How to Learn a Useful Critic? Model-based Action-Gradient-Estimator Policy Optimization

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Abstract

Deterministic-policy actor-critic algorithms for continuous control improve the actor by plugging its actions into the critic and ascending the action-value gradient, which is obtained by chaining the actor’s Jacobian matrix with the gradient of the critic w.r.t. input actions. However, instead of gradients, the critic is, typically, only trained to accurately predict expected returns, which, on their own, are useless for policy optimization. In this paper, we propose MAGE, a model-based actor-critic algorithm, grounded in the theory of policy gradients, which explicitly learns the action-value gradient. MAGE backpropagates through the learned dynamics to compute gradient targets in temporal difference learning, leading to a critic tailored for policy improvement. On a set of MuJoCo continuous-control tasks, we demonstrate the efficiency of the algorithm with respect to model-free and model-based state-of-the-art baselines.

1 Introduction

Reinforcement learning (RL) [32, 47] studies sequential decision making problems, in which an agent aims at maximizing the cumulative reward it collects in an environment. One of the most popular classes of algorithms for RL are policy gradient methods [9, 48], which involve differentiable control policies improved by gradient ascent. They feature suitability to environments with continuous state and action spaces, and compatibility with state-of-the-art deep learning methods. Policy gradient algorithms often employ the actor-critic [22] scheme: an actor, which determines the control policy, is evaluated using a critic. Thus, the degree of actor’s improvement is limited by the information provided by the critic, naturally raising the question of how the critic should be trained.

Typically, algorithms that use powerful function approximators [16, 24] learn the critic by temporal difference [45], optimizing for an accurate prediction of the expected return of the actor. For deterministic-policy continuous-control [24, 42], however, the value provided by the critic is neither used for improving the policy nor for acting in the environment [48]. Instead, only the action-gradient of the value function, i.e., the gradient of the critic w.r.t. the action performed by the actor, is employed during policy optimization. Specifically, the policy gradient is obtained through the computation of the action-value gradient, by chaining the actor’s Jacobian with the action-gradient of the critic.

Learning the critic by value rather than by action-gradient of the value relies on hazy smoothness assumptions on the real value function [42]. This means that, in conventional temporal difference learning, the critic learns action-value gradients implicitly, which could harm the performance of a deterministic policy gradient algorithm.

In this paper, we propose Model-based Action-Gradient-Estimator Policy Optimization (MAGE), a continuous-control deterministic-policy actor-critic algorithm that explicitly trains the critic to provide

1The PyTorch implementation of the algorithm is available at https://github.com/nnaisense/MAGE

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accurate action-gradients for the use in the policy improvement step. Motivated by both the theory on Deterministic Policy Gradients [42] and practical considerations, MAGE utilizes temporal difference methods to minimize the error on the action-value gradient. To this aim, the algorithm leverages a trained dynamics model as a proxy for a differentiable environment and techniques reminiscent of double backpropagation [11]. On challenging continuous control benchmarks [6, 50], we show that MAGE is significantly more sample-efficient than state-of-the-art model-free and model-based baselines.

The rest of the paper is organized as follows. In Section 2, we provide the notation and background on deterministic policy gradients. Our algorithm, together with its theoretical motivation, is introduced in Section 3 followed by empirical results in Section 4. In Section 5 we present some of the related work and its relationship with our approach.

2 Background

2.1 Preliminaries

Consider a discrete-time Markov Decision Process [32] (MDP), defined as $\mathcal{M} = (\mathcal{S}, \mathcal{A}, p, r, \gamma, \mu)$, where $\mathcal{S}$ is the space of possible states, $\mathcal{A}$ is the space of possible actions, $p : \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S})$ is the transition model, $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is the known and differentiable reward function, $\gamma$ is the discount factor, $\mu \in \Delta(\mathcal{S})$ is the initial state distribution. The behavior of the agent is described by a deterministic policy $\pi_\theta : \mathcal{S} \rightarrow \mathcal{A}$, belonging to a parametric space of policies $\Pi = \{\pi_\theta : \theta \in \Theta \subseteq \mathbb{R}^n\}$. Let $d_\mu^n(s) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \Pr(s_t = s | \pi, \mu)$. The total reward collected by an agent is quantified with the action-value function $Q_\pi(s, a) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) | s_0 = s, a_0 = a]$ and performance function $J(\theta) = \mathbb{E}_{s \sim \mu}[Q_\pi(s, \pi_\theta(s))]$.

