WEAK NEUTRAL CURRENTS IN ATOMIC PHYSICS
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1) INTRODUCTION:

With the discovery of high energy neutrino reactions induced by a weak interaction involving the product of neutrino and hadron weak neutral currents, the question of the existence of a similar interaction with the electron (or the muon) playing the role of the neutrino, arises quite naturally. Indeed the most popular gauge model of weak interactions, the Weinberg Salam model, does predict both electron and neutron weak currents. The best place to look for effects associated with these currents seems, a priori, the high energy muon scattering experiments, where an appreciable violation of parity or charge congregation is expected to show up when the momentum transfer of the charged lepton reaches values of the order of 100 GeV. Indeed several theoretical papers have appeared containing proposals in this spirit.

Here we would like to discuss a totally different approach, based on atomic physics measurements, i.e. involving energy transfers of only a few electron volts. We shall try to convince you that such an idea is, perhaps, not as stupid as it may look, for the following reasons: i) very accurate measurements are usually possible in atomic physics, so, tiny effects are often detectable; ii) the atomic physics effects, associated with neutral currents, exhibit a very strong enhancement in heavy atoms (they go roughly like $Z^3$); iii) recent progress in laser technology (dye lasers) allows the study of highly forbidden radiative transitions where a weak electron-nucleus interaction has the best chance to show up.

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II) PHENOMENOLOGY OF THE PARITY VIOLATING ELECTRON-NUCLEON INTERACTION INDUCED BY WEAK NEUTRAL CURRENTS:

The electronic weak neutral currents will be chosen among the bilinear covariants constructed from the Dirac field $e(x)$ of the electron $\dagger$.

\[
S_e(x) = \bar{e}(x) e(x); \quad P_e(x) = \bar{e}(x) i \gamma^5 \bar{e}(x); \quad T_e(x) = \bar{e}(x) \sigma^{\mu \nu} e(x)
\]
\[
V_e^\mu(x) = \bar{e}(x) \gamma^\mu e(x); \quad A_e^\mu(x) = \bar{e}(x) \gamma^\mu \gamma^5 e(x)
\]

(1)

We exclude from our discussion the currents which involve field derivatives, like the vector current associated with Pauli magnetic moment or an electric dipole moment ("second class currents").

The Dirac matrices have been defined in such a way that the five currents $S_e(x), P_e(x)$ ... are hermitian operators.

If we require the hadronic neutral currents to conserve strangeness (and eventually charm and color), we are led to the following currents "diagonal" in the quark field $q_i(x)$:

\[
S_h^i(x) = \sum_i a_{s,i} \bar{q}_i(x) q_i(x)
\]
\[
V_h^\mu(x) = \sum_i a_{v,i} \bar{q}_i(x) \gamma^\mu q_i(x) \quad \text{etc}...
\]

(2)

We shall restrict our attention to the part of the weak neutral current hamiltonian density $H_{\mu \nu}(x)$ which violates parity. It will be written as a sum of terms involving the products of the currents defined above:

\[
H_{\mu \nu} = H_{SP} + H_{PS} + H_{TT} + H_{AV} + H_{VA}
\]

(3)

For the partial hamiltonian densities $H_{SP}, H_{PS}, H_{VA}, H_{AV}$ the first index is relative to the electron, the second to the hadrons:

\[\{ \gamma^\mu, \gamma^\nu \} = \begin{cases} 2 & \text{if } \mu = \nu \\ 0 & \text{if } \mu \neq \nu \end{cases}, \quad \sigma^{\mu \nu} = -\gamma^\nu \gamma^\mu + \gamma^\mu \gamma^\nu, \quad g^{00} = -g^{11} = -g^{22} = -g^{33}, \quad g^{0i} = g^{1i} = g^{2i} = g^{3i} = 0, \quad \sigma^{\mu \nu} = \frac{i}{2} \{ \gamma^\mu, \gamma^\nu \}.\]
\[ H_{SP} = \frac{G_F}{\sqrt{2}} S_e(x) P_h(x) ; \quad H_{PS} = \frac{G_F}{\sqrt{2}} P_e(x) S_h(x) \]
\[ H_{VA} = \frac{G_F}{\sqrt{2}} V^\mu(x) A_{h\mu}(x) ; \quad H_{AV} = \frac{G_F}{\sqrt{2}} A^\mu_e(x) V_{h\mu}(x) \]

(4)

For the tensor interaction, we have only one partial hamiltonian density \( \tilde{H}^{\gamma T}_T = \tilde{H}^{\gamma T}_T \) where \( \tilde{T}_{\mu\nu} \) is obtained from \( T_{\mu\nu} \) by replacing \( \sigma^{\mu\nu} \) by \( i \sigma^{\mu\nu} \gamma_5 \). The electronic currents \( S_e(x), P_e(x) \) etc... are hermitian by construction and commute with the hadronic ones. The hermiticity of \( H_{\mu\nu} \) implies the hermiticity of hadronic currents i.e. the reality of the dimensionless constants \( \alpha_{5i}, \alpha_{pi}, \alpha_{vi} \) etc... From this, it follows that the hadronic currents have the transformation properties under time reflexion \( T \) and charge conjugation \( C \) characteristic of Dirac bilinear covariants. The partial hamiltonian densities \( H_{SP}, H_{PS}, H_{VA}, H_{AV}, H_{\gamma T} \) are even under \( C \) and odd under \( P \) and \( T \) while \( H_{\gamma T}, H_{VA}, H_{VA} \) are even under \( P \) and \( T \) and odd under \( C \) and \( P \)

\[
\begin{array}{cccccc}
H_{SP} & H_{PS} & H_{VA} & H_{AV} & H_{\gamma T} \\
P & - & - & - & - \\
C & + & + & - & + \\
T & - & - & + & + \\
\end{array}
\]

The wavelength of the atomic electron being always very large compared to the nuclear size, it is legitimate to make the kind of monopole approximation used in beta decay calculations.

