Quantum vortices near the superconductor-insulator transition in
Josephson junction arrays

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Abstract

The properties of vortices in Josephson junction arrays are investigated in
the quantum regime near the superconductor-insulator transition. We de-
rive and study an effective action for vortex dynamics that is valid in the
region where the charging energy is comparable to the Josephson coupling
energy. In the superconducting phase the onset of quantum effects reduces
the vortex mass and depinning current. In the case of long range Coulomb in-
teraction between Cooper pairs we find that as the transition is approached,
the velocity window in which ballistic vortex motion is possible grows. At
the superconductor-insulator transition the vortex mass vanishes and vortices
and spinwaves decouple. In the case of on-site Coulomb repulsion (which is
of relevance for superconducting granular films) the vortex mass it is sample-
size dependent in the superconducting phase, but stays finite at the critical
point where it is scale invariant. The relation of our work to experiment is
discussed.

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I. INTRODUCTION

In classical Josephson junction arrays (JJA) vortices are well known topological excitations that characterize both the dynamical and thermodynamical properties of these systems. At low temperatures these excitations are bound in dipoles of opposite vorticity and at a critical temperature they unbind in a Kosterlitz-Thouless-Berezinskii (KTB) phase transition \cite{1} leading the system from the superconducting (SC) to a resistive phase.

When an external driving current is applied to the array, free vortices (as induced by applying a magnetic field) move and a voltage drop across the sample appears. In classical JJA’s, in which the Josephson coupling $E_J$ is much larger than the charging energy $E_C$, vortex motion is diffusive. Some time ago it was realized that charging effects yield dynamics and a mass for the vortices in the SC phase \cite{2,3,4}. The electrostatic energy stored in the junction capacitances can be interpreted as a kinetic energy due to the vortex motion and as a consequence a mass can be attributed to the vortex. The mass in this regime was found to be proportional to the junction capacitance $C$ and in case of small damping ballistic vortex motion was predicted. Modern lithographic techniques allow for the realization of JJA’s with junction resistance much larger than $\hbar/e^2$, thereby making possible an experimental check of these predictions. Very recently, in an important experiment, van der Zant et al. \cite{5} demonstrated the existence of such ballistic vortices in triangular arrays.

The possibility of ballistic motion depends crucially on the dissipation mechanisms available to a vortex. Nowadays it is possible to fabricate high quality JJA’s in which no ohmic dissipation is present \cite{6}. If the vortex velocities are below a certain threshold (related to the superconducting gap), also the quasiparticles are frozen and do not contribute to the damping. It was pointed out in recent analytical \cite{7,8} and numerical \cite{9} studies that another mechanism by which a classical moving vortex can lose its kinetic energy is by emitting spinwaves. Depending on the spinwave spectrum, it has been shown that in triangular arrays there exists a small window of velocities for the vortex to move over the pinning potential provided by the lattice without suffering too much damping. For square classical arrays
ballistic vortex motion is not possible.

All the previous investigations were mainly concerned with the case in which the Josephson coupling $E_J$ is much larger than the charging energy $E_C$ (and larger than the temperature $T$). When these two energies become comparable quantum effects play a major role and, eventually, at a certain critical value of their ratio, a superconductor-insulator (S-I) transition at zero temperature takes place \cite{10,11,12}. This phase transition separates regions where either the Cooper pairs or the vortices are localized. In the insulating phase vortices are delocalized, as are the Cooper pairs in the SC phase. Thus, close to the S-I transition one expects vortex properties to reflect the presence and nature of the transition. One may also expect that experiments on vortices in this region \cite{13} yield more insight into the phase transition.

The analysis of vortex dynamics as one approaches the S-I transition will be the aim of this paper. We will consider a square JJA at $T = 0$ with a superimposed external magnetic field such that a small amount of vortices of one sign are induced in the system. If the magnetic frustration is very small one may suppose that the vortex motion is not influenced by their mutual interaction. By applying an external current the dynamics can be studied. When $E_J \gg E_C$ the description has been formulated in \cite{2,3,4,7,8}. Vortex motion is then described by a phenomenological equation of motion for the vortex coordinate $x$ of the form \cite{14}

$$M_v \ddot{x} + \eta \dot{x} = 2\pi E_J I / I_{cr} + \pi U_{bar} \sin(2\pi x),$$

where $M_v$ is the vortex mass, $\eta$ a phenomenological damping, $I$ the applied current, $I_{cr}$ the junction critical current, and $U_{bar}$ the height of the potential energy barrier for a vortex to go from one plaquette to a neighboring one in the potential landscape provided by the array. When we enter the quantum regime fluctuations of vortex anti-vortex pairs (dipoles) start to be relevant and they will interact with the moving vortex. As a result the vortex dynamics will be modified.

In order to describe this regime, we first derive an effective action for one single vortex...
in the next section. We will show that it depends on the charge-charge correlation function; Eq. (5) is the central result of our work. It implies that a description in terms of an equation of motion is still possible, however, with modified coefficients. General expressions for the vortex mass and the spin wave damping that are not constrained by the inequality $E_J \gg E_C$ and are also valid when the Josephson energy is comparable with the charging energy are then presented. After that we briefly show in Sect. III how to recover the previously known results in the limit of large Josephson coupling. Analytic approaches in the two opposite limits in which either the junction capacitance or the capacitance to the ground is dominant are discussed in Sects. IV and V respectively. For long range Coulomb interactions we find that closer to the S-I transition the vortex mass clearly deviates from the classical results and is not simply proportional to the junction capacitance. Furthermore a velocity window for ballistic motion opens if one approaches the transition, also for square arrays in which ballistic motion in the limit $E_J \gg E_C$ is impossible. The effect of the capacitance to the ground has until now been overlooked for JJA’s, but it gives a contribution to the vortex mass which is of the same order of magnitude (for a typical experimental setup) as the mass due to the junction capacitance and therefore, we argue, it should be taken into account. In the limit of short range Coulomb interactions we notice that the mass should be finite at the transition and the vortex dynamics is governed by the same correlation functions that lead to the universal conductance at the S-I transition. Therefore we feel that the understanding of vortex dynamics in this regime may shed light on the source of dissipation which is responsible for the metallic behavior at the S-I transition [15,16]. Sect. VI discusses Monte Carlo simulations that verify our conclusions. The main results of our work are then summarized in the last section.

