Primordial black holes as biased tracers

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Abstract

Primordial black holes (PBHs) are theoretical black holes which may be formed during the radiation dominant era and, basically, caused by the gravitational collapse of radiational overdensities. It has been well known that in the context of the structure formation in our Universe such collapsed objects, e.g., halos/galaxies, could be considered as bias tracers of underlying matter fluctuations and the halo/galaxy bias has been studied well. Employing a peak-background split picture which is known to be a useful tool to discuss the halo bias, we consider the large scale clustering behavior of the PBH and propose an almost mass-independent constraint to the scenario that dark matters (DMs) consist of PBHs. We consider the case where the statistics of the primordial curvature perturbations is almost Gaussian, but with small local-type non-Gaussianity. If PBHs account for the DM abundance, such a large scale clustering of PBHs behaves as nothing but the matter isocurvature perturbation and constrained strictly by the observations of cosmic microwave backgrounds (CMB). From this constraint, we show that, in the case a certain single field causes both CMB temperature perturbations and PBH formations, the PBH-DM scenario is excluded even with quite small local-type non-Gaussianity, $|f_{\text{NL}}| \sim O(0.01)$, while we give the constraints to parameters in the case where the source field of PBHs is different from CMB perturbations.

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I. INTRODUCTION

The identity of dark matters (DMs) is one of the most earnest questions for scientists. A lot of candidates have been proposed, but it still remains an open question. The primordial black hole (PBH) [1] is one of those candidates which is studied for a long time. PBHs are black holes which are theoretically suggested to be formed in the early universe, and distinguished from ordinal stellar black holes.

The constraint on the abundance of PBHs has been basically obtained from the non-detection of them [2], and recently, by the Kepler telescope, all of the mass windows for PBHs to be a dominant component of DMs would be closed and the possibility of PBH-DM scenario seems to be completely excluded if PBHs have a monochromatic mass function [3, 4]. However, there remain theoretical uncertainty of constraints, the possibility such that the mass region of PBHs broadens widely, and so on.

In this paper, we consider the large scale clustering behavior of PBHs and discuss the possibility of PBH-DM scenario in a way independent of the PBH mass. Here, we assume the case that PBHs are formed from the gravitational collapse of the radiational overdense regions, though there are several mechanisms for the PBH formation like a collapse of cosmic strings [5], domain walls [6] or bubble collisions [7].

In the context of the structure formation, the large scale clustering of such collapsed objects, e.g., halos/galaxies, has been studied well. For example, Bardeen et al. [8] showed that the collapsed objects tend to be formed intensively in widely energy-dense regions, in the peak-background split picture in which source density perturbations are divided into short- (peak) and long-wavelength (background) modes. In the case where the statistics of the initial primordial perturbations is Gaussian, the large scale clustering of the peak objects could be related to the distribution of the underlying matter density field by introducing a constant scale-independent bias parameter. On the other hand, due to the existence of the non-Gaussian feature in the statistics of the initial primordial perturbations, the bias parameter tends to have a scale-dependent component, which is known as scale-dependent bias, and this has been considered to be a powerful tool to hunt the primordial non-Gaussianity and extensively studied (see e.g., Ref. [9]).

Chisholm [10] considered the scale-independent bias of PBHs and concluded that though the bias factor is large in itself, the density perturbations of PBHs are hardly produced on
much larger scale like the scale of cosmic microwave backgrounds (CMB). Recent several papers \cite{11, 12} also agree with this result. This is because that PBHs are caused by the collapse of horizon scale overdensity and a super-horizon energy density which has little causality with physics inside the horizon hardly affect whether that horizon collapse or not.

In this paper, based on these previous works, we consider the case where the statistics of the primordial curvature perturbation is almost Gaussian, but with small non-Gaussian component, and investigate the large scale clustering of the PBHs. As a non-Gaussian component, we consider the so-called local-type non-Gaussianity, which is known to give a strong scale-dependence in the halo bias. We find that if PBHs account for the DM abundance, the large scale clustering of the PBH can be detected as the matter isocurvature perturbations and excluded by CMB observations even in the case with small non-Gaussianity.

This paper is organized as follows. In section II, we briefly review the bias effect and calculate the matter isocurvature perturbations from PBH bias in the case where some single source causes both observed CMB temperature perturbations and PBH formations. In section III, we discuss the other case where the source of PBH formations is different from that of CMB perturbations. Section IV is devoted to the conclusions. In appendix A, we discuss the case where the source field of PBH formations follows the chi-squared distribution as a simple example of fully non-Gaussian case.

