The vibroacoustic analogies method

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Abstract. The article deals with the main problem of computational dynamic modeling of building structures, which is associated with the computational modeling of dynamic parameters of structural elements. The methods and schemes of construction of dual analogies between such dynamic properties of designs elements as inertia, rigidity and pliability to dynamic loadings are presented. The Vibroacoustic analogies for Ohm's law, Kirchhoff's laws, Hooke and Newton and the mathematical analogies between acoustic and electrical impedances for the elements of building structures are considered. The patterns of Vibroacoustic analogies for the resonance properties simulation of structural elements are examined. It is shown that the elements of the design have a multi-mode acoustic impedance. It is shown that correct modal analysis of structural elements dynamic reaction is a necessary condition for the correct simulation dynamic reaction schemes of buildings and structures to the effects of dynamic loads as complex mechanical systems. The article shows the convenience of using vibroacoustic analogies both in calculations of dynamic load distribution on building structures elements and in dynamic monitoring data interpretation of buildings and structures technical condition. The dynamic models of design are analyzed taking into account their vibroacoustic interaction in the acoustically coupled system of structural elements. The article presents equivalent schemes and systems of equations calculation modeling of the distribution of dynamic stress and strain in the building structure space. As an example, the computational modeling principles of dynamically indeterminate mechanical systems of buildings and structures is considered.

1. Introduction

Currently, many software systems for design modeling of building structures provide possibility to calculate the buildings and structures dynamic characteristics [1,2].

However, subsequent full-scale studies of the buildings and structures dynamic characteristics show that computational modeling errors due to limited mathematical capabilities of the methods and software systems significantly exceed 10% [3-7].

When forming a package of initial data for dynamic modeling in these complexes, the vibroacoustic characteristics of structural elements and parameters of acoustic connections between them are not taken into account.

It is a good reason to believe that dynamic modeling in such complexes has limited application and is only suitable for quasi-static mathematical description of constructions.

These limitations of software systems (for example, Nastran, Abaqus, ANSYS, etc.) were confirmed at a number of specialists meetings from NASA, ESA and RSA. As a result, the Agencies focused on experimental methods and equipment for studying dynamic loads (microgravity) on Board.
the Space Station «Mir» and International Space Station [8]. In parallel, the development of algorithms for calculating vibration isolation methods for onboard technological equipment was carried out. The most promising of these methods was the vibroacoustic analogies method by the Honorary doctor M.S. Hlystunov.

In building design modeling complexes, this problem is complicated by the fact that vibroacoustic impedances of structural elements and the vibroacoustic connections between them are not certified by the suppliers of structural elements.

Further we will be considered the basic principles of construction and application of the Vibroacoustic analogies method by Hlystunov [9] more detail.

2. Dual spectral Vibroacoustical and Electromechanical analogies

For dynamic modeling of load distribution on elements of construction we will use the spectral vibroacoustical and electromechanical analogies method (developed by Prof. Hlystunov) which is based on the mathematical similarity (equivalence) characteristics, equations, formulas and laws of mechanics, acoustic and electrical engineering.

For the calculated dynamic modeling in the analogies system using the mathematical similarity (1) and (2), it is advisable to present them in the form of Fourier images:

\[
\begin{align*}
\text{Fourier-image of electrical engineering formulas} & \quad \text{Direct vibroacoustic and mechanical analogues} & \quad \text{Inverse vibroacoustic and mechanical analogues} \\
U(j\omega) & \quad P & F(j\omega) & \quad P & v = j\omega w(j\omega) \\
I(j\omega) & \quad P & v = j\omega w(j\omega) & \quad P & F(j\omega) \\
Z_L = j\omega L & \quad P & j\omega m & \quad P & j\omega K(j\omega) \\
Z_C = \frac{1}{j\omega C} & \quad P & \frac{1}{j\omega m} & \quad P & \frac{1}{\beta} \\
r & \quad P & \beta & \quad P & \beta \\
U(j\omega) = I(j\omega)Z(j\omega) & \quad P & F(j\omega) = u(j\omega)Z(j\omega) & \quad P & u(j\omega) = F(j\omega)Z(j\omega)
\end{align*}
\]