II. THE EFFECTIVE ACTION

We start from the well known Hamiltonian for a Josephson junction array
\[
H = \frac{1}{2} \sum_{ij} Q_i C_{ij}^{-1} Q_j - E_J \sum_{<ij>} \cos(\phi_i - \phi_j) + \sum_i \vec{I}_i \cdot \vec{\nabla} \phi_i ,
\]  

(2)

where \(Q_i\) and \(\phi_i\) denote respectively the charge and the phase of the superconducting order parameter of the i-th island. The external current is denoted by \(I\). The typical energy scales are the Josephson coupling \(E_J\) and the charging energy \(E_C = e^2/2C\) or \(E_o = e^2/2C_o\). \(C_o\) and \(C\) are, respectively, the ground capacitance and the nearest neighbor capacitance that constitute the capacitance matrix \(C_{ij}\). The range of the electrostatic interaction between Cooper pairs, which is described by the inverse capacitance matrix \(C^{-1}\), is \(\lambda^{-1} = \sqrt{C/C_o}\).

The quantum mechanical description is completed by the commutation relation \([Q_i, \phi_j] = 2e\delta_{ij}i\).

By means of well established duality transformations \([17]\) it is possible to recast the partition function in terms of the topological excitations of the system. For the Hamiltonian under consideration this was done in Refs. \([12,18]\), where a detailed derivation is presented. In order to make this paper self-contained we sketch the main steps in appendix A. The result is a description in terms of two discrete gasses \(q\) and \(v\) that denote charges and vortices respectively. In this formulation the partition function for a JJA is obtained summing over all configurations of charges and vortices \([12]\)

\[
Z = \sum_{\{q_i,\tau\}} \sum_{\{v_i,\tau\}} \exp\{-S[q, v]\}
\]

where the action is

\[
S[q, v] = e\pi E_J \sum_{ij,\tau} v_{ij,\tau} G_{ij} v_{ij,\tau} + \frac{2eE_C}{\pi} \sum_{ij,\tau} q_{ij,\tau} U_{ij} q_{ij,\tau} + i \sum_{ij,\tau} \dot{q}_{ij,\tau} \Theta_{ij} v_{ij,\tau} + i \sum_{ij,\tau} \vec{I}_i,\tau \cdot \vec{\nabla} \Theta_{ij} v_{ij,\tau} + \frac{1}{4\pi e E_J} \sum_{ij,\tau} \dot{\hat{q}}_{ij,\tau} G_{ij} \dot{\hat{q}}_{ij,\tau}
\]

(3)

where the kernels \(\Theta_{ij} = \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right)\) and \(G_{ij} = -\ln |r_i - r_j|\) were introduced. \(U_{ij}\) is related to the capacitance matrix as \(U_{ij} = 2\pi CC_{ij}^{-1}\), it has the explicit form \(U_{ij} = K_0(\lambda | r_i - r_j |)\), where \(K_0\) is a modified Bessel function \([12]\). Spinwaves are described by the last term in Eq.(3). We introduced \(N_\tau\) time-slices to describe the quantum dynamics of the system. The
lattice spacing in the time direction is denoted by $\epsilon$ and $N_\tau \epsilon = \beta$ is the inverse temperature (see App.A). The phase diagram for the model described by (3) was studied in ref. [12] and exhibits a zero temperature S-I transition (an analysis based on a Kosterlitz type RG procedure is in progress [13]).

Eq.(3) will be our starting point. We seek an effective action for a single vortex (with coordinate $r(\tau)$) that includes the effect of the interaction with fluctuating charges and other vortices (present in the system because of quantum fluctuations). It is obtained by a summation in the partition function over all configurations of the charges and other vortices. It turns out to be more transparent to introduce the trajectory of the vortex $u_{i,\tau} = v \delta(r_i - r(\tau))$. Here $v = \pm 1$ for a vortex or anti-vortex respectively. Formally the effective action can now be written as

$$S_{eff} = -\ln \left\langle \epsilon 2\pi E_{Jv} \sum_{ij,\tau} v_{i,\tau} G_{ij} \delta(r_j - r(\tau)) + iv \sum_{ij,\tau} \dot{q}_{i,\tau} \Theta_{ij} \delta(r_j - r(\tau)) + iv \epsilon \sum_{ij,\tau} \vec{I}_{i,\tau} \cdot \vec{\nabla} \Theta_{ij} \delta(r_j - r(\tau)) \right\rangle_{(3)} ,$$

where the average is to be taken with the action (3). The first term describes the static interaction with other vortices, whereas the second describes the dynamical interaction with charges. This expression is formally exact, but difficult to evaluate because of the nonlinearity of the action (3). As we are interested in the kinetic contribution (quadratic in the vortex velocities) it is, however, sufficient to expand the average in (4) in cumulants and stop at the second order in the vortex velocities $\dot{r}(\tau)$. For a uniform external current distribution we find

$$S_{eff} = \frac{1}{2} \sum_{\tau,\tau'} \dot{r}_a(\tau) \mathcal{M}_{ab}(r(\tau) - r(\tau'), \tau - \tau') \dot{r}_b(\tau') + 2\pi i \epsilon \sum_{\tau} \epsilon_{ab} I_{\tau}^a r^b(\tau) ,$$

$$\mathcal{M}_{ab} = \sum_{jk} \nabla_a \Theta(r(\tau) - r_j) \langle q_j q_k \rangle \nabla_b \Theta(r(\tau) - r(\tau')) ,$$

where $a, b = x, y$ and $\epsilon_{ab}$ is the anti-symmetric tensor. Thus, vortex dynamics is governed by the charge-charge correlation, which depends on the full coupled charge vortex gas (CCVG) Eq.(3). The effective action Eq.(3) describes the dynamical vortex properties for all values
of $E_J/E_C$ and is therefore a good starting point for the investigation of vortex properties near the S-I transition.