For instance the partial hamiltonian \( H_{AV} = \int H_{AV}(\vec{x},0) d^3x \)
can be replaced by the following expression:

\[ H_{AV} = \frac{G_F}{\sqrt{2}} \bar{e}(x) \gamma_\mu \gamma_5 e(x) \bigg|_{x=0} \int V^{\mu}_h(\vec{x},0) d^3x \]

(5)
We shall be interested here only in processes where the nucleus does not suffer any transition, so that only the time component of $V^\mu_h$ does contribute:

$$H_{AV} \approx \frac{g_F}{\sqrt{2}} \langle x \rvert \gamma_5 \langle x \rvert Q_h \rangle_{x=0}$$

where $Q_h$ is the "charge" operator associated with the vector current

$$Q_h = \int V^0_h (\vec{x}, 0) \, d^3 x \quad (7)$$

For a nucleus of mass $A$ and charge $Z$, it is convenient to express the "charge" of the nucleus in terms of the "charges" $C_{Vp}$ and $C_{Vn}$ of the proton and the neutron:

$$\langle p \rvert Q \rvert n \rangle_{p} = \langle p \rvert Q_h \rvert n \rangle \quad ; \quad \langle n \rvert Q \rvert n \rangle_{n} = \langle n \rvert Q_h \rvert n \rangle$$

The current $V^\mu_h (x) = \sum_i \tilde{a}_{VI} \gamma_5 q_i (x)$ is conserved if one neglects weak and electromagnetic interactions.

One can write the "charge" of the nucleus as:

$$\left\langle Q_h \right\rangle_{Z, A} = C_{Vp} Z + C_{Vn} (A - Z) \quad (8)$$

It is also possible to express the constants $C_{Vp}$ and $C_{Vn}$ in terms of the quarks coupling constants $\alpha_{VI}$ which are the relevant ones in the discussion of parity violation in deep inelastic electron scattering. In the simplest version of the Weinberg-Salam model which involves a quark isospin doublet, a singlet $\Delta$ of strangeness = 1 and a charmed quark $C$, one has:

$$C_{Vp} = 2 \, \alpha_{Vu} + \alpha_{Vd} = -\frac{i}{2} + 2 \sin^2 \theta_W \quad (9-a)$$

$$C_{Vn} = \alpha_{Vu} + 2 \alpha_{Vd} = \frac{i}{2} \quad (9-b)$$
For our discussion it is convenient to work in first quantized scheme for the electron. Going to the non relativistic limit, one can replace the hamiltonian $H_{AV}$ by a non relativistic potential:

$$V_{AV} = -\frac{G_F}{2\sqrt{2}} \left( \frac{\sigma \cdot \vec{p}_e}{m_e} \delta^3(\vec{r}) + \frac{3}{m_e} \frac{\sigma \cdot \vec{p}_e}{m_e} \right) \times$$

$$\left[ C_{V_\nu} Z + C_{V_n} (A - Z) \right]$$

(In the above expression $m_e$, $\vec{r}$, $\vec{p}_e$, $\sigma$ are the mass, position, momentum and spin of the electron, respectively).

A similar analysis can be performed for the partial hamiltonian $H_{pS}$

$$H_{pS} \approx \frac{G_F \beta \gamma_S e(x)}{\sqrt{2}} \left. \beta \gamma_S e(x) \right|_{x=0} \times \int s_n(\vec{x},0) \, d^3x$$

The additivity relation which gives the "scalar" charge of a nucleus in terms of the scalar "charge" of the nucleons $C_{5\nu}$ and $C_{5n}$, is only approximate but should hold as long as the nucleus can be described as a system of non relativistic nucleons. To get the relation of $C_{5\nu}$ and $C_{5n}$ with the quarks parameters one needs a specific model of the nucleon. Such calculations have been made by Adler et al. Using their results obtained with the M.I.T. Bag model of the nucleon one finds:

$$C_{5\nu} \approx 0.96 \, a_{5u} + 0.48 \, a_{5d}$$  \hspace{1cm} (12-a)$$

$$C_{5n} \approx 0.48 \, a_{5u} + 0.96 \, a_{5d}$$  \hspace{1cm} (12-b)$$
Going to the non relativistic limit one obtains as before a parity violating electron - nucleon potential $V_{PS}$:

$$V_{PS} = \frac{G_F}{2\sqrt{2}} \left( i \frac{\vec{\sigma} \cdot \vec{p}_e}{m_e} \delta^3(\vec{r}) - i \frac{\vec{d}^3(\vec{r})}{m_e} \right) \times \left[ C_{SP} Z + C_{SN} (A - Z) \right]$$  \hspace{1cm} (13)

The potential $V_{PS}$ looks very similar to $V_{AV}$, but now an $i$ is standing in front of $\frac{\vec{\sigma} \cdot \vec{p}_e}{m_e} \delta^3(\vec{r})$. It is a clear indication that $V_{PS}$ is odd under time reflection.

