$B^0 - \bar{B}^0$ Mixing in Gauge-Higgs Unification

Yuki Adachi, Nobuaki Kurahashi*, Nobuhito Maru** and Kazuya Tanabe*

*Department of Sciences, Matsue College of Technology, Matsue 690-8518, Japan.
**Department of Physics, Kobe University, Kobe 657-8501, Japan.

Abstract

We discuss flavor mixing and resulting Flavor Changing Neutral Current (FCNC) in a five dimensional $SU(3)_{\text{color}} \otimes SU(3) \otimes U'(1)$ gauge-Higgs unification. Flavor mixing is realized by the fact that the bulk and brane localized mass terms are not diagonalized simultaneously. As the concrete FCNC processes, we calculate the rate of $B^0_d - \bar{B}^0_d$ mixing and $B^0_s - \bar{B}^0_s$ mixing due to the exchange of non-zero Kaluza-Klein gluons at the tree level. We obtain a lower bound on the compactification scale of order $\mathcal{O}(\text{TeV})$ by comparing our prediction on the mass difference of neutral $B$ meson with the recent experimental data.
1 Introduction

In spite of the great success of the Standard Model (SM), the origin of electroweak gauge symmetry breaking is still unknown in particle physics. Though in the SM, Higgs boson is assumed to play a role for the symmetry breaking, it seems to have various theoretical problems such as the hierarchy problem and the presence of many theoretically unpredicted arbitrary coupling constants in its interactions.

Gauge-Higgs unification (GHU) [1] is one of the fascinating scenarios beyond the SM. It provides a possible solution to the hierarchy problem without supersymmetry. In this scenario, Higgs boson in the SM is identified with the extra spatial components of the higher dimensional gauge fields. A remarkable fact is that the quantum correction to Higgs mass is UV-finite and calculable due to the higher dimensional gauge symmetry regardless of the non-renormalizability of the theory. This has opened up a new avenue to solve the hierarchy problem [2]. The finiteness of the Higgs mass has been studied and verified in various models and types of compactification at one-loop level [3] and even at the two loop level [5]. The fact that the Higgs boson is a part of gauge fields implies that Higgs interactions are restricted by gauge principle and may provide a possibility to solve the arbitrariness problem of Higgs interactions as well.

From such point of view, it seems that the following issues are particularly important for the GHU to be phenomenologically viable. The first one is whether there are any characteristic prediction on the observables subject to precision tests. The second one is how CP violation is achieved since the Higgs interactions are given by gauge interactions with real couplings. The last one is how flavor mixing is generated since Yukawa coupling in GHU is given by gauge interactions which are universal for all flavors.

As for the first issue, it will be desirable to find finite (UV-insensitive) and calculable observables, in spite of the fact that the theory is non-renormalizable and observables are very UV-sensitive in general. Works on the oblique electroweak parameters and fermion anomalous magnetic moment from such a viewpoint have been already done in the literature [6–8]. The second issue has been addressed in our previous papers [9,10], where CP violation is claimed to be achieved spontaneously either by the VEV of the Higgs field or by the complex structure of the compactified extra space.

In this paper, we focus on the remaining issue concerning the flavor physics in the GHU scenario. It is highly non-trivial problem to explain the variety of fermion masses and flavor mixings in this scenario, since the gauge interactions should be universal for all matter fields, while the flavor symmetry has to be broken eventually in order to distinguish each flavor and to realize their mixings. In our previous papers [11,12], we addressed this issue and have clarified the mechanism to generate the flavor mixings by the interplay between bulk masses and the brane localized masses.

\[1\text{For the case of gravity-gauge-Higgs unification, see [4].}\]
Important point is that such introduced two types of mass terms generically may be flavor non-diagonal without contradicting with gauge invariance, which leads to the flavor mixing in the up- and down-types of Yukawa couplings \[13\]. We may start with the base where the bulk mass terms are diagonalized, since the bulk mass terms are written in the form of hermitian matrix, which may be diagonalized by suitable unitary transformations, keeping the kinetic and gauge interaction terms of fermions invariant \[11,12\]. Even in this base, however, the brane-localized mass terms still have off-diagonal elements in the flavor base in general. Namely, the fact that two types of fermion mass terms cannot be diagonalized simultaneously leads to physical flavor mixing.

Once the flavor mixings are realized, it will be important to discuss flavor changing neutral current (FCNC) processes, which have been playing a crucial role for checking the viability of various new physics models, as is seen in the case of SUSY model. This issue was first discussed in \[14\] in the context of extra dimensions. Since our model reduces to the SM at low energies, there is no FCNC processes at the tree level with respect to the zero mode fields. However, it turns out that the exchange of non-zero Kaluza-Klein (KK) modes of gauge bosons causes FCNC at the tree level, though the rates of FCNC are suppressed by the inverse powers of the compactification scale (“decoupling”) \[11,12\]. The reason is the following. The gauge couplings of non-zero KK modes of gauge boson, whose mode functions are \(y\)-dependent, to zero mode fermions are no longer universal since the overlap integral of mode function of fermion and KK gauge boson depends on the bulk mass \(M\) different from flavor by flavor in general.

In the previous papers, as typical processes of FCNC, we have calculated the \(K^0 - \bar{K}^0\) mixing and the \(D^0 - \bar{D}^0\) mixing amplitude at the tree level via non-zero KK gluon exchange and obtained the lower bounds for the compactification scale as the predictions of our model \[11,12\]. Interestingly, the obtained lower bounds of \(\mathcal{O}(10)\) TeV were much milder than what we naively expect assuming that the amplitude is simply suppressed by the inverse powers of the compactification scale, say \(\mathcal{O}(10^3)\) TeV. We pointed out the presence of suppression mechanism of the FCNC processes, which is operative for light fermions in the GHU model. In the analysis, we focused on the simplified two generation scheme in order to estimate the mass difference and the lower bound on the compactification scale.

On the other hand, these suppression mechanism in the third generation containing top and bottom quarks does not work so strongly by the absence of bulk masses as we will discuss in the main text. Then it is expected that the dangerous large FCNC containing the third generation such as \(B^0 - \bar{B}^0\) mixing arises and more stringent constraints will be obtained. Thus it would be more desirable to discuss the FCNC process in the three generation scheme.

In this paper, we discuss flavor mixings in the three generation model and especially consider the typical FCNC processes, i.e. \(B^0_d - \bar{B}^0_d\) mixing and \(B^0_s - \bar{B}^0_s\) mixing, which is caused by the mixing between down and bottom quarks or strange and bottom quarks.
We will calculate the dominant contribution to the $B_d^0 - \bar{B}_d^0$ mixing and the $B_s^0 - \bar{B}_s^0$ mixing at the tree level by the non-zero KK gluon exchange. The rate of the FCNC processes is suppressed by the small mixings between the third generation and lighter generations. Comparing the prediction of our model with the data, the lower bound on the compactification scale is obtained.