In order to obtain the vortex effective action in Eq.(5) we disregarded all higher order terms in the cumulant expansion. This is certainly correct in the $E_J \gg E_C$ limit where (as discussed in detail in the next section) the charges can be considered as continuous variables and vortex fluctuations can be disregarded. In general, however, the average defined in Eq.(4) is far from Gaussian ($q$ and $v$ are integer valued fields). As long as we are in the superconducting phase, however, the charges are strongly fluctuating and the vortices are still bound in dipoles. Therefore we expect that the higher order cumulants are still not important \cite{20}. A full description of the vortex motion in the resistive region, nevertheless, may require the analysis of a dynamical equation that contains also terms proportional to higher powers of the velocity.

III. THE CLASSICAL LIMIT

The expression (5) yields the known results \cite{3,7,8} in the classical limit where $E_J \gg E_C$. In this region of the phase diagram the Josephson coupling dominates over the electrostatic energy. Far from the transition vortex fluctuations due to quantum effects are suppressed and they may be neglected. In this regime the charges are wildly fluctuating (as to constitute the supercurrents that keep the SC phases well defined) and may be considered to be continuous variables.

Thus in the classical limit we may concentrate on the charge part of the action (3). It is rewritten conveniently as

$$S[q] = \sum_{ij,\tau,\tau'} q_{i\tau}Q_{ij,\tau,\tau'}q_{j\tau'}, \quad Q_{ij,\tau,\tau'} = \frac{2\epsilon E_C}{\pi} \left( U_{ij}\delta_{\tau,\tau'} + \frac{G_{ij}}{2\omega_p^2}\left(2\delta_{\tau,\tau'} - \delta_{\tau,\tau'+\epsilon} - \delta_{\tau,\tau'-\epsilon}\right)\right), \quad (6)$$

where the plasma frequency $\omega_p = \sqrt{8E_JE_C}$ was introduced. The charge-charge correlation is half of the inverse of the kernel $Q$. It is

$$\langle qq\rangle_{k,\omega_\mu} = \frac{(E_Jk^2/\epsilon)/(\omega_\mu^2 + \omega_k^2)}{\omega_k^2 = \omega_p^2U_k/G_k}, \quad (7)$$
The spinwave dispersion is described by $\omega_k$. It is optical, i.e. $\omega_k = \omega_p$, for long range Coulomb interactions, whereas for on-site interactions we have $\omega_k = \bar{\omega}_p k$. Here $\bar{\omega}_p = \sqrt{8E_J E_o}$ is the plasma frequency for the case of on-site Coulomb interactions.

The action (5) reduces to that of a free particle in the limit of small velocities $\dot{r}(\tau)$. Since the charge-charge correlation (7) is short range in time we may put $r(\tau) = r(\tau')$ in Eq.(5).

The corresponding adiabatic vortex mass $M_v$ is

$$M_v = \epsilon \sum_\tau M_{xx}(0, \tau),$$

which reduces in the classical limit to $M_v = M_{ES} + \ln(L) M_o$, where $M_{ES} = \pi^2/4E_C$ is the Eckern-Schmid mass and $M_o = \pi/8E_o$. Thus both self- and nearest neighbour capacitances yield a contribution to the mass. The self capacitance contribution depends on the system size $L$. For generic sample sizes and capacitance ratio’s the new contribution (which has been overlooked so far for JJA’s since it makes no sense in the thermodynamic limit, see ref. [21] for size dependent vortex masses in different systems) is somewhat smaller than the Eckern-Schmid mass.

The instanton action $S_{inst}$, related to a hop from one plaquette to a neighbouring one, determines tunnel rates and the depinning current. As vortex trajectory we now take $\dot{v}_{i\tau} = v_{i\tau + \epsilon} - v_{i\tau} = \delta_{i\tau \delta_{i,x+a} - \delta_{i,x}}$ for a hop from $x, t \rightarrow x + a, t + \epsilon$. This may be inserted in the CCVG action (3) to find the general result

$$S_{inst} = \frac{1}{2} M_{xx}(0,0).$$

In the classical limit we recover all known results [22], i.e. for general capacitance matrix

$$S_{inst} = \frac{\pi E_J}{4\bar{\omega}_p} \left[ \sqrt{\pi} \sqrt{\lambda^2 + 4\pi} + \frac{\lambda^2}{2} \ln \left( \frac{2\sqrt{\pi}}{\lambda} + \sqrt{1 + \frac{4\pi}{\lambda^2}} \right) \right],$$

which reduces to $S_{inst} = \pi^{3/2} E_J/4\bar{\omega}_p$ and $S_{inst} = \pi^2 E_J/2\omega_p$ for $C = 0$ and $C_o = 0$ respectively. The general form of an instanton action in the WKB approximation, which is proportional to the square root of mass times barrier height, $S_{inst} \sim \sqrt{M_v U_{bar}}$, determines the barrier $U_{bar}$ for a vortex to hop. The depinning current $I_{dep}$ is half the barrier height.
Since deep in the classical limit $U_{\text{bar}} = E_J/5$, we may establish $S_{\text{inst}} = \pi \sqrt{5M_v U_{\text{bar}}/2}$ for the $C_o = 0$ case.

