Prediction of the Rheological Properties of Invert Emulsion Mud Using an Artificial Neural Network

Abdelrahman Gouda, Samir Khaled,* Sayed Gomaa, and Attia M. Attia

ABSTRACT: Successful drilling operations require optimum well planning to overcome the challenges associated with geological and environmental constraints. One of the main well design programs is the mud program, which plays a crucial role in each drilling operation. Researchers focus on modeling the rheological properties of the drilling fluid seeking for accurate and real-time predictions that confirm its crucial potential as a research point. However, only substantial studies have real impact on the literature. Several AI-based models have been proposed for estimating mud rheological properties. However, most of them suffer from non-being field applicable attractive due to using non-readily field parameters as input variables. Some other studies have not provided a comprehensive description of the model to replicate or reproduce results using other datasets. In this study, two novel robust artificial neural network (ANN) models for estimating invert emulsion mud plastic viscosity and yield point have been developed using actual field data based on 407 datasets. These datasets include mud plastic viscosity (PV), yield point (YP), mud temperature (T), marsh funnel viscosity (MF), and solid content. The mathematical base of each model has been provided to provide a clear means for models’ replicability. Results of the evaluation criteria depicted the outstanding performance and consistency of the proposed models over extant ANN models and empirical correlations. Statistical evaluation revealed that the plastic viscosity ANN model has a coefficient of determination ($R^2$) of 98.82%, a root-mean-square error (RMSE) of 1.37, an average relative error (ARE) of 0.12, and an absolute average relative error of 2.69, while for yield point, this model has a coefficient of determination ($R^2$) of 94%, a root-mean-square error (RMSE) of 0.76, an average relative error (ARE) of −0.67, and an absolute average relative error of 3.18.

1. INTRODUCTION

Drilling mud in the well functions as blood in the body; it is simply a mixture of continuous phase (i.e., base) and other dispersed components in prespecified amounts (e.g., additives) that has the crucial ability to promote the chemical and mechanical stabilities of borehole, uplift drilling cuttings to surface, even to support partial weight of the drill-string or casing, transmit hydraulic HP to the bottom of borehole, obtain maximum information from the well, and many other functions that are executed concurrently.1,2 Drilling fluids could be classified regarding the continuous phase, namely, water-based mud (WBM), oil-based mud (OBM), and gas-based mud (GBM).3–6 To ensure a stable drilling process, fluid properties should be tracked and kept within the designed mud program as mud may encounter temperature ranges between 0 and 150 °C and pressure exceeds 5800 psi.7–9 Certainly, under these conditions, mud rheology would be altered and may result in borehole challenges including well control issues, hole cleaning, lost circulation, and more undesirable incidents;6,9–11 therefore, mud properties (i.e., rheological properties) including plastic viscosity (PV),

Received: September 9, 2021  
Accepted: November 16, 2021  
Published: November 24, 2021
apparent viscosity (AV), yield point (YP), gel strength besides, flow behavior index (n), and flow consistency (K) are normally measured once every 12 h, while for mud density, marsh funnel viscosity and solid content are ideally measured each 10–20 min.\textsuperscript{6,8,9,12,13} Rheological properties are conventionally estimated via a viscometer, which is tedious and time-consuming; however, its measurements are frequently required to ensure mud quality while drilling. Hence, robust real-time estimation of mud rheology is compelling. Traditionally, a marsh funnel was utilized to achieve relative, quick, and inaccurate viscosity estimation is seconds every 10–20 min.\textsuperscript{14} Humongous approaches have been developed for determining quick and accurate mud rheological properties including predictive models, which range from empirical to artificial intelligent models. Despite the humongous number of predictive models, a few suffer from complexity, uncertainty, and overparameterization.\textsuperscript{9}

Through the search for hydrocarbons, deep-drilled wells, unforgiving drilling terrains may be encountered for natural source exploitation; hence, oil-based mud may be recommended in such situations due to its high stability at a high temperature, providing shale stability besides eliminating formation damage. Invert emulsion is a low-toxicity version of the oil-based mud, which is commonly selected in oil and gas drilling operations;\textsuperscript{14} thus, this study aspires to build a robust, intelligent model that could estimate the rheological properties of invert emulsion mud using field measurements including mud density, marsh funnel viscosity, and solid content within the mud since sketchy estimations for mud status will always cause humongous defects and implications on drilling cost. This study also assesses the predictive performance and field utility of literature predictive models.

2. MUD RHEOLOGICAL PROPERTIES

Frequently, the rheology term is confused with the viscosity term; however, rheological properties include plastic viscosity and apparent viscosity. Rheology is the science of flow and deformation of matter that describes the interaction between force, deformation, and time, whereas viscosity means the fluid internal resistance to flow. There are two kinds of viscosity: plastic viscosity and apparent viscosity.\textsuperscript{15} Estimating rheological properties plays a vital role in determining whether the fluid will withstand the operating conditions or not.\textsuperscript{16–19}

2.1. Viscosity. Viscosity as mentioned previously is the internal resistance of the fluid to deformation forces,\textsuperscript{20} and for Newtonian fluids, it is the relation between shear stress and shear rate that could be employed solely for characterizing fluid properties. However, most drilling fluids are non-Newtonian fluids, which means that the relation between shear stress and shear rate is nonlinear and varies with the applied conditions; hence, it should be measured regularly. Basically, a viscometer is utilized to measure the mud viscosity in the laboratory, where a marsh funnel is a simple field apparatus that provides a rough estimation of any viscosity alteration that occurs during the drilling process.\textsuperscript{21}

2.1.1. Plastic Viscosity. Essentially, plastic viscosity is a good indicator of solid content within the mud. The optimum practice relevant to plastic viscosity is to have a high yield point—plastic viscosity ratio (YP/PV). In other words, a high yield point and a low plastic viscosity are desirable as by increasing plastic viscosity, the rate of penetration decreases and the required pumping power increases.\textsuperscript{22}

2.1.2. Apparent Viscosity. Apparent viscosity could be dealt with as the effective viscosity that reflects the status of both yield point and plastic viscosity,\textsuperscript{23,24} and as the name implies, the mud appears to have viscosity while measurement.\textsuperscript{25,26}

2.2. Yield Point. Yield value or yield point is the stress required to initiate the fluid flow or the fluid resistance to initial flow due to the electrical charges on the solid particles.\textsuperscript{27} It also reflects the ability of mud to suspend drilled cuttings during drilling.

