The Estimation of Watermarking Embedding Strength Based on PSNR in Orthogonal Domain

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Abstract. Based on the relation of points and vectors in Euclid Space, in this paper an image is regarded as a point in Euclid Space, and a watermarking as a vector. Then this paper presents how to estimate the embedding strength of watermarking according to the characteristic of the PSNR of the watermarked digital image in orthogonal domain. Experimental results show that the formula of the embedding strength is accurate and has minor errors.

Key words: digital watermarking, embedding strength, orthogonal transform, PSNR.

1. Introduction
With the rapid development of computer technology and networks communication technology, the problem of information security has become increasingly prominent [1], among which the issue of copyright has always been one of the hot spots of people's attention. Digital watermarking, as an effective supplementary method of traditional encryption methods, has been considered as an important method to solve the problem of copyright protection, and it is also a hot spot in the current research field of multimedia information security [2-6]. This technique is to verify the ownership of the data by embedding the discriminatory secret information in the original data without affecting the availability of the host data. This embedded watermark can be a piece of text, logo, serial number, etc. This watermark is usually invisible or imperceptible, it is tightly combined with the original data (such as image, audio, video data) and hidden, and can be saved after undergoing some operations that do not destroy the value or commercial value of the source data.

In terms of embedding strength, it is actually a compromise between imperceptibility and robustness. The embedding strength in the current watermarking algorithm is discussed above. It mainly depends on experiments or the use of JND of the image to determine the capacity and superposition of the watermark to be embedded intensity. Given an initial superimposed value of a watermark to obtain a watermarked image, this process is repeated until the obtained watermarked image to meets both imperceptibility and robustness, but an optimal quantified index cannot be given. An effective and reasonable watermarking algorithm should be able to make a reasonable and effective estimation of the embedding strength of the watermark according to some evaluation criteria of image quality. At present, PSNR is a mature and commonly used indicator that can be used to evaluate the degree of image degradation caused by embedding watermarks. Therefore, this paper mainly uses the relationship between N-dimensional Euclidean space points and vectors based on the nature of orthogonal transformation. Taking the grayscale image as an example (the color image can be deduced by analogy), an image watermark embedding formula based on PSNR is derived. The
algorithm is characterized by using a general superimposed watermark embedding strength estimation formula, that is, the determination criterion of a given image--PSNR, the strength of the embedded watermark can be accurately calculated. The experimental results show that the formula has minor errors and has certain practicability. Therefore, it can satisfactorily solve the energy estimation problem in the watermark algorithm.

2. The estimation of watermarking embedding strength in orthogonal transformation
For a particular watermarking algorithm, compromise of robustness and imperceptibility must be considered. This is actually a question of how to determine the strength of the watermark embedding. Therefore, given the evaluation criterion of image quality, the embedding strength of the watermark should be able to be calculated. Therefore, based on the characteristics of orthogonal transformation, this paper takes the grayscale image as an example to derive the general superimposed watermark embedding strength estimation formula, that is, given the PSNR of the image, the strength of the embedded watermark can be accurately calculated.

Property I The orthogonal transformation keeps the signal energy constant, that is
\[ \sum_{i=0}^{N-1} f^2(i) = \sum_{i=0}^{N-1} F^2(i). \]
Expand to two-dimensional space, that is
\[ \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f^2(i, j) = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} F^2(i, j). \]

Among them, \( f(i) \), \( f(i, j) \) represents the signal before transformation, \( F(i) \), \( F(i, j) \) represents the signal after transformation.

Proof process was omitted. The specific content of orthogonal transformation, please refer to [6].

Property II The orthogonal transform is an equidistant transformation while keeping the vector length constant.

Proof: Let \( A=(a_1, a_2, \cdots, a_N) \) and \( B=(b_1, b_2, \cdots, b_N) \) be two points in N-dimensional Euclidean space. \( A \) is orthogonally transformed to \( A'=(a_1', a_2', \cdots, a_N') \) and \( B \) is transformed to \( B'=(b_1', b_2', \cdots, b_N') \).

Then use the vector \( \alpha \) to represent the directed line segment with \( A \) as the starting point and \( B \) as the end, then \( \alpha=(b_1-a_1, b_2-a_2, \cdots, b_N-a_N) \), use the vector \( \beta \) to represent the directed line segment with \( A' \) as the starting point and \( B' \) as the end, then \( \beta=(b_1'-a_1', b_2'-a_2', \cdots, b_N'-a_N') \).

The vector \( \alpha \) undergoes orthogonal transformation, then vector \( \beta \) is obtained. According to property 1, the orthogonal transform keeps the signal energy constant, it can be known that
\[ \sum_{i=0}^{N-1} (b_i-a_i)^2 = \sum_{i=0}^{N-1} (b_i'-a_i')^2 \]
\[ \therefore |\alpha| = \sqrt{\sum_{i=0}^{N-1} (b_i-a_i)^2} = \sqrt{\sum_{i=0}^{N-1} (b_i'-a_i')^2} = |\beta| \]
\[ \therefore A \beta = |\alpha| = |\beta| = A' B' \]
Complete.

