Temperature Dependence of the Spin Polarization in the Fractional Quantum Hall Effects

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Using a Hamiltonian formulation of Composite Fermions that I recently developed with R. Shankar, I compute the dependence of the spin polarization on the temperature for the translationally invariant fractional quantum Hall states at $\nu = 1/3$ and $\nu = 2/5$. I compare my results to experiments at $\nu = 1/3$, and find reasonably good agreement.

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The fractional quantum Hall (FQH) effect has introduced us to new, highly correlated, incompressible states of electrons in high magnetic fields. A unified understanding of all fractions $\nu = p/(2sp + 1)$ was achieved by the Composite Fermion picture of Jain, in which the electrons are dressed by $2s$ units of statistical flux to form Composite Fermions (CFs). At a mean field level, the CFs see a reduced field $B^* = B/(2sp + 1)$, in which they fill $p$ CF-Landau levels (CF-LLs), and exhibit the integer quantum Hall effect.

Due to the small $g$ factor of electrons in GaAs, spins may not be fully polarized in FQH states. Transitions between singlet, partially polarized, and fully polarized states (based on gap measurements) have been observed for a number of fillings, which can be understood in terms of CF’s with a spin $S = 1/2$. The transitions happen when an unoccupied CF-LL of one spin crosses the occupied CF-LL of the opposite spin.

While these low temperature measurements are in satisfactory agreement with the ground states predicted in the CF picture, in order to understand the temperature dependence of the polarization $P(T)$ one has to consider all excited states as well. Detailed measurements of $P(T)$ for the $\nu = 1/3$ state have recently appeared in the literature. It is well-known that the $\nu = 1/3$ state is spontaneously polarized at $T = 0$, even when the Zeeman coupling $E_Z = \mu B_{tot}$ is zero. In this it is analogous to the $\nu = 1$ state, which has been extensively studied theoretically and experimentally. There are, however, significant differences between the two cases at finite $T$. The $P(T)$ curve for $\nu = 1/3$ has a different shape, and has empirically been fitted to a noninteracting form $P(T) = \tanh(\Delta/4k_B T)$ where $\Delta$ is found to approximately twice $E_Z$.

In a recent paper, MacDonald and Palacios identified a key qualitative feature that makes $\nu = 1/3$ very different from $\nu = 1$. In the $\nu = 1$ case the particle-hole excitations are very high in energy compared to $E_Z$, and are frozen out at all low temperatures of interest. Consequently, the $T$ dependence of $P$ comes mainly from spin wave excitations and their interactions. This is the reason why long-wavelength effective theories such as the continuum quantum ferromagnet approach are successful. However, for $\nu = 1/3$, particle-hole excitations are on the same scale of energy as $E_Z$, and cannot be ignored at any $T$. MacDonald and Palacios use a simplified model to illustrate this feature, but the model is not sufficiently detailed to enable a calculation of $P(T)$ for a realistic sample. Also, the model cannot be readily extended to non-Laughlin fractions.

The goal of this paper is to describe a general analytical method for approximately computing $P(T)$ for an arbitrary principal fraction for realistic samples. In the last few years, R. Shankar and I have developed a Hamiltonian formalism which allows us to carry out approximate calculations for any physical quantity in the fractional Hall regime. Our central result is a formula for the LLL-projected electronic charge density at small $q$:

$$\rho_e(q) = \frac{\sum_j e^{-iqx_j}}{2ps + 1} - i\lambda^2(\sum_j (q \times \Pi_j)e^{-iqx_j})$$

(1)

where $\hat{x}_j$ is a CF coordinate, $l = 1/\sqrt{eB}$ is the magnetic length, and $\Pi_j = \hat{P}_j + e\hat{A}(x_j)$ is the velocity operator of the CFs. The low-energy Hamiltonian is $H = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} v(q)\hat{P}(q)\hat{P}(-q)$ where $v(q)$ is the electron-electron interaction. To include the effects of finite sample thickness, and to stay within the limitations of our small-$q$ approach, we work with a modified Coulomb interaction of the form $v(q) = e^{-\lambda q^2\pi c^2/q}$, where the length $\lambda$ is connected to the thickness. A notable feature of the formalism is that energy dispersions arise entirely from interaction effects. The Hartree-Fock (HF) approximation has been applied to the above Hamiltonian, and reasonable success has been obtained in computing gaps and scaling relations between gaps for the principal fractions, and magnetoexciton dispersion.

