Coupled Mode Flutter of a Linear Compressor Cascade in Subsonic and Transonic Flow Conditions

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Abstract. Flutter onset in turbomachinery is typically investigated numerically via decoupled methods due to a high mass ratio of structure to air. The unsteady aerodynamic response of a forced motion vibration is evaluated for a positive or negative work entry. The forced motion simulations assume vacuum mode shape vibrations at certain amplitudes without modal coupling due to aerodynamic forces. This approach, also known as the energy method, is valid for a high blade mass ratio and small logarithmic decrement values. An aeroelastic study of a multi-passage linear compressor cascade was performed. In fluid-structure-coupled time-marching CFD, generic heave and pitch degrees of freedom are allowed to vibrate freely in reaction to any aerodynamic forces. For one subsonic and one transonic flow condition predicted to be stable by the classical energy method approach, aerodynamically coupled-mode flutter is observed. It is shown that variations in the starting conditions, i.e. the initial vibration and inter-blade phase angle of the system, can have a strong influence on the number of CFD iterations required until amplitudes grow. However, if coupled-mode flutter is present in the system, it will ultimately set in at a distinct inter-blade phase angle.

1. Introduction

The energy method is typically used for flutter analysis of turbomachinery components. Structural coupling in bladed disks or shrouded blades is accounted for during modal analysis [1, 2]. From the resulting orthogonal mode shapes, single degree-of-freedom (DOF) instabilities are the only mechanism usually considered for flutter onset. As turbomachinery bladeings typically have a high mass ratio of structure to air, the assumption is justified as vibrations will be close to the in-vacuo mode shapes and natural frequencies [3, pp. 527-532].

Even though the structural behavior can be linearized, aerodynamic nonlinearities can lead to fluid-structure-interaction (FSI) phenomena not predictable by the energy method [4] which are strongly influenced by small changes in mass ratio [5]. Hence, time-marching simulations with fluid-structure-coupling (FSC) may be necessary. Furthermore, the aeroelastic frequency shift can potentially destabilize the single-DOF system [6].

These previous researches only consider single-DOF flutter as a cause. From fixed-wing aeroelasticity it is well known, that flutter onset typically occurs at lower flow velocities than single-DOF analysis would predict. This so-called coupled-mode flutter is a combination of heave and pitch motions, that are coupled by aerodynamic forces and can be significantly off the in-vacuo mode shape frequencies [3]. The purpose of the presented research is to demonstrate that aerodynamically coupled-mode flutter is possible under certain conditions (low mass ratio) for turbomachinery geometries. Furthermore, the concept of traveling wave modes is still true.
2. Theory

2.1. Aeroelastic Modeling for Turbomachinery

From derivation in the manual of Platzer and Carta [7], the governing aeroelastic equations of motion in generalized blade coordinates $\tilde{u}_i$ yield

$$M \ddot{u}_i(t) + K u_i(t) = \phi^T \mathbf{f}_i(t)$$  \hspace{1cm} (1)

where $M$ and $K$ are generalized mass and stiffness respectively. Structural damping is neglected. On the right-hand side, the generalized aerodynamic forces (GAF) express the work done by the motion induced unsteady aerodynamic loads in the displacement of the individual mode shapes $\phi_r$ for $r = 1, 2, \ldots, R$.

For turbomachinery, the fundamental concept of traveling waves formulated by Lane [8] describes a symmetric rotor with $N$ identical segments that are oscillating harmonically at the same frequency and mode shape with a constant phase shift taken into account by the inter-blade phase angle (IBPA):$\sigma_n = 2\pi n / N$ with $n = 0, 1, \ldots, N - 1$  \hspace{1cm} (2)

Normalizing $\sigma_n$ into the range of $(-\pi, \pi]$, a forward or backward traveling wave can be defined in geometrical terms (see Fig. 1) by looking at the sign of $\sigma_n$. Each blade $k$ has a modal deflection $\tilde{z}_{k,i}$ in the mode shape $i$ at a point in time $t$, or analogously a physical deflection $z_{k,i}$ that, if mode shapes are real-valued only, can be expressed as:

$$\tilde{z}_{k,i}(t) = \cos (\omega_i t + (k - 1)\sigma_n)$$  \hspace{1cm} (3)

$$z_{k,i}(t) = \phi_i \cos (\omega_i t + (k - 1)\sigma_n)$$  \hspace{1cm} (4)

![Figure 1. Definition of inter-blade phase angle $\sigma_n$ with the example of $\sigma_n = 90^\circ$ (cascade stagger and dimensions not to scale). The positive direction used here defines a positive blade counting in the (suggested) rotational direction. The deflections shown on the left are at $\omega_i t = 0^\circ$, the arrows point towards the current movement direction.](image)

