Coarse grained short-range correlations

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We develop a scheme to take into account the effects of short-range nucleon-nucleon correlations in the nucleon-pair wave function by solving the Bethe-Goldstone equation for a coarse grained delta shell potential in S-wave configuration. The S-wave delta shell potential has been adjusted to reproduce the \( ^1S_0 \) phase shifts of the AV18 potential for this partial wave up to 2 GeV in the laboratory kinetic energy. We show that a coarse grained potential can describe the high momentum tail of the back-to-back correlated pairs and the G-matrix in momentum space. We discuss the easiness and robustness of the calculation in coordinate space and the future improvements and utilities of this model. This work suggests the possibility of using perturbation theory for describing the short-range correlations, and related to this, to substitute the G-matrix by an appropriate coarse-grained potential.

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I. INTRODUCTION

Historically the existence of a strong repulsive core was first pointed out by Jastrow in 1950 in his analysis of proton-proton scattering (for a historical overview covering up to 1989 we refer to \cite{1}). The assumption that this repulsive core dominates short-range correlations, preventing two nucleons to approach each other to distances closer than half a fermi, invalidates direct use of mean field methods applied directly to the NN interaction as inferred directly from NN scattering data. From a Wilsonian point of view the strength of the interaction, however, depends on the probing wavelength \( \Delta r \) (see e.g. Ref. \cite{2} and references therein) which is ultimately related to a high energy cut-off in the problem.

In this paper we want to focus on the description of high momentum components of the nuclear wave function. Since the core distance \( r_c \) and the Fermi momentum \( k_F \) at saturation fulfill \( 2k_Fr_c \sim 1 \) we will see that by judiciously tuning the corresponding length scale \( \Delta r \sim r_c \) it is possible to access high momentum pair distributions in nuclear matter with back-to-back momentum \( p \sim 2k_F \) while keeping scattering information for the same situation in the free space without need of a strong repulsive core. Going beyond this maximum momentum is possible within a NN potential model approach, but it also faces the problem that the NN interaction needs to take into account a substantial inelasticity which would require explicit consideration of nucleon resonance production, such as \( NN \rightarrow N\Delta \) or \( NN \rightarrow \Delta \Delta \) for \( p_\Delta \sim \sqrt{M(M_\Delta - M)} \sim 2k_F \) in the scattering problem.

There exist many possible ways and techniques to handle and analyze short range correlations (see e.g. \cite{2} and references therein). For our purposes we will follow the venerable Brueckner-Goldstone theory of nuclear matter. We believe this framework is a good starting point which provides a satisfactory method for finding the bulk properties of nuclear matter, in particular its saturation energy and equilibrium density.

The formal derivation and the mathematical framework to carry out calculations within this theory were also provided by Goldstone and Bethe \cite{3, 4} in terms of an integro-differential equation in configuration space. Excellent reviews on this topic can be found on Refs. \cite{9, 11}.

One of the main ingredients in solving the G-matrix in the Brueckner-Bethe-Goldstone formalism is the two-nucleon potential \( V_{ij} \). Most early and modern so-called realistic nucleon-nucleon (NN) potentials \cite{12} contain parameters that are usually fitted to available scattering information up to a certain energy (usually about pion production threshold), and they also provide the right static properties of the unique bound state of the two-nucleon system, namely the deuteron.

The AV18 potential \cite{12} is a popular and versatile choice which has attracted much attention from nuclear structure theorists since it described up and pp scattering with \( \chi^2/\text{dof} \sim 1.1 \) for LAB energies up to 350 MeV at the time of the Nijmegen analysis (the quality has worsened with the new Granada database to \( \chi^2/\text{dof} \sim 1.46 \) without refitting up to 300 MeV due to the 40% more new data \cite{19}). Besides being local, it presents both a repulsive core and turns out to provide a \textit{a fortiori} a qualitative high energy description of scattering data when the inelasticity effects are neglected. Our approach in this work will consist in taking a coarse grained delta shell potential whose strength parameters are fitted to obtain the same phase shifts as the AV18 potential \cite{12} up to a laboratory kinetic energy of 2 GeV. The coarse
graining is based on the idea that if one wants to determine some NN scattering observables in a limited energy range, then the interaction potential only needs to be known in a limited number of points. The average separation of these points is related to the maximum resolution power which can be achieved within this upper limited energy range implying a shortest de Broglie wavelength. The findings of this work are based on the early an insightful work by Avilés [20] which was rediscovered within a Wilsonian renormalization perspective [21] and fully exploited in NN scattering analysis [22,27] to which we refer for further details. We emphasize that this simplification takes place in configuration space and we will exploit this feature explicitly in our analysis.

Our aim is to study the properties of the coarse-grained (GR) potential in the nuclear medium. In the past we studied and fitted the GR to NN scattering data. Here we go beyond the bare NN interaction and study the influence of coarse-grain on the short-range correlations between nucleon pairs inside the nucleus, by analyzing the high-momentum components of the relative wave function, comparing with the potential AV18, which produce similar phase-shifts.

Since the appearance of the Bethe-Goldstone (BG) equation, several methods have been developed to solve it (for a critical review on some of them we refer the reader to Ref. [28]). Most of the early treatments analyzed the problem in configuration space until Haftel and Tabakin introduced a momentum space solution via a direct matrix inversion method after some smoothing of NN interaction was implemented [29]. It should be noticed that a repulsive core in configuration space generates long high momentum tails which inevitably lead to large matrices (typical dimensions are of the order of ∼ 50) [30]. While it is possible to carry out such a coarse grained analysis we will proceed here directly by using the original integro-differential version of the BG equation.

Besides, some inherent difficulties in the BG solution have been overcome by resorting to approximate solutions. Just to enumerate a few of them, we can mention: the treatment of the center-of-mass (CM) motion of the two-nucleon system; the handling of the Pauli blocking operator [31] or the necessity (or not) of a partial wave decomposition [32]. The first two of the above difficulties are easily overcome by choosing the kinematic configuration where the CM momentum of the nucleon pair is zero. This situation corresponds to a back-to-back configuration for the correlated nucleon pair, and recently it has drawn attention from the theoretical nuclear physics community [33,39] as well as from the experimental electron [40,44] and neutrino scattering physics communities [45,46].

In this work we try to keep the maximum simplicity as possible in the approach to this problem in order to properly understand the effects of all the ingredients involved and how they are mutually intertwined. Therefore we try to be pedagogical and refrain from going beyond the S-wave configuration in a partial wave expansion. We also restrict our calculation to the back-to-back configuration for a nucleon pair at rest ($P_{CM} = 0$) in order to avoid all the problems related to the CM motion, specifically the angular averaging of the Pauli blocking operator.

The structure of the paper is as follows. In section II we review the Bethe-Goldstone equation in coordinate representation for the correlated wave function. As originally recognized by Bethe and Goldstone this is particularly suited when there is a strong repulsive piece of the interaction. We also deal with the correlated wave function in momentum representation before entering, in section III, into the discussion of the results on the high momentum tail of the momentum distribution, the $G$-matrix, and how the repulsive and attractive components are reshuffled depending on the resolution wavelength. Finally we summarize our findings and outline our future plans in section IV.

