Coupled heat transfers in a three cavity hollow block subjected to an incident solar flux: Effect of heat conductivity

M Najjaoui¹, T Ait-Taleb¹, A Abdelbaki², Z Zrikem² and H Chaib¹

¹ ERME, Department of Physics- Chemistry, Polydisciplinary Faculty of Ouarzazate, Ibn Zohr University, P.O. Box 638, Ouarzazate, Morocco.
E-mail: taittaleb@gmail.com, mourad.najjaoui@gmail.com
² LMFE, Department of Physics, Cadi Ayyad University, Faculty of Sciences Semlalia, B.P. 2390, Marrakech, Morocco.

Abstract. Hollow blocks or bricks are widely used due to the good thermal insulation, lightweight, and acoustic insulation etc. In this paper, we present a numerical study of heat transfers coupled by natural convection and radiation conduction through a hollow block with three cavities used for the construction of ceilings of buildings. This hollow block is subjected to an incident solar flux and exchanging heat with the air ambient. The governing equations of the mathematical model used for the simulation are discretized by the finite volume method and solved by the SIMPLE algorithm. The main objective of this simulation is to study, in steady state, the effect of the thermal conductivity of the solid partitions on heat flux through this hollow block. The results are presented in terms of streamlines, the isotherms, temperature profiles on the actives surfaces and the obtained global heat flux.

Keywords: Heat transfer; Numerical simulation; Steady state; Thermal conductivity, Hollow block.

1. Introduction

Great and inefficient energy consumption intensifies the energy crisis and causes various environment issues. Buildings energy consumption (residential and tertiary) usually takes up a significant percentage of the total social energy consumption. And more than half of this part energy is eventually consumed for heating and cooling buildings. A significant portion of energy in buildings is consumed due to heat loss or heat gain in the building envelope. Indeed, heat losses through the building envelope are localized at all levels since a building behaves like a container pierced with different types of holes: doors and windows, external walls, ceilings, ventilation, including the envelope of roofing represents 25 to 30% of these thermal losses ... Therefore, the energy efficiency in this sector is very important and there is a great potential to do this. With this in mind, it is important to take this in consideration at the time of design.

Utilization of energy-efficient building envelope may be a good means for improving the building energy efficiency. In recent years, hollow block/brick, a new kind of prefabricated building construction material, is becoming more and more popular. This block has good thermal insulation, lightweight, acoustic insulation etc. (Zhang et al. [1], Gao et al. [2]). But, in general, the heat transfer in the hollow block as the main unit of this envelope is complex and coupled by conduction in solid
partitions, natural convection inside the cavities and radiation between the internal surfaces of the cavities.

Consequently, adequate study of the thermal behaviour of such structures must take into account the simultaneous existence of the three modes of heat transfer. Many investigations concerned about the hollow block/brick have been presented. Some of them focus on the thermal properties of this system mainly by using theoretical (Bouchair [3]), numerical (Del Coz Diaz et al. [4]) or experimental studies (Kus et al. [5], Ahmad et al. [6]).

Most of the work carried out in this sense has been devoted to the study of the effects of conduction and/or radiation on natural convection in a differentially heated rectangular block. A part of these works studied the effect of conduction in solid partitions on natural convection [7-8]. Whereas the other part studied the effect of radiation exchange between cavity surfaces on natural convection [9-10]. Recently, Ait-Taleb et al. [11-13] presented a series of numerical studies that take into account the three modes of heat transfer in alveolar blocks heated from below or above, but submitted to the imposed temperatures excitation. These studies play an important role in simulating the thermal performance and optimizing the configurations of the hollow block.

In this paper, we propose a numerical simulation, in steady state, the effect of thermal conductivity on heat transfer in a hollow block with three horizontal rectangular cavities with thick walls subjected to a solar flux imposed, and exchanging heat with ambient air at constant temperatures with surface exchanges coefficients \( h_e \) and \( h_i \). The main objective of this simulation is to examine the effect of the thermal conductivity of the solid partitions on the nature of flow and on the overall thermal behaviour of this hollow block.

2. Mathematical formulation

2.1. Studied configuration and governing equations

Figure 1 shows the schematic of a hollow block with one row of air cells. This hollow block has three rectangular cavities surrounded by solid walls. Each cavity has a width \( l \) and height \( h \) having vertical conductive walls of thickness \( e_x \) and other horizontal ones of thickness \( e_y \). The top horizontal face is subjected to an incident solar flux \( (G) \) and exchanges heat with the ambient air at the temperature \( T_e \) with a surface exchange coefficient \( h_e \). The lower surface exchanges heat with the indoor air of a room at temperature \( T_i \) and with a surface exchange coefficient \( h_i \), while the vertical surfaces are considered adiabatic.

