Improving reasoning ability through contextual teaching and learning in differential equations

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Abstract. One of important aspects in mathematics learning is reasoning. This research aims to improve students’ reasoning ability through contextual teaching and learning in differential equations. This is a research and development based on lesson study. The subjects are 34 students of the fifth semester of the Department of Mathematics Education, Universitas Muhammadiyah Surakarta. The data were collected through observation, field notes, test and documentation. The technique of data analysis was a flow-method that consisted of data reduction, data display and verification. The result showed that there were the increase in the average values of the indicators of reasoning ability as follows: 1) analyzing the problems of 3.5; 3.59; and 3.88, 2) organizing the settlement of 2.18; 3.31; and 3.41, 3) interpreting the results of 2.18; 3.0; and 3.32, and 4) explaining the problems of 1.44; 2.01; and 3.29. It can be concluded that the implementation of contextual teaching and learning is able to improve the student reasoning ability in differential equations.

1. Introduction
Differential Equations (DE) is one of the applied subjects widely used in various fields of mathematics. DE is also used in other fields of science, such as physics, chemistry, biology, engineering, medicine, economics, etc. It is often used to formulate real problems that arise in real-world. The material discussed in DE consists of: a) introduction: definition, classification and solution of DE, b) first order DE: separable DE, homogeneous DE, non-homogeneous DE, exact and non-exact DE, c) first order linear DE: Bernoulli method, Lagrange method and Bernoulli DE, d) high order Linear DE: homogeneous linear with constant coefficient, non-homogeneous linear with constant coefficient, homogeneous linear with variable coefficient, non-homogeneous linear with variable coefficient, and e) some examples of application of DE.

To solve the problems of DE, good reasoning skills are required. Kilpatrick et al. [1] stated that reasoning is an adhesive connecting the concepts and procedures in mathematics so that they can be used to solve problems. Ball and Bass [2] mentioned reasoning as a basic mathematical ability needed to understand mathematical concepts, use mathematical ideas and procedures, and construct mathematical understanding. Essentially, reasoning is an attempt to make a conclusion by using logic rules based on assumptions, principles, traits, and pre-existing evidence. It shows that in order to have good reasoning skills, an understanding of logical rules and linking knowledge of concepts, rules, principles, traits, or evidences that have been learned to form or make new conclusions or evidence is required.
In fact, in DE learning process, most students more focus on analytical and procedural calculations in solving the problems posed to them. Khotimah and Masduki [3] found that most students have not been able to associate the concepts learned in differential and integral calculus to solve the DE problems. Meanwhile, Muslich [4] argued that contextual teaching and learning (CTL) is a learning concept that helps educator associate learning material with the student’s real world situation, and encourage students to make connections between the prior knowledge and the application in their daily-life. The basic philosophy of CTL is constructivism, in which the learning process involves not merely memorizing, but also reconstructing or building new knowledge and skills through facts or propositions that they experience in their lives.

There are seven principles in developing contextual learning as stated by Hamruni [5]. These principles are as follows:

1. **Constructivism:** in building or compiling student knowledge based on the experiences undergone by students themselves. Learning is sought to encourage students to be able to construct their own knowledge through observation and real experience.

2. **Inquiry:** the learning process is based on search and discovery through a systematic process of thinking. Knowledge is not a number of facts from the results of remembering, but the results of the process of finding oneself. Thus in the learning process, the teacher does not prepare a number of material that must be memorized, but designs learning activities that enable students to find material themselves that must be understood.

3. **Asking:** the essence of learning is to ask and answer questions. Asking can be seen as a reflection of the curiosity of each individual, while answering questions reflects a person's ability to think. Through questions, the teacher can guide and direct students to find every material learned.

4. **Learning Community:** A person's knowledge and understanding is supported by a lot of communication with other people. The problem is impossible to solve alone, but requires the help of others. Collaboration between giving and receiving is needed in solving problems. In contextual learning, it is suggested that learning outcomes be obtained through collaboration with others.

