Gauge Hierarchy in $SU(3)_c \times SU(3)_L \times SU(3)_R$ and Low Energy Implications*

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Abstract

We explore the gauge hierarchy problem within the framework of supersymmetric $SU(3)_c \times SU(3)_L \times SU(3)_R$ with a minimal set of higgs supermultiplets. Imposition of a suitable discrete (alternatively $R$) symmetry ‘prevents’ the electroweak higgs doublets from becoming superheavy through renormalizable couplings. A full resolution of the problem requires consideration of the non-renormalizable couplings which play an essential role. Other key differences from the minimal supersymmetric $SU(5)$ model include the fact that the proton is stable and that an effective $5 + \bar{5}$ supermultiplet appears around the $TeV$ mass scale.

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The presently measured gauge couplings of the standard model, when extrapolated to higher energies with supersymmetry (SUSY) becoming relevant at scales $\sim 100 \, GeV$ - few $TeV$, appear to merge together at scales around $10^{16} \, GeV^{(1)}$. This certainly is a boost for ideas based on supersymmetric grand unification,$^{(2)}$ with SUSY $SU(5)$ or $SO(10)$ being the obvious candidates. However, a potential drawback for SU(5) type models is that they cannot be ‘embedded’ in any ‘straightforward’ superstring approach, which has led to renewed interest in gauge groups such as $G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$. The gauge group $G$ not only emerges from the simplest superstring theories$^{(3)}$ but has the potential, as was recently emphasized$^{(4)}$, to retain one of the outstanding features of minimal SUSY $SU(5)$ namely perturbative unification of the gauge couplings consistent with the measured value of $\sin^2 \theta_W (M_Z)$. Furthermore$^{(4)}$, in contrast to SUSY $SU(5)$, a simple discrete symmetry when appended to $G$, stabilizes the proton by eliminating both the dimension five and six baryon number violating operators.

The main purpose of this letter is to address the gauge hierarchy problem in the framework of $G^{(5,6,7)}$. We would like to resolve this normally difficult issue within the minimal scheme, doing away with the undesirable fine tuning in the process. This goal, it turns out, can be accomplished by introducing an additional discrete symmetry, which is compatible with the discrete symmetry needed to stabilize the proton and the lightest supersymmetric particle (LSP). [An alternative approach relies on the R-symmetry.] The presence of the discrete symmetry (or R-symmetry) provides for flat directions in the exact supersymmetry limit, which get slightly lifted after including the su-
pertsymmetry breaking effects. The full resolution of the gauge hierarchy problem, it turns out, necessitates a discussion of the non-renormalizable interactions. In one particular scenario the non-renormalizable couplings play the decisive role, both in generating the superheavy mass scale as well as resolving the gauge hierarchy problem. A particularly striking prediction is the presence of a relatively light (\(\sim T eV\) or less) supermultiplet which is an effective \(5 + \bar{5}\) of \(SU(5)\).

Under the gauge group \(G\) the left handed lepton, quark and antiquark superfields respectively transform as \((1, 3, 3), (3, 3, 1)\) and \(3, 1, 3)\). They are denoted as \(\lambda_a, Q_a\) and \(Q^c_a (a = 1, 2, 3)\):  

\[
\begin{align*}
\lambda_a &= \begin{pmatrix} H^{(1)} & H^{(2)} & L \\ E^c & \nu^c & N \end{pmatrix} \\
Q_a &= \begin{pmatrix} u \\ d \\ g \end{pmatrix} \\
Q^c_a &= \begin{pmatrix} u^c \\ d^c \\ g^c \end{pmatrix}
\end{align*}
\]

\(H^{(1)}, H^{(2)}, L\) denote \(SU(2)_L\) doublet superfields, \(N\) and \(\nu^c\) are standard model singlets, while \(g(g^c)\) is an additional down-type quark (antiquark) superfield.

