Ordered roughness effects on NACA 0026 airfoil

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Abstract. The effects of highly-ordered rough surface – riblets, applied onto the surface of a NACA 0026 airfoil, are investigated experimentally using wind tunnel. The riblets are arranged in directionally converging – diverging pattern with dimensions of height, \( h = 1 \text{ mm} \), pitch or spacing, \( s = 1 \text{ mm} \), yaw angle \( \alpha = 0^\circ \) and \( 10^\circ \). The airfoil with external geometry of 500 mm span, 600 mm chord and 156 mm thickness has been built using mostly woods and aluminium. Turbulence quantities are collected using hotwire anemometry. Hotwire measurements show that flows past converging and diverging pattern inherit similar patterns in the near-wall region for both mean velocity and turbulence intensities profiles. The mean velocity profiles in logarithmic regions for both flows past converging and diverging riblet pattern are lower than that with yaw angle \( \alpha = 0^\circ \). Converging riblets cause the boundary layer to thicken and the flow with yaw angle \( \alpha = 0^\circ \) produces the thinnest boundary layer. Both the converging and diverging riblets cause pronounced outer peaks in the turbulence intensities profiles. Most importantly, flows past converging and diverging pattern experience 30% skin friction reductions. Higher order statistics show that riblet surfaces produce similar effects due to adverse pressure gradient. It is concluded that a small strip of different ordered roughness features applied at a leading edge of an airfoil can change the turbulence characteristics dramatically.

1. Introduction

Studies of turbulent boundary layer have been an important focus in the past few decades. Reducing fuel usage would be monumental if these efforts could be commercially realized, where high ordered surface roughness called riblets are applied in aerospace, automotive and energy [1, 2, 3]. There have been a lot of technological advancements in getting energy efficient vehicles or systems, such as the lighter composite and more fuel-efficient jet engines for aviation industry, energy efficient vehicles by employing hybrid technology, using electric motors and hydrogen fuel cells for automotive industry and many more. These efforts have been helpful but there are more rooms for improvement to reduce drag by controlling surface turbulence.

Skin-friction drag comes from a thin region of slow moving fluid immediately adjacent to a solid surface. Energy consumption to overcome skin-friction drag is relatively costly, for example: around 50% of the total drag in modern aircraft is caused by skin-friction drag [4, 5, 6], in large ships such as container ship and Very Large Crude Carrier (VLCC), it is up to 80%. One area to control skin drag is by controlling the turbulence levels on the surface and one of the methods is by applying surface roughness. A few decades ago, scientists have realized that turbulent flows have repeating patterns or features known as coherent structures [7] and small-scale near-wall structures dominate turbulent production [8], which are responsible for drag. Most engineering applications deal with high geometry, high velocities, and therefore high Reynolds numbers. As Reynolds number increases, small-scale...
structures are replaced by large-scale features that reside further away from the wall [9, 10]. It is believed that large features may influence the near-wall structures, which are directly correlated to the skin friction drag. One of the surface roughness types is the directional riblets.

Originally, bio-inspired riblets placoid scales-like topology have been observed from nature [11], especially over the whole body of shark so-called "shark skin" [12]. Concepts of featured surfaces leading to skin friction reduction were studied to control turbulent boundary layers, for instance the herringbone riblets of birds is a passive method used to reduce drag up to 16% in turbulent flow [13]. Generally, the friction in the surface is proportional to total kinetic resistance at more than 60% for a cargo ship [12]. Riblets can contribute up to 10% drag reduction compared to flows over a flat surface [14]. More importantly, for a better understanding of boundary layer control, riblets arranged in a ‘herringbone’ pattern have been studied in zero pressure gradient (ZPG) with different parameters. Riblets could alter flow characteristics in the near-wall region [3, 8] similar to pressure gradient effects [16]. Nano-coating with microstructural riblets has already been applied in aerospace and maritime applications, resulting in drag reduction of 5.2% for torpedo-shaped and 6.2% for wing profile [17]. Furthermore, different types of riblets have been applied in very wide applications such as nature gas pipeline, racing car, high-speed train, windmill, swimming suit and airplane [11]. Last but not least, riblets have been tested on airfoils, namely NACA 0012 [18, 19]. In this research, high order riblet is applied by conducting the experiment with NACA 0026 airfoil [20].

2. Methodology

2.1. Flow facility

The experiment has been conducted at low-speed turbulent boundary layer wind tunnel in Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia (UKM), Malaysia. This wind tunnel is referred to as Pangkor-LST, which is an open-circuit blown type [21]. Main geometries of the wind tunnel are shown in Figure 1.

