Determining the relative phase in $\psi'$ and $J/\psi$ decays into baryon and antibaryon

Kai Zhu, Xiao-Hu Mo, and Chang-Zheng Yuan

Institute of High Energy Physics, CAS, Beijing 100049, China

Received 16 May 2015
Revised Day Month Year

With the recent measurements of $\psi'$ and $J/\psi$ decay into octet-baryon pairs, we study the relative phase between the strong and the electromagnetic amplitudes, and find a large phase by fitting the present data. The fits take into account the details of experimental effects, including energy spread and initial state radiation. We also predict some branching fractions of $\psi'$ decays and the continuum production rates at the $J/\psi$ mass based on the relative phase and absolute amplitudes obtained from the fits.

Keywords: Relative phase; charmonium decay; baryon and antibaryon.

12.38.Aw, 13.25.Gv, 13.40.Gp, 14.40.Gx

1. Introduction

Studying the relative phase between the electromagnetic (EM) and strong decay amplitudes, in addition to the magnitudes of them, provides us a new viewpoint to explore the quarkonium decay dynamics. Till now, no theory can give a satisfactory explanation of the origin of or a constraint on this relative phase, a better knowledge of it may lead to a better understanding of the quarkonium decay dynamics. Experimentally, the charmonium ($c\bar{c}$) states $J/\psi$ and $\psi'$ are especially suitable for such a study. First of all, they decay at the charm energy scale. Comparing with lighter resonances, these decays provide more comparable amplitudes between strong and EM processes. Comparing with heavier resonances, they provides larger fractions of the final states with simple topologies, then benefits to experimental analysis. Furthermore, $J/\psi$ and $\psi'$ are vector resonances and can be produced directly via $e^+e^-$ collision, and huge data samples were collected at many experiments. This means that very small statistical uncertainties or/and good experimental precisions can be achieved in measuring the decay branching fractions.

Studies have been carried out for many $J/\psi$ two-body mesonic decay modes with various spin-parities: $1^-0^-$ [1,2], $0^-0^-$ [3,4], and $1^-1^-$ [5], and baryon antibaryon pairs [6]. These analyses reveal that there exists a relative orthogonal phase between the EM and strong decay amplitudes in $J/\psi$ decays [1,7].

As to the $\psi'$, author of Ref. [7] argues that the only large energy scale involved in
the three-gluon decay of the charmonia is the charm quark mass, then one expects that the corresponding phases are not much different between $J/\psi$ and $\psi'$ decays. There is also a theoretical argument which favors the $\pm 90^\circ$ phase [8]. This large phase follows from the orthogonality of the three-gluon and one-photon virtual processes. Experimentally, some analyses [9–12] based on limited $1^{-0^{-}}$ and $0^{-0^{-}}$ data indicate that the large phase is compatible with the data.

Recently more measurements with much improved precision for baryon and antibaryon ($B\bar{B}$) final states have been presented by CLEO [13], BES [14–17] and BESIII [18, 19] Collaborations, at on- and off-resonance regions for $\psi'$ and $J/\psi$. These results provide a possibility for a more precise phase analysis. Comparing with analysis of mesonic decays the formalism for bayonic decay is a little more complicated, therefore in Sec. 2 the formalism we adopted is depicted, special attention is paid for experimental conditions, such as energy spread and initial state radiation (ISR) which have profound effect on the fit results. The fits and corresponding results are presented in Sec. 3 followed by a summary.

2. Description of the formalism

A general prescription of the parametrization of the amplitude for the mesonic decays is given in Refs. [20, 21], where the decay amplitudes are expressed in terms of the $SU(3)$-symmetric and $SU(3)$-symmetry breaking coupling strengths. The idea and technique of this scheme are extended to describe the bayonic decays below. Then by virtue of parametrization form, we obtain the Born cross sections for all possible modes of octet-baryon pairs, as well as the observed cross sections taking all the experimental details into account.

2.1. Parametrization

Under $SU(3)_{flavor}$ symmetry (the subscript “flavor” is omitted hereafter for briefness) the baryons can be arranged in singlet, octet, and decuplet irreducible representations:

$$3 \otimes 3 \otimes 3 = 1_A \oplus 8_{M_1} \oplus 8_{M_2} \oplus 10_S.$$  

The subscripts indicate antisymmetric ($A$), mixed-symmetric ($M_1$, $M_2$) or symmetric ($S$) multiplets under interchange of flavor labels of any two quarks. Each multiplet corresponds to a unique baryon number, spin, parity, and its members are classified by isospin, its third component, and strangeness. The ground octet and decuplet states, denoted as $B_8$ and $B_{10}$, correspond to $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively.

