Production and Detection of
Cosmic Gravitational Wave Background
in String Cosmology

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String cosmology models predict a cosmic background of gravitational waves produced during a period of dilaton-driven inflation. I describe the background, present astrophysical and cosmological bounds on it, and discuss in some detail how it may be possible to detect it with large operating and planned gravitational wave detectors. The possible use of smaller detectors is outlined.

I. INTRODUCTION

A robust prediction of models of string cosmology which realize the pre-big-bang scenario is that our present-day universe contains a cosmic gravitational wave background, with a spectrum which is quite different than that predicted by other early-universe cosmological models. In the pre-big-bang scenario the evolution of the universe starts from a state of very small curvature and coupling and then undergoes a long phase of dilaton-driven kinetic inflation reaching nearly Planckian energy densities, and at some later time joins smoothly standard radiation dominated cosmological evolution, thus giving rise to a singularity free inflationary cosmology.

In this paper I describe the cosmic gravitational wave background predicted by models of string cosmology and present numerical estimates for spectral parameters, I review astrophysical and cosmological bounds on the spectrum’s shape and strength, and show that
currently operating and planned large gravitational wave detectors could further constrain
the spectrum and perhaps even detect it. I discuss detection strategies and compare the ef-
ficiency of different types of detectors. Finally, I outline the possible use of small resonators
and the use of the “memory effect” to detect the background or constrain its parameters.

Because the gravitational interaction is so weak, a background of gravitational radiation
decouples from matter in the universe at very early times and carries with it information
on the state of the universe when energy densities and temperatures were extreme. The
weakness of the gravitational interaction makes a detection of such a background very hard,
and necessitates a strong signal. String cosmology provides perhaps the strongest source
possible: the whole universe, accelerated to nearly Planckian energy densities. Although in
this paper I use particular string cosmology models, the main conclusions will remain valid
for all models in which the universe spends a finite time at near Planckian energy densities.

A discovery of any primordial gravitational wave background, and in particular, the one
predicted by string cosmology, could provide unrivaled exciting information on the very early
universe. Such a discovery will confirm the basic principles used to theoretically derive the
background. For example, it will confirm the validity of quantum mechanics as we know it
all the way up to the Planck scale.

II. COSMIC GRAVITATIONAL WAVE BACKGROUND IN STRING
COSMOLOGY

In models of string cosmology [3] (see also [1,2]), the universe passes through two early
inflationary stages. The first of these is called the “dilaton-driven” period and the second
is the “string” phase. Each of these stages produces stochastic gravitational radiation by
the standard mechanism of amplification of quantum fluctuations [11]. Deviations from
homogeneity and isotropy of the metric field are generated by quantum fluctuations around
the homogeneous and isotropic background, and then amplified by the accelerated expansion
of the universe. The transverse and traceless part of these fluctuations are the gravitons. In
practice, we compute graviton production by solving linearized perturbation equations with vacuum fluctuations boundary conditions. The production strength of gravitons depends on the curvature and coupling. Since at the end of the accelerated expansion phase curvatures reach the string curvature, and the coupling reaches approximately the present coupling, graviton production is expected to be at the strongest possible level.

In order to describe the background of gravitational radiation, it is conventional to use a spectral function $\Omega_{GW}(f) = \frac{1}{\rho_{\text{critical}}} \frac{d\rho_{GW}}{d\ln f}$, where $d\rho_{GW}$ is today’s energy density in stochastic gravitational waves (GW) in the frequency range $d\ln f$, and $\rho_{\text{critical}}$ is the critical energy-density required to just close the universe, $\rho_{\text{critical}} = \frac{3c^2H_0^2}{8\pi G} \approx 1.6 \times 10^{-8}h_{100}^2$ ergs/cm$^3$. The Hubble expansion rate $H_0$ is the rate at which our universe is currently expanding, $H_0 = h_{100}100 \frac{\text{Km}}{\text{sec-Mpc}} = 3.2 \times 10^{-18}h_{100}Hz$. $h_{100}$ is believed to lie in the range $0.5 < h_{100} < 0.8$. The spectral function is related to the dimensionless strain $h$, $\Omega_{GW}(f) \approx 10^{36}h_{100}^{-2}(f/\text{Hz})^2h(f)^2$ and to the strain in units $1/\sqrt{Hz}$, $\sqrt{S_h}(f)$,

