Observation of nodal-line semimetal with ultracold fermions in an optical lattice

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Realization and observation of topological phases beyond two-dimension (2D) have been an open challenge for ultracold atoms. Here we realize for the first time a three-dimensional (3D) nodal-line semimetal phase for ultracold fermions with novel spin-orbit couplings in an optical Raman lattice, and observe the nodal lines of the 3D topological band. The realized 3D topological semimetal exhibits an emergent magnetic group symmetry, which has an important consequence that the nodal lines can be precisely determined through imaging the Dirac points in specific $k_z$-layer with different Zeeman splittings. We successfully observe in experiment such nodal lines embedded in the 3D topological semimetal by measuring the $k_z$-layer resolved spin-texture and reconstruct the 3D topological structure. Furthermore, in the quench from a deep trivial regime to topological semimetal phase, we observe the band inversion lines from quench dynamics, which are bulk counterparts of Fermi arc states and connect the Dirac points, reconfirming the realized topological band structure. Our results demonstrate the first approach to observe 3D band topology and open the way to probe exotic topological quantum physics for ultracold atoms in high dimension.

The past decade has witnessed great progresses in search for topological phases of matter. The remarkable new topological states, including topological insulators [1,2] and topological semimetals [3–8], have been discovered in various solid state materials which commonly have strong spin-orbit (SO) couplings. A topological insulator is characterized by a bulk gap, together with the symmetry-protected gapless boundary. The topological semimetals are new topological matter beyond insulators, with the bulk band exhibiting gapless nodes which are protected by symmetry and topology [9,10]. In particular, the Weyl semimetal bears discrete bulk nodal points called Weyl points, which are chiral and protected by Chern number of 2D gapless Fermi surface enclosing each node [11]. Moreover, the nodal-line semimetal, which has degenerate bulk quasiparticles extending one-dimensional (1D) line [12,13], can serve as a parent phase to further realize rich topological states including Weyl semimetals and topological insulators. Compared with the achievements made in topological insulators and Weyl semimetals, observation of nodal-line semimetals is more challenging, since the line-shape nodes of solids are usually embedded in the complicated 3D band structures and their direct imaging is unavoidably affected by the complexity of the system [14].

Recently, considerable efforts have been made in ultracold atoms to explore synthetic SO couplings and topological quantum phases beyond natural conditions [15–18]. The ultracold atoms are pristine platforms and have explicit advantages in the maneuverability over solids. A number of interesting phases have been reported in optical lattice experiments, including the Haldane model [19], the minimal 2D SO coupled model [20] for quantum anomalous Hall effect [21], a supersolid-like phase [22], a 1D symmetry-protected topological state [23], and SO interactions in synthetic dimension [24,25]. In particular, realizations of 2D SO couplings ignite lots of interests to explore high dimensional topological states with ultracold atoms [26–30]. However, to date only 1D and 2D topological phases were implemented in experiment, but no 3D topological states have been achieved experimentally. The great challenge is that to observe a 3D topological phase necessitates the experimental characterization of 3D band topology which cannot be measured by standard momentum-space tomography or band mapping, as used for detecting 1D and 2D phases.

Here, we realize for the first time a 3D topological semimetal with nodal lines for ultracold fermions in an optical lattice, and successfully observe the nodal lines embedded in the 3D topological band. The realization is based on 2D SO coupling proposed in an optical Raman lattice [31], which forms a 2D Dirac semimetal in the $x$-$y$ plane, together with a 1D linear SO coupling along the free space of $z$ direction [32–34]. A novel technique is developed to construct the 3D band topology by $k_z$-layer resolved measurement of spin texture for the 3D phase.

The experiment starts with a two-component degenerate $^{173}$Yb Fermi gas with number \(N_{↑,↓}=5\times10^3\) prepared at \(T/T_F\simeq0.5\), where \(|↑,↓\rangle=m_F=5/2, m_F=3/2\rangle\) are hyperfine states [35,36]. An optical AC Stark shift is applied to separate out an effective spin-$1/2$ subspace from other hyperfine levels [see Fig. 1(b)] [37]. We utilize the recently developed optical Raman lattices where periodic Raman potentials are imposed on and exhibit nontrivial relative symmetries with respect to the lat-
Figure 1: (color online) **Two-dimensional SO coupling in optical Raman lattices.** (a) A two-dimensional (2D) optical lattice potential, created by blue- and red-detuned lattice beams, induces 2D SO coupling in the $x-y$ plane with the Raman beam (green arrow). Additional linear SO coupling is synthesized along the $z$ direction by tilting the Raman beam by $68^\circ$ from the red-detuned lattice beam. (b) The electronic Stark shift caused by optical lattices separates out a spin-$1/2$ subspace from other spin states and simultaneously suppresses other irrelevant two-photon transitions. (c) The $|\uparrow\rangle$ and $|\downarrow\rangle$ atoms, adiabatically loaded into the optical Raman lattice, are expanded after all optical lattice potentials are switched off. The Raman potential along the $x$ direction is modulated as $\cos k_x z$ inducing symmetric momentum distribution along the $x$ direction. However, a running-wave Raman potential along the $y$ direction causes the asymmetric momentum distribution. The graph in (c) shows the differential momentum distribution $n_\uparrow(k_x, k_y) - n_\downarrow(k_x, k_y)$ obtained from spin-selective images after a time-of-flight expansion.

A square optical lattice is formed in the $x-y$ plane by two pairs of standing-wave lights, one blue- and one red-detuned from the principle resonant transition $F = 5/2 \rightarrow F' = 7/2$, with the lattice depth denoted by $V_{x,\sigma}$ and $V_{y,\sigma}$ ($\sigma = \{\uparrow, \downarrow\}$) for the two lights, respectively. The system is free of lattice in the $z$ direction. Raman coupling is created by applying an additional blue-detuned plan-wave beam (green arrow in Fig. 1(a)), polarized along $x$-axis and running in $y$-plane with a tile angle $\theta = 68^\circ$ to the $y$-axis, which together with the aforementioned blue-detuned lattice beam in $x$ direction generates Raman coupling between $|\uparrow\rangle$ and $|\downarrow\rangle$.

The key ingredient to realize the topological semimetal is that the generated Raman potential is of 3D form $V_R = M_R \cos k_0 y e^{i k_1 x} e^{i k_2 z} |\downarrow\rangle|H.C.|$, where $M_R$ is the Raman coupling strength, $k_1 = k_0 \cos(68^\circ)$, $k_2 = k_0 \sin(68^\circ)$, and $k_0 = 2\pi/\lambda$ for $\lambda = 556$ nm. The total Hamiltonian realized in the experiment then reads

$$H_{\text{total}} = \frac{p^2}{2m} + \sum_{\sigma = \uparrow, \downarrow} (V_{x,\sigma} \cos 2k_0 y + V_{y,\sigma} \cos 2k_0 z) + V_R,$$

where $m$ is the atom mass. In Fig. 1(c), the spin-dependent momentum shift of the atomic cloud is imaged in the $x-y$ plane after the time-of-flight (TOF) expansion revealing the 2D SO coupling in $x-y$ plane with a specific $k_z$. Such 2D SO coupling leads to the 2D Dirac semimetal in the $x-y$ plane \[31\], whereas the remaining plane-wave term $e^{i k_2 z}$ of the Raman potential induces an additional 1D linear SO coupling which modulates the 2D Dirac semimetals along $k_z$ axis. As a result, the full effect of the Raman coupling is depicted by the coupling between 2D spin components ($\sigma_{x,z}$) and 3D momentum, which realizes the 3D nodal-line topological semimetal.

