Leveraging the Influence of Power Grid Links in Renewable Energy Power Generation

WEI WANG1, HAIBO WANG2, BAHRAM ALIDAEE3, JUN HUANG4, AND HUIJUN YANG1

1College of Economics and Management, Chang’an University, Xi’an 710064, China
2A. R. Sanchez Jr. School of Business, Texas A&M International University, Laredo, TX 78041, USA
3School of Business Administration, University of Mississippi, Oxford, MS 38677, USA
4Norris-Vincent College of Business, Angelo State University, San Angelo, TX 76909, USA

Corresponding authors: Huijun Yang (584955463@qq.com), Wei Wang (wwang@chd.edu.cn), and Haibo Wang (hwang@tamiu.edu)

ABSTRACT  Smart grid construction provides the basic conditions for grid connection of renewable energy power generation. However, the grid connection of large-scale intermittent renewable energy sources increases the complexity of operational control of power systems. With the increase in intermittent renewable energy grid connections, distribution system operators must optimize and integrate these new participants to ensure the flexibility and stability of smart grids. Dividing the smart grid into logical clusters helps to overcome the problems caused by intermittent renewable energy grid connections. In this study, we propose a 2-step modeling approach that includes both link and leverage analyses to detect the network partitioning and assess the stability of the smart grids. Our experimental results of the link analysis show that, despite the identical scores in modularity and Silhouette Coefficients (SC), the total computational time of the linear programming model for linkage (CD1) is 29.8% shorter than that of the quadratic programming model for linkage (CD2) on 7 networks with fewer than 200 nodes, whereas CD2 is 29.5% faster than CD1 on 19 larger networks with more than 200 nodes. The leverage results of benchmark networks indicate that the computational time of each instance with the proposed linear programming model for leverage (ID1) and quadratic programming model for leverage (ID2) was substantially reduced, and the Critical Node Problem (CNP) results of medium- and large-scale networks were better than those reported in the literature, which play a significant role in smart grid optimization.

INDEX TERMS  Intermittent renewable energy, smart grid, link analysis, leverage analysis, grid partitions.

I. INTRODUCTION  The increasing demand for electricity has resulted in stricter requirements for reliable power supply. Thus, there is an urgent need to solve the problems of energy shortages and environmental pollution. The power industry regards the promotion of the development of renewable energy as an inevitable choice. With the rapid development of smart grids, renewable energy has attracted increasing social attention and the scale of development is increasing. The use of renewable energy has effectively promoted energy conservation and emission reduction, as well as alleviated the energy crisis [1]. However, the disadvantages of intermittent renewable energy power generation, such as fluctuations, intermittence, low energy density, and unstable output, increase the complexity of the operation and control of electrical grid systems [2]. In intermittent renewable energy power generation, electricity is generated by using recycled resources. Its advantages can only be exploited when it is effectively combined with a power grid.

Intermittent renewable energy power generation is a green, low-carbon, and less polluting power generation method that can save fossil energy and protect the environment. However, owing to the volatility and uncertainty of renewable energy power generation, the extensive use of renewable energy affects the security and stability of the power grid. The security of a power system is compromised to a certain extent [3]. In combination with relevant smart grid technologies and the grid connection of intermittent renewable sources, the grid division of large-scale renewable energy consumption has
Research on complex network community structures and their detection methods has been applied in many power grid analysis studies. Power systems are widely accepted as typical complex-network systems. To monitor the power grid operation state in real time and make dispatching decisions quickly, operators usually divide the power grid into several subregions to effectively improve the calculation speed and reduce the complexity of the power grid state analysis [8]. Generally, the subarea division of a power network can be realized based on the operator’s experience or the geographical area to which the subarea belongs. However, these methods cannot accurately reflect the situation and correlation of all parts of the power grid or adapt to rapid changes in the operation of the power grid. Therefore, some scholars have applied community detection methods to obtain grid partitions and facilitate partitioning.

In recent years, the complex network theory has been applied in research. Community network structures and community detection. It was found that many networks have local aggregation characteristics owing to uneven edge relationships. The network can be divided into multiple subnetworks. The internal connections of the sub-networks are relatively dense, and the connections between the sub-networks are relatively sparse. Accordingly, each subnetwork is called a community; an example of the community structure is shown in Figure 1.

FIGURE 1. Community structure sketch graph.

A community typically comprises of network nodes with similar functions or properties. Its essence is the regional coupling of the physical, chemical, and social interactions between network nodes. The study of community structure can help analyze the modules, functions, and evolution of an entire network in a divide-and-rule manner. As a typical artificial complex network system, modern power grids have complex systems and network characteristics. The operation, protection, and control of a traditional power grid are significantly different from those of large-scale intermittent renewable energy power generation. Furthermore, the development of large-scale intermittent renewable energy sources has increased the complexity and vulnerability of smart grids [9]. Therefore, the application of research on complex network community structures in power grid analysis is a general trend.

The current study focused on two research questions (see Figure 2): (1) How many stable grid partitions do power grids have? (2) How can the vulnerability and critical nodes of the smart grids be assessed? First, we used a linear programming model for linkage (CD1) and quadratic programming model for linkage (CD2) to detect the network structure. Then, CD1 and CD2 were employed for leverage analysis to identify the importance and influence of key players in the network. We selected modularity and Silhouette Coefficients (SC) to measure the partitioning quality of link analysis models (CD1 and CD2). The Critical Node Problem (CNP) and a measure of fragmentation (F-score) were used to evaluate the performance of the linear programming model for leverage (ID1) and quadratic programming model for leverage (ID2).

