Hagiotopeonyms in France: Saint popularity, like a herding phase transition

Marcel Ausloos$^{1,2,3}$

December 22, 2020

$^1$ GRAPES
Sart Tilman, B-4031 Angleur Liege, Belgium
marcel.ausloos@ulg.ac.be

$^2$ Department of Statistics and Econometrics
Bucharest University of Economic Studies
6 Piata Romana, 1st district, Bucharest, 010374 Romania
marcel.ausloos@ase.ro

$^3$ School of Business, University of Leicester
Brookfield, Leicester, LE2 1RQ, UK
ma683@le.ac.uk

Abstract

A spectacular order-order-like transition is presented in the distribution of hagiotopeonyms in France. Data analysis and displays distinguish male and female cases. The respective hapax values point to a very large variety of saints with a specific devotion. The most popular ones are St. Martin and the apostles. The less popular ones are not so well known. These features are explained in terms of herding in agent behaviors: people have either preferred popular saints with supposedly good links to God, whence a herding behavior, or (non-herding) agents have preferred to name their local human settlement through a reference to some holy person(s) with more local specificities, - yet with moral or religious leadership, and conjectured to have good contact with God, whence at least locally defined as a saint.

Keywords: hagiotopeonyms; France; power law; Bradford-Zipf-Mandelbrot law; herding; phase transition
1 Foreword

Dietrich Stauffer was very keen in propagating basic statistical mechanics ideas and models toward other branches of sciences [1] [2] [3] [4]. Here is a "sociological case", an interdisciplinary research, he might have liked, and enjoyed it, since it concerns some data analysis of some "exotic", unusual, data. The analysis seems to indicate that there is something like a phase transition, because, on a log-log plot, one finds two straight lines, with different slopes on both sides of a critical point, as are critical exponents in research on thermodynamic and geometric bona fide phase transitions.

This type of (so called) phase transition for which I will give some tentative explanation seems to bear upon sociological behavior of likely religious populations; at least, one can surely conjecture some religiosity affair in the historical and geographic context at work here. I call the finding a "herding behavior phase transition". The words have already been used for the behavior of investors on the financial markets [5] [6] [7] [8]. But there is not much relationship with such an investor phenomenon here below.

What I found, see Fig. 1, was briefly shown at some small gathering, some time ago, but has never been submitted for publication,

The origin of the data and analysis will be outlined in subsequent sections below. However, preliminary remarks seem appropriate. They explain the "why?" of this "research", as Dietrich would have surely liked. First, I should recall that I was educated in catholic schools, and interested in history and geography. Moreover, there are many festivals, e.g. in Western Europa at least, depending on days in which (so called) saints are honored and implored for transmitting to God some request or huge thanks from people. Devotion to saints is an integral part of catholicism (https://the−shrine.org/resources/catholic−devotion−to−the−saints/). In contrast to protestant movements, - which promote a direct connection with God, it is not wrong for catholics to pray a saint for his or her intercession to God. Saints play a major role in the Orthodox churches as well, but the examined data does not pertain to orthodox tradition, thereafter.

Another "detail" has to be noted. Up to recent times the first name of children were often saints’ names given at baptism. Thus famous Saints, like St. Peter, St. Paul, St. Jean (John), St. Etienne (Stephen), St. Marcel, St. Roch, and for women, St. Marie (Mary) , Ste. Élisabeth (Elizabeth), Ste. Catherine, Ste. Cécile, were quite popular "first names". Traveling in (Southern) France, I came across villages called St. Lions, St. Jurson, St. Usuge, St. Latier, St. Avit, and even St. Saturnin (sometimes called San Savournin).

It was appealing to consider that many other Saints of whom I was aware were somewhat

[1] Officially, the "Virgin Mary", St; Marie is not a "canonized" Sainte, because having been carried up into heaven, there is no available relics.
Figure 1: Log-log plot of the number \((N)\) of hagiotoponyms in France starting with either Saint-or Saint, and referring to a supposed or confirmed catholic saint from INSEE data bank; the best power law fits are shown when separating high and low rank cases.

famous (for not necessarily religious reasons), with some singularity, like jet set, wine, cheese, ..., as St. Tropez, St. Malo, St. Emilion, St. Estèphe, St. Véran, St. Nectaire, ... . Going to the touristic literature, I "came across" Saint-Remy-en-Bouzemont-Saint-Genest-et-Isson (F-51290), Saint-Germain-de-Talleyvende-la-Lande-Vaumont (F-14500), and Beaujeu-Saint-Vallier-Pierrejux-et-Quitteur (F-70100). I became curious about their specificity as intercessors to God, and their popularity.

This led me to (numerical) questions: the first being, how many cities, so called hagiotoponyms, in France bear the name of a saint. Is there a preferential name distribution?, for example as discussed in [9] for babies names.

I decided to download the list of all cities in France, also thereafter sometimes called "communes".
2 Introduction

Statistical mechanics literature is full of reports, discussions, measurements, about phase transitions in many varied systems, usually occurring between an ordered and a disordered phase. Most of the relevant phenomena occur as a function of the temperature \[10\], thus in thermodynamics. External fields, like pressure or magnetic field, can influence the phase transitions pattern(s). Other cases pertain to geometrical concepts or descriptions, like for "percolation phase transitions" \[11\].

