The tight empirical relation between dark matter halo mass and flat rotation velocity for late-type galaxies

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ABSTRACT

We present a new empirical relation between galaxy dark matter halo mass ($M_{\text{halo}}$) and the velocity along the flat portion of the rotation curve ($V_{\text{flat}}$), derived from 120 late-type galaxies from the SPARC data base. The orthogonal scatter in this relation is comparable to the observed scatter in the baryonic Tully–Fisher relation (BTFR), indicating a tight coupling between total halo mass and galaxy kinematics at $r \ll R_{\text{vir}}$. The small vertical scatter in the relation makes it an extremely competitive estimator of total halo mass. We demonstrate that this conclusion holds true for different priors on $M_{\text{b}}/L_{3.6\mu}$ that give a tight BTFR, but requires that the halo density profile follow DC14 rather than NFW. We provide additional relations between $M_{\text{halo}}$ and other velocity definitions at smaller galactic radii (i.e. $V_{2.2}$, $V_{\text{eff}}$, and $V_{\text{max}}$) which can be useful for estimating halo masses from kinematic surveys, providing an alternative to abundance matching. Furthermore, we constrain the dark matter analogue of the radial acceleration relation and also find its scatter to be small, demonstrating the fine balance between baryons and dark matter in their contribution to galaxy kinematics.

Key words: galaxies: evolution – galaxies: formation – galaxies: fundamental parameters – galaxies: haloes – galaxies: spirals

1 INTRODUCTION

The Tully–Fisher relation (TFR; Tully & Fisher 1977) was first formulated as a relation between optical luminosity and 21 cm line width in late-type galaxies as a way of accurately measuring distances to galaxies. Subsequently, it has been recognized that the line width is a proxy for the galaxy circular velocity (e.g. Verheijen 2001) while the luminosity is a proxy for the stellar mass of the system. This has led to the development of numerous alternative forms for the relation, replacing luminosity with stellar, gas, or total baryonic mass and line width by the velocity at specific radii on a rotation curve (RC), such as the maximum rotation velocity $V_{\text{max}}$, the velocity $V_{80}$ at the radius enclosing 80 percent of the light, or the velocity $V_{\text{flat}}$ where the RC plateaus (e.g. McGaugh et al. 2000; Verheijen 2001; Torres-Flores et al. 2011; McGaugh 2012). These relations have proven useful not only for calibrating the cosmic distance ladder at low redshifts (e.g. Tully & Pierce 2000), but also for providing a testing ground for models of galaxy formation (e.g. Eisenstein & Loeb 1996; Mo, Mao & White 1998; Courteau & Rix 1999; McGaugh et al. 2000; van den Bosch 2000; Mo & Mao 2004; Dutton et al. 2010, 2011; Trujillo-Gomez et al. 2011; Desmond & Wechsler 2015).

Since the luminosity of a galaxy is set by the baryonic matter and the rotation velocity is often dominated by the dark matter (e.g. Rubin, Ford & Thonnard 1980; de Blok, McGaugh & Rubin 2001; de Blok et al. 2008), the TFR provides a unique insight into the relation between these two components. Interestingly, this relation extends more than six decades in baryonic mass while the intrinsic scatter is small and may be consistent with zero (McGaugh 2012; Lelli, McGaugh & Schombert 2016b). This is considered a strong test of lambda cold dark matter ($\Lambda$CDM) due to the scatters expected between halo mass, concentration, and baryonic mass (Macciò, Dutton & van den Bosch 2008; Dutton & Macciò 2014; Desmond 2017b). Similarly, various dynamical processes can restructure the halo and gas distribution and impact $V_{\text{flat}}$, such as adiabatic contraction (e.g. Blumenthal et al. 1986), feedback-driven outflows (e.g. Navarro, Eke & Frenk 1996a; Pontzen & Governato 2012), and dynamical friction (e.g. El-Zant, Shlosman & Hoffman 2001; Weinberg & Katz 2002; Johannsen, Naab & Ostriker 2009). It is difficult to conceive of a scenario where the scatter in the baryonic Tully–Fisher relation (BTFR) remains consistent with zero when all of these processes non-linearly interact and impact $V_{\text{flat}}$. Semi-analytic models that aim to understand the scatter in the BTFR...
are only marginally consistent with the observations (Dutton 2012; Desmond 2017b). Since \( V_{\text{flat}} \) is often dominated by halo mass, one might expect that the scatter in the \( M_{\text{halo}}-V_{\text{flat}} \) relation would be less than that of the BTFR. If this holds true, the kinematics of the galaxy at \( r < R_{\text{vir}} \) can be used to accurately estimate the halo mass, providing an empirical alternative to other techniques such as abundance matching. Until now, the \( M_{\text{halo}}-V_{\text{flat}} \) relation has yet to be empirically determined, nor has the scatter been calculated. In this letter we measure this relation and compare it with the BTFR for the same galaxy sample.

