Recently, deep reinforcement learning (RL) methods have been applied successfully to multi-agent scenarios. Typically, these methods rely on a concatenation of agent states to represent the information content required for decentralized decision making. However, concatenation scales poorly to swarm systems with a large number of homogeneous agents as it does not exploit the fundamental properties inherent to these systems: (i) the agents in the swarm are interchangeable and (ii) the exact number of agents in the swarm is irrelevant. Therefore, we propose a new state representation for deep multi-agent RL based on mean embeddings of distributions. We treat the agents as samples of a distribution and use the empirical mean embedding as input for a decentralized policy. We define different feature spaces of the mean embedding using histograms, radial basis functions and a neural network learned end-to-end. We evaluate the representation on two well known problems from the swarm literature (rendezvous and pursuit evasion), in a globally and locally observable setup. For the local setup we furthermore introduce simple communication protocols. Of all approaches, the mean embedding representation using neural network features enables the richest information exchange between neighboring agents facilitating the development of more complex collective strategies.

1 Introduction

In swarm systems, many identical agents interact with each other to achieve a common goal. Swarm systems have been used for a variety of applications, including foraging, formation control, collective manipulation, or the localization of a common 'food' source [Bonabeau et al., 1999]. Each agent in a swarm typically has limited capabilities in terms of sensing and manipulation, such that the considered tasks need to be solved collectively by multiple agents.

A common strategy to obtain control strategies for swarm systems is to apply optimization-based approaches using the model of the agents or a graph abstraction of the swarm [Lin et al., 2004; Jadbabaie et al., 2003]. Optimization-based approaches can compute optimal control policies for tasks that can be well modeled, such as rendezvous or consensus problems [Lin et al., 2007] and formation control [Ranjbar-Sahraei et al., 2012], or to learn pursuit strategies to capture an evader [Zhou et al., 2016]. Yet, these approaches typically use simplified models of the agents and of the task and often rely on unrealistic assumptions, such as operating in a connected graph [Dimarogonas and Kyriakopoulos, 2007] or having full observability of the system state [Zhou et al., 2016]. Rule-based approaches use heuristics inspired by natural swarm systems, such as ants or bees [Handl and Meyer, 2007]. Yet, while the resulting heuristics are typically simple and can lead to complex swarm behavior, the obtained rules are difficult to adapt, even if the underlying task changes only slightly.

Recently, deep reinforcement learning (RL) strategies have become popular to solve multi-agent coordination problems. In RL, tasks are specified indirectly through a cost function, which is typically easier than defining a model of the task directly or a finding a heuristic for the controller. Having defined a cost function, the RL algorithm aims at finding a policy that minimizes the expected cost. Applying deep reinforcement learning within the swarm setting, however, is challenging due to the large number of agents that need to be considered. Instead of one observation about the state of an agent as in single-agent learning, each agent can perceive state information from neighboring agents
and we now have a set of observations for each agent populating the environment. Therefore, the challenges in the swarm setting are the following:

1. High dimensionality due to the set of state information received from many agents.
2. Changing size of the information set, either due to addition or removal of agents, or because the number of neighbors changes over time.

Most current multi-agent deep reinforcement learning methods either concatenate the set of information \cite{Lowe2017} or encode it in a multi-channel image, where the image channels contain different features based on a local view of an agent \cite{Sunehag2017,Zheng2017}. However, both types of methods bare major drawbacks. Since neural network policies assume a fixed input dimensionality, a concatenation of observation sets is unsuitable in the case changing agent numbers. Furthermore, a concatenation disregards the inherent permutation invariance of identical agents in a swarm system and scales poorly to large system sizes. Top-down image based representations alleviate the issue of permutation invariance, however, the information obtained from neighboring agents is of mostly spatial nature. While additional information can be "stacked" into additional image channels, the dimensionality of the representation increases linearly with each added feature. Furthermore, the discretization into pixels has limited accuracy due to quantization errors.

In this paper, we treat the state information perceived from neighboring agents as samples of a random variable and use mean feature embeddings (MFE) \cite{Smola2007} to encode the current distribution of the agents. Each agent gets a local view of this distribution, where the information obtained from the neighbors is encoded in the mean embedding. Due to the sample-based view of the collected state information, we achieve a permutation invariant representation that in addition is invariant to the number of agents within the swarm or the number of perceived neighbors. Mean feature embeddings have so far been mainly used for kernel-based feature representations \cite{Gretton2012}, but can also be applied to histograms or radial basis function networks. To the best of our knowledge, we are the first to use mean embeddings inside a deep neural network architecture where the feature space of the mean embedding as well as the policy are learned end-to-end.

We test our state representation on various rendezvous and pursuit evasion problems using Trust Region Policy Optimization (TRPO) \cite{Schulman2015} as underlying deep RL algorithm. We compare our representation to several deep RL baselines as well as to optimization-based solutions, if available. Herein, we perform our experiments in a global observability setting (i.e., all agents are neighbors) and a local observability setting, where only neighboring agents can be perceived. In the latter setting, we also evaluate different communication protocols \cite{Huttenrauch2018} that allow the agents to transmit additional information about the local agent graph they currently observe. For example, an agent might transmit the number of neighbors it is current neighborhood. Previously, such additional information could not be encoded efficiently due to the poor scalability of the histogram-based approaches. Our results show that agents using our representation can learn faster and obtain policies of higher quality, suggesting that the representation as mean embedding is an effective encoding of the global state configuration for swarm based systems. Moreover, mean embeddings are simple to implement inside existing neural network architectures and can be applied to any deep RL algorithm, which makes the approach applicable in a wide variety of scenarios.

We also provide source code to reproduce the results which can be found here:

https://github.com/LCAS/deep_rl_for_swarms

2 Related Work

The main contribution of this work lies in the development of a compact representation of state information in swarm systems, which can easily be used within arbitrary deep multi-agent reinforcement learning (MARL) settings. In fact, our work is mostly orthogonal to the other research in the field of MARL and the presented ideas can be incorporated into most existing approaches. To provide an overview, we begin with a brief survey of algorithms used in (deep) MARL, revisit the basics of mean embedding theory, and summarize some classic approaches to swarm control for the rendezvous and pursuit evasion task.

