A new calibration method of sub-halo orbital evolution for semi-analytic models

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

Understanding the non-linear dynamics of satellite halos (a.k.a. “sub-halos”) is important for predicting the abundance and distribution of dark matter substructures and satellite galaxies, and for distinguishing among microphysical dark matter models using observations. Typically, modeling these dynamics requires large N-body simulations with high resolution. Semi-analytic models can provide a more efficient way to describe the key physical processes such as dynamical friction, tidal mass loss, and tidal heating, with only a few free parameters. In this work, we present a fast Monte Carlo Markov Chain fitting approach to explore the parameter space of such a sub-halo non-linear evolution model. We use the dynamical models described in an earlier work and calibrate the models to two sets of high-resolution cold dark matter N-body simulations, ELVIS and Caterpillar. Compared to previous calibrations that used manual parameter tuning, our approach provides a more robust way to determine the best-fit parameters and their posterior probabilities. We find that jointly fitting for the sub-halo mass and maximum velocity functions can break the degeneracy between tidal stripping and tidal heating parameters, as well as providing better constraints on the strength of dynamical friction. We show that our semi-analytic simulation can accurately reproduce N-body simulations statistics, and that the calibration results for the two sets of N-body simulations agree at 95% confidence level. Dynamical models calibrated in this work will be important for future dark matter substructure studies.

Key words: cosmology: theory – cosmology: dark matter – galaxies: formation – galaxies: halos

1 INTRODUCTION

Exploring the physics behind galaxy and star formation is one of the major concerns of modern astrophysics. The simple cold dark matter (CDM) paradigm successfully explains large-scale cosmic properties, including the cosmic microwave background (Peebles 1982) and the large-scale structure (LSS) of galaxy distributions (Planck Collaboration et al. 2011, 2014a; Anderson et al. 2012). However, on galactic scales, several puzzles such as the core-vs.-cusp problem (Rubin et al. 1980; Bosma 1981; Persic & Salucci 1988; Persic et al. 1996; Salucci 2001; Donato et al. 2004, 2009; Newman et al. 2009, 2011; de Blok 2010; Donato et al. 2004, 2009; Newman et al. 2009, 2011; de Blok 2010; Kuzio de Naray & Spekkens 2013; Kuzio de Naray & Kaufmann 2011; Salucci et al. 2012; Wolf & Bullock 2012; Relatores et al. 2019a,b) and the missing satellite problem (Kauffmann et al. 1993; Klypin et al. 1999b; Bullock 2010; Boylan-Kolchin et al. 2011, 2012; Wang et al. 2012) still remain to be fully explained. Many possible solutions including baryonic feedback (Macciò et al. 2007; Brooks & Zolotov 2014; Kim et al. 2017) and modified dark matter (DM) models (Markevitch et al. 2004; Boehm & Schaeffer 2005; Ahn & Shapiro 2005; Randall et al. 2008; Lovell et al. 2012; Kaplinghat et al. 2016) have been proposed and tested via N-body and hydrodynamical simulations (Bullock & Boylan-Kolchin 2017; Robles et al. 2017; Bozek et al. 2019; Lovell et al. 2020), although whether any of the proposed models can fully explain the deviation of CDM expectations from observational results remains unclear. A variety of upcoming experimental measurements (Simon et al. 2005; Viel et al. 2009), especially future strong lensing surveys (Keeton & Moustakas 2009; Vegetti & Koopmans 2009; Vegetti et al. 2010, 2012, 2018; Hsu et al. 2016; Birrer et al. 2017; Spinogola et al. 2018; Gilman et al. 2019, 2020; Morningstar et al. 2019; Hsueh et al. 2020; Nierenberg et al. 2020)
is its modularity—different models that describe identical physical processes. One free and open source SAM—Galacticus—is developed by Benson (2012). The key feature of Galacticus is its modularity—different models that describe identical physical processes can be added and compared easily.

The original Galacticus was based on the CDM paradigm. Benson et al. (2013) generalized the EPS formalism, which is used in generating realizations of halo merging histories (merger trees), to the warm dark matter (WDM) model. Pullen et al. (2014; hereafter AP2014) then added models that describe the orbital evolution and mass loss of sub-halos within host halos by accounting for dynamical friction, tidal stripping, and tidal heating, and studied how these non-linear effects influence the sub-halo distribution under the CDM and WDM paradigms. The dynamical friction acts as a net drag force on the sub-halo while it orbits within its host, causing the sub-halo to gradually sink into the center of the host. Tidal forces from the host strip the outer parts of sub-halo, leading to a decrease in the remaining sub-halo bound mass. Finally, rapid changes in the tidal field seen by the sub-halo as it moves along its orbit act to “heat up” the particles in the sub-halo and cause expansion. This is the so-called tidal heating effect. The density of the sub-halo consequently drops, making it easier to be further tidally stripped (Taylor & Babul 2001).

AP2014 adopted the dynamical friction Coulomb logarithm proposed by Taylor & Babul (2001) and the tidal heating adiabatic index proposed by Gnedin & Ostriker (1999). The tidal effect models were then calibrated to the Aquarius CDM N-body simulation (Springel et al. 2008) through manual parameter tuning. The dynamical friction model and the calibrated tidal effect models were then applied to WDM halos. AP2014 showed qualitatively that these sub-halo-host interactions, especially the tidal effects, have significant influence on the sub-halo population. Varying the efficiency of tidal stripping and tidal heating can significantly change the amplitude and slope of the sub-halo mass function. AP2014 also showed evidence that DM halo statistical properties such as the halo mass function and density profiles differ between CDM and WDM models. These findings point to the importance of accurately modeling non-linear evolution of sub-halos. However, AP2014 did not vary the Coulomb logarithm for dynamical friction, or the adiabatic index for tidal heating. A full search of the parameter space through a Monte Carlo Markov Chain (MCMC) fit was also not performed. Therefore, reliable and accurate values of model parameters applicable for future studies are still unclear.

