Equivalent Sets of Histories
and Multiple Quasiclassical Realms

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Abstract

We consider notions of physical equivalence of sets of histories in the quantum mechanics of a closed system such as the universe. We show first how the same set of histories can be relabeled in various ways, including the use of the Heisenberg equations of motion and of alias (passive) transformations of field variables. In the contrasting case of the usual approximate quantum mechanics of a measured subsystem of the universe, two observables represented by different Hermitian operators (as opposed to the same operator relabeled) are physically distinguished by the different pieces of apparatus used to measure them. That is true even if they are related by a unitary transformation and the state of the system is such that the probabilities of ranges of values of the observables are the same. In the quantum mechanics of a closed system, however, any apparatus is part of the system and the notion of physically distinct situations has a different character. Making our previous suggestions more precise, we show that a triple consisting of an initial condition, a Hamiltonian, and a set of histories is physically equivalent to another triple if the operators representing these initial conditions, Hamiltonians, and histories are related by any fixed unitary transformation. We apply this result to the question of whether the universe might exhibit physically inequivalent quasiclassical realms (which we earlier called quasiclassical domains), not just the one that

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includes familiar experience. We describe, in more detail than we have before, how the probabilities of alternative forms, behaviors, and evolutionary histories of information gathering and utilizing systems (IGUSes) using the usual quasiclassical realm could in principle be calculated in quantum cosmology, although it is, of course, impractical to perform the computations. We discuss how, in principle, the probabilities of occurrence of IGUSes could be calculated in realms distinct from the usual quasiclassical one — realms such as Lloyd’s representation of the universe as a quantum computer. We discuss how IGUSes adapted mainly to two different realms could draw inferences about each other using a hybrid realm consisting of alternatives drawn from each.
I. INTRODUCTION

Quantum mechanics, in its most general form, predicts the probabilities of alternative decohering coarse-grained histories of the universe. The computation of these probabilities requires an initial condition, given by a density matrix \( \rho \), and a narrative description of the histories expressed in terms of suitable operators. Suppose, for simplicity that the density matrix is pure. What gives rise to probabilities in quantum mechanics is the mismatch between the state vector of the universe and the state vectors, orthogonal to each other, associated with the individual decoherent histories in a set of alternative histories. When the state vector of the universe is resolved into a sum of vectors corresponding to the histories in the set, the norms of those vectors are the probabilities. The specification of a set of alternative decoherent histories is just as important for the utilization of quantum mechanics as the characterization of the Heisenberg state vector of the universe (or the equivalent Schrödinger state vector and its time development). It is not just the state of the universe that gives quantum mechanics its meaning. It also matters which set of questions is asked of that state.

Among all the possible sets of alternative histories for which probabilities are predicted by the quantum mechanics of the universe, those describing a quasiclassical realm\(^1\), like the one that includes familiar experience, are of special importance. By a quasiclassical realm we mean roughly a set of histories (or a class of nearly equivalent sets) maximally refined, consistent with obeying a realistic principle of decoherence and with exhibiting patterns of approximately deterministic correlations governed by phenomenological classical laws connecting similar operators at different times \([1]\). (Those patterns are interrupted, of course, by frequent small fluctuations and occasional major branchings of histories.) Such quasiclassical realms are important for at least two reasons: (1) The existence of at least one quasiclassical realm appears to be a reasonable extrapolation from empirical fact and should therefore be a prediction in quantum cosmology from the fundamental theory of the elementary particles and the initial condition of the expanding universe. That usual quasiclassical realm is defined by alternative ranges of values of certain operators (called usual quasiclassical operators), which are particular kinds of local operators (such as electromagnetic fields or densities of conserved or nearly conserved quantities) averaged over small regions of space throughout the universe, at a sequence of times that span the whole history of the universe. (2) Coarse grainings of this usual quasiclassical realm are what we (humans and many other systems) use in the process of gathering information about the universe and making predictions about its future. We deal directly with values of usual quasiclassical operators, to some of which our senses are adapted. Other quantum-mechanical operators are accessible when correlated with these quasiclassical ones, i.e., in measurement situations.

There are very many of sets of histories that decohere, and trivial examples of exactly decohering sets can easily be exhibited that are nothing like a quasiclassical realm, let alone the quasiclassical realm that includes everyday experience \([1,2]\). To understand what quasiclassical realms are, we need to study more carefully the structure of the set of all possible histories and how the coherence between histories is altered by the process of decoherence.

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\(^1\)In our previous work we have referred to “quasiclassical domains”. We now suggest using instead the expression “quasiclassical realms” to avoid confusion with the usual meaning of “domains” in physics. We use the word “realm” as a synonym for “decohering set of alternative histories”. 

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sical realms are possible in quantum mechanics, it is desirable to make more mathematically precise the measure of classicality on the collection of all sets of decohering histories. The refinement of the definition \[1,3,4\] can help to answer the difficult and fundamental question of whether the quantum mechanics of the universe exhibits only an essentially unique quasiclassical realm or whether there are essentially different ones. It can help to give a general characterization of quasiclassical operators, the values of which specify the alternative histories of a quasiclassical realm and to, in principle, derive the form of the phenomenological deterministic laws that approximately govern a given quasiclassical realm as has already been done in some model problems \[3\]. (That form tends to be far removed from the form of the equations describing the underlying fundamental dynamics, e.g., heterotic superstring theory.) In connection with the phenomenological laws, it is important analyze the coarse graining necessary to achieve approximate classical predictability in the presence of the noise that typical mechanisms of decoherence produce. Finally, the coarse graining(s) used to define entropy in the second law of thermodynamics should be connected the coarse graining used to define the histories of a quasiclassical realm \[3\].

We shall not review here the efforts to achieve all of these objectives nor shall we investigate the details of the measure. Rather, we discuss, as a prerequisite, the nature of physical equivalence between sets of coarse-grained histories of a closed system, so as to understand better what it would mean for the universe to exhibit essentially inequivalent quasiclassical realms. We then examine some implications for information gathering and utilizing systems (IGUSes) of such inequivalent realms (and of certain other realms as well).

We mentioned in \[1\] that the notion of physically distinct sets of alternative histories has a different character for a closed quantum-mechanical system and for the approximate quantum mechanics of measured subsystems. We make this difference precise in Section II of this article. We first show how the description of a given set of histories, constructed from alternatives at a sequence of times, may be varied in several different ways without affecting the histories themselves. First, making use of the Heisenberg equations of motion to change the description of the alternatives in terms of fundamental fields, we can reassign the times of the alternatives (as long as their time-order is maintained.) Second, the alternatives may be relabeled by making alias (passive) transformations of the fundamental fields and conjugate momenta.

Under a relabeling of the above types, operators representing the histories remain unchanged. However, in the quantum mechanics of a closed system described in terms of quantum fields, even sets of histories represented by different operators may be physically equivalent. That is not true in the usual approximate quantum mechanics of measured subsystems, where two observables represented by different Hermitian operators are physically distinguished by the different kinds of apparatus (outside the subsystem) used to measure them. In the quantum mechanics of a closed system, however, any apparatus is part of the system and triples of Hamiltonians, initial conditions, and sets of histories represented by different operators may be physically indistinguishable. Suppose that a fixed unitary transformation acts on the Hamiltonian, the density matrix representing an initial condition, and on the projection operators representing alternatives at moments of time in a set of histories, but not on the fundamental fields in terms of which all these operators are described. We show in Section II that the resulting new initial conditions and sets of histories are physically equivalent to the old ones in the sense that they admit an identical description in
terms of fundamental fields, with the same probabilities for corresponding histories. Initial conditions and sets of histories related in this way should be identified with each other and placed together in physical equivalence classes.

