THERMAL TIMESCALE MASS TRANSFER AND THE EVOLUTION OF WHITE DWARF BINARIES

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ABSTRACT

The evolution of binaries consisting of evolved main-sequence stars (1 < \(M_d/M_\odot\) < 3.5) with white dwarf companions (0.7 < \(M_{wd}/M_\odot\) < 1.2) is investigated through the thermal mass-transfer phase. Taking into account the stabilizing effect of a strong, optically thick wind from the accreting white dwarf surface, we have explored the formation of several evolutionary groups of systems for progenitors with initial orbital periods of 1 and 2 days. The numerical results show that CO white dwarfs can accrete sufficient mass to evolve to a Type Ia supernova, and ONeMg white dwarfs can be built up to undergo accretion-induced collapse for donors more massive than about 2 \(M_\odot\). For donors less massive than \(\sim 2 M_\odot\), the system can evolve to form an He and CO or ONeMg white dwarf pair. In addition, sufficient helium can be accumulated (\(\sim 0.1 M_\odot\)) in systems characterized by mass ratios \(1.6 \leq M_d/M_\odot \leq 1.9\) and \(0.8 \leq M_{wd}/M_\odot \leq 1\) such that sub-Chandrasekhar-mass models for Type Ia supernovae, involving off-center helium ignition, are possible for progenitor systems evolving via the Case A mass-transfer phase. For systems characterized by mass ratios \(\geq 3\), the system likely merges as a result of the occurrence of a delayed dynamical mass-transfer instability. We develop a semianalytical model to delineate these phases that can be easily incorporated in population synthesis studies of these systems.

Subject headings: binaries: close — novae, cataclysmic variables — stars: evolution — stars: mass loss — supernovae: general

On-line material: color figures

1. INTRODUCTION

Close binary systems consisting of main-sequence–like (MS-like) stars with white dwarf companions have long been recognized as an important stage of evolution for understanding the formation of several diverse classes of objects. Such systems are products of an evolution of an MS-like star with a red giant or asymptotic giant branch companion in which significant mass and orbital angular momentum have been lost. This extremely nonconservative evolution facilitates the transformation of the system from a wide orbit with an orbital period of \(\sim 1\) yr to a narrow one with a period of several days via a common-envelope phase in which the orbital energy released in the spiralling in process is sufficient to eject the common envelope (for reviews see Iben & Livio 1993; Taam & Sandquist 2000). For orbital periods of post common envelope systems \(\lesssim 0.5\) days, angular momentum losses by magnetic braking are effective (see Pylser & Savonije 1988) in shrinking the orbit further, producing cataclysmic variable systems. On the other hand, for periods \(\gtrsim 0.5\) days, the MS-like stars with masses \(\gtrsim 1 M_\odot\) can evolve to the mass-transfer (MT) stage as a result of envelope expansion induced by the nuclear burning in the MS star’s interior. The future evolution of the nuclear evolved systems with donors in the mass range of 1–3 \(M_\odot\) may significantly contribute to the formation channels for the production of supersoft X-ray sources (massive white dwarfs accreting at rates sufficient for steady hydrogen burning; van den Heuvel et al. 1992), of a class of Type Ia supernovae models (Li & van den Heuvel 1997), of ultrashort-period (\(P \lesssim 30\) minutes) interacting double white dwarf (DWD) AM CVn systems (Podsiadlowski, Han, & Rappaport 2003), and of detached DWD systems (Nelemans et al. 2001).

Common to the evolution of these systems is the occurrence of a phase of MT on a thermal timescale. Until recently, this population of systems was generally neglected in population synthesis studies, since it was assumed that the accreting white dwarf would expand to red giant dimensions as a result of reactivation of hydrogen burning at high mass accretion rates (\(\geq 10^{-7} M_\odot\) yr\(^{-1}\)). This expansion was hypothesized to lead to the formation of a second common-envelope phase and to the eventual formation of a double degenerate system. However, it was pointed out by Kato & Hachisu (1994) that an optically thick wind can be driven from the surface of white dwarfs more massive than about 0.5 \(M_\odot\), thereby stabilizing the MT in the system and making new binary evolutionary channels possible. In this case, the photosphere of the accreting white dwarf lies within its corresponding Roche lobe, and the system can be stabilized, preventing evolution into the common-envelope phase. The existence of such solutions was made possible by a strong peak in the OPAL opacities at the temperatures of about 1.6 \(\times 10^5\) K (Iglesias, Rogers, & Wilson 1987, 1990; Iglesias & Rogers 1991, 1993; Rogers & Iglesias 1992) achieved in the envelope of white dwarfs accreting at high rates. Recently, observational evidence for the accretion wind picture has been suggested by Hachisu & Kato (2003) based on the long-term variability of the light curve of the transient supersoft X-ray source RX J0513.9–6951 (see Reinsch et al. 2000).