![Figure 1. Schematics of hollow blocks: (a) hollow block ceilings; (b) hollow block; (c) cross-section of the hollow block](image-url)

The heat transfer of the perforations (cavities) of the hollow tile as shown in Figure 1 is complex and coupled with heat conduction, convection and radiation. Therefore, non-linear characteristics exist within the heat transfer process of these perforations. In order to model this problem we adopt some
simplifying hypothesis such as the inner surfaces, in contact with the fluid, are assumed to be gray, diffuse emitters and reflectors of radiation with an emissivity $\varepsilon$. The flow is conceived to be laminar, two-dimensional and incompressible with negligible viscous dissipation. The fluid is assumed to be no participating to radiation and the heat transfer is two-dimensional. All the thermophysical properties of the solid and the fluid are assumed constant except the density in the buoyancy term which is assumed to vary linearly with temperature (Boussinesq approximation), such a variation gives rise to the buoyancy forces.

Taking into account the above-mentioned assumptions, the dimensionless equations representing the conservation of the mass, the momentum and the conservation of the energy inside cavities full of air are:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  
\[ (1) \]

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]  
\[ (2) \]

\[
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra \cdot Pr \cdot \theta
\]  
\[ (3) \]

\[
\frac{\partial \theta_s}{\partial \tau} + U \frac{\partial \theta_s}{\partial X} + V \frac{\partial \theta_s}{\partial Y} = \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2}
\]  
\[ (4) \]

Where $U$, $V$, $\tau$, $\theta$ and $P$ are the dimensionless parameters associated to the primitive quantities $u$, $v$, $t$, $T$ and $p$, respectively.

The dimensionless variables are obtained from the following expressions:

\[
U = \frac{u}{H}, \quad V = \frac{v}{H}, \quad \theta = \frac{T - T_i}{T_e - T_i}, \quad P = \frac{p + \rho_0 g y}{\rho_0 (\alpha / H)^2} \quad \text{and} \quad \tau = \frac{t}{H^2}
\]  
\[ (5) \]

The Rayleigh number (Ra) and the Prandtl number (Pr) are given successively by:

\[
Ra = \frac{g \beta \Delta \theta H^3}{\alpha \nu_a^2} \quad \text{and} \quad Pr = \frac{\nu_a}{\alpha_a}
\]  
\[ (6) \]

The dimensionless equation for heat conduction in solid walls is:

\[
\frac{\partial \theta_s}{\partial \tau} = \frac{\alpha_s}{\alpha_a} \left( \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} \right)
\]  
\[ (7) \]

The thermal boundary conditions imposed are:

Adiabaticity conditions :

\[
\left( \frac{\partial \theta_s}{\partial X} \right)_{X=0} = \left( \frac{\partial \theta_s}{\partial X} \right)_{X=1} = 0 \quad 0 \leq Y \leq 1
\]  
\[ (8) \]

On the external horizontal side:

\[
\frac{H}{k_a} \alpha G + \frac{h_e H}{k_i} (0e - 0se) = \frac{d \theta_s}{d Y}
\]  
\[ (9) \]

On the inner horizontal side:

\[
\frac{h_i H}{k_s} (0si - 0i) = \frac{d \theta_s}{d Y}
\]  
\[ (10) \]

To these conditions are added those of continuity of the temperature and the heat flux to the fluid-solid interfaces, which given by:

\[
\theta_s (X, Y) = \theta_s^f (X, Y)
\]  
\[ (11) \]
\[-\frac{\partial \theta_s}{\partial \eta} = -N_k \frac{\partial \theta_s}{\partial \eta} + N_r Q_r\]  

(12)

Where \(\eta\) is the dimensionless coordinate normal to the considered interface. The expression for calculating the dimensionless net radiative flux is given by the radiosity method of Siegel and Howell [14]:

\[Q_r(r_k) = \varepsilon_k \left(1 - \frac{1}{G}\right)^4 \left(\theta_k(r_k) + \frac{1}{G - 1}\right)^4 - \varepsilon_k \sum_{j=1}^{4} \int_{S_j}^{4} \int_{J_j(r_j)}^{4} dF_s dS_k - dS_j\]  

(13)

The dimensionless average heat flux across the structure is given by:

\[Q_a = -\frac{H}{L} \int_{Y=0}^{L} d\theta_s \]  

(14)

The dimensional average heat flux in (W.m\(^{-2}\)) is calculated by this equation:

\[Q = (Q_a \times k \times \Delta T) / H\]  

(15)

2.2. Method of solution

Governing equations are discretised using the finite differences method based on the control volumes approach (Patankar [15]) with a power law scheme and are solved by the SIMPLE algorithm. The resulting system of algebraic equations is solved by the Tri-Diagonal-Matrix-Algorithm. To accelerate the convergence of solutions, the governing equations are solved in their instationary form. To realize a compromise between accuracy and computation time, a study on the effects of both grid spacing and time step on the simulation results has been conducted. This study leads to the non-uniform grid size of 80 × 40. This number of mesh point used is sufficient to modeling accurately the heat transfer and fluid flow inside the hollow tile. The dimensionless time used is \(10^{-6}\). The convergence criterion is \(10^{-4}\) which is based on the relative change in the variables \(U, V, P,\) and \(Q_r\) at different nodes of the calculation domain:

\[\left|\frac{f^{n+1}(i,j) - f^n(i,j)}{f^n(i,j)}\right| \leq 10^{-4}\]  

(16)

Where \(f^n(i,j)\) is the variable \(f = (F, V, P, \theta, Q_r)\) value at node \((i,j)\) calculated at iteration \(n\).

