The BE-Higgs boson as spin-0 partner of the $Z$

in the Supersymmetric Standard Model

Pierre FAYET

Laboratoire de Physique Théorique de l’ENS (UMR 8549 CNRS)
24 rue Lhomond, 75231 Paris Cedex 05, France

Supersymmetric extensions of the standard model lead to superpartners for all particles, squarks and sleptons, gluinos, charginos and neutralinos, ... [1,2]. They differ from ordinary particles by 1/2 unit of spin and are distinguished by a $R$-parity quantum number related to baryon and lepton numbers, discrete remnant of a continuous $U(1)_R$ symmetry, making the lightest superpartner stable.

While the standard model [6,7] involves a single scalar doublet leading to one Brout-Englert-Higgs boson [8–11], spontaneous electroweak breaking is induced here by two doublets $h_1$ and $h_2$. They are responsible for charged-lepton and down-quark masses, and up-quark masses, respectively, leading to additional charged and neutral spin-0 BEH bosons. These theories also provide systematic associations between massive gauge bosons and spin-0 BEH bosons, a very non-trivial feature owing to their different gauge symmetry properties [1,2,12].

These relations were proposed in 1974 even before the standard model (SM) got considered as “standard”, and are at the basis of its supersymmetric extensions, even if they may often go unnoticed. Weak neutral currents were just recently discovered [13] with their structure unknown, and the $W^\pm$ and $Z$ hypothetical. Little attention was paid to fundamental spin-0 particles, the very possibility of their existence getting questioned and frequently denied for many years later.

Proposing relations between massive spin-1 mediators of weak interactions and spin-0 particles associated with electroweak breaking and mass generation [1,12] then amounted to

\[ \text{relate two classes of hypothetical particles, using an hypothetical symmetry!} \]  \( (1) \)

And this at a time when supersymmetry was viewed as an algebraic structure [14–17] very far from being able to describe nature, for many reasons including an obvious lack of similarities between known bosons and fermions.

Forty years later, the situation has improved considerably. With the introduction of $R$-odd superpartners and two spin-0 doublets for the electroweak breaking, supersymmetry could indeed be a symmetry of the fundamental laws of physics [2]. The discoveries in 1983 of the $W^\pm$ and $Z$ mediators of weak interactions [18,19], and in 2012 of a new boson considered as a spin-0 BEH boson [20,21] confirmed to a very large extent the validity of the standard theory (SM) or of a closely approaching one. This gives additional interest to the relations between spin-1 and spin-0 bosons provided by supersymmetric extensions of the standard model.

These relations may be more concretely discussed now that we know, with the $Z$ and $h$ bosons, at least one representative in each class of formerly hypothetical particles. The Supersymmetric Standard Model offers a way to view the 125 GeV/c^2 boson recently observed at CERN as a spin-0 partner of the $Z$, up to a mixing angle induced by supersymmetry breaking.

II. THE SPIN-0 $\pm$ PARTNER OF THE $Z$

Within supersymmetry two spin-0 doublets $h_1$ and $h_2$ are needed for the electroweak breaking, at first to avoid a massless chiral chargino, allowing for the construction of two Dirac winos associated with the $W^\pm$ within a massive gauge multiplet of supersymmetry [3]. These doublets $h_1 = (h^0_1, h^+)$ and $h_2 = (h^0_2, h^0_2)$ have weak hypercharges $Y = -1$ and +1. By leading to a negative mass^2 term for $h_2$, the term $-\xi D^2$ [22] associated with $U(1)_Y$ in the Lagrangian density plays a crucial role in triggering
spontaneous electroweak breaking and giving masses to the $W^\pm$ and $Z$, and to their spin-$\frac{1}{2}$ and spin-0 partners.

The auxiliary components $D$ and $D'$ associated with the $SU(2) \times U(1)_Y$ gauge group are expressed as

$$\begin{align*}
D &= -\frac{g}{2} (h_1^\dagger \tau h_1 + h_2^\dagger \tau h_2) + \ldots, \\
D' &= \xi + \frac{g'}{2} (h_1^\dagger h_1 - h_2^\dagger h_2) + \ldots.
\end{align*}$$

The resulting potential reads

$$V = \frac{1}{2} (D^2 + D'^2) + \ldots = \frac{g^2}{8} (h_1^\dagger \tau h_1 + h_2^\dagger \tau h_2 + \ldots)^2 + \frac{1}{2} (\xi + \frac{g'}{2} (h_1^\dagger h_1 - h_2^\dagger h_2 + \ldots)^2 + \ldots, (3)
$$

ignoring for the moment possible soft supersymmetry-breaking terms, considered at a later stage. Its quartic part, fixed by the electroweak couplings $g$ and $g'$ as

$$V_{\text{quartic}} = \frac{g^2 + g'^2}{8} (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2} |h_1^\dagger h_2|^2, (4)
$$

appears within supersymmetry as part of electroweak gauge interactions.

The potential is minimum for $<h_1^0> = v_1/\sqrt{2}$, $<h_2^0> = v_2/\sqrt{2}$. The correspondence with the notations of $1$, using two doublets $\varphi^0$ and $\varphi'$ with the same $Y = -1$, is as follows:

$$h_1 = \varphi^0 \equiv \left( \begin{array}{c} h_1^0 \\ h_2^- \end{array} \right), \quad h_2 = \left( \begin{array}{c} h_2^0 \\ h_2^+ \end{array} \right) = -\varphi'^0,$$

$$\tan \beta = v_2/v_1 \equiv \tan \delta = \tan \delta / \tan \beta. (5)
$$

The $\mu H_1 H_2$ superpotential term is first taken to vanish, as initially forbidden by a continuous $U(1)_Z$ and/or an extra $U(1)_R$ symmetry $1$. The latter acts according to $\varphi^0 \rightarrow e^{i \alpha} \varphi^0$, $\varphi' \rightarrow e^{-i \alpha} \varphi'$ as introduced in a pre-susy two-doublet model in $23$, i.e.

$$h_1 \rightarrow e^{i \alpha} h_1, \quad h_2 \rightarrow e^{i \alpha} h_2, (6)
$$

allowing to rotate the phases of the two doublets independently. Taking $\mu = 0$ in this first stage allows for gauge symmetry to be spontaneously broken with supersymmetry remaining conserved in the neutral sector, shedding light on the relations between massive gauge bosons and spin-0 BE-Higgs bosons provided by supersymmetry.

