On Reality of Tachyonic de Broglie Waves

Rajat K. Pradhan*† and Lambodar P. Singh

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Department of Physics, Utkal University, Bhubaneswar - 751 004, India.

Abstract

We investigate the tachyonic nature of the de Broglie matter waves associated with a free quantum object to show that granting reality to them would lend completeness to the quantum description of reality. Basing on the robustness of the well known Einstein-de Broglie reciprocal relation between the phase and the particle velocities, we extend the concept of complementarity to them and thereby propose a complementary relation between a bradyon and its corresponding tachyon (i.e. the associated matter wave) to endow the tachyons with a degree of reality, at least on par with the bradyons, within the current framework of quantum physics and extended relativity. The duality is used to argue that because of the observed localised nature of bradyons, tachyons should always be pervasive or global in character and thus, there can be no point-like tachyons. A common misconception regarding the nonrelativistic limit of the Einstein-de Broglie relation is pointed out and the consequent error of long standing is remedied.

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*Present address: V. Dev College, Jeypore - 764 001, India
†Corresponding author; E-mail: rajat@iopb.res.in
1. Introduction

Tachyons [1] have been the focus of attention in recent times in three different sectors. Firstly, the researches in String Theory have led unequivocally to their presence and currently there has been great deal of interest in tachyon condensation [2] on branes and tachyonic inflation [3] in String cosmology. Secondly, the tachyons have been investigated as candidates for the dark matter in relativistic cosmologies [4]. Thirdly, there have been analyses and experiments on superluminal transmission in evanescent photon tunneling as well as in quantum mechanical barrier penetration phenomena [5]. In this work, we look for a possible place for tachyons within the current framework of quantum theory following de Broglie’s original treatment of wave-particle duality [6].

The traditional description of a free quantum object has been in terms of a wave packet composed of an infinite number of plane waves so as to grant meaning to the observed localized nature of the object within the limits set by the uncertainty principle. The duality between a particle and its corresponding wave envisaged by de Broglie was eschewed in favour of the description in terms of a packet of probability waves in the interpretation of quantum theory developed by Born et al., which is the standard quantum theory.

The basic problem with a single de Broglie wave is its non-localized nature which gives an infinite extendedness to the free particle contrary to its observed localized nature. The uncertainty principle on the other hand, achieves the localization with a position uncertainty but, at the same time, introduces a simultaneous uncertainty in the momentum. A second problem with the de Broglie wave is the fact that its velocity, which is the wave or phase velocity, is not equal to the particle velocity and for material particles which always move with subluminal speeds (bradyons), it exceeds the speed of light. Therefore, these superluminal matter waves are undoubtedly tachyonic in nature. Some experiments with tunneling phenomena have been performed to ascertain actual superluminal transmission across a potential barrier [7].

In traditional relativity, superluminal motion is considered unphysical because of the restrictive second Einstein postulate and since tachyons have not been observed experimentally, the wave packet description is generally accepted to be the only valid
description of a free quantum object. However, it has got its own problems also as delineated below:

- **Spreading**: As is well known, the wave packet inevitably spreads in space as the particle moves due to the nonlinearity of the energy-momentum relation and the harmonic time dependence of the wave function. The description becomes quite inadequate and unsatisfactory at large times. The packet disperses with time occupying ever larger spatial domains.

- **Interpretation**: As per the Copenhagen interpretation, these waves are probability waves and we can only talk about the probability of finding the particle at a point in space. But, this is not accepted by the other proposed interpretations of quantum theory like the de Broglie-Bohm pilot wave interpretation [8]. In fact, both the path integral approach of Feynman [9], and the many worlds/minds interpretation [10], envisage the motion of the particle as a sum over all possible particle paths with appropriate amplitudes, not necessarily confined to the narrow tubelike region traversed by the linear movement of the centre of the wave packet from one point to the other as expected from Ehrnfest’s theorem for wave packet dynamics. Obviously, Feynman grants more conceptual reality to the particle itself compared to the wave packet, since the sum is over particle paths and not over wavepacket traversal tubes during the motion.

- **Irrationality** The bradyonic wave packet is made up of superposition of individual tachyonic waves which are themselves unphysical from the viewpoint of relativity. We may ask: How can we get a description of a physical bradyonic particle by superposing ‘unphysical’ tachyonic solutions? It is certainly not logically satisfying that we label the component waves as unphysical and then call their superposition as physical, for in that case, all physicality may be said to be rooted in utter unphysicality, which is neither acceptable nor tenable.

