A Naturally Small Cosmological Constant on the Brane?

C.P. Burgess\textsuperscript{a}, R.C. Myers\textsuperscript{a} and F. Quevedo\textsuperscript{b}

\textsuperscript{a} Physics Department, McGill University, Montréal, Québec, Canada H3A 2T8.
\textsuperscript{b} D.A.M.T.P., Silver Street, Cambridge, CB3 9EW, England.

There appears to be no natural explanation for the cosmological constant’s small size within the framework of local relativistic field theories. We argue that the recently-discussed framework for which the observable universe is identified with a p-brane embedded within a higher-dimensional ‘bulk’ spacetime, has special properties that may help circumvent the obstacles to this understanding. This possibility arises partly due to several unique features of the brane proposal. These are: (1) the potential such models introduce for partially breaking supersymmetry, (2) the possibility of having low-energy degrees of freedom which are not observable to us because they are physically located on a different brane, (3) the fundamental scale may be much smaller than the Planck scale. Furthermore, although the resulting cosmological constant in the scenarios we outline need not be exactly zero, it may be suppressed relative to the mass splittings of supermultiplets by weak coupling constants of gravitational strength, in accord with cosmological observations.

I. INTRODUCTION

There is no understanding, at present, of why the cosmological constant should be as small as is required to explain the enormous size of the observable universe. This lack of understanding is particularly vexing from the theoretical point of view because local quantum field theories appear to offer no way to account for the enormous disparity between cosmological scales on one hand, and the microscopic scales of elementary-particle physics on the other \cite{4,5}. If anything, this dismal situation only worsens should better data bear out present indications for a nonzero, but extremely tiny, cosmological constant \cite{6}, with high-redshift supernova surveys favouring

\[ \lambda \sim (3 \times 10^{-3} \text{ eV})^4, \]  

in units for which $\hbar = c = 1$.

The nature of the problem is this. The cosmological constant can be considered to be the energy density of the vacuum, and so samples the quantum zero-point energy density contributed by physics at any particular scale. However, as a rule, degrees of freedom at scale $m$ contribute $\delta \lambda = O(m^4)$, leading to unacceptably large results. This is true, in particular, for the theories which successfully describe all the well-understood physics associated with scales between $10^{-3}$ eV and 100 GeV.

Supersymmetric field theories are the only known examples which evade this general statement, since these theories can predict a vanishing cosmological constant even though they involve massive particles. They do so only if the supersymmetry is not spontaneously broken, since in this case bosons and fermions precisely cancel in their contributions to the vacuum energy. Unfortunately supersymmetry must be broken if it is to apply to the real world, and the absence of superpartners for the known elementary particles implies that the scale of this breaking must be at least of order $m \gtrsim 100$ GeV. However, the resulting failure in the bose-fermi cancellations implies a cosmological constant which is also $O(m^4)$, and so which is much too large.

The purpose of this paper is to indicate a possible way out of this dilemma, based on the recently much-discussed possibility that all observed nongravitational particles are confined to a domain-wall-like p-brane which sits within a larger $(4 + n)$-dimensional ‘bulk’ spacetime \cite{7}. The choice $p = 3$ gives the simplest picture, in which we are trapped on one of potentially many three-branes which sweep out a four-dimensional world volume within the larger-dimensional bulk space. Gravitational interactions, on the other hand, are not restricted to the wall, and so are responsible for any communication which takes place between different branes. This kind of picture is actually believed to be realized within string theory, where the branes involved can be $(p + 1)$-dimensional Dirichlet branes \cite{6}. We keep this particular realization in mind, not least since it has the advantage that the resulting brane and bulk properties are well-formulated and concrete.

Although we do not yet have a working model, we argue here that this framework has several new features which may offer a way out of the usual cosmological-constant conundrum, our purpose being to identify desirable features that might guide explicit model building. We believe the brane scenario may have something to offer for understanding the size of the cosmological constant at both the microscopic and macroscopic levels.

As viewed microscopically, it can do so because: (i) the extended supersymmetry of the bulk space permits

\begin{enumerate}
\item[1] Although a mechanism for obtaining supermultiplet splittings without a large cosmological constant has been proposed \cite{7} for three-dimensional field theories, its implementation in four dimensions is not clear. See [8] for other related discussions of the cosmological constant problem.
\end{enumerate}
the classical contributions of different branes to the cosmological constant to cancel, and (ii) the influence of supersymmetry breaking can be suppressed by powers of small gravitational couplings because any one brane can be arranged to only partially break these extended supersymmetries. Further, while similar factors arise in the mass splitting between superpartners, our brane-world scenarios may provide an enhanced suppression of the cosmological constant relative to these masses.

Regardless of what happens microscopically, a small cosmological constant must also be understandable on more macroscopic scales, right down to energies that are fractions of an eV. The brane picture can also help here, since it permits the existence of many low-energy degrees of freedom which we do not see because they are trapped on other branes to which we have no direct access. Any residual symmetries which relate low-energy states on these other branes to those on our own might ensure the cancellation of contributions to \( \lambda \) on scales large compared to the inter-brane separations.

We now elaborate on these ideas, focussing in turn on the macroscopic and microscopic points of view.

II. INTERBRANE SUPERSYMMETRY AND THE MACROSCOPIC PERSPECTIVE

Any intrinsically microscopic understanding of the smallness of the cosmological constant cannot be complete because it leaves open why well-understood lower-energy physics, does not ruin the story by contributing too strongly to \( \lambda \). We argue in this section that the brane picture offers a new perspective for this part of the problem.

The new possibility which the brane picture introduces at low energies is the potential it has for hiding low-energy degrees of freedom from us. The macroscopic part of the cosmological-constant problem states that the observed low-energy degrees of freedom themselves induce too large a cosmological constant. However, the difficulty vanishes if there are other low-energy degrees of freedom about which we have no nongravitational information which can enforce the low-energy cancellations required in \( \lambda \).

As an extreme example, suppose the branes which fill the universe repel one another and so in equilibrium arrange themselves into a lattice. Further, suppose there exist residual unbroken (or very weakly broken) discrete supersymmetry transformations which relate the bosons on one brane to the fermions on another. If such a graded lattice symmetry were to force bosons on one brane to be degenerate with fermions on another, and vice versa, then they can cancel in their contributions to the effective \( \lambda \) which is observed on scales much wider than the spacings between the branes.

