Measurement of Polarization and Triple-Product Correlations in $B \to \phi K^\pm$ Decays

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Abstract

We present a measurement of the decay amplitudes and triple-product correlations in $B \to \phi K^*$ decays based on 140 fb$^{-1}$ of data recorded at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB $e^+e^-$ storage ring. The decay amplitudes for the different $\phi K^*$ helicity states are measured from the angular distributions of final state particles in the transversity basis. The longitudinal and transverse complex amplitude moduli and angles are $|A_0|^2 = 0.51 \pm 0.06 \pm 0.04$, $|A_\perp|^2 = 0.24 \pm 0.06 \pm 0.03$, arg($A_0$) = $-2.21 \pm 0.22 \pm 0.05$ rad, and arg($A_\perp$) = $0.72 \pm 0.21 \pm 0.06$ rad. The $T$-violating asymmetries through triple-product correlations are measured to be consistent with zero.

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The decay $B \to \phi K^*$ involves the FCNC $b \to s$ transition that is forbidden to first order in the Standard Model (SM) but can proceed by second order penguin and box diagrams. This process therefore provides information on the Cabibbo-Kobayashi-Maskawa matrix element $V_{ts}$ and is sensitive to physics beyond the SM. The decay $B \to \phi K^*$ is a mixture of $CP$-even and $CP$-odd states, and polarization measurements allow us to project out the different $CP$ states statistically. Measurements of the $T$-odd triple-product correlations provide information about $T$-violating asymmetries as well as observables that are sensitive to New Physics (NP) \[3\]. Our previous measurement \[4\] and a recent report by BaBar \[5\] both suggest that the fraction of longitudinal polarization component differs from the naive SM prediction. This provides a strong motivation to update our study with a larger dataset.

In this paper, we present a measurement of the decay amplitudes in $B^+ \to \phi K^{**}$ and $B^0 \to \phi K^{*0}$ decays by a full three-dimensional angular analysis. The charge conjugate modes are included everywhere unless otherwise specified. We also report the measurements of triple-product correlations and related $T$-violating asymmetries, as well as the observables that are sensitive to NP.

This analysis is based on a data set with an integrated luminosity of 140 fb$^{-1}$ taken at the $Y(4S)$ resonance recorded by the Belle detector \[6\] at the KEKB $e^+e^-$ collider \[7\]. This luminosity corresponds to $152 \times 10^6$ produced $B\bar{B}$ pairs. The beam energies are 8 GeV for $e^-$ and 3.5 GeV for $e^+$.

The Belle detector is a general purpose magnetic spectrometer equipped with a 1.5 T superconducting solenoid magnet. Charged tracks are reconstructed in a Central Drift Chamber (CDC) and a Silicon Vertex Detector (SVD). Photons and electrons are identified using a CsI(Tl) Electromagnetic Calorimeter (ECL) located inside the magnet coil. Charged particles are identified using specific ionization ($dE/dx$) measurements in the CDC as well as information from Aerogel Cherenkov Counters (ACC) and Time of Flight Counters (TOF). A kaon likelihood ratio, $P(K/\pi) = \mathcal{L}_K / (\mathcal{L}_K + \mathcal{L}_\pi)$, has values between 0 (likely to be a pion) and 1 (likely to be a kaon), where $\mathcal{L}_K(\pi)$ is derived from the $dE/dx$, ACC and TOF measurements.

Candidate $\phi \to K^+K^-$ decays are found by selecting pairs of oppositely charged tracks that are not pion-like ($P(K/\pi) > 0.1$). The vertex of the candidate charged tracks is required to be consistent with the interaction point (IP) to remove poorly measured tracks. In addition, candidates are required to have a $K^+K^-$ invariant mass that is less than 10 MeV/$c^2$ from the nominal $\phi$ meson mass.

Pairs of oppositely charged tracks are used to reconstruct $K_S^0 \to \pi^+\pi^-$ decays. The $\pi^+\pi^-$ vertex is required to be displaced from the IP by a minimum transverse distance of 0.22 cm for high momentum ($> 1.5$ GeV/$c$) candidates and 0.08 cm for those with momentum less than 1.5 GeV/$c$. The direction of the pion pair momentum must agree with the direction defined by the IP and the vertex displacement within 0.03 rad for high-momentum candidates, and within 0.1 rad for the remaining candidates.

