Two-Pion Exchange Interaction Between Constituent Quarks

D. O. Riska\textsuperscript{1} and G. E. Brown\textsuperscript{2}

\textsuperscript{1}Department of Physics, 00014 University of Helsinki, Finland
\textsuperscript{2}Department of Physics, State University of New York, Stony Brook, NY 11794, USA

Abstract

The two-pion exchange interaction between constituent quarks is shown to enhance the effect of the the isospin dependent spin–spin component of the one-pion exchange interaction, and to cancel out its tensor component. It therefore provides a partial explanation for the phenomenological observation that the hyperfine interaction between constituent quarks is well described by a flavor dependent spin–spin interaction, which is attractive at short and repulsive at long range. The spin–orbit component of the two-pion exchange interaction is stronger than and has the opposite sign from that associated with the linear confining interaction in the $P$–shell multiplets.

PACS: 12.39.Pn, 12.39.-x, 14.20.-c

KEYWORDS: Quark-quark interaction, two–pion exchange
1. Introduction

The instanton liquid model of the vacuum \cite{1,2} implies pointlike quark–quark interactions, which when iterated in the $t$–channel, admit a meson exchange interpretation of the interaction between the constituent quarks that form the baryons \cite{3}. This description is supported by explicit QCD lattice calculations, which suggest that the chromomagnetic interaction between gluons is screened out at the length scales $\sim 0.3$ fm characteristic of the instanton fluctuations \cite{4,5,6}. Phenomenological investigation of the baryon spectrum, as presently known, does indeed reveal that main features of the baryon spectrum up to the spin–orbit splittings, may be described by a flavor-spin dependent interaction, with a form similar to the short range part of the flavor–spin component of $\pi, K$ and $\eta$–meson exchange interaction between constituent quarks alone \cite{7,8}. Even so there remain a number of gaps in this description.

To these gaps belongs the fact that, while the flavor–spin component of the pseudoscalar meson exchange interaction has the spectroscopically desired features, the corresponding tensor component by itself would cause small but empirically contraindicated spin–orbit splittings of the low lying negative parity multiplets in the baryon spectrum. Another unclosed gap is the need for a repulsive spin–orbit interaction component to cancel the attractive (Thomas term) spin–orbit interaction, which is associated with the linear scalar confining interaction. The former one of these problems may be solved qualitatively by invoking vector meson exchange interactions in addition to the pseudoscalar meson exchange interaction between the constituent quarks \cite{9}. An alternate source of cancellation for the tensor interaction is the tensor component that is associated with the irreducible $\pi$–gluon exchange interaction, which has significant strength even when the gluon coupling to constituent quarks is screened at large distances \cite{10}. Invocation of such interaction mechanisms does however, inevitably, raise the question of the relative importance of the related irreducible two–pion exchange interaction. This issue is addressed here by an explicit calculation of the two–pion exchange interaction between constituent quarks.

Before proceeding is should be noted that the latter spin–orbit problem may be avoided phenomenologically by postulation of a strong gluon exchange term in the hyperfine interaction \cite{11}, although only at the price of incorrect spectral ordering \cite{12} and conflict with the QCD lattice result that
the gluonic part of the chromomagnetic hyperfine interaction should be very weak at the relevant length scales \[4, 5, 6\].

Turning then to the two–pion exchange interaction between constituent quarks we note that the pseudovector pion–quark coupling constant \(f_{\pi qq}\) is smaller by a factor \(\sim 3/5\) than the corresponding pion–nucleon coupling constant \(f_{\pi NN} \approx 1\), while the constituent quark mass is about \(\sim 1/3\) of the nucleon mass. Therefore the \textit{a priori} expectation should be that the strength of the two–pion exchange interaction between constituent quarks should – on the average – be about as strong as the corresponding two–pion exchange interaction between nucleons, as the scaling factor implied by the coupling constant and the mass ratios is \((3/5)^4(1/3)^{-2} \approx 1.2\). We find this indeed to be the case, and that for quark separations smaller than \(\sim 0.6\) fm, which is the scale relevant for baryon structure, the components of the two pion exchange interaction do dominate over those of the one–pion exchange interaction.

The calculational tools required to construct a properly defined irreducible two–pion exchange interaction are available in the literature, having originally been developed for the two–pion exchange interaction between nucleons \[12, 13, 14\]. These methods may, with but minor modifications, be applied to the interaction between constituent quarks, provided that the effect of quark confinement on the quark propagators may be approximately incorporated in the effective constituent quark mass. The interaction is constructed from the Poincaré invariant 4–point function, which is used to define a quasipotential for the covariant Blankenbecler–Sugar equation \[15, 16\], from which the iterated one–pion exchange interaction is subtracted. The covariance requirement is essential in the case of light constituent quarks, which have velocities close to that of light, when confined within the baryons.

The spontaneous breaking of chiral symmetry requires the pion–quark coupling to have pseudovector form. The pions are assumed to decouple from the constituent quarks at the chiral symmetry restoration scale \(\Lambda_\chi \sim 1\) GeV, and therefore no ultraviolet divergence appears, although the calculated values of the potential at short range are sensitive to the way this cut–off is implemented. While the calculation of the two-pion exchange loop diagrams is straightforward – if both “lengthy and tedious” – those loop diagrams do not by themselves provide a realistic estimate of the two–pion exchange interaction. This is strongly influenced by the strong interaction between the exchanged pions in the \(S\)– and \(P\)–states in the \(t\)–channel, which has to be
taken into account. This may be done by separating out the contributions of those to partial waves from the loop amplitudes, and replacing them with amplitudes, which do contain the main resonance contributions, in addition to the Born terms \([13, 17]\).

The main features of the two–pion exchange interaction may be summarized as follows: (1) it has a strong spin and isospin dependent central interaction, which adds to the corresponding component of one–pion exchange, (2) its isospin dependent tensor component in effect cancels the tensor component of one–pion exchange, (3) its spin–orbit components combine in isospin antisymmetric quark pair states to a repulsive net spin–orbit interaction, which overwhelms the spin–orbit component of the linear scalar confining interaction in the \(P\)–shell multiplets. These features conform with the phenomenological observation that the baryon spectrum is well described by a linear confining interaction in combination with a flavor dependent spin–spin interaction, which is attractive at short and repulsive at long range. The one–plus–two–pion exchange hyperfine interaction is attractive in quark pair states with \(T = S = 0\), and repulsive in quark pair states with \(T = S = 1\).

This paper falls into 6 sections. In section 2 the calculational method is described. The method for treating the strong \(\pi\pi\) interaction in the \(J = 0, 1\) states in the \(t\)–channel is described in section 3. The numerical results are given in section 4. Section 5 contains a discussion of the phenomenological implications and the role of \(\omega\) and gluon exchange between quarks. Section 6 contains a summarizing discussion.

