Lepton Flavour Violation in the Constrained MSSM with Constrained Sequential Dominance

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Abstract

We consider charged Lepton Flavour Violation (LFV) in the Constrained Minimal Supersymmetric Standard Model, extended to include the see-saw mechanism with Constrained Sequential Dominance (CSD), where CSD provides a natural see-saw explanation of tri-bimaximal neutrino mixing. When charged lepton corrections to tri-bimaximal neutrino mixing are included, we discover characteristic correlations among the LFV branching ratios, depending on the mass ordering of the right-handed neutrinos, with a pronounced dependence on the leptonic mixing angle $\theta_{13}$ (and in some cases also on the Dirac CP phase $\delta$).
1 Introduction

Over the past decade neutrino physics has revealed the surprising fact not only that neutrinos have mass, but also that lepton mixing must involve two large mixing angles, commonly referred to as the atmospheric angle $\theta_{23}$ and the solar angle $\theta_{12}$ [1]. The latest neutrino oscillation data [2] is consistent with tri-bimaximal lepton mixing [3]. Theoretical attempts to reproduce this structure typically produce tri-bimaximal mixing in the neutrino sector [1], with charged lepton mixing giving important corrections to the physical lepton mixing. For example, in the see-saw mechanism [5], sequential dominance (SD) [6] is well known to provide a natural explanation of hierarchical neutrino mass together with large neutrino mixing angles. When certain constraints are imposed on the neutrino Yukawa matrix elements then tri-bimaximal neutrino mixing can result from such a constrained sequential dominance (CSD) [7]. Charged lepton corrections can provide calculable deviations from tri-bimaximal mixing, resulting in predictive neutrino mixing sum rules [7, 8, 9] which may be proved with future long baseline neutrino experiments [10].

When neutrino mass models are combined with supersymmetry (SUSY) then lepton flavour violation (LFV) is an inevitable consequence [11, 12, 13]. In the constrained minimal supersymmetric standard model (CMSSM), in which the soft scalar mass matrices are described by a single universal soft high energy parameter $m_0$, and a universal trilinear parameter $A_0$, then the only source of LFV is due to RGE running effects, and in this case the connection between LFV processes and neutrino mass models has received a good deal of attention [14]. In the case of SD models it has been shown that LFV could reveal direct information about the neutrino Yukawa couplings in the diagonal charged lepton basis, depending on the particular nature of the SD, for example whether the dominant right-handed neutrino is the heaviest one or the lightest one [15, 16]. For example if the dominant right-handed neutrino is the heaviest one, then large rates for $\tau \rightarrow \mu \gamma$ are expected [15, 16]. However even in this case, the amount of information one can deduce is limited due to the large number of unconstrained Yukawa couplings.

In this paper we consider LFV for the case of CSD, where the number of independent neutrino Yukawa couplings is reduced. In this case the LFV predictions are also sensitive to the charged lepton mixings, so some further assumptions are required in order to make predictions. In addition to tri-bimaximal mixing via CSD, we shall also additionally assume CKM-like charged lepton corrections. This will lead to interesting correlations in LFV muon and tau decays, independent of the SUSY mass parameters, and Yukawa couplings, providing quite specific predictions for LFV.
2 (Constrained) Sequential Dominance

Sequential Dominance (SD) [6] represents classes of neutrino models where large lepton mixing angles and small hierarchical neutrino masses can be readily explained within the see-saw mechanism. To understand how Sequential Dominance works, we begin by writing the right-handed neutrino Majorana mass matrix $M_{RR}$ in a diagonal basis as

$$
M_{RR} = \begin{pmatrix}
M_A & 0 & 0 \\
0 & M_B & 0 \\
0 & 0 & M_C \\
\end{pmatrix}.
$$

(1)

We furthermore write the neutrino (Dirac) Yukawa matrix $\lambda_\nu$ in terms of $(1,3)$ column vectors $A_i, B_i, C_i$ as

$$
Y_\nu = \begin{pmatrix} A & B & C \end{pmatrix},
$$

(2)

using left-right convention. The term for the light neutrino masses in the effective Lagrangian (after electroweak symmetry breaking), resulting from integrating out the massive right handed neutrinos, is

$$
L^{\nu}_{eff} = \frac{(\nu^T_i A_i)(A^T_j \nu_j)}{M_A} + \frac{(\nu^T_i B_i)(B^T_j \nu_j)}{M_B} + \frac{(\nu^T_i C_i)(C^T_j \nu_j)}{M_C}
$$

(3)

where $\nu_i$ ($i = 1, 2, 3$) are the left-handed neutrino fields. Sequential dominance then corresponds to the third term being negligible, the second term subdominant and the first term dominant:

$$
\frac{A_i A_j}{M_A} \gg \frac{B_i B_j}{M_B} \gg \frac{C_i C_j}{M_C}.
$$

(4)

In addition, we shall shortly see that small $\theta_{13}$ and almost maximal $\theta_{23}$ require that

$$
|A_1| \ll |A_2| \approx |A_3|.
$$

(5)

