Applicability of the continuum-discretized coupled-channels method to the deuteron breakup at low energies

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We re-examine the deuteron elastic breakup cross sections on $^{12}$C and $^{10}$Be at low incident energies, for which a serious discrepancy between the continuum-discretized coupled-channels method (CDCC) and the Faddeev–Alt-Grassberger-Sandhas theory (FAGS) was pointed out. We show the closed-channels neglected in the preceding study affect significantly the breakup cross section calculated with CDCC, resulting in good agreement with the result of FAGS.

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Introduction. Projectile breakup reactions have played a major role in studying the structure of loosely-bound nuclei [1]. Such a reaction contains at least three particles in the final state. Thus, one may say that the accurate description of the three-body breakup process is a minimum requirement for nuclear reaction theories. It is well known that the Faddeev theory [2], or, alternatively, the Alt-Grassberger-Sandhas (AGS) theory [3] gives the exact solution to such a three-body scattering problem. On the other hand, the continuum-discretized coupled-channels method (CDCC) [4–6] has widely been applied with high success to projectile breakup reactions at various incident energies. The theoretical foundation of CDCC was given in Refs. [7, 8] in connection with the distorted-wave Faddeev formalism [9]. Quite recently [10], invention of the treatment of the Coulomb interaction made the Faddeev-AGS theory (FAGS) applicable to various three-body breakup reactions, and the results of FAGS have directly been compared with those of CDCC. In many cases the two give very similar cross sections, which validates CDCC as an effective three-body reaction model, as predicted in Refs. [7, 8].

In a systematic comparison [11] between FAGS and CDCC, however, it was shown that at high incident energies $E_d$ of deuteron, $(d,p)$ transfer cross sections calculated with CDCC somewhat deviate from those with FAGS, i.e., the exact cross sections. More seriously, at $E_d$ below about 20 MeV, the deuteron elastic breakup cross sections obtained with CDCC overshoot those of FAGS by about a factor of three at most. The latter finding can particularly be a striking indication of the limitation of CDCC, suggesting that at low incident energies one has to rely on a more elaborated reaction model or exact FAGS for describing even elastic breakup processes. In Ref. [11], however, the so-called closed channels (see below) were not included. As mentioned in literature, e.g., Refs. [5, 8], inclusion of closed channels is crucial for quantitative discussion on observables, at low incident energies in particular. This has numerically been confirmed in Ref. [12] for a one-dimensional scattering problem, and in Ref. [13] for the scattering of $^{11}$Be. There exist several indications of the importance of closed channels also for transfer reactions [14–16]. Under the circumstances, in the present study, we revisit the problem reported on the low-energy elastic breakup cross sections for $^{10}$Be$(d,pn)^{10}$Be at $E_d = 21$ MeV and $^{12}$C$(d,pn)^{12}$C at $E_d = 12$ MeV, and discuss more in detail the convergence of CDCC results, putting emphasis on the closed channels.

CDCC and closed-channels. We give a brief review on CDCC; for more details, see, e.g., Refs. [4–6]. We describe the deuteron elastic breakup with the target nucleus $A$, on the basis of a $p + n + A$ three-body model. We do not explicitly take into account the excitation of $A$ during the breakup process. We neglect also the intrinsic spin of each of the three particles, following Ref. [11]. In CDCC the total three-body wave function of the total angular momentum $J$ and its projection $M$ is expanded in term of the complete set of the projectile wave function $\{\phi\}$:

$$\psi_{JM}(r, R) = \sum_{i=0}^{i_{max}} \sum_{\ell=0}^{\ell_{max}} \sum_{L=|J-\ell|}^{J+\ell} \phi_{i\ell}(r) \chi_c(R) \times \left[ i^L Y_\ell^J(\hat{r} \otimes \hat{R}) \right]_{JM},$$

where $r$ ($R$) is the coordinate of $p$ (the center-of-mass of $d$) relative to $n$ ($A$), $i$ is the energy index and $i = 0$ represents the ground state of $d$. The orbital angular momenta corresponding to $r$ and $R$ are denoted by $\ell$ and $L$, respectively; $Y_{im}$ is the spherical harmonics. We have put the channel indices of the scattering wave $\chi$ altogether in $c$, i.e., $c = \{J, i, \ell, L\}$. In the derivation of Eq. (1) we have discretized the $p$-$n$ continua with the so-called momentum-bin average method:

$$\phi_{i\ell}(r) = \frac{1}{\sqrt{\Delta k}} \int_{k_i}^{k_i+\Delta k} dk \ \varphi_{k,i}(r),$$

where $k_i = (i - 1) \Delta k$ and $\varphi_k$ is the partial wave of the $p$-$n$ scattering wave function under a $p$-$n$ interaction $V_{pn}$, with $k$ the absolute value of the asymptotic relative momentum. The discretized $p$-$n$ energy of the $i$th state ($i > 0$) is given by [4]

$$\epsilon_i = \frac{\hbar^2}{2\mu_{pn}} \sqrt{\frac{\Delta k^2 + (2k_i + \Delta k)^2}{4}},$$

where $\mu_{pn}$ is the $p$-$n$ reduced mass. The size $\Delta k$ of the momentum bin, the maximum linear momentum $k_{max} = \sqrt{\epsilon_i \mu_{pn}}$. 

