Fast scrambling without appealing to holographic duality

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Motivated by the question of whether all fast scramblers are holographically dual to black holes, we study the dynamics of a non-integrable spin chain model composed of two ingredients - a nearest neighbor Ising coupling, and an infinite range XX interaction. Unlike other fast scrambling many-body systems, this model is not known to be dual to a black hole. We quantify the spreading of quantum information using an out-of-time-ordered correlator (OTOC), and demonstrate that our model exhibits fast scrambling for a wide parameter regime. Simulation of its quench dynamics finds that the rapid decline of the OTOC is accompanied by a fast growth of the entanglement entropy, as well as a swift change in the magnetization. Finally, potential realizations of our model are proposed in current experimental setups. Our work establishes a promising route to create fast scramblers.

Introduction: The dynamics of thermalization in closed quantum systems has received immense attention in recent years [1–9]. A central focus of these studies has been the “scrambling” of quantum information [10–20]. Scrambling is the process by which locally encoded information gets spread over non-local many-body degrees of freedom during the time evolution of a complex quantum system. This paradigm has been used to address a diverse array of questions in areas ranging from quantum chaos to quantum gravity [21–30]. Several recent experiments in a variety of analog quantum simulator platforms have successfully probed quantum scrambling [31–38], thereby paving the path to answer fundamental questions about non-equilibrium quantum dynamics.

Black holes are the fastest scramblers known in nature. Motivated by advances in holography, researchers have studied quantum many-body systems that can exhibit fast scrambling. Perhaps, the most celebrated example of this is the Sachdev-Ye-Kitaev (SYK) model [39–43]. This model describes $N$ Majorana fermions interacting via random infinite range interactions. The SYK model can be exactly solved in the large $N$ limit, where it is conjectured to be holographically dual to the Jackiw-Teitelboim model of gravity in two dimensions [44–46], and it can scramble as fast as a black hole in the low temperature limit [47]. However, the randomness in the long range interactions is not necessary to produce fast scrambling. As Bentsen et al. have demonstrated in a recent paper, a non-disordered spin model describing sparsely connected spin-1/2 particles, can also be a fast scrambler [48]. Their proposal was motivated by the p-adic version of the anti-de Sitter/conformal field theory correspondence [49]. While their model is very elegant, its experimental realization can be very difficult when the system size becomes large. Thus, it is necessary to search for alternative approaches to realize fast scrambling without disordered interactions. Furthermore, all of these works raises a crucial question: are all fast scramblers holographically dual to black holes?

In this letter, we address this issue by proposing a fast scrambling many-body model, that is not inspired by holography. Our model essentially comprises two ingredients - a short range Ising interaction, and an infinite range XX interaction. Both of these features are crucial since short range interacting systems can not be fast scramblers [50], while uniform infinite range interactions can not induce quantum chaos [51]. Although our model is integrable in certain limits, we show that there is a large parameter regime, where the system exhibits fast scrambling. Furthermore, such a vanilla model may be easier to realize experimentally, even for large system sizes. Our results suggest that an appropriate combination of short and long range interactions can lead to fast scrambling. While, it is not clear whether our model is dual to a black hole, we believe that our work can serve as a promising starting point to search for fast scrambling models that may not be dual to black holes.

We study information scrambling by studying the dynamics of an out-of-time ordered correlator (OTOC). In quantum chaotic systems, the growth of the OTOC at early times is exponential ($\sim e^{\lambda t}$), where $\lambda$ is bounded ($\lambda \leq 2\pi k_BT/h$) [29]. Moreover, the fast scrambling conjecture states that the time it takes for local information to be thoroughly scrambled in a $N$ body quantum system obeys a lower bound ($t \geq \log(N)/\lambda$) [27]. We identify a large parameter regime where our model behaves as a fast scrambler. We also find that in this regime, our model is non-integrable, and the entanglement entropy grows very fast.