and

\[
\begin{align*}
\text{Fourier-image of electrical engineering formulas} & \quad \text{Direct vibroacoustic and mechanical analogues} & \quad \text{Inverse vibroacoustic and mechanical analogues} \\
U(j\omega) = \frac{q(j\omega)}{C} & \quad P & F(j\omega) = K(j\omega)w(j\omega) & \quad P & v(j\omega) = F(j\omega)j\omega m \\
I(j\omega) = j\omega q(j\omega) & \quad P & u(j\omega) = j\omega w(j\omega) & \quad P & F(j\omega) \\
U(j\omega) = I(j\omega)r & \quad P & F(j\omega) = \beta v(j\omega) & \quad P & v(j\omega) = F(j\omega)\beta \\
U(j\omega) = j\omega LI(j\omega) & \quad P & F(j\omega) = j\omega m u(j\omega) & \quad P & u(j\omega) = j\omega F(j\omega)K(j\omega)
\end{align*}
\]

where \( Z_C, Z_L \) - the vibroacoustic impedances of capacitor and inductor; \( F, w, v, a, K, m, \beta \) - force, absolute deformation (displacement), speed and acceleration of deformation (displacement), stiffness, mass and damping factor of the structural element dynamic reaction; \( U, q, I, C, L, r \) - electrical voltage, electrical charge, electrical current, electrical capacitance and inductance, electrical resistance equivalent to vibroacoustic losses in the design element.
Using systems of electromechanical analogies (1) and (2) in the vicinity of the main resonance and antiresonance, the original element of mechanical design in Figure 1a can be represented by electrotechnical analogue (direct Figure 1b or inverse Figure 1c) in the form of a double resonant circuit.

3. Similarity of the structural element resonant properties and its electrical model

Mechanical model in Figure 1a reflects the dynamic properties of the structural element only for one \( ij \)-th mode on \((2n+1)\) the harmonic of its vibroacoustic excitation (loading) and only in the vicinity of one of the resonances and the associated antiresonance corresponding to this harmonic.

The scheme of a similar electric model can be represented as an electrical circuit with two resonances in Figure 1b. In this electric scheme there is one resonant circuit, which consists of a capacitor \( C_1 \), connected in series with the inductance \( L_1 \) and the active resistance \( r_1 \). Another resonant circuit (antiresonance circuit) of this electric scheme consists of a capacitor \( C_2 \) connected in parallel with inductance \( L_1 \) through a capacitor \( C_1 \) and resistance \( r_1 \).

In Figure 1 it is shown the method of indexing the resonance and antiresonance of the structural element. The main vertical longitudinal mode of excitation has the index which corresponds to \( n=0 \) then \((2n+1)=1\). The main antiresonance mode of excitation has the index which corresponds to \( n=1 \) then \((2n)=2\).

Using this method for indexing the high harmonic of resonances and antiresonances, the complete equivalent electrical model of the structural element, taking into account all the harmonics of the considered (one) excitation mode, we will theoretically have an infinite number of double resonant electric circuits, as shown in Figure 2 (direct electromechanical analogies).

However, in real structural elements there is a large increase in attenuation with increasing frequency. In this regard, only the first resonances of the scheme in Figure 2 will be excited by the dynamic load, and the rest degenerate.

Figure 1. The vibroacoustic model (a) and an equivalent (b,c) electrical models of the structural element in the vicinity of \((2n+1)\) resonance and \(2n\) antiresonance \((n=0,1,2,3,\ldots)\) in the system of the direct and inverse Electromechanical analogies (b,c)
Figure 2. Complete equivalent electric model of the structural element, taking into account all of the harmonics of one mode of excitation in the direct electromechanical analogies system.

