A closer look at non-decoupling $D$-Terms

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Non-Decoupling $D$-Terms are an attractive possibility to enhance the tree-level mass of the standard model like Higgs boson in supersymmetric models. We discuss here for the case of a new Abelian gauge group two effects usually neglected in literature: (i) the size of the additional radiative corrections to the Higgs mass due to the presence of the new gauge coupling, and (ii) the impact of gauge kinetic mixing. It is shown that both effects reduce to some extent the positive effect of the non-decoupling $D$-terms on the Higgs mass.

I. INTRODUCTION

The discovery of the Higgs boson at the Large Hadron Collider (LHC) [1, 2] completes not only the Standard Model (SM) of particle physics, but gives also new constraints on any extension of it. The most studied extension of the SM to date is supersymmetry (SUSY), see Ref. [3, 4] and references therein. However, the minimal-supersymmetric standard model (MSSM) is facing increasing pressure over the last few years. One of the reasons is the measured Higgs mass of 125 GeV which is not the size of the SM to date is supersymmetry (SUSY), see [3, 4] and references therein. However, the minimal-supersymmetric standard model (MSSM) is facing increasing pressure over the last few years. One of the reasons is the measured Higgs mass of 125 GeV which is not trivial to accommodate in the MSSM. The tree-level mass of the Higgs has the strict upper limit $m_h^T \leq M_Z$, and one therefore needs large radiative corrections to push the mass to the required value. The two main possibilities within the MSSM are either heavy stops, and/or a large mixing among them. The first option raises immediately the question about naturalness [5], while the second one can cause charge and colour breaking minima [6–8].

For these reasons the interest in non-minimal SUSY models has increased in the last years. A widely studied approach to soften the need for large radiative corrections to the Higgs mass is to increase the mass already at tree-level. This is done for instance in different singlet extensions via $F$-terms [9–15]. The other option is to use $D$-terms [16–18].

The simplest realisations of these ideas show the expected decoupling behaviour: as soon as the new sector responsible for the enhanced tree-level mass becomes heavy, the positive effect of the Higgs mass shrinks. However, it has been shown that this decoupling can be circumvented at the price of introducing large soft-breaking masses. In fact, valid models with non-decoupling $F$-terms [19–21] or $D$-terms [22–27] have been successfully constructed.

We will consider in this letter the case of non-decoupling $D$-terms via a new Abelian gauge group $U(1)_X$. So far, only the leading order effect on the Higgs mass was studied in this scenario: the impact of the non-decoupling $D$-terms at tree-level in the case of vanishing gauge kinetic mixing was considered. We will show that (i) radiative corrections and (ii) the presence of gauge-kinetic mixing can give important corrections to this approximation. This letter is organised as follows: in section [1] a short excursion into gauge kinetic mixing in supersymmetric models is given, before in section [II] the origin of both effects in a concrete model is described. The impact on the Higgs mass is analysed numerically in section [IV] before we summarise in section [V].

II. GAUGE KINETIC MIXING IN SUPERSYMMETRIC MODELS

If more than one Abelian gauge group is present, mixing terms between the field strength tensors of the groups are allowed by gauge and Lorentz invariance [28]:

$$\mathcal{L} = -\frac{1}{4} \kappa F_{\mu\nu}^A F^{A,\mu\nu}, \quad A \neq B,$$

(1)

Here, $F^{\mu\nu}$ are the field strength tensors of two different Abelian groups $A, B$ and $\kappa$ is in general an $n \times n$ matrix if $n$ Abelian groups are present. For practical applications it is often useful to bring the propagators of the vector bosons back into their canonical form by the redefinition [29]

$$V \to \kappa^{1/2} V.$$

(2)

The trade-off for this redefinition is a change in the covariant derivative:

$$\partial_{\mu} - i Q_{\mu}^T \tilde{G} V \to \partial_{\mu} - i Q_{\mu}^T \tilde{G} \kappa^{-1/2} V,$$

