On the origins of extreme wealth inequality 
in the Talent vs Luck Model

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We introduce a simplified version (STvL) of the Talent versus Luck (TvL) model [1] where only lucky events are present and verify that its dynamical rules lead to the same very large wealth inequality as the original model. We also derive some analytical approximations aimed to capture the mechanism responsible for the creation of such wealth inequality from a Gaussian-distributed talent. Under these approximations, our analysis is able to reproduce quite well the results of the numerical simulations of the simplified model. On the other hand, it also shows that the complexity of the model lies in the stochastic transformation of lucky events into an increase of capital, so that, when the talent heterogeneity of the population increases, the task of finding a formal analytical relationship between the distributions of capital, talent and luck in either the TvL or the STvL models becomes very hard.

Keywords: Pareto law, Success, Talent, Luck, Randomness, TvL model, Wealth distribution

I. INTRODUCTION

It is well known that the wealth distribution in the world is highly asymmetric, with a large majority of poor people and a very small number of very rich individuals. Recently, this gap has become much greater than it had been thought: just eight men own the same wealth as the poorest half of humanity, i.e. about 3.6 billion people [2]. More generally, as originally discovered by Pareto [3], the tail of wealth distribution follows a heavily-tailed power-law distribution where 80% of people own only the 20% of the total capital and the 20% own the remaining 80%. These intriguing features of the wealth distribution have been largely studied in the last decades through many theoretical models developed in the context of statistical physics, game theory and complex networks theory [4–7].

On the other hand, it is equally well known that the human talent is normally distributed among a population [8–10]; the same applies for the efforts which an individual can invest during each single week of her life in the attempt of achieving success [11]. Finally, it is also accepted that randomness (good or bad luck) plays a not negligible role in determining the outcome of our efforts: for example, living in an environment rich of opportunities or being in the right place at the right time, are considered to be decisive incentives for becoming rich or successful [12]. But, again, fortune is blind by definition, thus, assuming the same external conditions, one should not expect, in principle, extreme differences in the occurrence of either lucky or unlucky events among the individuals in the everyday life. All such considerations done, why is the
wealth (i.e., the success) so unevenly distributed, given that talent and luck are much less differentiated than it?

In order to answer this question, three of us recently introduced an agent-based model called "Talent vs Luck" [1] (TvL thereafter), which leads to a heavy-tailed distribution of capital in a population of individuals, despite of the non-heavy distributions of both talent and random events (positive and negative). In the original TvL model, individuals were endowed with a normally distributed talent and the same initial amount of capital, and were exposed to the random action of both positive/lucky and negative/unlucky events. When a lucky event occurs, a person doubles the capital with a probability proportional to her talent; contrariwise, an unlucky event halves her capital with certainty.

At the end of the simulation, as a result of such a multiplicative dynamics, the probability distribution of capital is heavy-tailed: approximately a power-law with a negative exponent between 1 and 2 [1]. This mimics the well known "Matthew effect" or "rich get richer" effect, induced by the feedback mechanisms of the real socio-economic complex networks [13]. But the effects of randomness were more subtle: as a matter of fact, success and talent appeared to be not much correlated and richest individuals almost never were the most talented ones. On the other hand, the most successful individuals were always very lucky, even if their talent was very close to the mean. In other words, very lucky people, although moderately talented, appeared to have much more possibilities to reach the apex of the social success than very talented but unlucky persons – a finding in agreement with our perception of real life.

In this paper we explore the origin of the extreme wealth/success inequality in the TvL model. To this end, after introducing a simplified version of the original model, where only lucky events — i.e. opportunities — are taken into account, we propose an analytical approach that links distributions of lucky events and capital and shows why the latter is heavy tailed even if the former is not. This approach underlines that the distribution of capital is not simply log-normal because of the heterogeneity of the talent and of a finite number of iterations.

In the second section we describe in detail the simplified model and show, with the help of some numerical results, that despite its simplifications it is still able to reproduce the main stylized facts of the original model. In the third section we discuss the basis of our formal approach and present the analytical derivation of the capital/success inequality, under different assumptions related to the talent distribution among individuals. In the last section we present some conclusive remarks.

