Experimental multi-level quantum teleportation

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Quantum teleportation [1] provides a way to transmit unknown quantum states from one location to another via previously shared quantum entanglement and classical communications. Discrete variable states [2–8] and continuous variable states [9–12] in one degree of freedom have been transported. Recent work has also demonstrated the capability of teleporting multiple degrees of freedom of a single photon [13]. These advances [14] enable the quantum teleportation technique to transport the states of quantum systems reliably. However, in the quantum world, multi-level systems such as the single atom of quantized energy and the single photon of orbital angular momentum are more prevalent. Therefore, to completely rebuild the quantum states of a single particle remotely, one needs to teleport multi-level states. However, performing a deterministic multi-level Bell state measurement, a key step in quantum teleportation, remains a challenge. Here, we demonstrate the teleportation of multi-level states of a single photon in a three-dimensional six-photon system. We exploit the path mode of a single photon as the multi-level system, use two auxiliary entangled photons to realize a deterministic three-dimensional Bell state measurement. We thereby overcome these obstacles and demonstrate the teleportation of a three-level (three-dimensional) quantum state using the spatial mode of a single photon.

Suppose Alice wishes to teleport to Bob the quantum state of a single photon (photon 1, Fig. 1), encoded in angular momentum [15] [16], the temporal mode [17], the frequency mode [18] and the spatial mode [19] [20] of a single photon are all natural attributes of multi-level states, which are exploited as high-dimensional systems. However, to teleport multi-level quantum states is still a challenge for two reasons. One is the generation of high-quality multi-level entanglement feasible for quantum teleportation. There has been much work on high-dimensional entanglement generation [15–20], including attempts to observe interference between different high-dimensional entangled pairs [21] [22]. Nevertheless, the interference visibility between different pairs is still quite low at 63.5%. The other concerns performing a deterministic high-dimensional Bell state measurement (HDBSM). Here, we use the spatial mode (path) to encode the three-level states that has been demonstrated to extremely high fidelity [20] and use an auxiliary entangled photon pair to perform the HDBSM. We thereby overcame these obstacles and demonstrate the teleportation of a three-level (three-dimensional) quantum state using the spatial mode of a single photon.

FIG. 1. Scheme for quantum teleportation of the multi-level states of a single photon. Alice wishes to teleport the multi-level (three-dimensional) quantum state of single photon 1 to Bob. Initially, Alice and Bob share a three-dimensional entangled photon pair 2-3. Then, Alice performs a high-dimensional Bell state measurement (HDBSM) assisted by another entangled photon pair 4-5 and sends the results to Bob through a classical channel. Finally, according to the results of HDBSM, Bob applies the appropriate three-dimensional Pauli operations on photon 3 to convert it into the original state of photon 1.
the path mode as

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle,$$

where $|0\rangle$, $|1\rangle$, and $|2\rangle$ denote the path degree of freedom (DOF). This DOF exists in an infinite dimensional space of the photonic system; here, we take only three dimensions as an example. The coefficients $\alpha$, $\beta$, and $\gamma$ are complex numbers satisfying $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Alice and Bob initially need to share a high-dimensional Bell states, Bob needs to perform a three-dimensional unitary operation on photon 3 and to rotate the state of photon 3 to $|\varphi\rangle$ according to the measurement results of photons 1 and 2.

However, HDBSM is still a challenge with linear optics [23]. Although one can classify high-dimensional entangled states into several categories [24, 25], one cannot identify any of them. The possible solution to this HDBSM is to introduce an auxiliary system. Here, we introduce a pair of assistant entangled photons to complete the HDBSM.

The Bell state measurement (BSM) of a two-dimensional polarized state is divided into two steps [26]. The four Bell states are first divided into two categories ($|HH\rangle \pm |VV\rangle)/\sqrt{2}$ and ($|HV\rangle \pm |VH\rangle)/\sqrt{2}$) by a polarizing beam splitter (PBS) according to classical terms. Second, the two states are distinguished with different phases by projecting onto basis $|H \pm V\rangle/\sqrt{2}$. The structure of HDBSM in our system is similar to that of qubit polarized BSM. According to classical terms, nine three-dimensional Bell states are divided into three categories, then, the localized projection measurement is used to identify the three-dimensional Bell states.

