Research on Multi-objective Shared Parking Matching Decision Based on Two-side Matching

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Abstract. The purpose of this paper is to find a reasonable method for matching shared parking spaces. The travelers and the parking space sharing party are regarded as two separate subjects, and the matching of parking spaces is regarded as a two-side matching. The final parking space matching result belongs to the stable matching of the bipartite graph and the existence of Nash equilibrium is proved by the knowledge of graph theory. A multi-objective decision-making model for shared parking space matching based on two-side matching is established on the basis of considering the benefits of travelers and parking providers. The multi-objective model is transformed into a single-objective assignment model using weighting coefficients with adjustment parameters, and the corresponding solution algorithm is designed. Finally, an example is used to verify the rationality and feasibility of the established model. The results show that the parking space matching problem under sharing conditions belongs to the category of two-side matching. When the benefits of both parties are fully considered, the matching results obtained are relatively stable.

1. Introduction

At present, the phenomenon of parking difficulties is very common in many cities, which seriously restricts the development of social economy and people's normal travel and daily life. Finding a parking spot and walking to work often constitutes an appreciable fraction of the total travel time. If the free parking lots in residential areas during the daytime and commercial areas at night are reasonably shared and utilized, the problems of parking difficulties and traffic congestion will be alleviated. In recent years, as an effective measure to relieve parking pressure, shared parking has attracted the attention of scholars at home and abroad.

Through the research on the shared residential parking spaces between residents and public, it is found that these temporarily vacant parking lots can be efficiently utilized to meet the parking demand of other drivers who are working at nearby locations or drivers who come for shopping or other activities [1-2]. The reservation and income distribution plan for limited parking in urban areas is studied [3]. Studies have shown that shared parking spaces can reduce parking supply imbalances and reduce roadside parking congestion [4]. Based on the analysis of the vacant characteristics of parking spaces in residential areas, the shared service capabilities of parking spaces are evaluated [5]. The bi-level planning model is used to study the parking berth sharing strategy of colleges and universities in the central area [6]. An optimal allocation algorithm was designed to match the parking period that the residential parking space owner is willing to rent with the parking demand to improve the utilization of shared parking spaces [7]. Based on considering the personalized service and guidance service strategy of residents in residential area, the problem of shared parking allocation in residential area is studied [8].
A new multi-mode parking space matching algorithm is proposed to quickly reserve parking spaces under shared conditions [9]. According to the traveler’s preference, the allocation and pricing of parking spaces are studied [10]. Based on virtual parking berths, with the goal of improving berth utilization, an integer programming model is established to optimize the problem of berth supply and demand matching [11]. On the basis of considering the space and time conflict characteristics of parking space supply and demand, a multi-objective parking space allocation model is established and a particle swarm algorithm is designed to solve it [12]. On the basis of comprehensively considering the maximization of system benefits, the entropy weight method is introduced to study the allocation of shared berths among regional multiple parking lots [13]. With the goal of maximizing the profit of the shared platform, the problem of renting and distributing idle parking spaces is studied [14].

Judging from the above research status, most of the current research on shared parking focuses on the supply and demand capacity and berth allocation. In the research, the goal of maximizing platform revenue or high berth utilization is usually considered. It is failed to consider from the perspective of maximizing the benefits of both berth demanders and parking spaces. Therefore, based on state-of-art research, this paper uses the two-side matching theory to study the parking space matching problem based on the consideration of the benefits of both parties.

The remainder of this paper is organized as follows: Section II describes and defines the basic problem and proves that the matching result is a stable matching. The model establishment, simplification and algorithm design are described in Section III. An example is given to verify the correctness of the model and algorithm established above in Section IV, which is followed by the conclusion.

2. Basic problem description

Shared parking means that within a certain area, according to the difference of parking berth usage time between different users, berth sharing is adopted to alleviate the pressure of parking. A set of travelers $R = \{r_1, r_2, \ldots, r_i, \ldots, r_m\}$, for $i = 1, 2, \ldots, m$. Let a set of shared parking berth $P = \{p_1, p_2, \ldots, p_j, \ldots, p_n\}$, for $j = 1, 2, \ldots, n$. Any traveler $r_i (r_i \in R)$ have travel a OD pair $(o_i, d_i)$. Because the parking space at the destination is tight, a parking space reservation will be made before the traveler completes from $o_i$ to $d_i$. When there is no extra parking berth at the destination, the traveler will choose to be near area to find a satisfactory parking berth $p_j (p_j \in P)$, otherwise they will park illegally or travel without driving (assuming taxi travel). Due to the high cost of illegal parking, this paper considers that if you cannot find a satisfactory parking berth, you should take a taxi. If all travelers use the mode of "Driving + Sharing parking + Walking" to reach the destination, then the entire process can be represented by the following network figure 1.