What we have done for the two hamiltonians $H_{AV}$ and $H_{PS}$ can be generalized to the other types of coupling, but for $H_{VA}$ and $H_{TT}$ the nuclear part of the effective potential will involve the total spin operators of protons $\vec{S}_p = \sum_i \frac{1}{2} \vec{\sigma}_p i$ and the total spin of the neutrons $\vec{S}_n = \sum_i \frac{1}{2} \vec{\sigma}_n i$. One should also note the fact that with our monopole approximation the partial hamiltonian $H_{SP}$ does not contribute.

The final expression for the parity violating electron - nucleon potential induced by neutral weak currents is:

$$V^{r,v} = -\frac{G_F}{\sqrt{2}} \left\{ \frac{i \vec{\sigma} \cdot \vec{p}_e}{2 m_e} \delta^3(\vec{r}) \right\} \left[ Z (C_{VP} + i C_{SP}) + (A - Z) (C_{VN} + i C_{SN}) \right] + \frac{\vec{d}_{\vec{P}_e}}{m_e} \delta^3(\vec{r}) \times \left[ \vec{S}_p (C_{AP} + i C_{TP}) + \vec{S}_n (C_{Am} + i C_{Tm}) \right]$$  \hspace{1cm} (14)

The constants $C_{AP}$ and $C_{Tm}$ are defined in terms of the
matrix elements of $\int A_{h}^{\mu}(\vec{x},0)\,d^{3}x$ and $\int T_{h}^{\mu\nu}(\vec{x},0)\,d^{3}x$ between non-relativistic nucleon states:

$$\langle p'|\int A_{h}^{\mu}(\vec{x},0)\,d^{3}x|p\rangle = C_{A_{p}} \mathcal{X}_{p'}^{x} \sigma \mathcal{X}_{p} \quad (15 - a)$$

$$\langle p'|\int T_{\mu\nu}^{ij}(\vec{x},0)\,d^{3}x|p\rangle = C_{T_{p}} \epsilon_{ijk} \mathcal{X}_{p'}^{x} \sigma_{k} \mathcal{X}_{p} \quad (15 - b)$$

where $\mathcal{X}_{p}$, $\mathcal{X}_{p'}$, are two components Pauli wave functions.

Let us indicate the predictions of the Weinberg-Salam model when the contributions associated with currents of type $\mathcal{A}$ and $\mathcal{C}$ are neglected:

$$C_{A_{p}} = - C_{A_{n}} = 1.25 \left( 2 \sin^{2} \theta_{W} - \frac{1}{2} \right) \quad (16 - a)$$

$$C_{T_{p}} = C_{T_{n}} = 0 \quad (16 - b)$$

Looking at the potential (14), one can clearly see that, for heavy atoms the dominant terms will be those associated with the scalar and vector hadronic currents. The shell structure of the nucleus implies that $\mathcal{X}_{p}$ and $\mathcal{X}_{n}$ do not increase with the atomic number $Z$ but rather fluctuate like the magnetic moment of the nucleus between single particle values. Except in those cases where its contribution vanishes identically, the nuclear spin independent part of $V_{\pi\nu}$ will be dominant, as long as $Z > 50$

To end this section we would like to say a few words about possible effects of an electron-electron parity violating weak potential. In reference [5] it is argued that parity
mixing induced by such a potential is smaller than the one associated with the electron - nucleus potential by two orders of magnitude. For light atoms where the relative velocity of the electrons is of the order of $\alpha c$, the Coulomb repulsion suppresses any short range electron - electron interactions. For heavy atoms the Coulomb repulsion is no longer effective since the relative electron velocity is increased, but because of the enhancement of the electron wave function near the nucleus, the electron - nucleus potential remains dominant by a factor of order $Z$.

### III) Parity Mixing Between Atomic Levels of Opposite Parities:

To discuss the parity mixing in atoms we shall use a Hartree Fock approximation, which, for an atom like Cesium, reproduces, with 10% accuracy, properties like hyperfine splitting, optical transition probabilities, etc... Because of its zero - range nature the potential $V^{p,\nu}$ mixes only $\Delta_{1/2}$ and $\Pi_{1/2}$ states. The matrix element of the nuclear spin independent part of $V^{p,\nu}$ is given in terms of the radial wave functions $R_{n\ell}$ by:

$$
\langle n \Delta_{1/2} | V^{p,\nu} | n' \Pi_{1/2} \rangle = \frac{i \times 3 G_F}{8 \pi \sqrt{2}} \times \left. \frac{R_{n'\ell}(r)}{r} \frac{R_{n,\ell}(r)}{r} \right|_{r=0} \times \left[ Z (C_{1p} + i C_{3p}) + (A-Z) (C_{1n} + i C_{3n}) \right]
$$

In the literature there exists a semi - empirical formula (the Fermi - Segre formula) which leads to a determination of $R_{n,0}^{0}(0)$ based only on the knowledge of the energy spectrum of $\Delta$ levels. When used to compute hyperfine separation of atoms and ions with one $\Delta$ electron, the formula appears rather accurate, provided
relativistic corrections are included properly \[\text{[6]}\]:

\[
\begin{array}{cccccc}
\text{Fermi-Segre} & \text{Li} & \text{Na} & \text{K} & \text{Ca}^+ & \text{Rb} & \text{Cs} & \text{Au} \\
\text{Exp.} & 0.97 & 0.88 & 0.92 & 0.96 & 0.94 & 1.00 & 0.95
\end{array}
\]

The derivation of Fermi Segre has been improved by Foldy \[\text{[7]}\] using a refined WKB approximation. We have extended and improved the method of Foldy in order to get

\[
\frac{R_n \ell (r)}{r^l} \bigg|_{r=0} \text{ for } \ell > 0
\]

Although the final result is remarkably simple the derivation is rather involved. All the details are given in reference \[\text{[5]}\].

