Pseudoscalar mesons in asymmetric matter

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Abstract

The behavior of kaons and pions in hot non strange quark matter, simulating neutron matter, is investigated within the SU(3) Nambu-Jona-Lasinio [NJL] and in the Enlarged Nambu-Jona-Lasinio [ENJL] (including vector pseudo-vector interaction) models. At zero temperature, it is found that in the NJL model, where the phase transition is first order, low energy modes with $K^-, \pi^+$ quantum numbers, which are particle-hole excitations of the Fermi sea, appear. Such modes are not found in the ENJL model and in NJL at finite temperatures. The increasing temperature has also the effect of reducing the splitting between the charge multiplets.
1 Introduction

During the last few years, major experimental and theoretical efforts have been dedicated to heavy-ion collisions aiming at understanding the properties of hot and dense matter and looking for signatures of phase transitions to the quark-gluon plasma. As a matter of fact, it is believed that, at critical values of the density, $\rho_c$, and/or temperature, $T_c$, the system undergoes a phase transition, the QCD vacuum being then described by a weakly interacting gas of quarks and gluons, with restored chiral symmetry.

The nature of the phase transition is an important issue nowadays. Lattice simulations [1] provide information at zero density and finite temperature but for finite densities no firm lattice results are available and most of our knowledge comes from model calculations. The Nambu-Jona-Lasinio [2] (NJL) type models have been extensively used over the past years to describe low energy features of hadrons and also to investigate restoration of chiral symmetry with temperature or density [3, 23, 6, 5, 7].

Recently, it was shown by Buballa [8] that, with a convenient parameterization, the SU(2) and SU(3) NJL models exhibit a first order phase transition, the system being in a mixed phase between $\rho = 0$ and $\rho = \rho_c$, the energy per particle having an absolute minimum at $\rho = \rho_c$. This suggests an interpretation of the model within the philosophy of the MIT bag model. The system has two phases, one consisting of droplets of quarks of high density and low mass surrounded by a non trivial vacuum and the other one consisting of a quark phase of restored chiral symmetry. Similar concepts appear in NJL inspired models including form factors [9, 10]. The physical meaning of the mesonic excitations in the medium within this interpretation of the model is an interesting subject that will be analyzed in this paper.

The possible modifications of meson properties in the medium is an important issue nowadays. The study of pseudoscalar mesons, such as kaons and pions, is particularly interesting, since, due to their Goldstone boson nature, they are intimately associated with the breaking of chiral symmetry. Since the work of Kaplan and Nelson [11], the study of medium effects on these mesons in flavor asymmetric media attracted a lot of attention. Indeed, the charge multiplets of those mesons, that are degenerated in vacuum or in symmetric matter, were predicted to have a splitting in flavor asymmetric matter. In particular, as the density increases, there would be an increase of the mass of $K^+$ and a decrease of the mass of $K^-$; a similar effect would occur for $\pi^-$ and $\pi^+$ in neutron matter. The mass decrease of one of the multiplets raises, naturally, the issue of meson condensation, a topic specially relevant to Astrophysics.

Most theoretical approaches dealing with kaons in flavor asymmetric media, predict a slight raising of the $K^+$ mass and a pronounced lowering of the $K^-$ mass [12, 3, 8, 4], a conclusion which is supported by the analysis of data on kaonic atoms [13]. Experimental results at GSI seem to be compatible with this scenario [16, 17, 18].

Studies on pions in asymmetric medium are mainly related with the problem of the $u-d$ asymmetry in a nucleon sea rich in neutrons. Such flavor asymmetry has been established in SIS and DY experiments and theoretical studies show that there is a significant difference in $\pi^+, \pi^-$ distribution functions in neutron rich matter [19].

