Three dimensional visualisation and reconstruction of the luminosity distribution of a flame using digital imaging techniques

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Abstract This paper presents an algorithm for the three-dimensional (3D) visualisation and luminosity reconstruction of a combustion flame. A combination of image processing techniques and filtered-back projection algorithms is employed to reconstruct grey-scale sections of the flame from three 2D images taken by three identical CCD monochromatic cameras placed around the flame. The technique proposed is capable of reconstructing both cross- and longitudinal sections of the flame, and consequently a complete 3D luminous model of the flame.

1. Introduction
Advanced technology for monitoring and characterization of combustion flames has become increasingly important for improved understanding and subsequent optimization of combustion processes [1]. In recent years, a significant effort has been made to develop vision based instrumentation systems for quantitative monitoring and characterization of flames using digital imaging and image processing techniques [2-5]. These systems are capable of determining two-dimensional (2D) parameters of flames such as size and location, brightness and uniformity and temperature and flicker frequency under various operation conditions [2, 4, 5].

3D monitoring and characterisation of combustion flames have been a topical area of research in recent years [6-9]. Preliminary work for 3D temperature measurement of a candle flame using multi-spectral tomographic imaging techniques has been reported [6]. Theoretical aspects in determining 3D temperature distribution of a flame in a large-scale furnace using multiple cameras have also been studied [7]. Bheemul et al [8] proposed a system for 3D visualization and quantitative characterization of gas-fired flames. The work has focused on the 3D reconstruction of geometrical models of a flame from its 2D images. Brisley et al [9] also demonstrated a single camera system for 3D grey-level reconstruction of flame cross sections and temperature measurement of the flame [9]. Although a single camera approach is simple, robust and low cost which is particularly attractive for a practical use, it is based on the assumption that the flame is rotationally symmetrical. This assumption is not always valid, particularly for a flame in an industrial furnace. This paper presents an improved computing algorithm for the 3D grey-level reconstruction of a flame from three 2D images captured simultaneously by three CCD cameras from different locations around the flame. The filtered-back projection theory is applied to reconstruct the cross-sections of the flame. The information obtained can be directly used for determining the geometrical, luminous and temperature characteristics of a flame on a 3D basis.

2. Methodology
The images of the flame used in this study were captured using a multi-camera imaging system the details of which have been given elsewhere [8]. Three identical monochromatic cameras are
installed around the burner axis to obtain three 2D images. In order to reconstruct the grey-level section of the flame from the three 2D images so as to reveal the inner structure of the flame, a deconvolution process must be taken. The process of capturing the light from a combustion flame onto an imaging sensor is physically equivalent to a Radon transformation where a 2D flame cross-section undergoes transformation to produce a 1D section projection [9]. Consequently, the reconstruction of a flame cross-section from its 1D projection is essentially the inverse Radon transformation. The general expression of Radon transformation of a 2D function, \( g(\bar{x}) \), is given by,

\[
P_\theta(x') = \int_{\infty}^{\infty} g(\bar{x}) \delta(\bar{x} \cdot \bar{n} - x') d^2 \bar{x},
\]

where \( P_\theta(x') \) is the projection of \( g(\bar{x}) \) along particular angle \( \theta \) and through the rectilinear path determined by the Dirac delta function \( \delta(\bar{x} \cdot \bar{n} - x') \), \( \bar{n} = (\cos \theta, \sin \theta) \) is a unit vector normal to the projection beam, and \( \bar{x} \) is a vector on the projection beam. The Central Slice (also known as the Central Projection) Theorem states that the 2D Fourier transform of 2D function, \( \tilde{f} \), yields the same result as the successive execution of a Radon and a 1D Fourier transform in radial direction. Consequently, we can reconstruct the function \( g(\bar{x}) \) by back-projecting and adding successively the filtered projections \( Q_\theta \):

\[
g(\bar{x}) = \int_{\infty}^{\infty} Q_\theta(\bar{x} \cdot \bar{n}) d\theta,
\]

where

\[
Q_\theta(\bar{x} \cdot \bar{n}) = (h * P_\theta)(x')
\]

being \( h \) the filter impulse response and \(*\) denoting convolution. In practice, in a real discrete case, it is more efficient to implement this convolution in the frequency domain by using the Fast Fourier Transform (FFT). For a total discrete number of \( M_{\text{proj}} \) projections the reconstructed function can be approximated as:

\[
g(\bar{x}) \approx \frac{\pi}{M_{\text{proj}}} \sum_{i=1}^{M_{\text{proj}}} Q_\theta(\bar{x} \cdot \bar{n}).
\]

