Limits of the $D$-brane action

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ABSTRACT

For background geometries whose metric contain a scale $\gamma$ we reformulate the Born-Infeld $D$-brane action in terms of $\varepsilon \equiv \gamma/(2\pi\alpha')$. This may be taken as a starting point for various perturbative treatments of the theory. We study two limits that arise at zeroth order of such perturbations. In the first limit, that corresponds to the $g_s \to \infty$ with $\varepsilon$ fix, we find a ”string parton” picture, also in the presence of some background $RR$-fields. In the second limit, $\varepsilon \to 0$, we find a topological model.

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1 Introduction

In this brief letter we study two limits of the Born-Infeld $D$-brane action. We base the treatment on an expansion in a rescaled tension $\varepsilon$, and include a full background.

We first take a new look at a "strong coupling" limit ($g_s \to \infty$). This is intended as a supplement to the discussion in [1] and [2] where the $D$-brane world volume was shown to be foliated by string world sheets in this limit. The result is a corroboration of the foliation picture plus the modified equations involving the background $n$-forms. In particular, we give an example showing that the allowed "string partons" are either infinite (which might correspond to $D1$-branes), closed or open strings ending on other $D$-branes (sources of the $RR$-fields). We also show that the string theory must be of type $IIB$.

The second limit we present is $\varepsilon \to 0$. Here we give an $\varepsilon$ expansion to first order and discover that the leading term is topological, i.e. independent of the metric. For the case when the dilaton and the 0-form are constant it is proportional to the Chern class of the $U(1)$-field.

We suggest that both these limits might serve as suitable starting points for investigating $D$-branes order by order in a parameter expansion. To do so one needs to specify the background, however, and we leave this as a topic for future studies.

2 Background

The long wavelength limit of $D$-brane dynamics is described by the Born-Infeld action

$$S_{BI}(T_p) = T_p \int d^{p+1}\xi e^{-\Phi'} \sqrt{-\det(\gamma_{ij} + F_{ij})},$$

where $\gamma_{ij} \equiv \partial_i X^\mu \partial_j X^\nu G_{\mu\nu}(X)$ is the metric on the world-sheet induced from a background metric $G_{\mu\nu}$ and $F_{ij}$ is a world-volume two-form,

$$F_{ij} \equiv 2\pi \alpha' \partial_i A_j + B_{ij}. \tag{2.2}$$

Here $B_{ij} \equiv \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}$ is the pull-back of the (dimensionless) background Kalb-Ramond field. The $p$-brane tension $T_p$ is related to the fundamental
string tension \( T \equiv (2\pi\alpha')^{-1} \) and the string coupling constant \( g_s \equiv e^{\Phi_\infty} \) by

\[
T_p \equiv \frac{1}{(2\pi)^p g_s (\sqrt{\alpha'})^{p+1}},
\]

with \( \Phi_\infty \) being the dilaton expectation value and \( \Phi' \equiv \Phi - \Phi_\infty \).

We shall be interested in the case when the background metric \( G_{\mu\nu} \) contains a parameter \( \gamma \) of dimension length\(^2\). As described in \cite{3} this can then be used to introduce a perturbative scheme in the rescaled string tension

\[
\varepsilon \equiv \frac{\gamma}{2\pi\alpha'}.
\]

The parameter \( \gamma \) may, e.g., be (the square of) the radius of (anti) deSitter space. In general \( \gamma \) will be chosen from the parameters in the string moduli space, and this choice will not be unique.

