The determination of requirements for metrological parameters of geotechnical monitoring systems based on the bifurcation approach

N V Dorofeev¹, A V Grecheneva¹, E S Pankina¹

¹ Vladimir State University, 87, Gor’kogo str., Vladimir, 600000, Russia

E-mail: dorofeev nv@yandex.ru

Abstract. The article is devoted to substantiating the possibility of applying the bifurcation approach to determine the requirements for the metrological characteristics of the measuring part of geotechnical monitoring systems. It is noted that during the operation of the constructions, the change of physico-mechanical properties is in the “soil base-foundation-structure” system. It is caused by seasonal, climatic factors and the change in the industrial environment. Prospects of using the bifurcation approach to assessing the stability of constructions under the influence of exogenous factors are noted. The nonlinear model of the deformation of the construction is described using an example of the rod with one degree of freedom, and the bifurcation diagram of its stability is constructed. It is noted that at the development stage of the construction the application of the bifurcation approach allows one to carry out the modelling and to determine bifurcation points, requirements for the sensitivity and accuracy of measuring instruments. It is proved that the probability of occurrence of errors of the 1st and 2nd kind is much lower on the basis of the proposed approach to assessing stability. The possibility of estimating the permissible error of measuring instruments is proved based on data from bifurcation diagrams.

1. Introduction
Geotechnical monitoring is designed to assess the integrated stability and reliability of geotechnical systems, which include one or more subsystems of the form “soil foundation-foundation-structure”. The geotechnical monitoring is the system of instrumental observations and control that are carried out continuously during construction and operation to assess the technical condition, the timely detection of changes in functional ability due to static, seismic and dynamic loads, the prevention of emergency situations and assessing the accuracy of forecasts, accepted calculation methods and the design decisions. All this is a part of the solution of the problem of assessing the stability and the reliability of structures of buildings and constructions [1].

According to the current regulatory documents, the geotechnical monitoring of structures, which are under the effect of technogenic vibrational noise, provides the periodic inspection of soils, foundations and structures of the construction by visual and instrumental measurements of the group of local and integral physical and mechanical parameters. These parameters include [2]:

- slopes of the controlled surfaces of constructions;
- deformation of controlled points;
- movement of controlled points;
- acceleration and speed of controlled points;
- periods of natural vibrations of structures;
forms of vibrations;
• the speed of propagation of pulses between the controlled points.

The listed parameters provide the control of the strength of materials, their rigidity and the geometric compliance of the structural system with design solutions. To register the above parameters, sensor units and sensors, tensometers, accelerometers, inclinometers, etc., are introduced into the geotechnical monitoring systems [3]. Sensors are placed on the most loaded structural elements in accordance with the calculations of the stress-strain state of structures of constructions.

At the same time, the effectiveness of geotechnical monitoring is determined by metrological characteristics of measuring instruments which are a part of automated systems. However, statistics of the occurrence of disasters indicates the lack of effectiveness of applied methods and monitoring tools. These disasters are due to the loss of stability of engineering structures in both civil and industrial constructions. At least 26% of industrial and civilian accidents are due to unexplored processes of the geotechnical nature. It is according to the Russian National Society of Soil Mechanics, Geotechnics and Foundation Engineering [4]. This is due to the fact that even in seismically passive regions there is an increase in the intensity of microseismic and microgravity excitation of geodynamic resonances of industrial facilities and engineering structures, which accelerate the degradation of their geotechnical reliability. In addition, it is known that requirements are formed to the metrological support of measuring instruments of geotechnical monitoring systems of specific structures on the basis of initial design decisions. It is according to the classification of constructions and soil conditions which are described in regulatory documents [5]. However, during operation, the change of physical and of mechanical properties of the “soil base-foundation-structure” system is possible. It is caused not only by seasonal and climatic factors, but also by the change in the industrial environment (the introduction of new construction, underground and above-ground communications, etc.). This leads to the fact that the lack of sensitivity and accuracy of measuring instruments does not allow one to register the initial stage of loss of stability of the structure.

In this case, methods of the chaos theory and dynamical systems allow one to solve the problem of assessing the structural stability of the construction [6]. According to these methods, stability limits (bifurcation points) can be determined by introduction into the mathematical model of the values of mechanical parameters of the construction depending on parameters of the effect on the construction of vibration factors. If these limits will exceed, then the system goes into an unstable non-deterministic state. In reality it may indicate loss of stiffness and destruction of the construction. In this case a possibility of the formulating of requirements becomes the metrological characteristics (accuracy, sensitivity) of measuring instruments, included in geotechnical monitoring systems. It is the result of the constructing of bifurcation diagrams based on the numerical values of the mechanical parameters of the structure, which is in an extremely stable state.