### 2. The two–pion exchange potential

#### 2.1. The one–pion exchange interaction

The spontaneously broken approximate chiral symmetry of QCD suggests that to lowest order in \((p_\pi/m_\pi)\) the coupling of pions to constituent quarks have the form

\[
\mathcal{L}_{\pi qq} = -\frac{1}{f_\pi} \partial_\mu \bar{q} \gamma_\mu \sigma_\mu \cdot A, \quad (2.1)
\]

where \(\bar{A}_\mu = ig_A \bar{q} \gamma_\mu \gamma_5 \tau_q q/2\) is the axial current of the quark and \(f_\pi\) is the pion decay constant (93 MeV). The effective (pseudovector) pion–quark coupling
constant is then
\[ f_{\pi qq} = \frac{g_A m_\pi}{2 f_\pi}. \tag{2.2} \]

With \( g_A \simeq 0.87 \) for the quark \[18, 19\], this yields \( f_{\pi qq} \simeq 0.65 \), which is close to the static quark model value \( f_{\pi qq} = \frac{3}{5} f_{\pi NN} \simeq 0.6 \), where \( f_{\pi NN} \) is the pseudovector \( \pi \)-nucleon coupling constant.

The interaction (2.1) immediately yields the one–pion exchange interaction
\[ K_\pi = \frac{f_{\pi qq}^2}{m_\pi^2} \frac{\gamma^{(1)} \cdot k \gamma^{(1)} \gamma^{(2)} \gamma^{(2)}}{m_\pi^2 + k^2} f(k^2) \vec{\tau}_1 \cdot \vec{\tau}_2. \tag{2.3} \]

Here \( f(k) \) is a cut–off function, the role of which is to switch off the pion–quark coupling at the chiral symmetry restoration scale \( \Lambda_\chi \). We shall take this to have the simple monopole form
\[ f(k^2) = \frac{\Lambda_\chi^2 - m_\pi^2}{\Lambda_\chi^2 + k^2}, \tag{2.4} \]

where \( \Lambda_\chi \simeq 1 \, \text{GeV} \).

### 2.2. Definition of two–pion exchange interaction

The \( \pi \)-quark coupling (2.1) leads in 4th order to the two–pion exchange 4–point functions illustrated diagrammatically in Fig.1. For the construction of the corresponding irreducible interaction the iteration of the one pion exchange interaction (2.3) has to be subtracted from the amplitude with uncrossed pion lines (Fig.1a). For this purpose we employ the framework of the Blankenbecler–Sugar equation \[15, 16\], which may be written formally as the integral equation
\[ M = U + \tilde{g} M, \tag{2.5} \]

where \( M \) is the 4–point function, \( U \) the interaction (quasipotential) and \( \tilde{g} \) the 3–dimensional two–quark propagator
\[ \tilde{g} (\tilde{k}, W) = 2\pi i \delta (k_0) \frac{\Lambda^{(1)}_+ (\tilde{k}) \Lambda^{(2)}_+ (-\tilde{k})}{E^2 (\tilde{k}) - W^2 - i\epsilon}. \tag{2.6} \]
Here $W$ is the total energy and $\Lambda^{(i)}(\vec{k})$ ($i = 1, 2$) are positive energy projection operators \cite{12}. In (2.6) $E^2(\vec{k}) = m^2 + \vec{k}^2$, where $m$ is the constituent quark mass. For small quark momenta the Blankenbecler–Sugar equation adiabatically approaches the form of the Lippmann–Schwinger equation, and therefore the conventional phenomenological Hamiltonian approach to the quark model.

The relation between the interaction $U$ and the 4–point function $M$ is defined by the equation

$$U = K + K(G - \tilde{g})U; \quad (2.7)$$

where $K$ is an irreducible interaction kernel and $G$ is the product of two free fermion propagators for the quarks.

Given the one–pion exchange interaction $K_\pi$ (2.3), perturbative solution of (2.5) leads to the following formal expressions for the one– ($U_\pi$) and two–pion exchange ($U_{\pi\pi}$) components of the quasipotential:

$$U_\pi = K_\pi, \quad (2.8a)$$

$$U_{\pi\pi} = M_{\pi\pi} - K_\pi\tilde{g}K_\pi. \quad (2.8b)$$

Here $M_{\pi\pi}$ is the full two–pion exchange 4–point function.

The explicit expressions for the two–pion exchange interaction $U_{\pi\pi}$ have been given in ref.\cite{13} for the case of the nucleon–nucleon interaction, and may be applied to the case of the two–pion exchange interaction between constituent quarks with only minor modifications, which are described below. These include replacement of the pion–nucleon coupling constant by the appropriate pion–quark coupling constant, and of the nucleon mass by the pion mass in all expressions. The main modification is that required by introduction of the cutoff factor $f(k^2)$ (2.3) for the pion–nucleon coupling.

\section*{2.3. Calculational Details}

The two–pion exchange 4–point function $M_{\pi\pi}$ is calculated under the assumption that the external quarks satisfy the Dirac equation. The amplitude may then be decomposed into 5 linearly independent spin invariants, the choice \cite{20}:

$$P_1 = 1, \quad P_2 = i(\gamma^1 \cdot P + \gamma^2 \cdot N),$$
\[ P_3 = (i\gamma^1 \cdot N)(i\gamma^2 \cdot P), \quad P_4 = \gamma^1 \cdot \gamma^2, \quad P_5 = \gamma_5^1 \gamma_5^2, \quad (2.9) \]

being particularly convenient. Here \( P = (p_1 + p'_1)/2 \) and \( N = (p_2 + p'_2)/2 \), where \( p_1, p_2 \) and \( p'_1, p'_2 \) are the 4–momenta of the initial and final quarks, respectively.

The two–pion exchange amplitude then takes the form

\[ M_{\pi\pi} = \sum_{j=1}^{5} [3p_j^+(s, t, u) + 2p_j^-(s, t, u)\vec{\tau}_1 \cdot \vec{\tau}_2]. \quad (2.10) \]

Here \( s, t \) and \( u \) are the invariant variables defined as

\[ s = -(p_1 + p_2)^2, \quad t = -(p'_1 - p_1)^2, \quad u = -(p'_2 - p_2)^2. \quad (2.11) \]

The coefficient functions \( p_j^{\pm} \) admit the spectral representations

\[ p_j^{\pm}(s, t, u) = \frac{1}{\pi} \int_{4m_q^2}^{\infty} dt' \rho_j^{\pm}(s, t') \mp (\mp) \rho_j^{\pm}(u, t') \frac{t - t'}{t - t'}. \quad (2.11) \]

The explicit expressions for the spectral weight functions \( \rho_j^{\pm} \) for the two–pion exchange amplitudes in Fig.1 are given in ref.\[3\], and may be employed in the present case, once the fermion mass \( m \) is interpreted as the quark mass, and the pion–nucleon coupling constant \( g \) is replaced by the corresponding pion–quark coupling constant \( g = 2(m_q/m_\pi)f_{\pi qq} \). The spectral functions \( \rho_j^{\pm} \) are formed as a sum of loop diagram contributions and terms associated with the \( S- \) and \( P- \) wave intermediate states as described in section 3 below. This separation makes it possible to take into account the interaction between the exchanged pions as well as the constraints of chiral symmetry for the \( S- \) wave amplitude as shown in \[7\].