2.1 Deep RL

Recently, there has been increasing interest in deep reinforcement learning for swarms and multi-agent systems in general. For example, \cite{Zheng2017} provide a many-agent reinforcement learning platform based on a multi-channel image state representation, which uses Deep Q-Networks (DQN) \cite{Mnih2015} to learn decentralized
control strategies in large grid worlds with discrete actions. Gupta et al. [2017] show a comparison of centralized, concurrent and parameter sharing approaches to cooperative deep MARL, using TRPO [Schulman et al., 2015], DDPG [Lillicrap et al., 2015] and DQN. They evaluate each method on three tasks, one of which is a pursuit task in a grid world using bitmap-like images as state representation. A variant of DDPG for multiple agents in Markov games can be found in Lowe et al. [2017], utilizing a centralized action-value function. The authors evaluate the method on tasks like cooperative communication, navigation and others. The downside of a centralized action-value function is that the input space grows linearly with the number of agents, and hence, their approach scales poorly to large system sizes. A more scalable approach is presented in [Yang et al., 2018]. By employing mean field theory, the interactions within the population of agents are approximated by the interaction of a single agent with the average effect from the overall population, which has the effect that the action-value function input space stays constant. Experiments are conducted on a Gaussian squeeze problem, an Ising model, and a mixed cooperative-competitive battle game. Yet, the paper does not address the state representation problem for swarm systems.

Omidshafiei et al. [2017] investigate hysteretic Q-learning [Matignon et al., 2007] and distillation [Rusu et al., 2015]. They use DRQN [Hausknecht and Stone, 2015] to solve single and multi-task Dec-POMDP problems. Following this work, Palmer et al. [2017] add leniency [Panait et al., 2006] to the hysteretic approach to prevent “relative overgeneralization” of agents. The approach is evaluated on a coordinated multi-agent object transportation problem in a grid world with stochastic rewards.

Sunehag et al. [2017] tackle the “lazy agent” problem in cooperative MARL with a single team reward by training each agent with a learned additive decomposition of a value function based on the team reward. Experiments show an increase in performance on cooperative two-player games in a grid world. Rashid et al. [2018] further develop the idea with the insight that a full factorization of the value function is not necessary. Instead, they introduce a monotonocity constraint on the relationship between the global value function and each local value function. Results are presented on the StarCraft micro management domain.

Finally, Grover et al. [2018] show a framework to model agent behavior as a representation learning problem. They learn an encoder-decoder embedding of agent policies via imitation learning based on interactions and evaluate it on a cooperative particle world [Mordatch and Abbeel, 2018] and a competitive two-agent robo sumo environment [Al-Shedivat et al., 2018].

An application related to our approach can be found in Gebhardt et al. [2018], where the authors use mean embeddings to learn a centralized controller for object manipulation with robot swarms. Here, the key idea is to directly embed the swarm configuration into a reproducing kernel Hilbert space, whereas we use an embedding of the local view of an agent. Furthermore, using kernel-based feature spaces for the mean embedding scales poorly in the number of samples as well as the dimensionality of the embedded information.

2.2 Optimization-Based Approaches for Swarm Systems

To provide a concise summary of the most relevant related work, we concentrate on optimization–based approaches that derive decentralized control strategies for the rendezvous and pursuit-evasion problem considered in this paper. Ji and Egerstedt [2007] derive a control mechanism preserving the connectedness of a group of agents with limited communication abilities for the rendezvous and formation control problem. The method focuses on high-level control with single integrator linear state manipulation and provides no rules for agents that are not part of the agent graph. Similarly, Genaro and Jadabare [2006] present a decentralized algorithm to maximize the connectivity (characterized by an exponential model) of a multi-agent system. The algorithm is based on the minimization of the second smallest eigenvalue of the Laplacian of the proximity graph. An approach providing a decentralized control strategy for the rendezvous problem for nonholonomic agents can be found in Dimarogonas and Kyriakopoulos [2007]. Using tools from nonsmooth Lyapunov theory and graph theory, the stability of the overall system is examined. A control strategy for the pursuit evasion problem with multiple pursuers and single evader, which we will refer to in this paper later, can be found in Zhou et al. [2016]. The authors derive decentralized control policies for the pursuers and the evader based on the minimization of Voronoi partitions. Again, the control mechanism is for high-level linear state manipulation. Furthermore, the method assumes visibility of the evader at all times. A survey on pursuit-evasion in mobile robotics in general can be found in Chung et al. [2011].

3 Background

In this section, we give a short overview of Trust Region Policy Optimization and mean embeddings of distributions.
3.1 Trust Region Policy Optimization

Trust Region Policy Optimization is an algorithm to optimize control policies in single-agent reinforcement learning problems [Schulman et al., 2015]. These problems are formulated as Markov decision processes (MDP), which can be compactly written as a tuple \((S, A, P, R)\). In an MDP, an agent chooses an action \(a \in A\) via some policy \(\pi(a | s)\), based on its current state \(s \in S\), and progresses to state \(s' \in S\) according to the transition dynamics \(P\), i.e., \(s' \sim P(s' | s, a)\). After each step, the agent is assigned a reward \(r = R(s, a)\), provided by the reward function \(R\), which judges the quality of its decision. The goal of the agent is to find a policy that maximizes the cumulative reward achieved over a certain period of time.

In TRPO, the policy is parametrized by a parameter vector \(\theta\) containing the weights and biases of a neural network. In the following, we denote this parameterized policy as \(\pi_\theta\). The reinforcement learning objective is expressed as finding a new policy that maximizes the expected advantage function of the current policy, i.e., \(J_{\pi_{\text{old}}} = E_{s,a}|s'\sim P_{\pi_{\text{old}}}(s'|s,a),R} [A(s,a) - V_{\pi_{\text{old}}}(s)]\),

where \(\hat{A}\) is an advantage estimate of the current policy \(\pi_{\text{old}}\) defined as \(\hat{A}(s,a) = Q_{\pi_{\text{old}}}(s,a) - V_{\pi_{\text{old}}}(s)\). Herein, the state-action value function \(Q_{\pi_{\text{old}}}(s,a)\) is typically estimated via trajectory rollouts, while for the value function \(V_{\pi_{\text{old}}}(s)\) linear or neural network baselines are used that are fitted to the Monte-Carlo returns. The objective is to be maximized subject to a fixed constraint on the Kullback-Leibler (KL) divergence of the policy before and after the parameter update, which ensures that the updates to the policy parameters \(\theta\) are bounded, in order to avoid divergence of the learning process. The overall optimization problem is summarized as

\[
\max_{\theta} \mathbb{E} \left[ \pi_{\theta} \hat{A}(s,a) \right] \\
\text{subject to } \mathbb{E} [D_{KL}(\pi_{\theta||\pi_{\text{old}}})] \leq \delta.
\]

The problem is approximately solved using the conjugate gradient optimizer, after linearizing the objective and quadratizing the constraint.