II. SINGLE SOURCE CASE

Let us discuss the simplest case that the fluctuation of a certain single field causes both observed large scale CMB temperature perturbations and PBH formations, although, here, we do not mention the concrete inflationary model where the generated primordial adiabatic perturbations could be seeds of PBH formation. Let us denote a primordial adiabatic curvature perturbation as $R$ and define it on the comoving slice. The amplitude of $R$ is around $10^{-5}$ on CMB scale but it is possibly larger enough to produce PBHs on smaller scales. Also we assume $R$ has a small local-type non-Gaussianity which is represented as

$$R(x) = g(x) + f_{NL}(g^2(x) - \langle g^2 \rangle),$$

(1)

where $g$ is a Gaussian field.

Here we must mention that it is difficult to discuss the generic lower bound of the magnitude of non-linearity parameter $f_{NL}$. In Ref. \cite{13}, the author claimed that there is a
consistency relation between non-linearity parameter \( f_{NL} \) and the scale-dependence of the
adiabatic curvature perturbations, and hence even in the simplest single inflation model the
non-Gaussianity does not vanish as \( f_{NL} \approx \mathcal{O}(0.01) \). However, recently, some authors claimed
that the local-type non-Gaussianity evaluated by the consistency relation is nothing but an
unphysical gauge artifact and the physical value of \( f_{NL} \) is much smaller than \( \mathcal{O}(0.01) \) in the
single inflation model [14–16]. That may be so, but here we suppose \( |f_{NL}| \gtrsim \mathcal{O}(0.01) \) from
the scale dependence of \( \mathcal{R} \) on CMB scale.

A. PBH bias

Here we assume that PBH formation can be described by Press-Schechter approach [17]
as in the standard structure formation. Following the peak background split picture in the
structure formation [8], let us discuss the large scale clustering of PBH. According to [11],
when we discuss whether PBH is formed or not, we should use the density perturbations of
the radiation on the comoving slice \( \delta \), not \( R \). The Fourier transformed component of such
density perturbations of the radiation is given by

\[
\delta(k) = \frac{4}{9} \left( \frac{k}{aH} \right)^2 \mathcal{R}(k),
\]

on super-horizon scale. Following the Press-Schechter approach, we consider the density
perturbations coarse-grained on some scale. With some window function \( W(kR) \), let us
define the coarse-grained density perturbations as

\[
\delta_s(k) = \mathcal{M}_s(k)\mathcal{R}(k),
\]

where

\[
\mathcal{M}_s(k) = \frac{4}{9} \left( \frac{k}{aH|_{\text{PBH}}} \right)^2 W(kR_s) = \frac{4}{9} (kR_{\text{PBH}})^2 W(kR_s).
\]

\( R_s \) is the coarse-graining scale, and almost the same as the horizon scale at the PBH-
formation time \( R_{\text{PBH}} = (aH|_{\text{PBH}})^{-1} \) since overdense regions will collapse soon after entering
the horizon. Note that these scales should be much smaller than the present horizon scale.

In the Press-Schechter approach, if \( \delta_s \) of some coarse-grained region exceeds a certain
threshold \( \delta_c \), that region is assumed to collaps to be PBHs.\(^1\) Assuming the statistics of \( \delta_s \) is

\(^1\) In the case such that \( \mathcal{R} \) has the non-Gaussianity, the dynamics of collapse after the overdensity enter the
also nearly Gaussian, the probability of PBH formation is given by

\[
P_1(> \nu) = \frac{2}{\sqrt{2\pi \sigma_s^2}} \int_{\delta_c}^{\infty} \delta \exp \left( -\frac{\delta^2}{2\sigma_s^2} \right)
\]

\[
= \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right) \approx \sqrt{\frac{2}{\pi \nu}} e^{-\nu^2/2},
\]

(5)

where \( \nu = \delta_c/\sigma_s \) and \( \sigma_s \) is the standard deviation of \( \delta_s \) defined by

\[
\sigma_s^2 = \langle \delta_s^2 \rangle = \int \log k \mathcal{P}_{\delta_s}(k) = \int \log k \mathcal{P}_{\delta_l}(k) \mathcal{M}_s^2(k),
\]

