Filtration of 2-particles suspension in a porous medium

L I Kuzmina¹, Yu V Osipov²*, and M D Astakhov²

¹National Research University Higher School of Economics, Moscow, Russia
²National Research Moscow State University of Civil Engineering, Moscow, Russia

E-mail: yuri-osipov@mail.ru

Abstract During the construction of underground storage of hazardous waste, it is necessary to create waterproof walls in the ground. The grout is filtered in the rock, fills the pores and, when hardened, creates a reliable barrier to groundwater. A one-dimensional model of the flow of inhomogeneous particles in a porous medium is considered. The retained particles profiles formed during deep bed filtration are studied. It is shown that when filtering a 2-particle suspension, the deposit is distributed unevenly. The profile of large retained particles is always monotonous, and the profile of small retained particles is nonmonotonic. The monotonicity of the total deposit profile depends on the model parameters. The shape of non-monotonic profiles is time-dependent. At short times, the profile decreases monotonously. At some point, a maximum appears on the profile graph, which shifts from the inlet to the output with increasing time. When the maximum point reaches the outlet, the profile becomes monotonically increasing. With a further increase in time, the retention profiles remain monotonically increasing. Analytical solutions for a filtration model with particles of three or more different types are unknown. Analysis of the retention profiles of the polydisperse suspension requires further study.

1. Introduction

When constructing tunnels and underground structures, it is necessary to protect buildings from the penetration of groundwater. To create waterproof partitions, a reinforcing solution is pumped into the soil. The liquid grout is pumped in the pores of the soil and forms a waterproof layer during cementation. Mathematical modeling allows to optimize the construction technology [1, 2].

The transport of small particles in porous media is accompanied by the formation of deposit. During deep bed filtration the particles are retained in the whole porous medium. Depending on the physical and chemical properties of the particles, the structure and material of the porous medium, various mechanisms of particle capture can prevail: retention in the neck of narrow pores, formation of arch bridges by several particles at the inlet of the pores, attraction to the walls of the pore channels, diffusion into dead-end pores, etc. [3, 4]. As a rule, the deposited particles cannot be knocked out from retention sites by moving particles or by the flow of a carrier fluid and remain motionless.

Mass balance of suspended and retained particles and deposit growth equations form a hyperbolic system of deep bed filtration. Unknown functions are volume concentrations of suspended and retained particles. Retention profiles are the dependences of deposit concentration on the spatial coordinate at a fixed time. When filtering a monodisperse suspension, the retention profiles always monotonically decrease [5]. For a polydisperse suspension containing particles of various sizes, the retention profiles become nonmonotonic. This was first shown experimentally [6, 7]. Then theoretical
models with nonmonotonic profiles appeared considering particles which move with different velocities [8, 9].

In [10], a bicomponent filtration model with particles of different sizes moving at the same velocities is considered. The non-monotony of the profiles is associated with different particle sizes and depends on the model parameters. The retention profile of large particles always monotonously decreases, and the retention profile of small particles changes the monotony: decreases with a short time and increases with a long time. At intermediate times, a maximum point appears on the profile graph, which shifts from the inlet to the outlet of the porous medium. The monotony of the total retention profile depends on the model parameters. However, the conditions under which the retention profile of the total deposit is nonmonotonic were unknown.

This study contains simple mathematical conditions for the profile nonmonotonicity. It is shown that the total deposit profile monotonically decreases at short times and monotonically increases at a large time. At intermediate time, the profile has a maximum point. Numerical calculations and graphs illustrate the change in the monotony of the profiles.

