Positron Bunch Radiation in the System of Tightly-Packed Nanotubes

Koryun Gevorgyan1,3*, Lekdar Gevorgian2 and Hayk Gevorgyan1,2

1 Yerevan State University (YSU), 1 Alex Manoogian, Yerevan, 0025, Armenia
2 A.Alikhanyan National Laboratory (Yerevan Physics Institute), Br. Alikhanyans 2, Yerevan, 0036, Armenia
3 CANDLE Synchrotron Research Institute, 31 Acharyan, Yerevan, 0040, Armenia

E-mail: koryun.tsv@gmail.com

Abstract. The problem of channeling radiation of positron bunch in the system of packed nanotubes was investigated in the present work. Used the model of harmonic potential which is justified since on the one hand the number of positrons in the region near the axis of nanotube is small, and on the other hand their contribution to the formation of the total radiation is also small. The problem is solved in the dipole approximation. The radiation at first harmonic occurs at zero angle too. At zero angle are radiated both extremely hard and extremely soft photons due to the medium polarization. The frequency-angular distribution of number of emitted photons was received. The distribution does not depend on the azimuthal angle, since the task has cylindrical symmetry. Radiation at the zero angle is fully circularly polarized. For formation of radiation there is an energy threshold: lower threshold is due to the polarization of medium, the upper threshold depends on the oscillation amplitude of channelling positrons. When the bunch energy coincides with the upper threshold then in radiation contribute all channeled positrons. Each positron in average radiates one photon. Thus is formed intensive, quasi-monochromatic and circularly polarized X-ray photon beam which may have important practical application.

1. Introduction

In 1947, V.L. Ginzburg developed the radiation theory of relativistic oscillating electrons [1]. Under the leadership of Motz the experimental studies were conducted to detect of the radiation of relativistic electrons passing through periodic field generated by magnets (undulator) [2]. It was found high intensity radiation in the millimeter range. Such intensity was obtained due to coherent radiation of individual bunch which longitudinal dimension is smaller than the radiation wavelength [3]. As was shown in [4] for shorter wavelengths regardless of the radiation type the coherence factor can exceed unity when the electron distribution is asymmetrical in the longitudinal direction. The effect of partially coherent radiation of bunch was detected in the experimental work [5]. The coherent radiation of asymmetrical bunch will increase the efficiency of free electron laser (FEL) [6]. The gain of FEL as has been shown by Madey depends on the shape of the spontaneous emission [8]. Undulator spontaneous radiation in the X-ray frequency range was investigated by Korkhmazyan [9] and the experiment was carried out on Yerevan's accelerator [10] to detect this radiation. In the formation of X-ray undulator radiation has a significant role the medium polarization [11]. The radiation is
generated when the bunch energy is greater than the threshold energy. When the energy is close to the threshold then the frequency-angular spectrum is narrowed i.e. the emitted photon density increases [12]. X-ray FEL was observed recently in the experiment of SASE FEL [13]. Crystal can perform the role of peculiar microunstudentor for the channeled charged particles. As a result [14-15] of numerical modeling of the process when fast electrons penetrate into monocrystal it has been observed that in certain crystal orientations mean free path of ions increases abnormally (channeling). The phenomenon of channeling has been observed experimentally in [16-17] and was explained by Lindhard in the work [18] where the true potential of the crystal was replaced by the continuous potential averaged over atom coordinates. The theory of radiation channeling of charged particles has been developed by Kumakhov [19]. This topic has been the subject of much theoretical and experimental works [20]. The oscillation frequency of channeled particles in the crystalline or nanotube resonators with the harmonic potential depends on the particle energy. In the case of nonharmonic potential this frequency depends on the oscillation amplitude also [21]. In periodically curved crystals, besides channeling radiation, the undulator radiation is also formed due to the periodicity of average trajectory of particles [22]. The characteristics of the particle radiation, generated in a crystalline undulator, were investigated in [23]. The spontaneous and stimulated radiation in the crystalline or nanotube undulators has been studied taking into account the medium polarization [24]. Due to centrifugal force [25] the process of dechanneled positrons does not occur if the maximum curvature angle of crystalline undulator smaller than Lindhard angle [26].

