Supergravity corrections to the \((g - 2)\)\(_l\) factor by Implicit Regularization

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We apply Implicit regularization in the calculation of the one-loop graviton and gravitino corrections to the anomalous magnetic moment of the lepton in unbroken supergravity. This is an important test for any regularization method. We find a null result as it is expected from supersymmetry. We compare our results with the ones obtained by using Differential Regularization and Dimensional Reduction, which are known to preserve supersymmetry at one-loop order.

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I. INTRODUCTION

One fundamental test for any regularization method is its applicability in supersymmetric theories. This is because the dynamics of the best candidates of fundamental theories is believed to respect supersymmetry. Searching for ideal physical calculations to perform consistency tests in this area, one finds among others, the anomalous magnetic moment of the lepton in supergravity (SUGRA), the local version of supersymmetry (SUSY). In spite of being a non-renormalizable theory, gravity yields a finite result in one-loop correction to this observable \[1\]. More restrictively, its combination with SUSY imposes that the \((g - 2)\)\(_l\) factor vanishes \[2\], because there is no Pauli term in the Lagrangian of a chiral supermultiplet (in this context, there is a generalization to a set of sum rules for any charged SUSY multiplet \[3\], \[4\]). Hence, if we want a regularization technique to respect supersymmetry, it must implement, order by order, the cancellation of the quantum corrections.

Some methods were tested in this context \[5\], \[6\], \[7\], \[8\], \[12\], \[15\], \[18\], and it is well known that Dimensional Regularization \[5\] does not provide a null result, though being finite \[7\]. Dimensional Reduction \[11\], which is supersymmetric invariant, at least at one-loop order (some improvements are being established at higher order calculations \[11\]), as well as Differential Regularization \[12\]–\[18\] in its constrained version, were shown to give a cancellation between the contributions from the graviton and the gravitino sectors \[2\], \[7\], \[14\], \[15\], \[18\].

In this work, we intend to perform this important test in the context of Implicit Regularization (IR). The basic idea behind the method is to implicitly assume some (unspecified) regulating function as part of the integrand of divergent amplitudes and to separate their regularization dependent parts from the finite one. Symmetries of the model, renormalization or phenomenological requirements determine arbitrary parameters introduced by this procedure \[19\]–\[28\]. In fact, there is a special choice of the parameters that automatically preserves the symmetries in all non-anomalous cases we have studied. The possibility of these parameters being fixed at the beginning of the calculation is desirable, since it simplifies a lot the application of the method.

The technique has been shown to be tailored to treat theories with parity violating objects in integer dimensions. This is the case of chiral and topological field theories. The ABJ anomaly \[30\], and the radiative generation of a Chern-Simons-like term, which violates Lorentz and CPT symmetries \[21\], \[22\] are examples where the technique was successfully applied. Moreover the method was shown to respect gauge invariance in both Abelian and non-Abelian theories at one-loop order \[21\], \[22\], \[23\], \[24\], \[25\]. The calculation of the \(\beta\)-function of the massless Wess-Zumino model (at three loops) was also performed as a test of the procedure \[25\].

This paper is divided as follows: in the section II we make a short review of the Implicit Regularization method; in section III we establish the problem to be treated; in section IV, we make the calculation of the anomalous magnetic moment of the lepton in unbroken supergravity and compare ours with previous results. Finally, in section V some concluding comments

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are presented.

II. A SHORT REVIEW OF THE IMPLICIT REGULARIZATION TECHNIQUE

In order to implement the procedures prescribed by the Implicit Regularization technique, it is necessary that the amplitude be regularized. We begin by recalling the basic steps of the method:

- some (as yet unspecified) Regularization is applied to the amplitude, so as to allow for operations with the integrand. It will be indicated by the index \( \Lambda \) in the integrals;
- we judiciously use the identity,

\[
\frac{1}{(p-k)^2 - m^2} = \frac{1}{(k^2 - m^2)} - \frac{p^2 - 2 p \cdot k}{(k^2 - m^2)(p-k)^2 - m^2), \tag{1}
\]

until the regularization dependent integrals (those that would be divergent) do not depend on the external momentum;
- we can solve the finite part which is regularization independent and use a subtraction scheme such that the remaining regularization dependent integrals are eliminated.

