Evaluation and analysis of novel flux-adjustable permanent magnet eddy current couplings with multiple rotors

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Abstract
This article proposes a novel flux-adjustable permanent magnet eddy current coupling with multiple rotors. According to the different functions, the permanent magnet rotors are divided into fixed-flux and flux-adjustable rotors. By adjusting the relative position between two types of permanent magnet rotors, the magnetic field, torque, and output speed are controlled without changing the air-gap length. To quickly and easily analyse the torque performance, an analytical model for the torque of such devices is developed based on the non-linear magnetic equivalent circuit approach. In the modelling phase, a number of factors are considered, such as the skin effect, the inductance of eddy current field, the temperature effect, and so on. A three-dimensional finite element method is employed to validate the model results, and the comparative analysis shows the advantages and potential of the proposed topology. The further sensitivity analysis of the key parameters is also made, which offers helpful information for the preliminary design of such devices.

1 | INTRODUCTION

Permanent magnet eddy current coupling (PMECC) can transmit power through the air gap without any mechanical contact, thus have been widely used in the drive systems, especially the pump and fan loads. Compared with the other counterparts, such devices are in good stability and reliability, durable and green, and extensive applicability in harsh environments [1]. Accompany with the in-depth research and development, the technology has gradually expanded to the high-tech fields, for example, automobile [2], train [3], and wind power generation [4, 5].

From the perspective of the main structure, the PMECCs are usually divided into two broad categories, namely disk and cylindrical type, and each one will be equipped with own ways of regulating speed. In brief, the former is controlled by adjusting the air-gap length between permanent magnets and conductor disk, while the latter is controlled by changing the coupling area between them. But either topology, to implement these functions, necessary auxiliary facilities have to be added, which will increase the complexity and instability of the overall system. Therefore, how to reduce the complexity of regulating parts becomes a meaningful research topic in the development of PMECCs. In [6, 7], a movable stator ring is assembled on the outer side of the permanent magnet rotor, which will avoid the rotor moving along the axial direction. In [8], another novel PMECC with a double layer permanent magnet rotor (PMR) is proposed, and the key idea is the change of magnetic field by adjusting the relative positions between PMR’s poles. However, these novel topologies relate only to the cylindrical-type PMECC, and most have a serious problem of magnetic leakage. Compared with the cylindrical type, due to the good heat dissipation and ventilation effect, low installation requirement, stable governing performance, and high security, disk-type PMECCs have wider application in the industry. Therefore, based on the above adjustable magnetic technology, a novel disk-type flux-adjustable PMECC with multiple rotors is proposed and studied in this article, as shown in Figure 1.
The research and development of PMECCs are mainly realized by numerical or analytical methods. As the typical representative of numerical methods, finite element analysis (FEA) has been employed by scholars and researchers to solve the electromagnetic field problems of PMECC [9, 10]. With such methods, the complex geometric structure and the non-linear physical characteristic of the material can be concerned in the solving process. Considering the popularity of commercial finite element software and the continuous development of computer operation ability, the FEA method has a wider range of applications. However, its shortcomings are equally notable, the accuracy of the calculation has to be guaranteed by the quality of mesh subdivision, while the compact mesh needs higher computer hardware resources; the solution process is the continuous iteration until the results converge, and the calculation process is time-consuming. Therefore, the FEA method is more suitable for the result verification and performance analysis than for the design and optimization.

More attention, by contrast, has been paid to the analytical modelling methods of PMECCs. Among these methods, the two-dimensional (2-D) layer model is used to predict the 2-D distribution of magnetic field for disk-type PMECCs [11, 12], and cylindrical-type PMECCs [13, 14], but the approximate analytical values of torque model must be corrected; to overcome this problem, the three-dimensional (3-D) analytical model of the eddy current for the PMECC is established in 3-D Cartesian coordinates [15, 16] and 3-D cylindrical coordinates [17], especially the latter has taken into account the curvature effects and the radial edge effects, hence reached a quite high accuracy. In these methods, the solution domain is divided into different layers according to the material properties; then, to establish the control equation of every layer; finally, to solve these field equations by the variable separation method (VSM) and boundary conditions. However, for some complex topologies, such as slotted conductor disk [18], flux focussing PMR [19], and the mixture of both [20], these methods become ineffective. In order to solve the magnetic problems in such cases, the sub-domain model has been employed, although valid, the solution of the system is often quite complicated. In addition, equivalent magnetic charge method [21] and equivalent surface current method [22] have been used to evaluate the performance of PMECC. But the accuracy of the model is worth discussing because the eddy current effect in conductor disk is not considered.

Compared with the methods mentioned above, the magnetic equivalent circuit (MEC) method is simpler and more intuitive, and applicable to the analytic analysis of PMECC with a unique structure. Moreover, the researchers have made continuous efforts to enhance the precision of MEC models. In [23], the MEC model of PMECC is derived without considering the eddy current effects; in [24, 25], the free MEC models with the supplement of eddy current effects are established; in [26], the MEC model directly taking into account the eddy current effects is presented. In addition, MEC can be combined with other methods to solve some thorny issues of electromagnetic field [27].