In an $SU(3)$ symmetric world only the decays into final states $B_8 \bar{B}_8$ and $B_{10} \bar{B}_{10}$ are allowed, with the same decay amplitudes for a given decay family if the EM contributions are neglected. Nevertheless, the $SU(3)$ symmetry can be broken in several ways [5], so in the phenomenological analysis both symmetry-conserved and
symmetry-breaking terms are to be included. In this article we analyze the octet-baryon pair final states.

To describe the $SU(3)$ octet states, it is convenient to introduce the matrix notations

\[
\mathbf{B} = \begin{pmatrix}
\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\
\Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\
\Xi^- & -\Xi^0/\sqrt{2} + 2\Lambda/\sqrt{6} & -2\Lambda/\sqrt{6}
\end{pmatrix}
\]

(1)

and

\[
\mathbf{B} = \begin{pmatrix}
\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^- \\
\Sigma^+ & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Xi^- \\
p & -\Xi^0/\sqrt{2} + 2\Lambda/\sqrt{6} & -2\Lambda/\sqrt{6}
\end{pmatrix}
\]

(2)

for octet baryons and antibaryons, respectively. Note the fact that, to an extremely good approximation, the $\psi'$ and $J/\psi$ are $SU(3)$ singlets. For the decay $\psi' \rightarrow B\overline{B}$, to derive the consequences of the $SU(3)$ symmetry, the $SU(3)$ multiplets containing $B$ and $\overline{B}$ are combined in an $SU(3)$-invariant way. The only such a combination is

\[
\mathcal{L}_{eff}^0 = g \text{Tr}(\mathbf{B}\overline{\mathbf{B}}),
\]

(3)

where “Tr” represents the trace of the matrix and the effective coupling constant $g$ is proportional to the decay amplitude.

Now turn to the $SU(3)$-breaking effects. Following the recipe proposed in Ref. [20], $SU(3)$-breaking effect is simulated by constructing an $SU(3)$-invariant amplitude involving three octets and by choosing one of the octets (called a “spurion” octet) to point in a fixed direction of $SU(3)$ space particular to the desired breaking. Two types of $SU(3)$ breaking are considered. First, the quark mass difference. The $SU(2)$ isospin symmetry is assumed, that is $m_u = m_d$; but $m_s \neq m_u, m_d$ and this mass difference between $s$ and $u/d$ quarks leads to an $SU(3)$-breaking effect. By writing the quark mass term as

\[
m_d(\overline{dd} + \overline{uu}) + m_s\overline{ss} = m_0\overline{qq} + \frac{1}{\sqrt{3}}(m_d - m_s)\overline{q}\lambda_8q,
\]

where $q = (u, d, s)$; $m_0 = (2m_d + m_s)/3$ is the average quark mass; $\lambda_8$ is the 8th Gell-Mann matrix. It can be seen that this $SU(3)$ breaking corresponds to a spurion pointing to the 8th direction of the abstract space spanning by 8 Gell-Mann matrices. Explicitly, the matrix $\mathbf{S}_m$ is introduced to describe such a mass breaking effect

\[
\mathbf{S}_m = \frac{g_m}{3} \begin{pmatrix}
1 \\
1 \\
-2
\end{pmatrix},
\]

(4)

where $g_m$ is effective coupling constant due to the mass difference effect.
Second, the EM decay amplitude. The EM effect violates $SU(3)$ invariance since the photon coupling to quarks is proportional to the electric charge:

$$\frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s = \frac{1}{2} \left( \lambda_3 + \frac{\lambda_8}{\sqrt{3}} \right) q .$$

The above expression indicates that the EM breaking can be simulated by the spurion matrix $S_e$ as follows

$$S_e = \frac{g_e}{3} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} ,$$

where $g_e$ is effective coupling constant due to the EM effect.

To build an $SU(3)$ invariant out of three matrices, there are two different ways of combination $[22]$:

$$\text{Tr}(B\overline{B}S) , \quad \text{or} \quad \text{Tr}(\overline{B}BS) .$$

These are conventionally combined further into combinations involving the commutator and anticommutator of the two matrices and called $F$- and $D$-type, respectively. Therefore, the most general form of $SU(3)$ invariant effective Lagrangian for three matrices is

$$\mathcal{L}_{\text{eff}} = D \text{Tr}([B, \overline{B}]S) + F \text{Tr}(\{B, \overline{B}\}S) .$$

Notice that $S$ can be either $S_m$ or $S_e$, together with $\mathcal{L}_{\text{eff}}^0$ for symmetric part, the synthetic Lagrange reads

$$\mathcal{L}_{\text{eff}} = \sum g \text{Tr}(B\overline{B}) + d \text{Tr}([B, \overline{B}]S_e) + f \text{Tr}(\{B, \overline{B}\}S_e) + d' \text{Tr}([B, \overline{B}]S_m) + f' \text{Tr}(\{B, \overline{B}\}S_m) .$$

With the expressions in Eqs. (1), (2), (4), and (5) for $B$, $\overline{B}$, $S_m$, and $S_e$, respectively, the parametrization forms for octet-baryon-pair final state are worked out and presented in Table 1 where the coupling constants are recast as $A = g$, $D = dg_e/3$, $F = fg_e$, $D' = dg_m/3$, and $F' = fg_m$, following the conventions in Refs. [5] and [22].

