Optimal Auction Mechanism for Spectrum Allocation in Cognitive Radio Networks under Uncertain Spectrum Availability

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Abstract

In this paper, we consider the problem of dynamic spectrum allocation in cognitive radio (CR) networks and propose a new sealed-bid auction framework to address the spectrum allocation problem when the spectrum is not available with certainty. In our model, we assume that the moderator plays the dual role of being a fusion center (FC) for spectrum sensing and an auctioneer for spectrum allocation where it attempts to maximize its utility. We also consider the cost of collisions with the primary user (PU) and assign this cost to the FC, making it completely responsible for its allocation decision. With the help of CRs participating in the network, the FC makes a global inference on the availability of the spectrum followed by spectrum allocation. We investigate the optimal auction-based framework for spectrum allocation and also investigate the conditions under which such an auction is feasible. Note that the optimal auction forces the moderator to compensate the CRs for the sensing cost that they incur. Some numerical examples are presented for illustration.

Index Terms

Cognitive Radio Networks, Spectrum Availability Uncertainty, Auctions, Spectrum Allocation, Spectrum Sensing.

I. INTRODUCTION

With the overwhelming growth of new wireless devices and applications, spectrum available for communication has become scarce, thus, affecting the task of spectrum leasing severely when carried out by the FCC. Therefore, in order to accommodate the demand for spectrum, FCC has tried several approaches such as short-term spectrum leasing and spectrum-commons [1] on the unlicensed spectrum. Unfortunately, none of these approaches could satiate the demand for spectrum from the new wireless applications. Therefore, FCC conducted a survey [1] on how efficiently the leased spectrum is being utilized by the licensed (primary) users (in short, PU). This survey suggested that the licensed spectrum is under-utilized and therefore, the spectrum efficiency is very low. Due to the inefficient spectrum usage, FCC relaxed some of its stringent policies on spectrum usage, particularly in TV bands, and encouraged dynamic spectrum access (DSA) where PUs share licensed spectrum with
unlicensed users (or secondary users, in short, SU), under interference constraints. As a solution to DSA, intelligent CRs were proposed by Mitola in [2] which learn and adapt to the dynamically changing spectrum-usage. Therefore, in this paper, we hereafter use both the terms, SU and CR, interchangeably.

In order to utilize PUs’ spectra under interference constraints, two fundamental approaches have been proposed for CRs: overlay and underlay mechanisms. In the spectrum-overlay mechanism, CRs infer about the state of licensed bands by sensing the respective PU channels. Opportunities (or, idle bands) are determined based on the spectrum sensing observations and are utilized by CRs to satiate their communication needs. Once the CR tunes to a vacant band, it continues to communicate in the same channel as long as there is no PU in the channel. Once the CR senses the presence of a PU, the CR is forced to vacate and hop to another vacant band for communication. This task of the CRs is termed spectrum-handoff, and is one of the most fundamental tasks of CRs needed to satisfy interference constraints imposed by the PUs. On the other hand, in the case of spectrum-underlay mechanisms, CRs spread their energy over wide-spectra using spread spectrum techniques during communication, in such a way that the impact of CRs’ transmitted energy is below the prescribed interference-temperature at the corresponding PUs. Note that, in the case of spectrum underlay, there is no need for spectrum-handoff mechanisms as CRs communicate as long as necessary, without interfering with the PUs. On the other hand, in order to have a reasonable performance, the CRs need ultra-wide spectra to communicate, while simultaneously meeting the PU interference constraints.

In this paper, we focus on spectrum-overlay mechanisms, with just one narrow-band PU in the network. We assume the presence of multiple CRs which compete for spectrum opportunities.

Extensive literature is available on the design and analysis of dynamic spectrum allocation using different mechanisms. Among the different mechanisms proposed to address spectrum allocation, an effective technique has been the use of auctions [3]. There is substantial agreement among economists that auctions are the best way to assign scarce resources [4]. Furthermore, spectrum trading via auctions allows a more dynamic, competitive and efficient communications market than is possible under the traditional regimes implemented so far, primarily because spectrum users and wireless service providers (WSPs) have better knowledge than regulators about their spectrum requirements and valuations. For further details, the reader is referred to a survey of auction mechanisms designed for dynamic spectrum allocation in [5] and a tutorial paper in [6] which discuss the use of auctions for dynamic spectrum allocation in CR networks. Details on other schemes that have been proposed to address the problem of dynamic spectrum allocation, are available in the surveys [7] and [8].
There are two important variants of the problem of dynamic spectrum allocation, namely direct and indirect spectrum allocations (Refer to Figure [1]). The first variant is the problem of direct spectrum allocation where the PU itself serves as an auctioneer within the allocation framework, and directly auctions the spectrum to the CR network whenever it does not need its spectrum. Some examples are discussed in [9]–[13]. This not only improves the efficiency of spectrum utilization, but also provides revenue to the PUs. One example is the case of TV bands where there is negligible dynamics in the PU’s spectrum usage.

On the other hand, indirect spectrum allocation is the framework where the seller (PU) trades its spectrum to the CRs indirectly through a moderator. In practice, this framework allows CR networks to work in concert with legacy wireless systems, without any need for hardware replacements. Also, it is useful in scenarios where PUs may possess wide spectrum-bandwidths distributed over large geographic areas, resulting in a cumbersome management of spectrum, especially for SU usage. Therefore, in order to minimize the effort, the seller may outsource the task of selling the spectrum to a moderator, from whom revenue is collected. Depending on how much information the moderator has regarding the PU activity, two types of allocation mechanisms have been proposed in the context of indirect spectrum allocation, namely complete-information and incomplete-information mechanisms. In complete-information allocation mechanisms, the moderator has complete knowledge about the current state of the spectrum. These mechanisms are of practical importance in networks, where nodes with different priorities share a given spectrum (or, any resource, in general). Some of the applications of complete-information allocation mechanisms include military communication networks, where CRs are handled by military officials with varied ranks, or any commercial network where the spectrum is shared among users with different priorities (which, in turn, depend on the service that they pay for). For example, Kasbekar et al., in [14], considered the auction-based framework for a network with two different categories of nodes, namely primary (high-priority access) and secondary (low-priority access) nodes. The access allocation problem is solved based on the bids placed by the different users, so as to maximize the auctioneer’s revenue.

