Optimization design of gear reducer based on compound shape method

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Abstract. Following general design sequence of gear reducers, through establishment of target function, choice of parameters and confirmation of constraint conditions of the two-stage helical cylindrical gear reducer are systematically studied, its optimum mathematical model is set up. The reducer is designed by using the complex method; it reduces the weight of the gear reducer and enhances the design efficiency.

1. Introduction
Reducer is a common part in machinery, and cylindrical gear reducer is widely used in metallurgical mining, machine tools, automotive, material handling and other industries. Traditional reducer design generally through repeated trial and error, check to determine the design program, but most of the cylindrical gear reducer products have been standardized, although can also meet the given conditions of the design program, but the matching of its parameters is not necessarily the optimal, the program is not necessarily the best [1]. With the development of modern new technology and related software, product design quality requirements are increasingly improved, and the design cycle requirements are increasingly shortened, the traditional design methods have become increasingly unable to adapt to the needs of industrial development. Therefore, it is of practical value to optimize the cylindrical gear reducer.

2. Optimization objectives and design principles
2.1. Optimization objective
Gear design optimization can choose a variety of objectives, such as the weight of the lightest, minimum size, maximum strength, etc. This paper takes the transmission device of the conveyor belt in figure 1 as an example, and takes the minimum volume of the reducer as the optimization target, which requires the compact structure and the lightest weight, that is, the minimum center distance of the reducer.

2.2. Design principles
Gear bearing capacity is mainly determined by the contact strength of the tooth surface. This program is still designed according to the contact fatigue strength of tooth surface and checked according to the bending fatigue strength of tooth root [2]. According to the general steps of reducer design: according to the power, speed, total transmission ratio and other known conditions, the design and calculation of the total center distance, and then determine the constraint conditions, the establishment of the objective function, and its optimization.
3. Establish mathematical models

3.1. Parameters
It is known that the speed reducer is shown in the figure, its input power is \( P = 3.2 \text{kw} \), the high-speed shaft speed is \( n_1 = 1440 \text{r/min} \), and the total transmission ratio is \( I = 18.30 \). Gear material pinion selection of 40Cr, quenched and tempered, tooth surface hardness of 240~286HBS; The large gear is made of 45 steel. The hardness of tooth surface is 217~255HBS. The service life is 10 years. It is required that the center distance of the reducer should be minimum when the strength is satisfied.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig1.png}
\caption{Belt conveyor}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig2.png}
\caption{Structure diagram of reducer}
\end{figure}

3.2. Select design variables
As shown in FIG. 2, the center distance of the two-stage spur gear reducer is [3]:

\[ \Sigma a = a_1 + a_2 = 0.5[m_1 z_1 (1 + i_1) + m_2 z_3 (1 + i_2)] \]  \hspace{1cm} (1)

example: \( m_1, m_2 \) is Modulus of high and low speed gears; 
\( i_1, i_2 \) is High speed stage to low speed stage transmission ratio; 
\( z_1, z_3 \) is Number of pinion teeth for high and low speed classes; 
\( a_1, a_2 \) The center distance between the high and low speed stages

The independent parameters of computing center distance are \((m_1, m_2, i_1, z_1, z_3)\) \( i_2 = 18.30/i_1 \), 
Optimization design variables: \( X = [m_1, z_1, m_2, z_3, i_1] \) \( T = [x_1, x_2, x_3, x_4, x_5] \)

3.3. Establish the objective function
Formula (1) is expressed as a design scalar, and its objective function is [4]:

\[ f(x) = 0.5[x_1 x_2 (1 + x_3) + x_3 x_4 (1 + 18.30/x_5)] \] \hspace{1cm} (2)

3.4. Define constraints

3.4.1. Strength condition:
(1) Contact fatigue condition of tooth surface [5]:
\[ 668^2 k_1 T_1 (i_1 + 1) - \varphi_d (m_1 z_1)^3 i_1 [\sigma_{H1}]^2 \leq 0 \]
\[ 668^2 k_2 T_2 (i_2 + 1) - \varphi_d (m_2 z_3)^3 i_2 [\sigma_{H2}]^2 \leq 0 \]

(2) Fatigue condition of tooth root bending [6]:
\[ 2k_1 T_1 Y_{pa1} Y_{pda1} - \varphi_d m_1^3 z_1^3 [\sigma_{Fr}] \leq 0 \]
\[ 2k_1 T_1 Y_{pa2} Y_{pda2} - \varphi_d m_1^3 z_1^3 [\sigma_{Fp}] \leq 0 \]
The meanings of the above symbols are:

