Flagmill -A new power generator utilizing flexible sheet-

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Abstract
The present study proposes “a flagmill”, which is a new power generator utilizing the flutter of a flexible sheet such as a flag. The sheet flutter causes the angular oscillation of the supporting pole. Since the supporting pole is connected to the power generator axis, the angular oscillation produces the electromotive power. The flutter and the power generation characteristics were investigated experimentally and analytically. The flutter occurs by decreasing the relative stiffness or increasing the mass ratio. The electromotive force cause the increase of the critical flutter speed. The higher flutter frequency and the larger amplitude of the angular oscillation of the sheet leading edge can generate the larger electromotive power.

Key words: Flexible sheet flutter, Power generation, Flutter frequency, Amplitude of the angular oscillation

1. Introduction

The flexible sheet flutter is often observed such as a flag flapping in the wind. Once the flow speed exceeds a critical one, the fluid energy continuously pumps into the flag oscillation, which is called a negative damping, and accordingly the sheet oscillation is sustained. This phenomenon is called a self-excited oscillation. The flutter occurs in not only sheets such as flags, but also the plates like objects such as wings of airplanes, various blades in turbomachines and so on. It is quite well-known that the original Tacoma Narrows bridge (1940) was collapsed owing to the torsional flutter. In general, the flutter has been dealt with as problems which cause destructions of various machines and structures. On the other hand, the researches utilizing the flutter of flags or wings for the power generation have also been existed.

Adamko et al. (1978) and Mckinney et al. (1981) proposed the concept of the power generation utilizing a rigid airfoil flutter. The flutter is maintained by the mechanical link system so as to keep the heaving and pitching motions with an appropriate phase difference. Isogai et al. (2003) proposed a power generation concept by using a flapping wing. The heaving oscillation of a wing is excited by the lift induced by the forced pitching oscillation. Bryant et al. (2011) investigated the piezoelectric energy harvester driven by the flutter of a rigid wing pinned at a beam end. A pair of piezoelectric patches is pasted on the beam root upper and lower surfaces. When the rigid wing flutters, the bending deflection of the cantilevered beam occurs, and consequently produces the electromotive power by the piezoelectric patches. Tang et al. (2009) applied a flexible sheet flutter to the power generation. A flexible sheet with embedded conductors is placed between two magnetic panels. The change of the magnetic flux caused by the flutter motion generates the electromotive power.

The present study introduces “a flagmill”, which is a new power generator utilizing the flexible sheet flutter. The sheet is simply supported by a supporting pole. The sheet flutter causes the angular oscillation of the supporting pole. Since the supporting pole is connected to the power generator axis, the angular oscillation produces the electromotive power. Compared to the flutter power generators mentioned above, the present one might become simpler and more portable. By the simplicity and portability, the new applications of the wind power systems such as disaster areas and outdoor activities will be expected. In the present study, the flutter and the power generation characteristics were investigated experimentally and analytically.
2. Nomenclature

\( A_0 \): dimensional amplitude of sheet displacement at 20mm downstream of sheet leading edge [m]

\( B \): nondimensional complex coefficient matrix [-]

\( c_s \): nondimensional electromotive coefficient shown in Eq.(22) [-]

\( c_c \): nondimensional skin friction coefficient defined by Eq.(5) [-]

\( c_{m_s} \): nondimensional structure damping coefficient shown in Eq.(18) [-]

\( c_l \): nondimensional total damping coefficient defined by Eq.(25) [-]

\( E_s \): dimensional Young’s modulus [Pa]

\( E_{rms} \): dimensional root mean square of electromotive power [W]

\( e_{rms} \): root mean square of nondimensional electromotive power [-]

\( f_{c} \): dimensional flutter frequency of maximum amplitude of angular oscillation [s^{-1}]

\( H \): dimensional thickness of sheet [m]

\( I_c \): nondimensional current [-]

\( I_s \): dimensional second moment of inertia per unit width of sheet [m^3]

\( j \): imaginary unit in time [-]

\( k_s \): nondimensional electromotive force coefficient shown in Eq.(21) [-]

\( k_q \): nondimensional torque coefficient shown in Eq.(19) [-]

\( k_m \): nondimensional stiffness coefficient shown in Eq.(18) [-]

