Students’ thinking process in solving mathematical problems based on the levels of mathematical ability

A Sanjaya¹, R Johar¹, M Ikhsan¹ and L Khairi¹
¹Syiah Kuala University, Jl. Teuku Nyak Arief Darussalam, Banda Aceh 23111, Indonesia

E-mail: rahmahjohar@fkip.unsyiah.ac.id

Abstract. Problem-solving is one of the main skills in mathematics learning. In Indonesia, problem-solving skill has been embedded in the national education goals. Nevertheless, many students still have difficulties in solving mathematical problems that involve higher order thinking process. This study aimed to analyze students’ thinking process when solving mathematical problems according to the levels of mathematical ability. This study is descriptive research and uses a qualitative approach. The participants of the study were undergraduate students in one of the universities in Aceh province, Indonesia. The data were collected by a triangulation method, involving mathematical problem-solving test and semi-structured interviews. The data were analyzed by using by Miles and Huberman’s method, including the data reduction, data presentation, and conclusion drawing stages. The findings showed that the three groups of students had different thinking processes. While students with high mathematical ability thought conceptually when solving mathematical problems, students with low ability had computational thinking. Meanwhile, students with moderate ability used semi-conceptual thinking during the problem-solving process. The findings of this study suggested that the students’ mathematical ability might lead to different ways of students’ thinking processes when solving mathematical problems and different levels of communication skills. Thus, mathematics educators are supposed to consider this aspect when designing teaching instructions.

1. Introduction
Thinking ability is a crucial element during a learning process to understand and acquire the new knowledge. Thinking is a natural process in an individual’s mind that can help to solve various problems and improve the quality of life. Thinking involves a process of mental representation that is formed through the transformation of information with a complex interaction of mental attributes, such as assessment, abstraction, logic, imagination, and problem solving[1]. Furthermore, a student’s thinking process during a learning process is supposed to be optimized in order to help the student to improve problem-solving skill. As suggested by Gagne, that "the central point of education is to teach people to think, to use their rational powers, to become better problem solvers" [2].
In mathematics education context, problem-solving has become an inseparable part of mathematics learning. Problem-solving skill is considered strongly related to the success of mathematics education so that the integration of problem-solving during the learning process should be necessary [3]. Also, problem-solving is generally regarded as the most critical cognitive activity in the everyday learning
process [4]. In addition, it is believed that problem-solving skill is the only skill that significantly contributes to the development of students’ thinking ability, while other skills are only the parts of problem-solving [5]. Nevertheless, students often find mathematical problems challenging, and they cannot immediately find the steps or procedures to solve the problems [6]. A student might find mathematical questions problematic when the student does not know the requirements to get the answer of the questions, or the student is not challenged to answer the questions [7]. Besides, studies mentioned that many Indonesian students showed the poor understanding of the fraction topics [8] [9]. Whereas, fraction in mathematics is one of the topics that involve mathematical thinking skill and related to many other mathematical issues, such as decimal, the comparison of fractional functions, scale, inheritance, and measurement. These issues indicate that many students might still have the poor quality of mathematical thinking, which might contribute to the low problem-solving skill.

Most of higher education includes the mathematics subject as a compulsory course in their programs, even though the programs are not the mathematics-related programs. The aspect of problem-solving skill has been embedded into the main goals of the Indonesian mathematics curriculum, in most of higher education. The government expects that all university graduands in Indonesia could possess the problem-solving skill through mathematical thinking to meet the demands of the 21st-century skills in the globalization era. However, the studies on the quality of mathematical thinking and problem-solving skill among the university students, who are not from mathematics-related programs, have rarely been conducted. Therefore, there is a need to investigate the university students’ thinking process when solving mathematical problems, which could picture the quality of students problem-solving skill. This study focused on students with different levels of mathematical abilities since the intellectual ability of students could be instrumental in the mastery of facts and concepts [10]. The analysis on the different groups aimed to examine the how the relations between the mathematical conceptual understanding with students’ thinking process when solving mathematical problems. This study aimed to provide a better understanding about to what extent the levels of mathematical abilities could influence students thinking process when they solve mathematical problems. The findings of this study were expected to provide educators or researchers with the insight about the aspects that could contribute to the quality of students' problem-solving skills, which they need to focus on when design teaching instructions or conducting further studies.

2. Method

This study is descriptive research that refers to a study directed to describe the symptoms, facts or events systematically and accurately on the nature of research subjects [11]. Further, the qualitative approach was used in this research to understand the thinking process of university students whose different levels of mathematical abilities when solving mathematical problems. This study involved 49 students of Forestry Faculty of Agriculture Studies Program in one of the universities in Aceh. The participants were categorized into three groups, based on their final scores on mathematics course. The first group was students with high mathematical ability, the second group was students with medium mathematical ability, and the third group was students with low mathematical ability. This classification aimed to analyze the differences of students' thinking process according to different levels of mathematics ability. Further, the problem-solving skill of students was analyzed through the quality of stages when they solve the questions, including when understanding the problem; preparing a problem-solving plan; implementing a problem-solving plan; and re-examining the problem solving [12]. This study focussed on fraction topics, which are related to other materials such as decimal, the comparison of fractional functions, scale, inheritance, and measurement.