The initial $U(1)_R$ symmetry survives the electroweak breaking induced by $<h_1>$ and $<h_2>$. As long as it is present it allows us to benefit, in the absence of a $\mu$ term and of direct gaugino mass terms, from Dirac neutralinos as well as charginos. And more specifically two Dirac winos and a Dirac zino, carrying $\pm 1$ unit of the additive quantum number $R$.

Some attention may be useful in the presence of an extra $U(1)$ symmetry acting on $h_1$ and $h_2$ as in $23$, that became known later as a $U(1)_{PQ}$ symmetry. Indeed it might lead to a classically massless pseudoscalar $A$ (and associated scalar $s_A$), jointly described by $1, 2$

$$\varphi^0 \sin \delta + \varphi'^0 \cos \delta = h_1^0 \sin \beta + h_2^0 \cos \beta = \frac{s_A + i A}{\sqrt{2}}. (7)
$$

These particles, momentarily appearing as classically massless in the spectrum $2$, get a mass as in $1$ through an explicit breaking of the $U(1)_A$ symmetry $32$.

We see from $33$ that the term $-\xi D'$ in $\mathcal{L}$ generates a negative mass $^2$ for $h_2$, triggering spontaneous electroweak breaking. The origin is a saddle point of the potential, with $m^2(h_2) = -\xi g' < 0$. $m^2(h_1) = \xi g' > 0$. The would-be spin-0 Goldstone field (with $\delta = \beta$ as indicated in $33$)

$$z_g = -\sqrt{2} \text{ Im}(\varphi^0 \cos \delta + \varphi'^0 \sin \delta) = \sqrt{2} \text{ Im}(-h_1^0 \cos \beta + h_2^0 \sin \beta) \quad (8)
$$

is eliminated by the $Z$. The corresponding real part

$$z = \sqrt{2} \text{ Re}(-\varphi^0 \cos \delta + \varphi'^0 \sin \delta) = \sqrt{2} \text{ Re}(-h_1^0 \cos \beta + h_2^0 \sin \beta) \quad (9)
$$

describes a scalar BEH boson associated with the $Z$ under supersymmetry, with the same mass $m_Z$ as long as supersymmetry is unbroken $1, 2, 12$.

This results in the general association

$$Z \xrightarrow{\text{SUSY}} \text{2 Maj. zinos} \xrightarrow{\text{SUSY}} \text{spin-0 BEH boson } z \quad (10)
$$

with this description

$$z = \sqrt{2} \text{ Re}(-h_1^0 \cos \beta + h_2^0 \sin \beta). \quad (11)
$$

This is also made possible by the $U(1)_R$ symmetry remaining unbroken at this stage, allowing for the two Majorana zinos to combine into a Dirac zino of mass $m_Z$. It implies the existence of a spin-0 BE-Higgs boson of mass

$$m \approx 91 \text{ GeV}/c^2 \text{ up to susy-breaking effects.} \quad (12)
$$

This result, valid independently of $\tan \beta$, may now be compared with the recent CERN discovery of a new boson with a mass close to $125 \text{ GeV}/c^2$ $20, 21$.

The spin-0 field $z$ may also be compared with the SM-like BEH field

$$h_{SM} = \sqrt{2} \text{ Re}(h_1^0 \cos \beta + h_2^0 \sin \beta). \quad (13)
$$

This has Yukawa couplings “of the wrong sign” to down quarks and charged leptons, acquiring their masses through Yukawa couplings to $h_1$ $2, 33$. It becomes very close to $h_{SM}$ at large $\tan \beta$, with

$$<h_{SM}|z> = -\cos 2\beta. \quad (14)$$
We thus rotate neutral chiral superfields as indicated by (7,9), according to
\[
\begin{align*}
H_z &= -H_1^0 c_\beta + H_2^0 s_\beta = (z + i z_0)/\sqrt{2} + \ldots , \\
H_A &= H_1^0 s_\beta + H_2^0 c_\beta = (s_A + i A)/\sqrt{2} + \ldots. 
\end{align*}
\]
(15)

\(H_z\) describes the would-be Goldstone field \(z_0\) and spin-0 \(z\) associated with the \(Z\) as in (11), while \(H_A\) describes the scalar and pseudoscalar discussed more later.

### III. ELECTROWEAK BREAKING AND Z AND Z MASSES

From
\[
D_3 = \frac{g}{2} (-|h_0^1|^2 + |h_2^0|^2) + \ldots , \\
D' = \xi + \frac{g}{2} (|h_0^1|^2 - |h_2^0|^2) + \ldots ,
\]
we get for \(D_Z = D_3 c_\theta - D' s_\theta, D_\gamma = D_3 s_\theta + D' c_\theta, \)
\[
\begin{align*}
D_Z &= -\xi s_\theta + \frac{\sqrt{g^2 + g'^2}}{2} (|h_0^1|^2 - |h_2^0|^2) + \ldots , \\
D_\gamma &= \xi c_\theta + 0 + \ldots.
\end{align*}
\]
(18)

We express \(V = \frac{1}{2} (D_Z^2 + D_\gamma^2) + \ldots \) as a function of \(h_0^1\) and \(h_2^0\) as
\[
V = \frac{1}{2} (-\xi s_\theta + \frac{\sqrt{g^2 + g'^2}}{2} (|h_0^1|^2 - |h_2^0|^2) + \ldots)^2 + \ldots. \quad (19)
\]

Minimizing this term fixes only \(v_2^2 - v_1^2\), leading to a flat direction associated with \(s_A; v_2^2 - v_1^2\) adjusts so that
\[
< D_Z > = -\xi s_\theta + \frac{\sqrt{g^2 + g'^2}}{4} (v_2^2 - v_1^2) \equiv 0. \quad (20)
\]

Expanding \(D_Z\) in (18) at first order in \(h_0^1\) and \(h_2^0\) we have
\[
\begin{align*}
D_Z &= \frac{1}{2}\sqrt{g^2 + g'^2} (-v_1 \sqrt{2} \text{ Re} h_1^0 + v_2 \sqrt{2} \text{ Re} h_2^0) + \ldots \\
&= m_Z \sqrt{2} \text{ Re} (-h_0^1 c_\beta + h_2^0 s_\beta) + \ldots = m_Z z + \ldots, 
\end{align*}
\]
(21)

providing from \(D_Z^2/2 = \frac{1}{2} m_Z^2 z^2 + \ldots \) the supersymmetric mass term for the spin-0 field \(z\).

