1 Overview

While gerrymandering has been widely suspected in Georgia for years, it has been difficult to quantify. We understand a plan to be a partisan gerrymander if it produces election results whose outcomes and structures are not typically seen in maps drawn only with non-partisan considerations. In particular, our use of the term gerrymandering does not make reference to any concept of proportionality and includes how the spatial distribution of the state’s electorate interacts with the redistricting process.

Using historic voting data, we compare the Georgia congressional redistricting plan enacted in 2021 with a large collection of randomly generated non-partisan maps \(^1\) (see Section 1.1 for details). Similar methodologies have been employed in various states, including North Carolina \(^2\), \(^3\), Ohio \(^4\), Pennsylvania \(^5\), \(^6\), Virginia \(^7\), Maryland \(^8\), and Wisconsin \(^9\). In each state, the methods are adapted to account for the state’s specific redistricting requirements. We continue these explorations by examining the partisan behavior under Georgia’s enacted 2021 congressional map and a collection of maps that were sampled without partisan consideration. Our main findings are as follows:

1. Non-responsiveness: We find that the 2021 plan will likely be highly non-responsive to changing opinions of the electorate. The enacted map is structured so that it will reliably elect nine Republicans and five Democrats over a wide range of studied voting patterns, with statewide Democratic vote fraction percentages ranging from the mid 40s to the low 50s, as has been typical in recent Georgia elections (See Figure 1). In contrast, over this same range, the non-partisan maps in our ensemble react to the changing voter preferences by shifting the partisan make-up of those elected. Only 0.12% (186 out of 159,997) of the plans in the ensemble behave similarly to the enacted plan in that they produce a single election outcome over the seventeen elections considered (between 2016-2020). Moreover, when the statewide vote fractions start strongly favoring either party, the number of officials that would be elected for that party under the enacted map may be systematically smaller than what is projected under plans from our ensemble (See Figures 2, 10, 11). This effect is consistently seen when disadvantaging the Democratic party; it is only occasionally seen when disadvantaging the Republican party.

2. Polarization in Competitive Districts: We find the major cause of the non-responsive nature of the enacted 2021 map is that it polarizes voters across the more competitive districts. Specifically, there are five districts that would be significantly more competitive under maps with only our non-partisan considerations, but have been shifted to become more Republican; similarly, there are three districts that have more Democratic voters than is typical. In such a way, districts that could have been more

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\(^4\) This analysis was conducted using voting and precinct data from the Redistricting Data Hub and demographic data from the 2020 Census.
responsive to the changing opinion of the voters, have become more solidified in their partisan lean, and the enacted map is non-responsive to voting pattern shifts.

3. Packing in Democratic Strongholds and Cracking in one Republican Stronghold: Utilizing a combination of our ensemble statistics and spatial analysis, we find quantitative evidence that the heavily Democratic 4th, 5th and 13th Congressional Districts and the heavily Republican 9th District contain an unrepresentatively large number of Democrats. The consolidation or packing of Democratic voters in the 4th, 5th, and 13th Districts creates significant numbers of wasted votes and dilutes their voting power.

Furthermore, based on comparisons with both the previously enacted 2011 map and the ensemble of plans, we find that a significant number of Democratic voters have been safely added to the solidly Republican 9th District, replacing Republican voters who have been moved to the 6th and 10th Districts. This redrawing substantially weakens, or cracks, the potential influence of Democratic votes in these districts.

1.1 Overview of the ensemble

We place a probability distribution on congressional plans for Georgia that captures the non-partisan policy considerations and is in compliance with the legal considerations, as explained next. From this distribution, we produce a representative set of sampled plans, called an ensemble, to serve as a benchmark against which to compare particular maps. The ensemble is generated by sampling the distribution using the Metropolis-Hastings Markov chain Monte Carlo algorithm in a parallel tempering framework that employs the proposal from the Metropolized Multiscale Forest RECOM algorithm [2].