2 THE \( M_{\text{HALO}}-V_{\text{FLAT}} \) AND BARYONIC TULLY–FISHER RELATIONS

To determine the \( M_{\text{halo}}-V_{\text{flat}} \) relation\(^1\) and the BTFR, we employ the gas and stellar mass models from the SPARC data set (Lelli, McGaugh & Schombert 2016a) as well as the technique from Katz et al. (2017)\(^2\) to empirically determine halo masses. We have rerun all of the Markov Chain Monte Carlo (MCMC) simulations presented in Katz et al. (2017) and now directly sample in \( \log_{10}(M_{\text{halo}}) \), \( \log_{10}(\rho_{\text{G}}) \), and \( \log_{10}(M/L) \), but all further analysis remains the same. Halo masses were estimated using both the DC14 halo profile (Di Cintio et al. 2014), a result from cosmological hydrodynamics simulations that can exhibit cusps or cores depending on the ratio of \( M_{\text{halo}}/M_{\text{stellar}} \), and the NFW halo profile (Navarro, Frenk & White 1996b), which is derived from dark-matter-only simulations. Only the former is consistent with other observational constraints (Aller, Gentile & Baes 2017; Katz et al. 2017). The NFW model often provides unrealistically large halo masses for galaxies with slowly rising rotation curves (the cusp–core problem), so they fall very far from the stellar mass–halo mass relation derived from abundance matching (see fig. 3 of Katz et al. 2017). Of the 147 galaxies fit by Katz et al. (2017), only 120 have measured values for \( V_{\text{flat}} \) (according to the Lelli et al. 2016b definition) and thus, only these galaxies are used to derive the \( M_{\text{halo}}-V_{\text{flat}} \) relation and the BTFR.

An important consideration is the choice of prior on the mass-to-light ratio (\( M/L \)), which impacts the inferred stellar, baryonic, and fitted halo masses. For our fiducial model we assume a flat prior in the range 0.3–0.8 (McGaugh & Schombert 2014, designated as ‘Flat’), although we also show results for the case where \( M/L = 0.5 \) (‘05’), which minimizes the scatter in the BTFR (Lelli et al. 2016b).\(^3\)

In Fig. 1, we show the resulting \( M_{\text{halo}}-V_{\text{flat}} \) relation for both \( M/L \) priors and the corresponding BTFRs. Note how the higher mass galaxies have a higher ratio of stellar mass to total baryonic mass (see also Katz et al. 2018). In order to constrain the mean relations, we must account for the non-Gaussian and asymmetric uncertainties on \( M_{\text{halo}} \) and the observational errors bar on \( V_{\text{flat}} \). To do this, we create 10 000 resampled catalogues where we randomly draw a halo mass for each galaxy from the posterior distribution mapped out by the MCMC chains and a \( V_{\text{flat}} \) from a Gaussian distribution using the measured \( V_{\text{flat}} \) from the RC and its uncertainty.

