Crack resistance criterion of plane stress RC elements with prestressed reinforcement

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Abstract. An analytical condition for crack resistance of a specific plane stress reinforced concrete element with prestressed reinforcement is considered in this article. The specific concrete element is reinforced with orthogonally located prestressed rods that coincide with the directions of the coordinate axes. The crack resistance condition is constructed by generalizing of G.A. Geniev theory of plasticity of concrete and reinforced concrete on the region of existence of tensile stresses: “tension-tension” and “compression-tension”. A graphical interpretation of this condition is presented for a plane stressed element depending on the angle between the main force and the coordinate axes. As an example, the crack resistance condition of the support zone of a prestressed concrete beam of a monolithic reinforced concrete frame is considered.

1. Introduction
Whereas it is necessary to protection of buildings and structures from progressive collapse, which is provided for by the requirements of regulatory documents of various countries, including Russia, research is being conducted and various proposals are being developed to solve this problem. As such proposals, in relation to the RC frames of multi-storey buildings, various protection methods are recommended, which include increasing the cross-sections of the load-bearing elements of the building frame [1, 2], intensity and reinforcement schemes [3-5], setting up additional elements in the form of a links system [5, 6], the exclusion of one-way bonds in compounds of elements [7], etc. One of the effective methods of such protection can be the application of prestressing beams over the first, and if necessary, over other floors of the building. In this regard, there is a need to solve a number of theoretical problems, one of which is the task of determining the crack resistance condition of a plane stressed RC element with prestressed reinforcement.

2. Development of crack resistance criterion
An analytical form of the crack resistance criterion for a RC element with prestressed reinforcement can be constructed by generalizing the strength condition of G.A. Geniev the theory of plasticity of concrete and reinforced concrete [10] on the region of existence of tensile stresses: “tension-tension” and “compression-tension”. When constructing analytical dependencies, the following are accepted as initial hypotheses:
- the specific plane stressed concrete element is reinforced with orthogonally located prestressed rods that coincide with the directions of the coordinate axes of the XOY system (Fig. 1);
- only axis normal stresses in reinforcing rods and, in the general case, the dowel action is taken into account in a plane stress state. This assumption is due to the fact that before the formation of cracks in a characteristic element under the action of tangent forces, the dowel action in reinforcing rods is miniscule;
- the forces in the tensile reinforcement reach the forces of crack formation during crack formation in concrete of a plane stressed element equal to the total force acting in the section under consideration in the direction normal to the crack.

**Figure 1.** Scheme of a characteristic prestressed element in the axes of the XOY (a) and in the axes of the main forces (b).

When constructing the analytical crack resistance condition with positive stresses, we will take compressive stresses as negative. We express the total axial forces in a characteristic RC element of unit dimensions $N_i$ through the critical forces in concrete $\bar{N}_i$ and the forces reduced to concrete equivalent to the forces in the reinforcement at the cracking moment in concrete ($\mu_iN_{s,cr}$):

$$\begin{align*}
N_x &= \bar{N}_x + \mu_xN_{s,cr} \quad \text{(a)}; \\
N_y &= \bar{N}_y + \mu_yN_{s,cr} \quad \text{(b)}; \\
\tau_x &= \bar{\tau}_x; \tau_y &= \bar{\tau}_y.
\end{align*}$$

Where $\mu_i$ - the corresponding reinforcement coefficient, defined as the ratio of the area of reinforcement of the corresponding $i$ direction to the area of concrete; $N_{s,cr}$ - total conditional linear force reduced to concrete, equivalent to the total force in conventional and prestressed reinforcement at the time of crack formation.
Let us assume that the value \( \mu_i \) can be both positive and negative, determining the sign of the force in the reinforcement. Obviously, for the signs in concrete to match the corresponding reduced reinforcement stress, it is necessary that \( |N_i| \geq |\mu|N_{s,rc} \).

The crack resistance condition of RC can be represented by analogy with the plasticity condition \([10, 11]\) for a plane stress state in the XOX axes:

\[
\overline{N}_x^2 + \overline{N}_y^2 - (N_x \cdot N_y) + 3N_{xy}^2 - (R_b - R_{by}) (\overline{N}_x + \overline{N}_y) - R_b \cdot R_{by} \cdot t = 0,
\]

(2)

where \( t \) is the thickness of the specific plane stressed element (see Fig. 1).

Since the crack resistance criteria of a typical plane stressed element in the general case, it is convenient to present the analysis of the directions of crack formation in the components of the main forces \( N_1 \) and \( N_2 \). We express condition (2) in terms of the \( N_1 \) and \( N_2 \), using the known dependences of the theory of elasticity between the components of the forces when the axes rotate:

\[
\begin{align*}
N_x &= \frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2} \cos 2\beta; \\
N_y &= \frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2} \cos 2\beta; \\
N_{xy} &= \frac{N_1 - N_2}{2} \sin 2\beta,
\end{align*}
\]

(3)

where \( \beta \) is the angle between the direction of the larger main axial force \( N_1 \) and the positive direction of the X axis (see Fig. 1).

