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On topological aspects of degree based entropy for two carbon nanosheets

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Abstract: The entropy-based procedures from the configuration of chemical graphs and multifaceted networks, several graph properties have been utilized. For computing, the organizational evidence of organic graphs and multifaceted networks, the graph entropies have converted the information-theoretic magnitudes. The graph entropy portion has attracted the research community due to its potential application in chemistry. In this paper, our input is to reconnoiter graph entropies constructed on innovative information function, which is the quantity of different degree vertices along with the quantity of edges between innumerable degree vertices. “In this study, we explore two dissimilar curricula of carbon nanosheets that composed by $C_4$ and $C_8$ denoted by $T_1C_4C_8(S)[m, n]$ and $T_2C_4C_8(R)[m, n]$. Additionally, we calculate entropies of these configurations by creating a connection of degree-based topological indices with the advantage of evidence occupation.

Keywords: entropy; Zagreb kind indices; Balaban index; carbon nanosheets; $T_1C_4C_8(S)[m, n]$; $T_2C_4C_8(R)[m, n]$

1 Introduction

A branch of mathematical chemistry that uses the tools of graph theory to develop the organic phenomenon mathematically is called chemical graph theory (Ali et al., 2019). Additionally, for resolving molecular problems, chemical graph theory links to the nontrivial solicitations of graph theory. This theory has important applications in the domain of chemical sciences (Gao and Farahani, 2016; Gao et al., 2018). Chem-informatics (containing chemistry and information science) analysis (QSAR) and (QSPR) are used to anticipate the bioactivity and physiochemical possessions of organic mixtures (Wu et al., 2015).

$G$ is a graph in which the vertex and the edge set are represented by $V(G)$ and $E(G)$, respectively. The degree $\Delta(r)$ of a vertex $r$ is the quantity of edges of $G$ contiguous with vertex $r$. Let $G$ be a graph with $m$ vertices and $n$ edges, where $m$ embodies the order and $n$ refers to the size of the graph. A graph of order $m$ and size $n$ is characterized as $(m, n)$-graph see: Assaye et al. (2019), Basavanagoud et al. (2017), Hosamani et al. (2017), and Shirakol (2019).

In molecular graph, the vertices designate atoms and edges signpost as the substance bonds. A arithmetic value that is calculated arithmetically by using the molecular graph is characterized as a topological index. It is connected to chemical composition demonstrating for association of chemical structure with plentiful physical, chemical possessions and biological activities. For further details of formulas of topological indices and application points see: Siddiqui et al., (2016a, 2016b), and Gao et al. (2017) and Imran et al. (2018), respectively.

The basic idea of entropy was introduced in the following statement: “The entropy of a possible dissemination is known as a quota of the unpredictability of evidence content or a portion of the uncertainty of a coordination” (Shannon, 2001), which was developed for evaluating the mechanical evidence of graphs and chemical networks.

Afterward, it has been used significantly in graphs and chemical networks. Rashevsky (1955) introduced the graph entropy impression established on the classifications of vertex orbits in 1955. Currently, graph entropies have been widely pragmatic in an extensive assortment of questions, such as chemistry and sociology (Dehmer and Grabner, 2013; Ulanowicz, 2004).

There are many specific categories of aforementioned graph entropy procedures. They associate probability
distributions with elements (vertices, edges, etc.) of a graph that can be categorized as core and extrinsic dealings. Dehmer (2008) and Dehmer et al. (2012) presented information based function graph entropies, which imprisonment operational evidence and analyze their properties. For more details, see: Bonchev (2003), Dehmer and Mowshowitz (2011), Morowitz (1955), Rashevsky (1955), Shannon (2001), Solé and Valverde (2004), Quastler (1954), Tan and Wu (2004), and Trucco (1956).