Our study is, in part, similar to that conducted by Li & van den Heuvel (1997) but differs in that we map out the boundaries delineating the evolutionary channels leading to the formation of double degenerate dwarfs, neutron stars by accretion-induced collapse (AIC), and near–Chandrasekhar Type Ia supernova models from such progenitor binary systems. As such, we have carried out detailed binary evolutionary calculations of the Roche lobe–filling donor at a stage of evolution between the MS and the base of the giant branch at orbital periods for which nuclear evolution rather than angular momentum losses dominate. The evolution of the
system is carried through the thermal timescale MT phase to determine its ultimate outcome. To obtain a clear picture of the possible evolutionary histories, the mass and evolutionary state of the donor star and the mass of the white dwarf are systematically varied for systems characterized by initial orbital periods of 1 and 2 days. The assumptions and input physics underlying our calculations are described in § 2. The detailed binary sequences and numerical results are described and compared with a semianalytical picture for the boundaries of the various evolutionary channels in § 3. Finally, we summarize and discuss the implications of our results in § 4.

2. FORMULATION

The binary evolutionary sequences calculated in this investigation are based on a stellar evolution code developed by Kippenhahn, Weigert, & Hofmeister (1967) and updated as described in Podsiadlowski, Rappaport, & Pfahl (2002). The stellar models are computed using a reaction network with rates taken from Rauscher & Thielemann (2000, 2001; see also Thielemann, Truran, & Arnould 1986) and using OPAL opacities (Rogers & Iglesias 1992), supplemented with opacities at low temperatures (Alexander & Ferguson 1994), for a solar metallicity (Z = 0.02).

2.1. Mass Transfer

During phases of the evolution when the donor filled its Roche lobe, the MT rate, $\dot{M}_r$, was calculated in an implicit manner. In this case, $\dot{M}_r$ is found such that the radius of the donor is equal to its Roche lobe radius, $R_{RL}$, approximated as (see Eggleton 1983)

$$R_{RL} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln\left(1 + q^{1/3}\right)} A,$$

(1)

where $q$ is the ratio of the mass of the donor, $M_d$, to the mass of the white dwarf, $M_{wd}$, and $A$ is the orbital separation. We consider the radius-mass exponents of the Roche lobe $\zeta = d \ln R_{RL}/d \ln M$ and of the star itself $\zeta = d \ln R/d \ln M$ in our solution method. The response of the Roche lobe to the MT is solely a function of the mass ratio, whereas the response of the stellar radius to the MT is the function of the MT rate. For a given model, we tabulate values of $\zeta(M)$. By equating the Roche lobe radius and the predicted stellar radius, one can find the MT rate for further models. The MT rate solution is not necessarily unique, since it can be multivalued. In this case, we accept the smallest value of the MT rate. While the star evolves, the response of the stellar radius evolves as well, and we recalculcate the table of $\zeta(M)$ if the predicted stellar radius differs from the calculated one by $\delta \ln R = 10^{-4}$.

2.2. Growth of the White Dwarf

The ultimate fate of the material transferred to the white dwarf depends on the mass and composition of the white dwarf and the rate of MT. In this study, we consider CO white dwarfs with initial masses between 0.7 and 1.15 $M_\odot$ and ONeMg white dwarfs more massive than 1.15 $M_\odot$. The actual growth of the white dwarf is determined by the amount of accreted hydrogen that is eventually converted to helium and heavier elements. In the case of rapid MT, the white dwarf accumulates matter at the rate

$$\dot{M}_r \sim 7.5 \times 10^{-7}(M_{wd}/M_\odot - 0.4) M_\odot \text{ yr}^{-1},$$

(2)

with the remaining matter radiatively driven away in a strong optically thick superwind from the white dwarf surface (Hachisu, Kato, & Nomoto 1999). The accumulation ratio, $\eta_{\text{hi}} = M_{\text{eui}}/M_\text{eui} \leq 1$. For transfer rates less than $M_{\text{cr}}$, but greater than about $M_{\text{low}} \sim 10^{-7} M_\odot \text{ yr}^{-1}$, hydrogen burns steadily to helium ($\eta_{\text{hi}} \sim 1$). If the MT rate varies in the range between $\sim 3 \times 10^{-8} M_\odot \text{ yr}^{-1}$ and $M_{\text{low}}$, the white dwarf experiences mild recurrent hydrogen shell flashes. For MT rates below $\sim 3 \times 10^{-8} M_\odot \text{ yr}^{-1}$, strong unstable hydrogen shell flashes result in nova explosions, and we assume that the evolution is fully nonconservative, with all the transferred matter ejected from the system. In this regime, the white dwarf mass is constant (i.e., $\eta_{\text{hi}} = 0$).

In our calculations, $\eta_{\text{hi}}$ is based on the work of Prialnik & Kovetz (1995), who considered the accretion of hydrogen-rich matter onto white dwarfs of varying mass and central temperature. Since the accumulation ratios do not sensitively depend on the thermal state of the white dwarf, fits of their results were used for accretion rates ranging from $10^{-5}$ to $10^{-6} M_\odot \text{ yr}^{-1}$ for their hot white dwarf models.

