Flavor structure of generalized parton distributions from neutrino experiments

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The analysis of deeply virtual meson production is extended to neutrino-production of the pseudo-Goldstone mesons ($\pi$, $K$, $\eta$) on nucleons, with the flavor content of the recoil baryon either remaining intact, or changing to a hyperon from the $SU(3)$ octet. We rely on the $SU(3)$ relations and express all the cross-sections in terms of the proton generalized parton distributions (GPDs). The corresponding amplitudes are calculated at the leading twist level and in the leading order in $\alpha_s$, using a phenomenological model of GPDs. We provide a computational code, which can be used for evaluation of the cross-sections employing various GPD models. We conclude that these processes can be studied in the experiment 

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I. INTRODUCTION

During the last decade the notion of generalized parton distributions (GPDs) became a standard theoretical tool to describe the nonperturbative structure of the hadronic target. These new objects, being an special case of the general Wigner distributions, contain rich information about the nonperturbative dynamics of the target structure, such as form factors, ordinary parton distribution functions (PDF), fractions of the spin carried by each parton, etc. (see e.g. recent reviews in [1–6]). In hard exclusive reactions in the Bjorken kinematics, due to collinear factorization [7, 8] the amplitude of many processes may be represented as a convolution of the process-dependent perturbative coefficient functions with target-dependent GPDs. While a model-independent deconvolution and extraction of GPDs from data is in general impossible, nevertheless data help to constrain the available models of GPDs.

Currently the main source of information about GPDs are the electron-proton measurements done at JLAB and HERA, in particular deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) [3, 5, 7, 8, 12–23]. A planned CLAS12 upgrade at JLAB will help to improve our understanding of the GPDs [23].

Having only data on DVCS one cannot single out the flavor structure of GPDs. The process of DVMP potentially is able to disentangle the flavor structure of the GPDs, since different mesons are sensitive to different GPD flavor combinations [24, 25]. However, the practical realization of this program suffers from large uncertainties. In the HERA kinematics ($x_{Bj} \lesssim 10^{-2}$), one is close to the saturation regime, where gluons dominate, and as was discussed in [26], NLO corrections in this kinematical range become large due to BFKL-type logarithms. As a consequence, for the description of exclusive processes one should use models where the saturation is built-in [27–30]. At the same time, in the JLAB kinematics, the range of $Q^2$ is quite restricted, and the GPDs extracted from DVCS may be essentially contaminated by higher-twist effects. In case of the DVMP, an additional uncertainty comes from the distribution amplitudes (DA) of the produced mesons: while there is a lot of models, only the DAs for $\pi$ and $\eta$ were confronted with data [31] (see also the recent review in [34, 35]). For heavier mesons ($\rho, \omega, \phi$), the DAs are completely unknown, because their partonic structure is controlled by confinement, rather than by the chiral symmetry as for Goldstone mesons. In the general case it is not

1 An exception is the process of double deeply virtual Compton scattering discussed in [8, 11], however vanishingly small cross-sections make it unreachable at modern accelerators.
even known if the corresponding DAs should vanish at the endpoints of the fractional light-cone momentum distribution, as is required by collinear factorization, or if the amplitude gets a contribution from transverse degrees of freedom.

From this point of view, consistency checks of GPD extraction from JLAB data, especially of their flavor structure, are important. Neutrino experiments present a powerful tool, which could be used for this purpose. Up to recently, the high-precision exclusive neutrino-hadron differential cross-sections were available only in the low-energy region, where the physics is described by s-channel resonances \[36–40\]. In the high-energy regime, due to the smallness of the cross-sections and vanishingly small luminosities in the tails of the neutrino spectra, the experimental data have been available so far only either for inclusive or for integrated (total) exclusive cross-sections. The situation is going to change next year, when the high-intensity NuMI beam at Fermilab will switch on to the so-called middle-energy (ME) regime with an average neutrino energy of about 6 GeV. In this setup the MINERVA experiment \[41\] should be able to probe the quark flavor structure of the targets. Potentially, NuMI neutrino beam may reach energies up to 20 GeV, without essential loss of luminosity. Even higher luminosities in multi-GeV regime can be achieved at the planned Muon Collider/Neutrino Factory \[42–44\].

One can access the GPD flavor structure in neutrino interactions by studying the same processes as in \(ep\) collisions and employing the difference of the weak and electromagnetic couplings for the vector current. An example of such a process is the weak DVCS discussed in \[45\]. However, the weak DVCS alone is not sufficient to constrain the flavor structure. Moreover, the typical magnitude of the cross-sections for such processes is tiny, of the order \(~10^{-44} \text{cm}^2/\text{GeV}^4\).

The \(\nu\text{DVMP}\) measurements with neutrino and antineutrino beams are complementary to the electromagnetic DVCS. In the axial channel, due to the chiral symmetry breaking we have an octet of pseudo-Goldstone bosons which act as a natural probe for the flavor content. Due to the \(V−A\) structure of the charged current, in \(\nu\text{DVMP}\) one can access simultaneously the unpolarized GPDs, \(H, E\), and the helicity flip GPDs, \(\tilde{H}\) and \(\tilde{E}\). Besides, important information on flavor structure can be obtained by studying the transitional GPDs in the processes with nucleon to hyperon transitions. As was discussed in \[46\], due to \(SU(3)\) flavor symmetry, these GPDs can be related to the ordinary diagonal GPDs in the proton.