The formula for \(\ell > 0\) has been tested against Hartree Fock calculations and is expected to hold with an accuracy of 10\% for valence state of alkali atoms with \(Z \gg 1\).

Let us quote the final result:

\[
\left\langle n \, \ell \frac{1}{2} \left| \sqrt{\frac{\hbar^2}{2m}} \right| n' \, \ell' \frac{1}{2} \right\rangle = -i \, G_F \frac{Z^2}{2\pi \sqrt{2} m_e} a_0^4 \left( \gamma_n \gamma_{n'} \right)^{3/2} \left[ Z \left( C_{Vn} + i C_{Sn} \right) + (A - Z) \left( C_{Vn'} + i C_{Sn} \right) \right]
\]

\[(18)\]

In the above expression \(a_0\) is the Bohr radius; \(\gamma_n\)

and \(\gamma_{n'}\) are the effective radial numbers related to the binding energies \(E_n\) and \(E_{n'}\) written in Rydberg's units as

\[
E_n = -\frac{1}{\gamma_n^2} \quad \text{and} \quad E_{n'} = -\frac{1}{\gamma_{n'}^2}
\]

For small binding energies i.e. \(E_n, E_{n'} \ll 1\) and large \(Z\), \(\delta_0(E_n)\) and \(\delta_1(E_{n'})\) are of the order of a few per cent and are given explicitly in ref. \[\text{[5]}\].
Relativistic correction factors \( K_r \) have still to be applied. They are slightly different for PS and AV interactions. Approximate expressions in terms of the nuclear radius are given in ref (5). As an example let us quote the results obtained in the case of Cesium:

\[
K_r^{PS} \approx 2.3 \\
K_r^{AV} \approx 2.8
\]

The remarkable feature of expression \([18]\), beside its simplicity, is the \( Z \) dependence which obviously favours heavy atoms. One \( Z \) factor is connected with the proportionality of the hadronic current to the nucleon number; the second \( Z \) reflects the fact that in heavy atoms, the electronic density near the nucleus is proportional to \( Z \) (See Landau and Lifchitz for a simple argument); the third \( Z \) factor is associated with the rapid variation of the wave function due to the presence of a classical turning point \( r_c \) near the origin \( r_c \approx a_o Z^{-4} \).

IV) EXPERIMENTAL IMPLICATIONS OF PARITY MIXING IN ATOMIC WAVE FUNCTIONS

A) P AND T VIOLATIONS INDUCED BY SCALAR AND TENSOR NEUTRAL CURRENTS:

The experimental consequences of parity mixing are rather different depending on the behaviour of the parity violating potential under time reflection. Let us consider first the case of the potentials

\[
V_{PS}, \quad V_{TT}
\]

which are odd both under space and time reflections (the potential \( V_{SP} \) gives contributions smaller by a factor \( \frac{m_e}{m_p} \)). On the formula (17) one sees that the parity mixing amplitudes induced by \( V_{PS}, \quad V_{TT} \) are real. As expected, this fact implies the existence of a static electric dipole moment in the perturbed atomic states. In the simple case of the ground state of an alkali
atom, the diagonal matrix element of the $z$ component of the electric dipole $d_z$ moment, between perturbed $S$ states is found to be \cite{8}:

$$d = \langle \widetilde{n S_{1/2}} | d_z | \widetilde{n S_{1/2}} \rangle$$

$$= \frac{2}{\pi} \frac{G_F m_e^2 (\alpha)^2}{\eta^2} \left[ C_{sp} Z + C_{sn}(A-Z) \right] \mathcal{D}(E_n)$$

with

$$\mathcal{D}(E_n) = \sum_{n'} \frac{\langle n' P_{1/2} | d_z | n S_{1/2} \rangle (E_n E_{n'})^{3/4} \times \left[ 1 + \gamma(E_n) + \delta(E_n) \right]}{E_n - E_{n'}}$$

\(\mathcal{D}(E_n)\) involves a sum upon electric dipole amplitudes of allowed transitions and is expected to be of the order $e a_0$. In the case of atomic Cesium we have found $\mathcal{D}(E_n) \approx -2 e a_0$ with an uncertainty which is less than 15%. Using this result one gets the following value for $d_{Cs}$:

$$d_{Cs} = -1.33 \left( 0.41 C_{sp} + 0.59 C_{sn} \right) 10^{-10} e a_0$$

To this number one should add the contribution $\mathcal{R} d_e$ associated with an intrinsic electric dipole moment $d_e$ of the electron, $\mathcal{R}$ being the atomic shielding factor computed by Sandars \cite{9}

We have considered a simple renormalizable model involving a scalar boson field $\phi$ coupled to electrons and hadrons with similar coupling constants which violates $P$ and $T$. The ratio $\xi = \frac{\mathcal{R} d_e}{d_{Cs}}$ is found to be

$$\xi \approx 3.5 \times 10^{-4} \left( \frac{\log \frac{m_\phi}{m_e}}{0.75} - \frac{3}{4} \right)$$

We see that unless the mass $m_\phi$ of the scalar boson is assumed to have a totally unrealistic high value, the contribution
of the potential $V_{ps}$ associated with the exchange of the $\phi$ meson is the dominant one. In the model of spontaneous violation of $T$, proposed by T.D. Lee [16], the coupling constants of $\phi$ to the fermion is proportional to the fermion mass so that the ratio $\kappa$ is further reduced by a factor $10^{-3}$.