We do not have an explicit brane model which exhibits such a symmetry, but as a first step we can ask whether supersymmetric interactions can be devised in such a way as to be consistent with the related bosons and fermions living on different branes. Imagine, therefore, constructing a supersymmetric model consisting of a collections of scalars, \( \varphi_i \), living on one brane and a collection of spin-half superpartners of these bosons, \( \psi_i \), living on another brane. We imagine coupling these fields using a supermultiplet of ‘bulk’ scalars and fermions, \( X = \{ x_a, \chi_a \} \), which couple to both branes. Macroscopically we would have to consider a model with supersymmetry relating fields of different branes. Macroscopically, at scales larger than the brane separation, the requirement that the fields \( \varphi_i \) and \( \psi_i \) live on different branes amounts to asking the effective lagrangian to have the following additive form:

\[
L = L_b(\varphi, X) + L_f(\psi, X) + L_{\text{bulk}}(X). \tag{2}
\]

An example of a lagrangian of this form, and which has a supersymmetry relating the components of the supermultiplet, \( \phi_i = \{ \varphi_i, \psi_i \} \) (as well as of \( X_a = \{ x_a, \chi_a \} \)), is obtained by writing a globally-supersymmetric Wess-Zumino model with minimal kinetic term, \( K = \phi_i^* \phi_i + X_a^* X_a \), and superpotential

\[
W = \frac{m_{ij}}{2} \phi_i \phi_j + B_i(X) \phi_i + C(X), \tag{3}
\]

since this implies the component interaction terms:

\[
\begin{align*}
L_{\text{int}}^b(\varphi, X) + L_{\text{int}}^f(\psi, X) &= - |m_{ij} \varphi_j + B_i(x)|^2 \\
&- |B_{i,a}(x) \varphi_i + C_{,a}(x)|^2 \\
&- \left[ \frac{1}{2} \sum_a \gamma^a \chi_b \left( B_{i,ab}(x) \varphi_i + C_{,ab}(x) \right) + \text{c.c.} \right] \\
L_{\text{int}}^f(\psi, X) &= - \frac{1}{2} \gamma^a \gamma^b \psi_i B_{i,a}(x) - \frac{1}{2} \psi_i \gamma^a \gamma_b \psi_j m_{ij} + \text{c.c.} \tag{5}
\end{align*}
\]

In this model the explicit supersymmetry is not broken, so the \( \varphi_i \) and \( \psi_i \) are parforce degenerate in mass. It is this supersymmetry which keeps the vacuum energy precisely zero. On the other hand, for sufficiently weak couplings to the bulk fields, the observable universe on any one brane consists of either the fields \( \{ \varphi_i, x_a, \chi_a \} \) or \( \{ \psi_i, x_a, \chi_a \} \); which would look nonsupersymmetric since it has in either case a mismatched number of bosons and fermions.

In the simple toy model above, we see that the usual cosmological constant paradox is removed in a surprising way. There are superpartners degenerate in mass with all of the observed particles, however, they reside on a hidden brane, physically separated from our own, and so cannot be directly detected. With this new feature, which the brane-world scenarios can provide, the existence of these superpartners might not be in conflict with observations. A drawback of our toy model is that the separated superpartners only interact through bulk fields with gravitational strength couplings. At the moment it is not clear to us how this mechanism could be implemented for the case where the superpartners are charged.
under a gauge symmetry. Clearly, it would be interesting to find a realization of this scenario in an explicit example, in order to better explore its low-energy implications, as well as its connection with the microscopic picture, which we now describe.

III. EXTENDED SUPERSYMMETRY AND THE MICROSCOPIC PICTURE

The previously-described macroscopic scenario for understanding the smallness of the cosmological constant is pointless if the integration over more microscopic degrees of freedom does not also keep the cosmological constant acceptably small. We now describe what new features the world-as-a-brane picture might add to this part of the problem. We wish to argue that \( \lambda \) may be very close to zero because it is protected by more than one supersymmetry, with not all supersymmetries directly broken on our own brane.

Here we will have in mind a particular scenario involving a system of branes having the following properties:

1. **Extended Supersymmetry**: In the absence of the branes the bulk-space dimensionality reduces to a four-dimensional system having \( N \geq 2 \) supersymmetries.

2. **Multiple Brane Species**: The compactification involves several different kinds of branes separated in the bulk.

   Each brane will preserve some fraction of the \( N \) bulk-space supersymmetries, and one of these branes is imagined to be the three-brane on which we ourselves live.

3. **Partial Supersymmetry Breaking**: Although each of the different types of branes preserve some of the supersymmetry, we imagine that each of the bulk-space supersymmetries are broken by at least one brane. We denote by \( B \) the minimum number of branes which are required to break all of the supersymmetries. For instance, if the bulk has two supersymmetries, while each brane respects one supersymmetry, then two branes together could break both supersymmetries of the bulk, and in this case \( B = 2 \).

   Thus similar to previous discussions \cite{1000,1001}, supersymmetry breaking is only transmitted to our own world from a ‘distant’ brane. For our discussion of the cosmological constant, however, it will be important that (at tree-level) there exist linearly realized supersymmetries on each of the separate branes. In ref. \cite{1002}, Horava gives an M-theory realization of such a scenario with two separated branes.

   **A. The Relevant Scales**

   Microscopically, there are potentially three fundamental mass scales in the brane picture. The first of these is the scale, \( M_p \), associated with the inverse ‘width’ of the brane itself. The second is the scale, \( M_s \), set by the bulk-space gravitational couplings. In the Dirichlet-brane picture \( M_b \) and \( M_s \) are both set by the string scale. Third, there is the inverse radius, \( M_r = 1/r_s \), of \( n \) of the extra bulk-space dimensions. These dimensions are imagined to be small compared to macroscopic distance scales in the four-dimensional world, but still (much) larger than \( 1/M_b \) and \( 1/M_s \). For simplicity, we consider \( n \) extra dimensions all of roughly the same size. We disregard any further compact dimensions with sizes of the order of \( 1/M_s \), as they would play no role in the following analysis.

   The scales \( M_b \) and \( M_s \) are not independent because they are related to the four-dimensional Planck mass, \( M_p = (16\pi G)^{-1/2} = 1.72 \times 10^{18} \text{ GeV} \) (where \( G \) is Newton’s constant). For instance, in string theory this relation is given by

   \[
   M_p^2 \sim e^{2\phi} \left( \frac{M_s}{M_c} \right)^{n+2} M_c^2, \tag{6}
   \]

   where \( e^{-\phi} \) is the closed-string coupling, and all other dimensionless constants are taken to be \( O(1) \).

