Joint Communications and Sensing Employing Optimized MIMO-OFDM Signals

Kai Wu, Member, IEEE, J. Andrew Zhang, Senior Member, IEEE, Zhitong Ni, Member, IEEE, Xiaojing Huang, Senior Member, IEEE, Y. Jay Guo, Fellow, IEEE, and Shanzhi Chen, Fellow, IEEE

Abstract—Joint communications and sensing (JCAS) have the potential to improve the overall energy, cost and frequency efficiency of Internet of Things (IoT) systems. As a first effort, we propose to optimize the MIMO-OFDM data symbols carried by subcarriers (SCs) for better time- and spatial-domain signal orthogonality. This can reduce intertarget and interantenna interference, enabling high-quality sensing. We establish an optimization problem that modifies data symbols on SCs to enhance the above-mentioned signal orthogonality. We also develop an efficient algorithm to solve the problem based on the majorization–minimization framework. Moreover, we discover unique signal structures and features from the newly modeled problem, which substantially reduce the complexity of majorizing the objective function. We also develop new projectors to enforce the feasibility of the obtained solution. Simulations show that to achieve the same sensing performance, the optimized waveform can reduce the signal-to-noise ratio (SNR) requirement by 3–4.5 dB compared with the original waveform, while the SNR loss for the uncoded bit error rate is only 1–1.5 dB.

Index Terms—Joint communications and sensing (JCAS), integrated sensing and communications (ISACs), joint communications and sensing (JCAS), majorization–minimization (MM), multiple-input and multiple-output (MIMO), orthogonal frequency-division multiplexing (OFDM), waveform optimization, waveform orthogonality.

I. INTRODUCTION

Joint communications and sensing (JCAS), a.k.a., integrated sensing and communications (ISACs), have attracted extensive attention in Internet of Things (IoT) community recently [1], [2], [3], [4], [5], [6]. On one hand, it is because microwave sensing, which may also be known as radar/radio sensing or wireless sensing, has found extensive applications in numerous IoT use cases, including intelligent transport, smart cities/homes/farms/health care, human-activity sensing, etc. [1], [7], [8]. On the other hand, performing communications and sensing with one set of hardware and software resources can greatly improve energy-, cost- and frequency-efficiency [2]. These efficiencies are of critical significance to the sustainable development of IoT systems, as they increasingly penetrate almost all aspects of our lives.

There are mainly two types of JCAS: 1) passive [1], [7], [8]. While a passive JCAS tends to integrate sensing (communications) into an existing communications (sensing) system without modifying the way the original system works, we consider here the active JCAS that integrates sensing into a communication system with tolerable changes made to the latter. So far, such active JCAS has been performed through designing dual-functional waveforms in spatial, time and frequency domains. In the spatial domain, dual-functional precoders/beamformers are generally designed to, e.g., approach desired sensing waveforms subject to signal-to-interference-plus-noise ratio (SINR) requirements for multiuser downlink multiple-input and multiple-output (MIMO) communications [10]. In time and frequency domains, existing works mainly resort to designing the frame structure [11], subcarrier (SC) occupation [12], power allocation [13], and pilot/preamble signals [14]. Communication data waveforms are rarely optimized, yet have been widely used, for JCAS.

In [15], a classical sensing method using the orthogonal frequency-division multiplexing (OFDM) communication signals is developed. The method transforms the echo signal into the frequency domain, removes the data symbols through pointwise divisions, and generates the range-Doppler map through a 2-D Fourier transform. The method has been extensively used in the past decade, particularly in automotive sensing [16], [17]. In [18], the single-carrier OFDM (SC-OFDM) sensing is developed using communication data signals. The sensing is performed in a similar way to that in [15]. However, since SC-OFDM can have severely fluctuating amplitudes in the frequency domain, the pointwise division in [15] is replaced with the pointwise product to reduce noise enhancement. In [19], the methods [15], [18] are improved by relieving the constraints imposed by communication systems on sensing. In [20], [21], [22], [23], and [24], sensing using the data signals of orthogonal time-frequency space (OTFS) communication systems is developed.

The communication data signals-based JCAS designs [15], [18], [19], [20], [21], [22], [23] reviewed above mainly use a single antenna. The method [15] is later extended for MIMO cases. To ensure orthogonality of the signals transmitted by
different antennas, an equidistant SC interleaving scheme is developed in [25]. The scheme lets antenna $m$ use SCs $m + iM$ for $m = 0, 1, \ldots, M - 1$, where $M$ is antenna number, and $i \geq 0$ such that $(m + iM) \mod M$ is no greater than the overall SC number. Considering that the equidistant interleaving can reduce the unambiguously measurable distance, the work [26] develops a nonequidistant SC interleaving scheme. However, these interleaving schemes [25], [26] can reduce communication spectral efficiency, as many SCs need to be kept unused on each antenna.

This article is motivated to improve the sensing performance by using the widely available data symbols in a MIMO-OFDM system for JCAS without incurring substantial performance changes to communications. Similar to [25] and [26], we aim to enhance the orthogonality of data signals from all antennas. This is also to create a sensing scheme analogous to conventional orthogonal MIMO radars [27]. Such a sensing scheme can be quite attractive to IoT systems, as a single transmission can illuminate a large spatial region with a relatively uniform power distribution. Moreover, the waveform orthogonality can be exploited to virtually extend the array aperture (and hence the spatial resolution) to be much larger than the physical aperture that is typically small in IoT devices.

Different from [25] and [26], we do not perform the optimization through SC interleaving (which, as mentioned earlier, will reduce communication spectral efficiency). Instead, we consider a more common case in MIMO-OFDM communications, where all antennas occupy the same SCs [28], [29]. Moreover, we aim to introduce minimal changes to data symbols carried by all SCs to enhance the orthogonality of the corresponding time-domain signals from all antennas. Data symbols are in the frequency domain and need to be strictly constrained to maintain satisfactory communication performance. Thus, the proposed waveform design is distinct from the conventional MIMO radar waveform design that is directly performed and constrained in the time domain; see, e.g., [30], [31], [32], and [33]. The main contributions of our work are summarized as follows.

1) We propose to optimize the frequency-domain data symbols of a MIMO-OFDM communication system to enhance the time and spatial orthogonality of the signals transmitted by all antennas. To the best of our knowledge, this is the first work exploring the data symbol domain for JCAS waveform optimization. We model an optimization problem for the proposed waveform design. In particular, we stack all data symbols over SCs antenna-by-antenna into a long signal sequence; unitarily express all the cyclic auto- and cross-correlations of multiantenna time-domain signals as functions of the same long signal sequence; and minimize the peak sidelobe of the correlations subject to the power and similarity constraints.

2) We develop an efficient algorithm to solve the optimization problem using the majorization–minimization (MM) framework [34]. As suggested by the name, MM includes majorizing the objective function first and then minimizing the majorized function subject to certain constraints. For the first stage, we discover the unique signal structures and features from the newly modeled optimization problem, substantially reducing the computational complexity (CC) in majorizing the objective function. Taking a typical IoT system with four antennas and 128 SCs for instance, we reduce the majorization complexity from $\mathcal{O}(10^8)$ to $\mathcal{O}(10^2)$, significantly facilitating IoT devices to perform JCAS. More details will be given in Section II-D. For the second stage of MM, we develop new projectors for two of the most common communication constellations, i.e., phase shift keying (PSK) and quadrature amplitude modulation (QAM), effectively confining the optimized solutions in feasible regions.

We perform extensive simulations to validate the proposed waveform optimization by illustrating its impact on both communications and sensing performances. Key metrics include the uncoded bit error rate (BER) for communications and the detecting probability (DP) for sensing. For 4/8/16 PSK modulations, the proposed waveform optimization can reduce the signal-to-noise ratio (SNR) requirement by more than 3 dB, compared with the original communication waveform, to achieve the same DP that is greater than 0.75. Correspondingly, the BER loss is less than 1.5 dB. For the 16 QAM modulation, our design can reduce the SNR requirement by up to 4.5 dB than using the original communication waveform, to achieve the same DP of 0.85, while the BER loss is only up to 1.2 dB.

**Paper Structure:** The remainder of this article starts with illustrating the system model, motivation and problem formulation in Section II. The solution to the problem is developed by first majorizing the objective function in Section III, then developing projectors in Section IV to enforce the feasibility of the solution, and finally establishing the overall algorithm for the proposed waveform optimization in Section V with complexity analysis. Simulation results are provided in Section VI, and this article is concluded in Section VII.

**Notations and Symbols:** Throughout this article, we use bold upper case letters, e.g., $X$, for matrices; bold lower case letters, e.g., $x$, for vectors; calligraphic upper case letters, e.g., $\mathcal{C}$, for sets; $\mathbb{C}$ for optimization-independent constants and $\mathbb{C}$ denotes the set of complex numbers. We use $(\cdot)\mathbb{T}$ for transpose, $(\cdot)^*$ for conjugation, $(\cdot)\mathbb{H}$ for conjugate transpose, and $(\cdot)^\dagger$ for pseudo-inverse. Moreover, $\otimes$ is for the pointwise product, and $\otimes$ for the Kronecker product. We let $\Re\{x\}$ (or $\Im\{x\}$) take the real (imaginary) part of a complex number $x$.

While $\begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}_{m,n=0}^{n-1}$ generates a square matrix with the row index $m$, the column index $n$ and the size $N$, $[x]_n$ takes the $n$th entry of $x$; $[X]_{a,b}$ the $(a, b)$th entry of $X$; $[X]_{a,:}$ the $a$th row; $[X]_{:,b}$ the $b$th column. The operation $\text{vec}(X)$ vectorizes $X$ column-by-column; $\lambda_{\text{max}}(X)$ returns the maximum eigenvalue of $X$; $\text{diag}(x)$ generates a diagonal matrix with $x$ put on the diagonal;
by \( \mathbf{X}_o = [x_0, x_1, \ldots, x_{M-1}] \). As indicated by the block “Waveform Optimization” in Fig. 1, \( \mathbf{X}_o \) will be optimized over SCs and antennas, using the methods to be developed in Sections III and V, to improve the spatial and time-domain orthogonality. Let \( \mathbf{X} \) denote the optimized waveform. Then, a DFT is taken over SCs, i.e., each column of \( \mathbf{X} \). Denote the result by \( \mathbf{F} \mathbf{X} \in \mathbb{C}^{N \times M} \), where \( \mathbf{F} \) is an \( N \)-dimensional DFT matrix, as given by

\[
\mathbf{F} = \left[ e^{-j \frac{2\pi mn}{N}} \right]_{m,n=0}^{N-1}.
\]

Here, \( \left[ e^{-j \frac{2\pi mn}{N}} \right]_{m,n=0}^{N-1} \) generates an \( N \times N \) square matrix with the row index \( m \) and column index \( n \). Next, add CP by copying the bottom \( N_{CP} \) rows of \( \mathbf{F} \mathbf{X} \) to its top. Each column of signal samples is then transmitted by an individual antenna after being processed by a radio frequency chain (DAC, frequency conversion and power amplification). The signals go through a communication (sensing) channel and arrive at a communication (sensing) receiver. Next, we briefly describe the signal processing steps at the two different receivers.