In this paper we got the spectral distribution of total radiation of channeled positrons at normal incidence to the tightly-packed nanotube system.

2. The trajectory of a channeled positron in the nanotube potential well

Let a positron bunch having transverse uniform distribution moves parallel to the axis of the tightly-packed nanotubes. Then approximately 90 percent of positrons are channeling inside nanotube [21].

It has been shown that the potential of the form \( V(s) = U_0 s^6 \) is in good agreement with the calculated potential, where \( U_0 \) is the depth of the potential well in energy units, \( s = r/R \) (0 ≤ s ≤ 1), \( r \) is initial distance from the axis of the nanotube, \( R \) is the nanotube radius. In this work the trajectories of channeled positrons have been calculated. We are interested in the total spectrum of the radiation intensity of all channeled positron of bunch.

The choosing of potential type \( V(s) \) conditioned by that it also agrees with calculated potential also for the small values of \( s \). However, in this case, on the one hand it is complicate calculation of the spectral intensity and on the other hand is lost the monochromaticity of radiation.

Therefore, the contribution of positrons with small \( s \) to the total spectrum can be neglected, since both their number and emission intensity (\( \sim s^2 \)) are small.

In this paper, in order to find the spectral distribution of the radiation intensity of channeled positrons, we will use a harmonic potential, which will provide the radiation monochromaticity:

\[
U(s) = U_0 s^2.
\]

In the such potential well of nanotube the positrons are oscillate with the same frequency \( \Omega_{ch} \):

\[
\Omega_{ch} = \frac{\alpha_0}{\sqrt{\beta}}, \quad \Omega_0 = \frac{c\sqrt{e\beta}}{R}, \quad \nu = \frac{U_0}{m\nu^2},
\]

where \( \gamma \) is the relativistic Lorentz factor, \( c \) is a speed of light, \( m \) is a mass of the positron, \( \Omega_0 = 2\pi c/\lambda_0 \) and \( \lambda_0 = 2\pi R/\sqrt{2\nu} \) are the natural frequency and the spatial period of nanotube.

The trajectory of the positron with the initial coordinates \((s, \psi)\) has the following form:

\[
\vec{s}(s, \psi, z) = s\hat{e}_\psi \cos(k_{ch}z) + \frac{z}{R}\hat{e}_z, \quad k_{ch} = \frac{\alpha_{ch}}{\beta_z},
\]

where \( \hat{e}_\psi \) and \( \hat{e}_z \) are basis vectors in the transverse and longitudinal direction, \( \beta_z \) is the average longitudinal velocity in units of the light speed \( c \).
3. The radiation field of channeled positrons

The radiation field of the channeled positrons produced in the nanotube with length $L$, is represented by the following integral over trajectory:

$$
\vec{E}_{s,\psi}(\omega, \theta, \varphi) = \int_{-L/2}^{L/2} \{\vec{n} \times \vec{\beta}(s, \psi)\} \exp\{i\left[\frac{\omega}{\beta_{ch}c} z - \vec{k}(\theta, \varphi) \cdot \vec{r}(s, \psi)\right]\} dz,
$$

(4)

where $\vec{k}(\theta, \varphi) = \frac{\omega}{c} \vec{e}_{\psi}$ is the wave vector in the direction of wave vector $\vec{e}(\theta, \varphi, \varphi)$, $\theta$ and $\varphi$ are polar and azimuthal angles of radiation. The dielectric constant of the medium $\varepsilon(\omega)$ has the following dependence on the frequency $\omega$ which is much higher than the plasma frequency of medium $\omega_p$:

$$
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}.
$$

(5)

When the bunch positrons oscillates in nanotubes and the oscillation number $n$ is the large value:

$$
n = \frac{L}{l_{ch}} = \frac{L}{l_0 \sqrt{\gamma}} = \frac{n_0}{\sqrt{\gamma}} \gg 1,
$$

(6)

then the radiation field is different from zero only for small values of the argument of the exponential function in the integrand expression (4).

