Suppression of the TeV Pair-beam–Plasma Instability by a Tangled Weak Intergalactic Magnetic Field

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Abstract

We study the effect of a tangled sub-fG level intergalactic magnetic field (IGMF) on the electrostatic instability of a blazar-induced pair beam. Sufficiently strong IGMF may significantly deflect the TeV pair beams, which would reduce the flux of secondary cascade emissions below the observational limits. A similar flux reduction may result from the electrostatic beam–plasma instability, which operates the best in the absence of IGMF. Considering IGMF with correlation lengths smaller than a kiloparsec, we find that weak magnetic fields increase the transverse momentum of the pair-beam particles, which dramatically reduces the linear growth rate of the electrostatic instability and hence the energy-loss rate of the pair beam. We show that the beam–plasma instability is eliminated as an effective energy-loss agent at a field strength three orders of magnitude below that needed to suppress the secondary cascade emission by magnetic deflection. For intermediate-strength IGMF, we do not know a viable process to explain the observed absence of GeV-scale cascade emission.

Unified Astronomy Thesaurus concepts: Gamma-rays (637); Particle astrophysics (96); Intergalactic medium (813); Blazars (164)

1. Introduction

GeV–TeV gamma-ray signals from various blazars (z > 0.024) have been observed by the Fermi-LAT telescope and the imaging atmospheric Cerenkov telescopes (i.e. VERITAS, MAGIC, and HESS; Albert et al. 2008; H.E.S.S. Collaboration et al. 2010). Very high energy gamma rays annihilate with the extragalactic background light (EBL), producing a collimated beam of electron–positron pairs, which are expected to quickly lose their energies via the inverse Compton scattering on the cosmic microwave background (CMB; Gould & Schréder 1967; Blumenthal & Gould 1970). Primary gamma rays of a few TeV would produce an electromagnetic cascade in the GeV energy band, but that emission appears to be absent in the gamma-ray spectra from some blazars (Neronov & Semikoz 2009).

One possible explanation for the absence of the GeV cascade emission is significant magnetic deflection of the electrons and the positrons of the beam (Elyiv et al. 2009; Neronov & Semikoz 2009; Neronov & Vovk 2010; Takahashi et al. 2011; Taylor et al. 2011; Vovk et al. 2012). This deflection results in an extended emission or/and a time delay of the cascade emission. The field strength required to suppress the cascade emission due to the time delay is around $B_{\text{IGM}} > 10^{-16} \text{G}$ for intergalactic magnetic field (IGMF) with a correlation length similar to or larger than the energy-loss length of the beam, $\nu_{\text{IGM}} > 10 \text{kpc}$, and stronger than that for a small correlation length, for which the beam sees a fluctuating magnetic field and the deflection becomes diffusive (Ackermann et al. 2018).

Those fields might be the original form of seed fields that may be amplified to stronger magnetic fields in the galaxies and galaxy clusters (Durrer & Neronov 2013; Batista & Saveliev 2021; Vachaspati 2021).

If the magnetic field is strong enough to deflect by a radian or more, then the cascade emission from active galactic nuclei (AGNs) with oblique jets (jets more than 30° off of our line of sight) should become visible (Broderick et al. 2016; Tiede et al. 2020), but corresponding emission has not been found (Tiede et al. 2017; Ackermann et al. 2018; Broderick et al. 2018).

Another possibility are beam–plasma instabilities that work as an alternative energy-loss mechanism to the inverse Compton scattering of the pair beam (Broderick et al. 2012; Schlickeiser et al. 2012; Miniati & Elyiv 2013; Schlickeiser et al. 2013; Broderick et al. 2014; Chang et al. 2014; Sironi & Giannios 2014; Supsar & Schlickeiser 2014; Chang et al. 2016; Kempf et al. 2016; Rafighi et al. 2017; Vafin et al. 2018; Alves Batista et al. 2019; Vafin et al. 2019; Shalaby et al. 2020). The beam–plasma instabilities involve both electrostatic and electromagnetic modes, the two-stream instability ($k \times E_1 = 0$, where $E_1$ is the perturbed electric field), and transverse Weibel and filamentation modes ($k \cdot E_1 = 0$) (Bret et al. 2010).