2 Degree based topological indices of graph

Gutman with coauthors (Gutman and Das, 2004; Gutman and Trinajstic, 1972) defined first and second Zagreb index as:

\[ M_1 = \sum_{rs \in E(G)} (\Xi(r) + \Xi(s)), \quad M_2(G) = \sum_{rs \in E(G)} (\Xi(r) \times \Xi(s)) \]

Shirdel et al. (2013) introduced “hyper Zagreb index” as:

\[ HM = \sum_{rs \in E(G)} (\Xi(r) + \Xi(s))^2 \]

Furtula and Gutman (2015) defined F-index as:

\[ F(G) = \sum_{rs \in E(G)} \left( \Xi(r)^2 + \Xi(s)^2 \right) \]

Furtula et al. (2010) acquainted with a new topological index named “augmented Zagreb index” and is characterized as:

\[ AZI(G) = \sum_{rs \in E(G)} \left( \frac{\Xi(r)\Xi(s)}{\Xi(r) + \Xi(s) - 2} \right)^3 \]

Balaban (1982) and Balaban and Quintas (1983) announced a new topological index for a graph G of order p, size q as:

\[ J(G) = \frac{q}{q-p+2} \sum_{rs \in E(G)} \frac{1}{\sqrt{\Xi(r)\Xi(s)}} \]

For more details about these indices see: Akhter et al. (2016), Ali et al. (2019), Liu et al. (2020a, 2020b), Raza (2020a, 2020b), Raza and Ali (2020), and Raza and Sukaiti (2020).

3 General entropy of graph

In 2014, Chen et al. (2014) familiarized the definition of the entropy of edge partisan graph. The entropy of edge partisan graph is characterized in Eq. 1:

\[ ENT_{\psi}(G) = \sum_{rs \in E(G)} \log \left( \frac{\psi(r's')}{\sum_{rs \in E(G)} \psi(rs)} \right) \]

3.1 The first Zagreb entropy

If \( \psi(rs) = \Xi(r) + \Xi(s) \), then:

\[ \sum_{rs \in E(G)} \psi(rs) = \sum_{rs \in E(G)} (\Xi(r) + \Xi(s)) = M_1(G) \]

Now Eq. 1 is converted and called the first Zagreb entropy:

\[ ENT_{M_1}(G) = \log \left( M_1(G) \right) \]

\[ -\frac{1}{M_1(G)} \log \left( \prod_{rs \in E(G)} \left( \Xi(r) + \Xi(s) \right)^{\Xi(s) + \Xi(s)} \right) \]

3.2 The second Zagreb entropy

If \( \psi(rs) = \Xi(r) \times \Xi(s) \), then:

\[ \sum_{rs \in E(G)} \psi(rs) = \sum_{rs \in E(G)} (\Xi(r) \times \Xi(s)) = M_2(G) \]

Now Eq. 1 is converted and called the second Zagreb entropy:

\[ ENT_{M_2}(G) = \log \left( M_2(G) \right) \]

\[ -\frac{1}{M_2(G)} \log \left( \prod_{rs \in E(G)} \left( \Xi(r) \times \Xi(s) \right)^{\Xi(r) \times \Xi(s)} \right) \]
3.3 The hyper Zagreb entropy

If \( \Psi(rs) = \left[ \Xi(r) + \Xi(s) \right]^2 \), then:

\[
\sum_{nsc(G)} \Psi(rs) = \sum_{nsc(G)} \left[ \Xi(r) + \Xi(s) \right] = HM(G)
\]

Now Eq. 1 is converted and called the hyper Zagreb entropy:

\[
\text{ENT}_{hm}(G) = \log(HM(G))
\]

\[
- \frac{1}{\text{HM}(G)} \log \left( \prod_{nsc(G)} \left[ \left( \Xi(r) + \Xi(s) \right) \right] \right) \] (4)

3.4 The forgotten entropy

If \( \Psi(rs) = (\Xi(r)^2 + \Xi(s)^2) \), then:

\[
\sum_{nsc(G)} \Psi(rs) = \sum_{nsc(G)} \left[ \Xi(r)^2 + \Xi(s)^2 \right] = F(G)
\]

Now Eq. 1 is converted and called the forgotten entropy:

\[
\text{ENT}_{f}(G) = \log(F(G))
\]

\[
- \frac{1}{F(G)} \log \left( \prod_{nsc(G)} \left[ \left( \Xi(r)^2 + \Xi(s)^2 \right) \right] \right) \] (5)