Let the image size be \( M\times N \), the original image is \( I \), the watermarked image is \( V \), the orthogonal transform is \( D \), the inverse transform is \( D^{-1} \), and the transformed images are \( I' \), \( V' \) respectively. The watermarking embedding method used in this paper is a general superimposed watermark embedding method, that is,
\[ V'(i,j)=I'(i,j)+\alpha w_k, \quad k=1,2,\ldots,L \]
Among them, \( \alpha \) is the watermarking embedding strength, and \( L \) is the length of the watermarking sequence.

Then this paper derives the watermarking embedding strength of the orthogonal domain based on property II.

Property III The orthogonal watermark embedding formula based on PSNR is
\[ \alpha = 255 \sqrt{\frac{MN}{PSNR}} \]

Proof: Scanning \( I, I', V, V' \) from left to right and top to bottom to change the image from a two-dimensional sequence to a one-dimensional sequence, that is 
\[ g_i(k) = I(i,j), \quad g_i'(k) = I'(i,j), \quad g_j(k) = V(i,j), \quad g_j'(k) = V'(i,j). \]
Among them \( k = i \times (N-1) + j; \quad i=0,1,...,M-1; \quad j=0,1,...,N-1 \)
At this time, the original image \( I \), the watermarked image \( V \), and the transformed images \( I' \) and \( V' \) are regarded as four points in the \( MN \)-dimensional European space, that is
\[ I = (g_1(0), g_1(1), ..., g_1(M \times N-1)), \quad I' = (g_1'(0), g_1'(1), ..., g_1'(M \times N-1)), \]
\[ V = (g_2(0), g_2(1), ..., g_2(M \times N-1)), \quad V' = (g_2'(0), g_2'(1), ..., g_2'(M \times N-1)) \]
Let the vector \( \alpha \) represents the directed line segment with \( I \) as the starting point and \( V \) as the ending point, then
\[ \alpha = (g_2(0)-g_1(0), g_2(1)-g_1(1), ..., g_2(M \times N-1)-g_1(M \times N-1)) \]
Let the vector \( \beta \) represents the directed line segment with \( I \) as the starting point and \( V \) as the ending point, then
\[ \beta = (g_2'(0)-g_1'(0), g_2'(1)-g_1'(1), ..., g_2'(M \times N-1)-g_1'(M \times N-1)) \]

From property II, \[ \sum_{i=0}^{M \times N-1} (g_2(i) - g_1(i))^2 = \sum_{i=0}^{M \times N-1} (g_2'(i) - g_1'(i))^2 \]
is available.
According to the watermark embedding formula \( V'(i,j) = I'(i,j) + \alpha w_k, \quad k=1,2,...,L, \)
So only \( L \) components from \( I' \) to \( V' \) are modified, the remaining \( M \times N \times L \) components are not modified, and the modification amplitude of each component is \( \alpha \).
So
\[ \sum_{i=0}^{M \times N-1} (g_2(i) - g_1(i))^2 = L\alpha^2 \]
\[ \sum_{i=0}^{M \times N-1} (I(i,j) - V(i,j))^2 = \sum_{i=0}^{M \times N-1} (g_2(i) - g_1(i))^2 + \sum_{i=0}^{M \times N-1} (g_2'(i) - g_1'(i))^2 = L\alpha^2 \]
According to the formula of PSNR
\[ PSNR = 10 \log \frac{255^2 MN}{L\alpha^2} \]
\[ \alpha = 255 \sqrt{\frac{MN}{PSNR}} \]
\[ \therefore \quad \sqrt{\frac{L10}{10}} \]
Completed.

3. Experimental results
This experiment uses Peppers (512×512), Barbara (512×512), Couple (512×512) and a binary image (144×144) as the watermarking. WHT and DCT are performed respectively on the original image. Assume that the peak signal-to-noise ratio (PSNR) to be obtained is 51, 47, 41, 37, 33. It can be calculated by formula (1) that embedding strength is 2.5553, 4.0499, 8.0807, 12.8070, and 20.2977 respectively.
In order to verify the validity of the formula derived in this paper, this experiment also uses Boats (720×576), Goldhill (720×576), using binary image (64×64) as a watermarking. Assume that the peak signal-to-noise ratio (PSNR) to be obtained is 51, 47, 41, 37, 33. Assume that the peak signal-to-noise ratio (PSNR) to be obtained is 51, 47, 41, 37, 33. It can be calculated by formula (1) that embedding strength is 7.2317, 11.4614, 22.8686, 36.2441, and 57.4431.