Practical algorithms can employ an approximate action-value function $\hat{Q}$ and an approximate dynamics model $\hat{p}$, which, most commonly, are parametric function approximators specified by the spaces $\hat{Q} = \{Q_\phi : \phi \in \Phi \subseteq \mathbb{R}^h\}$ and $\hat{P} = \{p_\omega : \omega \in \Omega \subseteq \mathbb{R}^k\}$.

2.2 Deterministic Policy Gradients and TD-learning

Policy gradient methods optimize policy $\pi_\theta$ by ascending the direction of the gradient of its performance function $J(\theta)$. The Deterministic Policy Gradient Theorem [42] provides a practical way to calculate this gradient. It shows that, under some mild regularity conditions on the MDP, the gradient of the performance of a deterministic policy $\pi_\theta$ is given by:

$$\nabla_{\theta} J(\theta) = \frac{1}{1 - \gamma} \int_{\mathcal{S}} d_\mu^n(s) \nabla_a Q_\pi(s, a)|_{a = \pi_\theta(s)} \nabla_{\theta} \pi_\theta(s) ds. \tag{1}$$

This result can be interpreted through the lens of the chain rule applied to the action-value gradient $\nabla_{\theta} Q(s, \pi_\theta(s))$: the policy gradient does not directly depend on the gradient of $d_\mu^n$, and can be obtained by just chaining the actor’s Jacobian $\nabla_{\theta} \pi_\theta$ with the action-gradient of the value function $\nabla_a Q_\pi(s, a)$.

The theorem motivates a family of policy gradient actor–critic algorithms, such as DDPG [24] and TD3 [16]. Similarly to the classical policy iteration [47], the evaluation of a policy $\pi \in \Pi$ (called actor in this context) is interleaved with its improvement w.r.t the approximate action-value function $\hat{Q} \in \hat{Q}$ (called critic). Specifically, the typical desideratum consists in finding a critic $\hat{Q}$ which minimizes the policy evaluation error:

$$\hat{Q} \in \arg \min_{\hat{Q} \in \hat{Q}} \mathbb{E} \left[ \delta^\pi \hat{Q}(s, \pi(s)) \right], \tag{2}$$

where $\delta^\pi \hat{Q}(s, a) = Q_\pi(s, a) - \hat{Q}(s, a)$ is a deviation w.r.t the true state-action value. Given the lack of knowledge about the transition model, $Q_\pi$ needs to be approximated. A common approximation technique consists in employing the temporal-difference (TD) error [45], defined as $\delta^\pi \hat{Q}(s, a, s') = r(s, a) + \gamma \hat{Q}(s', \pi(s')) - \hat{Q}(s, a)$, giving rise to a bootstrapped optimization.
Gradient Theorem will be effective at solving the control problem. Instead, the following result provides foundations for effective
An actor can only be as good as allowed by its critic. Thus, obtaining an 
which relies on maximization in a discrete action space that cannot be easily carried out in continuous 
The proposition (see Appendix A for the proof) is a direct consequence of the 
Deterministic Policy 
value function \( \hat{Q} \) gating through the environment dynamics 
A better objective function for critic learning.

The above can be seen as a generalization of the policy improvement step in classical policy iteration, 
which uses the norm of the difference between the true policy gradient \( \nabla Q(\pi, \pi) \), can be upper bounded as:

\[
\| \nabla J(\theta) - \hat{\nabla} J(\theta) \| \leq \frac{L_p}{1 - \gamma} \mathbb{E}_{s \sim d^a_{\pi}} \left\| \nabla_a \delta^\pi \hat{Q}(s, \alpha) \right\|_{a=\pi(s)}
\]

The proposition (see Appendix A for the proof) is a direct consequence of the Deterministic Policy Gradient Theorem. The Lipschitz assumption for \( \pi \) is easily satisfied for many policy classes of practical use, e.g., neural networks [15].

