A possible origin of the endpoint anomaly in tritium $\beta$-spectrum

J.Řízek, V. Brabec, O. Dragoun, M. Ryšavý, A. Špalek

Nuclear Physics Institute, Acad. Sci. of Czech Republic,
CZ-250 68 Rež near Prague, Czech Republic

Abstract

The influence of the residual $T$ atoms appearing after the decay of $T_2$ molecule on the $\beta$-spectrum shape is considered. Recent experiments performed in Mainz, Troitsk, and Livermore are briefly reviewed from this viewpoint. Aspects connected with the possible time dependent change of the tritium source composition are discussed.

A. Introduction. — The recent attempts to determine the limits on the electron antineutrino mass from the endpoint part of the $\beta$-spectrum of tritium decay revealed a strange fact. When fitting the measured spectrum in all the experiments [1] – [6] performed in this decade, the negative (i.e. nonphysical) values of the antineutrino-mass squared were obtained. The reason for this is presently unknown. The recent review of the negative neutrino-mass squared problem and some considerations are given in [7].

In this note some remarks concerning the measurement and interpretation of the tritium $\beta$-spectrum are given. We consider the case when the radioactive source is composed of the $T_2$, $(R_0, T)$ and $(R_1, R_2)$ molecules. Here $R_i$ denote the nonradioactive parts of the molecules. We take into account that after the decay of one $T$ atom in the $T_2$ molecule the other atom can remain in the source and its decay can influence the shape of the measured $\beta$-spectrum.

Let us at first consider the experiments carried by the Mainz [5, 8] and Troitsk [6, 9] groups, and the result of Livermore (LLNL) experiment [10].

B. Results of Troitsk, Mainz and LLNL experiments. — The Troitsk experiment consists at present from two runs - 1994 and 1996. In the 1994 run the negative $m_{\bar{\nu}}^2$ was obtained [6]. As a reason for this nonphysical result, the excess of counts at 7.6 eV below the endpoint was claimed. When corrected for this excess, which was interpreted as a monoenergetical bump in the differential spectrum with branching ratio of about $7 \times 10^{-11}$, the negative value of $m_{\bar{\nu}}^2$ diminished. Thus in this 1994 experiment, instead of the puzzle of the negative $m_{\bar{\nu}}^2$, another one – a strange anomaly in the $\beta$-spectrum near the endpoint – appeared. The value of $m_{\bar{\nu}}^2$ was found to depend considerably on the size

*e-mail: ryasy@ujf.cas.cz
of the fitted energy interval below the endpoint. When the interval was sufficiently large, the negative antineutrino mass squared appeared again. This was explained by the excess of counts below 18300 eV energy. The energies and populations of molecular final states were taken into account using the results of theoretical calculations [11, 12].

The 1996 experiment [9] confirmed the main conclusions of the previous one and led to new problems. The interpretation of the data was improved in the sense that the effect of the trapping the electrons in the tritium source was treated more carefully (it turned out that this effect was underestimated earlier). Another set of final states was taken into the account in accordance with the newest theoretical results [13]. However, the position of the bump-like anomaly turned out to be time dependent: in the 1996 run it was situated at 12.3 eV below the endpoint while its intensity remained almost the same as in previous run. Taking into account all the corrections and the bump-like structure, the fitted value of the $m_\nu^2$ was positive. But again the result depended on the size of the energy interval considered. In Table 1 the results of Troitsk experiment are summarized.

### Table 1: Summary of the Troitsk experiment

| $m_\nu^2$ (eV)$^2$ | $m_\nu$ (eV) | endpoint (eV) | interval from (eV) | run | 2) |
|-------------------|--------------|---------------|-------------------|-----|----|
| $-22 \pm 4.8$     | 18350        | 1994          | no                |
| $-4.1 \pm 10.9$   | 18575        | 1994          | yes               |
| $-2.7 \pm 10.1 \pm 4.9$ | $\langle 4.35 \rangle$ | 18575.4      | 1994          | yes |
| $-11. \pm 5.4$    | 18300        | 1994          | yes               |
| $3.8 \pm 7.4 \pm 2.85$ | $\langle 4.4 \rangle$ | 18575       | 1996          | yes |
| $1.5 \pm 5.9 \pm 3.6$ | $\langle 3.9 \rangle$ | 18575       | 1994+1996      | yes |

1) corrected in 1996
2) indicates whether or not the spectrum was corrected for the near endpoint anomaly

As concerned the Mainz experiment, there were also two runs – in 1991 and in 1994. No endpoint anomaly was reported. However, as in the Troitsk experiment, an excess of counts at the energies of $\sim 100$ eV below the endpoint was observed. The results of Mainz experiment are summarized in Table 2.

