The problem of ultraviolet divergences is analysed in the quantum field theory. It was found that it has common roots with the problem of cosmological singularity. In the context of fibre bundles the second quantization method is represented as a procedure of the quantization for vector bundle cross-section. It is shown to be quite a different quantization way called as a fibre quantization which leads to an idea on existence of the non-standard dynamical system, i.e. the relativistic be-Hamiltonian system. It takes place on supersmall distances and is well described by the mathematical apparatus for the non-unitary quantum scheme using a dual pair of topological vector spaces in terms of the non-Hermitian form. The article contains the proof of the theorem on radical changes in space and in matter structure taking place for a very high density of matter: the phase transitions "Lagrangian field system (elementary particles) $\rightarrow$ relativistic bi-Hamiltonian system (Feynman’s partons)" and "continuum $\rightarrow$ discontinuum".

All required calculations in the framework of the proposed theory are published in the Russian periodicals. The purpose of this article is to replace the calculations by reasonings and concepts. The present article begins the systematic exposition of principles of the theory.

11.

I. INTRODUCTION

Perhaps, many physicists agree with the opinion that the elementary particle theory of today is in deep crisis. As will be shown below, it is caused by the early application of the second quantization procedure. To get it out of the state and to put an end to errors of the last decades, it is necessary first to formulate initial principles in their precise mathematical form. Then using strict mathematical structures we shall show that there is quite a different way in quantum field theory by means of a new quantization procedure, the quantization of Dirac fibre. As a result there appears an idea on existence of a non-standard dynamical system in Nature. Proceeding from the system, the consistent theory of elementary particles and their interactions can be constructed.

II. INITIAL STATEMENTS

a) As is well known, since the thirties the quantum field theory of particles confronted with the problem of ultraviolet divergences. The problem being unsolved so far has blocked the normal development of this theory. However, a cause of the difficulty arose earlier. It turns out to be rooted in the second quantization method using the distributions (for example, the Dirac $\delta$-function).

Up till now the physics uses the Newtonian concept of space-time treated as a continuum or differential manifold $M_4$ at each point $X \in M_4$ of which there is a pair of vector spaces — the tangent space $T_X$ (with the basis $\partial/\partial X_\mu$) and the cotangent space $T^*_X$ (with the basis $dX^\mu$). $M_4$ is considered to have the measure $d^4X$.

---

1 A sign of the crisis is the contradiction between the urgent necessity in a new physics related with supersmall distances or particule structure and the continued utilization of old mathematical instruments on the base of field concept (field is a function in the space-time continuum), i.e. geometry, whereas the supersmall distances belong to algebra rather than geometry.
Measure spaces are bad because they have sets of measure zero \(1\) (for example, isolated points in which the point-like particles\(^2\) can exist are the objects far from identical zero, but they are alike by occupying the position in space-time). For such sets will not quite lose in continuum, the notion of distributions is introduced to mathematics. Namely, the null sets are their supports. From the point of view of functional analysis the distributions are functionals or generalized functions.

If ordinary functions (in particular, trial functions) form not only a linear space (we may add them), but a ring too (we may multiply them), distributions forming a linear space have not a reasonable ring structure, i. e. possesses the functor properties. Thus two and only two physics theories — classical and quantum — of particles?

\(2\) It is important to note that with regard to the wave properties (the quantization \(1\) \(\rightarrow\) \(3\)), see below) the particles as though are smeared, and the degree of divergences is reduced. Therefore, for the divergences be eliminated, it is necessary to improve the quantum theory, rather than to change the classical aspect of a particle approaching, for example, to strings.

\(3\) The enveloping algebra \(h_{2n}\). In our opinion it should be error to consider, in connection with the microcosm, the nonlinear (curved) phase spaces and so-called geometric quantization.

The mathematical meaning of the first quantization is the following. We pass from the phase space \(PS^{2n}\), which is constructed over the differential manifold \(M\) (in a general case its dimension is \(n\)), to the tangent bundle \((M, T^*M)\). Here \(T^* = \bigcup_{X \in M} T^*_X\) is the cotangent bundle at the point \(X \in M\), with the basis \(dX_j\) (in this case the momenta \(P_j\), \(j = 1, 2, ..., n\) is defined as \(P_j \, dt = m \, dX_j\) where \(t\) is the time, \(m\) is the mass of a particle), and \(T = \bigcup_{X \in M} T_X\) is the tangent space at the point \(X \in M\) with the basis \(\frac{\partial}{\partial X_j}\) (in this case the momenta become the operators \(\hat{P}_j = -i\hbar \frac{\partial}{\partial X_j}\) where \(\hbar\) is Planck’s constant).

