Recent Developments in Optimal Placement of Phasor Measurement Units Considering in Incomplete Observability

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Abstract

Independent System Operator (ISO) monitors the SCADA system to continuously check the health of power system. SCADA system refreshing rate is several seconds which may cause delay or even miss important information and the ISO may not be able to take appropriate action when most needed. An emerging technology is the Phasor Measurement Unit (PMU) which takes the help of the Global Positioning System (GPS) and timestamps the whole power system network with much faster refreshing rates. In this paper difference of Phasor Measurement Unit (PMU) and SCADA is represented. Rules and conditions to optimally place the PMUs in power system are discussed. A review on methods used for placement of the PMU in the power system based on incomplete observability is given. Also the spanning tree search based optimal placement method is also discussed.

Keywords: Complete and Incomplete Observability, Optimal Placement, Phasor Measurement Unit (PMU), SCADA

1. Introduction

In power system control center state estimation has very important role. Critical power system securities and electrical market decisions are dependent on the results of state estimators. Now-a-days algorithms used for state estimations are majorly used for the measurement of the asynchronous measurement of real and reactive power, voltage magnitudes, load angle from the bus from where state vectors has been calculated. The algorithms are based on iterative solutions most widely used algorithm is weighted least square algorithm. Although these state estimator’s algorithms are capable to solve very large iterations, it affects the system's reliability and accuracy when power system is under stress.

PMU are able to improve current state estimators which is SCADA. The SCADA system can measure voltage and current waveforms but they cannot measure the voltage angle at every bus. This can be done by using the PMU technology. PMU has been under research from early 1980s. With the invention of Phasor Measurement Unit angle was directly measured for first time. A PMU is digital device that can measure synchronized voltage and current measurements. Here phasors are vector representation of magnitude of voltage and current. To determine phase angles from different sites, all PMU sites are synchronized with common time pulse which can be got from GPS in order to 1ms that can measure very precise voltage and current phasors. Reporting rate for SCADA system is once in 4 to 6 seconds so it means we get only 1 reading for estimation in 4 to 6 seconds where reporting rate for PMU is very fast it depends on frequency which has minimum 10 readings for calculation in 1 second.

Figure 1 shows the typical working block diagram of PMU. As analog inputs, voltage and current signals are provided to anti-aliasing filter from the measurement.
instruments CTs and PTs. Anti-aliasing filter is used to attenuate the frequency more than the nyquist frequency.

The phase-locked oscillator (PLL) converts GPS signal 1 pulse per second into a sequence of very high speed time pulses that will be used in sampling of the waveform. Next block the A/D converter is used to convert the analog current and voltage signals to digital signals, which are derived into the phasor microprocessor to perform the Discrete Fourier Transform (DFT) for phasor cal-

Figure 1. Block diagram of PMU.

Figure 2. Conceptual Diagram of Phasor Measuring System.
Calculations of signals. Then computed signals are send to Phasor Data Concentrator (PDC) which is then send to modems to transmit it.

Some application fields can be built for PMUs in order to study the interconnected grid in a active way. Such application fields include:

1. Dynamic state estimation
2. Power systems protection
3. Monitor the stress on the electric transmission system
4. Identifying the corrective actions (such as damping) needed in case of discrepancies
5. Various stability studies including angular stability and voltage stability.
6. Monitor system oscillations
7. Wide Area Monitoring systems (WAMS)
8. Available transmission capacity
9. Power systems control

Uses of PMUs are increasing day by day in recent years, it improves monitoring and controls the power networks. Proposed PMU applications, the relatively high cost of phasor measurement units as well as the facilities of communication cost. So to decrease the cost of PMU, optimal placement of PMU must be necessary and it is a big challenge. So some conventional optimization techniques have been introduced to solve the Optimal Placement Problem (OPP), such as linear programming (LP), dynamic programming, nonlinear programming (NLP) or combinational optimization. To overcome the problems raised due to optimization techniques, such as risk of difficulties in handling constraint, trapping at local optima or numerical difficulties, new techniques have been proposed such as depth first search (DeFS), simulated annealing (SA), tabu search (TS), differential evolution (DE), particle swarm optimization (PSO).

3. Formulation of Optimal Placement Problem

PMU can measure the voltage phasor where the PMU has been installed and current phasor of all the lines connected to that bus. There are some rules that can be used for the optimal placement of the PMU:

**Rule 1:** Specify a voltage measurement at a bus where PMU is placed with a current measurement to each line and branch connected to that bus.

