$e^+e^- \rightarrow 3$ jets and event shapes at NNLO

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We report on the calculation of NNLO corrections to the 3-jet cross section and related event shape distributions in electron-positron annihilation. The corrections are sizable for all variables, however the magnitude of the corrections is substantially different for different observables. We observe that inclusion of the NNLO corrections yields a considerably better agreement between theory and experimental data both in shape and normalization of the event shape distributions in the region where the perturbative result is expected to hold. A new extraction of $\alpha_s$ using the event shape variables up to NNLO yields a considerably better consistency between the observables indicating a stabilization of the perturbative corrections at this order.

1. Introduction

Jet observables in electron–positron annihilation play a pivotal role in studying the dynamics of the strong interactions, described by the theory of quantum chromodynamics (QCD). In addition to measuring multi-jet production rates, more specific information about the topology of the events can be extracted using variables which characterize the hadronic structure of an event. With the precision data from LEP and SLC, experimental distributions for such event shape variables have been extensively studied [1,2] and have been compared with theoretical calculations based on next-to-leading order (NLO) parton-level event generator programs [3,4], improved by resumming kinematically-dominant leading and next-to-leading logarithms (NLO+NLL) [5] and by the inclusion of non-perturbative models of power-suppressed hadronisation effects [6].

Up to now, the precision of the strong coupling constant determined from event shape data has been limited largely by the scale uncertainty of the perturbative NLO calculation. We report here on the first calculation of NNLO corrections to the 3-jet cross section and related event shape variables. The knowledge of the NNLO corrections to the event shape distributions has important phenomenological impact on the extraction of $\alpha_s$ from LEP data.

2. The 3-jet cross section at NNLO

Jets are defined using a jet algorithm, which describes how to recombine the momenta of all energetic hadrons in an event to form the jets. These algorithms are used in the experimental analysis and in the parton-level event generators to combine particles into jets. Here we present the first calculation of the NNLO corrections to the three-jet production rate at parton-level in $e^+e^-$ annihilation using the Durham measure [7].

The calculation of the $\alpha_s^3$ corrections for three-jet production is carried out using a recently developed parton-level event generator program EERAD3 [8] which contains the relevant matrix elements with up to five external partons. Besides explicit infrared divergences from the loop integrals, the four-parton and five-parton contributions yield infrared divergent contributions if one or two of the final state partons become collinear or soft. In order to extract these infrared diver-
extended to NNLO level \([10]\) and implemented for corrections, the antenna subtraction method \([9]\) was gences and combine them with the virtual corrections, the antenna subtraction method \([9]\) was extended to NNLO level \([10]\) and implemented for \(e^+e^- \rightarrow 3\) jets \([11]\) and related event-shape variables \([12]\) into EERAD3.

Figure 1 displays the three-jet rate at LEP1 energy \(Q = M_Z\), compared to data obtained with the ALEPH experiment \([2]\).

\[\delta = \frac{\max_\mu(\sigma(\mu)) - \min_\mu(\sigma(\mu))}{2\sigma(\mu = M_Z)}\]

at NLO and NNLO as an inset. The uncertainty on the LO calculation is constant at 10.2%.

As can be seen from the plot, the theoretical uncertainty is lowered considerably compared to NLO. Especially in the region \(10^{-1} > y_{\text{cut}} > 10^{-2}\), which is relevant for precision phenomenology, one observes a reduction by almost a factor three, down to below two per cent relative uncertainty.

For large values of \(y_{\text{cut}}, y_{\text{cut}} > 10^{-2}\), the NNLO corrections turn out to be very small, while they become substantial for medium and low values of \(y_{\text{cut}}\). The maximum of the jet rate is shifted towards higher values of \(y_{\text{cut}}\) compared to NLO, and is in better agreement with the experimental observation.

The fixed-order theoretical predictions for the three-jet rate become negative for small values of \(y_{\text{cut}}\), where fixed order perturbation theory is not applicable due to the emergence of large logarithmic corrections at all orders which require re-summation \([7,14]\). We therefore restrict our comparison to \(y_{\text{cut}} > 10^{-4}\). Even with this restriction, at low jet resolution, the fixed-order NNLO description lies above the data. The theoretical parton-level prediction is compared however to hadron-level data, thereby neglecting hadronisation corrections, which may account for part of the discrepancy.

The total hadronic cross section consists of the sum over all jet multiplicities. At \(\mathcal{O}(\alpha_s^3)\), this sum runs from two-jet through to five-jet final states, such that the corresponding fractional jet rates must add to unity. Consequently, our calculation yields the \(N^3\)LO expression for \(e^+e^- \rightarrow 2\) jets as a by-product.

Figure 2 shows the parton-level theoretical predictions for the jet fractions at first, second and third order in the strong coupling constant, compared to experimental hadron-level data from ALEPH \([2]\).

By comparing the three plots, we observe that the agreement for each of the jet rates becomes systematically better as the order of perturbation theory increases. At each order a new multi-jet channel opens up, e.g. the five-jet rate at \(\mathcal{O}(\alpha_s^3)\), which is positive definite and essentially monotonically increasing as \(y_{\text{cut}}\) decreases. Since all jet rates are normalized to unity, the new five-jet channel has the effect of reducing the contribution to the two-jet, three-jet and four-jet rates, in the region of \(\log_{10}(y_{\text{cut}})\) where the five-jet rate contributes. One very clear effect is to cause the turnover in the four-jet rate (which is not present at \(\mathcal{O}(\alpha_s^2)\)). A second effect is to add more structure to the shape of the two- and three-
jet rates, which lie much closer to the data for $\log_{10}(y_{\text{cut}}) < -2.5$. Of course, the effect of the higher order corrections also extends to larger values of $y_{\text{cut}}$ and, by adding more structure to the theoretical prediction, one obtains a better description of the data.

