On the origins of scaling corrections in ballistic growth models

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We study the ballistic deposition and the grain deposition models on two-dimensional substrates. Using the Kardar-Parisi-Zhang (KPZ) ansatz for height fluctuations, we show that the main contribution to the intrinsic width, which causes strong corrections to the scaling, comes from the fluctuations in the height increments along deposition events. Accounting for this correction in the scaling analysis, we obtained scaling exponents in excellent agreement with the KPZ class. We also propose a method to suppress these corrections, which consists in divide the surface in bins of size ε and use only the maximal height inside each bin to do the statistics. Again, scaling exponents in remarkable agreement with the KPZ class were found. The binning method allowed the accurate determination of the height distributions of the ballistic models in both growth and steady state regimes, providing the universal underlying fluctuations foreseen for KPZ class in 2+1 dimensions. Our results provide complete and conclusive evidences that the ballistic model belongs to the KPZ universality class in 2+1 dimensions. Potential applications of the methods developed here, in both numerics and experiments, are discussed.

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I. INTRODUCTION

Non-equilibrium dynamics of growing interfaces has attracted much interest in several scientific branches such as Physics, Chemistry, Biology and Engineering [1, 2]. A simple and widespread approach to the modeling of evolving surfaces considers particles in a random flux that irreversibly aggregate to the substrate following a given rule. Considering ballistic trajectories for particles that aggregate at a first contact with the deposit we have the celebrated ballistic deposition (BD) model [3], formerly proposed to simulate rock sedimentation, with applications to the modeling of thin film growth at low temperatures [2] and to describe colloidal particle deposition at the edges of evaporating drops [4]. A central characteristic of ballistic growth models is the lateral growth that produces a velocity excess. Other models exhibiting this property include the Eden model [5], a paradigm in the study of curved surfaces, and models where large grains are randomly deposited [6, 7].

The velocity excess is a hallmark of the Kardar-Parisi-Zhang (KPZ) universality class [8]. Therefore, in the hydrodynamic limit, one expects that the growth dynamics of ballistic models is described by the KPZ equation [8]

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h + \frac{1}{2} (\nabla h)^2 + \xi(x,t),$$

(1)

where terms in the right side accounts, respectively, for the surface tension, local growth in the normal direction and a delta-correlated noise, with $\langle \xi(x,t,t') \rangle = 0$ and $\langle \xi(x,t)\xi(x',t') \rangle = 2D \delta(t-t') \delta^d(x-x')$, associated to the randomness of the deposition process. In $d = 1 + 1$ dimensions, the surface height in KPZ systems asymptotically evolves according to the ansatz [9, 12]

$$h \simeq v_\infty t + s_\lambda (\Gamma t)^{\beta \chi},$$

(2)

where $v_\infty$ is the asymptotic growth velocity, $s_\lambda$ is the signal of $\lambda$ in the KPZ equation [Eq. (1)], $\Gamma$ is a non-universal constant associated to the amplitude of the interface fluctuations, $\beta$ is the growth exponent, and $\chi$ is a stochastic quantity given by Tracy-Widom distributions. This conjecture was confirmed in distinct KPZ systems [13–19] besides exact solutions of KPZ equation [20–23]. Recent numerical simulations have shown that the KPZ ansatz can be generalized to 2+1 [24] and higher [27] dimensions, but the exact forms of the asymptotic distributions of $\chi$ are yet not known.

Although the KPZ equation was initially proposed to explain ballistic deposition models, numerical simulations commonly fail to provide a reliable connection between them and the KPZ class, mainly in higher dimensions. For example, the interface width $W \equiv \langle \delta^2 \rangle_c$ (here $\langle X^n \rangle_c$ represents the nth cumulant of $X$) scaling with time $t$ in the growth regime ($W \sim t^\beta$ for $t \ll L^z$, where $z = \alpha/\beta$ is the dynamic exponent [1]) and with the system size $L$ in the steady state ($W \sim L^\alpha$ for $t \gg L^z$) leads to growth ($\beta$) and roughness ($\alpha$) exponents smaller than the KPZ values [28–30]. In particular, for the BD model in $d = 1 + 1$, exponents in agreement with the KPZ ones were obtained through appropriated extrapolations of effective exponents [23] and, more recently, from extremely large-scale simulations accessing the regimes where corrections become negligible [30]. Moreover, recent studies of height distributions have given additional proofs of the KPZ universality of Eden and BD models in $d = 1 + 1$ [16–19]. For Eden models, scaling exponents and height distributions consistent with KPZ class were also found in $d = 2 + 1$ [24, 31]. However, for the BD
model and also for a grain deposition (GD) model in $d = 2 + 1$ dimensions, strong corrections to the scaling were found and evidences of the KPZ class was limited to the collapse of interface width distributions in the steady state.