The spinwave damping that a moving vortex experiences may also be calculated from (3). Varying the vortex coordinate $r^a(\tau)$ in Eq.(3) yields the equation of motion

$$2\pi i \epsilon_{ab} \dot{I}^b/I_{cr} = \frac{1}{\epsilon} \partial_\tau \sum_{\tau'} \mathcal{M}_{ab}(r(\tau) - r(\tau'), \tau - \tau') \dot{r}^b(\tau')$$

(11)

and its constant velocity solutions in the presence of an external current determine the nonlinear relation between driving current and vortex velocity, once the charge-charge correlation is analytically continued (i.e. sending $i\omega \nu \rightarrow \omega + i\delta$) to real frequencies [8]. The relevant information is in the real part of Eq.(11), which reads in Fourier components and for a constant vortex velocity $\vec{r}(\tau) = (v, 0)$

$$I^v/I_{cr} = \frac{v}{4} \int d\omega \int_{-\pi}^{+\pi} dk \frac{k^2}{k_x^2} [\delta(\omega - \omega_k) + \delta(\omega + \omega_k)] \delta(\omega - v k_x) .$$

(12)

The delta functions express the spinwave dispersion (from the analytic continuation of the charge-charge correlation) and the vortex dispersion respectively. The overlap integral determines the amount of dissipation a moving vortex suffers from coupling to spinwaves. If we adopt the smooth momentum integration cut-off $\int d^2k \rightarrow 2\pi \int_0^{\infty} dk k \exp(-k/\sqrt{2\pi})$ that was introduced in Ref. [3], we recover in the classical limit the results of Refs. [7,8], see Fig.1 for the current vs. velocity relation for long range Coulomb interactions. Note that the minimum velocity that a vortex needs to move over the pinning potential of the lattice follows from the phenomenological equation of motion Eq.(1) by demanding that $\frac{1}{2} M_v v^2 \geq U_{\text{bar}}$ and is about $0.14 \omega_p$ (see the dotted line in Fig.1) which is also the velocity at which the spinwave damping sets in. Therefore ballistic vortex motion is almost impossible in classical square arrays. In triangular arrays, however, a similar analysis yields a somewhat wider velocity window [8]. In the next section we show how the inclusion of quantum effects contributes to the opening of a more robust velocity interval, also for square arrays, where ballistic motion can be observed.
IV. LONG RANGE COULOMB INTERACTIONS

When the ratio $E_J/E_C$ decreases the charge-charge correlation must be calculated beyond the classical approximation. We first consider the case of long range Coulomb interactions between Cooper pairs. According to the arguments given in Refs. [12,24], the zero temperature S-I phase transition is presumably of the KTB type, as is the finite temperature transition to the resistive phase. This means that no dimensional cross-over takes place at zero temperature, or in other words the dynamical critical exponent $z$ equals zero.

Thus we are led to conclude that the vortex fugacity in the superconducting phase scales to zero in the renormalization group sense also at zero temperature. This means that the charge-charge correlation function in the SC phase may still be evaluated in the absence of vortex fluctuations. Therefore we consider again only the charge part Eq.(6) of the action Eq.(3), but in contrast to the classical limit we now treat the charges as discrete variables.

The charge-charge correlation function may be rewritten as (see Appendix B1 for the derivation) the classical result minus a correction

$$\langle q_{j\tau} q_{k\tau'} \rangle = \frac{1}{2} Q_{jk\tau\tau'}^{-1} - \frac{\pi^2}{2} \sum_{mntt'} Q_{jmnt}^{-1} \langle \phi_{mt} \phi_{nt'} \rangle Q_{nktt'}^{-1},$$

where the correlation function of the dual variables $\phi$ is now to be calculated using the sine-Gordon-like action [25]

$$S[\phi] = \pi^2 \sum_{ij} \sum_{tt'} \phi_{it} Q_{ijtt'}^{-1} \phi_{jt'} - H \sum_{it} \cos(2\pi \phi_{it})$$

We will calculate the charge-charge correlation function in a self consistent harmonic approximation which is valid not too close to the transition point. It amounts to the replacement of the nonlinear cosine term in the Hamiltonian by a mass term (or inverse correlation length) which is determined selfconsistently by means of the Bogoliubov variational principle [26]. The constant $H$ is related to the fugacity for charges in the original model, it is $H^{-1} = 2\epsilon E_C$.

We take a trial action with the $\cos(2\pi \phi_{it})$ replaced by a mass term

$$S[\phi] = \frac{1}{2} \sum_{ij} \sum_{tt'} \phi_{it} [2\pi^2 Q_{ijtt'}^{-1} + \delta_{ij} \delta_{tt'} \xi^{-2}] \phi_{jt'}$$

(15)
and determine the correlation length $\xi$ (or inverse mass) from the selfconsistency equation

$$\frac{\xi^{-2}}{4\pi^2 H} = \exp(-2\pi^2 \langle \phi^2 \rangle \xi^2)$$  \hspace{1cm} (16)

in the usual way [28]. The result for the correlation length is

$$\xi^2 = \frac{\epsilon E_C}{2\pi^3} \left( \pi e^{-\epsilon E_C(c)/\pi} \right)^\frac{1}{1-\alpha}, \quad \alpha = \frac{\epsilon E_C}{\pi} \left( 1 + \frac{2}{\epsilon^2 \omega_p^2} \right)$$  \hspace{1cm} (17)

and the function $c(\epsilon)$ is of order one. The phase transition is at $\alpha = 1$, which corresponds to $E_J/E_C = 1/\pi^2$. Thus, without vortex fluctuations the phase transition is at a smaller $E_J/E_C$ value than the $2/\pi^2$ that follows from a duality argument [12].

