A Two-DOF Active-Passive Hybrid Vibration Isolator Based on Multi-Line Spectrum Adaptive Control

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Abstract: In order to effectively control the low-frequency vibration of ship machinery, based on the improved multi-line spectrum adaptive control algorithm, a two-degree-of-freedom (two-DOF) active-passive hybrid vibration isolator composed of an electromagnetic actuator, rubber spring, and the hydraulic device is proposed. The dynamic model of the two-DOF vibration isolation system is established and the main control force demand of the vibration isolation system at different damping forces is analyzed. By introducing the improved wavelet packet decomposition algorithm with the Hartley block least mean square algorithm to the filter-x least mean square (FxLMS) algorithm, an improved wavelet packet Hartley block filter-x least mean square (IWPHB-FxLMS) algorithm is established. The experimental results show that the IWPHB-FxLMS algorithm has better control performance. Compared with the traditional FxLMS algorithm, the IWPHB-FxLMS control algorithm improves the convergence speed by 91.7% and the line spectrum power spectrum attenuation by 58.1%. The active-passive hybrid vibration isolator is based on multi-line spectrum adaptive control and can achieve good control effects under the excitation of multi-frequency line spectrum and constant frequency line spectrum.

Keywords: low-frequency vibration; active-passive hybrid vibration isolator; adaptive control; IWPHB-FxLMS algorithm

1. Introduction

The mechanical system noise plays a major role of radiation noise when ships sail at low-mid speed [1]. Such noise is generally with low-frequency band, which propagates farther in water. Currently, low-frequency line spectrum is the main characteristic signal for modern passive sonar to detect, track, and identify targets in underwater acoustic countermeasures. It is harmful to the acoustic stealth performance of ships [2]. Therefore, the control of low-frequency line spectrum is currently a popular research field to improve ships’ acoustic stealth performance.

Traditional passive vibration isolation devices can suppress the level of broadband vibration, but can hardly eliminate the low-frequency vibration. Instead, active vibration isolation technology [3–8] is considered to be an effective means of attenuating low-frequency vibration. Meanwhile, active-passive hybrid vibration isolator can isolate the broadband vibration and the low-frequency line spectrum at the same time. Such hybrid vibration isolator basically consists of a passive vibration isolation element, an actuator, and a control system [9]. It has two types of structures: parallel and series. Although the active-passive hybrid vibration isolator can effectively isolate the low-frequency line spectrum, it has not been popularized on ships. The reasons are that the structure of vibration isolator, the bearing capacity of actuator, and the limitation of control algorithms need to be improved [10]. Currently, the research interests of active-passive hybrid vibration isolator are mainly in three aspects: new material [10–13], structural design [14–20], and control
algorithm [21–24]. Jason et al. [11] presented a new actuator employing magnetostrictive material Terfenol-D. A novel geometric arrangement of a Stewart platform, called the “cubic configuration” is developed. A six degree-of-freedom active vibration isolation system with the “cubic configuration” of the Stewart platform has been implemented, and about 30 dB of vibration attenuation is achieved in real-time experiments. Anderson et al. [12] designed an active vibration isolation device based on the piezoelectric actuator, which can provide ultrastable work surfaces. This type of actuator is a contact type, which is difficult to be used for ship machinery and equipment with heavy weight and severe vibration. Qiang et al. [13] proposed a type of metamaterial vibration isolator with a honeycomb structure and a multi-fidelity sequential optimization approach based on feasible region analysis. This work investigated the application of the MF surrogate model in the design of the metamaterial vibration isolator with a honeycomb structure. He et al. [9] developed the magnetic levitation active-passive composite airbag isolator, which can realize the dual control of broadband and low-frequency line spectrum. The test results show that the broadband passive vibration isolation effect can reach more than 30 dB, and the low-frequency line spectrum control effect can reach 17–30 dB. Jiang et al. [15] designed a six-degree-of-freedom (6-DOF) active vibration isolation system (AVIS) driven by the voice coil motors (VCMs). To promote the control performance, the composite nonlinear feedback (CNF) controller was designed based on the identification parameters. The CNF control law was introduced into the control of the AVIS for the first time and the performance was verified by the simulations and experiments. Chi et al. [16] designed a whole-star active-passive hybrid vibration isolator based on an electromagnetic actuator, and the low-frequency line spectrum vibration isolation effect on the Stewart platform can reach 12.8–24 dB. Zhang et al. [17] proposed an active-passive hybrid isolator based on a novel magnetic negative stiffness isolator, which significantly reduces the isolation initial frequency and suppresses the resonant response of the system. N. Alujević et al. [18] proposed an active-passive hybrid vibration isolator based on an inert. Zhang et al. [19] developed an active-passive integrated vibration isolation truss structure, and the strategy of active-passive integrated vibration control on the truss enveloping control moment gyroscopes (CMGs) was presented, and their characteristics of the time domain and frequency domain were analyzed. Xu et al. [20] designed an active-passive hybrid-driven prosthesis and a real-time control algorithm based on the feedforward compensation angle outer loop, which can improve the controllability of the active and passive ankle joint prostheses. Daley et al. [21] proposed a control algorithm based on the three-dimensional vibration acceleration of the base, which was applied to the active vibration isolation system of the diesel engine, effectively attenuating the low-frequency vibration response at the base of the diesel engine. An et al. [22] proposed an adaptive control algorithm based on a radial basis function network, but the model must be a high-order time-domain model of a physical system, which is difficult to be used for real-time control of vibration isolation system with long impact response time. Lee et al. [23] proposed an H-infinite and proportional–integral–derivative (PID) controller, which was applied to its 6-DOF vibration control system using six-velocity sensors to measure system vibrations. The transmissibility of the presented hybrid isolation system is in the range of −10 to −48 dB at its passive resonance frequency. Chi et al. [24] proposed a control solution based on the LADRC strategy, which is independent of the mathematical model, and the system parameters are tuned according to the operating bandwidth. With the properly tuned LADRC parameters, the vibration isolation system can attenuate the transmission of vibration for about 30 dB in low frequency.