The paper is organized as follows. In Section II we evaluate the Goldstone meson production by neutrinos on nucleon targets. The main result of this section is Table I and Eqns 6-10. In Section III for the sake of completeness we sketch the properties of the GPD parametrization which will be used for evaluations. In Section IV we present numerical results and make conclusions.

## II. CROSS-SECTION OF THE \(\nu\text{DVMP}\) PROCESS

The DVMP process in the vector channel has been studied in \[24, 25\]. In the leading order in \(\alpha_s\), the hard coefficient function gets contributions from the diagrams shown in Figure 1.

**FIG. 1:** Leading-order and leading twist contributions to the DVMP hard coefficient functions.

Evaluation of these diagrams is straightforward and yields for the amplitude of the process,

\[
T_M = \frac{8\pi i \alpha_s f_M}{9gQ} \left( \int dz \frac{\phi_M(z)}{z} \right) \sum_i H_M^i \bar{N}(p_2) \Gamma N(p_1),
\]

where \(N(p), \bar{N}(p)\) are the spinors of the initial/final state baryon, \(\phi_M(z)\) is the normalized to unity distribution amplitude.
of the produced meson, \( f_M \) is the decay constant of the corresponding meson, \( \sum_t H^T_{tq} \bar{N} (p_2) \Gamma N (p_1) \) is a symbolic notation for summation of all leading twist GPDs contributions (defined below), and \( H^T_M \) are the convolutions of the GPDs \( H^T \) of the target with the proper coefficient function. Currently, the amplitude of the DVMP is known up to NLO accuracy \([47, 48]\).

Extension of the analysis of \([24, 25]\) to neutrinos is straightforward. In contrast to electroproduction, due to \( V - A \) structure, the amplitudes acquire contributions from both the unpolarized and helicity flip GPDs.

In the leading twist, four GPDs, \( H, E, \tilde{H} \) and \( \tilde{E} \) contribute to this process. They are defined as

\[
\frac{\hat{P}^+}{2\pi} \int dz \, e^{i\hat{P}^+ z} \left\langle B (p_2) \right| \bar{\psi}_q \left( \frac{z}{2} \right) \gamma_+ \psi_q \left( \frac{z}{2} \right) \left| A (p_1) \right\rangle = \left( H_q (x, \xi, t) \bar{N} (p_2) \gamma_+ N (p_1) \right)
\]

\[
+ \frac{\Delta_\perp}{2m_N} E_q (x, \xi, t) \bar{N} (p_2) i\gamma_5 N (p_1) \right) \\right),
\]

where \( \hat{P} = p_1 + p_2, \Delta = p_2 - p_1 \) and \( \xi = -\Delta^+ / 2\hat{P}^+ \approx x_{Bj} / (2 - x_{Bj}) \) (see e.g. \([20]\) for details of kinematics). In the general case, when \( A \neq B \), in the right-hand side (r.h.s.) of Eqs. (2), (3) there might be extra structures which are forbidden by \( T \)-parity in the case of \( A = B \) \([20]\).

In what follows we assume that the initial state \( A \) is either a proton or a neutron, and \( B \) belongs to the same lowest \( SU(3) \) octet of baryons. In this case, all such terms are parametrically suppressed by the current quark mass \( m_q \) and vanish in the limit of exact \( SU(3) \), so we will disregard them. Since in neutrino experiments the target cannot be polarized due to its large size, it makes no sense to discuss contributions of the transversity GPDs \( H_T, E_T, \tilde{H}_T, \tilde{E}_T \). Also, in this paper we ignore the contributions of gluons, because in the current and forthcoming neutrino experiments the region of small \( x_{Bj} \ll 1 \) is not achievable, so the amplitude \( 1 \) simplifies to

\[
T_M = \frac{8\pi i \alpha_s f_M}{9 Q} \left( \int dz \, \phi_M (z) \right) \left[ \frac{\hat{H}_M \bar{N} (p_2) \gamma_+ N (p_1) + \Delta_\perp}{2m_N} \tilde{E}_M \bar{N} (p_2) \gamma_5 N (p_1) \right]
\]

In table I the corresponding amplitudes are listed for each final state \( M \). It is restricted to the cases of either protons or neutrons in the initial state, and only baryons from the lowest lying octet in the final state. We used ordinary \( SU(3) \) relations \([46]\) to relate the transitional GPDs \( \left\langle Y \left| \hat{O}_{q,q'} \right| \pi \right\rangle \) to the proton GPDs \( \left\langle \pi \left| \hat{O}_{q,q'} \right| \pi \right\rangle \). As was mentioned in \([46]\), these relations for the GPD \( \tilde{E} \) can be broken due to the different masses of pion and kaon in the \( t \)-channel. Also, the \( SU(3) \) relations can be inaccurate at small-\( x_{Bj} \) (high energy), due to different intercepts of the \( \pi / \rho \) and \( K / K^* \) Regge trajectories \([40, 50]\). Besides, as was discussed in \([26]\), in the small-\( x_{Bj} \) regime NLO corrections become large due to BFKL-type logarithms, and a lot of care is needed to make a systematic resummation and avoid double counting. For this reason, in what follows we restrict our consideration to the moderate energy range \( x_{Bj} \gtrsim 0.1 \). For a neutron target, in the right panel of table II we flipped \( H_{u/n} \rightarrow H_{d/p}, H_{d/n} \rightarrow H_{u/p}, \) so all the GPDs are given for a proton target. The corresponding constants \( g_f \) should be understood as neutral current couplings \( g_A \) and \( g_V \) for the DVMP form factors \( H, E, \tilde{H}, \tilde{E} \) respectively. Electroproduction data \([24, 25]\) correspond to the mere change of the charges, \( g_A \rightarrow 0, g_V \rightarrow e_f \).