For completeness, let it be recalled that one can distinguish static phase transitions (drastic, even discontinuous, changes in the stationary profiles of the system through some "order parameter") \[10\], and dynamical phase transitions (discontinuous changes in the characteristic "relaxation times" of the system) \[12\] \[13\]. One has also observed instabilities with inhomogeneous stationary states, needing extension of the theory of thermal fluctuations around equilibrium states to non-equilibrium situations \[14\] [15] [16] [17].

Nowadays, statistical mechanics has evolved from such major fields and is touching more "exotic subfields", like in econophysics and sociophysics, for which "agent based models" are relevant. One has found analogies with thermodynamics phase transitions in topics outside physics, as in financial crashes \[18\] [19] [20] or opinion formations \[21\] [22].

I have found, as demonstrated below, a (herding-like, as I will propose) system, with two quite distinct phases, as we usually observe in organized-disorganized societies, - similar to "ordering transitions" in liquid crystals \[23\], or in magnetic systems \[24\]. The matter concerns hagiopoynomials, the names of cities bearing a saint name in France.

France is a rather catholic country, with famous pilgrimages, in cities which do not necessarily carry a saint name. However, devotions to saints have led to many hagiopoynomials, since the 4th-5th centuries. Cities like St. Etienne or (Mont) St. Michel are rather well known, and the Saints also. In contrast, St. Tropez, St. Nazaire, St. Lo, St. Malo, St. Emilion occur more rarely, are also well known, but not much the Saints. Other cases, e.g., St. Saturnin, - yet sometimes called St. Sernin, or San Savournin, are not rare but are not "common". Moreover, the saints "having given" their names to cities, can be males, but such saints can also be females: indeed, there are many well known, like Ste. Marie cities: Saintes-Maries-de-la-Mer, Moustiers-Sainte-Marie, ... or a contrario not so well known, like Ste. Verge, Ste. Thorette, or Ste. Néomaye.

There are 34970 "communes" (including 17 in Mayotte, and 212 in "France d'Outre-Mer", DOM-TOM) on Jan. 01, 2019, according to one of the (most recent) official (INSEE) lists of
cities in France[^2]. I selected those which start with "Saint", see Sect. 3 as it seems to be a good choice \textit{a priori}. Soon, I realized that I should use "Saint" as a string, instead, - since the string and reference to some saint can occur inside (Coise-Saint-Jean-Pied-Gauthier) or at the end (Fleurey-lès-Saint-Loup; Pont-Sainte-Maxence) of some city name, - or in a "complex way" (Saint-Martin-Lars-en-Sainte-Hermine, Saint-Maximin-la-Sainte-Baume). Some clean up must be made visually, since for example "Saintes" is not the plural of "Sainte" (but refer to a tribe living in the area a long time ago), - thus should not be considered in the relevant list. Yet, looking for cities fully devoted to a saint or sainte, I felt the necessity of including those referring to "Our Lady" (Notre-Dame). Also "Saint-Sauveur" is included, since referring to "somebody", - who is found not to be necessarily Jesus-Christ, - the latter is not considered to be a Saint.

I did not consider "Dieu", as pertinent, though many cities contain such a string (La Chaise-Dieu, Dieulefi, Villedieu ...). I kept angels, e.g., St. Michel; other debatable cases are discussed in the Data gathering section (Sect. 3), and in a "technical" Appendix. However, it can be mentioned here that I have also decided that archangels and angels are "saints"[^3] and males.

However, for completeness, due to the existence of dialects[^4] one has to take into account, cities with names starting with "San", like San Savournin, who is nobody else that Saint Saturnin, pointing to some further work in order to select the right Saint set at counting time. For hyper-completeness, one should notice that in Corsica several cities have a saint name: in "Haute Corse", (Northern Corsica), there are 20 cities with "San"; but one can distinguish "San-" (9 cases) , "Santa-" (6 cases), and one should be aware that if the first letter of the saint is a vowel, one should consider "Sant’ " (3 cases) in the data gathering, and "Santo-" (2 cases), but there is also Sainte-Lucie-de-Tallano, thus 21 hagiotoponyms (out of 236 communes)

In contrast, in "Corse du Sud" (Southern Corsica), out of 124 communes, one finds San-Gavino-di-Carbini, Sant’Andréa-d’Orcino, Santa-Maria-Figaniella, Santa-Maria-Siché, but there is also Saint-Florent.

One should add that "saint(e)" is sometimes replaced by "don" or "dan", or "dame", for example in Dampierre, Dommartin, Dammarti, Dammarie, Dannemarie, Dame-Marie.

Notice also that I do not wish to distinguish \textit{bona fide} catholic saints, whom the Catholic Church has canonized as saints, as listed in some Catholic Encyclopedia or in The Oxford dictionary of Saints[^25], or elsewhere[^4] from so called saints who have been locally defined as such because of

[^2]: \url{https://www.insee.fr/fr/statistiques/4277602?sommaire=4318291}; one can also get some help from \url{https://fr.wikipedia.org/wiki/Listes_des_communes_de_France} and from the Michelin Guide

[^3]: even though they are said to have no sexual attribute

[^4]: e.g., occitan, breton, .. see Appendix on the latter

[^5]: \url{https://en.wikipedia.org/wiki/List_of_saints}; \url{https://en.wikipedia.org/wiki/List_of_early_Christian_saints}; \url{https://www.catholic.com/encyclopedia}
some specific recognition and for whom the recognition does not extend to the worldwide church.