   Eq. (6) permits the elimination of one of \( M_c \) or \( M_s \) in favour of the other and \( M_p \). A recently much-explored scenario daringly takes \( M_s \) to be as low as the TeV scale \( \tilde{M}_s \), in which case \( M_s \) ranges from \( \sim 10 \text{ MeV} \) (if \( n = 6 \)) to \( \sim 10^{-3} \text{ eV} \) (if \( n = 2 \)). The requirement for a small ratio \( M_c/M_s \ll 1 \) is necessary in this picture to ensure the existence of the hierarchy between \( M_p \) and the weak scale, \( M_w \). The question of explaining this large hierarchy is then transferred to understanding why the extra dimensions are so large.

   For the present purposes we must consider a different scenario, in which \( M_s \) is identified with the intermediate scale, \( M_s \sim 10^{10} \text{ GeV} \). We must do so because we envisage some supersymmetries to be breaking on distant branes, with the news of this breaking reaching our brane only through gravitational-strength bulk-space interactions. As we shall see, this generally produces supermultiplet mass splittings which are of order \( M_c^2/M_p \), which we identify with the electroweak scale, \( M_w \). As is described elsewhere \cite{1003}, having the string scale at \( 10^{10} \text{ GeV} \) has at least three other attractive features, namely: (i) The hierarchy problem is solved without introducing a small value for \( M_s/M_w \) as input, since (for \( n = 6 \)) \( M_w/M_p \sim M_c^2/M_p^2 \sim e^{-2\phi} (M_c/M_w)^n \) is acceptably small so long as \( M_c/M_w \sim e^{-\phi} \sim 1\% \); (ii) The strong CP problem can be naturally solved using the many axions found in string theory\cite{1004} since the decay constant for these axions is \( M_s \), which precisely fits within the allowed window set by astrophysical and cosmological bounds. (iii) Induced neutrino masses are generically

\[ \text{Even better, there usually exist ‘brane axions’ which, unlike the model independent string axion, only couple to the standard model gauge fields and not also to the hidden sector ones.} \]
of order $m_s^2/M_s \sim 10^{-1}$ eV, putting them in the right regime to account for neutrino-oscillations results.

Ultimately, our goal is to understand $\lambda$ as arising as a power of the ratio $M_c/M_s$ (or, equivalently, of $M_s/M_p$). For these purposes $M_c$ itself is taken to be a parameter which is given, with no attempt made to understand the dynamics which determines why the small dimensions stabilize at the desired radius. The focus instead is to understand the more difficult problem of how it can be that the large dimensions can remain large even once it is granted that the small dimensions are small.

**B. Cancellations at the Highest Energies**

Brane theories get off to a good start because the effective four-dimensional vacuum energy can naturally cancel at the very highest energies. We start with a reminder of how this cancellation works within the D-brane context.

Imagine integrating out all of the microstructure of the branes (such as string-scale physics for Dirichlet branes) to obtain the (higher-dimensional) effective field theory at scales below $M_b$. We require the low-energy effective action which governs long-distance gravitational effects once these higher-energy modes are integrated out. At scales just below $M_b$ but well above $M_c$ we have an effective $(4+n)$-dimensional field theory of the light closed-string-modes (collectively denoted by $\Phi$, say) of the bulk space coupled to the light brane modes (denoted by $\Psi_b$), localized on the various branes (which are labelled by $b$). The resulting effective action has the additive form:

$$S = S_{\text{bulk}}[\Phi] + \sum_b S_b[\Phi, \Psi_b].$$  \hspace{1cm} (7)

The form for the effective action at this point may be robustly stated, because the form of the lowest-derivative terms is dictated (in string theory) by supersymmetry. The leading terms in a low-energy expansion of the gravitational part of the bulk-space action are

$$S_{\text{bulk}} = - \int d^{(4+n)}x \sqrt{-g} \ e^{2\phi} \left( M_s^{4+n} \ R + \cdots \right),$$  \hspace{1cm} (8)

where $g_{mn}$ and $R$ are the $(4+n)$-dimensional metric and scalar curvature, respectively. $\phi$ here is the dilaton field, which is related to the metric by supersymmetry, and is normalized so that $e^{-\phi}$ is the closed string coupling strength. Notice that supersymmetry precludes the appearance here of a bulk-space cosmological term, $- \int d^{(4+n)}x \sqrt{-g} \ \Lambda$.

The analog of eq. (8) for the low-energy action for each brane is

$$S_b = - \int d^{(p+1)}x \sqrt{-g} \ e^\phi \left( \tau_b + \cdots \right),$$  \hspace{1cm} (9)

where $x^m(\xi)$ is a parameterization of the `$b$th' brane's position within the bulk space, and $\gamma_{\mu\nu} = g_{mn} \partial_\mu x^m \partial_\nu x^n$ is the brane's induced metric. The ellipses denote dependence on other fields, and on the metric’s curvature, while the constant $\tau_b$ denotes this brane’s tension. The stability of the modes which describe the overall motion of the brane’s centre-of-mass generally requires $\tau_b$ to be positive.

The action of eq. (9) is also dictated by the symmetries of the problem, being manifestly invariant with respect to all of the supersymmetries of the underlying string theory, provided that both brane and bulk fields are transformed. Those supersymmetries which are unbroken by the brane in question are realized in the usual way, with particle states grouping into degenerate supermultiplets. The broken supersymmetries are nonlinearly realized, however, with particles not grouped into degenerate supermultiplets, and some brane fermions, $\eta$, acquiring shifts under these transformations: $\delta \eta = \xi + \cdots$.

A second kind of brane-like quantity which can arise in these scenarios is a fixed surface which is not free to move. In string theory, for instance, orientifolds are obtained by identifying points in spacetime which are related by a parity transformation, giving rise to fixed surfaces at the fixed points of these transformations. The low-energy action acquires a contribution similar to eq. (9) from fields evaluated on these fixed surfaces, with the noteworthy property that the tension, $\tau_b$, for such surfaces may be negative.

Imagine now integrating out all but the most slowly-varying bulk-field configurations, $\varphi$:

$$e^{i\Gamma[\varphi]} = \int D\Phi \ e^{iS_{\text{bulk}}[\varphi + \Phi]} \prod_b D\Psi_b \ e^{iS_b[\varphi + \Phi, \Psi_b]}.$$  \hspace{1cm} (10)

The effective cosmological constant, $\lambda$, is obtained by focussing on the lowest-derivative terms in the dependence of this result on the four-dimensional metric, $g_{\mu\nu}$,

$$\Gamma = - \int d^4x \sqrt{-g} \left( \lambda + M_p^2 \ R + \cdots \right),$$  \hspace{1cm} (11)

where $R$ is the four-dimensional scalar curvature. Notice that for the purposes of identifying $\lambda$ it suffices to consider $g_{\mu\nu}$ infinitesimally close to flat space.

