RODA: Reverse Operation based Data Augmentation for Solving Math Word Problems

Qianying Liu,¹ Wenyu Guan, ² Sujian Li, ² Fei Cheng, ¹ Daisuke Kawahara ³ and Sadao Kurohashi ¹

Abstract—Automatically solving math word problems is a critical task in the field of natural language processing. Recent models have reached their performance bottleneck and require more high-quality data for training. We propose a novel data augmentation method that reverses the mathematical logic of math word problems to produce new high-quality math problems and introduce new knowledge points that can benefit learning the mathematical reasoning logic. We apply the augmented data on two SOTA math word problem solving models and compare our results with a strong data augmentation baseline. Experimental results show the effectiveness of our approach.

Index Terms—Math Word Problems, Question Answering, Data Augmentation

I. INTRODUCTION

SOLVING Math Word Problems (MWPs) is the task that infers a mathematical expression and the final answer from the natural language description of a math problem, which has been crucial due to its importance of numerically reasoning in natural language (NLP) processing [1], [2]. Figure 1 shows two examples which include math word problems and their corresponding solution equations and results.

Various manually constructed MWP datasets have advanced the development of MWP solving has been advanced, including Math23K [3] and Dolphin18k [3]. With these datasets as training data, the recent mainstream approaches are based on sequence-to-sequence (seq2seq) neural networks (NN), where a math word problem serves as the input sequence and a math equation as the output sequence. However, these approaches have reached their performance bottleneck as the size of the available data is still far from enough. Thus, how to augment data is the primary consideration for improving the performance of MWP solving.

Data augmentation for NLP has been a critical and challenging research topic, especially in the case of MWP solving. Traditional data augmentation methods are mainly based on paraphrase generation, such as EDA [4], which randomly edits words, and back translation [5], which translates the example to an auxiliary language and then back to the original language to paraphrase the example. These methods have two key drawbacks when applied to MWPs.

First, due to the preciseness of mathematics, the text description of each math word problem must be absolutely rigorous, so that even missing only one keyword could make the information incomplete and the problem unsolvable. As shown in Figure 1, all key information points of the problem marked in bold cover nearly one-third of the text content. With any of the key points missed, the problem would become meaningless. Therefore, in the task of MWP solving, paraphrase-based methods may potentially produce noise, mislead the model and degrade the performance. Second, the critical difficulty of MWPs solving lies in linking mathematical knowledge points and reasoning logic with examples. Here the mathematical knowledge points refer to formulae that can solve the MWP, such as \( \text{area}_{\text{circle}} = \pi \times \text{radius}^2 \), while the reasoning logic refers to the abstractive understanding of the relationship between these concepts, which is the desired ability of the MWP solving model. Paraphrase-based methods only introduce lexicon level variance but do not generate new examples on the mathematical level. Therefore, the performance improvement of paraphrase-based data augmentation methods in the task of MWP solving is limited. Recently, a new research line of logic-based data augmentation uses task-specific logic to construct new examples for Natural Language Inference and Question Answering [6]–[8]. These methods introduce reliable rule-based logic, which does not produce noise, to generate logic-level new examples. However, existing methods are task specifically designed and cannot be directly applied to MWP solving. Meanwhile, these methods are based on one or two-step simple logic operations and are relatively simple.

In this paper, unlike the previous practice of data augmentation, we instead simulate human double-checking and propose an MWP generation method to obtain more high-quality MWPs that are inferred through the reverse operation of the original problems. As we observe, when humans solve MWPs, a common technique to guarantee accuracy is to perform double-checking on the problem. As shown in Figure 1, MWP1 asks about the time that the two cars have spent before meeting each other, and its solution implicitly holds the mathematical knowledge point that \( \text{time} = \frac{\text{distance}}{\text{speed}} \). With this knowledge, we can get the equation \( x = \frac{660}{(32 + 34)} \), where 10 is the final answer of \( x \) and denotes the spent time. To verify the correctness of the solution of MWP1, humans conceive a reverse problem MWP2 with the mathematical knowledge point \( \text{distance} = \text{time} \times \text{speed} \), which takes the answer of MWP1 (i.e., \( x = 10 \)) as a known quantity and the distance \( x' \) as unknown. We can efficiently produce a new math word problem in the reverse operation and ensure its
solution quality. Concretely, we can seek one known quantity (e.g., 660 in MWP1) in the original problem and change its surrounding declarative description into an interrogative sentence (i.e., the last sentence in MWP2). At the same time, we change the original interrogative description about the unknown quantity into a declarative statement with the original solution (e.g., \( x = 10 \)) substituted (i.e., the next to last sentence in MWP2). Then, with most content unchanged, we can obtain a new math problem (e.g., MWP2) from the original problem (e.g., MWP1).