FIGURE 2. Framework of smart grids analysis.
in smart grid partitions. In Section III, we use three benchmark networks with different sizes to compare the results. Finally, Section IV provides concluding remarks and proposes future work.

II. LINK AND LEVERAGE ANALYSES

A. LINK ANALYSIS—COMMUNITY DETECTION

Community detection is an important research field in social network analysis. This refers to the process of identifying interacting node groups based on their structural characteristics [10]. Community detection reveals the structure of a specific network community within a specific time range. This enables us to analyze problems from a group-level perspective [11]. Therefore, it has many application areas such as biology, physics, social science, applied mathematics, and computer science, which can help complete group-level tasks. These application areas enhance the development of specific algorithms to create technology and tools for practical use [12]. The computational complexity of community detection is hindered by two main factors: the large scale of current social networks and their evolving structures [13]. Therefore, an increasing number of scholars are focusing on community detection in social network analysis and have conducted extensive research in this field. Modern network science has achieved many breakthroughs in the study of complex systems. In many cases, social networks can be represented by graphs, in which nodes are used to represent individuals in the network and edges represent interactive relationships. In sociology, biology, computer science, and other disciplines, community detection is important because systems in these disciplines are usually represented by graphics models. Although many interdisciplinary scientists have made great efforts in this field in the past few years, this problem remains difficult to solve satisfactorily in power-grid partitioning.

Communities in networks are local structures with dense and sparse internal connections, which help solve complex problems. Large-scale networks are divided into considerably smaller loosely coupled sub-networks for easy control. In power grids, community structures are often used in power system recovery [14], [15], reactive power network partitioning [16], and coherency-based dynamic equivalence [17]. Several methods have been commonly used in previous studies to detect communities in networks, such as hierarchical clustering [18], [19], modular optimization [20], [21], machine learning, and other algorithms [22], [23], [24]. Existing studies and algorithms may only focus on the physical structure of the power grid and ignore its functions [25], thereby failing to fully reflect the electrical characteristics of the power grid. The multi-attribute partitioning method combines the k-means algorithm and evolutionary algorithm to divide the power grid. However, this approach requires predefining the number and size of the clusters, which affects their adaptability. Moreover, existing research mainly focuses on static analysis, while the current study includes both static and dynamic power network analyses. We detected the actual grid structure and tracked how it changed over time.

As observed in the literature, community detection and key-player discovery are two distinct research topics with many applications. To improve the robustness of a smart grid system, this study focuses on the analysis of smart grid partitioning and the influence of key players. We conducted a leverage analysis on the influence and disruption of social networks. Next, we show a 2-step modeling approach with a common set of variables and constraints on community detection and key-player discovery. Both problems can be formulated as graphical problems.

B. LEVERAGE ANALYSIS—INFLUENCE AND DISRUPTION

Compared with traditional power grids, smart power grids have self-healing and disaster-resistant abilities. However, the destruction of critical nodes also causes vulnerability to smart grids. Leverage analysis improves the system analysis ability of the algorithm and identifies the importance and influence of the key players (critical nodes) in the network. We define the key players as nodes that generate the largest influence on smart power networks. When such nodes are removed from the network, the stability of the entire power network is significantly disrupted. A leverage analysis of social networks is generally conducted from two aspects: influence and disruption. The influence of key players plays a fundamental role in social networking. Key players in achieving optimal dissemination through the network were identified. The influence of key players in leverage analysis is analogous to the “Key Player Problem/Positive” (KPP-Pos) [26], in which planners quickly spread information, behaviors, or goods by searching a group of network nodes with the best location. Disruption analysis aims to identify key players by removing critical nodes that disrupt or fragment the network. This aspect of leverage analysis is similar to that of “Key Player Problem/Negative” (KPP-Neg) in [26], which depends on the extent to which the network relies on its key players to achieve cohesion.

Several studies have examined the identification of the key players in power networks. One way to identify key players is based on centrality measures [27], [28]. Traditional centrality measures for key player problems only consider the network structure and do not consider additional information [29]. A new approach uses entropy measures to detect key player sets within a social network, providing a simple solution to the KPP-Pos problem [30]. Entropy-based measures are employed to identify the key player sets, but the shortcoming of such methods is that they are limited to nondense heterogeneous networks [31], [32], [33].

Related studies have aimed to identify a set of key players rather than individual key players. The problem of maximizing influence was proposed to identify nodes that initially adopt new products or innovations, such as a greedy-based heuristic with a hill-climbing algorithm [34]. Analysis of diffusion in social networks has always been a research hotspot [35], [36]. Therefore, several algorithms have been proposed...
to identify the key players, and each method emphasizes a single object to be examined. However, in complex practical applications, a set of algorithms that can perform well for multiple interesting objects is required [37], [38], [39]. The current study proposes an improved algorithm with multiple objects to detect the actual smart grid structure and track its dynamic change.

C. LINK AND LEVERAGE MODELS
In this section, we present several models for community detection and measure the influence and disruption of linkages within a network after removing critical nodes.

