Oscillation Frequency Dependence of Non-Classical Rotation Inertia of Solid $^4$He

Y. Aoki, J. C. Graves, and H. Kojima
Serin Physics Laboratory, Rutgers University, Piscataway, NJ 08854 USA

The non-classical rotational inertia fraction of the identical cylindrical solid $^4$He below 300 mK is studied at 496 and 1173 Hz by a double resonance torsional oscillator. Below 35 mK, the fractions are the same at sufficiently low rim velocities. Above 35 mK, the fraction is greater for the higher than the lower mode. The dissipation peak of the lower mode occurs at a temperature $\sim$ 4 mK lower than that of the higher mode. The drive dependence of the two modes shows that the reduction of the fraction is characterized by critical velocity, not amplitude nor acceleration.

PACS numbers: 67.80.-s

Ever since the observation of "Non-Classical Rotation Inertia" (NCRI) of solid $^4$He contained in their torsional oscillators by Kim and Chan [2,3], apparent superfluidity in solid $^4$He has motivated numerous experimental and theoretical studies. A clear understanding of the observed apparent partial decoupling between solid $^4$He and the container walls of torsional oscillators is lacking at present. A number of interpretations of the observed decoupling has been introduced including glassy state [4], vortex liquid state [5], superglass [6], and a phenomenological two component model [7] (see [8] for a recent review). Many of these theoretical ideas contain predictions on frequency dependence of the magnitude of NCRI phenomenon depending on the history of oscillation velocity and thermal processing from the normal state above 300 mK to low temperature. Our observations put severe constraints on some of the theoretical notions on the interpretation of NCRI.

The conventional torsional oscillator having one resonant frequency is made of one solid $^4$He container attached to a single torsion rod. Our torsional oscillator as shown in Fig. 1 contains two masses, the upper "dummy" without solid $^4$He and the lower containing sample $^4$He, and two torsion rods. The torsion rods, a portion of the dummy mass and the lower flange fitted to the sample container are all machined from a single stock of BeCu rod. The first resonant mode (frequency $f_1 = 495.8$ Hz) occurs when the two masses move in phase and the second mode ($f_2 = 1172.8$ Hz) when the two masses move 180° out of phase. The phase relation of the modes are verified by measuring the oscillator response at both the upper and the lower detectors. Measurements of the oscillator response are made with the upper electrode fins. The lower electrode is used to calibrate oscillation amplitude at the sample cell against the upper detector response. Owing to the uncertainty in the capacitance between the lower fixed and movable electrodes, the absolute oscillator amplitude is accurate only to within $\pm$ 10 %. The relative accuracy in comparing the oscillator amplitudes between the two modes is $\pm$ 5 %. The temperature at the mixing chamber is measured by a calibrated $^3$He melting pressure thermometer. The temperature of the copper isolator block which supports the oscillator is monitored by a carbon resistance thermometer.

FIG. 1: Double resonance torsional oscillator. It contains two masses and BeCu torsion rods (a) (2 mm outer diameter). The upper mass includes a BeCu disc (b) with movable detector and generator electrode fins (c). The lower mass is made of a Stycast 1266 epoxy block (e) containing a hollow cylindrical solid $^4$He sample space (10.2 mm in diameter and 7.6 mm in height. A second movable detector electrode (f) attached to the bottom side of the lower mass is used to calibrate the upper electrode detector motion. $^4$He is filled through a central 0.8 mm diameter hole (d) drilled in the torsion rods and the upper mass. Fixed electrodes placed facing the movable electrodes are not shown.

The "background" characteristics [9] of the resonant frequencies, $f_0 \ (i = 1, 2)$, and the quality factors $Q_{ii}$ of our torsional oscillator with the sample cell empty were
measured as functions of temperature and drive level. The $Q_{0i}$ ($1.4 \times 10^6$ and $0.63 \times 10^6$ for $i = 1$ and 2, respectively, at 30 mK) was determined from the ring down time. The oscillation amplitude was calibrated against the measured $Q_{0i}$ by the ring down time. The temperature dependence of the resonant frequency and the quality factor of the empty cell were both slightly dependent on the drive level but qualitatively similar to those observed by others. After the sample cell is loaded with solid $^4$He at 37 bar, for example, the resonant frequencies measured at 300 mK decreased from the empty cell values by $\Delta f_1 = 0.667$ Hz and $\Delta f_2 = 2.049$ Hz for the first and second mode, respectively.

