A design method for absorption of low-frequency noise using acoustic metamaterials

Xi'an Xu¹, Xiaoming Wang¹, and Yulin Mei²*

¹ School of Mechanical Engineering, Dalian University of Technology, Dalian, Liaoning, 116024, China
² School of Automotive Engineering, Dalian University of Technology, Dalian, Liaoning, 116024, China
* Corresponding author’s e-mail: meiyulin@dlut.edu.cn

Abstract. This paper investigates the design method for absorbing and isolating low-frequency noise by utilizing the acoustic metamaterials, which consist of mass and membrane vibrators embedded into acoustic materials. Firstly, the unit cell model of the metamaterials is established to analyze characteristics of band-gaps, which is verified further by frequency domain response analysis. And then, the influence of geometric and material parameters of unit cells on the band-gap properties is investigated. Finally, the wave propagation in the metamaterials with finite periodic structures is simulated for the studies of the low-frequency noise isolation, and the effect of different group patterns of unit cells on the isolation of low-frequency noise is discussed. The results show that the greater the density difference between the mass and the base material is, the greater the number of band-gaps and each bandwidth become. And also, as the thickness of membrane increases, the number of band-gaps increases but the band-gaps are narrow and scattered. The finite periodic structures composed of uniform unit cells can hardly widen the low-frequency bandwidth by adjusting the parameters of the components. However, the non-uniform finite periodic structures composed of different unit cells can widen the band-gaps of the whole structure by adjusting parameters of different components.

1. Introduction
Low-frequency noise and vibration is characterized by its long penetrating power. It is difficult for traditional sound-insulating materials to effectively reduce low-frequency noise and vibration due to the law of mass action[1]. Therefore, it is especially important to broaden the low-frequency isolation bandwidth of structures. In general, vibration control methods have two categories: passive control and active control[2]. Considering the complexity of active control, more research is based on passive control[3]. The artificially modified acoustic metamaterials realize the integrated design of the structure and the noise isolation system. And the composite structures composed of the artificial micro-structure units can effectively control the low-frequency noise[4].

In the early research, the propagation of elastic waves in phononic crystal of infinite size received a lot of attention. The phononic crystal mainly applies the Bragg scattering mechanism[4]. In a dozen years, Liu studied the three-dimensional three-component phononic crystal composed of soft material wrapped lead-embedded epoxy resin, and proposed the locally resonant band-gap mechanism of phononic crystal [5]. Hou et al. calculated the band-gap curve of the Lamb waves in the phononic crystal plate by using the plane wave spectrum method[6]. Mei et al. proposed a lightweight...
membrane structure with a thickness as low as a millimeter based on local resonance theory, and obtained a good sound insulation capability of low-frequency [7]. Naify et al. proposed a membrane-type acoustic metamaterials based on the principle of local resonance, which can change the peak of noise insulation by adjusting the size of the resonance mass [8]. Sui et al. studied acoustic metamaterials with a single layer membrane fixed on the surface of a honeycomb panel, which has good properties of low-frequency noise insulation [9].

Based on the locally resonant mechanism of metamaterials, this paper proposes a variety of channel structures consisting of embedded mass-and-membrane acoustic metamaterials. Firstly, the band-gap characteristics of unit cells are calculated by FEM combined with Bloch boundary condition, and the influence of the parameters of unit cell upon the band-gaps of the metamaterials is investigated, and the metamaterial performance of noise isolation is analyzed. Furthermore, the effects of unit cell structure forms, cell size and cell distribution forms on the noise isolation of the channel structure filled in the metamaterials are also studied, in order to obtain the relationship between the acoustic characteristics of the channel structure and the design parameters. Finally, by adjusting the parameters of each part of structures, a structure for the insulation of low-frequency noise is shown with a property of wide bandwidth.

2. Calculation method of band-gaps
According to the elastic theory for the elastic, anisotropic, undamped and passive inhomogeneous medium, the elastic wave equation can be expressed as

$$\nabla \cdot [C(r) \nabla u] = \rho(r) \frac{\partial^2 u}{\partial t^2}$$  \hspace{1cm} (1)

Here, $r = (x, y, z)$ is the location vector; $u = (u_x, u_y, u_z)$ is the displacement vector; $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ is the differential operator; And “.” represents double dot multiplication; $C(r)$ and $\rho(r)$ represent the elastic tensor and the mass density tensor, separately.

