Imaging shear stress distribution and evaluating the stress concentration factor of the human eye

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Healthy eyes are vital for a better quality of human life. Historically, for man-made materials, scientists and engineers use stress concentration factors to characterise the effects of structural non-homogeneities on their mechanical strength. However, such information is scarce for the human eye. Here we present the shear stress distribution profiles of a healthy human cornea surface in vivo using photo-stress analysis tomography, which is a non-intrusive and non-X-ray based method. The corneal birefringent retardation measured here is comparable to that of previous studies. Using this, we derive eye stress concentration factors and the directional alignment of major principal stress on the surface of the cornea. Similar to thermometers being used for monitoring the general health in humans, this report provides a foundation to characterise the shear stress carrying capacity of the cornea, and a potential bench mark for validating theoretical modelling of stresses in the human eye in future.

Stress concentration occurs in materials when they possess structural discontinuity or anisotropy; either naturally or induced, under different loading environments. In engineering design practices, this is commonly characterised using stress concentration factors (SCFs) depending on the type of loading (normal and/or shear SCFs). SCFs are a key safety indicator in the design of materials and ensure the stability of structures under different functional environments. For example, window panels of the aircrafts are designed to avoid sharp edges to maintain the stress concentration factors within allowable limits. This has helped to reduce the number of air accidents due to structural failures quite dramatically. Unlike in the case of man-made materials, information on the shear stress distribution characteristics and estimates of shear stress concentration factors of the human eye is scarcely found in the literature.

Here we provide in-vivo distribution of shear stress and stress concentration characteristics on the surface of a healthy human eye. Tests are conducted on a typical left eye of a healthy, normal white male subject in the 40–45 years age group, with no other known defects. The study is based on measuring birefringence in the cornea using photo stress analysis tomography (PSAT), a method well known in engineering and medical applications for visualising stresses in birefringent materials. Recent developments in the digital imaging and processing of birefringent data provide the opportunity to visualise shear stress distribution whole-field in the eye. This is advantageous as the birefringent measures can be quantified for any point of interest on the surface of cornea in relation to its optical axes. Existing methods of measuring stress in the eye for example, measuring an intra-ocular pressure (IOP) only provides an average pressure (i.e., force required to applanate target area of the cornea). Furthermore, IOP provides the fluid pressure inside the eye, and hence is hydrostatic in nature (about 22 mm of Hg, and a slight variation of this is possible in the normal eye at different times of the day or seasons). However, under normal working conditions, biological materials such as the human cornea tissue could experience non-hydrostatic shear stress distribution and this aspect is not yet fully understood. Hence the focus of this current work is based on measuring the maximum shear stress (anisotropic) distribution on the surface of the cornea in vivo. Failure of the materials is known to show more dependence on the anisotropic state of stress fields than on the hydrostatic state of stress. However, hydrostatic loading on an anisotropic structural body, such as an eye cornea, could still experience anisotropic stress distribution. It is important to assess it to improve our fundamental understanding of the functioning and health of the human eye.

Birefringence in materials refers to having different indices of refraction at different crystallographic orientations at the point of interest under the mechanical stress. The cornea is the front window of the human eye (Figure 1). Ocular media of the human eye (i.e., the transparent substances of the eye including the cornea, the
aqueous humour, the crystalline lens and the vitreous humour) is naturally birefringent. Previous studies on birefringent measurements in the human eye have focused mainly on the medical treatment perspectives for example, to characterise retinal structures, their age effects and in vitro analysis of keratoconus. A number of studies based on X-ray diffraction have provided an insight into the structural architecture of the human eye. They report anisotropy in the orientation of collagen fibrils (diamond-like structure) which provides mechanical stability of the cornea. Computational and analytical studies are reported to estimate the biomechanical properties of eye tissues, but their model parameters are still being disputed. New experimental results on shear stress profiles of the eye could help to fine-tune the biomechanical simulation parameters of cornea.

