Implication of GW170817 for cosmological bounces

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Abstract

The detection of GW170817 and its electromagnetic counterpart has revealed the speed of gravitational waves coincides with the speed of light, $c_T = 1$. Inspired by the possibility that the physics implied by GW170817 might be related with that for the primordial universe, we construct the spatially flat stable (throughout the whole evolution) nonsingular bounce models in the beyond Horndeski theory with $c_T = 1$ and in the degenerate higher-order scalar-tensor (DHOST) theory with $c_T = 1$, respectively. Though it constricts the space of viable models, the constraint of $c_T = 1$ makes the procedure of building models simpler.

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I. INTRODUCTION

Inflation is a successful scenario of the early universe [1–4]. However, it is well-known that inflation suffered from the singularity problem [5, 6]. This suggests that our understanding about the gravity and the early universe is incomplete. Instead of going to the quantum regime and studying the physics of the “singularity”, one might construct classical nonsingular cosmological models alternative or complementary to the inflation scenario. Bouncing cosmology is a class of such models with different applications, see e.g. [7–9] for earlier studies, [10–12] for recent reviews.

Building nonsingular cosmological models in the scalar field theories has been still one of the endeavors. It had been observed that the spatially flat bounce models constructed in Horndeski theories [13] inevitably encounter instabilities (or else the singularity in Lagrangian) [14, 15], the so-called No-go Theorem, see also Refs.[16–18] for the attempts in the Horndeski theory. Recently, based on the effective field theory (EFT) of cosmological perturbations, it has been found that the solutions of fully stable (without ghosts, gradient instabilities, etc., throughout the whole evolution) cosmological bounce do exist if one goes beyond Horndeski [19, 20], see Ref.[21–23] for the corresponding bounce models performed in full covariant Lagrangians (of the beyond Horndeski theory [24]). The progress caused by "No-go" have also stimulated lots of studies, e.g.[25–32].

Beyond Horndeski theories are a subclass of the degenerate higher-order scalar-tensor (DHOST) theory [33–36]. Unlike in general relativity (GR), the propagating speed $c_T$ of gravitational waves (GW) in the DHOST theory might deviate considerably from the speed of light. Recently, the detection of GW170817 [37] and its electromagnetic counterpart has provided a precise measurement for the speed of GWs: it coincides with the speed of light with deviations smaller than a few $\times 10^{-15}$, i.e. $c_T = 1$. This measurement strictly constrained the scalar-tensor theories responsible for the acceleration of the current universe [38–45]. Though the physics implied by GW170817 seems not straightly related with that for the primordial universe, undeniably, such potential relevance will be interesting.

In this paper, inspired by the implication of GW170817, we will construct the stable cosmological bounce models with $c_T = 1$. Using the ADM metric, we replace the covariant $c_T = 1$ DHOST Lagrangian with its ADM form (Sect.II), and perform the perturbation calculations with it. We construct fully stable bounce models in the beyond Horndeski
theory with $c_T = 1$ (Sect.III A), which is a special subclass of the DHOST theory, and in
the full $c_T = 1$ DHOST theory (Sect.III B), respectively.

II. DHOST THEORY WITH $c_T = 1$

A. The Lagrangians

We begin with the covariant Lagrangian of the beyond Horndeski theory with $c_T = 1$ [40]

$$L^{bH}_{c_T=1} = \sqrt{-g} L^{bH}_{c_T=1} = \sqrt{-g} \left[ G_2(\phi, X) + G_3(\phi, X) \Box \phi + B_4(\phi, X) R 
- \frac{4}{X} B_{4, X}(\phi, X) (\phi^\mu \phi^\nu \phi_{\mu\nu} \Box \phi - \phi^\mu \phi_{\mu\nu} \phi_{\lambda\nu} \phi_{\lambda\nu}) \right],$$

where $\nabla_\mu \phi \equiv \phi_{\mu}, \nabla_\nu \nabla_\mu \phi \equiv \phi_{\mu\nu}$ and $X \equiv \phi_{\mu} \phi^\mu$.