Let us now compare with the experimental result of M.G. Weisskopf et al. [11]

$$d_{C_{\Delta}} = (0.8 \pm 1.8) \times 10^{-22} \text{ cm}$$
$$d_{C_{\Delta}} = (1.5 \pm 3.4) \times 10^{-14} \text{ cm}$$

We obtain the following limit for pseudoscalar - scalar interaction:

$$0.41 C_{SP} + 0.59 C_{SN} = (-1.1 \pm 2.6) \times 10^{-4}$$

Unless the ratio between the coupling constants $C_{SP}$ and $C_{SN}$ appears to be accidentally equal to $-\frac{78}{55}$, the effective coupling constant associated with a $S\bar{P}$ neutral currents interaction is smaller than the Fermi constant by more than three orders of magnitude.

Similar limits can be obtained from atomic electric dipole measurements performed on Thallium [12] and Xenon [13].

Using a formula similar to (19) we have found:

$$d_{TL} \approx (0.40 C_{SP} + 0.60 C_{SN}) 10^{-9} \text{ cm}$$

(24)
This number is to be compared with the experimental limit

\[ d_{\text{exp}}^{\text{Tl}} \approx (2.5 \pm 4.5) \times 10^{-13} \, e \cdot a_0 \]  

(25)

Calculations of Hinds et al. \cite{14} give for the \((5p^6 6s)^3P_2\) state of Xenon:

\[ d_{\chi_e} \approx (0.4 \, C_{5p} + 0.6 \, C_{5n}) \times 10^{-10} \, e \cdot a_0 \]  

while

\[ |d_{\chi_e}^{\text{exp}}| \lesssim 4 \times 10^{-14} \, e \cdot a_0 \]  

(26)

(27)

In all the above cases the contribution of an eventual tensor interaction is expected to give a contribution smaller by two orders of magnitude than a scalar interaction of similar strength.

In order to get information on tensor - pseudo tensor interactions, one has to reanalyze experiments originally performed in order to detect a proton electric dipole. Only one such experiment exists up to now; it involves the polar molecule \(\text{TlF}^{205}\) \cite{15}. (The ground state proton configuration of \(\text{TlF}^{205}\) is \(3\Delta_{1g}\).) We have estimated the ratio \(\kappa'\) between the contribution coming from a proton electric dipole moment given by a renormalizable model of scalar pseudoscalar currents involving an intermediate neutral scalar boson and that associated with a tensor pseudotensor interaction of similar strength. The ratio is approximately independent of the molecular wave function and has been found to be of the order of \(\frac{m_e}{m_p}\).

So it appears legitimate to analyse the \(\text{TlF}\) experiment in terms of a pseudotensor - tensor electron - nucleon potential only.
The molecule $\text{Tl} - F$ being a rather complex object, a reliable evaluation of the molecular wave function near the origin is not a simple affair. In order to get an order of magnitude, we shall describe the valence electron of the $\text{Tl}$ atom in the $\text{Tl} - F$ molecule assumed to be completely oriented in a static electric field along the $\gamma$ axis by the simple "hybrid orbital":

$$
\Psi_{\text{TlF}} = \frac{1}{\sqrt{2}} \left( \psi_{6s}(\vec{r}) + \psi_{6p_y}(\vec{r}) \right),
$$

$$
\psi_{6s}(\vec{r}) = \frac{1}{\sqrt{4\pi}} R_{60}(r) \text{ and } \psi_{6p_y} = R_{61}(r) \frac{\gamma}{r} \sqrt{\frac{3}{4\pi}}
$$

being the wave functions of the free atom.

The matrix element of the electron spin-independent tensor potential, using the method of ref [5] or available Hartree Fock results [16], is found to be:

$$
\langle \Psi_{\text{TlF}} | V_{\gamma\gamma} | \Psi_{\text{TlF}} \rangle \approx \frac{\sqrt{3}}{\sqrt{2}} \frac{G_F}{4\pi m_e^2} \frac{\langle \mathbf{S}_{p_y} \mathbf{S}_{p_y} \rangle}{\langle \mathbf{S}_{p_y} \mathbf{S}_{p_y} \rangle} \frac{R_{61}(r)}{r} X_{n=0} \left( \frac{R_{60}(r)}{r} \right) | C_{T\text{p}} |
$$

$$
\approx \frac{G_F m_e^2 \langle \mathbf{S}_{p_y} \mathbf{S}_{p_y} \rangle (2\alpha)^2 m_e \alpha^2 C_{T\text{p}}}{\sqrt{6} \pi (Y_{6s} Y_{6p})^{3/2}}
$$

If one takes for $\langle \mathbf{S}_{p_y} \mathbf{S}_{p_y} \rangle$ one half of the ratio of the magnetic moment to that of the free proton, one gets the final result:

$$
\langle \Psi_{\text{TlF}} | V_{\gamma\gamma} | \Psi_{\text{TlF}} \rangle \approx C_{T\text{p}} \times 10^2 H_\gamma
$$

(30)
In the nucleon magnetic resonance experiment of Harrison et al. the frequency shift associated to a reverse of direction of the electric field with respect to the magnetic field, was found to be less than 0.2 Hz. In this way, we get \( |C_{TP}| \leq 10^{-3} \).

