Neutrino Induced Electrostatic Waves in Degenerate Electron Plasmas

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ABSTRACT

The plasma theory of electrostatic waves generated collisionlessly when neutrinos pass through degenerate electrons is developed. It is consistent with quantum statistical field theory in the degenerate limit. The reactive case can also be treated. For neutrino distributions considered herein, neither case develops significantly during e.g. the neutrino burst in a supernova explosion.

1. Introduction

A number of recent papers have speculated on the collisionless heating of thermal plasma behind a Type II supernova shock by the intense flux of neutrinos emitted from the supernova core. The idea is attractive since the total energy radiated by a supernova in neutrinos (and antineutrinos, $\sim 10^{53} - 10^{54}$ erg) far exceeds that in the thermal plasma ($\sim 10^{51}$ ergs), and so a small transfer of energy from neutrinos to electrons and nuclei, in excess of that already accounted for by collisional processes alone, could go a long way to solving what has become known as the Type II supernova problem, i.e., that in models, the supernova shock following the core collapse and bounce loses energy to nuclear dissociation, stalls and eventually collapses back. Such an event would produce neutrinos, but not the optical emission for which supernovae are famed.

Bingham and coworkers [1] derived a growth rate for the neutrino analog of the reactive two stream instability in hot ($\sim 3 \times 10^9$ K) plasma, treating the coupling between the neutrinos and the plasma by an effective potential $\sqrt{2}G_F n_e$, following [2], where $G_F$ is the Fermi weak interaction constant and $n_e$ is the electron density. A number of the assumptions in this paper were criticized by Hardy & Melrose, who used an approach based on quantum statistical field theory to derive a growth rate for the kinetic instability. Aside from rather elementary assumptions like the use by [1] of a monoenergetic neutrino beam rather than a Fermi-Dirac distribution, and the treatment of the thermal plasma with a
cold electron dielectric tensor [3] show that their result for the growth rate is lower than that of [1] taken in the kinetic regime by a factor \((1 - \omega^2/|k|^2c^2)^2\), where \(\omega\) and \(k\) are the wave frequency and wavenumber respectively, and \(c\) is the speed of light. This is an important point, since for plasmas where the electrons are relativistic or quasi-relativistic, \(\omega \simeq |k|c\), and this extra factor gives a big reduction in the calculated growth rate. Below we will present a plasma theory treatment of the instability that reproduces the quantum statistical field theory result in the kinetic limit, and gives a result similar to [1] in the reactive limit.

Another important caveat to these works (also emphasised by [3]) is the collisional damping as electrons within the wave collide with ions. Even the most optimistic calculation gives a neutrino growth rate that is orders of magnitude lower than the collisional damping rates. Assuming that the wave mode actually exists, neutrino induced oscillations will be rapidly damped giving rise to plasma heating. In [1, 3] the plasma was assumed to be hydrogenic, at an electron temperature \(T_e = 3 \times 10^9\) K and density \(n_e = 10^{30}\) cm\(^{-3}\), giving of order \(\Lambda \sim 100\) electrons per Debye sphere. However heavy elements are often the dominant component of supernova plasma. For example Cas A is thought to be mainly O [4, 5]. In a heavy element plasma \(\Lambda\) scales as \(Z^{-3/2}\) where \(Z\) is the ion charge. For fully ionized O, \(\Lambda \sim 10\), and the so-called plasma approximation may be expected to break down. In this case no collective motions will exist, and no collisionless energy deposition can occur.

For this reason this paper will concentrate on degenerate electron plasma found at the supernova core. At densities \(10^{34} - 10^{38}\) cm\(^{-3}\) the electrons are highly degenerate (Fermi energies 10 - 300 MeV compared with \(kT_e \sim 0.3\) MeV) and collisional damping is strongly suppressed. The anisotropy in the neutrino distribution will be taken to come from the natal kick that a pulsar may acquire during the supernova event. Various authors ([2, 4] and references therein) have considered the role of neutrino scattering from partially polarized neutrons in a \(10^{15}\) G magnetic field that may be present as an explanation for this kick, (due to parity violation, the neutrinos preferentially scatter to directions along the magnetic field). However a recent paper (which also gives a convenient review of the observations of pulsar kicks) [8] suggests that this is unlikely to be the actual mechanism. However it is clear that if a pulsar is born moving with a particular velocity with respect to the supernova remnant, no matter what the precise mechanism for this is, then the neutrino emission must be anisotropic. This comes about simply because the momentum carried off by the visible supernova/supernova remnant is generally much smaller than that of the pulsar, and consequently cannot conserve momentum by itself. Such a scenario might plausibly result from an off-centre detonation. For a pulsar moving with velocity 1000 km sec\(^{-1}\) an anisotropy of order 1% [5] must be present in the neutrino emission.
2. The Neutrino-Plasma Interaction