Our sample solid $^4$He is grown from commercially available helium gas with nominal $^3$He concentration of 0.5 ppm by the blocked capillary method. The cell is loaded near 3 K to a desired pressure. The part of the fill line which is thermally anchored to the 1 K pot is quickly plugged with solid while $^4$He in the cell attached to the mixing chamber remains in the liquid state. The liquid pressures and temperatures, when the plug is formed and when the $^4$He cools down to the liquid/solid coexistence point, are monitored by a pressure sensor at the mixing chamber. The change in pressure within the cell itself is sensed from the oscillation amplitude of one of the modes during solidification. After the plug is formed, the total time elapsed to freeze $^4$He in the cell was varied from several minutes to four hours.

When our torsional oscillator with the sample cell filled with solid $^4$He is cooled, the two resonant frequencies ($f_i(T)$) increase below 300 mK and reach maxima near 35 mK. This increase in frequency has been interpreted to arise from a partial decoupling of solid $^4$He from the sample cell wall and has been called NCRI [1, 10]. The NCRI fraction (NCRIf) is evaluated from our measurements as $[f_i(T) - f_{0i}(T) - \Delta f_i]/\Delta f_i$. The temperature dependent changes in resonant frequencies are accompanied by decrease in quality factor. A measure of dissipation induced by the presence of solid $^4$He is evaluated as $\delta [1/Q_i(T)] = 1/Q_i(T) - 1/Q_{0i}(T)$. The data of resonant frequency and quality factor reported here are obtained after stabilizing the temperature and waiting for transient effects to diminish to negligible levels.

In all, we have grown and studied five solid samples with final solid pressures between 27 and 42 bar. The observed features were qualitatively similar in all solid samples. Among the samples we studied, we present results, as an example, of a 37 bar sample solidified over four hours. The temperature of this sample was maintained below 310 mK throughout all of the measurements reported here after the initial cool down so as to avoid any changes in the properties of our sample by some annealing process. The measured NCRI and the dissipation while keeping the rim velocity sufficiently low to avoid critical velocity effects (see below) are shown in Fig. 2

It should first be noted that the maximum NCRIf observed is only 0.1 %, which is considerably smaller than those found generally in annular cells [3, 11]. (The maximum NCRIf in another sample solidified deliberately rapidly over several minutes was larger at 0.12 %.) On the other hand, the maximum is not so different from some of those found in cylindrical cells in other laboratories [12, 13, 14]. At $T < 35$ mK, NCRIF is independent of both temperature and frequency. The dissipation is small in the same temperature range. In the higher temperature range $T > 35$ mK, both the NCRIF and the dissipation show significant dependence on frequency. The NCRIF measured with the first mode is lower than that with the second mode where $T > 35$ mK. At $T = 100$ mK, $\text{NCRIF}(f_2)/\text{NCRIF}(f_1) = 1.9$ and this ratio continues to increase at higher temperatures. The observed magnitude of $\delta Q^{-1}_1$ is also smaller than in other annular cells. In the temperature range, $35 \text{ mK} < T < 100 \text{ mK}$, $\delta Q^{-1}_1$ is greater than $\delta Q^{-1}_2$ contrary to what might be expected from the relative difference of NCRIF measured with the two modes. The dissipation maximum value of the first mode is greater than that of the second by a factor of 1.7. The temperature where the dissipation maximum occurs is slightly ($\sim 4 \text{ mK}$) lower in the first mode than the second. The temperature “width” of the dissipation peak is narrower in the first mode.

