Charged lepton contributions to bimaximal and tri-bimaximal mixing for generating $\sin \theta_{13} \neq 0$ and $\tan^2 \theta_{23} < 1$

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Abstract

Bimaximal (BM) and Tri-bimaximal (TB) mixings of neutrinos are two special cases of lepton mixing matrix, which predict the reactor angle $\theta_{13} = 0$ and the atmospheric angle $\tan^2 \theta_{23} = 1$. Recent precision measurements and global analysis of oscillation parameters, have confirmed a non-vanishing value of $\theta_{13}$ as well as deviations of $\theta_{12}$ and $\theta_{23}$ from their maximal values predicted by BM or TB mixing. In this work we mainly concentrate on $\theta_{13}$ and $\theta_{23}$ to assign $\theta_{13} \neq 0$ and $\tan^2 \theta_{23} < 1$ with the help of charged lepton corrections defined by $U_{PMNS} = U_{l}^\dagger U_{\nu}$. We first consider $U_{\nu}$ to be given separately by BM and TB mixing matrices and then find the possible forms of $U_{l}$ such that the elements of PMNS matrix, finally yield $\theta_{13} \neq 0$ and $\tan^2 \theta_{23} < 1$ in agreement with latest observational data. To compute the values of mixing angles we assume the charged lepton correction to be of Cabbibo-Kobayashi-Maskawa (CKM) like. All the mixing matrices involved in the calculation satisfy the unitarity condition to leading order of expansion parameter.

Key-words: Charged lepton correction, Bimaximal and Tri-bimaximal mixing.

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1 Introduction

Recent precision measurements\cite{1-4} and latest global $3\nu$ oscillation analysis\cite{5} of neutrino mixing parameters, have confirmed non-vanishing value of $\theta_{13}$ as well as deviation of atmospheric mixing angle from maximal value, $\theta_{23} < \pi/4$. One of the important aspects of neutrino physics is to understand such mixing patterns\cite{6}. Charged lepton corrections\cite{7} to neutrino mixing matrix is an attractive tool which can impart non-zero value of $\theta_{13}$ as well as deviation of $\theta_{23}$ from maximal value. We address the issue of charged lepton correction to both bimaximal(BM) and tri-bimaximal(TB) neutrino mixings to produce desired results.

To begin with we start with the lepton mixing matrix, known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix\cite{8},

$$U_{PMNS} = U_l^\dagger U_\nu, \quad (1)$$

which is analogous to CKM matrix, $V_{CKM} = U_{uL}^\dagger U_{dL}$ for quark sector\cite{9,10}. In relation (1), $U_l$ and $U_\nu$ are the diagonalizing matrices for charged lepton and left-handed Majorana neutrino mass matrices respectively which are defined as : $m_l = U_{lL} m_l^{diag} V_{lR}^\dagger$ and $m_\nu = U_\nu^* m_\nu^{diag} U_\nu^\dagger$. In the basis where charged lepton mass matrix is diagonal, $m_\nu$ is expressible as\cite{11}

$$m_\nu' = U_{lL}^\dagger m_\nu U_{lL}. \quad (2)$$

In the standard Particle Data Group (PDG) parametrization\cite{10}, with three mixing angles and three CP phases- one Dirac CP phase ($\delta$) and two Majorana CP phases ($\alpha$, $\beta$), PMNS matrix has the form,

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.P, \quad (3)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ with $\theta_{12}$ being the solar angle, $\theta_{23}$ being the atmospheric angle and $\theta_{13}$ being the reactor angle and $P = diag(1, e^{i\alpha}, e^{i\beta})$ contains the Majorana CP phases. In our present work we ignore all the CP phases. Then under $\mu - \tau$ symmetry, with $\theta_{13} = 0$, PMNS matrix takes the form\cite{12} :

$$U_{PMNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (4)$$
which predicts maximal value of the atmospheric angle ($\theta_{23} = \frac{\pi}{4}$) leaving solar angle ($\theta_{12}$) arbitrary.