On the other hand, in incomplete-information allocation mechanisms, the moderator does not have complete knowledge of the state of the PU’s spectrum (available/busy for use by SUs). Therefore, the moderator relies mainly on the prior distributions and the spectrum sensing capabilities of the CRs to gain knowledge about the PU’s spectrum usage. Note that this allocation mechanism is particularly useful in environments where spectrum sensing is employed and the SUs use the white-spaces (spectrum opportunities) without interfering with the PU. In this paper, we design an incomplete-information auction mechanism that maximizes the moderator’s utility. In the rest of this section, we focus on the related work on incomplete-information mechanisms and highlight our contributions in light of existing related work.

A. Related Work and Main Contributions

As mentioned earlier, in this paper, we consider the indirect spectrum allocation framework and propose a novel incomplete-information auction mechanism for spectrum allocation in CR networks. Here, the CRs are the buyers and the moderator is the auctioneer. We assume that the moderator does not have complete knowledge about PU’s spectrum availability. Therefore,

\[\text{Note that the moderator could still be owned by the PU. In such a case, the uncertainty about spectrum availability at the moderator can arise due to the lack of communication with the central control of the PU, requiring the CRs to sense the spectrum before transmitting any message.}\]
the moderator infers about the state of the PU’s channel by fusing all the spectrum-sensing messages collected from the 
CRs. Since the moderator’s decision may be erroneous sometimes, there is uncertainty about the state (idle/busy) of the 
PU’s channel at the moderator.

In our previous work in 2012, we made the first attempt to address an incomplete-information auction mechanism for 
spectrum allocation under spectrum uncertainty [15]. As discussed earlier, the moderator makes a global inference on the 
availability of the spectrum based on the received spectrum sensing decisions from the CRs, followed by spectrum allocation, 
if profitable. Since the moderator allocates the spectrum to the CRs based on the global inferences that it makes about the PU 
channel state, we assumed that the moderator is responsible for any collisions with the PU, and therefore, imposed a collision 
cost $c_{\text{coll}}$ on the moderator whenever a CR collides with the PU. In [15], we assumed that the CRs incur a participation cost 
$c_p$ due to sensing and transmission of CR messages to the moderator. However, CRs were compensated for their participation 
cost by the auction mechanism only when the moderator allocated spectrum to them resulting in negative utility for some 
CRs. This may discourage the CRs from participating in the auction and thus the model in [15] was not practical. In this 
paper, we consider a practical and a much more general mechanism where the moderator always compensates the CRs for 
their participation cost. Since the participation cost is now borne by the moderator, we investigate the feasibility conditions 
of the proposed auction, under which the moderator always acquires a non-negative utility. Thus, we design optimal, truthful 
and a sealed-bid auction that maximizes the expected utility of the moderator, while simultaneously guaranteeing the non-
negative expected utilities at all the players. We present simulation results to examine the behavior of the proposed auction 
mechanism in terms of the costs of sensing, collision and the moderator’s spectrum sensing performance.

Note that the proposed auction mechanism is not only a contribution to dynamic spectrum allocation in CR networks, but, 
significantly advances auction theory in general. In fact, the proposed auction is a generalized Myerson’s optimal auction 
[16] applicable for stochastic scenarios where the resource is not available with certainty. The proposed auction reduces to 
Myerson’s optimal auction if the moderator has complete knowledge about the state of the PU spectrum and $c_p = c_{\text{coll}} = 0$.

In 2013, Li et al. also investigated the problem of spectrum allocation in the presence of spectrum uncertainty [17], where 
they maximized the social-welfare function over a finite time-horizon. The authors restricted the CR payments by equating 
them to the total valuation of the allocated spectrum, thus optimizing the allocation mechanism over a restricted space of all 
auction mechanisms. In our proposed model here, we assume that the CR payments can take the form of any generalized 
function of both allocated spectrum and valuations. Thus, our proposed model results in a greater expected utility at the 
moderator than the one presented by Li et al., in [17]. Also in 2013, Tehrani et al. proposed a first-price auction mechanism 
[18] which does not guarantee truthful revelations (defined later, in Section II-C) at the CRs. Therefore, CRs may have an 
incentive to reveal false valuations to the moderator in order to improve their individual utilities. In contrast to the auction 
mechanism in [18], our auction mechanism is incentive-compatible since CRs maximize their utility only by revealing 
truthful valuations.

The remainder of the paper is organized as follows. In Section II the sensing and the auction models of the proposed system 
are presented, along with definitions of all the necessary concepts such as expected utilities, rationality, truthfulness and
feasibility. In Section III, the main results of this paper are presented along with a feasibility analysis. Simulation results are presented in Section IV. Concluding remarks are made in Section V.

II. SYSTEM MODEL

Consider a network of $N$ CRs which compete for a given PU’s spectrum, as shown in Figure 2. We assume the presence of a moderator which is responsible for the task of spectrum allocation in our proposed mechanism. In this paper, we assume that the moderator has no knowledge about the true state of the PU activity in the spectrum. Therefore, the moderator has two roles: fusion of sensing decisions and allocation of available PU spectrum among the CRs in the network.

In order to make reliable decisions about the spectrum allocations with minimal interference to the PU, we assume that the CRs transmit hard-quantized observations (binary decisions) regarding spectrum availability to the moderator, so that the moderator fuses all these sensor messages into a global inference about the PU activity. This role (subsystem) of the moderator is that of a fusion center (FC). If the moderator makes an inference that the spectrum is available, then it allocates the spectrum among the set of $N$ CRs in the network via an auction mechanism in which the moderator plays the role of an auctioneer.

In short, the auctioneer subsystem acquires the knowledge of spectrum availability from the FC and allocates the spectrum to the CRs only when it deems it profitable.