In this paper, an active-passive hybrid vibration isolator is proposed. Such isolator is based on multi-line spectrum adaptive control algorithm. It consists of an electromagnetic actuator, a rubber spring, and a hydraulic device with good loading capacity, compact structure, and good linearity. An adaptive fast convergence algorithm under multi-line spectral excitation is studied using the active-passive hybrid vibration isolator as an actuator, which breaks through the bottlenecks of slow convergence speed, large time delay,
and poor stability. Four of such hybrid isolators are installed in an active-passive isolation mounting loaded with four counterweights, and the vibration-isolation performance of this system is tested through a two-DOF vibration isolation experiment.

The rest of the paper is organized as follows. In Section 2, theoretical analysis of the dynamic characteristics of the two-DOF hybrid vibration isolation system is presented. In Section 3, the structure of a hybrid isolator of the two-DOF isolation system is designed and the dynamic characteristics of the hybrid isolator are analyzed. In Section 4, the establishment process of a multi-line spectrum adaptive control algorithm is introduced in detail. Finally, in Section 5, a prototype of the two-DOF active-passive hybrid vibration isolation system is established, and the vibration-isolation performance of this system is tested.

2. Theoretical Analysis of the Two-DOF Vibration Isolator

A multi-line spectrum adaptive active controller is introduced into the two-DOF vibration isolation system. The system consists of the hybrid vibration isolator, a passive vibration isolator, a power amplifier, and the control system. The structure and parameter design of the hybrid vibration isolator should be based on the dynamic characteristics of the two-DOF vibration isolation system itself. Therefore, on the premise of not considering the active control force, the hybrid isolator is simplified as a rigid damping structure (the effective mass of the hybrid isolator is small, and the elastic element can be regarded as a massless spring when it vibrates at a low frequency). The mechanism of the two-DOF vibration isolation system is shown in Figure 1.

![Figure 1. Mechanism of the two-DOF active-passive hybrid vibration isolation system.](image)

The dynamic equation of the hybrid vibration isolation system can be described as:

$$
\begin{align*}
M_r \ddot{x}_1 + C_r (\dot{x}_1 - \dot{x}_2) + K_r (x_1 - x_2) &= f_0 - f_r \\
M_p \ddot{x}_2 - C_r (\dot{x}_1 - \dot{x}_2) + C_p \ddot{x}_2 - K_r (x_1 - x_2) + K_p x_2 &= f_r
\end{align*}
$$

(1)

where $K_r$ and $C_r$ are the stiffness and damping of hybrid isolator respectively, $K_p$ and $C_p$ denote the stiffness and damping of passive isolator respectively, $M_r$ is the machinery’s weight, $M_p$ is the middle-device’s weight, $x_1$ and $x_2$ represent the vibration displacement of machinery and middle-device respectively, $f_0$ is the exciting force, and $f_r(t)$ refers to the active control force. Assuming $\omega$ is the angular frequency of the sinusoidal vibration, $f_0$ can be expressed as $f_0(t) = f_0 e^{j \omega t}$.