Two popular neutrino mixing matrices are the bi-maximal (BM) mixing[13] and the tri-bimaximal (TB) mixing[14], which can be obtained from equation (4) by setting $s_{12} = \frac{1}{\sqrt{2}}$ and $s_{12} = \frac{1}{\sqrt{3}}$ respectively and are given as:

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{6} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. (5)$$

Both these two neutrino mixing matrices predict $\tan^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = 1$ and $\sin^2 \theta_{13} = |U_{e 3}|^2 = 0$.

The paper is organized as follows: In section 2 we discuss charged lepton correction to BM neutrino mixing and present predictions of the mixing angles along with graphical representations. In a similar way section 3 is devoted to TB mixing. Then in section 4 we analyze both the schemes in presence of Dirac CP phase. Finally section 4 is devoted to summary and discussion.

## 2 Charged lepton correction to BM mixing

General forms of the lepton mixing matrix ($U_{\nu}$) and the neutrino mixing matrix ($U_{l}$) in equation (1) can be expressed as

$$U_{l} = \begin{pmatrix} c_{12} c_{13} & -s_{12} c_{13} & s_{13} \\ s_{12} c_{23} - c_{12} s_{23} s_{13} & c_{12} c_{23} - s_{12} s_{23} s_{13} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} & -c_{12} s_{23} - s_{12} c_{23} s_{13} & c_{23} c_{13} \end{pmatrix} \quad (6)$$

and

$$U_{\nu} = \begin{pmatrix} c_{12} c_{13} & -s_{12} c_{13} & s_{13} \\ -s_{14} c_{23} - c_{14} s_{23} s_{13} & c_{14} c_{23} - s_{14} s_{23} s_{13} & s_{23} c_{14} \\ s_{14} s_{23} - c_{14} c_{23} s_{13} & -c_{14} s_{23} - s_{14} c_{23} s_{13} & c_{23} c_{14} \end{pmatrix}. \quad (7)$$
where we have ignored the CP violating phases. For our case we first consider the neutrino mixing pattern to be of bi-maximal nature. Then $U_\nu = U_{BM}$ is given by equation (5). We then take the following form of the lepton mixing matrix[15],

$$U_l = \begin{pmatrix} \tilde{c}_{12} & \tilde{s}_{12} & 0 \\ -\tilde{s}_{12} & \tilde{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

where $\tilde{s}_{ij} = \sin \theta_{ij}$ and $\tilde{c}_{ij} = \cos \theta_{ij}$. This structure(8) had been studied earlier[15] but we study it again here in the light of latest observational data[5].

From equations (1), (5) and (8), we finally obtain the PMNS matrix $U_{PMNS} = U^\dagger_l U_{BM}$ as

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{c}_{12} + \frac{\tilde{s}_{12}}{\sqrt{2}}) & \frac{1}{\sqrt{2}}(\tilde{c}_{12} - \frac{\tilde{s}_{12}}{\sqrt{2}}) & -\frac{\tilde{s}_{12}}{\sqrt{2}} \\ -\frac{1}{2}(\tilde{c}_{12} - \sqrt{2}\tilde{s}_{12}) & \frac{1}{2}(\tilde{c}_{12} + \sqrt{2}\tilde{s}_{12}) & \frac{\tilde{s}_{12}}{\sqrt{2}} \\ 0 & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (9)$$

Let us now assume that the charged lepton corrections are Cabibbo-Kobayashi-Maskawa (CKM) like[10], which allows us to take

$$\tilde{s}_{12} = \sin \theta_{12} = \lambda, \quad (10)$$

where the Wolfestein parameter $\lambda$ is related to the Cabibo angle ($\theta_C$) by $\lambda = \sin \theta_C$. Under this consideration, PMNS matrix in equation (9), can be approximated to the form,

$$U_{PMNS} \approx \begin{pmatrix} \frac{1}{\sqrt{2}}(1 + \frac{\lambda^2}{2} - \frac{\lambda^2}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{2} - \frac{\lambda^2}{2}) & -\frac{\lambda}{\sqrt{2}} \\ -\frac{1}{2}(1 - \sqrt{2}\lambda - \frac{\lambda^2}{2}) & \frac{1}{2}(1 + \sqrt{2}\lambda - \frac{\lambda^2}{2}) & \frac{\lambda}{\sqrt{2}} (1 - \frac{\lambda^2}{2}) \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (11)$$