In the remainder of the section, we present the system model in two subsections, each dedicated to the tasks of sensing-data fusion and the auction of available PU spectrum at the moderator. For the sake of clarity in articulating the role of the moderator, henceforth, we use the terms ‘FC’ and ‘auctioneer’ in the place of moderator according to the context and need.
A. Sensing Model

Spectrum sensing is a binary hypothesis-testing problem where \( H_0 \) corresponds to the absence of the PU activity while \( H_1 \) corresponds to the presence of the PU activity in the licensed band. We denote the prior probabilities of the two hypotheses as \( \pi_0 = P(H_0) \) and \( \pi_1 = P(H_1) \). It is important to note that the secondary network is interested in identifying \( H_0 \) for spectrum opportunities, and \( H_1 \) to avoid collisions with the PU.

Let \( \mathbb{N} = \{1, 2, \cdots, N\} \) denote the set of CRs in the network. We assume that the CRs perform collaborative spectrum sensing by sending their sensing decisions to the FC. The \( i^{th} \) CR senses the spectrum and sends a binary decision \( u_i \in \{0, 1\} \) to the FC. The FC makes a global decision about the spectrum availability based on all the sensor decisions. In this paper, we assume that the probabilities of false alarm and detection (denoted \( P_{fa} \) and \( P_{d} \) respectively) of the \( i^{th} \) CR are known at the FC, for all \( i \in \mathbb{N} \).

Since the FC fuses all the participating CRs’ local decisions to obtain a reliable inference about the PU activity, the detection performance at the FC can be characterized by the global probabilities of false alarm and detection, denoted as \( Q_f \) and \( Q_d \) respectively. Both these quantities can be computed in terms of \( P_{fa} \) and \( P_{d} \), for all \( i \in \mathbb{N}_A \), for a given fusion rule. As an example, we present the computation of \( Q_f \) and \( Q_d \) for a \( k\)-out-of-\( n \) fusion rule in our simulation results in Section IV.

B. Auction Model

The moderator may not be willing to provide its service for free (e.g., an IEEE 802.22 BS [19]) and may be a profit seeking entity. Thus, we assume that the CRs have to pay the FC for the service. In any auction, before the process of auctioning starts, the auctioneer valuates the item to be auctioned. Since we assume that the moderator does not have any personal use for spectrum, in this paper, we assume that the moderator’s valuation for the available spectrum is zero. Note that the moderator finds it profitable to allocate the available spectrum only when the revenue from the auction is non-negative. Likewise, every CR valuates the available spectrum depending on its need for spectrum. We assume that the \( i^{th} \) CR has a truthful valuation (type) \( t_i \) for the available spectrum. We assume that each of the valuations \( t_i \in [a_i, z_i] \), for all \( i = 0, 1, \cdots, N \). Of course, when the spectrum is not available for secondary usage (hypothesis \( H_1 \) is true), we assume that the spectrum is valuated at all the nodes (\( N \) CRs, and the moderator) to be zero.

Having valuated the spectrum locally at the CR, the valuations are revealed to the auctioneer (through noiseless, orthogonal control-channels) for spectrum allocation. But, being rational entities, the CRs might reveal false valuations for selfish reasons, in order to acquire greater utility. So, we denote the valuation revealed by the \( i^{th} \) CR to the moderator as \( v_i \) (which may or may not equal to \( t_i \)). Of course, the choice of \( v_i \) depends on the true valuation \( t_i \) of the \( i^{th} \) CR. Once the spectrum auctioning is complete, the \( i^{th} \) CR gets a spectrum allocation \( \psi_i \in [0, 1] \) and pays a total amount of \( b_i \) to the moderator at the end, for having participated in the auction. Note that, if the spectrum is indivisible, \( \psi_i \) can be interpreted as the probability of allocating the spectrum to the \( i^{th} \) CR, while if the spectrum is divisible, \( \psi_i \) can be interpreted as the fraction of spectrum allocated to the \( i^{th} \) CR.
Figure 3 describes the payoffs corresponding to different strategies employed by different players in the auction model, where $c_{\text{coll}}$ is the cost of a collision with the PU, and $c_p$ is the cost of participation (includes cost of sensing and transmission) for a given CR. As shown in Figure 3, we assume that the CRs are risk-neutral and restrict the structure of the payoffs to be additively separable in terms of payments, spectrum and costs incurred.

Note that, in our model, we impose the penalty of collisions completely on the FC making it responsible for its erroneous decisions, while the CRs bear a participation cost $c_p$ if they participate in the spectrum sensing task, whether or not spectrum allocation is made to them. In addition, the term $\sum_{i=1}^{N} \psi_i c_{\text{coll}}$ represents the total cost that the FC bears due to the allocation $\psi$. Note that, if $\sum_{i=1}^{N} \psi_i = 1$, then the FC incurs the complete cost of collision, $c_{\text{coll}}$.

Figure 3 shows the interactions between the CRs and the moderator in our proposed mechanism. Once the CRs make their observations, they transmit their local decisions $u_i$ along with their valuations (revelations) $v_i$ to the moderator. With the sensing knowledge from $u_i$, the FC subsystem makes a global inference about the PU state and informs the auctioneer subsystem along with the corresponding $Q_f$ and $Q_d$ respectively. After the auctioning process, the moderator provides the allocation vector $\psi$ to the CRs, and the CRs pay $b_i$ to the moderator for its service.

It is worth mentioning that our auction-based mechanism reduces to several problems of interest as special cases in the
CR-i
Moderator
(FC / Auctioneer)

Acquire observations \( r_i \) from the PU’s channel
at CR-i and quantize
them into \( u_i \), decide the
valuation \( t_i \) of the
spectrum at CR-i
Use the allocated
spectrum \( \psi_i \), Give the
appropriate payment to
the FC
Fuse all the decisions \( u \) from the CRs and make
a global inference \( u_0 \).
Auction the spectrum
whenever it is available
based on the CRs’
valuations \( t \)
Accumulate all the
payments \( b \) from the
CRs

Fig. 4: Interactions between the CRs and the FC (Auctioneer)

context of dynamic spectrum access. Note that if \( c_{coll} = 0 \), \( \pi_0 = 1 \), \( Q_f = 0 \) and \( Q_d = 1 \), then the auction mechanism
reduces to the problem of spectrum commons, where the CRs share unlicensed spectrum that is available with certainty. On
the other hand, one can interpret \( \psi \) as a fractional or a stochastic allocation vector depending on whether the spectrum is
divisible or indivisible respectively. In the case of divisible spectrum, the CRs would employ an OFDM-based system so that
an appropriate waveform is designed within the allocated bandwidth. Otherwise, a CR employs a simple spectrum-overlay
system when the spectrum is indivisible.