### Table 2: Summary of Mainz experiment

| $m_\nu^2$ (eV)$^2$ | $m_\nu$ (eV) | endpoint (eV) | size of interval (eV) | run |
|-------------------|--------------|---------------|----------------------|-----|
| $-39 \pm 34 \pm 15$ | $\langle 7.2 \rangle$ | 18574.8      | 140          | 1991 |
| $-22 \pm 17 \pm 14$ | $\langle 5.6 \rangle$ |               | 143          | 1994 |
| $-5 \pm 27 \pm 5$  | $\langle 7.1 \rangle$ |               | 100          | 1994 |

Different tritium sources were used in the above experiments – gaseous one in Troitsk
and condensed molecular film in Mainz. According to ref. [6], the Troitsk source consisted in 1994 from the mixture of $T_2 + (HT) + H_2$ in the proportion of 6:8:2. Some admixture of free $T^-$ ions was also reported but considered to be negligible. The composition of the Mainz source was, in accordance with ref. [14], 73.9% $T_2$, 19.2% $(H, T)$ and 6.9% $H_2$.

Finally let us cite the results of ref. [10]. In that LLNL experiment, the gaseous source of molecular tritium was used, the purity of which was declared to be 98%. Anomalous structure in the last 55 eV below the endpoint was reported. This was interpreted as a narrow peak at $\sim 23$ eV below the endpoint with intensity $3 \times 10^{-9}$ of the total $\beta$-strength.

The question arises whether the mentioned anomalies are due to the fact that especially the tritium decay is studied or whether it can be observed in other decays, too. Unfortunately the problem of the final atomic or molecular states makes it very difficult to study this problem using more complicated atoms or molecules than the tritium ones. In addition, the interpretation has to rely only upon the theoretical calculations of the final states energies and populations. No experimental data on the final states are available. Many reasons that can cause these observed anomalies were qualitatively discussed – e.g. account of some missed final states, possible neutrino mixing, special neutrino interaction.

Especially intriguing is the observed time dependence of the near endpoint count excess. It seems that the composition of the source can change with time. In the above mentioned three experiments the radioactive sources of different compositions were used. A decay of one tritium atom of the $T_2$ molecule changes the composition of the source and this change is time dependent. In what follows we try to analyse qualitatively namely this point.

C. The decay of $T_2$. — The decay of molecular tritium proceeds as follows:

$$T_2 \rightarrow (^3He, T)^+ + e^- + \bar{\nu}_e$$

Here $n$ denotes the different molecular branches (final states) of the decay.

Probably in most cases the $(^3He, T)^+$ ion breaks and the tritium atom reacts with some residual atom or with the atoms of wall surrounding the initial $T_2$ molecules. It seems possible that some number of new molecules with the tritium atom as a part will remain in the radioactive source and will contribute to measured spectrum. In the next we consider the decay more quantitatively.

Let us suppose that at time $t = 0$ we have 100% of $T_2$ molecules in the source. Further on we will use the following notation:

$N_2 (t) \ldots$ the number of $T_2$ molecules at time $t$,

$N_1 (t) \ldots$ the number of the tritium atoms that are not a part of the $T_2$ molecule,

$N (t) \ldots$ the whole number of the tritium atoms at time $t$.

