Charged particle fluctuations and microscopic models of nuclear collisions

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I. INTRODUCTION

One of the main purposes of high energy heavy-ion collisions is to produce a macroscopic size of quark-gluon-plasma (QGP)\textsuperscript{[1,2]}. With hadrons as final observables, several signatures have been suggested: \textit{J/\psi} suppression\textsuperscript{[3]}, single event fluctuations measurements\textsuperscript{[4–10]}, and void and gap searches\textsuperscript{[11]}. It was also proposed in Refs.\textsuperscript{[12,13]} that the quantity

\begin{equation}
D(\Delta y) = \langle N_{ch}\rangle_{\Delta y} \langle \delta R^2 \rangle_{\Delta y} \sim 4 \frac{\langle \delta Q^2 \rangle_{\Delta y}}{\langle N_{ch}\rangle_{\Delta y}}, \tag{1}
\end{equation}

be used as a signature of QGP. Here \(N_{ch} = N_+ + N_-\) is the total number of charged particles, \(R = N_+/N_-\) is the ratio between positive charge and negative charge and \(Q = N_+ - N_-\) is the net charge. The second moments of \(R\) and \(Q\) are defined as

\begin{equation}
\langle \delta x^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2, \tag{2}
\end{equation}

where \(\langle ... \rangle\) means the average taken over all events and \(\Delta y\) is the rapidity window in which we calculate the above quantities. The last step in Eq.\textsuperscript{(1)} is correct to leading order in \(1/\langle N_{ch}\rangle\) and in the fluctuations. It is the observable defined by this last term of Eq.\textsuperscript{(1)} which we calculate throughout this paper. It has been found that for a pion gas, the \(4\langle \delta Q^2 \rangle /\langle N_{ch}\rangle\) is around 4 and for a QGP gas it is approximately 1\textsuperscript{[12,13]}; therefore, the \(D\)-measure has been proposed as a signature of QGP\textsuperscript{[12,13]}.

However, this observable (\(D\)-measure) has some caveats which have been discussed recently. Gazdzicki and Mrowczynski\textsuperscript{[14]} have argued that \(\langle \delta Q^2 \rangle\) in AA collisions could be determined by the number of participating protons. So the smallness of the \(D\)-measure may be just an indication of the smallness of \(\langle \delta Q^2 \rangle\). Fialkowski and Wit\textsuperscript{[15]} have commented that the \(D\)-measure is a rapidity dependent quantity and the prediction of the \(D\)-measure from PYTHIA/JETSET model is smaller than the estimated value for hadron gas and become much smaller (even less than the estimated value for QGP) when the rapidity region is very large.

In reply to the second criticism, Bleicher, Koch and one of us\textsuperscript{[16]} argued that the dependence of \(D\)-measure on rapidity is due to the following two facts: (1) In Ref.\textsuperscript{[2]}, the approximation \(\langle N_+ \rangle_{\Delta y} = \langle N_- \rangle_{\Delta y}\) was used which may not be fulfilled for heavy-ion collisions, so one needs to apply the correction \(\langle N_+ \rangle_{\Delta y} / \langle N_- \rangle_{\Delta y} \rangle^2\). (2) Also, it was assumed that the charge ratio fluctuated independently in each rapidity window\textsuperscript{[2]}. This is inappropriate due to global charge conservation, and this brings up another correction factor \(1 - \langle N_+ \rangle_{\Delta y} / \langle N_- \rangle_{\Delta y}\). After these corrections, it was found that the corrected measure \(D_{corr}(\Delta y)\)

\begin{equation}
D_{corr}(\Delta y) = \frac{D(\Delta y)}{\langle N_+ \rangle_{\Delta y}^2 (1 - \langle N_+ \rangle_{\Delta y} / \langle N_- \rangle_{\Delta y})}, \tag{3}
\end{equation}

predicted by UrQMD\textsuperscript{[7]} is around 2.5 ~ 3.1\textsuperscript{[16]} which is consistent with the estimated value of \(D\)-measure for resonance gas\textsuperscript{[16,17]}. In Ref.\textsuperscript{[16]}, no significant differences have been found for \(D\)-measure values at SPS and RHIC energies, meaning that the \(D\)-measure has little energy dependence\textsuperscript{[13,14]}