When the active control is not considered, the forced vibration caused by harmonic excitation is steady-state vibration, $x_1 = X_1 e^{j \omega t}, x_2 = X_2 e^{j \omega t}$. By analyzing the dynamic characteristics of the vibration isolation system, we can get:

$$
-\omega^2 \begin{bmatrix} M_r & 0 \\ 0 & M_p \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + j \omega \begin{bmatrix} C_r & -C_r \\ -C_r & C_r + C_p \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} K_r & -K_r \\ K_r & K_r + K_p \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} f_0 \\ 0 \end{bmatrix}
$$

(2)
Solving Equation (2) gives:

\[
\begin{align*}
X_1(\omega) &= \frac{-M_0\omega^2 + j\omega(C_r + C_c) + K_r + K_c}{\omega^2 - j\omega(\xi_c + \omega C + j\omega d + \varepsilon)} f_0 \\
X_2(\omega) &= \frac{j\omega C_r + K_r}{\omega^2 - j\omega(\xi_c + \omega C + j\omega d + \varepsilon)} f_0 \\
\end{align*}
\]  

(3)

With \( a = M_r M_p, b = M_r C_r + M_r C_p + M_p C_r, c = M_r K_r + M_r K_p + M_p K_r + C_p C_r, d = K_p C_r + C_p K_r, e = K_r K_p \).

When the active control is considered, assume the single frequency excitation force transmitted to the pedestal is completely canceled out. Then Equation (1) can be expressed as:

\[
\begin{align*}
M_0 \ddot{x}_1 + C_r \dot{x}_1 + K_r x_1 &= f_0 - f_r \\
- C_r \ddot{x}_1 - K_r x_1 &= f_r \\
\end{align*}
\]  

(4)

The solution of Equation (3) is “\( X_{1,2}(\omega) = \frac{-1}{M_0 \omega^2} f_0 \)” , the amplitude-frequency characteristic of the active control force is “\( f_r(\omega) = \frac{K_r + j\omega C_r}{M_0 \omega^2} f_0 \)”. Then the ratio (\( \Phi \)) of the demanded active control force over the excitation force can be expressed as:

\[
\Phi = |f_r(\omega)/f_0(\omega)| = \left| \frac{K_r + j\omega C_r}{M_0 \omega^2} \right|
\]  

(5)

In this paper, the main objective of active control is to attenuate the vibration transmitted to the pedestal. Meanwhile, the vibration amplitude of machinery (on the upper frame of the hybrid vibration isolation system) should not be increased. The evaluation index \( \Psi \), that is, the ratio of the origin amplitude admittance at the action point of the excitation force under the action of the active-passive control and the passive control, is introduced to study the effect of the active control force on the amplitude of the vibration isolation object. Assume \( \omega_r = \sqrt{K_r/M_r}, \omega_p = \sqrt{K_p/M_p}, \xi_r = C_r/2M_r \omega_r, \xi_p = C_p/2M_p \omega_r \). Where \( \omega_r \) and \( \omega_p \) are the natural angular frequency of machinery and middle-device respectively, \( \xi_r \) and \( \xi_p \) are the damping ratio of machinery and middle-device respectively. Assume that \( D_{\text{non}} = \frac{x_n(t)}{f_0(t)} \) is the displaceable admittance of the machinery at the point of the excitation force before active control, and \( D_{\text{hyb}} = \frac{x_n(t)}{f_0(t)} \) is the displaceable admittance after active control, then the amplitude ratio (\( \psi \)) of machinery vibration after active control over that before active control can be calculated as:

\[
\psi = \frac{D_{\text{hyb}}}{D_{\text{non}}} = \frac{1}{-M_r \omega^2} \times \frac{a \omega^4 - j\omega^3 b - \omega^2 c + j\omega d + \varepsilon}{-M_p \omega^2 + j\omega(C_p + C_r) + K_p + K_r} = \frac{|C + jD|}{|G + jI|} = \sqrt{\frac{C^2 + D^2}{G^2 + I^2}}
\]  

(6)

With \( u = M_p/M_r, f = \omega_p/\omega_r, g = \omega/\omega_r, G = g^2(u^2 - u f^2 - 1), I = -2g^3(\xi_r + \xi_p), C = u g^4 - g^2(1 + uf^2 + u + 4\xi_r^2 \xi_p), D = 2g(\xi_r uf^2 + \xi_p^2) - 2g^3(\xi_r + \xi_p + u \xi_r) \).

The demanded active control force estimated using Equation (5) is shown in Figure 2a, and the amplitude ratio (\( \psi \)) calculated using Equation (6) is shown in Figure 2b.