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2 Similar results may be obtained in the framework of the dipole model \([27, 30]\), which is valid for very small \( x \lesssim 10^{-2} \). The amplitude gets a substantial contribution from the endpoint region with \( \alpha \) or \( \tilde{\alpha} \sim m_q^2 / Q^2 \). This asymmetry in the \( \alpha \) contribution depends strongly on the quark mass and obviously breaks the \( SU(3) \) symmetry.
TABLE I: List of the DVMP amplitudes $\mathcal{H}_M$, $\mathcal{E}_M$, $\tilde{\mathcal{H}}_M$, $\tilde{\mathcal{E}}_M$ for different final states. For a neutron target, in the r.h.s. we flipped $H_{u/n} \rightarrow H_{d/p}$, $H_{d/n} \rightarrow H_{u/p}$, so all the GPDs are given for a proton target. To get $\mathcal{E}$, $\tilde{\mathcal{H}}$, $\tilde{\mathcal{E}}$, replace $H$ with $E$, $\bar{H}$, $E$ respectively. The corresponding constants $g_j$ should be understood as neutral current couplings $g_j^0$ for the DVMP form factors $\mathcal{H}$, $\mathcal{E}$, and as $g_j^f$ for $\tilde{\mathcal{H}}$, $\tilde{\mathcal{E}}$. $V_{ij}$ are the CKM matrix elements. $c_{\pm}$ is a shorthand notation $c_{\pm}(x, \xi) = 1/(x \pm \xi \mp i0)$ for the leading order coefficient function. NLO corrections to the coefficient functions may be found in \cite{15, 48}. For the sake of brevity, we did not show the arguments $(x, \xi, t, Q)$ for all GPDs and omitted the integral over the quark light-cone fraction $\int dx$ everywhere.