3.5 The augmented Zagreb entropy

If \( \Psi(rs) = \left( \frac{\Xi(r)\Xi(s)}{\Xi(r)+\Xi(s)-2} \right)^2 \), then:

\[
\sum_{nsc(G)} \Psi(rs) = \sum_{nsc(G)} \left( \frac{\Xi(r)\Xi(s)}{\Xi(r)+\Xi(s)-2} \right) = AZI(G)
\]

Now Eq. 1 is converted and called the augmented Zagreb entropy:

\[
\text{ENT}_{azi}(G) = \log(AZI(G))
\]

\[
- \frac{1}{AZI(G)} \log \left( \prod_{nsc(G)} \left( \frac{\Xi(r)\Xi(s)}{\Xi(r)+\Xi(s)-2} \right) \right) \] (6)

3.6 The Balaban entropy

If \( \Psi(rs) = q^{-p+2} \frac{1}{\sqrt{\Xi(r)\Xi(s)}} \), then:

\[
\sum_{nsc(G)} \Psi(rs) = \sum_{nsc(G)} \frac{1}{\sqrt{\Xi(r)\Xi(s)}} = J(G)
\]

Now Eq. 1 is converted and called the Balaban entropy:

\[
\text{ENT}_{b}(G) = \log(J(G))
\]

\[
- \frac{1}{J(G)} \log \left( \prod_{nsc(G)} \left( \frac{q}{\sqrt{\Xi(r)\Xi(s)}} \right) \right) \] (7)

4 Crystallographic structure of first carbon nanosheet

\( T^1C_4C_8(S)[m, n] \)

A \( C_4C_8(S) \) nanosheet is a trivalent ornamentation prepared by blinking tetragons \( C_4 \) and octagons \( C_8 \). There are two categories of nanosheets which canister be completed by \( C_4 \) and \( C_8 \) following the trivalent ornamentation which we mention to as \( T^1C_4C_8(S)[m, n] \) and \( T^2C_4C_8(R)[m, n] \). The \( T^1C_4C_8(S)[m, n] \) nanosheet is the two-dimensional lattice of \( TUC_4C_8(S)[m, n] \), where \( m \) and \( n \) are significant restrictions in Figure 1. In this segment, we deliberate the first kind of nanosheet i.e. \( T^1C_4C_8(S)[m, n] \) in which \( C_4 \) acts as a tetragonal and \( m \) and \( n \) are the quantity of octagons in any pillar and racket individually. Figure 2 portrays the Type-I \( C_4C_8(S) \) nanosheet \( T^1C_4C_8(S)[m, n] \). The vertex and edge cardinalities of this organic graph are 8mn and 12mn – 2(m + n) correspondingly.

The vertex barrier of \( T^1C_4C_8(S)[m, n] \) established on degrees of respectively vertex is portrayed in Table 1. Also the edge barrier of \( T^1C_4C_8(S)[m; n] \) centered on degrees of end vertices of an edge are depicted in Table 2.

![Figure 1: A TUC_4C_8(S)[m, n] nanotube.](image-url)
Table 1: Vertex partition of $T^{C_4}C_8(S)[m, n]$.

| $\Xi(r)$ | Frequency | Set of vertices |
|----------|-----------|-----------------|
| 2        | $4m + 4n$ | $V_1$           |
| 3        | $8mn - 4m - 4n$ | $V_2$ |

Table 2: Edge partition of $T^{C_4}C_8(S)[m, n]$.

| $(\Xi(r), \Xi(s))$ | Frequency | Set of edges |
|-------------------|-----------|-------------|
| (2, 2)            | $2m + 2n + 4$ | $E_1$ |
| (3, 2)            | $4m + 4n - 8$ | $E_2$ |
| (3, 3)            | $12mn - 8m - 8n + 4$ | $E_3$ |