1. When the frequency ratio \( \omega/\omega_r \) is smaller than 1, the required active control force is greater than the excitation force; when \( \omega/\omega_r = 1 \), the required force is close to the excitation force. When \( \omega/\omega_r > 1 \), the required force is smaller than the excitation force (\( \Phi < 1 \)), and the larger is \( \omega/\omega_r \), the smaller force will be required. In addition, “\( \Phi \)” is smaller when \( \xi_r \) is smaller in the same frequency band.

2. When \( \omega/\omega_r \geq 1 \) and \( \xi_r \) is small, not only the vibration displacement magnitude transmitted to the pedestal will be attenuated, but also the vibration displacement magnitude of machinery will not be increased after active control. When \( \omega/\omega_r = 1 \), the machinery vibration displacement magnitude is decreased significantly. However, when \( 2 < \omega/\omega_r < 3 \), the vibration displacement magnitude is increased slightly.
Considering the influence of comprehensive performance parameters on the vibration isolation performance, the two-DOF hybrid vibration isolation system has the following characteristics: (1) In order to obtain an ideal active control force, the damping ratio should be as small as possible; (2) after active control, the vibration of the controlled object will increase only in a small frequency band.

![Figure 2](image-url)  
Figure 2. Dynamic characteristics of the hybrid vibration isolation system: (a) The estimated demand of actuator force; (b) the amplitude admittance ratio.

3. Design of the Active and Passive Hybrid Vibration Isolator

The active-passive hybrid vibration isolator is the most important part of the two-DOF vibration isolation system. It consists of an electromagnetic actuator, a rubber spring, and a hydraulic device, and the basic structure of a hybrid isolator is shown in Figure 3. The hydraulic device consists of an upper liquid chamber, a lower liquid chamber, an inertial channel, a decoupling membrane which is connected with the mover, and a leather bowl which contains steel skeleton. The hydraulic device is located between the electromagnetic actuator and the rubber spring. Active unit electromagnetic actuator and passive unit rubber spring are combined in parallel.

The effective area of the decoupling film is less than half of the rubber spring, so that the electromagnetic force can be effectively amplified by the hydraulic device. When the load frequency is small, the upper liquid chamber is compressed, which makes the medium flow in the inertial channel, and the vibration energy is converted into internal energy. In this way, the vibration response will be reduced. When the load frequency is large, the force produced by electromagnetic actuator is used to the mover, the lower liquid chamber is compressed. The electromagnetic force is significantly amplified, and the vibration response will be reduced.

The structural parameters and composition materials of the hybrid vibration isolator are shown in Table 1.

The working characteristics of electromagnetic actuator were simulated by COMSOL MULTIPHYSICS software. The number of coil turns is 70, the excitation current amplitude I is 0–10 A, and the excitation current frequency is 0–200 Hz. Figure 4 shows the magnetic induction intensity inside the electromagnetic actuator at different amplitudes and frequencies of the excitation current. Figure 5 shows the electromagnetic forces under different excitation current amplitudes and frequencies.

As can be seen from Figure 4, when the excitation current amplitude is constant and the frequency increases, the magnetic induction line tends to the inner surface of the stator core. When the excitation current frequency is constant and the amplitude increases, the intensity of magnetic induction line increases obviously, which is consistent with the theory [25–28].
Figure 3. Electromagnetic-rubber-hydraulic integrated hybrid vibration isolator: (a) the physical model; (b) the schematic diagram.

Table 1. The structural parameters and composition materials.

| Parameters                                      | Value               | Parameters                                      | Value               |
|-------------------------------------------------|---------------------|-------------------------------------------------|---------------------|
| Equivalent area of rubber main spring           | 2180 mm$^2$         | Decoupling membrane equivalent area             | 650 mm$^2$          |
| Rubber main spring stiffness                     | 3026 N/mm           | Decoupled membrane equivalent stiffness         | 1048 N/mm           |
| Actuator outer diameter and height              | 100/60 mm           | Mover, Stator core                              | AISI 1020 Steel     |
| Mover outer diameter                             | 52 mm               | Energized coil                                  | Copper Wire         |
| Volume compliance of the upper liquid chamber   | $3.344 \times 10^{-6}$ mm$^2$/Pa | Permanent magnet                               | N48SH Rubidium Iron Boron |
| Lower chamber volume compliance                 | $1.438 \times 10^{-4}$ mm$^2$/Pa | Air gap                                         | Air                 |

It can be seen from Figure 5 that the electromagnetic force generated by the electromagnetic actuator has a positive linear correlation with the magnitude of the current amplitude. With the increase of frequency, the electromagnetic force decreases gradually.