2.3. Gel Strength. It demonstrates the ability of mud to suspend drilled cuttings while the drilling operation is in a static mood. Gel strength equals the shear stress readings at a low shear rate after the mud was under static condition for 10 s, 10 min, and 30 min in the viscometer.\textsuperscript{27} Gel strength plays a viable role in hole cleaning as, under low gel strength condition, drilled solids would accumulate and build up once the circulation is ceased, resulting in considerable downhole challenges.

3. CONVENTIONAL DRILLING FLUIDS MODELS

To date, several models have been developed for demonstrating the relation between shear stress and shear rate for drilling mud using experimental data.\textsuperscript{28,29} The following models could be considered as widely applicable for estimating the rheological properties of non-Newtonian drilling fluids.

3.1. Bingham Model. This two-parameter theoretical plastic model was developed in 1916 to describe fluid characteristics. Fluids that follow the Bingham model are characterized by a yield point and plastic viscosity that are independent of shear rate. The model could be represented mathematically as

\[
\tau = \tau_y + \mu \gamma
\]

(1)

\[
PV = \mu_p = \theta_{600} - \theta_{300}
\]

(2)

\[
YP = \tau_y = \theta_{300} - \mu_p
\]

(3)

\[
AV = \theta_{600}/2
\]

(4)

where \(\tau\) is the shear stress, \(\tau_y\) is the yield point (YP), \(\gamma\) is the shear rate, \(\mu_p\) is the plastic viscosity (PV), \(\theta_{600}\) is the viscometer dial reading at 600 rpm, \(\theta_{300}\) is the viscometer dial reading at 300 rpm, and AV is the apparent viscosity. The model has some deficiencies including instability at high temperatures and low shear rates.\textsuperscript{8,29,28} Furthermore, Huang\textsuperscript{24} indicated that the model tends to overestimate the mud yield stress.

3.2. Power Law Model. Ostwald de Waele\textsuperscript{8} or commonly known as power law model, this 2-parameter model was developed in 1923, 1925, respectively. It is widely employed with non-Newtonian fluids\textsuperscript{8} and expressed as

\[
\tau = K \gamma^n
\]

(5)

\[
n = 3.32 \log \left( \frac{\theta_{600}}{\theta_{300}} \right)
\]

(6)

\[
K = \frac{\theta_{600}}{1022^n}
\]

(7)

where \(n\) is the flow behavior index and \(K\) is the flow consistency index. However, it does not take yield stress into consideration that leads to erroneous estimations.

3.3. Herschel–Bulkley Model. Basically, it has been defined as a modified power law model, which coupled both
the Bingham model and power law, which result in calculation challenges\textsuperscript{2,24} and could be mathematically expressed as

\[ \tau = \tau_0 + K(\gamma)^n \]  \hspace{1cm} (8) 

3.4. Tscheuschner Model. This four-parameter model was developed in 2006 to predict the mud rheological properties; however, due to its relative complexity caused by the number of parameters, the model has a limited utility in oil and gas industries.\textsuperscript{30}

\[ \tau = \tau_0 + \eta_0 \gamma + \eta_\text{int} \frac{\gamma}{\gamma_\text{r}} \]  \hspace{1cm} (9)

4. LITERATURE MODELS

Modeling mud rheology is an open area for research over decades, and several predictive models for estimating mud rheological properties have been developed using field available measurement. Models vary in data size, input features, and mud type. Mostly studied input features are mud weight, marsh funnel viscosity, and solid content, which are all readily available field data. Different techniques have been developed in the last decade to utilize different AI techniques. Most models focused on OBM and WBM with less concern on foam-based mud.

4.1. Rheological Models for Foam-Based Mud. By reviewing the literature, most rheological predictive models are focusing on WBM and OBM and not much for aerated or foam-based mud. It may be due to the peculiar nature of foam, which makes it difficult to build a predictive model for it. Hatzschek\textsuperscript{31} developed a simple mathematical model for estimating foam viscosity as a function of liquid-phase viscosity and foam quality. Mitchell\textsuperscript{32} established a modified correlation that has the same characteristics as the Hatzschek model. Valko\textsuperscript{33} developed a detailed model for predicting the rheological properties of foam mud using liquid-phase density and foam density.

