FeynArts model file for MSSM transition counterterms from DREG to DRED

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Abstract

The FeynArts model file \texttt{MSSMdreg2dred.mod} implements MSSM transition counterterms which can convert one-loop Green functions from dimensional regularization to dimensional reduction. They correspond to a slight extension of the well-known Martin/Vaughn counterterms, specialized to the MSSM, and can serve also as supersymmetry-restoring counterterms. The paper provides full analytic results for the counterterms and gives one- and two-loop usage examples. The model file can simplify combining \textsc{ms}-parton distribution functions with supersymmetric renormalization or avoiding the renormalization of $\epsilon$-scalars in dimensional reduction.

PROGRAM SUMMARY

\textit{Manuscript Title:} FeynArts model file for MSSM transition counterterms from DREG to DRED
\textit{Authors:} Dominik Stöckinger, Philipp Varšo
\textit{Program Title:} \texttt{MSSMdreg2dred.mod}
\textit{Licensing provisions:} LGPL-License [1]
\textit{Programming language:} Mathematica, FeynArts
\textit{Operating system:} Any, with running Mathematica, FeynArts installation
\textit{Keywords:} FeynArts model file, MSSM, Dimensional regularization
\textit{Classification:} 4.4 Feynman Diagrams, 5 Computer Algebra, 11.1 General, High Energy Physics and Computing
\textit{Nature of problem:} The computation of one-loop Feynman diagrams in the minimal supersymmetric standard model (MSSM) requires regularization. Two schemes, dimensional regularization and dimensional reduction are both common but have different pros and cons. In order to combine the advantages of both schemes one would like to easily convert existing results from one scheme into the other.
\textit{Solution method:} Finite counterterms are constructed which correspond precisely to the one-loop scheme differences for the MSSM. They are provided as a FeynArts [2] model file. Using this model file together with FeynArts, the (ultra-violet) regularization of any MSSM one-loop Green function is switched automatically from dimensional regularization to dimensional reduction. In particular the counterterms serve as supersymmetry-restoring counterterms for dimensional regularization.
\textit{Restrictions:} The counterterms are restricted to the one-loop level and the MSSM.

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1. Introduction

The minimal supersymmetric standard model (MSSM) is a promising extension of the standard model. Many phenomenological investigations require the computation of quantum corrections and regularization of ultraviolet (UV) and infrared (IR) divergences. The two most common regularization schemes for the MSSM are dimensional regularization (DREG) \[1\] and dimensional reduction (DRED) \[2, 3\]. In both schemes all momentum integrals are formally continued to \(D\) dimensions, allowing for very efficient integration techniques. In DREG also gauge fields are treated as \(D\)-component quantities. This leads to a mismatch between the number of degrees of freedom of gauge fields and gauginos (\(D\) vs. 4) and the breaking of supersymmetry on the regularized level. In DRED gauge fields remain formally 4-component quantities, and therefore DRED is better suited for supersymmetric theories.

Nevertheless, as discussed e.g. in the report of the “Supersymmetry Analysis Project” \[4\], DREG should not be discarded as a regularization of the MSSM. In particular, if hadronic processes should be interfaced with the common \(\overline{\text{MS}}\)-parton distribution functions, or if existing building blocks or algorithms in DREG should be used, DREG can be advantageous. Refs. \[5, 6\] provide examples of MSSM computations where DREG has been used, and where the breaking of supersymmetry has been compensated by adding appropriate supersymmetry-restoring counterterms. Given that both DREG and DRED have specific advantages it would be optimal to be able to combine these advantages in practical computations.

In the present paper we introduce the model file \texttt{MSSMdreg2dred.mod} for the \texttt{FeynArts} \[7\] package for generating Feynman diagrams and amplitudes. The model file contains one-loop transition counterterms for the MSSM corresponding to switching the UV-regularization from DREG to DRED. In particular it thus automatically includes all necessary supersymmetry-restoring counterterms for DREG. It is fully compatible to the original \texttt{FeynArts} MSSM model file and to further processing of the generated amplitudes with \texttt{FormCalc} \[8\] or similar programs.

The basic equation satisfied by the transition counterterm action \(\Gamma_{\text{ct,trans}}^{(1)}\) implemented in our model file is

\[
\Gamma_{\text{ct,trans}}^{(1),\text{DRED}} = \Gamma_{\text{ct,trans}}^{(1),\text{DREG}} + \Gamma_{\text{ct,trans}}^{(1)} + \mathcal{O}(D - 4),
\]

where \(\Gamma_{\text{ct,trans}}^{(1),\text{RS}}\) is the generating functional for one-loop one-particle irreducible off-shell Green functions regularized in the scheme RS. In words, the transition counterterms translate off-shell (and IR-finite on-shell) Green functions from DREG to DRED. The terms of \(\mathcal{O}(D - 4)\) are meant to include Green functions with external so-called \(\epsilon\)-scalars, which exist only in DRED but not in DREG, and which are discussed further below. Analytical results for such transition counterterms have already been published.
at the one-loop level for physical parameters in general supersymmetric models in Ref. [9] and at the two-loop level for supersymmetric QCD in Ref. [10].

Since the IR-regularization is unaffected by the transition (11), on-shell Green functions with IR divergences do not become equal by adding the transition counterterms. This is desired since it makes possible to achieve UV regularization by DRED and IR regularization by DREG or vice versa. If e.g. hadronic MSSM processes are computed in this way, manifestly supersymmetric UV renormalization can be easily combined with using the customary \(\overline{\text{MS}}\)-parton distribution functions. The transition counterterms in our model file are thus complementary to the transition rules presented in Ref. [11], which correspond to switching the IR-regularization from DREG to DRED.

The outline of the present paper is as follows. In the remainder of this Introduction we briefly review the relevant status of DREG and DRED. Section 2 describes the installation and usage of the model file. In section 3 we explain the theory behind the transition counterterms and collect analytical results for generic supersymmetric theories. Section 4 is devoted to the specialization to the MSSM and the implementation. In section 5 we show which one- and two-loop tests we have carried out to validate the model file. In section 6 we conclude with further remarks on possible applications. The Appendix contains the full result of the MSSM transition counterterm Lagrangian.