Most recently Shankar has computed $P(T)$ for the compressible half-filled LL. The reason HF works so well is that our Hamiltonian is expressed directly in terms of CF coordinates. The CF’s have the right (fractional) charge and dipole moment in our formalism, and corrections to HF are expected to be small for most physical quantities.

We will compute $P(T)$ for $1/3$ and $2/5$ by simply carrying out the Hartree-Fock (HF) approximation for CFs at finite $T$. We will use a CF-LL cutoff to ensure that the correct number of electronic states occur in the Hilbert space. This implies that one must keep 3 CF-LLs for
ν = 1/3 and 5 CF-LLs for ν = 2/5. While this restriction is relatively unimportant at very low T, it becomes increasingly important as the temperature increases, and occupations of the excited CF-LLs become significant.

Let us proceed to the results. We first consider the 10W sample of Khandelwal et al. The sample parameters are \( B_\perp = 9.61T, B_{tot} = 12T \), and each quantum well has a thickness of 260 Å. This implies that the Coulomb energy scale is \( E_C = e^2/\varepsilon l_0 \approx 160K \) and the Zeeman energy is \( E_Z = 0.0175E_C \). The nominal thickness of the sample is \( l = 3l_0 \).

![FIG. 1. Polarization versus T for ν = 1/3. The circles are the data from the 10W sample of Khandelwal et al. The solid line is the prediction from our theory for \( \lambda = 2.2l_0 \), while the long dashed line is for \( \lambda = 3.0l_0 \).](image1)

Figure 1 shows the HF prediction from our theory for \( \lambda = 3l_0 \) compared to the experimental data (dashed line). The agreement is very gratifying. However, it must be regarded as fortuitous, since the simple model potential that we have assumed is unlikely to reproduce the complicated effects of density distributions in the actual sample with finite thickness. In other words, \( \lambda \) is related to the sample thickness only in a complicated and indirect way. Another way to determine the value of the effective \( \lambda \) is to go to the calculation of Shankar et al. for ν = 1/2, in which he found a reasonable fit to \( P(T) \) assuming \( \lambda \approx 1.75l_0 \) for the same 10W sample. Accounting for the fact that the magnetic length changes when one changes filling at constant density, we estimate \( \lambda \approx 2.2l_0 \) for ν = 1/3. Figure 1 also shows the prediction for this value (solid line). It can be seen that the predicted curve lies above the data over a range of intermediate \( T \). This is only to be expected since the simple HF does not include the effects of spin wave excitations, or of the modification of single-particle energies due to the interaction of CFs with spin waves. The agreement between theory and experiment shown in Figure 1 is reasonably good, even for \( \lambda = 2.2l_0 \). It can also be seen that changes in \( \lambda \) do not make huge changes in \( P(T) \).

![FIG. 2. Knight shift versus T for ν = 1/3. The circles are the data from the M242 sample of Melinte et al. The lines are the predictions from our theory for \( \lambda = 4l_0 \) and \( 2.0l_0 \), assuming two different values for the Knight shift that corresponds to \( P = 1 \).](image2)

Figure 2 shows the same type of comparison for the data of Melinte et al. for their M242 sample. Here the sample parameters are \( B_{tot} = B_\perp = 17T \), and the nominal well thickness is 250 Å. This implies that \( E_C \approx 210K, E_Z \approx 0.0186E_C \approx 4K \), and a nominal value for \( \lambda = 4.0l_0 \). Once again this value for \( \lambda \) is very likely an overestimate, so I have also calculated the prediction for \( \lambda = 2.0l_0 \). There is a lot of scatter in the data at low \( T \), due to the very long times needed to measure the Knight shift, and the error bars are also large at low \( T \). This gives us some latitude in defining what we mean by the Knight shift corresponding to \( P = 1 \). In any reasonable theory one expects to find that \( P = 1 \) for \( T \ll E_Z \), and expects to see this saturated value of \( P \) up to about \( T = 0.5E_Z \) or so.

Based on these considerations I have used two values of \( K_{s,P=1} = 21kHz \) and 19kHz, both of which lie within the error bars of the low \( T \) data. One possibility that can explain this spread is that spin-reversed quasiparticles are present in the ground state due to disorder, which can bring down the “saturated” value of the Knight shift. The 21kHz value was used by Melinte et al. in a phenomenological \( \tanh(\Delta/k_BT) \) fit to obtain \( \Delta = 1.7E_Z \). I believe that the fit for \( K_{s,P=1} = 19kHz \) is closer to the truth, since then the experimental saturation region is about \( 0.5E_Z \). The agreement between theory and experiment for this value of \( K_{s,P=1} \) is much better than for \( K_{s,P=1} = 21kHz \). Overall the agreement is somewhat worse than for the Khandelwal et al. data, but still adequate, considering the simple nature of the approximation.