As has been discussed, for example, in Open Standard Code and Routine (OSCAR) conferences\textsuperscript{[18]}, all MCGs used now are not simple codes, they contain different physical ingredients and assumptions; therefore, it is very interesting to study and compare theoretical predictions from some MCGs, which are based on different physics pictures. In this paper we calculate the \(D\)-measure using the VNIb\textsuperscript{[19]}, RQMD v2.4\textsuperscript{[21]}, HIJING v1.35\textsuperscript{[22]}, and HIJING/\Bar{B}\textsuperscript{[23]} models (See section II for a short discussion of these models). One of the striking results is that the values of \(D\)-measure from VNIb model (running with rescattering turned off) is much less than the values of \(D\)-measure from RQMD, HIJING, HIJING/\Bar{B} and UrQMD\textsuperscript{[14]} models. The reason for this difference could be the different number of gluons embedded in the model. In heavy-ion collision processes, if the degrees of freedom are partons or hadrons at the initial stage of collisions, we will expect to
have a different charged fluctuation if the rescattering effects do not play a key role in interactions. In this sense, the $D$-measure could be a signature of QGP. However, to be considered as a good signature of QGP, $D$-measure values must be compared also between nucleus nucleus (AA) and proton-proton (pp) collisions. If the $D$-measure is dominated by the physics just before hadronization, any differences between the values obtained from AA and pp collisions indicate that either rescattering effects are strong, or a signature of new physics (e.g. presumably QGP) in AA collisions.

One of the aims of this work is to perform a systematic study of the charged fluctuations using many of the available and popular event generators. We believe this exercise is valuable first to establish the fluctuations as a robust variable, then to interpret the physical information the measurements contain. We investigate the effects of rescattering and we also consider the impact parameter dependence of the signal.

This paper is arranged in the following way: In Sec. II, using VNIb, RQMD v2.4, HIJING v1.35, HIJING/Bb v1.10 models we calculate the $D$-measure for AA collisions at total centre of mass (c.m) energy $\sqrt{s} = 200A$ GeV and we find that the $D$-measure of VNIb (with rescattering turned off) is much less than the $D$-measure of other models and an explanation is given. In Sec. III, we study the rescattering effects on VNIb and RQMD. Our results show that rescattering effects may spoil the signature of physics in VNIb; on the other hand, the rescattering effects on the $D$-measure of RQMD are less dramatical. A comparison of $D$-measure between pp and AA are performed and the similar value between $D$-measure between pp and AA are explained within a “participant model”. Finally, our discussions and conclusions are given in Sec. IV.

II. THE $D$-MEASURE WITH DIFFERENT MCGS

As in Ref. [10], we will calculate the $D$-measure using VNIb [13], RQMD v2.4 [21], HIJING v1.35 [22], HIJING/BB v1.10 [24] models. In the following we briefly outline the main features of those models.

In HIJING [22], the physics of minijets is addressed explicitly in perturbative Quantum ChromoDynamics (pQCD). The cross sections for hard parton scattering are calculated at the leading order and a K-factor is invoked to account for higher-order corrections. Soft contributions are modeled by diquark-quark strings with gluon kinks induced by soft gluon radiation. Jet quenching and shadowing can also be treated in this approach. HIJING/BB [24] is based on HIJING and a baryon junction mechanism is introduced in order to understand the longitudinal distributions of anti-baryons from pA and AA collisions at SPS energies. The junction-antijunction loops that arise naturally in Regge phenomenology are also included in the calculation. Final state interactions among produced hadrons are implemented neither in HIJING nor in HIJING/BB. RQMD [21] is a transport approach for hadrons and resonances, with initial-state hadronic string generation. There, overlapping strings may fuse into colour-ropes. The fragmentation products from ropes, strings, and resonances may then interact with each other and with the original nucleons. In this model copious rescatterings lead to the development of collective flow and can drive the system towards local equilibrium.

As opposed to RQMD, HIJING and HIJINGBB, VNIb treats a nuclear collisions in terms of parton-parton interactions. It uses a transport algorithm to follow the evolution of the many-body system of interacting partons and hadrons in phase space. For hadronization, VNIb uses a parton-cluster formation and fragmentation approach. Rescattering among partons and hadrons is included in the code. One important feature of VNIb is that, at RHIC energy, it generates a substantial gluon population. Those then play an important role in the simulation of RHIC data in VNIb.