In order to give an example of the use of these steps, we apply the method to the simple logarithmically divergent one-loop amplitude below:

\[
I = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)(k^2 - m^2)^2} \tag{2}
\]

By applying identity \( \text{(1)} \) in the regularized amplitude above, we get

\[
I = I_{\text{log}}(m^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^2} \tag{3}
\]

where \( k \) stands for \( I \delta^4 k/(2\pi)^4 \) and we have defined the basic one-loop logarithmically divergent object,

\[
I_{\text{log}}(m^2) = \int \frac{1}{(k^2 - m^2)^2}. \tag{4}
\]

By applying the regularization independent relation

\[
I_{\text{log}}(m^2) = I_{\text{log}}(\lambda^2) + b \ln \left( \frac{m^2}{\lambda^2} \right), \tag{5}
\]

with \( b = (4\pi)^2 \). The mass parameter \( \lambda^2 \) is suitable to be used as the renormalization group scale, as it can be seen in refs. \( \text{[25, 28]} \). After solving the finite part, we are left with

\[
I = I_{\text{log}}(\lambda^2) - b Z_0(p^2, m^2, \lambda^2), \tag{6}
\]

where

\[
Z_0(p^2, m^2, \lambda^2) = \int_0^1 dx \ln \left( \frac{p^2 x (1-x) - m^2}{\lambda^2} \right). \tag{7}
\]

Next we would like to address one of the essential aspects of IR: its application to gauge theories and the kind of constraint that should be introduced in order to preserve this symmetry. There is also the question of how to deal with anomalous theories. These aspects are suitably discussed in the ref. \( \text{[22]} \), but we will shortly discuss it here, since it is closely related to the preservation of SUSY. Let us consider the vacuum polarization tensor of QED at one-loop order. In \( \text{[22]} \), where an arbitrary routing in the loop momenta was used, we obtained

\[
\Pi_{\mu\nu} = \Pi(p^2)(p_\mu p_\nu - p^2 \eta_{\mu\nu})
+ 4 \left( \frac{\eta_{\mu\nu}^2}{2} - \frac{1}{2}(k_1^2 + k_2^2) \right) \eta_{\mu\nu} + \frac{1}{3}(k_1^2 + k_2^2) \eta_{\mu\nu} \eta_{\alpha\beta} 
- (k_1 + k_2) \eta_{\mu\nu} \eta_{\alpha\beta}
- \frac{1}{2}(k_1^2 + k_2^2) \eta_{\mu\nu} \eta_{\alpha\beta}, \tag{8}
\]

with \( p = k_1 - k_2 \), the external momentum. In the equation above, \( \Pi(p^2) \) includes the divergent (regularization dependent) parts. We also have the following relations:

\[
\Pi_{\mu\nu} = \Pi(p^2) - 2 \int_k \frac{k_{\mu} k_{\nu}}{(k^2 - m^2)^2}, \tag{9}
\]

and

\[
\Pi_{\mu\nu} = \Pi(p^2) - 4 \int_k \frac{k_{\mu} k_{\nu}}{(k^2 - m^2)^2}, \tag{10}
\]

The above differences between integrals of the same degree of divergence are arbitrary constants. In ref. \( \text{[21]} \), these constants were shown to have to vanish if we want an amplitude independent of the choice of the momentum routing in the loops. This is also clear from eq. \( \text{(5)} \). Moreover, by setting them to zero, gauge invariance is assured. Nevertheless, even if momentum routing invariance is violated, it is possible to find...
another choice that preserves gauge invariance. Let us choose $k_2 = 0$ and parametrize these differences such that
\[
\gamma^2_{\mu
u} = \mu^2 \alpha_1 \eta_{\mu
u}, \quad \gamma^0_{\mu
u} = \alpha_2 \eta_{\mu
u}, \quad \gamma^0_{\mu\nu\alpha\beta} = \alpha_3 \eta_{\mu\nu} \eta_{\alpha\beta},
\]
so that
\[
p^\mu \Pi_{\mu
u} = 4 \left( \mu^2 \alpha_1 + (\alpha_3 - 2\alpha_2) \rho^2 \right) p_\nu.
\]
It is clear that if we take $(\alpha_1, \alpha_2, \alpha_3) = (0, \alpha_2, 2\alpha_2)$, gauge invariance is restored. The point here is the following: as long as we do not have any anomaly, we can simply set all those $\gamma$'s to zero and be sure of the conservation of gauge symmetry. We can call it the constrained version of Implicit Regularization.