The effect of including the cutoff factor \( f(k^2) \) in the one–pion exchange interaction \( (2.3) \) on the two–pion exchange amplitude may be taken into account by separation into partial fractions. In both loop amplitudes in Fig. 1 there appears a product of two pion propagators and two cut–off factors, which may decomposed as

\[ \frac{f(k_1^2)}{m_\pi^2 + k_1^2} \frac{f(k_2^2)}{m_\pi^2 + k_2^2} = \frac{1}{m_\pi^2 + k_1^2} \frac{1}{m_\pi^2 + k_2^2} - \frac{1}{\Lambda^2 + k_1^2} \frac{1}{m_\pi^2 + k_2^2} + \frac{1}{\Lambda^2 + k_1^2} \frac{1}{\Lambda^2 + k_2^2}. \quad (2.12) \]
This implies that the two–pion exchange interaction will be formed as a linear combination of 4 different terms, in which the masses of both the exchanged “mesons” take the values \( m_\pi \) and \( \Lambda \) in turn.

The first and last terms on the r.h.s. of (2.12) are calculable directly using the formulae in ref.[13]. As the interaction is here only considered in the limit \( W = 2m \), the two intermediate terms in (2.12) may be calculated using the same formulae, provided that (a) the lower limit in the dispersion integrals (2.11) is replaced by \( (m_\pi + \Lambda)^2 \), and (b) the variable combination \( \sqrt{t'/4 - m_\pi^2} = q \) (and its square) is replaced everywhere where it occurs in the integrand by the more general expression

\[
q = \frac{\sqrt{t'^2 - 2t' (\Lambda^2 + m_\pi^2) + (\Lambda^2 - m_\pi^2)^2}}{4t'}.
\]  

The expressions given in ref.[13] for the iterated one–pion exchange interaction term in (2.8b) may also be employed when the cut–off factor is taken into account as in (2.12). The modification required in those expressions (when \( W = 2m \)) is the replacement of the squared pion mass \( m_\pi^2 \) everywhere in the corresponding integrands by \( (m_\pi^2 + \Lambda^2)/2 \) for the second and third terms on the r.h.s. in (2.12) and by \( \Lambda^2 \) in the case of the fourth term.

3. The \( \pi\pi \) interaction in \( S- \) and \( P- \) waves

3.1 Partial wave projection in the \( t- \)channel

The two–pion exchange loop amplitudes (Fig.1), when calculated without account of the strong interaction between the two exchanged pions, do not provide a realistic description of the two–pion exchange interaction. The strong interaction between the pions in the \( S- \)state in the \( t- \)channel leads to a substantial enhancement of the attractive part of the potential. In the \( P- \)state the \( \pi\pi \) interaction so strong as to form the \( \rho- \)meson resonance. This has to be taken into account in order to obtain a spin–orbit component with the phenomenologically required sign.

The \( \pi\pi \) correlations in the \( S- \) and \( P- \)waves in the \( t- \)channel may be taken into account by subtracting the \( S- \) and \( P- \)wave components in the
Concretely this is implemented by decomposing the spectral functions $\rho_j^\pm$ (2.11) in the form \[13\]:

$$
\rho_j^\pm = d_j^\pm + b_j^\pm + c_j^\pm,
$$

(3.1)

where $d_j^\pm$ is the loop amplitude contribution (Fig.1), $b_j^\pm$ are the corresponding contributions from $S$- and $P$-wave $\pi\pi$ intermediate states multiplied by $-1$ and $c_j^\pm$ are corresponding amplitudes, which do take into account the $\pi\pi$ correlations. The explicit expressions for the $S$- and $P$-wave contributions $b_j^\pm$ are given in ref. [13], and may be employed here with the modifications listed in section 2.3 above.

### 3.2 The $\pi\pi$ $S$-wave interaction

The $I = 0$ $S$-wave $\pi\pi$ state only contributes to the amplitude $c_1^+$ as defined in (3.1). This contribution may be expressed in terms of the helicity amplitude $f_1^{(+0)}$ for $q\bar{q} \to \pi\pi$ defined in analogy with that for [21] as

$$
\begin{align*}
c_1^+(s, t) + c_1^+(u, t) &= \frac{2\pi}{\sqrt{t' t' - 4m^2}} |f_1^{(+0)}(t)|^2.
\end{align*}
$$

(3.2)

Here $q(t)$ is defined as in (2.13).

With the pseudovector coupling model (2.1) this amplitude is real and takes the form [17]

$$
\begin{align*}
f_1^{(+0)}(t)_B &= \frac{g^2}{4\pi} \left\{ \frac{\chi^2}{m} - m(1 - h \arctan \frac{1}{h}) \right\},
\end{align*}
$$

(3.3)

where $\chi^2 = m^2 - t/4$ and $h = (q^2 - t/4)/2q\chi$. Note that the function $h \arctan(1/h)$ should be analytically continued to $(H/2)\log[(H-1)/(H+1)]$, where $h = iH$ for $t > 4m^2$, and that $g = 2(m_q/m_\pi)f_{\pi qq}$.

The $I = 0$ $S$-wave $\pi\pi$ interaction is known to cause a large enhancement of the magnitude of the helicity amplitude $f_1^{(+0)}(t)$ for $t$-values in the range $10 - 20m_\pi^2$ [22], and possibly even to a wide resonance (”$\sigma$”), although the latter issue remains contentious [23, 24, 25]. Such a resonance contribution
may be added to the Born term helicity amplitude (3.3) as:

\[ f^{(+0)}_{\pm}(t) = f^{(+0)}_{\pm}(t)_B - \frac{\lambda^2}{4\pi} \frac{g_{\sigma q q} g_{\sigma \pi \pi}}{m_\sigma^2 - t - i\gamma q(t)}. \]  

(3.4)

Here \( m_\sigma \) represents the mass and of the resonance, and \( g_{\sigma q q} \) and \( g_{\sigma \pi \pi} \) denote the \( \sigma \)–quark and \( \sigma \pi \pi \) coupling strengths respectively. The parameter \( \gamma \) is defined as \( \gamma = m_\sigma \Gamma/q(m_\sigma) \).

For the scalar meson resonance parameters we employ the suggested values \( m_\sigma = 470 \) MeV and \( \Gamma = 250 \) MeV \[23\], from which it follows that \( \gamma = 620 \) MeV. To estimate the coupling constant product \( g_{\sigma q q} g_{\sigma \pi \pi} \) we fall back on the \( \sigma \)–model. When applied to constituent quarks, this yields \( g_{\sigma q q} = m_q/f_\pi \) and \( g_{\sigma \pi \pi} = m_\sigma^2/2f_\pi \). With a constituent quark mass value of 340 MeV \[7\] this then yields \( g_{\sigma q q} = 3.65 \) and \( g_{\sigma \pi \pi} = 2.5m_\sigma = 1.19 \) GeV.