The data were collected by the triangulation method, in this case, by using two different research instruments for the same participants to strengthen the validity of the data. The first data collection method was a mathematical test on fractions topics, in which the test questions were developed by the researchers and were validated by mathematics education experts as Figure 1. The semi-structured interviews were conducted to get the further descriptions about students’ thinking process when solving mathematics problems. The interviews were conducted with three students who represented
each level group. The three students were then given initial names based on the represented groups. MKT for the student with high level of mathematical ability, MKS for the student with the medium level of mathematical ability, and MKR for the student with low mathematical ability.

### Question 1
Mr. Said has a garden. Half of the garden area is planted with spinach in raised beds, while the water spinach is planted in half as much as the spinach area. The remaining area of the garden is planted with coffee trees.

a. What is the part of the garden area that is planted with water spinach?

b. What is the total part of the garden area that is planted with vegetables?

c. What is the part of the garden that is planted with coffee trees?

d. How many raised beds can be made to plant spinach if each bed has the area $1/20$ of the garden area?

### Question 2
Mr. Amru has a rectangular garden that has the area $90 \text{ m}^2$ and the length 15 m. To provide the water for the plants in the garden, he made a circular lake with radius 2 m in the middle of the garden as shown in the following picture.

![Diagram of garden and lake](image1.png)

A third of the garden area is planted with coffee trees, while in the opposite side, chocolate trees are planted as much as half of the area planted with coffee. The remaining garden area is planted with potatoes. Find each garden area that is planted with chocolate trees and the potatoes!

### Question 3
If the following picture has the value of $\frac{1}{2}$, draw a $\frac{7}{3}$ picture.

![Diagram of hexagon](image2.png)

Figure 1. The questions of the problem-solving test

The data were then analyzed by Miles and Huberman’s method that included the data reduction, data representation, and conclusion drawing [13]. The data reduction stage refers to summaries of the data, by selecting the main points and focusing on the important information, to be analyzed the common themes of the data. Regarding the interview data, the interview results were transcribed first before being reducing. The data representation stage involves the displays of the data, which includes grouping the data based on the themes and patterns obtained from the previous stage. The last stage is to draw the conclusion based on the groups of the data, in this case, about students’ thinking process when solving mathematical problems.
3. Results and Discussion
The data in this study showed the students with different levels of mathematical ability had different thinking processes. The data in this study showed that the students with high mathematical ability were able to analyze the test questions in details, including what was known and unknown in the questions. Also, they could solve the questions correctly. During the interviews, they were also able to re-explain the procedures they performed when solving the questions. They admitted that they knew the procedures from the concept of fractions they had learned before. When solving mathematics questions, students who are able to understand the problem with their own words correctly and able to see the relationship between the known and unknown information from the question are students who use their conceptual thinking [14]. These students tend to apply the concepts they have learned and are able to reveal the steps they have taken when solving the problem. In other words, the students with high ability in this study were students whose conceptual ability when solving mathematics education.

Meanwhile, students with medium mathematical ability in problem-solving were able to identify what was known from the questions. However, some information was missed in their descriptions. Similarly, when they were trying to figure out what was being asked from the questions, even they could determine the asked points, there was also some missing information from the participants of this moderate ability group. Also, they could not use the detailed procedures of the prior concepts they have learned before when solving the questions. Furthermore, during the interviews, some of them explained every step of their strategies incompletely and incorrectly. This group of students has semi-conceptual thinking when solving mathematical questions, in which they are less able to express with their own words about what is known and unknown of the posed problem [14]. In other words, these students could not fully understand when and how to apply the mathematical concepts they have learned when solving mathematical problems.

With regards to students with low mathematical skill, the students were unable to re-explain the problem with their own language, as well as were unable to identify what was known and unknown in the questions. They expressed the information of the questions incompletely and incorrectly. During the interview, the student with low mathematical skill was unable to justify her answers when answering the questions. Besides, her response to the interviewer sounded like she had guessed the procedures she had used to solve the questions. These students could be categorized in the group of students with computational thinking. A student with computational thinking is usually not able to express mathematical ideas in their own sentence [14]. They tend to forget the concept that has been learned, and not able to explain the steps taken in answering the question.