The parameter \(\xi\) associated with \(U(1)_Y\) determines \(m_Z\), given by \(m_Z^2 = (g^2 + g'^2) (v_1^2 + v_2^2)/4.\). For \(v_1 \simeq 0\) we would get for \(Z\) and \(z\) (described by \(\simeq \sqrt{2} \text{ Re} h_2\))
\[
m_Z^2 = m_z^2 \simeq -2 m^2 (h_2) = \xi g', \quad (22)
\]
up to radiative corrections, and supersymmetry-breaking effects for \(m_z\). With \(g' = g/\cos \theta \simeq 3.45\) the \(Z, W^\pm\) and spin-0 \(W^\mp\) boson masses get fixed by the \(\xi\) parameter associated with \(U(1)_Y\). This leads to
\[
\xi \approx \frac{m_Z^2}{g'} \simeq \frac{m_W m_Z}{e} \simeq 2.4 \times 10^4 \text{ GeV}^2, \quad (24)
\]
or equivalently
\[
\sqrt{\xi} \approx \frac{v}{2} \frac{1}{\sin \theta} \sqrt{\frac{e}{\cos \theta}} \simeq 150 \text{ GeV}, \quad (25)
\]
up to radiative corrections.

The \(\xi\) parameter \(22\) determines here the \(W^\pm\) and \(Z\) masses, a feature that may further persist when \(R\)-odd squarks and sleptons acquire large mass\(^2\), e.g. from the compactification of extra dimensions. More generally we have
\[
(- \cos 2\beta)) m_Z^2 = \xi g', \quad (26)
\]
reducing to (23) for large \(\tan \beta\). \(\xi = 0\) would be associated with \(\tan \beta = 1\), leaving at this stage \(m_Z\) and \(m_W\) unfixed, at the classical level \(31\). In such a situation the scalar \(s_A\) associated with this flat direction would describe a classically massless particle with dilatoniclike couplings.

We also have from (18,20)
\[
< D_\gamma > = \xi c_\theta = \frac{g}{4s_\theta} (v_2^2 - v_1^2) = \frac{m_W^2}{e} (-\cos 2\beta). \quad (27)
\]

Having at this stage the photino as the Goldstone fermion implies that charged particles only are sensitive to the spontaneous breaking of the supersymmetry. Neutral ones remain mass degenerate within massive \((Z)\) or massless \((\gamma)\) multiplets of supersymmetry, before the introduction of extra terms breaking the \(U(1)_A\) and \(U(1)_R\) symmetries, the latter reduced to \(R\)-parity.

We now discuss zinos, winos and charged spin-0 bosons within massive gauge multiplets, before returning to spin-0 bosons, and how they may be described by massive gauge superfields, in contrast with the usual formalism.

### IV. ZINOS AND OTHER NEUTRALINOS

The massive gauge multiplet of the \(Z\) includes a Dirac zino, obtained from chiral gaugino and higgsino components transforming under \(U(1)_R\) according to
gaugino \(\lambda_Z \to e^{-i \alpha} \lambda_Z\), higgsino \(\tilde{h}_Z \to e^{-i \alpha} \tilde{h}_Z\). \(28\)

It may be expressed as a massive Dirac zino with \(R = +1\),
\[
\lambda_{ZL} + (\tilde{h}_Z)_R = (\lambda_3 c_\theta - \lambda_1 s_\theta)_L + (\tilde{h}_1^c c_\beta - \tilde{h}_2^c s_\beta)_R. \quad (29)
\]
Or equivalently as two Majorana zinos, degenerate as long as \(U(1)_R\) is preserved, with a mass matrix given in the corresponding \(2 \times 2\) gaugino-higgsino basis by \(35\)
\[
\mathcal{M}_{\text{zinos}} = \begin{pmatrix}
0 & m_Z \\
m_Z & 0
\end{pmatrix}. \quad (30)
\]
TABLE I: Minimal content of the Supersymmetric Standard Model (MSSM). Gauginos $\lambda$, $\lambda_3$ mix with higgsinos $h_1^0$, $h_2^0$ into a photino, two zinos and a higgsino, further mixed into four neutralinos. The charged $w^\pm$ associated with $W^\pm$ is usually known as $H^\mp$. The scalars ($z$, $s_A$) mix into $h$ and $H$. The N/nMSSM also involves an extra singlet superfield $s$ with a trilinear superpotential coupling $\lambda H_1 H_2 S$, leading to an additional neutralino (singlino) and two singlet bosons.

| gluinos  | $\tilde{\psi}$ | photon  | $\tilde{\gamma}$ |
|----------|----------------|---------|------------------|
| $W^\pm$  | $\tilde{W}^\pm_{1/2}$ | $Z_{1/2}$ | higgsino $h_A$ |
|          | $w^\pm$          | $z$     | $s_A$, $A$       | BE-Higgs bosons |
|          | lepton           | $\tilde{l}$ | slepton           |
|          | quark            | $\tilde{q}$ | squark           |

Supersymmetry remains unbroken in this sector, in the absence of direct gaugino ($m_1$, $m_2$) and higgsino ($\mu$) mass terms.

This $2 \times 2$ zino mass matrix may be unpacked again into a $4 \times 4$ neutralino matrix expressed in the $(\lambda', \lambda_3, h_1^0, h_2^0)$ basis using (29). Including additional $\Delta R = \pm 2$ supersymmetry-breaking contributions from gaugino ($m_1$, $m_2$) and higgsino ($\mu$) mass terms it reads

$$M_{\text{inos}} = \begin{pmatrix}
m_1 & 0 & -s_\beta c_\beta m_Z & s_\beta \bar{s}_\beta m_Z \\
0 & m_2 & c_\beta \bar{s}_\beta m_Z & -c_\beta \bar{s}_\beta m_Z \\
-s_\beta c_\beta m_Z & c_\beta \bar{s}_\beta m_Z & 0 & -\mu \\
s_\beta \bar{s}_\beta m_Z & -c_\beta \bar{s}_\beta m_Z & -\mu & 0
\end{pmatrix}.$$ (31)

For equal gaugino masses $m_1 = m_2$ the photino $\lambda_* = \lambda' c_\theta + \lambda_3 s_\theta$ is a mass eigenstate. The remaining $3 \times 3$ mass matrix is expressed in the $(\lambda_Z, h_1^0, h_2^0)$ basis (with $\lambda_Z = -\lambda' s_\theta + \lambda_3 c_\theta$) as

$$M_{\text{inos}} = \begin{pmatrix}
m_2 & c_\beta m_Z & -s_\beta m_Z \\
c_\beta m_Z & 0 & -\mu \\
-s_\beta m_Z & -\mu & 0
\end{pmatrix},$$ (32)

as seen from (29). It further simplifies for $\tan \beta = 1$ into

$$M_{\text{inos}} = \begin{pmatrix}
m_2 & m_Z/\sqrt{2} & -m_Z/\sqrt{2} \\
m_Z/\sqrt{2} & 0 & -\mu \\
-m_Z/\sqrt{2} & -\mu & 0
\end{pmatrix}.$$ (33)

Next to a pure higgsino of mass $|\mu|$ corresponding to $(\gamma_5) (h_1^0 + h_2^0)/\sqrt{2}$, the two zinos constructed from $\lambda_Z$ and $(h_1^0 - h_2^0)/\sqrt{2}$ have the mass matrix

$$M_{\text{inos}} = \begin{pmatrix}m_2 & m_Z \\
m_Z & \mu
\end{pmatrix},$$ (34)

as obtained directly from (30).