We adopt the ADM metric

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

where $N$ is the lapse, $N_i$ is the shift, $h_{ij}$ is the spatial metric. We will use $\eta = \phi$ as the
time coordinate in the FRW metric, $ds^2 = -N(\eta)^2 d\eta^2 + a^2|d\vec{x}|^2$. Dynamics of $\phi$ has been
absorbed into $N(\eta)$, since $\phi' \equiv d\phi/d\eta = 1$.

In the unitary gauge $\delta \phi = 0$, the covariant Lagrangian $L^{bH}_{c_T=1}$ (1) may be rewritten in the
ADM form [24],

$$L^{bH}_{c_T=1} = P(N, \eta) + Q(N, \eta) K + A(N, \eta) (R - K_2),$$

where $R \equiv h^{ij} R_{ij}$ is the Ricci scalar on the spacelike hypersurface, $K \equiv h^{ij} K_{ij}$ is the
extrinsic curvature on the spacelike hypersurface and $K_2 \equiv K^2 - K_{ij} K^{ij}$. The coefficients
$P(N, \eta), Q(N, \eta)$ and $A(N, \eta)$ are related with $G_2, G_3$ and $B_4$ in (1) by

$$P(X, \phi) = G_2 - \sqrt{-X} \int \frac{G_{3, \phi}}{2\sqrt{-X}} dX,$$

$$Q(X, \phi) = - \int G_{3, X} \sqrt{-X} dX + 2(-X)^{3/2} \int \frac{XB_{4, X} - B_{4, \phi}}{X^2} dX,$$

$$A(X, \phi) = B_4.$$

The covariant Lagrangian $L^{DHOST}_{c_T=1}$ of the DHOST theory with $c_T = 1$ has been identified
in Ref.[45]. As pointed out in Ref.[40], $L^{DHOST}_{c_T=1}$ may be obtained by performing a conformal
rescaling $g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu}$ to $L^{bH}_{c_T=1}$. Since the light cone is not altered, the corresponding
DHOST theory will maintain $c_T = 1$. Therefore, the ADM Lagrangian of DHOST theories with $c_T = 1$ may be straightly calculated by rescaling

$$N \to \sqrt{C}N \quad h_{ij} \to Ch_{ij} \quad h^{ij} \to C^{-1}h^{ij}$$

where $C = C(N, \eta)$. Here, without loss of generality, we will set $N_i = 0$ in the calculation. Considering $K_{ij} = \frac{1}{2N}(h'_{ij} - \nabla_i N_j - \nabla_j N_i)$, after some integrations by parts and redefinition of coefficients, we have

$$L^{DHOST}_{c_T=1} = N\sqrt{h}L^{DHOST}_{c_T=1} = N\sqrt{h}\left[P + QK + A(\mathcal{R} - \mathcal{K}_2) - \frac{3AB^2}{2N^2}N'^2 - \frac{2AB}{N}N'K + \frac{B}{a^2}\left(\frac{2A}{N} + 2AN - \frac{AB}{2}\right)(\partial N)^2\right],$$

where $B(N, \eta) = \partial N(\log C)$. It can be checked that this ADM Lagrangian is equivalent to the covariant $L^{DHOST}_{c_T=1}$ showed in Ref.[45]. When $B = 0$ (or $C = \text{const.}$), $L^{DHOST}_{c_T=1}$ reduces to the beyond Horndeski Lagrangian $L^{bH}_{c_T=1}$ in (3). In order to write out (5), we have absorbed the term linear in $N'$ into $P + QK$ by

$$\frac{f(N, \eta)}{N}N' = n^\mu \nabla_\mu F - \frac{1}{N}\partial_\eta F = -FK - \frac{1}{N}\partial_\eta F$$

where $F \equiv \int f dN$.