II. THE SIMPLIFIED TVL MODEL (STVL)

A. Model description

Let us consider $N$ individuals randomly placed at fixed positions within a square world with periodic boundary conditions and surrounded by a given number $N_E$ of lucky event-points, corresponding to opportunities occurring by chance in the real world. These event-points are also initially randomly placed: their spatial distribution would be perfectly uniform only if $N_E \to \infty$. Thus, for finite $N_E$, at the beginning of each simulation there will be a greater random concentration of event-points in different areas of the world, while other areas will be more neutral. In addition, for a relatively small number of iterations, the random placement of individuals also induces small spatial correlations in the number of lucky events.
In Fig. [1] an example with $N = 10000$ agents and $N_E = 3000$ events is shown. At each time step, lucky events move according to an unbiased random walk, which neither depends on the presence of the individuals, nor on their intrinsic qualities. The further random movement of the points inside the world does not change this fundamental feature of the model, which exposes different individuals to different amount of opportunities during their life.

At the beginning of each simulation run, each agent is endowed with a given level of talent and with an initial capital. The talent of agent $i$ is represented by a real variable drawn in the interval $[0, 1]$ from a known symmetric distribution $P(T)$, e.g. a Gaussian $T_i \sim \mathcal{N}(\mu_T, (\sigma_T)^2)$, constant for the whole duration of the simulation. As in the original Tvl model, talent is meant to represent any kind of ability (including intelligence, skill, efforts, etc...) which allows an individual to transform a random opportunity into reality. Therefore, having a great/small talent represents a strong a-priori advantage/disadvantage for a given agent.

On the other hand, the initial capital $C_i$ of the agents, which represents their starting level of success/wealth (expressed in dimensionless units), is distributed according to a thin-tailed distribution in order to not offer any comparatively large initial advantage to anyone. We use a uniform distribution of $C \in [0.5, 1.5]$ for $P(C)$ in this section.

For a single simulation run, a given time period of $M$ time steps is considered. During the time evolution of the model, all event-points randomly move within the world and, thus, possibly intercept the position of some agent. Two possible situations may happen to a given agent $A_i$:

1. no event-points intercept the position of agent $A_i$, which means that the agent $A_i$ does not perform any action and her capital remains unchanged;

2. an event-point intercepts the position of agent $A_i$, which means that a lucky event (an opportunity) has occurred at that time step and, as a consequence, agent $A_i$ doubles her capital/success with a probability proportional to her talent $T_i \in [0, 1]$, i.e.,

$$ C_i(t) = 2C_i(t-1) \Leftrightarrow \text{rand}[0,1] < T_i$$

(meaning that the agent is smart enough to take advantage of the opportunity).

We denote with $n_i$ the total number of opportunities experimented by an agent $A_i$ and with $k_i$ the number of those ones successfully transformed in an increase of capital. At the end of each simulation, both these variables will result to be distributed among the agents according to the functions $P(n)$ and $P(k)$ respectively. We are interested in studying the final distribution of capital $P(C)$ and its relationship with $P(n)$ and $P(k)$.

B. Numerical results

Consider $N = 10000$ agents, with an initial amount of capital $C_i(0) \in [0.5, 1.5]$ $\forall i$ and with a talent $T_i \in [0, 1]$ following a normal distribution with mean $\mu_T = 0.6$ and standard deviation $\sigma_T = 0.1$. Further, initially consider $N_E = 5000$ lucky event-points and a time period of $M = 30$ simulated time steps.

At the end of the simulation, we find that the simplified dynamic rules of the STvl model are still able to produce a heavy tailed distribution $P(C)$ of capital/success, with a large amount of poor (unsuccessful) agents and a small number of very rich (successful) ones. The nature of $P(C)$ is clearly heavy-tailed: to be more precise, assuming that the tail of $P(C) \propto C^{-\alpha}$, we applied the method of [15] and its implementations in R [16].
and Python [17] to $P(C)$, which yields $\alpha \simeq 1.6$. However, as made clear by Fig. 2 the tail of the reciprocal cumulative function $P_S(c) = P(C > c)$, and thus that of $P(C)$, are not pure power-laws. Vuong likelihood ratio tests indicate that a truncated power-law is more likely (p-value of about 0.0006), while a log-normal distribution is probably better on average (p-value of about 0.02). This is in part due to the fact that the number of iterations is small.

As expected, in this simplified model, success and talent are not strongly correlated, success being mostly due to luck. In Fig. 3 the distribution $P(n)$ of the total number $n$ of lucky events occurred to the agents during the same simulation run is reported. It appears quite asymmetric (an effect of the small number of iterations), with a large majority of individuals who experienced a number of events included between 2 and 10, while only a very small number of them were so lucky to intercept more than 10 events. In any case, nobody experienced more than $n_{\text{max}} = 18$ events.