Fig. 2 illustrates our linear optical scheme for teleporting the three-level quantum states. The first step is to divide nine Bell states into three categories according to classical terms $|i, i\rangle$, $|i, i + 1\rangle$, and $|i, i + 2\rangle$, $i \in \{0, 1, 2\}$ under modulo-3 arithmetic. Photons 1 and 2 are sent to a PBS, which transmits horizontally polarized terms ($|H\rangle$) and reflects vertically polarized terms ($|V\rangle$). In
the three-dimensional path state, we control the polarization of each path to satisfy \((0) \rightarrow |H\rangle, |1\rangle \rightarrow |V\rangle, \text{ and } |2\rangle \rightarrow |H\rangle\). After the PBS, we post-select the event in which there is one and only one photon in each outport. For the nine classical terms of the three-dimensional Bell states \(|i,j\rangle, i, j \in \{0, 1, 2\}\), five of them are selected \(|i,j\rangle\) with \(i + j\) even.

The second step is to use a local projection measurement to determine which Bell state is post-selected through \(|\psi_{00}\rangle, |\psi_{10}\rangle \text{ and } |\psi_{20}\rangle\) (here, terms \(20\) and \(02\) are noise terms and are cancelled later). We can construct an arbitrary single qutrit basis (e.g., \(|0\rangle + |1\rangle + |2\rangle) / \sqrt{3}\) by half-wave plates (HWP's), beam displacers (BDs) and PBSs, so that we can determine whether the measured state is \(|\psi_{00}\rangle\) by measuring this basis on both sides [20].

To cancel the noise terms (\(20\) and \(02\)), we introduce another entangled photon pair (Methods). Hence, we can distinguish at least one Bell state deterministically. For Bell state \(|\psi_{10}\rangle \text{ and } |\psi_{20}\rangle\), we need to select different local projection measurements. Finally, we have teleported a three-level quantum state with a success probability of 1/54. To increase the success probability, we use the non-maximally entangled state \((2|00\rangle + |11\rangle + |22\rangle)/3\) to replace the maximally entangled state \(|\xi\rangle_{23}\), and adjust the measurement base on HDBSM correspondingly; this

FIG. 3. Experimental results for quantum teleportation of three-dimensional single photon states \(|\varphi_1\rangle-|\varphi_2\rangle\). a-c, Measurement results of the final state of the teleported photon 3 in bases \(|0\rangle, |1\rangle, \text{ and } |2\rangle\) for \(|\varphi_1\rangle-|\varphi_2\rangle\). d, The results for state \((|0\rangle + |1\rangle)/\sqrt{2}\), measured in bases \(|0\rangle \pm |1\rangle)/\sqrt{2}\) and \(|2\rangle\). e, The results for state \((|0\rangle + |1\rangle)/\sqrt{2}\), measured in bases \(|0\rangle \pm |i\rangle)/\sqrt{2}\) and \(|2\rangle\). f, The results for state \((|0\rangle + |i\rangle)/\sqrt{2}\), measured in bases \(|0\rangle \pm |i\rangle)/\sqrt{2}\) and \(|1\rangle\). g, The results for state \((|0\rangle + |i\rangle)/\sqrt{2}\), measured in bases \(|0\rangle \pm |2\rangle)/\sqrt{2}\) and \(|1\rangle\). h, The results for state \((|0\rangle + |2\rangle)/\sqrt{2}\), measured in bases \(|0\rangle \pm |2\rangle)/\sqrt{2}\) and \(|1\rangle\). i, The results for state \((|1\rangle + |2\rangle)/\sqrt{2}\), measured in bases \(|1\rangle \pm |i\rangle)/\sqrt{2}\) and \(|0\rangle\). Error bars are calculated from Poissonian counting statistics of the raw detection events.
increases the success probability of teleportation to 1/18 (Methods).

The implementation of the HDBSM requires Hong-Ou-Mandel (HOM)-type interference between indistinguishable single photons with good temporal, spatial, and spectral overlap. We use a narrow band interference filter (3 nm) and a single-mode fiber to improve the visibility of HOM interference. For photon 3 and photon t, we use a broad band interference filter (8 nm) to increase the coincidence efficiency.