(3)

where $\tilde{G}$ is the original diagonal matrix of $n$ individual gauge couplings associated to the $n$ Abelian gauge factors, and $Q_{\mu}$ is the vector of the relevant $U(1)$ charges. Thus, the $\kappa^{-1/2}$ factor can be translated into a new set of $\frac{1}{2} n(n-1)$ “effective” gauge couplings whose combinations populate off-diagonal entries of a gauge-coupling matrix

$$G \equiv \tilde{G} \kappa^{-1/2},$$

(4)

In supersymmetric models, gauge kinetic mixing appears not only for the vector bosons, but also for both, the $D$-terms and the gaugino soft-breaking terms. Of particular
importance in the following are the new $D$-terms which read for the Abelian sector

$$L_{D,U(1)} = \sum_{ij} (\phi_i^* \phi_j) (G^T Q_i) (Q_j) (\phi_i^* \phi_j)$$

(5)

while the non-Abelian $D$-terms remain unchanged. For the special case of two Abelian gauge groups, the most general form of $G$ reads

$$G = \begin{pmatrix} g_{XX} & g_{XY} \\ g_{YX} & g_{YY} \end{pmatrix}$$

(6)

In general, one has the freedom to perform a change in basis to bring $G$ into a particular form. The most commonly considered cases are the symmetric basis with $g_{XY} = g_{YX} = \tilde{g}$ and the triangle basis with $\tilde{g} = g_{XY}, g_{YX} = 0$. The triangle basis has the advantage that the new scalars do not contribute to the electroweak VEV, and the entire impact of gauge kinetic mixing is encoded in one new coupling $\tilde{g}$. The relations between $g_{ij}$ ($i,j = X,Y$) and the physical couplings called $g_Y$, $g_X$, $\tilde{g}$ are given by

$$g_Y = \frac{g_{XY} g_{XX} - g_{X} g_{YY}}{\sqrt{g_{XX}^2 + g_{YY}^2}}, \quad g_X = \sqrt{g_{XX}^2 + g_{YY}^2},$$

$$\tilde{g} = \frac{g_{XY} g_{XX} + g_{YX} g_{YY}}{\sqrt{g_{XX}^2 + g_{YY}^2}}.$$  

(7)

III. A MODEL FOR NON-DECOUPLING $D$-TERMS

A. The Higgs mass at tree-level

In order to show the impact of loop corrections and gauge kinetic mixing on the Higgs mass in the presence of non-decoupling $D$-terms, we take a model inspired by Ref. [27] as example\(^3\). In this model, the particle content of the MSSM is extended by a gauge singlet superfield $\hat{S}$, a superfield for the right-handed neutrino and two fields $\hat{\eta}, \bar{\eta}$ responsible for the breaking of $U(1)_X$. It has also been shown that this model, when extended in addition by three generations of vector-like leptons and quarks, provides an attractive explanation for the diphoton excess observed at 750 GeV by ATLAS and CMS [31]. The $U(1)_X$ charges of all superfields are summarised in table\(1\) while the most general superpotential in agreement with all global and local symmetries is given by

$$W = W_{\text{MSSM}} + Y_\nu \hat{\nu} \hat{\eta} + (\mu + \lambda \hat{S}) H_u \bar{H}_d + \hat{S} (\xi + \lambda X \bar{\eta} \hat{\eta}) + M_S \hat{S} \bar{S} + \frac{1}{3} \kappa \hat{S} \bar{S} \hat{S}. \quad (8)$$

For simplicity, we will neglect in the following $\lambda, M_S, \kappa$ as well as their soft-breaking terms. In addition, the soft-breaking term of $\xi$ can always be shifted away. The relevant soft-breaking terms in the scalar sector are

$$-\mathcal{L}_{SB} = \cdots + (B_\mu H_u \bar{H}_d + T_\lambda S \eta \bar{\eta} + \text{h.c.}) + m_\varphi^2 |\varphi|^2, \quad (9)$$

with $\varphi = \{ H_u, H_d, \eta, \bar{\eta}, S \}$. Electroweak symmetry breaking and the breaking of $U(1)_X$ are triggered by the following scalars receiving vacuum expectation values (VEVs)

$$H_u^0 = \frac{1}{\sqrt{2}} (\phi_d + v_d + i \sigma_d), \quad H_d^0 = \frac{1}{\sqrt{2}} (\phi_u + v_u + i \sigma_u),$$

$$\eta = \frac{1}{\sqrt{2}} (\phi_\eta + v_\eta + i \sigma_\eta), \quad \bar{\eta} = \frac{1}{\sqrt{2}} (\bar{\phi}_\eta + v_\bar{\eta} + i \bar{\sigma}_\eta).$$

