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To learn production of the scalar and tensor mesons in
\( \gamma\gamma^*(Q^2) \rightarrow \eta\pi^0 \) reaction

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Introduction

This work is the development of N.N. Achasov and G.N. Shestakov, Phys. Rev. D 81, 094029 (2010), where the high-statistical Belle data on the $\gamma\gamma \to \eta\pi^0$ cross-section were analyzed.

We will analyze the Belle data together with the KLOE data on the $\phi \to \eta\pi^0\gamma$ decay and predict the $\gamma^*(Q^2)\gamma \to \eta\pi^0$ cross-section.

References:
S. Uehara et al. (Belle Collaboration) Phys. Rev. D 80, 032001 (2009).
A. Aloisio et al. (KLOE Collaboration) Phys. Lett. B 536, 209 (2002).
Light scalar mesons

- Nonet of light scalar mesons: $a_0(980)$, $f_0(980)$, $\sigma(600)$, $\kappa(800)$

- Were discovered $\sim 50$ years ago and became hard problem for the naive quark model from the outset

- Elucidation of their nature can shed light on confinement and the chiral symmetry realization way in the low energy region

- Perturbation theory and sum rules don’t work

- The $\sigma(600)$, $a_0(980)$, and $f_0(980)$ are studied in $\phi \rightarrow S\gamma$ decays, $\pi\pi$ scattering, $\gamma\gamma \rightarrow \pi\pi$, $\eta\pi^0$ and other processes
Light scalars in $\gamma\gamma$ collisions

Let’s $S = \sigma(600)$, $a_0(980)$, $f_0(980)$ and $T = a_2(1320)$, $f_2(1270)$.

In the $q\bar{q}$ model $\Gamma_{S \to \gamma\gamma}$ are originated from direct $S\gamma\gamma$ coupling. From the experimental results

$$\Gamma_{f_2(1270) \to \gamma\gamma} \approx 3 \text{ keV}, \quad \Gamma_{a_2(1320) \to \gamma\gamma} \approx 1 \text{ keV}$$

it was found $\Gamma_{f_0(980) \to \gamma\gamma} \geq 3.4 \text{ keV}, \quad \Gamma_{a_0(980) \to \gamma\gamma} \geq 1.3 \text{ keV}$.

Four quark model: $\Gamma_{f_0(980) \to \gamma\gamma} \sim \Gamma_{a_0(980) \to \gamma\gamma} \sim 0.27 \text{ keV}$

(Achasov, Devyanin, Shestakov, 1982)

These widths are caused by rescatterings:

$f_0 \to K^+K^- + \pi^+\pi^- \to \gamma\gamma$, $a_0 \to K^+K^- + \eta\pi^0 + \eta'\pi^0 \to \gamma\gamma$
The $\gamma \gamma \rightarrow \eta \pi^0$ data description

$$\sigma(\gamma \gamma \rightarrow \eta \pi^0) = \sigma_0 + \sigma_2$$

$$\sigma_\lambda = \frac{\rho_{\pi \eta}(s)}{64\pi s} \int |M_\lambda|^2 d\cos \theta ; \rho_{\pi \eta}(s) = \sqrt{(1 - \frac{(m_\eta + m_\pi)^2}{s})(1 - \frac{(m_\eta - m_\pi)^2}{s})}$$
\[ M_0(\gamma\gamma \rightarrow \pi^0\eta; s, \theta) = M_0^{\text{Born}} V(\gamma\gamma \rightarrow \pi^0\eta; s, \theta) + iV_{\pi^0\eta}(s) T_{\pi^0\eta \rightarrow \pi^0\eta}(s) + iV_{\pi^0\eta'}(s) T_{\pi^0\eta' \rightarrow \pi^0\eta}(s) + \left( iK^{*+}_{K+K^-}(s) - iK^{*0}_{K^0K^0}(s) + iK^+_{K+K^-}(s, x_1) \right) \times T_{K^+K^- \rightarrow \pi^0\eta}(s) + M_{\text{res}}^{\text{direct}}(s), \] (1)

\[ M_2(\gamma\gamma \rightarrow \pi^0\eta; s, \theta) = M_2^{\text{Born}} V(\gamma\gamma \rightarrow \pi^0\eta; s, \theta) + 80\pi d_{20}^2(\theta) M_{\gamma\gamma \rightarrow a_2(1320) \rightarrow \pi^0\eta}(s), \] (2)

