Constraints on Supersymmetric $SO(10)$ GUTs with Sum Rules among Soft Masses

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Abstract

We study phenomenological aspects of supersymmetric $SO(10)$ GUTs with sum rules among soft SUSY breaking parameters. In particular, the sum rule related to the stau mass leads to the constraints from the requirements of successful electroweak breaking and the positivity of stau mass squared. The bottom quark mass is also estimated.

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The unification of force based on the minimal supersymmetric standard model (MSSM) has been hopeful from the data of precision measurements. The supersymmetric SO(10) grand unified theory (SUSY SO(10) GUT) is one of attractive candidates of realistic theory at and above the GUT scale $M_X$ because a simplest unification of quarks and leptons can be realized in each family. After the gauge symmetry breakdown, the remnant exists in the MSSM as specific relations among physical parameters at $M_X$, which are usually used as initial conditions on the analysis by the use of renormalization group equations (RGEs), e.g., $g_3 = g_2 = g_1 \equiv g$ for the gauge couplings of the SM gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ and $Y^\alpha{}_{\alpha\beta} = Y^\beta{}_{\alpha\beta} = Y^\tau{}_{\alpha\beta} \equiv Y^\alpha{}_{\alpha\beta}$ for the Yukawa coupling matrices of up-type quarks, down-type quarks and leptons. Here $\alpha, \beta$ are the family indices. The number of independent soft SUSY breaking parameters is also reduced by the $SO(10)$ symmetry, i.e., the parameters are given as $(m_{16}^{\alpha\beta}, m_H, M, A^{\alpha\beta}, B)$ at $M_X$ for soft squark and slepton masses, soft Higgs masses, gaugino masses, $A$-parameters and $B$-parameter up to the contributions of $SO(10)$ breaking including $D$-term contribution.

The magnitude of each parameter is expected to be determined by some underlying theory or new concept of SUSY-GUT. The supergravity (SUGRA) is regarded as a promising theory, describes the physics beyond SUSY-GUT effectively, and offers an interesting scenario of the origin of soft SUSY breaking parameters. The structure of the SUGRA model is reflected on the pattern of the parameters, e.g., the universal soft SUSY breaking parameters originate in models with a canonical Kähler potential. The analyses based on this type of initial conditions have been intensively carried out.

There is another interesting scenario to control the relations among physical parameters. The parameters can be reduced by the adoption of a new concept ‘coupling reduction’ by the use of RG invariant relations. The assumption in a model is that the Yukawa couplings are expressed RG-invariantly by the gauge coupling $g$ as

$$Y_{ijk} = g \rho_{ijk},$$

where $\rho_{ijk}$ are model-dependent constant independent of $g$ at the tree level. Higher order corrections are systematically calculated. It is called gauge-Yukawa unified (GYU) model. Recently, it is found that the following
relations of soft parameters are RG-invariant\[10, 11\],
\[
\sum m^2 \equiv m_i^2 + m_j^2 + m_k^2 = M^2, \tag{2}
\]
\[A_{ijk} = -M\] \tag{3}
where $Y_{ijk}$, $m_i$ and $A_{ijk}$ are Yukawa couplings, soft scalar masses and $A$-parameters. The indices $i, j, k$ denote the particle species. It has been known that the relations (2) and (3) are derived in other several theories, i.e., the finite field theories [13], a certain class of 4-dimensional models from superstring theory (SST) [14] and a non-minimal SUGRA model with a certain type of structure regarded as a generalization of models from SST [10]. Hence this type of relations can give a hint to high energy physics beyond the MSSM. Therefore it is important to study the phenomenological implications and low-energy consequences from this type of relations [15, 16].

The low-energy constraints from (2) and (3) are studied in Ref. [11] based on finite SUSY SU(5) GUT models and it is shown that eigenvalues of stau masses squared $m_{\tilde{\tau}}^2$ tend to be negative in some parameter region. The sbottom and stop fields always have heavier masses than the lightest stau. Thus the condition of the positivity $m_{\tilde{\tau}}^2 > 0$ as well as the electroweak symmetry breaking conditions constrains the parameter regions severely in this type of models.

In the sector where the above relations (1), (2) and (3) hold on, independent parameters are limited to $g$, $M$ and $m_i$. The radiative corrections of $m_i^2$ are given as functions of $g$ and $M$ because the contribution from Yukawa couplings contains soft scalar masses only as a combination of $\sum m^2$. In this sense, this type of soft SUSY breaking parameters are much more restrictive than a general non-universal one.

