A Dynamical Model of the Heliosphere with the Adaptive Mesh Refinement

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Abstract. A three-dimensional time-dependent heliospheric model is developed with the adaptive mesh refinement (AMR) technique. The AMR code, SFUMATO, is adopted, which has been originally developed for star-formation problems. The solar wind model is based on the space weather forecast system, SUSANOO, and it is imposed on the inner boundary at \( r = 25 \ R_\odot \). The solar wind model is derived from magnetogram synoptic maps, which are observed daily. Using the potential field source surface models, the WS formula, and a few empirical laws, the MHD parameters at the inner boundary are reproduced. The heliospheric model here extends up to 14 au, and the AMR grid resolves the heliospheric current sheets and co-rotating interaction regions with refined resolutions. The AMR grid brings considerable improvement in the fine features in the outer region of \( r \gtrsim 2 \ au \) compared to the concentric nested grid, of which resolution has the relationship \( \Delta x \propto r \), similar to spherical coordinates. The refinement criteria are based mainly on the distance from the Sun, the anti-parallel toroidal magnetic fields, and the density of the solar wind.

1. Introduction
Simulating a change in the heliospheric environment is important for space weather. The magnetic field structure and the speed of the solar wind affect the modulation of galactic cosmic rays (GCRs).
In order to calculate the modulation of GCRs, the simple Parker spiral models have been utilized [1, 2]. This approach is advantageous in that the heliospheric environment is described by several parameters, e.g., the solar wind speed, the tilt angle, and the polarity, and it is easy to perform numerical experiments according to observations of solar activity and cycle. The spiral should, however, have more complicated structures in dynamical and time-dependent phenomena. Several works [3, 4] have used magneto-hydrodynamics (MHD) models to reproduce the modulation of GCRs, while steady-state solutions have been adopted.

Since the heliospheric environment is affected by solar activity, we have been developing a framework for simulating the heliosphere by using MHD simulations. The GCRs are transported efficiently in the heliospheric current sheet (HCS) [5]. The gradient-curvature drift of the GCRs allows them to propagate along the HCS efficiently. The HCS should be reproduced with a fine resolution in the model. In the previous work [2], we solved the stochastic differential equations (SDEs) rather than the individual motion of the GCR proton. The timestep is restricted in the region close to the HCS so that the HCS is resolved at a high spatial resolution, e.g., $\Delta x = 0.1$ au at $r = 2$ au. The required resolution is larger at the points farther from the Sun. We therefore utilized the adaptive mesh refinement (AMR) technique for improving the local resolution. In this paper, we present the current status of the model development.

This paper is organized as follows. In Section 2, the models and methods are shown, and the prescription of the solar wind model and the refinement criteria are described. The results of the simulations are presented in Section 3, and they are summarized and discussed in Section 4.

2. Model and Method

2.1. Basic Equations

We assume the ideal MHD with a point source of gravitational force. The governing equations are given by,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0,$$

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot \left[ \rho vv^T + \left( P + \frac{|B|^2}{2} \right) I - BB^T \right] = \rho g,$$

$$\frac{\partial B}{\partial t} + \nabla \cdot (vB^T - BB^T) = 0,$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot \left[ \left( \rho E + P + \frac{|B|^2}{2} \right) v - B (v \cdot B) \right] = \rho g \cdot v,$$

$$E = \frac{|v|^2}{2} + \frac{P}{(\gamma - 1) \rho} + \frac{|B|^2}{2 \rho},$$

$$g = -\frac{GM}{r^3} r,$$

where $\rho$, $v$, $B$, $E$, $P$, $g$, and $I$ are the density, velocity, magnetic field, energy in the unit mass, pressure, gravity, and unit vector, respectively. The upper script $T$ denotes a transposed matrix. The adiabatic index is assumed as $\gamma = 1.46$ [6]. The pressure $P$ and temperature $T$ are related by $P = (\rho/\mu)k_B T$, where $k_B$ and $\mu (= 0.5)$ are the Boltzmann constant and mean molecular weight.

2.2. Computational Domain and Boundary Conditions

The computational domain is a cube with $x, y, z \in [-R_{out}, R_{out}]^3$, where $R_{out}$ denotes the size of the computational domain. We set $R_{out} = 3000R_\odot = 14$ au. The Sun is located at the origin (the center of the computational domain).
At the inner boundary condition, the solar wind is injected into the computational domain. The inner boundary condition is located at $r = R_{in}$ in spherical coordinates $(r, \theta, \varphi)$. We set $R_{in} = 25 R_\odot$, following [6]. Note that the solar wind velocity should be super-Alfvénic at $r = R_{in}$ in order to avoid inward propagation of the waves. The solar wind model imposed here is described in the next subsection.