The symmetry breaking of \(G\) to \(SU(3)_c \times U(1)_{em}\) requires at least two sets of higgs supermultiplets, denoted by \(\lambda + \bar{\lambda}\) and \(\lambda' + \bar{\lambda}'\). Note that \(\lambda, \lambda'\) transform the same way under \(G\) as \(\lambda_a\). The conjugate fields \(\bar{\lambda}, \bar{\lambda}'\) are necessary to ensure that the supersymmetry breaking scale is well below the GUT scale. The superheavy vacuum expectation values (vevs) are along the \(N(\bar{N})\) direction of \(\lambda(\bar{\lambda})\) and \(\nu^c(\bar{\nu}^c)\) direction of \(\lambda'(\bar{\lambda}')\), and break \(G\) to supersymmetric \(SU(3)_c \times SU(2)_L \times U(1)\). One of the main challenges posed
by the gauge hierarchy problem is to ensure the survival of a pair of ‘light’ 
$SU(2)_L$ scalar doublets (say $H^{(1)}, H^{(2)}$ from $\lambda$) to accomplish the electroweak 
breaking. As might be expected, this is not possible without postulating 
additional symmetries.

Let us begin by considering the single pair of higgs superfield $\lambda^A + \bar{\lambda}^A$, 
where Greek and Latin indices respectively refer to $SU(3)_L$ and $SU(3)_R$. The 
most general renormalizable superpotential invariant under $G$ is

$$W_1 = M\lambda^A \bar{\lambda}^A + c \epsilon^{\alpha\beta\gamma} \epsilon_{ABC} \lambda^A_\alpha \lambda^B_\beta \lambda^C_\gamma$$

Note that the constants $c$ and $\bar{c}$ are not equal, and there is no symmetry under 
the interchange $\lambda \leftrightarrow \bar{\lambda}$. There are two supersymmetric vacua corresponding 
to (2):

(i) $\langle \lambda \rangle = \langle \bar{\lambda} \rangle = 0$, with $G$ unbroken;

(ii) $\langle \lambda \rangle \propto \text{diag}(1, 1, 1)$ 

$\langle \bar{\lambda} \rangle \propto \text{diag}(1, 1, 1)$

with $G$ broken to the diagonal subgroup $SU(3)_{L+R}$.

Consequently, to obtain the desired breaking $G \rightarrow SU(2)_L \times SU(2)_R \times 
U(1)$ we must introduce an additional singlet superfield $S$. The new super-
potential takes the form (we suppress all indices and $\epsilon$ symbols from now 
on):

$$W_2 = fS(\lambda\bar{\lambda} - M^2) + \frac{\kappa}{2}S^2 + \frac{h}{3}S^3 + c\lambda^3 + \bar{c}\bar{\lambda}^3$$

There now exist three supersymmetric vacua:
Clearly it is (iii) which is of interest to us. The vevs are along the $N(\bar{N})$ directions of $\lambda(\bar{\lambda})$, and $G$ is broken to $SU(2)_L \times SU(2)_R \times U(1)$. Unfortunately, the presence of the term proportional to $\lambda^3$ in (4) guarantees (!) that the $SU(2)_L$ doublet pair $H^{(1)} - H^{(2)}$ in $\lambda$ is superheavy. This term gives rise to the coupling $H^{(1)} H^{(2)} N$, thereby eliminating the desired pair from the low energy spectrum.

In order to obtain the ‘light’ electroweak doublets the $\lambda^3$ term should therefore be eliminated. For instance, a $Z_2$ symmetry under which $\lambda \rightarrow -\lambda$ can accomplish this (An alternative approach relies on $R$ symmetry. We will discuss it after completing this case). Consider therefore the superpotential

$$W_3 = fS(\lambda\bar{\lambda} - M^2) + \frac{\kappa}{2}S^2 + \frac{h}{3}S^3$$

where, for the moment, $\bar{\lambda} \rightarrow -\bar{\lambda}$ and $S \rightarrow S$ under $Z_2$. This system possesses an ‘accidental’ global pseudosymmetry $SU(9)$, a subgroup $SU(3)_L \times SU(3)_R$ of which is gauged. The vevs in $\lambda(\bar{\lambda})$, which provide the correct breaking of the local gauge symmetry, also spontaneously break $SU(9)$ to $SU(8)$, resulting in some pseudo-Goldstone superfields, which include the $H^{(1)} - H^{(2)} (\bar{H}^{(1)} - \bar{H}^{(2)})$ pair from $\lambda(\bar{\lambda})$.

