The QCD Phase Diagram from Chiral Approaches

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Abstract

I show an updated QCD phase diagram with recent developments from chiral effective theories and phenomenological models. Expected signals of a QCD critical point accessible in heavy-ion collisions are also discussed. In particular, non-monotonic behavior of fluctuations associated with conserved charges is focused on.

1. Introduction

One of the main issues addressed in QCD is the phase structure of strongly interacting matter at finite temperature and baryon density. Remarkable progress in lattice QCD simulations provides a reliable description of bulk QCD matter for small chemical potentials, i.e. equations of state and order of phase transitions, which is an input to heavy-ion phenomenologies \[1\]. Model studies of dense baryonic and quark matter have suggested a rich phase structure of QCD at temperatures and quark chemical potentials being of order \(\Lambda_{\text{QCD}}\). Our knowledge on the QCD phase structure is however still limited: The physics around normal nuclear matter density has been empirically known and can be successfully described by chiral effective field theories guided by experimental data \[2\]. At asymptotically high density the Color-Flavor-Locked phase is preferred as the QCD ground state \[3\]. In intermediate densities where lattice QCD is not accessible, a description of dense matter still relies on effective theories and models.

The order of the QCD phase transition is neither established at low temperature or high density. First-order phase transitions for cold dense matter have been predicted in several approaches using chiral models \[4\], Schwinger-Dyson equations \[5\] and lattice QCD in strong coupling limit \[6\]. This along with the observation of a crossover at zero chemical potential from lattice QCD computations might suggest a critical point in the QCD phase plane \[4\]. The existence of a QCD critical point is still an issue under debate. In fact, the location of a critical point strongly depends on parameters, e.g. current quark masses and coupling strengths of hadronic interactions in model studies \[7\].

The presence of diquarks in dense matter leads to a possibility of new critical points at low temperature depending on couplings of the chiral condensate to diquarks \[8,9\]. A hypothetical phase diagram is shown in Figure 1 assuming multiple critical points A-C. Since the critical point C appears in the two-flavored Nambu–Jona-Lasinio (NJL) model with a fine-tuning of the relative strength of the chiral and diquark couplings \[8\], this might disappear from the phase diagram due to correlations via pion exchanges in dense medium. The appearance of the point B indicates a crossover from hadronic matter to color superconductor. This is a realization of hadron-quark continuity \[10\] in the Ginzburg-Landau description. Close similarities to the BEC-BCS crossover in ultra-cold atomic systems are also suggestive \[11\].
2. Chiral symmetry breaking vs. confinement

Strong interaction leads not only to dynamical breaking of chiral symmetry but also to confinement. Both features are characterized by strict order parameters associated with global symmetries of the QCD Lagrangian in two limiting situations: the quark bilinear $\langle \bar{q}q \rangle$ in the limit of massless quarks, and the Polyakov loop $\langle \Phi \rangle$ in the limit of infinitely heavy quarks. The NJL model with Polyakov loops (PNJL model) has been developed to deal with those features simultaneously [12]. The model describes that only three-quark states are thermally relevant below the chiral critical temperature, which is reminiscent of confinement. The two order parameters in the PNJL model, chiral condensate and Polyakov loop expectation values, are shown in Figure 2. Due to finite quark masses chiral and deconfinement transitions are crossover and the corresponding pseudo-critical temperatures, $T_{ch}$ and $T_{dec}$, are defined as the steepest points of derivatives of $\langle \bar{q}q \rangle$ and $\langle \Phi \rangle$. The interference of quark with gluon sectors makes both $T_{ch}$ and $T_{dec}$ mutually shifted. At zero chemical potential one sees that the two transitions take place almost simultaneously, as indicated in lattice QCD.

The phase transitions at higher chemical potential may have a close relation to the notion of Quarkyonic Phase which has been proposed as a novel phase of dense quarks based on the argument using large $N_c$ counting where $N_c$ denotes number of colors [14]: In the large $N_c$ limit there are three phases which are rigorously distinguished using $\langle \Phi \rangle$ and the baryon number density $\langle N_B \rangle$. The quarkyonic phase is characterized by $\langle \Phi \rangle = 0$ indicating the system confined and non-vanishing $\langle N_B \rangle$ above $\mu_B = M_B$ with a baryon mass $M_B$. The phase structure in large $N_c$ is shown in Figure 3. A possible deformation of the phase boundaries in Figure 3 together with chiral phase transition is illustrated using a PNJL model [15]. Finite $N_c$ corrections make the transition lines bending down. One finds that for $N_c = 3$ deconfinement and chiral crossover lines are on top of each other in a wide range of $\mu$. A critical point associated with chiral symmetry appears around the junction of those crossovers.