The verification of the teleportation results relies on the coincidence events of six photons. To suppress the statistical error, the data collection time is tens of hours. Hence, the stability of the whole system becomes a crucial aspect for the experiment. In our system, the HOM interference between the different photons is stable enough [30], whereas the interference between different spatial modes after passing through the single-mode fibers is not. Here, we use a fiber phase locking system (Methods) to maintain a phase-stable interferometer. The measured interference visibility remained above 0.98 in 45 hours (Methods).

|ϕ⟩ = (|0⟩ + i|2⟩)/√2, |ϕ2⟩ = (|1⟩ + |2⟩)/√2, |ϕ3⟩ = (|0⟩ + i|1⟩)/√2, |ϕ4⟩ = (|0⟩ + |1⟩)/√2, |ϕ5⟩ = ((0 + i|1⟩)/√2, |ϕ6⟩ = (|0⟩ + |2⟩)/√2.

We prepared ten different initial states to be teleported: |ϕ1⟩ = |0⟩, |ϕ2⟩ = |1⟩, |ϕ3⟩ = |2⟩, |ϕ4⟩ = (|0⟩ + |1⟩)/√2, |ϕ5⟩ = (|0⟩ + i|1⟩)/√2, |ϕ6⟩ = (|0⟩ + |2⟩)/√2, |ϕ7⟩ = (|0⟩ + i|2⟩)/√2, |ϕ8⟩ = (|1⟩ + |2⟩)/√2, |ϕ9⟩ = (|1⟩ + i|2⟩)/√2, and |ϕ10⟩ = (|0⟩ + |1⟩+ |2⟩)/√3. The first nine states (|ϕ1⟩-|ϕ9⟩) constitute a complete orthogonal basis in three-dimensional space; the last state |ϕ10⟩ is a linear-dependent superposition of quantum states in this space. All these states are prepared by the BDs, QWPs, and HWPs.

To evaluate the performance of the multi-level teleportation, we use fidelity F = Tr(ρ|ϕ⟩⟨ϕ|), representing the overlap of the ideal teleported state (|ϕ⟩) and the measured density matrix (ρ). Conditioned on the detection of the trigger photon and the four-photon coincidence after the HDBSM, we registered the photon counts of teleported photon 1. The first nine states are a set forming a complete basis for three-dimensional tomography. The measurement basis and raw data of states are shown in Fig. 3. The fidelity for each of |ϕ1⟩, ⋯, |ϕ9⟩ is, in numerical sequence: 0.740 ± 0.035, 0.715 ± 0.035, 0.707 ± 0.033, 0.673 ± 0.033, 0.6782 ± 0.0365, 0.665 ± 0.033, 0.645 ± 0.036, 0.630 ± 0.037, 0.643 ± 0.037.

In summary, we have reported the quantum teleportation of a multi-level state of a single photon.

Despite the experimental noise, the measured fidelities (summarized in Fig. 4) of the ten teleported states are all well above 0.50, the classical limit [27, 28], defined as the optimal state-estimation fidelity on a single qutrit system. These results prove the successful realization of quantum teleportation of a multi-level state of a single photon.

In summary, we have reported the quantum teleportation of multi-level quantum states of a single quantum particle, demonstrating the capability to control coherently and teleport simultaneously a high-dimensional state of a single object. The generation of high-quality high-dimensional multi-photon state will stimulate the research on high-dimensional quantum information tasks, and entanglement-assisted methods for HDBSM are feasible for other high-dimensional quantum information tasks.
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Methods

Protocol for teleporting three-dimensional quantum states. The combined state of photons 1, 2, and 3 are rewritten in terms of basis states formed from nine orthogonal three-dimensional Bell states as follows:

$$|\xi\rangle_{23}|\varphi\rangle_1 = \frac{1}{\sqrt{3}}(|00\rangle_{23} + |11\rangle_{23} + |22\rangle_{23}) (|0\rangle_1 + \beta |1\rangle_1 + \gamma |2\rangle_1)$$

$$= \frac{1}{3}(\psi_{00}^{12} \otimes |\varphi\rangle_3 + |\psi_{10}^{12} \otimes |\varphi\rangle_3 + |\psi_{02}^{12} \otimes |\varphi\rangle_3$$

$$+ |\psi_{01}^{12} \otimes |\varphi\rangle_3 + |\psi_{11}^{12} \otimes |\varphi\rangle_3 + |\psi_{21}^{12} \otimes |\varphi\rangle_3$$

$$+ |\psi_{02}^{12} \otimes |\varphi\rangle_3 + |\psi_{12}^{12} \otimes |\varphi\rangle_3 + |\psi_{22}^{12} \otimes |\varphi\rangle_3).$$