These remarks are made in order to indicate the complexity of sorting out such hagiotoponyms, and to suggest that one has also to dive into some relevant "saint" information. One should be aware that one should study city names before counting them when a "saint" string occurs in the name of a city; another example: "Les Saintes-Maries-de-la-Mer", which in fact also refers to several (female) saints.

Possible unintentional errors in counting, with their correction, sometimes based on assumptions, are discussed in the Appendix "Methodology and Materials". The problems (and "solutions") are so listed in a somewhat arbitrary order as sources of concern on the reliability of the final data which has been analyzed. Some arbitrary choice sometimes had to be introduced. Previous experience on identifying hagiotoponyms when examining financial aspects and demography in Italy has been of much help [26, 27], in fact, much trial and error procedure was part of the research work. There is no guarantee that all Saints are found nor distinguished. However, I have a strong "belief" that the sampling error bar is rather weak and does not drastically impair the qualitative findings.

Thereafter, one can use a classical (Zipf-like) rank-size plot, or more complicated (but better) empirical laws [28, 29].

3 Data

Several steps were to be taken when I downloaded the name of French cities; I saved names containing the string: "saint", "sainte", "saint-", "sainte-", "san", "sant", "santo", "santa", automatically including those like "saints", "saintes", "-saint-", "-sainte-", "-saints-" and "-saintes-". One has to distinguish city names containing or not a hyphen in their name or not.

I crosschecked the list with respect to the list of communes in each French Metropolitan department [7]. N.B. There are 101 departments in France: 96 in "Metropole", counting 2 departments in Corsica, and 5 away from the "Metropole", in DOM-TOM. In order to remain within a rather coherent framework, I decided not to take into account in the analysis the hagiotoponym cities in the 5 DOM-TOM departments, - where there are several hagiotoponyms in fact [8]. of course, May-

---

6https://en.wikipedia.org/wiki/List_of_Catholic_saints
7https://fr.wikipedia.org/wiki/Projet:Communes_de_France/Noms_des_articles_de_com commune
   for the first 45 departments; the 1997 list is more complete, and more useful, in :
https://fr.wikipedia.org/wiki/Projet:Communes_de_France/Noms_des_articles_de_com commune
#Liste_2019_(mise_%C3%A0_jour_le_9_septembre_2019)
8"for completeness", there are 7 female (5 different ones) and 21 male (16 different ones) hagiotoponyms, excluding Saint-Esprit
otte was also self-excluded, since it is a muslim department, - without saints and with specifically
different culture and laws.

I checked also the ”Mairie” ("City Hall") list. There are cities in France without inhabitants,
ence without a mayor, but having a City Hall, even though it is abandoned, but it exists legally.

I verified the ”postal code” and the ”INSEE communal code”; sometimes the postal code does
not correspond to the department; this demanded further crosschecking.

In so doing, multiple equivalent entries were expected to be removed, and missing entries in
some data bank were hopefully taken into account.

A comment is in order concerning data banks which were (or are) used:

- data bank 1: acp2 from [http://www.phpmyadmin.net](http://www.phpmyadmin.net); this is an absolute junk; a waste of
time; cities are missing; due to likely (stupid) encoding when two cities have the same name;

- data bank 2: official INSEE, giving the list of ”city halls”, but by alphabetical order at
first; thus it seems easy to get a list of cities commencing by S and within such a ”box”, to
get cities commencing by Sainte-, then Sainte ; then Saint-, and finally Saint. One should
possibility count and select combinations like ” -Saint- ”; however it is obviously not no easy
to get cities like ”Xyz-Saint-Lmn..”; the search being too tedious;

- data bank 3: Liste des communes de France, from a Wikipedia project gives the number of
cities per department, within an alphabetical order; thus more easy to select places like ”
Xyz-Saint- ”; however it is observed that several cities are sometimes missing; but since the
alphabetical order is combined with a list of INSEE codes which approximately follow the
alphabetical order, it is possible to guess what the missing city is, thereafter going to search
for the code on a complementary data bank of INSEE, like in the case of ”cities which have
changed names”, for example, or have fused. Indeed France cities change names once in a
while fused, defused, change departments, etc.

- data bank 4: Michelin Guide, of interest, but the cities listed in the index are not necessarily
those having a mayor nor a city hall, but the guide index is useful.

Notice that the expected alphabetical order is (surprisingly) different in such data banks, in brief
depending on abbreviations, e.g. Ste. B...., for Sainte B..., thereby appearing after Sainte-M..., 
due to alphabetical ”ordering”: a , b, ..., e; ...

Thereafter, came the treatment of several ambiguities, leading to several municipalities exclu-
sions. The criteria for exclusion have been basically due to (i) names which do not ”obviously”

[9see http://insee.fr/fr/methodes/nomenclatures/cog]/recherche_historique.asp?debut = 1930&fin = 2011
point to a specific human Saint and/or (ii) whether the toponym, though being a "sanctified location", is clearly derived from some Bible fact or event: e.g., Sainte-Eglise, Sainte Foi, Sainte-Baume, Sainte-Croix, ...

A related question concerns the "least popular" Saints, e.g. those occurring only once, so called hapaxes. I find interesting to ask how many of them exist? but "why are such Saints rare?" is outside the purpose of the present report. Nevertheless, how they geographically distributed is an unexplored and interesting question to be examined in further work.