According to the Bloch theorem, the displacement field $u(r)$ for a periodic structure can be expressed as

$$u(r) = e^{i(k \cdot r)} u_k(r)$$  \hspace{1cm} (2)

In equation (2), $i = \sqrt{-1}$, and wave vector $k = (k_x, k_y, k_z)$ in 3D; $k = (k_x, k_y)$, $k_z = 0$ in 2D; $k = k_x, k_y = k_z = 0$ in 1D. $u_k(r)$ is a periodic function with the same period as the unit cell cycle.

When using the finite element method to solve the wave equation, according to the Bloch theorem, the calculation can be performed on a single cell (figure 1) due to the periodicity of the acoustic metamaterials. The band-gaps characteristics analysis of the periodic large structure is simplified to the unit cell analysis. After meshing the unit cell with finite elements, the eigenvalue equation of the unit cell discrete form can be expressed as

$$(K - \omega^2 M) U = 0$$  \hspace{1cm} (3)

In formula, $K = \int B^T C(r) B dV_e$ is the unit cell stiffness matrix, $V_e$ is the entire area of unit cell; $M = \int \rho(r) N^T N dV_e$ is the unit cell mass matrix, $N$ is the shape function matrix; the unit cell displacement matrix is $U = [U_1 \ U_2 \ \cdots \ U_n]^T$, $U_i = [u_i \ v_i \ w_i]^T$ ($i = 1, 2, \cdots, n$) is the displacement of each node.
The outer boundary of the unit cell should satisfy the Bloch periodic boundary condition, so the external boundary displacement $U(r)$ should satisfy the following condition.

$$U(r + a) = e^{i(k \cdot a)} U(r)$$

(4)

where, $a$ is the lattice constant vector. Combined equation (3) and (4), the characteristics frequency can be solved when giving a wave vector $k$. Substituting the characteristics frequency into the equation (3), the eigenmode $U(r)$ corresponding to the characteristics frequency can be obtained. The band-gaps can be obtained by sweeping the irreducible Brillouin region with the wave vector $k$. This method is also called $w(k)$ method, it is generally necessary to sweep the boundary of the irreducible Brillouin area.

Since the Floquent-Bloch boundary condition involves the calculation of complex numbers, ANSYS can only solve the electromagnetic field problem in the complex domain; ABAQUS is more difficult to implement when directly dealing with complex boundary value problems. So, combining with parametric modeling, the solid mechanics module of the finite element analysis software COMSOL Multiphysics 5.2 is applied to calculate the band-gaps curve of the unit cell.

3. Calculation method of sound insulation performance

As shown in figure 2, when an acoustic wave encounters an obstacle in the process of propagation, part of the sound wave will be reflected by the obstacle, and part of the sound wave will be dissipated by the obstacle in the form of energy absorption, and the remaining part will pass through the obstacle. After reflection and absorption, the sound energy transmitted through the obstacle is only a part of the incident sound energy, thereby achieving vibration and noise reduction. The decibel difference between the transmitted sound wave and the incident sound wave is called as the sound insulation. The sound insulation is related to many factors, such as structure, material and sound frequency. Here, a channel structure composed of finite period unit cells is constructed and the vibration transmission spectrum of along the channel is calculated for the analysis of the isolation characteristics of the embedded mass-and-membrane type acoustic metamaterials. A line source displacement load of unit amplitude is applied to one end of the structure, and a displacement response signal is picked up at the
other end. The response value can be calculated by equation (5)

$$T = 20 \log \left( \frac{u_t}{u_i} \right)$$

(5)

where, $T$ is the displacement response amplitude ratio, representing amplitude-frequency characteristics; $u_t$ and $u_i$ are respectively the amplitudes of the transmitted and incident waves.

4. Simulation of acoustic performance of membrane

In this paper, the acoustic performance of the embedded mass-and-membrane type acoustic metamaterials is analyzed by the acoustic module of COMSOL Multiphysics simulation software. Firstly, modeling the membrane with thin plate elements, and the finite element model of the metamaterials is constructed and analyzed to investigate working mechanism of the metamaterials in the sound field. Figure 3 is a schematic diagram of a simulation model mainly composed of sound field parts and a metamaterials structural part.

