Finite Element Analysis of an Acoustic Metamaterial Plate Incorporating Tunable Shape Memory Cantilever Absorbers

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Abstract. Metamaterials are materials having artificially tailored internal structure and unusual physical and mechanical properties. Due to their unique characteristics, metamaterials possess great potential in engineering applications. This study proposes a tunable metamaterial for the applications in vibration or acoustic isolation. For the state-of-the-art structural configurations in metamaterial, the geometry and mass distribution of the crafted internal structure is employed to induce the local resonance inside the material. Therefore, a stopband in the dispersion curve can be created because of the energy gap. For the conventional metamaterial, the stopband is fixed and unable to be adjusted in real-time once the design is completed. Although the metamaterial with distributed resonance characteristics has been proposed in the literature to extend its working stopband, the efficacy is usually compromised. In this study, the incorporation of tunable shape memory materials (SMM) via phase transformation into the metamaterial plate is proposed. Its theoretical finite element formulation for determining the dynamic characteristics is established. The effect of the configuration of the SMM cantilever absorbers on the metamaterial plate for the desired stopband in wave propagation is simulated by using finite element model and COMSOL Multiphysics software. The result demonstrates the tunable capability on the stopband of the metamaterial plate under different activation controls of the SMM absorbers, and shows the ability to trap the vibration at the designed frequency and prevent vibration wave from propagating downstream in different absorber arrangement and alloy phase. To conclusion, this study should be beneficial to precision machinery and defense industries which have desperate need in vibration and noise isolation.

Keywords: Shape memory material, smart material, metamaterial plate, cantilever absorbers, finite element analysis, stopband

1. Introduction

Metamaterials are artificially designed and fabricated material structures which demonstrate special and/or peculiar properties in contrast to conventional engineering materials. Due to their special properties, such as negative reflectivity, negative mass and negative Poisson’s ratio, they have potential applications in electromagnetic wave cloaking, acoustic and vibration inhibition of structure [1–4]. The bandgap of the metamaterial in acoustic transmission presents a useful tool for isolating the targeted acoustic wave. Several tunable mechanisms for the bandgap of acoustic metamaterial have been proposed, such as the use of piezoelectric actuator [5], the volume control of Helmholtz resonator [6],
the change in the size parameters of a kagome-sphere lattice [7], the use of distributed vibration absorbers [8,9], and the incorporation of chiral elastic lattice [10], etc. They all demonstrate the wide bandgap characteristics of their proposed designs. Although by changing the frequency dispersion function of the structure or the parameters of the electric circuits, the changeable bandgap can provide more flexibility of the metamaterial to adapt to different design circumstances, the time-varying excitation source presents another challenge to real-time adaptability of the metamaterial. For example, the speed of the machine in operation can change which induces the variation in associated vibration and noise excitation. Smart materials, such as piezoelectric ceramics, magnetorheological fluid, electrorheological fluid, shape memory materials, are a class of materials which properties can be tuned according to the demand [11]. If smart material is incorporated into the metamaterial, the bandgap property can then be tuned by controlling the property of its constituent smart material. In our previous study, absorbers consisted of the shape memory material (shape memory alloy and shape memory polymer) as the structural member were employed in the design of an acoustic metamaterial beam [12-16]. The activation on the phase change of the shape memory material by controlling the heating current tuned the frequency of the bandgap and obtained the required isolation of the designated excitation. As a continuation of the previous work, the SMM absorbers were used in a plate structure. The design and dynamic characteristics of this acoustic metamaterial plate were simulated through finite element modeling.

2. Design

2.1 Design of the SMM Absorber

Figure 1 presents the proposed tunable absorber made of SMM spring. The spring can be in helical configuration or cantilevered beam configuration. The absorber with helical spring is able to provide more structural flexibility and more vibration modes at low frequency range [14]. Therefore, it can present the acoustic metamaterial plate with wide bandwidth of the stopband. Nevertheless, the fabrication of the helical spring is more complicated comparing with its cantilever counterpart. The SMM cantilever is close to a single degree of freedom system because its second natural frequency is nearly four times higher than its fundamental frequency [17]. For acoustic frequency, this SMM cantilever is easier to design in the required frequency span. However, due to its lower mass, the effect of vibration isolation can be limited [18]. As required by the design of metamaterial, these absorbers should be arranged and attached to basis plate with lattice configuration.