C. Some Definitions: Expected Utilities, Rationality, Truthfulness and Feasibility

Being a rational entity, the moderator would want to maximize its net utility. Since the moderator is also responsible for
allocating the spectrum, it also incurs the losses whenever erroneous decisions are made regarding the availability of the
spectrum, resulting in collisions with the PU. Obviously, if the PU’s presence is detected, then there is no motivation for the
moderator to allocate the spectrum to the CRs. On the other hand, when PU’s absence is detected, the moderator allocates
the spectrum to the CRs, if profitable. While collisions with the PU transmissions occur when there is a misdetection at
the FC with a probability \( q_1 = \pi_1(1 - Q_d) \), allocation without collision takes place with probability \( q_0 = \pi_0(1 - Q_f) \). Of
course, allocation does not take place with a probability \( 1 - q_0 - q_1 = \pi_1 Q_d + \pi_0 Q_f \).

From Figure 4 we define the expected utility of the moderator as follows.
Definition 1. The utility of the moderator, denoted as $U_0$, is given by Equation (1).

$$U_0(\psi, b) = \mathbb{E}_t \left[ \pi_0 (1 - Q_f) \sum_{i=1}^{N} b_i + \pi_1 (1 - Q_d) \sum_{i=1}^{N} \psi_i c_{coll} \right]$$

(1)

Note that the moderator is not aware of the truthful valuations $t$, since the CRs' revelations indicate that their valuations are $v$. Therefore, we assume that the truthful valuations $t$ form a random vector, and therefore, the expected utility of the moderator is computed by averaging over different values of $t$.

Since the valuations are revealed confidentially to the auctioneer, this mode-of-operation is termed direct-revelation [16]. We address the notion of direct revelation in greater detail, along with the notion of a given node’s rationality in the following subsection.

Note that the moderator’s utility depends on $b$ and $\psi$, which in turn, depend on the revelations $v$, $c_p$, $c_{coll}$, $q_0$ and $q_1$. If the moderator receives a negative expected utility, there is no motivation for the auction mechanism to be pursued by the moderator.

Since a CR does not have any direct-control in the decision-making process, the node will be satisfied as long as its expected utility is non-negative. We define the expected utility of the $i^{th}$ CR, from Figure 3, as follows.

**Definition 2.** The utility of the $i^{th}$ CR, denoted as $U_i$, is given by

$$U_i(\psi, b, t_i) = \mathbb{E}_{t_{-i}} \left[ q_0 \{ \psi_i t_i - b_i - c_p \} \right]$$

$$+ (1 - q_0) \{ -b_i - c_p \}$$

(2)

where $t_{-i} = \{ t_1, \cdots, t_{i-1}, t_{i+1}, \cdots, t_N \}$, $\psi_i$ is the fraction of bandwidth allocated to the $i^{th}$ CR, and $b_i$ is the price paid by the $i^{th}$ CR to the FC for participating in the auction.

Also, we define the expected amount of spectrum that the $i^{th}$ CR gets, in the proposed allocation, is defined as follows.

**Definition 3.** The expected fraction of bandwidth that node $i$ gets for a given valuation $v_i$, denoted as $\Psi_i(v_i)$ is defined as
follows.

\[ \Psi_i(v_i) = \int q_0 \psi_i(v_i, t_{-i}) p_{-i}(t_{-i}) dt_{-i} \] (3)

Since the valuation of the \( i^{th} \) CR is known locally, it is not treated as a random variable, and the expectation in the \( i^{th} \) CR’s utility is carried over the valuations of other CRs, namely \( t_{-i} = \{t_1, \cdots, t_{i-1}, t_{i+1}, \cdots, t_N\} \). In other words, we assume that the true spectrum valuation (personal preferences) of any given CR is not known to the other CRs.

In order to incentivize participation of CRs in the auction mechanism, there is a need to guarantee a non-negative utility to each of the nodes. Otherwise, the CR would not participate because of the loss that it may incur. This condition is termed \textit{individual rationality}, and is defined as follows.

**Definition 4** (Individual Rationality). \textit{Individual rationality criterion, which motivates the participation of CRs in the auction mechanism, is defined as}

\[ U_i(\psi, b, t_i) \geq 0, \quad \forall i \in \mathbb{N}. \] (4)

for any \( t_i \in [a_i, z_i] \).

Any selfish and rational entity tries to maximize its individual payoff. So can a CR lie in revealing its valuations and achieve personal gains. However, such a behavior on the part of the CRs can lead to inefficient auction outcomes. Therefore, in order to prevent the CRs from lying about their valuations, we design the optimal auction mechanism with truthful revelations from the CRs. But, in order to force all the CRs to reveal truthful valuations to the moderator, we need to ensure that there is no incentive for the CR to falsify its revelations. This is called the \textit{incentive compatibility} condition and is defined as follows.

**Definition 5** (Incentive Compatibility). \textit{A truthful revelation is ensured by the incentive compatibility condition, where no CR has the incentive to reveal false valuations to the moderator. This is given by}

\[ U_i(\psi, b, t_i) \geq \int [q_0 \psi_i(v_i, t_{-i}) t_i - b_i(v_i, t_{-i}) - c_p] p_{-i}(t_{-i}) dt_{-i} \] (5)

where, \( t_i \) is the true valuation of node \( i \), and \( v_i \neq t_i \) is any dishonest valuation declared.

Note that, if the moderator finds the auction mechanism unprofitable (\( U_0 \) is smaller than 0), for all theoretical purposes, we assume that the auction is infeasible. Of course, we consider the possibility of having a feasible auction that allows \( \psi = 0 \). Thus, we define the feasibility of an auction as follows.

**Definition 6** (Feasibility). \textit{We define that an auction is feasible if the following three conditions are satisfied.}

- \textit{Individual Rationality, as given by Equation (4)
• Incentive Compatibility, as given by Equation \[5\]

\[
\sum_{i \in N} \psi_i \leq 1, \text{ where } 0 \leq \psi_i \leq 1 \text{ for all } i \in \mathbb{N}.
\]

• \(U_0(\psi, b) \geq 0\).