Without loss of generality, then, we shall label the dominant right-handed neutrino and Yukawa couplings as $A$, the subdominant ones as $B$, and the almost decoupled (sub-subdominant) ones as $C$. Note that the mass ordering of right-handed neutrinos is not yet specified. Again without loss of generality we shall order the right-handed neutrino masses as $M_1 < M_2 < M_3$, and subsequently identify $M_A, M_B, M_C$ with $M_1, M_2, M_3$ in all possible ways. LFV in some of these classes of SD models has been analysed in [16].
Writing $A_\alpha = |A_\alpha|e^{i\phi_{A\alpha}}, B_\alpha = |B_\alpha|e^{i\phi_{B\alpha}}, C_\alpha = |C_\alpha|e^{i\phi_{C\alpha}}$ and working in the mass basis of the charged leptons, under the SD condition Eq. (4), we obtain for the neutrino mixing angles [6]:

\[ \tan \theta_{\nu 23}^\nu \approx \frac{|A_2|}{|A_3|}, \quad (6a) \]

\[ \tan \theta_{\nu 12}^\nu \approx \frac{|B_1|}{c_{23}|B_2| \cos \phi_2 - s_{23}|B_3| \cos \phi_3}, \quad (6b) \]

\[ \theta_{\nu 13}^\nu \approx e^{i(\tilde{\phi} + \phi_{B_1} - \phi_{A_2})} \frac{|B_1|(A_2^* B_2 + A_3^* B_3)}{|A_2|^2 + |A_3|^2} \frac{M_A}{M_B} + e^{i(\tilde{\phi} + \phi_{B_1} - \phi_{A_2})} \frac{|A_1|}{\sqrt{|A_2|^2 + |A_3|^2}}, \quad (6c) \]

and for the masses

\[ m_3 \approx \frac{(|A_2|^2 + |A_3|^2) v^2}{M_A}, \quad (7a) \]

\[ m_2 \approx \frac{|B_1|^2 v^2}{s_{12}^2 M_B}, \quad (7b) \]

\[ m_1 \approx \mathcal{O}(|C|^2 v^2 / M_C). \quad (7c) \]

As in [6] the PMNS phase $\delta$ is fixed by the requirement that we have already imposed in Eq. (6b) that $\tan(\theta_{12})$ is real and positive,

\[ c_{23}|B_2| \sin \tilde{\phi}_2 \approx s_{23}|B_3| \sin \tilde{\phi}_3, \quad (8) \]

\[ c_{23}|B_2| \cos \tilde{\phi}_2 - s_{23}|B_3| \cos \tilde{\phi}_3 > 0, \quad (9) \]

where

\[ \tilde{\phi}_2 \equiv \phi_{B_2} - \phi_{B_1} - \tilde{\phi} + \delta, \]

\[ \tilde{\phi}_3 \equiv \phi_{B_3} - \phi_{B_1} + \phi_{A_2} - \phi_{A_3} - \tilde{\phi} + \delta. \quad (10) \]

The phase $\tilde{\phi}$ is fixed by the requirement (not yet imposed in Eq. (6c)) that the angle $\theta_{13}$ is real and positive. In general this condition is rather complicated since the expression for $\theta_{13}$ is a sum of two terms. However if, for example, $A_1 = 0$ then $\tilde{\phi}$ is fixed by:

\[ \tilde{\phi} \approx \phi_{A_2} - \phi_{B_1} - \zeta \quad (11) \]

where

\[ \zeta = \arg (A_2^* B_2 + A_3^* B_3). \quad (12) \]
Eq. (12) may be expressed as
\[
\tan \zeta \approx \frac{|B_2| s_{23} s_2 + |B_3| c_{23} s_3}{|B_2| s_{23} c_2 + |B_3| c_{23} c_3}.
\] (13)

Inserting \( \tilde{\phi} \) of Eq. (11) into Eqs. (8), (10), we obtain a relation which can be expressed as
\[
\tan(\zeta + \delta) \approx \frac{|B_2| c_{23} s_2 - |B_3| s_{23} s_3}{-|B_2| c_{23} c_2 + |B_3| s_{23} c_3}.
\] (14)

In Eqs. (13), (14) we have written \( s_i = \sin \zeta_i, \; c_i = \cos \zeta_i, \) where we have defined
\[
\zeta_2 \equiv \phi_{B_2} - \phi_{A_2}, \quad \zeta_3 \equiv \phi_{B_3} - \phi_{A_3},
\] (15)

which are invariant under a charged lepton phase transformation. The reason why the see-saw parameters only involve two invariant phases rather than the usual six, is due to the SD assumption in Eq. (4) that has the effect of effectively decoupling the right-handed neutrino of mass \( M_C \) from the see-saw mechanism, which removes three phases, together with the further assumption (in this case) of \( A_1 = 0 \), which removes another phase.

2.1 CSD and tri-bimaximal neutrino mixing

Tri-bimaximal neutrino mixing \([3]\) corresponds to the choice \([7]\):
\[
|A_1| = 0, \hspace{1cm} |A_2| = |A_3|, \hspace{1cm} |B_1| = |B_2| = |B_3|, \hspace{1cm} A^\dagger B = 0.
\] (16) (17) (18) (19)

