The $\gamma\gamma \rightarrow \gamma Z$ process at high energies and the search for virtual SUSY effects.

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Abstract

We study the helicity amplitudes and the observables of the process $\gamma\gamma \rightarrow \gamma Z$ at high energy. As in the case of the $\gamma\gamma \rightarrow \gamma\gamma$ process studied before, the relevant diagrams in the standard model (SM) involve $W$, charged quark and lepton loops; while in SUSY we also have contributions from chargino, and charged sfermion or Higgs loop diagrams. Above 250 GeV, the dominant SM amplitudes are themselves dominated by the $W$ loop, and as for $\gamma\gamma \rightarrow \gamma\gamma$, they are helicity conserving and almost purely imaginary. We discuss the complementary information provided by $\gamma\gamma \rightarrow \gamma Z$ for the identification of possible non standard effects. This process, together with $\gamma\gamma \rightarrow \gamma\gamma$, should provide very useful information on the nature of possible new physics particles, above the threshold of their direct production.

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1 Introduction

In previous papers [1, 2] we studied the process $\gamma\gamma \rightarrow \gamma\gamma$. The most striking property of this process in the Standard Model (SM), is that its whole set of possible helicity amplitudes is strongly dominated by just the three helicity conserving ones; which moreover are almost purely imaginary. This simple property offers new possibilities for improving the search for new physics (NP) at high energy. Some of these possibilities related to $\gamma\gamma \rightarrow \gamma\gamma$ have been investigated in the aforementioned papers. We now extend this study to the process $\gamma\gamma \rightarrow \gamma Z$.

In more detail, this remarkable property of the $\gamma\gamma \rightarrow \gamma\gamma$ processes is due to the fact that the Standard Model (SM) amplitude first appears at the 1-loop level and at high energies it is dominated by the $W$ loop contribution, which mainly enhances the imaginary parts of the three helicity non flip amplitudes. Thus in SM, this process is dominated by just a few almost purely imaginary helicity conserving amplitudes. As we will see in the present work, similar properties are also valid for the process $\gamma\gamma \rightarrow \gamma Z$ studied here.

This suggests to use the $\gamma\gamma \rightarrow \gamma\gamma, \gamma Z$ processes as a tool for searching for types of new physics characterized by amplitudes with a substantial imaginary part that can interfere with the SM one; like e.g. effects due to chargino or charged slepton loop diagrams above the threshold; $s$-channel resonance production; or new strong interactions inducing unitarity saturating contributions to the NP amplitudes.

These studies could be achieved at a future $e^+e^-$ Linear Collider (LC) [3] operating as a $\gamma\gamma$ Collider ($LC_{\gamma\gamma}$) whose c.m. energy may be variable and as high as 80% of the initial $e^+e^-$ c.m. energy, by using the laser backscattering technique [4, 5]. Polarized $\gamma\gamma$ beams can also be obtained using initially polarized electron beams and lasers.

This search for NP through its virtual effects, is complementary to the direct production of new particles and it should help identifying their nature; since it avoids the model-dependent task of studying their decay modes, once they are actually produced. More explicitly: the charged sparticle loop contribution to $\gamma\gamma \rightarrow \gamma\gamma, \gamma Z$, is independent of the many parameters entering their decay modes and determining e.g. the soft SUSY breaking and the possible $R$-parity violating sectors.

In the present paper we study in detail the $\gamma\gamma \rightarrow \gamma Z$ amplitudes in the standard and SUSY models. The idea is to confirm and improve the searches for NP signatures that can be done through direct production and the measurements of the $\gamma\gamma \rightarrow \gamma\gamma$ process. The situation for such measurements should be more favorable in $\gamma\gamma \rightarrow \gamma Z$ than in $\gamma\gamma \rightarrow \gamma\gamma$, because the cross section is larger by about a factor 6. Then, if a signal suspected in $\gamma\gamma \rightarrow \gamma\gamma$ is also seen here; the detailed properties of the SM departures, which now depend on the occurrence of the $Z$ couplings, should allow some identification of the nature of the effect. In particular it should help identifying the $SU(2) \times U(1)$ quantum numbers of the new physics particle contributing virtually.

In Sec.2 we discuss the main properties of the $W$, fermion and scalar loop contributions at high energies, which had not been fully analyzed before. This allows us to predict the type of effects expected in case of New Physics (NP) contributions caused by new fermion or scalar particle loops. We consider SUSY, as an example of such an NP, and we discuss
the physical properties of the contribution to the above amplitudes from a chargino or a charged slepton, showing how the presence of the $Z$ coupling can distinguish them. For example the magnitude of the contribution changes notably when passing from a gaugino-like to a higgsino-like contribution. And for a slepton loop, even changes of sign appear, when passing from a slepton-L to a slepton-R case.

In Sec. 3, we study the $\gamma\gamma \rightarrow \gamma Z$ cross sections in the standard and SUSY models, for various polarizations of the incoming photons. We identify the sensitivity of these cross sections to various SUSY effects and we discuss their observability in unpolarized and polarized $\gamma\gamma$ collisions, realized through the present ideas of laser backscattering. Finally, in Sec. 4, we summarize the results and give our conclusions.

The explicit expressions for the $W$ and fermion loop contributions to the helicity amplitudes are given in Appendix A, using the non-linear gauge of [9]. We agree with the previous authors, apart from some slight corrections affecting the $W$ contributions to some small helicity amplitudes. In addition, we also give the 1-loop contribution induced by a single charged scalar particle. In Appendix B simple asymptotic expressions for the helicity amplitudes are given, which elucidate their physical properties at high energies.

2 An overall view of the $\gamma\gamma \rightarrow \gamma Z$ amplitudes.

The invariant helicity amplitudes $F_{\lambda_1\lambda_2\lambda_3\lambda_4}^{\lambda_5\lambda_6\lambda_7\lambda_8} (\hat{s}, \hat{t}, \hat{u})$ for the process $\gamma\gamma \rightarrow \gamma Z$ with $\lambda_j$ being the helicities of the incoming and outgoing particles, are given in Appendix A. Altogether there are $3 \times 2^3 = 24$ helicity amplitudes, which must of course satisfy the constraints from Bose (A.2). In SM or SUSY models, charge conjugation enforces parity invariance at the 1-loop level, which implies (A.3) and allows to express all helicity amplitudes in terms of nine analytic functions; six for transverse $Z$ and three for longitudinal $Z$:

$$
F_{++++}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}), \quad F_{+++}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}), \quad F_{++-}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}), \quad F_{+-+}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}), \\
F_{++-}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}) = F_{+-+}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{u}, \hat{t}), \quad F_{++-}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}) = F_{+-+}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{u}, \hat{t}), \\
F_{+++0}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}), \quad F_{++-0}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}), \quad F_{+-+0}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}) = F_{+-+0}^{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{u}, \hat{t}).
$$

In Appendix A, we reproduce the $W$ and charged fermion contributions of [7, 6, 8], and we also give the contributions to these amplitudes due to a scalar particle loop. The essential difference between these amplitudes and those of the $\gamma\gamma \rightarrow \gamma\gamma$ case presented in [1], (apart from the obvious $m_Z$ dependent kinematic terms) is the appearance of longitudinal $Z$ states, and the replacement of the factor $Q_4^x$ by $Q_{xg_{Vx}}^x$ for an $x$-particle loop contribution.

All results are given in terms of the standard 1-loop functions $B_0$, $C_0$ and $D_0$, first introduced in [10]. Explicit asymptotic expressions for these functions, relevant for the

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1 Certain corrections are found for the W contributions to some small helicity amplitudes, when comparing to [7].
The dominant $W$ contributions to the corresponding asymptotic helicity amplitudes are also given there; while the corresponding expressions for the fermion and scalar contributions can be easily written using the presented formulae.

In the Standard Model and for $\hat{s} \gtrsim (250\text{GeV})^2$, the only non-negligible amplitudes in both the $\gamma\gamma \rightarrow \gamma Z$ and $\gamma\gamma \rightarrow \gamma\gamma$ process, are $F_{\pm\pm\pm\pm}(\hat{s}, \hat{t}, \hat{u})$ and $F_{\pm\mp\mp\mp}(\hat{s}, \hat{t}, \hat{u}) = F_{\pm\mp\mp\mp}(\hat{s}, \hat{u}, \hat{t})$, which turn out to be completely dominated by the $W$-loop contribution and almost purely imaginary. They satisfy

$$ImF^W_{\gamma\gamma\rightarrow\gamma Z} \simeq \frac{c_W}{s_W} ImF^W_{\gamma\gamma\rightarrow\gamma\gamma}. \quad (1)$$

The fermion loop contribution to these same amplitudes is much smaller, its real and imaginary parts are comparable and roughly satisfies

$$F^f_{\gamma\gamma\rightarrow\gamma Z} \simeq \frac{g^Z_f}{Q_f} F^f_{\gamma\gamma\rightarrow\gamma\gamma}, \quad (2)$$

where for a standard quark or lepton $f$,

$$g^Z_f = \frac{t_f^3 - 2Q_f s^2_W}{2c_W s_W}. \quad (3)$$

The rest of the amplitudes turn out to be much smaller then the above dominant ones. For them, the real and imaginary parts are roughly on the same footing; as well the $W$- and fermion-loop contributions.

