Optomechanical conversion by mechanical turbines

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Liquid crystal elastomers are rubbers with liquid crystal order. They contract along their nematic director when heated or illuminated. The shape changes are large and occur in a relatively narrow temperature interval, or at low illumination, around the nematic–isotropic transition. We present a conceptual design of a mechanical turbine based on photo-active liquid crystal elastomers which extracts mechanical work from light. Its efficiency is estimated to be 40%.

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We propose a mechanical turbine to harness the contractions of soft, photo-responsive solids with a large stroke. We thus take photo-active nematic liquid crystal elastomers (LCEs) as our working material. Related engines have been proposed before, for instance based on the bend response of strips of nematic photo-glasses connecting two wheels [1, 2]. We believe that engines based on stretch rather than bend are much more efficient, particularly with soft solid working materials where distortions are great and energy storage at a given stress is huge; we proposed a two-wheel stretch engine [3] using a nematic photo-LCE. The engine had analogies to that of Steinberg et. al who studied chemical to mechanical energy conversion [4]. In this Letter, we extend the approach to mechanical turbines in order to get a much higher conversion of light to mechanical work, see Fig. 1.

There is a similarity to a turbine that converted chemical to mechanical energy [5]. Here we analyse the mechanics and losses involved in such turbines. Modelling the geometrical and material parameters of this engine, along with the known photo-response of typical LCEs, suggests that its efficiency can be as high as 40%.

Classical elastomers are cross-linked polymer melts exhibiting liquid characteristics locally, but are solid-like on a macroscopic scale. Incorporating molecular rods into the polymers of a simple elastomer leads to networks that combine orientational liquid crystal order with the extreme stretchiness of rubber, that is LCEs [6, 7]. The shape of a monodomain LCE is very sensitive to the change of the nematic order parameter $Q$; network polymers are elongated by the directional order and mechanical shape change ensues. The order decreases on increasing the temperature, which manifests as a uniaxial contraction, by a factor $\lambda_m$ (< 1) of the elastomer along the nematic director [7]. The contraction is especially rapid in the vicinity of the transition temperature to the isotropic state. Analogous shape changes occur in photoelastomers in which photoisomerizable dye molecules, rod-like in their trans ground state, are connected to the LCE structure [8, 9]. Here, illumination causes the creation of bent-shaped cis isomers of the dye which act as impurities that reduce the nematic order, in turn leading to a contraction of the photoelastomer. The presence of cis isomers raises the effective temperature of the photoelastomer from $T$ to a pseudo $\tilde{T} > T$ which depends on the cis concentration and mimics the disorder as if it were induced thermally [8, 10]. It is important to note that mechanical deformations of elastomers are reversible, that is, on removal of heat or light, recovery elongations by a factor of $1/\lambda_m$ occur. These elongations can be huge, up to 400% [11].

The optothermal cycle of our engine is shown in Fig. 2. For a nematic elastomer of shear modulus $\mu$, the free energy per length of unstretched band, $F$, and the tension $f$ depend on the stretch ratio (deformation gradient) $\lambda$ [7]:

$$F(\lambda, T) = \frac{1}{2} \mu A_0 \left( P_\parallel \lambda^2 + \frac{2 P_\perp}{\lambda} \right),$$

$$\tilde{f} = \frac{f}{\mu A_0} = \frac{1}{\mu A_0} \left( \frac{\partial F}{\partial \lambda} \right)_T = \left( P_\parallel \lambda - \frac{P_\perp}{\lambda^2} \right),$$

where $A_0$ is the cross sectional area of the unstretched elastomer, and $\tilde{f} = f/(\mu A_0)$ is the tension reduced by the natural force scale in the problem. The modulus $\mu$ is temperature dependent. A simple, freely-jointed rod model

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FIG. 1. Schematic of an optomechanical turbine.
FIG. 2. The optothermal cycle of the engine. Scaled force $\bar{f}$ against stretch $\lambda$. Upper curve — pseudo temperature $\bar{T}$ (illuminated, isotropic state), bottom — lower $T$ (nematic state). Dashed lines — two-wheel motor [3].