The topological band structure can be well understood with the Bloch Hamiltonian derived from $H_{\text{total}}$, which is given by (see supplementary material for details [31])

$$H = \left[ m_z + \frac{h^2 (k_x k_z)}{2m} + h^2 y \sigma_y + h_0 \sigma_0 \right].$$

Here $(h^2 y, h^2 z) \propto (\sin q_y a, \cos q_y a + \cos q_y a)$ and the linear $k_z$-term correspond to the SO coupling in the 2D lattice of $x-y$ plane and in the 1D free space along $z$ direction, respectively, with the lattice constant $a$ and the $\hbar$-$p_\tau$-term characterizes the spin-independent dispersion \[31\]. For $k_z = 0$, the above Hamiltonian describes a 2D Dirac semimetal with the Dirac points determined by $m_z = -h^2 h_0$ and $h^2 y = 0$ \[31\] [31]. Further, the 1D linear
is characterized by probing a spin texture in a variable \( k_z \) plane that is experimentally controlled by scanning \( m_z \). The theoretical spin textures are obtained with a good agreement for the experimental parameters. The blue curves are predicted nodal lines.

Band inversion is observed around \( k_z \sim 0 \) (denoted by \( \mathcal{Z} \)), when two bands are resonantly coupled by SO coupling, showing opposite spin polarizations along the \( \Gamma-X-M-\Gamma \). (c) From the spin textures measured in (a), the \( q_y \)-component of the nodal point in the \( k_z \) plane can be determined by identifying the topological phase transition of the quasi-1D spin texture obtained at each \( q_y \). (d) Finally, we can reconstruct nodal lines projected onto the \( q_y-k_z \) plane for parameters \( \{V_x, V_z, V_{x,z}, V_{y,z}, M_R\} = \{2.3(5), 1.6(3), 2.3(2), 3.0(2), 0.68(5)\} \times E_r \). The measured nodal lines is consistent with the theoretical prediction with measurement uncertainty (blue region).

SO term in the \( z \) direction effectively modulates the Zeeman energy \( m_z \), shifting the positions of the Dirac points for different \( k_z \) layers. The Dirac points at all \( k_z \) layers form the nodal lines of the 3D topological semimetal. The numerical results are shown in Fig. 2(b), where the nodal lines in a typical band structure are obtained. In Fig. 2(b) the band structure within \( q_x-q_y \) plane at \( k_z = 0 \) shows two Dirac points at \( (q_x, q_y) = (0, \pm q_y^D) \) along the \( \Gamma-X \) line, with an energy difference resulted from \( h_0 \) term and proportional to \( \sin(q_y^D / a) \). The 1D non-trivial spin texture is obtained in \( q_x \) dimension for \( |q_y| < q_y^D \), with the spin polarization changing sign at two momentum points between \( \Gamma \) and \( X_2 \) [Fig. 2(b,c)]. Such sign changing momenta is called band inversion points and is related to a nonzero winding number [24, 37]. In contrast, the spin texture is trivial for \( |q_y| > q_y^D \). The Dirac points at \( q_y = \pm q_y^D \) then mark the transition between topological and trivial regimes for the quasi-1D bands along \( q_x \) direction, and are characterized by the junctions of two band inversion lines formed by the band inversion points extending to \( q_y \) direction in the \( q_x-q_y \) plane.

We proceed now to probe the topological nodal lines by mapping out the 3D spin texture of the semimetal phase. A degenerate Fermi gas is adiabatically loaded into the optical Raman lattice, and the equilibrium spin texture in the \( q_x-q_y \) plane is obtained using the spin-sensitive imaging after time-of-flight [23]. During the expansion, we apply a resonant 556 nm light which selectively blasts a certain \( m_F \) hyperfine state [23]. The momentum distribution of atoms is then recorded by using a 399 nm light resonant to the \( ^1S_0 \rightarrow ^3P_1 \) transition (see supplementary materials for details). Our key observation is that the 2D momentum \( (q_x, q_y) \) resolved spin texture for different \( m_z \), with \( k_z \) layers being integrated out, matches well the spin texture for different \( k_z \) layer but fixed \( m_z \). The experimental data and theoretical simulation are shown in Fig. 3(a), with which the nodal lines are resolved. This remarkable result is followed by a novel theoretical mechanism as explained below. We notice that at the \( k_z = 0 \) layer, the Bloch states at two band inversion points are

Figure 3: (color online) **Measurement of nodal lines in the 3D momentum space.** (a) The 3D topological band is characterized by probing a spin texture in a variable \( k_z \) plane that is experimentally controlled by scanning \( m_z \). The theoretical spin textures are obtained with a good agreement for the experimental parameters. The blue curves are predicted nodal lines. (b) Band inversion is observed around \( k_z \sim 0 \) (denoted by \( \mathcal{Z} \)), when two bands are resonantly coupled by SO coupling, showing opposite spin polarizations along the \( \Gamma-X-M-\Gamma \). (c) From the spin textures measured in (a), the \( q_y \)-component of the nodal point in the \( k_z \) plane can be determined by identifying the topological phase transition of the quasi-1D spin texture obtained at each \( q_y \). (d) Finally, we can reconstruct nodal lines projected onto the \( q_y-k_z \) plane for parameters \( \{V_x, V_z, V_{x,z}, V_{y,z}, M_R\} = \{2.3(5), 1.6(3), 2.3(2), 3.0(2), 0.68(5)\} \times E_r \). The measured nodal lines is consistent with the theoretical prediction with measurement uncertainty (blue region).
given by the Hamiltonian \( H_{q_x,q_y,0} = 0 \sigma_z + 2t_{so} \sin q_x \sigma_y \) and have zero spin polarization along \( z \) axis. Here \( t_{so} \) denotes the spin flip hopping coefficient. On the other hand, for layers with \( k_z \neq 0 \), the Bloch states at \( q_x,y = q_{x0,0} \) is governed by the Hamiltonian \( H_{q_x,q_y,k_z} = \alpha k_z k_z \sigma_z + 2t_{so} \sin q_x \sigma_y + \alpha k_z^2 \sigma_0, \) (2)

where \( \alpha = \hbar^2/2m \). The Hamiltonian satisfies a magnetic group symmetry defined by \( M_z H_{q_x,q_y,k_z} M_z^{-1} = H_{q_x,-q_y,-k_z} \), with \( M_z = \sigma_z K \). For this the Bloch states at \( \pm k_z \)-momenta are degenerate but have opposite \( z \)-component spin polarizations, giving an important consequence that contributions to total spin polarization from \( \pm k_z \) layers exactly cancel out. Thus the total spin texture integrated over all \( k_z \) layers renders the spin texture of the \( k_z = 0 \) layer. Further, as the Zeeman term \( m_z \) is modulated by the \( k_z \) momentum, the spin texture for fixed \( m_z = m_0 \) but different \( k_z \) is identical to the spin texture for fixed \( k_z = 0 \) layer but different \( m_z \). This explicitly follows the relation (after neglecting a constant):

\[
H_{q_x,q_y,0,m_z} = H_{q_x,q_y,k_z,0} \text{ with the scanning parameters satisfying } m_z = \hbar^2 k_z^2 / 2m.
\]

Thus measuring the \( k_z \)-integrated spin texture by scanning \( m_z \) maps out the 3D spin texture for fixed \( m_z \), as considered here to probe the nodal lines of the 3D topological semimetal. Fig. 3(b) shows the spin polarization between symmetric momenta in the \( q_x-q_y \) space. A nontrivial spin texture is observed around \( m_z = 0 \), where two lowest energy bands are inverted and coupled by SO interaction. No band inversion occurs when two bands are far-detuned, in which the observed spin textures are trivial.