The following lemma, often called the revelation principle, discusses the existence of an auction design that ensures the incentive compatibility condition. This result was first presented by Myerson in \([20]\).

**Lemma 1** (Revelation Principle). Given any feasible auction mechanism, there exists an equivalent feasible direct revelation mechanism which provides the same utilities to all the players (in this context, CRs and the moderator).

This lemma motivates us to proceed further and investigate an optimal direct-revelation mechanism that is feasible.

### III. Optimal Design of Auction Mechanisms Under Spectrum Availability Uncertainty

In this section, we formulate the problem of designing an optimal auction-based mechanism, and design an auction-mechanism that maximizes the expected utility of the moderator \(U_0\), while simultaneously holding the incentive-compatibility condition. We formally state our problem of finding the optimal auction mechanism for DSA in CR networks, in Problem 1.

**Problem 1.**

\[
\text{arg max}_{\psi, b} \quad U_0(\psi, b) \quad \text{s.t.}
\]

1. \(U_i(\psi, b, t_i) \geq 0, \quad \forall i \in \mathbb{N}. \) (As given in Equation \[4\])

2. \(U_i(\psi, b, t_i) \geq \int [q_0 \psi_i(v_i, t_{-i})t_i - b_i(v_i, t_{-i}) - c_p] p_{-i}(t_{-i}) dt_{-i},\)

\[\forall v_i \in [a_i, z_i], \quad \forall i \in \mathbb{N}. \quad \text{(As given in Equation } \[5\]\text{)}

3. \(\psi_i \geq 0, \quad \forall i \in \mathbb{N} \text{ and } \sum_{i \in \mathbb{N}} \psi_i \leq 1.\)

Note that the conditions under which we maximize the utility of the moderator are the same as the ones presented in Definition \([6] \text{ on the feasibility of a given auction mechanism, except for one condition where } U_0 \geq 0. \text{ Therefore, we first present an auction mechanism that satisfies individual rationality and incentive compatibility conditions. Later, we investigate the feasibility of the proposed auction mechanism in Theorem } 2\).

We start our analysis of Problem Statement \([1]\) by investigating the structure of the CRs’ utility functions. For the sake of
tractability, we restrict our analysis to a certain class of utility functions which are monotonically increasing in terms of their true valuations.

**Lemma 2.** The expected utility function of the $i^{th}$ player, if monotonically increasing in its true valuation $v_i$, can be expressed as

$$U_i(\psi, b, t_i) = U_i(\psi, b, a_i) + \int_{a_i}^{t_i} \Psi_i(v_i) dv_i$$

(7)

**Proof:** Let us initially consider the case where $a_i \leq v_i \leq t_i \leq b_i$. Expanding the RHS of Equation (5), we have Equation (8).

In the case where $a_i \leq t_i \leq v_i \leq b_i$, Equation (8) reduces to

$$U_i(\psi, b, v_i) \geq U_i(\psi, b, t_i) + (v_i - t_i) \Psi_i(t_i)$$

(9)

Combining Equations (8) and (9), we have

$$(t_i - v_i) \Psi_i(v_i) \leq U_i(\psi, b, t_i) - U_i(\psi, b, v_i) \leq (t_i - v_i) \Psi_i(t_i)$$

(10)

If the expected utility function $U_i$ is a monotonically increasing function of the true valuation $t_i$, then Equation (10) allows $U_i$ to be Riemann-integrable, resulting in Equation (7).

Next, we focus our attention on the properties of $U_0$.

**Lemma 3.** The expected utility of the FC can be expressed as

$$U_0(\psi, b) = - \sum_{i=1}^{N} U_i(\psi, b, a_i) + T$$

(11)

where

$$T = \mathbb{E} \left[ \sum_{i=1}^{N} g_0 \psi_i \left\{ t_i - \frac{1 - F_i(t_i)}{p_i(t_i)} - \frac{q_1}{q_0} c_{coll} \right\} - N c_p \right],$$

and $F_i(t_i) = \int_{t_i}^{b_i} p_i(s_i) ds_i$. 

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\[ T = \mathbb{E} \left[ q_0 \sum_{i=1}^{N} \psi_i t_i - q_1 c_{coll} \sum_{i=1}^{N} \psi_i - N c_p \right] - \sum_{i=1}^{N} \int_{a_i}^{b_i} \left( \int_{s_i}^{b_i} \Psi(s_i) p_i(t_i) dt_i \right) ds_i. \]

\[ = \mathbb{E} \left[ q_0 \sum_{i=1}^{N} \psi_i t_i - q_1 c_{coll} \sum_{i=1}^{N} \psi_i - N c_p \right] - \sum_{i=1}^{N} \int_{a_i}^{b_i} (1 - F_i(s_i)) \Psi(s_i) ds_i \] (16)

\[ = \mathbb{E} \left[ q_0 \sum_{i=1}^{N} \psi_i t_i - q_1 c_{coll} \sum_{i=1}^{N} \psi_i - N c_p \right] - \sum_{i=1}^{N} \int q_0 \psi_i \left( \frac{1 - F_i(t_i)}{p_i(t_i)} \right) p(t_i) dt \]

**Proof:** Let us start from the definition of \( U_0 \), as given in Equation (1). Therefore, we have

\[ U_0(\psi, b) = \mathbb{E}_t \left[ \sum_{i=1}^{N} b_i - q_1 c_{coll} \sum_{i=1}^{N} \psi_i \right] \] (12)

Adding and subtracting \( \sum_{i=1}^{N} (q_0 \psi_i t_i - c_p) \) in the RHS of Equation (12), we have

\[ U_0(\psi, b) = \mathbb{E}_t \left[ \sum_{i=1}^{N} (c_p + b_i - q_0 \psi_i t_i) \right. \]

\[ - q_1 c_{coll} \sum_{i=1}^{N} \psi_i - N c_p + q_0 \sum_{i=1}^{N} \psi_i t_i \] \] (13)

\[ = - \sum_{i=1}^{N} \int U_i(\psi, b, t_i) p_i(t_i) dt_i \]

\[ + \mathbb{E} \left[ \sum_{i=1}^{N} \psi_i \{ q_0 t_i - q_1 c_{coll} \} - N c_p \right] \]

