Only in a very specific limit our CP transformation reduces to standard $\mu - \tau$ reflection, for which $\theta_{23}$ and $\delta_{\text{CP}}$ become both maximal. We have also analysed the phenomenological implications of our scheme for present as well as upcoming neutrino oscillation experiments T2K, NO$\nu$A and DUNE. In analogy to the case of $\mu - \tau$ reflection symmetry, we expect that in our generalized $\mu - \tau$ reflection symmetry approach it may be possible to predict the value of the angle $\Theta$. This may arise from some particular residual flavor symmetries which close, say, to a finite group [48], or in the context of a flavor symmetry combined with the generalized CP symmetry [26]. Detailed study of this possibility is left for future work.

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