This paper is organized as follows. After introducing our model in the next section, we summarize in section 3 how the flavor mixing is realized in the context of the gauge-Higgs unification, which was clarified and described in detail in our previous paper \[11,12\]. In section 4, as an application of the flavor mixing discussed in section 3, we calculate the mass difference of neutral $B$-mesons caused by the $B_d^0 - \bar{B}_d^0$ mixing and the $B_s^0 - \bar{B}_s^0$ mixing via non-zero KK gluon exchange at the tree level. We also obtain the lower bound for the compactification scale by comparing the obtained result with the experimental data. Our conclusion is given in section 5.

2 The Model

The model we consider in this paper is a five dimensional (5D) $SU(3)_{\text{color}} \otimes SU(3) \otimes U'(1)$ GHU model compactified on an orbifold $S^1/Z_2$ with a radius $R$ of $S^1$. The three generation model is basically obtained by extending our previous model, but top quark mass cannot be incorporating as it stands. It is known that the fermion masses have an upper bound in GHU,

$$m_q \leq \sqrt{n}M_W \quad \left( M_W : W\text{-boson mass} \right)$$

(2.1)

where $n$ is the number of indices of the representation the fermion belongs to \[16\]. Up-type quarks in our model belong to the totally symmetric tensor representation of $SU(3)$, i.e. $n = 2$, in our two generation model. Thus, we should modify our model to obtain the correct top mass $m_t \sim 2M_W$. Obviously, the simplest choice would be a 4-rank tensor representation. The representation of rank 4 of $SU(3)$ are known to be $15$, $24$, and $27$ \[17\]. We modify our model by using the smallest representation $15$. Although it is still remaining a small gap between top and twice of $W$-boson masses, it is attributed to the quantum correction of top Yukawa coupling. Focusing on the quark sector, we introduce three generations of bulk fermion in the $3$, two generations of them in the $\bar{6}$ and one generation of bulk fermion in the $15$ dimensional representations of $SU(3)$ gauge group,

$$\psi^i(3) = Q^i_3 \oplus d^i \quad \left( i = 1, 2, 3 \right),$$  

(2.2a)

$$\psi^i(\bar{6}) = \Sigma^i_6 \oplus Q^i_6 \oplus u^i \quad \left( i = 1, 2 \right),$$  

(2.2b)

$$\psi(15) = \Theta \oplus \Delta \oplus \Sigma_{15} \oplus Q_{15} \oplus t$$

(2.2c)
where all of the fermions are decomposed into those in the representations of $SU(2)$ subgroup of $SU(3)$ gauge group. An extra $U'(1)$ is required for $\psi(\mathbf{15})$ to fix the hypercharges. These sets of fermions contain ordinary quarks of the SM in the zero mode sector, i.e. $Q^i_3$ and $Q^i_6$ ($i = 1, 2$) corresponding to the first two generation quark doublets, $Q^{i=3}_3$ and $Q^{i=15}_3$ corresponding to the third generation quark doublet, and $d^i$ ($i = 1, 2, 3$), $u^i$ ($i = 1, 2$), $t$ corresponding to three generation down-type quark singlets, the first two generation up-type quark singlets, top quark singlet, respectively. $\psi^i(\mathbf{6})$ have $SU(2)$ triplet exotic states $\Sigma^i_6$ and $\psi(\mathbf{15})$ also does $SU(2)$ quintet, quartet, triplet exotic states $\Theta, \Delta$ and $\Sigma_{15}$.

The bulk Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2} \text{Tr}(F_{MN} F^{MN}) - \frac{1}{4} B_{MN} B^{MN} - \frac{1}{2} \text{Tr}(G_{MN} G^{MN})$$

$$+ \bar{\psi}^3 \{ i \partial_3 - M^i \epsilon(y) \} \psi^3 + \bar{\psi}^{i=3} \{ i \partial_3 \psi^{i=3} \}$$

$$+ \bar{\psi}^6 \{ i \partial_6 - M^i \epsilon(y) \} \psi^6 + \bar{\psi}(\mathbf{15}) i \partial'_y \psi(\mathbf{15})$$

(2.3)

where the gauge kinetic terms for $SU(3), U'(1), SU(3)_{\text{color}}$ and the covariant derivatives are

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig [A_M, A_N] ,$$

$$B_{MN} = \partial_M B_N - \partial_N B_M ,$$

$$G_{MN} = \partial_M G_N - \partial_N G_M - ig_s [G_M, G_N] ,$$

$$\bar{\psi} = \Gamma^M (\partial_M - ig A_M - ig_s G_M) ,$$

$$\bar{\psi}' = \Gamma^M (\partial_M - ig A_M - ig' B_M - ig_s G_M) .$$

(2.4a)

(2.4b)

(2.4c)

(2.4d)

(2.4e)

The gauge fields $A_M$ and $G_M$ are written in a matrix form, e.g. $A_M = A^a_M \lambda^a_2$ in terms of Gell-Mann matrices $\lambda^a$. It should be understood that $A_M$ in the covariant derivative $D_M = \partial_M - ig A_M - ig_s G_M$ acts properly depending on the representations of the fermions. $M, N = 0, 1, 2, 3, 5$ denotes indices of the bulk space-time. The five dimensional gamma matrices are given by $\Gamma^M = (\gamma^\mu, i \gamma^5) \ (\mu = 0, 1, 2, 3)$. $g, g'$ and $g_s$ are 5D gauge coupling constants of $SU(3)$, $U'(1)$ and $SU(3)_{\text{color}}$, respectively. $M^i$ ($i = 1, 2$) are generation dependent bulk mass parameters of the first two generations of fermion accompanied by the sign function $\epsilon(y)$. For the third generation, the bulk mass parameter should be taken to be zero to reproduce top quark mass.

The periodic boundary condition is imposed along $S^1$ and $Z_2$ parity assignments are taken for gauge fields as

$$A_\mu(-y) = PA_\mu(y) P^{-1}, \quad A_y(-y) = -PA_y(y) P^{-1} ,$$

$$G_\mu(-y) = G_\mu(y), \quad G_y(-y) = -G_y(y) ,$$

$$B_\mu(-y) = B_\mu(y), \quad B_y(-y) = -B_y(y)$$

(2.5a)

(2.5b)

(2.5c)

where the orbifolding matrix is defined as $P = \text{diag}(-, -, +)$ and operated in the same way at the fixed points $y = 0, \pi R$. We can see that the gauge symmetry $SU(3)$ is explicitly
broken to $SU(2) \times U(1)$ by the boundary conditions. The gauge fields with $Z_2$ odd parity and even parity are expanded by use of mode functions,

$$S_n(y) = \frac{1}{\sqrt{\pi R}} \sin \frac{n}{R} y, \quad C_n(y) = \frac{1}{\sqrt{2^{n,0} \pi R}} \cos \frac{n}{R} y,$$

(2.6)

respectively.