Numerical results for these amplitudes using the exact 1-loop functions are presented in Fig.1a,b, and they agree with the above expectations. Indeed the real part of the large amplitudes $F_{\pm\pm\pm\pm}(\hat{s}, \hat{t}, \hat{u})$ and $F_{\pm\mp\mp\mp}(\hat{s}, \hat{t}, \hat{u}) = F_{\pm\mp\mp\mp}(\hat{s}, \hat{u}, \hat{t})$, is always more than $4(15)$ times smaller than the imaginary part at $\sqrt{\hat{s}} \simeq 0.3(0.6)\text{TeV}$.

As in the $\gamma\gamma \rightarrow \gamma\gamma$ case [1], the asymptotic expressions in Appendix B are quite accurate in describing the large SM helicity amplitudes $F_{\pm\pm\pm\pm}(\hat{s}, \hat{t}, \hat{u})$ and $F_{\pm\mp\mp\mp}(\hat{s}, \hat{t}, \hat{u}) = F_{\pm\mp\mp\mp}(\hat{s}, \hat{u}, \hat{t})$, for the process $\gamma\gamma \rightarrow \gamma Z$ also. This is due to the fact that the double-log real contributions from (B.8, B.7) always cancel out for physical amplitudes, and the only important contributions remaining are the single-log imaginary one. For the rest of the helicity amplitudes, all log contributions either cancel out or they are strongly suppressed by $m_W^2/\hat{s}$ factors.

This confirms the fact that $\gamma\gamma \rightarrow \gamma Z$, much like $\gamma\gamma \rightarrow \gamma\gamma$ scattering, may provide a very useful tool for searching for types of New Physics (NP), with largely imaginary amplitudes [2].

We have thus computed the contributions of SUSY particles, i.e. the contributions from a chargino or a sfermion loop.
The contribution from the lightest positively charged chargino $\chi_1^+$ is obtained from the effective interaction (A.22) by using [11]

$$g_{V\chi}^Z = \frac{1}{2c_W s_W} \left\{ \frac{3}{2} - 2s_W^2 + \frac{1}{4} \{ \cos(2\phi_L) + \cos(2\phi_R) \} \right\} \right) ,$$

(4)

with

$$\cos(2\phi_L) = -\frac{M_Z^2 - \mu^2 - 2m_W^2 \cos(2\beta)}{\sqrt{(M_Z^2 + \mu^2 + 2m_W^2)^2 - 4[M_2\mu - m_W^2 \sin(2\beta)]^2} } ,$$

$$\cos(2\phi_R) = -\frac{M_Z^2 - \mu^2 + 2m_W^2 \cos(2\beta)}{\sqrt{(M_Z^2 + \mu^2 + 2m_W^2)^2 - 4[M_2\mu - m_W^2 \sin(2\beta)]^2} } ,$$

(5)

and

$$M_{\chi_1^+}^2 = \frac{1}{2} \left\{ M_Z^2 + \mu^2 + 2m_W^2 - \sqrt{(M_Z^2 + \mu^2 + 2m_W^2)^2 - 4[M_2\mu - m_W^2 \sin(2\beta)]^2} \right\} .$$

(6)

Using the formulae (A.27 -A.35) in Appendix A, together with the exact 1-loop calculation from [12], and (4, 6); we present in Fig.2 the results for two almost "extreme" situations corresponding to a light chargino of mass $M_{\chi_1^+} \approx 95 \text{ GeV}$ and $\tan \beta = 2$. In the first case the chargino nature is taken gaugino-like, by choosing (see Fig.2a,b)

$$M_2 = 0.081 \text{ TeV} , \quad \mu = -0.215 \text{ TeV} , \quad g_{V\chi}^Z = 1.72 ;$$

(7)

while in the second case it is taken "higgsino-like" by choosing (see Fig.2c,d)

$$M_2 = 0.215 \text{ TeV} , \quad \mu = -0.081 \text{ TeV} , \quad g_{V\chi}^Z = 0.73 .$$

(8)

We also consider the two L,R-slepton cases, with $M_{\tilde{l}} = 0.1 \text{ TeV}$, $Q_{\tilde{l}} = -1$ and

$$g_{V\tilde{l}}^Z = \frac{1}{c_W s_W} \left[ t_3^\prime - Q_{\tilde{l}} s_W^2 \right] .$$

(9)

For $t_3^\prime = -\frac{1}{3}$, these lead to $g_{V\tilde{l}}^Z = -0.65$ (case L); while for $t_3^\prime = 0$ we find $g_{V\tilde{l}}^Z = +0.54$ (case R). As a result, a change of sign appears between $L$ and $R$ slepton contributions. The corresponding results for a slepton, are derived using (A.40 - A.48) and presented in Fig.3a-d.

As seen for both cases of Fig.2a-d and Fig.3a-d, the real and imaginary parts of the fermion or scalar loop contributions to the $\gamma\gamma \to \gamma Z$ amplitudes above threshold, are more or less on the same footing. It is also seen that immediately above the threshold, an imaginary contribution to the $F_{\pm\pm\pm\pm}(s, \bar{t}, \bar{u})$ and $F_{\pm\mp\pm\pm}(s, \bar{t}, \bar{u}) = F_{\pm\mp\pm\pm}(s, \bar{u}, \bar{t})$ amplitude starts developing, which can interfere with the SM one and produce a measurable effect. The slepton contribution is smaller than the chargino one, by about a factor of seven though.

As compared to the $\gamma\gamma \to \gamma\gamma$ case, we also notice the presence of large longitudinal $Z$ amplitudes, for both the chargino and the slepton cases. However these are not easily
observable, since they do not find a corresponding large longitudinal SM amplitude to interfere with. So at the end, they will produce very small effect.

For transverse amplitudes, as a consequence of the $g_{\gamma f}^2/Q_f$ factor, the effect in the $\gamma\gamma \rightarrow \gamma Z$ case is larger (weaker) than the $\gamma\gamma \rightarrow \gamma\gamma$ effect in the gaugino (higgsino) cases. The corresponding effect in the slepton cases, changes sign when passing from the L-slepton to the R-slepton contribution. These properties will directly reflect in the threshold effects appearing in the cross sections that we study in the next section.

3 The $\gamma\gamma \rightarrow \gamma Z$ Cross sections

We next explore the possibility to use polarized or unpolarized $\gamma\gamma$ collisions in an LC operated in the $\gamma\gamma$ mode, through laser backscattering and the procedure described in [K]. The assumption of Parity invariance leads to the following form for the $\gamma\gamma \rightarrow \gamma Z$ cross section (note the factor 2 as compared to the $\gamma\gamma \rightarrow \gamma\gamma$ due to the presence of a non symmetric final state)

$$
\frac{d\sigma}{d\tau d\cos \vartheta^*} = \frac{dL_{\gamma\gamma}}{d\tau} \left\{ \frac{d\bar{\sigma}_0}{d\cos \vartheta^*} + \left( \xi_2 \xi_2' \right) \frac{d\bar{\sigma}_{22}}{d\cos \vartheta^*} + \left( \xi_3 \right) \cos 2\phi \frac{d\bar{\sigma}_3}{d\cos \vartheta^*} + \left( \xi_3' \right) \cos 2\phi' \frac{d\bar{\sigma}'_3}{d\cos \vartheta^*} + \left( \xi_3 \xi_3' \right) \sin 2\phi' \frac{d\bar{\sigma}_{33}}{d\cos \vartheta^*} - \left( \xi_2' \xi_2 \right) \sin 2\phi \frac{d\bar{\sigma}'_{23}}{d\cos \vartheta^*} \right\}, \tag{10}
$$

where

$$
\frac{d\bar{\sigma}_0}{d\cos \vartheta^*} = \left( \frac{\beta Z}{64\pi s} \right) \sum_{\lambda_3 \lambda_4} \left[ |F_{++\lambda_3\lambda_4}|^2 + |F_{+-\lambda_3\lambda_4}|^2 \right], \tag{11}
$$

$$
\frac{d\bar{\sigma}_{22}}{d\cos \vartheta^*} = \left( \frac{\beta Z}{64\pi s} \right) \sum_{\lambda_3 \lambda_4} \left[ |F_{++\lambda_3\lambda_4}|^2 - |F_{+-\lambda_3\lambda_4}|^2 \right], \tag{12}
$$

$$
\frac{d\bar{\sigma}_3}{d\cos \vartheta^*} = \left( \frac{-\beta Z}{32\pi s} \right) \sum_{\lambda_3 \lambda_4} \text{Re}[F_{++\lambda_3\lambda_4} F^{\ast}_{+-\lambda_3\lambda_4}], \tag{13}
$$

$$
\frac{d\bar{\sigma}'_3}{d\cos \vartheta^*} = \left( \frac{-\beta Z}{32\pi s} \right) \sum_{\lambda_3 \lambda_4} \text{Re}[F_{++\lambda_3\lambda_4} F^{\ast}_{+-\lambda_3\lambda_4}], \tag{14}
$$

$$
\frac{d\bar{\sigma}_{33}}{d\cos \vartheta^*} = \left( \frac{\beta Z}{64\pi s} \right) \sum_{\lambda_3 \lambda_4} \text{Re}[F_{++\lambda_3\lambda_4} F^{\ast}_{+-\lambda_3\lambda_4}], \tag{15}
$$

$$
\frac{d\bar{\sigma}'_{33}}{d\cos \vartheta^*} = \left( \frac{\beta Z}{64\pi s} \right) \sum_{\lambda_3 \lambda_4} \text{Re}[F_{++\lambda_3\lambda_4} F^{\ast}_{+-\lambda_3\lambda_4}], \tag{16}
$$

$$
\frac{d\bar{\sigma}_{23}}{d\cos \vartheta^*} = \left( \frac{\beta Z}{32\pi s} \right) \sum_{\lambda_3 \lambda_4} \text{Im}[F_{++\lambda_3\lambda_4} F^{\ast}_{+-\lambda_3\lambda_4}], \tag{17}
$$