From the theoretical point of view, the driving mechanism for the mass splitting is attributed mainly to the selective effects of the Pauli principle, although, in the case of $K^-$, the interaction with the $\Lambda(1405)$ resonance plays an important role as well. In the
study of the effects of the medium on hadronic behavior, one should have in mind that the medium is a complex system, where a great variety of medium particle-hole excitations occur, some of them with the same quantum numbers of the hadrons under study; the interplay of all these excitations might play a significant role in the modifications of hadron properties. In previous works we have established, within the framework of NJL models, the presence, in flavor asymmetric media, of low energy pseudoscalar modes, which are excitation of the Fermi sea [4, 6]. The combined effect of density and temperature, as well as the effect of vector interaction, was discussed for the case of kaons in symmetric nuclear matter without strange quarks [5, 23].

This paper addresses the following points: a) analyzes of the phase transition with density and temperature in neutron matter in the $SU(3)$ NJL model with two different parameterizations and within the ENJL model; b) behavior of kaonic and pionic excitations in these models and discussion of the meaning of the Fermi sea excitations, in connection with the nature of the phase transition; c) combined effect of density and temperature.

Formalism

We work in a flavor $SU(3)$ NJL type model with scalar-pseudoscalar and vector-pseudovector pieces, and a determinantal term, the 't Hooft interaction, which breaks the $U_A(1)$ symmetry. We use the following Lagrangian:

$$\mathcal{L} = \bar{q} \left( i \gamma^\mu \partial^\mu - m \right) q + \frac{1}{2} g_S \sum_{a=0}^8 \left[ (\bar{q} \lambda^a q)^2 + (\bar{q} i \gamma_5 \lambda^a q)^2 \right] - \frac{1}{2} g_V \sum_{a=0}^8 \left[ (\bar{q} \gamma_\mu \lambda^a q)^2 + (\bar{q} \gamma_\mu \gamma_5 \lambda^a q)^2 \right] + g_D \left\{ \det \left[ \bar{q} \left( 1 + \gamma_5 \right) q \right] + \det \left[ \bar{q} \left( 1 - \gamma_5 \right) q \right] \right\} \tag{1}$$

In order to discuss the predictions of different models we consider the cases: $g_V = 0$ with two parametrizations (NJL I and NJL II) and $g_V \neq 0$ (ENJL). The model parameters, the bare quark masses $m_d = m_u, m_s$, the coupling constants and the cutoff in three-momentum space, $\Lambda$, are essentially fitted to the experimental values of $m_\pi$, $f_\pi$, $m_K$ and to the phenomenological values of the quark condensates, $<\bar{u}u>$, $<\bar{d}d>$, $<\bar{s}s>$. The parameter sets used are, for NJL I: $\Lambda = 631.4$ MeV, $g_S \Lambda^2 = 3.658$, $g_D \Lambda^5 = -9.40$, $m_u = m_d = 5.5$ MeV and $m_s = 132.9$ MeV; for ENJL: $\Lambda = 750$ MeV, $g_S \Lambda^2 = 3.624$, $g_D \Lambda^5 = -9.11$, $g_V \Lambda^2 = 3.842$, $m_u = m_d = 3.61$ MeV and $m_s = 88$ MeV. For NJL II we use the parametrization of [20], $\Lambda = 602.3$ MeV, $g_S \Lambda^2 = 3.67$, $g_D \Lambda^5 = -12.39$, $m_u = m_d = 5.5$ MeV and $m_s = 140.7$ MeV, which underestimates the pion mass ($m_\pi = 135$ MeV) and of $\eta$ by about 6%.

The six quark interaction can be put in a form suitable to use the bosonization procedure (see [24, 22, 23]):

$$\mathcal{L}_D = \frac{1}{6} g_D D_{abc} (\bar{q} \lambda^c q) [(\bar{q} \lambda^a q)(\bar{q} \lambda^b q) - 3 (\bar{q} i \gamma_5 \lambda^a q)(\bar{q} i \gamma_5 \lambda^b q)] \tag{2}$$

with: $D_{abc} = d_{abc}, a, b, c \epsilon \{1, 2, ..8\}$, (structure constants of SU(3)), $D_{000} = \sqrt{\frac{2}{3}}, D_{0ab} = -\sqrt{\frac{1}{6}} d_{ab}$. 