To further confirm the nodal lines, we measure the projected nodal points onto the \( q_y-k_z \) plane. For this we construct a quasi-1D spin texture along \( q_y \) direction, obtained at each \( q_y \) from the 2D spin texture with fixed \( k_z \) [Fig. 3(c)]. The dimension reduction allows us to consider \( q_y \) as a parameter, and to measure the critical quasi-momentum \( q_y \) of topological transition for the reduced 1D band in \( q_y \) direction \( \text{[23].} \) Then the \( q_y \)-position of the Dirac points can be measured versus \( k_z \). Between the Dirac points, the quasi-1D spin texture exhibits nontrivial winding. Fig. 3(d) shows the projected nodal lines onto \( q_y-k_z \) plane, in agreement with theoretical prediction within the experimental uncertainty, and confirms the capability of resolving the 3D band topology.

We finally probe the far-from-equilibrium spin dynamics following a quench from a deep trivial regime to topological regime, which allows to identify the band inversion lines and Dirac points with relatively higher resolution. We begin with a fully spin-polarized Fermi gas with a large value of \( m_z = -9.2(3)E_r \). Then we suddenly change \( m_z \) to the semimetal regime with \( m_z = 0.20(7)E_r \), and measure quench spin dynamics as presented in Fig. 4. As shown recently, on the band inversion lines the complete spin-flip transitions occur and lead to a vanishing time-averaged spin polarization \( \text{[38].} \) The two band inversion lines are clearly observed in Fig. 4(c) from the zero time-averaged spin polarization measured over 1 ms–3 ms after quench [Fig. 4(e)]. Ending at the Dirac points, the two band inversion lines are the bulk counterparts of Fermi arc states in the semimetal, which is a peculiar measurement of the present study. The observed quench dynamics match well the numerical simulations given in Fig. 4(a,b,f) \( \text{[37].} \) The time-averaged spin dynamics are featureless when quenching the system to fully spin-polarized trivial bands with \( m_z = -0.74E_r \) or \( 0.74E_r \) as described in Fig. 4(d) or (e).

We have realized in experiment the first 3D topological semimetal with nodal lines for ultracold fermions, and successfully measure the bulk topology of the 3D phase. The nodal-line semimetal is achieved by stacking 2D SO coupled Dirac semimetals along \( z \) axis, which are modulated by a linear SO coupling along this direction. We develop a novel technique by tuning an emergent magnetic group symmetry to reconstruct the nodal lines in the 3D momentum space by directly imaging spin texture in specific \( k_z \) plane with different Zeeman splittings. Our work opens the way to explore novel topological quantum physics for ultracold atoms in high dimension.

Figure 4: (color online) Measuring band inversion lines from quantum quench dynamics. The spin-polarized gas initially prepared at \( m_z = -9.2(3)E_r \) is quenched to a semimetal band \( \text{[c],} \) \( m_z = 0.20(7)E_r \) and trivial bands \( \text{[d],} \) \( m_z = -0.74(7)E_r \) and \( \text{[e],} \) \( m_z = 0.74(7)E_r \), respectively. The overall spin polarization over the first Brillouin zone is monitored at different hold times after the quench to the topological regime as described in (g), the feature of which is consistent with theoretical prediction (f). The quasimomentum-dependent spin texture is then time-averaged from 1 ms to 3 ms after the quench resulting in the time-averaged spin textures (c,d,e). The band inversion line is observed as the zero spin-polarization only when the system is quenched to the topological regime. The band inversion lines measured through quantum quench dynamics well-consistent with the prediction calculated with experimental parameters in (b). The predicted spin textures at the \( k_z = 0 \) plane in (a) shows the capability of resolving the band topology for a specific \( k_z \)-plane with \( k_z \) layers being integrated out.
Acknowledgment We appreciate the valuable discussions with Lin Zhang. This work was supported by the Joint Research Scheme sponsored by the Research Grants Council (RGC) of the Hong Kong and National Natural Science Foundation of China (NSFC) (Project No. N-HKUST601/17 and No. 11761161003). G.-B. J. acknowledges the support from the RGC and the Croucher Foundation through ECS26300014, GRF16300215, GRF16311516, GRF16305317 and the Croucher Innovation grants respectively. X.-J. L. also acknowledges the support from the National Key R&D Program of China (2016YFA0301604), NSFC (No. 11574008), and the Strategic Priority Research Program of Chinese Academy of Science (Grant No. XDB28000000).

Competing interests The authors declare that they have no competing interests.

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Supplementary Material

S-1. Methods for numerical simulations

A. Plane wave expansion calculation

Without considering the trapping potential, the 2D optical lattice potential and Raman potential take the form

\[
H_{\text{Lattice}} = \sum_{\sigma = \uparrow, \downarrow} V_{x\sigma} \cos 2k_0 x + V_{y\sigma} \cos 2k_0 y
\]

\[
V_R = M_R \cos k_0 x \exp (ik_1 y + ik_2 z)|\downarrow\rangle\langle\uparrow| + H.c.,
\]

where \(k_1 = k_0 \cos \theta, k_2 = k_0 \sin \theta\), and \(\theta = 68^\circ\). The total Hamiltonian including the kinetic energy and Zeeman energy takes the form

\[
H_{\text{total}} = H_{\text{Lattice}} + V_R + \sum_{\sigma = \uparrow, \downarrow} \frac{p_{x\sigma}^2 + p_{y\sigma}^2 + p_{z\sigma}^2}{2m} + m_z (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|).
\]

The Hamiltonian can be diagonalized in the plane wave basis, labelled by the momentum and spin. The optical lattice potential couples the planes waves with momentum difference \((\pm 2k_0,0,0)\) or \((0,\pm 2k_0,0)\) and the conserves spin, while the Raman potential couples plane waves with momentum difference \((\pm k_0,k_1,k_2)\) and flips the spin. As a result, the Bloch states in nth band with lattice momentum \((q_x,q_y,k_z)\) take the form

\[
|\psi_{q_x,q_y,k_z,n}\rangle = \phi_{q_x,q_y,k_z,n}(M,N,\uparrow) |q_x + 2Mk_0,q_y + 2Nk_0,k_z + k_2/2,\uparrow\rangle + \phi_{q_x,q_y,k_z,n}(M,N,\downarrow) |q_x + 2Mk_0,q_y - k_1 + 2Nk_0,k_z - k_2/2,\downarrow\rangle.
\]

Here \(M,N\) are integers, and \(\phi_{q_x,q_y,k_z,n}(M,N,\sigma)\) denotes the coefficient of the plane waves. Spin polarization is measured along z direction. The eigenstate spin polarization is calculated from

\[
S_z(q_x,q_y,k_z,n) = \frac{\langle \psi_{q_x,q_y,k_z,n} | \hat{S}_z | \psi_{q_x,q_y,k_z,n} \rangle}{\langle \psi_{q_x,q_y,k_z,n} | \psi_{q_x,q_y,k_z,n} \rangle} = \sum_{M,N} |\phi_{q_x,q_y,k_z,n}(M,N,\uparrow)|^2 - \sum_{M,N} |\phi_{q_x,q_y,k_z,n}(M,N,\downarrow)|^2.
\]

The spin texture in equilibrium state is calculated from the density matrix \(\rho\) with matrix elements \(\rho(q_x,q_y,k_z,n;q_x',q_y',k_z')\) in the basis of Bloch eigenstates. For equilibrium states, \(\rho\) is diagonal and the matrix elements take the form \(\rho(q_x,q_y,k_z,n;q_x,q_y,k_z,n),\) and lowest four bands are taken account. The numerical results shown in Fig. 3(a,d) in main text are calculated with temperature \(T = 0.35E_r\) and chemical potential \(\mu = 0.5E_r\). For the quench dynamics, we have considered two cases. In the case without considering decay of \(k_z, k_z\) is taken as a good quantum number and the density matrix elements take the form \(\rho(q_x,q_y,k_z,n;q_x,q_y,k_z,n')\). The spin texture of a single \(k_z\) layer is calculated by

\[
S_z(q_x,q_y,k_z) = \frac{\text{Tr}_n(\hat{S}_z \rho)}{\text{Tr}_n \rho},
\]

and the observable spin texture with \(k_z\) integrated is calculated by

\[
S_z(q_x,q_y) = \frac{\text{Tr}_{k_z}(\text{Tr}_n(\hat{S}_z \rho))}{\text{Tr}_{k_z} \text{Tr}_n \rho}.
\]