3.2 Mean Embeddings

Our work is inspired by the concept of embedding distributions into reproducing kernel Hilbert spaces [Smola et al., 2007] from where we borrow the idea of mean embeddings. A probability distribution \(P(X)\) can be represented as an element in a reproducing kernel Hilbert space by its expected feature mapping (i.e., the mean embedding),

\[
\mu_X = \mathbb{E}_{X} [\phi(X)],
\]

where \(\phi(x)\) is a (possibly infinite dimensional) feature mapping. Given a set of observations \(\{x_1, \ldots, x_m\}\), drawn i.i.d. from \(P(X)\), the empirical estimate of the expected feature mapping is given by

\[
\hat{\mu}_X = \frac{1}{m} \sum_{i=1}^{m} \phi(x_i).
\]

Using characteristic kernel functions \(k(x, x') = \langle \phi(x), \phi(x') \rangle\), such as Gaussian RBF or Laplace kernels, mean embeddings can be used in, for example, two-sample tests [Gretton et al., 2012] and independence tests [Gretton et al., 2008]. A characteristic kernel is required to uniquely identify a distribution based on the mean embedding. However, this assumption can be relaxed to using finite feature spaces if we just want to extract relevant information from a distribution such as, in our case, the information needed for the policy of the agents.

4 Deep Reinforcement Learning for Swarms

The reinforcement learning algorithm presented in the last section has been originally designed for single-agent learning. In order to apply this algorithm to the swarm setup, we switch to a different problem domain and show the implications on the learning algorithm. Policies in this context are then optimized in a centralized–learning/decentralized–execution fashion.

4.1 Problem Domain

The problem domain for our swarm system takes the form of a swarm MDP environment [Šošić et al., 2017]. The swarm MDP can be regarded as a special case of a decentralized partially observable Markov decision process (Dec-POMDP) [Bernstein et al., 2002] and is constructed in two steps. First, an agent prototype is defined as a tuple
\( \mathbb{A} = (\mathcal{S}, \mathcal{O}, \mathcal{A}, \pi) \), determining the local properties of an agent in the system. Herein, \( \mathcal{S} \) denotes the set of the agent’s local states, \( \mathcal{O} \) is the set of possible local observations, \( \mathcal{A} \) is the set of actions available to the agent, and \( \pi : \mathcal{O} \times \mathcal{A} \to [0, 1] \) is the agent’s stochastic control policy. Based on this definition, the swarm MDP is defined as \( (N, \mathbb{A}, P, O, R) \), where \( N \) is the number of agents in the system and \( \mathbb{A} \) is the aforementioned agent prototype. The coupling of the agents is specified through a global state transition model \( P : \mathcal{S}^N \times \mathcal{S}^N \times \mathcal{A}^N \to [0, \infty) \) and an observation model \( O : \mathcal{S}^N \times \{1, \ldots, N\} \to \mathcal{O} \), which determines the local observation \( o^i \in \mathcal{O} \) for agent \( i \) at a given swarm state \( s \in \mathcal{S}^N \), i.e., \( o^i = O(s, i) \). Finally, \( R : \mathcal{S}^N \times \mathcal{A}^N \to \mathbb{R} \) is the global reward function, which encodes the cooperative task for the swarm by providing an instantaneous reward feedback \( R(s, a) \) according to the current swarm state \( s \) and the corresponding joint action assignment \( a \in \mathcal{A}^N \) of the agents. The specific state dynamics and observation models considered in this paper are described in Section 5.

The model encodes two important properties of swarm networks, which reflect the characteristics of natural swarm systems. First, all agents in the system are assumed to be identical, and hence, they use the same decentralized policy \( \pi \). This is an immediate consequence of the two-step construction of the model, which implies that all agents share the same internal architecture. Second, the agents are only partially informed about the global system state, as prescribed by the observation model \( O \). Note that both the transition model and the observation model are assumed to be invariant to permutations of the agents, in order to ensure the homogeneity of the system. For details, see [Šošić et al., 2017].

### 4.2 Local Observation Models

The local observation \( o^j \) introduced in the last section is a combination of observations \( o^j_{\text{loc}} \) an agent makes about local properties (like the agent’s current velocity or its distance to a wall) and observations \( O^j \) of other agents. In order to describe the observation model used for the agents, we use an interaction graph representation of the swarm. This graph is given by nodes \( V = \{v_1, v_2, \ldots, v_N\} \) corresponding to the agents in the swarm and an edge set \( E \subset V \times V \), which contains unordered pairs of the form \( \{v_i, v_j\} \) that indicate whether two agents are neighbors. The interaction graph is denoted as \( \mathcal{G} = (V, E) \). If both, the set of nodes and the set of edges, are not changing we speak of static interaction graphs (SIGs), if either of the set undergoes changes, we speak of dynamic interaction graphs (DIGs). The set of neighbors of agent \( i \) in the graph \( \mathcal{G} \) is given by

\[
\mathcal{N}_G(i) = \{ j \mid \{v_i, v_j\} \in E \}.
\]

Within this neighborhood, an agent can sense local state information of the neighbors, for example distance or bearing to each neighbor. We denote the information agent \( i \) receives from agent \( j \) as \( o^{i \rightarrow j} = f(s^i, s^j) \), which is a function of the local states of agent \( i \) and agent \( j \). The observation \( o^{i \rightarrow j} \) is only available for agent \( i \) if \( j \in \mathcal{N}_G(i) \). Hence, the complete state information agent \( i \) receives from all neighbors is given by the set \( O^i = \{ o^{i \rightarrow j} \mid j \in \mathcal{N}_G(i) \} \).

As the observations of other agents are summarized in form of sets \( \{O^j\} \), we require an efficient encoding that can be used as input to neural networks. In particular, this encoding must meet the following two properties:

- The encoding needs to be invariant to the indexing of the agents, respecting the unorderedness of the elements in the observation set. Using such invariance, we can reduce the curse of dimensionality for large system sizes.
- The encoding must be applicable to a varying set sizes, as not all agents might have the same number of neighbors and because the neighborhood can change dynamically at each time step. Even if we are in the globally observable case where each agent can observe the entire system, the encoding should be applicable for different swarm sizes.

