Universality classes of dense polymers and conformal sigma models

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In the usual statistical model of a dense polymer (a single space-filling loop on a lattice) in two dimensions the loop does not cross itself. We modify this by including intersections in which three lines can cross at the same point, with some statistical weight \( w \) per crossing. We show that our model describes a line of critical theories with continuously-varying exponents depending on \( w \), described by a conformally-invariant non-linear sigma model with varying coupling constant \( g^2 \). For the boundary critical behavior, or the model defined in a strip, we propose an exact formula for the \( \ell \)-leg exponents, \( h_\ell = g^2 \ell (\ell - 2)/8 \), which is shown numerically to hold very well.

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Loop models are ubiquitous in low dimensional statistical mechanics, and have been studied for decades [1]. They have recently grown to play a major role in topological quantum computing [3].

Most loop models studied so far have to forbid intersections to be solvable. Their critical exponents can then be calculated using techniques of conformal field theory [3], Coulomb gas, or stochastic Loewner evolution (SLE) [4, 5].

Self avoiding walks, whose long distance properties describe real polymers at interfaces, are the simplest of all loop models. In the ordinary, so-called dilute case, it is known that allowing intersections (hence obtaining self-avoiding trails) does not change the long-distance properties [6]. The dense case, where a single self avoiding loop on a lattice is forced to occupy a finite fraction of the sites (and thus resembles a real polymer in a melt), is different. Allowing intersections does take the model to another universality class [7], which, however, shares many features with ordinary Brownian motion, and does not seem to exhibit new families of critical exponents.

The first important result of this paper is that only allowing intersections where three lines cross simultaneously produces a very different behavior: a line of critical points is obtained, with central charge \( c = -2 \), and continuously varying critical exponents.

Our second important result concerns the nature of this critical line, which is very unusual in statistical mechanics.

A convenient way to describe many loop models is to use a supersymmetric (SUSY) formulation, in which the degrees of freedom can take bosonic or fermionic values [8], and the action is invariant under the action of a supergroup. The resulting field theories are however difficult to solve, in part due to the lack of unitarity, and of current algebra symmetry.

In the last few years, progress on one kind of such theories - \( \sigma \)-models on supergroups or supercosets - has been achieved in the framework of the AdS/CFT conjecture [8, 10]. An archetypal example is the principal chiral (PCM) model on \( PSL(2|2) \), which was found to be massless for a large range of values of the coupling constant \( g^2 \). This is very different from what happens in ordinary groups, such as \( SU(2) \), where the PCM exhibits asymptotic freedom and spontaneous mass generation. The presence of conformal invariance and (super)group symmetry is very interesting and potentially useful, yet, despite a lot of work, no complete solution of even the \( PSL(2|2) \) case, has been achieved [11].

We show in this paper that allowing crossings in dense polymers leads to close cousins of the \( PSL(n|n) \) models: \( \sigma \)-models on superprojective spaces (the super-analogs of ordinary projective spaces) \( U(n|n)/U(n-1|n) \times U(1) \). This identification has crucial consequences. It bridges the study of loop models with the one of \( \sigma \) models, it gives direct access to properties of the \( \sigma \)-models both numerically, and, potentially, analytically using the techniques developed in [12, 13]. It also allows the determination of the critical exponents in the original geometrical problem.

SUSY formalism for dense polymers. The universality class of dense polymers is generically obtained when one forces a finite number of self-avoiding loops or walks to fill up a fraction of space \( \rho > 0 \). When \( \rho = 1 \) for finite systems, one obtains Hamiltonian walks, the polymer limit of fully-packed loop models. For \( \rho = 1 \) the CFT is lattice dependent, i.e., universality breaks down [14]. Nevertheless, the loop model in Fig. 1 is in the generic dense polymer universality class.

The CFT of dense polymers has \( c = -2 \); we review it below. It was discovered a few years back [15] that if one allows four-leg crossings the model flows to a different universality class with \( c = -1 \), and trivial geometrical exponents. Such crossings imply that loops no longer conserve the lattice orientation of Fig. 1 indicating that a crucial symmetry is broken.