II. FORMALISM

The Bethe-Goldstone equation can be regarded as the in-medium scattering equation. This is an integral equation which, besides the effect of the inter-particle potential, also incorporates the effect of the surrounding medium by preventing the interacting nucleon pair from being scattered into the already occupied levels below the Fermi momentum $k_F$. Here we review it so that our conventions and notation as well as our coarse grained method of solution and analysis can be more easily introduced in configuration space. While the original BG discussion has become textbook material [47,48] only some simple cases were discussed in a rather sketchy fashion. We hope that our presentation will be useful both for newcomers as well as researchers familiarized with the more popular momentum space approach.

A. Correlated wave function

The Bethe-Goldstone equation for the wave function of an interacting nucleon pair in the independent-pair approximation [48] can be written in the form

$$|\Psi\rangle = |k_1 k_2\rangle + \int d^3k'_1d^3k'_2 \ |k'_1 k'_2\rangle \frac{\langle k'_1 k'_2 | V |\Psi\rangle}{E - (E_{k_1'} + E_{k_2'})} \times \theta(|k_1'| > k_F) \theta(|k_2'| > k_F) \quad (1)$$

where $\theta(x)$ is the Heaviside or unit step function, and $|\Psi\rangle$ is the correlated wave function. This is given in terms of the unperturbed plane wave solution $|k_1 k_2\rangle$ (with $|k_1| < k_F$) plus the high momenta components $|k'_1 k'_2\rangle$ (with $|k'_1| > k_F$), which are weighted by the transition matrix element of the potential and the energy denominator that damps the largest energy differences with respect to the exact eigenvalue $E$. The unperturbed energies $E_{k'_i}$
correspond to the eigenvalues of the unperturbed Hamiltonian $H_0 = T_1 + T_2$ and are given by

$$E_{k_l'} = \frac{k_l'^2}{2M_N} \quad (2)$$

where $M_N$ is the nucleon mass.

Note that in eq. (1) we have particularized the solution of the B-G equation for the case of nuclear matter or a Fermi gas, where the plane wave solutions with definite momenta are known to be the eigenfunctions of the uncorrelated system.

If we left-multiply eq. (1) by the bra $(x_1, x_2)$, we get the correlated wave function in coordinate representation as

$$\Psi(x_1, x_2) = \Phi_{k_1, k_2}(x_1, x_2) + \int d^3k_1' d^3k_2'$$

$$\times \Phi_{k_1', k_2'}(x_1, x_2) \theta(|k_1'| > k_F) \theta(|k_2'| > k_F)$$

$$\times \frac{\int d^3x_1' d^3x_2' \Phi_{k_1', k_2'}(x_1', x_2') V(x_1', x_2') \Psi(x_1, x_2)}{E - (E_{k_1} + E_{k_2})} \quad (3)$$

where it is easier to notice the self-consistent integral character of the B-G equation.

Now we use the trick of changing all coordinate and momenta from particles’ variables to CM and relative ones

$$R_{CM} = \frac{1}{2} (x_1 + x_2) : \quad K_{CM} = k_1 + k_2$$

$$x = x_1 - x_2 : \quad k = \frac{1}{2} (k_1 - k_2)$$

Furthermore, if the potential only depends on the relative coordinate $x$ and not on the CM one, further simplifications are possible. This kind of potentials preserve the CM motion and the correlated total wave function is separable into a product of a plane wave describing the free motion of the CM system and a correlated relative wave function $\Psi$ that fulfills a BG-like equation [48]

$$\Psi(x_1, x_2) = e^{i K_{CM} R_{CM}} \frac{\psi_{CM, k}(x)}{(2\pi)^{\frac{3}{2}}} \quad (4)$$

$$\Phi_{k_1, k_2}(x_1, x_2) = e^{i K_{CM} R_{CM}} \frac{\psi_{CM, k}(x)}{(2\pi)^{\frac{3}{2}}} \quad (5)$$

and analogously for primed variables.

If the above expressions (1) and (5) are substituted into eq. (3) then the CM wave functions factorize and cancel on both sides and we are left with the BG-like equation for the correlated relative wave function $\psi$,

$$\psi_{CM, k}(x) = e^{i k \cdot x} + \int d^3k' \frac{1}{(2\pi)^{\frac{3}{2}}} e^{i k' \cdot x} \theta\left(\left|\frac{K_{CM}}{2} - k'\right| - k_F\right) \theta\left(\left|\frac{K_{CM}}{2} + k'\right| - k_F\right)$$

$$\times \int d^3x' e^{-i k' \cdot x'} V(x') \psi_{CM, k}(x') \quad (6)$$

where $\mu = \frac{M_N}{2}$ stands for the reduced mass of the two-nucleon system; the energy denominator now only contains the difference between the initial and final relative kinetic energies (the initial and final CM energies cancel in the denominator). The two step functions impose the conditions $|k_1'|, |k_2'| > k_F$.

One of the main complications in solving eq. (6) involves the integration over the angles of $k'$ due to the presence of the step functions, which explicitly depend on the angle between $K_{CM}$ and $k'$. For that reason an angular average of the projection operator is usually performed [3] (see Ref. [49] for a full treatment of this problem in the general case). Another possibility is simply considering the situation where $K_{CM} = 0$, which corresponds to a back-to-back configuration for the two-nucleon system (where the effect of NN correlations should be maximized). Note that in the latter case the two step functions get reduced to a single one, simply implying that $|k'| > k_F$.

**B. Integro-differential Bethe-Goldstone equation**

Up to this point the discussion has been quite general on the correlated wave function problem. Next discussion can be also followed from Refs [48, 50], where the integral B-G equation for the relative wave function is transformed in an integro-differential equation by applying the operator $(\nabla^2 + k^2)$ on both sides of eq. (6) and taking the especial case when $K_{CM} = 0$. Indeed, we have then

$$(\nabla^2 + k^2) \psi_{k}(x) = \int \frac{d^3k'}{(2\pi)^3} e^{i k' \cdot x} \theta\left(\left|k'\right| - k_F\right)$$

$$\times \int d^3x' e^{-i k' \cdot x'} 2 \mu V(x') \psi_{k}(x') \quad (7)$$

$$= 2 \mu V(x) \psi_{k}(x) - \int \frac{d^3k'}{(2\pi)^3} \theta\left(k_F - |k'|\right) e^{i k' \cdot x}$$

$$\times \int d^3x' e^{-i k' \cdot x'} 2 \mu V(x') \psi_{k}(x') \quad (8)$$

where in the last steps we have used the property of the Heaviside function $\theta(x) = 1 - \theta(-x)$ and the orthonormality condition of the plane waves

$$\int d^3 k' e^{i k' \cdot (x - x')} = (2\pi)^3 \delta^3(x - x')$$

Now we expand the wave function $\psi_{k}(x)$ in partial waves. For simplicity in this work we assume that the nucleon pair is coupled to total spin $S = 0$. Then the NN potential do not mix different partial waves. The function $V_l(s)$ is the l-multipole of the potential in the $^1L_L$ channel, and each reduced radial wave function, $u_{k,l}(r)$,
verifies the decoupled integral-differential equation:

\[
\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \left( k^2 - 2 \mu V(r) \right) \right\} u_{k,l}(r) = - \frac{4 \mu r}{\pi} \int_0^{k_F} dk' k'^2 j_l(k'r) \int_0^\infty dr' r' j_l(k'r)V(r')u_{k,l}(r')
\]

where \( r = |x| \) is the relative distance, \( k = |k| \) and \( j_l(\rho) \) is a spherical Bessel function.