Substituting the result of Lemma 2 in Equation (13), we have

\[ U_0(\psi, b) = - \sum_{i=1}^{N} U_i(\psi, b, a_i) + T \] (14)

where

\[ T = \mathbb{E}_t \left[ q_0 \sum_{i=1}^{N} \psi_i t_i - q_1 c_{coll} \sum_{i=1}^{N} \psi_i - N c_p \right] \]

\[ - \sum_{i=1}^{N} \int_{a_i}^{b_i} \left( \int_{s_i}^{b_i} \Psi(s_i) ds_i \right) p_i(t_i) dt_i. \] (15)

Henceforth, we focus our attention to the structure of \( T \), as defined in Equation (15). Changing the order of integration in the second term of the R.H.S of Equation (15), and substituting \( \pi_1 \) in place of \( 1 - \pi_0 \), we have Equation (16).
Rearranging the terms in Equation 16, we have

\[ T = E \left[ \sum_{i=1}^{N} q_0 \psi_i \left\{ t_i - \frac{1 - F_i(t_i)}{p_i(t_i)} - \frac{q_1}{q_0} c_{coll} \right\} - N c_p \right] . \]  

(17)

Having used the incentive compatibility condition, given in Equation 5 in Lemma 3, we can rewrite Problem 1 as follows.

**Problem 2.**

\[
\arg \max_{\psi, b} \quad T - \sum_{i=1}^{N} U_i(\psi, b, a_i) \\
\text{s.t.} \\
1. \quad U_i(\psi, b, t_i) \geq 0, \quad \forall i \in \mathbb{N}.
2. \quad \psi_i \geq 0, \quad \forall i \in \mathbb{N}
3. \quad \sum_{i \in \mathbb{N}} \psi_i \leq 1.
\]

Note that the term \( N c_p \) can be interpreted as the compensation that the FC pays back to the CRs to incentivize their participation in the spectrum sensing task.

Now, we focus our attention on solving Problem 2. Using Lemmas 2 and 3, we prove the most important result of this section in the following theorem.

**Theorem 1.** For all \( i \in \mathbb{N} \), if the function \( w_i(t_i) = t_i - \frac{1 - F_i(t_i)}{p_i(t_i)} \) is strictly increasing in \( t_i \) (regularity condition), then the optimal allocation, that maximizes the moderator’s revenue, is given by

\[
\psi^*_i = \begin{cases} 
\Delta_i & \text{if } |M(t)| > 0, \quad \forall i \in M(t) \\
0 & \text{otherwise}
\end{cases}
\]

for any \( \Delta_i \) such that \( \sum_{i \in M(t)} \Delta_i = 1 \), and where \( M(t) = \left\{ i \mid i = \arg \max_{j \in \mathbb{N}} w_j(t_j) \geq \frac{q_1}{q_0} c_{coll} \right\} \).

Also, the optimal payments made by the CRs to the FC are given by

\[
b^*_i = q_0 \psi^*_i t_i - c_p - q_0 \int_{a_i}^{t_i} \psi^*_i(s_i, t_{-i}) \, ds_i. \]

(19)

**Proof:** Note that, the term \( U_i(a_i) \) has a negative contribution to the utility of the moderator. As we know that \( U_i(\psi, b, t_i) \) is non-negative for all \( t_i \in [a_i, z_i] \) and \( i \in \mathbb{N} \) from the individual rationality criterion, as given in Equation 4, the moderator will let \( U_i(\psi, b, a_i) = 0, \forall i \in \mathbb{N} \). Substituting this in Equation 7, we have

\[
U_i(t_i) = \int_{a_i}^{t_i} \left( \int q_0 \psi_i(v_i, t_{-i}) p_{-i}(t_{-i}) \, dt_{-i} \right) dv_i
\]

(20)
Substituting the definition of $U_i(t_i)$ (refer to Equation (2)) in Equation (20), we have

$$
\mathbb{E}_{t \leftarrow i} \left[ q_0 \psi_i(t_i - b_i - c_p) \right] = \mathbb{E}_{t \leftarrow i} \left[ q_0 \int_{a_i}^{t_i} \psi_i(v_i, t_{-i}) dv_i \right]
$$

(21)

Thus, one of the possible selection of optimal payments is given by

$$
q_0 \psi_i^* t_i - b_i - c_p = q_0 \int_{a_i}^{t_i} \psi_i^*(v_i, t_{-i}) dv_i
$$

(22)

Rearranging the terms in Equation (22), we have Equation (19).

Driving the term $U_i(a_i) = 0$, the utility of the moderator can be rewritten as follows.

$$
U_0(\psi, b) = T
$$

(23)

Therefore, we focus our attention on $T$, which is restated as follows.

$$
T = \mathbb{E} \left[ \sum_{i=1}^{N} q_0 \psi_i \left\{ t_i - \frac{1 - F_i(t_i)}{p_i(t_i)} - \frac{q_1}{q_0} c_{coll} \right\} - Nc_p \right].
$$

(24)

Let us denote $w_i(t_i) = t_i - \frac{1 - F_i(t_i)}{p_i(t_i)}$. Note that, if $w_i(t_i)$ is an increasing function of $t_i$ (regularity condition), then the moderator maximizes $T$ by allocating the spectrum to the CRs in the set $M(t) = \left\{ i \mid i = \arg \max_{j \in \mathbb{N}} w_j(t_j) \geq \frac{q_1}{q_0} c_{coll} \right\}$. Whenever $|M(t)| \geq 2$, the tie-breaker can be resolved in any manner without affecting the optimal value of $T$, as long as the allocation of the spectrum is restricted to the nodes in $M(t)$.

Since the allocation of the spectrum depends on $w_i(t_i)$, instead of the valuation $t_i$ at the $i^{th}$ CR, we call the term $w_i(t_i)$ as the virtual valuation of the $i^{th}$ CR.