Here we present an engineering approach to understand the shear stress distribution characteristics of the human eye using reflection type PSAT. The working principle of PSAT is outlined in Fig. 2. In brief, an achromatic, fully circularised polarised light falls on the surface of the cornea. The surface of the cornea has a curvature, although relatively small when compared with other parts of the eye. Hence using a circularly polarised light to ‘sweep’ the surface of cornea helps to minimise the error in the retardation measurements as it is orientation independent and allows for making birefringent measurement at all orientations simultaneously (more corrections would have been required if the falling light was plane-polarised only). The tomography is designed efficiently to minimise any such errors for practical purposes. Previous studies have also reported that the human cornea exhibits circular birefringence. We show later that the current birefringent measurements on the cornea are comparable with using other methods reported in the previous literature, for example an optical coherence tomography. The out-coming light is elliptically polarised and carries the signature of birefringent retardation of the cornea. The elliptically polarised light is examined using an analyser for a number of angular positions of the optical axis of analyser (α) with respect to the major axis of the ellipse (Fig. 2). The outputs are used to build the birefringent map of the cornea and used subsequently in the stress analysis (see the method section).

Results and Discussion
The components (τ₁ and τ₂) of the maximum shear stress (τₘₐₓ) are illustrated in Fig. 3. These, together with the direction of the major principal stress at any point of interest on the cornea, provide the temporal and spatial distribution of shear-bearing regions of the cornea. Figure 4 presents the in vivo sensing of shear stress acting on the horizontal and vertical planes of the cornea (τᵥ). The values of retardation reported here (and the mean value 45 nm) using light wavelength 650 nm are comparable to and within the range of previous studies on birefringence measurements of the human eye, using other methods in a different context: 0–95 nm using a wavelength of 585 nm (mean approx. 50 nm) and 30–69 nm at a wavelength of 828 nm using polarisation sensitive optical coherence tomography (PS-OCT). This supports the adequacy of using PSAT in the current research reported here. Interestingly, the shear stress profile quantitatively shown in Fig. 4 correlates well with previous studies reported on the diamond-like structural anisotropy of normal cornea using X-ray based studies and close to the diamond-like manual (qualitative) traces of fibril structures in the eye. Previous studies have shown that the diamond-like structure contributes to the structural stability of the eye. Hence the birefringent measurements made here are
consistent with previous literature but, more importantly, the current alternative approach is non-X-ray based and possesses no additional health risks to the eyes. From the stress analysis, it is evident that structural architecture of the cornea is active in sustaining the shear, and correlates to the distribution of shear along the horizontal and vertical planes (shear stress architecture). The measurement of shear stress provided here in Fig. 4 can also be calibrated to the Pa unit for which further mechanical properties of the cornea are required. For example, based on previous literature, the following parameters are considered: Young’s modulus and Poisson’s ratio of cornea are 1 MPa and 0.49 respectively\(^{17,18}\). Average cornea thickness 520 μm\(^{18}\) and maximum shear stress at the centre of the cornea \(\sim 62\) μPa\(^{25}\). Based on these, a multiplication factor of 3.175 is to be applied to scale the retardation to the μPa unit. However, this conversion would not affect the determination of the eye stress concentration factors reported here (see method section).

The stress-structural anisotropy correlation, as observed above, is also supported by probing the directional alignment of the major principal stress on the surface of the cornea. This is presented in Fig. 5. A detailed analysis was performed on the individual distributions of \(\tau_{\text{max}}\) and \(\tau_{\text{z}}\) and their profiles are also presented here. We observed that the profile of the maximum shear stress distribution displays diamond-like architecture. The shear stress distribution profile at 45° plane to the horizontal and vertical axes (\(\tau_{\text{z}}\)) displays a comparable level of activity but tends to possess relatively less anisotropy when compared with the \(\tau_{\text{z}}\) distribution. Overall, it is evident that the diamond-like architecture of shear stress profiles along the key planes viz., horizontal and vertical planes and the plane carrying maximum shear stress are similar to the structural architecture of the cornea reported earlier\(^{16,22}\). They enlighten the flexibility of the corneal tissues possess to enable the structural movements of the eye, and an in-built ability to distribute shear at orthogonal directions.