As stated earlier, the magnetic transmission technology is moving into more areas, which have resulted in the change of the application environment, especially in the relatively limited workspace, such as automobile, train, wind power generation, and so on. The high torque density and easy operation are the driving force and development tendency of PMECCs. Based on the background herebefore, a novel flux-adjustable permanent magnet eddy current coupling (FA-PMECC) has been proposed in this article, and the corresponding performance parameters are the rate power: 2.2 kW, the speed range: 0~2760 r/min, and the maximum torque: 5 N·m.

In the improved topology, the permanent magnet rotors are divided into fixed flux and flux-adjustable rotors. By adjusting the relative position of these rotors, the output torque and speed are easily under control. Due to the complexity of the magnetic circuits in this system, a quasi-analytical model method for the magnetic field is proposed by combining the static 2-D FEA with non-linear MEC. In the proposed method, the saturation and permeability of iron, the skin effect and inductance effect of eddy current, and the temperature effect of copper sheet are all taken into account. The validity of the proposed model is proved by 3-D FEM, and then the performance comparisons with the existing structure and the parametric studies are performed.

2 | TOPOLOGY AND PRINCIPLE

2.1 | Topology

The structure of the novel FA-PMECC is shown in Figure 1. As shown, it mainly consists of permanent magnet rotor (PMR) and conductor rotor (CR). The CR is composed of two rotors, each of which contains copper sheet (CS) and corresponding back iron. The PMR is divided into flux-adjustable rotor (FAR) and fixed flux rotor (FFR), each of which adopts spoke-mounted PMs magnetized circumferentially, and they are inserted into the iron cores. Besides the air gap between CR and FAR, there will be tiny air gaps between FFR and FAR. By

![Figure 1](image_url)
adjusting the mechanical angle of FAR, the relative position of poles located in FAR and in FFR can be in control, so is the magnetic path and the air-gap flux density. As the PMR and the CR are connected to the prime-mover and the load, respectively, the torque and speed will be conveniently adjusted.

2.2 | Principle

According to the Lenz’s law and the Faraday’s law of induction, the eddy currents will be induced in the CS by the relative motion between CR and PMR, and interact with the magnetic field; then the Lorentz force is generated to produce the mechanical torque, which will make the CR rotate at the expected speed.

When the relative position of poles located in FAR and FFR is changed, the effective air-gap flux density changes accordingly. In order to elaborate the working principle, all the operation cases are divided into three states, as shown in Figure 2, which is disposed with the linear 2-D model. At state 1, as shown in Figure 2a, there is no mechanical angle between the FAR and FFR's N-poles, most of the flux lines emitted from the magnetic sources will pass through the CS, and the air-gap flux density will be maximal. At state 2, as shown in Figure 2b, there is a certain mechanical angle between the FAR and FFR's N-poles, some flux lines will link between the FFR and FAR’ N- and S-poles to form the loop, the others will pass through the CS, which will be affected by the relative position between the FFR and FAR' N-poles, so is the air-gap flux density. At state 3, as shown in Figure 2c, there is no mechanical angle between the FAR's N-poles and the FFR's S-poles. Because all most of flux lines will link between the FFR and FAR' N- and S-poles to form the loop, few flux lines pass through the CS, and the air-gap flux density will be minimum, nearly to be zero.

In practice, only the FAR needs to be controlled with a servo motor to adjust the relative position, but the FFR is fixed with the output shaft and the load. Moreover, compared with the magnetic structure in [19], because of the multiple rotors, the flux leakage will be greatly reduced, and the utilization of magnetic energy will be improved.

3 | QUASI-ANALYTICAL MODEL

The simple and reliable theoretical model performs an important role in the initial design of electromagnetic devices. Considering the proposed FA-PMECC has distinguishing features in structure, the classic analytical modelling approach based on VSM is not an ideal choice. Therefore, the quasi-analytical model is proposed to analyse the electromagnetic and torque characteristics of FA-PMECC. The block diagram depicted in Figure 3 shows the flow of the model. According to the foregoing analysis, different positions of FAR will generate different flux paths, three representative cases shown in Figure 2 are modelled and analysed. During the subsequent modelling phases, some assumptions are adopted as follows:

1. The air-gap between FFR and FAR is fairly small, thus the length can be ignored, then the reluctances in these regions are not taken into account in the analytical model.
2. The relative position between FFR and FAR' N-poles (denoted as \( l_0 \)) is used as the case division, and the static and dynamic field correspond, respectively, to \( s = 0 \) and \( s \neq 0 \), the essential difference is whether there are eddy currents in CS or not.
3. The magnetic saturation effects of iron materials are considered. Because the flux density is not uniform in the different iron layers, even the same iron layer but different areas, the permeability of different iron regions will be different in the analytical model.
4. The material properties of permanent magnets in FFR and FAR are uniform. However, in order to force the air-gap field to change from zero, the magnet height of FAR is set to twice the size of the FFR magnet.