**C. Microscopic Supersymmetric Cancellations**

If all of the supersymmetries of the problem are not broken by the brane configuration of interest, then the contributions to the cosmological constant from all of the various branes and fixed surfaces are known to cancel quite generally. Since we will later argue that supersymmetry can suppress the final cosmological constant, even when broken, we first describe the supersymmetric cancellation in more detail.

Consider first integrating out $\Psi_b$ in the classical (tree-level) approximation within this low-energy theory. This
corresponds to simply eliminating these fields using their classical equations of motion. The cosmological constant, in this approximation, receives a contribution from the tension of each brane, which is typically not small for any one brane, being generically $O(M_p^{p+1})$. Our first goal is to see how supersymmetry can ensure that the contributions of the branes and fixed surfaces can robustly cancel in the cosmological constant, without fine tuning.

To see why this is possible, consider the case (in string theory) of several parallel branes which, taken together, do not break all of the supersymmetry. The exact cancellation of the classical cosmological constant is in this case related to the stability of these configurations due to the cancellation of the classical forces between the branes [7,14].

There are three steps involved in understanding this cancellation. First notice that the unbroken supersymmetry of any one brane configuration ensures that the tension of any particular brane is strictly related to the value of the nonzero Ramond charge which it carries. It is this relation which ensures the precise cancellation between the long-range gravitational attraction between parallel branes, and their long-range repulsion due to the force mediated by the skew-symmetric tensor gauge fields coupling to the Ramond charge.

Second, the generalization of Gauss’ law for the skew-tensor fields requires that the total Ramond charge carried by all branes must sum to zero when the transverse dimensions are compact. This is because there is no place for the flux of a nonzero charge to go in a compact space. In orientifold examples the total charge vanishes due to a cancellation between fixed orientifold surfaces (carrying negative Ramond charge), and branes (whose charges are positive). However, this cancellation of charges then automatically ensures the cancellation of the negative tension of the fixed surfaces against the positive tension of the branes, ensuring the sum $\sum b_\tau = 0$.

Third, supersymmetry ensures that this cancellation survives quantum corrections to this classical picture. This is because the BPS nature of the branes ensures the strict equality of their masses and Ramond charges even at the quantum level.

D. Partial Supersymmetry Breaking

The key question is how these arguments are modified when the brane configuration breaks all of the supersymmetries. Although we do not have an explicit model in hand [13], we now wish to argue that the partial breaking of supersymmetries on different branes opens a possibility that the supersymmetric vacuum energy cancellation might partially persist even after supersymmetry breaks.

Recall that in the scenario we wish to consider all of the supersymmetries of the bulk theory are broken, but that several of the separated branes are required to do so. In this case, because of the supersymmetry breaking, the forces between branes need no longer strictly cancel. If they do not, then the branes will move until they minimize their energy. For instance, if the resulting forces are repulsive the branes may try to maximize their distances from one another, perhaps by arranging themselves into a lattice within the compact $n$ dimensions. If they instead attract one another, they may form bound states, or simply be widely separated from one another.

Our assumption is that the resulting stable configuration continues to break all of the supersymmetries. (The possible role of slow brane motion of this type for generating inflation was recently considered in ref. [14].)

Now there are two important points. First, because supersymmetry is broken, the cosmological constant need no longer vanish. However, it cannot become nonzero without interactions amongst the branes which are mediated by the exchange of bulk states. Furthermore, no such contribution to the vacuum energy is possible from a bulk exchange that is not complicated enough to ‘know’ that there are enough branes to break all of the supersymmetries.

Consider, for example, the case where our brane preserves $N = 2$ supersymmetry, and that each of these supersymmetries is broken by a separate hidden brane. Hence diagrams such as those in figures 1a and 1b would not contribute to $\Lambda$, since they only involve interactions between two of the branes. Their contributions to the effective action would still preserve $N = 1$ supersymmetry. An effective cosmological constant is only induced by vacuum diagrams involving all three branes, as illustrated in figure 1c.

The second point is that supermultiplet mass splittings are also induced by the interbrane exchange of bulk modes, however, such masses would be induced by any such exchange (as long as the branes involved do not preserve precisely the same supersymmetries). Hence to leading order, interactions between only two branes will induce mass splittings. Still, these exchanges will be suppressed by at least two powers of the (small) bulk-space coupling, and so we find $\delta m \sim M_p^2/M_p^2$, in accord with previous supergravity analyses (see details below).

Hence in the example of $N = 2$ supersymmetry considered above, diagrams such as those in figures 1a and 1b (with appropriate insertions on the observable brane) will contribute to the mass splittings. While either diagram would individually induce a mass matrix which respects an $N = 1$ supersymmetry, these are different supersymmetries for each of the diagrams and so the total mass matrix induced at this order would split the masses of all of the superpartners on the observable brane. However, as discussed above, the vacuum energy remains zero at this order, and so the suppression of the cosmological constant may be enhanced relative to the mass splitting.
Figure 1: Some tree-level graphs which potentially contribute to the cosmological constant. Dotted lines represent the exchange of virtual bulk states while fat lines indicate branes (the magenta line being the observable brane). For the $N = 2$ scenario (see text), diagram $c$ contributes to the cosmological constant but is suppressed by powers of the bulk coupling. Diagrams $a$ and $b$ do not contribute to the cosmological constant, but do contribute to mass splitting between superpartner fields.

Two comments are noteworthy here: First, this kind of suppression because of partial supersymmetry breaking is an intrinsically braney mechanism. This is because, in four dimensions, it is difficult to construct models which partially break an extended supersymmetry, as is enunciated in a well-known apparent ‘no-go’ theorem \[10,17\]. The situation is different for supersymmetric theories containing extended objects such as domain walls or branes. In this case the no-go theorem is evaded \[12,18\], with a single brane typically breaking half of the bulk-space supersymmetries and leaving the others unbroken. More complicated configurations, involving several branes, can then break more of these bulk-space supersymmetries. Our second point is that there is the potential in these scenarios to suppress the cosmological constant relative to the supermultiplet mass splittings. This enhancement is possible because the number of hidden branes involved in bulk exchanges contributing to these low-energy couplings need not be the same, at least in scenarios where more than two branes are required to break all of the supersymmetries. We will consider this possibility in more detail below.

