Analytical Study on Characteristics of Dual Microring Resonators Combined with the Add-drop and All-pass Types

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Abstract. This paper presents an analytical study on characteristics of dual microring resonators combined with the add-drop and all-pass types, which can provide novel filtering and delaying characteristics. Analytical expressions of transmission spectra and normalized delays of the dual microring resonators are developed using the transfer matrix method. The effects of coupling coefficients and loss on both transmission spectra and delays are then investigated. The analytical study shows that two resonance peaks of transmission spectra are existent. Under lossless coupling, the resonance frequency of the transmission spectra increases as the ring-ring coupling coefficient increases. However, it is independent on the ring-bus coupling coefficient. The valley amplitude of the transmission spectra increases as the ring-bus coupling coefficient increases, which is also independent on the ring-ring coupling coefficient. Generally, three delay peaks are existent. The corresponding frequencies of delay peaks on either side of the centre frequency are partially coincident with the normalized resonance frequencies of transmission spectra. The normalized delays can develop from three delay peaks to two delay peaks or single delay peak under specified coupling coefficients and loss.

1. Introduction
Microring resonators can be widely used as basic building blocks for future high density optical circuits, such as optical filters [1], wavelength converters [2], modulators [3], multiplexers [4], Lasers [5], switches [6] and sensors [7]. The spectral response and delays of a single ring resonator are typical periodical Lorenz shapes. Dual microring resonators are the simplest structure of multiple microring resonators, which have particular characteristics compared with the single microring resonator. Series-cascaded [8, 9] and parallel-cascaded [10, 11] dual microring resonators were studied in previous works. There are some other schemes to cascade two microring resonators. We have proposed a novel structure of dual microring resonators coupled via 3×3 couplers [12], which can be found applications in fields such as sensors and filters. Another novel configuration of dual microring resonators has been proposed and studied [13-15] and demonstrated [16]. Experimental results show the good agreement with the theoretical results. This structure is also a combination of the add-drop and all-pass types. Hence, the novel configuration of dual microring resonators is also named as dual microring resonators combined with the add-drop and all-pass types (DMRCAAT) in this paper. However, previous study on this novel structure is not completely. Characteristics of delays are also not analyzed and transmission spectra can be further studied.

In this paper, DMRCAAT is systematically investigated. First, analytical expressions of transmission spectra and delays of DMRCAAT are developed using the transfer matrix method. Then,
characteristics of transmission spectra and delays of DMRCAAT are further investigated and discussed under different coupling coefficients and loss.

2. Theory
The DMRCAAT consists of a conventional add-drop microring resonator (ring1), with another all-pass microring (ring2) coupled to the add-drop microring, as shown in Fig.1.

![Figure 1. Schematic diagram of DMRCAAT.](image)

The structure has four ports, which are input, output, upload and download respectively. The arrows in the Fig.1 indicate the light wave propagation directions. For simplicity, we assume the coupling coefficients at the ring-bus waveguides to be identical as \( k_0 \), and the coupling coefficient between two rings is \( k_1 \). Corresponding transmission coefficients are \( t_0 = 1 - k_0^2 \) and \( t_1 = 1 - k_1^2 \) respectively. The radius is \( R \) and hence the perimeter is \( L = 2\pi R \). The effective index is \( n_{\text{eff}} \) and the optical wavelength is \( \lambda \). The normalized frequency is denoted as \( \theta = n_{\text{eff}} (2\pi/\lambda) L \) with \( \tau \) as the resonator round-trip transmission coefficient. Using the transfer matrix method [17], the output complex optical field \( q \), spectrum \( D \), effective phase shift \( \Phi \) and delay \( T \) at the download port are found as

\[
q = \frac{-k_0^2 \tau^2 \exp(i\theta/2)[t_t - \tau \exp(i\theta)]}{1 - t_t \tau \exp(i\theta) - t_0^2 \tau^2 \exp(i\theta) + t_0^2 \tau^2 \exp(i2\theta)}
\]

\[
D = |q|^2 = \frac{\sigma_1 (e_z - \cos \theta)}{1 + e_0 (e_z - \cos \theta)^2}
\]