4.2. Rheological Models for Water-Based Mud. For WBM, several models have been developed, including Politte\textsuperscript{34} who used the linear regression technique besides selecting temperature and pressure as correlating parameters to develop his predictive model for estimating plastic viscosity, whereas Alderman\textsuperscript{35} used the multiplicative factor method with selecting temperature, pressure, interaction term, compressibility term, induced volumetric, and volumetric change in fluid phase to develop such a complex model. Okumo\textsuperscript{36} used the factorial design method to develop a plastic viscosity predictive model based on eight datasets; the model is a function of temperature and amounts of starch and potash. Makinde\textsuperscript{37} built his model using regression analysis and aging time and temperature as input features for estimating mud plastic and apparent viscosities. Razi\textsuperscript{38} established an artificial neural network (ANN) model that consists of a single hidden layer with three neurons to estimate mud apparent viscosity using shear stress at a shear rate of 600 rpm, temperature, and concentration as input features, but he did not provide a mathematical model for his proposed model to be used with other datasets or to be compared with any upcoming models. Almahdawi\textsuperscript{39} used 34 datasets to build an apparent viscosity predictive model utilizing marsh funnel viscosity and mud weight temperature as input parameters. Mohammed\textsuperscript{40} developed a model for predicting maximum shear stress that could be applied on WBM modified with iron oxide nanoparticles, and 35 datasets were gathered and used to develop the model. Xu\textsuperscript{41} used the relative dial readings (RDR) method on 21 datasets to predict high-temperature high-pressure (HTHP) mud rheology using system pressure and temperature. Elkatatny\textsuperscript{42} used 3000 field measurements data points in building an ANN model that is capable of predicting mud apparent viscosity using marsh funnel viscosity, mud density, and solid content as input features. Elkatatny\textsuperscript{43} built another ANN model with 20 neurons within its single hidden layer to predict the NaCl WBM apparent viscosity using marsh funnel viscosity, mud density, and solid content as correlating parameters. da Silva Bispo\textsuperscript{44} used an artificial neural network to build an intelligent model using system temperature and additives concentrations (i.e., barite, xanthan gum, and bentonite concentrations) for predicting mud apparent viscosity. Al-Azani\textsuperscript{45} built his ANN model with single hidden layer using 383 datasets; the ANN model is capable of predicting mud. Plastic viscosity is a function of marsh funnel viscosity, mud density, and solid content. Abdelgawad\textsuperscript{46} established two models for predicting mud plastic viscosity and apparent viscosity using 1029 datasets and marsh funnel viscosity, mud density, and solid content were selected as input features. Huang\textsuperscript{47} used regression analysis to develop a predictive mathematical model for estimating poly-sulfonated drilling fluid apparent viscosity. Tomiwa\textsuperscript{48} established a single hidden layer with seven neurons ANN model using 65 actual field measurements. His proposed model could estimate the WBM yield point, plastic viscosity, and apparent viscosity based on the bentonite and solanum tuberosum biopolymer quantities within the mud. Al-Khdheeari\textsuperscript{49} used 142 actual datasets to develop the ANN model and nonlinear multiple regression correlation that both could estimate WBM apparent viscosity based on marsh funnel viscosity and mud weight. Tomiwa\textsuperscript{50} established an ANN model with a single hidden layer that contains 15 neurons; the model could be utilized for estimating apparent viscosity, plastic viscosity, and yield point of modified biopolymer WBM based on 100 actual datasets, water volume, biopolymer, and bentonite concentrations, which were selected as input features. Gowida\textsuperscript{51} developed another ANN model for predicting WBM plastic viscosity and apparent viscosity using mud weight and marsh funnel viscosity as input parameters, and 200 actual datasets were employed to develop such a model. Goma\textsuperscript{52} used 814 actual datasets to develop the WBM apparent viscosity ANN predictive model using mud density and marsh funnel viscosity as input parameters.

4.3. Rheological Models for Oil-Based Mud. Several models have been established for estimating the rheological characteristics of both OBM and inverted emulsion mud, including McMordie,\textsuperscript{53} who applied the multiplicative factor technique to develop a predictive model that can be easily used for estimating shear stress by knowing both temperature and pressure of the mud. Politte\textsuperscript{54} estimated a predictive model for estimating OBM plastic viscosity as a function of mud pressure and temperature. Houwen\textsuperscript{55} developed a plastic viscosity predictive model for the invert emulsion mud using the multiplicative factor method. The model assumes that the input features of mud pressure and temperature are under HPHT conditions. Minton\textsuperscript{56} used 200 actual measurements to develop his mathematical model for estimating OBM viscosity based on mud pressure, temperature, and initial viscosity. Rommetveit\textsuperscript{57} focused on building predictive correlations for estimating the OBM rheology using multiplicative factor...
technique, where mud pressure and temperature, besides shear rate were selected as correlating parameters. Ibeh\textsuperscript{50} gathered eight datasets for developing a predictive correlation for estimating OBM plastic viscosity as a function of mud pressure and temperature. Tchameni\textsuperscript{41} developed another correlation for predicting OBM apparent viscosity using regression analysis. Amani\textsuperscript{52} established his viscosity model in an exponential form using mud pressure and temperature as input parameters. Zhou\textsuperscript{53} established his correlation for estimating OBM HTHP shear stress as a function of pressure, temperature, and shear stress at normal conditions. Elkatatny\textsuperscript{14} developed two ANN models for predicting invert emulsion mud plastic viscosity and apparent viscosity. Oliveira\textsuperscript{54} established a nonlinear regression model for estimating the OBM plastic viscosity using mud pressure and temperature. Igwilo\textsuperscript{55} utilized least-square and Gaussian elimination for building an OBM plastic viscosity predictive model as a function of mud temperature. Elkatatny\textsuperscript{12} gathered 6000 datasets for establishing a predictive model for invert emulsion mud apparent viscosity as a function of marsh funnel viscosity and mud density. Al-Azani\textsuperscript{43} developed another OBM model for estimating apparent viscosity using 383 datasets. Alsabaa\textsuperscript{51} built two ANFIS models for predicting invert emulsion mud plastic viscosity and apparent viscosity using mud density and marsh funnel viscosity.

