Criteria of existence for bounce solutions in false vacuum decay with gravity

Nicholas W K Wong, Jiangbin Gong, Yen-Kheng Lim and Qing-hai Wang

1 Department of Physics, River Valley High School, 649961, Singapore
2 Ministry of Education, 138675, Singapore
3 Department of Physics, National University of Singapore, 117551, Singapore

E-mail: whye_khuin_nicholas_wong@moe.edu.sg, phygj@nus.edu.sg, phylyk@nus.edu.sg and qhwang@nus.edu.sg

Received 23 October 2017, revised 18 December 2017
Accepted for publication 3 January 2018
Published 25 January 2018

Abstract
The bounce solutions of self-interacting scalar fields coupled to gravity are studied using a semi-classical approach. We found that bounce solutions have a maximum required barrier curvature, in addition to the known minimum required barrier curvature. In particular, as the maximum barrier curvature is approached, the scale factor of the well-known Coleman–De Luccia (CDL) bounce solutions become divergent. Unlike the CDL or its more general oscillating bounce counterparts, this cannot be considered as a subset of the Hawking–Turok solution.

Keywords: inflation, instantons, false vacuum decay, early universe

(Some figures may appear in colour only in the online journal)

1. Introduction

The study of false vacuum decays in the presence of gravity [1] plays an important role in describing quantum effects in the early universe. In these models, the false vacuum is an initial meta-stable state with a constant scalar field. The scalar potential at this state corresponds to an effective cosmological constant. Transitions between the two vacua may describe the phase transition between any combination of de Sitter and anti-de Sitter geometries. The de Sitter case is of particular interest since it provides a useful model for the inflationary epoch in the early universe [2–4].

The semi-classical description of false vacuum decays in flat space has been well understood for some time [5–8]. A generalisation of the problem to include gravity was considered by Coleman and De Luccia (CDL) [1]. In both flat- and curved-space descriptions, the decay rate is calculated from the Euclidean action evaluated under the classical path which
satisfies the Euler–Lagrange equations. It follows that a full understanding of the vacuum decay requires knowledge of the classical solutions to the field equations.

To model the decay of the field, the potential function for the field \( \phi \) is required to have two minima separated by a barrier. Thus a natural choice that captures this essential features is a potential function quartic in \( \phi \), which depicts an asymmetric double well. The classical bounce\(^4\) solution under such a potential was found by CDL. It describes \( \phi \) being initially at the false vacuum, then crossing the barrier to reach the true vacuum. The original solution presented by CDL was obtained under the thin wall approximation, where the energies at the false and true vacua are infinitesimally close to each other. Since then, numerical solutions without the thin wall approximation have also been obtained [11–16]. Subsequent studies were built upon these results, with notable examples of [17–19], all of which found new classes of solutions, with the latter two only appearing in the presence of gravity. More recently, Dong and Harlow [20] presented exact solutions for the CDL bounce in higher dimensions, though such solutions require potentials that do not have simple closed-form expressions. In four dimensions, it is possible to obtain exact bounce solutions if the potential contains exponential terms [21]. The particular case where the scalar potential is symmetrical (i.e. with degenerate vacua) was considered in [22] and provides a possible braneworld interpretation. In [23], it was found that the presence of inhomogeneities enhances the tunnelling rate.

A class of solutions corresponding to multiple bounces was considered by Hackworth and Weinberg [19], where the field crosses the potential barrier multiple times before settling at either the false or true vacuum as its final state. A study of fluctuations around these higher-bounce solutions shows that they contain more than one negative mode [24] and thus thermal transitions dominate over those attributed to quantum tunnelling. Therefore, these higher-bounce solutions are more akin to the Hawking–Moss solutions [17]. The nature of the bounce solutions also depends on the size of the bubbles. It turns out that small bubbles behave similarly to its flat spacetime counterparts, and that large bubbles have no flat space analogues [25].

Using a semi-classical approach with numerical integration, we verified claims by [19, 24] that oscillating bounce solutions with more oscillations appear as the barrier curvature increases. We also verified claims by [24] that these new oscillating bounce solutions have more than one negative mode, which corresponds to thermalisation. On the other hand, Coleman–De Luccia (CDL) bounce solutions, also referred to as 1-bounce solutions, have only one negative mode and thus corresponds to tunnelling [24].