\[
\phi = \arg(q) = \frac{3\theta}{2} + a \tan \frac{t_t \sin \theta}{\tau - t_t \cos \theta} + a \tan \frac{t_t \tau (1 + t_0^2) \sin \theta - t_0^2 \tau^2 \sin 2\theta}{1 - t_t \tau (1 + t_0^2) \cos \theta + t_0^2 \tau^2 \cos 2\theta}
\]

\[
T = \frac{d\phi}{d\theta} = \frac{1}{e_z (e_z - \cos \theta)} + \frac{\sigma_4 (e_z - \cos \theta)}{1 + e_0 (e_z - \cos \theta)^2}
\]

where \( \sigma_1 = 2t_t \tau (1-t_0^2)/[(1-t_0^2)^2](1-t_0^2)^2 t_t^2/4t_0^2) \), \( \sigma_4 = t_t (1+t_0^2)/(1-t_0^2)^2 t_t^2/(4t_0^2) \), \( e_0 = 4t_0^2 \tau^2 / [(1-t_0^2)^2(1-t_0^2)^2 t_t^2/4t_0^2) \), \( e_4 = (1-t_0^2)^2/4t_t \). Hence the characteristics of transmission spectra and delays are related to parameters \( k_0 \), \( k_1 \) and \( \tau \).
3. Analysis

3.1. The effects of coupling coefficients on transmission spectra
Firstly we investigate the case of lossless resonators, i.e., \( \tau =1 \). Fig.2 (a) shows the transmission spectra with different ring-bus coupling coefficient \( k_0 \) under a fixed ring-ring coupling coefficient \( k_1 \). Fig.2 (b) shows transmission spectra for \( k_0=0.1, 0.5 \) and 0.99 respectively. The transmission spectra split into two symmetrical resonance peaks near the centre frequency of \( \theta = 0 \), due to the mode splitting [18] between the coupled microring resonators.

![Figure 2](image-url) Transmission spectra \( D \) as functions of coupling coefficients. (a) Transmission spectra as a function of \( k_0 \) under \( k_1=0.5 \), (b) transmission spectra for \( k_0=0.1, 0.5 \) and 0.99 respectively under \( k_1=0.5 \), (c) transmission spectra as a function of \( k_1 \) under \( k_0=0.5 \), (d) transmission spectra for \( k_1=0.01, 0.5 \) and 0.99 respectively under \( k_0=0.5 \).

As shown in Fig.2 (a) and (b), both the full width at half maximum (FWHM) and the amplitude at the centre frequency \( (\theta = 0) \) of transmission spectra \( (D_1) \) increase as the ring-bus coupling coefficient \( k_0 \) increases under a fixed ring-ring coupling coefficient \( k_1 \). On the contrary, Fig.2 (c) and (d) show that both the FWHM and \( D_1 \) of transmission spectra are almost not changed, while the free spectral range increases as the ring-ring coupling coefficient \( k_1 \) increases under a fixed ring-bus coupling coefficient \( k_0 \).

From Eq. (2) and \( dD/d\theta =0 \), the equation can be written as

\[
\sin \theta [\cos^2 \theta - 2v_2 \cos \theta (1/v_0 + v_1^2 - 2v_1 v_2)] = 0
\]

which suggests that there are off-resonance frequencies at \( \sin \theta = 0 \) (i.e., \( \theta = \pm \pi \) or \( \theta = 0 \)) and resonance frequencies \( \theta = \pm \arccos(\sqrt{v_2^2 + 1/v_0 + v_1^2 - 2v_1 v_2}) = \pm \arccos(t_1) \). This means that resonance frequencies are only dependent on \( t_1 \) and independent on \( t_0 \). When \( \theta_1 \) is very small, there is a near linear relationship \( \theta_1 \sim k_1 \). \( \theta_1 \) increases as \( k_1 \) increases in a near linear relationship as shown in the solid lines in Fig. 3. Then we analyze \( D_1 \) using numerical simulations. From Eq. (2), \( D_1 = [(1-t_0^2)/(1+t_0^2)]^2 \), which means that \( D_1 \) is only dependent on \( t_0 \) and independent on the \( t_1 \). \( D_1 \) increases as \( k_0 \) increases as shown in the dashed lines in Fig. 3.
3.2. The effects of coupling coefficients on delays

Under the lossless coupling, Fig.4 shows delays as functions of coupling coefficients. For simplicity, we make a normalized processing for delays as shown in Figs.4 (a) and (c). Moreover, we also limit the maximum delay as 15 in Figs.4 (b) and (d).

