Homomorphic Sortition – Secret Leader Election for Blockchain

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ABSTRACT

In a secret single leader election protocol (SSLE), one of the system members is chosen as a leader and no other member can know who the leader is, unless she decides to reveal herself. Leader-election oracles are known to be instrumental in designing efficient consensus protocols, and one can expect that secret leader election may help in developing long-lived blockchain systems that are secure with respect to the adaptive adversary.

In this paper, we introduce secret leader sortition (SLS) that we believe to perfectly match progress and security expectations of proof-of-stake (PoS) blockchains. An SLS protocol produces an unpredictable permutation of system members that can be fed to an accompanying blockchain protocol in order to ensure deterministic finality.

We describe Homomorphic Sortition, an SLS protocol based on Threshold Fully Homomorphic Encryption (ThFHE). An interesting novelty of our protocol is a cryptographic hash function evaluation under ThFHE and, somewhat surprisingly, we show that all ThFHE stimuli required for our protocol can be efficiently and proactively constructed in a parallelizable fashion. In contrast to existing secret leader election protocols, our SLS solution is purely asynchronous and it fairly leverages the stake distribution among the system members: the probability distribution for each position in the permutation is proportional to the stakes of remaining candidates. As we demonstrate, our SLS protocol seamlessly partners with a large family of PoS blockchain implementations.

CCS CONCEPTS

- Theory of computation → Design and analysis of algorithms
- Distributed algorithms;

KEYWORDS

Byzantine Fault Tolerance, Single Secret Leader Election, Sortition, Blockchain, Deterministic Termination, Proof of Stake

1 INTRODUCTION

Since the advent of Bitcoin [29], blockchain systems have grown to an entire field of study in computer science. At a high level, a blockchain is a tamper-proof ledger of blocks of data issued by the system members. As a baseline, the chain of data blocks keep record of asset transfers between the participants [29], but often it offers more general functionality [12]. To reach agreement on the order in which data blocks appear in the chain, blockchains resort to the fundamental problem of consensus.

Consensus is one of the most studied problems in distributed computing. Mainstream consensus implementations follow the classical leader-based approach [16, 25], where a single process is selected to be the leader whose task is to propose a value and orchestrate the agreement.

Secret leader election. Using a publicly known leader as a proposer of a new block opens the door to the Denial of Service (DoS) attack. The adversary can try bringing down the leader, which may slow down the consensus protocol, and thus the blockchain as a whole. This issue can be addressed in multiple ways.

First, we can allow every participant to propose a block (thus, avoiding a single target of attack) and then use consensus to choose one of them (this is done for example in RedBelly blockchain [19]). However, as the blocks are publicly known, the adversary may try alienating participants with undesired proposals.

Alternatively, following the approach of Algorand [23], one can use verifiable random functions (VRF), to select a set of participants as potential leaders and then pick one of them using a deterministic tie breaker (e.g., the lowest VFR output). In this approach, however, a process cannot know for sure that it is elected before every other candidate reveals its input, which incurs extra communication complexity and creates a potential security breach.

Thus, one may want to hide the identity of the leader until she makes her proposal, and then it is too late to mount a DoS attack against her. This functionality is captured by the single secret leader
The election protocol can be run on a blockchain system using the election abstraction (SSLE) introduced by Boneh et al. [6]. Besides avoiding the DoS attack, the SSLE-based approach also preserves the original structure of most blockchain implementations currently in use. However, repeatedly using SSLE for each optimistic attempt to solve consensus may only yield probabilistic progress guarantees. Even in synchronous executions, it may take an unbounded number of election rounds before a correct leader is picked up.

**Our contributions.** In this paper, we introduce a new secret leader-election abstraction, secret leader sortition (SLS), that perfectly matches blockchain systems relying on leader-based consensus by providing deterministic finality (e.g., Tezos based on Tenderbake [1], or Cosmos based on Tendermint [11]). The SLS abstraction ensures that every process gets elected in a bounded round and, thus, the counterpart consensus protocol is guaranteed to terminate in a bounded number of rounds. More precisely, instead of electing an independent leader for each round of consensus, SLS constructs a permutation of n system participants that can be used for n consecutive consensus rounds. Moreover, the construction is unpredictable: nobody can learn the position of a correct participant in the permutation before she reveals herself.

Following Boneh et al. [6], our SLS implementation that we call Homomorphic Sortition is based on Threshold Fully Homomorphic Encryption (ThFHE). Unlike [6], however, our protocol is purely asynchronous. Furthermore, our protocol can be efficiently used in proof-of-stake (PoS) blockchains that rely on the assumption that the participants controlled by the adversary can only hold a minor fraction of the total stake, typically less than one third. SLS fairly accounts for the stake distribution across the participants in the permutation: the probability of a participant to be assigned a position in the permutation is proportional to its share of stake among the remaining candidates.

On the practical side, Homomorphic Sortition was designed to benefit from multiprocessing capabilities of modern machines. Whenever possible, different stages of the ThFHE circuits were designed in a mutually-independent manner, which enables efficient parallelization of cryptographic computations. Furthermore, once the setup phase is complete, the protocol does not require any information from consensus when deployed in a blockchain, which allows us to run several instances of our protocol in a pipeline. By using precomputations, i.e., by electing the leaders in advance, the block creation latency can thus be made comparable to that of deterministic, insecure round-robin leader scheme.

The rest of the paper is organized as follows. In Section 2, we formally define the SLS problem. In Section 3, we describe our Homomorphic Sortition algorithm, explain how it can be used in the blockchain context, and give details on our implementation. In Section 4, we report on its performance. Section 5 overviews related work and Section 6 concludes the paper.

## 2 PROBLEM STATEMENT: SECRET LEADER SORTITION

The election protocol can be run on a blockchain system using leader-based consensus. It has a set of processes \( \Pi = \{ \pi_1, \pi_2, \ldots, \pi_n \} \) forming a committee with stakes \( S \). If the underlying system provides the means to do so, the membership of the committee might be dynamic, i.e. the set of processes that take part in it might change over time.

Each next data block in the blockchain defines publicly known stake distribution. More precisely, the total stake of the committee is \( s_t = \sum_{i=1}^{n} S[i] \), where \( S < s_t/3 \) is controlled by Byzantine committee members. Byzantine participants may exhibit arbitrary behaviour. In particular, they may collaborate, trying to prevent correct (non-Byzantine) processes from being elected. To this end, they might omit information, send different messages to different processes (equivocate), share not well-formed data, etc.