While doing so, we obtained a class of tunnelling bounce solutions with a non-vanishing scale factor, suggesting that the boundary conditions imposed by [1] and subsequent work may be relaxed. Unlike the CDL solutions or the more general oscillating bounce solutions, these cannot be considered a subset of Hawking–Turok solutions due to the non-vanishing scale factor. This suggests that bounce solutions in general are not a subset of Hawking–Turok solutions. In addition, like the CDL bounce solutions, these solutions have only one negative mode, which suggest that it is possible to have quantum tunnelling effects without the scale factor vanishing for finite Euclidean time. This solution was only found for initial conditions beginning near the true vacuum, and not for initial conditions beginning near the false vacuum.

In addition, our results suggest that the bounce solutions found by [19, 24] may vanish as barrier curvature increases, starting from solutions with the smallest number of oscillations. However there is one particular exception: for solutions corresponding to transitions from true

\(^4\) Here, the term ‘bounce’ refers to the oscillation of the scalar field, as opposed to a similar terminology used in the context of ‘Big Bounce’ cosmology which refers to time-evolution of the scale factor in Friedmann–Robertson–Walker cosmology, for instance studies in [9, 10].
to false vacua, the 1-bounce and 2-bounce solution vanish at the same value of barrier curvature. The 1-bounce solution has a further property where as the barrier curvature increases, it turns into a new class of bounce solution (which we will describe in section 4) before vanishing. Instead, it appears to vanish at the value of barrier curvature for which the new class of bounce solution begins, suggesting that the mechanism responsible for the vanishing solutions may be related to the new class of bounce solution.

This paper is organised as follows: In section 2 we review the Euclidean action and derive the classical equations of motion for the theory. In the same section we will also provide a description of the potential function used in this paper. Subsequently, we give a brief review of the known solutions in section 3. In section 4 we present a bounce solution that satisfies a different set of boundary conditions from the earlier known solutions. In section 5, we provide numerical evidence that the existence of oscillating bounce solutions require that the barrier curvature lie within a certain range of values. This paper concludes with some closing remarks in section 6.

2. Action and equations of motion

In the semi-classical limit, the decay rate per volume of the meta-stable state is given by [1, 7]
\[ \Gamma \propto e^{-S_E}, \]
where \( S_E \) is the Euclidean action of a four-dimensional self-interacting scalar field minimally coupled to gravity,
\[ S_E = \int d^4x\sqrt{-g} \left( \frac{1}{2} \left( \nabla \phi \right)^2 + V(\phi) - \frac{1}{2\kappa} \mathcal{R} \right), \]
where \( \kappa = 8\pi/M_{Pl}^2 \) gives the coupling to gravity via the Ricci scalar \( \mathcal{R} \). As mentioned in section 1, it is of interest to study the classical solutions to this action, which correspond to \( \delta S_E = 0 \). Varying the action with respect to the scalar field and metric leads to the Einstein-scalar equations
\[ R_{\mu\nu} = \kappa \left( \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} V(\phi) \right), \]
\[ \nabla^2 \phi = \frac{dV}{d\phi}. \]
Coleman et al has shown that bounce solutions are always \( O(4) \)-symmetric [26]. Therefore we take our geometry to be described by the metric
\[ ds^2 = d\sigma^2 + a(\sigma)^2 d\Omega^2_{(3)}, \]
where \( d\Omega^2_{(3)} \) is the metric of a round \( S^3 \). With this \( O(4) \) symmetry reflected in the geometry, \( \phi \) can be taken to depend only on ‘Euclidean time’ \( \sigma \), then the equations of motion (3) reduce to
\[ \ddot{\phi} = -\frac{3}{a} \dot{a} \phi + \frac{dV}{d\phi}, \]
\[ \ddot{a} = -\frac{1}{3} \kappa a \left( \dot{\phi}^2 + V \right), \]
along with a constraint
Here, over-dots denote derivatives with respect to $\sigma$.

