Research Article

A Novel Improved Maximum Entropy Regularization Technique and Application to Identification of Dynamic Loads on the Coal Rock

Chunsheng Liu¹ and Chunping Ren ²

¹Heilongjiang University of Science and Technology, Harbin 150022, China
²College of Mechanical and Electrical Engineering, Harbin Engineering University, Harbin 150001, China

Correspondence should be addressed to Chunping Ren; renchunpin@sina.com

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A new signal processing algorithm was proposed to identify the dynamic load acting on the coal-rock structure. First, the identification model for dynamic load is established through the relationship between the uncertain load vector, and the assembly matrix of the responses was measured by the machinery dynamic system. Then, the entropy item of maximum entropy regularization (MER) is redesigned using the robust estimation method, and the elongated penalty function according to the ill-posedness characteristics of load identification, which was named as a novel improved maximum entropy regularization (IMER) technique, was proposed to process the dynamic load signals. Finally, the load identification problem is transformed into an unconstrained optimization problem and an improved Newton iteration algorithm was proposed to solve the objective function. The result of IMER technique is compared with MER technique, and it is found that IMER technique is available for analyzing the dynamic load signals due to higher signal-noise ratio, lower restoration time, and fewer iterative steps. Experiments were performed to investigate the effect on the performance of dynamic load signals identification by different regularization parameters and calculation parameters, respectively. Experimental results show that the identified dynamic load signals are closed to the actual load signals using IMER technique combined with the proposed PSO-L regularization parameter selection method. Selecting optimal calculated parameters is helpful to overcome the ill-condition of dynamic load signals identification and to obtain the stable and approximate solutions of inverse problems in practical engineering. Meanwhile, the proposed IMER technique can also play a guiding role for the coal-rock interface identification.

1. Introduction

Identification problem of load sources, like the system parameter identification and dynamic characteristics modification, is an inverse problem of structural dynamic. Furthermore, the inverse problem has been developed as a hot subject which has been widely applied in the fields of many practical engineering problems, such as dynamic isolation, health monitoring, and fault diagnosis [1, 2]. The primary concern of mathematical workers and engineering technicians in the inverse problem derives from the development of the theory of solving the problems of universal existence and ill-posed problems in practical engineering applications in recent years [3, 4]. However, in many practical engineering problems today, for instance, the knowledge of dynamic loads in machinery dynamic systems is always required. Identification of dynamic loads on coal-rock structure has been a major concern in fatigue failure analysis of rotary machinery; once accurate dynamic load sources are obtained, it is possible to apply some advanced algorithms to analyze the characteristics of the dynamic load signals. In general, an input load signal can be directly measured by the force transducer, but sometimes it is difficult in calculating accurate results, and this would be mainly caused by the noise data of measured responses and the ill-conditioned characteristic of system; dynamic load
sources identification is classified as a kind of complex inverse problems with inherent ill-posedness and will not be resolved directly and accurately through some traditional mathematical methods [3, 5, 6]. Aiming to overcome this difficulty of instability, we consider that it is essential to develop a stabilization technique for effective solutions.

Thus, regularization techniques, which can deal with such inverse (ill-posed) problems, have been developed successfully since their appearance. The basic idea of the so-called regularization technique is to approximate the solution of the original problem by the solution of a family of well-posed problems close to the original problem [7–10]. In recent years, the maximum entropy regularization (MER) technique has been put forward one after another, which has been applied successfully in the wide areas of image reconstruction, signal processing, and load identification problem [11–13], and the advantages of using MER technique are as follows: (i) it can offer an unbiased way of obtaining information from the incomplete data; (ii) it implicitly has the nonnegativity constraint to the probability distribution [14]. Chiang et al. [15] adopted MER technique to solve the ill-posed nature of the dipolar signal in pulsed and to obtain the relatively stable solutions of certain inverse problems. Prot et al. [16] applied MER technique to the identification of wave distribution function for an ELF hiss event. You et al. [17] used MER technique for solving an ill-posed problem of moving force identification. Liu et al. [18] offered the dynamic load identification for stochastic structures based on the gegenbauer polynomial approximation and regularization method. Jackson et al. [19] adopted a known extension of the usual MER technique that can be applied to images consisting of pixels of unconstrained sign. Hofmann and Krämer [20] investigated the applicability of the method of MER technique to a specific nonlinear ill-posed inverse problem in a purely time-dependent model of option pricing. Gillet et al. [21] presented a maximum entropy technique to research on image reconstruction with the core-mantle boundary. Prot et al. [22] proposed an effective mollification MER technique to deal with a Cauchy problem of an elliptic equation in a multi-dimensional case. MER technique mentioned above can lay a firm foundation for the steady and accurate solutions of ill-posed problem [23].