While in the case with considering decay of \(k_z, k_z\) is not a good quantum number any more and the density matrix elements take the form \(\rho(q_x,q_y,k_z,n;q_x,q_y,k_z',n')\). In this case, the observable spin texture with \(k_z\) integrated still can be calculated by Eq. \(\text{(S6)}\). In both cases, without loss of accuracy, lowest five bands are taken account for both pre-quench and post-quench Hamiltonians.
B. Tight binding model

Different from the ideal two-dimensional model, atoms are actually distributed in finite size along the z direction. The spin-up plane waves with z direction kinetic energy \( h^2(k_z + k_x^2/2)/2m \) are coupled with spin-down plane waves with kinetic energy \( h^2(k_z - k_x^2/2)/2m \), so the kinetic energy difference in z direction can be viewed as an effective Zeeman term. The tight binding Hamiltonian in lattice site \((x, y)\) with momentum \(k_z + \sigma k_x/2\) in the z direction has the form \(^[4]\)

\[
H_{TB} = - \sum_{x,y,k_z} \left( t_{x\uparrow} c_{x,y,k_z,\uparrow}^\dagger c_{x+1,y,k_z,\uparrow} + t_{x\downarrow} c_{x,y,k_z,\downarrow}^\dagger c_{x+1,y,k_z,\downarrow} + t_{y\uparrow} c_{x,y,k_z,\uparrow}^\dagger c_{x,y+1,k_z,\uparrow} + t_{y\downarrow} c_{x,y,k_z,\downarrow}^\dagger c_{x,y+1,k_z,\downarrow} \right)
+ \sum_{x,y,k_z} \left( -i e^{ik_\perp y} s_0 (c_{x,y,k_z,\uparrow}^\dagger c_{x+1,y,k_z,\downarrow} - c_{x,y,k_z,\downarrow}^\dagger c_{x-1,y,k_z,\uparrow}) + H.c. \right)
+ \sum_{x,y,k_z,\sigma} \left[ \frac{\hbar^2 (k_z + \sigma k_x/2)^2}{2m} + \sigma m_z \right] c_{x,y,k_z,\sigma}^\dagger c_{x,y,k_z,\sigma}.
\]

(S7)

Under Fourier transformation \(c_{x,y,k_z,\sigma} = N_L^{-1/2} \sum_{q_x,q_y} c_{q_x,q_y,k_z} e^{iq_xx+iq_yy} c_{q_x,q_y,k_z,\sigma}\) where \(N_L\) denotes the number of lattice sites, the above Hamiltonian in quasi-momentum space has the form

\[
H_{TB} = - \sum_{q_x,q_y,k_z} \left[ (2t_{x\uparrow} \cos q_x a + 2t_{y\uparrow} \cos q_y b) c_{q_x,q_y,k_z,\uparrow}^\dagger c_{q_x,q_y,k_z,\uparrow} + (2t_{x\downarrow} \cos q_x a + 2t_{y\downarrow} \cos q_y b) c_{q_x,q_y,k_z,\downarrow}^\dagger c_{q_x,q_y,k_z,\downarrow} \right]
+ \sum_{q_x,q_y,k_z} 2i t_{so} \sin (q_z a - \pi) c_{q_x,q_y,k_z,\uparrow}^\dagger c_{q_x,q_y-k_z\uparrow,\downarrow} - 2i \sin (q_z a - \pi) c_{q_x,q_y-k_z\uparrow,\downarrow} c_{q_x,q_y,\downarrow}^\dagger + \sum_{q_x,q_y,k_z,\sigma} \left[ \frac{\hbar^2 (k_z + \sigma k_x/2)^2}{2m} + \sigma m_z \right] c_{q_x,q_y,k_z,\sigma}^\dagger c_{q_x,q_y,k_z,\sigma}.
\]

(S8)

The tight binding Hamiltonian can be expressed by the Bloch Hamiltonian

\[
H_{TB} = \sum_{q_x,q_y,k_z} \left( c_{q_x,q_y,k_z,\uparrow}^\dagger H_{q_x,q_y,k_z} c_{q_x,q_y-k_z\uparrow,\downarrow} + c_{q_x,q_y,k_z,\downarrow}^\dagger H_{q_x,q_y,k_z} c_{q_x,q_y-k_z\downarrow,\uparrow} \right),
\]

(S9)

where \(H_{q_x,q_y,k_z}\) can be expressed in terms of the Pauli matrices

\[
H_{q_x,q_y,k_z} = [m_z - 2t_{x\uparrow} \cos q_x a + 2t_{y\uparrow} \cos (q_y a - \phi_+) + \sin \theta k_z] \sigma_z
+ 2t_{so} \sin q_x a \sigma_y
+ [2 t_{x\downarrow} \cos q_x a + 2 t_{y\downarrow} \cos (q_y a + \phi_+ - k_z^2 + \sin^2 \theta/4)] \sigma_0.
\]

(S10)

Here \(2t_{x\pm} = t_{x\uparrow} \pm t_{x\downarrow}, 2t_{y\pm} = \sqrt{(t_{y\uparrow} \pm t_{y\downarrow} \cos k_1 a)^2}, \tan \phi_\pm = \frac{t_{y\downarrow} \sin k_1 a}{t_{y\uparrow} \pm t_{y\downarrow} \cos k_1 a}\), and we have assumed \(k_0 = 1, E_r = 1\). If the Bloch momentum \(q_y\) is shifted as \(q_y \rightarrow q_y + \phi_+/a\), one obtains the Bloch Hamiltonian Eq. (1) shown in the main text

\[
H_{q_x,q_y,k_z} = [m_z - 2t_{x\uparrow} \cos q_x a + 2t_{y\uparrow} \cos q_y a + \sin \theta k_z] \sigma_z
+ 2t_{so} \sin q_x a \sigma_y
+ [2 t_{x\downarrow} \cos q_x a + 2 t_{y\downarrow} \cos (q_y a + \phi_- + \phi_+) + k_z^2 + \sin^2 \theta/4] \sigma_0,
\]

(S11)

of which the positions of Dirac points (nodal lines) are symmetric in \(q_y\) direction.

C. Lindblad form master equations

For non-equilibrium case, in order to phenomenally simulate the decay effect from upper bands to lower bands during the quench dynamics, we introduce the Lindblad form master equation which describes the evolution of single
Here $\rho$ is the time dependent density matrix, $L_n = |n\rangle\langle n+1|$, and $n$ denotes the band index. In this case we neglect the momentum scattering caused by trapping potential and coupling with environment, lattice momentum $(q_x, q_y, k_z)$ are good quantum numbers and momentum indices are omitted in the formula. The initial density matrix is determined by the pre-quench Fermi distribution. The pre-quench single particle density matrix is determined by temperature $T$ and chemical potential $\mu$, values of which are similar to experiment settings. Time-dependent spin polarization in the case for a $k_z$ layer after quench is calculated by Eq. (S5) using the density matrix is calculated from Eq. (S12). In order to simulate the real experiment, we should calculate the $k_z$ integrated time-averaged spin texture. In principle it is difficult to calculate the exact distribution of $k_z$ dispersion minimum and the dominant dynamics is just the inter-band oscillation as shown in Fig. 4(g) in the main text. So we can reasonably approximate the time averaged spin texture with

$$\bar{S_z}(q_x, q_y) = \int_{t_i}^{t_f} dt \int_{-\delta}^{\delta} dk_z \frac{Tr\rho L_z}{Tr\rho},$$

where $k_z \in [-\delta, \delta]$ is a small $k_z$ integration interval around post-quench $k_z$ dispersion minimum as shown in Fig. S3 (b,c,d), $t \in [t_i, t_f]$ is the time integration interval for several final periods. We show in the main text the numerical result of the time averaged spin texture $\bar{S_z}(q_x, q_y)$ in Fig. 4(a,b), with parameters $m_z = 0.27E_r$, $\delta = 0$ and $0.32k_0$ for (a) and (b) respectively.