It is clear that the following relations hold ($\lambda$ denotes the tritium decay constant):

$$N_2 (0) e^{-\lambda t} = N_1 (t) + 2N_2 (t)$$

(1)

$$N_1 (t + \Delta t) - N_1 (t) = -\lambda N_1 (t) \Delta t + (N_2 (t) - N_2 (t + \Delta t))$$

(2)
From (1) we can get the relation between derivatives of \( N_1(t) \) and \( N_2(t) \), from the equation (2) we obtain (in the limit \( \Delta t \to 0 \)) another relation between these derivatives. As a result the differential equations for \( N_1(t) \) and \( N_2(t) \) are obtained. Explicitly this is written as

\[
\begin{align*}
-\frac{dN_2(t)}{dt} &= \frac{1}{2} \left[ \frac{dN_1(t)}{dt} + 2\lambda N_2(0) e^{-\lambda t} \right], \\
\frac{dN_1(t)}{dt} &= -\lambda N_1(t) - \frac{dN_2(t)}{dt}.
\end{align*}
\]

Using these relations we get the equation for \( N_1(t) \)

\[
\frac{dN_1(t)}{dt} = -2\lambda \left[ N_1(t) - N_2(0) e^{-\lambda t} \right], \quad N_1(0) = 0.
\]

The solution fulfilling initial condition is

\[
N_1(t) = 2N_2(0) \left[ 1 - e^{-\lambda t} \right] e^{-\lambda t}, \quad N_2(t) = N_2(0) e^{-2\lambda t}.
\]

Therefore we have

\[
\frac{N_1(t)}{N_2(t)} = 2 \frac{1 - e^{-\lambda t}}{e^{-\lambda t}}.
\]

This is the fraction of the tritium atoms with respect to the number of \( T_2 \) molecules that are not part of the tritium molecules. For illustration, this fraction amounts to 0.3% and 10% for \( t \) equal 10 days and one year, respectively. The question is where these atoms (which undoubtedly appear) are in a real experiment. Are they removed in some way in order not to influence the measurement or they remained within the source in a form different from the molecular tritium? They can be as well absorbed on the walls of the bottle where the original molecular tritium is stored. Of course the fraction will be different from that predicted by eq.(6) if some of the tritium atoms escape out of the source.

In the more general case when the initial number of tritium atoms is not zero we have

\[
N_1(t) = N_1(0) e^{-\lambda t} + 2N_2(0) \left( 1 - e^{-\lambda t} \right) e^{-\lambda t}, \quad N_2(t) = N_2(0) e^{-2\lambda t}.
\]

For completeness let us give the results for the source which composition is

\[
N_2(0) = p_2 N(0), \quad N_1(0) = p_1 N(0), \quad N_0(0) = p_0 N(0),
\]

where \( N_i(0) \) denotes the number of molecules \( T_2 (i=2) \), \( RT \ (i=1) \) and \( R_1 R_2 \ (i=0) \) at time \( t = 0 \). Then the following relations at time \( t \) hold

\[
\begin{align*}
N_2(t) &= p_2 N(0) e^{-2\lambda t}, \\
N_1(t) &= N(0) \left[ p_1 + 2p_2 \left( 1 - e^{-\lambda t} \right) \right] e^{-\lambda t}, \\
N_0(t) &= N(0) \left[ p_0 + p_1 \left( 1 - e^{-\lambda t} \right) + p_2 \left( 1 - e^{-2\lambda t} \right) \right].
\end{align*}
\]

In the two next sections we consider some aspects that can be connected with different composition of the tritium source.
D. Different endpoint energies. — From the above consideration we can expect that some, may be unknown, admixture of type \((T, R)\) will be present in the source.

Let us suppose that this is the case and consider the resulting \(\beta\)-spectrum as a superposition of two spectra. The essential here is that the energy endpoints for these two branches are different. We consider the situation near endpoint and suppose that the neutrino mass is zero. The expression

\[
F(Z, E) \sqrt{E^2 - 1}
\]

(here \(E\) denotes the full electron energy \((h = m = c = 1)\), \(F\) stands for the Fermi function) turns out to be almost constant on the 300 eV interval below endpoint. In order to simplify the calculation we suppose that it is a constant. Then the differential spectrum near the endpoint energy is written as follows:

\[
\frac{dP(E)}{dE} = C \begin{cases} 
A \frac{(W_0 - E)^2}{W_0 - E} & W_0 \geq E \geq W_0 - \Delta E, \\
A \frac{(W_0 - E)^2}{W_0 - E^2} + B \frac{(W_0 - E - \Delta E)^2}{W_0 - \Delta E} & E \leq W_0 - \Delta E.
\end{cases}
\]  
(9)

