An application of Galactic parallax: the distance to the tidal stream GD-1

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ABSTRACT

We assess the practicality of computing the distance to stellar streams in our Galaxy, using the method of Galactic parallax suggested by Eyre & Binney. We find that the uncertainty in Galactic parallax is dependent upon the specific geometry of the problem in question. In the case of the tidal stream GD-1, the problem geometry indicates that available proper-motion data, with individual accuracy \( \sim 4 \text{mas yr}^{-1} \), should allow estimation of its distance with about 50 per cent uncertainty. Proper motions accurate to \( \sim 1 \text{mas yr}^{-1} \), which are expected from the forthcoming Pan-STARRS PS-1 survey, will allow estimation of its distance to about 10 per cent uncertainty. Proper motions from the future Large Synoptic Survey Telescope (LSST) and Gaia projects will be more accurate still, and will allow the parallax for a stream 30 kpc distant to be measured with \( \sim 14 \) per cent uncertainty. We demonstrate the feasibility of the method and show that our uncertainty estimates are accurate by computing Galactic parallax using simulated data for the GD-1 stream. We also apply the method to actual data for the GD-1 stream, published by Koposov, Rix & Hogg. With the exception of one datum, the distances estimated using Galactic parallax match photometric estimates with less than 1 kpc discrepancy. The scatter in the distances recovered using Galactic parallax is very low, suggesting that the proper-motion uncertainty reported by Koposov et al. is in fact overstated. We conclude that the GD-1 stream is \( (8 \pm 1) \) kpc distant, on a retrograde orbit inclined 37° to the plane, and that the visible portion of the stream is likely to be near pericentre.

Key words: methods: data analysis – methods: numerical – astrometry – Galaxy: structure.

1 INTRODUCTION

Measuring distances in our Galaxy is critical to understanding its structure. However, line-of-sight distances can typically be measured with only relatively poor precision. This lack of precision is manifest in the most basic of Galactic parameters; the solar radius \( R_0 \) is hardly known to better than 5 per cent uncertainty (Gillessen et al. 2009), and this result renders the circular velocity at \( R_0 \) similarly uncertain (McMillan & Binney 2009). An accurate knowledge of distances is essential to create convincing models of the Milky Way, which in turn influence our understanding of the physics of galaxy formation in general.

Conventional trigonometric parallax has long been used to calculate accurate distances to nearby stars. The regular nature of the parallactic motion of a star, caused by the Earth’s orbit around the Sun, allows this motion to be decoupled from the intrinsic proper motion of the star in the heliocentric rest frame. Hence, the distance to the star can be calculated. However, the maximum baseline generating such parallaxes is obviously limited to 2 au. For a given level of astrometric precision, this imposes a fundamental limit to the observable distance. Indeed, the accuracy of parallaxes reported by the Hipparcos mission data (van Leeuwen 2007) falls to 20–30 per cent at best for distances \( \sim 300 \text{pc} \) and only then for the brightest stars. Upcoming astrometric projects such as Pan-STARRS (Kaiser et al. 2002), LSST (Tyson 2002) and the Gaia mission (Perryman et al. 2008) will achieve similar uncertainty for Sun-like stars as distant as a few kpc, and at fainter magnitudes than was possible with Hipparcos. This extended range will encompass less than 1 per cent of the total number of such stars in our Galaxy.

It is clear that it will not soon be possible to calculate distances to many of the stars in our Galaxy with conventional trigonometric parallaxes. Alternative means to compute distances to stars are therefore required. Photometry can be used to estimate the absolute magnitude of a star which, when combined with its observed magnitude, allows its distance to be computed. Unfortunately, all attempts to calculate such photometric distances are hindered by the same problems: obscuration by intervening matter alters both observed magnitude (Vergely et al. 1998) and colour (Schlegel, Finkbeiner & Davis 1998; Drimmel & Spergel 2001), and it is difficult to model appropriate corrections without a reference distance scale. The effects of chemical composition and age further complicate matters (Jurić et al. 2008). It is therefore difficult to compute photometric distances with an accuracy much better than 20 per cent, even for nearby stars, and distances to faint stars are less accurate still (Jurić et al. 2008).
Latter, it was realized (Eyre & Binney 2009, hereafter Paper I) that the orbital motion of the Sun about the Galaxy could be used to compute trigonometric distances to stars. In the general case, it is not possible to do this because the parallactic motion of a star and its intrinsic proper motion are inextricably mixed up. However, in the special case where the star can be associated with a stellar stream, its rest-frame trajectory can be predicted from the locations of the other associated stars. Using this trajectory, the proper motion in the Galactic rest frame (grf) can indeed be decoupled from the reflex motion of the Sun, and the component of its motion due to parallax can be computed.

