Systematics of Large Axial Vector Meson Production in Heavy Flavor Weak Decays

Harry J. Lipkin

Department of Particle Physics Weizmann Institute of Science, Rehovot 76100, Israel

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel

High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439-4815, USA

harry.lipkin@weizmann.ac.il

Abstract

Branching ratios observed for $D$ and $B$ decays to final states $a_1(1260)^\pm X$ are comparable to those for corresponding decays to $\pi^\pm X$ and $\rho^\pm X$ and much larger than those for all other decays. Implications are discussed of a “vector-dominance model” in which a $W$ is produced and immediately turns into an axial vector, vector or pseudoscalar meson. Data for decays to all such final states are shown to have large branching ratios and satisfy universality relations. Upper limits on small strong phase differences between amplitudes relevant to CP violation models are obtained from analysis of the predicted and observed suppression of $B^0$ decays into neutral final states $\pi^0 X^0$, $\rho^0 X^0$ and $a_1^0 X^0$. Branching ratios of $\approx 1\%$ are predicted for the as yet unobserved presence of the $D_{s1}(2536)$ charmed-strange axial vector in $B$ decays.

I. A VECTOR-DOMINANCE MODEL FOR HEAVY-FLAVOR DECAYS

*Supported in part by grant from US-Israel Bi-National Science Foundation and by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.
The large branching ratios observed \([1]\) for the appearance of the \(a_1(1260)^\pm\) in all quasi-two-body decays \(D \rightarrow a_1(1260)^\pm X\) and \(B \rightarrow a_1(1260)^\pm X\) are comparable to those observed for \(\pi^\pm X\) and \(\rho^\pm X\) and contrast sharply with the much smaller branching ratios observed to \(a_2 X\), \(b_1 X\), and \(a_0^0 X\). In the simple quark model the \(a_1\), \(a_2\) and \(b_1\) mesons are \(q\bar{q}\) p-wave excitations which differ only in their spin and orbital angular momentum couplings. However their weak couplings are very different. The charged \(a_1\) couples to the weak axial vector current in the same way that the \(\rho\) couples to the vector current. The \(b_1\) couples to a second-class axial vector current. The spin-2 tensor meson \(a_2\) and the neutral \(a_1\) cannot couple directly to the \(W\).

The experimental systematics imply a crucial role for weak couplings in these dominant decay modes and suggest a description by a “vector-dominance” model like the diagram shown in fig. 1 for \(D^o \rightarrow K^- M^+\) decays in which the initial hadron state \(i\) decays to a final state \(f\) by emitting a \(W^\pm\) which then hadronizes into a charged vector, axial-vector or pseudoscalar meson, denoted by \(M^\pm\)

\[ i \rightarrow f + W^\pm \rightarrow f + M^\pm \quad (1.1) \]

For the cabibbo-favored \(D\) and \(B\) decays the “vector-dominance” model gives:

\[
D(c\bar{q}) \rightarrow (W^+s)\bar{q} \rightarrow [s\bar{q} \rightarrow M(s\bar{q})]_S \cdot (W^+ \rightarrow M^+)_W \rightarrow M(s\bar{q})M^+ \quad (1.2)
\]

\[
B(\bar{b}q) \rightarrow (W^+c)q \rightarrow [(\bar{c}q) \rightarrow M(\bar{c}q)]_S \cdot (W^+ \rightarrow M^+)_W \rightarrow M(\bar{c}q)M^+ \quad (1.3)
\]

where the subscripts \(S\) and \(W\) denote strong and weak form factors, \(q\) denotes \(u, d, s\) or \(c\), \(M(s\bar{q})\)and \(M(q\bar{c})\) denote respectively mesons with the quark constituents \(s\bar{q}\) and \(q\bar{c}\), the three charmed mesons \(D^+, D^o\) and \(D_s\) which differ only by the flavor of the spectator quark are all treated on the same footing and similarly for the four \(B\) mesons \(B^+, B^o, B_s\) and \(B_c\).