As is seen we consider the superposition of two spectra with amplitudes \(A, B\) \((A + B = 1)\) and endpoint energies \(W_0, W_0 - \Delta E\), respectively. The normalization constant is denoted as \(C\). The integral spectrum from some \(E_{low} < W_0 - \Delta E\) up to \(W_0\) is

\[
I(E_{low}) = C \left\{ \frac{1}{3} y^3 - B \Delta E \Phi(y) \right\}
\]  
(10)

where

\[
y = W_0 - E_{low}, \quad \Phi(y) = y^2 - \Delta E y + \frac{1}{3} (\Delta E)^2.
\]

Usually several measurements are done that are separated from each other by some time interval. From our viewpoint the constants \(B\) are different in these measurements due to the different composition of the sources. If we normalize the two measurements to the same integral intensity at some point \(y_0\), we get for the normalization factors the relation

\[
\frac{1}{3} (C_1 - C_2) y_0^3 = (B_1 C_1 - B_2 C_2) \Delta E \Phi(y_0).
\]  
(11)

Using this relation we can write for the difference of two the spectra

\[
I_1(y) - I_2(y) = \frac{1}{3} (C_1 - C_2) \left\{ y^3 - \frac{y_0^3}{\Phi(y_0)} \Phi(y) \right\}.
\]  
(12)

This function has local extrema at the points

\[
y_{\pm} = \frac{y_0^3}{3 \Phi(y_0)} \left( 1 \pm \sqrt{1 - 3 \Delta E \frac{\Phi(y_0)}{y_0^2}} \right).
\]  
(13)

Only one point is feasible due to the condition \(y > \Delta E\). Of course the results obtained are qualitative only. We neglect the energy dependence arising from the common factor including the Fermi function (in accordance with our numerical calculations this dependence is linear but close to a constant in the interval considered). We also know nothing about the actual “fate” of the residuals after one of the tritium atom from the molecule decays. But if these residuals will in part create some impurities in the source there is a
possibility that these can serve as a reason of observed anomalies in the spectra near the endpoint.

If we consider the expression for the spectra with $B \neq 0$, we see that under the preposition of small both $B$ and $\Delta E$ the part of the spectrum influenced by this term is very narrow and concentrated somewhere below $W_0 - \Delta E$. This term will be quickly embedded in the statistical error of dominating part connected with $A$ branch of the spectrum when going sufficiently below this energy. Let us notice that the “effective position” where this term is in effect depends on $B$ and therefore it is – from our viewpoint – time dependent. The question is whether this influence can be simulated by some step function at some energy.

We can also point out another aspect. Namely, let us consider the case of nonzero neutrino mass and suppose that $B = 0$. Then the integral spectrum will be (we consider this spectrum from such electron energies that the neutrino mass may be considered small and the approximation $\sqrt{1 - \frac{m_\nu^2}{(W_0 - E)^2}} \simeq 1 - \frac{m_\nu^2}{2(W_0 - E)^2}$ can be used)

$$I(E) = C \left\{ \frac{1}{3} y^3 - \frac{1}{2} m_\nu^2 y + \frac{1}{2} m_\nu^2 \delta E \right\}, \quad y = W_0 - E. \quad (14)$$

Here $\delta E$ denotes such energy interval below $W_0$ that we can use the approximation of the square root written above (i.e. we integrate from $E$ to $W_0 - \delta E$). If we now compare this expression with the spectrum for the case $B \neq 0$, $m_\nu = 0$, eq. (10), we see that these spectra have common main term but differ in

$$- B \Delta E y^2 + B (\Delta E)^2 y - \frac{1}{3} B (\Delta E)^3 \quad \text{in (10)}$$

$$- \frac{1}{2} m_\nu^2 y + \frac{1}{2} m_\nu^2 \delta E \quad \text{in (14)} \quad (15)$$

If the spectrum measured is indeed that of (10) and fitted one that of (14) we see that we can expect to find the nonzero square of neutrino mass. It is probable that the fitted squared neutrino mass will be positive unless we have a term $\sim y^2$ to fit the background.