| Process         | type | $\mathcal{H}_M$ | Process         | type | $\mathcal{H}_M$ |
|-----------------|------|-----------------|-----------------|------|-----------------|
| $\nu p \rightarrow \mu^+ \pi^0 p$ | CC   | $V_{ud} (H_{d}c_+ + H_{u}c_+)$ | $\nu n \rightarrow \mu^- \pi^+ n$ | CC   | $V_{ud} (H_{u}c_- + H_{d}c_-)$ |
| $\bar{\nu} p \rightarrow \mu^+ \pi^0 \bar{p}$ | CC   | $V_{ud} (H_{u}c_- + H_{d}c_-)$ | $\bar{\nu} n \rightarrow \mu^- \pi^+ \bar{n}$ | CC   | $V_{ud} (H_{d}c_+ + H_{u}c_+)$ |
| $\nu p \rightarrow \mu^+ \pi^0 n$ | CC   | $V_{ud} (H_{u} - H_{d}) (c_+ - c_-)/\sqrt{2}$ | $\nu n \rightarrow \mu^- \pi^0 n$ | CC   | $V_{ud} (H_{d} - H_{u}) (c_- - c_+)/\sqrt{2}$ |
| $\nu p \rightarrow \nu \pi^0 n$ | NC   | $(H_{u} - H_{d}) (g_{u}c_- + g_{d}c_+)$ | $\nu n \rightarrow \nu \pi^0 \bar{n}$ | NC   | $(g_{u}H_{d} - g_{d}H_{u}) (c_- + c_+)/\sqrt{2}$ |
| $\bar{\nu} p \rightarrow \mu^+ \pi^- \bar{\Sigma}_{0}$ | CC   | $-V_{ua} (H_{d} - H_{u}) c_+$ | $\bar{\nu} n \rightarrow \mu^- \pi^- \bar{\Sigma}_{0}$ | CC   | $-V_{ua} (2H_{d} - H_{u} - H_{a}) c_+ /\sqrt{6}$ |
| $\bar{\nu} p \rightarrow \mu^+ \pi^0 \bar{\Sigma}_{0}$ | CC   | $V_{ua} (H_{d} - H_{u}) c_+ /\sqrt{2}$ | $\bar{\nu} n \rightarrow \mu^- \pi^0 \bar{\Sigma}_{0}$ | CC   | $V_{ua} (H_{d} - H_{u}) c_+ /\sqrt{2}$ |
| $\nu p \rightarrow \mu^- K^- p$ | CC   | $V_{ua} (c_+ H_{d} + c_- H_{a})$ | $\nu n \rightarrow \mu^- K^- n$ | CC   | $V_{ua} (c_+ H_{d} + c_- H_{a})$ |
| $\bar{\nu} p \rightarrow \mu^- K^- \bar{p}$ | CC   | $V_{ua} (H_{d} c_+ + H_{a} c_+)$ | $\bar{\nu} n \rightarrow \mu^- K^- \bar{n}$ | CC   | $V_{ua} (H_{d} c_+ + H_{a} c_+)$ |
| $\bar{\nu} p \rightarrow \mu^- K^0 \bar{\Sigma}_{0}$ | CC   | $-V_{ud} (H_{d} - H_{u}) c_- /\sqrt{2}$ | $\bar{\nu} n \rightarrow \mu^- K^0 \bar{\Sigma}_{0}$ | CC   | $-V_{ud} (H_{d} - H_{u}) c_- /\sqrt{2}$ |
| $\nu p \rightarrow \mu^- K^- \bar{K}^0$ | CC   | $-V_{ud} (2H_{d} - H_{u} - H_{a}) c_- /\sqrt{6}$ | $\nu n \rightarrow \nu K^0 \Lambda$ | NC   | $-g_{d} (2H_{d} - H_{u} - H_{a}) (c_- + c_+)/\sqrt{6}$ |
| $\nu p \rightarrow \mu^- K^- \bar{\Sigma}_{0}$ | CC   | $-V_{ud} (H_{d} - H_{u}) c_-$ | $\nu n \rightarrow \nu K^0 \bar{\Sigma}_{0}$ | NC   | $-g_{d} (H_{d} - H_{u} - H_{a}) (c_- + c_+)/\sqrt{6}$ |
| $\bar{\nu} p \rightarrow \mu^- K^- \bar{\Sigma}_{+}$ | CC   | $-V_{ud} (H_{d} - H_{u}) c_-$ | $\bar{\nu} n \rightarrow \mu^- K^- \bar{\Sigma}_{+}$ | CC   | $-V_{ud} (H_{d} - H_{u}) c_-$ |
| $\nu p \rightarrow \nu K^+ \bar{\Sigma}_{0}$ | NC   | $(H_{d} - H_{u}) (g_{u}c_+ + g_{d}c_+)/\sqrt{2}$ | $\nu n \rightarrow \nu K^+ \bar{\Sigma}_{0}$ | NC   | $(H_{d} - H_{u}) (g_{u}c_+ + g_{d}c_+)/\sqrt{2}$ |
| $\nu p \rightarrow \nu K^+ \bar{\Sigma}^+$ | NC   | $-g_{d} (H_{d} - H_{u}) (c_- + c_+)$ | $\nu n \rightarrow \nu K^+ \bar{\Sigma}^+$ | NC   | $-g_{d} (H_{d} - H_{u}) (g_{u}c_- + g_{d}c_-)$ |
| $\nu p \rightarrow \nu \eta p$ | NC   | $(g_{u}H_{d} + g_{d}H_{a} - 2g_{d}H_{a}) (c_- + c_+)/\sqrt{6}$ | $\nu n \rightarrow \nu \eta n$ | NC   | $(g_{u}H_{d} + g_{d}H_{a} - 2g_{d}H_{a}) (c_- + c_+)/\sqrt{6}$ |
| $\bar{\nu} p \rightarrow \mu^+ \eta \bar{n}$ | CC   | $V_{ud} (H_{d} - H_{u}) (c_- + c_+)/\sqrt{6}$ | $\bar{\nu} n \rightarrow \mu^+ \eta \bar{n}$ | CC   | $V_{ud} (H_{d} - H_{u}) (2c_- - c_+)/\sqrt{6}$ |
| $\bar{\nu} p \rightarrow \mu^+ \eta \bar{\Sigma}_{0}$ | CC   | $V_{ua} (H_{d} - H_{u}) (c_+ - 2c_-)/2\sqrt{3}$ | $\bar{\nu} n \rightarrow \mu^+ \eta \bar{\Sigma}_{0}$ | CC   | $V_{ua} (H_{d} - H_{u}) (c_+ - 2c_-)/\sqrt{6}$ |
| $\bar{\nu} p \rightarrow \mu^+ \eta \Lambda$ | CC   | $V_{ua} (2H_{d} - H_{u} - H_{a}) (c_+ - 2c_-)/6$ | $\bar{\nu} n \rightarrow \mu^+ \eta \Lambda$ | CC   | $V_{ua} (2H_{d} - H_{u} - H_{a}) (c_+ - 2c_-)/6$ |

While in the table we listed, for the purpose of reference, all 41 amplitudes, only 12 of them are independent due to the $SU(3)$ symmetry\footnote{This follows from number of irreps in $8 \times 8 = 1 + 8 + 8 + 10 + 10^* + 27$ for a given $J^P$.}. This agrees with the fact that all the amplitudes are linear combinations of 6 functions.
\[ dx H_{u,d,s} (x, \xi, t) c_\perp (x, \xi) \] and 6 functions \( \int dx H_{u,d,s} (x, \xi, t) c_\perp (x, \xi) \) for the axial and vector channels respectively. This implies a large number of relations between different cross-sections, some of which are obvious. For example, comparing different elements of the Table II, we may get:\n
\[ \begin{align*}
\frac{d\sigma_{\nu p \rightarrow \mu^- p \pi^-}}{d\tau_{Bj}} &= \frac{d\sigma_{\bar{\nu} p \rightarrow \mu^+ p \pi^+}}{d\tau_{Bj}}, \\
\frac{d\sigma_{\nu p \rightarrow \mu^- p K^0}}{d\tau_{Bj}} &= \frac{d\sigma_{\bar{\nu} p \rightarrow \mu^+ p K^0}}{d\tau_{Bj}}, \\
\frac{d\sigma_{\nu n \rightarrow \mu^- n K^0}}{d\tau_{Bj}} &= \frac{d\sigma_{\bar{\nu} n \rightarrow \mu^+ n K^0}}{d\tau_{Bj}}.
\end{align*} \]