In the following, we consider a scalar potential of the form

$$V(\phi) = V_{\text{top}} + \beta H_{\text{top}}^2 \nu^2 \left( -\frac{1}{2} \frac{\phi}{\nu} \right)^2 - \frac{b}{3} \left( \frac{\phi}{\nu} \right)^3 + \frac{1}{4} \left( \frac{\phi}{\nu} \right)^4.$$

This potential has two local minima, namely the false vacuum $\phi = \phi_{fv}$ and the true vacuum $\phi = \phi_{tv}$, where $V(\phi_{tv}) < V(\phi_{fv})$. The two minima are separated by a barrier at $\phi = 0$, as shown in figure 1. The parameter $b$ sets the spacing between $\phi_{fv}$ and $\phi_{tv}$, with $\phi_{fv} < 0 < \phi_{tv}$, while $\nu$ sets the scale for the scalar field, which we may parametrise more conveniently as $\epsilon = \sqrt{\kappa \nu^2/3}$. The parameter $\beta$ is the ‘barrier curvature’, as it is related to the second derivative of the potential at $\phi = 0$ by

$$\frac{d^2V}{d\phi^2}\bigg|_{\phi=0} = -H_{\text{top}}^2 \beta.$$

For later convenience, we further define

$$H(\phi) = \sqrt{\frac{\kappa}{3} |V(\phi)|}.$$

We will also find it convenient to introduce shorthands for the following quantities:

$$H_{fv} = \sqrt{\frac{\kappa}{3} |V(\phi_{fv})|}, \quad H_{tv} = \sqrt{\frac{\kappa}{3} |V(\phi_{tv})|}, \quad H_{\text{top}} = \sqrt{\frac{\kappa}{3} |V_{\text{top}}|}.$$

Throughout most of this paper, we will be concerned with finding numerical solutions to equation (5) under the potential (7). To do this we implement the standard shooting method where we begin the integration at $\sigma = 0$ with the relevant initial conditions which we will elaborate below. Our shooting parameter will be $\phi(0)$, which we adjust until we find a solution which satisfies the appropriate end-point boundary conditions. The constraint (6) is used to set the scale of $a(\sigma)$. 

Figure 1. Plots of $V(\phi)$ for several $\beta$ with $\epsilon = 0.23$ and $b = 0.25$. With these parameters, $\beta = 104.74$ corresponds to $V(\phi_{fv}) \simeq 0$. 

$$\dot{a}^2 + \frac{1}{3} \kappa a^2 \left( V - \frac{1}{2} \phi^2 \right) = 1.$$
3. Review of known solutions

In this section, we review the various solutions found in earlier literature. Aside from the trivial solutions, these can be categorised by the boundary conditions, namely the behaviour of $\phi$ and $a$ at the boundaries of the domain $0 \leq \sigma \leq \sigma_{\text{max}}$ for some positive $\sigma_{\text{max}}$.

3.1. Trivial solutions

The simplest solutions to equations (5a) and (5b) are the trivial solutions with $\phi$ being constant at 0, $\phi_{\text{tv}}$, or $\phi_{\text{fv}}$. The particular case $\phi = 0$ is known in the literature as the Hawking–Moss solution [17]. With a constant $\phi$, equation (5a) is trivially satisfied and equation (5b) is solved by

$$a(\sigma) = \begin{cases} \frac{1}{M(\phi)} \sin [H(\phi)\sigma], & V(\phi) > 0, \\ \frac{1}{M(\phi)} \sinh [H(\phi)\sigma], & V(\phi) < 0, \\ \sigma, & V(\phi) = 0. \end{cases} \tag{11}$$

**Bounce solutions:** These are solutions defined by the boundary conditions

$$\dot{\phi}(0) = 0, \quad \dot{\phi}(\sigma_{\text{max}}) = 0,$$

$$a(0) = 0, \quad a(\sigma_{\text{max}}) = 0, \tag{12}$$

where the last condition is taken as the defining equation for $\sigma_{\text{max}}$, particularly the point where $a(\sigma)$ returns to zero at some finite $\sigma$. The typical scenario of interest is where $\phi$ starts somewhere close to $\phi_{\text{tv}}$ at $\sigma = 0$, and ends near $\phi_{\text{fv}}$ at $\sigma = \sigma_{\text{max}}$. Over this interval of $\sigma$, $\phi$ may oscillate, or bounce, multiple times across the barrier at $\phi = 0$. A bounce solution can then be characterised by its number of bounces, $n$, where $n$ as the number of times the field crosses the $\phi = 0$ barrier. In particular, the single-bounce solution depicted in figure 2 was found by Coleman and De Luccia [1] (CDL) and we shall refer to it as the ‘CDL bounce solution’, as we have alluded to in section 1. Solutions with more bounces are those considered in [19, 24]. An example of $n = 13$ is shown in figure 3. We shall denote these $n \geq 2$ solutions as ‘oscillating bounce solutions’.

