Light propagation through a coiled optical fiber and Pancharatnam phase

Rajendra Bhandari

Raman Research Institute,
Bangalore 560 080, India.
email: bhandari@rri.res.in

The nature of changes in the interference pattern caused by the presence of polarization-changing elements in one or both beams of an interferometer, in particular those caused by an effective optical activity due to passage of a polarized beam through a coiled optical fiber are clarified. It is pointed out that for an incident state that is not circularly polarized so that the two interfering beams go to different polarization states, there is an observable nonzero Pancharatnam phase shift between them which depends on the incident polarization state and on the solid angle subtended by the track of the $\vec{k}$-vector at the centre of the sphere of $\vec{k}$-vectors. The behaviour of this phase shift is singular when the two interfering states are nearly orthogonal. It is shown that for zero path difference between the two beams, the amplitude of intensity modulation as a function of optical activity is independent of the incident polarization state. © 2008 Optical Society of America

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1. Introduction

The nature of changes in an interference pattern resulting from a change in the polarization of one or both beams in an interferometer have received considerable attention in recent years following the work of Berry [1] in which he discovered an interesting phase-shift effect on the wavefunction of a quantum mechanical system evolving under the action of a cyclic, adiabatically varying hamiltonian. Having their origin in the work of Pancharatnam [2], topological phase shifts arising from polarization transformations in a light wave with a fixed $\vec{k}$-vector were measured in interference experiments and shown to have many counter-intuitive properties [3–9]. Another interesting topological phase shift, arising from a cyclic change in the direction of propagation of a light beam along a three-dimensional curve
in space, for example that caused by the passage of a monochromatic beam through a monomode optical fibre, was predicted by Chiao and Wu [10]. The basic effect here is the parallel transport of the polarization of the wave as it traverses a three-dimensional curve in space such that the tip of its $\vec{k}$-vector goes around a closed curve on the sphere of $\vec{k}$-vectors. In such a transport, a circularly polarized wave propagates without a change in its polarization state but acquires a geometric phase shift $\pm \gamma$, where $\gamma$ is equal to the solid angle subtended by the track of the $\vec{k}$-vector at the centre of the sphere of $\vec{k}$-vectors; the sign of the phase shift being different for the two different circular polarizations. This has been known in literature as the "spin redirection phase". The difference between the phases acquired by the left and the right circularly polarized states in such a transport was experimentally demonstrated by Frins and Dultz [11] to be of magnitude $2\gamma$.

An arbitrarily polarized state propagating through the fiber, however, experiences a rotation of its polarization ellipse about the $\vec{k}$-vector through an angle equal to $\gamma$. This corresponds to a rotation of the representative point on the Poincaré Sphere about the polar axis of the sphere through an angle $2\gamma$; the poles representing the circularly polarized states. Such a rotation, for linearly polarized states, was experimentally demonstrated by Tomita and Chiao [12]. This effect, originating in the degeneracy of the two circularly polarized states, was interpreted by Anandan [13] as an example of the Wilczek-Zee phase [14]. This can be looked upon as an effective optical activity. A question may be asked: in addition to such a rotation, which represents a change in the polarization state of the beam, does the beam experience a phase shift? The answer is yes and this is the main subject of this paper. The omission of this phase shift may lead to incorrect conclusions about the results of interference experiments. An example is a recent paper by Senthilkumaran [15] in which a related experimental situation namely a "tunable fibre optic mirror" is analyzed and the absence of such a phase shift is assumed. The conclusions arrived at contradict the results of earlier experimental work on the tunable fiber optic mirror [16,17]. Since interference situations where the polarization states of one or both beams change are commonly encountered and since these are not treated correctly in existing books on interferometry (for example see ref. [18]), we attempt to present, in the following sections, a correct analysis of the situation.

2. Interference of polarized light

In this section we present an analysis of interference of polarized light in the presence of polarization-changing elements in the path of one or both interfering beams in the context of the simplest interference situation namely Young’s two-slit interference experiment (fig.1).

Since our focus in this paper is on optical activity, we shall use circularly polarized states as the basis states unless stated otherwise. The variable $n$ is an integer throughout the paper.