However, MER technique is not perfect, it has been improved in a series of ways, and most of the major improvements are focused on the entropy function [24]. Mohammad-Djafari et al. [25] applied the improved entropy function combining the Lagrange method and conjugate gradient to process mass spectrometry data problems. P Du and Cui et al. [26] discussed an improved maximum entropy modeling to handle with multiclass categorization problems using the Newton iteration algorithm. In [27], by combining maximum entropy and Bayesian, a new maximum entropy regularization is constructed to distributed load acting on boundary structure. From the perspective of the penalty function for the improved entropy function mentioned, it is only the steepest descent technique from the viewpoint of numerical optimization, and the rate of convergence is relatively slow. In fact, the entropy function exerts different penalty on the unknown variables to stabilize the numerical solution, and the entropy function can be designed flexibly according to the characteristics of identified objects. It is generally known that different identification results will be obtained when applying different entropy functions.

In our study, a novel improved maximum entropy regularization (IMER) technique that can identify dynamic load acting on coal rock is determined. The method we propose is new to our research field. Significant improvements are made compared with our previous work [28].

(i) The entropy item of IMER technique is redesigned using the robust estimation method and the 1-norm estimation. The proposed objective function considers the robustness of the estimation and uses a different stabilizing item to stabilize.

(ii) An improved Newton iteration algorithm was employed to solve the objective function and by combining particle swarm optimization (PSO) and L-curve method, which was named as PSO-L method, was proposed to select the regularization parameter. It can effectively overcome the ill-posedness of dynamic load identification.

(iii) The method proposed does not require any a priori information on the model for the dynamic load identification, and only the displacement response measured by experiment is adequate for the identification algorithm.

(iv) Global convergence and stability of the solution for identified load model is proved to improve the precision and detailed information of dynamic load identification.

(v) The applicability of the proposed algorithm is effectively demonstrated through a practical engineering example. It is found that the algorithm we propose is completely effective for solving the large-scale ill-posed problem, which is better than that of other previous works, and can effectively overcome the ill-posedness of dynamic load identification.

In the present work, we aimed at identifying dynamic load sources acting on coal-rock structure combining a novel improved maximum entropy regularization (IMER) technique and an improved Newton iteration algorithm. The rest of this paper is organized as follows. In Section 2, the identification model is established through the relationship between the uncertain load vector and the assembly matrix of the responses measured by the machinery dynamic system. In Section 3, a novel improved maximum entropy regularization (IMER) technique, an improved Newton iteration algorithm, and a PSO-L method was proposed, respectively. In Section 4, experimental arrangements and results discussion. Some important concluding remarks are made in Section 5.

2. Identification Model of Dynamic Load Signals

An abridged general view of interaction dynamic load between the pick and coal-rock structure is plotted in Figure 1.
For deterministic MDOF structures, the kinetic equation in case of subjecting to the dynamic random load signals can also be described as follows [29]:

$$[M][\ddot{y}(t)] + [C][\dot{y}(t)] + [K][y(t)] = [S(x, y, z, t)],$$  \tag{1}

where $M$, $C$, and $K$ represent the mass matrix, damping matrix, and stiffness matrix, respectively; $S(x, y, z, t)$ denotes the load column vector; $\dot{y}(t)$, $\ddot{y}(t)$, and $\dddot{y}(t)$ are displacement, velocity, and acceleration response vectors, respectively.