In the modelling and analysis process, the model parameters involved in this article are shown in Table 1. To simplify the analysis, a pole pitch is considered, and the effective length and pole pitch are defined as

\[
L = r_2 - r_1 \quad \tau_p = \pi(r_2 + r_1)/2p
\]  

(Figure 2) Effects of FAR on the effective air-gap magnetic field: (a) Maximum case, (b) General case, (c) Minimum case

(Figure 3) Block diagram of the proposed quasi-analytical model
TABLE 1 Parameters of the studied model

| Symbol | Quantity       | Value  |
|--------|----------------|--------|
| $r_1$  | Inner radius   | 30 mm  |
| $r_2$  | Outer radius   | 50 mm  |
| $b_1$  | Magnet thickness | 5 mm   |
| $b_2$  | Magnet height of FFR | 10 mm |
| $b_3$  | Magnet height of FAR | 20 mm |
| $p$    | Number of pole pairs | 4     |
| $b_4$  | Thickness of CS | 1 mm   |
| $b_5$  | Air-gap length  | 1 mm   |
| $b_6$  | Thickness of iron | 4 mm |
| $b_7$  | Over length of conductor disk | 10 mm |
| $\Sigma$ | Conductivity of conductor | 58 MS/m |
| $B_r$  | Remanence of the PM | 1.27 T |

3.1 | Topology

1) Case 1: when the relative position of magnetic poles between FFR and FAR is set as shown in Figure 2a, that is $l r = 0$, the effective air-gap flux density will achieve the maximum $q_{\text{max}}$.

The magnetic equivalent circuit model for case 1 is shown in Figure 4. According to assumption 4, the magneto-motive force and corresponding intrinsic reluctance of PM in FFR and FAR can be calculated as follows:

$$F_{m1} = H_c b_1 \quad R_{m1} = b_m / \mu_0 \mu_p b_2 L$$  \hspace{1cm} (2)

$$F_{m2} = H_c b_1 \quad R_{m2} = b_m / \mu_0 \mu_p b_3 L$$  \hspace{1cm} (3)

The equivalent reluctance of the air-gap and CS regions of the main magnetic path can be calculated as follows:

$$R_g = (b_4 + b_5) / \mu_0 L \left[ (\tau_p - b_1) / 2 \right]$$  \hspace{1cm} (4)

The leakage flux path through the air-gap region can be regarded as the superposition of a semicircle with a line segment. Thus the leakage reluctance of this region can be calculated as follows:

$$R_{lg} = 1 / \int_0^{b_4 + b_5} \frac{\mu_0 L dl}{\pi l + b_m} = \pi / (L \mu_0 \ln[1 + \pi (b_4 + b_5) / b_1])$$  \hspace{1cm} (5)

Considering the magnetic lines through the iron core in the FFR side have different paths, the average reluctance of this section can be divided into two groups. One stems from PM in FFR, can be calculated by

$$R_{ip1} = 0.5(\tau_p - b_1) / \mu_0 S_1$$  \hspace{1cm} (6)

where $\mu_0$ is the permeability of iron core in the FFR side; $S_1$ is the area average, and expressed by

$$S_1 = [Lb_2 + 0.5L(\tau_p - b_1)] / 2$$  \hspace{1cm} (7)

The other one stems from PM in FAR, and can be calculated by

$$R_{ip2} = b_2 / \mu_0 S_2$$  \hspace{1cm} (8)

where $S_2$ is the average area, and expressed by

$$S_2 = 0.5L(\tau_p - b_1)$$  \hspace{1cm} (9)

Owing to the neglect of the air gap around the FAR, the reluctance of iron core in the FAR side can be expressed by

$$R_{ia} = 0.5(\tau_p - b_1) / \mu_0 S_3$$  \hspace{1cm} (10)

where $\mu_0$ is the permeability of iron core in FAR; $S_3$ is the area average in FFR, and expressed by

$$S_3 = [b_3 L + 0.5L(\tau_p - b_1)] / 2$$  \hspace{1cm} (11)

The average reluctance of iron core in the CS side can be divided into two parts, that is

$$R_{i2} = 2R_{i1} + R_{i2}$$  \hspace{1cm} (12)

where $R_{i2}$ is the reluctance of iron core facing the PM and $R_{i1}$ is the reluctance of iron core in both sides. They are expressed respectively by

$$R_{i1} = 0.5(\tau_p - b_1) / \mu_0 S_4$$  \hspace{1cm} (13)

$$R_{i2} = b_1 / \mu_0 S_5$$  \hspace{1cm} (14)

FIGURE 4 Magnetic equivalent circuit model for case 1
where $\mu_{i1}$ and $\mu_{i2}$ are respectively the permeabilities of different iron core regions; $S_0$ and $S_1$ are respectively the average areas of associated regions, and expressed by

$$ S_4 = \left[ b_0 L + 0.5 L (\tau_p - b_1) \right] / 2 \quad S_5 = L b_i $$

According to the Kirchhoff’s voltage law (KVL), the governing equation of the static field for case 1 can be determined by

$$ \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \times \begin{bmatrix} \varphi_{i1} \\ \varphi_{i2} \\ \varphi_{i3} \end{bmatrix} = \begin{bmatrix} F_{m1} \\ F_{m2} \end{bmatrix} $$

where $R_{11} = R_{m1} + R_{i0}$; $R_{12} = R_{i0}$; $R_{13} = 0$; $R_{22} = R_{m1}$; $R_{32} = 0$; $R_{21} = R_{m1} + 2 R_{i0} + 2 R_i + 2 R_{i0} + R_{i0}$; $R_{23} = 2 R_i + 2 R_{i0} + 2 R_{i0} + 2 R_i + 2 R_{i0} + 2 R_{i0} + 2 R_{i0} + 2 R_{i0}$;