![Figure 1. Illustration of wind-tunnel geometry (side view)](image)

The wind tunnel consists of a 1.2 m wide, 0.5 m high and 3 m long test section. The traverse is newly built, with a ball screw pitch of 1.6 mm coupled with Vecta stepper motors model PK266, ensuring a traversing resolution well below 0.1 mm in the spanwise and vertical directions. The traverse is located on the test section and it can be moved manually along the test section (in the streamwise direction). Velmex model VXM-3 controllers used were sequenced automatically with the acquisition system. Further details of design and construction low-speed wind tunnel are given in [21].

2.2. Experimental procedure

The hotwire anemometry system used is the National Instrument’s Multi-Channel CTA system model 54N80. The hotwire data is collected using module NI 9215 while data of all other sensors such as temperature, static pressure (pitot-tube), atmospheric pressure, room humidity and dew point were collected using Comet, model H7331 [21]. This system allows measurement performed at very high frequency. It is important that high frequency is employed so that turbulence characteristics could be
gathered [16]. The hotwire calibration was performed in-situ. Some information on data acquisition are described in Table 1.

**Table 1. Experiment equipment**

| Equipment                        | Specification                                      |
|----------------------------------|----------------------------------------------------|
| Acquisition system               | National Instrument’s model NI 9215                |
| Hotwire anemometry Sensor        | Multi-Channel CTA 54N80, 8-channel                 |
|                                  | Handmade, Wollaston Pt core probes,                |
|                                  | Dantec 55P05, 5 µm, 1.5 mm, exposed section = 1.06 mm |
|                                  | Overheat ratio 1 – 1.5                              |
| Temperature, atmospheric pressure and room humidity sensor | Comet, model H7331                                   |
| Pressure differential            | LSI-Lastem model ESP024                            |

2.3. **Wollaston wire probes**

Experimenting with hotwires require hotwire fabricating bench and other related equipment; otherwise waiting for a repaired sensor can be frustrating. For this, a hotwire fabricating bench was fabricated and it consists of the small traverse to control the sensor position, etching set up, cutter, welding equipment and a microscope. Our newly built three-dimensional (3D) traverse system comes with mounting for etching feature. The 3D traverse jigs are used to traverse and hold both hotwire support with probe manually in three directions (x, y, and z). It provides ability and control for the movement under the microscope and this is especially important during etching and soldering processes. A fine grit sandpaper is required before soldering processes. Soldering flux is applied at the portion between the Wollaston wire and the prong tip. The primary purpose of flux is to prevent oxidation [22]. The in-house etching facility allowed the experiment to be conducted and have full control of the exposed portion of the sensor. This was especially important to address the attenuations of signal due sensor length [26, 27]. Distilled water was used to clean the newly-built sensor. A multimeter was used to measure the resistance of the new sensor and finally, the sensor was balanced to obtain an acceptable response during operation. For the experiment, Wollaston wire produced by Sigmund Cohn Corp. with platinum core diameter of 5 micron (µm) was used.

2.4. **NACA 0026 airfoil**

A purpose-built NACA 0026 airfoil has been fabricated using mostly wooden structure, plywood surface and stainless steel support [20]. The chord, c is 600 mm and the span, s_p is 500 mm. The thickness, t is 156 mm. The chord line falls on the mean camber line, therefore this is a symmetrical airfoil. Wind tunnel test section bottom and top surface work as plates to minimize two-dimensionality issues. A 16 mm diameter stand made of stainless steel is attached to one side of the airfoil to ensure a rigid support. The riblet surface was applied on one side of the airfoil using glue and this is shown by the black strip in Figure 2. The length of the riblet surface is 50 mm. The riblets’ surfaces were applied here because of the needs of a sufficient length where the after-effect could develop in the boundary layer towards the rest of the airfoil (until the trailing edge).

