Mathematical modeling and fault tolerant control of uav with wing layout

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Abstract: As the flying wing layout unmanned aerial vehicle (uav) extensive research and task environment increasingly complex, Yu Fei yi layout unmanned aerial vehicle (uav) for fault tolerant control gradually become the main technical means of the flight control, according to the established mathematical model of the flying wing layout of unmanned aerial vehicle (uav) set the actuator failure effect, with adaptive fault-tolerant algorithm, is given for unmanned aerial vehicle (uav) horizontal lateral movement in the MATLAB/simulink, realize the rapid and stable, the control command and response to complete the fault-tolerant control of flying wing uav s.

1. Introduction

The flying wing configuration is undoubtedly a classic configuration for UAV as it boasts simple structure, high mobility, stealth and wing-body fusion[1-4]. However, because the tasks performed by flying-wing UAV usually have large coverage, high endurance and harsh environmental conditions, it has a high demand for the reliability of the control system. Fault-tolerant control is necessary to ensure that UAV can successfully perform tasks and return safely[5-7].

Though there is no vertical tail in the flying wing, the stable flight of the aircraft is attained through the vector thrust device and multiple control surfaces[8].

As the operating mechanism of UAV, the control surface is extremely vulnerable as it’s the busiest component in the system. It is urgently needed to solve the problems in the control surface of flying wing UAV.

The basic configuration of the flying wing UAV in this study has four pairs of control surfaces: the elevon functions as the aileron and elevator to control the pitching channel of the UAV. The redundancy actuator is connected with the aileron for adjustment based on the actual needs, and act as the aileron and functions as the elevator that controls the rolling channel.

![Figure 1 Configuration of Fly-wing UAV](image-url)
Table 1 Performance Parameters of Fly-wing UAV

| Parameter                     | Value  |
|-------------------------------|--------|
| Fuselage Length               | 11.6m  |
| Maximum Height                | 3.2m   |
| Reference Wing Area           | 84.5   |
| Aspect Ratio                  | 4.23   |
| leading-edge Sweep Angle      | 55°/30°|
| Take-off Weight               | 20t    |
| Landing Weight                | 15.9t  |
| Zero fuel weight              | 12.5t  |
| Voyage                        | 3910Km |
| Speed                         | 0.75Ma |
| Maximum Flight Altitude       | 12000m |

The yaw motion of the UAV is accomplished by the outermost split drag rudder. See Table 1 for specific performance parameters of the UAV.

2. Mathematical modeling of flying wing UAV

2.1 Dynamic equations

The Newton's Second Law of Motion was used to derive the nonlinear dynamic equations and angular dynamic equations of UAV under the combined action of gravity, aerodynamic force and thrust force $F_\Omega$ and the force moment $M_\Omega$. The dynamic equations offer the relationship between the derivative $u, v, w$ of velocity component $V$ in the airframe coordinate system and the component $F_x, F_y, F_z$ of combined force $F_\Omega$ in the airframe coordinate system

$$
\begin{align*}
\dot{u} &= rv - wq + \frac{F_x}{m} - g \sin \theta \\
\dot{v} &= wp - ru + \frac{F_y}{m} + g \cos \theta \sin \phi \\
\dot{w} &= qu - pv + \frac{F_z}{m} + g \cos \theta \cos \phi
\end{align*}
$$

\hspace{1cm} (1)

where

$$
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
G_{xb} + X_b + T_x \\
G_{yb} + Y_b + T_y \\
G_{zb} + Z_b + T_z
\end{bmatrix}
$$

\hspace{1cm} (2)

Angular dynamics equations reveal the relationship between the derivative of component $p, q, r$ of angular velocity $\Omega$ in the airframe coordinate system and the component $M, L, N$ of $M_\Omega$

$$
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
(c_5r + c_3p)q + c_2L + c_4N \\
c_4pr - c_b(p^2 - r^2) + c_5M \\
(c_b p - c_5r)q + c_2L + c_4N
\end{bmatrix}
$$

\hspace{1cm} (3)

where
2.2 Kinematic equations

The relative position of the airframe to the ground coordinate system could be used to derive the kinematic equations. The dynamic movement of the UAV to the ground could be described by the angular motion equation.

