Vacuum and spacetime signature in the theory of superalgebraic spinors

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ABSTRACT: Formulas for vacuum state vector and operators of the Lorentz transformations and gauge charge transformations of spinors are derived in the superalgebraic representation of spinors. Five operator analogs of five Dirac gamma matrices exist in the superalgebraic approach as well as two additional operator gamma matrices. They are constructed from Grassmann densities and derivatives with respect to them. We show that second copy of gamma operators exist and they are Lorentz invariant. They are constructed from operators of creation and annihilation. We show that the condition for the existence of spinor vacuum imposes restrictions on possible variants of Clifford algebras of gamma operators: only real algebra with one timelike basis Clifford vector corresponding to the zero gamma matrix in the Dirac representation can be realized. In this case, the signature of the four-dimensional spacetime, in which there is a vacuum state, can only be (1, -1, -1, -1), and there are two additional axes corresponding to the inner space of the spinor, with a signature (-1, -1).
1 Introduction

The question of the origin of the dimension and the spacetime signature has long attracted the attention of physicists. At the same time, there are different approaches in attempts to substantiate the observed dimension and the spacetime signature.

One of the main directions is the theory of supergravity. It was shown in [1] that the maximum dimension of spacetime, at which supergravity can be built, is equal to 11. At the same time, multiplets of matter fields for supersymmetric Yang-Mills theories exist only when the dimension of spacetime is less than or equal to 10 [2].

Subsequently, the main attention was paid to the theory of superstrings and supermembranes. Various versions of these theories were combined into an 11-dimensional M-theory [3, 4]. In [5], the most general properties of the theories of supersymmetry and supergravity in spaces of various dimensions and signatures were analyzed. Proceeding from the possibility of the existence of majoram and pseudo-Maioran spinors in such spaces, it was shown that supersymmetry and supergravity of M-theory can exist in 11-dimensional and 10-dimensional spaces with arbitrary signatures, although depending on the signature the theory type will differ. Later, other possibilities were shown for constructing variants of M-theories in spaces of different signatures [6].

Another approaches are Kaluza-Klein theories. For example, in [7] it was shown that in the theories of Kaluza-Klein in some cases it is possible not to postulate, but to determine from the dynamics not only the dimension of the spacetime, but also its signature.

In [8–10], an attempt was made to find a signature based on the average value of the quantum fluctuating metric of spacetime.
An attempt was made in [11] to explain the dimension and signature of spacetime from the anthropic principle and the possibility of causality, in [12] from the existence of equations of motion for fermions and bosons coinciding with four-dimensional ones, in [13] from the possibility of existence in spacetime classical electromagnetism.

In all the above approaches, the fermion vacuum operator in the second quantization formalism is not constructed and the restrictions imposed by such a construction are not considered. Therefore, the possibility of the existence of a vacuum and fermions is not discussed. In particular, the vacuum should be a Lorentz scalar and have zero spin, but in the theory of algebraic spinors, which more generally describes spinors than the Dirac matrix theory, Clifford vacuum has the transformational properties of the spinor component, and not the scalar [14].

The author develops an approach to the theory of spacetime, allowing to solve this problem. It is based on the theory of superalgebraic spinors – an extension of the theory of algebraic spinors, in which the generators of Clifford algebras (Dirac gamma matrices) are composite.

In [15, 16], it was shown that using Grassmann variables and derivatives with respect to them, one can construct an analog of matrix algebra, including analogs of matrix columns of 4-spinors and their adjoint rows of conjugate spinors. But at the same time, the spinors and their conjugates exist in the same space – in the same algebra.

In [17, 18], this approach was developed – Grassmann densities \( \theta^a(p) \), \( a = 1, 2, 3, 4 \), and derivatives \( \frac{\partial}{\partial \theta^a(p)} \) with respect to them were introduced, with CAR-algebra

\[
\left\{ \frac{\partial}{\partial \theta^a(p)}, \theta^k(p') \right\} = \delta(p-p')\delta^k_i. \tag{1.1}
\]

Superalgebraic analogs \( \hat{\gamma}^\mu \) (1.2) are constructed for Dirac gamma matrices \( \gamma^\mu \) from these densities, we call them gamma operators.

\[
\hat{\gamma}^0 = \int d^3p \left[ -\frac{\partial}{\partial \theta^1(p)} \theta^1(p) + \frac{\partial}{\partial \theta^2(p)} \theta^2(p) + \frac{\partial}{\partial \theta^3(p)} \theta^3(p) + \frac{\partial}{\partial \theta^4(p)} \theta^4(p), \ast \right],
\]
\[
\hat{\gamma}^1 = \int d^3p \left[ -\frac{\partial}{\partial \theta^1(p)} \frac{\partial}{\partial \theta^4(p)} - \theta^4(p)\theta^1(p) + \frac{\partial}{\partial \theta^2(p)} \frac{\partial}{\partial \theta^3(p)} - \theta^3(p)\theta^2(p), \ast \right],
\]
\[
\hat{\gamma}^2 = i \int d^3p \left[ -\frac{\partial}{\partial \theta^1(p)} \frac{\partial}{\partial \theta^1(p)} - \theta^1(p)\theta^1(p) + \frac{\partial}{\partial \theta^2(p)} \frac{\partial}{\partial \theta^3(p)} + \theta^3(p)\theta^2(p), \ast \right], \tag{1.2}
\]
\[
\hat{\gamma}^3 = \int d^3p \left[ -\frac{\partial}{\partial \theta^1(p)} \frac{\partial}{\partial \theta^3(p)} - \theta^3(p)\theta^1(p) - \frac{\partial}{\partial \theta^2(p)} \frac{\partial}{\partial \theta^4(p)} + \theta^4(p)\theta^2(p), \ast \right],
\]
\[
\hat{\gamma}^4 = i\hat{\gamma}^5 = i \int d^3p \left[ -\frac{\partial}{\partial \theta^1(p)} \frac{\partial}{\partial \theta^3(p)} + \theta^3(p)\theta^1(p) + \frac{\partial}{\partial \theta^2(p)} \frac{\partial}{\partial \theta^4(p)} + \theta^4(p)\theta^2(p), \ast \right].
\]