From now on we particularize eq. (9) for the S-wave case \((l = 0)\). We have the BG equation which is solved in Sect. IIE,

\[
\left\{ \frac{d^2}{dr^2} + (k^2 - 2 \mu V(r)) \right\} u_k(r) = - \frac{4 \mu}{\pi} \int_0^\infty dr' \chi(r,r')V(r')u_k(r')
\]

where the kernel \( \chi(r,r') \) is given by 48

\[
\chi(r,r') = \int_0^{k_F} dk' \sin(k'r) \sin(k'r') = \frac{1}{2} \left[ \sin \left( \frac{k_F(r-r')}{r + r'} \right) - \frac{1}{r + r'} \right].
\]

C. Coarse graining vs fine graining

The novelty of the present work consists in solving eq. 10 for equally spaced delta shell potential. As such, this method can be regarded as a simple quadrature method for a given potential \( V(r) \), namely making the replacement

\[
V(r) \rightarrow \sum_{i=1}^N \delta(r - r_i)
\]

for an equidistant grid \( r_n = n\Delta r \), which stops when \( r_N \sim a \) with \( a \) the range of the interaction. Of course, for \( N \rightarrow \infty \) and \( \Delta r \rightarrow 0 \), we expect a better and more accurate solution. Here \( \Delta r \) plays the role of an integration step for this particular quadrature method. We will show below that this fine graining requires in practice a large number of mesh points for a potential \( V(r) \) such as the AV18 which has been determined from a fit to NN scattering data up to a maximum energy.

In contrast, the coarse graining approach already presented in Refs. 20 22 26 corresponds to take \( \Delta r \) not as an auxiliary integration step but as a physical parameter. Namely, we take it as the shortest de Broglie wavelength resolution \( \Delta \lambda_{\text{min}} = 1/p_{\text{CM}}^{\text{max}} \) with \( p_{\text{CM}}^{\text{max}} \) the maximum CM momentum and use the values of \( V_{\Delta r}(r_i) \) as fitting parameters themselves, thus we make the replacement

\[
V(r) \rightarrow \sum_{i=1}^N \Delta r V_{\Delta r}(r_i) \delta(r - r_i) \equiv \sum_{i=1}^N \frac{\lambda_i}{2\mu} \delta(r - r_i)
\]

Obviously the number \( N \) and values of these fitting parameters depend on the resolution wavelength, \( \Delta r \) and hence on the maximal fitting energy as well as the desired accuracy in the phase-shifts. For a potential with range \( a \) we expect \( N \sim a/\Delta r \approx p_{\text{CM}}^{\text{max}} \) points. Thus, if we take a potential \( V(r) \) with phase shifts \( \delta(p) \), its coarse grained representation \( V_{\Delta r}(r) \), corresponds to find \( V_{\Delta r}(r_i) \) such that \( \frac{\delta(p)}{\Delta p} \leq \Delta \delta(p) \) for \( p \leq 1/\Delta r \) and with \( \Delta \delta(p) \) the tolerated discrepancy. In practice we use the standard \( \chi^2 \) as a figure of merit

\[
\chi^2(\lambda_1, \ldots, \lambda_N) = \sum_{n=1}^N \left[ \frac{\delta \Delta r(p_n) - \delta(p_n)}{\Delta \delta(p_n)} \right]^2
\]

and determine the coarse grained parameters \( \lambda_i \) by minimization. An educated guess is to take \( \Delta \delta(p) \) as the expected systematic discrepancies 27 and \( p_n \) as the values corresponding to the tabulated LAB energies.

Of course, we expect that for \( \Delta r \rightarrow 0 \), \( V_{\Delta r}(r_i) \rightarrow V(r_i) \). These considerations hold equally well regardless of the value of the Fermi momentum \( k_F \). In order to illustrate the procedure we will discuss first the vacuum case, \( k_F = 0 \), corresponding to the scattering problem before coming to the BG solution.

D. Scattering Solution for \( \delta \)-shell potentials

The scattering problem in the S-wave corresponds to solving the equation 21 22

\[
\left\{ \frac{d^2}{dr^2} + \left( k^2 - \sum_{i=1}^N \lambda_i \delta(r - r_i) \right) \right\} u_k(r) = 0
\]

with the boundary conditions at the origin and at infinity

\[
u_k(0) = 0, \quad u_k(r) \rightarrow C_k \sin(kr + \delta(k))
\]

The solution outside anyone of the concentration radii \( (r_i) \) can be written as

\[
u_k(r) = A_k \sin(kr + \delta_i)
\]

for \( r_i < r < r_{i+1} \) \((i = 0, 1, \cdots N)\)

where \( \delta_i \) is the accumulated phase shift due to the delta shells potential up to \( r_i \) and \( A_k \) are amplitudes which are fixed by an arbitrary normalization condition (for instance \( A_N = 1 \)). Taking \( r_0 = 0 \) and \( \delta_0 = 0 \) the phase shift is given by the total accumulated one

\[
\delta(k) = \delta_N
\]

1 Note that this is not the same as fixing \( p_{\text{CM}}^{\text{max}} \) and increasing the wavelength resolution by taking \( \Delta \lambda_{\text{p}} \ll 1 \). In this case, the final potential, while continuous, does not reproduce the original one; oscillations of period \( 2\pi/p_{\text{CM}}^{\text{max}} \) develop exhibiting the physical resolution.
Now it is necessary to match the different branches of the reduced wave function on each interval around the \(i\)-th delta shell, and this is done by imposing continuity of the total wave function at the \(i\)-th concentration radius and discontinuity of the first derivative of the wave function due to the existence of an extremely singular potential at \(r = r_i\). These two conditions reduce to
\[
\begin{align*}
  u_k(r_i^+) &= u_k(r_i^-) \\
  u_k(r_i^+) - u_k(r_i^-) &= \lambda_i u_k(r_i) \quad (i = 1, 2, \cdots, N)
\end{align*}
\]  
(19)

whence the total accumulated phase-shift may be computed. The recurrence relation for the amplitudes of the homogeneous solution is then
\[
A_{i+1} = A_i \frac{\sin(kr_{i+1} + \delta_i)}{\sin(kr_i + \delta_i)}
\]  
(20)

E. BG Solution for \(\delta\)-shell potentials

We come here to the core of our construction. The Bethe-Goldstone equation \(10\) is a linear integral-differential equation. For the \(^1S_0\) delta shell potential it reads
\[
\left\{ \frac{d^2}{dr^2} + \left( k^2 - \sum_{i=1}^{N} \lambda_i \delta(r - r_i) \right) \right\} u_k(r) = F_k(r)
\]
(23)

where we have defined the source function, \(F_k(r)\), which depends on the values of the wave function \(u_k(r_i)\) at the points \(r_i\) where the delta shells are localized
\[
F_k(r) \equiv -\frac{2}{\pi} \sum_{i=1}^{N} \lambda_i u_k(r_i) \chi(r, r_i).
\]
(24)