$$\Omega_{GW}(f) = 1.25 \times 10^{36}h_{100}^{-2}(f/\text{Hz})^3S_h(f).$$  

The spectrum of gravitational radiation produced during the dilaton-driven (and string) phase was estimated in [3] (see also [12–15]). It is approximately given by

$$\Omega_{GW}(f) = z_{eq}^{-1}g_s^2 \left( \frac{f}{f_S} \right)^3 \left[ 1 + z_S^{-3} \left( \frac{g_1}{g_S} \right)^2 \right], f < f_S,$$  

where some logarithmic correction factors were dropped. The coupling $g_1$ is today’s coupling, assumed to be constant from the end of the string phase, $g_S$ is the coupling at the end of the dilaton-driven phase, and $f_S$ is the frequency marking the end of the dilaton-driven phase. The frequency $f_1 = f_S z_S$ is the frequency at the end of the string phase, where $z_S$ is the total red-shift during the string phase and $z_{eq} \sim 10^4$ is the red-shift from matter-radiation equality until the present. We will present a more quantitative and detailed estimate of spectral parameters later.

The spectrum can be expressed in a more symmetric form [13],

$$\Omega_{GW}(f) = z_{eq}^{-1}g_1^2 \left( \frac{f}{f_1} \right)^3 \left[ z_S^3(g_S/g_1)^2 + z_S^{-3}(g_S/g_1)^{-2} \right].$$  

(3)
Note that the spectrum is invariant under the exchange $z_s^3(g_s/g_1)^2 \leftrightarrow z_s^{-3}(g_s/g_1)^{-2}$ and that this implies a lower bound on the spectrum, $\Omega_{GW}(f) \geq 2z_{eq}^{-1}g_1^2 \left(\frac{f}{f_1}\right)^3$. The lower bound is obtained for the “minimal spectrum” with $z_s = 1$ and $g_s/g_1 = 1$ describing a cosmology with almost no intermediate string phase.

In the simplest model, which we will use to estimate the spectrum and prospects for its detection, the spectrum depends upon four parameters. The first pair of parameters are the maximal frequency $f_1$ above which gravitational radiation is not produced and $g_1$, the coupling at the end of the string phase. The second pair of these are $z_S$ and $g_S$. The second pair of parameters can be traded for the frequency $f_S = f_1/z_S$ and the fractional energy density $\Omega_{GW}^S = \Omega_{GW}(f_S)$ produced at the end of the dilaton-driven phase. At the moment, we cannot compute $g_S$ and $z_S$ from first principles, because they involve knowledge of the evolution during the high curvature string phase. We do, however, expect $z_S$ to be quite large. Recall that $z_S$ is the total red-shift during the string phase, and that during this phase the curvature and expansion rate are approximately string scale, therefore, $z_S$ grows roughly exponentially with the duration (in string times) of this phase. Some particular exit models \[16\] suggest that $z_S$ could indeed be quite large. I cannot estimate, at the moment, a likely range for the ratio $g_1/g_S$ except for the reasonable assumption $g_1/g_S > 1$. We prefer to concentrate on the features of the spectrum that can be computed theoretically as cleanly as possible. Since at the moment the best understood part of the spectrum is the part produced during the dilaton-driven phase, we concentrate our attention on parameters associated with this phase.

A useful approximate form for the spectrum in the range $z_S > 1$ and $g_1/g_S \gtrsim 1$ is the following \[17\]

$$\Omega_{GW}(f) = \begin{cases} 
\Omega_{GW}^S(f/f_S)^3 & f < f_S \\
\Omega_{GW}^S(f/f_S)^3 & f_S < f < f_1 \\
0 & f > f_1.
\end{cases}$$ (4)
where $\beta = \frac{\log \Omega_{GW}(f_1)/\Omega^S_{GW}}{\log [f_1/f_S]}$ is the logarithmic slope of the spectrum produced in the string phase (see also other models \[18\]). The corresponding spectral density $S_h$, in units of $Hz^{-1}$, is given by

$$S_h(f) = \begin{cases} 
S^S_h & f < f_S \\
S^S_h (f/f_S)^{\beta - 3} & f_S < f < f_1 \\
0 & f > f_1.
\end{cases} \quad (5)$$

Note that $S_h$ is constant during the dilaton-driven phase. The form (5) is particularly useful in comparing sensitivities of different detectors.