The $Z_2$ parities of fermions are assigned for each component of the representations as follows:

$$
\Psi^i(3) = \{Q^i_{3L}(+,+) + Q^i_{3R}(-,-)\} \oplus \{d^i_{L}(-,-) + d^i_{R}(+,+)\} \quad (i = 1, 2, 3),
\Psi^i(6) = \{\Sigma^i_{6L}(-,-) + \Sigma^i_{6R}(+,+)\} \oplus \{Q^i_{6L}(+,+) + Q^i_{6R}(-,-)\} \\
\oplus \{u^i_{L}(-,-) + u^i_{R}(+,+)\} \quad (i = 1, 2),
\Psi^{(15)} = \{\Theta_{L}(-,-) + \Theta_{R}(+,+)\} \oplus \{\Delta_{L}(+,+) + \Delta_{R}(-,-)\} \\
\oplus \{\Sigma_{15L}(-,-) + \Sigma_{15R}(+,+)\} \oplus \{Q_{15L}(+,+) + Q_{15R}(-,-)\} \\
\oplus \{t_{L}(-,-) + t_{R}(+,+)\}.
$$

Thus a chiral theory is realized in the zero mode sector by $Z_2$ orbifolding.

The fermions are also expanded by an orthonormal set of mode functions. Here we will focus on the zero-mode sector necessary for the argument of flavor mixing. The zero mode sector of each component of $\psi^i(3), \psi^i(6)$ and $\psi^{(15)}$ are written in the following way.

$$
Q^i_3 = Q^i_{3L}f^i_L(y), \quad d^i = d^i_{R}f^i_R(y) \quad (i = 1, 2, 3),
\Sigma^i_6 = \Sigma^i_{6R}f^i_R(y), \quad Q^i_6 = Q^i_{6L}f^i_L(y), \quad u^i = u^i_{R}f^i_R(y) \quad (i = 1, 2),
\Theta = \Theta f_R(y), \quad \Delta = \Delta f_L(y), \quad \Sigma_{15} = \Sigma_{15R}f_R(y), \quad Q_{15} = Q_{15L}f_L(y), \quad t = t R f_R(y).
$$

(2.7a-c)

The mode function for the zero mode of each chirality is given in [9]:

$$
f^i_L(y) = \sqrt{\frac{M^i}{1 - e^{-2\pi R M^i}}} e^{-M^i |y|}, \quad f^i_R(y) = \sqrt{\frac{M^i}{e^{2\pi R M^i} - 1}} e^{M^i |y|}.
$$

(2.8)

We notice that there are two left-handed quark doublets $Q_{3L}$ and $Q_{6L}(Q_{15L})$ per generation in the zero mode sector, which are massless before electro-weak symmetry breaking. In the one generation case, for instance, one of two independent linear combinations of these doublets should correspond to the quark doublet in the SM, but the other one should be regarded as an exotic state. Moreover, having an exotic fermion $\Sigma_{6R}, \Sigma_{15R}, \Delta_L$ and $\Theta_R$, we therefore introduce brane localized four dimensional Weyl spinors to form $SU(2) \times U(1)$ invariant brane localized Dirac mass terms in order to remove these exotic massless fermions from the low-energy effective theory [13][19].

$$
\mathcal{L}_{BLM} = \mathcal{L}_{BLM}^Q + \mathcal{L}_{BLM}^{\Sigma_6} + \mathcal{L}_{BLM}^{\Sigma_{15}} + \mathcal{L}_{BLM}^{\Delta} + \mathcal{L}_{BLM}^{\Theta}
$$

(2.9)
where for the first two generations,
\[ \mathcal{L}_{\text{BLM}}^{\Sigma} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\text{BLM}}^{\Sigma} \delta(y - \pi R) \Sigma^i \Sigma^i_6(x, y) + (\text{h.c.}) \] (2.10a)

and for the third generation,
\[ \mathcal{L}_{\text{BLM}}^{\Sigma_6} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\text{BLM}}^{\Sigma_6} \delta(y - \pi R) \Sigma^i_{15}(x, y) \Sigma^i_{15}(x) + (\text{h.c.}) , \] (2.10b)
\[ \mathcal{L}_{\text{BLM}}^{\Delta} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\text{BLM}}^{\Delta} \delta(y) \Delta_L(x, y) \Delta_R(x) + (\text{h.c.}) , \] (2.10c)
\[ \mathcal{L}_{\text{BLM}}^{\Theta} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\text{BLM}}^{\Theta} \delta(y - \pi R) \Theta_R(x, y) \Theta_L(x) + (\text{h.c.}) \] (2.10d)

and for three generations \( i = 1, 2, 3 \)
\[ \mathcal{L}_{\text{BLM}}^{Q} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} \delta(y) \bar{Q}^i_R(x) \left\{ \eta_{ij} Q^j_L(x, y) + \lambda_{ij} Q^j_L(x, y) \right\} + (\text{h.c.}) \] (2.10e)

where
\[ Q_L(x, y) = \begin{bmatrix} Q^1_{6L}(x, y) & Q^2_{6L}(x, y) & Q^3_{15L}(x, y) \end{bmatrix} \]

\( Q_R, \Sigma_{6,15L}, \Delta_R \) and \( \Theta_L \) are the brane localized Weyl fermions of doublet, triplet, quartet and quintet of \( SU(2) \) respectively. The \( 3 \times 3 \) matrices \( \eta_{ij}, \lambda_{ij} \) and \( m_{\text{BLMS}} \) are mass parameters. These brane localized mass terms are introduced at opposite fixed points such that \( Q_R, \Delta_R (\Sigma_{6,15L}, \Theta_L) \) couples to \( Q_{3,6,15L}, \Delta_L (\Sigma_{6,15R}, \Theta_R) \) localized on the brane at \( y = 0 \) \((y = \pi R)\). Let us note that the matrices \( \eta_{ij}, \lambda_{ij} \) can be non-diagonal, which are the source of the flavor mixing [11–13].

### 3 Flavor mixing

In the previous section we worked in the base where fermion bulk mass terms are written in a diagonal matrix in the generation space. Then Yukawa couplings as the gauge interaction of \( A_y \) is completely diagonalized in the generation space. Thus flavor mixing does not happen in the bulk and the brane localized mass terms for the doublets \( Q_{3L} \) and \( Q_{6L} \) \((Q_{15L})\) is expected to lead to the flavor mixing. We now discuss how the flavor mixing is realized in this model.

