Status of CP violation in Kaon systems

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CP violation is an important tool to test the Standard Model and his extensions. We describe kaon physics observables testing CP violation and more in general short distance physics. Channels under consideration will be $K \to \pi\nu\bar{\nu}$, $K \to \pi^+\pi^-$, $K^\pm \to 3\pi$, $K^\pm \to \pi^\pm\pi^\gamma$ and $K^\pm \to \pi^\pm\pi^0 e^+ e^-$. 

I. INTRODUCTION

The Standard Model (SM) is very successful phenomenologically; this success has been strengthened by Higgs discovery along the potential possibility to have discovered an ultimate theory up to almost the GUT scale [1]. Flavor physics has the possibility to test extensions of the SM in the two possible options, minimal flavor violation (MFV) discussed in the next section or adding new flavor structures. Particularly useful to this purpose are the $K^+ \to \pi^+\nu\bar{\nu}$ decays discussed in section III; in section IV we discuss the challenging $K_L \to \pi^0 e^+ e^-$ and the related channels $K \to \pi\gamma\gamma$ and others, all interesting as chiral tests too. In section V and VI we analyze CP violation and chiral tests in $K^+ \to 3\pi$, $K \to \pi\pi\gamma$ and $K \to \pi\pi e e$ decays.

II. MINIMAL FLAVOUR VIOLATION

Flavour physics has been crucial to dismantle arbitrary extensions of the SM, in fact soon after the discovery of technicolor, an interesting global symmetry, minimal flavour violation (MFV), was introduced to avoid large FCNC carried by the new flavour structures of techni-particles [2]. The SM lagrangian has an interesting symmetry in the limit that all the fermionic sector is massless: defining respectively with $Q$’s, $U$’s and $D$’s, the left-handed doublets, right-handed up singlets and right-handed down singlets, the global symmetry, $G_F = U(3)_Q \times U(3)_U \times U(3)_D$, is conserved. This global symmetry is broken by the mass terms

$$\mathcal{L}^\gamma_{SM} = \bar{Q}Y_D DH + \bar{Q}Y_U UH_e + \text{h.c.} \quad (1)$$

The Higgs generates the mass terms and the Yukawas furnish only sources of flavour group breaking $G_F$; then the effective FCNC hamiltonian is generated through this breaking

$$\mathcal{H}^{SM}_{\Delta F} = \frac{G_F^2 M_W^2}{16\pi^2} \left[ \frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4} (d_L^\mu \gamma^\mu b_L)^2 + \frac{(V_{ts}^* m_s^2 V_{ts})^2}{v^4} (d_L^\mu \gamma^\mu s_L)^2 \right] + \text{charm} \quad (2)$$

New flavour structures in susy generated for instance by soft breaking terms

$$\mathcal{L}_{\text{soft}} = \bar{Q}^l m_Q^2 \bar{Q} + \bar{L}^l m_L^2 \bar{L} + \bar{U}_a \bar{Q} H_u + \ldots \quad (3)$$

and for generic squark masses, the requirement not to alter the experimental FCNC status sets a severe limit ($\sim 100$ TeV) to SUSY scale. One then requires that the New Physics flavour structures have the same SM flavor breaking, i.e. the Yukawas, to an effective hamiltonian proportional to eq. (2). This effective approach to flavour physics beyond the Standard Model is the so called minimal flavor violation (MFV) [2,3].
III. THE ULTRA-RARE DECAY $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The SM predicts the $V - A \otimes V - A$ effective Hamiltonian

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi \sin^2 \theta_W} (V_{cb}^* V_{td} X_{NL} + \lambda x_c) \lambda^2 A^2 (1 - \rho - i \eta) x_t$$

where

$$x_q = \frac{m_q^2}{M_W^2}, \quad \theta_W \quad \text{the Weak angle and } X \text{‘s are the Inami-Lin functions with Wilson coefficients known at two-loop electroweak corrections}.$$