If we assume that there is no late entropy production and make reasonable choices for the number of effective degrees of freedom, then two of the four parameters may be determined in terms of the Hubble parameter $H_s$ at the onset of radiation domination immediately following the string phase.

We turn now to obtain numerical estimates of $f_S$ and $\Omega^S_{GW}$. Our assumptions are somewhat different than those used in \[19\], but the resulting range is similar. To obtain estimates for the spectral parameters we must assume some late time background cosmology. Here we assume standard cosmology in a flat universe without a cosmological constant. A different choice of late time background cosmologies will lead to calculable changes in these estimates. To obtain numerical estimates for the spectral parameters it is useful to consider the “minimal spectrum”, in which the dilaton-driven inflationary phase connects almost immediately to standard radiation-dominated evolution. For the minimal spectrum $z S = 1$, $g_s/g_1 = 1$, $f_1 = f_S$.

We start our discussion with the frequency axis. For the minimal spectrum, the frequency of the end-point $f_1$ today is given by the frequency which just reenters the horizon at the beginning of the radiation dominated phase, red-shifted to its present value,

$$f_1 = f_s / z_s = \frac{H_s}{2\pi} \frac{1}{z_s} \quad (6)$$
where $H_*$ is the Hubble parameter at the end of the string phase and $z_*$ is the red-shift since then, given as the ratio of the scale factors today and then $z_* = a_0/a_*$. We use entropy considerations to evaluate $z_*$. Let us first assume that entropy is approximately conserved during the evolution from the end of the string phase until today. This must be an approximation. Entropy cannot be absolutely conserved because some non-adiabatic processes, such as the relaxation of the dilaton and other moduli towards the minimum of their potential, are expected. If entropy is approximately conserved, we obtain $g_s(t_0)T_0^3a_0^3 = g_s(t_*)T_*^3a_*^3$ from which we may calculate

$$z_* = \left( \frac{T_*}{T_0} \left( \frac{g_s(t_0)}{g_s(t_*)} \right)^{1/3} \right) ,$$

where $g_s = \sum_{i_{bosons}} g_i(T_i/T)^3 + \frac{7}{8} \sum_{i_{fermions}} g_i(T_i/T)^3$ measures the effective number of degrees of freedom and should not be confused with the string coupling parameter at the beginning of the string phase $g_S$. In the previous equations, and in the rest of the paper, a subscript 0 refers to the present values of various quantities. Since $g_s(t_0) = 3.91$ and $T_0 = 2.74K$ are known (see, for example, [20]), $z_*$ is given by

$$z_* = \frac{T_*}{2.74K} \left( \frac{g_s(t_*)}{3.91} \right)^{1/3} .$$

Assuming local thermal equilibrium and radiation domination at $t_*$ we may relate $T_*$ to $H_*$ in a standard way

$$T_* = H_*^{1/2}m_{pl}^{1/2} \left[ \frac{90}{8\pi^3 g_\rho} \right]^{1/4} ,$$

where $G_N \equiv 1/m_{pl}^2$ and $g_\rho = \sum_{i_{bosons}} g_i(T_i/T)^4 + \frac{7}{8} \sum_{i_{fermions}} g_i(T_i/T)^4$. Substituting $T_*$ from eq.(8) into eq.(7)

$$z_* = \frac{H_*^{1/2} m_{pl}^{1/2}}{2.74K} g_\rho^{-1/4} g_s(t_*)^{1/3} \left[ \frac{90}{8\pi^3} \right]^{1/4} 3.91^{-1/3} ,$$

and substituting $z_*$ from eq.(9) into eq.(8) we obtain

$$f_1 = 1.2 \times 10^{11} Hz \times \left( \frac{H_*}{m_{pl}} \right)^{1/2} \left( g_\rho^{1/4} g_s(t_*)^{-1/3} \right) .$$