In simpler molecules of similar structure like B-F or Al-F, the above evaluation of \( \frac{R_\Delta(r) R_\mu(r)}{r} \bigg|_{r=0} \) tends to be ten times larger than the results obtained with more refined molecular wave functions, but this is partially compensated by the fact that a relativistic correction, \( K_\kappa \approx 6 \), has to be introduced, so our simple estimate is expected to be not very far from the truth. However, Hinds et al. [14] have made an apparently more refined calculation from which they concluded that \( |C_{TP}| \leq 6 \times 10^{-5} \) which means that their value of the matrix element of \( V_{TP} \) is at least ten times larger than our simple minded estimate.

In conclusion, we can say that the existing measurements of electric dipole moment of heavy atoms exclude weak electron-nucleon interaction involving scalar-pseudoscalar and tensor-pseudo tensor products of neutral currents with a coupling constant larger than \( 10^{-3} G_F \), the limits on the scalar being on much firmer grounds than that of the tensor.

B) CONSEQUENCES OF GAUGE MODELS IN ATOMIC PHYSICS.

The existence of weak neutral currents was considered as a rather unlikely theoretical possibility until the discovery that gauge theories of weak and electromagnetic interactions could be renormalized. The minimal gauge model which unifies strong and electromagnetic interactions without introducing new types of leptons—the so called Weinberg-Salam Model— involves necessarily weak neutral vector currents, hadronic and leptonic. Although other models
could account for the neutral current interactions observed in high energy neutrino scattering all the data are presently consistent with the Weinberg-Salam model. It is of course of interest to explore the consequences of the Weinberg model in atomic physics. We shall be mainly interested here in parity violating effects, but one should keep in mind that the Weinberg-Salam model predicts also short-range parity conserving effects which give rise to corrections to the predictions of conventional quantum electrodynamics.

For instance, the hyperfine structure splitting of hydrogen $\Delta W_H$ and muonium $\Delta W_\mu$ are modified by small quantities $\delta_H$ and $\delta_\mu$ given by:

$$\delta_H \approx \frac{3}{2\pi \sqrt{2} \alpha} \left(G_F M_H m_\mu m_e \right) \frac{g_A}{\mu_p} \approx 4.12 \times 10^{-7}$$

$$\delta_\mu \approx \frac{3}{2\pi \sqrt{2} \alpha} \left(G_F m_e m_\mu \right) \approx 0.28 \times 10^{-7}$$

But, in hydrogen, the incertitudes associated with the polarizability of the nucleon and, in muonium, the experimental errors prevent, up to now, any test of these predictions.

The best way to test the presence of neutral currents predicted by gauge theories is to look for the parity violating effects associated with the $P$-odd $T$-even electron nucleus potential $V_{AV}$ given by formula (10). This potential, in contradistinction with the potential $V_{PS}$ discussed above, does not give rise to any static electric dipole moment but can induce an electric dipole transition amplitude $E_1$ between states of same parity which will interfere
with the normal magnetic dipole amplitude $M_1$ in the simple case of radiative transitions. If the $n\, S_{1/2} \rightarrow n'\, S_{1/2}$ transition is excited by photons having alternatively right and left circular polarization, parity violation would manifest itself by a difference between the cross-sections relative to right and left polarized photon $\sigma_{R,L}$. One defines an asymmetry

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

which is given by:

$$A = \frac{\text{Im} \frac{E_1 M_1^*}{|E_1|^2 + |M_1|^2}}{2}$$

The quantity $A$ transforms like an helicity and consequently is odd under $P$ but even under $T$. So that $A$ receives contribution only from potentials of the type $V_{AV}$ or $V_{VA}$. Indeed the mixing amplitudes computed in formula (17) are purely imaginary for $A-V$, but purely real for $P-S$.

In references [17] [5] the case of the $6\, S_{1/2} \rightarrow 7\, S_{1/2}$ transition of Cesium was studied in details. The transition amplitude $E_1$ defined as the matrix element of the $d_{z^2}$ component of the electric dipole operator between the perturbed $S$ states,

$$E_1 = \langle 6\, S_{1/2} | d_{z^2} | 7\, S_{1/2} \rangle$$

is given by a formula similar to (19) involving a sum over allowed electric dipole amplitudes $\langle n\, P_{1/2} | d_{z^2} | 6\, S_{1/2} \rangle$ and $\langle n\, P_{1/2} | d_{z^2} | 7\, S_{1/2} \rangle$. Let us quote the result:

$$E_1 = -\epsilon \ 1.12 \ e\ a_0 \ T(z,A)$$
with
\[
\mathcal{F}(Z,A) = -\frac{G_F}{\sqrt{2}} m_e^2 \frac{(Z\alpha)^2}{\alpha} K_n(Z C_{\nu n} + (A-Z) C_{\nu m})
\]
(34-b)

In the Weinberg-Salam model with \( \sin^2 \theta_W = 0.35 \)
one finds:
\[
E_1 = i \frac{1.7 e a_o}{10^{-14}} = i \frac{0.47 \mu_B}{c} 10^{-8}
\]
(35)
(\( \mu_B \) is the Bohr magneton).