Model and Spectral Statistics: We study a one dimensional spin chain with $L$ sites described by the following
FIG. 1. Model and Spectral statistics: (a) Schematic representation of the model in Eq. (1). The model is characterized by a nearest neighbor Ising coupling ($J_1$), a nearest neighbor $XX$ coupling ($J_1 \alpha$), and an infinite range $XX$ coupling ($J_2$). (b) The spectral statistics for our model when the total z-magnetization is 0, as characterized by the averaged ratio of adjacency gaps (defined in the main text). We conclude that there is a large parameter regime, where $\langle r \rangle \sim 0.53$, and the system is non-integrable. We find that $\langle r \rangle \sim 0.39$, only when $\alpha \sim 1$, and the model is integrable. Fast scrambling is only expected when the system in non-integrable, and thus we focus on the $\alpha = 0$ regime in this paper.

Hamiltonian:

$$H = J_1 \sum_{i=1}^{L} [\sigma_i^z \sigma_{i+1}^z + \alpha (\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+)]$$

$$+ J_2 \sum_{i=1}^{L} \sum_{j>i}^{L} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+)$$

where $\sigma_i^\pm = \frac{1}{\sqrt{2}}(\sigma_i^x \pm i \sigma_i^y)$, $\sigma_i^z$ is the standard Pauli matrix at lattice site $i$. A schematic of this model is shown in Fig. 1a. In accordance with realistic experimental realizations of the all-to-all interaction, we do not rescale $J_1$ by $1/L$. When $J_1 \to 0$, this model reduces to a form of the Lipkin-Meshkov-Glick model, and it is mean field solvable in the thermodynamic limit \[52, 53\]. On the other hand, When $J_2 \to 0$, the model maps to the well-known $XXZ$ model, that can be solved by employing the Bethe Ansatz \[54\].

While cousins of our model (with $J_1, J_2 \neq 0$) have been studied extensively \[55–61\], to the best of our knowledge, neither the equilibrium phase diagram, nor the non-equilibrium dynamics of this precise model has been studied before. Consequently, we discover a trove of rich non-equilibrium physics that arises from the interplay of nearest neighbor and infinite range interactions. We use exact diagonalization to study this model with open boundary conditions. The total $z$-magnetization ($M_z = \sum_{i=1}^{L} \sigma_i^z$) is conserved during the time evolution of this system, and we examine the $M_z = 0$ sector in this work.

An important diagnostic that is used to distinguish quantum chaotic systems from integrable systems is the energy level statistics \[02, 64\]. We exami-
ine the level statistics of this model by sorting the energy eigenvalues $E_1 < E_2 < E_3 < \ldots$, computing the adjacent energy gaps $\Delta E_n = E_{n+1} - E_n$, and then calculating the ratio of the adjacent energy gaps, $r_n = \min(\Delta E_m, \Delta E_{m+1})/\max(\Delta E_m, \Delta E_{m+1})$. Integrable systems are typically characterized by a Poisson distribution of $r_n$, i.e. $P(r) = 2/(1 + r)^2$, with a mean value of $\langle r \rangle \approx 0.39$. In contrast, thermalizing systems are characterized by Wigner-Dyson distribution of energy eigenvalues, i.e. $\langle r \rangle \approx 0.53$. Figure 1b shows the energy level statistics for our model, we take $\alpha = 0$ in the remainder of this paper. We note that a small finite $\alpha$ does not alter the results qualitatively.

**Out-of-time-order correlations:** Information scrambling is typically diagnosed by analyzing the dynamics of out-of-time ordered correlators (OTOCs). OTOCs capture the spreading of quantum information in a system by measuring operator growth. In particular, for two unitary and Hermitian operators $A$ and $B$, the operator commutator can be quantified by examining the expectation value of a squared commutator:

$$C(t) = \langle [A(t), B(t)]^2 \rangle = 2 - 2\text{Re}[\langle A(t)BA(t)B \rangle],$$

where $A(t) = e^{itH}Ae^{-itH}$, and the OTOC is $\langle A(t)BA(t)B \rangle$. For our model, we take $A = \sigma^z_i$, $B = \sigma^z_j$, and we compute the following OTOC:

$$F(j, t) = \frac{1}{2} \left( 1 + \text{Re}[\langle \sigma^z_i(t)\sigma^z_j(t)\sigma^z_i(t)\sigma^z_j(t) \rangle] \right).$$