The three-dimensional Pauli matrices are:

$$U_{00} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{01} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad U_{02} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

By performing HDBSMs on photons 1 and 2, Alice determines which Bell state she has, for example, $|\psi_{00}\rangle$. When she tells Bob the measurement result, Bob applies an appropriate three-dimensional unitary operation on the qutrit entanglement source. The auxiliary entangled pairs are generated by our standard technology [39].

Generating three photon pairs. The ultraviolet pulse laser from the mode-locked Ti:sapphire laser is split into two beams by a PBS. One beam (with 170 mW power) is used to generate a qutrit entanglement source. The other beam (with 170 mW power) successively passes through two “beamlike” sources to generate the two-photon pairs in type-II spontaneous parametric down conversion (SPDC).

The entanglement source is different from the previous three-dimensional entanglement source we have prepared [29]. The source here is pumped by an ultrashort pulsed laser using a type II BBO crystal with a beam-like sandwich structure [30]. Hence, the whole optical path must be compensated strictly in the temporal domain and spatial domain (see Supplementary Figure 1). Finally, we used a BD20 to divide $|H\rangle$ and $|V\rangle$ polarization into two parallel beams to form a three-dimensional entanglement of photons. In this experiment, we ignored the $|VV\rangle$ photon pairs generated by the lower path. The interference visibility of any two-dimensional subspace $|\{0\} + |1\} \rangle/\sqrt{2}$, $|\{0\} + |2\} \rangle/\sqrt{2}$, $|\{1\} + |2\} \rangle/\sqrt{2}$ is greater than 0.98. To maximize the probability of successful teleportation, the entangled state we actually prepared is $(2|00\rangle + 2|11\rangle + |22\rangle)/3$.

The teleported high-dimensional quantum state is triggered by the detection of photon t. HWPs, QWPs, and BD20 are used to prepare our teleported quantum states. The reference light is then emitted from the other end of the optical fiber, and the two Mach–Zehner (MZ) interferometers are constructed by pairs BD1–BD2 and BD1–BD3. The phase between BD is stable, and all the phase changes are due to the instability of the two.
optical fibers. Hence, we only need to use the PZT to adjust the position of the mirror according to the signal of the MZ interferometer composed of BD1 and BD2 to lock the phase of the optical fiber. The other interferometer consisting of BD1 and BD3 is used to monitor the interference visibility of the optical fiber system. In this system, weak light is used as reference light, and a single photon detector is used for detection. We monitored the interference visibility of our system for 45 hours, and the results show that the average interference visibility remained around 0.98 (see Supplementary Figure 3).

Analysis of noise in the experiment. There are two main noise sources in our experiment: one is noise from higher-order photon-pair emissions and the other is from the temporal distinguishability between different photon pairs. The most efficient way of reducing the former is to improve photon detection efficiency \( \eta \). In our system, the coincidence efficiencies of the three sources are \( \eta_1 \sim 20\% \), \( \eta_2 \sim 26\% \), and \( \eta_3 \sim 14\% \).

Aside from choosing a narrow band filter (FWHM is 3 nm) for the interfering photons to minimize the temporal distinguishability noise, we also split the pump laser into two beams (see Fig. 1) to generate the three EPR sources. In this way, the dispersion of the pump pulse in passing through the BBO crystals is minimized and the temporal indistinguishability parameter \( \varepsilon/\Delta \) is maximized.