By solving the governing equation, the flux of air-gap region can be expressed by

$$ \varphi_{\text{max}} = \varphi_{i1} + \varphi_{i3} $$

The flux density of iron core in the FFR side can be expressed by

$$ B_{ip} = \varphi_{i1}/S_1 + \varphi_{i3}/S_2 $$

The flux density of iron core in the FAR side can be expressed by

$$ B_{i1} = \varphi_{i3}/S_3 $$

The flux densities of different iron core regions in the CS side can be expressed by

$$ B_{i4} = \varphi_{\text{max}}/S_4 \quad B_{i2} = \varphi_{\text{max}}/S_5 $$

In order to take into account the effects of saturation characteristics of ferromagnetic material on permeability, in the light of the B-H curve of ferromagnetic material, an iterative method is employed to determine the values of $\mu_{ip}$, $\mu_{i0}$, $\mu_{i1}$, and $\mu_{i2}$ through

$$ \mu_i(k) = \left[ \mu_i(k) \right] \left[ \mu_i(k - 1) \right]^{1-d} $$

where $k$ is the iteration time, $d$ refers to the damping constant and sets to 0.1. The end of the iteration is provided via the following criterion

$$ |B_i(k) - B_i(k - 1)| \leq 0.01 $$

The approximate axial component of static air-gap flux density on the surface of CS can be estimated as follows

$$ B_{i4} = \begin{cases} \frac{\varphi_{\text{max}}}{L} \left[ (\tau_p - b_1) / 2 \right] & |x| \leq (\tau_p - b_1) / 2 \\ 0 & \text{elsewhere.} \end{cases} $$

2) Case 2: when the relative position of magnetic poles between PMR and FAR is set as shown in Figure 2c, the effective air-gap flux density achieves its minimum, nearly zero, that is

$$ B_{i2} \approx 0 $$

3) Case 3: when the relative position of the magnetic poles between PMR and FAR is set as shown in Figure 2b, the effective air-gap flux density can be expressed as a controlled variable, which is in correlation to the value of $L_r$. Owing to the complexity of the magnetic path, it is difficult to formulate the MEC model. Through analysis, the air-gap flux density can be expressed as a function of $L_r$ and $B_{i4}$

where $B_{i4}$ can be calculated by Equation (23), but the coefficient function $f(\Delta)$ is hard to construct based on the analytical modelling strategy. Thus, the polynomial fitting and 2-D FEA are introduced to estimate $f(\Delta)$ as follows

$$ f(\Delta) = -11.485 \Delta^3 + 27.3091 \Delta^2 - 22.0379 \Delta^3 + 7.7619 \Delta^2 - 2.5032 \Delta + 1 $$

As shown in Figure 5, Equation (26) can achieve higher accuracy in forecasting the static air-gap flux density. Therefore, the static air-gap flux density in any case is calculated by

$$ B_i = \begin{cases} B_{i\text{max}} = f(\Delta) \varphi_{\text{max}} / L \left[ (\tau_p - b_1) / 2 \right] & |x| \leq (\tau_p - b_1) / 2 \\ 0 & \text{elsewhere.} \end{cases} $$

3.2 Dynamic field modelling

Figure 6 depicts the different fluxes under the dynamic states. According to the working principle of such devices, the relative speed between PMR and CR will develop the eddy current in the CS, the radial component of which can be given as
Ampere’s law, the relation equation between magnetic field and eddy current is established as follows:

\[ J = \sigma_1 E = \sigma_1 \nu \times B = R_{mc}\sigma_1 \omega B_i \]  

(28)

where \( \sigma_1, \omega_1, \) and \( \nu \) are the conductivity of the CS, the relative angular velocity, and the relative linear velocity, respectively. The total air-gap flux density along the axial direction can be expressed as

\[ B_t = B_s + B_i \]  

(29)

where \( B_i \) denotes the reaction flux density in the air gap, which is produced by the eddy current in the CS. Based on the Ampere’s law, the relation equation between magnetic field and eddy current is established as follows:

\[ \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_{x_1}^{x_2} J_{yi} J \, dy \, dx \]  

(30)

Therefore, the mathematics model of \( B_i \) is converted to resolve the first-order differential equation, and the general solution to Equation (33) is given as follows:

\[ B_i = \begin{cases} B_{il} = k_1 e^{mx} & -\tau_p/2 \leq x < (b_1 - \tau_p)/2 \\ B_{im} = k_2 e^{mx} - B_{max} & (b_1 - \tau_p)/2 \leq x < (\tau_p - b_1)/2 \\ B_{ir} = k_3 e^{mx} & (\tau_p - b_1)/2 \leq x < \tau_p/2. \end{cases} \]  

(34)

where the unknown constants \( k_1, k_2, \) and \( k_3 \) can be determined by the boundary conditions, which are given in Appendix. Thus, \( k_1, k_2, \) and \( k_3 \) can be calculated by

\[ \begin{align*} 
  k_1 &= B_{max} \left[ e^{-mx_0} - e^{-(\tau_p - b_1)/2} \right] \\
  k_2 &= B_{max} e^{-mx_0} \\
  k_3 &= B_{max} \left[ e^{-mx_0} - e^{b_1 - \tau_p}/2 \right] 
\]  