In order to have a clearer idea of what is the real shape of $P(n)$ we run three simulations with $N = 10000$, $M = 300$ time steps and $N_E = 5000$, 10000, and 20000. The three resulting distributions $P(n)$ are shown in the top panel of Fig. 4 while the corresponding q-q plots are reported just below, in the bottom panels of the same figure. It clearly appears that the distributions progressively tend to assume a Gaussian shape, which becomes very good for $N_E = 20000$.

On the other hand, showing in Fig. 5 the distribution $P(k)$ of the transformed opportunities obtained for the last simulation with $N_E = 20000$ (top panel), along with the corresponding q-q plot (bottom panel), a sensible deviation from Gaussian behavior is still observed, as the heterogeneity of $T$ makes $P(k)$ fatter than $P(n)$. These findings will be discussed in details in the next section, where an analytical derivation of $P(C)$ as function of $P(T)$, $P(n)$ and $P(k)$ is presented.

III. ANALYTICAL APPROACH TO THE STvL MODEL

The main result of [11] was that $P(C)$ has a heavy tail despite the fact that neither $P(T)$ nor $P(n)$ have one. In the previous section we verified that such a feature still holds true also in the simplified version of the model introduced in this work. This section is devoted to the analytical characterization of the relationship between these distributions in the STvL.
therefore a binomial distribution \(B\) steps of a simulation, and thus each of them transformed into a capital increase. As we have already seen, during \(n\) and with \(N\) the number of transformed lucky events for the figure 5: Upper panel: probability distribution \(P\) of the number of transformed lucky events for the \(N = 10000\) agents during a single run of \(M = 300\) time steps and with \(N_E = 20000\) lucky event points. Lower panel: the q-q plot of the same distribution shows a consistent deviation from normal behavior.

A. Formal link between lucky events and capital distributions

Let us first state a few simple relationships between variables. As we have already seen, during a simulation, agent \(i\) experiences \(n_i\) lucky events, each of them transformed into a capital increase with probability \(T_i\), thus producing \(k_i\) transformed events. Let us drop the indices \(i\): \(P(k|n,T)\) is therefore a binomial distribution \(B(n,T)(k)\).

Formally, if \(M\) denotes the total number of time steps of a simulation,

\[
P(k|T) = \sum_{n=k}^{M} P(k|n,T)P(n) = \sum_{n=k}^{M} B(n,T)(k)P(n). \tag{1}
\]

and thus

\[
P(k) = \int dTP(T) \sum_{n=k}^{M} B(n,T)(k)P(n). \tag{2}
\]

Assuming that all the agents have the same a-priori probability \(\rho\) to experience a lucky event in a single time step of the simulation, the total number of lucky events collected by a given agent should follow a binomial distribution, which can be approximated by a Gaussian distribution when the total number of time steps \(M\) is large enough: \(P(n) \sim N(\mu_n, \sigma_n^2)\) with \(\mu_n = \rho M\) and \(\sigma_n = M\rho(1 - \rho)\). As we have seen in the previous section, the results of the numerical simulations shown by Fig. 4 confirm that this condition is nicely satisfied when \(M = 300\) and the number of lucky event-points \(N_E\) is large enough to have also quite large values for both \(\mu_n\) and \(\sigma_n\) (since, for a given \(M\), as \(N_E\) increases, \(\rho\) evidently increases too).

Assuming that \(P(k)\) is known, let us discuss now how \(P(C)\) may gain its heavy tails. Using \(C_i(T) = C_i(0)2^{k_i}\), assuming for simplicity that \(C_i(0) = 1\) \(\forall i\), that \(C\) and \(k\) are continuous variables, and dropping the index \(i\), one has

\[
P(C) = P[k = \log_2 C, \frac{dk}{dC} \propto \frac{1}{C} P(\log_2 C). \tag{3}
\]

Thus, for \(P(C)\) to have a heavy tail, i.e., to decrease more slowly than any exponential, \(P(k)\) must have a tail which decreases more slowly than \(e^{-\alpha k}\). This is the case e.g. both for \(P(k) \propto \exp(-\lambda k)\), which leads to \(P(C) \propto C^{-\alpha}\) and \(P(k) \sim N(\mu_k, \sigma_k^2)\) which leads to a log-normal \(P(C)\). Both distributions are difficult to distinguish, as it is well known [15]. Let us examine in the following a few specific cases.