In addition, the electromagnetic force of the actuator under different excitation current amplitude and frequency was tested by indirect measurement method, as shown in Figure 6. When the frequency is constant, the electromagnetic force increases approximately linearly with the increase of the current; when the current amplitude is constant, the electromagnetic force decreases with the increase of the frequency, and the decay rate gradually decreases. The test results are basically consistent with the simulation results. Moreover, when the excitation current frequency is 20 Hz, the output efficiency of the actuator can reach up to 16.9 N/A.
Figure 4. Magnetic induction intensity at different current amplitudes and frequencies.

Figure 5. The variation curve of the electromagnetic force with the current: (a) the variation of current frequency; (b) current amplitude variation.
Moreover, when the excitation current frequency is 20 Hz, the output efficiency of the machines decreases. The compensation coefficient (ic) is set for the subband signal after threshold compression, and the coefficient is determined by the subband signals, and select the first 1 subband signals, which has the non-target line spectrum, in order from large to small, and the other signals as the new 1~Q subband in order from large to small, and the other signals as the Q + 1 subband after superposition. Assume $E_R(n)$ ($j = 1, \ldots J$) is the original subband energy, and $E_{ri}(n)$ $(i = 1, \ldots Q + 1)$ is the new subband energy.

If $j = k$, max $\{ E_R(n) \} = E_{R_k}(n)$, let $R_1(n) = R_k(n)$. Take out $P_k(n)$, let.

$$R_2(n) = P_k(n) \text{ max} \{ E_{R_{j\neq k}}(n) \} = E_{R_{k}}(n)$$ (7)

So, take out $r_3(n), r_4(n), \ldots r_Q(n)$, the other subbands are superimposed as $r_{Q+1}(n)$. The signal is decomposed into $Q + 1$ subband signals, which has the non-target line spectrum. In essence, non-target line spectrum is caused by energy mapping or leakage, and the energy of the line spectrum is small. The threshold compression method is adopted to eliminate the non-target line spectrum. Compared with $R_i(n)$, the energy of line spectrum signal $F_i(n)$ after threshold compression is smaller. The compensation coefficient $(c_i)$ is set for the subband signal after threshold compression, and the coefficient is determined by

4. Multi-Line Spectrum Adaptive Control Algorithm

4.1. Improved Wavelet Packet Decomposition Algorithm

The signal can be divided into frequency bands with equal intervals by wavelet packet decomposition and effectively solve the multi-frequency spectrum control problem. However, most subbands do not contain the information of the target line spectrum, controlling all subband signals will greatly increase the number of filters and the complexity of the algorithm, which is prone to other line spectrum vibrations after control. Therefore, it is necessary to improve the signal characteristics after wavelet packet decomposition.

In the process of wavelet packet decomposition, the data are divided according to different frequency bands, but the amount of data remain unchanged. Therefore, the original signal can be obtained only by superimposing the decomposed signals. According to the target line spectrum information, the energy selection method is used to transform the subband signal $P_1(n), P_2(n) \ldots P_J(n) (J = 2^m)$, redivided into $Q + 1$ subband signal $R_1(n), R_2(n) \ldots R_{Q+1}(n)$, where $Q$ is the number of target line spectrum. Sort the energy of the subband signals, and select the first $Q$ frequency bands as the new 1~Q subband in order from large to small, and the other signals as the $Q + 1$ subband after superposition. Assume $E_{P_i}(n)$ $(j = 1, \ldots J)$ is the original subband energy, and $E_{ri}(n)$ $(i = 1, \ldots Q + 1)$ is the new subband energy.

If $j = k$, max $\{ E_{P_i}(n) \} = E_{P_k}(n)$, let $R_1(n) = P_k(n)$. Take out $P_k(n)$, let.

$$R_2(n) = P_k(n) \text{ max} \{ E_{P_{j\neq k}}(n) \} = E_{P_k}(n)$$ (7)

So, take out $r_3(n), r_4(n), \ldots r_Q(n)$, the other subbands are superimposed as $r_{Q+1}(n)$. The signal is decomposed into $Q + 1$ subband signals, which has the non-target line spectrum. In essence, non-target line spectrum is caused by energy mapping or leakage, and the energy of the line spectrum is small. The threshold compression method is adopted to eliminate the non-target line spectrum. Compared with $R_i(n)$, the energy of line spectrum signal $F_i(n)$ after threshold compression is smaller. The compensation coefficient $(c_i)$ is set for the subband signal after threshold compression, and the coefficient is determined by