### 3.3 The \( \pi \pi \) P–wave interaction

The \( I = 1 \) P–wave \( \pi \pi \) state contributes to the amplitudes \( c_1^-, c_2^- \) and \( c_4^- \) in (3.1). These contributions may be expressed in terms of the helicity amplitudes \( f_{\pm}^{(-1)} \) for \( q \bar{q} \to \pi \pi \) as \[13\]:

\[ c_1^-(s,t) - c_1^-(u,t) = \frac{-\pi}{6} N(s-u)q^2(t)|\lambda^-|^2, \]  

(3.5a)

\[ c_2^-(s,t) + c_2^-(u,t) = \frac{-2\pi}{3} Nq^2(t)\text{Re}\{\eta^-\lambda^-\}, \]  

(3.5b)

\[ c_4^-(s,t) + c_4^-(u,t) = \frac{-2\pi}{3} Nq^2(t)|\eta^-|^2. \]  

(3.5c)

Here \( N = q(t)/32\pi^2\sqrt{t} \), and

\[ \lambda^- = \frac{12\pi}{m^2 - t/4} [f_{+}^{(-1)}(t) - \frac{m}{\sqrt{2}} f_{-}^{(-1)}(t)], \]

\[ \eta^- = 6\pi\sqrt{2} f_{-}^{(-1)}(t). \]  

(3.6)

The helicity amplitude combinations \( \lambda^- \) and \( \eta^- \) are expressed as combinations of Born terms and a \( \rho \)–meson resonance contribution:

\[ \lambda^- = \lambda^-_B + \lambda^-_R, \quad \eta^- = \eta^-_B + \eta^-_R. \]  

(3.7)
The Born term expressions are obtained as [17]:

\[ \lambda^- = \frac{3\pi mg^2}{2q\chi^3} \left\{ 3h - (1 + h^2)arctan\frac{1}{h} \right\}, \quad (3.8a) \]

\[ \eta^- = -\frac{3\pi g^2}{2q\chi} \left\{ h - (1 + h^2)arctan\frac{1}{h} \right\}. \quad (3.8b) \]

The resonance contributions that correspond to the \( \rho^- \)-meson pole are obtained as

\[ \lambda_R^- = -\frac{2\kappa g_{\rho qq}}{m} \frac{g_{\rho\pi\pi}}{m^2_\rho - t - i\gamma q^3(t)}. \quad (3.9a) \]

\[ \eta_R^- = 2g_{\rho qq}(1 + \kappa) \frac{g_{\rho\pi\pi}}{m^2_\rho - t - i\gamma q^3(t)}. \quad (3.9b) \]

Here \( m^2_\rho = 0.59 \text{ GeV} \) and \( \gamma = m_\rho \Gamma / q^3(m_\rho) = 2.5/\text{GeV} \) as determined from the empirical mass and width of the \( \rho^- \)-meson.

The quark model relation between the vector coupling constants of the \( \rho^- \)-meson to quarks and to nucleons respectively as obtained from the charge coupling term is

\[ g_{\rho qq} = g_{\rho NN}, \quad (3.10) \]

whereas as obtained from the current coupling term it is

\[ g_{\rho qq}(1 + \kappa_{\rho qq}) = \frac{3}{5} \left( \frac{m_q}{m_N} \right) g_{\rho NN}(1 + \kappa_{\rho NN}). \quad (3.11) \]

With \( g_{\rho\pi\pi}^2 / 4\pi \simeq 0.52 \) the first relation gives the value \( g_{\rho qq} = 2.6 \). Since for nucleons \( \kappa_{\rho NN} = 6.6 \) [20], it follows from the second relation that \( \kappa_{\rho qq} = 0.65 \) when \( m_q = 340 \text{ MeV} \).

The \( \rho\pi\pi \) coupling constant \( g_{\rho\pi\pi} \) is conventionally taken to be twice that of the \( \rho^- \)-nucleon coupling constant, but may also be determined from the \( \rho \to \pi\pi \) decay width. We shall take it to have the value \( g_{\rho\pi\pi} = 2g_{\rho NN} = 5.12 \). With these parameter values the model for the two-pion exchange interaction that takes into account the strong \( \pi\pi \) interactions in the \( S^- \) and \( P^- \)-waves in the \( t^- \)-channel is completely specified.

4. Numerical values of the two-pion exchange potential
4.1 The local part of the potential

The most transparent way to illustrate the two–pion exchange interaction between constituent quarks is to consider only the leading local components in (asymptotic) series an expansion in $\vec{p}/m$ of the interaction. Since the confined constituent quarks in the baryons have large velocities, the leading local components of the interaction can however at most give a qualitative description of the full interaction. In view of the large uncertainties pertaining to the coupling of pions to constituent quarks at short range, we shall here nevertheless be content with a calculation of only the local components of the interaction in order to obtain a qualitative understanding of the nature of the two–pion exchange interaction between constituent quarks.

In the local approximation the spin invariants $P_j$ are reduced to the potential operators

$\tilde{\Omega}_C = 1, \quad \tilde{\Omega}_{LS} = \frac{1}{2} i (\vec{\sigma}^1 + \vec{\sigma}^2) \vec{p}' \times \vec{p},$

$\tilde{\Omega}_T = \vec{\Delta}^2 (\vec{\sigma}^1 \cdot \vec{\sigma}^2) - 3 (\vec{\sigma}^1 \cdot \vec{\Delta}) (\vec{\sigma}^2 \cdot \vec{\Delta}),$

$\tilde{\Omega}_{SS} = \vec{\sigma}^1 \cdot \vec{\sigma}^2,$

where $\vec{\Delta} = \vec{p}' - \vec{p}$. When expanded to order $\vec{p}'^2/m^2$ the general two–pion exchange interaction then in the adiabatic limits takes the form

$V = \sum_\alpha [\tilde{v}_\alpha^+(t) + \tilde{v}_\alpha^-(t) \vec{\tau}^1 \cdot \vec{\tau}^2] \tilde{\Omega}_\alpha,$

(4.2)

where $\alpha$ runs over the set (4.1), and the potential coefficients $v_\alpha^\pm(t)$ only depend on (invariant) momentum transfer. These may be expressed as weighted integrals of the spectral functions $\rho_\alpha^\pm(t)$ and the corresponding weight functions for the iterated one–pion exchange interaction. Explicit expressions for these weight functions are given in ref. [13].

Once the interaction potential is expressed in the form (4.2), it may readily be Fourier transformed, and finally — to first order in order $\vec{p}'/m$ — takes the form

$V = \sum_\alpha [v_\alpha^+(r) + v_\alpha^-(r) \vec{\tau}^1 \cdot \vec{\tau}^2] \Omega_\alpha,$

(4.3)
where the set of spin operators $\{\Omega_\alpha\}$ is defined as

$$
\Omega_C = 1, \quad \Omega_{LS} = \vec{S} \cdot \vec{L},
$$

$$
\Omega_T = S_{12}, \quad \Omega_{SS} = \vec{\sigma}_1 \cdot \vec{\sigma}_2.
$$

(4.4)

The potential functions $v_\alpha^\pm(r)$ are then expressed as integrals over Yukawa functions \[13\]:

$$
v_\alpha^\pm(r) = -\frac{1}{4\pi^2 r} \int_{4m_\pi^2}^\infty dt' \tilde{\rho}_\alpha(t') R_\alpha(r \sqrt{t'}) e^{-r\sqrt{t'}}.
$$

(4.5)

Here the weight functions $R_\alpha(r \sqrt{t'})$ are defined as

$$
R_C(x) = 1, \quad R_{LS}(x) = -\frac{t'}{x}(1 + \frac{1}{x}),
$$

$$
R_T(x) = t'(1 + \frac{3}{x} + \frac{3}{x^2}), \quad R_{SS} = -t'.
$$

(4.6)

The spectral functions $\tilde{\rho}_\alpha$ here are formed as linear combinations of the spectral functions $\rho_\pm^j$ (2.11). The explicit expressions for these linear combinations are given in ref. \[13\].