The districting plans in the ensemble satisfy the following:

- **Contiguity:** All districts consist of one contiguous region.
- **Population Balance:** The total population in each district is within 1% of the ideal district population. We have verified in previous work the small changes needed to make districting plans have perfectly balanced populations do not have significant impact on the partisan results of our ensembles. (See the supplemental material of [6])
- **Maximum splits:** The maps split no more than 21 counties. The number of 21 is chosen in accordance with the number of county splits in the 2021 map.
- **Traversing Boundaries:** Districts traverse each county boundary at most once; when a district splits a county, it may not form two discontinuous regions when restricted to that county.
- **Compactness:** The distribution is concentrated on more compact plans; this reflects the General Assembly’s guidelines that the plans should be compact. We have tuned the distribution so that it yields plans of a similar compactness to those of the 2021 map [4]. We measure compactness with the Polsby-Popper score, which has been used to assess compactness in many states including North Carolina [12] and Virginia [16], to measure compactness. See Appendix C for formal definitions of the measure and a comparison with the enacted plan (Figure 9).

Our chosen probability distribution over redistricting plans, from which we draw plans in our ensemble, prioritizes the desired policy considerations and complies with legal considerations. To mathematically account for compactness of plans, we use a target measure which includes a compactness score. We target compactness scores that are in line with the enacted plan from 2021. It is known that not all distributions on possible redistricting plans are mathematically easy to sample from, and indeed including a target measure with compactness increases the complexity of sampling from the corresponding distribution. To overcome this difficulty and effectively generate a representative ensemble of maps, we use the established method of parallel tempering [1], which allows one to effectively sample from a possibly difficult distribution by using samples from “easier” distributions through a sequence of intermediate “interpolating” distributions. See Appendix A for more details.
We use Multiscale Forest RECOM proposals in the Metropolis-Hastings sampling scheme. We take chains of 4 million proposals, subsampling at every 25 steps, for a total of 160,000 plans. By running multiple chains and comparing them across observables of interest, we confirm that the observables have become independent of their initial conditions and agree in distribution across the multiple chains. This provides numerical evidence that the distribution has mixed and has been sufficiently sampled to provide stable statistics; see Appendix B for more details.

2 Non-responsiveness of the 2021 map

We see that the enacted Georgia map does not produce different partisan results, in terms of the representative elected, over range of recent voting patterns. In contrast, maps in the ensemble typically respond to the changing popular vote over these voting patterns. Furthermore, when the Democratic vote share grows, the enacted map systematically under-elects Democrats. Similarly, there is a smaller range of election environments where the map under elects Republicans.

To demonstrate this, we analyze the enacted Georgia map by evaluating how many Democrats it would elect under a number of statewide voting patterns and compare this with the ensemble of plans. We demonstrate this through a type of plot we call collected seat histograms. The election data we use is either a set of historical elections (Section 2.1) or data generated by applying a uniform swing to a particular historical election (Section 2.2)

Both kinds of collected seat histograms are effective at identifying maps that are non-responsive or that under-respond to changing voter opinions. A map that reacts by changing representation when the number of votes for a particular party changes sufficiently is a minimal requirement of a democratic process that is responsive to the changing will of the people.

2.1 Collected seat histograms under historic elections

Only 2.1% (3428 of the 159997) of the maps have no change in the number of elected officials from each party when the votes vary over a collection of historic elections with a statewide Democratic vote fraction between 46.5%-50.5%. There is a single election, US Senate 2016, outside of this range; when that election is included we find that only 0.12% (186 out of 159997) of the plans in the ensemble elect the same number of Democrats, and thus match the non-responsiveness of the enacted plan by generating the same number of seats for each party regardless of which of the seventeen elections we compare on. In short, the plans in the ensemble are nearly always more responsive than the enacted plan.

This result is visualized in Figure 1, which gives the collected seat histograms for the ensemble of districting plans sampled from our non-partisan probability distribution. Using historical elections, these illustrate the level of responsiveness to changes in the votes one should expect of maps drawn without a partisan bias. This figure also shows how many Democrats the enacted congressional plan would have elected under the votes from a variety of historic elections.

