First lattice calculation of charmed hadrons’ electromagnetic form factors

K U Can¹,², G Erkol¹, B Isildak¹, M Oka² and T T Takahashi³

¹ Department of Natural and Mathematical Sciences, Faculty of Engineering, Ozyegin University, Nisantepe Mah. Orman Sok. No:13, Alemdag 34794 Cekmekoy, Istanbul Turkey
² Department of Physics, H-27, Tokyo Institute of Technology, Meguro, Tokyo 152-8551 Japan
³ Gunma National College of Technology, Maebashi, Gunma 371-8530, Japan

E-mail: utku.can@th.phys.titech.ac.jp

Abstract. Electromagnetic form factors of D and D∗ mesons and Ξcc, Σc, Ωc and Ωcc baryons are calculated in 2+1 flavor lattice QCD. As a by product of this calculation electric/magnetic charge radii and magnetic moments are extracted. Compared to the PDG values of the light-sector, i.e. pion and proton, charmed hadron results are systematically smaller.

1. Introduction

Electromagnetic form factors are good instruments to probe the internal structure of the hadrons and extract information about their sizes and shapes. Lattice discretisation of QCD provides an ab initio scheme to perform non-perturbative calculations to extract form factors. In this proceedings we report our work on the electromagnetic form factors of charmed hadrons, namely D and D∗ mesons [1] and Ξcc, Σc, Ωc and Ωcc baryons [2]. Baryon calculations are expanded and improved upon previous Ξcc calculation [3].

2. Form Factors

Matrix element of the vector current can be written in the form

\[ \langle D(p')|V_\mu(q)|D(p)\rangle = (p+p')_\mu \left[ e_c F_c(Q^2) + e_q F_q(Q^2) \right] \]

for the spin-0 D meson. As for the spin-1 D∗ meson, we have

\[ \langle D^*(p',s')|V_\mu(q)|D^*(p,s)\rangle = \epsilon'_{\mu\nu}(p',s') \left\{ \tilde{G}_1(Q^2) (p_\mu + p'^\mu) \delta^{\nu\sigma} \ight. \\
\left. + \tilde{G}_2(Q^2) (g^{\mu\sigma} q^\tau - g^{\mu\tau} q^\sigma) - \tilde{G}_3(Q^2) q^\tau q^\sigma \frac{(p_\mu + p'^\mu)}{2m_{D^*}} \right\} \epsilon_\sigma(p,s), \]

where ε and ε’ are the polarization vectors of the initial and final vector mesons, respectively. The form factors \( \tilde{G}_{1,2,3} \) can be arranged in terms of Sachs electric form factor as follows [4]:

\[ F_C(Q^2) = \tilde{G}_1(Q^2) + \frac{2}{3} \eta F_Q(Q^2), \]

\[ F_M(Q^2) = \tilde{G}_2(Q^2) \]

\[ F_Q(Q^2) = \tilde{G}_1(Q^2) - \tilde{G}_2(Q^2) + (1 + \eta) \tilde{G}_3(Q^2), \quad \text{where } \eta = Q^2/4m_{D^*}^2. \]

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Published under licence by IOP Publishing Ltd
In the baryon sector, matrix element of the vector current interaction of a spin-1/2 baryon is written as

$$\langle B(p) | V_\mu | B(p') \rangle = \bar{u}(p) \left[ \gamma_\mu F_{1,B}(q^2) + i \frac{q_\nu q^\nu}{2m_B} F_{2,B}(q^2) \right] u(p),$$

where $q_\mu = p_\mu' - p_\mu$ is the transferred four-momentum. Here $u(p)$ denotes the Dirac spinor for the baryon with four-momentum $p^\mu$ and mass $m_B$. The Sachs electric and magnetic form factors are related to the Dirac, $F_{1,B}(q^2)$, and Pauli, $F_{2,B}(q^2)$, form factors by the relations

$$G_{E,B}(q^2) = F_{1,B}(q^2) + \frac{q^2}{4m_B^2} F_{2,B}(q^2), \quad G_{M,B}(q^2) = F_{1,B}(q^2) - F_{2,B}(q^2).$$

### 3. Lattice Formulation and Setup

One can extract the Sachs form factors in the large Euclidean time limit, viz., $t_2 - t_1$ and $t_1 \gg a$, by calculating the following ratio

$$R(t_2, t_1; p', p; \Gamma, \mu) = \left( \frac{\langle F^{\nu \mu} (t_2, t_1; p', p; \Gamma) \rangle}{\langle F^{\nu \mu} (t_2, t_1; p, p; \Gamma) \rangle} \right)^{1/2},$$

where $\langle F^{\nu \mu} (t_1; p; \Gamma_4) \rangle$ and $\langle F^{\nu \mu} (t_2; t_1; p', p; \Gamma) \rangle$ are the two-point and three-point correlation functions with $t_1$ and $t_2$ being the current insertion and the sink particle’s time slice respectively. In case of baryons $\Gamma_4 = (1 + \gamma_4)/2$ and $\Gamma_i = \gamma_i \gamma_5 \Gamma_4$, whereas for mesons $\Gamma = \Gamma_4 = 1$.