(6)

with the power spectrum of \( \mathcal{R} \), \( \mathcal{P}_{\delta_l}(k) = \frac{2\pi^2}{k^3} |\mathcal{R}(k)|^2 \). We have included conventional factor 2 of Press-Schechter formulation. Also we have assumed a high peak limit as \( \nu \gg 1 \), because PBHs are rare objects. However at this time if this region is on a long-wavelength density fluctuation \( \delta_l \), the probability should be slightly modified because the threshold is effectively reduced as \( \nu \to \nu - \delta_l/\sigma_s \) (see Fig. 1), namely

\[
P_1(> \nu|\delta_l) = P_1 \left( > \nu - \frac{\delta_l}{\sigma_s} \right)
\]

\[
\approx P_1(> \nu) \left[ 1 + \frac{1}{\sigma_s} \frac{dP_1(> \nu)}{d\nu} \delta_l \right].
\]

(7)

Therefore the number density fluctuations of PBHs on large scales, which is given by

\[
\delta_{\text{PBH}}(x) := \frac{P_1(> \nu|\delta_l(x))}{P_1(> \nu)} - 1,
\]

is proportional to \( \delta_l(x) \) by the factor of \( -\frac{1}{\sigma_s} \frac{dP_1(> \nu)}{d\nu} \delta_l \). This is just a scale-independent bias factor of the clustering of PBH and will be denoted by \( b_0 \) in this paper.

\[\text{horizon may also vary. Therefore we may not be able to say sweepingly the overdensity such that } \delta_s > \delta_c \text{ will collapse (For instance, Nakama et al. [18] analyzed the development of overdensities of generalized curvature profile beyond the Gaussian one and proposed two crucial parameters as thresholds of PBH formations.)}. \]

\[\text{However those effects are beyond the scope of this paper, and we just follow the standard Press-Schechter approach.}\)

\[\text{2 The effect of the non-Gaussian profile of } \delta_s \text{ is considered in [19–21] for example. However, since we assume small non-Gaussianity } |f_{\text{NL}}| \sim \mathcal{O}(0.01) \text{ here, that effect is expected to be negligibly small.}\]
FIG. 1. Schematic diagrams of a bias effect in the peak-background split formulation. Red dashed lines represent the threshold for PBH formation $\delta_c$ and black points show the regions which will be PBHs. In the first (top) figure, it can be seen that the threshold for the short-wavelength mode $\delta_s$ is effectively reduced to $\delta_c - \delta_l$. Therefore PBHs tend to be formed in $\delta_l > 0$ region, and this effect is the scale-independent bias. However, long-wavelength modes of the comoving radiational perturbations are practically quite suppressed due to the factor $(kR_{PBH})^2$ in Eq. (4) as the second (bottom) figure shows. Even in such a case, if $f_{NL} > 0$ (or $< 0$), the amplitude of $\delta_s$ in itself tends to be larger in $\delta_l > 0$ (or $< 0$) region via the non-Gaussianity of primordial curvature perturbations. Such a bias effect is called the scale-dependent bias.

Thus, following the peak-background split picture, we find that the number density fluctuations of PBH, $\delta_{PBH}$, on large scales is given by $\delta_{PBH} \simeq \frac{\nu}{\sigma_s} \delta$. However, although a biasing factor $\nu/\sigma_s$ is larger than order of unity, $\delta_{PBH}$ is much smaller than $\mathcal{R}$ on the CMB scale. This is because the CMB scale $k_{CMB}^{-1}$ is quite larger than the PBH scale $R_{PBH}$, namely $k_{CMB}R_{PBH} \ll 1$, and then $\delta_{PBH}$ is strongly suppressed as $\delta_{PBH}(k_{CMB}) \sim$
\[ \nu/\sigma_s \times (k_{\text{CMB}} R_{\text{PBH}})^2 \mathcal{R}(k_{\text{CMB}}) \ll \mathcal{R}(k_{\text{CMB}}). \] Indeed, this result is consistent with the claims of [10–12]. However, it will be dramatically changed if we consider the effect of non-Gaussianity, namely the scale-dependent bias [9].