- **Extrapolation** Before measurement (or any kind of interaction involving the object), we have got to assign equal a priori probability of its being anywhere in space. It is important to remember that “Our common sense notion of a point particle is always a post-measurement notion.” Because we cannot say anything definite about the pre-measurement state of a particle, we cannot also say anything about the pre-measurement
position of the particle, since position and momentum are the essential observables for state characterization.

This is as simple as it sounds and follows from common-sense knowledge of quantum mechanical principles, since before a measurement we cannot even know whether a quantum object is a point object or not. The very knowledge of the existence of an object presupposes a measurement of its state or position. We cannot imagine an object without the category of space or spatial location associated with it. Further, in view of the postulate of quantum measurement in regard to the collapse of the wave function, we may very well argue that it is the measurement (or interaction) that brings about the familiar individuated, localised particulate existence from a pervasive position wave function with constant amplitude everywhere.

Thus, we have to endow the individual de Broglie waves with some degree of reality, at least on par with their superposition i.e. the wave packet. This leads us to a further investigation of the issue with a view to clearly bringing out the essential pervasive character of a free quantum object, which we have all along been trying to circumvent by resorting to a confined, pointlike, localized description with the help of various adhoc assumptions in the standard quantum theory. We may ask, “when in the Feynman picture, we allow for all possible paths during the motion, why cannot we allow, a priori, all possible positions to the quantum object at rest?” In fact, if this is done, it would lead to a description quite in keeping with what the de Broglie relation would imply for the particle at rest (λ → ∞). Of course, Feynman’s interpretation of the Dirac factor $exp(iS/\hbar)$ as the probability amplitude for the path was to recover the Born probabilistic interpretation for the acceptability of his formulation. But here, in our proposed interpretation, ‘the particle actually follows all possible paths unless it is subjected to a measurement to determine its transit route’.

The paper is organised as follows: In Section - 2, we discuss the concept of the free quantum object as used in this paper and then derive the Einstein-de Broglie relation between the phase and the particle velocities. In Section - 3, we look at the issue of spreading of the free particle wave packet taking the relativistic formula for energy and study the implications. A common misconception in textbooks as well as in some recent works regarding the non-relativistic phase velocity being half the group velocity are
clarified in the light of this derivation in Section - 4 and the uncontradictability of the Einstein-de Broglie relation is established. This relation is rederived from a different perspective in Section - 5, wherein we bring out a reinterpretation of the relation. In Section - 6, we propose an extension and a generalization of the complementarity principle to bradyon-tachyon duality/complementarity on the basis of this re-interpretation. Finally, in Section - 7, we conclude with a discussion on the new interpretation of quantum theory presented here and point out some of its shortcomings and also its advantages.

2. Phase velocity and particle velocity

Consider a free quantum object at rest characterized by rest energy $E_0$. The Schrödinger equation $H\psi = i\hbar \psi/dt$ yields $\psi(t) = A \exp(-i\omega_0 t)$ where, $\omega_0 = E_0/\hbar$ is the frequency of vibration and $H = E_0$ is the Hamiltonian. Interestingly enough, since the Hamiltonian here is having no space dependence, the quantum object can be said to be either independent of space or to be equally pervading all space. If it is completely independent of space, then there is no possibility of our ever making any contact with it except through the time dimension, in which it is seen to be a harmonic vibration. The other alternative is to interpret it as a cosmic vibration pervading all space with equal amplitude for existence at all points. This has been used by de Broglie [11] to arrive at his famous relation expressing wave-particle duality.

For an observer moving with a velocity $v$ along the negative x-direction, this cosmic vibration will appear to have a velocity $v$ in the positive x-direction. Lorentz transforming to the moving frame and on employing

$t' = \gamma(t - vx/c^2)$

with $\gamma = (1 - \beta^2)^{1/2}$ and $\beta = v/c$, we have for the wave function:

$\psi(t') = \psi[\gamma(t - vx/c^2)] = A \exp [-i\omega_0 t'] = A \exp [-i\omega_0 \{\gamma(t - vx/c^2)\}]$ (2)

This wave function should have the generic form

$\psi(x, t) = A \exp [-i\omega_0 \{\gamma(t - x/v_w)\}]$ (3)

whence we get $v_w = c^2/v$ for the wave velocity or phase velocity.
This gives us the Einstein-de Broglie relation between the particle velocity $v$ and the wave velocity $v_w$ as:

$$v \cdot v_w = c^2 \quad (4)$$

Thus, the changed frequency of the traveling wave train is given by $\omega = \gamma \omega_0$ and the changed energy of the object is $E = \gamma E_0 = \gamma h \omega_0$. The wavelength is $\lambda_{dB} = v_w/\nu = c^2/\nu v$ and on using the general formula $p = vE/c^2$ for momentum, we get the relation $\lambda_{dB} = h/p$. As is well known, the de Broglie relation expressing wave-particle duality had an undoubted interpretation via the association of ‘some kind of a matter wave’ with a matter particle in the old quantum theory before the emergence of the new quantum theory with the standard Born probabilistic interpretation of the Schrödinger wave function. But, it was stripped of all its significance when the probability interpretation gained currency.