In this work, we introduce an MCMC fitting workflow to fully explore the parameter space with high efficiency. We apply this MCMC fitting method to calibrate the dynamical friction, tidal stripping, and tidal heating models introduced in AP2014 to the ELVIS (Garrison-Kimmel et al. 2014) and Caterpillar (Griffen et al. 2016) CDM N-body simulations of Milky Way-sized host halos. This MCMC fitting workflow is also applicable for non-linear evolution model refinements in the future.

The plan of this paper is as follows. In Section 2 we review the dynamical friction, tidal stripping, and tidal heating models implemented in Galacticus. In Section 3 we introduce ELVIS and Caterpillar—the two sets of Milky Way-sized N-body simulations we used in this work. We also present relevant parameter settings in the corresponding Galacticus simulations. We introduce our fast MCMC fitting strategy as well as the fitting results in Section 4. We discuss the physical meaning behind the MCMC results in Section 5 and conclude in Section 6.

2 NON-LINEAR EVOLUTION THEORY

In this section we give a brief review of the models for three key non-linear evolution processes—dynamical friction, tidal stripping, and tidal heating—implemented in Galacticus by AP2014. The geometry of a simplified system which consists of a host halo, a satellite, and a DM particle of the satellite is presented in Figure 1 to clarify different position vectors involved in the non-linear evolution models. We also refer readers to Taylor & Babul (2001); Benson et al. (2002); Zentner et al. (2005) for further details.

The DM halo evolution engine in Galacticus works as follows. First, merger trees are constructed (using the EPS formalism, specifically the algorithm proposed by Parkinson et al. 2008) backward in time until the required mass resolution is reached along each branch. The properties of halos are then evolved forward in time. When two halos encounter each other in a merger tree, the more massive becomes the host with the lighter one becoming a satellite (sub-halo) within that host. The satellite is initially placed isotropically on the sphere corresponding to the virial radius of the host, and is given an initial velocity drawn from a distribution obtained from cosmological simulations (Benson 2005), with the radial component directed inward, and the direction of the tangential component sampled isotropically at random. The position within the host, bound mass, and density profile of the satellite are then tracked until certain merging/disruption criteria are satisfied at which point the satellite is considered to be full disrupted (merged with the host) and is removed.

Several assumptions are made in Galacticus to achieve fast simulation. As Galacticus dynamically evolves the positions and
velocities of a satellite, masses of other satellites are treated as a part of the host halo and the detailed sub-halo–sub-halo interactions are ignored. Peña-Rubio & Benson (2005) shows that such interactions have negligible influence on the mass and spacial distribution of the substructures. In this work, GALACTICUS classifies a satellite being destroyed by its host if 1) the distance between the sub-halo and the host halo is smaller than a fraction $f$ of the host virial radius; or 2) the sub-halo mass falls below a specified mass resolution $M_{\text{res}}$. These criteria are adjustable in GALACTICUS and can be changed for different applications. In this work we take $f = 0.01$ and $M_{\text{res}} = 5 \times 10^7 M_\odot$. We have checked that these two criteria are sufficient for the sub-halo mass range we care about in this work. More details about the GALACTICUS mass resolution setting are presented in Section 3.

### 2.1 Dynamical Friction

We assume that as a DM sub-halo with mass $M$ and velocity $V_{\text{sat}}$ travels through the sea of host halo DM particles, the sub-halo will experience a steady deceleration, known as dynamical friction. Dynamical friction arises as the sub-halo deflects nearby DM particles through gravitational interaction, and thus creates an over dense region behind it. This accelerates the sub-halo opposite to its direction of motion, slowing it down. First proposed by Chandrasekhar (1943) to describe the motion of a body through a uniform medium, the dynamical friction equation can be applied to bodies traveling through finite media with only minor modification (Weinberg 1986). If we assume that the distribution of host particles is reasonably modeled with a Maxwell-Boltzmann distribution (Lewin & Smith 1996; Mao et al. 2013), the Chandrasekhar formula gives the acceleration of the sub-halo caused by dynamical friction $a_{\text{df}}$ as:

$$a_{\text{df}} = -4\pi G^2 \ln \Lambda M_{\text{sat}} \rho_{\text{host}}(r_{\text{sat}}) \frac{V_{\text{sat}}}{V_{\text{vir}}}$$

$$\times \left[ \text{erf}(X_v) - \frac{2 X_v}{\sqrt{\pi}} \exp(-X_v^2) \right],$$

where $r_{\text{sat}}$ is the sub-halo position within the host, $X_v = V_{\text{sat}} / \sqrt{2} \sigma_v$ with $\sigma_v$ the velocity dispersion of DM particles in the host. We assume the host halo has an NFW density profile $\rho_{\text{host}}$ (Navarro et al. 1997):

$$\rho_{\text{host}}(r_{\text{sat}}) \propto \left( \frac{r_{\text{sat}}}{R_s} \right)^{-1} \left( 1 + \frac{r_{\text{sat}}}{R_s} \right)^{-2},$$

where $R_s$ is the scale length. The NFW profile is normalized such that the total halo mass is enclosed within the virial radius $R_{\text{vir}}$. The halo concentration parameter $c \equiv R_{\text{vir}} / R_s$ is computed following Diemer & Kravtsov (2015). The Coulomb logarithm in Eq (1), $\ln \Lambda$, is treated as a free parameter.