2
The usual procedure to obtain a four quark effective interaction from this six quark interaction is to contract one bilinear \((\bar{q} \lambda_a q)\). Then, from the two previous equations, an effective Lagrangian is obtained:

\[
L_{\text{eff}} = \bar{q} (i \gamma^\mu \partial_\mu - \hat{m}) q + S_{ab} [(\bar{q} \lambda^a q)(\bar{q} \lambda^b q)] + P_{ab} [(\bar{q} i \gamma_5 \lambda^a q)(\bar{q} i \gamma_5 \lambda^b q)] - \frac{1}{2} g_V \sum_{a=0}^{S} [(\bar{q} \gamma_\mu \lambda^a q)^2 + (\bar{q} \gamma_\mu \gamma_5 \lambda^a q)^2] \tag{3}
\]

where:

\[S_{ab} = g_S \delta_{ab} + g_D D_{abc} <\bar{q} \lambda^c q>\]
\[P_{ab} = g_S \delta_{ab} - g_D D_{abc} <\bar{q} \lambda^c q>\] \tag{4}

By using the usual methods of bosonization one gets the following effective action:

\[
I_{\text{eff}} = -i \text{Tr} \ln (i \partial_\mu \gamma_\mu - \hat{m} + \sigma_a \lambda^a + i \gamma_5 \phi_a \lambda^a + \gamma^\mu V_\mu + \gamma_5 \gamma^\mu A_\mu) - \frac{1}{2} (\sigma_a S_{ab}^{-1} \sigma_b + \phi_a P_{ab}^{-1} \phi_b) + \frac{1}{2G_V} (V^a_\mu A^a_\mu) \tag{5}
\]

from which we obtain the gap equations and meson propagators.

In order to introduce the finite temperature and density, we use the thermal Green function, which, for a quark \(q_i\) at finite temperature \(T\) and chemical potential \(\mu_i\) reads:

\[
S(\vec{x} - \vec{x'}, \tau - \tau') = \frac{i}{\beta} \sum_n e^{-i \omega_n (\tau - \tau')} \int \frac{d^3 p}{(2\pi)^3} \frac{e^{-i \vec{p}(\vec{x} - \vec{x'})}}{\gamma_0 (i \omega_n + \vec{p}^2) - \gamma_\tau \vec{p} - M_i}, \tag{6}
\]

where \(\beta = 1/T, \vec{p}_i = \mu_i - \Delta E_i, \Delta E_i\) is the energy gap induced by the vector interaction, \(M_i\) the mass of the constituent quarks, \(E_i = (p^2 + M_i^2)^{1/2}\) and \(\omega_n = (2n + 1) \frac{\pi}{\beta}, n = 0, \pm 1, \pm 2, \ldots,\) are the Matsubara frequencies. The following gap equations are obtained:

\[
M_i = m_i - 2 g_S <\bar{q}_i q_i> - 2 g_D <\bar{q}_j q_j> <\bar{q}_k q_k> \tag{7}
\]
\[
\Delta E_i = 2 g_V <q_i^+ q_i> \tag{8}
\]

with \(i, j, k\) cyclic and \(<\bar{q}_i q_i>, <q_i^+ q_i>\) are respectively the quark condensates and the quark densities at finite \(T\) and \(\mu_i\).

The condition for the existence the poles in the propagators of kaons leads to the following dispersion relation:

\[
(1 - K_P J_{PP})(1 - K_A J_{AA}) - K_P K_A J^2_{PA} = 0 \tag{9}
\]
with:

$$\omega J_{PA} = (M_u + M_s) J_{PP} + 2 (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle).$$

$$\omega J_{AA} = (M_u + M_s) J_{PA} + 2 (\langle u^+u \rangle - \langle s^+s \rangle).$$