They convert and their linear combinations in the same way that Dirac matrices convert matrix columns and their linear combinations. The theory is automatically secondarily quantized and does not require normalization of operators.

In the proposed theory, in addition to analogs of the Dirac matrices, there are two additional gamma operators and , the rotation operator in whose plane (gauge transformation)
is analogous to the charge operator of the second quantization method [18]:

\[
\hat{\gamma}^6 = i \int d^3p \left[ \frac{\partial}{\partial \theta^1(p)} \frac{\partial}{\partial \theta^2(p)} + \theta^2(p)\theta^1(p) - \frac{\partial}{\partial \theta^3(p)} \frac{\partial}{\partial \theta^4(p)} - \theta^4(p)\theta^3(p) \right],
\]

\[
\hat{\gamma}^7 = \int d^3p \left[ \frac{\partial}{\partial \theta^1(p)} \frac{\partial}{\partial \theta^2(p)} + -\theta(p)\theta^1(p) + \frac{\partial}{\partial \theta^3(p)} \frac{\partial}{\partial \theta^4(p)} - \theta^4(p)\theta^3(p) \right],
\]

(1.3)

In [18], it was shown that transformations of densities and \( \rho \), while maintaining their CAR-algebra of creation and annihilation operators, provide transformations of field operators of the form:

\[
\Psi' = (1 + i\hat{\gamma}^a d\omega_a + \frac{1}{4} \hat{\gamma}^{ab} d\omega_{ab})\Psi,
\]

(1.4)

where \( \hat{\gamma}^{ab} = \frac{1}{2}(\hat{\gamma}^a\hat{\gamma}^b - \hat{\gamma}^b\hat{\gamma}^a) \); \( a, b = 0, 1, 2, 3, 4, 6, 7 \), and \( d\omega_{ab} = -d\omega_{ba} \) – real infinitesimal transformation parameters. The multiplier \( \frac{1}{4} \) is added in (1.4) compared to [18] to correspond to the usual transformation formulas for spinors in the case of Lorentz transformations.

## 2 Operators of pseudo-orthogonal rotation

Operators \( \hat{\gamma}^{ab} \) are generators of pseudo-orthogonal rotations of the form \( \exp(\hat{\gamma}^{ab} \omega_{ab}/4) \), where \( a, b = 0, 1, 2, 3, 4, 6, 7 \). We will call them gamma operators of rotations. They are generators of Lorentz rotations when \( a, b = 0, 1, 2, 3 \).

Operators of annihilation of spinors \( \hat{b}_\alpha(p), \alpha = 1, 2, \) and of antispinors \( \hat{b}_\tau(p), \tau = 3, 4, \) are obtained by Lorentz rotations from \( \frac{\partial}{\partial \theta^1(0)} \) and \( \frac{\partial}{\partial \theta^\tau(0)} \), and the Dirac conjugated to them operators of creation \( \hat{\bar{b}}_\alpha(p) \) and \( \hat{\bar{b}}_\tau(p) \) – by Lorentz rotations from \( \theta^\alpha(0) \) and \( \theta^\tau(0) \) [17, 18], while momentum in the argument is replaced from 0 to \( p \):

\[
\hat{b}_i(p) = (e^{\hat{\gamma}^{ab} \omega_{ab}/2} \frac{\partial}{\partial \theta^i(0)})|_{0\to p},
\]

\[
\hat{\bar{b}}_i(p) = (e^{\hat{\gamma}^{ab} \omega_{ab}/2} \theta^i(0))|_{0\to p},
\]

(2.1)

Anticommutation relations for \( \hat{b}_i(p) \) and \( \hat{\bar{b}}_k(p') \)

\[
\{ \hat{b}_i(p), \hat{\bar{b}}_k(p') \} = \delta(p - p')\delta^i_k.
\]

(2.2)

In (2.1), the particle momentum \( p \) depends on Lorentz rotation parameters \( \omega_{0k} \). For example, for rotation in the plane \( \hat{\gamma}^0, \hat{\gamma}^1 \) the transformation (4) for \( \hat{b}_1(p) \) and \( \hat{\bar{b}}_1(p) \) will look like

\[
\hat{b}_1(p) = \cosh \frac{\omega_{01}}{2} \frac{\partial}{\partial \theta^1(p)} + \sinh \frac{\omega_{01}}{2} \hat{\gamma}^{01} \frac{\partial}{\partial \theta^1(p)},
\]

\[
\hat{\bar{b}}_1(p) = \cosh \frac{\omega_{01}}{2} \theta^1(p) + \sinh \frac{\omega_{01}}{2} \hat{\gamma}^{01} \theta^1(p).
\]