If we assume that the values \(u_k(r_i)\) are known, the above equation \(23\) corresponds to a second-order ordinary differential equation (ODE) with a source term. Therefore \(u_k(r)\) can be written as the sum of a solution of the homogeneous ODE plus a particular solution of the whole ODE,
\[
u_k(r) = u_h(r) + u_p(r)
\]
(25)

where \(u_h(r)\) is a solution of the homogeneous equation
\[
\left\{ \frac{d^2}{dr^2} + \left( k^2 - \sum_{i=1}^{N} \lambda_i \delta(r - r_i) \right) \right\} u_h(r) = 0.
\]
(26)

The condition of regularity at origin imposes \(u_k(0) = 0\). The solution of eq. \(26\) outside anyone of the concentration radii \((r_i)\) can be written as in Eq. \(17\), i.e.
\[
u_h(r) = A_i \sin(kr + \delta_i)
\]
(27)

for \(r_i < r < r_{i+1}\) (\(i = 0, 1, \cdots, N\)).

A particular solution \(u_p(r)\) of eq. \(23\) is given by
\[
u_p(r) = \frac{1}{k} \int_0^r dr' F_k(r') \sin(k(r - r'))
\]
(28)

which can be proven by direct differentiation \([38]\). Therefore, we can write the complete solution of the Bethe-Goldstone equation \(23\) as the piecewise function
\[
u_k(r) = A_i \sin(kr + \delta_i) + u_p(r) \quad \text{if} \quad r_i < r < r_{i+1}
\]
(29)

Notice that \(u_p(r)\) vanishes at origin by construction, and that it is a continuous function because it is the integral of the product of two continuous functions. The regularity condition of \(u_k(r)\) at \(r = 0\) implies that the phase shift at origin \(\delta_0 = 0\).

By imposing the two conditions expressed in eqs. \(19\) and \(20\) at every point where the delta shells are localized, we can recursively relate the amplitude and phase shift of the wave function on the right of each delta shell with those of the wave function at the left of that delta shell. For the amplitudes the recurrence relation is again
\[
A_{i+1} = A_i \frac{\sin(kr_{i+1} + \delta_i)}{\sin(kr_i + \delta_i)}.
\]
(30)

The corresponding recurrence relation for the phase shifts is given by
\[
k \cot(kr_{i+1} + \delta_i) - k \cot(kr_i + \delta_i) =
\lambda_i \left( 1 + \frac{u_p(r_{i+1})}{A_i \sin(kr_i + \delta_i)} \right).
\]
(31)

It reduces to expression \(21\) when there is no medium and therefore \(u_p(r) = 0\), as it is the case for the free scattering problem.

Solving for \(\delta_{i+1}\) we get
\[
\delta_{i+1} = -kr_{i+1} + \frac{\arctan}{A_i k \sin(kr_{i+1} + \delta_i)} \left[ A_i k \cos(kr_{i+1} + \delta_i) + A_i \lambda_{i+1} \sin(kr_i + \delta_i) + \lambda_{i+1} u_p(r_{i+1}) \right]^{-1}
\]
(32)

Note that for the determination of the amplitudes \(A_i\) and phase shifts \(\delta_i\) of the BG solution, it is completely necessary to know the particular solution \(u_p(r_i)\) at the points \(r_i\) where the delta shells are located.

Therefore we cannot get rid of the self-consistency problem inherent to any solution of the B-G equation: to determine the whole solution we need some constants which have to be determined by means of recurrence relations provided the particular solution is already known;
and to determine the particular solution, eq. (23), we need to know the whole wave function $u_k(r_i)$, at the points $r_i$, in order to compute the source function $F_k(r)$ in eq. (24).

Here we solve this problem by carrying out an iterative procedure. For a given $k$ we take as starting points $u_k^{(0)}(r_i) = \sin(kr_i)$ to compute the source function and the particular solution. We then calculate the amplitudes $A_i$ and phase shifts $\delta_i$ to obtain the first-iteration $u_k^{(1)}(r_i)$ of the BG solution. With this complete solution, we calculate again the source function and iterate the procedure a number $N$ of times up to achieving convergence for the $N$-th iteration $u_k^{(N)}(r_i)$.

Because the BG equation is linear, its solution is determined up to a normalization constant. In our approach, in each iteration the value of $A_0$ is the global normalization constant that is fixed by imposing the long-distance condition

$$u_k(r) \rightarrow \sin(kr), \quad r \rightarrow \infty. \quad (33)$$

That is, the correlated wave function of the nucleon pair, $u_k(r)$, approach the free relative wave function, $\sin(kr)$, at large distances.

A convergence criterion for the iterations can be derived from the solution, Eq. (24), for $r > r_N$. In fact by expanding the particular solution in terms of sines and cosine functions, we get

$$u_k(r) = A_N \sin(kr + \delta_N) + u_p(r)$$

$$= \sin(kr) \left[ A_N \cos(\delta_N) + \frac{1}{k} \int_0^r \int_0^r d'F_k(r') \cos(kr') \right]$$

$$\quad + \cos(kr) \left[ A_N \sin(\delta_N) - \frac{1}{k} \int_0^r d'F_k(r') \sin(kr') \right]$$

$$\rightarrow \sin(kr) \quad \text{when} \quad r \rightarrow \infty. \quad (34)$$

The above expansion allows us to identify the coefficients of $\sin(kr)$ and $\cos(kr)$ for $r \rightarrow \infty$, thus providing the convergence conditions

$$\frac{1}{k} \int_0^{\infty} d'F_k(r') \cos(kr') = 1 - A_N \cos(\delta_N) \quad (35)$$

$$\frac{1}{k} \int_0^{\infty} d'F_k(r') \sin(kr') = A_N \sin(\delta_N) \quad (36)$$

In each iteration we check if the solution of the B-G equation simultaneously verify eqs. (35) and (36) within a given accuracy. This is our convergence criterion, which of course depends on "where" we have defined the large distances behavior. In our case we have set it to $r_{\text{max}} = 50$ to $100$ fm and we have found that our results do not depend on this choice. Typically we need less than six iterations to obtain convergence for all the NN potentials considered in this work.