In order to study DRED and its relation to DREG it is useful to decompose the metric tensor appearing e.g. in the propagator numerator of a regularized vector field as

\[ g^{\mu\nu} = \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu}, \]

where \( \hat{g} \) and \( \tilde{g} \) are the metric tensors of the \( D \)-dimensional and \((4 - D) = 2\varepsilon\)-dimensional subspaces. Accordingly, a vector field \( V^{\mu} \) can be decomposed into its \( D \)-dimensional and \( 2\varepsilon \)-dimensional parts \( \hat{V}^{\mu} = \hat{g}^{\mu\nu}V_{\nu}, \tilde{V}^{\mu} = \tilde{g}^{\mu\nu}V_{\nu} \). \( \hat{V}^{\mu} \) behaves as the \( D \)-dimensional gauge field. \( \tilde{V}^{\mu} \), on the other hand, has the interactions and gauge transformations of scalar fields in the adjoint representation with multiplicity \( 2\varepsilon \), hence the name \( \varepsilon \)-scalars [3]. Several subtle problems of DRED have been reported in the literature, most notably Siegel’s inconsistency [12], the violation of unitarity [13], and the violation of infrared factorization [14]. These problems are reviewed and stressed e.g. in Refs. [15, 4]. In recent years, significant progress in the understanding of DRED has been achieved in all desired directions. The current status can be summarized as follows:

- DREG and DRED can both be formulated in a mathematically consistent way, such that any calculation leads to an unambiguous answer [16, 17]. The consistent formulations justify the required formally \( D \)- and 4-dimensional algebraic operations such as \( g^{\mu\nu} = 4 \). But in DRED they forbid to use certain strictly 4-dimensional identities related to assuming that the l.h.s. of Eq. (2) has the explicit form diag(1, −1, −1, −1), thus avoiding Siegel’s inconsistency [12].

- DREG and DRED are equivalent, i.e. for any theory regularized in DREG with certain bare parameters there is a corresponding theory regularized in DRED
with suitably chosen bare parameters and fields for which the S-matrix and Green functions (ignoring IR divergences) are equal \[18\]. This theorem proves the existence of transition counterterms like in Eq. (11) at all orders for any theory. It is only valid if the masses and couplings of $\epsilon$-scalars are renormalized independently, but Ref. \[18\] also shows that the renormalized $\epsilon$-scalar masses and couplings can be chosen at will. It implies that DRED applied in this way preserves unitarity

- At the one-loop level DRED preserves supersymmetry. For a list of references and methods and recent results for higher orders, see e.g. \[15\], \[17\], \[23\], \[24\], \[25\].

- At the one-loop level DREG and DRED are both compatible with QCD factorization of IR singularities. In DRED, the $\epsilon$-scalar gluons have to be treated as independent QCD partons, which contribute to splitting functions of other partons and have their own splitting functions. The resulting difference between (UV-renormalized but IR divergent) DREG and DRED virtual and real next-to-leading order corrections can be cast into simple transition rules \[26\], \[11\].

2. Installation and usage of the model file

In order to use our model file, two files need to be copied into the `Models/` directory of a complete FeynArts installation:

- **LorentzTadpole.gen**: a replacement generic model file which differs from the original `Lorentz.gen` only by the possibility of tadpole (one-point) counterterm Feynman rules.

- **MSSMdreg2dred.mod**: the model file containing all the transition counterterms. It is realized as an Add-on model file, building on the original `MSSMQCD.mod` model file.

The two files can be obtained from the web page [http://iktp.tu-dresden.de/?theory-software](http://iktp.tu-dresden.de/?theory-software) either separately or in a `.tar` archive, together with documentation.

The model file is used just like any other, by the rule `Model->[MSSMdreg2dred]`, which must be accompanied by `GenericModel->[LorentzTadpole]` when using the

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1. The version of DRED used in Ref. \[18\] corresponds to the consistent version of Ref. \[17\] since the multiplicity of the $\epsilon$-scalars is kept arbitrary throughout the computation, which corresponds to the “quasi-4-dimensional” treatment of Ref. \[13\].

2. For recent explicit examples of the need for an independent renormalization of $\epsilon$-scalars see Refs. \[19\], \[20\], \[21\]. For an attempt to rescue a renormalization scheme with equal treatment of gauge fields and $\epsilon$-scalars see Ref. \[22\].

3. When comparing Refs. \[11\] and \[21\] one should note that the term “four-dimensional helicity scheme” (FDH) is used differently in both references. In the former reference it denotes a regularization scheme which differs slightly from DRED, while in the latter reference it implies a certain renormalization prescription which leads to incorrect results. The modifications to FDH proposed in the Conclusions of Ref. \[21\] are in fact in agreement with the way renormalization has been done in Ref. \[11\].
**FeynArts** `InsertFields` function. Since it uses the same naming conventions as the original MSSMQCD model file it is fully compatible not only with **FeynArts** but also with further programs such as FormCalc or TwoCalc.

We now give two code examples for a typical use of the model file, corresponding to the two validation tests described below in section 5. First, the contribution of the electron self energy to Eq. (1) can be computed as follows in a Mathematica session where **FeynArts** and FormCalc are loaded. The one-loop self energy and counterterm diagrams are generated by

```mathematica
top1L = CreateTopologies[1, 1->1, ExcludeTopologies->{Tadpoles, Internal}];
topCT = CreateCTTopologies[1, 1 -> 1, ExcludeTopologies -> {Tadpoles, Internal}];
amp1L = CreateFeynAmp[InsertFields[top1L, F[2,{1}]->F[2,{1}], Model -> MSSMdreg2dred, GenericModel -> LorentzTadpole]];
ampCT = CreateFeynAmp[InsertFields[topCT, F[2,{1}]->F[2,{1}], Model -> MSSMdreg2dred, GenericModel -> LorentzTadpole]];
```

Note that the same model file can be used for the one-loop and counterterm diagrams. For `amp1L` the choice `Model->MSSM` would lead to the same result. With these definitions we can compute

```mathematica
GammaDRED = Plus @@ CalcFeynAmp[amp1L, OnShell -> False, Dimension -> 4, FermionChains -> Dirac] //. Abbr[];
GammaDREG = Plus @@ CalcFeynAmp[amp1L, OnShell -> False, Dimension -> D, FermionChains -> Dirac] //. Abbr[];
GammaCTtrans = Plus @@ CalcFeynAmp[ampCT, OnShell -> False, Dimension -> D, FermionChains -> Dirac] //. Abbr[];
```

After identifying $\text{ME}$ with $M_f[2,1]$ we obtain that $\Gamma_{\text{DRED}} = \Gamma_{\text{DREG}} + \Gamma_{\text{CTtrans}}$, as prescribed by Eq. (1).

