Screening Single Phase Laminar Convective Heat Transfer of Nanofluids in a Micro-tube

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Abstract. Nano scale solid particles dispersed in base fluids are a new class of engineered colloidal solutions called nanofluids. Several studies reported enhancement of heat transfer by using nanofluids. This article reports convective single-phase heat transfer coefficients in an open 30 cm long, 0.50 mm internal diameter stainless steel test section. The setup is used for screening single phase laminar convective heat transfer with water and three different nanofluids: water based Al$_2$O$_3$, ZrO$_2$, and TiO$_2$ (all with 9 wt% of particles). A syringe pump with adjustable pumping speed is used to inject fluids into the test section. Thirteen T-type thermocouples are attached on the outer surface of the test section to record the local wall temperatures. Furthermore, two T-type thermocouples are used to measure inlet and outlet fluid temperatures. A DC power supply is used to heat up the test section and a differential pressure transducer is used to measure the pressure drop across the tube. Furthermore, the effective thermal conductivities of these nanofluids are measured using the Transient Plane Source (TPS) method at a temperature range of 20 - 50°C.

The experimental average values of heat transfer coefficients for nanofluids are compared with water. Enhancement in heat transfer of nanofluids is observed only when compared at constant Reynolds number (Due to higher viscosity for nanofluids, higher velocity or mass flow rate is required for nanofluids to reach the same Reynolds number). The other methods of comparison: equal mass flow rate, volume flow rate, pressure drop and pumping power did not show any augmentation of the heat transfer coefficient for the tested nanofluids compared to water.

1. Introduction

Conventional air cooling of electronics has reached its limits and in the future, even liquid cooling using extended surfaces, mini and micro-channels and other enhancement techniques may prove insufficient. As the possibility of cooling is already setting the limits to the performance of electronic devices, new methods of enhancing heat transfer is direly needed [1]. Dilute dispersions of nanoparticles (less than 5 vol%) in conventional heat transfer fluids (like water, ethylene glycol/water, oils etc) form a new class of heat transfer fluids which may be used for high heat flux applications. Such fluids are called nanofluids [2].

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Several investigators have claimed that nanofluids have better thermal properties than their base fluids. A first study of the influence of suspended solid particles on the thermal conductivity of liquids was presented already by Maxwell [3], who also suggested a correlation for estimation of the thermal conductivity of such fluids. Later studies focused on an extension of the Maxwell equation based on experimental results, e.g. [4]. Conventional approaches/correlations show a linear trend of increasing thermal conductivity with increasing particle concentration, however many experimental studies with nanofluids have shown a nonlinear trend [5].

Laminar convective heat transfer and pressure drop with aqueous alumina and zirconia nanofluids was experimentally studied by Rea et al. [6]. Experiments were conducted in the laminar flow regime. A resistively heated stainless steel tube (4.5 mm ID & 6.4 mm OD with 1.01 m length in vertical arrangement) was used as a test section. Traditional models/correlations for laminar flow were used for comparison, and the results showed 17 and 27% enhancement in average heat transfer coefficient with alumina (6% vol) in the entrance and fully developed regions, while the corresponding values were 2 and 3% for zirconia (1.32 vol%) nanofluids. Pressure drop was measured in an isothermal section and was compared with calculated pressure drop using Darcy-Weisbach equation. Pressure drop with nanofluids was found to be higher than for pure water due to the increased viscosities, and in agreement with the classical correlations within 20% error.

Heat transfer measurements in laminar flow under constant wall temperature conditions with water based Al\textsubscript{2}O\textsubscript{3} nanofluids (0.2-2.5 vol% concentration) were carried out by Heris et al. [7]. A concentric tube (with 6 mm ID and 32 mm OD) copper/stainless steel setup was used as the test section and constant wall temperature was maintained by saturated steam. Based on their study, the experimental results are much higher than the theoretical prediction by Sieder-Tate correlation. One possible stated reason was that the correlation used for estimating thermal conductivity is valid for static conditions and no correction was used for applying it to the dynamic case. For calculating effective thermal conductivity of nanofluids they used Yu and Choi correlation [8], which considers the interface between particles and fluid under stationary conditions while, they argued, in reality (due to fluid flow) dynamic characteristics need to be considered as well. Furthermore the plot of average based Nusselt number versus Peclet number shows higher Nusselt at equal Peclet number for nanofluids than for the base fluid. The difference increased with increasing particle concentration. This result may, however, be caused by erroneous values for thermal and transport properties used in the calculation of Nu and Re.

