Some new distance-4 constant weight codes

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Abstract

Improved binary constant weight codes with minimum distance 4 and length at most 28 are constructed. A table with bounds on the chromatic number of small Johnson graphs is given.

1 Introduction

A binary constant weight code of word length $n$ and weight $w$ and minimum distance $d$ is a collection of $(0,1)$-vectors of length $n$, all having $w$ ones and $n - w$ zeros, such that any two of these vectors differ in at least $d$ places. The maximum size of such a code is denoted by $A(n,d,w)$. In this note we give improved lower bounds for $A(n,d,w)$ for $d = 4$ and smallish $n$.

The standard reference for constructions of binary constant weight codes of length at most 28 is [3]. One of the constructions discussed there depends on the existence of partitions of all words of a given length and weight into codes with minimum distance at least 4 (that is, on proper colorings of the Johnson graph). Such partitions are typically found using some form of heuristic search, and today it is easy to improve on the results of [3]. For example, [3] says that $A(22, 4, 11) \geq 39688$, while we find $A(22, 4, 11) \geq 40624$ here. Earlier improvements have been given in [9] and [10]. The bounds here improve all but one of the bounds from [9] and all from [10]. For example, [3] gives $A(26, 4, 13) \geq 424868$, [10] gives $A(26, 4, 13) \geq 425950$, and we find $A(26, 4, 13) \geq 431672$.

Motivated by an application to frequency hopping lists in radio networks, the authors of [18] extended the tables of constant weight codes to word length 63. For the case of $d = 4$ they give bounds on $A(n, 4, 5)$. Table 2 below gives improvements.

Apart from codes obtained via this construction using partitions, we also give five direct constructions, showing that $A(15, 4, 6) \geq 399$, $A(16, 4, 5) \geq 322$, $A(16, 4, 6) \geq 616$, $A(18, 4, 5) \geq 544$, and $A(21, 4, 5) \geq 1113$.

Finally, this note contains (in §6) a discussion of the chromatic number of Johnson graphs and determines this number in a few new cases.

2 Direct constructions

2.1 $A(15, 4, 6) \geq 399$

We show $A(15, 4, 6) \geq 399$ using the group of order 21 (that permutes the 15 coordinate positions, numbered right-to-left 0–14) that fixes position 14, and
acts on positions 0–13 with the two generators $(0,2,1,4,5,3,6)(7,9,8,11,12,10,13)$ and $(0,2,4)(1,3,5)(7,9,11)(8,10,12)$. The 23 base blocks:

| Base Blocks | Description |
|-------------|-------------|
| 0000001111110 | 0000001111100011 | 1000000011000100 |
| 1000000011110100 | 0000010011101000011 | 00001010110000100 |
| 0000000011111000010 | 0000101100011000010 | 000010101110100000 |
| 1000000011000100 | 0000101100011000010 | 000010101110100000 |
| 0000000011111000010 | 0000101100011000010 | 000010101110100000 |
| 0000000011111000010 | 0000101100011000010 | 000010101110100000 |

2.2 $A(16, 4, 5) \geq 322$

We show $A(16, 4, 5) \geq 322$ using the group of order 21 that fixes the first two coordinate positions, and acts on positions 0–13 with the two generators $(0,2,1,4,5,3,6)(7,9,8,11,12,10,13)$ and $(0,2,4)(1,3,5)(7,9,11)(8,10,12)$. The 20 base blocks:

| Base Blocks | Description |
|-------------|-------------|
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |
| 0000000011101000 | 0000010010000011 | 0000001111100000 |

2.3 $A(16, 4, 6) \geq 616$

We show $A(16, 4, 6) \geq 616$ using a group of order 32 isomorphic to the direct product $C_2 \times D_{16}$, generated by the three permutations

$(0,1,2,3,4,5,6,7)(8,9,10,11,12,13,14,15)$,
$(0,1)(2,7)(3,6)(4,5)(8,9)(10,15)(11,14)(12,13)$,
$(0,8)(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)$.