Table 1: Best fit, 1$\sigma$ and 3$\sigma$ ranges of parameters for NH obtained from global analysis[22]
And the expression in equation (8) becomes

$$U_{iL} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & -\lambda & 0 \\ \lambda & 1 - \frac{\lambda^2}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  \hfill(12)

It can be emphasised here that both mixing matrices in equations (11) and (12) satisfy the unitarity condition as expected. Then equation (11) leads to

$$\tan^2 \theta_{12} = \left( \frac{1 - |U_{e3}| - |U_{\nu3}|^2}{1 + |U_{e3}| - |U_{\nu3}|^2} \right)^2,$$  \hfill(13)

$$\tan^2 \theta_{23} = (1 - |U_{e3}|^2)^2,$$  \hfill(14)

$$|U_{e3}|^2 = \sin^2 \theta_{13} = \frac{\lambda^2}{2}.$$  \hfill(15)

With $\lambda = 0.232$ corresponding to $|U_{e3}|^2 = 0.027$, we get $\tan^2 \theta_{12} \approx 0.50$ and $\tan^2 \theta_{23} = 0.946$. The variations of $\tan^2 \theta_{12}$ with $|U_{e3}|^2$ and $\tan^2 \theta_{23}$ with $|U_{e3}|^2$ are shown in Fig.1 and Fig.2 respectively for both $1\sigma$ and $3\sigma$ ranges (Table 1) of latest global observational data [22]. As expected $3\sigma$ range of
data can accommodate both $\tan^2 \theta_{12}$ and $\tan^2 \theta_{23}$ predictions. However, the 1σ range of data just marginally covers $\tan^2 \theta_{12}$ prediction at $\tan^2 \theta_{12} \approx 0.5$ (TB value) but not the $\tan^2 \theta_{23}$ prediction within the range. Certain theoretical refinements are needed in this front.

3 Charged lepton correction to TB Mixing

Tri-bimaximal neutrino mixing is a special case of mixing matrix with $\mu - \tau$ symmetry. It can give a very close description of the experimental data except the case: $\theta_{13} = 0$. The TB neutrino mixing matrix ($U_\nu = U_{TB}$) is given in equation (5). In order to account for the charged lepton correction to the TB neutrino mixing, we start with the lepton mixing matrix which satisfies unitarity condition,

$$
\tilde{U}_l = \begin{pmatrix}
1 - \frac{\lambda^2}{4} & -\frac{\lambda}{2} & -\frac{\lambda}{2} \\
\frac{\lambda}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\
-\frac{\lambda^2}{8} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8}
\end{pmatrix}.
$$

(16)
Using the form of $U_\nu$ for TB, given by equation (5), we have $U_{PMNS} = \tilde{U}_l^\dagger U_{TB}$ which reproduces the following PMNS matrix first proposed by King[16],

$$U_{PMNS} = \begin{pmatrix}
\sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}}(1 + \lambda) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \\
\frac{1}{\sqrt{6}}(1 - \lambda) & -\frac{1}{\sqrt{3}}(1 + \frac{\lambda}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4})
\end{pmatrix} . \tag{17}
$$

This PMNS matrix has unique property of unitarity to leading order, and also predicts $\tan^2 \theta_{23} = 1$. In order to have $\tan^2 \theta_{23} < 1$ in the light of present experimental data[5], we now modify the charged lepton mixing matrix (16) by the relation

$$U_l^\dagger = \tilde{R}_{23} \tilde{U}_l^\dagger , \tag{18}
$$

where $\tilde{R}_{23}$ has a structure similar to that of rotation matrix and is given by

$$\tilde{R}_{23} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \tilde{c}_{23} & \tilde{s}_{23} \\
0 & -\tilde{s}_{23} & \tilde{c}_{23}
\end{pmatrix} , \tag{19}
$$

with $\tilde{s}_{23} = \sin \theta_{23}'$ and $\tilde{c}_{23} = \cos \theta_{23}'$. 