There may also be additional neutralinos, as described by the extra N/nMSSM singlet $S$ with a $\lambda H_1 H_2 S$ superpotential coupling, leading through $<H_1> = v/\sqrt{2}$ to a $\lambda v \sqrt{2}/2$ Higgs superpotential mass term. Here $H_A = H_1^0 s_\beta + H_2^0 c_\beta$ is the same “left-over” chiral superfield as obtained in (15), now acquiring a mass by combining with $S$ [1].

V. THE SPIN-0 $w^\pm$ PARTNER OF THE $W^\pm$, AND ASSOCIATED WINOS

We have, in a similar way,

$$W^\pm \frac{\text{SUSY}}{2} \text{Dirac winos} \frac{\text{SUSY}}{\text{spin-0 boson}} w^\pm,$$ (35)

with $m_{w^\pm} = m_W$, also up to supersymmetry-breaking effects. This is why the charged boson now known as $H^\mp$ was called $w^\pm$ in [1].

The two doublets being expressed as $\varphi^a = (h_1^0, h_1^+)$ and $\varphi^c = (h_2^0, -h_2^+)$, as in (5) with $\delta = \beta$, the would-be Goldstone field

$$w_3^\pm = \varphi^a \cos \delta + \varphi^c \sin \delta = h_1^+ \cos \beta + h_2^+ \sin \beta$$ (36)

is eliminated by the $W^\pm$. The orthogonal combination

$$w^\pm = \varphi^a \sin \delta - \varphi^c \cos \delta = h_1^+ \sin \beta - h_2^+ \cos \beta$$ (37)

(approaching $h_1^\pm$ at large $\tan \beta$) describes a charged spin-0 BEH boson associated with the $W^\pm$ [1 2 12].

With

$$h_1^1 h_2 = h_1^0 h_2^+ + h_1^+ h_2^0 = \frac{v}{\sqrt{2}} (h_1^+ \sin \beta + h_2^+ \cos \beta) + ...$$

$$= \frac{v}{\sqrt{2}} w^+ + ...,$$ (38)

the quartic terms [41] in the potential,

$$V = \frac{g^2}{4} (h_1^+ h_2)^2 + ... = \frac{g^2 v^2}{4} |w^+|^2 + ...,$$ (39)

generate a mass $m_w = m_W = gv/2$ for

$$w^\pm \equiv H^\pm = h_1^\pm \sin \beta + h_2^\pm \cos \beta.$$ (40)

It is the same as for the $W^\pm$, to which it is related by two infinitesimal supersymmetry transformations.

The mass spectrum is given, at this first stage for which supersymmetry is spontaneously broken with the photino as the Goldstone fermion, by [1]

$$m_{w^\pm} = m_{W^\pm} = \frac{g^2 (v_1^2 + v_2^2)}{4},$$

$$m^2(\text{winos}_{1,2}) = \frac{g'^2 v_1^2}{2} = m_W^2 (1 \pm \cos 2\beta)$$ (41)

$$= m_W^2 \pm e <D_7 >.$$
Boson-fermion mass\(^2\) splittings are given by \(\pm \epsilon < D_\gamma >\) (as in \(^4\) in the absence of other sources of supersymmetry breaking), and fixed by \(^{27}\).

The two Dirac winos are \(R\) eigenstates carrying \(R = \pm 1\), with masses \(g v_1/\sqrt{2}\) and \(g v_2/\sqrt{2}\). The wino mass matrix would be supersymmetric (as for zinos in \(^{40}\)) for \(\xi = 0\) so that \(\beta = \pi/4\) and \(m(\text{wino}) = m_W\), with \(<D_\gamma > = <D_Z > = 0\) from \(^{27}\).

In the presence of additional \(\Delta R = \pm 2\) gaugino and higgsino mass terms further breaking the supersymmetry as well as \(U(1)_R\) for \(m_2\) and \(\mu\) and \(U(1)_A\) for \(\mu\), the wino mass matrix obtained from \(^{41}\) reads

\[
M_{\text{winos}} = \begin{pmatrix}
m_2 & \frac{g v_2}{\sqrt{2}} = m_W \sqrt{2} s_\beta \\
\frac{g v_1}{\sqrt{2}} = m_W \sqrt{2} c_\beta & \mu
\end{pmatrix}.
\]

\(m_2\) and \(\mu\) jointly allow for both winos to be heavier than \(m_W\) (as experimentally required \(^{24}\)).

For gaugino and higgsino mass terms related by \(m_1 = m_2 = m_3 = -\mu\) (up to radiative corrections), possibly also equal to the gravitino mass \(m_{3/2}\), with \(\tan \beta = 1\) \(^{25}\), we get from \(^{34,12}\) remarkable mass relations like, at the classical level,

\[
\begin{align*}
m^2(\text{wino}) &= m_W^2 + m_{3/2}^2, \\
m^2(\text{zino}) &= m_Z^2 + m_{3/2}^2, \\
m(\text{photino}) &= m(\text{glinos}) = m_{3/2}.
\end{align*}
\]

This also paves the way for more general situations involving \(N = 2\) extended supersymmetry with grand-unification groups \(^{26, 27}\). Similar mass relations like

\[
\begin{align*}
m^2(\text{xino}) &= m_X^2 + m_{3/2}^2, \\
m^2(\text{yino}) &= m_Y^2 + m_{3/2}^2 = m_X^2 + m_W^2 + m_{3/2}^2,
\end{align*}
\]

are then obtained for xinos, yinos, etc., with a grand-unification gauge group like \(SU(5)\) or \(O(10), \ldots\).