F. High-momentum components of the pair wave function

Once the Bethe-Goldstone has been solved in coordinate space it is straightforward to compute the high-momentum components of the correlated pair. We start by writing the BG equation for the relative wave function of a back-to-back correlated pair with total spin $S=0$, in the form

$$|\psi_k\rangle = |k\rangle + \int d^3p \theta(p - k_F) \frac{1}{2\mu - \frac{p^2}{2\mu}} |p\rangle \langle V| \psi_k\rangle \quad (37)$$

where $k < k_F$. Thus in presence of the medium the interacting pair acquires high momentum components given by the integrand in the above equation, namely, for $p > k_F$, 

$$\langle p|\psi_k\rangle = \frac{1}{2\mu - \frac{p^2}{2\mu}} |p\rangle \langle V| \psi_k\rangle. \quad (38)$$

in this equation a general multipole expansion can be done. Here we study the particular case of the $S$-wave contribution. The $l=0$ partial wave of the BG wave function is written as

$$\psi_{l=0}(r) = \frac{1}{(2\pi)^{3/2} k r} u_k(r) \quad (39)$$

where $u_k(r)$ is the BG solution in $S$-wave obtained in the last section, Eq. (24). Note that here we are using the same normalization as the plane wave $|k\rangle$ which for $l=0$ is

$$\int 1 \frac{e^{ik\cdot r}}{(2\pi)^{3/2}} = \frac{1}{(2\pi)^{3/2}} \frac{\sin(kr)}{kr}. \quad (40)$$

Therefore the matrix element of the NN potential between the correlated $S$-partial wave and the high-momentum state, $p > k_F$, is

$$\langle p|V|\psi_{l=0}\rangle = \int d^3r \frac{e^{-ip\cdot r}}{(2\pi)^{3/2}} \sum_i \frac{\lambda_i}{2\mu} \delta(r-r_i) \frac{1}{(2\pi)^{3/2}} \frac{u_k(r)}{kr} \quad (41)$$

where we have introduced the delta-shell potential in $S$-wave as a sum of delta functions. The angular integral selects the $l=0$ component of the plane wave through the general relation, for any radial function $f(r)$,

$$\int d^3r e^{-ip\cdot r} f(r) = 4\pi \int dr r^2 j_0(pr) f(r) \quad (42)$$

where $j_0(pr) = \sin(pr)/pr$ is a spherical Bessel function. Integrating over the radial coordinate using the Dirac delta functions we obtain

$$\langle p|V|\psi_{l=0}\rangle = \frac{4\pi}{(2\pi)^3} \sum_i \frac{\lambda_i \sin(pr_i)}{2\mu} \frac{u_k(r_i)}{k} \quad (43)$$
Using this result in Eq. (38) we obtain the high-momentum components of the $S$-wave correlated back-to-back pair in the analytical form

$$\langle p|\psi_{l=0}\rangle = \frac{4\pi}{(2\pi)^3} \frac{1}{k^2 - p^2} \frac{1}{pk} \sum_i \lambda_i u_k(r_i) \sin(pr_i)$$  \hspace{1cm} (44)$$

In the next section we present plots of the high-momentum radial wave function, $\tilde{\Phi}_k(p)$, defined as

$$\langle p|\psi_{l=0}\rangle = \frac{\tilde{\Phi}_k(p)}{k} Y_0(\tilde{p})$$  \hspace{1cm} (45)$$

while the high-momentum pair density will be proportional to the square $|\tilde{\Phi}_k(p)|^2$.

### G. G-matrix

The G-matrix is defined by

$$V|\psi_k\rangle = G|k\rangle.$$  \hspace{1cm} (46)$$

From Eq. (43) it is straightforward to obtain the G-matrix element for a back-to-back nucleon pair in the $^1S_0$ channel, as

$$G(p, k) = \langle p|G|^1S_0|k\rangle = \frac{\sum_{i=1}^{N} \lambda_i u_k(r_i) \sin(pr_i)}{2\pi^2 M_N pk}$$  \hspace{1cm} (47)$$

Note that this expression has been obtained for $p > k_F$, and then analytically extended to the $p < k_F$ branch. This matrix element quantifies the short-range correlations in the initial back-to-back nucleon pair state with relative momentum $k$, allowing a transition to a state with momentum $p$. As it is known, the diagonal elements $G(k, k)$ for $k \leq k_F$ contribute to the nuclear binding energy, whereas the off-diagonal elements with $k \leq k_F$ and $p > k_F$ correspond to induced high momentum components above the Fermi level produced by the NN interaction.

### III. NUMERICAL RESULTS

We come to our numerical results. In this initial exploratory study we are interested in the properties of the solution of the B-G equation for several NN potentials in $S$-wave. In particular we compare several coarse-grained potentials with the results obtained with the AV18 interaction [12]. In a previous study [22] we performed a coarse graining of the AV18 potential for $p_{max} \sim 2$ fm$^{-1}$ with $N = 5$ delta shells separated by $\Delta r \sim 0.5$ fm. Here we extend the analysis to higher energies. We adjust the strength of the delta shells below $r_{max} = 3$ fm in order to reproduce the phase-shifts of the AV18 potential for the $^1S_0$ partial wave, up to some maximum LAB kinetic energy $E_{max}$. By increasing $E_{max}$ we find that the number of delta shells incorporated has to be accordingly increased.

In figure 1 we show the fine grained version of the AV18 [12] potential for the $^1S_0$ partial wave compared to the coarse grained potentials for the same partial wave with 4, 5, 6 and 7 delta shells, respectively (these potentials which will be denoted as GR4, GR5, GR6 and GR7 respectively). The values of the strengths of the delta shells potentials can be read from table I. In each case, the values of $\lambda_i$ have been adjusted to reproduce the same phase shifts as those of the AV18 potential up to a certain kinetic energy in LAB, which is increasing with the number of delta shells between 0 and 3 fm. The AV18 potential has been sampled at 600 equally spaced points between 0 and 6 fm ($\Delta r = 0.01$ fm), and then the equivalent strengths of the 600 delta shells have been calculated.

It can be seen from figure 1 that the strength of the first delta shell (second row) increases when the number of delta shells increases, or equivalently when the distance between consecutive shells decreases. This is necessary in order to reproduce the same phase shifts as those of the AV18 with a small number of delta shells. The strength of the first delta shell mimics the repulsive properties of the NN potential at short distances. When we increase the number of deltas, we reveal the short-distance behavior of the potential, and the relevance of the repulsive short-distance region becomes more and more important.

All the potentials shown in figure 1 give the same real phase shifts up to a certain CM momentum or kinetic energy in LAB system. This can be observed in figure 2, where the real phase shifts for the $^1S_0$ neutron-proton (np) partial wave are plotted for all the potentials considered in this work. The delta shell strengths have been adjusted to reproduce the same phase shifts as the AV18 potential. The adjusted region in LAB energy is increased with the number of delta shells. For instance, for a potential with 4 delta shells (DS), we have fitted the phase shifts up to $T_{LAB} = 250$ MeV; while for 6 DS, the fitted region ranges up to $T_{LAB} = 921$ MeV. It can also be seen that a potential with 7 DS between 0 and 3 fm reproduces the phase-shifts of the AV18 potential up to almost $T_{LAB} = 2$ GeV. For completeness we also show in figure 2 the imaginary part of the phase-shifts which differs substantially from zero above $\Delta$ production threshold, $p_\Delta \sim 550$ MeV, whereas it is exactly zero in the case of the AV18 and GR potentials.