Here is a remark concerning the treatment of charge conjugate final states. Applying the operator for charge conjugation to a baryon-antibaryon system,

$$C\ket{B_n\overline{B}_m} = \pm \frac{B_{n\overline{m}}}{|B_{n\overline{m}}|} \text{ for } n = m \neq |B_{n\overline{m}}| \text{ for } n \neq m ,$$

generally leads to a different state. Charge conjugate states will be produced with the same branching fractions, therefore we adopt the convention that charge conjugate states are implicitly included in the measurement of branching fractions, and the parametrization in Table 1 has followed such an convention.
Determining relative phase in $\psi'$, J/$\psi$ decays into baryon and antibaryon

Table 1. Amplitude parametrization forms for decays of the $\psi'$ or J/$\psi$ into a pair of octet baryons (phase space is not included). General expressions in terms of SU(3)-symmetry-conserved ($A$), as well as symmetric and antisymmetric charge-breaking ($D, F$) and mass-breaking terms ($D', F'$).

| Final state         | Amplitude parametrization form                              |
|---------------------|-------------------------------------------------------------|
| p$p'$               | $A + D + F - D' + F'$                                        |
| n$n'$               | $A - 2D - D' + F'$                                            |
| $\Sigma^+ \Sigma^+$ | $A + D + F + 2D'$                                             |
| $\Sigma^0 \Sigma^0$ | $A + D + 2D'$                                                 |
| $\Sigma^- \Sigma^+$ | $A + D - F + 2D'$                                             |
| $\Xi^0 \Xi^0$       | $A - 2D - D' - F'$                                            |
| $\Xi^- \Xi^+$       | $A + D - F - D' - F'$                                         |
| $\Lambda \bar{\Lambda}$ | $A - D - 2D'$                                               |
| $\Sigma^0 \Lambda + \Sigma^0 \bar{\Lambda}$ | $\sqrt{3}s$                                                   |

2.2. Born cross section

For $e^+e^-$ colliding experiments, there is the inevitable continuum amplitude

$$e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$$

which may produce the same final state as the resonance decays do. The total Born cross section therefore reads [24–26]

$$\sigma_B(s) = \frac{4\pi\alpha^2}{3\sqrt{s}}[a_{3g}(s) + a_\gamma(s) + a_c(s)]^2 \mathcal{P}(s),$$

which consists of three kinds of amplitudes correspond to (a) the strong interaction ($a_{3g}(s)$) presumably through three-gluon annihilation, (b) the electromagnetic interaction ($a_\gamma(s)$) through the annihilation of $\tau\bar{\tau}$ pair into a virtual photon, and (c) the electromagnetic interaction ($a_c(s)$) due to one-photon continuum process. Notice $a_\gamma(s)$ corresponds to the contributions from resonance, J/$\psi$ or $\psi'$ here, then it will be much larger than $a_c(s)$ even both of them are via single virtual photon process. The phase space factor $\mathcal{P}(s)$ is expressed as

$$\mathcal{P}(s) = v(3 - v^2)/2, \quad v \equiv \sqrt{1 - \frac{(m_{B1} + m_{\bar{B}2})^2}{s}},$$

where $m_{B1}$ and $m_{\bar{B}2}$ are the masses of the baryon and anti-baryon in the final states, and $v$ velocity of baryon in the center-of-mass system (CMS).

For the octet-baryon-pair decay, the amplitudes have the forms:

$$a_c(s) = \frac{Y}{s},$$

$$a_\gamma(s) = \frac{3Y\Gamma_{ee}/(\alpha\sqrt{s})}{s - M^2 + iM\Gamma_t},$$
where $\sqrt{s}$ is the center-of-mass energy, $\alpha$ is the QED coupling constant; $M$ and $\Gamma_t$ are the mass and the total width of the $\psi'$ or $J/\psi$, respectively; $\Gamma_{ee}$ is the partial width to $e^+e^-$. $X$ and $Y$ are the functions of the amplitude parameters $A, D, F, D'$, and $F'$ listed in Table 1, viz.

