STABILIZING WAVE ATTENUATION EFFECTS IN HIGH-SPEED METAL CUTTING

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Abstract
Theory of chatter-suppressing wave attenuation effects in high-speed metal cutting process is presented in this work. Analysis shows that rise in natural frequency of cutting tool leads to rise in the wave attenuating forces. This result leads to the postulation that rise in natural frequency suppresses high-speed chatter instability. The theory is based on the assumption of very short free-flight of cutting tool, and shown to fail when this assumption is violated. The concluding opinion becomes that the resistance of a high-speed tool to chatter stems from both structural damping and wave attenuation effects.

Keywords: chatter; high-speed machining; wave attenuation effects; turning process; milling process.

1. Introduction

Machine tool chatter has been described by Taylor as the most obscure and delicate of all problems facing the machinist [1]. Chatter is a complex problem that stems from complex dynamical interplay of machine tool elements. It is a problem that must be solved because of cost on production but that is proving too difficult to be solved completely thus causing an intense ongoing research more than hundred years running after Taylor’s observation. Regenerative undulations on machined surface was discovered and showed to cause varying cutting forces that excite the vibrations in the cutting process [2]–[6]. Regenerative effect is presently accepted as the main cause of chatter and has been studied from the mid twentieth century till date [5], [8]. Low-speed milling process is known to be stabilized by process damping. The near continuous tool-workpiece engagement at process damping speed range causes the tool flank to interfere with machined surface waves. This interference is destructive to tool vibration thus making machining less prone to chatter. Excellent works are found in literature [9]–[11] on process damping. In high-speed milling, the cutting process becomes highly interrupted. Highly interrupted milling process is characterized by tool disengaging from the workpiece to go into a short-lived free sojourn and then remaking contact with the workpiece. A wave could get flattened or attenuated at the end of free sojourn due to impact of a cutting edge effectively depopulating the chatter-causing regenerative effects. The process of engagement, free sojourn and re-engagement of the high-speed tool is highly-intermittent meaning that many waves would be attenuated. This further means that the effects of wave attenuation on chatter instability of high-speed milling will be noticeable on a macro-scale when noticeable alteration are effected on the variables of the attenuating forces [12], [13].
Any factor that increases the mean value of the attenuating forces will increase attenuation effects and then should lead to chatter suppression. The suppressive effect on chatter instability of milling process by the per tooth cutting force-induced wave attenuation effects is recognized and given theoretical justification in [14], [15]. In this paper, the theory of chatter-suppressing wave attenuation effects is expanded to include the flattening effects of impact of the tool on the waves of machined workpiece after free flight in high-speed machining.

2. Theoretical postulation

Intuitively reasoning, the duration of impact will be very short meaning that the expected value of the attenuating forces will be proportional to the impulse

\[ F(t) \, dt = m \dot{z}(t), \] (1)

where \( m \) is the mass of the tool and \( \dot{z}(t) \) the velocity of free sojourn. The Dirac delta function can be introduced in equation (1) to give the impulsive force as

\[ F(t) = m \dot{z}(t) \delta(t). \] (2)

If a free sojourn terminates at an instant \( t_s \) the impulsive force \( F(t) \) becomes more appropriately represented as

\[ F(t) = m \dot{z}(t) \delta(t-t_s). \] (3)

It is seen from equation (3) that increase in \( \dot{z}(t) \) spells rise in the attenuating force \( F(t) \). The free sojourn of the tool is the damped natural vibration of the tool governed by the ordinary differential equation

\[ \ddot{z}(t) + 2\xi \omega_n \dot{z}(t) + \omega_n^2 z(t) = 0 \] (4)

where \( \omega_n \) is the natural frequency of the tool and \( \xi \) is the damping ratio of the tool. Putting the solution of form \( z(t) = Ke^{\lambda t} \) into equation (4) the characteristic equation becomes

\[ \lambda^2 + 2\xi \omega_n \lambda + \omega_n^2 = 0, \] (5)

with the roots \( \lambda_{1,2} = -\omega_n \xi \pm \omega_n \sqrt{\xi^2 - 1} \) such that the transient response becomes

\[ z(t) = K_1 e^{\lambda_1 t} + K_2 e^{\lambda_2 t}. \] (6)