6
\[
\frac{d\sigma'_ {23}}{d\cos \vartheta^*} = \left( \frac{\beta_Z}{32\pi s} \right) \sum_{\lambda_3\lambda_4} \text{Im}[F_{++}\lambda_3\lambda_4 F^*_{--}\lambda_3\lambda_4],
\]

are expressed in terms of the $\gamma\gamma \rightarrow \gamma Z$ amplitudes given in Appendix A. In (11 - 18), $\beta_Z = 1 - \frac{m_Z^2}{\hat{s}}$ is the $Z$ velocity in the $\gamma\gamma$ rest frame, while $\vartheta^*$ is the scattering angle, and $\tau \equiv s_{\gamma\gamma}/\hat{s}$. Note that $d\tilde{\sigma}_0/d\cos \vartheta^*$ is the unpolarized cross section and it is the only $\tilde{\sigma}_j$ quantity which is positive definite. We also note that $d\tilde{\sigma}'_j/d\cos \vartheta^*$ are forward-backward symmetric except those below which satisfy

\[
\left. \frac{d\tilde{\sigma}'_3}{d\cos \vartheta^*} \right|_{\vartheta^*} = \left. \frac{d\tilde{\sigma}_3}{d\cos \vartheta^*} \right|_{\pi-\vartheta^*},
\]

\[
\left. \frac{d\tilde{\sigma}'_{23}}{d\cos \vartheta^*} \right|_{\vartheta^*} = \left. \frac{d\tilde{\sigma}_{23}}{d\cos \vartheta^*} \right|_{\pi-\vartheta^*}.
\]

The results for the cross sections $\tilde{\sigma}_j$, integrated in the range $30^0 \leq \vartheta^* \leq 150^0$, are given in Fig.4a-f, for the standard model (SM); as well as for the cases of including the contributions from a single chargino or a single charged slepton with mass of about 95 or 100 GeV respectively.

As seen in Fig.4a, the effects in the unpolarized cross section $\tilde{\sigma}_0$ are consistent with those expected from the dominant imaginary amplitudes quoted in the previous section. In more detail, the (gaugino, higgsino) effects are of the order of $(10, 5)%$, respectively; while the slepton ones are an order of magnitude smaller.

The relative (NP versus SM) effects are somewhat reduced in $\tilde{\sigma}_{22}$, but they are largely enhanced in the other $\tilde{\sigma}_j$ cross sections; (compare Fig.4b with Fig.4a-f.) However most of these later cross sections have small absolute values, making their observation doubtful. Only $\tilde{\sigma}_3$ and $\tilde{\sigma}_{33}$ are of the order of a few $fb$. The SUSY effects are here notably enhanced as compared to the SM contributions, reaching the 25% level in some cases. Particularly striking is the SUSY effect for $\tilde{\sigma}_{33}$ and $\tilde{\sigma}'_{33}$ near but above threshold. For $\tilde{\sigma}'_{33}$, such a behaviour also occurred in the $\gamma\gamma \rightarrow \gamma\gamma$ case [1], and it may be useful for disentangling of the various SUSY examples.

In the present case, the option to select longitudinal polarization for the outgoing $Z$, is also available. In this case remarkable SUSY effects, strongly dependent on the type of particle running along the loop, are generated. However, the absolute values of the related cross sections are unfortunately less than 1 $fb$; rendering these effects unobservable. See Fig.5a-c where $\tilde{\sigma}_0(Z_L)$, $\tilde{\sigma}_{22}(Z_L)$ and $\tilde{\sigma}_{33}(Z_L)$ are presented; (the other ones are less than 0.1 $fb$.)

The angular distributions of the various $d\tilde{\sigma}_j/d\cos \vartheta^*$ are illustrated for $\sqrt{s} = 0.4$ TeV in Fig.6a-f. Restricting the discussion to those ”cross sections” whose absolute values are larger than 1 $fb$; one observes that forward-backward peaks arise only for $d\tilde{\sigma}_0/d\cos \vartheta^*$. Among the rest, the most interesting ones are $d\tilde{\sigma}_{22}/d\cos \vartheta^*$, $d\tilde{\sigma}_3/d\cos \vartheta^*$ (note [13]) and $d\tilde{\sigma}_{33}/d\cos \vartheta^*$; which in fact have a forward deep. At a weaker level, a similar result is also true for $\gamma\gamma \rightarrow \gamma\gamma$, where of course all ”cross sections” are forward-backward symmetric; (but no figures are shown in [1]). Fig.6a-f also show that SUSY effects often appear mostly pronounced at large angles.
To get a feeling on the observability of the various quantities $\bar{\sigma}_j$ appearing in (14), we next turn to the experimental aspects of the $\gamma\gamma$ collision realized through laser backscattering [1][2]. The quantity $dL_{\gamma\gamma}/d\tau$ in (14), describes the photon-photon luminosity per unit $e^-e^+$ flux, in an LC operated in the $\gamma\gamma$ mode [2]. The Stokes parameters $\xi_2$, $\xi_3$, and the polarization angle $\phi$ in (14), determine the normalized helicity density matrix of one of the backscattered photons $\rho_{\lambda\lambda}^{BN}$ through the formalism described in Appendix B of [1]; compare eq.(B4) of [1] and [13]. The corresponding parameters for the other backscattered photon are denoted by a prime. The numerical expectations for $dL_{\gamma\gamma}/d\tau$, $\langle \xi_j \rangle$, $\langle \xi_j' \rangle$ and $\langle \xi_i \xi_j' \rangle$ are given in Appendix B and Fig.4 of [1]. To estimate the expected number of events, one should multiply the cross sections in (10) by the $e^+e^-$ luminosity $L_{ee}$, whose presently contemplated value for the LC project is $L_{ee} \approx 500 - 1000 \, fb^{-1}$ per one or two years of running in e.g. the high luminosity TESLA mode at energies of 350 – 800 GeV [3].

We next turn to the expected statistical accuracies for the various $\bar{\sigma}_j$. The relative uncertainty for the unpolarized cross section $[\bar{\sigma}_0(\langle \cos \vartheta^* \rangle)]$, calculated by integrating the respective differential cross section over a certain reduced energy bin $\Delta \tau$ and an angular bin $\Delta \cos \vartheta^*$, is:

$$\frac{\delta [\bar{\sigma}_0(\langle \cos \vartheta^* \rangle)]}{\bar{\sigma}_0(\langle \cos \vartheta^* \rangle)} = \left[ L_{ee} (\Delta \tau) (\Delta \cos \vartheta^*) \frac{dL_{\gamma\gamma}}{d\tau} \right] \frac{d\bar{\sigma}_0}{d(\cos \vartheta^*)} \frac{1}{2} .$$  \hspace{1cm} (21)

For the other "cross sections", for whose measurement we need various combinations of longitudinal $e^\pm$, and longitudinal and transverse laser polarizations, we perform an analysis similar to the one in Section 3 of [1]. For simplicity, we define

$$R_j(\langle \cos \vartheta^* \rangle) \equiv \frac{[\bar{\sigma}_{ij}(\langle \cos \vartheta^* \rangle)]}{\bar{\sigma}_0(\langle \cos \vartheta^* \rangle)}_{SM} .$$  \hspace{1cm} (22)

Then, the absolute uncertainties $\delta [\bar{\sigma}_{ij}(\langle \cos \vartheta^* \rangle)]$ satisfy

$$\delta R_j(\langle \cos \vartheta^* \rangle) = \frac{\delta [\bar{\sigma}_{ij}(\langle \cos \vartheta^* \rangle)]}{\bar{\sigma}_0(\langle \cos \vartheta^* \rangle)}_{SM} = \frac{1}{c_j} \left[ L_{ee} (\Delta \tau) (\Delta \cos \vartheta^*) \frac{dL_{\gamma\gamma}}{d\tau} \right] \frac{d\bar{\sigma}_0}{d(\cos \vartheta^*)} \frac{1}{2} ,$$  \hspace{1cm} (23)

where $c_j = \sqrt{2}(\xi_2 \xi_2')$, $\sqrt{2}(\xi_3)$, $\sqrt{2}(\xi_3' \xi_3)$, $\sqrt{2}(\xi_2 \xi_3')$, for $R_{22}$, $R_3$, $(R_{33}$ or $R'_{33})$, $(R_{23}$ or $R'_{23})$ respectively.

