Polarized Synchrotron Foreground Assessment for CMB Experiments

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Abstract

Polarized Galactic synchrotron emission is an undesirable foreground for cosmic microwave background experiments observing at frequencies <150 GHz. We perform a combined analysis of observational data at 1.4, 2.3, 23, 30, and 33 GHz to quantify the spatial variation of the polarized synchrotron spectral index, $\beta^{\text{pol}}$, on ~3° scales. We compare results from different data combinations to address limitations and inconsistencies present in these public data, and form a composite map of $\beta^{\text{pol}}$. Data quality masking leaves 44% sky coverage (73% for $b > 45^\circ$). Generally $-3.2 < \beta^{\text{pol}} \lesssim -3$ in the inner Galactic plane and spurs, but the Fan Region in the outer galaxy has a flatter index. We find a clear spectral index steepening with increasing latitude south of the Galactic plane with $\Delta \beta^{\text{pol}} = 0.4$, and a smaller steepening of 0.25 in the north. Near the south Galactic pole the polarized synchrotron spectral index is $\beta^{\text{pol}} \approx -3.4$. Longitudinal spectral index variations of $\Delta \beta^{\text{pol}} \sim 0.1$ about the latitudinal mean are also detected. Within the BICEP2/Keck survey footprint, we find consistency with a constant value, $\beta^{\text{pol}} = -3.25 \pm 0.04$ (statistical) $\pm 0.02$ (systematic). We compute a map of the frequency at which synchrotron and thermal dust emission contribute equally to the total polarized foreground. The limitations and inconsistencies among data sets encountered in this work make clear the value of additional independent surveys at multiple frequencies, especially between 10 and 20 GHz, provided these surveys have sufficient sensitivity and control of instrumental systematic errors.

Unified Astronomy Thesaurus concepts: Observational cosmology (1146); Diffuse radiation (383)

1. Introduction

The cosmic microwave background (CMB) has been crucial to cosmology. All CMB experiments must ensure that foreground contamination is minimized and upper limits are well-estimated, as this often sets the ultimate limits on cosmological results. With the current and future observational focus on CMB polarization measurements, characterization of polarized foregrounds is a key aspect of CMB studies.

Galactic polarized foregrounds are dominated by synchrotron emission at lower frequencies and thermal dust emission at higher frequencies with a crossover at $\sim 80$ GHz, depending on sky location (e.g., Page et al. 2007; Bennett et al. 2013; Planck Collaboration X 2016). Other sources of low-frequency polarized emission are theoretically possible and must happen at some level, but have not yet been detected. This includes polarized spinning dust emission, which is usually considered to arise from radius $a \lesssim 1$ nm rapidly rotating dust grains with an electric dipole moment (Erickson 1957; Draine & Lazarian 1998; Hensley & Draine 2017). While spinning dust emission has been detected, its polarization has not (Génova-Santos et al. 2017; Dickinson et al. 2018). Draine & Hensley (2016) suggest that the spinning dust polarization will be negligible due to quantization of the vibrational energy levels in the emitting grains, which would exponentially suppress their alignment. In this paper, we attribute all of the low-frequency polarized emission to the synchrotron mechanism.

Synchrotron emission is generated by a population of ultra-relativistic electrons that spiral around magnetic field lines in our galaxy. For a power-law population distribution of electron energies $N(\gamma) \propto \gamma^{-p}$, where $\gamma$ is the relativistic Lorentz factor, the maximum degree of polarization (polarization fraction) in a uniform magnetic field is $P_{\text{max}} = (3p + 3)/(3p + 7)$ (Rybicki & Lightman 1986). The Galactic magnetic field is typically $\sim 6 \mu$G (Beck 2001) and for $p = 3.0$ the spectral index is $\beta = -(p + 3)/2 = -3.0$ and the maximum degree of polarization $P_{\text{max}}$ is 75%. However, this high degree of polarization is almost never observed for many reasons, including the pitch angle distribution of the emitting electrons along a line of sight that effectively depolarizes the observed emission. In units of antenna temperature, the spectral index $\beta$ is defined by $T_{\text{A}} \propto \nu^{\beta}$. We use the notation $\beta^{\text{pol}}$ in subsequent sections to emphasize that the spectral index specifically refers to the polarized signal. The spectral index may vary both in the frequency and spatial domains. Mechanisms such as synchrotron self-absorption, along with cosmic-ray electron propagation and energy loss, are contributing factors (Strong et al. 2011).

Single region foreground template removal methods have been adequate for cosmological analyses of large sky areas surveyed by previous experiments such as WMAP and Planck (e.g., Hinshaw et al. 2013; Planck Collaboration VI 2020). However, Errard & Stompor (2019) noted that spurious detections of the tensor-to-scalar ratio $r$ of order 0.01 could result from using a single set of spectral parameters to clean foregrounds over the entire observed sky. The demands on accurate foreground removal will only grow in the future with deeper measurements and more aggressive goals. Osumi et al. (2021) recently showed that the polarized dust spectral index variations are not currently well enough known for ambitious new experiments. We now consider how well we can characterize the polarized synchrotron foreground emission with current public observational data. This
study is intended not only to assess the current status of our knowledge of this foreground, but also to highlight those areas where additional observations would provide constraints that the current data do not. We cite previous work of particular relevance to this analysis in the body of the paper. Additionally, Table 1 of Jew & Grumitt (2020) provides a summary of references to earlier determinations of the polarized synchrotron spectral index.

This paper is organized as follows. We introduce the observational data we use in Section 2 along with a discussion of Faraday rotation and depolarization. We present our data analysis of the Galactic plane region together with the associated spur regions in Section 3. These regions have relatively higher signal-to-noise ratios (S/N) but also higher Faraday rotation than other sky regions. In Section 4, we present our data analysis of the higher Galactic latitude regions, which have lower S/N. We consider several types of data analyses using differing combinations of data. In Section 5 we create a composite polarized synchrotron spectral index map and use it to derive results, while also illustrating the substantial limitations of the current data for these purposes. Difficulties we encountered in connecting polarization and low-frequency intensity data are presented in Section 6. Section 7 discusses the implications of our findings for synchrotron foreground removal. Finally, we summarize our conclusions in Section 8.

2. Observational Data

2.1. Polarized Sky Data

At present, there are only a few publicly available polarized maps of the sky at frequencies dominated by synchrotron emission. The WMAP (Bennett et al. 2013) and Planck (Planck Collaboration I 2020) space missions provide full-sky coverage in the 23–44 GHz range. These offer the advantage of precise gain calibration with minimal Faraday rotation, counterbalanced by relatively low S/N in some high Galactic latitude directions. We limit our investigation to the three frequencies with the highest S/N: WMAP 23 GHz (K band), Planck Low Frequency Instrument (LFI) 30 GHz, and WMAP 33 GHz (Ka band). There are multiple versions of Planck LFI 30 GHz maps available. These include the 2016 Public Release 3 and 2020 Public Release 4, a.k.a. NIPIPE, Planck Collaboration Int. LVII 2020, and the BeyondPlanck (BeyondPlanck Collaboration et al. 2020) maps. We discuss our choices of which maps to use in the data analysis sections.

Ground-based observations at lower frequencies include S-PASS (2.3 GHz, $\delta < -1^\circ$, Carretti et al. 2019), DRAO5 (1.41 GHz, $\delta > -29^\circ$, Wolleben et al. 2006), Villa Elisa (1.435 GHz, $\delta < -10^\circ$, Testori et al. 2008). The latter two 1.4 GHz surveys may be merged to form a full-sky map, but were not originally projected in HEALPix6 (Górski et al. 2005). We use the DRAO-only HEALPix projection of La Porta et al. (2005), as well as the merged all-sky version from Centre d’Analyse de Données Etendues (CADE).7

Polarization angle conventions are not inherently consistent between all the above data sets. While the WMAP and Planck products follow the HEALPix convention, those of the S-PASS and 1.4 GHz surveys follow the IAU convention. The sign of Stokes $U$ changes when converting between the two conventions. Figures shown in this paper follow the HEALPix convention, and unless otherwise noted, maps are shown as Mollweide projections in Galactic coordinates, centered on $l = 0^\circ$, $b = 0^\circ$. Temperature units are also not inherently consistent between data sets. Both Planck and WMAP have adopted thermodynamic units, while the ground-based data are in antenna (Rayleigh–Jeans) temperature. In general, we show each in their native units, but the conversion factors between the two systems are relatively small (a few percent) for these frequencies.

In comparison to the lower frequency data sets, the noise characteristics of WMAP and Planck maps are somewhat complex. WMAP noise includes pixel–pixel covariance at lower resolutions as well as some large-scale modes with enhanced uncertainty (Jarosik et al. 2003, 2007, 2011). Planck noise includes non-Gaussian instrumental effects and systematic residuals (Planck Collaboration II 2020). We therefore also make use of ancillary products such as data splits, covariance matrices, and simulations provided by the instrument teams for noise characterization, as described in individual analysis sections.