To identify this symmetry we need to get into a bit of algebra. We consider the lattice in Fig. 1 and a transfer matrix \( T \) propagating vertically. We introduce a supersymmetric (SUSY) formulation [15, 16]: each edge carries a \( \mathbb{Z}_2 \)-graded vector space of dimensions \( m + n \) (resp. \( n \)) for the even bosonic (resp. odd fermionic) subspace \( (m + n, n \geq 0) \) are integers). We label edges \( i = 0, 1, \ldots, 2L - 1 \) for a system of width \( 2L \). The \( \mathbb{Z}_2 \) space is chosen as the fundamental \( i \) of
the Lie superalgebra $\text{gl}(m+n|n)$ for $i$ even (down arrow), and its dual $\square$ for $i$ odd (up arrow). $T$ acts on the graded tensor product $\mathcal{H} = \square \otimes \square \otimes L$ (this “Hilbert” space has in fact an indefinite inner product).

To construct $T$ for critical dense polymers we first observe that, for generic $m$, the tensor products $\square \otimes \square$ and $\square \otimes \square$ decompose as the direct sum of the singlet and the adjoint. The projectors on the singlet obey the Temperley-Lieb algebra relations $E_i^2 = mE_i$, $[E_i, E_j] = 0$ for $|i - j| > 2$, and $E_i E_{i+1} E_i = E_i$ (and here $m = 0$). The $E_i$ can be expressed as quadratic terms in the SUSY generators, and are closely related with the Casimir [18]. We have $T = T_1 T_2 \cdots T_2L-3T_0 T_2 \cdots T_{2L-2}$, where $T_1 = 1 + xE_i$. By taking either of the two terms in $T_i$ for each vertex, the expansion of Fig. 1 is obtained, with a power of $x$ for each vertex, and a factor $(n + m) - n = \text{str} 1 = m$ for each loop. The latter equals the supertrace in the fundamental representation (denoted str) of 1, since states in $\mathcal{H}$ flow around the loop. This holds whether the loop be topologically nontrivial or homotopic to a point. Isotropic dense polymers now correspond to $m = 0$ and $x = 1$. Note that when $m = 0$, the tensor product $\square \otimes \square$ is indecomposable; $E_i$ can then be defined as the unique invariant coupling (on two sites) other than the identity. The rest of the discussion is unchanged.

Letting $x \to 0$ allows one to extract the spin chain hamiltonian $H \propto -\sum_i E_i$, acting on $\mathcal{H}$; the scale of $H$ is chosen to ensure conformal invariance. The interaction is simply the invariant quadratic coupling (Casimir), providing a natural generalization of the Heisenberg chain to the $\text{gl}(n + m|n)$ case.

For such models, there is a corresponding continuum quantum field theory [14], which is a nonlinear $\sigma$-model with target space the symmetry supergroup [here $U(n + m|n)$], modulo the isotropy supergroup of the highest weight state (see [17, 18] for related non-SUSY examples, and [15, 19] for SUSY random fermion problems). Here we obtain $U(n + m|n)/U(1) \times U(n + m - 1|n) \cong \mathbb{C}P^{n+m-1|n}$, a SUSY version of complex projective space. Moreover, the mapping shows that this model has a topological angle $\theta = \pi$.

**Dense polymers and sigma models.** Let us now make things concrete: the fields can be represented by complex components $z^a = (a = 1, \ldots, n + m)$ and $\zeta^a = (a = 1, \ldots, n)$, where $z^a$ is commuting, $\zeta^a$ is anticommuting. In these coordinates, at each point in spacetime, the solutions to the constraint $z_i^a z^a_i + \zeta_i^a \zeta^a_i = 1$ (we use the conjugation $\dagger$ that obeys $(\eta | \xi) = \xi | \eta \dagger$ for any $\eta, \xi$, modulo $U(1)$) phase transformations $z^a \to e^{iB} z^a$, $\zeta^a \to e^{iB} \zeta^a$, parametrize $\mathbb{C}P^{n+m-1|n}$. The Lagrangian density in 2D Euclidean spacetime is

$$L = \frac{1}{2g_\sigma^2} \left[ (\partial_\mu - ia_\mu) z^a_\mu (\partial_\mu + ia_\mu) z^a - \frac{\theta}{2\pi} (\partial_\mu a_\nu - \partial_\nu a_\mu) \right] + \frac{\theta}{4\pi} (\partial_\mu a_\nu - \partial_\nu a_\mu),$$

where $a_\mu = \frac{1}{2}[z^a_\mu \partial_\nu z^a - \zeta^a_\mu \partial_\nu \zeta^a - (\partial_\mu z^a_i) z^a - (\partial_\mu \zeta^a_i) \zeta^a]$ for $\mu = 1, 2$. The fields are subject to the constraint, and under the $U(1)$ gauge invariance $a_\mu$ transforms as a gauge potential; a gauge must be fixed in any calculation. This set-up is similar to the non-SUSY $\mathbb{C}P^{n-1}$ model in [20, 21]. The coupling constants are $g_\sigma^2$, the usual $\sigma$-model coupling (there is only one such coupling, because the target supermanifold is a supersymmetric space, and hence the metric on the target space is unique up to a constant factor), and $\theta$, the coefficient of the topological term ($\theta$ is defined modulo $2\pi$).