We can see that this kind of reversion-based data augmentation has the following benefits: First, this way of generating new data is relatively simple and reliable so that the key information will not be lost; Second, the reverse operation can infer new knowledge points which helps to learn mathematical reasoning logic; Third, more high-quality data can be used to train the neural networks well. Next, we combine this Reverse Operation based Data Augmentation (RODA) with seq2seq models and conduct experiments on Math23K, the most influential large-scale dataset for MWPs. To be noticed, our method could be easily adapted to any supervised model on high-quality arithmetic MWP corpora with equation solutions. Experimental results show that our method could benefit various models and outperform previous state-of-the-art results on solving MWPs.

In summary, our contributions are three-fold:

- We follow the double-checking mechanism and propose the reverse operation based data augmentation method, which is easy to apply and accurate.
- We show how our method can introduce new mathematical knowledge points to the new examples and help to learn the reasoning logic.
- We effectively use data augmentation results in the MWP solving and achieve the state-of-the-art on the Math23K dataset.

II. RELATED WORK

A. Math Word Problem Solving

1) Models: Early approaches of solving MWPs mainly rely on predefined rules to map the problems into several predefined templates [9], [10]. Recent studies on solving MWPs adopt either semantic parsing techniques to parse the natural language questions into logic forms [3], [11]–[14], or question answering style end-to-end models to directly predict the solution equations [15], [16]. Wang et al. [17] first used seq2seq based models to generate the mathematical solution of MWPs directly. Wang et al. [18] further extended the method by performing equation normalization as preprocessing and generating the suffix notation of the equation. Zhang et al. [19] further studied the diverse solution problem via multiple-decoders. Various studies [16], [20]–[23] further improved the model with tree-structured information. Attention mechanism and Graph Neural Networks have been studied to capture the intra-relation of the numbers [24], [25].

2) Datasets: Early datasets of MWP solving were designed for rule-based methods and are relatively small [11], [26], [27]. As shown in Table I. These datasets range over a small variety of problem domains and answer equation templates.

With the advent of data-driven methods especially neural networks, larger datasets are constructed. These datasets can be divided into two categories according to the answer format. In the first category, the mathematical solution is expressed as a mathematical equation. The most influential dataset is Math23K [3], a Chinese dataset containing 23,162 elementary-school-level math word problems and equation solutions. The solution equation to each problem contains only one unknown variable. Another large scale dataset is Dolphin18K [3], which is an English dataset that contains 18,460 problems along with the solutions. The dataset contains multi-unknown variable problems, many minor typos, and wrong solutions. We do not use this dataset in our experiment that our data augmentation method pursuits are producing high-quality new data from high-quality data. In the second category, the mathematical solution is expressed as semantic parsing style or descriptive text style output instead of equations, such as AQuA [15] and...
MathQA [28]. Our method does not apply to these answer expressions since the new equation generation module could not be applied. Here we conduct experiments on a small, simple English dataset AllArith and the large-scale, complex Chinese dataset Math23K to show how our method is effective among different languages and datasets.

B. Data Augmentation

The two most popular methods for sentence-level data augmentation in NLP are back translation [5] and EDA [4]. Yu et al. [5] used a high-quality machine translation system to translate the original text into a new language and then backward to perform paraphrase style data augmentation. Wei and Zou [4] slightly modified the input sentence on the token level by performing four kinds of operations: synonym replacement, random insertion, random swap, and random deletion. The drawback of applying these two methods on MWPs is that MWPs are very sensitive to even slight modifications; any key information missed the problem would become unsolvable. In addition, these methods only provide lexical level variance but no new reasoning knowledge.

Recently due to the reliability of rule-based data augmentation in NLP, task-specific logic rules for data augmentation have been explored in Natural Language Inference (NLI) and Question Answering. Kang et al. [6] studied NLI-specific logic-based data augmentation, which generates new examples by replacing tokens or changing labels on the original training examples. They only used the logic operations of NOT(¬) and equivalence(⇔). Asai and Hajishirzi [7] further extended entailment(→) operation for common domain question answering data augmentation by measuring the transitive consistency of pairs of questions. Gokhale et al. [8] studied disjunction(∨) and conjunction(∧) operation for yes-no style visual question answering. All these studies involve only one or two steps of simple logic. By contrast, our method uses reversed operation of complex mathematical computation and can introduce new reasoning logic in the generated new examples.

III. METHODOLOGY

Given a high-quality MWP dataset, we propose RODA, producing new accurate math word problems to enlarge the data scale. There is a linear correlation between the number of new questions generated and the number of known variables in the question. Next, we can use the augmented dataset to improve the supervised MWP solving models.

