Axial-vector transitions and strong decays of the baryon antidecuplet in the self-consistent SU(3) chiral quark-soliton model

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Abstract

We investigate the axial-vector transition constants of the baryon antidecuplet to the octet and decuplet within the framework of the self-consistent SU(3) chiral quark-soliton model. Taking into account rotational $1/N_c$ and linear $m_s$ corrections and using the symmetry-conserving quantization, we calculate the axial-vector transition constants. It is found that the leading-order contributions are generally almost canceled by the rotational $1/N_c$ corrections. Thus, the $m_s$ corrections turn out to be essential contributions to the axial-vector constants. The decay width of the $\Theta^+ \rightarrow NK$ transition is determined to be $\Gamma(\Theta \rightarrow NK) = 0.71$ MeV, based on the result of the axial-vector transition constant $g^*_A(\Theta \rightarrow NK) = 0.05$. In addition, other strong decays of the baryon antidecuplet are investigated.

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I. INTRODUCTION

Since the LEPS collaboration announced the evidence of the $\Theta^+$ existence [1], which was motivated by Ref. [2] where its decay width was predicted to be very small with its mass 1540 MeV [3] as well, there has been a great deal of experimental and theoretical work on the $\Theta^+$ (see, for example, reviews [4, 5] for the experimental and theoretical status before 2006). However, a series of very recent experiments conducted by the CLAS collaboration reported null results of finding the $\Theta^+$ [6, 7, 8, 9] in various reactions. These CLAS experiments were dedicated ones with high statistics. The null results of the CLAS experiment imply that the total cross sections for photoproductions of the $\Theta^+$ are tiny. In fact, the 95% CL upper limits on the total production cross sections for the $\Theta^+$ at 1540 MeV lie mostly in the range of $0.3 - 0.8$ nb [6, 7, 9]. In Ref. [8] the 95% CL upper limit on the $\gamma d \rightarrow \Lambda \Theta^+$ total cross section is set to be 5 nb in the mass range between 1.52 and 1.56 GeV/$c^2$. The KEK-PS E522 collaboration [10] has performed the experiment searching for the $\Theta^+$ in the $\pi^- p \rightarrow K^- X$ reaction and found a bump at 1530 MeV/$c^2$ but with only $(2.5 - 2.7)\sigma$ statistical significance. The upper limit of $\Theta^+$ production cross section in the $\pi^- p \rightarrow K^- \Theta^+$ reaction was extracted to be 3.9 $\mu$b. A later sequential experiment at KEK, however, has observed no clear peak structure in the $K^+ p \rightarrow \pi^+ X$ reaction [11], giving a 95% CL upper limit of 3.5 $\mu$b/sr on the differential cross section averaged over $2^\circ$ to $22^\circ$.

In the meanwhile, the DIANA collaboration has continued to search for the $\Theta^+$ in the $K^+ n \rightarrow K^0 p$ reaction and has found a direct formation of a narrow $pK^0$ peak with mass of $1537 \pm 2$ MeV/$c^2$ and width of $\Gamma = 0.36 \pm 0.11$ MeV [12]. Note that the former measurement by the DIANA collaboration for the $\Theta^+$ has yielded the mass of the $\Theta^+$ $1539 \pm 2$ MeV/$c^2$ with the decay width $\Gamma \leq 9$ MeV [13]. The SVD experiment has also announced a narrow peak with the mass $M = 1523 \pm 2$ (stat.) $\pm 3$ (syst.) MeV/$c^2$ in the inclusive reaction $pA \rightarrow pK^0_s + X$ [14, 15]. The LEPS collaboration also brings about positive new results indicating the existence of the $\Theta^+$ [16, 17].

Theoretically, it is of great importance to understand why the $\Theta^+$ is rather elusive. Actually, the cross sections of the $\Theta^+$ photoproduction as well as of the mesonic production are known to be very small. One reason for this can be attributed to the fact that the $K^*N\Theta^+$ coupling constant should be tiny, as was pointed out in Ref. [11]. In fact, Ref. [18] has shown that the tensor coupling constant for the $K^*N\Theta^+$ vertex is indeed very small. In fact, Ref. [18] has predicted even before the CLAS null results that the cross section for the $\Theta^+$ photoproduction is below the upper limits given by the above-mentioned CLAS experiments [6, 7, 9]. Azimov et al. [19] has derived the even smaller value of the $K^*N\Theta^+$ tensor coupling constant, employing the vector meson dominance with SU(3) symmetry. Note that the vector coupling constant for the $K^*N\Theta^+$ vertex vanishes in SU(3) symmetry due to the generalized Ademollo-Gatto theorem [20]. In Ref. [20], the present authors have also shown that the vector and tensor coupling constants for the $K^*N\Theta^+$ vertex are very small within the framework of the chiral quark-soliton model (χQSM), taking into account SU(3) symmetry breaking effects. Note that exactly the same self-consistent formalism and numerical methods are used in the present work. With this formalism used, we can describe pentaquark observables on the same ground as those for the octet baryons without doing any additional conceptual changes and using the same parameter set.

In addition to the $\Theta^+$, a recent GRAAL experiment [21, 22, 23] has announced a new finding of the nucleon-like resonance around 1.67 GeV in the neutron channel, measuring the cross section of $\eta$ photoproduction off the deuteron. The corresponding width was found
to be around 40 MeV, which probably results from a small width being enlarged by Fermi-
motion. It was also shown in Refs. [21, 22, 23] that the resonant structure was not seen in the
quasi-free proton channel. This new resonance is consistent with the theoretical predictions
by Ref. [24, 25] of a new exotic nucleon like state in that mass region. In fact, the narrow
width and its dependence on the initial isospin state are the typical characteristics for the
photo-excitation of the non-strange anti-decuplet pentaquark [26, 27]. Very recently, a new
analysis of the free proton GRAAL data [28, 29] has been carried out, the beam asymmetry
being emphasized, and has revealed a resonance structure with a mass around 1685 MeV
with a width $\Gamma \leq 15$ MeV. Note, however, that the results of Ref. [28] do not agree with those
of Ref. [30]. For a detailed discussion of this discrepancy, we refer to Ref [29]. Furthermore,
the LNS-GeV-$\gamma$ collaboration [31, 32] reports a new resonance at 1670 MeV with a width
$\Gamma \leq 50$ MeV in the $\gamma d \rightarrow \eta pn$ reaction. It is consistent with the above-mentioned observation
that this resonance is enhanced in the $\gamma n \rightarrow \eta n$ reaction, while it was not observed in the
quasi-free proton channel as in the case of Refs. [21, 22, 23]. Moreover, the CB-ELSA
collaboration [33] has reported an evidence for this nucleon-like resonance compatible with
those of GRAAL and LNS-GeV-$\gamma$, which are studied theoretically in Ref [34]. All these
experimental facts are consistent with the results for the transition magnetic moments in the
$\chi QSM$ [26, 27] as well as with the phenomenological analysis for the non-strange pentaquark
baryons [35]. Furthermore, recent theoretical calculations of the $\gamma N \rightarrow \eta N$ reaction [36, 37]
describe qualitatively well the GRAAL data, based on the values of the magnetic transition
moments in Refs. [27, 35]. References [28, 41] have also investigated the anti-decuplet
focusing on the non-strange partners of the $\Theta^+$, results of which are comparable with those
in this work. Due to all these experimental and phenomenological results, we assume in the
present work the anti-decuplet pentaquarks to exist.

Since the first prediction [2] for the small width of the $\Theta^+$, several calculations have been
elaborated within the same framework, i.e. in the $\chi QSM$ in order to understand the narrow
decay width of the pentaquarks. A formulation of the $\chi QSM$ in the light-cone framework
various quark components in Fock space have been decomposed [38]. In the chiral limit
a decay width of around 2 MeV was derived [39]. A “model-independent approach” as
in Ref. [2] was extended to the axial-vector channel, based on the experimental data of
SU(3) baryon semileptonic decay constants and on the singlet axial-vector constant, and
has produced the decay width of the $\Theta^+$ even smaller than 1 MeV [40]. However, while
the “model-independent” approach is plausible in describing the smallness of the $\Theta^+$ decay
width, one has to understand the origin of its small width.

In the present work, we want to investigate the axial-vector transition constants and the
widths of the baryon antidecuplet within the framework of the $\chi QSM$ with the symmetry-
conserving quantization [42] employed, considering the rotational $1/N_c$ corrections and effects
of SU(3) symmetry breaking. In contrast to the “model independent” approach of the $\chi QSM$ in Ref. [40], the present calculations are based on a self-consistent calculation of the
solitonic profile function without any refitting of $\chi QSM$ parameters. The $\chi QSM$ has
been proved very successful not only in predicting the $\Theta^+$ but also even more in describing
various properties of SU(3) baryon octet and decuplet such as the mass splittings, form
factors and parton- and antiparton-distributions [43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53]
and fulfilling all relevant sum rules of these observables. In particular, the dependence of
almost all form factors on the momentum transfer is well reproduced within the $\chi QSM$. As
a result, the strange electromagnetic form factors [54] and the parity-violating asymmetries
of polarized electron-proton scattering, which require nine different form factors (six elec-
tromagnetic form factors $G_{E,M}^{(u,d,s)}(Q^2)$ and three axial-vector form factors $G_{A}^{(u,d,s)}(Q^2)$, are in good agreement with experimental data with one set of fixed parameters \cite{55}. Thus, in the present work, we extend the self-consistent $\chi$QSM to study the axial-vector properties of the $\Theta^+$ and of the other members of the baryon anti-decuplet. In this calculation, the resulting decay width of the $\Theta^+$ will be shown to be 0.71 MeV.

The structure of this work is as follows. In Section II, we present briefly the general formalism for the decay widths of the SU(3) baryons. In Section III, we review the $\chi$QSM and show how to calculate the axial-vector transition constants. We also explain how to derive the widths of the baryon antidecuplet within the $\chi$QSM. In Section IV, we discuss the numerical results. In the final Section, we summarize the present work and draw conclusions. Useful formulae are given in the Appendix.

