Near-Extremal Black Branes with $n^3$ Entropy Growth

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Abstract

In this paper, which is a companion to ArXiv:1204.2711[hep-th], we formulate a criterion for constructing 11D/10D maximal supergravity black-brane solutions with the near-extremal $S \sim n^3 T^5$ entropy-temperature relation. We present explicit examples of such solutions with special attention paid to thermodynamics of black M-waves (KK-waves), intersecting/interpolating black branes with KK-waves, M2/M5 branes and their descents. We find the conditions on charges and numbers of branes in intersections, under which the dielectric effect of brane polarization becomes manifest on the supergravity side. We also briefly discuss the shear viscosity per entropy bound of M5 and D4 black branes within the AdS/CFT hydrodynamical limit.
1 Introduction

One of the most intriguing puzzles of M-theory relates to counting its degrees of freedom. Sharply non-perturbative in nature, M-theory involves various non-perturbative, i.e., solitonic degrees of freedom. They range from higher spin fields of a hidden symmetry of M-theory [1,2], and “preonic” constituents [3] of M-algebra [4], to branes [5]. It has been hoped that 6D (2,0) theory, a mysterious theory associated with the effective worldvolume description of M5 branes [6–11], may provide a clue to the existence of new hidden non-perturbative constituents of M-theory [12–15].

As widely known, the entropy of \( n \) coincident M5 branes scales as \( n^3 \) [16]. The difference between the near-extremal entropy growth of coincident black \( n \) D3-branes and that of coincident black \( n \) M5 branes was emphasized. For the near-extremal D3-branes, \( S \sim n^2 T^3 \) in agreement with \( U(n) \) symmetry of \( n \) coincident D3-branes [17]. For \( n \) near-extremal M5 branes, the corresponding relation takes \( S \sim n^3 T^5 \); the \( n^3 \) entropy growth cannot be associated with any classical Lie group.\(^1\) It was taken as an indication of novel degrees of freedom of 6D (2,0) theory.

Part of the hidden constituents of 6D (2,0) theory may be related to the self-dual strings [19], living on the world-volume of the M5 branes (see [20] for a review). They contribute to the effective potential of 6D chiral supersymmetric theory via string junctions [14,21] and instanton transitions [12,13,15]. The \( n^3 \) contribution of the internal degrees of freedom was also viewed from anomaly inflow in 11D supergravity in the presence of dynamical M5 branes [22–24], as well as in pure 6D (2,0) superconformal theory [25–27]. In the AdS/CFT of holographic hydrodynamics [28,29] (see [30] for a review), inclusion of M5 branes led to the \( n^3 \) behavior of pressure and shear viscosity of 6D strongly-coupled supersymmetric plasma at non-zero temperature [28,29,31].

The close relation between 6D effective worldvolume theory of M5 branes

\(^1\)A system of \( n \) coincident D4 branes also exhibits \( n^3 \) near-extremal entropy behavior. See [18] for early notice of this phenomenon.
and 5D effective worldvolume description of D4 branes goes back to the equivalence of M-theory and the strongly-coupled type IIA string theory. This duality allows one to study M5 branes through D4 brane/open string techniques. Although the $n^3$ entropy growth was originally discovered in the supergravity setup [16], the role of open strings in closed string theory has been widely recognized. Different aspects of open string feedbacks to closed string/supergravity models include, for instance, the seminal KLT relation [32] between open and closed string tree amplitudes (see [33,34] for recent progress), as well as the CSW [35] and BCFW [36] recursive relations. The role of open string loops in D-brane curvature generation was proposed in [37,38] (and refs therein). Recently, the $n^3$ entropy growth was obtained [39] in a D0-based SYM setup by including Myers’ terms [40–43] and subsequently applying the localization technique of [44,45].

One of the goals of this work is to search for additional examples of black M-brane configurations that display the characteristic $S \sim n^3T^5$ near-extremal entropy-temperature relation of black M5 branes. Configurations of KKW/M2/M5 black branes are natural places for searching for the systems with the $n^3$ behavior: all of these constituents (KKW,M2,M5) enter M-algebra [46] in a democratic way. Below we observe the characteristic $n^3$ growth in a system of KKW/M2/M5 black branes, a generalization of the M2/M5 interpolating solution of [47,48], and formulate conditions under which the behavior arises. One novelty of the solution worth mentioning is that it has a single harmonic function in M2/M5 sector and certain “duality” in thermodynamics under the interchange of the roles of KKW and M5. We will comment on its implication for our recent work [39] in the conclusion.

As we will discuss, the pertaining configurations of lower dimensional branes, lying inside or intersecting with M5 branes, is thermodynamically equivalent to the stack of coincident M5’s. It should be a manifestation of the Myers’ effect [40,49] on the supergravity side, and provides a rationale for the setup of our work [39]. Recall that in the supergravity context, the dielectric effect should manifest itself as the dissolution of lower dimensional
branes into higher dimensional ones. The dielectric effect was observed in other supergravity setups that correspond to the bound states of D0/D4 branes [50], flux branes [51, 52], and tubular branes [53, 54].

The same $n^3$ scaling is found below for descents of our KKW/M2/M5 solution to lower dimensions. Examples of non-intersecting black branes with the $n^3$ entropy include D3-branes in 9D, D2 branes in 8D, D1-branes in 7D and D0 branes in 6D. We explicitly check their near-extremal entropy growths, and confirm their $n^3$ behaviors.

The rest of the paper is organized as follows. In Section 2 we review thermodynamics of non-extremal M5 branes and the equation of state of coincident M5 black branes in the near-extremal limit. We establish the $n^3$ entropy growth, and find the shear viscosity/entropy bound of M5 branes within the AdS/CFT hydrodynamical limit. The same section contains a near-extremal entropy analysis of non-extremal M5/KKW and M5/M2/KKW intersecting branes. The analysis leads to formulation of a criterion for constructing supergravity solutions that have the same equation of state as that of M5 branes. In Section 3 we construct the KKW/M2/M5 interpolating solution with the $n^3$ near-extremal entropy growth. Dimensional reduction to a type IIA configuration is considered in Section 4. There, we also construct the descents of the IIA D4 black-brane solution, and examine their thermodynamical properties. Section 5 contains discussion of the results and comments on further studies. For self-containedness of the paper, we have collected a brief summary of thermodynamical characteristics of black branes and the basics of toroidal dimensional reduction in two Appendices.

2 Black M branes and their intersections

In this section we consider various configurations of black branes of M-algebra [4, 46] – KK-waves, M2 and M5 branes – and their near-extremal thermodynamics. Some of these configurations exhibit the $n^3$ scaling while
others do not.