### 4.3 Local Communication Models

In addition to perceiving local state information of neighboring agents, the agents can also communicate additional information about the interaction graph \( \mathcal{G} \) [Hüttenrauch et al., 2018]. For example, agent \( j \) can transmit the number of perceived neighbors to agent \( i \). Furthermore, the agents can also perform more complex operations on their local neighborhood graph. For example, they could compute the shortest distance to a target point (such as an evader) that is perceived by at least one agent within their local sub-graph. Hence, by using local communication protocols, observation \( o^{i \rightarrow j} \) can contain information about both, the local states \( s^i \) and \( s^j \) as well as the graph \( \mathcal{G} \), i.e., \( o^{i \rightarrow j} = f(s^i, s^j, \mathcal{G}) \).
4.4 Mean Embeddings as State Representations for Swarms

In the simplest case, the local observation $o_{i,j}$ that agent $i$ receives of agent $j$ is composed of the distance and the bearing angle of agent $i$ to agent $j$. However, $o_{i,j}$ can also contain more complex information, such as relative velocities or orientations. The simplest way of representing the information set $O^i$ is by concatenating the local quantities $\{o_{i,j}\}_j$ into a single observation vector. However, as mentioned before, this representation has various drawbacks as it ignores the permutation invariance inherent to a homogeneous swarm. Furthermore, it grows linearly with the number of agents in the swarm and is, therefore, limited to a fixed amount of neighbors when used in combination with neural network policies.

To resolve these issues, we treat the elements in the information set $O^i$ as samples from a distribution that depends on the current swarm configuration, i.e., $o_{i,j} \sim p_i(\cdot|s)$. We can now use an empirical encoding of this distribution in order to achieve permutation invariance of the elements of $O^i$ as well as flexibility to the size of $O^i$. As highlighted in Section 3.2 a simple way is to use a mean feature embedding, i.e.,

$$\hat{\mu}_{O^i} = \frac{1}{|O^i|} \sum_{o_{i,j} \in O^i} \phi(o_{i,j}),$$

where $\phi$ defines the feature space of the mean embedding. The input dimensionality to the policy is given by the dimensionality of the feature space of the mean embedding and, hence, does not depend on the size of the information set $O^i$ any more. This allows us to use the mean embedding $\hat{\mu}_{O^i}$ as input to a neural network used in deep RL. In the following sections, we describe different feature spaces that can be used for computing the mean embedding.

4.4.1 Neural Network Feature Embeddings

In line with the deep RL paradigm, we propose to use a neural network as feature mapping $\phi^{NN}$, whose parameters are determined by the reinforcement learning algorithm. Using a neural network to define the feature space allows us to handle high dimensional observations, which is not feasible with traditional approaches such as histograms [Hüttenrauch et al., 2018]. In our experiments, a rather shallow architecture with one layer of RELU units already performed very well, but deeper architectures could be used for more complex applications. To the best of our knowledge, we present the first approach for using neural networks to define the feature space of a mean embedding.

4.4.2 Histograms

An alternative feature space are histograms, which can be related to image-like representations. In this approach, we discretize the space of certain features, such as the distance and bearing to other agents, into a fixed number of bins. That way, we can collect information about neighboring agents in the form of a fixed-size multi-dimensional histogram. Herein, the histogram bins define a feature mapping $\phi^{HIST}$ using a one-hot-coding per observed agent. A detailed description of this approach can be found in [Hüttenrauch et al., 2018]. While the approach works well in discrete environments where each cell is only occupied by a single agent, in the continuous case this representation can lead to blurring effects between agents. Moreover, the histogram approach does not scale well with the dimensionality of the feature space.

4.4.3 Radial Basis Functions

A specific problem of the histogram approach is the hard assignment of agents into bins, which results in abrupt changes in the observation space when a neighboring agent moves from one bin to another. A more fine-grained representation can be achieved by using radial basis functions with a fixed amount of basis functions evenly distributed over the observation space. The resulting feature mapping $\phi^{RBF}$ is then defined by the activations of each basis function and can be seen as a “soft-assigned” histogram. However, both representations (histogram and RBF) suffer from the curse of dimensionality, as the number of required basis functions typically increases exponentially with the number of dimensions of the observation vector.

4.5 Modifications to TRPO

Gupta et al. [2017] present a parameter-sharing variant of TRPO that can be used in a multi-agent setup. During the learning phase, the algorithm collects experiences made by all agents and uses them to optimize one policy with a single set of parameters $\theta$. The optimization problem is expressed using advantage values based on all agents’ observations. During execution, however, each agent only has access to its own perception. Hence, the terminology of centralized–learning/ decentralized–execution is chosen.
Since, in the swarm setup, we assume homogeneous agents that are potentially indistinguishable to each other, we omit the agent index introduced in [Gupta et al., 2017].

5 Experimental Results

Our experiments are designed to study the use of mean embeddings in a cooperative swarm setting. The two main aspects are:

1. How do the different mean embeddings (neural networks, histograms and RBF representation) compare when provided with the same state information content?
2. How does the mean embedding using neural networks perform when provided with additional state information while keeping the dimensionality of the feature space constant?

In this section, we first introduce the swarm model used for our experiments and present the results of different evaluations afterwards. During a policy update, a fixed number of $K$ trajectories are sampled, each yielding a return of $G_k = \sum_{t=1}^T r(t)$. The results are presented in terms of the average return, denoted as $\hat{G} = \frac{1}{K} \sum_{k=1}^K G_k$. Videos demonstrating the agents’ behavior in the different tasks can be found here: http://computational-learning.net/deep_rl_for_swarms

5.1 Swarm Models

Our agents are modeled as unicycles, a commonly used agent model in mobile robotics (see, for example, Egerstedt and Hu [2001]), where the control parameters either manipulate the linear and angular velocities $v$ and $\omega$ (single integrator dynamics) or the corresponding accelerations $\dot{v}$ and $\dot{\omega}$ (double integrator dynamics). In the single integrator case, the state of an agent is defined by its location $x = (x, y)$ and orientation $\phi$. In the double integrator case, the agent is additionally characterized by its current velocities. The exact state definition and kinematic models can be found in Appendix A. Note that these agent models are more complex than those typically considered in optimization-based approaches, which mostly assume single integrator dynamics on $x$. Depending on the task, we either opt for a closed state space where the limits act as walls, or a periodic toroidal state space where agents exceeding the boundaries reappear on the opposite side of the space. Either way, the state is bounded by $x_{\text{max}} = y_{\text{max}} = 100$.