A. Reversion based Data Augmentation

To perform data augmentation, we reverse the original problems in the dataset to new problems, and the reversion process consists of three steps: number identification, problem transformation, and equation generation. To ensure the quality of the new data, the main criteria of our reversion process is “quality first quantity second”, so that our method relies on some well-designed empirical rules in the three steps.

Number Identification: This stage consists of two steps. We first use an LSTM classifier to perform significant number identification and determine irrelevant numbers. Then we further filter out numbers that cannot perform reversed-based operation and then use exact match to map the numbers in the equation to the numbers in the question text. The statistics of this stage are given in Table IV.

A MWP might contain various numbers that are irrelevant to the solution, such as the date or description text such as ‘tenth grade student’. Following Wang et al. [17]’s work, we perform significant number identification by building an LSTM-based classifier to determine the significance of the numbers and filtering out the irrelevant numbers. The classifier uses single layer LSTMs with 128 nodes and a symmetric window of length 3. The classification performance can reach around 99% accuracy.

In addition, the numbers in a math word problem do not necessarily map one to one with the numbers in the corresponding solution equation. One number may appear in the solution equation but do not appear in the word problem, and vice versa. Thus, to conduct high-quality word problem reversion, we first need to identify the valid numbers which can be converted to an unknown quantity using the reverse logic of the original solution. Here, we propose a rule-based method to identify the possible numbers for reversion. Four key rules are explained as follows, and we also show their examples in Table II.

1) Problem Number Duplication If a number appears more than once in the problem, we filter this number out because we cannot map the numbers in the text to the equation with an injective function. As shown in Table II, we cannot know whether a separate ‘2’ is related to \(A\) or \(B\), and thus it is difficult for us to conduct a precise reversion.

2) Equation Number Duplication If a number appears more than once in the equation, we filter this number out because it may not be capable of being solved with a linear equation with one unknown variable. To solve higher-order polynomial functions, introducing new operators such as root operation would be essential. However, such operators are out-of-domain (OOD) with the original data and would introduce noise.

3) Power Operation If a number is involved with power operation, we filter it out because the inverse operation, which is logarithmic operation or root operation, is OOD. We filter out both the base number and the exponent number.

4) Constant Term Numbers Constant term numbers are not applicable for reverse operation, such as \(\pi\) and unit conversion terms.

We then perform exact match between the numbers in the question and the equation to align the numbers.

Problem Transformation: After collecting a set of valid number candidates, we perform problem reversion for each number to get a new transformed math word problem. The main work is to convert the original question sentence into a declarative sentence with a definite quantity and convert the sentence with the identified number into a question sentence.
with its question point on the number. Specifically, for the first conversion, we name the original question sentence as Q. From Q, we find the interrogative pronoun according to a list compiled in advance and replace the pronoun with the answer of the original problem. Next, we adjust the word order to make the sentence fluent and natural and get the declarative sentence D′.

For the second conversion, given the candidate number cᵢ, we get the sentence D, which contains cᵢ. Next, we take cᵢ as the question focus and change D into a question sentence Q′. For different languages, the conversion process is different. For example, for Chinese, the conversion is relatively simple and just replaces cᵢ with an interrogative pronoun such as “多(How many)” without the need of adjusting word order. We show further details of the interrogative pronouns in the appendices.

For English problem transformation, we follow the manually encoded transformation rules from Heilman et al. [29]. We extend the interrogative pronoun candidates to the following list: how many, how much, how far, how tall, how long, how fast, how old, how big, what fraction, what.

We then edit the original math word problem. We delete the two sentences D and Q, and add D′ and Q′ at the end of the text. Then we get the new transformed math word problem. Our problem transformation method is simple but effective, which is the basis of producing new high-quality MWPs.

**Equation Generation:** For each new transformed word problem, we need to generate its solution equation. Similar to problem transformation, we derive the new solution equation according to the solution equation of the original problem.

To make the process of generating the new equation clear, we take the two MWPs in Figure 1 as examples. For the pre-identified number 600 in MWP1, it will be changed into a variable (i.e., x′). At the same time, we substitute the original variable x with its answer 10. Then we can get an intermediate equation 10 = x′/(32 + 34).

