Theory status of four-fermion production at $e^-e^+$ colliders

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Abstract

The status of predictions for four-fermion production at $e^-e^+$ colliders is reviewed with an emphasis on the developments after the LEP2 era and an outlook to the challenges posed by the precision program at future colliders.

1 Introduction

After the discovery of a Higgs boson, the search for physics beyond the Standard Model (SM) of particle physics is one of the main objectives of run 2 of the LHC and of future colliders. In case new particles are not directly accessible at these colliders or in non-collider experiments, one can search for indirect evidence for new physics through precise studies of electroweak (EW) or flavour observables.

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and the couplings of the gauge bosons and the Higgs boson. Further, accurate measurements of input parameters of the SM such as the masses of the $W$ and $Z$ bosons and the top quark are required for the precision-physics program. Here future $e^-e^+$ colliders could play a particularly important role by revisiting the LEP precision measurements at higher statistics, and further measuring top-quark and Higgs-boson properties. Currently linear colliders such as ILC and CLIC as well as circular colliders such as FCC-ee or CEPS are investigated.\footnote{Four-fermion final states arising from Higgs-boson production with subsequent decay to $b$ quarks or $\tau$ leptons are not considered in this contribution.}

An important signature at high-energy $e^-e^+$ colliders is given by four-fermion production processes\footnote{Four-fermion final states arising from Higgs-boson production with subsequent decay to $b$ quarks or $\tau$ leptons are not considered in this contribution.} as shown in Figure 1. They have been explored at LEP2\footnote{Four-fermion final states arising from Higgs-boson production with subsequent decay to $b$ quarks or $\tau$ leptons are not considered in this contribution.} for centre-of-mass energies $\sqrt{s} = 161.3-206.6$ GeV, allowing precision tests of the SM through measurements of cross-sections, the mass, width and branching ratios of the $W$-boson in $W$-pair production (Fig 1 (a)), and triple-vector boson couplings in $W$-pair production, $Z\gamma$ and single-$W$ production (Fig 1 (c) and (d), respectively). At future $e^-e^+$ colliders the precision of these measurements could be increased, for instance by up to two magnitudes for the triple gauge boson couplings.\footnote{Four-fermion final states arising from Higgs-boson production with subsequent decay to $b$ quarks or $\tau$ leptons are not considered in this contribution.} For $M_W$, an accuracy of 3–4 MeV is projected for an ILC, while 1 MeV may be possible using a threshold scan of the $W$-pair production cross section at a future circular $e^-e^+$ collider.

![Figure 1: Classification of signatures in four-fermion production.](image-url)
Figure 2: Diagrams contributing at tree-level to the $e^- e^+ \rightarrow u \bar{d} \mu^- \bar{\nu}_\mu$ process.

In this contribution, the theoretical challenges and the methods used for four-fermion production are discussed in Section 2. Recent theoretical results are reviewed in Section 3 while an outlook to future developments needed to meet the requirements of planned colliders is given in Section 4.

2 Theoretical challenges and methods

In the theoretical description of four-fermion production, in general all diagrams contributing to a given final state must be taken into account for a consistent, gauge invariant result, resulting in a large number of contributing Feynman diagrams, in particular beyond leading order. These typically include topologies different from the resonant “signal” diagrams of the processes in Figure 1. For instance, as shown in Figure 2, ten tree-diagrams contribute to the final state $u \bar{d} \mu^- \bar{\nu}_\mu$, where only three diagrams include a resonant $W$-boson pair. Similarly, 20 diagrams contribute to the single-$W$ signature $u d e^- \bar{\nu}_e$.

The consistent treatment of the $W/Z$-boson decay-widths poses a further theoretical challenge. The Dyson series allows the resummation of the self-energy $\Sigma_V$ of the vector boson $V$ to all orders into the denominator of the $V$-boson propagator, $(\not{p}^2 - M_V^2 + \Sigma_V(p^2))$. The complex pole $\mu_V$ of the propagator defined by $\mu_V^2 - M_V^2 + \Sigma_V(\mu_V^2) = 0$ provides a gauge invariant definition of the mass $M_V$ and width $\Gamma_V$ of the vector bosons, $\mu_V^2 \equiv M_V^2 - iM_V\Gamma_V$.