A correction in the squared interface width $W^2$ for the Eden model was proposed long ago as a constant additive term in the Family-Vicsek ansatz, so that

$$W^2 \simeq L^{2\alpha} f\left(\frac{t}{L^2}\right) + w_i^2,$$  \hspace{1cm} (3)

where the first term at the right side accounts for long wavelength fluctuations and $w_i$ is called intrinsic width. Corrections consistent with an intrinsic width have been observed in many ballistic models. The intrinsic width was initially attributed to large steps at surface, but it was shown that large local height gradients is not a sufficient condition for intrinsic width since other KPZ models presenting local height differences comparable to those of Eden and ballistic deposition do not present a relevant intrinsic width.

Finite-time corrections observed in several KPZ systems lead to the modified ansatz

$$h \simeq v_\infty t + s_\chi (\Gamma t)^\beta \chi + \eta + \cdots,$$  \hspace{1cm} (4)

where $\eta$ is, in principle, a model-dependent stochastic quantity responsible by a shift in the mean of the scaled variable

$$q = \frac{h - v_\infty t}{(\Gamma t)^\beta}$$  \hspace{1cm} (5)

in relation to the $\chi$ distributions, which vanishes as $t^{-\beta}$. Corrections in higher order cumulants of $q$ and consequently, of $h$ were also observed but without universal schema.

In the present work, we perform a detailed study of BD and GD models on two-dimensional substrates and show that the intrinsic width $w_i$ can be suited in terms of the finite-time corrections of the KPZ ansatz, Eq. (4), and the leading contribution to $w_i$ is due to a stochastic component of the local columnar growth intrinsic to ballistic growth. More precisely, we show that $w_i^2$ is very close to the variance of the local height increments during the deposition process. Including this variance in the scaling analysis, exponents in striking agreement with the KPZ ones are found. Since large variances in height increments are due to narrow-deep valleys in the surface, we also propose a method where the surface is constructed considering only the maximal heights inside bins of size $\varepsilon$ and show that the intrinsic width is strongly reduced, leading to scaling exponents and height distributions in excellent agreement with the KPZ class. Our results providing a thorough confirmation of the KPZ universality of the ballistic deposition in $d = 2 + 1$ demystify a longstanding question which has been chased for decades. Applications of our methods to other important ballistic systems are discussed.

The sequence of this paper is organized as follows. The investigated models and the method to define the surface are presented in Sec. II. The determination of the non-universal parameters in the KPZ ansatz given by Eq. (3) is done in section III. The analysis of the scaling corrections and their consequences to the scaling exponents of ballistic growth models are presented in Sec. IV. Universality of the underlying fluctuations in height distributions is analyzed in section V. Final discussions and potential applications of the methods are presented in Sec. VI.

II. MODELS AND METHODS

In the ballistic deposition (BD) model, particles are randomly released perpendicularly to an initially flat substrate and permanently stick at their first contact with the deposit or the substrate. Therefore, porous deposits with large steps at the surface are formed, as shown in Fig. 1(a).

We also investigated the grain deposition (GD) model conceived to simulate grained surfaces. In this model, cubic grains of side $l$ (in units of the lattice parameter) are released in a trajectory perpendicular and with two faces parallel to the substrate, at randomly chosen positions. The grains permanently aggregate when their bottoms touch the top of a previously deposited grain or the substrate. The deposited grain is usually laterally shifted in relation to underneath grains, which also leads to a porous deposit and large steps are formed in the surface. We present results for grain sizes $l = 2$ and $l = 4$, hereafter named as GD2 and GD4, respectively.

Both models are defined on square lattices with periodic boundary conditions. A unity of time is defined as the deposition of $L^2$ particles (lattice unitary cells) in both models. Therefore, in GD model, $L^2/l^2$ grains are deposited during a time unity. We study these models on square lattices of lateral sizes up to $L = 214$.