Proposition [3.1] suggests that it is the norm of the action-gradient of the policy evaluation error instead of its value that should be minimized to reduce the bias introduced by the use of the approximate value function \( \hat{Q} \). To minimize the bound, a proxy for the unknown \( Q^\pi \) is needed. To this aim, it is possible to follow the approach of traditional TD-learning, substituting the evaluation error \( \delta^\pi, \hat{Q} \) with the TD-error \( \hat{\delta}^\pi, \hat{Q} \). This leads to the following optimization problem:

\[
\hat{Q} \in \arg \min_{Q, \theta} \mathbb{E}_{s \sim d^a_{\pi}} \left\| \nabla \hat{\delta}^\pi, \hat{Q}(s, \alpha, s') \right\|
\]

Notice that computing the gradient w.r.t. the action of the TD-error \( \hat{\delta}^\pi, \hat{Q} \) requires taking into account the effect of action \( \alpha \) on the transition to the subsequent state in the environment \( s' \), i.e., backpropagating through the environment dynamics \( p \). Since \( p \) is not available in typical RL settings, especially
in a differentiable form, it needs to be substituted with an approximate model \( \hat{p} \), as commonly done in model-based RL [7, 9, 20]. An environment model gives rise to imaginary transitions \((s, \pi(s), \hat{s})\), where \( \hat{s} \sim \hat{p}(\cdot|s, \pi(s)) \). Given differentiable model, policy, and action-value function, the action-gradient can be effectively computed by leveraging standard automatic differentiation tools [5]. The corresponding computational graph is depicted in Figure 1. This leads to a viable way to obtain \( \hat{Q} \):

\[
\hat{Q} \in \arg \min_{\tilde{Q} \in \mathcal{Q}} \mathbb{E}_{s \sim d^\pi, \hat{s} \sim \hat{p}(\cdot|s, \pi(s))} \left\| \nabla_a \delta^{\pi, \tilde{Q}}(s, \pi(s), \hat{s}) \right\|
\]

(6)

Even in the general case of a stochastic model, differentiating through the resulting computations is still possible for many commonly used model classes via the reparametrization trick [19]. Using an approximate model \( \hat{p} \) implies a tradeoff, since additional bias is injected into the estimation of the critic. Nonetheless, the use of \( \hat{p} \) is the most direct way to solve the optimization problem in Equation 6, and thus to obtain a \( \hat{Q} \) that provides a more accurate policy gradient w.r.t. the one obtained by training the critic using the TD-error.

### 3.2 Model-based Action-Gradient-Estimator Policy Optimization

The outlined procedure for learning the value function requires an approximate model \( p_\omega \), thus naturally suggesting its integration into a model-based policy optimization framework. A model-based actor-critic method involves three steps during each iteration: learning the model \( p_\omega \), updating the action-value function \( Q_\phi \) and improving the policy \( \pi_\theta \). In the following, we consider neural networks as function approximators to represent the three modules, although any class of differentiable models could be leveraged. Our approach is inspired by Dyna [46], and employs an approximate dynamics model for generating 1-step imaginary on-policy transitions starting from observed states stored in a replay buffer. Those transitions are then employed to learn \( Q_\phi \), and, in turn, leveraged for computing an improvement direction for the parameters of the policy \( \pi_\theta \).

In preliminary experiments, we have found that directly solving the minimization problem in Equation 6 is hard in practice. During the optimization, the parameters are prone to be trapped in local-minima, which leads to degenerate solutions. A demonstration of this effect is detailed in Appendix B.1. The root cause of this effect is unknown.

A remedy consists in introducing a constraint into the optimization problem. We argue that, among the possible solutions, a natural one is constraining the optimization landscape by bounding the traditional TD-error (see Equation 3), and thus solving the following optimization problem:

\[
\min_{\hat{\phi} \in \Phi} \mathbb{E}_{s \sim d^\pi, \hat{s} \sim \hat{p}(\cdot|s, \pi(s))} \left\| \nabla_a \delta^{\pi, \hat{Q}_\phi}(s, a, \hat{s}) \right\|_{a = \pi(s)} \\
\text{s.t.} \mathbb{E}_{s \sim d^\pi, \hat{s} \sim \hat{p}(\cdot|s, \pi(s))} \left\| \delta^{\pi, \hat{Q}_\phi}(s, \pi(s), \hat{s}) \right\| \leq \lambda.
\]

(7)
We plug our critic training method into a model-based Dyna-like algorithm, giving rise to Model-based Action-Gradient-Estimator Policy Optimization (MAGE).

