Modeling of short duration extreme rainfall events over Lower Yamuna Catchment

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ABSTRACT. Short duration rainfall estimates and their intensities for different return periods are required for many purposes such as for designing flood for hydraulic structures, urban flooding etc. An attempt has been made in this paper to Model extreme rainfall events of Short Duration over Lower Yamuna Catchment. Annual extreme rainfall series and their intensities were analysed using EVI distribution for rainstorms of short duration of 5, 10, 15, 30, 45 & 60 minutes and various return periods have been computed. The Self recording rainguage (SRRGs) data for the period 1988-2009 over the Lower Yamuna Catchment (LYC) have been used in this study. It has been found that EVI distribution fits well, tested by Kolmogorov-Smirnov goodness of fit test at 5 % level of significance for each of the station.

Key words – Extreme rainfall series, Short duration, Modeling, EVI, Gumbel distribution, Kolmogorov-Smirnov.

1. Introduction

Modeling of extreme rainfall along with their intensities of short duration ≤ 1 hour etc. for various return periods for any region are required for many purposes such as for fixing the design floods, designing of water related structures, hydrological design of railway, road bridges and their protection works, design of culverts, drainage & storm sewers in urban areas, irrigation, flood control and flood forecasting. The design engineers, requires estimates of rainfall extremes of short duration specially ≤ 1 hour and in this field there have been limited studies. A large amount of the variability of rainfall is due to the occurrence of extreme rainfall events. Therefore, the amount of extreme rainfall events over different parts of the area under study are needed. The study of spatial variability of extreme rainfall events helps to identify the areas of high and low values. The Rainfall frequency maps of India (Harihara Ayyar and Tripathi, 1974) published by India Meteorological department for different duration and also India Meteorological department 2009 published Atlas of State wise Genaralised Isopluvial (Return Period) maps for 24 hrs give estimate of rainfall for different return periods. But these maps do not show values specially for short duration of the order of ≤ 1 hr, whereas it is observed generally that high rainfall intensities are occurred for short duration. Studies on short duration rainfall analysis have been carried out by Dhar and Kulkarni [1970 (b), 1971] over U.P. & south India respectively, Dhar & Ramchandran [1970 (a)] over Kolkata. The Extreme value type I (Gumbel, 1958) distribution is one of the most widely used model for the probabilistic characterization of a variety of extreme
### TABLE 1 (a-f)

Statistical Rainfall intensity (RI) estimates (mm/hr) of annual extreme series

(a) RI for rainstorms of 5 minutes duration

| S. No. | Station (Dist.) Lat./ Long. | No. of years of record | Kolmogorov-Smirnov statistic D | Mean C.V. (%) | Maximum | Return periods in years |
|--------|-----------------------------|------------------------|--------------------------------|---------------|----------|------------------------|
|        |                             |                        |                                |               |          | 2         | 5         | 10        | 25        | 50        | 100       |
| 1.     | Poiyaghat (Agra) 27° 16' N  78° 02' E | 15                    | 0.25403                        | 120           | 14       | 144       | 108       | 132       | 131       | 144       | 156       | 168       |
| 2.     | Rajghat (Lalitpur) 24° 50' N  78° 12' E | 16                    | 0.2559                         | 132           | 32       | 240       | 132       | 168       | 204       | 228       | 264       | 288       |
| 3.     | Nautghat (Jhansi) 25° 22' N  78° 37' E | 16                    | 0.2559                         | 108           | 33       | 156       | 108       | 132       | 156       | 180       | 204       | 216       |
| 4.     | Mohana (Betwa) 25° 42' N  79° 25' E | 17                    | 0.2763                         | 120           | 28       | 180       | 120       | 144       | 168       | 198       | 216       | 228       |
| 5.     | Auraiya (Auraiya) 26° 26' N  79° 05' E | 19                    | 0.24468                        | 120           | 26       | 180       | 108       | 132       | 156       | 180       | 192       | 204       |
| 6.     | Kalpi (Jalaun) 26° 12' N  79° 42' E | 17                    | 0.24811                        | 108           | 33       | 168       | 96        | 132       | 156       | 180       | 192       | 216       |
| 7.     | Kaimaha (Mahoba) 25° 06' N  79° 50' E | 19                    | 0.2714                         | 90            | 36       | 132       | 84        | 108       | 126       | 150       | 168       | 186       |
| 8.     | Banda (Banda) 25° 29' N  80° 18' E | 18                    | 0.26063                        | 144           | 32       | 240       | 132       | 168       | 192       | 216       | 240       | 264       |
| 9.     | Chillaghat (Banda) 25° 47' N  80° 31' E | 22                    | 0.19782                        | 132           | 37       | 240       | 120       | 168       | 192       | 228       | 252       | 276       |