**B. Communication Receiver Processing**

We describe the MIMO-OFDM communications at the receiver, for the purpose of introducing some communication performance metrics. These metrics will be observed later to evaluate the impact of the proposed design on data communications. To serve the above purpose in a concise yet sufficient manner, we consider a digital MIMO communication receiver that has \( K \) antennas. Since \( M \) independent data streams are transmitted, \( K \geq M \) is necessary for spatial multiplexing.

The signals received at all antennas are digitized by the receiver chains. Removing CPs and taking the DFT over \( N \) samples of each antenna, we can write the received signal matrix as \( \mathbf{H} \mathbf{X}^\dagger + \mathbf{N} \in \mathbb{C}^{K \times N} \), where \( \mathbf{H} \in \mathbb{C}^{K \times M} \) is the channel matrix and \( \mathbf{N} \) is the noise matrix. Typically, \( [\mathbf{N}]_{k,n} \sim \mathcal{CN}(0, \sigma^2_n) \) \( \forall k, n \), where \( [\mathbf{N}]_{k,n} \) takes the \((k,n)\)th entry of a matrix, and \( \sigma^2_n \) denotes noise power. Applying the zero forcing combiner, the data symbol matrix can be estimated as \( \mathbf{\hat{X}} = \mathbf{H}^\dagger \mathbf{F}^\dagger \mathbf{X} \). Demodulation can then be performed based on \( \mathbf{\hat{X}} \).

**C. Sensing Receiver Processing and Motivation**

For ease of exposition, we consider a single-antenna sensing receiver and illustrate the receiving steps based on a single OFDM symbol. After some typical RF processing, such as low-noise power amplification, frequency conversion, and ADC, the CP in the digitized samples is first removed. Assuming that all target delays are smaller than the CP duration, an \( N \)-dimensional DFT can be taken over the time-domain samples to obtain the following signals over SCs:

\[
y = \sum_{i=0}^{N_t-1} \alpha_i (\mathbf{X}(\theta_i)) \odot \mathbf{b}(\tau_i) + \mathbf{n}
\]

where \( i \) denotes the target index, \( N_t \) the total target number, \( \alpha_i \) absorbs the scattering coefficient and RF processing gain, \( \theta_i \) denotes the Angle-of-Departure (AoD) of the \( i \)th target, \( \mathbf{a}(\theta_i) \) the spatial steering vector, \( \tau_i \) the target delay, \( \mathbf{b}(\tau_i) \) the
range steering vector, and \( n \) a complex Gaussian noise vector with identically and independently distributed entries. The \( m \)th entry of \( \mathbf{a}(\theta_i) \) and the \( n \)th of \( \mathbf{b}(\tau_i) \) can be, respectively, given by

\[
a_{mi} = a(\theta_i)_m = e^{-j\pi m \sin \theta_i}; \quad [\mathbf{b}(\tau_i)]_n = e^{-j2\pi m\tau_i}/\Delta_1.
\]

where the antenna spacing is half a wavelength, and \( T_s \) denotes the sampling interval.

As shown in Fig. 1, stacking the \( M \) copies of \( \mathbf{y} \) in the row dimension yields a matrix \( \mathbf{Y} \). Performing a pointwise product between \( \mathbf{Y} \) and \( \mathbf{X}^* \) and then taking the inverse DFT (IDFT) of each column, we obtain \( \mathbf{Z} = \text{IDFT}(\mathbf{Y} \odot \mathbf{X}^*) \), where \( \mathbf{X} \) is the matrix of all data symbols over SCs (rows) and antennas (columns) at the transmitter. The \( m \)th column of \( \mathbf{Z} \), as denoted by \( \mathbf{z}_m \), is the cyclic cross-correlation (CCC) between IDFT(\( \mathbf{y} \)) and IDFT(\( \mathbf{x}_m \)) [18]. Based on (2), we can express \( \mathbf{z}_m \) as

\[
\mathbf{z}_m = \sum_{i=0}^{N_t-1} a_{mi} \text{IDFT}([|\mathbf{x}_m|^2 \odot \mathbf{b}(\tau_i)]) + \sum_{m' \neq m} \sum_{i=0}^{N_t-1} a_{m'\tau_i} \text{IDFT}([\mathbf{x}_m^* \odot \mathbf{x}_m' \odot \mathbf{b}(\tau_i)] + \mathbf{n}
\]

where \( a_{i\tau} \) is the \( x \)th entry of \( \mathbf{a}(\theta_i) \).

Based on (3) and the circular shift property of IDFT [37], the shape of IDFT([\( |\mathbf{x}_m|^2 \odot \mathbf{b}(\tau_i) \)]) is only dependent on \( |\mathbf{x}_m|^2 \), while the peak location changes with \( \tau_i \). If two targets are located closely (i.e., their \( \tau_i \)'s are similar), they interfere with each other. To reduce the interference, we expect IDFT([\( |\mathbf{x}_m|^2 \)]) \( \forall m \) to have low sidelobes. This leads to the first goal of waveform design.

**Goal 1:** To reduce intertarget interference, IDFT(\( \mathbf{x}_m \)) needs to have low sidelobes in its cyclic auto-correlation.

The second term on the right-hand side (RHS) of (4) is the interference to antenna \( m \) caused by signals transmitted by other antennas. To reduce this interference, we expect IDFT([\( \mathbf{x}_m^* \odot \mathbf{x}_m' \)]) to be small. This results in another waveform design goal.

**Goal 2:** To reduce interantenna interference, IDFT(\( \mathbf{x}_m \)) \( \forall m \) and IDFT(\( \mathbf{x}_m' \)) \( \forall m \neq m \) need to have low CCC.

Since IDFT([\( |\mathbf{x}_m|^2 \odot \mathbf{b}(\tau_i) \)]) accumulates at a target location, e.g., \( \{\tau_iT_s\} \), in an approximately coherent manner, the \( \{\tau_iT_s\} \)th entry of \( \mathbf{z}_m \) differs over \( m \) mainly because of \( a_{mi} \), as seen from (3). Thus, taking another IDFT over each row of \( \mathbf{Z} \), the row dimension will then be transformed into the angular domain. Consequently, we obtain a range-angle map (RAM) that can be used for target detection and localization. To highlight the importance of the two waveform design goals, let us compare the RAMs that are obtained by different waveforms.

Fig. 2 plots the RAMs obtained using the original communication waveform in (a), the waveform optimized based on Goals 1 and 2 in (b), and the orthogonal waveform in (c), where the optimization methods will be developed shortly. Note that different waveforms lead to different \( \mathbf{X} \) in creating the echo signal, as modeled in (2). Moreover, the orthogonal waveform alternates the allocations of SCs over antennas such that each SC is only used by a single antenna in an OFDM symbol [25].

For Fig. 2, we set \( N = 128 \), \( M = 4 \) and \( N_{\text{CP}} = 32 \). Moreover, five targets are simulated with the range bins, i.e., \( \{\tau_iT_s\} \) in (3), are set as \( 1, 8, 75, 16.5, 24.25 \), and 32. Their AoDs are randomly generated in \([0, 2\pi]\) radians. The sizes of IDFTs over SCs and antennas for generating RAM, see Fig. 1, are 8 times their actual sizes. To highlight the impact of waveforms on sensing, noise is not added. In the RAMs, the five brightest elliptical patches correspond to the five targets. The background surrounding the targets is caused by intertarget and interantenna interference, as noise is not added here. As indicated by the color bar in Fig. 2, the brighter the background, the stronger the interference can be.

From Fig. 2(a), we see that the original communication waveform can lead to a strong interference background. Fig. 2(b) shows that the optimized waveform can substantially weaken interference over the whole range-angle domain. This intuitively validates the improvement of sensing that can be achieved by the proposed waveform optimization based on Goals 1 and 2. Furthermore, Fig. 2(c) shows that the orthogonal waveform [25] achieves the lowest interference background. This is expected, as the orthogonal waveform can fully suppress the interantenna interference. However, it should be noted that the sum rate of the orthogonal waveform [25] is reduced by 73.8%,\(^2\) compared with the original and proposed waveforms in Fig. 2(a) and (b).

While the RAMs presented in Fig. 2 intuitively demonstrate the reduction of sensing interference, as achieved by the proposed waveform design, we will employ more sensing metrics in Section VI, the DP and beam pattern in particular, to further validate the sensing performance improvement achieved by the proposed design.

\[\text{D. Problem Formulation}\]

Now that we have confirmed the feasibility of Goals 1 and 2 in improving sensing performance, we proceed to formulate an optimization problem to realize the two goals. In doing so,

\[\text{2For the original and proposed waveforms, each antenna transmits } (N - N_{\text{un}}) \text{ data symbols, where } N_{\text{un}} \text{, as will be detailed in Section IV, denotes the number of unused SCs per antenna. The orthogonal waveform only transmits } N/M \text{ data symbols per antenna. Thus, the sum rate loss is given by } 1 - (|N/M|/(N - N_{\text{un}})).\]
we propose to moderately modify $x_m$ ($\forall m$) to a level that the communication performance is only slightly affected.

Let $x$ collect all frequency-domain waveforms from the $M$ transmitter antennas, i.e.,

$$x = [x_0; \ x_1; \ \ldots; \ x_{M-1}] \in \mathbb{C}^{MN \times 1}$$

(5)

where $\mathbb{C}^{MN \times 1}$ denotes the set of $MN \times 1$ complex vectors, and “;” concatenates column vectors (as in MATLAB [38]). Corresponding to $x$, we let $x_r$ denote the vector of the original communication data symbols, each drawn independently from a communication constellation, e.g., PSK or QAM. Define $S_m$ as a selection matrix with the expression

$$S_m = [I_{Mm}]^T \otimes I_N$$

(6)

where $[\cdot]_m$ takes the $m$th column of the enclosed matrix, $\otimes$ denotes the Kronecker product, and $I_x$ is an $x$-dimensional identity matrix. With the aid of $S_m$, the time-domain waveform of antenna $m$ can be written as $\tilde{x}_m = F^H S_m x$ ($\forall m$), where the DFT matrix $F$ is given in (1). Define the following circulant matrix:

$$[U_i]n,n' = \begin{cases} 1, & \text{if } n' - n \equiv i \\ 0, & \text{otherwise} \end{cases}$$

(7)

where $(\cdot)$ denotes modulo-\(x\). Then we can express the periodic cross-correlation between $\tilde{x}_m$ and $\tilde{x}_k$ as

$$r_{mki} = (F^H S_k x)^H U_i (F^H S_m x)$$

(8)

where $(\cdot)^H$ performs conjugate transposes. Note that $m$ and $k$ ($m, k = 0, 1, \ldots, M - 1$) are indexes of transmitting antennas, and $i$ ($i = 0, 1, \ldots, N - 1$) is the index of range gate (or bin). Recall that the echo model (2) is obtained based on the assumption that the target delay is no greater than the CP duration. Thus, we only focus the first $N_{CP}$ range bins, where $N_{CP}$ is the CP length.