(35)

where

\[ x_0 = -1/m \ln \left[ \cosh (mb_1/2)/\cosh (m\tau_p/2) \right] \]  

(36)

Finally, the total air-gap flux density \( B_s \) is further expressed as

where \( \delta \) denotes the skin depth, which is not a constant value, and related to the slip, and the thickness and material property of the CS. Therefore, an improved skin depth is employed by [28].

\[ \delta_e = |1 - \exp(-b_1/\delta)| \delta^2/b_1 \]  

(32)

The differential of Equation (30) with respect to \( x \) can be obtained as follows:

\[ dB_s/dx - mB_s = mB_i \quad \text{with} \quad m = \mu_0 \sigma_1 \omega b_{ef}/2(b_1 + b_5) \]  

(33)

where the right-hand side denotes the total currents enclosed by the closed path \( C \), and it is worth noting that the magnetomotive force which drops across the iron is neglected.
\[
B_i = \begin{cases} 
B_{\text{max}} \left[ e^{-\tau h} - e^{-\tau h/2} \right] e^{\tau x} & \frac{-\tau_p}{2} \leq x < \frac{b_1 - \tau_p}{2} \\
B_{\text{max}} e^{-\tau h} e^{\tau x} & \frac{b_1 - \tau_p}{2} \leq x \leq \frac{\tau_p - b_1}{2} \\
B_{\text{max}} \left[ e^{-\tau h} - e^{-\tau h/2} \right] e^{\tau x} & \frac{\tau_p - b_1}{2} < x \leq \frac{\tau_p}{2}.
\end{cases}
\]

(37)

3.3 Accounting for inductance

Research has shown that the inductance changes a little with the working state (slip speed) [29]. It can be considered that the value of the inductance mainly depends on its own structural parameters. To simplify the analysis, a single eddy current ring is regarded as one-turn rectangular planer coil depicted in Figure 6, the accurate expression for the total inductance can be given by [30]

\[
L_T = L_1 + L_2 + L_3 + L_4 - 2(M_{1,3} + M_{2,4})
\]

(38)

where \(L_1, L_2, L_3,\) and \(L_4\) are the self-inductances of segments 1, 2, 3, and 4, respectively; \(M_{1,3}\) is the mutual inductance between segments 1 and 3; \(M_{2,4}\) is the mutual inductance between segments 2 and 4.

The self-inductances can be calculated by

\[
L_i = 2l_i \left[ \ln \left( \frac{2l_i}{b_{ef} + b_w} + 0.5 + \frac{b_{ef} + b_w}{3l_i} \right) \right]
\]

(39)

where \(b_w\) is the equivalent track width, which are expressed by

\[
b_w = \frac{(\tau_p - b_1)}{2}
\]

(40)

and \(l_i\) is the width of segment \(i\). According to the distribution of eddy current in CS [11], \(l_i\) can be estimated as follows:

\[
l_i = \begin{cases} 
l_1 = l_3 = \tau_p \\
l_2 = l_4 = \min\{L + L/2, L + b_l\}
\end{cases}
\]

(41)

The mutual inductance \(M = \{M_{1,3}, M_{2,4}\}\) can be calculated by

\[
M = 2l_i Q_i
\]

(42)

where \(Q_i (i = 1, 2)\) is the mutual inductance parameter, and can be calculated by [31].

\[
Q_i = \ln \left[ \frac{l_i}{l_{\text{md},i}} + \left( 1 + \frac{l_i^2}{l_{\text{md},i}^2} \right)^{0.5} \right] - \left( 1 + \frac{l_{\text{md},i}^2}{l_i^2} \right)^{0.5} + \frac{l_{\text{md},i}}{l_i}
\]

(43)

where \(l_{\text{md},i} (i = 1, 2)\) is the geometric mean distance between segment 1 (or 2) and segment 3 (or 4). The approximate value of \(l_{\text{md},i}\) can be calculated by

\[
l_{\text{md},i} \approx \exp \left( \ln d_i - \frac{b_{ef}^2}{12d_i^2} - \frac{b_w^4}{60d_i^4} - \frac{b_w^6}{168d_i^6} \right)
\]

(44)

where \(d_i (i = 1, 2)\) is the centre distance between conductor segment \(i\) and \(i + 2\).