The correlation function is modified to

$$\langle \phi \phi \rangle_{k,\omega} = \frac{k^2 E_J/\epsilon}{\omega_{\mu}^2 + \omega_k^2}, \quad \tilde{\omega}_k^2 = \omega_k^2 + 4\pi^2 E_J \xi^2 k^2/\epsilon.$$  \hspace{1cm} (18)

Thus the spinwave dispersion is affected at small distances (large $k$-vectors). This hardening of the spinwave dispersion close to the transition may be interpreted as resulting from the discreteness of the Cooper pairs, which makes fluctuations of phase and charge on short distances unfavourable. It leads to a mass

$$M_v = \frac{\epsilon}{8\pi\xi^2} \ln \left[ 1 + \frac{2\pi^3 \xi^2}{\epsilon E_C} \right]$$  \hspace{1cm} (19)

In the limit of small $\xi$ the Eckern-Schmid mass is recovered. An extrapolation to the S-I transition where $\xi \rightarrow \infty$ yields a mass that vanishes at the transition, see Fig.2. We find a similar result for the instanton action

$$S_{\text{inst}} = \frac{\epsilon \omega_p}{16\pi \xi^2} \left( \sqrt{1 + \frac{2\pi^3 \xi^2}{\epsilon E_C}} - 1 \right)$$  \hspace{1cm} (20)

Again, the classical result is recovered in the limit $\xi \rightarrow \infty$, whereas an extrapolation to the transition gives an instanton action that vanishes. From the WKB relation between mass, instanton action and barrier height $U_{\text{bar}}$ (as discussed in Sect.III), we find that the depinning current $I_{\text{dep}} \sim S_{\text{inst}}^2/M_v \sim 1/\ln(\xi)$ and thus vanishes algebraically close to the transition, i.e. $I_{\text{dep}} \sim (1 - \alpha)$, where $\alpha$ was defined in Eq.(17).
With the charge-charge correlation given in Eq. (18) we may calculate the spinwave damping of vortex motion due to the coupling to spinwaves beyond the classical limit. Replacing $\omega_k$ by $\tilde{\omega}_k$ in Eq. (12), the overlap integral over the delta functions only contributes for vortex velocities that are higher than a threshold velocity $v_t = 2\pi \xi \sqrt{E_J / \epsilon}$ (see Fig. 2). Note that this threshold velocity is independent of the momentum integration cut-off that is used. Thus, for vortex velocities $v \leq v_t$ there is no constant velocity solution to the equation of motion, unless the external driving current $I = 0$. Taking into account quantum effects changes the spinwave spectrum in such a way that the velocity window in which vortices move over the lattice potential without emitting spinwaves grows larger. The resulting relation between applied current and vortex velocity is shown in Fig. 1 for several values of $v_t$. An extrapolation to the S-I transition yields a diverging threshold velocity and vortices and spinwaves decouple.

V. SHORT RANGE COULOMB INTERACTIONS

In this section we consider the case of short range Coulomb interaction $U$, i.e. the junction capacitance is negligible compared with the capacitance to the ground. This limit is of more relevance for 2-dimensional superconducting films. The system undergoes a $T = 0$ phase transition which belongs to the 2+1-dimensional XY universality class [11]. The critical properties are well captured by a coarse-grained Ginzburg-Landau free energy. The effective free energy has been derived from the Hamiltonian (2) using a Hubbard-Stratonovich transformation [11,27].

It is therefore natural to express the charge-charge correlation function in terms of a Ginzburg-Landau coarse grained order parameter field. The charge-charge correlation can be expressed as a functional derivative of an appropriate generating functional as follows (see Appendix B 1 for more details)

$$\langle q_{j\tau} q_{k\tau'} \rangle = \frac{\delta^2}{\delta \mu_{j\tau} \delta \mu_{k\tau'}} \ln \left[ \int D\bar{\psi} D\psi \exp(-F[\bar{\psi}, \psi, \mu]) \right],$$

where
\[ F = \int d^3 x \left\{ \frac{1}{4} \left| \nabla \psi \right|^2 + r \left| \psi \right|^2 + u \left| \psi \right|^4 + \zeta \left| \psi \right|^2 \partial_\tau - \mu \psi \right|^2 \} , \]  

(21)

and the coefficients are \( r = 1/2E_J - 1/2E_o \), \( u = 7/128E_o^3 \), and \( \zeta = 1/32E_o^3 \). The charge-charge correlation is related to the response of the system to a twist of the boundary conditions in the time direction. Its Fourier transform is

\[ \langle qq \rangle_{k,\omega,\mu} = \zeta \left[ 2\langle \psi(\vec{r},\tau)\psi^*(\vec{r},\tau) \rangle - 4\zeta \int d^2 r d\tau \langle J^x(\vec{r},\tau)J^x(0,0) \rangle e^{ikr + i\omega\tau} \right] \]  

(22)

Where the current in the \( \tau \) direction \( J^\tau \) is defined as

\[ J^\tau = \frac{1}{2i} \left\{ \psi^*(\vec{r},\tau)\partial_\tau \psi(\vec{r},\tau) - \psi(\vec{r},\tau)\partial_\tau \psi^*(\vec{r},\tau) \right\} \]  

(23)

This type of correlation function was extensively investigated in [15]. Due to the isotropy of the model in space-time the \( k = \omega = 0 \) term in Eq. (22) is proportional to the superfluid density \( \rho_s \) of the system.

As discussed previously the adiabatic mass is related to the zero frequency component of the charge-charge correlation function. Therefore one obtains

\[ M_v \sim \rho_s \ln(L/a) + \text{ terms not divergent in } L , \]  

(24)

where the terms independent of \( L \), that do not diverge with the system size, may arise from the \( k \)-dependence of the zero frequency component of the charge-charge correlation function, however, the \( \ln(L/a) \) is dominant in the superconducting case. Close to the transition \( \rho_s \sim (E_J/E_o - 1)^{\beta} \), with \( \beta \approx 2/3 \). We stress that this result is independent on the particular approximation we may choose to evaluate the correlation functions.

More care it is needed right at the transition where it can be shown that the vortex mass does not vanish; we remind that it is determined by an integral of the charge-charge correlation function over the first Brillouin zone. Although \( \langle qq \rangle_{k=\omega=0} \) vanishes, there is an important contribution from the \( k \) dependence of the correlation function. Following [13] it may be calculated employing a 1/N expansion and to leading order

\[ \langle qq \rangle_{k,\omega,\mu} = \frac{1}{32\epsilon E_o} \sqrt{k^2 + \omega_\mu^2/4E_o^2} . \]  

(25)
The k-dependence at zero frequency will regularize the k-integral for the mass and, as a result it will not vanish at the transition but becomes independent of the system size

\[ M_{\text{trans}} = \frac{\pi^{3/2}}{32E_o} \approx 0.44M_o \]  

In the superconducting phase the charge-charge correlation function can be approximated to

\[ \langle qq \rangle_{k,\omega,\mu} \sim \rho_s k^2 / (k^2 + 4\zeta\omega^2_\mu) \]  

Spinwave damping may be calculated from the equation of motion. From Eqs. (12) and (27) a threshold velocity \( v_t = \bar{\omega}_p \sqrt{E_o/8E_j} \) is found, which for the short range Coulomb interacting does not diverge at the transition.