Rescaling the fields according to

\[
\tilde{\xi}^i = \gamma^{-\frac{1}{2}} \xi^i, \quad \tilde{X}^\mu = \gamma^{-\frac{1}{2}} X^\mu, \quad \tilde{A}_\mu = \gamma^{\frac{1}{2}} A_\mu,
\]

and including the additional coupling to the various (dimensionless) antisymmetric forms \( C_{\mu_1...\mu_k} \), the D-brane action in terms of dimensionless fields reads

\[
S = \tilde{T}_p \left[ \int d^{p+1}\tilde{\xi} e^{-\Phi'} \sqrt{-\det(\tilde{\gamma}_{ij} + \tilde{\mathcal{F}}_{ij})} + g_s \int \tilde{C} \wedge e^{\tilde{\Phi}} \right],
\]

where

\[
\tilde{T}_p \equiv \frac{\varepsilon^{\frac{p+1}{2}}}{g_s (2\pi)^{\frac{p-1}{2}}}, \quad \tilde{\mathcal{F}}_{ij} = \frac{1}{\varepsilon} \tilde{\partial}_j \tilde{A}_i + \tilde{B}_{ij}.
\]

The action \( \ref{2.5} \) is written in the string frame. The dilaton (coupling constant) dependence of the brane-action is frame dependent. We choose to consider the string frame since here the mass of fundamental strings is \( \mathcal{O}(1) \) in the string coupling and thus unaffected by our strong coupling limit. Other limits have been considered, however \cite{4}.

### 3 A strong coupling limit

In this section we will study a limit of the D-brane action when \( g_s \to \infty \) in the context of Type \( IIB \) string theory.

\footnote{Calling \( \varepsilon \) the tension is really a misnomer. It is a dimensionless entity and only related to the fundamental string tension via a rescaling. We hope that this will not cause any confusion.}
In general, to investigate the strong coupling limit of string- or M theory one would like to consider the limit of the action (2.5) for large $g_s$ in conjunction with a transformation of the background \[5\]. We will address the simpler problem of taking the limit $g_s \to \infty$, or more precisely $\varepsilon^{\frac{p+1}{2}} g_s << 1$, $\varepsilon = \text{const}$, in a fixed background.

The restriction to Type IIB theory is now evident from the following: In Type IIA string theory, because of its 11D M-theory origin, the string coupling $g_s$ is related to the radius $R_{11}$ of the compactified dimension through $g_s^2 \sqrt{\alpha'} = R_{11}$. Hence $g_s^2 = \varepsilon$, (identifying $\gamma = R_{11}^2 / 2\pi$ in the definition of $\varepsilon$ above), and it is inconsistent to let $g_s \to \infty$ keeping $\varepsilon$ fixed.

Using the method employed in \[1\] and \[2\], we find the desired limit to be (dropping tildes),

$$S = \frac{1}{2} \int d^{p+1} \xi \left[ V^i W^j (\gamma_{ij} + F_{ij}) \right] + \varepsilon^{\frac{p+1}{2}} \int C \wedge e^F, \quad (3.7)$$

where $V^i(\xi)$ and $W^j(\xi)$ are world volume vector densities. This may be considered as the zeroth order term in an expansion of the action (2.5) in powers of $g_s^{-1}$. We will study it in its own right, though.

The $V^i$ and $W^j$-field equations derived from (3.7) read

$$W^j(\gamma_{ij} + F_{ij}) = 0$$
$$V^i(\gamma_{ij} + F_{ij}) = 0, \quad (3.8)$$

and the integrability condition for these equations is

$$\det(\gamma_{ij} + F_{ij}) \equiv \det(\gamma_{ij} + B_{ij} + \varepsilon^{-1} F_{ij}) = 0. \quad (3.9)$$

To gain a first qualitative understanding of this relation, we set $B = 0$. In this case it is equivalent to

$$\det(\delta^i_j + \varepsilon^{-1} F^i_j) = 0. \quad (3.10)$$

In analogy to the usual treatment of the Born-Infeld action one might be tempted to employ a weak gradient expansion \[3\] and expand (3.10) in a power series in $|\partial A| << 1$. However, as is easily seen, this implies that

\[5\]We shall not be interested in the trivial solution $V^i = W^i = 0$. This corresponds to a theory with an WZ action given by the second term on the r.h.s. of (3.7).
$\varepsilon^2 \propto trF^2 << 1$, which is not true in general. E.g., when the background is Minkowski space we have $\varepsilon \propto \infty$. We thus learn that the weak gradient expansion is not in general viable.