\( L \): dimensional length of sheet [m]

\( L_s \): dimensional distance between sheet leading edge and measurement point for \( A_0 \) [m]

\( m \): total panel number [-]

\( n_m \): nondimensional inertia moment coefficient shown in Eq.(18) [-]

\( \Delta p \): nondimensional pressure difference between upper and lower surfaces of a sheet, calculated by Eq.(6) [-]

\( R \): nondimensional resistance of generator [-]

\( T \): nondimensional period of angular oscillation [-]

\( T_m \): nondimensional measurement time [-]

\( t \): nondimensional time [-]

\( U \): dimensional uniform wind speed [m/s]

\( V_c \): nondimensional electromotive force [-]

\( W \): dimensional width of sheet [m]

\( \nu_0 \): nondimensional perpendicular velocity on sheet, calculated by Eq.(8) [-]

\( x \): nondimensional coordinate in \( x \) direction [-]

\( x_s \): nondimensional coordinate in \( x \) direction at \( k \)-th panel [-]

\( y \): nondimensional sheet displacement [-]

\( y_s \): complex amplitudes of nondimensional sheet displacement at \( k \)-th panel [-]

\( \beta \): relative stiffness defined by Eq.(3) [-]

\( \gamma_s \): nondimensional complex amplitudes of distributed circulation of bound vortex at \( k \)-th panel [-]

\( \gamma_c \): nondimensional distributed circulation of bound vortex [-]

\( \gamma_f \): nondimensional distributing circulation of free vortex [-]

\( \xi \): nondimensional vortex location in \( x \) coordinate [-]

\( \eta_p \): nondimensional pivot moment due to electromotive force [-]

\( \eta_{eff} \): efficiency of power generation shown in Eq.(48) [-]

\( \eta \): nondimensional pivot moment due to sheet flutter at leading edge [-]

\( \theta \): angle of sheet at leading edge [rad], nondimensional parameter

\( \chi \): nondimensional tension force defined by Eq.(4) [-]

\( \mu \): mass ratio defined by Eq.(2) [-]

\( \rho \): dimensional density of fluid [kg/m^3]

\( \rho_s \): dimensional density of sheet [kg/m^3]

\( \tau \): dimensional skin friction [N/m^2]
Fig. 1 Sketch of the experimental setup. The supporting pole is arranged vertically. The uniform wind speed, the sheet displacement and the motion pictures were obtained by using the pitot tube, the laser displacement sensor and the high-speed camera, respectively.

Table 1 Material and geometrical properties of flexible sheets

| Material                      | Density [kg/m³] | Young’s modulus [GPa] | Width [mm] | Thickness [mm] | Length [mm] |
|-------------------------------|-----------------|-----------------------|------------|----------------|-------------|
| Polyethylene terephthalate (PET) | 1278            | 2.12                  | 100        | 0.5            | 100–300     |
| Polystyrene (PS)              | 1151            | 1.76                  |            |                |             |
| Polyvinyl chloride (PVC)      | 1369            | 1.44                  |            |                |             |
| Polypropylene (PPP)           | 904             | 1.61                  |            | 0.2            |             |

ω: complex number of nondimensional angular velocity [-]
ω₁: nondimensional angular velocity for first bending mode of pivoted-free beam in vacuum, calculated by Eq.(39) [-]
ω₂: nondimensional angular velocity of sheet flutter, calculated by Eq.(38) [-]
ωᵢₘ: imaginary part of complex number of nondimensional angular velocity [-]
ωᵣₑ: real part of complex number of nondimensional angular velocity [-]

Superscripts
~: infinitesimal complex amplitude
^: finite amplitude

3. Experimental Method

Figure 1 shows the experimental setup of the test section of the blowdown wind tunnel. The test section has a 600x600 mm cross section. The dimensional uniform wind speed $U$ was varied by the blower rotation controlled by an inverter. The outlet is located at 1700mm downstream of the supporting pole. The supporting pole is arranged vertically so that the gravitational effect on the sheet bending becomes minimum. A flexible sheet was clamped to the supporting pole. The supporting pole was held by two bearings so that it can rotate around its axis. As a result, the flexible sheet can also pivot around the supporting pole axis.