Figure 6. Electromagnetic force at different current amplitude and frequency.
the subband energy before and after filtering. In the calculation of line spectrum energy, de-correlation is used to reduce the influence of noise.

\[
c_i = K_i \cdot \frac{E_{R_i}(n)}{E_{P_i}(n)} = K_i \cdot \frac{E[R_i(n)R_i(n-1)]}{E[P_i(n)P_i(n-1)]} i = 1, 2, \ldots, Q + 1
\]

The improved wavelet packet decomposition (IWPD) algorithm no longer directly uses the decomposed signal \(P_1(n)\), \(P_2(n)\)…\(P_L(n)\) of the wavelet packet for the control algorithm. The specific improvement of the wavelet packet decomposition algorithm is shown in Figure 7.

![Figure 7. The structure schematic diagram of the IWPD algorithm.](image)

The IWPD algorithm is applied to the FxLMS algorithm, which can reduce the number of control filters and the algorithm complexity. Therefore, the influence of the virtual line spectrum in the sub-band on the control system is eliminated, and it is easier to implement in engineering.

### 4.2. Hartley Domain Block Algorithm

Hartley transform is an integral transform method in the real number domain proposed by electrical engineer Hartley in 1942 [29]. The Hartley transform uses real numbers as the transform kernel and is a unitary transform, which can only be completed in the real number domain. The fast Hartley transform (FHT) [30–32] can realize fast convolution and correlation operations in the frequency domain. Therefore, the block algorithm can be implemented in the Hartley domain. The FHT is as fast as or faster than the fast Fourier transform (FFT) and serves for all the uses such as spectral analysis, digital processing, and convolution to which the FFT is at present applied. However, the FHT computes convolutions and power spectra distinctly faster than the FFT [31].

Thus, the Hartley domain block algorithm is introduced to the adaptive control algorithm, which can improve its convergence speed. The fast convolution and linear operation in the frequency domain can be realized by the FHT. The Hartley domain block algorithm is described in detail.

For the subband signal \((f_k)\) after wavelet packet decomposition, the instantaneous estimation is replaced by the unbiased time average of input data, and the iteration step size can be set as:

\[
u(k + 1) = \alpha u(k) + P * \beta \left[1 - \frac{\text{eps}}{\frac{1}{P} \sum_{i=0}^{P-1} f_i^T(kP + i) f_i(kP + i) + \frac{\beta}{P} \sum_{i=0}^{P-1} E(kP + i) + 1}\right]
\]

The block length \((P)\) is set equal to the filter length \((L)\). The \(L\) tap weight of filter are filled with zero, and the FHT of 2\(L\) points is carried out, that is \(W_H(k) = FHT \begin{bmatrix} W(k) \\
0 \end{bmatrix}\).
The block length \( P \) is set equal to the filter length \( L \). The transmission process of the output signal to the error signal sensor.

\[ y_{II}(k) = FHT[y(kL - L), y(kL - L + 1), \ldots, y(kL + L - 1)]^T \]

\[ = F_{He}(k)W_{II}(k) + F_{Ho}(k)W_{II}(2L - k) \]  

Assume that the filtered reference signal \( F(k) \) is linearly related to the error signal \( e(k) \), and the error signal is zeroed forward, the FHT of 2L points is carried out to obtain the correction term of control coefficient \( e_{II}(k) = FHT\left[ 0 \ e(k) \right] \).

Then we can obtain the time domain correction term \( \Delta W(k) \):

\[ \Delta W(k) = F(k)e(k) = IFHT[F_{He}(k)e_{II}(2L - k) + F_{Ho}(k)e_{II}(k)] \]

The first \( L \) elements of \( \Delta W(k) \) are taken as the control coefficient correction term of time domain. In order to be consistent with the weight vector length of frequency domain, add zeros after \( \Delta W(k) \) to make its total length 2L. Then, the FHT is carried out to updated Hartley domain expression of weight vector.

\[ W_{II}(k + 1) = W_{II}(k) - \frac{\mu(k)}{L}FHT\left[ \Delta W(k) \right] \]

The Hartley block least mean square algorithm (HBLMS) is established by using Equations (9)–(13).

4.3. Multi-Line Spectrum Adaptive Control Algorithm

When the block algorithm is applied in the actual control system, it will cause at least one data block delay, and the overall system has a large time delay. The existence of time delay will affect the convergence and stability of the algorithm. Even if the adaptive algorithm has a certain ability to adapt to the time delay, the system time delay still needs to be predicted and compensated. The main sources of time delay are:

(1) The process of signal acquisition and transmission between hardware.