This relationship of physical equivalence is important for the problem of quasiclassical realms. Measures of classicality should be on physical equivalence classes. Given an initial density matrix $\rho$ and a set of histories constituting a quasiclassical realm, we do not exhibit another distinct quasiclassical realm by transforming the projection operators of the first realm using a constant unitary transformation that leaves $\rho$ invariant. The two sets are physically indistinguishable.

As observers of the universe, we (and all other IGUSes that we know of) make use of a particular quasiclassical realm (further coarse grained according to the limitations of our senses and instruments.) The explanation for this is not to be sought in some privilege conferred on quasiclassical realms by the quantum mechanics of closed systems, for quasiclassical realms are but a small subset of the collection of all sets of decoherent histories, and moreover IGUSes, including human beings, occupy no such special place and play no such preferred role in this formulation of quantum mechanics as they do in the “Copenhagen” interpretation(s). Rather, as we suggested in [1], it is plausible, in the context of quantum cosmology, that IGUSes evolve by exploiting realms with a high level of predictability such as quasiclassical realms, focusing on variables that present enough regularity over time to permit the generation of models (schemata) with significant predictive power.

It is, of course, an impractical task to compute the probabilities of alternative evolutionary tracks for IGUSes from the fundamental quantum-mechanical theory of the elementary particles and of the initial condition of the universe. Nevertheless, it is clarifying to investigate how such questions might be posed in principle in quantum cosmology even if we can only guess the answers. We offer some thoughts on this topic in Section III.

If only one collection of essentially equivalent sets of decoherent histories with high classicality emerges from the initial condition of the universe and the dynamics of the elementary particles, then the usual quasiclassical realm is essentially unique. However, if the quantum mechanics of the universe exhibits essentially inequivalent quasiclassical realms then it is possible that IGUSes evolve on branches of more than one of them. Moreover, there may be other realms, even more deterministic, in which IGUSes arise, for example, a realm in which the universe behaves as a quantum computer. We discuss these matters in Sections III, IV, and V.

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2 Similar notions of physical equivalence will be discussed in [2].

3 The question of the relationship between sets of histories related by unitary transformations that leave the initial density matrix invariant was first raised for one of the authors (JBH) in discussions with R. Penrose in 1989 where the question of whether they should be identified also arose.

4 Cf. the remarks in [3].
II. PHYSICALLY EQUIVALENT SETS OF HISTORIES

In this Section we shall describe a notion of physical equivalence between sets of alternative histories of a closed quantum-mechanical system, in the general formulation of quantum mechanics appropriate to such systems. Many readers will be more familiar with the “Copenhagen” formulation of quantum mechanics, usually explained in textbooks, which is concerned with predicting the outcomes of measurements on a subsystem. These two formulations are not in conflict with each other. The usual “Copenhagen” formulation is an approximation to the more general quantum mechanics of a closed system and is applicable to sets of histories describing measurement situations when certain approximate features of these histories can be idealized as exact [8]. We begin by describing physical equivalence in the quantum mechanics of a closed system and return in Section I to the more restrictive notion valid in the approximate quantum mechanics of measured subsystems.

A. The Quantum Mechanics of a Closed System

To establish some notation and clarify our assumptions, we give a brief review of the quantum mechanics of a closed system. We consider such a system, most generally and accurately the universe as a whole, including both observers and observed, both measuring apparatus and measured subsystems, if any. We work in the approximation in which the geometry of spacetime is approximately fixed and gross fluctuations in it are neglected. We also assume that spacetime is foliable by spacelike surfaces. Times are then well-defined and the usual formalism of Hilbert space, states, Hamiltonian, etc. can be used to describe quantum theory. We assume a fundamental quantum field theory. We shall usually indicate just a single scalar field \( \phi(x) \), hoping that the reader may make the straightforward generalization to the usual panoply of Fermi, tensor, and other fields (or to superstring theory, in which there is something like an infinite set of such fields). The dynamical evolution of the field through a family of spacelike surfaces is generated by a Hamiltonian which, on a spacelike surface labeled by \( t \), is a functional of the field on that surface \( \phi(x, t) \) and its conjugate momentum \( \pi(x, t) \). The canonically conjugate pair satisfy the fundamental commutation relations

\[
[\phi(x, t), \pi(x', t)] = i\delta(x, x'),
\]

where \( \delta(x, x') \) is the \( \delta \)-function on the spacelike surface. We use units for which \( \hbar = 1 \).

Various quantities represented by Hermitian operators \( O[\phi(x, t), \pi(x, t)] \) can be constructed from the fields and momenta on a spacelike surface. Projections onto an exhaustive set of alternative ranges of these quantities define alternatives at the moment of time \( t \).

We follow our earlier work, for example, [1], [8], [3].

It is a special virtue of our approach that these restrictions are not actually necessary. For a generalized quantum theory that can incorporate dynamical spacetime geometry see [11] and earlier references therein.
Giving a sequence of such sets of alternatives at times $t_1, \ldots, t_n$ defines a set of alternative histories for the closed system, although not of the most general kind, as we shall see. We denote the sets of projections by $\{P^k_{\alpha_k}(t_k)\}$, $k = 1, \ldots, n$ where the superscript $k$ distinguishes the quantity $O$ and the set of ranges employed at time $t_k$, and $\alpha_k$ the particular range represented by the projection. The operators $\{P^k_{\alpha_k}(t_k)\}$ satisfy

$$P^k_{\alpha_k}(t_k)P^k_{\alpha'_k}(t_k) = \delta_{\alpha_k\alpha'_k} P^k_{\alpha_k}(t_k), \quad \Sigma_{\alpha_k} P^k_{\alpha_k}(t_k) = I,$$

showing that they are projections representing an exhaustive set of mutually exclusive alternatives.

The Heisenberg equations of motion

$$\phi(x, t') = e^{iH(t'-t)} \phi(x, t) e^{-iH(t'-t)}, \quad (2.3a)$$

$$\pi(x, t') = e^{iH(t'-t)} \pi(x, t) e^{-iH(t'-t)}, \quad (2.3b)$$

allow an operator $O[\phi(x, t), \pi(x, t)]$ to be reexpressed in terms of the field and momentum at another time. Thus in the Heisenberg picture each exhaustive set of orthogonal projection operators may be regarded, for any time, as a set of projections on ranges of some quantity at that time. Given a set of projections satisfying (2.2), an arbitrary time can be assigned, and the projections expressed in a suitable manner in terms of the field and momentum operators at that time.

Each sequence of alternatives $(\alpha_1, \ldots, \alpha_n)$ at definite moments of time defines one member of a set of possible alternative histories of the closed system. Such histories are represented by the corresponding time-ordered chains of projection operators. A completely fine-grained set of histories would be defined by sets of one-dimensional projections (projections onto a basis for Hilbert space) at each and every time. There are infinitely many different sets of fine-grained histories corresponding to the different choices of basis at each time.

The most general notion of a set of alternative histories is a partition of one of these sets of fine-grained histories into exclusive classes $\{c_\alpha\}$. The individual classes are the individual coarse-grained histories and are represented by class operators, $C_\alpha$, which are sums of chains of the corresponding projections in the class:

$$C_\alpha = \sum_{(\alpha_1, \ldots, \alpha_n)\in\alpha} P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1) \quad (2.4)$$

Alternatives lacking a well defined time, such as averages of fields over ranges of time, are not represented in terms of projections onto ranges of the corresponding operators. To do so would generally leave the time ordering of such projections ambiguous. Rather, such spacetime alternatives are represented by sums of possibly continuous chains of projections. For fuller details see [12], [11].