This is called constrained sequential dominance (CSD) \([7]\). For example, a neutrino Yukawa matrix in the notation of Eq. (2), which satisfies the CSD conditions in Eqs. (16)-(19), may be taken to be:
\[
Y_\nu = \begin{pmatrix}
0 & be^{i\beta_2} & c_1 \\
-ae^{i\beta_3} & be^{i\beta_2} & c_2 \\
ae^{i\beta_3} & be^{i\beta_2} & c_3
\end{pmatrix}.
\] (20)
where $C$ is not constrained by CSD, since it only gives a sub-subdominant contribution to the neutrino mass matrix, so we have written it as $C = (c_1, c_2, c_3)$ above. CSD leads to tri-bimaximal mixing in the neutrino mass matrix $m_\nu$, i.e. to

$$
V_{\nu,\text{tri}}^\dagger = \begin{pmatrix}
\sqrt{2/3} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\
1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}.
$$

(21)

### 3 Charged lepton corrections

The form of the PMNS matrix will depend on the charged lepton Yukawa matrix whose diagonalisation will result in a charged lepton mixing matrix $V_{eL}$ which must be combined with $V_{\nuL}^\dagger$ to form $U_{\text{PMNS}}$. The resulting lepton mixing matrix will therefore not be precisely of the tri-bimaximal form, even in theories that predict precise tri-bimaximal neutrino mixing. We consider here the case that CSD holds in a basis where the charged lepton mass matrix is not exactly diagonal, but corresponds to small mixing. This is a situation, often encountered in realistic models [7, 9, 17].

In the presence of charged lepton corrections, the prediction of tri-bimaximal neutrino mixing is not directly experimentally accessible. However, this challenge can be overcome when we make the additional assumption that the charged lepton mixing matrix has a CKM-like structure, in the sense that $V_{eL}$ is dominated by a 1-2 mixing $\theta \equiv \theta_{e12}^c$, i.e. that its elements $(V_{eL})_{13}$, $(V_{eL})_{23}$, $(V_{eL})_{31}$ and $(V_{eL})_{32}$ are very small compared to $(V_{eL})_{ij}$ ($i, j = 1, 2$). In the following, we shall take these elements to be approximately zero, i.e.

$$
V_{eL} \approx P \begin{pmatrix}
\cos \theta & -\sin \theta e^{-i\lambda} & 0 \\
\sin \theta & \cos \theta e^{-i\lambda} & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(22)

where $\cos \theta \equiv c_\theta$, $\sin \theta \equiv s_\theta$, $\lambda$ is a phase required to diagonalise the charged lepton mass matrix [7], and $P$ is a diagonal matrix of phases $P = \text{diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3})$ which are chosen to remove phases from the product $V_{eL}V_{\nuL}^\dagger$ to yield the physical $U_{\text{PMNS}}$. In the present case it is convenient to choose $\omega_1 = 0$, $\omega_2 = \lambda$, $\omega_3 = 0$, to yield,

$$
V_{eL} \approx \begin{pmatrix}
\cos \theta & -s_\theta e^{-i\lambda} & 0 \\
s_\theta e^{i\lambda} & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(23)

With this choice, then by constructing $U_{\text{PMNS}}$ and comparing to the Standard PDG form of this matrix, one obtains, by comparing with Eq.(82) of [7],

$$
\lambda = \delta - \pi
$$

(24)
where $\delta$ is the Standard PDG CP violating oscillation phase. Also note that $\lambda \approx \delta_{22} - \delta_{12}$ where $\delta_{ij} = \text{arg} \ M^i_{ej}$.

We remark that the assumption that the charged lepton mixing angles are dominated by $(1, 2)$ Cabibbo-like mixing arises in many generic classes of flavour models in the context of unified theories of fundamental interactions, where quarks and leptons are joined in representations of the unified gauge symmetries [7, 9, 17]. Under this assumption, it follows directly from Eq. (A.3) that $(U_{\text{PMNS}})_{31}$, $(U_{\text{PMNS}})_{32}$ and $(U_{\text{PMNS}})_{33}$ are independent of $V_{\text{eL}}$, and depend only on the diagonalisation matrix $V_{\text{eL}}^\dagger$ of the neutrino mass matrix. This leads to the parameterization-independent relations [10]:

\begin{align}
| (V_{\text{eL}}^\dagger)_{31} | & \approx | (U_{\text{PMNS}})_{31} | , \\
| (V_{\text{eL}}^\dagger)_{32} | & \approx | (U_{\text{PMNS}})_{32} | , \\
| (V_{\text{eL}}^\dagger)_{33} | & \approx | (U_{\text{PMNS}})_{33} | .
\end{align}

(25a, 25b, 25c)

In addition to the assumption that $V_{\text{eL}}$ is of the form of Eq. (23) for tri-bimaximal neutrino mixing the 1-3 mixing in the neutrino mass matrix is zero,

\begin{equation}
(V_{\text{eL}}^\dagger)_{13} = 0 .
\end{equation}

(26)

Using Eq. (26) and applying the standard PDG parameterization of the PMNS matrix (see e.g. [18]), Eq. (25a) leads to the sum rule [7, 8, 9]:

\begin{equation}
\left| s_{23} s_{12} - s_{13} c_{23} c_{12} e^{i \delta} \right| \approx \left| s_{23} s_{12} - s_{13} c_{23} c_{12} \cos(\delta) \right| ,
\end{equation}

(27)

where the last step holds to leading order in $s_{13}$. This sum rule can be used to test tri-bimaximal ($\theta_{12} = \arcsin(\sqrt{2}/3)$) structure of the neutrino mass matrix in the presence of CKM-like charged lepton corrections.