Algorithm 1 Model-based Action-Gradient-Estimator Policy Optimization (MAGE)

Input: Initial buffer $B$, set of parameter vectors $\{\omega, \phi, \theta\}$

for each iteration do
    Collect transition $(s, a, s')$ acting according to exploratory version of $\pi_\theta$
    $B \leftarrow B \cup \{(s, a, s')\}$
    for each model learning step do
        $\omega \leftarrow \omega - \alpha_p \nabla \omega f(s, a, s'; \omega)$, $\quad (s, a, s') \sim B$
    end for
    for each policy optimization step do
        Extract state $s$ after sampling $(s, a, s') \sim B$
        $\phi \leftarrow \phi$
        $\delta(s, a, s; \phi) \leftarrow r(s, \pi_\theta(s)) + \gamma Q(\hat{s}, \pi_\theta(\hat{s})) - Q(\phi(s, \pi_\theta(s)))$, $\quad \hat{s} \sim p_\omega(\cdot | s, \pi_\theta(s))$
        $\phi \leftarrow \phi - \alpha_Q \nabla \phi \left(\| \nabla_a \delta(s, a, s; \phi) \|_a=\pi_\theta(s) \| + \lambda \| \delta(s, a, s; \phi) \| \right)$
        $\theta \leftarrow \theta + \alpha_\pi \nabla_\theta Q(\phi(s, \pi_\theta(s))$
    end for
end for

As the above expressions already require non-trivial gradient computations, we decided to avoid the use of complex and expensive methods for nonlinear programming. Instead, we resort to penalty function methods [24] by regularizing the original objective by using the TD-error. A similar approach has been used in the past, e.g., Proximal Policy Optimization (PPO, [41]) to approximately solve a different constrained optimization problem.

Eventually, the parameters of $Q_\phi$ are learned by descending the gradient

$$\nabla_\phi L(s, a, \hat{s}; \phi, \theta) = \nabla_\phi \left(\| \nabla_a \delta^{\pi_\theta, Q_\phi}(s, a, \hat{s}) \|_a=\pi_\theta(s) \| + \lambda \| \delta^{\pi_\theta, Q_\phi}(s, a, \hat{s}) \| \right)$$

(8)

on an imaginary transition $(s, a, \hat{s})$. This expression requires computing second-order gradients, which would be computationally expensive if computed w.r.t. to the high-dimensional space of parameters $\Phi$ of the $Q$-function. Here, however, the optimization is affordable since the gradients are computed w.r.t., typically low dimensional, actions. Notice also that the computational overhead of the second term in Equation 8 is minimal, since evaluating the TD-error $\delta^{\pi_\theta, Q_\phi}(s, a, \hat{s})$ is anyway, when using automatic differentiation, required to compute its gradient.

We plug our critic training method into a model-based Dyna-like algorithm, giving rise to Model-based Action-Gradient-Estimator Policy Optimization (MAGE), which is presented in Algorithm 1. At each iteration, the dynamics model $p_\omega$ is trained to maximize the likelihood of the transitions stored in the experience replay buffer $B$, or, equivalently, to minimize an appropriate loss function $\ell$:

$$\omega \in \arg \min_{\omega \in \Omega} \mathbb{E}_{(s, a, s') \sim B} \left[ \ell(s, a, s'; \omega) \right].$$

(9)

Then, for one or more steps, the TD-error for the current policy and action-value function is computed, and used together with its action-gradient to update $Q_\phi$, which in turn is leveraged to improve $\pi_\theta$.

4 Experiments

4.1 Sample-Efficient Continuous Control with MAGE

Algorithm settings. The general structure of MAGE is compatible with many actor-critic algorithms with deterministic policies. In this experiment, we employ TD3 [16], a popular, state-of-the-art extension to DDPG [24], as a base policy optimization method. This amounts to the addition of target policy smoothing, delayed policy updates, clipped double-Q learning and target functions. We call this version of our algorithm MAGE-TD3. After each step of environment interaction, we add the collected transition in the replay buffer $B$, train the approximate model $p_\omega$, and update critic and actor 10 times. We employ a single value of $\lambda = 0.05$ across all the experiments, since we found MAGE reasonably robust to the choice of this hyperparameter (see Appendix B). In order to reduce the impact of model bias, MAGE leverages an ensemble of 8 probabilistic Gaussian-output models, trained by maximum likelihood estimation.

As for simplicity of presentation, an abstract version of MAGE is considered in Algorithm 1. Any actor-critic algorithm can be then used to instantiate MAGE into a practical incarnation.
Baselines and environments. We consider one model-based and two model-free algorithms as baselines. The first one is Dyna-TD3, which uses a classical TD-error loss, otherwise being identical to MAGE-TD3. It resembles 1-step horizon Model-based Policy Optimization (MBPO [20]), but uses a deterministic policy optimized by TD3. Apart from that, we compared MAGE against TD3 and its sample-efficient variant [52], which employs multiple updates for each environment step and trades off computational efficiency and, potentially, stability [28] for sample-efficiency. Specifically, for a fair comparison with MAGE-TD3, we execute 10 critic and actor updates after each interaction with the environment. We employ environments from OpenAI Gym [6] and the MuJoCo physics simulator [50] as continuous control benchmarks, assuming, for all the environments, the availability of a differentiable reward function (we show in Appendix B.2 that MAGE behaves well also with a learned reward). Additional details concerning the experimental setting are reported in Appendix D.