5 Or equivalently, $n$ is the number of roots of the equation $\phi(\sigma) = 0$.

Figure 2. The CDL, or 1-bounce solution with $\beta = 45$, $b = 0.25$, and $\epsilon = 0.23$. The horizontal axis is plotted in units of $H_{\text{tv}}\sigma$, where $H_{\text{tv}}$ is defined in equation (10).
The solutions considered by Hawking and Turok [17] are those with the boundary conditions
\[
\dot{\phi}(0) = 0, \\
\dot{a}(0) = 0, \\
a(\sigma_{\text{max}}) = 0.
\] (13)

Between the discrete bounce solutions, there exist a continuous range of solutions where \(\phi(\sigma_{\text{max}}) \to \pm \infty\) instead of ending at some finite \(\phi(\sigma_{\text{max}})\) with \(\dot{\phi}(\sigma_{\text{max}}) = 0\). We shall also call these \(\phi\)-diverging solutions, in part to distinguish this class from a separate one we consider in the next section where \(a\) diverges instead of \(\phi\). It was suggested by [18] that these are valid classical solutions, but are ignored because they diverge and are non-discrete. We also observe that equation (13) is simply equation (12) with the condition \(\dot{\phi}(\sigma_{\text{max}}) = 0\) relaxed. Hence we may regard the bounce solution as a subset of the Hawking–Turok solutions.

4. Bounce solutions with diverging scale factor

In addition to the three classes of solutions outlined in the previous section, we report in this paper another class of solutions that, to our knowledge, has yet to be studied in detail. The boundary conditions are
\[
\dot{\phi}(0) = 0, \\
\dot{a}(0) = 0, \\
a(\sigma_{\text{max}}) = 0.
\] (14)

Similar to the Hawking–Turok solutions, these conditions are simply equation (12) with one condition relaxed. In the present case, the condition we relax is the requirement that \(a(\sigma)\) approaches zero within finite \(\sigma\). Nevertheless, our solution here is distinct from the Hawking–Turok in the sense that \(\phi\) is always finite, whereas \(\phi\) diverges in the Hawking–Turok case. Unlike all the known solutions outlined in the previous sections, the range of \(\sigma\) for the present class is semi-infinite, \(\sigma \in [0, \infty)\). Due to the boundary conditions on \(\phi\), the solutions of \(\phi\) do not diverge. Therefore it is similar to the bounce solutions outlined in section 3, except that \(a\)

\[\text{As equation (5) are non-linear equations, the number of boundary conditions is not always the same for special solutions. For our present purposes, it is only crucial that the initial conditions are fixed, and the set of solutions should be discretised. A continuous family of solutions are not helpful in calculating path integrals.}\]
diverges as $\sigma \to \infty$. Therefore, we shall henceforth refer to this class as the $a$-diverging solutions. For these solutions to occur, the potential is required to satisfy

$$V(\phi_{tv}) < 0 \quad \text{and} \quad V(\phi_{fv}) < 0.$$  

(15)

The boundary conditions (14) are distinct from those reviewed in section 3, thus one might regard this as a new class of solutions. However if viewed in terms of transitions between two AdS vacua (as is clear from equation (15)), these solutions are implicit in the constructions considered in, among others, [1, 27, 28], most of which were calculated under thin-wall approximations. More recently, numerical solutions in the case of Minkowski to AdS vacuum has been studied by Masoumi et al in [29].

An example of this $a$-diverging class is shown in figure 4, where we can see that it consists of two distinct regions of $\dot{\phi} \simeq 0$ and negative $V(\phi)$ connected by a very brief transition. In fact, it spends most of its time near the vacua such that $\phi$ and $a$ behave like their respective trivial solutions. As a result, the scale factor $a$ accordingly has two sinh-like regions, each of which well-approximated by the negative $V(\phi)$ case in (11). This is shown explicitly in figure 4 where we have also plotted $a_{fv}$ and $a_{tv}$, the trivial solutions given by

$$a_{fv} = \frac{1}{H_{fv}} \sinh (H_{fv} \sigma + 1.37), \quad a_{tv} = \frac{1}{H_{tv}} \sinh (H_{tv} \sigma),$$  

(16)

and we have exploited the invariance of the equations of motion under $\sigma \to \sigma + \text{constant}$ to see that the growth rate coincides with the new solution after the transition.