In order to further interpret the evolution of the oscillation centre of experimental result, we include the decay effect along $k_z$. From the pre- and post-quench equilibrium Fermi distribution in Fig. S3 one can see that the band minimum depends on $k_z$. When $|m_z|$ is not quite large, on the white curves (zero polarization curves) of the static spin texture the particle number distribution centre is around $k_z = 0$. While in the deep trivial case where $m_z \approx -9.2E_r$, there exist no white curves and the $k_z$ distribution centre is away from $k_z = 0$, as is shown in Fig. S3(a). Phenomenally, we add a decay term to previous Lindblad form master equation Eq. (S12).

$$\dot{\rho} = -i[H, \rho] + \sum_n \gamma_n [L_n \rho L_n^\dagger - \frac{1}{2} \{L_n^\dagger L_n, \rho\}]$$

$$+ \sum_{k_z, k_{z1}} \gamma_{k_z} [L_{k_z} \rho L_{k_z}^\dagger - \frac{1}{2} \{L_{k_z}^\dagger L_{k_z}, \rho\}]$$

Here discrete $k_z \in [k_{z1}, k_{z2}]$ is taken, $L_{k_z} = \sum_n |q_x, q_y, k_z, n\rangle\langle q_x, q_y, k_z, n|$, $L_n = \sum_{k_z} |q_x, q_y, k_z, n\rangle\langle q_x, q_y, k_z, n+1|$, and $q_x$, $q_y$ are still good quantum numbers. The initial density matrix is determined by the pre-quench Fermi distribution. Such Lindblad operator can be used to phenomenally simulate the decay of $k_z$ from dispersion minimum of the pre-quench Hamiltonian to that of the post-quench Hamiltonian. In this case, the time evolution for the observed spin polarization is calculated from Eq. (S6) using the density matrix calculated by Eq. (S14). We show in the main text the numerical result of the time evolution for spin polarization in Fig. 4(f) with parameters $m_z = 0.27E_r$, $\gamma = 0.01$, $\gamma_z = 0.05$, where the $(q_x, q_y)$ position of the white curve is taken from Fig. 4(a).

S-2. 2D Dirac semimetal phase and 3D nodal line semimetal phase

A. Equilibrium state

In this section, we will provide a theoretical description for both 2D Dirac semimetal and 3D nodal line semimetal phases, and show how to reconstruct 3D nodal line semimetal phases from a set of our 2D Dirac semimetal phase measurement. We first prove that the spin texture with $k_z$ integrated measured in experiment is equivalent to the spin texture with single $k_z = 0$ layer, and thus the set of measured $k_z$ integrated spin textures with various Zeeman energy $m_z$ is equivalent to a set of 2D spin textures with fixed $k_z = 0$ but various $m_z$. Then we prove that because of the features of our model, the Zeeman energy $m_z$ can be viewed as an effective $k_z$, so $m_z$ scan with fixed $k_z$ is equivalent to $k_z$ scan with fixed $m_z$. 3D spin textures therefore can be effectively measured in our experiment.
1. The two-dimensional Dirac semimetal phase

The Dirac points in a 2D layer with specific \( k_z \) can be characterized by the topological phase transition points of a 1D Hamiltonian \( H_q \), which is reduced from the original 2D Hamiltonian in a certain \( k_z \) layer as,

\[
H = [m_z - 2t_{x+} \cos q_x a + 2t_{y+} \cos (q_y a - \phi_+) + \sin \theta k_z] \sigma_z + 2t_{s0} \sin q_x a \sigma_y, \tag{S15}
\]

where \( \sigma_0 \) term has been discarded since it does not contribute to gap closing and has no effect on topology. Now the dimension reduction operation of the 2D Hamiltonian is performed by taking \( q_y \) as a parameter, thus the 1D Hamiltonian for a specific \( q_{y_0} \) reads

\[
H_{q_{y_0}} = \mathbf{\hat{h}} \cdot \mathbf{\hat{\sigma}} = h_z \sigma_z + h_y \sigma_y = [m_z'(q_{y_0}) - 2t_{x+} \cos q_x a] \sigma_z + 2t_{s0} \sin q_x a \sigma_y, \tag{S16}
\]

The above 1D static topological phase is classified by integer invariants in the Altland-Zirnbauer (AZ) symmetry classes [2]. 1D winding number to characterize the topology is defined as

\[
u_{q_{y_0}} = \frac{1}{4\pi} \int_{\text{BZ}} dq_x \text{Tr}[\sigma_z H_{q_{y_0}} \frac{dH_{q_{y_0}}}{dq_x}], \tag{S17}
\]

where \( H_{q_{y_0}} = \mathbf{\hat{h}} / |\mathbf{\hat{h}}| \cdot \mathbf{\hat{\sigma}} \) is normalized Hamiltonian. The winding number is formulated by a mapping from the BZ, which is a 1D torus \( T^1 \), to the 1D spherical surface \( S^1 \) through the unit vector field

\[
\mathbf{n}(k) = \mathbf{\hat{h}} / |\mathbf{\hat{h}}|. \tag{S18}
\]

The topological number counts the times that the mapping covers the spherical surface \( S^1 \). To determine the topology of the 1D Hamiltonian, here we use the concept “band inversion surface (BIS)” introduced in [3]. Here the BIS is defined by \( q_x \) points in the 1D FBZ by solving \( h_z(q_x, q_{y_0}) = 0 \). In our 1D model, the BISs are discrete \( q_x \) points, and
the non-trivial winding number requires the existence of two symmetric BISs along $q_x$ direction. After defining the $h'$ term

$$h'(q_x, q_y) = -2t_x + \cos q_x a + 2t_y + \cos (q_y a - \phi_+)$$ (S19)

contained in the $h_z$ term, it is easy to verify that when $-\max[h'(q_x, q_y)] < m_z + \sin \theta k_z < -\min[h'(q_x, q_y)]$, $q_x \in [-k_0, k_0]$, the winding number is non-zero as shown in the right hand side of Fig. S1(a,b). In experiment, the existence of BISs can be monitored by looking at spin-balanced points in the equilibrium spin texture.

In a single 2D $k_z$ layer, the BIS points form curves. The BIS curves in a 2D layer can be taken as a “bulk correspondence” of the Fermi arc in time-reversal breaking Weyl semimetals. The 3D time-reversal breaking Weyl semimetals can usually be viewed as stacked 2D Chern insulators in a third direction. The Fermi arc is formed by non-trivial chiral edge states varying as a function of the momentum in the third direction, and the Weyl nodes are exactly at the transition point of the 2D Chern insulators. Similar to the case for the Weyl semimetals, the existence of Dirac points in the 2D layer also reflects the topological phase transitions of the reduced 1D $H_{q_y}$ Hamiltonians.

As functions of parameter $q_y$, the BIS points of the 1D reduced Hamiltonians form white curves in 2D $(q_x, q_y)$ plane. If the two BISs touch and then disappear, the 1D winding number changes and the gap should close at the transition point. The gapless points are exactly the Dirac points in our model. Due to the reflection symmetry along $q_x = 0$ axis, the Dirac points can only appear at $q_x = 0$ or $\pm k_0$. With specific $m_z$, the Dirac points exist when $-\max[h'(sk_0, q_y)] < m_z + \sin \theta k_z < -\min[h'(sk_0, q_y)]$, $q_y \in [-k_0, k_0]$, where $s = 0$ or $\pm 1$ corresponds to Dirac points located at $q_x = 0$ or $\pm k_0$.