We study two different observability scenarios for the agents, i.e., global observability and local observability. In the global observability case, all agents are neighbors, i.e.

$$\mathcal{N}_{G}(i) = \{j \in \{1, \ldots, N\} \mid i \neq j\},$$

which corresponds to a fully connected static interaction graph (SIG). For the local observability case, we use $\Delta$-disk proximity graphs, where edges are formed if the distance $d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ between agents $i$ and $j$ is less than a pre-defined cut-off distance $d_c$ for communication, which corresponds to a dynamic interaction graph (DIG). The neighborhood set of the DIG is then defined as

$$\mathcal{N}_{G}(i) = \{j \in \{1, \ldots, N\} \mid i \neq j, d_{i,j} \leq d_c\}.$$

For a detailed description of all observational features available to the agents in the tasks, see Appendices B and C.

5.2 Rendezvous

In the rendezvous problem, the goal is to minimize the distances between all agents. The reason why we choose this experiment is because a simple optimization-based baseline controller can be defined by the consensus protocol,

$$\dot{x}^i = -\sum_{j \in \mathcal{N}(i)} (x^i - x^j),$$

where $x^i = (x^i, y^i)$ denotes the location of agent $i$. To make the solution compatible to the double integrator agent model, we make use of a PD-controller (see Appendix A for details). The reward function for the problem can be found in Appendix E.1.

We evaluate different observation vectors $o_{i,j}$ which are fed into the policy. To compare the histogram and RBF embedding with the proposed neural network approach, we restrict the basic observation model (see below) to a set of two features: the relative distance $d_{i,j}$ between two agents and the corresponding bearing $\phi_{i,j}$. This restriction
allows for a comparison to the optimization-based consensus protocol, which is based on displacements (an equivalent formulation of distance and bearing). To show that the neural network embeddings can be used with more informative observations, we further introduce an extended set and a communication (comm) set. These sets may include relative orientations $\theta^{i,j}$ or relative velocities $\Delta \nu^{i,j}$ (depending on the agent dynamics), as well as the own neighborhood size and those of the neighbors. An illustration of these quantities can be found in Figure 1.

5.2.1 Global Observability

First, we study the rendezvous problem with 20 agents in the global observability setting with double integrator dynamics to illustrate the algorithms ability to handle complex dynamics. To this end, we compare the performances of policies using histogram, RBF and neural network embeddings on the basic set, as well as neural network embeddings on the extended set. The observations $\phi^{i,j}$ in the basic set comprise the distance $d^{i,j}$ and bearing $\phi^{i,j}$. In the extended set, which is processed only via neural network embeddings, we additionally add neighboring agents’ relative orientations $\theta^{i,j}$ and velocities $\Delta \nu^{i,j}$. The local properties $\phi^{i}_{loc}$ consist of a shortest distance and orientation to the closest boundary, i.e., $d^{i}_{wall}$ and $\phi^{i}_{wall}$. The sets are summarized as follows:

\[
\begin{align*}
\text{Basic} : & \quad \phi^{i,j} = \{d^{i,j}, \phi^{i,j}\} \\
\text{Extended} : & \quad \phi^{i,j} = \{d^{i,j}, \phi^{i,j}, \theta^{i,j}, \Delta \nu^{i,j}\}
\end{align*}
\]

The results are shown in Figure 2a. On first sight, they reveal that all methods eventually find a successful strategy, with the histogram approach showing worst performance. Upon a closer look, it can be seen that the best solutions are found with the neural network embedding, in which case the learning algorithm also converges faster, demonstrating that this form of embedding serves as a suitable representation for deep RL.

Figure 3 show visualizations of the best policies found with each approach. We plot the evolution of the mean distance between all agents over 1000 episodes with equal starting conditions. We also include the performance of the PD-controller defined in Appendix A. It can be seen in Figures 3a and 3c that the policies using the neural network embeddings can decrease the mean distance most quickly and also find the best steady-state solutions of all learning approaches. While the optimization-based solution (PD) can drive the mean distance to zero, a small error remains for the learning-based approaches. However, the learned policies are faster in reducing the distance and therefore show a better average reward. Although the optimization-based approach is guaranteed to find the optimal solution, the approach is build for simpler dynamics and is, therefore, performing suboptimally. Note, that the controller gains for this approach have been tuned manually to maximize performance.

In order to show the generalization abilities of the embeddings, we finally evaluate the obtained policies (except for the concatenation) with 100 agents. The results are displayed in Figures 4a. Again, the neural network embedding of the extended set is quickest in reducing the inter-agent distances, resulting in the best overall performance.

5.2.2 Local Observability

The local observability case is studied with 20 agents and a communication cutoff distance of $d_c = 40$. Due to the increased difficulty of the task, we resort to single integrator dynamics for this experiment. Again, we evaluate the
Figure 2: Learning curves for the rendezvous task with different observation models. The curves show the median of the average return based on the top five trials on a log scale. **Legend:** NN++: neural network mean embedding of comm set, NN+: neural network mean embedding of extended set, NN: neural network embedding of basic set, RBF: radial basis function embedding of basic set, HIST: histogram embedding of basic set, concat: simple concatenation of extended set.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Learning curves for the rendezvous task with different observation models. The curves show the median of the average return based on the top five trials on a log scale. **Legend:** NN++: neural network mean embedding of comm set, NN+: neural network mean embedding of extended set, NN: neural network embedding of basic set, RBF: radial basis function embedding of basic set, HIST: histogram embedding of basic set, concat: simple concatenation of extended set.}
\end{figure}

Figure 3: Visualization of a learned policy for the rendezvous task. The policy is learned and executed by 20 agents using a neural network mean embedding of the extended set.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Visualization of a learned policy for the rendezvous task. The policy is learned and executed by 20 agents using a neural network mean embedding of the extended set.}
\end{figure}

While the received neighborhood sizes \( \{ |N(j)| \} \) are treated as part of agent \( i \)'s local observation of the swarm, the own perceived neighborhood size \( |N(i)| \) is considered as part of the local features \( o_{\text{loc}}^i \). The observation models for the local observability case are thus summarized as:

- **Basic**:
  \[ o^{i,j} = \{ d^{i,j}, \phi^{i,j} \} \]
  \[ o_{\text{loc}}^i = \{ d_{\text{wall}}^i, \phi_{\text{wall}}^i \} \]

- **Extended**:
  \[ o^{i,j} = \{ d^{i,j}, \phi^{i,j}, \theta^{i,j} \} \]
  \[ o_{\text{loc}}^i = \{ d_{\text{wall}}^i, \phi_{\text{wall}}^i \} \]

- **Comm**:
  \[ o^{i,j} = \{ d^{i,j}, \phi^{i,j}, \theta^{i,j}, |N(j)| \} \]
  \[ o_{\text{loc}}^i = \{ d_{\text{wall}}^i, \phi_{\text{wall}}^i, |N(i)| \} \]

For the experiment, we limit our comparison to RBF embeddings of the basic set and neural network embeddings of the extended set and the comm set. The results are illustrated in Figure 2b, which shows that the neural network embeddings lead to a quicker learning progress. Furthermore, by introducing the comm model, better performing
policies can be learned. Compared to the global observability case, however, the learning process exhibits an increased variance caused by the information loss in the reward signal (see Appendix E).

Figure 4c illustrates the performances of the learned policies. Again, the neural network embedding is quicker in reducing the inter-agent distances and converges to better steady-state solutions.

In order to test the communication protocol in the local observability case, we evaluate the learned policies with 10 agents. The results are displayed in Figure 4d. As expected, the performance decreases due to the lower chance of agents seeing each other but we can still see the benefit of the communication protocol.

5.3 Pursuit Evasion with a Single Evader

Our implementation of the pursuit evasion scenario is based on Zhou et al. (2016), from which we adapt the evader strategy. The strategy is based on Voronoi regions, which the pursuers try to minimize and the evader tries to maximize. While the original paper considers a closed world, we change the world type from closed to periodic, thereby making it impossible to simply trap the evader in a corner. In order to encourage a higher level of coordination between the agents, we allow the evader to have a higher maximum velocity than the pursuers, set to twice the pursuers’ maximum velocity. An episode ends after the evader is caught. The reward function for the problem can be found in Appendix E.2.

5.3.1 Global Observability

Again, we study the global observability case with ten agents. Since the pursuit of an evader is a more challenging task already, we reduce the movement complexity to single integrator dynamics. The basic and extended set are equal.
Figure 5: Learning curves for the pursuit evasion task with different observation models. The curves show the median of the average return based on the top five trials on a log scale. **Legend:** NN++: neural network mean embedding of comm set, NN+: neural network mean embedding of extended set, RBF: radial basis function embedding of basic set, HIST: histogram embedding of basic set, concat: concatenation of extended set

Figure 6: Visualization of a learned policy for the pursuit evasion task. The policy is learned and executed by 10 agents using a neural network mean embedding of the extended set. Pursuers are illustrated in blue, the evader is highlighted in red.

Figure 5a reveals that successful strategies can be obtained with all methods. However, this time, a clear advantage of policies using the neural network mean embedding with the extended set can be seen, both in terms of behavior quality but also in the number of samples necessary to find the solution.

Figure 6 illustrates the strategy a policy using the neural network mean embedding of the extended set exerts. After random initialization, the agents first spread in a way that there is no possibility for the evader to increase its Voronoi region anymore, thereby keeping it almost on the same spot. Once this configuration is reached, they surround the evader in a circular pattern and start to reduce the distance until one pursuer successfully catches the evader.

To investigate the performance of each method’s best policy (learned with 10 agents), we estimate the corresponding probabilities that the evader is caught within a certain time frame. For the sake of completeness, we also include the
method proposed in [Zhou et al., 2016], which was originally not designed for a setup with a faster evader, though. The results are plotted in Figure 7 as the fraction of episodes ending at the respective time instant, averaged over 1000 episodes. The plot in Figure 7b shows that the evader may be caught using all presented methods if run for enough time steps. As already indicated by the learning curves, using the neural network mean embedding representation ensures the quickest capture among all methods. The additional information in the extended set further increases performance.

Next, we examine the generalization capabilities of the learned policies, this time on scenarios with 5 (Figure 7a), 20 (Figure 7c) and 50 (Figure 7d) agents. Increasing the amount of agents leads to quicker capture for all methods with the best strategy still applied by the agents with the neural network embedding of the extended set. Interestingly, when using less agents than in the original setup, all methods struggle to capture the evader. After inspection of the behavior, we found that the strategy of establishing a circle around the evader leads to too large distances between individual agents through which the evader can escape.

5.3.2 Local Observability

The local observability case is studied with 20 agents and a communication cutoff distance of $d_c = 40$. Additionally, we introduce an observation radius $d_o = 20$ within which the pursuers can observe the distance and bearing to the evader. We reuse the basic and extended set from last section and modify the comm set to include the shortest path information of agents $j$ in the neighborhood of agent $i$ to the evader. That way, each agent $i$ can compute a shortest path to the evader over a graph of connected agents, such that the path $P = (v^1, v^2, \ldots, v^M)$ minimizes the sum
Figure 8: Performance comparison of the best policies in the pursuit evasion task with local observability. The curves show the probability that the evader is caught after $t$ time steps. All policies are learned and executed by 20 agents. Results are averaged over 1000 episodes with identical starting conditions.

$$d_{\text{min}}^{i,e} = \sum_{m=1}^{M-1} d^{m,m+1}$$ where $v^i$ is agent $i$ and $v^M$ is the evader. The observation sets are given as:

**Basic**: $o^{i,j} = \{d^{i,j}, \phi^{i,j}\}$  \hspace{1cm} $o^\text{loc} = \{d^{i,\text{wall}}, \phi^{i,\text{wall}}, d^{i,e}, \phi^{i,e}\}$

**Extended**: $o^{i,j} = \{d^{i,j}, \phi^{i,j}, \theta^{i,j}\}$  \hspace{1cm} $o^\text{loc} = \{d^{i,\text{wall}}, \phi^{i,\text{wall}}, d^{i,e}, \phi^{i,e}, d^{i,e}_{\text{min}}\}$

**Comm**: $o^{i,j} = \{d^{i,j}, \phi^{i,j}, \theta^{i,j}, d^{j,e}_{\text{min}}\}$  \hspace{1cm} $o^\text{loc} = \{d^{i,\text{wall}}, \phi^{i,\text{wall}}, d^{i,e}, \phi^{i,e}, d^{i,e}_{\text{min}}\}$

Note that in this case the distance and bearing to an evader are only available if $d^{i,e} \leq d_o$. Furthermore, the correct shortest path is only available if an agent and the evader are in the same sub-graph, otherwise, a pre-defined value is fed into the policy.