(2.3)

As a result, we get

\[
\hat{b}_1(p) = \cosh \frac{\omega_{01}}{2} \frac{\partial}{\partial \theta^1(p)} + \sinh \frac{\omega_{01}}{2} \theta^1(p),
\]

\[
\hat{\bar{b}}_1(p) = \cosh \frac{\omega_{01}}{2} \theta^1(p) - \sinh \frac{\omega_{01}}{2} \frac{\partial}{\partial \theta^1(p)}.
\]

(2.4)
Expressions for operators $\gamma^{ab}$ are given in (2.5)-(??) – they will be important later.

\[
\gamma_{01}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} \frac{\partial}{\partial \theta^{3}(p)} + \theta^{1}(p)\theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \frac{\partial}{\partial \theta^{3}(p)} + \theta^{3}(p)\theta^{2}(p), * \right], \\
\gamma_{02}^{4} = -i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} \frac{\partial}{\partial \theta^{3}(p)} - \theta^{1}(p)\theta^{1}(p) - \frac{\partial}{\partial \theta^{3}(p)} \frac{\partial}{\partial \theta^{3}(p)} + \theta^{3}(p)\theta^{2}(p), * \right], \\
\gamma_{03}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} \frac{\partial}{\partial \theta^{3}(p)} + \theta^{3}(p)\theta^{1}(p) - \frac{\partial}{\partial \theta^{3}(p)} \frac{\partial}{\partial \theta^{3}(p)} - \theta^{3}(p)\theta^{2}(p), * \right], \\
\gamma_{04}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} \frac{\partial}{\partial \theta^{3}(p)} - \theta^{3}(p)\theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \frac{\partial}{\partial \theta^{3}(p)} - \theta^{3}(p)\theta^{2}(p), * \right], \\
\gamma_{06}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} \frac{\partial}{\partial \theta^{2}(p)} + \theta^{2}(p)\theta^{1}(p) - \frac{\partial}{\partial \theta^{2}(p)} \frac{\partial}{\partial \theta^{2}(p)} + \theta^{2}(p)\theta^{3}(p), * \right], \\
\gamma_{07}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} \frac{\partial}{\partial \theta^{2}(p)} + \theta^{2}(p)\theta^{1}(p) - \frac{\partial}{\partial \theta^{2}(p)} \frac{\partial}{\partial \theta^{2}(p)} + \theta^{2}(p)\theta^{3}(p), * \right], \\
\gamma_{12}^{4} = -i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{2}(p)} - \theta^{1}(p)\theta^{1}(p) - \frac{\partial}{\partial \theta^{2}(p)} \frac{\partial}{\partial \theta^{2}(p)} + \theta^{2}(p)\theta^{2}(p), * \right], \\
\gamma_{13}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{1}(p)} - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{14}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{2}(p)} - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p), * \right], \\
\gamma_{16}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p), * \right], \\
\gamma_{17}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p), * \right], \\
\gamma_{23}^{4} = -i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p), * \right], \\
\gamma_{24}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p), * \right], \\
\gamma_{26}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{2}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p), * \right], \\
\gamma_{27}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{3}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{34}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{3}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{36}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{3}(p)} \theta^{3}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{37}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{3}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{3}(p)} \theta^{2}(p) - \frac{\partial}{\partial \theta^{3}(p)} \theta^{3}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{46}^{4} = \int d^{3}p \left[ \frac{\partial}{\partial \theta^{4}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{47}^{4} = i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{4}(p)} \theta^{1}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right], \\
\gamma_{67}^{4} = -i \int d^{3}p \left[ \frac{\partial}{\partial \theta^{4}(p)} \theta^{1}(p) + \frac{\partial}{\partial \theta^{4}(p)} \theta^{2}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{3}(p) - \frac{\partial}{\partial \theta^{4}(p)} \theta^{4}(p), * \right]. \]
Denote the integrands in (1.2)-(1.3) as $\hat{\gamma}^a(p)$ and in (2.5) as $\hat{\gamma}^{ab}(p)$. So, we can rewrite (1.2)-(1.3) as
\[
\hat{\gamma}^a = \int d^3p \hat{\gamma}^a(p),
\]
and (2.5) as
\[
\hat{\gamma}^{ab} = \int d^3p \hat{\gamma}^{ab}(p).
\]

3 Vacuum and discrete analogs of Grassmann densities

In [17], the author proposed a method for constructing a state vector of a vacuum. Let’s analyze it in more detail. We divide the momentum space into infinitely small volumes. We introduce operators
\[
B_k(p_j) = \frac{1}{\Delta^3p_j} \int d^3p b_k(p),
\]
\[
\bar{B}_k(p_j) = \frac{1}{\Delta^3p_j} \int d^3p \bar{b}_k(p).
\]
At the same time, given (2.2),
\[
\{\bar{B}_k(p_i), B_l(p_j)\} = \frac{1}{\Delta^3p_i\Delta^3p_j} \int d^3p \int d^3p' \{\bar{b}_k(p), b_l(p')\} = \frac{1}{\Delta^3p_j} \delta^i_j \delta^k_l.
\]

There is no silent summation over the index that numbers discrete volumes. For example, it does not exist at index $j$ in (3.1)-(3.2). For indexes enclosed in triangular brackets (for example, in (3.4)), there is also no silent summation.