The communication channels are reliable: messages exchanged by correct processes are eventually delivered. We assume, however, that communication is asynchronous, i.e., communication delays may not be bounded.\(^1\)

A Secret Leader Sortition (SLS) protocol outputs a list of \( n \) publicly known vouchers \( v_1, v_2, \ldots, v_n \) and \( n \) proofs \( \pi_1, \pi_2, \ldots, \pi_n \), where, for each \( i \), \( \pi_i \) is only revealed to \( p_i \) and no information about it is revealed to any other process or the adversary. For each \( r = 1, \ldots, n \), a process \( p_i \) is said to be elected in round \( r \), and we write \( l_r = p_i \), if \( \pi_i \) can claim the voucher \( v_i \), which is confirmed via a publicly known function verify.

An SLS protocol must satisfy the following properties:

- **Uniqueness:** \( \forall r, \exists ! i : \text{verify}(i, \pi_i, v_i) = \text{true} \). Moreover, it is computationally intractable for the adversary to produce a proof \( \pi \) such that for \( p_j \neq p_i \), \( \text{verify}(j, \pi, v_r) = \text{true} \), even if they know \( \pi_i \). This prevents potential front-running attacks.

- **Fairness:** The leader in round \( r \) is selected from \( \Pi \setminus \{l_1, \ldots, l_{r-1}\} \) with probability proportional to the un-electected amount of stakes. More formally, for all \( p_i \in \Pi \setminus \{l_1, \ldots, l_{r-1}\} \):

\[
\Pr[l_r = p_i] = \frac{S[i]}{\sum_{p_j \in \Pi \setminus \{l_1, \ldots, l_{r-1}\}} S[j]}
\]

- **Unpredictability:** if \( l_r \) is correct, unless the proof \( \pi_i \) is revealed, the adversary cannot guess the leader for the round \( r \) better than based on the information about the stakes, the set of already revealed leaders, and the positions of the Byzantine processes in the permutation.

- **Termination:** If all correct processes execute the protocol, then every correct process \( p_i \) eventually yields all vouchers and the proof \( \pi_i \).

Therefore, the SLS abstraction ensures that every process \( p_i \) computes the same list of \( n \) vouchers, one for each position (round) \( r \) in the cycle of election. In addition, \( p_i \) computes a private proof \( \pi_i \) that matches exactly one voucher \( v_i \) in the list, i.e., \( \text{verify}(i, \pi_i, v_i) \) evaluates to true. Unless, \( \pi_i \) is revealed, no other process can learn who is elected in rounds \( 1, \ldots, n - 1 \) (for the last round, revealing \( \pi_1, \ldots, \pi_{n-1} \) implies that the only remaining process is \( l_n \)). Finally, the probability of of a process to be elected in a round

\(^1\)Notice that an accompanying blockchain system that consumes the leader indications produced by our protocol and establishes setup for each election cycle may make stronger synchrony assumptions. For example, most leader-based consensus protocols assume partial synchrony: after an unknown amount of time GST all the messages must arrive within a certain upper bound of time \( \Delta \). This, however, does not affect our leader election mechanism.
\( r \) is proportional to its fraction of stake among its competitors, \( \Pi \setminus \{l_1, \ldots, l_{r-1}\} \).

### 3 SLS SOLUTION

**Notation.** First we introduce the notations used in the following sections: scalar values are written with a single lower case letter (e.g., \( f \)), vectors with a single upper case letter (e.g., \( K \)) and matrices with double upper case letters (e.g., \( RR \)). Values which are homomorphically encrypted are written with an underline (e.g., \( n \)). Subscripts might be added to variables to give more context when necessary. A list of values enclosed in square brackets (e.g., \( \{a, b, c\} \)) denotes a column vector, \((\cdot)_i\) denotes a value signed using \( SK_i \) and \( \{\cdot\}_i \) denotes a value encrypted using \( PEK_i \).

#### 3.1 Cryptographic primitives

First of all, we need an encryption and a digital signature scheme. Every process \( p_i \) has a pair of public and private encryption keys (\( PEK_i, SK_i \)) as well as a pair of public and private signature keys (\( PK_i, SK_i \)). The private keys are known only by the process who owns them and the public keys are globally known. Moreover, we also need a threshold fully homomorphic encryption system (TFHE) \([7]\). This type of encryption schemes allows computations over encrypted data and guarantees that a certain threshold stake of key holders (defined by a threshold value) are able to cooperate to decrypt a ciphertext.

After a setup is complete, each process \( p_i \) has access to a private key \( SThFHE_i \) used to partially decrypt a ciphertext encrypted with the joint public key \( PThFHE \), known to every process. An encryption operation is denoted \( ThFHE.Enc \) and a partial decryption operation is denoted \( ThFHE.Dec \). Once processes holding at least \( s_t - s_f \) stake issue a partial decryption operation for a given encrypted value, it is possible to retrieve the corresponding clear-text value. Shares corresponding to less than \( s_t - s_f \) stake reveal no information about the encrypted value. Additionally, before decrypting the final value using the operation \( ThFHE.Dec \), a process can check that the partial decryption it receives corresponds to an encrypted value by calling \( ThFHE.Ver \).

The operations over encrypted data are done via operation \( ThFHE.Eval \) using publicly known circuits presented in Table 1 along with the list of operations exported by the TFHE system.