As a second example, the one-loop counterterm diagrams for the two-loop selectron self energy such as the one in Fig. 3 are generated by

```mathematica
InsertFields[
    CreateCTTopologies[2, 1 -> 1, ExcludeTopologies -> {WFCorrections, Tadpoles, TadpoleCTs, Internal}],
    S[12,{1,1}] -> S[12,{1,1}],
    InsertionLevel -> Particles,
    Model -> MSSMdreg2dred,
    GenericModel -> LorentzTadpole
];
```

in a Mathematica session where **FeynArts** is loaded. Section 5 below discusses the result for the two-loop selectron self energy.
3. Computation and results for a generic theory

We begin with the transition counterterms in a generic softly broken supersymmetric gauge theory with simple gauge group. Using the same conventions as Ref. [9], the gauge fields, gauginos and chiral supermultiplets are denoted as $V_\mu^a$, $\lambda_a$, $\Phi_i$. The chiral supermultiplets consist of scalar fields $\phi_i$ and two-component Weyl fermions $\psi_i$ and transform under representations $r_i$ under the gauge group. The generators $T^a_{(i)}$ for the representation $r_i$ satisfy $\sum_a(T^a_{(i)} T^a_{(j)})_{kl} = C(r_i) \delta_{ij}$, and the structure constants of the gauge group satisfy $f_{abc} f_{dbc} = C(G) \delta_{ad}$. The superpotential is given by $W = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k$, and the soft breaking gaugino mass term by $L_{\text{soft}} = -\frac{1}{2} m_{\lambda a} \lambda_a \lambda_a + \text{h.c.}$. Further soft breaking terms turn out to be irrelevant for the analysis. The theory is quantized in the usual $R_\xi$-gauges, where $\epsilon$-scalars do not appear in the gauge fixing and ghost terms.

The computation of the transition counterterms differs from the ones in Refs. [9, 10] in that we require equality of all off-shell one-loop Green functions, not only of physical quantities. Fig. 1 shows sample one-loop diagrams which illustrate the structure of the regularization dependence. The only difference between DREG and DRED originates from the additional degrees of freedom of the vector fields in DRED, the $\epsilon$-scalars. In diagrams (a) and (b) the $\epsilon$-scalars give contributions to the numerator algebra of the order $\epsilon = (4 - D)/2$, which combine with the $\frac{1}{\epsilon}$ poles of the loop integral to a finite difference between DREG and DRED. In diagram (c) the scalar–vector coupling is proportional to momenta, which are always regularized in $D$ dimensions, and hence the $\epsilon$-scalars cannot contribute and there is no difference.

We have computed the difference between DREG and DRED for all one-loop one-particle irreducible Green functions and thus determined the transition counterterms, defined by Eq. (1) for the generic theory. They can be written as

$$\Gamma_{ct,\text{trans}}^{(1)} = \int d^4x L_{ct,\text{trans}}$$

where the counterterm Lagrangian $L_{ct,\text{trans}}$ can be obtained from the original Lagrangian by suitable field and parameter renormalizations. We find full agreement with Ref. [9] for the parameter renormalization, while the field renormalization transition counterterms have not been published before. For completeness and convenience we list all results in the following.
• The parameter counterterms respect gauge invariance but not supersymmetry and cannot be directly obtained by multiplicative renormalization of the original Lagrangian. The only parameters of the original Lagrangian are $g$, $Y_{ijk}$, $m_\lambda$ and further soft breaking and gauge fixing parameters. As shown in [9] the transition counterterms require us to distinguish the actual gauge coupling $g$, which appears in all couplings to gauge fields, from the couplings $\hat{g}_i$, which appear in the gaugino interactions with $\phi_i$ and $\psi_i$, and by treating the Yukawa couplings $Y^{ijk}\phi_i\psi_j\psi_k$ as non-symmetric in $(ijk)$. The transition counterterms can then be applied by renormalizing these parameters multiplicatively, in the form $p \rightarrow (1 + \delta Z_p)p$, with

\begin{align}
\delta Z_g &= \frac{g^2}{96\pi^2}C(G) \\
\delta Z_{\hat{g}_i} &= \frac{g^2}{32\pi^2}(C(G) - C(r_i)) \\
\delta Z_{Y^{ijk}} &= \frac{g^2}{32\pi^2}(C(r_j) + C(r_k) - 2C(r_i)) \\
\delta Z_{m_\lambda} &= \frac{g^2}{16\pi^2}C(G)
\end{align}

The quartic scalar interactions cannot be treated in a multiplicative way. Instead, we need to add the counterterm Lagrangian $-\frac{1}{4}\delta\lambda_{ij}^{kl}\phi_i^*\phi_j^*\phi_k^*\phi_l^*$ with

\begin{equation}
\delta\lambda_{ij}^{kl} = -\frac{g^4}{16\pi^2}\{T^a, T^\phi\}_i^k\{T^a, T^\phi\}_j^l + (i \leftrightarrow j).
\end{equation}

Here $T^a$ denotes the block matrices for the generators of the full, reducible representation consisting of all irreducible representations $r_i$. No counterterms corresponding to gauge fixing or soft breaking scalar mass parameters are required.

• The field renormalization counterterms arise from applying the renormalization transformation $\phi \rightarrow (1 + \frac{1}{2}\delta Z_\phi)\phi$ to all fields with

\begin{align}
\delta Z_V &= \frac{g^2}{48\pi^2}C(G) \\
\delta Z_\lambda &= \frac{g^2}{16\pi^2}C(G) \\
\delta Z_{\psi_i} &= \frac{g^2}{16\pi^2}C(r_i)
\end{align}

on the original gauge invariant Lagrangian of the generic theory. No such renormalizations are required for scalar or ghost fields. The field renormalization counterterms do not modify physical quantities but are required to obtain equality between Green functions.

Note that the simple abelian-like relation $\delta Z_g + \frac{1}{2}\delta Z_V = 0$ holds since $\epsilon$-scalars do not interact with gauge fixing and ghost terms. For the same reason, gauge fixing and
ghost terms do not require transition counterterms, and the renormalization transformations must not be applied to gauge fixing and ghost terms.

We close the section with a reminder of several subtleties related to the $\epsilon$-scalars in the equivalence theorem between DREG and DRED \[18\]. First, it is clearly impossible to define transition counterterms for Green functions with external $\epsilon$-scalars. In DREG there are no $\epsilon$-scalars, and in DRED the best we can do is to renormalize Green functions with external $\epsilon$-scalars such that they become finite. At the one-loop level Green functions with external $\epsilon$-scalars can be ignored if we do not desire to regularize IR singularities in DRED \[11\]. However, if Green functions with external $\epsilon$-scalars appear as subgraphs in higher-order graphs their renormalization is vital in order to obtain results consistent with renormalizability, unitarity, and equivalence to DREG \[18, 19, 21\]. In theories with softly broken supersymmetry, the diagram in Fig. 2(a) produces a divergent contribution which can only be cancelled by renormalization of an $\epsilon$-scalar mass. It thus exemplifies that in DRED, $\epsilon$-scalar masses (and in general also couplings) need to be renormalized independently of the corresponding gauge field parameters. Diagram (b) is an example of a one-loop diagram whose finite part depends on the tree-level $\epsilon$-scalar mass in DRED. As shown in \[18\], all choices of the $\epsilon$-scalar mass lead to equivalent results, and it is no restriction to set it to zero at the renormalized level, as done e.g. in the $\overline{\text{DR}'}$ scheme \[27\].

