Computational and experimental investigation of progressive
damage accumulation in I-plate composite structure

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Abstract. As the research object to the I-plate composite structure, a stiffness degradation
method is developed by combined continuum damage mechanics with the solid finite element
analysis method. The Tsai-Wu failure criterion is applied for structural damage initiation.
Additionally, exponential equation associated with stress is used to simulate damage
development. Based on the progressive damage model established, a 3D finite element
technique is proposed to perform the progressive failure process in ANSYS. The structural
damage development and simulated load–displacement curve are received, and compared with
experimental results under transverse bending load. The compare results show the validity of
progressive damage analysis method.

1. Introduction
The number of engineering applications using composite has grown in quantity in the aerospace
industry last decade, mainly because of many advantages such as high strength/stiffness-to-weight ratio,
resistance to fatigue and durability[1]. Accident analysis for the engineering structures shows that the
damage development is one of the important reasons to cause the structure failure. Therefore, the
research on damage propagation in continuous fiber reinforced composite structure is one of the main
research targets.

Nevertheless, the existing research has been mainly concentrated in the form of laminated plates.
This research takes progressive damage accumulation of the typical I-plate composite structure as its
study object, because of this engineering components widely used in tail or panel structure. The
Tsai-Wu failure criterion and exponential equation are adopted to simulate damage occurrence and
extension. Afterwards, damage constitutive relation and the program of progressive damage method
are achieved. In conclusion, based on the comprehensive application of numerical and experimental
investigation, the damage development and strength characteristics of the I-plate composite structure
are achieved.

2. Progressive damage method
The research priorities of progressive failure model are the failure criteria and computer simulation of
damage evolution. Failure initiation of fiber reinforced composite structure is predicated by the strength
failure criterions, related to the stress and strain values at the response point. Furthermore, damage
evolution equation is achieved by using experimental (or experience) method, irreversible
thermodynamics and the equivalent displacement method. And then the damage constitutive relations of
continuous fiber reinforced composite materials are derived. The research object is to establish
nonlinear progressive damage model.

Currently, various failure theories and criterions have been developed to predict the damage initiation [2][3]. Specially, there were lots of polynomial strength failure criteria, which were no difference in addition to the coefficient substantially. Tsai and Wu proposed the Tsai – Wu criteria based on a multiple of polynomial failure criterions, and the Tsai-Wu failure criterion is widely applied in engineering [4].

As a result, this work is based on Tsai-Wu criteria:

\[ \phi = F_i \sigma_i + F_j \sigma_j, i, j = 1, ..., 6 \]  

When \( \phi \geq 1 \), Damage occurs.

Afterwards, progressive damage process of composite structure needs to study. The stiffness degradation of fiber-reinforced composite structures under static loads is an important response of the damage evolution. There are three types of methods to receive damage evolution equation. There are three types of methods to obtained damage evolution equation: experimental method [ 5][6], irreversible thermodynamics[7][8] and the equivalent displacement method[9][10]. Mathematically, the function forms of damage evolution equation are linear, multidirectional or exponential generally.

Finally, Three-dimensional braided composite can be viewed as the orthotropic composite materials, relating the CDM theory with the finite element method, damage constitutive equation is achieved:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{12} \\
\tau_{13} \\
\tau_{23}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{12} \\
\gamma_{13} \\
\gamma_{23}
\end{bmatrix}
\]

Introduce the damage variable \( d \) to stiffness matrix:

\[
\begin{align*}
C_{11} &= (1-d_1)C_{11}^0, C_{22} = (1-d_2)C_{22}^0 \\
C_{33} &= (1-d_3)C_{33}^0, C_{12} = \sqrt{(1-d_1)(1-d_2)}C_{12}^0 \\
C_{13} &= \sqrt{(1-d_1)(1-d_3)}C_{13}^0, C_{23} = \sqrt{(1-d_2)(1-d_3)}C_{23}^0 \\
C_{44} &= (1-d_4)C_{44}^0, C_{55} = (1-d_5)C_{55}^0 \\
C_{66} &= (1-d_6)C_{66}^0
\end{align*}
\]

Where \( C_{ij} \) is component of damage stiffness matrix; \( C_{ij}^0 \) is component of the initial stiffness matrix; \( d_i \) is damage variable of each direction.

The damage variables \( d_i \) employ exponential equation [11], Related to the stress of response point.

\[
d_i = \begin{cases} 
0, \varphi_i \leq \varphi_{i0} \\
(\ln R)\frac{\varphi_i - \varphi_{i0}}{1-\varphi_{i0}} + 1, \varphi_i > \varphi_{i0}
\end{cases}, i = 1...6
\]

Where R (the value is 0.01) is the residual stiffness coefficient, indicating the lower limit of stiffness degradation coefficient; \( m \) is the shape factor of stiffness degradation curve. \( \varphi_i \) are the intensity factors of each direction in Tsai-Wu failure criteria; \( \varphi_{i0} \) are stress intensity factors of each direction, when composite damage at the beginning.
3. The finite element technique

Contacted the failure criteria and the finite element implementation in the progressive failure analysis, which predicts the stiffness degradation and failure strengths of composite structures, a 3D finite element technique is developed to perform the progressive failure analysis. The material model is implemented as a user-defined material subroutine linked to the ANSYS finite element software.

