Chapter

Turbulence Generation in Inhomogeneous Magnetized Plasma Pertaining to Damping Effects on Wave Propagation

Ravinder Goyal and R.P. Sharma

Abstract

The damping phenomenon is studied due to the collisions of ions and neutral particles and Landau approach on the turbulent spectra of kinetic Alfvén wave (KAW) in magnetized plasma which is inhomogeneous as well. The localization of waves is largely affected by inhomogeneities in plasma which are taken in transverse as well as parallel directions to the ambient magnetic field. There is significant effect of damping on the wave localization and turbulent spectra. Numerical solutions of the equations governing kinetic Alfvén waves in the linear regime give the importance of wave damping phenomena while retaining the effects of Landau (collisionless) damping and ion-neutral collisional damping. A comparative study of the two damping effects reveals that the Landau damping effect is more profound under similar plasma conditions.

Keywords: kinetic Alfvén wave, inhomogeneous plasma, ion-neutral collisional damping, Landau damping, laboratory plasma, turbulence

1. Introduction

As far as enormous space plasma phenomena like solar wind turbulence, acceleration of solar wind, heating of solar coronal loops, solar flares, etc. are concerned, kinetic Alfvén wave (KAW) has a vital role to play [1]. The dispersion characteristics of KAW make it distinguishable from its parent Alfvén wave [2]. When Alfvén wave propagation develops a large value for wave number perpendicular \( k_\perp \) to the ambient magnetic field, then it gives rise to kinetic Alfvén wave (KAW). This mode conversion may lead to transportation of large amount of electromagnetic energy across geomagnetic field lines [3]. In Tokamak/ITER plasma where KAW plays an important role, the anomalous diffusion coefficient which depends upon the turbulence level is inversely related to the confinement time. In the scenario where the electric fields are nonstationary, this wave plays a vital role in the phenomenon of particle acceleration [4]. The acceleration and heating [5] of plasma particles are dependent on perpendicular wave number and is attributed to KAW carrying finite perturbations parallel to electric field. Also, the plasma heating is caused by dissipation of turbulence and is essential to interpret the observations of most astronomical [6] and laboratory [7] systems. The turbulent behavior of KAW
can be thought of as the probable mechanism to describe the magnetic fluctuations which are observed near earth’s magnetopause [8]. The theoretical and experimental studies [9, 10] support the importance of KAW in solar wind turbulence energy cascade and particle heating. Earth’s magnetopause is also known to have a vital role played by KAW in the mechanism of stochastic ion heating [11]. Nykyri et al. [12] observed the turbulent spectra in high altitude cusp and revealed that break in observed spectra might be due to damping of obliquely propagating KAW.

In general, both space as well as laboratory plasmas incorporate various kinds of inhomogeneities, few of these being magnetic field [13], temperature, and density perturbations. The coronal heating has been theoretically studied by Davila [14] by assuming Alfvén absorption when Alfvén velocity is dependent on background plasma density in transverse direction to ambient magnetic field. Also, the magnetic field fluctuations due to shear Alfvén waves have been observed experimentally [15]. The Alfvén wave energy changes along radial direction have also been studied by Vincena et al. [16]. The present work is inspired from the experimental studies of formation of varying magnetic fields and electric fields by propagation of KAW in inhomogeneous plasmas [17].

A comparative study of experimental results with theoretical ones [18] fully support the launching of KAW in inhomogeneous plasma. There may be qualitative and quantitative deviation of ideal behavior [19] of KAW from that when inhomogeneities or nonlinearities are introduced due to kinetic theory. Therefore, it is always better to have understanding of magnetic fluctuations with an introduction of Landau damping, which is considered to be an important phenomenon as far as KAW propagation is concerned [20, 21]. Based on Landau fluid model, several theories [5, 22–24] have been proposed regarding the propagation of magnetohydrodynamic waves in collisionless plasma and Alfvén filamentation process due to density channels. Taking into consideration the experimental observations by Houshmandyar and Scime [17], Sharma et al. [25] have studied the effect of Landau damping on KAW propagation in inhomogeneous magnetized plasma and have concluded that there is considerable damping effect on wave propagation due to phenomenologically incorporated Landau damping factor.