Other relations can be extracted using the identity

\[ |A + B|^2 + |A - B|^2 = 2 \left( |A|^2 + |B|^2 \right). \tag{5} \]

For example, fixing

\[ A = (H_u - H_d) c_-, \quad B = (H_u - H_d) c_+, \]

we arrive at the relation between the Cabibbo-suppressed and Cabibbo-allowed cross-sections

\[ (d\sigma_{\nu p \rightarrow \mu^- p K^0 n} + d\sigma_{\nu n \rightarrow \mu^- K^0 p}) = \frac{|V_{us}|^2}{|V_{ud}|^2} (d\sigma_{\nu n \rightarrow \mu^- e^- p} + 3 d\sigma_{\bar{\nu} p \rightarrow \mu^+ e^- n}). \]

The corresponding neutrino-cross sections for charged and neutral currents read,

\[ \frac{d^3\sigma_{CC}}{dt d\ln x_{Bj} dQ^2} = \frac{G_F^2 x_{Bj}^2 \left( 1 - y - \frac{m_\pi^2 x_{Bj}^2}{Q^2} \right)}{32\pi^3 Q^2 (1 + Q^2/M_W^2)^2 \left( 1 + \frac{4m_\pi^2 x_{Bj}^2}{Q^2} \right)^3 |T_M|^2}, \tag{6} \]

\[ \frac{d^3\sigma_{NC}}{dt d\ln x_{Bj} dQ^2} = \frac{G_F^2 x_{Bj}^2 \left( 1 - y - \frac{m_\pi^2 x_{Bj}^2}{Q^2} \right)}{32\pi^3 \cos^4 \theta_W Q^2 (1 + Q^2/M_W^2)^2 \left( 1 + \frac{4m_\pi^2 x_{Bj}^2}{Q^2} \right)^3 |T_M|^2}. \tag{7} \]

In analogy to the electro- and photoproduction processes, it makes sense to introduce the cross-section of the subprocess \( W/Z + p \rightarrow M + p \), which has the form,

\[ \begin{align*}
\frac{d\sigma_{W}}{dt} &= \frac{G_F M_W^2}{\sqrt{2}} \frac{x_{Bj}^2 |T_M|^2}{16\pi Q^4 \left( 1 + \frac{4m_\pi^2 x_{Bj}^2}{Q^2} \right)}, \tag{8} \\
\frac{d\sigma_{Z}}{dt} &= \frac{\sqrt{2} G_F M_W^2}{\cos^2 \theta_W} \frac{x_{Bj}^2 |T_M|^2}{16\pi Q^4 \left( 1 + \frac{4m_\pi^2 x_{Bj}^2}{Q^2} \right)} = \frac{\sqrt{2} G_F M_Z^2 x_{Bj}^2 |T_M|^2}{16\pi Q^4 \left( 1 + \frac{4m_\pi^2 x_{Bj}^2}{Q^2} \right)}. \tag{9}
\end{align*} \]

As one can see from the GPDs \( E, \bar{E} \) always contribute in bilinear combinations \( H^* H - E^* E \) with the same coefficients as the GPDs \( H, \bar{H} \), so this does not change the total count of independent cross-sections

For example, the cross-sections of \( \nu p \rightarrow \mu^- p \pi^- \) and \( \bar{\nu} n \rightarrow \mu^+ p \pi^- \) are equal because the amplitudes of subprocesses \( W^+ p \rightarrow \pi^+ p \) and \( W^- n \rightarrow \pi^- n \) coincide due to isospin symmetry \( (I = 3/2 \) state).
In neutrino experiments the target is unpolarized, so $|T_M|^2$ can be simplified to

$$|T_M|^2_{unp} = \frac{64\pi^2 a^2 f_M}{81 Q^2(2 - x_{Bj})^2} \left( \int dz \frac{\phi_M(z)}{z} \right)^2 4 \left[ (1 - x_{Bj}) \left( H_M H_M^* + \tilde{H}_M \tilde{H}_M^* \right) - \frac{x_{Bj}^2 t \tilde{E}_M \tilde{E}_M^*}{4m_N^2} \right] + \frac{x_{Bj}^2}{4m_N^2} 2 \left( H_M E_M^* + E_M H_M^* + \tilde{H}_M \tilde{E}_M^* + \tilde{E}_M \tilde{H}_M^* \right) - \left( x_{Bj}^2 + (2 - x_{Bj})^2 \frac{t}{4m_N^2} \right) E_M E_M^* \right] \tag{10}$$