The work of [1] treated the neutrino plasma interaction by an effective potential \( \sqrt{2} G_F n_e \), following Bethe [2], who was considering the neutrino refractive index in the solar core and envelope, and hence was dealing with a non-relativistic plasma. For neutrino interactions with a relativistic plasma the effective potential should be generalized. The amplitudes \( F_W \), \( F_Z \) for neutrino-electron forward scattering for \( W \) and \( Z \) exchange are (in units with \( \hbar = c = 1 \)) [10, 11]

\[
F_W = - \frac{G_F}{\sqrt{2}} \bar{u}_\nu \gamma_\mu (1 - \gamma_5) u_\nu \bar{u}_e \gamma^\mu (1 - \gamma_5) u_e, \tag{1}
\]

\[
F_Z = - \frac{G_F}{\sqrt{2}} \bar{u}_\nu \gamma_\mu (1 - \gamma_5) u_\nu \bar{u}_e \gamma^\mu \left( \frac{1}{2} - 2 \sin^2 \theta_W - \frac{1}{2} \gamma_5 \right) u_e, \tag{2}
\]

where \( \bar{u}_a, u_a \) are final and initial wavefunctions for particles of species \( a \), \( (m_a \text{ their mass, } p_{a,\mu} \text{ their 4-momenta,}) \) and \( \sin^2 \theta_W \simeq 0.23 \) is the Weinberg mixing angle. A Fierz transformation has been performed in the expression for \( F_W \). Only electron neutrinos interact via \( W \) exchange, whereas all three neutrino flavours interact with electrons through \( Z \) exchange. The axial vector terms do not contribute to the excitation of longitudinal waves, so \( F_W >> F_Z \). Henceforward we consider only \( F_W \), putting

\[
\bar{u}_\nu \gamma_\mu (1 - \gamma_5) u_\nu = 2 \bar{u}_\nu u_\nu p_{\nu,\mu} / m_\nu \tag{3}
\]

where the last term follows from the Gordon identity. For electrons the same treatment gives \( \bar{u}_e u_e (p_e^\mu / m_e - \sigma) \), where \( \sigma \) is a 3-vector composed of the Pauli spin matrices. Including wavefunction normalization factors, \( \sqrt{m_a / E_a} \) and and integrating over a electron distribution \( f_e(p_e) \) \( (p_a \text{ is the 3-momentum for particle species } a) \) with no net polarization we get the following result for the effective potential for electron neutrinos

\[
V_{\text{eff}} = \sqrt{2} G_F \int f_\nu(p_\nu) \left( 1 - \frac{p_\nu \cdot p_e}{E_\nu E_e} \right) d^3 p_e. \tag{4}
\]

Obviously for non-relativistic plasma the Bethe result is recovered. Also for isotropic plasma, electrons with oppositely directed momenta will average to zero in equation (4) recovering \( \sqrt{2} G_F n_e \). However if a plasmon exists in the degenerate plasma, it must be carried by electrons moving at or very close to the Fermi velocity, and this scalar product will not necessarily average out to zero.

Following the development and notation in [12] (pages 19-20) we estimate instability growth rates. The force \( F \) acting on an electron becomes

\[
F = -q (E + v_e \times B) - \sqrt{2} G_F \nabla \int f_\nu(p_\nu) \left( 1 - \frac{p_\nu \cdot p_e}{E_\nu E_e} \right) d^3 p_\nu. \tag{5}
\]
where $E$ and $B$ are electric and magnetic fields, $\omega$ and $k$ are the frequency and wavevector of oscillating quantities, $-q$ is the electron charge and $v_e$ the electron velocity. This gives an extra current density $J_i$ due to the neutrino interaction

$$J_i = \sqrt{2} q G_F \int \nabla \int f_\nu (p_\nu) (1 - \frac{p_\nu \cdot p_e}{E_\nu E_e}) d^3 p_\nu \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} v_i d^3 p_e.$$  

(6)

Putting $\nabla f_\nu = \nabla \delta f_\nu = i k \delta f_\nu$, $p_a/E_a = v_a$ and

$$\delta f_\nu (p_\nu) = -\sqrt{2} G_F \int \delta f_e (p_e) (1 - v_\nu \cdot v_e) d^3 p_e \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e},$$  