As the wind speed is increased, the sheet suddenly starts to flutter at a wind speed. This wind speed indicates one flutter boundary. On the other hand, as wind speed is decreased, the sheet flutter suddenly ends at a wind speed. This
wind speed indicates another flutter boundary. The former flutter speed is larger than the latter one. In the present study, the latter flutter speed is adapted. Here, although the torsional flutter modes were observed for quite higher wind speeds, the torsional flutter modes were not dealt with in the present paper.

A miniature motor (Tamiya, Hyper Dash Motor Pro) was used for an electromotive power generator. Since the angular oscillation of the supporting pole can rotate the generator axis, it can generate the electromotive power. The electromotive force and current were measured by digital multimeters (ADCMT, ADCMT-7461A). The wind speed was measured by using a pitot tube located at 1400mm upstream of the supporting pole. The sheet displacement $A_d$ at $L_d$ of 20mm downstream of the supporting pole was measured by using a laser displacement sensor (Keyence LK-G150). The bottom wall of the test section was made of a transparent acrylic resin plate so that the temporal sheet deformation could be observed. To obtain the motion pictures of the sheet flutter, a high-speed camera (IDT JAPAN N4S3) was used with the frame rate of 1000fps. The sheet deformations were analyzed by the image processing of the motion pictures. The time histories of the electromotive force, the current, the displacement and the movie were synchronously obtained by using a digital measurement and controlling system (Micro Science, LabDAQ). The tested material and geometrical properties of the flexible sheets are shown in Table.1.

4. Analytical Method

4.1 Governing equation for sheet flutter

To predict the flutter characteristics including the flutter boundaries, flutter frequencies and flutter modes, the two-dimensional linear stability analysis was performed. This is similarly to the analysis of Yokota et al. (1999) which dealt with the sheet flutter for a cantilevered beam at the trailing edge in the uniform flow. In the present study, the aspect ratios of all tested sheets are larger than unity. It has been reported that the three-dimensional effects for the sheet flutter with aspect ratios larger than unity are not significant (Eloy et al., 2008). In addition, the sheet displacements observed in the experiment were almost uniform in the width direction. Moreover, the two dimensional sheet flutter analytical results obtained by Guo and Paidoussis (1999), Yamaguchi et al. (2000), Watanabe et al. (2002), and Tang and Paidoussis (2007) show the qualitative agreement with the experimental ones as well as the present results. Based on these three reasons, it is considered that the three-dimensional effects is small in the present study.

Figure 2 shows the schematic of the sheet deformation and the vortex sheet distributions in the present analysis. The following assumptions are adopted.

1. The fluttering motion of a sheet is uniform in the width direction, and consequently the flow is two dimensional.
2. The sheet deformation and the flow disturbance are infinitesimal, and therefore the second or higher order quantities of them are negligible.
3. The flow is potential, therefore the vortex sheet consists of a bound vortex representing the sheet and a free vortex representing the wake shed from the sheet trailing edge.
4. The viscous effect appears as a tension force acting on upper and lower surfaces of a sheet.

As a result, the deformation equation is the elastic one of a beam with a tension force and a pressure difference between the upper and lower surfaces of a sheet. Most of the variables are nondimensionalized by using the dimensional flow density $\rho$, the dimensional uniform wind speed $U$ and the dimensional sheet length $L$. The adjective “nondimensional”
is omitted for the nondimensional variables. The adjective “dimensional” is written for the dimensional variables. The fluttering motion and the flow are two dimensional, and therefore the sheet deformation equation per unit width, nondimensionalized by $\rho U^2$, is expressed as

$$
\mu \frac{\partial^2 y}{\partial t^2} + \beta \frac{\partial^4 y}{\partial x^4} - \frac{\partial}{\partial x} \left( \chi \frac{\partial y}{\partial x} \right) = \Delta p,
$$

(1)

where

$$
\mu = \frac{\rho_s H}{pL}, \quad \beta = \frac{E_s I_s}{\rho U^2 L^5}.
$$

Here, $y$ is the sheet displacement at $x$, $\chi$ the tension force, $\Delta p$ the pressure difference. In addition, $\rho_s, E_s, H$ and $I_s$ represent the dimensional sheet density, Young’s modulus, sheet thickness and second moment of the inertia per unit width, respectively. Correspondingly, the sheet exists in the region of $0 \leq x \leq 1$. The mass ratio $\mu$ means the ratio of the sheet inertia to the flow one and the relative stiffness $\beta$ the ratio of the sheet bending stiffness to the aerodynamic force.