The experimental branching ratios shown in Tables I and II suggest that quasitwobody \(D\) and \(B\) decays are dominated by the diagram in which a charged pseudoscalar, vector
or axial vector is produced from the weak vertex. No decays to the other p-wave mesons are within an order of magnitude of these values. Note in particular the difference between the $a_1$ and the $a_2$. All 24 decays of the form $B \to \bar{D}W^+ \to \bar{D}M^+$, where $M$ can denote $a_1, \rho, \pi, \ell^+\nu_\ell, D_s, D_s^*$, are dominant with branching ratios above 0.3%. Other $B$-decay modes have upper limits in the $10^{-4}$ ball park. The absence with significant upper limits of neutral decays $B^0 \to \bar{D}^o M^o$ which are coupled by strong final state interactions to $B^o \to D^- M^+$ also places stringent limits on values of strong relative phases crucial in some models for CP violation.

Some enhancement might be expected for color-favored decays also favored by factorization. But the absence of other equally-favored final states suggests something special about axial vectors.

Underlying this systematics is a deeper theoretical problem where this phenomenology may provide interesting input; namely the dichotomy, contradictions and interface between the chiral and constituent quark pictures, which remain to be hopefully resolved by QCD. The pion behaves sometimes like a Goldstone boson and sometimes like a $\bar{q}q$ pair just like the other eight pseudoscalars in the nonet, differing from the $\rho$ only by spin couplings and scattering like $2/3$ of a nucleon. The $a_1$ behaves sometimes like the chiral partner of the $\rho$ and sometimes like a $\bar{q}q$ pair with a completely different wave function from that of the $\rho$ and differing from the $b_1, a_2$ and the scalar only by spin and orbital angular momentum couplings.

The constituent quark picture is used in the heavy quark effective theory for heavy flavor hadrons. The strange mesons are somehow in the middle being classified in the same flavor-SU(3) multiplets as the light mesons, but with SU(3) breaking by the quark mass difference introducing some heavy quark effects. The strange axial vector mesons present particularly interesting challenges. In this context the production of these hadrons in heavy flavor decays can provide interesting experimental input.

Extending the systematics shown in Tables I and II to the charmed-strange sector suggests that the charmed-strange axial vector $D_{s1A}$ should be the strongest excited charmed-
strange state seen in B decays, with a dominant $D^*K$ decay mode analogous $a_1 \to \rho \pi$. The question remains open whether $B \to D_{s1A}X$ decays indeed have branching ratios in the 1% ball park while others are around $10^{-4}$. The only candidate listed is $D_{s1}(2536)$ and no upper limit has been reported for $B_q(\bar{b}q) \to (W^+\bar{c})q \to (\bar{c}q)D_{s1} \to M(\bar{c}q)D_{s1} \to M(\bar{c}q)D^*K$ \hfill (1.4)

\textbf{B. Universality of vector dominance couplings}

For all decays of the form \((1.1)\) in which the $W$ emitted in the transition $i \to f + W^\pm$ decays to an $a_1$, $\rho$ or $\pi$, the couplings to these three states should be universal. Thus

$$R(if\pi) \equiv \frac{BR[i \to f\pi^+]}{BR[i \to f\rho^+]} \approx \frac{|W^+ \to \pi^+|^2}{|W^+ \to \rho^+|} \hfill (1.5)$$

$$R(ifa) \equiv \frac{BR[i \to fa_1(1260)^+]}{BR[i \to f\rho^+]} \approx \frac{|W^+ \to a_1^+|^2}{|W^+ \to \rho^+|} \hfill (1.6)$$

for all states $i$ and $f$ with corrections for phase space. For six decays where data are available, the prediction \((1.3)\) gives

$$R(D^+K^0\pi) \approx R(D^0K^-\pi) \approx R(B^0D^-\pi) \approx R(B^0D^{*}\pi) \approx R(B^+D^0\pi) \approx R(B^+D^{*0}\pi) \hfill (1.7)$$

$$\quad .44 \pm .17 \approx .35 \pm .09 \approx .38 \pm .08 \approx .41 \pm .20 \approx .40 \pm .06 \approx .30 \pm .07 \hfill (1.8)$$

