Depinning at the initial stage of the resistive transition in superconductors with a fractal cluster structure

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Depinning of vortices in percolative superconductor containing fractal clusters of a normal phase is considered. Transition of the superconductor into a resistive state corresponds to the percolation transition from a pinned vortex state to a resistive state when the vortices are free to move. The motion of the magnetic flux transferred by these vortices gives rise to the region of initial dissipation on current-voltage (U-I) characteristic. The influence of normal phase clusters on distinctive features of U-I characteristics of percolative type-II superconductors is considered. It is found that an increase in the fractal dimension of the normal phase clusters causes the initial dissipation region to broaden out. The reason of this effect is an increase in the density of free vortices broken away from the pinning centers by the Lorentz force. Dependencies of the free vortex density on the fractal dimension of the normal phase cluster boundaries are obtained.

The dynamics of vortices in superconductors with fractal boundaries between the normal and superconducting phases has recently received much attention [1] - [3]. The study of their U-I characteristics enable to get new information on the electromagnetic properties as well as on the nature of a vortex state in such materials. New problems encountered in this field are of interest in view of their importance for the application of superconducting composites in electronics and power engineering, especially, for superconducting wire fabrication. Superconductors containing fractal clusters have specific magnetic and transport properties [4], [5]. The possibility to increase the critical current through the enhancement of pinning by the fractal normal phase clusters is of a particular interest [6], [7].

This paper is devoted to an analysis of the initial region of the resistive transition where the energy dissipation sets-in. Here the process of vortex depinning gradually accurs resulting finally in the destruction of superconducting state because of the thermo-magnetic instability. The problem of initial dissipation in high-temperature superconductors (HTS’s) has been studied by many authors [8], [9] - [11], [12], [13]. The residual resistance of bismuth-based HTS’s \( \text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+y} \) (BSCCO-2223) and \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{10+y} \) (BSCCO-2212) at small currents has been explained by deterioration of grain boundaries, by initiation of micro-cracks which can act as chains of serially connected weak links, as well as by the grain-to-grain misalignment or by the degradation of grains themselves [9], [10]. The ohmic behavior of U-I curves on the initial stage of resistive transition in BSCCO-2223 and BSCCO-2212 has been attributed to the local transfer of excess current into the normal metal inclusions [11]. The fractal regime in the initial stage of dissipation has been observed in BSCCO-2223, \( \text{YBa}_2\text{Cu}_3\text{O}_{7-x} \) (YBCO), and \( \text{GdBa}_2\text{Cu}_3\text{O}_{7-x} \) [8]. The fractal nature of the normal phase clusters in YBCO thin films has been found [2] and the effect of such clusters on vortex dynamics has been analyzed [3] - [5], [7].

Let us consider a superconductor containing inclusions of a normal phase, which are out of contact with one another. We will suppose that the characteristic sizes of these inclusions far exceed both the superconducting coherence length and the penetration depth. A prototype of such a structure is a superconducting wire.

The first generation HTS wires are fabricated following the powder-in-tube technique (PIT). The metal tube is being filled with HTS powder, then the thermal and deformation treatment is being carried out. The resulting product is the wire (tape) consisting of one or more superconducting cores armored by the normal metal sheath. The sheath endows the wire with the necessary mechanical (flexibility, folding strength) and electrical (the possibility to
release an excessive power when the superconductivity will be suddenly lost) properties. At the present time the best results are obtained for the silver-sheathed bismuth-based composites, which are of practical interest for energy transport and storage. In view of the PIT peculiarity the first generation HTS wire has a highly inhomogeneous structure. Superconducting core represents a dense conglomerate of BSCCO micro-crystallites containing normal phase inclusions inside [14], [15]. These inclusions primarily consist of a normal metal (silver) as well as the fragments of different chemical composition, grain boundaries, micro-cracks, and the domains of the reduced superconducting order parameter. The volume content of the normal phase in the core is far below the percolation threshold, so there is a percolative superconducting cluster that carries the transport current.

The second generation HTS wires (coated conductors) have multi-layered film structure consisting of the metal substrate (nickel-tungsten alloy), the buffer oxide sub-layer, HTS layer (YBCO), and the protective cladding made from the noble metal (silver). Superconducting layer, which carries the transport current, has the texture preset by the oxide sub-layer. In the superconducting layer there are clusters of columnar defects that can be created during the film growth process as well as by the heavy ion bombardment [16]. Such defects are similar in topology to the vortices, therefore they suppress effectively the flux creep that makes possible to get the critical current up to the depairing value [17].