quite accurately describes a wide range of LCEs [7, 12] and, in particular, the development of photoforce [10]. In this model, $P_\parallel$ and $P_\perp$ depend on the nematic order parameters $Q(T)$ and $\tilde{Q} = Q(\bar{T})$ as $P_\parallel = (1 + 2Q)/(1 + 2\tilde{Q})$ and $P_\perp = (1 - Q)/(1 - \tilde{Q})$. A free elastomer has $\bar{f} = 0$ in (2) (the force-free state D or A depending on the temperature). Changing from the formation temperature $T$ to $\bar{T}$ then gives a natural contraction $\lambda_m = (P_\perp/P_\parallel)^{1/3}$ along its director [7]. The illuminated state $\bar{T}$ is isotropic with $\tilde{Q} \approx 0$. Thus $\lambda_m = [(1 - Q)/(1 + 2\tilde{Q})]^{1/3}$. We choose the formation temperature $T$ to also be the operating temperature, meaning that $P_\parallel = 1$ and $P_\perp = 1$ along A→B, and for temperatures or pseudo-temperatures $\bar{T} > T$, along C→D, the parameters are $P_\parallel > 1$ and $P_\perp < 1$. The large change between $\lambda = \lambda_m$ and $\lambda = 1$ is what makes LCEs promising working materials. We henceforth describe optical response, thermal response being entirely analogous. To realize a continuously operating engine, we subject the elastomer to the ideal cyclical process through states A→B→C→D→A by changing the force and illumination.

It is easy to see from Fig. 2 that the net work delivered by the engine, per unit length of its LCE working material in its unstretched (formation) state as it is taken around the cycle, is:

$$W = W_{CD} - W_{AB} = \mu A_0 \left( \int_{\lambda_m}^{\lambda_h} f(\bar{T})d\lambda - \int_1^{\lambda_m} f(\bar{T})d\lambda \right) = F(\lambda_h, \bar{T}) - F(\lambda_h, T),$$

and is similar in form to the work done in the various gaseous $p$–$V$ cycles. Here, the moduli are assumed to be comparable, $\mu(T) \approx \mu(T)$. One already sees from Fig. 2 that the pre-stretch, $\lambda_h$, imposed before illumination, enhances work output considerably, just as a higher compression ratio improves conventional engines. Further, taking a turbine allows one to extend the cycle to zero force (at a contraction equal to the illuminated natural length, $\lambda_m$) which is impossible in two-wheel stretch engines [3]: see cycle A→B→C→A in Fig. 2. However for a soft solid, where much more work can be delivered than for hard solids, formidable problems exist in realising the above $W$. These include sliding and frictional losses when finite tension differences in soft solids exist across the engine. The remainder of this paper is concerned with overcoming these problems to get close to the above $W$ by using the turbine of Fig. 1. We will need to generalise the classical pulley result of Euler [13] to highly extensible belts.

The energy input per unit length of unstretched elastomer in the illumination process B→C is $\varepsilon n_{dye} A_0$, with $\varepsilon$ the appropriate photon energy and $n_{dye}$ the number density of dye molecules. For our elastic model of an isotropic elastomer, the internal energy is a function of temperature only and is unchanged along an isotherm [14]. Thus the heat input per unit length of unstretched elastomer during the isothermal process C→D equals the work done $W_{CD}$. The efficiency is the ratio of the work done to the optical energy plus heat invested per cycle:

$$\eta = \frac{W_{CD} - W_{AB}}{W_{CD} + \varepsilon n_{dye} A_0} = \left(1 - \frac{I_2}{I_1}\right)/\left(1 + \varepsilon n_{dye} \frac{1}{\mu} \frac{1}{I_1}\right),$$

where $I_1$ and $I_2$ are the reduced force integrals in Eq. (3) and depend on the order along the isotherms $T$ and $\bar{T}$. Obviously, as $I_1 > I_2$ the efficiency is always $\eta < 1$. The ratio of material constants has been estimated as $\varepsilon n_{dye}/\mu \approx 2$ [3, 15]. Thus the efficiency is determined by the values of the order parameter $Q$ and the value of the pre-stretch $\lambda_h$. Modest values $Q = 0.5$ and $\lambda_h = 3\lambda_m$ give $\eta \approx 40\%$.

