Research on multi-task command allocation method for flying wing aircraft

Ning Zhang 1,2, Lixin Wang 2 and Feng Li 1

1 China Academy of Aerospace Aerodynamics, Beijing 100074, China; 2 School of Aeronautical Science and Engineering, Beihang University, Beijing 100191, China

Abstract. Flying wing aircraft adopts redundant multiple control surfaces configuration, especially the new type of drag based yaw control device is used in directional axis. Therefore, its command allocation problem has strong nonlinear and multi-axis coupling characteristics. According to the command allocation characteristics of flying wing aircraft, a multi-objective optimization based nonlinear programming method is used for command allocation, and the differences of the control surface command allocation results under different optimization targets is analyzed. The results are also carried out in the attitude tracking simulation based on flight control system designed with nonlinear dynamic inversion. It demonstrates that the different optimization targets have great and quite different impacts on flight states. It proves that the multi-objective optimization based control allocation method is a highly adaptable and flexible command allocation method for flying wing aircraft.

1. Introduction

Flying wing aircraft generally has concise configuration, satisfactory lift and loading characteristics. Nevertheless, the flying wing differs greatly from conventional aircraft in aerodynamic configuration, flight dynamics and control characteristics [1]. Poor stability and difficulty in control become the main concern of flying wing aircraft design, which hinder the development and application of the flying wing for a long period. With the development of the fly-by-wire flight control technology and increasing stealth requirement for military aircraft since 1970s, especially inspired by the successful service of B-2 stealth bomber, the design and research of flying wing aircraft enter into a new boom period and receive more and more attention. But flying wing aircraft lack conventional stabilizing surfaces and the associated control surfaces, it is necessary to adopt novel control surfaces to ensure sufficient control power. More specifically, more attention is paid to improving yawing and pitching control power by adding novel effectors in the airframe design. Innovative drag rudders and elevons are common combination in flying wing, and redundant control surfaces are designed and used to meet the controllability requirements [2,3].

Conventional aircraft usually have three actuators, namely, elevator, aileron and rudder, which control the flight attitude of the aircraft by generating three-axis control moments [4]. Therefore, in the range where the three-axis control moment is reachable, the control input that the aircraft required for maneuvering is uniquely determined. Flying wing aircraft adopts multiple sets of control surface configurations. In order to solve the redundant problem that the number of control surfaces is more than the expected control axes and the control surface function is not unique, a command allocation
unit is required to optimally assign the desired control moment generated by the control law to the relevant actuators[5-7].

2. Control surfaces dynamic characteristics for flying wing aircraft

Due to the innovative control effector of the flying wing aircraft, especially the directional axis adopts the drag typed control effectors, such as the all-moving wing tip used on the tailless fighter, as shown in Figure 1, it is found that for flying wing aircraft not only the longitudinal and lateral-directional axes under different flight conditions have strong coupling effects, but also the generated large amount of drag cannot be ignored. Therefore, drag, lift and side force should also be considered in command control allocation.

![All moving wing tip](image)

Figure 1. Multi-axis control coupling effect of all-moving wing tip.

The control surface aerodynamic nonlinear characteristics of the flying wing aircraft are very strong, which leads to the traditional simple linear allocation method no longer applicable [4, 8]. Besides, the drag based control effector usually adopts a single-sided large deflection strategy, such as the all-moving wing tip deflection range is [0, 60] degree, the split drag rudder is even in the range [0, 90] degree, and so the dynamics of the actuator cannot be ignored, including the frequency and rate of deflection. Here only the speed limit $\dot{\delta}_i$ of the actuator is considered, namely,

$$\rho_{i\min} < \dot{\delta}_i < \rho_{i\max} (i = 1 \ldots n)$$

(1)

Where $\rho_{i\min}$ and $\rho_{i\max}$ are the minimum and maximum deflection rate of each control surface. Therefore, the command allocation problem for tailless aircraft is described as follows[9],

$$g(u(t)) = M_d(t)$$

s.t. $\delta_{i\min} \leq u_i \leq \delta_{i\max}$

(2)

Where $g(\cdot)$ represents the nonlinear function of mapping control surface deflection $u(t)$ into control moments, $M_d(t)$ are the desired three-axis control moments, $\delta_{i\min}$ and $\delta_{i\max}$ are the lower and upper position limits for the $i_{th}$ control surface. For a multi-control surface flying wing aircraft usually has $n$ control surface. And $\delta$ is,

$$u = [\delta_1, \delta_2, \ldots, \delta_{i}, \ldots, \delta_n] \quad (n > 3)$$

(3)

The control surface position and rate limit can be approximated as follows, the upper and lower limits of the position within a single sampling period,

$$\delta_{\max} = \max(\delta_{i\max}, \delta(t-T) + T\rho_{i\max})$$

$$\delta_{\min} = \min(\delta_{i\min}, \delta(t-T) - T\rho_{i\max})$$

(4)

Dynamic control allocation based on nonlinear conditions is a very challenging research direction [10-14].