We use Eq (14) of Lokas & Mamon (2001) to calculate $\sigma_v(r_{\text{sat}})$. This is slightly different from the one used in AP2014, where $\sigma_v$ is approximated by the virial velocity of the host halo $V_{\text{vir}}$. It is shown in Du et al. (in preparation) that using the accurate form of $\sigma_v$ is important for correctly computing the dynamical friction.

### 2.2 Tidal Stripping

While the satellite orbits its host, it is subjected to tidal forces, which pull the satellite material on the near side toward the host center and in the opposite direction on the far side. When the tidal force is larger than the gravitational force from the satellite itself, material in the satellite could become unbound, forming tidal tails. The radius at which the tidal force equals the self-gravity force is called the tidal radius. To first order, the tidal force is proportional to the gradient of gravitational force from the host at the satellite position and the distance to the satellite center. Thus, the satellite will be stripped outside-in as the pericenter of its orbit moves ever closer to the host center due to dynamical friction, and as the sub-halo’s density drops due to tidal heating. A summary of various definitions of tidal radius is presented in van den Bosch et al. (2018). Taking into account the extended sub-halo mass profile and the motion of particles within the satellite, GALACTICUS computes the tidal radius, $r_t$, as (King 1962; van den Bosch et al. 2018):

$$r_t = \left( \frac{GM_{\text{sat}}(<r_t)}{\omega^2 - \frac{d^2 \Phi}{dR^2} |_{r_{\text{sat}}}} \right)^{1/3}.$$

Here $M_{\text{sat}}(<r_t)$ is the satellite mass enclosed within the tidal radius, $\omega$ is the angular frequency of the satellite orbit, and $R$ is the distance from the center of the host halo to the satellite DM particle. Here we have assumed that the satellite and its DM particles are orbiting within the host with a common angular frequency. Since we assume a spherically symmetric NFW profile, $\rho_{\text{host}}$, for the host halo, the second derivative of the gravitational potential from the host $d^2 \Phi / dR^2$ is given by:

$$\frac{d^2 \Phi}{dR^2} |_{r_{\text{sat}}} = -2 GM_{\text{sat}}(<r_{\text{sat}}) r_{\text{sat}}^3 + 4\pi G \rho_{\text{halo}}(r_{\text{sat}}).$$

Following Zentner et al. (2005), GALACTICUS models the tidal stripping effect by assuming that the satellite mass outside $r_t$ is lost on
an orbital time scale:
\[
\frac{dM_{\text{sat}}}{dt} = -\alpha \frac{M_{\text{sat}}(>r_i)}{T_{\text{orb}}}.
\] (5)

Here we define the instantaneous orbital period as the minimum of the instantaneous angular and radial periods \(T_{\text{orb}} = \min(2\pi/\omega, 2\pi r_{\text{sat}}/V_{\text{sat}})\), and \(\alpha\) is treated as a free parameter.

### 2.3 Tidal Heating

The host halo not only strips off mass from the satellite through gravitational tides, but also introduces an additional velocity dispersion to the satellite particles. The extra random motion within the satellite caused by the rapidly varying tidal field heats up the satellite. As a result, tidal heating will cause the satellite to expand and a larger fraction of the satellite mass will extend outside the tidal radius and become subjected to tidal stripping.

Gallactic models tidal heating following Gnedin et al. (1997) and Taylor & Babul (2001). Under the impulse approximation, the heating rate introduced by this effect averaged over all the randomly distributed DM particle members can be modeled as (Taylor & Babul 2001):

\[
\frac{dE}{dt} = \frac{1}{3} r^2(t) g_{ab}(t) G_{ab}(t).
\] (6)

Here \(r\) is the distance between the satellite and the DM particle, \(G\) is the tidal tensor, and \(G\) is the time integral of \(g\):

\[
G_{ab} = \int_0^t dt' \left[ g_{ab}(t') - G_{ab}(t')/T_{\text{orb}} \right].
\] (7)

Here we have added a decaying term \(-G_{ab}(t')/T_{\text{orb}}\) in the integrand considering that the positions of DM particles have non-negligible changes in one satellite orbital time, thus the impulse approximation is not valid on time scales larger than \(T_{\text{orb}}\).

Gnedin & Ostriker (1999) points out that although the tidal heating in the sub-halo outskirts is well described by the impulse approximation, the effect in the inner part (where dynamical times in the sub-halo may be comparable to the shock timescale) is more complex. These more strongly bound satellite particles respond more adiabatically to the tidal heating process, and the conservation of the adiabatic invariant suppresses the heating shock. On the other hand, resonances of the system will strengthen the effects of the shock. Accounting for the breakdown of the impulse approximation where the shock duration becomes comparable to the orbital time scale as well as the high order heating effects, AP2014 modify Eq. (6) as:

\[
\frac{dE}{dt} = \frac{\epsilon_h}{3} \left[ 1 + (\omega_p T_{\text{shock}})^2 \right]^{-1} r^2(t) g_{ab}(t) G_{ab}(t).
\] (8)