We now follow the mechanical/thermal cycle in detail: A closed band of photo-LCE is wound around the pair of spindles of Fig. 1, the tops of which are rigidly coupled by a loop of inextensible wire, thus ensuring they have equal angular velocities. The spindles have slightly tilted axes which enables the elastomer to spontaneously follow a helical path down the spindles as they rotate. The horizontal cross-section of the spindles can be approximated by circles, the radii of which grow from the top to attain a largest value at the middle of the spindle, after which they decrease.

Initially, a free elastomer belt in the nematic, formation state A at temperature $T$ comes on to the upper part of the spindles with $\lambda = 1$ and hence $f = 0$. Isothermally, it passes helically from smaller to larger spindle radii, extending at each pass by a fraction equal to the ratio of the radii which material conservation requires as discussed later. It suffers multiple-step extension during the process A→B in Fig. 2. State B, with the largest radius $r$ (Fig. 1), has the highest (pre-)stretch $\lambda = \lambda_h$, and has force $f = f_B$. When in contact with the surface
of radius $r$, the elastomer is illuminated and assumed to reach $\bar{T}$ before it leaves the spindle; along $B$–$C$ the process is isometric ($\lambda = \lambda_h$, an assumption of no slip along $B$–$C$). At the point $C$, the force takes the highest value in the cycle $f_C$, and the elastomer is in the isotropic state. After passing the surface of maximum radius $r$, the elastomer spirals down the lower part of the spindles, $C$–$D$. It moves now repeatedly from larger to smaller radii, contracting at each pass by the ratio of the radii. It releases its elastic energy as work done on the spindles; in the final state $D$ the stretch reaches the minimum value $\lambda = \lambda_m$, the natural length of elastomer at $\bar{T}$. To remain with $\bar{T}$ along $C$–$D$, because of cis $\to$ trans back reaction one must continue some level of illumination. Equally, to remain isothermal one needs external heat (most of which in practice may come from the heat released in the back reaction) since entropy rises in a contracting elastomer. Finally, on removal of illumination, the elastomer gradually recovers and elongates at zero force from $\lambda_m$ and $\bar{T}$ to $\lambda = 1$ and $T$ (D–A of the cycle).

The engine converts into mechanical work part of the optical energy elevating the elastomer to the effective temperature $\bar{T}$. Work is extracted by turning one spindle, the left in Fig. 1, against the external torque, $G_{\text{ext}}$.

The useful work done per cycle is less than that of Eq. (3) because of sliding friction. We analyze each pass of the elastomer around a spindle, considering differences in the tensions $f_1 < f_2$ of the incoming and outgoing sections; see Fig. 3(a).

The beautiful classical solution [13] has exponential increase of tension along the belt from $f_1$ to $f_2$ over parts of $x = 0$ to $x = \pi R$: Since the surface is curved, there is an inward force per unit length on the spindle from the belt of $f/R$; see Fig. 3(b). If the surface is rough, the normal force gives a frictional shear force (per unit length on the circumference) of $\gamma \propto f/R$. The net shear force $\gamma dx$ balances the difference in tangential forces $f(x + dx) - f(x)$ across an element $dx$. Clearly $\frac{df}{dx} \propto f/R$ gives an exponential increase in $f(x)$. There is an obvious limitation to the classical analysis that is important in soft solids where substantial stretch $\lambda$ accompanies increasing $f$: the belt has to slide on the spindle to extend in order that $f$ increases with $x$. It cannot translate with the circumferential speed $v = \omega R$ for all its contact length with the spindle. Changing strain implies changing stored elastic energy. Sliding implies frictional losses during the transmission of power.