![Figure 3: Variation of $\tan^2 \theta_{23}$ with $\sin \tilde{\theta}_{23}$ for TB mixing after taking charged lepton correction. Dotted and Dashed lines represents 1σ and 3σ bounds respectively, obtained from the global analysis[22]](image)
Figure 4: Variation of $\tan^2 \theta_{23}$ with $U_{e3}^2$ for TB mixing after taking charged lepton correction. Dotted and Dashed lines represents 1σ and 3σ bounds respectively, obtained from the global analysis.[22]

Then equations (1),(16) and (18) give the following elements of the new PMNS matrix, $U_{PMNS} = U_l^T U_{TB}$,
\[ (U_{PMNS})_{11} = \sqrt{\frac{2}{3}} (1 - \frac{\lambda^2}{4}), \]
\[ (U_{PMNS})_{12} = \frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{4}), \]
\[ (U_{PMNS})_{13} = \frac{\lambda}{\sqrt{2}}, \]
\[ (U_{PMNS})_{21} = -\frac{1}{\sqrt{6}} [(\tilde{c}_{23} + \tilde{s}_{23}) + (\tilde{c}_{23} - \tilde{s}_{23})\lambda], \]
\[ (U_{PMNS})_{22} = \frac{1}{\sqrt{3}} [(\tilde{c}_{23} + \tilde{s}_{23}) - (\tilde{c}_{23} - \tilde{s}_{23})\frac{\lambda}{2}], \]
\[ (U_{PMNS})_{23} = \frac{1}{\sqrt{2}} (\tilde{c}_{23} - \tilde{s}_{23})(1 - \frac{\lambda^2}{4}), \]
\[ (U_{PMNS})_{31} = \frac{1}{\sqrt{6}} [(\tilde{c}_{23} - \tilde{s}_{23}) - (\tilde{c}_{23} + \tilde{s}_{23})\lambda], \]
\[ (U_{PMNS})_{32} = -\frac{1}{\sqrt{3}} [(\tilde{c}_{23} - \tilde{s}_{23}) + (\tilde{c}_{23} + \tilde{s}_{23})\frac{\lambda}{2}], \]
\[ (U_{PMNS})_{33} = \frac{1}{\sqrt{2}} (\tilde{c}_{23} + \tilde{s}_{23})(1 - \frac{\lambda^2}{4}). \]  

(A)

From these elements we calculate

\[ \tan^2 \theta_{23} = \left( \frac{1 - \tan \tilde{\theta}_{23}}{1 + \tan \tilde{\theta}_{23}} \right)^2, \]  

which is lesser than maximal value for non-zero \( \tan \tilde{\theta}_{23} \). Assuming that the charged lepton corrections are Cabbibo-Kobayashi-Maskawa (CKM) like, we can have\[10,17\]

\[ \tilde{s}_{23} = \sin \theta_{23}^l = A\lambda^2 \approx 0.041, \]  

leading to \( \tan^2 \theta_{23} = 0.85 \), where we have adopted \( \lambda = 0.2324 \) and \( A = 0.759 \). The variation of \( \tan^2 \theta_{23} \) with \( \sin \tilde{\theta}_{23} \) is shown in Fig.3. The prediction on \( \tan^2 \theta_{12} \) is fixed at TB value while the change is confined to \( \tan^2 \theta_{23} \) only and its variation with \( |U_{e3}|^2 \) along with 1\( \sigma \) and 3\( \sigma \) ranges of latest global observational data\[22\] is shown in Fig.4. At 3\( \sigma \) range the prediction on \( \tan^2 \theta_{23} \) is in fair agreement with global data as like BM case. However, in
TB case we notice an improvement of our prediction at 1σ range that it just passes through the 1σ region in the plot unlike the BM case.

4 Effects of Dirac CP phase

In this section we would like to discuss briefly the effects of CP violating phases in the proposed schemes. To observe the effects of the Dirac type CP phase in the BM scheme we follow two ways of introducing the phase. First case assumes a CP phase \( \phi \), coming from the charged lepton sector, with the unitary matrix [20]

\[
U_l = \begin{pmatrix}
\tilde{c}_{12} & \tilde{s}_{12}e^{-i\phi} & 0 \\
-\tilde{s}_{12}e^{i\phi} & \tilde{c}_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]  