Substituting (3) into (2), we obtain the crack resistance condition of a specific plane stressed element in the components of the main forces:

\[
\begin{align*}
&\left(\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2} \cos \beta\right)^2 + \left(\frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2} \cos \beta\right)^2 - \\
&\left(\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2} \cos \beta\right)\left(\frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2} \cos \beta\right) - t(R_b - R_{by}) - \\
&\left(\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2} \cos \beta\right)\left(\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2} \cos \beta\right) + 3\left(\frac{N_1 - N_2}{2} \sin \beta\right)^2 - \\
&- \mu_s N_{s,rc} ((N_1 + N_2) + (N_1 - N_2) \cos \beta) - \frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2} \cos \beta - \\
&- \mu_s N_{s,rc} ((N_1 + N_2) + (N_1 - N_2) \cos \beta) - \frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2} \cos \beta + \\
&+ N_{s,rc}^2 (\mu_s^2 - \mu_s \mu_y + \mu_y^2) + t(R_b - R_{by})(\mu_s + \mu_y)N_{s,rc} - t \cdot R_b \cdot R_{by} = 0,
\end{align*}
\]

(4)

and after the transformations we finally get.
Analyzing the crack resistance condition (5) for the considered prestressed reinforcement plane stressed element, it has been found, on conditions that $\mu_s \neq \mu_i$, the types of the crack resistance criterion depend on the angle made by the direction of the $N_1$ with the direction of reinforcement. In general, when $\mu_s \neq \mu_i$ and $|\beta| \neq \pi/4$ the crack resistance condition (4) in coordinate system $N_1, N_2$ represents an ellipse whose main axes do not pass through the origin and make angles $\pi/4$ and $3\pi/4$ with the direction of the force. At $|\beta| = \pm \pi/4$, the large major axis of the ellipse passes through the origin of the system $N_1, N_2$.

Let us analyze the specific points of the crack resistance criterion (5), assuming in the general case that crack formation in a specific element can arise from the main tensile stresses in the «tensile-tensile», «tensile-compressive» regions (see Fig. 1b).

3. Uniaxial state of stress

Consider the crack resistance condition of a specific prestressed concrete element during reinforcement only in the direction of the main tensile forces $\beta = 0; N_z = 0; \mu_i = 0$. For this case, condition (5) can be written as

$$N_i^2 - N_i N_z + N_z^2 \left[ t(R_b - R_{bi}) + \frac{\mu_s + \mu_i}{2} \right] (N_i + N_z) -$$

$$3 \frac{\mu_s + \mu_i}{2} N_{s, ecc} (N_i + N_z) \cos \beta + (\mu_s^2 - \mu_i \mu_s + \mu_i^2) N_{s, ecc}^2 +$$

$$t(R_b - R_{bi})(\mu_s + \mu_i) N_{s, ecc} - t \cdot R_b \cdot R_{bi} = 0. \quad (5)$$

For the case of uniaxial compression at $\mu_s = \mu > 0$ equation (6) determines the relation:

$$N_i^b = tR_b + \mu N_{s, ecc}, \quad (7)$$

and in the case of uniaxial tension at $\mu_s = -\mu$:

$$N_i^{bt} = -tR_b + \mu N_{s, ecc}, \quad (8)$$

where $\mu$ is the absolute value of the reinforcement coefficient.

4. Numerical analysis.

Let us determine the crack resistance of the support zone of the prestressed reinforced concrete beam of the double-span frame design, the diagram of which is shown in Figure 2. The choice of this construction is due to the fact that the authors conducted experimental studies, the results of which and, in particular, the obtained experimental value of the crack formation load can be a reliability criterion for the proposed analytical conditions of crack resistance.
Figure 2. The structural diagram of the experimental frame with prestressed crossbars: formwork (a), reinforcement diagram (b); a diagram for coupling a prestressed beam with a column with a distinguished characteristic element E1 at the support zone (c).
The frame construction is made of concrete B40. The beam above the first floor in order to protect the frame from progressive collapse is prestressed in the upper and lower zones from class A600 reinforcement from one rod with a diameter of 8 mm. Such reinforcement is associated with the need to exclude from the constructive nonlinear system under special effects of one-way connections. The cross reinforcement is made of wire with a diameter of 4 mm and a rod spacing in the support zone of 50 mm. The value of the primary controlled stress is \( \sigma = 570 \text{MPa} \). The prestressing force is \( P_{02} = 22\text{kN} \), taking into account all the losses, amounted to each reinforcing rod.

The following values of the calculated parameters for the considered element E1 are obtained owing to a static calculation of the frame:

\[
\mu_x = 1.92 \cdot 10^{-2}, \quad \mu_y = 0.48 \cdot 10^{-2}, \quad N_x = 5.1\text{kN}, \quad N_y = 0.2\text{kN}, \quad N_{xy} = 1.2\text{kN}, \quad \beta = 0.79 \text{ rad}.
\]

The value of \( N_{s,cr} \) is calculated according to the formulas of the current Russian standards [12], taking into account the combined stress state and the adopted reinforcement of the element. \( N_{s,cr} = 12.1\text{kN} \) (without of prestressing \( P_{02} = 0 \), \( N_{s,cr} = 5.03\text{kN} \)).

Taking \( N_{xy} = 0 \) from the system of equations (3) we obtain the values of the main forces:

\[
N_1 = 8.7\text{kN}; \quad N_2 = 6.3\text{kN}.
\]

A graphical interpretation of the crack resistance criterion (5) in the axes \( N_1, N_2 \) at \( \beta = 0.79 \text{ rad} \) is presented in Figure 3.

![Figure 3](image)

**Figure 3.** The crack resistance criterion of the prestressed element E1 (a) at the initial level of release in the x-axis direction (1); unstressed element (2); general view of the experimental evidence crack formation in the support zone of the beam of frame in the zone of the characteristic element E1 (b) under consideration (see Fig. 2, node 1).
This graph shows the experimental values of the coordinates \(N_1\) and \(N_2\) of the forces at the time of the formation of a diagonal cracking the support zone of the prestressed beam for the characteristic element E1 under consideration (Fig. 3b).

5. Conclusions
The formulated crack resistance criterion of a plane stressed reinforced concrete element with prestressed reinforcement on the region of existence of tensile stresses: “tension-tension” and “compression-tension” can be used in the crack resistance calculation of support zones of beams of frame structural systems, including when developing protection methods of frameworks from progressive collapse.

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