Fig. 3(a) assigns a $\lambda(x)$ corresponding to a $f(x)$. In particular $\lambda = \lambda_1$ for the belt incoming on to the spindle, and $\lambda = \lambda_2$ when it emerges with $f_2$. Temperature is $T$ throughout the belt; $f_2 > f_1$ then implies $\lambda_2 > \lambda_1$. Soft solids are essentially incompressible under extension, hence material conservation dictates volume conservation. The sectional area $A = A_0/\lambda$ must diminish as the belt extends by $\lambda$. The volume flux of belt on to the spindle is $v_1 A = v_1 A_0/\lambda_1$ and must be matched by $v(x) A_0/\lambda(x)$ at a general point. Cancelling $A_0$, we get

$$v_1/\lambda_1 = v(x)/\lambda(x) = v_2/\lambda_2,$$

where now we have the unstretched length of band per unit time passing. This flux of unstretched length is equivalent to $\omega R/\lambda_1$ when the band is not sliding. As $\lambda_2 > \lambda_1$ then it follows $v_2 > v_1$ — the belt starts going faster than the spindle. There is sliding friction. Neglecting inertia [16], we get:

$$\frac{df}{dx} = \mu_k f/R,$$

where $\mu_k$ is the coefficient of kinetic (sliding) friction giving $\gamma = \mu_k f/R$. Since $f_2 > f_1$, friction inhibits sliding in the $+x$ direction. Solving (6) gives

$$f(x) = f_2 \exp\left[\frac{\mu_k}{R}(x - \pi R)\right],$$

which differs from the classical result in its use of kinetic friction coefficient, $\mu_k$, and the passage of energy to friction and to elastic potential. The tension varies from $f = f_2$ at $x = \pi R$ down to $f = f_1$ at an $x = x_1$ given by

$$x_1 = R \left[\pi - \frac{1}{\mu_k} \ln\left(\frac{f_2}{f_1}\right)\right].$$

In the initial section $(0, x_1)$, the tension retains its incoming value $f = f_1$. It is gradients $df/dx$ that transfer force (torque) to the spindle in the region $(x_1, \pi R)$. There is a region without torque if $x_1 > 0$, that is if $f_2 < f_1 e^{\mu_k \pi}$. The power delivered to the spindle, in A–B of Fig. 2 with $f_2 > f_1$ and the spindle turning clockwise, is the speed of its surface, $v_1$, times the force exerted on it:

$$P_w = v_1 \int_{x_1}^{\pi R} \frac{f(x)}{R} dx \mu_k = v_1 \int_{x_1}^{\pi R} \frac{df}{dx} dx = v_1 (f_2 - f_1).$$

The term $-v_1 f_1$ is the power given by the spindle to the incoming band. The power $v_1 f_2$ is the portion of
the power $v_2 f_2$ delivered by the more tense band that actually finds its way to the spindle. Thus, all the useful power is delivered via the region that is slipping. When slip is complete, $x_1 = 0$ in (8), then $f_2 = f_1 e^{i \varphi_i \pi}$ takes its maximal value, as does $P_w$ in (9); $P_w = v_1 f_1 (e^{i \varphi_i \pi} - 1)$.

The power lost to friction in A–B is

$$P_l = \int_{x_1}^{\pi R} \frac{df}{dx} [v(x) - v_1],$$

with $x_1$ given by (8). The second part, involving $v_1$, is trivially $-v_1 (f_2 - f_1)$, Eq. (9). The first term we integrate by parts to get $(f_2 v_2 - f_1 v_1) - \int_{x_1}^{\pi R} dx f(x) d\lambda f(x)$.

The last term has $d/v/dx = (v_1/\lambda_1)(d\lambda/dx)$, and becomes:

$$-v_1 \int_{x_1}^{\pi R} dx f(x) \frac{d\lambda}{dx} = -\frac{v_1}{\lambda_1} \int_{\lambda_1}^{\lambda} d\lambda f(\lambda)$$

$$= -\frac{v_1}{\lambda_1} [F(\lambda_2, T) - F(\lambda_1, T)],$$

where we used Eq. (2) with $P_\parallel = P_\perp = 1$, since the elastomer is at $T$. Overall, the power lost to friction is:

$$P_l = f_2 (v_2 - v_1) - \frac{v_1}{\lambda_1} [F(\lambda_2, T) - F(\lambda_1, T)],$$

and does not involve the friction coefficient $\mu_k$, unless slippage is complete ($x_1 = 0$). In the process A–B of the engine, $\lambda_3 > \lambda_1$ is realized by moving the elastomer belt in each pass from a spindle surface of a smaller radius $r_1$ to a surface of a larger radius $r_2$. The velocities are $v_1 = \omega r_1$ and $v_2 = \omega r_2$. Then using (5) one gets $\lambda_2/\lambda_1 = r_2/r_1 > 1$, as required.