### 4.2 Numerical results for the potential components

The calculated components of the one and two–pion exchange interactions of the interaction between constituent quarks are shown in Figs.2–9 and are also listed in Table 1. The potential components are given separately for the contribution of the two–pion exchange loop amplitudes in Fig.2 and as obtained after the interaction between the exchanged pions in the $S$– and $P$–waves have been taken into account. The calculated values at short range are very sensitive to the choice of the value for the cut–off parameter $\Lambda$, which here has been taken to equal the nucleon mass, and therefore these values should be given no more than qualitative value. The sensitivity to this cut–off is illustrated in Figs.2–9 by the curves marked “800”, which show the results for the two-pion exchange interaction components when the value of $\Lambda$ is reduced from $m_N = 939$ MeV to 800 MeV. Several of the
potential components change notably at very short range by that reduction, and therefore the uncertainty range of those potential components for quark separations less than \( r \approx 0.2 \text{ fm} \) is large.

The general features of the calculated two–pion exchange interaction are reminiscent of the corresponding interaction for nucleons, although overall the interaction components are weaker in the present case. This relative weakness is mainly due to the smallness of the effective pion–quark coupling constant. The isospin dependent two–pion exchange tensor and spin–spin potentials are however much stronger than the corresponding one–pion exchange interactions in the range of relevance for baryon structure: \( 0.1 \leq r \leq 0.6 \text{ fm} \). Therefore any realistic meson exchange model for the hyperfine interaction between constituent quarks has to include the two–pion exchange interaction. The effect of the two–pion exchange interaction is to in effect cancel out the one–pion exchange tensor interaction in the relevant range, and to strongly enhance the effect of the one–pion exchange spin–spin interaction.

5. Phenomenological considerations

5.1 The components of the interaction

It is instructive to consider the two–pion exchange interaction between constituent quarks calculated here together with the other components of the interaction between constituent quarks that are required for a satisfactory description of the baryon spectrum. To these belong the confining interaction, which is the source of the unbounded discrete spectrum, and the one–pion exchange interaction, which appears naturally as an iteration of the instanton induced interaction in the \( t \)-channel. Finally there presumably remains a weak screened gluon exchange interaction. We propose that the conceptually simplest phenomenologically acceptable model for the baryon spectrum is obtained as a combination of a linear scalar confining interaction along with the one– and two–pion exchange interactions and complemented by an omega exchange interaction, which represents the most important component of the three–pion exchange interaction.

There are several reasons to believe that the confining interaction should have the form of a scalar flavor independent interaction, with leading linear
component \([27, 28, 29]\). That interaction would then have the central and spin–orbit components

\[
v_{c, \text{conf}}(r) = cr, \quad v_{LS, \text{conf}} = -\frac{c}{2m^2r},
\]

where \(c\) is the string tension, which for quarks has the magnitude \(\sim 500\) MeV/fm. While all realistic dynamical quark models contain a central linear confining interaction of this form, the spin–orbit interaction associated with the linear confining interaction has to be cancelled by another interaction component in view of the smallness of the spin–orbit splittings of the baryon multiplets in the \(P\)–shell \([30]\).

The spin–orbit components of the two–pion exchange interaction provide such a cancelling mechanism for the spin–orbit interaction in (5.1). Note first that it is only the spin–orbit interaction in quark pairs with antisymmetric flavor symmetry (or isospin 0), which plays a role in the case of the \(P\)–shell baryons \([7, 31]\). This implies that the effective spin–orbit interaction in the \(P\)–shell is the combination \(v_{LS}^+(r) - 3v_{LS}^-(r)\). Since the spin–orbit component \(v_{LS}^-(r)\) of the two–pion exchange interaction is much stronger than the component \(v_{LS}^+(r)\) in the case of the two–pion exchange interaction (Figs.4, 5), the combination \(v_{LS}^+(r) - 3v_{LS}^-(r)\) is positive. While its magnitude is very sensitive to the model for the \(\pi\pi\) interaction in the \(S\)– and \(P\)–waves, it is – as shown in Fig.10 – in any case much larger than that of \(v_{LS, \text{conf}}^+\), for quark separations in the relevant range, and therefore more than cancels out the effect of the spin–orbit interaction that is associated with the confining interaction (5.1). To show the parameter sensitivity of the combination of spin–orbit interactions the result as obtained with a 20 % weaker \(\rho\)–meson–quark coupling constant is also shown. The positive net spin–orbit potential in quark pair states with symmetric spin and antisymmetric flavor symmetry implies spin–orbit splittings in the empirically indicated direction in \(P\)–shell.

The spin-independent central component of the two–pion exchange interaction is attractive in flavor antisymmetric quark pair states and weak and repulsive in flavor symmetric states. These interaction components should be considered in combination with the spin–spin interaction potentials, which are much larger in magnitude. The isospin independent and dependent spin–spin interactions combine to a moderately strong repulsive interaction in spin 1 and isospin 1 quark pair states. That interaction adds to the net repulsion from the central interaction. In spin 0 and isospin 0 quark pair states
the net interaction is in contrast strongly attractive. This spin and flavor
dependence of the interaction suggests that the key component of the meson
exchange hyperfine interaction is indeed a flavor–spin dependent interaction
of the form \(-\vec{\tau}^1 \cdot \vec{\tau}^2 \sigma^1 \cdot \sigma^2\). That is exactly the operator form required for
the explanation of the empirical reversal of normal ordering in the baryon spectrum \cite{[7]}.

The isospin dependent tensor component of the two–pion exchange inter-
action (Fig.7) serves to in effect cancel out the corresponding one–pion ex-
change tensor interaction in the relevant range between 0.2 and 0.6 fm. This
is a phenomenologically desirable feature, as (a) the one–pion exchange ten-
sor interaction by itself would give rise to spin–orbit splittings of the \(P\)–shell
baryon multiplets, which – while small – typically go in the wrong direction
and (b) as the extant evidence on tensor interaction induced deformation of
the \(\Delta_{33}\) resonance suggests that to be very small \cite{[12]}.

The situation concerning the isospin dependent spin–spin interactio
is exactly the opposite: that component of the two–pion exchange interaction
strongly enhances the corresponding component of the one–pion exchange
interaction, and explains the origin of the required structure of the hyperfine
interaction.

5.2 Two-pion exchange and \(\rho\)–meson exchange

The enhancement of the isospin–dependent spin–spin component of the
one–pion exchange interaction and the cancellation of much of the tensor
component is well known in nuclear physics, where nucleons are the effective
degrees of freedom. It is instructive to outline the reasons for this enhance-
ment and cancellation in nonrelativistic notation, which is generally sufficient
for the nuclear case. The mechanisms are the same as those operating be-
tween constituent quarks.