However, for the blazar-induced TeV pair beams the electrostatic modes dominate the wave spectrum (Bret et al. 2005), and Weibel-type modes are likely suppressed (Rafighi et al. 2017). Hence, considering only the electrostatic oblique modes (wavevectors with finite angle to the beam propagation direction) recovers the essential physics (Chang et al. 2016).

Through their nonlinear feedback these electrostatic waves transfer energy from the beam particles to heat in the intergalactic medium (IGM). Cosmological simulations including this heating process can successfully reproduce the observed IGM temperature and the effective optical depth as a function of redshift (Puchwein et al. 2012). Perry & Lyubarsky (2021) argued otherwise, namely, that the back-reaction of the unstable waves on the pair-beam particle distribution moderately scatters the beam particles and does not impose a significant energy loss. We do not discuss the...
particulars of the nonlinear feedback here and leave this issue for future studies.

The beam–plasma electrostatic instability operates best in the absence of a magnetic field. Noting that magnetic deflection needs more than a femtogauss field amplitude, here we address the effect on the electrostatic instability that would be imposed by much weaker IGMFs with a correlation length much smaller than the beam energy-loss length. In particular, we investigate whether the plasma instability still is the dominant energy-loss process and how strongly the cascade emission is suppressed (Yan et al. 2019).

In this work, we consider an IGMF with small correlation length far below the energy-loss length of the pair beam, \( \lambda_B \ll \lambda_p \), which deflects the electrons and positrons equivalently. Note that this condition implies that we assume the IGMFs to have no large-scale (\( \gtrsim \text{kpc} \)) or homogeneous component. We only consider the fluctuation component. Magnetic fields with strengths of \( B_{\text{IGM}} \ll 10^{-12} \text{ G} \) do not modify the linear dispersion relation of the beam–plasma instability obtained by the electrostatic approximation. However, those fields may impact the instability linear growth rate by their effect on the beam distribution function. The case of large magnetic field correlation lengths involves a net current in the beam and will be considered in a future publication.

Our IGMF model is the same as that widely used in the analysis of deflection and time-delays (Elizy et al. 2009; Neronov & Semikoz 2009; Neronov & Vovk 2010; Takahashi et al. 2011; Taylor et al. 2011; Vovk et al. 2012). The focus lies on a weaker field strength and on small correlation lengths. In such magnetic fields the electrons and the positrons of the blazar-induced pair beam perform a random walk passing through many regions with different field orientations, resulting in an increased angular spread of the pair beam that scales with the mean field strength and the square root of the correlation length (Durrer & Neronov 2013).

We showed that this widening of the beam significantly slows the electrostatic instability, which decreases the energy-loss rate of the beam particles. At a certain limit in the parameter space (\( B_{\text{IGM}} \), \( \lambda_B \)), driving the waves becomes less effective than inverse Compton scattering the CMB, and the GeV cascade emission can no longer be suppressed. For the plasma instability model in Vafin et al. (2018), this limit is found to be around three orders of magnitude below the one that by magnetic deflection would impose a time delay of the cascade emission by 10 yr (Ackermann et al. 2018).

The structure of this paper is as follows. In Section 2, we present the linear growth rate spectrum of the electrostatic instability of realistic pair-beam distributions without and with weak IGMFs. In Section 3, we present the nonlinear instability saturation of the unstable electrostatic waves. Finally, we demonstrate our results in Section 4 and conclude in Section 5.

### 2. Linear Growth Rate of the Electrostatic Instability

In this section, we present the linear growth rate of electrostatic waves for a realistic blazar-induced pair beam with finite angular spread (kinetic instability) moving in an unmagnetized IGM. Then, we consider the magnetic fields in the IGM and find their impact on the beam distribution function and the implications for the growth rate of electrostatic waves.