A variety of finite elements such as shell181, solid185 and solid186 can be applied in the damage simulation in ANSYS. Specially, because the solid185 element has super elasticity, stress toughened, creep, large deformation and large strain capacity, it can simulate the progressive failure of three-dimensional entity structure. The New - Raphson method iterative algorithm can be widely used to solve the loss of element stiffness in the finite element analysis.

Combined the CDM methodology with the nonlinear finite element solution for failure problem, the ANSYS APDL language is applied to establish the damage analysis program. The Tsai-Wu failure criteria and stiffness degradation equation are written in the ANSYS software.

 Twelve different layer angles and geometric size of the typical composite laminates[12] to validate the analysis program. The tensile failure from is consistent with those from the tensile tests in table 1. The calculation accuracy is different for different types of composite laminates, but most of simulation errors are less than 20%. Compared with the related calculation, the calculation accuracy is reasonable.

| Serial number | Test σb/MPa | Chang-Chang σb/MPa | error /% | numerical simulation σb/MPa | error/% |
|---------------|-------------|---------------------|----------|----------------------------|---------|
| A1            | 226.1       | 206.8               | -8.5     | 184.5                      | -18.4   |
| A2            | 277.2       | 227.5               | -17.9    | 207.8                      | -25.0   |
| A3            | 235.8       | 179.3               | -24.0    | 191.8                      | -18.7   |
| A4            | 256.5       | 206.8               | -19.4    | 214.7                      | -16.3   |
| B1            | 177.9       | 165.5               | -7.0     | 159.8                      | -10.2   |
| B2            | 236.5       | 193.1               | -18.4    | 170.1                      | -28.1   |
| B3            | 185.5       | 151.7               | -18.2    | 147.3                      | -20.6   |
| B4            | 204.1       | 172.4               | -15.5    | 167.7                      | -17.8   |
| C1            | 134.4       | 124.1               | -7.7     | 153.6                      | 14.3    |
| C2            | 191.0       | 144.8               | -24.2    | 163.2                      | -14.6   |
| C3            | 160.0       | 103.4               | -35.3    | 143.7                      | -10.2   |
| C4            | 158.6       | 124.1               | -21.7    | 156.8                      | 1.1     |

The calculation precision proves that the effective of finite element technique. The next step is to establish the finite element model of the I-plate. Based on this model, the progressive damage and strength characteristics under bending load are simulated by using the finite element technique.

4. Progressive damage analysis of the I-plate

By combined the finite element method with experimental investigation, the damage development and structure bearing capacity of the fiber reinforced composite I-plate structure(C/SiC) are obtained under transverse bending load.

The experiment sample structure total length of about 300 mm, width of 60 mm, and thickness of 8 mm, in the condition of experiment, fixed and loading device are showed:
In the work, 8 node SOLID185 units are adopted to establish the finite element analysis model, as a result of 51296 nodes and 40153 units. Under transverse bending load, numerical simulation of the progressive damage process is shown in figure.2 (red shows damage location); Structural failure forms obtained from three repetitive bending experiments are showed in figure.3. Based on comparison and analysis of the results from computer simulation and experimental method, the structural damage location is similar and the validity of progressive damage method is proved.
The load–displacement curve and the ultimate load of I-plate structure obtained from the finite element simulation are compared with experimental results. Figure 4 (a) shows the numerical simulation of bending load–displacement curve and Figure 4 (b) shows the three experimental results of the I-plate structure under transverse bending loading. Because of limited experimental conditions, only the comparison of structural ultimate bearing is showed in Table 2. By quantitative analysis of the structural ultimate bearing from numerical simulation and three experiments, the relative errors can reach the order of 20% magnitude. Whereas, numerical calculation of the load is smaller than three experiments, and the difference of experimental ultimate loads have difference greatly. The reason may be the dispersion of structural material or the experimental conditions, etc.

![Figure 4](image)

**Table 2. Comparison between experimental load data and predicted value**

| Samples | Experimental load /N | Predicted value /N | Fractional error /% |
|---------|-----------------------|--------------------|---------------------|
| 1#      | 870.1                 | 776.34             | -10.78              |
| 2#      | 795.99                | 776.34             | -2.47               |
| 3#      | 940.17                |                    | -17.43              |

**5. Conclusion**

This work provides a progressive damage analysis method to simulate the progressive damage development of fiber reinforced composite structure, which includes the Tsai-Wu failure criteria and damage evolution equation exponentially connected with stress in response to point. The progressive damage accumulation of the I-plate structure is analyzed based on computational and experimental investigation. The results show that the damage development captured by the numerical predictions and the experimental observations are in reasonable agreement. Next, the above method will be expanded in a near future to research the material dispersion by probability method.
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