Because of uncertainties on galaxy distance, systematic features in the RCs,\(^4\) and the limited radial extent of the RCs,\(^5\) we expect that not all galaxies will be well fitted by the halo model and therefore outliers will be present in our data set. As these may bias our fits to the relations, we employ the RANSAC algorithm (Fischler & Bolles 1981). This is a robust estimation technique that uses an iterative procedure to determine whether a data point is an inlier or an outlier given the other data in the set, without relying on sigma clipping. We use RANSAC to fit the \( M_{\text{halo}}-V_{\text{flat}} \) relation and the BTFR for each of the 10 000 catalogues using the following equation:

\[
\log_{10}(M_{\text{halo}} \text{ or } b/M_{\odot}) = A \log_{10}(V_{\text{flat}}/\text{km s}^{-1}) + B.
\]

The mean and standard deviations of the fit parameters are calculated using the 10 000 realizations and the resulting relations and 1σ confidence intervals are shown as the black line and grey shaded regions in Fig. 1.

The best-fitting parameters for the \( M_{\text{halo}}-V_{\text{flat}} \) relation and the BTFR, and their uncertainties, are listed in Table 1 for both sets of priors. We also quote the scatter in the relations in the \( M \)-direction and the orthogonal (\( \perp \)) scatter quantified by 1.48 times the median absolute deviations (MADs) of the points from the best-fitting lines (McGaugh 2012), and the percentage of outliers identified by RANSAC.\(^6\) Interestingly, we find that the \( \perp \) scatter in the \( M_{\text{halo}}-V_{\text{flat}} \) relation is comparable to that of the BTFR for both \( M/L \) priors within 1σ (but see Section 4). The vertical scatter is then smaller in the \( M_{\text{halo}}-V_{\text{flat}} \) relation compared to the BTFR due to the shallower slope. The \( \perp \) scatter may indicate which relation is more fundamental while the vertical scatter can be used to determine the accuracy of the mass prediction from \( V_{\text{flat}} \). For comparison, we also present in Table 1 the results when using the NFW halo profile: it is evident that both the scatter in the \( M_{\text{halo}}-V_{\text{flat}} \) relation and the outlier fractions are significantly increased. This demonstrates that having a tight \( M_{\text{halo}}-V_{\text{flat}} \) relation is not guaranteed when fitting the RCs with any halo model.

Since the \( M_{\text{halo}}-V_{\text{flat}} \) relation has more outliers than the BTFR, we have also computed the \( \perp \) and vertical scatterings using all of the galaxies in each catalogue (i.e. assuming no outliers), which puts an upper limit on the scatter in the relations. Even in this extreme case, which almost certainly overestimates the scatter, the \( \perp \) scatters in the BTFR and \( M_{\text{halo}}-V_{\text{flat}} \) relation are comparable to within 2σ, and the vertical scatters are nearly identical, regardless of the prior on \( M/L \). However, this is not the case for NFW. Our estimated slopes and normalizations for the BTFR are unsurprisingly independent of the halo profile and are very consistent with the error-weighted fits from Lelli et al. (2016b), who use the same data set with a fixed mass-to-light ratio. Likewise, the slopes are slightly shallower, albeit still consistent within the uncertainties with the estimates from McGaugh (2012). The observed scatter we measure in the BTFR is also entirely consistent with that of McGaugh (2012) and Lelli et al. (2016b).

\(^1\)We define \( M_{\text{halo}} \) to be the dark matter mass within the radius \( R_{\text{vir}} \) that contains an average density of 93.6\( \rho_{\text{crit}} \), consistent with a WMAP3 cosmology (Spergel et al. 2007). \( M_{\odot} = M_{\text{halo}} + M_{\text{stellar}} \).

\(^2\)The fitting procedure of Katz et al. (2017) is also used to calculate the stellar mass-to-light ratio for each galaxy, given the priors, which determines the stellar mass. Only the models without the ‘\( \Lambda \)CDM’ priors are used in the work.

\(^3\)We have also investigated the use of Gaussian POPSYNTH (McGaugh & Schombert 2014) and DiskMASS (Martinsson et al. 2013) priors (see Katz et al. 2017); however, these result in BTFRs that have much larger scatter compared to the 05 prior, so we do not consider them in the remainder of our analysis. However, see Table 1 for the fitted values and scatter of the BTFR and \( M_{\text{halo}}-V_{\text{flat}} \) relation.