![Figure 1 shared parking process network](image-url)

Through the above analysis, it is not difficult to find that passengers travel in the process of berth sharing $r_i$ and Shared berth $p_j$ belong to two sets of $r_i \in R$ and $p_j \in P$ to realize the parking reservation...
success, it must be under the condition of satisfy certain premise to find a satisfactory matching pair \((r, p)\), for \(r \in R, p \in P\). This happens to coincide with the matching of the bipartite graph. In this paper, the bipartite graph definition and stable matching theory are first used to prove that the matching problem of shared parking berth is a stable matching problem of bipartite graph, and the follow-up research is carried out.

Figure 2 Schematic diagram of travelers sharing berth matching

**Definition 1**: \(E(G) = \{ (r, p), r \in R, p \in P \}\) is a set of all the edge of the bipartite graph, \(M\) is the set of independent edges of stable matching of \(G\). If \((r_1, p_2) \in E(G) - M\), then there exist \(p_3 \in P\), have \((r_1, p_3) \in M\), namely the traveler \(r_1\) choose shared berth \(p_3\), more than \(p_2\) advantage, or exist \(r_2 \in R\), have \((r_2, p_2) \in M\), namely shared berth \(p_2\) selection an appointment with \(r_2\) is more advantageous than \(r_1\).

According to definition 1, if the traveler \(r_1\) does not choose the shared berth \(p_2\), then the traveler \(r_1\) will choose a more favorable berth \(p_3\), or the shared berth \(p_2\) will choose more suitable for traveler than \(r_2\), otherwise, the traveler \(r_1\) will choose shared berth \(p_2\) and complete the trip.

Suppose \(M \cup \{ (r_1, p_2) \}\) is a matching of graph \(G\), and \((r_1, p_2) \in E(G) - M\), explain the traveler \(r_1\) cannot choose parking berth. Shared parking berth \(p_2\) is not chosen to park as well. In this case, the stable matching \(M\) is a maximum matching, not perfect matching. In the process of berth sharing, in order to meet the travel, purpose of travelers and at the same time alleviate the parking pressure, this paper assumed that the traveler \(r_1\) must choose a parking berth, and a car must park at the shared parking berth \(p_2\). The matching result of shared parking berth obtained is a perfect matching. Therefore, the matching process of passengers sharing berth can be defined as follows:

**Definition 2**: Suppose bipartite graph \(G = G(m, n)\), for \(R = \{ r_1, r_2, \cdots, r_m \}\) is a set of traveler, and \(|R| = m\), \(P = \{ p_1, p_2, \cdots, p_n \}\) is a set of Shared parking berth, and \(|P| = n\), \(u\) is a set of matching of graph \(G\), then the matching has the following properties: 1. \(r_w \in R, p_k \in P\), \((r_w, p_k) \in u\), and \(r_k, p_w\) can find better shared parking berth and the travelers of matching object in \(u\). Therefore, the matching \(M\) of the constructed bipartite graph \(G\) is a stable matching, if and only if \(M\) has no interrupt pairs.

**Proposition 1**: The stable matching of the traveler shared berth matching bipartite graph is Nash equilibrium.

**Proof**: It is assumed that there exists a stable match in graph \(G\), which is composed of traveler set and Shared parking berth set, and \(M\) has no interrupt pairs. According to the definition of stable matching, if the traveler \(r_1\) does not choose a parking berth \(p_2\) then \(r_1\) would definitely choose a parking berth better than \(p_2\), or a parking berth \(p_2\) will choose a traveler who is more favorable than \(r_1\), otherwise \(r_1\) will choose \(p_2\) to park. Namely: for any \((r_1, p_2) \in M\), \(r_1\) and \(p_2\) cannot exist at the same time better matching object. Nash equilibrium means that in the game process, a favorable strategy combination will be selected by each participant when the strategy space of other players is given [15]. This is identical to the exact matching given above in the bipartite graph. Thus, it can prove that the stable matching of the bipartite graph of shared parking berth established in this paper is Nash equilibrium, which can be used to study the shared parking matching of travelers. Problem Analysis and Modelling.
Shared parking space matching is a multi-scheme, multi-attribute and multi-objective complex problem involving berth demand side (i.e., traveler) and berth provider and shared platform. Different traffic participants will judge and select suitable matching objects according to different income evaluation, and finally achieve the matching between berth demanders and berth sharer to complete travel tasks. In this paper, the platform income is not considered for the time being, and the two-side matching decision model is established with the goal of the berth demand side and the berth supply side. The detailed steps are as follows.