III. GPD AND DA PARAMETRIZATIONS

As was mentioned in the introduction, in the case of the DVMP a large part of the uncertainty comes from the DAs of the produced mesons. In spite of many model-dependent estimates, so far DAs are poorly known. Experimentally, only the DAs of the pions and $\eta$-meson have been challenged, and even in this case the situation remains rather controversial. The early experiments CELLO and CLEO [51], which studied the small-$Q^2$ behavior of the form factor $F_{\gamma\gamma}$, found it to be consistent with the asymptotic form, $\phi_{\gamma\gamma}(z) = 6f_M z(1 - z)$. Later the BABAR collaboration [32] found a rapid growth of the form factor $Q^2 |F_{\gamma\gamma}(Q^2)|^2$ in the large-$Q^2$ regime. This observation drew attention to this problem and gave rise to speculations that the pion DA could be far from the asymptotic shape [51] (see also a recent review by Brodsky et. al. in [54, 55]). However, the most recent data from BELLE [33] did not confirm the rapid growth found by BABAR. As was found in [52, 53], the Gegenbauer expansion coefficients of the pion DA $\phi_{2,\pi}(z)$ are small and at most give a 10% correction for the minus-first moment, based on the fits of BELLE, CLEO and CELLO data. For kaons there is no direct measurements of the DAs, it is expected however that the deviations from the pion DA are parametrically suppressed by the quark mass $m_s/GeV$. Numerically this corresponds to a 10-20% deviation.

For this reason in what follows we assume all the Goldstone DAs to have the asymptotic form,

$$\phi_{2,\{\pi, K, n\}}(z) \approx \phi_{as}(z) = 6\sqrt{2} f_M z(1 - z).$$

For the decay couplings we use standard values $f_\pi \approx 93$ MeV, $f_K \approx 113$ MeV, $f_n \approx f_K$.

More than a dozen of different parametrizations for GPDs have been proposed so far [1, 10, 21, 54, 59]. While we neither endorse nor refute any of them, for the sake of concreteness we select the parametrization [54, 60, 61], which succeeded to describe HERA [62] and JLAB [54, 60, 61] data on electro- and photoproduction of different mesons, so it might provide a reasonable description of $\nu$DVMP. The parametrization is based on the Radyushkin’s double distribution ansatz. It assumes additivity of the valence and sea parts of the GPDs,

$$H(x, \xi, t) = H_{val}(x, \xi, t) + H_{sea}(x, \xi, t),$$

which are defined as

$$H_{val}^u = \int_{\alpha + |\beta| \leq 1} d\beta d\alpha \delta (\beta - x + \alpha \xi) \frac{3\beta}{4(1 - |\beta|)^2 \alpha \xi} q_{val}(\beta) e^{(b_i - \alpha_i \ln |\beta|) t},$$

$$H_{sea}^u = \int_{\alpha + |\beta| \leq 1} d\beta d\alpha \delta (\beta - x + \alpha \xi) \frac{3\alpha \xi}{8(1 - |\beta|)^2} q_{sea}(\beta) e^{(b_i - \alpha_i \ln |\beta|) t},$$

and $q_{val}$ and $q_{sea}$ are the ordinary valence and sea components of PDFs. The coefficients $b_i, \alpha_i$, as well as the parametrization of the input PDFs $g(x)$, $\Delta g(x)$ and pseudo-PDFs $e(x)$, $\tilde{e}(x)$ (which correspond to the forward limit of the GPDs $E$, $\tilde{E}$) are discussed in [54, 60, 61]. The unpolarized PDFs $q(x)$ in the limited range $Q^2 \lesssim 40$ GeV$^2$ roughly coincide with the CTEQ PDFs. Notice that in this model the flavor symmetry for asymptotically large $Q^2$, $H_{sea}^u = H_{sea}^d = \kappa (Q^2) H_{sea}^s$, 

$$H_{sea}^u = H_{sea}^d = \kappa (Q^2) H_{sea}^s, \tag{11}$$
\begin{equation}
\kappa(Q^2) = 1 + \frac{0.68}{1 + 0.52 \ln (Q^2/Q_0^2)}, \quad Q_0^2 = 4 \text{ GeV}^2.
\end{equation}

The equality of the sea components of the light quarks in (11) should be considered only as a rough approximation, since in the forward limit the inequality \( \bar{d} \neq \bar{u} \) was firmly established by the E866/NuSea experiment [63]. For this reason, predictions done with this parametrization of GPDs for the \( p \leftrightarrow n \) transitions in the region \( x_{Bj} \in (0.1...0.3) \) might slightly underestimate the data.

\section{IV. NUMERICAL RESULTS AND DISCUSSION}

In this section we perform numerical calculations of the cross-sections of the processes listed in Table I relying on the GPDs presented in the previous section. The results for neutrino-production of pions on nucleons are depicted in Figure 2.

In the left pane of the Figure 2 we grouped the pion production processes without excitation of strangeness. The results for the diagonal channels, \( p \rightarrow p \) and \( n \rightarrow n \) extend to large \( x_{Bj} \) our previous calculations [64] for diffractive neutrino-production of pions performed in the dipole approach, which assumes dominance of the sea. Differently from small-\( x_{Bj} \) diffraction, in the valence quark region we found that the production rate of \( \pi^+ \) on neutrons is about twice larger than on protons. This results from the fact that the handbag diagram in the proton probes the GPD \( H_d \), whereas larger \( H_u \) contributes via crossed handbag; in the case of neutron they get swapped. At large \( x_{Bj} \gtrsim 0.6 \) the corresponding cross-section is suppressed due to increase of \( |t_{min}(x,Q^2)| \).