To minimize the number of ‘light’ doublets, we restore the $\bar{\lambda}^3$ term to the superpotential. That is, we require that $\bar{\lambda} \rightarrow \bar{\lambda}$ and $S \rightarrow -S$ under $Z_2$. The
superpotential is now given by

$$W_4 = fS\lambda\bar{\lambda} + \frac{\kappa}{2}S^2 + c\bar{\lambda}^3$$  \hspace{2cm} (7)$$

In the supersymmetric limit, we have

$$< S > = < \bar{\lambda} > = 0$$

$$< \lambda > = \text{diag}(\lambda, \lambda, \lambda), \text{ with } \lambda \text{ undetermined}$$  \hspace{2cm} (8)$$

However, in the presence of supersymmetry breaking, the potential takes the form $V$

$$V = \sum_{Z_i = \lambda, \bar{\lambda}, S} \left| \frac{\partial W_4}{\partial Z_i} \right| + m_3\lambda_i^2[A - 3][W_4 + W_4^*] + D \text{ terms}$$  \hspace{2cm} (9)$$

where $m_3 (\sim \text{TeV})$ denotes the gravitino mass. For simplicity, let us put $A = 3$. There now exists a minimum given by

$$< S > = -m_3/f$$

$$< \lambda > = < \bar{\lambda} > = \sqrt{\frac{\kappa + m_3}{f^2}} \text{ diag}(0, 0, 1)$$  \hspace{2cm} (10)$$

For $\kappa \sim M_p (\sim 1.2 \times 10^{19} \text{ GeV})$, $|< \lambda >| \sim M_X \sim 10^{16} \text{ GeV}$ with $f \sim 10^{-5}$. Only a single pair of electroweak doublets (primarily from $\lambda$) is now ‘massless’, as desired.

Note that in order to generate fermion masses, we should allow at least some trilinear couplings of the form $\lambda_a\lambda_b\lambda_c$, where $\lambda_{a,b}$ denote the matter superfields. This is readily accomplished by embedding $Z_2$ in a larger symmetry, which we take to be $Z_4$. In contrast to $\lambda, \bar{\lambda}$, some of the matter fields
transform as faithful representations of $Z_4$. The $<\lambda>, <\bar{\lambda}>$ vevs, as well as $<S>$, spontaneously break the $Z_4$ symmetry to $Z_2$ which is just matter parity.

Next let us include the $\lambda' - \bar{\lambda}'$ sector. The superheavy vev here has to be along the $\nu'^c - \bar{\nu'^c}$ direction, and we also must ensure that the $\lambda - \lambda'$ couplings leave intact the ‘light’ pair found above. In the superpotential we therefore allow $\lambda'^3, \bar{\lambda}'^3$ couplings, but not for instance $\lambda'\lambda\lambda$. This is most simply achieved through a $Z_3$ symmetry under which

$$\lambda' \rightarrow \alpha\lambda', \bar{\lambda}' \rightarrow \alpha^2\bar{\lambda}'$$

with all other fields invariant. We therefore have a $Z_{12}(\simeq Z_4 \times Z_3)$ symmetry which acts as an identity on $\bar{\lambda}$, as $Z_2$ on $\lambda$ and $S$, as $Z_3$ on $\lambda', \bar{\lambda}'$, and as $Z_4$ on the matter fields $\lambda_a, Q_a, Q'_a$. The most general renormalizable higgs superpotential (with no $\lambda - \lambda'$ coupling) is given by

$$W(\lambda, \lambda', S, S') = fS\lambda\bar{\lambda} + \frac{\kappa}{2} S^2 + \bar{c}\lambda^3$$

$$+ f' S'(\lambda'\bar{\lambda'} - M_1^2) + \frac{M'}{2} S'^2 + \frac{4}{3} S'^3 + a\lambda'^3 + \bar{a}\bar{\lambda}'^3$$

$$+ \text{(possible } S^2S' \text{ term)}$$

Here $S'$ is another $G$ singlet field, analogous to the $S$ field, and in the absence of SUSY breaking it has zero vev. However, with the SUSY breaking switched on, $<S'> \sim m_3$. Note that a possible mass term proportional to $\lambda'\bar{\lambda'}$ is absorbed in the redefinition of $S'$.