### 4.2 Tension force

The tension force $\chi$ is due to the skin friction $\tau$ acting on upper and lower surfaces of a sheet. Therefore, it is given in an integral form of

$$
\chi = 2 \int_0^1 \frac{dx}{\rho U^2} = \int_0^1 c_{\tau} dx = c_{\tau}(1 - x),
$$

(4)

where

$$
c_{\tau} = \frac{\tau}{\frac{1}{2} \rho U^2}.
$$

### 4.3 Pressure difference between upper and lower surfaces

By integrating the $x$-direction Euler equation from infinite upstream of $x$, the pressure difference $\Delta p$ between the upper and lower surfaces of a sheet can be expressed as

$$
\Delta p = \Delta p(x, t) = \int_0^1 (\gamma_s(\xi, t) + \frac{\partial y}{\partial t} d\xi).
$$

(6)

Here, $\xi$ is the coordinate of the vortex location in $x$ direction and $\gamma_s$ represents the distributed circulation of the bound vortex representing the sheet.

### 4.4 Kinematic boundary condition

Since the flow is potential, the slip condition should be applied as the velocity boundary condition on a sheet surface. The slip condition on the moving boundary is also known as the kinematic boundary condition. Neglecting the second orders of infinitesimal variables, the slip condition on the deforming sheet is given by

$$
\frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} = v_0, \quad \text{for} \quad 0 \leq x \leq 1.
$$

(7)

Here, the subscript zero represents the value on the $x$-axis.

### 4.5 Induced velocity on sheet vortex and shedding vortex

The infinitesimal induced velocity $v_0$ on the $x$-axis can be expressed as the integral of distributed circulations on the $x$-axis representing the sheet and the wake as
Here, $w$ indicates the distributing circulation of the free vortex representing the wake. Based on the assumption (3), the wake consists of the vortex sheet, which is shed from the sheet trailing edge and carried away with the free stream speed. Therefore, the distributing circulation of the wake $w_s(x, t)$ was shed at the time of $t-(\xi-1)$ from the trailing edge at $\xi=1$. As a result, the relation between $s$ and $w$ is given by

$$\gamma_s(x, t) + \gamma_w(x, t) = \gamma_s(1, t-\xi+1)$$

(9)

### 4.6 Governing equation and kinematic boundary condition for sheet flutter

Expressing the governing equations by $y$ and $s$, Eq.(1) becomes

$$\mu \frac{\partial^2 y}{\partial t^2} + \beta \frac{\partial^4 y}{\partial x^4} + \gamma \frac{\partial y}{\partial x} + c_t (x-1) \frac{\partial^2 y}{\partial x^2} + \int_0^\infty \frac{\partial^2 y_s(\xi, t)}{\partial t^2} d\xi + \gamma_s(x, t) = 0$$

(10)

Similarly, the kinematic boundary condition becomes

$$\frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} = \frac{1}{2\pi} \int_0^1 \gamma_s(x-\xi, t) d\xi + \int_0^\infty \frac{\gamma_s(1, t-\xi+1)}{x-\xi} d\xi = 0.$$  

(11)

The governing equations (10) and (11) are the simultaneous equations with respect to $y$ and $s$.

### 4.7 Kutta’s condition

Since the kinematic boundary condition is a first order differential equation in terms of $x$, one boundary condition is necessary in order to determine one solution. For two dimensional objects with no separation and vortex shedding at the trailing edge, the Kutta’s condition is usually adopted. By putting $x=1$ and $\Delta p=0$ into Eq.(6), the Kutta’s condition can be easily obtained as

$$\int_0^1 \frac{\partial \gamma_s(\xi, t)}{\partial t} d\xi + \gamma_s(1, t) = 0.$$  

(12)

By substituting $\xi=1$ to Eq.(9), $\gamma_s(1, t)$ is the equal to $\gamma_w(1, t)$. Thus, Eq.(12) is expressed as

$$\frac{\partial}{\partial t} \int_0^1 \gamma_s(\xi, t) d\xi + \gamma_w(1, t) = 0.$$  

(13)

Since the first term represents the temporal variation of the circulation of the sheet and the second term represents the circulation of the vortex shed from the trailing edge, this equation means the circulation conservation, i.e. the Kelvin’s circulation theorem for the vortex shedding at the trailing edge.