2.2. Faraday Rotation Measures

Both the 1.4 and 2.3 GHz data are significantly affected by Faraday rotation and depolarization. The rotation measure (RM) defines a plane of polarization rotation angle $\theta$ per line of sight at a given frequency, assuming a wavelength-squared dependence: $\theta = \text{RM} \lambda^2$ (Burn 1966). A rotation matrix with $2\theta$ as the argument, applied to the true signal as originally emitted, describes the observed Faraday-rotated Stokes $Q$ and $U$ signals after passage through the ISM (Vinyajkin 2004; Oppermann et al. 2015; Fuskeland et al. 2021):

$$
\begin{bmatrix}
Q \\
U
\end{bmatrix}^{\text{true}} = \begin{bmatrix}
\cos 2\theta & \sin 2\theta \\
-\sin 2\theta & \cos 2\theta
\end{bmatrix} \begin{bmatrix}
Q \\
U
\end{bmatrix}^{\text{obs}}.
$$

The polarized intensity $P = \sqrt{Q^2 + U^2}$ is invariant under pure rotation. However, the construction of $P$ from $Q$ and $U$ maps with a significant noise component introduces a positive noise bias which must be accounted for. Propagation of uncertainties is more straightforward when using $Q$ and $U$ directly, with the necessary accompanying Faraday rotation corrections. In our analyses of $Q$ and $U$ maps, we make use of the all-sky Faraday depth ($\phi$) map of Hutschenreuter et al. (2022) determined from observations of extragalactic sources, and the RM map of Carretti et al. (2019) determined from S-PASS, WMAP, and Planck data for specific lines of sight. These two maps are shown in the top and bottom panels of Figure 1, respectively. Under thin-screen conditions, RM and $\phi$ are equivalent. However, in the case of cumulative multiple rotations along the propagation path, depolarization is likely to occur with $RM \neq \phi$ (Wolleben et al. 2010b; Hutschenreuter et al. 2022).

Faraday rotation and depolarization can greatly reduce the amount of detectable polarized emission, lowering the observed polarization fraction and producing a shallower synchrotron spectral index than would be observed in its absence. Throughout this paper, we either mask to avoid regions subject to depolarization, apply RM corrections under conditions where $\phi \approx RM$, or use frequencies for which Faraday effects are not significant.
3. Analysis of Galactic Plane and Spur Regions

The highest S/N regions available for determinations of $\beta_{\text{pol}}$ are located close to the Galactic plane and in the extensions (spurs) reaching north and south of the plane near the Galactic center. However, Faraday depolarization near the Galactic plane limits $\beta_{\text{pol}}$ analyses in these regions to frequencies $>5$–10 GHz, with Planck and WMAP data providing full-sky coverage and sub-percent absolute calibration. Some previous results using Planck LFI or WMAP data and specifically including plane and spur regions are Fuskeland et al. (2014), Vidal et al. (2015), Jew & Grumitt (2020), and Svalheim et al. (2020). When combined with WMAP K-band, QUIET\(^8\) 43 GHz observations of planar regions designated G-1 ($l = 329^\circ$), and G-2 ($l = 0^\circ$) result in $\beta_{\text{pol}} \sim -2.9$ on-plane ($|b| \leq 2.5^\circ$) and $\sim -3.07$ to $-3.14$ off-plane ($2.5^\circ < |b| < 10^\circ$) (Ruud et al. 2015).

We revisit a determination of $\beta_{\text{pol}}$ in these high S/N regions, using pairs of frequencies: (i) the 9 yr WMAP 23 GHz (K-band) and 33 GHz (Ka-band) maps, and separately (ii) the WMAP 23 GHz and Planck PR3 LFI 30 GHz maps. Ideally, one would wish to perform a combined three-frequency fit to optimally reduce instrumental noise contributions, but for the reasons we discuss below, this is not a straightforward exercise.

We use a linear correlation analysis method similar to that described by Fuskeland et al. (2014) for each data frequency pair. Stokes $Q$ and $U$ maps at each frequency are first smoothed to $2^\circ$ FWHM, and degraded to HEALPix $N_{\text{side}} = 32$ pixels, or about $1^\circ$8 on a side. We then fit a line of zero intercept through the combined $Q$ and $U$ data points within a $\sim$$T^2$ superpixel (HEALPix $N_{\text{side}} = 8$). Converted to antenna temperature units, the slope of this line, $m$, determines the spectral index $\beta_{\text{pol}} = \log(m) / \log(\nu_2/\nu_1)$, where $\nu_1$ and $\nu_2$ are the frequencies associated with the data plotted on the two axes. There are a total of 32 data pairs possible per superpixel (16 $Q$, 16 $U$). All pairs are used in each superpixel fit except for a small number of cases in which bright supernova remnants close to the plane were excluded (Tycho, 3C58, W51, W63, Pup A, Cas A, Tau A, and MSH15\(_{\text{._56}}\)). Since the instrumental noise uncertainties are comparable for both frequencies, statistical weighting along both axes points is applied. The uncertainty of the fit slope parameter $\sigma_m$ is used to compute the statistical uncertainty in $\beta_{\text{pol}}$: $\sigma(\beta_{\text{pol}}) = \sigma_m / [m \log(\nu_2/\nu_1)]$ as in, e.g., Equation (11) of Fuskeland et al. (2014).

Data uncertainties used in the linear fits are derived from the diagonal $QQ$ and $UU$ pixel variances appropriate to each map. However, these per-pixel uncertainties do not fully encompass the more complex noise characteristics of WMAP and Planck data (see Section 2). We use simulations to verify that the use of the diagonal uncertainties in the fit does not bias the recovery of the true parameters and their uncertainties. Simulated maps include realistic sky signals and independently generated instrument noise that more completely characterizes the full noise properties of the data.

The simulation suite consists of $Q$ and $U$ maps produced from the sum of synchrotron, CMB, and instrument noise components. We use the synchrotron sky model from the Planck Full Focal Plane (FFP10) simulations, adopting a 23 GHz $Q$ and $U$ signal level and then use a spatially constant $\beta_{\text{pol}}$ of either $-3.15$ or $-2.8$ to extrapolate to the other frequencies. Two choices of $\beta_{\text{pol}}$ were used to verify that results are not sensitive to the chosen spectral index value. The synchrotron component does not vary with realization, but the CMB and instrument noise components are different for each realization. The CMB component adopts a standard $\Lambda$ cold dark matter cosmology from the Planck PR3 public chains.\(^9\) The inclusion of the CMB component is not essential, since its contribution is completely subdominant to the foreground signal and noise. WMAP 23 and 33 GHz noise maps are computed from bootstrapped samples taken from the population of single-year null difference maps, scaled to 9 yr noise levels. The Planck 30 GHz noise is taken from the FFP10 simulations. We apply the sample smoothing and downsampling to the simulations as with the data. Uncertainties from the simulations generally agree with those derived from the data within $\sim$5%.

Results of the $\beta_{\text{pol}}$ determination using the linear correlation method for the WMAP 23 and 33 GHz bands are shown in the top panel of Figure 2. Only pixels with an uncertainty $\sigma(\beta_{\text{pol}}) \leq 0.2$ are shown; all others are excluded and shown in gray. We chose this limit for reliable determinations based on our simulations. A consequence is that at this pixel resolution, determinations are not included for fainter portions of the outer Galactic plane. Uncertainties are typically lowest close to the Galactic plane where the $S/N$ is highest, and those in the northern and southern spurs are more typically near 0.15–0.2. The top panel shows $\beta_{\text{pol}} \lesssim -3$ over much of the analyzed area, with a somewhat flatter index in the Fan Region (an extended bright region in the Galactic plane centered near

\(^8\) QU Imaging Experiment.

\(^9\) chain identifier: base_plikHM_TTTEEE_lowl_lowE_lensing.
The Astrophysical Journal, 936:24 (18pp), 2022 September 1

Figure 2. Top: polarized spectral index $\beta^{\text{pol}}$ computed from WMAP 23 GHz and 33 GHz 9 yr Q and U maps, in Galactic coordinates. White coordinate grid lines are in 30° increments, with the convention $l, b = (0°, 0°)$ is at the center, (90°, 0°) midway to the left, and (270°, 0°) midway to the right. Bottom: the $\beta^{\text{pol}}$ map computed from WMAP 23 GHz and Planck LFI 30 GHz from PR3. Pixel sizes correspond to HEALPix $N_{\text{side}} = 8$ (~7° on a side). Pixels for which $\sigma(\beta^{\text{pol}}) > 0.2$ are shown as the gray masked region. There are clear visual differences between the two determinations: those in the Fan Region (on the left at $l \sim 140°$, where the colors shift from mostly orange to mostly dark blue) are statistically significant, but are attributable to large-scale systematic differences between WMAP and Planck LFI maps. As discussed by Planck Collaboration II (2020), the large-scale systematic modes are primarily attributable to the 30 GHz map.

In the bottom panel of Figure 2, the $\beta^{\text{pol}}$ map derived using the same methodology is shown for the WMAP 23 and LFI 30 GHz frequency pair. There are similarities in the map with the [23, 33] result, but also clear differences. The two most visually striking differences are within the plane: a steeper [23, 30] spectral index near the Fan Region, and a string of shallower index pixels running across the plane near the inner galaxy. We quantify the significance of these differences through the use of four larger regions as defined in the top of Figure 3. These regions correspond to (1) an outer plane region dominated by the Fan Region, (2) an inner plane dominated region, and northern (3) and southern (4) extensions off the plane. For both the [23, 30] and [23, 33] analyses, weighted means and associated uncertainties for each region are given in Table 1. The results in Table 1 are plotted in the bottom panel of Figure 3. The figure illustrates the high statistical significance of the difference in $\beta^{\text{pol}}$ between the [23, 33] and [23, 30] determinations for region 1.

