Chiral anomalies in superfluid hydrodynamics

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Hydrodynamics

- Description of matter in local equilibrium. Focuses on conserved currents - $T^\mu{}^\nu$, $J_a^\mu$.
- Constitutive relations - formulas for $T^\mu{}^\nu$ and $J_a^\mu$ in terms of basic variables $u^\mu$, $T$, $\mu_a$.
- Gradient expansion.
  Zeroth order - ideal fluid. First order - transport terms.
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Examples of transport terms

- Shear viscosity - $T^\mu_\pi^{\nu} = -2\eta_\pi^{\mu\nu}$.
- Conductivity - $J^{a\mu}_E = \sigma^{ab} \left( E^\mu_b - T \nabla^\mu \frac{\mu_b}{T} \right)$
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Two approaches for finding transport terms

- Second law of thermodynamics - $\nabla_\mu s^\mu \geq 0$.
- Kubo formulas - 2-point correlators in thermal QFT.
Hydrodynamics and QFT

Field theory allows new features for currents

- Chirality - $\epsilon^{\mu\nu\rho\sigma}$.
- Anomalies - $\nabla_\mu J_\mu^a \neq 0$.
- Spontaneously broken currents - superfluidity.
- (Non-abelian currents.)
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- Nuclear and QCD fluids - neutron stars, early universe, heavy ion collisions...
- Condensed-matter shenanigans (probably not for anomalies).
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Task:

▶ Find the transport terms that follow from these new possibilities.
Anomalies in normal fluid
Son, Surowka 0906.5044, YN, Oz 1011.5107, Amado et.al. 1102.4577

Chiral magnetic effect
\[ J^a_\mu B^\mu = B^\mu_b \left(C^{abc} \mu_c - \frac{n^a}{\epsilon+p} \frac{1}{2} C^{bcd} \mu_c \mu_d \right) \]

Chiral vortical effect
\[ J^a_\omega = \omega^\mu \left(C^{abc} \mu_b \mu_c - \frac{n^a}{\epsilon+p} \frac{2}{3} C^{bcd} \mu_b \mu_c \mu_d \right) \]

\[ C_{abc} - JJJ \text{ anomaly coefficient. Omitted the terms from } JTT \text{ anomaly.} \]
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Kubo formulas

\[ J^{a\mu}_{B,\omega} = j^{a\mu} - \frac{n^{a}}{\epsilon + p} j'^{\mu} \]

\( j^{a\mu} \) and \( j'^{\mu} \) - combinations of QFT correlators \( \langle JJ \rangle \), \( \langle JT \rangle \), \( \langle TT \rangle \).
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- \( j^{a\mu} \) and \( j'^{\mu} \) - combinations of QFT correlators \( \langle JJ \rangle, \langle JT \rangle, \langle TT \rangle \).

Nice structure emerges
- \( \partial j^{a\mu} / \partial \mu_{b} = C^{abc} (B_{c}^{\mu} + 2 \mu_{c} \omega^{\mu}) \)
- \( \partial j'^{\mu} / \partial \mu_{a} = C^{abc} \mu_{b} (B_{c}^{\mu} + 2 \mu_{c} \omega^{\mu}) \)
Superfluid hydrodynamics

Framework

- Spontaneously broken symmetry.
- Additional variable - the vacuum phase gradient $\xi^a_\mu$.
- Josephson condition - $\xi^a_\mu u^\mu = \mu^a + \text{corrections}$.
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Transport terms in general case

- Work in progress, spurred by holography.
New chiral terms in superfluids

Bhattacharyya, Minwalla, Yarom 1105.3733

Results from second-law constraints

- Corrections to $T^{\mu\nu}$, $J^\mu_a$ and to the Josephson condition.
- The corrections involve $\epsilon^{\mu\nu\rho\sigma} u_\nu \xi_\rho$ - vanish in collinear limit.
- Proportional to $\pi_{\mu\nu}$ or to $E^a_\sigma - T \nabla_\sigma \frac{\mu_a}{T}$.
- Also, some unspeakable things.
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In particular - Chiral Electric Conductivity

$$J_E^{a\mu} = \chi^{abc} \epsilon^{\mu\nu\rho\sigma} u_\nu \xi_\rho (E^b_\sigma - T \nabla_\sigma \frac{\mu^b}{T}); \quad \chi_{abc} = \chi_{bac}$$
Identifying the anomaly in the new terms

Educated guess: \( J^a_\mu \) arises from the \( JJJ \) anomaly

Precise form of the coefficient strongly hinted at by the existing results:

\[
J^a_\mu = C^{cde} \left( \delta^a_d - \frac{n^a n_d}{h} \right) \left( \delta^b_e - \frac{n^b n_e}{h} \right) \epsilon^{\mu\nu\rho\sigma} u_\nu \xi^c_\rho \left( E^b_\sigma - T \nabla_\sigma \frac{\mu_b}{T} \right)
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\]

Structural components of the guess

- Upgrade \( \mu^a u_\mu \) in the anomalous normal-fluid result to \( \xi^a_\mu \).
- \( B^\mu_a + 2\mu_a \omega^\mu \) and
  \[
  E^a_\mu - \nabla_\mu \mu^a - \mu^a a_\mu \approx \left( \delta^a_b - \frac{\mu^a n^a_{b}}{h} \right) \left( E^b_\mu - T \nabla_\mu \frac{\mu^b}{T} \right)
  \]
  are the magnetic and electric fields for the gauge potential \( A^a_\mu + \mu^a u_\mu \).
Further directions

- Test the guess for the Chiral Electric coefficient $\chi_{abc}$ with a Kubo-formula calculation.
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