The superpotential in (12) possesses a global pseudosymmetry $[SU(3)_L \times SU(3)_R]^2$ associated with the $\lambda - \bar{\lambda}$ and $\lambda' - \bar{\lambda'}$ sectors. Consequently, there
are additional ‘pseudogoldstone’ superfields associated with the spontaneous breaking of this larger symmetry. They include $H^{(2)'}$, $H^{(2)''}$, $(E^c, N')$ and $(\bar{E}^c, \bar{N}')$. However, the $E^c$ and $\bar{E}^c$ superfields are absorbed in the breaking of the gauge symmetry $SU(2)_R$ by $<\nu^c> = <\bar{\nu}^c>^* \neq 0$.

In order to ensure that the superheavy vevs of $\lambda$ and $\lambda'$ are respectively along the ‘orthogonal’ directions $N$ and $\nu^c$, we supplement (12) with an additional term $Z\bar{\lambda}\lambda'$, where $Z$ denotes a $G$ singlet superfield carrying the appropriate $Z_4$ and $Z_3$ quantum numbers. Note that the presence of the singlet superfield $Z$ eliminates the $N'$ field from the low energy spectrum.

Let us summarize the discussion so far. In the lepton sector we have the three chiral matter superfields $\lambda_a$, while the (minimal) superhiggs sector consists of $\lambda + \bar{\lambda}$ and $\lambda' + \bar{\lambda}'$. We found that a discrete symmetry $Z_4 \times Z_3$ is necessary so that the $\lambda$ sector, which acquires a superheavy vev along $<N>$ can deliver a pair of ‘light’ electroweak doublets. The $\lambda' - \bar{\lambda}'$ sector acquires superheavy vev along the $\nu^c - \bar{\nu}^c$ direction. [These vevs leave unbroken a discrete $Z_2 \times Z_3'$ symmetry.] The ‘low energy’ lepton-higgs sector essentially coincides with the minimal supersymmetric standard model (MSSM), with one important difference. There is an additional pair of $SU(2)_L$ doublet pseudogoldstone superfields.

As indicated earlier (see remarks immediately preceding eq. (6)), a somewhat different way of arriving at a pair of ‘light’ higgs supermultiplets from the $\lambda$ sector relies on $R$-symmetry. Under the $R$-symmetry
$$S \rightarrow e^{iRS}$$
$$\lambda \rightarrow e^{-\frac{iR}{2}}\lambda$$
$$\bar{\lambda} \rightarrow e^{\frac{iR}{2}}\bar{\lambda}$$

the superpotential $W_2 \rightarrow e^{iR}W_2$, provided that $\kappa = h = 0$. The superpotential $W_2$ now reduces to

$$W'_2 = fS(\lambda\bar{\lambda} - M^2) + \bar{c}\bar{\lambda}^3$$

With SUSY unbroken, the ground state is given by

$$<\lambda> = \text{diag}(u, u, M^2/x)$$
$$<\bar{\lambda}> = \text{diag}(0, 0, x)$$
$$<S> = 0$$

where, from the $D$ terms, we have the constraint

$$|u|^2 = |x|^2 \left(\frac{M^4}{|x|^4} - 1\right)$$

The potential is flat in the direction $u \rightarrow \infty$ and the electroweak doublet pair is ‘massless’.

A complete discussion of this case, including the $\lambda' - \bar{\lambda}'$ sector, will not be attempted here. We do, however, wish to mention a notable difference from the previous ($Z_4 \times Z_3$) case. The gauge symmetry $SU(3)_L \times SU(3)_R$ has been broken at a superheavy scale (see 15), with SUSY unbroken. Among other things, this implies a superheavy mass for the excitation of the $N$ field about the minimum. [In the $Z_4 \times Z_3$ case the corresponding mass is on the order of
the SUSY breaking scale which, without proper care, may be cosmologically troublesome.]