(2) The operation process of preprocessing an iterative update of the acquired signal.

(3) The transmission process of the output signal to the error signal sensor.

In order to improve the convergence speed of the algorithm in the sub-band and reduce the computational complexity, by combining the improved wavelet packet decomposition algorithm with the Hartley block least mean square algorithm, an improved wavelet packet Hartley block filtered-x least mean square (IWHB-FxLMS) algorithm is established. Its algorithm structure is shown in Figure 8.

The reference signal \( x(n) \) is decomposed into a series of sub-band signals \( f_i(n) \) by IWPD. The frequency band signal \( f_i(n) \) is convolved with its corresponding HBLMS controller to obtain the control output signal \( y_i(n) \) of this frequency band. After superposition, the full-band control output signal \( y_0(n) \) is obtained. After \( y_0(n) \) is filtered by the secondary channel \( S(z) \), it is superposed with the desired signal filtered by the primary channel \( P(z) \) to form the error signal \( e(n) \). \( e(n) \) and IWPD are used to complete the iterative update of the controller HBLMS. Here, independent step size is set for each subband.
The maximum output force of the exciter is 1000 N, the middle stage mass is 40 kg, the total mass of four counterweights is 380 kg, and the RT controller bus width is 8 GB/s.

The single hybrid isolator mass is 10.8 kg, and the PCB sensor sensitivity is 9.87~9.96 mV/g.

5. Experiment

The vibration-isolation performance of the two-DOF active-passive hybrid isolator is tested using the prototype shown in Figure 9a,b.

![Figure 9a](image1.png)  ![Figure 9b](image2.png)

**Figure 9.** The prototype of the two-DOF hybrid vibration isolation system: (a) physical diagram; (b) schematic diagram.

The upper part of the device is a vibrator, which simulates the vibration characteristics of the ship’s machinery, and uses four mass blocks to increase the upper counterweight. Four hybrid vibration isolators are installed at the four bottom corners of the middle device, respectively, which are connected to the upper and lower layer of the device by bolts. The four acceleration sensors are installed at the bottom corners of the upper layer and the middle layer respectively. The upper sensor outputs monitoring signals, and the middle sensor outputs error signals. The middle layer of the device is connected with the pedestal through an elastic bracket.

As shown in Table 2, it is the design parameters of the two-DOF active-passive hybrid vibration isolation system.

| Parameter                                | Value       | Parameter                                | Value       |
|------------------------------------------|-------------|------------------------------------------|-------------|
| the maximum output force of the exciter  | 1000 N      | the middle stage mass                    | 40 kg       |
| the upper stage mass                     | 120 kg      | the NI data collector channels           | 16          |
| total mass of four counterweight         | 380 kg      | the RT controller bus width              | 8 GB/s      |
| the single hybrid isolator mass          | 10.8 kg     | the PCB sensor sensitivity               | 9.87~9.96 mV/g |

**Table 2.** Parameters of two-DOF active-passive hybrid vibration isolation system.
5.1. Passive Vibration Isolation Experiment

The passive vibration isolation performance of the hybrid vibration isolator is studied. The white noise is used to excite the vibration exciter. The upper layer signal and the error signal are collected respectively without the active control. The sampling time is 10 s and the sampling frequency is 1 KHz. Figure 10 shows the time history of the error signal and the upper layer signal and the changing trend of the acceleration vibration level under passive vibration isolation. It can be seen from the time history diagram that the error signal is attenuated compared with the upper acceleration signal. It can be seen from the acceleration vibration level diagram that there is a certain vibration isolation effect in the whole frequency range.

![Figure 10. Experimental results of passive vibration isolation: (a) time history diagram; (b) acceleration vibration level.](image)

5.2. Active Control Experiment of Constant Frequency Excitation

In order to verify the effectiveness of the improved adaptive control algorithm, a constant frequency excitation active control experiment was carried out. Two experimental schemes are adopted:

Control scheme 1: the active-passive hybrid vibration isolator adopts the traditional FxLMS control and the recording data duration is 85 s, and the active control is turned on at 17 s.

Control scheme 2: the active-passive hybrid vibration isolator adopts the IWP HB-FxLMS control, the recording data duration is 70 s, and the active control is turned on at 23 s.

The excitation signal is set to the frequency of 30, 37, 60, 110 Hz sinusoidal line spectrum excitation, the white noise (30–60 Hz, amplitude of 0.1 A) is superimposed, which simulate the frequency components of the multi-line spectrum of the ship machinery. Sampling frequency: 1 kHz, output current amplitude |y| < 9A, controller order: 256, and the third-order Butterworth filter is used for 200 Hz low-pass filtering. The evaluation index of the vibration-isolation performance is the power spectrum attenuation of the error signals before and after control.

