Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays

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As photons do not interact with each other, it is interesting to ask whether photonic systems can be modified to exhibit the phases characteristic of strongly coupled many-body systems. We demonstrate how a Mott insulator type of phase of excitations can arise in an array of coupled electromagnetic cavities, each of which is coupled resonantly to a single two level system (atom/quantum dot/Cooper pair) and can be individually addressed from outside. In the Mott phase each atom-cavity system has the same integral number of net polaritonic (atomic plus photonic) excitations with photon blockade providing the required repulsion between the excitations in each site. Detuning the atomic and photonic frequencies suppresses this effect and induces a transition to a photonic superfluid. We also show that for zero detuning, the system can simulate the dynamics of many body spin systems.

Introduction: Strongly coupled many-body systems described by the Bose-Hubbard models exhibit Mott insulating phases whose realization in optical lattices have opened varied possibilities for the simulation of many body physics. Can there be another engineered quantum many-body system which displays such phases? This will be especially interesting if the strengths of this system are “complementary” to that of optical lattices – for example if it allowed the co-existence of accessibility to individual constituents of a many-body system and a strong interaction between them, or if it allowed the simulation of arbitrary networks rather than those derivable from superposing lattices. Particularly arresting will be to find such phases by minimally modifying a system of photons which, by being non-interacting, are unlikely candidates for the studies of many-body phenomena. Here we propose such a system consisting of coupled electromagnetic cavities, doped with single two level systems. Using the nonlinearity generated from the corresponding photon blockade effect, we show the possibility of observing an insulator phase of total (atomic+photonic or simply polaritonic) excitations and its transition to a superfluid of photons. Compared to the optical lattices case, the different Hubbard-like model which describes the system involves neither purely bosonic nor purely fermionic entities, the transition from insulator to superfluid is also accompanied by a transition of the excitations from polaritonic to photonic, and that excitations, rather than physical particles such as atoms are involved. In addition, the possibility to simulating the dynamics of an XY spin chain with individual spin manipulation, using a mechanism different from that used in optical lattices is suggested.

System Description: Assume a chain of \( N \) coupled cavities. A realization of this has been studied in structures known as a coupled resonator optical waveguide (CROWs) or couple cavities waveguides (CCW) in photonic crystals, in tabered fiber coupled toroidal microcavities and coupled superconducting microwave resonators. This has already stimulated proposals to use such systems to implement optical and cluster state quantum computing, entangled photons and studying interacting polaritons. We will describe the system dynamics using the operators corresponding to the localized eigenmodes (Wannier functions), \( a_k \). The Hamiltonian is given by

\[
H = \sum_{k=1}^{N} \omega_d a_k^\dagger a_k + \sum_{k=1}^{N} A(a_k^\dagger a_{k+1} + H.C.)
\]

The Hamiltonian is free for the light and dopant parts, \( H^{\text{int}} \) the Hamiltonian describing the internal coupling of the photon and dopant in a specific cavity and \( H^{\text{hop}} \) for for the light hopping between cavities.

\[
H^{\text{free}} = \omega_d \sum_{k=1}^{N} a_k^\dagger a_k + \omega_0 \sum_k \langle e_k | e_k \rangle
\]

\[
H^{\text{int}} = g \sum_{k=1}^{N} (a_k^\dagger | g \rangle \langle e_k + H.C.)
\]

\[
H^{\text{hop}} = A \sum_{k=1}^{N} (a_k^\dagger a_{k+1} + H.C)
\]

where \( g \) is the light atom coupling strength. The \( H^{\text{free}} + H^{\text{int}} \) part of the Hamiltonian can be diagonalized in a basis of mixed photonic and atomic excitations, called polaritons. These polaritons, also known as dressed states,
The above implies (assuming the regime $A < g \sqrt{n}$) that the lowest energy state for a given number, say $\eta_-$, of net excitations at the $k$th site would be the state $|\eta_+\rangle$ (this is because $|\eta_+\rangle$ has a higher energy, but same net excitation $\eta$). Thus one need only consider the first, third and last lines of the above Hamiltonian $H$ for determining the lowest energy states. The first line corresponds to a linear spectrum, equivalent to that of a harmonic oscillator of frequency $\omega_d - g$. If only that part was present in the Hamiltonian, then it would not cost any extra energy to add an excitation (of frequency $\omega_d - g$) to a site already filled with one or more excitations, as opposed to an empty site. However, the term $g(n - \sqrt{n})P_{k}^{(-,n)}\Pi_{k}^{(-,n)}$ raises energies of uneven excitation distribution such as $|n+1\rangle_k|n-1\rangle_l$ among any two sites $k$ and $l$ relative to the uniform excitation distribution $|n\rangle_k|n\rangle_l$ among these sites. Thus the third line of the above Hamiltonian can be regarded as an effective, nonlinear “on-site” photonic repulsion, and leads to a Mott state of the net excitations per site being the ground state for commensurate filling. Reducing the strength of the effective nonlinearity, i.e., the blockade effect, should drive the system to the superfluid regime. For this, one should move the system away from the strong resonant interaction to a weaker dispersive regime. This could be done by Stark shifting and detuning (globally again) the atomic transitions from the cavity by an external field. The new detuned polaritons are not as well separated as before and their energies are merely shifts of the bare atomic and photonic ones by $\pm g^2(n+1)/\delta$ for the $|\eta, n\rangle$ and $|g, n+1\rangle$ respectively. In this case it costs no extra energy to add excitations (excite transitions to higher polaritons) in a single site, and the system moves to the superfluid regime.