First, we identify the SM quark doublet by diagonalizing the relevant brane localized mass term,
\[
\int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} \delta(y) \bar{Q}_R(x) \begin{bmatrix} \eta & \lambda \end{bmatrix} \begin{bmatrix} Q_{3L}(x, y) \\ Q_{6L}(x, y) \end{bmatrix} \supset \sqrt{2\pi R} \bar{Q}_R(x) \begin{bmatrix} \eta f_L(0) & \lambda f_L(0) \end{bmatrix} \begin{bmatrix} Q_{3L}(x) \\ Q_{6L}(x) \end{bmatrix}
\]
\[ = \sqrt{2\pi R} \bar{Q}_R'(x) \begin{bmatrix} m_{\text{diag}} & 0_{3\times3} \end{bmatrix} \begin{bmatrix} Q_{6L}(x) \\ Q_{SML}(x) \end{bmatrix} \]

(3.1)
where
\[
\begin{bmatrix}
U_1 & U_3 \\
U_2 & U_4
\end{bmatrix} \begin{bmatrix}
Q_{HL}(x) \\
Q_{SML}(x)
\end{bmatrix} = \begin{bmatrix}
Q_{3L}(x) \\
Q_{L}(x)
\end{bmatrix}, \quad U^Q Q_R(x) = Q'_R(x),
\] (3.2a)
\[
U^Q \begin{bmatrix}
\eta_f L(0) \\
\lambda_f L(0)
\end{bmatrix} \begin{bmatrix}
U_1 & U_3 \\
U_2 & U_4
\end{bmatrix} = \begin{bmatrix}
m_{\text{diag}} & 0 \\
3 \times 3 & 3 \times 3
\end{bmatrix}.
\] (3.2b)

In eq. (3.1), \( \eta_f L(0) \) is an abbreviation of a 3 \( \times \) 3 matrix whose \((i,j)\) element is given by \( \eta_{ij} f_j L(0) \), for instance. \( U_3, U_4 \) are 3 \( \times \) 3 matrices satisfying the unitarity condition
\[
U_3^\dagger U_3 + U_4^\dagger U_4 = 1_{3 \times 3},
\] (3.3)
which indicates how the quark doublets of the SM are contained in each of \( Q_{3L}(x) \) and \( Q_{6,15L}(x) \) and compose a 6 \( \times \) 6 unitary matrix together with \( U_1, U_2 \), which diagonalizes the brane localized mass matrix. The eigenstate \( Q_H \) becomes massive and decouples from the low energy processes, while \( Q_{SM} \) remains massless at this stage and is identified with the SM quark doublet. After this identification of the SM doublet, Yukawa couplings are read off from the higher dimensional gauge interaction of \( A_y \), whose zero mode is the Higgs field \( H(x) \):
\[
-\frac{g_4}{2} \left\{ \langle H^\dagger \rangle \bar{d}_R(x) I_{RL}^{(00)} U_3^{ij} Q_{SML}^j(x) + \langle H^t \rangle i\sigma^2 \bar{u}_R^i(x) (WI_{RL}^{(00)})^{ij} U_4^j Q_{SML}^j(x) \right\} + \text{h.c.}
\] (3.4)
where \( g_4 \equiv \frac{g}{\sqrt{2}\pi R} \) and the matrix \( W \) indicates the factor \( \sqrt{n} \) in (2.1):
\[
W \equiv \text{diag}(\sqrt{2}, \sqrt{2}, 2)
\] (3.5)
and \( I_{RL}^{(00)} \) is an overlap integral of mode functions of fermions with matrix elements \( (I_{RL}^{(00)})_{ij} = \delta_{ij} I_{RL}^{(00)} \):
\[
I_{RL}^{(00)} = \int_{-\pi R}^{\pi R} dy f_L^i f_R^j = \begin{cases}
\frac{\pi R M^i}{\sinh(\pi R M^i)} & (i = 1, 2) \\
1 & (i = 3)
\end{cases}
\] (3.6)
which behaves as \( 2\pi R M^i e^{-\pi R M^i} \) for \( \pi R M^i \gg 1 \), thus realizing the hierarchical small quark masses without fine tuning of \( M^i \). We thus know that the matrices of Yukawa coupling \( \frac{g_4}{2} Y_u \) and \( \frac{g_4}{2} Y_d \) are given as
\[
\frac{g_4}{2} Y_u = \frac{g_4}{2} W I_{RL}^{(00)} U_4, \quad \frac{g_4}{2} Y_d = \frac{g_4}{2} I_{RL}^{(00)} U_3.
\] (3.7)
These matrices are diagonalized by bi-unitary transformations as in the SM and Cabibbo-Kobayashi-Maskawa (CKM) matrix is defined in a usual way [18].
\[
\begin{cases}
\hat{Y}_d = \text{diag}(\hat{m}_d, \cdots) = V_{dR}^\dagger Y_d V_{dL} \\
\hat{Y}_u = \text{diag}(\hat{m}_u, \cdots) = V_{uR}^\dagger W Y_u V_{uL}
\end{cases}, \quad V_{CKM} \equiv V_{dL}^\dagger V_{uL}
\] (3.8)
where all the quark masses are normalized by the $W$-boson mass as $\hat{m}_f = \frac{m_f}{M_W}$. A remarkable point is that the Yukawa couplings $\frac{g}{2}Y_u$ and $\frac{g}{2}Y_d$ are related through the unitarity condition eq. (3.3), on the contrary those are completely independent in the SM.

For an illustrative purpose to confirm the mechanism of flavor mixing, we will see how the realistic quark masses and mixing are reproduced. Here we leave aside CP violation since the issue discussed in this paper is independent of it and assume that $U_3$, $U_4$ are real. Let us notice that $3 \times 3$ matrices $U_{3,4}$ can be parametrized because of (3.3) without loss of generality as

$$U_4 = R_u \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}, \quad U_3 = R_d \begin{bmatrix} \sqrt{1 - a_1^2} & 0 & 0 \\ 0 & \sqrt{1 - a_2^2} & 0 \\ 0 & 0 & \sqrt{1 - a_3^2} \end{bmatrix}$$

(3.9)

where $R_u$ and $R_d$ are arbitrary $3 \times 3$ rotation matrices parametrized as

$$R_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_2' & \sin \theta_2' \\ 0 & -\sin \theta_2' & \cos \theta_2' \end{bmatrix}, \quad R_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_3 & \sin \theta_3 \\ 0 & -\sin \theta_3 & \cos \theta_3 \end{bmatrix}$$

(3.10a, b)

Actually the most general forms of $U_3$ and $U_4$ have a common orthogonal matrix multiplied from the right, which can be however eliminated by suitable unitary transformations among the members of $Q_{SM}(x)$.

Now physical observables $\hat{m}_u$, $\hat{m}_c$, $\hat{m}_t$, $\hat{m}_d$, $\hat{m}_s$, $\hat{m}_b$ and the angles of the CKM matrix are expressed in terms of $a_i, b_i \equiv l R_l^{(00)}$ and 6 rotation angles in $R_u$ and $R_d$. Note that our theory has 2 free parameters which cannot be determined by the observables since 9 physical observables are written in terms of 11 parameters.

As we have discussed in the previous paper [12], if the large mixings between the 1-3 and 2-3 generations are introduced then the top quark mass decreases from 160 GeV $\sim 2M_W$. Thus, we expect that the mixing angles between the third generation and the first two generations are considered to be small to keep $m_t \sim 2M_W$. Also the relation between the masses of top and bottom quark $m_t^2 + (2m_b)^2 = (2M_W)^2$ for the $M_3 = 0$ holds and we must choose $a_3 \sim 1$. It implies that the rotation angles $\theta_2', \theta_3', \theta_2, \theta_3$ and parameter $\sqrt{1 - a_3^2}$ should be small, and also other 6 parameters $a_1, a_2, b_1, b_2$ and $\theta_1', \theta_1$ should take values close to those of 2 generation model [12].

