Adjustable Nonlinear Mechanism System for Wideband Energy Harvesting in Rotational Circumstances

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Abstract. Nonlinear energy harvesters have already been exhibited to draw energy from ambient vibration owing to their particular dynamic characteristics, and are feasible to desirable responses for broadband excitations of bistable and monostable systems. This study proposes an energy harvester for rotational applications, in which a cantilever beam pasted piezoelectric film and magnets with the same polarity are comprised as a nonlinear vibrating system. As the rotationally angular velocity gradually increases, the tensile stress to the cantilever beam is also self-adjusted with the increscent centrifugal force, causing the potential barriers of bistable type become shallow, so that the cantilever beam has the ability to maintain the high energy orbit motion from bistable hardening type to monostable hardening behavior. From the implemented results, the valid bandwidth of angular frequency can be improved from 26 rad/s \textendash{} 132 rad/s to 15 rad/s \textendash{} 215 rad/s, under the case of the effect of centrifugal force on nonlinear vibrating behavior. It demonstrates that the centrifugal force can significantly promote the performance of nonlinear energy harvesters.

1. Introduction

There are several approaches to covert mechanical vibration energy to electrical energy, termed as piezoelectric, electromagnetic and electrostatic [1]. One of the latest methods of energy harvesting using piezoelectric-type transducer can produce relatively high voltage and comprise of elegant structure, which offers the most promising potential for the application to micro energy harvesters. Among of these the single degree of freedom mechanical beam system is one of the most popular type [2,3]. Thereby, independent of this kind energy harvester delivers maximum output power, only when the natural frequency of the system is consistent with the environmental vibration frequency. Therefore, the performance dramatically decreases under the non-resonance conditions, many researchers have concentrated on overcoming such disadvantages by proposing resonance frequency tunable methods [4], multimodal energy harvesting [5], and frequency up-conversion covering linear energy harvester systems [6].

The nonlinearity of the system that can improve the performance of the energy harvester over a wider bandwidth is presented for enhancing energy harvesting over that of the conventional linear energy harvester [7]. It is classified into bistable [8], monostable hardening [9], and monostable...
softening cases [10]. Through adjusting the parameters of the above nonlinear systems, the systems can appear an enhanced high-energy orbit motion over a certain frequency range. Meanwhile, because of the existence of unstable and low-energy orbits, it is necessary to stabilize the high-energy orbit all the time by the adjustability of system parameters, with a widely variable frequency of environment excitations.

This paper reports a nonlinear energy harvesting model that is considered to apply to rotational environments, which is different from the previous researches that the centrifugal force is not exploited [11–13]. The linear stiffness of this system can be tuned to achieve the sustained high-energy orbit oscillating by stimulating the system of bistable into monostable hardening type under considering the increasingly centrifugal force effect. In comparison to a passive self-tuning linear and pendulum energy harvesters adopted to rotational motion [15,16], the apparent advantage in the nonlinear section is specifically analyzed in this study.

The content of paper is organized as follows. The next section elaborates the mathematic model for the possibility of the stiffness adjustability, and the system state variability by investigating the frequency response of the system under harmonic excitation. Then the simulation comparison of three conditions is implemented, it is confirmed that the effective bandwidth is extended to a broader range.

2. Mathematic Model

The energy harvester comprises a cantilever beam pasted piezoelectric film and two magnets with the same polarity, one of which is attached on the pointed end of the beam, and another one is fixed at frame of energy harvester. The proposed nonlinear energy harvester is radially oriented, at a distance of $r$ from the center axis of rotation $o$, as shown in figure 1(a).

![Figure 1. (a) Schematic of the rotating energy harvesting model and (b) Equivalent force analysis of magnetic tip mass generated owing to centrifugal force effect.](image-url)
While the device model rotates on the anticlockwise direction with the angular frequency of $\omega$, as analyzed in figure 1(b), the tensile stress generated from the centrifugal force of the tip mass is yielded as

$$F_r(t) = m\omega^2 l'(t),$$

(1)

in which $m$ is the tip mass, $l'$ is the distance from the center axis of rotation to center of gravity of the tip mass, and the $l'$ can be expressed as

$$l'(t) = \sqrt{(L+r)^2 - u(t)^2},$$

(2)

where $L$ is the length of the beam, $r$ is the distance from the root of the beam to the center axis of rotation, and $u$ is the vibrational amplitude of the cantilever beam at the tip position, by rearranging the equation (2), the approximate distance of equation (3) is obtained as

$$l'(t) \approx (L+r) \left[ 1 - \frac{1}{2} \left( \frac{u(t)}{L+r} \right)^2 \right],$$