Now, consider the isotropic elastomer with $T$, which in each turn around a spindle goes from a higher to lower force (process C–D), $f_1 > f_2$ and $\lambda_1 > \lambda_2$ in Fig. 3(a). Note that the spindle’s direction of turn is fixed in the clockwise direction by the engine operation as a whole, independently of the relative size of the forces. As before there is an $x_1$ with no slip for $x < x_1$, while slippage occurs for $x = (x_1, \pi R)$. Repeating the above, the power delivered and the frictional power lost are:

$$P_w = v_1 (f_2 - f_1),$$

$$P_l = f_2 (v_2 - v_1) + \frac{v_1}{\lambda_1} [F(\lambda_1, T) - F(\lambda_2, T)].$$

In this case the elastomer belt in each pass from one spindle to the next goes from a larger to a smaller radius, $r_1 > r_2$, leading to $v_1 > v_2$ and $\lambda_2/\lambda_1 = r_2/r_1 < 1$.

It remains to analyze the elastomer belt going around the surface of radius $r$ in the process B–C. Now the elastomer at force $f_B$ and temperature $T$ comes on the spindle where the illumination begins. In Fig. 3(a) we now have $f_1 = f_B$ and $\lambda_1 = \lambda_h$. We assume that the change $T$ to $T$ occurs without slippage and is complete before the elastomer reaches a point $x_1$. (Force rising from $f_B$ to $f_C$ without slippage assumes $df/dx < \mu_k f/r$ with $\lambda = \lambda_h$ and where variation with $x$ is from the changing $P_\parallel$ and $P_\perp$ factors in expression (2) for $f$.) After $x_1$, slippage then occurs and the force drops from $f_C$ to $f_S$ (in Fig. 3(a)) $f_2 = f_S$, $\lambda_2 = \lambda_S$.

The state $S$ is marked in Figs. 1 and 2. The net power delivered on the transit of $r$ is

$$P_w = \omega r (f_S - f_B).$$

For the frictional losses, expression (14) with the appropriate values of stretches, velocities and forces is still applicable since process C–S is part of C–D.

Excluding the middle spindle surface of radius $r$, the power delivered to the spindle in each extension step of A–B is given by (9) and is clearly positive as one expects if $f_2 > f_1$. However, the total power delivered in A–B is actually negative (see Fig. 2) since the band emerges stretched from A–B. This is due to the fact that one must add the highly negative contribution $-\omega r f_B$ of (15) to the set of positive contributions of the form (9). Similarly, in C–D the highly positive term $\omega r f_S$ of (15) is added to negative contributions (14), making the total power delivered in C–D positive since the elastomer contracts.

We now calculate the total power $P_{nk}$ delivered to the spindles by the elastomer for a $n$–stage contraction process C–D and $k$–stage extension process A–B. When the engine runs at a constant velocity $\omega$ the net torque acting on each of the spindles is zero. For simplicity we shall neglect frictional forces at the bearings. Then one can express the balance of torques on the each spindle separately. Beside the torques produced by the elastomer, one should take into account the torques due to the inextensible wire which couples the spindles, and the external torque $G_{ext}$ acting on the left spindle (see Fig. 1). In such a way, one obtains a pair of equations which enable one to determine $G_{ext}$, and thus the useful power $P_{nk} = \omega G_{ext}$:

$$P_{nk} = \omega \sum_{i=1}^{n} f_{i+1} (r_i - r_{i+1}) - \omega \sum_{j=1}^{k} f'_{j+1} (r'_{j+1} - r'_{j}).$$

