Comparative investigation on nonlinear transient vibrations of laminated composite shallow shell with active constrained layer damping treatment using PFRC and AFC patch material

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Abstract. Here, the comparative investigation for the performance of active constrained layer damping treatment using PFRC and AFC patch material for controlling the nonlinear transient vibrations of laminated composite shallow shell is carried out. The patch material used in this case for making the constraining layer of the ACLD treatment is PFRC and AFC. For modelling of ACLD in the time domain Golla-Hughes-McTavish (GHM) method is used. The Von Kármán type non-linear strain displacement relations along with a simple first-order shear deformation theory are used for deriving this electromechanical coupled problem. A finite model in 3-dimensions have been developed for smart composite shallow shell integrated with the ACLD treated patches of PFRC and AFC. The results of both the cases have been compared and it has been found that the improved performance of AFC patch over the PFRC in subsiding the transient nonlinear vibrations of symmetric cross ply laminated composite shallow shell.

1. Introduction
Recent trend shows the use of lightweight flexible structures in many applications such as spacecrafts, aircrafts and so on. This demands the laminated composite structures in use. But lighter laminated flexible structures are weak to withstand the vibrations having wide-ranging decay time since they possess little internal damping. To increase the strength and damping, an alternative active control system is designed with the help of sensors and actuators and integrated with the host structure to make it a “smart structure” [1]. Different piezoelectric materials’ distributed sensors and actuators are used in making of smart structures and many researchers have worked on this in recent years. Iman Fattahi and Hamid Reza Mirdamadi [2] have used piezoelectric smart structures in formulating the FE model based on equivalent single layer theory and all parameters were calculated for 3D active beam elements. M. Kerboua and et.al [3] have investigated the vibration control with the use of piezoelectric based smart materials and results showed that vibration reduction by using smart beam. Shaikh Tauseef and R. K. Agarwal [4] have studied the active vibration of cantilever beam. They used the lead zirconium titanate patches as actuators and by applying different voltages, amplitude of vibration
was observed. Ehsan Omidi and et al [5] have worked on vibration reduction using the velocity feedback control. The optimization problem was addressed by linear quadratic regulator (LQR). Ashok M H and et al [6, 7] worked on actively damped laminated composite shallow shells and plates with AFC actuators. The surplus vibrations of the host structure could be minimized by using ACLD treatment with PFRC/AFC patches to it. Though the work related to PFRC and AFC patch materials is reported in the papers but comparative investigations were not made. In this paper, comparison of performance of ACLD with PFRC and AFC in controlling the nonlinear transient vibrations is presented.

2. Finite element model
In this section, the FE model is generated for performance evaluation of ACLD treatment. Figure 1 shown here gives details of $N$ orthotropic layers’ laminated composite shallow shell. The parameters like thickness, shallowness angle, length, circumferential width, and average radius of the shell are represented by symbols $h$, $\phi$, $a$, $s$ and $R$ respectively. Figure 1 illustrates the details of ACLD technique using PFRC/AFC patches applied on the top surface of the shell. The thickness of viscoelastic constrained layer used in ACLD treatment is taken as $h_v$ whereas $h_p$ is considered as thickness of PFRC/AFC patch material. To find out whole response of the shallow shell combined with ACLD treated PFRC/AFC patches, FSDT (First Order Shear Deformation) theory is used.

The generalized displacements are denoted by $u$, $v$, and $w$ in different directions i.e. along $x$, $y$, and $z$ directions at any point in any coat for entire panel respectively. These are given as:

$$
{d} = {d_f} + [Z] {d_r}
$$

(1)

Where, ${{d}} = [u \ v \ w]^T$, ${{d_f}} = [u_v \ v_v \ w_v]^T$, ${{d_r}} = [\theta_x \ \theta_y \ \phi_x \ \phi_y \ \gamma_x \ \gamma_y]^T$.

![Figure 1. Composite panel with PFRC/AFC patch](image)
The governing equations of motion of a typical element of the overall panel with ACLD system can be obtained as follows:

\[
\begin{bmatrix}
M^e
\end{bmatrix}\ddot{d}_t^e + \begin{bmatrix}
K_n^e
\end{bmatrix}d_t^e + \begin{bmatrix}
K_t^e
\end{bmatrix}d_t^e + \begin{bmatrix}
K_{trsv}^e
\end{bmatrix} = \begin{bmatrix}
K_{n}^e
\end{bmatrix}\frac{\partial}{\partial \tau}d_t^e + \begin{bmatrix}
K_{t}^e
\end{bmatrix}\frac{\partial}{\partial \tau}d_t^e + G(t-\tau)\begin{bmatrix}
K_{trsv}^e
\end{bmatrix}\frac{\partial}{\partial \tau}d_t^e + \begin{bmatrix}
f^e
\end{bmatrix} + \begin{bmatrix}
f_{pn}^e
\end{bmatrix} = \begin{bmatrix}
F^e
\end{bmatrix} + \begin{bmatrix}
F_{ip}^e
\end{bmatrix}V + \begin{bmatrix}
F_{tr}^e
\end{bmatrix}n a
\]