### 3.1 Solving the VW-equations

Although the equation (3.9) says that the combined matrix $\gamma + \mathcal{F}$ is degenerate, this is not so for the induced metric $\gamma$ itself. We may thus introduce a vielbein $e^i_a$ and its inverse $e^a_i$ on the world-volume according to

$$e^a_i e^b_j \eta_{ab} = \gamma_{ij} a, b = 0, \ldots, p,$$

(3.11)

where $\eta_{ab}$ is the $p+1$-dimensional Minkowski metric. In [4] it was found that the first part of the action (3.7) has a two-dimensional Lorentz structure. This may be displayed by a redefinition of $V^i$ and $W^i$. With $e^{-1}$ being the determinant of the inverse vielbein, we use the $SO(p, 1)$-tangent space group to write\textsuperscript{4}

$$V^i = (e^i_0 + e^i_1) \sqrt{e^{-1}} \equiv (\tilde{e}^i_0 + \tilde{e}^i_1),$$
$$W^i = (e^i_0 - e^i_1) \sqrt{e^{-1}} \equiv (\tilde{e}^i_0 - \tilde{e}^i_1),$$

(3.12)

thus breaking the tangent-space group down to $SO(1, 1) \times SO(p-2)$. In this gauge, (3.8) reads

$$\mathcal{F}_{ij} e^i_0 = \gamma_{ij} e^i_1, \quad \mathcal{F}_{ij} e^j_1 = \gamma_{ij} e^i_0,$$

(3.13)

or, equivalently,

$$\left(\mathcal{F}^2\right)^i_j e^j_{0,1} = e^i_{0,1}$$

(3.14)

with the integrability condition

$$\det \left( \delta^i_j - \left(\mathcal{F}^2\right)^i_j \right) = 0.$$

(3.15)

Expanding the two-form $\mathcal{F}$ in the bivector basis

$$\mathcal{F} = \frac{1}{2} \mathcal{F}_{ij} d\xi^i \wedge d\xi^j = \frac{1}{2} \mathcal{F}_{ab} e^a \wedge e^b,$$

(3.16)

\textsuperscript{4}This results if we think of Minkowski space as the $R \rightarrow \infty$ limit of de Sitter space. In an alternative scheme discussed in [3] $\varepsilon$ is of order 1 which also violates our inequality.

\textsuperscript{5}We introduce the tilde notation for later convenience.
(3.13) is solved by
\[ \mathcal{F} = \mathcal{F}^\parallel + \mathcal{F}^\perp = 2e^0 \wedge e^1 + \mathcal{F}_{AB} e^A \wedge e^B, \quad A, B = 2, \ldots, p, \quad (3.17) \]
or, equivalently,
\[
\begin{align*}
\nabla_{[0A]} &= \varepsilon(1 - B_{01}), \quad \nabla_{[0A]} = -\varepsilon B_{0A}, \\
\nabla_{[1A]} &= -\varepsilon B_{1A}, \quad \mathcal{F}_{AB} = \varepsilon^{-1} \nabla_{[AB]} + B_{AB},
\end{align*}
\]
(3.18)

where \( \nabla \equiv \partial + \omega \cdot M \) is the covariant derivative, \( \omega \) being the spin-connection and \( M \) the Lorentz-group generator.

The integrability condition (3.15) is of course identically satisfied by (3.17).

We thus learn from the \( V \) and \( W \) equations that the 2-form \( \mathcal{F} \) splits into one part which lies in the 2D tangent space spanned by \( e^0 \) and \( e^1 \), and one part which lies entirely in the orthogonal part spanned by the \( e^A \)'s. As seen from (3.18), when \( B = 0 \), this implies that the \( D \)-brane electric field is constant and lies in the \( e_1 \) direction.

### 3.2 The A-equations

The \( A \) field equations that follow from (3.7) are
\[ \partial_i \left( V^i W^j \right) = \mathcal{J}^j \]
(3.19)

where \( \mathcal{J}^i \) represents the contribution from the background \( RR \) gauge fields and is given by
\[ \mathcal{J}^i \equiv -\varepsilon^{(p+1)/2} \frac{1}{(s-1)!2^{s-1}} \star H^i_{k_1 \ldots k_{s-1}} \mathcal{F}_{k_1 \ldots k_{s-1}}, \quad (3.20) \]

where \( \star H^i_{\ldots}^{\ldots} \) is the dual of the pull-back of the field strength of the \( C^{(n)} \) form.