![Fig. 2. Example of equation conversion.](image)

Next, we need to convert this equation to its equivalent format where the variable x′ is located on the right side of the equal-sign alone. Figure 2 displays the equation conversion result of our running example. To conduct equation conversion, we design a recursive conversion algorithm based on the syntax tree structure. We first construct a quasi-binary syntactic tree for the original math equation. We denote the part on the left to the equal sign as f₁(the formula in stage 1). For the part right to the equal sign, we build a binary tree with one operator op₁ as the root node with two child

![Fig. 3. Rules of equation reversion](image)

![Fig. 4. Illustration of recursive equation conversion.](image)

| Rule | MWP examples | Num | Equation |
|------|--------------|-----|----------|
| 1    | A has 4 piles of 2 apples and B has 2 apples. B gave 1 apple to A, how many does A have in total now? | 2   | x = 4 * 2 + 1 |
| 2    | The side length of a square is 2, what is the area? | 2   | x = 2 * 2 |
| 3    | The side length of a cube is 4, what is the volume? | 4 & 3 | x = 4³ |
| 4    | The diameter of a circle is 3, what is the perimeter? | π   | x = π * 3 |

**Algorithm 1** The algorithm of equation conversion

**Input:** Left part of medium equation l_tr and right part of equation r_tr

**Output:** Inverse equation in tree structure

1: root = r_tr.root
2: num_tr, var_tr = find_var(r_tr)
3: while root! = x′ do
4: l_tree = rule(l_tr, num_tr, root)
5: r_tr = var_tr
6: root = r_tr.root
7: num_tr, var_tr = find_var(r_tr)
8: end while
9: return l_tr = x′
trees. The child tree which has the new variable $x'$ is marked as $v_3$ (the child tree with a variable in stage 1) and the other one as $n_1$ (the child tree without any variables in stage 1), and this identifying process is named as function $\text{find\_var}$ in Algorithm 1. This step corresponds to the upper left part of Figure 4. Next, like the upper middle part of Figure 4, we move $n_1$ to the left side and get a new operator $op'_1$ according to equation revision rules, which are summarized from the basic mathematical computation. We show these rules in Figure 3. Regarding $f_1$ and $n_1$ as two new child trees and $op'_1$ as the new root node, we can get a new tree $f_2$, which is the black circular ring in the upper right part of Figure 4, and we just use a single node $f_2$ to represent it in the following step.

Then, $v_1$ is further broken down as a binary tree which is composed of the root node $op_2$ and the child trees $n_2$ and $v_2$ as shown in the lower-left part of Figure 4. If we ignore the dotted circular sign of $v_1$, in this state, the equation has the same structure as the beginning state, so the following process will repeat until $v_{n_3}$ has only one node $x'$. At this time, referring to the lower right part of Figure 4, all nodes of numbers and operators are moved to the left side and only the variable node $x'$ is left on the right side. The concrete process of equation conversion is shown in Algorithm 1.

Since one math equation can be written in several equivalent forms (e.g., $4 + 2 - 3$ and $4 - 3 + 2$), which brings noise to the model training, we conduct equation normalization for all the original and generated equations. We follow the criteria of Wang et al. [18] that if one equation could be converted into a shorter one, then it should be shortened, and the order of the numbers in one equation should follow their occurrence order in the problem text as much as possible. Wang et al. [18] performed equation normalization by applying a series of rules that only work on limited cases such as a sequence of multiplication. Here we increase the applicable scope. We also use the simplification algorithm supported by sympy [30] that heuristically simplifies the equations and matches with human writing regulations. We show further details of the algorithm in the appendices.

This equation normalization is essential for assuring the model performance since the equations generated by the reverse operation are written in a completely different style from that of the equations in the original data. The normalization can avoid domain shift of the prediction target equations.

### B. MWP Solving Model

Our data augmentation method can be combined with any preferred neural model. Here, we adopt two seq2seq models and one classification model, then examine their performance. First, we implement Liu et al. [20]'s prefix model, which is a light and effective baseline. At the same time, to show whether an advanced model can be further improved by our augmented data, we use the SOTA model named Goal-driven tree-structured MWP solver (GTS) [23], which is an extension of the prefix baseline model. We also build a classification model based on Transformers following Kushman et al. [26] and Robaidek et al. [31], which considers the equation template as the label of one MWP example. Here, we briefly introduce the three models.

For the prefix model, formally, the model takes a sequence of tokens $\{x_i\}_{i=0}^n$ as the input and embeds them into a sequence of word embedding representations $\{e_i\}_{i=0}^n$ which are fed into a bidirectional long short term memory network (BiLSTM) encoder. Then two context-aware representations $h_{enc}^{i\_1}$ and $h_{enc}^{i\_2}$ are calculated and concatenated as $h_{enc}^{2\_i}$ for each token. Then these representations are given to the decoder to decode the output equation.