When $\phi$ is initially near the true vacuum, the scale factor increases exponentially at a rate which is approximately $H \simeq H_{tv}$. When $\phi$ bounces over to near the false vacuum, $a$ is still exponentially increasing, but at a different rate $H \simeq H_{fv}$. At present, the $a$-diverging class of solution with the above-mentioned behaviour of $a$ has only been found for single-bounces of $\phi$. We believe that sinh-like scale factors for oscillating bounce solutions are not found because the transition region is too large.

We regard this class also as bounce solution since the scalar field continues to behave in a similar manner as the CDL bounce solutions. However the domain of this solution is $\sigma \in [0, \infty)$, instead of ending at a finite $\sigma$ as in the CDL case. Clearly, this cannot be considered

\footnote{We thank our anonymous referee for pointing this out.}
a subset of the Hawking–Turok solutions, since, as mentioned above, \( a \) diverges and thus do not obey Hawking–Turok boundary conditions in equation (13).

Additionally, we have studied the quantum fluctuation about these classical solutions using a method similar to [24]. The \( a \)-diverging solutions only have one negative mode, which is identical to the results obtained for the CDL solutions, suggesting that this class of solutions also correspond to quantum tunnelling effects [24]. This also suggests that the solutions are of a similar vein of bounce solutions with the currently known CDL and oscillating bounce solutions. It further reinforces the fact that bounce solutions, in general, are not subsets of a general Hawking–Turok solution, but rather the choice of which three boundary condition should be used is not restricted as long as we have \( \dot{\phi} = 0 \) at the two end-points.

5. Criteria of existence for bounce solutions

For the various classes of bounce solutions we described above, we will demonstrate that there is an upper bound of \( \beta \) for which bounce solutions can exist. When the value \( \beta \) goes beyond this bound, the bounce solutions no longer exist. This result supplements earlier arguments that \( \beta \) has a lower bound. In particular, [19] found that for the \( n \)-oscillating bounce solution to exist, the value of \( \beta \) must have a sufficiently large value, namely

\[
\beta > n(n + 3).
\]  \hfill (17)

This result was obtained by considering small perturbations about \( \phi_{\text{top}} \), and was verified numerically in the non-perturbative case. This continues to hold true for the non-perturbative oscillating bounce solutions in the large-\( n \)-limit [24].

Here, we demonstrate that there also exist an upper bound to \( \beta \), which we will denote as \( \beta_* \), in addition to demonstrating that the non-perturbative, oscillating bounce solutions are consistent with (17). We show in this section that the oscillating bounce solutions require

\[
n(n + 3) < \beta < \beta_*,
\]  \hfill (18)

for some value \( \beta_* \) which we will explore in this section.

In general, we observe that as we increase \( \beta \) further away from \( n(n + 3) \), the initial value \( \phi(0) \) required to obtain various bounce solutions approaches the minima \( \phi_{\text{tv}} \). In other words, as \( \beta \) increases, the difference \( \phi_{\text{tv}} - \phi(0) \) for the oscillating bounce solution and the \( a \)-diverging solution becomes smaller. This difference continues to diminish until a value of \( \beta = \beta_* \) where the difference effectively vanishes and only the trivial solution remains. We shall call this upper bound \( \beta_* \) the ‘vanishing point’.

We first consider solutions corresponding to the true vacuum to false vacuum transitions. We see that the \( n = 1 \) CDL solution vanishes first with the smallest \( \beta_* \) compared to the \( n = 3 \) and \( n = 5 \) solutions. Table 1 shows the vanishing points for bounces up to \( n = 10 \). We note that the \( n = 1 \) (the CDL bounce) and \( n = 2 \) bounce vanishes first, at \( \beta_* = 112.4 \), followed by the vanishing of the higher bounce solutions at larger \( \beta_* \). Therefore, beyond a certain \( \beta \), there are no longer any quantum transitions due to the CDL bounce, but rather thermal ones with \( n > 1 \). For these values of \( \beta_* \), the potentials are negative for both the false and true vacua. The vanishing points for bounces up to \( n = 40 \) are plotted in figure 5.

| \( n \) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \beta_* \) | 112.4 | 112.4 | 136.5 | 142.7 | 206.2 | 220.0 | 297.0 | 316.4 | 406.7 | 431.1 |
It is worth observing in figure 5 that the odd and even bounces can be considered to lie on separate, slightly different trend-lines, with $\beta_*$ for the odd bounces are always slightly higher than those of the even bounces. We may also estimate a functional relation for each trendline by performing a Richardson extrapolation on the data points. We find that $\beta_*$ has the form

$$\beta_* \sim \frac{9}{4} n(n + \zeta), \quad n \to \infty,$$

(19)

where the odd and even solutions have the parameter $\zeta$ given by

$$\zeta \approx \begin{cases} 8.61, & \text{odd } n, \\ 7.62, & \text{even } n. \end{cases}$$