The problem of the physical equivalence of sets of histories would be considerably simplified if there were a unique allowed set of fine-grained histories. A particular distinguished set, paths in the configuration space of quantum fields, is the starting point for a standard sum-over-histories formulation of quantum mechanics. However, for generality, we allow here all the other fine-grained histories that can be constructed by transformation theory.
where we allow the possibility of an infinite sequence of times. Although we have not indicated it explicitly, such sets of histories are generally branch-dependent — the set of operators \( \{ P_{\alpha_k}^k(t_k) \} \) may depend on the previous specific alternatives \( \alpha_1, \ldots, \alpha_{k-1} \) and times \( t_1, \ldots, t_{k-1} \) and should really be written \( \{ P_{\alpha_k}^k(t_k; \alpha_{k-1}, t_{k-1}, \alpha_{k-2}, t_{k-2}, \ldots, \alpha_1, t_1) \} \). (We have assumed causality, and so only previous alternatives matter.)

Probabilities are predicted for members of a set of alternative histories of a closed system when there is negligible quantum-mechanical interference between them. Interference between a pair of histories is measured by the decoherence functional

\[
D (\alpha', \alpha) = \text{Tr} \left( C_{\alpha'} \rho C_{\alpha}^\dagger \right),
\]

where \( \rho \) is a density matrix representing the initial condition of the closed system.\(^9\) When the “off-diagonal” elements of \( D \) are sufficiently small the set of histories is said to (medium) decohere\(^1\) the diagonal elements are then the probabilities \( p(\alpha) \) of the individual histories in the set; viz.

\[
D (\alpha', \alpha) \approx \delta_{\alpha' \alpha} p(\alpha).
\]

In particular, when the initial \( \rho \) is pure, \( \rho = |\Psi\rangle \langle \Psi| \), and the histories of the set are all (medium) decoherent, one has

\[
p(\alpha) = \| C_{\alpha} |\Psi\rangle \|^2.
\]

B. The Description of Histories and the Trivial Relabeling of Hilbert Space

As the preceding discussion shows, the objects of interest in quantum theory are triples \((\{ C_\alpha \}, H, \rho)\) consisting of operators \( \{ C_\alpha \} \) of the form \((2.4)\) representing a set of alternative coarse-grained histories of the closed system, a Hamiltonian \( H \) connecting field operators at different times through Heisenberg equations of motion, and a density matrix \( \rho \) representing the initial condition. Given a Hilbert space \( \mathcal{H} \), it is possible in principle to enumerate mathematically, without reference to the fundamental fields, all the operator triples that represent decohering sets of histories, Hamiltonians, and initial conditions. However, as we stressed in \((1)\), “it is clear that the mathematical problem of enumerating the sets of

\(^9\)We thereby restrict ourselves to the usual quantum cosmology in which a distinction is made between the past, with an initial condition represented by \( \rho \), and the future, with a condition of effective indifference with respect to final state. Similar notions of physical equivalence can be introduced in the time-neutral generalizations of the usual quantum mechanics of closed systems (with initial and final conditions) that have been discussed \((e.g., \) in \((13), (14)\)) largely as “straw man” theories.

\(^1\)We use medium decoherence for illustrative purposes. Our considerations would also apply to weak decoherence and to the still weaker consistency conditions of Griffiths \((13)\) and Omnès \((13)\). However, in realistic cases medium decoherence obtains, or an even stronger condition.
decohering histories in a given Hilbert space has no physical content by itself. No description of the histories has been given. ... No distinction has been made between one vector in Hilbert space as a theory of the initial condition and any other. The resulting probabilities, which can be calculated, are merely abstract numbers.”

As we further discussed in [1], the sets of possible triples acquire physical content when the operators corresponding to the fundamental fields $\phi(x, t)$ are specified in $\mathcal{H}$ so that, for example, the eigenvectors of the smeared field operators are fixed. Any set of orthogonal projections $\{P_{\alpha}(t)\}$ at time $t$ can be described as projections onto ranges of some operator $\mathcal{O}[\phi(x, t), \pi(x, t)]$ at that time. It is then possible to give a narrative describing each member of a set of alternative histories $\{C_{\alpha}\}$. Contact with the fundamental interactions is made when the Hamiltonian $H$ is expressed in terms of the field operators. Initial conditions are distinguished when $\rho$ is described in terms of fields. The probabilities thereby acquire physical meaning as the probabilities of alternative histories of the universe, with a particular Hamiltonian and initial condition.

There is some arbitrariness in the choice of subspaces of the mathematical Hilbert space identified as having definite values of the smeared fields. This identification can be changed by transforming all operators in the theory — the $C_{\alpha}$’s, $H$, $\rho$, and the fields $\phi(x)$ — by a fixed unitary transformation. The result is merely a relabeling of Hilbert space with no physical consequences. Sets of triples and fields so related are clearly physically equivalent. Only an alias (passive) transformation has been carried out.

While the quantum mechanics of closed systems clearly exhibits this trivial notion of physical equivalence, it also exhibits further, less trivial kinds of physical equivalence arising from the Heisenberg equations of motion and from the invariance of the theory under field redefinitions. In the remainder of this Section we explore those kinds of equivalence.

C. The Same Operators Described in Terms of Fields at Different Times

We mentioned earlier that in the Heisenberg picture a given projection operator could be assigned to an arbitrary time by using the equations of motion to determine its form in terms of the field operators. For example, in the case of a free particle with mass $m$ moving in one dimension, the Heisenberg equations of motion have the solutions

$$x(t) = x(0) + p(0)t/m , \quad (2.8a)$$
$$p(t) = p(0) . \quad (2.8b)$$

Thus, a projection onto a range $\Delta$ of the position operator $x(6)$ for time $t = 6$ might equally well be described as the projection onto the range $\Delta$ of the operator $x(0) + p(0)6/m$ referring to $t = 0$. These projection operators are equal as a consequence of the equations of motion.

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The analogous situation in classical physics may be clarifying. In the classical “Heisenberg picture,” in which the alternatives are regions of phase space that vary in time according to the equations of motion, the region of phase space in which the initial $(x(0), p(0))$ are such that $x(0) + p(0)t/m$ lies in a range $\Delta$ is the same as the region in which $x(t)$ lies in $\Delta$, as a consequence of the time evolution.
For a closed system the equality of such operators assigned to different times, arising from the Heisenberg equations of motion, is absolute, since there is no way to alter these Heisenberg equations by means of an external perturbation. Thus a set of projection operators described in terms of fields at two different times does not correspond to two different sets of alternatives for the closed system, but rather to the same set of alternatives described in two different ways. Put differently, in a closed system there are no external clocks to give an independent meaning to a moment of time. Of course, there may be good reason for us to prefer one description to another. In particular, the lengths of the two descriptions may be different, but there is no physical distinction between them.

Thus, in a set of alternative histories consisting of sequences of sets of alternatives at definite moments of time \([\text{cf.} (2.4)]\), the times of the alternatives may be assigned arbitrarily although it is convenient – and necessary to avoid ambiguity – to keep their order the same as the order of the sets so that the histories are narratives proceeding forward in time. The order of the projection operators is important, for different sets of projection operators do not necessarily commute. Since a change in the the values of the times merely corresponds to a different description of the histories, the decoherence and probabilities of the set of histories must be unaffected by such a change in description. In fact, the operators themselves are unchanged and hence the decoherence functional remains the same.