We use the following expansions in the argument of the exponential function in (4):

$$
\beta_{ch} = 1 - \frac{Q(s)}{2\gamma^2}, \quad Q(s) = 1 + \mu s^2, \quad \mu = \nu \gamma,
$$

(7)

$$
\sqrt{\varepsilon} = 1 - \frac{\omega_p^2}{\omega^2}, \quad \cos \vartheta = 1 - \frac{\vartheta^2}{2}.
$$

Then after integration we obtain

$$
\vec{E}_{s,\psi}(\omega, \theta, \varphi) = \frac{1}{2} \sqrt{\frac{2\pi}{\gamma}} \left[\vec{e}_\varphi \times \vec{e}_z\right] - \frac{\omega}{\beta_{ch}} \frac{\gamma^2}{\omega} \cos(\varphi - \psi) \left[\vec{e}_\varphi \times \vec{e}_z\right]
$$

$$
\frac{\sin(n_0 \gamma)}{\gamma}
$$

(8)

$$
Y = \frac{\pi \omega}{2 \alpha n_0} \left(\gamma^2 + \frac{Q(s)}{\gamma^2} + \frac{\omega_p^2}{\omega^2} \frac{2\Omega_p}{\omega_0 \sqrt{\gamma}}\right).
$$

4. The spectral distribution of emitted photons

For the frequency-angular distribution of the number of emitted photons we have

$$
\frac{dN_{ph}(s,\psi)}{dxdu^2d\varphi} = \alpha x^2 \left[1 - 2u^2 \cos^2(\varphi - \psi) + u^2 \cos^2(\varphi - \psi)\right] \frac{\sin^2(n_0 \gamma)}{\gamma^2},
$$

(9)

$$
Y = \frac{\pi x}{2 \sqrt{\gamma}} \left[u^2 - \varphi_s(x)\right], \quad \varphi_s(x) = \frac{1}{x} - Q(s) - \frac{r}{x^2}, \quad r = \frac{\gamma_0}{\gamma}, \quad \gamma_0 = \left(\frac{\lambda_p}{\lambda}\right)^2
$$

(10)

where $\alpha = e^2/\hbar c$ is a fine structure constant, $x = \omega/(\Omega_0 \gamma^{3/2})$ is the dimensionless frequency, $u = \theta \gamma$ is the radiation angle in units $1/\gamma$, $\lambda_p$ is the plasma wavelength.

Accurate to the small order $1/n_0$ ($n_0 \gg 1$), we can use the following known limit:

$$
\lim_{n_0 \to \infty} \frac{\sin^2(n_0 \gamma)}{\gamma^2} = \pi n_0 \delta\left(u^2 - \varphi_s(x)\right).
$$

(10)

After substituting (10) into (9) and integrating over the azimuthally angle $\varphi$ from zero to $2\pi$ we obtain the following expression:

$$
\frac{dN_{ph}(s)}{dxdu^2} = K\left[1 + (1 - x^2)^2\right] \delta\left(u^2 - \varphi_s(x)\right), \quad K = \frac{\pi \alpha n_0 \sqrt{\gamma}}{2},
$$

(11)
which is independent of coordinate $\psi$ due to cylindrical symmetry of the problem. The frequency distribution of the number of emitted photons is represented by the following expression:

$$
\frac{dN_{\text{ph}}(s)}{dx} = K \left[ 1 + \left( 1 - x \varphi_s^2(x) \right)^2 \right] = K s^2 \left[ f_0(x) + 2\mu f_1(x) s^2 + \mu^2 x^2 s^4 \right]
$$

(12)

$$
f_0(x) = 1 + \left( 1 - x - \frac{\gamma}{x} \right)^2, \quad f_1(x) = x^2 - x - r.
$$

The radiation of the channeled positron with an initial coordinate $s$ occurs in the frequency range:

$$
\frac{1 - \sqrt{1 - (1 + \mu^2 s^2)r}}{1 + \mu s^2} \leq x \leq \frac{1 + \sqrt{1 - (1 + \mu^2 s^2)r}}{1 + \mu s^2}.
$$

(13)