All previous torsional oscillator experiments have shown that the NCRIf decreases as the drive level is increased and the decrease has been ascribed as a “critical velocity” effect. It has not been clarified if the reduction in NCRIf was controlled instead by the amplitude of

![FIG. 2: Temperature dependence of non-classical rotational inertia fraction and change of dissipation (see text) at $f_1$ (○) and $f_2$ (△). Pressure of the solid is 37 bar. Rim velocities are $8.5 \sim 13.5 \mu m/\text{sec at } f_1$, and $7.2 \sim 16.0 \mu m/\text{sec at } f_2$.](image-url)
displacement or acceleration. To investigate the critical velocity effect, the oscillator is cooled down to 19 mK in the first mode with a low velocity near 10 μm/s. When the drive level is increased, to our surprise, the NCRIf does not diminish but remains constant up to 600 μm/s as shown by circles in Fig. 3. It is conceivable that the NCRIf decays if we wait long enough, but the time constant for this is estimated (assuming exponential decay for this purpose) to be more than 10^2 hours. Following the advice by A. Clark and M. Chan [12], the torsional oscillation is initiated in the first mode with a relatively high drive level at 300 mK. While keeping the drive level constant, the oscillator is then cooled down to 19 mK, where the rim velocity becomes 610 μm/s (shown by crosses). In this process, as the drive level is decreased, the NCRIf increases and eventually attains the low velocity limit consistent with Fig. 2. This increase in NCRIf as the rim velocity is decreased is similar to those velocity effects as observed by others [2, 3, 13, 15]. If the oscillation is initiated at any other rim velocity < 610 μm/s at 300 mK and cooled down to 19 mK, the observed NCRIf is that shown by the cross in Fig. 3 at the corresponding rim velocity. When similar procedures are followed with the second mode, the same NCRIf is obtained when plotted as a function of rim velocity (shown by triangles and diamonds). Cooling the oscillator at a low or high rim velocity is analogous to "zero field cooling" or "field cooling" process, as suggested to us by A. Clark and M. Chan [12], in the studies of superconducting and magnetic materials.

The reduction in NCRIf as the drive level is increased is observed to be the same for both modes when it is plotted against the cell rim velocity. If NCRIf is plotted against displacement amplitude or acceleration, the reduction does not coincide in the two modes. This observation demonstrates convincingly that it is the rim velocity, not displacement amplitude (strain) nor acceleration (stress) applied to the solid, that best parameterize the reduction in NCRIf. The observed history dependence of NCRIf on the initial state set by the rim velocity at low temperature is an important characteristics of the supersolid state.

Our measurements of critical velocity effects at 63 mK are shown in Fig. 4. The circles (crosses) and diamonds (triangles) show NCRIf when the rim velocity is increased (decreased) for the first and second mode, respectively. The NCRIf decreases as the rim velocity is increased. When the difference at this temperature at low velocity (see Fig. 2) is added to the NCRIf(\( f_1 \)), the reduction is best parameterized by the rim velocity just as at 19 mK. The saturation of the NCRIf below 15 μm/s as seen in Fig. 3 is absent. There is no history dependence at 63 mK! The measured NCRIf does not depend on how the initial state is reached. The border between the history dependent low temperature behavior and the reversible higher temperature behavior appears to be around 40 mK. This is close to the temperature where the NCRIf begins to decrease.

Anderson interprets the observed NCRI as a consequence of vortex liquid and supercurrents flowing around a thermally excited fluctuation of vortices. He suggests that there is a relaxation rate at which vortices can move in and out of the sample. The rate is thought to decrease as the temperature is lowered and as the \(^3\)He
impurity concentration is increased. $^3$He is thought to act promote pinning. When the torsional oscillator frequency matches the rate of vortex motion, a maximum in dissipation would be observed. According to the model, the dissipation peak should occur at higher temperature for higher oscillator frequency. Our data shows that the dissipation peak for the second mode occurs at a slightly higher (by 4 ± 1 mK) temperature than that of the first mode. The direction of change in the peak temperature is consistent with the Anderson model, but a quantitative comparison is not possible (see Fig. 1 of Ref. [5]). If the Anderson model is correct, the vortex flow rate is greater than 500 s$^{-1}$ at $T \geq 50$ mK. It would then be expected that the NCRI is reversible, as indeed seen in our experiment, at the time scale (of order $10^2$ s) of our measurements. At low temperatures, $T \leq 35$ mK, however, the observed relaxed rate is dependent on the history of velocity variation as illustrated in Fig. 3. The vortex motion rate is apparently dependent strongly on the existing number of vortices.