The reasons for this are twofold:

- First, It represented a tachyonic wave (associated with the bradyonic particle) which seemed not only to move ahead of the particle leaving the latter behind but also was unphysical from the point of view of the second postulate of relativity.

We wish to clarify regarding this objection that the matter wave being pervasive along the direction of motion is everywhere present and thus cannot “leave” the particle “behind” anywhere, for there is no position where the wave is not. So, we see that because of its tachyonic character the wave can, without any contradiction, be associated with the particle all through its motion. “Leaving behind” is a notion applicable only to bradyons (which always have finite extension) looked at from subluminally moving frames.

- Second, because of the infinitude of its extent, it did not aid the visualization of the point particle as a somewhat smeared-out existence in a finite region of space as

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1 Even for photons(luxons) the concept of “leaving behind” is not applicable because of the constancy and the maximality of the speed of light. For a photonic(or, infinite-momentum) frame, the entire line of motion shrinks to a point because of length contraction and all entities along the line are seen to be on top of each other simultaneously, unless of course, the line of motion is infinite in extent (in which case, it may contract to a finite length).
a satisfactory extension of the concept of the point particle as required to explain the various quantum phenomena.

Regarding this second point, we recall that the Born interpretation looks at the wave function as representing the “probability of existence” at a space-time point and not at “existence” as such. Thus, matter waves become moving probability waves and physicality or materiality of the waves is simply washed out with this interpretation and what is retained is, surprisingly enough, only our classical notion of the particle as a point object located at some point in space at any instant of time! Having a probability of being located at a certain point at any instant of time precisely means that the particle is a point object. The probability interpretation thus keeps intact the common-sense notion of the particulate existence of matter at the fundamental level, rendering our description of matter classical, but with the added factor of probability. Assuming that this widely accepted interpretation is correct, we still run into problems with maintaining the permanence of the free quantum object since the wave packet that we construct for it by superposing the component probability waves inevitably spreads in time as it moves. This is because of the nonlinear relation between energy and momentum which leads to dispersion.

Thus, there is no contradiction involved in de Broglie’s association of a tachyonic matter wave with a bradyonic particle. What this discussion brings out is that –

(a) the quantum object at rest is actually a pervasive existence and when it is in interaction with something else like a measuring instrument, it appears to be a finite, localised existence which we have all along tried to describe by constructing wave packets.

(b) when it is in motion, it is again a pervasive existence along the line of motion represented by the de Broglie wave train.

In either case, the de Broglie duality relation is a sufficient tool to understand its nature and the associated tachyonic matter wave is a reality not only to be reckoned with but also to be made use of in understanding issues where we have failed with the standard approach.
When a wave packet is artificially constructed to force finitude and a localised existence upon this quantum object, it inevitably spreads to regain its infinitude and pervasiveness, for the harmonic time dependence is at the back of both, the pervasiveness (when space is not involved) and the spreading of the wave packet (when there is motion in space).

This conclusion can also be inferred from the phenomenon of *zitterbewegung* in relativistic quantum mechanics, where the attempt to localize a particle beyond a certain minimal limit invites zittery oscillations making the particle highly unstable. So, it seems that the concept of a localized particle is an artifact of our classical outlook and is not in keeping with the nature of the reality as such. As a result, wherever we have tried to forcibly impose finitude anywhere in quantum theory in any manner whatsoever, we have run into insurmountable conceptual difficulties including those connected with renormalization.

We shall look at the issue of spreading a little more closely taking the relativistic mass energy formula in the following Section.