Next we justify why this choice seems to be the natural one. As long as the symmetry is manifest in the Lagrangian of the theory, a regularization technique that extends the properties of regular integrals to the regularized ones would naturally respect this symmetry. This is the spirit in which we have based the Dimensional Regularization (see the section 3 of [1]) and the Constrained Differential Regularization, for example. One of these important properties is the possibility of making shifts. This means that in regular integrals surface terms are null. By analyzing the equations above, we see that if we want to eliminate all the surface terms, then we must choose $\alpha$'s = 0, since the Consistency Relations listed above are, in fact, proportional to surface terms. We can write:
\[
\gamma^0_{\mu\nu} = \int_k \frac{\partial}{\partial k^\nu} \left( \frac{k_\nu}{(k^2 - m^2)^2} \right),
\]
and
\[
\gamma^2_{\mu\nu} = \int_k \frac{\partial}{\partial k^\nu} \left( \frac{k_\nu}{(k^2 - m^2)^2} \right).
\]
For $\gamma^0_{\mu\nu\alpha\beta}$, we have
\[
\int_k \frac{\partial}{\partial k^\alpha} \left[ \frac{4k_\mu k_\nu k_\alpha}{(k^2 - m^2)^3} - \frac{k_\alpha \eta_{\mu\nu} + k_\mu \eta_{\nu\alpha} + k_\nu \eta_{\alpha\mu}}{(k^2 - m^2)^2} \right] = \eta_{\mu\nu} \eta_{\alpha\beta} (\alpha_3 - 2\alpha_2),
\]
where we have made use of equations (10), (14) and (12). If the expression of eq. (15) vanish, we find the alternative condition to respect gauge invariance in the QED vacuum polarization tensor discussed above. In this particular case, it was not necessary to make also $\alpha_2 = 0$, but this situation will not hold in general. An important remark is that the parameters that come from logarithmically divergent integrals are finite. As surface terms, they can be calculated by symmetric integration. The procedure of making them zero corresponds, indeed, to automatically add local symmetry restoring counterterms in order to cancel surface terms. In Dimensional Regularization, they are null, as it would be all surface terms. This principle can be applied to higher order calculations [20]. At n-loop order, besides the CR of all the previous orders, some others will be needed. Generally, a Consistency Relation with $n$ Lorentz indices can be obtained from the integral of the $n$-th order derivative of the corrected fermion internal line.

The situation changes completely if we are treating anomalous processes as, e.g., the anomalous pion decay (axial–vector–vector triangle diagram) [30]. In this case, in order to have a “democratic” choice of the preserved identities, one is forced to violate momentum routing invariance, as shown in ref. [22].

III. SUGRA LAGRANGIAN AND THE STATEMENT OF THE PROBLEM

We are interested in the coupling of supergravity with matter, which has been vastly described in literature [51]. We are considering the linearized interaction Lagrangian density of superQED-SUGRA, which in Minkowski space is given by
\[
\mathcal{L}_I = \sum_{(\ell = 1, 2, \ldots, N_{SUGRA})} \sum_{(A_\mu, F_{\mu\nu}^A, \chi_\mu, \lambda)} \frac{i}{2} e A_\mu \bar{\Psi} \not{\nabla} \gamma_\mu \Psi
\]
\[
- e \sqrt{2} (\lambda \gamma_\mu P_L \bar{\Psi} + h.c.) + (L \leftrightarrow R)
\]
\[
- \frac{\kappa}{4} h^{\alpha\beta} \left[ (i \bar{\Psi} (\gamma_\alpha \partial_\beta + \gamma_\beta \partial_\alpha) \Psi + h.c.)
\]
\[
- 2e \bar{\Psi} (\gamma_\alpha A_\beta + \gamma_\beta A_\alpha) \Psi
\]
\[
- \frac{\kappa}{\sqrt{2}} \left[ \bar{\chi}^\nu P_L (i \not{\partial} - m) \gamma_\nu \gamma_\nu \Psi
\]
\[
+ e \bar{\chi}^\nu P_L A_\nu \gamma_\nu \Psi + h.c.) + (L \leftrightarrow R)
\]
\[
+ \kappa h^{\alpha\beta} (F_{\mu\nu} F_{\mu\nu}^A - \frac{1}{4} \eta_{\alpha\beta} F_{\mu\nu} F^{\mu\nu})
\]
\[
+ \left( \frac{i}{8} \lambda \gamma_\nu [\not{\partial}, \not{A}] \gamma_\mu + h.c. \right),
\]
where $\not{\partial} = \partial - \not{\partial}$, $P_{R,L} = \frac{1}{2} (1 \pm \gamma_5)$ are the usual chiral projectors and we have $\kappa^2 = 8\pi G_N$, $G_N$ being the Newton’s gravitational constant. The following notation is used for the fields: $\Psi$ for the lepton, $A_\mu$ for the photon, $h_{\mu\nu}$ for the graviton, $\varphi_{L,R}$ for the slepton, $\lambda$ for the photino and $\chi_\mu$ represents the gravitino.