Extra compact dimensions may then be responsible for supersymmetry and grand-unification breakings \(^{27}\), \(R\)-odd supersymmetric particles carrying momenta \(\pm m_{3/2}\) along an extra dimension. When \(R\)-parity is identified with the action of performing a closed loop along such a compact dimension, \(m_{3/2}\) and more generally superpartner masses get quantized in terms of its size, according to \(^{43, 44}\) with e.g. in the simplest case

\[
m_{3/2} = (2n + 1) \frac{\pi h}{L c} = (2n + 1) \frac{h}{2 R c}.
\]

But let us return, in a more conservative way, to 4 space-time dimensions.

VI. THE PSEUDOSCALAR \(A\) AND SCALAR \(s_A\)

The potential \(^3\) admits, at this initial stage excluding a \(\mu H_1 H_2\) superpotential term (both \(U(1)_R\) and \(U(1)_A\) symmetries being present) two classically flat directions corresponding to the scalar \(s_A\) and pseudoscalar \(A\) in \(^4\), both classically massless \(^2\). A then appears as an “axion” associated with the extra \(U(1)_A\) symmetry acting on \(h_1\) and \(h_2\) as in \(^{40, 23}\), extended to supersymmetry according to \(^{1}\)

\[
H_1 \to e^{i a} H_1, \quad H_2 \to e^{i a} H_2.
\]

Its scalar partner \(s_A\) is also associated with a flat direction, the minimisation of the potential \(^3\) fixing only \(v_2^2 - v_1^2\).

This “axion” \(A\) (a notion unknown at the time, that appeared in a different context several years later) and associated scalar \(s_A\) were given a mass in \(^1\) by breaking explicitly the \(U(1)_A\) symmetry \(^{44, 43}\), now often referred to as \(U(1)_{\rho Q}\). This was done by introducing a singlet \(S\) coupled through a trilinear superpotential \(\lambda H_1 H_2 S\), and transforming under \(U(1)_A\) according to

\[
S \to e^{-2 i a} S.
\]

Its \(f(S)\) superpotential interactions, that may include \(S, S^2\) and \(S^3\) terms as in the \(N/nMSSM\), break explicitly \(U(1)_A\), the presence of a quasimassless “axion” being avoided.

Explicitly, the potential includes an extra term \(V_\Lambda\), with a vanishing minimum still preserving the supersymmetry. It reads

\[
V_\Lambda = \left| \frac{\partial W}{\partial S} \right|^2 = | \lambda h_1 h_2 + \sigma + \ldots |^2 + \
= \frac{\lambda^2 v^2}{2} | h_1 s_\beta + h_2 c_\beta |^2 + \ldots = \frac{\lambda^2 v^2}{2} (s_A^2 + A^2) + \ldots
\]

It provides a mass term \((\lambda v/\sqrt{2})\) for the complex field

\[
h_A = \frac{s_A + i A}{\sqrt{2}} = h_1 s_\beta + h_2 c_\beta
\]

the would-be “axion” \(A\) (and associated scalar \(s_A\) acquiring a mass \(m_A = \lambda v/\sqrt{2}\) \(^3\)).

In terms of superfields, the \(\lambda H_1 H_2 S\) superpotential coupling of the \(N/nMSSM\) generates in \(^3\), from \(<H_1> = v_1/\sqrt{2}, <H_2> = v_2/\sqrt{2}\),

\[
\lambda H_1 H_2 S = \frac{\lambda v}{\sqrt{2}} (H_1 s_\beta + H_2 c_\beta) S + \ldots = \frac{\lambda v}{\sqrt{2}} H_A S + \ldots
\]

a supersymmetric mass term \((\lambda v/\sqrt{2})\) for \(H_A\) and \(S\), possibly to be combined with a \(\frac{1}{2} \mu S S^2\) singlet mass term, if present.
VI. $z$ YUKAWA COUPLINGS “OF THE
WRONG SIGN”

The new boson found at CERN close to 125 GeV/c$^2$
[20, 21] is considered as a Brout-Englert-Higgs boson [8-
11] associated with the electroweak breaking, as expected
in the standard model [5, 7] where this breaking involves
a single spin-0 doublet. But it may also be interpreted,
in general up to a mixing angle, as a spin-0 partner of the
$Z$ under two infinitesimal supersymmetry transforma-
tions. The $z$ field in (11) may be compared with the
SM-like scalar, obtained from the real part of the neutral
component of “active” doublet combination

$$\varphi_{sm} = \varphi^0 \cos \delta + \varphi' \sin \delta = h_1 \cos \beta + h_2 \sin \beta,$$  \hspace{1cm} (51)

such that $<\varphi^0_{sm}> = \frac{v}{\sqrt{2}}$, and

$$h_{SM} = \sqrt{2} \text{Re} \left( h_1^0 \cos \beta + h_2^0 \sin \beta \right)$$  \hspace{1cm} (52)

as in (13). We have $<h_{SM}> | z > = -\cos 2\beta$, the
two fields getting close for large $\tan \beta$, with the $z$ tending
to behave very much as the SM-like $h_{SM}$.

More precisely while $h_{SM}$ has standard Yukawa couplings
to quarks and charged leptons $m_{q,l}/v = 2^{1/4} G_F^{1/2}$
and $m_{q,l}$, the $z$ has almost-identical couplings

$$\frac{m_{q,l}}{v} 2 T_{3q,l} = 2^{1/4} G_F^{1/2} m_{q,l} 2 T_{3q,l}. $$  \hspace{1cm} (53)

They simply differ by a relative change of sign for $d$
quarks and charged leptons (with $2 T_{3d,l} = -1$) acquiring
their masses through $<h^0_1>$, as compared to $u$ quarks.

This may also be understood by deducing the scalar
couplings of the spin-0 $z$ from the axial couplings of the
spin-1 $Z$, as follows:

The $Z$ is coupled, with coupling $\sqrt{g^2 + g'^2}$, to the weak
neutral current $J^\mu_Z = J^\mu_l - \sin^2 \theta J^\mu_{sm}$, with an axial part
$J^\mu_{ax} = T_{3q,l}/2$. It gets its mass by eliminating
the Goldstone field $z_g$, pseudoscalar partner of the scalar
$z$. As seen from the global limit $g, g' \rightarrow 0$ for which the
$Z$ would become massless and behave like the spin-0 $z_g$,
this $z_g$ has pseudoscalar couplings to quarks and leptons
given by

$$\sqrt{g^2 + g'^2} \frac{T_{3q,l}}{2} \frac{2 m_{q,l}}{m_Z} =$$

$$= \frac{m_{q,l}}{v} 2 T_{3q,l} = 2^{1/4} G_F^{1/2} m_{q,l} 2 T_{3q,l}. $$  \hspace{1cm} (54)

This is the same argument as for relating the axial coupling
of a $U$ boson to the pseudoscalar coupling of the