If the primordial perturbations have local-type non-Gaussianity as given in Eq. (1), there is a correlation between short- and long-wavelength perturbations as shown later. Therefore the long-wavelength density perturbation \( \delta_l \) not only reduces the threshold effectively but also modifies the variance of short-wavelength perturbations, \( \sigma_s \). Thus the bias parameter

\[ b = \frac{d \log P_1}{d \delta_l} \bigg|_{\delta_l=0} \]

has two components as

\[ b = \left( \frac{\partial \delta_c}{\partial \delta_l} \frac{\partial \log P_1}{\partial \delta_c} \right)_{\delta_l=0} + \left( \frac{\partial \sigma_s}{\partial \delta_l} \frac{\partial \log P_1}{\partial \sigma_s} \right)_{\delta_l=0}. \] (8)

The former one is nothing but the scale-independent bias \( b_0 \) which was discussed above and the latter one is so-called scale-dependent bias factor and written as \( \Delta b \) in this paper.

To evaluate the second term, let us make the relation between scales of interest clear. Assuming \( R_s \) and \( R_l \) as the scales of the collapsed regions and the correlation lengths of the clustering of PBH (slightly smaller than the CMB scale) respectively, we will consider the PBH energy density in some region of size \( R \), with \( R_s \ll R \ll R_l \). From Eq. (1), \( \mathcal{R}(x) \) can be decomposed into long- and short-wavelength components as

\[ \mathcal{R}(x) = g_l(x) + g_s(x) + f_{\text{NL}} [(g_l(x) + g_s(x))^2 - \langle g_l^2 \rangle - \langle g_s^2 \rangle] \]

\[ = g_l(x) + f_{\text{NL}} (g_l^2(x) - \langle g_l^2 \rangle) \]

\[ + g_s(x) + f_{\text{NL}} (2g_l(x)g_s(x) + g_s^2(x) - \langle g_s^2 \rangle). \] (9)

In the second equation, the first line which is independent of \( g_s \) denotes the long-wavelength mode \( \mathcal{R}_l \) and the second line is regarded as the short-wavelength mode \( \mathcal{R}_s \). Therefore, from the expression of \( \mathcal{R}_s \), noting \( g_l(x) \) is almost constant on the scale of \( R \), we can write the Fourier transformed component of short-wavelength mode of density perturbations as

\[ \delta_s(k, x) = M_s(k) [g_s(k) + 2f_{\text{NL}}g_l(x)g_s(k)]. \] (10)

Here we have neglected the contribution from the \( g_s^2(x) \) because it is known that such a quadratic term does not affect the scale-dependent bias. Therefore, the standard deviation \( \sigma_s \) also increase by the factor \( 1 + 2f_{\text{NL}}g_l(x) \) and then

\[ \left. \frac{\partial \sigma_s}{\partial \delta_l} \right|_{\delta_l=0} = M_l^{-1}(k) \left. \frac{\partial \sigma_s}{\partial g_l} \right|_{g_l=0} = 2f_{\text{NL}} \sigma_s M_l^{-1}(k), \] (11)
where,
\[ \mathcal{M}_l(k) = \frac{4}{9} (kR_{PBH})^2 W(kR_l). \] (12)

Following the Press-Schechter approach, we have
\[
\frac{\partial \log P_1}{\partial \sigma_s} = -\frac{\delta_c \partial \log P_1}{\sigma_s^2} \frac{\partial \nu}{\sigma_s} = \frac{\delta_c}{\sigma_s} b_0, \tag{13}
\]
and then the scale-dependent bias can be written as
\[ \Delta b(k) = 2f_{NL} \mathcal{M}_l^{-1}(k) \delta_c b_0. \] (14)

It is noteworthy that \( \Delta b \) has a factor of \( \mathcal{M}_l^{-1} \). Since this factor cancels out \((kR_{PBH})^2\) in \( \delta_s, \delta_{PBH} \) from the scale-dependent bias is not negligible compared to the adiabatic curvature perturbation, \( \mathcal{R} \), even on much larger scale. This is because the scale-dependent bias is directly proportional to \( \mathcal{R} \) via the non-Gaussianity, while the scale-independent bias is proportional to \( \delta \).