As we mentioned in the introduction, the electrostatic approximation is valid for the blazar-induced pair beam, for which the electrostatic modes grow far more quickly than do the electromagnetic modes (Bret et al. 2010; Chang et al. 2016). A comparison of the Weibel growth rate for blazar-induced pair beams using a cold-beam distribution (Schlickeiser et al. 2012) and a Waterbag distribution (Rafighi et al. 2017) shows that the Weibel instability is suppressed for a realistic blazar-induced pair beam. Therefore, we will proceed with the electrostatic approximation in our analysis.

Linearizing the Vlasov–Maxwell equations for electrostatic waves leads to the following dispersion relation (Breizman 1990):

\[
1 - \frac{\omega_p^2}{\omega^2} - \sum_{b} \frac{4\pi n_b e^2}{k^2} \int d^2 p \frac{\partial f_b(p)}{\partial p} \cdot \mathbf{k} \cdot \mathbf{v} - \omega = 0, \tag{1}
\]

where \( f_b(p) = f_b(p, x) / n_b \) is the normalized momentum distribution function of the beam, \( n_b \) is the total number density of the beam, and \( \omega_p = (4\pi n_e e^2/m_e)^{1/2} \) is the plasma frequency of the intergalactic background plasma with density \( n_e \). The wavevector is chosen as \( \mathbf{k} = (k_\perp, 0, k_z) \), and the beam propagates along the \( z \)-axis with cylindrical symmetry.

In our analysis we consider the kinetic instability for which the beam temperature plays a significant role. The kinetic instability is applicable if the velocity spread times the wavenumber of the unstable waves is larger than the growth rate of the reactive instability (Chang et al. 2016)

\[
|\mathbf{k} \cdot \mathbf{v}| \gg \omega_{\text{ir}}, \tag{2}
\]

The peak reactive growth rate is (Bret et al. 2010)

\[
\omega_{\text{ir}} = \frac{\sqrt{5}}{2^{1/3}} \omega_p \left( \frac{n_b}{\gamma_b n_c} \right)^{1/3} \left( \frac{k_\parallel}{k} \right)^2 + \frac{1}{\gamma_B^2} \left( \frac{k_\perp}{k} \right)^2 \right)^{1/3}, \tag{3}
\]

where \( \gamma_b \) is the beam Lorentz factor, and the parallel wavenumber is fixed at the resonance, \( k_\parallel = \omega_p / c \). For a relativistic beam the perpendicular velocity spread is \( \Delta v_\perp \approx c / \gamma_B \) and the parallel velocity spread is \( \Delta v_\parallel \gtrsim c / \gamma_B \), resulting from the Lorentz boost of the beam from the center-of-momentum frame to the lab frame (Miniati & Elizy 2013). For a realistic blazar-induced pair beam with Lorentz factor \( \gamma_b \approx 10^3 - 10^5 \), Equation (2) is satisfied for essentially all oblique waves, meaning that we should consider the kinetic regime and not the reactive one (cold limit).

For a relativistic electron beam (\( \gamma_b \gg 1 \)) with a small angular spread (\( \Delta \theta \ll 1 \text{ rad} \)) traveling in a homogeneous background plasma with a number density \( n_c \), the dispersion relation, Equation (1), in the kinetic regime yields the following linear growth rate of electrostatic waves (Breizman 1990):

\[
\omega_1(k) = \pi \omega_p \frac{n_b}{n_c} \left( \frac{\omega_p}{k} \right)^3 \int_0^\theta d\theta \
\times \frac{-2g(\theta) \sin \theta + (\cos \theta - \frac{k_\perp}{k} \cos \theta_1 \cos \theta \sin \theta_1 e^{-\gamma_b \theta})}{\left[\cos \theta_1 - \cos \theta (\cos \theta - \cos \theta_1) e^{-\gamma_b \theta_1} \right]^{1/2}}, \tag{4}
\]

