On line material
Supplementary data. A metapopulation model to simulate West Nile virus circulation in the old world.

Model description

The model is a deterministic discrete time meta-population model with a daily time step. A population is defined here as a group of animals (vectors or hosts) that share the same location (or places for migratory populations) and that have the same annual life cycle. The epidemiological system is represented by a set $Y$ of host populations that share during their annual life cycle a set $X$ of locations where vector populations live. Some of the host populations are migratory and move between locations during the year.

1. State variables

A host population is described by the distribution of animals according to their health state (the set of which is denoted $Z$) and to their age class (the set of which is denoted $W$). Three health states are considered: $S$ (susceptible), $I$ (viraemic), and $R$ (immune). Several age classes are also distinguished:

- $n$ nestlings age classes numbered 1-$n$ ($N_i$: newly hatched animals, $N_n$: $n$-days old nestlings that will leave the nest the following day),
- juveniles ($J$) that have left the nest but do not participate in reproduction,
- reproductive adults ($A$).

At day $t$, the proportion of birds in age class $a$, health state $z$ of population $y$ is denoted $H_{y,a,z}(t)$. Regarding to species that reproduce all over the year, population size was assumed constant and, whatever the day $t$, $\sum_{aW,z\in Z} H_{y,a,z}(t) = 1$. For species with a seasonal reproduction, population size varies during the annual life cycle, between a minimal value at the beginning
of the hatch period and a maximal value at the end of this period. Population dynamic parameters were adjusted so that the preceding equality holds on the 1st day of the hatch period.

A vector population is described by the distribution of adults according to their health state: \( S \) (susceptible), \( E \) (latent, during the extrinsic incubation period) and \( I \) (infectious: when biting, the vector will transmit the virus to the host). At day \( t \), the proportion of vectors living at location \( x \) that have the health state \( z \) is denoted \( V_{x,z}(t) \): whatever the day \( t \), \( \sum_{z \in Z} V_{x,z}(t) = 1 \).

Vector abundance varies according to a site-specific parameter: \( \omega_x(\tau) \), that represents the size of the population at calendar day \( \tau \) (1-365), relatively to the maximal vector abundance at location \( x \) during the annual cycle: whatever the calendar day, \( \omega_x(\tau) \leq 1 \).

### 2. Population dynamic

#### 2.1. Birds

Let \( h_{y,a,z} \) denote the state of the bird population \( y \) after ageing, death and renewal processes. It is computed using Equations (1–3):

\[
h_{y,A,z} = \left[ H_{y,A,z}(t) + r_y(\tau)H_{y,J,z}(t) \right] (1 - \mu_A)
\]

where:

- \( \tau \) is the calendar day,
- \( -\mu_A \) is the daily adult mortality rate,
- \( -r_y(\tau) \) is the daily juvenile recruitment rate in the adult age class (transfer from the \( J \) to the \( A \) age class) for host population \( y \) and calendar day \( \tau \).

\[
h_{y,J,z} = \left[ (1 - r_y(\tau))H_{y,J,z}(t) + H_{y,N,z}(t) \right] (1 - \mu_J)
\]

where \( \mu_J \) is the daily juvenile mortality rate.

\[
h_{y,N,z} = H_{y,N,z+1}(t)(1 - \mu_N)
\]

where \( \mu_N \) is the daily nestlings mortality rate.
Nestling always hatch in the S state, as indicated by Eqs. 4 and 5, where \( b_y(\tau) \) is the daily per-capita hatch rate for host population \( y \) and calendar day \( \tau \).

\[
h_{y,N_1,S} = \sum_{z \in Z} (1 - \mu_y) b_y(\tau) H_{y,a,z}(t) \tag{4}
\]

\[
h_{y,N_1,I} = h_{y,N_1,R} = 0. \tag{5}
\]

2.2. Vectors

Let \( v_{x,z} \) denote the state of the vector population at location \( x \) after adults daily mortality and emergence. Neglecting vertical transmission of WNV, \( v_{x,z} \) is computed using Equations (6–8):

\[
v_{x,S} = V_{x,S}(t)(1 - \mu_v) + \mu_v \text{ where } \mu_v \text{ is the vector daily mortality rate} \tag{6}
\]

\[
v_{x,E} = V_{x,E}(t)(1 - \mu_v) \tag{7}
\]

\[
v_{x,I} = V_{x,I}(t)(1 - \mu_v). \tag{8}
\]

3. Infection dynamic

3.1. Birds

At each time step, infectious vectors bite susceptible birds that become infected (Eq. 9) according to a population- and age-specific force of infection \( \lambda_{y,a}(t) \). Transition rate from health state I to R is constant (Eqs. 10–12). Birds are assumed to acquire a lifelong immunity [26], there is thus no transition from R to S.