The 27 base blocks:

| Base Blocks | Description |
|-------------|-------------|
| 0000000111000111 | 0000011110010100 | 0000101010100100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |
| 0000000100111111 | 0000111110010000 | 0000101101010100 |

2.4 $A(18, 4, 5) \geq 544$

We show $A(18, 4, 5) \geq 544$ using a cyclic group of order 17 that fixes the first coordinate. The 32 base blocks:
2.5 A(21, 4, 5) ≥ 1113

We show A(21, 4, 5) ≥ 1113 using a group of order 63 that acts on positions 0–20 with the three generators (0,2,1,4,5,3,6)(7,9,8,11,12,10,13)(14,16,18,19,17,20), (0,2,4)(1,3,5)(7,9,11)(8,10,12)(14,16,18)(15,17,19), and (0,7,14)(1,8,15)(2,9,16)(3,10,17)(4,11,18)(5,12,19)(6,13,20). The 19 base blocks:

3 Tables with lower bounds from partitioning

Table 1 below gives lower bounds on A(n, d, w), the maximum size of a binary constant weight code of word length n, minimum distance d, and constant weight w, where d = 4 and 2w ≤ n. These lower bounds are obtained using the partitioning construction discussed below.

| n | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|----|----|----|----|----|
| 18 | 1260° | 2042 | 3186° | 3540° |
| 19 | 1620° | 3172° | 4667° | 6726° |
| 20 | 2304° | 4213° | 7730° | 10048° | 13452° |
| 21 | 2856° | 6161 | 10767 | 17177 | 20654 |
| 22 | 3927° | 8338 | 16527 | 25902 | 37127 | 40624 |
| 23 | 5313° | 11696 | 23467 | 41413 | 58659 | 76233 |
| 24 | 7084° | 15656° | 34914° | 59904 | 98852 | 118422 | 151484 |
| 25 | 7787 | 21220 | 47265 | 89736 | 142372 | 198386 | 231530 |
| 26 | 10010° | 27050 | 66352 | 129682 | 222723 | 320512 | 401937 | 431672 |
| 27 | 12012° | 35874 | 88604 | 188561 | 334834 | 517989 | 686152 | 791449 |
| 28 | 15288° | 44915 | 122685 | 262980 | 508952 | 818897 | 1167909 | 1420892 | 1535756 |

Legenda: °: shortened code. S: Steiner system S(5, 6, 24). #: a group code from [3]. °: idem from [17]. p: product construction from [3]. eb: idem from [9]. Unmarked entries are from this paper.
Next we give a table with lower bounds for $A(n, 4, 5)$ for $29 \leq n \leq 64$, to be compared with the table in [18]. It improves all bounds from that paper except for the three values for $n = 45, 46, 47$. The values marked $^a$ are derived from Steiner systems $S(5, 6, 36)$ and $S(5, 6, 48)$ ([1, 5]). The values marked with a dot are exact.

| $n$  | 29  | 30  | 31  | 32  | 33  | 34  | 35  | 36  | 37  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| bd   | 4423| 4901| 5697| 6582| 7656$^a$| 8976$^a$| 10472$^a$| 10948| 12473|
| $n$  | 38  | 39  | 40  | 41  | 42  | 43  | 44  | 45  | 46  |
| bd   | 13471| 15010| 17119| 19258| 20671| 22728| 25564| 28413| 31878$^a$|
| $n$  | 47  | 48  | 49  | 50  | 51  | 52  | 53  | 54  | 55  |
| bd   | 35673$^a$| 36809| 40560| 42920| 46612| 51420| 56251| 59293| 63973|
| $n$  | 56  | 57  | 58  | 59  | 60  | 61  | 62  | 63  | 64  |
| bd   | 69931| 75550| 79330| 85728| 93206| 100527| 105472| 112457| 121902|

4 The Partitioning Construction

A partition $\Pi(n, w) = (C_1, ..., C_m)$ is a partition of the set of all $\binom{n}{w}$ binary vectors of length $n$ and weight $w$ into codes $C_i$ that all have minimum distance at least 4. By definition, $C_j = \emptyset$ for $j > m$.