2.3. Parameters of simulation

The results presented in this study are obtained for a hollow block representing a type of hollow bricks. This block is formed by three rectangular cavities each one of the geometrical dimensions such as the width \(l = 13\) cm and the height \(h = 7\) cm (type of 50 × 11) and having vertical conductive walls of thickness \(e_x = 2.5\) cm. And other horizontal ones of thickness \(e_y = 2\) cm. The thermal conductivity \(k_s\) varies between 0.5 and 1W/mK, and its thermal diffusivity is \(\alpha_s = 4.25 \times 10^{-7}\) m\(^2\)/s. The cavities are full by air of thermal conductivity \(k_a\) and thermal diffusivity \(\alpha_a\) are equal to 0.0262W/mK and 1.57\(\times 10^{-5}\) m\(^2\)/s respectively. The Prandtl number is \(Pr = 0.71\). The emissivity of the internal sides of the cavities is equal 0.9. In accordance to real conditions, the average surface exchange coefficients were set at the values, \(h_e = 17\) W/m\(^2\)/°C and \(h_i = 8.3\) W/m\(^2\)/°C. The indoor air temperature is maintained at temperature \(T_i = 20^\circ C\), while the ambient temperature is set to \(T_e = 40^\circ C\). The incident solar flux \(G\) on the outside face of the structure is fixed at the value 1000W/m\(^2\).

3. Results and discussion
3.1. Streamlines and isotherms

Figure 2. illustrates the streamlines (at the left) and the isotherms (at the right) obtained for the considered three thermal conductivity values $k_s=0.5$; $k_s=0.8$ and $k_s=1$W/m.K. We are in a situation of heating from the top, due to the positions of the hot and cold facades. As expected, the flow within the three cavities is characterized by the existence of four small cells rotating in opposite directions. These small recirculation cells (at the streamlines) are due to low intensity flows ($\Psi_{\text{max}} = 0.86$) resulting from the temperature gradients created by conduction and by radiation at the corners of the various cavities. Concerning the effect of the thermal conductivity $k_s$, examination of the current lines shows that this does not affect the nature of the flow. Indeed, this nature of flow is similar in the various cavities of the block and so for the three values of $k_s$ considered: The air particles in direct contact with the upper surface of the various cavities, heated by conduction, are the lightest, and consequently their movements, remain limited in the vicinity of the active walls. Temperature field analysis shows that heat transfer is mainly through conduction and radiation. Indeed, the isotherms are almost parallel lines in the cavities (horizontally stratified temperature fields) with small distortions of the isotherms at the level of the vertical solid walls which are due to the difference between the thermal conductivities of the two solid and fluid environments.

![Streamlines and isotherms](image)

Figure 2. Streamlines (left) and isotherms (right) obtained for $G=1000$W/m$^2$ and different values of conductivities: (a) $k_s=0.5$W/m.K; (b) $k_s=0.8$W/m.K and (c) $k_s=1$W/m.K.

3.2. Temperature Profiles

Figure 3 shows the temperature profiles of $T_s(X)$ on both high and low horizontal surfaces of the studied hollow bloc. Through this figure, it is shown that the conductive thermal transfer is more important in solid partitions than via fluid environment. Furthermore, we notice that the temperature of the higher wall decreases noticeably as the conductivity $k_s$ increases. This situation is reversed in the higher horizontal side where the wall temperature is lower in the environment, and increases as $k_s$ increases but in more significant way. All this can be explained by the radiation transfer who is less important in the lower side than in the higher. The increase/(decrease) of temperature in the higher side/(lower) as $k_s$ increases can be explained simply by the increase of heat conduction in accordance with Fourier’s law, in the isolation characteristics of materiel that increases when $k_s$ decreases more precisely. We also notice that in the higher side, the temperature is low in the lower corner and increases in the function of $X$. It reaches its maximum inside every cavity as we decrease and move the higher corner. This behaviour matches the statements made while we analyse the isotherms, in which we noticed small distortions of isotherms at the level of solid vertical walls. This is due to the differences between thermal conductivities in both solid and fluid environments.
3.3. Global heat transfer

With respect to the total heat transfer in the vertical direction, figure 4 for $G = 1000\text{W/m}^2$, represents the distribution as a function of the width $X$, of the overall heat flux passing through the cold ($Y = 0$) and hot ($Y = 1$) hollow block, respectively for the values of the thermal conductivity $k_s = 0.5, 0.8$ and $1.0 \text{W/m.K}$.

