Chiral magnetic effect of light

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We study a photonic analogue of the chiral magnetic (vortical) effect in geometric optics. We discuss that the vector component of magnetoelectric tensors plays a role of gravitational fields, and its rotation causes the Coriolis force in a ray of light. The Coriolis force results in anomalous shift of a ray of light through an interplay with the Berry curvature of photons. The mechanism is the same as that of the chiral magnetic (vortical) effect of a chiral fermion. As a result, magnetoelectric material can act as a chiral (helical) prism, which separates lights according to their helicities.

I. INTRODUCTION

The Berry phase [1] and Berry curvature have attracted a lot of interests in subdisciplines of physics such as condensed matter physics, nuclear physics, and particle physics. They characterize topology of wave functions in momentum space and explain many properties of topological materials such as topological insulators [2, 3], topological superconductors [4], and Weyl/Dirac semimetals [5, 6] as well as those of relativistic Weyl/Dirac fermions such as quarks and neutrinos.

It has been known that photons have a nonzero Berry curvature, which originates from massless and helical nature of them [7–9]. The Berry curvature of photons leads to novel effects, which cannot be explained by the standard geometric optics according to Fermat’s principle. The famous example is the Hall effect of light [10, 11], which is also known as the optical Magnus effect [12–15]. A spatially varying refractive index can be understood as “electric field” for lights, and causes transverse shifts of wave packets of lights. This deviation from snell’s law in a finite beam has been confirmed by experiments [16, 17]. The phenomena can be explained by considering the anomalous group velocity of lights due to the interplay between “electric field” and Berry curvature, and the mechanism is the same as that of the anomalous Hall effect in electron systems [18, 19].

However such an analogy between geometric optics and semiclassical dynamics of electrons is not complete yet. This is because photons do not couple with magnetic fields, and no Lorentz force appears in the geodesic equation of a ray of light. Realizing effective “magnetic fields” and “Lorentz force” for lights not only deepens the analogy between wave mechanics and quantum mechanics, which has played an important role in the development of quantum mechanics, but also enriches the physics of photonics. In fact, there have been several works to study the physics of photons under artificial magnetic fields in the context of photonic crystals [20–26].

In this paper, we study a photonic analogue of another novel effect, that is, the chiral magnetic (vortical) effect [27–29], which originates from an interplay of magnetic field (rotation) and Berry curvature [30–33]. We discuss that rotation of the vector component of inhomogeneous magnetoelectric tensors is equal to gravitomagnetic field (Below we term it “magnetic field”), and causes anomalous shifts of wave packets of lights along the direction parallel to it through an interplay with the Berry curvature of photons. The mechanism is the same as that of the chiral magnetic (vortical) effect of chiral fermions [30–33], and we term the phenomena the chiral magnetic effect of a light. We show that because of the chiral magnetic effect of a light, magnetoelectric material can act as a chiral (helical) prism, which shifts circularly polarized lights in opposite directions according to their helicities. We also discuss an analogue of the spectral flow [33, 34], and its implication to geometric optics.

The same effect arising in rotating frame was discussed in Refs. [35, 36]. Ref. [35] calculated spin currents of vector bosons including photons on the basis of the Kubo formula and their analysis is applicable only to equilibrium states (of photons). Ref. [36] also calculated spin currents of photons on the basis of the kinetic theory with the Berry curvature correction, which is, in principle, applicable to nonequilibrium states. However the wave packet dynamics was not quantitatively studied in Ref. [36], which is relevant in geometric optics. In addition, our proposal does not consider noninertial frame under rotation, but the effective metric for electromagnetic fields is generated by materials. It may be tested in magnetoelectric materials such as LiCoPO$_4$, TbPO$_4$ [37, 38], and ZnCr$_2$Se$_4$ spinel with a conical spiral state [39]. We may be able to make spatial gradient of magnetoelectric tensors and experimentally control “magnetic fields,” by applying temperature gradient or preparing inhomogeneous ordering such as domain walls [40].