Let us now return to the lepton-higgs sector and discuss in more detail the ‘low energy’ spectrum. After taking into account the superhiggs mechanism, the pseudogoldstone states include a pair of $SU(2)_L$ doublets as well as two standard model singlets. More explicitly, they are the states $< \nu'_c > L - < N > H'_2$, $< \bar{\nu}'_c > \bar{L} - < \bar{N} > \bar{H}'_2$, $< \nu'_c > \nu'_c - < N > N'$ and $< \bar{\nu}'_c > \bar{\nu}'_c - < \bar{N} > \bar{N}'$. As indicated earlier, after SUSY breaking, the $S'$ field in (12) acquires a non-zero vev of order $m_3$. As a consequence, the fermionic components of the additional doublet pair acquire mass of order $m_3$. One linear combination of the doublet scalar fields also acquires mass of order $m_3$. However, the orthogonal component, which is the true pseudogoldstone field, acquires mass only through radiative corrections involving the gauge interactions. [Note that the soft SUSY breaking terms respect the pseudosymmetry.] The leading one loop contributions to the mass of this field turns out to be on the order of $100 - 150$ GeV (with $m_3 \sim$ TeV).

We next consider the all important issue related to the gauge hierarchy problem, to wit, the ‘$\mu$ term’ ($\mu H^{(1)} H^{(2)}$) of the minimal supersymmetric standard model. The absence of the $\lambda^3$ term ensures that, in the absence of SUSY breaking, $\mu$ is zero at tree level. After SUSY breaking, the ‘effective’ $\mu$ term turns out to be at most of order $10^{-3}$ GeV or less, a value too small to lead to viable low energy models. [For instance, there would be an unwanted axion. Also, constraints from LEP appear to require $|\mu| \gtrsim 50$ GeV.] In order to overcome this problem we must consider the contributions to $\mu$ from the non-renormalizable terms. [The reader may wonder about the contribution
to \( \mu \) from the ‘hidden’ sector. See for instance ref. (9). It turns out that with the minimal ‘hidden’ sector the problem is unresolved and we prefer to search for a solution within the ‘known’ sector.

Let us first consider the \( Z_4 \times Z_3 \) case. The leading (quartic) non-renormalizable terms include the following:

\[
\frac{\gamma_1 (\bar{\lambda}\lambda)^2}{M_P}, \frac{\gamma_2 (\lambda \bar{\lambda}\lambda \bar{\lambda})}{M_P}
\]

The presence of the first term in (17), in particular, gives rise to \( \mu \sim \gamma_1^2 M_X^3 / M_P^2 \sim TeV \) for \( \gamma_1 \sim 10^{-3.5} \). To implement the scenario including the non-renormalizable terms in the most economical way, it is convenient to consider the following superpotential

\[
W = \bar{c} \bar{\lambda}^3 + \frac{1}{2M_P} [\gamma_1 (\bar{\lambda}\lambda)^2 + \gamma_2 (\lambda \bar{\lambda}\lambda \bar{\lambda})] + M' \lambda \bar{\lambda}' + a \lambda'^3 + \bar{a} \bar{\lambda}'^3 + \frac{1}{2M_P} [\beta_1 (\lambda' \bar{\lambda}')^2 + \beta_2 (\lambda' \bar{\lambda}' \lambda' \bar{\lambda}')] \quad (18)
\]

Note that in the presence of non-renormalizable terms, the two singlets \( S, S' \) in eq. (12) can and indeed have been dropped! For a related approach see ref. (10).

In the SUSY limit, we find the minimum

\[
< \nu'_c >= < \bar{\nu}'_c >= \left( \frac{-M' M_P}{\beta_1 + \beta_2} \right)^{\frac{1}{2}} \sim M_X, \quad (19)
\]

provided \( M' \sim m_3^2 \) and \( \beta_i (i = 1, 2) \sim (M_P m_3^2)^{\frac{1}{2}} / M_X \). After SUSY breaking (see (9)) we obtain the correct minimum also in the \( \lambda - \bar{\lambda} \) sector:
\[ < N > =< \bar{N} > = \left\{ (-A - 1 - [(A + 1)^2 - 12]^\frac{1}{2}) \frac{m_3 M_P}{6(\gamma_1 + \gamma_2)} \right\}^{\frac{1}{2}} \sim M_X \]  

(20)

where \( \gamma_i \sim \beta_i(i = 1, 2) \). Note that in (20), \( A - 3 > 0 \) and \( m_3 M_P / (\gamma_1 + \gamma_2) < 0 \).

For the reader who is uncomfortable with \( M' \sim m_3^2 \) in (18), an alternative is to re-introduce in the superpotential a singlet \( S' \). One can now obtain the right minimum even with \( M' \sim M_X \).