The transition counterterms listed in the present section have been evaluated for zero tree-level $\epsilon$-scalar mass. They are sufficient to satisfy Eq. (1), and they form the basis of our FeynArts model file. $\epsilon$-scalar mass counterterms will be required for a consistent multi-loop renormalization of the MSSM in DRED, but these are not the focus of the present paper.

4. Specialization to the MSSM and the model file

The MSSM is a softly broken supersymmetric $\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)$ gauge theory with chiral superfields for quarks, leptons and two Higgs doublets. The generic result for the transition counterterms can be specialized to the MSSM after applying two generalizations \[9\]. First, the three gauge group factors $G_{1,2,3}$ with gauge couplings $g_{1,2,3}$ can be taken into account by applying the rules of Ref. \[28\], i.e. by replacing $g^2C(G) \rightarrow g_a^2C(G_a)$, $g^2C(r_i) \rightarrow \sum_b g_b^2C_b(r_i)$ in Eqs. (6,4), and replacing $gT^a$ by $g_r T^a_{(r)}$. 

![Figure 2: Two diagrams illustrating the independent role of $\epsilon$-scalars. Dashed lines denote the scalar component $\phi$ of a chiral supermultiplet; dotted lines denote $\epsilon$-scalars. (a) Scalar contribution to the $\epsilon$-scalar self energy, which requires renormalization of the $\epsilon$-scalar mass; (b) $\epsilon$-scalar contribution to a scalar self energy, which depends on the tree-level $\epsilon$-scalar mass.](image)
in Eq. (5), where \( T^a_{\alpha} \) is the \( G \) generator, and carrying out a double sum over the gauge groups. Second, the dimensionful \( \mu \)-parameter in the MSSM superpotential can be treated like a Yukawa coupling to a spurion field for which \( C(r) = 0 \).

We have written a Mathematica program which implements all these rules and generates the full transition counterterm Lagrangian for a generic model, and we have specialized this program to the MSSM. Using this program we have obtained the MSSM transition counterterm Lagrangian in interaction eigenstates, in a form appropriate for input to the package FeynRules \[26\]. This Lagrangian is explicitly reprinted in Appendix A.

As a second step we have implemented the spontaneous symmetry breaking and the mixing of interaction to mass eigenstate fields in the MSSM as an input file for FeynRules. Here we have used the same conventions for mixing matrices as the original FeynArts MSSM model file \[7\], and we have followed the restriction to neglect family mixing in the sfermion sector. In contrast to the original model file, we also neglect family mixing in the fermion sector, i.e. we set the CKM matrix to the unit matrix. Then we have used FeynRules to generate the transition counterterm Lagrangian expressed in terms of mass eigenstates.

After spontaneous symmetry breaking and inserting Higgs vacuum expectation values, 3- and 4-point interactions involving Higgs fields generate also 2- and 1-point interactions. Because of this, the MSSM transition counterterm Lagrangian contains not only gaugino but also fermion, vector and scalar self energy transition counterterms, as well as scalar tadpole transition counterterms. A complete transition from DREG to DRED requires all these counterterms. Even in a renormalization scheme where tadpoles are renormalized to zero, tadpole graphs and hence tadpole transition counterterms are needed as they contribute to self energy or other counterterm insertions.

In order to take these necessary counterterms into account, several modifications to FeynRules and FeynArts had to be made.

- The possibility to generate Feynman rules for self energy and tadpole counterterms has been implemented in FeynRules.
- The possibility to allow tadpole counterterms and generate tadpole counterterm diagrams has been implemented in FeynArts in terms of a replacement for the generic Lorentz.gen model file. We call our generic model file LorentzTadpole.gen.

Finally, with this procedure we have generated the FeynArts model file MSSMpreg2dred.mod for all the transition counterterms of the MSSM. It has the form of an Add-on model file, and it has to be used together with our LorentzTadpole.gen generic model file. The usage has been described in section \[2\]. A lengthy collection of transition Feynman rules contained in this model file can obtained from the web page [http://iktp.tu-dresden.de/?theory-software](http://iktp.tu-dresden.de/?theory-software).
5. Validation

We have validated our model file in two ways. As a first and direct test we have evaluated the contribution to Eq. (1) of every MSSM Green function for which transition counterterms exist. The computation is analogous to the one described in section 2 for the electron self energy, suitably generalized and automatized. On the one hand, FeynArts/FormCalc was used to compute the one-loop contributions both in DREG and DRED, and the difference between DREG and DRED in the limit $D \to 4$ was evaluated. On the other hand, the corresponding contributions of the transition counterterms were computed using FeynArts/FormCalc with our transition counterterm model file. We found full agreement, thus explicitly verifying Eq. (1). The source code of this program, validation.m, can be obtained from the web page [http://iktp.tu-dresden.de/?theory-software](http://iktp.tu-dresden.de/?theory-software). It is easily adaptable, provides a comprehensive check and illustrates the usage further. It requires only a FeynArts and FormCalc installation.

As a second, less comprehensive but more intricate test we have considered an MSSM two-loop computation with renormalization of the one-loop subdiagrams. We have used FeynArts/TwoCalc [30] to compute the selectron self energy $\Gamma^{(2)}_{\tilde{e}\tilde{e}}$ at the two-loop level in DREG and DRED. For simplicity we have carried out the calculation numerically, for an MSSM parameter point where all mixing matrices are non-trivial, and we have expanded in the external momentum being small. It turned out that the dependence of the difference on the external momentum was not a polynomial of second degree. Fig. 3(a) shows a sample diagram which contributes in this way. However, after adding one-loop diagrams $\Gamma^{(1+ct,DREG)}_{\tilde{e}\tilde{e}}$ with insertions of the one-loop transition counterterms, see Fig. 3(b), we obtain that

$$\Gamma^{(2,DRED)}(p^2) = \Gamma^{(2,DREG)}(p^2) + \Gamma^{(1+ct,DREG)}_{\tilde{e}\tilde{e}}(p^2) + a + bp^2 + O(D - 4).$$

(7)

Here, the polynomial $(a + bp^2)$ is of a form that could be absorbed by a local mass and field renormalization counterterm, corresponding to a two-loop transition counterterm $\Gamma^{(2)}_{ct,trans}$. This is exactly what is expected from the general statement that DREG and DRED are equivalent and one can find transition counterterms at all orders. Since many transition counterterms contribute to $\Gamma^{(1+ct,DREG)}_{\tilde{e}\tilde{e}}(p^2)$, and since the two-loop diagrams involve up to $1/(D - 4)^2$ divergences, this constitutes a non-trivial test of our counterterm model file.