The direct product $\Pi(n_1, w_1) \times \Pi(n_2, w_2)$ of two partitions $(C_1, ..., C_{m_1})$ and $(D_1, ..., D_{m_2})$ is the code $\bigcup_i C_i \ast D_i$ (of word length $n_1 + n_2$ and weight $w_1 + w_2$ and size $\sum |C_i||D_j|$), where for two codes $C$ and $D$ the code $C \ast D$ is the code consisting of all possible concatenations $c \ast d$ with $c \in C$ and $d \in D$.

The partitioning construction for codes of length $n$, weight $w$ and minimum distance 4 constructs the code $C = \bigcup_i \Pi(n_1, 2i + \epsilon) \times \Pi(n_2, w - 2i - \epsilon)$ where $n = n_1 + n_2$ and $\epsilon \in \{0, 1\}$ and the union is over all $i$ with $i \geq 0$ and $2i + \epsilon \leq w$.

It is usually nontrivial to construct the required ingredients $\Pi(n, w)$. However, for $w \leq 1$ the partition is trivial, namely the partition into singletons, and for $w = 2$ the optimal partition is that of the $n(n-1)/2$ pairs into $n-1$ parts of size $n/2$ if $n$ is even, and into $n$ parts of size $(n-1)/2$ if $n$ is odd. Partitions $\Pi(n, w)$ and $\Pi(n, n-w)$ are related by complementation. It is always possible to find a $\Pi(n, w)$ with at most $n$ parts, cf. [12].

Example We show $A(18, 4, 7) \geq 2042$. Take $n_1 = 8$, $n_2 = 10$, $\epsilon = 1$, using direct products $\Pi(8, 1) \times \Pi(10, 6)$, $\Pi(8, 3) \times \Pi(10, 4)$, $\Pi(8, 5) \times \Pi(10, 2)$, $\Pi(8, 7) \times \Pi(10, 0)$. From a $\Pi(10, 4)$ with sizes $(30, 30, 30, 28, 26, 23, 22, 20, 1)$ we find $\binom{10}{4} - 1 = 209$ for the first product, from $\Pi(8, 3)$ with 7 parts of size 8 and a $\Pi(10, 4)$ with sizes $(30, 30, 30, 30, 22, 22, 12, 2, 2)$ we find $8\binom{10}{4} - 16 = 1552$ for the second, then $5\binom{8}{3} = 280$ for the third, and 1 for the last, 2042 altogether.

4.1 Improvements by Etzion & Bitan

The code $C$ that results from the partitioning construction is not always maximal. Etzion & Bitan [9] gave a handful of examples of improvements. Let us redo two of their examples here (using improved ingredients).
Example We show $A(21,4,7) \geq 6161$. Take $n_1 = 10$, $n_2 = 11$, $\epsilon = 0$. The products $\Pi(10,0) \times \Pi(11,7)$, $\Pi(10,2) \times \Pi(11,5)$, $\Pi(10,4) \times \Pi(11,3)$, $\Pi(10,6) \times \Pi(11,1)$ contribute $A(11,4,4) + 5 \binom{11}{5} + 17 \binom{10}{5} + \binom{10}{4} = 6125$. For $\Pi(11,5)$ and $\Pi(10,4)$ we used partitions with 9 parts, for $\Pi(11,3)$ the Etzion-Bitan partition with 9 parts of size 17, 1 part of size 3, and 9 parts of size 1, where the 9 parts of size 1 are the triples covering the pair 0$^9$1$^2$. Only the first 9 parts were used, and of $\Pi(11,1)$ also only the first 9 parts were used, so that the vector 0$^9$1$^2$ has distance at least 3 to all second halves used so far, and $A(10,4,5) = 36$ vectors $u \ast 0^91^2$ can be added.