In our problem, the determination of the $(g-2)$ factor to one-loop order, we are interested in the quantum corrections to the vertex lepton-photon-lepton. By Lorentz invariance and current conservation, we must obtain something of the form
\[
e A_\mu \bar{\Psi} \left[ \gamma_\mu F_1(q^2) + i e_{\mu\nu} \frac{p - p'}{2m} F_2(q^2) \right] \Psi,
\]
\[ (\not p + m) \Delta_F (p^2) \]

\[ \Delta_F (p^2) \]

\[ g_{\mu \nu} \Delta_F (p^2), \quad \mu^2 = 0 \]

\[ \frac{1}{2} \{ g_{\mu \nu} g_{\rho \sigma} + g_{\mu \rho} g_{\nu \sigma} - g_{\mu \sigma} g_{\nu \rho} \} \Delta_F (p^2), \quad \mu^2 = 0 \]

\[ -1/2 \gamma_{\mu} \not p \not \gamma_{\mu} \Delta_F (p^2), \quad \mu^2 = 0 \]

\[ \frac{1}{2} \{ g_{\mu \nu} g_{\beta \gamma} - g_{\mu \beta} g_{\nu \gamma} - g_{\mu \gamma} g_{\nu \beta} + g_{\alpha \beta} g_{\gamma \nu} p_{\mu} - 1/2 g_{\alpha \beta} (p \cdot p' g_{\mu \nu} - p_{\mu} p_{\nu}') \}. \]

\[ \frac{1}{2} \{ g_{\mu \nu} g_{\beta \gamma} - g_{\mu \beta} g_{\nu \gamma} - g_{\mu \gamma} g_{\nu \beta} + g_{\alpha \beta} g_{\gamma \nu} p_{\mu} - 1/2 g_{\alpha \beta} (p \cdot p' g_{\mu \nu} - p_{\mu} p_{\nu}') \}. \]

\[ \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \]

\[ \tilde{g} \rightarrow e : i \frac{e}{\sqrt{2} P_{L,R}} (p' + m) \gamma_{\nu} \]

\[ e \rightarrow \tilde{g} : -i \frac{e}{\sqrt{2} P_{L,R}} (p' - m) \]

\[ \tilde{\gamma} \rightarrow \tilde{g} : -\frac{i}{4} \gamma_\alpha [\not p, \gamma_\mu] \]

\[ \tilde{g} \rightarrow \tilde{\gamma} : \frac{i}{4} [\gamma_\mu, \not p] \gamma_\alpha \]

\[ \tilde{g} \rightarrow \tilde{g} : \frac{i}{\sqrt{2}} \gamma_\mu \gamma_\alpha P_{L,R} \gamma_\nu \gamma_\mu P_{L,R} \]
can be suitably arranged such that we have

$$e \Lambda_{\mu} \bar{\Psi} \left[ \gamma^\mu (F_1 + F_2) + i \frac{(p + p')^\mu}{2m} F_2 \right] \Psi. \quad (19)$$

Since we are interested only in $F_2$, we will not consider the terms in $\gamma^\mu$ in the calculations to be done. The diagrams which contribute to the one-loop correction to the lepton-photon-lepton vertex are displayed in fig. 2. So, we have

$$\Lambda_{\mu} = \sum_{j=1}^{10} \Lambda_{\mu}^{(j)}. \quad (20)$$

IV. THE $(g - 2)_\mu$ FACTOR BY IMPLICIT REGULARIZATION

We will now determine the anomalous magnetic moment of the lepton as stated in the previous section. We will explicitly calculate here the contribution $\Lambda_{\mu}^{(2)}$, represented by the second diagram of figure 2. Using the Feynman rules of figure 1, we can write

$$\Lambda_{\mu}^{(2)} = \int_k^\Lambda \frac{1}{D} \frac{ik}{2} \gamma_\alpha i(p' - \bar{k} + m)(2p' - k)\beta \times \frac{i}{2} (\eta^\alpha \eta^\beta + \eta^\beta \eta^\alpha - \eta^\alpha \eta^\beta) \times (-i \epsilon \kappa \eta_{\rho \sigma} \gamma_\sigma) = -\frac{e \kappa^2}{4} \int_k^\Lambda N_{\mu} \frac{1}{D}, \quad (21)$$

where

$$D = [(p' - k)^2 - m^2]k^2 \quad (22)$$

and

$$N_{\mu} = \gamma_\mu (p' - \bar{k} + m)(2p' - k) + \gamma_\alpha (p' - \bar{k} + m)\gamma^\alpha (2p' - k)\mu - (2p' - \bar{k})(p' - \bar{k} + m)\gamma_\mu. \quad (23)$$