We note that $F$ is a bounded function ($0 \leq F \leq 1$). In quantum chaotic systems, $F(t)$ decays exponentially at early times, i.e. $F(t) \approx e^{-\lambda_L t}$, where $\lambda_L$ is analogous to the Lyapunov exponent. A salient characteristic of fast scramblers is that $F(j, t)$ starts deviating significantly from 1, at a time $t^* \propto \log(j)$. In order to compare the properties of our model to other fast scramblers that have previously been studied in the literature, we compute $F(j, t)$ for the infinite temperature state, where

$$\langle \sigma^z_i(t)\sigma^z_j(t)\sigma^z_i(t)\sigma^z_j(t) \rangle = \mathbb{D}^{-1} \text{Tr}[\sigma^z_i(t)\sigma^z_j(t)\sigma^z_i(t)\sigma^z_j(t)],$$

and $\mathbb{D}$ is the Hilbert space dimension. Our results for a 16 site chain is shown in Fig. 2b. It is clear from these figures that fast scrambling occurs in a wide parameter regime when $J_2/J_1 \sim O(1)$. As detailed in the supplementary material, we can obtain an analytical understanding of this behavior using a short-time expansion.

While all the numerical results that we have discussed so far are exact, our study has been limited to small system sizes. To overcome this limitation, we study the spin-$S$ version of our model, in the $S \to \infty$ limit, where it can be analyzed semi-classically. Following Ref. [68], we compute the averaged sensitivity:

$$C_{cl}(j, t) = \frac{1}{S^2} \left( \frac{dS_j^z(t)}{d\phi} \right)^2,$$

where $\phi$ is a small initial rotation of spin 1 about the $z$-axis. This quantity can capture the sensitivity to initial conditions in classical systems, and it can be derived from Eq. (2) by substituting the commutator with appropriate Poisson brackets. In order to compute the infinite temperature OTOC, we evaluate Eq. (5) for an ensemble in which each spin is initially aligned in a random direction. Similar to the $S = 1/2$ scenario, even in this case, fast scramblers exhibit exponential growth of $C_{cl}$, such that $C_{cl}(j, t^*) \sim 1$, at time $t^* \propto \log(j)$. We perform our calculation for a 100 site model. Our results are shown in Fig. 2b, for the case when $J_1 = J_2$. Our semi-classical calculations confirm that our model can exhibit fast scrambling.

**Quench Dynamics:** While the infinite temperature OTOC dynamics provides compelling evidence for fast scrambling in our model, preparing this state in an experiment can be challenging. To alleviate this concern, we also study quench dynamics of this system for different initial states in the $M_z = 0$ sector. In particular, we examine the OTOCs for unentangled product states of the form $|z_1; z_2; z_3 \ldots z_L \rangle$, where $z_i$ is a spin polarized in the $z$-direction at site $i \uparrow \downarrow$. Motivated by experiments on cold atoms, we study the time evolution of the system, when it is initially prepared in the classical
Néel initial state (|↑↑↑...↑↑⟩). As shown in Fig. 3, we find that signatures of fast scrambling can be seen in the quench dynamics. We observe qualitatively similar behavior for other initial states.

A complementary approach to study information propagation in a quantum many-body system is to examine the growth of the half-chain entanglement entropy,

\[ S_A = \text{Tr}[\rho_L \log(\rho_L)], \]

where \( \rho_L = \text{Tr}_R(\psi(\psi|) \psi) \) is the reduced density matrix obtained by tracing over the degrees of freedom of one half of the chain. As shown in Fig. 3, we find that \( S_A \) grows faster and saturates to higher values as \( J_2 \) increases. These results agree with our previous observation that the system exhibits faster scrambling when the strength of the infinite range interaction increases, as long as the system remains non-integrable.