B. The homogeneous case \(T = 1\)

Suppose that all agents have the same talent \(T = 1\). In this case \(k = n\), \(P(k) = P(n)\) is Gaussian and thus \(P(C)\) is a pure log-normal distribution. In other words, the heavy tails of \(P(C)\) are due to the combination of the stochastic nature of the number of lucky events and the multiplicative process which drives capital increases. While this case is relatively trivial, in the model \(P(n) \neq P(k)\) when \(T < 1\) for all agents.

C. The homogeneous case \(0 < T < 1\), with constant \(T\)

Let us assume now that all agents still have the same talent, but chosen in the interval \((0, 1)\), i.e. \(T': P(T) = \delta(T - T')\). It is worth to notice that
this special case resembles a situation often occurring in the real world. For example, it is realized when a few individuals with very high and very similar talent are drawn from a larger social group through any kind of selective test or competition (sportive, artistic, for a working place, etc...): in all such cases, the talent of everyone being almost identical, the final success is necessarily mostly a matter of luck.

Since \( n \) and thus \( k \) are proportional to \( M \), let us write \( k = \kappa M \) and \( n = \nu M \) and take the \( M \gg 1 \) case, which leads to, dropping the prime of \( T' \),

\[
P(\kappa) \approx \int_0^1 d\nu P(\kappa|\nu,T)P(\nu),
\]

where \( P(\kappa|\nu,T) \approx N[\nu T, \nu T(1 - T)/M](\kappa) \) and \( P(\nu) \approx N[\rho, \rho(1 - \rho)/M](\nu) \). Using the characteristic functions of these two distributions simplifies much the computations:

\[
P(\kappa) \approx \int_0^1 d\nu \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-is\kappa} e^{isT\nu - \frac{1}{2} \frac{\nu T(1 - T)}{M} s^2} \times e^{-it\nu} e^{it\rho - \frac{1}{2} \frac{(1 - \rho)^2}{M} \nu^2} \\
= \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-is\kappa} e^{isT\nu - \frac{1}{2} \frac{\nu T(1 - T)}{M} s^2} \times \int_0^1 d\nu e^{it\nu} e^{it\rho - \frac{1}{2} \frac{(1 - \rho)^2}{M} \nu^2} e^{-it\nu}.
\]

The integral on \( \nu \) can be readily performed as only linear terms in \( \nu \) appear in the exponential:

\[
\int_0^1 d\nu e^{[isT - it - \frac{1}{2} \frac{T(1 - T)}{M} s^2]} = \frac{1}{\lambda} (e^{\lambda} - e^{\kappa \lambda}),
\]

where \( \lambda = isT - \frac{1}{2} \frac{T(1 - T)}{M} s^2 - it \).

Setting

\[
f(t,\kappa) = -\frac{1}{2\pi i} e^{it(\rho - \kappa) - \frac{1}{2} \frac{\rho(1 - \rho)}{M} t^2} e^{-it\kappa},
\]

\( P(\kappa) \) can be shortened to

\[
P(\kappa) \approx \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-is\kappa} e^{isT\nu - \frac{1}{2} \frac{T(1 - T)}{M} s^2} \int_{-\infty}^{\infty} dt f(t,1) + \\
- \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-is\kappa} e^{is(T - \frac{1}{2} \frac{T(1 - T)}{M} s^2)} \int_{-\infty}^{\infty} dt f(t,\kappa).
\]

By Cauchy's theorem, as \( f(t,\kappa) \) has only one pole at \( t^*(s) = sT + \frac{1}{2} \frac{T(1 - T)}{M} s^2 \), thus

\[
\int_{\gamma} dt f(t,\kappa) = e^{it(\rho - \kappa) - \frac{1}{2} \frac{\rho(1 - \rho)}{M} t^2} \bigg|_{t=t^*(s)}.
\]

if \( \gamma \) encloses \( t^*(s) \) in an anticlockwise way. Let us assume that \( \gamma \) is the union of a line \( t \in [-a, a] \in \mathbb{R} \) and of the anti-clockwise semi-circle \( ae^{i\pi x}, x \in [0,1] \), denoted by \( \text{Arc}(a) \). For each \( \kappa, \gamma \) encloses \( t^*(s) \) if \( a > |t^*(s)| \). Because of the term \( e^{-\frac{t^*(\rho - \kappa)^2}{M}} \) in the integral, \( \lim_{a \to \infty} |f(t \in \text{Arc}(a))| = 0 \). Hence,