E. Numerology

Although we do not have a concrete brane model in hand, it is nonetheless instructive to estimate the suppression which might be expected for the cosmological constant due to these arguments, since the results can suggest the properties to which a more detailed construction should aspire. Imagine, then, integrating out scales between $M_s$ and $M_c$, for which the universe effectively has $4 + n$ dimensions. From a four-dimensional perspective this corresponds to integrating out all of the massive Kaluza-Klein (KK) modes whose masses are proportional to $M_c$.

Suppose that, within a particular brane scenario, the largest contribution with a nonvanishing interaction energy density requires $a$ couplings of the bulk fields to the various branes, and $b$ self-couplings amongst the bulk fields themselves. On dimensional grounds, we may estimate the resulting $(4 + n)$-dimensional vacuum energy density is of order

\[
\delta \Lambda \sim \left[ e^{-\phi} \left( \frac{M_c}{M_s} \right)^{1+n/2} \right]^{a+b} \left( \frac{\tau}{M_c^{p+1}} \right)^a M_c^{4+n},
\]

where we put a factor of the $(4 + n)$-dimensional gravitational coupling, $\kappa = e^{-\phi}/M_s^{(1+n/2)}$ for each gravitational interaction, i.e., for each of the bulk couplings and for each coupling of bulk fields to any brane. For each bulk-brane coupling, we also include a factor of the brane tension. The remaining powers of $M_c$ are included on dimensional grounds since this is the scale of the physics which has been integrated out. The corresponding four-dimensional cosmological constant induced in this way is of order $\lambda \sim \Lambda/M_c^n$.

At this point, we note that $a$ is at least as large as the total number, $B$, of branes required to break all of the supersymmetries, but to start we also entertain the possibility that $a$ might be larger than this. Furthermore, in order to produce a connected diagram, we must actually have $b \geq a - 2$.

More detailed use of eq. (12) requires an estimate for the brane scale, $\tau \sim M_b^{p+1}$. In a string theory scenario involving Dirichlet-branes, we would have $\tau \sim e^\phi M_b^{p+1}$. In this case, eq. (12) becomes

\[
\delta \lambda \sim (e^{-\phi})^\alpha \left( \frac{M_c}{M_s} \right)^\beta M_c^4,
\]

where the exponents are $\alpha = b$ and $\beta = (n/2 - p) a + (n/2 + 1) b$. Let us focus on the ratio of mass scales $M_c/M_s$ (which we will assume is smaller than the string coupling $e^{-\phi}$). We are interested in the most dangerous graphs, which make the largest contribution to $\delta \lambda$, and so will choose our parameters in order to minimize the exponent $\beta$. Here we see that we should choose the smallest possible value of $b$, i.e., $b_{\text{min}} = a - 2$, which yields

\[
\beta = (n + 1 - p) a - (n + 2).
\]

The results beyond this point depend crucially upon the parameters $n$ and $p$, which would be fixed for a particular scenario. There are three mutually-exclusive alternatives:

1. If $p > n + 1$, the coefficient of $a$ in eq. (14) is negative and our estimate for $\delta \lambda$ becomes arbitrarily
large as $a$ increases. Since this means that nominally higher-order graphs contribute larger contributions to $\delta \lambda$, we do not consider this case any further.

2. If $p = n + 1$, graphs with larger $a$ are suppressed only by powers of $e^{-\phi}$, and $\beta < 0$, implying that higher-order graphs can enhance $\delta \lambda$ by an $n$-dependent power of $M_s/M_c$. For instance, with $p = 3$ and $n = 2$, $\delta \lambda = (e^{-\phi})^a M_s^4$. Of course, for weak string coupling (i.e., $e^{-\phi} < 1$), the largest estimate comes from choosing the smallest possible value for $a$, which by assumption is the smallest number of branes which taken together break all supersymmetries, $a_{\text{min}} = B$.

3. The most interesting case is $p < n + 1$, for which the coefficient of $a$ in eq. (14) is positive. Thus the largest contribution to $\delta \lambda$ comes from setting $a$ to its minimum value of $a, a_{\text{min}} = B$, the total number of branes needed to break all of the supersymmetries. Note, however, the second term in eq. (14) is negative. Hence in order to produce suppression of $\delta \lambda$ with a positive $\beta$, we still need

$$B > \frac{n + 2}{n + 1 - p}. \quad (15)$$

Notice that this lower bound for $B$ is minimized when $n$ is maximized and $p$ is minimized, so when comparing models sharing the same value for $B$, we expect those having largest $n$ and smallest $p$ to enjoy the largest suppression to $\delta \lambda$.

From here on we restrict ourselves to option 3, for which $p < n + 1$, for which graphs with more gravitational interactions are more strongly suppressed by powers of $M_c/M_s$. As an example, consider $p = 3$ and $n = 6$, i.e., with three-branes and six extra dimensions. In this case, we find the simple results: $\beta = 4(B - 2)$ and

$$\delta \lambda \sim \left( e^{-\phi} M_s^4 \right)^{B - 2} M_c^4 \left( \frac{M_c}{M_p} \right)^{B - 2} M_s^4. \quad (16)$$

Hence for $B > 2$, we apparently find an enormous suppression of the vacuum energy density.

To establish the degree of the suppression more systematically, we can compare $\delta \lambda$ with an estimate of the mass splittings within the supermultiplets. Let us make this comparison for the case $p = 3$, but general $n$, for which one finds

$$\delta \lambda \sim \left( e^{-\phi} \right)^{B - 2} \left( \frac{M_c}{M_s} \right)^{(n - 2)B - (n + 2)} M_s^4. \quad (17)$$

The condition $p = 3 < n + 1$ forces us to focus on $n > 2$, for which the coefficient of $B$ in the exponent is positive.

In analogy to eq. (12), we may estimate the mass splittings on dimensional grounds as

$$\delta m^2 \sim \left[ e^{-\phi} \left( \frac{M_s}{M_s} \right)^{1 + n/2} \right]^{\tilde{a} + \tilde{b}} \left( \frac{\tau}{M_p^{e + 1}} \right)^{\tilde{a} - 1} M_c^4, \quad (18)$$

where, as above, a factor of $\kappa = e^{-\phi}/M_s^{1 + n/2}$ appears for each gravitational interaction. The latter includes $\tilde{a}$ couplings of bulk fields to a brane, and $\tilde{b}$ couplings of the bulk fields amongst themselves. For all but one of the bulk-brane couplings, we also include a factor of the brane tension. This factor is omitted for one of these couplings as it must involve an operator insertion, e.g., a world-volume fermion or boson bilinear, in order that this exchange contributes to the mass splittings for supermultiplets on our brane. As above, the remaining powers of $M_c$ are included on dimensional grounds.