In the case of $m = k$, $r_{mki}$ becomes the cyclic auto-correlation of the time-domain signal transmitted by antenna $m$. In the case of $m \neq k$, $r_{mki}$ becomes the CCC between signals transmitted by different antennas. With $r_{mki}$ introduced, Goals 1 and 2 can be approximated by minimizing $\max_{m, k, i \in \{1, N_{CP} - 1\}} |r_{mki}|$. The maximum can be further approximated by the $p$-norm with a large $p$ [30]. Therefore, the pursued waveform design is modeled as

$$\min_x f(x) = \sum_{m,k=0}^{M-1} \sum_{i=0}^{N-1} w_i |r_{mki}|^p$$

(9a)

s.t. $w_i = \begin{cases} 1, & \text{if } i = 1, \ldots, N_{CP} - 1 \\ 0, & \text{otherwise} \end{cases}$

(9b)

$$\|x\|^2 = 1$$

(9c)

$$x \approx x_r.$$  

(9d)

Note that the power constraint (9c) takes unit one on the RHS for convenience. When a different power value, say $P$, is set, the solution to (9) only needs to be scaled by $\sqrt{P}$. Also note that the similarity constraint (9d) is given in a general form for the moment. Based on the modulations used by the underlying communication system, we will rewrite (9d) in more specific forms in Section IV. As will be seen, the similarity constraint (9d) can constrain the amplitudes and phases of $x$ separately, complicating the feasible region substantially and differentiating it from a hypersphere [39].

Problems with similar form to (9) have been established for MIMO radar waveform design [30], [31], [32], [33], and the MM algorithm\(^3\) has shown great potential in efficiently solving such problems like (9). Enlightened by the prior success [30], [31], [32], [33], we choose to use the well-established MM optimization framework [34] for solving problem (9). We do not claim that MM is the best option, but we will show that employing MM does enable us to develop an efficient algorithm for solving (9). Moreover, as widely performed in previous works [30], [31], [32], [33], we will first solve the problem (9) without the similarity constraint (9d) and then project the solution onto the feasible region defined by the constraint.

We emphasize that, despite the similarity in form, our optimization problem (9) is very different from those for MIMO radar waveform design [30], [31], [32], [33]. A substantial difference lies in the expression of cyclic auto- and cross-correlations, i.e., $r_{mki}$ given in (8). Since we optimize the frequency-domain waveforms, $r_{mki}$ involves DFT and selection matrices; see (8). In contrast, the variable equivalent to $r_{mki}$ in [30], [31], [32], and [33] do not have these matrices, as $x$ directly denotes the time-domain waveforms therein. This difference prevents us from directly using the existing algorithms [30], [31], [32], [33]. Furthermore, we point out that, even though the MM framework is well developed, applying it to efficiently solve a problem generally involves problem-specific features that require nontrivial effort to discover and exploit. To further elaborate on this point, let us consider applying the following lemma.

**Lemma 1:** The quadratic form $x^H A x$ can be majorized at $x_0$ by $x^H B x + 2\Re\{x^H (A - B) x_0\}$, where $A$ and $B (\succeq A)$ are Hermitian matrices having matching sizes with $x$ for the above calculation, and $C$ absorbs irrelevant constants [33, Lemma 1].

The lemma provides a simple and useful tool for majorizing a quadratic function as will be often encountered later in Section III. However, applying the lemma requires us to find a suitable matrix $B (\succeq A)$. One popular option is to take $B = \lambda_{\max}(A) I$, where $\lambda_{\max}(A)$ denotes the maximum eigenvalue of $A$. However, in our problem, the equivalent matrices playing the role of $A$ have the size of $MN \times MN$, where $M$ is the antenna number and $N$ is the SC number. In typical IoT communication configurations, $MN$ can be about 500 [29], and hence getting $\lambda_{\max}(A)$ has a CC of $O(MN^2) = O(10^{10})$, too luxury for IoT devices. This strongly validates our point: MM indeed gives us useful tools (rules), such as Lemma 1, but simply applying them does not guarantee a practically usable solution (at least in our case).

In what follows, instead of a naive use of MM, we will disclose in Section III some unique signal structures and features embedded in our problem, and exploit them to majorize the objective function (9a). As such, we manage to reduce $O(10^6)$
in the above example to only $O(10^2)$. Furthermore, we will design effective projectors in Section IV to deal with the similarity constraint (9d). The overall algorithm for solving (9) will be established in Section V.

III. MAJORIZING OBJECTIVE FUNCTION (9a)

According to [33, Lemma 10], a general $p$-norm function $x^p$ with $p \geq 2$ and $x \in [0, \bar{x}]$ can be majorized at $x_0$ by $ax^2 + (p\bar{x}^{p-1} - 2\bar{x}_0)x + C$, where

$$a = \left(\bar{x}^p - x_0^p - p\bar{x}^{p-1}(\bar{x} - x_0)\right)/(\bar{x} - x_0)^2.$$

Without obfuscation, we use $C$ to absorb optimization-independent terms hereafter. Applying the majorization method, each summand $|r_{mkq}|^p$ given in (9a) can be majorized individually, leading to

$$f(x) \leq \tilde{f}(x|x^{(l)}) = \tilde{f}_1(x|x^{(l)}) + \tilde{f}_2(x|x^{(l)}) + C$$

s.t. \(\tilde{f}_1(x|x^{(l)}) = \sum_{m,k=0}^{M-1} \sum_{l=0}^{N-1} w_{l}(r_{mk})^2\)

\(\tilde{f}_2(x|x^{(l)}) = \sum_{m,k=0}^{M-1} \sum_{l=0}^{N-1} w_{l}(r_{mk})\)

(10)

where $x^{(l)}$ denotes the optimal solution after the $l$th iteration, $r_{mkq}_{mkq}$ is obtained by substituting $x = x^{(l)}$ into (8), $|r_{mkq}|^p (\forall m, k, l)$ is majorized at $r_{mkq}$ over $[0, \tilde{r}_{mkq}]$ with $r_{mkq} = \max_{m, k, l} |r_{mkq}|$, and the two coefficients are

$$a^{(l)}_{mkq} = \frac{\left(\tilde{r}_{mkq}^p - r_{mkq}^p - p\tilde{r}_{mkq}^{p-1}(\tilde{r}_{mkq}^p - r_{mkq}^p)\right)}{(\tilde{r}_{mkq}^p - r_{mkq}^p)^2}$$

$$b^{(l)}_{mkq} = p \left| \tilde{r}_{mkq}^{p-1} - 2a_{mkq}r_{mkq}\right|.$$

Both $\tilde{f}_1(x|x^{(l)})$ and $\tilde{f}_2(x|x^{(l)})$ in (10) can be further majorized. To proceed, the following properties [40] are useful:

$$\text{vec}(ABC) = (C^T \otimes A)\text{vec}(B)$$

$$\text{vec}(A^H \otimes B^H) = (A^H \otimes D)\text{vec}(C)$$

(13)

where $\otimes$ is Kronecker product, and $A$, $B$, $C$, and $D$ are general matrices with dimensions matching for product.

A. Majorizing $\tilde{f}_1(x|x^{(l)})$ Given in (10)

We start with the following lemma to simplify the expression of $r_{mkq}$; see Appendix-A for the proof.

**Lemma 2**: The periodic cross-correlation given in (8) can be simplified into

$$r_{mkq} = x^H A_{mkq} x, \text{ s.t. } A_{mkq} = S_k^H \text{diag}(N[F_i]_i) S_m$$

where $[F_i]_i$ denotes the $i$th column of the DFT matrix given in (1) and $(\cdot)^*$ takes conjugate.

Applying (13) and Lemma 2, we can have

$$\text{vec}(r_{mkq}) = \left(x^* \otimes x\right)^H \text{vec}(A_{mkq}).$$

Since $r_{mkq}$ is already a scalar, plugging $r_{mkq} = \text{vec}(r_{mkq})$ into (10) turns $\tilde{f}_1(x|x^{(l)})$ into

$$\tilde{f}_1(x|x^{(l)}) = \left(x^* \otimes x\right)^H \left(\sum_{m,k=0}^{M-1} P_{mk}\right) x$$

s.t. \(P_{mk} = \sum_{l=0}^{N-1} (\delta_{mk} w_{l}) \text{vec}(A_{mk}) \text{vec}(A_{mk})^H\).

(15)

Note that $\tilde{f}_1(x|x^{(l)})$ is in a quadratic form and, given real positive coefficients $\delta_{mk}$ and $w_{l}$, $(\sum_{m,k=0}^{M-1} P_{mk})$ is Hermitian. Applying Lemma 1, $\tilde{f}_1(x|x^{(l)})$ obtained in (15) can be further majorized by

$$\tilde{f}_1(x|x^{(l)}) \leq x^H \hat{M} x$$

$$+ 2\tilde{\lambda} \left\{\sum_{m,k=0}^{M-1} \sum_{l=0}^{N-1} P_{mk} \right\} + C$$

s.t. $\hat{M} = \sum_{m,k=0}^{M-1} P_{mk}$; \(x^{(l)} = (x^{(l)})^* \otimes x^{(l)}\).

(16)

The following $M$ is sufficient for the first constraint above:

$$M = \hat{\lambda} I, \text{ s.t. } \tilde{\lambda} \leq \lambda_{\max} \sum_{m,k=0}^{M-1} P_{mk}.$$

However, it is challenging to calculate $\hat{\lambda}$, as the size of $(\sum_{m,k=0}^{M-1} P_{mk})$ can be very large. Taking $N = 100$ and $M = 4$ for example, the dimension of $P_{mk}$ is $400 \times 400$. The following theorem greatly simplifies the calculation of $\hat{\lambda}$; see Appendix-B for the proof.

**Theorem 1**: The maximum eigenvalue of \(\sum_{m,k=0}^{M-1} P_{mk}\), as denoted by $\hat{\lambda}$, satisfies

$$\hat{\lambda} = \max_{m,k=0,...,M-1} \lambda_{\max} \sum_{m,k=0}^{M-1} P_{mk}.$$
Corollary 1: The majorization of \( \tilde{f}_1(x|x^{(l)}) \) given in (16) can be further developed into
\[
\tilde{f}_1(x|x^{(l)}) \leq x^H \left( \frac{Q^{(l)}_1 - 2\lambda}{x} \right) x + C
\]
s.t. \( Q^{(l)}_1 = \sum_{m,k=0}^{M-1} \sum_{i=0}^{N-1} \frac{a_{mki}^i w_i}{2} \left( r^{(l)}_{mki} A_{mki} + r^{(l)}_{mki} A_{mki}^H \right) \),
where \( Q^{(l)}_1 \) is Hermitian, i.e., \( (Q^{(l)}_1)^H = Q^{(l)}_1 \).

B. Majorizing \( \tilde{f}_2(x|x^{(l)}) \) Given in (10)

This majorization is relatively easier than the previous one. With reference to [31, eq. (22)], we can perform the following transformations:
\[
|\tilde{r}_{mki}|^2 \geq \left| \frac{a_{mki}^i x}{\tilde{r}_{mki}} \right|^2 = \sqrt{3\|x\|^2 + 3\|x\|^2}
\]
\[
\tilde{f}_2(x|x^{(l)}) \leq x^H Q^{(l)}_2 x
\]
s.t. \( Q^{(l)}_2 = \sum_{m,k=0}^{M-1} \sum_{i=0}^{N-1} \frac{w_i b_{mki}^i}{2 \tilde{r}_{mki}} \left( r^{(l)}_{mki} A_{mki} + r^{(l)}_{mki} A_{mki}^H \right) \),
where \( (Q^{(l)}_2)^H = Q^{(l)}_2 \).