### 4.8 Pivot support and free boundary conditions

Since the deformation equation is a fourth order differential one, four boundary conditions are necessary in order to determine one solution. The pivot support for the leading edge and the free end for the trailing edge were applied. Their conditions are expressed as

$$y(0, t) = 0,$$

(14)

$$\frac{\partial^2 y}{\partial x^2}(0, t) = \eta_1(t),$$

(15)

$$\frac{\partial^2 y}{\partial x^2}(1, t) = 0,$$

(16)

and
\[ \frac{\partial^3 y}{\partial x^3}(1,t) = 0. \]  

(17)

Here, \( \eta_t(t) \) represents the pivot moment.

### 4.9 Angular momentum equation of supporting pole

Based on the equation of the torsional oscillation with the electromotive torque \( \eta_e(t) \), \( \eta(t) \) is given by

\[
\eta_t(t) = n_m \frac{d^2 \theta(t)}{dt^2} + c_m \frac{d\theta(t)}{dt} + k_m \theta(t) + \eta_e(t). 
\]

(18)

Here, \( n_m, c_m \) and \( k_m \) are a inertia moment coefficient, a structure damping coefficient and a stiffness coefficient, respectively.

Based on the Lorentz force, the electromotive torque \( \eta_e \) is given by

\[
\eta_e(t) = k_q I_c. 
\]

(19)

Here, \( k_q \) and \( I_c \) are a torque coefficient and a current, respectively.

Applying Ohm’s law to Eq.(19),

\[
\eta_e(t) = k_q \frac{V_e(t)}{R}. 
\]

(20)

Here, \( V_e \) and \( R \) are an electromotive force and a resistance of generator, respectively.

The electromotive force \( V_e \) can be written by the Faraday’s law as

\[
V_e(t) = k_e \frac{d\theta(t)}{dt}. 
\]

(21)

Here, \( k_e \) is an electromotive coefficient. From Eqs. (19)–(21), we obtain

\[
\eta_e(t) = k_q k_e \frac{d\theta(t)}{dt} = \frac{k_q k_e}{R} \frac{d\theta(t)}{dt}. 
\]

(22)

Here, \( c_e \) is an electromotive coefficient.

Since an angle of the sheet leading edge \( \theta \) is infinitesimal, \( \theta \) can be expressed as

\[
\theta(t) = \tan^{-1} \frac{\dot{\gamma}_y}{\dot{\gamma}_x}(0) \approx \frac{\dot{\gamma}_y}{\dot{\gamma}_x}(0). 
\]

(23)

By putting Eqs.(22) and (23) into Eq.(18),

\[
\eta_t(t) = n_m \frac{\partial^3 y}{\partial x^3}(0) + c_1 \frac{\partial^2 y}{\partial t \partial x}(0) + k_m \frac{\partial y}{\partial x}(0) 
\]

(24)

where

\[
c_1 = c_m + c_e. 
\]

(25)

In the present study, the total damping coefficient \( c_1 \) is used for evaluation of the effect of the electromotive force on the flutter characteristics.

### 4.10 Procedure for calculation

The deformation, the distributed circulation and the pivot moment are expressed by the complex exponential function as

\[
y(x,t) = \tilde{y}(x) e^{i \omega t},
\]

(26)

\[
\gamma_s(x,t) = \tilde{\gamma}_s(x) e^{i \omega t},
\]