3. Group velocity and the inevitable spreading of wave packets

In standard quantum theory, the way out of the problem of the tachyonic phase velocity of the de Broglie waves is to superpose these very waves with slightly differing wavelengths so as to get some kind of an *average* velocity by looking at the stationarity of the phase around the central value \( \vec{p}_0 \) with respect to change in momentum within a small range of values \( \Delta p \leq |\vec{p} - \vec{p}_0| \). Such a wave packet can be written as:

\[
\psi(\vec{r}, t) = \int \frac{d^3p}{(2\pi \hbar)^3} \phi(\vec{p}) \exp \left[ \frac{i}{\hbar} (\vec{p} \cdot \vec{r} - E(\vec{p})t) \right] 
\]  

(5)

where \( \phi(\vec{p}) \) is the weight function for the momentum space distribution. Suppose \( \phi(\vec{p}) \) has a small range of non-zero values in a region \( \Delta p \leq |\vec{p} - \vec{p}_0| \) about a maximum at \( \vec{p}_0 \). The group velocity is calculated by demanding the stationarity of the phase:

\( \vec{\nabla}_\vec{r} \{ \vec{p} \cdot \vec{r} - E(\vec{p})t \}|_{\vec{p}_0} = 0 \), which yields \( \vec{r}(t) = v_0 t \) with \( \vec{v}_0 = \vec{\nabla}_\vec{p} E(\vec{p})|_{\vec{p}_0} \). This position \( \vec{r}(t) = \vec{v}_0 t \) corresponds to the maximum of \( \psi(\vec{r}, t) \). It describes the motion of the approximate center of the packet which we take to represent the classical motion of the particle.
with velocity $\vec{v}_0$. For an explicit evaluation of the spreading in time of the packet, we take the relativistic energy formula $E(p) = \sqrt{p^2c^2 + m^2c^4} = \gamma mc^2$ and expand the phase about $\vec{p}_0$ in a Taylor series to obtain:

$$
\vec{p}.\vec{r} - E(p)t = \vec{p}_0.\vec{r} - E(\vec{p}_0)t + (\vec{r} - \vec{\nabla}_p E(\vec{p})|_{\vec{p}_0})(\vec{p} - \vec{p}_0) \\
+ \frac{1}{2} \sum_{i,j} \left(-\frac{\partial^2 E}{\partial p_i \partial p_j} \right)_{\vec{p}_0} (p_i - p_{0i})(p_j - p_{0j}) + .......
$$

$$
= \vec{p}_0.\vec{r} - E(\vec{p}_0)t + (\vec{r} - \vec{r}(t)).(\vec{p} - \vec{p}_0) \\
- \frac{tc^2}{2E_0} \sum_{i,j} \left( \delta_{ij} - c^2 \frac{p_{0i}p_{0j}}{E_0^2} \right) \Delta p_i \Delta p_j + .......
$$

where

$$E_0 = E(\vec{p}_0) = \sqrt{\vec{p}_0^2c^2 + m^2c^4} = \gamma_0 mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}},$$

$$p_i - p_{0i} = \Delta p_i, \quad p_j - p_{0j} = \Delta p_j; \quad \vec{v}_g(t) = \vec{\nabla}_p E(\vec{p})|_{\vec{p}_0} \text{ and } \vec{r}(t) = \vec{r}_0(t) + \vec{v}_g t.$$

Substituting back in eq.(5) above we have,

$$\psi(\vec{r}, t) = \exp \left[ \frac{i}{\hbar} \left( \vec{p}_0.\vec{r} - E(\vec{p}_0)t \right) \right] \int \frac{dp}{2\pi \hbar^3} \phi(p) \exp \left[ \frac{i}{\hbar} \left( (\vec{r} - \vec{r}(t)).(\vec{p} - \vec{p}_0) \\
- \frac{tc^2}{2E_0} \sum_{i,j} \left( \delta_{ij} - c^2 \frac{p_{0i}p_{0j}}{E_0^2} \right) \Delta p_i \Delta p_j + ....... \right) \right] \quad (6)$$

Without any loss of generality, we now reorient our axes so that the motion of the packet is along the +ve x-direction. On writing

$$p = p_x, \quad p_{0x} = p_0 \text{ and } E(p_{0x}) = E(p_0) = E_0,$$

we have

$$\psi(x, t) = \exp \left[ \frac{i}{\hbar} \left( p_{0x} - E_0 t \right) \right] \int \frac{dp}{2\pi \hbar} \phi(p) \exp \left[ \frac{i}{\hbar} \left( (x - x(t))(p - p_0) \\
- \frac{tc^2}{2E_0} \left( 1 - \frac{c^2 p_0}{E_0^2} \right) (p - p_0)^2 + ....... \right) \right]$$

$$= \exp \left[ \frac{i}{\hbar} \left( p_{0x} - E_0 t \right) \right] \int \frac{dp}{2\pi \hbar} \phi(p) \exp \left[ \frac{i}{\hbar} \left( (x - v_0 t)(p - p_0) - \frac{it}{2\mu \hbar} (p - p_0)^2 + ....... \right) \right] \quad (7)$$

where,

$$\mu^{-1} = \frac{c^2}{E_0} \left( 1 - \frac{c^2 p_0^2}{E_0^2} \right)$$
For definiteness, we now choose a Gaussian form for $\phi(p)$:

$$\phi(p) = A \exp \left[ -\frac{d^2}{\hbar^2} (p - p_0)^2 \right]$$

(8)