| Method          | Inputs                              | Outputs                                               |
|-----------------|-------------------------------------|-------------------------------------------------------|
| Circuit \( C_0 \) | \([x_1, x_2, \ldots, x_n]\)         | \(\bigoplus_{i \in \{1, \ldots, n\}} x_i\)           |
| Circuit \( C_{Sym} \) | \([x_1, x_2, \ldots, x_n], [k_1, k_2, \ldots, k_n]\) | \([\{x_1\}_{k_1}, \{x_2\}_{k_2}, \ldots, \{x_n\}_{k_n}\}\) |
| Circuit \( C_{||} \) | \(pre, [x_1, x_2, \ldots, x_n], pos\) | \([pre|x_1||pos, pre|x_2||pos, \ldots, pre|x_n||pos]\) |
| Circuit \( C_{||2} \) | \([x_1, x_2, \ldots, x_n], [y_1, y_2, \ldots, y_n]\) | \([x_1||y_1, x_2||y_2, \ldots, x_n||y_n]\) |
| Circuit \( C_{H} \) | \([x_1, x_2, \ldots, x_n]\)         | \([H(x_1), H(x_2), \ldots, H(x_n)]\) |
| Circuit \( C_x \) | \(x, y\)                            | \([x < y_1, \ldots, x < y_n]\)                       |
| Circuit \( C_c \) | \([y_1, y_2, \ldots, y_n]\)         | \([-x_1, -x_2, \ldots, -x_n]\)                       |
| Circuit \( C_{Sel} \) | \([x_1, x_2, \ldots, x_n], [y_1, y_2, \ldots, y_n]\) | \(\exists i : y_i = 1 \land \forall j \neq i, y_j = 0\) |
| \(TFHE.Enc\) | \([x_1, x_2, \ldots, x_n]\)         | \([x_1, x_2, \ldots, x_n]\)                         |
| \(TFHE.Eval\) | \(C, Inputs\)                      | Evaluation of \(C\) applied to \(\text{Inputs}\)     |
| \(TFHE.PDec\) | \(i\)                               | 1 out of \(n - f\) shares of \(i\)                   |
| \(TFHE.Ver\) | \(j, i\)                            | 1 if \(j\) is a share of \(i\), 0 otherwise         |
| \(THF.H.Dec\) | \([y_1, y_2, \ldots, y_{n-f}]\) (shares of \(i\)) | \(i\)                                                 |

**Table 1: ThFHE methods**
The circuit $C_x$ takes two encrypted values and returns their product. The circuit $C_z$ takes an encrypted value $x$ and a list of encrypted values $[y_1, y_2, \ldots, y_n]$ and returns a list where whenever $x < y_i$ the $i$-th is set to 1, otherwise it is assigned a 0 value. Finally, the circuit $C_{Sel}$ receives an array such that only one position $i$ is set to 1 and all the others have a value 0 and a second array of encrypted values, returning the $i$-th element of this array.

### 3.2 The Homomorphic Sortition Protocol

**Algorithm 1** Homomorphic Sortition for process $p_i$

1. operation sortition($P$, $R$, $r$)
2. if $r = 1$ then
3. $U := \text{ThFHE.Enc}([S[j], \ldots, S[j]])$
4. $m := \text{ThFHE.Enc}([S[j]])$
5. $x := \text{the b MSB of ThFHE.Eval}(C_{x}, R[r], m)$
6. // MSB stands for most significant bits
7. // $m$ and $R[r]$ have $b$ bits, hence the product has $2b$ bits
8. $L := \text{ThFHE.Eval}(C_{x}, x, U)$
9. $G := \text{ThFHE.Eval}(C_{z}, L)$
10. $[1 := \text{L[1]}$
11. $[2 := \text{ThFHE.Eval}(C_{y}, L[2..], G[1..n-1])$
12. $s_x := \text{ThFHE.Eval}(C_{sel}, E, S)$
13. $\pi_t := \text{ThFHE.Eval}(C_{sel}, E, P)$
14. $i_r := \text{ThFHE.Eval}(C_{sel}, E, \pi_t)$
15. $\hat{v}_r := \text{ThFHE.Eval}(G, \pi_t)$
16. $\pi_r := \text{ThFHE.Eval}(G, \pi_t)$
17. $D := \text{ThFHE.Eval}(C_{x}, \pi_r, L)$
18. $U := \text{ThFHE.Eval}(C_{y}, U, D)$
19. $m := \text{ThFHE.Eval}(C_{y}, m, \pi_r)$
20. Send $\{PVoucher, \text{ThFHE.PDec}(\pi_r)\}$ to all
21. upon receiving $\{PVoucher, (\hat{v}_r)\}$ from $p_j$
22. if $\text{ThFHE.} \text{Ver}(\pi_r, \hat{v}_r) \land (\pi_r) \notin \Lambda_r$ then
23. $\Lambda_r := \Lambda_r \cup (\hat{v}_r)$
24. upon $\sum_{(\pi_r) \notin \Lambda_r} S[j] > s_r - s_f$ then
25. $\hat{v}_r := \text{ThFHE.Dec}(\Lambda_r)$
Overview. At a high level, in our Homomorphic Sortition protocol (presented in Algorithm 1), every process samples the vouchers one by one in order to create an ordered list. Once a voucher is sampled, the corresponding process’ information is updated in the sampling mechanism so that each user is selected exactly once. Once enough correct processes execute the protocol, the voucher is subsequently decrypted to its plain value.

Sortition. The vouchers are picked one by one following a weighted sample. To carry this sample, processes begin by building a cumulative sum of and total sum of the participant’s stake doing operations in clear data and then storing in their encrypted form (lines 3 and 4). After these computations are done the circuit shown in Figure 1 (for clarity, we illustrate the case when $n = 4$) is executed $n$ times.

Sampling. For the moment, assume that processes have access to an array $E$ of $n$ common random numbers of $b$ bits each. Importantly, the number of bits used to represent the total stake in the system $m$ is also $b$. By multiplying these two values and keeping only the $b$ most significant bits of the result in the variable $x$, we get a random number between 0 and $m$ (line 5).

The following lines (9 to 12) find the position in the cumulative sum array that contains the smallest value bigger than $x$, which is enough to obtain a result satisfying the randomness requirement. This is done by first creating a vector $L$ set to 1 in all positions such that $x$ is strictly less the value in $L$ in the same position. The desired index will be the first one set to TRUE, which can be obtained by making a logical and between $L$ and its inverse shifted by one position, i.e. $L[1], L[2] \land \neg L[1], L[3] \land \neg L[2], \ldots, L[n] \land \neg L[n-1]$. Once the process is selected, its stake $s_r$ is subtracted from the total unselected stake $m$, as well as from cumulative sums of its position and of the processes who follow it. This is equivalent to a new partial sum array in a scenario where $l_r$ has stake zero, and hence cannot be selected (lines 13, 18 and 19).