![D-measure vs. rapidity, $y_{cm}$ ± Δ$y$ for Au+Au central collisions ($b$ ≤ 2 fm) at total c.m energy $\sqrt{s} = 200A$ GeV. Full squares and diamonds denote the results predicted by UrQMD (from Ref. [16]) and VNIb respectively. Circles and triangles denote the results obtained when taking into account the correction factors (see text for explanations).](image)

In Fig. 1 the values of the $D$-measure from VNIb (rescattering turned off) vs. the rapidity window are shown. For comparison, the results from UrQMD [14] are also included in the plot. We notice that there are big differences between the values of the $D$-measure from VNIb and those from UrQMD. Applying the correction method given in Ref. [14], we calculate also the corrected values of $D_{corr}$ (see Eq. (3)) and we obtain a higher value for a large rapidity
window. For a rapidity window around ($-2, 2$) we find that the value of $D_{\text{corr}}$ is around one. For smaller rapidity window, the value of $D_{\text{corr}}$-measure is bigger than one, and this can be explained by the fact that small windows will not catch all the decay products of a resonance. If we analyze the correction factor, $1 - \frac{\langle N_q \rangle}{\langle N_{\bar{q}} \rangle}$, given in Ref. [10], we find that for the whole kinematic phase space this correction factor should be zero and can not be used for very larger rapidities, so we must overlook the results for larger rapidity windows ($\Delta y > 4$) in Fig. 1.

The corrected values $D_{\text{corr}}$ obtained from the predictions of VNIb, UrQMD, RQMD, HIJING, HIJING/$B\bar{B}$ models are shown in Fig. 2. The values from VNIb are lower than the values predicted by other MCG models. The main difference can be due to the different number of gluons embedded in VNIb, which is higher than in any other MCGs considered here.

The predictions obtained from all the above models, except VNIb, are consistent with each other in the limit of statistical errors. If rescattering among produced hadrons is not a dominant effect during heavy-ion collisions, then the $D$-measure should be determined by the physics just before hadronization as assumed in Ref. [21,22]. According to this picture, string model codes, like UrQMD, RQMD, HIJING, HIJING/$B\bar{B}$ (we note that UrQMD and RQMD include also hadronic picture in the code), form strings using the quarks or diquarks from two collided nucleons and there is no, or very few gluons [27]. So those quarks and antiquarks will dominate the final state charge fluctuations. On the other hand, for a model like VNIb which contains a large population of gluons, the observed $D$-measure should be different from the results calculated from RQMD and HIJING. It is known that if there are only gluons in the initial state of heavy-ion collisions and if we consider gluon fusion processes (like $gg \rightarrow q\bar{q}$), then the charge fluctuation in a larger rapidity window (for our case from $-2$ to $2$ for example)

$$\langle \delta^2 Q \rangle \approx 0,$$

as the charge is almost conserved in that window; those gluons also produce large number of charged particles. Thus the $D$-measure for a gluon gas should be very small. In the VNIb code, we have quarks, antiquarks and gluons. By examining the parton population in VNIb, we find that the ratio of the number of gluons to the number of quarks and antiquarks from the runs for $Au + Au$ collisions at 200 GeV is around 1.2. If we exclude the extra valence quarks (those valence quarks will mainly contribute to fragmentation regions) coming from nucleons (so that $\langle N_q \rangle = \langle N_{\bar{q}} \rangle$), then the ratio is 1.8. That is, the central rapidity region is the most gluon dominated region in VNIb code [28]. This could explain why the $D$-measure from VNIb is less than the $D$-measure from RQMD and HIJING.

This analysis shows that one obtains different values of $D$-measure owing to the different physics embedded in the MCG; however, to draw any final conclusion, we should have new theoretical predictions using models such as, for example, ARC [20] and compare them with predictions from VNIb and ZPC [26]. ARC is based on hadronic physics and pictures nuclear collisions in terms of nucleon-nucleon collisions. For nucleon-nucleon collisions, the model uses data from experiment. As opposed to RQMD and HIJING, there is no string picture in ARC. Because of this we expect that ARC should give a value of the $D$-measure around three. On the other hand, ZPC [20] is a versatile simulation program that can use initial parton distributions from any source as input, and can study parton evolution and rescattering. However, there is no hadronization algorithm implemented in the code. One could use the parton mode of ZPC to calculate directly the $D$-measure which should be less than one, following the reasoning in [12,13].

