BBR-induced Stark shifts and level broadening in a helium atom

T Zalialiutdinov\textsuperscript{1,3}, D Solovyev\textsuperscript{1} and L Labzowsky\textsuperscript{1,2}

\textsuperscript{1}Department of Physics, St. Petersburg State University, Ulianovskaya 1, Petrodvorets, St. Petersburg 198504, Russia
\textsuperscript{2}Petersburg Nuclear Physics Institute, 188300, Gatchina, St. Petersburg, Russia

E-mail: zalialiutdinov@gmail.com

Received 9 July 2017, revised 19 August 2017
Accepted for publication 22 September 2017
Published 28 November 2017

Abstract
The precise calculations of blackbody radiation (BBR)-induced Stark shifts and depopulation rates for low-lying states of a helium atom with the use of the variational approach are presented. An effect of the BBR-induced Stark-mixing of energy levels is considered. It is shown that this effect leads to a significant reduction in the lifetimes of helium-excited states. As a consequence, the influence of the Stark-mixing effect on the decay rates of metastable states in helium is discussed in the context of formation processes of the cosmic microwave background.

Keywords: quantum electrodynamics, helium, atomic transitions, stark shifts

1. Introduction
The influence of blackbody radiation (BBR) on the atom is an important topic of modern atomic physics. The interest in these investigations is stimulated by essential progress in the theoretical and experimental research of atomic clocks and the determination of frequency standards [1] that requires a comprehensive analysis of the influence of BBR on an atom [2].

Recently, the temperature-dependent one-loop self-energy (SE) correction of bound atomic electron states was considered in [3] within the framework of quantum electrodynamics (QED). According to the results of [3], an energy shift arises as the real part of SE correction, while the imaginary part represents the BBR-induced depopulation rate for a given atomic state. In particular, it was shown that the regularization of divergent energy denominators in thermal SE correction leads to an additional contribution to level width. This contribution has not been considered before and can be explained by the level mixing of atomic states with opposite parity.

The effect of level mixing produces a more significant broadening of the emission line than the well-known depopulation rates induced by BBR [4]. According to [3], the BBR-induced level mixing leads to a significant reduction of lifetimes even at room temperature. The results of the evaluation of BBR-induced level-mixing widths show that the width of a 2s state becomes comparable with the width of a 2p state at \( T \approx 3000 \) K. This value is about eight orders of magnitude higher than the natural width, \( \Gamma_{2s} = 8.229 \) s\(^{-1}\). Moreover, in that case a decay of the 2s state occurs with the emission of an E1 photon. Although this effect is negligible for low temperatures, \( T \approx 3 \) K, it could be important [5] for the ionization history of primordial plasma where the 2s state in a hydrogen atom plays a crucial role in the formation processes of the cosmic microwave background (CMB) [6, 7]. For the same reasons, the BBR-induced level mixing effect and corresponding line broadening should be significant for the helium atom since its recombination occurs earlier than hydrogen and takes place at temperatures of \( 4 \times 10^3 < T < 10^4 \) K [8, 9].

Another reason for the further investigation of the influence of BBR on atoms is the recent accurate measurements of light atoms by spectroscopic methods. With the exception of hydrogen, the most promising atomic system with a sufficiently accurate theory is the helium atom. The advanced precise methods for the solution of the three-body atomic problem were developed in [10–12]. The accuracy of the laser spectroscopy experiments of the fine-structure splitting in the \( 2p_f \) levels of \(^4\)He [13] reaches 0.13 kHz, allowing an independent determination of the fine-structure constant \( \alpha \) with a precision of \( 2 \times 10^{-9} \). Moreover, during the last few decades, the accuracy of experiments with helium has reached a level where the determination of nuclear parameters is possible and, in particular, the determination of the nuclear charge radius [14, 15].
In the present paper, we calculate BBR-induced Stark shifts, depopulation rates and BBR-induced level mixing for the helium atom within a variational approach developed in [10]. The paper is organized as follows. The theory of the BBR-induced Stark shift is presented in section 2. A contribution to the line broadening corresponding to the BBR-induced level mixing is considered in section 3. In section 4, a brief overview of the variational approach and computational details are considered. Section 5 is devoted to the conclusions and discussion of the mixing effect in a helium atom to the recombination processes in the early universe. Relativistic units (\(\hbar = c = m = 1\)) are used throughout this paper unless otherwise stated.