Suppose that a stream is part of a stellar stream, and has a location \( (x-x_0) = r \hat{r} \), where \( r \) is the distance to the star and \( x_0 \) is the position of the Sun. In the plane of the sky, the tangent to the trajectory of the stream, near the star, be indicated by the vector \( \hat{p} \). Assume the velocity of the Sun, \( v_0 \), in the grf is known. Paper I showed that if the measured proper motion of the star is \( \mu \hat{t} \), then

\[
\dot{u} \hat{p} = \mu \hat{t} + \frac{v_t}{r} = \mu \hat{t} + \Pi \hat{v},
\]

where \( \Pi \equiv 1/r \) is the Galactic parallax, \( \dot{u} \) is the proper motion as would be seen from the grf and \( v_t \) is the Sun’s velocity projected into the plane of the sky. We note that \( \dot{u} = v_t/r \), where \( v_t \) is that component of the star’s grf velocity perpendicular to the line of sight, and that

\[
v_t = (v_0 - \hat{F} \cdot v_0 \hat{F}).
\]

Equation (1) is a vector expression and can be solved simultaneously for both \( \mu \) and \( \Pi \) provided that \( \hat{p}, \hat{t} \) and \( v_t \) are not parallel. The stream direction \( \hat{p} \) will not typically be known outright, but must be estimated from the positions of stream stars on the sky. We can achieve this by fitting a low-order curve through the position data, the tangent of which is then taken to be \( \hat{p} \). The curve must be chosen to reproduce the gross behaviour of the stream, but we must avoid fitting high-frequency noise, because \( \hat{p} \) is a function of the derivative of this curve, which is sensitive to such noise.

### 2.1 Uncertainty in Galactic parallax calculations

We begin by rendering equation (1) into an orthogonal on-sky coordinate system, whose components are denoted by \((x, y)\). In this coordinate system, equation (1) can be solved for \( \Pi \):

\[
\Pi = \frac{\mu(t_y \sin \alpha - t_x \cos \alpha)}{v_{\alpha} \cos \alpha - v_{\beta} \sin \alpha},
\]

where the \((x, y)\) suffixes denote the corresponding components of their respective vectors, and where we have defined the angle \( \alpha = \arctan(p_x/p_y) \).

The choice of coordinates \((x, y)\) is arbitrary. We are therefore free to choose the coordinate system in which \( \alpha = 0 \), i.e. that system in which the \( x \)-axis points along the stream trajectory, \( \hat{p} \). Equation (3) becomes,

\[
\Pi = -\frac{\mu \hat{t}_x}{v_{\hat{t}_x}},
\]

where we now identify the \( y \)-component of the various vectors as that component perpendicular (\( \perp \)) to the stream trajectory, and the \( x \)-component as that component parallel (\( \parallel \)) to the trajectory. Equation (4) shows explicitly that the Galactic parallax effect is due to the reflex motion of stream stars perpendicular to the direction of their travel.

Uncertainties in the \((x, y)\) components of the measured quantities \( \mu \hat{t} \) and \( v_{\hat{t}} \), and uncertainty in \( \alpha \), can be propagated to \( \Pi \) using equation (3). When we set \( \alpha = 0 \), this equation becomes,

\[
\frac{\sigma_{\Pi}^2}{\Pi^2} = \frac{\sigma_\mu^2}{\mu^2} + \frac{\sigma_{v_{\alpha}}^2}{v_{\alpha}^2} + \frac{\sigma_{v_{\beta}}^2}{v_{\beta}^2} \left( v_{\|} + \frac{\mu \hat{t}_t}{\Pi} \right)^2,
\]

where we anticipate the uncertainty in \( \mu \hat{t} \) to be isotropic, and so we have set \( \sigma_{\mu t} = \sigma_{v_{\|}} = \sigma_{v_{\perp}} \).

We assume \( \sigma_{\mu} \) to be known from observations; it may contain any combination of random and systematic error. \( \sigma_{v_{\perp}} \) is calculated directly from the error ellipsoid on \( v_{\perp} \), which is assumed known. Any error on \( v_{\perp} \) affects all data in exactly the same way. However, the projection of error on \( v_{\perp} \) varies with position on the sky.
Hence, the effect of $\sigma_{p}$ is to produce a systematic error in reported distance that varies along the stream in a problem-specific way.

Uncertainty in $\alpha$ arises from two sources. First, because the on-sky trajectory $\hat{p}$ is chosen by fitting a smooth curve through observational fields, $\hat{p}$ need not be exactly parallel to the underlying stream. Further, since $\hat{p}$ depends on the derivative of the fitted curve, it is likely to be much less well constrained for the data points at the ends of the stream than for those near the middle.