Thus, the 100% identification of a catholically recognized Saint is admittedly sometimes not possible. A Saint having given his/her name to a city is not necessarily a bona fide catholic Saint, but only a "Saint by tradition" (see [25]). This ambiguity seems not relevant to my consideration.

Therefore, the Saint name in the studied list is assumed to point to a unique Saint who is then considered to be the representative element of the related set of the Saints with the equivalent name.

4 Data Analysis

The first concern is the search for the statistical distribution of names of Saints attached to city names. One ranks the saints by the number of towns using their names: \( N_1 \) is the number of towns associated to the saint who has the most towns; \( N_r \) decreases with \( r \); the alphabetical order is used if two saints have the same \( N_r \). Thereafter, a rank-size law is looked for.

Several sets of plots can be made available, with various axis types and scales, after counting the cities having various strings at various places in the hagiotoponyms.

Two cases illustrate the more interesting features. First, I present the log-log plot of the number \( (N_r) \) of hagiotoponyms in France, ranked in decreasing size order, with either a male or a female name, and their combination (black diamond), derived from INSEE data bank, on Fig. [2]. The data is not far from a power law, called Zipf law [30]

\[
N_r = \frac{N_1}{r^\alpha},
\]

but is better represented by a Zipf-Mandelbrot-like (ZM) law, sometimes called Bradford-Zipf-Mandelbrot-like (BZM) [31],

\[
N_r = \frac{J^*}{(\nu + r)^\zeta},
\]

might be considered as more realistic. It implies three parameters \( (J^*, \nu \text{ and } \zeta) \).

The fits are in each case made through a Levenberg-Marquardt Algorithm [32, 33, 34]. A mere power law for the female saint would be quite fair \( (\alpha \simeq 1.01; R^2 \simeq 0.989) \) and quasi not
Figure 2: Rank-size log-log plot of the number \( (N_r) \) of hagiotoponyms in France with either a male (blue, down triangle) or a female (red, up triangle) name, and their combination (black diamond), derived from INSEE data bank; indicative empirical fits are: based on a BZM law (blue dash line) or a mere power law (black tiny dot line) for males, and for females (red continuous line). Parameter values are given in the text. A -1 power law (black heavy dot line) is also shown for reference.
Figure 3: Rank-size log-log plot of the number \((N_r)\) of male saints having their name included in that of cities in France according to INSEE data bank; "best" fits at low (a BZM law; continuous line; black dots) and high (a power law; dotted line; blue dots) ranks, respectively are shown; an intermediary regime (red dots) is emphasized. Fit parameters are given in the text.
distinguishable from a BZM law, Eq. (2), with parameters \( \zeta \simeq 1.02 \pm 0.06; R^2 \simeq 0.992 \). The overall fits to a BZM law give (\( \zeta \simeq 0.978 \pm 0.04; R^2 \simeq 0.984 \)), and (\( \zeta \simeq 0.918 \pm 0.032; R^2 \simeq 0.989 \)), for the male saint and the overall combination, respectively. For completeness, let it be mentioned that the ”king” \[35\] is St. Martin, and the ”vice-roys” \[36\], St. John and St. Peter.

A \( r^{-1} \) power law is also shown on Fig. 2, reminiscent of a similar value found in a related problem with hagiotoponyms in Italy \[26, 27\].

A second case indicating the complexity of the resulting analysis is a rank-size log-log plot of the number \( (N_r) \) of (only) male saints having their name attached to that of cities in France according to INSEE data bank; the size is ranked in decreasing order on Fig. 3. One can observe two smooth regions: at low rank, a distribution, well described by a BZM law, and at high rank, by a mere power law. In between the behavior is more complex, as often in such plots, or systems. The main fit parameters are equal to (\( \zeta \simeq 1.579 \pm 0.091; \nu \simeq 5.21; R^2 \simeq 0.991 \)), for the low rank data, 8 data points, and (\( \alpha \simeq 1.0242 \pm 0.0007; R^2 \simeq 0.979 \)), for the high rank data, 904 data points. It is fair to indicate that the saints contributing to the low rank fit are St. Germain St. Julien, St. Laurent St. Hilaire and St. Aubin. For completeness, let it be mentioned that after the ”king” (!) \[35\], Ste. Marie, the ”vice-roy” \[36\], is Ste. Colombe.

However, it occurs to astute readers and tourists that one should distinguish hagiotoponyms not necessarily according to the linguistics aspects, but according to the ”relevance” of the saint in the commune name. One can propose to consider that if the name of the city begins with the string ”Saint”, the religious aspect of the localisation has a more relevant religious content, that if the commune name “ends” with some string ”-Saint-Xyz...”. In the latter case, the beginning of the name of the commune is the most relevant term, - the saint name, coming next is an additional information, in order to distinguish, e.g., previous parishes or localities. As an example, consider Nuits-Saint-Georges: ”Nuits” is ”more important” than St. Georges. Likewise, in St. Saturnin-lès-Apt and St. Saturnin-lès-Avignon, two nearby to each other cities in Vaucluse department, ”-lès-Apt” and ”-lès-Avignon” are mere indications of the area, - the emphasis being on St. Saturnin.

