Curvaton dynamics and the non-linearity parameters in the curvaton model

Qing-Guo Huang\textsuperscript{1} and Yi Wang\textsuperscript{2,3}

\textsuperscript{1} School of Physics, Korea Institute for Advanced Study, 207-43, Cheongryangri-Dong, Dongdaemun-Gu, Seoul 130-722, Korea
\textsuperscript{2} Institute of Theoretical Physics, CAS, Beijing 100080, People’s Republic of China
\textsuperscript{3} The Interdisciplinary Centre for Theoretical Study, USTC, Hefei, Anhui 230026, People’s Republic of China

E-mail: huangqg@kias.re.kr and wangyi@itp.ac.cn

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Abstract. We investigate the curvaton dynamics and the non-linearity parameters in the curvaton model with a potential slightly deviating from the quadratic form in detail. The non-linearity parameter $g_{NL}$ will show up due to the curvaton self-interaction. We also point out that the leading order of the non-quadratic term in the curvaton potential can be negative, for example in the axion-type curvaton model. If a large positive $g_{NL}$ is detected, the axion-type curvaton model will be preferred.

Keywords: axions, cosmology of theories beyond the SM, physics of the early universe

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1. Introduction

The inflation model [1] provides an elegant mechanism for solving many puzzles in the hot big bang model due to the quasi-exponential expansion of the universe. The quantum fluctuation generated during inflation naturally seeds the small anisotropies in the cosmic microwave background radiation and the formation of the large-scale structure. In general many scalar fields are present in the early universe. The dynamics of inflation is governed by inflaton(s), but the primordial curvature perturbation can be generated by inflaton(s) [2] or curvaton(s) [3], or both. However the feature of the primordial power spectrum caused by the inflaton is similar to that caused by the curvaton.

The CMB non-Gaussianity [4] opens a new window for probing the physics of the early universe. A well-discussed ansatz of non-Gaussianity has the local shape. This kind of non-Gaussianity can be characterized by some non-linearity parameters:

\[ \zeta(x) = \zeta_g(x) + \frac{3}{5} f_{\text{NL}} \zeta_g^2(x) + \frac{9}{25} g_{\text{NL}} \zeta_g^3(x) + \cdots, \] (1.1)

where \( \zeta \) is the curvature perturbation in the uniform density slice, \( \zeta_g \) denotes the Gaussian part of \( \zeta \). The non-linearity parameters \( f_{\text{NL}} \) and \( g_{\text{NL}} \) parametrize the non-Gaussianity from the irreducible three-point and four-point correlation functions respectively.

In the case of the single-field inflation model, \( f_{\text{NL}}^{\text{local}} \sim \mathcal{O}(n_s - 1) [5] \), which is constrained by WMAP \( (n_s = 0.960^{+0.014}_{-0.013}) [6] \) to be much less than unity. In some general inflation models [7] a large non-Gaussianity may be obtained, but the shape of the non-Gaussianity is different from the local shape. As we know, a curvaton model can easily generate a large local-type non-Gaussianity [3, 8]. Different groups have reported different results from data analyses for \( f_{\text{NL}}^{\text{local}} \) based on WMAP three-year data:

\[ 27 < f_{\text{NL}}^{\text{local}} < 147 \] (1.2)

in [9] and

\[ -70 < f_{\text{NL}}^{\text{local}} < 91 \] (1.3)
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in [10] at 95% CL. The WMAP group reported the five-year result [6]

\[-9 < f_{\text{NL}}^{\text{local}} < 111 \quad (95\%\text{CL}).\]  

(1.4)

Up to now, a Gaussian distribution of the primordial curvature perturbation is still consistent with experiments. However, the lower bound on $f_{\text{NL}}^{\text{local}}$ in equation (1.4) is greatly reduced from WMAP3 to WMAP5. If a large positive $f_{\text{NL}}^{\text{local}}$ is confirmed by more years of integration of WMAP, or the Planck mission, it could help us to distinguish the curvaton model from the inflation model. Recently, many issues related to the curvaton were widely discussed in [11]–[13].

