The Influence of Post-Local Buckling Mechanics on the Stress Variations, Axial Stiffness and Ultimate Failure Strength of Uniformly Compressed Thin-Walled I-Section Struts.

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Abstract. It is well known that thin-walled compression members are subject to the effects of local buckling and that due to these local effects the compressive carrying capability of short strut members can be significantly reduced. Finite element simulation is employed in this paper to examine the post-buckled response of thin-walled sections giving due consideration to the influence of geometric imperfections and to elasto-plastic material behaviour. The findings from this work highlight the complete loading history of the compression struts from the onset of elastic local buckling through the nonlinear elastic and elasto-plastic post-buckling phases of behaviour to final collapse and unloading. A detailed account of the growth and redistribution of stresses as well as the influence of yielding and yield propagation throughout loading is given in the paper. The results from the finite element simulations are shown to compare well with independent simulations using the finite strip method of analysis.

1. Introduction
The structural performance of thin-walled compression members is governed, essentially, by the effects of buckling and post-buckling behaviour. There are a number of possibilities with regard to the mode of buckling that can occur and this would depend, of course, on the geometrical details of the member cross-section, the member length, end conditions with regard to support and loading and also the member structural material. The buckling mode is characterised, in essence, by half-wavelength such that local buckling of the section walls is associated with a short half-wavelength, distortional buckling by an intermediate half-wavelength and torsional, flexural and torsional-flexural buckling are the overall modes at the longer half-wavelengths. Clearly, interaction is possible and coupled instabilities can be encountered in design whereby local-distortional buckling can occur or, indeed, the interaction between local buckling and overall flexural behaviour may be of influence. The buckling and post-buckling mechanics of thin-walled compression members has been studied by a great many investigators over the years and as a result, our knowledge and understanding of buckling phenomena is now at a fairly detailed level.

Column interaction buckling and post-buckling behaviour has been studied by numerous researchers over the years. The interaction of local buckling with overall column bending has been studied by Loughlan and Howe [1] and by Loughlan [2] for the case of plain channel, lipped channel and I-Sections using a semi-energy post-local buckling analysis.
procedure. In these works the influence of geometrical imperfections and of local form change during loading are shown to have special significance for columns designed with close local and overall buckling modes. Lau and Hancock [3] and Kwon and Hancock [4] have studied the distortional buckling behaviour of thin-walled channel section columns. Explicit analytical expressions were derived [3] to predict the distortional buckling stress of thin-walled channel section columns and consideration has been given to the post-buckling response [4] of such sections when undergoing local and distortional deformations. The work of Rasmussen [5] and Young and Rasmussen [6] considered the overall bifurcation of locally buckled thin-walled members. The appropriate variational equations developed in these works are applied to doubly symmetric columns [5] and to singly symmetric cross-sections [6] that locally buckle in the fundamental state. For the case of plain channel sections it is shown, using the fundamental state equations, that fixed-ended columns exhibit overall bifurcation behaviour while pin-ended columns do not.

Schafer [7] considered the local, distortional, and Euler buckling of thin-walled columns with regard to their treatment in design. At the time of this work the North American design specifications for cold-formed steel columns ignored local buckling interaction and did not provide an explicit check for distortional buckling. As a result of this, Schafer proposed a new method for design that incorporated, explicitly, the local, distortional and Euler buckling aspects of thin-walled columns. The distortional buckling of cold-formed steel columns is described in comprehensive detail in the AISI research report by Schafer [8]. The report details design methods which account for local and distortional interaction and which permit distortional and Euler interaction. Dinis et al [9] have examined the local-distortional mode interaction behaviour of cold-formed steel lipped channel columns. This work uses the ABAQUS finite element code and 4-node shell elements are employed to simulate behaviour. The numerical post-buckling results reported clearly indicate the evolution of the elastic-plastic column deformations during loading.