Charged tracks with $P(K/\pi) > 0.4$ ($< 0.9$) are considered to be kaons (pions). For $\pi^0 \to \gamma\gamma$, a minimum photon energy of 50 MeV is required and the $\gamma\gamma$ invariant mass must be less than 16 MeV/$c^2$ from the nominal $\pi^0$ mass. $K^*$ candidates are reconstructed in three decay modes: $K^{*0} \to K^+\pi^-$, $K^{*+} \to K^+\pi^0$ and $K^{*+} \to K_S^0\pi^+$. The invariant mass of the $K^*$ candidate is required to be less than 70 MeV/$c^2$ from the nominal $K^*$ mass.

A $B$ meson is reconstructed from a pair of $\phi$ and $K^*$ candidates and identified by the energy difference $\Delta E \equiv E_B^{\text{cms}} - E_{\text{beam}}^{\text{cms}}$, and the beam constrained mass $M_{bc} \equiv \sqrt{(E_{\text{beam}}^{\text{cms}})^2 - (P_B^{\text{cms}})^2}$. $E_{\text{beam}}^{\text{cms}}$ is the beam energy in the center-of-mass system (cms) of the
\( \Upsilon(4S) \) resonance, and \( E_B^{\text{cms}} \) and \( p_B^{\text{cms}} \) are the cms energy and momentum of the reconstructed \( B \) candidate. The \( B \)-meson signal window is defined as \( 5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2 \) and \( |\Delta E| < 45 \text{ MeV} \). The signal window is enlarged to \(-100 \text{ MeV} < \Delta E < 80 \text{ MeV} \) for \( B^+ \to \phi K^+(K^+ \to K^+\pi^0) \) because of the impact of shower leakage on \( \Delta E \) resolution. An additional requirement \( \cos \theta_{K^*} < 0.8 \) is applied to reduce low momentum \( \pi^0 \) background, where \( \theta_{K^*} \) is the angle between the \( K^* \) direction and its daughter kaon defined in the \( K^* \) rest frame. In the signal window, about 1\% of the events have multiple candidates. We choose the best candidate according to the goodness of fit of the \( B \) meson’s vertex.

The dominant background is continuum \( e^+e^- \to q\bar{q} \) production \((q = u, d, c, s)\). Several variables are used to exploit the differences between the event shapes for continuum \( q\bar{q} \) production (jet-like) and for \( B \) decay (spherical) in the cms frame of the \( \Upsilon(4S) \). These variables are combined into a single likelihood ratio \( R_s = \mathcal{L}_s / (\mathcal{L}_s + \mathcal{L}_{q\bar{q}}) \), where \( \mathcal{L}_s (\mathcal{L}_{q\bar{q}}) \) denotes the signal (continuum) likelihood. The requirements on the likelihood ratio are determined by a figure of merit study that includes a dependence on the \( B \)-tagging quality.

Backgrounds from other \( B \) decay modes such as \( B \to KKK^* \), \( B \to f_0(980)K^*(f_0 \to K^+K^-) \), \( B \to \phi K\pi \), \( B \to KKK\pi \), and feed-across between \( \phi K^* \) and \( \phi K \) decay channels are studied. The contributions from \( B \to KKK^* \) and \( B \to f_0(980)K^*(f_0 \to K^+K^-) \) are estimated from the \( K^+K^- \) invariant mass distribution. The \( K^+K^- \) mass distribution for \( B \to KKK^* \) is determined by Monte Carlo (MC) simulation assuming three-body phase space decay. The shape for \( f_0(980) \) is obtained from MC, where a \( S \)-wave Breit-Wigner with a 40 MeV/c\(^2\) intrinsic width is assumed. These contributions are estimated separately by fits to the events outside of the \( \phi \) mass region. The contribution from \( B \to KKK^* \) is estimated to be 5–10\%[10] of the signal yield depending on \( K^* \) decay modes. The \( B \to f_0K^+ \) contribution is estimated to be 4–11\%. Contamination from four-body \( B \to KKK\pi \) decays is checked by performing fits to the non-resonant region of \( K^+K^- \) and \( K\pi \) mass. It is found to be very small and is neglected. To remove the contamination of \( \phi K \) decays, they are explicitly reconstructed and removed from \( \phi K^* \) candidates.