The theoretical incertitude on the above result is believed to be of the order of, or less than 15%.

The magnetic dipole amplitude \( M_4 \) is more difficult to compute. \( M_4 \) vanishes in the non relativistic limit and is closely related to the \( q \) shift of atomic Cesium \( \Delta q = q_{Cs} - q_{c} \approx 1.4 \times 10^{-4} q_{c} \). In reference [5] we have given a discussion of the mechanisms which can contribute to \( M_4 \) and concluded that
\[
10^{-5} \mu_B/c < |M_4| < 10^{-4} \mu_B/c
\]
(36)

Preliminary measurements [19] indicate that
\[
| M_4 | < 3 \times 10^{-4} \mu_B/c
\]
(37)

The modulus \( |M_4| \) can be obtained through an absolute measurement of excitation cross sections. In ref. [20] we have proposed
a method to get the sign of $M_1$. We have shown that the interference of $M_4$ with the electric dipole amplitude induced by a static electric field $E_o$ which can be computed quite reliably can produce an electronic polarization in the final state along $\hbar L \cdot E_o$ ($\hbar$ is the photon momentum) as large as 64%.\[29\]

Adopting the above limits on $M_1$, the Weinberg-Salam model predicts an asymmetry $\Delta$ lying in the range $10^{-3} - 10^{-4}$. Such a large value of $\Delta$ is due to two facts:

a) the enhancement factors $K_r Z^3$ appearing in the matrix element of the weak electron-nucleon potential,

b) the high degree of forbiddenness of the transitions.

The excitation of such twice forbidden transitions raises, of course, many problems but seems feasible with the dye laser beams presently available. Experiments along the lines suggested here are in progress (Ecole Normale Supérieure Paris) or in project (University of California Livermore and Berkeley).

We would like to discuss another possible way to detect parity violation first suggested by F.C. Michel [21] in the case of nuclear physics. He considers the coherent forward scattering of thermal neutrons in matter. If the neutron interaction with matter violates parity, there should be a difference $\delta n = n_R - n_L$ between the refraction indices of neutrons with right and left helicity. The polarization of a transversely polarized neutron beam i.e. a coherent superposition of right and left helicity states, rotates of an angle $\gamma$ as the beam is transmitted through matter. The rotation angle $\gamma$ is given by:

$$\gamma = -\frac{2\pi}{\hbar} l \ Re(n_R - n_L)$$

(38)
where \( \lambda \) is the wavelength and \( l \) the length of the traversed matter. It will receive a contribution both from the parity violating neutron - nucleus forces and the parity violating neutron - electron interaction induced by the neutral currents.

The same analysis can be performed for optical photons \([22]\) the rotation \( \delta \) of the plane of polarization of photons transmitted through a unit length of matter is:

\[
\delta = -\frac{\pi}{\lambda} \Re(e^{i\pi R}(n_R - n_L))
\]

(39)

where \( n_R \) and \( n_L \) are now the indices relative to right and left circularly polarized light of wavelength \( \lambda \).

This number has to be compared to the absorption coefficient \( \kappa \) per unit length. In order to get measurable effects, it is necessary to be in the vicinity of a resonance associated with a mixed magnetic - electric dipole transition. Assuming that elastic scattering amplitude has a Breit - Wigner Shape, we get for a frequency \( \omega = \frac{2\pi c}{\lambda} \):

\[
\frac{\delta}{\kappa} = -\frac{\Re(e^{i\pi R}(n_R - n_L))}{2\Im(n_R + n_L)} = \frac{\omega - \omega_0}{\Gamma/2} \frac{\Im E_1 M_1^*}{|E_1|^2 + |M_1|^2}
\]

(40)

(\( \omega_0 \) and \( \Gamma \) are respectively the frequency and width of the transition)

From this formula it would seem that one should consider, as before, twice forbidden \( M_1 \) transitions in order to have the ratio \( \left| \frac{E_1}{M_1} \right| \) as large as possible. In fact, if one considers a twice forbidden transition like the \( 6S_{1/2} \rightarrow 7S_{1/2} \) in Cesium the rotation angle \( \delta \), being proportional to \( \Im E_1 M_1^* \), is far too small for reasonable values of the length \( l \). So one should turn to
ordinary magnetic dipole transitions like the $6\ ^5\!P_{1/2} \rightarrow 6\ ^3\!P_{3/2}$ transition in Thalium. In practice, because of Doppler effect the resonance has not a Breit-Wigner Shape, but is obtained by a convolution of the Breit-Wigner amplitude $\left(\omega - \omega_o + i \Gamma/2\right)^{-1}$ with a Gaussian probability density $\left(\sqrt{2\pi} \Gamma_D\right)^{-1} \exp\left(-\frac{(\omega - \omega_o)^2}{2\Gamma_D^2}\right)$

with \[ \Gamma_D = \omega_0 \sqrt{\frac{kT}{M\varepsilon^2}}. \]

( $T$ absolute temperature, $M$ atomic mass).