\(^4\)These may result from non-circular motion or asymmetries.

\(^5\)This can lead to large uncertainties in halo mass if the halo RC is still rising out to the last measured point.

\(^6\)We have checked that most outliers are a result of probabilistically sampling the wide posterior in halo mass because the halo RC is still rising out to the farthest observed radius.
The derived $M_{\text{halo}}-V_{\text{flat}}$ relation is considerably shallower than the BTFR, with a slope slightly less than 3 indicating that the $V_{\text{halo}}-V_{\text{flat}}$ relation has a slope near unity, where $V_{\text{halo}}$ is the circular velocity of the dark matter halo at $R_{\text{vir}}$. Directly measuring this relation, we find

$$V_{\text{flat}} = (0.904 \pm 0.216)V_{\text{flat}}^{0.947\pm0.051}$$ \hspace{1cm} (2a)$$

$$V_{\text{flat}} = (1.028 \pm 0.251)V_{\text{flat}}^{0.919\pm0.052}$$ \hspace{1cm} (2b)$$

with $\sigma_{V_{\text{flat}}} = 0.06, 0.07$ dex for the Flat and 05 priors, respectively, and hence $V_{\text{halo}} \sim V_{\text{flat}}$.

We also consider several common choices for the velocity measure used to define the relations. Although our fiducial choice is $V_{\text{flat}}$, we also investigate $V_{2.2}$ (measured at 2.2 disc scale lengths), $V_{\text{max}}$, and $V_{\text{eff}}$ (at the half-light radius). The disc scale length and the half-light radius are measured for each galaxy in the SPARC data set from the surface brightness profile (see section 3.1 of Lelli et al. 2016a). We show the BTFR and $M_{\text{halo}}-V$ relations for all of these velocity measures in Fig. 2, and we quantify their scatters in Fig. 3. The error bars are the standard deviations across the 10 000 Monte Carlo realizations. In nearly all cases, the scatters in the BTFR and $M_{\text{halo}}-V$ relations are consistent within $\sim1-1.5\sigma$ (but see Section 4). However, for these other velocity measures, the $\perp$ scatters for the $M_{\text{halo}}-V$ relations tend to fall systematically above the BTFRs. This is not surprising: $V_{\text{flat}}$ is generally measured farthest out in the galaxy where the halo often dominates the RC while the others are measured much closer to the centre of the galaxy.
The relation between halo mass and $V_{\text{flat}}$

The trends are qualitatively as expected: At lower $g_{\text{total}}$, dark matter accounts for almost all of the acceleration, so the uncertainty on $g_{\text{dark}}$ at fixed $g_{\text{total}}$ is small. In contrast, for the Flat $M_{\odot}/L$ prior, at high acceleration $g_{\text{total}}$ is a poor predictor of enclosed dark matter mass because baryons dominate. The relations cross over at $g_{\text{total}} \sim 10^{-10} \text{ m s}^{-2}$, indicating the transition between baryon and dark matter dominance. We see however that the choice of the $M_{\odot}/L$ prior has a large effect on the scatter of the relations, especially the hRAR at high $g_{\text{total}}$. For $M_{\odot}/L = 0.5$, $g_{\text{dark}}$ may in fact be estimated precisely from $g_{\text{total}}$ even in high-acceleration regions – almost as precisely as $g_{\text{baryon}}$. Although $g_{\text{dark}}$ must be inferred from the RCs themselves – and hence the hRAR does not relate observables as the RAR does – it does fill in the part of the galaxy dynamics that the RAR misses, and demonstrates more explicitly the relative role the two mass components play in setting the kinematics of different parts of galaxies.

Finally, we quantify the total scatter in the hRAR relative to the RAR (Desmond 2017a). For the Flat prior we find $\sigma_{\text{tot,RAR}} = 0.208$ and $\sigma_{\text{tot,RAR}} = 0.168$, while for the 05 prior we find $\sigma_{\text{tot,RAR}} = 0.201$ and $\sigma_{\text{tot,RAR}} = 0.138$.