To transform \((M, T^*M)\) and \((M, TM)\) in a canonical system, in the first (classical) case the Lie structure is known to be given by Poisson brackets \(\{f, g\} = \sum_{j=1}^n (\frac{\partial f}{\partial X_j} \frac{\partial g}{\partial P_j} - \frac{\partial f}{\partial P_j} \frac{\partial g}{\partial X_j})\) where \(f, g\) are functions in \((M, T^*M)\) as

\[
\{X_j, P_k\} = \delta_{jk}, \quad \{X_j, X_k\} = \{P_j, P_k\} = 0,
\]

and in the second (quantum) case the Lie brackets (commutators) are used

\[
[\hat{X}_j, \hat{P}_k] = i\hbar \{X_j, P_k\} = i\hbar \delta_{jk}, \quad [\hat{X}_j, \hat{X}_k] = [\hat{P}_j, \hat{P}_k] = 0.
\]

Relations \(1\) and \(3\) define the Heisenberg algebra \(h_{2n}\).

Being accompanied by the mapping \(1\) \(\rightarrow\) \(3\) the mapping \((M, T^*M) \rightarrow (M, TM)\) is uniquely, universal and the only possible mapping, i. e. possesses the functor properties. Thus two and only two physics theories — classical \((M, T^*M)\) and quantum \((M, TM)\) — can be related with the continuum \(M\). Then, the dynamical system is defined as a canonical system with a given dynamical group. In the case of small oscillations (plaining a particularly important role in the microcosm physics) the dynamical group is the group \(Sp(n)\) of automorphisms for the Heisenberg algebra \(h_{2n}\). In our opinion it should be error to consider, in connection with the microcosm, the nonlinear (curved) phase spaces and so-called geometric quantization.

We can now say that the first quantization is a functor in the category of differential manifolds from \((M, T^*M)\) to \((M, TM)\). The second quantization is mathematically empty (see below) as opposed to the first quantization.

c) Next, both classical and quantum theories consider the enveloping algebras \(U[[M, T^*M]]\) for \((M, T^*M)\) and \(U([[M, TM]])\) for \((M, TM)\) as over Heisenberg algebras \(h_{2n}\). Note that it is the pure algebraic structure. In this case the dynamical variables of one system or another belong to these algebras or to their certain topological closures.

The quantum theory deals with a representation of the associative algebra \(U[[M, TM]]\) \((U\) is the operation taking an enveloping algebra in the topological vector space \(\mathcal{F}\). The space is constructed as follows. The maximal commutative subalgebra — the Lagrange plane — is considered in the Heisenberg algebra \((M, TM)\). Usually \(M\) itself takes its role \(3\). The enveloping algebra \(U[M]\) is constructed for \(M\) as a maximal commutative subalgebra in \(U[[M, TM]]\). \(U[M]\) is taken to be a compact set in \(\mathcal{F}\), and \(\mathcal{F}\) itself is obtained from \(U[M]\) by means of the topological closure along the topology \(\tau\): \(\mathcal{F} = U[M]^\tau\). As a rule, the Hilbert topology and the Hilbert space \(H\) are considered. But the extended
Hilbert spaces $\mathcal{F} \subset H \subset \mathcal{F}'$ are also considered where $\mathcal{F}$ and $\mathcal{F}'$ are the spaces of trial and generalized functions (distributions in $M$ belong to $\mathcal{F}'$, but if $H$ is a ring, see [1], its expansion $\mathcal{F}'$ is no longer a ring).

The elements $\psi(X)$ ($X \in M$) of the space $H$ are called the wave functions or state vectors of the system $(M, TM)$. The Hermitian definite form is a scalar product in $H$

$$ (\psi, \psi') = \int \psi^*(X)\psi'(X) dX $$

where $dX$ is the measure for $M$. The representation $U[(M, TM)]$ in $H$ is called the Schrödinger one. Note that in this representation the state vectors of the system are functions in $M$. The Hermitian definite form (3) plays the most important role in the Heisenberg-Schrödinger quantum theory: it corresponds to the unitarity axiom defining the unitary nature of used symmetry group representations of the space $M$.