4.4. Artificial Neural Network. Since the petroleum industry deals with massive datasets in almost all its aspects including drilling, reservoir engineering, production, and even exploration,\textsuperscript{47} it was vital to employ artificial intelligent techniques to cluster, classify, or even in regression analysis to establish intelligent models that can function by itself with a high degree of accuracy due to their excellent ability to capture complex nonlinear patterns within any data bank.\textsuperscript{56–58} One of these techniques is artificial neural network, which proved its potential in many fields. ANN is a trial to mimic the biological neural system of human beings for building independent and intelligent models.\textsuperscript{59} The ANN architecture varies according to data size, required degree of accuracy, and a lot more reasons; however, the ANN model should comprise three layers at least, including an input layer where input features are fed into the model; a middle hidden layer, which is the core of the model; and an output layer to represent the estimated value. To link these layers, activation functions besides training and optimization algorithms are utilized, including logistic-sigmoid and tan-sigmoid functions as transfer functions in addition to the Levenberg–Marquardt, Bayesian regularization, and scaled conjugate gradient as optimization approaches. Ahmed\textsuperscript{60} provided a detailed comparison between machine learning algorithms in predicting the gas hydrate formation conditions. Each layer has a specific number of neurons that could be thought as embedded processing units in the ANN model. Connections between neurons have weights and biases, and adjustment of these connection has a direct impact on the model performance.\textsuperscript{51,53} Selecting the right number of neurons in the hidden layers is crucial as inadequate selection may cause overfitting or underfitting circumstances.\textsuperscript{62,63}

Several AI models had been developed for the accurate estimation of different petroleum aspects including PVT properties prediction of retrograde gases,\textsuperscript{64,58} estimating the flow assurance parameters such as predicting gas hydrate temperature\textsuperscript{65} and asphaltenes precipitation prediction,\textsuperscript{11} and predicting reservoir porosity and permeability.\textsuperscript{59,70} rather than the typical estimation techniques.

5. METHODOLOGY

In this study, 407 actual datasets were gathered from daily mud reports for invert emulsion mud. Basically, mud daily reports (MDR) for invert emulsion mud summarize mud characteristics including mud weight, marsh funnel viscosity (FV), plastic viscosity (PV), V-G meter readings, yield point (YP), gel strength, solid content percentage, and other relative features. Correlation coefficient analysis was performed, and results are shown in Figures 1 and 2. The results show an intimate relationship between plastic viscosity and yield point of invert emulsion mud with mud density, marsh funnel viscosity, and solid content percentage per volume.

It also implies considerable bond with mud temperature. Mud weight, marsh funnel viscosity, and mud temperature are readily available data, whereas solid content percentage, which is a non-ease-to-measure feature, will be employed in the proposed model for simulation and more accurate estimations.

5.1. Data Description and Analysis. Data would be split so that 70% of the data will be utilized in building and training the models while the remaining 15% (unseen data) will be employed to test the models’ performance; then, total data will be employed to check the models’ performance against the literature empirical correlations. Table 1 depicts full descriptive statistics for the compiled data, and the results clarify the applicability range of the proposed model and characteristics of the developing data.

5.2. Plastic Viscosity Model. For the plastic viscosity model, 407 datasets including plastic viscosity, mud density, mud temperature, marsh funnel viscosity, and solid content percent were extracted from actual invert emulsion mud daily

Figure 1. Correlation coefficients for plastic viscosity model.

Figure 2. Correlation coefficients for yield point model.
are calculated from the following expression

\[
S_{ij} = \sum_{j=1}^{N} (W_{ij}x_j) + b_i \tag{10}
\]

where \(x_i\) is the normalized mud density (ppg), mud temperature (F), funnel viscosity (s/qt) and solid content (%).

The normalized mud density, mud temperature, funnel viscosity, and solid content can be calculated using the following expressions

\[
\rho_{mn} = 6.589041 - 0.547945\rho_m \tag{11}
\]
\[
T_{mn} = 2.090909 - 0.018182T_m \tag{12}
\]
\[
\mu_F = 2.777778 - 0.044444\mu_F \tag{13}
\]
\[
c_m = 2.312217 - 0.090498C_s \tag{14}
\]

and finally, the plastic viscosity can be calculated using the following formula

\[
PV = 32.5 + 17.5 \left[ \sum_{i=1}^{N} \left( \frac{w_{hoi}}{1 + e^{-x_i}} \right) + b_{ho} \right] \tag{15}
\]

### 5.3. Yield Point Model.

For yield point model, 407 datasets including yield point, mud density, mud temperature, marsh funnel viscosity, and solid content percent were selected to develop the model. Several ANN architectures have been tried to optimize the model performance; then, using a three-layer model with 12 neurons in the single hidden layer was found to be the most vital structure based on its predictive performance. The Bayesian regularization training algorithm was selected as an optimum algorithm based on previously stated reasons. Logistic-sigmoid and pure-linear activation functions were coupled to obtain the optimum performance (Tables 3–5).

#### 5.3.1. ANN Mathematical Model.

For \(i = 1\) to no. of neurons and for \(j = 1\) to no of inputs, the inputs for the hidden are calculated using eq 10, and the inputs are normalized using

\[
S_{ij} = \sum_{j=1}^{N} (W_{ij}x_j) + b_i \tag{10}
\]