$$Y = Y(D, F) ,$$

$$X = X(A, D', F') e^{i\phi} .$$

The special form of $X$ or $Y$ depends on the decay mode, as examples, for $p\overline{p}$ decay mode, $X = A - D' + F'$ and $Y = D + F$ while for $\Xi^{-}\Xi^{+}$ decay mode, $X = A - D' - F'$ and $Y = D - F$, according to the parametrization forms in Table 1. In principle, the parameters listed in Table 1 could be complex arguments, each with a magnitude together with a phase, so there are totally ten parameters which are too many for nine octet-baryon decay modes. To make the following analysis practical, and referring to the analyses of measonic decays, it is assumed that there is no relative phases among the strong-originated amplitudes $A$, $D'$, $F'$, and no relative phase between EM amplitudes $D$ and $F$; the sole phase (denoted by $\phi$ in Eq. (15)) is between the strong and the electromagnet interactions, that is, between $X$ and $Y$ as indicated in Eqs. (15) and (14), where $A$, $D$, $F$, $D'$, and $F'$ are treated actually as real numbers.

### 2.3. Observed cross section

In $e^+e^-$ collision, the Born cross section is modified by the ISR in the way [27]

$$\sigma_{r.c.}(s) = \int_0^{x_m} dx F(x, s) \frac{\sigma_B(s(1 - x))}{|1 - \Pi(s(1 - x))|^2} ,$$

where $x_m = 1 - s'/s$. $F(x, s)$ is the radiative function which has been calculated to an accuracy of 0.1% [27][29], and $\Pi(s(1 - x))$ is the vacuum polarization factor. In the upper limit of the integration, $\sqrt{s'}$ is the experimentally required minimum invariant mass of the final state particles. In the following analysis, $x_m = 0.2$ is used which corresponds to an invariant mass requirement of greater than 3.3 GeV (2.8 GeV) for the $\psi'$ ($J/\psi$) analysis.

The $e^+e^-$ collider has a finite energy resolution which is much wider than the intrinsic width of narrow resonances such as the $\psi'$ and $J/\psi$. Such an energy resolution is usually a Gaussian distribution:

$$G(W, W') = \frac{1}{\sqrt{2\pi} \Delta} e^{-\frac{(W-W')^2}{2\Delta^2}} ,$$

where $W = \sqrt{s}$ and $\Delta$, a function of the energy, is the standard deviation of the Gaussian distribution. The experimentally observed cross section is the radiative
Determining relative phase in \( \psi' \), \( J/\psi \) decays into baryon and antibaryon

corrected cross section folded with the energy resolution function

\[
\sigma_{\text{obs}}(W) = \int_0^\infty dW' \sigma_{\text{r.c.}}(W') G(W', W) .
\]

Table 2. Breakdown of experiment conditions correspond to different detectors and accelerators. The data taking position is the energy which yield the maximum inclusive hadronic cross section. The data with star (\( * \)) are the equivalent luminosity calculated with relation \( \mathcal{L} = \frac{N_{\text{tot}}}{\sigma_{\text{max}}} \).

| Detector | Accelerator | Energy Spread (MeV) | Data Taking Position (GeV) | Total events \((\times 10^6)\) | Integrated luminosity (pb\(^{-1}\)) |
|----------|-------------|---------------------|-----------------------------|-----------------------------|-------------------------------------|
| BESIII   | BEPCII      | 1.112               | 3.097                       | 225.3                       | 79.6                                |
| CLEO [30]| CESR        | 1.5                 | 3.68625                     | 3.08                         | 2.74                                |
|          |             | 2.3                 | 3.68633                     | 3.08                         | 2.89                                |
|          |             | 2.28                | 3.671                       | –                            | 20.7                                |
| BESII [31]| BEPC       | 1.3                 | 3.68623                     | 14.0                         | 19.72                               |
|          |             | 1.27                | 3.650                       | –                            | 6.42                                |
|          |             | 0.85                | 3.09700                     | 57.7                         | 15.89*                              |
| MARKII   | SPEAR       | 2.40                | 3.09711                     | 1.32                         | 0.924*                              |
| DMII     | DCI         | 1.98                | 3.09711                     | 8.6                          | 5.053*                              |
| FENICE   | ADONE       | 1.24                | 3.09704                     | 0.15                         | 0.059*                              |

As pointed out in Ref. [25], the radiative correction and the energy spread of the collider are two important factors, both of which reduce the height of the resonance and shift the position of the maximum cross section. Although the ISR are the same for all \( e^+e^- \) experiments, the energy spread is quite different for different accelerators, even different for the same accelerator at different running periods. As an example, for the CLEO data used in this paper, the energy spread varies due to different accelerator lattices [30]: one (for CLEO III detector) with a single wiggler magnet and a center-of-mass energy spread \( \Delta = 1.5 \) MeV, the other (for CLEO-c detector) with the first half of its full complement (12) of wiggler magnets and \( \Delta = 2.3 \) MeV [12]. The two \( \Delta \)'s lead to two maximum total cross sections 635 nb and 441 nb, respectively, which differ prominently from BESII value of 717 nb for \( \Delta = 1.3 \) MeV [31]. All these subtle effects must be taken into account in data analysis. In the following analysis all data were assumed to be taken at the energy point which yields the maximum inclusive hadron cross sections instead of the nominal resonance mass [25,32]. Some experimental details are summarized in Table 2 and they are crucial for the data fitting preformed below.