The expression $\frac{1}{\Delta^3p_j} \delta^i_j$ in (3.1)-(3.2) is a discrete analogue of the delta function $\delta(p-p')$.

In addition, due to the anticommutativity of all $b_k(p)$ and $b_l(p')$ as well as all $\bar{b}_k(p)$ and $\bar{b}_l(p')$ it is obvious that
\[
(B_k(p_j))^2 = (\bar{B}_k(p_j))^2 = 0.
\]

We introduce operators
\[
\Psi_{B_{k,j}} = \Delta^3p_j B_{<k>}(p_j)\bar{B}_{<k>}(p_j),
\]
\[
\Psi_{V_j} = \Psi_{B_{1,j}} \Psi_{B_{2,j}} \Psi_{B_{3,j}} \Psi_{B_{4,j}}
\]
and determine through them the fermionic vacuum operator $\Psi_V$
\[
\Psi_V = \prod_j \Psi_{V_j},
\]
where the product goes over all physically possible values of $j$. In this case, we will assume that all volumes $\Delta^3p_j$ are formed by Lorentz rotations from the volume $\Delta^3p_j=0$ corresponding to $p = 0$, and the grid of angles $\omega_{\mu\nu}$ of these rotations is discrete.
Further, it will often be convenient to represent (3.5) in the form
\[ \Psi_V = \Psi_{V_j}^\prime \Psi_j^\prime, \]  
(3.6)

where
\[ \Psi_{V_j}^\prime = \prod_{i \neq j} \Psi_{V_i}, \]  
(3.7)

is the product of factors in (3.5), independent of \( p_j \).

Replace in the formulas with participation of \( \gamma^a \) and \( \gamma^{ab} \) continuous operators \( b_k(p) \) and \( \bar{b}_k(p) \) to discrete \( B_k(p_j) \) and \( \bar{B}_k(p_j) \), and the integral \( \int d^3 p \ldots \) to the sum \( \sum_j \Delta^3 p_j \ldots \). In this case, all formulas using continuous operators \( b_k(p) \) and \( \bar{b}_k(p) \) are replaced by completely similar ones using discrete ones, with the replacement of the delta function \( \delta(p - p') \) by \( \frac{1}{2 \pi p_i} \delta_{ij} \), where \( p_i \) corresponds to \( p \), and \( p_j \) corresponds to \( p' \). We will use for operators \( \gamma^a = \sum_j \Delta^3 p_j \gamma^a(p_j) \) and \( \gamma^{ab} = \sum_j \Delta^3 p_j \gamma^{ab}(p_j) \) after such a replacement the same notation as for the corresponding continuous ones, and we will call such \( \gamma^a \) as discrete gamma operators, and \( \gamma^{ab} \) as discrete gamma operators of rotations.

4 Action of gamma operators on the vacuum

Consider action of \( \gamma^0 \) on the vacuum (3.6). Since \( \gamma^0 \) is a commutator, we have
\[ \gamma^0 \Psi_V = \gamma^0 \prod_j \Psi_{V_j} = (\gamma^0 \Psi_{V_0}) \Psi_{V_1} \Psi_{V_2} \ldots + \Psi_{V_0} \gamma^0 \Psi_{V_1} \Psi_{V_2} \ldots + \Psi_{V_0} \Psi_{V_1} \gamma^0 \Psi_{V_2} \ldots + \ldots, \]  
(4.1)

Here brackets limit the scope of the commutator \( \gamma^0 \). In this case, from (3.4) it follows
\[ \Psi_{V_j} = (\Delta^3 p_j)^4 B_1(p_j) \bar{B}_1(p_j) B_2(p_j) \bar{B}_2(p_j) B_3(p_j) \bar{B}_3(p_j) B_4(p_j) \bar{B}_4(p_j) \]  
(4.2)

Taking into account the introduced notation for discrete operators and taking into account the fact that an arbitrary spatial momentum can be obtained from the state with \( p = 0 \) (2.1),
\[ B_1(p_j) = e^{i \theta \omega_{0k} / 2} B_1(0), \]  
(4.3)

At the same time \( B_1(p_j) \) means that the result of rotation of a state with \( p = 0 \) turns into the state with \( p = p_j \).

First consider action of \( \gamma^0(0) \) on . Easy to see that
\[ \gamma^0 \Psi_V = (\gamma^0 \Psi_{V_0}) \Psi_{V_1} \Psi_{V_2} \ldots = \]  
\[ \Delta^3 p_j [\frac{\partial}{\partial \theta^k(0)} \theta^k(0), \frac{\partial}{\partial \theta^1(0)} \theta^1(0), \frac{\partial}{\partial \theta^2(0)} \theta^2(0), \frac{\partial}{\partial \theta^3(0)} \theta^3(0), \frac{\partial}{\partial \theta^4(0)} \theta^4(0)] \Psi_{V_1} \Psi_{V_2} \ldots = 0 \]  
(4.4)

