Slew maneuver control of flexible spacecraft for vibration suppression

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Abstract. This paper presents the input shaping technique for reducing the vibrations of flexible structures under attitude control inputs applied to the flexible spacecraft. The input shaping profile is investigated for the effective suppression of transient vibrations of modal responses. It is applied to the input commands to minimize the residual vibrations and suppress the overshoot of the modal responses. The results of numerical simulations, using a simple dynamic model of a flexible spacecraft, show that the input shaping technique is useful for suppressing the residual vibrations caused by attitude maneuvers.

1. Introduction

One of the problems faced during the attitude control of a spacecraft is unsatisfactory precision control caused by vibrations of the flexible structures. It is very important to suppress the residual vibrations quickly to meet the mission requirements. The input shaping technique proposed by Smith [1], is one of the most effective methods for suppressing the transient vibrations of a flexible structure and it has been used in many applications [1]-[4]. This technique has also been applied to the concurrent design with feedback control of nonlinear coupled vibration systems such as the trajectory tracking control of a flexible manipulator [5] and the slewing maneuver of a flexible spacecraft [6].

The main disadvantage of the input shaping is that the equation used to reduce the residual vibrations requires the estimates of the natural frequencies and damping ratios. If these structural parameters are different from the real values, the method does not work and the effectiveness of the input shaping in vibration suppression is degraded. Furthermore, the input shaper is designed for suppressing a single vibration mode and it is not effective for other vibration modes.

This paper introduces an input shaping technique based on Nil-Mode-Exciting profiler (NME). The NME profiler has been studied for shaping the input command to attenuate the vibration of the spacecraft appendage structures [7]-[8]. The input command is designed such that not to excite all modes of vibration higher than the designated natural frequency of the flexible structure.

The Input Shaping technique and the NME profiler are studied in this paper for the suppression of residual vibrations due to the rest-to-rest maneuver of the spacecraft and they are integrated to achieve a superior performance in vibration damping.
2. Input shaping method for vibration suppression

Figure 1 shows the block diagram of a servo control system consisting of a feedback loop for pursuing the control objective, such as the attitude maneuver of a flexible satellite, and an input shaper for suppressing the excitation of vibrational responses.

![Block diagram of input shaper with a feedback controller.](image)

The amplitude of vibration response when $N$ impulses are fed to a one-degree-of-freedom vibration system is given as follows.

$$A_{\text{amp}} = \sqrt{\left(\sum_{j=1}^{N} B_j \cos \omega_d t_j\right)^2 + \left(\sum_{j=1}^{N} B_j \sin \omega_d t_j\right)^2}$$

(1)

where

$$B_j = A_j \frac{\omega_0}{\sqrt{1-\zeta^2}} \exp \left(-\zeta \omega_0 (t_N - t_j)\right)$$

(2)

In addition, $A_j$ denotes the amplitude of the $j$th impulse, $t_j$ is the time at which the impulse occurs, $\omega_0$ is the natural frequency of the system, and $\zeta$ is the damping ratio of the system. To eliminate the vibrational response after the input has been applied, the expression for $A_{\text{amp}}$ should be zero at time $t_N$ when the input ends. In other words, both the squared terms in Eq. (1) should be equal to zero.

$$\sum_{j=1}^{N} A_j \exp \left(-\zeta \omega_0 (t_N - t_j)\right) \cos \omega_d t_j = 0$$

(3)

$$\sum_{j=1}^{N} A_j \exp \left(-\zeta \omega_0 (t_N - t_j)\right) \sin \omega_d t_j = 0$$

(4)

For example, consider two impulse inputs at times $t_1$ and $t_2$ with amplitudes $A_1$ and $A_2$. The responses to the two impulse inputs cancel each other, and the vibrations after time $t_2$ vanish, only if the natural frequency and damping ratio of the system are known exactly. Figure 2 demonstrates the input shaping technique using two impulses.

![Basic idea of the input shaping technique.](image)

For the successful application of the input shaping technique in most real systems, the constraint equations, i.e., Eqs. (3) and (4) must ensure robustness to modeling errors. Singer and Seering [2] have developed a method of robust input shaping by using the condition that the derivative of Eqs. (3) and (4) with respect to the frequency of the residual vibrations equals zero. The resulting shaper is called a
zero vibration and derivative (ZVD) shaper. Further details may be obtained by referring to their study.

3. Nil-Mode-Exciting profiler

The concept of Nil-Mode-Exciting profiler is based on the use of the sinc function

\[\text{sinc}(t) = \frac{\sin(\omega_0 t)}{\omega_0 t}\]  

(5)

A Fourier transform of the sinc function shows the characteristics of an ideal low-pass filter with a cutoff frequency of \(\omega_0\) rad/s. The frequency response function and time history of the sinc function are shown in Fig. 3.

\[A = \sin(\omega_a(t-t_{\text{offset1}})) \omega_a(t-t_{\text{offset1}}) - \sin(\omega_a(t-t_{\text{offset2}})) \omega_a(t-t_{\text{offset2}})\]  

(6)

where

\[t_{\text{offset2}} - t_{\text{offset1}} = \frac{2\pi}{\omega_a}\]  

(7)

Equation (6) represents the sum of the accelerating and decelerating functions with time offsets. A typical synthesized sinc function is shown in Figure 4.
Practically, to modify the above infinite sinc function into a finite-time function, it is necessary to introduce a time window function such as a Hamming function:

\[ w(x) = 0.54 - 0.46 \cos 2\pi x \quad \text{if } 0 \leq x \leq 1 \]  \hspace{1cm} (8)

4. Integration of Input Shaping and Nil-Mode-Exciting profiler

The third method is introduced in this paper and shown in Fig. 5, where a single NME profile for the torque input (solid line) is divided into three inputs (dotted lines) according to the sequence of three inputs for the Input Shaping.