2. Stark shift and depopulation rates

To describe the effects induced by BBR, the quantum mechanical (QM) approach [4, 16] is usually applied. An extensive use of nonrelativistic QED theory was employed in [17, 18] in the search for the finite-temperature effects in bound states. Rigorous QED were applied to derive the Stark shift and widths of the atomic energy levels induced by BBR in [3], where a perfect agreement between the QED (in the nonrelativistic limit) and QM results was also demonstrated. The QED description of the BBR-induced Stark shift has shown that, besides the well-known ac-Stark shift and depopulation rates, the quadratic mixing effect of states with opposite parity also arises in the mean BBR electric field [3]. The latter can be estimated with the root mean squared (rms) value (in a.u.)

\[
\langle E^2(\omega) \rangle = \frac{8\alpha^3}{\pi} \omega^3 \frac{e^{3\omega T} - 1}{e^{3\omega T} - 1},
\]

where \(\beta = 1/k_B T\), \(k_B\) is the Boltzmann’s constant, \(T\) is the radiation temperature in Kelvin, \(\alpha\) is the fine structure constant and \(\omega\) is the field frequency. Then the averaged over frequency rms value of the electric field is

\[
\langle E^2 \rangle = \frac{1}{2} \int_0^\infty \langle E^2(\omega) \rangle d\omega = \frac{4\alpha^3}{15} (k_B T)^4 = \frac{8.319}{\mathrm{V/cm}^2} [T(K)/300]^4.
\]

(2)

Following [3, 4], the BBR-induced Stark shift for the atomic state of the two-electron atom \(a \equiv \{n_L L_a S_a M_a \}\) (\(n\) is the principal quantum number, \(L\) is the orbital angular momentum, \(S\) is the total electron spin, \(J\) is the total electron angular momentum and \(M\) is the \(z\) component of \(J\); an LS coupling scheme is assumed with notations \(n 2S + 1 J_L\), where \(2S + 1 = J\) is the multiplicity of the term \(LS\) and \(J\) enumerates the fine structure levels) in the nonrelativistic limit is

\[
\Delta E_a^{(2)} = \frac{4e^2}{3}\sum_{i,j=1,2} \sum_b \left[ \int_0^\infty d\omega \left( \frac{\omega^3}{e^{\omega\omega} - 1} - \frac{\omega^2}{\omega^2} \right) \right] \langle n_L L_b S_b M_b | r | n_L L_a S_a M_a \rangle^2,
\]

(3)

where \(r\) is the radius-vector of the corresponding electron, \(e\) is the electron charge, \(\langle a | ... | b \rangle\) denotes the matrix element with Schrödinger wave functions and \(\omega_{ab} = E(n_L L_a S_a M_a) - E(n_L L_b S_b M_b)\). Expression (3) is written in a general case but in the nonrelativistic limit within an LS coupling scheme and can be easily applied to an arbitrary multielectron atom with a substitution of corresponding wave functions [4].

The angular integration in the matrix elements in equation (3) can be performed using standard angular techniques [19]

\[
\langle n_L L_b S_b M_b | r | n_L L_a S_a M_a \rangle = (-1)^{J_a - M_b} \left( J_b \left[ \begin{array}{c} J_b \\ M_b \\ q \end{array} \right] \{n_L L_b S_b J_b| r | n_L L_a S_a J_a \},
\]

\[
\langle n_L L_b S_b | r | n_L L_a S_a \rangle = \delta_{S_b, S_a} \{2J_b + 1\}^{1/2} \left( J_b \left[ \begin{array}{c} J_b \\ S_b \end{array} \right] \{n_L L_b S_b| r | n_L L_a S_a \},
\]

where \(r^q\) is a spherical component of a radius-vector and \(\langle b| | a \rangle\) denotes the reduced matrix element.