2. Mathematical model

In the domain \( \Omega = \{0 \leq x \leq 1, \ t \geq 0\} \) consider a system [11]

\[
\frac{\partial c_i}{\partial t} + \frac{\partial c_i}{\partial x} + \frac{\partial s_i}{\partial t} = 0, \tag{1}
\]

\[
\frac{\partial s_i}{\partial t} = (1 - b) \lambda_i c_i, \quad i = 1, 2. \tag{2}
\]

Here \( b = B_i c_1^0 s_1 + B_2 c_2^0 s_2 \), \( \lambda_i \), \( B_i \), \( c_i^0 \), \( i = 1, 2 \) are positive constants, and \( c_1^0 + c_2^0 = 1 \).

In equation (2), the deposit growth rate is proportional to the linear filtration and concentration functions. The model uses the blocking filtration function \((1 - b)\) [12]. For low concentrations of suspended particles, the concentration function is proportional to the first degree of concentration [13].

Boundary and initial conditions determine the unique solution to the system:

\[
x = 0: \ c_1 = c_1^0, \ c_2 = c_2^0, \tag{3}
\]

\[
t = 0: \ c_1 = 0, \ c_2 = 0, \ s_1 = 0, \ s_2 = 0. \tag{4}
\]

Conditions (3) specify the injection of a suspension of constant concentration at the inlet of the porous medium \( x = 0 \); initial conditions (4) mean that at the initial moment \( t = 0 \) the porous medium does not contain any suspended and retained particles.

Assume that \( \lambda_1 > \lambda_2 \), i.e. particles of type 1 are larger than particles of type 2.

The solutions \( c_1(x, t), c_2(x, t) \) (concentrations of suspended particles) are discontinuous on the characteristic \( t = x \), the solutions \( s_1(x, t), s_2(x, t) \) (concentrations of retained particles) are continuous in the entire domain \( \Omega \). The straight line \( t = x \) is the concentration front of suspended and retained particles, dividing the domain \( \Omega \) into two subdomains. In the domain \( \Omega_1 = \{0 \leq x \leq 1, \ 0 \leq t \leq x\} \) before the front the system has a zero solution \( c_1 = 0, s_1 = 0, \ i = 1, 2 \); in the domain \( \Omega_2 = \{0 \leq x \leq 1, \ t \geq x\} \) after the front filtration occurs and the solution is positive.

At the concentration front \( t = x \), the solution \( s_1 = 0, s_2 = 0 \Rightarrow b = 0 \). Substitution of formula (2) into equation (1) yields the relation for the suspended particles concentrations on the front

\[
\frac{\partial c_i}{\partial t} + \frac{\partial c_i}{\partial x} + \lambda_i c_i = 0, \quad i = 1, 2. \tag{5}
\]

Solution of equations (5) with conditions (3)
Innovations and Technologies in Construction (BUILDINTECH BIT 2021)  
Journal of Physics: Conference Series 1926 (2021) 012001  
doi:10.1088/1742-6596/1926/1/012001

\[ c_i^*(x) = c_i(x, x) = c_i^0 e^{-\lambda_i x}, \quad i = 1, 2. \]  

An analytical solution to problem (1)-(4) is constructed similarly to [14-16]. Two types of the suspended particles concentrations are related by equations

\[ c_1 = c_1^0 \left( \frac{c_2^0}{c_2^1} \right)^{\lambda_1/\lambda_2}, \quad c_2 = c_2^0 \left( \frac{c_1^0}{c_1^1} \right)^{\lambda_1/\lambda_2}, \]  
and the concentrations of suspended and retained particles are related by Riemann invariants - by the relations between solutions on the characteristics of the system [17]

\[ s_i = \frac{c_i - c_i^*}{B_1 c_1^0 (c_1^0 - c_1^*) + B_2 c_2^0 (c_2^0 - c_2^*)}, \quad i = 1, 2. \]  

The condition of nonmonotonicity of the total deposit profiles follows from equations (7), (8)

\[ B_1 c_1^0 > B_2 c_2^0. \]  

3. Results

Numerical calculations are used in the study of complex models for which the analytical solution is either unknown or has a complex implicit form [18-20]. Solutions obtained by numerical methods allow the use of mathematical models to optimize technological processes [21, 22].