![Figure S2: Positions of the nodal lines with $(V_x, V_{x+}, V_y, V_{y+}, M, m_z) = (2.3, 1.6, 2.3, 3.0, 0.68, 0)E_r$ in 3D momentum space, and their projections on three 2D planes. The red and blue curves correspond to $q_x = \pm k_0$ and 0 respectively.](image)

To define the topological invariant of the Dirac points $(q_{x1}, q_{y1})$ in a $k_z$ layer, we choose a closed clockwise loop $C$ in the $q_x$-$q_y$ plane that only wraps the Dirac point $(q_{x1}, q_{y1})$. Expanding the Hamiltonian around the degenerate point, one obtains the continuously deformed Hamiltonian

$$H = \mathbf{q} \cdot \mathbf{\sigma} = s_1 \epsilon_x \sigma_y + s_2 \epsilon_x \sigma_y,$$ (S20)

where $(\epsilon_x, \epsilon_y) = \epsilon(\cos \beta, i \sin \beta)$ form the loop $C$ with radius $\epsilon$. $\beta$ is determined by $(\epsilon_x, \epsilon_y)$, and both $s_1$, $s_2$ can be + or −. The Dirac point can be characterized by the winding number

$$v_{\text{point}} = \frac{i}{2 \pi \hbar^2} \int_C \mathbf{q}^* d\mathbf{q} = \pm \int_0^{2\pi} (\cos^2 \beta + \sin^2 \beta) d\beta = \pm 1,$$ (S21)

where the winding numbers $\pm 1$ correspond to $s_1 s_2 = \pm 1$ respectively.

2. The three-dimensional nodal line semimetal phase

In our system, positions of Dirac points in $q_x$-$q_y$ plane depend on $k_z$, resulting nodal line structures. At $\sin q_x a = 0$ with $q_x = 0(\pm k_0)$, for any $q_y \in [-k_0, k_0]$, there exist a $k_z$ where the tight binding Hamiltonian has degenerate points.
When \( q_y \) varies from \(-k_0\) to \( k_0\) the degenerate points form curves, which is called the nodal line. We plot the positions of the nodal points in 3D momentum space together with their projections in 2D planes with \( m_z = 0 \) in Fig. S2.

To define the topological invariant of the nodal lines, similar to the former case for Dirac points, we select a degenerate point \((q_x, q_y, k_z)\) in the nodal lines with \( q_x = 0(\pm k_0)\), and choose a closed clockwise loop \( C \) in the \( q_x - k_z \) plane that wraps around the \((q_x, q_y, k_z)\) point but does not intersect with the nodal lines. After discarding the \( \sigma_0 \) terms and expanding the Hamiltonian near the \((q_x, q_y, k_z)\) point, one obtains the continuously deformed Hamiltonian

\[
H = \mathbf{q} \cdot \mathbf{\sigma} = \epsilon_z \sigma_z \pm \epsilon_x \sigma_y,
\]

where \((\epsilon_z, \epsilon_x) = \epsilon(\cos x, i \sin x)\) form the loop \( C \) with radius \( \epsilon \), \( \beta \) is determined by \((\epsilon_z, \epsilon_x)\), and \(+(-)\) correspond to nodal lines with \( q_x = 0(\pm k_0)\). The nodal line can be characterized by the winding number

\[
v_{\text{line}} = \frac{i}{2\pi \epsilon^2} \int_{C} \mathbf{q}^* d\mathbf{q} = \pm \int_{0}^{2\pi} (\cos^2 \beta + \sin^2 \beta) d\beta = \pm 1.
\]

The winding numbers \( +1(-1) \) correspond to nodal lines with \( q_x = 0(\pm k_0) \) respectively. The \( v_{\text{line}} \) defined here can be related to the \( v_{\text{point}} \) at the same degenerate point defined in former subsection via

\[
v_{\text{line}} = \pm v_{\text{point}}
\]

if the \( C \) loops defined in each cases can be continuously deformed to each other. The \( + \) sign is taken when the direction of the loop does not change after deformation, and the \( - \) sign is taken when the direction of the loop changes.

Figure S3: Particle number and spin polarization versus \( k_z \) at different \((q_x, q_y)\) points with parameters \((V_{xx}, V_{xy}, V_{yx}, V_{yy}, M) = (2.3, 1.6, 2.3, 3.0, 0.68)E_{\text{c}}\). (a) \( T = 0.42E_{\text{c}}, \mu = -8E_{\text{c}}, m_z = -0.2E_{\text{c}}, (q_x, q_y) = (0.35, 0.35)k_0\). Due to large \( m_z \), the \( k_z = 0 \) layer band is the deep trivial and there is no white curves structure in the spin texture. (b) \( T = 0.42E_{\text{c}}, \mu = 0.14E_{\text{c}}, m_z = 0, (q_x, q_y) = (-0.475, -0.275)k_0\). (c) \( T = 0.42E_{\text{c}}, \mu = 0.14E_{\text{c}}, m_z = 0.5kH_z, (q_x, q_y) = (0.225, -0.475)k_0\). (d) \( T = 0.42E_{\text{c}}, \mu = 0.14E_{\text{c}}, m_z = 1kH_z, (q_x, q_y) = (0.375, 0.275)k_0\). In the figure (b)-(c) spin polarizations with \( k_z \) integrated are almost zero, thus the spin texture of \( k_z = 0 \) layer and of \( k_z \) integrated have the same white curves structure.

3. *Equivalence between* \( k_z = 0 \) *layer spin textures and* \( k_z \) *integrated spin textures*

Here we show that the static \( k_z = 0 \) layer spin textures and static \( k_z \) integrated spin textures are equivalent in the sense that their positions of white (zero spin polarization) curves are exactly the same in tight binding regime, yielding that positions of Dirac points are the same. Although for a random single \((q_x, q_y)\) point the \( k_z \) distribution centre may be away from \( k_z = 0 \) as shown in Fig. S3(a), however, it turns out that such \((q_x, q_y)\) points are highly polarized and far away from white curves in the spin texture. Now we theoretically show that the positions of white
curves are exactly the same in both cases. From the tight binding model, the Hamiltonian at those spin-balanced momenta \((q_{x0}, q_{y0}, 0)\) in the \(k_z = 0\) layer reads

\[
H_{q_{x0}, q_{y0}, 0} = 0 \sigma_z + 2t_{so} \sin q_x a \sigma_y,
\]

where the irrelevant terms have been discarded. With fixed 2D lattice momentum \((q_{x0}, q_{y0})\), the Hamiltonian at \((q_{x0}, q_{y0}, k_z)\) points should be

\[
H_{q_{x0}, q_{y0}, k_z} = \sin \theta k_z \sigma_z + k_z^2 \sigma_0 + 2t_{so} \sin q_x a \sigma_y.
\]

It is clear that the Hamiltonian satisfies an emergent magnetic group symmetry defined by

\[
\mathcal{M}_z H_{q_{x0}, q_{y0}, k_z} \mathcal{M}_z^{-1} = H_{q_{x0}, q_{y0}, -k_z},
\]

where \(\mathcal{M}_z = \sigma_z K\). As a result the Bloch states at \(\pm k_z\)-momenta are degenerate but have opposite \(z\)-component spin polarizations. So after \(k_z\) is integrated the polarization will be kept zero at \((q_{x0}, q_{y0})\) point. Thus, the zero polarization \((q_x, q_y)\) points for the \(k_z = 0\) layer should also be zero polarized when \(k_z\) is integrated, resulting that the positions of white curves are the same in these two situations.