Assuming the initial conditions \( z(0) \) and \( \dot{z}(0) \), the constants become \( K_1 = \frac{z(0)-\lambda_2 \dot{z}(0)}{\lambda_1-\lambda_2} \) and \( K_2 = \frac{\lambda_1 z(0)-\lambda_2 \dot{z}(0)}{\lambda_1-\lambda_2} \) resulting in the response vector becoming

\[ \begin{align*}
\begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} &= \frac{1}{\lambda_1-\lambda_2} \begin{bmatrix} \lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t} & e^{\lambda_1 t} - e^{\lambda_2 t} \\ \lambda_1 \lambda_2 e^{\lambda_1 t} - \lambda_1 \lambda_2 e^{\lambda_2 t} & \lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t} \end{bmatrix} \begin{bmatrix} z(0) \\ \dot{z}(0) \end{bmatrix}. \end{align*} \] (7)

The velocity with which the tool impacts the workpiece at probably the crest of a certain wave becomes

\[ \dot{z}(t_s) = \frac{\lambda_1 \lambda_2}{\lambda_1-\lambda_2} \left( e^{\lambda_2 t_s} - e^{\lambda_1 t_s} \right) z(0) + \frac{1}{\lambda_1-\lambda_2} \left( \lambda_1 e^{\lambda_1 t_s} - \lambda_2 e^{\lambda_2 t_s} \right) \dot{z}(0). \] (8a)
Recognition of difference of complex conjugate numbers in equation (8a) gives the forms

\[ \ddot{z}(t_s) = \frac{-\lambda_1 \lambda_2}{i2\text{Im}(\lambda_1)} i2\text{Im}(e^{\lambda_1 t_s}) z(0) + \frac{1}{i2\text{Im}(\lambda_1)} i2\text{Im}(\lambda_1 e^{\lambda_1 t_s}) \dot{z}(0), \] (8b)

\[ \ddot{z}(t_s) = -\omega_n e^{-\omega_n t_s} \sin(\omega_n \sqrt{1-\xi^2} t_s) z(0) + \] 

\[ e^{-\omega_n t_s} \left\{ -\xi \sin(\omega_n \sqrt{1-\xi^2} t_s) + \sqrt{1-\xi^2} \cos(\omega_n \sqrt{1-\xi^2} t_s) \right\} \dot{z}(0). \] (8c)

3. Verification

In what follows the effect of variation of natural frequency \( \omega_n \) on the impact velocity \( \dot{z}(t_s) \) is investigated by considering the derivatives of \( \dot{z}(t_s) \) with respect to \( \omega_n \). Since the tool will most likely disengage with the workpiece in the same direction as the initial deflection, the initial conditions; \( z(0) \) and \( \dot{z}(0) \) are taken to be of same sign and are both considered positive in the ongoing analysis. For the simpler case when there is no damping (\( \xi = 0 \)) equation (8c) becomes

\[ \ddot{z}(t_s) = -\omega_n \sin(\omega_n t_s) z(0) + \cos(\omega_n t_s) \dot{z}(0). \] (9)

Differentiating equation (9) with respect to \( \omega_n \) gives

\[
\frac{d\dot{z}(t_s)}{d\omega_n} = -\omega_n \sin(\omega_n t_s) z(0) + \cos(\omega_n t_s) \dot{z}(0) - \sin(\omega_n t_s) \dot{z}(0) - \omega_n t_s \cos(\omega_n t_s) z(0) - t_s \sin(\omega_n t_s) \dot{z}(0).
\]

This is simplified to read

\[
\frac{d\dot{z}(t_s)}{d\omega_n} = [(-\omega_n - 1) \sin(\omega_n t_s) - \omega_n t_s \cos(\omega_n t_s)] z(0) + [\cos(\omega_n t_s) - t_s \sin(\omega_n t_s)] \dot{z}(0).
\] (10)

Making use of the simplifications that \( t_s \) is very small and \( \dot{z}(0) \gg z(0) \), equation (10) becomes

\[
\frac{d\dot{z}(t_s)}{d\omega_n} \approx \dot{z}(0). \] (11)

It is seen from equation (11) that velocity of impact of the undamped free tool \( \dot{z}(t_s) \) rises with rise in \( \omega_n \) since \( \frac{d\dot{z}(t_s)}{d\omega_n} > 0 \). In order to take into account the effect of damping, equation (8c) is differentiated with respect to \( \omega_n \) to give

\[
\frac{d\dot{z}(t_s)}{d\omega_n} = e^{-\omega_n t_s} \left\{ -\omega_n \sin(\omega_n \sqrt{1-\xi^2} t_s) z(0) + \right\} \dot{z}(0) - \sin(\omega_n \sqrt{1-\xi^2} t_s) z(0) + \]