An airfoil is chosen to run the effect of riblets because study on riblets has been documented mostly only on the ZPG turbulent boundary layer or flat surfaces. Vortex generators (VG) in the form of triangular shapes are most often used to delay flow separation. To accomplish this, VGs are often placed on the external surfaces of vehicles and wind turbine blades. They are usually installed quite close to the leading edge of the airfoil in order to maintain steady airflow over the control surfaces at the trailing edge. To replicate similar effects, riblet surfaces are applied near to the leading edge of the
airfoil. Furthermore, a symmetry airfoil such as NACA 0026 is chosen because two dimensionalities issues could be kept minimum. The length or chord, \( c = 60 \text{ cm} \) was chosen so that sufficient locations of measurement could be performed. In the earlier experiment [3], the thickness of boundary layer is approximately \( \delta \approx 5 \text{ cm} \) and therefore, it is possible to have approximately ten places of measurement points along the streamwise direction (chord, \( c / \text{boundary layer thickness}, \delta : 60 \text{ cm} / 5 \text{ cm} = 12 \), i.e. order of ten measurement points per streamwise direction).

Figure 2. NACA 0026 airfoil dimension and application of riblet

2.5. Converging – diverging riblets fabrication
The riblets were made from silicone rubbers. A mould fabricated from aluminium has been machined using a high-precision CNC machine [20]. Silicone rubber material and hardening materials were mixed to reduce hardening process. The mixing ratio was 450 g of silicon rubber to 5.5 ml hardener. The properly-mixed materials were transferred into a pressure chamber to remove air bubble [3] for one minute. For this experiment, a purpose-built pressure chamber connected to a vacuum pump was also constructed. The materials now, free of bubbles, were poured evenly onto the aluminum mould. This pouring process may produce some small bubbles on the surface. Applying a gentle brush on the surface was helpful to ensure the materials fill in triangular-shaped mould surface. It took four days for the material to be completely hardened and ready to be cut: these were called riblets. The riblets were cut by a cutter such that converging and diverging pattern with yaw angle \( 10^\circ \) could be formed. The thickness has been controlled so as to match with the rectangular cavity provided on the airfoil, hence riblet surface was flushed with the adjacent airfoil surface.

Prior to using silicone rubber, different riblet materials such as resin-epoxy mixtures have been tested. However, their surface quality was not as good as the silicone rubber. The resin-epoxy mixtures were more flexible but small particles could be found on the riblets’ surface. In summary, a riblet strip onto an airfoil was produced and applied, where the angle of flow is zero and \( 10^\circ \) (\( \alpha = 0^\circ \) and \( 10^\circ \)), relative to the air flow. The height of the riblet is \( h = 1 \text{ mm} \) and the spacing, \( s = 1 \text{ mm} \). Figure 3(a) shows \( s \) and \( h \), while Figure3(b) shows the riblets’ arrangement. Note that, in this figure, \( s \) appears longer than \( h \) due to the bending of the riblet surface to obtain a clear photograph. The general
arrangement of the experiments contains the NACA0026 airfoil, a strip of riblets with diverging, converging and zero alignment with the flow, a hotwire sensor and a Pitot tube attached to a fine-resolution traverse. This arrangement is shown in Figure 4.

![Figure 3](image_url)  
**Figure 3.** (a) Silicone rubber riblet dimension, (b) Converging–diverging riblet pattern arrangement

![Figure 4](image_url)  
**Figure 4.** General Arrangement of NACA 0026 airfoil in UKM-LST

3. Result and analysis

3.1. Hotwire experiment and experimental parameters

The locations of hotwire sensor, Pitot tube, two dimensional traverse system were properly arranged so that the flows were not obstructed. The distance of the rod of the traverse to the airfoil surface is at minimum 300 mm away. This way, sufficient areas were provided for boundary layer to develop without obstructions. Prior to performing the measurement, the hotwire was calibrated in-situ using the Pitot tube. The calibration was carried out by positioning the hotwire in the center of test section of the wind tunnel (free-stream). The temperature, along with the atmospheric pressure and room humidity, was recorded during the hotwire calibration. This is called the pre-calibration. Another calibration process at the end of the experiment was also performed, which is called post-calibration. The calibration curves were compared and, if necessary, temperature compensations might have to be implemented.

The parameters and key flow properties for the smooth and rough-wall case with a converging-diverging angle $\alpha = 10^\circ$ and zero angle. Table 2 contains skin friction velocity ($U_f$), friction Kármán number ($Re_f$) and the boundary layer thickness $\delta$. The experimental codes indicate converging, diverging $D$ and zero angle $Z$ for the flow exposed with rough/riblet surfaces, $S$ for smooth surface.
Table 2. Experimental parameters for smooth and rough surfaces, \( l' = lu/u = 35-40 \) for all cases