$SU(2)$ isospin symmetry relates hadronic matrix elements for $K \rightarrow \pi \nu \bar{\nu}$ to $K \rightarrow \pi \nu \bar{\nu}$ to a very good precision while long distance contributions and QCD corrections are under control and the main uncertainties is due to the strong corrections to the charm loop contribution. The structure in (3) leads to a pure CP violating contribution to $K_L \rightarrow \pi^0 \nu \bar{\nu}$, induced only from the top loop contribution and thus proportional to $\Im m(\lambda_t)$ ($\lambda_t = V_{tb}^* V_{td}$) and free of hadronic uncertainties. This leads to the prediction

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.22 \pm 0.69 \pm 0.29) \times 10^{-11} \quad \mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = (2.43^{+0.40}_{-0.37} + 0.06) \times 10^{-11}$$

where the first is the parametric uncertainty due to the error on $|V_{cb}|$, $\rho$ and $\eta$, $f_K$, and the second error summarizes the theoretical uncertainties on non-perturbative physics and QCD higher order terms. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ receives CP conserving contributions proportional to $\Re m(\lambda_c)$, and to $\Re m(\lambda_t)$ and a CP violating one proportional to $\Im m(\lambda_t)$. E949 Collaboration $^8$ and E391a Collaboration $^9$ have then measured

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.73^{+0.15}_{-1.05}) \times 10^{-10} \quad \text{E949}$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 2.6 \times 10^{-8} \text{ at 90\% C.L. \ E391aCollaboration}$$

The direct upper bound for the neutral decay can be improved with a theoretical analysis: the isospin structure of any $\overline{\tau}d$ operator (bilinear in the quark fields) leads to the model independent relation among $A(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $A(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ $^{10}$; this leads to

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 4 \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$$

The upcoming KOTO experiment $^7, 11$ for $K_L \rightarrow \pi^0 \nu \bar{\nu}$, NA62 $^{12, 13}$ and possibly ORKA experiment at Fermilab $^{14}$ for $(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ encourage theoretical investigations of extensions of the SM: these experiments probe deeply to the MFV scale. More aggressive NP models can furnish substantial enhancements and be either discovered or ruled out $^6, 13$.

IV. $K_L \rightarrow \pi^0 e^+ e^-$, THE RELATED CHANNELS $K \rightarrow \pi \gamma \gamma$ AND $K_S \rightarrow \pi^0 e^+ e^-$

The electroweak short distance contribution to $K_L \rightarrow \pi^0 e^+ e^-$, analogously to the one $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is a direct CP violating one, however there is long distance contamination due to electromagnetic interactions: i) a CP conserving contribution due to two-photon exchange and ii) an indirect CP violating contribution mediated by one photon exchange, i.e. the contribution suppressed by $\epsilon$ in $K_L \sim K_2 + e K_1 \rightarrow \pi^0 e^+ e^-$ determined by the CP conserving $A(K_S \rightarrow \pi^0 e^+ e^-)$ $^{16, 18}$. The CP-conserving decays $K^\pm (K_S) \rightarrow \pi^\pm (\pi^0) \ell^+ \ell^-$ are dominated by the long-distance process $K \rightarrow \pi \gamma \rightarrow \pi \ell^+ \ell^-$ $^{17, 18}$. Our ignorance in the long distance dominated $g A(K_S \rightarrow \pi^0 \ell^+ \ell^-)$ can be parametrized by one parameter $a_S$ to be determined experimentally, NA48, finds respectively in the electron $^{19}$ and muon final state $^{20}$

$$|a_S|_{ee} = 1.06^{+0.26}_{-0.21} \pm 0.07 \quad |a_S|_{\mu \mu} = 1.54^{+0.40}_{-0.32} \pm 0.06 \quad (9)$$

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These results allow us to evaluate the CP violating branching

\[
B(K_L \rightarrow \pi^0 e^+ e^-)_{CPV} = \left[ 15.3 a_S^2 - 6.8 \frac{3 \lambda_t}{10^{-4}} a_S + 2.8 \left( \frac{3 \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12},
\]

(10)

The first term and last terms are respectively the indirect and the direct contribution, the second one is the interference, expected constructive allowing a stronger signal [16].