The parameters and variables in the models are presented in Table 1:

| TABLE 1. Variable definitions. |
|-----------------------------|
| **G=(V,E)** | An undirected graph where V is the set of nodes and E is the set of edges |
| **w_{ij}** | A signed weight on edge (i, j) ∈ E |
| **x_{ij}** | Equal to 1 if nodes i and j are in the same cluster, 0 otherwise |
| **C** | Total number of possible nodes to be removed from V |
| **C=1** | The index of the cluster that all remaining nodes are assigned to |
| **x_{is}** | Equal to 1 if node i is moved to cluster s, s=1,...,C,C+1, 0 otherwise |
| **y_{is}** | Equal to 1 if an edge (i,j) is assigned to a cluster s, s=1,...,C, 0 otherwise; There cannot be an edge between two nodes in different clusters s≠ t, for s,t=1,...,C |
| **y_1** | Equal to 1 if node i is removed from V, 0 otherwise |

**Link/Leverage analysis**

- **Link analysis:**
  1. Assign all nodes of V to C_max clusters such that:
  2. Each node is assigned to one cluster
  3. The sum of the edge weights over all clusters formed is as large as possible

- **Community detection (CD):**
  1. Assign all nodes of V to C_max clusters such that:
  2. Each node is assigned to one cluster
  3. The sum of the edge weights over all clusters formed is as large as possible

- **Leverage analysis:**
  1. Assign all nodes of V to C+1 clusters such that:
  2. Nodes in each cluster can be more than C
  3. No edge can exist between two nodes in two different clusters i,...,C
  4. Minimize all edges remaining in clusters 1,...,C

We use a classical linear programming (LP) model [40] for social network link analysis and community detection (CD):

**CD1 for LP Model:**

\[
\begin{align*}
\text{max } x_0 &= \sum_{i,j=1}^{n-1} w_{ij} x_{ij} \\
\text{s.t. } x_{ij} + x_{ik} - x_{jk} &\leq 1 \text{ for all } i,j,k \in V
\end{align*}
\]

(2) where \( w_{ij} \) denotes the unrestricted edge weight. If edge \((i, j)\) is in partition, \( x_{ij} = 1 \); otherwise, it is 0. It is an edge-based formula. Even for medium-scale graphs, the model was expanded into sizes with \( C_2 \) and \( 3C_2 \) constraints.

In CD1, there are several major shortcomings: 1) the number of triangle inequality constraints increases with an increase in the size of nodes; and 2) the objective function cannot show the optimal number of clusters formed in the optimal solution directly because the objective function cannot provide the information to assign data points to each cluster.

An equivalent quadratic programming (QP) model [41] was employed to address the issues in CD1. CD2 for QP Model:

\[
\begin{align*}
\text{max } x_0 &= \sum_{i,j=1}^{n-1} \sum_{j=i+1}^{n} w_{ij} \sum_{k=1}^{c_{\text{max}}} x_{ik} x_{jk} \\
\text{s.t. } \sum_{k=1}^{c_{\text{max}}} x_{ik} &= 1 \text{ for } i = 1, n
\end{align*}
\]

(4) CD2 significantly reduces the number of constraints and directly provides an optimal number of clusters. CD2 is a node-oriented model with fewer variables than the CD1 model. The time complexity of CD2 is \( O(n^3) \) compared to that of CD1, which is \( O(n^4) \). Although CD1 is a linear model and CD2 is a quadratic model, the size difference makes the quadratic model suitable for larger problems in which the computational burden of CD1 hinders its practical application. The main advantage of the CD2 model over the CD1 model is that it can simultaneously determine the optimal number of communities as well as the optimal allocation of each member of the community.

After the optimal number of communities in a social network is detected, the stability of the formed communities can be assessed by removing the important nodes for leverage analysis. As in CD1, we first introduce an LP model for leverage analysis: influence and disruption (ID).

**ID1 for LP Model:**

\[
\begin{align*}
\text{Min } \sum_{i,j \in V} w_{ij} x_{ij} \\
\text{s.t. } x_{ij} + y_i + y_j &\geq 1, \forall (i,j) \in E \\
\sum_{i \in V} y_i &\leq C \\
x_{ij} + x_{ik} - x_{jk} &\leq 1 \text{ for all } i,j,k \in V
\end{align*}
\]

(8) In the ID1 model, there are \( C_2 \) variables and \( C_3 + 1 \) constraints.

**ID2 for QP Model:**

\[
\begin{align*}
\text{Min } \sum_{i,j \in V} \sum_{k=1}^{C} w_{ij} x_{ik} x_{jk} \\
\text{s.t. } \sum_{k=1}^{C+1} x_{ik} &= 1, \forall i \in V \\
\sum_{i \in V} x_{ik} C_{i+1} &\leq C \\
x_{ik} + x_{jl} &\leq 1, \forall (i,j) \in E, \text{ and } k \neq l = 1, \ldots, C
\end{align*}
\]

(12) In ID2, the number of variables is \( nC \) and the number of constraints is \( n+n(n-1)/2+1 \). ID2 had fewer variables and constraints than ID1. The time complexity of ID1 is \( O(n^3) \), whereas that of ID2 is only \( O((C)n^2) \). Both models can be solved using an exact solver, such as Gurobi.

D. PERFORMANCE MEASURES OF LINK AND LEVERAGE MODELS
It is difficult to identify the optimal number of communities by using clustering approaches. The most popular method for
detecting communities in a graph is to optimize the quality function, that is, modularization introduced by [42]. Modularity refers to the number of edges in a packet minus the expected number of randomly placed edges in an equivalent network. Modularization quantifies the deviation between the internal link density of a cluster and the density expected in the same vertex group in random graphs, with the same expectation as the network degree sequence. Vertices linked to each other randomly cannot form communities because high link density values cannot be obtained. Therefore, the high values of modularity must represent the “suspicious” high values of the internal link densities of the subgraphs. These subgraphs differ from the vertex groups of random links and can be regarded as real communities [43].