The curvaton is a light scalar field with weak self-interaction. Even though its energy density is subdominant during inflation, it can make the main contribution to the primordial curvature perturbation. In the curvaton model, $f_{\text{NL}}$ is inversely proportional to the fraction of the curvaton energy density in the energy budget at the epoch of curvaton decay, and the value of $g_{\text{NL}}$ is mainly contributed by the curvaton self-interaction. In the literature, the interaction term of the curvaton is assumed to be negligible and then $g_{\text{NL}}$ is small. Now the cosmological observations are becoming more and more accurate, and even $g_{\text{NL}}$ could be detected in the near future if it is not too small. So it is worth working out how $g_{\text{NL}}$ is related to the curvaton self-interaction.

In [13,14] the authors only considered the case with small positive interaction term and they concluded that $g_{\text{NL}} \sim \mathcal{O}(10^{-4}) - \mathcal{O}(10^{-5})$ for reasonable cases. In this paper we will see that it is also reasonable to consider the case with small negative interaction term for the curvaton model and discuss the dynamics of the curvaton more carefully.

This paper is organized as follows. In section 2, we discuss some possible forms of the curvaton potential. In section 3, the dynamics of the curvaton with a potential slightly deviating from the quadratic form is studied in detail. In section 4, we treat the non-quadratic potential as a perturbation, and analytically calculate $f_{\text{NL}}$ and $g_{\text{NL}}$ to leading order. How to determine the non-quadratic term from experiments is investigated in section 5. At the end, we give a summary, in section 6.

2. Estimator of small deviation from the quadratic potential

We consider the following form of the curvaton potential:

$$V(\sigma) = \frac{1}{2} m^2 \sigma^2 + \lambda m^4 \left(\frac{\sigma}{m}\right)^n.$$  

(2.1)

The mass term is assumed to be dominant, and the size of the interaction term compared with the mass term is measured by

$$s \equiv \lambda \left(\frac{\sigma_*}{m}\right)^{n-2},$$  

(2.2)

where $\sigma_*$ is the value of the curvaton at Hubble exit. In the curvaton scenario the curvaton mass $m$ is assumed to be much smaller than the Hubble parameter $H_*$ during inflation, and then the value of the curvaton at the end of inflation is also $\sigma_*$. Usually the stability of the system requires the coupling constant $\lambda$ to be positive. Otherwise the potential does not have a lower bound when the vacuum expectation value

\[\text{Here the value of } s \text{ is half of that in [13,14].}\]
of the field goes into infinity. The authors in [13,14] focused just on the case of $s > 0$. However we cannot expect the potential to still be reliable when $\sigma$ is very large. So we can expect $s$ to take a negative value.

On the other hand, to keep the mass of the curvaton small enough, some symmetries are called for, to prevent a large quantum correction to its mass. It seems natural to invoke supersymmetry. However, supersymmetry must be broken down during inflation and the mass square of each scalar field generically receives a correction of order $H^2$. An alternative curvaton candidate is the pseudo-Nambu–Goldstone boson [15]—the axion. The potential of the axion is given by

$$V(\sigma) = M^4 \left( 1 - \cos \frac{\sigma}{f} \right), \quad (2.3)$$

where $f$ is called the axion decay constant. The smallness of the axion mass is protected by the shift symmetry $\sigma \to \sigma + \delta$. In string compactifications, axion fields are popular, and even when all other moduli are stabilized, the axion potential remains rather flat as a consequence of non-renormalization theorems. In inflation driven by the axion, the decay constant is required to be larger than the Planck scale. However, such a large value of $f$ cannot be achieved in string theory [16,17]. But the axion is a nice curvaton candidate. For a small axion displacement $\sigma < f$, the potential is expanded as

$$V(\sigma) \simeq \frac{1}{2} \frac{M^4}{f^2} \sigma^2 - \frac{1}{24} \frac{M^4}{f^4} \sigma^4 + \cdots. \quad (2.4)$$

The mass and the coupling of the axion are respectively $m = M^2/f$ and $\lambda = -M^4/(24f^4)$. In this case the parameter $s$ is

$$s = -\frac{1}{24} \frac{\sigma^2}{f^2}, \quad (2.5)$$

which is negative. Since $\sigma_* < f$, $-1/24 < s < 0$.