2.1. Income Analysis and Quantification between Supply and Demand

2.1.1. Demand side benefits of berths
The traveler will choose to travel by taxi if the traveler \( r_i \) fail to match the parking space, as shown in figure 3. Remember \( C^0 \) travel cost when the traveller \( r_i \) fails to match the parking space and \( C^0 = t_i \times \alpha \). The travel cost \( C^0 \) when the traveler \( r_i \) successfully matched to the shared parking space \( p_j \), the \( C_{ij} \) is recorded as the travel cost when the traveller \( r_i \) successfully matched to the \( p_j \) of the shared parking space, and is record as \( C_{ij} = \alpha_1 t_{ij} + \alpha_2 w_{ij} + \alpha_3 t_{ij}' \). The \( t_{ij} \) indicates that the traveler \( r_i \) travel time by taxi, the \( t_{ij}' \) indicates that the traveler \( r_i \) the travel time driving to the shared parking space \( p_j \), and the walking time \( w_{ij} \) the traveler from the shared parking space \( p_j \) to the destination \( d_j \), \( \alpha_1 \), \( \alpha_2 \), \( \alpha_3 \) represent the cost coefficient of taxi, driving, parking and walking respectively.

![Figure 3: Traveler's travel time](image)

Therefore, this paper regards the demand-side revenue of berths as the cost savings after the successful matching of travelers to parking berths, that is: \( u_{ij} = C^0 - C_{ij} \).

2.1.2. Revenue from berth providers
The berth provider (parking berth owner), if not shared, will pay a portion of the fixed fee (such as parking rental fee, purchase fee and cleaning fee, etc.) Remember \( C^0 \) charge to the berth provider when the \( p_j \) does not share, if the berth provider chooses to share the parking space, it will receive a certain parking fee, which is recorded as \( C'_{ij} \) parking fee when the parking space is shared to the traveler \( r_i \) and \( C'_{ij} = \alpha_2 w_{ij} \).

Therefore, we consider the benefit of the berth provider as the cost savings after the successful matching of the shared parking berth to the traveler, notes: \( v_{ij} = C^0 - C'_{ij} \).

2.2. Model of Parking Space Sharing Matching
According to the above analysis, when the parking space of the travel destination is in short supply, the traveler stops by booking the matching parking space in advance, and then completes the travel task by walking to the destination. At the same time, the traveler chooses the satisfied traveler to make the matching reservation according to his own income. In the whole process, the goal of the participants is to maximize their respective benefits, set \( x_{ij} \) as decision variables, where \( x_{ij} \in \{0,1\} \); if the traveler \( r_i \) successfully matches the shared parking space \( p_j \) then \( x_{ij}=1 \), otherwise \( x_{ij}=0 \). This paper assumes that the shared parking space can meet the needs of the present situation, that is \( m \leq n \), according to the theory of two-side matching, the multi-objective model is established as follows:
\[ \text{max} \quad Z_R = \sum_{i=1}^{m} \sum_{j=1}^{n} u_{ij} x_{ij} \]  
\[ \text{max} \quad Z_P = \sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij} x_{ij} \]  
\[ \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, m \]  
\[ \sum_{i=1}^{m} x_{ij} \leq 1, \quad j = 1, 2, \ldots, n \]  
\[ x_{ij} \in \{0, 1\}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \]

The formula (1) indicates that all travelers save the most cost; formula (2) indicates that all shared parking berths save the most cost; formula (3) indicates that each traveler can find a satisfactory shared parking berth in the process of sharing berth matching; formula (4) indicates that each berth holder finds up to one satisfied traveler in the process of sharing berth matching; formula (5) indicates that the \( x_{ij} \) is 0-1 variable, that is, the traveler \( r_i \) successfully match with the shared parking space \( p_j \), then \( x_{ij} = 1 \), otherwise \( x_{ij} = 0 \).