To parallel the analysis above for $\delta \lambda$, we set $\tau \sim e^{\phi} M_p^{p + 1}$ in eq. (18) which yields

$$\delta m^2 \sim (e^{-\phi})^{\tilde{a}} \left( \frac{M_c}{M_s} \right)^{\tilde{b}} M_c^2, \quad (19)$$

where the exponents are $\tilde{a} = \tilde{b} + 1$ and $\tilde{b} = (n/2 - p)\tilde{a} + (n/2 + 1)\tilde{b} + p + 1$. The graphs which contribute the largest amount to $\delta m^2$ are those having the smallest value for $\tilde{b}$, i.e., $\tilde{b}_{\text{min}} = \tilde{a} - 2$, which yields

$$\tilde{b} = (n + 1 - p)(\tilde{a} - 1). \quad (20)$$

Because our interest is in the case $p < n + 1$, the largest $\delta m^2$ is achieved by choosing the smallest possible $\tilde{a}$.

Now comes the key point. As we have argued in the previous section, the smallest value for $\tilde{a}$ required to split superpartner masses is not $B$, the number of branes needed to break all of the supersymmetries. Rather, as argued above, supermultiplet masses can be split by adding the contributions of several graphs, each of which breaks just some of the supersymmetries and not all of them, although all supersymmetries are broken once all graphs are added together. So we can instead set $a_{\text{min}} = 2$, for which $\tilde{b} = n + 1 - p$. Focussing on $p = 3$, we have

$$\delta m^2 \sim e^{-\phi} \left( \frac{M_c}{M_s} \right)^{n - 2} M_c^2 \sim e^{\phi} \frac{M_s^4}{M_p^2}, \quad (21)$$

where the latter result is in accord with previous analyses involving the low-energy supergravity action.

Comparing the two results in eqs. (17) and (21), we find

$$\frac{\delta \lambda}{\delta m^2} \sim (e^{-\phi})^{B - 4} \left( \frac{M_c}{M_s} \right)^{(n - 2)B - (3n - 2)}. \quad (22)$$

Now recall that eq. (13) gives an inequality, $B > (n + 2)/(n - 2)$, which must be satisfied in order that the scale of the vacuum energy density $\delta \lambda^{1/4}$ be suppressed relative to the compactification scale $M_c$, down to which we are integrating out low-energy degrees of freedom. From
eq. (22), we can derive a more interesting (and restrictive) inequality
\[ B > \frac{3n - 2}{n - 2}, \] (23)
which must be satisfied if the vacuum energy density, \( \delta \lambda^{1/4} \), is actually suppressed relative to the superpartner mass splitting scale \( \delta m \). As an example, consider \( n = 6 \) (and \( p = 3 \)). Eq. (23) yields \( B > 2 \), while eq. (24) requires \( B > 4 \). The essential ingredient in the separation of scales appearing in eq. (24) was that the number of branes involved in exchanges contributing to \( \delta \lambda \) was \( a_{min} = B \) while that number for \( \delta m^2 \) was \( \delta a_{min} = 2 \).

While the bulk-brane couplings contributing in eqs. (13) and (14) produce a suppression through the insertion of a gravitational coupling factor \( M_s/M_p \), they also tend to enhance the result due to the brane tension factors. In particular, for the above calculations with \( \tau \sim M_s^{p+1} \), the net effect of these couplings is to enhance the result if \( p > n/2 \), which will hold in most cases. For example, if \( p = 3 \) and \( n = 6 \), these couplings have a neutral effect. Hence one can think of the suppressions discussed above as arising from the couplings of the bulk fields to themselves, which are necessary to ‘sew’ the inter-brane exchanges together.

An even more interesting suppression is possible if \( M_s < M_p \), i.e., \( \tau < M_s^{p+1} \). For example in string theory, the branes of interest could well be described by a number of D-branes (with positive tension) sitting on an orientifold plane (with negative tension) leaving a vanishing net tension. Then the leading term in the effective action (1) would be absent. As a result, the bulk-brane couplings for these particular branes also produce suppressions by the Planck scale in our estimates. If for simplicity we assume all of the branes share this property of a vanishing tension, the revised estimates are found by substituting \( \tau \sim M_s^{p+1} \) in eqs. (13) and (14), implying the absence of the leading order factors involving the brane tension. Eq. (13) for the cosmological constant is then replaced by
\[ \delta \lambda \sim \left[ e^{-\phi \left( \frac{M_c}{M_s} \right)^{1+n/2}} \right]^{a+b} M_c^4 \sim \left( \frac{M_c}{M_p} \right)^{a+b} M_c^4. \] (24)

Here, as usual, the largest contribution comes from choosing the minimum values for both \( a \) and \( b \): \( b_{min} = a - 2 \) and \( a_{min} = B \). With these parameters,
\[ \delta \lambda \sim \left[ e^{-2\phi \left( \frac{M_c}{M_s} \right)^{2+n}} \right]^{B-1} M_c^4 \sim \left( \frac{M_c}{M_p} \right)^{2(B-1)} M_c^4 \] (25)
and hence there is considerable suppression (relative to \( M_c \)) for any number of large extra dimensions and for branes of any dimension. Similarly, eq. (13) for the mass splitting is replaced by
\[ \delta m^2 \sim \left( e^{-\phi \left( \frac{M_c}{M_s} \right)^{1+n/2}} \right)^{a+b} \] \[ M_c^2 \sim \left( \frac{M_c}{M_p} \right)^{a+b} M_c^2. \] (26)

Again this result is maximized by choosing the smallest possible values for the exponents, i.e., \( b_{min} = a - 2 \) and \( a_{min} = 2 \). This choice yields
\[ \delta m^2 \sim e^{-2\phi \left( \frac{M_c}{M_s} \right)^{n+2}} M_c^2 \sim \left( \frac{M_c}{M_p} \right)^{2} M_c^2. \] (27)

which is also a considerable suppression compared to \( M_c \).