Let us compare the excluded regions of soft stau mass $m_{\tilde{\tau}}$ and gaugino mass $M$ at $M_X$ between the models with the universal type of soft masses $(M, m_0)$ inspired by a minimal SUGRA and the models with the relations (2) and (3) at $M_X$, based on the MSSM with large $\tan \beta$. From the requirement of successful electroweak symmetry breakdown, there is the constraint $m_{H_1}^2 - m_{H_2}^2 > M_Z^2$ at the weak scale. Here $m_{H_1}^2$ and $m_{H_2}^2$ are soft Higgs masses squared with the hypercharge $-1/2$ and $1/2$, respectively. Using the analysis of RGEs, we get the relation such that $m_{H_1}^2 - m_{H_2}^2 = c_1 M^2 - c_2 \sum m^2 / 3$ under the condition that $m_{H_1}^2 = m_{H_2}^2$ at $M_X$ [17] and the constant factors $c_1$ and $c_2$.

* We can calculate higher order corrections to RG invariant relations [12].
are of $\mathcal{O}(0.2)$. From these formulae, the region with $m_{\tilde{\tau}}(=m_0) > M$, $(m_{\tilde{\tau}} \gg M_Z)$ is excluded in the universal case. On the other hand, in the case with sum rules, the excluded region is $M < \mathcal{O}(200)$GeV independent of the value of $m_{\tilde{\tau}}$. Another requirement is the positivity of physical stau mass squared. The Yukawa coupling induces to a radiative correction with a negative sign to the mass squared. Thus the stau mass squared can be negative if the magnitude of $\sum m^2$ is sizable in the large tan $\beta$ scenario. This happens easily in the case with sum rules because of the existence of the relation $\sum m^2 = M^2$ and, in this case, the region with $M \gg m_{\tilde{\tau}}$ is excluded. In the universal case, the situation is different because the contribution including the factor $\sum m^2(=3m_0^2)$ becomes tiny when the value of $m_0 = m_{\tilde{\tau}}$ is small, i.e. $M \gg m_0$. Hence it is expected that the excluded regions of $(M,m_{\tilde{\tau}})$ are located at the opposite corners (besides $M < \mathcal{O}(200)$GeV in the latter case) each other from the above two phenomenological requirements.

In this paper, we study generic SUSY $SO(10)$ GUT model with sum rules among soft SUSY breaking parameters and make sure the above estimation of the parameter regions, quantitatively, imposing the conditions of successful electroweak symmetry breaking and the positivity of physical stau mass squared. The method of analysis is almost same as that made in Ref. [11]. SUSY corrections to the bottom quark mass are also estimated.

First we give a brief review on the derivation of the relations (1), (2) and (3) in GYU-models. We assume that parameters $Y_{ijk}$, $m^2_i$ and $A_{ijk}$ are expressed in terms of $g$ and $M$. The relations (1), (2) and (3) are obtained by solving so-called reduction equations perturbatively,

$$\beta_{Y_{ijk}} = \beta_g \frac{dY_{ijk}}{dg}, \quad (4)$$

$$\beta_{m^2_i} = \beta_M \frac{\partial m^2_i}{\partial M} + \beta_{M^t} \frac{\partial m^2_i}{\partial M^t} + \beta_g \frac{\partial m^2_i}{\partial g}, \quad (5)$$

$$\beta_{A_{ijk}} = \beta_M \frac{\partial A_{ijk}}{\partial M} + \beta_{M^t} \frac{\partial A_{ijk}}{\partial M^t} + \beta_g \frac{\partial A_{ijk}}{\partial g} \quad (6)$$

where $\beta_X$ denotes a $\beta$-function of parameter $X$. For the application on an explicit model, see Ref. [20].

† For the study on case with non-universal initial conditions for soft SUSY breaking parameters, see Refs. [18, 19].
Second we give some basic assumptions and relations on our analysis. The first assumption is that all of quarks and leptons in each family belong to one 16-plet under $SO(10)$ and this 16-plet has the Yukawa coupling such as $(16)^2H$ where $H$ is 10-plet including $H_1$ and $H_2$. We ignore the family mixing effects. The second one is that the relations (2) and (3) hold on at $M_X$ in the third family. The third one is that there are no extra contributions on the symmetry breaking $SO(10) \rightarrow G_{SM}$. (Later we relax this assumption by the introduction of $D$-term contribution.) Our initial conditions at $M_X$ are summarized as follows,