**B. PBH-DM isocurvature perturbation**

If DMs consist of PBHs, the matter isocurvature perturbation is defined as
\[ S = \delta_{PBH} - \frac{3}{4} \delta. \] (15)

By using Eq. (14), the Fourier transformed component of the matter isocurvature perturbation smoothed on the scale \( R \) is
\[
S_R(k) = \left( b_0 + \Delta b(k) - \frac{3}{4} \right) \delta_R(k)
= \left( b_0 \mathcal{M}_R(k) + 2f_{NL}\delta_c b_0 \mathcal{M}_l(k) - \frac{3}{4} \mathcal{M}_R(k) \right) \mathcal{R}(k), \tag{16}
\]
where \( \mathcal{M}_R(k) = \frac{4}{9} (kR_{PBH})^2 W(kR) \). As mentioned before, the factor \((k_{CMB}R_{PBH})^2\) in \( \mathcal{M}_R \) is so small that we can neglect the first and third terms in the parenthesis. Thus the power spectrum of the isocurvature mode is given by
\[
P_S(k_{CMB}) \simeq (2f_{NL}\delta_c b_0)^2 P_R(k_{CMB})
\simeq (2f_{NL} \nu^2)^2 P_R(k_{CMB}). \tag{17}
\]
Here we have approximated $\frac{M_R}{M_l}(k_{\text{CMB}})$ as unity because $k_{\text{CMB}}^{-1}$ is larger than both $R_l$ and $R$ now and then $W(k_{\text{CMB}}R_l) \simeq W(k_{\text{CMB}}R) \simeq 1$. Note that $S_R(k_{\text{CMB}})$ is similarly equal to $S(k_{\text{CMB}})$. Also we have used $\nu = \delta_c/\sigma_s$ and $b_0 \simeq \nu/\sigma_s$ in the second equation.

CMB observations can give tight constraints on the amplitude of the matter isocurvature perturbations and, according to the Planck collaboration [22], for the case of fully (anti-) correlated, namely $S \propto R$ (or $S \propto -R$), the matter isocurvature mode is constrained as

$$\frac{\mathcal{P}_S}{\mathcal{P}_S + \mathcal{P}_R} \lesssim \begin{cases} 0.0025. & (\text{fully correlated, } f_{\text{NL}} > 0) \\ 0.0087. & (\text{anti-correlated, } f_{\text{NL}} < 0) \end{cases}$$

(18)

Therefore, from Eq. (17) we must satisfy

$$|f_{\text{NL}}| \nu^2 \lesssim \begin{cases} \frac{\sqrt{0.0025}}{2} = 0.025. & (f_{\text{NL}} > 0) \\ \frac{\sqrt{0.0087}}{2} = 0.047. & (f_{\text{NL}} < 0) \end{cases}$$

(19)

Let us show that this constraint is too strong. Assuming the nearly monochromatic mass function for PBHs, the current abundance of PBHs is given by [2]

$$\Omega_{\text{PBH}} \sim 0.86 \times 10^8 P_1 \left( \frac{M}{M_{\odot}} \right)^{-1/2},$$

(20)

where $M_{\odot}$ denotes the solar mass $\sim 2 \times 10^{33}$ g. Inversely, if DMs consist of PBHs and $\Omega_{\text{PBH}} = \Omega_{\text{DM}} = 0.31$ [23], the probability $P_1$ should satisfy the following relation

$$P_1 \sim 0.36 \times 10^{-8} \left( \frac{M}{M_{\odot}} \right)^{1/2}.$$

(21)

Furthermore, from Eq. (5), $\nu$ can be written in terms of $P_1$ as

$$\nu = \sqrt{2} \text{erfc}^{-1}(P_1),$$

(22)

where $\text{erfc}^{-1}$ denotes the inverse function of the complementary error function. Therefore, if we assume the mass of PBH is less than $10^{40}$ g, we obtain the following relation

$$\nu^2 \gtrsim 2 \left[ \text{erfc}^{-1} \left( 0.36 \times 10^{-8} \left( \frac{10^{40}}{2 \times 10^{33}} \right)^{1/2} \right) \right]^2 \simeq 20.$$

(23)

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3 Here we respect the plot of Ref. [4] in which the PBH constraints were summarized up to $M_{\text{PBH}} = 10^{40}$ g. Moreover Afshordi et al. showed that for more massive PBHs than $\sim 10^{37}$ g, the Poisson-noise fluctuations deviating from the adiabatic perturbations become too large to be consistent with observed $\sigma_8$ [24]. Therefore we constrain the PBH mass to be less than around these values.
FIG. 2. The plot of the upper limit of the current PBH fraction $\Omega_{PBH}/\Omega_{DM}$ from the isocurvature constraints (18). Shaded regions represent existing constraints from various observations [3, 4]. The black solid and dotted lines show the upper limit in the case of $f_{NL} = \pm 0.01$ and the red solid and dotted ones exhibit the constraints in the case of $f_{NL} = \pm 0.1$. Also from this plot, it can be seen that PBHs cannot be the main component of DMs even with small non-linearity parameter $|f_{NL}| = 0.01$.