The conversion between the two coordinate systems suggests the following relationship between the airframe and the rotation angular velocity rate \( \dot{\phi} \), \( \dot{\theta} \), \( \dot{\psi} \) and the component \( p, q, r \) of corresponding rotation angular velocity \( \Omega \) in the ground coordinate system

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
p + r \cos \phi \tan \theta + q \sin \phi \tan \theta \\
q \cos \phi - r \sin \phi \\
r \cos \phi + q \sin \phi / \cos \theta
\end{bmatrix}
\]

The relationship between \( V \) and \( u, v, w \) is given by

\[
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} = L_{ab}^T \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V \cos \alpha \cos \beta \\ V \sin \beta \\ V \sin \alpha \cos \beta \end{bmatrix}
\]

Therefore, we have

\[
\begin{bmatrix}
\dot{V} \\
\dot{\alpha} \\
\dot{\beta}
\end{bmatrix} = \begin{bmatrix}
\frac{T \cos \alpha \cos \beta - D + G_s}{m} \\
-m \sin \alpha - L + n \phi [-p \cos \alpha + \cos \beta \beta - \sin \cos \alpha] + G_s \\
n \phi \cos \beta \\
-m \sin \alpha + Y - m \phi [-p \sin \alpha + \cos \alpha] + G_s \\
n \phi \sin \beta
\end{bmatrix}
\]

Twelve nonlinear differential equations for the flying wing UAV can be obtained from the above models, with a corresponding nonlinear functional correlation between the state variables and the control input vectors, which are given by

\[
u = \begin{bmatrix}
\delta_i \\
\delta_j \\
\delta_k \\
\delta_g
\end{bmatrix}
\]

3. Adaptive Fault-tolerant Control of Flying Wing UAV

3.1 Adaptive control technology

Adaptive technology was developed from the automatic flight control system in the 1950s, which was used against unknown parameters and interferences during flight.
The adaptive control system would make decisions based on differences between the actual and expected parameters of the operating system and adjust the control parameters and structure, thus ensuring the consistency of the subject with the target parameter.

3.1.1. Reference model of the adaptive control system
The reference model of the control system consists of a dynamic system based on expected performance, featuring parameter-based automatic adjustment that ensures consistency between the system and the reference system.

In literature [1], this system was integrated with the signal synthesis adaptive algorithm to develop a fault-tolerant control system with high robust structure, which solved the unknown interference and system failure, allowing the system to run according to the expected performance parameters. In literature [2], the state feedback adaptive fault-tolerant controller was designed by the backstepping method, in which the general error vector was the only variable. Literature [3] combined adaptive state feedback with output tracking and designed a fault-tolerant controller, which eliminated the stuck fault of actuator and attained fault-tolerant control.

3.1.2. Self-adaptive control
The self-adaptive control system features the control of input and output parameters and the identification of results to adjust control parameters. In literature [4], the dynamic system was monitored by the self-tuning adaptive device, which reported the input and failures to the fault-tolerant system. In literature [5], the adaptive Kalman filter was used to obtain information for inspection and fault identification before utilizing adaptive control for reconfiguration of the control device. In literature [6], the estimated state value was input as a state feedback, and the Kalman filter was used to estimate the state residual error for online control and develop the steady-state regulator.

3.2 Design of adaptive fault-tolerant controller
The adaptive fault-tolerant control system of the flying wing UAV is designed as follows.

The description of fault at the rudder surface satisfies the linear time-invariant system

\[
\begin{cases}
\dot{x}(t) = Ax(t) + Bu(t) & t < t_f (\text{nominal system}) \\
\dot{x}(t) = Ax(t) + B(u(t) + \Delta) & t > t_f (\text{failing system}) \\
y(t) = Cx(t)
\end{cases}
\]  

(8)

Faults at the rudder surface usually include floating, damage and jamming of rudder surface. Faults at the rudder surface of UAV exists in single or multiple rudder surfaces, given by

\[
\dot{x}(t) = ax(t) + b\delta(t) + Gf_a(t) \\
y(t) = cx(t)
\]  

(9)

where \( G \) is the distribution matrix of faults at the rudder surface, \( f_a(t) \) is a function of faults at the rudder surface, which may present different forms based on specific faults.

When the above faults occur on the rudder surface of the flying wing aircraft, other rudder surfaces are re-operated by the control system to compensate the faulty rudder surface so that the flight can continue.

If the rudder surface was the only component with faults, the structural controller of the fault-tolerant control system is as follows

\[
u(t) = \hat{K}_1(t)x(t) + K_2(t)
\]  

(10)

where \( \hat{K}_1(t) \) stabilizes the UAV system, and \( K_2(t) \) compensates the faulty rudder surface, which is characterized by gains. \( \hat{K}_1(t) \) can be obtained by the following adaptive law
In the formula, $\Gamma_j$ is a normal number with a boundary, and $b_{i}$ is the $i$th column of $B$.

The gain of the adaptive control system $K_2(t)$ is given by

$$K_2 = -\frac{(x^T PB)^T \gamma}{\alpha} \left\| x^T PB \right\| \hat{k}_i(t)$$

(12)

In the formula, the positive constants are $\alpha$ and $\gamma$ in turn, while $P$ is the positive definite matrix.

$\hat{k}_i(t)$ is adjusted by the following adaptive law

$$\frac{d\hat{k}_i(t)}{dt} = r \left\| x^T PB \right\|$$

(13)

$r$ is an arbitrary constant.

