Neural Partial Differentiation Based Nonlinear Parameter Estimation from Noisy Flight Data

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Abstract. This paper focuses on the application of neural partial differentiation (NPD) approach to estimate the longitudinal parameters of an aircraft HFB 320 from noisy flight data. By exciting both short period and phugoid modes of an aircraft with thrust variation, the aircraft system dynamics becomes highly nonlinear and aerodynamic parameters appears nonlinear to the state trajectories of velocity, AOA, pitch rate and pitch angle. This paper highlights the application of NPD for such a class of nonlinear dynamics; previously it was used only for the estimation of parameter appearing linear to the states. The extracted the nonlinear longitudinal parameters of HFB 320 aircraft are compared with the parameters estimated by using adaptive Unscented Kalman Filter (UKF) approach. Finally, the estimation results are validated by comparing with flight data and the responses obtained from the estimates by adaptive UKF.

1 introduction

Aircraft modeling and parameter estimation play a vital role in determining its stability and control characteristics of the complex aircraft system. Presently online aircraft modeling method has attracted more and more research attention for the real-time estimation of aircraft parameters in the design of an adaptive controller to maintain flight stability and high performance in an uncertain environment [1], [2], [3], [4]. However, the recently introduced neural partial differential approach is capable to provide theoretical insight into the statistical information (standard deviation and relative standard deviation) of parameter estimates from noisy data. In the presence of an adverse atmospheric condition, aircraft aerodynamic parameters can be estimated accurately from flight data using adaptive unscented Kalman filter [5][6][7]. But this algorithm requires high computational power and initial values of the parameters.

Online estimation of the aircraft aerodynamic parameters from noisy flight data is challenging since aircraft system dynamics are nonlinear and stochastic in nature. Most of the established algorithms like Filtering Methods (FM) and Output Error Method (OEM) are needed prior knowledge of initial values of the parameters and system dynamic models for estimating aircraft system parameters. The Equation Error Method (EEM) does not require initial values of parameters but it restricted to linear system dynamics. Neural networks are widely used to model the nonlinear aircraft system dynamics [8], [9], [10], [11], [12]. Consequently, Delta and Zero methods are used as estimation approaches from neural model [8], [13], [14]. Even though these approaches provide parameter values through the offline analysis, online estimation of aerodynamic parameters from noisy is not feasible with statistical information of the estimated parameters.

Neural Partial Differentiation (NPD) approach is recently introduced to extract derivatives in real time from noisy flight data with their statistics of Relative Standard Deviation (RSTD) [15], [16]. In the application of NPD to flight data, a neural model of an aircraft is primarily established from flight data.
and estimated aerodynamic derivatives from the neural model by partial differentiation. When we consider the excitation of both modes of longitudinal motion namely short period mode and phugoid mode, the velocity of an aircraft varies by the employment of thrust variation. In such a case, aircraft system parameters are appearing nonlinear to the state trajectories of the aircraft and estimating those parameters are hard in nature. In this paper, we explain how estimate aircraft aerodynamic derivatives accurately from noisy can flight data of short period and phugoid mode where aerodynamic derivatives considered as parameters appearing nonlinear to the state trajectories. The main contributions of this paper are:

- Longitudinal flight stability and control parameters of HFB-320 aircraft are estimated from flight data. Neural partial differentiation method is used for this purpose.
- For the precise excitation of the system dynamics, both short period and phugoid modes of aircraft are excited during the maneuvering of flight for the estimation of aircraft aerodynamic parameters in longitudinal dynamics.
- In the case of velocity variation, aircraft system dynamics are highly nonlinear and aerodynamic parameters are appearing nonlinear to the state trajectories of velocity, AOA, pitch rate and pitch angle. Neural partial differentiation method is applied to multi- input single out aircraft system (MISO) to extract the nonlinear parameters of HFB 320 aircraft. The results are compared with the estimates obtained from adaptive UKF.
- The estimated Neural Model of aircraft is verified by comparing the time history of a normal acceleration and pitch rate of flight data with their estimated model responses.

The rest of the paper is organized as follows. Section II presents the problem formulation for the aircraft parameter estimation using Neural Partial Differentiation. The estimation results of aerodynamic derivatives and their validation are discussed in section III. Section IV describes concluding remarks of the paper.

### 2 Aircraft parameter estimation

As the aircraft system dynamics are nonlinear and stochastic in nature, the linearized models without having accurate knowledge of noise characteristics to them may lead to inaccurate results and poor performance. Thus, the feasibility study of adaptive UKF for aircraft parameter estimation is carried out from flight data [5] to addressing the issues related to nonlinearity and unknown noise statistics of covariance. But adaptive UKF algorithm requires initial values of the estimates and shows high computational power. This motivates the use of neural partial differentiation (NPD) method that can provide accurate estimates of aircraft parameters without their initial values. The following section describes how can we apply NPD to estimate nonlinear aircraft parameters from noisy flight data of HFB 320 aircraft (Figure:1). The details of the NPD method are not given in this paper in order to avoid the repetition, and they are referred in [17].