It may be tempting to look at this figure and conclude that the enacted plan is actually fairly typical of the ensemble, as it is nearly always in the center of the histograms over the majority of the elections in Figure 1. However, the plans in the ensemble elect 4-6 Democrats under the 2018 Commissioner of Agriculture election (at a statewide Democratic vote fraction of just under 47%) and 5-7 Democrats under the 2020 Presidential election (at a statewide Democratic vote fraction of just over 50.2%). This shift reflects the idea that it is typical for the number of elected Democrats under a typical plan to change with respect to the vote. It is highly unlikely for the number for Democrats not to change over this range as seen in the enacted plan.

\[ \text{Democratic vote share} = \frac{\text{number of Democratic votes}}{\text{number of Democratic votes} + \text{number of Republican votes}}. \]

since the fraction of third party votes varies from election to election. To determine the number of seats won by each party, we compare this fraction to 0.5.

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2 In accordance with the uniform swing hypothesis, we take a single election and then uniformly increase or decrease the vote percentage for a given party across all the districts. This creates a new set of voting data with the same spatial structure but a different statewide partisan percentage for each party.

3 Throughout, we normalize election results by using the fraction
In particular, the enacted congressional plan is stuck electing five Democrats in the fourteen districts, despite similar shifts in the statewide vote fraction and the distribution of votes across the state. Over these elections, with a statewide vote Democratic vote ranging from 42.8% to 50.1%, the enacted map does not change the number of Republicans and the number of Democrats elected at all. This shows the enacted map to be highly non-responsive to the changing opinion of the electorate, and without holding the election, one largely knows that 9 Republicans and 5 Democrats will be elected.

2.2 Uniform swing analysis

In addition to using historical statewide votes to produce our collected seat histograms, we create a collection of collected seat histograms built from a single historical vote which is shifted in accordance with the “uniform swing hypothesis” to produce a new collection of votes [7]. This preserves the relative voting pattern across the state while allowing us to study the effect of shifting the partisan tilt of the election.

In Figure 2 for 2016 elections (and Figures 10 and 11, for 2018 and 2020 elections respectively), we see that the non-responsiveness phenomenon from Figure 1 is repeated much more severely. In many cases the enacted map fails to respond to the shifting will of the electorate, or is significantly less responsive than the maps produced by the ensemble. Our analysis shows that even if the Democratic vote share were to increase to greater than 54%, the enacted plan under-elects Democrats. For example, when the statewide Democratic vote fraction for the 2016 presidential election (16PR) and 2016 U.S. senate election (16USS) elections is swung to between 54% and 60%, the ensemble elects the same or fewer Democrats in 0.28% and 0.15% of the plans respectively. Though Georgia has not historically seen such large Democratic swings, this map may serve to solidify these districts against future demographic changes. Furthermore, there are regions in which the enacted plan elects more Democrats than expected; for example, when the 2016 Presidential votes are swung to a Democratic statewide vote share between 43%-45% (see left Figure 2). This non-responsiveness is due to polarized districts that abnormally separated Democratic and Republican voters; we demonstrate this
Figure 2: The individual histograms give the frequency of the Democratic seat count in the ensemble for each of the shown statewide elections, with a uniform swing. The histograms are organized vertically based on the swung statewide partisan vote fraction. The red dots denote the Democratic seat count for the enacted plan for each of the swung vote profiles.

in the next section. See Appendix D for additional plots for 2018 and 2020 statewide elections. Moreover, it underlines the increased polarization we see in many districts.
Figure 3: The red dots display the ordered Democratic vote share for each of the 14 districts in the enacted 2021 plan based on 2020 presidential (top) and 2020 United States Senate (bottom) statewide elections. The box plots display the range of Democratic vote share observed for plans in the ensemble, with the 14 vote shares for each plan ranked from least to most Democratic.