Let us notice that we can have the case when the endpoint energy of the $B$ branch lies higher than that of $A$ branch. The same considerations as given above result in the following conclusions:

1. The difference in spectra leads to the local extremum at the point

$$y_+ = \frac{y_0^3}{3 \Psi(y_0)} \left\{ 1 + \sqrt{1 + 3 \Delta E \frac{\Psi(y_0)}{y_0^3}} \right\} \quad (16)$$

where

$$\Psi(y) = y^2 + \Delta E y + \frac{1}{3} (\Delta E)^2.$$

2. The different term in analogy with the consideration of zero and nonzero neutrino mass as given above in equation (15) is:

$$B \left( \Delta E y^2 + (\Delta E)^2 y + \frac{1}{3} (\Delta E)^3 \right).$$
We see that in this case we obtain, when fitting neutrino mass, negative value for \( m^2_{\nu} \).

When the difference of the \( \beta \)-spectra taken in Troitsk in 1994 and 1996 were investigated some maximum was indeed found.

**E. The same endpoint.** — The theoretical spectrum is described by the following formula (again we suppose that the above mentioned factor is constant or can be eliminated as in the case when Kurie plot is considered)

\[
P(E) = \sum_n w_n \left( E_0 - E - \varepsilon_n \right) \sqrt{(E_0 - E - \varepsilon_n)^2 - m^2_{\nu}} \theta (E_0 - E - \varepsilon_n - m_{\nu}). \tag{17}
\]

Here \( E_0, E, w_n, \varepsilon_n \) denote the endpoint energy, the electron energy, probability of the excitation of \( n \)-th ion excited state and its energy, respectively. For small \( m_{\nu} \) we get

\[
P(E) = W_N \left\{ \left( E_0 - E - \overline{\varepsilon N} \right)^2 + W_N \frac{\overline{\varepsilon^2 N} - (\overline{\varepsilon N})^2}{W^2_N} - m^2_{\nu} \right\}, \tag{18}
\]

\[
W_N = \sum_{n=0}^N w_n, \quad \overline{\varepsilon N} = \sum_{n=0}^N w_n \varepsilon_n, \quad \overline{\varepsilon^2 N} = \sum_{n=0}^N w_n \varepsilon^2_n.
\]

When all the excited atomic states of the ionized molecule are taken into account, we have \( W_N = 1 \), and we denote

\[
\bar{\varepsilon} = \overline{\varepsilon N}, \quad \bar{\varepsilon^2} = \overline{\varepsilon^2 N}, \quad \sigma^2 = \bar{\varepsilon^2} - (\bar{\varepsilon})^2. \tag{19}
\]

Now we suppose that there are two components of the source. According to the calculations \[15\] the value of \( \bar{\varepsilon} \) does not change too much with the composition of the tritiated molecule but the value of \( \sigma^2 \) can differ in the range \( \sim 100 \text{ eV} \). So we suppose that

\[
\overline{\varepsilon_1} = \overline{\varepsilon_2} = \bar{\varepsilon}, \quad \sigma^2_1 \neq \sigma^2_2,
\]

and we can write for the energies far enough from the endpoint the following:

\[
\frac{dP(E)}{dE} = (E_0 - E - \varepsilon)^2 + \sigma^2_1 + B \left( \sigma^2_2 - \sigma^2_1 \right) - \frac{m^2_{\nu}}{2}. \tag{20}
\]

Here \( B \) denotes the amplitude of the second part of the source. In other words we suppose, for example, that the source composition is \( AT_2 + B (RT) \), \( A + B = 1 \). The second part with the amplitude \( B \), when not taken into account, can give rise to some counts when compared with the theoretical spectrum, calculated for \( B = 0 \). This will be more significant when being far enough from the endpoint i.e. when all excited states of ionized molecule are in a play. As concerned the tritium molecules, these excited states are restricted by \( \sim 170 \text{ eV} \). Therefore, in the integral spectrometer we can expect some rise of the counts if some – though small – admixture \((\text{R, T})\) is present in the source.
F. Conclusion. — The theme of present work was inspired by the recently published work of Troitsk group \[9\] on the time dependence of the near endpoint anomaly observed in the tritium $\beta$-decay. The possibility that the composition of the tritium source is time dependent due to the presence of free T atoms that are left in the source after the decay of $T_2$ molecule was considered. We suppose that these ions react with some residual molecules and remain in the source. It was shown qualitatively that this can explain some anomalies observed in the electron $\beta$ spectra near the endpoint.