Because the curvaton mass is small compared to the Hubble parameter $H_*$ during inflation, the Compton wavelength of the curvaton is larger than the background curvature radius $H_*^{-1}$, and the gravitational effects may play a crucial role in the behaviour of the curvaton. In [18] the authors explicitly showed that the quantum fluctuation of a light scalar field $\sigma$ with mass $m$ in de Sitter space gives it a non-zero expectation value of $\sigma^2$ as follows:

$$\langle \sigma^2 \rangle = \frac{3H_*^4}{8\pi^2m^2}. \quad (2.6)$$

So the typical value of the curvaton at the Hubble exit is given by

$$\sigma_* \simeq \sqrt{\frac{3}{8\pi^2}} \frac{H_*^2}{m}, \quad (2.7)$$

and then

$$s = \lambda \left( \sqrt{\frac{3}{8\pi^2}} \frac{H_*^2}{m} \right)^{n-2}. \quad (2.8)$$
For the axion-type curvaton, we have
\[ s = -\frac{1}{(8\pi)^2} \frac{H^4}{m^2 f^2} = -\frac{1}{(8\pi)^2} \frac{H^4}{M^4}. \]  

Here we assume \( \sigma_* > f \). Let us consider a concrete example of an axion from [16] where a five-dimensional U(1) gauge theory is compactified on a circle of radius \( R \). A non-local potential as a function of the gauge invariant Wilson loop
\[ e^{i\theta} = e^{i f A_5 dx^5} \]  
will be generated in the presence of charged fields in the bulk. The effective Lagrangian for \( \theta \) takes the form
\[ \mathcal{L} = \frac{1}{2g_4^2(2\pi R)^2} (\partial \theta)^2 - V(\theta), \]  
where the four-dimensional gauge coupling \( g_4 \) is related to that in five dimensions by
\[ g_2^4 = \frac{g_5^2}{(2\pi R)}, \]  
and the potential is roughly given by
\[ V(\theta) \sim \frac{c}{16\pi^6} \frac{1}{R^4}(1 - \cos \theta), \]  
and \( c \sim \mathcal{O}(1) \) depends on the number of charged fields in the bulk. In this case, \( f = 1/(2\pi g_4 R) \), \( M^4 = (c/16\pi^6)(1/R^4) \) and the parameter \( s \) becomes
\[ s = -\frac{\pi^4}{4c}(H_* R^4). \]  

For validity of the effective description of the four-dimensional scenario, the size of the extra dimension is required to be much smaller than \( H_*^{-1} \). If \( s \) can be determined by experiments, we can roughly establish the ratio between the size of the extra dimension and the Hubble radius during inflation in this model.

3. The dynamics of the curvaton after inflation

After inflation, radiation dominates our universe and the Hubble parameter drops over time as \( H = 1/(2t) \). The curvaton equation of motion after inflation is
\[ \ddot{\sigma} + 3 \frac{3}{2t} \dot{\sigma} = -m^2 \sigma \left( 1 + n \lambda \left( \frac{\sigma}{m} \right)^{n-2} \right). \]  

The correction in the equation of motion is small if \( |s| \ll 1/n \). We introduce a dimensionless time coordinate \( x = mt \) and the curvaton equation of motion becomes
\[ \sigma'' + \frac{3}{2x} \sigma' = -\sigma \left( 1 + n \lambda \left( \frac{\sigma}{m} \right)^{n-2} \right), \]  
where the prime denotes the derivative with respect to \( x \). The time when inflation ends is roughly given by \( t_{\text{end}} = 1/(2H_*) \) and thus the initial time coordinate \( x_{\text{ini}} = mt_{\text{end}} \simeq 0 \). Assume that \( x = x_0 = mt_0 \) at the time when the curvaton is starting to oscillate. Perturbatively solving equation (3.2) with initial conditions \( \sigma_{\text{ini}} = \sigma_* \) and \( \sigma'_{\text{ini}} = 0 \) at
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\[
\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 P_\zeta(k_1) \delta^3(k_1 + k_2), \tag{4.1}
\]
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3), \tag{4.2}
\]
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4) \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta^3(k_1 + k_2 + k_3 + k_4). \tag{4.3}
\]

Figure 1. The values of \(w(x_o)\) and \(g(n, x_o)\) for different values of \(x_o\).