\footnote{In some cases, gauge invariant subsets of diagrams can be identified.}
The Dyson summation of the self-energy includes only a subset of higher-order diagrams, but neglects other contributions of the same order. A naive application therefore can lead to inconsistencies such as violations of gauge invariance and unitarity, which can result in dramatically wrong predictions, in particular in the case of single-\(W\) production at high energies.\(^5\) A simple use of a Breit-Wigner propagator with a fixed width is sufficient in many leading-order applications, but does not respect electroweak gauge invariance. In the complex-mass scheme\(^6\), the replacement \(M_W^2 \rightarrow \mu_W^2\) is made in the propagator as well as in the Feynman rules, e.g. in the weak-mixing angle \(\cos \theta_w = M_W/M_Z \rightarrow \sqrt{\mu_W^2/\mu_Z^2}\). In this way, algebraic identities among vertices and propagators required by gauge invariance are satisfied also for a finite width. The fermion-loop scheme,\(^5\) applied in particular to the single-\(W\) process at LEP2,\(^7\) uses the fact that diagrams with a closed fermion loop form a gauge invariant subset of diagrams. Finally, the double-pole approximation (DPA) consistently splits the NLO corrections into factorizable corrections to on-shell vector-boson production and decay, and non-factorizable soft-photon corrections connecting vector-boson production, propagation and decay. The DPA has been applied to \(W\)- and \(Z\)-boson pair production at LEP2.\(^8\)

The methods summarized here have been used successfully to describe the LEP2 measurements of four-fermion production with a theoretical accuracy better than 1% for \(W\)-pair production and 2–5% for the other processes.\(^2\),\(^3\)

\section{Recent theoretical developments}

The high accuracy possible at future \(e^- e^+\) colliders makes it mandatory to improve the theoretical predictions of four-fermion cross sections beyond the level achieved for LEP2. The \(M_W\) measurement from a threshold scan requires a calculation of the \(W\)-pair production cross section with a precision of a few per-mille in the threshold region \(\sqrt{s} \sim 2M_W\), where the accuracy of the DPA degrades. A complete NLO calculation of charged-current 4-fermion production was performed in the complex mass scheme, including loop corrections to singly- and non-resonant diagrams.\(^6\) The DPA agrees well with the full \(e^- e^+ \rightarrow 4f\) calculation for energies \(200\text{ GeV} \lesssim \sqrt{s} \lesssim 500\text{ GeV}\) while the full calculation is required near threshold \(160\text{ GeV} \lesssim \sqrt{s} \lesssim 170\text{ GeV},\) and for \(\sqrt{s} > 500\text{ GeV},\) where off-shell effects become important. In a further development, effective-field theory (EFT) methods have been used for a dedicated
calculation of four-fermion production near the $W$-pair production threshold. This method has allowed to isolate and compute the subset of NNLO corrections that is enhanced near threshold due to Coulomb-photon effects. Combining these dominant NNLO corrections, which are of the order of 0.5%, with the full NLO result reduces the theoretical uncertainty of the $M_W$-measurement from a threshold scan to $\Delta M_W \lesssim 3$ MeV, below the ILC precision goal.

At centre-of-mass energies $\sqrt{s} \gtrsim 800$ GeV, which are particularly relevant for measurements of triple gauge couplings, higher-order EW corrections are enhanced by Sudakov logarithms. For $W$-pair production, NNLO corrections due to NNLL Sudakov logarithms $\alpha^2 \log^m(s/M_W^2)$ with $m = 2, 3, 4$ have been computed and are of the order of 5% (15%) for $\sqrt{s} = 1$ TeV (3 TeV), so they should be taken into account in the second phase of an ILC or at CLIC.

In addition to these precision calculations, the search for indirect signals of new physics requires a systematic treatment of deviations from the SM in an EFT framework, which has recently been applied to study the sensitivity to anomalous gauge boson couplings in $W$-pair production.

4 Outlook

Theoretical methods for higher-order calculations have seen remarkable progress after the LEP2 era. The theoretical uncertainty on charged-current four-fermion production has been reduced well below the percent level by a full NLO calculation. The extension of this calculation to the remaining processes of Fig. 1 will be simplified by recent progress on the automation of EW NLO calculations and may provide sufficient precision for future linear $e^-e^+$ colliders, if supplemented with dominant NNLO effects in special kinematic regions and an improved treatment of initial-state radiation. The precision goals of future circular $e^-e^+$ colliders may require a full NNLO calculation of EW corrections, where the current state of the art is given by $1 \to 2$ processes. The extension to $2 \to 2$ processes such as on-shell vector-boson pair production is beyond current methods, but may be feasible within several years. This would provide one of the building blocks of the extension of the double-pole approximation or the EFT approach to NNLO, which in addition requires the computation of two-loop soft-photon corrections with finite-width effects. Steps towards a decision on the construction of a future $e^-e^+$ collider would stimulate theoretical developments to meet these challenges.
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