(20)

This is one of the major results of this paper. The fitting estimated in (19) for $\zeta = 8.61$ (odd $n$) is plotted as the solid line in figure 5. The even-$n$ case is not shown as it is nearly indistinguishable from the odd-$n$ plot on a figure of that scale. Furthermore, we have verified that all the bounce solutions satisfies the condition (17) given by [19] under the thin-wall approximation.

We can also perform a similar analysis for the false to true vacuum transitions. The vanishing points of the bounce solutions up to $n = 10$ are shown in table 2. It is interesting to note that in this case, the vanishing point for the CDL $n = 1$ bounce is $\beta_* = 100.63 < 104.74$. For this value of $\beta$, the potential $V$ is positive at $\phi = \phi_{fv}$. The values of $\beta_*$ for the higher ($n > 1$) bounces all correspond to negative $V$ at both vacua. Performing the Richardson extrapolation for the false-to-true transitions, we find

$$\beta_* \sim \frac{9}{4} n(n + \lambda), \quad n \to \infty,$$

(21)

where

$$\lambda \approx \begin{cases} 8.667, & \text{odd } n, \\ 9.778, & \text{even } n. \end{cases}$$

(22)

As suggested by [19], the only bounce solution corresponding to quantum tunnelling is the $n = 1$ CDL bounce, as the higher-bounce solutions have much larger actions, thus does
not contributing greatly to the tunnelling effect. Furthermore, the higher-bounce solutions are those that approach the Hawking–Moss solution, which is a transition that is thermal in nature.

6. Conclusion

By exploring the numerical solutions to the classical Euclidean equations of motion, we have explored a class of bounce solutions with a diverging scale factor. The behaviour of the scalar field in this $a$-diverging class is similar to that of the CDL single-bounce solution, though the scale factor $a$ grows monotonically with Euclidean time.

The reason for a diverging $a$ is mainly due to this class of solution having vacua corresponding to $V < 0$, or a negative effective cosmological constant. Thus the model actually describes false vacuum decay from one anti-de Sitter space into another. While this may not be particularly relevant to inflation or cosmology, it might have potentially interesting holographic descriptions in the context of the AdS/CFT correspondence [30–32].

We have also numerically verified an argument made in Hackworth and Weinberg [19] that all bounce solutions must satisfy the condition $\beta > n(n + 3)$. This argument was originally made based on the thin-wall approximation. Here, our numerical solutions which are found beyond the thin-wall approximation continues to satisfy this bound. In addition, as the barrier curvature $\beta$ increases, the single-bounce CDL solution ceases to exist. Therefore we have provided an additional upper bound and the range of $\beta$ is now bounded to $n(n + 3) < \beta < \beta^*$. Since this is the solution with the negative mode that is the main contributor to tunnelling, that means that beyond this value of $\beta$, the remaining modes are those coming from the higher-bounce solutions which behave more similarly to Hawking–Moss thermal transitions.

ORCID iDs

Jiangbin Gong  
https://orcid.org/0000-0003-1280-6493

Yen-Kheng Lim  
https://orcid.org/0000-0002-0907-6904

Qing-hai Wang  
https://orcid.org/0000-0001-7952-7101

References

[1] Coleman S R and De Luccia F 1980 Gravitational effects on and of vacuum decay Phys. Rev. D 21 3305
[2] Guth A H 1981 The inflationary universe: a possible solution to the horizon and flatness problems Phys. Rev. D 23 347
[3] Linde A D 1982 A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems Phys. Lett. B 108 389
[4] Albrecht A and Steinhardt P J 1982 Cosmology for grand unified theories with radiatively induced symmetry breaking Phys. Rev. Lett. 48 1220
[5] Kobzarev I Y, Okun L B and Voloshin M B 1975 Bubbles in metastable vacuum Sov. J. Nucl. Phys. 20 644

Table 2. The vanishing points of $n$-bounce solutions up to $n = 10$ for the false vacuum to true vacuum transitions, with $\beta = 0.25$ and $\epsilon = 0.23$ [24].