Then the wave function on neglecting third order and higher becomes

$$\psi(x, t) = \exp \left[ \frac{i}{\hbar} (p_0 x - E_0 t) \right] \frac{A}{2\pi \hbar} \int dp \, \exp \left[ \frac{i}{\hbar} (x - v_g t)(p - p_0) - a(p - p_0)^2 \right]$$

(9)

where,

$$a = \frac{d^2}{\hbar^2} + \frac{it}{2\mu \hbar}, \quad v_g = \left. \frac{\partial E}{\partial p} \right|_{p_0} = \frac{p_0 c^2}{E_0}. \quad \quad (10)$$

To further simplify, we put $p_0 x - E_0 t = \phi_0$ and $x - v_g t = \delta x(t) = \delta x$ to obtain

$$\psi(x, t) = \exp \left[ \frac{i}{\hbar} \phi_0 \right] \frac{A}{2\pi \hbar} \int dp \, \exp \left[ \frac{i}{\hbar} (\delta x(p - p_0) - a(p - p_0)^2 \right]$$

$$= \exp \left[ \frac{i}{\hbar} \phi_0 - a p_0^2 - i \frac{\delta x p_0}{\hbar} \right] \frac{A}{2\pi \hbar} \int dp \, \exp \left[ -ap^2 + 2a \left( p_0 + \frac{i\delta x}{2ah} \right) p \right]$$

(11)

which, on evaluation yields

$$\psi(x, t) = \sqrt{\frac{\pi}{a}} \frac{A}{2\pi \hbar} \exp \left[ \frac{i}{\hbar} \phi_0 - a \left( \frac{\delta x}{2ah} \right)^2 \right]$$

$$= \sqrt{\frac{\pi}{a}} \frac{A}{2\pi \hbar} \exp \left[ \frac{i}{\hbar} (p_0 x - E_0 t) - a \left( \frac{\delta x}{2ah} \right)^2 \right]$$

The density is given by

$$|\psi(x, t)|^2 = \left( \frac{A}{2\pi \hbar} \right)^2 \frac{\pi}{a} \exp \left[ 2\text{Re} \left\{ \frac{-\delta x^2}{4ah^2} \right\} \right] = \left( \frac{A}{2\pi \hbar} \right)^2 \frac{\pi}{a} \exp \left[ 2\text{Re} \left\{ \frac{-\delta x^2}{2d^2(1 + \alpha^2)} \right\} \right]$$

(12)

where, $\alpha = \frac{\hbar}{2\mu \sigma^2}$ and the normalisation factor is $A = (8\pi d^2)^{1/4}$. Thus we get the normalised density:

$$|\psi(x, t)|^2 = \frac{1}{d \sqrt{2\pi(1 + \alpha^2)}} \exp \left[ -\frac{(x - v_g t)^2}{2d^2(1 + \alpha^2)} \right]$$

(13)

which is a Gaussian in space whose maximum moves with the group velocity

$$v_g = \left. \frac{\partial E}{\partial p} \right|_{p_0} = \frac{p_0 c^2}{E_0} \quad \quad (14)$$
Since the quantity
\[ \alpha = \frac{\hbar}{2\mu d^2} = \frac{\hbar c^2}{2d^2E_0} \left( 1 - \frac{v^2}{c^2} \right) \]  
increases linearly with time, the wave packet spreads.

To compare with the corresponding nonrelativistic (NR) result obtained by taking only the kinetic energy in the Hamiltonian we rewrite \( \alpha \) by using \( E_0 = \gamma mc^2 \) as
\[ \alpha = \frac{\hbar}{2md^2} \left( 1 - \frac{v_g^2}{c^2} \right)^{3/2} = \alpha_{NR} \left( 1 - \frac{v^2}{c^2} \right)^{3/2}, \]
where \( \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \) and \( \alpha_{NR} = \frac{\hbar}{2md^2} \) is the corresponding factor for the nonrelativistic packet. We see that for faster moving particles with \( \beta = \frac{v}{c} \gg \frac{v_{NR}}{c} \), the position uncertainty defined by
\[ < \Delta x > = \sqrt{< (x - < x >)^2 >} = d\sqrt{1 + \alpha^2} \]
grows at a lesser and lesser rate compared to the low velocity nonrelativistic regime. Nevertheless, spreading is there and it is clear that the spreading is an inevitable consequence of the harmonic nature of the time dependence of plane waves, whatever be our choice of the momentum distribution \( \phi(p) \).