**Rule 2:** Specify a pseudo measurement of Voltage to each node which can be reached by other equipped with a PMU.

**Rule 3:** Specify a pseudo measurement of current to each branch which is connecting to two buses where known voltage is there. It will allow to interconnect the observable zones.

**Rule 4:** Specify a pseudo measurement of current to each line and branch where current can be calculated indirectly by KCL (Kirchhoff current law).

But these rules can be applied only when there is known current balance at particular node.

The main principle of the optimal OPP problem is the right choice of the minimum no of PMUs to be installed \(n_p\) and to choose the optimal location \(S(n_p)\) to place the no of PMU \(n_p\) from where complete observability can be achieved. The OPP problem can be formulate as

\[
\min_{n_p} \left\{ \max_{S(n_p)} R(n_p, S(n_p)) \right\}^2
\]

(1)

Such that

\[
O_{bs}(n_p, S(n_p)) = 1
\]

(2)

where,

\[R(n_p, S(n_p)) = \text{Redundancy Measurement Index}\]

\[O_{bs} = \text{observability evaluation function}\]

The conditions for observability that have to be met for the selection of the placement of PMU are...
Condition 1: Bus voltages phasors and current phasors of all incident branches should be known at bus where PMU is installed.

Condition 2: If current phasor and voltage phasor of one end is known then voltage phasor of another end can be calculated easily.

Condition 3: If at both ends, voltage phasors of any branch is known then current phasor can be determined directly.

Condition 4: If system has zero injection bus which does not consist PMU and its current phasor of incident line are known, then current phasor can be calculated for unknown branch using KCL.

Condition 5: If zero injection bus has unknown voltage phasor and that voltage phasors of all other adjacent buses are known then voltage phasor at zero injection bus can be calculated by using nodal voltage equations.

Condition 6: If voltage phasor of all adjacent zero injection buses are unknown, but at the same time voltage phasors of all the adjacent buses to that set are known, then the voltage phasor at particular bus can be determined by using nodal voltage equations.

To get direct state measurement, PMU should be deployed at all the buses of the system. Measurements includes synchronized positive sequence current and voltage measurements. All measurement assumed to be zero mean, usually distributed noise component. Measured vector M can be formulated by

\[ M = \left[ \begin{array}{c} V \\ I \end{array} \right] + \left[ \begin{array}{c} \varepsilon_V \\ \varepsilon_I \end{array} \right] \]  

where, $V$ and $I$ = vectors of true values of branch current and bus voltage in rectangular form

$\varepsilon_V$ and $\varepsilon_I$ = Error Vectors

The errors are adopted to have a covariance matrix

\[ W = \begin{bmatrix} W_{VV} & 0 \\ 0 & W_{II} \end{bmatrix} \]  

If for branch circuit elements $\pi$ representation is assumed, then relationship between $I$ and $V$ can be shown as

\[ I = [yA + y_s] V \]  

where,

$A$ = Bus incident matrix for current measurement
$y_s$ = primitive matrix for shunt admittances at the measured ends

Substituting equation (5) into (3) results in

\[ M = \begin{bmatrix} I \\ y_sAT + y_s \end{bmatrix} V + \left[ \begin{array}{c} \varepsilon_V \\ \varepsilon_I \end{array} \right] \]  

\[ M = B \cdot V + \varepsilon \]  

Weighted least square method to determine the state vector $V$ can be shown as

\[ G \cdot V = B^* \cdot W \cdot M \]  

where,

$G$ is called as gain matrix which is given by

\[ G = B^* \cdot W \cdot B \]
3. PMU Optimal Placement for Incomplete and Complete Observability

All installed PMUs should be monitored by SCADA/EMS system. Different types of methods like graph theory and simulated annealing shows that minimum 1/5 to 1/4 of the system buses would be provided with PMUs to make the system completely observable. In optimization techniques required number of PMUs are minimized and observability of the system still remain as it is with those reduced number of PMUs.

Placement of PMU for incomplete observability method is a topology that systematically distributed PMUs in all the network resulting. When the number of PMUs is not sufficient and their location is not optimal at that time PMU cannot cover whole the system, it is called incomplete observability.