4.1 Results for carbon nanosheet $T^{C_4}C_8(S)[m, n]$  

4.1.1 The first Zagreb entropy of $T^{C_4}C_8(S)[m, n]$  

Now using Eq. 2 and Table 2, we computed following results. The first Zagreb index by using Table 2 is:

$$M(G) = 72mn - 20(m + n)$$

Now Eq. 2 (with Table 2) can take the following form:

$$\text{ENT}_{M_1} \left(T^{C_4}C_8(S)\right) = \log \left(M_1(G)\right) - \frac{1}{M_1(G)} \log \prod_{r \in E_1(G)} \left[\Xi(r) + \Xi(s)\right] \times \prod_{r \in E_1(G)} \left[\Xi(r) + \Xi(s)\right] \times \prod_{r \in E_1(G)} \left[\Xi(r) + \Xi(s)\right]$$

$$= \log \left(M_1\right) - \frac{1}{M_1} \log \left[\left(2m + 2n + 4\right) \times \left(256\right)\right] \times \log \left[\left(4m + 4n - 8\right) \times \left(3125\right)\right] \times \log \left[\left(12mn - 8m - 8n + 4\right) \times \left(46656\right)\right]$$

$$= \log \left(72mn - 20(m + n)\right) - \frac{\log \left[\left(2m + 2n + 4\right) \times \left(256\right)\right]}{72mn - 20(m + n)} - \frac{\log \left[\left(4m + 4n - 8\right) \times \left(3125\right)\right]}{72mn - 20(m + n)} - \frac{\log \left[\left(12mn - 8m - 8n + 4\right) \times \left(46656\right)\right]}{72mn - 20(m + n)}$$

4.1.2 The second Zagreb entropy of $T^{C_4}C_8(S)[m, n]$  

Now using Eq. 3 and Table 2, we computed following results. The second Zagreb index by using Table 2 is:

$$M_2(G) = 108mn - 40(m + n) + 4$$

Now Eq. 3 (with Table 2) can take the following form:

$$\text{ENT}_{M_2} \left(T^{C_4}C_8(S)\right) = \log \left(M_2(G)\right) - \frac{1}{M_2(G)} \log \prod_{r \in E_1(G)} \left[\Xi(r) \times \Xi(s)\right] \times \prod_{r \in E_1(G)} \left[\Xi(r) \times \Xi(s)\right] \times \prod_{r \in E_1(G)} \left[\Xi(r) \times \Xi(s)\right]$$

$$= \log \left(M_2\right) - \frac{1}{M_2} \log \left[\left(2m + 2n + 4\right) \times \left(256\right)\right] \times \log \left[\left(4m + 4n - 8\right) \times \left(46656\right)\right] \times \log \left[\left(12mn - 8m - 8n + 4\right) \times \left(9^4\right)\right]$$

$$= \log \left(108mn - 40(m + n) + 4\right) - \frac{\log \left[\left(2m + 2n + 4\right) \times \left(256\right)\right]}{108mn - 40(m + n) + 4} - \frac{\log \left[\left(4m + 4n - 8\right) \times \left(46656\right)\right]}{108mn - 40(m + n) + 4} - \frac{\log \left[\left(12mn - 8m - 8n + 4\right) \times \left(9^4\right)\right]}{108mn - 40(m + n) + 4}$$
4.1.3 The hyper Zagreb entropy of $T^4C_4(S)[m, n]$

Now using Eq. 4 and Table 2, we computed following result. By using Table 2, we have:

$$
\text{ENT}_{HM} \left( T^4C_4(S) \right) = \log \left( \frac{F(G)}{\text{HM}(G)} \right) = \log \left( \frac{216mn - 76(m + n)}{16} \times \left( \frac{4m + 4n - 8}{25} \times \frac{12m - 8m - 8n + 4}{36} \right) \right)
$$

$$
= \log \left( \frac{216mn - 76(m + n)}{16} \right) - \frac{1}{\text{HM}(G)} \log \left( \prod_{r \in E(G)} (\Xi(r) + \Xi(s))^2 \prod_{s \in E(G)} (\Xi(r) + \Xi(s))^2 \prod_{t \in E(G)} (\Xi(r) + \Xi(s))^2 \right)
$$

$$
= \log \left( \frac{216mn - 76(m + n)}{16} \right)
$$

4.1.4 The forgotten Zagreb entropy of $T^4C_4(S)[m, n]$

Now using Eq. 5 and Table 2, we got following expressions. By using Table 2, we have:

$$
\text{ENT}_{F} \left( T^4C_4(S) \right) = \log \left( \frac{F(G)}{\text{F}(G)} \right) = \log \left( \frac{216mn - 76(m + n)}{16} \times \left( \frac{4m + 4n - 8}{25} \times \frac{12m - 8m - 8n + 4}{36} \right) \right)
$$

