Puzzles of excited charm meson masses

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Abstract

We attempt a comprehensive analysis of the low lying charm meson states which present several puzzles, including the poor determination of masses of several non-strange excited mesons. We use the well-determined masses of the ground states and the strange first excited states to ‘predict’ the mass of the non-strange first excited state in the framework of heavy hadron chiral perturbation theory, an approach that is complementary to the well-known analysis of Mehen and Springer. This approach points to values for the masses of these states that are smaller than the experimental determinations. We provide a critical assessment of these mass measurements and point out the need for new experimental information.

1 Introduction

The Particle Data Group in its latest Review of Particle Properties \cite{PDG} lists several low-lying charm meson states, the ground state being the $J^P = 0^-, 1^-$ states, and the first set of excited states corresponding to $J^P = 0^+, 1^+$. The latter present some puzzles: the strange mesons known as $D_s(2317)$ and $D_s(2460)$ are said to be lighter than expected from constituent quark model predictions (for reviews, see e.g. \cite{reviews}). The observation of $D_s(2317)$ was first reported in \cite{Ds2317} and the confirmation reported in \cite{Ds2317_confirmation}, the latter also reporting the discovery of $D_s(2460)$. The non-strange counterparts of these states had been reported earlier in refs. \cite{D002308} \cite{D012438} which are denoted $D_{0}^{0}(2308)$ and $D_{1}^{1}(2438)$ (the last number resulting from an average over BELLE \cite{BELLE} and CLEO \cite{CLEO} numbers). The observation of charged non-strange mesons with $0^+$ quantum numbers were reported by the FOCUS collaboration \cite{FOCUS} at a mass of about 2460 MeV. In view of a conflict with the measurements of the strange meson masses, it has been disregarded in some theoretical treatments. No observation of the corresponding charged non-strange meson with $1^+$ has been reported so far. A summary is provided in the diagram given in Fig. 1.

The masses have been analyzed in great detail in the framework of heavy hadron chiral perturbation theory by Mehen and Springer \cite{Mehen_Springer}, including one-loop chiral and $O(m_c^{-1})$ corrections. This is a hybrid framework that exploits
Figure 1: Representation of the masses in MeV of the charm mesons with their quantum number assignments, and the names of the experiments for determinations associated with the non-strange excited states.

the chiral symmetry associated with the lightness of the u-, d- and s-quarks and the heavy quark symmetry that is realized in the limit of the mass of the heavier quark, \textit{viz.} b or c, becoming very large. Of the conclusions drawn in ref. \cite{9}, a notable one is that due to the scarcity of experimental information, it was hard to fix either the tree-level parameters or the the coupling constants entering the one-loop corrections to any significant accuracy.

More recently, best fit values for three of the coupling constants, $g, g', h$ have been reported, taking into account chiral corrections to the strong decays of these positive and negative parity states \cite{10}. These determinations are in agreement with the orders of magnitudes considered earlier in the literature \cite{11,12}. In particular, note however, $g'$ continues to be a parameter for which there is no direct experimental evidence and results only from a combined fit to the information on decays and other sources of information. While carrying out our numerical fits, we will confine these parameters to lie in the range compatible with the numbers presented in ref. \cite{10}.

Our main motivation is to find the masses of the non-strange excited states from the observed experimental values of all the ground states and excited strange mesons only. In the present work, we employ the expressions presented in the comprehensive work of Mehen and Springer \cite{9}. Our approach is in a sense complementary to that of ref. \cite{9}, where the masses of the excited non-strange states are also used in their fits. One of the main differences in our work stems not only from constraining the range of $g$ and $h$ to be 0 to 1, but also requiring other parameters to lie in a narrow range satisfying experimental mass determinations. Note however, that we will consider only those fits as proper in which $g'$ is significantly smaller in magnitude in comparison with $g, h$. This is in accordance with the observations presented in ref. \cite{13}, which are based on arguments coming from QCD sum rules and relativistic as well
as non-relativistic quark model. Another ingredient in which we differ from their work is that the mass of the s-quark is chosen to be 130 MeV, which differs from the value 90 MeV used in ref. [9]. The difference stems from the renormalization group evolution of the \( \overline{\text{MS}} \) mass from the scale 2 GeV down to the present renormalization scale of 1 GeV. In practice, it is found that the masses are not very sensitive to this choice.