Three-dimensional BSM In this section, we introduce the 3-HDBSM in detail. There are nine three-dimensional Bell states \( |\psi_{ij}\rangle \):

\[
\begin{align*}
|\psi_{00}\rangle &= (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3}, \\
|\psi_{10}\rangle &= (|00\rangle + e^{2\pi i/3}|11\rangle + e^{4\pi i/3}|22\rangle)/\sqrt{3}, \\
|\psi_{20}\rangle &= (|00\rangle + e^{4\pi i/3}|11\rangle + e^{2\pi i/3}|22\rangle)/\sqrt{3}, \\
|\psi_{01}\rangle &= (|01\rangle + |12\rangle + |20\rangle)/\sqrt{3}, \\
|\psi_{11}\rangle &= (|01\rangle + e^{2\pi i/3}|12\rangle + e^{4\pi i/3}|20\rangle)/\sqrt{3}, \\
|\psi_{21}\rangle &= (|01\rangle + e^{4\pi i/3}|12\rangle + e^{2\pi i/3}|20\rangle)/\sqrt{3}, \\
|\psi_{02}\rangle &= (|02\rangle + |10\rangle + |21\rangle)/\sqrt{3}, \\
|\psi_{12}\rangle &= (|02\rangle + e^{2\pi i/3}|10\rangle + e^{4\pi i/3}|21\rangle)/\sqrt{3}, \\
|\psi_{22}\rangle &= (|02\rangle + e^{4\pi i/3}|10\rangle + e^{2\pi i/3}|21\rangle)/\sqrt{3}.
\end{align*}
\]

Similar to the measurements of the two-dimensional Bell states, the first step is to classify Bell states according to classical terms. For three-dimensional Bell states, we need to classify them into three categories: \( \{ |\psi_{0i}\rangle ; i = 0, 1, 2 \} \), which contain \( |00\rangle \), \( |11\rangle \), and \( |22\rangle \); \( \{ |\psi_{1i}\rangle ; i = 0, 1, 2 \} \), which contain \( |01\rangle \), \( |12\rangle \), and \( |20\rangle \); and \( \{ |\psi_{2i}\rangle ; i = 0, 1, 2 \} \), which contain \( |02\rangle \), \( |10\rangle \), and \( |21\rangle \). We convert the path information to polarization-path hybrid encoded states (see Supplementary Figure 4), such that \( |0\rangle \rightarrow |H\rangle_0 \), \( |1\rangle \rightarrow |V\rangle_1 \), \( |2\rangle \rightarrow |H\rangle_2 \).

After the two photons enter the PBS1, only \( |00\rangle \), \( |11\rangle \), \( |22\rangle \), \( |02\rangle \), and \( |20\rangle \) can have one photon on each output port of PBS1; the other classical terms \( |01\rangle \), \( |12\rangle \), \( |10\rangle \), and \( |21\rangle \) have two photons on only one side. If we only collect the events for which two photons exit on the two sides of PBS1, then the latter four classical terms are ignored. Here we only consider three terms \( |00\rangle \), \( |11\rangle \), and \( |22\rangle \) as signals (there are still two noise terms \( |02\rangle \) and \( |20\rangle \), which can be cancelled by introducing an auxiliary entanglement to be explained later), and can distinguish the three Bell states \( |\psi_{00}\rangle \), \( |\psi_{10}\rangle \), and \( |\psi_{20}\rangle \) through local projection measurements from BD3–BD6, PBSs, and HWPs. For example, if we construct a measurement basis of \( (|0\rangle + |1\rangle + |2\rangle)/\sqrt{3} \) on both sides, then we can identify the quantum state of \( (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3} \).

We now explain the removal of the two noise terms \( |02\rangle \), \( |20\rangle \). In our scheme, we use an auxiliary entangled photon pair 4–5 to cancel the noise terms. If we consider the optical path between BD2 and BD4 as a process in changing the quantum state, we find that after BD3 and BD4, classical terms \( |00\rangle \), \( |11\rangle \), and \( |22\rangle \) convert to \( |HH\rangle \) or \( |VV\rangle \) whereas noise terms \( |02\rangle \) and \( |20\rangle \) convert to \( |HV\rangle \) and \( |VH\rangle \). If we only collect those events for which there is one and only one photon at each output of PBS3 and PBS4, then the two noise terms can be cancelled.