Since the mass matrix and stiffness matrix can be used to obtain the natural frequency and modal vector of the structure, the displacement of the structure can be expressed as

$$y(t) = \sum_{i=1}^{n} \Phi_i q_i(t) = \Phi Q,$$  \tag{2}

where $\Phi$ is matrix of dynamic mode and $\Phi = [\Phi_1, \Phi_2, \ldots, \Phi_n]$; and $Q$ is the displacement vector and time function in the generalized coordinates, $Q = [q_1, q_2, \ldots, q_n]^T$.

By means of equations (1) and (2), the dynamic equations represented by the physical coordinates are transformed into the modal coordinate system, and the $n$ decoupled modal equations are obtained in the following form:

$$q''_i + 2\xi_i\omega_i q'_i + \omega_i^2 q_i = \frac{F_i(t)}{M_i}, \tag{3}$$

where $\xi_i$ is the $i$th order modal damping ratio; $M_i$ and $F_i(t)$ are the generalized mass and the generalized load, respectively, $M_i = \Phi_i^T M \Phi_i$, and $F_i(t) = \Phi_i^T S(t)$.

The initial velocity and initial displacement of the system are assumed to be $x_i$ and $x_{id}$, respectively. So, $q_i$ in equation (2) can be expressed as follows:

$$q_i(t) = q_{i1}(t) + q_{i2}(t). \tag{4}$$

It is well known that $q_{i1}(t)$ can be obtained by the homogeneous equation of equation (1), which can be described as follows [30]:

$$q_{i1}(t) = e^{-\xi_i \omega_i t} \left( q_{i0}(t) \cos \omega_{id} t + \frac{q_{i0} + \xi_i \omega_i q_{id}(t)}{\omega_{id}} \sin \omega_{id} t \right), \tag{5}$$

where $q_{i0}$ and $q_{id}$ denote the corresponding values in modal coordinates, which can be expressed as follows:

$$q_{id} = \frac{\Phi_i^T M x_i}{M_i} \tag{6}$$

$q_{i2}$ can be expressed as follows form because it has nothing to do with the initial conditions of the system:

$$q_{i2}(t) = \frac{1}{M_i \omega_{id}} \int_0^t F_i(t) e^{-\xi_i \omega_i (t-t')} \sin \omega_{id} (t-t') dt'. \tag{7}$$

The displacement of the system can be expressed as follows:

$$y = y^0 + y^*, \tag{8}$$

where $y^0 = \sum_{i=1}^{n} \Phi_i q_{id}$ and $y^* = \sum_{i=1}^{n} \Phi_i q_{i2}$.

The system load is identified according to the known displacement, and the displacement is represented as follows:

$$y^* = y - y^0. \tag{9}$$

The following expressions can be obtained by substituting equations (5) and (7) into the following equation:

$$y^* = \int_0^t G(t-t') S(t') dt'. \tag{10}$$

According to the superposition principle of linear time invariant system, the forward model of inverse problem of load identification can be expressed as a convolution integral, namely, the modal response $y^*(t)$ can be expressed as a convolution integral of the modal load function and the corresponding Green’s kernel in time domain:

$$y^*(t) = \int_0^t G(t-t') S(t') dt'. \tag{11}$$

Hence, the specific form of the unit impulse response function can be obtained by comparing equations (1) and (2):

$$G(t) = \sum_{i=1}^{n} \frac{\Phi_i \Phi_i^T}{M_i \omega_{id}} e^{-\xi_i \omega_i t} \sin \omega_{id} t. \tag{12}$$

Let $\{y^*(t)\} = \{y^*(t) \cdot y^*(t) \cdot \ldots \cdot y^*(m)(t)\}^T$, $\{S(t)\} = \{S_1(t) \cdot S_2(t) \cdot \ldots \cdot S_m(t)\}^T$, and $\{G(t)\} = \{g_1(t) \cdot \ldots \cdot g_m(t)\}^T$, then we can discretize the convolution integral (11) into $n$ equally spaced sample points in the time domain. Finally, equation (11) is transformed into a matrix form that can be expressed as follows:

$$\begin{bmatrix} y_1^* \\ y_2^* \\ \vdots \\ y_m^* \end{bmatrix} = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 \\ g_2 & g_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_m & g_{m-1} & g_{m-2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{bmatrix}. \tag{13}$$