4. The calculation Algorithm of the vibroacoustic Impedance
Let us consider the method of application of electromechanical analogies on the example of the structure carcass, which consists of 8 columns (Figure 3).

Figure 3. The example of the structure carcass.

Their foundations are isolated from each other.
The columns are rigidly attached to the overlap panel. The dynamic load is applied to column foundations, e.g. in the form of a vertical vibration from vehicle or explosive shot or seismic shock. The equivalent vibroacoustic model of such carcass can be represented by the scheme in Figure 4a.
Figure 4. Vibroacoustic model of the dynamic loading of the carcass (a) and simplified model (b) in accordance with expressions (3) and (4).

Where:
- $F_{ij} = F_{ij} + jF_{ij}^*$ are Fourier images of force loads on the $ij$-th structural element in the form of a complex variable with active (real) $F_{ij}$ and reactive (imaginary) $F_{ij}^*$ components;
- $Z_{ij}^v(i = j)$ is vibro-acoustic impedance of attachment of the slab to the $i$-th column;
- $Z_{ij}^v(i \neq j)$ is the impedance of the vibroacoustic connection along panel between the $i$-th and $j$-th columns;
- $X_{ij}^v(i = j)$ is vibroacoustic impedance of the $i$-th column;
- $Y_{ij}^v(i = j)$ is vibro-acoustic impedance of the attachment of the $i$-th column to the Foundation;
- $V_{ij}^v(i = j)$ is the impedance of the vibroacoustic connection of the Fundament of the $i$-th column with the ground base;
- $V_{ij}^v(i \neq j)$ is the vibroacoustic impedance of connection along the ground between Fundaments of the $i$-th and $j$-th columns;
- $F_i$ is an external vertical dynamic force load entering on the Fundament of the $i$-th column.

We will simplify the scheme of the vibroacoustic model of the carcass structure, without losing the essence of the method, as follows:

1. The total acoustic impedance $P_i$ of the $i$-th column is presented in the form of amount its own impedance and the impedance $X_{ij}$ its nodes pair with overlap $Z_{ij}$ and with Fundament $Y_{ij}$ and also its Fundament with the soil $V_{ij}$:

$$P_i = Z_{ii} + X_{ii} + Y_{ii} + V_{ii} = R_{ii} + jQ_{ii},$$

where $R_{ii}$ and $jQ_{ii}$ respectively active (real) and reactive component of the total vibroacoustic impedance of the column and its nodes with other adjacent elements of the design.

2. We take the option with an ideal vibroacoustic isolation of the columns fundaments with each other. In this case, the impedance of this connection can be taken equal to infinity, that is

$$V_{ij} = \infty.$$

3. We consider the case when the external dynamic force load acts through the soil only on the base of column $N4$, that is only $F_4 \neq 0$.

Now the scheme in Figure 4a can be simplified as the view shown in Figure 4b.
Let us use inverse electromechanical analogies. This is due to the fact that the use of inverse analogies allows for no additional physical and mathematical manipulation. As a rule, such manipulations are necessary for calculations of dynamically undefined type of object structure.

The dynamic load $F_{ij}$ will correspond to the alternating current $I_{ij}$ in accordance with the inverse electromechanical analogies.

It will create on the $ij$-th structural element a strain rate (displacement rate) $\upsilon_{ij}$ equivalent to the electric voltage $U_{ij}$ in accordance with the Ohm's law:

$$F_{ij} = \frac{\upsilon_{ij}}{(R_{ij} + jQ_{ij})}. \quad (5)$$

On the other hand, according to Kirchhoff's law, the sum of currents in the nodes of the electric circuit must be equal to zero. The sum of the currents entering in the node must be strictly equal to the sum of the currents leaving it.