A unit intensity beam in polarization state $\eta$ can be represented by a two-component
Fig. 1. This figure shows the interference pattern on a screen formed by interference of two parts of a wavefront $W$ through slits $S_1$ and $S_2$ which have undergone polarization transformations $J_1$ and $J_2$ corresponding to an effective optical activity that causes a rotation about the beam axis through angles $\gamma$ and $-\gamma$ respectively. Intensity variation on the screen as a function of the distance $y$ along the screen (approximately proportional to the optical path difference $\alpha$ between the two beams) is shown for 5 different values of $\gamma$, i.e $\gamma = 0^\circ$, $\gamma = 45^\circ$, $\gamma = 90^\circ$, $\gamma = 135^\circ$ and $\gamma = 180^\circ$. As $\gamma$ changes, the visibility of the fringes changes and the fringes shift along the $y$-axis by an amount $\delta$ which is the Pancharatnam phase difference between the beams. Note, when $\alpha = n\pi$, the amplitude of the intensity modulation as a function of $\gamma$ is 1, whereas for an arbitrary value of $\alpha$, the modulation can be less than 1. The fringes shown in the figure correspond to a polarization state with $\theta = 60^\circ$. 
complex column vector

\[ \eta = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) \exp(i\phi) \end{pmatrix} \quad (1) \]

where \( \theta \) and \( \phi \) are the polar and azimuthal coordinates respectively of the point representing the polarization state on the Poincaré Sphere. To keep the discussion general at this point let the slits \( s_1 \) and \( s_2 \) have different widths so that the beams exiting \( s_1 \) and \( s_2 \) have different intensities. Beams with intensities \( I_1 \) and \( I_2 \) in polarization state \( \eta \) can be represented by the complex column vectors

\[ \eta_1 = \sqrt{I_1} \eta \quad \text{and} \quad \eta_2 = \sqrt{I_2} \eta \quad (2) \]

Let the beams after passing through the slits \( s_1 \) and \( s_2 \) pass through a box placed in front of each of the slits where unitary transformations \( J_1 \) and \( J_2 \) are performed on the states \( \eta_1 \) and \( \eta_2 \) respectively following which the beams travel to a point \( P \) on the screen where the intensity resulting from their interference is considered (fig.1). The transformations \( J_1 \) and \( J_2 \) can each be broken into two factors, the first representing an isotropic (polarization-independent) part that multiplies the state vectors by phase factors \( \exp(i\beta_1) \) and \( \exp(i\beta_2) \) and the second, an SU(2) part that multiplies the state vectors by 2 x 2 complex unitary matrices \( U_1 \) and \( U_2 \) with determinant +1. So that,

\[ J_1 = \exp(i\beta_1) U_1 \quad \text{and} \quad J_2 = \exp(i\beta_2) U_2 \quad (3) \]

The states \( \psi_1 \) and \( \psi_2 \) at \( P \) are given by,

\[ \begin{align*}
\psi_1 &= J_1 \exp(i\alpha_1) \eta_1 = \sqrt{I_1} \exp(i(\beta_1 + \alpha_1)) \tilde{\psi}_1 \quad \text{and} \\
\psi_2 &= J_2 \exp(i\alpha_2) \eta_2 = \sqrt{I_2} \exp(i(\beta_2 + \alpha_2)) \tilde{\psi}_2
\end{align*} \quad (4) \]

where

\[ \tilde{\psi}_1 = U_1 \eta, \quad \tilde{\psi}_2 = U_2 \eta \quad \text{and} \quad \tilde{\psi}_2^\dagger \tilde{\psi}_2 = \tilde{\psi}_1^\dagger \tilde{\psi}_1 = 1 \quad (5) \]

The phase factors \( \exp(i\alpha_1) \) and \( \exp(i\alpha_2) \) are due to propagation of the two waves from the slits to the point \( P \). The intensity \( I \) at \( P \) is given by,

\[ I = \psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2 + 2Re[\psi_2^\dagger \psi_1] \]
\[ I = I_1 + I_2 + 2(I_1I_2)^{\frac{1}{2}} \text{Re}[\tilde{\psi}_2^\dagger \tilde{\psi}_1 \exp(i(\beta + \alpha))] \]
\[ = I_1 + I_2 + 2(I_1I_2)^{\frac{1}{2}} \text{Re}[\xi \exp(i(\delta + \beta + \alpha))] \]  \hspace{1cm} (6)

where \( \beta = \beta_1 - \beta_2 \), \( \alpha = \alpha_1 - \alpha_2 \) and \( \tilde{\psi}_2^\dagger \tilde{\psi}_1 = \xi \exp(i\delta) \) \hspace{1cm} (7)