As \( b_{mki}^i \leq 0 \) for sure, \( \tilde{f}_2(x|x^{(l)}) \) given in (10) is then majorized by
\[
\tilde{f}_2(x|x^{(l)}) \leq x^H Q^{(l)}_2 x
\]
s.t. \( Q^{(l)}_2 = \sum_{m,k=0}^{M-1} \sum_{i=0}^{N-1} \frac{w_i b_{mki}^i}{2 \tilde{r}_{mki}} \left( r^{(l)}_{mki} A_{mki} + r^{(l)}_{mki} A_{mki}^H \right) \).

We emphasize that a necessary condition for the above result is that the matrix \( 2\lambda(x|x^{(l)}(x|x^{(l)})^H) \) has a non-negative minimum eigenvalue. Since \( 2\lambda(x|x^{(l)}(x|x^{(l)})^H) \) is a rank-one matrix, \( \lambda \) is positive for sure and the necessary condition is satisfied.

Again, we are facing a challenging problem of determining \( \bar{\mu} \). To solve it, we begin by disclosing some features of \( Q^{(l)} \); see Appendix-D for the proof.

Lemma 3: \( Q^{(l)} \) is real symmetric and satisfies
\[
Q^{(l)} = \sum_{i=0}^{N-1} Q^{(l)}_i \otimes \text{diag}(|M|), \text{s.t.} \quad \lambda_{\text{max}} \left[ Q^{(l)} \right] = \lambda_{\text{max}} \left[ \left| Q^{(l)} \right| \right]
\]

where \( Q^{(l)}_i \) is also real symmetric and the product of any two summand matrices is a zero matrix.

As a result of Lemma 3,
\[
\lambda_{\text{max}} \left[ Q^{(l)} \right] = \lambda_{\text{max}} \left[ A \otimes B \right]
\]

where the last result is based on the fact that \( \lambda_{\text{max}}(A \otimes B) = \lambda_{\text{max}}(\lambda_{\text{max}}(A) \lambda_{\text{max}}(B)) \).

Algorithm 1 Majorizing \( f(x) \) Given in (9)

\begin{itemize}
  \item [1)] Reshape \( x^{(l)} \) into an \( N \times M \) matrix \( X^{(l)} \);
  \item [2)] For each \( m \), \( \tilde{r}^{(l)}_{mk} = \text{IDFT} \{ \tilde{r}^{(l)}_{mk} \} \) \( k = 0, \cdots, M-1 \);
  \item [3)] Take the IDFT of \( \tilde{r}^{(l)}_{mk} \) giving \( r^{(l)}_{mk} \) and \( r^{(l)}_{mk} = \left[ \tilde{r}^{(l)}_{mk} \right] \);
  \item [4)] Find \( \lambda_{\text{max}} \left[ Q^{(l)} \right] \);
  \item [5)] Calculate \( a_{mki}^i \) in (11), \( b_{mki}^i \) in (12) and \( c_{mki}^i \) in (22);
  \item [6)] Find \( \lambda \) based on Theorem 1;
  \item [7)] Calculate \( v_{mki} \) in (27);
  \item [8)] Construct \( \tilde{r}^{(l)}_{mk} \) based on (61) and (27);
  \item [9)] Construct \( Q^{(l)} \) in (10) and \( \bar{\mu} \) and \( Q^{(l)} \) into (21) results in the final majorization function \( \tilde{f}(x|x^{(l)}) = \mathcal{N} \left[ xH y^{(l)} \right] + C \).
\end{itemize}
\( Q^{(i)} \). In particular, based on (60), we obtain

\[
\Lambda_{mk}^{(i)} = \text{diag}\left(2\|v_{mk}^{(i)}\|\right)
\]

s.t. \( v_{mk}^{(i)} = F[w_{mk}^{(i)}]_{m,k=0}^{N-1} \) \quad (27)

where \( \cdot \) generates an \( N \)-dimensional column vector.

Steps 10–12 efficiently calculate \( \hat{\mu} \) based on the derivations in Section III-C. Note that \( Q^{(i)} \) can be fast constructed based on \( v_{mk}^{(i)} \) obtained in (27). In particular, jointly inspecting (25) and (61), we have

\[
Q^{(i)} = \left[2\|\{v_{mk}^{(i)}\}\|\right]_{m,k=0}^{M-1} \quad (28)
\]

where the outer square brackets generate a matrix with row index \( m \) and column index \( k \). For the eigenvalue calculation in step 12, we can apply efficient algorithms, such as the Lanczos iteration, since \( Q^{(i)} \) is a real symmetric as a result of Lemma 3. Interested readers are referred to [40] for efficient eigendecomposition algorithms which shall not be further elaborated on here. In step 13, we let \( C \) adsorb \( x^{\dagger}N_{x}x \) based on the structure of \( N \) given in (24). We have also suppressed the constant scaling factor “2,” as it does not affect the minimization of majorized function.

IV. DEALING WITH THE SIMILARITY CONSTRAINT (9d)

Based on the majorization function obtained from step 12 in Algorithm 1, we can recast the waveform optimization problem (9) into

\[
x^{(l+1)} = \arg\min_{x} \{x^{H}y^{(l)}\}
\]

s.t. \( y^{(l)} = \left(Q^{(l)} - 2\hat{\mu}\left(x^{(l)}x^{(l)H}\right) - \hat{\mu}I\right)x^{(l)} \)

\[
\|x\|^2 = 1 \quad (9c); \quad x \approx x_{r} \quad (9d)
\]

where \( \arg\{x^{H}y^{(l)}\} \) is the majorization function obtained in step 12 of Algorithm 1, and the two constraints given in (9) are cited here for convenience. Without the similarity constraint (9d), the solution to problem (29) is given by

\[
x^{(l+1)} = -y^{(l)}/\|y^{(l)}\|.
\]

Next, we develop efficient projectors to make \( x^{(l+1)} \) feasible under the similarity constraint (9d). We consider two representative modulations, i.e., PSK and QAM, that are widely used in standardized communication systems.

A. PSK

We start with PSK which mainly uses phases of \( x_{r} \) to convey information bits. The amplitudes of \( x_{r} \) are less relevant and may bear more changes than phases. Therefore, the similarity constraint (9d) can be reformulated by treating the phases and amplitudes of \( x_{r} \) differently, as follows:

\[
0 \leq |[x_{r}]_{i}| - |[x^{(l+1)}]_{i}| \leq \varepsilon_{a}, \quad i = 0, 1, \ldots, MN - 1; \quad (31)
\]

\[
|\arg\{[x^{(l+1)}]_{i}\} - \arg\{[x_{r}]_{i}\}| \leq \varepsilon_{p}
\]

where \( \arg\{\cdot\} \) takes the phase of a complex number; \( \varepsilon_{a} \) and \( \varepsilon_{p} \) denote the maximum tolerable changes in amplitude and phase, respectively. Next, we develop projectors to make \( [x^{(l+1)}]_{i} \) in (30) satisfy (31) and (32) with minimal changes. We start with determining the feasible region defined by the two inequalities.

A complex number can be mapped onto the 2-D plane spanned by the real and imaginary axes, as illustrated in Fig. 3. Thus, the reference point \( [x_{r}]_{i} \) in (30) is drawn as a PSK constellation, can be represented by \( A \) in the figure with |\( OA \)| = 1, where \( OA \) denotes a vector from \( O \) to \( A \) and |\( \cdot \)| takes the length of a vector. The inequality in (31) indicates that in the direction of \( AO \), \( |[x^{(l+1)}]_{i}| \) can go from \( A \) to \( B \) at most, where \( |AB| = \varepsilon_{a} \) as noted in Fig. 3. The inequality in (32) implies that \( |[x^{(l+1)}]_{i}| \) can only move on the circular arc EF (centered at the original) when \( |[x^{(l+1)}]_{i}| = 1 \); and it can move only on the circular arc CD (centered at \( A \)) when \( |[x^{(l+1)}]_{i} - [x_{r}]_{i}| = \varepsilon_{p} \). Based on the two extreme cases considered above, we obtain that the border of the feasible region defined by (31) and (32) is CDEF. However, to simplify the projector, we replace the circular arc CD and EF with GH and QR, respectively. The two line segments are tangent to the two circular arcs. Namely, we use \( \text{GH}Q \) as the border of the feasible region. The approximation error is negligible, as \( \varepsilon_{p} \) is generally small.

Next, we consider three cases, as differentiated by the lengths of the projections of \( OK, OJ, \) and \( OL \) on \( OA \), where \( K, J, \) and \( L \) are three representative positions of \( [x^{(l+1)}]_{i} \) in the 2-D complex plane; see Fig. 3. The three projections share the same expression, as given by \( P \cdot OA \). Here, \( P \) denotes the inner product between \( OK (OJ \) or \( OL) \) and \( OA \). It can be calculated as

\[
P = \arg\{[x^{(l+1)}]_{i}^{H}[x_{r}]_{i}\}.
\]

Note that \( OA \) is already a unit vector; otherwise, a normalization is necessary in the above expression.

Case 1: \( P > 1 \). In this case, \( [x^{(l+1)}]_{i} \) can be represented by \( K \) in Fig. 3. Obviously, \( K \) is outside the feasible region. How to project \( [x^{(l+1)}]_{i} \) onto the border of the feasible region depends on the vertical distance between \( [x^{(l+1)}]_{i} \) and the ray \( OA \). A critical vertical distance, as denoted by \( d_{0} \), is achieved when \( K \) is located on the ray \( OF \). In Fig. 3, \( |KL| > d_{0} \) and we see that \( F \) is the closest point on the border of the feasible region to \( K \). Thus, the projector will replace \( [x^{(l+1)}]_{i} \) with \( [x_{r}]_{i}e^{\varepsilon_{p}} \). Note that \( |KL| > d_{0} \) can also happen when \( K \) locates below the ray \( OR \). Then \( R \), as on the border of the feasible region,
will be the closest point to K. On the other hand, if |KL| ≤ d₀, the angle of |x^{(l+1)}_i| would satisfy (32) and we only need to scale the amplitude of |x^{(l+1)}_i|, to one. Based on the geometric relation in Fig. 3, we have d₀ = P tan(εₚ). Summarizing the above analyses, we obtain the following projector to make \[x^{(l+1)}_i\] feasible in the case of \(P > 1\):

\[
\begin{align*}
[x^{(l+1)}_i] & = \begin{cases} 
|X_i| e^{jεₚ}, & \text{if } C_1, \\
|X_i| e^{-jεₚ}, & \text{if } C_1' \end{cases} 
\end{align*}
\]

s.t. \(C_1: |x^{(l+1)}_i| - |x_i| e^{jεₚ} < |x^{(l+1)}_i| - |x_i| e^{-jεₚ}\)

where the left-hand side of the inequality in \(C\) calculates the vertical distance between \(K\) and the ray \(OA\). Note that \(\dot{X}\) takes the negation of a condition \(X\). For example, \(\dot{C}\) is obtained by changing \(>\) in \(C\) to \(<\).