- $\sigma_{F_1}$ is the allowable bending stress of gear I (I = 1, 2, 3, 4);
- $[Y_{F_{al}}]$ and $[Y_{F_{al}d}]$ is gear I (I = 1, 2, 3, 4) is gear profile coefficient and stress correction coefficient;
- $k_1$, $k_2$ is load coefficients of high speed stage and low speed stage;
- $\varphi_d$ is the tooth width coefficient; take
- $[\sigma_{F_1}] = [\sigma_{F_3}] = 408.32 \text{MPa}, \quad [\sigma_{F_2}] = [\sigma_{F_4}] = 302.4 \text{MPa}, \quad Y_{F_{a1}} = 2.65, \quad Y_{F_{a2}} = 2.18, \quad Y_{F_{a3}} = 2.52, \quad Y_{F_{a4}} = 2.14, \quad Y_{F_{S1}} = 1.58, \quad Y_{F_{S2}} = 1.79, \quad Y_{F_{S3}} = 1.625, \quad Y_{F_{S4}} = 1.83, \quad T_1 = 21.2 \text{N.m}, \quad T_3 = 19.76 \text{N.m}, \quad i_2 = 18.30/11, \quad k_1 = k_2 = 1.2$

3.4.2. Boundary constraint
According to performance requirements such as transmission, the boundary constraint condition is[7]:

$$2 \leq m_1 \leq 5, \quad 2 \leq m_2 \leq 5, \quad 17 \leq z_1 \leq 30, \quad 17 \leq z_3 \leq 30, \quad 3 \leq i_1 \leq 5$$

3.4.3. Geometric interference constraint:

$$a_1 - E - d_{u2} \geq 0,$$

$$m_i(z_2 + 2) + E - m_2(z_3 + z_4) \leq 0$$

According to the above constraints [8],

$$-1.47 * 10^{-3} x_1^3 x_2 x_3 + x_3 + 1 \leq 0 \quad (3)$$

$$-3.67 * 10^{-4} x_1^3 x_4^3 + x_3 + 18.30 \leq 0 \quad (4)$$

$$x_1^3 x_2^2 \geq 0.87 \quad (5)$$

$$x_1^3 x_2^2 \geq 1.089 \quad (6)$$

$$x_3^3 x_4^2 \geq 1.23 \quad (7)$$

$$x_3^3 x_4^2 \geq 1.02 \quad (8)$$

$$x_1 x_2 x_3^2 + 2x_1 x_5 + 100x_5 + x_3 x_4 x_5 + 18.30 x_3 x_4 \geq 0 \quad (9)$$

$$2 \leq x_1 \leq 5, \quad 2 \leq x_3 \leq 5, \quad 17 \leq x_2 \leq 30, \quad 17 \leq x_4 \leq 30, \quad 3 \leq x_3 \leq 5 \quad (10)$$

Fortran language was used to build the optimization model:

$$\min f(x) = 0.5[x_1 x_2 (1 + x_5) + x_3 x_4 (1 + 18.30/ x_5)]$$

constraint condition:
According to the above constraints and the objective function, the compound shape method was used to optimize the algorithm, and Fortran language was used to write the program code. Finally, the optimal value of the minimum objective function was solved.

4. Analysis of optimization results

The optimal solution is obtained by calculation:
$$\tilde{X} = [X_1, X_2, X_3, X_4, X_5, X_6]^T = [2.006340, 17.13737, 4.158501, 20.43418, 3.641678, 3.579381]^T$$

The constraint function values are respectively:
- $G(1) = 2.006340$
- $G(2) = 17.13737$
- $G(3) = 4.158501$
- $G(4) = 20.43418$
- $G(5) = 3.641678$
- $G(6) = 5 - X(1) = 2.993660$
- $G(7) = 30 - X(2) = 12.86263$
- $G(8) = 5 - X(3) = 0.841499$
- $G(9) = 30 - X(4) = 9.56582$
- $G(10) = 5 - X(5) = 1.358322$

In the actual design of the gear, the modulus of the gear is taken as the standard value, the number of gear teeth should be rounded to integer, and the design parameters are modified as
$$m_1 = 2 \text{mm}; m_2 = 4 \text{mm}; z_1 = 17; z_2 = 20; i_1 = 3.64.$$

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| Item             | $m_1$ | $z_1$ | $m_2$ | $z_2$ | $i_1$ | $a$ |
|------------------|-------|-------|-------|-------|-------|-----|
| Conventional     | 2.5   | 20    | 5     | 25    | 4     | 473 |
| Optimal design   | 2     | 17    | 4     | 20    | 3.64  | 335 |

5. Conclusion

1) When the strength condition is satisfied, the volume of the gear is 29.1% lower than the traditional design volume after the design optimization of the compound shape method, which makes the structure more compact and saves space, so the use prospect is broader.
2) This optimization method is convenient and can flexibly change the design variables. It is usually used in the design of standard cylindrical gear reducer. The program runs fast, saves time, improves efficiency and shortens the design cycle.

3) In this paper, the parameters of the reducer are optimized and some results are obtained. But the actual gear application, also needs to consider such as the processing craft, the fixture and so on many aspects comprehensive factor.

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