| $n$ | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\beta^*$ | 100.63 | 131.71 | 136.5 | 193.9 | 206.2 | 280.1 | 297.0 | 383.6 | 406.7 | 507.2 |
Kobzarev I Y, Okun L B and Voloshin M B 1974 Bubbles in metastable vacuum Yad. Fiz. 20 1229
[6] Frampton P H 1976 Vacuum instability and Higgs scalar mass Phys. Rev. Lett. 37 1378
Frampton P H 1976 Vacuum instability and Higgs scalar mass Phys. Rev. Lett. 37 1716 (erratum)
[7] Coleman S R 1977 The fate of the false vacuum. 1. Semiclassical theory Phys. Rev. D 15 2929
Coleman S R 1977 The fate of the false vacuum. 1. Semiclassical theory Phys. Rev. D 16 1248
(erratum)
[8] Callan C G Jr and Coleman S R 1977 The fate of the false vacuum. 2. First quantum corrections Phys. Rev. D 16 1762
[9] Odintsov S D and Oikonomou V K 2017 Big-bounce with finite-time singularity: the $F(R)$ gravity description Int. J. Mod. Phys. D 26 1750085
[10] Nojiri S, Odintsov S D and Oikonomou V K 2017 Modified gravity theories on a nutshell: inflation, bounce and late-time evolution Phys. Rep. 692 1
[11] Konoplich R V and Rubin S G 1985 Decay probability for metastable vacuum in scalar theory Yad. Fiz. 42 1282 (in Russian)
[12] Isidori G, Ridolfi G and Strumia A 2001 On the metastability of the standard model vacuum Nucl. Phys. B 609 387
[13] Strumia A, Tetradis N and Wetterich C 1999 The region of validity of homogeneous nucleation theory Phys. Lett. B 467 279
[14] Baacke J and Lavrelashvili G 2004 One loop corrections to the metastable vacuum decay Phys. Rev. D 69 025009
[15] Dunne G V and Min H 2005 Beyond the thin-wall approximation: Precise numerical computation of prefactors in false vacuum decay Phys. Rev. D 72 125004
[16] Lee B-H and Lee W 2009 Vacuum bubbles in a de Sitter background and black hole pair creation Class. Quantum Grav. 26 225002
[17] Hawking S W and Moss I G 1982 Supercooled phase transitions in the very early universe Phys. Lett. B 110 35
[18] Hawking S W and Turok N 1998 Open inflation without false vacua Phys. Lett. B 425 25
[19] Hackworth J C and Weinberg E J 2005 Oscillating bounce solutions and vacuum tunneling in de Sitter spacetime Phys. Rev. D 71 044014
[20] Dong X and Harlow D 2011 Analytic Coleman–De Luccia geometries J. Cosmol. Astropart. Phys. JCAP11(2011)044
[21] Kanno S and Soda J 2012 Exact Coleman–De Luccia instantons Int. J. Mod. Phys. D 21 1250040
[22] Lee B-H, Lee C H, Lee W and Oh C 2012 Oscillating instanton solutions in curved space Phys. Rev. D 85 024022
[23] Gregory R, Moss I G and Withers B 2014 Black holes as bubble nucleation sites J. High Energy Phys. JHEP03(2014)081
[24] Dunne G V and Wang Q-h 2006 Fluctuations about cosmological instantons Phys. Rev. D 74 024018
[25] Koehn M, Lavrelashvili G and Lehners J-L 2015 Towards a solution of the negative mode problem in quantum tunnelling with gravity Phys. Rev. 92 023506
[26] Coleman S R, Glaser V and Martin A 1978 Action minima among solutions to a class of euclidean scalar field equations Commun. Math. Phys. 58 211
[27] Parke S J 1983 Gravity, the decay of the false vacuum and the new inflationary universe scenario Phys. Lett. B 121 313
[28] Brown J D and Teitelboim C 1988 Neutralization of the cosmological constant by membrane creation Nucl. Phys. D 297 787
[29] Masoumi A, Paban S and Weinberg E J 2016 Tunneling from a Minkowski vacuum to an AdS vacuum: a new thin-wall regime Phys. Rev. D 94 025023
[30] Maldaçena J 2010 Vacuum decay into anti de Sitter space (arXiv:1012.0274)
[31] Garriga J 2011 Holographic description of vacuum bubbles Prog. Theor. Phys. Suppl. 190 261
[32] Kanno S, Sasaki M and Soda J 2012 Holographic dual of de Sitter universe with AdS bubbles Nucl. Phys. B 855 361