To estimate these, we take the numerical values for the photon spectra and polarization degrees given in Appendix B and Fig.4 of [1]. For the $e^+e^-$ luminosity we assume $1000 \, fb^{-1}$. Using then bins of the order of $\Delta \tau \approx 0.4$, $\Delta \cos \vartheta^* \approx 1$, and $dL_{\gamma\gamma}/d\tau \gtrsim 1$, as well as an unpolarized differential cross section of the order of $30 \, fb$ (see Fig.3a); one obtains a relative uncertainty of the order of $1\%$, for the unpolarized cross section in (21).

For the ratios $R_j$ defined in (22), the factor $1/c_j$ will increase the absolute uncertainty in (23). According to Fig.4,5 of [1], this factor depends strongly on the backscattering configurations and on the reduced energy range. It can easily vary between 1 and 10. But if the kinematic range to be studied is known, then the backscattering configuration can be tuned to optimize the flux spectrum. Thus, for the time being, we can roughly
conclude that the accuracy at which the ratios $R_j$ can be measured, should lie between 1 and 10%. This means that it is reasonable to expect an absolute uncertainty of the order of $0.3 \, fb$ for $d\bar{\sigma}_0/d\cos \vartheta^*$ at large angles; and something in the range $(0.3 - 3) \, fb$, for the other $d\bar{\sigma}_{ij}/d\cos \vartheta^*$.

These values have to be compared with the NP effects expected on the corresponding observables. Thus, from Fig.4,6 one sees that the unpolarized integrated cross section should very sensitive to chargino effects, a sensitivity characterized by a statistical significance notably increased as compared with the $\gamma\gamma \rightarrow \gamma\gamma$ case; (up to 10 SD instead of 3 SD if the chargino is in the 100 GeV mass range). For slepton searches, the situation in $\gamma\gamma \rightarrow \gamma Z$ is similar to the one in $\gamma\gamma \rightarrow \gamma\gamma$, because of the small Z-slepton couplings.

The illustrations given in the present paper are for a chargino or slepton in 100GeV mass range. For higher masses, the relative merits of the $\gamma\gamma \rightarrow \gamma Z$ and $\gamma\gamma \rightarrow \gamma\gamma$ processes remain about the same. We expect therefore that $\gamma\gamma \rightarrow \gamma Z$ should be very helpful for sparticle searches with mass up to 300 GeV. As a final remark, we recall that in $\gamma\gamma \rightarrow \gamma\gamma$, if several SUSY particles exist within a given mass range, then their effects are all positive and cumulate in $\bar{\sigma}_0$. This is not necessarily the case in $\gamma\gamma \rightarrow \gamma Z$, because the $g^Z_{Vx}$ can have different signs as we have seen in Sect.2.

4 Conclusions

In this paper we have extended our previous analysis of the helicity amplitudes and observables in the process $\gamma\gamma \rightarrow \gamma\gamma$ at high energies, to the $\gamma\gamma \rightarrow \gamma Z$ case.

It appears that both processes share the spectacular property that in the Standard Model and at energies above 0.25 $TeV$, only three independent helicity conserving amplitudes are important, which moreover are almost purely imaginary. Exactly as it would had been predicted about 30 years ago, on the basis of Vector Meson Dominance and Pomeron exchange! These three amplitudes are $F_{\pm\pm\pm\pm}(\hat{s}, \hat{t}, \hat{u})$ and $F_{\pm\pm\pm\pm}(\hat{s}, \hat{t}, \hat{u}) = F_{\pm\pm\pm\pm}(\hat{s}, \hat{u}, \hat{t})$. Thus the $\gamma\gamma \rightarrow \gamma\gamma$ and $\gamma\gamma \rightarrow \gamma Z$ processes should be excellent tools for searching for virtual new physics contributions characterized by important imaginary contributions. This means that they should very helpful in identifying the nature of nearby new particles, which can also be directly excited. But they should not be of much use for studying high scale NP effects described by effective lagrangians, which naturally lead to real amplitudes.

This has been illustrated for the particular SUSY cases of a single chargino or charged slepton contribution. Clear threshold effects in the various observables appear due to the interference of the imaginary parts of the SUSY amplitudes with the SM ones. These contributions depend of course on the mass and quantum numbers of the SUSY partners, but are independent of the many model-dependent parameters entering their decay modes, contrary to the case of direct SUSY particle production. Thus, the study of the $\gamma\gamma \rightarrow \gamma\gamma$, $\gamma Z$ cross sections should offer complementary information, to the one obtained from direct SUSY production cross sections. We have indeed found that the unpolarized $\gamma\gamma \rightarrow \gamma\gamma$, $\gamma Z$ cross sections $\bar{\sigma}_0$, are most sensitive to a chargino loop contribution. For a light

\[ In [1] we gave some illustration for sparticles at 250 GeV in the $\gamma\gamma \rightarrow \gamma\gamma$ case. \]
chargino the signal is at most of 3 to 4 SD in $\gamma\gamma \rightarrow \gamma\gamma$, while it can reach 10 SD in the $\gamma\gamma \rightarrow \gamma Z$ case. For a single charged slepton with a 100 GeV mass, we have found that the corresponding effect on $\sigma_0$ is an order of magnitude smaller. Angular distributions are most sensitive at large angles ($|\cos \theta^*| < 0.5$) in both cases. Polarization should allow to test the nature of the particles involved in the loop. There are eight different observables in $\gamma\gamma \rightarrow \gamma Z$ and six in $\gamma\gamma \rightarrow \gamma\gamma$. Only five of them ($\sigma_0$, $\sigma_{22}$, $\sigma_3$, $\sigma_{33}$, $\sigma'_{33}$), will be measurable with sufficient accuracy to allow checks of the global picture. This requires an optimization of the laser backscattering procedure though.

The comparison of the situations in $\gamma\gamma \rightarrow \gamma\gamma$ and $\gamma\gamma \rightarrow \gamma Z$ is very instructive. It is first important to notice, that in $\gamma\gamma \rightarrow \gamma\gamma$, both, the charged fermion and the charged scalar particle loops, increase the SM prediction for $\sigma_0$. If, as seems quite plausible, a chargino, as well as all six charged sleptons and $\tilde{t}_1$, lie in the (100-250)GeV mass range; then a clear signal could be seen in $\sigma_0(\gamma\gamma \rightarrow \gamma\gamma)$. The study of $\gamma\gamma \rightarrow \gamma Z$, including also polarization effects, should then give information on the origin of the signal.

Similar type of effects could also appear for other virtual NP contributions of fermionic or scalar nature; like e.g. heavy fermions, technifermions, charged Higgses, pseudogoldstone bosons or even heavy charged vector bosons. In the $\gamma\gamma \rightarrow \gamma\gamma$ case, the effect is only controlled by the electric charge; while in $\gamma\gamma \rightarrow \gamma Z$, the $g_{\gamma Z}^\nu$ coupling also enters. There exist other process of this type, namely $\gamma\gamma \rightarrow ZZ$ and $\gamma\gamma \rightarrow HH$, $HZ$, which only receive SM contributions at 1-loop, and could be equally interesting for NP searches. However they are essentially controlled by the properties of the Higgs sector and deserve separate studies which are in progress.

In any case it appears to us that $\gamma\gamma \rightarrow \gamma\gamma$, $\gamma Z$ are very clean processes which should supply excellent tools for NP searches, and should add to the interest in providing for the eventual realization of the $\gamma\gamma$ mode in the high energy LC colliders.
Appendix A: The $\gamma\gamma \to \gamma Z$ amplitudes in SM and SUSY.

