Axial and Vector correlator mixing
in hot and dense hadronic matter

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Abstract

We study the manifestations of chiral symmetry restoration which have a significance for the parity mixing. Restricting to pions and nucleons we establish a formalism for the expression of the vector correlator, which displays the mixing of the axial correlator into the vector one and unifies the cases of the heat bath and of the dense medium. We give examples of mixing cross-sections. We also establish a link between the energy integrated mixing cross-sections and the pion scalar density which governs the quenching factors of coupling constants, such as the pion decay one, as well as the quark condensate evolution.

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1 Introduction

The consequences of chiral symmetry restoration in a dense or hot medium is one of the fascinating problems raised in the sector of non perturbative QCD physics. At normal nuclear density for instance the amount of restoration is large (about 35%) \footnote{Also at Theory Division, CERN, CH-1211 Geneva 23, Switzerland}. It is hardly conceivable that such a large effect does not show up in a way directly linked to the symmetry. These manifestations have been searched for either in the direction of mass changes or in parity related effects. For what concerns the second aspect, in the heat bath, Dey \textit{et al.} \footnote{See also Ref. \cite{ref3}} showed that
the thermal pions induce a mixing of axial and vector correlators accompanied by a depletion of the original correlator. At low temperature, this depletion is universal and equal to the mixing term. For a dense medium, Chanfray et al. [4] established the existence of a certain analogy with the thermal case. One can consider that in a certain sense chiral symmetry restoration induces a mixing of a response of axial nature into the vector one, and vice versa, accompanied by a dropping of coupling constants such as that of the axial current. However the pions responsible for this mixing are those of the virtual nuclear pion cloud. The fact that they belong to the medium itself and not to an external reservoir introduces a major difference with the heat bath situation. In Ref. [4] the quenching factor was expressed in terms of the scalar pion density, which also partly governs chiral symmetry restoration. In this letter we establish a formalism which treats on the same footing the heat bath and the dense case so as to reach a unified description when both effects are present, such as in heavy ions collisions. We illustrate some situations where the mixing pieces of the vector current act and we derive the mixing cross-sections that they generate. We will show through the study of the Compton amplitude that, in the case of the nuclear medium, it is possible to extract the quenching factor of the coupling constants from photoabsorption data. The importance of Compton scattering as a tool in relation to chiral symmetry was pointed out in Refs. [5, 6]. The existence of mixing cross-sections is linked to chiral symmetry restoration and can be viewed as one of its manifestations. Our conclusion is that such manifestations are numerous. It is only question to see familiar experimental data in a proper perspective.

2 Axial-Vector mixing in the nucleon and the nucleus

We start by rewriting the expression of the vector current as derived in Ref. [4] from a chiral Lagrangian in the Weinberg representation and in a world restricted to pions and nucleons:

$$\mathcal{V}_\mu = \frac{(\phi \times \partial_\mu \phi)}{(1 + \phi^2/4f^2_\pi)^2} + \frac{1}{2} \bar{\psi} \gamma_\mu \tau \psi + \frac{1}{4f^2_\pi} \frac{\bar{\psi} \gamma_{\mu}[(\tau \times \phi) \times \phi]\psi}{1 + \phi^2/4f^2_\pi} - \frac{g_A}{2f_\pi} \frac{\bar{\psi} \gamma_\mu \gamma_5 (\tau \times \phi) \psi}{1 + \phi^2/4f^2_\pi}. \tag{1}$$

We remind which terms of expression (1) can be viewed as mixing terms. They are those which lead to a part of the axial current once a pion field in their expression is taken care of by creation or annihilation of a thermal pion or a nuclear virtual one. An example is the Kroll-Ruderman term (last term of Exp. (1)) which leads (within a normalization factor $1/f_\pi$) to the nucleonic axial current. Another one is the photon coupling to the pion (term in $\phi \times \partial_\mu \phi$) which leads to the creation
or annihilation of a pion by the axial current, once the pion field in it (i.e. the one without derivative) is removed. If this pion attaches to a nucleon then these two pieces together give the full axial current, with its induced pseudoscalar piece. It is important to stress that our definition of mixing of the currents enlarges the notion of parity mixing. *Stricto sensu* this is the mixing with states of opposite parity, as occurs for instance in the heat bath at low temperature, where the pions are soft. For the mixing of the currents instead, the pions need not be soft, and indeed the nuclear pions are not (their momentum is a few hundred MeV/c).