(7)

with $J_i = \sigma_{ij} E_j$ and $K_{ij} = 1 + i \sigma_{ij}/\omega$ we arrive at an expression for the extra term in the dielectric tensor $K_{ij}$ due to the neutrino interaction;

$$\delta K_{ij} = \frac{2 G^2 q^2}{\omega^2} \int \int \left\{ \frac{\omega - k \cdot v_e'}{\omega - k \cdot v_e} \right\} \frac{\partial f_e}{\partial p_e} (1 - v_\nu \cdot v_e') d^3 p_e' \times \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} (1 - v_\nu \cdot v_e) d^3 p_\nu \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} v_i d^3 p_e.$$  

(8)

Specializing to longitudinal waves,

$$\delta K^L = \frac{2 G^2 q^2}{\omega^2} \int \int \left\{ \frac{k}{|k|} \cdot \frac{\partial f_e}{\partial p_e} \frac{\omega}{\omega - k \cdot v_e} \right\} (1 - v_\nu \cdot v_e') d^3 p_e' \times \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} (1 - v_\nu \cdot v_e) d^3 p_\nu \frac{k \cdot v_e}{|k|} \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} d^3 p_e.$$  

(9)

This expression is evaluated by noting that integrals of the form $\int v_e \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} d^3 p_e$ are parallel to $k$ for a spherically symmetrical electron distribution function $f_e (p_e)$, and those $\int v_e \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} d^3 p_e = 0$. Hence we may put

$$v_\nu \cdot v_e \rightarrow v_\nu \cdot k v_e \cdot k/|k|^2$$  

(10)

and perform the integration using the standard results in Table 1, putting

$$\frac{\partial f_\nu}{\partial |p_e|} = -2/(2\pi)^3 \times \delta (|p_e| - p_F)$$

and $\omega^2 = n_e q^2/\gamma F m_e$. We have neglected the variation of the relativistic $\gamma$ inside the integral and have approximated it by its value at the Fermi surface. The full result is

$$K^L = 1 + \frac{3}{2} \frac{\omega^2}{|k|^2 v_F^2} \left( 2 - \frac{\omega}{|k| v_F} \log \left( \frac{\omega + |k| v_F}{\omega - |k| v_F} \right) \right) (1 + \Delta)$$  

(11)

with

$$\Delta = 2 G^2 F \int \left( 1 - \frac{k \cdot v_e}{|k| |k|} \right)^2 \frac{3}{2} n_e \left( 2 - \frac{\omega}{|k| v_F} \log \left( \frac{\omega + |k| v_F}{\omega - |k| v_F} \right) \right) \frac{k \cdot \partial f_\nu}{\omega - k \cdot v_e} d^3 p_\nu.$$  

(12)
and the subscript $F$ denoting values of variables at the Fermi surface. If we let $\Delta \to 0$ we recover the usual plasmon dispersion relation for degenerate electrons (cf [13]). This dispersion relation is plotted in Figure 1 for an electron density of $10^{34}$ cm$^{-3}$. The vertical dotted line indicates the wavevector where $\omega = \omega_{\text{crit}} = |k|$. To the right of this line $\omega < |k|$ and neutrinos may excite electrostatic waves by a kinetic instability. To the left of this line they are kinematically forbidden from doing so, and only the reactive instability may operate. This line moves to higher wavenumber (in units of the plasma frequency) with increasing electron density.

2.1. The Kinetic Growth Rate

To derive the instability growth rate in the kinetic regime we let $\omega \to \omega + i\delta$ with $\omega >> \delta$ and put $\frac{1}{\omega - k \cdot v_\nu} \to P \left( \frac{1}{\omega - k \cdot v_\nu} \right) - i\pi \delta (\omega - k \cdot v_\nu)$, which gives, after taking imaginary parts,

$$\delta = \frac{2\pi G^2 \omega |k|^2}{q^2} \left( \frac{\omega^2 - |k|^2 v_F^2}{3\omega_p^2 - (\omega^2 - |k|^2 v_F^2)} \right) \left( 1 - \frac{\omega^2}{|k|^2} \right)^2 \times \int k \cdot \frac{\partial f}{\partial p} \delta (\omega - k \cdot v_\nu) d^3 p_\nu.$$ \hspace{1cm} (13)