d) In the case of elementary particles with spin a wholly different mathematical structure, namely the geometric structure of vector bundle over $M$, is used to construct the space of state vectors. The vector bundle is the trio $E = (M, S, \hat{L})$ where $M$ is the base (a differential manifold with a symmetry group $L$), the fibre $S$ is the vector space, and $\hat{L}$ is the structure group (covering for $L$) of the fibre $S$. In the particle theory $M = \mathcal{A}_{3,1}$ is the affine space-time with the inhomogeneous Lorentz group (or Poincaré group) as a symmetry group, and in the case of the most fundamental particles — fermions $\bar{\psi} = S$ is the space of the Dirac bispinors $S_8^{(2)} \ni \bar{\psi}_\alpha \psi_\alpha$ (where $\bar{\psi} = \psi^* \gamma_4$ is the Dirac adjoint bispinor to $\psi$, and $*$ is the involution connected with this adjoint or simply the complex adjoint operation).

The wave functions or particle fields $\psi(X), \bar{\psi}(X)$, satisfying the differential equations, are the cross-sections of the bundle $E = (M, S, \hat{L})$. Thus in the context of fibre bundles the field $\psi(X)$ in $M$ is the cross-section of the conformable vector bundle (all definitions used here can be found, for example, in [3]), and the Hilbert space $H$ is a space of bundle cross-sections.

e) Usually, the main development of the theory is associated with the quantization of the space $\mathcal{F}'$, i.e., the quantum postulate (second quantization method) is employed to bundle cross-sections. In our opinion the use of this procedure is early in the position.

What reasons proves the need of the second quantization? Only by that if the field $\psi(X)$ satisfies the Klein-Gordon equation $(\Box - m^2)\psi(X) = 0$ then its harmonics $\psi(P, t)$, in terms by which $\psi(X)$ is expressed as

$$ \psi(X) = \frac{1}{2\pi^{3/2}} \int \left( e^{ipX} \psi^+ (P, t) + e^{-ipX} \psi^-(P, t) \right) d^3P, $$

satisfy to the oscillator-like equation [3]: $\bar{\psi}^+ (P, t) + P_\mu^2 \psi^+ (P, t) = 0$ where $P_\mu = \sqrt{P^2 + m^2}$. Hence, the same commutation relations, which the first quantization put to variables of oscillator, could be applicable to $\psi^+ (P, t)$. But it is not, generally speaking, true. The form of commutation relations for fields must depend on their spin and statistics and does not always reduce to the commutation relations for oscillator. In this case, the analogy with oscillator is very superficial, and the quantization procedure of fields does not follow uniquely from it. And if the field $\psi(X)$ satisfies not the Klein-Gordon equation but, say, the Bopp equation $(\Box - M^2)(\Box - \mu^2)\psi(X) = 0$, generally, there is no analogy with an oscillator.

As we already known, the second quantization method is bad because of using distributions in commutation relations, and it yields ultraviolet divergences. We see that in the context of fibre bundles the second quantization method presents the quantization procedure of bundle cross-sections, i.e., the mapping $\psi(X) \rightarrow \hat{\psi}(X)$ which we write as $\mathcal{F}' \rightarrow \mathcal{F}'$ (elements of the set $\mathcal{F}'$ are local field operators $\psi(X)$). At the same time it is required for $\mathcal{F}'$ to be as a ring (algebra). It should be noted that since $\mathcal{F}'$ being a set of distributions (fields) in $M_4$ is not a ring, $\mathcal{F}'$ is not one too. And a smoothing of the form $\hat{\psi}(f) = \int \hat{\psi}(X)f(X) dX$ is no effective because the integration of bad (non-bounded) operators such as $\hat{\psi}(X)$ can make the operator $\hat{\psi}(f)$ more worse.

---

3It is important to note that the Dirac structure of extracting of the square root of $TM$ leads to a notion of the most fundamental Dirac (or spinor) fibre as to opening the spin variables of particles (they do not reduce to the space variables of $(T, TM)$ that is of great importance) [3]. As a result the mapping of the vector space $TM$ (or $M$) is the Clifford algebra $C$ for the Dirac matrices $\gamma_\mu : TM \rightarrow C$. Then the vectors, specifically current $j_\mu$, are construed by the formula $\psi(\gamma_\mu \psi$ ($\psi, \bar{\psi}$ are the elements of the Dirac fibre) describing the quantization of the current $j_\mu$.