We start with a brief review of [16] followed by comments on the hydrodynamics at the end. A stack of $n$ coincident black M5 branes is described by the following magnetic-type solution [55–57]

$$ds_{11}^2 = H_5^{-1/3}(r) \left[ -f(r)dt^2 + dx_1^2 + \ldots + dx_5^2 \right] + H_5^{2/3}(r) \left( \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \right),$$

(2.1)

$$H_5(r) = 1 + \frac{h_0^3}{r^3}, \quad f(r) = 1 - \frac{\mu^3}{r^3},$$

(2.2)

$$F[4] = 3nh_0^3 \left( 1 + \frac{\mu^3}{nh_0^3} \right)^{1/2} \epsilon_4, \quad F[4] = dC[3]$$

(2.3)

to 11D supergravity equations of motion [58]. $\epsilon_4$ is the volume form of the unit 4-sphere; $h_0$ is associated with the M5 charge. Calculating the entropy and temperature of the system by standard methods (cf. eqs. (A.2), (A.3)), one gets

$$S_5 = \frac{l^5 \Omega_4}{16\pi G_{11}} \left( 1 + n \frac{h_0^3}{\mu^3} \right)^{1/2} \mu^4,$$

(2.4)

$$T_5 = \frac{3}{4\pi} \left( 1 + n \frac{h_0^3}{\mu^3} \right)^{-1/2} \mu^{-1}.$$  

(2.5)

Another important characteristic of the system is its ADM mass. Adopting the general expression (A.4), we get

$$M_5 = \frac{l^5 \Omega_4}{16\pi G_{11}} \left[ 4\mu^3 + 3nh_0^3 \right].$$

(2.6)

Taking the extremal limit, $\mu \to 0$, results in the following mass-charge bound relation

$$M_{(0)}^{(0)} = \frac{l^5 \Omega_4}{16\pi G_{11}} 3nh_0^3.$$ 

(2.7)

(The entropy and the temperature in the extremal limit vanish as they should.) To compute $S(T)$ dependence in the near-extremal limit, we perturb, following [16], the ADM mass around its bound value (2.7),

$$M_5 = M_5^{(0)} \left( 1 + \frac{\delta M}{M^{(0)}} \right), \quad \delta M \ll M^{(0)}.$$ 

(2.8)
Therefore, $\mu$ satisfies $\mu \ll 1$ in the near-extremal limit. Roughly, $\delta M$ scales as $\delta M \sim \mu^3$; more precisely, one gets

$$\mu = \left( \frac{\delta M}{4} \right)^{1/3} \alpha_5^{1/3}, \quad \alpha_5 = \frac{16\pi G_{11}}{l^5 \Omega_4}, \quad (2.9)$$

Replacement of $\mu$ in (2.4) by its near-extremal value (2.9) leads to

$$S_5 = \frac{l^5 \Omega_4}{4G_{11}} \left( 1 + n \frac{h_0^3}{\mu^3} \right)^{1/2} \mu^4 \approx \frac{l^5 \Omega_4}{4G_{11}} h_0^{3/2} n^{1/2} \mu^{5/2} \sim n^{1/2} (\delta M)^{5/6}. \quad (2.10)$$

The Hawking temperature (2.5) takes

$$T_5 \approx \frac{3}{4\pi} h_0^{-3/2} n^{-1/2} \mu^{1/2} \sim n^{-1/2} (\delta M)^{1/6}. \quad (2.11)$$

Interpreting $\delta M$ as the energy $E$ (cf. [16]), we arrive at

$$S_5 \sim n^{1/2} E^{5/6}, \quad T_5 \sim n^{-1/2} E^{1/6}, \quad (2.12)$$

in agreement with the first law of thermodynamics. From the latter expressions we get the Klebanov-Tseytlin result

$$S_{M5} \sim n^3 T^5, \quad (2.13)$$

the near-extremal $n^3$ entropy growth.

As observed in [16], the near-extremal entropy of black M5 branes may be recovered in terms of a weakly-interacting ideal gas of 6D massless particles. Indeed, the explicit value of near-extremal entropy density (per unit volume) of black M5 branes [16, 28] is given by

$$S_5 = 2^7 3^{-6} \pi^3 n^3 T^5, \quad (2.14)$$

and is the same as the entropy of a gas of $N_5 = 2^{10} 3^{-6} 5n^3$ massless bosons and fermions, whose degrees of freedom match due to the supersymmetry of M5 branes.
In the context of AdS/CFT hydrodynamics [59,60], one of the test stone quantities is the ratio of the shear viscosity [28, 29, 31]
\[ \eta_5 = \frac{\pi^2}{2} \left( \frac{2}{3} \right)^6 n^3 T^5 \]  
(2.15)
to the entropy (2.14),
\[ \frac{\eta_5}{S_5} = \frac{1}{4\pi}. \]  
(2.16)
It precisely corresponds to the bound value of the universal \( \eta/S \) relation
\[ \eta/S \geq 1/4\pi, \]  
(2.17)
that was conjectured in [61].

2.1 Various other configurations containing M5’s

With the above review we pose the following question: could the characteristic \( n^3 \) entropy growth be observed in other coincident black brane systems? After examining various other configurations containing M5 branes below, we discuss the necessary conditions in section 2.2.

2.1.1 M5/KKW bound state in the near-extremal limit

To find the answer for the question above, let us consider a configuration of M5 branes with added M-waves (KKWs). The procedure is well-known [64–67], and the non-extremal KKW/M5 bound state solution has the following form
\[
\begin{align*}
 ds_{11}^2 &= H_5^{2/3} \left[ H_5^{-1} (-K^{-1} f dt^2 + K d\xi_1^2) + H_5^{-1} (dx_2^2 + \ldots + dx_5^2) \\
 & \quad + f^{-1} dr^2 + r^2 d\Omega_4^2 \right].
\end{align*}
\]  
(2.18)

\footnote{This bound was violated, e.g., in higher derivative gravity [62], or in anisotropic plasma [63].}
where
\[ H_5(r) = 1 + n_5 \frac{h_5^3}{r^3}, \quad K(r) = 1 + n_0 \frac{k_0^3}{r^3}, \quad f(r) = 1 - \frac{\mu^3}{r^3}, \quad d}\tilde{x}_1^2 = [dx_1 + (K^{-1} - 1)dt]^2, \]
(2.19)
\[ F_{[4]} = 3n_5 h_5^3 \left(1 + \frac{\mu^3}{n_5 h_5^3}\right)^{1/2} \epsilon_4. \]
(2.20)
The bound state is formed by \( n_0 \) black KK-waves that travel along the \( x_1 \) direction of the world-volume of \( n_5 \) coincident black M5 branes.