3 Polarization in competitive districts

In addition to looking at the number of elected representatives from each party, we examine the margins of victory within races between 2016-2020. To this end, we examine box plots that show the rank-ordered marginal distributions of the partisan vote fraction across the plans. These plots help identify when the map contains districts with abnormally many Democrats or Republicans. This is done by considering the partisan vote fraction for one of the political parties (Democrats) in each of the districts for a given redistricting plan. These marginal vote fractions are then ordered from smallest to largest, i.e., from the most Republican district to the most Democratic district. These ordered fractions are then tabulated over all of the plans in the ensemble and used to form order statistics over the ensemble (see Figure 3 and 4 for 2020 and 2016 elections). Qualitatively similar results are seen for 2018 elections, which we include in Appendix E.

The rank-ordered marginal box plots show the typical range of the most Republican district to the most Democratic district. Ranges are represented by box plots. In these box plots, 50% of all plans have corresponding ranked districts that lie within the box; the median is given by the line within the box; the ticks mark the 2.5%, 10%, 90% and 97.5% quartiles; and the extent of the lines outside of the boxes represent the range of results observed in the ensemble. Any box that lies above the 50% line on the vertical axis corresponds to a (ranked) district that will typically elect a Democrat; any box that lies below the 50% line corresponds to a (ranked) district that will typically elect a Republican (e.g., in Figure 3 (top), districts ranked 11-14 reliably elect Democrats, and districts ranked 1-6 reliably elect Republicans).

We evaluate the enacted plan with each set of votes and plot the ordered district results over the box
plots. If the districts of an enacted plan lie either far above or far below the ensemble at a particular ranking, this can indicate that the district was drawn to increase or decrease one party’s representation within it.

Figure 4: The red dots display the ordered Democratic vote share for each of the 14 districts in the enacted 2021 plan based on various 2016 statewide elections. The box plots display the range of Democratic vote share observed for plans in the ensemble, with the 14 vote shares for each plan ranked from least to most Democratic.

In Figures 3-4, we examine a variety of elections chosen from Figure 1 across 2016 to 2020 and consistently find that:

- the 5th-9th most Republican districts of the enacted 2021 map have significantly fewer Democratic votes than the corresponding 5th-9th most Republican districts of plans in the ensemble (e.g., see the pink highlighted regions),

- the 10th-12th most Republican districts of the enacted 2021 map have significantly more Democratic votes than the corresponding 10th-12th most Republican districts of plans in the ensemble (e.g., see the blue highlighted regions).

We consider the total Democratic votes in the 5th-9th most Republican districts from each plan in the ensemble and compare them to the total Democratic votes in the 5th-9th most Republican districts from the enacted plan. Of the 159,997 plans in our ensemble, across 17 elections from 2016 to 2020, no more than 0.017% of the plans (27 out of 159,997) have the same or fewer Democratic votes than the enacted plan, which suggests that the enacted 2021 map polarizes voters across the 5th-9th most Republican districts to make those districts more Republican.

On the other hand, when we consider the total Democratic votes in the 10th-12th most Republican districts from each plan in the ensemble, and compare them against the sum of the Democratic votes in the
We observe that, across all elections, no more than 0.2% of the plans (320 out of 159,997) would have the same or more Democratic votes than the enacted plan, which suggests that the enacted 2021 map polarizes voters across the 10th-12th most Republican districts to make those districts more Democratic instead. These statistics are summarized in Table 4 in Appendix E.

In this way, districts that could have been more responsive to the voters have been solidified in their partisan lean, and the enacted map is stable and non-responsive to voting pattern shifts.

4 A spatial analysis of packing and cracking in the 2021 map

The ranked ordered marginal distributions, in general, do not correspond to geographic regions. However, in Georgia, for the most Republican district and the three most Democratic districts in the ensemble, we are able to show that there are geographical consistencies across most maps in the ensemble and the enacted plan. We outline the enacted districts along with a heat map capturing typical locations of the corresponding districts in the ranked-marginals in the ensemble (see Figure 5).