Now consider action of \( \gamma^0(p) \) on \( \Psi_V \) for the case when continuous momentum \( p = p_1 \) with corresponding discrete \( p_j \), that is, it is directed along the axis \( \gamma^1 \). Let us present \( \Psi_V \) as a
product $\Psi_V = \Psi_{V,4} \Psi_{V,3} \Psi'_{V}$ where

$$
\Psi_{V,4} = (\Delta^3 p_j)^2 B_1(p_j) B_1(p_j) B_3(p_j)
\Psi_{V,3} = (\Delta^3 p_j)^2 B_2(p_j) B_2(p_j) B_3(p_j)
$$

(4.5)

Obviously,

$$
\hat{\gamma}^0(p_1) \Psi_V = ((\hat{\gamma}^0(p_1) \Psi_{V,4}) \Psi_{V,3} + \Psi_{V,4}((\hat{\gamma}^0(p_1) \Psi_{V,3})) \Psi'_{V}.
$$

(4.6)

Write useful relationships

$$
\frac{\partial}{\partial \theta^{<a>}(p_j)} \theta^{b}(p_j) \frac{\partial}{\partial \theta^{<a>}(p_j)} = \frac{1}{\Delta^3 p_j} \delta^b_a \frac{\partial}{\partial \theta^{<a>}(p_j)},
\theta^{<a>}(p_j) \frac{\partial}{\partial \theta^{a}(p_j)} \theta^{<a>}(p_j) = \frac{1}{\Delta^3 p_j} \delta^b_c \delta^{<a>}(p_j).
$$

(4.7)

Consider action of $\hat{\gamma}^0(p_1)$ on $\Psi_{V,4}$ and $\Psi_{V,3}$. From (4.3), (2.5) and (4.5), taking into account (4.7), we obtain with $p_j = p$

$$
\hat{\gamma}^0(p_1) \Psi_{V,4} = \frac{\Delta^3 p_j}{2} \sinh \omega_{01}(p_1) \left( \frac{\partial}{\partial \theta^{01}(p_1)} \frac{\partial}{\partial \theta^{01}(p_1)} + \theta^1(p_1) \theta^4(p_1) \right),
\hat{\gamma}^0(p_1) \Psi_{V,3} = \frac{\Delta^3 p_j}{2} \sinh \omega_{01}(p_1) \left( \frac{\partial}{\partial \theta^{01}(p_1)} \frac{\partial}{\partial \theta^{01}(p_1)} + \theta^2(p_1) \theta^3(p_1) \right).
$$

(4.8)

To understand the meaning of (4.8) we consider the action of the operator of creation of a fermion-antifermion pair $\Delta^3 p B_1(p) B_1(p) \approx \Delta^3 p \theta^1(p_1) \theta^4(p_1)$ on $\Psi_{V,4}$ when $p \to 0$. The multiplier $\Delta^3 p$ is necessary for normalization to the unit probability of finding spinors in the whole space.

That is, $\hat{\gamma}^0(p_1) \Psi_{V,4}$ contains a term corresponding to the creation of a fermion-antifermion pair $\theta^1(p_1) \theta^4(p_1)$, suppressed by a small multiplier $\sinh \omega_{01}(p_1)$ in the non-relativistic limit. And $\hat{\gamma}^0(p_1) \Psi_{V,3}$ corresponds to the creation of a pair $\theta^2(p_1) \theta^3(p_1)$ with different values of the spin.

Similarly, $\frac{\partial}{\partial \theta^{kl}(p_1)} \frac{\partial}{\partial \theta^{kl}(p_1)}$ are the creation operators of a fermion-antifermion pair for an alternative vacuum [14], where factors $\frac{\partial}{\partial \theta^{kl}(p_1)} \theta^1(p_1) \frac{\partial}{\partial \theta^{kl}(p_1)} \theta^4(p_1)$ in the vacuum state vector are replaced by $\theta^1(p_1) \frac{\partial}{\partial \theta^{kl}(p_1)} \theta^4(p_1) \frac{\partial}{\partial \theta^{kl}(p_1)}$, and similarly for operator $\frac{\partial}{\partial \theta^{kl}(p_1)} \frac{\partial}{\partial \theta^{kl}(p_1)}$ for a corresponding alternative vacuum.

So, $\hat{\gamma}^0(p_1) \Psi_V \to 0$ when $p \to 0$.

Carrying out the spatial rotations $exp(\hat{\gamma}^k \omega_{kl}/4)$, wher $k,l = 1, 2, 3$, of expressions (4.8) that do not affect the multiplier $\hat{\gamma}^0$, since $\hat{\gamma}^k$ commutes with $\hat{\gamma}^0$, we get a similar result when for arbitrary directions of the spatial momentum. Thus, in the non-relativistic limit $p \to 0$ can be considered $\hat{\gamma}^0 \Psi_V = 0$.

Similarly, we find the result of the action of $\hat{\gamma}^1(0)$ on the multipliers of $\Psi_V$:

$$
\hat{\gamma}^1(0) \Psi_{V,4} = (\Delta^3 p_j)^2 \left( \frac{\partial}{\partial \theta^{01}(0)} \frac{\partial}{\partial \theta^{01}(0)} + \theta^1(0) \theta^4(0) \right),
\hat{\gamma}^1(0) \Psi_{V,3} = (\Delta^3 p_j)^2 \left( \frac{\partial}{\partial \theta^{01}(0)} \frac{\partial}{\partial \theta^{01}(0)} + \theta^2(0) \theta^3(0) \right).
$$

(4.9)
That means the creation of fermion-antifermion pairs by the operator \( \hat{\gamma}^1 \) even at zero momentum, that is, without suppression of this process in the non-relativistic limit.

