Estimation of Stability Derivatives in Pitch for an Oscillating Wedge in Hypersonic Flow

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Abstract. A similitude has been obtained for an oscillating two-dimensional wedge in pitch with attached shock at high angle of incidence in hypersonic flow. A strip theory, in which flow at infinite span wise location is two dimensional and independent of each other is being used. Further this unites the similitude with piston theory to give one dimensional piston theory with large flow deflection. Closed form solution is obtained for stiffness and damping derivatives in pitch. The present theory is valid when the shock wave is attached with the nose of the wedge. With the increase in semi vertex angle of the wedge the stiffness and the damping derivatives are found to increase progressively, and with the increase in the Mach number both stiffness and damping derivatives are found to decrease then with increase in Mach number it becomes independent of Mach number. From the amalgamation of theory developed, we can evaluate the stiffness and damping derivatives for in pitch for a wide range of Mach numbers, for various pivot positions, and angle of attack. Significantly the same results are being validated with the analytical results of Liu and Hui [4] and Lighthill’s [3] theory for some cases.

1. Introduction

The analysis of hypersonic flow over wedge, cones or flat plate over wide range of angle of incidence and Mach number is of current interest in aerospace and aeronautical programs. At low hypersonic speeds, the molecular bonds vibrate, which changes the magnitude of the forces generated by the air on the aircraft. At higher hypersonic speeds, the molecules break apart producing electrically charged plasma around the aircraft. Large variations in air density and pressure occur because of shock waves, and expansion waves. Linearized theory was developed for the supersonic flow with the assumption of trailing edge and sharp leading edge at low angles of incidence. But these assumptions break down for similar bodies at hypersonic speeds. The hypersonic small disturbance theory is connected with classical hypersonic similitude but fails to provide hypersonic flow solution Binoy [5]. The idea of hypersonic similitude is due to Tsien [5], who analysed the 2-D and axisymmetric irrotational equation of motion. Lighthill [3] linked the unsteady problem to that of a gas flow in a tube driven by piston but gave solutions without considering the effects of secondary wave reflection. Lighthill developed a theory of oscillating airfoils at high Mach number. Here the pitching oscillation is taken into
consideration. Appleton [6] and McInthosh [7] have considered the effects of secondary wave reflection and considered the hypersonic flow past a slender wedge. Hui [8] analytically studied and gave exact formulae for stability of wedge of any thickness with attached bow shock for hypersonic flow. Sychev [1] considered 3-D hypersonic flow past bodies whose transverse dimensions are substantially smaller than their length and developed a number of general conclusions regarding the properties of 3D hypersonic flow past bodies at high angle of incidence. Ericsson’s [9] theory covers viscous and elastic effects for airfoil with large flow deflection. Orlik–Ruckemann [10] has included viscous effect and Mandl [11] has addressed small surface curvature effect for oscillating thin wedges. Ghosh’s [2] similitude and piston theory for the infinite span case with large flow deflection is valid for airfoil with planar or non-planar surfaces whereas Hui’s [9] theory is for plane wedges. The method to analyze the aerodynamic derivatives for stiffness and damping of oscillating hypersonic delta wings with curved leading edge is developed by Crasta et al [12].

This study is derive the mathematical expressions to enable us determine the stiffness and damping derivative for 2-D wedge. From the study, strip theory is used which combines with aerodynamic similitude to give the piston theory. With piston theory, we can evaluate the stiffness and damping derivatives in pitch for a range of hypersonic Mach number for various pivot positions and angle of incidence.

2. Experimental

2.1. Strip and Piston Theory

Ghosh’s [2] similitude and piston theory for the infinite span case with large flow deflection is valid only for aerofoils with planar surfaces. In the present work the Ghosh similitude has been extended for supersonic flows past a 2D wedge. This method can give approximate result to determine the stability derivatives but is only valid for attached shock wave case. In the present study an attempt has been made to calculate the stability derivatives in pitch for an oscillating wedge in hypersonic flow for wide range of Mach number and angle of incidence. The same theory can be extended for supersonic flow as well.
\[
\frac{p_1}{p_2} = 1 + \gamma \frac{U_p}{a_1} + \frac{\gamma(\gamma+1)}{4} \left( \frac{U_p}{a_1} \right)^2 + \frac{\gamma(\gamma+1)}{12} \left( \frac{U_p}{a_1} \right)^3
\]

(1)

The airfoil geometry and motion gives piston velocity \(U_p\) which is related to pressure \(p\). Since the piston Mach number \(M \gg 1\), instead of the approximate expression of Lighthill [3], the exact expression is used which can be written in quadratic form in pressure ratio,

\[
\frac{p}{p_\infty} = 1 + AM_1^2 (\sin \alpha)^2 + AM_1 \sin \alpha \sqrt{B} + M_1^2 (\sin \alpha)^2
\]

(2)

Where, \(A = \frac{\gamma(\gamma+1)}{4}\), \(B = \left( \frac{4}{\gamma+1} \right)^2\)