![HFB 320 aircraft](image)

**Figure 1. HFB 320 aircraft**

#### 2.1 Application of NPD to HFB 320 Aircraft.

The model structure of HFB 320 aircraft for the longitudinal dynamics is given by:
Where the lift, drag and pitching moment coefficients \( (C_L, C_D, C_m) \) are modeled as

\[
\begin{align*}
C_D &= C_{D0} + C_{DV} \frac{V}{V_0} + C_{D\alpha} \alpha \\
C_L &= C_{L0} + C_{LV} \frac{V}{V_0} + C_{L\alpha} \alpha \\
C_m &= C_{m0} + C_{mV} \frac{V}{V_0} + C_{mq} + \frac{q\rho}{2V_0} + C_{ms} \delta_r 
\end{align*}
\]

Where \( V_0 \) is initial value of \( V \). The dynamic pressure \( \tilde{\rho}(=0.5\rho V^2) \) contains state variable \( V \) which introduces nonlinearity in estimates process as it multiplies with all unknown parameters. To obtain aerodynamic parameters \( C_{m0}, C_{mV}, C_{mq}, C_{ms} \) and \( C_{m}\), Substitute \( C_m \) from (2) in \( \dot{q} \) in (1).

Thus,

\[
\dot{q} I_y = F_r \left( l_r \sin \sigma_T + l_c \cos \sigma_T \right) = V^2 C_{m0} + C_{mq} \alpha + \frac{q\rho}{2V_0} + C_{ms} \delta_r V^2
\]

\[\text{according to (3). For obtaining the parameters } C_{L0}, C_{LV}, C_{D0}, C_{DV} \text{ and } C_{D\alpha}, \text{ the equation for acceleration along z-axis is given below:}
\]

\[
a_{mz} = \frac{\tilde{q}S}{m} C_z + \frac{F_r}{m} \sin \sigma_T
\]

Where,

\[
C_z = -C_{l} \cos \alpha - C_{r} \sin \alpha
\]

Therefore, from (2) and (5) in (4):

\[
\frac{m a_{mz} - F_r \sin \sigma_T}{0.5\rho S} = -C_{l0} V^2 \cos \alpha - C_{LV} \frac{V^3}{V_0} \cos \alpha - C_{D0} V^2 \alpha \sin \alpha
\]

The left hand side of the above equation can be computed from flight data and used as the output of other neural network with model structure of the expression in right hand side of equation (6). With
use of the states are $V$ and $\alpha$, the inputs to the same neural network are $V^2, \alpha V^2, qV^2, \delta V^2$ and $V^3$ according to (6).

![Figure 2. The Schematics of Neural Network](image)

The schematics of such a neural network is given in fig.2 contain two hidden layers with inputs $A_0, A_1, A_2, A_3$ where $A_0$ is the bias [17].

\[
\begin{align*}
A_1 &= V^2 \\
A_2 &= \alpha V^2 \\
A_3 &= \frac{qV^2}{2V_0} \\
A_4 &= \delta V^2 \\
A_5 &= \frac{V^3}{V_0}
\end{align*}
\]  

From the Figure. 2, the output $D$ is the sum of products of total inputs from hidden layer 2 and their respective weights of each neuron. Thus,

\[ D = W^T C \]  

Where,

\[ W = \begin{pmatrix} b_{a1} \\ w_1 \\ \vdots \\ w_j \end{pmatrix} \]  

Similarly,

\[
\begin{align*}
C &= f \left(V^T B \right) \\
B &= g \left(U^T A \right)
\end{align*}
\]

$f$ and $g$ are activation functions of first and second hidden layers respectively. $f(x)$ is expressed as:

\[ f(x) = \frac{1-e^{-x}}{1+e^{-x}} \]  

The weight matrices are represented as:
\[ V = \begin{pmatrix} b_{i1} & \ldots & b_{il} \\ v_{i1} & \ldots & v_{il} \\ \vdots & \ddots & \vdots \\ v_{nl} & \ldots & v_{nl} \end{pmatrix} \] (12)

\[ U = \begin{pmatrix} b_{m1} & \ldots & b_{ml} \\ u_{m1} & \ldots & u_{ml} \\ \vdots & \ddots & \vdots \\ u_{nm} & \ldots & u_{nm} \end{pmatrix} \] (13)