$$J_{PP} = 2 N_c \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{M_u(M_s - M_u) - q_0 E_u}{(E_s^2 - (q_0 + E_u)^2)} \frac{\tanh \beta(E_u + \bar{\mu}_u)}{2} + \frac{M_u(M_s - M_u) + q_0 E_u}{(E_s^2 - (q_0 - E_u)^2)} \frac{\tanh \beta(E_u - \bar{\mu}_u)}{2} \right\}$$

$$\implies J_{PA} = \frac{1}{2} J_{PP} + \frac{1}{2} J_{PP},$$

$$\implies \omega J_{PA} = (M_u + M_s) J_{PP} + 2 (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle).$$

$$\omega J_{AA} = (M_u + M_s) J_{PA} + 2 (\langle u^+u \rangle - \langle s^+s \rangle).$$

We reanalyze the problem of the phase transitions in order to establish a connection between the vacuum state and its excitations. We consider here the case of asymmetric quark matter without strange quarks simulating neutron matter: $\rho_u = \frac{1}{2} \rho_d, \rho_s = 0$ and calculate the energy and pressure. At zero temperatures we found a first order phase transition in NJL model, exhibiting different characteristics according to the parameterization used. Within the parameterization NJL I, the pressure is negative for $0.8\rho_0 \leq \rho \leq 1.65\rho_0$ and the absolute minimum of the energy per particle is at $\rho = 0$ (dashed curves in Fig.1).

Figure 1: Pressure and energy per particle in NJL I (dashed curves) and NJL II (full curves).

The system, within the range of densities indicated above, is in a mixed phase consisting of droplets of massive quarks of low density and droplets of light quarks of high density and, for $\rho > 1.65\rho_0$, is in a quark phase with partially restored chiral symmetry.
(in the SU(2) sector). These droplets are unstable since the absolute minimum of the energy per particle is at \( \rho = 0 \). With parameterization II, the mixed phase starts at \( \rho \approx 0 \), because, although the zeros of the pressure are at \( \rho = 0.44\rho_0, \rho_c = 2.25\rho_0 \), the compressibility is negative in the low density region for \( \rho \approx 0 \); the energy per particle has an absolute minimum at the critical density of \( E/A = 1102 \text{ MeV} \), about three times the masses of the constituent non strange quarks in vacuum (Fig 1. full curves). The model may now be interpreted as having a hadronic phase — droplets of light \( u, d \) quarks with a density \( \rho_c = 2.25\rho_0 \) surrounded by a non trivial vacuum — and, above the critical density, a quark phase with partially restored SU(2) chiral symmetry. The model is not suitable to describe hadrons for \( \rho < \rho_c \). Since there is no definition of the density in the mixed phase, we study, in the following, the mesonic excitations only for \( \rho > \rho_c \).

The phase transition becomes second order at finite density and temperatures around 20\( \text{ MeV} \) and also in the ENJL model, for the set of parameters chosen. In these cases the system has positive pressure but the absolute minimum of the energy per particle is at zero density. Although a gas of quarks does not exist at low densities, the model has been used to study the influence of the medium in the mesonic excitations of the vacuum. Of course, an extrapolation of quark matter to hadronic matter should be made, since we do not have, at low densities, a gas of hadrons.

### 3 Behavior of pions and kaons in the medium

At zero temperature we observe a splitting between charge multiplets, both in NJL and ENJL models (see figs. 2-3).

![Figure 2: Masses of kaons and pions in NJL II. \( \omega_{up} \) and \( \omega_{low} \) denote the limits of the Fermi sea continuum.](image)