Based on equation (9) and (10), the closed-loop fault-tolerant control system is given by

$$\dot{x}(t) = (A + B\hat{k}_i(t))x(t) + B_iK_2(t) + B_i\Delta + B_i\hat{w}(t)$$

(14)

Assume

$$\hat{K}_{1,i}(t) = \tilde{K}_{1,i}(t) - K_{1,i}$$

$$\hat{k}_3(t) = \tilde{k}_3(t) - k_3$$

(15)

As $k_{1,i}$ and $k_i$ are unknown constants, the following error system can be obtained

$$\frac{d\hat{K}_{1,i}(t)}{dt} = -\Gamma_{i}^T b_{i}^T P x x^T$$

$$\frac{d\hat{k}_3(t)}{dt} = r \left\| x^T PB \right\|$$

(16)

Equation (15) and (16) represent the closed-loop system and error system respectively. For adaptive closed-loop system satisfying equation (10), (11) and (12), assume the existence of positive definite symmetric matrix $P$, and select equation (12) as the control gain equation and equations (11) and (13) as the adaptive law, the fault-tolerant system is endowed with gradual stability.

The design of the adaptive fault-tolerant control system for flying wing UAV was completed based on the above procedures. The third part of the paper simulated the control system by MATLAB/Simulink to analyze the control performance.

4. Simulation and Analysis of the Fault-tolerant Control System

4.1 Simulation by MATLAB/Simulink

The paper used MATLAB to construct the model to study the lateral movement of flying wing UAV with the following parameters:

The lateral movement of the flying wing UAV was expressed by system modelling (8) in Part I.

In the expression, each state quantity and correlation coefficient matrix were given by

$$x = [\beta \ r \ \phi]^T, \ u = [\delta_u \ \delta_f]^T, \ y = [\beta \ r \ \phi]^T$$

Where $\beta$ was the sideslip angle, $r$ was the roll angle velocity, $\phi$ was the yaw rate.
roll angle, $\delta_a$ was the aileron deflection angle and $\delta_r$ the rudder deflection angle. The flying wing UAV flew at a typical altitude of 15000m at a speed of 0.6Ma[2].

In system modelling (8), the values of the coefficient matrix were set at

$$
A = \begin{bmatrix}
-0.0008 & 0.03490 & -1.0000 & 0.0545 \\
-11.0737 & -1.6777 & 0.0552 & 0 \\
0 & 1.0000 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 & 0.0007 \\
-0.4109 & 0.6837 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

According to literature [8], the deflection angle of rudder was [-60°, +60°], and the deflection angle of aileron was [-45°, +45°]. According to literature [9], the simulation adopted the following parameters and initial conditions

$$
\Gamma_i = \text{diag}(10, 10, 10, 10)
$$

$$
r = 10, \alpha = 1, \gamma = 10
$$

$$
x(0) = [0.5, 1, -0.5, 0]
$$

$$
\hat{K}_i(0) = [0, 0, 0, 0]^T \quad i = 1, 2
$$

4.2 Analysis of simulation results

This section compared the lateral movement of the flying wing UAV with and without the fault-tolerant control using the same fault and initial parameters, and simulated the performance of fault-tolerant control under the two conditions.

At time=1s when there was fault at the rudder surface, the sideslip angle and roll angle at $\alpha_i=40^\circ$ were

![Figure 2 The Sideslip Angle Varied with Time](image)
As can be seen from the figure, under the fault condition, the sideslip and roll angle of the UAV responded immediately without stability, which means the UAV no longer had the ability to maintain stable flight.

Under rudder surface fault, the UAV state response adjusted by the fault-tolerant control system also had faults at the rudder surface at the time=1s and $\alpha_r=40^\circ$. The response of UAV sideslip angle and roll angle after time=1s is as follows:

According to the simulation results, faults of the rudder surface could be compensated by the well-designed adaptive fault-tolerant control system, allowing the UAV to maintain stable flight.
It can be seen from Figure 4 and Figure 5 that the fault-tolerant control system not only supports the static error-free response of the input command, but also keeps the response adjustment within a short duration.

To sum up, the fault-tolerant control system proposed and designed by this paper demonstrate good control performance and strong robustness, which meets the needs of lateral flight of flying wing UAV. In addition, the simulation results proved the efficiency of the model in trouble shooting for rudder surface of UAV, which is worth promoting and in-depth study.

5. Conclusion
The paper designed an adaptive fault-tolerant control system based on the motion equation of flying wing UAV and rudder surface fault model, and simulated the scenario for analysis. By comparing the responses with and without the adaptive fault-tolerant control system, the paper proved the reliability and efficiency of the fault-tolerant system.

Compared with the fault-tolerant model with fault detection and diagnosis, this paper designed an adaptive fault-tolerant control system, which skipped the above steps and lowered reliance on fault detection as well as risks brought by misdiagnosis.

The adaptive fault-tolerant control system of this paper could eliminate uncertainties in parameters and compensate faulty rudder surface. However, this paper presented only changes of constants in the linear time-invariant system. However, the practicality of this method in systems with varying parameters requires further study.

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