(2)

\[
\begin{bmatrix}
K_{n}^e
\end{bmatrix}\ddot{d}_t^e + \begin{bmatrix}
K_t^e
\end{bmatrix}d_t^e + \begin{bmatrix}
K_{trsv}^e
\end{bmatrix} = \begin{bmatrix}
K_{n}^e
\end{bmatrix}\frac{\partial}{\partial \tau}d_t^e + \begin{bmatrix}
K_{t}^e
\end{bmatrix}\frac{\partial}{\partial \tau}d_t^e + G(t-\tau)\begin{bmatrix}
K_{trsv}^e
\end{bmatrix}\frac{\partial}{\partial \tau}d_t^e = \begin{bmatrix}
f^e
\end{bmatrix} + \begin{bmatrix}
f_{ip}^e
\end{bmatrix}V + \begin{bmatrix}
F_{tr}^e
\end{bmatrix}
\]

(3)

Modelling of viscoelastic material is done by GHM (Golla Hughes McTavish) method in time domain. The equations obtained are as follows.

\[
[M]\dddot{X}_j + [K_n]X_j + [K_t]X_j - [K_{trsv}]G^{\alpha}\{Z_t\} - [K_{trsv}]G^{\alpha}\{Z_r\} = \{F\} + \{F_{pn}\} + \{F_{ip}\}V + \{F_{tr}\}
\]

(4)

\[
\dddot{Z}_r + 2\dddot{\omega}\{Z_r\} + \dddot{\omega}\{K_1\}X_j - \dddot{\omega}\{K_2\}\{Z_t\} + \dddot{\omega}\{K_3\}\{Z_r\} = \{F_{pc}\}V + \{F_p\}
\]

(5)

In above equations \{Z_t\} and \{Z_r\} are the dissipation coordinates introduced.

The following are the global open loop equations of motion of the overall panel/ACLD system in terms of the global translational degrees of freedom \{X_t\} and the dissipation coordinates \{Z_t\} and \{Z_r\}.

\[
[M]\dddot{X}_j + [K_n]\{X_j\} + [K_t]\{X_j\} - [K_{n}]\{Z_t\} - [K_{t}]\{Z_t\} - [K_{trsv}]G^{\alpha}\{Z_t\} - [K_{trsv}]G^{\alpha}\{Z_r\} = \{F\} + \{F_{pn}\} + \{F_{ip}\}V + \{F_{tr}\}
\]

(6)

\[
\dddot{Z}_r + 2\dddot{\omega}\{Z_r\} + \dddot{\omega}\{K_1\}\{X_j\} - \dddot{\omega}\{K_2\}\{Z_t\} + \dddot{\omega}\{K_3\}\{Z_r\} = \{F_{pc}\}V + \{F_p\}
\]

(7)

Finally, the global equations of motion governing the open loop behaviour of the laminated composite shallow shells integrated with the PFRC/AFC patches of ACLD treatment are obtained as follows:

\[
[M^*]\dddot{X} + [C^*]\dddot{X} + [K^*]X = \{F^*\} + \{F_p\}V + \{F_{ip}\}
\]

(8)

The necessary control voltage required for the activation of the patches is found out using a simple velocity feedback control law. Such voltage of each patch is represented by following equation.

\[
V^j = -K^j_d\dot{U}^j = -K^j_d[U^j][X]
\]

(9)

Where, control gain \(K^j_d\) is for the \(j^{th}\) patch. The unit vector \([U^j]\) defines the location of sensing the velocity signal and this would be fed back to \(j^{th}\) patch. By means of equation 9, the equation of motion for the dynamic response of the substrate plates treated by the patches of PFRC/AFC is given as follows:

\[
[M^*]\dddot{X} + [C^*][\dddot{X}] + [K^*]X = \{F^*\}
\]

(10)

Where, \([C^*]\) is an active damping matrix and is given by equation 11

\[
[C^*] = \sum_{j=1}^{m} K^j_d\{F_p^*\}[U^j]
\]

(11)

3. Numerical results

During the present work, Matlab software is used for FE modelling and analysis. The FE model is used to get numerical results for performance evaluation of the ACLD treated PFRC/AFC patches on attenuating the vibrations (which are geometrically nonlinear in nature) of laminated composite
panels. Numerical results are obtained for symmetric cross ply thin circular shallow shell. This shell is integrated with two rectangular patches of PFRC/AFC for ACLD treatment. Material properties of the orthotropic layers of the substrate panel, PFRC/AFC layer and viscoelastic layer are taken as follows [9]:

\[ E_L = 172.9 \text{GPa}, \quad E_T = 25, \quad G_{LT} = 0.5E_T, \quad G_{TT} = 0.2E_T, \quad v_{LT} = v_{TT} = 0.25, \quad \rho = 1600 \text{kg/m}^3 \]