Here, note that $\text{erfc}^{-1}(x)$ is a monotonic decreasing function.

Accordingly, we find that it is difficult to satisfy the isocurvature constraint (19) even with a small non-linearity parameter $|f_{NL}| \sim \mathcal{O}(0.01)$ and this means that the PBH-DM scenario is strictly constrained due to the existence of the small local-type non-Gaussianity in the single sourced case. This is a main result of this paper. Furthermore, we have also considered the case that PBHs are sub-components of DMs. We have plotted the upper limit of the PBH fraction in such a case to satisfy the isocurvature constraints (18) in Fig. 2. Note that the isocurvature power and the probability of PBH formation are given by $P_S = \left(\frac{\Omega_{PBH}}{\Omega_{DM}}\right)^2 P_{PBH}$ and $P_1 \sim 0.36 \times 10^{-8} \frac{\Omega_{PBH}}{\Omega_{DM}} \left(\frac{M}{M_\odot}\right)^{1/2}$ respectively in that case.
III. DIFFERENT SOURCE CASE

Even if the source of PBH-DMs is different from that of CMB fluctuations, the above discussion does not change so much. Let $\mathcal{R}$ and $\Sigma$ denote the curvature perturbations which cause CMB fluctuations and PBHs, respectively. In other words, $\mathcal{R}$ is dominant and has a nearly flat spectrum on CMB scale, while $\Sigma$ dominates on much smaller scale.

If we assume $\Sigma$ is almost Gaussian but has a small local-type non-Gaussianity as

$$\Sigma(x) = g(x) + f_{\text{NL,}\Sigma}(g^2(x) - \langle g^2 \rangle),$$

the power of the matter isocurvature perturbations is given by

$$\mathcal{P}_S \simeq \left(2 f_{\text{NL,}\Sigma} \nu^2 \right)^2 \mathcal{P}_\Sigma,$$

like as Eq. (17). By introducing an effective non-linearity parameter defined as

$$f_{\text{NL,eff}} := f_{\text{NL,}\Sigma} \sqrt{\frac{\mathcal{P}_\Sigma}{\mathcal{P}_\mathcal{R}}},$$

the power of the isocurvature perturbation can be written as

$$\mathcal{P}_S = \left(2 f_{\text{NL,eff}} \nu^2 \right)^2 \mathcal{P}_\mathcal{R}.\hspace{1cm}(27)$$

From the above expression, we can also obtain the constraints on the PBH-DM scenario in the different source case depending on the ratio between the power of $\mathcal{R}$ and that of $\Sigma$ on CMB scale. Note that in this case the correlation between adiabatic curvature perturbations and generated isocurvature perturbations is much suppressed, that is, almost uncorrelated-type isocurvature perturbations. The constraint for the uncorrelated isocurvature perturbations shown by Planck collaboration is [22]

$$\frac{\mathcal{P}_S}{\mathcal{P}_S + \mathcal{P}_\mathcal{R}} \lesssim 0.036, \quad \text{(uncorrelated)},$$

and therefore the constraint for $f_{\text{NL,eff}}$ is weakened up to

$$|f_{\text{NL,eff}}| \nu^2 \lesssim 0.095.\hspace{1cm}(29)$$

Furthermore, $\Sigma$ is not necessarily almost Gaussian but possibly quite non-Gaussian. Accordingly, we also discuss the chi-squared case in appendix A as a simple example.
IV. CONCLUSIONS

In this paper, we consider the clustering of the PBHs on CMB scale produced by the bias effect. Since there is a great gap between CMB scale and the horizon scale at the time of PBH formation during the radiation dominant phase, the distribution of PBH is hardly biased basically. However, if the curvature perturbation field which cause PBHs has even small local-type non-Gaussianity, we have showed non-negligible spatial fluctuations of PBH number density can be produced even on CMB scale because the source field has a correlation between large and small scale via the local-type non-Gaussianity. Specifically, the fluctuation of the number density of PBH is biased from the source curvature perturbation by the factor of non-linearity parameter $f_{\text{NL}}$ times $(\text{erfc}^{-1}(P_1))^2$ roughly, where $P_1$ is the PBH production probability which is commonly represented as $\beta$ in the literature. If DMs consist of PBHs, these extra density perturbations on CMB scale can be detected as the matter isocurvature perturbation and are constrained strictly by CMB observations.