Since we expect $H_*$ to be less than $m_{pl}$ and of the order of the string scale, $M_s \sim 5 \times 10^{17} GeV$ it is convenient to express $H_*$ as $\left( \frac{H_*}{m_{pl}} \right)^{1/2} = 0.20 \left( \frac{H_*}{5 \times 10^{17} GeV} \right)^{1/2}$. In addition, since both $g_\rho$
and $g_s(t_\ast)$ are expected to be approximately equal and much larger than the standard model values, $g_s \sim 100$, we assume for simplicity that they are equal, denote their common value as $g_\ast$ and parametrize them as $g_\ast^{1/4}g_s(t_\ast)^{-1/3} = 0.56 \ [g_s/1000]^{-1/12}$. Putting everything together we obtain

$$f_1 = 1.3 \times 10^{10}Hz \times \left[ \left( \frac{H_\ast}{5 \times 10^{17}GeV} \right)^{1/2} \left( \frac{g_\ast}{1000} \right)^{-1/12} \right]. \quad (11)$$

Equation (11) is the final result for the end-point frequency of the minimal spectrum assuming no entropy production and initial radiation domination just after the string phase (but not necessarily afterwards).

If the transition from the dilaton-driven phase to radiation domination is not immediate, as expected, and we neglect the effects of the backreaction of the produced particles on the background cosmology, then $f_S$ is simply red-shifted by the total amount of red-shift accumulated during the string phase, $f_S = f_1/z_S$.

If entropy was not even approximately conserved since the end of the stringy phase, then according to the second law of thermodynamics, entropy had to increase. This means that the value of $f_1$ in eq.(11) is an upper bound on $f_1$, as shown in [19]. The spirit of our model, in general, favors approximate entropy conservation.

The analysis of the amplitude axis depends more strongly on the details of the model, and therefore less accurate. Our starting point is the estimate of GW energy density [4,13–15]

$$\frac{d\rho_{GW}}{d^3 x d \ln k} = C k_1^4(g_s/g_1)^2(k/k_S)^3,$$  \quad (12)

where $C$ is a numerical coefficient of order 1, depending on the details of the matching procedure between phases. Deviding by the critical energy density $\rho_c$ we obtain

$$\Omega_{GW}(f) = C \left( \frac{2\pi f_1}{g_s} \right)^4 \left( \frac{g_s/g_1}{H_0^2 m_{pl}^2} \right)^2 \left( \frac{f}{f_S} \right)^3 \quad (13)$$

Substituting $f_1$ from eq.(11), setting $C$ to unity for the purpose of obtaining some definite answer, and using known numerical values we obtain

$$\Omega^S_{GW} = 1.3 \times 10^{-7} h_{100}^{-2} \left( \frac{g_\ast}{1000} \right)^{-1/3} \left( \frac{H_\ast}{5 \times 10^{17}GeV} \right)^2 \left( \frac{g_s/g_1}{1000} \right)^2 \quad (14)$$
Equation (14) reflects the absolute normalization of the amplitude provided by the uncertainty principle, however, it does involve an arbitrary numerical factor $C$, which was set to unity and which depends on details of the background evolution.

This completes the determination of the end-point coordinates for the minimal spectrum.

For non-minimal spectra the effect on the position of $\Omega^S_{GW}$ is more complicated. One important effect is that $g_S$ is no longer equal to $g_1$ and could be much smaller. There are some indications that $g_S$ could be a free parameter, depending on the initial conditions.

If entropy is not even approximately conserved, namely, if a phase of massive entropy production occurs at some later time then the amplitude of modes still outside the horizon does not change while the amplitude of modes inside the horizon decreases. Details of entropy production are important, for example, if GW are also produced by the entropy creation process then their spectrum gets modified, some parts are enhanced while other suppressed.

In summary, the estimated range of spectral parameters, if there is no substantial entropy production is as follows,

$$f_S = 1.3 \times 10^{10} \frac{1}{z_S} \left( \frac{H_*}{5 \times 10^{17} \text{ GeV}} \right)^{1/2} \left( \frac{g_*}{1000} \right)^{-1/12} \text{Hz},$$

$$\Omega^S_{GW} = 1.3 \times 10^{-7} h_{-2}^{-2} \left( \frac{H_*}{5 \times 10^{17} \text{ GeV}} \right)^2 \left( \frac{g_*}{1000} \right)^{-1/3} \left( \frac{g_S}{g_1} \right)^2,$$

and

$$S^S_h = 1.0 \times 10^{-43} \left( \frac{H_*}{5 \times 10^{17} \text{ GeV}} \right)^2 \left( \frac{g_*}{1000} \right)^{-1/3} \left( \frac{f_S}{H} \right)^{-3} \left( \frac{g_S}{g_1} \right)^2 H z^{-1}.$$
FIG. 1. Spectrum of GW background. The minimal spectrum discussed in the text and two other possible spectra are shown. Also shown are estimated spectra in other cosmological models.