In the simulation with COMSOL, the sound field consists of two parts: the incident sound field and the transmitted sound field. The middle area of the model is the embedded mass-and-membrane type acoustic metamaterials. The surrounding boundary of the structure is set as a fixed constraint, and the sides of two sound fields are defined as plane wave radiation conditions to eliminate the reflection of the sound waves, and the left side is set as the plane wave incident surface.

The length of both sound fields are 100mm, the width and the height are all of 50mm. The sound pressure of the plane wave is 1.0Pa, and the sound propagates in the positive direction along the $x$ axis. The fluid in the two sound fields is air, the speed of sound is 340m/s, and the density is 1.225kg/m$^3$. The remaining structure and material parameters are set as table 1. Here, the direct acoustic-vibration coupling method is used to solve the problem, and the influence of the parameters of acoustic metamaterials on the sound insulation is obtained. Figure 4 and figure 5 show the variation of sound insulation in accordance to membrane thicknesses or mass with frequency from 10 to 1000Hz, respectively.

It can be seen from the figures that the membrane of different thicknesses generally has the same trend of the amplitude-frequency characteristics curve, and the sound insulation performance increases...
as the thickness increases. When the thickness is increased to 1.2mm, a resonance phenomenon occurs at 650Hz; When the membrane is very thin, the sound insulation performance is small, but the resonance frequency is relatively low. After the mass is attached to the membrane, the resonance peak is increased, and a new peak is generated in the frequency band of 130 to 320Hz. Compared with a single membrane structure, the sound insulation of the whole structure has a large increase. Generally, the better sound insulation performance is obtained by the addition of mass, which provides guidance for the structure of the embedded mass-and-membrane unit cell.

5. Unit cell configuration and band-gaps analysis
Taking the 1D embedded mass-and-membrane type meta-materials as an example, according to the previous analysis of the sound insulation performance, the unit cell model is shown in figure 6. The structure is composed of frame, membrane, mass and matrix material. According to the selection of the matrix material in table 1, the longitudinal wave velocity \( c = 23m/s \). For the main research on low-frequency characteristics with \( f \leq 1000Hz \), According to the wave length \( \lambda = c/f, \lambda \geq 23mm \). Based on the principle of local resonance, the lattice constant of the unit cell \( a = 50mm \), the membrane thickness \( h = 1mm \), the width \( b = 30mm \), and the size of mass \( c = 10mm \). Define the horizontal direction of the membrane as the \( x \) direction, and the vertical direction as the \( y \) direction.

![Figure 6. Model of the embedded mass-and-membrane type metamaterials.](image)

The material of rigid frame, membrane, mass and matrix material are respectively aluminum, polyimide, steel and rubber, and the parameters of each part are shown in table 1.

| Material | Elastic modulus (Pa) | Poisson's ratio | Density (kgm\(^{-3}\)) | Thickness (mm) |
|----------|----------------------|----------------|------------------------|----------------|
| Membrane | Polyimide            | 6.4e9          | 0.45                   | 980            | 1              |
| Mass     | Steel                | 2.1e11         | 0.3                    | 7850           | 10             |
| Frame    | Aluminum             | 7.0e10         | 0.3                    | 2710           | 10             |
| Matrix   | Rubber               | 1.175e5        | 0.468                  | 1300           | -              |

The band-gaps are shown in figure 7. The vertical coordinate is the frequency, and the horizontal coordinates is the Brillouin zone scanned along the \( X - \Gamma - X \) direction. When the wave vector scans the entire Brillouin zone, the frequency in the band-gaps can't pass through the structure.

In order to further prove the calculation results of the band-gaps, using COMSOL to calculate the vibration transmission spectrum of the finite uniform periodic structure obtained by the array of different numbers of unit cells with the same composition along the \( x \) direction. It can be seen that the vibration isolation frequency band of the periodic structure is substantially identical to the band-gaps of the unit cell. However, there is a slight difference mainly due to the error of the two calculation methods. The straight line in the band-gap diagram indicates that resonance occurs at this frequency.
At the frequency where the straight band is concentrated, the local attenuation is obvious. As the number (N) of unit cell cycles increases, the vibration isolation increases. But, there is no significant change for the vibration isolation frequency band. Therefore, when constructing the low-frequency bandwidth of periodic structures, it can be selectively matched according to the band-gaps of unit cells.

Figure 7. Amplitude-frequency characteristics curve and band-gaps diagram of the embedded mass-and-membrane type meta-materials.