II. STRONG DECAYS OF THE SU(3) BARYONS

In the present Section, we briefly review the decay widths of the SU(3) baryons, based on the effective Lagrangian approach. Let us first consider the following decay modes:

$$
\Delta \to N + \pi, \quad \Theta^+ \to N + K, \quad N^* \to N + \pi(\eta), \quad N^* \to N + \pi
$$

where $\Delta$ denotes the $\Delta$ isobar, $N$ stands for the octet nucleon, $\pi$, $K$, and $\eta$ are the pion, kaon, and $\eta$ meson, respectively, and $N^*$ designates the anti-decuplet nucleon. In order to describe the above-given decays, we employ the following effective Lagrangian:

$$
\mathcal{L}_{\Theta NK} = i g_{\Theta NK} \bar{\Theta} \gamma^5 K N + h.c., \quad \mathcal{L}_{\Delta N\pi} = \frac{g_{\Delta N\pi}}{M_\Delta + M_N} \bar{\Delta} \gamma^\mu \pi^\mu N + h.c.
$$

where $g_{BBM}$ are the strong coupling constants for the baryon-baryon-meson vertices. In the rest frame of a decaying particle we have the formula for the strong decay as follows:

$$
\Gamma(B_1 \to B_2 + M) = \frac{4\pi}{32\pi^2} \frac{|k_m|}{2s_1 + 1} \frac{1}{M_i^2} \sum_{s_1, s_2} |\mathcal{M}_{BBM}|^2,
$$

where $M_1$ is the mass of the decaying particle, and $|k_m|$ the three momentum of the meson. The $s_1$ and $s_2$ denote the third components of the spin for the initial and final baryons, respectively. The invariant matrix elements $\mathcal{M}_{BBM}$ are therefore written as:

$$
\mathcal{M}_{\Theta NK} = i g_{\Theta NK} \bar{u}_N(p_N, s_N) \gamma^5 u_{\Theta}(p_\Theta, s_\Theta), \quad \mathcal{M}_{\Delta N\pi} = \frac{g_{\Delta N\pi}}{M_\Delta + M_N} \bar{\Delta} \gamma^\mu \pi^\mu N,
$$

$$
\mathcal{M}_{N^* N\pi} = i g_{N^* N\pi} \bar{u}_{N^*}(p_{N^*}, s_{N^*}) \gamma^5 \pi^\mu N, \quad \mathcal{M}_{N^* \Delta\pi} = \frac{g_{N^* \Delta\pi}}{M_{N^*} + M_\Delta} \bar{\Delta} \gamma^\mu \pi^\mu N^* .
$$

(4)
Using the results in Appendix A, we obtain the decay widths for the processes $B_1 \rightarrow B_2 + M$ as follows:

$$
\Gamma(B_{10} \rightarrow B_8 + M) = \frac{g_{B_{10}B_8M}^2}{M_{10}^2} \frac{|k_M|}{8\pi} \left( (M_{10}^2 - M_8^2 - m_M^2) \right),
$$

(5)

$$
\Gamma(B_{10} \rightarrow B_8 + \pi^0) = \frac{g_{B_{10}B_8\pi}^2}{(M_{10} + M_8)^2M_{10}^2} \frac{|k_\pi|}{24\pi} \left( (M_8 + M_{10}^2 - m_\pi^2) \right),
$$

(6)

$$
\Gamma(N^* \rightarrow \Delta + \pi^0) = \frac{g_{N^*\Delta\pi}^2}{(M_{N^*} + M_\Delta)^2M_\Delta^2} \frac{|k_\pi|}{12\pi} \left( (M_{N^*} + M_\Delta^2 - m_\pi^2) \right),
$$

(7)

with the meson momentum

$$
k_M^2 = \frac{(M_8^2 - M_3^2 + m_M^2)^2 - 4M_1^2m_M^2}{4M_1^2}.
$$

(8)

Using the generalized Goldberger-Treiman relations (see Eq. (10) given below), we can relate the axial-vector transition constants to the strong coupling constants. In the case of the $B_{10} \rightarrow B_8$ transitions, the axial-vector form factors are generally defined as the following matrix elements for the axial-vector current:

$$
\langle B_8(p', s')|A^{\mu\alpha}(0)|B_{10}(p, s)\rangle = \langle B_8(p', s')|\psi(0)\gamma^\mu\gamma^5\lambda^a\gamma^0\psi(0)|B_{10}(p, s)\rangle
$$

$$
= \bar{u}_8(p', s')[G_A(Q^2)\gamma^\alpha + G_F(Q^2)q^\alpha + G_T(Q^2)P^\alpha]\gamma^5\frac{\lambda^a}{2}u_{10}(p, s),
$$

(9)

where $Q^2 = -q^2 = -(p' - p)^2$ and $P = p' + p$. The $\lambda^a$ denotes the SU(3) flavor Gell-Mann matrices, satisfying $\{\lambda^a, \lambda^b\} = 2\delta^{ab}$. The axial-vector transition constant is defined as the value of $G_A(Q^2)$ at $Q^2 = 0$, i.e. $g_A^2 = G_A(0)$. The generalized Goldberger-Treiman relation connects the axial-vector transition constant to the corresponding strong coupling constant as follows:

$$
g_{B_1B_2M} = \frac{g_A^2(M_1 + M_2)}{2f_M},
$$

(10)

where $f_M$ stands for the corresponding meson decay constant.

As for the transitions from the baryon decuplet to the octet, we need to deal with the Lorentz structure of spin-3/2 particles, so that we have more form factors, i.e. Adler form factors \[57, 58, 59, 60\]. The axial-vector transition for the $\Delta^+ \rightarrow p + \pi^0$ process is then expressed in terms of four independent form factors:

$$
\langle \Delta^+(p', s')|A^{\mu\lambda}(0)|p(p, s)\rangle = \langle \Delta^+(p', s')|\psi(0)\gamma^\mu\gamma^5\frac{\lambda^3}{2}\psi(0)|p(p, s)\rangle
$$

$$
= \bar{u}_\Delta(p', s')\left[C_3^A(Q^2)g^{\mu\nu} + C_6^A(Q^2)q^\mu q^\nu + \left\{C_3^A(Q^2)\gamma_\lambda + C_4^A(Q^2)p^\lambda\right\}(q^\lambda g^{\mu\nu} - q^\nu g^{\mu\lambda})\right]u_p(p, s),
$$

(11)

where $u_{\Delta^+}$ and $u_p$ denote the Rarita-Schwinger and Dirac spinors for the $\Delta$ and proton, respectively. In this case, the axial-vector transition constant is defined as $C_3^A = C_5^A(0)$. In order to derive the Goldberger-Treiman relation for the spin-3/2 baryon, we first determine the divergence of the axial-vector current \[56\], which should vanish in the chiral limit:

$$
\bar{v}_\nu(p', s')q^\nu\left[C_5^A(Q^2) + C_6^A(Q^2)q^2\right]u_p(p, s) = 0.
$$

(12)
Thus, we obtain the following relation:

\[ C_5^A(q^2) + C_6^A(q^2)q^2 = 0. \]  

(13)

Since we have \( C_5^A(0) \neq 0 \), the term \( C_6^A(q^2) \) must have a pole at \( q^2 = 0 \). We can identify the pole term in Eq. (12) in the following way:

\[
\lim_{q^2 \to 0} \left( C_5^A(q^2) + C_6^A(q^2)q^2 \right) = \lim_{q^2 \to 0} \left[ C_5^A(q^2) - \frac{g_{\Delta N\pi}}{(M_\Delta + M_N)q^2} f(q^2) \right] = 0. 
\]  

(15)

Therefore, we get the Goldberger-Treiman relation for spin-3/2 baryon \([57, 58, 61]\) as follows:

\[ C_5^A(0) = f_\pi \frac{g_{\Delta N\pi}}{M_\Delta + M_N}. \]  

(16)

III. THE CHIRAL QUARK-SOLITON MODEL

A. General Formalism

The SU(3) \( \chi \)QSM is characterized by the following partition function in Euclidean space:

\[
Z_{\chi QSM} = \int D\psi D\psi^\dagger D\pi \exp \left[ -\int d^4x \bar{\psi} D(\pi) \psi \right] = \int D\pi \exp (-S_{\text{eff}}[\pi]),
\]  

(17)

where \( \psi \) and \( \pi \) denote the quark and pseudo-Goldstone boson fields, respectively. The \( S_{\text{eff}} \) stands for the effective chiral action expressed as

\[ S_{\text{eff}} = -N_c \text{Tr} \ln D(\pi), \]  

(18)

where Tr represents the functional trace, \( N_c \) the number of colors, and \( D \) the Dirac differential operator in Euclidean space:

\[ D(U) = \gamma_4 (i\not{\partial} - \hat{m} - MU^\gamma_5) = \partial_4 + h(U) + \delta m. \]  

(19)

Here, the \( \hat{m} \) denotes the current quark matrix \( \hat{m} = \text{diag}(\overline{m}, \overline{m}, m_s) \), isospin symmetry being assumed. The \( \partial_4 \) designates the derivative with respect to the Euclidean time and \( h(U) \) stands for the Dirac single-quark Hamiltonian:

\[ h(U) = -i\gamma_4 \gamma_i \partial_i + \gamma_4 MU^\gamma_5 + \gamma_4 \overline{m}. \]  

(20)

The \( \delta m \) is the the matrix of the decomposed current quark masses:

\[ \delta m = M_1 \gamma_4 1 + M_8 \gamma_4 \lambda^8, \]  

(21)

where \( M_1 \) and \( M_8 \) are singlet and octet components of the current quark masses defined as \( M_1 = (-\overline{m} + m_s)/3 \) and \( M_8 = (\overline{m} - m_s)/\sqrt{3} \). The \( \overline{m} \) is the average of up- and down-quark masses. The chiral field \( U^\gamma_5 \) is written as

\[ U^{\gamma_5} = \exp(i\gamma_5 \lambda^a \pi^a) = \frac{1 + \gamma_5}{2} U + \frac{1 - \gamma_5}{2} U^\dagger \]  

(22)
with \( U = \exp(i\lambda^a \pi^a) \). We assume here Witten’s embedding of SU(2) into SU(3):

\[
U_{SU(3)} = \begin{pmatrix}
U_{SU(2)} & 0 \\
0 & 1
\end{pmatrix}
\]

(23)

with the SU(2) hedgehog chiral field

\[
U_{SU2} = \exp[i\gamma_5 \hat{n} \cdot \tau P(r)].
\]

(24)

In order to solve the partition function in Eq.(17), we have to take the large \( N_c \) limit and solve it in the saddle-point approximation, which corresponds at the classical level to finding the profile function \( P(r) \) in Eq.(24). In fact, the profile function can be obtained by solving numerically the functional equation coming from \( \delta S_{eff} / \delta P(r) = 0 \), which yields a classical soliton field \( U_c \) constructed from a set of single quark energies \( E_n \) and corresponding states |\( n \rangle \) related to the eigenvalue equation \( h(U)|n\rangle = E_n |n\rangle \).

Since the classical soliton does not have the quantum number of the baryon states, we need to restore them by the semiclassical quantization of the rotational and translational zero modes. Note that the zero modes can be treated exactly within the functional integral formalism by introducing collective coordinates. The detailed formalism can be found in Refs. [43, 62]. Considering the rigid rotations and translations of the classical soliton \( U_c \), we can express the soliton field as

\[
U(x, t) = A(t)U_c(x - z(t))A^\dagger(t),
\]

(25)

where \( A(t) \) denotes a unitary time-dependent SU(3) collective orientation matrix and \( z(t) \) stands for the time-dependent displacement of the center of mass of the soliton in coordinate space.

Having introduced the zero modes as mentioned above, the Dirac operator in Eq.(19) is changed to the following form:

\[
D(U) = T_z(t)A(t) \left[ D(U_c) + i\Omega(t) - \hat{T}_z(t) T_z(t) - i\gamma^4 A^\dagger(t) \delta m A(t) \right] T_z^\dagger(t) A^\dagger(t),
\]

(26)

where the \( T_z(t) \) denotes the translational unitary operator and the \( \Omega(t) \) represents the angular velocity of the soliton that is defined as

\[
\Omega = -iA^\dagger \dot{A} = -i/2 \text{Tr}(A^\dagger \dot{A} \lambda^\alpha)\lambda^\alpha = 1/2 \Omega_\alpha \lambda^\alpha.
\]

(27)

Assuming that the soliton rotates and moves slowly, we can treat the \( \Omega(t) \) and \( \dot{T}_z(t) T_z(t) \) perturbatively. Moreover, since the flavor SU(3) symmetry is broken weakly, we can also deal with \( \delta m \) perturbatively.