Entropy production, roughly speaking, lowers $f_S$ and has a more complicated effect on $\Omega_{GW}^S$. For an example of a possible effect of entropy production see [19].

Spectra for some arbitrarily chosen parameters and possible backgrounds from other cosmological models are shown in Fig. [1]. The label PH denotes preheating after inflation [21], and the label BH denotes a possible background from accumulated black hole collapses [22]. It is clear from Fig. [1] that the string cosmology background can have a much higher amplitude than all the other possible astrophysical and cosmological sources of GW.
III. ASTROPHYSICAL AND COSMOLOGICAL BOUNDS

This section follows closely the discussion in [23]. At the moment, the most restrictive observational constraint on the spectral parameters comes from the standard model of big-bang nucleosynthesis (NS) [24]. This restricts the total energy density in gravitons to less than that of approximately one massless degree of freedom in thermal equilibrium. This bound implies that

$$\int \Omega_{GW}(f) d\ln f = \Omega_{GW}^S \left[ \frac{1}{3} + \frac{1}{\beta} \left( (f_{1}/f_{S})^{\beta} - 1 \right) \right] < 0.7 \times 10^{-5}h_{100}^{-2}. \quad (15)$$

where we have assumed an allowed $N_\nu = 4$ at NS, and have substituted in the spectrum (4). The NS bound and additional cosmological and astrophysical bounds are shown in Fig. 2, where $h_{100}$ was set to unity.

The line marked “Quasar” in Fig. 2 corresponds to a bound coming from quasar proper motions. A stochastic background of gravity waves makes the signal from distant quasars scatter randomly on its way to earth. This may cause quasar proper motions. An upper bound on quasar proper motions can be translated into an upper bound on a stochastic background [26]. A typical strain $h$ may induce proper motion $\mu$, $h/f \sim \mu$. The sensitivity reached was approximately micro arcsecond per year [26], corresponding to a dimensionless strain of about $h \sim 5 \times 10^{-9}$ at frequencies below the observation time: approximately $(20 \text{ years})^{-1} \sim 5 \times 10^{-9}Hz$, leading to $\Omega_{GW} < 0.1h_{100}^2$. Future improvement in astrometric measurements could improve this bound substantially [27].

The line marked “COBE” in Fig. 2 corresponds to the bound coming from energy density fluctuations in the cosmic microwave background, which can be expressed in terms of the measured temperature fluctuations $\Delta T/T$, and the fractional energy density in photons $\Omega_{\gamma}$, $\Omega(\text{perturbations}) \simeq (\Delta T/T)^2 \Omega_{\gamma} \sim 10^{-10} \times 10^{-4} = 10^{-14}h_{100}^{-2}$. Since it is known [7] that $\Omega_{GW} < 0.1\Omega(\text{perturbations})$, it follows that $\Omega_{GW}h_{100}^2 < 10^{-15}$ at frequencies $10^{-18}h_{100}Hz - 10^{-16}h_{100}Hz$.

The curve marked “Pulsar” represents the bound coming from millisecond pulsar timing [28]. Assuming known distance and signal emission times, the pulsar functions as a
giant one-arm interferometer. The statistics of pulse arrival time residuals $\Delta T$, puts an upper bound on any kind of noise in the system, including a stochastic background of GW. The typical strain sensitivity is $h \sim \frac{\Delta T}{T}$, where $T$ is the total observation time, reaching by now 20 years $\sim 6 \times 10^8 \text{sec}$ and $\Delta T \sim 10 \mu s$ is the accuracy in measuring time residuals. Translated into $\Omega_{\text{GW}}(f)$, this yields the bound shown in the figure, which is most restrictive at frequencies $f \sim 1/T \sim 5 \times 10^{-9} \text{Hz}$.