### B. The EFT of scalar perturbation

The quadratic order EFT of the DHOST theory is [35]

$$S^{quad} = \int d^3x d\eta a^3 M^2 \left\{ \delta K_{ij}\delta K^{ij} - \left(1 + \frac{2}{3}\alpha_L\right)\delta K^2 + (1 + \alpha_T)\left(\mathcal{R}\frac{\delta \sqrt{h}}{a^3} + \delta_2 \mathcal{R}\right) + \mathcal{H}^2\alpha_K \delta N^2 + 4\mathcal{H}\alpha_B \delta K \delta N + (1 + \alpha_H)\mathcal{R}\delta N + 4\beta_1 \delta K \delta N' + 2\beta_2 \delta N'^2 + \frac{\beta_3}{a^2}(\partial_\eta \delta N)^2 \right\}$$

(6)

Contracting (5) with (6), we can directly read off the effective coefficients in EFT (6),

$$\frac{M^2}{2} = NA, \quad \alpha_L = 0, \quad \alpha_T = 0,$$

$$\frac{M^2}{2}\mathcal{H}^2\alpha_K = L_N + \frac{1}{2}NL_{NN}, \quad \frac{M^2}{2}4\mathcal{H}\alpha_B = NL_{NK} + 2\mathcal{H}L_{NS},$$

$$\frac{M^2}{2}(1 + \alpha_H) = A + N\alpha_N,$$

$$\frac{M^2}{2}4\beta_1 = -2AB, \quad \frac{M^2}{2}\beta_2 = -\frac{3AB^2}{2N}, \quad \frac{M^2}{2}\beta_3 = NB\left(\frac{2A}{N} + 2AN - \frac{AB}{2}\right),$$

(7)
where $\mathcal{H}/N \equiv \frac{dn/d\eta}{aN} = H$, and $L_{cT=1}^{DHOST} = L$ is set for simplicity. Degenerate conditions have been checked

\[
\alpha_L = 0, \quad \beta_2 = -6\beta_1^2, \quad \beta_3 = -2N\beta_1 \left[2(1 + \alpha_H) + N\beta_1(1 + \alpha_T)\right].
\]

Compared with that in Ref.[35], the condition (9) has been slightly modified, since we have not necessarily $N(\eta) = 1$ here.

Use the scalar perturbation

\[
N_i \equiv \partial_i \psi, \quad h_{ij} \equiv a^2 e^{2\gamma} \delta_{ij}
\]

to expand (5) or EFT (6). In the corresponding result, $\delta N_i \zeta'$ is absorbed into $\tilde{\zeta}'^2$ by replacing $\zeta$ with a new variable $\tilde{\zeta} = \tilde{\zeta}(\zeta, \delta N)$. Using $\delta L/\delta \psi = 0$, and after some integrations by parts, we get the quadratic order Lagrangian of $\zeta$,

\[
\mathcal{L}_2 = a^3 M^2 \frac{2}{2} \left[ U \zeta'^2 - V \frac{(\partial \zeta)^2}{a^2} \right]
\]

with coefficients

\[
U = \frac{\Sigma}{\gamma^2} + \frac{6}{N^2}, \quad V = 2 \left[ \frac{N}{aM^2} \frac{d}{d\eta} (aM) - 1 \right],
\]

where

\[
\gamma \equiv \frac{\mathcal{H}}{N} + N\mathcal{H}\alpha_B - (N\beta_1)', \quad \Sigma \equiv \mathcal{H}^2 \left[ \alpha_K + 6 \left( \alpha_B^2 - \frac{\gamma^2}{\mathcal{H}^2 N^2} \right) - 18\alpha_B\beta_1 - \frac{6(\mathcal{H}M^2\alpha_B\beta_1)'}{\mathcal{H}^2 M^2} \right],
\]

\[
\mathcal{M} \equiv \frac{M^2}{\gamma} \left[ (1 + \alpha_H)/N + \beta_1 \right].
\]

The absence of ghost suggests

\[ U > 0. \]

The sound speed of scalar perturbation is

\[ c_S^2 = \frac{V}{U}. \]

Gradient stability suggests $c_S^2 > 0$.