However, the above statement is not accurate in real optical lattice system. The lowest two bands’ spin polarizations will not completely cancel, contrast to the two-band tight binding model, due to the spin-orbit coupling between \(s\) and higher \(p\) bands. At the zero polarization positions \((k_{x0}, k_{y0}, 0)\) with finite temperature, the symmetric property of the energy and anti-symmetric property of the spin polarization with respect to \(k_z\) are slightly broken but not much as shown in Fig. S3 and the positions of the white curves in the two cases still almost coincide in our plane-wave calculations as shown in Fig. S4.

![Figure S4: Comparison between the simulated results of the \(k_z\) integrated case and the \(k_z = 0\) case. Optical lattice and Raman potential parameters are set as \((V_{x\uparrow}, V_{y\uparrow}, V_{y\downarrow}, V_{y\uparrow}, M) = (2.3, 1.6, 2.3, 3.0, 0.68)E_r\) with \(T = 0.42E_r\), \(\mu = 0.14E_r\). Both the shape and the location of the white curves are almost the same between \(k_z\) integrated case and \(k_z = 0\) case.](image_url)

4. Equivalence between scanning of \(k_z\) and scanning of \(m_z\)

Here we show that \(m_z\) scan is equivalent to \(k_z\) scan due to the features of our model. The tight binding Hamiltonian \(H_{q_x, q_y, k_z, m_z}\) with parameters \((q_x, q_y, k_z, m_z)\) has the form

\[
H_{q_x, q_y, k_z, m_z} = [-2t_{x\uparrow} \cos q_x a + 2t_{y+} \cos (q_y a - \phi_+) + \sin \theta k_z + m_z] \sigma_z \\
\quad + 2t_{so} \sin q_x a \sigma_y \\
\quad + [-2t_{x\downarrow} \cos q_x a + 2t_{y-} \cos (q_y a + \phi_-) + k_z^2 + \sin^2 \theta/4] \sigma_0.
\]

One can immediately obtain

\[
H_{q_x, q_y, k_z, 0} = H_{q_x, q_y, 0, m_z} + \frac{m_z^2}{\sin^2 \theta} \sigma_0,
\]

\((S29)\)
with the condition \( m_z = \sin \theta k_z \), where both \( k_0 = 1 \) and \( E_r = 1 \) are assumed. The last constant \( \sigma_0 \) term can be discarded as an effective chemical potential. Finally one obtains

\[
H_{\vec{q}_x, \vec{q}_y, k_z, 0} = H_{\vec{q}_x, \vec{q}_y, 0, m_z},
\]

where \( m_z = \sin \theta k_z \). The above equivalence is not restricted to tight binding model and also holds in the plane-wave calculation as can be easily seen from Eq. (S2). We plot the spin textures of \( H_{\vec{q}_x, \vec{q}_y, k_z, 0} \) and \( H_{\vec{q}_x, \vec{q}_y, 0, m_z} \) with the condition \( m_z = \sin \theta k_z \) for both zero temperature and finite temperature cases to support the above statement in Fig. S5.

![Figure S5: Comparison of simulated spin textures between \( H_{\vec{q}_x, \vec{q}_y, k_z, 0} \) and \( H_{\vec{q}_x, \vec{q}_y, 0, m_z} \), where \( m_z \) and \( k_z \) are in units of \( E_r \) and \( k_0 \) respectively, and are related by \( m_z = \sin \theta k_z \). Optical lattice Raman potential parameters are set as \((V_x^\uparrow, V_x^\downarrow, V_y^\uparrow, V_y^\downarrow, M) = (2.3, 1.6, 2.3, 3.0, 0.68)E_r\). The left two columns correspond to finite temperature case with \( T = 0.42E_r, \mu = 0.14E_r \), while the right two columns correspond the spin texture of the lowest band without temperature effect. For the finite temperature case, the \( m_z \) (effective \( m_z \)) from top to bottom is \((0.27, 0.13, 0, -0.13, -0.27)E_r\); for the latter case, the (effective) \( m_z \) from top to bottom is \((0.40, 0.27, 0, -0.27, -0.40)E_r\).](image)

### B. Quench dynamics

As a counterpart to the equilibrium state study, we also explore the quench dynamics \([3]\) in our system, where the inter-band oscillation and time averaged spin texture in the quench dynamics also reveals the 2D Dirac semimetal phase. In experiment, the Zeeman energy \( m_z \) is quenched from a deep trivial regime with initial \( m_z^i = -9.2E_r \) to a non-trivial Dirac semimetal phase with final \( m_z^f = 0.20(7)E_r \). Consider a single \( k_z = 0 \) layer, the 1D tight binding
Figure S6: (colour online) Illustration of spin-sensitive detection with a blast light. From left to right, we obtain the momentum distribution of atoms without blasting, with blasting spin-up atoms and with blasting both spin-up and -down atoms, respectively. A blast light is applied at the beginning of time-of-flight expansion.

Hamiltonians $H_{q_x}(q_x)$ have the following form

$$H_{q_x}(q_x) = \vec{\sigma} \cdot \vec{q}_x = h_z(q_x)\sigma_z + h_y(q_x)\sigma_y = [m_x(q_y) - 2t_x + \cos q_x a]\sigma_z + 2t_x \sin q_x a \sigma_y.$$  \hspace{1cm} (S31)

Here the $\sigma_0$ term has been discarded as it only contributes a global phase. The initial state is prepared in deep trivial regime where all the spins are fully polarized up. Without considering inter-band decay, the time-dependent single particle wave function has the form \[4\]

$$|\psi(q_x, t)\rangle = e^{-iH_{q_y}(q_y) t} |\psi(q_x, 0)\rangle = [-i h_z \sin(E_q t) - E_q \cos(E_q t), h_y \sin(E_q t)]^T,$$  \hspace{1cm} (S32)

where $E_q = \sqrt{h_y^2 + h_z^2}$ is the eigenenergy of the Hamiltonian. The spin polarization is then \[4\]

$$S_z(q_x, t) = \frac{h_x^2 + \cos(2E_q t)h_y^2}{h_y^2 + h_z^2}.$$  \hspace{1cm} (S33)

It is clear that only at the BIS point where $h_z(q_x) = 0$ the time averaged spin polarization will vanish. Furthermore, with the inter-band decay included via the Lindblad form master equation in Eq. (S12), the BIS points still have zero time averaged spin polarization as shown in Fig. S1(c). The main effect of the inter-band decay is that the final steady state eventually approaches the ground state of the two band model while the inter-band oscillation can persist several periods. Thus one can determine the topology of the 1D model by the time averaged spin texture and verify the 2D Dirac semimetal phase. As a simulation to experiment, the single $k_z$ layer should be replaced by a $k_z$ domain around $k_z = 0$ for the final several periods. As a result, the time averaged spin texture looks similar to the pure $k_z = 0$ layer except that white points for zero spin polarizations become blurred as shown in Fig. 4(b) in the main text.

S-3. Experimental Sequence

A. Implementation of SO coupling in optical lattices

Experiments start with a degenerate Fermi gas of $N_{\uparrow, \downarrow} = 5 \times 10^3$ $^{173}$Yb atoms prepared at $T/T_F \approx 0.5$ in a far-detuned crossed optical dipole trap with the wavelength of 1064 nm, characterized by the trap frequencies of $\omega = (\omega_x, \omega_y, \omega_z)^{1/3} = 2\pi \times 126$ Hz [5, 6]. During the experiment, the magnetic field of 10 G is applied along the $\hat{x}$ direction (see Fig. 1(b) in the main text). The 2D optical Raman lattice forms a 2D square optical lattice potential in the $x$-$y$ plane and consists of two pairs of standing-wave lights using the 556 nm intercombination transition. They are detuned from the principle resonant transition $F = 5/2 \rightarrow F' = 7/5$ by +1.0 GHz (blue-detuned) and -1.3 GHz
so that the relevant Raman potential is given as
\[ \int \text{momentum distribution along the } y \text{ direction} \]
of ytterbium atoms is adiabatically loaded into the 2D Raman lattice with fixed two photon detuning 
are simultaneously ramped up to the final values with a time constant of 10 ms, by which a degenerate mixture 
expansion from the optical Raman lattice. Before the TOF, the intensities of 2D lattice beams and the Raman beam 
\[ k_0 \]
but our spin-sensitive detection is not susceptible to unwanted spin components.