Computing the ADM mass, entropy and temperature of the non-extremal configuration, one gets\(^3\)
\[ M = \frac{l^5 \Omega_4}{16\pi G_{11}} \left[4\mu^3 + 3n_0 k_0^3 + 3n_5 k_5^3\right], \]
(2.21)
\[ S = \frac{l^5 \Omega_4}{4G_{11}} \left(1 + n_5 \frac{h_5^3}{\mu^3}\right)^{1/2} \left(1 + n_0 \frac{k_0^3}{\mu^3}\right)^{1/2} \mu^4, \]
(2.22)
\[ T = \frac{3}{4\pi} \left(1 + n_5 \frac{h_5^3}{\mu^3}\right)^{-1/2} \left(1 + n_0 \frac{k_0^3}{\mu^3}\right)^{-1/2} \mu^{-1}. \]
(2.23)
In the near-extremal limit \( \delta M \sim \mu^3 \ll 1 \),
\[ S \sim \sqrt{n_0 n_5} \left(\delta M\right)^{1/3}, \quad T \sim (\sqrt{n_0 n_5})^{-1} (\delta M)^{2/3}. \]
(2.24)
Therefore, the entropy of black KKW/M5 branes obeys
\[ S \sim (n_0 n_5)^{3/4} T^{1/2}. \]
(2.25)
At first sight, the \( S(T) \) dependence of (2.25) is far from the \( n^3 \) dependence of (2.13). However, once \( n_0 \) in (2.21)–(2.23) is set to zero, (2.21)–(2.23) turn into (2.6), (2.4), (2.5); the near-extremal entropy will have the \( n^3 \) growth. The

\(^3\)Even though the metric (2.18) contains \( g_{x_1 t} \) non-diagonal term, it is diagonal in the \( \tilde{x}_1 \) coordinate; one may still use (A.2), (A.3), (A.4). There are two reasons behind this: the equality of the areas of black KKW/M5 bound-state configurations and the equality of the corresponding time-like Killing vectors in \( x_1 \) and \( \tilde{x}_1 \) coordinates. The former leads to the same entropy for both coordinates and the latter to the same values of temperature.
same thing happens when the number of KK-waves is restricted to \( n_0 k_0^3 \ll 1 \).\(^4\)

Tuning the parameters in this way, the horizon of the configuration is formed solely by M5 branes. It follows that the thermodynamics of the KKW/M5 solution in the \( n_0 k_0^3 \ll 1 \) limit is completely determined by M5 black brane characteristics. In other words, the near-extremal entropy grows as

\[
S_{KKW/M5} \sim n_5^3 T^5, \quad (n_0 k_0^3 \ll 1, \ n_0 \gg 1).
\] (2.26)

One thing worth pointing out is the fact that the respective “charges” \( h_5 \) and \( k_0 \) of the M5 branes and KKWs enter (2.21)–(2.23) in a symmetric way. Expressions for \( M, S, T \) are invariant under the interchange of \( n_0 \leftrightarrow n_5, h_5 \leftrightarrow k_0 \). This symmetry suggests the following interpretation upon considering the interchange of the roles of M5’s and KKWs. The bound state is formed with a number of KK-waves in the \( n_0 k_0^3 \gg 1 \) limit and a number of M5 branes in the \( n_5 h_5^3 \ll 1, n_5 \gg 1 \) limit. With this choice, the thermodynamics of KK-waves models that of the black M5 branes; the near-extremal entropy becomes

\[
S_{KKW/M5} \sim n_5^3 T^5, \quad (n_5 h_5^3 \ll 1, \ n_5 \gg 1),
\] (2.27)

It may be interpreted as a manifestation of the Myers’ effect [40] of polarization of KK-waves (M0 branes) into M5 branes.

### 2.1.2 M2/M5 intersection

Let us add M2 branes into the setup, and “blacken” the extremal solutions of M2/M5 and KKW/M2/M5 brane intersections respectively. The configuration that we consider in this subsection will not lead to \( n^3 \) behavior. Using [68], the number of common dimensions \( \bar{q} \) over which \( Mq \) and \( Mp \) branes intersect is evaluated as

\[
\bar{q} = \frac{(q + 1)(p + 1)}{9} - 1, \quad \bar{q} \in \mathbb{Z}, \quad \bar{q} > 0.
\] (2.28)

\(^4\)Note, however, that to keep the supergravity approximation, \( n_0 \gg 1 \) is still needed, and \( n_5 h_5^3 \gg 1 \).
Let us turn to the single M2/M5 branes intersection over a line \((\bar{q} = [(5 + 1)(2 + 1)/9] - 1 = 1)\), and set the line along the \(x_1\) direction. The corresponding intersection diagram looks as follows

\[
\begin{array}{ccccccccccc}
  t & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \\
  M2 & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
  M5 & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
\end{array}
\]

The line element\(^5\) is [64, 65]

\[
ds_{11}^2 = H_2^{1/3} H_5^{2/3} \left[ (H_2 H_5)^{-1} (-dt^2 + dx_2^2) + H_2^{-1} dx_6^2 \\
+ H_5^{-1} (dx_2^2 + dx_3^2 + \ldots + dx_5^2) + dr^2 + r^2 d\Omega_3^2 \right], \quad (2.29)
\]

where

\[
r^2 = x_7^2 + x_8^2 + x_9^2 + x_{10}^2, \\
H_i(r) = 1 + \frac{h_i^2}{r^2}, \quad i = 2, 5 \quad (2.30)
\]

are harmonic functions in the 4D space that is spanned by the radial coordinate \(r\) and three angles of \(d\Omega_3\). The generalization of (2.29), (2.30) to the non-extremal multiple intersecting M2/M5 branes takes

\[
ds_{11}^2 = H_2^{1/3} H_5^{2/3} \left[ (H_2 H_5)^{-1} (-fdt^2 + dx_2^2) + H_2^{-1} dx_6^2 \\
+ H_5^{-1} (dx_2^2 + dx_3^2 + \ldots + dx_5^2) + f^{-1} dr^2 + r^2 d\Omega_3^2 \right], \quad (2.31)
\]

\[
H_2(r) = 1 + n_2 \frac{h_2^2}{r^2}, \quad H_5(r) = 1 + n_5 \frac{h_5^2}{r^2}, \quad f(r) = 1 - \frac{\mu^2}{r^2}, \quad (2.32)
\]

where \(n_2\) and \(n_5\) are the M2 and M5 brane numbers respectively. The ADM mass, the entropy and the temperature of the system are

\[
M = \frac{l^6 \Omega_3}{16 \pi G_{11}} \left[ 3\mu^2 + 2n_2 h_2^2 + 2n_5 h_5^2 \right], \quad (2.33)
\]

---

\(^5\)Since we are mostly interested in computing \(M, S\) and \(T\), we skip the details on the gauge sector of the solution here and below.
\[ S = \frac{l^6 \Omega_3}{4G_{11}} \left( 1 + n_2 \frac{h_2^2}{\mu^2} \right)^{1/2} \left( 1 + n_5 \frac{h_5^2}{\mu^2} \right)^{1/2} \mu^3, \]  
(2.34)

\[ T = \frac{2}{4\pi} \left( 1 + n_2 \frac{h_2^2}{\mu^2} \right)^{-1/2} \left( 1 + n_5 \frac{h_5^2}{\mu^2} \right)^{-1/2} \mu^{-1}. \]  
(2.35)

They take

\[ S \sim n_2^{1/2} n_5^{1/2} (\delta M)^{1/2}, \quad T \sim n_2^{-1/2} n_5^{-1/2} (\delta M)^{1/2}. \]  
(2.36)

in the near-extremal limit \((\mu \sim (\delta M)^{1/2} \text{ for the present case})\). This result implies

\[ S \sim n_2 n_5 T. \]  
(2.37)

We conclude that it is impossible to recover the \(n^3\) entropy growth in the current case.