We find that the corresponding predictions for the non-strange states can be in conflict with experiment. Our fits are able to produce lower lying positive parity states with masses less than experimental observations. If we consider that masses of strange D-mesons are well determined then this would be the prediction of ref. [13]. In ref. [13] it is shown that the difference \( (m_{D^*_0} - m_D) - (m_{D^{*0}} - m_{D_s}) \) has to be less than zero, after the inclusion of chiral corrections. Here we explicitly calculate the masses of non-strange mesons, and show that they indeed satisfy the above relation. We must caution, however, that since there are a larger number of parameters than there are observables, fitting the masses of the (poorly determined) non-strange masses can also yield a perfectly natural set of parameters for the theory.

We then turn to a study of all the experiments that have reported the masses of the non-strange excited states. We find that the determination of the masses of the neutral non-strange mesons do not appear to be consistent. In addition, in the charged non-strange sector the experimental situation remains grossly unsatisfactory, as only the FOCUS collaboration has reported the observation of the \( 0^+ \) state with fairly large errors and a central value higher than the corresponding strange meson, in conflict with SU(3) predictions, while no experiment has reported the observation of the \( 1^+ \) state in this sector. A resolution of the situation could emerge if an independent experiment, e.g., BABAR could observe all these states and carry out a measurement of the masses of these mesons. Also the CLEO-c experiment could attempt to observe all the states discussed here at high precision.

2 Results from heavy hadron chiral perturbation theory

In the framework of heavy hadron chiral perturbation theory chiral corrections and corrections due to the finite mass of the charm quark are also accounted for. Our analysis uses the expressions for masses of the charm mesons presented in ref. [9]. The masses are expressed in terms of a formula which reads, for the residual masses,

\[
m^0_{R_a} = \delta_R + \frac{n_J}{4} (\Delta_R + \Delta^\prime_R m + (a_R m_a) + \sigma_R m + a_R m_a + \frac{g^2}{f^2} c^{R_a} K_1 + \frac{h^2}{f^2} c^{R_a} K_2,
\]

(1)

where \( R \) is an index that labels the ground state (H) and excited state (S), each of the ground and excited states having members corresponding to \( J = 0, 1 \), with \( n_0 = -3, n_1 = 1 \), the index \( a \) labels the light flavour and runs over
$u, d, s$, the functions $K_1$ and $K_2$ are the chiral loop functions, and the $c^{R_a}$ are coefficients listed in [9] and $g_H = g, g_S = g'$ in the notation therein. The relevant Lagrangian is reproduced for completeness in the Appendix.

Some insight can be obtained on the coefficients of interest in certain limits. For instance, in the heavy quark effective field theory limit, one can make the identification $\delta_R \rightarrow \Lambda_R + \lambda_R^1/(2m_Q), \Delta_R \rightarrow \lambda_R^2/(2m_Q)$. The heavy quark effective theory constants on the right-hand side of each of these relations have been estimated for the ground-state, namely for $R = H$. The reaction $b \rightarrow s\gamma$ measured by the CLEO collaboration [14] requires the constant $\Lambda_H \simeq 0.35\text{GeV}$. Fits to $b$-decays yield $\lambda_H^2 \simeq -0.20\text{GeV}^2$ [15], while measurements on the lattice yield for $\lambda_H^1 \simeq 0.10\text{GeV}^2$ [16]. Note that in the treatment of ref. [13], the hyperfine splitting of the odd-parity ground state, and that of the even-parity first excited state has to be equal. The lifting of the degeneracy of the hyperfine splitting is considered to be due a term arising from the next to leading order in the heavy quark effective theory. In the treatment of Mehen and Springer, the leading and next to leading order effects are both taken into account, both through the lumping of the effects into the $\delta_H$, as well as in the hyperfine splitting effects. The hyperfine splitting itself is of interest, since it seems to be equal in the ground as well as the first excited states. This is the subject of a recent study, ref. [17]. We now describe the results from our fits in the foregoing.

2.1 Results from constrained fits

In our numerical work we use the values for the quark masses of $m_u = m_d = 4, m_s = 130\text{MeV}$. The latter mass differs significantly from that used in ref. [9]; the change coming primarily from our having to use the mass of the $s$-quark at 1 GeV, related to the mass given as 95 MeV at 2 GeV, these being related by a factor of $\simeq 1.35$.

In our fit, we use six experimentally determined masses, viz. four from the strange sector (both negative and positive parity states) and two from the non-strange sector (negative parity states), in the iso-spin limit. The loop corrections depend upon 11 parameters: $g, g', h, \delta_H, \delta_S, \Delta_H, \Delta_S, a_H, a_S, \Delta_H^{(a)}, \Delta_S^{(a)}$. Since there is a surplus of parameters, a unique fit is not possible. Nevertheless, we could find good fits by constraining the parameters in well-motivated ranges. Special mention may be made to the values presented in ref. [10, 17]. Note that in ref. [17], heavy hadron chiral perturbation theory with only the ground states has been considered.

