Application of Numerical Optimization to Aluminum Alloy Wheel Casting

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Abstract. A method of numerically optimizing the cooling conditions in a low-pressure die casting process from the standpoint of maintaining good directional solidification, high cooling rates and reduced cycle times has been developed for the production of aluminum alloy wheels. The method focuses on the optimization of cooling channel timing and utilizes an open source numerical optimization algorithm coupled with an experimentally validated, ABAQUS-based, heat transfer model of the casting process. Key features of the method include: 1) carefully designed constraint functions to ensure directional solidification along the centerline of the wheel; and 2) carefully formulated objective functions to maximize cooling rate. The method has been implemented on a prototype production die and the results have been tested with plant trial test.

1. Introduction

The aluminum wheel industry is very competitive and manufacturers are constantly under pressure to improve quality and to reduce cost. This is achieved by increasing production rates while attempting to also maintain or reduce scrap rates. One of the defects that is a significant challenge to manufacturers is shrinkage porosity. Shrinkage porosity occurs when liquid or semi-solid metal is cut off from a source of liquid metal, which is needed to compensate for the volume change associated with solidification (the volume change is in the range of 5 to 7% depending on the aluminum alloy). In the case of wheel casting, achieving controlled progress of the solidification front, or “directional solidification”, is the key to avoiding shrinkage porosity. As illustrated in Figure 1, good directional solidification is achieved when solidification starts high in the inboard-rim flange, progresses down through the rim, and then across the spokes, ending just below the hub in the sprue. To achieve this solidification pattern, both proper die design and careful management of the cooling conditions within a cast cycle are required.

Historically, the design of a die and the associated operational parameters, were determined based on a combination of experience and trial-and-error optimization. The methodology typically involves long design lead times, prototyping, several preproduction trials and can result in high scrap rates and less than optimum production rates. The move to adopt water-cooling in place of air cooling in order to reduce cycle times, improve productivity and improve wheel fatigue performance represents a significant challenge to the foundry engineer as there is a limited experiential base to draw on. This manuscript describes a methodology in which the trial-and-error optimization of the operational parameters (die cooling timing) is eliminated from the preproduction trial process. It combines a casting process model with an advanced optimization algorithm that is applied to achieve the optimum set of operating conditions that produce a combination of minimum cycle time and limited or no shrinkage porosity in a given die design.
2. Background

Numerical design optimization provides a systematized and versatile procedure for arriving at a design solution. It can offer a reduction in design time, removes experiential biases from the design process and virtually always yields some design improvement [1]. Numerical optimization first found application in structural design [2, 3] and is now increasingly applied in other areas such as, cast product and casting process design [4-7].

The numerical optimization process is essentially a procedure for finding the minimum or maximum of an objective function, which mathematically describes some attribute of the process or product being optimized. Additionally, constraints are normally added to account for additional limitations imposed on the design. The general mathematical expressions of the objective function and the constraint functions are presented in Equations (1) and (2).

\[
\begin{align*}
\max (\text{or min}) & \quad f(\mathbf{x}) \quad \mathbf{x} = (x_1, x_2, \ldots, x_n) \\
\text{subject to} & \quad g_i(\mathbf{x}) = c_i \quad \text{for } i = 1, \ldots, n \\
& \quad h_j(\mathbf{x}) \leq d_j \quad \text{for } j = 1, \ldots, m
\end{align*}
\]

where \(\mathbf{x}\) is a vector containing all the design variables that may be modified in order to achieve the optimization goal, \(f(\mathbf{x})\) is the objective function, and \(g_i(\mathbf{x})\) and \(h_j(\mathbf{x})\) are equality and inequality constraints, respectively.