In the above consideration, we have not kept beyond the second order terms in the tailor series expansion of the phase, but, we can still get the ultrarelativistic behaviour by taking the limit \( v_g \to c \). We see that for photons there is no spreading, which is expected because of the equality of the phase velocity and the group velocity for them.

4. The nonrelativistic group velocity

Textbooks on nonrelativistic quantum mechanics (see e. g. [12]) describe the relation between phase and group velocities in the following manner:

The energy of the free particle is entirely kinetic i.e. \( E = \frac{1}{2} mv^2 \). The phase velocity of the wave is \( v_w = \omega/k = E/p = \frac{p}{2m} = v/2 \) while the group velocity of the wave packet is \( v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = p/m = v = \text{particle velocity} \). This implies that the group velocity is
twice the phase velocity.

However, the fact that the general relation \( v = pc^2/E \) holds for all velocities and that the most general expression for the energy is given by the relativistic formula \( E^2 = p^2c^2 + m^2c^4 \), we have, \( EdE = c^2pdp \) which gives the group velocity to be

\[
v_g = \frac{dE}{dp} = \frac{pc^2}{E} = v
\]

and the phase velocity as

\[
v_w = \frac{E}{p} = \frac{c^2}{v}
\]

Of course, from these two equations we get the general relation \( v.v_w = c^2 \) derived earlier in Section - 2. But, if \( v_w = v/2 \) as derived above, then there is a contradiction.

Our emphasis is that the matter waves are always tachyonic in character which means that we should have in all situations \( v_w > v = v_g \). It is not that this contradiction has gone unnoticed in the literature, although textbook authors have mostly gone by the above derivation and concluded unanimously that in the non-relativistic case the phase velocity is half the group/particle velocity. In a recent article [13], it has almost been figured out but not quite remedied in a proper manner.

To resolve it, we begin with the correct nonrelativistic expression for the energy of the free particle which must include the rest energy:

\[
E = mc^2 + \frac{p^2}{2m}
\]

which gives for the phase velocity

\[
v_w = \frac{E}{p} = \frac{p}{2m} + \frac{mc^2}{p} = \frac{v}{2} + \frac{c^2}{\gamma v} = \frac{v}{2} + \frac{c^2}{v} \left(1 - \frac{v^2}{c^2}\right)^{1/2}
\]
inclusive of the rest energy does not alter the result \( v_g = v \) obtained above, as expected because of the constancy of the rest energy.

The above analysis, in addition to removing a common misconception regarding the nonrelativistic limit of the relation \( v v_w = c^2 \), also gives us a hint that there is a much deeper connection of the relativity theory with the quantum theory [14], at least as far as de Broglie’s approach is concerned.

In the following Section, we further investigate this connection in a Compton effect-like situation and at the same time bring out from a quite different perspective the robustness of the expression \( v v_w = c^2 \) which leads us to a reinterpretation of our common sense notion of “the energy carried by a particle”.

5. The robustness of \( v v_w = c^2 \): a fresh approach

In a Compton Scattering experiment, let an energy \( \hbar \omega \) be absorbed by an electron (rest mass \( m \)) at rest. We note that this energy is not the whole energy of the incident photon but is only that part of the incident photon energy that is taken up by the electron in the process. As a result the electron would move with a kinetic energy \( K = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 \). This kinetic energy is entirely due to the energy \( \hbar \omega \) absorbed by it. Thus \( \hbar \omega = (\gamma - 1)mc^2 \). The total energy is then \( E = \hbar \omega + mc^2 = \hbar(\omega + \omega_0) \), where \( \hbar \omega_0 = mc^2 \) is the rest energy. Thus,

\[
E = \sqrt{p^2 c^2 + m^2 c^4} = \hbar \omega + mc^2
\]

whence,

\[
p = \frac{\hbar \omega}{c} \sqrt{1 + \frac{2mc^2}{\hbar \omega}} = \frac{\hbar \omega}{c} \sqrt{1 + \frac{\omega_z}{\omega}} \tag{23}
\]

where, \( \omega_z = 2mc^2/\hbar \) is the zitterbewegung frequency of the particle. Using this and the expression for the total energy of the particle we deduce for the phase velocity ,