If the on-shell condition and the Clifford algebra are used, we get

$$N_{\mu} = (-4m^2 + 4m\bar{k} - 2(p \cdot k))\gamma_\mu + (8m + 6\bar{k})p'_\mu + (-6m - 2\bar{k})k_\mu, \quad (24)$$

which, when substituted in eq. (21), gives

$$\Lambda_{\mu}^{(2)} = -\frac{e \kappa^2}{4} [8mp'_\mu I(p') + 6\gamma^\alpha p'_\mu I_\alpha(p') - 6mI_\mu(p') - 2m\gamma_\alpha I_{\alpha\mu}(p') + A\gamma_\mu]. \quad (25)$$

In the expression above, $A$ represents all the terms that multiply $\gamma_\mu$ which are irrelevant for the problem in question. There are also divergent integrals

$$I, I_{\mu}, I_{\mu\nu}(p') = \int_k^\Lambda \frac{1}{k^2 - (p' - k)^2} \frac{1}{m^2} \frac{1}{m^2} \frac{1}{m^2}, \quad (26)$$

which, on-shell evaluated, yield the following results:

$$I = I_{log}(\lambda^2) + b \ln \left( \frac{\lambda^2}{m^2} \right) + 2b, \quad (27)$$

$$I_{\mu} = \frac{p'_\mu}{2} \left\{ I_{log}(\lambda^2) + b \ln \left( \frac{\lambda^2}{m^2} \right) + b + \alpha_2 \right\}, \quad (28)$$

and

$$I_{\mu\nu} = \frac{1}{3} p'_{\mu} p'_{\nu} \left\{ I_{log}(\lambda^2) + b \ln \left( \frac{\lambda^2}{m^2} \right) + \frac{2}{3} b + \alpha_3 \right\} + \eta_{\mu\nu} \text{ terms}, \quad (29)$$

FIG. 2: SUGRA-QED contributions to the $ee\gamma$ vertex
with the $\alpha_1$'s being the arbitrary parameters defined in eq. (12). If we substitute the results of the integrals defined in the appendix, we obtain

$$
\Lambda_\mu^{(2)} = -\frac{e\kappa^2}{4}\left\{\frac{22}{3} I_{\text{log}}(\lambda^2) + \frac{22}{3} b \ln \left(\frac{\lambda^2}{m^2}\right) + \frac{140}{9} b \frac{2}{3} \alpha_3 \right\} m p'_\mu + A_{\gamma\mu}.
$$

This amplitude, when added up to $\Lambda_\mu^{(3)}$, obtained by the interchange $p \leftrightarrow p'$, yields for the coefficient of $(p + p')_\mu$:

$$
-\frac{e\kappa^2}{4} m \left\{\frac{22}{3} I_{\text{log}}(\lambda^2) + \frac{22}{3} b \ln \left(\frac{\lambda^2}{m^2}\right) + \frac{140}{9} b \frac{2}{3} \alpha_3 \right\}.
$$

So, given that $\kappa^2 = 8\pi G_N$, $b = i/(4\pi)^2$ and, by the definition of $F_2$, we must multiply the coefficient of $(p + p')_\mu$ by $2m/(ie)$, we find

$$
F_2^{(2+3)} = \frac{G N m^2}{\pi} \left\{-\frac{11}{6} \left(\frac{1}{b} I_{\text{log}}(\lambda^2) + \ln \left(\frac{\lambda^2}{m^2}\right)\right) - \frac{35}{9} \frac{8}{3} i \pi^2 \alpha_3 \right\}. 
$$