To justify the transition of the system from a Mott phase to a superfluid phase as detuning $\Delta = \omega_0 - \omega_d$ is increased, we have performed a numerical simulation using 3-7 sites using the Hamiltonian of Eq.(1)-(3)(numerical diagonalization of the complete Hamiltonian without any approximations) [34]. In the Mott phase the particle number per site is fixed and its variance is zero (every site is in a Fock state). In such a phase, the expectation value of the destruction operator for the relevant particles—the order parameter is zero. In the traditional mean field (and thus necessarily approximate) picture, this expectation value becomes finite on transition to a superfluid, as a coherent superposition of different particle numbers is allowed to exist per site. However, our entire system is a “closed” system and there is no particle exchange with outside. Superfluid states are characterized by a fixed “total” number of particles in the three site system and the expectation of a destruction operator in any given site is zero even in the superfluid phase. Thus this expectation value cannot be used as an order parameter for a quantum phase transition. Instead we use the variance of total number of excitations per site, the operator $N_k$, in a given site (we choose the middle cavity, but any of the other cavities would do) to characterize the Mott to superfluid phase transition. This variance $var(N_k)$ has been plotted in Fig.2 as a function of $\log_{10} \Delta$ for a filling factor of one net excitation per site. For this plot, we have taken the parameter ratio $g/A = 10^{2}$ ($g/A = 10^{1}$...
gives very similar results), with $\Delta$ varying from $\sim 10^{-3}g$ to $\sim g$ and $\omega_d, \omega_p \sim 10^3g$. We have plotted both ideal graphs (if neither the atoms nor the cavity fields underwent any decay or decoherence) and also performed simulations explicitly using decay of the atomic states and photonic states in the range of $g/\max(\kappa, \gamma) \sim 10^3$, where $\kappa$ and $\gamma$ are cavity and atomic decay rates.

These decay rates are expected soon to be feasible in toroidal microcavity systems with atoms \cite{14} and arrays of coupled stripline microwave resonators, each interacting with a superconducting qubit \cite{15}. For these simulations we have assumed that the experiment (of going from the Mott state to the superfluid state and back) takes place in a time-scale of $1/A$ so that the evolution of one ground state to the other and back is adiabatic. The simulations of the state with decay have been done using quantum jumps, and it is seen that there is still a large difference of $\text{var}(N_k)$ between the Mott and superfluid phases despite the decays. As expected the effect of dissipation reduces the final value of order parameter in the superfluid regime (population has been lost through decay) whereas in the Mott regime leads to the introduction of fluctuations again due to population loss from the $|1\rangle$ state. The Mott ($\text{var}(N_k) = 0$) to superfluid ($\text{var}(N_k) = 0.75$) transition takes place over a finite variation of $\Delta$ (because of the finiteness of our lattice) around $10^2g$ and as expected becomes sharper as the number of sites is increased.

In an experiment one would start in the resonant (Mott) regime with all atom-cavity systems initially in their absolute ground $|\langle g, 0 \rangle_{\otimes k}\rangle$ states and prepare the atom-cavity systems in the joint state $|1\rangle_{\otimes k}$ by applying a global external laser tuned to this transition. This is the Mott state with the total (atomic+photonic) excitations operator $N_k$ having the value unity at each site. One would then Stark shift and detune (globally again) the atomic transitions from the cavity by an external field and observe the probability finding $|1\rangle$ and the predicted decrease of this probability (equivalent to the increase in our order parameter, the variance of $N_k$) as the detuning is increased. For inferring the fluctuations in $N_k$ of our system, it is basically sufficient to check the population of the $|1\rangle$ state, as this is an eigenstate of $N_k$. For this, a laser is applied which is of the right frequency to accomplish a cycling transition between $|1\rangle$ and another probe third level. Through monitoring its fluorescence, accurate state measurements can be made in the standard atomic state measurement way \cite{29}.