The outer boundary condition is set at the surfaces $x, y, z = \pm R_{out}$, the edges of the computational domain. The zero-gradient boundary condition is imposed there so that the wind goes out of the computational domain.

2.3. Solar Wind Model

The solar wind model is based on the space weather forecast system, SUSANOO\(^1\) [7, 6]. The parameters of the solar wind are derived from observations of the solar magnetic field. Given time-series data of magnetogram synoptic maps, which are magnetic field distributions in two dimensions $(\theta, \varphi)$ at $r = R_\odot$, the model reproduces the solar wind as the inner boundary condition at $r = R_{in}$. To date, the Global Oscillation Network Group (GONG) project\(^2\) and the Wilcox Solar Observatory (WSO)\(^3\) are available as input observational data. In this paper, we use the photospheric magnetograms provided by the GONG project.

Based on the magnetogram synoptic map, the magnetic fields in the corona of $R_\odot \leq r \leq R_s$ are reconstructed by using the potential field source surface (PFSS) model, where $R_s$ denotes the source surface and is set at $2.5 R_\odot$. We thus obtain the radial component of the magnetic field at the source surface $B_r(R_s)$. The PFSS model assumes of the current-free magnetic field ($\nabla \times B = 0$), and the magnetic fields have only the radial component in the source surface ($B_\theta = B_\varphi = 0$), connecting to the open magnetic fields.

The solar wind velocity at the inner boundary ($r = R_{in}$) is obtained by the so-called WS formula as follows [8, 9],

$$v_r = 267.5 + \frac{410}{f_s^{1.4}} \text{ km s}^{-1},$$

where $f_s$ denotes the super-radial expansion factor defined as,

$$f_s = \left( \frac{R_\odot}{R_s} \right)^2 \frac{B_r(R_\odot)}{B_r(R_s)}.$$  \hspace{1cm} (8)

The super-radial expansion factor expresses a change in magnetic field strength along each field line, and the magnetic field strength $B_r(R_\odot)$ is measured at the foot point of the field line of $B_r(R_s)$. We assume that $v_\theta = v_\varphi = 0$ at the inner boundary. The WSA model [10, 11] is an alternative model for $v_r$, and improvement of the solar wind speed was discussed using this model [12, 13]. The WSA model will be incorporated into our model in future work.

The magnetic field at the inner boundary is extended by assuming a simple relationship,

$$B_r(R_{in}) = \left( \frac{R_{in}}{R_s} \right)^{-2} B_r(R_s).$$  \hspace{1cm} (9)

The azimuthal component of the magnetic field is set at,

$$B_\varphi(R_{in}) = -B_r(R_{in}) \frac{R_{in} \Omega_c \sin \theta}{v_r},$$  \hspace{1cm} (10)

\(^1\) http://cidas.isee.nagoya-u.ac.jp/susanoo/  
\(^2\) https://gong.nso.edu  
\(^3\) http://wso.stanford.edu
Figure 1. Grid configuration in the $y = 0$ slice plane. The squares with vertical and horizontal lines denote the AMR blocks. Each block contains $8^3$ cells. The left panel shows the concentric nested grid according to equation (13). The right panel shows the refined grid according to detection of the HCS.

According to the Parker spiral, and $B_\theta = 0$, where $\Omega_c (= 2\pi/25.38 \text{ days})$ is the angular velocity of the Sun.

The density and temperature at $r = R_{\text{in}}$ are obtained by following [14],

\begin{equation}
  n = \left( \frac{R_{\text{in}}}{50R_{\odot}} \right)^{-2} \left[ 62.98 + 866.4 \left( \frac{v_r}{100 \text{ km s}^{-1}} - 1.549 \right)^{-3.402} \right] \text{ cm}^{-3},
\end{equation}

\begin{equation}
  T = \left( \frac{R_{\text{in}}}{50R_{\odot}} \right)^{-2(\gamma - 1)} \left( -0.455 + 0.1943 \frac{v_r}{100 \text{ km s}^{-1}} \right) \times 10^6 \text{ K}.
\end{equation}

Because the original formula [14] estimates the density and temperature at $r = 50R_{\odot}$, the equations scale them up to $r = R_{\text{in}}$.