The important message is that experiments by studying the CP-violating branching 
\[
B(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10} \text{ at } 90\% \ CL.
\]

(11)

which also sets the interesting limit \(B(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}\) [22]. Still we have to show that we have under control the CP conserving contribution generated by two photon exchange. The general amplitude for \(K_L(p) \rightarrow \pi^0 \gamma(q_1) \gamma(q_2)\) can be written in terms of two Lorentz and gauge invariant amplitudes \(A(z, y)\) and \(B(z, y)\):

\[
A(K_L \rightarrow \pi^0 \gamma \gamma) = \frac{G_F \alpha}{4 \pi} \epsilon_1 \epsilon_2 \left[ A(z, y)(q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu \nu}) + \frac{2B(z, y)}{m_K^2}(p \cdot q_1 q_2^\mu p^\nu + p^\mu q_2 q_1^\nu - p \cdot q_1 q_2 g^{\mu \nu} - q_1 \cdot q_2 p^\mu p^\nu) \right],
\]

(12)

where \(y = p(q_1 - q_2)/m_K^2\) and \(z = (q_1 + q_2)^2/m_K^4\). Then the double differential rate is given by

\[
\frac{\partial^2 \Gamma}{\partial y \partial z} \sim \left[ z^2 |A|^2 + B^2 \right] + \left( y^2 - \frac{\lambda(z, y, \pi)}{4} \right) \left| B \right|^2,
\]

(13)

where \(\lambda(a, b, c)\) is the usual kinematical function and \(r_\pi = m_\pi/m_K\). Thus in the region of small \(z\) (collinear photons) the \(B\) amplitude is dominant and can be determined separately from the \(A\) amplitude. This feature is crucial in order to disentangle the CP-conserving contribution \(K_L \rightarrow \pi^0 e^+ e^-\). In fact the lepton pair produced by photons in \(S\)-wave, like an \(A(z)\)-amplitude, are suppressed by the lepton mass while the photons in \(B(z, y)\) are also in \(D\)-wave and so the resulting \(K_L \rightarrow \pi^0 e^+ e^-\) amplitude, \(A(K_L \rightarrow \pi^0 e^+ e^-)_{CPC}\), does not suffer from the electron mass suppression [16]. The important message is that experiments by studying the \(K_L \rightarrow \pi^0 \gamma \gamma\) \(z\)-spectrum have been able to limit \(B(K_L \rightarrow \pi^0 e^+ e^-) < 5 \cdot 10^{-13}\) at \(90\% \ CL\) [22, 24].

\[\text{FIG. 1: Unitarity contributions to } K \rightarrow \pi \gamma \gamma\]

\[\text{FIG. 2: } K^+ \rightarrow \pi^+ \gamma \gamma; \hat{c} = 0 \text{, full line, } \hat{c} = -2.3 \text{, dashed line, } [26]\]

Recently a related channel, \(K^+ \rightarrow \pi^+ \gamma \gamma\), has attracted attention: new measurements of this decay have been performed using minimum bias data sets collected during a 3-day special NA48/2 run in 2004 with 60 GeV \(K^\pm\) beams, and a 3-month NA62 run in 2007 with 74 GeV/c \(K^\pm\) beams [12].