The phase difference \( \alpha \) due to the waves from slits 1 and 2 having travelled different path lengths to the point P is given, for \( s/d \ll 1 \) and \( y/d \ll 1 \) by,
\[ \alpha = (2\pi s/\lambda)(y/d) \] \hspace{1cm} (8)

where \( \lambda \) is the wavelength of light, \( s \) is the distance between the slits, \( d \) is the distance between the planes containing the slits and the screen and \( y \) is the distance along the screen of the point P from the point of zero path difference between the waves. We define the quantity \( \delta = \text{arg}(\tilde{\psi}_2^\dagger \tilde{\psi}_1) = \text{arg}(\eta^\dagger U_2^\dagger U_1 \eta) \) to be the Pancharatnam phase shift between the two beams due to the SU(2) transformations \( U_1 \) and \( U_2 \). We have deliberately kept the isotropic phase shifts \( \beta_1 \) and \( \beta_2 \) out of the definition of the Pancharatnam phase so that \( \delta \) represents the effect of polarization transformations alone. The intensity given by eqn. (6) has maxima when \( (\delta + \beta + \alpha) = 2n\pi \) and minima when \( (\delta + \beta + \alpha) = (2n + 1)\pi \). These are given by,
\[ I_{\text{max}} = I_1 + I_2 + 2(I_1I_2)^{\frac{1}{2}} \xi \] \hspace{1cm} (9)
\[ I_{\text{min}} = I_1 + I_2 - 2(I_1I_2)^{\frac{1}{2}} \xi \] \hspace{1cm} (10)

so that the visibility \( V \) is given by,
\[ V = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}}) = [2(I_1I_2)^{\frac{1}{2}}/(I_1 + I_2)] \xi \] \hspace{1cm} (11)

when \( I_1 = I_2 \), \( V = \xi \). The quantity \( \xi = |\tilde{\psi}_2^\dagger \tilde{\psi}_1| \) is then a measure of the visibility of the interference fringes. The transformations \( J_1 \) and \( J_2 \) thus have two effects on the interference pattern: (i) the contrast of the pattern changes from 1 to \( \xi \) and (ii) the fringes shift along the \( y \)-axis (or say the \( \alpha \)-axis) by an amount \(- (\beta + \delta)\).

Let us next consider the case when the SU(2) transformations \( U_1 \) and \( U_2 \) correspond to optical activity so that the incident polarization states \( \eta_1 \) and \( \eta_2 \) are rotated about the beam axis through angles \( \gamma \) and \(-\gamma \) respectively without any change in the ellipticity of the polarization ellipse. Let us also assume that \( \beta_1 = \beta_2 = 0 \). Examples of such transformations are (i) passage of a polarized beam through a coiled optical fibre with integral number of turns and (ii) passage of a polarized beam with a fixed \( \vec{k} \) vector through a pair of halfwave plates whose principal axes make an angle \( \gamma/2 \) with each other. We then have,
\[ J_1 = U_1 = \begin{pmatrix} \exp(-i\gamma) & 0 \\ 0 & \exp(i\gamma) \end{pmatrix} \] and
\[ J_2 = U_2 = \begin{pmatrix} \exp(i\gamma) & 0 \\ 0 & \exp(-i\gamma) \end{pmatrix} \]  

so that,

\[ \mathbf{\tilde{\psi}}_2^\dagger \mathbf{\tilde{\psi}}_1 = \eta^\dagger U_2^\dagger U_1 \eta = \xi \exp(i\delta) = \cos^2(\theta/2)\exp(2i\gamma) + \sin^2(\theta/2)\exp(-2i\gamma) \]  

(13)

This gives,

\[ \xi \cos \delta = \cos(2\gamma) \quad \text{and} \quad \xi \sin \delta = \cos \theta \sin(2\gamma) \]  