The objective and constraint functions are generally evaluated based on the results of a numerical model. In this work, a comprehensive and experimentally validated model of a low pressure die casting (LPDC) process for the production of A356 automotive wheels has been used. Details related to the casting process model have been discussed previously in the literature [8, 9]. The current work presents the application of the optimization algorithm using two versions of the casting process model (see Sections 0 and 0) in order to highlight both the utility of the optimization algorithm and the importance of the process boundary conditions in the model utilized within the optimization algorithm.
3. Architecture of the optimization tool

Figure 2 shows the overall structure of the optimization algorithm or “tool”, which consists of three major modules. The first one is the casting process model, which has been developed in the commercial finite-element package ABAQUS™. The casting process model simulates the casting process and calculates the thermal field within the wheel and die. Due to the large thermal mass of the die, each change in cooling timing requires several casting cycles for the full effect of the change to propagate through the die and for a new cyclic steady-state condition to be achieved. Once the new steady state condition is achieved, the necessary data is obtained from the ABAQUS output file and processed with the analysis module developed in Python. Specifically, the analysis module evaluates the objective and the constraint functions and periodically, based on a set of criteria, evaluates the sensitivity of these functions to the design variables. This information is then passed to the optimizer. The optimizer is based on an open-source Python package (scipy.optimize), which has a FORTRAN-based core wrapped in Python. The optimizer determines if the design criteria, subject to the imposed constraints has been reached. If the criteria are not met, it then determines how to change the design variables, which are then passed back to the casting process model. If met, the optimal design parameters are output. The whole process iterates until an optimum solution is reached or until the user-defined maximum number of iterations is reached.

![Figure 2. Architecture of the optimization tool](image)

The strategy used to move from one iteration to the next in the design variable space distinguishes one algorithm from another. The package utilized in this study uses the line search method. The line search approach first finds a descent direction along which the objective function is reduced and then computes a step length that determines how far the update should move along that direction. In the line search method, the design variables are updated as follows [1]:

\[
x_{k+1} = x_k + \alpha S
\]

where \(x_k\) is the design at the \(k^{th}\) iteration and \(x_{k+1}\) is the new design at the \((k+1)^{th}\) iteration, \(S\) is the search direction, and \(\alpha\) is the step length.

The descent direction is determined using the quasi-Newton method in the current algorithm. The Quasi-Newton method is reported to have better convergence behavior than the Steepest Descent method, and it is more efficient than Newton’s method, as it does not require information on second-order derivatives. Detailed discussion of these three methods can be found in reference [1]. The step length can be determined either exactly, by finding the minima of the objective function in the given direction \(S\), or inexactely, by finding the value of \(\alpha\) that leads to an acceptable descent amount.

4. Application to a 2D Axisymmetric Prototype Die Model

The die structure used in the application presented in this paper is proprietary and cannot be disclosed. Hence, only the wheel and the approximate locations of the water-cooling channels are shown in
Figure 3. There are seven cooling channels in the die design: three in the top die (TD_CC_1 ~ 3), two in the side die (SD_CC_1 ~ 2), and two in the bottom die (BC_CC_1 ~ 2).

The design optimization strategy is based on a combination of maximizing the cooling rate in the wheel while achieving directional solidification. The former is desired to both increase productivity (reduce the overall cycle time) and produce a finer structure with better overall fatigue performance. The latter is needed to eliminate/reduce shrinkage porosity, which, if present, would negate the positive benefits of a reduced cycle time on fatigue performance.

The objective and constraint functions applied in the present work are given in Equations (4-5).

\[ f(x) = \sum_{i=1}^{6} a_i \times (-T_i^{\text{calculated}}) \]  

\[ h_j^1(x) = t_{j+1}^{575^\circ C} - t_j^{575^\circ C} \geq 0, \ j = 1,2,3,...35 \]  

\[ h^2(x) = (t_{\text{dieopen}} - t_{\text{solidification}})/t_{\text{dieopen}} \geq 0 \]  

where \( f(x) \) is the objective function, \( x \) is a vector containing the design variables (the on and off timings of the 7 water cooling channels), \( a_i \) are constants used to adjust the relative weights of the objective function, \( T_i^{\text{calculated}} \) is the calculated average cooling rate within the solidification range at the \( i^{\text{th}} \) point, \( h_j^1(x) \) is the first type of constraint imposed at the \( j^{\text{th}} \) constraint point, \( h^2(x) \) is a second the type of constraint function, \( t_{\text{dieopen}} \) is the time the die opens, and \( t_{\text{solidification}} \) is the time taken for the cast to solidify to the middle of the hub (a solid shell forms at this time to maintain the wheel structure).