From this figure it can be seen, as expected, that as the thermal conductivity $k_s$ increases there is a considerable increase in the heat flux, especially when $k_s$ increases from $0.5$ to $1.0 \text{W/m.K}$. Indeed, the average value of the total flux increases from value $161.56\text{W/m}^2$ for $k_s = 0.5 \text{W/m.K}$ to $191.32 \text{W/m}^2$ for $k_s = 1.0\text{W/m.K}$. This corresponds to an increase of about $18\%$. It is also noted that the heat flow is maximum at the solid partitions separating the three cavities due to the thermal conductivity of the solid $k_s$. Inside the cavities there is a moderation of the heat transfer, marking the absence of convection (because it is a heating from above) whatever the value of $k_s$. The comparison of the distributions of the total flux obtained on the two hot and cold sides of the block show that the range of
variation of the overall flux on the hot side (upper face) is more extensive than that on the cold side
(lower face) regardless of the value of the conductivity $k_s$ considered. However, and as expected, the
average heat flow is retained.

4. Conclusion
The coupled heat transfers by natural convection, conduction and radiation in a hollow block with
three cavities which is used in the construction the ceilings of buildings, subjected to an incident solar
flux and exchanging heat with the air ambient, have been studied numerically. The simulation results
show that heat transfer through this hollow block heated from above by the incident solar flux and
exchanging heat with the air ambient makes mainly by conduction and radiation. The intensity of flow
within the different cavities is very weak confirm that there is a very low convective heat transfer. It
has been found that thermal transfer through this system depends highly on the thermal conductivity
of solid partitions $k_s$. This parameter changes the structure of flow inside the cavity and contributes in the
global heat transfer growth in a significant way.

References
[1] Zhang Y, et al. 2014 Impact factors analysis on the thermal performance of hollow block wall, Energy and Buildings, 75, 330-341.
[2] Gao Y, et al. 2004 Reduced linear state model of hollow blocks walls, validation using hot box measurements, Energy and Buildings, 36 (11), 1107-1115.
[3] Bouchair A 2008 Steady state theoretical model of fired clay hollow bricks for enhanced external wall thermal insulation, Building and Environment, 43(10), 1603-1618.
[4] Del Coz Díaz J.J. et al. 2007 Analysis and optimization of the heat-insulating light concrete hollow brick walls design by the finite element method, Applied Thermal Engineering, 27, 1445-1456.
[5] Kus H, et al. 2013 Hot box measurements of pumice aggregate concrete hollow block walls, Construction and Building Materials, 38, 837-845.
[6] Ahmad A, et al. 2014 In situ measurement of thermal transmittance and thermal resistance of hollow reinforced precast concrete walls, Energy and Buildings, 84, 132-141.
[7] Kim D.M, R Viskanta R 1984 Study of the effects of wall conductance on natural convection in differentiallyoriented square cavities, J. Fluid. Mech. 144, 153-176,
[8] Zhang W, Zhang C.H, Xi G 2010, Conjugate conduction-natural convection in an enclosure with time-periodicsidewalltemperature and inclination, Int. Journal of Heat and Fluid Flow, 32, 52–64,
[9] Balaji C, Venkateshan S.P 1993 Interaction of surface radiation with free convection in a square cavity, Int. Journal of Heat and Fluid Flow, 14, 260-267,
[10] Wang H, Xin S, et Le Quéré P 2006 Etude numérique du couplage de la convection naturelle avec le rayonnement de surfaces en cavité carrée remplie d’air, C. R. Mécanique, 334, 48-57,
[11] Ait-Taleb T, Abdelbaki A, Zrikem Z 2008 Numerical simulation of coupled heat transfers by conduction, natural convection and radiation in hollow structures heated from below or above. Int. J. of Thermal Sciences 47, (4), 378-387,
[12] Ait-Taleb T, Abdelbaki A, Zrikem Z 2008 Transfer function coefficients for time varying coupled heat transfers. Application to hollow concrete brick”, Building Simulation Journal, 1, 303-310,
[13] Ait-Taleb T, Abdelbaki A, and Zrikem Z 2014, “Simulation of coupled heat transfers in a hollow tile with two vertical and three horizontal uniform rectangular cavities heated from below or above”. Energy and Buildings, 84, 628–632.
[14] Siegel R, Howell JR 1981 Thermal radiation heat transfer. 2nd ed. New York: McGraw-Hill.
[15] Patankar S.V 1980 Numerical Heat Transfer and Fluid Flow, Hemisphere, New York,