Our method of computing the baryon matrix element in Eq. 4, which was employed to extract the nucleon electromagnetic form factor, follows closely that of Ref. [5]. We extract the form factors $G_{E,B}(q^2)$ and $G_{M,B}(q^2)$ from Eq. (4) by choosing appropriate combinations of Lorentz direction $\mu$ and projection matrices $\Gamma$:

$$R(t_2, t_1; 0, -q; \Gamma_4; \mu = 4) \to \frac{1}{t_2 - t_1 \gg a} \left[ \frac{1}{2E_B(E_B + m_B)} \right]^{1/2} \left[ \epsilon_{ijk} q_k \right] G_{E,B}(q^2),$$

$$R(t_2, t_1; 0, -q; \Gamma_j; \mu = i) \to \frac{1}{t_2 - t_1 \gg a} \left[ \frac{1}{2E_B(E_B + m_B)} \right]^{1/2} \epsilon_{ijk} q_k \left[ \frac{1}{2E_B(E_B + m_B)} \right] G_{M,B}(q^2).$$

Interested reader can consult to the Ref. [2] and references therein for the details.

For D meson, the ratio in Eq. (4) reduces to

$$R(t_2, t_1; 0, p; 0) \to \frac{1}{t_2 - t_1 \gg a} \left[ \frac{E_D + m_D}{2\sqrt{E_D m_D}} \right] \left[ \epsilon_i F^{\nu}(Q^2) + \epsilon_q F^{\nu}(Q^2) \right],$$

where $m_D$ and $E_D$ are the mass and the energy of the initial baryon. As for the $D^*$ meson the ratio in Eq. (4) has different components for the different polarization vector directions of the initial and final particles and reduces to

$$R^{00}_{ii} = \left[ \frac{p_i^2}{3m_D \sqrt{E_D m_D}} \right] F_0(Q^2) + \left[ \frac{E_{D^*} + m_{D^*}}{2\sqrt{E_D m_D}} \right] F_1(Q^2),$$

$$R^{00}_{jj} \mid_{j \neq i} = \left[ \frac{p_i^2}{6m_D \sqrt{E_D m_D}} \right] F_0(Q^2) + \left[ \frac{E_{D^*} + m_{D^*}}{2\sqrt{E_D m_D}} \right] F_1(Q^2),$$

where $i$ and $j$ are the polarization directions. In order to single out the electric form factor we compute

$$\frac{1}{3} \sum_{i=1,2,3} R^{00}_{ii}(t_2, t_1; 0, p_i; 0) \to \left[ \frac{E_{D^*} + m_{D^*}}{2\sqrt{E_D m_D}} \right] \left[ \epsilon_i F^{\nu}(Q^2) + \epsilon_q F^{\nu}(Q^2) \right].$$
Compared to the experimental result, individual quark sector study shows that light and heavy quark spins are anti-aligned [1].

Of Ω, the values extrapolated to the physical point. We consider two fit forms, one being linear = 2 for the baryons. The charge radii, then, is calculated via,

\[
G = \sum \kappa_i \text{hopping parameters}
\]

For detailed discussions of our results we refer the reader to Refs. [2, 1].

Details can be found in Ref. [1] and references therein.

We run our simulations on 32^3 \times 64, 2+1-flavor configurations generated by the PACS-CS Collaboration [6] using the Clover quark action and the Iwasaki gauge action. We use the gauge configurations at \( \beta = 1.90 \) with the Clover coefficient \( c_{SW} = 1.715 \), which give a lattice spacing of \( a = 0.0907(13) \) fm (\( a^{-1} = 2.176(31) \) GeV). The simulations are carried out with four different hopping parameters for the sea and the u, d valence quarks, \( \kappa_{sea}, \kappa_{u,d} = 0.13700, 0.13727, 0.13754 \) and 0.13770, which correspond to pion masses of approximately 702, 570, 411, and 296 MeV. The hopping parameter for the strange valence is fixed to \( \kappa_s = 0.1364 \), same as the sea-quark’s. We use Clover action for charm quarks also, with \( c_{SW} = 1/u_0^3 \) where \( u_0 \) is the average link, and with hopping parameters \( \kappa_{c,\text{meson}} = 0.1224 \) and \( \kappa_{c,\text{baryon}} = 0.1246 \). Multiple gaussian-smeared source – wall-smeared sink pairs with \( t = 12a \) separation have been used to increase the statistics.