D. Fields as Coördinates

Fields and their conjugate momenta are canonical coördinates on the phase space of classical field theory, and classical field redefinitions mediate between different choices of these canonical coördinates. Classical mechanics may be formulated in a generally covariant manner, allowing arbitrary choices of canonical coördinates. In a similar way, the quantum mechanics we have been using allows arbitrary choices of the canonical pairs of field operators and their conjugate momenta satisfying \((2.1)\).

In classical mechanics it is possible to fix a coördinate system on phase space by requiring a sufficient number of physical quantities to have specified functional forms in terms of the coördinates. For example, in the classical mechanics of a system of particles, one could require that the Cartesian coördinates of the displacement of each particle from a fixed origin be equal to a coördinate set \(\{q^i\}\). Similarly, in quantum theory one could presumably eliminate the freedom to make field redefinitions by requiring certain physical quantities (e.g. the Hamiltonian, momentum, ... etc.) to have definite functional forms in terms of the field variables. The subspaces of Hilbert space are then labeled by particular physical quantities. As a result no issue of equivalent descriptions through field redefinitions arises — a particular kind of description has been singled out by convention. Even in that case, \footnote{In general relativity, which is invariant under reparametrizations of the time coördinate, the notion of alternatives at a moment of time requires careful examination, and in quantum gravity, where there is no fixed background spacetime geometry, we cannot expect to have the same notion of time as is described in this subsection, where spacetime geometry has been assumed fixed. For an approach to a generalized quantum mechanics of spacetime geometry see \([11]\).}


of course, the freedom remains to make transformations corresponding to exact symmetries of the Hamiltonian.

Labeling the Hilbert space by physical quantities is a convenient approach to the quantum mechanics of simple, particular systems where small numbers of physical quantities are readily identified, as in discussions in typical textbooks. Some authors prefer to assume that in any discussion Hilbert space has been implicitly labeled by physical quantities. That is possible, but inconvenient for discussing quantum mechanics in general rather than merely applying it to some specific system. For that reason we prefer to discuss the general form of the theory, where field redefinitions are not excluded by convention and different descriptions of the same physical situation are possible in terms of different fields. We leave the vectors of Hilbert space unlabeled until labels are explicitly assigned. Our subsequent discussion of physical equivalence should be understood in this context.

E. Different Descriptions of the Same Histories Through Field Redefinitions

A consequence of the general treatment of fields outlined above is that histories can be described in different ways through field redefinitions. We discussed above how projection operators representing alternatives at a moment of time could be described as projections onto ranges of values of operator functions of fundamental fields and their conjugate momenta. The Hamiltonian $H$ and the density matrix $\rho$ representing the initial condition may be similarly described. However, given one conjugate pair \((\phi(x), \pi(x))\) satisfying (2.1), it is possible to find other canonical pairs through field redefinitions

\[
\tilde{\phi}(x) = \tilde{\phi}(x; \phi(y), \pi(y)), \quad \tilde{\pi}(x) = \tilde{\pi}(x; \phi(y), \pi(y))
\]

such that $\tilde{\phi}(x)$ and $\tilde{\pi}(x)$ also satisfy (2.1). (The notation means that $\tilde{\phi}$ and $\tilde{\pi}$ are functions of $x$ but functionals of $\phi(y), \pi(y)$, thus allowing for non-local field redefinitions.)

Unitary transformations of the fields at one moment of time are an example of a field redefinition. Under such a transformation

\[
\tilde{\phi}(x) = U\phi(x)U^{-1}, \quad \tilde{\pi}(x) = U\pi(x)U^{-1},
\]

the fields so redefined satisfy the canonical commutation relations.\footnote{Indeed, were it not for the possibility of inequivalent representations of the commutation relations, (2.10) would be the most general field redefinition preserving those commutation relations.}

The operators in a triple \((\{C_\alpha\}, H, \rho)\) may be expressed as functions either of \((\phi(x), \pi(x))\) or \((\tilde{\phi}(x), \tilde{\pi}(x))\). In the absence of any external apparatus to give some objective meaning to the field operators, the description of a triple in terms of one set of fields and momenta is just as valid as the description in terms of another set, unless we use criteria such as algorithmic information content. Such criteria may lead us to prefer one description to another but are not intrinsic to quantum mechanics. The two different descriptions of the same triple...
— in terms of two different sets of fields and conjugate momenta — are thus physically equivalent. Triples of alternative histories, Hamiltonian, and initial condition may therefore be described in many different, physically equivalent ways.

Some discussion of the classical analog of this situation may be helpful. Classically, a fine-grained history is a curve in phase space. The curve may be described by introducing canonical coordinates \( (q^i, p_j) \) on phase space and giving the functions \( (q^i(t), p_j(t)) \). However, any other set of canonical coordinates \( (\tilde{q}^i, \tilde{p}_j) \), functions of \( (q^i, p_j) \) such that the Poisson bracket \( \{ \tilde{q}^i, \tilde{p}_j \} \) equals \( \delta^i_j \), would give equally good and physically equivalent ways of describing the history. Coarse-grained alternatives can be constructed using an exhaustive set of mutually exclusive regions of phase space analogous to projections. Again these can be described in many different ways.

**F. Equivalent Sets of Histories and Initial Conditions Represented by Different Operators**

Having in hand the above discussion of how the same operators may be described in various ways in terms of different quantum fields (or in terms of various functions of fields at different times, using the equations of motion), we now proceed to describe a notion of physical equivalence between sets of histories and initial conditions represented by distinct operators. Two triples \( (\{C_\alpha\}, H, \rho) \) and \( (\{\tilde{C}_\alpha\}, \tilde{H}, \tilde{\rho}) \) are physically equivalent if there are fields and conjugate momenta \( (\phi(x), \pi(x)) \) and \( (\tilde{\phi}(x), \tilde{\pi}(x)) \), respectively, in which the histories, Hamiltonian, and initial condition take the same form, for each triple.

Quantities invariant under field redefinitions are useful in identifying physically equivalent triples \( (\{C_\alpha\}, H, \rho) \). One such quantity is the decoherence functional, which is the same for two physically equivalent triples \( (\{C_\alpha\}, H, \rho) \) and \( (\{\tilde{C}_\alpha\}, \tilde{H}, \tilde{\rho}) \). They both decohere or not to the same accuracy, and, if they decohere, they have the same probabilities.