(b) RI for rainstorms of 10 minutes duration

| S. No. | Station Lat./ Long. | Kolmogorov-Smirnov statistic D | Mean C.V. (%) | Maximum | Return periods in years |
|--------|---------------------|-------------------------------|---------------|----------|------------------------|
|        |                     |                               |               |          | 2         | 5         | 10        | 25        | 50        | 100       |
| 1.     | Poiyaghat           | 0.17247                       | 96            | 27       | 132       | 90        | 114       | 132       | 150       | 162       | 174       |
| 2.     | Rajghat             | 0.1475                        | 108           | 31       | 180       | 102       | 132       | 150       | 174       | 192       | 210       |
| 3.     | Nautghat            | 0.2714                        | 90            | 36       | 132       | 84        | 108       | 126       | 150       | 168       | 186       |
| 4.     | Mohana (Betwa)      | 0.17728                       | 102           | 29       | 180       | 96        | 126       | 144       | 162       | 180       | 198       |
| 5.     | Auraiya             | 0.1264                        | 90            | 34       | 168       | 72        | 114       | 132       | 156       | 168       | 186       |
| 6.     | Kalpi               | 0.2636                        | 96            | 28       | 120       | 90        | 114       | 132       | 150       | 162       | 180       |
| 7.     | Kaimaha             | 0.24835                       | 108           | 23       | 150       | 102       | 126       | 138       | 156       | 174       | 186       |
| 8.     | Banda               | 0.11173                       | 108           | 36       | 180       | 102       | 138       | 162       | 192       | 216       | 234       |
| 9.     | Chillaghat          | 0.12885                       | 102           | 33       | 180       | 96        | 126       | 144       | 168       | 192       | 210       |

(c) RI for rainstorms of 15 minutes duration

| S. No. | Station Lat./ Long. | Kolmogorov-Smirnov statistic D | Mean C.V. (%) | Maximum | Return periods in years |
|--------|---------------------|-------------------------------|---------------|----------|------------------------|
|        |                     |                               |               |          | 2         | 5         | 10        | 25        | 50        | 100       |
| 1.     | Poiyaghat           | 0.1924                        | 80            | 28       | 124       | 76        | 96        | 108       | 124       | 136       | 148       |
| 2.     | Rajghat             | 0.1770                        | 96            | 27       | 160       | 92        | 116       | 132       | 152       | 164       | 180       |
| 3.     | Nautghat            | 0.1561                        | 80            | 37       | 120       | 76        | 104       | 120       | 144       | 160       | 174       |
| 4.     | Mohana (Betwa)      | 0.13461                       | 92            | 29       | 140       | 88        | 112       | 128       | 148       | 148       | 180       |
| 5.     | Auraiya             | 0.1439                        | 80            | 33       | 140       | 76        | 100       | 116       | 132       | 148       | 164       |
| 6.     | Kalpi               | 0.19354                       | 92            | 26       | 120       | 88        | 104       | 120       | 140       | 152       | 164       |
| 7.     | Kaimaha             | 0.17538                       | 100           | 26       | 140       | 96        | 116       | 132       | 152       | 168       | 180       |
| 8.     | Banda               | 0.18323                       | 92            | 36       | 152       | 88        | 116       | 136       | 160       | 180       | 196       |
| 9.     | Chillaghat          | 0.25182                       | 92            | 30       | 160       | 88        | 112       | 128       | 148       | 164       | 180       |
(d) RI for rainstorms of 30 minutes duration