The geographic locations highlighted by the heat map correspond quite well to the district locations of the enacted districts.

In particular, the most Republican district in the ensemble consistently corresponds to the 9th District, and the three most Democratic districts in the ensemble correspond to the 4th, 5th and 13th Districts.

Taking advantage of this geographic similarity, we are able to identify localized differences in partisan behavior of those districts in the 2021 plan and the ensemble.

We find that the 4th, 5th and 13th Districts have been packed with Democrats, while the 9th District has been used to crack the Democratic vote.

4.1 Packing in the three most Democratic districts

In all 17 historic elections across both the old 2011 and new 2021 enacted plans, we find that the three most Democratic districts are the 4th, 5th, and 13th Districts.

We also examine where the three most Democratic districts occur in the ensemble by examining the frequency with which each precinct exists within the three most Democratic districts over each of the 17 elections. We plot the resulting heat map in the ensemble and highlight the three most Democratic districts in the 2021 plan in Figure 5 (left). We find substantial similarity between the geographical location of the three most Democratic districts in the ensemble and those of the 2021 enacted plan.

We then compare the fraction of Democratic voters in the three most Democratic districts of the ensemble and the 2021 enacted plan. Despite the geographical similarities, we find that the 2021 enacted plan has the same or more Democrats than 99.41%-99.96% of the corresponding districts in the ensemble of plans. This demonstrates that these three districts, which correspond to a specific region of the state, have been artificially packed with Democrats.

4.2 Cracking in the most Republican district

In the 2021 enacted map, the 9th District, in the northeastern part of the state, is consistently the most Republican. We find that in the ensemble, the most Republican district is most often located in the northern part of the state and may either encompass precincts to the northwest or northeast (see Figure 5, right). As above, this analysis suggests a similar geographic location of the most Republican district across plans in the ensemble and 2021 enacted plan.

In Georgia, there is close alignment between the overarching structures of the 2011 and 2021 district plans; this is to say that the 1st District in 2011 “looks like” 1st District in 2021. We illustrate the similarity between the two plans in Figure 13 in the appendix.

Packing refers to concentrating atypically many voters of one type into a district to reduce their influence in other districts.

Cracking is dispersing voters of one type into many districts in order to deny them a dominant voting bloc in any particular district.

The 9th District is the most Republican across all but one (the 2020 presidential election, where it was the second most Republican district) of the 17 elections between 2016 and 2020 studied in this report.
We find that the most Republican district in the 2021 enacted plan contains fewer Republican voters than over 98.4% of the most Republican district from plans in the ensemble across all elections. This suggests that Democrats have been atypically introduced into this District and, correspondingly, Republicans have been removed. A direct consequence is that the newly included Democrats are removed from surrounding districts. In our analysis below, we show that the Republican voters who have been removed from the 9th District now dilute the voting power of Democrats in 6th and 10th Districts.

In order to investigate where the additional Republican voters have been moved, we begin with the following observations:

1. According to the 2020 census data, the 9th District as it was drawn in the 2011 map had a roughly 1% population deviation from being perfectly balanced in 2020. Other criteria, such as county boundaries, incumbent locations, and the number of seats assigned to Georgia, have not changed. Therefore, this district did not have to be substantially redrawn.

2. The 2011 9th District is consistently the most Republican district in the 2011 plan. However, its Democratic vote fraction is typical across all elections when compared to the most Republican district in each plan in the ensemble. In short, the 9th District could have been almost entirely unchanged, and if so would have had a typical vote fraction in the context of the ensemble. Instead, it has been modified and is now an outlier with an atypically small number of Republican voters. To determine where these Republican voters are removed, we contrast the 2011 map with the 2021 map in Figure 6 (left). We shade the regions that are no longer part of the 9th District with crosshatching and regions where the 9th District has expanded with dots. We also color the counties (and parts of counties) based on the Democratic vote fraction in the 2020 presidential election. The 9th District has been changed to shed Republican leaning regions to the west and south, and it has expanded to the southwest to pick up Democratic voters in northern Gwinnett County.

