Study of Λ hypernuclei in the quark mean field model

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Abstract

We extend the quark mean field model to the study of Λ hypernuclei. Without adjusting parameters, the properties of Λ hypernuclei can be described reasonably well. The small spin-orbit splittings for Λ in hypernuclei are achieved, while the Λ single particle energies in the present model are slightly underestimated as compared with the experimental values. About 3% deviation from the quark model prediction for the $\omega - \Lambda$ couplings is required in order to reproduce the experimental single particle energies.

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Over the past years hypernuclear physics has been attracting great interest. The most extensively studied hypernuclear system consists of a single Λ particle coupled to the nuclear core, which could be produced in \((\pi^+, K^+)\) or \((K^-, \pi^-)\) reactions. Λ hypernuclear spectroscopy indicates that Λ is weakly bound in nuclear medium and its spin-orbit splitting is quite small compared with the nucleon \([1, 2, 3, 4]\). Theoretical efforts have been devoted to understanding the properties of hypernuclei \([5]\). The relativistic mean field (RMF) models have been successfully applied to describe hypernuclei with adjustable meson-hyperon couplings and tensor couplings \([6, 7, 8, 9]\). Microscopic hypernuclear structure calculations have also been performed in the quark-meson coupling (QMC) model \([10, 11]\), where both the hyperon and the nucleon are the composites of quarks.

In this paper, Λ hypernuclei are investigated in the quark mean field (QMF) model, which has been successfully applied to study the properties of both nuclear matter and finite nuclei \([12, 13]\). The QMF model describes the nucleon in terms of the constituent quark model, while the MIT bag model was used in the QMC model \([14, 15, 16]\). The main purpose of the present work is to extend the QMF model to flavor SU(3) and further to the study of Λ hypernuclei. With the assumption that the \(\sigma\), \(\omega\), and \(\rho\) mesons couple only to the \(u\) and \(d\) quarks, the meson-hyperon couplings can be obtained from the quark model. It is very interesting to perform the self-consistent calculations for Λ hypernuclei in the QMF model without any freedom of adjusting parameters.

We start with the description of the Λ hyperon in nuclear medium. The nucleon and the Λ hyperon as composites of three quarks are described in terms of the constituent quark model, in which the constituent quarks satisfy the Dirac equations with confinement potentials \([13]\). The \(\sigma\) meson, which couples directly to the \(u\) and \(d\) quarks, provides a scalar potential to the quark and as a consequence reduces the constituent quark mass. The change of the nucleon properties under the influence of the \(\sigma\) mean field has been studied in Ref. \([13]\). We now treat the Λ hyperon in nuclear medium. According to the OZI rule, the non-strange mesons couple exclusively to the \(u\) and \(d\) quarks and not to the \(s\) quark. Therefore the state of the \(s\) quark inside the Λ hyperon will not be influenced due to the presence of the non-strange
meson mean fields. We follow Ref. [13] to take into account the spin correlations and remove the spurious center of mass motion, and then obtain the effective mass for Λ hyperon as \( M_{\Lambda}^* = \sqrt{(2e_q + e_s + E_{\text{spin}}^\Lambda)^2 - (2\langle p^2_q \rangle + \langle p^2_s \rangle)} \), while the effective mass for nucleon is expressed as \( M_N^* = \sqrt{(3e_q + E_{\text{spin}}^N)^2 - 3\langle p^2_q \rangle} \). Here the subscript \( q \) denotes the \( u \) or \( d \) quark. The energies \( (e_q \) and \( e_s \)) and momenta \( (\langle p^2_q \rangle \) and \( \langle p^2_s \rangle \)) can be obtained by solving the Dirac equations. We take the same two types of confinement as used in Ref. [13]: (1) scalar potential \( \chi_c = \frac{1}{2}kr^2 \) and (2) scalar-vector potential \( \chi_c = \frac{1}{2}kr^2(1 + \gamma^0)/2 \) with \( k = 700 \text{ MeV/fm}^2 \). The quark masses are taken as \( m_q = 313 \text{ MeV} \) and \( m_s = 490 \text{ MeV} \). The spin correlations \( (E_{\text{spin}}^N \) and \( E_{\text{spin}}^\Lambda \)) are fixed by fitting the nucleon and Λ masses in free space \( (M_N = 939 \text{ MeV}, \ M_\Lambda = 1116 \text{ MeV}) \).