Vouchers. The goal of the protocol is to determine the order in which processes become leaders, but because of the secrecy requirements, it must be done in a manner that only the elected process knows about the result. For the moment, let us assume that each participant is dealt its proof to claim the voucher and that every process has the list of all proofs $P$ encrypted with ThFHE. We use the information of the vector $E$ to determine the leader’s proof and ID. We then append these two values and take the hash of the result as the voucher. This guarantees that each voucher can be claimed by at most one process, thanks to the collision resistance of the hash function. Furthermore, since each process has access to its proof and, prior to its reveal, no other participant has knowledge of it, every user is able to see its position in the result but of no one else.

Decryption. The rest of the algorithm shows how each voucher is decrypted (lines 20 to 25). Processes first send to all processes their partial decryptions, collecting messages of this type from other participants. A message is only kept if it passes the partial decryption test. Once at least $s_f - s_i$ stake is validated, a final decryption yields the plaintext value of the voucher.

### 3.3 Integration with blockchain and consensus

#### Algorithm 2 Homomorphic Sortition + consensus for process $p_i$

26. The blockchain schedules consensus$_{h,c,r}$
27. before executing consensus$_{h,c,r}$
28. if $r = 1$ then
29. $P \leftarrow \text{ThFHE.Eval}(C_{[1]}|h, t, i, c)$
30. $P \leftarrow \text{ThFHE.Eval}(C_{HT, P})$
31. $K \leftarrow \text{ThFHE.Eval}(C_{[1]}|h, Q, c)$
32. $K \leftarrow \text{ThFHE.Eval}(C_{HT, K})$
33. $\text{sortition}_h(P, K, r)$
34. upon receiving PROPOSAL, $\pi_j \land v_{her} = H(\pi_j||j)$ from $p_j$
35. Accept proposal in consensus

Orchestration. So far we have considered how to obtain one sequence of non-repeating leaders. Nevertheless, in order to deploy the protocol in a blockchain, it is necessary to execute it several times. These executions are coupled with consensus by organising them into heights ($h$), cycles ($c$) and rounds ($r$). The position of a process is determined by the triple $h, c, r$ and all processes begin with $h = c = r = 1$.

Once a block is successfully appended to the blockchain, processes move to the position $h + 1, 0, 0$, maintaining the property that for every height only one block is accepted.

A height is divided into an arbitrary number of cycles, which, in turn, are divided into at most $n$ rounds each. A round retains the semantic given in the election protocol, having a unique proposer, while a cycle corresponds to an instance of sortition. The consensus protocol has an adaptive timer that stipulates how long a process waits for a block being accepted and if this event does not happen within the stipulated time period, the processes time out. In this case, participants move to the next round, corresponding to the position $h, c, r + 1$. However, in a partially synchronous network, it may happen that even after $n$ rounds, the consensus will still not be reached. In this case, the sortition protocol must be restarted, moving to position $h, c + 1, 0$.

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### Table 3: Extra variables used for integration in alphabetical order

| Variables | Description |
|-----------|-------------|
| $c$       | current cycle |
| $h$       | current height |
| $k_i$     | $p_i$'s symmetric key |
| $t_i$     | $p_i$'s current ticket |
| $T$       | processes’ tickets |
| $\Phi$    | partial decrypt. of tickets |
| $\pi_i$   | $p_i$'s proof of election |
| $Q$       | seed numbers |
| $QQ, TT$  | committed random values |
| $X$       | local rand. numbers |
| $Y$       | sym. encrypted tickets |

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5
Tickets and seeds. There are two types of randomness in Homomorphic Sortition: tickets and seeds. Tickets are used to generate proofs and vouchers, while the seeds are used to generate the randomness necessary in the sampling algorithm. We leave to the blockchain protocol the option to issue the algorithm that creates new tickets and seeds to be used in the Homomorphic Sortition (line 37). This should always be done at the beginning of the protocol (the first set of seeds and tickets can be generated in the setup of the ThFHE scheme) or if the membership of the committee changes.

Entering consensus. The proof of process $p_i$ in height $h$, cycle $c$ is $\pi_i = H(h||t||c||c)$, while the voucher is $H(\pi_i||i)$. Hence, once a process asserts that the current selected voucher is $H(h||t||c||c)||i$, it knows that it has been elected (line 40) and can use $\pi_i$ to convince others (line 47). Once the leader proposes a block, the other processes proceed normally with the consensus protocol if the election is verified or simply discard the proposal otherwise.

Producing the random tickets and seeds. Once a new set of random numbers is needed, every process generates $2n$ local random numbers (line 37). Half of them are used to produce tickets and the other half to produce seeds. Then the process encrypts each of them with the ThFHE scheme (line 38). The list of encrypted values is sent to all processes (line 39) and treated by the leader who picks $f + 1$ of these values corresponding to a stake of at least $s_f + 1$ and includes them in its proposal in the form of two $(f + 1) \times n$ matrices (line 44).

If there is also a Rand trigger, then the leader must also include two matrices containing the random numbers coming from $f + 1$ different processes which are used for producing seeds and tickets (lines 45 to 46). The other committee members check the integrity of the proof and random matrices and if the verification is successful, the proposal is treated in the consensus protocol, being discarded otherwise (lines 47 to 49).

Whenever a block is committed with the two random matrices aforementioned, these matrices are collapsed into two vectors by XORing them columnwise (lines 52 and 54). While the seeds are kept encrypted, the tickets are symmetrically encrypted (under ThFHE) with the matching process key elementwise, and then decrypted (lines 55 to 61), with every process sending a partial ThFHE decryption.

The vectors $P$ and $S$. Now it is possible to understand where the proofs and random numbers come from. At a first glance it might seem that processes can use their tickets as proofs; this would, however, mean that each ticket could be used only once and we would not be able to restart the protocol for cycles beyond 1. Instead, by prefixing the ticket by the current height and appending the current cycle (line 29) and taking the hash of the result (line 30), we obtain the proof used in our protocol. This proof can be easily generated for each new instance of Homomorphic Sortition and does not reveal any information about the ticket, making it possible to keep using the same value. The random numbers used in the sampling are generated from the seeds using the same strategy.

### 3.4 Example of a possible execution

Let us now describe in details a simple scenario with four processes $p_1, p_2$ (Byzantine), $p_3, p_4$. In a real implementation, the size of the random numbers and stake are in the order of billions, while the hash length is also sufficiently large for security, but for clarity we present them as 8 bit values. In our scenario, the participants have stake 42, 3C, 17 and 6A, respectively.