With this \( U_l \), \( U_{PMNS} = U_l^\dagger U_{BM} \) yields

\[
U_{PMNS} = \begin{pmatrix}
\frac{1}{\sqrt{2}}(\tilde{c}_{12} + \frac{\tilde{s}_{12}e^{i\phi}}{\sqrt{2}}) & \frac{1}{\sqrt{2}}(\tilde{c}_{12} - \frac{\tilde{s}_{12}e^{i\phi}}{\sqrt{2}}) & -\frac{\tilde{s}_{12}e^{i\phi}}{\sqrt{2}} \\
-\frac{1}{2}(\tilde{c}_{12} - \sqrt{2}\tilde{s}_{12}e^{-i\phi}) & \frac{1}{2}(\tilde{c}_{12} + \sqrt{2}\tilde{s}_{12}e^{-i\phi}) & \frac{\tilde{s}_{12}e^{-i\phi}}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & 1
\end{pmatrix}.
\]  

In the second approach we introduce the CP phase \( \delta \), originating from neutrino sector, by the following relation [21],

\[
U_{PMNS} = U_l^\dagger R_{23} \text{Diag}(e^{i\delta}, 1, e^{-i\delta}) R_{12},
\]  

where \( U_l \) is given by equation (8) and \( R_{23} \) and \( R_{12} \) are the 3 \times 3 orthogonal rotation matrices with \( \theta_{23} = \frac{\pi}{4} \) and \( \theta_{12} = \frac{\pi}{4} \) respectively. Then equation (24) gives,

\[
U_{PMNS} = \begin{pmatrix}
\frac{1}{\sqrt{2}}(\tilde{c}_{12}e^{i\delta} + \frac{\tilde{s}_{12}}{\sqrt{2}}e^{i\phi}) & \frac{1}{\sqrt{2}}(\tilde{c}_{12}e^{i\delta} - \frac{\tilde{s}_{12}}{\sqrt{2}}e^{i\phi}) & -\frac{\tilde{s}_{12}e^{-i\delta}}{\sqrt{2}} \\
\frac{1}{2}(\tilde{c}_{12} - \sqrt{2}\tilde{s}_{12}e^{-i\delta}) & \frac{1}{2}(\tilde{c}_{12} + \sqrt{2}\tilde{s}_{12}e^{-i\delta}) & \frac{\tilde{s}_{12}e^{i\delta}}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & 1
\end{pmatrix}.
\]  

Both the cases lead to a similar form of the rephasing invariant quantity defined as \( J_{CP} = \text{Im}\{U_{e2}U_{\mu3}U_{e3}^*U_{\mu2}^*\} \). For example, we get

\[
J_{CP}^{BM} = \frac{1}{4\sqrt{2}} \sin \tilde{\theta}_{12} \cos \tilde{\theta}_{12} \sin \phi,
\]  

(26)

9
and

\[ J_{CP}^{BM} = \frac{1}{4\sqrt{2}} \sin \tilde{\theta}_{12} \cos \tilde{\theta}_{12} \sin \delta, \]  

(27)

from equations (23) and (25) respectively. We further calculate

\[ \tan^2 \theta_{12} = \frac{2 - s_{12}^2 - 2\sqrt{2}c_{12}s_{12}\cos \phi}{2 - s_{12}^2 + 2\sqrt{2}c_{12}s_{12}\cos \phi} \]  

(28)

and

\[ \tan^2 \theta_{12} = \frac{2 - s_{12}^2 - 2\sqrt{2}c_{12}s_{12}\cos \delta}{2 - s_{12}^2 + 2\sqrt{2}c_{12}s_{12}\cos \delta} \]  

(29)

from equations (23) and (25) respectively, which show the dependence of solar angle on the CP phase. For maximal CP violation (\( \sin \delta = \pm 1 \)) we get 

\[ |J_{CP}^{BM}|_{\text{max}} \approx 0.03989. \]

From the relation \( \sin \tilde{\theta}_{12} = \sqrt{2} \sin \theta_{13} \) along with the approximation \( \cos \tilde{\theta}_{12} \approx 1 \) (from eq.(10)) equation (27) gives

\[ J_{CP}^{BM} \approx \frac{1}{4} \sin \theta_{13} \sin \delta \]  

(30)

which is consistent with the result of reference[7].