Actually, for the case of $R_u = 1_{3 \times 3}$ where the up type quark mixings vanish, this case gives almost the most stringent lower bound from $K^0 - \bar{K}^0$ mixing for example, these parameters are numerically found as

$$a_1^2 \approx 0.1023 \quad b_1^2 \approx 4.355 \times 10^{-9} \quad \sin \theta_1 \approx -2.587 \times 10^{-2}$$
\( a_2^2 \approx 0.9887 \quad , \quad b_2^2 \approx 1.302 \times 10^{-4} \quad , \quad \sin \theta_2 \approx 2.224 \times 10^{-2} \quad . \)  \( a_3^2 \approx 0.9966 \quad \sin \theta_3 \approx 2.112 \times 10^{-4} \quad . \)  

Also, for the another case of \( R_d = \mathbf{1}_{3x3} \) where the down type quark mixings vanish, these parameters are numerically found as

\[
\begin{align*}
    a_1^2 & \approx 0.0650 & b_1^2 & \approx 3.973 \times 10^{-9} & \sin \theta_1' & \approx 0.6704 \\
    a_2^2 & \approx 0.9931 & b_2^2 & \approx 2.235 \times 10^{-4} & \sin \theta_2' & \approx -3.936 \times 10^{-2} \\
    a_3^2 & \approx 0.9966 & & & \sin \theta_3' & \approx 1.773 \times 10^{-2} 
\end{align*}
\]

These two results show that the mixing angles \( \theta_2, \theta_3, \theta_2', \theta_3' \ll 1 \) and it is completely consistent with the above argument.

4 \textbf{\( B^0 - \bar{B}^0 \) mixing}

In this section, we apply the results of the previous section to representative FCNC processes, \( B_d^0 - \bar{B}_d^0 \) mixing and \( B_s^0 - \bar{B}_s^0 \) mixing responsible for the mass difference of two neutral \( B \) mesons.

We focus on the FCNC processes of zero-mode down-type quarks due to gauge boson exchange at the tree level. First let us consider the processes with the exchange of zero mode gauge bosons. If such type of diagrams exist with a sizable magnitude, it will easily spoil the viability of the model.

Concerning the \( Z \)-boson exchange, it is in principle possible to occur the tree-level FCNC. Since the mode function of the zero-mode gauge boson is \( y \)-independent, the overlap integral of mode functions is generation independent. Thus the gauge coupling of zero mode gauge boson depends on only the relevant quantum numbers such as the third component of weak isospin \( I_3 \). Therefore the condition proposed by Glashow-Weinberg \([15]\) to guarantee natural flavor conservation for the theories of 4D space-time is relevant.

Although there are right-handed down-type quarks belonging to different representations in our model, for example, the \( SU(2) \) singlet \( d_R \) in \( \psi(3) \) and one of components of the triplet \( \Sigma_R \) in \( \psi(6) \) or \( \psi(\mathbf{15}) \), these are known to have the same quantum number \( I_3 = 0 \), and thus the Glashow-Weinberg condition is satisfied in this sector \([11]\). However, the quintet \( \Theta_R \) in \( \psi(\mathbf{15}) \) also contains the right-handed down-type quark, and this has the different quantum number \( I_3 = 1 \) from that of \( d_R \) belonging to \( \psi(3) \).

What is worse, the quartet \( \Delta_L \) in \( \psi(\mathbf{15}) \) contains left-handed down-type quark with the different quantum number \( I_3 = -\frac{1}{2} \) from that of \( d_L \) belonging to the doublet \( Q_L \) in \( \psi(3) \), \( \psi(6) \) or \( \psi(\mathbf{15}) \) with the quantum number \( I_3 = -\frac{1}{2} \). Thus, the condition of Glashow-Weinberg is not satisfied in the down-type quark sector and FCNC process due to the

\footnote{For the studies of \( B^0 - \bar{B}^0 \) mixing in other new physics models, see for instance \([21]\).}
exchange of the zero mode $Z$-boson arises at the tree level\(^3\). However, the quintet $\Theta_R$ (quartet $\Delta_L$) is an exotic fermion and acquires large $SU(2)$ invariant brane mass. Thus the mixing between $d^i_R$ ($d^i_L$) and $\Theta_R$ ($\Delta_L$) is inversely suppressed by the power of $m_{BLM}$ and the FCNC vertex of $Z$-boson can be safely neglected. We may say that the condition of Glashow-Weinberg is satisfied in a good approximation in the processes via the zero mode gauge boson exchange.

One may worry that $Z'$ gauge boson exchange give rise to FCNC processes since an extra $U(1)$ gauge symmetry is indispensable for getting a realistic Weinberg angle. Note that the extra $U(1)$ gauge symmetry is explicitly broken by an anomaly and the gauge boson of the extra $U(1)$ gauge symmetry acquire a mass of the cutoff scale order. In our model, the cutoff scale is a 5D Planck scale which is larger than the intermediate scale $10^{13}$ GeV. Therefore, the FCNC effects by $Z'$ gauge boson exchange can be safely neglected comparing to the process by non-zero KK gluon exchanges considered later.

Hence, the remaining possibility is the process via the exchange of non-zero KK gauge bosons. In this case, the mode functions of KK gauge bosons are $y$-dependent and their couplings to fermions are no longer universal because of non-degenerate bulk masses, even if the condition of Glashow-Weinberg is met.

Therefore, such progresses lead to FCNC at the tree level. In our previous papers \(^{11,12}\), we have calculated $K^0 - \bar{K}^0$ mixing and $D^0 - \bar{D}^0$ mixing via the non-zero KK gluon exchange at the tree level and obtained a lower bound of the compactification scale as the prediction of our model. Along the same line of the argument as in our previous papers, we here study $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ mixings in the down-type quark sector by the non-zero KK gluon exchange at the tree level as the dominant contribution to these FCNC processes.