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1 When $N_i \neq 0$, $N'$ in $L_{cT=1}^{DHOST}$ (5) should be promoted to $N' - N_i \partial_i N$.

2 The $\gamma$ in (14) is related to the $\gamma$ in Refs.[16, 46] by $2A\gamma \rightarrow \gamma$. 
III. STABLE BOUNCE MODELS

We will construct the fully stable (pathology-free) bounce models in the beyond Horndeski theory (3) and DHOST theory (5) with $c_T = 1$, respectively. Both actually belong to the subclasses of the full DHOST theory. We will follow the method in Refs.[22, 23].

We first set the evolutions of background (the Hubble parameter $H$ and $N$). In our model, $H = \mathcal{H}/N$ follows

$$\mathcal{H}/N = \frac{\eta}{p(\eta)(1 + \eta^2)}$$

(17)

with

$$p(\eta) = p_i + \frac{1 + \tanh\left(\frac{\eta - \eta_p}{\tau_p}\right)}{2}(p_f - p_i),$$

(18)

where $p_f, p_i, \eta_p$ and $\tau_p = \text{const}$. Initially $\eta \ll -1, \mathcal{H} < 0$, the universe contracts with $p(\eta) = p_i$ ($p_i \gg 1$ corresponds to the ekpyrotic contraction [7]). Cosmological bounce occurs at $\eta = 0$. Hereafter, the universe expands, and $\mathcal{H} > 0$ has the desired asymptotic form $\sim 1/(p_f \eta)$, see Fig.1. Meanwhile $N$ follows

$$x(\eta) \equiv \frac{1}{N} = x_i + \frac{1 + \tanh\left(\frac{\eta - \eta_x}{\tau_x}\right)}{2}(x_f - x_i),$$

(19)

where $x_f, x_i, \eta_x$ and $\tau_x = \text{const}$. The choice of Refs.[22, 23] is equivalent to setting $p_i = p_f = 3$ and $x_i = x_f = 1$ (equivalently $\dot{\phi} = 1$) in (18) and (19), respectively.
A. In beyond Horndeski theory

We set \( M_p^2 = (8\pi G)^{-1} = 1 \), and write \( P(N, \eta), Q(N, \eta) \) and \( A(N, \eta) \) in (3) as

\[
P(N, \eta) = g_1(\eta) \frac{1}{2N^2} + g_2(\eta) \frac{1}{N^4} + g_3(\eta),
\]

\[
Q(N, \eta) = 0,
\]

\[
A(N, \eta) = \frac{1}{2} + f_1(\eta) \frac{1}{N^2},
\]

where the \( N \)-dependent part of \( A(N, \eta) \) sets the coefficient \( \sim B_{4, X}^{\phi, X} \neq 0 \) in \( L_{ct=1}^{BH} \) (1), and is required for the fully stable bounce [19–22]. \( Q(N, \eta) \) is related with the cubic Galileon \( G_3(\phi, X)\Box \phi \) in \( L_{ct=1}^{BH} \) (1), see [47–50] for the so-called G-bounce and [51] for super-bounce. However, \( G_3(\phi, X)\Box \phi \) only moves the period of \( c_S^2 < 0 \) to the outside of the bounce phase, but cannot dispel it completely, as pointed out in Refs.[16, 48]. Thus we set \( Q(N, \eta) = 0 \) for simplicity.

Since \( Q(N, \eta) = 0 \), (14) is simplified as

\[
\gamma = \frac{\mathcal{H}}{AN} (A - NA_N),
\]

noting \( \beta_1 = 0 \) in the beyond Horndeski theory (1). To avoid possible divergence of \( U \) induced by \( \gamma = 0 \) (usually called \( \gamma \)-crossing [16, 17, 46]), we choose \( \Sigma \) in Eq.(15) as

\[
\Sigma = c_1(\eta)\gamma^2.
\]

\( U > 0 \) (avoiding the ghost instability) can be insured by adjusting \( c_1(\eta) \). \( \gamma \)-crossing will bring a singularity in unitary gauge [46]. However, as pointed out in Ref.[23], this singularity does not affect the proof of the No-go Theorem [14, 15].