Figure 3 shows system which is incomplete observed. PMUs installed at bus 2 and bus 6 directly measures the voltage at bus 2 and 6, whereas voltages at bus 1, 3, 5 and 7 can be determined by using the measured values of voltages and currents. But at the same time bus 4 leaves unobservable. Here buses 1, 3, 5 and 7 are calculated buses and PMUs are installed at bus 2 and 6 so, these two buses are defined as directly measured buses. And here one bus is unobservable so it is defined as depth of one unobservability. If as shown in Figure 4 we move PMU 2 one bus away from its initial place, then two buses will be unobservable in the system. So it is defined as depth of two unobservability condition of the system. And these two unobservable buses are in between two PMUs.

Another technique for PMU placement is tree search which is described in Figure 5. Derive spanning tree from the parent graph of power system by eliminating the co-trees. Here it is assumed that 1 is reference node. In as

Figure 3. Depth of one unobservability.
shown in Figure 4B to have depth of one unobservability. PMU is put four buses away from previous PMU which is installed at bus A. Once tree is completely searched, PMU locations must be assigned as shown in Figure 4C. For optimal placement of PMU minimum no of PMUs should be installed.

**Figure 4.** Depth of two unobservability.

**Figure 5.** PMU placement for incomplete observability using tree search.
3.1 Illustration

The tree search techniques for the placement of PMU is explained in the Figure 6. As shown in figure there are 13 branches 14 nodes excuplated from a system graph 21 branches with including co-trees. Here we choose node 12 as a reference node. Logically PMU should be placed at node 6 (PMU-A) so to observe the reference node. We will find forward path along chosen path determined by the nodal sequence node 6 → node 5 → node 1 → node 2 → node 4. The next priority for the placement of the PMU (PMU-B) should be at node 4, which makes the node 1 as unobservable node with depth of one unobservability. Here mark that both PMUs are 4 node away from each other.

The next move, PMU should be move to terminal node 9. Now here we will backtrack PMU from terminal node 9 to terminal node 4 and from terminal node 4 to terminal node 7. The next movement of PMU should be to terminal node 8 and it is observable because current between node 7 and node 8 is known. Now we again backtrack till we don't reach node where we can move forward. Node 8

node 7 → node 4 → node 2. Now from node 2 we will move to node 6 with the path of node 2 → node 1 → node 5 → node 6. Now again we will move to node 10 in sequence followed by node 6 → node 11 → node 10.

Node 10 is good place for the placement of the PMU but we cannot place it here because node 10 has the depth of one unobservability and we can say this because node 11 and node 9 are observed branches. So now we will again backtrack from node 10 and we will further move to node 6 and move again forward to node 13. Node 13 has one depth of one unobservable bus which is node 14. So the last backtracking will be to reference node 12 to finish the search of optimal place. For the optimal placement with depth of one unobservability in 14 bus system as shown in Figure 6 we must install PMU at node 6 and node 4.

To confirm minimum number of PMU placements, it is needed to check another search by choosing different node as reference node.

Figure 6. Tree search PMU placement technique.
3.1 Flow Chart for the PMU Placement by Tree Search Technique for Incomplete Observability

Figure 7 explains the PMU placement techniques by using tree search method for incomplete observability. An outer loop, contains process box which is from box no 1 to decision box which is box no 5, iterates on a subset of spanning trees is added. The main objective of the flow chart is to find the tree which helps to find the optimal placement of PMU and reduce the number of PMU needed.

Remember that even a small part of the system has a large number of tree so prepared algorithm to find the

Figure 7. Algorithm flowchart to find the tree by using tree for incomplete observability.
tree must be comfortable to run on any no of tree and on any size of the system without getting fail. There so sev-
eral methods and algorithm to find the tree. In this paper authors have used incidence matrix A to find tree. Data
needed to find tree by this method is

1. systems’ P-Q line list with power system graph structure
2. user defined depth of unobservability v
3. Status of bus injection which is required by algo-
rithm used for observability

4. Conclusion
This paper has reviewed several developments in field of state estimation in power system using PMU. In this paper authors has reviewed in the following fields

- Comparison between PMU and SCADA system used for state estimation.
- Different rules and conditions for the optimal placement of the PMU.
- Optimal Placement techniques for incomplete and complete observability with PMU of power system.
- Algorithm to find the tree from the system for incomplete observability

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