The invariant helicity amplitudes for the process

$$\gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2) \to \gamma(p_3, \lambda_3)Z(p_4, \lambda_4),$$

(A.1)

are denoted as $F_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, \hat{t}, \hat{u})$, where the momenta and helicities of the incoming and outgoing photons are indicated in parenthesis, and $s = (p_1 + p_2)^2$, $\hat{t} = (p_1 - p_3)^2$, $\hat{u} = (p_1 - p_4)^2$. Bose statistics demands

$$F_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, \hat{t}, \hat{u}) = F_{\lambda_2\lambda_1\lambda_4\lambda_3}(s, \hat{u}, \hat{t})(-1)^{1-\lambda_4},$$

(A.2)

while, if parity invariance also holds, we get the additional constraint

$$F_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, \hat{t}, \hat{u}) = F_{-\lambda_2-\lambda_4-\lambda_3}(s, \hat{t}, \hat{u})(-1)^{1-\lambda_4}.$$ 

(A.3)

As a result, the 24 helicity amplitudes may be expressed in terms of just the nine amplitudes

$$F_{++++}(s, \hat{t}, \hat{u}),  F_{++-}(s, \hat{t}, \hat{u}),  F_{++-}(s, \hat{t}, \hat{u}),  F_{+++}(s, \hat{t}, \hat{u}),$$

$$F_{++-}(s, \hat{t}, \hat{u}) = F_{--}(s, \hat{u}, \hat{t}),  F_{--}(s, \hat{t}, \hat{u}) = F_{++-}(s, \hat{u}, \hat{t}),$$

$$F_{++-0}(s, \hat{t}, \hat{u}) = F_{++-0}(s, \hat{t}, \hat{u}) = F_{++-0}(s, \hat{t}, \hat{u}).$$

There are three different forms of contributions to these amplitudes arising from a $W$, fermion or scalar particle loop. To express them economically, we use the notation of [14] for the $B_0, C_0$ and $D_0$ 1-loop functions first defined by Passarino and Veltman [10], and we introduce the shorthand writing

$$B_0(s) \equiv B_0(s; m, m),$$

(A.4)

$$C_0(s) \equiv C_0(12) = C_0(0, 0, s; m, m, m),$$

(A.5)

and

$$B_2(s) \equiv B_0(s) - B_0(m_Z^2 + i\epsilon),$$

(A.6)

$$C_2(s) \equiv C_2(34) = C_0(m_Z^2, 0, s; m, m, m) = C_0(0, m_Z^2, s; m, m, m),$$

(A.7)

$$D_2(s, \hat{u}) \equiv D_2(123) = D_0(0, 0, 0, m_Z^2, \hat{s}; \hat{u}, m, m, m) = D_2(\hat{u}, \hat{s}).$$

(A.8)

The expressions

$$\tilde{F}(\hat{s}, \hat{t}, \hat{u}) \equiv D_2(\hat{s}, \hat{t}) + D_2(\hat{s}, \hat{u}) + D_2(\hat{u}, \hat{t}),$$

(A.9)

$$E(\hat{t}, \hat{u}) = E(\hat{u}, \hat{t}) \equiv \hat{t}C_0(\hat{t}) + \hat{u}C_0(\hat{u}) + \hat{t}_1C_2(\hat{t}) + \hat{u}_1C_2(\hat{u}) - \hat{t}\hat{u}D_2(\hat{t}, \hat{u}).$$

(A.10)

appear naturally in the amplitudes below, where $\hat{s}_1 = \hat{s} - m_Z^2$, $\hat{t}_1 = \hat{t} - m_Z^2$, $\hat{u}_1 = \hat{u} - m_Z^2$.

---

3Their sign is related to the sign of the $S$-matrix through $S_{\lambda_1\lambda_2\lambda_3\lambda_4} = 1 + i(2\pi)^4\delta(p_f - p_i)F_{\lambda_1\lambda_2\lambda_3\lambda_4}$. We use the Jacob-Wick convention.
The $W$ loop contribution to the helicity amplitudes may then be written as

$$F^W_{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u}) \equiv \alpha^2 \frac{C_W}{s_W} A^W_{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u})$$

where

$$A^W_{++\pm\pm}(\hat{s}, \hat{t}, \hat{u}) = \frac{16s_1\hat{s}}{s} E(\hat{t}, \hat{u}) + 4[2(\hat{s} - 4m_W^2)\hat{s}_1 - m_W^2(m_Z^2 - 6m_W^2)] F(\hat{s}, \hat{t}, \hat{u})$$

$$+ 2 \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ \frac{\hat{t}\hat{u} + m_W^2(\hat{s} + s_1)}{s\hat{s}_1} E(\hat{t}, \hat{u}) - \frac{2m_W^2}{s_1}[\hat{t}\hat{u}D_Z(\hat{s}, \hat{u}) + m_Z^2C_0(\hat{s})] \right\}$$

$$- (\hat{s} + m_Z^2)\frac{\hat{t}\hat{u}}{s_1\hat{u}_1} - \frac{2m_W^2m_Z^2\hat{s}}{s_1\hat{u}_1} C_Z(t) + \left( \frac{2\hat{t} + \hat{s}}{s_1} - \frac{m_Z^2s}{s_1\hat{u}_1^2} \right) B_Z(t)$$

$$- \frac{2m_W^2m_Z^2\hat{s}}{s_1\hat{u}_1} C_Z(\hat{u}) + \left( \frac{2\hat{u} + \hat{s}}{s_1} - \frac{m_Z^2s}{s_1\hat{u}_1^2} \right) B_Z(\hat{u}) \right\},$$

$$A^W_{++\mp\mp}(\hat{s}, \hat{t}, \hat{u}) = 2 \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ \frac{4m_Z^2 - s}\hat{s} D_Z(\hat{t}, \hat{u}) - \frac{s(\hat{u}^2 + \hat{t}^2)}{\hat{s}_1\hat{t}} C_0(\hat{s}) - \frac{s_1^2}{\hat{u}\hat{t}} C_Z(\hat{s}) \right\} + \left( \frac{\hat{s} + m_Z^2}\hat{t} \right)$$

$$D_Z(\hat{s}, \hat{t})$$

$$+ m_W^2 \left[ \frac{(4m_Z^2 - s)\hat{t}\hat{u}}{s\hat{s}_1\hat{t}_1} + \frac{2m_Z^2\hat{u} - \hat{s}\hat{t}_1}{s_1\hat{t}_1} \right] C_Z(\hat{t}) - \frac{(2m_Z^2\hat{u} + \hat{s}\hat{t})}{\hat{s}\hat{s}_1} C_0(\hat{t}) - \frac{\hat{s}\hat{t}\hat{u}}{\hat{u}\hat{t}} D_Z(\hat{s}, \hat{t})$$

$$+ m_W^2 \left[ \frac{(m_Z^2\hat{u} - \hat{s}\hat{t})\hat{s}}{s\hat{s}_1\hat{t}_1} + \frac{2m_Z^2\hat{t} - \hat{s}\hat{t}_1}{s_1\hat{t}_1} \right] C_Z(\hat{u}) - \frac{(2m_Z^2\hat{t} + \hat{s}\hat{u})\hat{u}}{\hat{s}\hat{t}\hat{s}_1} C_0(\hat{u}) - \frac{\hat{s}\hat{u}\hat{t}}{\hat{t}} D_Z(\hat{s}, \hat{u})$$

$$+ \frac{m_Z^2(2\hat{u} - \hat{s}\hat{t})}{s\hat{s}_1\hat{t}_1^2} B_Z(\hat{t}) + \frac{m_Z^2(2\hat{u}_1 - \hat{s}\hat{t})\hat{u}}{s\hat{s}_1\hat{u}_1^2} B_Z(\hat{u}) \right\},$$

$$A^W_{++\mp\mp}(\hat{s}, \hat{t}, \hat{u}) = 2 \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ \frac{4m_Z^2 - s}\hat{s} D_Z(\hat{t}, \hat{u}) - \frac{s(\hat{u}^2 + \hat{t}^2)}{\hat{s}_1\hat{t}} C_0(\hat{s}) - \frac{s_1^2}{\hat{u}\hat{t}} C_Z(\hat{s}) \right\} + \left( \frac{\hat{s} + m_Z^2}\hat{t} \right)$$

$$D_Z(\hat{s}, \hat{t})$$

$$+ m_W^2 \left[ \frac{(4m_Z^2 - s)\hat{t}\hat{u}}{s\hat{s}_1\hat{t}_1} + \frac{2m_Z^2\hat{u} - \hat{s}\hat{t}_1}{s_1\hat{t}_1} \right] C_Z(\hat{t}) - \frac{(2m_Z^2\hat{u} + \hat{s}\hat{t})}{\hat{s}\hat{s}_1} C_0(\hat{t}) - \frac{\hat{s}\hat{t}\hat{u}}{\hat{u}\hat{t}} D_Z(\hat{s}, \hat{t})$$

$$+ m_W^2 \left[ \frac{(m_Z^2\hat{u} - \hat{s}\hat{t})\hat{s}}{s\hat{s}_1\hat{t}_1} + \frac{2m_Z^2\hat{t} - \hat{s}\hat{t}_1}{s_1\hat{t}_1} \right] C_Z(\hat{u}) - \frac{(2m_Z^2\hat{t} + \hat{s}\hat{u})\hat{u}}{\hat{s}\hat{t}\hat{s}_1} C_0(\hat{u}) - \frac{\hat{s}\hat{u}\hat{t}}{\hat{t}} D_Z(\hat{s}, \hat{u})$$

$$+ \frac{m_Z^2(2\hat{u} - \hat{s}\hat{t})}{s\hat{s}_1\hat{t}_1^2} B_Z(\hat{t}) + \frac{m_Z^2(2\hat{u}_1 - \hat{s}\hat{t})\hat{u}}{s\hat{s}_1\hat{u}_1^2} B_Z(\hat{u}) \right\}. $
\[ A_{++,--}(\hat{s}, \hat{t}, \hat{u}) = A_{++,--}(\hat{s}, \hat{u}, \hat{t}) = 16 \frac{\hat{s}}{s_1} E(\hat{s}, \hat{t}) \]

\[ +4 \left( \frac{2\hat{s}\hat{u}(\hat{u} - 4m_W^2)}{s_1} - m_Z^2(m_Z^2 - 6m_W^2) \right) \hat{F}(\hat{s}, \hat{t}, \hat{u}) \]