We start by studying the propagation of photons in the nuclear medium. The case of the free proton will be discussed as well. It leads to an interesting possibility of an access to the pionic piece of the nucleon sigma commutator through photoabsorption data. In the vacuum the photon self-energy contains the two-pions intermediate states (Fig. 1). In the nuclear medium there is a direct excitation of particle-hole (p-h) states by the current (graph 2c). In addition the pions are modified by the dressing by p-h states. It is also necessary to attach the photon to the vertices in order to satisfy gauge invariance. In the one-nucleon loop approximation the corresponding graphs are those of Fig. 2d-f. The graphs 2c-f which can be cut and have an imaginary part represent the time ordered amplitude $T(\omega)$. Besides these terms the photon self-energy also contains seagull pieces: those where the two photons attach in a point-like fashion either to the nucleons or to the pions (graphs 2a-b). The piece 2a is given by the corresponding Thomson amplitude, $-e^2/M$ per proton with $e^2 = 1/137$ and $M$ the proton mass. For the pions, which are virtual in the nucleus, the seagull amplitude can be expressed [6, 7], to lowest order in the pion fields, as the expectation value of the squared pion field operator for charged pions: $-\frac{2}{3}e^2\int d\vec{x} \langle A|\phi(\vec{x})^2|A \rangle$, which is linked to the pion scalar density. Here and in the following $|A\rangle$ stands for a nuclear state and $|N\rangle$ will be used for a proton one. For a single proton the corresponding graphs for the forward photon-proton amplitude are shown in Fig. 3. We start by discussing this case. Summing the graphs of Fig. 3 we decompose the $\gamma N$ forward spin independent amplitude as:

$$f_N(\omega) = -e^2/M + S_\pi + T(\omega).$$

In the low energy limit, this amplitude should take the Thomson value, $-e^2/M$. Thus the dressing of the nucleon by the pion does not affect the low energy value. This constraint imposes the following relation between the time ordered amplitude and the seagull term: $S_\pi + T(0) = 0$. In the absence of form factors this relation is indeed satisfied by the calculated amplitudes from the graphs 3b-f, which is no surprise because the introduction of these graphs is necessary to satisfy gauge invariance, i.e., the low energy theorem. For instance in the static approximation, we have:
\[ S_\pi = -e^2 \int dx \langle N | [\phi(x)^2 - \phi_3(x)^2] |N \rangle = -\frac{2}{3} e^2 \int dx \langle N | \phi(x)^2 |N \rangle \]

\[ T(0) = \frac{e^2 g^2}{4\pi^2 M^2} \int_0^\infty dq \frac{q^4}{(q^2 + m_{\pi}^2)^2} \]

where the three terms in the last integrand correspond to the three graphs 3d-f and \( g \) is the \( \pi NN \) coupling constant. The positive energy nucleon Born term (graph 3c) does not contribute at \( \omega = 0 \). In the sum \( S_\pi + T(0) \) the divergent pieces cancel and the remaining integrals give an overall vanishing result.

More generally, writing a once subtracted dispersion relation for the proton Compton amplitude, we get:

\[ R e f_N(\omega) = R e f_N(0) + \frac{2\omega^2}{\pi} \int_0^\infty d\omega' \frac{I m f_N(\omega')}{\omega' (\omega'^2 - \omega^2)} = -\frac{e^2}{M} + \frac{\omega^2}{2\pi^2} \int_0^\infty d\omega' \frac{\sigma_\gamma(\omega')}{\omega'^2 - \omega^2}, \]

(4)

where \( \sigma_\gamma \) is the photoabsorption cross-section which corresponds to the graphs 3d-f. It arises from the Kroll-Ruderman and photoelectric terms which originate from the mixing pieces of the vector current. The corresponding cross-section thus represents the mixing of the axial correlator into the vector one. In the high energy limit, \( \omega \to \infty \) (defined as high compared to the excitation energies for the relevant degrees of freedom, see Ref. [6] for a detailed discussion), \( f_N \) reduces to its seagull parts:

\[ f_N(\infty) = -\frac{e^2}{M} - \frac{2}{3} \frac{e^2}{M} \int d\bar{x} \langle N | \phi(\bar{x})^2 |N \rangle . \]

(5)

Inserting (5) into (4) we obtain the relation:

\[ \int d\bar{x} \langle N | \phi(\bar{x})^2 |N \rangle = \frac{3}{2e^2} \frac{1}{2\pi^2} \int_0^\infty d\omega' \sigma_\gamma(\omega') . \]

(6)

It is then possible to evaluate the expectation value of the squared pion field from the part of the photoabsorption cross-section arising from the Kroll-Ruderman and pionic Born terms. The direct excitation of the \( \Delta \) resonance is not included in this description. It has to be subtracted from the cross-section. However the \( \Delta \) has to be included as an intermediate state in the graphs of Fig. 3 (except 3c).