\[ \sqrt{1-\xi^2} \cos(\omega_n \sqrt{1-\xi^2} t_s) \dot{z}(0) - \sin(\omega_n \sqrt{1-\xi^2} t_s) z(0) + \]
\[\begin{align*}
\omega_n \xi t_s \sin(\omega_n \sqrt{1 - \xi^2 t_s}) z(0) - \omega_n t_s \sqrt{1 - \xi^2} \cos(\omega_n \sqrt{1 - \xi^2 t_s}) z(0) - \\
\xi t_s \left[ \xi \sin(\omega_n \sqrt{1 - \xi^2 t_s}) + \sqrt{1 - \xi^2} \cos(\omega_n \sqrt{1 - \xi^2 t_s}) \right] \dot{z}(0) + \\
\left[ -\xi t_s \sqrt{1 - \xi^2} \cos(\omega_n \sqrt{1 - \xi^2 t_s}) - t_s (1 - \xi^2) \sin(\omega_n \sqrt{1 - \xi^2 t_s}) \right] \ddot{z}(0) + \\
\zeta t_s \left[ -\xi \sin(\omega_n \sqrt{1 - \xi^2 t_s}) - t_s (1 - \xi^2) \cos(\omega_n \sqrt{1 - \xi^2 t_s}) \right] \dot{z}(0) + \\
\right].
\end{align*}\]

(12)

Further simplification of the above gives

\[\begin{align*}
\frac{d\dot{z}(t_s)}{d\omega_n} &= e^{-\omega_n t_s \xi} \left\{ \left[ (-\omega_n - 1 + \omega_n \xi t_s) \sin(\omega_n \sqrt{1 - \xi^2 t_s}) - \\
\omega_n t_s \sqrt{1 - \xi^2} \cos(\omega_n \sqrt{1 - \xi^2 t_s}) \right] z(0) + \left[ (-\xi - t_s) \sin(\omega_n \sqrt{1 - \xi^2 t_s}) + \\
\sqrt{1 - \xi^2} (1 - 2 \xi t_s) \cos(\omega_n \sqrt{1 - \xi^2 t_s}) \right] \dot{z}(0) \right\}.
\end{align*}\]

(13)

Also making use of the simplifications that \( t_s \) is infinitely small and \( \dot{z}(0) \gg z(0) \), equation (13) becomes identical with equation (11) meaning that the conclusion that \( \dot{z}(t_s) \) rises with rise in \( \omega_n \) holds in real tools with damping.

An alternative way to achieve the foregoing analysis is to re-write \( \dot{z}(t_s) \) given in equation (8c) as

\[\begin{align*}
\dot{z}(t_s) &= A \sin \left[ (\omega_n \sqrt{1 - \xi^2 t_s}) + \phi \right],
\end{align*}\]

(14)

such that the amplitude and phase difference of the impact velocity read

\[\begin{align*}
A &= e^{-\omega_n t_s \xi} \left\{ \left[ \omega_n^2 [z(0)]^2 + 2 \omega_n \xi z(0) [z(0)]^2 \right] \right\}^{1/2},
\end{align*}\]

(15)

\[\begin{align*}
\phi &= \tan^{-1} \left[ \frac{-\dot{z}(0) \sqrt{1 - \xi^2}}{\omega_n \xi \dot{z}(0) + \xi \ddot{z}(0)} \right].
\end{align*}\]

(16)

When the damping ratio is zero, the amplitude of the impact velocity becomes \( A_{\xi=0} = \{ \omega_n^2 [z(0)]^2 + [\dot{z}(0)]^2 \}^{1/2} \) which is seen to rise with rise in \( \omega_n \). Making use of the simplifications that \( t_s \) is infinitely small and the knowledge that that machine tools usually have low damping ratio that lie in the range \( \xi \approx 0.005 - 0.02 \) [16] gives

\[\begin{align*}
A \approx \left\{ \frac{\omega_n^2 [z(0)]^2 + 2 \omega_n \xi z(0) [z(0)]^2 + [\dot{z}(0)]^2}{1 - \xi^2} \right\}^{1/2}.
\end{align*}\]

(17)

It is obvious that amplitude of the impact velocity \( \dot{z}(t_s) \) as given in equation (17) rises with rise in \( \omega_n \). Adopting typical initial values of \( z(0) = 0.00001 \) and \( \dot{z}(0) = 1 \), a plot of amplitude \( A \) given in equation (15) as a function of \( \omega_n \) is given in figure 1 for the damping ratios \( \xi = 0.005 \) and \( \xi = 0.02 \). Figure 1(left) which agrees with theory is for the case \( t_s = 0.00001 \). This value of \( t_s \) is low enough to satisfy the assumption of very short free flight thus the corresponding plot which shows rise of \( A \) with the rise of \( \omega_n \) verifies the postulated wave
attenuation theory. When the assumption on $t_s$ is not satisfied, relatively, then the theory fails as Figure 1(right) shows for $t_s=0.001s$.