Heat transfer and pressure drop with TiO\textsubscript{2} nanofluids (water based with 0.6 - 2 vol% concentrations) was measured by Duangthongsuk and Wongwises [9] in the turbulent flow regime. The test section was a double tube heat exchanger in which nanofluids were flowing through the inner tube while hot water was circulated in the annular section for heat supply. Tests were conducted within 3000-18000 Reynolds number range. Thermal conductivity was measured at 15, 20 and 25°C. Results showed increased heat transfer (22-30% enhancement in heat transfer coefficient and Nusselt number) at 1vol% concentration and lower enhancement in heat transfer at higher concentration (up to 14% compared with the base fluid) at 2 vol% concentration. However the comparison was done in the same Reynolds number and as a result this heat transfer enhancement might be due to the effect of viscosity increase. The authors mention that increased viscosity may have affected the boundary layer thickness with the higher concentration, which may then have deteriorated the heat transfer.

An experimental study on convective heat transfer in a microtube with three water based nanofluids (Al\textsubscript{2}O\textsubscript{3} 3vol%, CuO 4vol% and CNT 0.2 vol%) was reported by Lee et al. [10]. A resistively heated stainless steel tube (0.5 mm ID) test section was utilized in this study, the obtained results showed 5, 13.3 and 11.2% increment in the heat transfer at the same volume flow rate adjusted by an injection pump with Al\textsubscript{2}O\textsubscript{3}, CuO and CNT water based nanofluids respectively. When pressure drops were also included only CNT based nanofluids seemed to be the good choice for microtube heat exchangers as addition of particles improves heat transfer but at the same time also increases viscosity and thereby pressure drop. They concluded that CNT based nanofluids might be promising.
In this study single-phase convective heat transfer coefficients were measured in an open 30 cm long, 0.50 mm internal diameter stainless steel test section for water based Al\textsubscript{2}O\textsubscript{3}, ZrO\textsubscript{2}, and TiO\textsubscript{2} (all with 9 wt\% of particles). Different methods for comparing heat transfer performance of these nanofluids to water are used. The comparisons have been done at equal Reynolds number, mass flow rate, volume flow rate, pressure drop and pumping power. Moreover, the effective thermal conductivities of these nanofluids were measured using the Transient Plane Source (TPS) method, and the viscosities were measured by a rotating concentric cylinders viscometer in the temperature range of 20-50˚C. The properties and characteristic of the tested nanofluids in addition to the results of these nanofluids’ viscosity were presented in the same conference [11].

2. Experiments
The thermal conductivity instrument and the open flow loop including the microtube test section used to measure the heat transfer coefficient are explained in this section.

2.1. Thermal conductivity
To determine the thermal conductivity of the tested nanofluids, the transient plane source (TPS) method was used. The method is similar to transient hot wire (THW) and based on analysis of the transient response due to a short heat pulse to a plane sensor immersed in the still liquid. Both methods can be found in the literature for measuring thermal conductivity of nanofluids [12]. In TPS method two other thermal properties, thermal diffusivity and specific heat capacity can be measured as well, but if specific heat is known, a more exact value for the thermal conductivity can be achieved.
A TPS-analyser (HotDisk model 2500) was used for the measurements. The TPS-analyser was connected to a computer and to a Kapton insulated sensor. Moreover, a thermal bath was used to keep the temperature of the samples constant. Figure 1 illustrates how the sensor and the sample holder were assembled. As can be observed, the sensor is located in the centre of the volume where the fluids are injected.

![Figure 1. Sample holder and a sensor.](image)

To measure thermal conductivity, a small electric current (0.01 or 0.02 watt) was supplied to the sensor for a very short time (4-5 seconds). From the temperature increase of the sensor with time, the thermal conductivity was computed by the TPS analyser. It is worth mentioning that the electric current and the time of heating are crucial for the measurement since applying high power and/or long time can result in convection, and erroneous results. On the other hand, low values for power and time can lead to fluctuations in the temperature increase versus time response graph and possibly lead to
non-trustable values for the thermal conductivity. To prevent the aforementioned problems, based on testing different combinations of power and time, the following values presented in Table 1 were used:

| Temperature (°C) | Power (W) | Measurement time (s) |
|-----------------|-----------|----------------------|
| 20              | 0.02      | 5                    |
| 30              | 0.02      | 5                    |
| 40              | 0.01      | 5                    |
| 50              | 0.01      | 4                    |

Maxwell [3] introduced a correlation about 140 years ago for calculation of effective thermal conductivity of uniform mixtures of spherical solid particles and a liquid

$$k_{eff} = \frac{k_p + 2k_f + 2(k_p - k_f)\varnothing}{k_p + 2k_f - (k_p - k_f)\varnothing} k_f$$ (1)

In this correlation, $k_{eff}$, $k_p$ and $k_f$ are the thermal conductivity for nanofluids, nanoparticles, and base fluid. Moreover $\varnothing$ is volume fraction of particles in the mixture. For the nanofluids in this study the thermal conductivities predicted by Maxwell’s equation are compared with the experimental results.