(14)

or,

\[ \xi = \left[ \cos^2(2\gamma) + \cos^2(\theta)\sin^2(2\gamma) \right]^{1/2} \quad \text{and} \quad \tan \delta = \cos \theta \tan(2\gamma) \]  

(15)

The visibility \( \xi \) and the Pancharatnam phase shift \( \delta \) can be determined from the pair of equations (14) or (15). Figure 1 shows the intensity variation on the screen along the y-axis for 5 different values of \( \gamma \) for an incident state corresponding to \( \theta = 60^\circ \) and for \( I_1 = I_2 = 1/2 \). As \( \gamma \) changes, there are two changes in the interference pattern. The amplitude of the intensity variation, i.e. the visibility \( \xi \) given by eqn. (15) changes and the intensity curve shifts along the y-axis by an amount \( \delta \), also given by eqn. (15) or eqn. (14). In fig.1, \( \xi = 1 \) for \( \gamma = 0^\circ, 90^\circ, 180^\circ \) and \( \xi = 1/2 \) for \( \gamma = 45^\circ, 135^\circ \). The integrated phase shift \( \int d\delta \) determined from these equations for a typical set of values of the polar angle \( \theta \) of the incident polarization state on the Poincaré Sphere are shown in fig.2.

The curves in fig.2 represent the phase shift that would be measured by an interferometer that can keep track of the phase shift continuously as the parameter \( \gamma \) representing the optical activity is changed. As seen from equations (14) and (15) and from fig.2, at the values \( (\theta = 90^\circ, \gamma = (2n + 1)(\pi/4)) \), the visibility \( \xi \) of the interference pattern becomes zero and the phase shift becomes singular. The two interfering states are orthogonal at these points. The phase varies sharply near these points with an abrupt change in sign and a closed circuit in the parameter space \( (\gamma, \theta) \) around one of these points results in a total integrated phase shift equal to \( \pm 2n\pi \). Such a behaviour of the phase shifts caused by polarization transformations was first predicted using a gedanken polarization experiment in ref. [4] and has been observed in interference experiments using quarterwave and halfwave retarders as the SU(2) elements [5–7]. In the context where \( \gamma \) originates in transport through a coiled optical fibre, the phase shift shown in fig.2 corresponding to \( \theta = 0 \) has been seen in experiments reported by Frins and Dultz [11]. A simple extension of this experiment that allows the incident polarization state to be varied would enable the full behaviour of the phase shift to be observed except very near the singularities where the interference contrast becomes too low. Let us note that for \( \gamma = 90^\circ \), the polarization states of one of the two
Fig. 2. This figure shows the Pancharatnam phase shift $\delta$ in degrees as a function of the optical activity parameter $\gamma$ which in the case of propagation through a fiber loop is equal to the solid angle subtended by the track of the $\vec{k}$-vector at the centre of the sphere of $\vec{k}$-vectors. The 6 curves A, B, C, D, E and F correspond to incident polarization states with polar angle $\theta = 30^\circ, 75^\circ, 89.9^\circ, 90.1^\circ, 105^\circ$ and $150^\circ$. The curves for $\theta = 0^\circ$ and $\theta = 180^\circ$ are straight lines nearly coincident with the curves A and F respectively and are not shown separately. For $\gamma = n\pi/2$ sterradians, the polarization of the two beams undergoes rotation through $\pi$ and $-\pi$ on the Poincaré sphere and the total phase shift has magnitude $\pi$ irrespective of $\theta$. Also note the singular behaviour for $\theta = 90^\circ$ when $\gamma$ has the values $(2n + 1)\pi/4$ sterradians.
beams rotates through $\pi$ on the Poincaré sphere and that of the other beam rotates through $-\pi$. For this relative rotation of $2\pi$, the total phase shift is of magnitude $\pi$ irrespective of the value of $\theta$. This is analogous to the sign change of spin-$1/2$ wavefunctions under $2\pi$ rotations in real space. It may also be pointed out that the phase shift defined as above is the total phase shift due to the SU(2) transformation and not just the geometric part of the phase. This is elaborated further in section 5.