(10)

In addition, a VEV for the scalar singlet is induced in general

$$S = \frac{1}{\sqrt{2}} (\phi_s + v_S + i \sigma_s).$$

(11)

We define $\tan \beta = \frac{v_u}{v_d}$, $v = \sqrt{v_d^2 + v_u^2}$ as well as $\tan \beta_x = \frac{v_u}{v_\eta}$, and $x = \sqrt{v_\eta^2 + v_\bar{\eta}^2}$. One can use the five tadpole equations

$$\frac{\partial V}{\partial v_i} = 0 \quad i = u, d, S, \eta, \bar{\eta}$$

(12)

to eliminate five free parameters. We are doing this in the following for $m_{H_u}^2, m_{H_d}^2, m_{\eta}^2, m_{\bar{\eta}}^2$ and $\xi$. Thus, the remaining free parameters in the scalar sector are $B_\mu, \mu, \kappa, T_\lambda$ and the VEVs. In all numerical studies we are going to fix $\mu = \sqrt{T_\lambda}$, $T_\lambda/\lambda = 1$ TeV, and $\lambda = -0.2$. The neutral Higgs mass matrix can be written in the basis $(h, H, N, N', S)$ as

$$\begin{pmatrix}
\frac{x^2 v^2}{4} & \frac{x^2 v^2}{2} & 0 & \frac{1}{2} g_Y g_T v x \\
\frac{x^2 v^2}{2} & \frac{x^2 v^2}{2} & 0 & \frac{2 g_Y g_T v x}{t_\beta} \\
0 & 0 & \frac{\lambda_2 v^2}{2} & \lambda_2 x v S - \frac{x T_\lambda}{\sqrt{2}} \\
0 & 0 & \frac{\lambda_2 v^2}{2} & m_{44} \\
\end{pmatrix}$$

(13)

with the abbreviations

$$m_{44} = 2 m_\eta^2 + \lambda_2 v_\eta^2 + \frac{1}{2} g_X \left(2 g_X x^2 - \frac{g_Y v^2}{t_\beta^2} + g_T v^2\right).$$

(14)

\(^3\) In contrast to Ref. [27] we changed the $U(1)_X$ charges of the new fields to allow for a seesaw type-I via the term $Y_\nu \hat{\nu} \hat{\eta} \hat{\bar{\eta}}$.
and \( g_T = g_X + \tilde{g}, \) \( g^2 = g_1^2 + g_2^2 + g_7^2. \) We have used the relations \( h = \cos \beta \phi_d + \sin \beta \phi_u, \) \( H = \cos \beta \phi_u - \sin \beta \phi_d, \) \( N = \cos \beta_s \eta + \sin \beta_X \tilde{\eta}, \) \( N' = \cos \beta_s \tilde{\eta} - \sin \beta_X \eta, \) and took the limit \( \tan \beta \gg 1, \) \( \tan \beta_s = 1. \) The lightest eigenstate of the \((h,N')\) sub-system is approximated by

\[
m_h^2 \simeq \frac{2(g_1^2 + g_2^2 + (\tilde{g} + g_X)^2)m_h^2 + (g_1^2 + g_2^2)M_Z^2}{4(2m_h^2 + M_Z^2)}.
\]

(15)

Here, we made use of \( x = \frac{\sqrt{4M_Z^2 - (\tilde{g} + g_X)^2}}{g_x} \) and assumed \( M_Z \gg v. \) In addition, we replaced \( g_T \) as well as \( g^2 \) for clarity by the full expressions. We find the expected limits

- \( M_Z' \gg m_h; \) \( m_h = \frac{1}{2} (g_1^2 + g_2^2) v^2 \)
- \( M_Z' \ll m_h; \) \( m_h = \frac{1}{2} (g_1^2 + g_2^2 + (\tilde{g} + g_X)^2) v^2 \)