4 LFV in CSD with charged lepton corrections

When dealing with LFV it is convenient to work in the basis where the charged lepton mass matrix is diagonal. Let us now discuss the consequences of charged lepton corrections of the form of Eq. (23) for the neutrino Yukawa matrix with CSD. After re-diagonalising the charged lepton mass matrix, resulting in the assumed charged lepton mixing matrix in Eq. (23), $Y_{\nu}$ in Eq. (20) becomes transformed as:

\begin{equation}
Y_{\nu} \rightarrow Y'_{\nu} = V_{\text{eL}} Y_{\nu} .
\end{equation}

(28)
In the diagonal charged lepton mass basis the neutrino Yukawa matrix therefore becomes:

\[ Y'_\nu = \begin{pmatrix} A' & B' & C' \end{pmatrix} = \begin{pmatrix} a s_\theta e^{-i \lambda} e^{i \beta_3} & b (c_\theta - s_\theta e^{-i \lambda}) e^{i \beta_2} & (c_1 c_\theta - c_2 s_\theta e^{-i \lambda}) \\ -a c_\theta e^{i \beta_3} & b (c_\theta + s_\theta e^{i \lambda}) e^{i \beta_2} & (c_1 s_\theta e^{i \lambda} + c_2 c_\theta) \\ a e^{i \beta_3} & b e^{i \beta_2} & c_3 \end{pmatrix}. \tag{29} \]

where the column vectors \( A', B', C' \) are now defined in the diagonal charged lepton basis according to Eq. (29). Thus the results in Eqs. (6) with the redefined column vectors \( A', B', C' \) now yield the physical lepton mixing angles since these are equal to the neutrino mixing angles in the diagonal charged lepton basis of Eq. (29).

At leading order in a mass insertion (MI) approximation \cite{11,12} the branching fractions of LFV processes are given by

\[ Br_{ij} \equiv Br(l_i \rightarrow l_j \gamma) \approx \frac{\alpha^3}{G_F^2} f(M_2, \mu, m_{\tilde{\nu}}) |m_{L_{ij}}^2| \xi_{ij} \tan^2 \beta, \tag{30} \]

where \( l_1 = e, l_2 = \mu, l_3 = \tau \), and where the off-diagonal slepton doublet mass squared is given in the leading log approximation (LLA) of the CMSSM by

\[ m_{L_{ij}}^{2(\text{LLA})} \approx \frac{3(m_0^2 + A_0^2)}{8\pi^2} K_{ij}, \tag{31} \]

with the leading log coefficients given by

\[ K_{21} = A'_2 A'_1^* \ln \frac{\Lambda}{M_A} + B'_2 B'_1^* \ln \frac{\Lambda}{M_B} + C'_2 C'_1^* \ln \frac{\Lambda}{M_C}, \]

\[ K_{32} = A'_3 A'_2^* \ln \frac{\Lambda}{M_A} + B'_3 B'_2^* \ln \frac{\Lambda}{M_B} + C'_3 C'_2^* \ln \frac{\Lambda}{M_C}, \]

\[ K_{31} = A'_3 A'_1^* \ln \frac{\Lambda}{M_A} + B'_3 B'_1^* \ln \frac{\Lambda}{M_B} + C'_3 C'_1^* \ln \frac{\Lambda}{M_C}. \tag{32} \]

The factors \( \xi_{ij} \) in Eq. (30) represent the ratio of the leptonic partial width to the total width,

\[ \xi_{ij} = \frac{\Gamma(l_i \rightarrow l_j \nu_{\bar{j}} \nu_{\bar{j}})}{\Gamma(l_i \rightarrow \text{all})}. \tag{33} \]

Clearly \( \xi_{21} = 1 \) but \( \xi_{32} \) is non-zero and must be included for correct comparison with the experimental limit on the branching ratio for \( \tau \rightarrow \mu \gamma \). This factor is frequently forgotten in the theoretical literature.
If LFV is only induced by RG effects from $Y'_\nu$ on the soft breaking terms, as in the CMSSM, then in the LLog and MI approximation, the branching ratios for LFV charged lepton decays, like $\ell_i \rightarrow \ell_j \gamma$, are proportional to

$$Br_{ij} \propto |K_{ij}|^2 = |(A'A'^\dagger)_{ij} \ln(\Lambda/M_A) + (B'B^{\dagger})_{ij} \ln(\Lambda/M_B) + (C'C^{\dagger})_{ij} \ln(\Lambda/M_C)|^2. (34)$$

We have only assumed so far that the right-handed neutrino mass matrix has the diagonal form shown in Eq. (1), $M_{RR} = \text{diag}(M_A, M_B, M_C)$ with the dominant right-handed neutrino labelled by $A$, the leading subdominant one labelled by $B$, and the decoupled one labelled by $C$. However the masses of the right-handed neutrinos are not yet ordered, and we have not yet specified which one is the lightest and so on. After ordering $M_A, M_B, M_C$ according to their size, there are six possible forms of $Y'_\nu$ obtained from permuting the columns, with the convention always being that the dominant one is labelled by $A$, and so on. In particular the third column of the neutrino Yukawa matrix could be $A'$, $B'$ or $C'$ depending on which of $M_A, M_B$ or $M_C$ is the heaviest.