The nuclear one–pion exchange interaction, that corresponds to \(K_\pi\) in eq.
(2.3), is

\[
V_\pi(\vec{k}) = -\frac{f^2}{m^2_\pi}(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k}) \frac{k^2}{k^2 + m^2_\pi} \tag{5.2}
\]

where retardation has been neglected. The interaction \(V_\pi(\vec{k})\) can be separated
into spin–spin and tensor components as:

\[ V_\pi(\vec{k}) = -\frac{1}{3} f_\pi^2 (\vec{\tau}_1 \cdot \vec{\tau}_2) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left\{ 1 - \frac{m_\pi^2}{k^2 + m_\pi^2} \right\} \]

\[ -\frac{f_\pi^2}{m_\pi^2} (\vec{\tau}_1 \cdot \vec{\tau}_2) \left\{ \frac{\vec{\sigma}_1 \cdot \vec{k} \vec{\sigma}_2 \cdot \vec{k} - \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 k^2}{k^2 + m_\pi^2} \right\}, \quad (5.3) \]

where we have rewritten \( k^2/(k^2 + m_\pi^2) \) as \( 1 - m_\pi^2/(k^2 + m_\pi^2) \) in the first term.

The term with the 1 in the curly brackets upon transformation to configuration space represents a zero–range interaction which will be strongly modified in the presence of short–range repulsive interaction resulting, e.g., from vector–meson exchange, which keep the interacting particles apart.

The spin– and isospin–dependent part of the two–pion exchange interaction consists, as noted above, of a strongly correlated system of pions in a relative \( P \)–state, essentially a \( \rho \)–meson of distributed mass. The form of such an interaction between nucleons is obtained from the transverse coupling of the \( \rho \)–meson to the nucleon:

\[ L_\rho = f_\rho \bar{\psi}(x) (\vec{\sigma} \times \nabla) \cdot \vec{\rho} \cdot \vec{\tau}\psi(x). \quad (5.4) \]

The nucleon–nucleon – or quark–quark – interactions resulting from \( \rho \)–exchange with this tensor coupling is

\[ V_\rho(\vec{k}) = -\frac{f_\rho^2}{m_\rho^2} (\vec{\tau}_1 \cdot \vec{\tau}_2) \left( \vec{\sigma}_1 \times \vec{k} \right) \cdot \left( \vec{\sigma}_2 \times \vec{k} \right). \quad (5.5) \]

Rearrangement into spin–spin and tensor components yields

\[ V_\rho(\vec{k}) = -\frac{2}{3} \frac{f_\rho^2}{m_\rho^2} (\vec{\tau}_1 \cdot \vec{\tau}_2) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left\{ 1 - \frac{m_\rho^2}{k^2 + m_\rho^2} \right\} \]

\[ +\frac{f_\rho^2}{m_\rho^2} (\vec{\tau}_1 \cdot \vec{\tau}_2) \left\{ \frac{(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k}) - \frac{1}{3} (\vec{\sigma}_1 \cdot \vec{\sigma}_2) k^2}{k^2 + m_\rho^2} \right\}. \quad (5.6) \]

Comparison of eq.(5.6) with eq.(5.3) reveals that the spin–spin terms from \( \pi \)– and \( \rho \)– exchange have the same sign, whereas the tensor terms have opposite signs. This argument concerning the relative signs carries over directly to the case of the quark-quark interaction.
Although the $\rho$–meson couples transversely to the spin in eq. (5.4), modification of the zero–range part of the interaction to take into account short–range interactions, as discussed following eq. (5.3), introduces an effective longitudinal coupling which turns out to be important in pionic excitations in nuclei.

**5.3 $\omega$ meson and gluon exchange**

The isospin independent tensor component of the two–pion exchange interaction (Fig.6) is strong and attractive. Since the baryon spectrum reveals no phenomenological indication for such an interaction, this interaction component has to be cancelled by a tensor potential of opposite sign from another mechanism. In view of the significance of the $\rho$–meson resonance component of the two–pion exchange interaction it is natural to invoke an $\omega$ meson exchange mechanism between quarks to bring about this cancellation, as that has about the same range and strength as the $\rho$–meson exchange interaction.

The $\omega$ meson exchange mechanism forms the most significant component of the three–pion exchange interaction. This is described by the following potential components:

$$v^+_{C,\omega}(r) = m_\omega \frac{g^{2}_{\omega qq}}{4\pi} \frac{e^{-m_\omega r}}{m_\omega r},$$

$$v^+_{LS,\omega}(r) = -\frac{3m_\omega}{4\pi} \frac{g^{2}_{\omega qq}(m_\omega)}{m_q} \frac{e^{-m_\omega r}}{m_\omega r},$$

$$v^+_{T,\omega}(r) = -\frac{m_\omega}{12} \frac{g^{2}_{\omega qq}(m_\omega)}{4\pi} \frac{e^{-m_\omega r}}{m_\omega r},$$

$$v^{SS}_{\omega}(r) = \frac{m_\omega}{6} \frac{g^{2}_{\omega qq}(m_\omega)}{4\pi} \frac{e^{-m_\omega r}}{m_\omega r}. \{ \frac{4\pi}{m_\omega^3} \delta^{(3)}(r) \}. \quad (5.7)$$

Here $m_\omega$ is the $\omega$–meson mass (783 MeV) and $g_{\omega qq}$ is the $\omega$–quark vector coupling constant. In the presence of a monopole form factor for the $\omega - N$ interaction, the terms on the r.h.s. should be amended by corresponding terms with opposite sign and with $m_\omega$ replaced by the factor mass scale $\Lambda$.

The quark model relation between the $\omega$–quark and $\omega$–nucleon coupling constants is $g_{\omega qq} = g_{\omega NN}/3$ in the case of the spin–independent part of the
coupling and \( g_{\omega qq} = \left( m_q/m_N \right) g_{\omega NN} \) in the case of the spin–dependent part of the coupling. As \( m_q/m_N \) is close to 1/3 we shall use the former relation here. With \( g_{\omega NN}^2/4\pi \simeq 20 \) \(^{[33]}\) we then have \( g_{\omega qq} = 5.3 \).

The components of the \( \omega \)–meson exchange interaction between constituent quarks are shown in Fig. 11, as obtained with \( g_{\omega qq} = 5.3 \) and with the cut off mass scale of \( \Lambda = m_N \) also used in the calculation of the two–pion exchange potential above. The \( \omega \) exchange potential has the same functional form as a screened gluon exchange interaction, but with opposite overall sign. This makes it difficult to separate phenomenologically the effects of omega and gluon exchange on the baryon spectrum, as a stronger \( \omega \)–quark coupling may be compensated by a correspondingly stronger effective quark–gluon coupling. The expression for a screened one-gluon exchange interaction may be obtained from the expressions (5.2) for the omega exchange interaction by the replacements

\[
\frac{g_{\omega qq}^2}{4\pi} \to -\frac{2}{3}\alpha_S, \quad m_{\omega} \to m_G.
\]

Here \( \alpha_S \) is the effective color hyperfine constant, and \( m_G \) is a screening mass for the gluon exchange interaction. The screening mass should fall somewhere between \( \Lambda_{QCD} \sim 250 \text{ MeV} \) and \( \Lambda_\chi^{[10]} \).