The signal yields \((N_s)\) are extracted by extended unbinned maximum-likelihood fits performed simultaneously in \( \Delta E \) and \( M_{bc} \). The signal probability density functions (PDFs) are products of Gaussians in \( \Delta E \) and \( M_{bc} \). The means and widths are verified using \( B \to J/\psi K^* \) decays. Additional bifurcated Gaussians (Gaussians with different widths on either side of the mean) are used to model the tails in the \( \Delta E \) distribution.

The PDF shape for the continuum events are parameterized by an ARGUS function in \( M_{bc} \) and a linear function in \( \Delta E \). The parameters of the functions are determined by a fit to the events in the signal sideband. The sideband region is defined by \( M_{bc} < 5.265 \text{ GeV}/c^2 \) and \( |\Delta E| > 0.08 \text{ GeV} \) \((\Delta E > 0.10 \text{ GeV} \text{ and } \Delta E < -0.12 \text{ GeV})\) for the modes without (with) a \( \pi^0 \) in the final state, respectively. The signal and background yields are allowed to float in the fit while other PDF parameters are fixed. The measured signal yields and purities are summarized in Table[4] The distributions of \( \Delta E \) and \( M_{bc} \) are shown in Fig.[1]

The decay angles of a \( B \)-meson, to the two vector mesons \( \phi \) and \( K^* \) defined in the transversity basis[13], are shown in Fig.[2] The \( x-y \) plane is defined to be the decay plane of \( K^* \) and the \( x \) axis is in the direction of the \( \phi \)-meson. The \( y \) axis is perpendicular to the \( x \) axis and is on the same side as the kaon from the \( K^* \) decay. The \( z \) axis is perpendicular to the \( x-y \) plane according to the right-hand rule, \( \theta_{\phi\pi} \) \((\phi_{tr})\) is the polar (azimuthal) angle with respect to the \( z \)-axis of the \( K^+ \) from \( \phi \) decay in the \( \phi \) rest frame, and \( \theta_{K^*} \) is defined above.
The presence of FSI results in phases that differ from either 0 or \( \pm \pi \). The imaginary phases of the amplitudes are sensitive to final state interactions (FSI).

The complex amplitudes are determined by performing an unbinned maximum likelihood fit with the \( B \rightarrow \phi K^* \) candidates in the signal window. The combined likelihood is given

\[
\frac{d^3\Gamma(\phi_{tr}, \cos \theta_{tr}, \cos \theta_{K^*})}{d\phi_{tr} d\cos \theta_{tr} d\cos \theta_{K^*}} = \frac{9}{32\pi} |A_\perp|^2 |A_\parallel|^2 \cos^2 \theta_{tr} \sin^2 \theta_{K^*} \\
+ |A_\parallel|^2 \sin^2 \theta_{tr} \sin^2 \phi_{tr} \sin^2 \theta_{K^*} \\
+ |A_\parallel|^2 \cos^2 \theta_{tr} \cos^2 \phi_{tr} \cos^2 \theta_{K^*} \\
+ \sqrt{2} \text{Re}(A_\parallel^* A_0) \sin^2 \theta_{tr} \sin 2\phi_{tr} \sin 2\theta_{K^*} \\
- \eta \sqrt{2} \text{Im}(A_\parallel^* A_\perp) \sin 2\theta_{tr} \cos \phi_{tr} \sin 2\theta_{K^*} \\
- 2\eta \text{Im}(A_\parallel^* A_\perp) \sin 2\theta_{tr} \sin \phi_{tr} \sin^2 \theta_{K^*},
\]