As an illustration we present the results from one of our fits by requiring that the parameters $g, h$ lie in the range $(0,1)$ and $g'$ in the range $(-0.5,0.5)$. This yields, in the notation of ref. [9],

$\delta_H = 314.7\text{MeV} \quad \delta_S = 688.6\text{MeV}$
$\Delta_H = 149.9\text{MeV} \quad \Delta_S = 725.1\text{MeV}$
$a_H = -5.071 \quad a_S = -6.03$
$\Delta_H^{(a)} = -8.883 \quad \Delta_S^{(a)} = -9.544$
with \( g, g', h \) being 0.905, 0.001, 0.998 respectively. From this fit, we obtain for the non-strange \( 0^+ \) state the mass of 2155.7 MeV and for \( 1^+ \) 2395.1 MeV.

From the experimental values of non-strange ground states and strange ground as well as excited states, we can observe that \( \Delta_H \) and \( \Delta_S \) are of the order of 140 MeV. Therefore, we allow each of them to vary from 100 to 200 MeV. From the difference between different parity states in the strange sector we can see that \( \delta_H - \delta_S \leq 400 \) MeV, and therefore each of them is allowed to vary between 100 and 500 MeV. Moreover the mass difference between strange and non-strange ground states, which is of the order of 100 MeV, gives \( a_H(m_s - m_{u/d}) \sim 100 \) leading to \( a_H \) to be of order of unity. Similarly we consider all other chiral contribution to be of the order of unity. Consequently, we allow these parameters to vary from -2 to 2. The values of \( g, g', h \) are also chosen to be of the orders of magnitude given in ref. [10]. With the above constraints to the parameters we get several fits and a few of them are given below.

- \( \delta_H = 169.2 \pm 0.5 \) MeV \hspace{1cm} \( \delta_S = 345.4 \pm 0.7 \) MeV
  \( \Delta_H = 200.3 \pm 0.5 \) MeV \hspace{1cm} \( \Delta_S = 120.4 \pm 1.1 \) MeV
  \( a_H = 1.522 \pm 0.005 \) \hspace{1cm} \( a_S = 0.508 \pm 0.018 \)
  \( \Delta^{(a)}_H = -1.231 \pm 0.005 \) \hspace{1cm} \( \Delta^{(a)}_S = 0.193 \pm 0.015 \)
  \( |g| = 0.66 \pm 0.01 \) \hspace{1cm} \( |g'| = 0.03 \pm 0.01 \) \hspace{1cm} \( |h| = 0.42 \pm 0.01 \)

- \( \delta_H = 184.9 \pm 3.6 \) MeV \hspace{1cm} \( \delta_S = 368.1 \pm 2.4 \) MeV
  \( \Delta_H = 196.5 \pm 3.2 \) MeV \hspace{1cm} \( \Delta_S = 119.4 \pm 1.4 \) MeV
  \( a_H = 1.534 \pm 0.006 \) \hspace{1cm} \( a_S = 0.453 \pm 0.039 \)
  \( \Delta^{(a)}_H = -1.242 \pm 0.027 \) \hspace{1cm} \( \Delta^{(a)}_S = 0.189 \pm 0.022 \)
  \( |g| = 0.68 \pm 0.01 \) \hspace{1cm} \( |g'| = 0.01 \pm 0.04 \) \hspace{1cm} \( |h| = 0.32 \pm 0.02 \)

- \( \delta_H = 159.2 \pm 3.7 \) MeV \hspace{1cm} \( \delta_S = 350.6 \pm 2.7 \) MeV
  \( \Delta_H = 194.4 \pm 3.2 \) MeV \hspace{1cm} \( \Delta_S = 145.9 \pm 1.4 \) MeV
  \( a_H = 1.524 \pm 0.008 \) \hspace{1cm} \( a_S = 0.423 \pm 0.043 \)
  \( \Delta^{(a)}_H = -1.148 \pm 0.028 \) \hspace{1cm} \( \Delta^{(a)}_S = -0.01 \pm 0.02 \)
  \( |g| = 0.65 \pm 0.01 \) \hspace{1cm} \( |g'| = 0.05 \pm 0.03 \) \hspace{1cm} \( |h| = 0.45 \pm 0.02 \)

All these fits give mass values for \( 0^+ \) state to be in the range 2200-2250 MeV, and \( 1^+ \) to be in the range 2335-2375 MeV. Even though there are yet other regions of parameter space in which these mass values can change, here it is our main aim to point out that these values do not contradict any present day theory. In such regions of parameter space, the independent parameters of the theory would be in very different ranges compared to those considered here.