In hierarchical models, the $(3,3)$ elements of the Yukawa matrices describing quarks and charged leptons are amongst the largest elements in the Yukawa matrices. In unified models this will also be the case for the neutrino Yukawa matrix. If the heaviest right-handed neutrino mass is $M_A$ then the third column of the neutrino Yukawa matrix will consist of the $A'$ column, and since $Y_{33}' = A_3'$ and $A_1' \sim A_2' \sim A_3' \sim a$ then we conclude that all elements of $A'$ must dominate over those of $B', C'$, and hence all LFV processes will be determined approximately by $(A'A'^\dagger)_{ij}$. Similarly if the heaviest right-handed neutrino mass is $M_B$ then the third column of the neutrino Yukawa matrix will consist of the $B'$ column, and since $Y_{33}' = B_3'$ and $B_1' \sim B_2' \sim B_3' \sim b$ then we conclude that all elements of $B'$ must dominate over those of $A', C'$, and hence all LFV processes will be determined approximately by $(B'B^{\dagger})_{ij}$. Finally if the heaviest right-handed neutrino mass is $M_C$ then the third column of the neutrino Yukawa matrix will consist of the $C'$ column which contains the large element $Y_{33}' = C_3'$. However in this case we cannot conclude that all elements of $C'$ must dominate over those of $A', B'$ for the determination of LFV processes since the elements $c_1, c_2$ are undetermined by the seesaw mechanism and could even be set equal to zero. Nevertheless it is possible that in this case all elements of $C'$ could dominate over those of $A', B'$ and hence all LFV processes could be determined approximately by $(C'C^{\dagger})_{ij}$. In the following we consider the LFV predictions arising from the three cases

$$M_3 = M_A, \quad M_3 = M_B, \quad M_3 = M_C, \quad (35)$$

corresponding to the dominant Yukawa columns being $A'$, $B'$, $C'$, respectively.
5 Predictions for the ratios of LFV branching ratios

After ordering $M_A, M_B, M_C$ according to their size, there are six possible forms of $Y'_\nu$ obtained from permuting the columns, with the convention always being that the dominant one is labeled by $A'$, and so on. In particular the third column of the neutrino Yukawa matrix could be $A', B'$ or $C'$ depending on which of $M_A, M_B$ or $M_C$ is the heaviest. If the heaviest right-handed neutrino mass is $M_A$ then the third column of the neutrino Yukawa matrix will consist of the (re-ordered) first column of Eq. (29) and assuming $Y'_{33} \sim 1$ we conclude that all LFV processes will be determined approximately by the first column of Eq. (29). Similarly if the heaviest right-handed neutrino mass is $M_B$ then we conclude that all LFV processes will be determined approximately by the second column of Eq. (29). Note that in both cases the ratios of branching ratios are independent of the unknown Yukawa couplings which cancel, and only depend on the charged lepton angle $\theta \equiv \theta_{12}$ (and in some cases on $\lambda$), which in the case of tri-bimaximal neutrino mixing is related to the physical reactor angle by $\theta_{13} = \theta_{12}/\sqrt{2} \equiv \theta/\sqrt{2}$ [7, 9].

Also note that $\lambda = \delta - \pi$ where $\delta$ is the Standard PDG CP violating oscillation phase. The predictions for these two cases will now be discussed in detail. We will also comment on the third case $M_3 = M_C$, which is less predictive, and give an explicit minimal example.

5.1 $M_3 = M_A$

In this case, assuming that the third column of the neutrino Yukawa matrix (associated with the heaviest right-handed neutrino and hence the largest Yukawa couplings) is the dominant column $A'$ associated with the atmospheric neutrino of mass $m_3$, one can read off from Eq. (34) and Eq. (29) that the $Br_{ij} \equiv Br(\ell_i \to \ell_j \gamma)$ now satisfy

\begin{align}
Br_{\mu e} & \propto |a^2 s_\theta c_\theta|^2 \xi_{\mu e} , \\
Br_{\tau e} & \propto |a^2 s_\theta|^2 \xi_{\tau e} , \\
Br_{\tau \mu} & \propto |a^2|^2 \xi_{\tau \mu} .
\end{align}

Note that $\theta \equiv \theta_{12} = \sqrt{2}\theta_{13}$, so there is a direct (and simple) connection to the measurable lepton mixing angle $\theta_{13}$ in neutrino oscillation experiments in this case. In particular,
Figure 1: The left panel shows the ratios of branching ratios $B_{\ell_i \gamma}$ of LFV processes $\ell_i \rightarrow \ell_j \gamma$ in CSD for $M_3 = M_A$ with right-handed neutrino masses $M_1 = 10^8$ GeV, $M_2 = 5 \times 10^8$ GeV and $M_3 = 10^{14}$ GeV. Here the solid lines show the (naive) prediction, from the MI and LLog approximation and with RG running effects for the other parameters neglected, while the dots show the explicit numerical computation (using SPheno2.2.2 [19] extended by software packages for LFV branching ratios and neutrino mass matrix running [20, 21]) with universal CMSSM parameters chosen as $m_0 = 750$ GeV, $m_{1/2} = 750$ GeV, $A_0 = 0$ GeV, $\tan \beta = 10$ and $\text{sign}(\mu) = +1$. The right panel shows the predictions (from full computation) for $B_{\mu e} = B_{\mu \rightarrow e \gamma}$ in the CMSSM extended by the see-saw mechanism with CSD for the case $M_3 = M_A$ with $\theta_{13} = 3^\circ$ and $\delta = 0$. In this panel we have chosen the CMSSM parameters to satisfy $A_0 = 0$ GeV, $\tan \beta = 10$ and $m_{1/2} = 5m_0$, which approximately corresponds to the successful stau co-annihilation region of LSP neutralino dark matter (DM) giving $\Omega_{DM}$ within the current WMAP limits.