Figure 1: A schematic of interaction dynamic load between the pick and coal-rock structure.
or simply noted as
\[ Y = GS. \]  

Considering the noisy responses, equation (13) can be rewritten as follows:
\[ Y^d = GS_{\text{true}} + e, \]  

where \( S_{\text{true}} \) is the true load, \( e \) is the unknown noise, and \( e = l \cdot \text{std}(f(t)) \cdot \text{rand} \), in which \( l \) is a parameter to control the level of noise disturbance, \( \text{std}(f(t)) \) is the standard deviation of \( f(t) \), and \( \text{rand} \) denotes a random number within \([-1, 1]\).

To reconstruct \( S_{\text{true}}(t) \), we need to get \( Y^d(t) \) and \( G(t) \); moreover, it was found that formula (15) is the first kind of integral equation with parameters, which is a typical ill-posed problem. In some cases, small changes in the right-hand side \( e \) can cause arbitrarily large changes in the solution. In other words, they are usually sensitive to errors, such as data errors, discretization error, and so on, which can cause large deviations of identified results in inverse problems, thus attempt to solve equation (15) directly yielding solution vectors that are almost hopelessly disturbed with noise using some traditional mathematical methods [31, 32]. Hence, in the following section, a new combined regularization technique will be established to solve this ill-posed problem.

3. Algorithm Analysis

It is well known that identification of dynamic random load signals is a typical ill-posed problem, and its solution is very unstable [18, 33]. The methods that ensure the stability of a solution while enhancing the quality of the identified load should be applied to obtain a more meaningful identification result [34]. The regularization techniques are effective methods to deal with the ill-posed problems. In the past few years, maximum entropy regularization (MER) technique has been put forward one after another, which has been applied successfully in the wide area of practical engineering problem. However, MER technique is not very perfect; it has some disadvantages. Hence, in this work, an improved maximum entropy regularization (IMER) technique is studied.

3.1. Maximum Entropy Regularization (MER). It is generally known that maximum entropy regularization (MER) technique has in the past few years been widely adopted. The essence of the technique is to transform the solving of equation (15) into an optimization problem [35]:
\[ \min J(S) = \|GS - Y\|^2 + \lambda \sum_{i=1}^{n} S_i \ln S_i, \]  

where \( \lambda \) represents the regularization parameter.

Let \( \Omega(S) = \sum_{i=1}^{n} S_i \ln(S_i) \), and it is called the entropy item [36]. Thus, equation (16) can be replaced by the following:
\[ \min J(S) = \|GS - Y\|^2 + \lambda \Omega(S). \]  

3.2. Determination of a Novel Improved Maximum Entropy Regularization (IMER). In some circumstances, such as complex identified objects, MER technique is not very perfect, and it has some disadvantages of poor quality of identification. Hence, in this paper, the entropy item of MER technique is redesigned using the robust estimation method, and the elongated penalty function according to the ill-posedness characteristics of load identification, which was named as a novel improved maximum entropy regularization (IMER) technique, was proposed to process the dynamic random load signals.

It is obvious that different identification results are obtained when different entropy items are adopted. In this section, a function is used to design the entropy item:
\[ \Omega(S) = \sum_{i=1}^{n} S_i \ln\left(1 + \ln(1 + |S_i|^p)\right), \]  

where \( 0 < p_i < 2 \).

Since the absolute value function is not differentiable at points where its value is zero, the following expression is used to approximate [37, 38]:
\[ |S| = (S^2 + \xi^{1/2}). \]

Hence, equation (18) can be approximated by
\[ \Omega(S) \approx \sum_{i=1}^{n} S_i \ln\left(1 + \ln\left(1 + (S_i^2 + \xi)^{p/2}\right)\right), \]  

where \( \xi \) is a predetermined small parameter, and in this paper, it has been given a value of \( 10^{-10} \).