We present in Fig. 1 the variations of the effective masses for the nucleon and the Λ hyperon, \( \delta M_i^* = M_i^* - M_i \) \( (i = N, \Lambda) \), as functions of the quark mass correction due to the presence of the \( \sigma \) mean field, \( \delta m_q = m_q - m_q^* = -g^q_\sigma \sigma \) \( (q = u, d) \). The results with the scalar potential are shown by solid curves, while those with the scalar-vector potential by dashed ones. The behavior of the effective nucleon mass has been discussed extensively in our previous work [13]. We now focus on the effective mass for the Λ hyperon. It is obvious that the reduction of \( M_\Lambda^* \) is smaller than that of \( M_N^* \), since only two of the three quarks in the Λ hyperon are influenced by the \( \sigma \) mean field. We note that the dependence of the effective masses on the \( \sigma \) mean field must be calculated self-consistently within the quark model, therefore the ratio of the variation of the effective mass for the Λ hyperon to that for the nucleon, \( \delta M_\Lambda^*/\delta M_N^* \), is not so simple as a constant as in the RMF models [3, 4].

We treat a single Λ hypernucleus as a system of many nucleons and a Λ hyperon which interact through exchange of \( \sigma, \omega, \) and \( \rho \) mesons. The effective Lagrangian can be written as

\[
\mathcal{L} = \bar{\psi} \left[ i\gamma_\mu \partial^\mu - M_N^* - g_\omega \omega \gamma^0 - g_\rho \rho \tau_3 \gamma^0 - e \frac{(1 + \tau_3)}{2} A \gamma^0 \right] \psi \\
+ \bar{\psi}_\Lambda \left[ i\gamma_\mu \partial^\mu - M_\Lambda^* - g_\omega^\Lambda \omega \gamma^0 \right] \psi_\Lambda \\
- \frac{1}{2} (\nabla \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} (\nabla \omega)^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{4} c_3 \omega^4 \\
+ \frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} (\nabla A)^2,
\]

where \( \psi \) and \( \psi_\Lambda \) are the Dirac spinors for the nucleon and the Λ hyperon. The mean field
approximation has been adopted for the exchanged $\sigma$, $\omega$, and $\rho$ mesons, while the mean field values of these mesons are denoted by $\sigma$, $\omega$, and $\rho$, respectively. $m_\sigma$, $m_\omega$, and $m_\rho$ are the meson masses. $A$ is the electromagnetic field which couples to the protons. Since the $\Lambda$ hyperon is neutral and isoscalar, it only couples to the $\sigma$ and $\omega$ mesons. The influence of the $\sigma$ meson on the $\Lambda$ hyperon is contained in $M^*_\Lambda$, while the $\omega$ meson couples to the $\Lambda$ hyperon with the coupling constant $g_\omega^3 = 2g_\omega^2$ (for the nucleon, $g_\omega = 3g_\omega^2$, $g_\rho = g_\rho^2$). In the QMF model, the basic parameters are the quark-meson couplings ($g_\sigma^3$, $g_\sigma^2$, and $g_\rho^2$), the nonlinear self-coupling constants ($g_3$ and $c_3$), and the mass of the $\sigma$ meson ($m_\sigma$), which has been determined by fitting the properties of nuclear matter and finite nuclei in Ref. [13]. Therefore, no more adjustable parameters exist when it is extended to the calculation of $\Lambda$ hypernuclei. From the Lagrangian given in (1), we obtain the following Euler-Lagrange equations

\[
\begin{align*}
&\left[i\gamma_\mu \partial^\mu - M_N^* - g_\omega^2 \gamma^0 - g_\rho^3 \tau_3 \gamma^0 - e \frac{(1 + \tau_3)}{2} A \gamma^0 \right] \psi = 0, \\
&\left[i\gamma_\mu \partial^\mu - M_\Lambda^* - g_\omega^3 \gamma^0 \right] \psi_\Lambda = 0, \\
\left(-\Delta + m_\sigma^2\right) \sigma = -\frac{\partial M_N^*}{\partial \sigma} \rho_s - \frac{\partial M_\Lambda^*}{\partial \sigma} \rho_\Lambda^s - g_3 \sigma^3, \\
\left(-\Delta + m_\omega^2\right) \omega = g_\omega \rho_v + g_\omega^2 \rho_\lambda^A - c_3 \omega^3, \\
\left(-\Delta + m_\rho^2\right) \rho = g_\rho \rho_3, \\
-\Delta A = e \rho_p,
\end{align*}
\]