Here, $f_{i+1}$ and $f'_{j+1}$ are the forces in the $i$–th step of C–D and in the $j$–th step of A–B process, respectively, while $r_i$ and $r'_{j}$ are the corresponding spindle radii. In particular, $f_2 = f_B$ and $r_1 = r$ (force $f_1$ would be $f_C$). Clearly, since $r_i > r_{i+1}$ for all $i$, the first sum on the right hand side of (16), $P_n = \omega \sum_{i=1}^{n} f_{i+1} (r_i - r_{i+1})$, is positive. Similarly, we have $f'_{k+1} \equiv f_B$ and $r'_{k+1} \equiv r$ (force $f'_1$ would be zero). On the other hand, $P_k = -\omega \sum_{j=1}^{k} f'_{j+1} (r'_{j+1} - r'_{j})$ is negative since $r'_{j+1} > r'_{j}$.

Using relations (12) and (14) the useful power is

$$P_{nk} = \frac{\omega}{\lambda_h} \sum_{i=1}^{n} [F(\lambda_i, T) - F(\lambda_{i+1}, T)] - \frac{\omega}{\lambda_h} \sum_{j=1}^{k} [F'(\lambda'_{j+1}, T) - F'(\lambda'_{j}, T)] - \sum_{i=1}^{n} P_{\parallel,i}$$

$$- \sum_{j=1}^{k} P'_{\parallel,j},$$

where $\lambda_i$ and $\lambda'_j$ are the stretches, and $P_{\parallel,i}$ and $P'_{\parallel,j}$ are the frictional power losses in the contraction and extension.
tion steps respectively. We now sketch a geometrical interpretation of (17). The first term of the last line is the maximum useful mechanical power \( P = W \omega r / \lambda h \), with \( \omega r / \lambda h \) being the flux of unstretched length (5). The maximum mechanical work \( W \) done per unit length of unstretched elastomer in one cycle is given by the area in the solid line A–B–C–D–A in Fig. 2. The useful work done per unit length of unstretched elastomer in one cycle \( W_{nk} = P_{nk} / (\omega r / \lambda h) \) is the gray shaded area enclosed in Fig. 2 (where for simplicity the change in radii in each contraction and extension step is taken to be the same; see comments below). Thus, the work done in one cycle to overcome friction during the contraction process is equal to the area of the white regions between the solid curve C–D and the gray shaded area (corresponding frictional power losses are \( \sum_{i=1}^{n} P_{i,i} \)). Similarly, the area of regions between the gray shaded area and the solid curve A–B corresponds to the term \( \sum_{j=1}^{k} P'_{i,j} \) of (17).

By contrast, the two-wheel cycle a–B–C–s–a dashed in Fig. 2 has larger frictional losses: the areas between the isotherms a–B and C–s and the horizontal dashed lines respectively above and below them are much larger relative to the enclosed area (the maximum theoretical work). The inaccessibility of an \( \dot{f} = 0 \) state means the enclosed area is smaller than the turbine’s, even though the same optical energy input to go from B to C is required, further underscoring the superior efficiency of a turbine.

Notice that the more steps \( n \) and \( k \), the more useful work per cycle is done. We shall compare the power \( P \) corresponding to an infinite number of extension and contraction steps, with the power \( P_{nk} \) given by expression (16), and determine how many steps \( n \) and \( k \) one needs to achieve, say, \( P_{nk} / P = 0.9 \).

We take the change of radii in contraction process C–D to be the same for each step and equal to \( \Delta r = (r - r_{n+1}) / n \), where \( r_{n+1} \) is the final radius in the contraction process. Similarly, for the extension process A–B we take \( \Delta r' = (r' - r'_{1}) / k \), where \( r'_{1} \) is the initial radius in the extension process. From relations (5) it follows: \( r/\lambda h = r_{i}/\lambda i = r_{n+1}/\lambda m \) and \( r'_{1} = r'/\lambda j = r'/\lambda h \).

Here we used the facts that the stretch that corresponds to the radius \( r_{n+1} \) is actually the natural stretch \( \lambda m = (P_{L} / P_{h})^{1/3} \), and that the stretch corresponding to radius \( r'_{1} \) is \( \lambda = 1 \). The stretch steps in contraction and extension processes are then \( \Delta \lambda = \Delta r_{h}/r = (\lambda_{i} - \lambda_{m})/n \) and \( \Delta \lambda' = \Delta r'_{h}/r = (\lambda_{h} - 1)/k \), respectively. Specially the choice \( \Delta \lambda = \Delta \lambda' \) was made in Fig. 2 for simplicity.