We can quantify this effect. At the endpoints, the fitted curve is likely to depart from the stream by at most $\Delta \psi$, the angular width of the stream on the sky. For a low-order curve, this departure is likely to have been gradual over approximately half the angular stream length, $\Delta \theta$, giving a contribution to $\sigma_{p}$ from fitting of

$$\sigma_{p,f}^2 = \frac{4\Delta \psi^2}{\Delta \theta^2}. \quad (6)$$

The second contribution to $\sigma_{p}$ arises as follows. Since the stream has finite width, at any point, the stars within it have a spread of velocities, corresponding to the spread in action of the orbits that make up the stream. If the stars in a stream show a spread in velocity $\sigma_{\alpha,v}$, this effect contributes,

$$\sigma_{\alpha,v}^2 = \frac{1}{v_i^2} \left( \sigma_{\alpha}^2 \cos^2 \alpha + \sigma_{\alpha}^2 \sin^2 \alpha \right), \quad (7)$$

to the uncertainty in $\alpha$ for a single star. Again we can choose $\alpha = 0$, such that $\sigma_{\alpha,v} = \sigma_{\alpha,\perp}$, the velocity dispersion perpendicular to the stream direction. Equation (7) becomes

$$\sigma_{\alpha,\perp}^2 = \frac{\sigma_{\parallel}^2}{v_i^2} = \frac{\sigma_{\parallel}^2}{(v \sin \beta)^2}, \quad (8)$$

where we have introduced $v_i$, the grf speed of the stream and $\beta$, the angle of the stream to the line-of-sight. $\sigma_{\alpha,\perp}$ has its origin in the random motions of stars that existed within the progenitor object. In fact, if we assume the stream has not spread significantly in width, then the width and the velocity dispersion (section 8.3.3 Binney & Tremaine 2008) are approximately related by

$$\frac{\sigma_{\perp}}{v} \sim \frac{w}{R_p} = \frac{r \Delta \psi}{R_p}, \quad (9)$$

where $w$ is the physical width of the stream and $R_p$ is the radius of the stream’s perigalacticon. This gives

$$\sigma_{\alpha,v} = \frac{r \Delta \psi}{R_p \sin \beta}. \quad (10)$$

If secular spread has made the stream become wider over time, then this relation will overestimate $\sigma_{\alpha,v}$, since $\sigma_{\perp}/v$ is roughly constant. $\Delta \psi$ therefore represents an upper bound on the true value of $\sigma_{\alpha,v}$ through this relation. This argument also assumes that the stream was created from its progenitor in a single tidal event. Real streams do not form in this way. However, repeated tidal disruptions can be viewed as a superposition of ever younger streams, created from a progenitor of ever smaller $\sigma_{\alpha,\perp}$. Equation (10) holds for each of these individually. Thus, $\Delta \psi$ remains a good upper bound for $\sigma_{\alpha,v}$ through this relation.

In reality, we do not measure the proper motion of individual stream stars, but rather the mean motion of a field of $N$ stars. The contribution to $\sigma_{\alpha}$ is from the error on this mean. Putting this together with equation (6) gives our final expression for $\sigma_{\alpha}$,

$$\sigma_{\alpha}^2 = \frac{\sigma_{\alpha,v}^2}{N} + \sigma_{\alpha,f}^2 = \frac{r^2 \Delta \psi^2}{N R_p^2 \sin^2 \beta} + \frac{4 \Delta \psi^2}{\Delta \theta^2}. \quad (11)$$

We note that the first term represents a random error, and the second term represents a systematic error that will vary with position down the stream. In general, $\sin \beta$ and $R_p$ are a priori unknown. We can infer $\sin \beta$ from radial velocity information, either directly where the measurements exist, or indirectly from Galactic parallax distances. Guessing $R_p$ requires assumptions to be made about the dynamics, but in general we expect the ratio $r/R_p \approx 1$ or less.

Explicit evaluation of $\sin \beta$ and $R_p$ is not necessary to evaluate the uncertainty if $\sigma_{\alpha}$ is dominated by the error from fitting, $\sigma_{\alpha,f}$. We can see this will be the case when the number of observed stars per field,

$$N > \left( \frac{r \Delta \theta}{2R_p \sin \beta} \right)^2. \quad (12)$$

We expect this to be true in almost all practical cases.