\[
v_w = \frac{E}{p} = c \frac{1 + \frac{\omega_z}{\omega}}{\sqrt{1 + \frac{\omega_z}{\omega}}} = \frac{c^2}{v} \geq c
\]

where, the equality holds only for massless particles. Writing the Compton wavelength as \( \lambda_c = \frac{h}{mc} \) and the incident photon wavelength as \( \lambda_\gamma = 2\pi c/\omega \), we can explicitly
verify that the de Broglie wavelength $\lambda_{dB} = h/p$ is related to it by:

$$\lambda_{dB} = \lambda_\gamma \left(1 + \frac{\omega_z}{\omega}\right)^{-1/2} = \lambda_\gamma \sqrt{1 + \frac{2\lambda_\gamma}{\lambda_c}}$$

(25)

which is always less than the absorbed photon wavelength while the frequency of the matter wave $\nu_{dB} = \nu_\gamma(1 + \omega_z/2\omega)$ is always greater than the frequency of the absorbed photon so that $v_w = \lambda_{dB} \nu_{dB} = c^2/v \geq c$ holds always i.e. the matter wave is tachyonic.

Thus, we have derived the relation from the point of view of a particle absorbing an amount of energy $\hbar\omega$ and in the process undergoing a change of state from rest to motion with velocity $v$.

This discussion gives us a novel understanding of the motional energy or kinetic energy. Usually, we express ourselves by saying that the particle is possessing or carrying kinetic energy. But, here we clearly see that we can very well say from a more fundamental viewpoint that it is not that the particle is possessing or carrying the (photon)kinetic energy, rather, it is the photon (which is always in motion with speed $c$) that is carrying or possessing the particle and that has become burdened by the particle of mass $m$. As a result of which, it is able to carry the latter only with a speed less than its original speed $c$ which we identify as the bradyonic particle. When this occurs, we have the pervasive tachyon corresponding to the particle at rest going over to the tachyonic matter wave or the de Broglie wave train along the line of motion.

Accordingly, we can define a single-particle refractive index $\eta$ as follows:

$$\eta = \frac{c}{v_w} = \sqrt{1 - \frac{\omega_0^2}{\omega'^2}}$$

(26)

where, $\omega' = E/h$ with $E = \hbar\omega + mc^2$ as the total energy. We see further that this refractive index is always less than one, which means that the matter wave moves with velocity $v_w > c$ when the photon is loaded with the particle of mass $m$.

Moreover, we wish to point out that the phase velocity can never be less than the particle velocity and that the several consistent ways of deriving the same reciprocal relation between the two go to prove without doubt that the matter waves are always tachyonic for a bradyonic particle. We note also that the quantum object at rest being
an all-pervading existence, has actually no classical point particle-like character because it is a tachyonic wave of infinite wavelength before measurement. The manifestation of this may be taken to be in the form of the the transverse longitudinal fields (like the gravitational or electrostatic) [15, 16] associated with the particle, which have been found to propagate with infinite velocities tallying fully well with the “perverse existence interpretation” of the quantum object at rest proposed in this article.

For example, for an electron at rest, its electric field extends up to infinity and no matter where in space another charge is situated, it interacts with the latter- in principle at least- though the strength decreases as $1/(\text{distance})^2$. Even this inverse square field dependence may very well be a fact only for interacting sources, since we’ve no way of experimentally determining the field of a free source i.e. without subjecting it to an interaction. The assigning of the $1/(\text{distance})^2$ field dependence to the free particle is only an extrapolation of the interacting field-dependence to the non-interacting case. Here, in this classical extrapolation lies the crux of the matter, the root of all the problems faced by us in tackling self-interaction etc. which forced us to resort to renormalization in QFT. In fact, it has already been proposed in the literature[17] that the longitudinal and the transverse aspects of the electromagnetic field may be thought of as dual, and hence complementary, to each other and together they are to be taken to make up the full Reality of the electromagnetic field.

Thus we may say that the concept of a localised particle is an artefact of our classical imagination and is deduced from everyday experience with localized macroscopic objects and therefore, is divorced from the microscopic reality of the quantum world.

6. Bradyon-Tachyon Duality/Complementarity

Basing on the above discussion we extend the complementarity principle of Bohr to say that “the bradyonic and the tachyonic aspects of a quantum object are complementary to each other”. They may not be simultaneously observable in a single experiment and we may need appropriately designed experiments separately for the observation of any one aspect. Locality and non-locality, causality and teleology may be similarly thought of as making up a fuller reality, more complete than hitherto accepted in Physics. We may be just passing through a similar phase of reconciling dualities via a
complementary principle as happened in the early days of Quantum theory a century ago.