The aim of the work is to justify the possibility of the applying of the bifurcation approach to determine the requirements to metrological characteristics of the measuring part of geotechnical monitoring systems by the example of assessing the boundaries of the structural stability of the influence of external vibration loads.

2. Methods and approaches
The displacement vector will contain only horizontal displacement parameters that are indirectly manifested in the change in the acceleration of structural elements of the structure. It is a part of the solution to the problem of detecting negative changes at a local point in the geotechnical system using bifurcation diagrams of vibration factors:

\[ \mathbf{v}(t) = \int a(t) = [y_1, \ldots, y_n], \quad (1) \]

where \( a(t) = [a_1, \ldots, a_n] \) - acceleration vector of elements of the construction.

The load \( p(t) \), applied to the construction, is the combination of inertial forces \( f_a \) (for example, accelerations obtained from the vibrational effects of transports), stiffness forces \( f_c \) and damping forces \( f_s \). Then, the vector of vibrational effects can be represented in the form [7]:
Performing the increment of equation (4), we obtain:

$$\Delta p(t) = \Delta f_a(t) + \Delta f_c(t) + \Delta f_s(t)$$

(3)

where \(\Delta f_a(t) = m\Delta V(t)\), \(\Delta f_c(t) = c\Delta V(t)\), \(\Delta f_s(t) = k(t)\Delta V(t)\), \(\Delta p(t) = p(t + \Delta t) - p(t)\).

There is the large class of problems in the dynamics of constructions in which systems cannot be considered as linearly deformable objects. For example, in the concrete structure which is a part of the structure of the foundation of the building, the presence of defects of various configurations (micropores, microcavities) leads to a redistribution of the stress field to the primary (which propagate in the thickness of the material) and secondary ones (which is concentrated in the zones of localization of microinhomogeneities). This is the result of exposure to vibrational factors [8, 9]. The secondary stress field is characterized by variability in each of the loading cycles, since microcracks are the most vulnerable areas of stress concentration from the point of view of load distribution. Each cyclic impact below the fatigue limit leads to the accumulation of plastic deformations and the slow growth of microcracks. This process is accompanied by the convergence of the peaks of growing microcracks to a certain threshold (a certain critical distance), beyond which microcracks merge with the further growth of the crack. Thus, the vibrational loading creates favourable conditions for stress concentration at the weakest points of not only the soil base, but also the elements of the reinforced concrete structure of the building’s foundation. It leads to its destruction under significantly lower cyclic loads than under static conditions adopted in construction calculation models. As a result, the stiffness coefficients are able to change due to the appearance of fluidity in structural elements. In this case, the equation of motion of the system, taking into account (3) and nonlinearity properties, can be written as:

$$m\Delta V(t) + c\Delta V(t) + k(t)\Delta V(t) = \Delta p(t),$$

(4)

where \(m\) is the mass matrix; \(c\) is the attenuation matrix; \(k\) is the stiffness matrix; \(V\) is the displacement vector; \(p(t)\) is the vector of vibrational impact.

In this case, the damping \(c(t)\) and stiffness \(k(t)\) functions change according to certain laws and can be set at the end of the iteration stage, expressing the nonlinear nature of the materials of the structure:

$$c(t) = \frac{d f_c}{d V}, \quad k(t) = \frac{d f_s}{d V}.$$  \hspace{1cm} (5)

In the process of solving the problem, the nonlinear nature of the work of the structure was determined by a change in stiffness in accordance with the accepted elastoplastic dependence (5). In the numerical integration of equation (4), it was assumed that, within each time interval, the acceleration varies linearly, while the attenuation and stiffness remain unchanged. When considering motion, the change in acceleration was assumed to be linear, and the change in speed and displacement was assumed to be quadratic and cubic. As a result, equation (4) can be written as:

$$\tilde{k}(t)\Delta V(t) = \Delta p(t),$$

(6)

wherein:

$$\tilde{k}(t) = k(t) + \frac{a}{t} m + \frac{3}{t} c(t); \quad \Delta p(t) = p(t) + m\left(\frac{a}{\Delta t} m + 3V(t)\right) + c(t)\left(3V(t) + \frac{\Delta t}{2} V(t)\right)$$

(7)

where \(a\) is the acceleration of structural elements.