Let \( G = (V, E, \omega) \) be an undirected weighted graph without parallel edges and with non-empty sets Bader et al. [44] defined the following quality measures for partitioning.

\[
\text{modularity}(C) := \frac{\sum_{C \in c} \sum_{\{u, v\} \in E} \omega(u, v)}{\sum_{e \in E} \omega(e)} - \frac{\sum_{C \in c} \sum_{v \in C} \omega(v)^2}{4 \left( \sum_{e \in E} \omega(e)^2 \right)}
\]

This function shows the difference between the number of edges inside the module and the number of edges between modules.

The current study used the standard Silhouette Coefficient (SC) [45] to measure the partitioning quality. The SC value is a measure of the ratio of the distance between a member and other members in its own community compared to those in adjacent communities. This shows how similar that observation is to other observations in a neighboring cluster. The SC value of member \( i \) is defined as

\[
S(i) = \frac{b(i) - a(i)}{\max \{a(i), b(i)\}}
\]

where \( b(i) \) is the average distance between member \( i \) and all other members in the closest adjacent community and \( a(i) \) is the average distance between member \( i \) and all other members in its own community.

To measure the influence and disruption of social networks, we adopt two measures:

1. Minimize the total number of CNP using [46]:

\[
\text{Minimize} F(n_1, \ldots, n_L, L) = \frac{1}{2} \sum_{l=1}^{L} n_l \cdot (n_l - 1)
\]

denoted by \( n_l, l = 1, \ldots, L, \) and the number of nodes in each cluster (i.e., CNP) is equal to the objective function value of ID1 (5) or ID2 (9). Critical nodes and edges can characterize the vulnerability and robustness of a given network system. The removal of nodes and edges caused by adversarial attacks, random failures due to operating conditions, or natural disasters can damage the entire network system. The CNP shows the influence of the critical nodes removed from the network.

2. The F-measure is similar to the diversity measure, such as heterogeneity or the Hérfing-Findahl index. The F-score measure indicates network fragmentation. The maximum fragmentation occurs when each node is independent, creating the same number of components as the nodes. Measurement of fragmentation (F-score) proposed in [26] as

\[
F = 1 - \frac{\sum_k s_k (s_k - 1)}{n (n - 1)}
\]
TABLE 3. Illustration of links and disruption of critical nodes (600 s time limit).

| Benchmark networks | #node | C_max | Modularity | SC | Time | Modularity | SC | Time |
|--------------------|-------|-------|------------|-----|------|------------|-----|------|
| Karate             | 34    | 4     | 0.42       | 0.53 | 0.13 | 0.42       | 0.53 | 33   |
| chesapeake         | 39    | 3     | 0.27       | 0.32 | 15   | 0.27       | 0.32 | 4    |
| dolphins           | 62    | 5     | 0.53       | 0.46 | 3    | 0.53       | 0.46 | 114  |
| Lea Mis            | 77    | 6     | 0.56       | 0.47 | 12   | 0.56       | 0.47 | 76   |
| Books              | 105   | 5     | 0.52       | 0.48 | 119  | 0.52       | 0.48 | 55   |
| Football           | 115   | 10    | 0.60       | 0.24 | 47   | 0.60       | 0.24 | 102  |
| Jazz               | 198   | 4     | 0.44       | 0.60 | 437  | 0.44       | 0.60 | 518  |
| ER235              | 235   | 9     | 0.62       | 0.51 | 219  | 0.62       | 0.26 | 163  |
| FF250              | 250   | 67    | 0.62       | 0.59 | 316  | 0.62       | 0.59 | 206  |
| WS250              | 250   | 12    | 0.69       | 0.57 | 131  | 0.69       | 0.57 | 144  |
| US Air332          | 332   | 6     | 0.19       | 0.54 | 516  | 0.19       | 0.54 | 480  |
| Netscience         | 379   | 19    | 0.96       | 0.47 | 584  | 0.96       | 0.47 | 386  |
| ER466              | 466   | 27    | 0.65       | 0.51 | 413  | 0.65       | 0.51 | 227  |
| Erdos 472          | 472   | 9     | 0.51       | 0.51 | 578  | 0.51       | 0.51 | 424  |
| FF500              | 500   | 65    | 0.78       | 0.58 | 428  | 0.78       | 0.58 | 249  |
| BA500              | 500   | 16    | 0.89       | 0.66 | 512  | 0.89       | 0.66 | 376  |
| WS500              | 500   | 20    | 0.78       | 0.55 | 179  | 0.78       | 0.55 | 167  |
| ER941              | 941   | 77    | 0.66       | 0.52 | 447  | 0.66       | 0.52 | 375  |
| BA1000             | 1000  | 15    | 0.91       | 0.69 | 563  | 0.91       | 0.69 | 315  |
| FF1000             | 1000  | 61    | 0.80       | 0.55 | 515  | 0.80       | 0.55 | 322  |
| WS1000             | 1000  | 25    | 0.80       | 0.54 | 265  | 0.80       | 0.54 | 195  |
| WS1500             | 1500  | 30    | 0.87       | 0.53 | 439  | 0.87       | 0.53 | 238  |
| FF2000             | 2000  | 69    | 0.88       | 0.55 | 587  | 0.88       | 0.55 | 392  |
| ER2344             | 2344  | 93    | 0.68       | 0.50 | 579  | 0.68       | 0.50 | 572  |
| BA2500             | 2500  | 23    | 0.95       | 0.57 | 591  | 0.95       | 0.57 | 297  |
| BA5000             | 5000  | 63    | 0.96       | 0.55 | 584  | 0.96       | 0.55 | 426  |

FIGURE 4. Silhouette plot of Karate club social network.

where $s_k$ is the size of the connected sub-network after the node removal. $(\sum k [s_k (s_k - 1)]) / m(n - 1)$ measures the connectivity of the remaining network when nodes are removed.