### 4. Results and Discussion

Charge radii can be extracted from the slope of the form factors at zero momentum transfer by \( \langle r^2 \rangle = -\frac{6}{G(0)} \frac{\partial^2}{\partial Q^2} G(Q^2) \bigg|_{Q^2=0} \). We choose to model the form factors by \( G(Q^2) = G(0)/(1+Q^2/\Lambda^2)^n \), where \( G(Q^2) \) denotes the form factor and \( \Lambda \) is a fit parameter. We take \( a = 1 \) for mesons and \( a = 2 \) for the baryons. The charge radii, then, is calculated via, \( \langle r^2 \rangle = 6a/\Lambda^2 \). Magnetic moments of baryons are obtained from the magnetic form factor at zero momentum transfer by evaluating, \( \mu_B = G_M(0) \langle e/2mB \rangle = G_M(0) (m_N/m_B) \mu_N \).

Plateau, form factor and chiral extrapolation plots can be found in Refs. [2, 1], here we only quote the values extrapolated to the physical point. We consider two fit forms, one being linear in \( m^2_\pi \), \( f(m^2_\pi)_{lin} = a + bm^2_\pi \) and the other being quadratic as, \( f(m^2_\pi)_{quad} = a + bm^2_\pi + c(m^2_\pi)^2 \), where \( a, b \) and \( c \) are fit parameters. The results are compiled in Table 1.

In contrast to their light counterparts, i.e. \( < r^2_{\pi,E} > = 0.452 \) fm^2 and \( < r^2_{\pi,P,E} > = 0.770 \) fm^2 [7], charmed hadron results are observed to be smaller. Comparison of individual quark sector contributions given in Refs. [2, 1] shows that the decrease in values is mainly due to the small contribution of the charm quark. In the baryon sector, comparison of \( \Xi_{cc} \) and \( \Omega_{cc} \) shows that replacing a d-quark by an s-quark seems to have a negligible effect on the magnetic charge radius. \( \Sigma^{++}_c \) has the largest electric/magnetic charge radius amongst others. Peculiar \( m^2_\pi \) dependence of \( \Omega_c \) and \( \Omega_{cc} \) is a possible indication of the sea-quark effects [1].

Magnetic moments are dominantly determined by the doubly represented quarks and individual quark sector study shows that light and heavy quark spins are anti-aligned [1]. Compared to the experimental result, \( \mu_p = 2.793 \mu_N \) [7], the values are systematically smaller with \( \Sigma^{++}_c \) being closest to the proton’s value. In contrast to the charge radii, magnetic moments of \( \Omega_c \) and \( \Omega_{cc} \) seems independent of the sea-quark effects. Lattice results reproduce the signs of magnetic moments correctly but underestimates the values compared to the other models [1]. For detailed discussions of our results we refer the reader to Refs. [2, 1].
5. Conclusions
We have calculated the electromagnetic form factors of the D, D* mesons and Σc, Ξcc, Ωc and Ωcc baryons on 2 + 1-flavor lattices and as a by product extracted the electric and magnetic charge radii and the magnetic moments. Comparison with pion/nucleon data shows that charmed hadrons are more compact. This is believed to be due to the valance charm quarks. Magnetic moments are dominantly determined by the doubly represented quarks. Signs match with other models even though values are underestimated. Simulations on physical-point configurations are ongoing.

Acknowledgements
All the numerical calculations in this work were performed on National Center for High Performance Computing of Turkey (Istanbul Technical University) under project number 10462009. The unquenched gauge configurations employed in our analysis were generated by PACS-CS collaboration [6]. We used a modified version of Chroma software system [8]. This work is supported in part by The Scientific and Technological Research Council of Turkey (TUBITAK) under project numbers 110T245, 114F261 and in part by KAKENHI under Contract Nos. 22105503, 24540294 and 22105508.

References
[1] Can K, Erkol G, Oka M, Ozpineci A and Takahashi T 2013 Physics Letters B 719 103 – 109
[2] Can K, Erkol G, Isildak B, Oka M and Takahashi T 2014 J. High Energy Physics JHEP05(2014)125
[3] Can K, Erkol G, Isildak B, Oka M and Takahashi T 2013 Physics Letters B 726 703 – 709
[4] Brodsky S J and Hiller J R 1992 Phys. Rev. D 46 2141 – 2149
[5] Alexandrou C, Brinet M, Carbonell J, Constantinou M, Harraud P et al. 2011 Phys. Rev. D 83 094502
[6] Aoki S, Ishikawa K I, Ishizuka N, Izubuchi T, Kadoh D, Kanaya K, Kuramashi Y, Namekawa Y, Okawa M, Taniguchi Y, Ukawa A, Ukit a N and Yoshié T (PACS-CS Collaboration) 2009 Phys. Rev. D 79 034503
[7] Olive K A et al. (Particle Data Group) 2014 Chin. Phys. C 38 090001
[8] Edwards R G and Joo B (SciDAC Collaboration, LHPC Collaboration, UKQCD Collaboration) 2005 Nucl. Phys. Proc. Suppl. 140 832