3. Event shape variables

In order to characterize hadronic final states in electron-positron annihilation, a variety of event shape variables have been proposed in the literature. For a review see e.g. [15,16]. In our study we considered only variables for three-particle final states which are thus closely related to three-jet final states. Among these event shapes, six variables were studied in great detail [1]: the thrust $T$, the normalized heavy jet mass $\rho$, the wide and total jet broadenings $B_W$ and $B_T$, the $C$-parameter and the transition from three-jet to two-jet final states in the Durham jet algorithm called $Y_3$.

The perturbative expansion for the distribution of a generic observable $y$ up to NNLO at $e^+e^-$ centre-of-mass energy $\sqrt{s}$, for a renormalization scale $\mu^2$ involves perturbative coefficients [12] which only depend on the event shape variable $y$ itself and the strong coupling constant $\alpha_s$. Those coefficients are computed by a fixed-order parton-level calculation, which includes final states with three partons at LO, up to four partons at NLO and up to five partons at NNLO.

The precise size and shape of the NNLO corrections depend on the observable in question. Common to all observables is the divergent behaviour of the fixed-order prediction in the two-jet limit (corresponding to small values of the event shape variable $y$), where soft-gluon effects lead to an enhancement of the fixed-order coefficients by powers of $\ln(1/y)$. In order to obtain reliable predictions in the region of $y \ll 1$ it is necessary to resum entire sets of logarithmic terms at all orders in $\alpha_s$. A detailed description of the predictions at next-to-leading-logarithmic approximation (NLLA) can be found in Ref. [16].

For several event shape variables (especially $T$
Figure 3. Thrust distribution at $Q = M_Z$ at LO (blue), NLO (green) and NNLO (red). The solid lines represent the prediction for renormalisation scale $\mu = Q$ and $\alpha_s(M_Z) = 0.1189$, while the shaded region shows the variation due to varying the renormalisation scale between $\mu = Q/2$ and $\mu = 2Q$. The data is taken from [2].

and $C$) the full kinematical range is not yet realised for three partons, but attained only in the multijet limit. For the thrust distribution $1-T$, as seen in Fig. 3 the multi-jet limit corresponds to $1-T > 0.5$. Consequently, the fixed-order description is expected to be reliable in a restricted interval bounded by the two-jet limit on one side and the multi-jet limit on the other side.

In the intermediate region, we observe that inclusion of NNLO corrections (evaluated at the Z-boson mass, and for fixed value of the strong coupling constant) typically increase the previously available NLO prediction. The magnitude of this increase differs considerably between different observables [12], it is substantial for $T$ (18%), $B_T$ (17%) and $C$ (15%), moderate for $\rho$ and $BW$ (both 10%) and small for $Y_3$ (6%). For all shape variables, we observe that the renormalization scale uncertainty of the NNLO prediction is reduced by a factor 2 or more compared to the NLO prediction. We observe that the NNLO prediction describes the shape of the measured event shape distributions over a wider kinematical range than the NLO prediction, both towards the two-jet and the multi-jet limit.

4. Determination of the strong coupling constant

Using the newly computed NNLO corrections to event shape variables, we performed [17] a new extraction of $\alpha_s$ from data on the standard set of six event shape variables measured by the ALEPH collaboration [2] at centre-of-mass energies of 91.2, 133, 161, 172, 183, 189, 200 and 206 GeV. The combination of all NNLO determinations from all shape variables yields

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008 \text{(stat)}$$
$$\pm 0.0010 \text{(exp)} \pm 0.0011 \text{(had)}$$
$$\pm 0.0029 \text{(theo)}.$$  

We observe a clear improvement in the fit quality when going to NNLO accuracy. Compared to NLO, the value of $\alpha_s$ is lowered by about 10%, but still higher than for NLO+NLLA [2], which shows the obvious need for a matching of NNLO+NLLA for an even more precise result. Work is in progress in this direction [18].

As can be seen in Figure 4, the scatter among the $\alpha_s$-values extracted from different shape variables is lowered considerably when going to NNLO accuracy and the theoretical uncertainty is decreased by a factor 2 (1.3) compared to NLO (NLO+NLLA). The different sizes of the NNLO corrections for different observables is responsible for these large improvements. One infers that the scatter present at next-to-leading order was largely due to missing higher order perturbative corrections.

5. Outlook

Our results for the NNLO corrections to the 3-jet cross section and related event shape distributions in electron-positron annihilation open up a whole new range of possible comparisons with the LEP data. The potential of these studies is illustrated by comparisons of the NNLO fixed order results with jet and event-shape data from ALEPH. The corrections are sizable for all variables, but yield a considerably better consistency between the observables indicating a stabilization
Figure 4. The measurements of the strong coupling constant $\alpha_s$ for the six event shapes, at $\sqrt{s} = M_z$, when using QCD predictions at different approximations in perturbation theory. The blue band indicates the uncertainty due to renormalisation scale variation in each theoretical description.

of the perturbative corrections at this order. A fit to event shape data yielded a new determination of $\alpha_s$. We anticipate a further improvement by matching of the fixed order NNLO calculation with NLLA resummations [18].

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