In [1] and [2] the structure of the theory was shown to be simplified in the following diffeomorphism gauge:
\[ \partial_i V^i = \partial_i W^i = 0. \quad (3.21) \]
In that gauge (3.19) may be rewritten as

\[ [V, W]^i = \mathcal{J}^i \]  

(3.22)

where \([\ldots]\) denotes the Lie-bracket.

The equations (3.19) are highly non-linear. To proceed further we need to make some simplifying assumptions, and we first consider the case when the background \(RR\) gauge fields are zero \((C^{(n)} = 0)\). In that case (3.21) says that \([V^i \partial_i, W^j \partial_j] = 0\) which means that those directions may be chosen as coordinate-directions \([\cdot]\). In view of (3.12) we choose to use the \(\tilde{e}\) directions instead. Explicitly, we thus use a diffeomorphism gauge where

\[ e_a^i = \left\{ e^1_0 \delta^i_0, e^1_1 \delta^i_1, e_A^i \right\}, \quad (3.23) \]

which has the inverse

\[ e_i^a = \left\{ e^{-\frac{1}{2}} \delta^a_0, e^{\frac{1}{2}} \delta^a_1, \delta^a_M \right\}, \quad M = 2, \ldots, p \]

\[ e_A^M e_A^N = \delta^N_M, \quad e_A^M e_M^B = \delta^B_A, \quad e_0^1 = -e^{\frac{1}{2}} e_A^A e_0^1. \quad (3.24) \]

Again setting \(B = 0\) and returning to equation (3.13), we find

\[ F_0^i = \varepsilon \delta^i_0, \quad F_1^i = \varepsilon \delta^i_1. \quad (3.25) \]

We see that (the mixed components of) the \(U(1)\) curvature now splits into one part in the 2D-plane spanned by \(\xi^0\) and \(\xi^1\), and one part in the complementary directions spanned by the \(\xi^M\)'s. Alternatively, the covariant components are, (setting \(e = 1\),)

\[ F_{i0} = \varepsilon \partial_i X^\mu \partial_0 X^\nu G_{\mu \nu}, \quad F_{i1} = \varepsilon \partial_i X^\mu \partial_1 X^\nu G_{\mu \nu}, \quad (3.26) \]

with \(F_{AB}\) arbitrary. This is the \(\mathcal{F}\)-solution. This gauge also leads the following expressions that result from (3.11):

\[ X^\mu \dot{X}^\nu G_{\mu \nu} + X'^\mu X'^\nu G_{\mu \nu} \equiv \gamma_{00} + \gamma_{11} = \eta_{00} + \eta_{11} = 0 \]

\[ \dot{X}^\mu X'^\nu G_{\mu \nu} \equiv \gamma_{01} = \eta_{01} = 0 \quad (3.27) \]

i.e., the Virasoro constraints, (parametrized by \(\xi^i, i \neq 0, 1\), in conformal gauge.

\[ ^8\text{We may also use the residual symmetry preserving (3.21) to set } e = 1. \]
3.3 The $X^\mu$-equations

The $X^\mu$ field equations that follow from (3.7) are

$$\begin{align*}
\partial_i \left( (V^i W^j) G_{\mu\nu} + V^i [W^j] B_{\mu\nu} \partial_j X^\nu \right) \\
- V^i W^j \partial_i X^\rho \partial_j X^\nu (G_{\rho\nu} + B_{\rho\nu})_{,\mu}
\end{align*}$$

$$= I_\mu + K_\mu,$$  \hspace{1cm} (3.28)

where $I_\mu$ contains the contribution from the RR gauge fields

$$I_\mu \equiv \varepsilon^{(p+1)/2} \sum_{s=0}^{(p+1)/2} \frac{1}{(p + 1 - 2s)!2^s s!} \partial_{i_1} X^{\mu_1} \partial_{i_2} X^{\mu_2} \ldots \partial_{i_{(p+1-2s)}} X^{\mu_{(p+1-2s)}}$$