A further simplification in equation (3) follows when we can neglect the fine structure intervals compared to the value \(k_B T\). Then we can ignore the dependence of \(\omega_{ab}\) on \(J_b\) and sum over \(J_a\) in equation (3), using equality

\[
\sum_{J_a} (2J_a + 1) \left( J_a \left[ \begin{array}{c} J_a \\ L_a \\ S_a \end{array} \right] \right)^2 = \frac{1}{2L_a + 1},
\]

(6)

and substituting equations (4)–(6) into equation (3) we obtain

\[
\Delta E_a^{(2)} = \frac{4e^2}{3\pi} \frac{1}{2L_a + 1} \sum_{i,j=1,2} \sum_b \int_0^\infty d\omega \omega^3 e^{i\omega \omega} - \frac{\omega^2}{\omega^2} \langle n_L L_b S_b | r | n_L L_a S_a \rangle^2.
\]

(7)

According to [4], BBR efficiently redistributes the population among excited states, shortens the atomic lifetimes, and causes corresponding line broadening. The effective level width is

\[
\Gamma_a^{\text{eff}} = \Gamma_a + \Gamma_a^{\text{BBR}},
\]

(8)

where \(\Gamma_a\) is the natural decay width and \(\Gamma_a^{\text{BBR}}\) is the BBR-induced width. The natural level width of the two-electron atomic state \(a\) is given by

\[
\Gamma_a = \sum_{n=1}^\infty \sum_{E_a < E_b} W_{ab}(n).
\]

(9)

Here, the summation extends over all states with \(E_b < E_a\), and \(W_{ab}(n)\) is the decay rate for the transition \(a \rightarrow b + n\gamma\), where \(n\) is the number of emitted photons [20]. In the ‘length’ form, one-photon transition rate is (within the dipole approximation)

\[
W_{ab}^{(1)} = \frac{4e^2}{3} \frac{\omega_{ab}}{2L_a + 1} \sum_{i=1,2} \left( \langle n_L L_b S_b | r | n_L L_a S_a \rangle \right)^2.
\]

(10)

The corresponding equation in the ‘velocity’ form can be obtained with the relation \(\langle a | p | b \rangle = i\omega_{ab} \langle a | r | b \rangle\).

The derivation of the \(\Gamma_a^{\text{BBR}}\) with the QM approach was considered in [4], and in the dipole approximation the final
result is
\[
\Gamma^\text{BBR}_a = \frac{4e^2}{3} \frac{1}{2L_a + 1} \sum_{i,j=1,2} \sum_b \left| n_i L_b S_d | r_i | n_j L_a S_b \right|^2 \times \left| (n_i L_b S_d | r_i | n_j L_a S_b) \right|^2.
\]
(11)
The corresponding partial width $\Gamma^\text{BBR}_{aa'}$ connected with the transition to the state $b = a'$ is
\[
\Gamma^\text{BBR}_{aa'} = \frac{4e^2}{3} \frac{1}{2L_a + 1} \sum_{i,j=1,2} \sum_b \left| n_i L_b S_d | r_i | n_j L_a S_b \right|^2 \times \left| (n_i L_b S_d | r_i | n_j L_a S_b) \right|^2.
\]
(12)
Expression (12) represents the BBR-induced decay rate if $E_{a'} < E_a$ and the absorption rate if $E_{a'} > E_a$.

3. BBR-induced level mixing

It is known that an external electric field leads to the Stark mixing of states with opposite parity [21, 22]. This effect is most pronounced for close-lying states (2s and 2p states in hydrogen, for example). As was shown in [3], the QED derivation of the BBR-induced Stark shift and decay rates require an accurate regularization of divergent energy denominators. Then the Stark shift modifies slightly and includes the Lamb shift. The most interesting result arises with the account of the imaginary part of the energy denominators (level widths). In this case, the BBR-induced mixing effect for the states with opposite parity can be obtained. We should note that this effect can be rigorously derived within the QED theory only (within the QM approach the level widths are regarded phenomenologically).