\(x = 0\), we find that the value of the curvaton at \(x_o\) is
\[
\sigma_o = \sigma_s \left[ w(x_o) + ng(n, x_o)\lambda \left( \frac{\sigma_s}{m} \right)^{n-2} + O(s^2) \right], \tag{3.3}
\]
where
\[
w(x_o) = 2^{1/4} \Gamma \left( \frac{5}{4} \right) x_o^{-1/4} J_{1/4}(x_o), \tag{3.4}
\]
\[
g(n, x_o) = \pi 2^{(n-5)/4} \Gamma \left( \frac{5}{4} \right) n^{-1} x_o^{-1/4} \times \left[ J_{1/4}(x_o) \int_0^{x_o} J_n^4(x) Y_{1/4}(x) x^{(6-n)/4} \, dx \right. \\
\left. - Y_{1/4}(x_o) \int_0^{x_o} J_n^4(x) x^{(6-n)/4} \, dx \right], \tag{3.5}
\]
and \(J_{1/4}\) and \(Y_{1/4}\) are respectively the 1/4-order Bessel \(J\) function and the Bessel \(Y\) function. The curvaton starts to oscillate when the Hubble parameter drops below the order of magnitude of the curvaton mass \(m\). Usually we have \(H_o = m\) and then \(x_o = \frac{1}{2}\). In [14] the authors assumed \(x_o = 1\). The values of \(w(x_o)\) and \(g(n, x_o)\) for different \(x_o\) are shown in figure 1. For \(x_o = \frac{1}{2}\), \(w(x_o) \simeq 0.95\) and \(g(2, x_o) \simeq -0.049\), \(g(4, x_o) \simeq -0.047\), \(g(6, x_o) \simeq -0.046\). For \(x_o = 1\), \(w(x_o) \simeq 0.81\) and \(g(2, x_o) \simeq -0.179\), \(g(4, x_o) \simeq -0.16\), \(g(6, x_o) \simeq -0.145\). We see that \(w(x_o)\) and \(g(n, x_o)\) depend on the choice of the time when the curvaton starts to oscillate.

4. Non-linearity parameters in the curvaton model

The primordial curvature perturbation was calculated in [3, 8]. The primordial power spectrum, bispectrum and trispectrum are defined by

\[
\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 P_\zeta(k_1) \delta^3(k_1 + k_2), \tag{4.1}
\]
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3), \tag{4.2}
\]
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4) \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta^3(k_1 + k_2 + k_3 + k_4). \tag{4.3}
\]
The bispectrum and trispectrum are respectively related to the power spectrum by

\[ B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}[\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + 2 \text{ perms}], \tag{4.4} \]

\[ T_\zeta(k_1, k_2, k_3, k_4) = \tau_{\text{NL}}[\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_3)\mathcal{P}_\zeta(k_4) + 11 \text{ perms}] \]

\[ + \frac{54}{25} g_{\text{NL}}[\mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3)\mathcal{P}_\zeta(k_4) + 3 \text{ perms}]. \tag{4.5} \]

Here the non-linearity parameter \( \tau_{\text{NL}} \) depends on \( f_{\text{NL}} \):

\[ \tau_{\text{NL}} = \frac{36}{25} f_{\text{NL}}. \tag{4.6} \]