For such purpose, let us derive the four dimensional effective QCD interaction vertices for the zero modes of down-type quarks relevant for our calculation:

$$\mathcal{L}_s \supset \frac{g_s}{2\sqrt{2\pi R}} C_{\mu}^{a}(\bar{d}_{R}^{a} \gamma_{\mu} d_{R}^{a} + \bar{d}_{L}^{a} \gamma_{\mu} d_{L}^{a}) + \frac{g_s}{2} C_{\mu}^{a(n)}(\bar{d}_{R}^{a} \gamma_{\mu} d_{R}^{i} V_{dR}^{\dagger} I_{RR}^{(0n)}(0_{R}) V_{dR}^{(0n)})_{ij} + \frac{g_{s}}{2} C_{\mu}^{a(n)}(\bar{d}_{L}^{a} \gamma_{\mu} d_{L}^{i}(-1)^{n} V_{dL}^{\dagger}(U_{3}^{1} I_{RR}^{(0n)} U_{3} + U_{4}^{1} I_{RR}^{(0n)} U_{4})_{ij}) . \quad (4.1)$$

where $I_{RR}^{(0n)}$ and $I_{LL}^{(0n)}$ are overlap integrals relevant for gauge interaction,

$$I_{RR}^{(0n)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy (f_{R})^{2} \cos \frac{n}{\pi R} y = \frac{1}{\sqrt{\pi R}} \frac{(2RM^{i})^{2}}{(2RM^{i})^{2} + n^{2}} \frac{(-1)^{n} e^{2\pi RM^{i}} - 1}{e^{2\pi RM^{i}} - 1}, \quad (4.2)$$

$$I_{LL}^{(0n)} = I_{RR}^{(0n)} \bigg|_{M^{i} \to -M^{i}} = (-1)^{n} I_{RR}^{(0n)} \quad (4.3)$$

since the chirality exchange corresponds to the exchange of two fixed points. We can see from (4.1) that the FCNC appears in the couplings of non-zero KK gluons due to the

\(^{3}\)The FCNC due to the exchanges of zero-mode photon and gluon trivially vanish because the fermions of our interest have the same electric charge and color.
The diagrams of $B_0^d - \bar{B}_0^d$ mixing via KK gluon exchange. Those of $B_0^s - \bar{B}_0^s$ mixing via KK gluon exchange are obtained by the replacements $d \leftrightarrow s$.

Figure 1: The diagrams of $B_0^d - \bar{B}_0^d$ mixing via KK gluon exchange. Those of $B_0^s - \bar{B}_0^s$ mixing via KK gluon exchange are obtained by the replacements $d \leftrightarrow s$.

The Feynman rules necessary for the calculation of $B_0^d - \bar{B}_0^d$ mixing can be read off from (4.1).

\[ G_{\mu}^{a(n)} \]

\[ d_R \hspace{1cm} G_{\mu}^{a(n)} \hspace{1cm} b_R \]

\[ \frac{g_s}{2} \left( V_{dL}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{31} \lambda^a \gamma^\mu R , \]  

(4.4a)

\[ d_L \hspace{1cm} G_{\mu}^{a(n)} \hspace{1cm} b_L \]

\[ \frac{g_s}{2} (-1)^n \left\{ V_{dL}^\dagger \left( U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4 \right) V_{dL} \right\}_{31} \lambda^a \gamma^\mu L , \]  

(4.4b)

\[ G_{\mu}^{a(n)} \hspace{1cm} G_{\nu}^{b(n')} \]

\[ = \delta_{nn'} \delta_{ab} \frac{\eta^{\mu\nu}}{k^2 - M_n^2} \left( \text{t Hooft-Feynman gauge} \right) . \]  

(4.4c)

Those for $B_0^s - \bar{B}_0^s$ mixing are easily obtained by the replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ in the matrix element of the vertices. The non-zero KK gluon exchange diagrams providing the dominant contribution to the process of $B_0^d - \bar{B}_0^d$ and $B_0^s - \bar{B}_0^s$ mixing are depicted in Fig.[]

By noting the fact $k^2 \ll \left( \frac{n}{\bar{R}} \right)^2$ for $n \neq 0$ being the mass of $n$-th KK gluon and $k^\mu$ being internal momentum, the contributions from each type diagram of the $B_0^d - \bar{B}_0^d$ mixing in Fig.[] are written in the form of effective four-Fermi lagrangian obtained by use of Feynman rules listed above.

\[ d_L \hspace{1cm} G_{\mu}^{a(n)} \hspace{1cm} b_L \]

\[ \sim - \sum_{n=1}^{\infty} \frac{g_s^2}{4} \frac{1}{M_c^2} \left( \bar{b}_L \lambda^a \gamma^\mu d_L \right) \left( \bar{b}_R \lambda^a \gamma_\mu d_R \right) \times \]

\[ \left\{ V_{dL}^\dagger \left( U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4 \right) V_{dL} \right\}_{31} \left( V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{31} , \]  

(4.5a)
Similarly, those for $B_s^0 - \bar{B}_s^0$ mixing are obtained by the replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ in the matrix element of the vertices. The sum over the integer $n$ is convergent and the coefficients of the effective lagrangian (4.5) for the $B_d^0 - \bar{B}_d^0$ mixing and (4.5) after the replacements of $d \leftrightarrow s$ and $31 \leftrightarrow 32$ for the $B_s^0 - \bar{B}_s^0$ mixing are suppressed by the compactification scale as $1/M^2_c = R^2$.

Comparing the results with the experimental data, we can estimate a lower bound on the compactification scale. The most general effective Hamiltonian for $\Delta B = 2$ processes due to some “new physics” at a high scale $\Lambda_{NP} \gg M_W$ can be written as follows;

$$H_{\text{eff}}^{\Delta B=2} = \frac{1}{\Lambda_{NP}^2} \left( \sum_{i=1}^{5} z_i^q Q_i^q + \sum_{i=1}^{3} \tilde{z}_i^q \tilde{Q}_i^q \right)$$

(4.6)

where the 4-Fermi operators relevant for $B_d^0 - \bar{B}_d^0$ mixing are given as,

$$Q_1^d = \bar{d}_R^\mu \gamma_\mu d_L^\alpha \bar{d}_L^{\alpha 3} b_L^{\beta} b_L^\beta, \quad Q_2^d = \bar{d}_R^\mu b_L^\alpha \bar{d}_R^{\alpha 3} b_L^{\beta}, \quad Q_3^d = \bar{d}_R^\mu b_L^{\alpha 3} \bar{d}_L^\beta b_L^\beta,$$

$$Q_4^d = \bar{d}_R^\mu \gamma_\mu b_L^\alpha \bar{d}_L^{\alpha 3} b_L^\beta, \quad Q_5^d = \bar{d}_R^\mu b_L^\alpha \bar{d}_R^{\alpha 3} b_L^\beta$$

(4.7a)

and for $B_s^0 - \bar{B}_s^0$ mixing,

$$Q_1^s = \bar{s}_L^\mu \gamma_\mu b_L^\alpha \bar{s}_L^{\alpha 3} b_L^{\beta} b_L^\beta, \quad Q_2^s = \bar{s}_R^\mu b_L^\alpha \bar{s}_L^{\alpha 3} b_L^{\beta}, \quad Q_3^s = \bar{s}_R^\mu b_L^{\alpha 3} \bar{s}_L^\beta b_L^\beta,$$

$$Q_4^s = \bar{s}_R^\mu \gamma_\mu b_L^\alpha \bar{s}_L^{\alpha 3} b_L^\beta, \quad Q_5^s = \bar{s}_R^\mu b_L^\alpha \bar{s}_R^{\alpha 3} b_L^\beta.$$  