2.2. Test of convective heat transfer

An open flow loop utilizing a microtube test section was designed to measure heat transfer performance. The designed test rig comprised of a syringe pump having two parallel syringes (Legato 200 KD Scientific), a DC power supply (GW Instek PSP-405) to provide constant heat flux through the test section, one differential pressure transducer (UNIK 5000) to measure the pressure drop along the test section, a storage tank located on the scale which records mass flow rate of the nanofluid, and a stainless steel microtube test section (with 0.50 mm ID, 0.80 mm OD and 30 cm length). 13 thermocouples (in 2 cm distance with each other) were well attached on the outer surface of the horizontal microtube to record the wall temperature of the test section, and a data logger was used to collect and send the data to a computer (Figure 2). The flow rate can be adjusted by selecting different syringes and changing the setting of the pump. Therefore, different Reynolds numbers in laminar flow regime were achievable. Moreover, a constant heat flux along the test section was achieved using DC power applied by two electric connectors attached to the ends of test section.

Figure 2. Convective test setup.
3. Heat transfer calculation

The local heat transfer coefficient can be calculated by dividing the heat flux by the local temperature difference between the inside of the tube wall and the fluid

\[ h_x = \frac{q^*}{T_{S-in,x} - T_{f,x}} \]  

(2)

In eq. (2), \( q^* \) is the thermal heat flux at the inside of the tube wall. Moreover, \( T_{S-in,x} \) and \( T_{f,x} \) are the inner wall and the fluid temperatures at position \( x \) respectively.

The heat flux is determined from the following relation:

\[ Q = q^* A = \dot{m} C_p (T_{out} - T_{in}) \]  

(3)

\( T_{S-in,x} \), the inner wall temperature at position \( x \) is calculated for circular tubes as

\[ T_{S-in,x} = T_{S-out,x} + \frac{Q}{4\pi L k_{tube}} \left[ \frac{\varphi (1-\varphi)-1}{\varphi-1} \right] \]  

(4)

\[ \varphi = \left( \frac{D_{out}}{D_{in}} \right)^2 \]  

(5)

Above, \( T_{S-out,x} \) is the outer wall temperature, \( Q \) is the thermal power, \( L \) is the tube length, \( k_{tube} \) is the thermal conductivity of the tube, \( D_{in} \) and \( D_{out} \) are the inside and the outside diameter of the tube respectively.

Moreover, the fluid temperature at position \( x \) is calculated as

\[ T_{f,x} = T_{in} + \frac{q^* \pi D_{in} x}{\dot{m} C_p} \]  

(6)

where \( T_{in} \) is the inlet temperature. In this study, heat transfer coefficients for water and nanofluids are calculated based on the raw data and compared at equal Reynolds numbers, mass flow rate, volume flow rate, pressure drop and pumping power. The pumping power was calculated as the product of the volume flow rate and the pressure drop. Moreover, effective density and specific heat were calculated from the two phase mixture formulas

\[ \rho_{eff} = \varphi \rho_p + (1 - \varphi) \rho_f \]  

(7)

\[ C_p_{eff} = \frac{\varphi \rho_p C_p (1-\varphi) \rho_f C_p}{\rho_{eff}} \]  

(8)

where \( \rho \) and \( C_p \) are density and specific heat respectively. Because generally solids have higher density and lower specific heat than water, these formulas indicate that the higher the solid nano particles volume concentration in a water based solution, the higher the density and the lower the specific heat of the solution.

The non-dimensional Nusselt number can be calculated from the measured heat transfer coefficients and compared with the classical correlations. The local Nusselt number was calculated as a function of the Graetz number by the Shah-London correlation [13]

\[ Nu_x = \left\{ \begin{array}{ll}
1.302/(Gz)^{\frac{1}{3}} - 1 & \text{for } 1/Gz \leq 0.00005 \\
1.302/(Gz)^{\frac{1}{3}} - 0.5 & \text{for } 0.0005 \leq 1/Gz \leq 0.0015 \\
4.364 + 8.68/(1000Gz)^{0.506} e^{-41/Gz} & \text{for } 1/Gz \geq 0.0015 
\end{array} \right. \]  