where

\[
g(\theta) = m_e c \int_0^\infty dp \, \rho B(p, \theta), \tag{5}
\]
and

\[
\cos \theta_{1,2} = \frac{\omega_p}{k c} \left( \cos \theta' \pm \sin \theta' \sqrt{\left( \frac{k c}{\omega_p} \right)^2 - 1} \right),
\]

where \( k = \sqrt{k_{\perp}^2 + k_{\parallel}^2} \) is the module of the unstable electrostatic waves’ wavenumber vector (\( k_{\perp} \) and \( k_{\parallel} \) are the perpendicular and parallel components to the beam propagation direction, respectively), \( \theta' \) is the angle between the wavevector and the beam propagation direction, and \( \theta \) is the angle between the particle momentum and the beam direction axis (\( z \)-axis). The beam is azimuthally symmetric around the propagation axis.

The momentum distribution function, \( f_\theta(p, \theta) \), of the beam is normalized as

\[
2\pi \int_0^\infty dp p^2 \int_0^\pi d\theta \sin \theta f_\theta(p, \theta) = 1
\]

and can be factorized into parallel and perpendicular components

\[
f_\theta(p, \theta) = f_{\theta,\parallel}(p, \theta) f_{\theta,\perp}(p, \theta),
\]

where for the parallel momentum distribution \( f_{\theta,\parallel}(p) \) we used Equations (26) and (56) in Vain et al. (2018), which are obtained for a realistic pair beam at a distance of 50 Mpc from the blazar. The angular distribution, \( f_{\theta,\perp}(p, \theta) \), depends on whether or not we have IGMFs.

### 2.1. Electrostatic Instability for a Pair Beam in a Nonmagnetized Intergalactic Medium

In the case of a nonmagnetized IGM, the angular spread of the beam is due to the angular energy spread only. In this case, the angular distribution function of the beam, \( f_{\theta,\perp}(p, \theta) \), can be approximated by a Gaussian (Miniati & Elyiv 2013)

\[
f_{\theta,\perp}(p, \theta) \approx \frac{1}{\pi \Delta \theta^2_\perp} \exp \left( -\frac{\theta^2}{\Delta \theta^2_\perp} \right),
\]

where the angular energy spread is approximated as (Broderick et al. 2012)

\[
\Delta \theta_\perp \approx \frac{m_e c}{\theta},
\]

Substituting Equation (9) into Equations (8) and (4), we found the numerical solution for the linear electrostatic growth rate as shown in Figure 1, and we see that most of the unstable modes are in the oblique and parallel directions. The maximum linear growth rate is found to be

\[
\omega_{\perp,\max} = (3.83 \times 10^{-7}) \omega_p \frac{n_{p20}}{n_e7},
\]

\[
= (1.15 \times 10^{-8}) \omega_p,
\]

where \( \omega_p = 17.8 \text{ Hz} \) is the plasma frequency of the intergalactic background electrons for the unit density \( n_e = n_e710^{-7} \text{ cm}^{-3} = 10^{-7} \text{ cm}^{-3} \). For the fiducial pair-beam parameters, the number density of the pair beam is \( n_p = n_{p20}10^{-20} \text{ cm}^{-3} = 3 \times 10^{-22} \text{ cm}^{-3} \). Note that the maximum growth rate we found here is twice that reported in Vain et al. (2018) because the maximum growth rate in Vain et al. (2018) was computed for the parallel wavenumbers down to \((k_{\parallel} c/\omega_p - 1) \approx 10^{-7} \); however, we found that smaller parallel wavenumbers down to \((k_{\parallel} c/\omega_p - 1) \approx 10^{-14} \) have larger growth rates, as shown in Figure 1.

Vain et al. (2018) demonstrated that for a blazar with a redshift \( z = 0.2 \) those unstable waves drain the pair-beam energy around 100 times faster than does inverse Compton scattering on the CMB, taking into account the modulation instability as a damping process. The main uncertainties in that work are the assumptions on the spectrum and gamma-ray flux from the blazar and the approximation of the nonlinear saturation level.