Having quantized collectively, we obtain the following collective Hamiltonian

\[
H_{\text{coll}} = H_{\text{sym}} + H_{\text{sb}},
\]

(28)

where \( H_{\text{sym}} \) and \( H_{\text{sb}} \) represent the SU(3) symmetric and symmetry-breaking parts, respectively:

\[
H_{\text{sym}} = M_c + \frac{1}{2I_1} \sum_{i=1}^3 J_i J_i + \frac{1}{2I_2} \sum_{a=4}^7 J_a J_a + \frac{1}{m} M_1 \Sigma_{SU(2)},
\]

\[
H_{\text{sb}} = \frac{1}{m} \sum_{n=1}^8 \sum_{\lambda} (\tau_\lambda |n\rangle \langle n| \lambda).
\]
\[ H_{sb} = \alpha D_{88}^{(8)}(A) + \beta Y + \frac{\gamma}{\sqrt{3}} D_{88}^{(8)}(A)J_i. \]  

(29)

The \( M_s \) denotes the mass of the classical soliton and \( I_i \) and \( K_i \) are the momenta of inertia of the soliton \( \Sigma_{SU(2)} \), of which the corresponding expressions can be found in Ref. \[63\] explicitly. The components \( J_i \) denote the spin generators and \( J_a \) correspond to those of right rotations in flavor SU(3) space. The \( \Sigma_{SU(2)} \) is the SU(2) pion-nucleon sigma term. The \( D_{88}^{(8)}(A) \) and \( D_{88}^{(8)}(A) \) stand for the SU(3) Wigner \( D \) functions in the octet representation. The \( Y \) is the hypercharge operator. The parameters \( \alpha, \beta, \) and \( \gamma \) in the symmetry-breaking Hamiltonian are expressed, respectively, as follows:

\[ \alpha = \frac{1}{m} \frac{1}{\sqrt{3}} M_8 \Sigma_{SU(2)} - \frac{N_c}{\sqrt{3}} M_8 \frac{K_2}{I_2}, \quad \beta = M_8 \frac{K_2}{I_2} \sqrt{3}, \quad \gamma = -2 \sqrt{3} M_8 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right). \]  

(30)

The collective wave-functions of the Hamiltonian in Eq. (28) can be found as SU(3) Wigner \( D \) functions in representation \( \mathcal{R} \):

\[ \langle A|\mathcal{R}, B(YI_3, Y'J_3) \rangle = \Psi^{(\mathcal{R}^\ast, YI_3)}(A) = \sqrt{\text{dim}(\mathcal{R})} (-)^{I_3 + Y'/2} D_{\mathcal{R}^\ast}^{YI_3}(A). \]  

(31)

The \( Y' \) is related to the eighth component of the angular velocity \( \Omega \) that is due to the presence of the discrete valence quark level in the Dirac-sea spectrum. Its presence has no effect on the chiral field, so that it is constrained to be \( Y' = -N_c/3 = -1 \). In fact, this constraint allows us to have only the SU(3) representations with zero triality.

The effects of flavor SU(3) symmetry breaking having been taken into account, the collective baryon states are not in a pure representation but start to get mixed with other representations. This can be treated by considering the first-order perturbation for the collective Hamiltonian:

\[ |B_{\mathcal{R}}\rangle = |B_{\mathcal{R}}^{\text{sym}}\rangle - \sum_{\mathcal{R}' \neq \mathcal{R}} |B_{\mathcal{R}'}\rangle \frac{\langle B_{\mathcal{R}'}| H_{sb} |B_{\mathcal{R}}\rangle}{M(\mathcal{R}') - M(\mathcal{R})}. \]  

(32)

Solving Eq. (32), we obtain the collective wave functions for the baryon octet, decuplet, and anti-decuplet:

\[ |B_8\rangle = |8_1/2, B\rangle + c_{10}B^{10}_8|10_1/2, B\rangle + c_{27}B^{27}_8|27_1/2, B\rangle, \]  

(33)

\[ |B_{10}\rangle = |10_3/2, B\rangle + a_{27}B^{27}_8|27_3/2, B\rangle + a_{35}B^{35}_8|35_3/2, B\rangle, \]  

(34)

\[ |B_{35}\rangle = |35_1/2, B\rangle + d_{8}B^{8}_8|8_1/2, B\rangle + d_{27}B^{27}_8|27_1/2, B\rangle + d_{35}B^{35}_8|35_1/2, B\rangle, \]  

(35)

with the mixing coefficients

\[ c_{10}^B = c_{10} \left( \begin{array}{c} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{array} \right), \quad c_{27}^B = c_{27} \left( \begin{array}{c} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{array} \right), \]  

\[ a_{27}^B = a_{27} \left( \begin{array}{c} \sqrt{15/2} \\ 2 \sqrt{3/2} \\ 0 \end{array} \right), \quad a_{35}^B = a_{35} \left( \begin{array}{c} 5/\sqrt{14} \\ 2 \sqrt{5/7} \\ 3 \sqrt{5/14} \\ 2 \sqrt{5/7} \end{array} \right), \]  

\[ d_{8}^B = d_{8} \left( \begin{array}{c} 0 \\ \sqrt{5} \\ \sqrt{5} \\ 0 \end{array} \right), \quad d_{27}^B = d_{27} \left( \begin{array}{c} 0 \\ \sqrt{3/10} \\ 2/\sqrt{5} \\ \sqrt{3/2} \end{array} \right), \quad d_{35}^B = d_{35} \left( \begin{array}{c} 1/\sqrt{7} \\ 3/(2\sqrt{14}) \\ 1/\sqrt{7} \\ \sqrt{5/56} \end{array} \right), \]  

(36)
in the bases $[N, \Lambda, \Sigma, \Xi], [\Delta, \Sigma_{10}, \Xi_{10}, \Omega]$, and $[\Theta, N^*, \Sigma_{10}^*, \Xi_{10}^*]$, respectively. The coefficients $c_i$, $a_i$ and $d_i$ are expressed as

\[
\begin{align*}
    c_{27} &= -\frac{I_2}{15} \left( \alpha + \frac{1}{2} \gamma \right), \\
    a_{27} &= -\frac{I_2}{8} \left( \alpha + \frac{5}{6} \gamma \right), \\
    d_8 &= -\frac{I_2}{15} \left( \alpha + \frac{1}{2} \gamma \right), \\
    d_{27} &= -\frac{I_2}{8} \left( \alpha + \frac{7}{6} \gamma \right), \\
    d_{35} &= -\frac{I_2}{4} \left( \alpha + \frac{1}{6} \gamma \right).
\end{align*}
\]  

(37)

Now, we are in a position to evaluate the baryonic matrix elements given in Eqs. (9,11) within the framework of the $\chi$QSM. In general, the baryonic matrix element of the axial-vector current $A^\mu = i\bar{\psi} \gamma^\mu \lambda^a / 2\psi$ can be expressed as the following correlation function in the functional integral:

\[
\langle B'(p')|A^\mu_B(p)\rangle = \frac{1}{Z} \lim_{T \to -\infty} e^{-ip_4 T + ip_4 T} \int d^3 x' d^3 x e^{ip_{x'} - ip_{x}} \times \int D\psi D\psi' J_B^\dagger \left( \frac{T}{2}, x' \right) A^\mu_B(0) J_B^\dagger \left( -\frac{T}{2}, x \right) e^{-\int d^4 x' \bar{\psi} D(U) \psi}.
\]

(38)

with the baryonic current that consists of $N_c$ quarks and the baryon state:

\[
\begin{align*}
    J_B(x) &= \frac{1}{N_c!} \epsilon_{i_1 \cdots i_{N_c}} \Gamma_{\alpha_1 \cdots \alpha_2}^{\alpha_3} J_{\alpha_3} T_{\alpha_1} Y \psi_{\alpha_{i_1} i_{N_c}}(x), \\
    |B(p)\rangle &= \lim_{x_4 \to -\infty} \frac{1}{\sqrt{Z}} e^{ip_{x_4}} \int d^3 x e^{ip_{x}} J_B^\dagger(x) |0\rangle.
\end{align*}
\]

(39)

Here, $\alpha_1 \cdots \alpha_{N_c}$ denote spin-flavor indices, whereas $i_1 \cdots i_{N_c}$ represent color indices.

We can solve Eq. (38) in the saddle-point approximation justified in the large $N_c$ limit. In this approximation and with the help of the zero-mode quantization, the functional integral over the chiral field turns out to be the integral over the rotational zero modes. Since we will consider the rotational $1/N_c$ corrections and linear $m_s$ corrections, we expand the quark propagators in Eq. (38) with respect to $\Omega$ and $\delta m$ to the linear order and $\tilde{J}_z^{\dagger} \tilde{T}_{z(t)}$ to the zeroth order.

Having carried out a tedious but straightforward calculation (see Refs. [13, 62] for details), we finally can express the baryonic matrix element such as Eq. (9) as a Fourier transform in terms of the corresponding quark densities and collective wave-functions of the baryons:

\[
\langle B'(p')|A^\mu_B(0)|B(p)\rangle = \int dA \int d^3 z \ e^{iQ \cdot z} \Psi_B^* (A) F^a_\mu (z) \Psi_B (A),
\]

(40)

where $\Psi (A)$ denote the collective wave-functions and $F^a_\mu$ represents the quark densities corresponding to the current operator $A^\mu_B$.

Following the formalism presented above, we arrive at the final expressions for the axial-vector form factors:

\[
G_A(Q^2) = G_A^{(0^0, m_s^0)}(Q^2) + G_A^{(1^1, m_s^0)}(Q^2) + G_A^{(m_s^1), op}(Q^2) + G_A^{(m_s^1), wf}(Q^2)
\]

(41)

where the first term corresponds to the leading order $(0^0, m_s^0)$, the second one to the first $1/N_c$ rotational correction $(1^1, m_s^0)$ and the later to linear $m_s$ corrections coming from the operator and the wave-function corrections, respectively.
In the $\chi$QSM Hamiltonian of Eq.(20) the constituent quark mass $M$ is the only free parameter and $M = 420$ MeV is known to reproduce very well experimental data [49, 55, 62, 64]. Though the $M = 420$ MeV yields the best results for the baryon octet, we will present also those for $M = 400$ MeV and $M = 450$ MeV to see the $M$ dependence of the results in this work. Throughout this work the strange current quark mass is fixed to $m_s = 180$ MeV. In order to tame the divergent quark loops, we employ in this work the proper-time regularization. The cut-off parameter and $m$ are fixed for a given $M$ to the pion decay constant $f_\pi$ and $m_\pi$. The numerical results for the moments of inertia and mixing coefficients are summarized in Table I for $M = 420$ MeV. The results in Table I are obtained with the same parameters used in previous works [55, 65, 66]. We want to emphasize that all model parameters are the same as before. In previous works, the axial-vector form factors for the nucleon were already calculated. The axial-vector constants $g_A^3, g_A^8$ were found to be $g_A^3 = 1.176$ and $g_A^8 = 0.36$ which is in very good agreement with experimental data $g_A^3 = 1.267 \pm 0.0029$ [67] and $g_A^8 = 0.338 \pm 0.15$ [68].