| S. No. | Station         | Kolmogorov-Smirnov statistic D | Mean (cm) | C.V. (%) | Maximum (cm) | Return periods in years |
|-------|----------------|-------------------------------|-----------|----------|--------------|------------------------|
| 1     | Poiyaghat      | 0.13837                       | 60        | 30       | 100          | 58 74 84 96 106 116   |
| 2     | Rajghat        | 0.1401                        | 70        | 32       | 132          | 66 86 98 114 128 140  |
| 3     | Nautghat       | 0.1382                        | 62        | 42       | 96           | 58 82 96 116 130 144  |
| 4     | Mohana (Betwa) | 0.12009                       | 72        | 27       | 112          | 68 84 96 110 120 130  |
| 5     | Auraiya        | 0.1153                        | 60        | 39       | 110          | 56 76 90 108 120 134  |
| 6     | Kalpi          | 0.1739                        | 70        | 31       | 100          | 66 84 98 114 126 138  |
| 7     | Kaimaha        | 0.19577                       | 72        | 27       | 120          | 68 86 96 110 122 132  |
| 8     | Banda          | 0.18002                       | 66        | 39       | 136          | 62 86 100 120 134 148 |
| 9     | Chilaghat      | 0.09548                       | 68        | 31       | 120          | 64 82 94 110 120 132  |

(e) RI for rainstorms of 45 minutes duration

| S. No. | Station         | Kolmogorov-Smirnov statistic D | Mean (cm) | C.V. (%) | Maximum (cm) | Return periods in years |
|-------|----------------|-------------------------------|-----------|----------|--------------|------------------------|
| 1     | Poiyaghat      | 0.15903                       | 39        | 34       | 94           | 47 61 72 84 91 102    |
| 2     | Rajghat        | 0.1234                        | 53        | 34       | 105          | 51 67 77 90 101 110   |
| 3     | Nautghat       | 0.2150                        | 53        | 41       | 84           | 49 69 81 97 109 121   |
| 4     | Mohana (Betwa) | 0.10440                       | 56        | 28       | 87           | 53 67 75 87 94 102    |
| 5     | Auraiya        | 0.1288                        | 51        | 38       | 93           | 48 65 77 92 102 113   |
| 6     | Kalpi          | 0.12031                       | 60        | 30       | 93           | 58 73 84 98 108 118   |
| 7     | Kaimaha        | 0.15432                       | 57        | 27       | 100          | 55 69 75 89 97 106    |
| 8     | Banda          | 0.13813                       | 53        | 42       | 115          | 49 69 81 98 110 122   |
| 9     | Chilaghat      | 0.19782                       | 55        | 35       | 126          | 52 69 80 94 105 116   |

(f) RI for rainstorms of 60 minutes duration

| S. No. | Station   | Kolmogorov-Smirnov statistic D | Mean (cm) | C.V. (%) | Maximum (cm) | Return periods in years |
|-------|-----------|-------------------------------|-----------|----------|--------------|------------------------|
| 1     | Poiyaghat | 017952                         | 42        | 35       | 80           | 40 53 61 72 80 88     |
| 2     | Rajghat   | 0.1390                         | 47        | 32       | 81           | 44 58 67 78 86 94     |
| 3     | Nautghat  | 0.1822                         | 47        | 43       | 80           | 44 62 74 90 101 112   |
| 4     | Mohana (Betwa) | 0.23304                     | 46        | 30       | 73           | 44 56 64 75 82 90     |
| 5     | Auraiya   | 0.1190                         | 42        | 35       | 70           | 39 52 60 71 79 87     |
| 6     | Kalpi     | 0.13694                        | 50        | 31       | 75           | 47 61 70 82 90 98     |
| 7     | Kaimaha   | 0.19328                        | 49        | 29       | 92           | 46 59 67 78 86 93     |
| 8     | Banda     | 0.1294                         | 45        | 38       | 88           | 42 57 68 80 90 99     |
| 9     | Chilaghat | 48                              | 37        | 110      | 45           | 60 71 84 93 103       |
hydro meteorological rainfall series. Rao et al. (1983) in his paper over lower Godavari basin presented a technique to estimate short duration extreme rainfall for a given return periods from daily (24 hr) rainfall records for any area. Harihara Ayyar and Tripathi (1973) suggested in his study that a daily rainfall estimate should be converted in to any 24 hr rainfall estimate by multiplying by a factor of 1.15 and then a relationship already established should be used to deduce short duration rainfall estimate for each station. Singh (1989) found EVI as a better distribution than gamma in his study at Colaba, Bombay, for extreme series using estimated parameters by moments and maximm likelihood method. Goel and Kathuria (1984) applied the Gumbel Fisher & Tipper type II distribution to annual extreme rainfall series over Krishna basin, Mukherjee et al. (1991) for extreme rainfall concluded that extreme value type I distribution fits adequately the annual maximum (daily) rainfall series over most of the country except for west Rajasthan and Saurashtra & Kutch. In this paper, EVI distribution has been fitted to rainfall extreme series and their intensities (mm/hr) of short duration 5, 10, 15, 30, 45 and 60 minutes using method of moments and the spatial variation is also presented.

