Reversing the Asymmetry of the Geometrical Resonances of Composite Fermions

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We propose an experiment to test the uniform-Berry-curvature picture of composite fermions. We show that the asymmetry of geometrical resonances observed in a periodically modulated composite fermion system can be explained with the uniform-Berry-curvature picture. Moreover, we show that an alternative way of modulating the system, i.e., modulating the external magnetic field, will induce an asymmetry opposite to that of the usual periodic grating modulation which effectively modulates the Chern-Simons field. The experiment can serve as a critical test of the uniform-Berry-curvature picture, and probe the dipole structure of composite fermions initially proposed by Read.

	extbf{Introduction} A two-dimensional electron system (2DES) subjected to a strong perpendicular magnetic field exhibits exotic many-body states, in particular, the fractional quantum Hall states at odd-denominator filling factors\textsuperscript{[1] [2]}, and the Fermi-liquid-like states at even-denominator fillings\textsuperscript{[3] [4]}. Jain’s composite fermion (CF) theory provides a unified understanding to these states\textsuperscript{[5]}. A CF can be regarded as an electron attached with 2\textit{p} quantum vortices and feels an effective magnetic field \textit{B}∗ = \textit{B} − \textit{b}CS\textit{p} with \textit{B} being the external magnetic field and \textit{b}CS\textit{p} = 2\textit{p}\textit{me}\phi_{0} the emergent Chern-Simons field, where \textit{ne} is the density of electrons and \phi_{0} = \textit{h}/\textit{e} is the quanta of magnetic flux\textsuperscript{[2] [3]}. The Halperin-Lee-Read (HLR) theory treats the CF as an electron-like particle and predicts that CFs form a Fermi liquid at even-denominator filling \nu = 1/2\textit{p} for which the effective magnetic field \textit{B}∗ = 0\textsuperscript{[4]}. The Fermi liquid state is confirmed by various experiments\textsuperscript{[6]}. Though the HLR theory achieves great success in explaining many observed phenomena, it does not yield a correct CF Hall conductivity \sigma_{xy} = −\textit{e}^{2}/2\hbar at half-filling as required by the particle-hole symmetry\textsuperscript{[7]}. Motivated by the difficulty, Son proposes that the CF is a Dirac particle\textsuperscript{[8]}. In the Dirac theory, the CF is considered as a vortex dual of a Dirac electron coupling to an emergent gauge field. However, its microscopic basis is not yet clear\textsuperscript{[9]}. On the other hand, Shi et al. derive the dynamics of the CF Wigner crystal and find that CFs are subjected to a Berry curvature uniformly distributed in momentum space\textsuperscript{[10]}. Based on that, they propose the uniform-Berry-curvature picture of CFs\textsuperscript{[11]}. A calculation of the Berry phase of CFs from a microscopic wave function by Geraedts et al. seems to lend a support to the Dirac picture\textsuperscript{[12]}. However, a refined calculation suggests otherwise\textsuperscript{[13]}. Actually, the Berry curvature is analytically shown to be uniform for the Rezayi-Read wave function\textsuperscript{[14]}. Although the two pictures look quite different, both predict that a CF accumulates a \pi Berry phase when it moves around the Fermi circle.

The manifestations of the \pi Berry phase have been observed in a number of experiments and numerical calculations. In the numerical simulations of the infinite-cylinder density matrix renormalization group, the suppression of 2\textit{K}_{\text{F}} backscattering off particle-hole symmetric impurities is interpreted as a result of the \pi Berry phase\textsuperscript{[15]}. In the Shubnikov–de Haas oscillation experiments of CFs at a fixed magnetic field, the \pi Berry phase is shown to appear in the magneto-resistivity formula\textsuperscript{[16]}. In the geometrical resonance experiments of CFs with periodic grating modulations, the asymmetry of the commensurability condition on the two sides about half filling observed in Ref.\textsuperscript{[17]} can also be explained as a result of the \pi Berry phase (see below). Though these studies convincingly show the presence of the \pi Berry phase, they can not differentiate the Dirac picture and the uniform-Berry-curvature picture.

In this paper, we propose an experiment to test the uniform-Berry-curvature picture. First, we show that the uniform-Berry-curvature picture predicts a Fermi wave vector different from the HLR theory but same as the Dirac theory\textsuperscript{[8]}. The asymmetry of the commensurability conditions in Ref.\textsuperscript{[17]} can be explained with the modified Fermi wave vector. Next, we show that the uniform-Berry-curvature picture is equivalent to the dipole picture initially proposed by Read\textsuperscript{[18] [19]}. In the dipole picture, it becomes obvious that the external magnetic field \textit{B} and the Chern-Simons field \textit{b}CS are coupling to different degrees of freedom in a CF, i.e., the electron and the quantum vortices respectively (see Figure.\textsuperscript{[1]a}). We show that a geometrical resonance experiment with a periodically modulated external magnetic field will yield an asymmetry opposite to that of the usual periodic grating modulation. This experiment can serve as a critical test to the uniform-Berry-curvature picture, and at the same time, probe the dipole structure of CFs.