Consider an initial density matrix \( \rho \) and a set of histories \( \{C_\alpha\} \) made up of sums of chains of projections \( \{P^k_{\alpha_k}(t_k)\} \) at times \( t_1, \ldots, t_n \). Let

\[
\tilde{\rho} = U \rho U^{-1}, \quad (2.11a)
\]

\[
\tilde{H} = U H U^{-1}, \quad (2.11b)
\]

and, for each time \( t_k \),

\[
\tilde{P}^k_{\alpha_k}(t_k) = U P^k_{\alpha_k}(t_k) U^{-1}, \quad (2.11c)
\]

for some unitary fixed transformation \( U \), the same for all times \( t_k \). The transformed values of the class operators \( \{\tilde{C}_\alpha\} \) are defined by \( (2.4) \) with the \( P \)'s replaced by the corresponding \( \tilde{P} \)'s. The operators in the transformed triple \( (\{\tilde{C}_\alpha\}, \tilde{H}, \tilde{\rho}) \) may be regarded either as functions of the fields and momenta \( (\phi(x), \pi(x)) \) or as functions of any other set of fields and momenta \( (\tilde{\phi}(x), \tilde{\pi}(x)) \) satisfying the canonical commutation relations. The same is true for the untransformed \( (\{C_\alpha\}, H, \rho) \). The important point is that generally the operators in the triple \( (\{\tilde{C}_\alpha\}, \tilde{H}, \tilde{\rho}) \) will be different functions of a given set of fields and momenta from those in \( (\{C_\alpha\}, H, \rho) \). But the former have the same form in terms of fields \( (\phi(x), \pi(x)) \) as the
latter do in terms of the fields \((\tilde{\phi}(x), \tilde{\pi}(x))\), where \((\tilde{\phi}(x), \tilde{\pi}(x))\) are defined by (2.10) and obey the same canonical commutation relations as the \((\phi(x), \pi(x))\). Moreover, the decoherence functional is the same for the old triple and the new. Thus, the two triples are physically equivalent (in the sense defined above) and we propose that they be identified with each other.

The analogous classical situation may be helpful in understanding the notions of physical equivalence and identification that we have introduced. As we mentioned earlier, the classical analog of a fine-grained history is a curve in phase space and the analog of a projection is a region of phase space. The analog of an initial density matrix is an initial phase space distribution. A canonical transformation may be used to transform these into new curves, new regions, and new initial distributions. However, for a closed system, in the absence of an imposed labeling by physical quantities of the points in phase space, the new triple of histories, Hamiltonian, and initial condition is physically indistinguishable from the old triple because it has the same description in terms of the canonically transformed coordinates and momenta that the old one did in terms of the original coordinates and momenta. Again we can identify the two triples. Instead of distinguishing triples we then distinguish equivalence classes of triples. Of course, those theorists who prefer to impose — even for a closed system — a labeling by physical quantities of the rays in Hilbert space or the points in phase space (as discussed in Section D) have, by that convention, selected one member of each equivalence class.

G. Unitary Transformations That Leave the Initial Density Matrix Fixed

Of all the unitary transformations (2.11) yielding physically equivalent sets of histories and initial conditions, the ones that leave the initial density matrix fixed

\[ \rho = U \rho U^{-1} \]  

(2.12)

are of special importance for the problem of quasiclassical realms. If the initial density matrix is pure or close to pure, there will be a great variety of such \(U\)’s because out of the Hilbert space of the universe only one vector or a small subset of vectors needs to be left fixed.

Suppose we have an initial \(\rho\) and seek to compute the sets of histories representing any quasiclassical realms that emerge from \(\rho\) and \(H\). We might be tempted to think that if we found one sequence of sets of \(P\)’s representing a quasiclassical realm we could find many others simply by acting on the \(P\)’s with a \(U\) that preserves \(\rho\). However, all those sets are physically equivalent and identified with one another. They represent the same quasiclassical realm. The universe may also exhibit essentially inequivalent quasiclassical realms, but they are not to be found in this manner — simply by redefining fields. Any measure of classicality should be defined on equivalence classes of physically equivalent histories.

H. The Information Content of a Physical Equivalence Class

Having pointed out some transformations that leave unchanged a set of decohering histories such as a quasiclassical realm, we should now discuss where the information actually
lies that characterizes such a decohering set. Consider, for simplicity, the case of a pure 
\( \rho = |\Psi\rangle \langle \Psi| \). Then, what it means for a set of histories \( \{C_\alpha\} \) to decohere exactly is that 
all non-vanishing vectors \( C_\alpha |\Psi\rangle \) are orthogonal to one another, with their norms giving the 
probabilities of the alternative histories labeled by the index \( \alpha \). Since the histories are ex-
haustive, we have \( |\Psi\rangle = \sum_\alpha C_\alpha |\Psi\rangle \). The state vector \( |\Psi\rangle \) is thus resolved into components, 
corresponding to branches, in a basis consisting of the non-vanishing vectors \( C_\alpha |\Psi\rangle \) (nor-
malized to unity) and any other set of unit vectors, orthogonal to one another and to all the 
non-vanishing \( C_\alpha |\Psi\rangle \), that make the basis complete.

Since, however, sets of branches \( C_\alpha |\Psi\rangle \) that are related by any fixed unitary transfor-
mation \( U \) are physically equivalent, the only physical information contained in the relations 
among the vectors is that a normalized state \( |\Psi\rangle \) is resolved into a set of components (labeled 
by those \( \alpha \)'s such that \( C_\alpha |\Psi\rangle \neq 0 \)) with particular norms, which are the probabilities, and 
that zero probability is assigned to the basis vectors that are orthogonal to all the remaining 
non-vanishing \( |C_\alpha\rangle \). Besides the list of norms (probabilities) and zeros, there is no invariant 
information in the relation among the vectors. What does carry information, other than just 
probabilities, is the explicit narrative content of the \( \{C_\alpha\} \) — sums of chains of projections 
expressed in terms of field operators — compared with the form of the initial condition \( |\Psi\rangle \)
expressed in terms of the same field operators.

In that connection, it is interesting to remark that at one time \( \text{[16]} \) we introduced an 
entirely different set of equivalence classes from the ones discussed in this article. In the 
earlier work, two sets of histories \( \{C_\alpha\} \) and \( \{C'_\alpha\} \) are treated as equivalent if \( C_\alpha |\Psi\rangle = C'_\alpha |\Psi\rangle \)
for every \( \alpha \). Incorporating the results obtained here, we see that the list of probabilities of 
histories is the only invariant property of an equivalence class of the type we defined then. 
All the other properties of histories relate to variation within one of those equivalence classes, 
that is, variation of the different operators \( \{C_\alpha\} \), with their different narratives, leading to 
the same resolution of the state vector \( |\Psi\rangle \) of the universe into orthogonal branches \( \{C_\alpha |\Psi\rangle\} \).
That resolution, together with the content of the \( |\Psi\rangle \) and of the \( \{C_\alpha\} \) — both expressed in 
terms of a given language of field operators — is the basis of the interpretation of quantum 
mechanics, at least if \( \rho \) is pure.

**I. The Approximate Quantum Mechanics of Measured Subsystems**

Measurement situations are most accurately described in the quantum mechanics of a 
closed system that contains both measuring apparatus and measured subsystem. How-
ever, those situations can also be treated to an excellent approximation by the approximate 
quantum mechanics of measured subsystems (AQMMS) (aka the “Copenhagen” formul-
a­tion), which is so familiar from textbooks. This Section discusses the connection between 
the notions of physical equivalence that hold in these two formulations.

In the approximate quantum mechanics of measured subsystems, the joint probability of 
a sequence of “ideal” measurements carried out on a subsystem with a pure (for simplicity)
initial state \( |\psi\rangle \) is 
\[
p(\alpha_n, \ldots, \alpha_1) = \| s_{\alpha_n}^n(t_n) \cdots s_{\alpha_1}^1(t_1) |\psi\rangle \|^2.
\]
(2.13)
Here, for a given value of \( k \), the set \( \{s_{\alpha_k}^k(t_k)\} \) consists of projection operators (in the 
Heisenberg picture) representing the possible outcomes, enumerated by the index \( \alpha_k \), of
the measurement carried out at time $t_k$. Thus, if the subsystem consisted of a single particle and the measurement at time $t_k$ localized the particle to one of a set of position intervals $\Delta_{\alpha_k}^k, \alpha_k = 1, 2, \ldots$, then the operators $\{s_{\alpha_k}^k(t_k)\}$ would be projections onto those intervals at time $t_k$. State vectors, projections, etc. in (2.13) all refer to the Hilbert space $\mathcal{H}_s$ of the measured subsystem.