\[ d_{20}^2(\theta) = (\sqrt{6}/4) \sin^2 \theta \]
\[ T_{\eta\pi^0 \rightarrow \eta\pi^0} = \frac{e^{2i\delta_B^\eta\pi} - 1}{2i\rho_{\eta\pi}(m)} + e^{2i\delta_B^\eta\pi} \sum_{S,S'} \frac{g_S \eta\pi^0 G_{SS'}^{-1} g_{S'} \eta\pi^0}{16\pi} \]

\[ G_{SS'}(m) = \begin{pmatrix} D_{a_0'}(m) & -\Pi_{a_0a_0'}(m) \\ -\Pi_{a_0a_0'}(m) & D_{a_0}(m) \end{pmatrix} \]

\[ T_{ab\rightarrow cd} = e^{i\delta_B^a} e^{i\delta_B^c} T_{ab\rightarrow cd}^{res} \]

\[ T_{ab\rightarrow cd}^{res} = \sum_{S,S'} \frac{g_{Sab} G_{SS'}^{-1} g_{S' cd}}{16\pi} \]

\[ S, S' = a_0, a'_0 \]
Kaon form factor

\[ M_{00}^{\text{Born}} K^+(s', x_1) = 4\pi \alpha \frac{1 - \rho_K^2(s)}{\rho_K(s)} \left( \ln \frac{1 + \rho_K(s)}{1 - \rho_K(s)} - \ln \frac{1 + \frac{\rho_K(s)}{1+2x_1^2/s}}{1 - \frac{\rho_K(s)}{1+2x_1^2/s}} \right) \]

\[ I_{K^+ K^-}^+(s, x_1) = \frac{s}{\pi} \int_{4m_K^2}^{\infty} \frac{\rho_K(s') M_{00}^{\text{Born}} K^+(s', x_1)}{s'(s' - s - i\epsilon)} \]

New data description: we get rid of kaon formfactor.
Рис. 1: The $\gamma\gamma \rightarrow \eta\pi^0$ cross-section, "as is" (a) and averaged taking into account 20 MeV bins (b). Points are the Belle data.

Рис. 2: The $\eta\pi^0$ mass spectrum in $\phi \rightarrow \eta\pi^0\gamma$ decay. Points represent the KLOE data.
The $\gamma\gamma \rightarrow \eta\pi^0$ and $\phi \rightarrow \eta\pi^0\gamma$ description

The invariant mass spectrum of $\eta\pi^0$ in $a_0 \rightarrow \eta\pi^0$

$$\frac{dN_{\eta\pi^0}}{dm} \sim \frac{2m^2\Gamma(a_0 \rightarrow \eta\pi^0, m)}{\pi |D_{a_0}(m)|^2}.$$  \hfill (3)

