Denoising-based Vector AMP

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Abstract

The D-AMP methodology, recently proposed by Metzler, Maleki, and Baraniuk, allows one to plug in sophisticated denoisers like BM3D into the AMP algorithm to achieve state-of-the-art compressive image recovery. But AMP diverges with small deviations from the i.i.d.-Gaussian assumption on the measurement matrix. Recently, the VAMP algorithm has been proposed to fix this problem. In this work, we show that the benefits of VAMP extend to D-V AMP.

Consider the problem of recovering a (vectorized) image \( x_0 \in \mathbb{R}^N \) from compressive (i.e., \( M \ll N \)) noisy linear measurements

\[
y = \Phi x_0 + w \in \mathbb{R}^M,
\]

known as “compressive imaging.” The “sparse” approach to this problem exploits sparsity in the coefficients \( v_0 \equiv \Psi x_0 \in \mathbb{R}^N \) of an orthonormal wavelet transform \( \Psi \). The idea is to rewrite [1] as

\[
y = A v_0 + w \quad \text{for} \quad A \equiv \Phi \Psi^T,
\]

recover an estimate \( \hat{v} \) of \( v_0 \) from \( y \), and then construct the image estimate as \( \hat{x} = \Psi^T \hat{v} \).

Although many algorithms have been proposed for sparse recovery of \( v_0 \), a notable one is the approximate message passing (AMP) algorithm from [1]. It is computationally efficient (i.e., one multiplication by \( A \) and \( A^T \) per iteration and relatively few iterations) and its performance, when \( M \) and \( N \) are large and \( \Phi \) is zero-mean i.i.d. Gaussian, is rigorously characterized by a scalar state evolution.

A variant called “denoising-based AMP” (D-AMP) was recently proposed [2] for direct recovery of \( x_0 \) from [1]. It exploits the fact that, at iteration \( t \), AMP constructs a pseudo-measurement of the form \( v_0 + \mathcal{N}(0, \sigma_t^2 I) \) with known \( \sigma_t^2 \), which is amenable to any image denoising algorithm. By plugging in a state-of-the-art image denoiser like BM3D [3], D-AMP yields state-of-the-art compressive imaging.

AMP and D-AMP, however, have a serious weakness: they diverge under small deviations from the zero-mean i.i.d. Gaussian assumption on \( \Phi \), such as non-zero mean or mild ill-conditioning. A robust alternative called “vector AMP” (VAMP) was recently proposed [4]. VAMP has similar complexity to AMP and a rigorous state evolution...
that holds under right-rotationally invariant $\Phi$—a much larger class of matrices. Although VAMP needs to know the variance of the measurement noise $w$, an auto-tuning method was proposed in [5].

In this work, we integrate the D-AMP methodology from [2] into auto-tuned VAMP from [5], leading to “D-VAMP” (For a matlab implementation, see http://dsp.rice.edu/software/DAMP-toolbox).

To test D-VAMP, we recovered the $128 \times 128$ lena, barbara, boat, fingerprint, house, and peppers images using 10 realizations of $\Phi$. Table I shows that, for i.i.d. Gaussian $\Phi$, the average PSNR and runtime of D-VAMP is similar to D-AMP at medium sampling ratios. The PSNRs for $\nu$-based indirect recovery, using Lasso (i.e., “$\ell_1$”)-based AMP and VAMP, are significantly worse. At small sampling ratios, D-VAMP behaves better than D-AMP, as shown in Fig. 1.

To test robustness to ill-conditioning in $\Phi$, we constructed $\Phi = JSPFD$, with $D$ a diagonal matrix of random $\pm 1$, $F$ a (fast) Hadamard matrix, $P$ a random permutation matrix, and $S \in \mathbb{R}^{M \times N}$ a diagonal matrix of singular values. The sampling rate was fixed at $M/N = 0.1$, the noise variance chosen to achieve SNR=32 dB, and the singular values were geometric, i.e., $s_i/s_{i-1} = \rho \ \forall i > 1$, with $\rho$ chosen to yield a desired condition number. Table II shows that (D-)AMP breaks when the condition number is $\geq 10$, whereas (D-)VAMP shows only mild degradation in PSNR (but not runtime).

![Fig. 1. PSNR versus iteration at several sampling ratios $M/N$ for i.i.d. Gaussian $A$.](image)

| TABLE I | AVERAGE PSNR AND RUNTIME FROM MEASUREMENTS WITH I.I.D. GAUSSIAN MATRICES AND ZERO NOISE AFTER 30 ITERATIONS |
|---------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| sampling ratio | 10% PSNR time | 10% runtime | 20% PSNR time | 20% runtime | 30% PSNR time | 30% runtime | 40% PSNR time | 40% runtime | 50% PSNR time | 50% runtime |
| $\ell_1$-AMP | 17.7 | 0.5s | 20.2 | 1.6s | 22.4 | 1.6s | 24.6 | 2.3s | 27.0 | 3.1s |
| $\ell_1$-VAMP | 17.6 | 0.5s | 20.2 | 0.9s | 22.4 | 1.4s | 24.8 | 1.8s | 27.2 | 2.3s |
| BM3D-AMP | 25.2 | 10.1s | 30.0 | 8.8s | 32.5 | 8.6s | 35.1 | 9.1s | 37.4 | 9.8s |
| BM3D-VAMP | 25.2 | 10.4s | 30.0 | 8.5s | 32.5 | 8.2s | 35.2 | 8.5s | 37.7 | 8.8s |
TABLE II
AVERAGE PSNR AND RUNTIME FROM MEASUREMENTS WITH DHT-BASED MATRICES AND SNR=32 DB AFTER 10 ITERATIONS

| condition no. | PSNR time | PSNR time | PSNR time | PSNR time | PSNR time |
|---------------|-----------|-----------|-----------|-----------|-----------|
| ℓ₁-AMP       | 17.3 0.02  | <0 <0     | <0 <0     | <0 <0     | <0 <0     |
| ℓ₁-VAMP      | 17.4 0.04  | 17.4 0.04  | 15.6 0.03  | 14.7 0.03  | 14.4 0.03  |
| BM3D-AMP     | 24.8 5.2s  | 8.0 —      | 7.2 —      | 7.1 —      | 7.2 —      |
| BM3D-VAMP    | 24.8 5.4s  | 24.3 5.5s  | 22.6 5.3s  | 21.4 4.9s  | 20 4.5s    |

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