### Table 1. Descriptive Statistical Analysis for Invert Emulsion Mud Properties

| property       | density, ppg | mud temp, °F | FV, sec/qt | PV, cP | YP, lbs/100 ft² | solid % by vol |
|----------------|--------------|--------------|------------|--------|----------------|---------------|
| mean           | 12.56        | 131.37       | 63.49      | 34.58  | 17.9           | 25.87         |
| standard error | 0.069        | 1.52         | 0.47       | 0.617  | 0.162          | 0.335         |
| median         | 13.5         | 140          | 65         | 39     | 17             | 27.3          |
| mode           | 13.5         | 80           | 68         | 18     | 15             | 15.5          |
| kurtosis       | −1.14        | −1.02        | −0.38      | −1.63  | 0.124          | −1.259        |
| skewness       | −0.78        | −0.59        | −0.34      | −0.33  | 0.66           | −0.247        |
| range          | 3.65         | 110          | 45         | 35     | 19             | 22.1          |
| minimum        | 10.2         | 60           | 40         | 15     | 12             | 14.5          |
| maximum        | 13.85        | 170          | 85         | 50     | 26             | 36.6          |
| count          | 407          | 407          | 407        | 407    | 407            | 407           |

#### 5.2.1. ANN Mathematical Model.

For \(i = 1\) to no. of neurons and for \(j = 1\) to no of inputs, the inputs for the hidden are calculated from the following expression

\[
S_{ij} = \sum_{j=1}^{N} (w_{ij}x_j) + b_i \tag{10}
\]

### Table 2. Characteristics of Plastic Viscosity Model

| character                          | value               |
|------------------------------------|---------------------|
| number of layers                   | 3                   |
| number of input layer neurons      | 4                   |
| number of hidden layer neuron      | 10                  |
| training algorithm                 | Bayesian regularization |
| transfer function of the hidden layer | logistic-sigmoid   |
| transfer function of the output layer | pure-linear        |

#### 5.3. Yield Point Model.

For yield point model, 407 datasets including yield point, mud density, mud temperature, marsh funnel viscosity, and solid content percent were selected to develop the model. Several ANN architectures have been tried to optimize the model performance; then, using a three-layer model with 12 neurons in the single hidden layer was found to be the most vital structure based on its predictive performance. The Bayesian regularization training algorithm was selected as an optimum algorithm based on previously stated reasons. Logistic-sigmoid and pure-linear activation functions were coupled to obtain the optimum performance (Tables 3–5).

#### 5.3.1. ANN Mathematical Model.

For \(i = 1\) to no. of neurons and for \(j = 1\) to no of inputs, the inputs for the hidden are calculated using eq 10, and the inputs are normalized using

\[
S_{ij} = \sum_{j=1}^{N} (w_{ij}x_j) + b_i \tag{10}
\]

### Table 3. Weights and Biases between Input and Hidden Layers

| neuron | \(W_{i1}\) | \(W_{i2}\) | \(W_{i3}\) | \(W_{i4}\) | \(b_i\) |
|--------|------------|------------|------------|------------|--------|
| 1      | 5.938      | 0.81931    | 6.7126     | −2.7269    | −2.4736 |
| 2      | −3.7801    | 0.21827    | 1.1694     | 1.6706     | 0.76685 |
| 3      | 3.5548     | −0.98164   | −9.5609    | 0.98687    | 1.5138  |
| 4      | −1.573     | 0.023633   | −5.4149    | 8.2483     | −2.7095 |
| 5      | 2.7983     | 1.1317     | 0.90761    | −5.0711    | 1.5898  |
| 6      | −6.1276    | 2.1556     | −4.6358    | 8.0914     | 2.0412  |
| 7      | −0.040181  | −1.3807    | −4.2203    | −0.036472  | 1.5239  |
| 8      | 1.2209     | 1.3106     | −6.6714    | 7.8163     | −4.4876 |
| 9      | 3.6553     | −0.97893   | 1.8834     | −3.4854    | −1.0436 |
| 10     | −1.5146    | 0.80529    | 5.7719     | −0.92451   | −1.3723 |

where \(x_i\) is the normalized mud density (ppg), mud temperature (F), funnel viscosity (s/qt) and solid content (%).
temperature, and biodiesel content were utilized in refs 37 and 51. These models that require lengthy procedures to compile their input features are mainly recommended for design and simulation studies not for quick field calculations as these models seek for highly accurate results.

Table 4. Weights and Bias between Hidden and Output Layers

| neuron | $W_{21}$ | $b_{2}$ |
|--------|----------|---------|
| 1      | 2.0509   | 3.0882  |
| 2      | −3.5619  |         |
| 3      | 3.2376   |         |
| 4      | −4.4862  |         |
| 5      | −3.8813  |         |
| 6      | −2.7273  |         |
| 7      | 2.5194   |         |
| 8      | 2.3934   |         |
| 9      | −5.5651  |         |
| 10     | 6.9079   |         |

Table 5. Characteristics of Yield Point Model

| parameter                          | value                        |
|------------------------------------|------------------------------|
| number of layers                   | 3                            |
| number of input layer neurons      | 4                            |
| number of hidden layer neuron      | 12                           |
| training algorithm                 | Bayesian regularization      |
| transfer function of the hidden layer | Tan-sigmoid                  |
| transfer function of the output layer | Pure-linear                  |

Table 6. Weights and Biases between Input and Hidden Layers

| neuron | $W_{11}$ | $W_{12}$ | $W_{13}$ | $W_{14}$ | $b_{1}$ |
|--------|----------|----------|----------|----------|---------|
| 1      | 0.84623  | 1.3087   | −1.5345  | −1.7181  | 0.58863 |
| 2      | 1.9593   | −0.42878 | 0.64826  | −4.2705  | 0.94691 |
| 3      | −0.06918 | −0.52436 | 1.8872   | 2.166    | 0.0483  |
| 4      | 3.2878   | −0.09263 | 1.8621   | −1.5884  | −2.7109 |
| 5      | −0.7594  | 0.14914  | −4.9405  | −0.11377 | 2.1779  |
| 6      | −1.5886  | 1.6567   | 1.0881   | 0.6795  | 1.3864  |
| 7      | 1.5797   | −0.26607 | 1.6493   | 0.84165  | −2.9628 |
| 8      | −4.6602  | −0.81426 | 0.52975  | 4.0379   | 2.2418  |
| 9      | −1.2915  | 1.0305   | 0.5488   | 1.6308   | 0.059048|
| 10     | −0.7312  | 0.07191  | 3.8341   | 0.55874  | −0.86494|
| 11     | −1.1637  | 1.8294   | 1.2528   | 1.0997   | 0.57114 |
| 12     | −1.9281  | −0.16744 | 0.013065 | 2.2553   | 1.6459  |

Gomaa\textsuperscript{13} and Alsabaa\textsuperscript{21} provided a detailed description for their developed models (Tables 6 and 7).