4 CAVEATS

It should be emphasized that although we have directly compared the BTFR with the $M_{\text{halo}}-V_{\text{flat}}$ relation for the same set of galaxies, the two variables of the BTFR have been determined by completely independent observations while both $V_{\text{flat}}$ and $M_{\text{halo}}$ are measured from the same RCs. It is very difficult to determine how much the estimate of $M_{\text{halo}}$ is dependent on $V_{\text{flat}}$. We find that the dark matter contribution to the square of the galaxy circular velocity at $R_{\text{vir}}$, the innermost radius, where the RC becomes flat (see section 2.2 of Lelli et al. 2016b for details on the derivation of $V_{\text{flat}}$), is 56 per cent ± 24 per cent (1σ standard deviation) with a weak correlation for higher mass galaxies having a smaller contribution. Furthermore, the fraction of halo mass that exists at $r < R_{\text{vir}}$ can range between < 1 per cent and 35 per cent with a mean of ∼ 7 per cent, indicating that the majority of the halo mass is at much larger radii than where $V_{\text{flat}}$ is measured. The outer slope of the DC14 profile, which helps set the virial mass, is also dependent on the stellar content of the galaxy. While there is indeed a covariance between the two quantities, which may reduce the scatter of the $M_{\text{halo}}-V_{\text{flat}}$ relation, we stress again that this exercise does not work for the NFW halo (as shown in Table 1) and is unlikely to work for any arbitrary density profile (i.e. pseudo-isothermal, logarithmic, etc.). The DC14 model not only provides good fits to the RCs while simultaneously being in agreement with estimates of the $M_{\odot}-M_{\text{halo}}$ relation from abundance matching and the mass-concentration relation from dark matter-only simulations (Katz et al. 2017), but now also produces an $M_{\text{halo}}-V_{\text{flat}}$ relation with scatter comparable to the BTFR. The goal of our experiment is not to determine conclusively which relation is more fundamental, but rather to constrain the parameters and scatter of what appears to be a new and tight relation.

5 CONCLUSIONS

We present the $M_{\text{halo}}-V_{\text{flat}}$ relation, empirically determined from the SPARC data set of late-type galaxies (Lelli et al. 2016a) and the corresponding dark matter halo fits from Katz et al. (2017). This has a similar form to the baryonic Tully–Fisher relation and uses $M_{\text{halo}}$ rather than $M_{\odot}$, but it is more fundamental in the context of ΛCDM where galaxy dynamics in the outer regions are typically...
modelled by a power law over many decades in mass with a slope be accurately determined from measurements along the flat part of for the Flat (upper) and 05 (bottom) priors. The dashed line indicates MNRASL

Figure 4. Relation between $g_{\text{total}}$ and $g_{\text{dark}}$ (radial acceleration relation; blue) and between $g_{\text{total}}$ and $g_{\text{baryon}}$ (halo acceleration relation; green) for the Flat (upper) and 05 (bottom) priors. The dashed line indicates $x = y$.

Table 1. Best-fitting parameters and their 1σ uncertainties for the BTFR and $M_{\text{halo}} - V_{\text{flat}}$ relations as given in equation 1. For the DC14 profile we show results for both the Flat and 05 priors (as well as DISKMASS and POPSYNTH for $V_{\text{flat}}$), and for the NFW profile only the former. $\sigma$ denotes scatter orthogonal to the best-fitting line, derived by multiplying the mean of the MADs of the 10 000 fits by 1.48. We present the scatters both with (all) and without (inliers) outliers for both the orthogonal and vertical scatters to the relation.