### 2.1.3 boosted non-extremal M2/M5 intersection

Finally, let us check the near-extremal entropy growth in the system of non-extremal KKW/M2/M5 intersecting branes. The solution that corresponds to KKW/M2/M5 configuration can be obtained from (2.31) by inclusion of M5 branes boosted along the intersecting direction (it is the \(x_1\)-direction in our case) \([64,66,67]\). The boosted form of (2.31) is

\[ ds_{11}^2 = H_2^{1/3} H_5^{2/3} \left[ (H_2 H_5)^{-1} (-K^{-1} f dt^2 + K \delta x_1^2) + H_2^{-1} dx_6^2 \right. \]
\[ + H_5^{-1} (dx_2^2 + dx_3^2 + \ldots + dx_5^2) + f^{-1} dr^2 + r^2 d\Omega_3^2 \bigg], \]  
(2.38)

\[ H_2(r) = 1 + n_2 \frac{h_2^2}{r^2}, \quad H_5(r) = 1 + n_5 \frac{h_5^2}{r^2}, \quad f(r) = 1 - \frac{\mu^2}{r^2}, \]  
(2.39)

\[ K(r) = 1 + n_0 \frac{k_0^2}{r^2}, \quad \delta x_1^2 = \left[ dx_1 + (K^{-1} - 1) dt \right]^2, \]  
(2.40)

where \(n_0, n_2\) and \(n_5\) count the number of KK waves, M2 and M5 branes respectively. After some algebra, one can obtain

\[ M = \frac{l^6 \Omega_3}{16\pi G_{11}} \left[ 3\mu^2 + 2n_2 h_2^2 + 2n_5 h_5^2 + 2n_0 k_0^2 \right], \]  
(2.41)
\[ S = \frac{r^6 \Omega_3}{4G_{11}} \left( 1 + n_2 \frac{h_2^2}{\mu^2} \right)^{1/2} \left( 1 + n_5 \frac{h_5^2}{\mu^2} \right)^{1/2} \left( 1 + n_0 \frac{k_0^2}{\mu^2} \right)^{1/2} \mu^3, \quad (2.42) \]

\[ T = \frac{2}{4\pi} \left( 1 + n_2 \frac{h_2^2}{\mu^2} \right)^{-1/2} \left( 1 + n_5 \frac{h_5^2}{\mu^2} \right)^{-1/2} \left( 1 + n_0 \frac{k_0^2}{\mu^2} \right)^{-1/2} \mu^{-1}. \quad (2.43) \]

In the near-extremal limit of \((\delta M) \sim \mu^2\), the entropy and temperature scale according to

\[ S \sim n_2^{1/2} n_5^{1/2} n_0^{1/2} (\delta M)^0, \quad T \sim n_2^{-1/2} n_5^{-1/2} n_0^{-1/2} (\delta M). \quad (2.44) \]

In this case, one encounters the near-extremal entropy independence of energy. Also, as in the previous subsection, the desired \(n^3\) entropy does not result in this setup.

### 2.2 Condition for reproduction of \(n^3\) scaling

Let us pause and summarize the results so far. With appropriately chosen limits of the parameters (see (2.26) and (2.27)), we could observe the \(n^3\) entropy scaling in the boosted M5 solution (2.18) above. With a different set of scalings, the thermodynamics of \(n\) coincident black M5 branes could be completely modeled by the KK-waves that propagate inside the stack of M5’s. We have interpreted this phenomenon as a manifestation of the Myers’ effect on the supergravity side. Meanwhile, we have concluded that the \(n^3\) near-extremal entropy growth cannot be reached with the M2/M5 (eq.(2.31)) or the KKW/M2/M5 intersecting systems (eq.(2.38)) of delocalized M black branes.\(^6\) We would like to elaborate on this further here.

It is well-known that thermodynamics of black branes configurations is completely encoded in the structures of the harmonic functions and the “blackening” factor(s) that appear in the solution. (One can see this just by considering (A.1), (A.2), (A.3) and (A.4)). To reproduce the same thermodynamics of a given system (say, the M5 brane system in the present case)

\(^6\)In this paper we only focus on the delocalized solutions. The localized versions of the systems considered here (see, e.g., [69–72]) are not the subject of the paper.
by another system (say, the boosted M5 branes, or their intersections with
KK waves and M2 branes), it is necessary to assure that the two configura-
tions under consideration have the same entropy/temperature dependence on
the non-extremality parameter \( \mu \) in the blackening factor \( f(r) \). Comparing
(2.22) and (2.23) with (2.4) and (2.5), one can see that it is indeed the case:
the entropy and the temperature of two systems have the same dependence
on \( \mu \). Meanwhile, the entropy of M2/M5 branes given in (2.34) shows differ-
ent dependence on \( \mu \) compared with (2.4). It leads to a completely different
entropy-temperature equation of state in the end, and, consequently, to dif-
ferent thermodynamics. Although in the present paper we do not consider
solutions of partially localized branes, this observation remains true for the
latter case as well.

Therefore, it is clear that compelling candidates for intersecting solutions
of M branes that could potentially reproduce (exactly or within some special
choice of the parameters) the \( n^3 \) growth of the black M5 branes are those
whose harmonic functions have \( 1/r^3 \) dependence. Put differently, \( d\Omega_4 \) volume
element must be present in the solution.

In what follows we will focus on the M2/M5 interpolating solution of
[47, 48] and its boosted non-extremal version [66]. They both fall into the
criterion just drawn. Afterwards, we will show that solutions obtained by
dimensional reduction of M interpolating branes and double dimensional re-
duction of M5 branes exhibit the \( n^3 \) near-extremal entropy behavior.

3 M-branes interpolating solutions

Consider the following solution that interpolates between M2 and M5 branes
[47, 48, 66]

\[
ds_{11}^2 = (H \tilde{H})^{1/3} \left[ H^{-1}(-f dt^2 + dx_1^2 + dx_2^2) + \tilde{H}^{-1}(dx_3^2 + dx_4^2 + dx_5^2) + f^{-1}dr^2 + r^2d\Omega_4^2 \right],
\]

\[
\tilde{F}_4 = \frac{1}{2} \cos \zeta \ast dH + \frac{1}{2} \sin \zeta dH^{-1} \epsilon_3 + \frac{3}{2} \sin 2\zeta H^{-2}dH \epsilon_3,
\]

where

\[
H = H(x_1, x_2, x_3), \quad \tilde{H} = \tilde{H}(x_4, x_5).
\]
\[ H(r) = 1 + n \frac{h^3}{r^3}, \quad \tilde{H} = \sin^2 \zeta + H \cos^2 \zeta, \quad f(r) = 1 - \frac{\mu^3}{r^3}, \quad (3.3) \]

where \( \epsilon_3 \) and \( \bar{\epsilon}_3 \) are volume forms on \( M^3 \) and \( \mathbb{E}^3 \) parameterized respectively by \( (t, x_1, x_2) \) and \( (x_3, x_4, x_5) \); \( * \) is the Hodge dual of \( \mathbb{E}^5 \) that is transverse to the M5 branes. This solution is completely different from the standard intersection of M2/M5 (2.31). In contrast to (2.31) whose intersection diagram was given in Sec. 2.3, the solution (3.1)–(3.3) describes M2 branes entirely lying within M5 branes:

\[
\begin{array}{cccccccccccc}
 t & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \\
 M2 & \times & \times & \times & & & & & & & & \\
 M5 & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times
\end{array}
\]

Unlike (2.31) which preserves 1/4 of the original supersymmetry, (3.1) preserves 1/2. It is constructed using a single harmonic function \( H(r) \); \( \sin \zeta = 0 \) corresponds to the black M5 brane solution (2.1)–(2.3) and \( \cos \zeta = 0 \) gives a configuration of black M2 branes. As we will see shortly, dimensional reduction of (3.1) leads to type IIA interpolating solutions of D2 branes within D4 branes, or F1s within D4s when reduced along any of \( (x_3, x_4, x_5) \).