3. Fit to data

Since our analyses involve the experimental details as indicated in the preceding section, some measurements are not adopted in the following study due to the lack
of necessary information of the detectors and/or accelerators. In addition, the status of the accelerators are also different, so the fits to $\psi'$ and $J/\psi$ decays are discussed separately for the sake of clarity.

### 3.1. $\psi'$ decays

The experiment measurements were reported in Refs. [13, 14] and [33–35]. The results of Refs. [34] and [35] were presented four decades ago and only one branching fraction (for $p\bar{p}$) and two upper limits (for $\Lambda\bar{\Lambda}$ and $\Xi^-\bar{\Xi}^+$) were given. The results of Ref. [33] were obtained based on the data taken 15 years ago. Therefore, only the results acquired recently are utilized, which are quoted in Table 3.

| Mode          | $N_{\text{obs}}$ (peak) | $N_{\text{obs}}$ (continuum) | Efficiency (%) | Detector |
|---------------|--------------------------|-------------------------------|----------------|----------|
| $p\bar{p}$    | 556.5 ± 23.3             | 15.9 ± 4.0                    | 66.6           | CLEO     |
|               | 1618.2 ± 43.4            |                               | 34.4           | BESII    |
| $\Sigma^0\Sigma^-$ | 34.2 ± 5.86             | 0 ± 2.3                       | 4.1            | CLEO     |
| $\Sigma^0\Sigma^+$ | 58.5 ± 7.7              | 0 ± 2.3                       | 7.2            | CLEO     |
|               | 59.1 ± 9.1               |                               | 4.4            | BESII    |
| $\Xi^0\Xi^0$  | 19.0 ± 4.4               | 2.0 ± 2.7                     | 2.4            | CLEO     |
| $\Xi^-\Xi^+$  | 63.0 ± 8.0               | 1.8 ± 2.7                     | 8.6            | CLEO     |
|               | 67.4 ± 8.9               |                               | 3.9            | BESII    |
| $\Lambda\bar{\Lambda}$ | 203.5 ± 14.3          | 3.4 ± 2.9                     | 20.1           | CLEO     |
|               | 337.2 ± 19.9             |                               | 17.4           | BESII    |

It should be noted that for the results from CLEO Collaboration, the number of continuum ($N_{\text{con}}$) is not subtracted from the signal events at the $\psi'$ peak. The continuum data are all from CLEO and scaled by a factor $f_s = 0.2547$ for all decay modes. $f_s = 0.2547$ is calculated taking into account the differences in luminosity and efficiency, and $1/s^5$ correction [13]. The scaled results are shown in Table 3. In addition, the CLEO data were taken at two distinctive running states of the accelerator, which corresponds to different energy spread, so the data are treated separately. If denoting the number of events taken at CLEOIII as $N_1$ and at CLEO-c as $N_2$, then

$$ N_1 = \mathcal{L}_1 \cdot \sigma_{\text{obs}}^1 \cdot \epsilon, $$

$$ N_2 = \mathcal{L}_2 \cdot \sigma_{\text{obs}}^2 \cdot \epsilon. $$

Here the efficiencies ($\epsilon$) at the continuum and resonance are considered to be the same [13], $\mathcal{L}$ is the integrated luminosity of the data sample, and $N$ the number of observed signal events. So one gets

$$ N = (\mathcal{L}_1 \cdot \sigma_{\text{obs}}^1 + \mathcal{L}_2 \cdot \sigma_{\text{obs}}^2) \cdot \epsilon. $$
Determining relative phase in $\psi'$, $J/\psi$ decays into baryon and antibaryon

where $N = N_1 + N_2$ is the total number of signal events in the two data sets.

Chi-square method is used to fit the experimental data. The estimator is defined as
\[
\chi^2 = \sum_i \frac{[N_i - n_i(\vec{\eta})]^2}{(\delta N_i)^2},
\]
where $N_i$ denotes the experimentally measured number of events while $n_i$ the theoretically calculated number of events. The sum runs over all the final states at the $\psi'$ peak and the measurements at the continuum energy. The five continuum channels other than $p\bar{p}$ are combined to increase the statistics. The observed cross section is calculated according to Eq. (17), which contains the parameters to be fit, such as $A$, $D$, $F$, $D'$, $F'$, and the phase $\phi$. All these parameters are denoted by the parameter vector $\vec{\eta}$ in Eq. (20). It should be noticed that $n$ consists of two parts for CLEO data and should be calculated by Eq. (19).