The stiffness and damping derivatives are, respectively:

\[ -C_{m_2} = \frac{1}{\rho_1 U_1^2 L} \left( -\frac{\partial m}{\partial \alpha} \right) \]

and

\[ -C_{m_3} = \frac{1}{\rho_1 U_1 L^3} \left( -\frac{\partial m}{\partial q} \right) \]

Evaluated at \(\alpha = \theta\) and \(q = 0\), piston Mach number is equal to:

\[
M_p = \frac{1}{\alpha_\infty} [U_\infty (\sin \alpha)^2 + (x - x_\alpha) q]
\]

(5)

Assuming that two flat plates have formed a sharp wedge, close form formulae for stiffness and damping coefficients have been obtained for it. Viscous effects and wave reflections are not taken into account in this study. Results are obtained for hypersonic flow of a perfect gas over the wedges of different semi-vertex angles, Mach numbers, and pivot positions.

Fig. 2. Geometry of the wedge.

So in the flow past an airfoil at high Mach number, the perturbation and gradients are much larger in the lateral direction than those in the axial direction. Consider the flat plate aerofoil with the length.
L at mean angle of incidence $\theta$ and oscillating in pitch with small amplitude about a pivot point $01$ at a distance $x_0$.

$$\frac{P}{P_\infty} = 1 + AM_\infty^2 \tan^2 \alpha + AM_\infty \tan \alpha \sqrt{B + M_\infty^2 \tan^2 \alpha}$$

where $P_\infty$ is free stream pressure.

The aerodynamic stiffness is the coefficient of $\alpha$ and damping is of $q$ in the nose down moment,

$$-m = \int_0^L (x - x_0) p \, dx$$

of the load distribution about the axis $x = x_0$. $p$ is the pressure on the windward surface.

From equation (3), substituting the value of $p$ into equation (4) and obtain

$$-m = \int_0^L (x - x_0) p_\infty \left[ 1 + AM_\infty^2 \tan^2 \alpha + AM_\infty \tan \alpha \sqrt{B + M_\infty^2 \tan^2 \alpha} \right] \, dx$$

Final stiffness derivative can be written as,

$$-Cm_{\alpha_0} = \left[ \frac{(y+1)}{M_\infty^2 \cos \alpha_0 \cos \phi} \right] F(S_1) \left( \frac{1}{2} - h \cos^2 \alpha_0 \right)$$

Final Damping derivative can be written as,

$$-Cm_q = \left[ \frac{(y+1)}{M_\infty^2 \cos \phi \cos^3 \alpha_0} \right] \left[ \frac{1}{3} - h \cos^2 \alpha_0 + h^2 \cos^4 \alpha_0 \right]$$

To relate the piston velocity with pressure on the face of piston velocity with pressure on the face of piston, Light Hill suggested the use of three terms in the isentropic expression for the pressure on a piston as a power series in its velocity. So a condition that the piston velocity should be less than or equal to free stream sound velocity is imposed to satisfy isentropic assumption. This is consistent with hypersonic small disturbance theory on which Light Hill’s (3) piston analogy is based.

3. Results and discussion

3.1. Comparison of Lighthill’s Theory with the present theory

The Fig. and Fig. for Mach number 12 and 17 it is clearly shown to be valid up to angle of incidence of 15 degrees. The figure shows that the Lighthill’s curve is rapidly moves away from the oblique shock results whereas the present theory shows a good accuracy throughout.
The stiffness coefficients and damping derivatives were also compared with the results of the oblique shock theory as shown in Fig. 3 and Fig. 4 and the present theory is found to be in a good agreement with oblique shock at $M = 17$. 

Fig. 3: Comparison graph of pressure ratio vs. angle of incidence for $M = 12$.

Fig. 4: Comparison graph of pressure ratio vs. angle of incidence for $M = 17$. 

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Fig. 5: Comparison graph of stiffness derivative vs. angle of incidence for $M = 17$.

Fig. 6: Comparison graph of damping derivative vs. angle of incidence for $M = 17$. 
4. Conclusion

Based on the analytical method, it is evident that with the increase in angle of attack at \( h = 0.0 \) there is a continuous increase in the values of the damping derivatives. It is also observed that the stiffness derivative decreases linearly with pivot position. In addition, it is seen that the stiffness derivatives are linearly increases with the increase of angle of incidence which is very true and the trend is on the expected lines. Further, from the graph another observation is made that with the increase in the angle of incidence there is a continuous shift of the centre of pressure towards the aft position of the wedge. Due to this phenomenon, the method of continuous increasing the angle of incidence could be used to stabilize the aerodynamic vehicle for some special cases; the requirement of large stabilizing surface could be avoided. The present theory is valid only when the shockwave is attached with the nose of the wedge. Meanwhile, it is observed that the pressure distribution is significantly large at \( h = 0 \) and it is increasing abruptly as the Mach number increased up to 20. At this stage, aerodynamic heating is the main concern as extreme pressure could lead to extreme temperature.

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