Figures 1-3 show the partial and total retained concentration profiles for \( \lambda_1 = 25, \lambda_2 = 5 \) and various values of the parameters \( B_1, B_2, c_1^0, c_2^0 \). The curve’s maximum points are marked with dots.
Figure 1. Retained concentration profiles at $B_1 = 0.125$, $B_2 = 0.025$, $c_1^0 = 0.5$, $c_2^0 = 0.5$.

- a – $t=0.2$
- b – $t=2$
- c – $t=10$
- d – $t=40$

Figure 2. Retained concentration profiles at $B_1 = 0.025$, $B_2 = 0.125$, $c_1^0 = 0.5$, $c_2^0 = 0.5$.

- a – $t=0.2$
- b – $t=10$
- c – $t=30$
- d – $t=40$
4. Discussion

The calculations revealed patterns of change in the monotony of the profiles. Profiles \( s_1 \) are always monotonously decreasing, and profiles \( s_2 \) are not monotonous. At \( B_1 c_1^0 < B_2 c_2^0 \), the total retained profile \( s \) monotonically decreases (figure 1), and is not monotonous at \( B_1 c_1^0 > B_2 c_2^0 \) (figures 2, 3). The nature of nonmonotonic profiles changes with time. At short times, the profiles monotonically decrease (figures 2a, 3a), with increasing time, the profiles lose monotony, and a maximum point appears on the graphs (figures 2b, 2c, 3a, 3b, 3c). At large times, the maximum point disappears and the profiles become monotonically increasing (figures 2d, 3d).

With increasing time, the maximum points of the profiles move to the right along the coordinate axis. Before the maximum point at the inlet \( x = 0 \), the profiles monotonously decrease. When the maximum point reaches the outlet \( x = 1 \), the profiles become monotonically increasing. The maximum point on the partial retained concentration profile \( s_2 \) always exceeds the maximum point of the total retained concentration profile \( s \), it appears earlier near the inlet and disappears earlier, reaching the outlet of the porous medium.

Figure 3. Retained concentration profiles at \( B_1 = 0.125, B_2 = 0.025, c_1^0 = 0.9, c_2^0 = 0.1 \).

\[ a - t = 0.2, \ b - t = 1.7, \ c - t = 5, \ d - t = 15. \]
5. Summary
The profiles of the retained particles formed during the filtration of a 2-particle suspension in a porous medium are studied. Retained particles profiles of a monodisperse suspension always decrease monotonously. The profiles of large retained particles of a 2-particle suspension also decrease monotonously. Retention profiles of small particles are nonmonotonic.

The monotonicity of the total retention profile depends on the sign of the expression $Z = B_1 c^0 - B_2 c^2$. For $Z < 0$, the profiles monotonically decrease, for $Z > 0$, the profiles are nonmonotonic. The parameters $B_1, B_2$ determine the size of the regions occupied by the deposited particles on the porous medium frame.

The shape of non-monotonic profiles is time-dependent. At short times, the profile decreases monotonously. At some point, a maximum appears on the profile graph, which shifts from the inlet to the output with increasing time. When the maximum point reaches the outlet $x = 1$, the profile becomes monotonically increasing. With a further increase in time, the retention profiles remain monotonically increasing.

Analytical solutions for a filtration model with particles of three or more different types are unknown. Analysis of the retention profiles of the polydisperse suspension requires further study.