\[
H_{y,a,S}(t + 1) = h_{y,a,S}(1 - \lambda_{y,a}(t)) \tag{9}
\]

\[
H_{y,a,I}(t + 1) = h_{y,a,I}(1 - 1/Tv) + h_{y,a,S} \lambda_{y,a}(t) \tag{10}
\]

where \( Tv \) is the duration of viraemia (health state I) in days.

\[
H_{y,a,R}(t + 1) = h_{y,a,R} + h_{y,a,I}(1/Tv) \tag{11}
\]
The force of infection is computed according to Equation (12):

\[ \lambda_{x,a}(t) = 1 - \exp\left(-RR_a K_x \omega_x(\tau) v_{x,I}(1/T_{gs})\right) \]  

(12)

where:
- \( x \) is the location of population \( y \) at time step \( t \),
- \( K_x \) is the vector-host ratio for location \( x \), when vector abundance reaches its maximal value, \textit{i.e.} \( \omega_x(\tau) = 1 \),
- \( RR_a \) is the bite relative risk for age class \( a \),
- \( T_{gs} \) is the duration of the gonotrophic cycle for vectors living at location \( x \) (days).

In Equation (12) the number of infectious bites per host is assumed to follow a Poisson distribution. The expression \( 1/T_{gs} \) denotes the proportion of vectors that bite a host at a given day, and \( v_{x,I} \) is the proportion of infectious vectors. Thus, taking into account age-specific bite relative risk as well as the yearly variations of vector abundance, the expression \( RR_a K_x \omega_x(\tau) v_{x,I}(1/T_{gs}) \) denotes the number of infectious bites per host of age class \( a \). Considering this number as the parameter of the Poisson distribution, Equation 12 gives the probability that, for a given bird of population \( y \) and age class \( a \), the number of infectious bites is not zero.

### 3.2. Vectors

At each time step, susceptible vectors bite viraemic birds and become infected (Eq. 14) according to a site-specific force of infection \( \lambda_x(t) \). Transition rate from health state \( E \) to \( I \) is constant (Eqs. 13–15).

\[ V_{s,S}(t+1) = v_{s,S}(1-\lambda_x(t)) \]  

(13)

\[ V_{s,E}(t+1) = v_{s,E}(1-1/T_x) + v_{s,S} \lambda_x(t) \]  

(14)
where $T_{x}$ is the duration of the extrinsic incubation period of vectors living at location $x$ (state $E$).

$$V_{x,t}(t + 1) = v_{x,t} + v_{x,E}(1/T_{x}),$$

(15)

The force of infection applied to vectors living at location $x$ is defined in Equation (16):

$$\lambda_{x}(t) = \frac{1}{T_{g} x} \sum_{y \in Y} \sum_{a \in W} q_{y,a} p_{y,a},$$

(16)

where:

- $q_{y,a}$ is the proportion of bites that occur on birds of population $y$ and age class $a$:

$$q_{y,a} = \frac{\omega_{x,y}(t)RR_{a} \sum_{z \in Z} h_{y,a,z}}{\sum_{y \in Y} \sum_{a \in W} \sum_{z \in Z} \omega_{x,y}(\tau)RR_{a} h_{y,a,z}},$$

with $\omega_{x,y}(\tau)$ the demographic importance of population $y$ at location $x$ at the calendar day $\tau$ (zero if the population is not present);

- $p_{y,a}$ is the proportion of viraemic birds in population $y$ and age class $a$:

$$p_{y,a} = \frac{h_{y,a,l}}{\sum_{z \in Z} h_{y,a,z}}.$$  

In Equation (16), the force of infection is the product of the proportion of vectors that bite $1/T_{g}$ by the proportion of bites that occur on a viraemic bird. To compute the latter term, because of age-specific bite relative risk and because several populations may be present, we sum the prevalence rates of viraemic birds $p_{y,a}$ in the different populations and age classes, and weight each prevalence by the proportion of bites that occur on the corresponding birds $q_{y,a}$.

3.3. Incidental hosts

The daily infection rate of incidental hosts is assumed proportional to that of adult birds living
at the same place, the proportionality factor being host-specific. The incidence rate of infections in incidental hosts over a given time period from day $t_1$ to day $t_2$ may then be computed using Equation (17):

$$p_{x,k} = 1 - \exp\left( -\phi_h \sum_{t=t_1}^{t_2} \lambda_{j,h}(t) \right)$$

(17)

where:
- $y$ is the resident bird population living at location $x$,
- $\phi_h$ is the bite relative risk for host $h$, taking as a reference the force of infection for adult birds.