The equivalent impedance of CS is expressed by

\[
Z_E = R_E + jX_E
\]

(45)

where impedance and inductive reactance are calculated by

\[
R_E = \sum_{i=1}^{4} \frac{l_i}{2\alpha_i b_{ef} b_{ef}}
\]

(46)

\[
X_E = \pi p\omega L_T / 15
\]

(47)

The equivalent conductivity of CS and the power factor of FA-PMECC can be respectively expressed as

\[
\sigma_{eq} = \left[ \frac{1 - \sigma_c^2 + (2b_{ef} b_{ef} \pi p\omega L_T)^2}{\left( 15 \sum_{i=1}^{4} l_i \right)^2} \right]^{-0.5}
\]

(48)

\[
\cos \phi = R_E / \sqrt{R_E^2 + X_E^2}
\]

(49)

3.4 Developed torque

Ideally, the mechanical and the stray losses of such devices can be ignored. Thus, the energy equation for the system can be written as

\[
\begin{cases} 
T_{in} \omega_{in} = T_{out} \omega_{out} + Q_{\text{eddy}} \\
T_{in} = T_{out}
\end{cases}
\]

(50)

where \(T_{in}, T_{out}, \omega_{in}, \omega_{out},\) and \(Q_{\text{eddy}}\) denote the input torque, the output torque, the input speed, the output speed, and the eddy current loss in CS, respectively. Thus the developed torque of the coupling can be determined as
\[ T = \frac{Q_{\text{eddy}} \cos \phi}{\omega} \]  

(51)

Considering the periodicity of the eddy current distribution, the total eddy current loss of double-sided CS can be calculated as

\[ Q_{\text{eddy}} = 4pL/\sigma_{\text{eq}} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{r}{2}}^{\frac{r}{2}} |f|^2 \partial x \partial y \]  

(52)

Substituting Equations (28) and (37) into Equation (51) and simplifying yield

\[ T = \frac{2\sigma_{\text{eq}} vb_{\text{eff}} R_{\text{ct}} B_{\text{mec}}^2 L\cos \phi}{\mu} \left[ 2\cosh^2 \left( mb_1/2 \right) \right. 
\times \tanh \left( m\tau_p/2 \right) - \sinh (mb_1) \]  

(53)

However, it is not advisable to use Equation (53) immediately to evaluate the torque performance. The fundamental reason lies in only the radial flow of the eddy current in the CS which is considered in the above model, having lost sight of the fact that the eddy current is a group of closed loops, as shown in Figure 6. In order to resolve this problem, the so-called Russell’s 3-D correction factor is used, which can be expressed by [27]

\[ k_r = 1 - \tanh \gamma_1/\gamma_1 (1 + \tanh \gamma_1 \cdot \tanh \gamma_2) \]  

(54)

where

\[ \gamma_1 = \frac{\pi L}{\tau_p} \gamma_2 = \frac{\pi b_7}{\tau_p} \]  

(55)

As a result, the relatively accurate expression for the torque of the proposed FA-PMECC is

\[ T_{\text{M}} = k_r T \]  

(56)

3.5 | Effect of temperature on CS

According to Equation (53), the conductivity \( \sigma \) is an important parameter, and is sensitive to the working temperature \( t \). To improve the accuracy and breadth of the model, the conductivity of CS is considered as a function of the temperature by

\[ \sigma_e(t) = \sigma_0 (1 + at) \]  

(57)

where \( \sigma_0 \) denotes the conductivity at 20°C, \( t_0 = 20 \), \( a = 0.004 \) for copper and aluminium.

The eddy current Joule losses of CS are regarded as the only heat source. Based on the heat transfer theory and the available literature [32], the temperature of the CS \( t \) is calculated by

\[ t = t_0 + \frac{Q_{\text{eddy}}}{h_0 S_6 + h_5 S_7} \]  

(58)

where \( S_6 \) and \( S_7 \) denote the areas of the sides and the front, and expressed by

\[ \begin{cases} 
S_6 = 2\pi (r_2 + b_7)(b_4 + b_6) \\
S_7 = \pi (r_2 + b_7)^2 
\end{cases} \]  

(59)

and the convective heat transfer coefficients \( h_0 \) and \( h_5 \) are computed by [33].

\[ \begin{align*}
  h_0 &= \frac{0.0844 \left[ \frac{\rho c_p \omega_{\text{out}} (r_2 + b_7)^2}{\kappa \omega} \right]^{0.35}}{2(r_2 + b_7)} \\
  h_5 &= \frac{0.01846 \left[ \frac{\rho c_p \omega_{\text{out}} (r_2 + b_7)^2}{\kappa \omega} \right]^{0.6}}{(r_2 + b_7)}
\end{align*} \]  

(60, 61)

where \( \rho \), \( \kappa \), \( \mu \) and \( c_p \) denote the thermal properties of air: the density \( \rho = 1.177 \) kg m\(^{-3}\), the thermal conductivity \( \kappa = 0.026 \) W m\(^{-1}\) K\(^{-1}\), the dynamic viscosity \( \mu = 1.845 \times 10^{-5} \) kg m\(^{-1}\) s\(^{-1}\), the specific heat capacity \( c_p = 1006 \) J \cdot kg\(^{-1}\) K\(^{-1}\).

4 | RESULTS AND DISCUSSION

To verify the advantage of the proposed topology and the validity of the analytical model, the 3-D FEA and prototype experiment are used to evaluate the calculation results. The design parameters of FA-PMECC are shown in Table 1. For ease of comparison studies, the primary parameter values are derived from the similar research in [19]. The full 3-D model is meshed into 1,000,285 triangular elements, and the corresponding mesh of overall finite element model is shown in Figure 7. In addition, Figure 8 shows the 3D-FEA results for different \( \Delta \) values at the slip speed of 400 r/min, including

**FIGURE 7** The mesh of 3-D finite element model
the flux density and the eddy current distributions on the CS surface. The experiment platform system is designed and established as shown in Figure 9. Along the pedestal mounting plate, the experimental setups include three-phase asynchronous AC motor (2.2 kW; 2820 r/min); eddy-current coupling; torque/speed metre; load (DC motor combined DC governor),
and the AC motor is controlled by the frequency converter. Still it must be noted that the prototype experiment is only used to study the accuracy of the torque model, while the 3-D FEA is the preferred tool to verify the model of the air-gap flux density.