In the mass accretion rate regime in which hydrogen-rich matter is retained, helium will naturally be accumulated. For a sufficiently massive helium layer, the helium will be ignited. The critical mass necessary for helium ignition is dependent on the rate at which helium is processed in the hydrogen-burning shell $\dot{M}_{\text{H}}$ (Kato & Hachisu 1999). For rates less than $1.3 \times 10^{-6} M_\odot \text{ yr}^{-1}$, helium shell burning is unstable in the white dwarf envelope. The accumulation ratio, defined as the ratio of the processed material remaining after one cycle of a helium flash to the ignition mass, has been estimated by Kato & Hachisu (1999) as

$$\eta_{\text{hi}} = \begin{cases} 1, & -5.9 \leq \log \dot{M}_{\text{H}} \leq -5, \\ -0.175(\log \dot{M}_{\text{H}} + 5.35)^2 + 1.05, & -7.3 < \log \dot{M}_{\text{H}} < -5.9, \\ \end{cases}$$

(3)

where $\dot{M}_{\text{H}}$ is in units of $M_\odot \text{ yr}^{-1}$. We note that the He shell is always unstable whenever a significant amount of hydrogen is processed to helium.

If $\dot{M}_{\text{H}}$ falls below the lower range in equation (3), the helium layer is not sufficiently hot to ignite helium shell burning in a thin mass layer. In this case, helium is accumulated in the layer until the increasing density leads to helium ignition in a thick mass layer of about $0.1 M_\odot$. This ignition of the helium layer likely leads to the ignition of the CO or ONeMg core, disrupting the star as a sub-Chandrasekhar-mass Type Ia supernova (see Taam 1980; Livne & Glasner 1991; Woosley & Weaver 1994; Garcia-Senz, Bravo, & Woosley 1999).

2.3. Orbital Evolution

The evolution of the orbital separation is dependent on the prescription for mass and angular momentum loss. Since we consider systems characterized by orbital periods ($P \geq 1$ day) exceeding the bifurcation period (Pylyser & Savonije 1988), the only loss of orbital angular momentum from the system reflects that carried by the ejected matter. In this study we assume that the mass lost in the radiatively driven wind carries the specific orbital angular momentum of the white dwarf. Hence, the rate of angular momentum loss is given by

$$J = \frac{M_d}{M_{wd}} \frac{J}{M_d + M_{wd}} \dot{M}_{\text{wind}},$$

(4)
where the orbital angular momentum \( J \) of the system is

\[
J = \frac{M_d M_{\text{wd}}}{M_d + M_{\text{wd}}} A^2 \frac{2\pi}{P}
\]

and \( \dot{M}_{\text{wind}} \) is the mass-loss rate from the system.

The orbital and stellar evolution coupled to the stellar response to mass loss implicitly provides the constraint imposed on the system for determining the rate of MT as described above.

3. RESULTS

In order to facilitate an understanding of the detailed numerical results, we first approximately estimate the boundaries delineating the possible outcomes of the evolution in \( \S \) 3.1. A description of the detailed binary evolutionary sequences follows in the subsequent subsections.

3.1. Semianalytical Picture

The evolutionary fate of a given initial binary system is critically dependent on the rate of MT during the Roche lobe overflow phase. As a result, an estimate for the average MT rate is required to determine the approximate boundaries separating the possible outcomes. In the following, we take the average MT rate as

\[
\bar{M} = \Delta M / t_{\text{th}},
\]

where \( t_{\text{th}} \) is the thermal timescale on which the MT operates and \( \Delta M \) is the mass lost by the donor during this MT event. The thermal MT from an MS star or a giant star takes place when the accretor is the more massive star and terminates when \( M_d(t) = M_d - \Delta M = M_{\text{wd}} + \eta_{\text{H}}(M) \eta_{\text{He}}(M_{\text{He}}) \Delta M \). In this description, only \( \Delta M = (M_d - M_{\text{wd}}) / (1 + \eta_{\text{H}}(M) \eta_{\text{He}}(M_{\text{He}})) \) can be lost through the MT phase. In the case of the star evolving through the Hertzsprung gap (HG), the thermal time MT is also driven by the star’s own expansion, and complete envelope can be lost \( (\Delta M = M_{\text{env}}) \) during this phase; however, we do not treat it separately in our simplified picture.

The thermal timescale \( t_{\text{th}} \) on which \( \Delta M \) is lost, is approximately given as

\[
t_{\text{th}} = \frac{M_d/M_d \Delta M / M_{\odot}}{(R_d/R_{\odot}) L_d / L_{\odot}} = t_{h, \odot} \left( \frac{M_d/M_{\odot}}{R_d/R_{\odot}} \right)^{2.25} \frac{\Delta M / M_{\odot}}{\dot{M}}.
\]

Here we used the mass-luminosity relation for MS stars, \( L \propto M^{2.25} \) (this is also correct for stars at HG, since they maintain the same luminosity as at the end of the MS), and \( t_{h, \odot} \) is the thermal timescale of the Sun. In this simplified estimate, we do not follow the evolution of the star through the MT, and therefore use only the value of \( t_{\text{th}} \) at the onset of the MT. The stellar radius, \( R \), is identified then with the radius of the Roche lobe and is a function of the orbital period of the binary and the mass ratio of the system. This yields a very simple description for the average MT rate as

\[
\bar{M} = k \frac{R_d/R_{\odot} (M_d/M_{\odot})^{2.25}}{t_{h, \odot}},
\]

where \( k \) is some factor of the order of unity, which can be taken from the detailed calculations.