Results. Figure 2 shows the learning curves for the average return of all the approaches. Since our primary interest is MAGE’s sample efficiency, we show the first $10^5$ steps of environment interaction. The results show that MAGE is able to learn at least as fast as all the baselines on all the environments, confirming the intuitive advantage of directly optimizing for the accuracy of the estimated action-value gradient. Interestingly, no superiority of the vanilla Dyna-TD3 on its simple data-efficient version can be observed: this demonstrates that there is no intrinsic advantage in terms of sample-efficiency for model-based reinforcement learning, but it is instead highly environment- and algorithm-dependent. On the other hand, increasing the number of offline updates for model-free algorithms can cause strong instabilities in some environments, as it is the case, for instance, on the Pusher-v2 environment. Note that, in contrast with Dyna-TD3, that only leverages the model as a generator for additional transitions w.r.t. the ones that can be obtained in the environment, MAGE makes deeper use of the learned model of the dynamics in order to unlock a peculiar learning modality that would be impossible in a model-free setting.

4.2 Understanding MAGE

Action-Gradient Estimation. MAGE was designed to obtain a critic that is maximally useful for policy improvement by yielding accurate action-value gradients. How much better does it predict them compared to the traditional TD-learning? To investigate this question, we employ the Pendulum-v0 environment, using a differentiable oracle in place of the approximate dynamics model.
We fix a randomly initialized actor, and train only its critic with both MAGE-TD3 and its Dyna counterpart. During the training, for each transition on a trajectory, we estimate the true action-gradient by computing
\[
\nabla_a Q^\pi(s_t, a_t) = \nabla_a \left[ \sum_{t'=t}^{t+H-1} \gamma^{t-t'} r(s_{t'}, a_{t'}) \right] = \nabla_a \left[ \sum_{t'=t}^{t+H-1} \gamma^{t-t'} \pi(a_{t'} | s_{t'}) \right]
\]
and compare it to the action-gradient \( \nabla_a \hat{Q} \) provided by a learned critic. The results, shown in Figure 3 indicate that the MAGE’s critic progressively learns an accurate estimate of the action-gradient; by contrast, the one trained using traditional temporal difference difference completely fails in predicting it. The results undermine the common assumption that minimizing the TD-error yields also a minimization of the error on the gradients. The difference can explain the superior sample-efficiency of MAGE over classical TD-learning.

**Reward Availability.** Throughout the presentation and evaluation of MAGE, we assumed complete knowledge of the reward function \( r \) of the underlying Markov Decision Process. While this assumption is natural in many real-world settings \[9\] and thus commonly employed in other model-based reinforcement learning methods \[7,10,19\], its role is particularly crucial in our algorithm. In traditional temporal difference learning, given a transition \((s, a, s')\), the reward \( r(s, a) \) constitutes the only grounding element in the objective function. The reward function plays an even stronger role as a grounding element for bootstrapping in MAGE, since both its value \( r(s, a) \) and its action-gradient \( \nabla_a r(s, a) \) are needed: while the former can be usually observed in the environment, the latter can only be computed with complete knowledge of the underlying function. In our experiments on the sample-efficiency of MAGE, we employed the ground-truth reward function (with ground-truth gradients): a natural question is whether MAGE still performs reasonably well if an estimated reward function \( \hat{r} \) learned from data is used in place of the real \( r \). To answer this question, we evaluate a version of MAGE in which an approximate reward function \( \hat{r} \) is learned by using a neural network approximator and minimizing the mean squared error on the rewards observed in the environment. The results, perhaps surprising, are reported in Figure 4 for the Pusher-v2 environment (see Appendix B.2 for the complete results). They show that, for the commonly employed continuous control benchmarks, the performance of our method is only minimally degraded by the use of an approximate reward function in place of the real one, thus suggesting inherent robustness to inaccurate evaluations of the reward function as well as its action-gradient.

### 5 Related Work

Policy gradients are among the most popular methods in reinforcement learning. A variety of algorithms have been proposed for the estimation of the policy gradient, either involving only the policy \[4, 28, 55\] or also the value function \[29, 40, 41\]. The latter category of algorithms is referred to as actor-critic methods \[22, 31\]. Among them, the ones based on the Deterministic Policy Gradient \[23, 42\] leverage the action-gradient of the critic. When using function approximation, the quality of the learned critic is of paramount importance \[3\]: for instance, enforcing on the critic the compatibility conditions \[42\] ensures an unbiased estimate of the policy gradient.