\[ +2 \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ \left( \frac{\hat{s}t}{u^2} + \frac{2m_W^2}{u} \right) E(\hat{s}, \hat{t}) - m_W^2 \left[ \frac{2\hat{s}\hat{u}}{s_1} D_Z(\hat{s}, \hat{t}) + \frac{m_Z^2}{ss_1} E(\hat{u}, \hat{t}) \right] + \frac{2m_Z^2(2\hat{t}_1 + \hat{s})}{s_1\hat{t}_1} C_Z(\hat{t}) \right\} + \frac{\hat{s}(\hat{s} - \hat{t})}{s_1\hat{u}} B_Z(\hat{s}) - \frac{\hat{s}\hat{t}(2\hat{s}\hat{t}_1 + \hat{u})}{s_1\hat{u}\hat{t}_1^2} B_Z(\hat{t}) - \frac{\hat{s}}{s_1\hat{t}_1} \right\}, \quad (A.16) \]

\[ A_{++,--}(\hat{s}, \hat{t}, \hat{u}) = A_{++,++}(\hat{s}, \hat{u}, \hat{t}) \]

\[ = \frac{8m_Z^2\hat{u}}{s_1\hat{t}} \left[ 2E(\hat{s}, \hat{u}) - \hat{t}(4m_W^2 - \hat{t})\hat{F}(\hat{s}, \hat{t}, \hat{u}) \right] \]

\[ -2 \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ \frac{m_W^2}{s_1} \left[ \left( \frac{\hat{s}\hat{u}}{\hat{t}} + \frac{2m_W^2}{\hat{u}} \right) D_Z(\hat{s}, \hat{u}) + \left( \frac{\hat{s}\hat{t}}{\hat{u}} + \frac{2m_W^2}{\hat{u}} \right) D_Z(\hat{s}, \hat{t}) \right] + \frac{\hat{t}\hat{u}}{s_1} C_0(\hat{s}) + \frac{\hat{u}^2}{s_1\hat{t}} C_0(\hat{u}) + \frac{\hat{u}^2 + s_1^2}{s_1\hat{u}} C_0(\hat{t}) \right\}

\[ + \frac{\hat{t}^2}{\hat{u}} C_Z(\hat{u}) + \frac{2m_Z^2\hat{t}^2 + \hat{u}(\hat{t}\hat{t}_1 + \hat{s}s_1)}{s_1\hat{u}\hat{t}} C_Z(\hat{u}) + \frac{\hat{t}\hat{t}_1}{s_1\hat{u}} C_Z(\hat{t}) \]

\[ + \frac{m_Z^2\hat{u}\hat{u}_1}{s_1\hat{u}_1} B_Z(\hat{u}) - \frac{\hat{s}\hat{u}}{s_1\hat{u}_1} \right\}, \quad (A.17) \]

\[ A_{++00}(\hat{s}, \hat{t}, \hat{u}) = p_t \sqrt{2} m_Z \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ \left( \frac{\hat{t} - \hat{u}}{s_1} \right) \frac{3m_W^2}{s_1} D_Z(\hat{t}, \hat{u}) - \frac{E(\hat{t}, \hat{u})}{s_1} \right\}

\[ + \frac{2m_Z^2\hat{s}}{s_1\hat{t}\hat{u}} C_0(\hat{s}) + \frac{2s}{s_1\hat{t}_1\hat{u}_1} + m_W^2 \left[ \frac{\hat{s}}{\hat{u}} D_Z(\hat{s}, \hat{t}) + \frac{2}{s_1} C_0(\hat{t}) + \frac{2(2s^2 - \hat{t}^2)}{s_1\hat{t}_1\hat{u}} C_Z(\hat{t}) \right] \]

\[ + \frac{2(2\hat{t}_1 + m_Z^2\hat{u})}{s_1\hat{t}_1^2} B_Z(\hat{t}) - m_W^2 \left[ \frac{\hat{s}}{\hat{t}} D_Z(\hat{s}, \hat{u}) + \frac{2}{s_1} C_0(\hat{u}) + \frac{2}{s_1} C_Z(\hat{u}) \right] \]

\[ - \frac{2(\hat{u}\hat{u}_1 + m_Z^2\hat{t})}{s_1\hat{u}_1} B_Z(\hat{u}) \right\}, \quad (A.18) \]

\[ A_{++-0}(\hat{s}, \hat{t}, \hat{u}) = p_t \sqrt{2} m_Z \left( m_Z^2 - 6m_W^2 \right) \left\{ \left( \frac{\hat{t} - \hat{u}}{s_1} \right) \left( D_Z(\hat{t}, \hat{u}) - \frac{2s}{u} C_0(\hat{s}) \right) \right\}

\[ - \frac{\hat{s}}{\hat{u}} D_Z(\hat{s}, \hat{t}) + \frac{2}{s_1} C_0(\hat{t}) - \frac{2s\hat{u}_1}{s_1\hat{u}} C_Z(\hat{t}) + \frac{\hat{s}}{\hat{t}} D_Z(\hat{s}, \hat{u}) - \frac{2}{s_1} C_0(\hat{u}) + \frac{2\hat{u}_1}{s_1\hat{t}} C_Z(\hat{u}) \right\}, \quad (A.19) \]

\[ A_{++00}(\hat{s}, \hat{t}, \hat{u}) = A_{++00}(\hat{s}, \hat{u}, \hat{t}) = p_t \sqrt{2} \left\{ \frac{8m_Z\hat{s}}{s_1} \left[ (\hat{u} - 4m_W^2)\hat{F}(\hat{s}, \hat{t}, \hat{u}) + \frac{2E(\hat{s}, \hat{t})}{\hat{u}} \right] \right\}

\[ + m_Z \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ m_W^2 \left[ \left( \frac{\hat{t} - \hat{u}}{s_1} \right) D_Z(\hat{t}, \hat{u}) + \frac{\hat{s}}{\hat{t}} D_Z(\hat{s}, \hat{u}) + \frac{3s}{\hat{u}} D_Z(\hat{s}, \hat{t}) \right] \right\} \]

\[ + m_Z \left( \frac{m_Z^2}{m_W^2} - 6 \right) \left\{ m_W^2 \left[ \left( \frac{\hat{t} - \hat{u}}{s_1} \right) D_Z(\hat{t}, \hat{u}) + \frac{\hat{s}}{\hat{t}} D_Z(\hat{s}, \hat{u}) + \frac{3s}{\hat{u}} D_Z(\hat{s}, \hat{t}) \right] \right\} \]
\[ + \frac{2m_W^2}{\hat{s}} \left[ C_0(\hat{t}) + C_0(\hat{u}) - \frac{\hat{s}}{\hat{u} t} C_0(\hat{s}) + \frac{\hat{u}}{\hat{t}} C_Z(\hat{u}) + \frac{\hat{t}}{\hat{u}} C_Z(\hat{t}) - \frac{2\hat{u}}{\hat{t}} C_Z(t) \right] \]
\[- \frac{\hat{s}}{\hat{u} t} E(s, t) + \frac{2\hat{s}}{\hat{s} \hat{u}} B_Z(s) - 2 \left( \frac{1}{\hat{u}} + \frac{m_s^2 \hat{u}}{\hat{s} \hat{t}^2} \right) B_Z(t) + \frac{2\hat{s}}{\hat{s} \hat{t}} \right), \] (A.20)

where
\[ p_t = \sqrt{\hat{t} \hat{u}}. \] (A.21)

When comparing these results with those of \[7\], where helicity amplitudes are also given, we identify some discrepancies in \(A.13, A.14, A.18, A.19, A.20\), which are of minor importance though, since the affected amplitudes are very small. In addition there is a difference in sign for the longitudinal \(Z\) amplitudes, since \[7\] does not use the Jakob-Wick convention.\(^5\)

We next turn to the fermion loop contribution. Writing the effective \(Zf \bar{f}\) interaction as
\[ L_{Zff} = -eZ^\mu \bar{f}(\gamma^\mu g_{Zf}^Z - \gamma^5 \gamma^5 g_{Af}^Z) f, \] (A.22)
we remark that, due to charge conjugation, only the vector coupling \(g_{Zf}^Z\) gives a non-vanishing contribution. For ordinary quarks and leptons this is
\[ g_{Zf}^Z = \frac{t_f^3 - 2Qf s_W^2}{2s_W c_W}, \] (A.23)
where \(t_f^3\) is the fermion third isospin component, and \(Qf\) its charge. Denoting then the fermion mass as \(m_f\), its contribution to the helicity amplitudes is written as \[8\]
\[ F_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^f(s, \hat{t}, \hat{u}) \equiv \alpha^2 Q_f^3 g_{Zf}^Z A_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^f(s, \hat{t}, \hat{u}) \] (A.24)