There is no likely physical mechanism that suggests that the region 1 results for [23, 30] and [23, 33] are both correct. Faraday depolarization is not a significant factor at these frequencies, and furthermore would act to produce a shallower [23, 30] index compared to [23, 33] rather than the steeper index shown in Figure 3. The main beam FWHM of the 30 and 33 GHz radiometers are similar (33° and 40°, respectively), minimizing resolution-induced changes in the sampled physical conditions. Beam depolarization and spectral decoherence differences, rooted in magnetic field and synchrotron spectral energy distribution (SED) variations across the beam, are minimized given that the 30 and 33 GHz bandpasses overlap between 29 and 31 GHz, and the frequency lever arm between bands is not large. Abrupt changes in the electron energy spectrum are also unlikely at these frequencies (Strong et al. 2011; Orlando & Strong 2013).

A nonphysical origin for the region 1 discrepancy is more compelling. Large-scale systematic differences between WMAP and Planck LFI polarization maps have been noted in previous publications, e.g., Weiland et al. (2018), Planck Collaboration X

Table 1 $\beta^{\text{pol}}$ for Plane and Spur Regions

| Region Numbera | [23, 33] | [23, 30] |
|----------------|----------|----------|
| 1              | $-2.90 \pm 0.020$ | $-3.27 \pm 0.022$ |
| 2              | $-3.05 \pm 0.010$ | $-3.01 \pm 0.011$ |
| 3              | $-3.11 \pm 0.036$ | $-3.09 \pm 0.037$ |
| 4              | $-3.04 \pm 0.062$ | $-3.15 \pm 0.059$ |

Note. a See Figure 3 for region definition.
(2016). Planck Collaboration II (2020) shows that these large-scale systematic differences arise primarily from an unconverged iterative (gain calibration + sky model) solution in the PR3 LFI processing pipeline. That portion of the large-scale gain uncertainty modes induced in the LFI polarization maps that intersects with region 1 directly correlates with the area of highest \( \beta \) discrepancy, and is of correct sign to produce the steeper \([23, 30] \beta_{\text{pol}}\).

Although we have primarily discussed results from either \([23, 33]\) or Planck PR3 data in combination with 23 GHz, we also performed some exploratory computations in which either the NPIPE or BeyondPlanck 30 GHz maps were substituted for the PR3 30 GHz maps in the \([23, 30]\) combination. We show these results in Figure 4. The higher latitude regions 3 and 4 have mean values near \(-3\) to \(-3.1\), i.e., consistent with those already shown in Table 1 and Figure 3. However, results within the plane regions 1 and 2 are variable and depend solely on the version of 30 GHz maps used in the computation, since the same 23 GHz data are used in each computation. Using either one of these alternate LFI processings indicates substantially steeper \( \beta_{\text{pol}} \) values in the inner plane region 2, with means close to \(-3.4\) or \(-3.5\). Arguments against a physical interpretation, presented earlier for region 1, also apply to region 2. Furthermore, the independent \([23, 43]\) \( \beta_{\text{pol}} \) determination of Ruud et al. (2015) using QUIET data (see the beginning of this section) supports a spectral index between \(-2.9\) and \(-3.1\) for this region, in agreement with the \([23, 33]\) result. With the continued presence of large-scale modes of similar morphology to those in PR3, along with reliance on a sky model for certain aspects of LFI processing (e.g., temperature to polarization leakage) and the complex nature of emission within the Galactic plane, we suspect that \( \beta_{\text{pol}} \lesssim -3.3 \) in region 2 does not reflect true sky variations but rather data processing inconsistencies. Given that the polarized spectral index results depend significantly on processing differences of the same 30 GHz data set, we favor the use of the more stable and reliable 33 GHz data. We adopt the \( \beta_{\text{pol}} \) determined from the \([23, 33]\) frequency pair as our best estimate for these high S/N regions.

As a function of longitude within the Galactic plane, the \([23, 33]\) regional map shows a trend toward a slightly steeper spectral index in the inner galaxy compared to the Fan Region, at a significance of \( \lesssim 3\sigma \). A similar trend has been noted in previous analyses using different regional segmentations of the plane, e.g., Fuskeland et al. (2014) and BeyondPlanck Collaboration et al. (2020). These trends address large scales and do not represent the full complexity of the Galactic plane. The spatial resolution of the \( \beta_{\text{pol}} \) map has been constrained primarily by S/N in the WMAP 23 and 33 GHz bands. While the 23 GHz beam would allow a common resolution of \(~1^\circ\), the main effect of using \(~1^\circ\) pixels on this paper’s results would be one of increased uncertainty. Variations in electron energy spectral index and polarization fraction across a pixel that could be detectable at this higher resolution (e.g., Padovani et al. 2021) require additional data for a meaningful result.

In terms of latitudinal dependence, there is no clear trend based solely on the results for regions 3 and 4. Emission from regions 3 and 4 is likely dominated by local structures (e.g., the North Polar Spur; Vidal et al. 2015; West et al. 2021). Mean \( \beta_{\text{pol}} \) values in these regions are consistent with mean high-latitude values derived over very large sky fractions using WMAP and/or Planck data. We more completely address the higher latitude data in Section 4.

4. Analysis of Lower S/N Off-plane Regions

WMAP and Planck data in the 20–70 GHz range provide high-quality all-sky maps of the polarized synchrotron signal (with subdominant additional contributions from CMB and thermal dust components). Instrument noise and the relatively steep decrease in synchrotron signal with increasing frequency combine to cause the S/N of these maps at high latitudes to be lower than needed to produce tightly constrained maps of \( \beta_{\text{pol}} \) on few-degree (or smaller) scales (Planck Collaboration IV 2020). Numerous investigations have produced WMAP-only, Planck-only, or combined WMAP and Planck estimates on larger patches (e.g., Kogut et al. 2007; Dunkley et al. 2009; Fuskeland et al. 2014; Choi & Page 2015; Svalheim et al. 2020; Jew & Grumitt 2020; Planck Collaboration XI 2020; Martire et al. 2022). Methods have included both pixel-space and harmonic-space (power-spectrum) based techniques. As an example of the uncertainties achieved using up to 71% of the sky excluding planar regions, and the full range of WMAP and Planck frequencies in a power-spectrum analysis, Planck Collaboration XI (2020) computed a weighted mean and standard deviation of \( \beta_{\text{pol}} = -3.13 \pm 0.13 \).

Supplementing WMAP and/or Planck observations with data at lower (<20 GHz) frequencies is one option for the production of a finer spatial resolution map of \( \beta_{\text{pol}} \), as lower frequency observations have the advantage of a brighter intrinsic synchrotron signal. There are, however, challenges: (1) depending on the frequency, Faraday rotation, and depolarization along
the line of sight can be substantial, (2) for ground-based or balloon experiments, full-sky coverage is the exception, and calibration challenges are introduced from the atmospheric and ground environment, and (3) a greater frequency *lever arm* from target cosmological frequencies increases the error arising from potential spectral curvature deviations from a pure power-law dependence.

Another analysis option is to determine $\beta_{\text{pol}}$ using only lower frequency data ($\nu < 20$ GHz). Although the synchrotron signal is considerably higher at these frequencies, the challenges listed above also apply in this case. Absolute gain calibration uncertainties of order 5% can also play a larger role in this case than when the frequency lever arm is longer.

At present, the only publicly available lower frequency polarization data with substantial sky coverage are those at 1.4 and 2.3 GHz. In the following subsections, we discuss high-latitude $\beta_{\text{pol}}$ results obtained from three different combinations of frequency bands: (1) 1.4 and 2.3 GHz, (2) 2.3, 23, and 33 GHz, and (3) 1.4, 23, and 33 GHz. We analyze frequencies $\lesssim 40$ GHz to avoid the increasing fractional contribution of CMB and dust components at higher frequencies, and omit Planck 30 GHz in order to simplify our search for systematic signatures in fit residuals.

### 4.1. 1.4 and 2.3 GHz

The merged full-sky 1.4 GHz DRAO and Villa Elisa surveys overlap the footprint of the southern S-PASS 2.3 GHz survey. However, Faraday depolarization effects are more significant at 1.4 GHz than at 2.3 GHz, limiting the useful sky overlap between the two surveys to those portions for which $|b| \geq 45^\circ$ (Carretti et al. 2010). A further complexity is introduced by the few visually apparent artifacts in the DRAO northern survey (Wolleben et al. 2006) and scan striping in the southern Villa Elisa 1.4 GHz data (Testori et al. 2008). Figure 5 compares the polarized intensity $P = \sqrt{Q^2 + U^2}$ at the two frequencies for a cap about the southern Galactic pole. The 2.3 GHz survey data on the right has been smoothed to match the $\sim 36^\prime$ FWHM of the 1.4 GHz map on the left. Since instrumental noise is low compared to the signal, noise bias in $P$ is not prominent at either frequency, and the noise seen in the 1.4 GHz map is calibration related. Testori et al. (2008) note that the scan striping smooths out over scales of several degrees. Visual examination indicates the region shown presents the 1.4 GHz surveys at their worst in terms of these calibration-related features. We will discuss further consequences of these features in Section 4.2.

Figure 6 shows a correlation plot between 1.4 and 2.3 GHz polarized intensity data for $b \lesssim -80^\circ$. We have used $P$ rather than $Q$ and $U$ in this specific case because the high S/N mitigates against noise bias concerns normally present when working with $P$ and because the data are not sufficiently constraining to independently solve for RM. The correlation slope obtained using statistical uncertainties corresponds to $\beta_{\text{pol}} = -3.30 \pm 0.03$. This is a value consistent with other analyses, but the scan striping noise requires the use of larger sky areas than the native survey resolution. Furthermore, the presence of artifacts and gain discontinuities can skew slope determinations. The impact is higher for determinations using shorter frequency lever arms. For example, a 5% gain variation at 1.4 GHz translates to an error in $\beta_{\text{pol}}$ of $\sim 0.1$ if the second frequency is 2.3 GHz, but only $\sim 0.02$ if the second frequency is 23 GHz. It is more advantageous to proceed with computing separate fits of [1.4, 23, 33] and [2.3, 23, 33] at a lower spatial resolution and to compare results for those pixels in common.