This result is identical to that derived in [3] by the methods of quantum statistical field theory, with the substitution of the appropriate residue factor for degenerate electrons [13]

$$Z(|k|) = \left[ 1 + \frac{1}{2\omega} \frac{\partial \alpha}{\partial \omega} \right]^{-1}_{\omega = \omega(|k|)} = \frac{2 \left( \omega^2 - |k|^2 v_F^2 \right)}{3\omega_p^2 - (\omega^2 - |k|^2 v_F^2)}$$ \hspace{1cm} (15)

where $\omega^2 = -\alpha(\omega)$ is the plasmon dispersion relation, and we have substituted from the dispersion relation to eliminate the logarithm. Similar results for spontaneous Cherenkov emission by neutrinos have been obtained by [14, 15]. It is apparent the the factor \(\left(1 - \omega^2/|k|^2\right)^2\) by which this expression differs from the kinetic limit of the growth rate derived by [4] is not related to electro-weak parity violation effects as implied by [3], but to a proper treatment of the relativistic plasma. In fact it is straightforward to redo
the calculation of the neutrino-plasmon vertex in \[3\] admitting right-handed as well as left-handed neutrinos (i.e. replacing \(u_{\nu}\gamma_\mu(1 - \gamma_5)u_{\nu}\) with \(2\bar{u}_{\nu}\gamma_\mu u_{\nu}\)) The result is the same as for the pure left-handed neutrinos except for an extra factor of 2.

2.2. The Reactive Growth Rate

Turning to the reactive instability, considered in \[1\], where the pole in the integrand lies in a region where \(\partial f_{\nu}/\partial p_{\nu} = 0\). This will be the case in the region of phase space where the plasmon dispersion relation satisfies \(\omega >> |k|\). Using the dispersion relation to eliminate the log inside the integral and putting \(\omega = |k|v_F\) for \(\omega >> |k|\) elsewhere, we solve equations \(11\) and \(12\) with \(K^L = 0\) for to get

\[
\omega^2 - \omega_p^2 \left(1 + 3 \frac{|k|^2v_F^2}{\omega^2} \cdots + \right) \times \left\{1 - \frac{2G_F^2|k|^2}{\eta^2} \int (1 - \frac{k \cdot v_{\nu}}{|k|})^2 \frac{k \partial f_{\nu}}{\omega - k \cdot v_{\nu}} d^3p_{\nu}\right\} = 0
\]

This is similar to the expression derived in \[1\], but different by the factor \(\left(1 - \frac{k \cdot v_{\nu}}{|k|}\right)^2\) in the integral. The integral in equation \(16\) can be further simplified by taking the limit \(\omega >> |k|\). The reactive growth rate is given by the complex roots (if any) to equation \(16\). Assuming that \(\int k \cdot \partial f_{\nu}/\partial p_{\nu} d^3p_{\nu} = 0\) (this is true for the distribution functions considered below), then to lowest order in \(|k|/\omega\) a quadratic equation in \(\omega\) with two real roots results, giving no reactive growth. Going to higher order a cubic equation in \(\omega^2\) results, again with no complex roots. A negative real root for \(\omega^2\) would give an imaginary \(\omega\), implying reactive growth at zero frequency. However such solutions should not be over-interpreted. At low frequencies electrons will couple to protons, which in turn will couple to neutrons by the strong interaction, and these couplings would need to be included in the dispersion relation to get a physical result.

3. The Neutrino Distribution Function and Numerical Estimates

To get numerical estimates of the magnitudes of the various growth rates we will consider a neutrino distribution function of the form

\[
f_{\nu}(p_{\nu}) = \frac{1}{(2\pi)^3} \exp \left(\frac{|p_{\nu}|}{k_B T_{\nu}}\right) \frac{1}{1 + \tau P_n \cos \theta_{\nu}}.
\]

which comes from a thermal Fermi-Dirac distribution (the first term, with the neutrino chemical potential assumed zero) modified by scattering with an approximate differential
cross section \[3\]

\[\frac{d\sigma}{d\Omega} = \sigma_0 (1 + P_n \cos \theta)\]  \hspace{1cm} (18)

with \(\sigma_0 = G_F^2 E_\nu^2 / 4\pi^2\). The neutrino opacity, \(\tau = \tau_0 E_\nu^2 \sim (E_\nu / \text{MeV})^2\) cm\(^2\) for typical neutron star parameters. The neutron polarization \(P_n\) is given approximately by \[3\]