The damping refers to gradual decrease in wave intensity and wave amplitude. Not only collisionless damping (like Landau damping) but collisional damping also plays an important role in wave energy decay phenomenon and hence marks its role in Tokamak/ITER fusion plasma processing. The electron-neutral and ion-neutral damping is dominant in scrape-off region of these fusion plasma devices where energy transportation of wave takes place, and hence, neutral particles are the one among the important factors playing a vital role [26] in drawing the turbulence level. Also, the wave dispersion characteristics in space plasmas may get affected by the inclusion of neutral particles in the wave dynamics. Some of these characteristics include particle heating, spicules formation [27], solar wind turbulence and acceleration [28], and chromospheric energy balance [29]. As a consequence of the charged particles of plasma colliding with the neutral ones, the plasma waves may get absorbed by interstellar clouds, photosphere, and chromosphere [30–33]. As far as effect of neutral-dominated collisions on Alfvén waves is concerned, several research groups have worked in this direction and have given their fruitful theories in their respective domains. Based on the theory of chromospheric heating by Alfvén waves, the ion-neutral collisional damping has been studied analytically by De Pontieu et al. [29] in partially ionized chromosphere. Alfvénic turbulence has been considered as the base by Krishan and Gangadhara [34] to get intercoupling mechanism among three plasma species which are neutrals, ions, and electrons. The mass loading of ions by neutral particles has been used by Houshmandyar and
Scime [17] to experimentally study the collisional impact due to collisions between ion and neutral particles on the KAW propagation in plasma. Recently, Goyal and Sharma [35] have studied the effect of ion-neutral collisional damping on KAW turbulence.

The present research deals with a comparative study of the two damping phenomena discussed above viz. ion-neutral collisional damping and Landau (collisionless) damping on the propagation of KAW which have been studied separately by Sharma et al. [25] and Goyal and Sharma [35] in the light of inhomogeneous magnetized plasma. This comparison is very much important with respect to scrape-off regions of the fusion energy devices like Tokamak and ITER. The plasma inhomogeneity is taken into account by considering spatial (in x-z plane) inhomogeneities having scale lengths both in longitudinal and transverse directions to background magnetic field. Distinct model equations for KAW have been taken by considering damping factors corresponding to collisional and collisionless damping, and the results have been drawn, respectively, using two fluid simulation technique.

2. Equations governing the propagation of KAW

The governing equations of KAW come into their present form by taking into account the propagation of the wave in two dimensions (x-z surface), which imitates the propagation vector to be \( \vec{k}_0 = k_{0x} \hat{x} + k_{0z} \hat{z} \) where \( k_{0x}(k_{0z}) \) symbolizes the component of the wave vector in perpendicular (parallel) direction to ambient magnetic field \( \vec{B}_0 \) along z direction. Also, the plasma having background density \( n_0 \) is considered to be inhomogeneous in x as well as in z directions and hence is supposed to follow the density profile \( n_0 \exp \left(-x^2/L_x^2 - z^2/L_z^2\right) \), where \( L_x \) implies the inhomogeneity scale length perpendicular to ambient magnetic field \( \vec{B}_0 \) while \( L_z \) implies the same in parallel direction. Initially, the wave field profile is considered to follow hollow Gaussian behavior [36]. As the current numerical study is inspired from the experimental technique [17] where the plasma environment was developed experimentally to study the wave propagation, we have used the same laboratory parameters in our model which are as follows: \( B_0 \approx 560 \, \text{G} \), plasma density \( n_0 \approx 6 \times 10^{12} \, \text{cm}^{-3} \), neutral particle density \( n_n \approx 5 \times 10^{12} \, \text{cm}^{-3} \), ion density \( n_i = n_n/2 \approx 2.5 \times 10^{12} \, \text{cm}^{-3} \), \( T_e = 81,200 \, \text{K} \), \( T_n \approx T_i = 2900 \, \text{K} \). Using these experimental values, the other characteristic plasma parameters may be obtained like: \( v_A \approx 2.5 \times 10^7 \, \text{cm/s} \), \( c_i = 1.3 \times 10^6 \, \text{cm/s} \), \( \beta = (c_i^2/v_A^2) \approx 0.002 \), \( v_{te} \approx 1.1 \times 10^8 \, \text{cm/s} \), \( v_{ti} \approx 2.445 \times 10^8 \, \text{cm/s} \), \( \omega_{ci} = 2.68 \times 10^6 \, \text{rad/s} \), \( \lambda_c = 0.1 \, \text{cm} \), \( \rho_i \approx 0.09 \, \text{cm} \), \( \rho_\parallel \approx 0.49 \, \text{cm} \), \( k_{0x} \approx 0.09 \, \text{cm} \), and \( k_{0z} \approx 0.16 \, \text{cm}^{-1} \) for \( \omega/\omega_{ci} = f/f_{ci} \approx 0.9 \).