The decoder adopts a unidirectional LSTM to generate the output in an autoregressive manner. At each decoding time step $t$, the decoder calculates the attention weight distribution $\{a_i^t\}$ on $\{h_{enc}^{2\_i}\}$ with the embedding $e_{dec}^t$ of the output of the previous time step $y_{t-1}$ and the current hidden state $h_{dec}^t$ of the decoder LSTM, and assigns them to the encoder outputs $\{h_{enc}^{2\_i}\}$ to form an attention-aware representation $s_t$ which is finally fed to a Multi-layer Perceptron (MLP) layer to generate the output token $y_t$.

$$h_{dec}^t = \text{LSTM}(h_{dec}^{t-1}, e_{dec}^t)$$  

$$s_t = \sum_{i=1}^{n} \alpha_i \cdot h_{enc}^{i\_1} = \sum_{i=1}^{n} \exp(h_{enc}^{i\_1} \cdot h_{dec}^t) \cdot h_{enc}^{i\_2}$$  

The GTS model [23] further extends the prefix baseline with subtree representations which can provide more information for the decoding process. A recursive neural network is used to encode subtrees of the equation in a bottom-up manner. The subtree representation of one token $y_i$ is calculated based on its children nodes with the gate mechanism. With the subtree representations, this model can also well use the information of the generated tokens to predict a new token.

It is noted, for the seq2seq models, directly generating the solution equation with numbers suffers from a serious out-of-vocabulary (OOV) problem since the vocabulary of numbers is enormous. To address this problem, we follow Kushman et al. [26], which used equation templates instead of actual equations as the prediction target of the model. The numbers in one MWP are notated as $\text{temp}_i$, where $i$ denotes the order of the numbers that appear in the problem. Extra constant numbers such as $\pi$ and 1 are also added to the decoder vocabulary. Then the OOV problem can be solved.

For the classification model, we follow Kushman et al. [26] and Robaidek et al. [31], which encodes the question text and then classifies the corresponding equation template. We add a [CLS] token to the question text and use a Transformer to encode the question text. We feed the feature vector of the [CLS] position to a Multi-layer Perception network to classify the equation template.

To further improve the models, we mix and shuffle the augmented data with the original data as the new training set of the models. It is noticed that our method does not limit to these three models.
IV. EXPERIMENTS

A. Experiment Setup

In our experiments, we mainly experiment on Math23K which is the most influential large scale dataset for MWP solving in the Chinese language. It contains 23,162 one unknown variable elementary school level MWP with the corresponding equation solutions. In train-test setting, the dataset is split to 21,162 training examples, 1,000 validation examples and 1,000 test examples.

For the classification model, we use Transformer base with 12 layer, 768 hidden state size and 12 attention heads. The classification model has 5693 labels. We train with a 16 batch size for 20 epochs. We use the SGD optimizer with a learning rate of 2e-5 and weight decay of 0.01.

For the two seq2seq MWP solving models, we fix their embedding size to be 128. For the prefix baseline model, the dimension of decoder hidden state is set to 512 while the dimension of decoder hidden state is 1024. For the GTS model, the hidden state of both encoder and decoder is 512. We use Adam optimizer to optimize these parameters. The batch size is 128. To compare the performance with baselines fairly, GTS model is tested by both train-test setting same as the other models, and also 5-fold cross validation setting same as the original paper, while others are tested on the test set. The experiments are done on GTX 1080Ti GPU, with a runtime of 10 hours for prefix baseline, 110 hours for GTS and 15 hours for the classification model.

For data augmentation, we perform the reverse operation on the training set of Math23K, which includes 21,162 MWP. From these problems, we can get 58,699 numbers by using the LSTM-based classifier. At the same time, we filter out 1,490 problems which are composed of only numbers and operators such as 'Please calculate 5+7*10', because they can not give effective supervision to the MWP solving models. We also exclude 7,399 problems in the problems which are not easy to be reversed, because they do not map one to one with the numbers in the solution equations as stated in Section 3.1. Finally we totally get 47,318 new problem-equation pairs. We can see, our data augmentation method successfully generates new data whose size is more than two times the original data, demonstrating the method's ability for performing large-scale data augmentation cheaply. We illustrate the statistics of the data augmentation results in Table IV.

We also show the statistics of template coverage on the development set of the original training set and the augmented training set. As we can see our data augmentation method increased the template coverage of the development set, reducing the rate of uncovered templates by 21.5%. This can show how our method introduces new mathematical knowledge points and benefit the model.

B. MWP Solving Results

We first evaluate the MWP solving performance by comparing our methods with other baseline methods. Here we name our Reverse Operation based Data Augmentation method RODA. In this experiment, we use all the 47,318 augmented data for training the prefix and GTS models. Table V shows the results of our methods and other novel systems on the Math23k evaluated by the final answer accuracy. In this table, we classify the MWP solving models as retrieval-based, classification-based, generation-based and ensemble models. The retrieval-based models mainly calculate a similarity score for questions in the test set and the questions in the training set and assign the template that has the highest similarity. The classification-based models train a classifier to predict an equation template for each problem in a multi-class classification manner. For retrieval and classification models, we use the results from Robaidek et al. [31].