\text{equivalent axionlike pseudoscalar $A$ or $a$, with the $U$,
replaced by the $Z$, considered in the small mass and small
coupling limit [28]. The scalar partner $z$, described by
the same chiral superfield $H_z$ as the would-be Goldstone $z_g$,
has scalar couplings to quarks and leptons also given by

$$<h_{SM}> | z > = -\cos (\beta - \beta') = \sin (\beta + \alpha),$$  \hspace{1cm} (58)

At the same time

$$<h_{SM}> | z > = -\cos (\beta + \beta') = \sin (\beta - \alpha),$$  \hspace{1cm} (59)

the factor $\cos^2 \beta$ affecting the $Z^* + W^* + W^*$ decay rates
of a $z$ being replaced by $\cos^2 (\beta + \beta') = \sin^2 (\beta - \alpha)$. The
physical mass eigenstate $h$ is very close to the $z$ in (11)
for $\beta = \beta'$ i.e., $\beta + \alpha \approx \frac{\pi}{2}$, then justifying an almost
complete association of this 125 GeV/c$^2$ boson with the
spin-1 $Z$. This may be remembered as

$$\sqrt{g^2 + g'^2} \frac{T_{3q,l}}{2} \frac{2 m_{q,l}}{m_Z} = 2^{1/4} G_F^{1/2} m_{q,l} 2 T_{3q,l}. $$  \hspace{1cm} (55)
VIII. MASSIVE GAUGE SUPERFIELDS FOR SPIN-0 BOSONS

Supersymmetric theories thus allow for associating spin-0 with spin-0 particles within massive gauge multiplets of supersymmetry, leading to gauge/BE-Higgs unification, BEH bosons appearing as extra spin-0 states of massive spin-1 gauge bosons. We can even use the superfield formalism \[29\] to jointly describe these massive spin-1, spin-0 and now also spin-0 particles with massive gauge superfields \[12\].

Quite remarkably, this is possible in spite of their different electroweak properties, spin-1 fields transforming as a gauge triplet and a singlet with spin-0 BEH fields transforming as electroweak doublets. And although gauge and BE-Higgs bosons have very different couplings to quarks and leptons, which may first appear very puzzling but is elucidated in \[12\], using appropriate changes of field and superfield variables.

We make the generalized gauge choice (60), so that

\[
H_1 = \left( \begin{array}{c} v_2 \\\n\end{array} \right) , \quad H_2 = \left( \begin{array}{c} 0 \\
\end{array} \right) ,
\]

(62)

the ... involving the left-over superfield \(H_A\). A second order expansion of \(L\) along the lines of \[12\], with

\[-\xi D' = \xi \sin \theta D_Z - \xi \cos \theta D_\gamma . \quad (63)\]

generates superfield mass terms for \(W^\pm\) \((x, \theta, \bar{\theta})\) and \(Z(x, \theta, \bar{\theta})\). The term linear in \(Z(x, \theta, \bar{\theta})\), which appears with the coefficient

\[
\frac{\sqrt{g^2 + g'^2}}{4} (v^2_1 - v^2_2) + \xi \sin \theta = - < D_Z > \equiv 0 , \quad (64)
\]

vanishes identically owing to \(20\).

We get at second order

\[
L = \frac{1}{2} \left[ \left( \frac{g^2 + g'^2}{v^2} \right) (v_1^2 + v_2^2) \right] (W_3 \cos \theta - B \sin \theta)^2 + \frac{g^2(v_1^2 + v_2^2)}{4} (W_1^2 + W_2^2) \right]_D + ..., \quad (65)
\]

so that

\[
L = \frac{1}{2} m^2_Z (Z^2)_D + m^2_W |W^+|^2 + ...
\]

\[
= \frac{1}{2} m^2_Z (2 C_Z D_Z - \partial_\mu C_Z \partial^\mu C_Z - Z_\mu Z^\mu + ...)
\]

\[-\frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} + ... + \frac{D^2}{2} + ... + ... \quad (66)\]

After elimination of auxiliary fields through

\[
D_Z = -m^2_Z C_Z + ... = m_Z z + ..., \quad \text{etc.,} \quad (67)
\]

it includes the kinetic and mass terms for the gauge boson \(Z\) and associated spin-0 boson \(z\).

\[
L = \frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} - \frac{m^2_z}{2} Z_\mu Z^\mu - \frac{1}{2} \partial_\mu z \partial^\mu z - \frac{m^2_z}{2} z^2 + ...
\]

(68)

And similarly for the \(W^\pm\) and spin-0 partner \(w^\pm (\equiv H^\pm)\), keeping also in mind that supersymmetry is spontaneously broken for this superfield when \(\tan \beta \neq 1\).

In this picture these spin-0 bosons get described by the lowest \((C)\) spin-0 components of massive \(Z\) and \(W^\pm\) superfields, expanded as \(Z(x, \theta, \bar{\theta}) = C_Z + ... - \theta \sigma^\mu \bar{\theta} Z^\mu + ...\), \(W^\pm (x, \theta, \bar{\theta}) = C^\pm + ... - \theta \sigma^\mu \bar{\theta} W^\mu \pm + ...\). Their \(C\) components now describe, through non-polynomial field transformations linearized as \(z = -m_Z C_Z + ...\), \(w^\pm = m_W C^\pm + ...\), the spin-0 fields \(z\) and \(w^\pm\) in the usual formalism (with signs depending on previous choices for the definitions of \(z\) and \(w^\pm\)). We thus have

\[\begin{align*}
Z(x, \theta, \bar{\theta}) &= \left( \frac{z}{m_z} + ... \right) + ... - \theta \sigma_\mu \bar{\theta} Z^\mu + ..., \\
W^\pm (x, \theta, \bar{\theta}) &= \left( \frac{w^\pm}{m_W} + ... \right) + ... - \theta \sigma_\mu \bar{\theta} W^\mu \pm + ..., \quad (69)
\end{align*}\]
massive gauge superfields now describing spin-0 fields usually known as BEH fields! Their subcanonical \( \chi \) spin-$\frac{1}{2}$ components, instead of being gauged-away as usual, now also correspond to physical degrees of freedom describing the spin-$\frac{1}{2}$ fields previously known as higgsinos.