\textbf{Fermi wave vector} The uniform-Berry-curvature picture predicts a Fermi wave vector different from the HLR theory. In the HLR theory, the CF is treated as an electron-like particle. It predicts a Fermi wave vector \textit{k}_{\text{F}} = \sqrt{4\pi\textit{n}_{e}}. In the uniform-Berry-curvature picture,
the equations of motion of CFs are modified to

\[ \dot{x} = \frac{p}{m^*_{\text{CF}}} + \frac{1}{eB} \dot{z} \times p, \quad (1) \]

\[ \dot{p} = -eB^* \dot{z} \times \dot{x}, \quad (2) \]

where \( x, p, \) and \( m^*_{\text{CF}} \) are the position, momentum, and effective mass of a CF, respectively \[10\]. Due to the presence of the Berry curvature \( \Omega_z = 1/eB \) in Eq. (1), the phase-space density of states is modified by a factor \( D = 1 - B^*/B \) \[20\]. The Fermi wave vector \( k_F \) of CF can be determined through the condition \( \pi k_F^2 D/(2\pi)^2 = n_e \), and is

\[ k_F = \sqrt{eB/h}, \quad (3) \]

which is different from the prediction of the HLR theory and independent of \( n_e \). This result is the same as the Dirac theory. The coincidence is not surprising because both the pictures have a \( \pi \)-Berry phase along the Fermi circle. To differentiate the two pictures, one has to probe deeper.

**Dipole structure** The uniform-Berry-curvature picture is actually equivalent to the dipole picture initially proposed by Read \[18,19\]. To see that, we can rewrite the EOMs with the new variables \( x^e \equiv x \) and \( x^v = x - \dot{z} \times p/eB \):

\[ -eB \dot{z} \times x^e = \frac{\partial \epsilon}{\partial x^e}, \quad (4) \]

\[ eb^{CS} \dot{z} \times x^v = \frac{\partial \epsilon}{\partial x^v}, \quad (5) \]

where \( x^e \) and \( x^v \) are interpreted as the position of the electron and quantum vortices in a CF respectively, and \( \epsilon \propto |x^e - x^v|^2 \) is the binding energy between the electron and the quantum vortices \[10\]. The momentum \( p \) of a CF is interpreted as \( p = eB \dot{z} \times d \) with the displacement \( d \equiv x^v - x^e \) (see Figure 1(a)). From the EOMs, it is clear that the electron is only coupled to the external electromagnetic field \( B \) while the quantum vortices are only coupled to the emergent Chern-Simons field \( b^{CS} \). Moving a CF in the momentum space is equivalent to fixing the quantum vortices and moving the electron in the real space. The Aharonov-Bohm phase accumulated by the electron is nothing but the Berry phase expected from the uniform Berry curvature in Eq. (1) \[11\]. It also becomes obvious that the external magnetic field and the Chern-Simons field are not equivalent microscopically since they are coupling to different internal degrees of freedom. Therefore, we anticipate that the external magnetic field \( B \) and the Chern-Simons field \( b^{CS} \) have different effects on CFs.

**Periodic grating modulation** In 1989, Weiss et al. observed that when a 2DES was patterned with a weak one dimensional periodic grating modulation, its magnetoresistance showed an oscillation with respect to \( 2R_c/a \) at low magnetic field, where \( R_c \) is the cyclotron radius and \( a \) is the modulation period \[21\]. When the 2DES is at an even-dominator filling factor \( \nu = 1/2p \), CFs feel zero effective magnetic field \( B^* = 0 \). It is nature to expect that the Weiss oscillation can also be observed in CF systems when the effective magnetic field deviates slightly from zero. This has been confirmed by a number of geometrical resonance experiments for CFs \[22,25\].

The periodic grating modulation is equivalent to a modulation of the Chern-Simons field for CFs. In periodic grating modulation experiments, CFs are subjected to a weak electrostatic potential modulation \( \delta V_{\text{ext}}(x) = V_{\text{ext}} \cos(2\pi x/a) \) through the piezoelectric effect. The electrostatic potential will induce a modulation of the electron density \( \delta n_e \), which in turn induces a modulation of the Chern-Simons field \( \delta b^{CS} = 2\delta n_e \epsilon \). The energy changes due to the electrostatic potential and the Chern-

![Figure 1. Cyclotron orbits of a CF under various conditions:](image-url)
Simons field are \( -e\delta V^{\text{ext}} \) and \(-e\mathbf{x} \cdot \delta \mathbf{A}^{\text{CS}}\) respectively. By assuming a non-interacting CF model, the ratio of these two contributions is \( \pi/ak_F \ll 1 \) (e.g., for \( B = 14 \text{T} \) and \( a = 200 \text{nm} \), \( \pi/ak_F \approx 0.1 \) \[20\]). As a result, the modulation of the Chern-Simons field dominates in this case.