The GONG project provides a daily magnetogram synoptic map with $1^\circ \times 1^\circ$ resolution in the $\theta - \phi$ plane. By using the magnetogram synoptic maps and the prescription described above, we derive the daily data of the two-dimensional distributions of $\mathbf{B}$, $\mathbf{v}$, $n$, and $T$ at $r = R_{\text{in}}$ in the $\theta - \phi$ plane with the same resolution of $1^\circ \times 1^\circ$. We performed a temporal linear interpolation of daily data in order to obtain the data at a specific time level for the simulation. The temporal interpolated data is remapped from the spherical grid to the Cartesian grid by performing a spatial linear interpolation. Because we calculate the heliosphere in the stationary frame, the inner boundary condition is shifted in the $\phi$-direction in accordance with the rotation of the Sun.

The inner boundary condition described here does not take into account the divergence-free treatment of the magnetic field like [15]. Although a change in $B_r$ at the inner boundary condition produces $\nabla \cdot \mathbf{B}$ in the computational domain, the hyperbolic divergence cleaning method [19] suppresses the instability which arises from the solenoidality of the magnetic field.

2.4. Grid Refinement

We used the MHD-AMR code, SFUMAT0 [16], which has been originally developed for star-formation problems. SFUMAT0 utilizes the block-structured grid with an octree arrangement.
The refinement is performed in units of blocks. Each block contains $8^3$ cells.

We adopted AMR mainly for the following two purposes.

The first purpose is to set a grid suitable for radial flow. We adopted the concentrically nested grid shown in Figure 1 (left). This grid has the relationship $\Delta x \propto r$, which is the same as that of the azimuthal resolution of spherical coordinates. When the radial grid spacing is proportional to the azimuthal resolution, as is often done, the relationship $\Delta x \propto r$ holds for every dimension of the spherical coordinates. Each block is therefore refined up to the grid level of

$$\ell = \text{int} \left[ \log_2 \left( \frac{R_{\text{out}}}{d_{\text{max}}} \right) \right]$$

(13)

where $\ell \in [0, \ell_{\text{max}}]$ is the grid level, $\ell_{\text{max}}$ is the maximum grid level, and $d_{\text{max}}$ is the maximum distance of the cells within that block from the origin. In this paper, we set $\ell_{\text{max}} = 6$. Hereafter, this grid is called the fixed mesh refinement (FMR) because the resolution depends only on the location of the blocks.

The second purpose is to resolve the HCS with fine resolutions. In order to detect the HCS, we measure the anti-parallel magnetic field in the $\varphi$ component and a high density as well. A block is refined when the following two criteria are satisfied,

$$\max_{\text{block}} (B_\varphi) \times \min_{\text{block}} (B_\varphi) < 0,$$

(14)

$$\max_{\text{block}} (n) \left( \frac{r}{1 \text{ au}} \right)^2 \geq 10 \text{ cm}^{-3},$$

(15)

where $\max_{\text{block}}()$ and $\min_{\text{block}}()$ mean the maximum and minimum values in a block, respectively. Equation (14) detects the blocks in which two adjacent cells have $B_\varphi$ with different signs, e.g., $B_{\varphi,i,j,k} B_{\varphi,i+1,j,k} < 0$, showing that the HCS exists at the grid point $(i + 1/2, j, k)$. As shown in section 3.2, equation (14) detects noisy structures in the magnetic field as well. In order to exclude these structures, equation (15) is additionally evaluated. Finally, because of the geometrical effect of $B_\varphi \rightarrow 0$ as $\theta \rightarrow 0$, some blocks near the poles are satisfied with equation (14) although these blocks do not include the HCS. We therefore exclude the blocks near the poles after equations (14) and (15) are examined. If a block is satisfied with these criteria, the block is refined to two more levels than the FMR shown in Figure 1 (left). The resultant grid is shown in Figure 1 (right). Hereafter, this grid is called the AMR grid.

2.5. Scheme

The MHD method has third-order accuracy in space with MUSCL and second-order accuracy in time with the predictor-corrector method. We adopted the HLLD–Riemann solver [17] for the numerical flux. This solver is based on the HLLD Riemann solver [18], but it includes measures for curing the carbuncle phenomenon.

The hyperbolic divergence cleaning method [19] was adopted for suppressing numerical instability caused by $\nabla \cdot B$. The method proposes several formulations, and the mixed EGLM formulation [19] was adopted here.

3. Results

3.1. Overall structure

We followed the evolution of the heliosphere from October 1, 2016 to June 1, 2017. Figure 2 shows the solar wind speed distribution in the heliosphere on March 1, 2017. This figure illustrates that the time-dependent solar wind is reproduced by the AMR grid.