Ovesy et al [10-12] have studied the geometric nonlinear compressive behaviour of short struts using the finite strip method of analysis. These works cover channel [10], box [11] and I-sections [12] and give accurate representation of the effects of local buckling on the post-buckling compressional stiffness of the sections in the elastic range of behaviour. In this paper, finite element simulation is employed to examine the post-buckled response of thin-walled I-section struts giving due consideration to the influence of geometric imperfections and to elasto-plastic material behaviour. The findings from this work highlight the complete loading history of the compression struts from the onset of initial compressional loading through the nonlinear elastic and elasto-plastic phases of behaviour to final collapse and unloading of the sections.

2. Finite Element Modelling

The finite element method is used to simulate the compressional post-local buckling behaviour of the I-Section struts. The CQUAD4 quadrilateral shell element of the MSC/PATRAN/NASTRAN finite element software is used to discretise the struts and to formulate the finite element models. The CQUAD4 shell element is particularly well suited for dealing with the post-local buckling behaviour of thin-walled sections. It is readily able to account for the interaction of the local bending and membrane stretching of the section walls which takes place during the post-buckling process. The element has four connecting nodes and is elastically connected to only five of the six degrees of freedom at each of its nodes. It is connected to three translational and two rotational degrees of freedom at each node but does not provide direct elastic stiffness to the sixth degree of freedom which corresponds to the rotation about the normal to the surface of the element. The capability of being able to perform different types of analysis is provided for in MSC/NASTRAN through the appropriate selection of the different solution sequences made available in the software and designed for specific applications. The post-local buckling behaviour of the I-Section struts was determined using the nonlinear static solution sequence involving geometric nonlinearity as well as elasto-plastic material nonlinearity using the simplified elastic-perfectly plastic stress-strain model of the I-Section material.
Suitably refined finite element models with appropriate loading and boundary conditions were developed to simulate the complete compressional loading history of the I-section struts. This covers the onset of initial compression through the nonlinear elastic and elasto-plastic post-buckling phases of behaviour to final collapse and then through the plastic unloading phase of behaviour. The refined models utilised some seven to eleven thousand elements on the full section, depending on cross-sectional geometry, to accurately capture the post-buckling performance of the struts. The use of symmetry in the modelling process does, of course, halve the size of the structural models and reduces, considerably, the computation time involved in solving the appropriate post-buckling equilibrium equations. Results have been obtained in this work using symmetry for computational efficiency. In finite element modelling, boundary conditions and loading conditions are of great importance. Measurably different outcomes can be determined in structural analysis as a result of modifications to the compatibility boundary conditions of a structure. Changes in the boundary conditions cause stiffness enhancement or stiffness relaxation of the structure and thus the deformations and stresses may be quite different for a given level of load application.

2.1 Boundary Conditions
The boundary conditions applied to the I-Section strut models are such that the ends of the constituent flat plate components of the sections are simply supported in nature and thus all nodes at the section ends are constrained in the direction which is normal to the mid-surface of the flat plate component to which they belong. In addition, all nodes on one section end are constrained from moving axially whilst those at the other end are compressed uniformly by the same amount and in an incremental manner. The finite element nonlinear static solution procedure, which utilises the modified Newton-Raphson method, is then able to solve the assembled global equilibrium equations of the strut models for all nodal displacements and this is carried out for each increment of uniform end compression (end displacement) applied to the struts. Knowing the nodal displacements then permits the evaluation of the stress variations within the section walls and the magnitude of the constraint forces at the axial constraint nodes corresponding to each increment of applied end compression. Summing the axial nodal constraint forces then permits the evaluation of the axial load on the strut at each compression increment and thus the complete post-local buckling loading history can be detailed in a step by step manner. The application of end compression (end displacement) increments to the struts and then determining the axial load on the struts at each compression increment by summing the axial nodal constraint forces is a process that lends itself most suitable at the ultimate load condition of the struts when dealing with the plastic unloading phase of behaviour.