We checked the main conclusions of this section performing simulations that are presented in the next section.

VI. MONTE CARLO SIMULATIONS

We now turn to the Monte Carlo results that allow for a check of the previous calculations and provide information for the region where the self consistent harmonic approximation for the long range case is not valid. Since the CCVG described by Eq. (3) contains an imaginary coupling and long range interactions, it is more convenient \[16\] to simulate the system in the equivalent current loop representation Eq. (A5). The correspondence is simply \( \langle qq \rangle = \langle J^0 J^0 \rangle \). The condition that the currents \( J^\mu \) be divergenceless is taken into account by making Monte Carlo steps that preserve the property \( \nabla_\mu J^\mu = 0 \). Thus, we create or annihilate small current loops, as well as ‘periodic current loops’ that go through the whole system which is taken to have periodic boundary conditions in all three directions. In the case of long range Coulomb interaction periodic current loops in the time direction are forbidden since they violate charge-neutrality as demanded by the logarithmic Coulomb interaction.

The size in the time direction was taken to be 8, which corresponds to a temperature \( T \) equal to one eight of the plasma frequency, which is quite low for a JJA. The simulations
were done on lattices of linear dimension varying from 4 to 12 and the standard Metropolis algorithm was used, with typically 5000 sweeps through the lattice for equilibration and the same amount for measurement. In all cases the full charge-charge correlation function was measured and with the help of Eq.(8) the mass was determined. Reliable results for the correlation length would require larger systems than we were able to simulate.

For logarithmic Coulomb interaction data for the vortex mass is shown in Fig.3. The instanton action $S_{\text{inst}}$ for a vortex hop from a plaquette to a neighboring one and therefore the depinning current behave in a similar way. The critical point is at $E_J/E_C \approx .6$, which agrees with experimental findings [10]. Close to the S-I transition the vortex mass depends strongly on the ratio $E_J/E_C$. Note that apart from small corrections the mass is system size independent.

The results for the vortex mass for onsite Coulomb interaction are shown in Fig.4. The critical point is at $E_J/E_o \approx .85$. The collapse of the curves for different system sizes (see Fig.4b) demonstrates that the vortex mass indeed scales with $\ln(L)$ in the SC phase, whereas in the insulating phase it scales to zero (see the inset of Fig.4b). If the logarithm of the system size is not scaled out, the curves for the mass in systems of different size approximately cross at the transition between 0.4 and 0.5 times $M_o$, which is in good agreement with $M_{\text{trans}}/M_o \approx 0.44$. The region were the mass is strongly dependent on the ratio of the couplings is somewhat larger than in the long range case.

VII. CONCLUSION

We have investigated vortex motion, mass and spinwave damping in Josephson junction arrays. We first derived a one-vortex effective action from which it became clear that dynamical vortex properties are governed by the charge-charge correlation function. We showed how to recover all known results for classical arrays. The Eckern-Schmid mass being proportional to the junction capacitance, was found to be correct in the limit $E_J \gg E_C$, but also the ground capacitance contributes to the classical mass, which becomes system-size
dependent.

In the generic situation vortex motion is affected also by the presence of other vortices (or more precise dipoles in the superconducting state) which are present because of thermal or quantum (this is the case we considered in detail) fluctuations. In the quantum regime close to the S-I transition we investigated the vortex properties analytically and by means of Monte Carlo simulations.

We considered specifically the two extreme cases in which either the self-capacitance $C_o$ or the junction capacitance $C$ was set two zero. In the case of long range Coulomb interactions ($C_o$ equal to zero) the main conclusions are that the mass and depinning current vanish at the phase transition in a way that reflects the nature of the S-I transition, whereas the velocity window in which vortices can move without exciting spinwaves grows. Our predictions for the depinning current are in qualitative agreement with experiment \textsuperscript{29}. Our results suggest that ballistic vortex motion may be seen best in arrays that are close to the S-I transition.

The limit of on-site Coulomb interaction seems more appropriate for granular or uniform films. In this case the mass has a logarithmic dependence on the size of the sample in the superconducting region. It is proportional to the superfluid density, and therefore shows critical behaviour approaching the superconductor-insulator transition. At the transition it does not vanish, but becomes scale independent.

Using Monte Carlo simulations for determining the charge-charge correlation function numerically, we were able to verify several conclusions of the self consistent harmonic approximation (for long range Coulomb interactions) and the coarse-graining approach (for short range Coulomb interactions).

In all the results we presented we were mainly concerned with the superconducting side of the S-I transition, allthough the equations derived in section II are in principle valid throughout the phase diagram. We might also investigate the dynamical properties of the vortices in the insulating region. This is, however, not useful since the vortices are massless, delocalized, and strongly fluctuating in the insulating phase. In particular when $E_C \gg E_J$ the charge on the superconducting islands is a good quantum number and it is more useful to
describe the quantum dynamics of charges in the resistive (or insulating) region, which may be investigated using the same techniques as presented here for vortices. In the resistive (high temperature) phase the concept of vortex is also not useful as the system is not globally superconducting and the superconducting phases of the islands are disordered. We may also add that the main approximation that leads to the vortex effective action, namely the truncation of the cumulant expansion to the terms quadratic in the velocities may not be justified in the disordered (both resistive and insulating) phases. Probably in this case a more complicated equation of motion that includes terms proportional to higher powers of the velocity should be included.