### 3 The Experiments

We present here a discussion on the experiments that have reported the excited even-parity states. The first reports for the non-strange states come from CLEO [6] which saw the \( l = 1, 1^+ \) state. The central value reported here is 2461 MeV, which renders state heavier than the corresponding strange state.
The situation was mitigated by the measurement of the mass of this state with a central value of 2427 MeV by the BELLE collaboration [7]. The average of the two experiments has been used to label the state as $D^0_0(2438)$ [2]. The latest Review of Particle Properties [1] does not use the CLEO data, and instead labels the state as $D_1(2420)$. The corresponding $l = 0$ state which does not conflict the predictions of SU(3) symmetry is seen only by the BELLE collaboration which reports a central value for the mass 2308 MeV [7]. On the other hand, the FOCUS collaboration has reported signals for both the charged as well as neutral non-strange $0^+$ states, a little above 2400 MeV, and are thus heavier than their strange counterparts. These measurements, therefore, have been rejected on theoretical grounds. [Despite this, the latest Review of Particle Properties labels this state $D^*_0(2400)$ with a mass of 2352 MeV averaged over the BELLE and FOCUS masses, albeit with an uncertainty of 50 MeV.] The approach adopted in the work here points to a conclusion that that the masses 2308 and 2420 are still somewhat large to be the non-strange counterparts of the strange states at 2317 and 2460. Nevertheless, since the theory has more parameters than observables, successful fits can still be found with these masses, ref. [9].

The FOCUS experiment is based on the interaction of high energy photons with a fixed target. It has reported mass and width measurements of both the neutral as well as the charged non-strange $0^+$ states, the masses of both begin nearly degenerate as required by iso-spin invariance, but at a mass higher than the strange counterpart. This paradoxical situation has not been rectified and theorists have simply rejected these measurements without comment. It would desirable for the collaboration to carry out a reanalysis of their data to assist in sorting out the difficulties of the charm meson mass spectra discussed here.

The CLEO experiment has determined the mass of the neutral non-strange $1^+$. The data is gathered from $e^+e^-$ collisions and yields a mass that is not been included in the latest PDG. To our knowledge there is no further data from the collaboration in the past six years which clarifies the situation, since their measurement is significantly higher than the central value from BELLE, and is slightly greater than the mass of the strange counterpart.

Thus, we are left only with the measurement of the neutral non-strange $0^+$ and $1^+$ masses from the BELLE collaboration. This data gathered from the asymmetric B-factory is the only set that does not openly come into conflict with SU(3) predictions. Despite this, the mass determinations of these indicate that they cannot be comfortably accommodated into the existing theories as the expected splitting from the strange counterparts would be of the order of 100 MeV. Our finding with the constrained fits also suggest that this can be the order of the splittings, while the BELLE measurements show that the splitting is much less.

To summarize, the experimental determinations of the non-strange excited neutral mesons remain too high and are not necessarily self-consistent, although all the predicted states have been observed. In the corresponding charged sector, only the $0^+$ has been seen, but with a mass in contradiction with SU(3) prediction, while the $1^+$ has not been seen at all. It is our view that an inde-
pendent experiment, such as the BABAR collaboration could contribute to the resolution of the discrepancies here by searching for the states of interest.

4 Discussion and Conclusions

In this work we have revisited the issue of the masses of lowest-lying even-parity excited states in the open charm system. We have worked in the framework of heavy hadron chiral perturbation theory, including the $O(m_c^{-1})$ corrections, namely the framework employed in ref. [9]. The key difference is that we now constrain the values of the parameters $g, g', h$ to the values that are determined from the decays. Furthermore we have imposed the requirement that $m_s \approx 130$ MeV, and have required the parameters determining the tree-level hyperfine structure to be in a range determined by the well-established states. We find that the masses for the non-strange states that we determine can be lower than the numbers obtained by the BELLE collaboration. We conclude by noting that a corroboration of these numbers from other experiments is imperative. It must be added that as the number of parameters exceeds the number of observables, it would be possible to find natural fits to the parameters even with the present numbers, as in ref. [9]. Also of interest is the possibility of understanding these masses in Regge framework, see ref. [18]. Finally, an experimental determination of the charged non-strange even parity states would significantly clarify the situation. A determination of the constants $\lambda^S_1$ and $\lambda^S_2$ on the lattice would also contribute significantly to the understanding of the spectrum[1].