3. The tunable fiber optic mirror

In refs. [16,17], Senthilkumaran et al. reported a fiber optic device in which they use a coiled fiber in a Sagnac interferometer configuration in combination with a halfwave retarder to produce equal and opposite phase shifts $\gamma$ and $-\gamma$ on the propagating and counter-propagating circularly polarized beams respectively which result in a complimentary modulation of intensity at the two output ports of the interferometer as a function of the phase shift introduced; the latter being equal to the solid angle subtended by the track of the tangent vector to the fiber coil at the centre of the sphere of directions in space. It was observed in these experiments that when an arbitrary linear superposition of the two circularly polarized states is incident on the interferometer, the modulation index of intensity at the output port is independent of the incident polarization state. In this section we try to understand this result in the light of the discussion in the previous section.

Let a halfwave retarder oriented with its fast axis making an angle $\tau$ with the x-axis be represented by $H(\tau)$ and an optical rotator that rotates any polarization through an angle $\gamma$ about the beam axis be represented by $R(\gamma)$. In the Sagnac configuration used in refs. [16,17], the propagating beam undergoes an SU(2) transformation $U_1$ corresponding to a product of $H(0)$ and $R(\gamma)$ i.e.

$$U_1 = H(0)R(\gamma) = -i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \exp(-i\gamma) & 0 \\ 0 & \exp(i\gamma) \end{pmatrix}$$

$$= -i \begin{pmatrix} 0 & \exp(i\gamma) \\ \exp(-i\gamma) & 0 \end{pmatrix}$$

The counterpropagating beam on the other hand sees a transformation $U_2$ which is a product of the same two transformations in the reverse order, i.e.

$$U_2 = R(\gamma)H(0) = -i \begin{pmatrix} 0 & \exp(-i\gamma) \\ \exp(i\gamma) & 0 \end{pmatrix}$$

We have assumed as before that $\beta_1 = \beta_2 = 0$. Let us now assume that a unit intensity is incident on the beam splitter of the interferometer and that the beam splitter divides the
intensity between the transmitted and reflected waves in the ratio 1 : 1 so that \( \eta_1 = \eta_2 = \eta \).

Since the beam splitter acts twice before the beams exit the output ports, \( I_1 = I_2 = 1/4 \) and the intensity \( I \) at one of the output ports is given, following eqn. (6), by

\[
I = (1/2)[1 + \text{Re}(\bar{\psi}_2 \psi_1^\dagger \exp i\alpha)]
\]

where

\[
\bar{\psi}_2 \psi_1^\dagger = \eta U_2 \eta^\dagger U_1 \eta = \xi \exp(i\delta) = \cos^2(\theta/2)\exp(-2i\gamma) + \sin^2(\theta/2)\exp(2i\gamma)
\]

(19)

Notice the similarity of eqn. (19) with eqn. (13). Except for the sign of \( \gamma \), the two expressions are the same. The interference in this case is also between two states with the same ellipticity, but rotated with respect to each other about the beam axis through an angle \( 2\gamma \). However in this case due to the presence of H(0) there is a change in the helicity of both the states with respect to the initial state which also accounts for the change in the sign of \( \gamma \). Therefore the Pancharatnam phase in this case also has the same general behaviour as shown in fig. 2, with the appropriate sign changes.

Let us substitute eqn. (19) in eqn. (18). This gives,

\[
I = (1/2)[1 + \xi \cos(\delta + \alpha)]
\]

\[
= (1/2)[1 + \cos^2(\theta/2)\cos(\alpha - 2\gamma) + \sin^2(\theta/2)\cos(\alpha + 2\gamma)]
\]

\[
= (1/2)[1 + \cos \alpha \cos(2\gamma) + \sin \alpha \sin(2\gamma) \cos \theta]
\]

(20)

Rewriting the right hand side in eqn. (20),

\[
I = (1/2)[1 + M \cos(2\gamma - \chi)], \quad \text{where}
\]

\[
M \cos \chi = \cos \alpha,
\]

\[
M \sin \chi = \cos \theta \sin \alpha \quad \text{and}
\]

\[
M = [\cos^2 \alpha + \cos^2 \theta \sin^2 \alpha]^{1/2}
\]

(21)