The final mass of the accretor is \( M_{\text{wd}, f} = M_{\text{wd}} + \eta_{\text{He}}(M) \eta_{\text{He}}(M_{\text{He}}) M_{\text{th}} \) (for the complete description for \( \eta_{\text{H}} \) and \( \eta_{\text{He}} \), see \( \S \) 2.2). This simple approximate approach permits an exploration of the parameter space for the final product of the binary evolution as a function of initial donor mass, white dwarf mass, and orbital period. As representative examples, we illustrate the boundaries for the possible evolutionary fates of systems characterized by initial orbital periods of 1 and 2 days in Figures 1 and 2, respectively. For \( P = 1 \) day, where the thermal MT occurs near the MS, we adopt \( t_{h, \odot} \approx 3 \times 10^7 \) yr. On the other hand, a 1 \( M_{\odot} \) star in the HG has a thermal timescale of \( 1.8 \times 10^7 \) yr. This corresponds to \( t_{h, \odot} \approx 6 \times 10^7 \) yr in equation (7). For both Figures 1 and 2 a value of \( k = 1.5 \) was used based on the results of our detailed calculations.

The parameter range for which a white dwarf can accumulate sufficient mass to evolve to a 1.4 \( M_{\odot} \) white dwarf required for either a Type Ia supernova event (SN Ia) or an AIC is enclosed within the solid boundaries. The initial mass of the white dwarf is the distinguishing characteristic in these evolutionary channels, since CO white dwarfs with initial masses \( \lesssim 1.15 \ M_{\odot} \) explode upon the ignition of carbon, whereas ONeMg white dwarfs with initial masses \( \gtrsim 1.15 \ M_{\odot} \) collapse to form neutron stars as a result of electron capture processes. The mass and composition of the initial white dwarf component of the system is necessary, but not sufficient to ensure these evolutionary fates, since the donor must supply matter at a rate such that the white dwarf can grow significantly. Only donors of an

![Figure 1](image-url)
intermediate mass range can satisfy such a condition, since the evolution is truncated for massive donors by the onset of a delayed dynamical MT instability (Webbink 1985; Hjellming 1989) and for low-mass donors by the occurrence of the nova phenomenon. Specifically, the upper limit to the donor’s mass is a consequence of the change in the donor’s response when the steep entropy profile of the outer envelope layers is removed. In this case stellar contraction is replaced by stellar expansion as the flat entropy profile of the interior layers is exposed when sufficient matter is removed on a thermal timescale. Since the Roche lobe is contracting during this phase, the MT process becomes dynamically unstable. Based on the results of our detailed calculations we adopt a critical mass ratio of 2.9 (for $P = 1$ day) and 3.1 (for $P = 2$ days) above which the delayed dynamical instability (DDI) is found. For systems that enter the DDI phase, it is highly likely that the system will merge, leaving behind a rapidly rotating remnant. This upper limit, based on the DDI criterion, overestimates the donor’s mass for systems characterized by white dwarfs less massive than 1.05 $M_\odot$, since either donors in the mass range between 1.8 and 3.3 $M_\odot$ are not sufficiently massive to build up the white dwarf to 1.4 $M_\odot$ (even assuming an accumulation efficiency of unity) or because the efficiency of accretion is strongly reduced by the radiatively driven wind. A lower limit on the donor mass for the SN Ia and AIC evolutionary scenario is determined by the occurrence of strong unstable hydrogen shell flashes in the envelope of the white dwarf. In particular, the average MT rate as determined by the thermal timescale of the donor is insufficiently high for stars $\lesssim 1.7 M_\odot$ to prevent the ejection of significant mass from the system via nova explosions.

The remaining combinations of donor and white dwarf masses primarily lead to the formation of a DWD system. In these cases the MT rate is insufficient to lead to the growth to the SN Ia or AIC phase. We note that there exists a narrow range of parameter space ($M_d \sim 1.7 M_\odot$ and $M_{wd} \sim 0.8-1 M_\odot$) for which a significant layer of helium may be accumulated, giving rise to an evolutionary channel in which a sub–Chandrasekhar-mass supernova model may be viable.

The range in donor and white dwarf masses delineating the evolutionary fates of systems in Figure 2 for an initial orbital period of 2 days at the onset of MT are similar to those described in Figure 1, allowing for the possibility that white dwarfs can be significantly built up. For this greater orbital period, the MS-like donors are evolved to a greater extent, and the onset of MT occurs after the formation of a helium core when the donor is in the HG or at the base of the giant branch. As is evident from Figure 2, there exists a region where the system can enter into a common-envelope phase as a result of evolution to the giant stage. The lower dashed line denotes this region, corresponding to the condition that the mass ratio equals 1.3, as based on our detailed numerical calculations. For mass ratios greater than 1.3 the system enters into the common-envelope phase. In this case the donor has a well-defined core-envelope structure typical of red giant stars. A double degenerate system consisting of a He white dwarf with a CO or ONeMg white dwarf in a short-period orbit ($P \lesssim 1$ hr) may result, depending on the particular evolutionary state of the donor at the onset of the common envelope stage (Sandquist, Taam, & Burkert 2000). An additional difference between the systems depicted in Figure 1 and in Figure 2 is the absence of systems that can evolve to a sub–Chandrasekhar-mass SN. This reflects the fact that the regime in which a sufficient helium mass layer accumulates on the white dwarf occurs for systems in which the MT takes place only when the donor is close to the MS. For donors of more advanced evolutionary stages in mass transferring systems at initial orbital periods of 2 days, the timescale for evolution is so short that significant accumulation of helium required for a sub–Chandrasekhar-mass model is not found.

