Unification of Matter and Dark Matter with Radiative Neutrino Mass

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Abstract. Neutrino mass may be linked to dark matter through a well-studied radiative (scotogenic) mechanism. The new particles required are just right for the extension of the well-known SU(5) unification of quarks and leptons to include dark matter. Two examples are discussed: SU(6) which incorporates a discrete $Z_2$ symmetry for dark matter, and SU(7) which allows for a gauge $U(1)_D$ symmetry. In either case, just as the proton is not absolutely stable within the context of SU(5), dark matter is also not absolutely stable within SU(6) or SU(7).

1. Introduction
Physics beyond the standard model (SM) of quarks and leptons should include neutrino mass and dark matter (DM). Are they related? I proposed [1] in 2006 that neutrino mass is due to the existence of dark matter (scotogenic = caused by darkness). I will discuss the prototype model based on the discrete symmetry $Z_2$, and its recent variation [2] with a gauge $U(1)_D$ symmetry.

Just as quarks and leptons may be unified in $SU(5)$, I will show how matter and dark matter may be unified in $SU(6)$ and $SU(7)$.

2. Neutrino Mass: Six Generic Mechanisms
In 1979, it was first pointed out by Weinberg [3] that a unique dimension-five operator exists for Majorana neutrino mass in the SM, assuming that new physics occurs at a mass scale higher than all the masses of the SM. This effective interaction

$$\mathcal{L}_5 = -\frac{f_{\alpha\beta}}{2\Lambda}(\nu_\alpha\phi^0 - l_\alpha\phi^+)(\nu_\beta\phi^0 - l_\beta\phi^+)$$

became the starting point of most models of neutrino mass. In 1998, I discussed [4] for the first time altogether three tree-level realizations of this operator, as well as three possible one-loop (one-particle-irreducible) realizations. To summarize, the three tree-level realizations are mediated by (I) a heavy neutral singlet Majorana fermion $N$, (II) a heavy scalar triplet $(\xi^+, \xi^0, \xi^0)$, and (III) a heavy Majorana fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$. Ever since, these three mechanisms have been referred to as Type I, Type II, and Type III seesaw respectively.

As for the three generic one-loop (one-particle-irreducible) realizations, the loop is assumed to consist of a scalar line and a fermion line linking $\nu_\alpha$ with $\nu_\beta$. The three realizations are then classified according to (IV) one $\phi^0$ attached to the scalar line and one $\phi^0$ attached to the fermion line, (V) both attached to the scalar line, and (VI) both attached to the fermion line.
3. Scotogenic Neutrino Mass

In 2006, I proposed [1] a specific realization of (V) using a neutral fermion singlet $N$ per family which is odd under an imposed discrete $Z_2$ symmetry. The interaction $(\nu \phi^0 - l \phi^+) N$ is then forbidden and $\nu$ does not combine with $N$ in the usual way to obtain a Dirac mass, although a Majorana mass for $N$ itself is allowed. To generate a nonzero neutrino mass, I assumed a second scalar doublet $(\eta^+, \eta^0)$ which is also odd under $Z_2$, so that the interaction $(\nu \eta^0 - l \eta^+) N$ is allowed, but $Z_2$ is assumed to be unbroken (for dark matter) so $\eta^0$ has zero vacuum expectation value and again there is no Dirac neutrino mass. However, the quartic scalar term $(\lambda_5/2)(\Phi^\dagger \eta^0)^2 + H.c.$ is allowed, and the one-loop diagram (V) exists for a radiatively generated Majorana neutrino mass, as shown in Fig. 1. The one-loop integral is exactly calculated as

$$ (M_{\nu})_{\alpha \beta} = \sum_i \frac{h_{\alpha i} h_{\beta i}}{16 \pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)], $$

where $f(x) = -\ln x/(1-x)$. Let $m_R^2 - m_I^2 = 2\lambda_5 v^2 << m_0^2 = (m_R^2 + m_I^2)/2$, then

$$ (M_{\nu})_{\alpha \beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} \lambda_5 v^2}{8 \pi^2 M_i} \left[ \frac{x_i}{1-x_i} \right] \left[ 1 + \frac{x_i \ln x_i}{1-x_i} \right], $$

where $x_i = M_i^2/m_0^2$.

4. New Scotogenic $U(1)_D$ Model

The notion that dark matter may have its own interaction, not shared by ordinary matter, has become increasingly relevant as a means of understanding the details of the dark-matter density profile of (dwarf) galaxies, i.e. the cusp-core discrepancy. To support a gauge $U(1)_D$ symmetry [2], two scalar doublets $(\eta^+, \eta^0)$ are used together with three Dirac singlet neutral fermions $N_{1,2,3}$ which transform as +1 together with $\eta_1$, whereas $\eta_2$ transforms as $-1$. The allowed interaction $(\Phi^\dagger \eta_1)(\Phi^\dagger \eta_2) + H.c.$ results in two neutral mass eigenstates

$$ \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_1^0 \\ \eta_2^0 \end{pmatrix}. $$

Hence a one-loop neutrino mass may be generated as shown in Fig. 2 without breaking the gauge $U(1)_D$ symmetry. In that case, a massless dark photon would appear.