$$
= \log \left( \frac{216mn - 76(m + n)}{16} \right) - \frac{1}{\text{F}(G)} \log \left( \prod_{r \in E(G)} (\Xi(r) + \Xi(s))^2 \prod_{s \in E(G)} (\Xi(r) + \Xi(s))^2 \prod_{t \in E(G)} (\Xi(r) + \Xi(s))^2 \right)
$$

$$
= \log \left( \frac{216mn - 76(m + n)}{16} \right)
$$

4.1.5 The augmented Zagreb entropy of $T^4C_4(S)[m, n]$

Now using Eq. 5 and Table 2, we got following expressions. By using Table 2, we have:

$$
\text{AZI}(G) = \frac{2687}{16} \text{mn} - \frac{3453}{5} (m + n) + \frac{217}{16}
$$

Now Eq. 6 (with Table 2) becomes:
ENT_{AZI} \left(T^C_4 C_8 (S) \right) = \log \left( AZI(G) \right)

\begin{align*}
- \frac{1}{AZI(G)} \log \left[ \prod_{m \in E_4 (G)} \left( \frac{\Xi(t)\Xi(s)}{\Xi(t)+\Xi(s)-2} \right) \right] \\
- \frac{1}{AZI(G)} \log \left[ \prod_{m \in E_2 (G)} \left( \frac{\Xi(t)\Xi(s)}{\Xi(t)+\Xi(s)-2} \right) \right] \\
= \log \left( AZI(G) \right) - \frac{1}{AZI(G)} \log \left[ \frac{2m+2n+4}{8} \times \frac{4m+4n-8}{8} \times \frac{12mn-8m-8n+4}{4} \times \frac{9}{4^3} \right]
\end{align*}

4.1.6 The Balaban entropy of T^C_4 C_8(S)[m, n]

Now using Eq. 7 and Table 2, we move in the following way. By using Table 2, we have:

\begin{align*}
\text{ENT}_{AZI} \left(T^C_4 C_8 (S) \right) = \log \left( J(G) \right)
\end{align*}

\begin{align*}
\begin{pmatrix}
\prod_{m \in E_4 (G)} \left[ \frac{q}{q-p+2} \right] \\
\prod_{m \in E_2 (G)} \left[ \frac{q}{q-p+2} \right]
\end{pmatrix}
\end{align*}

\begin{align*}
\begin{pmatrix}
\frac{q}{2(q-p+2)} \\
\frac{q}{\sqrt{6}(q-p+2)}
\end{pmatrix}
\times
\begin{pmatrix}
\frac{q}{\sqrt{6}(q-p+2)} \\
\frac{q}{3(q-p+2)}
\end{pmatrix}
\times
\begin{pmatrix}
\frac{q}{\sqrt{6}(q-p+2)} \\
\frac{q}{3(q-p+2)}
\end{pmatrix}
\times
\begin{pmatrix}
\frac{q}{\sqrt{6}(q-p+2)} \\
\frac{q}{3(q-p+2)}
\end{pmatrix}
\end{align*}

Equation 7 (with Table 2) takes the following form:

\begin{align*}
J(G) = \frac{12mn-2m-2n}{4mn-2m-2n+2} \times \left( \frac{4m}{\sqrt{6}} - \frac{5}{3} \right) (m+n) + \frac{10}{3} - \frac{8}{\sqrt{6}}
\end{align*}
5 Carbon nanosheet $T^2C_4C_8(R)[m, n]$

A $C_4C_8(R)$ nanosheet is a trivalent decoration prepared by ashing rhombus $C_4$ and octagons $C_8$. We talk about this nanosheet as $T^2C_4C_8(R)[m, n]$. This nanosheet is the two-dimensional lattice of $TUC_4C_8(R)[m, n]$, where $m$ and $n$ are essential limitations in Figure 3. In this section we discourse $T^2C_4C_8(R)[m, n]$ in which $C_4$ entertainments as a rhombus and $m$ and $n$ are quantity of octagons in any pillar and row correspondingly. Figure 4 depicts the type-II $C_4C_8(R)$ nanosheet $T^2C_4C_8(R)[m, n]$. The vertex and edge cardinalities of this organic graph are $4mn + 4(m+n) + 4$ and $6mn + 5(m+n) + 4$ correspondingly.