Thus, we have the desired \( D \)-term enhancement for very large soft-terms, which is, however, modified by the presence of the off-diagonal gauge coupling \( \tilde{g}. \)

**B. The running of gauge kinetic mixing**

As long as \( \tilde{g} \) is small, the impact of gauge kinetic mixing will be negligible. Thus, as long as \( \tilde{g} \) is treated as free parameter, it can be set to zero at a given scale. However, via renormalisation group effects (RGE) it will be induced radiatively if the two Abelian gauge groups are not orthogonal. Orthogonality in this respect means that the anomalous dimension matrix defined by

\[
16\pi^2 \gamma_{ab} = \text{Tr} Q_a Q_b
\]

is diagonal. Here, \( a \) and \( b \) run over all \( U(1) \) groups and the trace runs over all superfields charged under the corresponding group. In many commonly studied \( U(1) \) extension as \( G_{SM} \times U(1)_{B-L}, \) \( \gamma \) is not diagonal [22]. Even if the two Abelian groups can be embedded in a single GUT group, gauge kinetic mixing might appear for the remaining light degrees of freedom [23, 24]. Calculating the anomalous dimension matrix for the model under consideration, one finds large off-diagonal entries

\[
16\pi^2 \gamma = N \begin{pmatrix} 11 & 7 \\ 7 & 9 \end{pmatrix} N,
\]

(17)

where \( N \) is a diagonal matrix containing the GUT normalisation. Since we don’t assume any GUT embedding here, we use \( N = \text{diag}(1,1) \) in the following. At the one-loop level one can calculate the value of \( \tilde{g} \) at the SUSY scale as function of (i) the scale \( \Lambda \) where \( \tilde{g} \) is assumed to vanish and (ii) the value for \( g_X \) at this scale. The running of the gauge coupling matrix \( G \) at one-loop is given by [25, 37]

\[
\beta_G^{(1L)} \propto GG^T \gamma G
\]

(18)

FIG. 1. \( \frac{\Delta \Phi}{\Phi} \) at 1 TeV as function of the scale \( \Lambda \) where gauge kinetic mixing vanishes and \( g_X \) at the scale.

One has to ensure to bring \( G \) to the particular basis defined in eq. (7) after the running. The result is shown in fig. [1] One finds that \( \tilde{g} \) is in general negative, and only for a very low cut-off \( \Lambda \) is the new coupling \( \tilde{g} \) is much smaller than \( g_X. \) For \( \Lambda = 10^6 \) GeV and large \( g_X, |\tilde{g}| \) can already be as large as 0.1, while for a GUT motivated scenario with \( \Lambda \approx 10^{16} \) and \( g_X(\Lambda) = 0.72, \tilde{g} \) can even be as large as \( 0.4-0.5 \times g_X \). For completeness, we want to mention that the possibility of positive ratios \( \tilde{g}/g_X \) exists: this can appear for very large, but negative values of \( g_X \) at the scale \( \Lambda. \)

**C. Loop corrections**

Because of the non-vanishing charge of the Higgs doublets under the new gauge group, many additional radiative contributions to the Higgs mass arise already at one-loop. The Feynman diagrams with possible contributions proportional to \( g_X \) are depicted in fig. [2] Since the right-sfermions carry also a \( U(1)_x \) charge, they give also non-vanishing diagrams at one-loop. However, it can be shown that the tadpole and self-energy corrections cancel exactly for them. The loop corrected scalar masses are the eigenvalues of the radiatively corrected mass matrix \( m_h^{2, (1L)} \) given by

\[
m_h^{2, (1L)}(p^2) = m_h^{2, (T)} + \Delta M^2(p^2)
\]

(19)

where \( \Delta M \) is the sum of tadpole- and self-energy contributions. The pole masses \( m^2_h \) are then associated with

\[\text{footnote 2 Even if small values of } \tilde{g} \text{ are considered, it can be wrong to neglect gauge-kinetic mixing. Although the impact on the Higgs mass might not be important, crucial effects for instance concerning } Z' \text{ searches [26, 39] or dark matter properties [39] could be missed.} \]
FIG. 2. One-loop diagrams contributing to the radiative corrections to \( m_\phi \) proportional to \( g_X^2 \). The scalar contributions are summarised by \( \Phi = \{ h, A^0, H^+ \} \), but for the case where only pseudo-scalars \( (A^0) \) can appear. \( m_n \) label the generations in the loop.