where $\rho_s$ ($\rho_\Lambda^s$), $\rho_v$ ($\rho_\lambda^A$), $\rho_3$, and $\rho_p$ are the scalar, vector, third component of isovector, and proton densities, respectively. The above coupled equations are solved self-consistently with the effective masses ($M_N^*$ and $M_\Lambda^*$) obtained at the quark level.

If we consider the spin of $\Lambda$ is carried exclusively by the $s$ quark, the spin-orbit interaction should come entirely from the Thomas precession [10, 14]. The correction, $\frac{1}{M_\Lambda^*} \left( \frac{d}{dr} g_\omega^3 \right) \vec{l} \cdot \vec{s}$, could be added perturbatively to the $\Lambda$ single particle energies, so that the spin-orbit interaction corresponds to the formula from Thomas precession [14]. With this correction, small spin-orbit splittings are obtained while the single particle energies are not much altered. For instance, the spin-orbit splitting for the $1f$ states ($1f_{5/2} - 1f_{7/2}$) in $^{91}_\Lambda$Zr decreases to 0.03 MeV, while
the value is 1.5 MeV without the perturbative correction. The recent experimental result [4] seems to reveal the splitting of 1.6 ± 0.15 MeV for the $1f$ states in $^{89}\Lambda Y$.

We present in Fig. 2 the calculated $\Lambda$ single particle energies in several hypernuclei consisting of a closed-shell nuclear core and a single $\Lambda$ hyperon, while the results in the QMC model [10] and the experimental values [1, 2, 3] are also shown for comparison. Here the QMC results do not contain the effect of the Pauli blocking, which has been included phenomenologically in Ref. [11] in order to reproduce the experimental single particle energies. The QMF I and QMF II denote the models with confinements $\chi_c = \frac{1}{2}kr^2(1 + \gamma^0)/2$ and $\chi_c = \frac{1}{2}kr^2$, respectively. The parameters in the QMF models have been determined in our previous work [13], therefore no free parameter in the present calculation. We notice that the final results are insensitive to the choice of the $s$ quark mass. It is found that small spin-orbit splittings for the $\Lambda$ in those hypernuclei are obtained in the present model. For instance, the spin-orbit splittings for the $1d$ states ($1d_{3/2} - 1d_{5/2}$) in $^{41}\Lambda Ca$ and $^{209}\Lambda Pb$ decrease to 0.02 and 0.01 MeV with the perturbative correction, while those values are about 1.4 and 0.5 MeV without the perturbative correction. The small spin-orbit splittings were mostly worked out by adding tensor interactions in the RMF models [6, 7, 8]. In Fig. 3, we plot the scalar and vector potentials ($U_S$ and $U_V$) for the $1s_{1/2} \Lambda$ state in $^{41}\Lambda Ca$ and $^{209}\Lambda Pb$. The results with two types of confinement shown by solid and dashed curves are almost identical. The attractive scalar potential is mostly canceled by the repulsive vector potential, and their difference at the center of the hypernuclei is about $20 - 25$ MeV.