Therefore, the previous method can be reformulated on such a filtered sample, taking into account only cities ”starting with a saint name”. This leads to the (amazing) Fig. 1. The power law exponent is (\( \alpha \simeq 0.454 \pm 0.001; R^2 \simeq 0.983 \)) and (\( \alpha \simeq 1.30 \pm 10^{-10}; R^2 \simeq 0.999 \)) at low rank and high rank respectively. The critical rank is \( r_c \simeq 16 \pm 1 \).

Finally, a numerical value on hapaxes is of interest: there are 628 ”male hapaxes”, for a total of 962 different male saints, and 77 ”female hapaxes”, for a total of 296 female saints. This points to a very large variety of specific devotions and intercessors to God, often by persons not recognized as duly sanctified along catholic rules.
5  Agent based model sketch

The features of Fig. 1, power laws, the critical rank value, and the "huge" number of hapaxes, suggest some reasoning, whence to sketch a socio-psychological model. One has a set of agents which (must) have an opinion, the name of a preferred saint; such agents (devoted citizens or their political or religious leaders) prefer to be with a saint, well known, who has a good reputation as being a fair intercessor to God; on the other hand, there are other agents who prefer to have a more personal saint, less popular, but more to their living place, often a sort of local hermit, who was a healer or benefactor; those agents are prompt to indicate that in their living place they have a specific (say hermit-like) "holy" benefactor, also with good connection to God since he is such a practical proof doing something good to them. In fact, this might be a reason why visually (and practically "touristically") one observes that the hapaxes saint are closely distributed. One can consider that a rivalry exists between holy persons, saints by extension, being links to God, - or by the local villagers.

This herd behavior is in line with the model first proposed by Banerjee in 1992 [5]. He proposed a herding model of decision making: in which people are inclined to mimic others' actions, collectively, but sometimes at a price. One may also follow a leader, whence becoming a "follower" [37]. However, it is possible that one accepts others' idea(s) even though one's own information tells to do something else [7, 8, 38, 39, 40].

More recently, Eguiluz and Zimmermann, [6] proposed an important application of herding model in information transmission with application to financial markets. The $N$ agents are on $N$ vertices of a network. Several agents connect with each other to form a cluster for "some reason", but sharing the same information or opinion, in the present case about the interest of having a saint name for a location. The degree heterogeneity of the network would affect the herding behavior and would lead to an order-(dis)order transition [11], as observed here.

Observe that one observes the endogeneous emergence of "leaders" in populations, like for the name of babies, as already mentioned [9]. In standard opinion dynamics models, herding behavior is usually considered to be obeyed at some local scale, due to the interaction of single agents with their neighbors. At more global scales, such models are governed by purely diffusive processes, as for example observed in universe creation discussion [12]. In the present study, some (somewhat external) agents influence the others and induce a sort of phase transition in diffusive phases. This leads to a herding phase where a fraction of the agents self-organizes into a regime leading to a rather global opinion, and to another less herding phase in which the whole population prefers another self-organization.

A more grounded historical study should take into account that the time scale is rather known,
since most of the hagiotoponyms appeared in early christianization time (4th or 5th century) of the Western Europe. Of course, city names are changing throughout centuries in France, but as far as I could see, do not much modify the numerical inputs in recent times (after Napoleon organization of France into Departments).

Should one say that the population opinion for or against a given saint, at the early times, could lead to challenges and lead to different tails in the saint name opinion distribution [43], according to the preference "herding or not herding, that is the question". Notice that for being closer to reality, one should generalize the considerations of Crokidakis and others [44, 45, 46] on opinion dynamics to networks rather than remaining on regular lattices.

Thus, the ad hoc modelization of the findings should result from a highly complex generalization of the Banerjee-Eguiluz-Zimmermann (BEZ) model. This is outside the present aim, and is left for future work.

6 Conclusions

On one hand, a phase transition, in thermodynamics, admittedly occurs, or is usually defined, when some "order parameter", usually a local variable, shows some drastic change; often it vanishes. At a usual second order phase transition one finds similar "critical exponents" on both sides of a "critical point" for quantities measuring correlations of the system variables, because of "fluctuations". Usually such a "critical point" is a specific value of some continuous variable, like the temperature. Nevertheless, one may have a succession of "critical points". in between these, various regimes may occur. They maybe characterized by power laws [47]. The same is seen in percolation studies [11].

On the other hand, the practice of venerating holy figures (and their relics) is a cultural phenomenon that engaged all sections of society [48]. Saints enjoyed high levels of popularity through their cults, leading to defining cities by the name of the saint whose cult is of interest. Not all instances were the same. Each city may have several parishes; each church is dedicated to a saint; communes bear the name of one or several saints. There is such an important tradition in catholic France. This leads to a very complex survey. In this research, one has been looking for cities, "fully devoted" to a "Saint" or a "Sainte", - not parishes.

The most popular Saints set is made among 16 individuals or so. In contrast, there is a huge set of Saints which are hapaxes, i.e. their name occurs only once.

In the present work, one reports measures of the intensity ("size") of an "order parameter", i.e. the number of times a "saint popularity" has led to a hagiotoponym in France. One looks for the intensity decay as a function of its "probability of occurrence", personified by its rank in
the list. One obtains some straight line on a log-log plot, with values of power law exponents ($\zeta$), different on both sides of a "critical rank" ($r_c$), i.e. $\zeta \simeq 0.45$ and $\zeta \simeq 1.30$, at low and high rank respectively, with $r_c = 16$.