2.2 Uncertainty in tangential velocity calculations

Equation (1) can also be used to solve for $u$,

$$\dot{u} = \mu(t_i + t_p + \Pi(v_{\parallel}, v_{\perp})) \cos \alpha + \sin \alpha \quad (13)$$

which becomes,

$$\dot{u} = \mu(t_i + t_p + \Pi(v_{\parallel}, v_{\perp})) = \mu t_i + \Pi v_{\parallel}, \quad (14)$$

when we set $\alpha = 0$. Equation (13) combined with equation (3) can be used to explicitly propagate uncertainties in the measured quantities to $\dot{u}$. When $\alpha = 0$, the uncertainty in $\dot{u}$ is

$$\sigma_{\dot{u}}^2 = \frac{\sigma_{\mu}^2}{\mu^2} (t_i v_{\parallel} - t_p v_{\perp})^2 + \frac{v_{\parallel}^2 \sigma_{\mu}^2 + v_{\perp}^2 \sigma_{\mu}^2}{v_{\parallel}^2 (t_i v_{\parallel} - t_p v_{\perp})^2} + \frac{v_{\parallel}^2 \sigma_{\mu}^2}{v_{\perp}^2}. \quad (15)$$

$\sigma_{\mu}$ and $\cos(v_{\parallel}, v_{\perp})$ are calculated directly from the error ellipsoid on $v_0$, which we have assumed known.

2.3 Practicality of Galactic parallax as a distance measuring tool

Using equation (4) to eliminate $\mu_{\perp}$ from equation (5), and taking the dot product of $\hat{p}$ with equation (1) to simplify the last term, we obtain

$$\sigma_{\mu}^2 = \frac{1}{v_{\perp}^2} \left\{ (r \sigma_{\mu})^2 + \sigma_{\mu}^2 + (r \dot{u})^2 \sigma_{\alpha}^2 \right\}$$

$$= \frac{1}{v_{\perp}^2} \left\{ (r \sigma_{\mu})^2 + \sigma_{\mu}^2 + v_{\perp}^2 \left( \frac{r^2 \Delta \psi^2}{R_p N} + \frac{4 \Delta \psi^2}{\Delta \theta^2} \right) \right\}, \quad (16)$$

where we have noted that $r \dot{u} = v_i = v \sin \beta$ and we have related the observed stream length, $\Delta \theta$, to the deprojected length, $\Delta \theta = \Delta \psi \sin \beta$. We note that the last term is independent of $r$, since $r \Delta \psi = w$ and $\Delta \psi / \Delta \theta$ are both constant, and that for a stream of given physical dimension, the uncertainty in $\Pi$ has no dependence upon the angle of the stream $\beta$ to the line of sight.

What level of uncertainty does equation (16) predict, when realistic measurement errors are introduced? The answer to this is dependent upon both the physical properties of the stream ($R_p$, $\Delta v_t$, $\Delta \psi$, $\nu$) and the geometry of the problem in question ($r$, $v_{\perp}$).

We progress by assuming ‘typical’ values for some of these quantities. The average magnitude of $v_t$ taken over the whole sky is $v_0 \pi/4$. The average perpendicular component, for a randomly oriented stream, is $2/\pi$ of this value. We therefore assume a typical
value for \( v_{\perp, \mathrm{sun}} \) of \( v_0/2 \sim 120 \, \text{km \ s}^{-1} \). We also assume a typical grf velocity equal to the circular velocity, \( v = v_0 \sim 220 \, \text{km \ s}^{-1} \).

McMillan & Binney (2009) recently summarized the current state of knowledge of \( v_0 \). The uncertainty quoted is typically \( \pm 5 \) per cent on each of \( U, V, W \). Correspondingly, we estimate a typical value for the uncertainty \( \sigma_{v_{\perp, \mathrm{sun}}} \), of 5 per cent of \( v_{\perp, \mathrm{sun}} \), or 6 \( \text{km \ s}^{-1} \).

The GD-1 stream that we consider below is exceptionally thin and long, with \( \Delta \psi \sim 0.1^\circ \) and \( \Delta \theta \sim 60^\circ \). The Orphan stream (Grillmair 2006; Belokurov et al. 2007) is of similar length, but about 10 times thicker. Both of these streams are near apsis, so \( \Delta \theta = \Delta \Theta \). We therefore take \( \Delta \psi \sim 1^\circ \), \( \Delta \Theta \sim 60^\circ \), as typical of the streams to which one would apply this method. If hundreds of stars are observed for each proper-motion datum, then equation (12) is true for all realistic combinations of \( (r, R_p) \), so we can ignore the contribution of \( \sigma_{u, v} \) to \( \sigma_\mu \). The contribution from \( \sigma_{u, v} \) gives \( \sigma_\mu \simeq 1.9^\circ \).

The individual United States Naval Observatory (USNO)/Sloan Digital Sky Survey (SDSS) proper motions (Munn et al. 2004) used by K09 have a random uncertainty \( \sigma_\mu \sim 4 \, \text{mas \ yr}^{-1} \). After averaging over hundreds of stars and accounting for a contribution from non-stream stars, K09 report a random uncertainty of \( \sigma_\mu \sim 1 \, \text{mas \ yr}^{-1} \) on their GD-1 data. For a stream 10 kpc distant, with these proper motions and the typical values mentioned, equation (16) reports an uncertainty of \( \sigma_\pi/\pi \sim 40 \) per cent. By far the greatest contribution comes from the first term in equation (16), hence, the error on proper-motion measurement is dominating our uncertainty.