II. GEODESIC EQUATION OF LIGHT

We discuss the propagation of a monochromatic electromagnetic wave with frequency $\omega$ in anisotropic, inhomogeneous, and lossless medium exhibiting linear mag-
netoelectric effect in Gaussian units:

\[ 4\pi P = \chi_e E - g \times H, \]
\[ 4\pi M = \chi_\mu H + g \times E, \]

where \( P \) and \( M \) (\( E \) and \( H \)) are polarization, and magnetization (electric and magnetic fields), respectively. \( \chi_e, \chi_\mu, \) and \( g \) represent electric susceptibility, magnetic susceptibility, and a vector component of magnetoelectric tensors [37, 39], respectively. We consider a situation where \( g \) slowly varies on space, e.g., \( g = g_0 + (-by, 0, 0) \) or \( g = g_0 + (0, bx, 0) \), with constant vector \( g_0 \) and small gradient \( b \). As shown below, \( \nabla_x \times g \) plays a role of “magnetic field” in the geodesic equation of a ray of light. We however note that \( g \) differs from vector potentials. Different \( g \) such as \( g = g_0 + (-by, 0, 0) \) and \( g = g_0 + (0, bx, 0) \) represent different physical systems, and these are not gauge degrees of freedom.

By taking the polarization and magnetization in Eqs. (1) and (2) into account, the Maxwell equations read

\[ \nabla_x \times (\epsilon E - g \times H) = 0, \]
\[ \nabla_x \times (\mu H + g \times E) = 0, \]
\[ \nabla_x \times E + \frac{1}{c} \frac{\partial}{\partial t} (\mu H + g \times E) = 0, \]
\[ \nabla_x \times H - \frac{1}{c} \frac{\partial}{\partial t} (\epsilon E - g \times H) = 0, \]

where \( \epsilon \) and \( \mu \) are the dielectric permittivity and magnetic permeability, and \( c \) is the speed of light in vacuum. Those equations become the same as the Maxwell equations in rotating frame [41] if \( g = \Omega \times x/2 \) with angular velocity \( \Omega \), and the centrifugal force terms \( O(\Omega^2) \) terms are neglected. Then for the monochromatic wave electric and magnetic fields, \( E = \tilde{E}(\omega)e^{-i\omega t} \) and \( H = \tilde{H}(\omega)e^{-i\omega t} \), we obtain the two eigen equations:

\[ D \times \mu^{-1} (D \times \tilde{E}) - i\tilde{E} = 0, \]
\[ D \times \epsilon^{-1} (D \times \tilde{H}) - \mu\tilde{H} = 0, \]

where \( D = \lambda \nabla_x - ig \), and \( \lambda = \lambda/(2\pi) = c/\omega \) (\( \lambda \) is the wave length of light in vacuum). Because of the inhomogeneous magnetoelectric effect, \( D \) no longer commute with each other and satisfy

\[ [D_i, D_j] = -i\lambda \epsilon_{ijk} b_k, \]

where \( b = \nabla_x \times g \), and \( \epsilon_{ijk} \) is the totally antisymmetric tensor. The noncommutativity leads to the Coriolis force in the dynamics of electromagnetic waves. The Coriolis force acts on only electromagnetic fields, i.e., electrons propagating through the same material do not interact with it. This might be the important difference from the standard Coriolis force in noninertial frame.

We consider the eikonal approximation up to the linear order in \( \lambda \), and derive the geodesic equation of a ray of light with the Berry curvature correction. When \( g = 0 \), and the anisotropy and inhomogeneity of \( \epsilon \) and \( \mu \) are small and treated perturbatively, as was performed e.g., in Refs. [42–44], by introducing the dimensionless momentum operator \( \hat{q} = -i\lambda \nabla_x \) and rewriting Eq. (7) into the Schrödinger-type equation \( \hat{H}\tilde{E} = 0 \), we can diagonalize the \( 3 \times 3 \) matrix \( \hat{H} \), and obtain three eigenvectors. Two of them almost correspond to transverse modes, and the other corresponds to the longitudinal (resonant) mode. Then when the system is off-resonant \((\epsilon \neq 0) [43]\), by neglecting contributions from the resonant mode, we can introduce the Berry connection \( \Lambda \) and Berry curvature \( \Omega \) to describe the noncommutative dynamics in the projected space spanned by the transverse eigenvectors in the same way with quantum mechanics:

\[ [\hat{x}_i, \hat{x}_j] = i\lambda \epsilon_{ijk} \Omega_k, \]

where \( \hat{x} = i\lambda \nabla_q + \Lambda \) are the covariant coordinate operator [42, 43]. In the geodesic equation of a ray of light, the noncommutativity is implemented as the spin-orbit interaction [42–44].