The vertex barrier of $T^2C_4C_8(R)[m, n]$ grounded on degrees of each vertex is portrayed in Table 3. Also the edge dividing wall of $T^2C_4C_8(R)[m, n]$ centered on degrees of expiration vertices of respectively edge are showed in Table 4.

5.1 Results for carbon nanosheet $T^2C_4C_8(R)[m, n]$

5.1.1 The first Zagreb entropy of $T^2C_4C_8(R)[m, n]$

Now using Eq. 2 and Table 4, we have:

$$M_1(G) = 36mn + 26(m+n) + 16$$

Equation 2 (with Table 4) is converted into the following form:

$$\text{ENT}_{M_1}(T^2C_4C_8(R)) = \log(M_1) - \frac{1}{(M_1)} \sum_{\text{res E}(G)} \prod \left[ \Xi_{\text{res E}(G)}(\tau) + \Xi_{\text{res E}(G)}(s) \right] + \prod_{\text{res E}(G)} \left[ \Xi_{\text{res E}(G)}(\tau) + \Xi_{\text{res E}(G)}(s) \right] + \prod_{\text{res E}(G)} \left[ \Xi_{\text{res E}(G)}(\tau) + \Xi_{\text{res E}(G)}(s) \right].$$

Figure 3: A $TUC_4C_8(R)[m, n]$ nanotube.

Figure 4: Type-II $C_4C_8(R)$ nanosheet $T^2C_4C_8(R)[m, n]$. 
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Table 3: Vertex partition of \( T^2 C_4 C_8(R)[m,n] \).

| \( \Xi(r) \) | Frequency | Set of vertices |
|----------------|-----------|-----------------|
| 2              | \( 2m + 2n + 4 \) | \( V_1 \) |
| 3              | \( 4mn + 2m + 2n \) | \( V_2 \) |

Table 4: Edge partition of \( T^2 C_4 C_8(R)[m,n] \).

| \( \Xi(r), \Xi(s) \) | Frequency | Set of edges |
|---------------------|-----------|--------------|
| (2, 2)              | 4         | \( E_1 \)    |
| (3, 2)              | \( 4m + 4n \) | \( E_2 \)    |
| (3, 3)              | \( 6mn + m + n \) | \( E_3 \) |

5.1.2 The second Zagreb entropy of \( T^2 C_4 C_8(S)[m,n] \)

Now using Eq. 3 and Table 4, we computed the first Zagreb entropy in the following way:

\[
\text{ENT}_{M_1} \left( T^2 C_4 C_8 (R) \right) = \log \left( M_1 \right) - \frac{1}{M_1} \log \left[ \left( 4 \right) \times \left( 256 \right) \times \left( 4mn + 2m + 2n \right) \times \left( 6mn + m + n \right) \times \left( 46656 \right) \right]
\]

\[
= \log \left( 36mn + 26 \left( m + n \right) + 16 \right) - \log \left( \left( 4 \right) \times \left( 256 \right) \right) - \log \left( \left( 4m + 4n \right) \times \left( 3125 \right) \right) - \log \left( \left( 6mn + m + n \right) \times \left( 46656 \right) \right)
\]

5.1.3 The hyper Zagreb entropy of \( T^2 C_4 C_8(R)[m,n] \)

Now using Eq. 4 and Table 4, we computed the following result:

\[
\text{ENT}_{HM} \left( T^2 C_4 C_8 (R) \right) = \log \left( HM \right) - \frac{1}{HM} \log \left[ \left( 4 \right) \times \left( 16 \right) \times \left( 4m + 4n \right) \times \left( 25 \right) \times \left( 6mn + m + n \right) \times \left( 36 \right) \right]
\]
Using Table 4, we obtained:

\[
\log \left( \frac{216mn + 136(m+n) + 64}{216mn + 136(m+n) + 64} \right) - \log \left( \frac{4 \times 16^{[16]}}{4m+4n \times 25^{[21]}} \right) - \log \left( \frac{6mn + m+n \times 36^{[36]}}{216mn + 136(m+n) + 64} \right)
\]