where \( A_0, A_\parallel, \) and \( A_\perp \) are the complex amplitudes of the three helicity states in the transversity basis with the normalization condition \( |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1 \), and \( \eta \equiv +1 \) (\( -1 \)) corresponds to \( B (\bar{B}) \) mesons. \( A_0 \) denotes the longitudinal polarization of the \( \phi \rightarrow K^+K^- \) system and \( A_\perp (A_\parallel) \) is the transverse polarization along the \( z \)-axis (\( y \)-axis). The value of \( |A_\perp|^2 \) (\( |A_\parallel|^2 + |A_\perp|^2 \equiv 1 - |A_\parallel|^2 \)) is the \( CP \)-odd (\( CP \)-even) fraction in the decay \( B \rightarrow \phi K^* \) \[14\]. The imaginary phases of the amplitudes are sensitive to final state interactions (FSI). The presence of FSI results in phases that differ from either 0 or \( \pm \pi \).
of efficiency function may be combined if both decays are dominated by $b$-annihilation contribution to $K^0$ background ($2 - 3 \%$). The systematic uncertainty in the angular resolution is estimated by MC simulation and found to be less than 1%. Uncertainties in the background PDFs, the signal yields, and the modeling systematic uncertainty in the angular resolution is estimated by MC simulation and found to be less than 1%. The value of $\eta$ is determined from the charge of the kaon in the $K^*$ decay, $R_{q\pi}$ is determined from sideband data and $R_{KKK^*}$ is assumed to be flat. The detection efficiency function ($\epsilon$) is determined by MC. The fractions of $\phi K^*$ ($f_{\phi K^*}$), $q\pi$ ($f_{q\pi}$) and $B \rightarrow KKK^*$ ($f_{KKK^*}$) are parameterized as a function of $\Delta E$ and $M_{K\pi}$. The value of $\arg(A_0)$ is set to zero and $|A_\parallel|^2$ is calculated from the normalization constraint in the fit. The four parameters ($|A_0|^2$, $|A_\perp|^2$, $\arg(A_\parallel)$, and $\arg(A_\perp)$) are determined from the fit. The results from $B^0 \rightarrow \phi K^{*0}$ and $B^+ \rightarrow \phi K^{*+}$ may be combined if both decays are dominated by $b \rightarrow s$ penguin transitions and the $b \rightarrow u$ annihilation contribution to $B^+ \rightarrow \phi K^{*+}$ can be neglected. Figure 3 shows the angle distributions with projections of the fit superimposed. The amplitudes obtained from the fit are shown in Table III.

The systematic uncertainties include the slow pion detection efficiency ($6 - 7 \%$), the background from higher $K^*$ states ($2 - 8 \%$), and the $B \rightarrow f_0 K^*$ background ($2 - 3 \%$). The systematic uncertainty in the angular resolution is estimated by MC simulation and found to be less than 1%. Uncertainties in the background PDFs, the signal yields, and the modeling of efficiency function ($\epsilon$) are estimated to be $1 - 3 \%$.

The triple-product for a $B$ meson decay to two vector mesons takes the form $\vec{q} \cdot \left(\vec{e}_1 \times \vec{e}_2\right)$, where $\vec{q}$ is the momentum of one of the vector mesons. The other two vectors $\vec{e}_1$ and $\vec{e}_2$ are the polarizations of the two vector mesons. The presence of the triple product can be demonstrated by measuring a non-zero value of the asymmetry $A_T = \Gamma[\vec{e}_1 \cdot (\vec{v}_2 \times \vec{v}_3)] - \Gamma[\vec{e}_2 \cdot (\vec{v}_1 \times \vec{v}_3)] < 0$. In the SM, $A_T$'s are very small. Experimentally, the following two $T$-odd quantities are used instead of $A_T$,

$$A_T^0 = \frac{\text{Im}(A_\perp A_0^\ast)}{A_0^2 + A_\perp^2 + A_\parallel^2}, \quad A_T^\parallel = \frac{\text{Im}(A_\perp A_\parallel^\ast)}{A_0^2 + A_\perp^2 + A_\parallel^2}.$$  