Now we further consider the effect of \( g \) on the basis of the derivative expansion. In fact this can be achieved in the same way with the vector potential in the wave packet dynamics in electron systems [45–48]. The modification is straightforward: We replace the canonical momentum \( q = -i\lambda \nabla_x \) by the covariant momentum \( p = i\lambda \) as \( q = p + g \) in the effective Lagrangian given in Ref. [44]. For simplicity, we hereafter consider locally isotropic medium in which the dynamics becomes abelian [10, 11, 42–44]. Two transverse modes can propagate independently and we do not need to take care of the dynamical change of polarization. Then the geodesic equation reads, in the linear order of \( \lambda \) and \( g \),

\[ \tilde{x}_c = \tilde{p}_c - \tilde{p}_c \times \langle \eta_c | \lambda \Omega | \eta_c \rangle, \]
\[ \tilde{p}_c = e + \tilde{x}_c \times b, \]
\[ (\eta_c) = i\tilde{p}_c \cdot \lambda |\eta_c\rangle, \]

where \( x_c \) and \( p_c \) are the coordinates and dimensionless wave vectors of a ray of light (Wave vectors are given as \( p_c/\lambda \)). For wave packets of lights, \( x_c \) and \( p_c/\lambda \) correspond to position of the center of wave packets and associated wave vector, respectively. \( |\eta_c\rangle = [\eta_+, \eta_-]^\dagger \) represents polarization states of lights. We use right-handed (+) and left-handed (−) circularly-polarized waves as a basis of polarization. The dot means the derivative with respect to the ray length \( l \), namely, the derivative along the trajectory, not time [42–44]. \( n(x_c) \) is isotropic and slowly varying refractive index, and \( e = \nabla_n x_c \). \( \lambda \) and \( \Omega = \nabla_{\hat{p}_c} \lambda - i\lambda \times \lambda \) are the Berry connection and Berry curvature, and given explicitly as [10, 11, 42–44]

\[ \Lambda = \frac{1}{p \tan \theta} e_p \sigma_z, \quad \Omega = \frac{\hat{p}}{p^3} \sigma_z, \]

where \( p, \theta \) and \( \varphi \) are spherical coordinates in \( p \) space, and \( \sigma_z \) is the Pauli matrix.

In the above geodesic equation, \( b \) is the same as the...
magnetic field in the classical equation of motion of electrons, so that we term $b$ “magnetic field.” However it originates from the effective metric induced by magnetoelectric materials, and strictly speaking, is equal to gravitomagnetic field [49]. The same is true for “electric field” $e = \nabla_x \cdot n(x,c)$, that is, it is equal to gravitoelectric field [49]. In fact, if we formulate the classical equation of motion for the wave packets of lights in the same way with Refs. [10, 11]. The coupling constant to “electromagnetic fields” $e = -\nabla_x (c/n)$ [10, 11] and $b = \nabla_x \times (g/n)$ in the associated “Coulomb” and “Lorentz” forces becomes $p$, so that $e$ and $b$ are nothing but the gravitoelectromagnetic fields.

We note that we neglected a self-rotation of a wave packet and the associated “Zeeman energy,” which may not be negligible in a beam with intrinsic angular momentum such as optical vortices. We also note that an idea that is similar but not exactly the same, i.e., to use rotation of the toroidal moment $\nabla_x \times T (T = P \times M)$ as “magnetic fields” of a light was proposed in Ref. [40]. Its relation to our proposal is explicitly shown in supplemental material. The effect of the Berry curvature was not discussed in Ref. [40], which is the main subject of this work.