5. Energy threshold depending on the oscillation amplitude

The frequency range depends on the bunch energy. There is an energy threshold for the formation of radiation: $\gamma_{\text{th}}(s) = (1 + \mu s^2)\gamma_0 = Q(s)\gamma_0$ depending on the oscillation amplitude $s$. The expression (13) implies that the radiation is produced when the bunch energy of positrons is larger than the energy threshold value ($\gamma \geq \gamma_{\text{th}}(s)$), and for the formation of the radiation it is necessary that the bunch energy is larger than the value $\gamma_0$. This threshold is due to the medium polarization. When $\gamma = \gamma_0$ the radiation doesn’t formed because positrons with amplitude $s = 0$ don’t radiate. There is an amplitude threshold. And finally for $\gamma_{\text{th}}(s)$ we have:

$$
\gamma_{\text{th}}(s) = \gamma_0(1 - \mu_0 s^2)^{-1}, \quad \mu_0 = \nu\gamma_0,
$$

(14)

$$
\gamma_0 = \gamma_{\text{th}}(0) < \gamma_{\text{th}}(s) \leq \gamma_{\text{th}}(1) = \gamma_1 = \gamma_0(1 - \mu_0)^{-1}.
$$

The larger the amplitude of positron oscillation in the channel, the greater the threshold energy. The frequency range expands with increasing of $\gamma$. When the bunch energy is greater than the energy value $\gamma_1$, then all positrons of the bunch contribute to the radiation.

a) In the energy range $\gamma \in (\gamma_0, \gamma_1)$ contribute not all positrons of bunch to the radiation. The positrons with the oscillation amplitude less than $s$, emit at frequencies $x_T$ which are defined by the equation:

$$
x_T = \frac{1 + \sqrt{1 - (1 + \mu^2 s^2)r}}{1 + \mu s^2}.
$$

(15)

This behavior is a consequence of the fact that the oscillator in a dispersive medium at a given angle radiates both hard and soft photons. Irrespective of the sign of the root in (15), the amplitude $s$ depends on the frequency $x$ as follows:

$$
s(x) = \sqrt{\frac{\varphi_0(x)}{\mu}}, \quad \varphi_0(x) = \frac{2}{x} - 1 - \frac{r}{x^2}.
$$

(16)

The function $s(x)$ takes maximum value $\sqrt{1/\mu_0 - 1/\mu}$ at the frequency $x = r$. When $\gamma = \gamma_1$ we have $\mu = \mu_1 = \mu_0/(1 - \mu_0)$ (s = 1). When the parameter $\mu_0 \ll 1$, which is typical for nanotube, the emission spectrum is formed in the frequency range (accurate to small order $\mu_0$):

$$
1 - \sqrt{\mu_0} < x < 1 + \sqrt{\mu_0}.
$$

(17)

The all positrons of bunch with energy $\gamma_1$ radiate in this frequency range. As follows from (17), the frequency spectrum has a width $\sqrt{\mu_0}$ up to a small order $\mu_0$. The width of angular spectrum is $\sqrt{\mu_0}/\gamma_0$.

b) The bunch energy is in the range $(\gamma_1; \gamma_c)$, where $\gamma_c = \sqrt{\gamma_0/2}$. In this case the range of emitted frequencies is divided into three intervals: $[x_-(0); x_-(1)]$, $[x_-(1); x_+(1)]$ and $[x_+(1); x_+(0)]$, where the boundary frequencies are:
\[ x_\pm(0) = 1 \mp \sqrt{1 - \frac{y_0}{y}}, \quad x_\pm(1) = \frac{1}{1+\mu} \left( 1 \mp \sqrt{1 - \frac{y_1}{y}} \right). \] (18)

The boundary frequencies \( x_\pm(1) \) of the second interval, coinciding when \( y \leq y_1 \), differ from each other with increasing \( y \). The width of this interval achieves the maximal value \( 2 - \sqrt{2\mu_0} \) when \( y = y_c \). It should be noted that the boundary frequency \( x_-(1) \) is gradually reduced with increasing \( y \) (\( y \leq y_c/\sqrt{2} \)) and the frequency \( x_+(1) \) increases. The width of the third interval \([x_+(1); x_+(0)]\) is increased by law \( 2\mu/(1+\mu) \).