6. Conclusions

In the present paper we have introduced a FeynArts model file MSSMreg2dred.mod, which implements MSSM one-loop transition counterterms from DREG to DRED. This model file in particular contains all required

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4In principle, such a test requires a subloop renormalization in DRED, rendering subdiagrams involving external $\epsilon$-scalars finite. In the MSSM this amounts to adding one-loop counterterm diagrams with appropriate $\epsilon$-mass counterterm insertions to the DRED result $\Gamma^{(2,DRED)}_{\tilde{e}\tilde{e}}(p^2)$. However, in the case of the selectron self energy these extra counterterm diagrams do not contribute since there is no selectron–selectron–$\epsilon$-scalar coupling and since we set the tree-level $\epsilon$-scalar mass to zero.
Figure 3: Sample two-loop diagram for the selectron self energy for which the DREG and DRED results differ by more than a polynomial of the form \((a + bp^2)\), and a corresponding one-loop counterterm diagram with insertion of a transition counterterm.

supersymmetry-restoring counterterms for DREG, and it thus allows to use DREG for MSSM loop calculations without violating supersymmetry. It allows to easily compare calculations, or to combine building blocks computed in different regularization schemes.

The present paper also contains complete results for the transition counterterms both in a generic supersymmetric model and in the MSSM, including field renormalization counterterms. We have shown that the gauge fixing and ghost sectors do not require such counterterms. These results and the model file should be valuable for a detailed theoretical understanding of DREG and DRED and as tools for practical calculations.

There are two main areas where the use of the transition counterterm model file can simplify calculations. One is the computation of MSSM processes in the presence of IR divergent QCD corrections. Here it is advantageous to regularize the IR divergences in DREG, both because of the existing MS-parton distribution functions (PDFs) and the complicated structure of factorization in DRED [14, 11, 26]. In the past, e.g. in Refs. [5, 6], DREG has been used for the supersymmetric QCD-part of the calculations, so that MS-PDFs could be used, and the required supersymmetry-restoring counterterm [9] has been added by hand. This procedure required a mixed DRED/DREG regularization in Ref. [6], where also electroweak corrections were taken into account.

With our model file at hand, one can now regularize the full MSSM process in DREG and then automatically add the transition counterterms. In this way UV-regularization by DRED and IR-regularization by DREG are combined in a straightforward way. Supersymmetry is manifest and e.g. the \(\overline{\text{DR}}\)-definition of MSSM parameters can be used, and simultaneously the IR divergences have the usual, simple DREG-structure, MS-PDFs can be employed and real corrections can be simply computed in DREG.

A second area where the transition counterterm model file can be helpful are multi-loop calculations which would require the renormalization of \(\epsilon\)-scalars in DRED. As mentioned above, even though DRED preserves supersymmetry, \(\epsilon\)-scalar masses have to be renormalized independently in order to guarantee correct results beyond the one-loop level. For a recent three-loop example and a description of the subtleties involved see Ref. [20]. If instead DREG is used together with the transition counterterms, the correct DRED result corresponding to zero renormalized \(\epsilon\)-scalar mass is directly
obtained. To the extent that no genuine two-loop transition counterterms are needed this constitutes a potentially simpler alternative procedure.

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Appendix A. MSSM transition counterterm Lagrangian

In the following we provide the explicit form of the MSSM transition counterterm Lagrangian, expressed in terms of interaction eigenstate fields.

The MSSM chiral superfields are denoted as $Q_{f,i}, \bar{u}_{f,c}, \bar{d}_{f,c}, l_{f,i}, \tilde{e}_i, H_{di}, H_{ui}$, where $f, i, c$ are family indices, SU(2) doublet indices and colour indices, respectively. The scalar and fermionic components are denoted as $\phi$ and $\Psi$, with appropriate indices. The vector fields and gauginos are denoted as $V_a$ and $\tilde{\lambda}$, with indices $G, W, B$ for SU(3), SU(2), U(1), respectively, and an adjoint group index $a$, if appropriate. $T^a$ and $\tau^a/2$ are the fundamental SU(3) and SU(2) generators, and $f^{abc}$ and $\varepsilon^{abc}$ are the structure constants. The gauge and family-diagonal Yukawa couplings are denoted as $g_{3,2,1}$, $y_{f_1,f_2}^{c,d,u}$, and the gaugino masses as $m_{G,W,B}$.

$$\mathcal{L}_{\text{ct,trans}}^{\text{MSSM}} = \mathcal{L}_{\text{2-pt}}^{\text{trans}} + \mathcal{L}_{\text{gaugino}}^{\text{trans}} + \mathcal{L}_{\text{gauge}}^{\text{trans}} + \mathcal{L}_{\text{Yukawa}}^{\text{trans}} + \mathcal{L}_{\text{quartic}}^{\text{trans}} \quad (A.1)$$

$$\mathcal{L}_{\text{2-pt}}^{\text{trans}} = \frac{g_2^2}{16\pi^2} \left( i \tilde{\lambda}_W^a \gamma^\mu \partial_\mu \tilde{\lambda}_W^a - 2m_{\lambda_W} \tilde{\lambda}_W^a P_L \tilde{\lambda}_W^a - 2m_{\lambda_W}^* \tilde{\lambda}_W^a P_R \tilde{\lambda}_W^a \right)$$

$$+ \frac{3g_2^2}{32\pi^2} \left( i \tilde{\lambda}_G^a \gamma^\mu \partial_\mu \tilde{\lambda}_G^a - 2m_{\lambda_G} \tilde{\lambda}_G^a P_L \tilde{\lambda}_G^a - 2m_{\lambda_G}^* \tilde{\lambda}_G^a P_R \tilde{\lambda}_G^a \right)$$

$$+ i \left( g_1^2 + 27g_2^2 + 48g_3^2 \right) \frac{576\pi^2}{1024} \left( \bar{\Psi}_{Q_{f_1},i_1,c_1} \gamma_\mu P_L \Psi_{Q_{f_1},i_1,c_1} \right)$$

$$+ i \left( g_2^2 + 3g_3^2 \right) \frac{64\pi^2}{1024} \left( \bar{\Psi}_{l_{f_1},i_1} \gamma_\mu \partial_\mu P_L \Psi_{l_{f_1},i_1} \right)$$

$$+ i \left( g_2^2 + 3g_3^2 \right) \frac{64\pi^2}{1024} \left( \bar{\Psi}_{u_{d_1},i_1} \gamma_\mu \partial_\mu P_L \Psi_{u_{d_1},i_1} \right)$$

$$+ i \left( g_1^2 + 12g_3^2 \right) \frac{36\pi^2}{144\pi^2} \left( \bar{\Psi}_{e_{f_1},c_1} \gamma_\mu \partial_\mu P_L \Psi_{e_{f_1},c_1} \right)$$

$$+ i \frac{g_1^2}{16\pi^2} \left( \bar{\Psi}_{e_{f_1}} \gamma_\mu \partial_\mu P_L \Psi_{e_{f_1}} \right)$$

$$+ \frac{g_1^2 + 3g_3^2}{32\pi^2} \left( \mu \epsilon_{1i_2} \overline{\Psi}_{H_d}^a_{i_1} P_L \Psi_{H_u i_2} + \mu^* \epsilon_{1i_2}^* \overline{\Psi}_{H_u i_2} P_R (\Psi_{H_d}^a)^*_{i_1} \right)$$