3. Control surface aerodynamic model for flying wing aircraft

Different from the conventional aircraft, as shown in Figure 2, a small aspect ratio flying wing combat aircraft named ICE is equipped with multiple control surfaces. The function of each control surface
has no obvious axial direction. The all-moving wing tip and the spoiler slot deflector are innovative drag type yaw control effector.

\[ \Delta C = \sum_{j=1}^{n} k_j(Ma, \alpha, \beta)\delta_j \] (5)

\( \Delta C \) represents the control coefficient for each control surface, \( k_j(Ma, \alpha, \beta) \) is function of different Mach numbers, angles of attack and side-slip angles, \( n \) is the dimension of the polynomial space determined based on the experimental data.

Taking the all-moving wing-tip as an example, the space function base only takes three dimensions \([\delta, \delta^i, \delta^j]\), in \( Ma = 0.6 \), trim angle of attack (AOA) 4.2 degree, the control aerodynamic model can be fitted as shown in Figure 3. It can also be seen that the all-moving wing tip will produce non-negligible tri-axial forces.
4. Multi-task command allocation method

Under different mission requirements, multi-objective nonlinear programming method is adopted to achieve different performance under different flight phases. The mission requirements can be described as follows,

$$\min J = \sum_{i=1}^{n} \omega_i f_i = \sum_{i=1}^{n} \omega_i \frac{f_i - f_{i,\text{min}}}{f_{i,\text{max}} - f_{i,\text{min}}}$$

$$\text{S.t} \quad g(\delta) = M_d \quad \delta_{\text{min}} \leq \delta \leq \delta_{\text{max}}$$

In above equations, $\omega_i$ is the weighting factor, satisfying $\sum_{i=1}^{n} \omega_i = 1$, which is used to characterize the importance of each individual optimization goal; $f_i$ represents the first $i$ positive value equalization indicators for optimization objective, $f_{i,\text{max}}$ and $f_{i,\text{min}}$ are the maximum and minimum values for the chosen variable in the feasible range respectively, which shall be determined according to the chosen variables and practical physical constraints of the actual aircraft.

Assume that the expected control moment coefficients of the aircraft at a certain time is $[0.02, -0.02, -0.02]$, and different optimization objectives for control allocation is adopt to verify the above multi-objective nonlinear programming allocation method.

It is common that the lift and drag of a certain aircraft should satisfy certain constraints during flight, and different flight missions have different requirements. For example, if only the optimization of minimum drag and maximum lift is considered, then the optimization target and weighting coefficients should be set to the form as is shown in equations (8) and (9).

$$f_1 = \Delta C_D, \quad \omega_1 = a$$

$$f_2 = -\Delta C_L, \quad \omega_2 = 1 - a$$

As shown in Figure 4, with the increasing of the weight indicator $a$ ($a \in [0, 1]$), the drag increment and the lift increment will decrease simultaneously. When $a$ is 0, the allocation result indicates that the additional lift increment of the control surface deflection is the largest, and the pitch flap is deflected to the maximum deflection as is shown in Figure 5; when $a$ is 1, the allocation result indicates that drag increment of the deflected control surface is minimal. As shown in Figure 6, when the optimization target of minimal drag and maximum lift is adopted, the drag rudder automatically adopts the single-sided deflection policy to participate in the manipulation.

When considering the optimization goal of maximum drag and maximum lift, the optimization objectives and weighting coefficients are set to the form in equations (10) and (11).

$$f_1 = -\Delta C_D, \quad \omega_1 = a$$

$$f_2 = -\Delta C_L, \quad \omega_2 = 1 - a$$

![Figure 4](image-url)  
**Figure 4.** Lift and drag optimization results with weighting coefficients varying in [0, 1].
Figure 5. Longitudinal and lateral control surface assignment results with weighting coefficients varying in [0, 1].

Figure 6. Directional control surface assignment results with weighting coefficients varying in [0, 1].

As shown in Figure 7, with the variation of the weight indicator \( a \) \((a \in [0, 1])\), the drag increments increases and the lift increments decreases. When \( a \) is 0, the allocation result indicates that the additional lift increment of the control surface deflection is the largest, and the pitch flap is deflected to the maximum deflection as shown in Figure 8; when \( a \) is 1, the result indicates that the drag increments of the control surface is maximum, and both sides of the drag rudder are deflected, resulting in a large additional drag increments, as shown in Figure 9.