Similar to the study of transmission spectra, there are three resonance peaks in delays due to the mode splitting and combinations of two optical paths. We define corresponding frequencies of resonance peaks on both sides of the centre frequency (θ = 0) as θ₂. Meanwhile, the corresponding amplitude of resonance peaks at the centre frequency (θ = 0) is denoted as T₁. As k₀ increases under a fixed k₁, T₁ increases and the amplitude of the resonance peak on both sides of the centre frequency decreases. Finally, three resonance peaks are degenerated into one peak at the centre frequency. On the contrary, T₁ decreases as the k₁ increases under a fixed k₀, and the single resonance peak splits into three peaks near the centre frequency, as shown in Fig. 4 (c) and (d).

From Eq. (4), the delays can be written as

\[ T = \varsigma_1 + \varsigma_2 \]
where \( \varsigma_1 = 1/\varepsilon_3 (\varepsilon_2 - \cos \theta) \), \( \varsigma_2 = \sigma_2 (\varepsilon_4 - \cos \theta)/[1 + \varepsilon_0 (\varepsilon_1 - \cos \theta)^2] \), which suggests that \( \varsigma_1 = f(t_1, \tau, \theta) \) and \( \varsigma_2 = f(t_0, t_1, \tau, \theta) \). This means that \( T_1 \) is only determined by \( k_1 \) under the lossless case, i.e., by ring 2. The ring 2 confines most light when \( k_1 \) is small. The light confined by ring 2 decreases as \( k_1 \) increases. Thus the delay at the center frequency decreases gradually. The delay peaks on both sides of the centre frequency are mainly decided by \( \varsigma_2 \). It is caused by the mode splitting. Fig. 5 (a) shows \( T_1 \) increases with \( k_0 \) increasing under a fixed \( k_1 \). \( \theta_2 \) changes very small when \( k_0 < 0.5 \), and reduces to 0 rapidly near \( k_0 = 0.6 \). Fig. 5 (b) shows that \( T_1 \) decreases and \( \theta_2 \) increases with \( k_1 \) increasing under a fixed \( k_0 \).

3.3. Frequency detuning between spectra and delays
The optical path difference of forward and backward light waves induces interferes in optical resonators. Light field distributions in DMRCAAT decide the characteristics of transmission spectra and delays. However, there are two resonance peaks in both spectra and delays on both sides of the centre frequency, and the corresponding peak frequencies (i.e., \( \theta_1 \) and \( \theta_2 \)) are different, also named as the peak frequency detuning. We obtain the relationship between \( (\theta_1 - \theta_2)/\pi \) and \( k_0 \) (or \( k_1 \)) by numerical simulations. From Fig. 6 (a), \( \theta_1 \) is coincident with \( \theta_2 \) when \( k_0 \) is small under a fixed \( k_1 \), with \( \theta_1 \) slightly higher than \( \theta_2 \). After \( \theta_2 \) decreases to 0 near \( k_0 = 0.6 \), the peak frequency difference will keep as a constant. Fig. 6 (b) shows that the peak frequency difference increases as \( k_1 \) increases to a maximum and then decreases under a fixed \( k_0 \).

Figure 5. (a) \( \theta_2/\pi \) (solid) and \( T_1 \) (dashed) as functions of \( k_0 \) under \( k_1 = 0.5 \) and \( \tau = 1 \); (b) \( \theta_2/\pi \) (solid) and \( T_1 \) (dashed) as functions of \( k_1 \) under \( k_0 = 0.5 \) and \( \tau = 1 \).