![Cosmological and astrophysical upper bounds on the cosmic gravitational wave background.](image)

FIG. 2. Cosmological and astrophysical upper bounds on the cosmic gravitational wave background.

Notice that all the existing bounds, except from the NS bound which bounds the total energy density, are in the very low frequency range, while the expected signal from string cosmology is in a higher frequency range. The bounds are therefore not very restrictive.
IV. DETECTING A STRING COSMOLOGY STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

The principles of using a network of two or more gravitational wave antennae to detect a stochastic background of gravitational radiation are by now well known [8,29,30]. The basic idea is to correlate the signals from separated detectors, and to search for a correlated strain produced by the gravitational wave background, which is buried in the instrumental noise. After correlating signals for time $T$ the ratio of signal to noise is given by

$$\left(\frac{S}{N}\right)^2 = \frac{9H_0^4}{50\pi^4} T \int_0^\infty df \frac{\gamma^2(f)\Omega_{\text{GW}}^2(f)}{f^6 P_1(f)P_2(f)}.$$  \hspace{1cm} (16)

The instrument noise in the detectors is described by the one-sided noise power spectral densities, in units of $1/\text{Hz}$, $P_i(f)$. The dimensionless overlap reduction function $\gamma(f)$ is determined by the relative locations and orientations of the two detectors [8].

It is useful to consider the approximation in which $P_1(f)$, $P_2(f)$ have a maximum sensitivity at a common frequency $f_{\text{ms}}$ and a bandwidth $\Delta f$. Then

$$\left(\frac{S}{N}\right)^2 = \frac{9H_0^4}{50\pi^4} T \Delta f \frac{\gamma^2(f_{\text{ms}})\Omega_{\text{GW}}^2(f_{\text{ms}})}{f_{\text{ms}}^6 P_1(f_{\text{ms}})P_2(f_{\text{ms}})}.$$ \hspace{1cm} (17)

An obvious remark is that both detectors need to have an overlapping frequency range around their maximum sensitivity frequency, otherwise it is impossible to perform a meaningful correlation experiment. To ensure a common frequency range, some amount of tuning flexibility is very important.

We would like to highlight a few specific points about the string cosmology background. In my opinion, one should look for the dilaton-driven signal even though the string phase signal could be higher. The dilaton-driven spectrum has the advantage that the spectrum is theoretically clean, and therefore, if the $f^3$ dependence of the spectrum could be established it can provide a clean experimental signal for detection. Looking specifically at the sensitivity for detecting the spectrum produced during the dilaton-driven phase we obtain

$$\left(\frac{S}{N}\right)^2 = \frac{9H_0^4}{50\pi^4} T \Delta f \frac{\gamma^2(f_{\text{ms}})\Omega_{\text{GW}}^2}{f_{\text{ms}}^6 P_1(f_{\text{ms}})P_2(f_{\text{ms}})}.$$ \hspace{1cm} (18)
provided $f_{ms} < f_s$.

From equation (18) we can draw the following lessons. An obvious conclusion is that it pays to increase the observation time $T$. For a given $S/N$ the reach in $\Omega$ increases as $\sqrt{T}$. Another observation is that it pays to increase sensitivity even if it comes at the expense of bandwidth \[31\]. This is because the signal to noise ratio goes up linearly with the maximal sensitivity of each detector $P_1(f_{ms})$, $P_2(f_{ms})$ but only increases as the square root of the bandwidth. A conclusion that is perhaps not obvious is that it is better to search at the highest frequency, if the same sensitivity in $1/Hz$ can be obtained. This is because the background from astrophysical sources is smaller at higher frequencies, so a detection at higher frequency provides a cleaner signal. Finally, it is helpful to have as many detectors as near by as possible, without introducing correlated noise. Additional pairs of detectors do not add sensitivity because the background is Gaussian \[32\], therefore there is no additional information in higher-point correlation functions. They do however increase the level of confidence in the case of detection and provide a good way of reducing local sources of noise. Tunable detectors could provide an opportunity to verify the spectral shape and are therefore essential.