Of particular interest are the conditions prevailing at the junctions between the constituent flange and web plates of the cross-sections along the lengths of the struts. Using the finite element method, the nodes at the section junctions are permitted to be free from constraints, in which case, the coupling of the in-plane and out-of-plane deflections of the section walls meeting at the junctions is able to satisfy the natural compatibility and equilibrium conditions at the junctions, leading to waviness of the junctions along the length of the struts. In previous analytical studies and numerical simulation procedures using the finite strip method of analysis it has been assumed that the in-plane and out-of-plane deflections at the section junctions are uncoupled. This means that the plate components meeting at the junctions are assumed to be free to wave in their own plane across their width along the junctions whilst the out-of-plane deflection of each of the plates vanishes. These assumptions clearly violate the compatibility and equilibrium conditions along the junctions and impose stress free in-plane conditions at the junctions in the width direction of each plate. The stress free in-plane approach has been implemented by a great many researchers in the past as an aide to simplifying their analysis and solution procedures when dealing with the post-local buckling behaviour of thin-walled sections. It is an approach whose assumptions are considered to represent fairly valid approximations to actual behaviour up to about three times the critical local buckling load of the section wherein the movements at the section junctions are considered to be negligibly small. The effects of the boundary conditions along
the junctions on the compressional behaviour of the I-Section struts have been examined in this paper. Comparisons are given corresponding to the three conditions of natural waviness of the junctions, in-plane stress free conditions at the junctions and immovable straight junctions, not allowed to wave, respectively.

2.2 Local Imperfections and Yielding
Geometric imperfections are taken into account in the finite element modelling process by assuming that the section walls are not perfectly flat. This, of course, leads to a nonlinear compressive response from the onset of loading which would not be the case for the initial loading of the geometrically perfect strut. A fairly common approach to dealing with the local out of flatness of the section walls is to assume that such local imperfections are similar in form to the locally buckled mode shape of the perfect cross-section. This approach has been adopted in the present work due to the fact that it has been shown to give realistic predictions of actual behaviour and also for the ease and simplicity of its application when using the finite element method of analysis. The locally buckled mode shapes are readily determined by subjecting the strut finite element models to the linear static buckling solution sequence of the MSC/PATRAN/NASTRAN finite element software. The local mode shapes thus obtained are then mapped on to the nodal grid system of the finite element model to form the imperfect strut which is then subsequently analysed using the nonlinear static solution sequence of the software to determine the effects of geometric nonlinearity. The magnitude of the imperfection applied to the finite element nodal grid system can be readily altered and thus it is a simple matter of setting the maximum amplitude of the buckled mode shapes to a specific value which is normally taken to be a percentage of the section wall thickness, i.e. maximum imperfection magnitude is set to 0.1t, 0.2t etc. This process allows the examination of the effects of local geometric imperfections in a fairly straightforward manner. With regard to the effects of material nonlinearity, the finite element model is further extended to include the elasto-plastic material constitutive equations into the post-local-buckling analysis of the doubly-symmetric I-section struts. The I-section struts are thus modelled using an isotropic, elastic-perfectly plastic material in conjunction with the Von Mises failure criterion.

3. Findings from Simulations and Discussion
Some findings are presented for a typical doubly-symmetric I-section strut with b/f/bw shape factor equal to 0.5. Figure 1 shows a typical diagram of the geometry for the I-section struts. The struts have a length, L of 100 mm, the width of the web, bw is 100 mm and the thickness is 1.5 mm for both the flanges and the web.

![Fig. 1. Geometry of the doubly-symmetric I-section strut.](image-url)
Figure 2 shows the non-dimensional load, P*, versus the non-dimensional end-shortening, u*, for a doubly-symmetric I-section strut with b/fw shape factor equal to 0.5. It clearly shows that the load-end shortening curve of the in-plane stress-free edge boundary condition at the section junctions bears a close resemblance to the curve of the natural waviness boundary condition for an initial post-local buckling analysis (i.e. up to about 1.5-2.0 times the buckling load). But as the end-compression progresses further the load-end shortening curves move away from each other. Therefore it can be concluded that the assumption of in-plane stress free edges at the junctions is a fairly valid approximation for an initial elastic post-local-buckling analysis. As a comparison with the finite element result, the independent load–end shortening curve [13] of the finite strip method of analysis is plotted. The curve from the finite element solution is seen to be more accurate than that from the finite strip solution. This is due to the fact that the finite element model is a fully converged fine-mesh model which provides a high degree of shape flexibility across the section and along the length of the struts in the post-buckling range. Figure 2 also shows that for the immovable straight edge boundary condition the strut is over constrained and it is clear that this condition does not resemble the actual behaviour of the I-section struts but results in an over-stiff post-local-buckling response. The immovable edge boundary condition was included in this work merely for comparative purposes.
Figure 3 shows the longitudinal membrane stress distribution across the quarter cross section, i.e. BOA as indicated in Figure 1, for the same I-section strut. The stress distributions are at the half length of the strut at the crest of the buckle. The loads at which the stress distributions are obtained are at twice and three times the buckling load as indicated. As expected, the stress concentrations are towards the junctions of the section in which the total load on the strut is carried more by elements close to the junctions. In addition, the maximum membrane stress occurs at the junctions at the crest of the buckle. The figure also shows good agreement between the finite element method and the finite strip method [13]. The load shedding depicted in Figure 3 is a typical characteristic of post-local buckling mechanics.