### 4.2. [1.4, 23, 33] and [2.3, 23, 33] Fits

There are results in the literature that use combinations of 1.4 GHz and WMAP frequencies, and separately 2.3 GHz...
combined with WMAP and/or Planck LFI data. Carretti et al. (2010) determined a mean full-sky value of \( \beta_{\text{pol}} = -3.2 \) using the combined DRAO and Villa Elisa 1.4 GHz data and an older WMAP 5th yr release 23 GHz map, but provided no commentary on uncertainties. Krachmalnicoff et al. (2018) analyzed 2.3, 23, and 30 GHz polarized intensity maps with a Galactic latitude cut \( |b| < 20^\circ \) to avoid regions affected by Faraday depolarization. On 2° scales, they determined \(-4.4 < \beta_{\text{pol}} < -2.5 \), but with only 12% of the sample found to differ at high significance from the mean of \(-3.2 \). Fuskeland et al. (2021) used a correlative analysis between WMAP 23 GHz and S-PASS to determine \( \beta_{\text{pol}} \) within 15° patches in the southern sky. They applied Faraday rotation corrections to the S-PASS \( Q \) and \( U \) maps, choosing the S-PASS, WMAP, and Planck-based RM map of Carretti et al. (2019) over an older extragalactic source-based map Hutschenreuter & Enßlin (2020).

4.2.1. Fitting Method

Our fits to \([1.4, 23, 33]\) and \([2.3, 23, 33]\) frequency combinations have similar analysis elements to those described above. However, there are differences in details, including chosen spatial resolution, sky fraction, selected frequencies, and treatment of Faraday rotation. We work at HEALPix \( N_{\text{side}} = 16 \), or roughly a pixel resolution of 3°. Working at this spatial scale increases the S/N of the polarization data over native resolution and allows us to use other products that are available at the same resolution, in particular covariance matrices and the WMAP loss-imbalance templates.

The following model is fit to the \( Q \) and \( U \) data in pixel-space for the three frequencies:

\[
\begin{align*}
[Q, U]_{\text{obs}} &= f(\text{RM}, Q_0, U_0), \\
[Q, U]_{\text{obs}}^{23} &= 1.014 [Q_0, U_0] (22.5/\nu_0)^{\beta_{\text{pol}}}, \\
[Q, U]_{\text{obs}}^{33} &= 1.026 [Q_0, U_0] (32.65/\nu_0)^{\beta_{\text{pol}}},
\end{align*}
\]

where \( \nu_0 \) is either 1.41 or 2.303 GHz, and the free parameters \( \text{RM}, Q_0, U_0, \) and \( \beta_{\text{pol}} \) are evaluated for each \( N_{\text{side}} = 16 \) pixel. \( Q_0 \) and \( U_0 \) are the intrinsic (unrotated) Stokes parameter maps at frequency \( \nu_0 \). The function \( f \) transforms the RM parameter to a rotation angle \( \theta \) assuming a \( \lambda^{-2} \) dependence and applies the corresponding rotation matrix (see Section 2.2) to the intrinsic synchrotron amplitudes \( Q_0 \) and \( U_0 \) to model the observed 1.4 or 2.3 GHz \( Q \) and \( U \) maps. Faraday rotation is ignored for the 23 and 33 GHz bands. However, the 23 and 33 GHz maps are in thermodynamic temperature units, so the factors 1.014 and 1.026 provide the conversion from model RJ units to thermodynamic (Bennett et al. 2013). The model assumes the spectral index has no frequency dependence.

We use the SciPy nonlinear fitting routine \textit{curvefit} for parameter estimation with weighting specified by \( QQ, QU, \) and \( UU \) variances per pixel for each frequency (there are no \( QU \) variances available at 1.4 or 2.3 GHz). The Faraday depth map of Hutschenreuter et al. (2022) is used for the initial RM guess to the iterative fit.

Simulations are used to help understand errors and potential systematic effects in the real data. We generate simulated \( N_{\text{side}} = 16 \) sky maps containing CMB, synchrotron, and instrument noise at the same frequencies. We use the same sky signal model as that described in Section 3. The simulated 2.3 GHz noise is based on the delivered \( Q \) and \( U \) noise maps, and the instrument noise for 1.4 GHz is generated based on the single value provided for each survey. Pixel–pixel covariance matrices for the WMAP 23 and 33 GHz bands are used to generate noise realizations for these frequencies. Use of the full pixel–pixel covariance incorporates the off-diagonal terms and large-scale modes of enhanced uncertainty discussed in Section 2. Faraday rotation under the assumption of thin-screen conditions is included in the simulation of the 1.4 and 2.3 GHz synchrotron emission, using the Faraday depth map of Hutschenreuter et al. (2022) (see Section 2.2). As described in the next section, we do not analyze regions expected to strongly violate the thin-screen approximation.

4.2.2. Pixel Selection

We fit the parameterized model described in the previous section to all available pixels: full sky for the [1.4, 23, 33] frequency set and the S-PASS southern footprint for the [2.3, 23, 33] fit. We then apply a set of pixel quality masks based on criteria that we derive from fit residuals, parameter errors, and estimates of potential systematic effects.

Figure 7 shows the Faraday rotation corrected amplitudes \( Q_0 \) and \( U_0 \) fit for 1.4 GHz (left column) and 2.3 GHz (middle), in comparison with the observed WMAP 23 GHz maps (right). Display scales are chosen such that if the sky synchrotron signal behaved as \( \nu^{-3.2} \) over the entire sky, the \( Q \) and \( U \) maps at all three frequencies would look the same, which they do not. The clearest visual difference takes the form of weaker emission near the Galactic plane for the 1.4 and 2.3 GHz maps compared to 23 GHz. This is the result of Faraday depolarization in these regions, which causes a shallower spectral index. This is an example of pixels that should be masked from the fits because of a systematic effect, but there are additional reasons for masking.

The following criteria are applied to mask pixels. The masks are not necessarily exclusive of each other, in that some effects show up in more than one set of criteria:

1. Pixels containing emission from strong polarized sources are masked. This includes extragalactic sources such as Fornax A, Cen A, and 3C279. In many cases, the source masking was not necessary because those pixels were masked by other criteria as well.
2. Outliers in the low-frequency fit residual maps are masked. Figure 8 shows the residuals (data minus model) for both the (a) [1.4, 23, 33] and (b) [2.3, 23, 33] fits. Compared to the 23 and 33 GHz data, the relative S/N for 1.4 and 2.3 GHz is very high and the fit essentially treats these frequencies as noiseless. This causes the $Q_0$ and $U_0$ parameters to be highly dominated by the $Q$ and $U$ maps of the lower frequencies, modulo the RM correction. The residuals at these frequencies are below the noise level, but show some structure. In particular, outliers in the 1.4 and 2.3 GHz residual maps coincide with areas of strong disagreement between, e.g., [1.4, 23] and [23, 33]. This includes regions of strong depolarization and potential map artifacts.

3. Pixels with high statistical uncertainties in $\beta^{\text{pol}}$ are masked. The statistical uncertainties in fit parameters $\beta^{\text{pol}}$ and RM follow the same pattern, illustrated in the top panel of Figure 9. This pattern strongly correlates with the S/N of the 23 GHz polarized intensity. This is expected given the relative S/N of the low frequency in each three-frequency set, and because the 33 GHz band has a lower S/N than that of 23 GHz. Simulations show the same effect, but because the simulated sky model is not identical to the data, the precise pattern is not replicated. Instead, the bottom panel of Figure 9 shows our estimate of the lowest S/N pixels in $P$ at 23 GHz. The estimate is based on evaluating $P^2$ from data splits and noting those pixels that can achieve negative values. We mask pixels for which $\sigma(\beta^{\text{pol}}) > 0.2$.

4. Pixels likely affected by Faraday depolarization are masked. We employ both a Galactic latitude cut and a cut on Faraday depth. The latitude cuts are $|\theta| < 40^\circ$ for the [1.4, 23, 33] fits, and $|\theta| < 15^\circ$ for the [2.3, 23, 33] combination. For both frequency combinations, pixels with Faraday depth $\phi > 40$ rad m$^{-2}$ are excluded. This value was chosen as an approximate threshold where the values of $\phi$ and RM shown in Figure 1 are in agreement within uncertainties, thus selecting lines of sight where the thin-screen assumption is most likely to be valid, and depolarization of least concern.

4.2.3. Comparison of [1.4, 23, 33] and [2.3, 23, 33] Results

In this section, we discuss systematic spatial differences between the $\beta^{\text{pol}}$ and RM parameter maps obtained from separate fits to [1.4, 23, 33] and [2.3, 23, 33] GHz. Since both of the three frequency fits each share the same 23 and 33 GHz data, differences in recovered parameters must result from either the model fit assumptions as a function of frequency (e.g., no curvature in the spectral index) or the low-frequency data themselves (e.g., calibration systematics). For example, the three-frequency model described by Equations (2)–(4) in Section 4.2.1 computes a Faraday rotation term only for the lowest frequency. The model fit adjusts the RM value in each pixel to enforce agreement with the 23 GHz polarization angle (with 33 GHz subdominant because of its lower S/N). Artifacts in the lowest frequency $Q$ and $U$ maps affecting polarization angle will result in an error in the recovered RM parameter, and thus affect the recovered $Q_0$ and $U_0$ values as well. Artifacts affecting the polarized intensity will bias the recovered value of $\beta^{\text{pol}}$. The two effects are not necessarily mutually exclusive.