The derivation and detailed analysis of the level mixing effect induced by BBR were made in a general case in [3] and can be applied to the helium atom with the substitution of corresponding wave functions into expression
\[
\Gamma^\text{mix}_a = \frac{2e^2}{3\pi} \frac{1}{2L_a + 1} \sum_{i,j=1,2} \sum_b \left| n_i L_b S_d | r_i | n_j L_a S_b \right|^2 \times \int_0^\infty d\omega n_j(\omega) \omega^3 \left[ \Gamma_{ba} \left( \omega_{ba}^2 + \omega^2 \right) + \frac{1}{4} \Gamma_{ba}^2 \right]
\]
(13)
where
\[
n_j(\omega) = \frac{1}{e^{\omega/\beta} - 1},
\]
(14)
and $\omega_{ba} = E_b - E_a + \Delta E^L_{ba}, \Delta E^\omega_{ba}$ is the corresponding Lamb shift, $\Gamma_{ba} = \Gamma_b + \Gamma_a$. The summation in equation (13) is extended over all states of parity opposite to the parity of state $a$.

In [3] it was noted that the most intriguing result arises for the metastable 2s state in a hydrogen atom. In particular, the level mixing effect leads to the additional one-photon electric dipole emission channel of this state. Then the corresponding magnitude of the level width significantly exceeds the natural level width and the BBR-induced depopulation rate, even at room temperature. The same situation arises for the 2S and 2S states in helium in which decay via two-photon transitions $2S \to 1S + 2\gamma(E1)$ and $2S \to 1S + 2\gamma(E1)$, respectively. The last one is allowed only due to spin-orbit interaction and was considered in [23-25]. Numerical values of the two-photon transition rates $2S \to 1S + 2\gamma(E1)$ and $2S \to 1S + 2\gamma(E1)$ in the absence of external fields are given in table 1.

The partial width $\Gamma^\text{mix}_{aa'}$ can be introduced as
\[
\Gamma^\text{mix}_{aa'} = \frac{2e^2}{3\pi} \frac{1}{2L_a + 1} \sum_{i,j=1,2} \left| (n_i L_a S_d | r_i | n_j L_a S_b) \right|^2 \times \int_0^\infty d\omega n_j(\omega) \omega^3 \left[ \Gamma_{a'a} \left( \omega_{a'a}^2 + \omega^2 \right) + \frac{1}{4} \Gamma_{a'a}^2 \right]
\]
\[
+ \frac{\Gamma_{a'a}}{\omega_{a'a}^2 + \omega^2 + \frac{1}{4} \Gamma_{a'a}^2},
\]
(15)
where $a'$ is a first nonvanishing term in the sum over $b$ in equation (13). Of particular interest to equation (15) is in the case where $a = 2^{15}S$ and $a' = 2^{13}P$, i.e. partial width $\Gamma^\text{mix}_{2^{15}S,2^{13}P \to 2S}$. The latter represents the one-photon decay of the mixed $2^{15}S$ state [3, 5]. It is important to note that the frequencies of the photons emitted in transitions $2S \to 1S + 2\gamma(E1)$ and $2S \to 1S + 2\gamma(E1)$ are $\omega_{2S,1S} = E_{2S} - E_{1S}$ and $\omega_{2S,1S} = E_{2S} - E_{1S}$, respectively.

