Light time calculations in high precision deep space navigation

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During the last decade, the precision in the tracking of spacecraft has constantly improved. With the recent discovery of few astrometric anomalies, such as the Pioneer and Earth flyby anomalies, it becomes important to deeply analyze the operative modeling currently adopted in Deep Space Navigation (DSN). Our study shows that some traditional approximations can lead to neglect tiny terms that could have consequences in the orbit determination of a probe in specific configurations such as during an Earth flyby. Here we suggest a way to improve the light time calculation used for probe tracking.

I. INTRODUCTION

Deep space data processing during the last decade has revealed the presence of some anomalies in the trajectory of probes \textsuperscript{2,3}. Lots of hypothesis have been made trying to solve this puzzle, which can be summarized in two main approaches: whether these anomalies are the manifestation of some new physics, or something is mismodeled in the data processing. We investigated Moyer’s book \textsuperscript{4}, which describes the relativistic framework used by space agencies for data processing. We came to the conclusion that the modeling is correct up to the first post-Newtonian approximation of General Relativity. However, some terms, a priori of very small amplitude, have been neglected in the light time calculation.

We know that the ephemeris of a space mission is built from subsequent measures involving the light time of a signal traveling between Earth and the probe and the solution of the inverse problem. Since the ephemeris is used for both operational goals (space probe navigation) and scientific goals (measurements for testing fundamental physics), a well defined model is then mandatory for both the interpretation of physical data and the orbit reconstruction.

This paper is structured as follows. In section 2, we give a brief overview of light time computation as described by Moyer’s book; we show that the transponder’s delay (i.e. the time delay between the reception and retransmission of the light signal on board the satellite) is not correctly taken into account in this model. In section 3 we present an alternative, more precise, modeling. Finally, in section 4, we compare both modelings to highlight their differences and give some conclusions in section 5.

Throughout this work we will suppose that space-time is covered by some global barycentric coordinates system $x^\alpha = (x^0, x)$, with $x^0 = ct$, $c$ being the speed of light in vacuum, $t$ a time coordinate and $x = (x^i, i = 1, 2, 3)$. Greek indices run from 0 to 3, and Latin indices from 1 to 3. Here $x^i_b$/$v^i_b$ represents the position/velocity of body $b$ at time $t$, where $b$ can take the value GS (ground station) or SC (spacecraft). Primed values are related to the Moyer’s modeling, while we will generally use non-primed values for the improved modeling.

II. MOYER’S NAVIGATION MODEL

Deep space navigation is based on the exchange of light signals between a probe and at least one observing ground station. The calculation of a light time, as described in Moyer’s book \textsuperscript{4}, is quite simple: a clock starts counting as an uplink signal is emitted from ground at $x^1_{GS}$. The signal is received by the probe at $x^2_{SC}$ and then, after a short delay, reemitted towards the Earth where it is received by a ground station at $x^3_{GS}$. The clock stops counting and gives the round-trip light time $\rho'$

$$\rho' = \frac{R_{12'}}{c} + \frac{R_{23'}}{c} + \frac{\Delta(x^1_{GS}, x^2_{SC})}{c} + \frac{\Delta(x^2_{SC}, x^3_{GS})}{c},$$

(1)

where $R_{ab} = \|x_a - x_b\|$, $c$ is the speed of light and $\Delta(x_a, x_b)$ is the Shapiro delay \textsuperscript{5}. The light time $\rho'$ is then used to compute two physical quantities:

- the Ranging, describing the distance between the probe and the ground station can be computed as follow:

$$\mathcal{R}' = \rho' + \delta t + \delta C,$$

(2)

where $\delta t$ and $\delta C$ are corrections due to the relativistic effects and $\delta t$ is the clock error.
where \( \delta t \) and \( \delta C \) are the 2.5 \( \mu s \) transponder delay and other corrections (ex: atmospheric delay ...) described by Moyer, respectively.

- the Doppler, related to the radial velocity of the probe with respect to the Earth, is obtained by differentiating two successive light time measurements, \( \rho'_s = t_{3s} - t_{1s} \) and \( \rho'_c = t_{3c} - t_{1c} \), during a given count interval \( T_e = t_{3e} - t_{3s} \). Moyer showed that

\[
\mathcal{F}' = \frac{\Delta \nu'}{\nu'} = M_2 f_T(t_1) \frac{\rho'_c - \rho'_s}{T_e} = M_2 f_T(t_1) \rho'
\]

where \( M_2 \) is a transponder’s ratio applied to the downlink signal when it is reemitted towards the Earth and \( \rho' = \frac{d \rho}{dt} \).

Since the Doppler signal results from the differentiation of the Ranging signal, all constant or slowly changing terms like \( \delta t \) and \( \delta C \) obviously cancel out in this modeling.

### III. OUR IMPROVED NAVIGATION MODEL

Nevertheless, the electronic delay of some microseconds \( \delta t \) due to on board processing of the incoming signal requires to consider a different position of the spacecraft at reemission time. We have studied its consequences on light time modeling for Ranging and Doppler calculations.