For presenting the fermion loop contribution it is convenient to introduce the definitions
\[ x_f = \frac{4m_f^2}{m_Z^2 - 6m_f^2}, \] (A.25)
and\[ 6\]
\[ G_f(s, \hat{t}, \hat{u}) = \frac{(10m_f^2 + m_Z^2)}{m_f^2} E(\hat{t}, \hat{u}) + 2\hat{s}[4\hat{s} - 10m_f^2 - m_Z^2] F(\hat{s}, \hat{t}, \hat{u}), \] (A.26)
which allow us to write \[8\]
\[ A_{+++}^f(s, \hat{t}, \hat{u}) = x_f A_{+++}^W(s, \hat{t}, \hat{u}; m_W \rightarrow m_f) - \frac{x_f \hat{s}}{\hat{s}} G_f(s, \hat{t}, \hat{u}), \] (A.27)
\[ ^5\text{Note also that the definitions of } \hat{t} \text{ and } \hat{u} \text{ used here and in } [1, 2] \text{ should be interchanged when comparing with } [7]. \]
\[ ^6\text{The functions } E(\hat{t}, \hat{u}) \text{ and } F(\hat{s}, \hat{t}, \hat{u}) \text{ are defined in } (A.10) \text{ and } (A.9) \text{ respectively, with } m = m_f. \]
\[ A_{+++}^f(\hat{s}, \hat{t}, \hat{u}) = x_f A_{+++}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_f), \quad (A.28) \]
\[ A_{++-}^f(\hat{s}, \hat{t}, \hat{u}) = x_f A_{++-}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_f), \quad (A.29) \]
\[ A_{+-+}^f(\hat{s}, \hat{t}, \hat{u}) = x_f A_{+-+}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_f), \quad (A.30) \]

\[ A_{++-}^f(\hat{s}, \hat{t}, \hat{u}) = A_{++-}^f(\hat{s}, \hat{u}, \hat{t}) = \frac{x_f \hat{s}}{\hat{s}_1} G_f(\hat{u}, \hat{t}, \hat{s}), \quad (A.31) \]
\[ A_{++-}^f(\hat{s}, \hat{t}, \hat{u}) = A_{++-}^f(\hat{s}, \hat{u}, \hat{t}) = \frac{x_f \hat{u} m^2}{\hat{s}_1} G_f(\hat{t}, \hat{s}, \hat{u}), \quad (A.32) \]
\[ A_{++0}^f(\hat{s}, \hat{t}, \hat{u}) = x_f A_{++0}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_f), \quad (A.33) \]
\[ A_{++-}^f(\hat{s}, \hat{t}, \hat{u}) = x_f A_{++-}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_f), \quad (A.34) \]
\[ A_{++0}^f(\hat{s}, \hat{t}, \hat{u}) = A_{++0}^f(\hat{s}, \hat{u}, \hat{t}) = x_f A_{++0}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_f) = \frac{x_f m_z}{\hat{s}_1} \sqrt{\frac{2 \hat{s}}{\hat{u}}} G_f(\hat{u}, \hat{t}, \hat{s}). \quad (A.35) \]

Finally the contribution to the helicity amplitudes arising from a loop due to a scalar particle\(^\text{7}\) of charge \(Q_S\), mass \(m_S\), and third isospin component \(t_3^S\), is

\[ F_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^S(\hat{s}, \hat{t}, \hat{u}) \equiv \alpha^2 Q^3_S g^Z S A_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^S(\hat{s}, \hat{t}, \hat{u}), \quad (A.36) \]

where

\[ g^Z_S = \frac{t_3^S - Q_S s^2_W}{s_W c_W}. \quad (A.37) \]

Using then the definitions

\[ x_S = \frac{2 m^2_s}{6 m^2_S - m^2_Z}, \quad (A.38) \]
\[ G_S(\hat{s}, \hat{t}, \hat{u}) = 2 E(\hat{t}, \hat{u}) + \hat{s}(\hat{s} - 4 m^2_s) \hat{F}(\hat{s}, \hat{t}, \hat{u}), \quad (A.39) \]

we obtain

\[ A_{+++}^S(\hat{s}, \hat{t}, \hat{u}) = x_S A_{+++}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_S) - \frac{8 x_S \hat{s}_1}{\hat{s}} G_S(\hat{s}, \hat{t}, \hat{u}), \quad (A.40) \]
\[ A_{++-}^S(\hat{s}, \hat{t}, \hat{u}) = x_S A_{++-}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_S), \quad (A.41) \]
\[ A_{++-}^S(\hat{s}, \hat{t}, \hat{u}) = x_S A_{++-}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_S), \quad (A.42) \]
\[ A_{++-}^S(\hat{s}, \hat{t}, \hat{u}) = x_S A_{++-}^W(\hat{s}, \hat{t}, \hat{u}; m_W \to m_S), \quad (A.43) \]

\(^7\)Like e.g. a slepton.
\[ A^S_{++-}(\hat{s}, \hat{t}, \hat{u}) = A^S_{-+++}(\hat{s}, \hat{u}, \hat{t}) = x_s A^W_{++-}(\hat{s}, \hat{t}, \hat{u}; m_W \to m_S) \]
\[ = \frac{8x_s\hat{s}}{\hat{s}_1} G_S(\hat{u}, \hat{s}, \hat{s}), \quad (A.44) \]

\[ A^S_{++-}(\hat{s}, \hat{t}, \hat{u}) = A^S_{-+++}(\hat{s}, \hat{u}, \hat{t}) = x_s A^W_{++-}(\hat{s}, \hat{t}, \hat{u}; m_W \to m_S) \]
\[ = -\frac{8x_s\hat{s}m_Z}{\hat{s}_1} \sqrt{\frac{2i\hat{s}}{\hat{u}}} G_S(\hat{u}, \hat{t}, \hat{s}). \quad (A.48) \]

**Appendix B: The asymptotic $\gamma\gamma \to \gamma Z$ amplitudes in SM.**

At high energies the 1-loop functions simplify considerably. Such asymptotic expressions are very useful in elucidating the physical properties of the amplitudes at high energies; as can be seen from [1] for the $\gamma\gamma \to \gamma\gamma$ case. In this Appendix, we present therefore the asymptotic expression for 1-loop functions relevant for the $\gamma\gamma \to \gamma Z$ amplitudes.

Using thus the well known asymptotic expression for the $B_0$ function of (A.4)
\[ B_0(\hat{s}) \simeq \Delta + 2 - \ln \left( \frac{\hat{s} - i\epsilon}{\mu^2} \right), \quad (B.1) \]
where $\Delta$ is the usual infinite term entering the calculation of the divergent integral [14], we obtain for the $B_Z(\hat{s})$ function defined in (A.6)
\[ B_Z(\hat{s}) \simeq -\ln \left( \frac{\hat{s} - i\epsilon}{-m_Z^2 - i\epsilon} \right), \quad \text{for} \quad |\hat{s}| \gg (m^2, m_Z^2) . \quad (B.2) \]

For the $C_0(\hat{s})$ function defined in (A.5), a useful form is [15]
\[ C_0(\hat{s}) = \frac{1}{2\hat{s}} \left[ \ln \left( \frac{-\hat{s} - i\epsilon}{m^2} \right) \right]^2 . \quad (B.3) \]

We next turn to $C_Z(\hat{s})$ and $D_Z(\hat{s}, \hat{u})$ of (A.7, A.8), which also depend on $m/m_Z$. Simple asymptotic expression are derived for them, for arbitrary $m/m_Z$, using the results of [15]. To present them we introduce the quantity
\[ a_Z \equiv \sqrt{1 - \frac{4m^2}{m_Z^2} + i\epsilon} \quad (B.4) \]
Then, for \(|s| \gg (m^2, m_Z^2)\),
\[
C_Z(s) \simeq \frac{1}{s} \left( \frac{1}{2} \ln^2 \left( \frac{-s - i\epsilon}{m^2} \right) + \frac{\pi^2}{2} - \frac{1}{2} \left[ \ln \left( \frac{1 + a_Z}{2} \right) - \ln \left( \frac{1 - a_Z}{2} \right) \right]^2 \right) + i\pi \left[ \ln \left( \frac{1 + a_Z}{2} \right) - \ln \left( \frac{1 - a_Z}{2} \right) \right] + O \left( \frac{1}{s} \right),
\]
while for \((|s|, |\hat{u}|) \gg (m^2, m_Z^2)\),
\[
D_Z(s, \hat{u}) \simeq \frac{2}{s\hat{u}} \left\{ \ln \left( \frac{-s - i\epsilon}{m^2} \right) \ln \left( \frac{-\hat{u} - i\epsilon}{m^2} \right) - \frac{1}{2} \left[ \ln \left( \frac{1 + a_Z}{2} \right) - \ln \left( \frac{1 - a_Z}{2} \right) \right]^2 \right\} + i\pi \left[ \ln \left( \frac{1 + a_Z}{2} \right) - \ln \left( \frac{1 - a_Z}{2} \right) \right] + O \left( \frac{1}{s}, \frac{1}{\hat{u}} \right).
\]

In all cases the principal value of the logarithm is understood, which has its cut is along the negative real axis.