The input and output values are scaled to the range of -0.9 to 0.9. From the above equations, the output can be expressed as:

\[ D = \left[ W^T f\left[ V^T g\left( U^T A \right) \right] \right] \] (14)

Partial differentiation of output of each layers are given by

\[ \frac{\partial D}{\partial C} = W^T \] (15)

\[ \frac{\partial C}{\partial B} = f'(V^T) \] (16)

\[ \frac{\partial B}{\partial A} = g'(U^T) \] (17)

Multiplying eq. (15), eq. (16) and eq. (17) gives,

\[ \begin{align*}
\begin{pmatrix}
\frac{\partial D}{\partial C} & \frac{\partial C}{\partial B} & \frac{\partial B}{\partial A} \\
\end{pmatrix} &= \begin{pmatrix}
W^T & f'(V^T) & g'(U^T) \\
\end{pmatrix} \\
\frac{\partial D}{\partial A} &= \begin{pmatrix}
W^T & f'(V^T) & g'(U^T) \\
\end{pmatrix}
\end{align*} \] (18)

Where, \( f' = \text{diag}\left[ 0, f'_1, \ldots, f'_l \right] \) and \( g' = \text{diag}\left[ 0, g'_1, \ldots, g'_m \right] \)

If the input and output of neural network are normalized, then

\[ \frac{\partial D}{\partial A} = \frac{\partial D}{\partial D_{\text{norm}}} \times \frac{\partial D_{\text{norm}}}{\partial A_{\text{norm}}} \times \frac{\partial A_{\text{norm}}}{\partial A} \] (19)

The normalized output of neural network can be denormalized by eq. (19). Where,

\[ \frac{\partial A}{\partial A_{\text{norm}}} = \begin{pmatrix}
1 & 0 & \ldots & 0 \\
0 & \partial A_{1} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \partial A_{l}
\end{pmatrix} \] (20)

The terms associated of eq. (15) to (20) be intermediate terms of neural networks while getting it trained. Therefore, there is no extra computation required to compute the aerodynamic derivatives, and they are directly given as:

\[ \frac{\partial D}{\partial A} = \begin{pmatrix}
\frac{\partial D}{\partial A_{1}} & \ldots & \frac{\partial D}{\partial A_{l}}
\end{pmatrix} \] (21)
From eq. (3) and eq. (7), the output of neural network is:

\[ D = C_{\alpha 0}A_{1} + C_{\alpha 1}A_{2} + C_{\alpha 2}A_{3} + C_{\alpha 3}A_{4} + C_{\alpha 4}A_{5} \] (22)

Therefore,

\[ \frac{\partial D}{\partial \alpha} = \begin{bmatrix} C_{\alpha 0} & C_{\alpha 1} & C_{\alpha 2} & C_{\alpha 3} & C_{\alpha 4} \end{bmatrix} \] (23)

Similarly, the parameters \( C\alpha_0, C\alpha_1, C\alpha_2, C\alpha_3, C\alpha_4 \) can be estimated easily by designing a neural network based on eq. (6). The standard deviation of estimated parameters in eq. (21) is computed by

\[ STD = \sqrt{\frac{1}{P} \sum_{p=1}^{P} \sum_{m=1}^{M} \left( \sum_{i=1}^{L} C_{\alpha 0}w_{i}D_{i}B_{\alpha 0}u_{m} - AVG \right)^{2}} \] (24)

Where,

\[ AVG = \frac{1}{P} \sum_{p=1}^{P} \sum_{m=1}^{M} \left( \sum_{i=1}^{L} C_{\alpha 0}w_{i}D_{i}B_{\alpha 0}u_{m} \right) \] (25)

Where, STD and AVG are standard deviation and average of data points, respectively. The relative standard deviation of estimates is given by:

\[ RSTD = \frac{STD}{AVG} \times 100\% \] (26)

2.2 Adaptive Unscented Kalman Filter

Parameter estimation of a nonlinear system can be accomplished using Unscented Kalman Filter (UKF) [18]. But the process noise covariance is computed beforehand through Filter Error Method (FEM) [5]. FEM is an offline algorithm which has the potential to estimate noise covariance in addition to system parameters. Whereas, the selection of process noise covariance is carried out in an adaptive UKF algorithm, which consists of two parallel filters referred to as master and slave. The master UKF estimates both states and aerodynamic parameters of aircraft while the slave one estimates the
diagonal elements the noise covariance matrix for the master UKF. The adaptive UKF approach has been introduced for the purpose of estimating aerodynamic derivatives from flight data of HFB 320 [5].