4Conversely, the derivation $F(\frac{\psi}{2\pi})\psi(X)$, see [3] can improve the operator. It is surprising that in the axiomatic approach the so-called smoothing operators are used everywhere see, e.g., [3].
The mapping $\mathcal{F} \to \hat{\mathcal{F}}$ is very contradictory. If the unitarity axiom holds true then the local field operator $\hat{\psi}(X)$ does not take place at all as a basis of the theory of quantized fields (Wightman’s theorem [3]). It means that any quantization procedures of infinite-number-of-degrees-of-freedom system is inadmissible in principle. Nevertheless, for example, in the case of the most fundamental Dirac fields $\psi(X)$, the equal-time commutation relations are written in the form

$$\left\{ \hat{\psi}(X, t), \hat{\bar{\psi}}(X', t) \right\} = \gamma_4 \delta^4(X - X').$$

(4)

Such relations define the continual Clifford algebra (in the case of bosons it will be the continual Heisenberg algebra). At $X = X'$ (i. e. when $\psi, \bar{\psi}$ are taken from the same fibre) we have $\{\psi, \bar{\psi}\} = \gamma_4 \delta^4(0) = \infty$.

Heisenberg was a first who understood [8] that the cause of all ultraviolet divergences was the $\delta$-function on the right-hand side of (4).

We should put an end to ultraviolet divergences once and for all if, following Heisenberg [8], we rejected the $\delta$-function on the right-hand side of (3). Finally, we should arrive at such relations

$$\left\{ \hat{\psi}(X, t), \hat{\bar{\psi}}(X', t) \right\} = 0$$

(5)

which are valid for $X = X'$ too, i. e. at each isolated point of the space $M_4$. Notice that relations (5) for fixed $X$ define the so-called finite dimensional Grassmann algebra. Regretfully (rather fortunately), as Pauli has noted, relations (5) contradict the differential equations of the first order (it does not matter linear or nonlinear) for the field $\psi(X)$. The real way to overcome this contradiction is only one: to give up any of differential equations in $M_4$. But then the space in which we may not differentiate and integrate of course is not now the Newtonian space. Next, we shall show that the algebraic contraction (3)$\to$ (5) is associated with the rejection from the Newtonian concept of space-time as a differential manifold in favour of the Riemannian idea on the completely spatial non-connection. The contraction occurs if and only if the space-time as a continuum is transformed into discontinuum, i. e. if there takes place the phase transition “continuum $\to$ discontinuum” which must come under certain extremal physical conditions, see section [10].

### III. PHASE TRANSITION “CONTINUUM $\to$ DISCONTINUUM”

Since in obvious way the problem of ultraviolet divergences is related with small distances now we pay attention mainly to them. Apparently, the supersmall distances must obey their particular physics which is incorrectly described by the local field theory. But what a specific character have supersmall distances compared, for example, with atomic or nuclear distances (including also distances related to dimensions of elementary particles)?

To answer this question we consider the state of matter characterized by its supercompact packing of elementary particles (in natural conditions the situation arises for collapsing Universe, i. e. in the neighbourhood of the singular point $R_{cr} = 0$, when its density of mass begins to exceed the density of nuclear matter $\rho_{nuc} \approx 10^{15}$ g/cm$^3$ by many orders and, as calculations have been shown in [3], reaches the value $\rho_{cr} \approx 10^{40}$ g/cm$^3$, whereas in laboratory conditions this is also attained for collisions of particles with very high energies $\varepsilon \approx 10^5$ GeV, [3]).

This state is characterized by the space between particles becoming less and less (in the critical point $R_{cr} = 0$ it vanishes at all, thus the main sign of the Hamiltonian or Lagrangian property of the system disappears completely: the Lagrangian plane $M$ concentrates in one point).

In such extremal conditions (since there are no free seats) the space-time translations defined on fields of particles (i. e. in quantum theory) by the formula $\psi(X) \to \psi(X + a) = e^{a \frac{\partial}{\partial X}} \psi(X)$ become impossible transformations, whatever $a$. This means that the operator $e^{a \frac{\partial}{\partial X}}$ does not exist exactly as generators of these transformations $\frac{\partial}{\partial X_\mu}$. Since $\frac{\partial}{\partial X_\mu}$ form a basis in the tangent space at point $X \in M$ from this it follows that in the situation under consideration the tangent spaces (and hence the cotangent spaces of vectors $dX_\mu$) do not exist. All this means that the space $M$ loses its former structure (of differential manifold) postulated by Newton, disintegrates on its isolated points, becomes a completely non-connected set of its points (in the case it continues to consist as consisted of infinite uncountable numbers of points), i. e. continuum is transformed into discontinuum (the specific character of supersmall distances lies in this).