In the curvaton model, the amplitude of the primordial power spectrum and the non-linearity parameters can be written as

\[ P_\zeta = \frac{1}{9\pi^2} f_D^2 q^2 \frac{H_*^2}{\sigma_\zeta^2}, \tag{4.7} \]

\[ f_{\text{NL}} = \frac{5}{4f_D} (1 + h) - \frac{5}{3} \frac{f_D}{6}, \tag{4.8} \]

\[ g_{\text{NL}} = \frac{25}{54} \left[ \frac{9}{4f_D^2} (\tilde{h} + 3h) - \frac{9}{f_D} (1 + h) + \frac{1}{2} (1 - 9h) + 10f_D + 3f_D^2 \right], \tag{4.9} \]

where

\[ f_D = \frac{3 \Omega_{\sigma,D}}{4 - \Omega_{\sigma,D}}, \quad q = \frac{\sigma_\sigma'}{\sigma}, \tag{4.10} \]

\[ h = \frac{\sigma_\sigma''}{\sigma'^2}, \quad \tilde{h} = \frac{\sigma_\sigma'''}{\sigma'^3}. \tag{4.11} \]

According to equation (3.3), the parameters \( q, h \) and \( \tilde{h} \) take the form

\[ q = \frac{w(x_o) + n(n - 1)g(n, x_o)s}{w(x_o) + ng(n, x_o)s}, \tag{4.12} \]

\[ h = \frac{w(x_o) + ng(n, x_o)s}{(w(x_o) + n(n - 1)g(n, x_o)s)^2} n(n - 1)(n - 2)g(n, x_o)s, \tag{4.13} \]

\[ \tilde{h} = \frac{(w(x_o) + ng(n, x_o)s)^2}{(w(x_o) + n(n - 1)g(n, x_o)s)^3} n(n - 1)(n - 2)(n - 3)g(n, x_o)s. \tag{4.14} \]

We see that both \( h \) and \( \tilde{h} \) are proportional to the size of the non-quadratic term \( s \). If the curvaton potential deviates from the exactly quadratic form, the value of \( f_{\text{NL}} \) can be small even when \( f_D \ll 1 \) if 1 is cancelled by \( h \), but \( g_{\text{NL}} \) is still very large.

Here we also want to stress that the choice of the time when the curvaton starts to oscillate has a great effect on the quantitative results. For \( n = 4 \), we illustrate the quantitative results for \( q, h \) and \( \tilde{h} \) for different choices of \( x_o \) in figure 2.

In [13] the authors only consider the case of \( s > 0 \) and they concluded that \( f_{\text{NL}} \) can be small, but \( g_{\text{NL}} \) will be \( \mathcal{O}(-10^4) - \mathcal{O}(-10^5) \) for reasonable cases. In section 2 we argued that it is reasonable to extend the discussion to the case of \( s \leq 0 \). For \( s < 0 \), both \( h \) and \( \tilde{h} \) are positive, \( f_{\text{NL}} \) always gets a large positive value if \( f_D \ll 1 \), and \( g_{\text{NL}} \sim f_{\text{NL}}^3 \), which is roughly of the same order of magnitude as \( \tau_{\text{NL}} \). See the numerical results for \( f_{\text{NL}} \) and \( g_{\text{NL}} \) in figure 3.
5. A proposal for measuring the non-quadratic term

In [13] the authors pointed out that any deviation from the relation

\[ g_{\text{NL}} \simeq -\frac{10}{3} f_{\text{NL}} \tag{5.1} \]

will indicate that the curvaton potential does not take the exactly quadratic form. Here we will take a closer look at the measurement of the curvaton self-interaction term.

From equations (4.8) and (4.9), we find

\[ g_{\text{NL}} + \frac{10}{3} f_{\text{NL}} = \frac{25}{24} \frac{\tilde{h}}{f_D^2} + \frac{25}{108} (-23 - 9h + 8f_D + 6f_D^2). \tag{5.2} \]

Here we focus just on the case with large absolute values of \( f_{\text{NL}} \) and \( g_{\text{NL}} \), because the accuracy of experiments is limited. Or equivalently, we have \( f_D \ll 1 \). If \(|s| \ll 1\), we have \( f_{\text{NL}} \simeq 5/(4f_D) \) and

\[ g_{\text{NL}} + \frac{10}{3} f_{\text{NL}} \simeq \frac{2}{3w(x_o)} f_{\text{NL}}^2 n^2(n - 1)(n - 2) g(n, x_o)s. \tag{5.3} \]
Figure 3. The non-linearity parameters $f_{NL}$ and $g_{NL}$ for different values of $n$, $s$ and $x_o$ are plotted. Here we take $f_D = 0.01$.