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Appendix: Effective Lagrangian for Heavy Hadron Chiral Perturbation Theory

In the heavy quark limit, spin of the heavy quark decouples from that of light degrees of freedom. Hence the pseudoscalars and vector mesons in the ground state of D-meson system become degenerate. In this case it is convenient to introduce a single field for the ground state doublet as

$$H_a = \frac{1 + \gamma'}{2} (H^a_{\mu} \gamma_\mu - H^a_5) ,$$

Similarly for the first excited douplet

$$S_a = \frac{1 + \gamma'}{2} (S^a_{\mu} \gamma_\mu \gamma_5 - S_a) ,$$

1We thank R. R. Horgan for a discussion on this subject.
Now the Lagrangian that describes the dynamics of mesons with heavy-light combination is given by

\[ \mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{axial}} + \mathcal{L}_{\text{ct}} \]

The kinetic part of lagrangian

\[ \mathcal{L}_{\text{kinetic}} = - \text{Tr} \left[ \overline{H}_a \left( iv \cdot D_{ba} - \delta H_{ab} \right) H_b \right] + \text{Tr} \left[ \overline{S}_a \left( iv \cdot D_{ba} - \delta S_{ab} \right) S_b \right] \]

where \( \delta H \) and \( \delta S \) are residual masses of H and S fields. The axial coupling Lagrangian is:

\[ \mathcal{L}_{\text{axial}} = g \text{Tr} \left[ \overline{H}_a H_b A_{ba} \gamma_5 \right] + g' \text{Tr} \left[ \overline{S}_a S_b A_{ba} \gamma_5 \right] + h \text{Tr} \left[ \overline{H}_a S_b A_{ba} \gamma_5 + h.c. \right] \]

where \( g, g' \) are coupling constants in the ground state and in excited state doublets respectively, and \( h \) is the coupling between mesons belonging to different doublets. The mass counterterm Lagrangian is:

\[ \mathcal{L}_{\text{ct}} = \text{Tr} \left[ (a_H \overline{H}_a H_b - a_S \overline{S}_a S_b) (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ab} \right] + \text{Tr} \left[ (\sigma_H \overline{H}_a H_a - \sigma_S \overline{S}_a S_a) (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{bb} \right] \]

where \( m_q = \text{diag} (m_u, m_d, m_s) \), \( \xi^2 = \exp(2i\phi/f) \) with \( \phi \) being usual matrix of pseudo-Goldstone bosons and \( f \approx 130\text{MeV} \). In terms of heavy quark symmetry conserving and symmetry violating terms the above lagrangian can be written as

\[ \mathcal{L}_{\text{ct}}^v = - \frac{\Delta_H}{8} \text{Tr} \left[ \overline{H}_a \sigma^\mu\nu H_a \sigma_{\mu\nu} \right] + \frac{\Delta_S}{8} \text{Tr} \left[ \overline{S}_a \sigma^\mu\nu S_a \sigma_{\mu\nu} \right] + a_H \text{Tr} [\overline{H}_a H_b] m^\xi_{ba} - a_S \text{Tr} [\overline{S}_a S_b] m^\xi_{ba} + \sigma_H \text{Tr} [\overline{H}_a H_a] m^\xi_{bb} - \sigma_S \text{Tr} [\overline{S}_a S_a] m^\xi_{bb} - \frac{\Delta_H^{(\sigma)}}{8} \text{Tr} [\overline{H}_a \sigma^\mu\nu H_b \sigma_{\mu\nu}] m^\xi_{ba} + \frac{\Delta_S^{(\sigma)}}{8} \text{Tr} [\overline{S}_a \sigma^\mu\nu S_b \sigma_{\mu\nu}] m^\xi_{ba} - \frac{\Delta_H^{(\sigma)}}{8} \text{Tr} [\overline{H}_a \sigma^\mu\nu H_a \sigma_{\mu\nu}] m^\xi_{bb} + \frac{\Delta_S^{(\sigma)}}{8} \text{Tr} [\overline{S}_a \sigma^\mu\nu S_a \sigma_{\mu\nu}] m^\xi_{bb} , \]

where \( m^\xi_{ba} = \frac{1}{2} (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ba} \). Here \( \Delta_H, \Delta_S \) are symmetry(spin) violating operators giving rise to hyperfine splitting and \( a_H, a_S, \sigma_H, \sigma_S \) preserve spin-symmetry while other operators violate heavy quark spin symmetry.
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