For this purpose, we introduce an improved light time model \( \rho \) taking into account four events (one more with respect to Moyer’s model): the emission from the ground station at \( x_1^{GS} \), the reception by the probe at \( x_2^{SC} \), the reemission at \( x_3^{SC} \) and the reception at ground at \( x_4^{GS} \). The additional event \( x_3^{SC} = x_2^{SC} + \delta t_{23} \) accounts for this small delay of \( \delta t_{23} = 2.5 \, \mu s \) (at least for modern spacecraft):

\[
\rho = \frac{R_{12}}{c} + \frac{R_{34}}{c} + \frac{\Delta (x_1^{GS}, x_2^{SC})}{c} + \frac{\Delta (x_3^{SC}, x_4^{GS})}{c} .
\]

We can use this light time \( \rho \) to compute Ranging \( \mathcal{R} \) and Doppler \( \mathcal{F} \) observables as for Moyer’s model. We get:

\[
\mathcal{R} = \rho + \delta t_{23} + \delta C, \quad \mathcal{F} = \frac{\Delta \nu}{\nu_1} = M_2 f_T(t_1) \frac{\rho_c - \rho_s}{T_e} = M_2 f_T(t_1) \rho
\]

It’s easy to see that \( \mathcal{R} - \mathcal{R}' \neq 0 \) and \( \mathcal{F} - \mathcal{F}' \neq 0 \), since primed and non-primed events are a priori separated.

### IV. COMPARISON OF THE TWO MODELINGS

Let’s analyze the supplementary event \( x_3^{SC} = x_2^{SC} + \delta t_{23} \). This term is implicitly related to \( \delta t_{23} \) by the first order development

\[
x_3^{SC} = x_2^{SC} + \delta t_{23} v_2^{SC} + O(\delta t_{23}^2).
\]

In practice, this \( \delta t_{23} \) delay is calibrated by space agencies: it is added to \( \rho \) when computing \( \mathcal{R} \) and seems to have no consequence on the differential Doppler \( \mathcal{F} \) since it’s a constant. However, as \( \delta t_{23} \) also appears implicitly in the expression of \( x_3^{SC} \), this is not true.

Let’s analyze the implications of this mismodeling. So far, we computed the difference between \( \rho \) and \( \rho' \) using Eq. (6) and found:

\[
\Delta \rho = \rho - \rho' = \delta t_{23} \frac{1 - \frac{1}{c} (v_2^{SC} \cdot N_{12})}{1 - \frac{1}{c} (v_1^{GS} \cdot N_{12})} \text{ with } N_{12} = \frac{x_2^{SC} - x_1^{GS}}{\| x_2^{SC} - x_1^{GS} \|},
\]

\( N_{12} \) being the Minkowskian direction between the ground station and the probe.

The last equation highlights the presence of an extra non-constant term, directly proportional to the transponder delay and neglected in Moyer’s model. This term also depends on the position and velocity of both the probe and the ground station. Neglecting it would actually lead to a wrong determination of the epoch \( t_1 \) and to an error in both Ranging and Doppler. In fact, extracting the downlink signal, we get \( |x_2^{SC} - x_1^{GS}| \neq |x_2^{SC} - x_1^{GS}| \) since \( t_2 \neq t_2' \) and \( t_1 \neq t_1' \).
V. APPLICATION TO REAL SPACECRAFT ORBITS

In order to evaluate the magnitude of the effect introduced by Moyer's model, we computed $\Delta \rho = \rho - \rho'$ (giving the difference between the Ranging calculated with the two models) and $\Delta \dot{\rho} = \dot{\rho} - \dot{\rho}'$ (related to the difference of the Doppler calculated by the two models) for the observation of a probe. We used the real orbit of some probes (Rosetta, NEAR, Cassini, Galileo) during their Earth flyby, which is an optimal configuration for the determination of $\Delta \rho$ and $\Delta \dot{\rho}$. The ephemeris for probes and planets to be used in the computation were provided by the NAIF/SPICE toolkit [1].

![FIG. 1: Ranging difference $\Delta \rho$ (meters - days from flyby) during NEAR flyby.](image)

Computing Eq. (7) and its time derivative for the NEAR probe during its Earth flyby on the 23 January 1998, we found a difference of the order of some cm in the probe distance $c\Delta \rho$ calculated by the two models and a difference up to some cm/s at the instant of maximum approach in radial velocity. These results are shown in Figs. 1-2.

In order to highlight the high variability of the transponder delay effect on Doppler measures, we computed $\Delta \dot{\rho}$ for different probes in different configurations with respect to the observing station. The results are exposed in Fig. 3 and show that this delay cannot be simply calibrated at the level of light time calculation nor neglected in the Doppler calculation.
FIG. 2: Doppler difference $\Delta \rho$ (mm/s - days from flyby) during NEAR flyby.

FIG. 3: Doppler difference $\Delta \rho$ (Hz - seconds from flyby) for different probe-station configurations.
VI. CONCLUSIONS

It seems obvious from our results that the influence of the transponder delay cannot be reduced to a simple calibration without taking some precautions. It is responsible for a tiny effect on the computation of light time and has an impact on both **Ranging** and **Doppler** determination. We represent it by a more complete modeling, considering four epochs instead of three. In order to test the amplitude and variability of this effect on real data, we calculated its influence on some real probe-ground station configurations during recent Earth flybys (NEAR, Rosetta, Cassini and Galileo). The observables calculated using Moyer’s model and our improved model showed differences not so far from the measuring precision available to space agencies.

Such an error is largely acceptable for most operational goals at present. Anyway, we would like to highlight the fact that this error is directly proportional to the transponder delay and that for past missions, whose data are still largely used for scientific purposes, transponders were more than $10^3$ times slower that today. In the future too, increasing ephemeris precision should be followed by the development of faster transponders or by the use of a more precise model.

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