The prediction \((1.4)\) gives

$$R(D^+K^0a) \approx R(D^0K^-a) \approx R(B^0D^-a) \approx R(B^0D^{*}-a) \approx R(B^+D^0a) \approx R(B^+D^{*0}a) \hfill (1.9)$$

$$\quad 1.2 \pm .5 \approx .68 \pm .12 \approx .8 \pm .4 \approx 1.9 \pm 1.0 \approx .37 \pm .30 \approx 1.2 \pm .4 \hfill (1.10)$$

This impressive agreement for such widely different decays suggests further investigation.

Obvious other cases to examine with this vector dominance approach are in $\tau$ decays, where there are no final state interactions \(\text{II}\).
\[ BR[\tau^+ \to \pi^+\nu] = 11.09 \pm 0.12 \quad (1.11) \]

Branching ratios for \( \tau \to a_1\nu \) and \( \tau \to \rho\nu \) are not quoted in the tables [1] nor in the extensive experimental investigations of these decays [2].

C. Further Analysis of Charm Decays

Most other decays not describable by vector dominance diagrams have much lower branching ratios. The \( D^o \) and \( B^o \) decays into two neutral mesons in the same isospin multiplets as the observed charged final states are coupled to the charged modes by final state interactions like charge exchange. Decays to \( \rho^o \) and \( a_1^o \) final states are observed to be suppressed, suggesting the absence of appreciable final state interactions.

\[
\frac{BR[D^o \to \bar{K}^o a_1(1260)^o]}{BR[D^o \to K^- a_1(1260)^+] < 1.9\%}{7.3 \pm 1.1\%} \quad (1.12)
\]

\[
\frac{BR[D^o \to K^- \rho^+]}{BR[D^o \to K^o\rho^0] + BR[D^o \to K^o\omega]} = \frac{10.8 \pm 1.0\%}{1.21 \pm 0.17\% + 2.1 \pm 0.4\%} = 3.3 \pm 0.4\% \quad (1.13)
\]

\[
\frac{BR[D^o \to K^+(892)^-\rho^+]}{BR[D^o \to K^+(892)^0\rho^0] + BR[D^o \to K^+(892)^0\omega]} = \frac{6.1 \pm 2.4\%}{1.47 \pm 0.33\% + 1.1 \pm 0.5\%} = 2.6 \pm 0.6\% \quad (1.14)
\]

In decays to neutral pion final states a similar suppression is observed for \( B \) decays but not in \( D \) decays, suggesting appreciable final-state charge-exchange scattering at the \( D \) mass but not at the \( B \) mass.

\[
\frac{BR[D^o \to K^-\pi^+]}{BR[D^o \to K^o\pi^0]} = 3.85 \pm 0.9\% \quad \frac{BR[D^o \to K^+(892)^-\pi^+]}{BR[D^o \to K^+(892)^0\pi^0]} = 5.1 \pm 0.6\% \quad (1.15)
\]

Strong form factors for final axial vector states are predicted to be suppressed with respect to those for pseudoscalars and vectors, because of the node in the axial vector wave functions. This is seen in two decays differing by these form factors,

\[
\frac{BR[D^o \to (s\bar{u} \to K^-)_{S} \cdot (W^+ \to a_1^+)_{W} \to K^- a_1(1260)^+]}{BR[D^o \to [s\bar{u} \to K_1^-]_{S} \cdot (W^+ \to \pi^+)_{W} \to \pi^+ K_1(1270)^-]} = \frac{7.3 \pm 1.1\%}{1.06 \pm 0.29\%} \gg 1 \quad (1.16)
\]
Equal branching ratios are predicted and observed for two decays differing only by the strong factors of the scalar $K_o^+(1430)$ and the axial $K_{1}^-(1270)$ which have very similar $3^P$ wave functions.