B. Axial-Vector Form Factors in the $\chi$QSM

The axial-vector form factors for baryons are generally expressed in terms of the quark matrix elements given in Eq.(9). Since we are using an explicit self-consistent soliton profile derived from an action principle minimizing the nucleon energy, we calculate $G_A(Q^2)$ via the baryonic matrix element. In order to extract the form factor $G_A(Q^2)$ it is helpful to make the vector products to the spacial component of this current by the vector $q$ and to perform an average over the angular momentum transfer orientation, $\int d^3 \Omega q$. Taking the rest frame for the initial baryon $(p = (M_B, 0), p' = (E_{B'}, -q))$, we get

$$\frac{d^3 q}{4\pi} q \times \left( q \times \langle B'_s(p') | A \rangle | B_s(p) \rangle \right) = -\frac{2}{3} q^2 \sqrt{\frac{E_{B'} + M_{B'}}{2M_{B'}}} G_A(Q^2) \phi_s^\dagger \sigma \phi_s. \quad (42)$$

Choosing equal initial and final baryon spins and using Eq.(38) with Eq.(42), we derive from the third spacial component the $\chi$QSM expression for the axial-vector constant as follows [55]:

$$G_A^a(0) = \int d^3 z \langle B_{s\uparrow} | G_A^a(z) | B_s \uparrow \rangle. \quad (43)$$
The axial-vector density $G^a(z)$ for a certain flavor part $a$ is given by

$$G^a(z) = -\sqrt{\frac{1}{3}} D^{(8)}_{a3} A(z) + \frac{1}{3\sqrt{3}} I_1 D^{(8)}_{a8} J_3 B(z) - \sqrt{\frac{1}{3}} I_2 D^{(8)}_{a6} J_d p^{q3} C(z)$$

$$- \frac{1}{3\sqrt{2} I_1} D^{(8)}_{a3} D(z) - \frac{2 K_1}{3\sqrt{3} I_1} M_8 D^{(8)}_{s8} D^{(8)}_{a3} B(z)$$

$$+ \frac{2 K_2}{\sqrt{3} I_2} M_8 D^{(8)}_{s8} D^{(8)}_{a8} d^{pq3} C(z) - \frac{2}{\sqrt{3}} M_1 D^{(8)}_{a3} + \frac{1}{\sqrt{3}} M_8 D^{(8)}_{s8} D^{(8)}_{a3} ] H(z)$$

$$+ \frac{2}{3\sqrt{3}} M_8 D^{(8)}_{s8} D^{(8)}_{a8} \mathcal{I}(z) - \frac{2}{\sqrt{3}} M_8 D^{(8)}_{s8} D^{(8)}_{a8} d^{pq3} J(z),$$

where the densities $A(z), B(z), \cdots$ are given in Appendix B. In the case of the Adler form factors Eq. (11) we have

$$\int \frac{d\Omega_q}{4\pi} \left[ q \times (q \times \langle \Delta^+(p', s') | A^{a=\delta} | p(p, s) \rangle \right] z$$

$$= \int \frac{d\Omega_q}{4\pi} \left[ q \times (q \times \pi_{\nu} \delta^+(p', s') \left[ C_3^A(Q^2) g^{k\nu} + C_4^A(Q^2) p^{\nu} q^{k} g^{\nu} \right] u_p(p, s) \epsilon^k \right] z,$n

where the form factor $C_3^A$ is taken to be zero \cite{57, 58} and we can neglect $C_4^A$. This treatment is similar to that for the $N^*(1440) \to \Delta\pi$ decay in Ref. \cite{69}. The Rarita-Schwinger spinor $u^k(p', \frac{1}{2})$ with its third component +1/2 is expressed as follows \cite{71}:

$$u^k(p', \frac{1}{2}) = \frac{1}{\sqrt{3}} u(p', -\frac{1}{2}) \epsilon^k (1) + \frac{2}{3} u(p', +\frac{1}{2}) \epsilon^k (0)$$

$$\bar{u}^k(p', \frac{1}{2}) u(p, \frac{1}{2}) = \sqrt{\frac{2}{3}} \sqrt{\frac{E_N + M_N}{2M_N}} \epsilon^k,$n

and we arrive at the expression for $C_5^a$:

$$C_5^a(0) = \sqrt{\frac{3}{2}} \int d^3z \langle B_{10}' \uparrow | G^a(z) | B_8 \uparrow \rangle$$

where $G^a_{10}$ is the axial-vector density for the $B_{10}' \to B_8$ transition.

The baryonic matrix element of the $D$ functions can be expressed in terms of SU(3) Clebsch-Gordan coefficients \cite{56, 70}:

$$\langle B_{R'}' | D^{n}_{am}(A) | B_R \rangle = \sqrt{\frac{\dim R'}{\dim R}} (-1)^{\frac{1}{2} Y_{R'} + S_{R'}'} (-1)^{\frac{1}{2} Y_{R} + S_{R}}$$

$$\times \sum_{\gamma} \left( \frac{R'}{Q'} n \frac{R}{Q} \right) \left( \frac{R'}{m} n - \frac{R}{S_3} \right) \left( -Y_{S} S_{R'}' - S_{R} - S_{3} - S_{3} \right),$$

with $Q = Y H_3$. The relevant results are listed in Appendix C.
IV. RESULTS AND DISCUSSION

A. Mass Splittings

The $\chi QSM$ in the present form is not able to calculate absolute masses since rotational and translational quantum corrections are not calculated [73]. However, the mass splittings are accessible. The mass splittings between the baryon octet and the antidecuplet have been already studied in detail [2, 72]. Note that there have been also some discussions about the applicability of the collective quantization due to the rigid rotation of the chiral soliton [74, 75, 76, 77]. The symmetry-breaking part $H_{sb}$ of the Hamiltonian in Eq.(28) enables us to calculate baryon mass splittings for various representations as done in [43, 63]. The mass splittings between the baryon octet and the decuplet and anti-decuplet are given in terms of the soliton moments of inertia $I_1$ and $I_2$ [72]:

$$\Delta M_{10-8} = \frac{3}{2} \frac{1}{I_1}, \quad \Delta M_{10-\bar{8}} = \frac{3}{2} \frac{1}{I_2}. \quad (50)$$

The center of the baryon octet is just the average of the $\Lambda$ and $\Sigma$ masses, i.e. $M_8 = 1151.5$ MeV, whereas the center of the baryon decuplet is determined by the $\Sigma^*$, i.e. $M_{10} = 1385$ MeV [67]. In general, the baryon octet must satisfy the Gell-Mann-Okubo mass formula [78]:

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma. \quad (51)$$

In the $\chi QSM$, we obtain for the baryon decuplet and anti-decuplet the equal-spacing mass formulae as follows:

$$m_{\Sigma^*} - m_\Delta = m_\Xi - m_{\Sigma^*} = m_\Omega - m_{\Xi^*},$$

$$m_{\Sigma^*} - m_\Theta = m_{\Sigma^*} - m_{N^*} = m_{\Xi^*} - m_{\Sigma^*}. \quad (52)$$

Since, in the $\chi QSM$, all baryons emerge from one classical configuration, we also have the Guadagnini relation that connects baryon masses of the octet with those of the decuplet [79]:

$$8(m_\Xi + m_N) + 3m_\Sigma = 11m_\Lambda + 8m_{\Sigma^*}. \quad (53)$$

Using the numerical results listed in Table I and wave functions in Eq.(35), we obtain the mass differences within a multiplet:

$$\Delta M_{B_1B_2} = M_{B_1} - M_{B_2} = \langle B_1|H_{sb}|B_1\rangle - \langle B_2|H_{sb}|B_2\rangle. \quad (54)$$

In Table II the results for the mass splittings are listed, where the values with $M = 420$ MeV are

| $M$ [MeV] | 400 | 420 | 450 | exp. |
|-----------|-----|-----|-----|-----|
| $\Delta M_{10-8}$ | 257 | 279 | 308 | 234 |
| $\Delta M_{10-\bar{8}}$ | 558 | 617 | 673 | - |

TABLE II: Mass splittings between the baryon octet and the baryon decuplet and anti-decuplet with three different values of the constituent quark mass $M$. The preferred value is $M = 420$ MeV.
TABLE III: The baryon octet and decuplet mass splittings in the \( \chi \) QSM given in MeV for the constituent quark mass of \( M = 420 \text{MeV} \).

| \( \langle B|H_{sb}|B \rangle \) | \( \Lambda - N \) | \( \Sigma - N \) | \( \Sigma - \Delta \) | \( \Xi - \Sigma \) | \( \Sigma_{10}^* - \Delta \) | \( \Sigma_{10}^* - \Sigma_{10}^* \) | \( \Omega - \Xi_{10}^* \) |
|-----------------|-----------|-----------|------------|------------|-----------------|-----------------|-----------------|
| exp.            | 123       | 177       | 55         | 96         | 103             | 103             | 103             |

MeV are the relevant ones for all applications of the \( \chi \) QSM. The octet-decuplet splitting deviates from the experimental data by about 52 MeV.

In Table III the results of the mass splittings in the baryon octet and decuplet are listed. They are obtained by calculating the matrix elements of the symmetry-breaking part of the collective Hamiltonian in Eq.(28). Generally, also for other constituent quark masses, the results of the \( \chi \) QSM with the parameter given in Table II underestimate the mass splittings for the hyperons by up to 73 MeV. Note that the deviation to the experimental data in this work are larger than that in Ref. [43]. It is due to the facts that in the present work we do not consider the quadratic \( m_s \) corrections for the Hamiltonian and, moreover, different numerical settings such as the size of the box for solving the Dirac equation and regularization parameters end up with slightly different results such as the \( \Sigma_{\pi N} \) term: For example, in Ref. [63] it is obtained to be 56.14 MeV while in the present work we get \( \Sigma_{\pi N} = 41 \text{MeV} \). In particular, the \( \Sigma_{\pi N} \) term is rather sensitive to the scheme of the regularization because of its quadratic divergence.

Turning now to the anti-decuplet. We can calculate the masses and splittings of the anti-decuplet baryons by

\[
M_{B_{10}} = M_{B_{8}}^{\text{exp}} + \Delta M_{\chi \text{QSM}}^{10-8} + \Delta M_{\Delta \chi \text{QSM}}^{10}. \tag{55}
\]

The hypercharge splittings of the anti-decuplet are listed in Table IV. The experimental data are taken from the GRAAL experiment [21] for the \( N_{10}^* \) and from the NA49 experiment [80] for \( \Xi_{10}^* \), though the NA49 data is still controversial. With the set of parameters given in Table II we obtain the results for the baryon masses in a qualitative agreement with the data.

With the \( \chi \) QSM values given in Tab. III we calculate also the Gell-Mann-Okubo and Guadagnini relations. Even though the absolute masses of the \( \chi \) QSM are off by \((8 - 97)\) MeV both relations are well satisfied, Table VI.