Finally, we mention that one can account for all final state particles in a MCG model, which is not the case in heavy-ion experiments because of the fact that detectors can not detect all charged particles. So, we can imagine that there is no charge conservation among the detected particles. Here, we will discuss detector efficiency for two cases: Case I: if we assume that the detector efficiency is the same for both positive and negative particles in each event, then the $\langle R^2 \rangle$ and $\langle R \rangle$ should remain the same. We notice that as the measured charged particles $f(N_{ch})$ becomes smaller (here $f$ is the detector efficiency which represents the ratio of the measured particles to the produced particles, $N_{ch}$ is the production particles), the efficiency for the production of particles decreases. The measured particles are protons, deuterons, and charged pions. The efficiency for the production of particles is small. The measured particles are protons, deuterons, and charged pions. The efficiency for the production of particles is small.
Thus from above we have
\[ P(N_i) = \frac{(N_i)^{N_i}}{N_i!} \exp(-\langle N_i \rangle) \quad i = \pm. \] (5)

We further assume that due to the detector efficiency, the observed particle number \( S_i \) follows a Binomial distribution \( (S_\pm \leq N_\pm) \)
\[ P(S_i|N_i) = \frac{N_i!}{S_i!(N_i-S_i)!} f^{S_i} (1-f)^{N_i-S_i} \quad i = \pm. \] (6)

Then one can easily verify that the observed charged particles have again a Poisson distribution
\[ P(S_i) = \sum_{N_i=S_i}^{\infty} P(N_i)P(S_i|N_i) \]
\[ = \frac{((N_i)f)^{S_i}}{S_i!} \exp(-\langle N_i \rangle f) \quad i = \pm. \] (7)

From above we have
\[ \langle \delta Q^2 \rangle = f\langle N_+ \rangle + f\langle N_- \rangle - 2f^2\langle \delta N_+\delta N_- \rangle. \] (8)

Thus
\[ D = 4 - 8f \frac{\langle \delta N_+\delta N_- \rangle}{\langle N_+ \rangle + \langle N_- \rangle}. \] (9)

This indicates that when \( f \) becomes smaller then the \( D \)-measure will become bigger. This is different from the conclusion in the Case I. In Case I, there is strong correlation between the detector efficiencies of positive and negative charge particles in each event; on the other hand, there is no correlation between detector efficiencies of positive particles and negative charge particles in Case II. The practical case can be more complex. However, as shown here that \( D \)-measure is sensitive to the detector efficiency and we need to exercise caution when comparing theoretical predictions with data.

III. RESCATTERING EFFECTS ON THE \( D \)-MEASURE

A. Rescattering effects on the \( D \)-measures of VNIb and RQMD

Large rescattering effects can destroy the physical correlations which originate from the QGP phase. Then we will only get a hadronic resonance gas signature, \( D \sim 3 \) \[29\]. Rescattering effects depends on two factors, one is the time that particles need to go through the collision region, another one is the density in the collision region. Those two effects will determine the mean free path of particles in the interaction region. For high energy collisions, the time that particles needed to pass through the collision regions is short, since the density is higher. No simple relation exist to determine the effects of rescattering on the \( D \)-measure yet.

We note that \( D \)-measure values from UrQMD model have no impact parameter dependence up to very peripheral collisions and we know that if the impact parameter of AA collisions is very large the nuclei-nuclei collision will be only a superposition of pp collisions (may be one or several pp collisions) and rescattering effects will become smaller. On the other hand, if the impact parameter is smaller, then rescattering effects could play an important role. Most MCGs use the following scheme
\[ A + A = \sum (\text{nucleon} + \text{nucleon}) + \\
(\text{secondary particle} + \text{secondary particle}) \\
+(\text{secondary particle} + \text{nucleon}). \] (10)

UrQMD model predictions show that \( D \)-measure is almost impact parameter independent, and this indicates that rescattering effects do not play a key role for the values of \( D \)-measure at RHIC energy \[16\]. The physics should then be dominated by the simple \( nn \) collisions if the model employed the scheme described by Eq. (10). We also remark that the predictions from UrQMD at SPS energy are larger than the predictions at RHIC energy. The main differences could perhaps be attributed to the mix of hadronic degrees of freedom and string degrees of freedom. When energy is higher the string formation dominate the collisions process, while when energy is lower the hadronic picture does. This may explain why the values of \( D \)-measure at SPS energy are slightly higher than the values at full RHIC energy.