The off-diagonal processes with \( p \leftrightarrow n \) transitions are suppressed at small \( x_{Bj} \) because they probe the GPD difference \( H_u - H_d \). In the small-\( x_{Bj} \) regime \( (x_{Bj} \lesssim 0.1) \) the density of light sea quarks become equal, \( \bar{d} \approx \bar{u} \), and cancel. The valence quark PDFs and the invariant amplitude \( T_M \) behave like \( \sim 1/\sqrt{x_{Bj}} \), so that the cross-section vanishes as \( \sim x_{Bj} \).
This result agrees with the Regge phenomenology, which predicts this cross-section to fall as $x_{Bj}^2 \approx x_{Bj}^{2-2\alpha_s(0)}$. The cross-sections of the neutral $\pi^0$ production on the proton and neutron (processes $\nu p \to \nu \pi^0 p$ and $\nu n \to \nu \pi^0 n$) coincide under the assumption of $H$-dominance, however in the general case they differ, with effects $\sim H_u - H_d$. Numerically in the considered parametrization of GPDs these effects are of order 1%, so the difference between the two curves is invisible in the plot. A similar result holds for the processes $\nu p \to \nu \pi^+ n$ and $\nu n \to \nu \pi^- p$: the corresponding cross-sections exactly coincide under the assumption of $H$-dominance but differ in the general case. As in neutral pion production, the difference is controlled by a small $\sim H_u - H_d$, however, due to suppression of the GPD $H$ in the small-$x_{Bj}$ region the effects proportional $\sim H_u - H_d$ are relatively large, and the difference between the two cross-sections becomes visible in the plot.

In the right pane we show the cross-sections of pion production with nucleon to hyperon transition. These cross-sections are Cabibbo-suppressed and hardly can be detected in the Minerva experiment. In contrast to $p = n$ processes, at small $x_{Bj}$ the sea contributes to the difference $H_u - H^s$, $H_d - H^s$. First of all, the sea flavor asymmetry appears due to the presence of proton nonperturbative Fock components, like $p \to K\Lambda$. This asymmetry vanishes in the invariant amplitude at small $x_{Bj}$ as $x_{Bj}^{\alpha_{K^*}}$, where the intercept of the $K^*$ Reggeon trajectory is $\alpha_{K^*} \approx 0.25$. Correspondingly, this contribution to the cross section is suppressed as $\sim x_{Bj}^{1.5}$.

At very small $x_{Bj}$ a more important source of flavor asymmetry is the Pomeron itself. This is easy to understand within the dipole approach, in which the sea-quark PDF probed for instance by a virtual photon, corresponds to the transition $\gamma^* \to q\bar{q}$ and interaction of the $q\bar{q}$ dipole with the target proton (in the target rest frame). This cross section is different for $\bar{s}s$ and $\bar{u}u(\bar{d}d)$ dipoles due to so called aligned-jet configurations, in which the dominant fraction of the dipole momentum is carried by the quarks. Such a non-perturbative contribution to the PDF is known to persist even at high $Q^2$ [63]. Such a flavor asymmetry of the sea rises in the cross section as $x_{Bj}^{2\epsilon(Q^2)}$, controlled by the value of the Pomeron intercept, $\epsilon(Q^2) = \alpha_F - 1$, which was found in the experiments at HERA [66] to grow steeply with $Q^2$.

At medium-small values of $x_{Bj}$ in the transition region between the two above regimes the interference of these two mechanisms, which behaves as $x_{Bj}^{1-\alpha_{K^*}-\epsilon(Q^2)}$, is also important. This region of $x_{Bj} \gtrsim 0.1$ is the domain of our main interest, while small $x_{Bj} \ll 1$ is beyond the scope of this paper. At $x_{Bj} \gtrsim 0.2$, the dominant contribution comes from the valence quarks.

In Figures [3][4] we show the results for kaon production. Figure [3] presents the results for Cabibbo-allowed processes ($\Delta S = 0$), which involve a nucleon into the hyperon transition. For moderate values of $x_{Bj} \gtrsim 0.2$ the dominant contribution to these processes comes from light quarks, and in the region of smaller $x_{Bj}$ from $s$-quarks. In the left and right panes of Figure [3][4] we show the cross sections for neutral and charge kaon production respectively. The order of magnitude of these cross sections is comparable with that for pions, so should be easily measured at Minerva. However, all these processes are suppressed at high energies (small-$x$ regime). As was discussed in Section [11] there are two parametrization-independent relations between the kaon-hyperon production cross-sections, $d\sigma_{\nu p \to p K^+K^-} = 4d\sigma_{\bar{\nu}p \to p K^+K^-}$ and $d\sigma_{\nu n \to n K^+K^-} = d\sigma_{\bar{\nu}n \to n K^+K^-}$. In Figure [4] one can see the cross-section without hyperon formation. Such diagrams are Cabibbo suppressed (have $\Delta S = 1$) and have too small cross-sections, hardly detectable in current experiments.

