Calculation of Screening Masses in a Chiral Quark Model

Xiangdong Li

Department of Computer System Technology
New York City College of Technology of the City University of New York
Brooklyn, New York 11201

Hu Li, C. M. Shakin and Qing Sun

Department of Physics and Center for Nuclear Theory
Brooklyn College of the City University of New York
Brooklyn, New York 11210

(Dated: May, 2004)

Abstract

We consider a simple model for the coordinate-space vacuum polarization function which is often parametrized in terms of a screening mass. We discuss the circumstances in which the standard result for the screening mass, \( m_{sc} = \pi T \), is obtained. In the model considered here, that result is obtained when the momenta in the relevant vacuum polarization integral are small with respect to the first Matsubara frequency.

PACS numbers: 12.39.Fe, 12.38.Aw, 14.65.Bt

*email address: casbc@cunyvm.cuny.edu
In a number of recent works [1-3] we have calculated various hadronic correlation functions and compared our results to results obtained in lattice simulations of QCD [4-6]. The lattice results for the correlators, \( G(\tau, T) \), may be used to obtain the corresponding spectral functions, \( \sigma(\omega, T) \), by making use of the relation
\[
G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) K(\tau, \omega, T),
\]
where
\[
K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \tag{2}
\]
The procedure to obtain \( \sigma(\omega, T) \) from the knowledge of \( G(\tau, T) \) makes use of the maximum entropy method (MEM) [7-9], since \( G(\tau, T) \) is only known at a limited number of points.

In our studies of meson spectra at \( T = 0 \) and at \( T < T_c \) we have made use of the Nambu–Jona-Lasinio (NJL) model. The Lagrangian of the generalized NJL model we have used in our studies is
\[
\mathcal{L} = \bar{q}(i\gamma^\mu - m^0)q + \frac{G_S}{2} \sum_{i=0}^8 [(\bar{q}\lambda^i q)^2 + (\bar{q}i\gamma_5 \lambda^i q)^2] - \frac{G_V}{2} \sum_{i=0}^8 [(\bar{q}\lambda^i \gamma_\mu q)^2 + (\bar{q}i\gamma_5 \gamma_\mu q)^2] + \frac{G_D}{2} \det[\bar{q}(1 + \lambda_5)q] + \det[\bar{q}(1 - \lambda_5)q] + \mathcal{L}_{conf}. \tag{3}
\]
Here, \( m^0 \) is a current quark mass matrix, \( m^0 = \text{diag}(m^0_u, m^0_d, m^0_s) \). The \( \lambda_i \) are the Gell-Mann (flavor) matrices and \( \lambda^0 = \sqrt{2/3}1 \), with \( 1 \) being the unit matrix. The fourth term is the 't Hooft interaction and \( \mathcal{L}_{conf} \) represents the model of confinement used in our studies of meson properties.

In the study of hadronic current correlators it is important to use a model which respects chiral symmetry, when \( m^0 = 0 \). Therefore, we make use of the Lagrangian of Eq. (3), while neglecting the 't Hooft interaction and \( \mathcal{L}_{conf} \). In order to make contact with the results of lattice simulations we use the model with the number of flavors, \( N_f = 1 \). Therefore, the \( \lambda^i \) matrices in Eq. (3) may be replaced by unity. We then have used
\[
\mathcal{L} = \bar{q}(i\gamma^\mu - m^0)q + \frac{G_S}{2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5 q)^2] - \frac{G_V}{2} [(\bar{q}\gamma_\mu q)^2 + (\bar{q}\gamma_5 \gamma_\mu q)^2]. \tag{4}
\]
in order to calculate the hadronic current correlation functions in earlier work [1-3].

In order to present our results in the simplest form, we consider only the scalar interaction proportional to \((\bar{q}q)^2\). We also extend the definition of \(\sigma(\omega, T)\) of Eq. (1) to include a dependence upon the total moment of the quark and antiquark appearing in the polarization integral. Thus we consider the imaginary part of the correlator, \(\sigma(\omega, \vec{P})\). Since we place \(\vec{P}\) along the \(z\)-axis this quantity may be written as \(\sigma(\omega, 0, 0, P_z)\) in accord with the notation of Ref. [10]. In this work we will present our result for the coordinate-dependent correlator \(C(z)\) which is proportional to the correlator defined in Eq. (1) of Ref. [10],

\[
C(z) = \frac{1}{2} \int_{-\infty}^{\infty} dP_z e^{iP_z z} \int_{0}^{\infty} d\omega \frac{\sigma(\omega, 0, 0, P_z)}{\omega}.
\]

(5)

We may also use the form

\[
C(z) = \frac{1}{4} \int_{-\infty}^{\infty} dP_z e^{iP_z z} \int_{0}^{\infty} dP^2 \frac{\sigma(P^2, 0, 0, P_z)}{P^2}.
\]

(6)