(9)

where the Graetz number can be seen as a non-dimensional inverted distance from the inlet (\( Gz = 1/X^* = (d/x)RePr \)). The Nu numbers are infinite when \( X^* = 0 \) and decrease gradually to approach a constant value of 4.364 at fully developed laminar flow (at constant heat flux conditions).
Moreover, in laminar flow for hydrodynamically fully developed flow condition, the Darcy equation for the friction factor is used:

\[ f = \frac{64}{Re} \]  
\[ Re = \frac{\rho u L}{\mu} \]  

For calculating the Re number, the measured values for the viscosity of the tested nanofluids was used [11]. In this study the Darcy- equation for the friction factor is compared with the friction factor calculated from the pressure drop:

\[ f = \frac{2D}{\rho L} \frac{\Delta P}{V^2} \]

where \( \Delta P \) is the measured pressure drop over the tube and \( V \) is the velocity of the fluid.

4. Uncertainty Analysis
If \( z \) is a function of several variables, \( x_i \), each with their own uncertainty, \( \Delta x_i \) the overall uncertainty in \( z \) is [14]

\[ \Delta z = \pm \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 (\Delta x_i)^2} \]

Table 2 shows the uncertainty range of the variables in percent.

| Variable name | Uncertainty range (%) |
|---------------|-----------------------|
| Nu            | 4.4 - 5.6             |
| X*            | 0.5 - 1.6             |
| h, avg        | 3.4 - 4.9             |
| h, local      | 11.6 - 16.9           |
| Re            | 6.9 - 8.7             |
| Q and \( m \) | 1.8 - 5.5             |
| \( \Delta P^1 \) | 5.8 - 34.3         |
| P^2           | 1.9 - 6.5             |
| f             | 8.8 - 13.8            |

Table 2. Uncertainty ranges.

1- Pressure drop  
2- Pumping power

5. Results and discussion
To check the validity of the TPS instrument, the thermal conductivity of distilled water was measured and compared with IAPWS-95 reference, which is a standard source for thermodynamic properties of water [15]. Figure 3 shows the results of the thermal conductivity measurement for water compared with IAPWS-95 reference. As shown the accuracy of measurement for water is within 2% compared with the IAPWS-95 data. The TPS instrument was used to measure thermal conductivity of the nanofluids as well. Moreover, the thermal conductivity of nanofluids is compared with the values predicted by the Maxwell equation, and the results are shown in the Figure 4. The error bars on the water data shows the maximum deviation of the measurements from the IAPWS-95 reference. As indicated, the thermal conductivity of nanofluids, similar to base fluid, increases with the temperature. Furthermore, the results show that adding nanoparticles to the base fluid improve its thermal conductivity. This increment in the highest case is around 7% (for Al\(_2\)O\(_3\)-water nanofluid at T=20°C Figure 5). As can be seen from Figure 4, for both Al\(_2\)O\(_3\)-water nanofluid and TiO\(_2\) nanofluid the Maxwell equation over-predicts the experimental results at low temperature and the prediction is improved at higher temperatures. The best prediction among these three nanofluids was for ZrO\(_2\). Generally the experimental results with the exception of ZrO\(_2\)-water nanofluids are over predicted by
the Maxwell equation. The results also reveal minimum enhancement in thermal conductivity (with same concentration) with water-ZrO$_2$ nanofluids. As shown in Figure 5 the increment in thermal conductivity is almost independent of temperature in the temperature range 20-50°C.

![Figure 3. Thermal conductivity of distilled water.](image)

![Figure 4. Thermal conductivity of the nanofluids.](image)

![Figure 5. Thermal conductivity increase for the nanofluids.](image)
As explained above, heat transfer coefficients were calculated based on the measured values for wall temperatures, inlet and outlet temperatures, and the mass flow rate. Moreover, pressure drop over the tube was measured and from this value the friction factor was calculated. The measurements were done at different flow rates. For water and water-TiO\textsubscript{2} nanofluids the volume flow rates were 10, 15, 20, 25 and 30 ml/min. However, the Al\textsubscript{2}O\textsubscript{3} and ZrO\textsubscript{2} nanofluids were not measured at 30 ml/min due to high pressure drop. The experimental results of the three nanofluids and distilled water are compared with the Shah correlation for the flow rate of 20 ml/min in Figure 6. As can be seen most of the experimental results are within 15% of the values predicted by the Shah correlation, indicating that the classical theory is valid also for the tested nanofluids. The average Nusselt numbers (dimensionless heat transfer coefficients) over the test section are plotted at different Reynolds numbers for water and the nanofluids in Figure 7. To facilitate the interpretation of the data the results for water (+/-) 10% is plotted in the same diagram. As shown, the Nusselt number at any given Reynolds number increase up to roughly 30% for water-Al\textsubscript{2}O\textsubscript{3} and water-ZrO\textsubscript{2} nanofluids, and by around 10% for water-TiO\textsubscript{2}. The alumina and zirconia nanofluids shows higher heat transfer coefficients at the same Reynolds number. As nanofluids have higher viscosity, higher flow velocity is necessary to achieve the same Reynolds number. Comparisons of Nusselt numbers or heat transfer coefficients at equal Reynolds numbers is therefore deceiving and not interesting from a practical point of view. Unfortunately, comparison at the same Reynolds number is very common in the literature in the field of nanofluids.