5. **Modelling:** in CTL, the teacher is not the only model in implementing learning. Models can be designed by involving students.

6. **Reflection:** the process of deposition of experiences that have been learned by reordering the events or learning events that have been passed. Through reflection, learning experiences will be included in the cognitive structure of students which will eventually become part of their knowledge.

7. **Authentic Assessment:** learning success is not only determined by the development of intellectual abilities, but the development of all aspects must be part of the assessment of the success of learning. Therefore, the assessment of learning success is not only determined by the results of the test, but also through the learning process with real assessment.

Suryawati [6] asserted that by using contextual teaching and learning, students can use what is learned into a new situation, and they will enjoy the problem-solving process. Kamaruddin and Amin [7] concluded that the use of contextual videos in learning can help students of engineering faculty at statistical classes, particularly to understand statistical material. Khotimah and Masduki [3] have designed a discovery-based contextual teaching and learning model that can improve the students' problem-solving ability in the course of DE. In this paper, the developing of student reasoning ability will be discussed.

2. **Method**

The present study is the second year of a research and development project. In the first year, CTL discovery-based models have been successfully developed for the subject of differential equations. In the second year, a measurement of students' reasoning ability was carried out through the implementation of CTL discovery-based model in the DE course based on lesson study. Lesson study is a learning enhancement approach originally developed in Japan [8]. Stepanek explained that lesson study is a collaborative process in which a group of teachers identify learning problem and design a learning
scenario (plan), teach students according to the scenario done by one teacher, while others observe (do), reflect and evaluate (see), and revise the learning scenario.

The CTL model implemented in the present study refers to the steps as presented in Table 1.

| Learning Phase      | Learning Activity                                                                 | CTL Principle                        |
|---------------------|----------------------------------------------------------------------------------|--------------------------------------|
| Stimulation         | To provide illustrations of real world problem related to the concepts.          | Asking                               |
| Problem Orientation | To organize the class into several groups, provide opportunities for students to  | Learning community, asking.          |
|                     | collaborate in the groups to identify or analyze the contextual problems given.   |                                      |
| Data Collection     | To encourage each group to find the information or collect required data from     | Learning community, asking,         |
|                     | various sources (books, internet, and lecturers) to solve problems.               | constructivism.                      |
| Data Processing     | To facilitate the students in groups to process information or data obtained for  | Learning community, asking,         |
|                     | solving the problems.                                                            | constructivism, and inquiry.         |
| Verification        | To facilitate the students to check the results of problem solving by comparing   | Learning community, asking,          |
|                     | answers between groups and verification with the lecturer.                         | modelling.                           |
| Generalization      | Drawing conclusions from the solution and make generalizations for the same problem | Constructivism, learning             |
|                     | or case.                                                                          | community.                           |

The study was carried out in the Department of Mathematics Education, Universitas Muhammadiyah Surakarta in 2015/2016 academic year. The subjects were 34 students who took Differential Equations course as the subjects. The data were collected through observation, field note, test and documentation. Observation was used to observe the implementation of models in classroom. In the present study, observation was carried out by colleagues as an observer team by using observation guidelines. The findings that had not been mentioned in the observation guidelines were written in the field note sheet. Test was used to collect the data of students’ reasoning abilities, while documentation was used to document the process and the results of research. The validity of the data used the method and investigator triangulation. Data analysis techniques were carried out descriptively using a flow method that consisted of data reduction, data presentation and conclusion drawing.