### 3.2. Detailed Calculations

We have computed the evolution of binary systems initially consisting of MS-like stars of masses 1–3.8 $M_\odot$ and white dwarfs of masses 0.7–1.2 $M_\odot$ through the thermal MT phase. The evolutionary state of the donor stars was chosen such that the Roche lobe overflow phase was initiated at an orbital period of either 1 or 2 days. To sample the parameter space adequately, a total of 65 evolutionary sequences were calculated, with 28 and 37 sequences calculated for systems at initial orbital periods of 1 and 2 days, respectively. The systems were evolved until the accreting white dwarf had reached 1.4 $M_\odot$, the donor had evolved to a point at which it was clear that the system would evolve into a DWD system, or the system had entered into a common-envelope phase as a result of either a dynamical or delayed dynamical MT instability.

A visual summary of the fate of all the calculated evolutionary sequences is presented in Figures 3 and 4 for initial orbital periods of 1 and 2 days, respectively. Upon inspection of the boundaries displayed in Figures 1–4, it is clear that the results based on the semianalytical approach compare very favorably with the detailed binary evolutionary computations. The quantitative results of representative evolutionary sequences are listed in Tables 1 and 2 for initial orbital periods of 1 and 2 days, respectively. Here, the initial mass of the white dwarf in units of $M_\odot$.
dwarf, \( M_{\text{wd},0} \), the mass of the white dwarf at the end of the calculations, \( M_{\text{wd},f} \), the initial mass of the donor, \( M_{\text{f},0} \), the mass of the donor at the end of the calculations, \( M_{\text{f},f} \), the minimum orbital period that the binary system has reached during the computed evolution, \( P_{\text{min}} \), the orbital period at the end of the calculations, \( P_{f} \), the evolution time from the onset of the MT phase, \( t_{e} \), and the average MT rate, \( \dot{M}_{\text{e}} \), are listed.

In the following, we divide the description of the detailed numerical results into those sequences for which growth of the accreting white dwarf is sufficient to lead to a near-Chandrasekhar- or sub-Chandrasekhar-mass model for SN Ia and to an AIC phase and sequences leading to the formation of double degenerate systems or to a merged object.

### 3.2.1. Growth of the White Dwarf to an SN Ia or AIC Phase

The progenitor systems that favor the growth of the white dwarf to a near Chandrasekhar mass are characterized by massive white dwarfs (\( M_{\text{wd},0} \gtrsim 0.8 M_{\odot} \)). It can be inferred from Tables 1 and 2 that the efficiency of mass accretion by the white dwarf, as defined by the ratio of the mass accreted by the white dwarf to the mass lost by the donor, is a strong function of the mass and evolutionary state of the donor. Specifically, the efficiency of white dwarf growth ranges from about 10% to 70% over the calculated grid with the highest efficiencies obtained for donors with masses \( \sim 2 M_{\odot} \). Clearly the MT rates for these donors are in the regime in which steady hydrogen burning takes place, but yet the supernova is not so effective in driving a significant amount of matter from the system. That is, the thermal timescale MT associated with donors of \( \sim 2 M_{\odot} \) favors a higher efficiency of material accumulation on the white dwarf in comparison to other donors. In contrast, the progenitor systems that lose a significant fraction of their mass via the radiatively driven wind from the white dwarf surface during the evolution generally are characterized by systems with massive donors \( \gtrsim 2.5 M_{\odot} \). In fact, the degree to which systems lose mass is exemplified by the systems that evolve to the SN Ia or AIC phase. For these systems, the total mass loss from the system can lead to the presence of \( \sim 0.4\)–1.9 \( M_{\odot} \) surrounding the system at the time the white dwarf has been built up to 1.4 \( M_{\odot} \).

An example of such an evolution characterized by an ONeMg white dwarf of \( M_{\text{wd},0} = 1.2 M_{\odot} \) and a donor of \( M_{\text{f},0} \approx 3.2 M_{\odot} \) at an initial orbital period of 2 days is illustrated in Figure 5. In this case the MT rates rise rapidly to \( \sim 10^{-5} M_{\odot} \) yr\(^{-1} \) within 10\(^5\) yr and averages about \( 6.8 \times 10^{-6} M_{\odot} \) yr\(^{-1} \) over the entire evolution. The efficiency of the mass accretion onto the white dwarf is low in this sequence amounting to \( \sim 10\% \). The orbital period initially decreases by more than a factor of 2 reaching a minimum period of 0.8 days after 2 \times 10\(^5\) yr before increasing to a period of 1.1 day at which point the white dwarf has increased to 1.4 \( M_{\odot} \) and the donor has decreased to 1.1 \( M_{\odot} \). The mass loss in such a sequence has been extensive with about 1.9 \( M_{\odot} \) enveloping the system. It is expected that the system would undergo an AIC further widening the orbit as the gravitational mass of the compact object is reduced by about 0.1–0.2 \( M_{\odot} \). With the additional expansion of the donor, the system will evolve to longer orbital periods and enter the low mass X-ray binary phase.