First we note a well-known important point about the SUSY models: the physics is the same for all $n$, in the following sense. For example, in the present model, correlation functions of operators that are local functions (possibly including derivatives) of components $a \leq n_1 + m$, $\alpha \leq n_1$ for some $n_1$ are equal for any $n \geq n_1$, due to cancellation of the “unused” even and odd index values. This can be seen in perturbation theory because the unused index values appear only in summations over closed loops, and their contributions cancel, but is also true nonperturbatively (it can be shown in the lattice constructions we discuss below). In particular, the renormalization group (RG) flow of the coupling $g_\sigma^2$ is the same as for $n = 0$, a non-SUSY $\sigma$-model. For the case of $\mathbb{C}P^{n+m-1|n}$, the perturbative $\beta$-function is the same as for $\mathbb{C}P^{n-1}$, namely (we will not be precise about the normalization of $g_\sigma^2$)

$$\frac{dg_\sigma^2}{d\xi} = \beta(g_\sigma^2) = mg_\sigma^4 + O(g_\sigma^6),$$

where $\xi = \log L$, with $L$ the length scale at which the coupling is defined [see e.g. [22], eq. (3.4)]. (The $\beta$-function for $\theta$ is zero in perturbation theory, and that for $g_\sigma^2$ independent of $\theta$. For $m > 0$, if the coupling is weak at short length scales, then it flows to larger values at larger length scales. For $\theta \neq \pi \pmod{2\pi}$, the coupling becomes large, the $U(n + m|n)$ symmetry is restored, and the theory is massive. However, a transition is expected at $\theta = \pi \pmod{2\pi}$ for $m > 2$. This transition is believed to be first order, while it is second order for $m \leq 2$ [17]. In the latter case, the system with $\theta = \pi$ flows to a conformally-invariant fixed-point theory. At the fixed point, a change in $\theta$ is a relevant perturbation that makes the theory massive.

For $m = 0$, the perturbative $\beta$-function vanishes identically. This can be seen either from direct calculations, which
have been done to at least four-loop order \cite{22}, or from an argument similar to that in \cite{9}: for \( n = 1 \), the \( \sigma \)-model reduces to the massless free fermion theory \cite{23} \( \mathcal{L} \propto \frac{1}{2 g^2} \partial_\mu \zeta \partial_\nu \zeta \) and further the \( \theta \)-term becomes trivial in this case. Thus, for all \( \sigma \)-model couplings \( g^2 > 0 \), the \( n = 1 \) theory is non-interacting. The free-fermion theory is conformal with \( c = -2 \), and \( \theta \) is a redundant perturbation, as it does not appear in the action (a similar argument appeared in Ref. \cite{24}). By the above argument, conformal invariance with \( c = -2 \) should hold for all \( n \), and also for all \( g^2 \) and \( \theta \), though the action is no longer non-interacting in general. Thus the \( \beta \)-function also vanishes non-perturbatively. In general, the scaling dimensions will vary with the coupling \( g^2 \), so changing \( g^2 \) is an exactly marginal perturbation, though for \( n = 1 \) the coupling can be scaled away, so there is no dependence on the coupling in the exponents related to those multiplets of operators that survive at \( n = 1 \). Hence for \( n = 1 \), the exactly-marginal perturbation that changes \( g^2 \) is redundant.

**Introducing the six-leg crossings.** For \( n = 1 \), the \( \sigma \)-model thus does not exhibit very interesting physics. It also describes very few observables in the dense polymer problem. Indeed, the underlying algebra \( psl(1|1) \) does not admit any non-trivial invariant tensor, so the only \( \ell \)-leg operators present have \( \ell = 0, 2 \), and they are moreover degenerate—and part of an indecomposable block. These observables are expected to be present in all theories with \( n > 1 \) as well, and to not depend on the coupling constant \( g^2 \). However, for \( n > 1 \), more observables are possible. E.g., \( \ell \)-leg operators for all even \( \ell \) exist, and correspond to fully symmetric invariant tensors of \( psl(n|n) \); there is no reason why the corresponding conformal dimensions should not depend on \( g^2 \), and indeed we will shortly see that they do.