In 3-DBSM, we identify only one of the nine Bell states deterministically, and we use only one measurement basis for the two sides, so the probability of success is only \( 1/9 \times 1/3 = 1/27 \). In addition, the probability of success for auxiliary entangled photons is \( 1/2 \). The overall efficiency of the HDBSM combining all steps is \( 1/9 \times 1/3 \times 1/2 = 1/54 \).

To improve the success probability, we do not use maximum entanglement \( (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3} \), but use \( (|00\rangle + 2|11\rangle + |22\rangle)/3 \) as the entangled channel for teleportation. Correspondingly, \( (|H\rangle \pm |V\rangle)/\sqrt{2} \) is used for the projection measurement on four photons in the HDBSM. In this way, we improve the success probability of teleportation to \( 1/18 \).

Suppose we prepare a three-dimensional entangled state \( |\xi_{23}\rangle = (2|H_0\rangle|H_0\rangle + 2|V_1\rangle|V_1\rangle + |H_2\rangle|H_2\rangle)/3 \) on photon 2–3. The teleported state (photon 1) and the trigger state (photon t) is prepared on \( |\varphi_1\rangle \otimes |H\rangle_t = (\alpha|H_0\rangle + \beta|V_1\rangle + \gamma|H_2\rangle) \otimes |H\rangle_t \). Therefore, the quantum state of photon 1, 2, 3, and t is
When photons 1 and 2 pass through PBS1, we neglect the instance when two photons go to the same side and hence obtain the quantum state

\[
\left( \frac{2}{3} |H_0\rangle |H_0\rangle |H_0\rangle |H_0\rangle |H_0\rangle \right) + \frac{1}{3} |H_2\rangle |H_0\rangle |H_2\rangle |H_0\rangle |H_0\rangle + \frac{2}{3} |\beta\rangle |V_1\rangle |V_1\rangle |V_1\rangle |V_1\rangle + \frac{1}{3} |\gamma\rangle |H_2\rangle |H_2\rangle |H_2\rangle |H_0\rangle \otimes |H_t\rangle.
\]  

Then, after BD1 and BD3, the quantum state becomes

\[
\left( \frac{2}{3} |H_0\rangle |V_0\rangle |V_0\rangle |V_0\rangle |V_0\rangle \right) + \frac{1}{3} |\alpha\rangle |H_2\rangle |V_0\rangle |V_0\rangle |V_0\rangle + \frac{2}{3} |\beta\rangle |V_1\rangle |H_0\rangle |H_0\rangle |V_0\rangle + \frac{1}{3} |\gamma\rangle |H_2\rangle |H_2\rangle |V_0\rangle |V_0\rangle \otimes |H_t\rangle.
\]  

After the HWP, the quantum state becomes

\[
\left( \frac{2}{3} |H_0\rangle |V_0\rangle |V_0\rangle |V_0\rangle |V_0\rangle \right) + \frac{1}{3} |\alpha\rangle |H_2\rangle |V_0\rangle |V_0\rangle |V_0\rangle + \frac{2}{3} |\beta\rangle |V_1\rangle |H_0\rangle |H_0\rangle |V_0\rangle + \frac{1}{3} |\gamma\rangle |H_2\rangle |H_2\rangle |V_0\rangle |V_0\rangle \otimes |H_t\rangle.
\]  

After BD2 and BD4, the quantum state becomes

\[
\left( \frac{1}{3} |\alpha\rangle |H_0\rangle |H_0\rangle |H_0\rangle |H_0\rangle \right) + \frac{\sqrt{3}}{6} |\alpha\rangle |H_2\rangle |V_0\rangle |V_0\rangle |V_0\rangle + \frac{\sqrt{3}}{3} |\beta\rangle |V_1\rangle |H_0\rangle |H_0\rangle |V_0\rangle + \frac{1}{3} |\gamma\rangle |H_2\rangle |H_2\rangle |V_0\rangle |V_0\rangle \otimes |H_t\rangle.
\]  