The surface of ballistic models is conventionally defined as the highest points of the deposit at each lattice position. With this standard definition the resulting surface have many narrow-deep valleys, as shown in Fig. 1(a) for BD model in $d = 1 + 1$. These valleys are more pronounced in $d = 2 + 1$, as shown in Fig. 1(c). In section IV we will show that the fluctuations in the height increments during the deposition process are responsible by the strong corrections to the scaling in ballistic models. We propose that the leading contribution to these fluctuations is due to these narrow-deep valleys. In order to check this hypothesis, we introduce an alternative definition of the surface considering only the largest local heights. More precisely, we divide the surface in boxes (bins) of size $\varepsilon$ and take only the maximal height inside each box to form a coarse-grained surface with $(L/\varepsilon)^2$ sites. Figures 1(b) and (d) show typical height
FIG. 1: (Color online) (a) A typical deposit of the BD model in \(d = 1+1\) and its height profile for \(\varepsilon = 1\). (b) Height profile built using local maximal heights in boxes of size \(\varepsilon = 4\) for the same deposit of (a). The vertical dashed lines indicate the separation between boxes used to construct the coarse-grained profile. Typical cross-sections of standard and binned (\(\varepsilon = 2\)) surfaces for the BD model in \(d = 2 + 1\), with \(L = 800\) and \(t = 1000\), are shown in panels (c) and (d), respectively. Arrows indicate a few narrow-deep valleys removed with the binning method.

Profiles obtained with binned surfaces for BD model in \(d = 1+1\) and \(2 + 1\), respectively. As expected, smoother surfaces are obtained since many narrow-deep valleys are discarded. The values of \(\varepsilon\) must be small when compared with the typical size of the large wavelength fluctuations, which are responsible by the universality class of the system. Notice that, in fact, the binning procedure preserves the long wavelength fluctuations.

III. NON-UNIVERSAL PARAMETERS

The scaling analysis based on the KPZ ansatz, Eq. [4], requires accurate estimates of the non-universal parameters. The growth velocity \(v = d\langle h \rangle/dt\) against \(t^{\beta-1}\) is shown for all studied models in Fig. 2(a), using both standard and binned surfaces definitions. Here, we adopt \(\beta = 0.241\) as the KPZ growth exponent in \(d = 2 + 1\) [39]. For the standard surface (\(\varepsilon = 1\)), strong corrections are found and the linear regime expected in the KPZ ansatz is observed only for very long times. However, for \(\varepsilon > 1\) the linear regime is much more evident. The central point is that the convergence is faster for \(\varepsilon > 1\), but the asymptotic velocity does not depend on the value of \(\varepsilon\). The values of \(v_\infty\) for the investigated models are shown in Table I.

The non-universal parameter controlling the amplitude of fluctuations in the KPZ ansatz can be obtained by the relation \(\Gamma = |\lambda|A^{1/\alpha}\) [10], where \(\alpha = 0.393(4)\) is adopted as the roughness exponent for the KPZ class in \(d = 2 + 1\) [39]. The parameter \(\lambda\) can be determined using deposition on tilted large substrates with an overall slope \(s\), for which a simple dependence between velocity and

| Model | \(v_\infty\) | \(\lambda\) | \(\Gamma\) |
|-------|-----------|-----------|--------|
| BD    | 3.3396(3) | 2.15(10)  | 57(7)  |
| GD2   | 3.6925(1) | 0.35(3)   | 3.5(3)\times10^3 |
| GD4   | 5.1124(1) | 0.76(3)   | 4.3(7)\times10^4 |

**TABLE I**: Non-universal parameters for ballistic models.
slope,
\[ v \simeq v_\infty + \frac{\lambda}{2} \xi^2, \tag{6} \]
is expected for the KPZ equation \[40\]. The parameter \( A \) is obtained from the asymptotic velocity \( v_L \) for finite systems of size \( L \) \[40\]. Using the relation
\[ \Delta v = v_L - v_\infty \simeq -\frac{A\lambda}{2} L^{2\alpha - 2}. \tag{7} \]
This approach is commonly called Krug-Meakin method \[40\] and the estimated values of \( \lambda \) and \( \Gamma \) parameters are shown in Table \[I\]. Notice that, as mentioned before, asymptotic velocities are independent of the coarse-graining parameter \( \varepsilon \) but the convergence is faster for larger \( \varepsilon \). Thus, the parameters shown in Table \[I\] were calculated for \( \varepsilon = 4l \), where \( l \) is the particle/grain size, which are the most accurate we obtained. Notice that we have \( l = 1 \) for BD model, even though the GD model with \( l = 1 \) is a random deposition \[1\].