III. CASE STUDY
A. DATA
This study uses the same set of benchmark networks for both the link and leverage analyses. We used the same datasets as in [46], [47], and [48] (see Table 2). The majority of [46] benchmark networks can be found in this resource. Five real-life medium-scale networks were employed by [47], and [48] proposed heuristics of 16 benchmark random graph structures based on four different types of complex network models: Barabasi–Albert, Erdos–Renyi, forest fire, and Watts–Strogatz. For each model of link and leverage analyses, commercial software such as Gurobi 9.0, was implemented on a set of test problems. The computational results were summarized and visualized using R software.

B. RESULTS
1) LINK ANALYSIS
The modeling of complex networks can be static or dynamic; therefore, for these two networks, community detection can
be performed. A static network can be considered as a frozen network within a specific time interval. However, over time, the communities in the network may continue to expand or shrink, and new communities may emerge, while some existing communities may disappear. Dynamic community detection can reveal and process dynamic communities. Therefore, the purpose of static community detection is to identify the actual community structure, whereas that of dynamic community detection is to detect and track how community structure changes over time.

This study proposed linear and quadratic programming models for clique partitioning in link analysis, which are robust compared to traditional approaches. This algorithm divides vertices into g groups, and the size of the g groups is predefined to obtain the minimum number of connections between identified communities [54]. The number of vertices between clusters is called cut size. If the number of clusters is not provided in advance and the minimum cut size is used for partitioning, the output will be a trivial solution. The links and disruptions of the critical nodes are listed in Table 3.

From Table 3, we can observe that despite the identical scores in modularity and SC, the computational times are different. For each problem instance, the optimal computational times are highlighted in bold. The total computational time of CD1 is 29.8% shorter than that of CD2 on 7 networks with fewer than 200 nodes, whereas CD2 is 29.5% faster than CD1 on 19 larger networks with more than 200 nodes. Taking the Karate network as an example, computational time of CD1 is 0.13 s, which is much smaller than that of CD2 (33 s). However, for large networks such as the BA2500 network, the computational time of CD2 (297 s) is much shorter than that of CD1 (591 s).

2) LEVERAGE ANALYSIS
Leverage analysis was used in this study to determine the importance and influence of critical nodes in a smart grid.

| C  | Best known solution (BKS) in the literature | ID1 | ID2 |
|----|---------------------------------------------|-----|-----|
|    | CNP | F-score | Node Removed | CNP | F-score | Node Removed | CNP | F-score | Node Removed |
| 1  | 62  | 0.542 | 1 | 61 | 0.642 | 34 | 61 | 0.642 | 34 |
| 2  | 54  | 0.683 | 1,2 | 45 | 0.684 | 1,34 | 45 | 0.684 | 1,34 |
| 3  | 34  | 0.797 | 1,33,34 | 34 | 0.797 | 1,33,34 | 34 | 0.797 | 1,33,34 |
| 4  | 26  | 0.884 | 1,3,33,34 | 26 | 0.884 | 1,3,33,34 | 26 | 0.884 | 1,3,33,34 |
| 5  | 19  | 0.926 | 1,2,3,33,34 | 19 | 0.926 | 1,2,3,33,34 | 19 | 0.926 | 1,2,3,33,34 |
| 6  | 16  | 0.930 | 1,2,3,6,33,34 | 16 | 0.930 | 1,2,3,6,33,34 | 16 | 0.942 | 1,2,3,24,33,34 |
| 7  | 13  | 0.940 | 1,2,3,6,26,33,34 | 13 | 0.940 | 1,2,3,6,26,33,34 | 13 | 0.943 | 1,2,3,4,26,33,34 |
| 8  | 10  | 0.959 | 1,2,3,6,24,25,33,34 | 10 | 0.959 | 1,2,3,6,24,25,33,34 | 10 | 0.962 | 1,2,3,4,24,25,33,34 |
| 9  | 7   | 0.971 | 1,2,3,4,6,24,32,33,34 | 7 | 0.971 | 1,2,3,4,6,24,32,33,34 | 7 | 0.971 | 1,2,3,4,7,24,32,33,34 |
| 10 | 5   | 0.980 | 1,2,3,4,5,6,24,32,33,34 | 5 | 0.980 | 1,2,3,4,5,6,24,32,33,34 | 5 | 0.980 | 1,2,3,4,5,6,24,25,33,34 |
We compared the results for the datasets of the Karate Club social network [46], [49]. The literature identified clusters of 34 participants aligned and reported the number of social contexts in which each pair of participants interacted. The data were displayed in the form of a network with an edge attribute context, which provided the number of interaction contexts for a pair of participants. Each member of the club is represented by nodes and the connections between members...
are represented by edges. The leverage results compared to those in [46] are presented in Table 4.