![Figure 2: Example of random generation](image-url)
Rand Trigger. As shown in Figure 2, the correct processes start by generating \(2n = 8\) random numbers each, while the Byzantine process can generate any amount of numbers and picking fixed values instead. These numbers are sent to every process and once one of them become a leader, it can include a subset of these numbers in its proposal. The proposal will be discarded if more than 42.

Once a valid proposal is put forward and is subsequently committed in the ThFHE scheme, where it will be updated by step computations for the system, the tickets are eventually decrypted and the processes continue with the next round.

The seeds are only kept inside the ThFHE encryption, but the first eight bits \((7E, 95, FF)\) are conserved, while the last eight are discarded. 78 is greater or equal to 42, but it is less than \(7E, 95\) and \(FF\), hence the vector \(L\) is 0, 1, 1, 1. We conserve the first set index by creating the array \(G\), which negates \(L\) to 1, 0, 0, 0. We build \(E\) by conserving the first bit of \(L\) for the other set index by taking the AND of \(L\) with \(G\) shifted one position to the right (see Figure 1): \(0 \land 1 = 1, 1 \land 0 = 0, 1 \land 0 = 0\). \(E\) can be used to select the proof of the elected process for this round, which in our example is the process \(p_2\), whose proof is \(C_2\), then the result of the election is the voucher \(v_1 = H(C22) = E3\). It is also used to select \(p_2\)'s stake, and subtracting it of the partial sum of all process with indices greater or equal than 2 via the vector \(D = 0, 3C, 3C, 3C\).

Therefore, the state of the partial sum array for the second round is \(42 - 0 = 42, 7E - 3C = 42, 95 - 3C = 59, FF - 3C = 3\) and the total unselected stake is \(3\). The second position of \(R\) is 57, hence the random number between 0 and the unselected stake for this round is \(3 \times 57 = 424\). Notice that 42 is greater than equal to 42 and less than 59 and 3, hence the values of \(L\) and \(G\) for this round are 0, 0, 1, 1 and 1, 0, 0, 0, resulting in \(E = 0, 0, 1, 0\). This means that process \(p_3\) is selected and the proof for this round is \(2B\), resulting in the voucher \(v_2 = H(2B3) = 47\) and the new partial sum for the next round shall be \(42 - 0 = 42, 42 - 0 = 42, 59 - 17 = 42, 3C - 17 = AC\). The same reasoning is applied for the last two rounds.

3.5 Pipelining

One of the features of Homomorphic Sortition is the complete independence between different heights and cycles, which combined to a vectorised approach adopted in the circuit design results into a highly parallelisable protocol which can be executed in a pipeline. A blockchain that deploys our protocol must guarantee that a voucher is produced for a specific height, cycle and round combination is produced before executing consensus.

Let us consider a scenario where the system has already stabilised and that the delays in communication and the performance of the local computations create a situation where executing one round
of HS takes roughly the same amount of time as executing three rounds of consensus. Then, an adaptive scheduler can manage its threads in the manner depicted in Figure 4. By executing the sortition protocol in parallel for every position possibly reached in three consensus instances, the consensus protocol can be run non-stop, yielding the same latency as a round-robin leader implementation would.

In the figure we represent different threads vertically and time elapsed horizontally. Timed out consensus operations are shown in light green, positions that generate a commit in dark green, aborted HS instances in yellow and completed sortitions in violet.

3.6 Computing Hashes in FHE

Let us first mention that even if there exists several "FHE-friendly" keystreams (e.g. Trivium [13], Kreyvium [14]), and blockciphers (e.g. Flip [28]), to the best of our knowledge, the hash functions were not analysed and conceived for an efficient homomorphic execution. In order to obtain a secure hash function that could be used on encrypted data with homomorphic encryption, we studied various approaches. The first one was to implement hash functions from the lightweight cryptography standards [30]. This approach seemed interesting at first because with homomorphic encryption, the algorithms we want to use need to be represented by a number of Boolean gates as low as possible especially in terms of multiplicative depth. It turned out that most of the hash functions candidates in lightweight cryptography had a quite high multiplicative depth and required the use of many Bootstrapping operations to reset the noise. With the candidates that we studied, it required several hours to execute their hash functions on encrypted data with TFHE [18] scheme (and even more with other FHE schemes). This result was not efficient enough for a election protocol for the blockchain (or any application at all) so we tried to find an approach that differs from most hash functions.

Most hash functions nowadays use a block permutation several times in some mode of operation. This generally implies that there is at least one operation in the block permutation with a certain multiplicative depth which grows as the permutation is called. In order to obtain a more efficient hash function, we looked for a construction which wouldn’t imply this behaviour and more specifically one that would allow us to do most of the computations in the clear. To achieve this goal, we looked for hash functions based on stream ciphers and found Grain-128a [26] which is a stream cipher with optional authentication.

The stream cipher Grain-128a is designed such that a MAC is computed in parallel of the cipher computation. The computation of a MAC is basically the computation of a keyed hash function so the idea was to fix the key used by the MAC in order to obtain a hash function. The construction we obtain is given below.

\[
\begin{align*}
&\left(\begin{array}{c}
\mathbf{b}_0 \\
\mathbf{b}_1 \\
\mathbf{b}_2 \\
\vdots \\
\mathbf{b}_{(m-1)}
\end{array}\right) \times \left(\begin{array}{c}
\mathbf{KS}_0 \\
\mathbf{KS}_1 \\
\mathbf{KS}_2 \\
\vdots \\
\mathbf{KS}_{m-1}
\end{array}\right) + \\
&\left(\begin{array}{c}
\mathbf{H}_0 \\
\mathbf{H}_1 \\
\mathbf{H}_2 \\
\vdots \\
\mathbf{H}_{n-1}
\end{array}\right) = \\
&\left(\begin{array}{c}
\mathbf{KS}_{m_0} \\
\mathbf{KS}_{m_1} \\
\mathbf{KS}_{m_2} \\
\vdots \\
\mathbf{KS}_{m_{n-1}}
\end{array}\right)
\end{align*}
\]

Figure 5: Homomorphic computation of a hash function derived from the Grain stream cipher

Let a message \( M \) of \( m \) bits, \( M = (b_0, b_1, ..., b_{m-1}) \) where each bit \( b_i \) is associated to \( n \) bits of a keystream of size \( n \times m \). To compute our hash, we multiply each bit of the keystream (0 or 1) by the corresponding message, then we perform the XOR of all the resulting multiplications.