The width averaged over the resonance mass distribution

$$\langle \Gamma_{a_0 \rightarrow \gamma\gamma} \rangle_{\eta\pi^0} = \frac{1.1 \text{ GeV}}{0.9 \text{ GeV}} \int_{0.9 \text{ GeV}}^{1.1 \text{ GeV}} \frac{s}{4\pi^2} \text{res}(\gamma\gamma \rightarrow \pi^0\eta; s) d\sqrt{s}$$  \hfill (4)
| Fit                                                                 | 1        | 2        | 3        | 4        |
|--------------------------------------------------------------------|----------|----------|----------|----------|
| $m_{a_0}$, MeV                                                      | 993.86   | 994.50   | 995.34   | 988.69   |
| $g_{a_0}^2 K^+ K^- / 4\pi$, GeV$^2$                                | 0.60     | 0.47     | 1.19     | 1.10     |
| $g_{a_0}^2 \eta \pi / 4\pi$, GeV$^2$                              | 0.60     | 0.76     | 1.09     | 1.04     |
| $g_{a_0 \gamma \gamma}$, $10^{-3}$ GeV$^{-1}$                     | 1.8      | 2.7810   | 4.5919   | 3.2520   |
| $\Gamma_{a_0 \rightarrow \gamma \gamma}^{(0)}$, keV               | 0.063    | 0.151    | 0.414    | 0.203    |
| $< \Gamma_{a_0 \rightarrow \gamma \gamma}^{\text{direct}} >_{\eta \pi^0}$, keV | 0.019    | 0.031    | 0.030    | 0.024    |
| $< \Gamma_{a_0 \rightarrow (K \bar{K} + \eta \pi^0 + \eta' \pi^0) \rightarrow \gamma \gamma} >_{\eta \pi^0}$, keV | 0.126    | 0.113    | 0.141    | 0.129    |
| $< \Gamma_{a_0 \rightarrow (K \bar{K} + \eta \pi^0 + \eta' \pi^0 + \text{direct}) \rightarrow \gamma \gamma} >_{\eta \pi^0}$, keV | 0.225    | 0.236    | 0.273    | 0.243    |
| $\Gamma_{a_0}(m_{a_0})$, MeV                                      | 116.76   | 140.81   | 218.65   | 186.75   |
| $\Gamma_{a_0}^{\text{eff}}$, MeV                                  | 34.6     | 57.24    | 38.76    | 44.76    |
| $m_{a_0'}$, MeV | 1400 | 1400 | 1300 | 1500 |
|-----------------|------|------|------|------|
| $g_{a_0'K^+K^-}/4\pi$, GeV$^2$ | 0.21 | 0.005 | 0.07 | 0.44 |
| $g_{a_0'\eta\pi}/4\pi$, GeV$^2$ | 0.77 | 0.18 | 0.52 | 0.63 |
| $g_{a_0'\eta'\pi}/4\pi$, GeV$^2$ | 1.80 | 4.12 | 2.15 | 2.64 |
| $\Gamma_{a_0'\rightarrow\gamma\gamma}(m_{a_0'})$, keV | 1.65 | 5.38 | 2.96 | 5.21 |
| $\Gamma_{a_0'}(m_{a_0'})$, MeV | 330.91 | 399.4 | 271.0 | 453.40 |
| $\chi_{\gamma\gamma}^2 / 36$ points | 12.4 | 4.8 | 5.3 | 6.7 |
| $\chi_{sp}^2 / 24$ points | 24.5 | 24.7 | 24.1 | 24.3 |
| $(\chi_{\gamma\gamma}^2 + \chi_{sp}^2) / n.d.f.$ | 36.9/46 | 29.5/46 | 29.4/46 | 31.0/46 |
Non-zero $k_2^2 = -Q^2$
The effective Lagrangian of the $a_2 \rightarrow V(1)V(2)$ transition (N.N. Achasov and V.A. Karnakov, Z. Phys. C 30, 141 (1986))

\[
L = g a_2 V(1)V(2) T_{\mu\nu} F_{\mu\sigma}^{V(1)} F_{\nu\sigma}^{V(2)},
\]

\[
F_{\mu\sigma}^{V(i)} = \partial_\mu V(i)_\sigma - \partial_\sigma V(i)_\mu; \ i = 1, 2
\]

\[
\{V(1), V(2)\} = \{\rho, \omega\}, \{\rho', \omega'\}, \{\rho'', \omega''\}
\]
\[ M_2(\gamma^* \gamma \rightarrow a_2(1320) \rightarrow \pi^0 \eta; s, Q, \theta) = A(s, Q) \sin^2 \theta \]

\[ M_1(\gamma^* \gamma \rightarrow a_2(1320) \rightarrow \pi^0 \eta; s, Q, \theta) = -\sqrt{2} A(s, Q) \sqrt{\frac{Q^2}{s}} \sin \theta \cos \theta \]

\[ M_0(\gamma^* \gamma \rightarrow a_2(1320) \rightarrow \pi^0 \eta; s, Q, \theta) = -A(s, Q) \frac{Q^2}{s} (\cos^2 \theta - \frac{1}{3}) \]

\[ A(s, Q) = 20\pi F_{a_2}(Q) \sqrt{\frac{6s \Gamma_{a_2 \rightarrow \gamma\gamma}(s) \Gamma_{a_2 \rightarrow \eta\pi^0}(s)}{\rho_{\eta\pi^0}(s)}} \frac{1}{D_{a_2}(s)} (1 + \frac{Q^2}{s}) \]

QCD: At $Q \rightarrow \infty$ $M_0 \sim 1/Q^2$, $M_1 \sim 1/Q^3$, $M_0 \sim 1/Q^4$. It is reached if $F_{a_2}(Q) \sim 1/Q^4$.

V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984).