According to (3), we have the equations of \( \mathcal{H} \) and \( N \) as follows,

\[
6\frac{\mathcal{H}^2}{N^2}(A - NA_N) = -P - NP_N - 3\frac{\mathcal{H}}{N}(NQ_N),
\]

\[
4\frac{a'}{aN} = -P - 2\frac{\mathcal{H}^2}{N^2}A + \frac{1}{N}(Q_N + Q_NN').
\]

One can solve out \( g_1(\eta), g_2(\eta) \) and \( g_3(\eta) \) in \( P(N, \eta) \) algebraically by considering Eqs.(22) and (23), which are showed in Appendix A 1.

Substituting the corresponding solutions into (16), we have

\[
\mathcal{M} = \frac{1 - 4f_1^2x^4}{2\mathcal{H}(1 + 6f_1x^2)}.
\]
We choose $f_1(\eta)$ as
\[
f_1(\eta) = c_2(\eta) \frac{c_3(\eta) \mathcal{H}(\eta) + 1}{2x^2(\eta)}, \quad c_2(0) = 1,
\] (24)
to make $\mathcal{M}$ not divergent at $\mathcal{H} = 0$. $V > 0$ (avoiding the gradient instability) can be insured by adjusting $c_2(\eta)$ and $c_3(\eta)$, noting $1 - 4f_1^2x^4 = 0$ at $\mathcal{H} = 0$.

Therefor, with $c_1(\eta)$, $c_2(\eta)$ and $c_3(\eta)$ satisfying certain conditions, we will have a fully stable bounce model. As a concrete example, setting
\[
c_1(\eta) = k_1 \left[ 1 - \tanh \left( \frac{\eta}{\tau_1} \right) \right],
\]
\[
c_2(t) = \exp \left( -\frac{\eta^2}{\tau_2} \right),
\]
\[
c_3(\eta) \equiv k_2,
\] (25)
we plot Figs.2 and 3 with the parameters $p_i = 8$, $p_f = 3$, $\tau_p = 1$, $\eta_p = 0.7$ and $-\eta_x = \tau_x = 3$ in (17) and (19), as well as $k_1 = 0.06$, $k_2 = 2$, $\tau_1 = 2$ and $\tau_2 = 0.6$ in (25). Fig.2 shows that the coefficients $g_1(\eta), g_2(\eta), g_3(\eta)$ and $f_1(\eta)$ in (20) have been fixed. Fig.3 shows that the model is indeed gradient-stable and ghost-free.

That the gravity should asymptotically approach GR requires $f_1 \to 0$ in the asymptotic future. The asymptotic behavior of $f_1$ is controlled by $c_2(\eta)$. As a result, the sound speed squared $c_S^2(+\infty)$ is (assume $c_1(+\infty)$ vanishes)
\[
c_S^2(+\infty) = \frac{-xH' + Hx'}{3H^2x^2}.
\]
Require $c_S^2(+\infty) = 1$ and insert background (19), one finds
\[
x_f = \frac{p_f}{3},
\] (26)
Similarly, $x_i$ is related to $p_i$ by requiring $c_S^2(-\infty) = 1$. 
FIG. 2: Coefficients of the beyond Horndeski Lagrangian (20) in our bounce model.

FIG. 3: The model is ghost-free and gradient-stable since $U > 0$ and $c_S^2 > 0$. During the expansion and contraction far from the bounce phase, $c_S^2 = 1$. 