(27)

and
Here, \( j, \omega, \tilde{y}(x), \tilde{s}(x) \) and \( \tilde{\eta} \) denote the imaginary unit, the complex angular velocity of the harmonic oscillation, the infinitesimal complex amplitudes of the deformation at \( x \), the infinitesimal complex amplitudes of the circulation at \( x \) and the infinitesimal complex amplitudes of the pivot moment, respectively. By putting \( \alpha_k = \omega \alpha_m \) into \( \alpha_k \), Eqs. (26)–(28) become, respectively,

\[
y(x,t) = \tilde{y}(x)e^{-\alpha_m t}e^{j\alpha_m t},
\]

\[
\gamma_s(x,t) = \tilde{\gamma}_s(x)e^{-\alpha_m t}e^{j\alpha_m t},
\]

and

\[
\eta(t) = \tilde{\eta}e^{-\alpha_m t}e^{j\alpha_m t}.
\]

These notations clearly indicate that the real and imaginary parts of \( \omega \) represent the angular velocity and the damping rate of the oscillation, respectively. For \( \alpha_m < 0 \), the infinitesimal amplitudes grow, which means the flutter. The zero damping rate indicates the flutter boundary. By substituting Eqs. (26)–(28) to the governing equations and the boundary conditions, they are transformed into

\[
-\mu \omega^2 \tilde{y} + \beta \frac{\partial^4 \tilde{y}}{\partial x^4} + c_r \frac{\partial \tilde{y}}{\partial x} + c_t (x - 1) \frac{\partial^2 \tilde{y}}{\partial x^2} + j \omega \int_{0}^{1} \gamma_s(\xi)d\xi + \tilde{\gamma}_s = 0,
\]

\[
j \omega \tilde{y} + \frac{\partial \tilde{y}}{\partial x} - \frac{1}{2\pi} \int_{0}^{1} \left( \frac{\partial^2 \gamma_s}{\partial x^2}(0) \right) - \tilde{\gamma}_s(1) \frac{e^{-j\omega(\xi-1)}}{x-\xi}d\xi = 0,
\]

\[
\tilde{y}(0) = 0, \quad \frac{\partial^2 \tilde{y}}{\partial x^2}(0) = (-\omega^2 n_m + j \omega c_t + k_m) \frac{\partial \tilde{y}}{\partial x}(0), \quad \frac{\partial^2 \tilde{y}}{\partial x^2}(1) = \frac{\partial^2 \tilde{y}}{\partial x^2}(1) = 0,
\]

and

\[
j \omega \int_{0}^{1} \gamma_s(\xi)d\xi + \tilde{\gamma}_s(1) = 0.
\]

Due to the last term in Eq. (33), the solutions can be obtained only for the negative damping and the zero damping.

In order to solve these equations numerically, the sheet is divided into \( m \) elements, which are called the panel for the circulation in the vortex method. The differential terms are discretized by the fourth order finite difference method and the integral terms by the Euler’s trapezoidal rule. As a result, the homogeneous simultaneous equations with the \( 2(m+1) \) variables are derived as

\[
B(\omega) \begin{bmatrix} y_k \\ \gamma_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

Here, \( y_k = \tilde{y}(x_k) \) and \( \gamma_k = \tilde{\gamma}_s(x_k) \) are complex values of \( \tilde{y} \) and \( \tilde{\gamma}_s \) at \( x = x_k = k/m \), \( 0 \leq k \leq m \). In the present study, the panel number of \( m = 100 \) is adopted.

In order that there exists a non-trivial solution, the determinant of the complex coefficient matrix \( B(\omega) \) must be zero, i.e.

\[
\det B(\omega) = 0.
\]

This represents the algebraic equation in terms of \( \omega \).
5. Results and Discussions