V.N. Baier and A.G. Grozin, Fiz. Elem. Chast. At. Yad. 16, 5 (1985) [Sov. J. Part. Nucl. 16, 1 (1985)].
Let's \( F_{a_2}(Q) = \bar{F}_{a_2}(Q)/\bar{F}_{a_2}(0) \),

\[
\bar{F}_{a_2}(Q) = \frac{g_{a_2 \rho \omega}}{f_\rho f_\omega} \left( \frac{1}{1 + Q^2/m_\rho^2} + \frac{1}{1 + Q^2/m_\omega^2} \right) + \frac{g_{a_2 \rho' \omega'}}{f_\rho' f_\omega'} \left( \frac{1}{1 + Q^2/m_\rho'^2} + \frac{1}{1 + Q^2/m_\omega'^2} \right) \\
+ \frac{g_{a_2 \rho'' \omega''}}{f_\rho'' f_\omega''} \left( \frac{1}{1 + Q^2/m_\rho''^2} + \frac{1}{1 + Q^2/m_\omega''^2} \right) = \frac{g_{a_2 \rho \omega}}{f_\rho f_\omega} \left( \frac{1}{1 + Q^2/m_\rho^2} + \frac{1}{1 + Q^2/m_\omega^2} \right) \\
+ a \left( \frac{1}{1 + Q^2/m_\rho^2} + \frac{1}{1 + Q^2/m_\omega^2} \right) + b \left( \frac{1}{1 + Q^2/m_\rho''^2} + \frac{1}{1 + Q^2/m_\omega''^2} \right)
\]

The condition \( m_\rho^2 + m_\omega^2 + a(m_\rho'^2 + m_\omega'^2) + b(m_\rho''^2 + m_\omega''^2) = 0 \) leads to asymptotics, based on QCD.

The loop function \( I_{K^+K^-}^{K^+}(s, Q) \to 8\alpha \ln \frac{Q^2}{m_K^2} \) at \( Q \to \infty \).
Рис. 3: The cross-section $\sigma_0(\gamma\gamma^*(Q^2) \rightarrow \eta\pi^0, s) + \sigma_2(\gamma\gamma^*(Q^2) \rightarrow \eta\pi^0, s)$, $|\cos \theta| < 0.8$, for Fit1 parameters, $a = -0.082$ and $b = -0.15$.

a) Solid line - $Q^2 = 0$, point line - $Q^2 = 0.25 \text{ GeV}^2$, dashed line - $Q^2 = 1 \text{ GeV}^2$; b) Solid line - $Q^2 = 2.25 \text{ GeV}^2$, point line - $Q^2 = 4 \text{ GeV}^2$, dashed line - $Q^2 = 6.25 \text{ GeV}^2$.

Рис. 4: The same cross-section averaged over bins.
Рис. 5: $\sigma_1(\gamma\gamma^*(Q^2) \to \eta\pi^0, s), \, |\cos \theta| < 0.8$ for the same parameters. Solid line - $Q^2 = 0.25$ GeV$^2$, point line - $Q^2 = 1$ GeV$^2$, dashed line - $Q^2 = 6.25$ GeV$^2$. 
Рис. 6: The $\sigma_0(\gamma\gamma^*(Q^2) \to \eta\pi^0, s) + \sigma_2(\gamma\gamma^*(Q^2) \to \eta\pi^0, s)$, for Fit 1, $a = 3.835$ and $b = -3$.

Рис. 7: The $\sigma_0(\gamma\gamma^*(Q^2) \to \eta\pi^0, s) + \sigma_2(\gamma\gamma^*(Q^2) \to \eta\pi^0, s)$, $|\cos \theta| < 0.8$, for Fit 1, $a = -4.413$ and $b = 3$. 
Conclusion

1. Belle data Данные on the $\gamma\gamma \rightarrow \eta\pi^0$ where analyzed simultaneously with the KLOE data on the $\phi \rightarrow \eta\pi^0\gamma$ decay without kaon formfactor, $G_{K^+}(t,u) = 1$. The data supports scenario based on the four-quark model of $a_0(980)$.

2. The prediction of the $\sigma(\gamma\gamma^*(Q^2) \rightarrow \eta\pi^0)$ is presented, different variants are considered.

3. The QCD-based asymptotics of $\sigma(\gamma\gamma^*(Q^2) \rightarrow \eta\pi^0)$ is reached after taking into account vector excitations.