5.1.4 The forgotten Zagreb of \( T^2 C_4 C_8 (R) \)[m, n]

Using Table 4, we obtained:

\[
\text{F(G)} = 108mn + 70(m+n) + 32
\]

Equation 5 (with Table 4) becomes:

\[
\text{Ent}_f \left( T^2 C_4 C_8 (R) \right) = \log \left( \text{F(G)} \right)
\]

\[
= \log \left( \frac{2187}{32} \right) - \log \left( \frac{4 \times 8^{[8]}}{4m+4n \times 11^{[11]}} \right) - \log \left( \frac{6mn + m+n \times 18^{[18]}}{108mn + 70(m+n) + 32} \right)
\]

5.1.5 The augmented Zagreb of \( T^2 C_4 C_8 (S) \)[m, n]

Using Table 2, we have:

\[
\text{AZI(G)} = \frac{2187mn}{32} - \frac{2777}{64}(m+n) + 32
\]

\[
\text{Ent}_\text{AZI} \left( T^2 C_4 C_8 (R) \right) = \log \left( \text{AZI(G)} \right)
\]

\[
= \frac{2187}{32} \text{mn} - \frac{2777}{64}(m+n) + 32
\]

\[
= \log \left( \frac{4 \times 8^{[8]}}{4m+4n \times 8^{[8]}} \right) - \log \left( \frac{6mn + m+n \times \frac{729}{64}}{108mn + 70(m+n) + 32} \right)
\]
### 5.1.6 The Balaban entropy of $T^2C_4C_8(R)[m, n]$

By using Table 4, we obtained:

$$\text{ENT}_{T^2C_4C_8(R)} = \log(J(G)) - \frac{1}{J(G)} \log \left( \prod_{r,s \in \mathbb{E}(G)} \frac{q}{q - p + 2} \frac{1}{\sqrt{\Xi(r)\Xi(s)}} \right)$$

Equation 7 (with Table 4) takes the following form:

$$J(G) = \frac{6mn+5}{2mn+(m+n)+2} \left[ \frac{1}{3} + \frac{4}{\sqrt{6}} \right] (m+n)+2$$

### 6 Comparisons and discussion for $T^2C_4C_8(S)[m, n]$

Since the degree based entropy has part of utilization in various parts of science, in particular pharmaceutical, science, organic medications and software engineering. So the numerical and graphical portrayal of these determined outcomes are useful to researcher. So in this area, we have registered numerically all degree based entropies for various estimations of $m, n$ for $T^2C_4C_8(S)[m, n]$. Furthermore, we develop Tables 5 and 6

| [m, n] | $\text{ENT}_{M_1}$ | $\text{ENT}_{M_2}$ |
|-------|----------------|----------------|
| [2, 2] | 2.25 | 2.37 |
| [3, 3] | 2.69 | 2.84 |
| [4, 4] | 2.98 | 3.14 |
| [5, 5] | 3.19 | 3.35 |
| [6, 6] | 3.36 | 3.53 |
| [7, 7] | 3.51 | 3.67 |
| [8, 8] | 3.63 | 3.79 |
| [9, 9] | 3.74 | 3.90 |
| [10, 10] | 3.83 | 4.01 |

| [m, n] | $\text{ENT}_{\text{sum}}$ | $\text{ENT}_x$ | $\text{ENT}_{\text{ord}}$ | $\text{ENT}_y$ |
|-------|----------------|----------------|----------------|----------------|
| [2, 2] | 2.94 | 2.66 | 2.51 | 1.74 |
| [3, 3] | 3.43 | 3.14 | 2.96 | 2.08 |
| [4, 4] | 3.73 | 3.44 | 3.25 | 2.33 |
| [5, 5] | 3.95 | 3.36 | 3.47 | 2.51 |
| [6, 6] | 4.13 | 3.83 | 3.64 | 2.67 |
| [7, 7] | 4.27 | 3.97 | 3.78 | 2.80 |
| [8, 8] | 4.40 | 4.10 | 3.90 | 2.91 |
| [9, 9] | 4.50 | 4.20 | 4.01 | 3.01 |
| [10, 10] | 4.60 | 4.30 | 4.11 | 3.10 |
for little estimations of \( m, n \) for degree based entropy to numerical correlation for the structure of \( T^2C_4C_8(S) \) \([m, n]\). Presently, from Tables 5 and 6, we can without much of a stretch see that all the estimations of entropy are in expanding request as the estimations of \( m; n \) are increments. The graphical portrayals of registered outcomes are delineated in Figures 5-7 for specific estimations of \( m, n \).