The left panels of Figure 2 show that the Parker spiral exhibits complex structures, indicating that it is not in a steady state. In the middle-left panel, the fast winds catch up with the slow winds, producing the shock waves. These features correspond to the formation of co-rotating
Figure 2. Velocity distributions of the heliosphere on March 1, 2017. Left and right panels are the cross sections in the $z = 0$ and the $y = 0$ planes, respectively, on larger scales from top to bottom. The color denotes the velocity. The squares with vertical and horizontal lines denote the AMR blocks. The white circle at the center denotes the inner boundary in each panel.
Figure 3. Ratios between the toroidal and poloidal components of magnetic fields, $B_\phi/(B_r^2 + B_\theta^2)^{1/2}$ in the $z = 0$ plane (left) and the $y = 0$ plane (right). The squares with vertical and horizontal lines denote the AMR blocks. The contour curve denotes $B_\phi = 0$. The stage is the same as in Figure 2.

Figure 4. Distributions of the refinement flags in the $z = 0$ plane (left) and the $y = 0$ plane (right). The grids satisfied with equation (14) have 1 bit (blue), and those satisfied with equation (15) have 2 bits (green). The grids satisfied with both the criteria have 3 bits (red). The flag of 3 bits is a necessary condition for refinement. The stage is the same as in Figure 2.

interaction regions (CIRs). The CIRs are resolved with the fine blocks because they are located near the HCSs.

The right panels of Figure 2 clearly show that the HCSs are resolved with the fine blocks. The HCSs are associated with low velocity winds (blue regions). The ballerina skirt structures are clearly reproduced on all the scales.

3.2. Grid Refinement

Figure 3 shows distributions of the ratio between the toroidal and poloidal components of magnetic fields, $B_\phi/(B_r^2 + B_\theta^2)^{1/2}$. The HCS exhibits boundaries between the positive (red) and negative (blue) toroidal component of the magnetic fields. Such boundaries exist not only near the mid-plane but also near the polar regions. The former corresponds to the HCSs, while the latter is caused by the geometrical effect. Only former regions are refined according to the
Figure 5. Solar wind speed (left) and temperature (right) distributions on the x-axis at the stage of June 1, 2017. The black and red lines denote the distributions solved by the FMR and AMR grids, respectively. The dots associated with lines denote the grid points.

criteria described in Section 2.4. Note that $B_\phi$ changes rapidly across the HCSs compared to the other boundaries.

Figure 4 illustrates how refinement criteria work. The grids satisfied with the refinement criterion of equation (14) have 1 bit flags (blue). These grids are distributed on the HCS and the polar regions. Because of the additional criterion of equation (15) (green), the refined grids (red) are selected appropriately except for the several grids in the northern polar region. In the right panel of Figure 4, the refined regions are not in contact with each other at $x \sim 500$ au because the blocks here are not satisfied with equation (15) (criterion with 2 bits). However, the 1-level coarser blocks safely cover this region because of the margin of refinement.

3.3. Effects of AMR

Figure 5 compares the solutions between the AMR and FMR grids. Both the solar wind speed and temperature in $r \lesssim 2$ au exhibit little differences because the CIRs are not developed within this radius. In contrast, in $r \gtrsim 2$ au, they exhibit significant differences between the AMR and FMR. The wind speed in the AMR exhibits sharp discontinuities, which corresponds to shock waves associated with the CIRs. The shock waves compressed the gas, leading to high temperature there.

4. Summary and Discussion

We developed a numerical model for the heliosphere in $r \leq 14$ au. The solar wind model is based on daily observations of solar magnetic fields, and the numerical model reproduces the time-dependent inner heliosphere. The AMR technique is adopted for resolving the HCS. A concentric nested grid is arranged, and the grid is refined further according to (1) the anti-parallel magnetic fields in the toroidal direction, (2) the high-density region or plasma sheet, and (3) exclusion of the polar region.

We examined an alternative refinement criteria as follows. We replaced the second criterion above by that on the low velocity, e.g., $|v| \leq 400$ km s$^{-1}$. Although the HCS has a low velocity, the amplitude of the $v_r$ decreases as the wind flows outward, as shown in Figure 5. Figure 2 also shows a similar tendency. The criterion based on low velocity therefore does not work in the outer region.
In addition, we examined another criterion based on currents, e.g., \( r^{-2} \nabla \times B \), where \( r^{-2} \) is for scaling according to the distance from the Sun. Although the HCS has currents, they are weaker in the HCS than in the shock waves associated with CIRs. The shock waves sharply bend the magnetic field lines, leading a strong current. This suggests that the criterion based on the currents is more suitable for detecting CIRs rather than the HCS.

Our model does not take into account a coronal mass ejection (CME). CMEs have a strong impact on the heliospheric environment, especially near the maximum in the solar cycle. We reported here modeling of the heliosphere near the solar minimum, in which the role of CMEs is relatively small.

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