### B. Axial-Vector Transition Constants

We first present the results for the nonstrangeness (\( \Delta S = 0 \)) \( B_{10} \rightarrow B_{8} \) transitions, using Eq.(48) with \( a = \frac{\lambda^2}{2} \). In Table VII we list the \( \Delta S = 0 \) axial-vector transition constants.
TABLE V: Masses of the baryon octet, of the decuplet, and of the anti-decuplet in unit of MeV. We started from the experimental octet center $M_8 = 1151.5$ and used the $\chi$QSM mass-splitting for $M = 420$MeV. Experimental values are written in parentheses and are taken from the PDG [67], from the GRAAL [21] and from the NA49 collaboration [80].

|                | Octet | Decuplet | Anti-decuplet |
|----------------|-------|----------|---------------|
| $\Theta^-$     | 1538  | 1329     | 1653          |
| $N/\Delta$     | 1001  | 1179     | 1329          |
| $\Lambda/\Sigma$ | 1124  | 1431     | 1768          |
| $\Xi$          | 1275  | 1533     | 1883          |
| $\Omega^-$     | 1635  |          |               |

TABLE VI: The Gell-Mann-Okubo and Guadagnini relations in the self-consistent $\chi$QSM given in unit of GeV and for the constituent quark mass $M = 420$MeV.

|                | $2N + 2\Xi$ | $3\Lambda + \Sigma$ | $8(m_{\Xi^0} + m_N) + 3m_\Sigma$ | $11m_\Lambda + 8m_{\Sigma^0}$ |
|----------------|-------------|----------------------|----------------------------------|--------------------------------|
| $\chi$QSM      | 4.55        | 4.54                 | 23.81                            | 23.81                          |
| experiment     | 4.51        | 4.54                 | 23.32                            | 23.36                          |

$C_A^*$ for the $B_{10} \rightarrow B_8$. Since the leading ($\Omega^0, m_0^0$)-order and the $1/N_c$ rotational corrections ($\Omega^1, m_0^0$) in Eq.(48) do always constructively interfere, the effects of the $m_s$-corrections turn out to be rather small, i.e. they contribute to the axial-vector transition constants by about 7%.

TABLE VII: Axial-vector transition constants for the $B_{10} \rightarrow B_8 + \pi^0$ processes with $\Delta S = 0$ and using $M = 420$ MeV. In the last line the final results are listed with the SU(3) symmetry breaking included.

|                | $\Delta^+ \rightarrow \pi p$ | $\Sigma_{10}^0 \rightarrow \Lambda$ | $\Sigma_{10}^+ \rightarrow \Sigma^+$ | $\Xi_{10}^0 \rightarrow \Xi^0$ |
|----------------|-------------------------------|---------------------------------------|-----------------------------------|----------------------------------|
| $m_0^0$        | -0.89                         | -0.77                                 | 0.45                              | 0.45                             |
| $m_s^0 + m_1^1$| -0.96                         | -0.82                                 | 0.45                              | 0.46                             |

We will present now the results of the anti-decuplet transitions results. The anti-decuplet axial-vector transition constants $g_A^*$ for $B_{10} \rightarrow B_8 + m$ are listed in Table VIII. In the case of pion- and eta-transitions, the corresponding operators in Eq.(13) are with $a = \lambda^3$ and $a = \lambda^8$, respectively, and for kaon transitions $a = \frac{1}{2}(\lambda^4 \pm i\lambda^5)$. We find that the leading-order contributions ($\Omega^0, m_0^0$) and the rotational corrections ($\Omega^1, m_0^0$) interfere always destructively, so that the axial-vector transition constants turn out to be rather small in the chiral limit. Due to this cancellation, the $m_s$ corrections become relevant. As for the $\Theta^+ \rightarrow n$ transitions, the $m_s$ corrections reduce even further $g_A^* (\Theta^+ \rightarrow n)$ by about 40% still being corrections in the
The mixing of the octet wave functions in the $p_{10}^+$ transitions are, compared to other transitions, large. The total $m_s$ corrections reduce $g_A^{* (p_{10}^+ \to n\pi^+)}$ but increase $g_A^{* (p_{10}^+ \to \Lambda K^+)}$ and $g_A^{* (p_{10}^+ \to \eta K^+)}$. However, for $g_A^{* (p_{10}^+ \to n\pi^+)}$, various parts cancel each other, sometimes almost completely.

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TABLE VIII: Axial-vector transition constants for the $B_{70} \to B_s$ using the self-consistent $\chi$QSM with $M = 420\text{MeV}$. Each column shows each contribution. The $m_s$ corrections are listed separately: The wave-function corrections from the mixing with the octet, those from the 27-plet mixing, and the operator corrections, respectively.

| $g_A^*$ | $\Omega^0, m_0^2$ | $\Omega^1, m_1^2$ | $m_s^*, \text{wf}(8)$ | $m_s^*, \text{wf}(27)$ | $m_s^*, \text{op}$ | total   |
|--------|------------------|------------------|------------------|------------------|------------------|---------|
| $\Theta^+ \to n\pi^+$ | 0.310 | -0.220 | -0.013 | -0.024 | 0.001 | 0.053 |
| $p_{10}^+ \to n\pi^+$ | 0.180 | -0.130 | -0.102 | 0.033 | 0.002 | -0.017 |
| $p_{10}^+ \to \Lambda K^+$ | -0.220 | 0.160 | -0.068 | 0.034 | -0.004 | -0.098 |
| $p_{10}^+ \to \Lambda K^+$ | -0.310 | 0.220 | -0.042 | 0.024 | 0.0003 | -0.107 |
| $p_{10}^+ \to \Delta^+ \pi^0$ | 0 | 0 | 0.074 | -0.154 | 0.005 | -0.075 |
| $\Xi^0 \to \Xi^0 \pi^+$ | -0.310 | 0.220 | 0 | -0.016 | -0.014 | -0.120 |
| $\Xi^- \to \Sigma^- K^+$ | -0.310 | 0.220 | -0.013 | -0.008 | -0.007 | -0.118 |

One should note that in SU(3) flavor symmetry, the transitions from the baryon antidecuplet to the decuplet are strictly forbidden, because the direct product of the baryon antidecuplet and the octet current does not contain the decuplet in its irreducible representation. However, if we turn on the $m_s$ corrections, the transitions from the anti-decuplet to the decuplet are allowed, since the anti-decuplet mixes with the octet and 27-plet according to Eq.(35). We will concentrate in the present work on the $N_{70} \to \Delta$ transition only. In the last line in Table VIII, each contribution to the axial-vector transition constant for the $p_{10}^+ \to \Delta^+$ process is listed. As shown in the last row of Table VIII, there is no contribution from the leading and rotational $1/N_c$ orders, but we get small contributions from the $m_s$ corrections. It is found that the wave-function correction due to the 27-plet mixing turns out to be large and has the opposite sign to the octet mixing one. The $m_s$ corrections from the operators are negligible.

C. Decay Widths

Now, we are in a position to calculate the decay widths for the transitions between different baryons, using Eq.(17). We first consider the nonstrangeness transitions from the baryon decuplet to the octet. In Table IX, we list the corresponding results with $M = 420$ MeV. We have assumed here isospin symmetry. Calculating relative isospin factors, we can evaluate the total decay width for each channel. Summing all possible transitions and averaging over the initial states, we obtain

$$\Gamma(\Delta \to N\pi) = \frac{3}{2} \Gamma(\Delta^+ \to p\pi^0), \quad \Gamma(\Sigma^* \to \Lambda\pi) = \Gamma(\Sigma^{*+} \to \Lambda\pi^0),$$

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TABLE IX: Decay widths of the $\Delta S = 0$ transitions from the baryon decuplet to the octet with $M = 420$ MeV.

| $\Delta^+ \to p\pi^0$ $\Sigma^{*+} \to \Lambda\pi^0$ $\Sigma^{++} \to \Sigma^{+}\pi^0$ $\Xi^* \to \Xi^0\pi^0$ | $\Gamma$ [MeV] |
|---|---|---|---|
| 48.7 | 30.3 | 2.2 | 3.9 |

$\Gamma(\Sigma^* \to \Sigma\pi) = 2\Gamma(\Sigma^{*+} \to \Sigma^+\pi^0)$, $\Gamma(\Xi^* \to \Xi^0\pi^0) = 2\Gamma(\Xi^{*0} \to \Xi^0\pi^0)$. \hspace{1cm} (57)

The total decay widths for $\Delta S = 0$ transitions are listed in Table X.

TABLE X: Total decay widths for the decuplet to octet transitions with $\Delta S = 0$ using self-consistent $\chi$QSM with $M = 420$ MeV. For the $\Xi^*$ transitions, we take the experimental data from Ref. [67]: $\Gamma(\Xi^- \to \Xi^0) + \Gamma(\Xi^- \to \Xi^0)$ and $\Gamma(\Xi^0 \to \Xi^-)$.

| $\Delta \to N\pi$ $\Sigma^* \to \Lambda\pi$ $\Sigma^* \to \Sigma\pi$ $\Xi_{10} \to \Xi\pi$ | $\Gamma^{\chi\text{QSM}}$ [MeV] |
|---|---|---|---|
| 73.1 | 30.3 | 4.4 | 7.9 |

| $\Gamma^{\text{PDG}}$ [MeV] | 111.5 | 36.1 | 16.2(1)(11.6(2)) |

Even though the values of the present work underestimate the decuplet widths they are comparable to the decuplet widths given in Ref. [2] by taking the presented formulae literally. A clarification of the given numbers in Ref. [2] can be found in Ref. [82].

It is also of great interest to compare the ratio of the decay widths for $\Sigma^* \to \Sigma$ and for $\Sigma^* \to \Lambda$. In the present work, we obtain the ratio as follows:

$$\frac{\Gamma(\Sigma^* \to \Sigma)}{\Gamma(\Sigma^* \to \Lambda)} = 0.145,$$ \hspace{1cm} (58)

which is in good agreement with the data from the particle data group [67]: $\Gamma(\Sigma^* \to \Sigma)/\Gamma(\Sigma^* \to \Lambda) = 0.135 \pm 0.011$.

We now consider the decays from the baryon anti-decuplet to the octet. Based on the results of the axial-vector transition constants listed in Table VIII, we can immediately calculate the decay widths for the $B_{10} \to B_8$ transitions. The results are presented in Table XI. Since the axial-vector transition constants for the $B_{10} \to B_8$ turn out to be rather small, we get consequently the small decay widths for the baryon anti-decuplet to the octet transitions.