By reviewing the literature, only three studies provided a comprehensive description of invert emulsion rheological properties models, which are Elkatatny\textsuperscript{12} (the first model), Elkatatny, Tariq, and Mahmoud\textsuperscript{14} (the second model), and Alsabaa, Gamal, Elkatatny, and Abdulaheem\textsuperscript{21} (the third model). However, by revising the Alsabaa, Gamal, Elkatatny, Abdulaheem\textsuperscript{21} models, a crucial defect had been recognized in their proposed models as despite stating using tan-sigmoid activation function, mathematical formulas of their models do not reflect it; in other words, they missed in expressing their activation function mathematically. Tables 8 and 9 demonstrate the statistical evaluation of the proposed model and two models. Results show poor predictive strength of and Elkatatny, Tariq & Mahmoud\textsuperscript{14} ANN yield point models which -may be- due to that, the used data in this study is out of its applicability range as Elkatatny\textsuperscript{12} and Elkatatny, Tariq & Mahmoud\textsuperscript{14} did not clarify the applicability range for their models.

6. RESULTS AND DISCUSSION

By reviewing the literature, one crucial obstacle that encounters field applicability of most of extant models is the selection of input features. A great percentage of extant models employ the results of lengthy experimental tests as input features in the case of utilizing solid content in works of refs 14, 41, 43, 71, 72. Other parameters such as aging time, aging temperature, and biodiesel content were utilized in refs 37 and 51. These models that require lengthy procedures to compile their input features are mainly recommended for design and simulation studies not for quick field calculations as these models seek for highly accurate results.

Many studies\textsuperscript{21,45,72,73} did not provide a sufficient description for the developed models unlike others.\textsuperscript{14,51,4}

eqs 2–5. Finally, the yield point can be calculated using the following formula

$$YP = 21.5 + 9.5\left[\sum_{i=1}^{N}\left(\frac{2}{1 + e^{-2b_{hoi}}} - 1\right) + b_{bo}\right]$$

(16)

6.1. Evaluation of Model Performance. Statistical and graphical indicators such as coefficient of determination ($R^2$), root-mean-square error (RMSE), average percent relative error (APRE), and absolute average relative error (AAPRE) as statistical indicators, and results are listed in Tables 8 and 9 for plastic viscosity model and yield point model respectively. In addition to cross-plot and error distribution as graphical indicators, results are shown through Figures 3–18.
Tables 8 and 9 depict the outstanding performance of the proposed models compared to the three published models.

**Table 8. Statistical Comparison between the Proposed Plastic Viscosity Model and Literature Models**

| correlations | $R^2$ | SD  | RMSE | ARE  | AARE |
|--------------|-------|-----|------|------|------|
| Elkatatny $^{12}$ | 55.81 | 28.62 | 10.04 | −13.74 | 23.2 |
| Elkatatny, Tariq, and Mahmoud $^{14}$ | 85.36 | 356.26 | 111.38 | −59.28 | 318.06 |
| Alsabaa, Gamal, Elkatatny, and Abdulraheem $^{21}$ | 0.75 | 256.4 | 60.8 | −207.9 | 211.8 |
| this study | 98.82 | 3.73 | 1.37 | 0.12 | 2.7 |

**Table 9. Statistical Comparison between the Proposed Yield Point Model and Literature Models**

| correlations | $R^2$ | SD  | RMSE | ARE  | AARE |
|--------------|-------|-----|------|------|------|
| Elkatatny $^{12}$ | 3.54 | 51.8 | 8.14 | −42.5 | 44.14 |
| Elkatatny, Tariq, and Mahmoud $^{14}$ | 2.48 | 52.42 | 8.29 | −39.46 | 43.51 |
| Alsabaa, Gamal, Elkatatny, and Abdulraheem $^{21}$ | 18.72 | 2559.8 | 454.3 | 2427 | 2458 |
| this study | 94 | 4.31 | 0.76 | −0.67 | 3.18 |

**Figure 3.** Cross-plot for the proposed plastic viscosity model.

**Figure 4.** Cross-plots for training, validating, and testing phases for the proposed plastic model.

**Figure 5.** Testing the first previous published model of plastic viscosity.
The proposed ones provide the highest coefficient of determination and the lowest root-mean-square error (RMSE), average percent relative error (APRE), and absolute average relative error (AAPRE), where Figures 3–8 demonstrate the reliability of the proposed models against extant models as shown in the coherence between actual and predicted values with ±20% error distribution around the zero line. Cross-validation for the proposed models has been provided using the 30% unseen data splitting into testing and validation as shown in Figures 4 and 17 to ensure that there is not any overfitting or underfitting problem through training, validation, or testing phases.