To incorporate the Dirac type CP effects in TB scheme we first adopt the Tri-bimaximal-Cabbibo mixing matrix \( U_{TBC} \) proposed by King[16].

\[
U_{TBC} = \begin{pmatrix}
\sqrt{\frac{2}{3}}(1 - \lambda^2 / 4) & \frac{1}{\sqrt{3}}(1 - \lambda^2 / 4) & \frac{\lambda}{\sqrt{2}}e^{-i\delta} \\
-\frac{1}{\sqrt{6}}(1 + \lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1 - \lambda^2 / 4) & \frac{1}{\sqrt{2}}(1 - \lambda^2 / 4) \\
\frac{1}{\sqrt{6}}(1 - \lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1 + \lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1 - \lambda^2 / 4)
\end{pmatrix}.
\]  

(31)

For \( \delta = 0 \) equation (30) reproduces the mixing matrix given by equation (17). Then the relation \( U_{PMNS} = \tilde{R}_{23}^U U_{TBC} \) produces the following desired elements of the PMNS matrix, given in the set of equations (A), modified by the CP phase \( \delta \).
\[
\begin{align*}
(U_{PMNS})_{11} &= \sqrt{\frac{2}{3}} (1 - \frac{\lambda^2}{4}), \\
(U_{PMNS})_{12} &= \frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{4}), \\
(U_{PMNS})_{13} &= \frac{\lambda}{\sqrt{2}} e^{-i\delta}, \\
(U_{PMNS})_{21} &= -\frac{1}{\sqrt{6}} \left[ (\bar{c}_{23} + \bar{s}_{23}) + (\bar{c}_{23} - \bar{s}_{23}) \lambda e^{i\delta} \right], \\
(U_{PMNS})_{22} &= \frac{1}{\sqrt{3}} \left[ (\bar{c}_{23} + \bar{s}_{23}) - (\bar{c}_{23} - \bar{s}_{23}) \frac{\lambda}{2} e^{i\delta} \right], \\
(U_{PMNS})_{23} &= \frac{1}{\sqrt{2}} \left( \bar{c}_{23} - \bar{s}_{23} \right) (1 - \frac{\lambda^2}{4}), \\
(U_{PMNS})_{31} &= \frac{1}{\sqrt{6}} \left[ (\bar{c}_{23} - \bar{s}_{23}) - (\bar{c}_{23} + \bar{s}_{23}) \lambda e^{i\delta} \right], \\
(U_{PMNS})_{32} &= -\frac{1}{\sqrt{3}} \left[ (\bar{c}_{23} - \bar{s}_{23}) + (\bar{c}_{23} + \bar{s}_{23}) \frac{\lambda}{2} e^{i\delta} \right], \\
(U_{PMNS})_{33} &= \frac{1}{\sqrt{2}} \left( \bar{c}_{23} + \bar{s}_{23} \right) (1 - \frac{\lambda^2}{4}).
\end{align*}
\]

The set of equations (B) predicts the rephasing invariant quantity as

\[
J_{CP}^{TB} = \frac{1}{6} \lambda \left( 1 - \frac{\lambda^2}{4} \right)^2 (\bar{c}_{23}^2 - \bar{s}_{23}^2) \sin \delta.
\]  

(32)

We also examine the structure of the PMNS matrix under the parameterization described in equation (24) where \( U_l^\dagger \) is now given by equation (18) and \( R_{23} \) and \( R_{12} \) are respectively described by \( \theta_{23} = \frac{\pi}{4} \) and \( \theta_{12} = \arcsin \frac{1}{\sqrt{3}} \).