\[
g_3 = g_2 = g_1 = g, \quad Y_t = Y_b = Y_\tau = Y,
\]
\[
m^2_\tilde{Q} = m^2_\tilde{t} = m^2_\tilde{b} = m^2_\tilde{\tau}_L = m^2_\tilde{\tau}_R = m^2_{16},
\]
\[
m^2_{H_1} = m^2_{H_2} = m^2_H,
\]
\[
A_t = A_b = A_\tau = -M, \quad 2m^2_{16} + m^2_H = M^2
\]

where $\tilde{Q}$, $\tilde{t}$, $\tilde{b}$, $\tilde{\tau}_L$ and $\tilde{\tau}_R$ denote the $SU(2)_L$ doublet squark of the third family, the singlet stop, the singlet sbottom, the stau in the doublet slepton of the third family and the singlet stau. Here and hereafter we omit the index representing the third family.

Next we parametrize the Yukawa coupling $Y$ using $g$ as $Y = \rho g$. The value of $\rho$ gives an important information on the matter content and its interactions in GYU-models and/or the structure of superpotential in SUGRA. We take the following input parameters,

\[
M_\tau = 1.777\text{GeV}, \quad M_Z = 91.188\text{GeV},
\]
\[
\alpha^{-1}_{EM}(M_Z) = 127.9 + \frac{8}{9\pi} \log \frac{M_t}{M_Z},
\]
\[
\sin^2 \theta_W(M_Z) = 0.2319 - 3.03 \times 10^{-5} T - 8.4 \times 10^{-8} T^2
\]

where $T = M_t/\text{GeV} - 165$. Here $M_\tau$ and $M_t$ are physical tau lepton and top quark masses. The Yukawa unification condition gives the predicted top quark mass from the above experimental value of $M_\tau$ for each value of $\rho$. Fig. 1 shows the predicted value of the physical top quark mass for $k \equiv \rho^2$. Thus we find the realistic region such that $0.7 \leq k \leq 1.4$ to obtain the present experimental value of the top mass, $M_t = 175.6 \pm 5.5\text{GeV}$. For example, the value $k = 1.0$ leads to $M_t = 175$ GeV and $\tan \beta = 53$, while $\tan \beta = 50$ and 55 for $k = 0.7$ and 1.4, respectively.
Similarly we can calculate the bottom quark mass at the tree level for each value of $k$. However, SUSY corrections to the bottom quark mass is sizable in the large tan $\beta$ scenario [21] and that leads to another constraint [17, 18]. Thus, we will estimate the predicted bottom quark mass with SUSY corrections after calculations of the SUSY mass spectrum.

We determine the values of $\mu$ and $B$-parameters by using the following two minimization conditions of the Higgs potential at the weak scale,

$$m_1^2 + m_2^2 = -\frac{2\mu B}{\sin 2\beta},$$

$$m_1^2 - m_2^2 = -\cos 2\beta (M_Z^2 + m_1^2 + m_2^2),$$

where $m_{1,2}^2 = m_{H_1,H_2}^2 + \mu^2$.

Soft SUSY breaking parameters can be constrained from requirements. One of the most important constraints is the realization of electroweak symmetry breaking. To this end, the following condition should be satisfied,

$$m_1^2m_2^2 < |\mu B|^2.$$  \hfill (13)

In addition, the bounded-from-below condition along the $D$-flat direction in the Higgs potential requires

$$m_1^2 + m_2^2 > 2|\mu B|.$$  \hfill (14)
Another important condition is the positivity of physical scalar mass squared \( \tilde{m}^2 \). For example, two stau masses squared, \( m_{\tilde{\tau}_1}^2 \) and \( m_{\tilde{\tau}_2}^2 \), are obtained as eigenvalues of the following (mass)\(^2\) matrix:

\[
\begin{pmatrix}
  m_{\tilde{\tau}_L}^2 + M_Z^2 \cos 2\beta \left( -\frac{1}{2} + \sin^2 \theta_W \right) & vY_\tau (A_\tau \cos \beta - \mu \sin \beta) \\
vY_\tau (A_\tau \cos \beta - \mu \sin \beta) & m_{\tilde{\tau}_R}^2 - M_Z^2 \cos 2\beta \sin^2 \theta_W
\end{pmatrix}
\]

(15)

where \( v^2 \equiv \langle H_2 \rangle^2 + \langle H_1 \rangle^2 \). Here we neglect the SUSY stau mass squared \( M_{\tilde{\tau}}^2 \).