$$\frac{BR[D^o \to [s\bar{u} \to K^-_o]_S \cdot (W^+ \to \pi^+)_W \to \pi^+K_o^+(1430)^-]}{BR[D^o \to [s\bar{u} \to K^-_1]_S \cdot (W^+ \to \pi^+)_W \to \pi^+K_{1}^-(1270)^-]} = 1.04 \pm 0.26\% \approx 1$$ (1.17)

Interesting predictions analogous to but opposite to (1.16) arise for the doubly-cabibbo suppressed decays,

$$\frac{BR[D^o \to K^+a_1(1260)^-]}{BR[D^o \to \pi^-K_{1}^+(1270)^+]} \ll 1; \quad \frac{BR[D^+ \to K^+a_1(1260)^0]}{BR[D^+ \to \pi^0K_{1}^+(1270)^+]} \ll 1$$ (1.18)

Here the axial $K_{1}^+(1270)^+$ is produced by a weak form factor; the axial $a_1(1260)^-$ must be produced by a strong form factor. Since the DCSD for the $D^o$ leads to the same final state as a cabibbo-favored decay for the $\bar{D}^o$ and the two initial states are mixed, it may be difficult to check this prediction. On the other hand, a decay mode like $\pi^-K_{1}(1270)^+$ may be useful in studies of $D^o - \bar{D}^o$ mixing by observing decays with time dependence produced by the interference between Cabibbo-favored for one and DCSD amplitudes $[10]$. Here the form factor difference enhances the interference by enhancing the doubly forbidden and suppressing the favored amplitudes.

In $D_s$ decay this model predicts the large branching ratios observed to the final states $\rho^+\eta$, $\rho^+\eta'$, $\pi^+\eta$ and $\pi^+\eta'$. However, the large and unexplained $\eta'/\eta$ ratio does not fit the production of both via their approximately equal $s\bar{s}$ components. The decays $D_s \to a_1^+\eta$, and $D_s \to a_1^+\eta'$, have not been reported, but the reported ratio

$$\frac{BR[D_s \to \pi^+\pi^+\pi^-\pi^-\pi^0]}{BR[D_s \to \rho^+\eta]} = \frac{4.9 \pm 3.2\%}{10.8 \pm 3.1\%}$$ (1.19)

suggests that the $a_1^+\eta$ might be present in the $\pi^+\pi^+\pi^-\pi^-\pi^0$ final state to the extent predicted by the vector-dominance picture.

II. SYMMETRY CONSIDERATIONS
A. Relations from isospin invariance

The strong form factors \([s\bar{q} \to M(s\bar{q})]_S\) and \([(\bar{c}q) \to M(\bar{c}q)]_S\) both conserve isospin. Thus the partial widths of corresponding neutral and charged decays are equal in any model with the \(W\) completely separated from the transition in the hadron recoiling against the \(W\).

\[
\Gamma[D^+(c\bar{d}) \to M(s\bar{d})^0 M(ud)^+] = \Gamma[D^0(c\bar{u}) \to M(s\bar{u})^- M(ud)^+]
\] (2.1)

\[
BR[B^+(\bar{b}u) \to M(\bar{c}u)^0 M(ud)^+] \approx BR[B^0(\bar{b}d) \to M(\bar{c}d)^- M(ud)^+]
\] (2.2)

\[
BR[B^+(\bar{b}u) \to M(\bar{c}u)^0 M(c\bar{s})^+] \approx BR[B^0(\bar{b}d) \to M(\bar{c}d)^- M(c\bar{s})^+]
\] (2.3)

where \(\Gamma\) denotes the partial width of the given decay mode. Approximate equalities of branching ratios are obtained for the \(B^0\) and \(B^+\) decays where the ratio of the charged and neutral meson lifetimes is sufficiently close to unity.

The relations (2.1) and (2.2) can be violated by final state interactions between the produced isovector meson and the other hadron. However the relations (2.3) where the produced meson is isoscalar are exact consequences of isospin invariance. Thus comparing the experimental validity of these two types of transitions can provide insight on the strength of final state interactions.