In the simulation and test, some of the instructions are as follows: (1) In the 3-D FEA simulation, the temperature of CS is set according to the analytical predictions; (2) In the prototype experiment, the FAR is offline to adjust; (3) All results are obtained under stable conditions, regardless of transient values.

4.1 | Flux density distribution

Figure 10 shows the axial components of the total air-gap flux density of FA-PMECC for the slip $s = 0.1$. As the increase in the value of $\Delta$ from 0 to 0.4, the amplitude of the air-gap flux density decreases from 0.6 to 0.3 T, which indicates the feasibility of the regulation of the air-gap flux density by the proposed approach. Moreover, there are good agreements between the model and 3-D FEM results. When the value of $\Delta$ is fixed, the total air-gap flux density of FA-PMECC for different values of slip is also investigated in Figure 11. As shown in Figure 10, with the increase of the value of slip from 0.1 to 0.4, the distortions of the total air-gap flux density curves are getting worse, which is caused by the increasingly serious eddy current effects. Figure 12 shows the total air-gap flux density of FA-PMECC for the low slip values. It is clear that the amplitude of the total air-gap flux density is not changing much, and the distortion is not obvious compared with the static field ($s = 0$).

4.2 | Torque characteristics

Figures 13 and 14, respectively, show the comparison of torque-slip speed curves between FEM and analytical results.
for the global slip case and low slip case. As different governing states, the values for $\Delta = 0, 0.1, 0.2,$ and 0.3 are considered. As indicated in Figure 12, whatever the value of $\Delta$ is, the torque values will increase first and then stabilize, and the stability locations of the curves are relatively similar, which are about 1300 r/min; with the increase in the value of $\Delta$, the torque will rapidly decrease for the same slip speed. Moreover, the deviation between FEM and analytical results is very small, it's feasible and effective to study the performance of devices by using the proposed analytical model.

For such devices, the customers are more concerned about the torque in low slip areas. As can be seen form Figure 14, the torque-slip speed curves are approximately a series of lines within 300 r/min, of which the slope is decreasing with the increase in the value of $\Delta$, therefore, the device can be controlled conveniently and steadily. In these working areas, the prediction results of analytical model are highly close to the FEM results.

Figures 15 and 16 show the comparison between the analytically predicted torque–slip speed characteristics and the measured data under the overall slip and low slip states. As shown in Figures 15 and 16, two values for govern state were considered ($\Delta = 0$ and $\Delta = 0.3$). It can be noticed that the experimental measurements are in good agreement with the one obtained with the proposed torque formula, especially at the low slip state. Moreover, the deviations between the analytical model and the real values are not greater than 10%.

Although the proposed model is developed taking into account various factors, individual factors remain out of reach, such as the inconsistencies in the radial distribution of

![Figure 13](image13.png)  
**Figure 13** Comparison of torque-slip speed curves between FEM and model results for the global slip case

![Figure 14](image14.png)  
**Figure 14** Comparison of torque-slip speed curves between FEM and model results for the low slip case

![Figure 15](image15.png)  
**Figure 15** Comparison of torque-slip speed curves between experiment and model results under the overall slip state

![Figure 16](image16.png)  
**Figure 16** Comparison of torque-slip speed curves between experiment and model results under the low slip state
temperature. As shown in Figures 13–16, with the increase of slip speed, the temperature of CS rises gradually, then the deviations gradually increase. On the other hand, the test system, the physical properties of materials, and the sensors’ precision will lead to bias.

### 4.3 Advantages analysis of model

In addition to the general advantages of the analytical model, compared with the existing model, more factors are taken into account in the proposed model. To highlight its advantages, the comparison results are given in Figure 17. To be fair, the 3-D FEA results are set as a benchmark, the deviation rates obtained from different models are given in Figure 17. The model without Equation (31) means the neglect of skin effect. We can infer that the skin effect has great effect on the torque model in the case of high slip speed, however, in the case of $s < 0.1$, the skin effect can be ignored. The model without Equation (48) means the neglect of the inductance effect. It can be discovered that the inductance effect is relatively obvious in the case of low slip speed, while it can be negligible in the case of high slip speed. The model without Equation (57) means the neglect of the temperature effect. Due to the small power of the case, in the case of low slip speed, the temperature has little effect on the accuracy of the model, and with the increase in slip speed, the temperature factor has to be taken into account in the analytical model. It can be seen that the proposed comprehensive analytical model has the most accurate prediction because many factors are involved.