The students' reasoning abilities was measured based on four indicators, namely: 1) analyzing problem, 2) organizing the solution, 3) interpreting the results, and 4) explaining the problem. The measurement of four indicators was done using a rubric as presented in Table 2 below:

| Indicator          | Score | Description                                      |
|--------------------|-------|--------------------------------------------------|
| Analyzing the      | 2     | Writing down the mathematical model correctly.    |
| problem            | 1     | Some formulas / mathematical models are correct   |
|                    | 0     | Blank or there is no formula / model or there is  |
|                    |       | a model but it is wrong                          |

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|                    | 0     | Blank or there is no formula / model or there is  |
|                    |       | a model but it is wrong                          |
### Indicators Score Description

| Organizing the solution | 2 | Steps of solution and the answer are correct. |
|-------------------------|---|---------------------------------------------|
|                         | 1 | Steps of solution are correct but the answer is incorrect, or incorrect steps. |
|                         | 0 | No answer. |

| Interpreting the results | 2 | Interpretation, conclusion and proof are correct. |
|--------------------------|---|---------------------------------------------|
|                         | 1 | Interpretation, conclusion and proof are incorrect. |
|                         | 0 | Do not carry out interpretation, conclusion and proof. |

| Explaining the problem | 2 | Give the explanation correctly. |
|------------------------|---|--------------------------------|
|                        | 1 | Part of the explanation is correct. |
|                        | 0 | Blank or there is no explanation, or there is an explanation but it is incorrect. |

Furthermore, a scale of 0 – 4 was employed to calculate the average value of student reasoning for each indicator and the overall indicator. The criteria for the average value of reasoning ability are presented in Table 3 below.

#### Table 3. Average value criteria

| No. | Average | Criteria |
|-----|---------|----------|
| 1   | Value ≤ 1.33 | Less |
| 2   | 1.33 < Value ≤ 2.33 | Moderate |
| 3   | 2.33 < Value ≤ 3.33 | Good |
| 4   | 3.33 < Value ≤ 4.00 | Very Good |

### 3. Results and Discussion

The implementation of CTL discovery-based model in the classroom is carried out in four cycles, in which each cycle consists of three stages of activity, namely plan-do-see. In the “plan” stage, the researcher and colleagues discussed the lesson plan, including teaching materials, student worksheets, and the assessment instruments. The guidelines of observation and field note instruments were also discussed. This stage generates a revised lesson plan and observation instruments. The next stage is “do”, which is the implementation of the revised lesson plan after “plan”. At this stage, the researcher as lecturer model carried out the learning model while being observed by the colleagues as observers. The final stage of the action is “see”, used to reflect what has succeeded and not succeeded in learning process. The results of reflection are used to improve the next actions for next cycle. The implementation of plan-do-see stages in fourth cycle is presented in Figure 1 below.

In the present study, the four cycles with the different learning materials in each cycle were conducted. In the first cycle, the learning material was the first cycle DE: separable DE and initial value problem. The second is the second cycle DE: to identify homogeneous DE, non homogeneous DE, exact and non-exact DE. The material in the third cycle was to solve non homogeneous DE, and the last cycle is to solve the first cycle linear of DE. To measure students’ reasoning abilities, three tests were conducted. The first test was carried out after the implementation of the first and second cycles, the second test was carried out after the implementation of the third cycle, and the third test was carried out after the implementation of the fourth cycle.
The reasoning test results showed that the implementation of CTL discovery-based in Differential Equation courses could increase the students' reasoning ability. The average value of reasoning ability on each indicator and the total average value of reasoning ability are presented in Table 4 below.

### Table 4. The average value of student reasoning

| Test | Indicator | Total Average Value |
|------|-----------|---------------------|
| I    | 3.5 | 2.18 | 2.18 | 1.44 | 2.32  |
| II   | 3.59 | 3.41 | 3 | 2.01 | 3.01  |
| III  | 3.88 | 3.41 | 3.32 | 3.29 | 3.48  |

**Notes:**
1 : Analyzing the problem
2 : Organizing the solution
3 : Interpreting the result
4 : Explaining the problem

From the result, for the first indicator, of analyzing problem, the average value of student reasoning in the first test was 3.5, 3.59 in second and 3.88 in the last test. The assessment criteria for the first indicator from the first test to the third test were in the category of "Very Good". Furthermore, for the second indicator of organizing the solution, the average value of student reasoning increased from 2.18 (Moderate) on the first test, to 3.41 (Good) on the second and third tests. For the third indicator of
interpreting the results, the average value increased from 2.18 (Moderate) on the first test, to 3.0 (Good) on the second test, and 3.32 (Good) on the third test. For the fourth indicator of explaining the problem, there was an increase from 1.44 (Moderate) in the first test to 2.01 (Moderate) on the second test and 3.29 (Good) on the third test.