\[
\int_{-\infty}^{\infty} dt f(t,\kappa) = \int_{-\infty}^{\infty} dt f(t,\kappa) = e^{it(\rho - \kappa) - \frac{1}{2} \frac{\rho(1 - \rho)}{M} t^2} \bigg|_{t=t^*(s)}.
\]

which is an exponential of a fourth-degree polynomial. The right-hand-side of Eq. (3) is therefore a sum of two integrals of exponentials of fourth-degree polynomials. Let us write the argument of the exponential of the second line of Eq. (3) as \( ia_1 s + a_2 s^2 + ia_3 s^3 + a_4 s^4 \). A straightforward computation yields

\[
a_1 = T(\rho - \kappa) - \kappa T = T\rho - \kappa T - \kappa = T\rho - \kappa - \kappa T \\
a_2 = -\frac{1}{2} \frac{\rho T(1 - \rho T)}{M} \\
a_3 = -\frac{1}{2} \frac{\rho(1 - \rho) T^2(1 - T)}{M^2} \\
a_4 = \frac{1}{8} \frac{\rho(1 - \rho) T^2(1 - T)^2}{M^3}
\]

The polynomial in the exponential of the first line of Eq. (3) has the same \( a_3 \) and \( a_4 \), while its first to two coefficients are

\[
a_1' = T(\rho - 1) - \kappa + T = T\rho - \kappa \\
a_2' = -\frac{1}{2} \frac{\rho T(1 - \rho T)}{M}
\]

Thus, as expected, the distribution is a Gaussian plus corrections due to the third and fourth degree terms in the exponential. For large \( M \), these terms have a vanishing influence for small \( s \), i.e., for the tails of \( P(\kappa) \). Let us therefore neglect them:

\[
P(\kappa) \approx \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-is\kappa} e^{isT\rho - \frac{1}{2} \frac{T(1 - T)}{M} s^2} \int_{-\infty}^{\infty} \frac{dt}{dr} f(t,1) + \\
- \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-is\kappa} e^{is(T - \frac{1}{2} \frac{T(1 - T)}{M} s^2)} \int_{-\infty}^{\infty} \frac{dt}{dr} f(t,\kappa).
\]

\[
=N \left( \rho T, \frac{\rho T(1 - \rho T)}{M} \left( \frac{1}{1 + T} \frac{\rho T(1 - T\rho)}{M(1 + T)^2} \right) \right)(\kappa).
\]
hold, which gives heavier tails to \( P(n) \).

### D. Heterogeneous case

The complexity of the model lies in the heterogeneity of talent, which leads to non-trivial distributions. Let us therefore generalize Eq. (3) by averaging \( T \) over its distribution. The aim of the original Talent vs Luck one, is to show that a thinned distribution of \( T \) leads to heavy-tailed \( P(C) \) and accordingly uses a Gaussian distribution for \( T, \mathcal{N}(\mu_T, \sigma_T^2) \) with a small \( \sigma_T \). Analytical computations however, are much simpler for a uniform distribution. Let us take a simple uniform distribution of \( T \) over \([T_0 - \frac{a}{2}, T_0 + \frac{a}{2}]\):

\[
P(k) \simeq \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-ik\rho s} M \int_{T_0 - \frac{a}{2}}^{T_0 + \frac{a}{2}} dTe^{T\rho s - \frac{1}{2} \rho^2 T \frac{(1 - \rho T)}{M}} + \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-ik\rho s} M \int_{T_0 - \frac{a}{2}}^{T_0 + \frac{a}{2}} dTe^{T\rho s - \frac{1}{2} \rho^2 T \frac{(1 - \rho T)}{M}}.
\]

The relevant case is the small heterogeneity limit \( a \ll 1 \), in which case

\[
P(k) \simeq P(k|T = T_0) + a \int_{-\infty}^{\infty} ds e^{-iks} \left[ e^{(T_0 + \frac{a}{2}) \rho s - \frac{1}{2} \rho (T_0 + \frac{a}{2}) (1 - \rho (T_0 + \frac{a}{2}))} - e^{(T_0 - \frac{a}{2}) \rho s - \frac{1}{2} \rho (T_0 - \frac{a}{2}) (1 - \rho (T_0 - \frac{a}{2}))} \right] + a + \int_{-\infty}^{\infty} ds e^{-iks} \left[ e^{i(T_0 + \frac{a}{2}) \rho s - \frac{1}{2} \rho (T_0 + \frac{a}{2}) (1 - \rho (T_0 + \frac{a}{2}))} - e^{i(T_0 - \frac{a}{2}) \rho s - \frac{1}{2} \rho (T_0 - \frac{a}{2}) (1 - \rho (T_0 - \frac{a}{2}))} \right] + a N \left( \rho |T_0 + a/2|, \frac{\rho |T_0 + a/2| (1 - \rho |T_0 + a/2|)}{M} \right) (k) \]