Actually, in Ref. [11], it is shown these conditions constrain severely the parameter space in SU(5) models with large \( \tan \beta \). Because, in the large \( \tan \beta \) scenario, the stau mass squared receives as sizable negative corrections due to the large tau Yukawa coupling as the soft Higgs masses squared \( m_{H_1}^2 \) and \( m_{H_2}^2 \) do. Large values of \( m_{\tilde{\tau}_L}^2 \) and \( m_{\tilde{\tau}_R}^2 \) at \( M_X \) are favorable to avoid \( m_{\tilde{\tau}_{1,2}}^2 < 0 \). Here \( m_{\tilde{\tau}_1} \) denotes the lightest mass of them. For example, it is impossible to satisfy these conditions in explicit SU(5) models in a small value of \( M \). It is shown that the case with a common soft scalar mass, \( m_i^2 = M^2/3 \), is not allowed in some finite SU(5) models.

Now we discuss these constraints in generic SO(10) model. Fig. 2 shows excluded regions by these constraints for \( k = 1.0 \) (\( \tan \beta = 53 \)). In this figure, the dotted region in the left side denotes the region forbidden by the electroweak breaking conditions. On the other hand, the place with asterisks correspond to the region with \( m_{\tilde{\tau}_1}^2 < 0 \) and squares denote the region where the light stau mass squared \( m_{\tilde{\tau}_1}^2 \) is smaller than the lightest neutralino mass squared \( m_{\chi_1^0}^2 \). Note that, in the case with the initial condition \( m_{\tilde{\tau}_L}^2 = m_{\tilde{\tau}_R}^2 \), the lightest stau \( \tilde{\tau}_1 \) almost originates in \( \tilde{\tau}_R \). Because the mass squared \( m_{\tilde{\tau}_L}^2 \) has sizable positive radiative corrections due to SU(2) gauginos, and a half size of negative contribution from \( \tau \) Yukawa coupling compared with that to \( m_{\tilde{\tau}_R}^2 \). In the whole parameter space, sbottom and stop are heavier than the lightest stau.

\(^\dagger\) It would be necessary to calculate the decay rate to the unbounded-from-below direction, e.g. corresponding to \( \hat{m}_{\tilde{\tau}_1}^2 < 0 \) in order to exclude completely such parameter region [22].
Fig. 2: The allowed region by the electroweak breaking condition and the constraint $m^2_\tilde{\tau} > 0$ for $k = 1.0$.

In Fig. 2, we find $m^2_{\tilde{\tau}_1} < 0$ for $m_{16} < 0.4M$ and $m^2_{\tilde{\tau}_1} < m^2_{\chi_1^0}$ for $m_{16} < 0.6M$. In the universal case with $m^2_{16} = m^2_H = M^2/3$, the lightest superpartner (LSP) is the lightest stau. For other values of $k$, we obtain similar results.

In the open region of Fig. 2, the mass of the lightest neutral CP-even Higgs boson is 90 GeV and the lightest neutralino and the other superpartners as well as the other Higgs fields are heavier than 170 GeV.

Next we estimate the bottom quark mass at the weak scale. The present experimental value of the bottom mass includes uncertainties. For example, in Ref. [23], it is shown

$$m_b(M_Z) = 2.67 \pm 0.50 \text{ GeV}. \quad (16)$$

On the other hand, the analysis of the $\Upsilon$ system [24] and the lattice result [25] give $m_b(m_b) = 4.13 \pm 0.06 \text{ GeV}$ and $4.15 \pm 0.20 \text{ GeV}$, respectively, which translate into

$$m_b(M_Z) \approx 2.8 \pm 0.2 \text{ GeV}. \quad (17)$$

\[^5\text{See also Ref. [26].}\]
For example, the case with $k = 0.7$ in our model predicts $m_b(M_Z) = 3.4$ GeV at the tree level. However, the large tan $\beta$ scenario, in general, leads to large SUSY corrections, i.e., $m_b = \lambda_b(H_1)(1 + \Delta_b)$. Dominant contributions to $\Delta_b$ are given \[ \Delta_b = \frac{2\alpha_3}{3\pi}M_\tilde{g}\mu \tan \beta I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_\tilde{g}^2) + \frac{Y_t^2}{16\pi^2}A_t\mu \tan \beta I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2) \] (18)

where $M_\tilde{g}$, $m_{\tilde{b}_i}$ and $m_{\tilde{t}_i}$ are the gluino, sbottom and stop eigenstate masses, respectively. The integral function $I(a, b, c)$ is given by

$$I(a, b, c) = \frac{ab\ln(a/b) + bc\ln(b/c) + ac\ln(c/a)}{(a - b)(b - c)(a - c)}.$$ (19)