For this, we need to be able to tune \( g^2 \) in the lattice model. We propose doing so by allowing not four-leg but six-leg crossings. This can be described most conveniently by going to the hamiltonian formalism, and adding interactions that preserve the symmetry. Four-leg crossings would then translate into a perturbation of the type \( \Pi_{i,i+1} \) which exchanges spaces at position \( i \) and \( i + 1 \). Since by construction our chain has alternating representations this is not possible within \( gl(n + m|n) \) symmetry, so forcing such crossings breaks the symmetry down to the orthosymplectic subgroup. On the other hand, six-leg crossings correspond to exchanging representations at position \( i \), \( i + 2 \) while the one at \( i + 1 \) just goes through, and is perfectly compatible with the \( gl(n + m|n) \) symmetry (notice however that it breaks the extended symmetry discussed in \cite{12}). The hamiltonian then becomes

\[
H \propto - \sum_i (E_i + w P_{i,i+2}).
\]  

Our first claim is that the continuum limit of \cite{3} is described by the superprojective \( \sigma \)-model \( CP^{n-1|n} \) with coupling \( g^2(w) \), at \( \theta = \pi \). Note that we could more generally study the spectrum of the hamiltonian \( H \propto - \sum_i (E_i + w P_{i,i+2} + w_2 (E_i E_{i+1} + E_{i+1} E_i)) \). The symmetries are unchanged, and one expects the continuum limit to be described by the same \( \sigma \)-model, with now \( g^2(w, w_2) \). This is confirmed by numerical calculations. Finally, a more pleasant realization of the same physics is provided by a model of dense polymers on the triangular lattice, where six-leg crossings can naturally take place; see Fig. 2. We shall call the Boltzmann weight of these vertices \( w \) as well, and the same conclusions will hold for this model as for the spin chain \( 3 \).

We first check what happens for \( n = 1 \), where everything can be reformulated in terms of free fermion operators and their adjoints \( f_i, f^\dagger_i \), obeying \( \{ f_i, f_j \} = 0, \{ f_i, f^\dagger_j \} = \delta_{ij} \); through \( E_i = (f_i + f^\dagger_{i+1}) (f_i + f_{i+1}) \) and \( P_{i,i+2} = - (f_{i+1} - f_{i+2}) (f_{i+1} - f_{i+2}) \). Since both are quadratic it is easy to show that the continuum limit of \( 3 \) is unchanged, with \( w \) only affecting the sound velocity and the fine structure of the Jordan blocks.

One can easily argue that the ground state energy is the same for the \( n = 1 \) and \( n > 1 \) models, whence \( c = -2 \) independently of \( w \). This is confirmed by transfer matrix calculations for the model in Fig. 2. The \( \ell = 2 \) exponent is conjugate to the fractal dimension of the loop, hence zero.

Numerical study of the \( \ell > 2 \) leg exponents then clearly shows that they are non-trivial, decreasing functions of \( w \). To discuss this some more we place ourselves in the simplest case of free boundary conditions. The exponents at the special point \( w = 0 \) are well known to be \( h_{0}^\ell = h_{1,1+\ell} = \frac{6(6-2\ell)}{12} \). We next assume that \( w \to \infty \) corresponds to the weak-coupling limit of the \( \sigma \)-model, \( g^2 \to 0 \). This is qualitatively very reasonable: in the limit of large \( w \), the system almost splits into two subsystems with \( gl(n|n) \) symmetry involving only the fundamental or only its dual, with in both cases a simple interaction of the type \( \Pi_{i,i+1} \). Such models are well-known to be integrable, and their physics to be described by a weak-coupling limit not unlike the XXX ferromagnetic spin chain. In such a limit, we can analyze the spectrum using the minusperspace approach, that is, by analyzing quantum mechanics on the target manifold. The spectrum of the Laplacian on the ordinary projective space \( CP^{n-1} = U(m)/U(1) \times U(m-1) \) is well-known to be of the form \( E_l \propto 4(l + m - 1) \), so, setting \( m = 0 \), we find that \( 26) \) \( h_{0}^{\infty} = g^2 \frac{1}{2} (l-1) \). Here \( l \) an integer, which we can identify using \( psl(n|n) \) representation theory with \( \ell/2 \). Remarkably, \( h_{0}^{\infty} \) coincides with the known result \( h_{0}^\ell \) at \( w = 0 \) (ordinary dense polymers) if we identify \( g^2 = 1 \) in that case.