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This channel starts at $O(p^4)$, with pion (and kaon) loops and a local term $\hat{c}$. Due to the presence of the pion pole, there is a new helicity amplitude, $C[25]$; the unitarity contributions at $O(p^6)$ in Fig.1 enhance the amplitude $A$ by 30%-40%, along with the generation of $B$-type amplitude [26]; the differential decay rate is

$$\frac{d^2\Gamma}{dydz} \sim z^2(|A+B|^2 + |C|^2) + \left(y^2 - \left(\frac{1 + r^2 - z^2}{4} - r^2\right)\right)^2 |B|^2$$

(14)

The constant $\hat{c}$ can be fixed by a precise determination of the rate and the spectrum as shown in Fig.2 [26]; this constant, combination of strong and weak counterterm, is predicted to have contributions from the axial spin-1 contributions.

$$\hat{c} = \frac{128\pi^2}{3} \left[3(L_9 + L_{10}) + N_{14} - N_{15} - 2N_{18}\right]^{FM} = 2.3 (1 - 2 k_f),$$

with $k_f$ is the factorization factor in the FM model or the weak axial vector coupling of Ref. [27]. BNL 787 got 31 events leading to $B(K^+ \rightarrow \pi^+\gamma\gamma) \sim (6 \pm 1.6) \cdot 10^{-7}$[28] and a value of $\hat{c} = 1.8 \pm 0.6$. Recently NA48 has presented preliminary results normalizing $K^+ \rightarrow \pi^+\gamma\gamma$ with the channel $K^+ \rightarrow \pi^+\pi^0$: $B(K^+ \rightarrow \pi^+\gamma\gamma) = (1.01 \pm 0.04 \pm 0.06) \cdot 10^{-6}$ and $\hat{c} = 2.00 \pm 0.24_{stat} \pm 0.09_{syst}$ [12].

V. CP ASYMMETRIES IN $K^+ \rightarrow 3\pi$-DECAYS

Direct CP violation in charged kaons is subject of extensive researches at NA48/2 [13]. Studying the $K \rightarrow 3\pi$ Dalitz distribution in $Y, X$ [29, 30]

$$|A(K \rightarrow 3\pi)|^2 \sim 1 + \lambda Y + j X + O(X^2, Y^2)$$

and determining both charged kaon slopes, $g_\pm$, we can define the slope charge asymmetry:

$$\Delta g/2g = (g_+ - g_-)/(g_+ + g_-).$$

(15)

There are two independent $I = 1$ isospin amplitudes $(a, b)$,

$$A(K^+ \rightarrow 3\pi) = ae^{i\alpha_0} + be^{i\beta_0} Y + O(Y^2, X^2)$$

(16)

with corresponding final state interaction phases, $\alpha_0$ and $\beta_0$. The hope is that $\Delta g$ in (15) does NOT need to be suppressed by a $\Delta I = 3/2$ transition. The strong phases, generated by the $2 \rightarrow 2$ rescattering, actually have their own kinematical dependence [31] and can be expressed in terms of the Weinberg scattering lengths, $a_0$ and $a_2$. It is particularly interesting to estimate the Standard Model (SM) size for $\Delta g/2g$, valid if there is a good chiral expansion for the CP conserving/violating $a, b$ amplitudes [29, 31, 32]:

$$\frac{\Delta g}{2g} \sim 22\epsilon'(\alpha_0 - \beta_0) \sim 10^{-5}.$$

The $K^+ \rightarrow \pi^+\pi^0\pi^0$ NA48/2 result [33] and New Physics (NP) scenarios [34]

$$\frac{\Delta g}{2g}^{NA48/2} = (1.8 \pm 2.6) \cdot 10^{-5} \leq 10^{-4}.$$

can then be compared to the SM.

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CP violation has been also studied in the $K \to \pi\pi\gamma$ and $K \to \pi\pi\text{ee}$ decays. We can decompose $K(p) \to \pi(p_1)\pi(p_2)\gamma(q)$ decays, according to gauge and Lorentz invariance, in electric ($E$) and magnetic ($M$) terms \[35\]. In the electric transitions one generally separates the bremsstrahlung amplitude $E_B$, predicted by the Low theorem in terms of the non-radiative amplitude and enhanced by the $1/E$ behavior. Summing over photon helicities: $d^2\Gamma/(dz_1dz_2) \sim |E(z_1)|^2 + |M(z_1)|^2$. At the lowest order, $(p^2)$, one obtains only $E_B$. Magnetic and electric direct emission amplitudes can be decomposed in a multipole expansion. In Table 2 we show the present experimental status of the DE amplitudes and the leading multipoles.