The thermodynamics pertaining to (3.1) are

\[
M = \frac{l_5^5 \Omega_4}{16 \pi G_{11}} \left[ 4 \mu^3 + 3nh^3 \right], \quad (3.4)
\]

\[
S = \frac{l_5^5 \Omega_4}{4G_{11}} \left( 1 + n \frac{h^3}{\mu^3} \right)^{1/2} \mu^4, \quad (3.5)
\]

\[
T = \frac{3}{4 \pi} \left( 1 + n \frac{h^3}{\mu^3} \right)^{-1/2} \mu^{-1}. \quad (3.6)
\]

These expressions are precisely those of the stack of black M5 branes (cf. (2.4)–(2.6)). One can easily check that

\[
S \sim n^3 T^5 \quad (3.7)
\]

in the near-extremal limit. The solution describes a configuration of the M2 branes completely dissolved in the M5 branes.
The boosted version of (3.1) involving the KK-wave along, say, the $x_1$ direction is described by the following ansatz

$$ds_{11}^2 = \left( H \tilde{H} \right)^{1/3} \left[ H^{-1}(-K^{-1}f dt^2 + Kdx_1^2 + dx_2^2) + \tilde{H}^{-1}(dx_3^2 + dx_4^2 + dx_5^2) + f^{-1}dr^2 + r^2d\Omega_3^2 \right], \quad (3.8)$$

with

$$H(r) = 1 + n\frac{k_0^3}{r^3}, \quad \tilde{H} = \sin^2 \zeta + H \cos^2 \zeta, \quad f(r) = 1 - \frac{\mu^3}{r^3},$$

$$K(r) = 1 + n_0\frac{k_0^3}{r^3}, \quad dx_1^2 = [dx_1 + (K^{-1} - 1)dt]^2. \quad (3.9)$$

The gauge part of the solution coincides with (3.2). The solution (3.8) with the harmonic functions and the blackening factor of (3.9) is a generalization of the boosted solution of [66]. Its thermodynamics is quite similar to the boosted M5 branes solution considered in Sec. 2.1.1. Therefore, tuning the parameters of (3.9) to $nh_0^3 \ll 1$, $n \gg 1$, $n_0k_0^3 \gg 1$, the KKW/M2/M5 interpolating solution becomes thermodynamically equivalent to the stack of $n_0$ black M5 branes.\(^\text{7}\) We interpret this phenomenon as a realization of the Myers’ effect of polarization of branes.

### 4 Dimensional reduction of black M-branes solutions to D=10 IIA ones

Let us consider the dimensionally reduced versions of the previously obtained solutions to 11D supergravity. Basics of the toroidal dimensional reduction are reviewed in Appendix B.

The $n^3$ near-extremal entropy growth can be found in the 10D type IIA $n$ D4 black branes that is obtained by dimensional reduction of (2.1).\(^\text{8}\) Since

\(^\text{7}\)The fact that only a single harmonic function appears together with this “duality” in thermodynamics may have an interesting implication related to our recent work [39]. We will comment on it in the conclusion.

\(^\text{8}\)This was noticed early in [18].
general expressions for the ADM mass, the entropy and the temperature are typically written in the Einstein frame, it is convenient to reduce (2.1) by use of the Einstein frame reduction ansatz

\[ g^{(11)}_{MN} dx^M dx^N = e^{-\phi/6} g^{(10)}_{mn} dx^m dx^n + e^{4\phi/3} (dx^b + A_m dx^m)^2, \]  

(4.1)

where \( x^b \) denotes the direction of the reduction. Reducing (2.1) along one of \( x_1, x_2, x_3, x_4, x_5 \) (say, \( x^5 = x_5 \)) one gets

\[ ds_{10}^2 = H^{-3/8} \left[ -f(r) dt^2 + dx_1^2 + \ldots + dx_4^2 \right] + H^{5/8} \left( \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \right), \]  

(4.2)

\[ H_4(r) = 1 + \frac{h_0^3}{r^3}, \quad f(r) = 1 - \frac{\mu^3}{r^3}, \quad e^\phi = H^{-1/4}_4. \]  

(4.3)

The \( C_3 \) gauge field (other gauge fields are set to zero) is given by

\[ F_4 = 3n h_0^3 \left( 1 + \frac{\mu^3}{nh_0^3} \right)^{1/2}. \]  

(4.4)

The solution (4.2)–(4.4) describes the background of \( n \)-coincident black D4 branes of type IIA theory (see, e.g., [73]).

Computing the ADM mass, entropy and temperature, one gets

\[ M_4 = \frac{l^4 \Omega_4}{16\pi G_{10}} \left[ 4\mu^3 + 3nh_0^3 \right], \]  

(4.5)

\[ S_4 = \frac{l^4 \Omega_4}{4G_{10}} \left( 1 + \frac{h_0^3}{\mu^3} \right)^{1/2} \mu^4, \]  

(4.6)

\[ T_4 = \frac{3}{4\pi} \left( 1 + \frac{h_0^3}{\mu^3} \right)^{-1/2} \mu^{-1}. \]  

(4.7)

Comparing the latter expressions with (2.4)–(2.6) it is easy to notice, after identifying \( G_{11} = lG_{10} \), the similarity between (4.5)–(4.7) and the corresponding quantities of \( n \)-coincident black M5 branes. Hence, one may conclude that the thermodynamics of \( n \)-coincident black D4 branes leads to the entropy-temperature dependence

\[ S_{D4} \sim n^3 T^5 \]  

(4.8)
in the near-extremal limit.