| $M/L$ | Halo model | Mass measure | Velocity measure | $\sigma_A$ | $\sigma_B$ | $\sigma_{\perp}$ (inliers) | $\sigma_{\perp}$ (all) | $\sigma_{M_{\text{halo}},\text{inliers}}$ | $\sigma_{M_{\text{halo}},\text{all}}$ | Outlier fraction |
|-------|------------|--------------|------------------|------------|------------|---------------------------|-----------------|--------------------------|-----------------|------------------|
| Flat  | DC14 $M_b$ | $V_{\text{flat}}$ | 3.623 0.023 | 2.406 0.051 | 0.062 0.062 | 0.234 0.234 | 0.001 |
| Flat  | DC14 $M_{\text{halo}}$ | $V_{\text{flat}}$ | 2.902 0.138 | 5.439 0.292 | 0.064 0.075 | 0.195 0.231 | 0.124 |
| 05    | DC14 $M_b$ | $V_{\text{flat}}$ | 3.647 0.024 | 2.374 0.055 | 0.062 0.063 | 0.236 0.237 | 0.004 |
| 05    | DC14 $M_{\text{halo}}$ | $V_{\text{flat}}$ | 2.947 0.136 | 5.334 0.283 | 0.067 0.082 | 0.207 0.256 | 0.139 |
| Flat  | NFW $M_b$ | $V_{\text{flat}}$ | 3.630 0.028 | 2.400 0.062 | 0.060 0.060 | 0.225 0.226 | 0.003 |
| Flat  | NFW $M_{\text{halo}}$ | $V_{\text{flat}}$ | 2.216 0.208 | 6.907 0.471 | 0.116 0.193 | 0.237 0.466 | 0.347 |
| Flat  | DC14 $M_b$ | $V_{\text{eff}}$ | 2.083 0.211 | 7.218 0.450 | 0.121 0.147 | 0.277 0.336 | 0.151 |
| 05    | DC14 $M_b$ | $V_{\text{eff}}$ | 2.859 0.104 | 4.108 0.211 | 0.080 0.084 | 0.241 0.254 | 0.042 |
| Flat  | DC14 $M_{\text{halo}}$ | $V_{\text{eff}}$ | 2.276 0.162 | 6.800 0.340 | 0.107 0.134 | 0.266 0.332 | 0.146 |
| Flat  | DC14 $M_{\text{halo}}$ | $V_{\text{eff}}$ | 2.474 0.051 | 5.016 0.110 | 0.102 0.104 | 0.271 0.277 | 0.020 |
| 05    | DC14 $M_{\text{halo}}$ | $V_{\text{eff}}$ | 1.877 0.166 | 7.707 0.353 | 0.130 0.167 | 0.275 0.353 | 0.172 |
| Flat  | DC14 $M_b$ | $V_{\text{max}}$ | 3.204 0.080 | 3.214 0.177 | 0.072 0.074 | 0.242 0.249 | 0.024 |
| Flat  | DC14 $M_{\text{halo}}$ | $V_{\text{max}}$ | 2.439 0.196 | 6.333 0.424 | 0.093 0.113 | 0.244 0.297 | 0.139 |
| 05    | DC14 $M_b$ | $V_{\text{max}}$ | 3.253 0.096 | 3.123 0.211 | 0.072 0.074 | 0.246 0.252 | 0.019 |
| 05    | DC14 $M_{\text{halo}}$ | $V_{\text{max}}$ | 2.348 0.277 | 6.538 0.595 | 0.106 0.131 | 0.267 0.328 | 0.150 |
| DISKMASS | DC14 $M_b$ | $V_{\text{flat}}$ | 3.259 0.063 | 2.986 0.127 | 0.086 0.089 | 0.293 0.302 | 0.032 |
| DISKMASS | DC14 $M_{\text{halo}}$ | $V_{\text{flat}}$ | 2.596 0.067 | 5.982 0.147 | 0.062 0.067 | 0.173 0.187 | 0.052 |
| POPSYNTH | DC14 $M_b$ | $V_{\text{flat}}$ | 3.509 0.157 | 2.583 0.307 | 0.072 0.076 | 0.261 0.277 | 0.054 |
| POPSYNTH | DC14 $M_{\text{halo}}$ | $V_{\text{flat}}$ | 2.773 0.071 | 5.652 0.153 | 0.062 0.076 | 0.182 0.224 | 0.128 |

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