7. CONCLUSIONS

In conclusion, accurate prediction of invert emulsion mud rheological properties is crucial for building robust drilling program since sketchy estimations for mud status will always cause humongous defects and implications on drilling cost. The current study provides two robust ANN models for plastic viscosity and yield point for invert emulsion mud with their mathematical base for results’ replicability. Comprehensive statistical and graphical evaluation criteria were conducted to demonstrate the predictive performance of the model against exact ANN models and simple multiple regression models. Mud density (ppg), mud temperature (F), marsh funnel viscosity (s/qrt), and solid content percentage (%) were
selected as input features after conducting a correlation coefficient analysis to measure the influence of each feature on the output. Results of the evaluation criteria depicted the outstanding performance and consistency of the proposed models over extant ANN models and empirical correlations. Statistical evaluation revealed that the plastic viscosity ANN model has a coefficient of determination ($R^2$) of 98.82%, a root-mean-square error (RMSE) of 1.37, an average relative error (ARE) of 0.12, and an absolute average relative error of 2.69, and the yield point model has a coefficient of determination ($R^2$) of 94%, a root-mean-square error (RMSE) of 0.76, an average relative error (ARE) of $-0.67$, and an absolute average relative error of 3.18. This study has a vital importance in establishing accurate invert emulsion drilling mud programs for providing the optimum mud performance, which may mitigate or even eliminate many

Figure 12. Cross-plot for the proposed yield point model.

Figure 13. Cross-plots for training, validating, and testing for the proposed yield point model.

Figure 14. Testing the first previous published model of yield point.

Figure 15. Testing the second previous published model of yield point.
drilling problems relevant to mud rheological defects due to inadequate design for mud program.

**AUTHOR INFORMATION**

**Corresponding Author**

Samir Khaled — Petroleum Engineering and Gas Technology Department, Faculty of Energy and Environmental Engineering, The British University in Egypt, El Shorouk City, Cairo 11837, Egypt

Sayed Gomaa — Petroleum Engineering and Gas Technology Department, Faculty of Energy and Environmental Engineering, The British University in Egypt, El Shorouk City, Cairo 11837, Egypt; Mining and Petroleum Engineering Department, Faculty of Engineering, Al-Azhar University, Nasr City, Cairo 11371, Egypt

Attia M. Attia — Petroleum Engineering and Gas Technology Department, Faculty of Energy and Environmental Engineering, The British University in Egypt, El Shorouk City, Cairo 11837, Egypt

Complete contact information is available at: https://pubs.acs.org/10.1021/acsomega.1c04937

**Notes**

The authors declare no competing financial interest.

**REFERENCES**

(1) Caenn, R.; Chillingar, G. V. Drilling fluids: State of the art. J. Pet. Sci. Eng. 1996, 14, 221–230.

(2) Andaverde, J.; Wong-Loya, J.; Vargas-Tabares, Y.; Robles, M. A practical method for determining the rheology of drilling fluid. J. Pet. Sci. Eng. 2019, 180, 150–158.

(3) Bleier, R. Selecting a drilling fluid. J. Pet. Technol. 1990, 42, 832–834.

(4) Abdelgawad, K.; Elkatatny, S.; Moussa, T.; Mahmoud, M.; Patil, S. Real-time determination of rheological properties of spud drilling fluids using a hybrid artificial intelligence technique. J. Energy Resour. Technol. 2019, 141, No. 032908.

(5) Al-Khdeereawi, E. A.; Mahdi, D. S. Apparent viscosity prediction of water-based muds using empirical correlation and an artificial neural network. Energies 2019, 12, No. 3067.

(6) Alakbari, F. S.; Mohyaldin, M. E.; Ayoub, M. A.; Muhsan, A. S.; Hassan, A. Apparent and plastic viscosities prediction of water-based drilling fluid using response surface methodology. Colloids Surf., A 2021, 616, No. 126278.

(7) Herzhaft, B.; Peysson, Y.; Isambourg, P.; Delépouille, A.; Abdoulaye, T. In Rheological Properties of Drilling Muds in Deep Offshore Conditions, SPE/IADC Drilling Conference; OnePetro, 2001.

(8) Shah, S. N.; Shanker, N. H.; Ogugbue, C. C. In Future Challenges of Drilling Fluids and Their Rheological Measurements, ADE Fluids Conference and Exhibition; American Association of Drilling Engineers: Houston, Texas, 2010.

(9) Agwu, O. E.; Akpabio, J. U.; Ekpenyong, M. E.; et al. A critical review of drilling mud rheological models. J. Pet. Sci. Eng. 2021, 203, No. 108659.

(10) Paiaman, A. M.; Al-Askari, M.; Salmani, B.; Alanazi, B. D.; Mashi, M. Effect of drilling fluid properties on rate of penetration. Nafta 2009, 60, 129–134.

(11) Rafati, R.; Smith, S. R.; Haddad, A. S.; Novara, R.; Hamidi, H. Effect of nanoparticles on the modifications of drilling fluids properties: A review of recent advances. J. Pet. Sci. Eng. 2018, 161, 61–76.

(12) Elkatatny, S. In Determination the Rheological Properties of Invert Emulsion Based Mud on Real Time Using Artificial Neural Network, SPE Kingdom of Saudi Arabia Annual Technical Symposium and Exhibition; OnePetro, 2016.

(13) Gomaa, I.; Elkatatny, S.; Abdurrahem, A. Real-time determination of rheological properties of high over-balanced drilling fluid used for drilling ultra-deep gas wells using artificial neural network. J. Nat. Gas Sci. Eng. 2020, 77, No. 103224.
Calibrations and Models of synthetic based drilling mud. Publishing, 2011.

Gulf Professional Properties of Drilling and Completion Fluids

Optimal determination of rheological parameters for Herschel–Bulkley drilling fluids and impact on pressure drop, velocity profiles and penetration rates during drilling. J. Pet. Sci. Eng. 2013, 53, 203−224.