In Figure 10, we show the $\beta^{\text{pol}}$ parameter maps from each of the three-frequency fits, and in Figure 11 we show results for the RM parameter. In both figures, the top plot corresponds to results from the [1.4, 2 3, 33] fit, and the middle plot shows those for [2.3, 23, 33]. The bottom plot in each panel of Figures 10 and 11 shows the per-pixel difference between the [1.4, 22, 33], and [2.3, 22, 33] parameter maps, and expresses the differences $\Delta$ as a fraction of an individual parameter’s uncertainty $\sigma(\Delta)$. The use of $\sigma(\Delta)$ here is not precise, as we use the root sum square of the uncertainties from the two fits even though we are not dealing with completely independent data sets. However, it is a useful representation in terms of the magnitude of deviation. These $\Delta/\sigma(\Delta)$ maps have been binned with color demarcations at $[-3.5, -3.0, -2.5, -2.0, 2.0, 2.5, 3.0, 3.5]$. Pixels with $\Delta/\sigma(\Delta)$ that exceed $-3$ (3) show as dark
blue (dark brown). In the case of the $\beta^{\text{pol}}$ uncertainty, the statistical uncertainty derived from the fit has been summed in quadrature with an additional 0.02 uncertainty that assumes an absolute gain uncertainty of 5% (Carretti et al. 2019).

In these figures, pixels with the largest disagreement in the $\Delta/\sigma(\Delta)$ maps are spatially clustered, rather than randomly scattered over the sky. The cluster of blue pixels in the $\Delta/\sigma(\Delta)$ map near $l \sim 30^\circ$, $45^\circ < b < 70^\circ$ is associated with a region of high polarized intensity emission in the 1.4 GHz map, which is not well correlated with emission in the 2.3 GHz map. The feature can be seen as the region of red in the lower-left corner of the left panel of Figure 5 and is also most easily seen in the $U_0$ map in Figure 7. This roughly wedge-shaped region lies in the decl. overlap strip ($-29^\circ < \delta < -10^\circ$) between the DRAO and Villa Elisa 1.4 GHz surveys. We have been showing results from the merged 1.4 GHz survey map, but also performed the same parameter model fit for the individual surveys to determine if there were significant differences in the parameter results in this region. Although there are clear differences in smaller-scale morphology and intensity, both surveys see the same general feature, which produces a steeper $\beta^{\text{pol}}$ and higher RM in this region than seen at 2.3 GHz. A bright feature is clearly not the result of depolarization, nor does it fit with the physical picture of synchrotron spectral index flattening with decreasing frequency. However, Wolleben et al. (2006) note the possibility of features from uncorrected beam sidelobes in the DRAO survey. The RM computed for this region echoes the wedge shape, and is not as morphologically consistent with the Faraday depth map as the RM derived from the 2.3 GHz data in the same location. For this region, evidence points to the 1.4 GHz map as the less accurate observation. Given the known presence of artifacts in the 1.4 GHz maps, it is likely that the 1.4 GHz data are the origin of the remaining discrepant pixels as well.

The above discussion raises the question of the accuracy of the northern portion of the [1.4, 23, 33] parameter maps, where there is as yet no complementary survey at a similar frequency for comparison. In this case, we must rely on secondary indicators of potentially compromised regions: general (but not

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Figure 8. (a) Residual (data minus model) Stokes $Q$ (top) and $U$ (bottom) maps for the [1.4, 23, 33] frequency combination. (b) As with (a), but residual maps are for the [2.3, 23, 33] frequency combination. Residuals at 23 and 33 GHz are visually dominated by instrument noise in most locations. However, poor fits resulting from strong depolarization in the low-frequency channel produce noticeable systematics in the residuals for all frequencies. Systematics from depolarization effects extend up to $|b| \sim 45^\circ$ in the [1.4, 23, 33] residuals and are particularly clear as dark red and blue shadings in the 1.4 GHz residuals. Pixel masking criteria include the rejection of outliers in the low-frequency residual maps.
from the feature in the we enlarged the pixel exclusion radius around one particular by the types of effects seen in the 1.4 and 2.3 GHz comparison, appear unusual compared to those of 23 and 33 GHz. Guided and visual detection of features in the comparison of the published. For those pixels meeting the selection criteria we also compare the $\delta$ primarily governed by the 23 GHz $S_{23}$ polarized intensity is at the level of instrument noise ($\sigma S_{23}$) we exclude these low $S_{23}$ locations from the parameter maps.

effect) morphological agreement with the Faraday depth map, and visual detection of features in the $Q$ and $U$ maps that appear unusual compared to those of 23 and 33 GHz. Guided by the types of effects seen in the 1.4 and 2.3 GHz comparison, we enlarged the pixel exclusion radius around one particular feature in the $[1.4, 23, 33]$ residual map centered around $l \sim 100^\circ$, $b \sim -50^\circ$ (see Figure 8). We did not find grounds to suspect additional regions of the 1.4 GHz survey for which $\delta > 0$, and thus proceed with our analysis.

Most of this section’s discussion has centered on a comparison of the $[1.4, 23, 33]$ and $[2.3, 23, 33]$ results, but we also compare the $[2.3, 23, 33]$ results with those previously published. For those pixels meeting the selection criteria described in Section 4.2.2, the $[2.3, 23, 33]$ $\beta_{pol}$ map presented here is consistent with those of Krachmalnicoff et al. (2018) and Fuskeland et al. (2021).

We find small differences between the RM values we obtained for these selected pixels with those of Carretti et al. (2019) (shown at full resolution in Figure 1). In the top left of Figure 12, we show the RM map of Carretti et al. (2019), degraded from the original $N_{side} = 32$ resolution to $N_{side} = 16$; the top right is the RM map we compute from $[2.3, 23, 33]$. For both maps, we use a less stringent pixel selection criterion that allows a greater percentage of sky to be visible. As mentioned in Section 2.2, Carretti et al. (2019) computed the RM from S-PASS, 23 and 30 GHz data, but noted systematic differences between the 23 and 30 GHz polarization angles, which resulted in their choice to exclude pixels for which the 23 and 30 GHz polarization angles differed by more than $15^\circ$. In the figure, we have excluded these same pixels, as well as pixels within the Galactic plane subject to depolarization ($|b| < 15^\circ$). The two maps in the top row of the Figure look very similar, but the difference between them (bottom left) shows indications of low-level large-scale differences. Since one analysis method includes 30 GHz data whereas the other does not, it is reasonable to posit that the systematic differences between the LFI and WMAP polarization data (see Section 3) are the source of this difference.

We demonstrate that this explanation is likely by creating a synchrotron sky model based on S-PASS polarized intensity, 23 GHz polarization angles, an assumed constant $\beta_{pol} = -3.2$, nominal instrument noise, and a CMB realization. Faraday rotation is applied to the simulated 2.3 GHz maps, assuming RM values from the Faraday depth map. The same model fitting routines used for the data are run on this simulated data set for three separate combinations: $[2.3, 23, 33]$, $[2.3, 23, 30, 33]$, and $[2.3, 23, 30, 33]$ where the superscript $s$ in the last combination indicates a systematic signature has been added to the 30 GHz $Q$ and $U$ maps. The systematic signature is computed from the data difference 30 GHz$-0.47 \times 23$ GHz (see Weiland et al. (2018) for further discussion).

The difference between RM parameters for the $[2.3, 23, 33] - [2.3, 23, 30, 33]$ simulation is consistent with a null map, whereas the $[2.3, 23, 33] - [2.3, 23, 30, 33]$ simulated RM map difference, shown at the bottom right of Figure 12, is very similar to that of the data difference on the bottom left.
imbalance-related modes may be suppressed through the use of dependent on the signed uncertainty for each radiometer. Loss-bands, the amplitude of the resultant large-scale morphology is between the assumed and actual correction values applied to each scan geometry, their amplitude is determined by the difference is still a subject of study.

Figure 11. RM results for individual fits after pixel selection. Top: RM from the fit to [1.4, 23, 33] GHz, in units rad per square meter. Middle: RM from fit to [2.3, 23, 33] GHz, same units. Bottom: The difference between the top and middle maps, divided by its $\sigma$ uncertainty. This is shown only for pixels in common between the top and middle maps. There are pixel clusters in the south for which $|\Delta RM/\sigma(\Delta RM)$ indicates significant disagreement. These clusters occur in the same general location as those pixels for which $|\Delta \beta pol|/\sigma(\Delta \beta pol)$ is high (see Figure 10).

Since the modeled systematic signature is generated from a difference between LFI and WMAP maps, this by itself does not isolate its origin. However, as discussed in Section 3, the Planck Collaboration associated the dominant contribution to the systematic modes with the PR3 30 GHz maps. In the next section, we discuss simulations designed to evaluate additional sources of uncertainty.

### 4.3. Simulations of Potential Additional Uncertainties

Although the large-scale differences between WMAP and Planck PR3 LFI polarization maps are dominated by the LFI gain uncertainty modes, how much WMAP contributes to this difference is still a subject of study.