Developed around such conditions, GProp \[23\] is, to the best of our knowledge, the only method that explicitly optimizes for the accuracy of the learned action-value gradient. It is significantly different w.r.t. MAGE, being model-free and based on gradient estimation via noisy perturbations together with an additional deviator network. Importantly, while GProp’s deviator network is a function approximator that outputs an estimate for the action-gradient, recent theoretical \[55\] and
practical insights outside of RL suggest that learning the action-gradient by second-order differentiation, as we propose in MAGE, is not only simpler to implement, but also fundamentally more effective when using neural network approximators.

The technique we use for learning the action-gradient relies on the differentiation of the TD-error and, thus, of the Bellman equation. This is related to a broad class of methods called value gradients, in which the policy is improved by backpropagating through the unrolled Bellman equation. Those approaches, however, learn the value function by standard temporal difference. Another approach, named Dual Heuristic Programming, learns the gradient of the state-value function in a model-based setting, leading to a TD-learning procedure that resembles our approach. However, the method has the main goal of improving generalization of the value function and exploration, and is fundamentally different from MAGE, that aims at learning an accurate action-gradient of the critic and is motivated by the Deterministic Policy Gradient Theorem.

More broadly, inside and outside of reinforcement learning, several algorithms incorporate gradient penalties into the loss function used for training a neural network. This technique, known as double backpropagation, has been employed in a number of applications, for instance increasing generalization capabilities, enforcing Lipschitz constants, or encouraging robustness to adversarial examples. Particularly related to our approach is Sobolev training, which leverages the availability of the derivatives of a target function to explicitly try to learn both value and gradient of it during supervised training; in our case, no ground-truth gradient is available and we use the action-gradient of the TD-target as a proxy.

Our method learns the action-gradient in the context of model-based policy optimization. We mainly build upon the classical Dyna framework, in which a learned model is used for generating imaginary transitions, then employed for training a value function. Our algorithm, which learns a Q-function from model-generated data but only optimizes the policy by using real data, is related to the approaches that compute the policy gradient by using a model-based value function together with trajectories sampled in the environment. In practice, we leverage an ensemble of models, which has been shown to improve performance in a variety of contexts.

Finally, our work is related in spirit to decision-aware model learning (DAML). In DAML approaches, the model of the dynamics of the environment is learned by explicitly considering how it will be used for improving the control policy: this is the same rationale behind the learning objective used in MAGE for the critic, focused on how it will be useful for policy optimization, and not merely on how it will be similar to the true value function.

6 Conclusion

In this paper, we presented MAGE, a model-based actor-critic algorithm with deterministic actor that leverages an approximate dynamics model to directly learn the action-value gradient via temporal-difference learning. MAGE employs second-order differentiation to obtain a critic tailored for policy improvement. The empirical evaluation of MAGE demonstrated its superiority over model-based and model-free baselines on challenging high-dimensional continuous control tasks.

A limitation of our method is of computational nature: in addition to the cost of model learning, paid also by other model-based actor-critic algorithms, we incur the expense of computing the gradient w.r.t. the critic parameters of the action-gradient of the TD-error, in result, approximately doubling the running time in comparison to the Dyna-based policy gradient approach. This can potentially be alleviated by the development of more efficient automatic differentiation tools, which is currently an active area of research.

While it is often hard to determine under which circumstances the addition of an approximate learned model to a model-free algorithm is beneficial, we have shown that model-based techniques such as MAGE’s gradient-learning procedure, can unlock novel learning modalities, inaccessible for model-free algorithms. This can actually be the true power of model-based reinforcement learning. Therefore, apart from improving MAGE (e.g., by investigating the unconstrained critic learning problem) and applying it in other model-based approaches (e.g., value gradients with real trajectories), we hope that future work along this direction will reveal other innovative learning schemes that are infeasible in model-free settings.
**Broader Impact**

The method presented in this paper is a reinforcement learning algorithm that can be used to control a system executing real-valued actions in an environment. Therefore, a natural application of it is in robotics, with positive (e.g., elderly care, resource-efficiency in manufacturing) and negative (e.g., military) applications. Alongside many deep reinforcement learning algorithms, our method is computationally intensive and its training can thus require considerable resources (i.e., hardware and electricity).