The above verification results show that taking different targets or index weights has a large difference in the additional drag or lift caused by the command allocation result. Therefore, in actual flight, different optimization objectives should be selected according to certain task requirements.

5. Simulation verification
An attitude tracking control law based on nonlinear dynamic inversion is used to verify the applied nonlinear programming command allocation method [15-17]. The designed control system is very suitable for intercepting air combat, ground attack, landing and other mission modes, and can be directly applied to advanced control system. The dynamic inversion method is shown in Figure 10.

Figure 7. Lifting and drag optimization results with weighting coefficients varying in [0, 1].
Figure 8. Longitudinal and lateral control surface assignment results with weighting coefficients varying in $[0, 1]$.

Figure 9. Directional control surface assignment results with weighting coefficients varying in $[0, 1]$.

Figure 10. Dynamic inversion process.

The tri-axial moment is calculated by inverse dynamics from the tri-axial angular velocity as shown in equation (12)

$$
\begin{align*}
M_x &= I_x \omega_x + (I_y - I_z) \omega_y \omega_z + I_{xy} (\omega_y \omega_z - \omega_x) \\
M_y &= I_y \omega_y + (I_z - I_x) \omega_z \omega_x - I_{yz} (\omega_z \omega_x + \omega_y) \\
M_z &= I_z \omega_z + (I_x - I_y) \omega_x \omega_y - I_{xz} (\omega_x \omega_y - \omega_z)
\end{align*}
$$

(12)

The tri-axial angular velocity is inversed from the tri-axial attitude angle as shown in equation (13)

$$
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} =
\begin{bmatrix}
1 & \sin \theta & 0 \\
0 & \cos \gamma \cos \theta & \sin \gamma \\
0 & -\sin \gamma \cos \theta & \cos \gamma
\end{bmatrix}^{-1}
\begin{bmatrix}
\dot{\gamma} \\
\dot{\psi} \\
\dot{\theta}
\end{bmatrix}
$$

(13)

The dynamic model between the desired attitude angle response and the instruction conforms to a simple first-order hysteresis system as follows
\[
\frac{\phi}{\phi_c} = \frac{1}{T_\theta s + 1}, \quad \frac{\psi}{\psi_c} = \frac{1}{T_\psi s + 1}, \quad \frac{\theta}{\theta_c} = \frac{1}{T_\theta s + 1}
\]  

(14)

The time constants of the three axes $T_\theta, T_\psi, T_\theta$ are chosen as 0.33s, 0.5s and 0.2s respectively.

Based on the wind tunnel test data, the nonlinear aerodynamic model of small aspect ratio flying wing aircraft ICE is adopted. The full-order six-degree-of-freedom aircraft dynamics and motion model are built, and the dynamic characteristics of the eight control surfaces are considered as one order system, as is shown in Table 1.

| Control surface       | Elevon for pitch control (ELEV) | Pitching flap (PF) | All-moving wing tip (AMT) | Spoiler slot deflector (SSD) |
|-----------------------|---------------------------------|-------------------|---------------------------|------------------------------|
| Deflection range      | -30°~30°                         | 0°~90°            |                           |                              |
| Rate limit            | ±100°/s                         |                   |                           |                              |
| Time constant         | 0.05s                           |                   |                           |                              |

Fast attitude tracking control has certain practical significance in actual air combat. As shown in Figure 11, when the intercepting target is attacking the upper right of the aircraft, the target is going to turn around after the attacking the aircraft, so the pilot of intercepting aircraft needs to quickly point the nose at the target and keep track and lock at the target aircraft. This fast-pointing maneuver can shorten the air combat response time without changing the track to lock the target.

Using the attitude command tracking control system described above, the above air combat action is simulated, and different trapezoidal command signals are simultaneously applied to the three directions of roll angle, yaw angle and pitch angle, so that the aircraft is maneuvered from the cruise attitude to a roll angle of -10 degree, the pitch angle is 15 degree and yaw angle is 20 degree.

Three optimization targets with minimum drag, maximum lift and maximum drag are used for command allocation, and Figure12 shows the results of attitude tracking. It can be seen that the attitude tracking characteristics are good, there is no overshoot and oscillation in the two important attitude directions of pitch and yaw axes, and the roll angle is accompanied by a small oscillation at the beginning and end of the maneuver.

Figures 13 and 14 depict the changes in aerodynamic angle and flight state. It can be seen that the trimmed angle of attack is the smallest when the maximum lift is used as optimization objective, and the maximum drag is corresponding to the maximum angle of attack; the change of the side slip angle is basically the same. Most importantly, the aircraft has the fastest increase in altitude when it is optimized for maximum lift, and rises by about 210 meters in 10 seconds. When the maximum drag is
used as optimization objective, the speed of the aircraft drops very fast, and it decreases by 90 m/s in 10 seconds.