![Diagram 1](image1.png)

![Diagram 2](image2.png)

![Diagram 3](image3.png)

![Diagram 4](image4.png)

The eigenvalues of \( m_H^{2,11} (p^2 = m_{h_n}^2) \).

For the calculation of the diagrams it is necessary to diagonalise the mass matrices of the particles in the loop, which have dimensions up to \( 8 \times 8 \). Thus, most contributions can only be calculated numerically. Only for the charged Higgs contribution we can find an analytical estimate for the impact on the SM-like Higgs mass which is mainly given by the change in the \((2,2)\)-element of \( \Delta M^2 \):

\[
\delta \Delta M^2 (p^2) = \Delta M^2 (p^2) |_{g_X=0} - \Delta M^2 (p^2) |_{g_X \neq 0} = \frac{g_X^2 v^2 \left((g_X^2 - 2g_\eta^2) \log \left( \frac{t_{\eta,\eta}^{2}}{M_{\eta,\eta}} \right) + (2g_\eta^2 + g_X^2) \log \left( \frac{M_{\eta,\eta}^2}{g_X^2} \right) \right)}{64\pi^2}
\]

(20)

\( Q^2 \) is the renormalisation scale and we used the limit \( p^2 \to 0 \). In order to give an impression of the importance of all other loop corrections, we group all diagrams into four sets: (i) diagrams involving only neutralinos \( (\chi^0) \), (ii) diagrams involving only neutral CP-even scalars \( (h) \), (iii) diagrams involving only charged Higgs scalars \( (H^+) \), and (iv) diagrams with CP-odd scalars, the new gauge boson and the corresponding ghost \( (A^0, \eta^0, \eta^-) \). One has to consider the fields in the last group together, in order to have a gauge invariant set of diagrams. In fig. 3 we show the values of the corrections to the \((2,2)\)-element of the Higgs mass matrix as function of \( g_X \) for the case of vanishing kinetic mixing, \( M_{Z'} = 3 \) TeV and \( m_{\eta^-} = 2 \) TeV. One can see that the main loop corrections come from the charged and CP-even scalar sector. Since both are negative, one can expect that the new loop corrections reduce the SM-like Higgs mass, i.e. they might compensate to some extent the positive effect at tree-level. The corrections stemming from neutralinos are comparable small, while the ones involving the \( Z' \), its Goldstone boson and its ghost are even smaller.

There is also an indirect effect of the extended gauge sector on the MSSM-like corrections to the Higgs mass: the new force changes the relation between the measured SM parameters and the running \( \overline{\text{DR}} \) values which enter the loop calculations. The most important parameter in this respect is the top Yukawa coupling \( Y_t \). As it can be seen from fig. 4, the change in the running coupling \( Y_t \) is only very small when turning up \( g_X \). Thus, the change in the loop corrections is only very tiny.

**FIG. 3.** Size of \( \Delta M_{Z'}^2 \) as function of \( g_X \) for the diagrams involving only (i) neutralinos (green, dot-dashed), (ii) CP-even scalars (blue, dashed), (iii) charged Higgs (orange, dotted), (iv) CP-odd scalars, \( Z' \) and its ghost (red, full).

![Diagram 5](image5.png)

**FIG. 4.** Running top Yukawa coupling \( Y_t \) at the SUSY scale \( Q = 1.5 \) TeV as function of \( g_X \). The red, dashed line is for \( M_{Z'} = 3 \) TeV and the blue, full line for \( M_{Z'} = 4 \) TeV.