The bracketed factor is the adiabatic correction discussed in Gnedin & Ostriker (1999), \(T_{\text{shock}} = r_{\text{sat}}/V_{\text{sat}}\) is the shock time scale, \(\omega_p\) is the angular frequency of particles at the half-mass radius of the satellite \(^1\). The heating coefficient, \(\epsilon_h\), which accounts for the higher-order heating effects is treated as a free parameter. AP2014 sets the adiabatic index \(\gamma = 2.5\) following Gnedin & Ostriker (1999). However, it has been shown that when \(T_{\text{shock}} \gg T_{\text{dyn, h}}\), the suppression from adiabatic correction is shallower with \(\gamma\) approaching 1.5 (Weinberg 1994a,b; Gnedin & Ostriker 1999). There is also evidence that ignoring the adiabatic correction does not have a significant influence on sub-halo statistics when applied to cosmological simulations (van den Bosch et al. 2018). Du et al. (in preparation) shows that neglecting the adiabatic correction gives a better description for the density evolution of sub-halos in their idealized N-body simulations. In our MCMC simulation, we consider two limiting cases, \(\gamma = 0\) and \(\gamma = 2.5\). We will present the MCMC fitting results for both \(\gamma\) values later in Section 4. Energy injected into the satellite through tidal heating will cause the density profile to change. Under the assumption that each mass shell within the satellite stays virialized, and that there is no shell-crossing, AP2014 show that the satellite density profile can be modified as:

\[
\rho_{\text{sat}}(r_f) = \left[ 1 - \frac{2r_f^2 Q(r_i)}{GM_{\text{sat}}(<r_i)} \right]^{\frac{4}{3}} \left[ 1 + \frac{4r_f^2 Q(r_i)}{GM_{\text{sat}}(<r_i)} \right]^{-1} \rho_{\text{sat}}(r_i).
\] (9)

Here \(r_i\) and \(r_f\) are the initial and final radii of a mass shell, \(Q(r_i) = E(r_i)/r_i^2\).

### 2.4 Statistics for model constraint

The sub-halo mass function is sensitive to satellite mass loss caused by tidal stripping and is therefore widely used to constrain DM phenomenology and clustering properties (Peter & Benson 2010; Wang & Zentner 2012; Kennedy et al. 2014; Marković & Viel 2014). In this work we not only calibrate the three nonlinear evolution models with the sub-halo mass function at redshift \(z = 0\), but also consider the present time maximum circular velocity statistics. We define sub-halo mass, \(M\), as the sub-halo's gravitationally bound mass at \(z = 0\). To minimize the amplitude fluctuation of the sub-halo mass function caused by the variation of host halo mass, we use the ratio between sub-halo mass and host halo mass as the mass variable of the sub-halo mass function. The advantages of a joint fit to \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) are shown below.

The parameters \(\ln \Lambda, \alpha, \gamma, \omega_p\) effectively control the strength of dynamical friction, tidal stripping, and tidal heating in our semi-analytic simulation. Increasing \(\ln \Lambda\) while fixing \(\alpha\) and \(\omega_p\) leads to greater deceleration of DM sub-halos caused by dynamical friction, thus more satellites merge into the host and \(dN/d\log(M/M_{\text{host}})\) decreases over the entire mass range. Since \(\epsilon_h \propto M,\) massive halos are more sensitive to dynamical friction, leading to a steeper slope of \(dN/d\log(M/M_{\text{host}})\) as \(\ln M\) increases. The maximum circular velocity of a DM halo with an NFW density profile at infall is:

\[
V_{\text{max}} = 0.465 \times \sqrt{\frac{GM_{\text{(infall)}}}{R_{\text{(infall)}}}} \frac{\sqrt{c_{\text{(infall)}}}}{f(c_{\text{(infall)}})},
\] (10)

Here \(M_{\text{(infall)}}, c_{\text{(infall)}}, R_{\text{(infall)}}\) are the mass, concentration, and virial radius of the satellite when it first enters the host's virial radius. After infall, the maximum circular velocity is computed from the heated density profile Eq. (9). Therefore, sub-halos with larger initial mass and concentration have larger \(V_{\text{max}}\). Since sub-halos with large initial mass stay in the host for longer before they reach the disruption mass, and are more sensitive to dynamical friction, as \(\ln M\) increases, the number of massive sub-halos with large \(V_{\text{max}}\) decreases, leading to a lower averaged \(V_{\text{max}}\) in the system. Semi-analytically simulated variations of \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) caused by varying \(\ln \Lambda\) are shown in the first column of Figure 2.

\(^1\) Here we follow the same definition as in Gnedin & Ostriker (1999), while AP2014 takes the orbital frequency of the satellite around the host.
Increasing $\alpha$ while fixing $\epsilon_h$ and $\ln \Lambda$ corresponds to higher efficiency for the host halo to strip off satellite mass distributed outside of the tidal radius of the sub-halos, thus $dN/d\log(M/M_{\text{host}})$ decreases over the entire mass range. However, the density profile of the satellites within the tidal radius is not influenced inside the tidal radius, such that a satellite with smaller mass can maintain its $V_{\text{max}}$ under strong tidal stripping. As a result $V_{\text{max}}(M)$ increases as $\alpha$ increases. The influences of $\alpha$ on $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ are shown in the second column of Figure 2.

Finally, increasing $\epsilon_h$ while fixing $\alpha$ and $\ln \Lambda$ corresponds to stronger tidal heating. A larger fraction of mass within the satellite will extend out of tidal radius and get stripped off by the host halo, this decreases $dN/d\log(M/M_{\text{host}})$ over the entire mass range. Since the density profile of satellite becomes less compact, $V_{\text{max}}(M)$ also decreases as $\epsilon_h$ increases. This phenomenon is presented in the third column of Figure 2.

Notice that $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ vary differently as a result of increases in $\alpha$ and $\epsilon_h$. Thus a joint fit to $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ can break the degeneracy between $\alpha$ and $\epsilon_h$. However, $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ vary in similar ways with increases in $\epsilon_h$ and $\ln \Lambda$, thus we expect to see the negative correlation in the posterior distribution of $\epsilon_h$ and $\ln \Lambda$. Although $\epsilon_h$ only influences the amplitude of the sub-halo mass function while $\ln \Lambda$ also changes its slope, the limited size of the ELVIS and Caterpillar N-body simulations we use in this work mean that there are too few of the most massive satellites to fully break the $\epsilon_h - \ln \Lambda$ degeneracy. We expect this to also lead to a weak constraint on $\ln \Lambda$.