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APPENDIX A: DUALITY TRANSFORMATIONS

This appendix reviews shortly how to pass from Eq.(2) to Eq.(3). For more details and the treatment of the external current contribution we refer to Ref. [18]. We start from the basic expression for the partition function

$$Z = \text{Tr} \exp(-\beta \hat{H}),$$

(A1)

where $\beta$ is the inverse temperature $T$ and $\hat{H}$ the Hamiltonian Eq.(2). We go over to a Euclidean path-integral formulation by introducing time-slices, i.e. dividing $\beta$ in $N_\tau$ intervals of size $\epsilon$, such that $N_\tau \epsilon = \beta$. Inserting complete sets of states at each time slice we arrive at

$$Z = \sum_{\{q_{i\tau}\}} \int \mathcal{D}\phi_{i\tau} \exp \left\{ -\frac{2\epsilon E_C}{\pi} \sum_{i,j\tau} q_{i\tau} U_{ij} q_{j\tau} + i \sum_{i\tau} q_{i\tau} \dot{\phi}_{i\tau} + \epsilon E_J \sum_{<ij>\tau} \cos(\phi_{i\tau} - \phi_{j\tau}) \right\} $$

(A2)

In Fourier components, the electrostatic interaction between the charges is $U_k = 2\pi/(k^2 + \lambda^2)$, where the inverse range of the interaction is $\lambda = \sqrt{C_o/C}$. It is related to the inverse
capacitance matrix by $U_{ij} = 2\pi C C_{ij}^{-1}$. Now we make the Villain approximation \[30\] for the cosine term

$$\exp(\epsilon E_J \cos(\phi_{i\tau} - \phi_{j\tau})\epsilon E_J) \approx \sum_{\{n_{ij,\tau}\}} \exp\left(-\frac{\epsilon E_J}{2} f(\epsilon E_J)(\phi_{i\tau} - \phi_{j\tau} - 2\pi n_{ij,\tau})^2\right), \quad (A3)$$

where $n$ is a directed discrete field that lives on the bonds between lattice sites. The function $f$ equals unity if its argument is large, i.e. when the product $\epsilon E_J$ is not too small. After a subsequent Poisson resummation, i.e. writing

$$\sum_{\{n_{ij,\tau}\}} F[n_{ij,\tau}] = \sum_{\{J_{ij,\tau}\}} \int DnF[n_{ij,\tau}] \exp(2\pi i \sum_{ij,\tau} n_{ij,\tau} J_{ij,\tau}), \quad (A4)$$

an integration over the fields $n_{ij,\tau}$ and the phases $\phi_{i\tau}$ yields a representation in terms of divergenceless discrete current loops

$$Z = \sum_{\{J_{i,\tau}^a\}} \delta(\nabla_{\mu} J_{i,\tau}^\mu) \exp\left\{-\frac{2\epsilon E_C}{\pi} \sum_{ij,\tau} J_{ij,\tau}^0 U_{ij} J_{ij,\tau}^0 - \frac{1}{2\epsilon E_J} \sum_{i,\tau,a} (J_{i,\tau}^a)^2\right\}, \quad (A5)$$

where $a = x, y$ and $\mu = x, y, 0$. Here the time components $J^0_{ij}$ of the current are simply the charges. The vortex degrees of freedom may now be extracted by solving the constraint in Eq.(A4) by writing $J_{i,\tau}^a = \epsilon_{ab} \nabla_b \psi_{i\tau} - O^a q_{i\tau}$ (the operator $O^a$ denotes the line integral in direction $a$, i.e. it is the inverse of $\nabla_a$) and making a final Poisson resummation on the discrete field $\psi$. Making use of the identity $\Theta_{ij} = \epsilon_{ab} \nabla^b G_{ij}$ Eq.(3) results.

Note that we did not keep track of determinants, as they are irrelevant for the present purpose. The requirements that both the factorization of $\exp(-\beta H)$ and the Villain approximation are valid restricts $\epsilon$ to be of the order of the inverse of the plasma frequency $\omega_p = \sqrt{8E_J E_C}$ for long and $\bar{\omega}_p = \sqrt{8E_J E_o}$ for short range Coulomb interactions, i.e. we take $\epsilon = p/\omega_p$ or $\epsilon = p/\bar{\omega}_p$. Details and numerical factors may depend on the exact choice for $p$. We take $p = 1$, except for the self consistent harmonic approximation where we take $p = \sqrt{2}$. The plasma frequency is the natural frequency for spinwaves.
APPENDIX B: THE CHARGE-CHARGE CORRELATION FUNCTION

In general the charge-charge correlation function may be expressed as a functional derivative of the free energy in the following way

\[ \langle q_{jt} q_{kt'} \rangle = \frac{\delta^2}{\delta \mu_{jt} \delta \mu_{kt'}} \ln \left[ \int D \xi \exp \left( -S[\xi] + \sum_{i\tau} \mu_{i\tau} q_{i\tau} \right) \right]_{\mu=0}. \]  

(B1)

Here \( \xi \) denotes the field or fields that are integrated (or summed) over, \( S[\xi] \) a corresponding action and \( \int D \xi \) the appropriate measure.