The solution (2.18), once dimensionally reduced along the KK waves propagation direction, leads to the D0/D4 bound state. Indeed, by using the reduction ansatz (4.1) and reducing along the \( x^b = x_1 \), we arrive at\(^9\)

\[
d s_{10}^2 = H_4^{5/8} K_0^{1/8} \left[ -(H_4 K_0)^{-1} f dt^2 + H_4^{-1}(dx_2^2 + \ldots + dx_5^2) + f^{-1} dr^2 + r^2 d\Omega_4^2 \right],
\]

(4.9)

\[
A_{[1]} = (K_0^{-1} - 1) dt, \quad e^{4\phi/3} = H_4^{-1/3} K_0,
\]

(4.10)

\[
H_4(r) = 1 + n_4 h_4^3 r^3, \quad K_0(r) = 1 + n_0 k_0^3 r^3, \quad f(r) = 1 - \mu^3 r^3.
\]

(4.11)

The near-extremal \( S(T) \) dependence of the D0/D4 brane intersection

\[
S \sim (n_0 n_4)^{3/4} T^{1/2}
\]

(4.12)

matches with (2.25). Therefore, the results of Sec. 2.1.1 are directly applied to the D0/D4 case, and one may scale the parameters appropriately so that that thermodynamics of black D0 branes will reproduce that of D4 branes in the near-extremal limit,

\[
S_{D0/D4} \sim n_0^3 T^5 \quad (n_0 k_0^3 \gg 1, \; n_4 h_4^3 \ll 1, \; n_4 \gg 1).
\]

(4.13)

Again, it should be a manifestation of the Myers’ effect of polarization of D0 branes to D4 branes on supergravity side.

### 4.1 D0/D2/D4 interpolating solution

Dimensional reduction of (3.8) along any of \( x_{3,4,5} \) directions leads to the D0/D2/D4 interpolating solution, D2/D4 part of which (in the string frame) was constructed in [48]. We will reduce along \( x_3 \) coordinate, which is associated with the traveling direction of the KK-waves. The eleven dimensional metric is

\[
d s_{11}^2 = (H \bar{H})^{1/3} \left[ H^{-1}(-K^{-1} f dt^2 + dx_1^2 + dx_2^2) + \bar{H}^{-1}(K dx_3^2 + dx_4^2 + dx_5^2) \right]
\]

\(^9\)See, e.g., [64,65,68,74] for a general discussion on Dq/Dp intersecting branes.
\[+ f^{-1}dr^2 + r^2d\Omega_4^2], \quad (4.14)\]

where

\[H(r) = 1 + \frac{n}{r^3}, \quad \tilde{H} = \sin^2 \zeta + H \cos^2 \zeta, \quad f(r) = 1 - \frac{\mu^3}{r^3},\]

\[K(r) = 1 + n_0 \frac{k_0^3}{r^3}, \quad \tilde{x}_3^2 = [dx_3 + (K^{-1} - 1)dt]^2. \quad (4.15)\]

Reducing along \(x_3\), one gets

\[ds_{10}^2 = H^{3/8} \tilde{H}^{1/4} K^{-3/8} \left[ H^{-1}(-K^{-1}f dt^2 + dx_1^2 + dx_2^2) + \tilde{H}^{-1}(dx_4^2 + dx_5^2) + f^{-1}dr^2 + r^2d\Omega_4^2 \right], \quad (4.16)\]

\[e^\phi = H^{1/4} \tilde{H}^{1/2} K^{-3/4}, \quad A_{[1]} = (K^{-1} - 1)dt. \quad (4.17)\]

in the Einstein frame. In the \(K = 1\) setting, (4.16), (4.17) reduce to the D2/D4 interpolating solution of [48]. One can show that

\[M = \frac{l_4^4 \Omega_4}{16\pi G_{10}} \left[ 4\mu^3 + 3nh_0^3 \right], \quad (4.18)\]

\[S = \frac{l_4^4 \Omega_4}{4G_{10}} \left( 1 + \frac{n}{\mu^3} \right)^{1/2} \mu^4, \quad (4.19)\]

\[T = \frac{3}{4\pi} \left( 1 + \frac{n}{\mu^3} \right)^{-1/2} \mu^{-1}. \quad (4.20)\]

These expressions coincide with those of the interpolating solution of M2/M5 (compare (4.18)–(4.20) with (3.4)–(3.6), identifying \(G_{11} = lG_{10}\)). In other words, the reduced solution with D0 branes inherits features of the KKW/M2/M5 solution (4.14). On the other hand, the thermodynamics of the latter coincides with thermodynamics of the boosted M5’s solution of Sec. 2.1.1. Therefore, one can tune the parameters so as to get the \(n^3\) entropy growth in the near-extremal limit.
4.2 Black branes solutions with the $n^3$ entropy growth in $9 \geq D \geq 6$

By dimensionally reducing the solutions of $n^3$ entropy, one may construct other near-extremal black branes with the $n^3$ scaling in lower dimensions. Here we focus on the solution (4.2), and obtain 9D D3, 8D D2, 7D D1 and 6D D0 black branes from it. The low-dimensional descents of D0/D2/D4 interpolating solution (4.16)–(4.17) can be obtained in a similar way.

To construct descents of (4.2) by toroidal dimensional reduction to 9D, 8D, 7D and 6D, we use the standard $D$-dimensional Kaluza-Klein ansatz
\[
g^{(D)}_{MN} dx^M dx^N = e^{2\alpha \phi(D-1)} g_{mn}^{(D-1)} dx^m dx^n + e^{2\beta \phi(D-1)} (dx^b + A_m^{(D-1)} dx^m)^2. \tag{4.1}
\]

In the Einstein frame, $\alpha$ and $\beta$ are given by
\[
\alpha = -\sqrt{\frac{1}{2(D-2)(D-3)}}, \quad \beta = -(D-3)\alpha, \quad D = 10, 9, 8, 7 \tag{4.2}
\]

Explicitly, they are given by
\[
\begin{align*}
\alpha &= -\frac{1}{4\sqrt{7}}, \quad \beta = \frac{\sqrt{7}}{4}, \quad D = 10; \\
\alpha &= -\frac{1}{2\sqrt{21}}, \quad \beta = \sqrt{\frac{3}{7}}, \quad D = 9; \\
\alpha &= -\frac{1}{2\sqrt{15}}, \quad \beta = \frac{1}{2}\sqrt{\frac{5}{3}}, \quad D = 8; \\
\alpha &= -\frac{1}{2\sqrt{10}}, \quad \beta = \sqrt{\frac{2}{5}}, \quad D = 7.
\end{align*}
\]

Proceeding in the standard manner and reducing along $x^b = x_4, x_3, x_2, x_1$ successively, one gets, for the graviton-dilaton part of solutions,

- 9D $n$D3 coincident black branes
\[
ds^2_9 = H_3^{-3/7} \left( -f dt^2 + dx_1^2 + \ldots + dx_3^2 \right) + H_3^{1/7} \left( f^{-1} dr^2 + r^2 d\Omega_4^2 \right), \tag{4.3}
\]
\[
e^{-2\sqrt{\frac{7}{3}} \phi(0)} = H_3; \tag{4.4}
\]

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• 8D \(n\)D2 coincident black branes

\[
ds_8^2 = H_2^{-1/2} (-f dt^2 + dx_1^2 + dx_2^2) + H_2^{1/2} (f^{-1} dr^2 + r^2 d\Omega_4^2),
\]
\[e^{-2\sqrt{7/3} \phi} = H_2 ; \] (4.5)