3 N-BODY SIMULATION AND GALACTICUS SETTINGS

In this work, we calibrate the three free parameters introduced in the dynamical friction and tidal effect models in the last section, to two independent CDM N-body simulations—ELVIS and Caterpillar. When calibrating GALACTICUS to Caterpillar we use Planck cosmological parameters, $\Omega_m = 0.32$, $\Omega_{\Lambda} = 0.68$, $\sigma_8 = 0.83$, $n_s = 0.96$, and $h = 0.6711$ (Planck Collaboration et al. 2014b), while for ELVIS we use cosmological parameters set by Wilkinson Microwave Anisotropy Probe 7 $\Omega_m = 0.266$, $\Omega_{\Lambda} = 0.734$, $\sigma_8 = 0.801$, $n_s = 0.963$, and $h = 0.71$ (Larson et al. 2011).

As described in Sec. 2.4, we use the sub-halo mass function and maximum circular velocity functions at redshift $z = 0$ from these simulations as the constraints on our model. We expect to constrain tidal mass loss and dynamical friction through the sub-halo mass function $dN/d\log M$. Since tidal heating effects will extend the density profile of satellites and decrease the maximum circular velocity of satellites, we use the maximum circular velocity function $V_{\text{max}}(M)$ to constrain tidal heating. Although $dN/d\log M$ is self-similar for CDM, the amplitude of $dN/d\log M$ is sensitive to the host halo mass. Each host halo in the N-body simulation has a slightly different mass, and the host halo mass distributions for ELVIS and Caterpillar differ. Averaging $dN/d\ln M$ over all the simulated host halos will introduce uncertainties to the sub-halo mass function amplitude and will further influence the parameter fitting accuracy. In order to minimize the effects of the distribution of host halo masses, we compute and calibrate the number of satellite in fractional mass bin $dN/d\log(M/M_{\text{host}})$ instead. The maximum circular velocity is directly determined by the satellite mass $M$ and is independent of the host halo mass $M_{\text{host}}$, so we fit AP2014 model on $V_{\text{max}}(M)$ instead of $V_{\text{max}}(M/M_{\text{host}})$.

In this work we only include satellites within the host halo virial radius for the $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ statistics. Since the Caterpillar simulation does not include host halos which experienced major mergers (1:3 infall mass ratio) below redshift $z < 0.05$, we also exclude halos of this type in GALACTICUS simulations for our Caterpillar-matched simulations.

Figure 3 shows the sub-halo mass function, $dN/d\log(M/M_{\text{host}})$, and maximum circular velocity function, $V_{\text{max}}(M)$, averaged over the 34 (24) host halos in Caterpillar (ELVIS isolated) respectively. The dots show the mean $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ among all catalogs. Error bars show the error on the mean $\sigma_d = \sigma_d/\sqrt{N}$, where $\sigma_d$ is the standard deviation of N-body data over all host halos, and $N$ is the number of host halos. The host halo mass range for ELVIS and Caterpillar simulation are $7 \times 10^{11}M_{\odot} \leq M_{\text{host}} \leq 3 \times 10^{12}M_{\odot}$ and $10^{12}M_{\odot} \leq M_{\text{host}} \leq 3 \times 10^{12}M_{\odot}$ respectively, we therefore set identical host mass range for GALACTICUS when generating merger trees. The halo mass resolution of the ELVIS simulation is $2 \times 10^7M_{\odot}$, while Caterpillar has a much higher resolution of $6 \times 10^7M_{\odot}$.

We find that for Caterpillar extending the mass resolution of GALACTICUS down to $5 \times 10^6M_{\odot}$ does not result in significantly stronger constraints on the parameters of our model, but does make the semi-analytic merger tree construction more computationally expensive. We therefore set the mass resolution of GALACTICUS to be $M_{\text{res}} = 5 \times 10^7M_{\odot}$ for both ELVIS and Caterpillar fits. We calibrate the non-linear models to $dN/d\log(M/M_{\text{host}})$ over fractional mass range $\log_{10}(M_{\text{min}}/M_{\text{host}}) \leq \log_{10}(M/M_{\text{host}}) < -1$, where $M_{\text{min}}$ is the lower limit of the host halo mass distribution. We calibrate models by $V_{\text{max}}(M)$ in sub-halo mass range $\log_{10}(M_{\text{res}}) \leq \log_{10}M < 11$. ELVIS is complete for sub-halos with $V_{\text{max}} \geq 8\, \text{km/s}$, while Caterpillar is complete to about $V_{\text{max}} \geq 4\, \text{km/s}$. To ensure that $V_{\text{max}}(M)$ is not biased by the incompleteness at low masses, we exclude all sub-halos with $V_{\text{max}} < 8\, \text{km/s}$ in both GALACTICUS and N-body simulations when computing the maximum circular velocity function. The blue (red) shaded regions in Figure 3 show the mass ranges we fit for ELVIS (Caterpillar).

In order to ensure the statistical errors from the GALACTICUS simulation are small comparing to those contributed by the N-body simulations, we set GALACTICUS to generate 381 (505) merger trees for ELVIS (Caterpillar), which is about 16 times larger than the corresponding number of N-body simulation merger trees. We therefore ignore the statistical uncertainty contributed by GALACTICUS simulations when constructing the likelihood function introduced in the following section.