Figure 4. Time history of input signals to the neural network

3 Parameter estimation results and discussion
Aircraft parameter estimation was carried out with open accessible flight test data of the research aircraft HFB-320 aircraft. The flight tests were conducted to excite the longitudinal motion of research aircraft through a multistep elevator input of 3-2-1-1 resulting in short period motion and a long duration pulse input leading to phugoid motion [5], [19]. The resulting flight data is corrected the flow angle sensor corrections through the flight path reconstruction [20], and ready to use for estimating aircraft longitudinal parameters to quantify the aircraft lift, drag, and pitching moment coefficients.

| Parameters | NPD | Adaptive UKF |
|------------|-----|---------------|
| $C_{m0}$  | 0.1227 (1.2757)* | 0.1115 (4.29) |
| $C_{ma}$  | -1.067 (0.9246) | -0.9703 (1.54) |
| $C_{mq}$  | -38.899 (1.1909) | -35.3628 (2.82) |
| $C_{mse}$ | -1.6934 (0.930) | -1.5395 (1.65) |
| $C_{mV}$  | 0.0051 (7.3542) | 0.0045 (92.19) |
| $C_{l0}$  | -0.0783 (4.1629) | -0.0871 (22.91) |
| $C_{lV}$  | 0.1321 (2.0504) | 0.1468 (11.42) |
| $C_{la}$  | 3.8604 (0.4076) | 4.2894 (1.14) |
| $C_{d0}$  | 0.1103 (1.4113) | 0.1226 (2.64) |
| $C_{dV}$  | -0.0578 (3.3394) | -0.0642 (4.27) |
| $C_{dla}$ | 0.2877 (0.8204) | 0.3197 (2.40) |

* The values in parenthesis denote relative standard deviation values in percentage.
A prior information of the postulated model structure of aircraft longitudinal dynamics is required to extract the parameters using neural partial differentiation method from a neural model of an aircraft. For estimating aerodynamic derivatives of $C_{m0}$, $C_{mV}$, $C_{m\alpha}$, and $C_{m\delta}$, we can use eq.(22) as the model structure for estimation algorithm, and neural model of aircraft can be established with inputs of $A_i, i = 1, 2, 3, 4, 5$ and output of network computed from the left hand side of the eq.(3). Similarly, we can establish another neural model of the aircraft with output computed from left hand side of eq.(6) and inputs of $-V^2 \cos \alpha, -V^2 \cos \alpha/V_0, -V^2 \sin \alpha, -V^2 \sin \alpha/V_0$ and $-V^2 \alpha \sin \alpha$. Subsequently, aerodynamic parameters $C_{L0}, C_L V, C_{L\alpha}, C_{L\delta}, C_{D0}, C_{DV}$ and $C_{D\alpha}, C_{D\delta}$ are estimated by the application of NPD from the established neural model.

Figure 3 shows the established neural models with respective eq. (3) and (6), and the extracted aircraft longitudinal parameters are tabulated in Table 1 with their relative standard deviations. The estimated parameter values were found to attain a stable value as the number of iterations increased as shown in figures 7, 8, where the variation of parameters with number of iterations is given. Aircraft estimated parameter variation with respect to the flight data points are also plotted in figures 5, 6. The estimated results are compared with those obtained from the adaptive UKF algorithm to demonstrate the efficacy of proposed approach. Figure 9 shows the comparative responses of pitch rate and normal acceleration measurements with those reconstructed from the parameters estimated by NPD and adaptive UKF. This estimation results confirms that NPD approach has potential to estimate aerodynamic derivatives from flight data where parameters are appearing nonlinear to the aircraft states in nonlinear dynamics.

![Figure 5. Variation of Parameters w.r.t. data points](image-url)
4 Conclusions
The NPD method is applied to the flight data of high performance HFB 320 aircraft for the estimation of the longitudinal aerodynamic derivatives. For this, the aircraft was inspired by the multistep input and long duration pulse to the elevator for the excitation of short period and phugoid modes of aircraft. Therefore, the velocity of the aircraft varies with thrust variation to accommodate the phugoid mode of aircraft. This makes the aircraft system dynamics highly nonlinear and aerodynamic parameters appear nonlinear to the state trajectories of velocity, AOA, pitch rate and pitch angle.
We have established the neural model of an aircraft with a suitable selection of appropriate inputs and output of the network. Moreover, a separate consideration of neural network is made for the estimation of parameters from the expression of $\dot{q}$ and $a_z$. Longitudinal derivatives of Aircraft are estimated from flight data using the NPD method, and comparable with estimates obtained from the adaptive UKF. This shows that the NPD method has become the preferred approach to estimate parameters as it does not require the initial value of estimates and less computation. As the initial values of parameters are not available in a practical situation as well as filtering approach requires these initial parameters with high computational cost, application of the NPD approach gives an advantage over adaptive UKF to estimate aerodynamic parameters from flight data.

5. References

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