5.1. Effect of mass size on band-gaps.

The control variable method is used only to change the size of the mass, the other structure and material parameters are consistent with the previous calculation of the band-gaps. The size is set as shown in table 2, and figure 8 is the bandwidth curve. Since the low-frequency vibration is mainly focused on, the band-gaps near 200Hz are mainly studied.

Table 2. Mass size and the first band-gap.

| First band-gap parameters | Steel mass size c (mm) |
|---------------------------|-----------------------|
|                           | 6         | 8         | 10        | 12        | 14        | 16        |
| Starting frequency (Hz)   | 188       | 175       | 165       | 160       | 155       | 152       |
| Termination frequency (Hz)| 188       | 190       | 192       | 195       | 200       | 200       |
| Band-gap (Hz)             | 0         | 15        | 27        | 35        | 45        | 48        |

As can be seen from figure 8, the size of the mass has a certain influence on the band-gaps of the metamaterials. When the size of mass is 6mm, there is no band-gap at the range of 150 to 200Hz. As the size increases, the band-gap of the frequency band gradually expands to the entire frequency band. At the same time, other frequency bands begin to appear. Also, the width and the number of band-gaps gradually increase. Finally, when the size of mass is increased to 16 mm, except for a small portion of the frequency band, several band-gaps occurs from 150 to 325 Hz. Therefore, as the size of the mass increases, the number of band-gaps increases gradually, and the bandwidth becomes larger and larger. By properly adjusting the size of the mass, a relatively ideal band-gap can be obtained to achieve a better damping effect.
5.2. Effect of mass material on band-gaps
The effect of mass of different materials on the band-gaps of unit cells is studied here, by changing the mass material of unit cells according to table 3. The other parameters of the unit cell are shown in table 1. The calculated band-gaps position curve of unit cells according to the different mass material is shown in figure 9.

| Material | Elastic modulus (Pa) | Poisson's ratio | Density (kgm⁻³) |
|----------|----------------------|----------------|-----------------|
| Aluminum | 7.0e9                | 0.3            | 2710            |
| Steel    | 2.06e11              | 0.3            | 7850            |
| Lead     | 4.08e10              | 0.37           | 11600           |
| Tungsten | 4.11e11              | 0.28           | 19250           |
| Gold     | 85.1e9               | 0.42           | 19500           |

It can be seen from the figure 9 that the density of the mass material from aluminum to gold is increasing. As the density of the mass increases, the density difference between the matrix material and the mass block increases. Also, the number of band-gaps and the bandwidth will increase. When the material is gold, a low-frequency band-gap from 120 to 300 Hz can be obtained. Therefore, the greater the mass per unit area of the mass block is, the better the sound insulation effect is. That means, under the same structural size conditions, the greater the density of the vibration isolating member is, the better the vibration isolation effect becomes, especially for the low-frequency bandwidth.

5.3. Effect of the thickness of the membrane on band-gaps
The effect of different thickness of the membrane on the band-gaps of unit cell is studied here. Keep the unit cell size constant, and the material parameters of each component of unit cell are shown in table 1. Only the thickness of the membrane is changed, and the resulted band-gaps is shown in figure 10.

As can be seen from figure 10, when the thickness of the membrane is gradually increased, the change of the band-gaps is not simply positively correlated. As the membrane thickness becomes larger, the width and number of individual band-gap below 225Hz will gradually decrease or even disappear, and the band-gaps moves toward high frequency. The trend of band-gaps above 225Hz is not very obvious. When the thickness of the membrane is small, the single band-gap in the low-frequency band is wider and the distribution is relatively concentrated. Therefore, the thickness of the membrane can be comprehensively selected according to actual needs and durability of the membrane.
6. Noise isolation analysis of finite periodic structure

6.1. Analysis of noise isolation characteristics of straight channel

In practical engineering applications, the finite periodic structure is more practical. By periodically arraying the unit cells along the $x$ direction, the straight channel with 9 unit cells is obtained, and its overall size is $50 \times 450 \text{mm}$. The mass is steel, the frame is aluminum, and the filling material is rubber. Their parameters are shown in table 1. The plane wave load is applied to the left side of the channel, and wave propagates in the positive direction along $x$ axis. The upper and lower boundaries are fixed, a PML absorption boundary is applied to the input and output sides of the structure. The structural model is shown in figure 11, and the displacement in $x$ direction is observed.
6.1.1. Analysis of frequency domain characteristics of straight channel
Firstly, the structure is analyzed in frequency domain. A plane wave whose amplitude is 0.001m is applied to the left side of the channel, and the frequency is changing from 10 to 600Hz. The amplitude-frequency characteristics of the channel are obtained, and the displacement field at 220 Hz is shown in figure 13.