There are two other alternatives. A scalar $\zeta \sim 2$ under $U(1)_D$ may be used to break it to $Z_2$. If $\zeta \sim 3$ , then $U(1)_D$ becomes a global symmetry. In either case, there will be a massive dark photon and a massive dark Higgs particle.
5. *SU*(6) Unification

To unify matter and dark matter, the well-known *SU*(5) grand unification [5] of quarks and leptons may be simply extended [6] to *SU*(6). The fundamental $5^*_F = (d^c, d^c, d^c, e, \nu)$ of *SU*(5) is extended to the $6^*_F = (d^c, d^c, d^c, e, \nu, N)$ of *SU*(6). The 10$_F$ of *SU*(5) is extended by a heavy 5$_F$ to form a 15$_F$ of *SU*(6). Together with the similar extensions of the scalar multiplets, the *SU*(5) Yukawa terms

$$5^*_F \times 10_F \times 5^*_S, \quad 10_F \times 10_F \times 5^*_S,$$

are extended to

$$6^*_F \times 15_F \times 6^*_S, \quad 15_F \times 15_F \times 15_S.$$  

Hence two different Higgs doublets are needed for quark and lepton masses.

Whereas $5^*_F + 10_F$ is anomaly-free in *SU*(5), the corresponding combination in *SU*(6) is $6^*_F + 6^*_F + 15_F$. The $5^*_F$ contained in the extra $6^*_F$ is heavy and pairs up with the 5$_F$ contained in the 15$_F$ through the 1$_S$ of 6$_S^*$. 

Consider now the *SU*(6) scalar multiplet $21_S$. It decomposes into $15_S + 5_S + 1_S$ of *SU*(5). It has then the second scalar doublet $(\eta^+, \eta^0)$ and the interactions

$$6^*_F \times 6^*_F \times 21_S, \quad 15_S \times 15_S \times 21_S \times 21_S.$$  

Hence N gets a Majorana mass through the 1$_S$ of 21$_S$. The scotogenic interaction $(\nu \eta^0 - e \eta^+)$N is now possible as well as the quartic $(\Phi^0 \eta)^2$ interaction. These three terms support a $Z_2$ symmetry as required for Fig. 3, but it is not absolute. Just as the proton is unstable at the scale of quark-lepton unification, dark matter is expected to be unstable at a similar scale. Matter and dark matter are thus unified through scotogenic neutrino mass.

The heavy color triplet gauge bosons contained in the adjoint $24_V$ of *SU*(5) mediate proton decay. The extra heavy color triplet gauge bosons contained in the adjoint $35_V$ of *SU*(6) will
connect $N$ to the quarks, resulting in $N \rightarrow p\pi^-, n\pi^0$. Another possibility is the mixing of the heavy scalar color triplet $\zeta^{-1/3}$ in $21_S$ with its counterpart $\xi^{-1/3}$ in $15_S$ through the adjoint $35_S$ of $SU(6)$. The dark $Z_2$ is thus broken at a high scale, just as baryon number is supposed to be broken in grand unified theories.

In the breaking of $SU(6)$ through the adjoint $35_S$, let the $6 \times 6$ matrix representing $35_S$ develop diagonal vacuum expectation values proportional to $(1, 1, 1, 0, 0, -3)$; then the residual symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$. The dark $U(1)_N$ is broken by $21_S$ at a lower energy scale for $N$ to acquire a Majorana mass. This choice of $35_S$ breaking mixes $\zeta$ with $\xi$, but not $\eta$ with $\Phi$. It is only possible here because of $SU(6)$. For $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$, it must be along the $(1, 1, 1, -3/2, -3/2)$ direction.

6. $SU(7)$ Unification

In the decomposition $SU(5) \times SU(2)_N$, the $7^*_F$ of $SU(7)$ contains an extra $SU(2)_N$ doublet $(N_1, N_2)$. The $10_F$ of $SU(5)$ is now extended to $21_F$ of $SU(7)$ containing two $SU(5)$ $5_F$s transforming as a doublet under $SU(2)_N$. The anomaly-free combination is $7^*_F \times 7^*_F \times 21_F$. The corresponding Yukawa couplings are

$$7^*_F \times 21_F \times 7^*_S, \quad 21_F \times 21_F \times 35_S. \quad (8)$$

The $28_S$ of $SU(7)$ now contains a bidoublet

$$\left( \begin{array}{c}
\eta_1^+ \\
\eta_2^+
\end{array} \right), \quad (9)$$

where $SU(2)_L$ applies vertically and $SU(2)_N$ applies horizontally. To complete the scotogenic loop as shown in Fig. 4 with $U(1)_D$ as the diagonal subgroup of $SU(2)_N$, $28_S$ is added with the interactions

$$7^*_F \times 7^*_F \times 28_S, \quad 21^*_S \times 21^*_S \times 28_S \times 28_S. \quad (10)$$

Figure 4. Scotogenic neutrino mass in $SU(7)$.

7. Conclusion

The evidence of dark matter signals a new class of particles at the TeV scale, which may manifest themselves indirectly through loop effects. They may be responsible for radiative neutrino mass and facilitate the unification of matter and dark matter.

Two prototype models based on $SU(6)$ and $SU(7)$ are presented. The former applies naturally to the original scotogenic model of neutrino mass with $Z_2$ symmetry. The latter fits well with a recent $U(1)_D$ extension.
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