I do not claim that one should distinguish between a mean field and a critical regime, where $r_c$ would rather be corresponding to the Landau point separating the mean field and critical regimes. The rank values are discrete points; this leads to a distinctive difference with thermodynamical phase transitions. However, the rank distribution of such a saint importance follows simple power laws, as those found in many phase transitions aspects. This has allowed some reasoning in order to propose that the features are those of a social phenomenon which can be called a herding phase transition.

One could somewhat validate such a claim showing clustering for at least certain names, i.e. the most popular ones. For the saints in the core, one could be extending the notion of popularity à la Hirsch [49, 50] through a core index, the h-index, being $r_c$. Below the critical rank, one could do a Voronoi tessellation [51] for each main saint, - obtaining the average distance between the nearest neighbor hagiotoponyms, for further conviction. This can also be left for future work.

In conclusion, one should not be afraid of limitations, and assumptions in pursuing this work. Nevertheless, there are interesting open questions. For example, what kind of network is the embedding set? What is the distribution through departments. Can this be related to geographical, sociological, political, historical facts? And in fine, it seems a hard job but an interesting challenge to look at parishes and church dedications, - and their life time.
7 Appendix: Methodology and Materials

In this Appendix, I list "problems" and solutions I found when selecting the sample for further analysis.

For example, it occurred to me at once that one should be aware that words containing "saint" were not referring to some individual. The most obvious (to me) was the "Saintes" city (see below). Moreover,

- "Saintes" is not referring to several female saints, but to the local tribe 2000 years ago in the area; so Saintes has not been counted; idem for "Saints" (-en-Puisaye) (89520)
- idem Saints F-77120 and F-89520 have not been counted
- but Saints-Geosmes makes life complicated, because it is only one city, but referring in one word to three (twin) saints; Geosmes being an alteration of jumeaux; this exceptional case (city) has been kept as referring to only one saint; not a drastic point
- the mention of some "saint(e)" is sometimes replaced by "don" or "dan", or "dame", for example in Dampierre, Dommartin, Dammarti, Dammarie, Dannemarie, Dame-Marie; but that increases much the number of possible strings, and demand to research many cities; I have neglected such strings
- I have neglected the breton toponyms, containing lok, but not meaning "saint" even though sometimes referring to a name like that of a saint: Locmaria (4 cases), Locronan surely referring to St. Ronan), Loctudy (likely referring to St. Tudy); notice that St. Tudy is himself sometimes identified with St. Tugdual (1 case), St. Pabu (1 case), St. Pabut (0 case), or St. Paban (0 case); St. Tugdual and St. Pabu were conserved;
- cases like St. Saturnin = St. Savournin = St. Sernin, have to be identified as only one individual
- one sometimes has to group hagiotoponyms; Saint Calais (F-72120) with Marolles-lès-Saint-Calais, also F-72120; notice two different city names, thus cities, in France list but same postal code; there are other same saints most likely Saint-Calais-du-Désert (F-53140);
- one has sometimes to "ungroup" hagiotoponyms: Saint-Remy-en-Bouzemont-Saint-Genest-et-Isson (F-51290) into two different saints; N.B. the longest name of a city in France;
- one can encounter a full name with some other information : Nuits-Saint-Georges, Moustiers-Sainte-Marie, Granges-Sainte-Marie; thus the string "saint" must be allowed to appear in various positions in a hagiotoponym
• too bad also for Saintry-sur-Seine F-91250, which is not related to Saint Ry; idem for Sainteny F-50500,

• but, Saint-Genis-de-Saintonge F-17240 should not be counted twice; Saint Onge does not exist! (see https://fr.wikipedia.org/wiki/Saint–Onge, for an illuminating comment)

• a problem occurs without specific decision for case like Mas-Saintes-Puelles and Les-Saintes-Maries de la Mer? ... how many saints have to be taken into account?

• "La Chapelle-aux-Saints’ F-19120 and "Longpré-les-Corps-Saints", F-80510 are also removed because they do not pertain to specific saints

• nor Nieul-lès-Saintes (F-17810); in french, "lès" means "next", here "next to the city" Saintes;

• nor Mas-Saintes-Puelles (F-11400); they buried Saint Sernin (St. Saturnin), but their name and number is unknown!

• finally, "interestingly", with respect to the model idea, in counting communes, several contain "saint" twice (8 cases); recall the "spectacular" case of Saint-Remy-en-Bouzemont-Saint-Genest-et-Isson (F-51290) mentioned here above; but, in addition, this shows that the number of cities with a saint name is related but not identical to the total number of used names.

Other warnings seem also relevant!

• (I) Eloy and Eloi are considered to be two different saints, but (II) Andre and André are the same saint, though there might be different Andre and André, and (III) local names have been used, identified or not, depending on my knowledge of the saint, i.e. Savournin and Saturnin; that does not seem to be a very drastic approximation though;

Improving on such three approximations would request to search about the saint itself, dive into his/her life, if any, and compare informations much outside those presently looked for.

NOTE ADDED AFTER ACCEPTANCE, - added on galley proofs at production time:

Interesting support literature, in particular pertinent references on hagiotoponyms in France, can be found in Ch. Higournet’s work [52, 53], and in subsequent reports [54, 55].

These studies are also to be found on the Internet site http://mnytud.arts.unideb.hu/onomural/

Acknowledgements
Thanks to A. Pękalski and K. Kulakowski, for various comments; special thanks go to C. Berman and D. Berman.