2.3. Model Solution and Algorithm

The benefits of both the traveler and the shared parking space provider in the above model are cost. If the objective function \( Z_R \) and \( Z_P \) are combined according to certain rules, and then make \( m = n \), the model can be represented as follows:

\[ \text{max} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \]  
\[ \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, m \]  
\[ \sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, 2, \ldots, n \]  
\[ x_{ij} \in \{0, 1\}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \]

Taking into account the fact that the complementarity of cost coefficient \( u_{ij} \) and \( v_{ij} \) in the original objective function will lead to the bias of matching results to one side, the optimization model which considers the complementarity and consistency of each objective coefficient and has adjustment parameters is proposed in [16], the optimization model proposed in reference \( c_{ij} = \lambda (u_{ij} + v_{ij})/2 + (1 - \lambda) \sqrt{u_{ij} v_{ij}} \). The above model belongs to the generalized assignment model \((m \leq n)\), can be constructed into a standard assignment model and solved by Hungarian algorithm, the detailed solution steps are as follows:

**Step 1:** Virtual \( n - m \) travelers based on the number of travelers and shared parking spaces and their cost factors \( u_{ij} = 0, v_{ij} = 0 \).

**Step 2:** Give the cost matrix \((u_{ij})_{n \times n}\) and \((v_{ij})_{n \times n}\) of both sides according to the income of traveler and berth.

**Step 3:** The \((c_{ij})_{n \times n}\) of cost coefficient matrix under different adjustment coefficients is calculated according to \( c_{ij} = \lambda (u_{ij} + v_{ij})/2 + (1 - \lambda) \sqrt{u_{ij} v_{ij}}, 0 \leq \lambda \leq 1 \), given calculation formula of cost coefficient.

**Step 4:** Since this model is the maximum of the objective function and the minimum of the objective function in the standard assignment model, the \( c_0 = \text{max}\{c_{ij}\} \), reconstructs the cost coefficient matrix \((c'_{ij})_{n \times n}\), of which \( c'_{ij} = c_0 - c_{ij} \).

**Step 5:** Use the MATLAB to design the Hungarian algorithm to solve the assignment model of cost coefficient as matrix \((c'_{ij})_{n \times n}\), and get the matching result and target value \( Z' \).

**Step 6:** Calculate the objective function value \( Z = nc_0 - Z' \) of the original model.

3. Numerical experiments

In this section, we conduct numerical experiments to illustrate the models and results and try to gain some useful insights. There are 6 travelers from different regions, denoted as \( r_1, r_2, \ldots, r_6 \). They will go
to 6 different destinations in a nearby area, and the destination does not have enough parking spaces. They need to choose a satisfactory parking from the 7 nearby shared parking spaces and walk to the destination. The detailed parameters are shown in Table 1.

| Travelers | Taxi travel time $t_{ij}/min$ | Driving time to parking lots $t_{ij}/min$ | Walking time $t_{ij}/min$ |
|-----------|-------------------------------|------------------------------------------|--------------------------|
| $r_1$     | 45                            | 25                                       | 18                       |
| $r_2$     | 40                            | 25                                       | 18                       |
| $r_3$     | 38                            | 26                                       | 18                       |
| $r_4$     | 42                            | 20                                       | 18                       |
| $r_5$     | 40                            | 18                                       | 18                       |
| $r_6$     | 48                            | 35                                       | 18                       |

Assuming that the cost coefficients for different travelers taking taxis, driving, parking, and walking are known, and the various coefficients for different travelers are the same, they are $\alpha = 1.2$ yuan/min, $\alpha_1 = 0.5$ yuan/min, $\alpha_2 = 0.05$ yuan/min, $\alpha_3 = 0.2$ yuan/min, parking time $w_{ij} = 600 + 2t_{ij}$, virtual traveler $r_7$, and make its income $u_i = v_i = 0$. Thus, the income matrices $(u_{ij})_{n \times n}$ and $(v_{ij})_{n \times n}$ of pedestrians and shared berth providers are calculated.

In order to illustrate the influence of linear weighting, multiplier weighting, and adjustment parameters on the matching results of parking space sharing, the following cases are classified and compared.

**Case 1:** Let $c_{ij} = u_{ij}$, and calculate the calculation example according to the algorithm given above;

**Case 2:** Let $c_{ij} = v_{ij}$, and calculate the calculation example according to the algorithm given above;

**Case 3:** Let $c_{ij} = (u_{ij} + v_{ij})/2$, and calculate the calculation example according to the algorithm given above;

**Case 4:** Let $c_{ij} = u_{ij}v_{ij}$, and calculate the calculation example according to the algorithm given above;
Case5: Let $c_{ij} = \lambda(u_{ij} + v_{ij})/2 + (1 - \lambda)\sqrt{u_{ij}v_{ij}}$, take different $\lambda$ range is $(0 \leq \lambda \leq 1)$, and perform calculation examples according to the algorithm given above.