It is convenient to express $\delta$ and $\kappa$ in terms of the maximum excitation cross section $\sigma_{max}$. ($J_i \rightarrow J_f$)

\[ \sigma_{max} = \frac{1}{4\sqrt{2\pi}} \frac{\Gamma_{M_1}}{\Gamma_D} \lambda_o^2 \left( \frac{2J_f + 1}{2J_i + 1} \right) \]

where $\Gamma_{M_1}$ is the rate associated with the spontaneous radiative transition given by:

\[ \Gamma_{M_1} = 2\omega_o^3 |M_1|^2 \]

We have the two useful relations:

\[ \delta(\omega) = -\frac{\text{Im}(E_1 M_1^*)}{|E_1|^2 + |M_1|^2} \sigma_{max} \text{Im} \omega(z) \]

\[ \kappa(\omega) = N \sigma_{max} \text{Re} \omega(z) \]

$N$ is the number of atoms per cm$^3$. The complex number $\omega(z)$ related to the error function $\text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_{-z}^{\infty} e^{-t^2} dt$ by

\[ \omega(z) = e^{-z^2} \text{erfc}(-iz) \]

The complex number $z = x + iy$ is given by:
The rate of increase of \( \frac{\delta(\omega)/\kappa(\omega)}{|\omega - \omega_0|} \) with \( \omega - \omega_0 \) is extremely sensitive to the value of \( \Gamma/\Gamma_D \). Under practical conditions \( \Gamma \) is not the natural width but the collision width \( \Gamma_c \), which is hard to evaluate with precision but easily accessible to experiment.

Let us quote our results for the transition

\[ 6p_{1/2} \rightarrow 6p_{3/2} \] of Thallium:

\[
\begin{align*}
\kappa & \simeq 0.95 \left( \frac{273}{T} \right)^{3/2} \rho_{\text{torr}} \quad \text{Re} \, w(y) \quad \text{cm}^{-1} \\
\delta & \simeq -0.27 \left( \frac{273}{T} \right)^{3/2} \rho_{\text{torr}} \quad \text{Im} \, w(y) \times 10^{-6} \text{ cm}^{-1}
\end{align*}
\]

We have used to get this number \( M \equiv -\frac{\sqrt{2}}{3} \mu_B \) (obtained by assuming the same radial functions for \( 6p_{1/2} \) and \( 6p_{3/2} \)) and the estimate:

\[
E_1 = \langle 6p_{1/2} | d_y | 6p_{3/2} \rangle \simeq -4.9 \times 10^{-10} \text{ eV a.u.}
\]

Taking \( \alpha = 2 \) and \( \gamma = 0 \) (i.e. \( \Gamma \ll \Gamma_D \)) we get:

\[
\begin{align*}
\delta & = -0.9 \left( \frac{273}{T} \right)^{3/2} \rho_{\text{torr}} \times 10^{-7} \text{ cm}^{-1} \\
\kappa & = 1.7 \left( \frac{273}{T} \right)^{3/2} \rho_{\text{torr}} \times 10^{-2} \text{ cm}^{-1}
\end{align*}
\]

These numbers have to be corrected for hyperfine structure effect. They are in reasonable agreement with that of reference [22].

We would like to point out that \( \delta \) does not increase indefinitely with the pressure. When the collision width \( \Gamma_c \) which
is proportional to the pressure becomes larger than $\Gamma_D$, $\text{Im} \, \psi(y)$ decreases like $1/\Gamma_c$; it implies that $\delta$ goes to a constant.

A rough estimate of $\Gamma_c$ shows that equality $\Gamma_c = \Gamma_D$ is obtained for a pressure of the order of 10 torr.

So nothing is gained by going to a pressure much higher than 10 torr.

In fact, it does not seem much easier to measure such a small rotation rather than to look for a circular polarization of the transmitted light when the absorption coefficient is of the order of unity. For an unpolarized incident beam of intensity $I_0$, the circular polarization of the transmitted light is given by:

$$P_c^{\text{trans.}} \approx -2 \, \text{Im} \left( \frac{E_1}{\mathcal{M}_1^x} \right) \log \frac{I_0}{I^{\text{trans.}}}.$$

In the simple case of Thallium at a pressure of one torr with $l = 10 \, \text{cm}$, we have

$$P_c^{\text{trans.}} \approx -0.65 \times 10^{-6}.$$

We have heard of two experiments (University of Oxford and University of Washington at Seattle) being set up in order to measure the rotatory power of a vapor of bismuth. The effect is expected to be of the same order of magnitude as for Thallium case.

Our discussion of the experimental implications of neutral currents is far from complete. For instance, the muonic atoms offer interesting possibilities which have been discussed extensively by many authors [5] [23] [24] [25]. The predicted asymmetries in the radiative transitions $^2S_{1/2} \longrightarrow ^4S_{1/2} + \gamma$ are in the case of light atoms ($Z < 10$) larger than the ones discussed before. Unfortunately,
since they have to be measured in spontaneous transitions the problem
of the branching ratio becomes crucial. There is a large number of compe-
ting processes: two photons decay, Auger effect, one photon decay in-
duced by a Stark effect involving the surrounding electrons etc... It
is only in the region $20 < Z < 40$ that the branching ratio reaches rea-
sonable value but the asymmetries are then reduced to the level of
$10^{-3} - 10^{-4}$. 
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