The physical consequences of AQMMS are left unchanged by a trivial relabeling of the Hilbert space $\mathcal{H}_s$ of the kind described in Section B for closed systems. Such a relabeling is implemented by an alias (passive) unitary transformation of all operators and vectors. However, the notions of physical equivalence of the kinds discussed in Sections C and F have a different character for AQMMS.

The probabilities (2.13) are unchanged by a reassignment of the times to the sequences of measurements as long as the operators representing those measurements are unchanged. However, in AQMMS one presumes that there are clocks external to the subsystem that give a physical meaning to time, so that sets of projection operators on the Hilbert space of the subsystem that are assigned to different times correspond to physically distinct alternatives. Specifically, they correspond to measurements on the subsystem carried out at different times as determined by the external clock. Reassignment of the times, therefore, does not lead to a physically equivalent set of histories in AQMMS — in contrast to the quantum mechanics of closed systems, where there are no external clocks. (Of course, even for a closed system one could arbitrarily specify time labels and thus remove the freedom to reassign the times.)

Next, consider a unitary transformation $u$ of the kind discussed in Section F, acting only on the Hilbert space $\mathcal{H}_s$ of the measured subsystem. The values of the probabilities (2.13) are left unchanged by the substitutions

$$|\psi\rangle \rightarrow |\tilde{\psi}\rangle = u|\psi\rangle, \quad (2.14a)$$
$$s_{\alpha_k}^k(t_k) \rightarrow \tilde{s}_{\alpha_k}^k(t_k) = us_{\alpha_k}^k(t_k)u^{-1}, \quad (2.14b)$$

where $u$ is the same for all $t_k$. However, in AQMMS, different sets of orthogonal projections $\{s_{\alpha_k}^k(t_k)\}$ are presumed to describe the alternative outcomes of distinct measurements, with distinct kinds of apparatus. Given a set of projections, it is in principle possible to construct an apparatus that measures the represented alternatives and distinguishes them from those represented by any other set of projections [9]. The measurements represented by the $\{s_{\alpha_k}^k(t_k)\}$, and the $\{\tilde{s}_{\alpha_k}^k(t_k)\}$ are different, despite the fact that they have the same probabilities, unless, of course, $u$ commutes with all the $\{s_{\alpha_k}^k(t_k)\}$, so that the set $\{s_{\alpha_k}^k(t_k)\}$ is the same as the set $\{\tilde{s}_{\alpha_k}^k(t_k)\}$. The only unitary operators that commute with all projections are multiples of the identity

$$u = e^{i\alpha}I, \quad (2.15)$$

or, when there are superselection rules, multiples of the identity with different phases on different superselection sectors. Thus, $|\psi\rangle$ is physically equivalent to $e^{i\alpha}|\psi\rangle$, or, in other words, physical states in quantum mechanics are represented by rays in Hilbert space [10].

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14See [10] for an insightful discussion of rays.
AQMMS is an approximation to the more general quantum mechanics of closed systems. Examination of models of measurement situations to which this approximation applies shows how the more restrictive notions of physical equivalence of AQMMS emerge in the more general context.

In a standard kind of closed system model of a measurement situation, the Hilbert space $H$ is assumed factored into two parts: a Hilbert space $H_s$ representing the measured subsystem and a Hilbert space $H_r$ representing the rest, including the measuring apparatus. It is convenient to think of separate canonical pairs of coordinates and momenta $z^a = \{y^i, p_i\}$ that act on $H_s$, and others $Z^A = \{Y^I, P_I\}$ that act on $H_r$. The interaction between the two subsystems is described by an interaction Hamiltonian $H_{\text{int}}$ that is a function of both kinds of variables as well as the time. Appropriate kinds of initial states in which the variables of $H_r$ and $H_s$ are uncorrelated evolve under the action of the total Hamiltonian containing $H_{\text{int}}$ into states in which the value of some physical quantity $f(z, t)$ on $H_s$ at time $t$ becomes tightly correlated with the value of a physical quantity $F(Z, T)$ on $H_r$ at a possibly distinct time $T$. In that way $f(z, t)$ is “measured” by the subsystem represented by $H_s$. Under suitably idealized conditions eq. (2.13) — representing the approximate quantum mechanics of measured subsystems — approximates the probabilities of the outcomes of successions of such measurements.

The connection between the notions of physical equivalence in AQMMS and in the closed system measurement model adumbrated above can be understood in the following way: We mentioned earlier that subspaces of Hilbert space could be labeled by requiring some physical quantities to have particular functional forms. Suppose that this is done in such a way as to label the subspaces of $H_r$ and further to specify the form of the interaction Hamiltonian $H_{\text{int}}(z, Z, t)$. The rules of AQMMS are not affected by such a choice since they refer neither to $H_r$ nor to $H_{\text{int}}$.

In the closed system, two triples $(\{C_\alpha\}, H, \rho)$ and $(\{\tilde{C}_\alpha\}, \tilde{H}, \tilde{\rho})$ are physically equivalent if the operators in the two triples are related by a constant unitary transformation or by reassignment of the times. However, except in special cases, such a transformation or reassignment can be expected to change the form of operators on $H_r$ or the form of $H_{\text{int}}(z, Z, t)$, with the exception of trivial unitary transformations that are multiples of the identity on each subspace. For instance, unitary transformations of the form $u \otimes I$, with $u$ unitary on $H_s$, would not affect the form of the operators on $H_r$, but would generally change the form of $H_{\text{int}}(z, Z, t)$. Thus the choice of the functional form of $H_{\text{int}}$ and of a sufficient number of operators on $H_r$ fixes an essentially unique representative of the physical equivalence class of descriptions of the measurement model. The remaining freedom will generally consist of unitary transformations that can be written $e^{i\alpha}I_s \otimes e^{i\beta}I_r$. We see that when AQMMS is viewed as an approximation to the quantum mechanics of closed systems with a fixed form of $H_{\text{int}}$ and a physical labeling of the subspaces of $H_r$, we recover the appropriate notion of physical equivalence for AQMMS that was described earlier in this Section.

III. IGUSES AND QUASICLASSICAL REALMS

As we mentioned in the Introduction, sets of histories constituting quasiclassical realms are important in quantum mechanics because they are utilized by IGUSes. We human observers, for instance, most often describe the world about us using coarse-grained histories
that distinguish ranges of values of familiar quantities of classical physics. In this Section, we discuss, not completely, but in more detail than we have before, how IGUSes are characterized and how the probabilities of their existing and behaving in certain ways are in principle predictable from quantum cosmology.

Human beings, bacteria, and computers equipped with certain kinds of hardware and/or software are all examples of IGUSes at various levels of complexity. IGUS is our name for a complex adaptive system in the context of quantum mechanics. Roughly, an IGUS is a subsystem of the universe that makes observations and thus acquires information, makes predictions on the basis of that information using some approximation (typically very crude) to the true quantum-mechanical laws of nature, and exhibits behavior based on those predictions. In general a complex adaptive system has following features: (1) It identifies and records regularities in an input data stream. (2) It compresses these regularities into a schema, which can be thought of as a model or theory. (There are typically variant schemata in competition with one another.) (3) A schema, enriched by further data, is used for describing the world, for predicting the future, and for prescribing behavior of the complex adaptive system as well as regulating the acquisition of further information. (4) These interactions with the world give rise to selection pressures exerted back on the competition of schemata, resulting in evolutionary adaptation.