In this conditions the Newtonian concept of space as a continuum (as a differential manifold) does not work any more and must be replaced by the Riemannian discrete concept (more precisely, of completely non-connection) of space in a little (i. e. for supersmall distances or superhigh energies).

Mathematically, the transition “continuum $\to$ discontinuum” signifies that we consider the greatest discrete topology for $M$ which is defined as the neighbourhood of any point does not contain the others.
In the theory of particles and fields the space-time continuum is an affine space $A_{3,1} = (A_{3,1}, R_{3,1})$ where $R_{3,1}$ is the vector Minkowski space (associated with $A_{3,1}$) with the Poincaré group $\mathcal{P} = L \times T_{3,1}$ as a symmetry group where $L$ is the inhomogeneous Lorentz group (the symmetry group of the space $R_{3,1}$), and $T_{3,1}$ is the group of translations in $A_{3,1}$. When $A_{3,1}$ disintegrates on its isolated points the group $T_{3,1}$ collapses only, the symmetry comes lower from $\mathcal{P}$ to $L$. By analogy we may say that for the superhigh density of energy “the mathematical fluid” — continuum — is coming to the boil changes into “the gas” — discontinuum.

When the base $M$ becomes the completely non-connected space, the vector bundle $E = (M, S, \bar{L})$ disintegrates in its isolated fragments — fibres $S_X$ given at each isolated point $X \in M$. Here it is important to note that a point $X$ with its fibre $S_X$ is more fundamental (and profound) object than the point $X$ without one. If $M$ may be identified by an empty space then $E$ should be a space with matter.

Let us see now in what are transformed the commutation relations $[\hat{\psi}(X, t), \hat{\psi}(X', t)]$ for the transition “continuum $\rightarrow$ discontinuum” (we shall call it by the phase transition). In new conditions the Dirac $\delta$-function $\delta^3(X - X')$ having the dimension cm$^{-3}$ passes to the dimensionless Kronecker symbol $\delta_{XX'}$. Since the separated points have the null measure and size, both continuum and discontinuum are not characterized by any fundamental lenght. Outgoing from physical reasons, we put

$$\left\{ \hat{\psi}(X, t), \hat{\psi}(X', t) \right\} = \gamma_4 \left( \frac{mc}{\hbar} \right)^3 \delta_{XX'}$$

where $m$ is the mass of the particle described by the field $\psi(X)$. But since the mass of either particle for superhigh energies may be ignored (the modulus square of field on the left of $[\hat{\psi}]$ is much more than the right side of this formula) and because in the completely non-connected space the matter does not exist as a particles we have nevertheless another relations, namely

$$\left\{ \hat{\psi}(X, t), \hat{\psi}(X', t) \right\} = 0.$$

Thus we have arrived at the Grassmann algebra, so we may say that the phase transition “continuum $\rightarrow$ discontinuum” is accompanied by the contraction of algebra $[\hat{\psi}]$ to algebra $[\hat{\phi}]$.

Hence, in the case when the space-time $M$ becomes a discontinuum the Dirac fibre $S_8^{(s)}$ must be considered for the Grassmann algebra $G$: $S_8^{(s)} = S_8^{(s)}(G)$.

The phase transition “continuum $\rightarrow$ discontinuum” with certain care can be called the quantization of space. At the same time it should be noted that if the quantum nature of matter shows itself enough early, for moderate energies $\sim 1\ eV \div 10^9\ eV$ (the quantum ladder has several steps: molecules, atoms, nuclei, elementary particles) the quantum nature of space begins to show itself much later, only for energies $\sim 10^{14}\ eV$. In the quantized space — discontinuum — there is no both the measure $dX$ and Hermitian form $[\hat{\phi}]$. Thus in this case the unitarity axiom loses validity, and now we must use the non-unitary symmetry group representations of the space $M$, which were discovered in $[\hat{\phi}]$ for the physically important case of spaces $R_3, R_{3,1}$ (i. e. for groups $SO(3)$ and $SO(3,1)$).

IV. PHASE TRANSITION “LAGRANGIAN FIELD SYSTEM $\rightarrow$ RELATIVISTIC BI-HAMILTONIAN SYSTEM”

As shown in $[\hat{\phi}]$, the Dirac-Grassmann fibre $S_8^{(s)}(G)$ has a complex internal, inherently dynamical, structure. Now we reconstruct the internal evidence.