The results for the semileptonic and the charmed-strange decays satisfy the exact isospin relations (2.3) as expected, the \(a_1\) decays satisfy with large errors the approximate isospin relations (2.1) and (2.2) which require the vector dominance diagram. Disagreements are shown for the \(\rho\) and \(\pi\) decays. Reducing the experimental errors sharpen any such disagreements and shed light on the relative importance of different contributions.

The charged and neutral final states of the \(B^o\) and \(D^o\) decays \(B^o \to \bar{D}M\) and \(D^o \to \bar{K}M\) are mixtures of the same isospin (1/2) and (3/2) amplitudes [3]. Failure to observe the neutral state places an upper limit on the strong phase difference between these amplitudes which constrains models of CP violation. To obtain a quantitative limit for the relative phase \(\phi\), we write the amplitudes in terms of their isospin (1/2) and (3/2) amplitudes, denoted
respectively as $A_1$ and $A_3$, and set $\sqrt{2} \cdot A_3 = (1 + \delta)e^{i\phi} \cdot A_1$ so that $B^o \rightarrow \bar{D}^o \rho^o = 0$ when $\phi = \delta = 0$. Then for the example of $B^o \rightarrow \bar{D}\rho$ decays,

$$\frac{A(B^o \rightarrow \bar{D}^o \rho^o)}{A(B^o \rightarrow D^- \rho^+)} = \frac{\sqrt{2} \cdot A_3 - A_1}{A_3 + \sqrt{2} \cdot A_1} = \frac{(1 + \delta) \cdot e^{i\phi} - 1}{2 + (1 + \delta) \cdot e^{i\phi}}$$  \hspace{1cm} (2.4)$$

Since the isospin couplings are the same for all the related decays of the neutral $B$ and $D$ mesons into their charged and neutral decay modes, this relation gives a value for the relative strong phase between the two isospin amplitudes for all cases where the neutral mode is appreciably suppressed.

$$\sin^2 \frac{\phi}{2} \leq \sin^2 \frac{\phi}{2} + \frac{2\delta^2}{9(1 + \delta)} = \frac{9}{8} \cdot \frac{BR(B^o \rightarrow \bar{D}^o \rho^o)}{BR(B^o \rightarrow \bar{D}^o \rho^o) + BR(B^o \rightarrow D^- \rho^+)} \cdot \left[ 1 - \frac{\delta}{3} \cdot \frac{1 - \delta}{1 + \delta} \right]$$ \hspace{1cm} (2.5)$$

In the case above the experimental the upper limit [1] for the right hand side of eq. (2.5) is 0.06. Better upper limits on these neutral decays can provide better upper limits on strong phases.

**B. Axial vector meson doublets and mixing**

The observed [2] appreciable weak decay $\tau \rightarrow W + \nu \rightarrow a_1 + \nu$ indicates an appreciable weak form factor for the $a_1$. The weak form factor for the $b_1$ is expected to be zero because it would be produced by a second-class current. Experiment seems to confirm that

$$\frac{BR[D^o \rightarrow K^- b_1(1235)^+]}{BR[D^o \rightarrow K^- a_1(1260)^+]} \approx 0$$ \hspace{1cm} (2.6)$$

But better upper limits on decays to the $b_1(1235)^+$ are of interest.

The simple quark model describes the $a_1$ and $b_1$ states as p-wave excitations in $L - S$ coupling with the two quark spins coupled to spin $S = 1$ or $S = 0$ and then to the orbital angular momentum $L=1$ to make two axial states, a scalar and a tensor. These states are also the eigenstates of SU(3) flavor symmetry and $G$-parity.

The $^3P_1$ and $^1P_1$ $u\bar{s}$ and $s\bar{u}$ states in the same SU(3) octets respectively as the $a_1$ and $b_1$, often denoted as $K_A$ and $K_B$, are not mass eigenstates but are mixed by flavor symmetry
breaking due to the \( u - s \) quark mass differences. One suggested mechanism for \( K_A - K_B \) mixing breaks flavor symmetry by the mass difference between the \( K^*\pi \) and \( K\rho \) propagators in the loop diagrams \([4,6]\)

\[
K_A \leftrightarrow K^*\pi \leftrightarrow K_B; \quad K_A \leftrightarrow K\rho \leftrightarrow K_B \quad (2.7)
\]

Another follows the heavy-quark-symmetry approach of neglecting the spin-dependent interaction of the heavier quark \([7]\). The spin of the light quark is coupled with the orbital angular momentum \( L=1 \) to make states with \( j=1/2 \) and \( j=3/2 \). These then couple to the heavy quark spin \( 1/2 \) to make four states as two doublets, rather than a triplet and a singlet.