Example We show $A(21,4,8) \geq 10767$. Take $n_1 = 10$, $n_2 = 11$, $\epsilon = 1$. The products $\Pi(10,1) \times \Pi(11,7)$, $\Pi(10,3) \times \Pi(11,5)$, $\Pi(10,5) \times \Pi(11,3)$, $\Pi(10,7) \times \Pi(11,1)$ contribute $\binom{11}{4} + 13 \binom{11}{5} + 17 \binom{10}{5} + 9.13 = 10737$. For $\Pi(11,7)$ we used a partition with 10 parts. For $\Pi(10,7)$ one with 9 parts of size 13 and one part of size 3, that is not used to leave room for $A(10,4,6) = 30$ vectors $u \ast 0^91^2$.

4.2 Varying the split

Instead of keeping $n_1$ and $n_2$ fixed in the partitioning construction, one can use a varying split. For example, one can show that $A(23,4,8) \geq 23467$ using the union of $\Pi(12,0) \times \Pi(11,8)$, $\Pi(12,2) \times \Pi(11,6)$, $\Pi(12,4) \times \Pi(11,4)$, $\Pi(12,6) \times \{0\} \times \Pi(10,2)$, $\Pi(13,8) \times \Pi(10,6)$. Because $\Pi(12,6)$ can be taken to have 9 parts, nothing is lost by taking $\Pi(10,2)$ instead of $\Pi(11,2)$, but something is gained taking $\Pi(13,8)$ instead of $\Pi(12,8)$.

5 Partitions used

Partitions $\Pi(n,0)$ and $\Pi(n,1)$ are trivial, and it is easy to see what the best partitions $\Pi(n,2)$ are (cf. [3]). Nowadays also optimal partitions $\Pi(n,3)$ are known. If $n \equiv 1,3 \pmod{6}$, $n \neq 7$, then a partition of all triples on $n$ points into Steiner triple systems exists ([15, 19, 13]), so that we have a $\Pi(n,3)$ consisting of $n^2$ parts, each of size $n(n-1)/6$. Shortening these we find that for $n \equiv 0,2 \pmod{6}$, $n \neq 6$, there is a partition $\Pi(n,3)$ consisting of $n-1$ parts, each of size $n(n-2)/6$. In [6, 7] partitions $\Pi(n,3)$ are constructed for $n \equiv 4 \pmod{6}$, consisting of $n$ parts, $n-1$ of size $(n^2-2n-2)/6$ and 1 of size $(n-1)/3$. Finally, [14] constructs partitions $\Pi(n,3)$ for $n \equiv 5 \pmod{6}$, $n \neq 5$, with $n-1$ parts, $n-2$ of size $(n^2-n-8)/6$ and 1 of size $4(n-2)/3$. All of these are optimal.

For Table 2 we used only the obvious partitions: for $w \leq 3$ the above ones, for $w = 4$ the Graham-Sloane partitions ([3], Theorem 14), and finally for $w = 5$ the partition with one part as large as possible (the best lower bound known for $A(n,4,w)$) and all other parts arbitrary, for example of size 1. It will be easy to improve these bounds a little.

For Table 1 we spent some effort to find good partitions. In Table 3 below we give the vector of part sizes for the partitions used. The actual partitions can be found near [2].

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## Table 3: Partitions used

| n  | w  | #  |
|----|----|----|
| 8  | 4  | 6  |
| 8  | 14 | 12 |
| 8  | 12 | 10 |
| 9  | 8  | 8  |
| 9  | 18 | 15 |
| 9  | 15 | 8  |
| 10 | 22 | 12 |
| 10 | 22 | 2  |
| 10 | 30 | 20 |
| 10 | 20 | 1  |
| 10 | 30 | 15 |
| 10 | 22 | 2  |
| 10 | 30 | 27 |
| 10 | 26 |
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We give a partition $\Pi(11, 4)$ with 10 parts explicitly (in the notation of [3]).

We also give a partition $\Pi(11, 5)$ with 9 parts. No part has two disjoint 5-sets, so extending by a point and adding complements yields a partition $\Pi(12, 6)$ with 9 parts.

