Approaching the Bottom Using Fine Lattices With Domain-Wall Fermions

Brendan Fahy

JLQCD Collaboration

Lattice 2016 Southampton UK
Lattice discretization effects are significant at large quark masses as some cutoff effects go as $a m$.

The JLQCD collaboration has recently produced very fine Domain Wall Lattices $a = 0.080$ to $0.044\text{fm}$.

We look at the charmed mesons and find that the cutoff effects are only a few percent.

How far can we push the limits beyond charm and extrapolate to the bottom?
JLQCD Lattices

- $N_f = 2 + 1$ simulations on 15 Ensembles with 10,000 MD times for each.
- Simulations at three lattice spacing $a^{-1} \approx 2.4, 3.6$ and $4.5 \text{GeV}$
- $m_\pi \approx 230, 300, 400, 500 \text{ MeV}$
- Domain-Wall (Möbius) fermions
- Stout link-smearing
- $m_{\text{res}} \approx 1 \text{MeV}$ on our coarsest lattice;
- $m_{\text{res}} \approx 0$ on the finer lattices.
### JLQCD Lattices

| Lattice Spacing | $L^3 \times T$ | $L_5$ | $am_{ud}$ | $am_s$ | $m_\pi$ [MeV] | $m_\pi L$ |
|-----------------|----------------|-------|-----------|--------|----------------|-----------|
| $\beta = 4.17, a = 0.080\, \text{fm}$<br>$a^{-1} = 2.453(4)\, \text{GeV}$ | $32^3 \times 64$ | 12 | 0.0035 | 0.040 | 230 | 3.0 |
|                  |                |      | 0.0070 | 0.030 | 310 | 4.0 |
|                  |                |      | 0.0070 | 0.040 | 310 | 4.0 |
|                  |                |      | 0.0120 | 0.030 | 400 | 5.2 |
|                  |                |      | 0.0120 | 0.040 | 400 | 5.2 |
|                  |                |      | 0.0190 | 0.030 | 500 | 6.5 |
|                  |                |      | 0.0190 | 0.040 | 500 | 6.5 |
|                  | $48^3 \times 96$ | 12 | 0.0035 | 0.040 | 230 | 4.4 |
| $\beta = 4.35, a = 0.055\, \text{fm}$<br>$a^{-1} = 3.610(9)\, \text{GeV}$ | $48^3 \times 96$ | 8 | 0.0042 | 0.018 | 300 | 3.9 |
|                  |                |      | 0.0042 | 0.025 | 300 | 3.9 |
|                  |                |      | 0.0080 | 0.018 | 410 | 5.4 |
|                  |                |      | 0.0080 | 0.025 | 410 | 5.4 |
|                  |                |      | 0.0120 | 0.018 | 500 | 6.6 |
|                  |                |      | 0.0120 | 0.025 | 500 | 6.6 |
| $\beta = 4.47, a = 0.044\, \text{fm}$<br>$a^{-1} = 4.496(9)\, \text{GeV}$ | $64^3 \times 128$ | 8 | 0.0030 | 0.015 | 280 | 4.0 |
Measurements

- Correlators calculated on each lattice for both smeared and unsmmeared $Z_2$ sources
- Measurements were produced on 100 configurations with 6 – 8 source points each.
- Combined fit to Axial and Pseudoscalar correlators
$D$ decay constant

- Chiral and Continuum extrapolation of $f_D$

- The lattice spacing dependence is small

- $f_D = 212.8 \pm 1.7 \pm 3.6$ MeV
$D_s$ decay constant

- Chiral and Continuum extrapolation of $f_{D_s}$
- Fit does not go through the lines due to miss tuning of $m_s$
- Interpolated using $2m_K^2 - m_\pi^2$
- $f_{D_s} = 244.0 \pm 0.84 \pm 4.1$ MeV
Comparison of $f_{D(s)}$ to existing results

(PRELIMINARY)
Since cutoff effects at the charm are reasonably controlled, how far above the charm mass can we go?