2.2. The Energy Method

The basic assumption of the energy method is that RHS values in (1) are very small compared to $K$. As a consequence, the structure is effectively vibrating in vacuum [3, pp. 527-532] and time-harmonic pressure variations $\tilde{p}$ due to blade motion can be evaluated for the amount of aerodynamic work per cycle $W_c$. For each individual surface cell $\eta$, the local $W_{c,\eta}$ is summed up to the global work:

$$W_c = \sum_{\eta} W_{c,\eta} = \sum_{\eta} - \int_0^{2\pi/\omega_i} \tilde{p}_\eta \phi_\eta \, dt$$  \hspace{1cm} (5)

A positive work entry ($\Re(W_c) > 0$) indicates that energy is transferred from the fluid into the structure and exciting the blade. Vice versa ($\Re(W_c) < 0$) the blade releases energy so
that vibrations are damped. With the modal mass $\tilde{m}_i$ and angular frequency $\omega_i$, the resulting damping value in terms of logarithmic decrement $\Lambda$ can be stated as

$$\Lambda = -\Re(W_c)/(\tilde{m}_i\omega_i^2) \quad (6)$$

2.3. Mass Ratio

The mass ratio $\mu$ is defined as

$$\mu = \frac{m_{\text{blade}}}{\rho_\infty \pi (c_b/2)^2 h} \quad (7)$$

with $m_{\text{blade}}$ as the mass of the blade without the root or disk, the upstream fluid density $\rho_\infty$, the chord length $c_b$ and the channel height $h$.

2.4. Numerical Flow Solver

The results presented in this study were performed with the in-house CFD solver for turbomachinery flows TRACE of the German Aerospace Center (DLR). TRACE is a hybrid-grid (structured/unstructured) RANS solver and includes a nonlinear solver in the time domain as well as a linear and nonlinear (harmonic balance, HB) solver in the frequency domain [9, 10]. All computations were carried out with a Wilcox $k-\omega$ turbulence model. At inlet and outlet boundaries the nonreflecting formulation according to Giles is applied.

For the unsteady flow solutions with prescribed motion in frequency domain, the harmonic balance (HB) technique is used, considering the first harmonic with mean flow coupling. To reduce computational effort, periodic respectively phase-lag boundary conditions allow the simulation of only one blade passage.

Time-marching simulations with fluid-structure coupling were performed with the modal FSI module of TRACE [11]. The modal dynamic equations are integrated in time with a Newmark scheme and solved with a Newton method. The coupling of flow and structure is performed with a serial Gauss-Seidel scheme [12].

3. Geometry

3.1. Model Setup

The NACA3506 linear cascade is based on an existing non-rotating annular compressor cascade with 20 blades. The geometry is sliced at midspan and arranged as a linear cascade in a Q3D mesh. General parameters are listed in Table 1 and the geometry is equal to previous publications [4, 5].

The structured CFD grid is wall-resolved with $y^+ < 1$ and contains roughly 26,000 volume cells. In the generic structural model as shown in Fig. 2, the elastic axis and the center of gravity are at the same physical coordinates of 50% chord length. Thus, heave and pitch motions are structurally decoupled and any coupling is solely aerodynamic. The spring constants $k_h, k_\alpha$, in combination with the mass and mass moment of inertia of the blade, can be trimmed to achieve

| Table 1. NACA3506 linear cascade |
|----------------------------------|
| Chord length $c_b = 77$ mm      |
| Stagger angle LE-TE $\beta_g = 40^\circ$ |
| Solidity $s = c_b/\tau = 1.362$  |
| Natural frequencies† $f_{\text{heave}} = 100$ Hz, $f_{\text{pitch}} = 200$ Hz |

† Trimmed via spring constants $k_h, k_\alpha$ dep. on blade material density

Figure 2. Structural model of each blade for the NACA3506 cascade
a certain natural frequency and mass ratio. Blade densities were adjusted to achieve the desired mass ratio and the springs were tuned for the natural frequencies.

The two flow conditions investigated are depicted in Fig. 3. The total conditions at the inlet are kept the same with a total pressure $p_t = 170$ kPa total temperature $T_t = 313.15$ K. The cascade is non-rotating, so an inflow angle of $\beta_0 = 48.3^\circ$ in the subsonic and $\beta_0 = 51.3^\circ$ in the transonic case is given. The back pressure is varied to adjust to the current flow condition.

For the time-marching coupled simulations, four blade passages are modelled. As a periodic setup, this restricts the system to a certain selection of IBPAs, allowing $\sigma_n$ to be $0^\circ$, $\pm 90^\circ$, or $180^\circ$. Each blade is coupled via the modal approach described above and allowed to vibrate freely in a heave and/or pitch motion.