From the point of view of vortex liquid model, the irreversible increase in NCRIIf as seen in Fig. 2 from the high to low rim velocity may be interpreted as a process in which vortices can easily escape presumably from the cell surface. Once the vortices are absent, or nearly absent, they are apparently excluded from entering solid $^4$He even when the rim velocity is increased. These observations are reminiscent of Meissner effect in superconductors. A theoretical discussion of vortex nucleation in solid $^4$He has been given by Saslow [16].

According to the phenomenological two component model developed by Huse and Khandker [2], if a solid sample is homogeneous, the change (δf) in the torsional oscillator frequency in the $T = 0$ limit is related to the maximum damping at the apparent supersolid transition as $δf/f ∼ δ(1/Q)_{max}$. Evaluating the quantities from the data shown in Fig. 2 we find the ratio $(δf/f)/(δ(1/Q)_{max})$ to be 2.7 and 6.5 for the first and second mode, respectively. The ratios are comparable to those of the "best" among the samples studied by Kim and Chan [15]. One might expect that the ratio should be independent of frequency if it is simply a measure of homogeneity. The temperature width of the dissipation peak is also thought to indicate the degree of sample inhomogeneity. It is apparent in Fig. 2 that the second mode shows a broader temperature width in $δ(1/Q)$ than the first.

Nussinov et al. [4] proposed that the observed increase in frequency and the peak in dissipation of torsional oscillators containing solid $^4$He might be explained in terms of solidification process of liquid-like component into a glass at low temperature. In their model of the glass state, there is a characteristic relaxation time $s$ which increases as temperature is decreased. When the inverse relaxation time matches the oscillator frequency, a peak in the oscillator dissipation appears. The change in frequency-dependent dissipation in their model naturally leads to a monotonic increase in frequency as temperature is decreased. If $s$ is assumed to take the simple form [4], $s = s_0 e^{\Delta/k_B T}$, where $s_0$ and $\Delta$ are parameters, the measured dissipations and frequency shifts of the two modes cannot be reconciled. Other forms of $s$ might reduce discrepancy. Our observations place severe constraints on the glass model to explain the origin of observed NCRI effects of solid $^4$He.

We thank A.C. Clark and M.H.W. Chan for discussing their experimental results and the relevance to vortex motion prior to publication. We are grateful to W.M. Saslow, P.W. Anderson, W.F. Brinkman, D.A. Huse, J.D. Reppy, K. Shirahama and Z. Nussinov for stimulating discussions. Y. A. thanks the Japan Society for Promotion of Science for providing postdoctoral fellowship and JCG Aresty Foundation for support of undergraduate research. This work has been supported by the National Science Foundation through grant DMR-0456862.

[1] A. J. Leggett, Phys. Rev. Lett. 25, 1543 (1970).
[2] E. Kim and M. H. W. Chan, Nature 427, 225 (2004).
[3] E. Kim and M. H. W. Chan, Science 305, 1941 (2004).
[4] Z. Nussinov, A. V. Balatsky, M. J. Graf, and S. A. Trugman, ArXiv:cond-mat/p. 0610743 (2006).
[5] F. W. Anderson, Nat Phys 3, 160 (2007).
[6] M. Boninsegni, N. Prokof’ev, and B. Svistunov, Phys. Rev. Lett. 96, 105301 (2006).
[7] D. A. Huse and Z. U. Khandker, ArXiv:cond-mat/0702243.
[8] N. Prokof’ev, ArXiv:cond-mat/0612499.
[9] Y. Aoki, J. C. Graves, and H. Kojima, To be published.
[10] E. Kim and M. H. W. Chan, J. Low Temp. Phys. 138, 859 (2005).
[11] A. S. C. Rittner and J. D. Reppy, Phys. Rev. Lett. 97, 165301 (2006).
[12] A. C. Clark and M. H. W. Chan, Private communication.
[13] K. Shirahama, M. Kondo, S. Takada, and Y. Shibayama, in APS March Meeting 2007 (Denver, CO, 2007), abstract P31.007.
[14] A. Penzev, Y. Yasuta, and M. Kubota, ArXiv:cond-mat/0702632.
[15] E. Kim and M. H. W. Chan, Phys. Rev. Lett. 97, 115302 (2006).
[16] W. M. Saslow, Phys. Rev. B 71, 092502 (2005).