IX. THE BE-HIGGS BOSON AS SPIN-0 PARTNER OF Z, IN THE (N/n)MSSM

A. MSSM

This applies to the spin-0 sector of the MSSM. The scalar potential may be expressed by adding to \( V \) obtained from (3) the soft dimension-2 supersymmetry-breaking term

\[
-m_A^2 |h_1 s_\beta - h_2^2 c_\beta|^2 = -m_A^2 |\varphi_{\text{in}}|^2
\]

including in particular the \( \mu \)-term contribution. This term, which vanishes for \( h_0 = v_1/\sqrt{2} \), is a mass term for the doublet \( \varphi_{\text{in}} \), which has no v.e.c. and thus do not direct trilinear couplings to gauge boson pairs (only to quarks and charged leptons). It does not modify the vacuum state considered, initially taken as having a spontaneously-broken supersymmetry in the gauge-Higgs sector. It breaks explicitly the \( U(1)_A \) symmetry \([3][4]\), lifting the two previously-flat directions associated with \( s_A \) and \( A \). With

\[
|\varphi_{\text{in}}|^2 = |h_1 \sin \beta - h_2^2 \cos \beta|^2 = |H^+|^2 + \frac{1}{2} A^2 + \frac{1}{2} \sqrt{2} \text{Re}(h_0^2 s_\beta - h_2^4 c_\beta)|^2,
\]

it provides an extra contribution \( m_A^2 \) to \( m_{\tilde{H}^\pm} \), so that

\[
m_{\tilde{H}^\pm}^2 = m_W^2 + m_A^2.
\]

Adding the supersymmetric \( m_Z^2 \) contribution associated with the \( z \) in \([11]\) and supersymmetry-breaking contribution \( m_A^2 \) from \([70]\) we get the scalar mass square matrix

\[
\mathcal{M}_o^2 = \begin{pmatrix}
  c^2_\beta m_Z^2 + s^2_\beta m_A^2 & s_\beta c_\beta (m_Z^2 + m_A^2) \\
  -s_\beta c_\beta (m_Z^2 + m_A^2) & s^2_\beta m_Z^2 + c^2_\beta m_A^2
\end{pmatrix},
\]

verifying

\[
\text{Tr} \mathcal{M}_o^2 = m_{H^+}^2 + m_h^2 = m_Z^2 + m_A^2, \quad \det \mathcal{M}_o^2 = m_{H^+}^2 m_h^2 = m_Z^2 m_A^2 \cos^2 2\beta,
\]

so that

\[
m_{H^+, h}^2 = \frac{m_Z^2 + m_A^2}{2} \pm \sqrt{\left(\frac{m_Z^2 + m_A^2}{2}\right)^2 - m_Z^2 m_A^2 \cos^2 2\beta}.
\]

It implies \( m_h < m_Z |\cos 2\beta| \) at the classical level, up to radiative corrections which must be significant if one is to reach \( \sim 125 \text{GeV}/c^2 \) from a classical value below \( m_Z \). These mass eigenstates behave for large \( m_A \) as

\[
\begin{align*}
H &\to \sqrt{2} \text{Re}(h_0^0 s_\beta - h_2^0 c_\beta) \quad \text{(large} m_H \gtrsim m_A) , \\
h &\to h_{SM} = \sqrt{2} \text{Re}(h_1^0 c_\beta + h_2^0 s_\beta) \quad \text{(SM-like)} .
\end{align*}
\]

The \( h \) field, presumably associated with the 125 GeV/c^2 boson observed at CERN, is then also very close to the \( z \) in \([11]\) for large \( \tan \beta \), justifying an almost complete association of this 125 GeV/c^2 boson with the spin-1 Z.

B. N/nMSSM

This also applies to extensions of the minimal model, as with an extra N/nMSSM singlet \( S \) with a trilinear \( \lambda H_1 H_2 S \) coupling, making it easier to get from \( \lambda \) large enough spin-0 masses \([36]\). In the N/nMSSM, first considered without a \( \mu \) term, the supersymmetric contributions to spin-0 masses are \([1]\)

\[
\begin{align*}
\delta m_{s_A}^2 = m_{W}^2, \quad &m_z = m_Z, \\
m (\text{scalar } s_A, \text{ pseudoscalar } \Lambda) = m_A = \lambda v/\sqrt{2}.
\end{align*}
\]

They correspond, already in the absence of supersymmetry breaking, to the neutral scalar doublet mass \( m_A^2 \) matrix

\[
\mathcal{M}_o^2 = \begin{pmatrix}
  c^2_\beta m_Z^2 + s^2_\beta m_A^2 & s_\beta c_\beta (m_Z^2 - m_A^2) \\
  -s_\beta c_\beta (m_Z^2 - m_A^2) & s^2_\beta m_Z^2 + c^2_\beta m_A^2
\end{pmatrix},
\]

where \( m_A = \lambda v/\sqrt{2} \).

Adding as in \([70]\) the supersymmetry-breaking term

\[
-\delta m_A^2 |h_1 s_\beta - h_2^2 c_\beta|^2 = -\delta m_A^2 |\varphi_{\text{in}}|^2
\]

does not modify the vacuum state, while shifting the \( A \) and \( w \) mass \( \delta m_A^2 \) by the same amount \( \delta m_A^2 \), so that

\[
m_A^2 = \frac{\lambda^2 v^2}{2} + \delta m_A^2, \quad m_w^2 = m_W^2 + \delta m_A^2 = m_W^2 + m_A^2 - \frac{\lambda^2 v^2}{2}.
\]

It provides as in the MSSM an extra contribution to the neutral scalar doublet mass \( m_A^2 \) matrix, shifted by

\[
\delta \mathcal{M}_o^2 = \begin{pmatrix}
  s^2_\beta \delta m_A^2 & -s_\beta c_\beta \delta m_A^2 \\
  -s_\beta c_\beta \delta m_A^2 & c^2_\beta \delta m_A^2
\end{pmatrix}.
\]

From this shift \( m_A^2 = \frac{\lambda^2 v^2}{2} \to \frac{\lambda^2 v^2}{2} + \delta m_A^2 \), the mass square matrix for the scalar doublet components reads

\[
\mathcal{M}_o^2 = \begin{pmatrix}
  c^2_\beta m_Z^2 + s^2_\beta m_A^2 & s_\beta c_\beta (\lambda^2 v^2 - m_A^2 - m_Z^2) \\
  s_\beta c_\beta (\lambda^2 v^2 - m_A^2 - m_Z^2) & s^2_\beta m_Z^2 + c^2_\beta m_A^2
\end{pmatrix}.
\]

For \( \lambda \to 0 \) \( S \) decouples and the spectrum \([79][81]\) returns to the usual MSSM one. For \( \lambda \neq 0 \) further contributions involving also a possible singlet mass term \( \mu_s^2 S^2 \) lead in general to a mixing between neutral doublet and singlet components, with \( \mathcal{M}_o^2 \) embedded into a \( 3 \times 3 \) matrix.