Finally, we also explore the quench dynamics, when the total magnetization \( M_z \) is finite. In particular, we compute the localization magnetization, since this order parameter is accessible to experimental measurements. Furthermore, this quantity can be related to the recently proposed fidelity out of time order correlators, and can hence be used to quantify scrambling \[69\]. We note that in the one-magnon sector, i.e. when there is one spin-up (spin-down), and \( L - 1 \) spin-down (spin-up) in the initial state, then the system is integrable, and the system exhibits localized dynamics. This localization can be traced to the presence of localized eigenstates of the form: \( |\psi_i| = |\phi_i| + \frac{1}{\sqrt{L}} \sum_{j \neq i} |\phi_j| \), where \( |\phi_i| = |↑↑...i↑i+1...↑⟩ \). While this model is integrable in the one-magnon sector, and it can be non-integrable for a large parameter regime in the \( L/2 \) magnon sector. We carefully study the crossover from slow to fast scrambling can be seen as \( |M_z| \) decreases. As shown in Fig. 4, we find that the fastest scrambling occurs when \( M_z = 0 \).

**Experimental Realizations**: The most natural way to realize our model is to couple an Ising spin chain to a single mode cavity \[70\]. By adiabatically eliminating the cavity degrees of freedom in the dispersive limit, we can obtain an effective infinite range coupling of the form described in Eq. \[1\] \[71\] \[72\]. A promising scheme to realize our spin model using Rydberg dressed atoms in an optical cavity has been proposed in Ref. \[74\]. Another feasible route is to place a trapped ion crystal in the cavity \[75\] \[76\]. Alternatively, it is possible to engineer this model by employing photon-mediated interaction between spins trapped in a photonic crystal waveguide \[77\], or by performing digital-analog simulations with trapped ions \[78\]. Several experimental protocols have been proposed to measure OTOCs in the experimental platforms described above. The infinite temperature OTOC can be determined by examining statistical correlations between measurements on randomized initial states \[79\]. Furthermore, some recent investigations have shown that it is be possible to probe the scrambling dynamics after a quantum quench by measuring two point correlation functions \[80\] \[81\]. Alternatively, interferometric techniques can also be used to measure OTOCs in different experimental platforms.

**Summary and Outlook**: The paradigm of fast scrambling is of fundamental importance in understanding the dynamics of highly chaotic quantum systems. Fast scramblers can also be harnessed for performing quantum information processing tasks, and is thus of great practical use \[82\] \[83\]. In this letter, we have demonstrated a novel route for creating a fast scrambler in an experimentally realizable spin model. Our proposal exploits the interplay of short and long range interactions to make the system highly chaotic.

By studying the level statistics, we have first demonstrated that the system is non-integrable for a wide range of parameters. We have then studied the infinite temperature OTOC of the system in this chaotic regime, using both exact diagonalization, and a semiclassical approximation technique, and concluded that the system exhibits fast scrambling. Next, we have examined the quench dynamics of the system, when it is initially prepared in the classical Néel state, and found that the OTOC and the half chain entanglement entropy grows very fast. Similar results are found for other non-
entangled initial states when the total magnetization, $M_z$ is 0. We have systematically explored how the scrambling rate depends on the total magnetization, and found that the system exhibits a crossover from slow to fast scrambling as the total magnetization decreases from $|M_z| = L$ (i.e. the fully polarized state) to $M_z = 0$. Finally, we have proposed possible experimental realizations of our model. Thus, our work presents a rare example of a many-body model where the fast scrambling is not induced by random long range interactions.

An extremely important feature of our work is that unlike other proposals studied in the literature, our model is not motivated by holography. This leads us to conjecture that fast scrambling is a necessary, but not sufficient condition for quantum many-body systems to be holographically dual to black holes. A rigorous proof of this conjecture can lead to the discovery of new classes of fast scramblers, thereby shedding light on some fundamental questions in non-equilibrium quantum dynamics. Future work can examine other models with both short and long range interactions, and determine general conditions under which quantum many-body systems can exhibit fast scrambling.