Eqn. (21) is the main result of this section and shows that at points in the interference pattern where \( \alpha = 0 \) or \( n\pi \), \( M = 1 \). This result is independent of \( \theta \), i.e. of the incident polarization state. This is indeed what was observed in refs. [16, 17], where the path difference \( \alpha \) between the two interfering beams is zero. The conclusion in ref. [15] that \( M \) depends on \( \theta \) results from the assumption therein that \( \delta = 0 \), which is incorrect. When \( \theta = 0 \) or \( \pi \), i.e. when the incident state is circularly polarized, eqn. (21) gives,
\[ I = (1/2)[1 + \cos(2\gamma \pm \alpha)], \quad (22) \]

so that \( M \) in this case is 1 irrespective of the value of \( \alpha \). In ref. [15] it was concluded that \( M = 0 \) for this case. For an arbitrary \( \theta \), however, \( M \) is a function of \( \alpha \), being 1 for \( \alpha = 0 \) as in refs. [16, 17].

A simple equivalent of the tunable fiber optic mirror in the context of conventional, non-fiber interferometry is shown in fig.3. BS is a 50:50 beam splitter that reflects incident circularly polarized light with a change of helicity and \( M_1, M_2, M_3 \) are mirrors that do the same. Modulation of the intensity at either of the two output ports in this case can be achieved by rotation of the halfwave plate \( \mathbf{H} \) about the beam axis; rotation of \( \mathbf{H} \) through an angle \( \gamma/2 \) being equivalent to the solid angle \( \gamma \) of the fiber loop. This equivalence can easily be seen if we note that (i) optical activity is equivalent to passage through a pair of halfwave plates with their principal axes making a certain angle with each other and (ii) any sequence of three halfwave plates is equivalent to a single halfwave plate oriented at an appropriate angle [8].

4. A pair of halfwave plates

A pair of aligned halfwave plates, i.e. a fullwave plate is an interesting device in that any polarization state passing through it acquires a topological phase shift of magnitude \( \pi \) in addition to any isotropic phase shift due to an extra path length. In the basis of circularly polarized states, the Jones matrix (the SU(2) part) of a halfwave plate with its fast axis making an angle \( \tau/2 \) with the x-axis is given by,

\[
\mathbf{H}(\tau/2) = -i \begin{pmatrix} 0 & \exp(-i\tau) \\ \exp(i\tau) & 0 \end{pmatrix} \quad (23)
\]

In the basis of x and y linearly polarized states, the SU(2) matrix for \( \mathbf{H}(\tau/2) \) is given by,

\[
\mathbf{H}(\tau/2) = -i \begin{pmatrix} \cos\tau & \sin\tau \\ \sin\tau & -\cos\tau \end{pmatrix} \quad (24)
\]

From either of eqns. (23) or (24), it can easily be verified that

\[
\mathbf{H}(\tau/2)\mathbf{H}(\tau/2) = - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \exp(\pm i\pi) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (25)
\]

This proves the above assertion. Experimentally, this can be verified in an interference experiment in which a pair of identical halfwave plates is placed in one arm of a Mach-Zhender interferometer in the configuration \( \mathbf{H}(\tau/2)\mathbf{H}(\tau/2 - 90^\circ) \) and the second halfwave
Fig. 3. This figure shows a conventional Sagnac interferometer configuration that would act as a tunable mirror equivalent to the tunable fiber optic mirror. A rotation of the halfwave plate $H$ about the beam axis results in a complimentary modulation of intensity at the two output ports. Rotation through an angle $\gamma/2$ is equivalent to passage through a fiber loop with solid angle $\gamma$ in $\vec{k}$-space.
plate is rotated through 90° so that the final configuration of the pair is \( H(\tau/2)H(\tau/2) \), while no change is made in the reference arm of the interferometer. The initial Jones matrix of the pair is 1 and the final matrix is -1. Consequently, for any incident polarized wave, the total phase shift measured will be +\( \pi \) or -\( \pi \) if the phase shift is continuously monitored. Another way to see that the phase shift is \( \pm \pi \) is to realize that a pair of halfwave plates is equivalent to an optical rotator and apply the considerations in section 3. How does one distinguish the topological phase shift of magnitude \( \pi \) from a possible isotropic phase shift due to a change in optical path during rotation of the halfwave plate? This is simple. If the halfwave plate were rotated through 360° while the phase shift is being continuously integrated, the net phase shift due to a change in optical path must be equal to zero, whereas the phase shift of topological origin would integrate to \( \pm 4\pi \). A related experiment that demonstrates this "unbounded" nature of topological phase shifts was reported in ref. [3]. Another distinction between the topological and the isotropic phase in the above experiment is that in case of the former some polarization states get a phase +\( \pi \) and some get a phase -\( \pi \), while in case of the latter all states get the same phase. Note that the factor \( i \) multiplying the matrices in eqns. (23) and (24) is nontrivial and cannot be omitted.