Thus, the operator \( \hat{\gamma}^1 \) in the nonrelativistic limit \( p \to 0 \) (and, therefore, in general) cannot have eigenvalues on state vectors.

We get the same situation for acting on a vacuum and on state vectors for operators \( \hat{\gamma}^a \), \( a = 1, 2, 3, 4, 6, 7 \) – they do not annul the vacuum in the nonrelativistic limit and cannot have eigenvalues on state vectors.

5 **Action of gamma operators of rotations on a vacuum**

We get the same situation for acting on the vacuum and on state vectors for boosts \( \hat{\gamma}^{ba}(0) \), \( a = 1, 2, 3, 4, 6, 7 \) – they do not annihilate the vacuum and cannot have eigenvalues on the state vectors.

But the rotation operators \( \hat{\gamma}^{kl}(0) \), \( k, l = 1, 2, 3, 4, 6, 7 \), have the same features as \( \hat{\gamma}^0(0) \) – they annihilate the vacuum and can have their own eigenvalues ( ) on the state vectors.

The invariance of the vacuum during Lorentz rotations \( \exp(\hat{\gamma}^{\mu\nu}/4) \), where \( \mu, \nu = 0, 1, 2, 3 \), is ensured by the fact that each volume \( \Delta^3 p \) passes into another volume \( \Delta^3 p \), and its place is occupied by the third volume \( \Delta^3 p \). Which only leads to a change in the order of the factors \( \Psi_{\nu} \) in (3.5). These factors commute, so the Lorentz rotations leave the vacuum \( \Psi_{\nu} \) invariant.

From (4.8), (4.9) and similar formulas for all \( \hat{\gamma}^a \) and \( \hat{\gamma}^{ab} \), \( a, b = 0, 1, 2, 3, 4, 6, 7 \), it follows that these operators are not Lorentz-invariant (which is obvious), and their eigenvalues on state vectors can be spoken only in the non-relativistic limit \( p \to 0 \).

6 **Lorentz-invariant gamma operators**

It is easy to construct Lorentz-invariant analogs \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \) of superalgebraic representations \( \hat{\gamma}^a \) of Dirac matrices and rotation generators \( \hat{\gamma}^{ab} \). To do this, it is enough in formulas (1.2)-(1.3), (2.5) replace all operators \( \frac{\partial}{\partial \theta(p)} \) by \( b_k(p) \), and operators \( \theta^k(p) \) by \( \bar{b}_k(p) \).

For example,

\[
\hat{\Gamma}^0 = \int d^3 p \left[ b_1(p)\bar{b}_1(p) + b_2(p)\bar{b}_2(p) + b_3(p)\bar{b}_3(p) + b_4(p)\bar{b}_4(p), \# \right], \tag{6.1}
\]

\[
\hat{\Gamma}^1 = \int d^3 p \left[ b_1(p)\bar{b}_4(p) - b_4(p)\bar{b}_1(p) + b_2(p)\bar{b}_3(p) - b_3(p)\bar{b}_2(p), \# \right], \tag{6.2}
\]

\[
\hat{\Gamma}^{67} = -i \int d^3 p \left[ b_1(p)\bar{b}_1(p) + b_2(p)\bar{b}_2(p) - b_3(p)\bar{b}_3(p) - b_4(p)\bar{b}_4(p), \# \right], \tag{6.3}
\]

and so on.

In the discrete version of the theory, in the operators \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \), as before, continuous operators \( b_k(p) \) and \( \bar{b}_k(p) \) are replaced by discrete \( B_k(p_j) \) and \( \bar{B}_k(p_j) \), and integrals \( \int d^3 p ... \) by sums \( \sum_j \Delta^3 p_j ... \).

The operators \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \) are constructed by summing (integrating in the continuous case) over spatial momentums the results of all possible Lorentz rotations of the operators.
\( \gamma^a(0) \) and \( \gamma^{ab}(0) \). As a result of such rotations, \( \frac{\partial}{\partial \theta^k(0)} \) goes to \( b_k(p) \), and \( \theta^k(0) \) to \( \bar{b}_k(p) \) as in the field operators, as in \( \gamma^a(0) \) and \( \gamma^{ab}(0) \).

In contrast to \( \hat{\gamma}^a \) and \( \hat{\gamma}^{ab} \), in the Lorentz transformations the operators \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \) do not change either, since, like for the vacuum, the sum element for some momentum goes into the sum element for another momentum, and the sum element for the third momentum takes its place. As a result, these operators are Lorentz-invariant (and therefore also Lorentz-covariant). For the same reason, if for some values of \( a \) and \( b \) the operator \( \hat{\gamma}^a(0) \) or \( \hat{\gamma}^{ab}(0) \) annihilates the vacuum, then \( \hat{\Gamma}^a \) or \( \hat{\Gamma}^{ab} \) annihilates the vacuum, and if \( \hat{\gamma}^a(0) \) or \( \hat{\gamma}^{ab}(0) \) annihilates the vacuum, then \( \hat{\Gamma}^a \) or \( \hat{\Gamma}^{ab} \) under the action on the vacuum do not give zero. And for the same reason, if \( \hat{\gamma}^a(0) \) or \( \hat{\gamma}^{ab}(0) \) has eigenvalue for the state with \( p = 0 \), then \( \hat{\Gamma}^a \) or \( \hat{\Gamma}^{ab} \) has corresponding eigenvalue for states with any momentums. That is why operators \( \hat{\Gamma}^a \) have the same signature as \( \hat{\gamma}^a(0) \) and, hence, the same signature as \( \hat{\gamma}^a \).