4 MCMC FITTING STRATEGY AND RESULTS

To perform a full search in the $[\alpha, \epsilon_h, \ln \Lambda]$ 3D parameter space, ideally we would want the MCMC chains to call GALACTICUS to compute $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ for each new proposed state in the parameter space. However, in this work we use GALACTICUS to generate 505 (381) merger trees with mass resolution $M_{\text{res}} = 5 \times 10^7M_{\odot}$ for ELVIS (Caterpillar) in each simulation, and it takes about 10 CPU hours to evolve the satellites according

\footnote{In the ELVIS simulation, a halo is considered to be resolved when it contains more than 100 particles. In the Caterpillar simulation, an improved halo finder is used and a halo containing more than 20 particles is considered to be resolved. Applying the same criteria used in ELVIS to Caterpillar, the halo mass resolution of the Caterpillar simulation is $3 \times 10^8M_{\odot}$.}
to the nonlinear evolution models in each simulation. It is not practical to conduct a standard MCMC fitting process in which each walker may take thousands of steps before convergence is reached. We therefore take an alternative approach. We first select multiple grid points in the 3D parameter space \([\alpha, \epsilon_h, \ln \Lambda]\), here \(i, j \) and \(k\) are indexes which run from 1 to \(N_x\), with \(N_x\) chosen for each parameter \(x\), giving a total of \(N\alpha N\epsilon N\ln \Lambda\) grid points in the parameter space. We then use Galacticus to compute \(dN/d\log(M/M_{\text{host}})\) as well as \(V_{\text{max}}(M)\) for each grid point. Galacticus simulation results for \([\alpha, \epsilon_h, \ln \Lambda]\) located between grid points are then estimated through linear interpolation. Since \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) change continuously and smoothly under \([\alpha, \epsilon_h, \ln \Lambda]\) variation, in the limit that the parameter space is gridded infinitely finely the linearly interpolated statistics will be identical to the semi-analytic simulation results.

We conduct multiple reduced \(\chi^2\) tests to ensure that our grid-

Figure 2. \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) simulated by Galacticus under different \([\alpha, \epsilon_h, \ln \Lambda]\) combinations. Galacticus simulations are made with the Caterpillar cosmology, and setting the tidal heating parameter \(\gamma = 0\). Parameter combinations used in the plots are chosen such that the \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) changes are easy to see. \(\alpha\) and \(\epsilon_h\) variation influence the sub-halo mass function in the same direction, while \(V_{\text{max}}(M)\) varies in the opposite way. Therefore a joint fit for \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) can break the degeneracy between \(\alpha\) and \(\epsilon_h\). However, \(dN/d\log(M/M_{\text{host}})\) and \(V_{\text{max}}(M)\) change in the same direction under \(\ln \Lambda\) and \(\epsilon_h\) enlargement, thus we still see negative correlation in \(\ln \Lambda - \epsilon_h\) contour in Figure 3.
parameter combinations by computing the reduced $\chi^2$ values:

$$\chi^2 = \sum_i \frac{(D_i - D_i')^2}{\sigma_i^2},$$

(11)

here $\chi^2$ is the reduced $\chi^2$ value, $D$ is the $dN/d\log(M/M_{\text{host}})$ or $\nu (M)$ for the removed $N, N, N, \Lambda$ set of parameter combinations directly simulated by Galacticus, $D'$ is the corresponding $dN/d\log(M/M_{\text{host}})$ or $\nu (M)$ linearly interpolated based on statistics of the remaining $(N - 1)N, N, N, \Lambda$ grid points, $\sigma_D$ is the error of the mean directly simulated by Galacticus, $\sigma_{D'}$ is estimated through linear interpolation, $i$ is the $M/M_{\text{host}}$ or sub-halo mass bin index, $n_i$ is the number of bins. We repeat the above tests for all parameter grid values except those on the boundaries. We find that $\chi^2$ for all the tested grid points are less than 1.9. 99% of the $\chi^2$ are below 1. We therefore confirm that our interpolator is a good description of the full model. Distributions of the $\chi^2$ for different statistics and cosmologies are presented in Figure 4.

According to Lu et al. (2016) and Boylan-Kolchin et al. (2010), the distribution of sub-halo mass functions as well as $\nu (M)$ is non-Gaussian. However, since we compute the average $dN/d\log(M/M_{\text{host}})$ and $\nu (M)$ over all host halos in each

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**Table 1.** Summary of uniform prior bounds used in different satellite nonlinear evolution models.

| Parameter | $\gamma = 0$ | $\gamma = 2.5$ |
|-----------|-------------|-------------|
| $\alpha$  | $1.5, 3.0$  | $2.0, 4.0$  |
| $\epsilon_h$ | $0.1, 1.5$ | $1.0, 8.0$ |
| $\ln \Lambda$ | $0.0, 5.0$ | $0.0, 8.0$ |
| $\ln f_1$  | $-10.0, 0.0$ | $-10.0, 0.0$ |
| $\ln f_2$  | $-10.0, 0.0$ | $-10.0, 0.0$ |

N-body simulation suite, with 34 (24) host halos in ELVIS (Caterpillar), the central limit theorem suggests that a normal distribution for the mean will be approximately valid.

The priors we use in this work are uniform over the range of our gridded parameter space. To locate the prior ranges for the three parameters, we use Galacticus to compute $dN/d\log(M/M_{\text{host}})$ or $\nu (M)$ for several points widely distributed throughout the parameter space. Through comparing Galacticus predictions with N-body data we can then roughly determine ranges of individual parameters that produce $dN/d\log(M/M_{\text{host}})$ or $\nu (M)$ comparable to N-body statistics. We then take finer grids within the prior ranges and repeat the former process until the prior ranges are narrow but fully cover the potential posteriors of the three parameters. A summary of the prior ranges we use in this work is presented in Table 1.