The same procedure is applied to obtain the remaining contributions to the total vertex $\Lambda_\mu$. The on shell results in terms of the integrals defined in the appendix are:

$$
\tilde{\Lambda}_\mu^{(1)} = \frac{\kappa^2 e}{8} \left\{2(p'_\mu - m \gamma_\mu) \gamma^\alpha (4m^2 J_\alpha + I_\alpha(p)) + 2 \gamma^\alpha (2p'_\mu - m \gamma_\mu) (4m^2 J_\alpha + I_\alpha(p')) - 24m^2 \gamma^\alpha J_{\alpha p} + 8m (2I\mu(p, p') - I_\mu(p) - I_\mu(p')) - 8m (p + p')_\mu (p, p') \right\},
$$

$$
\tilde{\Lambda}_\mu^{(4)} + \tilde{\Lambda}_\mu^{(5)} = \frac{\kappa^2 e}{2} \left\{-p_\mu \gamma^\alpha (I_\alpha(p) - I_\alpha(p')) + \gamma^\alpha (2p'_\mu - m \gamma_\mu) (4m^2 J_\alpha + I_\alpha(p')) + m (\gamma^\alpha \gamma_\mu) (I_\alpha(p) - I_\alpha(p')) + 2 (p + p')_\mu (\gamma^\alpha I_\alpha(p, p')) - 2m (I_\mu(p) + I_\mu(p')) + 4m I\mu(p, p') + 8m^2 J'_{\mu p} + 8m^2 I_\mu(p, p') - 8m^2 \gamma^\alpha J_{\alpha p} + p \leftrightarrow p' \right\},
$$

$$
\tilde{\Lambda}_\mu^{(6)} = -\frac{\kappa^2 e}{4} \left\{4(p + p')_\mu (m I(p) + I(p')) + \gamma^\alpha I_\alpha(p) + I_\mu(p')\right\] - 8 [m (I_\mu(p) + I_\mu(p')) + \gamma^\alpha (I_\mu(p) + I_\mu(p'))] + 12m^2 \gamma^\alpha J_{\alpha p} + 24m^2 \gamma^\alpha J_{\alpha p},
$$

$$
\tilde{\Lambda}_\mu^{(7)} + \tilde{\Lambda}_\mu^{(8)} = 2\kappa^2 e \left\{p'_\mu \gamma^\alpha I_\alpha(p') - m I_\mu(p') - \gamma^\alpha I_\mu(p') + p \leftrightarrow p' \right\},
$$

$$
\tilde{\Lambda}_\mu^{(9)} + \tilde{\Lambda}_\mu^{(10)} = \kappa^2 e \left\{2p'_\mu [m I(p') - m I(p, p')] + \gamma^\alpha I_\alpha(p, p') + 4m^2 \gamma^\alpha J_{\alpha p} - 2m^3 J\right\} + (p - p')_\mu [\gamma^\alpha (p, p') - m I(p, p')] + m \gamma^\alpha (I_\alpha(p) - I_\alpha(p')) \gamma_\mu + 2m [-I_\mu(p) + I_\mu(p, p') + 2m^2 J'_\mu - 2m \gamma^\alpha J_{\alpha p}] + p \leftrightarrow p' \right\}.
$$

In the expressions above, the tilde indicates that we have explicitly neglected terms with coefficients $\gamma_\mu$. Adding all the on-shell results obtained above we get for the contributions to the $F_2$ form factor:

$$
F_2^{(1)} = \frac{G N m^2}{\pi} \left\{-\frac{1}{6} \left(\frac{1}{b} I_{\text{log}}(\lambda^2) + \ln \left(\frac{\lambda^2}{m^2}\right)\right) - \frac{29}{18} - 2i \pi^2 \left(\frac{2}{3} \alpha_3 + 6 \alpha_2\right) \right\}
$$

$$
F_2^{(4+5)} = \frac{G N m^2}{\pi} \left\{\frac{2}{3} \left(\frac{1}{b} I_{\text{log}}(\lambda^2) + \ln \left(\frac{\lambda^2}{m^2}\right)\right) + 6 - 64i \pi^2 \alpha_2 \right\}
$$

$$
F_2^{(6)} = \frac{G N m^2}{\pi} \left\{-\frac{4}{3} \left(\frac{1}{b} I_{\text{log}}(\lambda^2) + \ln \left(\frac{\lambda^2}{m^2}\right)\right) - \frac{37}{18} - \frac{32}{3} i \pi^2 \alpha_3 \right\}
$$

$$
F_2^{(7+8)} = \frac{G N m^2}{\pi} \left\{-\frac{2}{3} \left(\frac{1}{b} I_{\text{log}}(\lambda^2) + \ln \left(\frac{\lambda^2}{m^2}\right)\right) - \frac{4}{9} x + \frac{32}{3} i \pi^2 \alpha_3 \right\}
$$

$$
F_2^{(9+10)} = \frac{G N m^2}{\pi} \left\{\frac{2}{3} \left(\frac{1}{b} I_{\text{log}}(\lambda^2) + \ln \left(\frac{\lambda^2}{m^2}\right)\right) + 2 - 64i \pi^2 \alpha_2 \right\}
$$