2. Methodology

The Cumulative function is given by

\[ F(x) = \exp \left[ - \exp \left\{ \frac{(x-u)}{\alpha} \right\} \right], \quad \alpha, u > 0, \quad -\infty < x < \infty \]
Where \( x \) represents the extreme rainfall series and \( \alpha \) & \( u \) are the shape and location parameters of the distribution respectively. In terms of the reduced variate \( Y = (x-u) / \alpha \), Eqn. (1) becomes:

\[
F(x) = \exp \left[ - \exp \left(-\frac{Y}{\alpha}\right) \right]
\]

(2)

If \( X_T \) is the \( T \)-year event value of the variable \( x \), and \( Y_T \) the corresponding value of reduced variate \( Y \), Eqn. (2) becomes:

\[
F(x) = 1 - \frac{1}{T} = \exp \left[ - \exp \left(-Y_T\right) \right]
\]

(3)

Then, \( Y_T = \ln \ln \left[ \frac{T}{(T-1)} \right] \)

(4)

and \( X_T = u + \alpha Y_T \)

(5)

The rainfall estimate for a particular return period (\( T \)) may be calculated by the equations 4 & 5. The standard error for the rainfall estimate is given by:

\[
\text{SE} (X_T) = \frac{1}{\alpha} \sqrt{\frac{6}{\pi}} \left( 1.11 + 0.52 Y_T + 0.61 Y_T^2 \right)
\]

(6)

Among from the several methods, in this paper selected the method of moments to fit EVI distribution to the extreme of SRRG hydro-meteorological observatories (HMO) listed in Table 1 in the LYC region. The moments estimators of the parameter \( \alpha \) and \( u \) of the EVI distribution are as follows:

\[
\alpha = \text{S.D.} \sqrt{\frac{6}{\pi}} = 0.7797 \times \text{S.D.}
\]

\[
u = - \frac{\text{S.D.} \sqrt{\frac{6}{\pi}}}{x -0.45 \times \text{S.D.}}
\]

(\( x \) = Euler’s constant = 0.5772)

Where \( \bar{x} \) and SD are the mean and standard deviation of the extreme series.

3. Data and analysis

3.1. Fitting of annual extreme rainfall series

The annual extreme rainfall series have been constructed by extracting the highest rainfall by analyzing the daily autographic 24 hourly rainfall data for durations of 5, 10, 15, 30, 45, 60 minutes at 9 HMO stations under LYC region listed in Tables 1(a-f) for the period (1988-2009), the period of data varied from 15 to 22 years which are shown in the Table 1 (a) stating the no. of years of records. The location of these Self Recording Rain Gauges (SRRGs) in LYC are shown in Fig. 1. These annual extreme series for the above mentioned durations at each station were subjected to frequency analysis by EVI using the method of moments and the rainfall estimates of different return periods \( T \) viz., 2, 5, 10, 25, 50 & 100 years for different durations were obtained by using Eqs. 4 & 5 and the corresponding standard errors \[\text{SE} (X_T)\] computed by using the Eqn. 6.