B. In DHOST theory

The procedure is similar to that in Subsection III A. We write \( P(N,\eta), Q(N,\eta), A(N,\eta) \) and \( B(N,\eta) \) in (5) as

\[
\begin{align*}
P(N,t) &= g_1(\eta) \frac{1}{2N^2} + g_2(\eta) \frac{1}{N^4} + g_3(\eta), \\
Q(N,t) &= 0, \\
A(N,t) &= \frac{1}{2} + \frac{g_4(\eta)}{N^2}, \\
B(N,t) &= b_0,
\end{align*}
\]

(27)

with \( b_0 \neq 0 \) constant. So \( \beta_1, \beta_2, \beta_3 \neq 0 \) in the quadratic order EFT of the DHOST theory (6).

Substituting (27) into (14), we have

\[ \gamma \sim 2\mathcal{H} + b_0 N', \]

Considering (19), we have \( N'(\pm \infty) = 0 \). This suggests \( \gamma(-\infty) \sim \mathcal{H} < 0 \) and \( \gamma(+\infty) > 0 \). In other words, the existence of \( b_0 \) only shifts the \( \gamma \)-crossing point to \( \eta_0 \neq 0 \) instead of eliminating it. Therefore, the condition (22) is still needed. To make \( \mathcal{M} \) not divergent at \( \eta_0 \), we might choose \( g_4(\eta) \) as

\[
g_4 = c_2(\eta) c_3(\eta) \frac{(b_0 N' + 2\mathcal{H}) + N^2(2N - b_0)}{2(b_0 + 2N)}, \quad c_2(\eta_0) = 1.
\]

(28)

Thus with \( c_1(\eta) \) required in Eq.(22), \( c_2(\eta) \) and \( c_3(\eta) \) in Eq.(28), we could have a fully stable bounce model based on the DHOST theory (5).

As a concrete example, setting

\[
\begin{align*}
c_1(\eta) &= k_1 e^{-\eta^2/\tau_1^2}, \\
c_2(\eta) &= e^{-(\eta-\eta_0)^2/\tau_x^2}, \\
c_3(\eta) &\equiv k_2,
\end{align*}
\]

(29)

we plot Fig.4 with the parameters \( p_i = 8, p_f = 3, \eta_p = \eta_x = 0, \tau_p = 1 \) and \( \tau_x = 3 \) in (17) and (19), as well as \( \tau_1 = 20 \) and \( k_1 = 40 \) in (29), and \( b_0 = 0.5 \). Fig.4 shows that the model is indeed gradient-stable and ghost-free.
IV. DISCUSSION

We have constructed the spatially flat stable cosmological bounce models with GR asymptotics in the $c_T = 1$ beyond Horndeski theory and in the full $c_T = 1$ DHOST theory, respectively. In Ref. [23], the stable bouncing solution with $c_T = 1$ has also been built in the beyond Horndeski theory (but not in the full DHOST theory). Here, since we start straightly from the Lagrangians with the constraint $c_T = 1$, the procedure of building models (even in full DHOST theory) is simpler.

It is well-known that the solutions of fully stable cosmological bounce do exist in theories
beyond Horndeski. Though the simplest implementing is to work in the beyond Horndeski theory \cite{21, 22}, the stable bounce in a full DHOST theory is still interesting for study, which might bring unexpected results. In our implementing, we set the parameter $B(\phi, X) = \text{const.}$ in the full $c_T = 1$ DHOST theory (5), see (27). Generally, it is not this case. The relevant issue will be studied elsewhere.

The singularity of inflation implies that a bounce preceding inflation might occur \cite{8}, see also \cite{52–56}. Recently, it has been showed in Ref.\cite{57} that the bounce inflation scenario can explain the power deficit of CMB TT-spectrum at low multipoles, specially the dip at multipole $l \sim 20$. Thus it is interesting to embed the bounce models built here into the corresponding scenario, which might bring distinct imprint of DHOST terms in the CMB spectrum.

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\textbf{Appendix A: On $g_1, g_2, g_3$}

We give the explicit algebraic solutions of $g_1, g_2, g_3$ here.