According to the above analysis, a novel improved entropy item for dynamic random load signal identification can be obtained, and it can be described as follows:
\[ \min J(S) = \|GS - Y\|^2 + \lambda \sum_{i=1}^{n} S_i \ln\left(1 + \ln\left(1 + (S_i^2 + \xi)^{p/2}\right)\right). \]  

3.3. Solving of the Optimized Objective Function. Equation (21) is considered as a class of unconstrained optimization problem; moreover, a series of optimization algorithms, which can deal with such optimization problems, have been developed successfully since their appearance. In this work, an improved Newton iteration algorithm was proposed to minimize equation (21); according to the basic idea of Newton iteration algorithm, new iteration sequences are constructed:
\[ S_{k+1} = S_k - \mu \left( \frac{j S_k}{j^2 S_k} \right), \]  

where \( \mu \) is the undetermined coefficient.
The undetermined coefficient \( \mu \) is determined as follows: the iterative function corresponding to the iterative sequence (22) is replaced by the following:

\[
\phi(S) = S - \mu \frac{J(S)}{J'(S)}
\]  

(23)

In order to improve the convergence rate, let \( \phi'(S) = 0 \). The undetermined coefficient \( \mu \) is obtained:

\[
\mu = \frac{\left[ J'(S) \right]^2}{\left[ J'(S) \right]^2 - J(S)J''(S)}
\]  

(24)

Then, equation (23) can be replaced by the following:

\[
\varphi(S) = S - \frac{J'(S)J(S)}{\left[ J'(S) \right]^2 - J(S)J''(S)}
\]  

(25)

So, an improved Newton iteration formula is obtained by

\[
S_{k+1} = S_k - \frac{f'(S_k)S_k}{\left[ f'(S_k) \right]^2 - f(S_k)f''(S_k)}, \quad (k = 0, 1, 2, \ldots).
\]  

(26)

The IMER technique was proposed to identify the dynamic random load signal. Hence, IMER technique procedure is described as follows:

**Step 1.** The initial approximation \( S_0 \) of the root of the equation and its accuracy \( \varepsilon \) are obtained

**Step 2.** Calculating \( S_1 \)

**Step 3.** Determining whether to meet the accuracy requirements: \( |S_1 - S_0| < \varepsilon; \) if it is established, which will transfer to step (4), if it is not established, and \( S_0 = S_1 \), which will transfer to step (2)

**Step 4.** The output can satisfy the root of the precision and the computation will be finished

### 4. Experimental Results and Discussion

#### 4.1. Experimental Setup for Machinery Dynamic System

In this paper, an improved method for selecting regularization parameters by combining particle swarm optimization (PSO) and L-curve method, which was named as PSO-L method, was proposed to select the regularization parameter \( \lambda \).

It is well known that the L-curve method is a parametric plot of \( (\|GS - Y\|, \|S\|) \), where \( \|S\| \) and \( \|GS - Y\| \) can measure the size of the regularized solution and the corresponding residual [41, 42]. The optimal value for the regularization parameter is considered to be the one that corresponds to the corner of the curve, but calculation of L-curve method is very large and consuming more CPU. Hence, in this section, an improved method for selecting regularization parameters was proposed to select the regularization parameter \( \lambda \). Particle swarm optimization (PSO) was invented by Dr. Eberhart and Dr. Kennedy, which is an iterative optimization tool [43, 44]:

\[
v = \omega \cdot v + c_1 \cdot \text{rand} \cdot (p_{\text{best}} - x) + c_2 \cdot \text{rand} \cdot (g_{\text{best}} - x),
\]  

(27)

\[
x = x + v,
\]  

(28)

where \( v \) is the velocity of particle, \( x \) is the current position of particle, \( c_1 \) and \( c_2 \) represent learning factor, respectively, \( c_1 = c_2 = 2 \), \( \omega \) represents the weighting factor, \( p_{\text{best}} \) and \( g_{\text{best}} \) represents the individual extreme and global extremum, respectively, and \( \text{rand} \) is random number of setting 0.1 to 0.9.