More generally, let \( b_i \) a bit of the message \( M \) and let denote \((KS_{i,0}, KS_{i,1}, ..., KS_{i,n-1})\) the keystream of size \( n \) associated to \( b_i \). Then, \( H(M) = \bigoplus_{i=0}^{m-1} b_i \times (KS_{i,0}, KS_{i,1}, ..., KS_{i,n-1}) \).

**Homomorphic evaluation of the hash.** We recall that the hash function is a keyed hash function with a fixed public key. In this construction, the keystream is also public and \( M = (b_0, b_1, ..., b_{m-1}) \) is encrypted homomorphically.

Therefore, the hash function we obtain is particularly convenient for a homomorphic execution because most of the computation relies on the generation of the keystream bits which can be done in the clear when the key is public like in our case. Also, when using homomorphic encryption, one must specifically pay attention to the multiplicative depth of the circuit they want to use. In our case, the multiplicative depth of the construction is in fact \( 0 \) because the only multiplications we perform \( b_i \times (KS_{i,0}, ..., KS_{i,n-1}) \) is a clear/cipher bit multiplication. Hence, if the clear bit is 0, the multiplication will be 0 and if the clear bit is 1, the multiplication will be the cipher and no additional noise is added to the cipher.

This construction can be performed on any homomorphic encryption scheme that allows to perform XOR between ciphers without requiring multiplications. To compute a hash of \( n \) bits from a \( m \) bits message, we need to perform \( m \) XOR of \( n \) bits for a total of \( n \times m \) XOR. To compute a hash of 256 output bits from a 256...
bits input, this gives a maximum of $2^{16} = 65536$ XOR (in fact, only around 32000 because about half of the keystream bits are zero so the corresponding XOR will not be necessary).

**Implementation** We implemented the hash function on Cingulata, a open-source bitwise compiler designed for homomorphic encryption. It allows to use the BFV [20] and TFHE [18] schemes. We naturally chose BFV because we don’t require the use of the Bootstrapping operation which would increase the execution time for no reason. The execution parallelized on 4 cores took approximately 5 seconds which is way better than the hours it takes to perform a more classic hash function on encrypted data.

The particular optimisation that allows us to do so little computations in the encrypted domain is the ability to generate the keystream bits in the clear and if we had to do this under homomorphic encryption the multiplicative depth would explode with a secure stream cipher and the performances would be nowhere near 5 seconds.

\[
\begin{align*}
M \ast KS_0 & \quad KS_1 \quad KS_2 \\
b0 & = KS[0,0] \quad KS[0,1] \quad KS[0,2] \\
b1 & = KS[1,0] \quad KS[1,1] \quad KS[1,2] \\
b2 & = KS[2,0] \quad KS[2,1] \quad KS[2,2]
\end{align*}
\]

**Figure 6: Example of batching by columns**

**Further optimisations.** With the BFV scheme, we can use the batching technique which consists of regrouping the rows or columns (not both) into one vector then allowing to perform the operations between rows or columns (respectively) all at once and optimize the space required by the homomorphic execution. When batching by columns, we end up with $O(n)$ operations and $O(m)$ when batching by rows ($n$ the size of the hash and $m$ is the size of the message). Besides, batching by columns gives us one encrypted batch and batching by rows gives us $m$ encrypted batches.

\[
\begin{align*}
M \ast KS_0 & \quad KS_1 \quad KS_2 \\
\text{totalSum} & = M \ast KS_0 \quad M \ast KS_1 \quad M \ast KS_2 \\
& \quad MKS_0 \ast + \ldots + MKS_2\quad MKS_0 \ast + \ldots + MKS_2
\end{align*}
\]

**Figure 7: Calculation of the hash using batching by columns**

Batching over the hash function hasn’t been implemented yet but gives perspectives that could lead to even better performances for the hash function.

**Security of the construction.** The construction is derived from the Grain family of stream ciphers whose security has been widely studied for example in [26].

**3.7 Implementation**

To implement the sortition algorithm we used as for the homomorphic hash from Section 3.6 the Cingulata compiler. In this section we dive a little bit into its operating mode in order to discuss the optimizations it carries on and their impact on global performances.

**The Cingulata Compiler.** Cingulata relies on instrumented C++ types to denote private variables, e.g., C11int for integers and C11bit for Booleans. Integer variables are dynamically sized and are internally represented as arrays of C11bit objects. The Cingulata environment monitors/tracks each bit independently. Integer operations are performed using Boolean circuits, which are automatically generated by the toolchain. Another option is to use the bit-tracker in order to build a circuit representation of the application. The later allows to use circuit optimization modules in order to further reduce the Boolean circuit representation. The hardware synthesis toolchain ABC² is used to minimize circuit size. It is an open-source environment providing implementations of state-of-the-art circuit optimization algorithms. These algorithms are mainly designed for minimizing circuit area or latency but, currently, none of them is designed for multiplicative depth minimization. In order to fill this gap, several heuristics for minimizing the multiplicative depth are available in Cingulata (please refer to [2, 15] for more details).

The optimized Boolean circuit is then executed using Cingulata’s parallel run-time environment (RTE). This RTE is generic, meaning that it uses a HE library wrapper, i.e. a “bit execution” object as defined earlier, in order to execute the gates of the circuit. The scheduler of the run-time allows to fully take advantage of many-core processors. Besides, a set of utility applications are provided for parameter generation (given a target security level), key generation, encryption and decryption. These applications are also generic, in the same vein as the parallel RTE.

**3.8 Threshold overhead**

We realised our implementation using non-threshold homomorphic encryption schemes (i.e. the single-key versions of homomorphic cryptosystems). However this is quite representative of a real-world scenario with threshold HE. Indeed in [8], authors develop a general approach for adding a threshold functionality that allows to split the secret key into a number $n$ of shares for LWE-based (Learning with errors) homomorphic schemes, with the property that the full decryption requires a number $t \leq n$ of partial decryptions (using secret key shares). The overhead due to a threshold scheme, as shown by Boneh et al., comes from the set up phase and the decryption operations while the performances of homomorphic operations are unaffected by the “thresholdizing”.