4. Variational approach and computational details

For the numerical calculations in the two-electron atom, we use trial wave functions with quasi-random nonlinear parameters developed in [10, 12]. The wave function with certain values of electron angular momentum $L$, its projection $M$ and parity $\pi = (-1)^L$ is
\[
\Psi_{LM}(r_1, r_2) = \sum_{l_1, l_2 = M} [\gamma_{L_1}^L(n_1, n_2) G_{L_1}^L(r_1, r_2) \pm (1 \leftrightarrow 2)],
\]
(16)
where $G_{L_1}^L$ is the radial part and $\gamma_{L_1}^L$ is the corresponding angular part [19]. The sign $+$ or $-$ in equation (16) refers to
the singlet or triplet state, respectively. Following the procedure in [10], the radial part \( G_{\ell s}^{\ell} \) is expanded into the exponential basis set with complex coefficients \( a_i, \beta_i \) and \( \gamma_i \):

\[
G_{\ell s}^{\ell}(r_1, r_2) = \sum_{i=1}^{N} \left[ U_i \text{ Re}[e^{-a_i r_1 - \beta_i r_2 - \gamma_i r_1 r_2}]ight. \\
+ \left. W_i \text{ Im}[e^{-a_i r_1 - \beta_i r_2 - \gamma_i r_1 r_2}] \right],
\]

where \( r_{12} = |r_1 - r_2| \), \( U_i \) and \( W_i \) are the linear parameters requiring optimization. The choice of nonlinear parameters for the helium states is discussed in [10]. Then the reduced matrix elements in equations (3), (11) and (13) with the wave functions (16) can be calculated in a closed analytical form (see [26]).

As a first step in testing the methods of calculation, the nonrelativistic energies of helium states were evaluated (see table 2), which are in good agreement with the values given in [11]. Variational parameters for the initial states were optimised to reach a precision of ten decimal digits in eigenvalues, which is quite enough for calculations of Stark shifts and transition probabilities. For the numerical calculations of Stark shifts, depopulation rates and BBR-induced level mixing of the different sets of basis states were employed. All the initial states \( a \) were evaluated with the basis length of \( N = 500 \). In order to test the convergence of the results for the Stark shifts, the basis of intermediate states was employed with two different lengths, \( N = 150 \) and \( N = 300 \) (see table 3). Calculations of depopulation rates were performed in the \('length'\) and \('velocity'\) forms. This also justifies the obtained values.

5. Conclusions

The evaluation of Stark shifts, depopulation rates and BBR-induced level mixing widths of helium states with the use of precise variational wave functions was performed. The results of the calculations of dynamic Stark shifts and depopulation rates are in sufficient agreement with the values presented in [4]. In [4], the method of the quantum defect was used for the calculations of Stark shifts and depopulation rates. This is more suitable for the evaluation of Rydberg states than low-lying ones. Therefore the variational approach applied in the present calculations provides more precise results.

The values of the BBR-induced Stark shift in table 3 can be important for the precise determination of transition frequencies. The results for depopulation rates \( \Gamma_{a_{\text{BBR}}}^\text{mix} \) are presented in table 4. These values were calculated in the \('length'\) and \('velocity'\) forms to check the numerical methods. The partial depopulation widths \( \Gamma_{a_{\text{BBR}}}^{\text{mix}} \) are given in table 5. These results show that the BBR-induced widths are headed by the corresponding partial decays to the ground state at room temperature. An increase in temperature leads to a more significant role of transition rates to upper states. It is important to note that in the present work, we do not consider other radiative corrections that depend on different powers of \( T \). This requires a separate study and we leave it for future works.

The BBR-induced level mixing widths \( \Gamma_{a_{\text{BBR}}}^\text{mix} \) and corresponding partial widths \( \Gamma_{a_{\text{BBR}}}^{\text{mix}} \) are given in tables 6 and 7, respectively. A comparison of these two magnitudes reveals that the leading contribution to the \( \Gamma_{a_{\text{BBR}}}^\text{mix} \) arises from the decay of the mixed state to the ground one. The most important result is that \( \Gamma_{a_{\text{BBR}}}^\text{mix} \) significantly exceeds \( \Gamma_{a_{\text{BBR}}}^\text{mix} \) at all temperatures (see tables 4 and 6). The reason is an additional one-photon decay channel which is allowed due to the mixing of states with opposite parity.