Figure [5] demonstrates the results for $\eta$-production, both with $\Delta S = 0$ and $\Delta S = 1$. The cross-sections for all such processes at moderate values of $x_{Bj}$ are found to be alike and to have similar dependences on $x_{Bj}$. Numerically, they differ at most by an order of magnitude. At small $x_{Bj}$ however, only the neutral current cross-sections $\nu p(n) \to \nu np(n)$ survive, which depends on $x_{Bj}$ with the chosen parametrization of GPDs as $\sim x_{Bj}^{0.2}$.

The $t$-integrated cross section of pion production, calculated for diagonal transitions and plotted in the left pane of Figure [6] demonstrates the features similar to the forward cross section depicted in Figure [2].

The cross section is steeply falling toward large $x_{Bj}$ due to increasing $|t_{\min}|$, whereas at small $x_{Bj}$ it behaves similarly

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6 The pion intercept $\alpha_s(0) \approx 0$, so the pion exchange contribution is suppressed as $x_{Bj}^2$
to the unintegrated cross section. Although in the valence region the cross-sections may differ up to a factor of two, all diagonal channels for charged and neutral currents unify at the same production rate at small $x_{Bj}$, confirming the results of the dipole description \cite{64}.

The $t$-dependence of the differential cross-section is controlled by the underlying parametrization of GPDs we rely upon, and our results for the differential cross section of neutrino-production of pions are plotted in the right pane of Figure 6.
FIG. 5: (color online) η-production on the nucleon. In the upper right corner are the Cabibbo-allowed processes ($\Delta S = 0$), in the lower left corner are Cabibbo-suppressed processes ($\Delta S = 1$). Kinematics $t = t_{\text{min}}$ ($\Delta_\perp = 0$) is assumed.

FIG. 6: (color online) (a) $t$-integrated two-fold cross-section $d^2\sigma/d\nu dQ^2$. (b) $t$-dependence of the differential cross-sections for selected processes.

It can be roughly approximated by the exponential $t$-dependence $d\sigma \sim \exp(B_{eff} t)$, with the slope

$$B_{eff} \equiv \frac{1}{\sigma_W} \left. \frac{d\sigma_{WN \rightarrow MN}}{dt} \right|_{t=0},$$

which decreases with $x_{Bj}$ from about 6 down to 2 GeV$^{-2}$, as is shown in the left pane of Figure 7. To a good extent the
FIG. 7: (color online) Effective slope $B_{\text{eff}}$ which controls the $t$-dependence of the differential cross-section, as a function of $x_{Bj}$ (left) and virtuality $Q^2$ (right).

calculated $x_{Bj}$-dependence of the slope is described by $B(x) = B_0 - \beta \ln(x)$, where the coefficient $\beta$ may vary between 0.3 ($\approx 2\alpha_{\text{sea}}'(0)$) and 1.8 ($\approx 2\alpha_{\text{val}}'(0)$), depending on the process and value of $Q^2$.

The $Q^2$ dependence of $B_{\text{eff}}$ depicted in the right pane of Figure 7 is rather mild, which is due to the weak dependence of the shape of PDFs on $Q^2$ under DGLAP evolution.

Finally, in order to demonstrate explicitly the effect of skewness we compare in Figure 8 the cross-sections of several processes calculated in the model [54, 60, 61], and with the simple zero-skewness parametrization

$$H_f(x, \xi, t) \approx q_f(x) F_N(t),$$

where $q_f(x)$ is the parton distribution, and $F_N(t)$ is the nucleon form factor. We see that the results of the two parametrizations differ up to a factor of two.

V. SUMMARY

We evaluated the cross-sections of deeply virtual meson production for pions, kaons and eta-mesons in neutrino-nucleon interactions. The production rate of the Cabibbo-allowed processes in the Bjorken regime is found to be sufficiently large to be detected at the current level of statistics of neutrino experiments, in particular in the Minerva experiment at Fermilab, with accuracy, which allows to disentangle between different models of GPDs. For this purpose we provided detailed information on the distributions of the production rate versus different variables, $x_{Bj}$, $t$, $Q^2$. For further practical applications we provide a computational code. We also evaluated the cross sections of Cabibbo-suppressed channels ($\Delta S = 1$), but found them too weak to be detected by any of forthcoming experiments.
\[ \nu p \rightarrow \mu^- \pi^+ p \ (KG) \]
\[ \nu p \rightarrow \mu^- \pi^+ p \ (ZS) \]
\[ \nu n \rightarrow \mu^- \pi^0 p \ (KG) \]
\[ \nu n \rightarrow \mu^- \pi^0 p \ (ZS) \]

Note added in proof

After this manuscript was submitted, we learned that neutrino-production of pions was also studied in [67] within the approach proposed in [68]. The central assumption of [67] is that the pion coupling has a \( \bar{q}\gamma_5 q \) structure, i.e. the dominant contribution comes from the subleading twist pion DA \( \phi_p \). This contribution represents a \( \mathcal{O}(1/Q^2) \) correction to the longitudinally polarized cross-sections discussed in this paper, however it may be numerically important for the transversely polarized current. Then, the amplitude gets contributions from transversity GPDs \( H_T, E_T, \tilde{H}_T, \tilde{E}_T \). We did not include such corrections, because apart from the uncertainty in the chiral odd GPDs, this requires modelling of the poorly known twist-three pion DA \( \phi_p \). No model-independent estimate for this quantity is available so far, while model-dependent results vary considerably [69, 70].

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