Continuum Limit of Scalar Masses and Mixing Energies

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We evaluate the continuum limit of the valence approximation to the mass of scalar quarkonium and to the scalar quarkonium-glueball mixing energy for a range of different quark masses. Our results answer several questions raised by the proposed identification of $f_0(1710)$ as composed primarily of the lightest scalar glueball.

Evidence that $f_0(1710)$ is composed mainly of the lightest scalar glueball is now given by a calculation [1], on a $16^3 \times 24$ lattice at $\beta$ of 5.70, yielding 108(29) MeV as the width for the lightest scalar glueball to decay to all possible pseudoscalar pairs, and by three independent calculations [2,3] with a combined prediction [4] of 1632(49) MeV as the infinite volume, continuum limit of the lightest scalar glueball mass. All four calculations were done in the valence (quenched) approximation. The decay width result combined with any reasonable guesses for the effect of finite lattice spacing, finite lattice volume, and the remaining width to multibody states yields a total width small enough for the lightest scalar glueball to be seen easily in experiment. The mass prediction combined with the expectation [5] that the valence approximation will underestimate the scalar glueball mass then points to $f_0(1710)$ as composed primarily of the lightest scalar glueball [1].

Among established resonances the only plausible alternative to $f_0(1710)$ is $f_0(1500)$. Refs. [1,6] propose that $f_0(1500)$ consists mainly of $s\bar{s}$ scalar quarkonium. A problem with this suggestion, however, is that $f_0(1500)$ apparently does not decay mainly into states containing an $s$ and an $\bar{s}$ quark [1]. Ref. [6] thus interprets $f_0(1500)$ as the lightest scalar glueball and $f_0(1710)$ as $s\bar{s}$ scalar quarkonium. A further problem for any of these identifications of $f_0(1500)$ and $f_0(1710)$ is that the Hamiltonian of full QCD couples quarkonium and glueballs so that physical states should be linear combinations of both. In the extreme, mixing could lead to $f_0(1710)$ and $f_0(1500)$ each half glueball and half quarkonium.

For a fixed lattice period $L$ of about 1.6 fm and several different values of quark mass, we have now found the valence approximation to the continuum limit of the mass of the lightest $q\bar{q}$ states and of the mixing energy between these states and the lightest scalar glueball. Continuum predictions are extrapolated from calculations at four different lattice spacings. For the two largest lattice spacings we have results also for lattice period 2.3 fm. Earlier stages of this work are reported in Refs. [1-4].

Our results provide answers to the questions raised by the identification of $f_0(1710)$ as composed largely of the lightest scalar glueball. With $L$ of 1.6 fm, the valence approximation to the continuum limit of the mass of the lightest $s\bar{s}$ scalar we find is significantly below the valence approximation to the infinite volume, continuum limit of the scalar glueball mass. Our calculations with $L$ of 2.3 fm show that as the infinite volume limit is taken, the mass of the lightest $s\bar{s}$ scalar will fall still further. Thus it appears to us that the identification of $f_0(1500)$ as mainly glueball with $f_0(1710)$ as mainly $s\bar{s}$ is quite improbable. Our values for mixing energy combined with the simplification of considering mixing only among the lightest discrete isosinglet scalar states, then support a set of physical mixed states with $f_0(1710)$ composed of 73.8(9.5)% glueball and $f_0(1500)$ consisting of 98.4(1.4)% quarkonium, mainly $s\bar{s}$. The glueball amplitude which leaks from $f_0(1710)$ goes almost entirely to the state $f_0(1390)$, which remains mainly $n\bar{n}$, normal-antinormal, the abbreviation we adopt for $(u\bar{u}+d\bar{d})/\sqrt{2}$. In addition, $f_0(1500)$ acquires an $n\bar{n}$ amplitude with

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sign opposite to its $s\bar{s}$ component. Interference between the $n\bar{n}$ and $s\bar{s}$ components of $f_0(1500)$ suppresses the state’s decay to $K\bar{K}$ final states by a factor consistent, within uncertainties, with the experimentally observed suppression.

An alternative calculation of mixing between valence approximation quarkonium and glueball states is given in Ref. [10]. We show in Ref. [11], however, that the calculation of Ref. [10] is not correct.

Our calculations, using Wilson fermions and the plaquette action, were done with ensembles of 2749 configurations on a lattice $12^3 \times 10 \times 24$ with $\beta$ of 5.70, 1972 configurations on $16^3 \times 14 \times 20$ with $\beta$ of 5.70, 2328 configurations on $16^3 \times 14 \times 20$ with $\beta$ of 5.93, 1732 configurations on $24^2 \times 20 \times 32$ with $\beta$ of 6.17, and 1003 configurations on $32^2 \times 28 \times 40$ with $\beta$ of 6.40. The smaller lattices with $\beta$ of 5.70 and 5.93 and the lattices with $\beta$ of 6.17 and 6.40 have periods in the two (or three) equal space directions of 1.68(5) fm, 1.54(4) fm, 1.74(5) fm, 1.66(5) fm, respectively, permitting extrapolations to zero lattice spacing with nearly constant physical volume. Conversions from lattice to physical units in this paper are made using the exact solution to the two-loop zero-flavor Callan-Symanzik equation for $\Lambda^{(0)}_{MS}$ with $\Lambda^{(0)}_{MS}$ of 234.9(6.2) MeV.