Evaluating relative isospin factors, we get from these transitions the total decay widths. Summing all transitions and averaging over initial states, we obtain

$$\Gamma(\Theta \to NK) = 2\Gamma(\Theta \to nK^+), \hspace{1cm} \Gamma(N_{10} \to N\pi) = \frac{3}{2}\Gamma(p_{10} \to n\pi^+),$$

$$\Gamma(N_{10} \to \Lambda K) = \Gamma(p_{10} \to \Lambda K^+), \hspace{1cm} \Gamma(N_{10} \to N\eta) = \Gamma(p_{10} \to p\eta),$$

$$\Gamma(\Xi_{10} \to \Xi\pi) = \Gamma(\Xi^{*+}_{10} \to \Xi^0\pi^+), \hspace{1cm} \Gamma(\Xi_{10} \to \Sigma K) = \Gamma(\Xi^{*+}_{10} \to \Sigma^0K^-),$$ \hspace{1cm} (59)

and for the $N_{10} \to \Delta$ transitions

$$\Gamma(N \to \Delta\pi) = 3 \times \Gamma(p_{10} \to \Delta^+\pi^0).$$ \hspace{1cm} (60)
The total decay widths are presented in Table XI. In Ref. \[82\], the formulae for the decay width of the $\Delta \rightarrow N\pi$ and $\Theta \rightarrow NK$ transitions are given as

$$\Gamma(\Delta \rightarrow N\pi) = \frac{3(G_0 + \frac{1}{2}G_1)^2}{2\pi(M_\Delta + M_N)^2}|k_\pi|^3 \frac{M_N}{M_\Delta} \frac{1}{5},$$

$$\Gamma(\Theta \rightarrow NK) = \frac{3(G_0 - G_1)^2}{2\pi(M_\Theta + M_N)^2}|k_K|^3 \frac{M_N}{M_\Theta} \frac{1}{5},$$

and for the $g_{\pi NN}$ constant in the $\chi$QSM as

$$g_{\pi NN} = \frac{7}{10}(G_0 + \frac{1}{2}G_1),$$

where terms proportional to $G_2$ and $c_{\pi NN}$ were dropped. In Ref. \[2\], the coupling constant of $G_0 + \frac{1}{2}G_1 = 19$ is used, which follows from inverting Eq.(62) with the experimental value $g_{\pi NN} = 13.6$. In order to separate $G_0$ and $G_1$, Ref. \[2\] has used the parameter $G_1/G_0 = 0.4$. We will comment on this ratio later in detail. In Ref. \[81\], $\Gamma(\Delta N\pi)$ in Eq.(61) is inverted by using the experimental value $\Gamma(\Delta \rightarrow N\pi) = 110\text{MeV}$ in order to obtain $G_0 + \frac{1}{2}G_1 = 25$, which would give a large $g_{\pi NN} = 17.5$. Reference \[81\] claimed that the decay widths should be $\Gamma(\Delta \rightarrow N\pi) \approx 68\text{MeV}$ and $\Gamma(\Theta \rightarrow NK) \leq 30 \text{MeV}$ compared to the in Ref. \[2\] given values of $\Gamma(\Delta \rightarrow N\pi) \approx 110 \text{ MeV}$ and $\Gamma(\Theta \rightarrow NK) \leq 15 \text{ MeV}$. Reference \[82\] clarifies the situation. Furthermore, in Ref. \[82\], the authors emphasized that the value of $\Gamma(\Theta NK) \leq 15\text{MeV}$ is the most conservative prediction and that by changing the ratio $G_1/G_0$ from 0.4 – 0.6 the decay width varies between $(11.2 – 3.6)\text{MeV}$. 

**TABLE XI:** Partial decay widths for the $B_{\pi\pi} \rightarrow B_s$ transitions in the self-consistent $\chi$QSM using $M = 420 \text{ MeV}$. The $m_s$ corrections from the operators are added to yield the total results.

| $\Gamma$ [MeV] | $\Omega^0$ | $\Omega^0 + \Omega^1$ | $\Omega^0 + \Omega^1 + \text{wf}(8)$ | $\Omega^0 + \Omega^1 + \text{wf}(8+27)$ | total |
|----------------|-----------|-------------------|---------------------|---------------------|------|
| $\Theta^+ \rightarrow nK^+$ | 12.23 | 1.04 | 0.77 | 0.36 | 0.36 |
| $p_{\pi\pi} \rightarrow n\pi^+$ | 53.38 | 4.11 | 4.45 | 0.59 | 0.47 |
| $p_{\pi\pi} \rightarrow \Lambda K^+$ | 3.16 | 0.23 | 1.07 | 0.58 | 0.63 |
| $p_{\pi\pi} \rightarrow p\eta$ | 16.92 | 1.43 | 3.07 | 2.05 | 2.04 |
| $p_{\pi\pi} \rightarrow \Delta^+\pi^0$ | 0 | 0 | 4.52 | 5.28 | 4.64 |
| $\Xi^+_0 \rightarrow \Xi^0\pi^+$ | 80.33 | 6.77 | 6.77 | 9.39 | 12.03 |
| $\Xi^-_{10} \rightarrow \Sigma^- K^-$ | 31.32 | 2.64 | 3.46 | 4.02 | 4.54 |

**TABLE XII:** Final result for the total decay widths for the $B_{\pi\pi} \rightarrow B_s$ transitions in unit of MeV, as varying $M$ from 400 to 450 MeV in the self-consistent $\chi$QSM. The results for $M = 420 \text{ MeV}$ are our preferred values.

| $M$ [MeV] | $\Theta \rightarrow NK$ | $N_{10} \rightarrow N\pi$ | $N_{10} \rightarrow \Lambda K$ | $N_{10} \rightarrow N\eta$ | $N_{10} \rightarrow \Delta\pi$ | $\Xi_{10} \rightarrow \Xi_8\pi$ | $\Xi_{10} \rightarrow \Sigma_8 K$ |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 400       | 0.46          | 1.54          | 0.63          | 1.85          | 17.06         | 11.60         | 4.23          |
| 420       | 0.71          | 0.71          | 0.63          | 2.04          | 13.92         | 12.03         | 4.54          |
| 450       | 1.01          | 0.09          | 0.63          | 2.09          | 11.45         | 12.24         | 4.75          |
The ratio $G_1/G_0$ is the only input depending of a certain model and originally this ratio was taken from the $\chi$QSM calculations Refs. [33, 34]. The present work is based on the same formalism as used and developed in Refs. [33, 34], however, several parts have been optimized since then. The symmetry conserving quantization [42] was established after the publication of Ref. [2] and has been applied since then for all octet baryon observables within the present formalism. The ratio $G_1/G_0$ corresponds to the ratio $a_2/a_1$ of the present work, where $a_i$ are defined in Eq. (C1). In the present work this ratio is $a_2/a_1 = -0.68$, i.e. $G_1/G_0 = 0.68$.

The difference of these ratios lies in the fact that Ref. [2] has only considered the wave-function corrections due to the mixing with the octet, while in the present work we consider all possible mixings. If we keep only the mixing with the octet, then we obtain the decay width $\Gamma(\Theta NK)$ as given in this work as follows:

$$\Gamma(m_0^2 \Theta NK) = 2.08 \text{ MeV}, \quad \Gamma(m_0^2 + m_1) \Theta NK = 0.71 \text{ MeV}. \quad (63)$$

without and with $SU(3)$ symmetry breaking effects taken into account, respectively. The corresponding decay widths for the $\Theta \rightarrow NK$ transition are then evaluated as

$$\Gamma(m_0^2) (\Theta NK) = 2.08 \text{ MeV}, \quad \Gamma(m_0^2 + m_1) (\Theta NK) = 0.71 \text{ MeV}.$$ 

It is interesting to compare this final result of $\Gamma(m_0^2 + m_1)(\Theta NK) = 0.71 \text{ MeV}$ with the experimental data of the DIANA collaboration $\Gamma(\Theta NK) = 0.36 \pm 0.11 \text{ MeV}$ [12]. The $\Delta/\Theta$ decay ratios are given in this work as follows:

$$\left. \frac{\Gamma(\Delta N\pi)}{\Gamma(\Theta NK)} \right|_{\chi QSM} = \frac{73.1 \text{ MeV}}{0.71 \text{ MeV}} = 103 \quad , \quad \left. \frac{\Gamma(\Delta N\pi)}{\Gamma(\Theta NK)} \right|_{\exp.} = \frac{111.5 \text{ MeV}}{0.36 \text{ MeV}} = 310. \quad (64)$$

The results of the $\chi$QSM are therefore compatible with the smallness of the $\Theta$ decay width, compared to the $\Delta$ decay width. Considering the fact that in the present work we do not have any adjustable free parameter except for the constituent quark mass $M$ that is also
fixed to be 420 MeV \(^1\), it is remarkable for the present results to be in such agreement with the DIANA data.

Projecting the \(\chi\)QSM-soliton upon its 3− and 5−quark components in the light-cone basis, a value of \(\Gamma(\Theta NK) = 2.26\) MeV is yielded for the SU(3)-symmetric case (i.e. \(m_s = 0\)) \(^{39}\). Imposing the condition of the energy-momentum conservation in the same method as in Ref. \(^{39}\), Ref. \(^{85}\) has shown the decay width to be \(\Gamma(\Theta NK) \sim 0.4\) MeV. A ”model-independent approach” in the \(\chi\)QSM gives the decay width \(\Gamma(\Theta NK) = 2.26\) MeV with the singlet axial-vector constant \(g_A^0 = 0.36\) and \(\Sigma_{\pi N} = 45\) MeV \(^{40}\). Note that these values are quite similar to those computed in the \(\chi\)QSM with a self-consistent soliton profile \(^{49}\). Thus all calculations based on the \(\chi\)QSM produce a small value of the decay width, i.e. \(\Gamma(\Theta NK) \leq 1\) MeV, which is consistent with the recent DIANA measurement \(^{12}\).

In Ref. \(^{25}\), the decay widths of non-strange partners of the \(\Theta^+\) have been investigated, where the decay widths are found to be

\[
\Gamma(N_{10}^{\Lambda}K) = 0.70, \quad \Gamma(N_{10}^{\Xi}N\eta) = 1.80, \quad \Gamma(N_{10}^{\Xi}N\pi) = 2.10, \quad \Gamma(N_{10}^{\Xi}\Sigma\pi) = 2.80, \quad (65)
\]

in unit of MeV. However, taking the values of the present work and taking only the mixing with the baryon octet into account, we get

\[
\Gamma(N_{10}^{\Lambda}K) = 1.07, \quad \Gamma(N_{10}^{\Xi}N\eta) = 3.07, \quad \Gamma(N_{10}^{\Xi}N\pi) = 6.67, \quad \Gamma(N_{10}^{\Xi}\Sigma\pi) = 13.92, \quad (66)
\]

while considering all \(m_s\) corrections, we obtain our final results as follows:

\[
\Gamma(N_{10}^{\Lambda}K) = 0.63, \quad \Gamma(N_{10}^{\Xi}N\eta) = 2.04, \quad \Gamma(N_{10}^{\Xi}N\pi) = 0.71, \quad \Gamma(N_{10}^{\Xi}\Sigma\pi) = 13.92. \quad (67)
\]

At this point we want to stress that the \(N\eta\) channel is stronger than the \(N\pi\) channel. It is found that the inclusion of the 27-plet mixing has a large influence on the \(\Gamma(N_{10}^{\Xi}N\pi)\) and \(\Gamma(N_{10}^{\Xi}\Sigma\pi)\). Therefore, these two transitions are more sensitive to the multiplet-mixing angles than the other. For the \(N_{10}^{\Sigma}\rightarrow N\pi\) transition, the 27-plet mixing contributes destructively to the axial-vector constant with the combined flavor SU(3)-symmetric part and octet mixing correction. The 27-plet mixing correction, being small compared to the \(\Omega^0\) order, turns out to be sizeable, since the \(\Omega^0 + \Omega^1\) contributions and the octet mixing are almost canceled. In the case of the \(\Gamma(N_{10}^{\Xi}\Sigma\pi)\), which is only finite with flavor SU(3)-breaking effects, the correction of the 27-plet mixing changes the sign of the axial-vector constant, though the decay width happens to be the same as that without it. When it comes to the \(\Gamma(N_{10}^{\Xi}\Sigma\pi)\) transition, we find that the decay formula used in this work is different from that in Ref. \(^{25}\). In Eq. (7) there is the mass of the \(\Sigma\) in the denominator, which comes due to the mass factor in Eq. (A4), whereas in other formulae there are the corresponding mass of the decaying particle in the denominator. The difference yields about 50% for the \(\Gamma(N_{10}^{\Xi}\Sigma\pi)\) decay width.