By comparing these three results, it can be observed that the CNP values of ID1 and ID2 outperformed the results of [46] when $C=1$ (CNP=61 compared with the best-known CNP=62 in the literature) and $C=2$ (CNP=45 compared with the best-known CNP=54 in the literature). For $C=9–10$, there were multiple optimal solutions for fragmentation with the same objectives, and their F-scores were the same. For example, when $C=9$, nodes removed in ID1 include 1, 2, 3, 4, 6, 24, 32, 33, and 34 (same as the BKS in the literature), while nodes removed in ID2 include 1, 2, 3, 4, 7, 24, 32, 33, and 34. For $C=6–8$, ID2 has a better F-score than ID1, even though both have the same CNP, but different nodes are removed from the graph. For example, when $C=6$, the F-score was 0.942 in ID2, which is larger than ID1 and BKS in the literature (F-score=0.930). The nodes removed in ID2 include 1, 2, 3, 24, 33, and 34, which is different from the nodes removed (1, 2, 3, 6, 33, and 34) in ID1 and BKS in the literature. Figure 3 shows the partition of the Karate Club social network. Nodes with different colors represent different sub-networks partitioned by the proposed algorithm, and edges represent the interactive relationships between these sub-networks. For example, nodes 5, 6, 7, 11, and 17 are classified in a sub-network, while nodes 24, 25, 26, 28, 29, and 32 belong to another sub-network. Figure 4 presents a Silhouette plot of the Karate Club social network. The average Silhouette width was 0.53, and the plot indicates that the number of clusters = 4 generated the best partitioning quality, which is consistent with the literature [46]. Table 5 shows a graphical comparison of the best results in ID2 based on the F-score with the results from the literature. We can see obvious differences in the graphical results.

Table 6 presents a comparison of the results to those in [47], which indicates that our study obtained better results for medium-scale networks. The computational time and CNP were significantly shorter than those reported in the literature.

### Table 7. Leverage results comparison to [48] (large-scale network).

| Benchmark networks | C | Time | CNP | Time | CNP | F-Score |
|--------------------|---|------|-----|------|-----|--------|
| BA500              | 50 | 66   | 195 | 3.06 | 137 | 0.9814 |
| BA1000             | 75 | 172  | 558 | 28.83 | 335 | 0.9984 |
| BA2500             | 100 | 840 | 3704 | 90.59 | 1271 | 0.9919 |
| BA5000             | 150 | 3154 | 16196 | 1799 | 2851 | 0.9956 |
| ER235             | 50 | 38   | 295 | 3.08 | 122 | 0.9473 |
| ER466             | 50 | 110  | 1542 | 28.29 | 301 | 0.9905 |
| ER941             | 140 | 361  | 5120 | 102.41 | 659 | 0.9940 |
| ER2344            | 200 | 1933 | 997839 | 12545 | 2313 | 0.9971 |
| FF250             | 37 | 37   | 194 | 3.29 | 129 | 0.8881 |
| FF500             | 25 | 156  | 257 | 39.63 | 185 | 0.9741 |
| FF1000            | 50 | 410  | 1260 | 277.06 | 626 | 0.9953 |
| FF2000            | 125 | 1723 | 4545 | 1913.3 | 1643 | 0.9964 |
| WS250             | 70 | 70   | 3186 | 3.05 | 547 | 0.9559 |
| WS500             | 125 | 173 | 2678 | 17.22 | 688 | 0.9171 |
| WS1000            | 200 | 548 | 113638 | 97.63 | 2862 | 0.8902 |
| WS1500            | 265 | 1816 | 13167 | 1355.7 | 2727 | 0.9389 |

### Table 8. Graphical results of [48].

- **BA500 original graph**: 50 node removal
  - Largest connected component: 500
  - Largest connected component: 5
  - Number of clusters: 1
  - Number of clusters: 363

- **BA1000 original graph**: 75 node removal
  - Largest connected component: 1000
  - Largest connected component: 9
  - Number of clusters: 1
  - Number of clusters: 665

- **BA5000 original graph**: 150 node removal
  - Largest connected component: 5000
  - Largest connected component: 32
  - Number of clusters: 1
  - Number of clusters: 2149

- **ER235-50 original graph**: 50 node removal
  - Largest connected component: 233
  - Largest connected component: 19
  - Number of clusters: 2
  - Number of clusters: 113
TABLE 8. (Continued.) Graphical results of [48].

| Graph | Node Removal | Largest Connected Component | Number of Clusters |
|-------|--------------|----------------------------|--------------------|
| Er466 | 80 node removal | 459                        | 4                  |
|       |              | 94                         | 2199               |
| ER941 | 140 node removal | 919                        | 12                 |
|       |              | 421                        | 293                |
| ER2344| 200 node removal | 2314                       | 14                 |
|       |              | 1915                       | 358                |
| FF500 | 50 node removal | 250                        | 1                  |
|       |              | 15                         | 132                |
| FF250 | 70 node removal | 250                        | 1                  |
|       |              | 6                          | 177                |

TABLE 8. (Continued.) Graphical results of [48].

| Graph | Node Removal | Largest Connected Component | Number of Clusters |
|-------|--------------|----------------------------|--------------------|
| FF500 | 110 node removal | 500                        | 1                  |
|       |              | 7                         | 319                |
| FF1000| 150 node removal | 1000                       | 1                  |
|       |              | 20                        | 466                |
| FF2000| 200 node removal | 2000                      | 1                  |
|       |              | 916                       | 334                |
| FX250 | 70 node removal | 250                        | 1                  |
|       |              | 179                       | 72                 |
| FX500 | 125 node removal | 500                        | 1                  |
|       |              | 374                       | 137                |
identification using both link and leverage analyses. It contributes to the literature by identifying the optimal grid partitions and critical nodes in the power network to alleviate the vulnerability of the smart grids.