**Case 2**: \(P \in [1 - εₐ, 1]\). In this case, \(|x^{(l+1)}_i|\) can be represented by \(J\) in Fig. 3. As in case 1, the vertical distance between \(J\) and the ray \(OA\) determines how to project \(|x^{(l+1)}_i|\) to make it feasible. The critical vertical distance shares the same expression as that in case 1 and hence is also denoted by \(d₀\). If \(|\dot{J}| ≤ d₀\), it is feasible; otherwise, \(J\) will be replaced by the intersection between \(JN\) and \(GQ\) (or \(HR\)). Based on the geometric relation shown in Fig. 3, the intersection can be given by \(P/[\cos(εₚ)]|X_i| e^{jεₚ}\) or \(P/[\cos(εₚ)]|x_i| e^{-jεₚ}\), where \(|x_i| e^{jεₚ}\) is the complex representation of \(OF\) and \(|x_i| e^{-jεₚ}\) is that of \(OE\). To summarize, the following projector can make \(|x^{(l+1)}_i|\) feasible in the case of \(P \in [1 - εₐ, 1]\):

\[
\begin{align*}
[x^{(l+1)}_i] & = \begin{cases} 
\frac{P}{\cos(εₚ)} |X_i| e^{jεₚ}, & \text{if } C_2, \\
\frac{P}{\cos(εₚ)} |X_i| e^{-jεₚ}, & \text{if } C_2' \end{cases} 
\end{align*}
\]

s.t. \(C_2: |X_i| e^{jεₚ}|x^{(l+1)}_i| > |X_i| e^{-jεₚ}|x^{(l+1)}_i|\)

where \(C\) is given in (34). Note that, geometrically, the left-hand side of \(C_2\) gives the projection length of \(OJ\) on \(OF\) and the RHS is that of \(OJ\) on \(OE\). We remind readers that \(J, \dot{J}\) as shown to be above \(GF\) in Fig. 3, can locate below \(HR\) as well, which leads to the two cases \(C_2\) and \(C_2'\) given above.

**Case 3**: \(0 ≤ P < 1\). In this case, \(|x^{(l+1)}_i|\) can be represented by \(I\) in Fig. 3. Considering the similarity of this case to case 1, we give the following projector without restating the details:

\[
\begin{align*}
[x^{(l+1)}_i] & = \begin{cases} 
\frac{(1 - εₐ)|x_i| e^{jεₚ}}{\cos(εₚ)}, & \text{if } C_3, \\
\frac{(1 - εₐ)|x_i| e^{-jεₚ}}{\cos(εₚ)}, & \text{if } C_3' \end{cases} 
\end{align*}
\]

s.t. \(C_3: |x^{(l+1)}_i| - (1 - εₐ)|x_i| e^{jεₚ} < |x^{(l+1)}_i| - (1 - εₐ)|x_i| e^{-jεₚ}\)

where \(C\) is given in (34). Geometrically, \([(1 - εₐ)|x_i| e^{jεₚ}]/[\cos(εₚ)]\) and \([(1 - εₐ)|x_i| e^{-jεₚ}]/[\cos(εₚ)]\) are \(OG\) and \(OH\), respectively.

Besides the three cases discussed above, there is a special case of \(P < 0\), as represented by \(P\) in Fig. 3. In this case, we can project \(P\) to \(G, H\) or the vertical projection point from \(P\) to the line segment \(GH\), as denoted by \(S\), whichever one is closest to \(P\). Note that the vertical projection point needs to be defined; otherwise, it is infeasible. This further leads to three cases. If \(GP \cdot GH < 0\), then we know that \(G\) is closest to \(P\); if \(HP \cdot HG < 0\), then \(H\) is closest to \(P\); otherwise, we need to calculate the vertical projection point. Based on Fig. 3, we see that \(OS = OG + GS\). Note that \(GS\) can be calculated as \((OP - OG) \cdot (GH/GH))\). The complex representations of \(OP, OG,\) and \(OH\) are \(|x^{(l+1)}_i|\), \([(1 - εₐ)|x_i| e^{jεₚ}]/[\cos(εₚ)]\), and \([(1 - εₐ)|x_i| e^{-jεₚ}]/[\cos(εₚ)]\), respectively. Thus, \((GH/GH) = [(OH - OG)/(OH - OG)]\) = \(-j|x_i|/|x_i|\). The above analyses can be summarized in the following projector:

\[
[x^{(l+1)}_i] = \begin{cases} 
(1 - εₐ)|x_i| e^{jεₚ}/\cos(εₚ), & \text{if } C_4, \\
(1 - εₐ)|x_i| e^{-jεₚ}/\cos(εₚ), & \text{if } C_5 \end{cases}
\]

s.t. \(C_4: \frac{|x_i|}{|x_i|} - |x^{(l+1)}_i| > 0\)

\[
C_5: \frac{|x_i|}{|x_i|} - |x^{(l+1)}_i| < 0.
\]

**B. QAM**

Different from PSK, QAM conveys information bits using both amplitudes and phases of \(x_i\). Therefore, without separating phases and amplitudes as done above, we now treat each entry of \(x_i\) as a circle center and translate the similarity constraint (9d) into the following feasible regions:

\[
|x^{(l+1)}_i| - |x_i| ≤ εₗ, \quad i = 0, \ldots, MN - 1.
\]

Geometrically, the above inequality defines a circular area centered at \(|x_i|\) with the radius of \(εₗ\). Therefore, the following projection can be performed, making the minimum change to each entry of \(x^{(l+1)}\) to ensure its feasibility:

\[
[x^{(l+1)}_i] = \begin{cases} 
|x_i| + εₗ |z_i|/|z_i|, & \text{if } |z_i| > εₗ, \\
|x^{(l+1)}_i|, & \text{otherwise} \end{cases}
\]

s.t. \(|z_i| = |x^{(l+1)}_i| - |x_i|\)

where \((|z_i|/|z_i|)\) is the unit direction vector and \(|x_i| + εₗ(|z_i|/|z_i|)\) is on the border of the region defined in (38).

**C. Unused Subcarriers**

In practice, there are generally some SCs not used for carrying data symbols. They can be reserved for pilot signals or just
Algorithm 2 Overall Algorithm for Waveform Optimization

\textbf{Input}: $\mathbf{x}_r, \mathbf{x}^{(0)}, p, w_i, Q, \epsilon_r, \rho, \mathcal{I}_{\text{used}}, \mathcal{I}_{\text{un}}, L_{\text{max}}$

1) Initialize $l = 0$ and $\mathbf{x}^{(0)} = \mathbf{x}_r$;
2) Run Algorithm 1 based on $\mathbf{x}^{(l)}$ to obtain $\mathbf{y}^{(l)}$. Based on the intermediate results after Step 4), set $\eta_l = \max_{k,i,l} |w_{r,ik}^{(l)}|$;
3) If $l = 1$, check whether $\eta_1 > \eta_{l-1}$ or not. If so, stop;
4) Calculate $\mathbf{x}^{(l+1)}$ as per (30);
5) If PSK is used for communications:
   a) Set $\epsilon_p = \frac{2\sqrt{p}}{\sqrt{\rho}}$;
   b) For each $i \in \mathcal{I}_{\text{used}}$:
      i) Calculate $\mathcal{P}$ given in (33);
      ii) If $\mathcal{P} > 1$, perform the projector in (34);
      iii) If $\mathcal{P} \in [1 - \epsilon_r, 1]$, perform (35);
      iv) If $\mathcal{P} \in (0, 1)$, perform (36);
      v) If $\mathcal{P} < 0$, perform (37);
   c) For each $i \in \mathcal{I}_{\text{un}}$, perform (40);
6) If QAM modulation is used for communications:
   a) Set $\epsilon_p = \frac{2\pi\rho}{Q}$;
   b) For each projector (39) on $[\mathbf{x}^{(l+1)}]$ at $\forall i \in \mathcal{I}_{\text{used}}$;
   c) For each projector (41) on $[\mathbf{x}^{(l+1)}]$ at $\forall i \in \mathcal{I}_{\text{un}}$;
7) If $l = L_{\text{max}} - 1$, stop; otherwise, go to Step 2) with $l = l+1$.

V. OVERALL ALGORITHM AND ANALYSIS

Joining the majorization in Section III and the minimization in Section IV, we can formulate the overall optimization algorithm for solving (9). The overall waveform optimization is summarized in Algorithm 2. In the overall list, $\mathbf{x} \in \mathbb{C}^{NM \times 1}$ stacks the communication data symbols carried by all SCs antenna-by-antenna; $\mathbf{x}^{(0)} \in \mathbb{C}^{NM \times 1}$ is the initial sequence of the algorithm; $p$ is the norm order in (9); $w_i$ is a boolean weight as given in (9); $Q$ is the modulation order of either PSK or QAM; $\epsilon_r$ as given in (31) is the maximum tolerable changes on the amplitude of each entry of $\mathbf{x}_r$ when PSK modulation is used; $\mathcal{I}_{\text{used}}$ is the index set of SCs used for communications; $\mathcal{I}_{\text{un}}$ unused due to deep fading [29]. These unused SCs are actually beneficial to sensing, as they are not subject to the similarity constraint and hence provide more degrees of freedom for waveform optimization. However, we need to constrain the amplitudes of signals carried by unused SCs to prevent severe power imbalance under the power constraint (9c). To this end, we require that the signal strength of unused SCs is not greater than the maximum amplitude of the communication constellation in use. If PSK modulation is used, the above requirement can be enforced as follows:

$$[\mathbf{x}^{(l+1)}]_i = \frac{[\mathbf{x}^{(l+1)}]}{[\mathbf{x}^{(l+1)}]_i]} \quad \text{if} \quad [\mathbf{x}^{(l+1)}]_i > 1 \quad \forall i \in \mathcal{I}_{\text{un}}$$

Algorithm 3 Waveform Optimization Accelerated by SQUAREM

\textbf{Input}: $\mathbf{x}_r, \mathbf{x}^{(0)}, p, w_i, Q, \epsilon_r, \rho, \mathcal{I}_{\text{used}}, \mathcal{I}_{\text{un}}, L_{\text{max}}$

1) Initialize $l = 0$ and $\mathbf{x}^{(0)} = \mathbf{x}_r$;
2) Run Steps 2)-6) of Algorithm 2 with $\mathbf{x}^{(l)}$, giving $x_1$ and $\eta_l$;
3) Run Steps 2)-6) of Algorithm 2 with $\mathbf{x}^{(l)} = x_1$, giving $x_2$;
4) $\mathbf{r} = x_1 - x^{(l)}$; $\mathbf{v} = x_2 - x_1 - \mathbf{r}$; $\alpha = -||\mathbf{r}||/||\mathbf{v}||$;
5) $\mathbf{x} = x^{(l)} - 2\alpha\mathbf{r} + \alpha^2\mathbf{v}$;
6) Make $\mathbf{x}$ feasible via Steps 5)-6) of Algorithm 2, leading to $\mathbf{x}$;
7) Run Steps 1)-3) of Algorithm 1 with $\mathbf{x}^{(0)} = \mathbf{x}$ and set $\eta_l = \max_{k,i,l} |w_{r,ik}^{(l)}|$ where $r^{(l)}$ is obtained in Step 4);
8) If $\eta_l > \eta_j$, take $\alpha = (\alpha - 1)/2$ and perform Step 5)-7) above;
9) Repeat Step 8) until the condition does not hold.
10) Set $\mathbf{x}^{(l+1)} = \mathbf{x}$ and $l = l+1$. Run Step 2)-3) of Algorithm 2.
11) Go to Step 2), if $l \leq L_{\text{max}}$.

where $Q$ denotes the modulation order, and the scaling coefficient two in $\epsilon_r$ is the minimum distance of QAM constellation points (±1, ±3, ±5, ±(\sqrt{Q} - 1)) in the real or imaginary axis.