The growth rate is calculated for a pure electron beam moving in a background plasma of electrons and ions. Schlickeiser et al. (2012) demonstrated that having separate distribution functions for electrons and for positrons yields the same growth rate as do calculations that assume only an electron beam (Broderick et al. 2012).

### 2.2. Electrostatic Instability for a Pair Beam with a Weak Intergalactic Magnetic Field

We address in this section the effects of weak IGMFs on the electrostatic plasma instability. If the electron gyromagnetic frequency, \( \omega_B = e B_{\text{IGM}}/m_e \), is much smaller than their plasma frequency, \( \omega_p \ll \omega_p \), then an external magnetic field does not change the electrostatic dispersion relation used to derive the linear growth rate (Fainberg 1961). The corresponding upper limit for the strength of the IGMF is \( B_{\text{IGM}} \leq 10^{-9} \text{ G} \), where we again assumed the number density to be \( n_e = 10^{-7} \text{ cm}^{-3} \).

The magnetic field correlation lengths we consider, \( \lambda_B \sim 10^{-7} - 10^{-13} \text{ pc} \), are much larger than the intergalactic plasma skin length, \( \lambda_D \sim 5 \times 10^{-16} \text{ pc} \), meaning that even the variations of the IGMF have no direct impact on the beam–plasma dispersion relation. However, the directional changes of the magnetic field affect the equilibrium beam distribution function, which in turn impacts the linear electrostatic growth rate (Equation (4)). In other words, the blazar-induced pair beam that triggers the instability travels through many correlations lengths in the IGMF. For example, the blazar-induced pair-beam distribution function we are considering in this work is calculated at a distance of 50 Mpc in the IGM from...
the blazar (Vafin et al. 2018), whereas the pair production starts at distances smaller than 1 Mpc (Miniati & Elyiv 2013).

More importantly, we can take the inverse Compton scattering length, \( \lambda_{\text{IC}} \approx 75 \text{kpc}(10^7/\gamma_b) \), as an upper limit on the energy-loss length of the beam particles, which gives around 188 kpc for a Lorentz factor of \( \gamma_b = 4 \times 10^9 \). This means that the pair-beam distribution function carries the effects of the magnetic fields over a large number of directional changes, since most of the particles in the beam have traveled many correlation lengths at least, \( \lambda_{\text{IC}} \gg \lambda_B \). This propagation of the pair beam over many correlation lengths imposes an additional angular spread on its momentum distribution, which in turn significantly affects the linear electrostatic growth rate.

Those fields lead to stochastic deflections of the electrons and positrons that diffusively widen the angular distribution function of the pair beam as shown in Appendix A. Adding in quadrature the energy angular spread \( \Delta \theta_e \) (Equation (10)) and the magnetic widening \( \Delta \theta_{\text{IGMF}} \) (Equation (A8)) gives the following distribution of the angular spread of the pair beam after traveling many correlation lengths in the IGM:

\[
\frac{dN}{d\Omega dE} = \frac{D = +}{\pi a^2} \exp \left( - \left( \frac{\theta}{\Delta \theta} \right)^2 \right), \quad 0 \leq \theta \leq \pi,
\]

where

\[
\Delta \theta = \frac{m_e c}{p} \sqrt{1 + \frac{2}{3} \lambda_B \lambda_{\text{IC}} (e B_{\text{IGMF}} / m_e c)^2}.
\]

Note that the result in Appendix A for the magnetic deflection, \( \Delta \theta_{\text{IGMF}} \), is consistent with the diffusion angle used in the IGMF deflection analyses, e.g., Equation (31) in Neronov & Semikoz (2009).

Finally, substituting Equation (12) into Equations (8) and (4), we numerically found the linear growth rate spectrum for a few values of the IGMF strength, \( B_{\text{IGMF}} \), and the correlation length, \( \lambda_B \), and displayed it in Figure 2. The main impact of the IGMF is a general reduction of the growth rate. Figure 3 shows the peak growth rate as a function of \( B_{\text{IGMF}} \) and \( \lambda_B \). To be noted from the figure is that specific values of the peak growth rate are found on a characteristic \( B_{\text{IGMF}} \propto \lambda_B^{0.5} \). The reduction of the instability growth rate increases the energy-loss time owing to the plasma instability, as we will see in the next section.