• 7D \(n\)D1 coincident black branes

\[
ds_7^2 = H_1^{-3/5} (-f dt^2 + dx_1^2) + H_1^{2/5} (f^{-1} dr^2 + r^2 d\Omega_4^2),
\]
\[e^{-2\sqrt{5/3} \phi} = H_1 ; \] (4.7)

• 6D \(n\)D0 non-extremal branes

\[
ds_6^2 = -f H_0^{-3/4} dt^2 + H_0^{1/4} (f^{-1} dr^2 + r^2 d\Omega_4^2),
\]
\[e^{-2\sqrt{10/3} \phi} = H_0 . \] (4.9)

Everywhere in (4.3)–(4.10)

\[
H_i(r) = 1 + \frac{h_0^3}{r^3}, \quad f(r) = 1 - \frac{\mu^3}{r^3}, \quad i = 3, 2, 1, 0 ; \] (4.11)

\[
F^{(D-1)}_4 = 3 n h_0^3 \left(1 + \frac{\mu^3}{n h_0^3}\right)^{1/2} \epsilon_4, \quad D - 1 = 9, 8, 7, 6 .\] (4.12)

Other scalars and tensors are set to zero. For all of these systems, the ADM mass, entropy, and temperature are given by

\[
M_p = \frac{l^p \Omega_4}{16\pi G_{[p+6]}} \left[4\mu^3 + 3n h_0^3\right], \] (4.13)

\[
S_p = \frac{l^p \Omega_4}{4G_{[p+6]}} \left(1 + n \frac{h_0^3}{\mu^3}\right)^{1/2} \mu^4, \] (4.14)

\[
T_p = \frac{3}{4\pi} \left(1 + n \frac{h_0^3}{\mu^3}\right)^{-1/2} \mu^{-1}. \] (4.15)

A comparison of these with (4.5)–(4.7) reveals that the solutions (4.3)–(4.12), in the near-extremal limit, have the \(n^3\) near-extremal entropy scaling.
5 Conclusions

In this paper we have constructed a series of black brane solutions of 11D/10D IIA supergravities that exhibit the \( n^3 \) near-extremal entropy behavior with appropriately chosen scalings of the parameters. They include the boosted M5 black branes (M5/KKW bound state of [64, 66, 67]) and the boosted M2/M5 interpolating black branes of [47,48,66]. The identical near-extremal equation of state can be assigned to their dimensionally reduced versions, IIA D0/D4 black branes bound state and the solution of boosted D2/D4 interpolating black branes.

We have proposed a simple criterion for a solution to have the \( n^3 \) near-extremal entropy. The compelling candidates are those whose harmonic functions depend on five coordinates transverse to the host brane worldvolume. In the spherical coordinate system it means that the harmonic functions will have \( r^{-3} \)-dependence.

Implementing dimensional reduction on the type IIA black D4 branes solution and taking the criterion as a guideline, we have established that black-brane solutions corresponding to 9D D3, 8D D2, 7D D1 and 6D D0 satisfy the criterion and all share the \( n^3 \) entropy growth. In fact, some of these solutions were previously known. For instance, the solution of 8D D2 black branes is similar to the purely magnetic D2 brane solution of [47].

Although some of the black-brane solutions constructed here share the \( n^3 \) entropy with the M5 solution of Klebanov-Tseytlin, the former differ from the latter in the following aspect. As noted in [16], the equations of state of non-dilatonic black branes in the near-extremal limit can be reproduced\(^{10}\) in terms of a weakly-interacting ideal gas of massless particles that are associated with massless excitation of \( p \)-brane modes. This is not the case for the descents of black M5 brane solutions constructed here, starting with the D4 black branes of 10D IIA. These descents are dilatonic branes; non-perturbative corrections become important in \( r \to 0 \) limit.

\(^{10}\)This is with an exception of the M2 case for which one cannot find an agreement between the Bekenstein-Hawking and statistical entropy.
One novelty of the KKW/M2/M5 solution is thermodynamics “duality”; there are two ways to scale the parameters such that the thermodynamics of the two scalings toggles under the interchange of $n_5 \leftrightarrow n_0$. The novelty may be related to the fact that it was constructed out of a M2/M5 solution that has only a single harmonic function. Perhaps the “duality” is an indication toward the supergravity analogue of the fact that the SYM account of D-branes physics admit “dual” description, one in terms of the lower dimensional branes and the other in terms of higher (see, e.g., [39] and refs therein).

We have also noticed that the ratio of the shear viscosity to entropy density of black M5 branes saturates the bound of the universal $\eta/S$ relation conjectured in [61]. The same bound value is expected for the strongly-coupled 5D supersymmetric plasma at nonzero temperature due to the similarity of $S(T)$ near-extremal dependence and the hydrodynamical limit [31] shear viscosity of D4 branes with those of M5 branes. It would be interesting to establish the $\eta/S$ ratio for descents of D4 branes with the $n^3$ growth, and to verify the universal relation conjecture.

String corrections to the Einstein gravity action may provide additional information on the subleading terms in the universal $\eta/S$ relation of D4 branes, reflecting, in particular, the contribution of the hidden degrees of freedom of 5D supersymmetric gauge theory as well as its 6D cousin. Since a D4-brane is an example of non-conformal theory, string corrections on the SYM side will provide a new test on AdS/CFT correspondence. In our opinion, this is a test of fundamental importance, and deserves further studies. We hope to report on this and other issues in the future.

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Appendix A: The ADM mass, the entropy and the Hawking temperature of black branes

A most general line element used throughout the paper has the following form

$$ds^2 = -B^2(r)dt^2 + C^2(r)\sum_{k=1}^{p-\bar{q}} dx_k^2 + D^2(r)\sum_{i=1}^{\bar{q}} dx_i^2 + E^2(r)\sum_{j=1}^{q-\bar{q}} dx_j^2 + F^2(r)dr^2 + G^2(r)r^2d\Omega_{d+1}^2,$$

(A.1)

where \( p + (q - \bar{q}) + d + 3 = D \), \( D \) is the number of space-time dimensions, \( p \), \( q \), and \( \bar{q} \) are the number of spatial dimensions of intersecting \( p \)- and \( q \)-branes (we implicitly assume that \( p \geq q \)), and \( \bar{q} \in \mathbb{Z} \) is the number of dimensions within the intersection. Configurations of coincident non-intersecting black branes correspond to \( q = \bar{q} = 0 \). Functions \( B(r), C(r), D(r), E(r), F(r), G(r) \) are some powers of the harmonic function(s) \( H(r) = 1 + h^d/r^d \) of the space transverse to the \( p \)-brane \( ^{11} \). \( B(r), F(r) \) also contain the “black factor” \( f(r) = 1 - \mu^d/r^d \), setting the horizon at \( r_H = \mu \).