This research would not have been taking place if Dietrich Stauffer had not shown me his interest in exotic applications of statistical physics, and approved my working on sidelines. This paper would subsequently never have been written, - nor published! Thanks again Dietrich for illuminating scientific frontiers roads, with humour. See you soon.
References

[1] Stauffer, D. (2004). Introduction to statistical physics outside physics. Physica A: Statistical Mechanics and Its Applications, 336(1-2), 1-5.

[2] Stauffer, D. (2000). Grand unification of exotic statistical physics. Physica A: Statistical Mechanics and its Applications, 285(1-2), 121-126.

[3] Stauffer, D., de Oliveira, S. M., and de Oliveira, P. M. C. (1999). Evolution, money, war and computers: non-traditional applications of computational statistical physics. Teubner.

[4] Stauffer, D. (1998). Can percolation theory be applied to the stock market?. Annalen der Physik, 7(5-6), 529-538.

[5] Banerjee, A.V. (1992). A simple model of herd behavior. The Quarterly Journal of Economics, 107(3), 797-817.

[6] Eguiluz, V. M. & Zimmermann, M. G. (2000). Transmission of information and herd behavior: an application to financial markets. Physical Review Letters, 85(26), 5659.

[7] Guo, M. (2009) Herd Behavior and Phase Transition in Financial Market.

https://guava.physics.uiuc.edu/~nigel/courses/569/Essays_Fall2009/files/guo.pdf

[8] Kononovicius, A. & Gontis, V. (2013). Three-state herding model of the financial markets. EPL (Europhysics Letters), 101(2), 28001.

[9] Krawczyk, M. J., Dydejczyk, A., & Kulakowski, K. (2014). The Simmel effect and babies’ names. Physica A: Statistical Mechanics and its Applications, 395, 384-391.

[10] Stanley, H.E. (1971). Introduction to phase transitions and critical phenomena. Oxford University Press, Oxford)

[11] Aharony, A., & Stauffer, D. (2003). Introduction to percolation theory. Taylor & Francis.

[12] Roshani, F., Aghamohammadi, A., & Khorrami, M. (2004). Static and dynamic phase transitions in multidimensional voting models on continua. Physical Review E, 70(5), 056128.

[13] Riego, P., Vavassori, P., & Berger, A. (2018). Towards an understanding of dynamic phase transitions. Physica B: Condensed Matter, 549, 13-23.

[14] Bedeaux, D., Mazur, P., & Pasmanter, R. A. (1977). The ballast resistor; An electro-thermal instability in a conducting wire I; The nature of the stationary states. Physica A: Statistical Mechanics and its Applications, 86(2), 355-382.
[15] Ausloos, M. (1981). Continuously forced ballast resistor model for superconducting hot spots. Physica B+C, 108(1-3), 969-970.

[16] Ausloos, M. (1982). Electrothermal Instabilities at Magnetic Critical Points. In Nonlinear Phenomena at Phase Transitions and Instabilities (pp. 337-341). Springer, Boston, MA.

[17] Riego, P., Vavassori, P., & Berger, A. (2017). Metamagnetic anomalies near dynamic phase transitions. Physical Review Letters, 118(11), 117202.

[18] Vandewalle, N., Boveroux, Ph., Minguet, A. & Ausloos, M. (1998). The crash of October 1987 seen as a phase transition: amplitude and universality, Physica A 255, 201-210.

[19] Ausloos, M., Ivanova, K., & Vandewalle, N. (2002). Crashes: symptoms, diagnoses and remedies. In Empirical science of financial fluctuations (pp. 62-76). Springer, Tokyo.

[20] Sornette, D. (2017). Why stock markets crash: critical events in complex financial systems (Vol. 49). Princeton University Press.

[21] Stauffer, D. (2002). Percolation and Galam theory of minority opinion spreading. International Journal of Modern Physics C, 13(07), 975-977.

[22] Galam, S. (2013). Modeling the Forming of Public Opinion: an approach from Sociophysics. Global Economics and Management Review, 18(1), 2-11.

[23] Singh, S. (2000). Phase transitions in liquid crystals. Physics Reports, 324(2-4), 107-269.

[24] Belov, K. P., Zvezdin, A. K., Kadomtseva, A. M., & Levitin, R. Z. (1976). Spin-reorientation transitions in rare-earth magnets. Soviet Physics Uspekhi, 19(7), 574-596.

[25] Farmer, D.H. (2011). The Oxford dictionary of Saints, Cambridge Univ. Press (1987).

[26] Cerqueti, R. & Ausloos, M. (2015). Socio-economical Analysis of Italy: the case of hagiography cities. The Social Science Journal, 52, 561-564.

[27] Ausloos, M. & Cerqueti, R. (2016). Religion-based Urbanization Process in Italy: Statistical Evidence from Demographic and Economic Data. Quality & Quantity, 50(4), 1539-1565.

[28] Ausloos, M. & Cerqueti, R. (2016). A universal rank-size law. PLoS ONE 11(1), e0166011.

[29] Ausloos, M. (2014). Toward fits to scaling-like data, but with inflection points & generalized Lavalette function. Journal of Applied Quantitative Methods 9, 1-21.