![Figure 12. Command tracking results.](image1)
![Figure 13. Simulation results of angle of attack and side slip angle.](image2)
![Figure 14. Speed and height simulation results.](image3)

Therefore, the simulation results further illustrate that adopting different command allocation optimization objectives has a greater impact on aircraft states. Under normal tasks, the minimum drag can be selected as main optimization objective. Under this condition, the required energy of the aircraft is the smallest. When flight task is climbing and tracking, the maximum lift shall be selected
as main optimization objective; when flight task requires deceleration tracking, the optimization objective of maximum drag and minimum lift should be used.

Figure 15 to 18 show the deflection of each control surface under the above three optimization objectives. It can be seen that the deflection of the pitch flap is maximized under the maximum lift optimization objective, and under the maximum drag optimization objective both sides of the drag rudder (all-moving wing tip and spoiler slot deflector) simultaneously deflected, therefore generating very large drag increment.

Figure 15. Elevon deflection results.

(a) Left elevon deflection results  
(b) Right elevon deflection results

Figure 16. Pitch flap deflection results.

(a) Left pitch flap deflection results  
(b) Right pitch flap deflection results

Figure 17. Right all moving wing tip deflection results.

(a) Left all moving wing tip deflection results  
(b) Right all moving wing tip deflection results

Figure 18. Right spoiler slot deflector deflection results.

(a) Left spoiler slot deflector deflection results  
(b) Right spoiler slot deflector deflection results
6. Conclusion

A multi-objective optimization based nonlinear programming method is used for flying wing command allocation. The differences of the control surface command allocation results under different optimization objectives is analyzed. The proposed method is also analyzed in the attitude tracking simulation based on flight control system designed with nonlinear dynamic inversion. It proves that the multi-objective optimization based control allocation method is a highly adaptable and flexible command allocation method for flying wing aircraft.

References

[1] Yang Enquan, Key Technology 2006 Research of Flight Control System for Unmanned Combat Aircraft Beijing University of Aeronautics and Astronautics 5
[2] Kenneth A. Bordignon, Wayne C. Durham Closed-Form solution to the constrained control allocation problem Virginia Polytechnic Institute and State University AIAA-94-3552-CP
[3] Zhan Zhengyong, Liu Lin 2006 Research on Aircraft Control Distribution Technology for Advanced Layout of Multi-Control Surfaces Flight Mechanics 24(01)
[4] Teel A. R, Buffington J M 1996 Anti-Windup for an F-16’s daisy chain control allocation Journal of Guidance, Control and Dynamics 19(6) 1226-1230
[5] Wayne C. Durham, Constrained control allocation Virginia Polytechnic Institute and State University AIAA-92-4550-CP
[6] Wayne C. Durham 1994 Computationally efficient control allocation Journal of Guidance control and Dynamics 17(6) 1371~1373
[7] Durham W C 2001 Attainable moments for the constrained control allocation problem Journal of Guidance control and Dynamics 24(3) 519~524
[8] Li Weiqi, Wei Chen, Chen Zongji 2005 New algorithm for direct allocation of restricted control Journal of Beijing University of Aeronautics and Astronautics 31(11)
[9] Shen Jinglai, Gao Jinyuan 1999 Optimal allocation of linear control for constrained control quantities of flight control systems Flight Mechanics 17(2)
[10] Marc Bodson. Evaluation of optimization methods for control allocation University of Utah Electrical Engineering AIAA-2001-4223
[11] Wayne C. Durham, John G. Bolling, Kenneth A. Bordignon. Minimum drag control allocation Virginia Polytechnic Institute and State University AIAA-96-3410-CP
[12] Wang Meixian, Li Ming 2006 Review of Research on Control Method of Advanced Fighter Control Aircraft Design 17(3)
[13] Mark D. Nelson, Wayne C. Durham. A comparison of two methods used to deal with saturation of multiple redundant aircraft control effectors Virginia Polytechnic Institute and State University AIAA-2002-4498
[14] Ola Härkegård. Dynamic control allocation using constrained quadratic programming AIAA-2002-4761
[15] Liu Yanbin, Lu Yuping 2006 Application of nonlinear adaptive control in tailless flight control system Acta Aeronautica Sinica 27(5)
[16] Ram Venkataraman, Michael Oppenheimer, and David Doman 2004 A new control allocation method that accounts for effector dynamics IEEE Aerospace Conference, Big sky, MT
[17] Liu Jiarun, Shen Gongyu 2005 Aircraft attitude control based on inverse dynamics and online parameter identification Journal of Beijing University of Aeronautics and Astronautics 31(2)