\[P_n = 2 \times 10^{-5} \left( \frac{B}{10^{13} \text{G}} \right) \left( \frac{3 \text{MeV}}{T_n} \right)\]  \hspace{1cm} (19)

where \(T_n\) is the temperature of the neutron star matter. Consequently \(\tau P_n << 1\) for reasonable parameter values and equation (17) is justified. As remarked above this neutrino distribution function based on elastic scattering from partially polarized neutrons is unlikely to be the cause of pulsar kicks \[3\]. The reason for this is that the anisotropy in the neutrino distribution is wiped out by neutrino absorption by neutrons, unless a difference in the magnetic field of at least \(10^{16}\) G exists between the two opposite poles of the newly formed neutron star. However the neutrino distribution function that results when a pulsar receives a kick is likely to be of this form, so we proceed assuming equation (17). For the plasma instabilities, the important feature is that neutrino scattering cross section rise with energy as \(E_\nu^2\). Thus the scattered (i.e. anisotropic) part of the neutrino distribution function will have a positive value of \(\partial f_\nu / \partial p_\nu\) giving rise to positive kinetic growth rates in this region of phase space.

3.1. The Kinetic Case

The integral over the neutrino distribution function may be carried out putting

\[\mathbf{k} \cdot \partial f_\nu / \partial \mathbf{p}_\nu = |\mathbf{k}| \cos \beta \partial f_\nu / \partial |\mathbf{p}_\nu| - |\mathbf{k}| \sin \beta / |\mathbf{p}_\nu| \partial f_\nu / \partial \theta_\nu\]  \hspace{1cm} (20)

with \(\cos \beta = \mathbf{k} \cdot \mathbf{p}_\nu / |\mathbf{k}| |\mathbf{p}_\nu|\). For emission along \(\theta_\nu = 0\), \(\beta = -\theta_\nu\) and using the standard result \(\int_0^\infty x^{n-1} / (\exp (x) + 1) \, dx = \Gamma (n) \zeta (n) (1 - 1/2^{n-1})\) the integral evaluates to

\[\int \mathbf{k} \cdot \frac{\partial f_\nu}{\partial \mathbf{p}_\nu} \delta (\omega - \mathbf{k} \cdot \mathbf{v}_\nu) \, d^3 \mathbf{p}_\nu = -\frac{(k_B T_\nu)^2}{24} + \frac{7\pi^2}{240} (k_B T_\nu)^4 \tau_0 P_n\]  \hspace{1cm} (21)

for \(\omega \sim |\mathbf{k}|\). The first term gives a negative contribution due to neutrino Landau damping by the thermal part of the distribution function. The second term gives rise to the instability. Thus \(0.7\pi^2 (k_B T_\nu)^2 \tau_0 P_n > 1\) is required. This evaluates to \(k_B T_\nu \sim 85\) MeV, comparable to the gravitational binding energy per nucleon released in the collapse, for \(P_N \sim 2 \times 10^{-5}\). At higher magnetic field strengths such as those thought to be present in magnetars \[16, 17\] \((B \sim 10^{15}\text{G})\) the required neutrino temperature drops to 8.5 MeV. This
temperature (though probably not the magnetic field) is more consistent with neutrinos actually observed from SN 1987A [18].

However as pointed out by [3] the kinetic growth rate will still be very small. The reason is that neutrinos may only excite waves with phase velocities in the range $v_F \to 1$. Reinstating the necessary factor of $\hbar^3 c^5$ in equation (14) and evaluating gives an approximate expression for the maximum growth rate

$$\delta \sim 1.8 \times 10^{-53} n_e \omega_p \gamma_F (1 - v_F)^3.$$  

(22)

for a neutrino temperature of 10 MeV, and the value of the integral in equation (21) taken to be $+(k_B T_\nu)^2/24$. This equation is evaluated for a variety of densities in Table 2. These values are far too small to be important during the few second neutrino burst during a supernova explosion. The lower densities in Table 2 may occur outside the neutrinosphere (the central region where neutrinos are trapped and are essentially isotropic, see [3]). In this case a beam instability of the type considered in [1, 3] would be more realistic.