The predictions for the ratios of branching ratios as a function of $\theta_{13}$ as well as for $B_{\mu e}$, for some sample choice of parameters, are shown in Fig. 1. We predict

\[
\frac{B_{\mu e}}{B_{\tau \mu}} = (s_{\theta} c_{\theta})^2 \xi_{\mu e} \xi_{\tau \mu} = \left[ \frac{1}{2} \sin(2\sqrt{2}\theta_{13}) \right]^2 \frac{\xi_{\mu e}}{\xi_{\tau \mu}},
\]

(39)

\[
\frac{B_{\mu e}}{B_{\tau e}} = (c_{\theta})^2 \xi_{\mu e} \xi_{\tau e} = \left[ \cos(\sqrt{2}\theta_{13}) \right]^2 \frac{\xi_{\mu e}}{\xi_{\tau e}},
\]

(40)

\[
\frac{B_{\tau e}}{B_{\tau \mu}} = (s_{\theta})^2 = \sin(\sqrt{2}\theta_{13})^2.
\]

(41)
5.2 $M_3 \equiv M_B$

In this case, assuming that the third column of the neutrino Yukawa matrix (associated with the heaviest right-handed neutrino and hence the largest Yukawa couplings) is the leading subdominant column $B'$ associated with the solar neutrino of mass $m_2$, one can read off from Eq. (34) and Eq. (29) that the $Br_{ij} \equiv Br(\ell_i \rightarrow \ell_j \gamma)$ now satisfy

\begin{align}
Br_{\mu e} &\propto |b^2(c_\theta - s_\theta e^{-i\lambda})(c_\theta + s_\theta e^{i\lambda})|^2 \xi_{\mu e}, \\
Br_{\tau e} &\propto |b^2(c_\theta - s_\theta e^{-i\lambda})|^2 \xi_{\tau e}, \\
Br_{\tau \mu} &\propto |b^2(c_\theta + s_\theta e^{i\lambda})|^2 \xi_{\tau \mu}.
\end{align}

Since $\theta \equiv \theta_{12}^* = \sqrt{2}\theta_{13}$, there is again a connection to the measurable lepton mixing angle $\theta_{13}$ in neutrino oscillation experiments. Furthermore, the branching ratios also depend on the phase $\lambda$, which is related to the Standard PDG CP violating oscillation phase $\delta$ by $\lambda = \delta - \pi$. The ratios of branching ratios are predicted as

\begin{align}
\frac{Br_{\mu e}}{Br_{\tau \mu}} &= \frac{|c_\theta - s_\theta e^{-i\lambda}|^2 \xi_{\mu e}}{\xi_{\tau \mu}} = |\cos(\sqrt{2}\theta_{13}) + \sin(\sqrt{2}\theta_{13})e^{-i\delta}|^2 \xi_{\mu e}, \\
\frac{Br_{\mu e}}{Br_{\tau e}} &= \frac{|c_\theta + s_\theta e^{i\lambda}|^2 \xi_{\mu e}}{\xi_{\tau e}} = |\cos(\sqrt{2}\theta_{13}) - \sin(\sqrt{2}\theta_{13})e^{i\delta}|^2 \xi_{\mu e}, \\
\frac{Br_{\tau e}}{Br_{\tau \mu}} &= \frac{|c_\theta - s_\theta e^{-i\lambda}|^2}{|c_\theta + s_\theta e^{i\lambda}|^2} = \frac{|\cos(\sqrt{2}\theta_{13}) + \sin(\sqrt{2}\theta_{13})e^{-i\delta}|^2}{|\cos(\sqrt{2}\theta_{13}) - \sin(\sqrt{2}\theta_{13})e^{i\delta}|^2}.
\end{align}

Fig. 2 shows the predictions for the ratios of branching ratios as a function of $\theta_{13}$, for the example $\delta = 0$, as well as the prediction for $Br_{\mu e}$ for some sample choice of parameters.

5.3 $M_3 \equiv M_C$

In this case, assuming that the third column of the neutrino Yukawa matrix (associated with the heaviest right-handed neutrino and hence the largest Yukawa couplings) is the most subdominant column $C'$ associated with the lightest neutrino of mass $m_1$, assuming that $c_3 \approx 1$, one can see from Eq. (34) and Eq. (29) that the $Br_{ij} \equiv Br(\ell_i \rightarrow \ell_j \gamma)$ now depend on undetermined coefficients $c_1, c_2$. Hence we cannot make definite predictions. Moreover, in some cases, the subdominant column of Yukawa coupling also contributes at the same order as the dominant one. Nevertheless, charged lepton corrections also have an impact here. Let us therefore generalize $V_{e_L}$ to include also a small $\theta_{23}^e \ll \theta_{12}^e$. As a minimal case, let us furthermore consider

\[ C = (0, 0, c)^T. \]
This may be viewed as minimal scenario regarding LFV, since typically (barring cancellations) the zeros are replaced by small entries and since, as mentioned above, the subdominant column of $Y_\nu$ can not in general be neglected. For a more accurate treatment of this scenario with respect to the charged lepton corrections, the (typically) even smaller $\theta_{e13} \ll \theta_{e23}$ can be included analogously.

