Agmon-type decay of eigenfunctions for a class of Schrödinger operators with non-compact classically allowed region

An important result by Agmon implies that an eigenfunction of a Schrödinger operator in $\mathbb{R}^d$ with an eigenvalue below the bottom of the essential spectrum decays exponentially if the associated classically allowed region is compact.

We extend this result to a class of Schrödinger operators with eigenvalues, for which the classically allowed region is not necessarily compactly supported: We show that integrability of the characteristic function of the classically allowed region with respect to an increasing weight function of bounded logarithmic derivative leads to $L^2$-decay of the eigenfunction with respect to the same weight.

Here, the decay is measured in the Agmon metric, which takes into account anisotropies of the potential. In particular, for a power law (or, respectively, exponential) weight, our main result implies that power law (or, respectively, exponential) decay of "the size of the classically allowed region" allows to conclude power law (or, respectively, exponential) decay, in the Agmon metric, of the eigenfunction.

The work presented in this talk is the result of joint research with my honors student, Hengrui Zhu (Oberlin '21).