1. The beyond Horndeski model

Recall that $x \equiv 1/N$ and $H = \mathcal{H}/N$.

$$
g_1 = -\frac{1}{2x^2} \left( 288c_1f_1^2H^2x^6 + 96c_1f_1H^2x^4 + 8c_1H^2x^2 + 12Hx^3f_1' \\
+ 36f_1H^2x^2 + 12f_1x^3H' + 24f_1Hx^2x' + 6H^2 + 6xH' \right)$$

$$
g_2 = -\frac{1}{8x^4} \left( -288c_1f_1^2H^2x^6 - 96c_1f_1H^2x^4 - 8c_1H^2x^2 - 4Hx^3f_1' \\
- 60f_1H^2x^2 - 4f_1x^3H' - 8f_1Hx^2x' - 6H^2 - 2xH' \right)$$

$$
g_3 = \frac{1}{8} \left( -18H^2 + 8c_1H^2x^2 - 36f_1H^2x^2 + 96c_1f_1H^2x^4 + 288c_1f_1^2H^2x^6 \\
- 12Hx^3f_1' - 6xH' - 12f_1x^3H' - 24f_1Hx^2x' \right)$$

2. The DHOST model

\[
g_1 = \frac{1}{8N^3 (2g_4 + N^2)} \times \left( -36b_0^2 c_1 g_4^2 N^3 (N')^2 - 12b_0^2 c_1 g_4 N^5 (N')^2 - b_0^2 c_1 N^7 (N')^2 + 432b_0^2 g_4^2 H N^2 N' \\
+ 144b_0^2 g_4^2 N^2 N'' - 504b_0^2 g_4^2 N (N')^2 + 72b_0^4 N^4 g_4' N' + 144b_0^2 g_4 N^2 g_4' N' \\
+ 360b_0^2 g_4 H N^4 N' + 120b_0^2 g_4 N^4 N'' - 288b_0^2 g_4 N^3 (N')^2 + 72b_0^2 H N^6 N' \\
+ 24b_0^2 N^6 N'' - 18b_0^2 N^5 (N')^2 - 144b_0^2 c_1 g_4^2 H N^3 N' - 48b_0^2 c_1 g_4 H N^5 N' \\
- 4b_0 c_1 H N^7 N' + 720b_0 g_4^2 H^2 N^2 + 96b_0 g_4^2 N^2 H' - 816b_0 g_4^2 H N N' \\
- 48b_0 g_4^2 N N'' + 144b_0 g_4^2 (N')^2 + 120b_0 H N^4 g_4' + 240b_0 g_4 H N^2 g_4' \\
- 48b_0 g_4 N g_4 N' - 24b_0 N^3 g_4^2 N' + 576b_0 g_4 H^2 N^4 + 120b_0 g_4 N^4 H' - 456b_0 g_4 H N^3 N' \\
- 48b_0 g_4 N^3 N'' + 96b_0 g_4 N^2 (N')^2 + 108b_0 H^2 N^6 + 36b_0 N^6 H' - 24b_0 H N^5 N' \\
- 12b_0 N^5 N'' + 12b_0 N^4 (N')^2 - 144c_1 g_4^2 H^2 N^3 - 48c_1 g_4 H^2 N^5 - 4c_1 H^2 N^7 \\
- 288g_4^2 H^2 N - 96g_4^2 N H' + 288g_4^2 H N' - 48H N^3 g_4' - 96g_4 H N g_4' - 192g_4 H^2 N^3 \\
- 96g_4 N^3 H' + 192g_4 H N^2 N' - 24H^2 N^5 - 24N^5 H' + 24H N^4 N') \right) \\
\]