The time dependence of the near endpoint anomaly as reported by Troitsk group seems to support such a point of view. The presence of this anomaly as reported in LLNL experiment and the excess of counts in the lower energy below the endpoint that is observed in all three experiments can also been qualitatively explained. The differences between the positions or no observation of the near endpoint anomaly in Mainz experiment can be also understood. Different tritium sources used (gaseous in Troitsk and LLNL, condensed film in Mainz) can be the reason. From our viewpoint the impurities that can be added little by little depend on the type of the source due to the different conditions under which the residual molecules react with tritium ions.

Unfortunately, we are not able to say anything about the fate of the tritium ions, nor to determine in what a molecular state supposed impurities can be. Some experimental investigation of this problem is desirable. From this viewpoint the recently \[8\] announced intention to study experimentally the final states in the tritium $\beta$-decay can be regarded as a first step that can help to understand this problem.

It seems also desirable to study experimentally another $\beta$-decay than that of tritium. Unfortunately, the account of the final states will be here more difficult than in the case of the tritium $\beta$-decay.

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References

[1] R.G.H. Robertson, T.J. Bowles, G.J. Stephenson, D.L. Wark, J.F. Wilkerson, D.A. Knapp: Phys. Rev. Lett. 67 (1991) 957.

[2] H. Kawakami, S. Kato, T. Oshima, S. Shibata, K. Ukai, N. Morikawa, N. Bogawa, K. Haga, T. Nagafuchi, M. Shigeta, Y. Fukushima, T. Taniguchi: Phys. Lett. B256 (1991) 105.

[3] W. Stoeffl: Bull. Am. Phys. Soc. 37 (1992) 1286.

[4] E.Holzschuh, M.Fritschi, W. Kündig: Phys. Lett. B287 (1992) 381.

[5] Ch. Weinheimer, M. Przyrembel, H. Backe, H. Barth, J. Bonn, B. Degen, Th. Edling, H. Fischer, L. Fleischmann, J.U. Grooss, R. Haid, A. Hermanni, G. Kube, P. Leiderer, Th. Loecken, A. Molz, R.B. Moore, A. Osipowicz, E.W. Otten, A. Pickard, M. Schrader, M. Steininger: Phys. Lett. B300 (1993) 210.

[6] A.I. Belesev, A.I. Bleule, E.V. Geraskin, A.A. Golubev, N.A. Golubev, O.V. Kazachenko, E.P. Kiev, Yu.E. Kuznetsov, V.M. Lobashev, B.M. Ovchinnikov, V.I.
Parfenov, I.V. Sekachev, A.P. Solodukhin, N.A. Titov, I.E. Yarikin, Yu.I. Zakharov, S.N. Balashov, P.E. Spivak: Phys. Lett. B350 (1995) 263.

[7] I.G. Kaplan: J. Phys. G 23 (1997) 683.

[8] H. Backe, H. Barth, A. Bleile, J. Bonn, B. Degen, L. Fleischmann, M. Gundlach, E.W. Otten, M. Przyrembel, Ch. Weinheimer: presented at: The XVII Conference on Neutrino Physics and Astrophysics, Helsinky, 1996.

[9] V.M. Lobashev, A.I. Belesev, A.I. Berlev, E.V. Geraskin, A.A. Golubev, N.A. Golubev, O.V. Kazachenko, Yu.E. Kuznetsov, V.S. Pantuev, L.A. Ryvkis, B.E. Stern, N.A. Titov, I.E. Yarikin, S.V. Zadorozhny, Yu.I. Zakharov: Report, Institute for Nuclear Research, Moscow, 1996.

[10] W. Stoeffl, D.J. Decman: Phys. Rev. Lett. 75 (1995) 3237.

[11] O. Fackler, B. Jeziorski, W. Kolos, H.J. Monkhorst, K. Szalewicz: Phys. Rev. Lett. 55 (1985) 1388.

[12] W. Kolos, B. Jeziorski, J. Rychlevski, K. Salewicz, H.J. Monkhorst, O. Fackler: Phys. Rev. A37 (1988) 2297.

[13] S. Jonsell, H.J. Monkhorst: Phys. Rev. Lett. 76 (1996) 4476.

[14] M. Przyrembel: Doctoral Thesis, J.G.-Universit"at Mainz, 1995.

[15] I.G. Kaplan: Revista Mexicana de Física 39, Suplemento 2 (1993) 123.