Careful selection of the objective and constraint functions is crucial to obtaining good results. In Equation (4), the cooling rate is maximized when the optimization algorithm minimizes \( f(x) \). The calculated cooling rates are assessed at six points distributed at different regions within the wheel, indicated by points P1 through P6 in Figure 3. The first type of equality/inequality constraints are based on the time to reach 575°C, which is the eutectic temperature of A356 [10]. As shown in Equation (5), the constraints are formulated to require that the difference in time between two adjacent points, \( j+1 \) and \( j \), to reach 575°C is greater than or equal to zero. The constraint functions are evaluated at 36 points, which are shown as the white dots in Figure 3. \( j = 1 \) is located at the top of the inboard rim flange, the first place to solidify, and \( j = 36 \) is located at bottom of the hub, the preferred last place to solidify. Referring back to Equation 5, the \( j+1 \) point is located further along the path than...
the \(j^{th}\) point in the desired direction for progress of the solidification front – i.e. upstream in the direction of the compensatory flow necessary to feed solidification shrinkage. This constraint is utilized to ensure directional solidification.

A second type of equality/inequality constraint is applied that requires the wheel to be solidified before the die opens (Equation (6)). The solidification requirement constraint is normalized to bring it into the same magnitude range as the other constraints as numerical difficulties often arise when one constraint function is of a different magnitude, or changes more rapidly, than the others and dominates the optimization process [1].

4.1 First Application – Base-Case Version of the Model

The optimization tool was first applied to a 2-D axisymmetric base-case version of the casting process model. The large computational overhead associated with repetitively running the thermal model necessitated that the initial application be based on a 2-D axisymmetric version of the LPDC process model in order to minimize computational time during development of the optimizer. The model includes the different stages in the casting process – i.e. die filling, wheel cooling and solidification, die open, wheel removal and die closing. A few additional features are listed below:

1. The wheel and die material properties are temperature dependent;
2. Die filling is not simulated however, the interfacial cooling between the die and the wheel is activated based on the expected filling sequence, to approximate the effect of filling on heat transfer;
3. The initial distribution of temperature in the wheel is assumed to vary from 620 °C at the top of the rim to 700 °C at the inlet from the sprue (this is done to approximate the heat loss from the wheel to the die as the liquid metal is filling the die cavity);
4. The die/wheel interface boundary conditions vary with temperature to account for gap formation during solidification and are also different for the different die sections – top, side and bottom dies – to account for differences in the evolution of the gap at the interface;
5. The heat transfer in the water cooling channels is assumed to be constant when on and is described using a single-phase convection heat transfer coefficient; and
6. Each model is capable of being run in a cyclic mode – i.e. the initial condition for the die at the beginning of each cycle uses the results from the previous one – and therefore can obtain the cyclic steady state condition for a given set of process cooling conditions.

The results showing the solidification patterns obtained for two sets of cooling timings are shown in Figure 4. Each row of images shows color contours of the temperature on a cross-sectional slice of the wheel at different times in the cycle for a given set of cooling conditions. The 572 and 575 °C isotherms are added on each contour to delineate the lines representing approximately 75 and 50% solid fraction for A356, respectively. The top row shows the results using the initial cooling times, which are based on trial-and-error optimization using the model. As can be seen at 49s there is a region of potential liquid/semi-solid encapsulation in the top of the rim indicated by the 50% fraction solid contour, which could result in solidification shrinkage. This is a well-known problem area in the water-cooled version of the LPDC process (note: this is occurring in an area that will be accurately described using the 2-D axisymmetric model). There is however, no liquid encapsulation predicted in the rim-spoke junction (73s), which is also an area prone to solidification shrinkage [9,11]. The results obtained with the numerical optimizer are presented in the lower row of images. As can be seen, these results show the elimination, or reduction, in the tendency to form shrinkage porosity in the top of the rim while maintaining similar solidification time.
Based on the optimized cooling timing, a casting trial was run at a commercial wheel plant (*note: the water-cooling timing actually used in the industrial casting trial was further refined using a 3-D model of the LPDC casting process to address limitations in the 2-D axi-symmetric model*). The results from this trial revealed fine, distributed, shrinkage porosity persistent in the upper rim – see Figure 5.