$$\varepsilon^{i_1 \ldots i_{(p+1-2s)} j_1 k_1 \ldots j_s k_s} H_{\mu_1 \ldots \mu_{(p+1-s)}} F_{j_1 k_1} \ldots F_{j_s k_s},$$  \hspace{1cm} (3.29)

and $K_\mu$ contains the $B$-field strength. Setting the $C^n$'s to zero and using the gauges (3.12), (3.23), the equation (3.28) becomes

$$\partial_0^2 X^\mu - \partial_1^2 X^\mu + \Gamma_{\nu\rho}^\mu \partial_0 X^\nu \partial_0 X^\rho - \Gamma_{\nu\rho}^\mu \partial_1 X^\nu \partial_1 X^\rho = 0,$$  \hspace{1cm} (3.30)

i.e., the equation of motion of a string in the 01-coordinates in conformal gauge. Since we also have the Virasoro constraints (3.27), we see that the stringy picture is complete.

3.4 Comments

The interpretation of the strong coupling limit that arises from the above considerations when $C^{(n)} = 0$ is as follows: The world-volume is foliated by world-sheets of (generally $(p, q)$ charged) strings. These strings are either closed or infinite. The form $\mathcal{F}$ is likewise split into one component in the world-sheet direction and the rest in the complementary directions. When $B = 0$ this splitting applies to the $U(1)$ field-strength itself: The component in the world-sheet is then determined by the string tension $\varepsilon$ and those in the complementary directions are not determined by the field equations, in agreement with the role of the A-field as a Lagrange multiplier (in this limit).

To gain some further insight into the "string parton" picture one may argue as follows: If instead of the gauge choice (3.23) we only make a partial choice $\bar{e}_0^i = \delta_0^i$, we find from the $A$-equations (3.19) that (in the absence of $RR$-fields),

$$\partial_0 \bar{e}_m^i = \partial_m \bar{e}_0^i = 0, \quad m = 1, \ldots, p.$$  \hspace{1cm} (3.31)
Thus $\tilde{e}_i^m$ is independent of $\xi^0$ and divergence-free in the spatial indices $m$. This means that $\tilde{e}_i^m$ has no sources and that its field lines are either closed or go off to infinity. Since the relations (3.21) still hold in this gauge, we retain the string interpretation with the $\tilde{e}_i^m$-direction representing the spatial string direction. We thus conclude that the strings are either closed or infinite. The strings going off to infinity might possibly have an interpretation as $D1$-branes (corresponding to branes within branes).

When $C^{(n)} \neq 0$, the situation will in general be more complicated. The one exception is when only $C^{(p+1)} \neq 0$, since this field does not enter in (3.22). The above analysis thus goes through, but now there is a $C^{(p+1)}$-dependent source on the right-hand side of (3.28). When $C^{(n)} \neq 0, n < p + 1$, the equations become very hard to analyze, in general. There are special cases, however, where an analysis similar to the one above is possible. We now discuss one such example.

Staying in the gauge $\tilde{e}_0 = \delta_0^i$ and making the further gauge choice $\partial_0 \tilde{e}_1^0 = 0$, we find from the $A$-equations (3.19) that

$$[\tilde{e}_0, \tilde{e}_1]_0^0 = 0, \quad [\tilde{e}_0, \tilde{e}_1]^m = J^m.$$  \hspace{1cm} (3.32)

Hence only the spatial components of the current (3.20), $J^m$, enter the commutation relation, not $J^0$. Clearly, in backgrounds where $J^m = 0$ we may again use the coordinates (3.23) (with $e = 1$ for simplicity), and the string interpretation is again viable.