A passage of electric current through a superconductor is linked with the vortex dynamics because the vortices are subjected to the Lorentz force created by the current. In its turn, the motion of the magnetic flux transferred by vortices induces an electric field that leads to the energy losses. In HTS’s the vortex motion is of special importance because of large thermal fluctuations and small pinning energies [18]. Here we will consider the simplified model of 1D pinning when a vortex filament is trapped on the set of pinning centers [19]. Superconductors containing separated clusters of a normal phase provides for effective pinning, because the magnetic flux is locked in these clusters, so the vortices cannot leave them without crossing the superconducting space. These clusters present the sets of normal phase inclusions, united by the common trapped flux and surrounded by the superconducting phase [2], [3]. The magnetic flux can be created both by an external source (e.g., at the magnetization in the field cooling regime) and by the transport current (in the self-field regime). In the latter case the flux is concentrated along irregular-shaped rings, which are deformed in such a way that the normal phase clusters would be most captured. When the current is increased, the vortices start to break away from the clusters of pinning force weaker than the Lorentz force created by the transport current. As this takes place, the vortices will first pass through the weak links, which connect the normal phase clusters between themselves. In such a system depinning has a percolative character [20], [21], [22], because unpinned vortices move through the randomly generated channels created by weak links. Weak links join the adjacent cells and enable the vortices to pass from one cluster to another. Weak links form readily in HTS’s due to the intrinsically short coherence length [23]. Depending on the specific weak link configuration each normal phase cluster has its own current of depinning, which contributes to the total statistical distribution of critical currents.

One the other hand, weak links do not only provide for the percolation of magnetic flux, but they also connect superconducting domains between themselves, maintaining the electrical current percolation. As the transport current is increased, the local currents flowing through ones or other weak links begin to exceed the critical values, therefore some part of them become resistive. Thus, the number of weak links involved in the superconducting cluster is randomly reduced so the transition of a superconductor into a resistive state corresponds to breaking of the percolation through a superconducting cluster. The transport current acts as a random generator that changes the relative fractions of conducting components in classical percolative medium [24], hence the resistive transition can be treated as a current-induced critical phenomenon [8].

The depinning current of each cluster is related to the cluster size, because the larger cluster has more weak links over its boundary with the surrounding superconducting space, and thus the smaller current of depinning [3]. As a measure of the cluster size we will take the area of its cross-section, and in the subsequent text we will call this value simply “the cluster area”. An important feature of normal phase clusters is that they can have fractal boundaries [2], i.e. the perimeter of their cross-section and the enclosed area obey the scaling law: \( P^{1/D} \propto A^{1/2} \), where \( D \) is the fractal dimension of the cluster boundary [25].

After the vortices start to move, a superconductor passes into a resistive state. The voltage \( U \), arising across the superconductor when the transport current \( I \) is passed through, can be expressed as a convolution integral, in which the contributions from depinning currents of all the clusters are taken into account:
\[ U = R_f \int_0^I (I - I') f(I') dI' \]  

(1)

where \( R_f \) is the flux flow resistance, \( f(I) \) is the distribution function of the depinning currents.

In most practically important cases the distribution of the cluster areas may be described by gamma distribution \([4]\) with the probability density

\[ w(a) = \frac{(g+1)^{g+1}}{\Gamma(g+1)} a^g \exp\left(- (g+1) a\right) \]  

(2)

where \( \Gamma(\nu) \) is Euler gamma function, \( a \equiv A/A_0 \) is the dimensionless area of the cluster, \( A \) is the area of the cross-section of the cluster by the plane, transversally to which the vortices are moving, \( A_0 > 0 \) and \( g > -1 \) are the parameters of gamma distribution that control the mean area of the cluster \( A = (g+1)A_0 \) and its variance \( \sigma_A^2 = (g+1)A_0^2 \). The mean dimensionless area of the cluster is equal to unity, whereas the variance is determined by \( g \)-parameter only: \( \sigma_a^2 = 1/(g+1) \).