Thus, we propose that Quantum Reality has a complementary tachyonic aspect associated with every bradyon as follows:

- Bradyon at rest $\Leftrightarrow$ pervasive tachyon.
- Bradyon in motion $\Leftrightarrow$ tachyonic wave train along the line of motion.

Therefore, we rewrite the Einstein-de Broglie relation in the form

$$v_b.v_t = c^2$$

(27)

where, $v_b$ and $v_t$ are the velocities of the bradyonic particle and its associated tachyonic matter wave. This relation is to be taken as the starting point for the investigation of the possible role of tachyons in explaining quantum entanglement and other related nonlocal phenomena. The reason why we do not observe tachyons is because of our presumption that they are localised objects like bradyons, which they are not. They being fundamentally pervasive in character, we cannot detect them experimentally in the traditional sense, but the duality relation above may be taken to be give us indirect proof of their existence.

We see that if we accept the viewpoint advocated in this paper, we immediately grant a reality to the tachyons which is long overdue. In fact, we have all along been working with them since de Broglie’s original work in the form of matter waves! Though the above relation looks like a restatement of the wave-particle duality and the conjugate-variable complementarity in old quantum theory, the conceptual shift in the paradigm is a huge one as discussed in this work. In fact, with the interpretation proposed here, the old bradyon-tachyon complementarity which was earlier required of the relativity theory for its completeness in the scheme of extended relativity proposed by Recami and others [18] becomes now a point of conformity with quantum theory, in the sense that both theories have the tachyons included as essential constituents.

We note that while in all the quantum mechanical duality/complementarity relations it is Planck’s constant $h$ which is the fundamental constant linking the two aspects (e.g. $\lambda.p = h$), here $c^2$ is the reciprocality constant connecting $v_b$ and $v_t$. Interestingly, the equation (27) can also be cast in terms of the dimensionless boost parameters:

$$\beta_b.\beta_t = 1$$

(28)
The reciprocality is such that in order not to be at loggerheads with Relativity, we have to accept that tachyons, by their very nature, are pervasive and that there can be no point-like tachyons. The notion of localised point-like particles is applicable only to bradyons. Thus, rather than trying to rule out quantum non-locality due to its conflict with relativity, we now rest on a solid ground of unification on the basis of the bradyon-tachyon complementarity where both the theories have non-locality as a common characteristic inbuilt into their structure through the tachyons.

7. Discussion and conclusion

We’ve shown that the pre-measurement state of a free quantum object at rest is not the same as the post-measurement state of a localized particle-like existence, but is a pervasive existence. The localization that we are familiar with from our classical observation of particulate existences can be interpreted to be the result of the interaction of the measuring apparatus with the free quantum object which brings about the collapse of its pervasiveness to a pointlike existence by the very design of our experiments or other interactions. We’ve shown that the wave packet description is not free from the problem of spreading even when the relativistic energy formula is employed. The wave packet description is shown to be a post-measurement description incorporating our classical common-sense notion of a localised pointlike particle and thus is not the true description of the free quantum object as it is i.e. before measurement. The relation between the group and the phase velocities is derived using different approaches and its robustness is established.

A long-standing conceptual error in textbooks is pointed out in connection with the nonrelativistic limit of this relation. In our approach we have shown that including the rest energy in the expression for total energy remedies the situation and proves once again the robustness of the relation beyond any element of doubt as well as its universal validity.

Basing on this relation we have proposed a reinterpretation of the notion of the kinetic energy carried by a particle. We have also proposed to extend Bohr’s complementarity principle to bradyon-tachyon complementarity thereby giving the tachyons their rightful
place in the scheme of quantum mechanics. The familiar point particle is but the ‘tip of the iceberg’ of pervasive tachyonic existence associated with it.

It is worth noting that the nonlocal EPR-like quantum correlations and entanglement effects become easy to understand, once a pervasive existence is granted to the quantum objects at rest and an associated tachyonic matter wave is granted to the moving quantum particles. The basic objection to the existence of tachyonic waves that if they exist they would carry signals faster than light can be met by the equally basic fact that a tachyonic matter wave being extended and encompassing in nature does not need to carry any information between two points as it simultaneously touches both ends!

The present work vindicates the efforts in some of the recent work[19] where the authors have also argued for the reality of the de Broglie waves. However, there remain many issues still unsolved regarding the exact description of the interaction of tachyons and bradyons and amongst tachyons themselves; the connection of the tachyonic matter waves with the antiparticles, spacelike measurements, and finally, with consciousness which need to be explored in future work.

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