To remove the two noise terms, we introduce auxiliary entanglement \(|\varepsilon\rangle_{45} = (|H\rangle |H\rangle + |V\rangle |V\rangle)/\sqrt{2}\) on photon 4–5; the total six-photon state then becomes

\[
\left( \frac{1}{3} |\alpha\rangle |H_0\rangle |H_0\rangle |H_0\rangle |H_0\rangle \right) + \frac{\sqrt{2}}{6} |\alpha\rangle |H_2\rangle |V_0\rangle |V_0\rangle |V_0\rangle + \frac{1}{3} |\beta\rangle |V_1\rangle |H_0\rangle |H_0\rangle |H_0\rangle + \frac{1}{3} |\gamma\rangle |H_2\rangle |V_0\rangle |V_0\rangle \otimes \frac{1}{\sqrt{2}}(|H_H\rangle + |V_V\rangle) \otimes |H_t\rangle.
\]

After PBS2 and PBS3, we only retain the six-fold coincidence quantum states

\[
\frac{\sqrt{2}}{6} \left( \frac{1}{3} |\alpha\rangle |H_0\rangle |H\rangle |H\rangle |H\rangle |H\rangle + \beta |V_1\rangle |H\rangle |H\rangle |H\rangle |H\rangle + \alpha |H_2\rangle |V\rangle |V\rangle |V\rangle |V\rangle \right) \otimes |H_t\rangle.
\]  

After setting HWP1–4 to 22.5°, the quantum state changes to

\[
\left( \frac{\sqrt{2}}{6} |\alpha\rangle |H_0\rangle + \beta |V_1\rangle + \gamma |H_2\rangle \right) \left( \frac{1}{4} (|H_HH_H\rangle + |H_HH_V\rangle + |H_VH_H\rangle + |H_VV_H\rangle + |V_HH_H\rangle + |V_HH_V\rangle + |V_VH_H\rangle + |V_VV_H\rangle) \right) \otimes |H_t\rangle.
\]  

Finally, we completed the teleportation according to the six-fold coincidence between parts 1, 2, 3, 4 (see Supplementary Figure 1) and trigger photon t, photon 3. The probability of a successful teleportation is 1/18.

**Single photon measurement on photon 3.** For photon 3, we use different single-observable measurements to determine the fidelity (see Supplementary Figure 5).
Supplementary Figure 1. **Experimental device for three-dimensional entanglement preparation.** The pump beam is split into two beams by two 390-nm BD20s and HWPs and focused on a type-II beamlike cut BBO crystal at two different spots to generate photon pairs via SPDC. The two BD20 axes are opposite to keep the upper and lower light paths equal. The upper path generates polarization entanglement $(|HH⟩ + |VV⟩)/\sqrt{2}$ and therefore needs time and space compensation. The beam splitting ratio of upper and lower pumping beams is 4:1. Finally, the $|H⟩$ and $|V⟩$ beams are divided into parallel beams by BD20 so we can generate the state $(2|00⟩ + 2|11⟩ + |22⟩)/3$.

Supplementary Figure 2. **Optical fiber phase locking system.** To reduce the influence of the reference light (orange solid line) on the signal light (red solid line) of the system, their propagation directions oppose each other. The light from both are weak. The whole system consists of two MZ interferometers, BD1–BD2 and BD1–BD3. The first interferometer is used to provide the phase-locked signal, and the other interferometer is used to monitor the phase-locked interference visibility.
Supplementary Figure 3. **Fiber lock-in phase stabilization.** Within 45 hours, the average interference visibility is above 0.98. The fluctuation of interference visibility is mainly affected by ambient temperature and power stability of reference light.
Supplementary Figure 4. **3-DBSM scheme.** When two three-dimensional photons are incident on the PBS, we only retain terms $|00\rangle$, $|11\rangle$, $|22\rangle$, $|02\rangle$, and $|20\rangle$ where there is only one photon on each outport. Noise terms $|02\rangle$, $|20\rangle$ are filtered out by the auxiliary entanglement.
Supplementary Figure 5. **Typical single-observable measurement devices.** Experimental setups for measuring: a, $(|0\rangle + |1\rangle + |2\rangle)/\sqrt{3}$, b, $(|0\rangle \pm |1\rangle)/\sqrt{2}$, c, $(|0\rangle \pm |2\rangle)/\sqrt{2}$, and d, $(|1\rangle \pm |2\rangle)/\sqrt{2}$. Here, $|0\rangle$ and $|2\rangle$ are horizontally polarized, and $|1\rangle$ is vertically polarized.