Gamma distribution of the cluster area of Eq. (2) gives rise to exponential-hyperbolic distribution of depinning currents \([4]\)

\[ f(i) = \frac{2G^{g+1}}{D\Gamma(g+1)} i^{-(2/D)(g+1)-1} \exp\left(-Gi^{2/D}\right) \]  

(3)

for which integral of Eq. (1) has the form:

\[ u = \frac{r_f}{\Gamma(g+1)} \left( i \Gamma\left(g+1, Gi^{2/D}\right) - G_i^{D/2}\Gamma\left(g + 1 - \frac{D}{2}, Gi^{2/D}\right) \right) \]  

(4)

where \( G \equiv (\theta/\theta_i) - (D/2) \exp(\theta) \Gamma\left(g, \theta_i\right) \)^{2/D}, \( \theta \equiv g + 1 + D/2 \), \( i \equiv I/I_i \) is the dimensionless electrical current normalized to the critical current of the transition into a resistive state \( I_i = \alpha (A_0G)^{-D/2} \), \( \alpha \) is the form factor of the cluster, \( D \) is the fractal dimension of the cluster boundary, \( \Gamma(\nu, z) \) is the complementary incomplete gamma function. The voltage across a superconductor \( U \) and flux flow resistance \( R_f \) are related to the corresponding dimensionless quantities \( u \) and \( r_f \) by the relationship: \( U/R_f = I_i(u/r_f) \).

In the simplest case of \( g = 0 \), when gamma distribution of Eq. (2) is reduced to the exponential one, \( w(a) = \exp(-a) \), the \( U-I \) characteristics of Eq. (4) can be written as

\[ u = r_f \left( i \exp\left(-Ci^{2/D}\right) - C_i^{D/2}\Gamma\left(1 - \frac{D}{2}, Ci^{-2/D}\right) \right) \]  

(5)

where \( C \equiv ((2 + D)/2)^{2/D+1} \) is the constant depending on the fractal dimension.

The corresponding \( U-I \) characteristics are shown in Fig. 1. The inset in this figure demonstrates that in the range of the currents \( i > 1 \) the fractality of the clusters reduces the voltage arising from the magnetic flux motion. Meanwhile, the situation is quite different in the neighborhood of the resistive transition below the critical current. When \( i < 1 \), the higher the fractal dimension of the normal phase cluster, the larger is the voltage across a sample and the more stretched is the region of initial dissipation in \( U-I \) characteristic. For further consideration it is convenient to introduce the onset current \( i_{on} \), starting from which this region spreads away. The magnitude of this current is set by the resolution of voltage measurement. In Fig. 1 the arrows indicate the onset current values \( i_{on} \) corresponding to the resolution level of \( 10^{-5}u/r_f \). The inset in the same figure shows the dependence of the onset current \( i_{on} \) on the fractal dimension. Figure 1 demonstrates that the value of the onset current decreases with increasing the fractal dimension, that is to say that the region of initial dissipation widens.

A significant difference in \( U-I \) characteristics before and after the resistive transition is related to the dependence of the density of free vortices on the fractal dimension for various transport currents. The free vortex density determines the resistance, because the more vortices are free to move the higher electric field is induced, and therefore, the greater
is the voltage at the same magnitude of a transport current. The density of vortices broken away from pinning centers by the transport current $i$ can be found from the distribution of the depinning currents of Eq. (3),

$$n = \frac{B}{\Phi_0} \int_0^i f(i') \, di' = \frac{B}{\Phi_0} F(i) \tag{6}$$

where $F(i)$ is the cumulative probability function of the depinning currents, $B$ is the magnetic field, $\Phi_0 \equiv hc/(2e)$ is the magnetic flux quantum, $h$ is the Planck constant, $c$ is the speed of light, and $e$ is the electron charge. The differential resistance of a superconductor (which gives the slope of the $U-I$ characteristic) is proportional to the density of free vortices: $R_d = R_f(\Phi_0/B)n$.

For the exponential-hyperbolic critical current distribution of Eq. (3), in the case of exponential distribution of the cluster areas ($g = 0$), the cumulative probability function has the form $F(i) = \exp(-C i^{-2/D})$. Therefore, the dependence of the free vortex density on the fractal dimension can be written as

$$n(D) = \frac{B}{\Phi_0} \exp \left(- \left(\frac{2 + D}{D} \right)^{2/D+1} i^{-2/D} \right) \tag{7}$$

In the special case of Euclidean clusters ($D = 1$), the formula of Eq. (7) becomes: $n(D = 1) = (B/\Phi_0) \exp(-3.375/i^2)$.