We then obtain the following elements of the PMNS matrix:
(\(U_{PMNS}\))_{11} = \sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4})e^{i\delta},
(\(U_{PMNS}\))_{12} = \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4})e^{i\delta},
(\(U_{PMNS}\))_{13} = \frac{\lambda}{\sqrt{2}}e^{-i\delta},
(\(U_{PMNS}\))_{21} = -\frac{1}{\sqrt{6}}[(\tilde{c}_{23} + \tilde{s}_{23}) + (\tilde{c}_{23} - \tilde{s}_{23})\lambda e^{i\delta}],
(\(U_{PMNS}\))_{22} = \frac{1}{\sqrt{3}}[(\tilde{c}_{23} + \tilde{s}_{23}) - (\tilde{c}_{23} - \tilde{s}_{23})\frac{\lambda}{2}e^{i\delta}],
(\(U_{PMNS}\))_{23} = \frac{1}{\sqrt{2}}(\tilde{c}_{23} - \tilde{s}_{23})(1 - \frac{\lambda^2}{4})e^{-i\delta},
(\(U_{PMNS}\))_{31} = \frac{1}{\sqrt{6}}[(\tilde{c}_{23} - \tilde{s}_{23}) - (\tilde{c}_{23} + \tilde{s}_{23})\lambda e^{i\delta}],
(\(U_{PMNS}\))_{32} = -\frac{1}{\sqrt{3}}[(\tilde{c}_{23} - \tilde{s}_{23}) + (\tilde{c}_{23} + \tilde{s}_{23})\frac{\lambda}{2}e^{i\delta}],
(\(U_{PMNS}\))_{33} = \frac{1}{\sqrt{2}}(\tilde{c}_{23} + \tilde{s}_{23})(1 - \frac{\lambda^2}{4})e^{-i\delta}.

The set of equations (C) yields the same rephasing invariant quantity in equation (32). From the relation \(\lambda = \sqrt{2}\sin\theta_{13}\) along with equations (10) and (21), equation (32) gives

\[J_{CP}^{TB} \approx \frac{1}{3\sqrt{2}}\sin\theta_{13}\sin\delta.\]  

Further, for maximal CP violation, we calculate \(|J_{CP}^{EB}|_{max} \approx 0.0374\) from equation (32). The expression for \(J_{CP}\) in equation (33) is consistent with the result of reference[7].

5 Summary and Discussion

We have studied two possible forms of the lepton mixing matrix \(U_l\) which can produce desired deviations from the bimaximal (BM) and tri-bimaximal(TB) mixings of neutrino sector under charged lepton corrections. The lepton
mixing matrices have basically been derived from rotation matrices and hence the conditions of unitarity of all diagonalising matrices including the final form of PMNS matrices discussed here, are satisfied at leading order. In such situation PMNS matrix proposed by King[16] is a pointer to the right direction. Assuming the charged lepton correction is CKM-like and taking $\lambda = 0.232$ we get $\sin^2\theta_{13} = 0.027$ for both BM and TB cases. For the same value of $\lambda$ we calculate $\tan^2\theta_{12} \approx 0.50$ and $\tan^2\theta_{23} = 0.946 < 1$ for BM case. After the introduction of Dirac CP phase we observe that $\tan^2\theta_{12}$ is affected by the phase, but not $\tan^2\theta_{23}$. We also find that predictions on $\tan^2\theta_{12}$ and $\tan^2\theta_{23}$ in terms of $|U_{e3}|^2$ are consistent with the 3$\sigma$ range of latest global observational data. However, at 1$\sigma$ range the predictions are not comfortable. In case of TB mixing, the charged lepton correction only deviates the atmospheric angle. The solar angle remains fixed at its TB value ($\tan^2\theta_{23} = 0.5$). For $\lambda = 0.232$ and $A = 0.759$ we get $\tan^2\theta_{23} = 0.85 < 1$. The variation of $\tan^2\theta_{23}$ with $|U_{e3}|^2$ shows that at 3$\sigma$ range the prediction on $\tan^2\theta_{23}$ is smoothly consistent with global data. However, in TB case we get better agreement of our prediction with 1$\sigma$ range of global data than that in BM case. Unlike the BM case, the inclusion of Dirac CP phase in TB mixing does not affect $\tan^2\theta_{12}$ and $\tan^2\theta_{23}$. Finally we obtain two important expressions for the rephasing invariant quantity: $J_{CP}^{BM} \approx \frac{1}{4} \sin \theta_{13} \sin \delta$ and $J_{CP}^{TB} \approx \frac{1}{3\sqrt{2}} \sin \theta_{13} \sin \delta$ which are consistent with the results of reference[7].

The deviation of solar mixing angle $\tan^2\theta_{12}$ below the value of 0.50, can be introduced in realistic $\mu - \tau$ symmetric neutrino mass matrices with specific choices of value of flavour twister term[15,18,19] present in the texture of the mass matrices, without affecting the good predictions on reactor and atmospheric mixing angles.

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