III. CHIRAL MAGNETIC EFFECT OF LIGHT

We here discuss anomalous shift of a ray of light caused by the interplay between the “magnetic field” and Berry curvature. We find that the geodesic equation in Eqs. (11) and (12) is diagonal for right- and left-handed polarizations, and obtain

$$\sqrt{J}\dot{x}_x = \frac{P_x}{p_x} + \chi \Omega_{++} \times e - \chi b \left( \Omega_{++} \cdot \frac{p_x}{p_x} \right),$$

(15)

$$\sqrt{J}\dot{p}_x = e - b \times \frac{p_x}{p_x} - \chi \Omega_{++} (e \cdot b),$$

(16)

where $\sqrt{J} = 1 - \chi b \cdot \Omega_{++}$, and $\chi = \pm 1$ for right- and left-handed polarization, respectively. The second term in Eq. (15) leads to the optical Hall effect [10–13]. The last terms in Eqs. (15) and (16), respectively, lead to the chiral magnetic effect of a light, and a photonic analogue of the spectral flow as shown below.

Let us first discuss the chiral magnetic effect of a light. We consider the propagation of wave packets of lights. As we can see in Eq. (15), the anomalous group velocity along the direction parallel to the “magnetic field” is generated by the Berry curvature. Since the sign is opposite between the right- and left-handed polarizations, those states propagate along the opposite direction as schematically shown in Fig. 1. We term the phenomena the chiral magnetic effect of a light since $b$ in the geodesic equation is the same as the magnetic field in the classical equation of motion of electrons. We note again that $b$ is rigorously gravitomagnetic field and the phenomena is the same as the photonic chiral vortical effect proposed in Refs. [35, 36], although we do not consider rotating frame, and the metric and gravitomagnetic field are effectively generated by magnetoelectric materials. Because of the chiral magnetic effect of a light, by controlling the magnetoelectric effect, we can make a chiral (helical) prism, which shifts circularly-polarized lights in opposite directions depending on their helicities as schematically shown in Fig. 1.

To confirm the idea, we consider a physical set up similar to Ref. [40], and numerically calculate the trajectory of a ray of light (or equivalently the trajectory of the center position of a wave packet) propagating through thin films at $b\lambda = (0, 0, 2 \times 10^{-3})$ [50] and uniform $n = 0.50, 2.0$ ($e = 0$). We assume that the sample is infinitely large in the $yz$ plane and thin along the $x$ direction. Then we consider the incident light along the $x$ direction with right-handed polarization. Without $b$ field, the light propagates along the $x$ direction. Shifts due to $b$ field are schematically shown in Fig. 2. The shifts along the $y$ and $z$ directions occur because of the “Lorentz force” (Coriolis force) and chiral magnetic effect of a light, respec
contributes to the phase of the outstate as \(|\eta^\text{out}_e| = [e^{i\Theta_+} \eta^+_e, e^{i\Theta_-} \eta^-_e]^t\), where the polarization of the initial state is \(|\eta^0_e| = [\eta^+_e, \eta^-_e]^t\). When \(e \cdot b\) is nonzero, the line element of the integrals becomes different between right- and left-handed polarization states because of the aforementioned helicity-dependent acceleration/deceleration, so that the phase difference \(\Theta_+ \neq \Theta_-\) arises at nonzero \(e \cdot b\). This is one approach to discuss a photonic analogue of the spectral flow.

IV. SUMMARY

We have studied the photonic analogue of the chiral magnetic (vortical) effect in geometric optics. We proposed that the vector component of spatially inhomogeneous magnetoelectric tensors \(g\) behaves as metric, and generates the Coriolis force in the dynamics of electromagnetic fields. Since the Coriolis force in the geodesic equation of a ray of light in Eqs. (11) and (12) has the same form with the Lorentz force in the classical equation of motion of electrons, we term \(b = \nabla_x \times g\) “magnetic field” for lights (Strictly speaking, \(e = \nabla_x \eta\) and \(b\) correspond to gravitoelectromagnetic fields). The interplay between the “magnetic field” and Berry curvature of photons causes anomalous shifts of wave packets of lights along the direction parallel to the “magnetic field.” As a result, magnetoelectric material can play a role of a prism, which separates lights according to their helicities. We have also discussed the spectral flow induced by the pseudoscalar product of “electromagnetic fields” for lights, \(e \cdot b\), which causes the momentum separation of lights (Lights are accelerated/decelerated depending on their helicities). This effect may be observed as the helicity-dependent phase shift. Such a phase shift due to inhomogeneous magnetoelectric effect has also been discussed in topological insulators [2, 52–55], which involves the scalar component of magnetoelectric tensors [37, 39].