6. The frequency spectrum of the bunch radiation of the channeled positrons

Integrating the expression (12) for the parameter \( s \) in the range from \( 0 \) to \( s(x) \), we obtain the spectral distribution of the total number of photons emitted by all channeled positrons in nanotube:

\[
\frac{dN_{\text{tot}}}{dx} = KF(x), \quad K = \frac{\pi a n_n a \nu n_b}{2},
\]

\[
F(x) = f_0(x) \frac{s^5(x)}{5} + 2\mu f_1(x) \frac{s^4(x)}{4} + \mu^2 x^2 \frac{s^3(x)}{3},
\]

\[
s(x) = \begin{cases} 
\frac{1}{\mu} \left( \frac{x^2}{\mu} - 1 - x \right), & \text{for } x \in \left[x_-(0); x_-(1)\right] \cup \left[x_+(1); x_+(0)\right] \\
1, & \text{for } x \in \left[x_-(1); x_+(1)\right].
\end{cases}
\]

We investigate the spectral distribution of radiation of channeled positrons with the bunch energy \( E \in [E_0, E_1] \). For the frequency distribution of emitted photons, taking into account that \( \mu = \nu r \ll 1 \) and \( f_0(x) \approx 2 \), we have following expression:

\[
F(x) = \frac{2}{3} \left( \frac{1}{\mu} \left( \frac{x^2}{\mu} - 1 - x \right) \right)^{3/2}, \quad r = \frac{y_0}{y},
\]

\[
1 - \sqrt{1 - r} \leq x \leq 1 + \sqrt{1 - r}.
\]

The maximum of spectrum takes place at the frequency \( x = r \). We note that in this case the frequency boundaries \( x_-(1) \) and \( x_+(1) \) coincide. The radiation generated around at the frequency \( x = 1 \). The spectrum expands and its intensity increases when the value of parameter \( y \) is approaching to \( y_1 \).

We obtain the total spectrum of emitted photons at all frequencies. With an increase of the bunch energy the radiation spectrum is expanding.

We shall find the threshold value of the bunch energy for nanotube having the radius \( R = 7 \) Å, natural spatial period \( l_0 = 3.93 \cdot 10^{-5} \) cm and the potential well depth \( U_0 = 32 \) eV \((\nu = 6.26 \cdot 10^{-5})\). If the polarization of nanotube medium is characterized by the plasma wavelength \( \lambda_p = 4 \cdot 10^{-6} \) cm or \( h\omega_p = 31 \) eV, then for the energy thresholds we have: \( E_0 = 49.3 \) MeV, \( E_1 = 49.63 \) MeV or \( \gamma_0 \approx 96.48, \gamma_1 \approx 97 \).

In the Figure 1 shows the frequency distributions of radiation for two values of the bunch energy.

The maximum of spectra correspond to the frequencies \( r_b = 0.996 \) and \( r_a = 0.9946 \) and are equal to \( F(r_b) = 0.36 \) and \( F(r_a) = 0.76 \). Using the positron bunch with the energy \( E = E_a = 49.63 \) MeV in the system of nanotubes with length \( L = 1 \) cm generated the monochromatic circularly polarized photon beam with energy 3 KeV and the line width 0.08. As a result for the number of radiated photons we have \( N_{\text{ph}} \approx 0.06 N_b \), where \( N_b \) is the positron number.
7. CONCLUSION
The problem about the channeling radiation of the positron bunch in nanotube is solved in the dipole approximation. The dipole approximation legitimately is applied for the large values of the oscillation parameter also, if the emission occurs at very small angles.

The main contribution into the radiation makes the first harmonic of radiation which is formed at a zero angle too that it is important from the practical point of view. The soft photons are emitted at a zero angle also because of the medium polarization. The numbers of soft and hard photons are of the same order of magnitude. There are two energy thresholds, namely, the lower threshold of the medium polarization and the upper threshold for positrons oscillating in the channel with the maximum amplitude: the higher the initial radial coordinate of a positron, the greater the threshold energy. If the bunch energy is equal to the maximum value of the threshold energy, then the all channelled positrons contribute to the radiation in nanotube.

Hence, the monochromatic beam of high energy photons with circular polarization is formed in nanotube due to the cylindrical symmetry. The radiation beam of soft X-ray photons has important practical significance.