The function $I(a, b, c)$ is of order $1/m_{\text{max}}^2$ where $m_{\text{max}}$ is the largest mass in the particles running in the corresponding loop. The first term of R.H.S. in eq.(18) is expected to be sizable. Since the tree level predicted value, $m_b = 3.4$ GeV, is larger than the values given in (16) and (17), SUSY corrections should be negative. That corresponds to $\mu < 0$. This region is also favorable for the constraint due to the $b \rightarrow s\gamma$ decay because this region, $\mu < 0$, can lead to smaller branching ratio in the large tan $\beta$ scenario than the prediction by the SM [27]. Thus we consider only the case with $\mu < 0$. Fig. 3 shows prediction of $m_b(M_Z)$ including the correction $\Delta_b$ for $k = 0.7$. The curves in the figure correspond to $m_b(M_Z) = 2.1$ GeV and 2.6 GeV, which are lower bounds given in (16) and (17), respectively. We have small SUSY corrections $|\Delta_b|$ in two regions, where $M$ is larger than $m_{16}$ and $m_{16}$ is much larger than $M$. These regions for $\mu < 0$ lead to the large bottom quark mass, e.g. $m_b(M_Z) \geq 2.6$ GeV. This behavior is easy to see since $|\Delta_b|$ is suppressed when $M \gg m_b$ or $M \ll m_b$ due to the factor $I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_\tilde{g}^2)$. Also dotted lines in the figure show the boundaries for $m_{\tilde{\tau}_1}^2 \leq 0$ and $m_{\tilde{\tau}_2}^2 \leq m_{\chi_1^0}^2$, which are almost same as those in Fig. 2. The constraint $m_{\tilde{\tau}_1}^2 \leq 0$ excludes the region with $m_b(M_Z) \geq 2.6$ GeV for $m_{16} < M < 1.5$ TeV. Furthermore, the stau is the LSP in the region with $m_b(M_Z) \geq 2.6$ GeV for $1.5\text{TeV} < m_{16} < M < 3$ TeV. To realize $m_b(M_Z) \geq 2.6$ GeV and the neutral LSP, it is needed that $M > 3$ TeV or $M \ll m_{16}$ \[ \text{[4]} \]. The region with $M \ll m_{16}$ can be more constrained

\[ \text{[4]} \] For more precise prediction of the bottom mass, it is necessary to take into account
by the requirement that the LSP should not overclose the universe, i.e., \( \Omega_\chi h^2 \geq 1 \), in the case where sfermions of first and second families are also much heavier than the gauginos [30]. A bigger value of \( k \), e.g. \( k = 1.0 \) or \( 1.4 \), leads to larger SUSY corrections \( |\Delta b| \) and predicts smaller values of the bottom mass for \( \mu < 0 \).

![Graph showing predicted values of \( m_b(M_Z) \) for \( k = 0.7 \).](image)

Fig.3: Predicted values of \( m_b(M_Z) \) for \( k = 0.7 \).

We have discussed the case that squarks and sleptons in the third family have the same soft scalar mass \( m_{16} \) at \( M_X \), which is required by unbroken \( SO(10) \) gauge symmetry. However, if a gauge symmetry breaks reducing its rank like \( SO(10) \to G_{SM} \), additional sizable contributions to soft scalar masses can appear at the breaking scale, which is called \( D \)-term contributions [4, 5]. These \( D \)-term contributions are, in general, proportional to quantum numbers of broken diagonal generators. If we specify the model, one can calculate their magnitudes [31]. Here we study the effect on parameter space keeping them free parameters. The soft scalar masses at \( M_X \) are written as

\[
\begin{align*}
    m_{Q}^2 &= m_{i}^2 = m_{\tilde{\tau}_R}^2 = m_{16}^2 - m_D^2, & m_{\tilde{\tau}_L}^2 &= m_{16}^2 + 3m_D^2, \\
    m_{H_1}^2 &= M^2 - 2m_{16}^2 - 2m_D^2, & m_{H_2}^2 &= M^2 - 2m_{16}^2 + 2m_D^2
\end{align*}
\]

For example, the quasi fixed point of the bottom Yukawa coupling as well as the top coupling is raised due to SUSY threshold effects in most of cases [28].
in the presence of the $D$-term contributions, $Q_{10}m_D^2$, where $Q_{10}$ denotes a broken diagonal charge up to a normalization factor. Note that the above soft masses satisfy the sum rule (2) even with taking into account $D$-term contributions. Because $D$-term contributions are proportional to broken charges and these charges should conserve in allowed couplings.