Three benchmark networks of different sizes were used to compare the results. Furthermore, linear and quadratic programming models were proposed for link and leverage analyses, and were found to be robust compared with traditional approaches. Our experimental results of the link analysis show that, despite the identical scores in modularity and SC, the total computational time of CD1 is 29.8% shorter than that of CD2 on 7 networks with fewer than 200 nodes, whereas CD2 is 29.5% faster than CD1 on 19 larger networks with more than 200 nodes. Therefore, CD1 is more suitable for small-scale networks whereas CD2 is more efficient for large-scale networks with more than 200 nodes. The leverage results of benchmark networks [46], [47], [48] indicate that the computational time of each instance with the new proposed models (ID1 and ID2) was substantially reduced, and the CNP results of medium- and large-scale networks were better than those reported in the literature, which play a significant role in smart grid optimization.

The proposed dimensionality reduction algorithm reduces time complexity and significantly shortens computational time. The methods proposed in this study can contribute to the real-time planning and operation of smart grids. The application of partitioning consumers according to their energy demand and supply can be explored. In addition, the link and leverage analyses of smart grids can connect future decentralized intermittent renewable energy communities to smart grids. By obtaining optimal grid partitions and identifying the critical nodes in the smart grids, we can alleviate the vulnerability of smart grids and improve the security and stability of large-scale power grids.

There are some limitations of the current study, and we plan to expand this research further. The effectiveness of the proposed algorithm in an actual power grid is worthy of further study. Meanwhile, additional electrical characteristics of the grid will be considered to detect energy communities in future studies. How to integrate more electrical characteristics into the model and apply the algorithm to larger networks remains to be explored further. In addition, the relationship between the modularization index and the power network partition process should be studied further, and multiple objectives should be considered simultaneously using the multi-objective method.

REFERENCES

[1] M. Z. I. Sarkar, L. G. Meegahapola, and M. Datta, “Reactive power management in renewable rich power grids: A review of grid-codes, renewable generators, support devices, control strategies and optimization algorithms,” IEEE Access, vol. 6, pp. 41458–41489, 2018.

[2] B. B. Adetokun, J. O. Ojo, and C. M. Muriithi, “Reactive power-voltage-based voltage instability sensitivity indices for power grid with increasing renewable energy penetration,” IEEE Access, vol. 8, pp. 85401–85410, 2020.

[3] S. P. Bihari, P. K. Sadhu, K. Sarita, B. Khan, L. D. Arya, R. K. Saket, and D. P. Kothari, “A comprehensive review of microgrid control mechanism and impact assessment for hybrid renewable energy integration,” IEEE Access, vol. 9, pp. 88942–88958, 2021.
[4] H. Dorotić, B. Doračić, V. Dobravec, T. Pukšec, G. Krajačić, and N. Dušić, “Integration of transport and energy sectors in island communities with 100% intermittent renewable energy sources,” Renew. Sustain. Energy Rev., vol. 99, pp. 109–124, Jan. 2019.

[5] N. Ganganath, J. Y. Wang, Z. X. Xu, C. T. Cheng, and C. K. Tse, “Agglomerative clustering-based network partitioning for parallel power system restoration,” IEEE Trans. Ind. Informat., vol. 14, no. 6, pp. 3325–3333, Aug. 2018.

[6] C. Zhao, J. Zhao, C. Wu, X. Wang, F. Xue, and S. Lu, “Power grid partitioning based on functional community structure,” IEEE Access, vol. 7, pp. 152624–152634, 2019.

[7] I. Kamwa, A. K. Pradhan, G. Joos, and S. R. Samantaray, “Fuzzy partitioning of a real power system for dynamic vulnerability assessment,” IEEE Trans. Power Syst., vol. 24, no. 3, pp. 1356–1365, Aug. 2009.

[8] M. Guerrero, F. G. Montoya, R. Baños, A. Alcaide, and C. Gil, “Community detection in national-scale high voltage transmission networks using genetic algorithms,” Adv. Eng. Inform., vol. 38, pp. 232–241, Oct. 2018.

[9] H. Karimipour, A. Dehghantanha, R. M. Parizi, K. K. R. Choo, and H. Leung, “A deep and scalable unsupervised machine learning system for cyber-attack detection in large-scale smart grids,” IEEE Access, vol. 7, pp. 80778–80788, 2019.

[10] J. Yang, J. McAuley, and J. Leskovec, “Community detection in networks with node attributes,” in Proc. IEEE 13th Int. Conf. Data Mining, Dec. 2013, pp. 1151–1156.

[11] A. Karatas and S. Sahin, “Application areas of community detection: A review,” in Proc. Int. Cong. Big Data, Deep Learn. Fighting Cyber Terrorism (IBIGDELFT), Dec. 2018, pp. 65–70.

[12] R. Chen, Q. Hua, Y. Chang, L. Zhang, and B. Wang, “A survey of collaborative filtering-based recommender systems: From traditional methods to hybrid methods based on social networks,” IEEE Access, vol. 6, pp. 64301–64320, 2018.

[13] M. Arazouli, D. Rhouma, and L. Ben Romdhane, “Community detection in large-scale social networks: State-of-the-art and future directions,” SocioNet. Anal. Mining, vol. 9, no. 1, pp. 1–32, Dec. 2019.

[14] A. Kavousi-Fard, M. Wang, and W. Su, “Stochastic resilient post-hurricane power system recovery based on mobile emergency resources and reconfigurable microgrid systems,” IEEE Access, vol. 6, pp. 72311–72326, 2018.

[15] Z. Lin, F. Wen, and H. Zhou, “A new algorithm for restoration subsystem division based on community structure of complex network theory,” Autom. Electr. Power Syst., vol. 33, no. 12, pp. 12–16, 2009.

[16] H. Ruan, H. Gao, Y. Liu, L. Wang, and J. Liu, “Distributed volt-amp control in active distribution network considering renewable energy: A novel network partitioning method,” IEEE Trans. Power Syst., vol. 35, no. 6, pp. 4220–4231, Nov. 2020.