**Conjecture for the exact exponents.** We conjecture that the boundary conformal dimensions in our model are simply linear in the Casimir of the associated representation of

![FIG. 2: Vertices and weights for dense polymers on the triangular lattice. When \( w = 0 \) this is equivalent \cite{25} to a Potts model with spins on the circles and arbitrary interactions within the gray triangles.](image-url)
psl(n|n). This is due to the structure of the perturbation theory where the vanishing of the dual Coxeter number—the Casimir in the adjoint—suggests exactness of the minisuperspace approximation (see [27] for a related case). A more thorough study of this perturbation theory, together with non-perturbative arguments, will appear elsewhere [28]. For now, we simply propose that the exponents be given by

$$h_{\ell} = g_{\sigma}^2 \frac{\ell(\ell - 2)}{8},$$

where $g_{\sigma}^2$ is a decreasing function of $w$, equal to unity when $w = 0$, and vanishing at large $w$.

This conjecture is compared with the results of exact diagonalizations in Fig. 3 (lower panels), where we have represented the function $g_{\sigma}^2(w)$ as extracted from [4] and various $\ell$. The different estimates collapse on a single curve over the whole range of $w$ values, in agreement with the conjecture.

For the model of Fig. 4 it is technically difficult to study operators with $\ell$ even. The $\sigma$-model formality can however be extended to $\ell$ odd, and the arguments leading to (4) extended to this case [28]. Exact diagonalization of the spin chain hamiltonian on $2L = 18$ sites yields results for $\ell$ even which look like the lower left panel of Fig. 3 except that $w$ now has a different meaning. The sound velocity is determined from analytical results for the $n = 1$ case.

While the $\ell$-leg exponents for the usual dense polymers ($w = 0$) agree with the general conjecture for $g_{\sigma}^2 = 1$, the fine structure of the spectrum at that point [13] differs from the one of the sigma model. The situation seems similar to the one encountered in [29] for the supersphere sigma model, where the point $w = 0$ is in fact singular.

A related question concerns periodic boundary conditions. In this case, the known values of the bulk polymer exponents at $w = 0$ are $h_{\ell} = \frac{\ell^2 - 1}{2\ell}$ and $h_{\ell} = \frac{\ell^2 - 1}{4\ell}$. The fact that $h_{\ell} = 1$ provides an independent argument for the marginality of the $w$ perturbation. Meanwhile, note that $h_{\ell}$ does not have the minisuperspace form. For large $w$, one can however argue that the minisuperspace form remains valid, as is confirmed numerically. This suggests again that the point $w = 0$ is singular. It could also be that in the periodic case, the arguments that the minisuperspace should be exact for any $w$ fail, which agrees with the expectations for a related model in [27]. More work is needed to clarify this point.

We checked that, within numerical accuracy, staggering the chain produces similar results but with a coupling constant that now depends on $w$ and the staggering parameter—that is, the $\theta$ angle in the continuum limit. We e also studied the effect of coupling additional $\Box$ or $\bar{\Box}$ representations on the boundary, which can be interpreted in terms of boundary $\theta$ angle [28]. All the results are compatible with the $\sigma$-model picture.

In conclusion, we have shown that allowing intersections where three lines cross profoundly modifies the dense polymer problem. It gives rise to a critical line of conformal field theories, with central charge $c = -2$, which can be identified with the long distance limit of a conformal sigma model such as those studied in the AdS/CFT correspondence. Our identification leads moreover to the proposal of an exact formula [4] for the $\ell$-leg polymer exponents in the boundary case, and opens the way to tackling the sigma model using lattice techniques.

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FIG. 3: The two upper panels show the central charge as a function of the intersection weight $w$ for width $L$ strips of the triangular lattice. In the text we show analytically that $c = -2$ always. The lower left panel represents the effective coupling constant $g_{\sigma}^2$ extracted from (4) using different values of $\ell$. The collapse on a single curve is quite striking. The lower right panel shows details of the exponents, in particular the region close to $w = 0$ where convergence appears actually less good than on the previous curves.

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