In Fig. 5 we plot the values of \( D \)-measure from RQMD (rescattering turned on) in order to study the impact parameter dependence of rescattering, for two different impact parameters regions. Analyzing the results from Fig. 5 we note that rescattering is slightly higher for central collisions (\( b < 5 \) \( fm \)) in comparison with peripheral ones (\( 5 < b < 10 \) \( fm \)) at low \( \Delta y \). Also, the results seems to indicate that rescattering effects are negligible for a rapidity window \( \Delta y > 1.0 \).

We also calculate the \( D \)-measure value from RQMD model at 130 GeV with rescattering turned on and off. The results are shown in Fig. 6. It is clear that rescattering effects on the value of \( D \)-measure are within 10%. This result is consistent with those in Fig. 5.
FIG. 3. \( D \sim 4\langle \delta Q^2 \rangle /\langle N_{ch} \rangle \) values from RQMD vs. rapidity \( y_{cm} \pm \frac{\Delta y}{2} \) for Au+Au collisions at total c.m. energy \( \sqrt{s} = 200 \text{ A GeV} \). The circles are the results for impact parameter range \( b \leq 5 \text{ fm} \) and the squares are the results for \( 5 \leq b \leq 10 \text{ fm} \). The full and empty symbols are corrected and uncorrected values, respectively.

FIG. 4. \( D(\Delta y) \) values from RQMD vs. rapidity \( y_{cm} \pm \frac{\Delta y}{2} \) for Au+Au collisions at total c.m. energy \( \sqrt{s} = 130 \text{ A GeV} \). The circles are the results with rescattering turned on while the squares are the results for the case without rescattering. The full and empty symbols are the corrected and uncorrected values, respectively.

In Fig. 3 the values of the \( D \)-measure from VNIb (rescattering turned on and off) are shown. It is found that the values of the \( D \)-measure are around 2.3 for rescattering turned on and are smaller with rescattering turned off (\( \approx 1.0 \)). These results show the different effects of rescattering in VNIb and RQMD v2.4 (see Fig. 3). Those could be related to the different densities of hadronic matter at the beginning stage of hadronization. As the density of hadronic matter of VNIb is higher than the density of RQMD, rescattering plays a more important role in VNIb.

FIG. 5. \( D \sim 4\langle \delta Q^2 \rangle /\langle N_{ch} \rangle \) values from VNIb vs. rapidity \( y_{cm} \pm \frac{\Delta y}{2} \) for Au+Au central collisions at total c.m. energy \( \sqrt{s} = 200 \text{ A GeV} \). The circles corresponds to the run with rescattering turned off while the squares corresponds to the run with rescattering turned on. The full and empty symbols are corrected and uncorrected values, respectively.

From above we conclude that the values of the \( D \)-measure from the RQMD and UrQMD models have no impact parameter dependence. Therefore we strongly suggest that the RHIC experiments must determine the impact parameter dependence of \( D \)-measure to verify the above results. If the experimental values indicate a different trend in comparison with theoretical predictions, we may consider that the idea of Eq. (10) is too simple and one needs to involve other effects, such as the fact that the parton distributions functions in nuclei are potentially different from the parton distributions functions in nucleon.

If \( D \)-measure for AA collisions is dominated by single nn interactions, one can imagine that at lower energy, the single nn collisions is dominated by hadronic picture (cluster picture), and at higher energy, nn collisions can see the content of nucleon. When energies increase, it is expected that gluon should have also higher contribution. Based on the above assumption, if we plot the \( D \)-measure of \( pp \) collision as the function of collision energy there should exist a drop from three to one. Even if there is no such drop, one needs to get the trend that \( D \)-measure is really high at lower energy and becomes smaller at high energy. Similar analyses should be performed for heavy-ion collisions too, in order to obtain energy dependence of \( D \)-measure from Bevalac to LHC energies.