References
[1] Ginzburg V L 1947 Izv. Akad. Nauk. SSSR Ser. Fiz. 11 165
[2] Motz H 1951 J. Appl. Phys. 22 527
[3] Motz H, Thon W and Whitehurst R N 1953 J. Appl. Phys. 24 826
[4] Korkhmanian N A, Gevorgian L A and Petrosian M L 1977 Zh. Tekh. Fiz. 47 1583
[5] Ishi K, Shibata Y, Takahashi T, Hasebe S, Ikezawa M, Takami K, Matsuyama T, Kobayashi K and Fujita Y 1995 Phys. Rev. E 51 R5212
[6] Gevorgian L A and Zhevago N K 1982 Coherent radiation of electron bunches in a free electron
laser Dokl. Akad. Nauk SSSR 267(3) 599

[7] Madey J M and Deacon D A 1977 Cooperative effects in matter and radiation ed Bouden C M and Hougate D W (New York) p 317

[8] Deacon D A G, Elias L R, Madey J M J, Raman C J, Schwetman H A and Smith T I 1977 First Operation of a Free-Electron Laser Phys. Rev. Let. 38 892

[9] Korkhmazian N A 1970 Izvestya Akad Nauk Arm SSR Fizika 5 287
Korkhmazian N A 1970 Izvestya Akad Nauk Arm SSR Fizika 5 418

[10] Alikhania n A I, Esin S K, Ispirian K A and et al 1972 Pisma Zh. Eksper. Teor. Fiz. 15 142
Ispirian K A and Oganesian A G 1971 Lectures on VII Intern. School on Exp. and Theor. Physics Yerevan Preprint YerPhI ME-4(71)

[11] Gevorgian L A and Korkhmazian N A 1979 Zh. Eksp, Teor. Fiz. 76 1226

[12] Gevorgian L A and Korkhmazian N A 1979 Hard Undulator Radiation in Dispersive Medium in Dipole Approximation Phys. Lett. 75A 453 (Copyright certificate № 784729)
Gevorgian L A and Korkhmazian N A 1977 Hard Undulator Radiation in Dispersive Medium in Dipole Approximation YerPhI Scientific Reports 273 66

[13] Emma P and et al 2009 Proc. FEL 397
Emma P and et al 2010 Nature Photonics 4 641
Emma P and et al 1998 LCLS Design Study Report SLAC-R-521

[14] Robinson M T and Oen O S 1963 Phys. Rev. 132 N5 2385
Beeler J R and Besko D G 1963 J. Appl. Phys 83 2873

[15] Lindhard J 1965 Dansk. Vid. Selsk. Mat. - Fys. Medd. 34 N14 30

[16] Piercy G R, Brown F, Davies J A and et al 1963 Phys. Rev. Let. 10 N4 399

[17] Lutz H and Sizmann R 1963 Phys. Lett. 5 N3 113

[18] KumakhoV M A 1976 On the theory of electromagnetic radiation of charged particles in crystal Phys. Lett. 57A 17

[19] Bazilev V A and Zhevago N K 1987 The radiation of fast particles in medium and in external fields (Moscow: Nauka Press)

[20] Gevorgian L A, Ispiryan K A and Ispiryan R K 1998 High energy particle channeling in nanotubes NIM B 145 155

[21] Kaplin V V, Plotnikov S V and Vorobiev S A 1980 Zh. Tekh. Fiz. 50 1079

[22] Korol A V, Soloviev A V and Greiner W 1999 Int. J. Mod. Phys. 8 49

[23] Avakian R O, Gevorgian L A, Ispiryan K A and Ispiryan R K 1998 Pisma Zh. Eksp. Teor. Fiz. 68 437
Avakian R O, Gevorgian L A, Ispiryan K A and Ispiryan R K 2001 Nucl. Instr. Meth. B 173 112

[24] Tsyganov E N 1976 Preprint FNAL-TM-682
Tsyganov E N 1976 Preprint FNAL-TM-684

[25] Gevorgian L A Proc. of SPIE vol 5974, ed Sultan Dabagov (Bellingham: WA) p 59740X-1

[26] Jackson J D 1999 Classical Electrodynamics (New York: Wiley)