3.2. Flutter Analysis Using the Energy Method

A classical flutter analysis in the frequency domain with one blade passage was performed considering all 20 possible IBPAs. For both flow conditions, heave and pitch motions are predicted to be stable as shown in Fig. 4. To support the frequency domain results, time-marching simulations of a multi-passage setup and forced motions are added in the diagrams for a selection of IBPAs. In subsonic flow both results match perfectly, but some differences in the transonic regime can be observed, especially for the heave motion. These differences are likely due to light flow nonlinearities that are not represented in the HB results using only the fundamental harmonic.

Please note: under the assumption that the natural frequency does not change, for any different mass ratio or blade density, the values of the damping diagram scale with the modal mass variation.
4. Results of Fluid-Structure-Coupled Time-Marching Simulations

4.1. General Observations
The time history of displacements is logged for the modal displacements of the heave and pitch DOF. With this information, the physical displacement at any blade position can be extracted. The physical displacement $z_{LE}(t)$ presented in this study is the distance of the leading edge at a given time $t$ compared to the resting position.

A generic time history of physical displacements is given in Fig. 5. This behavior is representative for all performed computations: Depending on the starting condition (i.e. initial deflection and/or velocity), some blades have a significantly higher energy level. If the blades are excited randomly, there is an initial phase with a transient behavior of the blades that is due to an uneven distribution of energy in the cascade. The cascade distributes energy evenly in the cascade now and during this process a distinct IBPA pattern is formed. After this transient phase, a periodic phase sets in where the blades are vibrating harmonically in damped or excited oscillations. Furthermore, the mean displacement of the blades are not equal to the resting position as indicated by the envelopes and mean displacement. This phenomenon can be separated from the coupled-mode flutter as described later.

4.2. Subsonic Flow Condition
Starting with the original material density of steel blades ($\mu = 280$), neither single-mode nor coupled-mode flutter is observed. For the purpose of the study, the density is reduced to $\rho = 5200 \text{ kg m}^{-3}$ ($\mu = 185$). As shown in Fig. 6, only allowing heave or pitch results in a damped cascade. The energy constantly decreases to amplitudes so small that only numerical artifacts are retained as noise (simulation A for $t > 0.5 \text{ s}$).

Once both modes were allowed in simulation C with random starting conditions, the two-DOF system first appears to be damped, but a distinguished vibration starting at a very small amplitude sets in for $t > 0.3 \text{ s}$. Knowing how the resulting vibration should look like, a new computation with adjusted starting conditions is started (simulation D). The initial transient phase is virtually non-existent and amplitudes grow right away with the already established vibration pattern known from C.

For simulations E and F, the starting conditions were randomized with a lower initial level of structural energy than C. Again, the simulation yield a large delay for the amplitudes to grow and in the end, the final vibration pattern is identical to the already known one from simulation C and D.

4.3. Transonic Flow Condition
In the transonic case, coupled-mode flutter can be observed with the original blade density, although the mass ratio $\mu = 339$ is much higher than in the subsonic case. As depicted in Fig. 7, simulations A and B include only single-DOF and yield a stable cascade. The system with
two-DOF in simulation C with random starting conditions has a long delay until amplitudes are growing.

Analogous to the subsonic case, simulation D is started with the formerly identified vibration pattern from C and amplitudes are growing immediately. Simulation E and F start with a lower initial level of structural energy compared to C. All two-DOF simulations (C-F) end up with the same vibration pattern.

5. Post-processing of Results

For all simulations, but especially visible for e.g. simulation B or C in the transonic case, the mean displacement of the blades is not equal to the resting position and changes over time resembling a very low frequency pattern. This phenomenon may result from a slight difference in the treatment of static loading when going from steady to unsteady flow simulation and thus not be physical. As this effect occurs to be only of relevance when amplitudes of the higher frequency oscillations where low or damped, it is not investigated further. Nevertheless, the local mean displacement has to be removed from any further evaluations to separate low and high frequency oscillations.

From the peaks in the time history two splines are created, representing the envelope limits in Fig. 5 as a lower $z_{\text{env},l}$ and upper $z_{\text{env},u}$, respectively. The mean displacement is the average of both envelope splines. To get the to-be-evaluated displacement $z$ at a vibration peak time $t_i$ the envelopes are utilized:

$$z(t_i) = z_{\text{raw}}(t_i) - (z_{\text{env},l}(t_i) + z_{\text{env},u}(t_i))/2$$ (8)

More sophisticated signal processing tools like high or low-pass filters might be used, but the described procedure is straightforward and turns out to deliver reasonable data. The technique is applied on modal and physical displacements before any further processing.
After reducing the time history to the relevant part (after the initial transient phase), the frequency, logarithmic decrement and IBPA can be determined by looking at the peaks of the physical displacements. As an example in Fig. 8, a segment of the displacement results of the time-marching simulations in subsonic flow are plotted. The frequency is directly extracted from the period $T$. The IBPA needs to be interpreted with the help of (3). The logarithmic decrement is calculated via:

$$\Lambda = \ln \left( \frac{z(t_i)}{z(t_{i+1})} \right)$$  \hspace{1cm} (9)