(3)

in equation (3), due to the amplitude of cantilever beam at tip position is insignificant compared to the distance of $L+r$, leading to $(u(t)/(L+r))^2 \approx 0$, thereby, the truncated equation of centrifugal force can be written as

$$F_r(t) = m\omega^2 (L+r).$$

(4)

Then tangentially component of the centrifugal force is resolved as

$$F_t(t) = m\omega^2 (L+r)\sin\theta(t),$$

(5)

where $\theta$ is the angular of the direction of centrifugal force to the direction of central component $T_c$. Assuming $\theta$ is tiny, the equation (5) is described as

$$F_t(t) = m\omega^2 (L+r)\theta(t).$$

(6)

With the existence of the repulsive force of magnets, a dynamic force of $P$ is loaded on the tip position of cantilever beam as

$$P(t) = \frac{3EI}{L^3}u(t),$$

(7)

in which $E$ is its Young’s Modulus, $I$ is the moment of inertia. The static deflection of the cantilever beam is generated as

$$w(x) = \frac{L^4}{3EI} \left( 1 - \frac{3x}{2L} + \frac{x^3}{2L^3} \right) P(t).$$

(8)

$x$ is the distance from the tip position of cantilever beam to the point of the load force $P$. Hence, the deflection angular $\theta(t)$ at the tip position of the cantilever beam, by differentiating the static deflection with respect to $x$, is given as

$$\theta(t) = -\frac{\partial w(x)}{\partial x} \bigg|_{x=0} = \frac{L^2}{2EI} \frac{P}{u(t)} = \frac{3}{2L} u(t),$$

(9)
Substituting the equation (9) into equation (6), tangential component of the centrifugal force is obtained as
\[
F_t(t) = \frac{3m\omega^2(L + r)}{2L} u(t).
\] (10)

When gravity force of tip mass is also included, the specific expression of the tangentially resultant force of tip mass is given as
\[
F_t(t)' = \frac{3m\omega^2(L + r)}{2L} u(t) - G\sin(\omega t + \eta_0).
\] (11)

In equation (11), \(G\) is the gravity force of tip mass, \(\eta_0\) is the initial angular of center axis of beam to the vertical axis of \(y\). Therefore, the motion equation of the bistable oscillator can be expressed in dimensional form as
\[
m\ddot{u} + c\dot{u} + (k - \frac{F_M}{d})u + bu^3 = -F_t(t)'.
\] (12)

where \(c\) is the viscous damping coefficient, \(k\) is the stiffness of the cantilever beam. When harvester model revolves around the center axis of rotation, as the variation of the distance \(d\) between the two magnets, the repulsive magnet force \(F_M\) changes to make the mechanism vibrate between two stable states and constant of \(b\) is the nonlinear coefficient of two magnets. While substituting equation (11) into equation (12), the rearranged dynamic motion equation under rotational system can be yielded as
\[
m\ddot{u} + c\dot{u} + \left[k + \frac{3m\omega^2(L + r)}{2L} - \frac{F_M}{d}\right]u + bu^3 = G\sin(\omega t + \eta_0).
\] (13)

Hence, the expression of equivalent stiffness coefficient of cantilever beam \(k'\) under rotational circumstance is given as
\[
k' = k + \frac{3m\omega^2(L + r)}{2L}.
\] (14)

By setting the parameter values, \(k = 152.7\) N/m, \(m = 8\) g, and \(L = 4.2\) cm, the equivalent stiffness coefficient as an function of \(\omega\) is plotted as figure 2, which indicates that as the angular velocity is increased up to 250 rad/s, the equivalent stiffness coefficient is self-tuned from 152.7 N/m to 628.9 N/m. From the third term coefficient of equation (13), the linear stiffness expression of system leads to
\[
a = \frac{F_M}{d} - \frac{3m\omega^2(L + r)}{2L} - k, \quad (a > 0).
\] (15)