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A Proof of Proposition 3.1

The proof follows directly from the Deterministic Policy Gradient Theorem. Therefore, the Proposition inherits all of its smoothness assumptions about the Markov Decision Process [42].

Proposition 3.1. Let $\Pi$ be a parametric space of $L_\pi$-Lipschitz continuous differentiable deterministic policies, $Q$ a space of approximate value functions and $\|\cdot\|$ any $p$-norm. Given $\pi \in \Pi$ and $\hat{Q} \in Q$, the norm of the difference between the true policy gradient $\nabla_{\theta} J(\theta)$ and its approximation $\hat{\nabla}_{\theta} J(\theta)$, which uses $\hat{Q}$, can be upper bounded as:

$$\|\nabla_{\theta} J(\theta) - \hat{\nabla}_{\theta} J(\theta)\| \leq \frac{L_{\pi}}{1 - \gamma} \mathbb{E}_{s \sim d^\pi_n} \left[ \nabla_a \delta^\pi(s,\hat{Q})_{\|a=\pi(s)\|} \right].$$

Proof.

$$\|\nabla_{\theta} J(\theta) - \hat{\nabla}_{\theta} J(\theta)\| = \frac{1}{1 - \gamma} \int_S d^\pi_n(s) \left( \nabla_a Q^\pi(s,a)_{\|a=\pi(s)\|} - \nabla_a \hat{Q}(s,a)_{\|a=\pi(s)\|} \right) \nabla_{\theta} \pi(s) ds \tag{10}$$

$$= \frac{1}{1 - \gamma} \int_S d^\pi_n(s) \left\| \nabla_a \delta^\pi \hat{Q}(s,a)_{\|a=\pi(s)\|} \nabla_{\theta} \pi(s) ds \right\| \tag{11}$$

$$\leq \frac{1}{1 - \gamma} \int_S d^\pi_n(s) \left\| \nabla_a \delta^\pi \hat{Q}(s,a)_{\|a=\pi(s)\|} \right\| \cdot \| \nabla_{\theta} \pi(s) \| \| ds \tag{12}$$

Equation (10) follows from the Deterministic Policy Gradient Theorem. To obtain Equation (11) we exploit the definition of $\delta^\pi$ and linearity of differentiation. Finally, in Equation (12) we use the Lipschitz policy assumption.

B Additional Experiments

B.1 Unconstrained Action-Value Gradient learning

Proposition 3.1 directly encourages training the critic by minimizing the bound on the error of the policy gradient, i.e., the norm of the action-gradient of the policy evaluation error. However, we found a direct optimization of this bound, by means of the TD-error, difficult in the context of Dyna-like algorithms. We analyze this behavior in the Pendulum-v0 environment [6], instantiating a version of MAGE based on DDPG [24] (MAGE-DDPG). To understand the learning dynamics of the action-value gradients in a way that is not affected by the model bias, we employ the differentiable version of the real environment dynamics and test MAGE without the TD-error regularization (i.e., with $\lambda = 0$). Therefore, at each step, $Q$ is improved by minimizing the norm of $\hat{\delta}$ computed on transitions whose next state is sampled from $p$. Unfortunately, no useful learning can be achieved in this setting: a degenerate solution consisting of $\hat{Q}$ such that $\| \nabla_a \hat{Q}(s,a) \| \approx 0$, $\forall s \in S$, $\forall a \in A$ is rapidly reached, as shown in Figure 5. We employ exactly the settings and hyperparameters that are successfully employed in the full version of MAGE.

We believe that understanding whether, or under which circumstances, the direct minimization of the bound in Proposition 3.1 is possible is an interesting open question.
Step Average Return

HalfCheetah-v2

0 50,000 100,000

Pendulum-v0

−1,500

CartPole-v1

−500

Pusher-v2

−500

Swimmer-v2

Dyna-TD3 (ours) MAGE-TD3 with \( \hat{r} \) Dyna-TD3 Dyna-TD3 with \( \hat{r} \)

Figure 6: Performance in terms of average return of MAGE-TD3 and Dyna-TD3 with and without the use of an estimated reward function \( \hat{r} \) (5 runs, 95% c.i.).

B.2 MAGE with Trained Reward Function

As discussed in Section 4, MAGE is able to achieve good performance even with an estimated reward function. We report in Figure 6 the full results of this experiment on all the considered environments. For reference, we test MAGE and Dyna-TD3 as well as their versions in which the ground-truth reward function is substituted with one trained on the experience replay data using the MSE loss.