Indeed it is known that a proper description of the pion cloud needs the \( \Delta \)-pion intermediate state. This is no difficulty. The seagull (pionic) term remains formally the same: \(-\frac{2}{3} e^2 \int d\bar{x} \langle N | \phi(\bar{x})^2 |N \rangle \). As for the cross-section it has to include the pion photoproduction accompanied by \( \Delta \) excitation of the nucleon. This cross-section has been measured [8] and we use it for our experimental input. It is strongly suppressed
above 1-1.2 GeV, where other channels, not incorporated in our description, open. This provides the scale at which a description in terms of one pion, eventually accompanied by $\Delta$ excitation ceases to be valid. We use the same cut off for the non-$\Delta$ part. We thus evaluate a total pion number (defined as the volume integral of the scalar pion density) $N_\pi = m_\pi \int d\vec{x} \langle N|\phi(\vec{x})^2|N\rangle$ of about 0.35. Another idea of this magnitude is provided by transforming this number into the pionic piece of the nucleon sigma commutator, using the relation: $\Sigma_N^{(\pi)} = \frac{1}{2} m_\pi^2 \int d\vec{x} \langle N|\phi(\vec{x})^2|N\rangle \approx 25$ MeV (out of this, 9 MeV come from the $\Delta$). This value well agrees with theoretical estimates [4, 14], showing the consistency of our overall concept of the meaning of mixing cross sections. As the sigma commutator provides a measure of the amount of chiral symmetry restoration brought in by nucleon, according to the relation:

$$\Sigma_N = 2m_q \int d\vec{x} \langle [N|\bar{q}q(\vec{x})|N\rangle - \langle 0|\bar{q}q(\vec{x})|0\rangle \rangle,$$  

the equation (3) amounts to a link between the amount of chiral symmetry restoration of pionic origin and the mixing cross-sections.

We now turn to the nuclear case. In the one-nucleon loop approximation the graphs which enter in the photon self-energy are those of Fig. [2]. Those which correspond to mixing terms of the current are those of Fig. [2]-f where the photon attaches to the $NN\pi$ vertex or to a pion in flight. Their contribution can be concisely introduced in the vector correlator, as explained below. We restrict ourselves to the pure space components, the other one can be obtained through current conservation. Let us first consider the axial correlator $A_{ij}(i,j = 1,2,3)$ as represented in Fig. [4]. Since our illustration of the mixing is made on that of the axial correlator into the vector one, we have ignored the mixing terms of the axial current. The axial current then couples only to the nucleon and the pion. In the non relativistic limit and for a spin saturated system the correlator $A_{ij}$ thus truncated writes:

$$\frac{1}{f_\pi^2} A_{ij}(k) = k_i k_j D(k) + 2k_i k_j \Pi_0(k)D(k) + \hat{k}_i \hat{k}_j \Pi_L(k) + (\delta_{ij} - \hat{k}_i \hat{k}_j) \Pi_T(k)$$

$$= k_i k_j (1 + \Pi_0(k))^2 D(k) + \hat{k}_i \hat{k}_j \Pi_0(k) + (\delta_{ij} - \hat{k}_i \hat{k}_j) \Pi_T(k).$$

(8)

As for the vector correlator we skip the third term in the expression (11) of the vector current which is a mixing term but would lead to a part of the axial current that we have ignored (i.e. the Weinberg-Tomozawa term). We also neglect the pion fields in the denominators. The correlator then follows from the graph of Fig. [2]-f and writes (see also Ref. [11]):

$$V_{ij}(q) = \Pi V_{ij}(q) + i \int \frac{d^4k_1}{(2\pi)^4} [(1 + \Pi_0(k_1))k_{1i} - (1 + \Pi_0(k_2))k_{2i}] \times D(k_1)D(k_2)[(1 + \Pi_0(k_1))k_{1j} - (1 + \Pi_0(k_2))k_{2j}]$$

$$+ i \int \frac{d^4k_1}{(2\pi)^4} [\hat{k}_{1i} \hat{k}_{1j} \Pi_0(k_1)D(k_2) + (\delta_{ij} - \hat{k}_{1i} \hat{k}_{1j})\Pi_T(k_1)D(k_2)]$$

$$+ \hat{k}_{2i} \hat{k}_{2j} \Pi_0(k_2)D(k_1) + (\delta_{ij} - \hat{k}_{2i} \hat{k}_{2j})\Pi_T(k_2)D(k_1)],$$