On the other hand, if the rescattering effects play a key role as in VNIb model, the signature of the initial stage of
heavy-ion collisions will be lost. However, combined analysis of $D$-measure with other signatures of QGP probably could still give us some more information about the unknown matter created in the early stages of AA collisions.

**B. $D$-measure for pp and AA**

We compare the values of $D$-measure for $pp$ and AA collisions obtained from VNIb (rescattering turned off), VNIb (rescattering turned on), HIJING v1.35, HIJING/$\bar{B}B$ v1.10, RQMD v2.4 (rescattering turned on) in Fig. 6(a-e). We note that the $D$-measure for AA collisions from VNIb without rescattering, HIJING, HIJING/$\bar{B}B$ and RQMD are all consistent with the $D$-measure for $pp$ interactions. On the other hand, the values of $D$-measure for AA from VNIb with rescattering are larger than the predictions for $pp$ due to rescattering effects.

FIG. 6. $D$-measure values from (a) HIJING v1.35; (b) HIJING/$\bar{B}B$ v1.10; (c) RQMD v2.4 (with rescattering); (d) VNIb (rescattering turned off); (e) VNIb (rescattering turned on) models vs. rapidity $y_{cm} \pm \Delta y$ for Au+Au central collisions (full symbols) and and pp collisions (empty symbols) at total nucleon-nucleon c.m. energy $\sqrt{s_{NN}}=200$ GeV.
The interesting result is that $D$ values are similar for $pp$ and $Au+Au$ collisions for all MCGs when rescattering effects are neglected. In the following, we try to explain this in the framework of a “participant model” [3]. As in Ref. [3], we write

$$Q = \sum_{i=1}^{N_p} Q_i.$$  \hspace{1cm} (11)

Here $Q$ is the total charge of $AA$ collisions, $Q_i$ is the charge produced by each nucleon + nucleon $(n+n)$ collisions in a specific rapidity window and $N_p$ is the number of $nn$ collisions for each $AA$ collisions. Taking the average over a number of events we have

$$\langle Q \rangle = \langle N_p \rangle \langle Q_i \rangle$$  \hspace{1cm} (12)

and

$$\langle Q^2 \rangle = \langle N_p \rangle \langle Q_i^2 \rangle + \langle N_p(N_p - 1) \rangle \langle Q_i \rangle^2.$$  \hspace{1cm} (13)

In the derivation we have used $\langle Q_i Q_j \rangle = \langle Q_i \rangle \langle Q_j \rangle$. The mean charged multiplicity for $AA$ collisions can be expressed as

$$\langle N_{ch} \rangle = \langle N_p \rangle \cdot \langle n_i \rangle.$$  \hspace{1cm} (14)

Here $n_i$ is the charged particles produced by each $n+n$ collisions. Finally we get the following equation:

$$\frac{\langle \delta Q^2 \rangle}{\langle N_{ch} \rangle} = \frac{\langle Q_i^2 \rangle}{\langle n_i \rangle} + \frac{\langle Q_i \rangle^2}{\langle n_i \rangle} \cdot \frac{\langle N_p \rangle}{\langle N_{ch} \rangle}.$$  \hspace{1cm} (15)

If the $P(N_p)$ distribution is Poissonian, then $\langle \delta N_p \rangle / \langle N_p \rangle = 1$. In Ref. [3], the author estimated the above value to be around 1.1. $\langle Q_i \rangle^2 / \langle n_i \rangle$ should be much less than one for very high energy $n+n$ collisions, that is

$$\langle n_i \rangle >> \langle Q_i \rangle.$$  \hspace{1cm} (16)