5.1 Flutter boundaries and critical frequencies on boundaries

Figure 3 shows the mass ratio and the relative stiffness on the flutter boundaries. The abscissa and the ordinate show the mass ratio and the relative stiffness, respectively. The uncertainties of the relative stiffness caused by the sheet bending stiffness are shown by the error bar. The solid line shows the flutter boundary calculated by neglecting the friction and damping coefficients. The short and long dashed lines indicate the flutter boundaries calculated by considering the friction and damping coefficients, respectively. The open and closed symbols mean the 1st and 2nd flutter boundaries obtained from the experiment, respectively. The upper and lower regions of all symbols and lines mean stable and flutter, respectively. Both flutter boundaries of the experimental and analytical results indicate the positive slopes in $\mu<8.0$. This means that the flutter occurs by decreasing the relative stiffness or increasing the mass ratio. In $\mu<0.61$, the 1st mode is seen. In $0.13<\mu<0.61$, the 2nd mode is found. In $\mu<0.13$, the 3rd mode for the analytical result appears. The present flutter boundaries obtained by using the pivot boundary condition at the leading edge are similar to those of the reference results (Yamaguchi et al., 2000) for a cantilevered one at the leading edge. This means that the boundary condition at the leading edge does not significantly affect the flutter boundary. Comparing the short dashed line with the solid line, the skin friction coefficient $c_t$ enlarges the stability region of the 2nd mode. Due to the skin friction coefficient, the analytical results approach the experimental ones. Comparing the long dashed line with the solid line, the damping coefficient $c_t$ corresponding to the electromotive force makes the 1st flutter mode stable. This means that the electromotive force causes the increase of the critical flutter speed. Although the inertia momentum coefficient $n_m$ and the stiffness coefficient $k_m$ shown in Eq. (24) were also examined, their effects on the flutter boundary were not significant, which is not shown here.

Figure 4 shows the frequency ratios $\omega_k/\omega_1$ and $\omega_k/\omega_3$ against the mass ratio. The angular velocity $\omega_k$ is expressed as

$$\omega_k = 2\pi f_e \frac{L}{U}. \tag{38}$$

Here, $f_e$ is the dimensional frequency where the amplitude of the angular oscillation of the sheet leading edge is maximum. The angular velocity for the 1st bending mode of a pivoted-free beam in vacuum $\omega_1$ is calculated by

$$\omega_1 = \frac{3.9^2}{L^2} \sqrt{\frac{E I_s}{\rho_s H_s}} \times \frac{L}{U} = 3.9^2 \sqrt{\frac{\beta}{\mu}}. \tag{39}$$
All symbols and lines are the same ones as shown in Fig. 3. As the mass ratio decreases, the flutter ratios increase due to the mode transition to the higher flutter mode.

5.2 Flutter frequency and amplitude of angular oscillation

The flutter frequency and the amplitude of the angular oscillation of the sheet leading edge are discussed at the wind speeds higher than the critical flutter speed. The results of the polypropylene (PPP) in the tested materials are shown in Figs 5–8.

Figure 5 shows the flutter frequencies $\omega_b$. The reciprocals of the mass ratio and the square root of the relative stiffness are expressed as

$$\frac{1}{\mu} = L \times \left( \frac{\rho}{\rho_s H} \right)$$

and

$$\frac{1}{\sqrt{\beta}} = U \times \left( \frac{\rho L^2}{E_s I_s} \right)$$

(40)

(41)

respectively. Thus, $1/\mu$ means a sheet length and $1/\sqrt{\beta}$ a wind speed. For $1/\mu=0.67$, as the wind speed increases, the sheet starts to flutter as the 1st mode, and the large increase of the flutter frequency is seen near “A” in Fig. 5. The visualization results for $1/\sqrt{\beta}=21$ and 22 near “A” are shown in Fig. 6. It can be seen that the sheet oscillates similar to the 1st and the 2nd modes as shown in Figs. 6(a) and (b), respectively. The mode transition causes the increase of the
flutter frequency.

For $1/\mu = 1.0$, the sheet starts to flutter as the 1st mode, similar to $1/\mu = 0.67$. The 1st mode is turned into the 2nd near “B”. The mode transition from the 1st mode to the 2nd mode is also confirmed by visualization.

For $1/\mu = 1.3$, the sheet starts to flutter as the 2nd mode near “C”. The 2nd mode is sustained within the tested wind speeds. This tendency is different from the results for $1/\mu = 0.67$ and 1.0.

The angle $\theta$ is obtained by

$$\theta = \tan^{-1}\left(\frac{A_d}{L_d}\right). \quad (42)$$

Here, $A_d$ is the dimensional amplitude of the sheet displacement at $L_d$ of 20mm downstream of the sheet leading edge.

Figure 7 shows the amplitude of the angular oscillation $\hat{\theta}$ at the flutter frequency $\omega_e$. As the wind speed increases, $\hat{\theta}$ drops near “A” for $1/\mu = 0.67$ and “B” for $1/\mu = 1.0$ in Fig.7. This is caused by the mode transition mentioned in Fig.5.