The generation-based models are the recent mainstream for MWP solving and use end-to-end seq2seq models to directly generate an equation template. Wang et al. [17] proposed the DNS model, which used seq2seq with significant number identification to generate an equation template. Wang et al. [18] improved their model and proposed the Suf+EN model, which extends the DNS model by decoding the suffix notation and performs equation normalization for preprocessing. TreeLSTM [20] uses a top-down tree-structured decoder to predict the equations. Group-Attention [24] uses various attention methods to capture the intra-relation of the numbers. We also use two ensemble models DNS+Retrieval and Suf+EN Ensemble for comparison, which uses bagging to combine the results of different models.

### Table IV

| Type              | #     | Prop. |
|------------------|-------|-------|
| Original Problems| 21,162| -     |
| Filtered Problems| 1,490 | 0.07  |
| Original Numbers | 58,699| 2.77  |
| Candidate Numbers| 54,717| 2.59  |
| Irreversible Numbers| 7,399| 0.35  |
| Augmented Problems| 47,318| 2.24  |

### Table V

| Type             | #     | Prop. |
|------------------|-------|-------|
| Dev Set Templates| 377   | -     |
| Original Covered | 307   | 81.4  |
| + RODA           | 322   | 85.4  |

### Table VI

| Type             | #     | Prop. |
|------------------|-------|-------|
| Template Coverage on Development Set. "Prop." stands for the proportion of the item compared with original problems. |
Our data augmentation method is exerted on two generation models Prefix [20] and GTS [23] which have been introduced in Subsection 3.2. We choose these models as they can somewhat be representative of generation-based methods, especially GTS achieves the SOTA performance. From Table V, we can see that generation-based methods generally outperform retrieval-based and classification-based methods. To show the efficiency of our method, we conduct experiments on both the classification models and the SOTA generation models. We can see that our method gains 8.8 points of improvement over the original data, we also control the size of our augmented data for the reported results. This can demonstrate the reverse operation can infer new knowledge points, which helps to learn the mathematical reasoning logic while more high-quality data can be used to well train the neural networks.

We also consider combining the original data with different proportions of augmentation data as training data whose performance is shown in Table VI. We can see that, as the size of the augmented data increases, the performance stably increases, which shows how the size of the training data affects the performance. The model performs similarly when the proportion is 1.5 and using the whole augmented data. Because the performance of the model has not decreased along with the increase of augmented data, we use the full augmented data for the reported results. This can demonstrate the reverse operation can infer new knowledge points, which helps to learn the mathematical reasoning logic while more high-quality data can be used to well train the neural networks.

We also compare our data augmentation method with another novel data augmentation method, back-translate (BT) [5]. For the BT method, we translate the problem text into English and then back to Chinese by Google Translate⁵. As BT can only perform data augmentation with the 1:1 proportion of the original data, we also control the size of our augmented data. Table VII compares the MWP solving results. As shown in Table VII, We can see that the performance of BT has a significant gap with RODA, even when the data augmentation proportion is the same. There are two reasons for such a performance gap. First, BT may introduce more noise into the training data through the translation-based paraphrase and degrade the model performance, while our method can better ensure data quality. Second, BT only paraphrases the same meaning on the lexicon level, while our data augmentation method can introduce new knowledge points that helps to learn the mathematical reasoning logic.

⁵https://translate.google.com/

| Model                  | Acc   |
|------------------------|-------|
| Retrieval Cosine [31]  | 23.8% |
| Jaccard [31]           | 47.2% |
| Classification Transformer [31] | 56.8% |
| Bi-LSTM [31]           | 57.9% |
| Generation DNS [17]    | 58.1% |
| BiLSTM+Suffix+EN [18]  | 66.7% |
| TreeLSTM [20]          | 69.0% |
| Group-Attention [24]   | 69.5% |
| Ensemble DNS+Retrieval [17] | 64.7% |
| DNS+suffix+EN Ensemble [18] | 68.4% |

**TABLE V**
Math word problem solving accuracy on Math23K. † denotes that the result is 5-fold cross validation performance. All other models are tested on the test set.

| Data                  | Data Prop. | Acc   |
|-----------------------|------------|-------|
| Original only         | —          | 68.0% |
| RODA Only(2.24 times) | —          | 50.0% |
| Orig+RODA             | 1.0:5      | 69.5% |
| Orig+RODA             | 1:1        | 69.8% |
| Orig+RODA             | 1:1.5      | 70.9% |
| Orig+RODA(All)        | 1:2.24     | 71.0% |

**TABLE VI**
Effects of data augmentation by adjusting the augmentation proportion on the development set. The middle column denotes the proportion of original data and augmented data.