X. CONCLUSIONS

Independently of specific realisations (MSSM, N/nMSSM, USSM, ...) supersymmetric theories provide spin-0 bosons as extra states for massive spin-1 gauge bosons, despite different symmetry properties and different couplings to quarks and leptons \[1, 12\]. This further applies to supersymmetric grand-unified theories with extra dimensions \[26, 27\]. By connecting spin-1 mediators of gauge interactions with spin-0 particles associated with symmetry breaking and mass generation, supersymmetry provides an intimate connection between the electroweak gauge couplings and the spin-0 couplings associated with symmetry breaking and mass generation.

The 125 GeV/c^2 boson recently observed at CERN may also be interpreted, up to a mixing angle induced by supersymmetry breaking, as the spin-0 partner of the Z under two supersymmetry transformations,

\[
\text{spin-1 } Z \xleftrightarrow{\text{susy}} \text{spin-0 BEH boson}, \tag{82}
\]

i.e. as a Z deprived of its spin.

This provides within a theory of electroweak and strong interactions the first example of two known fundamental particles of different spins related by supersymmetry, in spite of different electroweak properties. This is a considerable progress as compared to the initial situation in [1], bringing further confidence in the relevance of supersymmetry for the description of fundamental particles and interactions.

Supersymmetry may thus be tested in the gauge-and-\(BE\)-Higgs sector at present and future colliders, in particular through the properties of the new boson, even if \(R\)-odd superpartners were still to remain out of reach for some time.

[1] P. Fayet, Nucl. Phys. B 90, 104 (1975).
[2] P. Fayet, Phys. Lett. B 64, 159 (1976); B 69, 489 (1977).
[3] G. Farrar and P. Fayet, Phys. Lett. B 76, 575 (1978).
[4] P. Fayet, Phys. Lett. B 84, 416 (1979).
[5] S. Martin, A Supersymmetry Primer, hep-ph/9709356 (2011).
[6] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
[7] A. Salam, Proc. Nobel Symp. on El. Part. Th., ed. Swartholm, p. 367 (1968).
[8] J. Goldstone, Nuovo Cim. 19, 154 (1961).
[9] F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964).
[10] P. Higgs, Phys. Lett. 12, 132 (1964); Phys. Rev. Lett. 13, 508 (1964).
[11] G. Guralnik, C. Hagen and T. Kibble, Phys. Rev. Lett. 13, 585 (1964).
[12] P. Fayet, Nuovo Cim. A 31, 626 (1976); Nucl. Phys. B 237, 367 (1984).
[13] P. H. Hasert et al., Phys. Lett. B 46, 138 (1973).
[14] Yu. Gol’fand and E. Likhtman, ZhETF Pis. Red. 13, 452 (1971) [JETP Lett. 13, 323 (1971)].
[15] D. Volkov and V. Akulov, Phys. Lett. B 46, 109 (1973).
[16] J. Wess and B. Zumino, Nucl. Phys. B 70, 39 (1974).
[17] P. Ramond, Eur. Phys. J. C 74, 5, 2698 (2014) arXiv:1401.5977.
[18] A. Arison et al., Phys. Lett. B 122, 103 (1983); M. Bannier et al., Phys. Lett. B 122, 476 (1983).
[19] A. Arison et al., Phys. Lett. B 126, 398 (1983); P. Bagnaia et al., Phys. Lett. B 129, 130 (1983).
[20] ATLAS coll., Phys. Lett. B 716, 1 (2012).
[21] CMS coll., Phys. Lett. B 716, 30 (2012).
[22] P. Fayet and J. Iliopoulos, Phys. Lett. B 51, 461 (1974).
[23] P. Fayet, Nucl. Phys. B 78, 14 (1974).
[24] J. Beringer et al., Particle Data Group, Phys. Rev. D 86, 010001 (2012).
[25] P. Fayet, Phys. Lett. B 125, 178 (1983).
[26] P. Fayet, Nucl. Phys. B 246, 89 (1984).
[27] P. Fayet, Phys. Lett. B 159, 121 (1985); Nucl. Phys. B 263, 649 (1986); Proc. 2nd Nobel Symp. on El. Part. Phys., Phys. Scripta T 15, 46 (1987); Eur. Phys. J. C 74, 5, 2837 (2014) arXiv:1403.5951.
[28] P. Fayet, Nucl. Phys. B 187, 184 (1981); Phys. Lett. B 675, 267 (2009).
[29] A. Salam and J. Strathdee, Nucl. Phys. B 76, 477 (1974); S. Ferrara, J. Wess and B. Zumino, Phys. Lett. B 51, 239 (1974).
[30] S. Ferrara and B. Zumino, Nucl. Phys. B 131, 413 (1978); A. Salam and J. Strathdee, Phys. Lett. B 51, 353 (1974).
[31] Such a classically-massless or light pseudoscalar \(A\) associated with a \(U(1)_A\) symmetry is usually referred to as an axion or axionlike particle.
[32] This definition \[\phi\] of \(z\) includes a change of sign as compared to \[\psi\], so that it behaves as \(\sqrt{2}\) Re \(h^0\) for large \(\tan \beta\). The other sign would give, equivalently, Yukawa couplings of the “wrong sign” to up quarks. The choice \[\phi\] subsequently leads to \(D_2 = +m_Z z + \ldots\) in \(\phi\), and to identify, from \(D_2 = -m_Z^2 C_2 + \ldots\) in \(\phi\), \(C_2\) with \(-z/m_Z + \ldots\) within the massive gauge superfield \(\phi\).
[33] In a supersymmetric grand-unified theory with a semi-simple gauge group \(\xi\) must vanish at the GUT scale, requiring for two spin-0 doublets \(v_1 = v_2\) i.e. \(\tan \beta = 1\) at this scale.
[34] If \(\psi = a_L + b_R\) is a Dirac spinor constructed from two Majorana ones \(a\) and \(b\), its mass term may be expressed through a non-diagonal matrix, as \(-i m \psi\psi = -i m (b \frac{1}{2a} a + \bar{a} \frac{1}{2b} b) = -i m (ab + ba)\).
[35] \(\lambda\) was denoted \(\sqrt{\lambda} m_Z^2\) in \[\psi\], with \(\lambda v/\sqrt{2} = h v/2 \geq m_Z\) for \(\lambda \geq (g^2 + g'^2)/2\). This allows for all spin-0 masses to be \(\geq m_Z\) even before any breaking of the supersymmetry, independently of \(\tan \beta\), in contrast with the MSSM.