The exchange, and consequent removal of more Republican voters from the 9th District, has a cascading effect. First, it adds Republican voters to the northern part of the 10th District. The 10th District also recedes from the south as shown in Figure 6 (middle). This motion causes the 10th District to gain Republican voters in Jackson, Madison, Elbert, and Clarke counties (once in the 9th District) and give up Democratic voters in Gwinnett County, to the 7th District, and majority-African American Warren, Washington, and Jefferson counties, to the 12th District. We remark that the 10th District is within the cluster of plans that have been depleted of Democrats, as presented in the polarization analysis in Section 2.

Second, and similarly, Republican voters removed from the 9th District are added to the 6th District. This causes the 6th to move northward as shown in Figure 6 (right), picking up Republican voters in Dawson,
Figure 6: We display how the 2021 map has been updated from the 2011 map in the 9th (left), 10th (middle) and 6th (right) Districts, outlined in gold. In each plot, dots show area gained by the district from 2011 to 2021 and crosshatching shows area lost by the district from 2011 to 2021. The vote fraction of the 9th District in the 2011 map is typical. However, in the 2021 map, the 9th District absorbs Democratic voters (blue areas of the map) away from more competitive districts and shed Republican voters (red areas of the map) toward these districts. This move has a cascading effect that directly contributes to the changes in the 10th and 6th Districts.

northern Forsyth, and eastern Pickens counties (once in the 9th District) and shed Democratic voters from part of Cobb, Fulton, DeKalb and Gwinnett counties.

This shift dramatically increases the Republican vote fraction in the 6th District. In the 2011 map, the 6th District was consistently either the fourth- or fifth-most Democratic district; in the 2021 map it is now one of the group of districts that has atypically few Democrats and would stably elect a Republican representative, according to historic voting trends.

We remark that localized analysis of gerrymandering is still a developing field. A new tool has recently emerged to match plans spatially between the ensemble and enacted plans [11]. We plan to utilize this tool in a follow-up study to further test and explore the above findings.

5 Conclusion

This report shows that the 2021 enacted congressional districting plan in Georgia is likely to be highly non-responsive to the changing opinions of the electorate. Moreover, there is mathematical evidence of polarization of competitive districts in Georgia, which has been caused in part by the redrawing of the 6th, 9th, and 10th Districts. All our data and analysis is available on request.

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Appendices

A The Target Measure

We run the sampling method summarized in the main report for up to four million steps under different random seeds. We use parallel tempering to get an appropriate compactness weight with ten replicas with compactness weight ranging from 0.00 to 0.09. The compactness weight enters the probability distribution on plans via an exponential distribution on the compactness score (i.e. the sum of the isoperimetric ratios for each district). The distribution is then given as $\pi(T) = p(T)/Z$, where $T$ is a spanning forest that defines a redistricting plan, $Z$ is a normalization constant, and

$$p(T) = \begin{cases} e^{-wJ(T)} & \text{if } T \text{ meets constraining criteria} \\ 0 & \text{o.w.} \end{cases},$$

and $J$ is the sum of the isoperimetric ratios of the districts and $w$ is the weight which varies from 0 to 0.09. See [2, 1] for more details. The weight of $w = 0.09$ is the ‘target’ distribution which yields districts with comparable compactness levels to the enacted plan.

B Convergence

We run the sampling method summarized in the main report for up to four million steps under different random seeds. We use parallel tempering to get an appropriate compactness weight with ten replicas with compactness weight ranging from 0.00 to 0.09. To test for convergence, we compare the ensemble distribution from different random seeds, different running steps within the same random seed, and different replicas within the same random seeds and running steps, using the 2020 presidential election votes to see if the distributions mostly agree. See Figures 7 and 8 for results.

\[8\] In tempering, we use the hyper-parameter $\gamma = 0$ from the Metropolized Forest RECOM algorithm; see [1] for details.

\[9\] In tempering, we use the hyper-parameter $\gamma = 0$ from the Metropolized Forest RECOM algorithm; see [1] for details.
(a) Comparison of the ranked 2020 presidential Democratic vote share in each district of each plan across two different sets of 79,997 plans. The two sets of plans are taken from two runs of two million steps each, starting from two different initial random seeds, demonstrating that choice of initial seed was not significant.