These modes are excitations of the Dirac sea modified by the presence of the medium. The increase of \( K^+ \) and \( \pi^- \) masses with respect to those of \( K^- \) and \( \pi^+ \) is due to Fermi blocking and is more pronounced for kaons than for pions because, there are \( u \) and \( d \) quarks in the Fermi sea and therefore there are repulsive effects due to the Pauli principle acting on \( \pi^+ \). In the case of kaons, we do not have strange quarks at these densities.
and so there is more phase space available to create $\bar{u}s$ pairs of quarks and there are less repulsive effects on $K^-$. An interesting feature in NJL model is that, besides these modes, low energy modes with quantum numbers of $K^-$ and $\pi^+$ appear. These are particle-hole excitations of the Fermi sea which correspond to $\Lambda (1106)$-particle-proton-hole for kaons and to a proton-particle-neutron-hole for the case of pions. A similar effect is found for kaons in symmetric nuclear matter \cite{8, 9}. For the case of pions, the low energy modes is less relevant and exist only for $\rho = \rho_c$, merging in the Fermi sea continuum afterwards. We notice that when the ’t Hooft interaction is not included \cite{6} or in SU(2) \cite{10} the low energy mode for pions is more relevant.

The sum rules are a very important tool to analyze the collectivity and relative importance of the modes \cite{11, 6, 7, 12}. One can derive a generalization of the PCAC relation in the medium from the Energy Weighted Sum Rule (EWSR), well known from Many Body Theories. For the mesonic state $|r\rangle$ with energy $\omega_r$ associated with the transition operator $\Gamma$ the strength function $F_r = \omega_r |<r|\Gamma|0>|^2$ satisfies the EWSR which reads

$$m_1 = \sum_r \omega_r |<r|\Gamma|0>|^2 = \frac{1}{2} <\Phi_0|[\Gamma, [H, \Gamma]]]|\Phi_0>, \quad (11)$$

the transition operator being defined in the present case by $\Gamma = \Gamma_+ + \Gamma_-$, with $\Gamma_\pm = \gamma_5 (\lambda_4 \pm i \lambda_5)/\sqrt{2}$, for kaons, and, $\Gamma_\pm = \gamma_5 (\lambda_1 \pm i \lambda_2)/\sqrt{2}$, for pions. We obtain therefore the GMOR relation in the medium:

$$\sum_\alpha m_{K,\alpha}^2 f_{K,\alpha}^2 \simeq -\frac{1}{2} (m_u + m_s) [<\bar{u}u> + <\bar{s}s>] . \quad (12)$$

and

$$\sum_\alpha m_{\pi,\alpha}^2 f_{\pi,\alpha}^2 \simeq -\frac{1}{2} (m_u + m_d) [<\bar{\pi}u> + <\bar{d}d>] . \quad (13)$$

We verified that in the medium the degree of satisfaction of the sum rule is good, provided, naturally, that all the bound state solutions are considered. The strength associated to the low energy mode can not be neglected as the density increases \cite{9, 10}. 

![Figure 3: Masses of kaons and pions in ENJL model.](image-url)
In the ENJL model the low energy mode does not appear, which is consistent with the fact that the analysis of the EOS shows that there is no stable Fermi sea. The attractive effects concentrate on $K^-, \pi^+$. The splitting between the charge multiplets is even larger in this model.

The influence of the temperature is, on one side, to inhibit the occurrence of the low energy mode (this mode is not seen above very low temperatures) and, on the other side, to reduce the splitting of between the upper energy modes (see Fig. 4).

We notice that temperature has an effect on the low energy mode similar to the vector pseudovector interaction in vacuum. This is meaningful since, as it has been mentioned above, as the temperature increases the phase transition is second order and the minimum of energy per particle is at $\rho = 0$ and consequently the Fermi sea is not stable. So, it is reasonable that the excitations of the Fermi sea are not seen.

In conclusion, we have discussed the behavior of kaons and pions in neutron matter in NJL and ENJL model, in connection with the nature of the phase transition, at finite density with zero or non zero temperature. In the NJL model where the phase transition is first order and a stable Fermi sea exists at the critical density, we find low energy particle hole excitations of the Fermi sea, besides the usual splitting of charge multiplets which are excitations of the Dirac sea. This last effect does not occur in the ENJL model, where the transition is second order. The temperature inhibits this effect and reduces the splitting between the charge multiplets.

Figure 4: Masses of kaons and pions for $T = 50$ MeV and $T = 100$ MeV, in NJL II.
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