![Figure 6](image6.png)

**Figure 6.** Local Nusselt no vs X* (non-dimensional distance from inlet) for water and nanofluids at the nominal flow rate of 20 ml/min.

![Figure 7](image7.png)

**Figure 7.** Average Nusselt no for water and nanofluids vs Reynolds number.
Alternative ways for presentation of the results should be considered, besides comparison at the same Reynolds numbers. Comparing the heat transfer at the same volume flow rates, mass flow rates, pressure drops and (theoretical) pumping power (equal to pressure drop multiplied by volume flow rate) are considered as more appropriate options in this study shows these comparisons. Figure 8-A shows the average heat transfer coefficient versus volume flow rate. As indicated in the Figure the difference between the values are small and in this case no benefit was observed in using nanofluids over the base fluid. Figure 8-B shows the heat transfer coefficients versus mass flow rates for different fluids, and obviously nanofluids are not better than distilled water, in this comparison. In Figure 8-C heat transfer coefficients are plotted versus pressure drop. A consequence of higher viscosity for the nanofluids compared to the distilled water is higher pressure drop, which yields lower performance for nanofluids in this representation. As expected, the heat transfer coefficients at the same pumping power (calculated by multiplying the pressure drop and the volume flow rates), shown in Figure 8-D, is lower for the nanofluids than for water. In general as Figure 8 shows, the alternative presentations of the experimental results do not show any advantage of using nanofluids for cooling applications.

The friction factors calculated from the measured pressure drop are compared with the conventional Darcy friction factor for laminar flow in Figure 9. Based on the results in Figure 9 for distilled water, titania and alumina nanofluids, the difference is within 10%, however for zirconia the difference is
slightly larger. In general, as indicated in Figure 9, the Darcy friction factor seems to be valid for nanofluids.

![Figure 9](image)

Figure 9. Experimental values for the friction factor for water and the nanofluids compared with Darcy equation

6. Conclusion
This article presents results from measurements of thermal conductivity, convective heat transfer in developing laminar flow and pressure drop in laminar flow for water and three nanofluids prepared by dispersing $\text{Al}_2\text{O}_3$, $\text{ZrO}_2$ and $\text{TiO}_2$ in water to a concentration of 9% by weight. The following conclusions may be drawn from this study:

By adding the nano particles to water, the thermal conductivity of the water is increased. This increment can for the tested fluids be reasonably well predicted by the Maxwell equation. A maximum of 7% enhancement in thermal conductivity compared to water was observed for the water based alumina with 9 wt% particle loading at $T=20^\circ\text{C}$. The thermal conductivity of the nanofluids is temperature dependent, and as for the base fluids increases with increasing temperature. However, the relative thermal conductivity of the tested nanofluids (ratio between thermal conductivity of nanofluids to base fluid) is almost independent of temperature.

Conventional correlations for predicting heat transfer locally in laminar flow such as Shah correlation is also valid for the tested nanofluids. The criteria for comparison of heat transfer coefficient between base fluid and nanofluids should be carefully selected. Although generally in the literature comparisons have been done at the same Reynolds number, this might be completely misleading. Compared in this way, an increase around 10-30% was observed for the tested nanofluids. However, comparing heat transfer coefficients for two fluids at the same Reynolds number, requires higher volume flow rate for the fluid with higher viscosity. Therefore, the higher heat transfer coefficient at equal Reynolds number is not because of better nanofluids performance, but due to higher volume flow rate. Other criteria for comparison besides equal Reynolds number are equal volume flow rate, mass flow rate, pressure drop and pumping power. In this study, no benefit was observed for using any of the tested nanofluids over base fluid if either of these criteria are taken into consideration.

Acknowledgement
The financial support of the EU project NanoHex (Enhanced Nano-fluid Heat Exchange) for this study is highly appreciated.
The authors would like to thank ItN Nanovation AG, Germany and Dispersia Ltd, England for the effort in the preparation of the nanofluids.
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