A potential contributing signature in WMAP data arises from uncertainty in the applied correction for imbalance in the transmission efficiencies between the two sides of the differential WMAP instrument. Within the mapmaking framework, an error in the loss-imbalance correction results in the presence of large-scale modes in the WMAP maps (Jarosik et al. 2003, 2007, 2011; Bennett et al. 2013). While the spatial modes are well defined by scan geometry, their amplitude is determined by the difference between the assumed and actual correction values applied to each radiometer. With two radiometers in each of the 23 and 33 GHz bands, the amplitude of the resultant large-scale morphology is dependent on the signed uncertainty for each radiometer. Loss-imbalance-related modes may be suppressed through the use of the full covariance matrix available for this purpose or through selective filtering of the WMAP maps. However, since this analysis uses per-pixel fitting and unfiltered maps, simulations are necessary to estimate any bias in $\beta pol$ resulting from the presence of loss-imbalance modes. We use the simulations to derive a spatial template corresponding to loss-imbalance-related bias in $\beta pol$, and to determine the significance of any correlation between this template and the data.

The estimated maximum amplitude of loss-imbalance modes at 23 and 33 GHz is lower than the instrument noise, but could possibly produce biases in the recovered $\beta pol$ of up to $\Delta \beta \sim 0.05$. The 23 and 33 GHz mode morphologies are quite similar between the two bands, and attempting to individually estimate contributions at each frequency with the data here is not possible. However, the frequency with the largest potential uncertainty contribution is 23 GHz because the sky signal is larger at that frequency compared to 33 GHz.

We generate 1000 baseline realizations of 1.4, 2.3, 23, and 33 GHz $Q$ and $U$ maps with instrument noise, CMB, and synchrotron signal as estimated from the Planck FFP10 simulations (see Section 3). Then a complementary set of 1000 loss-imbalance simulations are generated, consisting of the same noise and signal components as the baseline set, plus a contribution$^{10}$ to the simulated 23 GHz $Q$ and $U$ maps from loss imbalance.

Fits to the [1.4, 23, 33] and [2.3, 23, 33] frequency combinations are performed as described in Section 4.2 for the data, separately for the 1000 baseline simulations that do not include the loss-imbalance signature, and for the 1000 simulations

$^{10}$K-band template1 from https://lambda.gsfc.nasa.gov/product/map/dr5/loss_imbal_template_rd_get.html.
that include the effect. The spectral index map recovered from the simulations that include the loss-imbalance term has a large-scale systematic difference from that recovered from the baseline simulations. From the mean of the difference between $\beta_{\text{pol}}$ maps recovered from these two simulation sets, we derive a map of the expected morphology that would be introduced in $\beta_{\text{pol}}$ if a loss-imbalance signature were present. Recovered $\beta_{\text{pol}}$ maps are then fit with the linear model $AT + c$, where $A$ scales the template $T$, and $c$ is a constant.

For the baseline simulations, the expected value is $A = 0.0$. For the simulations including loss imbalance, we expect to recover $A = 1$. However, this expectation only holds if the spectral index is constant over the whole sky, and the linear model is a good description of the data. In the case of a spatially varying $\beta_{\text{pol}}$, the recovered value of $A$ will be biased because of chance correlations with the spectral index morphology, and the partial sky coverage. For simulations in which the input $\beta_{\text{pol}}$ is the same as that we derive for the data (see Section 5), we recover $A = 1.43 \pm 0.21$ for the loss-imbalance set, and $A = 0.47 \pm 0.21$ for the null baseline set. When we employ the same fitting procedure to the data, we obtain $A = 0.25 \pm 0.07$.

The relatively low value of $A$ obtained for the data implies a low contribution from the WMAP loss-imbalance uncertainty $\Delta \beta \lesssim 0.01$, or alternatively that the fitting template obtained from simulations does not adequately match the data signature. In either case, we do not have sufficient evidence of a systematic bias in $\beta_{\text{pol}}$ resulting from loss-imbalance uncertainty, and do not include it in the estimated uncertainties. Residual quantification of the level of loss-imbalance signatures in the WMAP bands will benefit from the acquisition of additional independent data.

As described in Section 4.2, we have performed the [1.4, 23, 33] and [2.3, 23, 33] model fitting on a per-pixel basis, and ignored pixel–pixel covariances in the uncertainty estimation. Our simulations, which include the pixel–pixel correlations, confirm that we are not underestimating fitting uncertainties or biasing results because of this.

5. Composite Synchrotron $\beta_{\text{pol}}$ Map

We create a composite $\beta_{\text{pol}}$ map at $N_{\text{side}} = 16$ resolution by selectively populating pixels using the analysis results in Sections 3 and 4. The selection process follows a hierarchy: (1) fill all available pixels from the [2.3, 23, 33] GHz analysis, (2) next fill remaining unpopulated pixels from the [1.4, 23, 33] GHz analysis, and (3) fill any remaining unpopulated pixels from those in the [23, 33] GHz analysis. In the case of the [23, 33] analysis, which was performed at $N_{\text{side}} = 8$, we replicated the value of each of the lower resolution pixels to fill the four corresponding pixels at the one step higher pixel resolution. A color-coded map showing which of the three analyses was used to populate each pixel is shown in the top panel of Figure 13. Total sky coverage is $\sim 44\%$, with $\sim 73\%$ coverage of pixels for which $|b| > 45^\circ$.

The composite $\beta_{\text{pol}}$ map is shown in the middle panel of Figure 13, with the uncertainty map in the bottom panel. Pixels in the uncertainty map are populated in the same manner as for the $\beta_{\text{pol}}$ map. Uncertainties computed for the [1.4, 23, 33] and [2.3, 23, 33] analyses include an absolute gain uncertainty contribution added in quadrature with the statistical uncertainties.

The mean $\beta_{\text{pol}}$ latitudinal profile derived from the composite map is given in Table 2 and plotted in the top panel of

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**Figure 13.** Top: color-coded map showing where the three separate analyses presented in Sections 3 and 4 contribute to the final composite $\beta_{\text{pol}}$ map. Pixels from the high S/N region analysis using the WMAP 23 and 33 GHz bands are shown in red. Those from the higher latitude [1.4, 23, 33] are in blue, and pixels from the [2.3, 23, 33] region are in cyan. Regions in gray are not included in the final map. Middle and bottom: composite map of polarized synchrotron spectral index (middle) and associated uncertainty (bottom) based on analyses of the 1.4, 2.3, 23, and 33 GHz maps. Pixels in gray are not analyzed. Section 5 describes the process by which the composite maps are created.

**Figure 14.** Broad Galactic latitude bins (20° wide) are used in view of the partial sky coverage. Uncertainties are computed from the standard deviation within each bin, rather than using a weighted average. Because the $\beta_{\text{pol}}$ map lacks complete sky coverage, the latitude profile, particularly at midlatitudes, is likely biased. We suspect that any bias is toward larger values of $\beta_{\text{pol}}$ since coverage is weighted toward inner galaxy regions. The profile in Figure 14 shows the same index steepening at $b = -80^\circ$ seen by Fuskeland et al. (2021), which is expected
since S-PASS and 23 GHz data dominate the results for this region in both cases. For contrast, latitudinal profiles for two synchrotron spectral index models from PySM (Python Sky Model; Thorne et al. 2017) are shown in blue. The solid blue line is Model 1, which has been used for example in Planck FFP10 model simulations. The dashed blue line is for Model 2, a symmetric profile based on WMAP polarized foreground findings. Neither model captures the detail seen in the black trace of the data.

For further context, in the bottom panel of Figure 14 we show the latitudinal profile of the synchrotron intensity spectral index of the edge-on galaxy NGC 891, often cited as a Milky Way analog (Hummel et al. 1991). We compute the NGC 891 profile from the 1.5–6 GHz nonthermal spectral index map of Irwin et al. (2019), binning as a function of perpendicular distance $z$ above and below the midplane of that galaxy. Bins are in increments of $0\degree1$, where $1\degree$ corresponds to $\sim$2.6 kpc (as a guide, an approximate scale height for cosmic-ray electrons in the Milky Way is 1 kpc; Page et al. 2007). Although the NGC 891 profile is for intensity and not polarization, there are similarities in high-latitude spectral index behavior, including similar spectral index values and a north/south asymmetry.

6. Connecting Polarization and Intensity Data

It is reasonable to consider augmenting the gaps in the composite $\beta^{\text{pol}}$ map with $\beta^i$ values derived using temperature (intensity) maps of Galactic synchrotron emission. Full-sky temperature observations such as those from WMAP and Planck LFI include contributions from multiple components in addition to synchrotron, including CMB, free–free, thermal dust, and spinning dust emission. Component separation studies limited to these frequencies are affected by degeneracies between multiple component SEDs. At frequencies $\lesssim5$ GHz, foreground emission is much stronger than the CMB, and thermal and spinning dust emission are expected to be subdominant to synchrotron and free–free (Harper et al. 2022), which mitigates the component separation challenge. Unfortunately, calibration of ground-based observations at these lower frequencies is a difficult enterprise, and currently available data limit effective use of these frequencies for diffuse synchrotron spectral index determination, despite a number of efforts in the literature.

To illustrate the problem, we construct a synchrotron intensity model from a low-frequency template and extrapolate that template to other frequencies assuming that the $\beta^{\text{pol}}$ we derived above is applicable to temperature observations. We then compare that model to publicly available temperature maps at 1.4 and 2.3 GHz. As with previous investigations (e.g., Bennett et al. 2003, 2013; Planck Collaboration X 2016; Planck Collaboration IV 2020), we choose the Haslam 408 MHz map (Haslam et al. 1982) as the synchrotron template. For consistency with the Planck Collaboration X (2016) component separation, we use the destriped version of Remazeilles et al. (2015), from which we remove an 8.9 K extragalactic background offset. We also remove an estimate of free–free emission using the model of Planck Collaboration X (2016), but our results are not substantially affected by that choice. Single masked pixels in the $\beta^{\text{pol}}$ map have been inpainted with the mean from neighboring pixels in this simulation, for the purpose of allowing a more contiguous visual exposition of data-model residuals.