In the SU(3) symmetry limit the two loop diagrams \((2.7)\) exactly cancel and the states \( K_A \) and \( K_B \) remain exact mass eigenstates. If these loops are the dominant symmetry breakers, a 45° mixing angle results with one of the mass eigenstates decoupled from the \( K^*\pi \) mode and the other decoupled from \( K\rho \) \([4,6]\). The heavy-quark-symmetry \( j=1/2 \) state decays to the S-wave \( K^*\pi \) and \( K\rho \); the \( j=3/2 \) state to the D-wave. Thus loop diagrams like \((2.7)\) do not connect these two states.

A new complication arises from the newly reported \([8]\) \( \sigma(\pi\pi) \) and \( \kappa(K\pi) \) scalar resonances with a \( \sigma \) mass and width of \( 478 \pm 24 \pm 17 \text{ MeV}/c^2 \) and \( 342 \pm 42 \pm 21 \text{ MeV}/c^2 \) and a \( \kappa \) mass and width of \( 815 \pm 30 \text{ MeV}/c^2 \) and \( 560 \pm 116 \text{ MeV}/c^2 \). A \( \pi\kappa \to \pi\pi K \) and a \( \sigma K \to \pi\pi K \) would show up in the \( \pi\pi K \) Dalitz plot as an apparent nonresonant background with the \( \pi K \) or the \( \pi\pi \) system in an S-wave.

Differences in the way the \( K_1(1400)^+ \) and \( K_1(1270)^+ \) appear in heavy flavor decays can give information on the mixing angles. In particular, all diagrams producing the strange axial vector meson via the coupling to the \( W \) should produce the two states in the same ratio as in \( \tau \) decay, and give a value for the mixing angle. These arise in Cabibbo forbidden decays via the forbidden \( W \to K_A \) vertex, where it is assumed that the \( K_A \) and \( K_B \) are coupled respectively like the \( a_1 \) and \( b_1 \) to weak first and second class currents. For example,

\[
\frac{BR[D(c\bar{q}) \to (W^+s)\bar{q} \to (s\bar{q})K^+_A \to M(s\bar{q})K_1(1400)^+]}{BR[D(c\bar{q}) \to (W^+s)\bar{q} \to (s\bar{q})K^+_A \to M(s\bar{q})K_1(1270)^+]} = \frac{BR[\tau^+ \to (W^+\nu) \to \bar{\nu}K^+_A \to \bar{\nu}K_1(1400)^+]}{BR[\tau^+ \to (W^+\bar{\nu}) \to \bar{\nu}K^+_A \to \bar{\nu}K_1(1270)^+]}
\]

\[(2.8)\]
However we note that in the charged decays it is the $K_1(1400)^+$ that is seen at approximately the same level as the $K_1^*(1430)^+$ while the $K_1(1270)^+$ is not seen and its upper limit $7 \times 10^{-3}$ is down by almost an order of magnitude.

$$BR[D^+ \to \pi^+ K_1(1400)^+] = 4.9 \pm 1.2\% \approx BR[D^+ \to \pi^+ K_1^*(1430)^+] = 3.7 \pm 0.4\% \quad (2.9)$$

Differences in the way the two axial vector states $K_1(1400)^+$ and $K_1(1270)^+$ appear in heavy flavor decays; e.g. one appearing in charged D decays and the other in neutrals may provide interesting information about the structures and mixing of these states.