Then for each connection node in the figure 4b should be performed as follows:

$$F_{i} = F_{i(j-1)} + F_{i(j+1)} \quad (6)$$

The ultimate goal of this dynamic task is to determine the values of dynamic force loads and strain rates (displacements) on the elements of the carcass. Then, with the known vibroacoustic impedance of these elements, the number of unknowns in this task will be 32 (16 unknown dynamic force loads and 16 unknown dynamic deformation velocities). To do this, use the expressions (5, 6) and the diagram in Figure 3b, we obtain the following system of equations:

$$\begin{cases}
F_{i} = F_{i(j+1)} - F_{i(j-1)} & \forall (i \neq j, j \neq 4) \quad \text{in accordance with Kirchhoff's law} \\
F_{i(i+1)} = F_{i(i+1)} & \text{the condition of the external load action} \\
\upsilon_{ij} = U_{ij} (R_{ij} + jQ_{ij}) & \text{in accordance with the Ohm's law} \\
F_{i} = F_{i1} + F_{i2} + F_{i3} + F_{i5} + F_{i6} + F_{i7} + F_{i8} & \text{forces balance condition} \\
\upsilon_{ij} = \upsilon_{i(j+1)} + \upsilon_{i(j+2)} & \text{the continuity condition of the connections at the carcass nodes} 
\end{cases} \quad (7)$$

Such a system (7) has a solution because it contains more than 32 equations.

A similar approach for solving the dynamic task under consideration can be implemented not only for the simplified model, but also for the real case when, for example, microseismic, shock or vibration load are onload simultaneously to the foundations of all columns.

In this most real case, it is necessary to set or measure all the external dynamic loads $F_i \neq 0 \forall (i=1,2,3,\ldots,8)$ acted on the construction columns foundations. The assessment of force loads on the structure elements is carried out in two stages.

At the first step we calculate the distribution of forces on the elements of the structure separately for each external load. The load on the rest foundations of the carcass is conventionally assumed to be zero.

At the second stage of calculations we summarize for each element of the structure all the loads obtained at the first stage.

5. Conclusion
The possibilities of the method of vibroacoustic and electromechanical analogies for solving the problem of computational dynamic modeling of building structures showed that the correctness of its solution is associated with the correct computational modeling of dynamic parameters of structural elements.

It is shown that the spectral vibroacoustic analogies adequately reflect the mathematical equivalence of the basic equations, laws, formulas and physical quantities of mechanics, vibroacoustic and electrical engineering.
The method mentioned above as well as the equivalent circuits make it possible to effectively applying vibroacoustic analogies in the calculations of dynamic properties of structural elements (inertia, stiffness and compliance to dynamic loads).

High calculations efficiency is provided by the use of mathematical similarity of Ohm’s, Kirchhoff’s, Hooke’s and Newton’s laws, as well as the presence of similarity between electrical and vibroacoustic impedances.

The correctness of analogies for vibroacoustic and electrical simulation of the structural elements resonance properties was confirmed by physical modeling. Also it is necessary to consider that the design elements have multiple-mode acoustic impedance for the vibroacoustic loads.

It is shown that correct modal analysis of dynamic reaction of structural elements is a necessary condition to develop correct schemes of dynamic reaction of complex mechanical systems of buildings and structures. The calculation modeling principles of dynamically indeterminate mechanical structures considered in the article confirm the importance of the choice of conformal type (direct or inverse) analogies by Hlystunov.

The article shows for the first time that ensuring the adequacy of dynamic models of structures it is important to consider vibroacoustic interaction in acoustic connections (in connection nodes) in the system of structural elements of buildings and structures.

Along with this, it is confirmed that the design schemes using impedances of the structure elements can be effectively used in the interpretation of dynamic monitoring data of the buildings and structures technical condition [10,11].

The equivalent schemes and systems of the equations for settlement modeling of explosive dynamic loadings and deformations distribution in a building structure space presented in the article can be used for modernization of widespread program complexes of the automated design of buildings and constructions and other objects of a technosphere.

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