In the experiment the corresponding embedding strengths are calculated according to the PSNR firstly. Secondly, the original images are divided into some blocks, and each block size is 8×8. Thirdly DCT (or WHT) is performed in each block, then the embedded strengths are used in embedding watermark. Finally the PSNR of the watermarked image are calculated in order to compare the errors. The experimental results are as follows:

Table 1. DCT Experimental Results of Figure 1.

| Embedding Strength | α = 2.5553 | α = 4.0499 | α = 8.0807 | α = 12.8079 | α = 20.2977 |
|--------------------|------------|------------|------------|-------------|-------------|
| Peppers        | 50.4910    | 46.7526    | 41.0140    | 37.1999     | 33.3292     |
| Barbara       | 50.4529    | 46.7472    | 41.0289    | 37.1087     | 33.2045     |
| Couple       | 50.4700    | 46.7323    | 40.9907    | 37.0775     | 33.1480     |

Table 2. DCT Experimental Results of Figure 2.

| Embedding Strength | α = 7.2317 | α = 11.4614 | α = 22.8685 | α = 36.2441 | α = 57.4431 |
|--------------------|------------|------------|-------------|-------------|-------------|
| Boats            | 50.4431    | 46.7690    | 41.0389     | 37.1023     | 33.1722     |
| Goldhill         | 50.5672    | 46.8272    | 41.0585     | 37.1562     | 33.2001     |

Table 3. WHT Experimental Results of Figure 1.

| Embedding Strength | α = 2.5553 | α = 4.0499 | α = 8.0807 | α = 12.8079 | α = 20.2977 |
|--------------------|------------|------------|------------|-------------|-------------|
| Peppers        | 50.4887    | 46.7622    | 41.0946    | 37.3072     | 33.4322     |
| Barbara       | 50.4493    | 46.8138    | 41.1111    | 37.1825     | 33.2828     |
| Couple       | 50.4417    | 46.7214    | 41.0792    | 37.1506     | 33.2107     |
Table 4. WHT Experimental Results of Figure 2.

| Embedding Strength | $\alpha = 7.2317$ | $\alpha = 11.4614$ | $\alpha = 22.8685$ | $\alpha = 36.2441$ | $\alpha = 57.4431$ |
|--------------------|------------------|------------------|------------------|------------------|------------------|
| Boats              | 50.2317          | 47.4920          | 40.7182          | 36.8347          | 33.1601          |
| Goldhill           | 50.2845          | 47.6528          | 40.7374          | 36.8801          | 33.1265          |

Table 5. Errors of DCT Experiments.

| PSNR   | 51 | 47 | 41 | 37 | 33 |
|--------|----|----|----|----|----|
| Peppers| 0.5090 | 0.2474 | 0.0140 | 0.1999 | 0.3292 |
| Barbara| 0.5471 | 0.2528 | 0.0289 | 0.1087 | 0.2045 |
| Couple | 0.5300 | 0.2677 | 0.0093 | 0.0775 | 0.1480 |
| Boats  | 0.5569 | 0.2310 | 0.0389 | 0.1023 | 0.1722 |
| Goldhill| 0.4328 | 0.1728 | 0.0585 | 0.1562 | 0.2001 |

Table 6. Errors of WHT Experiments.

| PSNR   | 51 | 47 | 41 | 37 | 33 |
|--------|----|----|----|----|----|
| Peppers| 0.5223 | 0.2378 | 0.0946 | 0.3072 | 0.4322 |
| Barbara| 0.5507 | 0.1862 | 0.1111 | 0.1825 | 0.2828 |
| Couple | 0.5593 | 0.2786 | 0.0792 | 0.1506 | 0.2107 |
| Boats  | 0.7683 | 0.4920 | 0.2812 | 0.1653 | 0.1601 |
| Goldhill| 0.7155 | 0.4628 | 0.2626 | 0.1199 | 0.1265 |

Table 1-Table 2 show the PSNR of the DCT experiment, and Table 3-Table 4 show the PSNR of the WHT experiment. Each column shows the PSNR of the same embedding strength on different images. As can be seen from Tables 1 to 4, there is not much difference between the experimental results and the theoretically obtained values.

Tables 5 and 6 show the errors of the experimental values and theoretical values in the DCT and WHT domains, respectively. It can be seen that the largest absolute error is 0.5569 and the smallest error is 0.0093 in the DCT domain, and the largest absolute error is 0.7683 and the smallest error is 0.0792.

4. Conclusion

As an important application technology in copyright protection, this paper focuses on the systematic research on the embedding strength of digital watermarks. Based on the properties of orthogonal transformation, the relationship between points and vectors in Euclidean space is used to theoretically derive orthogonality. Domain image watermark embedding formula. From the derivation process, the formula is also applicable to other orthogonal transformations. The experimental results show that the embedded strength estimation formula proposed in this paper has small error in the orthogonal domain, has certain accuracy, and has certain practicability in controlling the watermark embedding strength.

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