Ignoring the adiabatic correction factor in the tidal heating model, for $\gamma = 0$ we use a likelihood function:

$$\ln L_1(x|\sigma_x, \alpha, \epsilon_b, \ln \Lambda) = -\frac{1}{2} \sum_b \left[ \frac{(x_b - x_b' (\alpha, \epsilon_b, \ln \Lambda))^2}{\sigma_x^2} + \ln (2\pi \sigma_x^2) \right],$$

$$\ln L_2(y|\sigma_y, \alpha, \epsilon_b, \ln \Lambda) = -\frac{1}{2} \sum_d \left[ \frac{(y_d - y_d' (\alpha, \epsilon_b, \ln \Lambda))^2}{\sigma_y^2} + \ln (2\pi \sigma_y^2) \right],$$

$$\ln L = \ln L_1 + \ln L_2$$

(12)
here $\ln L_1$ ($\ln L_2$) is the likelihood function that constrains the sub-halo non-linear evolution models only through the sub-halo mass function (maximum velocity function) statistics. $\ln L$ is the total likelihood function used for a joint $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ fit. $x$ and $y$ are $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ given by N-body simulation. $x'$ and $y'$ are the interpolated $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ given by Galacticus semi-analytic simulation. $\sigma_x(\sigma_y)$ is the error of the mean of the sub-halo mass function (maximum velocity function) given by N-body simulation. $b$ and $d$ are the index of the fractional mass and sub-halo mass bin located in the MCMC fitting mass range that we discussed in section 3.

For the $\gamma = 2.5$ tidal heating model, we find the MCMC fit reduced $\chi^2$ value under the likelihood function of Eq. (12) is much higher than 1, indicating a severe underestimation of the errors, or that the $\gamma = 2.5$ model is not a good description for the N-body data. To study how much the error bar of $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ should be enlarged to provide a good fit, we replace $\sigma_x$ and $\sigma_y$ in Eq. (12) by $s_x$ and $s_y$, defined as:

$$
(s^2_x)_b = (\sigma_x)_b^2 + f^2_x (\alpha, \epsilon_b, \ln \Lambda)^2,
$$

$$
(s^2_y)_d = (\sigma_y)_d^2 + f^2_y (\alpha, \epsilon_b, \ln \Lambda)^2,
$$

(13)

here we introduce two extra free parameters $f_1$ and $f_2$ to probe the error bar underestimation for $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ respectively.

We use emcee (Foreman-Mackey et al. 2013) to conduct the MCMC sampling. We run 10 MCMC walkers with initial position randomly distributed in the gridred parameter space.

An example to show the advantages of combining satellite mass and maximum circular velocity statistics together, we first present the MCMC fitting results with ELVIS cosmology and $\gamma = 0$ model constrained only by $dN/d\log(M/M_{\text{host}})$ or $V_{\text{max}}(M)$ alone in Figure 5. As discussed in section 3, $\alpha$ and $\epsilon_b$ are negatively correlated in $dN/d\log(M/M_{\text{host}})$ because tidal stripping and tidal heating effects are both channels for sub-halo mass loss. However, $\alpha$ and $\epsilon_b$ are positively correlated in $V_{\text{max}}(M)$ because $V_{\text{max}}(M)$ is determined by the density profile of the satellite, and only the tidal heating effect influences satellite density profiles. The $\{\alpha, \epsilon_b, \ln \Lambda\}$ posteriors of $\gamma = 0$ and $\gamma = 2.5$ jointly fitted by $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ are shown in Figure 6. Comparing with Figure 5, the degeneracy between $\alpha$ and $\epsilon_b$ is effectively weakened, and $\ln \Lambda$ is better constrained. The best-fit parameter values and reduced $\chi^2$ test results of Figure 6 are summarized in Table 2. We show the comparison between Galacticus interpolation at best-fit parameters and the N-body data in Figure 7.

| $\gamma$ = 0 | Caterpillar | $\gamma$ = 2.5 | Caterpillar |
|--------------|-------------|--------------|-------------|
| $\alpha$     | $2.34\pm0.28$ | $2.17\pm0.22$ | $2.62\pm0.44$ |
| $\epsilon_b$ | $0.46\pm0.41$ | $0.49\pm0.25$ | $2.98\pm1.02$ |
| $\ln \Lambda$| $2.3\pm1.9$  | $1.18\pm1.40$ | $3.8\pm2.2$  |
| $\ln f_1$    | $-6.4\pm3.4$ | $-6.4\pm3.5$ | $2.9\pm1.9$  |
| $\ln f_2$    | $-3.47\pm0.88$ | $-3.26\pm0.65$ |
| $\chi^2_{\nu}$| $0.88$ | $1.54$ | $1.08$ |

Table 2. Summary of best-fit parameter values and reduced $\chi^2$ of MCMC results shown in Figure 6. The upper and lower limit for the best-fit parameter values shows 95% c.l. $\downarrow$ means the lower limit of the 95% c.l. reaches the lower bound of prior.

5 DISCUSSION

While AP2014 calibrated the non-linear evolution models for sub-halo orbital evolution using only the sub-halo mass function, we add $V_{\text{max}}(M)$ as a further constraint on the free parameters describing dynamical friction and tidal effect models. The advantage of jointly fitting for $dN/d\log(M/M_{\text{host}})$ and $V_{\text{max}}(M)$ can help to break the tidal stripping and tidal heating model degeneracy, as shown in Fig. 6.