Why is HF so good in this case while it was so poor for \( \nu = 1 \)? To answer this question let us turn to the spin wave dispersions. These can be computed in the manner described in my magnetoexciton calculation, and are shown in Figure 3 for \( \lambda = 2.2l_0 \) and 3.0l_0. For \( E_Z = 0.0175E_C \) and \( T = 0 \). The \( q \rightarrow 0 \) limit is required to be \( E_Z \) by Larmor’s theorem, while the \( q \rightarrow \infty \) limit
is the spin-reversed particle-hole gap $\Delta_{SR}$. Figure 3 explicitly illustrates the feature that the spin-flip particle-hole excitations are at the same energy scale as $E_Z$. This in turn points to the need for a more sophisticated theory of these ferromagnets which includes particle-hole excitations, spin waves, and their interactions with themselves and each other. Such a complete theory does not yet exist.

Therefore, as the temperature increases, our theory becomes more weakly interacting, and our HF becomes more accurate. This trend can be expected to continue until a temperature scale when CFs cease to exist. There are no obvious signs of such a scale in the data.

Our theory is very general, and can be applied to any fractional Hall state. To illustrate this Figure 5 shows the $P(T)$ curves for $\nu = 2/5$ for $\lambda = 1.5l_0$ for a range of Zeeman couplings. Note the nonmonotonicity of the curves that start from the singlet ground state at $T = 0$. There is a transition to the fully polarized state around $E_Z = 0.01E_C$. Note also that I have allowed only translationally invariant HF states, which ignores possible partially polarized states that I have recently proposed to explain intriguing observations by Kukushkin et al. of a state with half the maximal polarization for $\nu = 2/5$, which is not allowed as a translationally invariant CF state. I plan to explore the temperature dependence of the polarization, and other properties of this inhomogeneous state more thoroughly in a future publication.

Finally, let us compare our results to the only other method that can compute $P(T)$ for arbitrary fractions, which is exact diagonalization (keeping all the excited states) and subsequent calculation of thermodynamic quantities. Due to computational limitations, this method is restricted to fairly small systems. For example, the largest system studied by Chakraborty and Pietilainen for $\nu = 1/3$ has 5 electrons, and for $\nu = 2/5$ has 4 electrons. For $\nu = 1/3$ the exact diagonalization result lies above our predictions (and the experiment) for $T > 4K$. In fact, at $T = 0.09E_C \approx 14K$, the exact diagonalization prediction seems to be almost a factor of two above our prediction, which essentially coincides with the experiment (Figure 1). This discrepancy might be the result of finite thickness or finite size corrections. However, at low $T$ the exact diagonalization result follows the data more closely than our HF approximation (in all the above comparisons I have used the $q = 0.5$ line in Figure 2 of ref[27] and compared to the 10W samp-
ple of Khandelwal et al. This sample has the closest parameters to those used in ref[27]). For \( \nu = 2/5 \), our results reproduce the nonmonotonicity of \( P(T) \) for those values of \( E_Z \) where the singlet state is the ground state, and the peaks in \( P(T) \) occur at roughly the same \( T \) in our results and the exact diagonalization results. However, the same overall pattern holds for \( \nu = 2/5 \), namely, the results of Chakraborty and Pietiläinen are below ours for low \( T \), but are higher for \( T > 0.02E_C \), where they once again see a \( 1/T \) tail with a large coefficient. It would be interesting to explore the finite size systematics to see if the large \( T \) tail is suppressed for larger sizes.

In summary, I have presented an approximate analytical method for computing the temperature dependence of the polarization for an arbitrary fractional quantum Hall state. An important open problem is the development of a formalism that can successfully compute corrections to HF by including particle-hole excitations, spin waves, and their interactions in a self-consistent way at finite \( T \). While the problem is not pressing for \( \nu = 1 \), where large-\( N \) treatments give quite good agreement with experiment, it is sorely needed for fractional Hall ferromagnets. It will be interesting to see whether such corrections to HF can improve the agreement between our predictions and experiment.