In overall, the total average values of student reasoning from the first to the third test were 2.32 (Moderate), 3.01 (Good), and 3.48 (Very Good). Thus, the implementation of CTL discovery-based model in DE can improve the student reasoning ability.

The example of student answer that fulfils the rubric of student reasoning for each indicator is as follows:

**Figure 2. A sample of students answer**

Figure 2 shows that the student has fulfilled the first indicator of reasoning, which is analyzing the problem as indicated by the ability to determine relationships between variables or objects in mathematical situations by writing down the mathematical model correctly. After making a model, the student can give the explanation correctly as demonstrated in the Figure 3.

**Figure 3. A sample of students’ explanation**

For the third indicator, which is organizing of solution, the student has been capable in organizing solutions, including counting and doing mathematical manipulations. Figure 4 shows that the step of solution and the answer of student are correct, and the student can interpret the result of solution correctly. It means that the student’s answer has fulfilled the rubric of reasoning.
Graphically, the improvement of student reasoning through implementation of CTL discovery-based in DE is illustrated in Figure 5.

The improvement in student reasoning ability through contextual learning is in line with the results of Saleh et al. [9] in which the students’ reasoning skills in learning mathematics by PMRI are better than those in the conventional learning. Khotimah and Masduki [3] found that the problem-solving abilities of student has enhanced after the implementation of contextual teaching and learning model in DE course.

This result is also in line with Kurniati et al. [10] that the implementation of contextual teaching and learning has successfully increased student mathematical critical thinking ability, which is higher than those with traditional teaching and learning. Vello et al [11] aimed at identifying the effects of RME learning on mathematical reasoning ability analogy and generalization on linear and simultaneous equations. The research design was quasi-experimental with a sample of 35 students in experimental class and 34 students in control class. The results of the research revealed that the effects of analogy reasoning abilities and experimental class/RME generalization were better than conventional class. The same result also reported by Surdin [12] and Simamora [13].
The present study also provides information that learning using learning phases in CTL discovery-based models can be received well by students as indicated by a positive response to the implementation of the model. The learning atmosphere is more dynamic with the discussion and mutual exchange of opinions between students. In addition to the positive response, students also demonstrate the development of attitude aspects in learning, namely responsibility, critical thinking, courage to express opinions, systematic thinking, self-knowledge, cooperation, courage to ask questions, confidence, be more active in learning, self-reliance, creativity, respect to the opinions of others, and curiosity. It is supported by Hamruni [5] who stated that contextual learning is learning that emphasizes the process of involvement of students to be able to find relationships between the material being studied and real life situations. In contextual learning, students are fully involved in the learning process. Students learn by listening and taking notes, but also learning to experience firsthand in real situations around them. Through the process of direct experience, students will experience comprehensive development. Instead of only cognitive aspects, it will elevate the psychomotor and affective aspects of students.

4. Conclusion
Based on the result of this research, it can be concluded that the implementation of CTL in Differential Equation course can improve the student reasoning abilities. The improvement of the reasoning can be seen from the average value of each indicator of test results: 1) in the indicator of analyzing the problem, the value increased from 3.5 on the first test into 3.88 on the final test. It is classified in the "very good" category, 2) in the indicator of organizing the solution, the value increased from 2.18 on the first test become 3.41 (good) on the final tests, 3) in the indicator of interpreting the results, the value increased from 2.18 (moderate) on the first test to 3.32 (good) on the third test, 4) in the indicator of explaining the problem, the value enhanced from 1.44 (moderate) in the first test become 3.29 (good) on the third test. In overall, the improvement of the average values of student reasoning from the first to the third test are 2.32 (moderate), 3.01 (good), and 3.48 (very good). It can be concluded that the contextual teaching and learning is able to improve the reasoning ability of the students.

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