Note that the $i^{th}$ CR’s payment, as given in Equation (19), has an integral term $\int_{a_i}^{t_i} \psi_i^*(s_i, t_{-i}) ds_i$, which needs to be computed in order to find the payments of the $i^{th}$ CR. The computation of this term is very interesting since it requires us to investigate four cases, as follows. Within our analysis, we denote $t_*$ as the valuation that corresponds to the second highest element (in value) within the array of virtual valuations $w(t)$.

a) **CASE-1:** $[i \notin M(t)]$ From Equation (18) in Theorem 1, we have $\psi_i^*(t_i, t_{-i}) = 0$. Therefore, $\int_{a_i}^{t_i} \psi_i^*(s_i, t_{-i}) ds_i = 0$, and consequently,

$$
b_i = -c_p.
$$

(25)

b) **CASE-2:** $[i \in M(t), |M(t)| \geq 2]$ As shown in Equation (18), we have $\psi_i^*(t_i, t_{-i}) = \Delta_i$. But, if the $i^{th}$ CR chooses to deviate to a valuation smaller than $t_i$, then $i \not\in M(t)$ and $\int_{a_i}^{t_i} \psi_i^*(s_i, t_{-i}) ds_i = 0$. Therefore,

$$
b_i = q_0 \Delta_i t_i - c_p.
$$

(26)
c) CASE-3: \( i \in \mathbb{M}(t), |\mathbb{M}(t)| = 1, t_s \geq a_i \) If \( t_s \geq a_i \), then the \( i^{th} \) CR gets the spectrum as long as \( t_i \geq t_s \). In other words,

\[
\int_{a_i}^{t_i} \psi_i^* (s_i, t-1) \, ds_i = \int_{t_s}^{t_i} 1 \cdot ds_i = (t_i - t_s)
\]

. Therefore, the payment at the \( i^{th} \) CR can be calculated as

\[
b_i = q_0 t_i - c_p - q_0 (t_i - t_s)
\]

\[
= q_0 t_s - c_p
\]

(27)

d) CASE-4: \( i \in \mathbb{M}(t), |\mathbb{M}(t)| = 1, t_s < a_i \) Since \( t_s < a_i \), then the \( i^{th} \) CR gets the spectrum for any \( t_i \in [a_i, b_i] \). In other words,

\[
\int_{a_i}^{t_i} \psi_i^* (s_i, t-1) \, ds_i = (t_i - a_i)
\]

. Therefore, the payment at the \( i^{th} \) CR can be calculated as

\[
b_i = q_0 t_i - c_p - q_0 (t_i - a_i)
\]

\[
= q_0 a_i - c_p
\]

(28)

Combining Equations (25)-(28), the \( i^{th} \) CR’s payment to the moderator is summarized as follows.

\[
b_i = \begin{cases} 
-c_p, & \text{if } i \notin \mathbb{M}(t) \\
q_0 \Delta_i t_i - c_p, & \text{if } i \in \mathbb{M}(t) \text{ and } |\mathbb{M}(t)| \geq 2 \\
q_0 t_s - c_p, & \text{if } i \in \mathbb{M}(t), |\mathbb{M}(t)| = 1 \text{ and } t_s \geq a_i \\
q_0 a_i - c_p, & \text{if } i \in \mathbb{M}(t), |\mathbb{M}(t)| = 1 \text{ and } t_s < a_i
\end{cases}
\]

(29)

Having computed the CRs’ payments, in the case where \( |\mathbb{M}(t)| \geq 2 \), if the allocation is chosen to be \( \Delta_i = \frac{1}{|\mathbb{M}(t)|} \), then, the proposed auction mechanism can be summarized as an algorithm, denoted Optimal Auction under Uncertain Spectrum Availability (in short, OAUSA), in Figure 5. In case, the designer would like to choose a different allocation \( \Delta \), appropriate changes can be made by replacing Line 16 in Figure 5.

Discussion: Feasibility of the Proposed Auction

Note that the proposed auction in Theorem 1 is feasible only if \( U_0 \geq 0 \). Otherwise, it is more profitable for the moderator to have the spectrum for itself. In other words, from Equation (25), we need \( T \geq 0 \). Therefore, if

\[
w_{max} = \begin{cases} 
w_i(t_i); & \text{if } i = \arg \max_{j \in \mathbb{N}} w_j(t_j) \\
0; & \text{otherwise,}
\end{cases}
\]

then it is expected that

\[
T = q_0 \left[ \mathbb{E}[w_{max}] - \frac{q_1}{q_0} c_{coll} \right] - N c_p \geq 0.
\]
1: procedure OAUSA(t, u)
2: Fuse all the sensing messages u to make a global inference $u_0$
3: if $u_0 = 1$ then
4: for all $i \in \mathbb{N}$ do
5: $\psi_i \leftarrow 0$
6: $b_i \leftarrow -c_p$
7: end for
8: else
9: $w_i(t_i) \leftarrow t_i - \frac{1 - F_i(t_i)}{p_i(t_i)}$, for all $i = 1, \cdots, N$.
10: Find $M(t) = \{i | i = \arg\max_{j \in \mathbb{N}} w_j(t_j)\}$.
11: for all $i \in \mathbb{N}$ do
12: if $i \notin M(t)$ then
13: $\psi_i \leftarrow 0$
14: $b_i \leftarrow -c_p$
15: else if $i \in M(t), |M(t)| \geq 2$ then
16: $\Delta_i \leftarrow \frac{1}{|M(t)|}$
17: $\psi_i \leftarrow \Delta_i$
18: $b_i \leftarrow q_0 \Delta_i t_i - c_p$
19: else if $i \in M(t), |M(t)| = 1$ then
20: $\psi_i \leftarrow 1$.
21: Find the second largest value $t_*$ in the vector $w(t)$.
22: if $t_* \geq a_i$ then
23: $b_i \leftarrow q_0 t_* - c_p$
24: else
25: $b_i \leftarrow q_0 a_i - c_p$
26: end if
27: end if
28: end for
29: end if
30: return ($\psi$, $b$)
31: end procedure

Fig. 5: Pseudo-code for the proposed algorithm to find the optimal spectrum allocation at the moderator

Or, equivalently,

$$\mathbb{E}[w_{\text{max}}] \geq \frac{1}{q_0} N c_p + \frac{q_1}{q_0} c_{\text{coll}}$$ (30)
In other words, the proposed auction is feasible only when the expected value of the maximum valuation among all the CRs is greater than $\frac{1}{q_0} N c_p + \frac{q_1}{q_0} c_{coll}$. Note that the instantaneous value of $w_{max}$ need not necessarily be greater than $\frac{1}{q_0} N c_p + \frac{q_1}{q_0} c_{coll}$. Therefore, we have the following theorem.