To obtain an uncertainty of \( \sigma_\pi/\pi < 20 \) per cent with Munn et al. (2004) proper-motion measurements, we would need to restrict ourselves to streams less than 5 kpc distant. 20 per cent error is also possible at 10 kpc given optimum problem geometry. This is clearly competitive with the \( \sim 20 \) pc at which one could observe a standard trigonometric parallax, with similar accuracy, using astrometry of this quality. However, previous work (Willett et al. 2009; K09) shows that SDSS photometry combined with population models produce distance estimates accurate to \( \sim 10 \) per cent for stars in streams at 8 kpc. The accuracy of Galactic parallax is therefore not likely to be as good as that of photometric distances for distant streams, using data this poor, unless the problem geometry is favourable.

Proper-motion data from the Pan-STARRS telescope is expected to be accurate to \( \sim 1 \, \text{mas \ yr}^{-1} \) for Sun-like stars at 10 kpc (Magnier et al. 2008). K09 reduce raw data with accuracy \( \sim 4 \, \text{mas \ yr}^{-1} \) to processed data accurate to \( \sim 1 \, \text{mas \ yr}^{-1} \), even though the expected proper motion of the stars is of the same size as the errors. It is not unreasonable to expect a similar analysis applied to Pan-STARRS raw data, where the relative error would be much less than unity, to yield processed data accurate to \( \sim 0.2 \, \text{mas \ yr}^{-1} \). In truth, the ability of Pan-STARRS to detect very faint stars will increase the number of stars identifiable with a stream, and thus reduce the uncertainty in the mean proper motion further than this, but we use 0.2 mas \( \text{yr}^{-1} \) as a conservative estimate.

The same 10 kpc distant stream would have a parallax error of \( \sigma_\pi/\pi \simeq 11 \) per cent with data this accurate. An error of less than 20 per cent is possible for a typical stream less than \( \sim 23 \) kpc distant, and for a stream with favourable geometry less than 50 kpc distant. Jurić et al. (2008) report that SDSS photometric distances for dwarf stars have \( \sim 40 \) per cent error at 20 kpc. Thus, the accuracy of Galactic parallax derived from Pan-STARRS data should be at least comparable to distance estimates from photometric methods, even in the typical case.

Future projects such as LSST and Gaia will each obtain proper motions accurate to \( \sim 0.2 \, \text{mas \ yr}^{-1} \) for Sun-like stars 10 kpc distant (Ivezić et al. 2008; Perryman et al. 2001). These data would allow a distance estimate for our typical stream accurate to 8 per cent, and a stream with favourable geometry accurate to 4 per cent. Error in the proper motion no longer dominates the uncertainty in these calculations. We might expect such accurate astrometric surveys to reduce the uncertainty in the solar motion; in this case, the error in parallax would be lower still.

Gaia will not observe Sun-like stars beyond 10 kpc, but LSST will, with accuracy of 0.4 mas \( \text{yr}^{-1} \) for dwarf stars 30 kpc distant (Ivezić et al. 2008). The accuracy of the parallax to our typical stream at this distance would be about 14 per cent with these data, and 6 per cent is achievable with optimum geometry. A typical stream could be measured to 20 per cent accuracy out to 40 kpc, and a stream with favourable geometry out to 54 kpc; this range approaches the limit of LSST’s capability for detection of dwarf stars. Such data will put the Orphan stream, which is about 20–30 kpc distant (Grillmair 2006; Belokurov et al. 2007; Sales et al. 2008), in the range of accurate trigonometric distance estimation. For comparison, photometric distances from SDSS data are hardly more accurate than 50 per cent for this stream (Belokurov et al. 2007).

3 TESTS

To test the method, pseudo-data was prepared from an orbit fitted to data for the GD-1 stream by K09. The orbit is described by the initial conditions \( x = (-3.41, 13.00, 9.58) \, \text{pc}, v = (-200.4, -162.6, 13.9) \, \text{km \ s}^{-1} \), where the \( x \)-axis points towards the Galactic centre and the \( y \)-axis points in the direction of Galactic rotation. The orbit was integrated in the logarithmic potential,

\[
\Phi(x, y, z) = \frac{v_c^2}{2} \log \left[ x^2 + y^2 + \left( \frac{z}{q} \right)^2 \right].
\]

where \( v_c = 220 \, \text{km \ s}^{-1} \) and \( q = 0.9 \). The resulting trajectory was projected on to the sky, assuming a solar radius \( R_\odot = 8.5 \, \text{kpc} \). Several points were sampled, and each was taken to be a separate datum in the pseudo-data set. The proper motion for each datum was computed by projecting the difference between its grf motion and the solar motion on to the sky.