Table 3 Analysis of matching results in different situations

| Situation   | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | $r_6$ | Objective function value $Z$ |
|-------------|-------|-------|-------|-------|-------|-------|-----------------------------|
| Case1       | $p_5$ | $p_3$ | $p_7$ | $p_1$ | $p_4$ | $p_6$ | $p_2$ | 80.500                      |
| Case2       | $p_2$ | $p_7$ | $p_5$ | $p_6$ | $p_3$ | $p_1$ | $p_4$ | 28.100                      |
| Case3       | $p_5$ | $p_3$ | $p_7$ | $p_1$ | $p_4$ | $p_6$ | $p_2$ | 52.750                      |
| Case4       | $p_3$ | $p_5$ | $p_2$ | $p_1$ | $p_6$ | $p_4$ | $p_5$ | 368.560                     |

Available from Table 4, The matching result of the product of the revenue of both two-side coefficients is different from the results when the traveler or parking space owner is considered separately. However, relatively stable matching results appeared in the pairwise comparison. Compared with Case3, 3 sets of results were stable, and compared with Case2, 1 set of results was stable. Compared with case 3, the bias of the results is weakened when only one party's income is considered. Therefore, comprehensively consider the complementarity and consistency of the product in the linear weighting of the returns of both parties to analyze the results. In order to obtain a satisfactory matching result, it is necessary to comprehensively consider the complementarity and consistency of the product in the linear weighting of both sides.

Since $(u_{ij} + v_{ij})/2 \geq \sqrt{u_{ij}v_{ij}}$, the objective function should show an upward trend with the increase of the adjustment parameter $\lambda$. According to the all matching results in Case5, the objective function value increases almost linearly with the $\lambda$ increase. When $0 \leq \lambda \leq 0.6$, the matching result remains
unchanged and is exactly the same as the matching result of Case4, the result is \( (r_1, p_7), (r_2, p_3), (r_3, p_5), (r_4, p_1), (r_5, p_4), (r_6, p_6), (r_7, p_2) \). At the same time, when \( 0.6 \leq \lambda \leq 1 \), the matching results of Case3 and Case3 are the same, which is \( (r_1, p_5), (r_2, p_3), (r_5, p_7), (r_4, p_1), (r_5, p_4), (r_6, p_6), (r_7, p_2) \). Analyzing the matching results of the adjustment parameter \( a \) in Case5 in different value ranges, it is found that the matching result will change in a small range with the change of the adjustment parameter value. However, the matching result is stable, and reasonable adjustment parameters are selected. A satisfactory matching result can be obtained. In the same way, through the verification of large-scale calculation examples, the stability of the matching result is better as the number of matching parties’ increases.

From the calculation example, it can be seen that the matching results of the shared parking space will be affected by the importance of different participants. A stable matching result that is satisfactory to both parties can be obtained when the interests of travelers and berth sharers are reasonably considered. At the same time, the stability of the result will be related to the value of the selected adjustment parameter and the scale of the problem.

4. Conclusion

The problem of parking space matching between travelers and parking providers under parking sharing conditions is studied in this paper. Firstly, the result of parking space matching composed of travelers and berth sharers is that the stable matching of the bipartite graph is proved by the stable matching theory of the bipartite graph, and it is proved that the stable matching is a Nash equilibrium. Then, a two-side matching multi-objective model is established, and the multi-objective problem simplification method of adjusting parameters is introduced at the same time. Finally, the matching results are finally obtained through the analysis of examples. The conclusions reached are as follows.

- For parking space matching under shared conditions, the matching results obtained when both the traveler and the parking space sharer are considered are different from the results obtained when only considering or focusing on the benefits of either party.
- The matching results of shared parking spaces will be seriously affected by the difference in the income of both parties. It is necessary to introduce different multi-objective model simplification coefficient processing methods, and fully consider the income of both parties to obtain a relatively satisfactory and stable matching scheme.
- The matching results of shared parking spaces will partially change with the change of revenue adjustment parameters, but it can ensure that the matching results of most travelers are stable, and the stability will increase as the number of participants increases.

In general, the shared parking space matching model based on the two-side matching theory established in this paper can be used to study the parking space matching problem between travelers and parking space sharers under the condition of parking space sharing. The time cost when the two parties choose each other in different situations is only considered and the benefits of the shared platform are ignored in this article. Factors such as multiple attributes and psychological avoidance of both parties will be considered in the further research, and a multi-objective solution algorithm will be designed to solve the problem.

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