So, we have in the graviton sector,

$$
F_2^{(1)} + F_2^{(2+3)} + F_2^{(4+5)} = \frac{G N m^2}{\pi} \left\{\frac{1}{2} - 4i \pi^2 (19 \alpha_2 + \alpha_3) \right\}
$$

which is finite for any value of $\alpha_2$ and $\alpha_3$, since they are finite surface terms. In the gravitino sector, we obtain

$$
F_2^{(6)} + F_2^{(7+8)} + F_2^{(9+10)} = \frac{G N m^2}{\pi} \left\{-\frac{1}{2} - 64i \pi^2 \alpha_2 \right\}
$$

For the total one-loop correction, we have

$$
\sum_{i=1}^{10} F^{(i)} = \frac{G N m^2}{\pi} \left\{-4i \pi^2 (35 \alpha_2 + \alpha_3) \right\}.
$$
As required by supersymmetry, the contributions from the two sectors cancel out if we set \( \alpha_2 = \alpha_3 = 0 \). Of course this is not the only choice for this calculation. For instance, \( \alpha_3 = -35\alpha_2 \) is compatible with supersymmetry. Although for any value of the \( \alpha \)'s we do not have terms proportional to the photon momentum (this would explicitly violate gauge symmetry) this choice would violate gauge invariance, as we have seen in section II. On the other hand, the choice \( \alpha_3 = 0 \) enforces momentum routing invariance and consequently gauge invariance, as long as we do not have anomalies.

We would like to compare our results with the ones obtained by using Differential Regularization and Dimensional Reduction (DRed). The SUGRA \((g - 2)_l\) factor by Constrained Differential Regularization (CDR) was calculated in [14], [15] and [18] (in ref. [18] some minimal corrections are made in the results of refs. [14] and [15]). We can say that we have got equivalent results. If we choose a scheme of subtraction such that the term that comes from the traceless part of the two sectors cancel out if we set \( \alpha_2 = \alpha_3 = 0 \) enforces momentum routing invariance and consequently gauge invariance, as long as we do not have anomalies.

The CR were first obtained by constraining the Feynman amplitudes to be momentum routing invariant. As a consequence, this assures gauge invariance. An example was discussed in section II. Also in section II we have seen that setting the CR to zero is not the unique choice that implements gauge invariance. It is possible to get this by finding, for each specific amplitude, a relation between the local arbitrary constants that parametrize the Consistency Relations. Nevertheless, this relation would be different for different situations. This was the case we have faced when calculating the \((g - 2)_l\) factor for SUGRA-QED: we could respect susy by choosing \( \alpha_3 = -35\alpha_2 \), but this relation is not compatible with, for instance, the manifest gauge invariance of the vacuum polarization tensor for QED discussed in section II. Thus, the constrained version of Implicit Regularization \((\alpha_3 = 0)\) is the direct and natural way for obtaining supersymmetric and gauge invariant amplitudes.

Although we have verified it in a particular case, we believe that Implicit Regularization is supersymmetric invariant. The reason is that the rules of the constrained version of IR were tailored in such a way that symmetries that are manifest in the Lagrangian, and consequently gauge invariant, are not spoiled. This is done by extending the properties of regular in-
tegrals to the regularized ones. These properties permits: shifts in the momentum of integration; cancellation between factors of the numerator and the denominator; algebraic manipulations in the integrand. This is in the same spirit of the Constrained Differential Regularization and Dimensional Regularization. We can justify our procedure of giving up surface terms by arguing that it corresponds to automatically adding local symmetry restoring counterterms.

The results obtained in the present work were compared with previous calculations performed with Differential Regularization and Dimensional Reduction. It was found that the results are equivalent. In DRed the $\gamma$-matrix algebra is performed in four dimensions and the subsequent use of dimensional regularization automatically sets all regularization dependent (arbitrary) parameters to zero. This is explained by the fact that in Dimensional Regularization all the surface terms are null. Therefore this method respects the CR and we can say that the two procedures will always yield identical results at one-loop order. Concerning the Constrained Differential Regularization, there are some evidences that the Consistency Relations are the momentum-space version of some of its rules. In a work in progress \[32\], the relation between the two techniques will be discussed.