The greatest advantage of particle swarm optimization algorithm is to find the global optimal solution. In this paper, by combining particle swarm optimization (PSO) and L-curve method, the PSO-L method is used to select the regularization parameter, and the optimization model is described as follows:

\[
\min_{PSO} L(\lambda) = \|GS - Y\|^2 \|S\|^2.
\]  

(29)

Hence, the detailed steps of the proposed PSO-L method are described as follows:

**Step 1.** Initializing regular parameters and setting the initial position and velocity of particle \( \lambda \) with random generate from 0 to 1

**Step 2.** Calculating the fitness value according to formula (29)

**Step 3.** Updating the position and velocity of particles according to formulas (27) and (28)

**Step 4.** Checking the termination condition and outputting the best regular parameters, then, terminating the iteration, or going back to step (2)

To sum up the above discussion, we have provided the flow chart of the whole computational algorithm, which is shown in Figure 2.
useful for rotary machinery systems, and it has another difference in comparison with the other measured devices that data measurement is more convenient because the cutting arm, the installation angle, and the cutting speed can be changed according to the actual conditions. However, there exists difficulty of environmental reflect effect, and data measurement contains a certain amount of noise during experiments, and the actual responses cannot be obtained precisely. In the following section, the identified loads are obtained based on the corresponding identified modal model, and then it can be compared with the experimental load.

4.2. The Comparisons of Identification Techniques. For performance analysis of dynamic load signal identification techniques used, the performance measurement metrics for dynamic load signals are defined as following:

Root mean-square-error (RMSE) [45] can be calculated by

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{n} [s_{\text{mea}} - s_{\text{iden}}]^2}, \quad (30)$$

where $s_{\text{mea}}$ is the measured load signals, $s_{\text{iden}}$ is the identified load signals, and $N$ is number of data points.
The signal-noise ratio (SNR) \[31\] can be calculated by
\[
\text{SNR} = 10 \log_{10} \left( \frac{P_{\text{signal}}}{P_{\text{noise}}} \right),
\]
where \(P_{\text{signal}} = 1/N \sum_{i=1}^{N} s_{\text{iden}}^2\) and \(P_{\text{noise}} = 1/N \sum_{i=1}^{N} (s_{\text{meas}} - s_{\text{iden}})^2\).

In this section, first, Figure 5 shows that displacement response was obtained by the dynamic testing system. Then, identified dynamic load signals can be given based on measured displacement response using the identified modal model and the proposed IMER technique.

The identified results were compared with the measured results by experiments, and the measured dynamic load signals acting on coal-rock structure are obtained in this study, and the experimental conditions are shown in Table 2.

Hence, the identification results are formed in Figures 6 and 7 by assembling the analysis results of each technique.

It was obtained that the dynamic load signals are easily identified with application of above identification techniques, and the identified dynamic load signals are obviously observed using the regularization parameter selected by L-curve and PSO-L method as seen in Figures 6 and 7, respectively.

Moreover, in Figures 6 and 7, it can be seen that two regularization techniques can identify the dynamic load signals. To some extent, the performance of dynamic load signals identification has a certain difference.

In this investigation, it was found that SNR using the IMER technique is obviously higher compared with the MER technique, but RMSE calculated by the MER technique is higher than the IMER technique from Table 3.

However, Table 3 shows that it can be observed that SNR using PSO-L method is higher than that of using L-curve. But RMSE, restoration time, and iteration steps using PSO-L method are lower than that of using L-curve. To sum up, we can know that IMER technique is associated with minimum RMSE, restoration time, and iteration steps and maximum SNR in comparison to other schemes.

Hence, we can conclude that IMER technique combined with PSO-L method is available to identify the characteristic information of dynamic load signals.

4.3. The Effect of the Different Calculated Parameter.
Equation (21) shows the calculated parameter \(p_i\) directly affects the performance of dynamic load signals identification using the proposed regularization technique.

The IMER technique combined with PSO-L method was used to identify the dynamic load signals acting on the coal-rock structure. Parameter \(\xi\) was set up \(10^{-10}\), and parameter \(p_i\) was set up 0.01, 0.05, 0.1, 0.15, respectively. And examples of the identified load signals are shown in Figure 8.

In Figure 8, it is found that the identified load signals are obviously unsatisfactory when the different parameter \(p_i\) becomes larger.