(4.7b)

Indices $\alpha, \beta$ stand for the color degrees of freedom. The operators $\tilde{Q}_{1,2,3}$ are obtained from the $Q_{1,2,3}$ by the chirality exchange $L \leftrightarrow R$. Since the SM contribution is poorly known, we can get the constraint on the new physics directly from the experimental data assuming that there is no accidental cancellation between the contributions of the SM and new physics. If we assume one of these possible operators gives dominant contribution to the mixing, each coefficient is independently constrained as follows, with the same
constraints for $z_i^q$ as those for $z_i^q$ ($i = 1, 2, 3$) [20]:

$$|z_1^d| \leq 2.3 \times 10^{-5} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2, \quad |z_1^s| \leq 1.1 \times 10^{-3} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2$$

$$|z_2^d| \leq 7.2 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2, \quad |z_2^s| \leq 5.6 \times 10^{-5} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2$$

$$|z_3^d| \leq 2.8 \times 10^{-6} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2, \quad |z_3^s| \leq 2.1 \times 10^{-4} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2$$

$$|z_4^d| \leq 2.1 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2, \quad |z_4^s| \leq 1.6 \times 10^{-5} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2$$

$$|z_5^d| \leq 6.0 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2, \quad |z_5^s| \leq 4.5 \times 10^{-5} \left( \frac{\Lambda_{NP}}{1 \text{TeV}} \right)^2$$

where the new physics scale $\Lambda_{NP}$ is regarded as the compactification scale in our case. All we have to do is to represent (4.5) and its replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ of (4.5) by use of (4.7) and to utilize these constraints (4.8).

We can rewrite the each type effective lagrangian for $B_d^0 - \bar{B}_d^0$ mixing (4.5) in terms of effective Hamiltonian by using the Fierz transformation and the completeness condition for Gell-Mann matrices;

$$\mathcal{H}_{\text{eff}, LL}^{\Delta B=2} = \frac{z_1^d Q_1^d}{R^2}, \quad \mathcal{H}_{\text{eff}, RR}^{\Delta B=2} = \frac{z_1^d Q_1^d}{R^2}, \quad \mathcal{H}_{\text{eff}, LR}^{\Delta B=2} = \frac{z_2^d Q_2^d + z_3^d Q_3^d}{R^2}$$

where

$$z_1^d = \frac{8\pi \alpha_s}{3} \pi R \sum_{n=1}^{\infty} \left\{ \frac{1}{n^2} \left( V_{dL}^\dagger U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4 \right) V_{dL} \right\}_{31}^2, \quad (4.10a)$$

$$z_1^s = \frac{8\pi \alpha_s}{3} \pi R \sum_{n=1}^{\infty} \left\{ \frac{1}{n^2} \left( V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right) \right\}_{31}^2, \quad (4.10b)$$

$$z_4^d = -8\pi \alpha_s \pi R \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n}{n^2} \left( V_{dL}^\dagger U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4 \right) V_{dL} \right\}_{31} \left( V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{31}, \quad (4.10c)$$

$$z_5^d = \frac{8}{3} \alpha_s \pi R \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n}{n^2} \left( V_{dL}^\dagger U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4 \right) V_{dL} \right\}_{31} \left( V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{31}. \quad (4.10d)$$

The four-dimensional $\alpha_s$ is defined by

$$\alpha_s = \frac{(g_s^{4D})^2}{4\pi} = \frac{1}{2\pi R} \frac{g_s^2}{4\pi} \quad (4.11)$$

The constant $\alpha_s$ should be estimated at the scale $\mu_b = m_b = 4.6$ GeV where the $\Delta B = 2$ processes are actually measured [20]. So we have to take into account the renormalization group effect from the weak scale down to $\mu_b$:

$$\alpha_s^{-1}(m_b) = \alpha_s^{-1}(M_Z) - \frac{23}{6\pi} \ln \frac{M_Z}{m_b} \quad \longrightarrow \quad \alpha_s(m_b) \approx 0.207 \quad (4.12)$$
where $\alpha_s(M_Z) \approx 0.1184$ has been put \cite{22}.

Similarly, the each type effective Hamiltonian for $B_0^s - \bar{B}_s^0$ mixing are respectively rewritten by replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ of (4.9) and (4.10).

Combining these results, we obtain the lower bounds for the compactification scale from the constraint (4.8). First let us assume that only one of the three types of diagrams (LL, RR, LR) gives dominant contribution to the mixing. Then we get lower bound on the compactification scale by use of the upper bound on the relevant coefficients $z_1^d$, $\tilde{z}_1^d$ and $z_4^d$ given in (4.8) in the unit of TeV:

\[
\begin{align*}
\text{LL} : \quad & \frac{1}{R} \gtrsim \sqrt{\frac{|z_1^d|}{2.3 \times 10^{-5}}} \quad \text{[TeV]} \quad \text{LL} : \quad & \frac{1}{R} \gtrsim \sqrt{\frac{|z_1^s|}{1.1 \times 10^{-3}}} \quad \text{[TeV]} \\
\text{RR} : \quad & \frac{1}{R} \gtrsim \sqrt{\frac{|\tilde{z}_1^d|}{2.3 \times 10^{-5}}} \quad \text{[TeV]} \\
\text{LR} : \quad & \frac{1}{R} \gtrsim \sqrt{\frac{|z_4^d|}{2.1 \times 10^{-7}}} \quad \text{LR} : \quad & \frac{1}{R} \gtrsim \sqrt{\frac{|\tilde{z}_4^d|}{1.6 \times 10^{-5}}} \quad \text{[TeV]}
\end{align*}
\]

Let us note that LR type diagrams yield both of $Q_4^d$ and $Q_5^d$ operators as is seen in (4.9). We can however safely ignore the contribution of $Q_5^d$ to the mixing, because the coefficients of the operator (4.10d) are smaller than that of $Q_4^d$ and also because the magnitude of the hadronic matrix element of $Q_4^d$ is known to be greater than that of $Q_5^d$, as the constraint for $z_4^d$ is more severe that that for $Q_5^d$ in (4.8). This is why we used the constraint for $z_4^d$ alone to get the lower bound for the case of LR type diagrams.