The top left panel in Figure 15 shows the 1.4 GHz intensity map from the combined northern and southern surveys of Reich & Reich (1986) and Reich et al. (2001), available from CADE. The quoted absolute gain uncertainty is roughly 5%, with a zero-point uncertainty of 500 mK. The middle left panel shows the corresponding model sky emission consisting of a

Table 2

| Bin Center | $\beta^{\text{pol}}$ | $\beta^i$ |
|------------|---------------------|----------|
| 80°        | $-3.238 \pm 0.011$  | $-3.245 \pm 0.007$ |
| 60°        | $-3.217 \pm 0.007$  | $-3.245 \pm 0.007$ |
| 40°        | $-3.217 \pm 0.012$  | $-3.245 \pm 0.007$ |
| 20°        | $-3.088 \pm 0.016$  | $-3.245 \pm 0.007$ |
| 5°         | $-2.964 \pm 0.014$  | $-3.245 \pm 0.007$ |
| $-5°$      | $-2.960 \pm 0.013$  | $-3.245 \pm 0.007$ |
| $-20°$     | $-3.166 \pm 0.026$  | $-3.245 \pm 0.007$ |
| $-40°$     | $-3.198 \pm 0.010$  | $-3.245 \pm 0.007$ |
| $-60°$     | $-3.245 \pm 0.007$  | $-3.245 \pm 0.007$ |
| $-80°$     | $-3.378 \pm 0.010$  | $-3.245 \pm 0.007$ |
synchrotron component computed from $T_{\text{synch}}^{408}(1.41/0.408)^{\beta_{\text{pol}}}$, a free–free component estimated using the model of Planck Collaboration X (2016), and an added monopole of 3300 mK, which accounts for the extragalactic background and zero-point uncertainties. The two panels have visual similarities, but it is clear that there is a high-latitude north–south asymmetry in the observations compared to the model. The difference between the observations and model is shown in the bottom left panel. The magnitude of the high-latitude differences in the north would require a difference of order 0.5 between $\beta_{\text{pol}}$ and $\beta_{\text{free}}$ such that a spectral index map produced from 0.408 and 1.4 GHz would deviate significantly compared to that seen in $\beta_{\text{pol}}$. We note that the original northern sky determination of $\beta_{\text{pol}}$ using these two frequencies (Reich & Reich 1988) indicated a typical $\beta_{\text{pol}} \sim -2.5$ at $b \sim 40^\circ$, as opposed to $\beta_{\text{pol}}$ near $-3.2$.

The right half of Figure 15 compares the more recent southern sky 2.3 GHz intensity map from the S-PASS survey (Carretti et al. 2019) against a model prediction. The 2.3 GHz model includes synchrotron and free–free, as did the 1.4 GHz model, but not an extragalactic component, since the monopole of the S-PASS map is calibrated to Galactic emission levels with an uncertainty of 70 mK. There is a significant discrepancy in brightness between the model and observations, at a level greater than the 5%–10% absolute gain uncertainty quoted for the S-PASS and Haslam surveys. A linear correlation between the model and the observations indicates that the 2.3 GHz observations are $\sim 2$–3 times brighter than the model; we do not give an exact number because the correlation is neither tight nor strictly linear. The linear correlation derives an offset near 50 mK, which is consistent with the zero-point uncertainty. We adjust the model based on the correlation slope and offset, and subtract it from the observations, with the result shown in the bottom right panel. This residual has a large-scale spatial pattern, but one quite different from that seen at 1.4 GHz. The scaling factor is unexpected, but as we can reproduce the polarization fraction values in Figure 27 of Carretti et al. (2019), there does not seem to be an error in our use of the delivered data files. The high scaling factor implies a calibration inconsistency between the older (408 MHz, 1.4 GHz) intensity data, and the newer 2.3 GHz intensity survey. The recent independent comparison of $\beta_{\text{pol}}$ derived using either Haslam 408 MHz or preliminary C-BASS$^{12}$ 5 GHz data as the synchrotron template (Harper et al. 2022) would seem to indicate that the S-PASS intensity data are in discord with the other surveys. There is no indication that S-PASS polarization data are in substantial disagreement on large scales with 1.4 GHz polarization data, however (Section 4.1).

In short, new polarization and intensity data from ongoing experiments such as C-BASS (Jones et al. 2018) and QUIJOTE$^{13}$ (Génova-Santos et al. 2015), as well as upcoming experiments, promise to provide valuable new constraints on the CMB foregrounds.

### 7. Foreground Removal Implications

Current data limitations prevent a well-constrained determination of polarized synchrotron foreground contributions over the entire sky. As noted by Ade et al. (2021) and Fuskeland et al. (2021), contributions at larger spatial scales ($l < 20$) of interest to e.g., LiteBIRD$^{14}$ (Hazumi et al. 2020) and CLASS$^{15}$ (Dahal et al. 2022) are of most concern, where the foreground power is greatest. The primary issue of cosmological importance is the accuracy to which foregrounds can be characterized and removed in relation to the CMB signal. In that respect, the spatial scale and range of $\beta_{\text{pol}}$ variations is a key factor. A significant large-scale $\beta_{\text{pol}}$ gradient with Galactic latitude is demonstrated in Figure 14. Longitudinal variations are more subtle, but present. Two examples of selected regions taken from the composite $\beta_{\text{pol}}$ map follow.

In the first example, we investigated the extent to which we could detect spectral variations in the BICEP2$^{16}$ (BICEP Collaboration 2014) field, based on the 25 unmasked $N_{\text{side}} = 16$ pixels that lie within the survey footprint (a 400 deg$^2$ patch centered near $l \sim 316^\circ$, $b \sim -58^\circ$, as taken from the LAMBDA$^{17}$ Footprint Tool$^{18}$). We plot the $\beta_{\text{pol}}$ values and uncertainties for these pixels in Figure 16. Uncertainties increase with decreasing Galactic longitude across the field, but the spectral index distribution is consistent with no variation, with a probability to exceed (PTE) = 0.21. The mean value we compute for this region is $-3.25 \pm 0.04$ (statistical) or 0.02, systematic). This determination is consistent with the $\beta_{\text{pol}} = -3.22 \pm 0.06$ value adopted by the BICEP Collaboration for the entire field (Ade et al. 2021). The determination is also consistent with the mean latitudinal variation in $\beta_{\text{pol}}$ predicted for this field from the Figure 14 data points. We compute the expected range in $\beta_{\text{pol}}$ based on the range of latitudes in the BICEP2 field and a linear interpolation.

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12 C-band All Sky Survey.  
13 QUI JOnT Tenerife.  
14 (Lite) B-mode polarization and Inflation from cosmic background Radiation Detection.  
15 Cosmology Large Angular Scale Surveyor.  
16 Background Imaging of Cosmic Extragalactic Polarization 2.  
17 Legacy Archive for Microwave Background Data Analysis.  
18 https://lambda.gsfc.nasa.gov/toolbox/footprint/
of the values in Table 2. The computed peak-to-peak latitudinal variation in $\beta_{\text{pol}}$ is $\sim 0.05$, which is not distinguishable from a constant given the data uncertainties.

The second example illustrates longitudinal spectral index behavior at four fixed Galactic latitudes. We choose high latitudes for which the composite $\beta_{\text{pol}}$ map has a reasonably large range of longitude coverage. These fixed latitude slices, shown in Figure 17, are centered on $b = \pm 52.5^\circ$ and $b = \pm 72.5^\circ$, and include all unmasked $\beta_{\text{pol}}$ map pixels within each of the $5^\circ$ wide latitude bins. For each of the four slices, we test the null hypothesis that spectral indices at all longitudes are consistent with a constant value given by their weighted mean (indicated by the red horizontal line in the Figure). The PTE for the top panel is $2.5 \times 10^{-4}$, and those for the remaining three panels are all of orders of magnitude lower and thus show strong statistical evidence for longitudinal variation.

The role that these variations play in synchrotron foreground removal depends upon the range of frequencies being analyzed, the selected sky region, the specific removal technique being employed, and the potential removal accuracy possible in the context of instrumental noise. The scale of spatial variations shown in this paper favor techniques adaptable to variations of order 0.05–0.1 on scales of several degrees, and tend to disfavor removal methods that rely on an assumed constant SED over large areas. For example, a simulated noiseless case that extrapolates a 5 GHz template to determine the 90 GHz synchrotron component over $|b| > 50^\circ$, but uses a fixed spectral index of $-3.2$, would produce a median residual value corresponding to a tensor-to-scalar ratio $r = 0.05$, with some residuals exceeding $r = 0.5$. With the recent BICEP2/Keck upper limit $r < 0.036$ (Ade et al. 2021), such residuals conflict with the goal of detecting B modes at 90 GHz. However, as we saw for the smaller BICEP2 footprint, the use of a constant $\beta_{\text{pol}}$ was within the uncertainties derived in this paper.