![Amplitude-frequency characteristics curve of the straight channel.](image)

It can be seen from the figure 12 that the effective frequency band for the noise isolation is mainly in the ranges of 150~250Hz, 270~315Hz and 330~550Hz, which is generally consistent with the band-gaps in the theoretical calculation of metamaterials. Compared with the structure without embedded the mass-and-membrane vibrators, the noise isolation performance of the channel with metamaterials is better. According to the displacement field at 220Hz, it can be seen that the plane wave inputting at this frequency is gradually absorbed after passing through this structure, and the response of the output end is small.

6.1.2. Analysis of time domain characteristics of straight channel
Then, time domain analysis of periodic structures with or without mass-embedded membrane are carried out. A plane wave \( y = 0.001 \sin (400 \pi t) \text{(m)} \) along x axis is applied to the left end of the structure. Figure 14 shows the displacement field obtained after the calculation is completed. The displacement data of the input and output ends are extracted, and the displacement-time curves of the mass, the input and the output are plotted in figure 15.

![Displacement field with or without embedded mass-and-membrane.](image)
Combined with the displacement field and the curve graph, it can be seen that the embedded mass-and-membrane periodic structure has better absorption and damping effects for plane wave of a given frequency than the structure without mass-embedded membrane. In figure 15, the displacement of the mass fluctuates greatly, and the displacement of the output end is small. This indicates that the structure does dissipate and absorb the energy in the sound wave through the resonance of the embedded mass-and-membrane vibrators to achieve a certain damping effect.

### 6.1.3. Analysis of noise isolation characteristics of non-uniform straight channel

According to the influence of the parameters of unit cells on the band-gaps, the band-gaps can be widened by adjusting the parameters of the membrane or mass along the channel. Dividing straight channel into four segments, and each segment contains three unit cells. Only one parameter of membrane thickness, mass material and mass size is adjusted while the other parameters remain unchanged, and the parameters of each part can refer to table 4. Figure 16 shows the displacement field of the channel at different frequencies with adjusting the material of the mass, and figure 17 shows the amplitude-frequency characteristics of the three models after integration.

| Distribution method | Membrane thickness h (mm) | Mass Material | Size c (mm) |
|---------------------|--------------------------|---------------|------------|
| Mass size           | 1                        | Steel         | 10 12 14 16 |
| Mass material       | 1                        | Steel Lead Tungsten Gold | 10 |
| Membrane thickness  | 1 1.2 1.4 1.6            | Steel         | 10         |

According to three adjusting methods: (a) mass material, (b) mass size and (c) membrane thickness. It can be seen that the main band-gaps of the non-uniform model is 110~240Hz (250~300Hz, 310-450Hz), 150~550Hz and 152~450Hz respectively. Their band-gap is 3 times wider than that of the channel composed of uniform unit cells, and the noise isolation effect is enhanced. By adjusting the thickness of the membrane, a relatively complete noise isolation band at low-frequency can be obtained, and the overall noise isolation effect is better in the low-frequency band. However, the continuity of the noise isolation band obtained by adjusting the mass material is relatively poor, mainly because that the density of the mass material cannot be continuously changed. According to the above analysis, the structure composed of uniform unit cells has the band-gaps which are fragmented and discontinuous. Adjusting the parameters of unit cell along the channel can significantly increase the low-frequency bandwidth, though it is difficult to increase the band-gaps of unit cells.
6.2. Analysis of noise isolation characteristics of spiral channel

Although the straight channel with non-uniform unit cells can widen the bandwidth, it will make the whole model thick. In order to make the whole structure smaller, the spiral channel, multi-channel and labyrinth channel are researched in this studies.

The unit cells is arranged along a spiral channel with an overall size of $400 \times 400 \text{mm}$ in diameter. Different from the model composed of uniform unit cells, the spiral channel is also divided into four
segments similar to straight channel. Each segment parameters of structure and material are set according to the corresponding band-gaps or table 4. The final results are shown in figure 18.