According to the extended KPZ ansatz, Eq. (4), \( \Gamma \) can also be obtained using
\[ \Gamma = \lim_{t \to \infty} \left[ \frac{\langle h^2 \rangle_c}{t^{2\beta} \langle \chi^2 \rangle_c} \right]^{1/2\beta}, \tag{8} \]
where \( \langle \chi^2 \rangle_c = 0.235 \) was adopted \[25, 26\]. In the case of \( \eta \) independent of \( \chi \), it is easy to check that Eq. (4) implies
\[ \Gamma(t) \equiv \left[ \frac{\langle h^2 \rangle_c}{t^{2\beta} \langle \chi^2 \rangle_c} \right]^{1/2\beta} = \Gamma(\infty) + ct^{-2\beta} + \cdots. \tag{9} \]

Fig. 2(b) confirms that \( \Gamma(\infty) \) is independent of \( \varepsilon \) and also that the correction in \( \Gamma(t) \) is consistent with \( t^{-2\beta} \).

The asymptotic \( \Gamma \) values obtained using this approach are the same, inside errors, as those found using the Krug-Meakin analysis shown in Table \[I\]. In summary, we conclude that the role played by corrections in Eq. (4) is being suppressed in surfaces for \( \varepsilon > 1 \).

### IV. SCALING CORRECTIONS AND THE INTRINSIC WIDTH

The finite-time corrections in Eq. (2) are non-universal and its nature, deterministic or stochastic for example, will depend on the investigated model \[18, 38\]. The second cumulant is, for the most general case, given by
\[ \langle h^2 \rangle_c = (\Gamma t)^{2\beta} \langle \chi^2 \rangle_c + 2(\Gamma t)^{\beta} \text{cov}(\chi, \eta) + \langle \eta^2 \rangle_c + \cdots \tag{10} \]
where \( \text{cov}(\chi, \eta) \equiv \langle \chi \eta \rangle - \langle \chi \rangle \langle \eta \rangle \) is the covariance. In order to determine the relevant corrections in \( \langle h^2 \rangle_c \), we plot \( \langle h^2 \rangle_c - (\Gamma t)^{2\beta} \langle \chi^2 \rangle_c \) against time, as shown in Fig. 3. The corrections reach a constant value - the squared intrinsic width \( w_i^2 \) - at relatively short times, ruling out a statistical dependence between \( \chi \) and \( \eta \), i.e., \( \text{cov}(\chi, \eta) = 0 \). At first glance, one would identify \( w_i^2 = \langle \eta^2 \rangle_c \) from Eq. (10) but, in principle, the contribution of higher order corrections to \( w_i^2 \) cannot be disregarded.

We define the squared intrinsic width as
\[ w_i^2 = \lim_{1 < t < L^2} \left[ \langle h^2 \rangle_c - (\Gamma t)^{2\beta} \langle \chi^2 \rangle_c \right]. \tag{11} \]

The values \( w_i = 3.6(2), 10.0(5), 26(2) \) were obtained for BD, GD2 and GD4 models, respectively. These estimates are in good agreement with the intrinsic width determined for these models using the collapse of the interface width distributions in the steady state \( t \gg L^2 \) \[32\], implying that the intrinsic width formed in the dynamic regime \( t \ll L^2 \) lasts indefinitely.