These findings imply that students’ thinking process could be influenced by the levels of the students’ mathematical ability. A strong understanding of the mathematical concepts may lead to conceptual thinking, which has a strong correlation with the ability to solve mathematical problems, as well as could effectively communicate the procedures how to get the solutions of the mathematical problems. In contrast, students with low mathematical skill were unable to analyze the questions and only apply the computational thinking skill when dealing with mathematics questions. Students with low mathematical skill might find difficult to identify the information of the questions correctly. The students could have misconception due to the poor quality of their conceptual understanding. The lack of conceptual understanding might also have led to a poor quality of students’ skill to communicate mathematical ideas. In other words, besides contributing to the thinking process, mathematical ability, which is indicated by the quality of conceptual understanding, might also influence students’ mathematical communication skill. Considering this importance, there appears a need for teachers to pay attention to the level of students’ conceptual understanding when designing mathematics teaching instructions. Teachers are supposed to build the learning instruments that could foster students’ mathematical thinking and communication, at the same time could respond to the needs of students at all levels. Improving the quality of mathematics tasks could be the starting point for teachers since the mathematical task is considered as important vehicles to build student capacity for mathematical thinking [15]. The factors of the task features and cognitive demands, such as the problem variations, number and kind of representations, the connection to the mathematical concept, and communication
requirements of the task could be the basic considerations for teachers when developing the mathematical tasks [15]. Furthermore, the mathematical tasks could be effective once the tasks are understandable and challenging for students to answer, and cannot easily be solved by routine procedures [16]. Thus, it would need the efforts of teachers to actively explore the insight about the mathematical tasks that are suitable to be posed to their students, which could match the different levels of mathematical abilities and various student needs.

4. Conclusion
This study showed that students with different levels of mathematical ability tend to have different thinking processes. The students with high mathematical ability tended to think conceptually when solving the mathematical problem. They were able to analyze the information and requirements of the questions, including to determine what is known and unknown, to solve the questions correctly, as well as to re-explain the steps to solve the questions verbally. Moreover, the students with moderate mathematical ability tended to have semi-conceptual thinking. They had a middle level of ability to generate the mathematical information from the questions. They sometimes had the misconception when re-explaining their procedures how to solve the questions. Meanwhile, the students with low mathematical ability tended to have computational thinking when solving mathematical knowledge. They were not able to express with their own sentence what was known and asked in the questions. Also, the students with low mathematical ability tended to forget the basic concepts that are prerequisite of the questions. The findings indicated that it is crucial to consider the levels of students’ mathematical ability when teachers develop teaching instructions. Enhancing the quality of the mathematical tasks could be the possible aspect that teachers should focus on.

References

[1] Solso R L 1995 Cognitive Psychology. Boston. Allyn and Bacon.
[2] Gagne R M 1980 The condition of learning (New York: Holt, Rinehart, & Winston) p 85
[3] Shadiq F 2004 Pemecahan Masalah, Penalaran dan Komunikasi Masalah disajikan Instruktur/ Pengembangan Matematika SMA Jenjang Dasar. PPPG Matematika Yogyakarta
[4] Jonannsen D H2000 Toward A Design Theory of Problem Solving ETR&D 48(4) 63-84
[5] Carso J 2007 A ProblemWith Problem Solving: Teaching Thinking Without Teaching Knowledge The Mathematics Educator 17(2) 7-14.
[6] Hudojo H 1979 PengembanganKurikulum Matematika dan Pelaksanaanya di Depan Kelas. Surabaya: Usaha Nasional.
[7] Rizal M 2011Proses Berpikir Siswa Sekolah Dasar Melakukan Estimasi dalam Pemecahan Masalah Berhitung Ditinjau dari Kemampuan Matematika dan Jenis Kelamin. Disertasi. Surabaya: PPs-UniversitasNegeri Surabaya.
[8] Netriwati 2016 Analisis Kesulitan Mahasiswa tentang Pembelajaran Pecahan pada Kitab Faroid. Al-Jabar: Jurnal Pendidikan Matematika 7(2) 21-28.
[9] Sugiatno 2010 Model Sajian Verbal-Model-Abstrak dan Model-Verbal-AbstrakUntuk Materi Penjumlahan Dan Pengurangan Pecahan di Sekolah Menengah Pertama. Pontianak. Jurnal Cakrawala Kependidikan: FKIP-PMIPA, Universitas Tanjungpura.
[10] Suryanto S 2005 Dasar-dasar Pendidikan Anak Usia Dini. Yogyakarta: Hikayat Publishing
[11] Riyanto Y 2008 Paradigma Pembelajaran. Surabaya: Unesa University Press.
[12] Polya G 1973 How to solve It. New Jersey: Princeton University Press.
[13] Miles M B, Huberman A M 1992 Analisis Data Kualitatif. Terjemahan oleh Tjetjep R. Rohidi. Jakarta: Universitas Indonesia (UI-Press).
[14] Maulana 2008 Dasar-dasar Keilmuan Matematika. Subang: Royyan Press

5
[15] Stein M K, Grover B W and Henningsen M 1996 Building student capacity for mathematical thinking and reasoning: an analysis of mathematical tasks used in reform classrooms *American Educational Research Journal*. 33 455-488

[16] Hudojo H 2001 *Pengembangan Kurikulum dan Pengembangan Matematika* Malang: Universitas Negeri Malang