As was found in [5], the effect of BBR-induced level mixing significantly influences the processes of radiation escape from matter in the cosmological recombination epoch of the early universe. The ionization fraction undergoes modification up to a level of 20% for the \( 2s \) state in a hydrogen atom, accounting for the mixing effect. Despite the fact that the period of recombination is almost the same, the essential changes in the CMB temperature fluctuations map is expected in the far tail of multipole expansion. In these aspects, the helium atom should be considered also [8].

The period of helium recombination in primordial plasma refers to the redshift \( 1600 < z < 3000 \), where the two-photon transition \( 2S \to 1S + 2\gamma(\text{E1}) \) plays an important role in the helium atom. Since the CMB has a blackbody spectrum, the effect of BBR-induced level mixing can be taken into account in the same way as described above. The presence of BBR makes the metastable \( 2S \) state in the helium atom decay with the emission of a one-photon electric dipole photon. This becomes possible due to the effect of BBR-induced level mixing [3]. According to the results presented in [3], the BBR-induced electric field leads to the mixing of states with opposite parity in an atom—\( 2S \) and \( 2P \) states in our case. Consequently, the admixed state \( 2S \) decays via electric dipole transition \( 2S \to 1S + \gamma(\text{E1}) \). The probability of this process dominates over the probability of spontaneous two-photon transition \( 2S \to 1S + 2\gamma(\text{E1}) \) even at room temperature (see tables 1 and 6). With an increase in temperature to thousands of kelvin, the decay rates of the \( 2S \) and \( 2P \) states become comparable.

The modern theory of CMB [8] was developed without taking the mixing effect into account, and also without the forbidden two-photon decay of the \( 2S \) helium state. The last transition is about ten orders less than the allowed \( 2S \to 1S + 2\gamma(\text{E1}) \) decay channel (see table 1) and occurs via
Table 3. The BBR-induced dynamic Stark shifts (in Hz) of the energy levels of helium at different temperatures $T$. The first line in each column represents the values calculated with the length of intermediate state basis $N = 150$, while the second one corresponds to values calculated with $N = 300$. In the second column, the lower line for each state indicates the results obtained in [4].

| State | $T = 300$ K | $T = 1000$ K | $T = 3000$ K | $T = 5000$ K | $T = 10^4$ K |
|-------|-------------|-------------|-------------|-------------|-------------|
| $1^2S$ | -0.0118308 | -1.47020 | -119.288 | -923.580 | -15021.1 |
|       | -0.0118519 | -1.47017 | -119.334 | -923.941 | -15032.9 |
|       | -0.16 | | | | |
| $2^2S$ | -7.14049 | -118.650 | 3133.29 | 78224.9 | 620851 |
|       | -7.14156 | -118.626 | 3125.79 | 78148.0 | 619594 |
|       | -3.15 | | | | |

Table 4. The values of depopulation rates $\Gamma_{\text{BBR}}^\text{first}$ (in s$^{-1}$) at different temperatures $T$. The 'length' form corresponds to the first subline, while the 'velocity' form corresponds to the second one. The third subline in the second column indicates the results obtained in [4]. The number in parentheses indicates the power of ten. Calculations were performed with the intermediate basis length $N = 300$.

| State | $T = 300$ K | $T = 1000$ K | $T = 3000$ K | $T = 5000$ K | $T = 10^4$ K | $\Gamma_{\text{f}}$ |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|
| $2^2S$ | 0.000452739 | 5469.89 | 641844 | 2.10029(6) | 9.91913(6) | 51.02 $[23]$ |
|       | 0.000452828 | 5470.97 | 641970 | 2.10067(6) | 9.91973(6) |            |
|       | 0.0001 | | | | |            |
| $2^2S$ | 1.08706$(-12)$ | 52.1895 | 370546 | 2.33433(6) | 1.23342($10^3$) | 3.17($-9$) $[23]$ |
|       | 1.08697$(-12)$ | 52.1869 | 370528 | 2.33420(6) | 1.23331(6) |            |
|       | $6(-11)$ | | | | |            |
| $2^2P$ | 1.50913$(-4)$ | 1823.40 | 388130 | 3.96617(6) | 3.90781(7) | 1.80089(9) |
|       | 1.50913$(-4)$ | 1823.41 | 388908 | 3.98178(6) | 3.92800(7) |            |
|       | $4(-4)$ | | | | |            |
| $2^2P$ | 6.0252$(-13)$ | 17.4167 | 203731 | 2.92180(6) | 3.54150(7) | 1.02164(7) |
|       | 6.0252$(-13)$ | 17.4167 | 203872 | 2.92598(6) | 3.54858(7) |            |
|       | $2(-11)$ | | | | |            |