Including charged lepton corrections from $\theta_{e12}^c$ and $\theta_{e23}^c$ (by $Y_\nu \to V_{eL}Y_\nu$) leads to approximately

$$C' = \left( c_{s_{23}}^2 s_{12}, c_{s_{23}}, c \right)^T,$$

and thus to the following relations for the branching ratios:

$$Br_{\mu e} \propto |c_{e1}^2 s_{23}^2 s_{12}|^2 \xi_{\mu e}, \quad (50)$$

$$Br_{\tau e} \propto |c_{e2} s_{23}^2 s_{12}|^2 \xi_{\tau e}, \quad (51)$$

$$Br_{\tau \mu} \propto |c_{e3} s_{23}|^2 \xi_{\tau \mu}. \quad (52)$$

As in the cases $M_3 = M_A$ and $M_3 = M_B$, the relation $\theta_{e12}^c = \sqrt{2} \theta_{13}$ holds under the considered assumption about the charged lepton corrections. For the ratios of the branching
The predictions for the ratios of branching ratios as a function of $\theta_{13}$ as well as for $Br_{\mu e}$ as a function of $\theta_{13}$ and $m_{1/2}$ (set equal to $5m_0$ as an example) are shown in Fig. 3. To give an explicit example, we have chosen $s_{23} = 2.36^\circ$ and other parameters as stated in the caption of Fig. 1. We would like to stress again that, in contrast to the cases $M_3 = M_A$ and $M_3 = M_B$ discussed above, the shown results are no definite predictions for the case $M_3 = M_C$, but rather order of magnitude examples for certain classes of models of CSD where the LFV branching ratios are significantly smaller than for CSD with $M_3 = M_A$ and $M_3 = M_B$. As can be seen from Fig. 4, this scenario can be readily distinguished from the cases $M_3 = M_A$ and $M_3 = M_B$. 

Figure 3: The left panel shows the ratios of branching ratios $Br_{ij}$ of LFV processes $\ell_i \to \ell_j \gamma$ for a minimal example with CSD and $M_3 = M_C$ described in the text. The solid lines in the left panel show the (naive) prediction, from the MI and LLog approximation and with RG running effects neglected, while the dots show the explicit numerical computation. The right panel shows the predictions (from full computation) for $Br_{\mu e} = Br(\mu \to e\gamma)$ in the CMSSM extended by the see-saw mechanism with CSD for the case $M_3 = M_C$ in the scenario with $m_1 = 10^{-3}$ eV and $\delta = 0$. The other parameters are chosen as in Fig. 1.

ratios we obtain

$$\frac{Br_{\mu e}}{Br_{\tau e}} = \left[ s_{12}^2 s_{23}^2 \right] \frac{\xi_{\mu e}}{\xi_{\tau e}} = \left[ \sin(\sqrt{2} \theta_{13}) s_{23} \right] \left[ \frac{\xi_{\mu e}}{\xi_{\tau e}} \right],$$

(53)

$$\frac{Br_{\mu e}}{Br_{\mu \tau}} = \left[ s_{23}^2 \right] \frac{\xi_{\mu e}}{\xi_{\mu \tau}},$$

(54)

$$\frac{Br_{\tau e}}{Br_{\tau \mu}} = \left[ s_{12}^2 \right] = \left[ \sin(\sqrt{2} \theta_{13}) \right]^2.$$

(55)
6 Conclusions

We have considered charged Lepton Flavour Violation (LFV) in the Constrained Minimal Supersymmetric Standard Model, extended to include the see-saw mechanism with Constrained Sequential Dominance (CSD), where CSD provides a natural see-saw explanation of tri-bimaximal neutrino mixing. When Cabibbo-like charged lepton corrections to tri-bimaximal neutrino mixing are included, this leads to characteristic correlations among the LFV branching ratios $Br_{\tau\mu}$, $Br_{\mu\tau}$ and $Br_{\tau\nu}$ which may be tested in future experiments.

There are two main differences between the study here and that in [16] where predictions for LFV were also presented for the CMSSM with SD. The first difference is that here we have focused on the special case of CSD, corresponding to tri-bimaximal neutrino mixing, where the neutrino Yukawa couplings are very tightly constrained compared to the general SD case. The second difference is that we have considered the effect of charged lepton corrections, which were not included in [16]. In particular we have mainly considered Cabibbo-like charged lepton corrections, which when combined with CSD leads to a very tightly constrained scenario in which ratios of branching ratios depend on $\theta_{13}$, which is related to the charged lepton mixing angle $\theta_{12}$. The predictions also depend crucially on which column of the Yukawa matrix is associated with the heaviest right-handed neutrino $M_3$, since this column will have the largest Yukawa couplings.