4 ANÁLISIS DE LA PROTOCOLO

4.1 Choice of the homomorphic scheme

As briefly discussed in Section 3.6, all homomorphic cryptosystems that enable both addition and multiplication of ciphertexts are designed with a built-in noise parameter included in the ciphertexts. This noise is an essential security element, but grows throughout homomorphic operations. The decryption requires the noise to be below a certain bound in order to output the correct plaintext. Two

²http://people.eecs.berkeley.edu/alanmi/abc/
approaches to deal with the noise growth gave birth to two families of homomorphic schemes. A first one is Fully homomorphic encryption (FHE) in which one can perform an unlimited number of multiplications and additions at the price of an expensive bootstrapping operation which allows to bring back a ciphertext to a nominal noise level while keeping its security intact. Another class of homomorphic cryptosystems are the Levelled HE. They require to know in advance the multiplicative depth of the homomorphic circuit so the cryptosystem’s parameters are set appropriately in order to ensure a ciphertext (with acceptable noise level) that decrypts correctly at the output of the circuit. However selecting high parameters affect the size of the ciphertexts as well as the time it takes to perform homomorphic operations. In this work we investigate the use of both types of homomorphic cryptosystems using TFHE [18] and BFV [20] with a special focus on TFHE. In addition, both cryptosystems are LWE-based and can be efficiently "thresholdized" as described in [8].

- Even though our hash function construction is perfectly suited for levelled HE such as BFV since it has a multiplicative depth of 0, our SSLE solution should be able to support up to 200 participants which would make the overall multiplicative depth become hard to manage in a levelled scheme. Consequently, we chose to implement our solution in TFHE which scales linearly to higher numbers of participants with the expense of TFHE’s bootstrapping operation after every Boolean gate.

4.2 Correctness

**Proposition 4.1.** Homomorphic Sortition ensures uniqueness.

**Proof.** First, let us show that in each round at most one process can be elected.

Suppose there are more than one voucher for any given round. These vouchers would have to be decrypted at some point, which requires at least \( n - f \) partial decryptions. Therefore, at least \( f + 1 \) participants would have to have decrypted two vouchers, which is a contradiction since at least one of the them must be correct and honest processes only decrypt a single voucher per round in line 25.

Moreover, because the protocol computes the voucher specifically for a single user \( i \) by appending it to the proof \( p_i \) and taking the hash of the result, only process \( i \) can claim the voucher (assuming the intractability of finding collisions of the hash function).

Now, let us prove that in each round there exists at least one process that can claim the voucher.

In order to compute the proofs, all processes perform the same deterministic computation in the homomorphic domain based on the same information obtained from the consensus. Therefore, all correct processes will compute the same vector of proofs inside the ThFHE scheme, thereafter also computing the same voucher array. All correct processes will then issue consistent partial decryptions for the same voucher. By the construction of the voucher, the elected process will be able to claim it with its ticket, reproducing the proof that was used to yield it.

The following three lemmata follow directly from the algorithm:

**Lemma 4.3.** The values in the partial sum array \( U \) are increasing. \( L[1] = 1 \implies \forall j \geq i, L[j] = 1 \).

**Lemma 4.4.** \( L_r = p_i \implies \forall j < i, L[j] = 0 \).

**Proposition 4.5.** Homomorphic Sortition ensures fairness.

**Proof.** The variable \( m \) keeps track of the total unselected state in the system. This variant is maintained by initialising it with the total amount of stake \( \sum s_j \) and subtracting in the end of the round the stake of the elected process in line 19.

The window of a process \( p_i \) is the range of values \( [U[i - 1], U[i)] \) for \( i \in [2..n] \) and \( [0, U[1]] \) for \( i = 1 \). Initially, the window of every process \( p_i \) has length \( S_i \), by construction. At every round \( r \), the length of the elected process’ \( p_j \) window becomes zero, while the other processes’ remain unchanged. This invariant holds because the partial sum is updated by subtracting the elected stake \( S_j \) from every position where \( L = 1 \). This means that \( U \) remains unchanged in every position less than \( j \) (cf. Lemma 4.2), keeping the corresponding processes windows unchanged, while for positions greater than \( j \), both sides of the window are shifted by the same amount (cf. Lemma 4.2), conserving the length. As for the process \( p_j \), the length of its window has been kept unchanged until it is elected, following the previous reasoning. However, at round \( r \), the lower end of its window is fixed, while the right is moved \( S_j \) to the left, so its length becomes \( S_j - S_j = 0 \).

Since a process is selected when a uniform random number between 0 and \( m \) falls within its window, the probability of a process being selected is the size of its window divided by the total amount of unselected state. For processes which have already been elected this probability is therefore zero, while for the others \( p_i \) it is \( \frac{S_j}{\sum s_j} \).

**Proposition 4.6.** Homomorphic Sortition ensures unpredictability.

**Proof.** We have already shown that our algorithm ensures fairness with respect to the probability distribution

\[
\mathbb{P}(L_{h,c,r} = p_i) = \frac{s_j}{\sum s_j - \sum s_j' \in [1..r-1]} s_{h,c,r}
\]

An adversary is able through the protocol to collect a subset of \( \{H(\pi_j j) : j \in [1..c_k]\} \) where \( c_k \) is the number of executed cycles at height \( h \). We rely on the diffusion assumption of our hash function construction from Section 3.6 i.e., a single bit flip in input flips about half output bits. This assumption ensures that the adversary, within the time between election and reveal, would not be able to guess better than a guess on the original distribution. Also, this security property is parameterised by the frequency of the Rand Trigger operation, since tickets (and consequently proofs) are changed after this operation, and thus, it becomes meaningless for an adversary to mount an attack based on storing a table of multiple hash outputs from inputs having a relatively small hamming distance from each other.
Proof. Since the arrays \( P \) and \( R \) are shared among all correct processes, as they are computed information given by the consensus protocol, all of them will obtain the same values for \( V \) and will sample the vouchers in the same order. Therefore, as the progress of the individual processes is only halted for waiting these partial decryptions and since there will always be at least \( s_L = s_F \) correct shares of them, the decryption will always be successful and the protocol will always terminate.