1. long range Coulomb interactions

For instance, if one takes the action to be the charge part of the CCVG action, as is done in section [IV], one has

\[ \langle q_{kt} q_{lt'} \rangle = \frac{\delta^2}{\delta \mu_{kt} \delta \mu_{lt'}} \ln \left[ \sum_{l_{i\tau}} \exp \left( - \sum_{ij\tau\tau'} q_{i\tau} Q^{-1}_{ij\tau\tau'} q_{j\tau'} + \sum_{i\tau} \mu_{i\tau} q_{i\tau} \right) \right]_{\mu=0}. \]  

(B2)

The kernel \( Q \) was defined in Eq.(3). Now the partition function may be rewritten in terms of different (dual) fields and if we keep track of the 'currents' \( \mu \) during the transformation we may express the charge-charge correlation function in terms of the correlation function of the new fields. Applying this strategy to Eq.(B2), we find after a Poisson resummation

\[ \langle q_{kt} q_{lt'} \rangle = \frac{\delta^2}{\delta \mu_{kt} \delta \mu_{lt'}} \ln \left[ \sum_{l_{i\tau}} \exp \left( - \pi^2 \sum_{ij\tau\tau'} l_{i\tau} Q^{-1}_{ij\tau\tau'} l_{j\tau'} + i\pi \sum_{ij\tau\tau'} l_{i\tau} Q^{-1}_{ij\tau\tau'} \mu_{j\tau'} + \frac{1}{4} \sum_{ij\tau\tau'} \mu_{i\tau} Q^{-1}_{ij\tau\tau'} \mu_{j\tau'} \right) \right]_{\mu=0}, \]  

(B3)

from which we read of immediately that

\[ \langle q_{kt} q_{lt'} \rangle = \frac{1}{2} Q^{-1}_{ktlt'} - \pi^2 \sum_{mn\tau\tau'} Q^{-1}_{kmn\tau} \langle l_{m\tau} l_{n\tau'} \rangle Q^{-1}_{nlt'} \]  

(B4)

This has simplified the problem of calculating the correlation function considerably, since the new field \( l \) interacts with the kernel \( Q^{-1} \) which is, in contrast to the original kernel \( Q \), short range in both the space and time directions. This discrete Gaussian model is therefore convenient for Monte Carlo simulations.
Another representation for the correlation function is found by applying an inverse Villain approximation, see chapter 11 of Ref. [25] for details. This enables one to rewrite the partition function as a path integral over a continuous field with sine-Gordon action. The result is very similar to Eq.(B4)

\[\langle q_{j\tau} q_{k\tau'} \rangle = \frac{1}{2} Q_{jk\tau\tau'} - \pi^2 \sum_{m\neq t} Q_{jm\tau\tau'}^{-1} \langle \phi_{mt} \phi_{nt'} \rangle Q_{nk\tau\tau'}^{-1}, \]  

but now the correlation function of the dual variables \( \phi \) is now to be calculated using the action [25]

\[S[\phi] = \pi^2 \sum_{ij} \sum_{tt'} \phi_{it} Q_{ijtt'}^{-1} \phi_{jt'} - H \sum_{it} \cos(2\pi\phi_{it}) \]  

In this formulation we may employ the self consistent harmonic approximation [26].

2. short range Coulomb interactions

Here we outline the derivation of Eq.(21) in the text. For details we refer to Ref. [27]. In the case of short range interaction, the critical properties of the system are well described by a coarse grained free energy. The generating functional that we consider is the partition function Eq.(A2) with a coupling to a current \( \mu \) as in Eq.(B1). After performing the integration over \( Dq \) we have in terms of phase-variables only

\[\langle q_{jit} q_{ks} \rangle = \frac{\delta^2}{\delta \mu_{ji} \delta \mu_{ks}} \ln \left[ \int D\phi \exp \left( \frac{1}{8e^2} \sum_{ij} \int d\tau \left[ \dot{\phi}_i(\tau) + i\mu_i(\tau) \right] C_{ij} \left[ \dot{\phi}_j(\tau) + i\mu_j(\tau) \right] \right) - E_J \sum_{<ij>} \int d\tau \cos(\phi_i - \phi_j) \right], \]  

where the measure \( D\phi \) still contains a summation over winding numbers \( m_i \) (i.e. \( \phi_i(\beta) = \phi_i(0) + 2\pi m_i \)) in order to account for the discreteness of the charges. Decoupling the Josephson term by means of a Hubbard-Stratonovich transformation one gets:

\[\langle q_{jit} q_{ks} \rangle = \frac{\delta^2}{\delta \mu_{ji} \delta \mu_{ks}} \ln \left[ \int D\psi D\bar{\psi} \exp \left\{ \frac{1}{2E_J} \sum_{ij} \int d\tau \left[ \bar{\psi}_i(\tau) t_{<ij>}^{-1} \psi_j(\tau) \right] \right\} \right] \left\{ \exp \left\{ \sum_{i} \int d\tau \left[ \bar{\psi}_i(\tau) e^{i\phi_i(\tau)} \right] \right\} \right], \]  

In this formulation we may employ the self consistent harmonic approximation [26].
where the matrix $t_{<ij>}$ is one for nearest neighbors and zero otherwise. The average in the last factor is with respect to the remainder of the action in (B7). Close to the transition point we may expand the average in powers (cumulants) of the fields $\psi$; this expansion yields the generating functional used in the paper. We note that the currents $\mu$ enter the time derivatives in the generating functional Eq.(21) in a gauge invariant way.
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Figure Captions:

Fig. 1 : Driving-current vs. vortex velocity relations for long range Coulomb interactions and several couplings. From left to right: \( v_t = 0 \) (for \( E_J/E_C \gg 1 \), the classical limit, see [7]), \( v_t/\omega_p = 0.5 \) and \( v_t/\omega_p = 1 \). The vertical dotted line indicates the minimum velocity a vortex needs to move ballistically over the pinning potential.

Fig. 2 : Mass, threshold velocity and coherence length as from the self consistent harmonic approximation.

Fig. 3 : Monte Carlo result for the mass with long range interactions. Shown are a 6x6x8 (crosses) and a 10x10x8 (diamonds) system. The statistical errors are of the order of the symbol size.

Fig. 4 : Monte Carlo results for the mass with on-site interactions. a) Shown are a 4x4x8 (lower curve), 6x6x8, 8x8x8, 10x10x8 and a 12x12x8 (upper curve) system. The statistical errors are of the order of the symbol size.  

b) The mass for on-site interactions with the logarithmic of the system size scaled out. Clearly the curves collapse on one line in the SC phase, but \emph{not} in the critical region. Inset: The size dependence of the mass with the logarithm scaled out for \( E_J/E_o = 1.45, 1.1, .95, .85, .75 \) (from top to bottom).