We have also found that systems with donors and white dwarfs in a narrow range \( (1.6 \lesssim M_{\text{wd},0}/M_{\odot} \lesssim 1.9, \ 0.8 \lesssim M_{\text{f},0}/M_{\odot} \lesssim 1) \) can lead to the accumulation of a sufficient layer of helium mass on the CO white dwarf required for the initiation of an off-center helium detonation. The conditions required for the growth of the helium layer are a function of the evolutionary state of the donor, since the strong helium

![Fig. 3.—Evolutionary fates of the binary sequences calculated for an initial orbital period of 1 day and donors of initial mass \( M_{\text{f}} \) and white dwarfs of initial mass \( M_{\text{wd}} \) in units of \( M_{\odot} \). The symbols represent the fate of the system as a DWD, a near-Chandrasekhar mass SN Ia model, a helium ignition sub-Chandrasekhar-mass model (He ignition), AIC, and an evolution into the common envelope stage via a delayed dynamical mass-transfer instability (CE DDI).](image-url)

![Fig. 4.—Evolutionary fates of the binary sequences calculated for an initial orbital period of 2 days and donors of initial mass \( M_{\text{f}} \) and white dwarfs of initial mass \( M_{\text{wd}} \) in units of \( M_{\odot} \). The symbols represent the fate of the system as a DWD, a near-Chandrasekhar-mass SN Ia model, AIC, an evolution into the common envelope stage via a dynamical instability (CE) leading to merger and possibly to a DWD system (CE → DWD), and delayed dynamical mass-transfer instability (CE DDI).](image-url)
flash regime was found to only occur for donors that undergo the MT phase near the MS at short orbital periods \((P \sim 1 \text{ day})\).

### 3.2.2. Formation of a DWD or Single Merged Object

Although the evolutionary sequences were carried out only through the thermal timescale MT phase, the outcome of an evolution as a DWD system can be inferred once the MT rates in the system have decreased to the point when strong hydrogen shell flashes are expected to occur in the white dwarf envelope. Figures 3 and 4 illustrate that DWD systems can form from progenitor systems with either a low mass \((\leq 1.4 M_\odot)\) or high mass \((\geq 2.6 M_\odot)\) donor. For the lower mass donors, the temporal evolution of the MT rates lead to nova explosions over part of the evolution such that the white dwarf does not build up to 1.4 \(M_\odot\). On the other hand, there is a narrow range for high mass donors where, although the white dwarf can be built up significantly, the donor becomes less massive than the white dwarf during the evolution such that the MT rates have decreased to the extent that accretion is prevented by the occurrence of nova explosions. The DWD systems formed via this channel are expected to have long orbital periods \((P \geq 1 \text{ days})\).

The evolutionary channel involving a common-envelope phase may also produce DWD systems. The calculations reveal that this evolutionary channel is restricted to the MT taking place after the donor has evolved through the HG. The response of the star with a deep convective envelope leads to an MT instability and to evolution into the common-envelope phase. The outcome of this evolution is not well understood, but if the system survives, it is likely to have a short orbital period \((P \leq 1 \text{ hr})\). The common envelope calculations carried out by Sandquist et al. (2000) indicate that evolution into the common envelope near the base of the giant branch may lead to the successful ejection of the common envelope if the degenerate helium core is more massive than \(-0.2–0.25 M_\odot\).

For more massive donors, the thermal timescale MT phase enters into a period in which the MT rate accelerates rapidly, indicating the onset of the delayed dynamical MT instability. For example, a sequence characterized by \(M_d = 3.4 M_\odot\), \(M_{\text{wd}} = 1 M_\odot\) at an initial orbital period of 2 days is illustrated in Figure 6. The MT rate accelerates by 3 orders of magnitude after \(~4 \times 10^4 \text{ yr}\), with the rapid increase occurring over a timescale of \(\lesssim 200 \text{ yr}\). In these sequences, the MT initially takes place on the thermal timescale of the donor until the outer radiative envelope layer has been removed. Further mass loss leads to the exposure of the layers characterized by a flat entropy profile and expansion of the donor results. Since the Roche lobe during this phase is contracting, the system evolves into a common-envelope phase. In this case, merger of the system is likely. The occurrence of this instability

### Table 1

**Binary Parameters and Outcomes of Representative Model Sequences for Systems in which the Onset of Mass Transfer Occurs at an Orbital Period of 1 Day**