Symbols used above give the common meaning. The piezoelectric fibres used here are made of lead zirconate titanate (PZT5H) and matrix made of epoxy is used for patches. For PFRC, fibre volume fraction is considered as 40% and the effective elastic coefficients are taken as [10]

\[ C_{11} = 32.6 \text{GPa}; \quad C_{12} = 4.3 \text{GPa}; \quad C_{22} = 7.2 \text{GPa}; \quad C_{44} = 1.05 \text{GPa}; \quad C_{55} = C_{66} = 1.29 \text{GPa}; \quad \rho = 3640 \text{kg/m}^3 \]

and the piezoelectric coefficient of the PFRC material is given by

\[ e_{31} = -6.76 \text{C/m}^2 \]

For AFC, fibre volume fraction is considered as 40% and the effective elastic coefficients are taken as [10]

\[ C_{11} = 131.8 \text{GPa}; \quad C_{12} = 71.15 \text{GPa}; \quad C_{22} = 148.9 \text{GPa}; \quad C_{44} = C_{55} = 32.35 \text{GPa}; \quad C_{66} = 39.14 \text{GPa}; \quad \rho = 3640 \text{kg/m}^3 \]

and the piezoelectric coefficients of the AFC material are given by

\[ e_{12} = e_{13} = 3.34 \text{C/m}^2; \quad e_{16} = 0 \]

The thickness of PFRC/AFC patch is considered as 250µm, thickness of viscoelastic patch is taken as 50.8µm and the substrate panel thickness is taken as 3mm. The shallowness angle (ϕ) is taken as 20º, the axial length (a) is taken as 0.5m. The piezoelectric fiber orientation angle (θ) in the PFRC/AFC patches is taken as 0º. The thickness of each orthotropic layer of the substrate panel is same. A uniformly distributed mechanical load \( P \) acting upward is considered here. For evaluating the numerical results, the simply supported (SS) boundary conditions at the ends of the overall panels are considered and specified by

\[ v_0 = w = \theta_y = \phi_y = \gamma_y = 0 \text{ at } x = 0, a \quad \text{and} \quad u_0 = w = \theta_x = \phi_x = \gamma_x = 0 \text{ at } y = 0, s \]

Here, the comparative investigation is carried out for the performance of ACLD treatment using PFRC and AFC patch material for controlling the nonlinear transient vibrations of laminated composite shallow shell. Figure 2 represents the comparison of the responses for symmetric cross ply (0º/90º/0º) laminated panel under passive mode using ACLD treatment made of PFRC and AFC patch material. From figure 2 we can see the improved performance of the AFC patch over the PFRC in attenuating the transient vibrations. Figure 3 shows the attenuation capabilities of AFC and PFRC patch material in active mode. It can be observed from figure 3 that, the amplitude of is less for AFC material compared to PFRC and we can also compare these responses under active mode with responses under passive mode. Figure 4 compares the control voltages requirement for AFC and PFRC for active mode which is nearly same. It means that, there is not much change in the control voltage required for activation of AFC or PFRC patches. Figure 5 gives the phase plot comparison for PFRC and AFC patch material.
Figure 2. Comparison of responses using PFRC and AFC patch material under passive mode

Figure 3. Comparison of responses using PFRC and AFC patch material under active mode
**Figure 4.** Comparison of control voltages requirement for AFC and PFRC for active mode

**Figure 5.** Comparison of phase plot of PFRC and AFC patch material
4. Conclusions
After the comparative investigation on the responses of laminated composite shallow shell integrated with the ACLD treated PFRC/AFC patches under active and passive modes, it can be concluded that the AFC material gives better results over the PFRC patches in terms of control voltage, attenuating capabilities and modes of operations. This also recommends the use of AFC patches in the ACLD treatment instead of PFRC patches to suppress the nonlinear transient vibrations.

5. References
[1] Chopra I 2002, AIAA Journal, 40 2145-2187
[2] Fattahi I, Mirdamadi H R 2017, Composite Structures, 179 161-171
[3] Kerboua M, Megnounif A, Benguediab M, Benrahou K H and Kaoulala F, 2015, Composite Structures, 123 430-442
[4] Shaikh T, Agarwal R K, 2016, Imperial Journal of Interdisciplinary Research, 2 1362-1365
[5] Omidi E, Mahmooodi S N, Shepard Jr W S 2015, Aerospace science and Technology, 45 408-415
[6] Ashok M H, Shivakumar J, Nandurkar S, Khadakbhavi V and Pujari S, 2018, IOP Conference Series: Materials Science and Engineering, 310 012101
[7] Shivakumar J, Ashok M H, Khadakbhavi V, Pujari S and Nandurkar S, 2018, IOP Conference Series: Materials Science and Engineering, 310 012100
[8] McTavish D J, Hughes P C 1993, Journal of Vibration Acoustics, 115 103-113
[9] Shivakumar J, Ray M C 2008, Journal of Reinforced Plastics and Composite, 28 525-541
[10] Sarangi S, Ray M C 2010, Smart Materials and Structures, 19 875-890