The scan for each parameter discloses the two minima of $\phi$ with opposite sign, while all the other parameters have the same values up to the significant digits listed below:

\[
\phi = (-98 \pm 25)^\circ, \text{ or } (+134 \pm 25)^\circ; \quad A = 2.857 \pm 0.066; \quad D' = -0.055 \pm 0.044; \quad F' = 0.060 \pm 0.066; \quad D = 0.142 \pm 0.033; \quad F = 0.027 \pm 0.052.
\]

The phase determined from $\psi' \to BB$ decay is fairly consistent with the analysis for $\psi' \to K_S^0K_L^0$ [11], where $\phi$ is determine to be $(-82 \pm 29)^\circ$ or $(+121 \pm 27)^\circ$. Here the solution $-98^\circ$ is more favorable for the universal assumption proposed in Ref. [43]. The results of Eq. (21) show that for $\psi' \to BB$ decays the $SU(3)$-symmetric amplitude ($A$) dominates while the other amplitudes are weaker by at least one order of magnitude.

With the above fit results, the ratios of the branching fractions $Br(\psi' \to n\bar{n})/Br(\psi' \to p\bar{p})$ and $Br(\psi' \to \Sigma^0\bar{\Lambda} + c.c.)/Br(\psi' \to p\bar{p})$ are predicted to be $1.31 \pm 0.14$ and $0.007 \pm 0.004$, respectively. Till now, there is no signal reported in experiments for $\psi' \to n\bar{n}$ and $\psi' \to \Sigma^0\bar{\Lambda} + c.c..$ We propose to measure them at experiments such as BESIII. Notice that even the branching fraction of $\psi' \to \Sigma^0\bar{\Lambda} + c.c.$ is only about $4 \times 10^{-6}$ as we predicted, with an assumption of 8% reconstruction efficiency, about 80 events can be observed with the 450 M $\psi'$ events collected at BESIII.

### 3.2. $J/\psi$ decays

There are lots of measurements at $J/\psi$ region. However, many of measurements were performed almost ten or twenty years ago [35]- [42]. The recent experimental
results were mainly from BES \cite{15,17} and BESIII \cite{18,19} Collaborations. Besides the data from them, the data from MARKII \cite{38} and DMII \cite{39,40} are adopted, since the numbers of events from these two experiments are considerable large and more information of distinctive decay modes are provided. All data used in this analysis are summarized in Table 4.

| Mode | \(N_{\text{obs}}^{\text{peak}}\) | Efficiency (%) | Detector |
|------|--------------------------------|---------------|----------|
| \(\psi'\) | 63316 ± 281 | 48.53 | BESII \cite{16} |
| \(\psi'\) | 1420 ± 46 | 49.7 | MARKII \cite{38} |
| \(\psi'\) | 314651 ± 561 | 66.1 | BESIII \cite{18} |
| \(\Lambda\bar{\Lambda}\) | 35891 ± 211 | 7.7 | BESIII \cite{18} |
| \(\Sigma^+\bar{\Sigma}^0\) | 8887 ± 132 | 7.59 | BESII \cite{16} |
| \(\Sigma^+\bar{\Sigma}^0\) | 365 ± 19 | 17.6 | MARKII \cite{38} |
| \(\Sigma^+\bar{\Sigma}^0\) | 1847 ± 67 | 15.6 | DMII \cite{39} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 1779 ± 54 | 2.32 | BESII \cite{16} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 90 ± 10 | 4.3 | MARKII \cite{38} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 884 ± 34 | 9.70 | DMII \cite{39} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 399.9 ± 26.7 | 0.462 | BESII \cite{18} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 203.6 ± 21.0 | 0.280 | BESII \cite{18} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 194 ± 14 | 12.9 | MARKII \cite{38} |
| \(\Sigma^0\Lambda + \Sigma^0\Lambda\) | 132 ± 12 | 2.20 | DMII \cite{39} |

The minimization estimator for \(J/\psi\) is similar to that of \(\psi'\) as defined in Eq. (20). However, for \(J/\psi\) data there is only limited information about each detector, especially the integrated luminosity. Therefore, it is difficult to deal with all data consistently and accurately. To alleviate the systematic biases among the data from different experiments, three scale factors are introduced. They are normalized with respect to the BESII experiment and are floated in the fit. It should be noted that no continuum data are available around \(J/\psi\) mass, so we have less constraint on the EM amplitudes than in the \(\psi'\) case. The fitted parameters are listed as follows:

\[
\phi = (-85.9 \pm 1.7)^\circ, \quad \text{or} \quad (+90.8 \pm 1.6)^\circ ; \\
A = 1.760 \pm 0.012 ; \\
D' = -0.067 \pm 0.006 ; \\
F' = 0.102 \pm 0.013 ; \\
D = 0.181 \pm 0.005 ; \\
F = 0.168 \pm 0.088 ;
\]

\[
f_{mk2} = 0.904 \pm 0.024 ; \\
f_{dm2} = 0.704 \pm 0.021 ; \\
f_{bes3} = 0.922 \pm 0.004 .
\]