In this work we find that ignoring the adiabatic correction factor in the tidal heating model, i.e. setting $\gamma = 0$, better describes the tidal heating process in CDM N-body simulations. Besides the evidence from idealized simulations by Du et al. (in preparation), the posterior of $\ln f_2$ presented in Figure 6 and the fractional error of the $V_{\text{max}}(M)$ function compared with N-body simulations presented in Figure 7 also indicate that Galacticus cannot provide a good fit to the N-body $V_{\text{max}}(M)$ statistics for $\gamma = 2.5$. We identify two possible explanations for the poor performance of the adiabatic correction factor in the tidal heating model. First, since it may take several orbital periods $T_{\text{orb}}$ before a satellite merges to its host, the position of the satellite DM particle member could gain a non-negligible change after multiple tidal shocks and breaks the impulse approximation. To account for the break down of the impulse approximation on time scales larger than $T_{\text{orb}}$ we introduce a decaying term $-G_M(t')T_{\text{orb}}$ in the time integral of Eq. (7). The decaying term effectively suppresses the tidal heating rate and serves similarly to the adiabatic correction factor, therefore the presence of the decaying factor might be the cause of a trivial adiabatic correction factor i.e. $\gamma = 0$. We leave a more careful comparison between the decaying term of the tidal tensor time integral and the adiabatic correction factor in future works. As a second possible explanation, van den Bosch et al. (2018) show that in the cosmological Bolshoi simulation the overall impact of the adiabatic correction factor on the energy injected to sub-halos by tidal heating effect is negligible.
Moreover, for sub-halos with orbital circularity $\eta \gtrsim 0.2$, the impulse approximation combined with the adiabatic correction factor underestimates the sub-halo mass fraction stripped off by the tidal effects. Therefore, setting $\gamma = 0$ effectively enhances tidal heating and helps to compensate the underestimation of tidal effects.

6 CONCLUSION

In this work we develop a fast MCMC fitting strategy for Galaxticus sub-halo orbital evolution models. We apply this new MCMC method to fit three parameters related to dynamical friction, tidal stripping, and tidal heating models introduced to Galaxticus by AP2014. We show that sub-halo statistics predicted by Galaxticus are in good agreement with ELVIS and Caterpillar N-body simulations.

Since both tidal stripping and tidal heating effect increase the mass loss from satellites, we find that using the sub-halo mass function alone for model calibration leads to a degeneracy between tidal effects. We show that including $V_{\text{max}}(M)$, which is sensitive to the sub-halo density profile, can break this degeneracy.

Limited by a lack of massive substructures in ELVIS and Caterpillar N-body simulations, we fail to put a strong constraint on the dynamical friction model. Second, dynamical friction can be probed in more detail through placing a massive sub-halo in the idealized simulation. Moreover, strong dynamical friction increases the concentration of sub-halos toward the host halo center. Therefore, the radial distribution of sub-halos may help to place stronger constraints on the dynamical friction model. We plan to explore these possibilities in the future. Du et al. (in preparation) will present more discussions about using idealized simulation to constrain the sub-halo non-linear evolution models.

We find evidence from our MCMC $\chi^2$ tests that ignoring the adiabatic correction factor in the tidal heating model fits the cosmological simulation data better than the original $\gamma = 2.5$ model of Gnedin & Ostriker (1999). It is possible that the decaying term we introduce to the time integral of tidal tensor in the tidal heating model effectively acts to replace some of the adiabatic correction factor. Alternatively, tidal heating with non-zero adiabatic correction may only be a good description for sub-halos with more radial orbits and may therefore underestimate the averaged tidal heating effects throughout the sub-halo population. Extracting the tidal heating energy directly from N-body simulation will be helpful to break the degeneracy between the tidal tensor decaying term and adiabatic correction factor. A more detailed study about the tidal heating model will be presented in Du et al. (in prep). For $\gamma = 0$, MCMC gives the best fit strength of dynamical friction, tidal stripping, and tidal heating effects as $\ln \Lambda = 2.3^{+1.6}_{-1.8}$, $\alpha = 2.34^{+0.28}_{-0.26}$, $\epsilon_h = 0.46^{+0.41}_{-0.24}$ for ELVIS cosmology and $\ln \Lambda = 1.18^{+1.40}_{-1.40}$, $\alpha = 2.17^{+0.24}_{-0.22}$, $\epsilon_h = 0.49^{+0.25}_{-0.24}$ for Caterpillar at 95% c.l. These posteriors agree within the 95% c.l.

A good understanding about the DM substructure evolution

Figure 6. $\alpha$, $\epsilon_h$, $\ln \Lambda$ posteriors under adiabatic index $\gamma = 0$ (left panel) and $\gamma = 2.5$ (right panel) from MCMC.
is crucial for constraining DM properties with future observations. The best-fit results of this work can make accurate and fast predictions for the sub-halo populations based on physics models and provide priors for future DM substructure studies and measurements. Orbital evolution models for DM sub-halos are still under intensive study and the best fit values of the parameters may vary with additional model refinements. Our fast MCMC fitting framework will be applicable to more sophisticated sub-halo and satellite evolution models in the future.

7 ACKNOWLEDGEMENT

We wish to thank Shenglong Wang for the IT support. We thank Brendan Griffen for providing $z = 0$ Caterpillar simulation catalog with $L_X = 14$ resolution. X.D. thanks Ethan Nadler for beneficial discussions on sub-halo mass function and maximum velocity function in N-body simulations and SAMs. This material is based on work supported by NASA under award numbers 80NSSC18K1014 and NNH17ZDA001N. This work was supported by the Simons Foundation.

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