Again, we limit the comparison for the local observability case to the more promising methods of neural network and RBF mean embeddings. The results in Figure 5b show that the performance gain of the neural network mean embeddings is even more noticeable than in the global observability case, with a clear advantage in the presence of the local communication protocols. The inspection of the termination probabilities in Figure 8 confirms that the neural network mean embedding results in a significantly improved policy.

### 5.4 Pursuit Evasion with Multiple Evaders

Lastly, we study a pursuit evasion scenario with multiple evaders, i.e., we assume that agent $i$ receives observation samples $\{o^i,e\}$ from several evaders, which are processed using a second mean embedding to account for the variable set size. Where in the previous experiment the agents had precise information about the evader in terms of distance and bearing, they now have to extract this information from the respective embedding. An additional level of difficulty results from the fact that the reward function no longer provides any guidance in terms of the distances to the evaders since it only counts the number of evaders caught in each time step. The reward function for the problem can be found in Appendix E.3.

We study a scenario with 50 pursuers and five evaders using the global observability setup in Section 5.3.1, except that we respawn caught evaders to a new random location instead of terminating the episode. The observation sets, containing the same type of information but arranged according to the inputs of the neural networks, are designed as follows:

**Basic**: $o^{i,j} = \{d^{i,j}, \phi^{i,j}\}$  \hspace{1cm} $o^i,e = \{d^{i,e}, \phi^{i,e}\}$  \hspace{1cm} $o^i_{\text{loc}} = \{d^{i,\text{wall}}, \phi^{i,\text{wall}}\}$

**Extended**: $o^{i,j} = \{d^{i,j}, \phi^{i,j}, \theta^{i,j}\}$  \hspace{1cm} $o^i,e = \{d^{i,e}, \phi^{i,e}\}$  \hspace{1cm} $o^i_{\text{loc}} = \{d^{i,\text{wall}}, \phi^{i,\text{wall}}\}$

Figure 9 shows the learning curves for policies with neural network and RBF mean embeddings and for the concatenation approach. The return directly relates to the number of evaders caught during an episode. Again, the neural network mean embedding performs significantly better than the RBF embedding. The curves clearly indicate the positive influence of the additional information present in the extended set. With this amount of agents, the dimensionality of the concatenation has increased to a point where learning is not feasible anymore.
6 Conclusion

In this paper, we proposed the use of mean feature embeddings as state information to overcome two major problems in deep reinforcement learning for swarms: The high and possibly changing dimensionality of information perceived by each agent. We introduced three different approaches to realize such a mean embedding. Two manually designed approaches using histograms and radial basis functions, and an end–to–end learned neural network feature representation. We evaluated the approaches on different variations of the rendezvous and pursuit evasion problem and compared their performance to classic approaches found in literature, as well as a naive concatenation of each agent’s information set. Our evaluation revealed that learning an embedding end–to–end using neural network features scales well with increasing numbers of agents and leads to better performing policies and, often, quicker convergence compared to all other approaches. As expected, the naive concatenation approach fails once the number of agents increases too much.

A Agent Kinematics

In the single integrator case, the state of an agent is given by $s^i = [x^i, y^i, \phi^i] \in \mathcal{S} = \{ [x, y, \phi] \in \mathbb{R}^3 : 0 \leq x \leq x_{\text{max}}, \ 0 \leq y \leq y_{\text{max}}, \ 0 \leq \phi < 2\pi \}$, and the linear velocity $v$ and angular velocity $\omega$ can be directly controlled by the agent. The kinematic model is given by

\begin{align*}
\dot{x} &= v \cos \phi \\
\dot{y} &= v \sin \phi \\
\dot{\phi} &= \omega.
\end{align*}

In the double integrator case, the state is given by $s^d = [x^d, y^d, \phi^d, v^d, \omega^d] \in \mathcal{S} = \{ [x, y, \phi, v, \omega] \in \mathbb{R}^5 : 0 \leq x \leq x_{\text{max}}, \ 0 \leq y \leq y_{\text{max}}, \ 0 \leq \phi < 2\pi, \ |v| \leq v_{\text{max}}, \ |\omega| \leq \omega_{\text{max}} \}$ and the agent can only indirectly change its velocity by acceleration. With the control inputs $a_v$ and $a_\omega$, the model is then given by

\begin{align*}
\dot{v} &= a_v \\
\dot{\omega} &= a_\omega \\
\dot{x} &= v \cos \phi \\
\dot{y} &= v \sin \phi \\
\dot{\phi} &= \omega.
\end{align*}

For the experiments, we use finite differences to model the system in discrete time.
B Observation Model

Irrespective of the task, an agent $i$ can sense the following properties about other agents $j \in \mathcal{N}(i)$ within its neighborhood:

- $d^{i,j}$ distance to neighboring agents
- $\phi^{i,j} = \arctan\left(\frac{y^j - y^i}{x^j - x^i}\right) - \phi^i$ bearing to neighboring agents
- $\theta^{i,j} = \arctan\left(\frac{y^j - y^i}{x^j - x^i}\right) - \phi^j$ relative orientation
- $\Delta v^{i,j} = v^i \begin{bmatrix} \cos \phi^i, \sin \phi^i \end{bmatrix} - v^j \begin{bmatrix} \cos \phi^j, \sin \phi^j \end{bmatrix}$ relative velocity

Furthermore, each agent has access to the following local properties:

- $d_{\text{wall}}^i = \min(x^i - x_{\text{min}}, y^i - y_{\text{min}}, x_{\text{max}} - x^i, y_{\text{max}} - y^i)$ distance to closest wall
- $\phi_{\text{wall}}^i = \phi_{\text{wall}}^i - \phi^i$ orientation to closest wall
- $v^i, \omega^i$ own velocity

where $\phi_{\text{wall}}^i$ denotes the absolute bearing of agent $i$ to the closest wall segment.