The two charmed-strange axial vector meson states, denoted as $D_{s1A}$ and $D_{s1B}$ are expected to be strongly mixed by the large $c-s$ mass difference. So far only the one charmed-strange axial vector state $D_{s1}(2536)$ is listed [1] and little is known about its properties. The loop-diagram-mixing diagrams are:

$$D_{s1A} \leftrightarrow D^*K \leftrightarrow D_{s1B}; \quad D_{s1A} \leftrightarrow K^*D \leftrightarrow D_{s1B} \quad (2.10)$$

However, the masses of the particles in the intermediate states show that the $D_{s1}(2536)$ can decay into $D^*K$ but not into $K^*D$. If the loop diagram (2.10) dominates the mixing the resulting mass eigenstates can have one state completely decoupled from the $D^*K$ mode.

$$M(K) \approx 490 MeV; \ M(K^*) \approx 895 MeV; \ M(D) \approx 1870 MeV; \ M(D^*) \approx 2010 MeV,$$

$$M(K) + M(D^*) \approx 2500 MeV; \ M(K^*) + M(D) \approx 2765 MeV$$

But there is now the possibility of a $D\kappa \to DK\pi$ final state which would show up in the $DK\pi$ Dalitz plot as an apparent nonresonant background with the $K\pi$ system in an S-wave.

**III. ADDITIONAL DECAYS DESCRIBED BY VECTOR DOMINANCE**

**A. Cabibbo-suppressed decays**

Diagrams including the Cabibbo-suppressed W-decay and W-production vertices, $W^+ \to u\bar{s}$ and $c \to W^+d$ describe the singly-Cabibbo-suppressed charm decays,

$$D(c\bar{q}) \to (W^+s)\bar{q} \to [s\bar{q} \to M(s\bar{q})]s \cdot (W^+ \to K^+)w \to M(s\bar{q})K^+ \quad (3.1)$$
\[ D(c\bar{q}) \rightarrow (W^+d)\bar{q} \rightarrow [d\bar{q} \rightarrow M(d\bar{q})]_S \cdot (W^+ \rightarrow M^+) \rightarrow M(d\bar{q})M^+ \quad (3.2) \]

where \( K^+ \) also denotes any strange resonance; e.g. \( K^*(892)^+ \) and \( K_A^+ \). The doubly-Cabibbo-suppressed \( c \rightarrow W^+d \rightarrow (us)d \) can also produce the following “vector-dominance” diagrams for doubly-cabibbo-suppressed decays:

\[ D(c\bar{q}) \rightarrow (W^+d)\bar{q} \rightarrow [d\bar{q} \rightarrow M(d\bar{q})]_S \cdot (W^+ \rightarrow K^+) \rightarrow M(d\bar{q})K^+ \quad (3.3) \]

In a mixed \( D^0 - \bar{D}^0 \) state the doubly-forbidden \( D^0 \rightarrow \pi^- K_A^+ \) interferes with the cabibbo-favored \( \bar{D}^0 \rightarrow \pi^- K_A^+ \). But the \( K_A^+ \) in the favored decay is created by a combination of the strange antiquark from the weak vertex and the spectator \( u \) quark. This amplitude is expected to be suppressed because it involves a hadronic form factor overlap between the initial nodeless wavefunction and the p-wave, while the doubly-forbidden amplitude has the enhanced \( W \rightarrow A \) vertex. The two interfering amplitudes may therefore be more nearly equal than in the \( K\pi \) case discussed in ref. [9,10]. The decays to the \( \pi^- K_A^+ \) final state may be particularly advantageous for studies of this interference and measurement of the relative strong phase [10].

**B. Vector-Dominance Decays of the \( B_c \)**

The \( B_c \) meson is identified against a large combinatorial background by decay modes including a \( J/\psi \). Vector dominance decay modes including the \( J/\psi \) are expected to have relatively large branching ratios. These include: \( J/\psi \rho^+, J/\psi a_1^+, J/\psi \pi^+, J/\psi D^*_s, J/\psi D_{s1A}, \) and \( J/\psi D_s \). The corresponding modes with a \( \psi' \) instead of a \( J/\psi \) are expected to have comparable branching ratios.