There are several generalizations of our work. One direction is to consider photonic crystals with effective magnetic fields [20–26], in which the chiral magnetic effect of a light might be enhanced as in the case of the optical Hall effect [10, 11]. Another direction is a generalization to include another form of the Berry curvature, which involves the temporal derivative, and is understood as emergent electric fields in momentum space [19]. It causes anomalous transport effects such as the Thouless pumping [56]. It will be interesting to discuss a photonic analogue of the Thouless pumping, which may give a sizable effect due to the massless nature of photons as in the case of Weyl semimetals [57].

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Chiral magnetic effect of light: Supplemental material

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I. NONRECIPROCAL REFRACTIVE INDEX

We here clarify the relation between our proposal of the “Lorentz force” of a light and the one given in Ref. [1]. For this purpose, we derive the dispersion relations of electromagnetic waves satisfying the Maxwell equations in the presence of magnetoelectric effects, whose explicit forms are given Eqs. (3)-(6) in the main text. We consider the case that the electric susceptibility $\epsilon$ and magnetic susceptibility $\mu$ are isotropic and spatially uniform, and the vector component of magnetoelectric tensors $g$ is spatially uniform. Then the Maxwell equations (11) and (12) in the main text are written as

$$(-D^2 - \epsilon \mu) E = 0,$$

$$(-D^2 - \epsilon \mu) H = 0,$$

where $E = \tilde{E} e^{i|\mathbf{q}| z - \omega t}$ and $H = \tilde{H} e^{i|\mathbf{q}| z - \omega t}$, and $D = i(\epsilon / \omega) \mathbf{q} - i \mathbf{g}$. The on-shell condition $D^2 + \epsilon \mu = 0$ is the quadratic equation with respect to $\omega$, so that we can obtain

$$\omega_\pm = (c')^2 \left( -1/c \mathbf{q} \cdot \mathbf{q} \pm \sqrt{\mathbf{q}^2 / (c')^2 + (1/c \mathbf{g} \cdot \mathbf{q})^2} \right),$$

where $c' = c / \sqrt{\epsilon \mu - \mathbf{g}^2}$. The dispersion relations are nonreciprocal, namely, depend on the propagating direction if $\mathbf{g} \cdot \mathbf{q} \neq 0$. We study Eq. (3) in detail for two cases: (I) $|\mathbf{g}| \ll \sqrt{\epsilon \mu}$ ($c'/c \gg 1$). In this case, we have

$$\omega_+ = \frac{c}{2g} q_x + \frac{c}{2g q_x} (q_y^2 + q_z^2),$$

$$\omega_- = -2(c')^2 \frac{c}{2g q_x} - \frac{c}{2g q_x} (q_y^2 + q_z^2),$$

where we assumed $\mathbf{g} = (g, 0, 0)$, and $g q_x > 0$. Photon shows the quadratic (nonrelativistic) dispersion relations along the direction perpendicular to $\mathbf{g}$, while it shows the linear dispersion relations along the direction parallel to $\mathbf{g}$. Next let us consider the case: (II) $|\mathbf{g}| \ll \sqrt{\epsilon \mu}$. In this case, we have

$$\omega_\pm = \pm \tilde{c} |\mathbf{q}| \left( 1 \mp \frac{\tilde{c} \mathbf{g} \cdot \mathbf{q}}{c^2} \right),$$

where $\tilde{q} = q / |\mathbf{q}|$, and $\tilde{c} = c / \sqrt{\epsilon \mu}$. The effective refractive index $n_\pm$ is defined as $\omega_\pm = \pm (c / n_\pm) |\mathbf{q}|$, and we get

$$n_+ - n_- = 2 \mathbf{g} \cdot \tilde{\mathbf{q}}.$$

This is nothing but the optical magnetoelectric effect discussed in Ref. [1], and the origin of the “Lorentz force” in their proposal. Therefore our proposal to use the inhomogeneous magnetoelectric effect is essentially the same as the one in Ref. [1].