Table 5. The partial widths $\Gamma_{\text{BBR}}^\text{second}$ (in s$^{-1}$) of helium energy levels at different temperatures $T$. The 'length' form corresponds to the first subline, while the 'velocity' form corresponds to the second one. The number in parentheses indicates the power of ten.

| State | $T = 300$ K | $T = 1000$ K | $T = 3000$ K | $T = 5000$ K | $T = 10^4$ K |
|-------|-------------|-------------|-------------|-------------|-------------|
| $2^2S$ | 0.000452739 | 5469.89 | 638864 | 1.94492(6) | 5.85697(6) |
|       | 0.000452828 | 5470.97 | 638990 | 1.94530(6) | 5.85813(6) |
| $2^2S$ | 1.80706$(-12)$ | 52.1895 | 370414 | 2.31340(6) | 1.10458(7) |
|       | 1.80697$(-12)$ | 52.1869 | 370396 | 2.31329(6) | 1.10453(7) |
| $2^2P$ | 1.37982$(-23)$ | 0.09354 | 161741 | 2.39645(6) | 2.78705(7) |
|       | 1.38615$(-23)$ | 0.09397 | 162483 | 2.39074(6) | 2.79984(7) |
| $2^2P$ | 6.0252$(-13)$ | 17.4033 | 190841 | 2.54616(6) | 2.59884(7) |
|       | 6.0252$(-13)$ | 17.4033 | 190976 | 2.54972(6) | 2.60332(7) |

the spin–orbit mixing of states [27]. However, the results of the calculations (second line in table 7) reveal that in the presence of BBR, the $2^2S$ state in the helium atom decays due to the admixture of the $2^1P$ state via the E1 transition. The corresponding transition rate significantly exceeds the natural width, even at room temperature. Thus, we conclude that the level mixing in the helium atom induced by BBR can significantly affect the ionization history of primordial plasma.
Table 6. The BBR-induced level-mixing widths $\Gamma_{\text{mix}}^a$ (in s$^{-1}$) of helium energy levels at different temperatures $T$. Calculations were performed with the intermediate basis length $N = 300$.

| State | $T = 300$ K | $T = 1000$ K | $T = 3000$ K | $T = 5000$ K | $T = 10^4$ K |
|-------|-------------|--------------|--------------|--------------|--------------|
| $2^3S$ | 238.879     | 2.18242(7)   | 2.54447(9)   | 7.81302(9)   | 2.71061(10)  |
| $2^3S$  | 0.842754    | 1917.88      | 1.27973(7)   | 8.08185(7)   | 1.35344(9)   |
| $2^3P$ | 112.408     | 7.27742(6)   | 4.53458(9)   | 7.01676(10)  | 9.9988(11)   |
| $2^3P$  | 0.995895    | 795.448      | 9.17896(8)   | 2.4130(10)   | 3.04126(11)  |

Table 7. The partial widths $\Gamma_{\text{mix}}^{a\text{tot}}$ (in s$^{-1}$) of energy levels of helium at different temperatures $T$.