The squared interface width against time obtained for BD model using different binning parameters are shown in Fig. 4. A quick convergence to the KPZ scaling is found when \( \varepsilon > 1 \) is considered and a large intrinsic width seems to be absent. This result is corroborated by the effective growth exponents \( \beta_{eff} \), defined as the local slopes in double-logarithmic plots of \( W \) against \( t \), shown in the inset of Fig. 4. Indeed, the intrinsic width

![Fig. 3: Determination of the intrinsic width for ballistic models using the KPZ ansatz. These curves were obtained using \( \langle \chi^2 \rangle_c = 0.235 \) and \( \Gamma = 57, 3500 \) and 43000 for BD, GD2 and GD4 models, respectively.](image)

![Fig. 4: Squared interface width for BD surfaces using different binning parameters. Dashed line is a power law with exponent \( 2\beta_{kpz} \). Effective growth exponents are shown in the inset. The horizontal line represents \( \beta_{kpz} = 0.241 \).](image)
determined in plots equivalent to Fig. 3(a) results in a reduction from $w_i = 3.6(2)$ for standard surface to $w_i = 1.5(2)$ and $0.8(2)$ when $\varepsilon = 2$ and $4$ are used, respectively. Similar results were found for the GD models.

We conclude that the presence of narrow-deep valleys is, in fact, a necessary condition to observe strong corrections and, consequently, the intrinsic width. In these valleys with large steps, the heights are incremented, in average, by $(\delta h) \rightarrow l v_{\infty} > 1$, where the factor $l$ (particle/grain size) is required to account for the time step definition in GD models. However, these increments are not deterministic and we propose that their stochastic fluctuations are the leading contributions to the intrinsic width. In order to validate this conjecture, we determined $(\langle \delta h \rangle)^2$, where $\delta h = h(x, t + \delta t) - h(x, t)$ is the height increment in a time step. The results are compared with the squared intrinsic width, obtained via the KPZ ansatz, in Fig. 3(a). For all models, the $w_i^2$ is slightly larger than $(\langle \delta h \rangle)^2$. Since the binning method reduces the intrinsic width, we conclude that the leading contribution to $w_i^2$ comes from the fluctuations in narrow-deep valleys.

A central contribution to the mechanism behind the leading corrections in ballistic growth models is therefore elicited. However, it is not exclusivity of ballistic models. For example, an intrinsic width can also be determined for the RSOS model [11]. In this model, at each time step, the height of a randomly selected column is incremented by a unity if the height difference between nearest neighbors obeys the constraint $|h_i - h_j| \leq m$, otherwise, the deposition attempt is refused. This model produces an asymptotic growth velocity $v_{\infty} < 1$ independently of the substrate dimension [17, 19]. Since only increments $\delta h = 1$ or $0$ are allowed, we have that deposition and refusal for long times occur with probabilities $v_{\infty}$ and $1 - v_{\infty}$, respectively. Therefore, we have $(\langle \delta h \rangle)^2 = v_{\infty} (1 - v_{\infty})$. In $d = 1 + 1$ and $2 + 1$, the RSOS asymptotic velocities for $m = 1$ are $v_{\infty} \approx 0.419$ [17] and $0.3127$ [21] resulting in small variances $(\langle \delta h \rangle)^2 = 0.24$ and $0.21$, respectively. We simulated the RSOS model and found $w_i^2 \approx 0.20(15)$ using Eq. 1 for both dimensions. The RSOS model also helps to understand why the intrinsic width cannot be solely associated to large steps in surface. One can chose a large value of $m$ such that steps of the same order of the ones in the BD are present in RSOS interfaces. However, one still has $(\langle \delta h \rangle)^2 \approx 1$ irrespective of $m$, which introduces a small correction in the scaling.

Figure 5(b) shows the corrections in the third cumulant of heights, $(\langle h^3 \rangle_c - (\Gamma_t)^{3/2} \langle \chi^3 \rangle_c)$, against time, where $(\langle \chi^3 \rangle)_c = 0.049$ was estimated using data given in Refs. [24, 25]. Again, the main correction is a constant that is smaller than $(\langle \delta h \rangle)^2$. The same behavior was found in the fourth order cumulants. So, based on these data we clearly show a strong correlation between finite-time corrections in the KPZ ansatz and $\delta h$ but we could not infer a simple functional dependence.

The interface width against time for $\varepsilon = 1$, discounting or not the $(\langle \delta h \rangle)^2$, is shown in Fig. 5(a). While the original curves do not scale as a power law for the investigated times (solid lines), the subtraction of $(\langle \delta h \rangle)^2$, leads to an excellent accordance with the growth exponent of the KPZ class, even for relatively short times. This analysis is confirmed through the effective growth exponents, obtained from either $W^2$ vs. $t$ or $W^2 - (\langle \delta h \rangle)^2$ vs. $t$, shown in Fig. 5(b). Similar plots are found for GD models. The obtained exponents are shown in Table I and are in remarkable agreement with the best estimates of the KPZ growth exponent $\beta = 0.2415(10)$ in $d = 2 + 1$ [39]. The exponents are, inside errors, the same as those obtained for binned surfaces built with $\varepsilon > 1$.