**C. Analysis of Data Augmentation**

Here we further investigate how the augmented data improves the MWP solving model. In consideration of the balance of experiment time and accuracy, we use the Prefix model on the development set of Math23K for analysis.

Previous studies on data augmentation [5] show that too much augmented data might harm the performance of the model. Thus, we experiment with what percentage of our augmented data can best improve the MWP solving model.


### Table VIII

**Examples of the output from the Data Augmentation Module.** "Coh" and "Cor" stand for Coherence and Correctness in Table X.

| Original Text | BT English | Origin & BT Equation | Coh | Cor |
|---------------|------------|----------------------|-----|-----|
| In the middle of a river, the water velocity of the middle stream and seacoast are very different. The middle stream velocity is \( t_b \) miles per hour and the seacoast velocity speed is \( t_d \) miles per hour. Today, a steamer sails down the river and travels for \( t_d \) miles in \( t_b \) hours. How long does it take to return to the starting point from the seacoast? | \( x = t_d / (t_d / t_b - t_a) \) | 2 | 0 |

### Table IX

**Comparison of Back Translation and Reversed Operation Based Data Augmentation.** "Coh" and "Cor" stand for Coherence and Correctness in Table X.

| Model         | Coherence | Correctness |
|---------------|-----------|-------------|
| Original      | 4.27      | 0.92        |
| BT            | 3.24      | 0.55        |
| RODA          | 3.86      | 0.84        |

### D. Human Evaluation

To further examine the data quality of our augmented data, we perform human evaluation to examine the original data, back-translate augmented data, and RODA data. We sample 100 MWP from each dataset. 25 examples have two numbers, 25 examples have three, 25 examples have four, and the last 25 have five or more numbers. The sampling in each category is random. The evaluation involves two aspects. The first is the coherence which is ranked between 1-5. This examines whether the generated text is coherent. The second is correctness which is classified as either 0 or 1. This examines whether the equation matches the problem text. Two annotators participate in ranking the data. Table X lists the average scores for each dataset with respect to the two metrics. From Table X, we can see our method can generate new data which has only a small performance gap with the original data. Compared to the BT method, our augmented data is of higher quality in both coherence and correctness. This can demonstrate how our method is reliable so that the key information is not lost, which is also why our augmented data can well boost the model performance.

Here we show two augmented examples in Table VIII. In case 1, this newly produced example has coherent text and correct solution, even if the original mathematical logic and description text is fairly complex. The original example involves the numbers that the time used by \( A, B \), and the dog is the same so that it can form the problems for each of the 5 variables: the speed of \( A, B \), and the dog, the distance between \( A \) and \( B \) and the distance that the dog has run. In this example, the original training data can be augmented into 4 new high-quality question-answer pairs, and each of them holds a new mathematical knowledge point, which demonstrates the effectiveness of our model. We also show an example where our method failed in case 2. When swapping the order of the sentences, the coherence resolution of apples in the final
question has changed that it no longer asks about the origin variable; therefore, the new reversed question would no longer be natural nor correct. In the case that the sentence order swapping would influence the coreference resolution of natural text, our method would no longer work. Such error are caused since our question generation module does not consider the dependency between discourses. Such kind of error could be reduced with an additional discourse analysis module, which could be left for further work.

We show one example of comparison between BT and RODA in Table IX. As we can see, during the paraphrase BT translates 中流 into 中游 and 沿岸 into 海岸, which makes the question no longer natural and reasonable. Meanwhile, it leaves out a key information point 顺中流而下 that the question meaning has completely changed and the equation no longer matches with the question. Such kind of minor errors during the paraphrase would be fatal for MWP data augmentation that the key information is changed. The noisy new examples would reduce the performance of the model. Meanwhile, our method RODA correctly formed a natural reversed question with a new mathematical knowledge point corresponding to the reversed equation.

E. A Study on English Dataset

| Type                  | #  | Prop. |
|-----------------------|----|-------|
| Original Problems     | 831|       |
| Filtered Problems     | 1  | 0.00  |
| Original Numbers      | 1953| 2.35  |
| Candidate Numbers     | 1951| 2.35  |
| Irreversible Numbers  | 1236| 1.49  |
| Augmented Problems    | 715 | 0.86  |

TABLE XII
THE STATISTICS OF THE DATA AUGMENTATION ON THE FULL SET OF ALLARITH. "PROP." STANDS FOR THE PROPORION OF THE ITEM COMPARED WITH ORIGINAL PROBLEMS.