Figure 1. The schematic diagram of the proposed design of SMM absorbers.
The design of an exemplary SMM cantilever absorber is shown in Figure 2. An insulated fixture pad was employed to mount the cantilevers and to secure them on the base plate. The cantilevers were arranged in parallel and symmetric configuration. The parallel cantilevers were to complete an electrical circuit for the heating current and the symmetric arrangement was to minimize the rotation loading on the fixture pad. On the sides of the fixture pad there were two conducting end tabs for introducing the electric current and forming the control circuit for the SMM absorbers. If a NiTinol cantilever with diameter \(d\) and length \(l\) was used, the following material properties can be applied: mass density \(\rho=6450\) kg/m\(^3\), Young's modulus of martensite phase \(E_m=24\) GPa, and Young's modulus of austenite phase \(E_A=76\) GPa [11]. The fundamental frequency of this cantilever absorber can be written as:

\[
\omega_u = \left(1.875\right)^2 \frac{EI}{\rho Al^4} \tag{1}
\]

Accordingly, the fundamental frequencies at low temperature (martensite) and elevated temperature (austenite) are 1499 Hz and 2668 Hz, respectively. For the following numerical analyses, the fundamental frequencies for the SMM absorber in the martensitic and austenitic phases were taken as 1499 Hz and 2668 Hz for simplicity, respectively.

2.2 Design of the Metamaterial Plate

2.2.1 Plate

To understand the metamaterial plate’s dynamic characteristics, the dynamic analysis on the base plate needs to be conducted first. The material of the base plate is aluminum with dimensions of 187.5 mm \(\times\) 150 mm and thickness of 3 mm, and mass density \(\rho=2780\) kg/m\(^3\), Young’s modulus \(E=72.4\) GPa, Poisson's ratio \(\nu = 0.33\). To simulate the cantilever configuration, we fixed one of the width side of the plate (Figure 3.). Subsequently, the equation of eigenfrequencies for the corresponding vibration modes can be written as [19]:

\[
f_{ij} = \frac{\lambda_{ij}^2}{2\pi a^2} \left[ \frac{E h^3}{12\gamma(1-\nu^2)} \right]^{1/2} \tag{2}
\]

In the above equation, \(\lambda_{ij}\) is a dimensionless parameter which is generally a function of the mode indices. In the cantilever condition, \(\lambda_{ij}^2\) are 3.4845, 10.10, 21.525 for first three eigenfrequencies sequentially. \(a\) is the length of plate, \(h\) is the thickness of plate, \(\gamma\) is the mass density per unit area. The first three eigenfrequencies for the natural vibration modes are listed in Table 1.
Table 1. The first three eigenfrequencies for the natural vibration modes

| mode | eigenfrequency (Hz) |
|------|---------------------|
| 1st  | 73.85               |
| 2nd  | 214.07              |
| 3th  | 456.22              |

Figure 3. (a) The cantilevered metamaterial plate; (b) The 2-DOF system model of the metamaterial plate

2.2.2 Metamaterial Plate

After understanding the dynamic characteristics of the absorber and the plate, we combined each other and built a Mathematica® model to make the analysis of displacement of the absorber and the base plate, as seen in Figure 3(b). The following notations were used in the following equations: \( m_1, m_2 \) the plate and absorber mass, \( k_1, k_2 \) the plate and absorber stiffness, \( c_2 \) the absorber damping coefficient, \( F \) the excitation force. By assuming \( u_i \) is the displacement of mass in Eq. 3 which can be written as \( u_i = X_i e^{j\omega t} \), \( \dot{u}_i = \frac{du_i}{dt} \) and \( \ddot{u}_i = \frac{d^2u_i}{dt^2} \), the system’s dynamic equations become:

\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_1 \\
    \ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
    c_2 & -c_2 \\
    -c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_1 \\
    \dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
    k_1 + k_2 & -k_2 \\
    -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
= \begin{bmatrix}
    F \\
    0
\end{bmatrix}
\]  

(3)

If the absorber’s damping coefficient \( c_2 \) was negligible, the transfer function relating the input force \( F \) and the resulted displacement \( X_1 \) of \( m_1 \):

\[
X_1 = \frac{k_2 - \omega^2 m_2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2} F
\]  

(4)

can re-written as:

\[
X_1 = \frac{\delta_st \left[ 1 - \left( \frac{\omega}{\omega_2} \right)^2 \right]}{1 + \mu \left( \frac{\omega}{\omega_1} \right)^2 - \left( \frac{\omega}{\omega_2} \right)^2 \left[ 1 - \left( \frac{\omega}{\omega_2} \right)^2 \right] - \mu \left( \frac{\omega}{\omega_2} \right)^2}
\]  

(5)

In Eq. (5), \( \delta_st = \frac{F}{k_1} \), which denotes the static displacement of \( m_1 \), \( \mu = \frac{m_2}{m_1} \), the mass ratio between the absorber and the plate, \( \omega_1, \omega_2 \) the absorber and the base plate natural frequencies. According to the Eq. (5), when the external excitation frequency \( \omega \) was equal to the natural frequency of the absorber, the
displacement of plate $X_1$ could approach zero. It can stop the wave propagation in the plate by transferring completely the vibration energy from the plate to the absorber. Figure 4 presents the metamaterial plate which was discretized into 20 elements and 25 nodes. At each node there was an installed absorber which is capable of transferring the vibration energy from the plate to stop the wave propagation.

3. Results and Discussion

This study used COMSOL Multiphysics commercial software to build the numerical model and analyzed the dynamic characteristics of the metamaterial plate. The wave propagation and suppression of vibration would also be discussed when an external harmonic force was applied at a specific node (Fig. 4). To reach the most effective suppression of vibration, the absorbers’ arrangement would also be optimized in this study.