Therefore, in quantum relativistic field theory, the eigenvalues of the operators \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \) are meaningful on the state vectors, and the operators \( \hat{\gamma}^a \) and \( \hat{\gamma}^{ab} \) cannot have eigenvalues at all, since they do not annul the vacuum. Operators \( \hat{\gamma}^a \) and \( \hat{\gamma}^{ab} \) have eigenvalues only in the non-relativistic limit \( p \to 0 \).

Since the commutation relations (2.2) for \( b_k(p) \) and \( \bar{b}_k(p) \) are the same as for \( \frac{\partial}{\partial \theta^k(p)} \) and \( \theta^k(p) \), the commutation relations for \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \) are the same as for \( \hat{\gamma}^a \) or \( \hat{\gamma}^{ab} \). That is, \( \hat{\Gamma}^a \) are analogs of Dirac matrices \( \gamma^\mu \), \( \mu = 0, 1, 2, 3, 4 \), but \( \hat{\Gamma}^{ab} \) and \( \hat{\Gamma}^{ab} \) also expand the set of analogs of Dirac matrices as \( \hat{\gamma}^6 \) and \( \hat{\gamma}^7 \).

We introduce the superalgebraic analogues [17] of the operators of the number of particles \( \hat{N}_1, \hat{N}_2 \) and antiparticles \( \hat{N}_3, \hat{N}_4 \) and the charge operator \( \hat{Q} \) in the method of second quantization:

\[
\hat{N}_k(p) = \left[ b_{<k>} (p) b_{<k>} (p), * \right] = -\left[ b_{<k>} (p) \bar{b}_{<k>} (p), * \right],
\]

\[
\hat{Q} = \int d^3 p (\hat{N}_1(p) + \hat{N}_2(p) - \hat{N}_3(p) - \hat{N}_4(p)), \tag{6.4}
\]

Then the physical meaning of \( \hat{\Gamma}^0 \) and \( \hat{\Gamma}^{67} \) is obvious, since (6.1) and (6.3) can be rewritten in the form:

\[
\hat{\Gamma}^0 = -\int d^3 p (\hat{N}_1(p) + \hat{N}_2(p) + \hat{N}_3(p) + \hat{N}_4(p)), \tag{6.5}
\]

\[
\hat{\Gamma}^{67} = i \int d^3 p (\hat{N}_1(p) + \hat{N}_2(p) + \hat{N}_3(p) + \hat{N}_4(p)) = i \hat{Q}.
\]

That is, \( -\hat{\Gamma}^0 \) is the operator of the total number of spinors and antispinors, and \( \hat{\Gamma}^{67} \) is related to the charge operator \( \hat{Q} \) by the ratio \( \hat{\Gamma}^{67} = i \hat{Q} \). Similarly, \( \hat{\Gamma}^{jk} = i \hat{\gamma}^l \), where \( j, k, l \) is cyclic permutation of 1, 2, 3. Moreover, \( \hat{\gamma}^l \) are Lorentz-invariant spin operators, which are analogs of the Pauli matrices. Operators \( \hat{W}^k = -i m \hat{\Gamma}^{k4} \) are components of Lorentz invariant analog of spin components of the Pauli-Lyubansky vector. However, physical meaning of operators \( \hat{\Gamma}^{jk} \), \( \hat{W}^k \) and \( \hat{\Gamma}^{\mu a} \), where \( \mu = 0, 1, 2, 3 \), \( a = 4, 6, 7 \), is incomprehensible.

Under the action on the vacuum (3.5)

\[
\hat{\Gamma}^0 \Psi_V = \hat{\Gamma}^{mn} \Psi_V = 0, \tag{6.6}
\]
where \( m, n = 1, 2, 3, 4, 6, 7 \), and the other gamma operators do not give zero under the action on \( \Psi_V \). Therefore, you can measure only the eigenvalues of the operators \( \hat{\gamma}^0(0) \), \( \hat{\gamma}^{mn}(0) \), \( \hat{\Gamma}^0 \), and \( \hat{\Gamma}^{mn} \) with \( m, n = 1, 2, 3, 4, 6, 7 \).

It is useful to note that the matrix formalism does not provide the possibility of zero eigenvalues of gamma matrices, in contrast to the proposed theory.