For convenience, we define a new quantity $\alpha(s)$ as follows:

$$\alpha(s) \equiv \frac{w(x_o)(3g_{NL} + 10f_{NL})}{2f_{NL}^2 n^2(n-1)(n-2)g(n,x_o)}.$$  \hspace{1cm} (5.4)

If $|s| \ll 1$, we can expect $\alpha(s) = s$. Since there are three free parameters ($f_D$, $n$, $s$) from the theoretical viewpoint, but only two observed quantities ($f_{NL}$, $g_{NL}$), we cannot model-independently determine the value of $s$ from experiments. For instance, we consider the interaction term taking the form $\lambda \sigma^4$ and show $\alpha(s)$ as a function of $s$ in figure 4. Even though we set $f_{NL} = 50$, one can check that $\alpha(s)$ is quite insensitive to $f_{NL}$. In this case $\alpha(s) \simeq s$ if $|s| < 0.01$ and this result is roughly independent of the choice of $x_o$.

Before the end of this section, we also want to estimate the order of magnitude of the dimensionless coupling constant $\lambda$ in the case where $n = 4$. The typical value of the curvaton during inflation is $\sigma_* \sim H_*^2/m$, and then $s \sim \lambda(H_*/m)^4$ and the amplitude of the primordial power spectrum is $P_\zeta \sim f_D^2 m^2/H_*^2$. For large $f_{NL}$ and small $s$, we have $f_{NL} \sim 1/f_D$. Therefore $m/H_* \sim \sqrt{P_\zeta f_{NL}}$ and $\lambda \sim s(\sqrt{P_\zeta f_{NL}})^4$. Taking into account the WMAP normalization $P_\zeta = 2.457 \times 10^{-9}$ \cite{6}, $\lambda \sim 10^{-18} f_{NL}^3 s$. For $f_{NL} \sim 10^2$ and $s \sim \pm 10^{-2}$, $g_{NL} \sim -10s f_{NL}^2 \sim \mp 10^3$ and $\lambda \sim \pm 10^{-12}$. Such weak interactions could possibly be detected by the cosmological experiments in the near future!
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Figure 4. This figure illustrates $\alpha(s)$ as a function of $s$. In fact, for large values of $f_{\text{NL}}$, it is insensitive to $f_{\text{NL}}$.

6. Conclusions

The fluctuations of a curvaton field evolve non-linearly on superhorizon scales if the curvaton potential deviates from the exactly quadratic form. In this paper, we suggest that the leading order of the non-quadratic term for the curvaton field can be negative, for example in the axion-type curvaton model. We also investigate the curvaton dynamics and the non-linearity parameters in the curvaton model with non-quadratic curvaton potential in detail. For a positive non-quadratic term, $f_{\text{NL}}$ can be very small even when $f_D \ll 1$. But this does not happen in the case with a negative non-quadratic term. We also see that the second-order non-linearity parameter $g_{\text{NL}}$ measures the size of the self-interaction of the curvaton field. If $g_{\text{NL}}$ is positive, the leading order of the self-interaction term should be negative, and the axion-type curvaton model is preferred. The next generation of experiments such as Planck will improve the accuracy to $\Delta f_{\text{NL}} \sim 6$ and $\Delta \tau_{\text{NL}} \sim 560$ [19]. We hope that the non-linearity parameters will be detected soon.

In this paper, the self-interaction term is treated as a perturbation. The case in which the self-interaction term dominates the curvaton potential during inflation will be discussed in [20]. On the other hand, a multiplicity of curvaton fields are expected in the fundamental theories going beyond the standard model. In particular, axions are typically present in large numbers in string compactifications. In [12], it is suggested that these axion fields are taken as curvatons. It would be worth investigating the non-linearity parameters in the $N$-vaton model [12] carefully in the future.

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