We are mainly interested in computing the entropy-temperature dependence in the near-extremal limit \( \mu \ll 1 \). The entropy and the Hawking temperature for (A.1) are given by (see, e.g., [73])\(^{12} \)

$$S = \frac{[D-(d+3)]\Omega_d+1C^{p-\bar{q}}D^{\bar{q}}E^{q-\bar{q}}(Gr)^{d+1}|_{r=r_H}}{4G_D},$$

(A.2)

$$T = \frac{1}{2\pi}\sqrt{g_{rr}}\frac{d}{dr}\sqrt{-g_{tt}}|_{r=r_H}. \quad \text{(A.3)}$$

In addition, one needs to compute the ADM mass (see, e.g., [55, 76])

$$M = \frac{[D-(d+3)]\Omega_d+1}{16\pi G_D} \left[ (d+1)r^d(F^2-G^2) - (d+1)r^{d+1}(G^2)' \right].$$

\(^{11}\)We choose the spherical coordinate system to parameterize the transverse space; the coordinates consist of the radial coordinate \( r \) and the \( d + 1 \) angles.

\(^{12}\)Recall that the Hawking temperature expression (A.3) is correct only for metrics with flat space-time limit (under \( r \to \infty \)). A case of a non-flat space-time limit and computation of its Hawking temperature can be found in [75].
$$-i^{d+1}\{ (p - \bar{q})(C^2)' + \bar{q}(D^2)' + (q - \bar{q})(E^2)' \}_{|r \to \infty}.$$  \hfill (A.4)

**Appendix B: Basics of toroidal dimensional reduction**

We give a brief summary of toroidal dimensional reduction from $D$ to $(D-1)$-dimensional space-time (see, e.g., [77–79]).

The Kaluza-Klein ansatz for a $D$-dimensional metric is

$$ds_D^2 \equiv g^{(D)}_{MN} dx^M dx^N = e^{2\alpha\phi(D-1)} g^{(D-1)}_{mn} dx^m dx^n + e^{2\beta\phi(D-1)} \left( dx^b + A^{(D-1)}_m dx^m \right)^2,$$

(B.1)

where $M, N, \ldots$ run over $0, 1, \ldots, D-1$; $m, n, \ldots$ run over $0, 1, \ldots, D-2$; $x^b$ is the direction of the reduction. $\phi(D-1)$ and $A^{(D-1)}_m$ are the dilaton and the Kaluza-Klein vector field in $(D-1)$ space-time dimensions. They come from $D$-dimensional metric $g^{(D)}_{MN}$. The signature of the KK metric (B.1) is chosen to be mostly positive.

Dimensional reduction of the Einstein part of a $D$-dimensional supergravity action in the Einstein frame leads to

$$\frac{1}{2k_D^2} \int d^Dx \sqrt{-g^{(D)}} R^{(D)} = \frac{1}{2k_{D-1}^2} \int d^{D-1}x e^{(D-3)\alpha + \beta} \phi \sqrt{-g^{(D-1)}} \times$$

$$\times \left[ R^{(D-1)} + \alpha(D-2) \left[ \alpha(D-3) + 2\beta \right] (\partial\phi)^2 - \frac{1}{4} e^{2(\beta-\alpha)} \left( F^{(D-1)}_{[2]} \right)^2 \right], \hfill (B.2)$$

$$F^{(D-1)}_{[2]} = dA^{(D-1)}_{[1]}.$$  \hfill (B.3)

Both to get the gravity part of $(D-1)$-dimensional action and to reproduce the right factor $-1/2$ in front of the $(D-1)$-dimensional dilaton kinetic term, one fixes

$$(D-3)\alpha + \beta = 0, \quad \alpha^2 = \frac{1}{2(D-2)(D-3)}.$$  \hfill (B.4)

In the string frame, $(D-1)$ reduced action is associated with the following choice

$$(D-3)\alpha + \beta = -2, \quad \alpha(D-2) [4 + \alpha(D-3)] = -4.$$  \hfill (B.5)
To avoid ambiguities we always choose the negative $\alpha$ first, and fix the corresponding $\beta$ afterwards.

On account of the KK ansatz (B.1), it is easy to recover the dimensionally reduced M-brane solutions. Let us describe the first descent of $D = 11$ $n$ M5 black brane solution as an example; other lower dimensional brane configurations considered in the paper can be obtained in a similar manner.

Start with the M5s non-extremal solution (2.1)

$$ds_{11}^2 = H^{-1/3}(r) \left[ -f(r)dt^2 + dx_1^2 + \ldots + dx_5^2 \right] + H^{2/3}(r) \left( \frac{dr^2}{f(r)} + r^2d\Omega_4^2 \right),$$  

and substitute it into the l.h.s. of (B.1). Fixing the reduced coordinate to be $x_5 = x^9$ (which corresponds to the double dimensional reduction of M5 branes), one gets

$$e^{2\beta\phi_{(10)}} = H^{-1/3}, \quad A_{[1]}^{(10)} = 0.$$  

Another part of the ten-dimensional solution can be recovered from

$$ds_{10}^2 = e^{-2\alpha\phi_{(10)}} \left[ H^{-1/3}(r) \left[ -f(r)dt^2 + dx_1^2 + \ldots + dx_4^2 \right] + H^{2/3}(r) \left( \frac{dr^2}{f(r)} + r^2d\Omega_4^2 \right) \right].$$  

From (B.4), (B.5), (B.7), one gets

$$ds_{10}^2 = H^{-3/8} \left[ -f(r)dt^2 + dx_1^2 + \ldots + dx_4^2 \right] + H^{5/8}(r) \left( \frac{dr^2}{f(r)} + r^2d\Omega_4^2 \right),$$  

$$e^{\phi_{(10)}} = H^{-1/4}$$  

in the Einstein frame, and

$$ds_{10}^2 = H^{-1/2} \left[ -f(r)dt^2 + dx_1^2 + \ldots + dx_4^2 \right] + H^{1/2}(r) \left( \frac{dr^2}{f(r)} + r^2d\Omega_4^2 \right),$$  

$$e^{\phi_{(10)}} = H^{-1/4}$$  

in the string frame. Harmonic functions $H(r)$ and $f(r)$ are the same as in (2.2). The reduction of the gauge sector of the M5s solution (2.3) goes as
follows

\[ C_{[3]}^{(11)} = C_{[3]}^{(10)} + B_{[2]}^{(10)} \wedge dx^b \rightarrow F_{[4]}^{(11)} = F_{[4]}^{(10)} + F_{[3]}^{(10)} \wedge (dx^b + A_{[4]}^{(10)}), \quad (B.13) \]

\[ F_{[4]}^{(10)} = dC_{[3]}^{(10)} - F_{[3]}^{(10)} \wedge A_{[1]}^{(10)}, \quad F_{[3]}^{(10)} = dB_{[2]}^{(10)}, \]

and leads, with non-trivial ansatz for \( C_{[3]}^{(10)} \) gauge field, to

\[ F_{[4]}^{(10)} = 3h_0^3 \left( 1 + \frac{\mu^3}{h_0^3} \right)^{1/2} \epsilon_4. \quad (B.14) \]

Solutions (B.9)–(B.12), (B.14) determine the background of 10D \( n \) coincident black D4 planes in different supergravity frames (cf., e.g., [73]).
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