Table 2 $DE_{exp}$

| $K_S \to \pi^+\pi^-\gamma$ | $\frac{9}{10} \cdot 10^{-5}$ | $E1$ |
| $K_L \to \pi^+\pi^-\gamma$ | $(2.92 \pm 0.07) \cdot 10^{-5}$ | $M1, E1$ |

Particularly interesting are the recent interesting NA48/2 data regarding $K^+ \to \pi^+\pi^0\gamma$ decays \[36\]. Due to the $\Delta I = 3/2$ suppression of the bremsstrahlung, interference between $E_B$ and $E1$ and magnetic transitions can be measured. Defining $z_i = p_i \cdot q/m_K^2$ $z_3 = p_k \cdot q/m_K^2$ and $z_3z_+^2 = \frac{m_3^2}{m_K^2}W^2$ we can study the deviation from bremsstrahlung from the decay distribution

$$
\frac{\partial^2\Gamma}{\partial T^*_c \partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T^*_c \partial W^2} \left[ 1 + \frac{m_3^2}{m_K^2}2Re\left(\frac{E_{DE}}{eA}\right)W^2 + \frac{m_3^2}{m_K^2}\left(\left|\frac{E_{DE}}{eA}\right|^2 + \left|\frac{M_{DE}}{eA}\right|^2\right)W^4 \right],
$$

where $A = A(K^+ \to \pi^+\pi^0)$; we plot in Fig. 3 this experimental deviation from bremsstrahlung. The Dalitz plot distribution of the interference term is shown in Fig. 4. Study of the Dalitz plot has lead NA48 to these results \[36\].

Table 3

| $T^*_c \in [0.80] \text{ MeV}$ | $\text{Frac}(DE)$ | $\pm 3.3 \times 10^{-2}$ |
| $\text{Frac}(INT)$ | $\pm 2.35 \times 10^{-2}$ |

Also the interesting CP bound was obtained \[36\]:

$$
\frac{\Gamma(K^+ \to \pi^+\pi^0\gamma) - \Gamma(K^- \to \pi^-\pi^0\gamma)}{\Gamma(K^+ \to \pi^+\pi^0\gamma) + \Gamma(K^- \to \pi^-\pi^0\gamma)} < \frac{1}{15} \cdot 10^{-3} \text{ at } 90\% \text{ CL.} \tag{17}
$$
Presented at Flavor Physics and CP Violation (FPCP 2012), Hefei, China, May 21-25, 2012

\[
\begin{align*}
K^+ &\rightarrow \pi^+ \pi^0 e^+ e^- \\
&\text{decay with our kinematical conventions. The blob represents the hadronic tensor} \\
&H_\mu.
\end{align*}
\]

With more statistics the Dalitz plot analysis in Fig. 4 will be more efficient.

We have studied also the decay \(K^\pm \rightarrow \pi^\pm \pi^0 e^+ e^-\) in Fig. 5 [37]. Historically kaon four body semileptonic decays, \(K_{e4}\) have been studied as a tool to tackle final state rescattering effects in \(K \rightarrow \pi\pi\)-decays: crucial to this goal has been finding an appropriate set of kinematical variables which would allow i) to treat the system as two body decay in dipion mass \(M_{\pi\pi}\) and dilepton mass \(M_{\ell\ell}\) [38] and ii) to identify appropriate kinematical asymmetries to extract observables crucially dependent on final state interaction. In Fig. 6 we show the traditional kinematical variables

\[
\begin{align*}
&\text{FIG. 5: Photon-mediated} \\
&K^+ \rightarrow \pi^+ \pi^0 e^+ e^- \text{ decay with our kinematical conventions. The blob represents the hadronic tensor} \\
&H_\mu.
\end{align*}
\]