Furthermore, similar to the evaluating stress concentration factors as a design parameter in engineering materials and structures\(^{6,7}\), a strength analysis is performed by examining the signatures of \(\tau_{\text{max}}\) at different sections of the cornea to quantify eye shear stress concentration factor \(K_e\) defined as follows:

\[
\frac{\text{Maximum shear stress } \tau_{\text{max}}}{\text{Nominal value of maximum shear stress } \tau_0} = K_e
\]

\(K_e = \) eye shear stress concentration factor (eSSCF)
\(\tau_0 = \) Nominal value of maximum shear stress

Figure 6 presents the variation of eSSCF in the cornea along the superior-inferior axis (section 1–2), nasal-temporal axis (section 3–4) and the 45° orthogonal axes (sections 5–6 and 7–8). The maximum value of eSSCF (w.r.t. point1) along these axes is found to be 2.60, 2.50, 2.76 and 2.38 respectively. By and large, the eSSCF profiles tend to present symmetry along the superior-inferior axis from the

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Figure 2 | Schematic diagram of the working principle of PSAT. See the method section for more details (Thanks: J. Drewes for the eye image insert).

Figure 3 | Components of maximum shear stress \(\tau_{\text{max}}\) marked on the Mohr’s circle\(^{2,24}\) representing shear stress state at a point of interest on the cornea. \(\tau_1\) represents shear stress acting in horizontal and vertical planes whereas \(\tau_2\) represents shear stress acting at 45° to the horizontal and vertical planes. \(\tau_{\text{max}}\) is the maximum shear stress acting at 45° to the principal stress direction. \(\sigma_1\) and \(\sigma_2\) are the major and minor principal stresses respectively.
The stress of the cornea is shown to have a good level of correlation with the diamond-like architecture of corneal tissues\textsuperscript{16} (Figs. 4, 5). The results show that the cornea is effective in distributing shear in the horizontal and vertical planes, as well as at 45° to the principal stress direction (where maximum shear stress acts). Overall, the present stress analysis demonstrates the effectiveness of the structural architecture of the cornea (diamond-like at\textsuperscript{16} or close to\textsuperscript{22} 45° angles of superior-inferior and nasal-temporal directions) in distributing shear loading. This is a naturally favourable stress distribution mechanism for easing the movements of the eye in daily life. The stress concentration factors show non-uniform variation along the different sections of the cornea as studied here, but the profiles tend to be relatively more symmetric along the superior-inferior section, passing through the centre of the cornea. The stress profiles presented here could aid modellers to compare with, and calibrate the material parameters of the cornea in their simulations. This is urgently required, as bio mechanical model parameters of eye elements are still uncertain in existing literature\textsuperscript{20}. For example, studies have shown that the measurement of the cornea thickness could depend on the type of equipment used\textsuperscript{27}. Measurements of material parameters, such as elastic modulus, could vary within the corneal tissues at molecular and micro scales\textsuperscript{26,31}. Their effects on the collective ability of the cornea to sustain shear is still not well known. However, the present whole-field and in vivo stress measurements and analysis provide a novel foundation with an engineering approach to characterising a complex biological problem. By employing the methodology used here for mapping out the shear stress distribution profiles at different planes using Mohr’s circle, the stress measurements could be linked with new optimisation and inverse calculation methods\textsuperscript{23} to fine tune the material parameters (e.g. elastic modulus) and model architecture in such a way that the simulations could initially reproduce the experimentally measured shear stress distribution characteristics of a healthy cornea. Thereafter, the sensitivity analysis of the material parameters on the shear stress distribution characteristics of other environmental conditions of the eye could be examined using simulations with relatively more confidence.