\[
g_2 = \frac{1}{32N (2g_4 + N^2)} \times \left( 36b_0^2 c_1 g_4^2 N^3 (N')^2 + 12b_0^2 c_1 g_4 N^5 (N')^2 + b_0^2 c_1 N^7 (N')^2 - 288b_0^2 g_4^2 H N^2 N' - 96b_0^2 g_4^2 N^2 N'' \\
+ 456b_0^2 g_4 N (N')^2 - 48b_0^2 N^4 g_4' N' - 96b_0^2 g_4 N^2 g_4' N' - 216b_0^2 g_4 H N^4 N' - 72b_0^2 g_4 N^4 N'' \\
+ 264b_0^2 g_4 N^3 (N')^2 - 36b_0^2 H N^6 N' - 12b_0^2 N^6 N'' + 18b_0^2 N^5 (N')^2 + 144b_0 c_1 g_4^2 H N^3 N' \\
+ 48b_0 c_1 g_4 H N^5 N' + 4b_0 c_1 H N^7 N' - 432b_0 g_4^2 H^2 N^2 + 816b_0 g_4^2 H N N' + 16b_0 g_4^2 N N'' \\
- 48b_0 g_4^2 (N')^2 - 72b_0 H N^4 g_4' - 144b_0 g_4 H N^2 g_4' + 16b_0 g_4 N g_4' N' + 8b_0 N^3 g_4' N' \\
- 288b_0 g_4 H^2 N^4 - 24b_0 g_4 N^4 H' + 456b_0 g_4 H N^3 N' + 16b_0 g_4 N^3 N'' - 32b_0 g_4 N^2 (N')^2 \\
- 36b_0 H^2 N^6 + 12b_0 N^6 H' + 24b_0 H N^5 N' + 4b_0 N^5 N'' + 4b_0 N^4 (N')^2 + 144c_1 g_4^2 H^2 N^3 \\
+ 48c_1 g_4 H^2 N^5 + 4c_1 H^2 N^7 + 480g_4^2 H^2 N + 32g_4^2 H N' - 96g_4^2 H N' + 16H N^3 g_4 \\
+ 32g_4 H N g_4' + 288g_4 H^2 N^3 + 32g_4 N^3 H' - 64g_4 H N^2 N' + 24H^2 N^5 + 8N^5 H' \\
- 8H N^4 N') \right) \\
\]

13
\[ g_3 = \frac{1}{32N^5 (2g_4 + N^2)} \times (36b_5^2c_1g_4N^3(N')^2 + 12b_0^2c_1g_4N^5(N')^2 + b_0^2c_1N^7(N')^2 - 576b_0^2g_4HN^2N' - 192b_0g_4^2N^2N'' + 648b_0^2g_4N(N')^2 - 96b_0^2N^4g_4N' - 192b_0^2g_4N^2g_4N' - 504b_0^2g_4HN^4N' - 168b_0g_4N^4N'' + 408b_0g_4N^3(N')^2 - 108b_0^2H^2N^6N' - 36b_0^2N^6N'' + 192b_0g_4^2HN^3N' + 48b_0c_1g_4HN^5N' + 4b_0c_1HN^7N' - 1008b_0g_4^2H^2N^2 - 192b_0g_4^2N^2H' + 816b_0g_4^2HN^4N'' - 48b_0g_4^2N^5N'' + 144b_0g_4^2(N')^2 - 168b_0HN^4g_4 - 336b_0g_4HN^2g_4 - 48b_0g_4N^2g_4N'' - 24b_0N^3g_4H^2N^4 - 216b_0g_4N^4H' + 456b_0g_4HN^3N' - 48b_0g_4N^3N'' + 96b_0g_1N^2(N')^2 - 180b_0H^2N^6 - 60b_0N^6H' + 24b_0HN^5N' - 12b_0N^5N'' + 12b_0N^4(N')^2 + 144c_1g_4^2H^2N^3 + 48c_1g_4H^2N^5 + 4c_1H^2N^7 - 288g_4^2H^2N - 96g_4^2NH' + 288g_4^2H^3N' - 48H^3g_4 - 96g_4HN^4g_4 - 288g_4H^2N^3 - 96g_4H^3N' + 192g_4HN^2N'' - 72H^2N^5 - 24N^5H' + 24HN^4N')) \]

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