### 3.2. The Reactive Case

With the same notation as above, the integral over the second term in equation (20) gives zero, while the first term evaluates to $(k_B T_\nu)^2/36$. As expected this term gives no wave growth, since it represents the Landau damping of waves by the thermal neutrino distribution. We remark here that our analysis of the reactive instability is necessarily rather superficial. It would not be difficult to assume a neutrino distribution function that would give wave growth. However the question of whether such a distribution function is realistic for a supernova would still not be answered. For this reason the subsequent discussion will be concerned only with the kinetic instability.

| $n_e$ (cm$^{-3}$) | $1 - v_F/c$ | $\gamma$ | $\omega_p$ (rad/s) | $\omega_{crit}$ ($\omega_p$) | $\delta$ (rad/s) |
|-----------------|-------------|---------|----------------------|-----------------------------|-----------------|
| $10^{32}$       | 0.016       | 5.6     | $2.4 \times 10^{20}$ | 2.11                        | $3 \times 10^{-7}$ |
| $10^{34}$       | $7.5 \times 10^{-4}$ | 26      | $1.1 \times 10^{21}$ | 2.92                        | $3 \times 10^{-9}$ |
| $10^{36}$       | $3.2 \times 10^{-5}$ | 120     | $5.2 \times 10^{21}$ | 3.64                        | $3 \times 10^{-11}$ |
| $10^{38}$       | $1.6 \times 10^{-6}$ | 550     | $2.4 \times 10^{22}$ | 4.21                        | $3 \times 10^{-13}$ |
4. Discussion and Conclusions

The fundamental reason that keeps the kinetic instability from developing significant wave growth is that the phase velocity of the waves supported by the plasma is very close to unity (the speed of light). Hence the factor of \(\left(1 - \omega^2/|k|^2\right)^2\) in the growth rate becomes very small. The physical reason for this is that when the dispersive properties of the plasma approach those of the vacuum, longitudinal waves will cease to exist. With this in mind we will briefly consider the effect of the magnetic field on the plasmon dispersion relation. Again working from ref \[12\] (page 166, equation 10.21) we can derive an expression for \(K_L\) with an ambient magnetic field. For simplicity we consider the case \(k \parallel B\), the problem is identical to the non-magnetic case discussed above. Putting \(\partial f_e/\partial p_{e\perp} = -2/(2\pi)^3 \times \delta \left(p_{e\perp} - \sqrt{p_F^2 - p_{e\parallel}^2}\right)\), \(J_{s-1}(x) + J_{s+1}(x) = \frac{2}{x} J_s(x)\), and rewriting the sum over Bessel functions as \(\sum_{\infty} \to \sum_0\) the expression is

\[
K_L = 1 - \int_0^\pi \sum_{s=0}^\infty \frac{3\omega p_s^2}{\omega^2 - s^2\Omega^2} J_s^2 \left(\frac{|k|v_F \sin \theta}{\Omega}\right) \frac{\Omega^2}{|k|^2 v_F^2} \sin \theta d\theta.
\] (23)

The electron cyclotron frequency is \(\Omega = q|B|/\gamma_F m_e\). For \(\Omega << |k|v_F\) the sum over \(s\) can be replaced by an integral, whereupon its evaluation recovers the non-magnetic longitudinal dispersion relation. It is more interesting to consider the behaviour of individual harmonics over the range \(|k|v_F << \Omega \to |k|v_F >> \Omega\). Then a behaviour something like the Bernstein modes discussed in \[12\] appears. For example for \(s = 1\), as \(|k| \to 0\), \(\omega \to \Omega_{UH}\) where \(\Omega_{UH}^2 = \omega_p^2 + \Omega^2\) is the upper hybrid frequency. However as \(|k| \to \infty\), \(\omega \to \Omega \neq |k|\), so a kinetic instability could have significantly higher growth rate than in the estimates above.

Another possibility if \(\Omega > \omega_p\) is a kinetic instability exciting electromagnetic waves, since for low frequencies \(K_T = |k|^2/\omega^2 > 1\). The discovery of pulsars with magnetic fields \(\sim 10^{15}\) G \([14, 17]\) opens up this possibility for degenerate electrons at densities \(10^{33} - 10^{34}\) cm\(^{-3}\). These novel scenarios will be investigated further in future papers.

In conclusion then we have given a plasma theory for the so-called neutrino instability, that reproduces the results of quantum statistical field theory in the kinetic limit, and which gives more accurate expressions to use in the development of the reactive instability. The growth rates for longitudinal waves are insignificant in, e.g. the neutrino mist during a supernova explosion, but we have been able to identify plausible conditions in which such wave growth might occur.

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Fig. 1.— The dispersion relation for electrostatic waves in a degenerate electron plasma of density $10^{34} \text{ cm}^{-3}$. The vertical dotted line indicates the wavevector where $\omega = |k|$, which is the onset of the kinetic instability regime.