(b) Comparison of the ranked 2020 presidential Democratic vote share in each district of each plan across two different sets of plans. The two sets of plans are taken from two runs of two million steps each, starting from two different initial random seeds, demonstrating that two million steps was sufficient for the run to converge.

Figure 7

C Compactness

For each district, the isoperimetric ratio is the square of its perimeter divided by its area. The total isoperimetric ratio of a districting plan is the sum of the isoperimetric ratios of each district. We use this total isoperimetric ratio as the measure of compactness in the parallel tempering process, with each interpolating distribution having a different weight $[1]$. To compare the ensemble with the enacted map, we use the Polsby-Popper score of each district, defined as $4\pi$ times the reciprocal of its isoperimetric ratio. Hence, a smaller isoperimetric ratio corresponds to larger Polsby-Popper scores. We observe that tempering up to compactness weight 0.04 produces an ensemble that converges (section B) and matches the 2021 map well, as measured by the ranked Polsby-Popper scores of the districts. See Figure 9 for this comparison.
Figure 8: Comparison of the ranked 2020 presidential Democratic vote share in each district of each plan across two different sets of plans. The two sets of plans are taken from two sets of four different replicas from an eight-replica run of two million steps each, demonstrating that choice of replica was not significant.

Figure 9: The red dots display the ordered Polsby-Popper score of the 14 districts in the enacted 2021 plan, compared to the ranges of Polsby-Popper scores observed for plans in the ensemble, ranked from least to most compact.

D Additional uniform swing plots

Figures 11 and 10 contain additional uniform swing analysis for 2018 and 2020 statewide elections. Again, as in Figure 2, we observe that the enacted plan would result in nine Republican seats until the statewide Democratic vote share is swung to 55%. When statewide Democratic votes are swung to range from 54% to 60%, the ensemble elects the same or fewer Democrats in 5.07% for the 2018 gubernatorial election (18GOV); for the 2018 lieutenant gubernatorial election (18LTG), 4.55%; for the 2020 presidential election (20PR), 8.91%, and for the 2020 United States Senate election (20USS), 4.81%.
Figure 10: The individual histograms give the frequency of the Democratic seat count in the ensemble for each of the shown statewide elections, with a uniform swing. The histograms are organized vertically based on the swung statewide partisan vote fraction. The red dots denote the Democratic seat count for the enacted plan for each of the swung vote profiles.

Figure 11: The individual histograms give the frequency of the Democratic seat count in the ensemble for each of the shown statewide elections, with a uniform swing. The histograms are organized vertically based on the swung statewide partisan vote fraction. The red dots denote the Democratic seat count for the enacted plan for each of the swung vote profiles.

### E Rank-ordered marginal box plots

Figure 12 contains additional rank-ordered marginals for 2018 statewide elections which are typical of the larger range of 2018 elections we consider. As in Figures 3 and 4, we observe greatly increased polarization in the enacted map, with the 5th-9th most Republican districts having far more Republicans than is typical in the ensemble and the 10th-12th most Republican districts having far more Democrats than is typical in
In Table 1, we display the number of plans in the ensemble with the same or fewer Democratic voters in the 5th-9th most Republican districts and the number of plans in the ensemble with the same or more Democratic voters in the 10th-12th most Republican districts.

The 2021 map and the 2011 map

Figure 12 shows the 2011 map and 2021 map. Comparing both maps, we can find that there is close alignment between the overarching structures of the 2011 and 2021 district plans. For example, the 1st District in 2011 looks like the 1st District in 2021.