7 Spacetime signature in the presence of a spinor vacuum

The reason for the difference between the action on the vacuum and the state vectors of the operators \( \hat{\gamma}^0(0) \) and \( \hat{\gamma}^{mn}(0) \), on the one hand, and \( \hat{\gamma}^m(0) \), \( \hat{\gamma}^{0m}(0) \), on the other, is related to the structure of these operators in (1.2)-(1.3), (2.5), (6.1)-(6.3). Since the vacuum state vector has a multiplier \( B_l(0) \overline{B}_l(0) \), the action on vacuum of operators consisting only of terms of the form \( [\overline{B}_l(0)B_k(0), \ast] \) will always give zero, since, by virtue of (3.2) and (3.3)

\[
[\overline{B}_l(0)B_k(0), \ast] = 0.
\]

(7.1)

But the terms of the form \( [B_k(0)B_l(0), \ast] \) and \( [\overline{B}_k(0)\overline{B}_l(0), \ast] \) will give a non-zero result. Summing the results of Lorentz rotations leads to similar conclusions for \( \hat{\Gamma}^0 \), \( \hat{\Gamma}^{mn} \), on the one hand, and \( \hat{\Gamma}^m \), \( \hat{\Gamma}^{0m} \), on the other.

Expansion (1.4) generates the expansion of field operators in momenta and leads to the implementation of the Dirac equation [18]. The question arises of what kind of Clifford bases such decomposition is possible.

If, as in the considered case, \( \hat{\gamma}^0 = (\hat{\gamma}^0)^+ \), \( \hat{\gamma}^m = -(\hat{\gamma}^m)^+ \), there is one time-like Clifford vector.

Multiplying \( \hat{\gamma}^0 \) by an imaginary unit will lead to the appearance in the expansion in momenta [18] of exponentially increasing terms, that is, to the impossibility of the existence of normalized solutions. Therefore, Clifford vectors \( \hat{\gamma}^0 \) and \( \hat{\Gamma}^0 \) are time-like and have signature +1 for spacetime where spinors can exist as physical particles.

Multiplication of any of the operators \( \hat{\gamma}^m \) (and, consequently, \( \hat{\Gamma}^m \)) by the imaginary unit due to the presence of the vacuum (3.5) will lead to asymmetry between Clifford vectors \( \hat{\Gamma}^0 \) and \( i\hat{\Gamma}^m \), since \( \hat{\Gamma}^0 \Psi_V = 0 \) and \( i\hat{\Gamma}^m \Psi_V \neq 0 \), and \( \hat{\Gamma}^0 \) can have eigenvalues on the state vectors but \( i\hat{\Gamma}^m \) cannot. The space of Clifford vectors with the same signature must be isotropic, however in this case we obtain a preferred direction. Therefore, other than \( \hat{\Gamma}^0 \) Clifford vectors could not have the same signature as \( \hat{\Gamma}^0 \). Consequently, the condition for the existence of the vacuum imposes restrictions on the possible variants of Clifford algebras: neither complex algebra nor algebras in which at least one of the base vectors \( \hat{\Gamma}^m \) (and hence \( \hat{\gamma}^m \) ) is timelike is suitable. Therefore, all Clifford vectors \( \hat{\Gamma}^m \) are spacelike (and hence \( \hat{\gamma}^m \) ) – they have a signature of -1, and there is only one basic timelike Clifford vector \( \hat{\Gamma}^0 \) (and hence \( \hat{\gamma}^0 \)).

Only 16 of the 28 operators \( \hat{\Gamma}^a \) and \( \hat{\Gamma}^{ab} \) in (1.4) annul the vacuum and therefore can have eigenvalues on the state vectors. Therefore, if we require the existence of a decomposition in momenta, that is, the existence of spinors as physical particles, out of seven gamma matrices \( \hat{\Gamma}^a \) (and hence \( \hat{\gamma}^a \)), one must have a positive signature, and the other six must have a negative signature.
Thus, in the superalgebraic theory of spinors, the signature of a four-dimensional space-time can only be \((1, -1, -1, -1)\), and there are two additional axes \(\hat{\gamma}^6\) and \(\hat{\gamma}^7\) with a signature \((-1, -1)\) corresponding to the inner space of the spinor. The reason why they and the axis \(\hat{\gamma}^4\) are not additional spatial axes is not yet clear.

8 Conclusion

The proposed theory has a number of interesting consequences.

- Expansion (1.4) ensures for spinors the existence of decomposition in momenta [18].
- The theory is free from divergences, leading to the need for the normalization of operators [17].
- It leads to an unambiguous signature of spacetime, which coincides with the observable.
- Part of the decomposition terms (1.4) corresponds to the usual field theories available in the framework of the general theory of relativity [19], as well as to theories of bundles [20]. An operator \(\hat{\Gamma}^{67} = i\hat{Q}\) and gauge transformation \(\exp(i\hat{Q}_\omega^{67})\) automatically arises, where \(\hat{Q}\) is the charge operator in the second quantization formalism, \(\hat{Q}\Psi = \Psi\) for the spinor \(\Psi\), and \(\hat{Q}\bar{\Psi} = -\bar{\Psi}\) for its antiparticle \(\bar{\Psi}\).
- The proposed approach to constructing a discrete vacuum is fundamentally different from theories in which the discreteness of spacetime is considered, leading to the loss of Lorentz covariance [21]. The proposed theory is Lorentz covariant and combines the features of discrete and continuous theories.
- We can construct operators \(\hat{\Gamma}^a\) and \(\hat{\Gamma}^{ab}\), \(a, b = 0, 1, 2, 3, 4, 6, 7\) from operators of creation and annihilation of spinors independently on superalgebraic representation \(\hat{\gamma}^\mu\) of Dirac gamma matrices \(\gamma^\mu\). However this representation makes interconnection between Dirac gamma matrices and operators \(\hat{\Gamma}^a\) obvious.

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