| \(M_{\text{wd},0}\) | \(M_d\) | \(M_{\text{wd},f}\) | \(M_d,f\) | \(P_{\text{min}}\) (days) | \(P_f\) (days) | \(\Delta t\) (yr) | \(M_f\) (\(M_\odot\) yr\(^{-1}\)) | Outcome |
|-----------------|-----|--------|------|-----------------|---------|---------|-----------------|-------|
| 0.7             | 2.0 | 1.34   | 0.89 | 0.42            | 0.5     | \(5.5 \times 10^6\) | 2 \times 10^{-7} | DWD   |
| 0.8             | 1.5 | 0.81   | 0.75 | 0.82            | 1.02    | \(7.9 \times 10^7\) | 9.5 \times 10^{-9} | DWD   |
| 0.8             | 1.7 | 0.9    | 1.25 | 0.69            | \(0.69\) | \(9.5 \times 10^6\) | 4.7 \times 10^{-8} | sub-Ch |
| 0.8             | 2.0 | 1.4    | 1.17 | 0.52            | 0.55    | \(3.9 \times 10^6\) | 2.2 \times 10^{-7} | SN Ia |
| 1.0             | 1.0 | 1.02   | 0.90 | 1.0             | 1.02    | \(1.1 \times 10^7\) | 9 \times 10^{-9}   | DWD   |
| 1.0             | 2.5 | 1.4    | 1.37 | 0.51            | 0.51    | \(1.7 \times 10^6\) | 6.6 \times 10^{-7} | SN Ia |
| 1.2             | 2.0 | 1.4    | 1.70 | 0.86            | 0.86    | \(3.3 \times 10^6\) | 9.1 \times 10^{-8} | AIC   |
| 1.2             | 3.0 | 1.4    | 1.64 | 0.5             | 0.5     | \(2.9 \times 10^6\) | 4.7 \times 10^{-7} | AIC   |

Notes.—The columns give the mass of the white dwarf, \(M_{\text{wd},0}\) and of the donor \(M_d\) at the onset of the MT \((M_d)\); the mass of the white dwarf \(M_{\text{wd},f}\) \((M_d)\), and the period \(P_f\) (days) at the end of the computation; \(P_{\text{min}}\) (days) is the minimum period of the binary system during the computed interval of evolution, \(\Delta t\) is the time of the MT phase in yr, and \(M_f\) \((M_\odot\) yr\(^{-1}\)) is the corresponding average MT rate. The possible outcomes are as follows: DWD, double white dwarf system; SN Ia, supernova Type Ia; sub-Ch, sub–Chandrasekhar-mass supernova; and AIC, accretion-induced collapse.

### Table 2

**Binary Parameters and Outcomes of Representative Model Sequences for Systems in which the Onset of Mass Transfer Occurs at an Orbital Period of 2 Days**

| \(M_{\text{wd},0}\) | \(M_d\) | \(M_{\text{wd},f}\) | \(M_d,f\) | \(P_{\text{min}}\) (days) | \(P_f\) (days) | \(\Delta t\) (yr) | \(M_f\) (\(M_\odot\) yr\(^{-1}\)) | Outcome |
|-----------------|-----|--------|------|-----------------|---------|---------|-----------------|-------|
| 0.8             | 2.0 | 1.4    | 1.25 | 1.05            | 1.08    | \(2. \times 10^6\) | 3.8 \times 10^{-7} | SN Ia |
| 0.8             | 2.4 | 1.32   | 0.32 | 0.58            | 4.2     | \(9.1 \times 10^6\) | 2.3 \times 10^{-7} | DWD   |
| 1.0             | 1.4 | 1.0    | 0.2  | 0.02            | 0.2     | \(2.6 \times 10^6\) | 2.1 \times 10^{-7} | SN Ia |
| 1.0             | 2.0 | 1.4    | 1.46 | 1.44            | 1.44    | \(6.4 \times 10^6\) | 1.2 \times 10^{-6} | SN Ia |
| 1.0             | 2.4 | 1.4    | 1.4  | 1.1             | 1.1     | \(8.4 \times 10^6\) | 1.2 \times 10^{-6} | SN Ia |
| 1.0             | 2.8 | 1.4    | 0.6  | 0.7             | 0.7     | \(7.7 \times 10^6\) | 2.9 \times 10^{-6} | SN Ia |
| 1.2             | 2.0 | 1.4    | 1.67 | 1.7             | 1.7     | \(2.5 \times 10^6\) | 1.3 \times 10^{-7} | AIC   |
| 1.2             | 2.6 | 1.4    | 1.97 | 1.4             | 1.4     | \(3.6 \times 10^6\) | 1.7 \times 10^{-6} | AIC   |
| 1.2             | 3.2 | 1.4    | 1.1  | 0.8             | 0.8     | \(3.1 \times 10^6\) | 6.8 \times 10^{-6} | AIC   |

Note.—The columns are the same as for Table 1; the additional possible outcome, CE—DWD, corresponds to systems that could possibly survive the common-envelope phase and form a short-period DWD system.

\(^a\) Determined by equating the orbital energy to the binding energy of the donor’s envelope.
occurs at a mass ratio of about 3.1 for donors in the HG and about 2.9 for donors near the MS, confirming earlier estimates by Hjellming (1989).

4. SUMMARY

The fate of binary systems that undergo a phase of mass transfer (MT) on a thermal timescale has been investigated for binaries composed of MS-like (MS-like) donors with white dwarf companions. Allowing for the possibility of an optically thick wind driven from the white dwarf surface gives rise to evolutionary channels in which the white dwarf accretes sufficient mass as required for either a near–Chandrasekhar-mass or sub–Chandrasekhar-mass model for Type Ia supernovae or the formation of neutron stars via an accretion-induced collapse (AIC) model. The range for donor and white dwarf masses delineating the formation of these objects from those producing a pair of degenerate dwarfs has been determined by detailed computations, and reproduced by a simple semi-analytic model, for progenitor systems characterized by initial orbital periods of 1 and 2 days.