Here, for all tested cases, it is confirmed that $\hat{\theta}$ for the 1st mode is larger than that for the 2nd one.

Fig. 5 Flutter frequencies for PPP with various lengths. The reciprocal of the mass ratio and the reciprocal of the square root of the relative stiffness mean a sheet length and a wind speed, respectively. The mode transition causes the increase of the flutter frequency.

Fig. 6 Pictures during one period of flutter for PPP. The sheet oscillates like the first flutter mode at $1/\sqrt{\beta} = 21$ for $1/\mu = 0.67$. On the other hand, the sheet oscillates like the second flutter mode at $1/\sqrt{\beta} = 22$ for $1/\mu = 0.67$. 
3 Evaluation of electromotive power

Figure 8 shows the Root Mean Square (RMS) of the electromotive power obtained by

\[ e_{\text{rms}} = \sqrt{\frac{1}{T_m} \int_0^{T_m} (V_e(t)I_e(t))^2 \, dt} \]  \hspace{1cm} (43)

Here, \( T_m \) is the measurement time and the variation of resistance \( R \) is about 0.65\%. It can be observed that the RMS of the electromotive power for the 1st mode is larger than that for the 2nd one, corresponding to \( \dot{\theta} \) in Fig.7. This result indicates that the large amplitude of the angular oscillation can generate the large electromotive power.

Equation (43) is also expressed as
Since the angle of the sheet leading edge oscillates similar to the sinusoidal wave, the angular oscillation is assumed that

\[ \theta(t) = \hat{\theta} \sin \omega_c t. \]  

Here, \( \hat{\theta} \) is determined as the maximum amplitude of the angular oscillation in the spectrum such as Fig. 9(b) and \( \omega_c \) is its frequency.

By putting Eq. (46) into Eq. (45), the RMS of the electromotive power is rewritten as

\[ e_{\text{rms}} = \frac{k_e^2}{R} \left( \omega_c^2 \hat{\theta}^2 \right) \frac{1}{T} \int_0^T \left( \cos \frac{2\omega_c t + 1}{2} \right)^2 dt \approx \frac{k_e^2}{R} \left( \omega_c^2 \hat{\theta}^2 \right) \frac{1}{T} \int_0^T \left( \cos \frac{4\omega_c t + 4\cos 2\omega_c t + 3}{8} \right) dt. \]  

By the period integration, Eq. (47) becomes

\[ e_{\text{rms}} = \frac{3}{8} \frac{k_e^2}{R} \left( \omega_c^2 \hat{\theta}^2 \right). \]  

Figure 10 shows the relation between \( \omega_c^2 \hat{\theta}^2 \) and \( e_{\text{rms}} \) for all tested cases. The solid line indicates \( k_e^2 \sqrt{3/8} / R = 4.0 \times 10^6 \), which is obtained by the least squares approximation. It can be seen that \( e_{\text{rms}} \) is roughly proportional to \( \omega_c^2 \hat{\theta}^2 \).

The flutter frequency and the amplitude of the angular oscillation is related with the electromotive power.

Figure 11 shows the RMS of the dimensional electromotive power and the efficiency for PPP with \( L=0.1m \) and \( W=0.1m \) as the representative case. The efficiency of the power generation is defined as

\[ \eta_{\text{eff}} = \frac{E_{\text{rms}}}{\rho U^3 LW}. \]  

Here, \( E_{\text{rms}} \) is the RMS of the dimensional electromotive power and \( W \) is the dimensional width of the sheet. The maximum RMS of dimensional electromotive power and efficiency are about 0.74mW and 0.0025%, respectively. Since the present electromotive power is the same order as the piezo bimorph generator by Pobering and Schwesinger (2004), it is considered that the present results are reasonable with respect to the electromotive power.

6. Conclusions

The power generator by using the flexible sheet flutter was investigated by varying uniform flow speeds, sheet
lengths and sheet materials. The following conclusions are obtained.

(1) The flutter occurs by decreasing the relative stiffness or increasing the mass ratio.

(2) The electromotive force causes the increase of the critical flutter speed.

(3) The higher flutter frequency and the larger amplitude of the angular oscillation of the sheet leading edge can generate the larger electromotive power.

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