In figure 3 we plot the solution, $u_k(r)$, of the BG equation for two values of the relative momentum of the nucleon pair. These results have been obtained using the GR7 potential corresponding to 7 DS. From the figure we observe that the BG wave function and the free one are almost identical because the effects of the potential get damped at large distances. The BG solution do not present any phase-shift because the presence of the medium prevents the scattering into already occupied states with $k < k_F$. The effect of the interaction is to distort the wave function at short distances, producing a bending with rapid oscillations of small amplitude. These small, fast oscillations are a direct signal of the presence of high-momentum components and the leading
FIG. 1: Comparison between the fine grained AV18 potential [12] for the \(^1\)S\(_0\) partial wave with 600 delta shells and the corresponding coarse graining with 4, 5, 6 and 7 (from left to right and top to bottom, respectively) equally spaced delta shells between 0 and 3 fm.

TABLE I: Strengths \(\lambda_i\) of the delta-shell (DS) potential (in fm\(^{-1}\)) fitted to reproduce the same phase shifts as the AV18 potential [12] for the \(^1\)S\(_0\) partial wave. The number of DS in the range 0 – 3 fm increases from left to right. In each column the separation (\(\Delta r\)) between consecutive delta shells is also displayed.

| \(\lambda_i\) (fm\(^{-1}\)) | 4 DS (\(\Delta r = 0.75\) fm) | 5 DS (\(\Delta r = 0.6\) fm) | 6 DS (\(\Delta r = 0.5\) fm) | 7 DS (\(\Delta r = 0.43\) fm) |
|-----------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| \(\lambda_1\)              | 0.19                          | 1.39                          | 4.64                          | 9.82                          |
| \(\lambda_2\)              | -0.636                        | -0.81                         | -1.011                        | -0.937                        |
| \(\lambda_3\)              | 0.019                         | -0.13                         | -0.231                        | -0.433                        |
| \(\lambda_4\)              | -0.044                        | -0.023                        | -0.081                        | -0.107                        |
| \(\lambda_5\)              | -                             | -0.028                        | -0.022                        | -0.052                        |
| \(\lambda_6\)              | -                             | -                             | -0.018                        | -0.018                        |
| \(\lambda_7\)              | -                             | -                             | -                             | -0.016                        |

The spikes are due to the singular delta shell potential which introduces discontinuities in the derivative at the points \(r_i\). One can also observe the oscillatory and amplitude-decreasing behavior of the defect wave function, making the BG solution to oscillate around the free solution with decaying amplitude for long distances.

feature of short-range correlations. The bending of the wave function at short distances can be better appreciated in the lower panels of fig. 3. The rapid oscillations can be observed as “spikes” in the BG wave function and in the defect wave function defined by

\[ \Delta u_k(r) = u_k(r) - \sin(kr). \] (48)
From our results we can also obtain the value of the healing distance, defined as the point where $\Delta u_k(r)$ first vanishes [52]. It is almost independent on the relative momentum of the pair. By inspection of the lower panels of figure 3 it is $\approx 0.75$ fm. This value for the healing distance is smaller than the one of a hard-core potential at $r_c = 0.4$ fm, which is $\approx 1.34$ fm [48, 52]. This makes sense because in a hard-core potential at $r_c = 0.4$ fm the wave function is forced to be zero at $r_c$ and not at $r_0 = 0$ as here. Therefore, one would expect the necessary distance to heal to the unperturbed wave function to be larger for the hard-core potential case.

In the upper panels of figure 3 we also show the solution $u_k(r)$ for the homogeneous ODE (24), and the medium effect or particular solution $u_p(r)$ of eq. (28). It can be seen that both solutions are shifted with respect to the free wave function and that they have different amplitudes, but in such a way that their sum has no phase shift with respect to the free solution at large distances.

In figure 4 we show the correlation function, defined as the quotient between the correlated and uncorrelated wave functions

$$f_{\text{corr}}(r) = \frac{u_k(r)}{\sin(kr)},$$

for two different relative momenta of the nucleon pair, $k = 40$ and $k = 140$ MeV/c, respectively. We only show the short-distance region region before the first zero of the uncorrelated wave function $\sin(kr)$, reached for $r = \pi/k$. While there is no phase-shift between both wave functions, this is the case for very large distances only. In figure 4 it can be seen that the deviations from unity of the correlation function primarily occur at very short distances, for $r < 0.5$ fm, where it takes small values. For larger values, around 1 fm, the correlated wave function heals and shows a trend to oscillate around the unity with damping amplitude. Thus the correlation function approaches oscillatory to unity.

In figure 5 we show the high-momentum radial $^1S_0$ wave function $\tilde{\Phi}_k(p)$ of the back-to-back correlated pair, for relative momenta $k = 40$ and $k = 140$ MeV/c, respectively, computed with several coarse grained potentials, and compared to the AV18 potential. The square of this function will contribute to the high-momentum tail induced by the short-range correlations in the momentum distribution.

From the figure we observe that, above the Fermi momentum, which has been fixed in this work to $k_F = 250$ MeV/c, and below $p = 400$ MeV/c, all the potentials basically give the same high-momentum tail. Moreover the results between $p = 400$ and 600 MeV are very similar, and above $p = 600$ their differences start to be more pronounced. Going back to figure 2 we can see that a common feature of these potentials is that they produce the same phase-shifts for LAB energy below 250 MeV. From these results, we conclude that the phase-shift information above 250 MeV in kinetic LAB energy is irrelevant to properly describe the high-momentum tail up to 400 MeV/c.

A similar case arises when comparing the results of the potentials GR6, GR7 and AV18. These three interactions give essentially the same high-momentum tail up to 1000 MeV/c. When looking at figure 3 one can observe that these three potentials describe the same phase

\[0\text{ MeV}\]
FIG. 3: Top panels: Comparison between the reduced wave functions $u_k(r)$ for two different ($k = 40$ MeV/c on the left panel and $k = 140$ MeV/c on the right one) initial relative momenta of the nucleon pair. We use the GR7 interaction corresponding to 7 DS potential with parameters given in the last column of table I. The curves correspond to the Bethe-Goldstone wave function $\sin(kr)$, the free wave solution $\sin(kr)$, the medium correction or particular solution $\Delta u_k(r)$, the homogeneous wave function given by eq. (27) and, finally, the defect wave function, which is defined as $\Delta u_k(r) = u_k(r) - \sin(kr)$. Bottom panels: detail of the BG, free and defect functions in the region below 5 fm. Note that the BG and the free wave functions are almost identical except for short distances and they cannot be disentangled in the upper panels. Their difference can be appreciated in the bottom panels for short distances.

shifts up to 1000 MeV in LAB energy, thus suggesting again that additional information contained in the phase-shifts beyond that energy has no influence in the high-momentum tail of the correlated wave function below $p = 1000$ MeV/c.

A closer view of the differences between the high momentum tail induced by the different interactions is provided in figure 3, where we show the square of the momentum wave function $|\tilde{\Phi}_k(p)|^2$, plotted in logarithmic scale. This function is proportional to the high momentum distribution of the back-to-back nucleon pair in relative $^1S_0$ state, where the well known minimum at $\sim 400$ MeV/c, produced by the node of the wave function, is apparent.