Figures 11 and 12 reflect the time history of the error signal and upper layer signal under the two experimental schemes and the changing trend of the error signal power spectrum before and after control. As shown in Figure 11, it can be seen from the time history diagram that the active control is turned on at 17 s, the signal tends to be stable at 38 s, and the convergence time is 21 s. 30 and 37 Hz line spectrum attenuation is obvious, 60 and 110 Hz line spectrum has no obvious effect or even aggravated vibration, and the single-frequency line spectrum attenuation is up to 25.8 dB. As shown in Figure 12, the active control is turned on at 23 s, the convergence is basically completed in only 25.5 s. Compared with a control scheme 1, the convergence speed is increased by 88.1%. All four line spectrums are effectively suppressed, and the single-frequency line spectrum...
attenuation is up to 35.5 dB. Compared with control scheme 1, the amplitude attenuation of the single-frequency line spectrum power spectrum is improved by 34.5%.

![Figure 11](image1.png)

**Figure 11.** The test results of constant frequency excitation active control (control scheme 1): (a) time history diagram; (b) power spectrum of error signal.

![Figure 12](image2.png)

**Figure 12.** The test results of constant frequency excitation active control (control scheme 2): (a) time history diagram; (b) power spectrum of error signal.

5.3. **Active Control Experiment of Multi-Frequency Line Spectrum Excitation**

In order to verify the effectiveness of the improved adaptive control algorithm, a multi-frequency excitation active control experiment was carried out. Two experimental schemes are adopted:

Control scheme 1: the active-passive hybrid vibration isolator adopts the traditional FXLMS control, the recording data duration is 100 s, and the active control is turned on at 18 s.

Control scheme 2: the active-passive hybrid vibration isolator adopts the IWPHB-FxLMS control, the recording data duration is 80 s, and the active control is turned on at 24 s.

The period of the vibrator output signal is set as 1 s, and the frequency ranges of the multi-line spectrum are 28–31, 35–38, 58–61, 108–111 Hz (four swept-frequency chirp signals), the white noise (30–60 Hz, amplitude of 0.1 A) is superimposed, which simulates the frequency components of the multi-line spectrum of the ship machinery. Sampling frequency: 1 kHz, output current amplitude \(|y| < 9A\), controller order: 256, and the third-order Butterworth filter is used for 200 Hz low-pass filtering. The evaluation index of the vibration-isolation performance is the power spectrum attenuation of the error signals before and after control.

Figures 13 and 14 reflect the time history of the error signal and upper layer signal under the two experimental schemes and the changing trend of the error signal power
Machines 2022, 10, x FOR PEER REVIEW... The hydraulic device of the passive unit of the hybrid isolator is proposed, which is mainly composed of electromagnetic actuators, rubber springs, and hydraulic devices. It has a compact structure, strong bearing capacity, and good linearity. The hydraulic device of the passive unit of the hybrid isolator is located between the electromagnetic actuator and the rubber spring, which can effectively amplify the electromagnetic output force. In addition, based on theFxLMS algorithm and

Figure 13. The test results of multi-frequency line spectrum excitation active control (control scheme 1): (a) time history diagram; (b) power spectrum of error signal.

Figure 14. The test results of multi-frequency line spectrum excitation active control (control scheme 2): (a) time history diagram; (b) power spectrum of error signal.

The test results prove that the multi-line spectrum adaptive control algorithm (IWP HB-FxLMS algorithm) has improved the convergence performance and control accuracy of theFxLMS algorithm. The active-passive hybrid vibration isolator based on multi-line spectrum adaptive control can achieve good control effects under the excitation of multi-frequency fluctuation line spectrum.

6. Conclusions
Based on the improved multi-line spectrum adaptive control algorithm, a two-DOF active-passive hybrid isolator is proposed, which is mainly composed of electromagnetic actuators, rubber springs, and hydraulic devices. It has a compact structure, strong bearing capacity, and good linearity. The hydraulic device of the passive unit of the hybrid isolator is located between the electromagnetic actuator and the rubber spring, which can effectively amplify the electromagnetic output force. In addition, based on the FxLMS algorithm and
the wavelet packet decomposition algorithm, the IWP HB-FxLMS algorithm is established, which can eliminate the data delay and improve the convergence speed. Moreover, the experimental results of passive and active vibration isolation verify the excellent vibration isolation performance of the active-passive hybrid vibration isolator and the effectiveness of the improved algorithm.

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