For the case $M_3 = M_A$, also known as Heavy Sequential Dominance (HSD) since the dominant right-handed neutrino is the heaviest one, we find the characteristic ratios in Fig. 1. Compared to the results in [16], the hierarchy between $Br_{\mu\tau}$ and $Br_{\tau\mu}$ is much milder. This can be understood from the fact that in [16] it was assumed that $|A_1| \ll |A_2| \approx |A_3| \sim 1$ (ignoring charged lepton corrections) which led to large $Br_{\tau\mu}$ but small $Br_{\mu\tau}$. However, including charged lepton corrections, we see that $|A'_1| \sim |A'_2| \sim |A'_3| \sim 1$, leading to both large $Br_{\tau\mu}$ and large $Br_{\mu\tau}$. In the present case we focus on tri-bimaximal neutrino mixing, which before charged lepton corrections are included implies that $|A_1| \ll |A_2| = |A_3| \sim 1$, corresponding to the CSD explanation of tri-bimaximal neutrino mixing. Then, after Cabibbo-like charged lepton corrections are included, this leads to well defined predictions for the each of the couplings $|A'_1|, |A'_2|, |A'_3|$, and hence rather precise predictions for ratios of $Br_{\tau\mu}$, $Br_{\mu\tau}$ and $Br_{\tau\nu}$, which depend on $\theta_{13}$, as shown in Fig. 1. We reemphasize that, after charged lepton corrections are included, $|A'_1| \sim |A'_2| \sim |A'_3| \sim 1$, and hence both $Br_{\tau\mu}$ and $Br_{\mu\tau}$ are large in this case, unlike [16] where charged lepton corrections were ignored.

In the case $M_3 = M_B$, where the leading subdominant right-handed neutrino responsible for the solar neutrino mass is the heaviest one, the predicted ratios of branching ratios are even milder, corresponding to the fact that all the Yukawa coupling in this col-
umn are equal before the inclusion of charged lepton corrections, $|B_1| = |B_2| = |B_3| \sim 1$, again corresponding to the CSD explanation of tri-bimaximal neutrino mixing. When Cabibbo-like charged lepton corrections are included this again leads to characteristic predictions for ratios of $B_{\tau\mu}$, $B_{\mu e}$ and $B_{\tau e}$, depending on $\theta_{13}$ (shown in Fig. 2 for $\delta = 0$) as well as on the Dirac CP phase $\delta$, as given in Eqs. (45) - (47).

The least predictive case is $M_3 = M_C$, which includes the case where the dominant right-handed neutrino is the lightest one known as light sequential dominance (LSD). In this case the generic prediction from [16] was that the $B_{\tau\mu}$ was generally quite small, typically of order $B_{\mu e}$, due to the small neutrino Yukawa couplings. In particular the neutrino Yukawa couplings of the third column were not considered relevant due to the large mass of the associated right-handed neutrino, which was assumed to exceed the GUT scale from which the RGEs were run down. Then the relevant Yukawa couplings were those from the second column, which all take similar (small) values leading to $B_{\tau\mu} \sim B_{\mu e}$. This may also be the case here, since including charged lepton corrections will not change this result, and CSD will only strengthen this conclusion. However in other cases, for example if the RGEs are run from the Planck scale, the third column of the neutrino Yukawa matrix should not be ignored. In Fig. 3 we considered an example of this, in which the LFV arises solely from the third column, and the Yukawa couplings in this column are again determined from charged lepton corrections, assuming that $C = (0, 0, c)^T$, which may be approximately true in practice, but which is by no means guaranteed.

In summary, the results presented here once again confirm that $B_{\tau\mu}$, $B_{\mu e}$ and $B_{\tau e}$ are all expected to be observed in the (near) future. If they are observed in the ratios predicted here, for some value of $\theta_{13}$, then this may be an indication of a high energy theory with the characteristics of the CMSSM extended to include the see-saw mechanism with CSD, corresponding to tri-bimaximal neutrino mixing corrected by Cabibbo-like charged lepton mixing angles.

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Appendix

A Conventions

In general, the mixing matrix in the lepton sector, the PMNS matrix $U_{\text{PMNS}}$, is defined as the matrix which appears in the electroweak coupling to the $W$ bosons expressed in terms of lepton mass eigenstates. With the mass matrices of charged leptons $M_e$ and neutrinos $m_\nu$ written as

$$
\mathcal{L} = -\bar{e}_L M_e e_R - \frac{1}{2} \bar{\nu}_L m_\nu \nu_L + \text{H.c.},
$$

and performing the transformation from flavour to mass basis by

$$
V_{eL} M_e V^\dagger_{eR} = \text{diag}(m_e, m_\mu, m_\tau), \quad V_{\nu L} m_\nu V^T_{\nu R} = \text{diag}(m_1, m_2, m_3),
$$

the PMNS matrix is given by

$$
U_{\text{PMNS}} = V_{eL} V^\dagger_{\nu L}.
$$

Here it is assumed implicitly that unphysical phases are removed by field redefinitions, and $U_{\text{PMNS}}$ contains one Dirac phase and two Majorana phases. The latter are physical only in the case of Majorana neutrinos, for Dirac neutrinos the two Majorana phases can be absorbed as well.

The standard PDG parameterization of the PMNS matrix (see e.g. [18]) is:

$$
U_{\text{PMNS}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} P_{\text{Maj}},
$$

which is used in most analyses of neutrino oscillation experiments. Here $\delta$ is the so-called Dirac CP violating phase which is in principle measurable in neutrino oscillation experiments, and $P_{\text{Maj}} = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 0)$ contains the Majorana phases $\alpha_1, \alpha_2$. In the following we will use this standard parameterization (including additional phases) also for $V^\dagger_{\nu L}$ and denote the corresponding mixing angles by $\theta^\nu_{ij}$, while the mixing angles $\theta_{ij}$ without superscript refer to the PMNS matrix.

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