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Supplementary material

SHORT TIME EXPANSION

In the main text, we have computed the OTOC employing exact diagonalization. However, at early times, it is possible to obtain an analytical expression for the decay of the OTOC. To do this we expand the operator $\sigma^z_1(t)$ in the form

$$\sigma^z_1(t) = \sigma^z_1(0) - it[\sigma^z_1, H] - \frac{1}{2}t^2[[\sigma^z_1, H], H] + \ldots,$$ 

(7)

Using Eq. (7), we can express the OTOC given in Eq. (3) of the main text as a polynomial in $t$. Fig. 5 shows the comparison between the numerically calculated OTOC and analytical expression (upto $O(t^3)$). We find that for the non-integrable spin chain ($\alpha = 0$), there is reasonably good agreement between both approaches at short times. However, the analytical and numerical results diverge in the fast scrambling regime at longer times. The analytical expression is valid upto longer times, when the spin chain becomes integrable (i.e. $\alpha = 1$).

![Graph comparing analytical and exact results](image)

FIG. 5. Comparison of an analytic short time expansion and exact diagonalization results for the OTOC, $F(8, t)$: The circles represent numerical data from the exact diagonalizaton calculation, while the lines represent the analytical expression. Both approaches agree at short times, even though they differ at longer times in the fast scrambling regime.

EXPERIMENTAL REALIZATION

We have mentioned in the main text that the model in Eq. (11) can be realized by coupling a spin chain to a single mode cavity. In this section, we explicitly derive the effective spin Hamiltonian that arises when this scenario is realized.

The dynamics of an Ising chain interacting with a cavity can be described by the master equation:

$$\frac{d\hat{\rho}}{dt} = -i[H_{SL}, \hat{\rho}] + L_c[\hat{\rho}],$$ 

(8)

where $\hat{\rho}$ is the density matrix of the system.

The Hamiltonian describing the unitary evolution of the system is

$$\hat{H}_{SL} = -\Delta c\hat{a}^+\hat{a} + J_1 \sum_{i=1}^{L} \hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + g \sum_{i=1}^{L} (\hat{a}\hat{\sigma}_i^- + \hat{a}^+\hat{\sigma}_i^+),$$ 

(9)
FIG. 6. **Schematic of the experimental realization of the spin model:** The fast scrambling model that we have studied can be realized when a one dimensional spin chain is collectively coupled to an optical cavity.

where $\Delta_c$ is the effective cavity frequency, $g$ is the coupling between the spins and the cavity field, $J_1$ is the Ising interaction strength, and the Lindblad term capturing the photon loss from the cavity at a rate $\kappa$ is given by:

$$\mathcal{L}_c[\hat{\rho}] = \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^+\hat{a}).$$

(10)

We can eliminate the cavity mode adiabatically in the bad cavity limit ($\kappa \gg g$), and obtain a master equation for the reduced density matrix $\hat{\rho}_s$ of the spin chain,

$$\frac{d\hat{\rho}_s}{dt} = -i[\hat{H}_{\text{eff}}, \hat{\rho}_s] + \mathcal{L}_\Gamma[\hat{\rho}_s],$$

(11)

where the effective Hamiltonian is given by:

$$\hat{H}_{\text{eff}} = \frac{4g^2\Delta_c}{4\Delta_c^2 + \kappa^2} \sum_{i,j} \hat{\sigma}_i^+\hat{\sigma}_j^- + J_1 \sum_{i=1}^{L} \hat{\sigma}_i^z\hat{\sigma}_{i+1}^z,$$

(12)

and

$$\mathcal{L}_\Gamma[\hat{\rho}_s] = \frac{2g^2\kappa}{4\Delta_c^2 + \kappa^2} \sum_{i,j} (2\hat{\sigma}_i^-\hat{\rho}_s\hat{\sigma}_j^+ - \hat{\sigma}_i^+\hat{\sigma}_j^-\hat{\rho}_s - \hat{\rho}_s\hat{\sigma}_i^+\hat{\sigma}_j^-).$$

(13)

When $\Delta_c \gg \kappa/2$, the dynamics is approximately unitary.