Bare quark masses chosen $m_i = (1.25)^i m_c$:

All heavy quarks treated with DW

| Beta | $m_0 = m_c$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ |
|------|------------|-------|-------|-------|-------|-------|
| 4.17 | 0.4404     | 0.5505| 0.6881| 0.8600|       |       |
| 4.35 | 0.2729     | 0.3411| 0.4264| 0.5330| 0.6661| 0.8327|
| 4.45 | 0.2105     | 0.2631| 0.3289| 0.4111| 0.5139| 0.6423|
Heavy-light and heavy-strange results

For both h-l and h-s for each of our heavy quark masses. Contains large discretization effects.
Global fit to \((1 + C_1/m + C_2/m^2)\) excluding \(m_q > 0.7\) with \(\gamma_1(a^2m^2), \gamma_2(a^2)\) and linear chiral and \(m_s\) corrections.
Account for the leading discretization effects

- Adjust the meson masses using $m_1$ and $m_2$ from
  \[ E = m_1 + \frac{p^2}{2m_2} + \ldots \]

- In the Continuum
  \[
  S(p) = \frac{1}{p + m} \quad \Rightarrow \quad C(t, \vec{p} = 0) = \int \frac{dp_0}{2\pi} S(p) e^{ip_0 t} = \frac{1 + \gamma^0}{2} e^{-mt}
  \]

- On the lattice this is not a simple exponential due to the non-locality of 4D effective Dirac operator of DW.

- In order to eliminate the leading discretization effects, we divide the correlator by the tree-level heavy quark propagator of DW and multiply back the corresponding continuum exponential. This is an extension of the Fermilab approach for DW.
Account for the leading discretization effects

- Matching between QCD and HQET. This allows $1/m$ expansion.

- $A^{\text{QCD}}_\mu = C(\mu)A^{\text{HQET}}_\mu(\mu)$

- Perturbative calculation available\(^1\) up to three loops ($\alpha_s^3$)

- Global fit to with continuum limit ($A + B/m + C/m^2$) excluding $m_q > 0.7$

- Fit function accounts for $\gamma_1 \alpha_s(a^2m^2)$, $\gamma_2(a^2)$ and linear chiral and $m_s$ corrections. Note tree level $(am)^2$ is already removed.

\(^1\)Bekavac et al. arXiv:0911.3356
Corrected $f_{hl} \sqrt{m_{hl}}$

$1/(m_{hl} + m_2 - m_1) [1/\text{GeV}]$

$\sqrt{m_{hl}} / C(\mu) [\text{MeV}^{3/2}]$

Continuum and Chiral Limit

fit $\beta$: 4.17

fit $\beta$: 4.35

fit $\beta$: 4.47

$f_B : 195.5 \pm 3.2 \pm 3.3 \text{ MeV}$

Check: at the charm this gives $f_D : 215.5 \pm 2.0 \text{ MeV}$ consistent with the charm only analysis
Corrected $f_{hS} \sqrt{m_{hS}}$

$$f_{hS} \sqrt{m_{hS}} [\text{MeV}^{3/2}]$$

$$1/(m_{hS} + m^2 - m_1) [\text{1/GeV}]$$

Continuum and Chiral Limit

fit $\beta$: 4.17

fit $\beta$: 4.35

fit $\beta$: 4.47

$f_{B_s} : 218.2 \pm 1.9 \pm 3.7 \text{MeV}$

Check: at the charm $f_{D_s} : 244.7 \pm 1.0 \text{MeV}$ consistent with the charm only analysis
Comparison of $f_{B(s)}$ to existing results

(PRELIMINARY)
Conclusions and Future work

- Results of heavy mesons seem promising and the cutoff effects for heavy domain wall fermions can be partially understood.

- Leading $a^2$ effects seem to be identifiable and corrected for.

- Extrapolation to the B using standard DW fermions seems somewhat reasonable.

- Investigate $f_{B_s}/f_B$.

- Further explore the “ratio method” using ratios of successive heavy masses to constrain the extrapolation.
Thank You.