The \( \omega \)–exchange potential components shown in Fig. 11 should be combined with the isospin independent two–pion exchange interaction components in Figs. 2–9. The central component of the \( \omega \)–exchange interaction cancels most of the corresponding component of the two–pion exchange interaction (Fig. 2). This suggests that the only significant spin– and isospin–independent interaction between two constituent quarks is the central component of the confining interaction, as shown in Fig. 10, where the combined meson exchange and confinement contributions to the isospin independent central interaction are shown.

The spin–orbit component of the \( \omega \)–exchange interaction adds to that generated by two–pion exchange (Fig. 4), but it brings no qualitative change. That interaction, while strong, is overwhelmed by the contribution of the isospin dependent two–pion exchange spin–orbit interaction in the flavor antisymmetric quark pair states, which determine the spin–orbit splitting of the \( P \)–shell baryon resonances. This is shown in Fig. 10, where the net spin–orbit interaction for such pair states is shown. Because the net spin–orbit interactions in the \( P \)–shell is weighted by a factor \( r^2 \) in the matrix elements,
there is a strong cancellation between the long range spin-orbit interaction associated with the confining interaction combined with that of \( \omega \)-exchange and that due to two-pion exchange.

The \( \omega \)-exchange tensor interaction adds to the isospin independent part of the two-pion exchange tensor interaction (Fig. 6), but it is weaker in magnitude. As the net isospin dependent tensor interaction is very weak (Fig. 7), the isospin independent tensor interaction is the main tensor component of the mesonic part of the hyperfine interaction between quarks. This tensor component has opposite sign to that of one-gluon exchange or one-pion exchange in isospin symmetric quark pair states, but it is comparable in magnitude, and has the same sign as the one-pion exchange tensor interaction in isospin antisymmetric quark pair states. The strong isospin dependent tensor interaction, which is associated with irreducible \( \pi \)-gluon exchange \([10]\), even in the case of a weak screened quark–gluon coupling, would however cancel out most of this tensor interaction in the flavor antisymmetric quark pair states, which determine the spin–orbit splitting of the \( P \)-shell baryon multiplets.

Finally the spin–spin component of the \( \omega \) exchange interaction has the opposite sign to the corresponding component of the two-pion exchange interaction (Fig. 8). The role of \( \omega \) exchange is therefore to reduce the strength of the isospin independent spin–spin interaction. This supports the role of the isospin dependent one- and two-pion exchange spin–spin potential as the dominant source of hyperfine splitting between constituent quarks.

6. Discussion

The instanton liquid model of the QCD vacuum, which appears to be supported by comprehensive lattice calculations \([4, 5]\), implies pointlike interactions between quarks. To obtain a realistic description of the hyperfine interaction between constituent quarks, that interaction has to be iterated (at least once) in the \( t \)-channel, to overcome the restriction to flavor–antisymmetric quark pairs. This is a necessary requirement, as the hyperfine interaction has to have about the same strength in the (completely) flavor symmetric \( \Delta \)-spectrum as in the nucleon spectrum. The \( t \)-channel iteration of the pointlike instanton induced interaction has an obvious meson exchange interpretation. In the pseudoscalar channel the iteration corresponds
to Goldstone boson exchange with long range – i.e. the pion exchange interaction. Once pion exchange is operative, with a relatively strong coupling strength, it follows that two-pion exchange also plays a significant role, as indeed found in the present calculation.

Phenomenological analysis of the baryon spectrum reveals that it is simplest to describe in the constituent quark model with a central linear confining interaction in combination with an attractive flavor-spin hyperfine interaction \[^7, 34\]. The present results show that the latter has a strong two-pion exchange contribution at scales commensurate with the range of the baryon wave functions, if its longest range term is due to one-pion pion exchange. The isospin dependent tensor components of the one- and two-pion exchange interactions largely cancel, which is a phenomenologically desirable feature. Moreover the combination of the spin-orbit components of the two-pion exchange interactions that appears in the \(P\)-shell multiplets is strong and overwhelms the long range spin-orbit interaction that would be associated with a linear scalar confining interaction. These qualitative conclusions are not hinged on the strength of the interaction between the exchanged pions in the \(S\)- wave – i.e. whether it is resonant or the strength of the coupling of that resonance to quarks.

The meson exchange description of the hyperfine interaction between constituent quarks has an obvious similarity to the meson exchange description of the nucleon–nucleon interaction. The similarity is not complete, however, as in the case of constituent quarks, the coupling to mesons should disappear beyond the chiral restoration scale \(\Lambda_{\chi}\). This implies that the meson exchange interaction should vanish at ranges shorter than \(\Lambda_{\chi}^{-1}\). Consequently only few-pion exchange mechanisms need to be invoked. Here we have considered one- and two-pion exchange and have approximated the main three-pion exchange interaction by \(\omega\)-meson exchange. Uncorrelated three-pion exchange should be weak, and may in principle be calculated by the methods in ref.[35].

Another notable difference between the two-pion exchange interactions between nucleons and quarks follows from the absence of excited states of quarks. The two-pion exchange interaction between quarks therefore has no analogue to the strongly attractive terms that arises from loop diagrams with intermediate nucleon resonances – first and foremost the \(\Delta_{33}\) – which provide the bulk of the attraction between nucleons.

In the present calculation of the two-pion exchange interaction the con-
constituent quarks have been treated as Dirac particles, with constant mass. As the effect of confinement is not taken into account in the quark propagators, the results should be viewed as qualitative and suggestive rather than quantitative. The confining interaction may in principle be taken into account by treating the constituent mass as a running mass, which grows linearly with separation of the quarks. At the length scales relevant for the structure of the baryons $\sim 0.2$–$0.6$ fm, the treatment of the constituent quark mass as an average constant should not be expected as very unrealistic, as all quark model descriptions of the baryon spectrum are based on that approximation.

Acknowledgments

We are indebted to Professor R. McKeown and W. Haxton for hospitality at the W. K. Kellogg Radiation Laboratory at the California Institute for Technology and the Institute for Nuclear Theory at the University of Washington respectively for hospitality during the completion of this work. DOR thanks Dr. L. Ya. Glozman for instructive correspondence. Research supported in part by the Academy of Finland by grants No. 34081, 43982 and the U. S. Department of Energy under grant DE-FG02-88ER40388.
References

[1] E. Shuryak, Rev. Mod. Phys. 65 (1993) 1

[2] M.A. Nowak, M. Rho and I. Zahed, Chiral Nuclear Dynamics, World Scientific (1996)

[3] L. Ya. Glozman and K. Varga, eprint hep-ph/9901439

[4] M. C. Chu et al., Phys. Rev. D49 (1994) 6039

[5] J. W. Negele, eprint hep-lat/9810053

[6] K. F. Liu et al., LANL eprint hep-ph/9806491

[7] L.Ya. Glozman and D.O. Riska, Phys. Rept. 268 (1996) 263

[8] L. Ya. Glozman et al., Phys. Rev. C57 (1998) 3406

[9] L. Ya. Glozman et al, “Quark Model with Vector Meson Exchanges”, Preprint, November (1998)