Since there is no bulk mass of third generation in this model, the “GIM-like” suppression mechanism from the large bulk masses which acts much more severe on the contribution of the LR type diagram \cite{12} does not occur. Thus the contribution of the LR-type diagram is not expected to be smaller than those of the LL and RR diagram in general. Actually, for the case of $R_u = 1_{3 \times 3}$ in (3.9), which gives almost the most stringent lower bound from $K^0 - \bar{K}^0$ mixing, the LR type contribution is dominant for $B_0^d - \bar{B}_0^d$ mixing while the LL type contribution is dominant for $B_0^s - \bar{B}_s^0$ mixing:

\[
\begin{align*}
R^{-1} \gtrsim 1.71 \text{ [TeV]} & \quad \text{for } B_0^d - \bar{B}_0^d \text{ mixing} , \\
R^{-1} \gtrsim 2.54 \text{ [TeV]} & \quad \text{for } B_0^s - \bar{B}_s^0 \text{ mixing} .
\end{align*}
\]

In the second case $R_d = 1_{3 \times 3}$, the contributions from the LR and RR type diagram vanish. This is because the down-type Yukawa coupling becomes diagonal: $V_{dL} = V_{dR} = 1_{3 \times 3}$, namely the mixings in the down quark sector disappear. Note, however, that the lower bound obtained from the LL type contribution, which does not vanish even though $V_{dL} = 1_{3 \times 3}$. Actually, we obtain the lower bound on $R^{-1}$ for $R_d = 1_{3 \times 3}$:

\[
R^{-1} \gtrsim 0.92 \text{ [TeV]} \quad \text{for } B_0^d - \bar{B}_0^d \text{ mixing} ,
\]

\[
R^{-1} \gtrsim 1.20 \text{ [TeV]} \quad \text{for } B_0^s - \bar{B}_s^0 \text{ mixing} .
\]
This is because $V_{uL}$ relevant for up-type quark mixing also contributes to the left-handed FCNC current. Namely, because of the mixing between $Q_{3L}$ and $Q_{6L}$ ($Q_{15L}$), $U_4$ also contributes to the FCNC vertex (4.4b) Thus even in the case of $V_{dL} = 1_{3 \times 3}$ we get a meaningful lower bound on $M_c$.

A comment is given. The obtained lower bounds are smaller than what we naively expect assuming that the tree level diagram relevant for the FCNC process is simply suppressed by $1/M_c^2$\textsuperscript{20};

\begin{align}
M_c &\gtrsim \mathcal{O}(10^3) \text{ [TeV]} \quad \text{for } B_d^0 - \bar{B}_d^0 \text{ mixing ,} \quad (4.16a) \\
M_c &\gtrsim \mathcal{O}(10^2) \text{ [TeV]} \quad \text{for } B_s^0 - \bar{B}_s^0 \text{ mixing .} \quad (4.16b)
\end{align}

which is much more stringent than the lower bound we obtained, in spite of the absence of the suppression by the large bulk masses. The obtained lower bounds also are milder than those from $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings. This apparent discrepancy may be attributed to the very small mixing between the third generation and the first two generations.

5 Summary

In this paper, we have discussed the flavor mixing and the resulting FCNC processes in the framework of five dimensional $SU(3)_{\text{color}} \otimes SU(3) \otimes U'(1)$ gauge-Higgs unification scenario. As the concrete FCNC processes, we have calculated the contributions to $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings by the non-zero KK gluon exchange at the tree level in the light of the recent progress in the measurements of $B^0 - \bar{B}^0$ mixing. For the processes with respect to the third generation, the “GIM-like” suppression mechanism, which is operative for the light first two generation quarks, does not work since their bulk mass has to be vanished to realize top quark mass. Therefore, we can anticipate large FCNC effects to arise and we are likely to obtain strong constraints for B-physics. The prediction of our model is that the lower bounds of compactification scale have been found to be of order $\mathcal{O}(\text{TeV})$ which is milder than those obtained from our study of $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings in our previous paper \textsuperscript{11,12} and from a naive expectation ($\sim 1000 \text{ TeV}$) where the dimension six operator is simply suppressed by $1/M_c^2$ in spite of the absence of the GIM-like suppression by the large bulk masses. This is because the smallness of the mixings between 1-3 and 2-3 generations, \emph{i.e.} $\theta_2, \theta_3, \theta'_2, \theta'_3 \ll 1$. In our model, they should be small to reproduce the realistic top quark mass $\sim 2M_W$, and then the induced $\Delta B = 2$ effective hamiltonian are strongly suppressed. Thus the lower bound of compactification scale becomes small.
Acknowledgments

We would like to thank Professor C.S. Lim for fruitful discussions and valuable suggestions at various stages of this work. This work was supported in part by the Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture, No. 21244036 and in part by Keio Gijuku Academic Development Funds (N. M.).

References

[1] N. S. Manton, Nucl. Phys. B 158, 141 (1979); D. B. Fairlie, Phys. Lett. B 82, 97 (1979); J. Phys. G 5, L55 (1979); Y. Hosotani, Phys. Lett. B 126, 309 (1983), Phys. Lett. B 129, 193 (1983), Annals Phys. 190, 233 (1989).

[2] H. Hatanaka, T. Inami and C. S. Lim, Mod. Phys. Lett. A 13, 2601 (1998).

[3] I. Antoniadis, K. Benakli and M. Quiros, New J. Phys. 3, 20 (2001); G. von Gersdorff, N. Irges and M. Quiros, Nucl. Phys. B 635, 127 (2002); R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B 671, 148 (2003); C. S. Lim, N. Maru and K. Hasegawa, J. Phys. Soc. Jap. 77, 074101 (2008).

[4] K. Hasegawa, C. S. Lim and N. Maru, Phys. Lett. B 604, 133 (2004).

[5] N. Maru and T. Yamashita, Nucl. Phys. B 754, 127 (2006); Y. Hosotani, N. Maru, K. Takenaga and T. Yamashita, Prog. Theor. Phys. 118, 1053 (2007).

[6] C. S. Lim and N. Maru, Phys. Rev. D 75, 115011 (2007).

[7] N. Maru and N. Okada, Phys. Rev. D 77, 055010 (2008); N. Maru, Mod. Phys. Lett. A 23, 2737 (2008).

[8] Y. Adachi, C. S. Lim and N. Maru, Phys. Rev. D 76, 075009 (2007); Phys. Rev. D 79, 075018 (2009).

[9] Y. Adachi, C. S. Lim and N. Maru, Phys. Rev. D 80, 055025 (2009).

[10] C. S. Lim, N. Maru and K. Nishiwaki, Phys. Rev. D 81, 076006 (2010).

[11] Y. Adachi, N. Kurahashi, C. S. Lim and N. Maru, JHEP 1011 150 (2010).

[12] Y. Adachi, N. Kurahashi, C. S. Lim and N. Maru, arXiv:hep-ph/1103.5980.

[13] G. Burdman and Y. Nomura, Nucl. Phys. B 656, 3 (2003).

[14] A. Delgado, A. Pomarol and M. Quiros, JHEP 0001, 030 (2000).

[15] S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977).
[16] G. Martinelli, M. Salvatori, C. A. Scrucca and L. Silvestrini, JHEP 0510, 037 (2005).

[17] G. Cacciapaglia, C. Csaki and S. C. Park, JHEP 0603, 099 (2006).

[18] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973);

[19] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005).

[20] M. Bona et al. [ UTfit Collaboration ], JHEP 0803, 049 (2008).

[21] K. Huitu, S. Khalil, A. Moursy, S.K. Rai and A. Sabanci, arXiv:hep-ph/1105.3087.

[22] S. Bethke, Eur. Phys. J. C64 (2009) 689–703.