Algorithm 2 is detailed next. Step 1 sets the initial waveform as the original communication waveform. Step 2 majorizes the objective function and obtains the peak sidelobe as denoted by $\eta_l$. Since the waveform design is to reduce the peak sidelobe level (PSL), step 3 stops the algorithm if the opposite happens. Step 4 returns the waveform without the similarity constraint and then the waveform is projected onto the feasible region in step 5/6. Note that Algorithm 2 can be accelerated using the squared iterative method (SQUAREM) [30]. The method, in essence, performs the gradient descent using a two-point objective function and obtains the peak sidelobe as denoted by $\eta_l$.

Algorithm 1 has a CC of $O(M^2N\log N)$. Specifically, step 2 has a CC of $O(M^2\log N)$. Step 3 has a CC of $O(M^2N\log N)$, where $O(N\log N)$ is the CC of an $N$-dimensional (inverse) fast Fourier transform (FFT). Step 4 has a complexity of $O(M^2NC_{\text{CP}})$, where $MN$ is the number of $(m, k)$ value pairs and $NC_{\text{CP}}$ is the number of $i$ values. Step 5 has a CC of $O(M^2NC_{\text{CP}})$, as each of three equations only involves computing $M^2NC_{\text{CP}}$ scalars. Step 6 also has a CC of $O(M^2NC_{\text{CP}})$, as $w_i$ is only nonzero for $i = 0, 1, \ldots, NC_{\text{CP}} - 1$. Step 7 has a CC of $O(N\log N)$, which applies the fast FFT. Steps 8 and 9 only rearrange existing results. Step 10 has a CC of $O(M^2)$, as each matrix $Q^{(l)}$ therein is $M \times M$. Finally, step 11 has a CC of $O(N)$. From the above analysis, steps 3 and 10 dominate
the overall CC. However, in IoT applications, the number of transmitting antennas, i.e., \( M \) is generally small, while that of \( N \) is \( O\left(10^2\right) \). Thus, we conclude that the CC of Algorithm 1 is dominated by that of step 3, i.e., \( O\left(M^2N\log N\right) \). Such a complexity incurred by the proposed waveform design can potentially be handled by modern and future IoT devices, particularly gateway devices.

Algorithm 2 has a CC of \( O\left(L_{\text{max}}M^2N\log N\right) \), which is an upper limit due to the maximum number of iterations. Enforced by step 3, the actual number of iterations can be much smaller. The core of Algorithm 2 includes steps 2–6 that have a CC of \( O\left(M^2N\log N\right) \). In particular, step 2 runs Algorithm 1 and hence has a CC of \( O\left[M^2N\log N\right] \). Step 4 has a CC of \( O\left[MN\right] \). Step 5 has complexity of \( O\left[MN\right] \); despite that many conditions, all projectors only involve scalar operations. Similarly, step 6 also has the complexity of \( O\left[MN\right] \). The CC of step 2 obviously outweighs the CCs of other steps.

When the same number of iterations are performed, Algorithm 3 at least doubles the CC of Algorithm 2. The least amount is achieved when the backtracking in steps 8 and 9 does not happen. The CC of the two steps cannot be determined, as how many times we need to perform them is unpredictable in practice. However, observed from our extensive simulations, Algorithm 3 requires much smaller number of iterations than Algorithm 2, using the same stopping criteria, as given in steps 10 and 11 of Algorithm 3 and steps 3 and 7 of Algorithm 2. As will be shown shortly, Algorithm 3 mostly stops after only three iterations. Furthermore, we note that due to the projection in step 6, there is no theoretical guarantee on the convergence of Algorithm 3. So we stop the algorithm once the objective function starts increasing (which is enforced by step 10, and the results in the second last iteration are returned. As shown next, nontrivial improvements in sensing performance can be well achieved even with only three iterations.\(^4\)

VI. SIMULATION RESULTS

Simulation results are provided in this section to validate the proposed designs. The following simulation settings are mainly used: the number of SCs \( N = 128 \), the number of transmitting antennas \( M = 4 \), the CP length \( N_{\text{CP}} = N/4 \), the initial waveform \( \mathbf{x}^{(0)} \) is set as the original communication waveform \( \mathbf{x}_r \), the norm order \( p = 50 \), the number of unused SCs per antenna is \( \lfloor 0.05N \rfloor \), the number of maximum iterations \( L_{\text{max}} = 10 \), and the maximum amplitude error of a unit PSK constellation point is \( \varepsilon_y = 0.2 \); see Fig. 3.

As mentioned in Section II-D, a large norm order is used to approximate the infinite norm. From extensive observations, the impact of \( p (\geq 50) \) on the proposed waveform optimization is negligible and will not be further illustrated for brevity. Moreover, we note that \( N = 128 \) and \( M = 4 \) are relatively small yet practical for IoT systems, as they generally have low-profile transceivers and use small-packet communications [29]. Extension to other values of \( N \) and \( M \) is straightforward and does not provide much insight. Thus, we shall not vary these two values and will focus on demonstrating the impact of more critical parameters: \( Q, \rho, \varepsilon_p, \varepsilon_y \), and constellation types.

The first set of simulations demonstrates how the proposed algorithm performs and the changes it brings to communications and sensing. QPSK is considered first, i.e., the modulation order \( Q = 4 \). The ratio between the maximum phase change and the angular interval of adjacent constellation points is \( \rho = 0.15 \); see (42). Other values of \( Q \) and \( \rho \) will be illustrated shortly. Moreover, we perform \( 10^3 \) independent trials, each resetting the communication waveform \( \mathbf{x}_r \) by randomly drawing data symbols from the QPSK constellation set. Recall that \( \mathbf{x}_r \) is a column vector stacking data symbols on all SCs antenna-by-antenna. In each trial, we run Algorithm 3 based on the parameters specified above.

Fig. 4(a) plots the value of the objective function, namely, the PSL in the auto- and cross-correlations of waveforms from communication-transmitting antennas, against the number of iterations. We see that the PSL of the original communication waveforms can be as high as about \(-10 \text{ dB} \) and no less than \(-15 \text{ dB} \). In contrast, the proposed waveform optimization reduces the PSL of all \( 10^3 \) trials to below \(-12.5 \text{ dB} \) and even reduces the lowest PSL to around \(-18.5 \text{ dB} \). Fig. 4(a) also shows that Algorithm 3 converges after three iterations for most independent trials. This is further confirmed by Fig. 4(b), where the y-axis on the right is the CDF of numbers of iterations, as denoted by \( n \) in the figure. We see that 78% independent trials have the algorithm stop after three iterations. The y-axis on the left is the complementary CDF (CCDF) of the PSL improvement the proposed design obtains. As highlighted in the figure, more than 60% independent trials have their PSL improved by at least 3 dB.

Fig. 4(c) illustrates how communication waveform is changed by the proposed waveform design. Centered around the four QPSK constellation points are four trapezoid-like point clouds. They are formed by the projectors derived in Section IV, which validates the effectiveness of those projectors in constraining the signal changes as desired. This result also shows the flexibility of the proposed projectors in individually constraining amplitudes and angles of constellation points. The “untethered” points seen in Fig. 4(c) are from the

\(^4\)Since the results from the last iteration are abandoned, we actually only need to run Algorithm 3 for two iterations.
unused SCs. We see that their amplitudes are upper bounded well.

Earlier in Section II-C, Fig. 2 in particular, we have demonstrated the improvement on the RAM quality brought by the proposed waveform design. Here, we further translate the improvement to extensively used performance metrics: BER for communications and the DP for sensing. As illustrated in Section II-B, the communication receiver performs the zero-forcing equalization, leading to the estimated data symbol matrix \( \hat{X} \). In the following simulation results, the uncoded BER under the hard-decision demodulation is evaluated based on \( \hat{X} \). To evaluate DP, we perform the 1-D cell-average constant false-alarm rate (CA-CFAR) detection along the range dimension of the obtained RAM. CA-CFAR is one of the most practically used radar detection algorithms [41]. Its implementations are briefly described as follows.

Let \( z_{mi} \) denotes the \( ith \) entry of the CCC result \( z_m \) obtained in (4). CA-CFAR checks whether a target locates at the \( ith \) range grid by comparing \( |z_{mi}|^2 \) with a threshold calculated based on the surrounding range grids. The \( ith \) range grid is called the grid under test (GUT). The \( N_{gap} \) grids adjacent to the GUT are gap grids (GGs), and the \( N_{ref} \) grids adjacent to the GGs are the reference grids (RGs), as exemplified by

\[
\begin{align*}
\underbrace{z_{mi}(i-8), \ldots, z_{mi}(i-3), z_{mi}, \ldots, z_{mi}(i+3)}_{N_{ref}=7 \text{ RGs}} \\
\underbrace{z_{mi}(i+4)}_{N_{gap}=1 \text{ GG}}
\end{align*}
\]

The power of the signals on the RGs is averaged. The result is multiplied by the following \( \beta \) to produce the above-mentioned threshold [41]:

\[
\beta = 2N_{ref} \left( P_{fa}^{-1/2N_{ref}} - 1 \right) \tag{44}
\]

where \( P_{fa} \) denotes an expected false alarm rate.

As shown in (43), we take \( N_{ref} = 7 \) and \( N_{gap} = 1 \) for the following simulations. To calculate the DP, we perform \( 2 \times 10^4 \) independent trials, each with randomly generated communication waveform and target ranges. The proposed Algorithm 3 is performed in each trial to generate the optimized waveform. All other parameters are kept unchanged as in the previous result unless otherwise specified. Since we only perform 1-D CFAR, the angle information will be irrelevant. All targets then have the zero AoD to simplify echo generation.

Fig. 5(a) observes DP against SNR, comparing the original communication waveform, the optimized one and the orthogonal one [25]. We take \( P_{fa} = 10^{-5} \) for this simulation and will show other values later. Consistent with what is observed in Fig. 2, the optimized waveform achieves greater detecting probabilities than the original waveform over the whole SNR region observed. More specifically, to achieve a DP of 0.87, the proposed waveform design reduces the SNR by 4 dB. For the same DP, the orthogonal waveform further reduces the SNR by 4 dB on top of our scheme.