During the vibrational impact on the foundations of structures, the main type of deformations is roll deformations, which are manifested in the loss of stability upon reaching the limit value of the angle of inclination of the structure. Moreover, the limiting stable value of the angle of inclination is determined by the elastic properties of the material and the limiting stable value of the angle of inclination changes according to the exponential law until the elastic limit of the materials of the structure is reached [10].

As a result of the research, the dynamic behaviour of the construction was modelled with one degree of freedom (figure 1). Model parameters are \(m = 200\) tons, \(k = 30000\) kN/m, \(h = 1.5\) m. In the first
approximation this model is the simplest link in the construction. Moreover, it is possible to use the modularity method in the case of modeling bifurcations of structures which have a more complex design scheme. In this case the scheme (in figure 1) replaces each of the basic elements of construction, and the interconnection of structural elements is ensured by the introduction of additional differentials of type (5).

In accordance with the theory of dynamical systems, the bifurcation of codimension 1 (fusion) is valid for this case. The bifurcation diagram of the change in the overall stability of the structure is shown in figure 2. This diagram is obtained during an increase in load $P$, taking into account the vibration effect.

![Figure 1. The design model of the frame building model with one degree of freedom](image1)

![Figure 2. The bifurcation diagram](image2)

According to the analysis of the bifurcation diagram, the bifurcation point is the displacement value for a structure with a static load of $P \approx 7500$ kgF. At this point, the displacement $\Delta \nu$ of the construction unit is $3.89$ cm. After the bifurcation point $\Delta \nu = 4.53$ cm the displacement of the assembly corresponds to the unstable state of the geotechnical system, which leads to the formation of the roll of the construction. As can be seen from the diagram, the bifurcation process occurs under a load $P < P_{cr}$. This suggests that under vibration exposure, the static load $P$ (which is the mass of the floors of the building) has a greater effect on reducing the stability of structures than that in the absence of vibration noise. To assess the effectiveness of the proposed approach, the rigidity of the structure was calculated with one degree of freedom under the influence of forces, according to figure 1, by a standard technique based on the use of a dynamic coefficient $Kd$ [11].

3. Results and discussion

As a result of structural calculations, the maximum strain $\nu_{max}$ was obtained at which the maximum displacement of the structure $\Delta \nu$ while maintaining its stability was $5.76$ cm. The difference of obtained limit values is $\Delta \nu = 5.76 - 3.89 = 1.87$ cm, which is significant in the theory of stability of constructions. According to the results obtained, when using the bifurcation approach, the confidence area of monitoring of the stability state becomes much smaller (figure 3). In figure 3 the red line is stability calculation; using a dynamic coefficient the system is not stable at displacement values $\Delta \nu = (-\infty; -5.76); (5.76; +\infty)$ and green line is the stability calculation using the bifurcation approach. The system is not stable at displacement values $\Delta \nu = (-\infty; -3.89); (3.89; +\infty)$. 
Figure 3. The identification of critical areas for assessing the system stability.

Nomograms (figure 4 and figure 5) are constructed to determine the probabilities of occurrence of errors of the first and second kind. It is for the case of the normal distribution of the controlled parameter of the displacement of the construction under the action of the load within the tolerance range of $T_1 = \pm 5.76$ and $T_2 = \pm 3.89$ with the standard deviation (technological dispersion) $a_1$ and the normal distribution of the measurement error with the standard deviation $a_2$.

Figure 4. Nomograms for determining the probability of errors of the first and second kind for the stability control method and the use of calculated dynamic coefficients.

Figure 5. Nomograms for determining the probabilities of errors of the first and second kind for the proposed bifurcation stability control approach.

Nomograms are constructed as the set of dependences of the probabilities $\alpha$ (errors of the first kind) and $\beta$ (errors of the second kind) on the ratio of standard deviations $\sigma_{\Delta}/\sigma_x$ with the parameters $T_1/\sigma_x$ and $T_2/\sigma_x$. In both figures probabilities are plotted on the logarithmic scale.
4. Conclusion
As can be seen from figures 4 and 5, the probability of occurrence of errors of the 1st and 2nd kind (which are determined on the basis of confidence intervals and bifurcation approach) is significantly lower. This is due to the fact that, according to this approach, the zone of uncertainty decreases (the zone of intersection of the states of stability and instability of the system). Thus, using the above nomograms, it is possible to estimate the permissible error of the measuring instruments for the given probabilities of control errors with a known tolerance to the technological dispersion ratio. This proves the possibility of applying the bifurcation approach to determine the requirements for the metrological characteristics of the measuring part of geotechnical monitoring systems.

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