\[
\begin{align*}
&\text{FIG. 6:} \\
&K^+ \rightarrow \pi^+ \pi^0 e^+ e^- \text{ kinematical planes:} \\
&\text{N. Cabibbo and A. Maksymowicz definition of the angles} \\
&[38]
\end{align*}
\]

for the four body kaon semileptonic decay which allow to write the four body phase space \(\Phi\) in terms of the two two-body phase space \(\Phi_{\pi}\) \(\Phi_{\ell}\) from [38]

\[
d\Phi = \frac{1}{4m_K^4}(2\pi)^5 \int ds_\pi \int ds_\ell \lambda^{1/2}(m_K^2, p_\pi^2, q^2)\Phi_{\pi}\Phi_{\ell}.
\]

Then defining \(q^2 = M_{\ell\ell}^2\) and \(p_\pi^2\) the \(\pi\pi\) invariant mass we can write

\[
d^5\Phi = \frac{1}{2^{14}\pi^6 m_K^2 s_\pi} \sqrt{1 - \frac{4m_K^2}{q^2} \lambda^{1/2}(m_K^2, p_\pi^2, q^2)\lambda^{1/2}(p_\pi^2, m_{\pi^+}^2, m_{\pi^0}^2)} dp_\pi^2 dq^2 d\cos \theta_\pi d\cos \theta_\ell d\phi.
\]

\[
\text{FIG. 6:} \\
K^+ \rightarrow \pi^+ \pi^0 e^+ e^- \text{ kinematical planes:} \\
\text{N. Cabibbo and A. Maksymowicz definition of the angles} \\
[38]
\]

\[
\text{FIG. 7:} \\
q^2 \frac{dT}{dq^2} \\
\text{q dependence of the different contributions. The} \\
\text{solid line represents the Bremsstrahlung. The dashed lines} \\
\text{(from bigger dash to smaller dash) are 100×M, 100×BE and} \\
300×E, \text{ respectively.}
\]

\[
\text{FIG. 7:} \\
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300×E, \text{ respectively.}
\]

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Then the $K_{e4}$ amplitude is written as

$$\mathcal{M}_{4e} = \frac{G_F}{\sqrt{2}} V_{us} \left[ \bar{u}(p_e)\gamma^\mu(1 - \gamma^5)v(p_\mu) \right] H_\mu(p_1, p_2, q),$$

(20)

where $H_\mu$ is the hadronic vector, which can be written in terms of 3 form factors $F_{1,2,3}$:

$$H^\mu(p_1, p_2, q) = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta.$$

(21)

The goal was to obtain some asymmetry strongly dependent on the final state $\delta^e_j(s)$ in the form factors

$$F_j(s) = f_j(s) e^{i\delta^e_j(s)} + ..$$

Indeed

$$\frac{d\delta^e_i}{dE_\gamma dT_e dl^2 d\cos \theta_l d\phi} = A_1 + A_2 \sin^2 \theta_t + A_3 \sin^2 \theta_t \cos^2 \phi + A_4 \sin 2\theta_t \cos \phi + A_5 \sin \theta_t \cos \phi + A_6 \cos \theta_t + A_7 \sin \theta_t \sin \phi + A_8 \sin 2\theta_t \sin \phi + A_9 \sin^2 \theta_t \sin 2\phi,$$

(22)

where $\theta_t$ and $\phi$ are two variables for $K_{e4}$ decays. $A_8, A_9$ are dynamical functions that can be parameterized in terms of 3 form factor. $A_{8,9}$, odd in $\theta_t$ are also linearly dependent on the final state, allowing a clear way to determine them; while $A_{5,6,7}$ are generated by interference with the axial leptonic current.