Although the loads can be identified, the effect is not the same. In order to analyze the influence of parameter \(p_i\) on the identification results, SNR and RMSE between the identified dynamic load signals and the measured dynamic load signals as function under different parameter \(p_i\) are plotted in Figures 9(a) and 9(b).

The SNR of load signals between the identified loads and the measured loads is first decreased with parameter \(p_i\) increasing from Figure 9(a). However, in Figure 9(b), we can observe that RMSE is increased with parameter \(p_i\) increasing, which illuminates that the calculated parameters \(p_i\) using IMER technique have some influence on the identified results. Therefore, in the later application, the value of the calculated parameters \(p_i\) is smaller, which will be beneficial to the identified result.
4.4. The Effect of the Parameter $\xi$. Equation (20) shows that the parameter $\xi$ directly affects the performance of dynamic load signals identification using the proposed regularization technique.

The IMER technique combined with PSO-L method was used to identify the dynamic load signals acting on the coal-rock structure. Parameter $p_i$ was set up 0.0001, and parameter $\xi$ was set up $10^{-1}$, $10^{-5}$, $10^{-10}$, $10^{-15}$, $10^{-20}$, respectively, and examples of the identified load signals are shown in Figure 10.

In Figure 10, although the loads can be identified, the effect is not the same. And it is found that the identified load signals are obviously unsatisfactory when the different parameter $\xi$ was set up $10^{-1}$, $10^{-5}$, and $10^{-15}$.

In order to analyze the influence of the parameter $\xi$ on the identification results, SNR and RMSE between the
identified dynamic load signals and the measured dynamic load signals as function under different parameter $\xi$ are plotted in Table 4.

In Table 4, the SNR of load signals between the identified loads and the measured loads is first increased when $\xi \in (10^0, 10^{-10})$, then it is decreased when $\xi \in (10^{-10}, 10^{-20})$. However, we can observe that RMSE is decreased when $\xi \in (10^0, 10^{-10})$, and then it is increased when $\xi \in (10^{-10}, 10^{-20})$, which illuminates that the parameter $\xi$ using IMER technique has the biggest SNR and the smallest

Figure 8: The identified results under different parameter $p_i$: (a) 0.01; (b) 0.05; (c) 0.1; (d) 0.15.

Figure 9: The effect of different parameter $p_i$: (a) SNR and (b) RMSE.
Figure 10: The identified results under different parameter $\xi$: (a) $\xi = 10^0$; (b) $\xi = 10^{-1}$; (c) $\xi = 10^{-5}$; (d) $\xi = 10^{-10}$; (e) $\xi = 10^{-15}$; (f) $\xi = 10^{-20}$. 
RMSE when $\xi = 10^{-10}$. That is, there is an optimal parameter ($\xi = 10^{-10}$).

5. Conclusions

In this paper, the identification model for dynamic load acting on coal-rock structure is established through the relationship between the uncertain load vector and the assembly matrix of the responses measured by machinery dynamic system. Then, the entropy item of maximum entropy regularization (MER) is redesigned using the robust estimation method and the elongated penalty function according to the ill-posedness characteristics of load identification, which was named as a novel improved maximum entropy regularization (IMER) technique, was proposed to process the dynamic load signals. Finally, the load identification problem is transformed into an unconstrained optimization problem and an improved Newton iteration algorithm was proposed to solve the optimized objective function. The results show that the proposed IMER technique is associated with minimum RMSE, minimum restoration time, and minimum iterative steps and maximum SNR in comparison to other schemes, and the PSO-L method is available for selecting the regularization parameter. Thus, we can safely conclude that the IMER technique and PSO-L method are combined to overcome the ill-condition of dynamic load identification and to obtain the stable and efficient solutions of inverse problems in practical engineering.

The different regularization parameters and calculation parameters $p_i$ directly influence on SNR and RMSE using the proposed IMER technique. The results show that the calculated parameters $p_i$ using IMER technique have some influence on the identified results. Therefore, in the later application, the value of the calculated parameters $p_i$ is smaller, which will be beneficial to the identified result. Hence, experimental examples demonstrate the validity and accuracy of the proposed IMER technique.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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