V. Conclusion

In this paper, we propose the reverse operation based data augmentation for MWP solving, which converts the question and equation via reverse operation. Enlightened by how humans perform double-checking during calculation, the method can perform cheap and accurate data augmentation that could be adapted to any model. The augmented data also provides supervision of new mathematical knowledge points that could benefit the model beyond paraphrasing the text. We evaluate our method on Math23K and achieve state-of-the-art performance. In comparison with a strong baseline Back Translation, we show how our method significantly outperforms on a complex and large-scale dataset Math23k and achieve comparable performance on a simple and small dataset AllArith.

Acknowledgments

The authors would like to thank Haoran Zhang for his helpful advice for this research. This work was partially supported by National Key Research and Development Project (2019YFB1704002) and National Natural Science Foundation of China (61876009).

Appendix A
Removing Negative Terms

To be noticed, sympy would put negative numbers in the first position of one equation during simplification, which causes the problem that one equation no longer can form a binary abstract syntax tree. We also design an algorithm to avoid this problem by moving the first addition term to the first position. We show an example in Table XIII.

| Step | Equation |
|------|----------|
| Equation | \(x = c - a - c + (c \times a) + (b/b)\) |
| Sympy  | \(x = -a + 1 + (a \times c)\) |
| Adjusted | \(x = 1 - a + (a \times c)\) |

TABLE XIII
ONE EXAMPLE OF EQUATION NORMALIZATION

We show the algorithm for removing the negative terms in the front of the equation during equation normalization in Algorithm 2. To be noticed, this process does not change the mathematical value of the equation but only how it is written.

Appendix B
Interrogative Pronouns

We show the list of interrogative pronouns used for question conversion here in Table XIV. Some of the interrogative
Algorithm 2 The algorithm of removing negative terms

**Input:** Original Equation  
**Output:** New Equation Written Form

1: **while** True **do**  
2:  
3: **if** Brackets in Equation **then**  
4:     
5: **for** all sub-equation in brackets **do**  
6:     
7: **end** for  
8: **end** if  
9: **if** Equation[0] == `-` **then**  
10:     
11: **Find** the first add token;  
12: **Move** the token to the front;  
13: **end** if  
14: **end** while  
15: **return** Equation

pronouns also have other meanings in the declarative discourse unit, so we only detect them if they are in the last discourse unit of the sentence.

| Chinese | English Translation |
|---------|---------------------|
| 几分之几 | the fraction number is |
| 几 | How many |
| 求 | Please solve |
| 多 | how much more than |

A LIST OF INTERROGATIVE PRONOUNS. † DENOTES THIS PRONOUN IS ONLY VALID WHEN IT IS IN THE LAST DISCOURSE UNIT OF THE QUESTION.

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Qianying Liu received her B.S. in Information and Computing Science from Peking University, China in 2018 and M.S. in Intelligence Science and Technology from Kyoto University, Japan in 2020, respectively. She is currently an Ph.D. candidate and research assistant at Kyoto University. Her major research interests include information extraction, text generation and question answering.

Wenyu Guan received the BSc degree from the Xian Jiaotong University and MSc degree from Peking University, China, in 2017 and 2020, respectively. He worked as a software engineer in the Institute of Information Engineering - Chinese Academy of Science from 2017 to 2019. He served as an intern research assistant in the Institute of Computational Linguistics, Peking University from 2019 to 2020. His research interest includes math word problem solving, semantic parsing, and information retrieval.

Sujian Li received the Ph.D. degree from the Institute of Computing Technology, Chinese Academy of Sciences, Beijing, China, in 2002. She is currently a tenured Associate Professor at the Institute of Computational Linguistics, Peking University, Beijing. Her main research interests include natural language processing, information extraction, and document summarization.

Fei Cheng received his B.S. in Applied Physics from Donghua University in 2005 and M.S. & Ph.D. in Informatics from NARA Institute of Science and Technology in 2013 and 2018. His research interests include morphological analysis, information extraction, and a broad range of natural language processing topics. Currently, he is a program-specific assistant professor at Kyoto University.

Daisuke Kawahara received his B.S. and M.S. in Electronic Science and Engineering from Kyoto University in 1997 and 1999, respectively. He obtained his Ph.D. in Informatics from Kyoto University in 2005. He is currently a professor of the department of communications and computer engineering, Waseda University. His research interests center on natural language processing, particularly knowledge acquisition and text understanding.

Sadao Kurohashi received the B.S. and M.S. in Electrical Engineering from Kyushu University in 1989, 1991 and 1994, respectively. He has been a visiting researcher of IRCS, University of Pennsylvania in 1994. He is currently a professor of the Graduate School of Informatics at Kyoto University.