Figure 2 demonstrates dependence of the relative density of free vortices $n(D)/n(D = 1)$ (relatively to the value for clusters with Euclidean boundary) on the fractal dimension for different values of transport currents. The vortices are broken away from pinning centers mostly when ($i > 1$), that is to say, above the resistive transition. Here the free vortex density decreases with increasing the fractal dimension. Such a behavior can be explained by the fact that the critical current distribution of Eq. (3) broadens out, moving towards greater magnitudes of current as the fractal dimension increases. It means that more and more clusters of high depinning current, which can trap the vortices best, are being involved in the game. The smaller part of the vortices is free to move the smaller the induced electric field. An important feature of the superconductors with fractal clusters is that fractality of the cluster boundary enhances pinning and, hence, a current-carrying capability of the superconductor. The relative change in free vortex density depends on the transport current (see inset in Fig. 2) and in the limiting case of the most fractal boundary ($D = 2$) reaches a minimum for $i = 1.6875$ (curve 6 in Fig. 2 goes below others). That corresponds to the maximum pinning gain and the minimum level of dissipation. As may be seen in Fig. 1, the voltage across a sample carrying the same transport current decreases with increasing fractal dimension.

In the range of transport currents below the resistive transition ($i < 1$), the situation is different: resistance, as well as the free vortex density, increases for the clusters of greater fractal dimension. Such a behavior is related to the fact that the critical current distribution of Eq. (3) broadens out, covering both high and small currents, as the fractal dimension increases. In spite of sharp increase in relative density of free vortices (Fig. 2), the absolute value of vortex density in the range of the currents involved is very small (much smaller than above the resistive transition). So the vortex motion does not lead to destruction of superconducting state yet, and the resistance remains very low. The low density of vortices at small currents is related to the peculiarity of exponential-hyperbolic distribution of Eq. (3). The cumulative probability function $F(i)$ for this distribution is so “flat” at small currents, that all its derivatives are equal to zero at the point of $i = 0$: $d^k F(0)/di^k = 0$ for any value of $k$. Even the expansion of $F(i)$ into Taylor’s series at this point tends to zero, rather than to $F$. This behavior has a clear physical meaning. Indeed, so small a transport current does not significantly affect the trapped magnetic flux because there are scarcely any pinning centers of such small critical currents, so that nearly all the vortices are still pinned. Significant breaking of the vortices away begins only after the resistive transition when $i > 1$.

For any hard superconductor (of type-II, with pinning centers) the dissipation in the resistive state does not mean the destruction of phase coherence yet. Some dissipation always accompanies any motion of a magnetic flux that can happen in a hard superconductor even at low transport current. The superconducting state collapses only when a growth of dissipation becomes avalanche-like as a result of thermo-magnetic instability.

Thus, the fractal properties of the normal phase clusters significantly affect the vortex dynamics in superconductors. The $U-I$ characteristics of superconductors with a fractal cluster structure have two distinctive regions - before and after the resistive transition. Each of them has different dependence of free vortex density of on the fractal dimension.
of normal phase clusters boundaries. After the resistive transition the fractality of the clusters suppresses the vortex depinning, thus increasing the current-carrying capacity of the superconductor.

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FIG. 1. The current-voltage characteristics of a superconductor with fractal clusters of different fractal dimensions $D = 1$ (1), $D = 1.5$ (2), and $D = 2$ (3). The arrows indicate the dissipation onset currents $i_{on}$ found on the level of $10^{-5}u/r_f$ and the critical current $i_c$ of the resistive transition.
FIG. 2. Dependence of the free vortex density on the fractal dimension of the normal phase clusters for different values of a transport current $i = 0.4 \; (1)$, $i = 0.5 \; (2)$, $i = 0.6 \; (3)$, $i = 0.7 \; (4)$, $i = 1 \; (5)$, $i = 1.6875 \; (6)$. The inset shows the free vortex density versus current for two values of a fractal dimensions $D = 1.5$ and $D = 2$. 