A. Large detectors

The LIGO project is building two identical detectors, the “initial” detectors. These detectors will be upgraded to so-called “advanced” detectors. Since the two detectors are identical in design, $P_1(f) = P_2(f)$. The design goals for the detectors specify these functions \[33\]. The design noise power spectrum for the Virgo detector \[34\] and of other large interferometers, GEO 600 \[35\] and TAMA 300 \[36\] and the noise power spectral densities of operating and planned resonant mass GW detectors (“bars”) \[37,38\] are also known. The overlap reduction function $\gamma(f)$ is identical for both the initial and advanced LIGO detectors, and has been determined for many pairs of GW detectors \[38,39\].
FIG. 3. Detection sensitivity of relic GW by operating and planned GW detectors. The interesting region of parameter space is below the “NS bound” lines.

Making use of the prediction from string cosmology (4), we may use equation (16) to assess the detectability of this stochastic background. For any given set of parameters we may numerically evaluate the signal to noise ratio $S/N$; if this value is greater than 1.65 then with at least 90% confidence, the background can be detected by a given pair of detectors. The regions of detectability in parameter space are shown in Fig. 3. The region below the NS bound lines and above the advanced LIGO curve is the region of interest. Two NS bounds are shown, the upper, more relaxed bound, assumes no GW production during the string phase [25]. The points at 1 KHz come from operating and planned resonant mass detectors. Some are taken from real experiments, an upper bound from a single detector run [39], and the first modern 12.5 hours correlation experiment between Nautilus and Explorer [40]. The arrow points to a hollow triangle showing by how much the correlation experiment can be improved if Nautilus works properly and the experiment could be done for one year. Other
points are from theoretical calculations \cite{30}. For Fig. 3 we have assumed $h_{100} = 0.65$ and $H_* = 5 \times 10^{17}$ GeV.

### B. Small detectors

The expected GW signal from string cosmology could have substantial power at high-frequencies. It is therefore tempting to explore the possibility to detect it with small devices, which could be more sensitive at high frequencies. The idea, in principle, is very simple. Build the most accurate long-lived two-level system possible. A practical way to do this is to take two identical resonators with the highest finesse, and couple them weakly. The weak coupling splits the resonance level into two near by levels. Then load the lower “pump” level with as many target particles, photons, atoms etc. (they have to be bosons, of course) and wait for a gravity wave to come along and knock one of the particles up to the upper level. The resonators can be electromagnetic \cite{41}, in the microwave or optical bandwidth, or perhaps a coherent atomic system.

#### 1. Microwave cavity detectors

A practical design of a two-level system is achieved by taking two superconducting microwave cavities and coupling them weakly \cite{42}. The lower symmetric level is the pump level, and the upper antisymmetric level is the “output” level. For a monochromatic GW of frequency $f_{GW}$, the sensitivity is the following \cite{42},

$$h(f_{GW}) = \frac{\Delta \ell}{\ell} \sim 2 \left( \frac{U_a}{U_s} \right)^{1/2} \frac{1}{Q} \left[ 1 + 4Q^2 \left( \frac{\Delta - f_{GW}}{f_a} \right)^2 \right]^{1/2}$$

(19)

where $\Delta = f_a - f_s$. $f_a$, $f_s$ and $U_a, U_s$ are the frequencies and energies stored in the antisymmetric and symmetric levels respectively. For a burst of duration $1/f_{GW}$ smaller than the lifetime of the resonance $Q/f_a$ the sensitivity in eq. (19) becomes

$$h(f_{GW}) \sim 2 \left( \frac{U_a}{U_s} \right)^{1/2} \frac{1}{Q} \left[ 2Q \frac{f_{GW}}{f_a} \right] = 4 \left( \frac{U_a}{U_s} \right)^{1/2} \frac{f_{GW}}{f_a}$$