The separation of the quarkyonic from hadronic phase is not clear any more in a system with finite $N_c$. Nevertheless, a rapid change in the baryon number would be interpreted as the
Figure 2: The chiral condensate normalized to its vacuum value (solid line) and the Polyakov loop (dashed dotted line) in a PNJL model [12]. The data from lattice calculations for the Polyakov loop in pure gauge and in full QCD are shown in [13].

Figure 3: The phase diagram in large $N_c$ proposed in [14].
Figure 4: The phase diagram of a PNJL model for different $N_c$ [15]. Two straight lines indicate the deconfinement and chiral phase transitions for $N_c = \infty$ and the lower curves for $N_c = 3$.

quarkyonic transition which separates meson dominant from baryon dominant regions. This might appear near the boundary for chemical equilibrium where one would expect a rapid change in the number of degrees of freedom [14]. More realistic models including mesons and baryons, rather than chiral quarks, are indispensible to further understanding of the physics of dense baryonic matter and possible appearance of the quarkyonic “phase” in $N_c = 3$ QCD. In particular, chiral symmetry restoration for baryons must be worked out. Although two alternatives for chirality assignment to baryons have been known, it remains an open question which scenario is preferred by nature [16].

3. Fluctuations and critical points

Modifications in the magnitude of fluctuations or the corresponding susceptibilities are considered as a possible signal for deconfinement and chiral symmetry restoration. In this context, fluctuations related to conserved charges play an important role since they are directly accessible in experiments [17]. Especially, non-monotonic behavior of baryon number fluctuations could be a clear indication for the existence of a critical point in the QCD phase diagram. Higher moments and their signs have also been proposed as more sensitive probes to the phase transitions [18]. If the critical region is sufficiently large, the critical point could be an attractor of isentropes and lead to a change in the transverse velocity dependence of proton-antiproton ratio [19]. However, a model study has recently suggested that such strong modifications are washed out by quantum fluctuations [20] and this indicates no focusing. Besides, isentropes are not universal since entropy and baryon number densities have no derivatives of the order parameter which are responsible for the universality.
Figure 5: The net quark number susceptibility $\chi_q$ in the stable and meta-stable regions [21]. Here $\chi_q$ is normalized to the three-momentum cutoff $\Lambda = 587.9$ MeV used in the $N_f = 2$ NJL model.

The suppression of density fluctuations along the first-order transition appears under the assumption that this transition takes place in equilibrium. This is modified when there is a deviation from equilibrium [21]. When entering the coexistence region, a singularity in the net quark number susceptibility $\chi_q$ appears at the isothermal spinodal lines, where the fluctuations diverge and the susceptibility changes sign. In between the spinodal lines, the susceptibility is negative. This implies instabilities in the baryon number fluctuations when crossing from a meta-stable to an unstable phase. The above behavior of $\chi_q$ is a direct consequence of the thermodynamics relation, $\left(\frac{\partial P}{\partial V}\right)_T = -\frac{\mu_q}{V} - \frac{1}{\chi_q}$. Along the isothermal spinodals the pressure derivative vanishes. Thus, for non-vanishing density $n_q$, $\chi_q$ must diverge to satisfy this relation. The evolution of the singularity at the spinodal lines in the $T$-$n_q$ plane under the mean field approximation is shown in Figure 5. The critical exponent at the isothermal spinodal line is found to be $\gamma = 1/2$, with $\chi_q \sim (\mu - \mu_c)^{-\gamma}$, while $\gamma = 2/3$ at the critical point [21]. Thus, the singularities at the two spinodal lines conspire to yield a somewhat stronger divergence as they join at the critical point. The critical region of enhanced susceptibility around the critical point is found to be fairly small in and beyond the mean field approximation [21], while in the more realistic non-equilibrium system one expects fluctuations in a larger region of the phase diagram, i.e. over a broader range of beam energies, due to the spinodal instabilities.