All known complex adaptive systems on Earth are related in some way to life. They range from the prebiotic chemical reactions that produced life, through biological evolution, the functioning of individual organisms and ecological systems, the process of thinking in humans (and other animals), and the operation of mammalian immune systems, to the functioning of computers programmed to evolve strategies for playing games. All of them utilize the usual quasiclassical realm. That is, the input data stream and the consequences of a schema in the world are all describable in essentially classical terms by ranges of values of usual quasiclassical operators. We might call such IGUSes entirely usual quasiclassical IGUSes (EUQUIGUSes).

Sets of alternative histories for EUQUIGUSes are necessarily coarse grainings of the usual quasiclassical realm. The predictions of quantum cosmology for the individual histories in these coarse-grained sets are the probabilities of those histories. For example, the histories of the universe associated with the usual quasiclassical realm might be partitioned into those that exhibit IGUSes at certain times and places and those that never do. In this way the probability of existence of EUQUIGUSes becomes in principle a calculable question in quantum cosmology. The histories of the usual quasiclassical realm might be partitioned according to different evolutionary tracks of classes of IGUSes; in this way their evolution could be discussed. For example, by using an appropriate coarse graining, one might ask whether IGUSes evolve preferentially near type G stars. One could in principle calculate the conditional probabilities for alternative behaviors of an individual IGUS given its input data, or for the alternative evolutionary tracks of species given different selection pressures. (Such conditional probabilities, while depending on the initial condition of the universe, may be especially sensitive to the information about the specific past history that sets the conditions.) We should not pretend that it is practical to calculate the probabilities of histories

\[^{15}\text{For a more complete discussion of complex adaptive systems, see [7], [8], [9].}\]
such as we are discussing. However, it is in this manner that the nature, behavior, and evolution of EUQUIGUSes would be predictable in principle, from a fundamental theory of the elementary particles and the initial condition of the universe, in the form of probabilities arising from the quantum mechanics of closed systems.

IV. OTHER REALMS

Alternative histories referring to IGUSes need not be restricted to coarse grainings of the usual quasiclassical realm as they were in the preceding Section. By relaxing the assumption that all features of an IGUS are describable in usual quasiclassical terms, we may investigate a broader class of questions pertaining to IGUSes. Such questions involve decohering sets of alternative coarse-grained histories of the universe defined by operators other than those of the usual quasiclassical realm. We can say that they are defined by realms other than the usual quasiclassical realm.

As we have emphasized before [1], quantum theory itself does not discriminate between different sets of alternative decohering histories of a closed system (different realms), except by measures of their coarse graining, classicality, etc. As stressed by Griffiths [13], Omnès [15], and – more recently – by Dowker and Kent [2], great care is therefore needed in the use of ordinary language in dealing with quantum mechanics. In particular, different language should be used for discussions of the properties of a single realm from that used to discuss the relationships between different realms. We recommend in particular that words like “exist”, “happen”, “occur” etc. should be used only to refer to alternatives within a single realm, or else to projections that are perfectly correlated with such alternatives, as when a quantum-mechanical operator is measured by a classical apparatus. That way these words would have meaning in terms of quantum-mechanical probabilities, as expected. For example, we have discussed elsewhere [1,8], what is meant by an event having “happened” in the past. In a decohering set of histories describing certain present data as well as alternatives in the past that include the event, the conditional probability for the occurrence of the event in the past — given the present data — is near unity, while the conditional probability for alternatives to the event is near zero. When discussing different realms as features of the theory, we recommend not using words (such as “exist”, “happen”, or “occur”) that could be expected to have a probabilistic meaning. The reason is that quantum mechanics does not assign probabilities to different realms. Rather we suggest using phrases like “the theory exhibits a realm with this or that property” or the theory “allows a realm....”. Adhering to these usages will be especially important for clarity in the discussions in the remainder of this Section of IGUSes in realms different from the usual quasiclassical one and for those of the relationships between realms in Sections V.

The four defining properties of an IGUS introduced in the previous Section may be applicable to more general realms than the usual quasiclassical one considered there. With such a definition, the probability of occurrence of IGUSes — and their nature, evolution, and behavior — could be investigated in realms very different from the usual quasiclassical one. Input data, selection pressures, etc would be described in terms of correspondingly different alternatives. For those realms in which one can meaningfully distinguish the branches on which IGUSes evolve from those on which they have not evolved, one may ask for the total probability of the branches that do exhibit IGUSes. This probability for IGUSes may then
be compared between different realms. If the probability is high mainly for quasiclassical realms or coarse grainings of such realms, that is one way to give a meaning to the to the conjecture that IGUSes evolve primarily on branches of quasiclassical realms.\footnote{The fact that known IGUSes have evolved to utilize mostly quasiclassical alternatives in their input data stream provides an answer to questions such as: “If the universe is in a superposition of quasiclassically distinct histories, why isn’t it seen in a superposition?” That sort of question is particularly relevant for cosmology. A postulated pure initial quantum state of the universe is typically a superposition of quasiclassical branches with differing positions of individual stars (among many other things) at a given time. The eyes of birds, for example, have evolved to distinguish such quasiclassical alternatives, rather than to discriminate between alternative superpositions of quasiclassical branches. In each branch, certain registrations of what birds see are correlated with fairly definite positions of bright stars. Thus they detect stars in particular places in the sky at a given time (and sometimes use them for navigation) even though the universe may be said to be in a superposition of branches in which the stars have various positions at that time — in the sense that its initial Heisenberg state vector is a sum of the corresponding branch state vectors.} Even if a high probability for IGUSes is not restricted to quasiclassical realms, one might establish the requirements for IGUSes by comparing different realms. For example, one could investigate the level of determinism necessary for IGUSes by comparing the probabilities for their evolution in realms exhibiting varying levels of determinism, that is, by comparing the probability of IGUSes in the usual quasiclassical realm with that in realms that are much less deterministic or much more deterministic.

The representation of the universe as a quantum computer, as discussed by Seth Lloyd and in earlier work cited by him, comes close to exhibiting a realm totally different from the usual quasiclassical one but much more deterministic. The coarse graining consists entirely of restriction to equally spaced instants of time. In a particular basis in Hilbert space, the different vectors represent different initial conditions of a computer consisting of the entire universe. From any of these initial states, the universe proceeds deterministically from one state to another as the equal intervals of time “tick” by. The equations of motion here are discrete and exactly deterministic instead of continuous and approximately deterministic, but there is some resemblance to a quasiclassical realm despite the absence of branching after the first tick. Presumably computer-based IGUSes are to be found with probability near one within this representation of the universe.

Apart from the somewhat artificial example of the universe as a quantum computer, there is the possibility that the quantum mechanics of the universe as a consequence of the initial condition and Hamiltonian could exhibit a realm that is essentially different from the usual quasiclassical one but characterized by a high measure of classicality. Such a distinct quasiclassical realm would be a set of alternative histories, obeying a realistic principle of decoherence, displaying with high probability patterns of deterministic correlation described by effective equations of motion, and maximally refined subject to those conditions. It would differ from the usual quasiclassical realm because its alternatives would not be describable (even by means of a unitary transformation preserving $\rho$) in terms of ranges of values of integrals over small spatial volumes of the familiar field and density operators of classical physics, but would have to be described in terms of other operators instead. Thus the
approximate and phenomenological deterministic laws would be different from those of the usual classical physics. Although in pointing out some features of the usual quasiclassical operators that crudely characterize them, we raised the suspicion that the quasiclassical realm of everyday experience might be essentially unique, it seems possible that the universe exhibits truly distinct quasiclassical realms.