Figure 18 shows the displacement field of spiral channel with different distributions at 160 Hz, where (a), (b), (c) and (d) represent the different parameter distributions of unit cells, including uniform unit cells, unit cells with variation of mass size, unit cells with variation of membrane thickness and unit cells with variation of mass density, respectively. As can be seen from the figures 18, compared with the structure composed of uniform unit cells, the wave can be effectively suppressed in a non-uniform structure, resulting in a large band-gap. Like the non-uniform straight channel, the overall noise isolation effect has been significantly improved.

6.3. Analysis of noise isolation characteristics of labyrinth channel
In addition to the spiral channel, the following four labyrinth structures are proposed, which are obtained from periodic arrays of unit cell with different parameters, as shown in figure 20. Among them, A, B, and C are three labyrinth structures, and D is a simple multi-channel structure. The overall sizes are 350×260mm, 720×330mm, 720×400mm, 720×330mm respectively. The results of frequency domain analysis are shown in the figure 20 below. It can be seen that these structures can obtain good noise isolation.
7. Conclusions

A design method of low-frequency noise isolation structure based on acoustic metamaterials is presented. And a structure model of unit cell with imbedded mass-and-membrane vibrators is established. The band-gaps of unit cells are analyzed, and the influence of unit cell parameters on the band-gaps is investigated further. Meanwhile, the wave propagate in the finite periodic structure composed of the unit cells is simulated, in order to obtain the low-frequency noise isolation property of the metamaterials.

The acoustic metamaterials, which is constructed by embedding mass-and-membrane vibrators into matrix material, can generate band-gaps by local resonance. The band-gaps within 0~600Hz is affected by structural sizes and material parameters of unit cells. Firstly, with the increase of the size or density of the mass block, the number and bandwidth of band-gaps increase gradually. When the thickness of the membrane increases gradually, the variation of band-gaps is not a simple linear relationship.

For a finite periodical structure composed of uniform unit cells, with the increase of the number of unit cells, the noise isolation enhances, but the bandwidth of noise isolation does not change obviously. When the number of unit cells reaches a certain amount, the noise isolation effect is not significantly improved. In contrast, the finite periodic structure composed of non-uniform unit cells, with increasing the number of each unit cell and appropriately distributing the parameters of unit cells, the band-gaps of the structure can be enlarged to realize effective noise isolation with a wide bandwidth. Meanwhile, in order to reduce the size of isolation structures, the channel for arranging non-uniform mass-and-membrane vibrators can be designed as spirals, labyrinths or multi-channel, etc. in this way, the noise isolation effect with low-frequency and wide bandwidth can also be obtained.

Acknowledgement:
This research was financially supported by the National Science Foundation No.51775080, No.11372059 and No.11272073.
References

[1] Wu, J.H., Ma, F.Y., Zhang, S.W., Shen, L. (2016) A review of the application of acoustic metamaterials in low-frequency vibration and noise reduction. Journal of Mechanical Engineering, 52(13): 69–78.

[2] Gu, Z.Q. (1997) Active Vibration Control. National Defense Industry Press.

[3] Shu, G.Q., Hao, Z.Y. (2000) Active control technology of elastic wave in structural vibration control. Journal of Vibration and Shock, 7(5): 29–31.

[4] Wen, J.H., Han, X.Y., Wang, G., Zhao, H.G., Liu, Y.Z. (2003) Overview of phononic crystal research. Functional Materials, 4(34): 364–367.

[5] Liu, Z., Zhang, X., Mao, Y., et al. (2000) Locally resonant sonic materials. Science, 289(5485): 1734–1736.

[6] Hou, Z.L., Assouar, B.M. (2009) Numerical investigation of the propagation of elastic wave modes in a one-dimensional phononic crystal membrane coated on a uniform matrix. Journal of Physics D: Applied Physics, 42(8): 085103.

[7] MEI, J., YANG, W., YANG, Z.Y., et al. (2008) Membrane-type acoustic meta-material with negative dynamic mass. Physical Review Letters, 101(20): 204301.

[8] Naify, C.J., Chang, C.M., McKnight, G., et al. (2010) Transmission loss and dynamic response of membrane-type locally resonant acoustic meta-materials. Journal of Applied Physics, 108(11): 114905.

[9] Sui, N., Yan, X., Huang, T.Y., et al. (2015) A lightweight yet sound-proof honeycomb acoustic meta-material[J]. Applied Physics Letters, 106(17): 171905.