$$- \frac{g_2^2}{48\pi^2} \left( (\partial_\mu V_W^a_{\mu})(\partial^\nu V_{W,\nu}^a) - (\partial^\nu V_W^a_{\nu})(\partial_\mu V_{W,\mu}^a) \right)$$
\[ L_{\text{trans}} = - \frac{g_3^3}{32\pi^2} \left( (\partial_{\mu} V_{G_{\mu}}^{a}) (\partial^{\nu} V_{G_{\mu}}^{\nu,a}) - (\partial_{\mu} V_{G_{\mu}}^{a}) (\partial^{\nu} V_{G_{\mu}}^{a}) \right) \]

\[ L_{\text{gaugino}} = - \frac{g_3^3}{4\sqrt{2\pi}} \left( \bar{\Psi}_{f_{1,1}} P_R \tilde{\lambda}_W \phi_{f_{1,1}} + \bar{\lambda}_W P_L \Psi_{f_{1,1}} \phi_{f_{1,1}}^{*} \right) \tau^{a}_{i_{1}i_{2}} \]

\[ - \frac{g_3^3}{4\sqrt{2\pi}} \left( \bar{\Psi}_{H_{a1}} P_R \tilde{\lambda}_W \phi_{H_{a1}} + \bar{\lambda}_W P_L \Psi_{H_{a1}} \phi_{H_{a1}}^{*} \right) \tau^{a}_{i_{1}i_{2}} \]

\[ - \frac{g_3^3}{4\sqrt{2\pi}} \left( \bar{\Psi}_{H_{d1}} P_R \tilde{\lambda}_W \phi_{H_{d1}} + \bar{\lambda}_W P_L \Psi_{H_{d1}} \phi_{H_{d1}}^{*} \right) \tau^{a}_{i_{1}i_{2}} \]

\[ - \frac{g_3^3}{4\sqrt{2\pi}} \left( \bar{\lambda}_W P_L \Psi^{C}_{u_{f_{1,1}} c_{1}} \phi_{u_{f_{1,1}} c_{1}} + \tilde{\lambda}_W P_R \Psi^{C}_{f_{1,1} c_{1}} \phi_{f_{1,1} c_{1}}^{*} \right) T^{a}_{c_{1}c_{2}} \]

\[ + \frac{3g_3^3}{8\sqrt{2\pi}} \left( \bar{\lambda}_U P_L \Psi^{C}_{u_{f_{1,1}} c_{1}} \phi_{u_{f_{1,1}} c_{1}} + \tilde{\lambda}_U P_R \Psi^{C}_{f_{1,1} c_{1}} \phi_{f_{1,1} c_{1}}^{*} \right) T^{a}_{c_{1}c_{2}} \]

\[ + \frac{3g_3^3}{8\sqrt{2\pi}} \left( \bar{\lambda}_W P_L \Psi^{C}_{d_{f_{1,1}} c_{1}} \phi_{d_{f_{1,1}} c_{1}} + \tilde{\lambda}_W P_R \Psi^{C}_{f_{1,1} c_{1}} \phi_{f_{1,1} c_{1}}^{*} \right) T^{a}_{c_{1}c_{2}} \]

\[ L_{\text{trans}} = - \frac{g_1}{3456\pi^2} \left( \bar{\Psi}_{Q_{f_{1,1} c_{1}} \gamma_{\mu}} P_{L} \Psi_{Q_{f_{1,1} c_{1}} \gamma_{\mu}} V_{B}^{\mu} \right) \]

\[ + \frac{g_1^3}{128\pi^2} \left( \bar{\Psi}_{f_{1,1} c_{1}} \gamma_{\mu} P_{L} \Psi_{f_{1,1} c_{1}} V_{B}^{\mu} \right) \]

\[ - \frac{g_1}{128\pi^2} \left( \bar{\Psi}_{H_{a1} c_{1}} \gamma_{\mu} P_{L} \Psi_{H_{a1} c_{1}} V_{B}^{\mu} \right) \]

\[ + \frac{g_1}{128\pi^2} \left( \bar{\Psi}_{H_{d1} c_{1}} \gamma_{\mu} P_{L} \Psi_{H_{d1} c_{1}} V_{B}^{\mu} \right) \]

\[ + \frac{3g_1^3}{54\pi^2} \left( \bar{\Psi}^{C}_{u_{f_{1,1} c_{1}}} \gamma_{\mu} P_{L} \Psi^{C}_{u_{f_{1,1} c_{1}}} V_{B}^{\mu} \right) \]

\[ - \frac{g_1}{432\pi^2} \left( \bar{\Psi}^{C}_{d_{f_{1,1} c_{1}}} \gamma_{\mu} P_{L} \Psi^{C}_{d_{f_{1,1} c_{1}}} V_{B}^{\mu} \right) \]

\[ - \frac{g_1^3}{16\pi^2} \left( \bar{\Psi}^{C}_{e_{f_{1} c_{1}}} \gamma_{\mu} P_{L} \Psi^{C}_{e_{f_{1} c_{1}}} V_{B}^{\mu} \right) \]

\[ - \frac{g_2}{3456\pi^2} \left( \bar{\Psi}_{Q_{f_{1,1} c_{1}} \gamma_{\mu}} P_{L} \Psi_{Q_{f_{1,1} c_{1}} \gamma_{\mu}} V_{W}^{\mu} \right) \]

\[ + \frac{g_2}{64\pi^2} \left( \bar{\Psi}_{f_{1,1} c_{1}} \gamma_{\mu} P_{L} \Psi_{f_{1,1} c_{1}} V_{W}^{\mu} \right) \]

\[ - \frac{g_2}{64\pi^2} \left( \bar{\Psi}_{H_{a1} c_{1}} \gamma_{\mu} P_{L} \Psi_{H_{a1} c_{1}} V_{W}^{\mu} \right) \]

\[ - \frac{g_2}{64\pi^2} \left( \bar{\Psi}_{H_{d1} c_{1}} \gamma_{\mu} P_{L} \Psi_{H_{d1} c_{1}} V_{W}^{\mu} \right) \]

\[ - \frac{g_2}{3456\pi^2} \left( \bar{\Psi}_{Q_{f_{1,1} c_{1}} \gamma_{\mu}} P_{L} \Psi_{Q_{f_{1,1} c_{1}} \gamma_{\mu}} V_{G_{\mu}}^{\mu} \right) \]

\[ + \frac{g_3}{576\pi^2} \left( \bar{\Psi}^{C}_{u_{f_{1,1} c_{1}}} \gamma_{\mu} P_{L} \Psi^{C}_{u_{f_{1,1} c_{1}}} V_{G_{\mu}}^{\mu} \right) \]