[17] G. Chicco, “Review of clustering methods for social networks,” Energy Syst. Res., vol. 4, no. 2, pp. 5–20, Jul. 2021.

[18] X. Su, S. Xue, F. Liu, J. Wu, J. Yang, C. Zhou, W. Hu, C. Paris, S. Nepal, D. Jin, Q. Z. Sheng, and P. S. Yu, “A comprehensive survey on community detection with deep learning,” IEEE Trans. Neural Netw. Learn. Syst., early access, Mar. 9, 2022, doi: 10.1109/TNNLS.2021.3157396.

[19] C. Zhou, L. Feng, and Q. Zhao, “A novel community detection method in bipartite networks,” Phys. A, Stat. Mech. Appl., vol. 492, pp. 1679–1693, Feb. 2018.

[20] S. Tahmasab, P. Moradi, S. Ghodsi, and A. Abdollahpouri, “An ideal point based many-objective optimization for community detection of complex networks,” Inf. Sci., vol. 502, pp. 125–145, Oct. 2019.

[21] X. Ma, D. Dong, and Q. Wang, “Community detection in multi-layer networks using joint nonnegative matrix factorization,” IEEE Trans. Knowl. Data Eng., vol. 31, no. 2, pp. 273–286, Feb. 2019.

[22] F. Liu, S. Xue, J. Wu, C. Zhou, W. Hu, C. Paris, S. Nepal, J. Yang, and P. S. Yu, “Deep learning for community detection: Progress, challenges and opportunities,” 2020, arXiv:2005.08225.

[23] P. Chunava, “Community detection in node-attributed social networks: A survey,” Comput. Sci. Rev., vol. 37, Aug. 2020, Art. no. 100286.

[24] H.-J. Li, L. Wang, Y. Zhang, and M. Perc, “Optimization of identifiability for efficient community detection,” New J. Phys., vol. 22, no. 6, Jun. 2020, Art. no. 063035.

[25] Z. Chen, Z. Xie, and Q. Zhang, “Community detection based on local topological information and its application in power grid,” Neurocomputing, vol. 170, pp. 384–392, Dec. 2015.

[26] S. P. Borgatti, “Identifying sets of key players in a social network,” Comput. Math. Org. Theory, vol. 12, no. 1, pp. 21–34, 2006, doi: 10.1007/10588-006-7084-x.
[52] D. Aloise, S. Cafieri, G. Caporossi, P. Hansen, S. Perron, and L. Liberti, “Column generation algorithms for exact modularity maximization in networks,” Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top., vol. 82, no. 4, Oct. 2010, Art. no. 046112, doi: 10.1103/PhysRevE.82.046112.

[53] T. A. Davis and Y. Hu, “The University of Florida sparse matrix collection,” ACM Trans. Math. Softw., vol. 38, no. 1, pp. 1–25, Nov. 2011.

[54] A. Pothen, “Graph partitioning algorithms with applications to scientific computing,” in Parallel Numerical Algorithms. Cham, Switzerland: Springer, 1997, pp. 323–368.

WEI WANG received the B.S. degree in communication engineering from the Xi’an Communication Institute, Xi’an, China, in 2006, and the M.B.A. and Ph.D. degrees in business administration from Texas A&M International University, Laredo, TX, USA, in 2016.

Since 2017, she has been an Assistant Professor with the School of Economics and Management, Chang’an University. She has published in the Journal of Cleaner Production, International Journal of Production Research, Communications in Statistics-Simulation and Computation, and other journals. Her main research interests include applied management science interests of businesses, data mining, and human resource management.

HAIBO WANG received the B.S. degree in biochemical engineering from the South China University of Technology, in 1991, and the M.S. degree in chemistry, the M.S. degree in computer science, and the Ph.D. degree in business administration from the University of Mississippi, in 1997 and 2004, respectively.

He is currently a Radcliff Killam Distinguished Professor in decision science and operations research with Texas A&M International University. His current research focuses on prescriptive analytics of big data in logistics, public transportation planning, information security, and health care. He has publications in such outlets as IEEE Transactions journals, OMEGA, EJOR, and other major OR journals.

BAHRAM ALIDAEE received the B.S. degree in business administration from the University of Tehran, in 1975, the M.B.A. degree from the University of North Texas, in 1981, and the Ph.D. degree in mathematical sciences from The University of Texas at Arlington, in 1988.

He is currently a Professor of operations and supply chain with the School of Business Administration, University of Mississippi. He has published in Management Science, Production and Operations Management, Transportation Science, various IEEE Transactions, and other major operations journals. His main research is in applied management science interests of businesses.

JUN HUANG received the B.S. degree in accounting from the Guangdong University of Technology, Guangzhou, China, in 2004, the M.S. degree in international management from Oxford Brookes University, Oxford, U.K., in 2006, and the Ph.D. degree in business administration with a specialization in quantitative management from Texas A&M International University, Laredo, TX, USA, in 2014.

From 2015 to 2016, he was a Visiting Assistant Professor with the Skema Business School. Since 2017, he has been an Assistant Professor with the Department of Management and Marketing, Angelo State University. His research interests include the integration of data mining and feature selection techniques and its application in providing decision support in a business context, optimization, and applying structural equation modeling (SEM) in management studies.

HUIJUN YANG received the Ph.D. degree from Xi’an Jiaotong University.

He is currently a Teacher with the School of Management, Chang’an University, China. His research interests include innovation, leadership, and corporate governance.