We next turn to the functions \(\tilde{F}(s, \hat{t}, \hat{u})\) and \(E(\hat{t}, \hat{u})\) defined in (A.9, A.10), which together with \(B_Z(s)\) (compare (B.2)), determine the high energy behaviour of the various \(\gamma\gamma \to \gamma Z\) amplitudes. Using (B.5, B.6), they can be expressed for \((|s|, |\hat{t}|, |\hat{u}|) \gg (m^2, m_Z^2)\) as
\[
\tilde{F}(s, \hat{t}, \hat{u}) \simeq \frac{2}{s\hat{u}} \ln \left( \frac{-s - i\epsilon}{m^2} \right) \ln \left( \frac{-\hat{u} - i\epsilon}{m^2} \right) + \frac{2}{s\hat{t}} \ln \left( \frac{-s - i\epsilon}{m^2} \right) \ln \left( \frac{-\hat{t} - i\epsilon}{m^2} \right) + \frac{2}{\hat{t}\hat{u}} \ln \left( \frac{-\hat{t} - i\epsilon}{m^2} \right) \ln \left( \frac{-\hat{u} - i\epsilon}{m^2} \right),
\]
\[
E(\hat{t}, \hat{u}) \simeq \frac{a^2}{m^2} + \left[ \ln \left( \frac{-\hat{t} - i\epsilon}{m^2} \right) - \ln \left( \frac{-\hat{u} - i\epsilon}{m^2} \right) \right]^2.
\]

It is worth remarking that no \(a_Z\) term appears in (B.2), (B.7) and (B.8). This implies that the asymptotic \(\gamma\gamma \to \gamma Z\) amplitudes do not depend on the ratio \(m_Z/m\); as opposed to the situation for the asymptotic \(C_Z\) and \(D_Z\) functions.

The corresponding asymptotic expressions of the \(W\) loop contributions obtained from (A.12, A.20) for \((|s|, |\hat{t}|, |\hat{u}|) \gg (m_W^2, m_Z^2)\), by neglecting terms of \(O(m_W^2/s)\) but keeping terms of \(O(m_W/s)\) are
\[
A_{+++}^W(s, \hat{t}, \hat{u}) \simeq 16 E(\hat{t}, \hat{u}) + 8s^2 \tilde{F}(s, \hat{t}, \hat{u})
+ 2 \left( \frac{1}{c_W^2} - 6 \right) \left[ \frac{\hat{t}\hat{u}}{s^2} E(\hat{t}, \hat{u}) - 1 + \frac{(\hat{t} - \hat{u})}{s} [B_Z(\hat{t}) - B_Z(\hat{u})] \right],
\]
\[
A_{++-}^W(s, \hat{t}, \hat{u}) = A_{+-+}^W(s, \hat{u}, \hat{t}) \simeq 16 E(\hat{s}, \hat{t}) + 8\hat{u}^2 \tilde{F}(\hat{s}, \hat{t}, \hat{u})
+ 2 \left( \frac{1}{c_W^2} - 6 \right) \left[ \frac{\hat{s}\hat{u}}{\hat{u}^2} E(\hat{s}, \hat{t}) - 1 + \frac{(\hat{s} - \hat{t})}{\hat{u}} [B_Z(\hat{s}) - B_Z(\hat{t})] \right],
\]
\[
A_{++0}^W(s, \hat{t}, \hat{u}) = p_t \sqrt{2} m_Z \left( \frac{1}{c_W^2} - 6 \right) \left[ (\hat{t} - \hat{u}) \left( - \frac{E(\hat{t}, \hat{u})}{s^2} + \frac{2}{\hat{t}\hat{u}} \right) \right].
\]
\[ +\frac{4}{\hat{s}}[B_Z(\hat{t}) - B_Z(\hat{u})] \], \quad \text{(B.11)}

\[ A^W_{++-0}(\hat{s}, \hat{t}, \hat{u}) = A^W_{+-0}(\hat{s}, \hat{u}, \hat{t}) = p_t \sqrt{2} m_Z \left\{ 8\hat{u} \hat{F}(\hat{s}, \hat{t}, \hat{u}) + \frac{16}{\hat{u}} E(\hat{s}, \hat{t}) \right. \]
\[ + \left( \frac{1}{c_W^2} - 6 \right) \left[ -\frac{\hat{s}}{u^2} E(\hat{s}, \hat{t}) + \frac{2}{\hat{u}} [B_Z(\hat{s}) - B_Z(\hat{t})] + \frac{2}{\hat{t}} \right] \left\} , \quad \text{(B.12)} \]

\[ A^W_{++-}(\hat{s}, \hat{t}, \hat{u}) \simeq A^W_{++-0}(\hat{s}, \hat{t}, \hat{u}) \simeq A^W_{+-++}(\hat{s}, \hat{t}, \hat{u}) \]
\[ \simeq A^W_{+-+-}(\hat{s}, \hat{t}, \hat{u}) \simeq A^W_{+-+-0}(\hat{s}, \hat{t}, \hat{u}) \simeq 2 \left( \frac{1}{c_W^2} - 6 \right) \], \quad \text{(B.13)}

\[ A^W_{++0} \simeq 0 \] \quad \text{(B.14)}

It is easy to see that at energies above 250 GeV, the Sudakov-like log-squared terms in (B.8, B.7) largely cancel out, when substituted in these asymptotic amplitudes. Essentially only the single logarithm large imaginary terms remain contributing to the dominant amplitudes \( A^W_{++-0}(\hat{s}, \hat{t}, \hat{u}) \) and \( A^W_{+-0}(\hat{s}, \hat{u}, \hat{t}) \). Almost negligible are, the amplitudes in (B.11, B.12), while the rest are even smaller.

Similar asymptotic expression can also be obtained for the fermion loop contributions appearing in (A.27 - A.35), by taking \( |\hat{s}|, |\hat{t}|, |\hat{u}| \gg (m_f^2, m_Z^2) \), and using (B.7, B.8). It turns out that at energies above 250GeV, the fermion loop contribution in SM to the large imaginary parts of the amplitudes \( F_{++-0}(\hat{s}, \hat{t}, \hat{u}) \) and \( F_{+-0}(\hat{s}, \hat{t}, \hat{u}) = F_{+-++}(\hat{s}, \hat{u}, \hat{t}) \) is completely negligible. Only to the other very small amplitudes, the W and fermion loop contributions are comparable.

¿From Fig.2 and Fig.3, it can also be concluded that the real and imaginary contributions of a fermion or scalar loop, are on an equal footing. As said already, the large imaginary contributions to \( F_{++-0}(\hat{s}, \hat{t}, \hat{u}) \) and \( F_{+-0}(\hat{s}, \hat{t}, \hat{u}) = F_{+-++}(\hat{s}, \hat{u}, \hat{t}) \) comes from the dominant W loop only.
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Figure 1: SM contribution to the dominant $\gamma\gamma \rightarrow \gamma Z$ helicity amplitudes at $\vartheta^* = 30^0$ and $\vartheta^* = 90^0$. All other amplitudes are predicted to be smaller or about equal to $F_{++0}$ or $F_{+-0}$.
Figure 2: Chargino contribution to $\gamma\gamma \rightarrow \gamma Z$ helicity amplitudes for the gaugino (a,b) and higgsino (c,d) cases at $\vartheta^* = 30^0$ and $\vartheta^* = 90^0$. The parameters used are indicated in the figures and $Q_{\chi_1^+} = 1$. 
Figure 3: Contribution to $\gamma\gamma \rightarrow \gamma Z$ helicity amplitudes from an isodoublet (a,b) and an isosinglet (c,d) slepton at $\vartheta^* = 30^0$ and $\vartheta^* = 90^0$. The parameters used are indicated in the figures and the slepton mass is taken $M_\tilde{l} = M_s = 100 GeV$. 
Figure 4: $\bar{\sigma}_0$, $\bar{\sigma}_{22} = \bar{\sigma}_{23}$ and $\bar{\sigma}_3 = \bar{\sigma}'_3$ for SM (solid) and in the presence of a chargino (dash, dash-circle) or a charged slepton (box, rhombus) contribution, using the same parameters as in Fig.2 or Fig.3 respectively.
Figure 4: $\bar{\sigma}_{33}$ and $\bar{\sigma}'_{33}$ for SM (solid) and in the presence of a chargino (dash, dash-circle) or a charged slepton (box, rhombus) contribution, using the same parameters as in Fig.2 or Fig.3 respectively.
Figure 5: Cross sections $\bar{\sigma}_0(Z_L)$, $\bar{\sigma}_{22}(Z_L)$, and $\bar{\sigma}_{33}(Z_L)$ for $Z_L$ production in SM (solid) and in the presence of a chargino or a charged slepton contribution denoted as in Fig.4.
Figure 6: Angular distributions for $d\tilde{\sigma}_0/d\cos \vartheta^*$, $d\tilde{\sigma}_{22}/d\cos \vartheta^*$, $d\tilde{\sigma}_{23}/d\cos \vartheta^*$, $d\tilde{\sigma}_3'/d\cos \vartheta^*$. 
Figure 6: Angular distributions for $d\bar{\sigma}_{33}/d\cos\vartheta^*$, $d\bar{\sigma}'_{33}/d\cos\vartheta^*$. 