The present study is not without limitations. More studies are also required to obtain a wider representation of the stress intensity profiles in the human eye. Future collaborative studies with the participation of engineering and medical researchers, could investigate
the corneal stress distribution characteristics for a wider representation of subjects, age groups, gender and ethnicity and in other stress-related environments. Such efforts could lead to developing additional post-operative eye health compliance criteria by comparing the stress concentration factors at different sections of the structural elements of the normal and abnormal cornea (in addition to corneal topography) in vivo. Abnormal stress raisers, using the stress concentration profiles of the cornea, could be identified in treating patients where stress effects could change the birefringence in the cornea, for example, in keratoconus patients. The present study has not considered any variations in the IOP on the shear stress distribution characteristics and the shear stress concentration factors of the eye. Minor variations in the IOP of normal eyes are less likely to change the values of eSSCFs significantly as they are relative measures. However, more detailed studies could be conducted in future taking into account the potential effects of IOP variations on anisotropic stresses during different periods of the day especially where the cornea is abnormal. In addition, development of eSSCFs due to infectious eye irritations, injuries and fatigue due to aging are also required. Fatigue failures usually originate from stress raisers, so removing such defects could improve the failure strength and quality of the imaging ability of the human eye. As the present method does not involve using X-rays, the methodology is particularly suited for making repetitive stress measurements in the cornea for example, measuring stresses at different time intervals without any significant risk in the future. The potential opportunities are extensive.

Methods

The working principle of PSAT, which is a grey field polariscope (Fig. 2), is based on photoelastic birefringent properties of materials. PSAT is a hybrid of a plane-polariscope and a circular polariscope. It illuminates the target sample (cornea)
with circularly polarized light but analyses it with a linear polarizer (Fig. 2). A comprehensive mathematical treatment of its optics can be found elsewhere but the comprehensive mathematical treatment of its optics can be found elsewhere but the with circularly polarized light but analyses it with a linear polarizer (Fig. 2). A

Captured image of the cornea (comprising birefringent fringes of different orders), the phase lag with respect to the fast axis. As in the case of previous studies on corneal birefringent measurements, individual contribution of other ocular media to retardation is ignored. \( A_{\text{maj}} \) and \( A_{\text{min}} \) are the light components along the major and minor axes of the ellipse respectively. The major axis of the ellipse is shifted by \( \pi/4 \) for odd fringes and \( -\pi/4 \) for even fringes. The out-going polarised light intensity \( I_{\text{out}} \) is analysed using an analyser (which is optically identical to a polariser), for different digital camera and computer allows completion of the retardation measurements in a minute) and with high precision lenses so that even a fraction of retardation can be recorded. From this captured image of the cornea (comprising birefringent fringes of different orders), the retardation can be resolved along the major and minor principal stress axis at any point of interest on the cornea. This optical information can be related to stress information using the stress-optic law, which has been used for more than a century in birefringent stress measurements: \( \sigma_1 - \sigma_2 = n_F \cdot s \), in which \( \sigma_1 \) and \( \sigma_2 \) are the major and minor principal stresses respectively, \( n \) is the fringe order (at the point of interest as a result of \( A \) ) and \( F \), is the stress fringe constant determined by calibration either numerically (for example as described in Fig. 4), or experimentally. However, these represent values in terms of the retardation unit does not require this calibration. Also determining the direction of principal stress components and the shear stress intensity factor (which is a relative measure) does not require this calibration. Furthermore, the stresses can be expressed using the well-known Mohr’s circle representation. Using this shear stress in the vertical and horizontal planes, together with the maximum shear stress and the direction of principal stresses, can be represented easily. The tomography setup is a powerful tool to measure sub fringe shear stress in the vertical and horizontal planes, together with the maximum shear stress and the direction of principal stresses, can be represented easily. The tomography setup is a powerful tool to measure sub fringe measurements accurately (less than 1 nm). The integrated experimental setup with a digital camera and computer allows completion of the retardation measurements in the subject at an adequate speed (about 20 revolutions of analyser positions within a minute) and with high precision lenses so that even a fraction of retardation can be recorded.

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