[10] C. Helminen and D.O. Riska, Phys. Rev. C58 (1998) 2928

[11] S. Capstick and N. Isgur, Phys. Rev. D34 (1986) 2809

[12] M. H. Partovi and E. L. Lomon, Phys. Rev. D2 (1970) 1999

[13] M. Chemtob, J. W. Durso and D. O. Riska, Nucl. Phys. B38 (1972) 141

[14] G. E. Brown and A. D. Jackson, The Nucleon–Nucleon Interaction, North–Holland Publ. Co., Amsterdam (1976)

[15] A. A. Logunov and V. Tavkhelidze, Nuovo Cimento 29 (1963) 380

[16] R. Blankenbecler and R. Sugar, Phys. Rev. 142 (1996) 1051

[17] G.E. Brown and J.W. Durso, Phys. Lett. B35B (1971) 120

[18] S. Weinberg, Phys. Rev. Lett. 65 (1990) 1181

[19] D. A. Dicus et al., Phys. Lett. B284 (1992) 384
[20] D. Amati, E. Leader and B. Vitale, Nuovo Cimento 17 (1960) 68

[21] W. Frazer and J.R. Fulco, Phys. Rev. 117 (1960) 1603

[22] H. Nielsen and G.C. Oades, Nucl. Phys. B49 (1972) 586

[23] M. Roos and N. Törnqvist, Phys. Rev. Lett. 76 (1996) 1575

[24] N. Isgur and J. Speth, Phys. Rev. Lett. 77 (1996) 2332

[25] M. Harada, F. Sannino and J. Schechter, Phys. Rev. Lett. 78 (1997) 1603

[26] G. Höhler and E. Pietarinen, Nucl. Phys. B95 (1975) 216

[27] D. Gromes, Phys. Lett. B202 (1988) 262

[28] G.S. Bali, K. Schilling and A. Wachter, Phys. Rev. D56 (1997) 2566

[29] T.A. Lähde, C. Nyfält and D.O. Riska, eprint hep–ph/9808438

[30] J. Carlson, J. Kogut and V.R. Pandharipande, Phys. Rev. D28 (1983) 2807

[31] F. Coester and D.O. Riska, eprint hep–ph/9707388

[32] C. Papanicolas, Acta Phys. Polonica B29 (1998) 2437

[33] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rept. 149 (1987) 1

[34] H. Collins and H. Georgi, Phys. Rev. D59 (1999) 094010

[35] J. Hamilton, Nordita preprint 81/45 (1981)
Table 1

| $r$ (fm) | $v^+_C$ | $v^-_C$ | $v^+_L$ | $v^-_L$ | $v^+_T$ | $v^-_T$ | $v^+_S$ | $v^-_S$ |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.1      | -420     | 1800     | -88700   | -64300   | -24800   | 3930     | 7380     | -4180    |
| 0.2      | -201     | 655      | -7270    | -9360    | -2100    | -80.6    | 1060     | -458     |
| 0.3      | -108     | 317      | -1477    | -2740    | -422     | -122     | 276      | -68.7    |
| 0.4      | -61.4    | 172      | -439     | -1050    | -121     | -72.4    | 92.7     | 0.14     |
| 0.5      | -36.6    | 99.9     | -161     | -465     | -41.9    | -40.9    | 35.8     | 11.7     |
| 0.6      | -22.6    | 59.1     | -68.4    | -224     | -16.6    | -23.3    | 15.2     | 11.1     |
| 0.7      | -14.4    | 36.2     | -32.0    | -115     | -7.25    | -13.6    | 6.96     | 8.42     |
| 0.8      | -9.4     | 22.6     | -16.2    | -61.4    | -3.41    | -8.07    | 3.38     | 5.90     |

Table 1. The components (in MeV) of the two–pion exchange interaction between quarks as defined in (4.3). The numerical values correspond to the case, when both the $S$– and $P$–wave interactions between the exchanged pions have been taken into account.

Figure Captions

Fig. 1 Two–pion exchange loop amplitudes. The fermion lines represent $u$ and $d$ quarks.

Fig. 2 Isospin independent central interaction $v^+_C$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1, the curve “S-wave interaction” is the result obtained after the interaction in the $S$–state of the $\pi\pi$ system is taken into account. The curve “800” shows the latter result when the cut–of mass is taken to be 800 MeV instead as $m_N$.

Fig. 3 Isospin dependent central interaction $v^-_C$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1, the curve “P-wave interaction” is the result obtained after the interaction in the $P$–state of the $\pi\pi$ system is taken into account. The curve “800” shows the latter result when the cut–of mass is taken to be 800 MeV instead as $m_N$.

Fig. 4 Isospin independent spin–orbit interaction $v^+_L$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1, the
curve “S-wave interaction” is the result obtained after the interaction in the $S-$state of the $\pi\pi$ system is taken into account. The curve “800” shows the latter result when the cut–of mass is taken to be 800 MeV instead as $m_N$.

Fig. 5 Isospin dependent spin–orbit interaction $v_{LS}^-$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1, the curve “P-wave interaction” is the result obtained after the interaction in the $P-$state of the $\pi\pi$ system is taken into account. The curve “800” shows the latter result when the cut–of mass is taken to be 800 MeV instead as $m_N$.

Fig. 6 Isospin independent tensor interaction $v_T^+$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1 and the curve “800” shows the result when the cut–of mass is taken to be 800 MeV instead as $m_N$.

Fig. 7 Isospin dependent tensor interaction $v_T^-$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1, the curve “P-wave interaction” is the result obtained after the interaction in the $P-$state of the $\pi\pi$ system is taken into account. The curve “800” shows the latter result when the cut–of mass is taken to be 800 MeV instead as $m_N$ and the curve “OPEP” is the one-pion exchange component.

Fig. 8 Isospin independent spin–spin interaction $v_{SS}^t$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1 and the curve “800” shows the result when the cut–of mass is taken to be 800 MeV instead as $m_N$.

Fig. 9 Isospin dependent spin–spin interaction $v_{SS}^-$. The curve “box” is the result obtained from the two-pion exchange loop diagrams in Fig.1, the curve “P-wave interaction” is the result obtained after the interaction in the $P-$state of the $\pi\pi$ system is taken into account. The curve “800” shows the latter result when the cut–of mass is taken to be 800 MeV instead as $m_N$ and the curve “OPEP” is the one-pion exchange component.

Fig. 10 The contributions to the spin–orbit interaction for quark pairs with antisymmetric flavor symmetry, which is the active part of the spin–orbit interaction in the $P-$shell baryons. The curve TWOPI1 represents the combination $v_{LS}^t - 3v_{LS}^- \omega$ of the two isospin components of the two–pion exchange interaction while the curve TWOPI2 gives the corresponding results when the $\rho-$meson-quark coupling has been reduced by 20 %. The curve CONF is the spin–orbit component of the confining interaction, and the curve OMEGA is the spin-orbit component of the $\omega-$meson exchange interaction.

Fig. 11 The components of the $\omega-$meson exchange interactions between
constituent quarks. The curves C, LS, T and SS represent $v_{C,\omega}^+$, $v_{LS,\omega}^+$, $v_{T,\omega}^+$ and $v_{SS,\omega}$ respectively.