7 Comparisons and discussion for \( T^2C_4C_8(R)[m, n] \)

Since the degree based entropy has parcel of utilization in various parts of science, in particular pharmaceutical, science, natural medications and software engineering. So the numerical and graphical portrayal of these determined outcomes are useful to researcher.

![Figure 5: (a) The first Zagreb entropy, (b) the second Zagreb entropy.](image1)

![Figure 6: (a) The hyper Zagreb entropy, (b) the forgotten entropy.](image2)

![Figure 7: (a) The augmented Zagreb entropy, (b) the Balaban entropy.](image3)
So in this area, we have figured numerically all degree based entropies for various estimations of \( m; n \) for \( \mathcal{T}_C C_4 \mathcal{C}_8(R)[m, n] \). Moreover, we build Tables 7 and 8 for little estimations of \( m, n \) for degree based entropy to numerical correlation for the structure of \( \mathcal{T}_C C_4 \mathcal{C}_8(R)[m, n] \). Presently, from Tables 7 and 8, we can without much of a stretch see that all the estimations of entropy are in expanding request as the estimations of \( m; n \) are increments. The graphical portrayals of processed outcomes are delineated in Figures 8-10 for specific estimations of \( m, n \).

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### Table 7: Comparison of first Zagreb entropy and second Zagreb entropy for \( \mathcal{T}_C C_4 \mathcal{C}_8(S)[m, n] \).

| \([m, n]\) | \(\text{ENT}_{M_1}\) | \(\text{ENT}_{M_2}\) |
|---------|----------------|----------------|
| \([2, 2]\) | 2.37           | 2.51           |
| \([3, 3]\) | 2.67           | 2.82           |
| \([4, 4]\) | 2.88           | 3.04           |
| \([5, 5]\) | 3.06           | 3.22           |
| \([6, 6]\) | 3.20           | 3.36           |
| \([7, 7]\) | 3.32           | 3.49           |
| \([8, 8]\) | 3.43           | 3.60           |
| \([9, 9]\) | 3.53           | 3.69           |
| \([10, 10]\) | 3.61          | 4.78           |

### Table 8: Comparison of \(\text{ENT}_{M_1}, \text{ENT}_{M_2}, \text{ENT}_{AZ}, \text{ENT}_{J}\) entropies for \( \mathcal{T}_C C_4 \mathcal{C}_8(S)[m, n] \).

| \([m, n]\) | \(\text{ENT}_{M_1}\) | \(\text{ENT}_{M_2}\) | \(\text{ENT}_{AZ}\) | \(\text{ENT}_{J}\) |
|---------|----------------|----------------|---------------|---------------|
| \([2, 2]\) | 3.09           | 2.81           | 2.62          | 1.72          |
| \([3, 3]\) | 3.41           | 3.12           | 2.92          | 1.99          |
| \([4, 4]\) | 3.64           | 3.34           | 3.15          | 2.19          |
| \([5, 5]\) | 3.82           | 3.52           | 3.32          | 2.35          |
| \([6, 6]\) | 3.96           | 3.67           | 3.47          | 2.49          |
| \([7, 7]\) | 4.09           | 3.79           | 3.59          | 2.60          |
| \([8, 8]\) | 4.20           | 3.90           | 3.70          | 2.70          |
| \([9, 9]\) | 4.30           | 3.99           | 3.80          | 2.79          |
| \([10, 10]\) | 4.38          | 4.08           | 3.88          | 2.88          |

**Figure 8:** (a) The first Zagreb entropy, (b) the second Zagreb entropy.

**Figure 9:** (a) The hyper Zagreb entropy, (b) the forgotten entropy.
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