To strengthen our case, we have also calculated the probabilities of populating the lowest polaritonic state of the first and second manifolds $|1\rangle_{k}$, $|2\rangle_{k}$ of a middle cavity out of an array of 3, 5 and 7 cavities. The initial state is the polaritonic state $|1\rangle_{k}$ excited at the right and the left cavities, with the middle cavity being in the ground state $|g, 0 \rangle_k$. Figure 3 shows as expected from our discussion above, on resonance the photon blockade is preventing any excitation to any state higher than the first (Mott insulator phase). However, by simply varying the atomic frequency and inducing some detuning (of the order of $100g$ in our simulation), the weakening of the blockade effect results in the probability of exciting to the second manifold to become increasingly strong.
photon like polaritonic particles now as we are in the dispersive regime) can hop together in bunches of two or more from cavity to cavity (superfluid regime).

Simulating spin models: We will now show that in the Mott regime the system simulates a XY spin model with the presence and absence of polaritons corresponding to spin up and down. Let us assume we initially populate

the presence and absence of polaritons corresponding to \( \omega_0 \) in the lattice initially with energy \( \omega_0 \). Secondly, if we create only the polaritons convert between polaritons, are fast rotating and they as the inter-converting terms are vanishing. Thus the

\[ P_k^{(\pm)} P_{k+1}^{(\mp)} + P_k^{(\mp)} P_{k+1}^{(\pm)} + H.C. \]  

where \( P_k^{(\pm)} = P_k^{(\pm,1)} \) is the polaritonic operator creating excitations to the first polaritonic manifold (Fig. 1). In the rotating wave approximation, Eq. (7) reads (in the interaction picture).

\[ H_I = A \sum_{k=1}^{N} \left[ P_k^{(-)} P_{k+1}^{(+)} + P_k^{(+)} P_{k+1}^{(-)} + H.C. \right] \]

In deriving the above, the logic is two step. Firstly note that the terms of the type \( P_k^{(-)} P_{k+1}^{(+)} \) which interconvert between polaritons, are fast rotating and they vanish. Secondly, if we create only the polaritons \( P_k^{(-)} \) in the lattice initially with energy \( \omega_0 - g \), then the polaritons corresponding to \( P_k^{(+)} \) will never even be created, as the inter-converting terms are vanishing. Thus the term \( P_k^{(+)} P_{k+1}^{(+)} \) can also be omitted. Note that because the double occupancy of the sites is prohibited, one can identify \( P_k^{(-)} \) with \( \sigma_k^z = \sigma_k^x + i \sigma_k^y \), where \( \sigma_k^x \) and \( \sigma_k^y \) are standard Pauli operators. Then the Hamiltonian becomes

\[ H_I = \sigma_k^z \sigma_{k+1}^z + \sigma_k^x \sigma_{k+1}^y. \]

The latter is the standard XY model of interacting spins with spin up/down corresponding to the presence/absence of a polariton. Note that although this is different with optical lattice realizations of spin models, where instead, the internal levels of a two level atom are used for the two qubit states, the measurement could be done using very similar atomic state measurement techniques (utilizing the advantage of larger distances between sites as well).

Some simple applications of XY spin chains in quantum information processing such as quantum information transfer \[22, 23, 24\] can thus be readily implemented in our system. Very recently a novel idea for efficient cluster state quantum computation was proposed in this system where the database search and factoring quantum algorithms could be implemented using just two rows of cavities \[25\].

In conclusion we showed that a range of many body system effects, such Mott transitions for polaritonic particles obeying mixed statistics could be observed in optical systems of individual addressable coupled cavity arrays interacting with two level systems. We also proposed possible implementations using photonic crystals, toroidal microcavities and superconducting systems. Finally we discussed the capability and advantages of simulating XY spin models using our scheme and noted the ability of these arrays to simulate arbitrary quantum networks.

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Finite numbers of sites is often used in studying the physics of the transition from Mott to superfluid phases, especially as analytic methods from mean-field theory are invalid for one-dimensional or network geometries. In the pioneering optical lattice Mott transition paper, for example, 7 lattice sites were used, and we use the same though our model is computationally more exhaustive than just having bosonic occupation numbers per site because of the extra atomic degree of freedom at each site that cannot be eliminated.