To gain an understanding of the circumstances that might yield such a result, let us consider the current (3.20) for the special case when only $C^{(p+1)}$ and $C^{(p-1)} \neq 0$. The first of these will not affect (3.32), as discussed above. The statement that $J^m = 0$ is then tantamount to $*H^m = 0$. A sufficient condition for this to be true is that

$$\partial_0 X^{\mu_1} H_{\mu_1 \ldots \mu_p} = 0.$$ \hspace{1cm} (3.33)

This last condition has a rather clear physical meaning: Since we may now go to coordinates (3.23) where the string interpretation holds, we can restate it as $P^{\mu_1} H_{\mu_1 \ldots \mu_p} = 0$, i.e., the fieldstrength $H^{(p)}$ is orthogonal to the momentum-density $P$ of the "parton" string. Since furthermore our gauge choice $\partial_0 \tilde{e}_1^0 = 0$ implies that $\partial_m \tilde{e}_1^m = J^0$, the argument below (3.31) extends to say that in this case the "string partons" are either infinite strings, closed strings or open strings that end on the $D$-branes that are sources for $C^{(p+1)}$ and $C^{(p-1)}$. The
last case requires that these sources intersect with the world-volume of the
$D$-brane under discussion \[7\].

\section{The $\varepsilon \to 0$ limit.}

In this section we turn to the $\varepsilon \to 0$ limit of the action (2.3). This may be
interpreted as either a small tension or a large curvature limit. The latter
interpretation is of particular interest for the AdS/CFT correspondence \[8\],
where the limit away from weak gravitation corresponds to a limit away
from large $N$ in the boundary theory. In any case $\varepsilon \to 0$ represents a high-
energy limit where higher derivative terms, i.e., derivatives of the $U(1)$ field
strength, are expected to become important. Since we are studying a low
energy effective action this limit thus seems irrelevant at first sight. Here
we take the attitude, however, that there may be features of the high energy
theory that are of topologic and algebraic nature and may be captured in
such a limit. Alternatively, since we do not prove this, one may also study
the limit of the model considered in its own right (i.e. independent of its
relation to string theory).

We again consider the TypeIIB theory, which means that $p$ is restricted
to be odd. We are interested in the first few terms in an $\varepsilon$- expansion of the
action. To that end we rewrite the Born-Infeld part of (2.3) as

\begin{equation}
S = g_s^{-1} \int d\xi^1 \wedge \ldots \wedge d\xi^{p+1} e^{-\Phi} \left[ \det \gamma \det \{(\varepsilon B + F)_\alpha^\beta} \right]^1 \frac{\sqrt{\det \{(\delta + \varepsilon(\varepsilon B + F)^{-1})_\gamma^\beta}}}{2}.
\end{equation}

Making use of the expansion\[^9\]

\begin{equation}
d\xi^1 \wedge \ldots \wedge d\xi^{p+1} \sqrt{\det\{(\varepsilon B + F)\}} = \frac{1}{l!}(\varepsilon B + F) \wedge \ldots \wedge (\varepsilon B + F),
\end{equation}

where $2l = p + 1$, we expand the full action (2.3) as follows
\[^9\]This corresponds to the expansion of the determinant of a $2l \times 2l$ skew matrix as the
square of the Pfaffian.
\[ S = \frac{g_s^{-1}}{n!} \int \left\{ (e^{-\Phi} + g_s C^{(0)}) F \wedge \ldots \wedge F \\
+ \lambda \varepsilon ((e^{-\Phi} + g_s C^{(0)}) B + g_s C^{(2)}) F \wedge \ldots \wedge F + O(\varepsilon^3) \right\}. \] (4.36)

We see that the leading term corresponds to a coupling to a $D$-instanton. With \( q(X) \equiv e^{-\phi} + g_s C^{(0)} \) it has the form

\[ \int q F \wedge \ldots \wedge F, \] (4.37)

which displays its topological character: When \( q = \text{const} \) it is proportional to the Chern class of the $U(1)$ field, and it is always independent of a metric.

The subleading is a (generalized) Chern-Simons type term. Further we note that the form of the action (4.36) preserves $SL(2, \mathbb{Z})$-invariance. In fact, had we only considered the Born-Infeld action in this limit and then imposed $SL(2, \mathbb{Z})$-invariance, we would have discovered the $C^{(0)}$ and $C^{(2)}$ couplings.