The phase lag of pitch in reference to heave is evaluated from the peaks of the modal displacements

$$\varphi_{\alpha_{\text{hev}},\alpha} = 360^\circ \Delta t_{\text{lag}} / T$$  \hspace{1cm} (10)

Finally, the modal participation factors $\Gamma$ are extracted using the envelopes of the modal displacements $\tilde{z}$.

$$\Gamma_h = \tilde{z}_h / (\tilde{z}_h + \tilde{z}_\alpha)$$ \hspace{1cm} and \hspace{1cm} $$\Gamma_\alpha = \tilde{z}_\alpha / (\tilde{z}_h + \tilde{z}_\alpha)$$  \hspace{1cm} (11)

The evaluation is done for each period of the lower and upper peaks and the standard deviation is calculated from that data set. In the given example in Fig. 8, the IBPA of $-90^\circ$ is identified. The modal displacements show a participation of heave and pitch in the resulting vibration pattern.

A deeper insight into the aeroelastic behavior can be given by the total structural energy provided by the sum of kinetic and potential energy [11]:

$$E_{\text{str},\text{tot}} = \sum_{k=1}^{N} \sum_{i=1}^{m} \left( \frac{1}{2} \tilde{m}_{0,i} \tilde{z}_{k,i}^2 + \frac{1}{2} \tilde{k}_{0,i} \tilde{z}_{k,i}^2 \right)$$  \hspace{1cm} (12)

with the number of blades $N$ and the number of mode shapes $m$. The mode shapes have been scaled to a modal mass $\tilde{m}_{0,i} = 1$, so that the modal stiffness is $\tilde{k}_{0,i} = (2\pi f_i)^2$. This variable increases, if the aeroelastic system is unstable. Furthermore, once the plot becomes a steady line in the logarithmic plot (e.g. in Figures 6 and 7), it is typically a good indicator on where to evaluate the aeroelastic system.

6. Discussion

The full post-processing analysis as described above is done for all time-marching simulations and a summary is given in Table 2. For all simulations with growing amplitudes, it can be observed that once the excited system is established, the amplitudes increased until they became too large for the mesh deformation process to handle them. As no fixed IBPA is set in the simulations, the phase shift between the blades is of natural occurrence in the coupled system.

Regardless of the initial condition, i.e. modal displacements and velocities of the cascade at $t = 0$, if an unstable combination of heave-pitch ratio, heave-pitch phase lag and IBPA is present
Table 2. Aeroelastic results of FSI simulations with four passages, standard deviation 1

| Case       | σ_n      | f [Hz]   | Λ   | φlag,α | TH [%] | TA [%] |
|------------|----------|----------|-----|--------|--------|--------|
| Subs., heave | 0.0°     | 100.3 ± 0.2 | 0.064 ± 0.010 | -     | -      | -      |
| Subs., pitch | -90.0°   | 151.3 ± 1.4 | 0.209 ± 0.051 | -     | -      | -      |
| Subs., 2-dof | -90.0°   | 152.6 ± 0.1 | -0.095 ± 0.006 | 176.8° ± 0.2° | 54.7 ± 0.5 | 45.3 ± 0.5 |
| Trans., heave | 0.0°     | 100.3 ± 0.02 | 0.036 ± 0.002 | -     | -      | -      |
| Trans., pitch | -90.0°   | 155.9 ± 1.0 | 0.199 ± 0.034 | -     | -      | -      |
| Trans., 2-dof | 180.0°   | 120.1 ± 0.2 | -0.112 ± 0.009 | 200.0° ± 2.6° | 86.0 ± 0.1 | 14.0 ± 0.1 |

in the system, the cascade will ultimately vibrate in this pattern. This is shown by running different random initial starting conditions (sim. C-F in Figures 6c and 7c).

Although the single-DOF with pitching motions experience a significant frequency shift away from the natural frequency, the cascade does not become unstable. Only two-DOF systems are observed to be unstable.

7. Conclusion and Outlook

The energy method can be non-conservative for flutter analysis of turbomachinery components under certain conditions, i.e. a low mass ratio. The numerical results should be completed and compared with experimental data, but no measurement data was available for the configuration considered.

Time-marching simulations with a fully coupled setup require a lot of effort and computational time, thus a frequency domain method is preferable for design evaluation. Over the decades, frequency domain methods that solve an aeroelastically coupled eigenvalue problem have been established in fixed-wing analysis. The most prominent choice is the p-k method as described by Hassig [13]. It has been adapted for turbomachinery usage by the authors and will be presented in an upcoming publication [14].

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