The commensurability condition can be derived semiclassically \[20\] \[27\]. For a CF on the Fermi circle, the solutions of Eqs. \( 1, 2 \) are

\[
\mathbf{x}(t) = \mathbf{x}_0 + R^*_e [-\cos(\omega^*_e t + \phi) , \sin(\omega^*_e t + \phi)], \quad (6)
\]

\[
\mathbf{p}(t) = \hbar k_F \left[ \sin(\omega^*_e t + \phi) , \cos(\omega^*_e t + \phi) \right], \quad (7)
\]

where \( \mathbf{x}_0, R^*_e = \hbar k_F/eB^* \) and \( \omega^*_e = eB^*/Dm^*_e \) are the center coordinate, radius and frequency of the cyclotron orbit, respectively, and \( \phi \) is a phase factor. Without the periodic modulation, all orbits have a degenerate energy. In the presence of the periodic modulation, the degeneracy is split. The average energy change of a CF caused by the Chern-Simons field modulation during a period of the cyclotron motion \( T = 2\pi/\omega^*_e \) is \( \langle \delta U \rangle = (1/T) \int_0^T dt (-e\mathbf{x} \cdot \delta \mathbf{A}^{\text{CS}}) = (2e\hbar k_F V^{\text{ext}}/q)J_1(qR^*_e) \cos qx_0, \)

where \( q = 2\pi/a \) and \( J_1(x) \) is the first Bessel function \[26\]. In the weak effective magnetic limit \( qR^*_e \gg 1 \), \( \langle \delta U \rangle \approx -\sqrt{2/\pi qR^*_e(2e\hbar k_F V^{\text{ext}}/q)} \cos qx_0 \cos(qR^*_e + \pi/4) \). This means that the broadening caused by the modulation is proportional to \( \cos(qR^*_e + \pi/4) \), which vanishes when the commensurability condition \( 2R^*_e/a = i + \gamma \) with \( \gamma = 1/4 \) is fulfilled. One may assume that the conductivity along the direction transverse to the modulation is proportional to the broadening \[20\] \[28\]. As a result, the commensurability condition is manifest in experiments as a series of the minimum of the longitudinal magnetoresistance. For the Fermi wave vector shown in Eq. \[3\], the commensurability condition can be written as:

\[
\frac{B_0}{B^*_i} \approx \frac{a}{2} \sqrt{\frac{eB_0}{\hbar}} (i + \gamma) + \left\{ \begin{array}{c} \frac{1}{2} \ B^* > 0 \\ -\frac{1}{2} \ B^* < 0 \end{array} \right. \quad (8)
\]

for \( |B^*_i| \ll B_0 \), where \( B_0 = 2n_e\phi_0 \) is the magnetic field at the half-filling, and \( B^*_i \) is the effective magnetic field for the \( i \)-th magnetoresistance minima. We see that the commensurability condition shows an asymmetry between the particle (\( B^* > 0 \)) and hole (\( B^* < 0 \)).

The asymmetry had actually been observed in periodic grating modulation experiments. We adapt and fit the experimental results of Ref. \[17\] in Figure. \[2\] One can see that for all index \( i \)'s, the value of \( \frac{B_0}{|B^*_i|} \) with \( B^*_i > 0 \) (hole) sits above that with \( B^*_i < 0 \) (particle). The vertical shift of the two lines is \( \Delta \left( \frac{B_0}{|B^*_i|} \right) = -1.33 \pm 0.39 \), close to the prediction \( \Delta \left( \frac{B_0}{|B^*_i|} \right) = -1 \).

**Periodic external magnetic field modulation** Next, we show that a weak periodic modulation of the external magnetic field will induce an asymmetry opposite to that of the periodic grating modulation. When a weak periodic modulation of the external magnetic field \( \delta \mathbf{B}(\mathbf{x}) = \delta B \cos \mathbf{q} \cdot \mathbf{x} \hat{z} \) is applied to a 2DES, it couples to the electron in the CF and the energy change is \( \delta U = -e\mathbf{x} \cdot \delta \mathbf{A}(\mathbf{x}) \) with \( \delta \mathbf{A}(\mathbf{x}) \) being the vector potential with respect to \( \delta \mathbf{B}(\mathbf{x}) \). Note that \( \delta U \) in current case is related to the electron coordinate \( \mathbf{x}^e \) instead of \( \mathbf{x} \) as in the case of periodic grating modulation. From Eqs. \( 6, 7 \), we determine \( \mathbf{x}^e(t) = \mathbf{x}(t) - \hat{z} \times \mathbf{p}(t)/e\mathbf{B} = \mathbf{x}_0 + R^e_c [-\cos(\omega^*_e t + \phi) , \sin(\omega^*_e t + \phi)] \) with

\[
R^e_c = DR^*_e. \quad (9)
\]