(9)
where \( k_1 \) and \( k_2 \) are the two pions four-momenta, (their sum is the incident momentum \( q = k_1 + k_2 \)) and \( \Pi_{ij} \) is the RPA response to (longitudinal) vector excitations (Fig. 2c). In the two expressions (8,9) \( D_0 \) is the free pion propagator, \( D \) is the fully dressed one with p-h insertions and \( \Pi_L \) and \( \Pi_T \) are the RPA responses to the isospin and spin excitations, respectively for the spin longitudinal and spin transverse ones. The index 0 for the quantities \( \Pi \) corresponds to the irreducible pion self-energy (including effects of short range correlations which can be parameterized by the usual \( g' \) parameter) and we have: \( D = D_0 + k^2 D_0 \Pi_0 D \). With these expressions for the correlators, we do not restrict to the one-nucleon loop approximation but the full RPA chains are included. This means that in the graphs 2 and 4, the pion is considered as a quasi-particle with a full dressing of p-h insertions. In the vector correlator, we separate out the part with only one nucleon-hole propagator (Fig. 2c) and its RPA chain which has nothing to do with mixing. The remainder can be expressed in terms of the axial correlator, in a form which displays the mixing effect:

\[
V_{ij}(q) - \Pi_{ij}(q) = i \int \frac{d^4k_1}{(2\pi)^4} \left[ \frac{1}{f_\pi} A_{ij}(k_1)D(k_2) + A_{ij}(k_2)D(k_1) \right] - (1 + \Pi_0(k_1))(1 + \Pi_0(k_2))(k_{1i}k_{2j} + k_{2i}k_{1j})D(k_1)D(k_2) \quad . \tag{10}
\]

Indeed, if one of the pions 1 or 2 is a thermal one, or a pion from the nuclear pion cloud (in this case the pion propagator \( D \) has to be dressed by at least one p-h insertion), one is left with the axial correlator, taken at the momentum of the other pion. On the right hand side of Eq. (10) there is an extra term which does not reduce to the product of the axial correlator with the pion propagator. Its existence is due to the interaction of the photon with the pion (term in \( \phi \times \partial_\mu \phi \)). The derivative can act on either of the two pions. The action on the thermal or the nuclear one leads to this extra piece. Indeed, as well known, in the photoelectric part of pion production on a nucleon, the momentum dependence is not that of the exchanged pion \( k \), but it involves the sum of the two outgoing and exchanged pions, \( q = 2k - q \), while the pseudoscalar piece of the axial current contains only the exchanged pion momentum \( k \). However the extra term of the vector correlator, which bars the simple factorization, does not alter the basic mixing concept. Indeed, when one pion is taken as a thermal or nuclear one, the remainder is still of axial nature. In the large incident momentum limit, when \( q \) is larger than the momentum distribution of the thermal or nuclear pion, the other pion practically carries the external momentum which is larger and the terms in \( k_{1i}k_{2j} \) can be ignored. In this case the factorization holds.

The expressions (8-10) unify the dense and thermal cases. In the latter one, the polarization propagators \( \Pi \) have to be ignored and the integration over \( k_1 \) has to be understood as a sum over Matsubara frequencies. The formalism applies equally well in the mixed case where both density and temperature effects are present, such as in heavy ion collisions.
Coming back to the nuclear case the spin independent Compton amplitude \( f_A \) \((e \cdot e' f_A = -\epsilon_i \epsilon'_j V_{ij})\), is also constrained by the low energy theorem: \( f_A(\omega = 0) = -(Ze)^2/MA \). We make the decomposition into the seagull terms \( S \) (graphs 1a-b) and the time ordered part \( T \) (graphs 1c-f), where \( S = SN + Spion \) with

\[
SN = \frac{-Ze^2}{M} \quad \text{and} \quad Spion = -\frac{2}{3} e^2 \int d\vec{x} \langle A|\phi(\vec{x})^2|A \rangle.
\]  

(11)

In the high energy limit, only the seagull terms survive. A once subtracted dispersion relation then provides the pionic seagull term:

\[
-S_{pion} = S_N - f_A(0) + \frac{1}{2\pi^2} \int_0^\infty d\omega' \sigma_{\gamma A}(\omega'),
\]  