\[ - \frac{g_3}{576\pi^2} \left( \bar{\Psi}_{H_{a1} c_{1}} \gamma_{\mu} P_{L} \Psi_{H_{a1} c_{1}} V_{G_{\mu}}^{\mu} \right) \]

\[ - \frac{g_3}{576\pi^2} \left( \bar{\Psi}_{H_{d1} c_{1}} \gamma_{\mu} P_{L} \Psi_{H_{d1} c_{1}} V_{G_{\mu}}^{\mu} \right) \]

\[ - \frac{g_3}{36\pi^2} \left( \bar{\Psi}^{C}_{u_{f_{1,1} c_{1}}} \gamma_{\mu} P_{L} \Psi^{C}_{u_{f_{1,1} c_{1}}} V_{G_{\mu}}^{\mu} \right) \]
\[
\mathcal{L}^\text{Yukawa}_{\text{trans}} = \frac{1}{32\pi^2} \left\{ \bar{\epsilon}_{i_1i_2} y_{f_2f_3} \left[ (g_1^2 - 3g_2^2) \overline{\psi}_{f_2i_2} P_R(\Psi_{H_d})_{i_1}^C \phi_{f_3} + 2g_1^2 \left( \overline{\psi}_{f_2i_2} P_R(\Psi_{H_d})_{i_1}^C \phi_{f_3} + \overline{\psi}_{f_2i_2} P_R \psi_{f_2i_2} \phi_{H_d}^* \right) \right] \right.
\]
\[
- 2g_1^2 \left( \overline{\psi}_{f_2i_2} P_R(\Psi_{H_d})_{i_1}^C \phi_{f_3} + \overline{\psi}_{f_2i_2} P_R \psi_{f_2i_2} \phi_{H_d}^* \right) \right\}
\]
\[
- \frac{1}{288\pi^2} \left\{ \bar{\epsilon}_{i_1i_2} y_{f_2f_3} \left[ 3 (g_1^2 + 9g_2^2) \overline{\psi}_{f_2i_2} P_R(\Psi_{H_d})_{i_1}^C \phi_{f_3} + 2g_1^2 \left( \overline{\psi}_{f_2i_2} P_R(\Psi_{Q})_{i_1}^C \phi_{f_3} + \overline{\psi}_{f_2i_2} P_R \psi_{f_2i_2} \phi_{H_d}^* \right) \right] \right.
\]
\[
- 2 (g_1^2 - 24g_2^2) \left( \overline{\psi}_{f_2i_2} P_R(\Psi_{Q})_{i_1}^C \phi_{f_3} + \overline{\psi}_{f_2i_2} P_R \psi_{f_2i_2} \phi_{H_d}^* \right) \right\}
\]
\[
- \frac{1}{288\pi^2} \left\{ \bar{\epsilon}_{i_1i_2} y_{f_2f_3} \left[ 4 (g_1^2 + 12g_3^2) \overline{\psi}_{f_2i_2} P_R(\Psi_{H_u})_{i_1}^C \phi_{f_3} + 12g_1^2 \overline{\psi}_{f_2i_2} P_R(\Psi_{H_u})_{i_1}^C \phi_{f_3} + 3 (g_1^2 - 9g_2^2) \overline{\psi}_{f_2i_2} P_R(\Psi_{Q})_{i_1}^C \phi_{f_3} \right] \right.
\]
\[
+ \epsilon_{i_1i_2} y_{f_2f_3} \left[ 4 (g_1^2 + 12g_3^2) \overline{\psi}_{f_2i_2} P_R(\Psi_{Q})_{i_1}^C \phi_{f_3} \right] \right\}
\]
\[
\mathcal{L}^\text{trans}_{\text{quartic}} = \frac{1}{5184\pi^2} \phi_{d_1f_1, e_1} \phi_{d_2f_2, e_2} \left[ 9g_3^2 (8g_1^2 + 15g_3^2) \phi_{d_1f_1, e_2} \phi_{d_2f_2, e_1} + (8g_1^4 - 24g_1^2 g_2^2 + 99g_3^2) \phi_{d_1f_1, e_1} \phi_{d_2f_2, e_2} \right]
\]
\[
+ \frac{g_1^4}{144\pi^2} \phi_{d_1f_1, e_1} \phi_{d_2f_2, e_2} \phi_{H_d, i_1} \phi_{H_d, i_2} + \frac{g_1^4}{144\pi^2} \phi_{d_1f_1, e_1} \phi_{d_2f_2, e_2} \phi_{H_d, i_1} \phi_{H_d, i_2} \]
\[\begin{align*}
&+ \frac{g_1^4}{144\pi^2} \phi_{d f_1, c_1} \phi_{d f_1, c_1} \phi_{f_2, c_2} \phi_{f_2, c_2} \\
&+ \frac{1}{2592\pi^2} \phi_{d f_1, c_1} \phi_{Q f_2, c_2} \left[9g_5^2 \left(15g_3^2 - 4g_1^2\right) \phi_{d f_1, c_1} \phi_{Q f_2, c_2} + (2g_1^4 + 12g_3^2g_1^2 + 99g_4^4) \phi_{d f_1, c_1} \phi_{Q f_2, c_2}\right] \\
&+ \frac{1}{2592\pi^2} \phi_{d f_1, c_1} \phi_{a f_2, c_2} \left[9g_5^2 \left(15g_3^2 - 16g_2^2\right) \phi_{d f_1, c_2} \phi_{a f_2, c_2} + (32g_1^4 + 48g_3^2g_1^2 + 99g_4^4) \phi_{d f_1, c_2} \phi_{a f_2, c_2}\right] \\
&+ \frac{g_1^4}{36\pi^2} \phi_{f_2, c_2} \phi_{f_2, c_2} \phi_{e f_1} \phi_{e f_1} + \frac{g_1^4}{8\pi^2} \phi_{e f_1} \phi_{e f_1} \phi_{e f_2} \phi_{e f_2} \\
&+ \frac{g_1^4}{16\pi^2} \phi_{e f_1} \phi_{e f_1} \phi_{H_{a i_2}} \phi_{H_{a i_2}} + \frac{g_1^4}{16\pi^2} \phi_{e f_1} \phi_{e f_1} \phi_{H_{a i_2}} \phi_{H_{a i_2}} \\
&+ \frac{g_1^4}{16\pi^2} \phi_{e f_1} \phi_{e f_1} \phi_{Q f_2, c_2} \phi_{Q f_2, c_2} + \frac{g_1^4}{144\pi^2} \phi_{e f_1} \phi_{e f_1} \phi_{Q f_2, c_2} \phi_{Q f_2, c_2} \\
&+ \frac{g_1^4}{9\pi^2} \phi_{e f_1} \phi_{e f_1} \phi_{a f_2, c_2} \phi_{a f_2, c_2} \\
&+ \frac{1}{128\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_4} \delta_{i_2 i_4} + 4g_2^2 \left(2g_1^2 \left(\tau_{i_1 i_4} \tau_{i_2 i_4} \right) + g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_4} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{64\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_3} \delta_{i_2 i_4} + 4g_2^2 \left(2g_1^2 \left(\tau_{i_1 i_3} \tau_{i_2 i_4} \right) + g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_3} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{64\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_5} \delta_{i_2 i_4} + 12g_2^2 \left(-2g_1^2 \left(\tau_{i_1 i_5} \tau_{i_2 i_4} \right) + 3g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_5} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{128\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_3} \delta_{i_2 i_4} + 4g_2^2 \left(2g_1^2 \left(\tau_{i_1 i_3} \tau_{i_2 i_4} \right) + g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_3} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{64\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_3} \delta_{i_2 i_4} + 12g_2^2 \left(-2g_1^2 \left(\tau_{i_1 i_3} \tau_{i_2 i_4} \right) + 3g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_3} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{128\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_5} \delta_{i_2 i_4} + 12g_2^2 \left(2g_1^2 \left(\tau_{i_1 i_5} \tau_{i_2 i_4} \right) + g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_5} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{64\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_3} \delta_{i_2 i_4} + 12g_2^2 \left(-2g_1^2 \left(\tau_{i_1 i_3} \tau_{i_2 i_4} \right) + 3g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_3} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{128\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \phi_{Q f_2, c_2} \\
&\times \left[g_4^4 \delta_{i_1 i_5} \delta_{i_2 i_4} + 12g_2^2 \left(2g_1^2 \left(\tau_{i_1 i_5} \tau_{i_2 i_4} \right) + g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_5} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{64\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_3} \delta_{i_2 i_4} + 12g_2^2 \left(-2g_1^2 \left(\tau_{i_1 i_3} \tau_{i_2 i_4} \right) + 3g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_3} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{128\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \phi_{Q f_2, c_2} \\
&\times \left[g_4^4 \delta_{i_1 i_5} \delta_{i_2 i_4} + 12g_2^2 \left(2g_1^2 \left(\tau_{i_1 i_5} \tau_{i_2 i_4} \right) + g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_5} \tau_{i_2 i_4}\right)\right)\right] \\
&+ \frac{1}{64\pi^2} \phi_{H_{a i_3}} \phi_{H_{a i_3}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \phi_{H_{a i_4}} \\
&\times \left[g_4^4 \delta_{i_1 i_3} \delta_{i_2 i_4} + 12g_2^2 \left(-2g_1^2 \left(\tau_{i_1 i_3} \tau_{i_2 i_4} \right) + 3g_2^2 \left(\{\tau^a, \tau^b\}_{i_1 i_3} \tau_{i_2 i_4}\right)\right)\right]
\end{align*}\]
\begin{align*}
&\times \left[ g_1^4 \delta_{i1,i3} \delta_{i2,i4} + 4g_2^2 \left( g_2^2 \left( \tau^a_{i1,i3} \tau^a_{i2,i4} \right) + g_2^2 \left\{ \left( \tau^a, \tau^b \right)_{i1,i3} \left( \tau^a, \tau^b \right)_{i2,i4} \right\} \right) \right] \\
&+ \frac{1}{576\pi^2} \phi^{*}_{f_1,i3} \phi^{*}_{f_1,i1} \phi^{*}_{Q_f_{2,i4},c_2} \phi^{*}_{Q_f_{2,i2},c_2} \\
&\times \left[ g_1^4 \delta_{i1,i3} \delta_{i2,i4} + 12g_2^2 \left( \left( \tau^a_{i1,i3} \tau^a_{i2,i4} \right) + 3g_2^2 \left\{ \left( \tau^a, \tau^b \right)_{i1,i3} \left( \tau^a, \tau^b \right)_{i2,i4} \right\} \right) \right] \\
&+ \frac{g_1^4}{36\pi^2} \phi_{H_{i1,2}} \phi^{*}_{H_{i1,2}} \phi_{u_{f_1,c_1},c_1} \phi_{u_{f_1,c_1},c_1} \\
&+ \frac{g_1^4}{36\pi^2} \phi_{f_2,i2} \phi^{*}_{f_2,i2} \phi_{u_{f_1,c_1},c_1} \phi_{u_{f_1,c_1},c_1} \\
&+ \frac{1}{2592\pi^2} \phi_{u_{f_1,c_1},c_1} \phi_{Q_f_{2,i2},c_2} \left[ \left( 8g_2^4 - 24g_2^3g_1^2 + 99g_4^4 \right) \phi^{*}_{f_{1,c_2},c_2} \phi_{Q_f_{2,i2},c_2} \\
&\quad + 9g_3^2 \left( 8g_1^4 + 15g_5^2 \right) \phi^{*}_{f_{1,c_2},c_2} \phi_{u_{f_2,c_2}} \right] \\
&+ \frac{1}{5184\pi^2} \phi_{u_{f_1,c_1},c_1} \phi_{u_{f_2,c_2}} \left[ 9g_3^2 \left( 32g_1^4 + 15g_5^2 \right) \phi^{*}_{u_{f_1,c_2}} \phi_{u_{f_2,c_1}} \\
&\quad + 128g_1^4 - 96g_2^3g_1^2 + 99g_4^4 \right) \phi^{*}_{u_{f_1,c_1}} \phi_{u_{f_2,c_2}} \right] \\
&+ \frac{1}{10368\pi^2} \phi_{Q_f_{1,i3},c_3,c_3} \phi^{*}_{Q_f_{1,i1},c_1} \phi_{Q_f_{2,i4},c_4} \phi^{*}_{Q_f_{2,i2},c_2} \\
&\times \left[ 18g_3^2 \left( 2g_1^2 + 15g_5^2 \right) \delta_{i1,i3} \delta_{i2,i4} \delta_{c_2,c_3} \delta_{c_1,c_4} \\
&\quad + \left( g_1^4 - 12g_1^2g_3^2 + 198g_4^4 \right) \delta_{i1,i3} \delta_{c_2,c_3} \delta_{c_1,c_4} \\
&\quad + 18g_3^2 \left( 48g_2^3g_1^2 \delta_{c_2,c_3} \delta_{c_1,c_4} \tau^a_{i1,i3} \tau^a_{i2,i4} + 4g_1^2g_2^3 \delta_{c_2,c_3} \tau^a_{i1,i3} \tau^a_{i2,i4} \right) \\
&\quad + 9g_2^6 \delta_{c_1,c_2} \delta_{c_2,c_4} \delta_{c_2,c_4} \left\{ \left( \tau^a, \tau^b \right)_{i1,i3} \left( \tau^a, \tau^b \right)_{i2,i4} + 2 \left( \tau^a, \tau^b \right)_{i2,i4} \left( \tau^a, \tau^b \right)_{i1,i3} \right\} \right] \right]
\end{align*}

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