The effective roughness exponents for ballistic deposition including or not the $(\langle \delta h \rangle)^2$ are shown in Fig. 5(c) and the estimates are given in Table I. The results for BD are again in very good agreement while those for GD models are slightly below the best estimates for KPZ exponents in $d = 2 + 1$, $\alpha = 0.393(3)$ [39], in sharp contrast with a very poor accordance obtained when the intrinsic width is disregarded. Indeed, if we neglect intrinsic width and use only $1024 \leq L \leq 2048$, the exponents are $\alpha \approx 0.33$, $0.32$ and $0.26$ for BD, GD2 and GD4, respectively. The exponents constitute a strong evidence that these models belongs, in fact, to the KPZ universality
class in $d = 2 + 1$.

V. HEIGHT DISTRIBUTION ANALYSIS

The presence of narrow-deep valleys at surface of ballistic deposition model is a hindrance to check the universality of the stochastic quantity $\chi$ in the KPZ ansatz. Thus, height distributions were analyzed using surfaces built with $\varepsilon > 1$. The first and second cumulants of $\chi$ can be obtained analyzing the asymptotic value of $\langle q \rangle$ and $\langle q^2 \rangle_c$, where $q$ is defined by Eq. (5). The results obtained for $\varepsilon = 2l$ are shown in Figs. 7(a) and 7(b). The results are essentially the same for $\varepsilon = 4l$. Since the standard correction $t^{-\beta}$ is present in the first cumulant, an extrapolation in time using the proper power law is imperative for a reliable estimate in finite time simulations [24]. The extrapolated values are shown in Table I and agree, inside errors, with the best estimates known for the KPZ class in $d = 2 + 1$ [26, 27]. For sake of comparison, results for BD model with $\varepsilon = 1$ are also shown in Figs. 7(a) and 7(b). In the former, we observe that the asymptotic estimate of $\langle \chi \rangle$ is almost independent of $\varepsilon$, but the mean value of the correction $\langle \eta \rangle$ is strongly affected by the choice of $\varepsilon$, resulting the values $\langle \eta \rangle = -1.70, 0.94$ and 2.60 for $\varepsilon = 1, 2$ and 4, respectively. In the latter, the curve for $\varepsilon = 1$ apparently converges towards the KPZ value, but still far from it even at the longest analyzed time.

The dimensionless cumulant ratios skewness $S = \langle h^3 \rangle_c / \langle h^2 \rangle_c$ and kurtosis $K = \langle h^4 \rangle_c / \langle h^2 \rangle_c^2$, calculated in the growth regime, are universal and in agreement with the known values for the KPZ class in $d = 2 + 1$ [24, 26], as one can see in Table I. We also analyzed the skewness of the height distributions in the steady state. For $L \geq 1024$ and $\varepsilon \geq 2l$, we found $S$ in agreement, within the uncertainties, with the value $S = 0.26(1)$ estimated from other KPZ models that have small corrections to the scaling [26, 42, 43]. Notice that negative skewed stationary distributions were reported in Ref. [24] using the standard surface definition ($\varepsilon = 1$) and the same sizes considered here. However, $S > 0$ is expected in ballistic models since $\lambda > 0$, which was indeed found in our analysis with $\varepsilon > 1$. Our results show that our method is also able to strongly reduce the finite-size corrections to scaling in the steady state.

The height distributions rescaled according to the KPZ ansatz are shown in Fig. 7(c) for the BD model after a growth time $t = 10^4$ using different binning parameters. This figure also shows the distribution obtained for the RSOS model that has small corrections to the scaling and exhibits excellent agreement with the KPZ ansatz in $d = 2 + 1$ dimensions [24, 26].
standard surface exhibits strong deviations in the left tail associated to fluctuations below the mean height (since \( \lambda > 0 \)), where deep valleys contributions are present. The rescaled distributions for binned surfaces are very close to the RSOS one. Therefore, we show that ballistic growth models in \( d = 2 + 1 \) dimensions obey the KPZ ansatz with the expected universal stochastic term \( \chi \), which would be practically impossible with the currently computer resources if the strong finite-time corrections were not explicitly taken into account in the analysis.