**IV. NUMERICAL RESULTS**

We turn now to the discussion of the impact on the SM-like Higgs mass of the two effects described in the
last section. For this purpose, we make use of the  SPheno  [11, 12] output of SARAH  [13, 14] . The SARAH generated Fortran code provides a fully fledged spectrum generator for the model which enables a calculation of the entire mass spectrum at the one-loop level without any approximation: any loop contribution and the full momentum dependence is included. In principle, SPheno includes also new two-loop corrections for the model under consideration  [49, 50] . However, these are so far only available in the gaugeless limit, i.e. the corrections we are mainly interested in are not covered.

We show in fig. 5 the shifts of the SM-like Higgs mass at tree-level and at the one-loop level for \( M_{Z'} = m_{\tilde{q}} = 3 \) TeV in the \(( g_X, \tilde{g}/g_X )\)-plane compared to the case \( g_X = \tilde{g} = 0 \). One sees that the enhancement in the Higgs mass when including loop effects and/or gauge-kinetic mixing is always smaller than the push at tree-level when neglecting these effects. Assuming a fixed ratio \( \tilde{g}/g_X \), gauge kinetic mixing becomes more important for larger \( g_X \). In contrast the (relative) impact of the loop corrections has only a mild dependence on the considered value of \( g_X \). This can be seen in fig. 6 where the shifts \( \delta m^{(T)} \) and \( \delta m^{(1L)} \) are shown as function of \( g_X \) and for different combinations of \( M_{Z'} \) and \( m_{\tilde{q}} \). One finds, that the loop effects always reduce the overall enhancement of the Higgs mass by 25–30\%. The radiative corrections are usually more important for small \( g_X \), but also depend to some extent on the hierarchy between \( M_{Z'} \) and \( m_{\tilde{q}} \): for larger \( m_{\tilde{q}} \), which corresponds to a large enhancement at tree-level, the relative importance of the loops decreases faster with increasing \( g_X \).

V. SUMMARY

We have revisited here the possibility to push the tree-level mass of the SM-like Higgs by non-decoupling \( D \)-terms. It has been shown that a pure study at tree-level can overestimate the positive effect on the Higgs mass significantly. Namely, new radiative corrections triggered by the extended gauge sector can reduce the Higgs mass by several GeV. In addition, it has been shown that gauge kinetic mixing, which is a natural effect if the new Abelian gauge group is not orthogonal to \( U(1)_Y \), reduces the positive shift of the Higgs mass at tree-level to some extent depending on the assumed cut-off scale \( \Lambda \).

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\[ \delta m_h^{(T)} = m_h^{(T)}(g_X) - m_h^{(T)}(g_X = 0) \]  \hspace{1cm} (21)