We have made a study of the screening mass in a simple model in order to understand the origin of exponential behavior for the correlator. To that end we make use of Ref. [11]. We consider the Matsubara formalism and note that the quark propagator may be written, with \(\beta = 1/T\),

\[
S_{\beta}(\vec{k}, \omega_n) = \frac{\gamma^0 (2n + 1)\pi/\beta + \overrightarrow{\gamma} \cdot \overrightarrow{k} - M}{(2n + 1)^2\pi^2/\beta^2 + \overrightarrow{k}^2 + M^2}.
\]

(7)

For bosons the vacuum polarization function is given as Eq. (1.51) of Ref. [11],

\[
\Pi(\vec{p}, p^0) = \frac{g^2}{2\beta} \sum_n \frac{d^3k}{(2\pi)^3} \frac{1}{\frac{4n^2\pi^2}{\beta^2} + \overrightarrow{k}^2 + M^2} \cdot \frac{1}{\left(\frac{2n\pi}{\beta} + p^0\right)^2 + (\overrightarrow{k} + \overrightarrow{p})^2 + M^2}.
\]

(8)

We modify Eq. (8) to refer to fermions. In this case the Matsubara frequencies are

\[
\omega_n = \frac{(2n + 1)\pi}{\beta}
\]

(9)

and we have

\[
\Pi(\vec{p}, p^0) = \frac{g^2}{2\beta} \text{Tr} \int \frac{d^3k}{(2\pi)^3} \left[ \left(\frac{\gamma^0 \pi/\beta + \overrightarrow{\gamma} \cdot \overrightarrow{k}}{\frac{\pi^2}{\beta^2} + \overrightarrow{k}^2} \right) \left(\frac{\gamma^0 (p^0 + \pi/\beta) + \overrightarrow{\gamma} \cdot (\overrightarrow{k} + \overrightarrow{p})}{\left(\frac{\pi}{\beta} + p^0\right)^2 + (\overrightarrow{k} + \overrightarrow{p})^2 + M^2} \right) \right].
\]

(10)
FIG. 1: The function $C(z)$ of Eq. (6) is shown for a sharp cutoff of $k_{\text{max}} = 0.1$ GeV. The dotted line represents an exponential fit to the curve using $m_{\text{sc}} = 1.23$ GeV. (We recall that $\pi T$ is equal to 1.27 GeV.)

if we keep only the first term in the sum, where $\omega_0 = \pi/\beta$. As a next step we drop $p^0$, so that we have

$$
\Pi(\vec{p}', 0) = \frac{g^2}{2\beta} \text{Tr} \int \frac{d^3k}{(2\pi)^3} \frac{\left( \gamma^0 \pi/\beta + \vec{\gamma} \cdot \vec{k} \right) \left( \gamma^0 \pi/\beta + \vec{\gamma} \cdot (\vec{k} + \vec{p}') \right)}{\left( \frac{\pi}{\beta} \right)^2 + \vec{k}^2 \left( \frac{\pi}{\beta} \right)^2 + (\vec{k} + \vec{p}')^2}.
$$

(11)

We then take $\vec{p}'$ along the $z$ axis and write $\Pi(p_z) = \Pi(\vec{p}', 0)$. We define

$$
C(z) = \int dp_z e^{ip_z z} \Pi(p_z).
$$

(12)

In our calculation we replace $g^2/2\beta$ by unity and use a sharp cutoff so that $|\vec{k}| < k_{\text{max}}$.

The results of our calculation of $C(z)$ of Eq. (6) are given in Figs. 1 and 2. In Fig. 1 we use $k_{\text{max}} = 0.1$ GeV and in Fig. 2 we put $k_{\text{max}} = 0.4$ GeV. For our calculations, we have $m_{\text{sc}} = \pi T = 1.27$ GeV when $T = 1.5 T_c$ and $T_c = 0.27$ GeV. Thus, the $k_{\text{max}}$ values considered here are less than $m_{\text{sc}}$ and that feature leads to the exponential behavior seen in Figs. 1 and 2. If $k_{\text{max}}$ is made larger than 0.4 GeV we begin to see deviations from exponential behavior for $C(z)$. (Since in our calculations reported in Refs. [1-3], the integrals were regulated with a Gaussian regulator $\exp[-\vec{k}^2/\alpha^2]$ with $\alpha \simeq 4$ GeV, we can see that the $\vec{k}$
FIG. 2: The function $C(z)$ of Eq. (6) is shown for a sharp cutoff of $k_{max} = 0.4$ GeV. The dotted line represents an exponential fit to the curve using $m_{sc} = 0.961$ GeV. (We recall that $\pi T$ is equal to 1.27 GeV.)

values in those calculations are so large as to preclude obtaining exponential behavior for our coordinate-space correlator.)

The goal of this work was to consider a simple quark model for the calculation of a hadronic current correlation function and to determine the conditions under which the coordinate-space correlator is dominated by the screening mass which is given by the first Matsubara frequency. We have found that the standard result is obtained if the quark and antiquark momenta in the vacuum polarization calculation are small compared to that frequency.

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