The results indicate that learning the reward function when it is not directly accessible does not produce any catastrophic harm to the performance of the algorithm. Therefore, our approach remains competitive even when the assumption of a known differentiable reward function is not satisfied.

B.3 Importance of \( \lambda \)

Our practical solution to viably minimize the norm of the action-gradient of the TD-error involves a constrained optimization problem, that limits the magnitude of the traditional TD-error. We approximately solve this problem by transforming it into an unconstrained one, introducing a new hyperparameter \( \lambda \). \( \lambda \) can be seen as a weight that is given to the traditional TD-error, assigning more or less importance to it compared to the error on the action-gradient. In the main experiment shown in the paper, we used \( \lambda = 0.05 \), which was chosen arbitrarily. How sensitive is MAGE to this parameter?

To study that, we carried out an experiment on the environment HalfCheetah-v2, by testing the TD3-based version of MAGE using four different values of \( \lambda \). The results are shown in Figure 7 and demonstrate that, regardless of the value of \( \lambda \), MAGE is significantly better than the baseline Dyna-TD3. MAGE is therefore robust to the choice of this hyperparameter. Notice also that the \( \lambda = 0.05 \) we used is not optimal for HalfCheetah-v2: thus, the absolute returns obtained by MAGE could be improved for particular tasks if careful parameter search is executed, which we leave for a future work. Nonetheless, we decided to...
Figure 8: Alternative view of the computational graph constructed during the computation of the TD-error \( \hat{\delta} \), following the notation from [39]. Round nodes represent stochastic variables, squares represent deterministic variables. Nodes with incoming dashed edges also depend on the state \( s \).

report in Figure 2 results for a fixed value of \( \lambda \) across all the environments to show the robustness and ease of use of MAGE.

C Action-Gradient of the TD-error

In this section, we present some additional information about the computation of the action-gradient of the TD-error, carried out during the critic learning step of MAGE. To implement MAGE, we employed PyTorch [30] and its automatic differentiation tools in order to compute the second-order gradient required by our method. In this way, we did not need to explicitly derive a closed form expression for a given model class or neural network architecture. Nonetheless, we report here the general expression for the action-gradient of the TD-error:

\[
\begin{align*}
\frac{\partial \hat{\delta}}{\partial a} &= \frac{\partial r(s,a)}{\partial a} + \gamma \frac{\partial \hat{p}(s'|s,a)}{\partial a} \left( \frac{\partial \hat{Q}(s',\pi(s'))}{\partial s'} + \frac{\partial \pi(s')}{\partial a'} \frac{\partial s'}{\partial a'} \right) - \frac{\partial \hat{Q}(s,a)}{\partial a}.
\end{align*}
\]

In MAGE, we employ a Gaussian stochastic model \( \hat{p} \): therefore, its action-gradient \( \frac{\partial \hat{p}(s'|s,a)}{\partial a} \) can be obtained by reparameterizing this distribution using randomly drawn unit Gaussian noise together with the learned mean and standard deviations. In our experiments, we only deal with continuous state and action spaces; however, by leveraging appropriate approximations (e.g., concrete distributions [27]), similar techniques can be employed also in the case of a discrete state space \( S \).

To further visualize the constructed computational graph, it is possible to employ a different view, inspired recent work on stochastic computational graphs [39], w.r.t. the one leveraged in Figure 1 (see Figure 8). In our case, the only possibly stochastic entity is the approximate model.

D Experimental details

D.1 Instantiating MAGE

We presented in Algorithm 1 a generic version with MAGE, whose structure can be adapted to many model-free actor-critic algorithms. In most of our experiments, we use TD3 [16] as a reference algorithm, due to its stability and performance, giving birth to MAGE-TD3. In Algorithm 2, we report pseudocode for this version of our method. Unfortunately, while the use of the model is unchanged w.r.t. the abstract version, the addition of a second value function implies the computational overhead of using second-order differentiation twice.

D.2 Hyperparameters

We employ 1000 (100 for the Pendulum and Cartpole environments) warmup steps of interaction with the environment before starting to update the critic and the actor. We use an ensemble of 8
We average this value across the 10 different trajectories.

Across all the experiments, despite the formulation we used throughout the paper, we employ a reward \( r(s, a, s') \), which is thus also a function of the next state. For generating the performance plots, we evaluate, after every 1000 steps of environment interaction, the actor for 10 episodes and...
average the result. To improve presentation, we then uniformly smooth the resulting curves with a window size of 25.