(20)
during the lifetime $Q/f_a$ of the resonance there are $N = (Q/f_a)f_{GW}$ such independent short bursts and the sensitivity increases by a factor $\frac{\Delta \ell}{\ell} = (f_a/f_{GW}Q)^{1/2}$. The sensitivity estimate now becomes

$$\frac{\Delta \ell}{\ell} \sim 4 \left( \frac{U_a}{U_s} \right)^{1/2} \frac{f_{GW}}{f_a} (f_a/f_{GW}Q)^{1/2}$$

$$= 4 \left( \frac{U_a}{U_s} \right)^{1/2} \frac{1}{\sqrt{Q}} \left( \frac{f_{GW}}{f_a} \right)^{1/2}$$

which, very optimistically, at $f_{GW} \sim 10KHz$, with a $Q \sim 10^{12}$, $U_a \sim 10^{-22}$ Watt, $U_s \sim 100$ Watt, $f_a \sim 10GHz$ will give $h \sim 10^{-21}$, yielding a respectable sensitivity of $\Omega_{GW} \sim 10^{-4}h_{100}^{-2}$ for a correlation experiment lasting one year, assuming a bandwidth of about $10KHz$ without considering the thermal noise and selectivity criterion. Thermal noise, in particular, can be the real killer for such a detector.

A similar result is obtained by integration of the response function of the two level system

$$1 + 4Q^2 \left( \frac{\Delta - f_{GW}}{f_a} \right)^2$$

against the density of gravitational energy $\rho(f) \sim f^2 h^2(f)$ to estimate $U_a$ [43].

A prototype is being built presently, aiming to prove the feasibility of building a real GW detector of this type, and to verify that sensitivity estimates are indeed reasonable [44].

2. Motion of free masses: the memory effect

A massive object, initially at rest, will start to move in a random motion under the influence of a stochastic background of GW, performing a sort of Brownian motion. Monitoring the position of the object over a length of time can therefore be used to detect the existence of a GW background. A possible setup can consist of two masses that are free to move, and a device that measures their relative distance, similar to an interferometer, with the important difference that masses are allowed to move freely. The idea was discussed by Braginskii and Grishchuk [45]. Here we use a different method of obtaining the estimated sensitivity.
We would like to compute $\Delta \ell_{r.m.s.} = \langle \langle \Delta \ell^2 \rangle \rangle^{1/2}$, the average taken over random realizations, or equivalently a time average. To evaluate $\Delta \ell_{r.m.s.}$ we use the relation $h^2_{r.m.s.} = \langle \frac{\Delta \ell^2}{\ell^2} \rangle$ and evaluate $h_{r.m.s.}$:

$$h^2_{r.m.s.} = \frac{1}{2} \int_{-\infty}^{\infty} df S_h(f) = \frac{3H_0^2}{4\pi^2} \int_0^{\infty} df \frac{\Omega_{GW}(f)}{f^3}. \tag{22}$$

Using the explicit form (4) of the spectral density, taking into account only the contribution from the dilaton-driven phase we obtain

$$h^2_{r.m.s.} = \frac{3H_0^2}{4\pi^2} \frac{\Omega^S_{GW}}{f^3}. \tag{23}$$

Plugging in some reasonable numerical values we obtain

$$h^2_{r.m.s.} \approx 10^{-54}(1MHz/f_S)^2(\Omega^S_{GW}/5 \times 10^{-7}h_{100}^{-2}), \tag{24}$$

leading to motions $\Delta \ell \sim 10^{-25}cm$ for $\ell \sim 100cm$. The surprise/disappointment is that the average displacement does not grow with time as in ordinary Brownian motion.

The same result can be obtained by computing the force exerted by the stochastic background on the massive object using Newtonian mechanics, $\vec{F} = m\vec{a}$. The force is given by the geodesic equation,

$$\frac{d^2x^j}{dt^2}_{waves} = -R_{j0i0}waves x^i \tag{25}$$

where $R_{j0i0} = -\frac{1}{2}h^T_{ji,00}$.

Looking for simplicity at the motion caused by a single TT component in one direction, $\ddot{\ell} \sim \ell \ddot{h}$, and the equation can be integrated

$$\frac{\Delta x^2}{x^2} = \langle \int^t \int \dddot{h}_{ij}(x, s) \int^t \int \dddot{h}_{ij}(x, s') \rangle \tag{26}$$

leading up to some geometry factors to the same answer, in particular, showing that the r.m.s. displacement does not grow with time.

This approach does not look too promising, but perhaps could be improved.

Our sensitivity analysis of detectors clearly points in favor of large detectors, at least at this point of technological development. However, using small detectors is still a very
interesting enterprise, because a discovery of high frequency GW is a unique cosmological signature of a high curvature universe \[6,9\], and could allow the cleanest detection of the string cosmology background.

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