Therefore, the electron has a cyclotron radius different from that of the CF. As a result, the commensurability condition becomes \( 2R^e_c/a = i + \gamma \), and can be written as:

\[
\frac{B_0}{|B^*_i|} \approx \frac{a}{2} \sqrt{\frac{eB_0}{\hbar}} (i + \gamma) + \left\{ \begin{array}{c} \frac{1}{2} \ B^* > 0 \\ -\frac{1}{2} \ B^* < 0 \end{array} \right. . \quad (10)
\]

We see that the value of \( B_0/|B^*_i| \) with \( B^*_i > 0 \) (electron) now sits above that with \( B^*_i < 0 \) (hole). The asymmetry is opposite to the asymmetry induced by the Chern-Simons field modulation.

However, there is still a complexity for the proposed experiment. This is because the energy of the lowest Landau level (LLL) is proportional to \( B \), and the modulation of \( B \) will introduce a modulation of the effective potential felt by CFs \[29\]. While the direct effect of the effective potential is negligible, the Chern-Simons field induced by modulation may not be small. To estimate the modulation amplitude of the induced Chern-Simons field, we apply the density functional theory \[30\] \[32\]. The grand canonical energy functional \( \mathcal{E} \) of the system can be
\[ \mathcal{E}[n] = \int dr \left[ -\mu n(r) + \left( \frac{\hbar e}{2m_b} + \frac{g \mu_B}{2} \right) B(r) n(r)\right. \\
+ \nu_{\text{xc}} \left[ n(r) \right] n(r) + \frac{e^2}{8\pi} \int dr' \frac{\Delta n(r) \Delta n(r')}{|r - r'|} \] \tag{11} \]

where \( \mu \) is the chemical potential, the second term is the kinetic and Zeeman energy of electrons in the LLL with \( m_b \), \( g \) and \( \mu_B \) being the band mass, the effective Landé factor and the Bohr magneton, respectively, \( \nu_{\text{xc}}[n(r)] \equiv (e^2/4\pi\epsilon_l) u(\nu) \) is the exchange-correlation energy per particle with \( u(\nu) \) determined by the interpolation formula presented in Ref. [33], \( l_B \equiv \sqrt{\hbar/eB} \) is the magnetic length and \( \epsilon \) is the static permittivity, the last term is the Coulomb energy due to the density modulation \( \Delta n(r) \).

To determine the density modulation due to the modulation of the external magnetic field, we minimize the energy functional with respect to the density. By assuming that both \( \delta n \) and \( \delta B \) are small quantities, we obtain \( \delta B_{\text{CS}}/\delta \nu_{\text{CS}} = \delta n/n = -\alpha (\delta B/B) \) with

\[ \alpha = \frac{a_c/l_B - 2}{a/a_B} \]

where \( a_c \equiv 2\pi (1 + g\mu_B/2m_e) a_B \) with \( a_B \) being the effective Bohr radius and \( m_e \) being the bare electron mass. \( \beta_1 \equiv 2\pi [(\nu u(\nu))'' - (\nu u(\nu))']_{\nu = 1/2} = 2.3 \) and \( \beta_2 = -2\pi [(\nu u(\nu))']_{\nu = 1/2} = 1.6 \).

To observe the asymmetry reversal predicted in Eq. (10), we require \( |\alpha| \ll 1 \). It is not difficult to fulfill the requirement in a GaAs-based 2DES, for which \( a_c \approx 62 \text{ nm} \). For the experimental parameters of Ref. [17], i.e., \( a = 200 \text{ nm} \) and \( B = 14 \text{ T} \), the value of \( \beta_2 \) is 0.24, fulfilling the requirement. In the strong field limit \( B \to \infty, l_B \to 0 \), we have \( \alpha = a_c/2 \). Therefore, one can always fulfill the requirement by choosing a modulation period \( a \) much larger than 62 nm. We further note that the modulation of the external magnetic field had already been achieved for electrons by placing a ferromagnet or superconductor microstructure on top of a 2DES [34] [39]. We expect that the same techniques can be employed for our proposed experiment for CFs.

**Summary** In summary, we theoretically study the manifestations of the uniform-Berry-curvature picture in the geometrical resonance experiments for CFs. We show that the modulation of an externally applied magnetic field will induce an asymmetry opposite to that induced by a periodic grating modulation. This experiment can serve as a critical test to the uniform-Berry-curvature picture. Since the effect originates from the dipole structure of CFs, its successful observation will also provide an experimental confirmation to the dipole picture of CFs initially proposed by Read.

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