We have showed, if the source curvature perturbation of PBHs also cause CMB temperature fluctuations, the PBH-DM scenario is excluded even with quite small non-linearity parameter $|f_{\text{NL}}| \sim \mathcal{O}(0.01)$. On the other hand, in the case that the source curvature perturbation of PBHs $\Sigma$ is different from that of CMB fluctuations $R$, we can avoid the isocurvature constraints if $\Sigma$ is sub-dominant enough on CMB scale. Even such a case, the PBH-DM scenario should satisfy $|f_{\text{NL,}}\Sigma| \sqrt{\frac{P_\Sigma}{P_R}}(k_{\text{CMB}}) \lesssim \mathcal{O}(0.001)$. These constraints are almost independent of the PBH mass, and moreover they may not change a lot even if the PBH-mass spectrum is quite non-monochromatic unlike many other constraints. This is because even if the isocurvature perturbations of each mass range are small enough, they will be piled up and the total amplitude of them will be determined only by $f_{\text{NL}}$ and $\Omega_{\text{PBH}}$, hardly affected by the shape of mass function. We leave more precise discussion about the effect of non-monochromaticity for future issue. Further, here we consider simple local-type non-Gaussianity. We consider that it is interesting and important to discuss the bias effect for other types of non-Gaussianity which would be generated in the concrete PBH-formation scenarios. We also leave such study for future issue.
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Appendix A: Chi-squared type

Let us consider the simple chi-squared $\Sigma$ which is characterized as

$$\Sigma(x) = g^2(x) - \langle g^2 \rangle,$$  \hspace{1cm} (A1)

as an example of fully non-Gaussian case. Suppose the coarse-grained density perturbations $\delta_s$ also follow the chi-squared distribution, the probability of the PBH formation is given by

$$P_1(> \delta_c) = 2 \int_{\delta_c}^{\infty} d\delta \frac{1}{\sqrt{\pi \sigma_s (\sqrt{2} \delta + \sigma_s)}} \exp \left( - \frac{\sqrt{2} \delta + \sigma_s}{2 \sigma_s} \right)$$

$$= 2 \text{erfc} \left( \frac{\sqrt{2} \nu + 1}{2} \right)$$

$$\approx 2 \sqrt{\frac{2}{\pi (\sqrt{2} \nu + 1)}} \exp \left( - \frac{\sqrt{2} \nu + 1}{2} \right),$$  \hspace{1cm} (A2)

where $\nu = \delta_c / \sigma_s$ again. The scale-independent bias can be calculated similarly to section II as

$$b_0 = - \frac{1}{\sigma_s P_1} \frac{\partial P_1}{\partial \nu} \approx \frac{1}{\sqrt{2} \sigma_s}.$$  \hspace{1cm} (A3)

On the other hand, to calculate the scale-dependent part of the bias parameter, we have to discuss the dependence of $\sigma_s$ on $\delta_l$. From Eq. (A1), we decompose $\Sigma$ into long- and short-wavelength components like as Eq. (9) [25]

$$\Sigma(x) = \Sigma_l(x) + \Sigma_s(x) + 2g_l(x)g_s(x)$$

$$\Sigma_l(x) := g_l^2(x) - \langle g_l^2 \rangle$$

$$\Sigma_s(x) := g_s^2(x) - \langle g_s^2 \rangle. $$  \hspace{1cm} (A4)
Similarly to the discussion in section II, the second and third term in the expression of \( \Sigma(x) \) denotes the short-wavelength mode of \( \Sigma \). Noting that third moments of \( g_s \) should vanish, the power spectrum of \( \delta_s \) on R-scale is given by

\[
P_{\delta_s}(k, x) = M_s^2(k)[4(g_s^2(x) - \langle g_s^2 \rangle)P_{g_s}(k) + P_{\Sigma}(k)|_{\Sigma(x) = 0}] = M_s^2(k)[4\Sigma_i(x)P_{g_s}(k) + P_{\Sigma}(k)|_{\Sigma(x) = 0}] .
\]  

(A5)