The different estimates \( (X_T) \) so computed for various short duration were plotted on probability paper for EVI distribution along with confidence limit (± 2SE) for each station. The observed extreme rainfall values for each HMO were also plotted on the probability paper by using Gringorton’s (1963) plotting position using empirical expression for return period \( T \) as

\[
T = N + 0.12 / m - 0.44
\]

(7)

Where \( m \) is the rank of the observed series arranged in descending order and \( N \) is the total no. of observations. It is observed from these plots as depicted in Figs. 2 (a-r) that EVI distribution fits satisfactorily because the observed rainfall values are lying between the confidence intervals for each station.

3.2. Descriptive rainfall intensities statistics

From the daily rainfall series annual maximum rainfall intensities (RI) in mm/hr of rainstorms of duration 5, 10, 15, 30, 45, 60 minutes are constructed for each of the 9 stations, Tables 1 (a-f) show their Maximum, Mean & C.V. (%) values.

(a) For 5 minutes duration of annual extreme series at various stations, Maximum RI over the stations shows variation from 144 mm/hr at Poyyaghat to 240 mm/hr over Banda and Chillaghat. This variation decreases as the duration of rainstorm increases.

(b) The lowest C.V. of 14% is observed at station Poyyaghat and highest C.V. of 37% is observed at station Chillaghat (Banda) of 5 minutes duration rainstorms. Thus variability of RI of rainstorm of 5 minutes duration is high in the area of high value of extreme RI and vice-versa.

(c) Generally C.V. value increases as duration of rainstorms increases at station Poyyaghat, Naughtghat, Kaimah and Banda while at Chillaghat station, first it decreases from duration 5 minutes to 15 minutes and then it increases as duration increases, for remaining stations its values fluctuate.
Figs. 2(a-f), Contd.
Figs. 2(g-l). Contd.
Figs. 2 (a-r). Fitting of EVI distribution to the annual extreme rainfall series of duration (a-i) 5, 15 & 45 minutes and (j-r) 10, 30 & 60 minutes respectively for different stations.
3.3. Spatial distribution of Rainfall intensities of extreme rain events

The estimates of RI have been computed up to 100 years return period. Using the method of moments, the rainfall intensities (RI) for each station for different return periods (R.P.) of 2, 5, 10, 25, 50 and 100 years are obtained for rainstorms of short durations viz., 5 minutes, 10 minutes, 15 minutes, 30 minutes, 45 minutes, 1 hour, these are presented in Tables 1 (a-f). From these tables, emerge that:

(a) The magnitude of RI decreases as the duration of the rainstorms increases. The station Poiyaghat (Agra) has the minimum R.P. value and the station Banda has maximum R.P. value for most of the duration of rainstorms. This resembles with the pattern of normal rainfall features i.e., the amount of rainfall increases from west to east along LYC region.

(b) The highest RI (mm/hr) of 132 for rainstorm of 5 minutes duration has been observed over stations Banda, Kaimah & Rajghat for 2 year return period while for 100 years R.P. value, it enhances to 288 (mm/hr).

(c) The variation in the value of RI over LYC decreases as duration increases and it increases as R.P. value increases.

(d) The Kolmogorov-Smirnov goodness of fit test was used for testing the fitting of EVI distribution. Tables 1 (a-f) also depicts the Kolmogorov-Smirnov goodness of fit statistic D computed from the time series of annual extreme RI of duration 5, 10, 15, 30, 45 minutes & 1 hour. The values are compared with the tabular values and these supported the hypothesis that EVI distribution can not be rejected at 5% level of significance for each of station in the LYC. Thus EVI distribution is a good fit for series of annual extreme of RI of various duration discussed above.

4. Conclusion

Modeling of extreme rainfall and their intensities for short duration (5, 10, 15, 30, 45 & 60 minutes) have been estimated using EVI distribution. The following points concluded:

(a) Generally the high value of extreme RI of rainstorms of various duration are observed along the eastern side of the catchment and low value along western side of LYC. This resembles with normal rainfall pattern. Also the low CV observed in western side and high CV value are toward eastern side in LYC region.

(b) The variation in extremes RI decreases as duration increases.

(c) Annual extreme rainfall and their intensities have been analysed for all stations by EVI distribution over LYC and concluded that EVI distribution fits adequately well over LYC as tested by Kolmogorov-Smirnov goodness of fit test.

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