The comparison of these triple product asymmetries ($A_T^0$ and $A_T^\parallel$) with the corresponding...
The subscript $\rho$ where $\Sigma$, $\Lambda$, and $A$ parameters are expressed as additional variables that can be measured through angular analysis are suggested in Ref. [3]. By introducing a time-dependent decay rate that can be written as follows

$$\Gamma(B(\bar{B}) \to V_1V_2) = e^{-Tt} \sum_{\lambda\leq\sigma} (\Lambda_{\lambda\sigma} \pm \Sigma_{\lambda\sigma} \cos(\Delta M t) \mp \rho_{\lambda\sigma} \sin(\Delta M t)) g_\lambda g_\sigma ,$$ (4)

where $\Sigma$, $\Lambda$, and $\rho$ are expressed as

$$\Lambda_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 + |\bar{A}_\lambda|^2) , \quad \Sigma_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 - |\bar{A}_\lambda|^2) ,$$ (5)

$$\Lambda_{i\perp} = -\text{Im}(A_{i\perp} A_{i\perp}^* - \bar{A}_{i\perp} \bar{A}_{i\perp}^*) , \quad \Sigma_{i\perp} = -\text{Im}(A_{i\perp} A_{i\perp}^* + \bar{A}_{i\perp} \bar{A}_{i\perp}^*) ,$$ (6)

$$\Lambda_{i0} = \text{Re}(A_{i\parallel} A_{i\parallel}^* + \bar{A}_{i\parallel} \bar{A}_{i\parallel}^*) , \quad \Sigma_{i0} = \text{Re}(A_{i\parallel} A_{i\parallel}^* - \bar{A}_{i\parallel} \bar{A}_{i\parallel}^*) ,$$ (7)

$$\rho_{\perp\perp} = \text{Im} \left( \frac{q}{p} A_{i\perp}^* \bar{A}_{i\perp} \right) , \quad \rho_{i0} = -\text{Im} \left( \frac{q}{p} A_{i\perp}^* \bar{A}_{i\parallel} \right) ,$$ (8)

$$\rho_{i\perp} = \text{Re} \left( \frac{q}{p} [A_{i\perp}^* \bar{A}_{i\perp} + A_{i\perp}^* \bar{A}_{i\parallel}] \right) , \quad \rho_{i\parallel} = -\text{Im} \left( \frac{q}{p} [A_{i\parallel}^* \bar{A}_{i\parallel} + A_{i\parallel}^* \bar{A}_{i\perp}] \right) .$$ (9)

The subscript $i$ is one of the $\{0, \parallel\}$ amplitudes, $\Lambda_{i0}$ and $\Lambda_{i\parallel}$ are equal to the T-violation parameters $A_{T}^0 = \bar{A}_{T}^0 - A_{T}^0$ and $A_{T}^\parallel = \bar{A}_{T}^\parallel - A_{T}^\parallel$, respectively. Since the parameters $\rho_{\lambda\sigma}$ appear in terms that are proportional to $\sin(\Delta M t)$, which vanish in a time integrated measurement,
we only report results for the other twelve \( \Lambda \) and \( \Sigma \) parameters. The following equations should hold in the absence of NP:

\[
\Sigma_{\lambda\lambda} = 0, \quad \Sigma_{||0} = 0, \quad \Lambda_{\perp\perp} = 0.
\] (10)

Any violation of these relations would be evidence for NP.

By separating \( B \)-meson and \( \bar{B} \)-meson samples and a rearranging the fitting parameters in the unbinned maximum likelihood fit, we obtain the helicity amplitudes for the \( B \) and \( \bar{B} \) samples, the triple-product correlations, and other NP-sensitive observables, which are given in Tables III and IV.

**TABLE III:** The measured helicity amplitudes and triple-product correlations in the \( B \)-meson and \( \bar{B} \)-meson samples.