**VI. APPENDIX**

We give below the on-shell results ($p^2 = m^2$, $(p - p')^2 = 0$) of the integrals which were necessary to the calculations of the previous sections:

\[
I(p) = \int_k^\Lambda \frac{1}{(p - k)^2 - m^2} = I_{\log}(\lambda^2) + b \ln \left(\frac{\lambda^2}{m^2}\right) + 2b, \tag{49}
\]

\[
I_\mu(p) = \int_k^\Lambda \frac{k_\mu}{(p - k)^2 - m^2} = \frac{p_\mu}{2} \left\{ I_{\log}(\lambda^2) + b \ln \left(\frac{\lambda^2}{m^2}\right) \right\}, \tag{50}
\]

\[
I_{\mu\nu}(p) = \int_k^\Lambda \frac{k_\mu k_\nu}{(p - k)^2 - m^2} = \frac{1}{3} p_\mu p_\nu \left\{ I_{\log}(\lambda^2) + \frac{2}{3} b + \alpha_3 \right\} + \eta_{\mu\nu} \text{ terms}, \tag{51}
\]

\[
I(p, p') = \int_k^\Lambda \frac{1}{((p - k)^2 - m^2)((p' - k)^2 - m^2)} = I_{\log}(\lambda^2) + b \ln \left(\frac{\lambda^2}{m^2}\right), \tag{52}
\]

\[
I_\mu(p, p') = \int_k^\Lambda \frac{k_\mu}{((p - k)^2 - m^2)((p' - k)^2 - m^2)} = \frac{(p + p')_\mu}{2} \left\{ I_{\log}(\lambda^2) + b \ln \left(\frac{\lambda^2}{m^2}\right) \right\} + \alpha_2, \tag{53}
\]

\[
I_{\mu\nu}(p, p') = \int_k^\Lambda \frac{k_\mu k_\nu}{((p - k)^2 - m^2)((p' - k)^2 - m^2)} = \frac{1}{6}(2 p_\mu p_\nu + 2 p'_\mu p'_\nu + p_\mu p'_\nu + p'_\mu p_\nu) \left\{ I_{\log}(\lambda^2) + b \ln \left(\frac{\lambda^2}{m^2}\right) \right\} + \eta_{\mu\nu} \text{ terms}, \tag{54}
\]

\[
J = \int_k^\Lambda \frac{1}{(p - k)^2 - m^2} \frac{1}{((p' - k)^2 - m^2)} = -\frac{b}{m^2} \int_0^1 dx \int_0^{1-x} dy \frac{1}{(x + y)^2}, \tag{55}
\]

\[
J_\mu = \int_k^\Lambda \frac{k_\mu}{(p - k)^2 - m^2} \frac{1}{((p' - k)^2 - m^2)} = -\frac{b}{m^2} (p + p')_\mu, \tag{56}
\]

\[
J_{\mu\nu} = \int_k^\Lambda \frac{k_\mu k_\nu}{(p - k)^2 - m^2} \frac{1}{((p' - k)^2 - m^2)} = -\frac{b}{6m^2} \left[ p_\mu p_\nu p'_\nu p'_\mu + \frac{1}{2} (p_\mu p'_\nu + p'_\mu p_\nu) \right] + \eta_{\mu\nu} \text{ terms}, \tag{57}
\]

\[
J' = \int_k^\Lambda \frac{1}{(p - k)^2 (p' - k)^2} = -\frac{b}{m^2} \int_0^1 dx \int_0^{1-x} dy \frac{1}{(x + y - 1)^2}, \tag{58}
\]

\[
J'_\mu = \int_k^\Lambda \frac{k_\mu}{(p - k)^2 (p' - k)^2} = -\frac{b}{m^2} \int_0^1 dx \int_0^{1-x} dy \frac{x (p + p')_\mu}{(x + y - 1)^2}, \tag{59}
\]

\[
J'_{\mu\nu} = \int_k^\Lambda \frac{k_\mu k_\nu}{(p - k)^2 (p' - k)^2} = -\frac{b}{m^2} \int_0^1 dx \int_0^{1-x} dy \frac{1}{(x + y - 1)^2} \times \left[ (p_\mu p_\nu + p'_\mu p'_\nu) x^2 + \frac{1}{2} (p_\mu p'_\nu + p'_\mu p_\nu) xy \right] + \eta_{\mu\nu} \text{ terms}, \tag{60}
\]

In the equations above, the on-shell integrals $J$, $J'$, $J'_\mu$ and $J'_{\mu\nu}$ are infrared divergent. They appear in the diagrams $\Lambda_{\mu}^{(4)}$, $\Lambda_{\mu}^{(5)}$, $\Lambda_{\mu}^{(9)}$ and $\Lambda_{\mu}^{(10)}$ and these singularities are cancelled pair by pair.
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