II. CHIRAL MAGNETIC EFFECT AT AN INTERSURFACE

We here compute transverse displacements of a wave packet in a physical setup discussed in Ref. [2], with taking the magnetoelectric effect into account. We consider the transmission of a monochromatic wave packet through the $z = 0$ boundary from vacuum ($z < 0$) to a half-infinite magnetoelectric medium ($z > 0$) [See Fig. 1(a)]. At a sharp surface between them, $n = \sqrt{\epsilon \mu}$ and $\mathbf{g}$ rapidly change like the step-function, and the delta-function-like effective electric field $\mathbf{e} = \nabla n \sim \delta(z) \hat{\mathbf{z}}$ and magnetic field $\mathbf{b} = \nabla \times \mathbf{g} \sim \delta(z) \hat{\mathbf{y}}$ are induced (Hereafter we consider $\mathbf{g} = (g, 0, 0) [g > 0]$). Then we expect shifts of a wave packet due to the spin Hall/chiral magnetic effect of a light.

The Maxwell equations (3)-(6) in the main text have the same form with those in moving dielectrics [3] with the velocity $\beta = v/c = -\mathbf{g} / (n^2 - 1)$ when $O(\beta^2)$ terms are negligible, so that transverse displacements can easily be computed by considering the spin Hall effect in the “rest” frame, which moves with the surface of a virtual dielectric material. Following Ref. [2], we consider a polarized incident beam with finite distribution along the $y$ direction, and the wave vector of the incident beam is represented as $\mathbf{q}_i = \mathbf{q}_i (\hat{\mathbf{z}}_i + \nu_y \hat{\mathbf{y}})$, where $\nu_y = q_y / q_i$ ($q_y / q_i \ll 1$), and $q_y$ has a distribution around zero. As a model of polarization of the incident beam, we employ [2, 4]

$$\tilde{E}_i = \frac{E_0}{\sqrt{1 + |m|^2}} (\tilde{x}_i + m \hat{\mathbf{y}} - m \nu_y \hat{\mathbf{z}}_i),$$

FIG. 1. Schematic figure of the incident, and refracted lights in the laboratory frame (a) and “rest” frame (b). Blue arrows denote the coordinates defined by the central wave vectors.
where ̂x is the unit vector along the x direction, and (x, y, z) is the coordinate defined by using the central wave vector \( \mathbf{q}_c = q \hat{z} \) of the incident beam (See Fig. 1(a)). We can change the frame by considering the Lorentz transformation with the boost parameter 
\[ -\beta = -\beta \hat{x} - \beta \cos \theta \hat{x}_1 - \beta \sin \theta \hat{z}_1 \] 
[\( \beta = q/(\eta^2 - 1) > 0 \)]. Then the incident electric field in the “rest” frame is given as \( \hat{\mathbf{E}}_1 = \hat{\mathbf{E}}_1 - \beta \times \hat{\mathbf{H}}_1 \), with \( \hat{\mathbf{H}}_1 = c \mathbf{q} \times \hat{\mathbf{E}}_1/\omega \):

\[
\hat{\mathbf{E}}_1 = \frac{\mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( (1 + m\beta \cos \theta \nu_y \nu_0) \hat{x}_1 \right) + (m - \beta \cos \theta \nu_y \nu_0 \theta) \hat{y} - m \nu_y \hat{z}_1 \tag{9}\]

where \( \mathbf{E}_0 = (1 + c\beta \cdot \mathbf{q}_i/\omega) \mathbf{E}_0, \nu_y = (1 - \beta \sin \theta) \nu_0, \) and we neglected \( O(\beta^2) \) and \( O(\nu_0^2) \) terms. The new coordinates \((x_1', y_1', z_1')\) are tilted by the angle \( \delta \theta = \beta \cos \theta \) from the co-moving coordinates \((\hat{x}_1, \hat{y}_1, \hat{z}_1)\) [See Fig. 1(b)]:

\[
x_1' = \hat{x}_1 - \beta \cos \theta \hat{z}_1, \tag{10}\]
\[
y_1' = \hat{y}_1 - \beta \cos \theta \hat{x}_1, \tag{11}\]

where we neglected \( O(\beta^2) \) terms. Because of the transverse Lorentz boost (the Lorentz transformation along the x1 direction), the momentum is shifted along the x1 direction:

\[
\mathbf{p}_1' = \mathbf{p}_1 + \nu_y \hat{y} \theta, \tag{12}\]

where \( \mathbf{p}_1' = (1 + \beta \sin \theta) \mathbf{p}_1 \), so that the polarization is also rotated to satisfy the transversality condition \( \mathbf{q}_1 \cdot \hat{\mathbf{E}}_1 = 0 \) as in Eq. (9). In the boosted frame, the electromagnetic field with the polarization (9) is incident upon a dielectric medium with the modified incident angle \( \theta_1 = \theta + \delta \theta \).