If a quasiclassical realm different from the usual one is exhibited then there is no reason to suppose that it might not possess coarse grainings describing the evolution and behavior of complex adaptive systems acquiring and utilizing information. That is, the universe might exhibit IGUSes in distinct quasiclassical realms, and the laws of quantum mechanics would not prefer one to another. Quantum theory supplies in principle the probability of such IGUSes in each realm, although those probabilities are far beyond our ability to compute in practice. In the next Section we discuss possible relationships between such realms.

V. RELATIONS BETWEEN REALMS

If different realms exhibit IGUSes, we may investigate certain relations between them. Probabilistic predictions concerning the relationships between IGUSes in two different realms may be made by using a decohering set of histories containing alternatives referring to IGUSes in one realm and also alternatives referring to IGUSes in the other realm, provided the decoherence of the hybrid set follows from the initial condition and Hamiltonian. The problem of drawing inferences in one realm concerning IGUSes using a distinct realm is then not so very different from that involved in ordinary searches for extraterrestrial intelligence. There, we observe projections accessible to us and try to infer from their particular values that they are signals from IGUSes, say because they can be seen to be an encoding of $\pi$ or of the periodic table of chemical elements. Such discussions are most often carried out using the usual quasiclassical realm. However, one could also use a realm containing alternatives from the usual quasiclassical realm that described our observations, and in addition alternatives from the other realm describing other IGUSes. Using such a hybrid realm, one could compute probabilities for alternatives referring to those other IGUSes. In the following we describe some kinds of hybrid realms.

The simplest example refers to alternatives describing IGUSes in one realm that are highly correlated with histories constituting a coarse graining of another realm. This is very much like an ordinary measurement situation in which a value of a non-quasiclassical operator such as a spin becomes almost exactly correlated with a range of values of a usual quasiclassical one. Then IGUSes making use of one realm could conceivably draw inferences about IGUSes in another by seeking or creating “measurement situations” in which an alternative of one realm is correlated almost perfectly with an alternative from the other.

Even in the absence of nearly exact correlations, it is possible that probabilistic inferences about some features of IGUSes evolving in different realms could be drawn making use of

17 A. A. Starobinsky tells us that such distinct quasiclassical realms are called “goblin worlds” by some science fiction writers. Using terminology such that “realm” refers to a set of alternative histories, we are discussing goblin realms.
this kind of hybrid realm with some alternatives drawn from both.

To give a very simple example of how questions may be asked in this way about IGUSes using realms different from the usual quasiclassical one, we may investigate the probabilities that IGUSes, otherwise described in quasiclassical terms, evolve to use an input data stream containing, in part, alternative ranges of values of operators essentially different from the usual quasiclassical ones. IGUSes like ourselves that measure the values of highly quantum-mechanical variables are examples. Sets of histories describing such alternative evolutionary tracks contain mostly operators of the usual quasiclassical kind, but occasionally non-quasiclassical operators describing measured alternatives, which are highly correlated with certain of the usual quasiclassical ones. A low probability that IGUSes evolve to make direct use of non-quasiclassical alternatives is another way in which our conjecture that IGUSes evolve to exploit the regularities of a quasiclassical realm can be given a meaning within quantum mechanics.

No particular IGUSes, such as human beings, play any distinguished role in the formulation of quantum mechanics that we are using. If the universe exhibits IGUSes in realms essentially different from the usual quasiclassical one, that does not constitute a paradox, but rather an intriguing example of the richness of possibilities that may be shown by a quantum universe.

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REFERENCES

[1] M. Gell-Mann and J.B. Hartle in *Complexity, Entropy, and the Physics of Information*, Santa Fe Institute Studies in the Sciences of Complexity, Vol. VIII, ed. by W. ˙Zurek, Addison Wesley, Reading (1990) or in *Proceedings of the 3rd International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology* ed. by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, Physical Society of Japan, Tokyo (1990).

[2] H.F. Dowker and A. Kent (to be published).

[3] M. Gell-Mann and J.B. Hartle, *Phys. Rev. D* **47**, 3345, (1993)

[4] J.P. Paz and W.H. ˙Zurek *Phys.Rev.D* **48**, 2728, (1993).

[5] M. Gell-Mann and J.B. Hartle (to be published).

[6] J.B. Hartle, *Quasiclassical Domains in a Quantum Universe*, in *Proceedings of the Lanczos Centenary Conference*, North Carolina State University, December, 1992, ed. by J.D. Brown, M.T. Chu, D.C. Ellison, and R.J. Plemmons, SIAM, Philadelphia, (1994).

[7] S. Lloyd, private communication and *Phys. Rev. Lett.* **71**, 943, (1993).

[8] J.B. Hartle, *The Quantum Mechanics of Cosmology*, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, ed. by S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991).

[9] W. Lamb, *Physics Today* **22**, 23, (1969).

[10] J.C. Solem and L.C. Biedenharn, *Found. Phys.* **23**, 185, (1993).

[11] J.B. Hartle, *Spacetime Quantum Mechanics and the Quantum Mechanics of Spacetime* in *Gravitation and Quantizations: Proceedings of the 1992 Les Houches Summer School*, ed. by B. Julia and J. Zinn-Justin, North Holland Publishing Co, Amsterdam, (1994), grqc/9304006.

[12] J.B. Hartle, *Phys. Rev.* **D44**, 3173, (1991).

[13] R. Griffiths, *J. Stat. Phys.* **36**, 219, (1984).

[14] M. Gell-Mann and J.B. Hartle, in *Proceedings of the NATO Workshop on the Physical Origins of Time Asymmetry*, Mazagón, Spain, September 30-October 4, 1991 ed. by J. Halliwell, J. Pérez-Mercader, and W. ˙Zurek, Cambridge University Press, Cambridge (1994), grqc/9304023.

[15] R. Omnès, *J. Stat. Phys.* **53**, 893, (1988), *ibid* **53**, 933, (1988); *ibid* **53**, 957, (1988); *ibid* **57**, 357, (1989); *Rev. Mod. Phys.* **64**, 339, (1992).

[16] M. Gell-Mann and J.B. Hartle in the *Proceedings of the 25th International Conference on High Energy Physics, Singapore*, August 2-8, 1990, ed. by K.K. Phua and Y. Yamaguchi (South East Asia Theoretical Physics Association and Physical Society of Japan) distributed by World Scientific, Singapore (1990).

[17] M. Gell-Mann, *The Quark and the Jaguar*, W. H. Freeman, New York (1994).

[18] M. Gell-Mann, *Complexity and Complex Adaptive Systems*, in *The Evolution of Human Languages*, Santa Fe Institute Studies in the Sciences of Complexity, Proc. Vol. X, ed. by M. Gell-Mann and J.A. Hawkins, Addison-Wesley, Reading, MA (1992).

[19] M. Gell-Mann, *Complex Adaptive Systems* in *Complexity: Metaphors, Models, and Reality*, Santa Fe Institute Studies in the Sciences of Complexity, Proc. Vol. XIX, ed. by G.A. Cowan, D. Pines, and D. Meltzer, Addison-Wesley, Reading, MA, (1994).