A positive value of $m_D^2$ is unfavorable for successful electroweak breaking and the constraint $m_{\tilde{\tau}_1}^2 > 0$ since it reduces the values of $m_{H_u}^2$ and $m_{H_d}^2$. Recall that $m_{\tilde{\tau}_R}$ is dominant to $m_{\tilde{\tau}_1}$ with the initial condition $m_{\tilde{\tau}_L}^2 = m_{\tilde{\tau}_R}^2$. For example, in the case with $m_D^2 = M^2/3$, the whole parameter space of $(M, m_{16})$ is ruled out from the electroweak symmetry breaking condition. On the other hand, a negative value of $m_D^2$ is favorable to increase both $m_{H_u}^2$ and $m_{H_d}^2$, while it reduces $m_{\tilde{\tau}_L}^2$. The region excluded by the electroweak breaking condition easily becomes narrow for $m_D^2 < 0$. The constraint due to the lightest stau is also relaxed as $m_D^2$ decreases from $m_D^2 = 0$. However, that reduces $m_{\tilde{\tau}_L}^2$ and below a critical value of $m_D^2$ the value of $m_{\tilde{\tau}_L}^2$ becomes dominant to $m_{\tilde{\tau}_1}^2$. Thus below such a critical value of $m_D^2$, the allowed region becomes narrow due to the constraint for $m_{\tilde{\tau}_1}^2$. Around $m_D^2/M^2 = -0.1$ we obtain the widest allowed region. Fig. 4 shows the case with $m_D^2/M^2 = -0.1$ and $k = 0.7$. In this case, the constraint due to the electroweak breaking is less important, and actually the excluded region by the electroweak breaking is out of the region shown in Fig. 4. Dotted lines correspond to boundaries for $m_{\tilde{\tau}_1}^2 \leq 0$ and $m_{\tilde{\tau}_1}^2 \leq m_\chi^2$. The region with $m_{\tilde{\tau}_1}^2 \leq 0$ becomes narrow. The predicted values of the bottom quark mass is shown as curves corresponding to 2.1 GeV and 2.6 GeV in Fig. 4. For $M > m_{16}$, we have large SUSY corrections $|\Delta_b|$ compared with the $m_D^2 = 0$ case.
If $m_D^2/M^2 < -0.1$, wider region is excluded by the stau mass constraint. On top of that, SUSY corrections $|\Delta_b|$ become large in the region $M > m_{16}$, although the opposite region with $M \ll m_{16}$ leads to slightly small SUSY corrections compared with the $m_D = 0$ case. For example, almost half of the parameter space is forbidden in the case with $m_D^2/M^2 = -0.3$ by the stau mass constraint. Thus we can not obtain $m_b(M_Z) \geq 2.6$ GeV for $M > m_{16}$, while the region with $m_b(M_Z) \geq 2.6$ GeV and $M \ll m_{16}$ becomes slightly wider compared with the $m_D = 0$ case. Therefore small values of $|m_D^2|$ are favorable for realistic models. Our result is expected to agree with that in SUSY $SU(5)$ GUT models with a large $\tan \beta$ because the soft scalar mass spectrum in the presence of $D$-term contribution in SUSY $SO(10)$ GUT models has the same pattern as that in SUSY $SU(5)$ GUT. In fact, our result is consistent with that in Ref. [1].

To summarize, we have considered phenomenological implications of relations (3) and (5) within the framework of SUSY $SO(10)$ GUT. We have investigated constraints due to successful electroweak symmetry breaking and the positivity of the stau mass squared. These forbid the parameter region with small gaugino mass and $m_{16} < 0.4M$. We have also estimated the bottom mass in the allowed region. Further, the allowed regions are more
constrained by other requirements, e.g., the lightest superparticle should be neutral and this particle should not overclose our universe. Also we have taken into account $D$-term contributions to soft scalar masses. Small $D$-term contributions are favorable to realistic models. It is an interesting subject to construct a realistic SUSY $SO(10)$ GUT with sum rules referring to our result.

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\[\text{\textsuperscript{\textcopyright} The gauge-Yukawa unification is studied by the use of explicit SUSY SO(10) gauge-Yukawa models in Ref. \textsuperscript{[3]}.}\]
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