### 4.4 Improvement of performances

Compared with the topology presented in the literature [19], there are some performance enhancements besides the speed regulation. To avoid overheating, the average eddy current density is generally limited to 50 $\text{A/mm}^2$, which corresponds to the maximum torque. Thus, the maximum torque, torque, and the utilization of permanent magnets are used to evaluate quantitatively the performance enhancements. The average eddy current density can be calculated by

$$J_{av} = \frac{\int_0^{\frac{b_d}{2}} \int_{r_p/2}^{r_p} j(x) \, dx \, dz}{b_d \, r_p}$$

and the utilization of permanent magnets is defined as

$$U_{PM} = \frac{F_{sd}}{M_{PM}}$$

where $M_{PM}$ denotes the total mass of PM.

Figure 18 shows the comparison of average eddy current density curves with different operating conditions that $\Delta = 0$, 0.1, 0.2, and 0.3. As indicated in Figure 18, with the increase of slip speed, the average eddy current density increases first and then decreases, which is similar to the torque-slip speed curve; with the increase in the value of $\Delta$, the average eddy current density decreases for the same slip speed, however, the slip speed corresponding to the maximum torque increases. The maximum torque of the original topology in [19] is about 2 N·m, while it is about 5 N·m for FA-PMECC for $\Delta = 0$, therefore, it makes significant improvement on the maximum torque.

Figure 19 shows the comparison of torque ($F_{sd}$) and the utilization of permanent magnets ($U_{PM}$) between the proposed FA-PMECC and the original topology for $\Delta = 0$. As indicated in Figure 19, at the low slip speed, the growth rate of torque is more than 300%, of which the maximum torque reach up to 500%; the performance indicator $U_{PM}$ shows different trends that the enhancement ratio is about 30% when the slip speed is

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**Figure 17** Comparison of the accuracy of different analytical models

**Figure 18** Comparison of average eddy current density curves with different operating conditions
less than 200 r/min, but the reduction ratio is less than 20% when the slip speed is more than 200 r/min. One of the main reasons is that part of the magnetic energy is used to regulate the magnetic circuit, instead of generating torque.

4.5 Influence of geometry parameters

As the major structural parameters, the influences of the fixed air-gap length ($b_0$), the number of pole-pairs ($p$), the thickness of CS ($h_4$), and the material of CS ($\sigma_c$) on the torque performance in case 1 will be further explored. During the analysis, the analytical results with the parameters in Table 1 are used as the basis for comparison employing the proposed model. Moreover, only the low slip values ($s < 0.1$) are heavily considered, which correspond to the high-efficiency working range areas.

Figure 20 shows the comparison of torque-slip speed curves with different values of air-gap length. As indicated in Figure 20, with the increase of air-gap length, the torque decreases rapidly, and the decrement increases from 22% to 85%. Therefore, in the design of the time, the smaller the air-gap length is at the available process level, the better the devices are.

Figure 21 shows the comparison of torque-slip speed curves with different pole-pairs. As indicated in Figure 21, with the increase in the numbers of pole-pairs, the torque presents different change, however, it goes up and stabilizes on the whole. The reduction ratio of the torque is over 20% when $p = 2$, while the increasing rates are under 10% when $p = 5$ and $p = 6$. Actually, with the increase in the number of pole-pairs, the area of iron yoke facing every pole is decreasing, and the ferromagnetic material becomes saturated.

Figure 22 shows the comparison of torque-slip speed curves with different values of thickness of CS. As indicated in Figure 22, with the increase in the thickness of CS, the torque increases firstly, and then decreases, and we can find that the optimal value of thickness of CS is $h_4 = 2$ mm. The root cause lies in the increase in reluctance will exceed the increase of effective eddy current with the increase of thickness of CS.

Figure 23 shows the comparison of torque-slip speed curves with different conductor materials. Here, copper, aluminum and brass are used and studied, whose conductivities are, respectively, 58, 38, and 14.5 MS/m. As indicated in Figure 23, when working in the low slip speed, the torque performance of the device used copper is the best, whose slope is maximum; when working in the whole slip area, the torque-slip speed curve of copper material reaches the inflection point
Further analysis reveals that the performance has been greatly improved compared with the existing structure. With the same structural parameters, the maximum torque is increased to 2.5 times for $\Delta = 0$ when the average eddy current density is less than $50 \text{ A/mm}^2$, the growth rate of torque is more than 300% at the low slip speed, and the utilization ratio of permanent magnets increases by 30% when the slip speed is less than 200 r/min.

**CONFLICT OF INTEREST**

The authors declare that they have no conflict of interest.

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Appendices

To determine the coefficient values of Equation (34), some interface conditions are necessary. According to the continuity of magnetic flux density distribution, Equation (34) will be satisfied as follows:

\[
B_{il}\left(-\frac{\tau_p - b_1}{2}\right) = B_{im}\left(-\frac{\tau_p - h_1}{2}\right) \quad (A.1)
\]

\[
B_{ir}\left(\frac{\tau_p - b_1}{2}\right) = B_{im}\left(\frac{\tau_p - h_1}{2}\right) \quad (A.2)
\]

In addition, in the interval \([-\tau_p/2, x_0]\) and \([x_0, \tau_p/2]\) are equal. Another interface condition can be given by

\[
B_{im}(x_0) = 0 \quad (A.3)
\]

\[
\int_{-\tau_p/2}^{x_0} J_0 dydx + \int_{0}^{\tau_p/2} J_{\tau_p/2} dydx = \int_{x_0}^{\tau_p/2} J_0 dydx \quad (A.4)
\]