For equation (4) to be true it is necessary that the projections are equiangular and its total number \( M_{\text{proj}} \) to be sufficiently large. If the flame is assumed geometrical symmetric [9], all the projections \( Q_\theta(\bar{x} \cdot \bar{n}) \) are to be equal. In this study, however, such an assumption can be avoided as availability of additional projections. The requirement of the high number of projections is overcome by generating a series of intermediate ones through a linear progression. For example, if \( Z_A \) is the projection from location \( A \), \( Z_B \) is the projection from location \( B \) and \( N \) is the number \( M_{\text{proj}} \) divided by the total number of cameras, i.e. 3, the luminosity \( Z_{ij} \) of the \( j \)th pixel of the \( i \)th intermediate projection between \( Z_A \) and \( Z_B \) is:

\[
Z_{ij} = Z_A + \left( \frac{Z_B - Z_A}{N} \right) \times i, \quad i = 0,1,\ldots,N-1
\]

Figure 1 shows a logical procedure of the algorithm developed. After the image acquisition and digitalization, each of the projections \( Z_A, Z_B \) and \( Z_C \) consist of a 1D row of 128 8-bit grey scale pixels. The left and right edges of the luminous region for a given row are calculated using a
threshold, $\delta$, that distinguishes between the luminous intensity of the flame and the background. The luminous region is shifted to the centre of the 128 pixels long row. Projections $Z_i$'s are then produced using equation (5). After this stage the back projection process commences. The projections $Z_i$'s are extended to 2D square matrices that are rotated by the corresponding angles $\theta_i$'s. A centred final result is obtained by the calculation of the average of the $M_{proj}$ 2D matrices. Finally, this resulting matrix is shifted back to its actual position.

$$\text{Figure 1. Algorithm for the 3D reconstruction of flame cross sections.}$$

3. Preliminary Results

To evaluate the effectiveness of the algorithm, a series of 2D images was taken on a gas fired combustion rig by using the multi-camera imaging system. The constituent elements of the system are illustrated in figure 2. The detailed description of the system can be found in [8]. Figure 3 shows a typical example of the 2D images of the flame taken using the three cameras. The flame is partially pre-mixed so as to obtain a reasonably stable and bright flame.

$$\text{Figure 2. System set-up}$$

$$\text{Figure 3. Flame images captured from positions A, B and C.}$$

Figure 4 shows the typical images of reconstructed flame cross-sections corresponding to projection numbers 160, 230, 300 and 350 as illustrated in figure 3. It is possible to notice the correspondence between the results and the original 2D images given in figure 3. For example, slice 160 shows a bright and circular pattern at its centre with various more intense vortices to the right. These patterns are also notable from the flame image observed at position B (figure 3). Furthermore, slices 300 and 350 show circular patterns that are less bright at the periphery and even darker at the centre as clearly shown in the images.
The reconstructed flame cross-sections illustrated in figure 4 correspond to four intermediated 1D projections. However, the same procedure can be applied to each 1D projection from the upper to the lower limit of the flame for every pixel row. Consequently, a large number of cross-sections are obtained that are combined to form a complete 3D reconstructed model of the flame. When this is accomplished, an exhaustive internal examination can be performed by viewing the longitudinal sections of the flame. Figure 5 shows examples of reconstructed longitudinal-sections of the flame. It should be noticed that the visual representation of the reconstructed longitudinal-sections depends on a point of view. The results presented in Figure 5(b) refer to the sections viewed from camera location A (Figures 2 & 3) and the location of each section are indicated in Figure 5(a). The four segments are very close around the burner axis and therefore show similar luminous profiles. This result, however, corresponds well with what have been observed from original 2D images (figures 3).

**Figure 4.** Grey-level reconstruction of cross-sections

4. Conclusion

An algorithm for the 3D visualisation and reconstruction of the luminosity distribution of a combustion flame has been proposed. The back projection theory has been employed to reconstruct the grey-level sections of the flame from the 2D images of the flame captured simultaneously by three monochromatic CCD cameras at different locations around the flame. Preliminary experimental results have demonstrated that it is possible to use a limited number of projections for the 3D reconstruction of flame sections and therefore ultimately the complete model of the flame can be determined. The information obtained can directly be used for determining the geometrical, luminous and temperature characteristics of a flame on a 3D basis. Future work will focus on the optimisation between the suitable number of proportions and the reliability of the reconstructed flame model.

**Figure 5.** Reconstructed longitudinal sections viewed from ‘position A’.
Acknowledgements
The authors would like to acknowledge the British Coal Utilization Research Association (BCURA), E.ON plc and RWE npower plc for their financial and technical support for the project.

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