In Ref. \(^{41}\) the same authors have argued that a larger value of the mixing angle between the octet and the anti-decuplet is more probable, which would increase the decay width to be \(\Gamma(N_{10}^{\Xi}\Sigma\pi) = 15\) MeV. Thus, in this case, the final result of this work is compatible with that of Ref. \(^{41}\), though in the present work we take into account 27-plet mixing corrections and a different decay-width formula. The authors of \(^{41}\) suggested a nucleon state \(N(1680)\)

\(^1\) One should note that the four parameters (cut-off mass \(\Lambda\), current quark mass \(m\), constituent quark mass \(M\), and strange current quark mass \(m_s\)) of the effective chiral action in the \(\chi\)QSM have been adjusted many years ago to \(f_\pi\), \(m_\pi\), the proton charge radius, and SU(3) baryon mass splittings.
as a pentaquark and concluded that the decay channels $N_{10} \to \Delta \pi$, though being flavor-SU(3) forbidden, and $N_{10} \to N \eta$ are the largest one. The anti-decuplet nucleon state in the $\chi$QSM of this work has a mass of $M = 1654$ MeV and the decay channels $N_{10} \to \Delta \pi$ and $N_{10} \to N \eta$ are also more noticeable than the other.

V. SUMMARY AND CONCLUSION

In the present work, we investigated the mass-splittings and strong decays of the baryon anti-decuplet within the framework of the self-consistent SU(3) chiral quark-soliton model ($\chi$QSM). We took linear rotational $1/N_c$ and linear $m_s$-corrections into account and employed the symmetry conserving quantization which is crucial in producing the small width of the $\Theta^+$. All parameters used in the present work have been already fixed in reproducing the pion and nucleon properties. The general formalism of this work corresponds to that in the $\chi$QSM done for many years, having been successfully applied to the baryon octet and decuplet regime since the development and publication of the symmetry conserving quantization. No additional parameter has been adjusted in the calculation for the baryon anti-decuplet. We used in the present work the self-consistent soliton profile in order to solve numerically single-particle Dirac Hamiltonian in the chiral quark-soliton model for the corresponding eigenvalues and eigenstates that are used in order to compute all observables.

Having considered the $1/N_c$ rotational corrections, we were able to calculate the centered mass differences between the baryon octet and the decuplet and anti-decuplet. In addition, having taken strange current quark masses into account, we also calculated the mass splittings within a baryon multiplet. We computed the absolute masses of the baryon anti-decuplet in the $\chi$QSM, starting from the experimental octet center. The final results of this work are given in Table V. Although this work does not reproduce well the experimental octet and decuplet splittings, compared to previous $\chi$QSM works in which quadratic $m_s$ corrections were considered, the results are still in qualitatively good agreement with experimental data.

We also calculated the axial-vector constants for the baryon decuplet to octet transitions, see Table VII. Applying the generalized Goldberger-Treiman relation those constants are used to calculate the strong couplings and evaluated the corresponding decay widths, see Table X for the final results. All decuplet to octet decays obtained in the $\chi$QSM are in agreement with earlier published results in the chiral solitonic picture and agree qualitatively with experimental data given by the particle data group [67].

Finally we applied the same techniques to the decays of the baryon anti-decuplet, see Table VIII and Table XII for the final results. In general, we found that those terms of the axial-vector transition constants associated with the $\Omega \to \Delta$ decays are very small. This occurs due to a numerical cancelation of the leading contributions ($\Omega^0$, $m_0^0$) with the rotational corrections ($\Omega^1$, $m_0^0$). Those terms are constructively interfering in the $10 \to 8$ transitions, while destructively interfering in the $\Omega \to 8$ transitions. Therefore, the axial-vector constants for the $10 \to 8$ transitions become large, while those for the $\Omega \to 8$ ones turn out to be small. These results immediately give the decay widths: $\Gamma \sim 70$ MeV for $\Delta \to N \pi$ and $\Gamma \leq 2$ MeV for $N_{10} \to N \pi$. The decay width of the $\Theta^+ \to NK$ transition is found to be $\Gamma = 0.71$ MeV which is comparable to the latest DIANA result: $\Gamma = 0.36 \pm 0.11$ MeV. Because of the fact that the leading contributions are almost canceled with the subleading rotational corrections, the $m_s$ corrections give sizeable effects on the axial-vector transition constants for the $10 \to 8$, so that SU(3) symmetry-breaking effects turn out to be about
50% for the decay widths from the baryon antidecuplet to the octet. We also investigated the SU(3)-forbidden decays from the baryon anti-decuplet to the decuplet. The transitions from the anti-decuplet to the decuplet are entirely due to $m_s$ corrections. However, it turns out that the forbidden decay width for $N_{10} \to \Delta \pi$ and the width for the SU(3) allowed channel $N_{10} \to N\eta$ are larger than that for the decay $N_{10} \to N\pi$, respectively.

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**APPENDIX A: DECAY FORMULAE**

Using the explicit expression for the Dirac and Rarita-Schwinger spinors and notations of [86]

$$
\sum_4 u_i^\mu(p) \bar{\pi}_\nu(p) = (p^\mu \gamma_\mu + m) \left[ -g_\mu\nu + \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{1}{3m} (\gamma_\mu p_\nu - \gamma_\nu p_\mu) + \frac{2}{3m^2} p_\mu p_\nu \right], \quad (A1)
$$

we get the following invariant matrix elements

$$
\sum_{s,s'} |M_{B_{10}B_{8m}}|^2 = g_{B_{10}B_{8m}}^2 2 \left( (M_{10} - M_8)^2 - m^2 \right) \quad (A2)
$$

$$
\sum_{s,s'} |M_{\Delta N\pi}|^2 = g_{\Delta N\pi}^2 \frac{4}{3} \left( (m_N + m_\Delta)^2 - m_\pi^2 \right) k_\pi^2 \quad (A3)
$$

$$
\sum_{s,s'} |M_{N^*\Delta\pi}|^2 = g_{N^*\Delta\pi}^2 \frac{4 M_{N^*}^2}{3 M_{\Delta}^2} k_\pi^2 \left( (M_{N^*} + M_\Delta)^2 - m_\pi^2 \right). \quad (A4)
$$

The input masses for the mesons and baryons, and for the meson decay constant are listed in Table XIII in unit of MeV.

**TABLE XIII: Input for the baryon and meson masses and for the meson decay constants.**

| Octet | Decuplet | Anti-decuplet | Mesons |
|-------|----------|---------------|--------|
| $M_N = 939$ | $M_\Delta = 1232$ | $M_\Theta = 1540$ | $m_\pi = 139$ |
| $M_\Lambda = 1116$ | $M_{N_{10}} = 1675$ | $m_K = 495$ |
| $M_\Sigma = 1189$ | $M_{\Sigma_{10}} = 1385$ | $m_\eta = 545$ |
| $M_\Xi = 1318$ | $M_{\Xi_{10}} = 1530$ | $M_{N_{10}} = 1862$ | $f_\pi = 93, \ f_K = f_\eta = 1.2 f_\pi$ |
APPENDIX B: AXIAL-VECTOR DENSITIES IN THE $\chi$QSM

The densities $A(z), B(z), \cdots$ given in the first part of Eq. (43) for $G^X_A(z)$ have to be evaluated explicitly. The corresponding densities in Eq. (44) are given as follows:

$$A(z) = N_c \langle v | z \rangle \{\sigma_1 \otimes \tau_1\}_0(z | v) + N_c \sum_n \sqrt{2G + 1R_1(\epsilon_n)} \langle n | z \rangle \{\sigma_1 \otimes \tau_1\}_0(z | n), \quad (B1)$$

$$B(z) = N_c \sum_{\epsilon_n \neq \epsilon_v} \frac{1}{\epsilon_v - \epsilon_n}(-)^G_m \langle m | z \rangle \sigma_1(z | v) \cdot \langle v | \tau_1 | n)$$

$$C(z) = N_c \sum_{n^0} \frac{1}{\epsilon_v - \epsilon^{n^0}_v} \langle v | z \rangle \{\sigma_1 \otimes \tau_1\}_1(z | v) \cdot \langle v | \tau_1 | n)$$

$$D(z) = N_c \sum_{n, m} \text{sign}(\epsilon_n) \langle v | z \rangle \{\sigma_1 \otimes \tau_1\}_1(z | v) \cdot \langle v | \tau_1 | n)$$

$$H(z) = -\frac{N_c}{2} \sum_{n, m} R_2(\epsilon_n, \epsilon_m) \sqrt{2G + 1} \langle m | z \rangle \{\sigma_1 \otimes \tau_1\}_1(z | n) \cdot n \cdot \langle m | \tau_1 | m)$$

$$I(z) = -\frac{N_c}{2} \sum_{n, m} R_2(\epsilon_n, \epsilon_m) \langle v | z \rangle \{\sigma_1 \otimes \tau_1\}_1(z | n) \cdot \langle v | \tau_1 | m)$$

$$J(z) = -\frac{N_c}{2} \sum_{n, m} R_2(\epsilon_n, \epsilon_m) \langle v | z \rangle \{\sigma_1 \otimes \tau_1\}_1(z | n) \cdot \langle v | \tau_1 | m)$$

where we have used a standard notation for the irreducible tensor algebra \[87\]. The proper-time regularization functions are defined as

$$R_1(\epsilon_n) = -\frac{1}{2\sqrt{\pi}} \int_{1/\Lambda^2}^{\infty} du \left\{\frac{\epsilon_n u^{2}}{\sqrt{u}} e^{-u\epsilon_n^2} \right\}, \quad (B8)$$

$$R_2(\epsilon_n, \epsilon_m) = \int_{1/\Lambda^2}^{\infty} du \frac{1}{2\sqrt{\pi}u} \left\{\frac{\epsilon_m e^{-u\epsilon_m^2}}{\epsilon_n - \epsilon_m} - \frac{\epsilon_n e^{-u\epsilon_n^2}}{\epsilon_n - \epsilon_m} \right\}, \quad (B9)$$

$$R_4(\epsilon_n, \epsilon_m) = \frac{1}{2\pi} \int_{1/\Lambda^2}^{\infty} du \int_{0}^{1} d\alpha e^{-\epsilon_n^2 u(1 - \alpha)}\alpha \epsilon_m e^{-\epsilon_m^2 u} \left\{\frac{\epsilon_n(1 - \alpha) - \alpha \epsilon_m}{\sqrt{\alpha(1 - \alpha)}} \right\}, \quad (B10)$$

$$R_5(\epsilon_n, \epsilon_m) = \frac{1}{2} \text{sign} \epsilon_n - \text{sign} \epsilon_m. \quad (B11)$$
The \( |v\rangle \) denotes the valence quark state and \( |n\rangle \) are the quark eigen-states of the \( \chi \)QSM Hamiltonian \( H(U) \). \( \varepsilon_n \) and \( \varepsilon_{n^0} \) are the corresponding eigen-energies, respectively. \( |n^0\rangle \) and \( \varepsilon_{n^0} \) are the eigen-states and eigen-energies of the Hamiltonian for the vacuum \( H(U = 1) \) (See, for example, Ref. [88]).