**Figure 4.** The initial (top) and the numerically optimized (bottom) solidification sequence in a water-cooled die.

**Figure 5.** X-ray image showing distributed shrinkage porosity in the top rim at the top of the wheel rim.

4.2 Second Application – Updated Wheel/Die Interface and Water Channel Boundary Conditions

To better understand why the industrial trial failed to produce a shrinkage free wheel, the thermal model used in the optimization tool was re-examined to assess the veracity of some of the key boundary conditions. On going work at UBC has revealed that: 1) the behaviour of the wheel die interface during a casting cycle is complex with gap formation in some regions, the development of pressure (no gap) in others and both gap formation followed by gap closure and pressure development in still others[12]; and 2) the heat transfer behaviour in the cooling channels is also complex exhibiting
transient periods of two phase boiling channel flow when the water first enters the channel and again when the flow is terminated to the channel [13]. The results of re-running the model with improved boundary conditions are presented in Figure 6. The upper set of images in Figure 6 (labeled “initial”) shows the results obtained with the previously optimized timing applied in the model with the updated boundary conditions. The lower image shows the results of re-running the optimizer to obtain a new optimum (labeled “optimized”). Focusing first on the initial results, the model now predicts shrinkage porosity in the upper rim consistent with what was observed in the industrial trial – i.e. the encapsulation of the 75% solid fraction isotherm would lead to distributed, relatively fine, shrinkage porosity. Note: some shrinkage was also predicted in the spoke approximately mid-way between the rim and the hub, which was not observed in the wheel. This is an artefact of the 2D axi-symmetric analysis, as the shrinkage is not predicted using a 3D version of the model. Thus, the initial boundary conditions lacked the sophistication needed for the optimizer to reach a valid solution. Turning to the optimized results, re-running the optimizer failed to produce a solution that was free of encapsulation / shrinkage in the upper rim owing to the elongated 75% solid fraction isotherm; however, it did reduce the severity of the encapsulation. The prediction of mid-spoke shrinkage was also eliminated. The conclusion from this result was that the current design of the dies from the standpoint of size and placement of the cooling channels in the vicinity of the top rim would not yield a wheel free of porosity without the addition of a riser to the top of the rim.

Figure 6. The initial (the upper row) and the optimized solidification conditions (the lower row) for the model run with the updated boundary conditions.

5. Conclusions
This paper discusses the development and the application of an optimization methodology to an LPDC process used for the commercial production of A356 automotive wheels. The optimization methodology has been successfully applied to optimize the on and off cooling timings of up to 7 water-cooling channels both from the standpoint of eliminating shrinkage based porosity associated with encapsulation of liquid and reducing the overall solidification time (process cycle time). The initial application of the results of the optimizer in an industrial setting revealed an inconsistency in the prediction of porosity in the upper rim area with what was observed in the cast wheel. One of the primary reasons for the inconsistency was the overly simplified boundary conditions used in the thermal model that was utilized within the optimizer. However, reapplication of the optimizer on a
model with updated boundary conditions revealed limitations in the underlying design of the dies with respect to the placement and size of some of the cooling channels. A key and perhaps obvious finding of the study is that the utility of numerical optimizers applied to casting processes in an industrial setting hinges critically on an accurate description of the process conditions. Looking forward, design of dies for the production of wheels will ultimately require the development and application of topological optimization algorithms in conjunction with process timing optimization, a formidable computational challenge.

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