![Diagram](image)

*Figure 4. The positions of the input force and output deflection.*

3.1 Convergence analysis

To make sure the accurate result could be calculated by using minimal simulation time, finding the suitable element size was necessary before building the full scale numerical model.

3.1.1 Accuracy of the COMSOL mode

Firstly, the COMSOL model of base plate was built, and simulated results about the first three eigenfrequencies were compared with analytical ones. Along with these finite element results, the analytical ones calculated from the formula of vibration textbook [19] were also included. The percentage errors of these first three eigenfrequencies from the finite element simulation were all smaller than 2%. The mode shapes were also consistent to the calculated results. Therefore, the accuracy of the finite element simulation was validated.
Table 3. The first three eigenfrequencies of a cantilevered rectangular plate

| Mode         | 1st     | 2nd     | 3rd     |
|--------------|---------|---------|---------|
| Analytical results | 73.85   | 214.07  | 456.22  |
| COMSOL results    | 73.34   | 209.73  | 452.94  |
| Error          | -0.69%  | -2.02%  | -0.72%  |

3.1.2 Suitable element size

Secondly, this section used the metamaterial plate’s first eigenfrequency to do the convergence analysis. The results are presented in Table 2. When the number of elements reached 143387, the calculation had approached to convergence. If there were too many elements used in simulation which occurred with nominal element size smaller than 0.3 mm, it would waste the computational resource without significant improvement in solution accuracy. According to the results, it was deemed that the suitable choice for nominal element size in this simulation was 0.5 mm.

Table 2. The convergence of the first eigenfrequency with respect to nominal element size in discretization

| Nominal element size (mm) | Number of elements | DOF       | Calculating time (s) | First eigenfrequency (Hz) |
|---------------------------|--------------------|-----------|-----------------------|----------------------------|
| 0.7                       | 55913              | 334839    | 26                    | 69.045                     |
| 0.6                       | 84027              | 488484    | 36                    | 69.032                     |
| 0.5                       | 117913             | 648822    | 52                    | 68.976                     |
| 0.4                       | 143387             | 805371    | 60                    | 68.978                     |
| 0.3                       | 309740             | 1584855   | 152                   | 68.976                     |

Figure 5. The first eigenfrequency result under different element size and simulation time
3.2 Absorbers configuration

3.2.1 Full configuration with one state

There were 25 nodes for the installation of one absorber at each node. For the absorbers activated in martensite or austenite state, the frequency response spectra, as seen in Figure 6, showed a bandgap at 1500 Hz and 2500 Hz, matching the results which predicted in the previous section, respectively. These results prove the absorbers on metamaterial plate were able to stop wave propagation at designated frequencies.

3.2.2 Full configuration with composite state

In addition, if the absorbers were activated differently in two grouped regions (martensite and austenite state) evenly, their frequency response spectra were also presented in Figure 6. In these cases, both stopbands at 1500 Hz and 2500 Hz occurred simultaneously but with less effectiveness. More absorbers at right frequency installed could demonstrate more damping on the targeted signal.

![Figure 6](image)

**Figure 6.** The frequency response spectra of a cantilevered plate with 25 absorbers activated in different phase states.

3.2.3 Partial configuration

The distribution control effect of the absorbers on the stopband characteristics were further explored in this cantilevered plate. With only 9 to 10 austenite absorbers installed at the discrete nodes, Figure 7(a) presents the distributions of 3 different configurations. Their corresponding frequency response spectra near the stopband were also shown in Figure 7(b). Because only 9 or 10 out of 25 available nodes were installed with absorbers, the stopband at designated 2500 Hz was not as effective as that presented in Figure 6. for all nodes installed. Nevertheless, among the 3 different distribution configurations, the more grouped arrangement demonstrated better stopband characteristics. For the alternate arrangement of the absorbers, nearly no stopband was observed in the frequency response spectrum, but the X shape configuration had the same effectiveness like full configuration because the configuration matched the wave propagation, which was also mentioned in Figure 6. previously.
Figure 7. A cantilevered plate with 9-10 absorbers installed in different distributions: (a) the configurations of absorber distribution; (b) the corresponding frequency response spectra.

4. Conclusion

A simulation on the design and dynamic characteristics of the proposed acoustic metamaterial plate with installation of SMM absorbers was performed in this study. The cantilevered beam configuration of the SMM absorber was easy to implement on an existing structure. The simulation using COMSOL Multiphysics finite element commercial software demonstrated the tunable stopband of this metamaterial plate in different absorber configurations. In other words, by controlling the cantilever absorber in martensite or austenite phase the working stopband frequency can be tuned in lower or higher frequency domain. The arrangement of the SMM absorbers on the base plate also plays an important role in demonstrating the function of this metamaterial plate. A more grouped arrangement of the absorbers was better in enforcing the stopband. The experimental measurement on this tunable metamaterial plate can be subject for further study.
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