(12)

from which we get \(1\), per nucleon:

\[
\frac{1}{A} \int d\vec{x} \langle A|\phi(\vec{x})^2|A \rangle = \frac{3}{2e^2} \left[ 1 - \frac{ZN}{MA^2} + \frac{1}{2\pi^2} \int_0^\infty d\omega' \frac{\sigma_{\gamma A}(\omega')}{A} \right].
\]  

(13)

As mentioned previously, the direct \( \Delta \) excitation should be removed from the photoabsorption cross-section. In the energy integrated cross-section we assume its contribution to be the same as in the nucleon, as suggested in Ref. [13]. Using the same cut-off \((\approx 1 \text{ GeV})\) as for the free proton we find, as in Ref. [14], a slightly increased value as compared to the free nucleon corresponding to a (scalar) pion number of 0.38 \((\text{vs} \ 0.35 \text{ in the free case})\) at the normal nuclear density \(\rho_0\). A certain amount of increase is indeed expected from the spin-isospin correlations in the nucleus. We remind that the expectation value defined by relation (13) governs, in the nuclear medium, the quenching factor \( r \) of coupling constants such as the pion decay one \(\frac{1}{4}\), for which we obtain \( r(\rho_0) \approx 0.86 \).

3 Conclusion

We conclude with some remarks about the manifestations of chiral symmetry restoration. We have seen that the cross-sections where the photon attaches to a pion or to a pion-nucleon state through the Kroll-Ruderman term can be viewed as mixing cross-sections. The interpretation of certain cross-sections as mixing ones is reinforced by the relations that we have established in Eqs. (6,13) between them and the pion scalar density. The latter governs the condensate evolution as well as the quenching factor of certain coupling constants. A natural question is whether these manifestations of chiral symmetry restoration, mixing and depletion effects, can be identified in experiments. In the nucleus the nucleon which is the source of the pion field responsible for the mixing, has to be ejected. This automatically leads to

\footnote{Similar ideas have been put forward by Gerasimov [12]}

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final states in the continuum, such as 2p-2h excitations, with the possibility for one particle to be a $\Delta$. The mixing cross-sections thus have a broad energy distribution. Associated with the mixing is a quenching effect, which can apply to discrete states, or a group of them. An example is the Gamow-Teller sum rule where the states of the giant Gamow-Teller (GT) resonance are excited. The depopulation of these states for the benefit of the continuum of the mixing cross-section is thus a manifestation of chiral symmetry restoration. This implies that the states of the giant GT resonance do not exhaust the axial strength. The vector correlator which is introduced by the mixing term of the axial current provides some additional broad strength which compensates, partly or totally, the depletion of strength of the giant GT states linked to chiral symmetry. Both signal the mixing and chiral symmetry restoration (however other nuclear processes such as core polarization can have a similar effect, for a review see Ref. [14]). It is possible that the missing strength of the longitudinal response in $(e,e')$ scattering follows the same pattern. It was indeed suggested [15] that the quasi-elastic longitudinal strength is depopulated in favour of the nucleon-hole $\Delta$-hole continuum. Another case is that of the heavy ion collisions where the inclusion of p-h excitations, i.e. mainly of the graphs of Fig. 2d, is responsible for a large part of the excess at low mass electron pairs [11, 16]. This also represents a mixing cross-section i.e. a manifestation of chiral symmetry restoration. The consistency of the description implies the corresponding depletion of the $\rho$ meson excitation by the vector current due to pion loops to be simultaneously taken into account. This quenching should not be only the thermal one considered for instance in Ref. [17], but it has to incorporate also the presence of the baryonic background. We are pursuing our work in this direction.

To summarize our findings we can conclude that in the nuclear medium, the mixing of axial and vector correlators, enlarged to the nuclear case and defined in Eq. (10), has a signature because the nuclear pions have a broad energy and momentum distribution. We have given a formalism where the analogy between the dense case and the heat bath is apparent. In the case where the original correlator concerns a group of narrow states, chiral symmetry restoration implies their depopulation and the appearance of the mixed correlator which has a broad energy distribution.

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Figure captions

Figure 1: Pionic contribution to the photon self-energy in the vacuum.

Figure 2: The Compton amplitude on the nucleus to lowest order in nucleon loops: seagull amplitudes (a-b), direct excitation of p-h by the vector current (c), mixing terms of the vector current (d-f).

Figure 3: The same as Fig.2 for the free nucleon.

Figure 4: The axial correlator $A_{ij}$. 
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