Within this scenario of tensionless branes, the ratio \( \delta \lambda/\delta m^2 \) is then replaced with
\[ \frac{\delta \lambda}{\delta m^4} \sim \left[ e^{-2\phi \left( \frac{M_c}{M_s} \right)^{2+n}} \right]^{B-3} \sim \left( \frac{M_c}{M_p} \right)^{2(B-3)} \] (28)

Hence this scenario yields a remarkable suppression of the cosmological constant relative to the scale of the supermultiplet mass splitting for any \( B > 3 \). Again the key ingredient in the separation of scales produced here was that the number of branes involved in exchanges contributing to \( \delta \lambda \) was \( a_{min} = B \) while that number for \( \delta m^2 \) was \( \delta a_{min} = 2 \).

**IV. DISCUSSION**

We see that partial supersymmetry breaking in the world-as-a-brane framework provides a natural mechanism by which the vacuum energy is systematically suppressed by weak coupling constants of gravitational strength. There is the further potential to suppress the scale of the cosmological constant relative to that for the mass splittings amongst supermultiplets.

Even more tantalizing, the potential exists for naturally generating phenomenologically interesting nonzero values for the cosmological constant. This is because the suppression typically comes as a power of \( M_w/M_p \), and comparatively few powers are needed to produce an acceptably small result. In fact, a relatively small power like \( \lambda \sim (M_w^2/M_p)^4 \) is already numerically roughly of order the experimental value of eq. (1).

To see what is required to reproduce such a size for \( \lambda \), we re-express the ratio (28) in terms of \( \delta m \sim M_w \) and \( M_p \), by using eq. (27),
\[ \frac{\delta \lambda}{\delta m^4} \sim \left( \frac{\delta m}{M_p} \right)^{B-3} . \] (29)

A power like \( \lambda \sim (M_w^2/M_p)^4 \) is obtained in eq. (29) for \( B = 7 \). In this case we have \( M_s \sim 10^{13} \) GeV and \( M_c \sim 10^{14} \) GeV, for \( n/2 = p = 3 \) therefore we have a concrete illustration of the fact that starting from a relatively small hierarchy of \( M_s/M_c \sim 10^{-2} \) we can simultaneously obtain \( M_w/M_p \sim 10^{-16} \) (the hierarchy problem).
and $\lambda^{1/4}/M_{\text{Planck}} \sim 10^{-16}$ (the cosmological constant problem) \cite{footnote1}.

Similarly in the case of equation (22) both hierarchies are reproduced if $B = 10$ branes are required to break all supersymmetries, and with branes communicating only with gravitational strength. Both again require: $M_\lambda = 10^{11}$ GeV, $M_c/M_\lambda \sim 10^{-2}$. Even though smaller values of $B$ would be more desirable, it is very encouraging that with such simple setting this scenario has the potential to ‘explain’ the large hierarchies. It would be of considerable interest to explore models of these types in more detail.

Of course, as described in section II, the branes must also provide some mechanism to maintain the suppression of $\lambda$ down to lower scales. One of the key ingredients in the latter separation of scales is that the number of branes involved in exchanges contributing to $\delta \lambda$ and $\delta m^2$ need not be the same. If a scenario like this is at work at the microscopic level, it is ensured down to the compactification scale, $M_c$, and then it may be perpetuated to lower energies by a macroscopic mechanism, such as the interbrane symmetry scenario given in section II. If no such mechanism were to come into play, the remaining contributions to the cosmological constant would be expected to be unacceptably large, of order $\delta \lambda \sim M_s^4$.

Needless to say, the discussion and analysis presented here falls far short of providing a complete solution of the cosmological constant problem. Our primary shortcoming is that we cannot offer an explicit example which implements the mechanisms discussed above. Explicit models, probably on similar lines as those of [11], will be needed in order to be more specific, and in particular, improve upon the naive estimates provided above using simple dimensional analysis. Our purpose then is to point out that the brane-world scenarios do offer a number of exciting new possibilities for suppressing the cosmological constant, and in particular, suppressing it relative to the superpartner mass scales. Our hope is that our observations may stimulate further model building along these lines.

Horava [11] has given an explicit M-theory realization of part of our scenario. However, this construction only involves two separated branes, $i.e.$, boundary nine-branes. These branes are distinguished by the appearance of a gluino condensate on one of the branes, and not the other. Locally each of the branes still preserves half of the supersymmetries, but supersymmetry is still broken as a ‘global effect.’ While this construction involving only two branes would not give an enhanced suppression of the cosmological constant relative to the superpartner masses, it would still be interesting to examine the model in more detail in the context of the questions posed here.

A number of remarks bear emphasis:

1. First and foremost, it is remarkable that the world-as-a-brane scenario may potentially provide loopholes to the conceptual roadblocks which have thwarted an understanding of the cosmological constant’s small size.

2. Next, it is striking that a mechanism for producing a naturally small cosmological constant also points to nonzero values, possibly in the range currently being favoured by high-$z$ supernova measurements.

3. Since individual branes tend to break half of the bulk-space supersymmetries at once, the mechanism described here suggests that our own brane might have more than one unbroken supersymmetry, $N_{\text{us}} \geq 2$. Of course, this is precisely the situation in which the cosmological constant may be suppressed relative to the superpartner masses. A careful exploration of the number of possible unbroken supersymmetries implied by these ideas for our world bears further scrutiny in view of the phenomenological problems extended supersymmetric models have, such as the difficulty obtaining chiral fermions. We are not daunted by these issues since we believe them to be easier to deal with than has proven to be the case with the cosmological-constant problem itself.

We believe that the world-as-a-brane framework furnishes numerous novel possibilities which may ultimately explain what keeps the cosmological constant small without the need of fine tuning. Notice that the mechanism crucially involves an interplay between the nonlocality of brane configurations and supersymmetry. Of course, a key test will be whether models constructed using the mechanism we are proposing can produce a sufficiently small $\lambda$ while still producing a large enough splitting amongst supermultiplets on our own brane. We believe a concrete realization of this idea (or variations on our theme) in terms of explicit string models to be well worth pursuing.

We acknowledge helpful discussions with Luis Ibáñez and Michael Green. Our research is supported in part by N.S.E.R.C. of Canada and F.C.A.R. of Québec and PPARC. C.B. would like to thank the Departament d’Estructura i Constituents de la Materia of the University of Barcelona and D.A.M.T.P. of Cambridge University, and R.C.M. would like to thank the Institute for Theoretical Physics at UCSB, for their kind hospitality during the final stages of this work. Research at the ITP was supported by NSF Grant PHY94-07194.