The powers \( P_{nk} \) and \( P \) can be expressed as

\[
\begin{align*}
P_{nk} &= \mu A_{0} \frac{\omega r}{\lambda h} \left[ \Delta \lambda \sum_{i=1}^{n} \left( P_{i} (\lambda h - i \Delta \lambda) - \frac{P_{L}}{(\lambda h - i \Delta \lambda)^{2}} \right) - \Delta \lambda' \sum_{j=1}^{k} \left( 1 + j \Delta \lambda' - 1/(1 + j \Delta \lambda')^{2} \right) \right], \\
P &= \mu A_{0} \frac{\omega r}{\lambda h} \left[ \int_{0}^{\lambda h} \int_{0}^{\lambda m} \int_{0}^{\lambda m} \frac{d\lambda P_{i}}{(\lambda - \frac{P_{L}}{h})^{2}} - \int_{0}^{\lambda m} \int_{0}^{\lambda m} \frac{d\lambda P_{i}}{(\lambda - \frac{1}{\lambda'}^{2})} \right].
\end{align*}
\]

Taking \( Q = 0.5 \) and \( \lambda h = 3\lambda m \), one finds \( P_{nk} / P = 0.9 \) for \( n = 15, k = 15 \).

The maximum mechanical work done per unit length of unstretched elastomer in one cycle \( W = P / (\omega r / \lambda h) \) is given in Eq. (3). The simple integrals above at constant temperatures return us to the free energies in Eq. (2).

In summary, the optical contraction of photo-LCEs can be used to harness optical energy to generate mechanical energy. Our mechanical turbine utilizes more effectively the optothermal cycle than the two-wheel engine [3]. Soft, extensible photo-solids do more work than hard solids, but their extensions and contractions lead to sliding and frictional losses. We analyzed such losses and calculated the fraction of work lost due to them.

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[1] M. Yamada, M. Kondo, J. Mamiya, Y. Yu, M. Kinoshita, C. J. Barrett, and T. Ikeda, Angew. Chem. Int. Ed. 47, 4986 (2008).
[2] Y. Geng, P. L. Almeida, S. N. Fernandes, C. Cheng, P. Pallivy-Muhoray, and M. H. Godinho, Sci. Rep. 3, 1028 (2013).
[3] M. Knežević and M. Warner, Phys. Rev. E 88, 040501(R) (2013).
[4] I. Z. Steinberg, A. Oplatka, and A. Katchalsky, Nature 210, 568 (1966).
[5] M. V. Sussman and A. Katchalsky, Science 167, 45 (1970).
[6] P. G. de Gennes, C. R. Acad. Sci. B 281, 101 (1975).
[7] M. Warner and E. M. Terentjev, Liquid Crystal Elastomers (Oxford University Press, Oxford, 2007).
[8] H. Finkelmann, E. Nishikawa, G. G. Pereira, and M. Warner, Phys. Rev. Lett. 87, 015501 (2001).
[9] P. M. Hogan, A. R. Tajbaksh, and E. M. Terentjev, Phys. Rev. E 65, 041720 (2002).
[10] M. Knežević, M. Warner, M. Čopić, and A. Sánchez-Ferrer, Phys. Rev. E 87, 062503 (2013).
[11] A. R. Tajbaksh and E. M. Terentjev, Eur. Phys. J. E 6, 181 (2001).
[12] H. Finkelmann, A. Greve, and M. Warner, Eur. Phys. J. E 5, 281 (2001).
[13] M. L. Euler, Mem. Acad. Sci., pp. 265 (1762).
[14] L. R. G. Treloar, The Physics of Rubber Elasticity (Oxford University Press, Oxford, 2005).
[15] M. Knežević and M. Warner, Appl. Phys. Lett. 102, 043902 (2013).
[16] M. S. Bechtel, S. Vohra, K. I. Jacob, and C. D. Carlson, J. Appl. Mech. 67, 197 (1999).