One can easily show that the Bremsstrahlung, direct emission and electric interference terms contribute to $A_{1-4}$. In contrast, $A_{8,9}$ receive contributions from the electric-magnetic interference terms (BM and EM) and therefore capture long-distance induced P-violating terms. $A_{5,6,7}$ are also P-violating terms but generated through the interference of $Q_{7A}$ with long distances.

Essentially two groups applied the $K_{e4}$ decays to the decay $K_L \to \pi^+\pi^- e^+ e^-$, here the targets are mainly short distance physics, i.e. $A_{5,6,7}$ and the diplane angular asymmetry proportional to $A_{8,9}$. This last observable is large and has been measured by KTeV and NA48; however this observable is proportional to electric (bremsstrahlung) and magnetic interference, both contributions known already from $K_L \to \pi^+\pi^-\gamma$; in fact these known contributions are large and they may obscure smaller but more interesting short distance physics effects.

We have performed a similar analysis for the decay $K^+ \to \pi^+\pi^0 e^+ e^-$ trying to focus on i) short distance physics and ii) all possible Dalitz plot analyses to disentangle all possible interesting long and short distance effects.

This decay has not been observed yet, and the interesting physics is hidden by bremsstrahlung. This decay has not been observed yet, and the interesting physics is hidden by bremsstrahlung.

$$\mathcal{B}(K^+ \to \pi^+\pi^0 e^+ e^-)_B \sim (330 \pm 15) \cdot 10^{-8}$$

$$\mathcal{B}(K^+ \to \pi^+\pi^0 e^+ e^-)_M \sim (6.14 \pm 1.30) \cdot 10^{-8},$$

(23)

and so Dalitz plot analysis is necessary in order to capture the more interesting direct emission contributions. The $K^+ \to \pi^+\pi^0 e^+ e^-$-amplitude is written as

$$\mathcal{M}_{LD} = \frac{e}{q^2} \left[ \bar{u}(k_-)\gamma^\mu v(k_+) \right] H_\mu(p_1, p_2, q),$$

(24)

We may wonder also what it is the advantage to study this 4-body decay, $K^+ \to \pi^+\pi^0 e^+ e^-$, versus $K^+ \to \pi^+\pi^0\gamma$; in fact there are two reasons to investigate this channel, i) first trivially there are more short distance operators and also more long distance observables (for instance interfering electric and magnetic amplitudes) and ii) going to large dilepton invariant mass there is an extra tool compared to $K^+ \to \pi^+\pi^0\gamma$ to separate the bremsstrahlung component. For instance at large dilepton invariant mass the bremsstrahlung can be even 100 time smaller than the magnetic contribution. In our paper we give practically all the distributions in eq. (22), here as example we show in Figs. 8 and 9 the Dalitz plot distribution for the novel electric magnetic interference. This decay has been analyzed by NA48/2-NA62.
FIG. 8: Dalitz plot in the \((E^*_\gamma, T^*_c)\) plane at \(q^2 = (50 \text{ MeV})^2\) for the P-violating BM contribution

FIG. 9: Dalitz plot BM contribution: two-dimensional density projection

VII. CONCLUSIONS

We are looking forward to the upcoming \(K_L \rightarrow \pi^0 \nu \bar{\nu}\) KOTO \[11\] and \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) \[12\] NA62 experiments probing deeply the flavour structure of the SM and we hope ORKA will join this enterprise \[14\]. We have also shown that there are other decay modes like \(K_L \rightarrow \pi^0 e^+ e^-\), \(K^+ \rightarrow \pi^+ \gamma \gamma\) and \(K^+ \rightarrow \pi^+ \pi^0 e^+ e^-\) which are very useful, in particular these last two have been studied recently by NA62. I would like also to mention CPT tests in kaon decays \[12\] through Bell-Steinberger relations, recently updated in \[30\]; these leads to best CPT limit and an accurate determination of the CP violating parameter \(\epsilon\).

Acknowledgements

I thanks the organizers of the FPCP 2012 workshop for their kindness.

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