**Theorem 2.** The auction proposed in Theorem [7] is feasible only if

$$E[w_{max}] \geq \frac{1}{q_0} N c_p + \frac{q_1}{q_0} c_{coll}.$$  

Note that, if the $i^{th}$ CR does not get any spectrum, then the optimal choice of the payment is $b_i = -c_p$ which is negative. This simply means that the FC pays an amount $c_p$ back to the $i^{th}$ CR node, as a compensation for sensing. Consequently, the $i^{th}$ CR participates in the spectrum sensing task with no loss or gain locally. But, it improves the sensing performance at the FC and reduces the probability of collision of the moderator with the PU. This allows the moderator to accumulate a non-negative utility, on an average.

**IV. Simulation Results**

In our simulation results, we consider a CR network with 10 identical nodes, where we assume the prior distribution of the PU spectrum usage as $\pi_0 = 0.8$ (or equivalently, $\pi_1 = 0.2$). Also, let the probabilities of false-alarm and detection be denoted as $P_{f_i} = P_f$ and $P_{d_i} = P_d$. We also assume that the moderator employs a $k$-out-of-$N$ fusion rule in order to make a global inference about PU’s spectrum availability, where $k$ is chosen to be optimal in the Bayesian sense [21], as follows.

$$k_{opt} = \left\lfloor \log \left( \frac{\pi_1}{\pi_0} \right) + N \log \left( \frac{1 - P_f}{1 - P_d} \right) \right\rfloor - \log \left( \frac{P_d(1 - P_f)}{P_f(1 - P_d)} \right)$$  

(Fig. 6: Utility of the moderator vs. local false alarm probability, for a fixed $P_d$. "
Then, the global probabilities of false alarm and detection, denoted $Q_f$ and $Q_d$ respectively, can be found as follows.

\[
Q_f = \sum_{i=k_{opt}}^{N} \binom{N}{i} P_f^i (1 - P_f)^{N-i} \tag{32a}
\]

\[
Q_d = \sum_{i=k_{opt}}^{N} \binom{N}{i} P_d^i (1 - P_d)^{N-i} \tag{32b}
\]

We also assume that the valuation $t_i$ of $i^{th}$ CR is uniformly distributed over $[0,1]$, i.e. $\mathcal{U}[0,1], \forall i = 1, \cdots, N$. Also, since both the optimal allocation and CRs’ payments depend on $t$, we obtain the results for the proposed auction based on 10000 Monte-Carlo runs.

First, we investigate the behavior of the proposed auction in terms of the false-alarm and detection probabilities, $P_f$ and $P_d$ at the spectrum sensors embedded within the CRs. On order to illustrate this behavior, we assume $c_p = 0.02$ and $c_{coll} = 5$ in our simulation results. In Figure 6, we plot the expected utility of the moderator in terms of $P_f$, when the CRs’ detection probability is fixed at $P_d = 0.5, 0.7, 0.9$. As per our intuition, the moderator’s expected utility decreases with increasing $P_f$, since the moderator misses to detect available opportunities even though the PU channel is idle. Note that the staircase pattern is due to the change in the optimal value of $k$, which is given in Equation (31). Similarly, in Figure 7, we plot the variation of the moderator’s expected utility with respect to the CRs’ detection probability $P_d$, for different values of $P_f$. Note that the plot resembles an improving sawtooth curve, where the sudden drop in moderator’s utility occurs whenever the value of the optimal $k$ decreases with increasing $P_d$. In other words, if $k$ decreases by one unit, a new term appears in $Q_d$, as given in Equation (32). Since $q_0$ is a decreasing function of $Q_d$, the expected utility of the moderator abruptly drops and then improves with increasing $P_d$ as long as the value of $k$ remains fixed. Also, note that the moderator’s utility becomes negative whenever the CRs have very low sensing performance (low $P_d$, and high $P_f$) in both Figures 6 and 7. This behavior can be explained by the feasibility condition presented in Theorem 2.
In our next set of simulations, we fix the probabilities of false alarm and detection at the CRs as $P_f = 0.1$ and $P_d = 0.9$. In Figure 8 we present the variation of the moderator’s utility $U_0$, with respect to the sensing cost $c_p$. In our simulation, we assume that the cost of collision $c_{coll} = 5$, and vary the cost of sensing $c_p$ over a range of $(0, 1)$. Similarly, in Figure 9 we present simulation results of the moderator’s utility for the proposed auction in terms of the cost of collision $c_{coll}$. In our results, we assume $c_p = 0.02$. As expected, we observe that the moderator’s utility decreases linearly with increase in $c_{coll}$. Similar to our simulation results in Figures 6 and 7 as shown in Theorem 2, we observe that the proposed auction becomes infeasible for high $c_p$ and $c_{coll}$.
V. CONCLUSION AND FUTURE DIRECTIONS

In summary, we designed a novel auction-based framework for CR networks, where the performance parameters from the spectrum sensing task are utilized to make decisions in the spectrum allocation task. We addressed the case where the moderator collects payments from the CRs at every time-instant. We showed that the moderator’s utility in the proposed mechanism is non-negative if \( E[w_{\text{max}}] \geq \frac{1}{q_0} N c_p + \frac{q_1}{q_0} c_{\text{coll}} \), and the amount of spectrum allocated is influenced by the CRs’ sensing performance, costs of sensing and collision, in addition to the individual CR valuations. In our future work, we will investigate the problem of spectrum markets under uncertain spectrum availability where multiple PUs are present in the network. In addition, we will also investigate the case where a given spectrum can be spatially reallocated to different CRs such that there is no interference between SUs.

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