5. Dynamics of a flexible spacecraft

A simplified dynamic model of a satellite with flexible appendages is shown in Figure 6. The model of the flexible structure is considered as a one-degree-of-freedom-vibration system, as defined in literature [4].
where \( J_b \) is the moment of inertia of the satellite body, \( m \) is the mass of the adjunct, \( R \) is the mounting position from the center of the satellite, \( \theta \) is the attitude angle, \( \theta_a \) is the angle of the arm, \( k \) is the spring constant of torsion, \( T_c \) is the torque of the reaction wheel, and \( T_d \) is the torque of the thrusters.

The dynamics of the attitude of the satellite with a flexible structure is derived by Lagrange’s equations of motion as follows.

\[
\begin{align*}
[J_b + 2m(R + L)^2]\ddot{\theta} + 2m(R + L)L\ddot{\theta}_a &= T_d + T_c \\
2m(R + L)L\ddot{\theta} + 2mL^2\ddot{\theta}_a &= -2k\theta_a
\end{align*}
\]  \tag{9} \tag{10}

where the angular velocity of the satellite body is ignored, because it is very small when compared to the angular velocity of the reaction wheel.

The torque of the thruster is considered as a pulse in one direction, and it functions with unit amplitude, changing sign at a specified time sequence. The impulses of the thruster torque are shown in Table 1, where \( A_j \) denotes the normalized amplitude of a single impulse, \( t_j \) is the switching time; \( t_1 = 0 \), because the first impulse is applied at the initial time.

**Table 1.** Amplitudes and switching times of thruster torque

| \( A_j \) | \( t_j \) |
|---|---|
| 1 | \( t_1 \) |
| -1 | \( t_2 \) |
| 1 | \( t_3 \) |
| -1 | \( t_4 \) |
| \vdots | \vdots |
| -1 | \( t_n \) |

In order to apply the ZVD, Eqs. (3) and (4), and their derivatives with respect to the natural frequency are used for the constraint equations. The number of switching times, \( n \), of the impulses to suppress the tangent vibration with frequency robustness is determined by solving the following equations.
\begin{align*}
\sum_{j=1}^{N} (-1)^{j+1} \sin \omega d t_j &= 0 \\
\sum_{j=1}^{N} (-1)^{j} \cos \omega d t_j &= 0 \\
\sum_{j=1}^{N} (-1)^{j+1} t_j \sin \omega d t_j &= 0 \\
\sum_{j=1}^{N} (-1)^{j+1} t_j \cos \omega d t_j &= 0
\end{align*}

(11) \quad (12) \quad (13) \quad (14)

We consider the torque of the reaction wheel. The attitude angle of the satellite is controlled by a simple proportional-integral-derivative (PID) controller.

\[ T_c = -I_w \ddot{\theta}_w = -k_p \theta - k_i \int_0^t \theta \tau - k_d \dot{\theta} \]  

(15)

where \( I_w \) is the moment of inertia of the reaction wheel and \( \ddot{\theta}_w \) is the angular acceleration of the reaction wheel.

6. Result of numerical simulation

The following numerical values are used in simulation.

\[ J_b = 12800 \text{ kg} \cdot \text{m}^2, \quad m = 25 \text{ kg}, \quad R = 2 \text{ m}, \quad L = 10 \text{ m}, \quad k = 5685 \text{ Nm} \]

The amplitudes and switching times of the thruster are obtained by using the damped natural frequency \( \omega_d \). These are listed in Table 2, where the time for the ignition of the thruster is set to \( t_1 = 0 \text{ s} \) and \( t_4 = 100 \text{ s} \).

| \( A_t \) | \( t_j(s) \) |
|----------|-------------|
| 1        | 0           |
| -1       | 0.55        |
| 1        | 1.11        |
| -1       | 100         |
| 1        | 100.55      |
| -1       | 101.11      |

Table 2 Amplitudes and switching times of thruster torque.

The results of numerical simulations are shown in Figures 7-11. Figure 7 shows the time response of the flexible structure when input shaping is not used, and Figure 8 shows the response with input shaping. The vertical axis represents \( \dot{\theta}_a \), which is the amplitude of the angular displacement of the flexible structure, and the horizontal axis represents time. These figures show that the convergence time is shorter, and the maximum amplitude of displacement is smaller, when input shaping is applied for the input torque.
The input torque $T_d$ of the thruster with input shaping, and the PID feedback control torque $T_c$ of the reaction wheel are shown in Figure 9. Figures 10 and 11 show the effect of zero vibration and derivative (ZVD) shaper for enhancing the robustness against the parameter errors. The vibrational response is excited due to the 15% error in the natural frequency of the flexible structure (Figure 10), whereas it is remarkably improved by using the robust input shaping (Figure 11). Figure 12 shows the input torque corresponding to the robust input shaping. It is observed from these numerical simulations that the input shaping technique and the ZVD version are very effective in suppressing the residual vibrations of the flexible structures excited by the attitude control torque.
Figure 9. Torque of the reaction wheel and thruster with input shaping.

Figure 10. Displacement with ordinary input shaping (15% error in natural frequency).
Figure 11. Displacement with robust input shaping (15% error in natural frequency).

Figure 12. Torque of the reaction wheel and thruster with robust input shaping.

7. Conclusion

Input shaping is very effective in suppressing the residual vibrations of flexible structures. The results of the numerical simulation demonstrate that the input shaping method is highly effective for agile attitude maneuvers of a flexible spacecraft. Future work involves comparing the input shaping method with other command shaping methods such as Null-Mode Excitation (NME) profiler, and experimental demonstration of the input shaping method with a simple laboratory model.

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