Figure 6. (a) \( (\theta_1 - \theta_2)/\pi \) as a function of \( k_0 \) under \( k_1 = 0.5 \) and \( \tau = 1 \), (b) \( (\theta_1 - \theta_2)/\pi \) as a function of \( k_1 \) under \( k_0 = 0.5 \) and \( \tau = 1 \).
3.4. The effects of loss on spectra and delays

To consider the case of loss, equal transmission coefficients $t_0 = t_1$ are adopted. This discussion can be divided into three cases by different relations between $\tau$ and $t_0$. They are the critical coupling ($t_0 = t_1 = \tau$), over coupling ($t_0 = t_1 < \tau$), and under coupling ($t_0 = t_1 > \tau$) conditions. From Eqs. (2) and (4), the change regularities of $D$ and $T$ with $\tau$ can be obtained. Fig. 7 (a) shows that the amplitude of $D$ decreases as $\tau$ decreases (i.e., the loss increases). As shown in Fig. 7 (b), $\theta_1$ increases and then decreases to 0 as $\tau$ decreases. Meanwhile, $D_1$ decreases to 0 at the critical coupling condition. Then $D_1$ increases and decreases, as $\tau$ decreases.

Figure 7. When $t_0 = t_1 = 0.8$, (a) transmission spectra under $\tau$ adopted as 1.0, 0.9, 0.7 and 0.4 respectively, (b) $\theta_1/\pi$ (solid) and $D_2$ (dashed) as functions of $\tau$.

To analyze the delays, the normalized processing by $|T|_{\text{max}}$ is carried out because of negative delays existing. We can obtain the relations between normalized delays $T/|T|_{\text{max}}$ and $\tau$, as shown in Fig. 8 (a). There are large negative and positive delays near the critical coupling region. As shown in Fig. 8 (b), the delays are positive when $\tau$ is larger than 0.8, while the delays at the centre frequency are always negative when $\tau$ is smaller than 0.8. The details are also shown in Fig. 8 (c). Fig. 8 (d) shows $\theta_2$ decreases slightly as $\tau$ decreases. Then $\theta_2$ increases to a maximum and finally decreases to 0.

Figure 8. When $t_0 = t_1 = 0.8$, (a) normalized delays $T/|T|_{\text{max}}$ as functions of $\tau$; (b) delays $T$ under $\tau$ adopted as 1.0, 0.9, 0.8, 0.7, 0.6 and 0.4 respectively; (c) delays $T$ under $\tau$ adopted as 0.82, 0.81, 0.8, 0.79 and 0.78 respectively, (d) $\theta_2/\pi$ (solid) and $T_2$ (dashed) as functions of $\tau$.

4. Conclusions

In conclusion, we have presented a comprehensive study on characteristics of DMRCAAT, which can provide novel filtering and delaying characteristics. Analytical expressions of transmission spectra and
delays of DMRCAAT are developed using the transfer matrix method. Then, the characteristics of
transmission spectra and delays of DMRCAAT are investigated in detail.

Under lossless coupling, the transmission spectra and delays split into multiple symmetrical
resonance peaks near the centre frequency because of the mode splitting induced by the two coupled-
microring resonators. Corresponding resonance frequencies of the transmission spectra increase with
the ring-ring coupling coefficient increasing, which are independent on the ring-bus coupling
coefficients. The valley amplitude of the transmission spectra increases with the ring-bus coupling
coefficients increasing, which is independent of the ring-ring coupling coefficient. Three delay peaks
exist near the centre frequency, which can vary from three delay peaks to two delay peaks or single
delay peak under specified coupling coefficients and loss. Corresponding frequencies of delay peaks
on either side of the centre frequency are partially coincident with the resonance frequencies of
transmission spectra. Therefore, there is a frequency detuning. The loss mainly affects the amplitude
of the spectra and delays, with peak frequencies also changed. There is an interesting phenomenon at
the critical coupling condition. The valley amplitude of the transmission spectra reaches to 0 at the
critical coupling condition, while there are large negative and positive delays near the critical coupling
region.

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