A running-wave Raman potential along the \( y \) direction.
The momentum distribution in the quasi-momentum space, we perform spin-resolved absorption imaging after a time-of-flight (TOF) expansion. At the beginning of the TOF expansion, we blast unwanted atoms in a spin component \( \sigma \) by using a 556 nm light resonant to the \( ^1S_0(F = 5/2) \rightarrow ^3P_1(F' = 7/2) \) transition. Three absorption images, \( I \), \( I_{\uparrow,\downarrow} \) and \( I_{\uparrow} \) are recorded using a 399 nm light resonant to \( ^1S_0 \rightarrow ^1P_1(F' = 7/2) \) transition, where the subscript \( \sigma \) stands for certain \( \sigma \) spin components removed by the blast light. Finally, a momentum distribution of the atomic cloud for each spin, \( D_{\sigma}^{TOF}(k_x, k_y) \) and \( D_{\uparrow}^{TOF}(k_x, k_y) \), are extracted from \( I - I_{\uparrow} \) and \( I_{\uparrow} - I_{\uparrow,\downarrow} \) respectively. We note that a tiny fraction of atoms may occupy other spin states due to the imperfect isolation of the spin-\( 1/2 \) subspace but our spin-sensitive detection is not susceptible to unwanted spin components.

B. Spin-sensitive Detection

To reconstruct a spin texture in the quasi-momentum space, we perform spin-resolved absorption imaging after a time-of-flight (TOF) expansion. Before the TOF, the intensities of 2D lattice beams and the Raman beam are simultaneously ramped up to the final values with a time constant of 10 ms, by which a degenerate mixture of ytterbium atoms is adiabatically loaded into the 2D Raman lattice with fixed two photon detuning \( \delta \). Since we integrate the momentum distribution along the \( z \) direction, the \( z \)-component term of the Raman potential is irrelevant so that the relevant Raman potential is given as \( \mathcal{M} = M_R \cos k_x \exp(i k_1 y) |\downarrow \rangle \langle |\uparrow| + H.C. \), where \( k_1 = k_0 \cos(68^\circ) \) and \( k_0 = 2\pi/\lambda \) for \( \lambda = 556 \) nm. This Raman potential effectively describes 2D SO coupling in the optical Raman lattice. A running-wave Raman potential along the \( y \) direction causes the asymmetric momentum distribution whereas the symmetric momentum distribution is induced along the \( x \) direction, which is shown in the differential momentum distribution

\[ n_{\uparrow}(k_x, k_y) - n_{\downarrow}(k_x, k_y) \] (Fig. 1(c) in the main text).

C. Spin-momentum locking in a 2D optical Raman lattice

Two-dimensional spin-momentum locking within an optical Raman lattice imparts spin-dependent momentum kick on atoms in the \( x-y \) plane. To reveal it, we record the momentum distribution of the atomic cloud after a TOF expansion from the optical Raman lattice. Before the TOF, the intensities of 2D lattice beams and the Raman beam are simultaneously ramped up to the final values with a time constant of 10 ms, by which a degenerate mixture of ytterbium atoms is adiabatically loaded into the 2D Raman lattice with fixed two photon detuning \( \delta \). Since we integrate the momentum distribution along the \( z \) direction, the \( z \)-component term of the Raman potential is irrelevant so that the relevant Raman potential is given as \( \mathcal{M} = M_R \cos k_x \exp(i k_1 y) |\downarrow \rangle \langle |\uparrow| + H.C. \), where \( k_1 = k_0 \cos(68^\circ) \) and \( k_0 = 2\pi/\lambda \) for \( \lambda = 556 \) nm. This Raman potential effectively describes 2D SO coupling in the optical Raman lattice. A running-wave Raman potential along the \( y \) direction causes the asymmetric momentum distribution whereas the symmetric momentum distribution is induced along the \( x \) direction, which is shown in the differential momentum distribution

\[ n_{\uparrow}(k_x, k_y) - n_{\downarrow}(k_x, k_y) \] (Fig. 1(c) in the main text).

D. Reconstruction of Spin Textures

The momentum distribution in the quasi-momentum \( D_{\sigma}(Q_x, Q_y) \) is constructed by folding \( D_{\sigma}^{TOF}(k_x, k_y) \) into the first Brillouin zone through shifting the integer number of \( 2k_0 \), 

\[ D_{\sigma}(Q_x, Q_y) = \sum_{M,N} D_{\sigma}^{TOF}(Q_x - 2Mk_0, Q_y - 2Nk_0) \]
for $M, N \in \mathbb{Z}$. Next, considering the momentum shift via the Raman process accompanied with the spin flip, we define states based on spin-$\uparrow$, $D_1(q_x, q_y) = D_1(Q_x, Q_y)$ and $D_2(q_x, q_y) = D_2(\text{mod}(Q_x - k_0, 2k_0), \text{mod}(Q_y - \cos\theta k_0, 2k_0))$, where mod is the modulo operator. Finally the spin texture $P(q_x, q_y)$ is determined by $(D_1(q_x, q_y) - D_2(q_x, q_y))/D_1(q_x, q_y)$. From the spin texture, the spin polarization at four critical momenta are taken account as $P(q_x, q_y) = \int_{\Lambda_0}^{\Lambda_+ + 0.1k_0} P(q_x, q_y)dq_xdq_y/\int_{\Lambda_0}^{\Lambda_+ + 0.1k_0} dq_xdq_y$. When we extract Dirac points, in the case of positive $m_z$, the spin polarization along the $X - \Gamma$ trajectory line, is calculated as $P(q_x = 0, q_y) = \int_{-0.1k_0}^{0.1k_0} P(q_x = 0, q_y)dq_x/0.2k_0$; while in the case of negative $m_z$, $P(q_x = 0, q_y) = [\int_{-1}^{-1+0.1k_0} P(q_x = 0, q_y)dq_x + \int_{1-0.1k_0}^{1} P(q_x = 0, q_y)dq_x]/0.2k_0$

S-4. Determination of the position of the Dirac points

We apply two methods to determine the momentum positions of the Dirac points from the result of spin texture measurement, illustrated in Fig. S7. First method is based on the topological phase transition points along $q_y$ direction in the spin textures. We first calculate the spin polarization along $q_x = 0$ and $q_y = k_0$ direction at different $m_z$, $P(q_x = 0, q_y)$ and $P(q_x = k_0, q_y)$ respectively. Next, we calculate a product of sign, $S = \text{sign}(P(q_x = 0, q_y) \cdot P(q_x = 1k_0, q_y))$ (example shown in Fig. S7(a)). To be noted, here the value of $S$ distinguishes different phases. Finally the positions of the Dirac points are determined by the sign-flip position $q_{D1}$ along $k_y$ direction for each $m_z$. Second method is based on the boundary between spin-$\uparrow$ and $\downarrow$ domain in the spin textures. Spin-flip positions $q_{D2}$ along $q_x = 0$ and $q_x = k_0$, determine the locations of the Dirac points for $m_z > 0$ and $m_z < 0$ respectively. The Dirac point position extracted from these two methods are consistent within the experimental uncertainty.

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