Table 4: Bounds on the chromatic number of Johnson graphs

| $n$ | $w$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|-----|-----|----|----|----|----|----|----|----|----|
| 5   | 5   | 5  | 5  | 5  | 5  | 5  | 5  | 5  | 5  |
| 6   | 6   | 5  | 6  | 6  | 6  | 6  | 6  | 6  | 6  |
| 7   | 7   | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  |
| 8   | 8   | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  |
| 9   | 9   | 9  | 7  | 8  | 8  | 8  | 8  | 8  | 8  |
| 10  | 10  | 9  | 10 | 8-9| 8-9| 8-9| 8-9| 8-9| 8-9|
| 11  | 11  | 10 | 10 | 8-9| 8-9| 8-9| 8-9| 8-9| 8-9|
| 12  | 12  | 11 | 10-11| 10-11| 8-9| 8-9| 8-9| 8-9| 8-9|
| 13  | 13  | 13 | 11-13| 10-13| 10-13| 10-13| 10-13| 10-13| 10-13|
| 14  | 14  | 13 | 11-13| 10-14| 10-14| 10-14| 10-14| 10-14| 10-14|
| 15  | 15  | 15 | 13-14| 12-15| 11-15| 10-15| 10-15| 10-15| 10-15|
| 16  | 16  | 16 | 13-14| 12-15| 11-16| 10-15| 10-15| 10-15| 10-15|

6 Chromatic numbers

The Johnson graph $J(n, w)$ is the graph on the binary vectors of length $n$ and weight $w$, adjacent when they have Hamming distance 2. The graphs $J(n, w)$ and $J(n, n - w)$ are isomorphic. The chromatic number $\chi = \chi(n, w) = \chi(J(n, w))$ is the smallest number of distance-at-least-4 codes its vertex set can be partitioned into. One has $\max(w + 1, n - w + 1) \leq \chi(n, w) \leq n$, where the lower bound is the size of a maximal clique, and the upper bound is due to [12]. One also has the monotonicity inequalities $\chi(n, w) \geq \chi(n - 1, w - 1)$ and $\chi(n, w) \geq \chi(n - 1, w)$. Table 4 below gives lower and upper bounds for $\chi$. 
Discussion
The cases $w = 1$ and $w = 2$ are trivial. For $w \geq 3$ and $n \leq 14$ the upper bounds were already given in [3] (Table VI), except that $\chi(11, 4) \leq 10$ and $\chi(11, 5) \leq \chi(12, 6) \leq 9$ were given above, and $\chi(12, 5) \leq 11$ follows from a partition $\Pi(12, 5)$ with parts $72^{11}$ given in [9]. That $\chi(15, 3) = 13$ follows from the existence of a large set of STS(15) ([4]). Optimal partitions $\Pi(11, 3)$ with parts $17^9 12$ and $\Pi(17, 3)$ with parts $44^{15} 20$ were given in [14]. An optimal partition $\Pi(16, 3)$ with parts $37^{15} 5$ was constructed by Doron Cohen and given in [6]. In particular, $\chi(16, 3) = 16$, in spite of the claim in [9] that $\chi(16, w) \leq 15$ for $2 \leq w \leq 6$. A partition $\Pi(16, 4)$ with parts $140^6 136^6 116$ 48 was given in [8]. That $\Pi(16, w) \leq 15$ for $w = 6, 8$ follows from [11]. A partition $\Pi(16, 5)$ with parts $302^{14} 140$ is found by the product construction from [11], since one can take the union of the last two parts of size 70 each.

Concerning the lower bounds for $w \geq 4$, all except two follow from monotonicity. We verified explicitly that $\chi(9, 4) > 7$—there are five, not seven mutually disjoint codes of word length $n = 9$, constant weight $w = 4$ and size 18. That $\chi(15, 5) > 11$ follows since there is no Steiner system $S(4, 5, 15)$ ([16]), let alone eleven mutually disjoint ones.

In the case of $J(10, 4)$ there exists a coloring with 9 colors where the last color is used only once. So $J(10, 4)$ minus a vertex has chromatic number 8.

It would be interesting to give more general constructions for colorings of $J(n, w)$ with fewer than $n$ colors.

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