The pseudo-data were transformed into the rotated coordinate system used by K09 to facilitate comparison with their data; the transformation rule is given in the appendix to K09. The stream is very flat in this coordinate system, so the dependence of \( \phi_2 \) on \( \phi_1 \) is relatively weak. This helps to increase the quality of the fitted curve and minimizes the corresponding error in \( \sigma_{\phi_1} \).

To simulate the observed scatter in the real positional data, the pseudo-data were each scattered in the \( \phi_2 \) coordinate according to a randomly sampled Gaussian distribution with a dispersion \( \sigma_{\phi_2} = 0.1^\circ \). The resulting positional pseudo-data are plotted in Fig. 1, along with the orbit from which they were derived (full curve). A cubic polynomial representing \( \phi_2(\phi_1) \) was least-squares fitted to the pseudo-data, the tangent of which was used to estimate \( \dot{\phi} \). In the case of the pseudo-data, uniform weights were applied to each datum for the fitting processes. The resulting curve is also shown in Fig. 1 (dotted curve).

When the correct orbit is used to calculate \( \dot{\phi} \) and precise values for the measured proper motion \( \mu \) and solar reflex motion \( v_\odot \), are used, the distance is recovered perfectly from equation (4). Fig. 2 compares the recovered distance when \( \dot{\phi} \) is estimated using the polynomial fit to the pseudo-data, but still using accurate values for \( \mu \) and \( v_\odot \). Our pseudo-data stream is \( \Delta \psi \simeq 0.1^\circ \) wide and
Δθ ≃ 60° long. Equation (6) therefore estimates σ_a/ ≃ 0.38°. The recovered distances in Fig. 2 are in error by only ≃ 2 per cent across most of the range, which is the approximate uncertainty predicted by equation (5) for this value of σ_a/. Thus, the estimation of 〈p from the observed stream is good, and contributes little error to the distance calculations.

The K09 observational data for the GD-1 stream, discussed below, have a similar uncertainty σ_a/ ≃ 0.38° due entirely to the fitting process, and proper-motion uncertainties σ_μ/ ≃ 1 mas yr⁻¹. Fig. 3 shows the recovered distances from Fig. 2 with error bars for the expected uncertainty in recovered distance, given these measurement uncertainties and the uncertainty in μ/ quoted in Section 1. Also plotted for each datum are the distances recovered from 60 Monte Carlo realizations of the pseudo-data input values, convolved with the errors given above.

Equation (5) is found to be a good estimator for the uncertainty, with approximately 80 per cent of the Monte Carlo realizations falling within the error bars. The error in parallax for the K09 data is thus predicted to be about 50 per cent, of which the greatest contribution comes from the uncertainty in proper motion.

### 4 DISTANCE TO THE GD-1 STREAM

Fig. 4 shows the on-sky position data for the GD-1 stream, as published in K09. Also shown in Fig. 4 is a linear least-squares fit of a cubic polynomial, φ/ (μ/), to these data; the inverse-square of the uncertainties was used to weight the fit.

K09 provide measured proper-motion data for five fields of stars, spanning the range φ/ ≃ (−55, −15); along with uncertainties for these measurements. Uncertainty in the stream direction is σ_μ/ ≃ 0.38°, which is entirely contributed by the curve fit to the stream; since hundreds of stars contributed to the calculation of the proper motions, the contribution from the first term in equation (11) is negligible. The uncertainty in μ/ is computed for each individual field from the uncertainty in μ/ given in Section 1.

Fig. 5 shows the Galactic parallax distances for each of these data, along with the K09 photometric distances. The dotted error bars represent the expected error in distance for the uncertainties given. The small solid error bars are the uncertainties reported by K09 for their photometric distances. The K09 orbit used to compute the earlier pseudo-data is plotted for comparison.
With the exception of the datum at $\phi_1 \sim -55^\circ$, the parallax distances and the K09 distances are in remarkable agreement. However, the dotted error bars vastly overestimate the true error in the results. If we ignore the datum at $\phi_1 \sim -55^\circ$, the scatter in the distance, $\sigma_\mu \sim 1$ kpc, is similar to that of the photometric distances, and consistent with a true random error of $\sigma_\mu \sim 0.3$ mas yr$^{-1}$, and negligible systematic error. We cannot explain this discrepancy, except by suggesting that the K09 proper-motion measurements are more accurate than the published uncertainties suggest. This is corroborated by the top right-hand panel of fig. 13 from K09 in which the $\mu_\phi$ data, with the exception of the datum at $\phi_1 \sim -55^\circ$, show remarkably little scatter within their error bars.