An important quantity for the assessment of the short range correlations is the value of the correlation function at the origin, provided in our formalism by the constant $A_0$

$$A_0 = f_{corr}(0) = \lim_{r \to 0} \frac{u_k(r)}{kr}$$

In figure 4 we show the values of $A_0$ for the different potentials considered in this work. These values are plotted against the initial relative momentum $k$ of the back-to-back nucleon pair. In our approach the value of $A_0$ is computed in each iteration by imposing the condition that the correlated wave function $u_k(r)$ goes to the uncorrelated one $\sin(kr)$ for large values of $r$. The values plotted in fig. 4 correspond to the 6th iteration to solve the BG equation, after convergence is reached. The importance of these curves lies in the fact that they correspond to the values that should be given as the initial
guess in the iterative procedure to reach convergence in only one iteration.

By inspection of figure [we see that for all the potentials considered here, the $A_0$ values are quite stable in the whole range of relative momenta $k$ up to the Fermi momentum, $k_F = 250$ MeV/c. There is only a small enhancement at the end of the curves when one is approaching the Fermi momentum.

The value $A_0$ measures the hardness of the potential at short distances. The larger the value of $\lambda_1$ (the amplitude of the first repulsive delta shell) the smaller the value of $A_0$, and the smaller the wave function at short distances.

It is instructive to separate in Fig. 8] the contribution of the attractive and repulsive parts of the potential to the high momentum tail of the wave function. For the coarse grained potentials, the first delta shell is positive and repulsive, while the others are negative and attractive. According to Eq. (44) the potential enters into the momentum tail through the sum $\sum_i \lambda_i u_k(r_i) \sin(pr_i)$. We thus can separate this sum into two terms including only the repulsive $\lambda_i > 0$ and attractive $\lambda_i < 0$ parts of the potential.

The results of this separation are shown in Fig. 9 for $k = 200$ MeV. As can be seen in the case of the AV18 potential, where $V(r) > 0$ for $r < 0.75$ fm and $V(r) < 0$ otherwise, the repulsive short-distance piece including the core gives a positive and large momentum tail, which is partly compensated by the negative contribution stemming from the attractive longer range potential. This effect is reproduced with the coarse grained GR7 potential which is equivalent, as shown before, to the AV18 potential both for the phase-shift and for the high-momentum wave function.

However, this feature depends strongly on the resolution scale $\Delta r$. In the bottom panel of the figure we observe that, in the case of the GR4 potential, the repulsive contribution is much smaller than the attractive contribution which gives alone almost the same total momentum tail as the AV18 or GR7 potentials. In fact most of the high-momentum tail is here dominated by the second, attractive delta located at $r = 1.5$ fm. This remarkable feature suggests that the short-range correlations can be traced back primarily not to the traditional repulsive core phenomenology but rather to the attractive mid-range part of the interaction. This opens a gateway to a perturbative treatment of short-range correlations which will be exploited elsewhere.

Our analysis also explains a feature which has been systematically found in large scale calculations, namely the appearance of an universal diffraction minimum at $p = 400$ MeV in the $^1S_0$ momentum distribution [55]. Although this is known to be produced by the mid-range part of the interaction and not to the repulsive core, we believe its diffractive origin can be understood in simple terms; our analysis shows that it is due to the fact that in the GR4 potential the high-momentum tail in the wave function is dominated by the second delta shell, located at $r_2 = 1.5$ fm. According to Eq. (44) the contribution in the $S$-wave is proportional to $\sin(pr_2)$ which vanishes for $p = \hbar c \pi / r_2 \approx 400$ MeV.

Let us mention that the momentum representation given in Eq. (44), suggests a simple way of parameterizing the high momentum components of the wave function. The short range correlations are encoded into the quantities $c_i(k) \equiv \lambda_i u_k(r_i), i = 1, \ldots, N$, which in principle could be parameterized with smooth functions which do not depend strongly on $k$. This could open an interesting line of research to investigate if information about these “coarse-graining correlation functions”, $c_i(k)$, could be experimentally extracted for example in two-nucleon knock-out experiments such as $(e,e'NN)$.

The coarse-grained approach allows us to compute easily the $G$-matrix in the $^1S_0$ channel using Eq. (47). In Fig. 9 we show the momentum space representation $G(k', k)$ for fixed momentum $k = 200$ MeV/c, as a function of $k'$. As we see the AV18 and GR7 potentials give...
essentially the same $G$-matrix up to $k' = 1000$ MeV/c. The GR4 potential agrees well with both of them for $k' \leq 500$ MeV $= 2k_F$, and start to considerably differ for $k' \sim 600$ MeV/c.

It is a striking result that, being the three potentials (AV18, GR7 and GR4) so different in momentum space, they generate essentially the same $G$-matrix for momenta below $2k_F$. This is because the $G$-matrix embodies the short-range correlations through the already mentioned coarse-grained correlation functions $c_i(k) = \lambda_i u_k(r_i)$. As we have seen the information encoded in these functions can be separated into repulsive and attractive contributions that, in the average, are very similar for all the potentials considered here.

A deeper insight is provided in fig. 10 where we show the separate repulsive and attractive parts of the NN potential in momentum space $V(k', k)$, and its corresponding contributions to the $G$-matrix. In the case of the AV18 potential the repulsive and attractive $G$-matrices are large and opposite in sign and partially cancel in the total $G$-matrix. The same can be said in the case of the GR7 potential, that gives essentially the same repulsive and attractive $G$-matrices as the AV18 potential.

However the GR4 potential provides a different case, where the repulsive contribution to the $G$-matrix is negligible, while the attractive part of the potential generates alone almost the full $G$-matrix. Note also that in the case of the AV18 and GR7 interactions, $G(k', k)$ is very dif-
For momenta below $\sim k_F$, one expects that the diagonal $G$-matrix, $G(k,k)$, to be identified with the effective NN interaction inside the nucleus, be also quite stable for all the interactions. This is shown in Fig. 11, where we compare $G(k,k)$ and the NN potential $V(k,k)$ for the three interactions AV18, GR7 and GR4 in the $^1S_0$ channel. We have multiplied these functions by a normalization factor so that these curves can be compared with the usual definition of the $V_{\text{lowk}}$ potential [22, 54], which has already been proposed as a convenient effective interaction in the nucleus, and that in fact is very similar to our results in figure 11.

Finally, in figure 12 we show the diagonal $G$-matrix
computed for \( k = k_F \) as a function of the Fermi momentum, in a range that covers the Fermi momentum of finite nuclei, nuclear matter, and extends well above the \( k_F \) values expected for neutron stars. The \( G \)-matrix close the Fermi surface is related to the effective interaction of the Landau-Migdal finite Fermi systems, and it should describe the particle-hole interaction near the Fermi surface. The GR4 potential give the same results as the ones of GR7 and AV18 for \( k_F < 350 \text{ MeV} \), while they start to disagree for larger \( k_F \)-values.

