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Authors
Kim, Arnold D
Hayakawa, Carole
Venugopalan, Vasan

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Estimating optical properties in layered tissues by use of the Born approximation of the radiative transport equation

Arnold D. Kim

School of Natural Science, University of California, Merced, P. O. Box 2039, Merced, California 95344

Carole Hayakawa and Vasan Venugopalan

Laser Microbeam and Medical Program, Beckman Laser Institute, Department of Chemical Engineering and Materials Science, University of California, Irvine, Irvine, California 92697-2575

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Determining optical properties of layered tissue structures is important for biomedical diagnostics. Several methods to determine layered tissue properties have been developed, but the vast majority of them use the diffusion approximation of the radiative transport equation (RTE). These methods are limited to systems whose reduced scattering coefficients are much larger than their absorption coefficients and that have a minimum layer thickness of the order of a few transport mean free paths [l' = 1/(μa + μs')]. However, several tissue systems exist that do not satisfy these requirements. Of prime importance is epithelial tissue, which consists of a cellular layer supported by an underlying stroma. The cellular layer has a characteristic thickness of ≤500 μm, which is significantly smaller than a transport mean free path. Hence methods based on the diffusion approximation cannot be applied credibly for evaluation of epithelial–stromal transformations. Specialized techniques such as elastic scattering spectroscopy and differential path-length spectroscopy have been developed to measure properties of the epithelium alone. While they are valuable, such techniques are unable to measure and isolate concomitant changes in the underlying stroma.

Using the Born approximation (BA) of the RTE, we formulate a novel method with which to determine optical properties of layered media. This method applies over a broad range of optical properties and to layer thicknesses of the order of a single scattering mean free path. Using the BA, we recover simultaneously the absorption and scattering coefficients in a single layer of a two-layer medium from spatially resolved reflectance data. In the calculations that follow, we assume that the optical properties of the other layer are known a priori. However, this method extends readily for more general problems. The estimates for the optical properties are the solution of an N×2 linear, least-squares problem with N≥2 denoting the number of measurements. Using Monte Carlo simulations for measured data, we show that this BA method is effective over a large range of parameter values.

Continuous-wave light transport in tissues is governed by the time-independent RTE

\[ \omega \cdot \nabla \Psi + \mu_a \Psi - \mu_s \mathcal{L} \Psi = Q. \]

The radiance \( \Psi \) depends on direction \( \omega \), a vector on unit sphere \( S^2 \), and position \( r \). Here, \( \mu_a \) and \( \mu_s \) are the absorption and scattering coefficients, respectively. Scattering operator \( \mathcal{L} \) is defined as

\[ \mathcal{L} \Psi = -\Psi + \int_{S^2} f(\omega' \rightarrow \omega) \Psi(\omega', r) d\omega'. \]

The scattering phase function \( f \) gives the fraction of light traveling initially in direction \( \omega' \) that scatters into direction \( \omega \). In Eq. (1), \( Q \) denotes an interior source.

Green’s function \( G(\omega, r; \omega', r') \) for half-space \( D = \{ z>0 \} \) composed of a uniform turbid medium is the solution of Eq. (1) with \( \mu_a \) and \( \mu_s \) constants and \( Q(\omega, r) = \delta(\omega - \omega') \delta(r - r') \), with \( r, r' \in D \). In addition, we prescribe that \( \Psi \) satisfy the boundary condition

\[ G|_{a_s,z=0} = R(\omega)G|_{a_s,z=0}, \quad \omega \cdot \hat{z} > 0. \]

Here \( R(\omega) \) denotes the directional variation of the Fresnel reflection coefficient that is due to the refractive-index mismatch at the boundary. For this study we use the analytical representation of this half-space Green’s function that is computed in terms of plane-wave solutions.

When the absorption and scattering coefficients are spatially heterogeneous, we represent them as

\[ \mu_{a,s}(r) = \mu_{a,s,0} + \delta \mu_{a,s}(r), \]

respectively. According to the BA,
Using relation (5), we model the reflectance for the \( /H9254/H9262 \) spectrally, with the mal layer (Fig. 1). The baseline optical properties in the \( /H9254/H9262 \) perturbed, so its optical properties are representing constant perturbations. We assume that the other layer is unperturbed, so its optical properties are \( /H9254/H9262 \) and \( /H9254/H9262 \). Using relation (5), we model the reflectance for the two cases, \( /H9254/H9262 \) and \( /H9254/H9262 \), by
\[
F^{t,b}_{a}(p) = F_{a}(p) - \delta_{a} F^{t,b}_{a}(p), \quad \delta_{a} > 0 \quad \text{and} \quad \delta_{b} > 0,
\]
respectively. The quantities \( F_{a} \) and \( F^{t,b}_{a} \) are defined as
\[
F_{a}(p) = \int_{S_{\text{NA}}} \hat{\omega} \cdot \hat{z} \Psi_{0}(\omega, p, 0) d\omega,
\]
\[
F^{t,b}_{a}(p) = \int_{S_{\text{NA}}} \hat{\omega} \cdot \hat{z} \Psi_{0}(\omega, p, 0) d\omega.
\]
In Eqs. (8), \( S_{\text{NA}} \) denotes the set of directions within the numerical aperture of the detector. Notice that in Eqs. (8b) and (8c) the spatial integration is restricted to either the top or the bottom layer.