Hatschek, J. Englisches Staatsrecht: Handbuch des öffentlichen Rechts der Gegenwart/unter Mitw. von... hrg. v. Heinrich von Marquardt. Bd. 4. Das Staatsrecht der außerdeutschen Staaten; Die Verwaltung: Mohr, 1906.
(52) Amani, M. The rheological properties of oil-based mud under high pressure and high temperature conditions. *Adv. Pet. Explor. Dev.* 2012, 3, 21–30.

(53) Zhou, H.; Wang, G.; Fan, H.; Niu, X.; Ye, Y. A novel prediction model for rheological properties of drilling fluids at HTHP conditions and its evaluation. *SOCAR Proc.* 2015, 2, 13–23.

(54) Oliveira, F.; Sodrê, C.; Marinho, J. Numerical investigation of non-Newtonian drilling fluids during the occurrence of a gas kick in a petroleum reservoir. *Braz. J. Chem. Eng.* 2016, 33, 297–305.

(55) Igwilo, K.; Godspower, I.; Nnanna, O.; Oseuke, G.; Jude, O.; Anawe, P. Modeling the Effects of Temperature on Oil Base Mud Viscosity Using Polynomial equation. *Int. J. Pet. Petrochem. Eng.* 2017, 3, 16–22.

(56) Monjezi, M.; Dehghani, H. Evaluation of effect of blasting pattern parameters on back break using neural networks. *Int. J. Rock Mech. Min. Sci.* 2008, 45, 1446–1453.

(57) Al-AbdulJabbar, A.; Elkatatny, S.; Mahmoud, M.; Abdelgawad, K.; Al-Majed, A. A robust rate of penetration model for carbonate formation. *J. Energy Resour. Technol.* 2019, 141, No. 032902.

(58) Gouda, A.; Gomaa, S.; Attia, A.; Emara, R.; Desouky, S.; El-hoshoudy, A. Development of an artificial neural network model for predicting the dew point pressure of retrograde gas condensate. *J. Pet. Sci. Eng.* 2021, 123, No. 109284.

(59) Angelini, E.; Ludovici, A. CDS Evaluation model with neural networks. *J. Serv. Sci. Manage.* 2009, 2, No. 15.

(60) Ahmadi, M.; Chen, Z.; Clarke, M.; Fedutenko, E. Comparison of kriging, machine learning algorithms and classical thermodynamics for correlating the formation conditions for CO2 gas hydrates and semi-clathrates. *J. Nat. Gas Sci. Eng.* 2020, 84, No. 103659.

(61) Hinton, G. E.; Osindero, S.; Teh, Y.-W. A fast learning algorithm for deep belief nets. *Neural Comput.* 2006, 18, 1527–1554.

(62) Rao, S. S.; Ramamurti, V. In A Hybrid Technique to Enhance the Performance of Recurrent Neural Networks for Time Series Prediction, IEEE International Conference on Neural Networks; IEEE, 1993.

(63) Van der Aalst, W. M.; Rubin, V.; Verbeek, H.; van Dongen, B. F.; Kindler, E.; Günther, C. W. Process mining: a two-step approach to balance between underfitting and overfitting. *Software Syst. Model.* 2010, 9, 87–111.

(64) Ahmadi, M. A.; Ebadi, M.; Marghnaeleki, P. S.; Fouladi, M. M. Evolving predictive model to determine condensate-to-gas ratio in retrograded condensate gas reservoirs. *Fuel* 2014, 124, 241–257.

(65) Khamechhi, E.; Shamohammadi, E.; Yousefi, S. H. Predicting the hydrate formation temperature by a new correlation and neural network. *Gas Process.* 2013, 1, 41–50.

(66) Ahmadi, M. A.; Golshadi, M. Neural network based swarm concept for prediction asphaltene precipitation due to natural depletion. *J. Pet. Sci. Eng.* 2012, 98–99, 40–49.

(67) Ahmadi, M.-A.; Ahmadi, M. R.; Hosseini, S. M.; Ebadi, M. Connectionist model predicts the porosity and permeability of petroleum reservoirs by means of petro-physical logs: application of artificial intelligence. *J. Pet. Sci. Eng.* 2014, 123, 183–200.

(68) Ahmadi, M.; Chen, Z. Machine learning-based models for predicting permeability impairment due to scale deposition. *J. Pet. Explor. Prod. Technol.* 2020, 10, 2873–2884.

(69) Alramahi, B. A.; Alshibli, K. A.; Attia, A. M. In *Influence of Grain Size and Consolidation Pressure on Porosity of Rocks*, Site Characterization and Modeling; American Society of Civil Engineers, 2005; pp 1–13.

(70) Alshibli, K. A.; Alramahi, B. A.; Attia, A. M. Assessment of spatial distribution of porosity in synthetic quartz cores using microfocus computed tomography (μCT). *Part. Sci. Technol.* 2006, 24, 369–380.

(71) Elkatatny, S.; Mousa, T.; Mahmoud, M. In *A New Approach to Determine the Rheology Parameters for Water-Based Drilling Fluid Using Artificial Neural Network*, SPE Kingdom of Saudi Arabia Annual Technical Symposium and Exhibition; OnePetro, 2018.

(72) Abdelgawad, K. Z.; Elzenary, M.; Elkatatny, S.; Mahmoud, M.; Abdulraheem, A.; Patil, S. New approach to evaluate the equivalent circulating density (ECD) using artificial intelligence techniques. *J. Pet. Explor. Prod. Technol.* 2019, 9, 1569–1578.

(73) Samnejad, M.; Gharib Shirangi, M.; Etehadi, R. In *A Digital Twin of Drilling Fluids Rheology for Real-Time Rig Operations*, Offshore Technology Conference; OnePetro, 2020.