With the aim of investigating the effects of material nonlinearity, the work is further extended to include the elasto-plastic material constitutive model into the post-local-buckling analysis of the doubly-symmetric I-section struts. Thus, the I-section strut is modelled as an isotropic, elastic-perfectly plastic material model in conjunction with the Von Mises failure criterion. The yield stress is varied from 250 N/mm$^2$ to 550 N/mm$^2$. Figure 4 presents the non-dimensional load-end shortening curves for different levels of yield stress. At all levels of yield stress, the ultimate load of the struts is clearly defined with a subsequent elasto-plastic unloading phase of behaviour. It has been found that the ultimate loads of the struts are closely associated with membrane surface yielding at the section junctions. The influence of local geometrical imperfections is indicated in Figure 4 by the dotted lines for the case of an imperfection maximum amplitude of 0.4$t_w$ and corresponding to the yield stress levels of $\sigma_y = 250$ N/mm$^2$ and 550 N/mm$^2$ respectively. It is of note that the imperfection curves tend asymptotically to their ideal perfect counterparts far into post-buckling during the plastic unloading phase of behaviour. Of particular interest is the effect of local imperfections on the ultimate carrying capability of the sections. It is clear that the effect is not significant for the higher yield stress section which has a range of stable elastic post-buckling reserve. In this case the loss in carrying capability is less than 3%. For the section designed with near simultaneous buckling and yielding, the effect of imperfections on ultimate load is noted to be significant. For this design the loss in carrying capability is just over 13% and thus the imperfection sensitivity of simultaneous interaction is noted to be an area requiring critical attention.
4. Concluding Remarks
The compressional stiffness of thin-walled I-section struts is noted to be significantly altered as a result of local buckling of the section walls. The loss in axial stiffness of short strut compression members will depend, to a large extent, on cross-sectional shape and more particularly on section wall thickness. For sections manufactured from high yield materials it is possible to design the struts with a fairly wide range of the ratio of yield stress to local buckling stress $\sigma_y/\sigma_{cr}$ of the sections. For the higher values of this ratio considered in this paper it is shown that the post-buckling response involves an initial range of elastic behaviour whereby the nonlinear loss in compressional stiffness is associated with large rotations and local form change during loading.

As loading progresses, surface yielding occurs as a result of the high through-the-thickness bending stresses caused by local buckling. At this point the struts enter into an elasto-plastic phase of behaviour and further loss in compressional stiffness is experienced as a result of the effects of material nonlinearity. The growth of yielding in the elasto-plastic phase of behaviour leads eventually to collapse of the sections at ultimate conditions and subsequently to a post-collapse unloading phase of behaviour. The in-plane stress free boundary condition at the section junctions has been shown to give a response which reflects good agreement with the behaviour of the sections when the junctions are allowed to wave naturally. This is noted to be true in the initial nonlinear elastic stages of behaviour up to about twice the initial buckling load. The load shedding to the section junctions as a result of local buckling is noted to result in junction yielding at the membrane surface of the section walls at, or in the near vicinity, of the ultimate collapse loads of the sections. The influence of local imperfections have been shown to be significant for simultaneous buckling and yielding designs as a result of the unstable equilibrium behaviour associated with simultaneous interaction.

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