Using the well-established methods [26–28], Maxwell’s equations and Faraday’s law, the two dynamical equations for finite amplitude KAW incorporating the effects of ion-neutral collisional damping [35] and Landau damping [25] can, respectively, be written as follows:

\[
\frac{\partial^2 B_y}{\partial t^2} + 2y_n \frac{\partial B_y}{\partial t} = k_0^2 \frac{\partial^2 B_y}{\partial x^2 \partial t^2} - \left( u_e^2 \lambda_c^2 + u_A^2 \rho_i^2 \right) \frac{\partial^4 B_y}{\partial x^2 \partial z^2} + v_A^2 \exp \left( x^2/L_x^2 + z^2/L_z^2 \right) \frac{\partial^2 B_y}{\partial z^2} \nonumber
\]

\[
\frac{\partial^2 B_y}{\partial t^2} + 2y_n \frac{\partial B_y}{\partial t} = k_0^2 \frac{\partial^2 B_y}{\partial x^2 \partial t^2} - \left( u_e^2 \lambda_c^2 + u_A^2 \rho_i^2 \right) \frac{\partial^4 B_y}{\partial x^2 \partial z^2} + \frac{v_A^2}{\omega_{ci}^2} \left( 1 + \frac{2(v_n u_n/v_i v_n)}{1 + \omega^2/\nu_{ni}} \right) \frac{\partial^4 B_y}{\partial z^2} \nonumber
\]

\[
(1)
\]
where $\gamma_n$ is damping factor representing ion-neutral collisions and $\gamma_L$ is phenomenologically incorporated Landau damping factor obtained using the dispersion relation used by Lysak and Lotko [20] and Hasegawa and Chen [21]. The perturbations being very small in comparison to other terms in the above equations; their spatial derivatives may not be considered. Now, in order to study the damping mechanism, wave localization phenomenon is very much important which is done by using an envelope solution [36] into Eqs. (1) and (2) to obtain a set of normalised equations [25, 35] which look mathematically similar to nonlinear Schrödinger (NLS) equation. So, on solving these equations individually by following the standard algorithm which has been developed in order to solve NLS equation, one can infer about the wave localization, energy dissipation, and turbulent cascading of wave energy. Post examining the accuracy for NLS invariants up to $10^{-5}$, these normalized equations are accommodated keeping in view the modifications being done in order to solve NLS equation. By considering hollow Gaussian profile (initial simulation condition) before the start of simulation process (at $t = 0$), these equations are solved numerically. The hollow Gaussian profile at $t = 0$ is given by [35]:

$$B(x, z, 0) = a_0 (x^2/r_{10}^2 + z^2/r_{20}^2) \exp \left(-x^2/r_{10}^2 - z^2/r_{20}^2\right)$$

(3)

Here, $a_0$ and $r_{10}$, respectively, dictate the amplitude of KAW and the scale size of wave field in perpendicular direction to the ambient magnetic field $B_0$ before start of the simulation and $r_{20}$ is the same in parallel direction. $r_{10}(r_{20})$ may be termed as the system length in 2-D geometry. These system lengths while spreading in 2-D geometry expand even beyond the stature of hollow Gaussian profile in transverse as well as longitudinal directions to $B_0$. Also, as $k_x(k_z)$ is the normalized propagation constant in transverse (longitudinal) direction to $B_0$, one can infer $r_{10} = k_x^{-1}$ and $r_{20} = k_z^{-1}$. The regulation of KAW evolution is done by initially taking the system constants to be $a_0 = 0.02$ and $k_x = k_z = 0.2$ where $a_0$ is computed by the use of experimental parameters [17] and the wave amplitude is normalized by ambient magnetic field $B_0$. Sharma and Singh [37] have established a distinct algorithm in order to numerically solve Eqs. (1) and (2). This algorithm is framed in such a way that it relies upon a pseudo spectral mechanism for spatial integrals having recurring lengths $l_x = r_{10}$, $l_z = r_{20}$, and a grid having squared dimensions (grid points) given by $2^6 \times 2^6$. And, the temporal evolution is studied by the use of the predictor corrector mechanism besides finite difference method where the evolution advances with a step size $\Delta t \approx 10^{-4}$.

3. Outcomes and results

The damping effects on the wave propagation are plainly detectable by having a view of Figure 1 where the time-dependent variations in normalized magnetic field intensity for specific values of parallel and transverse inhomogeneity scales are
shown. The figure depicts the formation of localized structures or normalized magnetic field intensity peaks due to density inhomogeneity in plasma. The wave gets focused and defocused due to these inhomogeneities. At a fixed time, the normalized magnetic field intensity has spatial spreads (in x-z plane), in almost similar manner in undamped as well as damped (ion-neutral collisions) cases but its peak values differ for different cases viz. undamped, ion-neutral collisions, and Landau damping. As expected, the field intensity has lower peak values for damped cases (ion-neutral collisions and Landau damping). A comparative picture of ion-neutral collisional damping and Landau damping reveals that the Landau damping effect seems to be more dominating due to smaller magnetic field intensity peaks but it can also be seen that the spatial spread is also more in Landau damping case. We can make an inference that due to Landau damping effect, the spatial energy spread in smaller peaks is more pronounced than that in ion-neutral damping case where the intensity peaks resemble almost the ones in undamped case. Also, the damping phenomena get more pronounced as we scale up the time domain.