Fig. 6 shows the grf proper motions, $\mu$, calculated from equation (14) along with their error bars, from equation (15). In the background are plotted the data from Fig. 9 of K09, which show the density of stars with a given grf proper motion in the sample of stars chosen to be candidate members of the stream, and after subtraction of a background field. The K09 grf proper motions have been calculated by correcting measured proper motion for the solar reflex motion, using an assumed distance of 8 kpc (Koposov, private communication); this assumption will cause a systematic error in the K09 proper motions, of the order of the distance error, which changes with position down the stream. The apparently large width of the stream in this plot is due to uncertainty in the underlying Munn et al. (2004) proper-motion data.

The stream is clearly visible in this plot as the region of high-density spanning $\phi_1 \sim (0, -60)^\circ$ with $\mu_\phi \simeq 0$ mas yr$^{-1}$ and $\mu_\phi$ falling slowly between $(-6, -10)$ mas yr$^{-1}$. Despite the expected systematic error, the estimates of $\mu$ from the parallax calculation are consistent with these data, with the exception of the same datum at $\phi_1 \sim -55^\circ$ that also reports an anomalous distance.

We explain this suspect datum as follows. From inspection of the top right-hand panel of fig. 13 from K09, it is apparent that the $\mu_\phi$ measurement for this datum is not keeping with the trend. Conversely, the corresponding $\mu_\phi$ measurement is not obviously in error. If the magnitude of $\mu_\phi$ for this datum has been overestimated by the K09 analysis, then equation (4) will overestimate the parallax, and hence underreport the distance. Fig. 5 indicates that the distance for this datum is indeed underreported.

The effect of such an error in $\mu_\phi$ on the grf proper-motion, $\dot{\mu}$, can be understood by considering equation (14). If $\Pi$ is overestimated, $\dot{\mu}$ will be either overestimated or underestimated, depending on the relative sign of the two terms. In the case of GD-1, $\mu_{i \, 1}$ and $v_{1 \, 1}$ have opposite signs, so an overestimated $\Pi$ will result in an underestimated $\dot{\mu}$. This too corresponds with the behaviour of the suspect datum in Fig. 6.

It is unknown why this particular datum should be significantly in error while the other data are not. There are no obvious structures in the lower panel of Fig. 6 which might cause the fitting algorithm in K09 to mistakenly return an incorrect value for $\mu_\phi$. None the less, if the scatter in the other data are accepted as indicative of their true statistical error, it is clear that the datum at $\phi_1 \sim -55^\circ$ cannot represent the proper motions of GD-1 stars at that location. We therefore predict that an appropriate re-analysis of the proper-motion data, taking care to ensure that a signal from GD-1 stream stars is properly detected, will return a revised proper motion of $\mu_\phi \sim -3$ mas yr$^{-1}$.

In summary, it seems that Galactic parallax measurements confirm the K09 photometric analysis, and predict that the stream is approximately $(8 \pm 1)$ kpc distant, where the uncertainty denotes the scatter in the results. Since Galactic parallax and photometric estimates are fundamentally independent, it seems unlikely that systematic errors in either would conspire to produce the same shift in distance; this implies that no systematic error is present.

We also calculate a grf proper motion for the stream of $\mu_{\phi} = (-7 \pm 2)$ mas yr$^{-1}$, corresponding to a grf tangential velocity of $(265 \pm 80)$ km s$^{-1}$ in a direction ($\mu_1 \cos b$, $\mu_\phi$) $\simeq (0.8, -0.6)$. This implies that the stream is on a retrograde orbit, inclined to
the Galactic plane by ~37°, which is in accordance with previous results (Koposov et al. 2009; Willett et al. 2009).

The galactocentric radius of ~14.5 kpc does not seem to be changing rapidly along stream’s length, which subends ~12° when viewed from the Galactic centre. This implies that the observed stream is at an apsis. The grf velocity of the stream is faster than the circular velocity, \( v_r \sim 220 \, \text{km} \, \text{s}^{-1} \). This implies that the stream is at pericentre, although the large uncertainty prevents a firm conclusion from being drawn. We note that the radial velocity data in K09 would also imply that the stream is observed at pericentre.

5 CONCLUSIONS

We have demonstrated the practical application of a technique for computing Galactic parallax, as described by Paper I. This technique utilizes the predictable trajectories of stars in a stream to identify the contribution of the reflex motion of the Sun to the observed proper motion. The parallax and the grf proper motion follow from this.

The only assumption made is knowledge of the grf velocity of the Sun. It is also a requirement that the observed stars are part of a stream. Recent evidence (Odenkirchen et al. 2002; Majewski et al. 2003; Yanny et al. 2003; Belokurov et al. 2006; Grillmair 2006; Grillmair & Dionatos 2006; Grillmair & Johnson 2006; Grillmair 2009; Newberg et al. 2009) indicates that tidal streams are a common constituent of the Galactic halo, and so this technique should have widespread application.