In JCAS, how much gain in sensing is generally linked to how much loss in communications. For the orthogonal waveform, its sum rate is reduced by 73.8%, as illustrated in Section II-C. The proposed scheme has the same sum rate as the original communications. However, our scheme can degrade the BER performance, as our sensing gain is essentially achieved by altering the data symbols on SCs. Fig. 5(a) shows that the BER loss increases with \( \rho \), an intermediate parameter determining the maximum angle change on a constellation point; see (42). As also shown in the figure, the loss is up to 2 dB at \( \rho = 0.25 \). Note that \( \rho \) monitors the trade-off between sensing and communication performances in our design.

Fig. 5(b) plots DP and BER against \( \rho \) to further investigates the tradeoff. As expected, the DP increases with \( \rho \), while the BER performances degrade increasingly with \( \rho \). However, it is interesting to notice that the increasing rate also changes with \( \rho \). In particular, the increasing rate of the DP changes slowly when \( \rho \) is over 0.15. On the other hand, the BER degrading rate for \( \rho > 0.15 \) is much larger than that for \( \rho < 0.15 \). These two results suggest that \( \rho = 0.15 \) is a good option, as it trades only a small BER loss for almost the maximum improvement on the DP.

Fig. 5(c) observes the DP and BER performance against \( \epsilon_a \), a parameter determining the maximum amplitude change on a constellation point; see Fig. 3. We see from Fig. 5(c) that increasing \( \epsilon_a \) does not help obviously in improving sensing performance. Also, we see that the BER loss increases substantially after \( \epsilon_a \) is over 0.2. This explains why we use \( \epsilon_a = 0.2 \) in Fig. 5(a). In fact, the above observation applies to other PSK modulation orders, as observed from extensive simulations. Thus, we continue using \( \epsilon_a = 0.2 \) next.

To show the wide applicability of the proposed design, we further evaluate the BER and detecting performance of 8 PSK and 16 PSK with a different \( P_{fa} = 10^{-5} \). The results are shown in Fig. 6. As stated earlier, we use \( \epsilon_a = 0.2 \) and \( \rho = 0.15 \). From Fig. 6(a), we see that the proposed waveform optimization nontrivially improves the DP over the original communication waveforms. To achieve a DP of about 0.75, our design reduces the SNR requirement by 3.5 and 3 dB for 8 PSK and 16 PSK, respectively. The BER loss is about 1 dB for both cases. Fig. 6(b) plots the BER and detecting performance against \( \rho \) for 8 PSK and 16 PSK. We see similar trends as those observed in Fig. 5(b) (for QPSK).
So far, we have shown the effectiveness of the proposed design for PSK modulations. Next, we further validate our design using 16 QAM. Fig. 7(a) illustrates the BER and detecting performance of QAM. We note that \( \rho \) is different from that in Fig. 5. Here, it defines the radius of a circular area to confine the changes made to a QAM constellation point; see (42). We see from Fig. 7(a) that the proposed design improves the detecting performance as \( \rho \) increases. To achieve a DP of about 0.87, the proposed scheme reduces the SNR requirement by up to 4.5 dB at \( \rho = 0.45 \). On the other hand, the BER loss increases with \( \rho \). The maximum loss is about 1.2 dB.

An interesting result from Fig. 7(a) is that the sensing performance gap between the orthogonal waveform and the proposed design is smaller than the gap for QPSK in Fig. 5. This is caused by the nonconstant amplitudes of QAM constellation points. The data symbols randomly selected from the 16 QAM constellation act as a random weighting when performing the matched filtering at the sensing receiver. This weighting can lead to unpredictable sidelobe changes around target peaks. In contrast, PSK constellations do not have this problem due to their constant modulus. Fig. 7(b) illustrates the range cut of the RAM, where five targets evenly distribute over range bins and have zero AoD. Complied with the above illustration, the orthogonal waveform random varying sidelobes. Moreover, we see from Fig. 7(b) that the proposed scheme reduces sidelobes levels overall, hence enabling better detecting performance.

**VII. CONCLUSION**

To reduce interantenna and intertarget interference for high-quality sensing, we propose to introduce tolerable changes to data symbols modulated on MIMO-OFDM SCs to enhance the time- and spatial-domain signal orthogonality. To realize the desired waveform, we establish an optimization problem and develop an efficient solution based on the MM framework. The high efficiency is achieved by the signal structures and features discovered by us. Extensive simulations are provided to validate the effectiveness of the proposed design and its superiority over the original communication waveform, specifically: 1) in general the proposed algorithm only needs a few iterations to achieve nontrivial improvement in the above-mentioned signal orthogonality; the complexity of each iteration is only \( \mathcal{O}(M^2N\log(N)) \) with \( M \) \( (N) \) being the number of antennas (SCs); 2) for 4/8/16 PSK modulations, using the optimized waveform can reduce the SNR requirement by more than 3 dB to achieve the same DP as using the original communication waveform, while the range cut of the RAM, where five targets evenly distributed over range bins and have zero AoD. Complied with the

![Image](https://example.com/image.png)

Fig. 6. BER and sensing DP against SNR in (a) and versus \( \rho \) in (b). The legend applies to the same line styles despite colors.

![Image](https://example.com/image.png)

Fig. 7. (a) BER and sensing DP versus SNR using the original communication waveforms based on 16 QAM, the optimized one, and the orthogonal design [25]. (b) Range cut of the RAM obtained using the three types of waveforms. The legend applies to the same line styles despite colors.

**APPENDIX**

**A. Derivation of \( \text{FU}F^H = \text{diag}(\{F\}_N) \)**

The relation is based on a nice property of a circulant matrix. In (8), \( U_l \) \( (i = 0, 1, \ldots, N-1) \) is obviously a circulant matrix which can be diagonalized via Fourier transforms. In particular, we have

\[
F^H U_l F = \text{diag}(\{\lambda F [U_l]_i\}^T).
\]

Appeared in (8) is \( \text{FU}_l F^H \) which is equal to \( (F^H U_{N-l} F)^T \). Two relations are used for the above equality: \( F^T = F \) and \( U_{N-l}^T = U_l \). Based on (45), we have

\[
\text{FU}_l F^H = (F^H U_{N-l} F)^T = \text{diag}(\{\lambda F [U_{N-l}]_i\}^T)^T = \text{diag}(\{F\}_N)_{1, i}
\]

where the last equality is because \( [U_{N-l}]_{1, i} \) takes one at \( N - i \) and zeros elsewhere.

**B. Proof of Theorem 1**

Based on the expression of \( A_{mki} \) given in (8), we can apply (13) and obtain

\[
P_{mk} = (S_m^T \otimes S_k^H) \sum_{i=0}^{N-1} a_{mki}^l w_i \text{vec}\{\text{diag}(N[F]_{N-i})\} \times \text{vec}\{\text{diag}(N[F]_{N-i})\}^H (S_m^T \otimes S_k).
\]

Based on the expression of \( S_m \) given in (6), we can validate that for \( \forall (m', k') \neq (m, k) \) we have

\[
(S_m^T \otimes S_k)(S_m^T \otimes S_k^H) = (S_m S_m^T) \otimes (S_k S_k^T) = 0.
\]
Combining (47) and (48), we further have

$$P_{m'k'}^H P_{mk} = 0 \quad \forall (m', k') \neq (m, k). \quad (49)$$

Since $P_{mk}$ ($\forall m, k$) is Hermitian, the above result implies that the eigenvectors of $P_{mk}$ and $P_{m'k'}$ ($\forall (m', k') \neq (m, k)$, are orthogonal. Then we can conclude that the eigenvectors of $P_{mk}$ ($\forall m, k$) are also those of $\left( \sum_{m,k=0}^{M-1} P_{mk} \right)$. Finally, we obtain

$$\lambda_{\text{max}} \left\{ \sum_{m,k=0}^{M-1} P_{mk} \right\} = \max_{m,k=0,1,\ldots,M-1} \lambda_{\text{max}} \{ P_{mk} \}. \quad (50)$$

Applying the property that $\text{eig}\{AB\} = \text{eig}\{BA\}$, $P_{mk}$ given in (47) satisfies

$$\text{eig}(P_{mk}) = \begin{cases} \text{eig} \left( (S_m^* \otimes S_k) (S_m^T \otimes S_k^H) P_{mk} \right) \\ \text{eig} \left( \tilde{P}_{mk} \right) \end{cases}$$

s.t. $\tilde{P}_{mk} = \sum_{i=0}^{N-1} d_{mk}^{(i)} w_i \text{vec}(\text{diag}(N[F]_{N-i}))$.

$$\text{vec}(\text{diag}(N[F]_{N-i}^H))^H \quad (51)$$

where $\text{eig}$ is because $(S_m^* \otimes S_k) (S_m^T \otimes S_k^H) = I_{MN}$. This turns our next task into calculating the eigenvalues of $\tilde{P}_{mk}$.

To do so, let us first investigate $\text{vec}(\text{diag}(N[F]_{N-i}^H))$. We notice that it satisfies

$$[\text{vec}(\text{diag}(N[F]_{N-i}^H)); 0_N] = N[F]_{N-i} \otimes \{1; 0_N\}. \quad (52)$$

Applying this result, we can have

$$\left[ \text{vec}(\text{diag}(N[F]_{N-i}^H))^H \right] \left[ \text{vec}(\text{diag}(N[F]_{N-i}^H)); 0_N \right] = \left[ \text{vec}(\text{diag}(N[F]_{N-i}^H))^H \text{vec}(\text{diag}(N[F]_{N-i}^H) 0_N^H) \right]$$

$$= N[F]_{N-i} \otimes \{1; 0_N\}; N[F]_{N-i} \otimes \{1; 0_N\}^H$$

$$= N^2 \left( [F]_{N-i}^H [F]_{N-i}^H \right) \otimes \{1; 0_N\} [1; 0_N]^H. \quad (53)$$

This further leads to

$$\left[ \tilde{P}_m^H \left[ 0_N^2 \otimes 0_N \right] \right] = N^2 \left( \sum_{i=0}^{N-1} d_{mk}^{(i)} w_i \text{vec}(\text{diag}(N[F]_{N-i}^H)) \right)$$

$$\otimes \{1; 0_N\} [1; 0_N]^H. \quad (54)$$

According to [33, Lemma 3], we have $\lambda_{\text{max}} \{ A \otimes B \} = \lambda_{\text{max}} \{ A \} \lambda_{\text{max}} \{ B \}$ given any two square matrices $A$ and $B$. Then, based on (54), we have

$$\lambda_{\text{max}} \{ \tilde{P}_{mk} \} = N^2 \lambda_{\text{max}} \{ Q_{mk} \}. \quad (55)$$

Based on the definition of $Q_{m,k}$ given in (54), it is obvious that $Q_{m,k}[F]_{N-i} = N a_{mk}^{(i)} w_i [F]_{N-i}$. Thus, the eigenvalues of $Q_{m,k}$ is $Na_{mk}^{(i)} w_i$ ($i = 0, 1, \ldots, N-1$). This result, substituted into (55), leads to the expression of $\tilde{\lambda}$ given in Theorem 1.