6. References
[1] Zhang Q et al 2015 Exploration and Grouting of Large-Scale Water Capsule in the Fault Fracture Zone of Yonglian Tunnel The Open Civil Engineering Journal 9 32-43
[2] Li S, Liu R, Zhang Q, Zhang X 2016 Protection against water or mud inrush in tunnels by grouting: A review Journal of Rock Mechanics and Geotechnical Engineering 8 753-766
[3] Chrysikopoulos C V, Sotirelis N P, Kallithrakas-Kontos N G 2017 Cotransport of graphene oxide nanoparticles and kaolinite colloids in porous media Transport in Porous Media 119 181-204
[4] Johnson W P, Rasmuson A, Pazmiño E, Hilpert M 2018 Why variant colloid transport behaviors emerge among identical individuals in porous media when colloid-surface repulsion exists Environmental Science & Technology 52 7230-7239
[5] Yuan H, Shapiro A 2010 Modeling non-Fickian transport and hyperexponential deposition for deep bed filtration Chemical Engineering Journal 162 974-988
[6] Bolster C H, Mills A L, Hornberger G M, Herman J S 1999 Spatial distribution of deposited bacteria following Miscible Displacement Experiments in intact cores Water Resources Research 35 1797-1807
[7] Harter T, Wagner S, Atwill E R. 2000 Colloid transport and filtration of cryptosporidium parvum in sandy soils and aquifer sediments Environmental Science & Technology 34 62–70
[8] Bradford S A, Torkzaban S, Simunek J 2011 Modeling colloid transport and retention in saturated porous media under unfavorable attachment conditions Water Resources Research 47 W10503
[9] Kuzmina L I, Osipov Yu V, Galaguz Y P 2017 A model of two-velocity particles transport in a porous medium International Journal of Non-Linear Mechanics 93 1-6
[10] Malgaresi G, Collins B, Alvaro P, Bedrikovetsky P 2019 Explaining non-monotonic retention profiles during flow of size-distributed colloids Chemical Engineering Journal 375 121984
[11] Pires A P, Bedrikovetsky P G 2007 First-Order Hyperbolic Systems of Quasilinear Equations. Systems of Conservation Laws of Gas Dynamic Type In: Polyanin A D, Manzhirov A V Handbook of Mathematics for Engineers and Scientists 780-795
[12] Bedrikovetsky P 2008 Upscaling of Stochastic Micro Model for Suspension Transport in Porous Media Transport in Porous Media 75 335-369
[13] Civan F 2015 Reservoir Formation Damage 741
[14] Kuzmina L I, Osipov Y V, Zheglova Y G 2018 Analytical model for deep bed filtration with multiple mechanisms of particle capture *International Journal of Non-linear Mechanics* **105** 242–248

[15] Vyazmina E A, Bedrikovetskii P G, Polyanin A D 2007 New Classes of Exact Solutions to Nonlinear Sets of Equations in the Theory of Filtration and Convective Mass Transfer *Theoretical Foundations of Chemical Engineering* **41(5)** 556-564

[16] Galaguz Yu P, Kuzmina L I, Osipov Yu V 2019 Problem of Deep Bed Filtration in a Porous Medium with the Initial Deposit *Fluid Dynamics* **54(1)** 85-97

[17] Kuzmina L I, Nazakinisii V E, Osipov Y V 2019 On a Deep Bed Filtration Problem with Finite Blocking Time *Russian Journal of Mathematical Physics* **26(1)** 130-134

[18] Safina G 2019 Numerical solution of filtration in porous rock *E3S Web of Conference* **97** 05016

[19] Galaguz Y, Safina G 2018 Calculation of colloids filtration in a porous medium *IOP Conference Series: Materials Science and Engineering* **365** 042005

[20] Fayziev B, Ibragimov G, Khuzhayorov B, Alias I A 2020 Numerical Study of Suspension Filtration Model in Porous Medium with Modified Deposition Kinetics *Symmetry* **12(5)** 696

[21] Bedrikovetsky P 2013 *Mathematical theory of oil and gas recovery: with applications to ex-USSR oil and gas fields* 575

[22] Badalyan A, You Z, Aji K, Bedrikovetsky P, Carageorgos T, Zeinjahromi A 2014 Size exclusion deep bed filtration: Experimental and modelling uncertainties, *Review of Scientific Instruments* **85(1)** 015111