Now we discuss the transmission of a light. The eigenstates of transmission are s- and p-polarization states defined by using the local beam coordinates of \( \mathbf{q}_1' = (x_1', y_1', z_1') \), not those of the central wave vector \( \mathbf{q}_1 \). The central wave vector coordinates \((x_1', y_1', z_1')\) is related with the local beam coordinates \((x_1'', y_1'', z_1'')\) by the following relations:

\[
x_1'' = x_1' + \nu_y \cos \theta_1 \hat{y}, \tag{13}\]
\[
y_1'' = y_1' - \nu_y \cos \theta_1 \hat{x}_1' - \nu_y \hat{z}_1, \tag{14}\]
\[
z_1'' = z_1' + \nu_y \hat{y}. \tag{15}\]

Now Eq. (9) reads

\[
\hat{\mathbf{E}}_1 = \frac{\mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \cos \theta_1 \hat{y} \right) + m \frac{\mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \left( \cos \theta_1 + \beta \cos \theta \right) \hat{y}_1'' \right) \tag{16}\]

Then the amplitude of the transmitted electric field reads

\[
\hat{\mathbf{E}}_T = \frac{t_p \mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( (\nu_y \cos \theta_1 \hat{y} \right) + \frac{m t_s \mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \left( \cos \theta_1 + \beta \cos \theta \right) \hat{y}_1'' \right) \tag{17}\]

where \( t_s \) and \( t_p \) are the Fresnel coefficients. In terms of the central wave vector coordinates, it reads

\[
\hat{\mathbf{E}}_T = \frac{t_p \mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \hat{y} \right) + \frac{m t_s \mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \left( \cos \theta_1 + \beta \cos \theta \right) \hat{y}_1'' \right) \tag{18}\]

where we used Snell’s law: \( \sin \theta_1 = n \sin \theta_T \), and \( \nu_y = n \nu_y \). By considering the inverse Lorentz transformation with the boost parameter \( \beta = \beta \hat{x} = \beta \cos \theta \hat{x}_1 + \beta \sin \theta \hat{z}_1 \), we obtained the transmitted wave in the laboratory frame (in a magnetoelectric material) as

\[
\hat{\mathbf{E}}_T = \frac{t_p \mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \cos \theta_1 \hat{y} \right) + \frac{m t_s \mathbf{E}_0}{\sqrt{1 + |\mathbf{m}|^2}} \left( \nu_y \cos \theta_1 \hat{y}_1'' \right) \tag{19}\]

where \( \hat{x}_T = \hat{x}_T' + n \beta \cos \theta \hat{x}_1 ' + \hat{z}_T = \hat{z}_T' - n \beta \cos \theta \hat{x}_1 ' \). This rotation is again needed to satisfy the transversality condition.

Now we compute the transverse displacements of a wave packet upon refraction. First, we consider a horizontally-polarized incident beam \( m = 0 \) with finite distribution along the y direction [2]:

\[
\Psi_{initial} = \frac{\Psi(y)}{\sqrt{2}} (1, 1) = \int dp_y e^{ip_yy} \Phi(p_y) \frac{1}{\sqrt{2}} (1, 1), \tag{20}\]

where \( (\eta_+, \eta_-)^T \) represents the state of polarization in the spin basis. The right- and left-handed states rotate in an opposite way upon refraction to satisfy the transversality condition, which leads to the transverse displacement of a central position via the spin-orbit coupling [2]:
Electrodynamics of Continuous Media, 2nd ed. (Pergamon, Amsterdam, 1984).

K. Y. Bliokh and Y. P. Bliokh, Phys. Rev. E 75, 066609 (2007).