![Figure 1. Variations of the amplitude of impact velocity $A$ as a function of $\omega_n$](image)

The arising postulation becomes that rise in $\omega_n$ will cause rise in mean attenuating force which in turn is proportional to $F(t)$ given in equation (3). The postulation fundamentally means that rise in natural frequency of a tool will improve the effectiveness of wave attenuation thus suppress high-speed chatter instability. The foregoing analysis suggests that the reluctance of a tool to chatter does not completely stem from structural damping capacity of the tool as classically thought but stems partly from structural damping and partly from wave attenuation effects.

4. Conclusion

In this paper, the theory of chatter-suppressing wave attenuation effects which was introduced in an earlier another is expanded to include the flattening effects of impact of cutting tool on the waves of machined workpiece after free flight in high-speed machining. It is demonstrated that rise in natural frequency $\omega_n$ causes rise in mean attenuating force. Postulation becomes that rise in natural frequency of a tool will improve the effectiveness of wave attenuation thus suppress high-speed chatter instability. The theory is based on the assumption of very short free-flight of cutting tool, and shown by numerical simulation to fail when this assumption is violated. The opinion that arises in this work is that the resistance of a high-speed tool to chatter stems from both structural damping and wave attenuation effects.

References

[1] Taylor, F. W. (1906). *On the Art of Cutting Metals.* (Vol. 23). American society of mechanical engineers.

[2] Ozoegwu C. G. and Omenyi, S. N. (2012) “Stability characterization of a turning process,” *J. Eng. Appl. Sci.*, vol. 8, no. 1, pp. 68–75.
[3] Okokpujie, I. P., Salawu, E. Y., Nwoke, O. N., Okonkwo, U. C., Ohijeagbon, I. O., & Okokpujie, K. O. (2018). Effects of Process Parameters on Vibration Frequency in Turning Operations of Perspex Material. In Proceedings of the World Congress on Engineering (Vol. 2), 700-707.

[4] Nwoke, O. N., Okonkwo, U. C., Okafor, C. E., & Okokpujie, I. P. (2017). Evaluation of Chatter Vibration Frequency In Cnc Turning of 4340 Alloy Steel Material. International Journal of Scientific & Engineering Research, 8(2), 487-495.

[5] Ezugwu, C. A., Okonkwo, U. C., Sinebe, J. E., & Okokpujie, I. P. (2016). Stability Analysis of Model Regenerative Chatter of Milling Process Using First Order Least Square Full Discretization Method. International Journal of Mechanics and Applications, 6(3), 49-62.

[6] Okonkwo Ugochukwu, C., Nwoke Obinna, N., & Okokpujie Imhade, P. (2018). Comparative Analysis of Chatter Vibration Frequency in CNC Turning of AISI 4340 Alloy Steel with Different Boundary Conditions. Journal of Covenant Engineering Technology (CJET) Vol, 1(1).

[7] Siddhpura, M., & Paurobally, R. (2012). A review of chatter vibration research in turning. International Journal of Machine tools and manufacture, 61, 27-47.

[8] Ozoegwu, C. G., Ofochebe, S. M., & Omenyi, S. N. (2016). A method of improving chatter-free conditions with combined-mode milling. Journal of Manufacturing Processes, 21, 1-13.

[9] Stépán, G. (1989). Retarded dynamical systems: stability and characteristic functions. Longman Scientific & Technical.

[10] Quintana, G., & Ciurana, J. (2011). Chatter in machining processes: A review. International Journal of Machine Tools and Manufacture, 51(5), 363-376.

[11] Ahmadi, K., & Ismail, F. (2010). Experimental investigation of process damping nonlinearity in machining chatter. International Journal of Machine Tools and Manufacture, 50(11), 1006-1014.

[12] Turkes, E., Orak, S., Neseli, S., & Yaldiz, S. (2011). A new process damping model for chatter vibration. Measurement, 44(8), 1342-1348.

[13] Moradi, H., Vossoughi, G., & Movahhedy, M. R. (2013). Experimental dynamic modelling of peripheral milling with process damping, structural and cutting force nonlinearities. Journal of Sound and Vibration, 332(19), 4709-4731.

[14] Ozoegwu, C. G., & Omenyi, S. N. (2013). Wave attenuation effects on the chatter instability of end-milling. Noise Control Engineering Journal, 61(4), 436-444.

[15] Ozoegwu, C. G. (2014). Stabilizing wave attenuation effects in turning process. Production & Manufacturing Research, 2(1), 2-10.

[16] Insperger, T. (2002). Stability analysis of periodic delay-differential equations modeling machine tool chatter.