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1. D. Tsui, H. Stromer and A. Gossard, Phys. Rev. Lett. 48, 1599, (1982).
2. R. B. Laughlin, Phys. Rev. Lett. 50, 1395, (1983).
3. J. K. Jain, Phys. Rev. Lett. 63, 199, (1989); Phys. Rev. B 41, 7653 (1990); Science 266, 1199 (1994); J. K. Jain and R. K. Kamilla, in “Composite Fermions”, Olle Heinonen, Editor (World Scientific, 1998).

4. B. I. Halperin, Helv. Phys. Acta 56, 75 (1983).
5. T. Chakraborty and F. C. Zhang, Phys. Rev. B 29, 7032; F. C. Zhang and T. Chakraborty, ibid 30 7320; M. Rasolt, F. Perrot, and A. H. MacDonald, Phys. Rev. Lett. 55, 433 (1985).
6. R. G. Clark, S. R. Haynes, A. M. Suckling, J. R. Mallett, P. A. Wright, J. J. Harris, and C. T. Foxon, Phys. Rev. Lett. 62, 1536 (1989).
7. J. P. Eisenstein, H. L. Stormer, L. Pfieffer, and K. W. West, Phys. Rev. Lett. 62, 1540 (1989); J. E. Furneaux, D. A. Syphers, and A. G. Swanson, Phys. Rev. Lett. 63, 1098 (1989).
8. A. Buckthought, R. Boulet, A. Sachrajda, Z. Wasilewski, P. Zawadski, and F. Guillon, Sol. St. Comm. 78, 191 (1991).
9. R. R. Du, A. S. Yeh, H. L. Stormer, D. C. Tsui, L. N. Pfieffer, and K. W. West, Phys. Rev. Lett. 75, 3926 (1995); Phys. Rev. B55, R7351 (1997).
10. X. G. Wu, G. Dev, and J. K. Jain, Phys. Rev. Lett. 71, 153 (1993); K. Park and J. K. Jain, Phys. Rev. Lett. 80, 4237 (1998).
11. P. Khandelwal, N. N. Kuzma, S. E. Barrett, L. N. Pfieffer, and K. W. West, Phys. Rev. Lett. 81, 673 (1998).
12. S. Melinte, N. Freytag, M. Horvatic, C. Berthier, L. P. Levy, V. Bayot, and M. Shayegan, Phys. Rev. Lett. 84, 354 (2000).
13. S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B 47, 16419 (1993).
14. N. Read and S. Sachdev, Phys. Rev. Lett. 76, 3204 (1996); C. Timm, S. M. Girvin, P. Henlius, and A. W. Sandvik, Phys. Rev. B 58, 1464 (1998).
15. M. Kasner and A. H. MacDonald, Phys. Rev. Lett. 76, 3204 (1996).
16. R. Haussmann, Phys. Rev. B 56, 9684 (1997).
17. R. Tycko, S. E. Barrett, G. Dabbagh, L. N. Pfieffer, and K. W. West, Science 268, 1460 (1995); S. E. Barrett, G. Dabbagh, L. N. Pfieffer, K. W. West, and R. Tycko, Phys. Rev. Lett. 74, 5112 (1995); M. J. Manfra, E. H. Aifer, B. B. Goldberg, D. A. Broido, L. N. Pfieffer, and K. W. West, Phys. Rev. B 54, 17327 (1996).
18. A. H. MacDonald and J. J. Palacios, Phys. Rev. B 58, 10171 (1998).
19. R. Shankar and G. Murthy, Phys. Rev. Lett. 79, 4437 (1997); G. Murthy and R. Shankar, in “Composite Fermions”, Olle Heinonen, Editor (World Scientific, 1998).
20. G. Murthy and R. Shankar, Phys. Rev. B 59, 12260 (1999).
21. G. Murthy, K. Park, R. Shankar, and J. K. Jain, Phys. Rev. B 58, 15363 (1998).
22. G. Murthy, Phys. Rev. B 60, 13702 (1999).
23. R. Shankar, cond-mat/9911285, Phys. Rev. Lett. 84, 3946 (2000).
24. J. K. Jain, private communication.
25. G. Murthy, Phys. Rev. Lett. 84, 350 (2000).
26. J. V. Kukushkin, K. v Klitzing, and K. Eberl, Phys. Rev. Lett. 82, 3665 (1999).
27. T. Chakraborty and P. Pietiläinen, Phys. Rev. Lett. 76, 4018 (1996).