| Exp Code | Location (mm) | \( U_\infty \) (m/s) | \( h \) (m) | \( s \) (m) | \( \delta \) (m) | \( U_\tau \) (m/s) | \( Re_\tau \) | \( C_f = \frac{2(u/U_\infty)^2}{(u/U_\infty)^2} \) |
|----------|----------------|----------------------|--------------|-------------|----------------|-----------------|----------------|-----------------------------|
| C        | converging     | 270                  | 9.38         | 0.001       | 0.001          | 0.0120          | 0.36           | 280             | 0.00147                     |
| D        | diverging      | 270                  | 8.93         | 0.001       | 0.001          | 0.0120          | 0.36           | 280             | 0.00162                     |
| Z        | zero degree    | 270                  | 9.21         | 0.001       | 0.001          | 0.0089          | 0.41           | 240             | 0.00197                     |
| S        | smooth         | 200                  | 10.22        | 0.00073     | 0.52           | 0.0089          | 0.41           | 240             | 0.00258                     |

3.2. Mean velocity

The traverse was set with logarithmic spatial movement. Figure 5 illustrates the application of the Clauser method to obtain skin-friction velocity over the rough and smooth surfaces. The vertical axis represents velocity, the overbar indicates mean value. The velocity is scaled with the friction velocity \( U_\tau \) obtained from the Clauser method while the surface-normal position is scaled with inner variables \( U_\tau/\nu \) to produce \( z' = zU_\tau/\nu \). In this case, it was found that \( \kappa = 0.25 \) and \( B = -1.5 \). The value of \( \kappa \) seems to deviate from the normally accepted values \( 0.38 < \kappa < 0.41 \). The product of Kármán constant, \( \kappa \) and intercept, \( B \) here is \( \kappa B = 0.25 \times -1.5 = -0.375 \), appears in the lower end of the proposed empirical relationship of \( \kappa B \) vs. \( B \) [24]. This is however acceptable because all flows considered here are equivalent the adverse pressure gradient flow i.e. as the fluids move pass the 146 mm from the leading edge of the airfoil (refer Figure 2), the fluids experience adverse pressure conditions as the results of thickening flow area. The exposed sensor part for all measurements is \( l = 1.5 \) mm, results in non-dimensionalised sensor length \( l' = lu/u = 35-40 \) for all flows.

**Figure 5.** Mean velocity profile, the dashed-line indicates \( 1/\kappa \ln(z') + B \), here \( \kappa = 0.25, B = -1.5 \)

The skin friction is calculated by a generalized form \( C_f = (u/U_\infty)^2 \). The skin friction produced by \( C \) and \( D \) flows are approximately the same i.e. \( C_f \approx 0.00150 \). Skin frictions are however 30% and 70% higher for zero degree, \( Z \) and smooth surface, \( S \) flows, respectively. Higher drag (results of higher skin friction) attributed to converging surface and vice versa as reported earlier on flat plate wind tunnel [3] was not observed here. The shifting of mean velocity profiles in the logarithmic region for \( C \) and \( D \) was also not observed as reported before [3]. These results however are in agreement with other surface roughness studies, i.e. a riblet surface lowers the drag by 6.2% as compared with smooth surface [6] and simulation with shark skin surface produced drag reduction up to 27% as compared with smooth surface [23]. The latter hypothesized that "the existence of the riblets makes the viscous sub-layer thicker than that of the flat surface, and the shear stress and local friction coefficient on riblet surface
are reduced. Besides, drag reduction of riblet surface mainly concentrates on the bottom of riblets” [23].

3.3. Turbulence intensities
The Reynolds number is approximately the same for all flows. Therefore, the turbulence intensities shown in Figure 6 do not contain Reynolds number effects. In Figure 6(a), the turbulence intensities, $\bar{u}^2$ are normalised with $U_\tau$ and the wall normal distance, $z^+$. In the near wall region, both flows exposed to converging $C$ and diverging $D$ riblets exhibit similar features i.e. maximum turbulence intensities in the near wall i.e. $\bar{u}^2/\bar{u}^2_{\text{max}} \approx 12$, this occurs at $z^+ = 20$. It is possible that the position at which the inner peak occurs higher than the normally accepted values of $z^+ = 12 – 15$ due influences of the large scale features in the outer region for thin boundary layer called imprint [27].