| State | $T = 300$ K | $T = 1000$ K | $T = 3000$ K | $T = 5000$ K | $T = 10^4$ K |
|-------|-------------|--------------|--------------|--------------|--------------|
| $2^3\Sigma$ | 237.873     | 2.18241(7)   | 2.54316(9)   | 7.74522(9)   | 2.33423(10)  |
| $2^3\Sigma$  | 0.256627    | 1845.36      | 1.27869(7)   | 7.98911(7)   | 3.81636(8)   |
| $2^3\Pi$  | 28.3746     | 4239.14      | 4.64835(8)   | 8.30987(9)   | 7.99245(10)  |
| $2^3\Pi$  | 0.80590     | 770.401      | 9.16819(8)   | 2.40758(10)  | 3.02379(11)  |

More detailed research on this subject should include the solution of rate equations, and we omit this for further investigations.

Acknowledgments

This work was supported by the Russian Science Foundation (grant 17-12-01035). The authors are indebted to V I Korobov for permission to use the Fortran code for the construction of the He variational wave functions.

References

[1] Riehle F 2004 *Frequency Standards: Basics and Applications* (Weinheim: Wiley)
[2] Safronova M S, Kozlov M G and Clark C W 2011 *Phys. Rev. Lett.* 107 143006
[3] Solovyev D, Labzowsky L and Plunien G 2015 *Phys. Rev. A* 92 022508
[4] Farley J W and Wing W H 1981 *Phys. Rev. A* 23 2397
[5] Zalaliutdinov T, Solovyev D, Labzowsky L and Plunien G 2017 *Phys. Rev. A* 96 012512
[6] Zel’dovich Y B, Kurt V G and Sunyaev R A 1968 *Zh. Eksp. Teor. Fiz., JETP* 28 146
[7] Peebles P E 1968 *Astrophys. J.* 153 1
[8] Seager S, Sasselov D D and Scott D 2000 *ApJS* 128 407
[9] Wong W W 2008 *PhD Thesis* University of British Columbia, Canada (arXiv:0811.2826)
[10] Korobov V I 2000 *Phys. Rev. A* 61 064503
[11] Drake G W F 1996 *Atomic, Molecular and Optical Physics Handbook* (New York: AIP)
[12] Korobov V I, Bakalov D and Monkhorst H J 1999 *Phys. Rev. A* 59 R919–21
[13] Zheng X, Sun Y R, Chen J-J, Jiang W, Puchucki K and Hu S-M 2017 *Phys. Rev. Lett.* 118 063001
[14] Patkos V, Yerokhin V A and Puchucki K 2017 *Phys. Rev. A* 95 012508
[15] Lu Z-T, Mueller P, Drake G W F, Ntershuser W, Pieper S C and Yan Z-C 2013 *Rev. Mod. Phys.* 85 1383
[16] Cooke W E and Gallagher T F 1980 *Phys. Rev. A* 21 588
[17] Escobedo M A and Soto J 2008 *Phys. Rev. A* 78 032520
[18] Escobedo M A and Soto J 2010 *Phys. Rev. A* 82 042506
[19] Varshalovich D A, Moskalev A N and Khersonskii V K 1988 *Quantum Theory of Angular Momentum* (Singapore: World Scientific)
[20] Berestetskii V B, Lifshitz E M and Pitaevskii L P 1982 *Quantum Electrodynamics* (Oxford: Pergamon)
[21] Azimov Y I, Anselm A A, Moskalev A N and Ryndin R M 1974 *Zh. Exp. Teor. Fiz.* 67 17
[22] Azimov Y I, Anselm A A, Moskalev A N and Ryndin R M 1975 *Sov. Phys. JETP* 40 8
[23] Solovyev D, Sharipov V, Labzowsky L and Plunien G 2010 *J. Phys. B: At. Mol. Opt. Phys.* 43 074005
[24] Derevianko A and Johnson W R 1997 *Rev. Mod. Phys.* 69 85
[25] Derevianko A and Johnson W R 1997 *Rev. Mod. Phys.* 69 85
[26] Derevianko A and Johnson W R 1997 *Rev. Mod. Phys.* 69 85
[27] Drake G W F and Morton D C 2007 *ApJS* 170 251