We have derived an expression for the uncertainty in Galactic parallax calculations. We include contributions from measurement errors in proper motion and solar motion, error in the estimation of stellar trajectories from the stream direction and algorithmic error in the estimation of stream direction itself.

The uncertainty for calculations involving a particular stream is depend upon the size, location and orientation of the stream, as well as upon measurement errors. We estimate that using individual proper motions accurate to 4 mas yr\(^{-1}\), available now in published surveys (Munn et al. 2004), the parallax of a 10 kpc distant stream with typical geometry could be measured with an uncertainty of 40 per cent. The parallax of a stream with optimum geometry could be measured with approximately half this uncertainty.

Proper-motion data from the forthcoming Pan-STARRS PS-1 survey (Kaiser et al. 2002; Magnier et al. 2008) will yield the distance to a typical 10 kpc distant stream with 11 per cent accuracy, or the distance to a stream at 23 kpc with 20 per cent accuracy; with favourable geometry this accuracy could be achieved for a stream as distant as 50 kpc. With data of this quality, the uncertainty in distances from Galactic parallaxes will be considerably lower than those of photometry for distant streams.

The LSST (Tyson 2002; Ivezić et al. 2008) and the Gaia mission (Perryman et al. 2001) will produce proper-motion data that are more accurate still. Such data would allow the distance to stars in a 10 kpc typical stream to be computed to an accuracy of the order of 8 per cent, where the limitation is now imposed by uncertainty in the solar motion and in the stream trajectory. It is likely that LSST and Gaia data will allow the uncertainty in the solar motion to be significantly reduced, so in reality much better precision can be expected at this distance. For streams 30 kpc distant, LSST proper motions will allow distance estimates as accurate as 14 per cent to be made in the typical case, and 6 per cent with optimum geometry. Thus, the high-quality astrometric data that is expected to be available in the next decade will allow parallax estimates for very distant streams to be made with unparalleled accuracy.

To test the method presented, we have created pseudo-data simulating the GD-1 stream (Grillmair & Dionatos 2006). When the method is provided with error-free pseudo-data, the correct parallax is computed perfectly. When errors are introduced into the pseudo-data, the reported parallax degrades in line with the uncertainty estimates.

We applied the method to the astrometric data for the GD-1 stream in K09. With the exception of a single datum, the Galactic parallax is remarkably consistent with the photometric distances quoted by K09. Indeed, the uncertainty in the measured proper motions quoted by K09 should produce significant error in the Galactic parallax. However, the scatter in the results is consistent with a random error of only ~0.3 mas yr\(^{-1}\), and if the photometric distances of K09 are believed, no systematic offset. This is at odds with the typical uncertainty in the proper motion of ~1 mas yr\(^{-1}\) reported by K09. We cannot explain this discrepancy, other than to suggest that the K09 method for estimating error in the proper motions is producing significant overestimates.

The grf proper motions predicted for the stream are also consistent with observational data from K09, with the exception of the same datum that also reports an inconsistent distance. We conclude that the proper motion associated with this datum is erroneous, and we predict that the re-analysis of the stream stars near this datum will reveal a reduced proper-motion measurement of \( \mu_\alpha \sim 3 \, \text{mas} \, \text{yr}^{-1} \).

Photometry and Galactic parallax produce fundamentally independent estimates of distance. The quality of the corroboration of the K09 photometric distance estimates for GD-1 by the Galactic parallax estimates presented here therefore lends weight to the conclusion that the predicted distance, in both cases, is correct. On this basis, we conclude that the GD-1 stream is about (8 ± 1) kpc distant from the Sun, on a retrograde orbit that is inclined 37° to the Galactic plane with a rest-frame velocity of \((265 \pm 75) \, \text{km} \, \text{s}^{-1}\). We also conclude that the visible portion of the stream is probably at the pericentre.

The prospect of being able to map trigonometric distances in the Galaxy to high accuracy at tens of kiloparsecs of range is indeed exciting. The distances generated using this method, although limited to stars in streams, could be used to calibrate other distance measuring tools, such as photometry, that would be more widely applicable. The technique is immediately applicable to any stream for which proper-motion data are currently available, although we anticipate limited accuracy until better proper-motion data are available.

Given enough parallax data points along a given stream, an orbit can be constructed by connecting those points. This orbit is predicted independently of any assumption about the Galactic potential, which it must strongly constrain. Constraints on the Galactic potential impose constraints on theories of galaxy formation and cosmology. It would seem that the combination of dynamics and Galaxy-scale precision astrometry, such as provided by this method, could well have profound implications for astrophysics in the future. At present, however, it is not obvious how to combine all sources of astrometric and dynamical information, to produce the tightest constraints on the potential. We therefore encourage the exploration of methods for combining this information, in anticipation of the arrival of higher quality astrometric data in the next few years.

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