Specifically, we have found that CO white dwarfs can accumulate sufficient matter to produce conditions ripe for initiation of a central carbon deflagration/detonation supernova explosion provided that their initial masses are greater than about 0.8 $M_\odot$ and their donor is more massive than about 2 $M_\odot$. An upper limit to the donor mass is set by the onset of a delayed dynamical instability that occurs at a mass ratio of $\sim$3. In addition, the ONeMg white dwarfs (with masses $\geq 1.15 M_\odot$) may accrete sufficient matter to collapse to form a neutron star by electron capture processes for binary systems with donors in the same mass range. During these phases, the MT rates are sufficiently high that hydrogen burning provides the bulk of the energy generation such that the sources are likely to be observed as supersoft X-ray sources (van den Heuvel et al. 1992; Kahabka & van den Heuvel 1997). These evolutionary results are similar to those reported by Li & van den Heuvel (1997) and extend the lower bound of the progenitor white dwarf masses from 0.9 to about 0.8 $M_\odot$ and less, confirming similar results obtained by Langer et al. (2000).\footnote{In a very recent study on SN Ia progenitors, an even lower bound for WD masses was reported: 0.67 $M_\odot$ (Han & Podsiadlowski 2003). This value, however, was based upon the full accretion of hydrogen-rich material, even for $M < M_{\text{Edd.}}$.}

Although Li & van den Heuvel (1997) did not consider AIC in their study, their results are also applicable for this neutron star formation path as well. In these cases, the amount of matter surrounding the system as a result of the wind loss from the white dwarf surface may be as high as 1–2 $M_\odot$. The existence of such circumbinary material is not dissimilar to that expected for complementary evolutionary channels leading to
an SN Ia involving MT from an asymptotic giant branch star to its massive white dwarf companion. We note that additional hydrogen-rich matter may also be lost from the system resulting from the interaction of the supernova shell with the donor (Marietta, Burrows, & Fryxell 2000). In the past, one of the arguments put forth against such accreting models was the lack of hydrogen emission in the spectra of SN Ia. However, the recent discovery of hydrogen emission in the observations of the Type Ia supernova SN 2002ic by Hamuy et al. (2003) provides some support for such models. Although Hamuy et al. (2003) attribute the hydrogen emission to circumstellar matter surrounding an asymptotic giant branch star, the wind present in the accreting white dwarf model in the short period systems investigated here suggests that sufficient matter surrounding the system may be a characteristic of these models as well.

Our detailed systematic investigation has also led to an identification of a channel in which a sufficient helium-rich mass layer accumulates ($\frac{M_{He}}{M_{H}} \gtrsim 0.1$) below the hydrogen-rich layer such that an off-center helium detonation may be ignited in the white dwarf envelope. The propagation of the nuclear burning front into the core region, enhanced by geometrical focusing, may lead to the incineration of the entire white dwarf (Woosley & Weaver 1994) for a sub–Chandrasekhar-mass star. The range of masses of the progenitor systems, as well as the evolutionary state of the donor, that follow this evolutionary channel are narrow in the parameter range studied in this paper. That is, only donors in the mass range between 1.6 and 1.9 $M_{\odot}$ that transfer matter to their white dwarf companion (with masses in the range $0.8 \lesssim M_{\text{wd}}/M_{\odot} \lesssim 1$) while close to the MS are viable. Such limited properties of the progenitor systems may be consistent with the fact that such models are expected to be minor contributors to the SN Ia rate, since these models are subluminous compared to models based on the carbon deflagration/detonation of near–Chandrasekhar-mass stars.

For donors less massive than $\sim 2 M_{\odot}$, less matter is transferred to the white dwarf, leading to systems composed of a He white dwarf with a CO or ONeMg white dwarf companion. Of these systems, those that undergo a dynamical MT instability evolve into the common-envelope phase. For those systems that survive, the systems are likely to emerge at short orbital periods ($P \lesssim 1$ hr), providing an alternative evolutionary channel for the formation of AM CVn binary systems in addition to the channel involving the stable MT evolution of evolved secondaries in cataclysmic variable systems (see Podsiadlowski et al. 2003). On the other hand, the systems that do not enter into the common-envelope phase produce double degenerate systems characterized by orbital periods that are greater than about 1 day.

The systems that undergo an AIC likely evolve to a low-mass X-ray binary phase in which an MS-like star with an evolved core transfers mass to its newly formed neutron star companion. Investigations of this phase have recently been carried out by Sutantyo & Li (2000), who considered the AIC scenario, as well as the intermediate-mass X-ray binary
scenario (also studied by Podsiadlowski et al. 2002). The system evolves to become a binary millisecond pulsar with a helium white dwarf companion in a short period system. Our results suggest that such systems that form via AIC processes are likely to have orbital periods greater than about 1 day.

Finally, we point out that the semianalytical model for the white dwarf binaries that we have developed in this paper can be easily incorporated into population synthesis investigations. We plan to carry out such calculations in the future in order to assess the importance of the formation channels associated with the thermal timescale MT phase in these systems.

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