It is assumed that the linear coefficient of the magnets \(F_M/d = 212.7\) N/m, nonlinear stiffness coefficient of the magnets \(b = 4.8 \times 10^7\) N/m\(^3\), the linear coefficient of system as a function of \(\omega\) is depicted as figure 3. It demonstrates that when the angular velocity is smaller than 88.7 rad/s, the value of system linear stiffness coefficient is plus and after that the coefficient declines to negative. Then, based on equations (13) and (15), the potential energy of system is obtained under the condition of the centrifugal force on the cantilever beam as
\[
U_0'(x,t) = -\frac{1}{2}a u^2 + \frac{1}{4} b u^4.
\] (16)
As shown in figure 4, with the variation of angular velocity $\omega$, when it is smaller than 88.7 rad/s, the system remains the bistable state, and when $\omega$ exceeds 88.7 rad/s, the performance of system is tuned to monostable hardening type behavior. The steady state solution of equation (13) is that of the form

$$ u = B + U' \cos \omega t, $$

where $B$ is a constant, when $B=0$, the frequency-amplitude response equation of the bistable hardening system can be obtained by using the harmonic balance method, which is expressed as
\[
\frac{9}{16} b^2 U^6 - \frac{3}{2} b (m\omega^2 + a) U^4 + \left[ \left( \frac{m\omega^2 + F_m}{d} - \frac{3m\omega^2 (L+r)}{2L} - k \right) + c^2 \omega^2 \right] U^2 - m^2 g^2 = 0. \tag{18}
\]

![Figure 4. Potential energy against the displacement of the magnetic tip mass.](image)

3. Numerical Simulation

In this section, a simulation study is investigated to verify that the performance characteristic under the condition of exploiting the centrifugal force and the stiffness coefficient is tunable along with the change of rotational frequency. A comparison study is implemented under following three cases: (1) Monostable hardening type without considering centrifugal force; (2) Bistable hardening type in the absence of centrifugal force; (3) Bistable hardening type in the existence of centrifugal force.

The system parameters is given in table 1, from the velocity response curves shown in figure 5, it can be seen that the oscillating point continues maintaining high energy motion from the high energy orbit of bistable system to high energy orbit of monostable system after 88.7 rad/s. Furthermore, as expressed in equation (19), the investigation of linearly increasing sweep with gravitational acceleration amplitude of \( B \) is implemented, it is confirmed that the high energy orbit motion can be maintained for a longer period of time shown in figure 6, and the effective broadband is widened from 26 rad/s – 132 rad/s to 15 rad/s – 215 rad/s by self-adjusting stiffness shown in figure 7.

\[
A = B \sin \omega t, \quad (0 < \omega < 250 \text{ rad/s}, \ 0 < t < 400 \text{ s}). \tag{19}
\]

| Types     | \( m \) | \( a \)       | \( b \)       | \( c \)       | \( k \)       | \( L \)      |
|-----------|---------|---------------|---------------|---------------|---------------|-------------|
| Monostable| 8 g     | 60 N/m        | \( 4.8 \times 10^6 \) N/m\(^3\) | 0.08 N/m/s    | 500 N/m      | 4.2 cm      |
| Bistable  | 8 g     | 60 N/m        | \( 4.8 \times 10^6 \) N/m\(^3\) | 0.08 N/m/s    | 152.7 N/m    | 4.2 cm      |
**Figure 5.** Comparison of frequency response curves under the three different conditions.

**Figure 6.** Time domain velocity responses of linearly increasing sweep excitation under the three conditions: (a) monostable hardening type without considering centrifugal force, (b) bistable hardening type in the absence of centrifugal force, and (c) bistable hardening type in the existence of centrifugal force.
4. Conclusion

This research presents an adjustable nonlinear energy harvester for rotational circumstances. As the rotationally angular velocity increases, the generated centrifugal force of the magnetic tip mass can adjustably tune the stiffness of the cantilever beam; thereby, the energy harvesting system can transform from initiative bistable system to monostable system. By means of the numerical analysis, it was validated that the energy harvester can remain the high energy orbit motion, and this self-adjustable approach significantly broadened the bandwidth of the rotating angular velocity. In conclusion, this adjustable phenomenon is potentially applicable to nonlinear energy harvesting systems for rotational circumstances.

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