![Figure 6. Turbulence intensities (a) scaled with inner variables, vertical dotted line indicate $z^+ = 20$, (b) scaled with outer variables, the vertical dotted line indicates $z/\delta = 0.45$](image)

Figure 6 (b) shows turbulence intensities scaled with outer variables free-stream velocity, $U_\infty$ and boundary layer thickness, $\delta$. Nearer to the wall, in contrast to inner variable-scaled data, this scaling suggests that turbulence intensities decrease with pressure gradient [16, 25, 28] as the smooth wall profile was acquired at $x \approx 200 \text{ mm}$ from the leading edge and the $C$, $D$ and $Z$ flows were located at $x \approx 270 \text{ mm}$ from the leading edge (pressure gradient increases from $x \approx 200 \text{ mm}$ to $x \approx 270$, as the flow area increases).

3.4. Higher-order statistics
In this section, more turbulence quantities are analysed through observations in the third and fourth order moments. The third order moment shows the skewness (asymmetry) while the fourth order moment describes the kurtosis (flatness) of the velocity signal. Figure 7 shows the skewness of streamwise velocity fluctuations for the entire fluid flow layer for the smooth-wall, converging, diverging regions and riblet with zero angle. Skewness is defined as $S_k = \frac{u^3}{(u^2)^{3/2}}$. A positive value of $S_k$ implies that large positive values of $u$ are more frequent than large negative values.

To start with, sensor length and Reynolds number effects [29] are ruled out because the experiment has been conducted within the range of controlled $l^*$ and $Re_c$. The wall normal distance is normalised with their respective inner variable $z^*$. For all four cases, they exhibit a non-Gaussian distribution, particularly in the near wall and wake region. All four flow cases exhibit a similar pattern at the edge of the boundary layer, the skewness plot shows that they have sharp negative skew and then change suddenly to positive. The $C$ and $D$ flows share similar trend from near the wall until the outer region. The $Z$ flow however shows lower $S_k$ value, especially in the logarithmic region. In general, the plot reveals that in the logarithmic region, the fluctuating velocities are skewed positively indicating that
the velocity distribution is concentrated on the right of PDF distribution, meaning that majority of the velocity fluctuations are high. High $S_k$ values indicate higher degree of interactions between the small and large-scales [30]. It is proposed here that positive $S_k$ in the logarithmic region is due to increased interactions of the small-scales in the inner region with the large-scales in the outer region, especially for the $C$ and $D$ cases, which is similar to pressure gradient effects [17].

Figure 8 shows kurtosis factors of the streamwise velocity component for all four cases. Flatness is defined as $F = u^4(\bar{u}^2)^{-2}$. A rise in flatness is often attributed to a rise in intermittency. Similar with skewness, the plot shows a departure from the Gaussian characteristics in the near-wall and wake region. The kurtosis figure shows that they have very sharp peak at the edge of the boundary layer. The sharp change at this particular location is associated with the intermittency of the turbulent and non-turbulent interface.

4. Conclusion
The effect of converging–diverging riblet-type surface roughness (riblets arranged in a ‘herringbone’ pattern) is investigated experimentally on NACA 0026 airfoil. For this initial parametric investigation, three different parameters of the surface are analysed in detail: the converging–diverging riblet with yaw angle $\alpha = 10^\circ$, riblet $\alpha = 0^\circ$ and smooth surface. Hotwire anemometry, with single wire, is used for experiment. The mean velocity profile is fitted to the logarithmic law with a constant $\kappa = 0.25$ and an intercept of $B = -1.5$. In the inner region, the mean velocities appear jagged due to the effects of the rough surface. In the wake region, the mean velocity scaled with friction velocity, $U_t$ for flow exposed to converging riblet pattern is higher than that exposed to the diverging pattern. In the near wall region, the inner wall maximum value is 12 and it occurs at $z^+ = 20$. For turbulence intensities scaled with $U_\infty$ and $\delta$, the effect of surface roughness is visibly more pronounced in the outer region. The turbulence intensities produced by the flow past the converging and diverging riblet show clear outer hump at $z/\delta = 0.45$. It is clear that the large scale features dominate in the outer region for the converging and diverging cases in comparison with the zero degree case. It is also shown that the skin friction for flows past the converging and diverging pattern have 25 – 30% less than that of the flow past riblets aligned with the flow (\(\alpha = 0^\circ\)). The results are consistent with vortex generator studies, which have reported an increase in turbulent intensities in rough surface of the flow field and a decrease in turbulent intensities within the smooth region. The higher order turbulence statistics also show marked different between these flows. These results are very encouraging in the sense that a small strip of vortex generator applied at a leading edge of an engineering application can have up to 20 – 25% reduction in skin friction, hence reduction in total drag, and more importantly reduction in energy or fuel.
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