**IV. ACKNOWLEDGMENTS**

It is a pleasure to thank J. Appel, E.L. Berger, S. Bergmann, Y. Grossman, U. Karshon, Z. Ligeti, Y. Nir and Richard Stroynowski, for helpful discussions and comments.
### TABLE I

Branching Ratios for D Decays into Vector Dominance Modes

|          | $D^0$ Decay | $D^+$ Decay | $D^0$ Decay | $D^+$ Decay |
|----------|-------------|-------------|-------------|-------------|
| $M^+$    | $BR(K^-M^+)$ | $BR(\bar{K}^0M^+)$ | $BR(K^*-M^+)$ | $BR(\bar{K}^{*0}M^+)$ |
| $a_1(1260)^+$ | 7.3 ± 1.1%  | 8.0 ± 1.7% |             |             |
| $a_2(1320)^+$ | < 0.3%      | < 0.2%      |             |             |
| $\rho^+$ | 10.8 ± 1.0% | 6.6 ± 2.5%  | 6.1 ± 2.4%  | 2.1 ± 1.3%  |
| $\pi^+$  | 3.85 ± 0.9% | 2.89 ± 0.26% | 5.0 ± 0.4%  | 1.90 ± 0.19% |
| $e^+\nu_e$ | 3.66 ± 0.18% | 6.7 ± 0.9% | 2.02 ± 0.33% | 4.8 ± 0.5% |
| $\mu^+\nu_\mu$ | 3.23 ± 0.17% | 7.0 ± 3.0% |             | 4.4 ± 0.6% |

### TABLE II

Branching Ratios for B Decays into Vector Dominance Modes

|          | $B^0$ Decay | $B^+$ Decay | $B^0$ Decay | $B^+$ Decay |
|----------|-------------|-------------|-------------|-------------|
| $M^+$    | $BR(D^-M^+)$ | $BR(\bar{D}^0M^+)$ | $BR(D^*-M^+)$ | $BR(\bar{D}^{*0}M^+)$ |
| $a_1(1260)^+$ | 0.60 ± 0.33% | 0.5 ± 0.4% | 1.30 ± 0.27% | 1.9 ± 0.5% |
| $\rho^+$ | 0.79 ± 0.14% | 1.34 ± 0.18% | 0.68 ± 0.34% | 1.55 ± 0.31% |
| $\pi^+$  | 0.3 ± 0.04% | 0.53 ± 0.05% | 0.276 ± 0.021% | 0.46 ± 0.04% |
| $\ell^+\nu_\ell$ | 2.10 ± 0.19% | 2.15 ± 0.22% | 4.60 ± 0.27% | 5.3 ± 0.8% |
| $D_s$    | 0.8 ± 0.3% | 1.3 ± 0.4% | 0.96 ± 0.34% | 1.2 ± 0.5% |
| $D_s^*$  | 1.0 ± 0.5% | 0.9 ± 0.4% | 2.0 ± 0.7% | 2.7 ± 1.0% |
REFERENCES

[1] Particle Data Group, Eur. Phys. J. C 15 (2000) 1
[2] CLEO collaboration, Phys. Rev.D61 (2000) 052004, Phys.Rev.D61 (2000) 112002, Phys. Rev. Lett. 72(1994)3762, 70(1993)1207 and 75(1995)3809
[3] Harry J. Lipkin, Phys. Rev. Lett. 46 (1981) 1307
[4] Harry J. Lipkin, Physics Letters B303 (1993) 119
[5] Mahiko Suzuki, Phys. Rev. D47 (1993) 1252
[6] Harry J. Lipkin, Phys. Lett. B 72 (1977) 249
[7] N. Isgur and M. B. Wise, Phys. Rev. Lett. 66 (1991) 1130
[8] Carla Gobel, E791 Collaboration, hep-ex/0012009
[9] L. Wolfenstein, Phys. Rev. Lett. 75 (1995) 2460
[10] S. Bergmann, Y.Grossman, Z. Ligeti, Y.Nir and A.A.Petrov, hep-ph/0005181
FIG. 1.
Weak vector dominance diagram.