\[ \delta m_h^{(1L)} = m_h^{(1L)}(g_X) - m_h^{(1L)}(g_X = 0) - \delta m_h^{(T)} \]  \hspace{1cm} (22)
[1] CMS Collaboration, S. Chatrchyan et. al., Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys.Lett. B716 (2012) 30–61
[2] ATLAS Collaboration, G. Aad et. al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys.Lett. B716 (2012) 1–29
[3] H. P. Nilles, Supersymmetry, Supergavity and Particle Physics, Phys. Rept. 110 (1984) 1–162.
[4] S. F. Martin, A Supersymmetry primer, [hep-ph/9709356 [Adv. Ser. Direct. High Energy Phys.18,1(1998)].
[5] B. Batell, G. F. Giudice and M. McCullough, Natural Heavy Supersymmetry, JHEP 12 (2015) 162
[6] J. Camargo-Molina, B. O’Leary, W. Porod and F. Staub, Stability of the CMSSM against sfermion VEVs, JHEP 1312 (2013) 103
[7] J. Camargo-Molina, B. Garbrecht, B. O’Leary, W. Porod and F. Staub, Constraining the Natural MSSM through tunneling to color-breaking vacua at zero and non-zero temperature, Phys.Lett. B737 (2014) 156–161
[8] U. Chattopadhyay and A. Dey, Exploring MSSM for Charge and Color Breaking and Other Constraints in the Context of Higgs@125 GeV, JHEP 1411 (2014) 161
[9] M. Bastero-Gil, C. Hugonie, S. F. King, D. P. Roy and S. Vempati, Does LEP prefer the NMSSM?, Phys. Lett. B489 (2000) 359–366 [hep-ph/0006198].
[10] R. Dermisek and J. F. Gunion, Consistency of LEP event excesses with an $h \rightarrow aa$ decay scenario and low-fine-tuning NMSSM models, Phys. Rev. D73 (2006) 111701 [hep-ph/0510322].
[11] U. Ellwanger, C. Hugonie and A. M. Teixeira, The Next-to-Minimal Supersymmetric Standard Model, Phys.Rept. 496 (2010) 1–77
[12] U. Ellwanger and C. Hugonie, The Upper bound on the lightest Higgs mass in the NMSSM revisited, Mod.Phys.Lett. A22 (2007) 1581–1590 [hep-ph/0612133].
[13] A. Delgado, C. Kolda, J. P. Olson and A. de la Puente, Solving the Little Hierarchy Problem with a Singlet and Explicit $\mu$ Terms, Phys. Rev. Lett. 105 (2010) 091802
[14] G. G. Ross and K. Schmidt-Hoberg, The Fine-Tuning of the Generalised NMSSM, Nucl.Phys. B862 (2012) 710–719 [1108.1284].
[15] L. J. Hall, D. Pinner and J. T. Ruderman, A Natural SUSY Higgs Near 126 GeV, JHEP 04 (2012) 131 [1112.2703].
[16] E. Mä, Exceeding the MSSM Higgs Mass Bound in a Special Class of $U(1)$ Gauge Models, Phys.Lett. B705 (2011) 320–323 [1108.4029].
[17] Y. Zhang, H. An, X.-d. Ji and R. N. Mohapatra, Light Higgs Mass Bound in SUSY Left-Right Models, Phys.Rev. D78 (2008) 011302
[18] M. Hirsch, M. Malinsky, W. Porod; L. Reichert and F. Staub, Hefty MSSM-like light Higgs in extended gauge models, JHEP 1202 (2012) 084 [1110.3037].
[19] X. Lu, H. Murayama, J. T. Ruderman and K. Tobioka, A Natural Higgs Mass in Supersymmetry from Non-Decoupling Effects, Phys.Rev.Lett. 112 (2014) 191803
[20] A. Kaminska, G. G. Ross, K. Schmidt-Hoberg and F. Staub, A precision study of the fine tuning in the DiracNMSM, JHEP 1406 (2014) 153 [1401.1816].
[21] R. Ding, T. Li, F. Staub, C. Tian and B. Zhu, The Supersymmetric Standard Models with a Pseudo-Dirac Gluino from Hybrid $F$– and $D$–Term Supersymmetry Breakings, 1502.03614.
[22] P. Batra, A. Delgado, D. E. Kaplan and T. M. P. Tait, The Higgs mass bound in gauge extensions of the minimal supersymmetric standard model, JHEP 02 (2004) 043 [hep-ph/0309149].
[23] A. Maloney, A. Pierce and J. G. Wacker, D-terms, unification, and the Higgs mass, JHEP 06 (2006) 034 [hep-ph/0509127].
[24] K. Babu, I. Gogoladze and C. Kolda, Perturbative unification and Higgs boson mass bounds, hep-ph/0410085
[25] R. Martinez, N. Poveda and J. A. Rodriguez, Upper bound of the lightest higgs boson in a supersymmetric $SU(3)_C \times U(1)_X$ gauge model, Phys. Rev. D69 (2004) 075013.
[26] E. Bertuzzo and C. Frugiuele, Natural SM-like 126 GeV Higgs boson via nondecoupling D terms, Phys. Rev. D93 (2016), no. 3 035019 [1412.2765].
[27] R. M. Capdevilla, A. Delgado and A. Martin, Light Stops in a minimal $U(1)_X$ extension of the MSSM, Phys. Rev. D92 (2015), no. 11 115020 [1509.02472].
[28] B. Holldom, Two U(1)’s and Epsilon Charge Shifts, Phys. Lett. B166 (1986) 196.
[29] R. M. Fonseca, M. Malinsky and F. Staub, Renormalization group equations and matching in a general quantum field theory with kinetic mixing, Phys. Lett. B726 (2013) 882–886 [1308.1674].
[30] B. O’Leary, W. Porod and F. Staub, Mass spectrum of the minimal SUSY B-L model, JHEP 05 (2012) 042 [1112.4600].
[31] F. Staub et. al., Precision tools and models to narrow in on the 750 GeV diphoton resonance, 1602.05581
[32] L. Basso, S. Moretti and G. M. Pruna, A Renormalisation Group Equation Study of the Scalar Sector of the Minimal B-L Extension of the Standard Model, Phys. Rev. D82 (2010) 055018 [1004.3039].
[33] M. Hirsch, W. Porod, L. Reichert and F. Staub, Phenomenology of the minimal supersymmetric $U(1)_{B-L} \times U(1)_X$ extension of the standard model, Phys. Rev. D86 (2012) 093018 [1206.3516].
[34] M. E. Krauss, W. Porod and F. Staub, SO(10) inspired gauge-mediated supersymmetry breaking, Phys.Rev. D88 (2013), no. 1 015014 [1304.0769].
[35] P. Athron, D. Harries, R. Nevzorov and A. G. Williams, $E_6$ Inspired SUSY Benchmarks, Dark Matter Relic Density and a 125 GeV Higgs, 1512.07040.
[36] F. del Aguila, G. D. Coughlan and M.Quiros, Gauge Coupling Renormalization With Several $U(1)$ Factors, Nucl. Phys. B307 (1988) 633. [Erratum: Nucl. Phys.B312,751(1989)].
[37] R. M. Fonseca, M. Malinsky, W. Porod and F. Staub, Running soft parameters in SUSY models with multi-
ple $U(1)$ gauge factors, Nucl.Phys. B854 (2012) 28–53 [1107.2670].