5. Discussion

The main issue dealt with in this paper is the phase acquired by a light wave when its polarization state changes under the action of a polarization transforming device. While the focus in this paper is on SU(2) transformations that conserve total intensity, the discussion applies to situations where the Jones matrices \( J_1 \) and \( J_2 \) have factors representing isotropic absorption and/or dichroism. In a paper written in 1956, Pancharatnam made two important contributions to this problem [2].

(1) It was suggested that the phase difference \( \delta \) between two waves in different states of polarization be defined such that they are in phase when the intensity resulting from their superposition is maximum. This leads to the definition \( \delta = \arg(\tilde{\psi}_2 \dagger \tilde{\psi}_1) \) if we adopt the convenient convention that isotropic phase shifts which are not due to polarization changes are excluded from the definition of \( \delta \).

(2) The second result can be re-stated as follows: if the polarization state of a light wave is taken along a closed geodesic polygon on the Poincaré sphere by means of a sequence of transformations each of which takes the state along a geodesic arc on the sphere, the wave acquires a phase shift equal to half the solid angle subtended by the polygon at the centre of the sphere. This phase is now known in literature as the "geometric phase" or "Berry’s

\footnote{When the Jones matrix of a halfwave plate is written without the factor \( i \) as done in ref. [15] and in some textbooks on optics, it implies an isotropic phase factor \( \exp(\pm i\pi/2) \) multiplying the SU(2) part. This must be removed before applying the considerations of this section.}
phase”.

When a wave undergoes an arbitrary sequence of transformations which does not result in a geodesic evolution on the sphere, it acquires a total phase which is a sum of two terms: (a) the geometric phase equal to half the solid angle of the closed curve on the Poincaré sphere as stated in (2) above and (b) a dynamical phase which is determined by the evolving state and the Jones matrix of the transformation. We have called the total phase the ”Pancharatnam phase” in this paper. It is important to note that in a general evolution the geometric part of the phase is not zero. It is a piece of the total phase.

In a simple evolution of the kind considered in this paper, namely a state undergoing rotation through a full circle about a fixed axis on the Poincaré sphere, the geometric phase is equal to ±π(1 − cosθ) and the dynamical phase is equal to ±πcosθ, their sum being equal to ±π. While the decomposition of the phase in a general evolution in a geometric and a dynamical part, first done by Aharonov and Anandan [19], is theoretically very interesting, the phase that is measured in an experiment is always the total phase. This is the reason for our choice of the total phase as the interesting quantity in this paper.

It also needs to be pointed out that the definition of Pancharatnam phase requires two waves; one or both of which may undergo polarization changes. In a situation where the polarization state of only one wave changes, the definition of the acquired phase δ still needs a reference state and the measured phase depends on this reference state. In the example considered in section 2, if \( J_2 = 1 \), i.e. only the wave 1 sees the transformation \( R(\gamma) \), the acquired phase \( \delta \) has the same behaviour as that shown in fig.2 with \( 2\gamma \) replaced by \( \gamma \). Such a phase shift can be seen in a Mach Zhender interferometer.

**Note Added:**

After submitting this manuscript we became aware of the work of Tavrov et al. [20] in which they use the geometric spin redirection phase due to out of plane propagation of light to realise an achromatic π- phase shift between the two beams of an astronomical interferometer for ”nulling interferometry”. In our judgement, the linear phase shift between the beams for circular polarization shown in fig. 2a and the highly nonlinear phase shift for linear polarization shown in fig. 2b of their paper correspond approximately to the curves A and C shown in fig. 2 of this paper.

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