---

\footnote{Notice the weak-scale-string case — $M_s \sim 1$ TeV and $n = 2$, for which $M_c \sim 10^{-3}$ eV — is not an equally successful option (even though $\lambda \sim M_s^2$ is the right size, as remarked in the first article of [11], for example) because in this scenario all supersymmetries on our brane must be broken to ensure particle multiplets are split by the weak scale. In this case no supersymmetry remains to forbid $\lambda \sim (1 \text{TeV})^4$.}

[1] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.
S. Kachru and E. Silverstein, JHEP 11 (1998) 1, hep-th/9808056; JHEP 01 (1999) 4, hep-th/9810125.

S. Kachru, J. Kumar and E. Silverstein, Phys. Rev. D59 (1999) 106004, hep-th/9807073.

J.A. Harvey, Phys. Rev. D59 (1999) 26002, hep-th/9807213.

C. Shi and H. Tye, Nucl. Phys. B542 (1999) 45, hep-th/9808095.

E. Kiritsis, JHEP 10 (1999) 010, hep-th/9906206.

P.J. Steinhardt, Phys. Lett. B462 (1999) 41, hep-th/9907080.

N. Kaloper, hep-th/9905210.

N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263, hep-ph/9803315, Phys. Rev. D59 (1999) 086004, hep-ph/9807344.

I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257, hep-ph/9804398; P. Horava and E. Witten, Nucl. Phys. B475 (1996) 94, hep-th/9603142; Nucl. Phys. B460 (1996) 506, hep-th/9510203; E. Witten, Nucl. Phys. B471 (1996) 135, hep-th/9602070; J. Lykken, Phys. Rev. D54 (1996) 3693, hep-th/9603133; I. Antoniadis, Phys. Lett. B246 (1990) 377.

See, for example: J. Polchinski, hep-th/9611050.; C. Bachas, hep-th/9806193 and C.V. Johnson hep-th/9812196.

T. Banks and M. Dine, Nucl. Phys. B479 (1996) 173, hep-th/9605136;

E. Dudas and C. Grojean, Nucl. Phys. B507 (1997) 553, hep-th/9704177;

I. Antoniadis and M. Quirós, Phys. Lett. B416 (1998) 327, hep-th/9707203;

E. Dudas, Phys. Lett. B416 (1998) 309, hep-th/9709043;

T.I.J. López and D. Nanopoulos, Mod. Phys. Lett. A12 (1997) 2647, hep-th/9702237.

K. Choi, Phys. Rev. D56 (1997) 6588, hep-th/9706171;

K. Choi, H.B. Kim and C. Muñoz, Phys. Rev. D57 (1998) 7521, hep-th/9711158;

A. Lukas, B. Ovrut and D. Waldram, Nucl. Phys. B532 (1998) 43, hep-th/9701204.

E. Mirabelli and M. Peskin, Phys. Rev. D58 (1998) 065002, hep-th/9712214.

L. Randall and R. Sundrum, Nucl. Phys. B557 (1999) 79, hep-th/9810155.

K.A. Meissner, H.P. Nilles and M. Olechowski, Nucl. Phys. B561 (1999) 30, hep-th/9905135.

H.P. Nilles, M. Olechowski and M. Yamaguchi, Nucl. Phys. B530 (1998) 43, hep-th/9801030; Phys. Lett. B415 (1997) 24, hep-th/9707143.

A. Lukas, B.A. Ovrut and D. Waldram, Phys. Rev. D57 (1998) 7529, hep-th/9711197.

Z. Lalak and S. Thomas, Nucl. Phys. B515 (1998) 55, hep-th/9707223.

P. Horava, Phys. Rev. D54 (1996) 7561, hep-th/9608019.

C.P. Burgess, L.M. Ibáñez and F. Quevedo, Phys. Lett. B447 (1999) 257, hep-ph/9810535.

See also: K. Benakli, Phys. Rev. D60 (1999) 104002, hep-ph/9809582.

J. Hughes and J. Polchinski, Nucl. Phys. B278 (1986) 147.

S. Kachru, J. Kumar and E. Silverstein, Class. Quant. Grav. 17 (2000) 1139, hep-th/9907038.

G. Dvali and S.H. Tye, Phys. Lett. B450 (1999) 72, hep-th/9812483.

For recent reviews, with references to the literature, of explicit brane constructions which break supersymmetry see: A. Sen, hep-th/9904207.

A. Lerda, R. Russo, hep-th/9905006.

J. Schwarz, hep-th/9908144.

O. Bergman and M. Gaberdiel, Class. Quant. Grav. 17 (2000) 961, hep-th/9908129.

E. Witten, Nucl. Phys. B188 (1981) 513.

J. Bagger and J. Wess, Phys. Lett. B138 (1984) 105; J. Bagger, Physica 15D (1985) 198.

M. de Roo and P. Wagemans, Phys.Lett. B177 (1986) 352.

J.P. Gauntlett, Phys.Lett. B228 (1989) 188.

I. Antoniadis, H. Partouche and T.R. Taylor, Phys. Lett. B337 (1996) 83; S. Ferrara, L. Girardello and M. Porrati, Phys. Lett. B376 (1996) 275; J. Bagger and A. Galperin, Phys. Rev. D55 (1997) 1091; H. Partouche and B. Pioline, Nucl. Phys. (Proc. Suppl.) 56B (1997) 322, hep-th/9702119.

E. Kiritsis and Costas Kounnas, Nucl. Phys. B503 (1997) 117, hep-th/9703059.

S. Bellucci, E. Ivanov and S. Krivonos, Phys. Lett. B460 (1999) 348, hep-th/9811244.

I. Antoniadis, G. D’Appollonio, E. Dudas and A. Sagnotti, Nucl. Phys. B553 (1999) 133, hep-th/9812118.

R. Altendorfer and J. Bagger, Phys. Lett. B460 (1999) 127, hep-th/9904213.

M. Rocek and A.A. Tseytlin, Phys.Rev. D59 (1999) 106001, hep-th/9811233.

I. Antoniadis, E. Dudas, A. Sagnotti, Phys. Lett. B464 (1999) 38, hep-th/9908023.

G. Aldazabal and A. M. Uranga, JHEP 10 (1999) 24, hep-th/9908072.

G. Aldazabal, L. E. Ibáñez, F. Quevedo, JHEP 1 (2000) 31, hep-th/9909124.

M. Gaberdiel, A. Sen, JHEP 11 (1999) 8, hep-th/9908060.

N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124.