The single particle energies in the present model seem to be slightly underestimated, which is opposite to the tendency in the QMC model. It is well known that the properties of $\Lambda$ hypernuclei are very sensitive to the effective coupling constants on the hadronic level, especially the two relative couplings $R_\sigma = g_\sigma^{\Lambda}/g_\sigma$ and $R_\omega = g_\omega^{\Lambda}/g_\omega$ [17]. The quark model value, $R_\sigma = R_\omega = 2/3$, usually gives large overbinding of $\Lambda$ single particle energies. Some effects, like correlated $\pi\pi$ exchange, may cause the deviations of $R_\sigma$ and $R_\omega$ from the quark model value of $2/3$. Most studies of the hypernuclei in the RMF models are performed by treating both $R_\sigma$ and $R_\omega$ (or only one of them) as phenomenological parameters, which are fitted by using
experimental data [3, 4, 5, 7]. In the present model, $R_\sigma = g^\Lambda_\sigma/g_\sigma = \left(\frac{\partial M^*_\Lambda}{\partial \sigma}\right) / \left(\frac{\partial M^*_\sigma}{\partial \sigma}\right)$ must be calculated self-consistently on the quark level, while $R_\omega = 2/3$ is based on the quark model. Comparing with the RMF models, $R_\sigma$ in the QMF model depends on the $\sigma$ mean field, and could not be a constant again. The resulting $\Lambda$ single particle energies are slightly underestimated in comparison with the experimental values as shown in Fig. 2. The results can be largely improved if we use the scaled coupling constant, $0.97 \times g^\Lambda_\omega$, which gives the $1s_{1/2}$ single particle energy in $^{209}\Lambda$Pb to be $-27.2$ MeV in QMF$^I$ and $-26.0$ MeV in QMF$^II$. On the contrary, the results in the QMC model (without Pauli blocking effect) are overestimated in comparison with the experimental data [10, 11]. The scaled coupling constant, $1.10 \times g^\Lambda_\omega$ (or $0.93 \times g^\Lambda_\sigma$), is required in order to reproduce the experimental single particle energies in the QMC model.

It is very interesting to discuss the origin of this difference, because of the similarity of the QMC and QMF models. $R_\omega = 2/3$ has been used in both models, while $R_\sigma$ has to be calculated with different approaches. $R_\sigma$ obtained in the QMC model was very close to $2/3$. This is related to its expressions for the effective masses, where the center of mass correction was assumed to be independent of the $\sigma$ mean field, and parametrized into the term of $Z_0$. In our calculation, we keep the center of mass correction, which is found to decrease with increasing $\sigma$ mean field, and then we get smaller values than $2/3$ for $R_\sigma$. This seems to be the dominant origin of the underbinding in the QMF model, to be contrasted with the overbinding in the QMC model.

In summary, we have reported the results of the first application of the QMF model to the description of $\Lambda$ hypernuclei. With the parameters determined by the properties of nuclear matter and finite nuclei [13], the calculated results for $\Lambda$ hypernuclei are acceptable. The small spin-orbit splittings for the $\Lambda$ in hypernuclei are obtained in the present model, while the single particle energies are slightly underestimated in comparison with the experimental values. We notice that about 3% deviation from $R_\omega = 2/3$ is required in order to reproduce the experimental single particle energies. The 3% reduction in $R_\omega$ provides $\sim 6$ MeV less repulsion, so that the $\Lambda$ potential decreases to the empirical value ($U_\Lambda \approx 25 - 30$ MeV). In the present calculation, the effective masses for the nucleon and the $\Lambda$ hyperon, which play important roles in getting the final results, are obtained with the assumptions that the spin correlations and
the confining potentials are not modified in nuclear matter. It is a challenging work to study
the variations of those values in nuclear medium.

Acknowledgments

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Figure captions

**Figure 1:** The variations of the effective masses for the nucleon and the Λ hyperon, $\delta M^*_i = M^*_i - M_i$ ($i = N, \Lambda$), as functions of the quark mass correction, $\delta m_q$. The results in the QMF model with $\chi_c = \frac{1}{2} kr^2$ are shown by solid curves, while those with $\chi_c = \frac{1}{2} kr^2(1 + \gamma_0)/2$ are shown by dashed curves.

**Figure 2:** Single particle energies in $^{41}_{\Lambda}$Ca, $^{91}_{\Lambda}$Zr, and $^{209}_{\Lambda}$Pb. QMF\textsuperscript{I} and QMF\textsuperscript{II} denote the models with $\chi_c = \frac{1}{2} kr^2(1 + \gamma_0)/2$ and $\chi_c = \frac{1}{2} kr^2$, respectively. The results in the QMC model [10] are also shown for comparison. The experimental data are taken from Refs. [1, 2, 3].

**Figure 3:** The scalar and vector potentials, $U_S$ and $U_V$, for the $1s_{1/2}$ $\Lambda$ state in $^{41}_{\Lambda}$Ca and $^{209}_{\Lambda}$Pb.
\[ \delta M_i = M_i - M_i^* = M_i - M_i^* \text{[MeV]} \]
Scalar and vector potentials [MeV]

$r$ [fm]

$U_V$

$U_S$

$^{41}\Lambda Ca$

$^{209}\Lambda Pb$