The dynamical structure of the fibre $S_8^{(s)}(G)$ is established by the splitting of skew-symmetric (symplectic) quadratic form for $S_8^{(s)}(G)$ in linear forms. It is interesting to note that this structure adjoins the number theory in the spirit of Galois’s and Kummer’s investigations corresponding to search of the most fundamental numbers which control the Universe. On the way to the realization of this idea the Grassmann numbers were discovered. Grassmann was right in regard to the applicability field of his numbers, this is the microcosm. However, other numbers control the submicrocosm.

A sympletic form on $S_8^{(s)}(G)$ is written as $[\chi, \chi] = \gamma E \chi = -2 \bar{\psi} \psi \neq 0$ where $\chi = (\hat{\phi})$ is the 8-spinor, $E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $1$ (0) is the unit (null) 44-matrix. The form $[\chi, \chi]$ as a polinomial of second degree for $\chi$ can be factorized into linear forms, by writting $[\chi, \chi] = \chi^2$ where $\chi = \sqrt{2} A_{\alpha} X_\alpha$ is the linear form. The coefficients $A = (\hat{\phi})$ of the linear form should take values from Heisenberg algebra. Since if $X \in G$ then, evidently, the relations $A_\alpha A_\beta - A_\beta A_\alpha = E_{\alpha\beta}$ must be valid as commutation relations

$$[\phi_\alpha, \bar{\phi}_\beta] = \delta_{\alpha\beta}, \quad [\phi_\alpha, \phi_\beta] = [\bar{\phi}_\alpha, \bar{\phi}_\beta] = 0$$

(7)
which define the Heisenberg algebra \( h_8^{(*)} \) with the involution \(*\) (one among real forms of the complex algebra \( h_8(G) \) which we have called the Dirac form). The mapping \( \psi_\alpha \rightarrow \phi_\alpha, \bar{\psi}_\alpha \rightarrow \bar{\phi}_\alpha \) in writing \( S_8^{(*)} \rightarrow h_8^{(*)} \) is called the quantization of Dirac fibre. Thus, we arrive at the Heisenberg algebra as well as to the canonical system for which its canonical variables are the generators \( \phi_\alpha, \bar{\phi}_\alpha \) of the algebra \( h_8^{(*)} \).

Notice that in this case the Heisenberg algebra plays the same role as algebraically closed rings in Galois’s theory and represents an analogy of Clifford algebra which arises by means of the Dirac operation employed to the vector space for the ring of usual Euclidean numbers. At the same time the elements of the representation space of the algebra \( h_8^{(*)} \) (we called the semispinors, see below) play the role of ideal Kummer’s numbers from which are constructed the tensorial values: spinors, scalars, vectors and so on.

Taking into account that canonical variables \( \phi, \bar{\phi} \) exist inside a spinor fibre which is considered at an isolated space-time point (i.e. for supersmall distances where space-time is discontinuum), and hence they should be placed into submicrocosm, we formulate the dynamical principle: the dynamical group of our system is the group of automorphisms for the algebra \( h_8^{(*)} \), which is denoted by \( S_p^{(*)}(4, \mathbb{C}) \). The generators of the dynamical group are the every possible bilinear forms of canonical variables: \( \phi_\alpha \phi_\beta, \phi_\alpha \bar{\phi}_\beta, \bar{\phi}_\alpha \phi_\beta \) (quadratic Hamiltonians) forming the semisimple Lie algebra which is isomorphic to the Cartan algebra \( sp^{(*)}(4, \mathbb{C}) \). To study the properties of the dynamical group and its linear representation we may say much about the properties of the dynamical system itself. We pay attention to the most important ones of them.

The real dynamical variables which we can write by means of sixty Dirac matrices \( \gamma_N \) as \( \bar{\phi} \gamma_N \phi \) play an especially important role. They form the Lie algebra which is isomorphic to the algebra \( u(2, 2) \) (this isomorphism is given by the mapping \( \gamma_N \rightarrow \bar{\phi} \gamma_N \phi \)). Among them there is the pair of 4-vectors \( \Gamma_\mu = i \bar{\phi} \gamma_\mu P_\tau \phi \), \( \bar{\Gamma}_\mu = -i \bar{\phi} \gamma_\mu P_\tau \bar{\phi} \) where \( P_\pm = \frac{1}{2}(1 \pm \gamma_5) \) having all properties of 4-momenta (excepting their dimension; \( \phi, \bar{\phi} \) are the dimensionless values)

\[
[\Gamma_\mu, \Gamma_\nu] = [\bar{\Gamma}_\mu, \bar{\Gamma}_\nu] = 0,
\]

but non-commuting with each other:

\[
[\Gamma_\mu, \bar{\Gamma}_\nu] = 2iI_{\mu\nu} + \delta_{\mu\nu}B
\]

where \( I_{\mu\nu} = \bar{\phi} \Sigma_{\mu\nu} \phi \) (\( \Sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu] \)), and \( B = \bar{\phi} \gamma_5 \phi \). The values \( I_{\mu\nu}, B \) and \( A = \bar{\phi} \phi \) form the Lie algebra which is isomorphic to the Lie algebra \( gl(2, \mathbb{C}) \). We define the 4-momenta \( p_\mu, \bar{p}_\mu \) as \( p_\mu = \hbar \Gamma_\mu, \bar{p}_\mu = \hbar \bar{\Gamma}_\mu \) where \( \hbar \) is Planck’s constant, and \( k \) is the third (after \( c \) and \( \hbar \)) fundamental constant having the dimension cm\(^{-1}\). It follows from the definition of \( \Gamma_\mu \) and \( \bar{\Gamma}_\mu \) that \( \Gamma_\mu \) and \( \bar{\Gamma}_\mu \) are the isotropic 4-vectors, i.e. \( p_\mu^2 = \bar{p}_\mu^2 = 0 \).

Thus the group \( U(2, 2) \) consists of two different Poincaré subgroups \( \mathcal{P} = GL(2, \mathbb{C}) \times T_{3,1} \) and \( \hat{\mathcal{P}} = GL(2, \mathbb{C}) \times \hat{T}_{3,1} \) crossing along the inhomogeneous Lorentz group \( GL(2, \mathbb{C}) \). In the dynamical group the operators \( I_{\mu\nu}, A, B \), \( p_\mu \) and \( \bar{p}_\mu \) are the generators for \( GL(2, \mathbb{C}), T_{3,1} \) and \( \hat{T}_{3,1} \).

The systems the dynamics of which is not described by one 4-momentum (as in the case of Hamiltonian systems), but a pair of those non-commuting with each other are called as relativistic be-Hamiltonian ones. Our analysis of the Dirac fibre structure allows to say that in extremal conditions when the elementary particles are in the highly compressed state and when all space is transformed from continuum into discontinuum (see section \( \text{III} \)) the matter changes radically: it is transformed from the Lagrangian system into the bi-Hamiltonian one.

The dynamics of bi-Hamiltonian matter is written by a pair of Hamiltonian flows non-commuting one with the other. In the Heisenberg picture (in the enveloping algebra \( U[h_8^{(*)}] \)) these flows are written by equations

\[
\begin{align*}
- i\frac{\partial}{\partial x_\mu} F &= [p_\mu, F] \\
- i\frac{\partial}{\partial \bar{x}_\mu} F &= [\bar{p}_\mu, F]
\end{align*}
\]

where \( F \in U[h_8^{(*)}], x \) and \( \bar{x} \) are the coordinates on groups \( T_{3,1} \) and \( \hat{T}_{3,1} \) (resulting from the consideration of the automorphisms \( e^{ipx} F e^{-ipx} \) and \( e^{i\bar{p}x} F e^{-i\bar{p}x} \)). It is easy to see that system (8) is not integrable in terms of

\[
\left( \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial \bar{x}_\nu} - \frac{\partial}{\partial \bar{x}_\nu} \frac{\partial}{\partial x_\mu} \right) F \neq 0,
\]

so that the operator \( F(x, \bar{x}) \) does not exist as a function on the manifold \( U(2, 2)/GL(2, \mathbb{C}) \).

The same result is obtained in the Schrödinger picture when it is considered in the unitary scheme using the self-adjoint space \( H \): the system
where \( f \in H \) is not integrable; the vector \( f(x, \dot{x}) \) does not exist as a function of two variables \( x \) and \( \dot{x} \). Such functions are indispensable in connection with reconstruction of the manifold \( M = \mathbb{A}_{3,1} \).

As is shown above, the unitary scheme postulated by von Neumann at his time in connection with needs of the Heisenberg-Schrödinger quantum theory does not work in our case. The cause lies in the motion equations being not integrated on a semisimple Lie group. On a solvable group the equations are integrated only. But a semisimple noncompact group as \( Sp^{(1)}(4, \mathbb{C}) \) has always solvable groups. As is well known, these are Borel subgroups associated with the Gaussian decomposition \( N_- H N_+ \) for group. For this reason if instead of the total group \( Sp^{(1)}(4, \mathbb{C}) \) we consider its open subgroups \( B_+ = H N_+ \) and \( B_- = N_- H \) (this means that the total symmetry is spontaneously broken and reduced up to its open subgroups \( B_+ \) and \( B_- \); by the way, the existence of open subgroups was always some mystery which now clears little by little) then we shall be able to integrate the equations on subgroups \( B_+ \) and \( B_- \).