Here the three factors \(f_{mk2}, f_{dm2}, \) and \(f_{bes3}\) reflect the possible systematic bias in
Determining relative phase in $\psi'$, $J/\psi$ decays into baryon and antibaryon

MARKII, DMII and BESIII, respectively, relative to the BESII experiment. The fit values indicate that the inconsistencies of these experiments from that of BESII vary from 10% to 30%. This effect is small on the determination of the phase, and is ignored in the discussion below.

The phase determined from $J/\psi \rightarrow B \overline{B}$ decays is similar to that from $\psi' \rightarrow B \overline{B}$ in this analysis and the magnitudes of amplitudes are similar too. We also notice that our results are consistent with those in Ref. [19], in which the “reduced branching ratio” method [5, 44] was applied, and the $\phi$ is determined to be $(+76 \pm 11)\,^\circ$. It should be emphasized that with the “reduced branching ratio” method the continuum contribution is simply subtracted from the data on the resonance peak, the interference between them has not been considered properly and it can only provide relative strengths of the different amplitudes.

With the EM amplitudes determined from the fit, one can calculate the continuum production cross sections of all the final states listed in Table 4. As a byproduct, we predict

$$\sigma(e^+e^- \rightarrow p\overline{p}) = 11.5 \pm 5.8 \text{ pb}, \quad \sigma(e^+e^- \rightarrow n\overline{n}) = 12.3 \pm 1.5 \text{ pb},$$
$$\sigma(e^+e^- \rightarrow \Lambda\overline{\Lambda}) = 2.8 \pm 0.4 \text{ pb}, \quad \sigma(e^+e^- \rightarrow \Sigma^0\overline{\Sigma^0}) = 2.7 \pm 0.3 \text{ pb},$$
$$\sigma(e^+e^- \rightarrow \Xi^0\overline{\Xi^0}) = 9.2 \pm 1.1 \text{ pb}, \quad \sigma(e^+e^- \rightarrow \Sigma^+\overline{\Sigma^-}) = 10.0 \pm 5.1 \text{ pb},$$
$$\sigma(e^+e^- \rightarrow \Sigma^0\overline{\Lambda} + \Sigma^0\overline{\Lambda}) = 8.3 \pm 1.0 \text{ pb} \quad (23)$$

at a center-of-mass energy corresponding to the $J/\psi$ mass; while the cross sections of $\sigma(e^+e^- \rightarrow \Sigma^+\overline{\Sigma^-})$ and $\sigma(e^+e^- \rightarrow \Xi^0\overline{\Xi^0})$ are at a few fb level. These can be tested with the data samples at the BESIII experiment.

4. Summary

The relative phase between the strong and the EM amplitudes of the charmonium decays is studied based on the recent experimental data of $\psi'$, $J/\psi \rightarrow B \overline{B}$ decays.

For $\psi'$ decays the phase is found to be $(-98 \pm 25)^\circ$ or $(+134 \pm 25)^\circ$ while for $J/\psi$ decays the phase is fitted to be $(-85.9 \pm 1.7)^\circ$ or $(+90.8 \pm 1.6)^\circ$. The relative phases are similar between $\psi'$ and $J/\psi$ decays into baryon and anti-baryon final states, also are consistent with previous results with meson final states [1,7,9,12], that should be updated with recent CLEO-c and BESIII measurements. For the phase study in this work, the detailed experimental conditions, such as energy spread and ISR, are taken into account. However, due to the limited precision of the data, only the strength of the dominate $SU(3)$-symmetric amplitude is determined reasonably well. In order to fix all parameters which describe the octet-baryon-pair decays, more accurate experimental measurements are needed.

With the fit results, we also predict and propose to measure more $\psi'$ decays modes as well as the continuum production of the baryon pairs at the $J/\psi$ mass region. Additional experimental information will be helpful to draw a final conclusion on the relative phase.
Acknowledgment

This work is supported in part by National Natural Science Foundation of China (NSFC) under contracts Nos. 11375206, 10775142, 10825524, 11125525, 11235011, 11475187; the Ministry of Science and Technology of China under Contract Nos. 2015CB856701, 2015CB856706, and the CAS Center for Excellence in Particle Physics (CCEPP).