**IV. CONCLUSIONS**

In this work we have explored the role of coarse grained interactions in the study of short-range correlations in nuclei, in terms of the high momentum components in the back-to-back nucleon pair wave functions. We remind that these components have traditionally been linked to the existence of a strong repulsive core between nucleons located at about \( r_c \sim 0.3 - 0.4 \text{ fm} \), preventing any kind of perturbative treatment, and promoting large scale calculations to befit the complexity of the problem. On the other hand, for a maximum back-to-back momentum \( p \) there corresponds a limiting resolution wavelength \( \Delta r \) which effectively samples the nuclear wave function as well as the corresponding NN interaction above the repulsive core scale \( r_c \). Therefore, we expect some cancellation between the repulsive and attractive parts of the interaction in the region around the core for \( pr_c \lesssim 1 \), which means \( p \lesssim 600 \text{ MeV} \). Previous experience fitting and selecting NN data in high precision partial wave analysis suggests that this “moderately” high momentum and core-blind regime might exist. Conversely, this also seems to imply that genuine repulsive-core features start being visible for \( p \gtrsim 600 \text{ MeV} \).

Therefore, as a first step in this exploratory work, we have addressed the problem of solving the Bethe-Goldstone equation in the \( ^1S_0 \) partial wave for several delta-shell potentials, including the fine-grained version of the AV18 potential. The strengths of the delta-shell potentials have been fitted to reproduce the same phase shifts as the AV18 up to a certain kinetic energy in the LAB frame.

We have first developed a very efficient method to solve the Bethe-Goldstone equation in coordinate space for this \( ^1S_0 \) channel, obtaining convergence for the reduced wave function \( u_k(r) \) after six iterations. With this solution we have studied the role of short-range correlations to induce high momentum components in the two-nucleon wave function. We have also obtained two-nucleon momentum distributions for this \( S \)-wave component and we have analyzed the values \( A_0 \) of the radial wave function at the origin. We have compared our coarse-grained results with those obtained with AV18 potential, which embodies many of the traditionally assumed features of the NN interaction, such as the repulsive core, and provides a good description of NN phase-shifts up to np LAB energy of 2 GeV.

We find a few appealing features of the present approach.

- A huge reduction in the number of mesh points needed to solve the Bethe-Goldstone equation in coordinate space. In the case of the AV18 we need to sample the interaction with 600 deltas (fine grain) to solve it, while in the coarse-grain case we just need a few mesh points (4–7) with almost the same
FIG. 8: Separation of the attractive and repulsive high momentum components of the wave function for $k = 200$ MeV/c for different NN potentials.

FIG. 9: $G$-matrix in the $^1S_0$ channel for a back-to-back pair with initial relative momentum $k = 200$ MeV/c. We compare the results corresponding to the three potentials AV18, GR4 and GR7.

result. The coarse-grain interactions embodies all needed information on high momentum components of the nuclear wave function up to $p = 600–1000$ MeV, while accurately reproducing the AV18 $^1S_0$ phase-shifts up to LAB energies of 2 GeV. We plan to investigate this feature in more involved problem of the peripheral partial waves, including the effects of the tensor interaction in the $S = 1$ channel.

- We also witness a marginal contribution of the repulsive core so that, quite unexpectedly, a perturbative treatment of short-range correlations as observed in terms of high momentum components is envisaged. This is due to a suitable choice of the coarse graining scale which allows a dominance of the attractive contribution due to a reduction of the repulsive one.

- We provide a simple explanation of the universally observed diffraction minimum in the high momentum distribution in the $^1S_0$ channel.

A pertinent comment triggered by figure 2 is that the input is dictated by the phase-shifts, but it is not clear up to which maximum energy they should be described by the potential to solve the Bethe-Goldstone equation without ambiguities. The reason is because above 500–600 MeV in relative momentum, inelasticities due to production of nucleon resonances start to contribute, producing a complex phase shift which cannot be described with a real potential.

Future plans include to solve the problem for higher partial waves, including coupled-channels induced by the nucleon-nucleon potential. This formalism could also be useful in the construction of $G$-matrices once the higher partial waves have been addressed.
FIG. 10: G-matrix in the $^1S_0$ channel for a back-to-back pair with initial relative momentum $k = 200$ MeV/c (left panels). We compare the results corresponding to the three potentials AV18, GR4 and GR7. In the right panels we compare corresponding potential matrix element in momentum space. In all cases we also show the separate contribution from the repulsive and attractive parts of the potential.

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[1] R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989).
[2] E. Ruiz Arriola, Symmetry 8, no. 6, 42 (2016).
[3] P. Ring and P. Schuck The nuclear many-body problem, (2004) Springer Science
FIG. 11: Upper panel: diagonal G-matrix (multiplied by $2\pi^2 M_N$) in the $^1S_0$ channel for a back-to-back pair with initial relative momentum $k = 200$ MeV/c. We compare the results corresponding to the three potentials AV18, GR4 and GR7. Bottom panel: the same for the potential matrix element in momentum space.

FIG. 12: Diagonal G-matrix (multiplied by $2\pi^2 M_N$) in the $^1S_0$ channel for a back-to-back pair with $k = k_F$ as a function of $k_F$. We compare the results corresponding to the three potentials AV18, GR4 and GR7.
[36] K. Niewczas and J. T. Sobczyk, Phys. Rev. C 93 (2016) no.3, 035502.
[37] U. Mosel and K. Gallmeister, Phys. Rev. C 94 (2016) no.3, 034610.
[38] T. Van Cuyck, N. Jachowicz, R. González-Jiménez, M. Martini, V. Pandey, J. Ryckebusch and N. Van Dessel, Phys. Rev. C 94 (2016) no.2, 024611.
[39] L. B. Weinstein, O. Hen and E. Piasetzky, Phys. Rev. C 94 (2016) no.4, 045501.
[40] C. Marchand et al., Phys. Rev. Lett. 60 (1988) 1703 Erratum: [Phys. Rev. Lett. 60 (1988) 2704].
[41] C. J. G. Onderwater et al., Phys. Rev. Lett. 81 (1998) 2213.
[42] K. I. Blomqvist et al., Phys. Lett. B 421 (1998) 71.
[43] K. S. Egiyan et al. [CLAS Collaboration], Phys. Rev. Lett. 96 (2006) 082501.
[44] R. Shneor et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 99 (2007) 072501.
[45] R. Acciarri et al. [ArgoNeuT Collaboration], Phys. Rev. D 90 (2014) no.1, 012008.
[46] F. Cavanna, O. Palamara, R. Schiavilla, M. Soderberg and R. B. Wiringa, [arXiv:1501.01983 [nucl-ex]].
[47] A. L. Fetter and J. D. Walecka, J. D., Quantum theory of many-particle systems. 2003. Dover books.
[48] J. D. Walecka, Theoretical Nuclear and Subnuclear Physics, Oxford University Press, Inc. (1995).
[49] E. Werner, Nucl. Phys. 10 (1959), 688-697.
[50] H. A. Bethe and J. Goldstone, Proc. Roy. Soc. Lond. A 238 (1957) 551.
[51] R. A. Arndt, I. I. Strakovsky and R. L. Workman, Int. J. Mod. Phys. A 18 (2003) 449.
[52] A. L. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill Publishing Company, Inc., (1971).
[53] R. B. Wiringa, R. Schiavilla, S. C. Pieper and J. Carlson, Phys. Rev. C 89 (2014) no.2, 024305.
[54] S.K. Bogner, T.T.S. Kuo, A. Scwenk, Phys. Rep. 386 (2003), 1.