The damping effect may also be observed on the temporal variation of magnetic field in Figure 2 for a specific spatial location (x, z). It can be clearly observed from Figure 2 that at the initial times, all the fluctuations are of high amplitude (magnetic field) but with the passage of time, the damping effects come into play as fluctuations die down for both ion-neutral collisional as well as Landau damping effect. This behavior is indicated from Figure 1 as well. Comparatively, Landau damping effect is more pronounced than the collisional effect. The Fourier transform of undamped as well as damped magnetic field fluctuations as given in Figure 2 results into the magnetic power spectra of the respective cases, which are indicated as the $|B_0|^2$ varying with frequency $\omega$ in Figure 3. These spectra have been obtained using a magnetic field fluctuations time window $\sim 10 \mu$s and indicate a clear difference due to damping effects for some specified values of inhomogeneity scale lengths. Once again, it is depicted from Figure 3 that the Landau damping
The effect is more pronounced than ion-neutral collisional effect on the propagation of KAW which is indicative from the initial spectral structures. Also, it is clearly observed from Figure 3 that on inclusion of damping effects, the spectral behavior (almost) retains its trend in the initial frequency regimes which indicates the linear propagation of KAW. But at the later stage (higher frequencies), the turbulent effects come into play and KAW propagation ceases to follow linear trend (undamped oscillations). Or this behavior might also be due to some unidentified nonlinearity or due to drawback of the numerical model used. The alteration in wave localization occurs due to the involvement of damping effects as is observed in Figure 1. The rates at which the spectral components are associated with spatially localized filamentary structures might emerge with time at enunciated rates. So, at a specific spatial location, the wave magnetic field is supposed to have fluctuation behavior with time, as is indicated in Figure 2.

In the present study, we deal only with the comparison of Landau damping and collisional damping on kinetic Alfvén waves in which plasma is considered to be inhomogeneous and magnetized. The purpose of the study is to look for the effect of
these damping effects on wave propagation. In case of ion-neutral collisions, the inclusion of dust particles could have further affect the damping phenomena. As the size of the dust particles is much larger than the neutral atoms, their collisional cross section would be much larger and hence the collisional damping would have been enhanced. Also, the frequency of collisions would be increased by incorporating dust particles into the plasma, their negative charges may also put adverse effect on the wave propagation and hence, damping phenomenon. In this way, the dust particles’ role is unavoidable when we talk about collisional effects on plasma, but in the present study, we have concentrated only on ion-neutral collisions. The effect of dust particles will be studied in our future work.

4. Conclusion

A comparative study about relative profoundness of damping effects has been done when KAW propagates in two-dimensional inhomogeneous magnetized plasma. The study is based on the numerical simulation technique that has been employed to solve the dynamical equations using two-fluid approach. These equations serve as the numerical representations of KAW propagation when ion-neutral collisional damping and Landau damping effects are separately incorporated into the KAW dynamics. In collisionless plasma, the prominent dissipation phenomenon, i.e., Landau damping is due to electrons unlike neutral particle-dominated plasma where the dissipation is primarily by ion-neutral collisions. Likewise, the neutral atoms colliding with other plasma species may cause dissipation of interplanetary magnetic field. Also, in scrape-off region of Tokamak/ITER, the major cause of particle energization is due to ion-neutral collisions because of presence of neutral atoms in plasma. But it is longitudinal component of electric field that causes particle heating in collisionless plasma because KAW follows this field due to its large transverse wavenumber and hence small wavelength. Due to dependency of Landau damping on wavenumber, it affects the localization amplitude, and hence, the localization of KAW happens due to wave packets which have varying wave numbers. The formation of these wave packets is attributed to the inclusion of inhomogeneities in KAW dynamics. It is clear from the figures that there is appreciable effect of damping phenomena on magnetic field fluctuations as well as on intensity profiles and the energy spread over a wide frequency range can be interpreted from the turbulence analysis. The collisional dynamics which considers ions and neutral particles to be at same temperatures has clear effect on wave dissipation but it is the Landau damping which shows relatively weighty effect as indicated by the wave amplitudes and the resultant frequency spectra. Furthermore, there is no nonlinearity included in KAW dynamics. Some interesting features of the dynamics are expected to be observed when temperature difference between ion and neutral particle dynamics are considered and/or when the nonlinear dynamics are included.

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Author details

Ravinder Goyal* and R.P. Sharma
Centre for Energy Studies, Indian Institute of Technology, Delhi, India

*Address all correspondence to: ravig.iitd@gmail.com

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