### C. Proof of Corollary 1

Substituting $M = \tilde{\lambda} I$ into (16), $\tilde{x}^H \tilde{M} \tilde{x}$ become $x$-independent under the power constraint (9c). Thus, let us focus on the first term in the second line of (16). In particular, based on the expressions of $P_{mk}$ and $\tilde{x}$ given in (15), we have

$$\tilde{x}^H P_{mk} \tilde{x} = \sum_{i=0}^{N-1} d_{mk}^{(i)} w_i (x^T \otimes x^H) \text{vec}(A_{mk})$$

$$= \sum_{i=0}^{N-1} d_{mk}^{(i)} w_i (x^T \otimes x^H) \text{vec}(A_{mk})$$

$$= \tilde{x}^H A_{mk} \tilde{x}. \quad (56)$$

To get the second result, the following calculations are performed:

1. Applying (13) in a reverse way, we obtain

$$[x^T \otimes x^H] \text{vec}(A_{mk}) = x^H A_{mk} x \quad (57)$$

where the vectorization of the RHS is suppressed as it is already a scalar.

2. Similarly, we have

$$\text{vec}(A_{mk})^H \left( (x^T \otimes x)^H \right) = \left( (x^T \otimes x)^H \right)^{\text{H}} = \left( (x^T \otimes x)^H \right)^{\text{H}} \quad (58)$$

where the second result is based on the definition of the periodic cross-correlation given in (8).

Similar to (56), we can calculate

$$\tilde{x}^H \tilde{M} \tilde{x} = \tilde{\lambda} \tilde{x}^H \tilde{x} = \tilde{\lambda} (x^T \otimes x^H) \text{vec}(x^T (x^H)^T)$$

$$= \tilde{\lambda} x^H \left( (x^T \otimes x^H)^T \right) \text{vec}(x^T (x^H)^T)$$

$$= \tilde{\lambda} x^H \left( (x^T \otimes x^H)^T \right) \text{vec}(x^T (x^H)^T) \quad (59)$$

Substituting (56) and (59) into (16) and rearranging terms, we obtain the majorization given in (19). Note that the first term on the RHS of the inequality in (19) corresponds to the real-taking term in (16). This implies that $Q_{1}^{(l)}$ is Hermitian, i.e., $Q_{1}^{(l)} \text{H} = Q_{1}^{(l)}$.

### D. Proof of Lemma 3

As the sum of two Hermitian matrices, $Q_{1}^{(l)}$ is also Hermitian. Thus, we only need to prove the matrix is real. Based on the expression of $A_{mk}$ given in (8), $Q_{1}^{(l)}$ can be written into $Q_{1}^{(l)} = \sum_{m',k'=0}^{M-1} w_{m'k'} [A_{mk}]^T$, where

$$A_{mk}^{(l)} = \sum_{i=0}^{N-1} \sum_{m',k'=0}^{M-1} w_{m'k'} [A_{mk}]^T \sum_{m',k'=0}^{M-1} w_{m'k'} [A_{mk}]^T \sum_{m',k'=0}^{M-1} w_{m'k'} \text{vec}(\text{diag}(N[F]_{N-i}^H))^H$$

$$\otimes \{1; 0_N\} [1; 0_N]^H. \quad (54)$$

According to [33, Lemma 3], we have $\lambda_{\text{max}} \{ A \otimes B \} = \lambda_{\text{max}} \{ A \} \lambda_{\text{max}} \{ B \}$ given any two square matrices $A$ and $B$. Then, based on (54), we have

$$\lambda_{\text{max}} \{ \tilde{P}_{mk} \} = N^2 \lambda_{\text{max}} \{ Q_{mk} \}. \quad (55)$$

Based on the definition of $Q_{m,k}$ given in (54), it is obvious that $Q_{m,k}[F]_{N-i} = N a_{mk}^{(i)} w_i [F]_{N-i}$. Thus, the eigenvalues of $Q_{m,k}$ is $Na_{mk}^{(i)} w_i$ ($i = 0, 1, \ldots, N-1$). This result, substituted into (55), leads to the expression of $\tilde{\lambda}$ given in Theorem 1.

This can be proved by enumerating the sub-blocks of $Q_{1}^{(l)}$. In particular, with the aid of $\Lambda_{mk}^{(i)}$ and the definition of $S_m$ ($\forall m$),
given in (6), we can express $Q^{(l)}$ as

$$
Q^{(l)} = \begin{bmatrix}
\Lambda_{10}^{(l)} & \Lambda_{11}^{(l)} & \cdots & \Lambda_{1(M-1)}^{(l)}
\end{bmatrix}
$$

which then leads to (25). Applying the property given in (14), we have

$$
Q_l \otimes \text{diag}(\|M_l\|) = Q_f \otimes \text{diag}(\|M_l\|).
$$

According to Lemma 3, we have $\Lambda^{(l)}_{mkl} = \Lambda^{(l)}_{kmn}$. This makes $Q^{(l)}(Y_l)$ given in (25) also real and symmetric.

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Kai Wu (Member, IEEE) received the B.E. degree from Xidian University, Xi’an, China, in 2012, and Ph.D. degree from the University of Technology Sydney (UTS), Sydney, NSW, Australia, in 2020. He is a Lecturer with UTS. He has published an authored book on Joint Communications and Sensing Employing Optimized MIMO-OFDM Signals, in December 2022. He has also delivered tutorials on JCA5 in WCNC’20, ICC’20, and ISCIT’23. His research interests include antenna array signal processing and its applications in radar, communications, and their joint designs.

Dr. Wu’s Ph.D. was awarded “Chancellor’s List 2020.” He has been a (co-)chair for numerous international conferences, including ICC’20 and ISCIT’23. He is serving as the Editor-in-Chief Assistant for the IEEE ISAC-ETI Newsletter.

J. Andrew Zhang (Senior Member, IEEE) received the B.Sc. degree from Xi’an Jiaotong University, Xi’an, China, in 1996, the M.Sc. degree from Nanjing University of Posts and Telecommunications, Nanjing, China, in 1999, and the Ph.D. degree from The Australian National University, Canberra ACT, Australia, in 2004.

He is currently a Professor with the School of Electrical and Data Engineering and the Director of the Radio Sensing and Pattern Analysis Laboratory, University of Technology Sydney, Sydney, NSW, Australia. He was a Researcher with Data61, CSIRO, Eveleigh, NSW, Australia, from 2010 to 2016; the Networked Systems, NICTA, Alexandria, NSW, Australia, from 2004 to 2010; and ZTE Corp., Nanjing, China, from 1999 to 2001. He has published over 270 journal and conference papers. His research interests are in signal processing for wireless communications and sensing with the current focus on integrated sensing and communications.

Dr. Zhang is a recipient of CSIRO Chairman’s Medal and the Australian Engineering Innovation Award in 2012 for exceptional research achievements in multigigabit wireless communications. He has won five best paper awards, including in IEEE ICC 2013.

Zhitong Ni (Member, IEEE) received the B.E. degree in information engineering from the Beijing Institute of Technology (BIT), Beijing, China, in 2017, the first Ph.D. degree from the University of Technology Sydney (UTS), Sydney, NSW, Australia, in 2022, and the second Ph.D. degree from BIT in 2023.

He is currently a Research Associate with the Global Big Data Technologies Centre, UTS. His research interests include array signal processing, channel parameter estimations, as well as precoding techniques in various applications, including the sixth-generation cellular systems and integrated radio sensing and communication systems.

Xiaojing Huang (Senior Member, IEEE) received the B.Eng., M.Eng., and Ph.D. degrees in electronic engineering from Shanghai Jiao Tong University, Shanghai, China, in 1983, 1986, and 1989, respectively.

He was a Principal Research Engineer with the Motorola Australian Research Center, Botany, NSW, Australia, from 1998 to 2003, and an Associate Professor with the University of Wollongong, Wollongong, NSW, Australia, from 2004 to 2008. He had been a Principal Research Scientist with the Commonwealth Scientific and Industrial Research Organisational (CSIRO), Sydney, NSW, Australia, and the Project Leader of the CSIRO Microwave and mm-Wave Backhaul Projects since 2009. He is currently a Professor of Information and Communications Technology with the School of Electrical and Data Engineering and the Program Leader for Mobile Sensing and Communications with the Global Big Data Technologies Center, University of Technology Sydney, Sydney, NSW, Australia. With over 34 years of combined industrial, academic, and scientific research experience, he has authored over 400 book chapters, refereed journal and conference papers, major commercial research reports, and filed 31 patents. His research interests include high-speed wireless communications, digital and analog signal processing, and synthetic aperture radar imaging.

Prof. Huang was a recipient of the CSIRO Chairman’s Medal and Australian Engineering Innovation Award in 2012 for exceptional research achievements in multigigabit wireless communications.

Y. Jay Guo (Fellow, IEEE) received the bachelor’s and master’s degrees from Xidian University, Xi’an, China, in 1982 and 1984, respectively, and the Ph.D degree from Xi’an Jiaotong University, Xi’an, in 1987.

He was named one of the most influential engineers in Australia in 2014 and 2015, and Australia’s Research Field Leader in Electromagnetics by the Australian Research Report for four consecutive years since 2020. He has published six books and over 700 research papers, including over 340 IEEE transactions papers, and he holds 26 international patents. He is a Distinguished Professor and the Director of Global Big Data Technologies Centre, The University of Technology Sydney (UTS), Sydney, NSW, Australia.

He is the Founding Technical Director of the New South Wales Connectivity Innovation Network. Prior to joining UTS in 2014, he served as a Director of CSIRO, Canberra, ACT, Australia, for over nine years. Before joining CSIRO, he held various senior technology leadership positions with Fujitsu Siemens, Munich, Germany, and NEC, London, U.K. His current research interests include 6G antennas, mm-wave and THz communications and sensing systems, as well as big data technologies.

Prof. Guo has won a number of the most prestigious Australian National Awards, including the Engineering Excellence Awards in 2007 and 2012, respectively, and the CSIRO Chairman’s Medal in 2007 and 2012, respectively.

Together with his students and Postdoctoral Fellows, he has won numerous best paper awards. In 2023, he received the prestigious IEEE APS Sergei A. Schelkunoff Transactions Paper Prize Award. He was a member of the College of Experts of Australian Research Council (ARC, 2016–2018). He has chaired numerous international conferences and served as a guest editor for a number of IEEE publications. He was the Chair of International Steering Committee and International Symposium on Antennas and Propagation (2019–2021). He has been the International Advisory Committee Chair of IEEE VTC2017, the General Chair of ISAP2022, ISAP2015, IWAT2014, and WPMC2014, and the TPC Chair of 2010 IEEE WCNC, and 2012 and 2007 IEEE ISCIT.

He was named Guest Editor of special issues on “Low-Cost Wide-Angle Beam Scanning Antennas,” “Antennas for Satellite Communications,” and “Antennas and Propagation Aspects of 60–90 GHz Wireless Communications,” all in IEEE Transactions on Antennas and Propagation, Special Issue on “Communications Challenges and Dynamics for Unmanned Autonomous Vehicles,” IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, and Special Issue on “5G for Mission Critical Machine Communications,” IEEE Network Magazine.

He is a Fellow of the Australian Academy of Engineering and Technology.