5 Discussion

We have studied two limits the Born-Infeld action for $D$-branes in a given fixed background. As a basis for this we used an action with rescaled fields and tension ($\varepsilon$). The strong coupling limit we took to be defined by $\varepsilon^{p+1}/g_s << 1$ with $\varepsilon$ fixed and the high energy limit we studied was $\varepsilon \to 0$.

In the first limit we were able to solve the $V^i, W^i, A_i$ and $X^\mu$ equations for $C^{(n)} = 0, n = 0, \ldots, p$ and recover the picture of the world volume as foliated by strings from [1]. Restrictions on the geometry led to the possible constituent strings going off to infinity ($D1$-branes) or being closed. The general case proved to be considerably more complicated due to the presence of the current built from the $C^{(n)}$'s, although we saw that the ”string parton” picture survived in a particular example. In that example we found the additional option of the constituent strings being open but ending on the $D$-brane sources of the $C^{(n)}$ background fields.

In the second limit, we found an $\varepsilon$-expansion whose leading term is topological. We also noted that $SL(2, \mathbb{Z})$-invariance predicts the form of the $C^{(0)}$
and $C^{(2)}$ couplings in the next to leading term. We also noted that interpreting this limit as taking the radius $R \to 0$ in the AdS picture, this limit should probe physics away from the large $N$ limit.

To proceed further with both these expansions one would need to specify the background. In fact, solving the $A_i$ and $X^\mu$ equations in the second limit order by order in $\varepsilon$ requires knowledge of the $\alpha'$-dependence of the background. This is a possible direction for future research. It would also be interesting to compare our results to other ways of extracting string dynamics from Born Infeld theory, e.g. the Born-Infeld strings of [9].

Acknowledgement: The research of UL was supported in part by NFR grant No. F-AA/FU 04038-312. AZ was supported in part by a grant from the Royal Swedish Academy of Sciences. We are grateful to Rikard von Unge for comments on the manuscript and to S. Thiesen for discussions.
References

[1] "A picture of D-branes at strong coupling", U. Lindström and R. von Unge, *Phys. Lett.* **B403** (1997) 233, [hep-th/9704051](http://arxiv.org/abs/hep-th/9704051).

[2] "A picture of D-branes at strong coupling II, spinning partons", H. Gustafsson and U. Lindström, *Phys. Lett.* **440B** (1998) 43, [hep-th/9807064](http://arxiv.org/abs/hep-th/9807064).

[3] "Tension as a perturbative parameter in non-linear string equations in curved space", A. A. Zheltukhin, *Class. Quantum Grav.* **12** (1996) 2357, [hep-th/9606013](http://arxiv.org/abs/hep-th/9606013); "Variational principle and a perturbative solution of non-linear string equations in curved space", S. N. Roshchupkin and A.A. Zheltukhin, *Nucl. Phys.* **B543** (1999) 365, [hep-th/9806054](http://arxiv.org/abs/hep-th/9806054).

[4] "Super D-branes revisited" E. Bergshoeff and P.K. Townsend *Nucl. Phys.* **B531** (1998) 226, [hep-th/9804011](http://arxiv.org/abs/hep-th/9804011).

[5] "String dynamics at strong coupling", C. M. Hull, *Nucl. Phys.* **B468** (1996) 113, [hep-th/9512181](http://arxiv.org/abs/hep-th/9512181).

[6] "Lectures on D-branes, gauge theory and M(atrices)" W. Taylor IV, Trieste summer school 1997, hept-th/9801182.

[7] "Branes within Branes" M. R. Douglas, [hep-th/9512077](http://arxiv.org/abs/hep-th/9512077).

[8] "The Large N limit of superconformal field theories and supergravity" J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231, [hep-th/9711200](http://arxiv.org/abs/hep-th/9711200).

[9] "Brane dynamics from the Born-Infeld action" C. Callan and J. Maldacena *Nucl.Phys.* **B513** (1998) 198, [hep-th/9708147](http://arxiv.org/abs/hep-th/9708147).