What about the higher order non-renormalizable terms? The dominant quintic contribution to \( \mu \) allowed by the \( Z_{12} \) symmetry arises from the coupling \( \delta(\lambda\bar{\lambda})(\lambda^3)/M_P^2 \). The constraint \( \mu \lesssim \mathcal{O}(T eV) \) requires that \( \delta \lesssim \gamma_i^2, \beta_i^2 \sim 10^{-7} \). The constraints on higher order non-renormalizable terms turn out to be much less restrictive.

The R-symmetry case offers the intriguing possibility of eliminating the quartic terms in the superpotential, thus leaving only the \( \delta \) term above as the dominant contribution to the ‘\( \mu \) term’. The idea would be to have the renormalizable part of the superpotential respect the full R-symmetry, while the non-renormalizable contributions are required to be invariant only under its discrete ‘non-anomalous’ subgroup (the well-known R-parity). We will not pursue this any further here, but focus instead on an intriguing new possibility obtained by combining \( Z_4 \times Z_3 \) with R-parity.

Consider then a general superpotential, including all possible non-renormalizable terms, which is invariant under \( Z_4 \times Z_3 \times \) R-parity. All superfields, as well as the superpotential, change sign under the action of R. The other charges of the superfields remain as before except for \( \bar{\lambda} \), which now transforms into \( \alpha^2\bar{\lambda} \) under \( Z_3 \). The vacuum structure of this theory is most
unusual. It is readily checked that there exists simultaneously F-flat and D-flat directions which correspond to the desired symmetry breaking pattern. To wit,

\[
| < \lambda > | = | < \bar{\lambda} > | = N \ \text{(undetermined)} \\
| < \lambda' > | = | < \bar{\lambda}' > | = \nu_c' \ \text{(undetermined)}
\]

Furthermore, the lowest dimensional operator in this theory which contributes to the ‘effective \( \mu' \) term of the electroweak doublets takes the form \( \lambda^3(\lambda\bar{\lambda})^3/M^6 \), where \( M \) denotes an appropriate superheavy scale. With \( < \lambda > = < \bar{\lambda} > \sim 10^{16} \text{ GeV} \) and \( M \sim 10^{18} \text{ GeV} \) (reduced Planck mass), one obtains a value for \( \mu \) in the right ball park, without assuming any small coefficients!

To summarize, a \( Z_4 \times Z_3 \times \text{R-parity} \) invariance appended to \( (SU(3))^3 \) is an example of a theory with a unique (although flat in the SUSY limit) vacuum, with the right symmetry breaking and light doublets. This is based on the most general allowed superpotential including non-renormalizable terms. It is unlikely that \( SU(5) \) or \( SO(10) \) can share this property. The qualitative difference is that for \( (SU(3))^3 \), all of the vevs can be extracted from the fundamental representation, and also that its epsilon tensor is odd.

Before concluding, we briefly address the issue of the unification of the gauge couplings in this scheme and an important low energy consequence.

The strongly interacting sector consists of the usual quark superfields as well as additional charge \(-\frac{1}{3}\) color triplets (one pair of \( g - g_c \) per chiral family). We expect that two of them, corresponding say to the second and third families, acquire masses on the order of or close to the GUT scale. However, in order to compensate in the renormalization group equations for
the additional doublet pair found above, one pair of $g - g_c$ should remain light, of order TeV or so. (The discrete symmetries introduced above can accomplish this.) The successful $SU(5)$ prediction of $\sin^2 \theta_W$, consistent with unification of the three couplings at $M_X \sim 10^{16} \text{GeV}$, would then be retained. These as well as other issues, including the general problem of fermion masses and mixings, will be addressed in more detail elsewhere.

In conclusion, we have considered a supersymmetric $SU(3)_c \times SU(3)_L \times SU(3)_R$ framework which retains the ‘good’ features of standard supersymmetric GUTs, perturbative unification of the gauge couplings consistent with a successful prediction of $\sin^2 \theta_W$. Furthermore, it allows for a resolution of the gauge hierarchy problem with the minimal number of Higgs supermultiplets. The proton and the LSP are stable in this approach, and one expects to find new particles in the $\text{TeV}$ range beyond those predicted in the minimal supersymmetric standard model.

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