4. Conclusions

Our understanding of the QCD phases remains inadequate to make a firm statement about the phase diagram and the existence of one or more critical points. Model studies carried out under the mean field approximation are expected to capture some essential features associated
with chiral dynamics. However, since under this approximation one omits important effects, e.g. quantum fluctuations, from a theory with a limited number of degrees of freedom, the models could lead to even qualitatively a different result from QCD.

Some lessons for more realistic modeling can be found in chiral effective field theories (EFTs) applied to normal nuclear matter: The nuclear equation of state (EoS) for various nuclei and the liquid-gas phase transition have been explored within the chiral approaches which allow us to make quantitative science\cite{2}. Figure 6 shows the in-medium chiral condensate normalized to its vacuum value calculated in the chiral EFT constrained by nuclear EoS\cite{23}. One observes that the in-medium condensate strongly relies on the pion mass. This comes from two-pion exchange correlations with virtual \( \Delta (1232) \) excitations and stabilizes the dropping of the condensate for physical pion mass with increasing density. This indicates that the chiral symmetry restoration is delayed and would take place much higher density than \( 2\rho_0 \). This also could suggest that in-medium correlations make a phase transition obscure and eventually first-order phase transition might disappear from the phase diagram. On the other hand, in the chiral limit the condensate vanishes already at \( \sim 1.5\rho_0 \). It is remarkable that small quark masses, \( m_{u,d} \sim 5 \text{ MeV} \), influence over thermodynamics so strongly. Obviously, a more realistic description for hadronic matter at higher densities and the critical point(s) should be provided in systematic approaches with mesons and baryons.

In condensed matter physics strongly coupled systems can be described by EFTs where the notion of e.g. quasi-particles and BCS pairings has been successful. One would thus expect a similarity in nuclear many-body systems in high densities. In fact, the spectroscopic factor for various nuclei indicating a deviation from the fully occupied mean-field orbits clearly shows that single-particle-ness of excitations reaches \( \sim 70 \% \), see e.g. Figure 11 in \cite{24}. This would encourage modeling dense baryonic matter in a quasi-particle picture.
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References