\[k_{max} = \sqrt{\epsilon_i \mu_{pn}}\]
\[ i_{\text{max}} \Delta k \text{ (in the unit of } \hbar) \text{, and } \ell_{\text{max}} \text{ are key values for determining the reaction model-space of CDCC.} \\
\text{The asymptotic form of } \chi_c \text{ is given by} \\
\chi_c \rightarrow U^{(+)}_{L, \eta^i_i}(K, R)R_{cc0} - \sqrt{K_0/K_i} S_{cc0} f^{(+)}_{L, \eta^i_i}(K, R) \]  
for \( E_i > 0 \), and
\[ \chi_c \rightarrow -S_{cc0} W_{-\eta^i_i, L+1/2}(-2\text{i}K_i R) \]  
for \( E_i \leq 0 \), where \( E_i = E - \epsilon_i \) and \( K_i = \sqrt{2\mu E_i / \hbar^2} \); \( \epsilon_0 \) represents the incident channel. \( U^{(-)}_{L, \eta^i_i} \) \((U^{(+)}_{L, \eta^i_i})\) is the incoming (outgoing) Coulomb wave function with the Sommerfeld parameter \( \eta_i \) and \( W_{-\eta^i_i, L+1/2} \) is the Whittaker function. Channels having \( E_i > 0 \) and \( E_i \leq 0 \) are called open channels and closed channels, respectively. \( S_{cc0} \) for open channels are scattering matrix elements, with which physics observables are calculated in a standard manner. On the other hand, \( S_{cc0} \) for closed channels are not related to observables, at least directly. It is obvious, however, that the closed channels can affect the breakup observables through mainly continuum-continuum couplings \([8]\).

**Results and discussion.** In the CDCC calculation shown below, we disregard the intrinsic spins of \( p \) and \( n \) as mentioned, and also the Coulomb breakup. For \( V_{pn} \), we adopt the one-range Gaussian interaction of Ref. \([17]\), and for the nucleon-nucleus optical potential, we employ the CH89 global potential \([18]\). These are the same model setting as in Ref. \([11]\). We use \( \Delta k = 0.05 \text{ fm}^{-1} \) and \( \ell_{\text{max}} = 8 \) for all the calculation shown below. As for \( \ell_{\text{max}} \), we take 0.9 fm\(^{-1}\) for \(^{12}\text{C}(d, pn)^{12}\text{C} \) at \( E_d = 12 \text{ MeV} \) (Fig. 1) and 1.1 fm\(^{-1}\) for other two reactions (Figs. 2 and 3). We have checked the convergence of the breakup cross sections by further increasing the model space, and thereby convergence with 98\% accuracy has been confirmed. In the multipole expansion of the nucleon-nucleus optical potential, we take the multipoles \( \lambda \) up to 16; it turned out that the multipoles for \( \lambda > 8 \) have no effect on the results shown below.

Figure 1(a) shows the angular distribution of the deuteron breakup cross section on \(^{12}\text{C} \) at \( E_d = 12 \text{ MeV} \) integrated over the \( p-n \) breakup energy \( \epsilon \). The horizontal axis is the scattering angle \( \theta \) of the center-of-mass of the \( p-n \) system. The solid line is the converged result of CDCC that agrees well with the result of FAGS (dash-dotted line) taken from Fig. 9(a) of Ref. \([11]\). The dashed line in Fig. 1(a) is the CDCC result calculated with including open channels only, as in Ref. \([11]\), which seems to be inside the hatched band in Fig. 9(a) of Ref. \([11]\). One sees in Fig. 1(a) a significant reduction of the cross section due to the coupling with the closed channels. Although still a small difference remains between the
FIG. 3: (Color online) Same as Fig. 1 but for $^{10}$Be at $E_d = 21$ MeV.

The disagreement found in the high $\epsilon$ region will need further investigations.

Next we show in Fig. 2 the results for $^{12}$C at $E_d = 56$ MeV. For this reaction, no significant difference between CDCC and FAGS was reported in Ref. [11]. It is quite natural that the coupling to the closed-channels is less important at higher incident energy. One can clearly see this for both angular distribution (Fig. 2(a)) and breakup energy distribution (Fig. 2(b)). In fact, the adopted $k_{\text{max}}$ (0.9 fm$^{-1}$) for this reaction that gives convergence is very close to the threshold of the open channels, 1.05 fm$^{-1}$. It is thus quite trivial that the two lines agree with each other in Fig. 2(a) and Fig. 2(b). In any case, checking the convergence with respect to $k_{\text{max}}$ is necessary.

Figure 3 is the result for $^{10}$Be($d$, $pn$)$^{10}$Be at $E_d = 21$ MeV. The role of the closed channels and the agreement between the converged CDCC and FAGS are the same as in Fig. 1 although the role of the odd partial waves is appreciable in this reaction.

Summary. We have reinvestigated deuteron elastic breakup reactions on $^{12}$C and $^{10}$Be at low incident energies, in which significant difference in the cross sections between CDCC and FAGS was reported [11]. We checked carefully the convergence of CDCC, with respect to the maximum $p$-$n$ breakup momentum $k_{\text{max}}$ in particular. The crucial importance of the closed channels was shown, and the converged CDCC results agree well with the FAGS results shown in Ref. [11]. At higher energy, the closed channels turned out to less important, as expected.

In conclusion, we have demonstrated the applicability of CDCC to elastic breakup reactions $^{10}$Be($d$, $pn$)$^{10}$Be at $E_d = 21$ MeV and $^{12}$C($d$, $pn$)$^{12}$C at $E_d = 12$ MeV, by confirming the convergence of the CDCC model space with respect to $k_{\text{max}}$. As a next step, a more systematic investigation on the role of closed channels, in transfer reactions in particular, will be important.

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