Clearly, the spontaneous breaking is not combined with the unitarity using one self-adjoint space of representation. But this phenomenon arises as a matter of course in the non-unitary scheme using a dual pair of spaces \( (\mathcal{F}, \mathcal{F}') \), \( \mathcal{F} \neq \mathcal{F}' \) with a non-Hermitian form \( \langle \cdot, \cdot \rangle \). In this case the group variables may be separated, so that the subgroup \( B_+ \) will be represented only (its Lie algebra \( b_+ \) will be integrable) in one from the spaces, say in \( \mathcal{F} \), and \( B_- \) (and \( b_- \) will only in \( \mathcal{F}' \) (their topology in \( \mathcal{F} \) and \( \mathcal{F}' \) is selected such that the nilpotent algebra \( n_- \) is not integrated in \( \mathcal{F} \) as \( n_+ \) in the case of \( \mathcal{F}' \)). Here the Gaussian decomposition \( N_- H N_+ \) for the group \( Sp^{(1)}(4, \mathbb{C}) \) is selected as \( p_+ \in N_+, \hat{p}_- \in N_- \). Thus the operators \( p_+ \) and \( \hat{p}_- \) non-commuting with each other generate flows in different spaces: respectively in \( \mathcal{F} \) and in \( \mathcal{F}' \) being dual to \( \mathcal{F} \), so that the equations of fluxes are written in the form

\[
-i \frac{\partial}{\partial x_\mu} f(x) = p_+ f(x), \quad -i \frac{\partial}{\partial \hat{x}_\nu} \hat{f}(\hat{x}) = \hat{p}_- \hat{f}(\hat{x})
\]

where \( f(x) \in \mathcal{F}, \hat{f}(\hat{x}) \in \mathcal{F}' \). The infinite-component fields \( f(x) \) and \( \hat{f}(\hat{x}) \) existing in a fibre \( (x \text{ and } \hat{x} \text{ are coordinates in the fibre}) \) are transformed by infinite-dimensional representations of the group \( SU(2) \) with quarter-integer spins and are called as semispinor ones [4]. It is of vital importance that in the non-unitary theory the Lorentz group \( GL(2, \mathbb{C}) \subseteq Sp^{(1)}(4, \mathbb{C}) \), too, disintegrates in its Borel subgroups as \( N_\pm (GL(2, \mathbb{C})) \subseteq N_\pm (Sp^{(1)}(4, \mathbb{C})) \). In this way the Lorentz symmetry of relativistic bi-Hamiltonian system turns out to be spontaneously broken.

The decomposition \( (B_-, B_+) \) co-ordinated to the pair of spaces \( (\mathcal{F}, \mathcal{F}') \) (in [1] the correspondence is called the polarization of dynamical system) is responsible for the spontaneous symmetry (time boost) breaking. In the non-unitary theory the functions of two variables \( x \) and \( \hat{x} \) exist only as sesquilinear non-Hermitian forms \( \langle \hat{f}(\hat{x}), f(x) \rangle \) or matrix elements being the finite-component bilocal fields satisfying the condition

\[
\left( \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial \hat{x}_\nu} - \frac{\partial}{\partial \hat{x}_\mu} \frac{\partial}{\partial x_\nu} \right) \langle \hat{f}(\hat{x}), f(x) \rangle = 0.
\]

In the theory such fields correspond to the non-point-like fundamental particles. The manifold \( M = \mathbb{A}_{3,1} \) will be reconstructed by them, and if the quantization procedure applies to them we arrive at the theory without ultraviolet divergences [1]. Thus before using the second quantization procedure it is necessary to carry out changing the theory proposed here.

For the present we have given the elementary introduction (from authors’ standpoint) to the class of ideas which would taken as a basis for construction of a new theory of elementary particles. As seen from the above, the proposed theory is related to the fundamental break-up of usual notions (space-time continuum, unitarity axiom, Hamiltonian systems) and their change by others (discontinuum, non-unitary representations, non-standard dynamical systems). The new theory is based on the notion of the relativistic be-Hamiltonian system which was succeeded in constructing the Lagrangian field system characterized by a certain mass spectrum and interactions. The questions of reconstruction will be considered elsewhere.

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