From each ensemble of configurations, we evaluated, following Ref. [2], the lightest pseudoscalar quarkonium mass, scalar quarkonium mass, scalar glueball mass and quarkonium-glueball mixing energy $E$. These calculations were done for a range of quark masses starting a bit below the strange quark mass and running to a bit above twice the strange quark mass.

For $L$ near 1.6 fm, Figure 1 shows the $s\bar{s}$ scalar mass in units of $\Lambda^{(0)}_{MS}$ as a function of lattice spacing in units of $1/\Lambda^{(0)}_{MS}$. A linear extrapolation of the mass to zero lattice spacing gives 1322(42) MeV, far below our valence approximation infinite volume continuum glueball mass of 1632(49) MeV. Figure 1 also shows values of the $s\bar{s}$ scalar mass at $\beta$ of 5.70 and 5.93 with $L$ of 2.24(7) and 2.31(6) fm, respectively. The $s\bar{s}$ mass with $L$ near 2.3 fm lies below the 1.6 fm result for both values of lattice spacing. Thus the infinite volume continuum $s\bar{s}$ scalar mass should lie below 1322(42) MeV. For comparison with our data, Figure 2 shows a linear extrapolation to zero lattice spacing of quarkonium-glueball mixing energy $E(m_s)$. The extrapolation uses the points with $L$ near 1.6 fm. The points with larger lattice period suggest that $E(m_s)$ rises a bit with lattice volume, but the trend is not statistically significant. As a function of quark mass with lattice spacing fixed, we found the mixing energy to be extremely close to linear. We were thereby able to extrapolate our
data reliably down to the normal quark mass \( m_q \). Figure 2 shows also a linear extrapolation to zero lattice spacing of the ratio \( E(m_n)/E(m_g) \). The fit is to the set of points with \( L \) near 1.6 fm but is also consistent with the points for larger \( L \). Thus the continuum limit we obtain for \( E(m_n)/E(m_g) \) is also the infinite volume limit. The limiting value of \( E(m_n) \) is 43(31) MeV and of \( E(m_g)/E(m_s) \) is 1.198(72).

The infinite volume continuum value for \( E(m_n)/E(m_g) \) we now take as an input to a simplified treatment of the mixing among valence approximation glueball and quarkonium states which arises in full QCD from quark-antiquark annihilation. The Hamiltonian coupling together the scalar glueball, the scalar \( s\bar{s} \) and the scalar \( n\pi \) isosinglet is

\[
\begin{pmatrix}
  m_g & E(m_s) & \sqrt{2}rE(m_s) \\
  E(m_s) & m_s & 0 \\
  \sqrt{2}rE(m_s) & 0 & m_{n\pi} 
\end{pmatrix}
\]

where \( r \) is \( E(m_n)/E(m_g) \), and \( m_g \), \( m_s \) and \( m_{n\pi} \) are, respectively, the glueball mass, the \( s\bar{s} \) quarkonium mass and the \( n\pi \) quarkonium mass before mixing.

The three unmixed mass parameters and \( E(m_s) \), for which our measured value has a large fractional error bar, we determine from four observed masses. To leading order in the valence approximation, with valence quark-antiquark annihilation turned off, corresponding isoscalar and isosinglet states composed of \( u \) and \( d \) quarks will be degenerate. For \( m_{n\pi} \) we thus take the observed isovector value of 1470(25) MeV. The three remaining unknowns we tune to give the mixing Hamiltonian eigenvalues of 1697(4) MeV, 1505(9) MeV and 1404(24) MeV, respectively the Particle Data Group’s masses for \( f_0(1710) \) and \( f_0(1500) \), and the weighted average of Refs. \( \text{[13]} \) masses for \( f_0(1390) \).

We find \( m_g \) becomes 1622(29) MeV, \( m_s \) becomes 1514(11) MeV, and \( E(m_s) \) becomes 64(13) MeV, with error bars including the uncertainties in the four input physical masses. The unmixed \( m_g \) is consistent with the world average valence approximation glueball mass 1632(49) MeV. \( E(m_s) \) is consistent with our measured value of 43(31) MeV, and \( m_s \) is about 13% above the valence approximation value 1322(42) MeV for lattice period 1.6 fm. This 13% gap is comparable to the largest disagreement, about 10%, found between the valence approximation and experimental values for the masses of light hadrons. In addition, the physical mixed \( f_0(1710) \) has a glueball content of 73.8(9.5)%%, the mixed \( f_0(1500) \) has a glueball content of 1.6(1.4)% and the mixed \( f_0(1390) \) has a glueball content of 24.5(10.7)%. These predictions are supported by a recent reanalysis of Mark III data \( \text{[13]} \) for \( J/\Psi \) radiative decays. Finally, the state vector for \( f_0(1500) \) we find has a relative negative sign between the \( s\bar{s} \) and \( n\pi \) components leading, by interference, to a suppression of the partial width for this state to decay to \( K\bar{K} \) by a factor of 0.39(16) in comparison to the \( K\bar{K} \) rate for an unmixed \( s\bar{s} \) state. This suppression is consistent with the experimentally observed suppression.

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