Figure 14(a) shows the precinct-level shifts in votes between the 2016 and 2020 presidential elections. Most precincts in the suburban Atlanta area are colored blue, indicating a 3% or greater increase in Democratic vote fraction. This shift is especially prominent within the old 6th and 7th Districts, which elected Republicans in 2016 and 2018 and Democrats in 2020 (see Figure 14(b) and (c)). However, the 2021 map is much less responsive in suburban Atlanta, and the seat pattern remains the same (shown in Figure 14(d)) across 17 elections from 2016 to 2020.
Elections | % Ensemble plans with same or **fewer** Dem. votes than 2021 enacted map in the 5th-9th most Rep. districts | % Ensemble plans with same or **more** Dem. votes than 2021 enacted map in the 10th-12th most Rep. districts
---|---|---
20PR | 0.000% (11/159,997) | 0.200% (320/159,997)
20USS | 0.000% (0/159,997) | 0.158% (252/159,997)
20PSC1 | 0.000% (0/159,997) | 0.078% (124/159,997)
20PSC2 | 0.000% (0/159,997) | 0.085% (136/159,997)
18GOV | 0.000% (0/159,997) | 0.154% (246/159,997)
18LTG | 0.000% (0/159,997) | 0.144% (231/159,997)
18ATG | 0.000% (0/159,997) | 0.088% (140/159,997)
18LAB | 0.000% (0/159,997) | 0.086% (138/159,997)
18AGR | 0.000% (0/159,997) | 0.082% (131/159,997)
18PSC3 | 0.000% (0/159,997) | 0.109% (174/159,997)
18PSC5 | 0.000% (0/159,997) | 0.082% (131/159,997)
18SOS | 0.001% (1/159,997) | 0.091% (146/159,997)
18SORS | 0.017% (27/159,997) | 0.129% (207/159,997)
18INS | 0.000% (0/159,997) | 0.084% (134/159,997)
18SPI | 0.000% (0/159,997) | 0.089% (107/159,997)
16USS | 0.000% (0/159,997) | 0.016% (25/159,997)
16PR | 0.000% (0/159,997) | 0.046% (78/159,997)

Table 1: Starting from the left, the first column gives the statewide election whose abbreviation is given in Figure 1. The total number of plans in the ensemble considered is 159,997. The second column gives the percentage of the number of the maps out of those 159,997 plans which have the same or **fewer** Democratic votes than the enacted 2021 Map for the 5th-9th-most Republican districts. These extremely low percentages show that the 5th-9th-most Republican districts in the enacted plan are significantly more Republican than they typically are in the ensemble. The third column gives the percentage of the number of the maps out of those 159,997 plans which have the same or **more** Democratic votes than the enacted 2021 Map for the 10th-12th most Republican districts. These extremely low percentages show that the 10th-12th-most Republican districts in the enacted map are significantly more Democratic than they typically are in the ensemble. Together, these demonstrate the strong polarization of the enacted map.

![Figure 13: The 2011 (left) and 2021 (right) congressional plans, with each district labelled by its name.](image)
Precinct-level voting changes from the 2016 presidential election to the 2020 presidential election. Precincts that saw at least a 3% Republican swing are colored red, while precincts that saw at least a 3% Democratic swing are colored blue.

(c) The pattern of 6 Democratic seats elected for 20PR, 20USS, 20PSC1, 20PSC4, 18GOV, 18SPI, 18SOS, 18PSC3, and 18PSC5 under the 2011 map.

(b) The pattern of 4 Democratic seats elected for 16PR, 16USS, 16INS, 18LTG, 18ATG, and 18AGR under the 2011 map.

(d) The pattern of 6 Democratic seats elected for 20PR, 20USS, 20PSC1, 20PSC4, 18GOV, 18LTG, 18SOS, 18SOSro, 18PSC3, 18PSC3ro, 18PSC5, 18ATG, 18AGR, 18INS, 18LAB, 18SPI, 16PR and 16USS under the 2021 map.