References

1. J. Jousset et al., [DMII Collab.], Phys. Rev. D 41, 1389 (1990).
2. D. Coffman et al., [Mark III Collab.], Phys. Rev. D 38, 2695 (1988).
3. M. Suzuki, Phys. Rev. D 60, 051501 (1999).
4. G. López, J. L. Lucio M. and J. Pestieau, arXiv:hep-ph/9902300.
5. L. Köpke and N. Wermes, Phys. Rep. 174, 67 (1989).
6. R. Baldini et al., Phys. Lett. B 444, 111 (1998).
7. M. Suzuki, Phys. Rev. D 63, 054021 (2001).
8. J.-M. Gérard and J. Weyers, Phys. Lett. B 462, 324 (1999).
9. C. Z. Yuan, P. Wang, X. H. Mo, Phys. Lett. B 567, 73 (2003).
10. P. Wang, C. Z. Yuan and X. H. Mo, Phys. Rev. D 69, 057502 (2004).
11. J. Z. Bai et al. [BES Collaboration], Phys. Rev. Lett. 91, 052001 (2004).
12. S. Dobbs et al. [CLEO Collaboration], Phys. Rev. D 74, 011105 (2006).
13. T.K. Pedlar et al. [CLEO Collaboration], Phys. Rev. D 72, 051108 (2005).
14. M. Ablikim et al. [BES Collaboration], Phys. Lett. B 648, 149 (2007).
15. J. Z. Bai et al. [BES Collaboration], Phys. Lett. B 591, 42 (2004).
16. M. Ablikim et al. [BES Collaboration], Phys. Lett. B 632, 181 (2006).
17. M. Ablikim et al. [BES Collaboration], Phys. Rev. D 78, 092005 (2008).
18. M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 86, 032014 (2012).
19. M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 86, 032008 (2012).
20. H. E. Haber and J. Perrier, Phys. Rev. D 32, 2961 (1985).
21. N. Morisita, I. Kitamura and T. Teshima, Phys. Rev. D 44, 175 (1991).
22. H. Georgi, “Weak interactions and modern particle theory” (The Benjamin/Cummings Publishing Company, 1984), p65.
23. H. Kowalski and T. F. Walsh, Phys. Rev. D 14, 852 (1976).
24. S. Rudaz, Phys. Rev. D 14, 298 (1976).
25. P. Wang, C. Z. Yuan, X. H. Mo and D. H. Zhang, Phys. Lett. B 593, 89 (2004).
26. P. Wang, X. H. Mo and C. Z. Yuan, Int. J. Mod. Phys. A 21, 5163 (2006).
27. E. A. Kuraev and V. S. Fadin, Yad. Fiz. 41, 733 (1985) [Sov. J. Nucl. Phys. 41, 466 (1985)].
28. G. Altarelli and G. Martinelli, CERN 86-02, 47 (1986); O. Nicrosini and L. Trentadue, Phys. Lett. B 196, 551 (1987).
29. F. A. Berends, G. Burgers and W. L. Neerven, Nucl. Phys. B 297, 429 (1988); ibid. 304, 921 (1988).
30. CLEO-c/CESR-c Taskforces & CLEO-c Collaboration, Cornell University LEPP Report No. CLNS 01/1742 (2001) (unpublished).
31. BES Collaboration, J. Z. Bai et al., Phys. Lett. B 550, 24 (2002).
32. P. Wang, C.Z. Yuan and X.H. Mo, HEP & NP 27, 463 (2003).
33. J. Z. Bai et al. [BES Collaboration], Phys. Rev. D 63, 032002 (2001).
34. G. J. Feldman and M. L. Perl, Phys. Rept. 33, 285 (1977).
35. R. Brandelik et al. [DASP Collaboration], Z. Phys. C 1, 233 (1979).
Determining relative phase in $\psi'$, $J/\psi$ decays into baryon and antibaryon

36. I. Peruzzi et al., Phys. Rev. D 17, 2901 (1978).
37. H. J. Besch et al., Z. Phys. C 8, 1 (1981).
38. M.W. Eaton et al. [MARKII Collaboration], Phys. Rev. D 29, 804 (1984)
39. D. Pallin et al. [DMII Collaboration], Nucl. Phys. B 292, 653 (1987).
40. P. Henrard et al. [DMII Collaboration], Nucl. Phys. B 292, 670 (1987).
41. A. Antonelli et al., Phys. Lett. B 301, 317 (1993).
42. J. Z. Bai et al. [BES Collaboration], Phys. Lett. B 424, 213 (1998)
43. P. Wang, C. Z. Yuan and X. H. Mo, Phys. Lett. B 574, 41 (2003).
44. G. Lopez Castro, J. L. Lucio M. and J. Pestieau, AIP Conf. Proc. 342, 441 (1995)