C Task Specific Communication Protocols

In the rendezvous task, agent $i$ additionally can sense information about neighborhood sizes:

$$|\mathcal{N}(i)|$$ own neighborhood size
$$|\mathcal{N}(j)| : j \in \mathcal{N}(i)$$ neighborhood size of neighbor $j$

In pursuit evasion, we additionally have one or multiple evaders with states $s^e = [x^e, y^e] \in \{[x, y] \in \mathbb{R}^2 : 0 \leq x \leq x_{\text{max}}, 0 \leq y \leq y_{\text{max}}\}$. Agents can sense the distance and bearing to an evader, given that the evader is within an observation distance $d_o$:

- $d^{i,e} = \sqrt{(x^i - x^e)^2 + (y^i - y^e)^2}$ if $d^{i,e} \leq d_o$ distance to evader
- $\phi^{i,e} = \arctan\left(\frac{y^e - y^i}{x^e - x^i}\right) - \phi^i$ if $d^{i,e} \leq d_o$ bearing to evader

Furthermore, we assume that each agent $i$ can compute a shortest path to the evader over a graph of connected agents, such that the path $P = (v^1, v^2, \ldots, v^M)$ minimizes the sum $\sum_{m=1}^{M-1} d^{m,m+1}$ where $v^1$ is agent $i$ and $v^M$ is the evader.

D Controller for Double Integrator Dynamics

We use a simple PD-controller to transform the consensus protocol with high-level direct state manipulation to the unicycle model with double integrator dynamics. It is given by

$$a_v = K_1(v_d - v)$$
$$a_\omega = K_2(\phi_d - \phi) + D_2(\omega_d - \omega)$$
$$v_d = \|\dot{x}\|$$
$$\phi_d = \arctan\left(\frac{\dot{y}}{\dot{x}}\right)$$
$$\omega_d = 0,$$

where the parameters $K_1$, $K_2$ and $D_2$ are tuned manually to give good performance on the problem.
E  Reward Functions

E.1 Rendezvous

The reward function is defined in terms of the inter-agent distances \( \{d_{i,j}\} \) as

\[
R(s, a) = \alpha \sum_{i=1}^{N} \sum_{j=i+1}^{N} \min(d_{i,j}, d_c) + \beta \|a\|,
\]

where in the global observability case we set the cut-off distance \( d_c = \max(x_{\text{max}}, y_{\text{max}}) \) to the maximum possible inter-agent distance in the respective environment. The factor \( \alpha = -\left( \frac{N(N-1)}{2} \right)^{-1} \) serves as a reward normalization factor and \( \beta = -1 \times 10^{-3} \) controls how strongly high action outputs of the policy are penalized.

E.2 Pursuit Evasion

For the case of a single evader, the pursuit evasion objective may be expressed in terms of the distance to the closest pursuer. More specifically, the reward function is given as

\[
R(s, a) = -\frac{1}{d_o} \min(d_{\text{min}}, d_o),
\]

where \( d_{\text{min}} = \min(d_{1,e}, \ldots, d_{N,e}) \). For the global observability case, we set \( d_o \) to the maximum possible distance of \( d_{i,e} \).

E.3 Pursuit Evasion with Multiple Evaders

In the case of multiple evaders, we use a sparser reward function that simply counts how many evaders are caught per time step, with no additional guidance of inter-agent distances. An evader \( e \) is assumed to be caught if the closest pursuer’s distance \( d_{\text{min},e} = \min(d_{1,e}, \ldots, d_{N,e}) \) is closer than a threshold distance \( d_t = 3 \). The reward function is given by

\[
R(s, a) = \sum_{e=1}^{E} 1_{[0,d_t]}(d_{\text{min},e}),
\]

where \( E \) is the number of evaders and

\[
1_{[a,b]}(x) = \begin{cases} 
1 & \text{if } x \in [a, b] \\
0 & \text{else}
\end{cases}
\]

is the indicator function.

F  Policy Architectures

This section briefly summarizes the chosen policy architectures. Illustrations can be found in Figure 10.

F.1 Neural Network Embedding Policy

In the experiments, the neural network mean feature embedding for agent \( i \), i.e.

\[
\phi^{\text{NN}}(O^i) = \frac{1}{|O^i|} \sum_{o^{i,j} \in O^i} \phi(o^{i,j}),
\]

is realized as the empirical mean of the outputs of a single layer feed-forward neural network,

\[
\phi(o_{i,j}) = h(W o_{i,j} + b),
\]

with 64 neurons and a RELU non-linearity \( h \). Figure 10a shows a block diagram of the proposed model.

F.2 Histogram Embedding Policy

The histogram embedding is achieved with a two-dimensional histogram over the distance and bearing space to other agents. We use eight evenly spaced bins for each feature, resulting in a 64 dimensional feature vector.
Figure 10: Illustration of (a) the proposed policy network, (b) the network architecture used for the RBF and histogram representation, and (c) for the simple concatenation. The numbers inside the boxes denote the dimensionalities of the hidden layers. The color coding in (a) highlights which layers share the same weights. The plus sign denotes the mean of feature activations.

F.3 RBF Embedding Policy

The RBF embedding is given by a vector \( \psi_{RBF}(O^i) = [\psi_1(O^i), \ldots, \psi_M(O^i)] \) of \( M^2 \) contributions from \( M = 8 \) radial basis functions whose means are evenly distributed in the distance and bearing space. With \( o^{i,j} = [d^{i,j}, \phi^{i,j}] \), \( \mu_m = [\mu_d, \mu_\phi] \), and \( \sigma = [\sigma_d, \sigma_\phi] \) its components are given by

\[
\psi_m(O^i) = \sum_{o^{i,j} \in O^i} \rho_m(o^{i,j}),
\]

where we choose

\[
\rho_m(o^{i,j}) = \exp \left( -\frac{1}{2} \left[ \frac{(d^{i,j} - \mu_d)^2}{\sigma_d^2} + \frac{(\phi^{i,j} - \mu_\phi)^2}{\sigma_\phi^2} \right] \right).
\]

The policy network structure used for both, the histogram and the RBF representations, is illustrated in Figure 10b.

F.4 Concatenation Policy

For the concatenation method, we first concatenate agent \( i \)'s neighborhood observations contained in the set \( O^i \) and process them with one hidden layer of 64 neurons and a RELU non-linearity. The resulting feature vector is then concatenated with the local properties \( o_{loc}^i \) and fed into a second layer of same size. Finally, the output of the second layer is mapped to the action. The corresponding policy network structure can be seen in Figure 10c.

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