[1] P. Petreczky, these proceedings.
[2] see e.g., P. Finelli, N. Kaiser, D. Vretenar and W. Weise, Nucl. Phys. A 735, 449 (2004); Nucl. Phys. A 770, 1 (2006), S. Fritsch, N. Kaiser and W. Weise, Nucl. Phys. A 750, 259 (2005).
[3] M. Alford, these proceedings.
[4] M. Asakawa and K. Yazawa, Nucl. Phys. A 504, 668 (1989), J. Berges and K. Rajagopal, Nucl. Phys. B 538, 215 (1999), A. M. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and J. J. M. Verbaarschot, Phys. Rev. D 58, 096007 (1998), O. Scavenius, A. Mocsy, I. N. Mishustin and D. H. Rischke, Phys. Rev. C 64, 045202 (2001), H. Fujii, Phys. Rev. D 67, 094018 (2003), H. Fujii and M. Ohtani, Phys. Rev. D 70, 014016 (2004).
[5] M. Alford, these proceedings.
[6] S. Klimt, M. Lutz and W. Weise, Phys. Lett. B 158, 239 (1985), F. Karsch and K. H. Mutter, Nucl. Phys. B 313, 541 (1989), N. Kawamoto, K. Miura, A. Ohnishi and T. Ohnuma, Phys. Rev. D 75, 014502 (2007).
[7] S. Klimt, M. Lutz and W. Weise, Phys. Lett. B 249, 386 (1990), Y. Hatta and T. Ikeda, Phys. Rev. D 67, 014028 (2003), B. J. Schaefer and J. Wambach, Nucl. Phys. A 757, 479 (2005), P. de Forcrand and O. Philipsen, JHEP 0701, 077 (2007).
[8] M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, Prog. Theor. Phys. 108, 929 (2002), T. Hatsuda, M. Tachibana, N. Yamamoto and G. Baym, Phys. Rev. Lett. 97, 122001 (2006), N. Yamamoto, M. Tachibana, T. Hatsuda and G. Baym, Phys. Rev. D 76, 074001 (2007).
[9] Z. Zhang, K. Fukushima and T. Kunihiro, Phys. Rev. D 79, 014004 (2009).
[10] T. Schafer and F. Wilczek, Phys. Rev. Lett. 82, 3956 (1999).
[11] G. Baym, T. Hatsuda, M. Tachibana and N. Yamamoto, J. Phys. G 35, 104021 (2008).
[12] K. Fukushima, Phys. Lett. B 591, 277 (2004), C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 014019 (2006), E. Megias, E. Ruiz Arriola and L. L. Salcedo, Phys. Rev. D 74, 065005 (2006), S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa and R. Ray, Phys. Rev. D 73, 114007 (2006), S. Roessner, C. Ratti and W. Weise, Phys. Rev. D 75, 034007 (2007), C. Ratti, S. Roessner and W. Weise, Phys. Lett. B 649, 57 (2007), C. Sasaki, B. Friman and K. Redlich, Phys. Rev. D 75, 074013 (2007), H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi and C. Ratti, Phys. Rev. D 75, 065004 (2007), Z. Zhang and Y. X. Liu, Phys. Rev. C 75, 064910 (2007), S. Roessner, T. Hell, C. Ratti and W. Weise, Nucl. Phys. A 814, 118 (2008), K. Kashiwa, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Lett. B 662, 26 (2008), Y. Sakai, K. Kashiwa, H. Kouno and M. Yahiro, Phys. Rev. D 77, 051901 (2008); Phys. Rev. D 78, 036001 (2008), H. Abuki, M. Cininalae, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Rev. D 77, 074018 (2008); Phys. Rev. D 78, 034034 (2008), K. Fukushima, Phys. Rev. D 77, 114028 (2008) [Erratum-ibid. D 78, 039902 (2008)]; arXiv:0901.0783 [hep-ph], H. Abuki, M. Cininalae, R. Gatto and M. Ruggieri, Phys. Rev. D 79, 034021 (2009), T. Hell, S. Roessner, M. Cristoforett and W. Weise, Phys. Rev. D 79, 014022 (2009), H. Abuki and K. Fukushima, Phys. Lett. B 676, 57 (2009).
[13] O. Kaczmarek and F. Zantow, Phys. Rev. D 71, 114510 (2005).
[14] L. McLerran and R. D. Pisarski, Nucl. Phys. A 796, 83 (2007), Y. Hidaka, L. D. McLerran and R. D. Pisarski, Nucl. Phys. A 808, 117 (2008).
[15] L. McLerran, K. Redlich and C. Sasaki, Nucl. Phys. A 824, 86 (2009).
[16] C. E. Detar and T. Kunihiro, Phys. Rev. D 59, 2805 (1999), Y. Nemoto, D. Jido, M. Oka and A. Hosaka, Phys. Rev. D 57, 4124 (1998), D. Jido, Y. Nemoto, M. Oka and A. Hosaka, Nucl. Phys. A 671, 471 (2000), H. c. Kim, D. Jido and M. Oka, Nucl. Phys. A 640, 77 (1998), D. Jido, T. Hatsuda and T. Kunihiro, Phys. Rev. Lett. 84, 3252 (2000).
[17] M. A. Stephanov, K. Rajagopal and E. Y. Shuryak, Phys. Rev. Lett. 81, 4816 (1998), S. Jeon and V. Koeh, Quark Gluon Plasma 3, Eds. R. C. Hwa and X. N. Wang, World Scientific Publishing, 2004.
[18] S. Ejiri, F. Karsch and K. Redlich, Phys. Lett. B 633, 275 (2006), B. Sticke, B. Friman and K. Redlich, Phys. Lett. B 673, 192 (2009), M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009), M. Asakawa, S. Ejiri and M. Kitazawa, arXiv:0904.2089 [nucl-th].
[19] C. Nonaka and M. Asakawa, Phys. Rev. C 71, 044904 (2005), M. Asakawa, S. A. Bass, B. Muller and C. Nonaka, Phys. Rev. Lett. 101, 122302 (2008).
[20] E. Nakano, B. J. Schaefer, B. Stokic, B. Friman and K. Redlich, arXiv:0907.1344 [hep-ph].
[21] C. Sasaki, B. Friman and K. Redlich, Phys. Rev. Lett. 99, 232301 (2007); Phys. Rev. D 77, 034024 (2008).
[22] B. J. Schaefer and J. Wambach, Phys. Rev. D 75, 085015 (2007); C. Sasaki, B. Friman and K. Redlich, Phys. Rev. D 75, 054026 (2007).
[23] N. Kaiser, P. de Homont and W. Weise, Phys. Rev. C 77, 025204 (2008).
[24] W. H. Dickhoff and C. Barbieri, Prog. Part. Nucl. Phys. 52, 377 (2004).