[30] Zipf, G.K. (1949). Human Behavior and the Principle of Least Effort : An Introduction to Human Ecology, Cambridge, Mass.: Addison Wesley.
[31] Fairthorne, R.A. (1969). Empirical hyperbolic distributions (Bradford-Zipf-Mandelbrot) for bibliometric description and prediction. Journal of Documentation 25, 319-343.

[32] Levenberg, K. (1944). A method for the solution of certain problems in least squares. Quarterly of Applied Mathematics, 2, 164-168.

[33] Marquardt, D.W. (1963). An Algorithm for Least-Squares Estimation of Nonlinear Parameters. Journal of the Society for Industrial and Applied Mathematics 11(2), 431-441.

[34] Lourakis, M.I.A. (2011). A Brief Description of the Levenberg-Marquardt Algorithm Implemented by levmar. Foundation of Research and Technology 4, 1-6.

[35] Laherrère, J., and Sornette, D., (1998). Stretched exponential distributions in nature and economy: fat tails with characteristic scales. European Physical Journal B 2, 525-539.

[36] Cerqueti, R., & Ausloos, M. (2015). Evidence of economic regularities and disparities of Italian regions from aggregated tax income size data. Physica A: Statistical Mechanics and its Applications, 421, 187-207.

[37] Mosquera-Donate, G., & Boguná, M. (2015). Follow the leader: Herding behavior in heterogeneous populations. Physical Review E, 91(5), 052804.

[38] Dong, L. (2008). A self-adapting herding model: The agent judge-abilities influence the dynamic behaviors. Physica A: Statistical Mechanics and its Applications, 387(23), 5868-5873.

[39] Kononovicius, A., & Ruseckas, J. (2014). Continuous transition from the extensive to the non-extensive statistics in an agent-based herding model. The European Physical Journal B, 87(8), 169.

[40] Schweitzer, F., Mavrodiev, P., & Tessone, C. J. (2013). How can social herding enhance cooperation?. Advances in Complex Systems, 16(04n05), 1350017.

[41] Lambiotte, R. (2007). How does degree heterogeneity affect an order-disorder transition?. EPL (Europhysics Letters), 78(6), 68002.

[42] Ausloos, M. (2015). Slow-down or speed-up of inter- and intra-cluster diffusion of controversial knowledge in stubborn communities based on a small world network, in On the Frontiers Research Topic Opinions, Choices and Actions: Applications of Sociophysics to the diffusion of ideas. A. Martins & S. Galam, Eds., Front. Physics 3, pp. 43-.

[43] Dhesi, G. & M. Ausloos, M. (2016). Modelling and Measuring the Irrational behaviour of Agents in Financial Markets: Discovering the Psychological Soliton. Chaos, Solitons & Fractals 88, 119-125.
[44] Crokidakis, N. (2017). Non-Equilibrium Phase Transitions Induced by Social Temperature In Kinetic Exchange Opinion Models on Regular Lattices. Reports in Advances of Physical Sciences, 1(01), 1740001.

[45] Schwämmle, V., González, M. C., Moreira, A. A., Andrade Jr, J. S., & Herrmann, H. J. (2007). Different topologies for a herding model of opinion. Physical Review E, 75(6), 066108.

[46] Xie, Y., Wang, B. H., Quan, H., Yang, W., & Hui, P. M. (2002). Finite-size effect in the Eguiluz and Zimmermann model of herd formation and information transmission. Physical Review E, 65(4), 046130.

[47] Ausloos M., Hubert, L., Dorbolo, S., Gilabert, A., and R. Cloots, R. (2002). Magnon-polaron and spin-polaron signatures in the specific heat and electrical resistivity of $La_0.6Y_{0.1}Ca_0.3MnO_3$ in zero magnetic field and the effect of Mn-O-Mn bond environment. Physical Review B 66, 174436.

[48] Morris, C.H. (2009). The Concept of Territory in the late Anglo-Saxon and Early Medieval Cult of Saints in England, Master of Philosophy thesis submitted to The University of Birmingham.

[49] Hirsch, E (2005). An index to quantify an individual’s scientific output. Proceedings of the National Academy of Sciences of the United States of America, 102, 16569–16572.

[50] Ausloos, M. (2013). A scientometrics law about co-authors and their ranking: the co-author core. Scientometrics, 95(3), 895-909.

[51] Ausloos, M., Bartolacci, F., Castellano, N.G, & Cerqueti, R. (2017). Exploring how innovation strategies at time of crisis influence performance: a cluster analysis perspective. Technology Analysis & Strategic Management, 30(4), 484-497.

[52] Higounet, Ch. (1953). Les saints mérovingiens d’Aquitaine dans la toponymie. In: Études mérovingiennes (Actes des Journées de Poitiers, 1er–3 mai 1952). Paris, pp. 157–167.

[53] Higounet, Ch. (1958). Hagionymie et histoire. Sainte-Eulalie dans la toponymie de la France. In: Ve Congrès International de Toponymie et d’Anthroponymie (Salamanca, 1955). Salamanca, pp. 105–113.

[54] Billy, P.H. (2011). Patrocinio Settlement Names in France, in Patrocinio Settlement Names in Europe, Valeria Toth (Ed.), Onomastica Uralica 8, 7-28.

[55] Taverdet, G. & Gendron, St. (2011). Patrocinio Settlement Names in France, in Patrocinio Settlement Names in Europe, Valeria Toth (Ed.), Onomastica Uralica 8, 29-54.