Then the standard deviation of \( \delta_s \), which is the square root of the integration of \( P_{\delta_s} \), can be approximated by

\[
\sigma_s \simeq \sigma_s|_{\Sigma_i(x) = 0} \left( 1 + \frac{2\sigma_{g,s}^2}{\sigma_s^2} \Sigma_i \right) ,
\]

where

\[
\sigma_{g,s}^2 = \int \text{d log } k P_g(k)M_s^2(k).
\]

(A6)

Therefore

\[
\frac{\partial \sigma_s}{\partial \delta_i(k)} \bigg|_{\delta_i = 0} = \frac{2\sigma_{g,s}^2}{\sigma_s^2} M_i^{-1}(k),
\]

(A7)

and with use of Eq. (13), we obtain the scale-dependent bias as

\[
\Delta b = \frac{2\sigma_{g,s}^2}{\sigma_s^2} M_i^{-1} \delta_i b_0 \simeq \frac{\sqrt{2}\sigma_{g,s}^2}{\sigma_s^2} M_i^{-1} \nu.
\]

(A8)

Since it has the factor of \( M_i^{-1} \) again, the number density fluctuation of the PBH can be produced through this scale-dependent bias even on CMB scale and the power of matter isocurvature is given by

\[
P_S(k_{\text{CMB}}) \simeq \left( \frac{\sqrt{2}\sigma_{g,s}^2}{\sigma_s^2} \nu \right)^2 P_{\Sigma}(k_{\text{CMB}}).
\]

(A9)

Introducing the effective non-linearity parameter as \( f_{\text{NL},\text{eff}} = \frac{\sqrt{2}\sigma_{g,s}^2}{\sigma_s^2} \sqrt{P_{\Sigma}(k_{\text{CMB}})/P_R(k_{\text{CMB}})} \), we obtain the constraint as

\[
f_{\text{NL},\text{eff}} \nu \lesssim 0.095.
\]

(A10)

Note that \( \nu \) is not squared unlike the case of small non-Gaussianity and this fact just comes from the apparent difference of \( P_1 \), Eq. (5) and (A2). The lower bound for \( \nu \) given by

\[
\nu = \frac{2 \left[ \text{erfc}^{-1}(P_1/2) \right]^2 - 1}{\sqrt{2}} \gtrsim 14,
\]

(A11)
is not so different from that of $\nu^2$ for the case of local-type non-Gaussian, Eq. (23).

We can not calculate $f_{\text{NL},\text{eff}}$ without determining the detail profile of $\Sigma$ (or $g$) like a spectrum index, but can understand this result qualitatively. First, let us approximate $\sigma^2_{g,s}$ by $\frac{\mathcal{P}_g}{\mathcal{P}_\Sigma}(R_s^{-1})$. In fact, the contribution around smoothing scale $R_s$ is dominant for the integrations for $\sigma^2_{g,s}$ and $\sigma^2_s$ due to the factor of $M_s$. Moreover $\mathcal{P}_\Sigma$ can be approximated by $\mathcal{P}^2_g$. Therefore $\sigma^2_{g,s}/\sigma^2_s$ is roughly given by $\mathcal{P}^{-1}_g$. On the other hand, for the case of nearly scale-independent $g$, local non-linearity parameter $f_{\text{NL},\Sigma}$ for $\Sigma$ can be written as $B_\Sigma(k_1,k_2,k_3)/(P_\Sigma(k_1)P_\Sigma(k_2) + 2 \text{ perms.})$, where $(2\pi)^3\delta^{(3)}(k_1 + k_2 + k_3)B_\Sigma(k_1,k_2,k_3) = \langle \Sigma(k_1)\Sigma(k_2)\Sigma(k_3) \rangle$ and $P_\Sigma(k) = |\Sigma(k)|^2$. According to Wick’s theorem, we can obtain $B_\Sigma(k_1,k_2,k_3) \sim \mathcal{P}_g(P_g(k_1)P_g(k_2) + 2 \text{ perms.})$. Moreover $(P_g(k_1)P_g(k_2) + 2 \text{ perms.})/(P_\Sigma(k_1)P_\Sigma(k_2) + 2 \text{ perms.}) \sim \mathcal{P}^{-2}_g$ if $g$ is almost scale-independent. Therefore $\sigma^2_{g,s}/\sigma^2_s \sim \mathcal{P}^{-1}_g$ indeed denotes the effective local non-linearity parameter $f_{\text{NL},\Sigma}$ for the case of chi-squared field.
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