**APPENDIX C: BARYON MATRIX ELEMENTS**

Eq. (43) can be expressed in the following way:

\[
G_A^a(0) = a_1 D_{a3}^{(8)} + a_2 d_{pq3} D_{ap}^{(8)} J_q + \frac{a_3}{\sqrt{3}} D_{aj}^{(8)} J_3
+ \frac{a_4}{\sqrt{3}} d_{pq3} D_{ap}^{(8)} D_{8q}^{(8)} + a_5 \left[ D_{a3}^{(8)} D_{88}^{(8)} + D_{a8}^{(8)} D_{83}^{(8)} \right] + a_6 \left[ D_{a3}^{(8)} D_{88}^{(8)} - D_{a8}^{(8)} D_{83}^{(8)} \right], \tag{C1}
\]

where the numerical results for the axial-vector parameters \( a_i \) with \( M = 420 \text{MeV} \) are listed in Table C. The axial-vector parameter \( a_1 \) contains not only the leading order in rotation and \( m_s \) but also a part of the rotational as well as the \( m_s \) operator corrections. Without \( m_s \) corrections, we have \( a_1 = -3.64 \). All the axial-vector constants presented in this work can be also reproduced by these values. We have the following expressions from Refs. [2, 89], respectively:

\[
\frac{F}{D} = \frac{5}{9} G_0 + \frac{1}{2} G_1 + \frac{1}{6} G_2, \quad \frac{F}{D} = \frac{5}{9} - a_1 + \frac{1}{2} a_2 + \frac{1}{2} a_3, \tag{C2}
\]

and from Ref. [72]

\[
G_1 = g a_{i+1}, \quad G_2 = \frac{2m_N}{3f_\pi} g_A^0 = g a_3 \tag{C3}
\]

with \( g = \frac{1}{a_3} \frac{2m_N}{3f_\pi} g_A^0 \). In the \( \chi \)QSM, we obtain \( g_A^0 = 0.367 \) [49] which yields \( g = 2.74 \) and the following results:

\[
G_0^{\chi\text{QSM}} = 9.98, \quad G_1^{\chi\text{QSM}} = 6.86, \quad G_2^{\chi\text{QSM}} = 2.47. \tag{C4}
\]

The baryonic transition matrix elements for the collective operators given in Eq. (C1) are listed in Tables XV, XVII. We use the octet-basis according to Ref. [56], which means that there is a factor of \( \sqrt{2} \) involved in calculating the strong coupling constants from the axial-vector constants for off-diagonal transitions.

The transition matrix elements of the \( p_{10} \rightarrow \Delta^+ \) process for the wave-function corrections are given as

\[
\langle \Delta^+ | \frac{1}{2} D_{33}^{(8)}_{a} | p_{10} \rangle = d_{8}^{B} \frac{1}{3} \sqrt{\frac{1}{5}} + d_{27}^{B} \frac{1}{9} \sqrt{\frac{1}{30}} + a_{27}^{B} \frac{5}{9} \sqrt{\frac{1}{30}},
\]

\[
\langle \Delta^+ | \frac{1}{2} D_{38}^{(8)} J_3 | p_{10} \rangle = 0,
\]

\[
\langle \Delta^+ | \frac{1}{2} d_{pq3} D_{3q}^{(8)} J_p | p_{10} \rangle = -d_{8}^{B} \frac{1}{6} \sqrt{\frac{1}{5}} + d_{27}^{B} \frac{2}{9} \sqrt{\frac{1}{30}} - a_{27}^{B} \frac{5}{9} \sqrt{\frac{1}{30}}.
\]

| TABLE XIV: Numerical values for the axial-vector parameters \( a_i \) with \( M = 420 \text{MeV} \). |
|---|---|---|---|---|---|---|
| \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) | \( a_6 \) |
| -3.70 | 2.50 | 0.90 | -0.18 | 0.02 | 0.04 |
TABLE XV: The transition matrix elements between the baryon anti-decuplet and the octet for the operators \( D^{(8)} \) and \( D^{(8)}_{a_3} \). Those for the operator \( d_{pq3} D_{ap}J_q \) are simply the same as those for \( D^{(8)}_{a_3} \), correspondingly.

| \( D^{(8)}_{a_3} \) | \( 0^+n \) | \( p_{10}n \) | \( p_{10}A \) | \( p_{10}p \) | \( \Xi_1^+ \Xi_1^0 \) | \( \Xi_1^- \Xi_1^0 \) | \( \Sigma^- \) | \( p_{10}A^+ \) |
|----------------|-------------|-------------|-------------|-------------|----------------|----------------|-------|-------------|
| \( D^{(8)}_{a_8} \) | \( -\frac{1}{2} \sqrt{\frac{1}{15}} \) | \( -\frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | 0 | 0 | 0 | 0 |

TABLE XVI: The transition matrix elements between the baryon anti-decuplet and the octet for the operators \( D^{(8)}_{a_3} \), \( D^{(8)}_{a_8} \), \( D^{(8)}_{a_8} \), and \( d_{pq3} D_{sp} D_{aq} \).

| \( D^{(8)}_{a_3} \) | \( 0^+n \) | \( p_{10}n \) | \( p_{10}A \) | \( p_{10}p \) | \( \Xi_1^+ \Xi_1^0 \) | \( \Xi_1^- \Xi_1^0 \) | \( \Sigma^- \) | \( p_{10}A^+ \) |
|----------------|-------------|-------------|-------------|-------------|----------------|----------------|-------|-------------|
| \( D^{(8)}_{a_8} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | 0 | 0 | 0 | 0 |
| \( D^{(8)}_{a_8} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | 0 | 0 | 0 | 0 |
| \( D^{(8)}_{a_8} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | \( \frac{1}{2} \sqrt{\frac{1}{15}} \) | 0 | 0 | 0 | 0 |

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TABLE XVII: The transition matrix elements for the wave-function corrections with the baryons states defined in Eqs. (33, 34, 35).

| $D_{a3}^{(8)}$ | $\Theta^+n$ | $p_{10}^n$ | $p_{10}^\Lambda$ | $p_{10}^p$ | $\Xi^+\Xi^-$ | $\Xi^-\Sigma^-$ |
|---------------|-----------|-----------|------------|-----------|-----------|-----------|
| $c_{B_{10}}$  | $-\frac{1}{8}\sqrt{\frac{1}{3}}$ | $-\frac{1}{24}$ | 0 | $-\frac{1}{8}\sqrt{\frac{1}{3}}$ | 0 | $-\frac{1}{8}\sqrt{\frac{1}{3}}$ |
| $c_{B_{27}}$  | $-\frac{7}{24}\sqrt{\frac{1}{10}}$ | $-\frac{49}{72}\sqrt{\frac{1}{30}}$ | $-\frac{7}{24}\sqrt{\frac{1}{10}}$ | $-\frac{7}{8}\sqrt{\frac{1}{90}}$ | $\frac{37}{10}$ | $\frac{7}{15}$ |
| $d_{8}^{B}$   | 0 | $-\frac{7}{30}$ | $-\frac{7}{6}$ | $-\frac{1}{15}\sqrt{\frac{1}{2}}$ | $\frac{1}{10}$ | $-\frac{1}{5}\sqrt{\frac{1}{10}}$ |
| $d_{27}^{B}$  | 0 | $-\frac{1}{15}\sqrt{\frac{1}{6}}$ | $\frac{1}{30}$ | $-\frac{1}{15}\sqrt{\frac{1}{2}}$ | $\frac{1}{10}$ | $-\frac{1}{5}\sqrt{\frac{1}{10}}$ |

| $D_{a8}^{(8)}J_{3}^{c}$ | $\Theta^+n$ | $p_{10}^n$ | $p_{10}^\Lambda$ | $p_{10}^p$ | $\Xi^+\Xi^-$ | $\Xi^-\Sigma^-$ |
|-------------------------|-----------|-----------|------------|-----------|-----------|-----------|
| $c_{B_{10}}$  | $\frac{1}{16}$ | $\frac{1}{16}\sqrt{\frac{1}{3}}$ | 0 | $\frac{1}{16}$ | 0 | $\frac{1}{16}$ |
| $c_{B_{27}}$  | $-\frac{3}{16}\sqrt{\frac{1}{30}}$ | $\frac{7}{50}\sqrt{\frac{1}{10}}$ | $\frac{1}{8}\sqrt{\frac{1}{10}}$ | $\frac{3}{16}\sqrt{\frac{1}{30}}$ | $-\frac{1}{8}\sqrt{\frac{1}{30}}$ | $-\frac{1}{8}\sqrt{\frac{1}{30}}$ |
| $d_{8}^{B}$   | 0 | $\frac{1}{20}\sqrt{\frac{1}{7}}$ | $\frac{7}{10}$ | $\frac{1}{20}\sqrt{\frac{1}{7}}$ | $\frac{3}{20}$ | 0 |
| $d_{27}^{B}$  | 0 | $\frac{1}{20}\sqrt{\frac{1}{7}}$ | $\frac{7}{10}$ | $\frac{1}{20}\sqrt{\frac{1}{7}}$ | $\frac{3}{20}$ | 0 |

| $d_{a3}^{(8)}J_{3}^{c}$ | $\Theta^+n$ | $p_{10}^n$ | $p_{10}^\Lambda$ | $p_{10}^p$ | $\Xi^+\Xi^-$ | $\Xi^-\Sigma^-$ |
|-------------------------|-----------|-----------|------------|-----------|-----------|-----------|
| $c_{B_{10}}$  | $-\frac{5}{16}\sqrt{\frac{1}{3}}$ | $\frac{5}{48}$ | 0 | $-\frac{5}{16}\sqrt{\frac{1}{3}}$ | 0 | $-\frac{5}{16}\sqrt{\frac{1}{3}}$ |
| $c_{B_{27}}$  | $\frac{11}{48}\sqrt{\frac{1}{10}}$ | $\frac{77}{144}\sqrt{\frac{1}{30}}$ | $\frac{11}{24}\sqrt{\frac{1}{10}}$ | $\frac{33}{144}\sqrt{\frac{1}{30}}$ | $\frac{11}{16}\sqrt{\frac{1}{10}}$ | $\frac{9}{16}\sqrt{\frac{1}{10}}$ |
| $d_{8}^{B}$   | 0 | $\frac{1}{5}\sqrt{\frac{1}{5}}$ | $\frac{1}{5}$ | $\frac{1}{5}\sqrt{\frac{1}{5}}$ | 0 | 0 |
| $d_{27}^{B}$  | 0 | $-\frac{2}{25}\sqrt{\frac{1}{3}}$ | $\frac{1}{15}$ | $-\frac{2}{25}\sqrt{\frac{1}{3}}$ | $\frac{2}{9}\sqrt{\frac{1}{10}}$ | $-\frac{2}{9}\sqrt{\frac{1}{10}}$ |

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