To confirm this, we plot the ratios $\frac{\langle Q_i \rangle^2}{\langle n_i \rangle}$ vs. rapidity window for $pp$ collisions at 200 GeV in Fig. 7. One sees clearly, that for smaller rapidity windows the ratio is near zero, while for whole window the value is around 0.2. The later is due to charge conservation effects. For larger rapidity regions, the particles are produced mainly near the leading valence quarks, so we notice that there is a sharp increase of the value $\frac{\langle Q_i \rangle^2}{\langle N_{ch} \rangle}$ for larger rapidity window. For the inner part of the rapidity region, due to the charge conservation, the mean charge $\langle Q_i \rangle \sim 0$. From above figures we can safely say that $D$-measure for $AA$ collisions should be roughly the same as for $pp$ case when the rescattering effects are negligible. Any deviation between the $D$-measure of $AA$ and $pp$ may indicate a signature of new physics in $AA$ collisions. Thus, it is necessary to check the consistency between $pp$ collisions and $AA$ collisions results before we may conclude that $D$-measure is a signature of QGP.

**IV. COMMENTS AND CONCLUSIONS**

Theoretical predictions of $D$-measure from VNIb, HIJING v1.35, HIJING/B$B$ v1.10, RQMD v2.4, indicate that the fluctuation of charge is sensitive to the parton number embedded in the model if the rescattering effects are not essential; therefore, $D$-measure can be a signature of QGP.

However, if the charge fluctuation shows no impact parameter dependence, then we have to slightly change our views. If we observe similar signal for $pp$ and for peripheral collisions, the charge fluctuation could be only a signature of the fundamental degrees of freedom that we need to take into consideration in the collision processes. In other words, charge fluctuation can tell us when we should treat the heavy-ion collisions as simple hadronic cascade or when is necessary to use QCD, or some model in between. This idea has been used in $e^+e^-$ collisions to see when one should use a cluster picture and when one needs to use a parton picture [10]. If the $D$-measure for $AA$ is bigger than the $D$-measure for $pp$, there could exist a stronger rescattering effects in heavy-ion collisions; on the other hand, if the $D$-measure for $AA$ is lower than the $D$-measure for $pp$, some new physics in $AA$ collisions should be involved. Upcoming experiments at RHIC will allow us to draw more definite conclusions.

To consider $D$-measure as a signature of QGP, one must certify that the model from Ref. [12] and Ref. [13] can not be applied for $pp$ collisions. If a single thermal model is valid for both $AA$ and $pp$ collisions, then the conclusions that $D$-measure is a signature of QGP may be questionable. If the statistical model for parton degrees of freedom can also be used in $pp$ collisions we can not
see any reason why the predictions from pp collisions should be different from the prediction of AA [22], but we totally agree that the $D$-measure can tell us if we need to consider partonic or hadronic degrees of freedom in the collisions.

Our theoretical predictions using different MCG models show that $D$-measure is sensitive to different parton content encoded in the model if the rescattering effects is not dominant. We find that the $D$-measure values do not depend on impact parameter for RQMD v2.4 model and also we obtain similar results for AA and pp collisions, and we explain this using the participant model. On the other hand, we find that the values of $D$-measure from VNIb model are strongly dependent on rescattering effects which spoil the original signature from the initial state of collisions. However, any deviation among the prediction of $D$-measure for different impact parameter in AA collisions and pp collisions may indicate that the rescattering effects play a key role in interactions, or a signature for new physics (e.g. presumably QGP) in AA collisions. Note that a recent paper [24] was concerned about the specific effects of rescattering on the $D$-measure. Within the framework of existing empirical models, our work can be seen as a quantitative answer to those questions. Also it will be crucial to repeat the calculations done here with the soon-to-be-release next version of the parton cascade code [33].

Recently, the STAR collaboration has analyzed the $D$-measure at RHIC energy ($\sqrt{s_{NN}} = 130$ GeV) and has found that the $D$-measure value is around three and has no centrality dependence [24]. This results are consistent with our prediction and those of Ref. [16]. In the calculation of the STAR collaboration [24], they did not use the correction which accounts for the net charge and global charge conservation; if we consider this correction, the $D$-measure will be around 3.9. However, the high value of $D$-measure does not imply that QGP is not formed at RHIC, this high value of $D$-measure may still be explained by final state rescattering.

Acknowledgment

This work was partly supported by the Natural Science and Engineering Research Council of Canada and the Fonds FCAR of the Quebec Government. The authors would like to thank U. Heinz, V. Koch, S. Mrowczynski, S. Voloshin and R. Wit for helpful communications.

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