| Mode | \( B \)-meson combined (\( \phi K^{*0} \) and \( \phi K^{*+} \)) | \( \bar{B} \)-meson combined (\( \phi \bar{K}^{*0} \) and \( \phi K^{*-} \)) |
|------|-------------------------------------------------|-------------------------------------------------|
| \( |A_0|^2 \) | 0.41 \( \pm \) 0.10 \( \pm \) 0.03 | 0.59 \( \pm \) 0.08 \( \pm \) 0.06 |
| \( |A_1|^2 \) | 0.24 \( \pm \) 0.10 \( \pm \) 0.02 | 0.26 \( \pm \) 0.08 \( \pm \) 0.04 |
| \( \text{arg}\,(A_\|) \) (rad) | \( -2.29 \) \( \pm \) 0.35 \( \pm \) 0.13 | \( -2.05 \) \( \pm \) 0.31 \( \pm \) 0.04 |
| \( \text{arg}\,(A_\perp) \) (rad) | 0.74 \( \pm \) 0.31 \( \pm \) 0.10 | 0.81 \( \pm \) 0.31 \( \pm \) 0.06 |
| \( A_0^0 \) | 0.21 \( \pm \) 0.08 \( \pm \) 0.03 | 0.28 \( \pm \) 0.09 \( \pm \) 0.05 |
| \( A_\| \) | 0.04 \( \pm \) 0.08 \( \pm \) 0.02 | 0.06 \( \pm \) 0.06 \( \pm \) 0.02 |

**TABLE IV:** The observables sensitive to NP extracted from angular analysis.

| \( \Lambda \) Observables | \( \Sigma \) Observables |
|--------------------------|--------------------------|
| \( \Lambda_{00} \) | 0.50 \( \pm \) 0.06 \( \pm \) 0.04 | \( \Sigma_{00} \) | \( -0.09 \) \( \pm \) 0.06 \( \pm \) 0.02 |
| \( \Lambda_{\|\|} \) | 0.25 \( \pm \) 0.06 \( \pm \) 0.02 | \( \Sigma_{\|\|} \) | 0.10 \( \pm \) 0.06 \( \pm \) 0.02 |
| \( \Lambda_{\perp\perp} \) | 0.25 \( \pm \) 0.06 \( \pm \) 0.03 | \( \Sigma_{\perp\perp} \) | \( -0.01 \) \( \pm \) 0.06 \( \pm \) 0.02 |
| \( \Lambda_{\perp0} \) (\( = A_0^0 \)) | 0.07 \( \pm \) 0.11 \( \pm \) 0.04 | \( \Sigma_{10} \) | \( -0.49 \) \( \pm \) 0.12 \( \pm \) 0.07 |
| \( \Lambda_{\perp\|} \) (\( = A_\| \)) | 0.02 \( \pm \) 0.10 \( \pm \) 0.03 | \( \Sigma_{\perp\|} \) | \( -0.09 \) \( \pm \) 0.10 \( \pm \) 0.02 |
| \( \Lambda_{\|0} \) | \( -0.39 \) \( \pm \) 0.13 \( \pm \) 0.06 | \( \Sigma_{00} \) | \( -0.11 \) \( \pm \) 0.13 \( \pm \) 0.04 |

In summary, the decay amplitudes for \( B \rightarrow \phi K^* \) are determined by fitting the angular distributions in the transversity basis. The longitudinal polarization component (\( |A_0|^2 \)) is in agreement with Ref. [4], but differs from the naive SM prediction. The measured value of \( |A_\perp|^2 \) shows that both \( CP \)-odd (\( |A_\perp|^2 \)) and \( CP \)-even (\( |A_0|^2 + |A_\||^2 \)) components are present in \( \phi K^* \) decays with a ratio of 1:3. The phase of \( A_\perp \) and \( A_\| \) differ from zero or \( -\pi \) by 3.6 and 3.7 standard deviations [17], respectively. Thus, our data indicate the presence of final state interactions. The triple products \( A_\|^0 \) and \( \bar{A}_\|^0 \) differ from zero by 2.4\( \sigma \) and 2.8\( \sigma \), respectively [17]. This may also be an indication of the presence of final state interactions. However, the differences between \( A_T \)'s and \( \bar{A}_T \)'s (\( = A_0^0 = \Lambda_{10,0} \)) that are sensitive to \( T \)-violating
asymmetry are consistent with zero as well as\cite{5}. The equations, $\Sigma_{\lambda\lambda} = 0$, $\Sigma_{\parallel\parallel} = 0$, and $\Lambda_{\perp\perp} = 0$, should hold in the absence of NP. Our data does not show any significant violation of those relations.

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