4.3 Communication Complexity

Let us first analyse the Homomorphic Sortition protocol from a distributed systems perspective, that is, by measuring the number of message round trips necessary per cycle (time complexity) and the number of words needed to be exchanged between processes (communication complexity). The only messages exchanged in the system are the partial decryption messages, which have all size \( O(\lambda^2) \), where \( \lambda \) is the size of the hash function used, because each bit is encrypted into \( \lambda \) bits. Therefore the latency of the sortition per round is of one message delay and the number of words exchanged in total is \( O(n^2\lambda^2) \). Since each cycle has at most \( n \) rounds, the time complexity is \( O(n) \) and the communication complexity is \( O(n^2\lambda^2) \) per cycle.

4.4 Performance

The main performance bottleneck comes from the homomorphic computations. Computing the hashes as mentioned in Section 3.6 allows to minimise the number of effective homomorphic operations. As for the sortition circuit, the main homomorphic operations are: the multiplication of the random numbers by the total stake (line 5); the computation of the \( L \) vector (line 9) which is equivalent to \( n \) homomorphic comparisons; the computation of the \( E \) vector which consists in \( n - 1 \) homomorphic evaluations of the and gate; the selection circuit which is \( B \times n \) evaluations of and gates, where \( B \) is the bit-size of the selected element (a voucher and a partial sum); finally the update of the list of partial sums of the stakes (homomorphic substractions). So the complexity is parameterised by \( n \) as well as the size of the data on which the homomorphic operations are performed. The cost of these operations varies from the use of Levelled FHE or the use of bootstrapping-based FHE namely TFHE.

We ran the TFHE version of our circuit in a Intel(R) Xeon(R) Gold 6154 CPU @ 3.00GHz machine with 72 cores.

5 RELATED WORK

Algorand. To the best of our knowledge, Gilad et al. [23] were the first to apply a secret election protocol to protect the consensus leader. In Algorand blockchain, two subsets of the users are selected: one to participate in the committee of users that will agree on which leader. In Algorand blockchain, two subsets of the users are selected: one to participate in the committee of users that will agree on which leader. In Algorand.

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Boneh et al. [6] formally defined the problem of a single secret leader election (SSLE), [6] and described three way of solve SSLE: Obfuscation [5, 22], Threshold Fully Homomorphic Encryption (ThFHE) [7], and Shuffling [24]. The first technique was not proposed to demonstrate theoretical feasibility of SSLE, while the other two were designed to be implemented in practice.

In order to employ secret leader election in Ethereum [12], Azouvi et al. [4] proposed a synchronous protocol which could be implemented by paying \( 0.108 \) in gas per leader election once deployed in the targeted blockchain. In this approach, the protocol advances in rounds from 1 to \( \text{round}_{\text{max}} \), with each participant creating a sequence of hashes \( h_1, h_2, \ldots, h_{\text{round}_{\text{max}}} \) where \( h_i = H(h_{i-1}) \), publicly committing \( H(h_{\text{round}_{\text{max}}}) \). Every round \( \text{round} \) has a threshold \( \text{target}_{\text{round}} \) and the processes generate together a common unpredictable random number \( R_{\text{round}} \). Any process for which \( (h_{\text{round}} \oplus R_{\text{round}}) < \text{target}_{\text{round}} \) is declared a leader of the round and can reveal its election by publishing \( h_{\text{round}} \). In

![Figure 8: Performance on multicore machine.](image-url)
In this paper we introduced the SLS problem and described Homomorphic Sortition, an SLS protocol that can be seamlessly integrated in blockchains built atop leader-based consensus protocols. To validate the premises of our protocol, we implemented and measured the performance of fully homomorphic circuits used in our constructions. Our solution faithfully applies the PoS approach, maintaining that the number of blocks created by a user are proportional to the amount of her stake. We also showed how to compute hashes more efficiently inside FHE schemes and how our protocol can be pipelined.

6 CONCLUSION AND FUTURE WORK

In subsequent work, Azouvi and Cappelletti [3] performed an in-depth analysis of potential performance gains of using SSLE. They compared protocols that elect one leader on expectation (that is, some rounds might elect none or several leaders) which are known as Probabilistic Leader Election – PLE – with SSLE. They found that compared to PLE, SSLE reduces the latency of block creation by 25% when the blockchain is under a private attack. In this attack, the adversary creates its own private chain in parallel, not sharing it with the rest of miners and trying to seize the longest chain.

Secrecy of stakes. Ganesh et al. [21] formulated a variation on traditional Proof-of-Stake called Private Proof-of-Stake (PPoS) where the identities and stake of participants are not publicly available. Their work focuses on implementations based on VRF functions and shows how a participant can achieve the desired results by splitting its stake into several virtual accounts and by using an anonymised version of VRF (AVRF). In AVRF, proofs are still verifiable but have the property that it is extremely hard to tell whether two proofs have been created by the same secret key or not.

Clusters of secrecy. An interesting approach to the problem was suggested by Tan et al. [32]. In their system, the processes are organised in clusters, so that within a cluster, the identity of the leader is known, but processes outside it can only state which cluster contains the leader. In this manner, processes can choose trustworthy nodes they wish to form a cluster and form a resilient system using simple primitives relying only on local computations.

Post-Boneh synchronous protocols. Catalano et al. [17] built upon [6] and proposed their own protocol using a cryptographic primitive known as functional encryption [9][31] in which given a ciphertext $c$ encrypting a keyword $w$ and a secret key $sk$ associated to another keyword $w'$, the decryption allows one to learn if $w = w'$ and nothing more. Their SSLE protocol is based on the idea that for every election a small committee of users generates a ciphertext $c$ that encrypts a random keyword $j \in \{1, \cdots, N\}$, every user is given a secret key $sk_j$ associated to an integer $i$, and can claim victory by giving a non interactive zero knowledge (NIZK) proof that they can decrypt the election’s ciphertext. They achieved a protocol that allows stake slashing of misbehaving nodes and, if executed whithout faults, the complexity of the protocol becomes sublinear on the number of participants.
In future work, we envision a solution for the SLS problem relying on Decisional Diffie-Hellman (DDH) assumption and random shuffles, similar to Boneh et al. [6] that can be implemented in partial synchrony, as well as integrating our protocol to Tendermint and making it public.

Finally, our current hash computation appears to suit better BFV executions, executing it within the TFHE scheme can be more expensive. An interesting question is whether we can use a combination of several Ring-LWE-based homomorphic encryption schemes as proposed in Chimera [10]. This would allow us to switch from BFV (in which the hash computation is more affordable) to TFHE for the rest of the circuits required by the homomorphic sortition are more efficient.

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