[38] L. Basso, A. Belyaev, S. Moretti, G. M. Pruna and C. H. Shepherd-Themistocleous, $Z'$ discovery potential at the LHC in the minimal $B-L$ extension of the Standard Model, Eur. Phys. J. C71 (2011) 1613 [1002.3586].

[39] M. E. Krauss, B. O'Leary, W. Porod and F. Staub, Implications of gauge kinetic mixing on $Z'$ and slepton production at the LHC, Phys. Rev. D86 (2012) 055017 [1206.3513].

[40] L. Basso, B. O’Leary, W. Porod and F. Staub, Dark matter scenarios in the minimal SUSY $B-L$ model, JHEP 09 (2012) 054 [1207.0507].

[41] W. Porod and F. Staub, SPheno 3.1: Extensions including flavour, CP-phases and models beyond the MSSM, Comput. Phys. Commun. 183 (2012) 2458–2469 [1104.1573].

[42] W. Porod, SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at $e^+ e^-$ colliders, Comput. Phys. Commun. 153 (2003) 275–315 [hep-ph/0301101].

[43] F. Staub, SARAH, 0806.0538

[44] F. Staub, From Superpotential to Model Files for FeynArts and CalcHep/CompHep, Comput.Phys.Commun. 181 (2010) 1077–1086 [0909.2863].

[45] F. Staub, Automatic Calculation of supersymmetric Renormalization Group Equations and Self Energies, Comput.Phys.Commun. 182 (2011) 808–833 [1002.0840].

[46] F. Staub, SARAH 3.2: Dirac Gauginos, UFO output, and more, Comput.Phys.Commun. 184 (2013) pp. 1792–1809 [1207.0906].

[47] F. Staub, SARAH 4: A tool for (not only SUSY) model builders, Comput.Phys.Commun. 185 (2014) 1773–1790 [1309.7223].

[48] F. Staub, Exploring new models in all detail with SARAH, [1503.04200]

[49] M. D. Goodsell, K. Nickel and F. Staub, Two-Loop Higgs mass calculations in supersymmetric models beyond the MSSM with SARAH and SPheno, Eur.Phys.J. C75 (2015), no. 1 32 [1411.0675].

[50] M. Goodsell, K. Nickel and F. Staub, Two-loop Higgs mass calculation from a diagrammatic approach, 1503.03098.