\[
+ a N \left( \rho |T_0 - a/2|, \frac{\rho |T_0 - a/2| (1 - \rho |T_0 - a/2|)}{M} \right) (k) \]

\[
- a \left( \frac{\rho |T_0 + a/2| (1 - \rho |T_0 + a/2|)}{M} \right) (k) \]

\[
- a \left( \frac{\rho |T_0 - a/2| (1 - \rho |T_0 - a/2|)}{M} \right) (k) \]

\[
+ a \left( \frac{\rho |T_0 + a/2| (1 - \rho |T_0 + a/2|)}{M} \right) (k) \]

\[
+ a \left( \frac{\rho |T_0 - a/2| (1 - \rho |T_0 - a/2|)}{M} \right) (k).
\]

Figure 7 shows that: i) the approximation is too coarse, but better than the homogeneous case; ii) the effect of heterogeneity of talent is to make \( P(k) \) wider; iii) locally, the superposition (mixture) of
Gaussian distributions may approximate an exponential over a given range of \( \kappa \) (which must be multiplied by \( M \)), which may lead to a power-law part of \( P(T) \).

Alternatively, by reverting the point of view, is it possible to find the distribution of \( P(T) \) that yields an approximately exponential distribution of \( \kappa \)? The answer is simple: assuming that one wishes to obtain \( P(\kappa) = \lambda e^{-\lambda \kappa} \) for \( \kappa \in [0, 1] \), one sets \( \lambda e^{-\lambda \kappa} = \int_0^1 dT P(\kappa|T)P(T) \) and, since Eq. (3) shows that \( P(\kappa|T) \) is approximatively a sum of two Gaussian distributions which become very peaked and tend to a Dirac function for large \( M \), one has

\[
P(\kappa) = \lambda e^{-\lambda \kappa} \approx \int_0^1 dT P(T)N \left( \rho T, \frac{\rho T(1 - \rho T)}{M} \right)(\kappa)
\]

\[
- \frac{1}{1+T} N \left( \rho \frac{T}{1+T}, \frac{\rho T(1 - T \rho)}{M(1+T)^2} \right)(\kappa)
\]

\[
\simeq \int_0^1 dT P(T) \left[ \delta(\rho T - \kappa) - \frac{1}{1+T} \delta \left( \rho \frac{T}{1+T} - \kappa \right) \right].
\]

The first Dirac selects \( T = \kappa/\rho \) and the second one \( T = \kappa/(\rho - \kappa) \), thus

\[
P(\kappa) \approx \frac{1}{\rho} P(T = \kappa/\rho) - \frac{1}{|\rho - \kappa|} P[T = \kappa/(\rho - \kappa)].
\]

If \( P(T) \propto e^{-T/\rho} \), the second term becomes negligible when \( \rho - \kappa \ll 1 \), i.e., for the large \( \kappa \) region from which the tails of \( P(C) \) originate. Thus the leading contribution to \( P(\kappa) \) and \( P(C) \) comes from the first term of the right hand side of the above equation, which leads to roughly exponentially-distributed \( \kappa \) and, mechanically, to an approximately power-law distributed \( C \).

IV. CONCLUSIONS

The main reason for the emergence of heavy-tails in the simplified Talent vs Luck model is the coupling of a multiplicative process for the capital dynamics and a stochastic occurrence of lucky events. However, the emergence of non-trivial distributions of capital comes from the stochastic nature of transformation of lucky events into a capital-increasing events: when talent is distributed homogeneously among the agent population, the capital distribution is log-normal only when talent equals 1 for all the agents, i.e., when the transformation of lucky events is not stochastic. Quite remarkably, a small heterogeneity in talent makes the final distribution of capital much more complex and very similar to a power-law.

On one hand, extending this result to the original Talent vs Luck model should be relatively simple, as the latter adds the occurrence of negative events, whose effect would be to reduce the effective number of lucky events occurring to each individual. On the other hand, the complexity of the analytical approach increases even further in the most general case, with greater talent heterogeneity, thus making the task of finding a formal analytical relationship between the distributions of capital, talent and luck in either the TvL or the STvL models a really hard problem.

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