### VI. FINAL DISCUSSIONS AND CONCLUSIONS

In summary, we have showed that the leading corrections to the scaling of ballistic growth models in \( d = 2 + 1 \) arise from the large stochastic fluctuations of the height increments \( \delta h \) during the deposition process, which is expressed in the form of an intrinsic width \( w_i \) in the Family-Vicsek scaling, Eq. (3). We observed that \( w_i^2 \approx \langle (\delta h)^2 \rangle_c \). This intrinsic width also exists in solid-on-solid KPZ models, but in this case \( w_i^2 \approx \langle (\delta h)^2 \rangle_c < 1 \), so that corrections to scaling are negligible. Anyway, since the variance \( \langle (\delta h)^2 \rangle_c \) can be easily computed in numerical simulations, we propose that it should be calculated together with the squared interface width \( W^2 \) and the standard analysis of \( W^2 \) against time \( t \) or substrate size \( L \), used in hundreds of previous works, should be replaced by \( W^2(t) - \langle (\delta h)^2 \rangle_c \) vs. \( t \) or \( L \), respectively. This procedure is able to eliminate the intrinsic width from the scaling analysis and to access the universal scaling exponents with affordable computer resources. We believe that this recipe must be a standard in numerical studies of growing interfaces, helping to uncover the universality of models where strong corrections play an important role.

The large fluctuations in the height increments \( \delta h \) arises mainly from the aggregation at narrow-deep valleys in the surface. Therefore, we also propose a simple method that eliminates these valleys at the surface, where the leading contributions to these fluctuations take place. Basically, the original surface is binned in boxes of lateral size \( \varepsilon \) and only the highest point inside each box is used to construct a coarse-grained surface and to perform statistics. We showed that, for \( \varepsilon \) larger than the typical particle/grain size, the intrinsic width is strongly reduced.

Both methods yield scaling exponents for ballistic growth models in excellent agreement with the KPZ class in \( d = 2 + 1 \). Despite of ballistic growth models present the requisites for KPZ class, to our knowledge, we provide the first convincing observation of KPZ scaling exponents in these models in two-dimensions. Moreover, the power of the binning method is not restricted to scaling exponents. Indeed, the effects of finite-time corrections in the KPZ ansatz become negligible if surfaces are built with \( \varepsilon > l \), while the fundamental non-universal parameters [growth velocity \( v_\infty \) and amplitude of fluctuations \( \Gamma \) in Eq. (2)] as well the universal quantities [growth exponent \( \beta \) and \( \chi \) in Eq. (2)] remain unchanged. So, we showed that the rescaled height distributions for the growth regime of ballistic growth models are the same as those obtained for other KPZ models in \( d = 2 + 1 \). Furthermore, the skewness of height distributions in the steady state also shows a good agreement with the value accepted for the KPZ class in this dimension. Therefore, we show that the ballistic growth models in \( d = 2 + 1 \) belongs to the KPZ universality class. In particular, our result ends a longstanding discussion about the validity of the KPZ class in the classic BD model [28,30].

The binning method can, in principle, be easily applied in the analysis of experimental surfaces. As an example, consider the recent experiment by Yunker et al. [4], where particles from a colloidal suspension were deposited at the edges of evaporating drops. For small anisotropy of the particles, the system was observed to be in the KPZ class. On the other hand, for highly anisotropic particles, exponents different from the KPZ class were found and attributed to the quenched KPZ class. However, this conclusion have been questioned [44, 45]. In Ref. [4], a transient anomalous scaling was proposed as a possible explanation for the deviation from the KPZ regime while in Ref. [45] an advection-diffusion model with strong corrections to the scaling due to a large intrinsic width was used to explain the deviation. In particular, we applied the binning method to advection-diffusion model of Ref. [45] and observed excellent agreement with KPZ exponents (data not shown). Depending on the parameters, both model and experimental surfaces present a large number of narrow-deep valleys. Thus, we believe that our binning method can be very useful to solve controversial issues as the colloidal deposition problem and others related systems.

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