Not all experiments have the option to survey the entire sky and must choose cosmologically interesting fields. In the top panel of Figure 18, we have computed the frequency at which polarized synchrotron and thermal dust contribute equally to the total foreground emission, for those regions of the sky that we have analyzed. The synchrotron model is based on the WMAP 23 GHz polarized intensity and the map of $\beta_{\text{pol}}$ shown in Figure 13. The thermal dust polarized contribution is modeled using a modified blackbody with spectral index 1.55 and dust temperature 19.6 K (Planck Collaboration IV 2020), and with the amplitude corresponding to the NPIPE 353 GHz full-mission polarized intensity. For off-plane regions, we find $\nu_{\text{eq}} = 79 \pm 13$ GHz. A similar mean value was found by Planck Collaboration IV (2020) based on rms amplitudes computed over sky fractions from 0.27–0.83. In the top panel of Figure 18, pixels with $\nu_{\text{eq}} > 90$ tend to have lower polarized dust emission. The bottom panel of the Figure shows the total foreground polarized intensity at $\nu_{\text{eq}}$ in thermodynamic temperature units. Most, but not all, higher latitude regions have lower foreground contributions. While we have estimates for uncertainties in $\beta_{\text{pol}}$, uncertainties in the
polarized dust spectral index are not well characterized (Osumi et al. 2021), and thus these maps only serve as a guide rather than a complete picture.

8. Conclusions

Well-constrained characterization of the amplitude and SED of polarized synchrotron emission is a necessity for future high-sensitivity cosmological experiments observing frequencies <150 GHz. This paper explores the extent to which current publicly available data sets with significant sky coverage can advance that goal. The analysis reaches three main conclusions:

1. The polarized synchrotron spectral index, $\beta_{\text{pol}}$, is not a constant over the entire sky, and the variations we derive are not well matched by frequently used models such as those in PySM.

2. Current public data are insufficient to characterize $\beta_{\text{pol}}$ for the future, for three reasons: a lack of full-sky coverage; conflicting results between experiments; and a lack of sensitivity to support future B-mode experiments.

3. Some sky regions are well enough measured to provide some guidance for some experiments, such as in the BICEP2/Keck analysis (Ade et al. 2021).

We summarize our findings in greater detail below.

The public data we analyze comprise Stokes $Q$ and $U$ maps from the 1.4 GHz surveys of DRAO and Villa Elisa (Testori et al. 2008; Wolleben et al. 2010a), the S-PASS 2.3 GHz southern sky survey (Carretti et al. 2019), the WMAP 23 and 33 GHz 9 yr maps (Bennett et al. 2013), and in specific cases, Planck 30 GHz maps from PR3 (Planck Collaboration I 2020), PR4, a.k.a. NPIPE (Planck Collaboration Int. LVII 2020), and BeyondPlanck (BeyondPlanck Collaboration et al. 2020).

A significant part of our analysis involves data selection and consistency tests between independent data sets that have differing sensitivities and calibration uncertainties. The presence of Faraday depolarization in both the 1.4 and 2.3 GHz maps requires that we divide the analysis into two spatial domains: the high S/N regions of the Galactic plane and spurs (Section 3), and lower S/N off-plane regions (Section 4). Within these two regimes, we found additional calibration discrepancies that required further selection choices:

1. In the high S/N Galactic plane and spurs (Figures 2 and 3), we found significantly different results for $\beta_{\text{pol}}$ in the plane when using pairs of frequencies from WMAP only (23 and 33 GHz) and from WMAP 23 GHz in combination with any of the Planck 30 GHz processing pipeline versions. The discrepancy traces to large-scale systematic differences between the WMAP 23 GHz and LFI 30 GHz maps, which have been shown (Planck Collaboration Int. LVII 2020) to change as a function of the mapmaking algorithm used to process the 30 GHz data. For this reason, we sacrifice the S/N advantages that would be gained from a combination of WMAP and Planck LFI data, and our results are based on the WMAP [23, 33] GHz maps only.

2. In the lower S/N off-plane domain, WMAP and Planck data lack the requisite S/N to determine $\beta_{\text{pol}}$ on all but very large sky patches. It is therefore desirable to include lower frequency radio data (in our case, 1.4 and 2.3 GHz) where the synchrotron S/N is much higher. In Section 4.1, we show that systematic calibration differences exist between the 1.4 and 2.3 GHz surveys for $b < -70^\circ$ (Figure 5), and argue against using these two frequencies alone for a southern sky determination of $\beta_{\text{pol}}$.

3. In light of the above two points, our off-plane analysis fits a parametrized sky model to two three-frequency map combinations: [1.4, 23, 33] GHz and [2.3, 23, 33] GHz (Section 4.2). Although we fit the entire sky, not all pixel fits have the same quality. We find some pixels for which there is strong disagreement between [1.4, 23, 33] and [2.3, 23, 33]. We further downselect pixels based on depolarization, systematic measurement errors, and/or low S/N regions.

Following our data quality assessment, we construct a composite map of the polarized synchrotron spectral index $\beta_{\text{pol}}$ (Section 5). The composite map is populated following a hierarchy that first fills all available pixels from the [2.3, 23, 33] GHz analysis, next fills the remaining unpopulated pixels from the [1.4, 23, 33] GHz analysis, and finally fills any remaining unpopulated pixels from those in the [23, 33] GHz analysis (top panel of Figure 13). This results in $\beta_{\text{pol}}$ coverage over 44% of the sky (73% for $|b| > 45^\circ$), with a pixel resolution of $\sim$3°7 ($\sim$7°3 in the plane). The maps and associated uncertainties are shown in Figure 13. Uncertainties include statistical noise and instrument absolute gain uncertainties. We searched for potential bias in the composite $\beta_{\text{pol}}$ map arising...
from WMAP mirror transmission efficiency differences, but did not definitively detect any (Section 4.3).

Variation in $\beta_{\text{pol}}$ is an important factor in synchrotron foreground removal. Because of data limitations, we are unable to discern spectral curvature, and have assumed a pure power-law frequency dependence in constructing the composite spectral index map. Based on the composite map, we characterize spectral-spatial variations in both Galactic latitude and longitude:

1. In the Galactic plane and spurs, we find $-3.2 < \beta_{\text{pol}} \lesssim -3$ for much of the region, but with a flatter value for the Fan Region in the outer galaxy.
2. We find a clear gradient in $\beta_{\text{pol}}$ with Galactic latitude, but the gradient is not symmetric between the northern and southern hemispheres. The mean $\beta_{\text{pol}}$ latitude profile indicates spectral index steepening with increasing latitude south of the Galactic plane with $\Delta \beta_{\text{pol}} \approx 0.4$, and a smaller steepening of 0.25 in the north. Near the south Galactic pole the polarized synchrotron spectral index is $\beta_{\text{pol}} \approx -3.4$. As discussed in the text, indications of a gradient have previously been reported in the literature. We note that the latitude profile we derive has the potential to bias high, particularly in the midlatitude regions where spatial coverage is predominantly from inner galaxy regions.
3. For those high-latitude sky regions included in our composite map, we find longitudinal variations in $\beta_{\text{pol}}$ of order 0.05–0.10 about the mean latitudinal value. This result has a greater dependence on the accuracy of the data uncertainties used in the analysis, and leaves unanswered the applicability to fainter high-latitude regions for which we could not sufficiently constrain $\beta_{\text{pol}}$.
4. We find $\beta_{\text{pol}}$ within the BICEP2/Keck survey footprint to be consistent with a constant value, $\beta_{\text{pol}} = -3.25 \pm 0.04$ (statistical) $\pm 0.02$ (systematic), in accord with the value adopted in Ade et al. (2021).

Since data selection criteria did not allow a full-sky determination of $\beta_{\text{pol}}$, we assessed the possibility of filling coverage gaps using a spectral index determined from 0.408, 1.4, and 2.3 GHz intensity data. At these frequencies, the diffuse Galactic sky signal is dominated by synchrotron emission, with some contribution from free-free. Unfortunately, a preliminary comparison between these data sets only served to emphasize systematic differences between currently available observations at these frequencies (Section 6, Figure 15), and no $\beta_{\text{pol}}$ determinations were included in our analysis. 

At $N_{\text{side}} = 16$ resolution, the $\beta_{\text{pol}}$ map is of most interest to ongoing and future experiments that survey large sky areas and target low multipole CMB reionization and recombination signatures ($2 \lesssim f \lesssim 100$), such as LiteBIRD and CLASS. For those portions of the sky that it covers, the $\beta_{\text{pol}}$ map may be used directly, although resolution and sensitivity limitations restrict its applicability. For those regions lacking coverage, the map serves to anticipate the level of synchrotron spectral variation that future instrument design and sky cleaning algorithms must account for. We express this roughly as a latitudinal variation overlaid with longitudinal variations of order 0.05–0.1 on few-degree scales.

Ultimately, however, the limitations and inconsistencies among data sets encountered in this work make clear the value of additional independent surveys at multiple frequencies. These additional surveys are necessary to provide increased sensitivity at low S/N high-latitude locations and provide sufficient frequency coverage to assess, e.g., spectral curvature. The frequency window of most utility for high latitudes is $2.3 < \nu < 30$ GHz, and especially between 10 and 20 GHz where depolarization is minimized while still ensuring synchrotron signal dominance. There are ongoing ground-based projects working to augment frequency coverage in this window. Calibration from the ground is substantially more difficult than in space, however, and many analyses will still rely on WMAP and Planck data as key frequencies. Although the absolute calibration for these two surveys is sub-percent, the S/N at some high-latitude locations is insufficient for precision determination of $\beta_{\text{pol}}$, and additional high-quality data are needed.

We will make the composite $\beta_{\text{pol}}$ map available through LAMBDA upon publication.

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