Suppose that data are collected at \( N \geq 2 \) detectors located at \( /H11568/H20849/H20850/H11032 \). Rearranging terms in Eq. (7), we obtain the following linear system:
\[
A_{t,b} \begin{bmatrix} \delta_{a} \\ \delta_{b} \end{bmatrix} = y.
\]
Row \( /H20849/H20850/H20851/H11032 \) of the \( /H20849/H20850/H20851/H11032 \times /H20849/H20850/H20851/H11032 \) matrices \( A_{t,b} \) is given by \( \{-F_{a}^{t,b}(p_{n}), F_{a}^{t,b}(p_{n})\} \). \( /H20849/H20850/H20851/H11032 \times /H20849/H20850/H20851/H11032 \) vector \( y \) has entries \( /H20849/H20850/H20851/H11032 \). In fact, only \( /H20849/H20850/H20851/H11032 \) measurements are needed to compute the perturbations. However, we incorporate more measurements to help stabilize this system under the influence of instrument noise. Hence we seek the least-squares solution of Eq. (9) to obtain estimates for both \( /H20849/H20850/H20851/H11032 \) and \( /H20849/H20850/H20851/H11032 \).

To demonstrate the utility of our inversion scheme, we generate measured data \( /H20849/H20850/H20851/H11032 \) with Monte Carlo simulations of radiative transport in a two-layer model system representative of cervical epithelium. The epithelium is represented by a \( /H20849/H20850/H20851/H11032 \) thick cellular layer supported by a semi-infinite stromal layer (Fig. 1). The baseline optical properties in both layers are taken to be those of normal cervical stromal tissue at \( /H20849/H20850/H20851/H11032 = 849 \text{ nm} : \mu_{a} = 0.034/\text{mm} \) and \( /H20849/H20850/H20851/H11032 \).
tively. In Fig. 2(a) we show results for seven simulated measurements in which \( \mu_a \) is held fixed and \( \mu_s \) varies from 33% to 300% of \( \mu_{a0} \). The recovered optical properties are shown in Fig. 3(a). In Fig. 3(a) we show results from seven simulated measurements in which \( \mu_a \) is held fixed and \( \mu_s \) varies from 33% to 300% of \( \mu_{a0} \). The diagonal line indicates the true \( \mu_s \) values, and the horizontal line indicates the true values of \( \mu_a \). For the data in which \( \mu_s \) is perturbed, 80% to 120% of \( \mu_{a0} \), the recovered values of \( \mu_s \) and \( \mu_a \) have relative errors within 15% and 18%, respectively. Outside this range, the recovered values for \( \mu_s \) are of similar quality. However, the recovered values of \( \mu_a \) are poorer, with the maximum relative error exceeding 40%.

For \( \mu_a \) and \( \mu_s \) perturbations in the bottom layer [Fig. 1(b)], the recovered optical properties are shown in Fig. 3. In Fig. 3(a) we show results from seven simulated measurements in which \( \mu_a \) is held fixed and \( \mu_s \) varies from 33% to 300% of \( \mu_{a0} \). The recovered values of \( \mu_a \) and \( \mu_s \) have relative errors within 25% and 5%, respectively. The recovered values of \( \mu_a \) degrade systematically as the perturbation grows. In Fig. 3(b) we show results from five simulated measurements in which \( \mu_s \) is held fixed and \( \mu_a \) varies from 60% to 140% of the baseline value. The recovered values of \( \mu_a \) and \( \mu_s \) have relative errors within 25% and 3%, respectively. The bottom-layer results are poorer than those of the top layer. This is so because the BA is less able to account for perturbations distributed over a large volume. Nonetheless, application of the BA to recover the optical properties of the bottom layer provides accurate trends of the perturbed variables. Moreover, it does so without significant coupling between the optical parameters.

We have demonstrated the use of the Born approximation of the radiative transport equation to estimate optical properties in a single layer of a twolayer medium. This method applies to systems with length scales smaller than \( l^* \) because it is based on the RTE rather than on the diffusion approximation. The BA of the RTE has been used in imaging applications of photon migration. Our implementation of the BA of the RTE for the two-layer problem uses an analytical representation of Green’s function and yields an \( N \times 2 \) linear, least-squares problem. Hence the associated computational costs are small.

This method requires knowledge of the optical properties of some reference medium that is close to the true medium. This \textit{a priori} information is contained implicitly in the computation of the half-space Green’s function. Nonetheless, this method yields results that are accurate for a broad range of optical parameters \( 6 \leq \mu'_s / \mu'_a \leq 25 \) that are not close to the reference medium. These data suggest that the method does not depend strongly on this \textit{a priori} knowledge. Moreover, we have assumed here knowledge of asymmetry parameter \( g \) and layer thickness \( z_0 \) and of which layer is perturbed. However, the BA is not restricted by these assumptions and can be generalized to take them into account. These issues require more attention and will be subjects of future research that includes validation by use of experimental measurements of layered tissue phantoms.

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