3. Nonlinear Instability Saturation

The unstable electrostatic waves grow exponentially with the linear growth rate, accumulating at the resonant parallel wavenumber \( k_{||} \approx \omega_p / c \). Depending on their nonlinear interactions, the waves could drain the kinetic energy of the pair beam and heat the IGM. The first type of those nonlinear interactions is the scattering of the electrostatic waves on the background plasma, known as nonlinear Landau interactions. The second nonlinear interaction is a wave–wave interaction between the electrostatic waves known as the modulation instability. The first process operates at any wave intensity, whereas the second occurs only above a certain threshold.

Simulations of the evolution of the beam/plasma system are impossible right now for realistic parameters. However, there are various analytical estimates in the literature concerning the energy density that the waves reach in an equilibrium state (Broderick et al. 2012; Schlickeiser et al. 2012; Miniati & Elyiv 2013; Vafin et al. 2018). The inverse energy-loss time of the pair beam due to the electrostatic instability is given by (Vafin et al. 2018; Miniati & Elyiv 2013)

\[
\tau_{\text{loss}}^{-1} = 2 \delta \omega_{\gamma, \text{max}},
\]

where \( \omega_{\gamma, \text{max}} \) is the peak linear growth rate and \( \delta = U_{\text{IGM}} / U_{\text{beam}} \) is the normalized wave energy density at the equilibrium level. The reduction of the linear growth rate due to the IGMF translates into an increase of the energy-loss time. At some limit the beam–plasma instability becomes less relevant than the inverse Compton scattering. We will find this limit in the next section.

The wave intensity, \( \delta \), depends on the nonlinear evolution of the electrostatic waves, for which we have different estimates. In the next section, we are going to first include the result given in Vafin et al. (2018) and then discuss the implications of changing the value of the intensity of the waves to that found by Broderick et al. (2012).

4. Results

We found the maximum linear growth rate of the unstable electrostatic 2D spectrum for each IGMF strength, \( B_{\text{IGMF}} \), and correlation length, \( \lambda_B \), as shown in Figure 3. Then, we calculated the approximated energy-loss time of the beam based on the maximum linear growth rate as in Equation (14), using the intensity of the waves given in Vafin et al. (2018), \( \delta = 10^{-5} \). This time should be smaller than the inverse Compton scattering energy-loss time; otherwise, the beam–plasma instability cannot suppress the secondary cascade. The energy-loss time of the inverse Compton scattering is given by

\[
\tau_{\text{IC}} = (7.7 \times 10^{13} \text{ s}) (1 + z)^{-4} \left( \frac{10^6}{\gamma_b} \right).
\]

which at redshift \( z = 0.2 \) and for a pair-beam Lorentz factor \( \gamma_b = 4 \times 10^9 \) gives the following ratio for the beam–plasma instability loss time given in Vafin et al. (2018):

\[
\frac{\tau_{\text{loss}}}{\tau_{\text{IC}}} = 0.026,
\]

if the IGMF is zero.

Using Equation (14), we infer that a reduction by a factor 40 of the instability growth rate is sufficient to render it inefficient. The dependence of the growth rate on \( B_{\text{IGMF}} \) and \( \lambda_B \) (see Figure 3) can then be turned into a limit in the \( B_{\text{IGMF}} - \lambda_B \) parameter space, above which inverse Compton scattering provides the dominant energy loss of the pair beam. We show this limit in Figure 4. It is at the same time an exclusion limit, because the then unavoidable inverse Compton emission is not seen, and so the cyan shaded area in the figure is excluded for the IGMF. For weaker fields the oblique instability may drain the beam energy sufficiently quickly, and for stronger fields the time delay of the cascade emission causes substantial uncertainty in the interpretation of the Fermi-LAT data of GeV-scale cascade emission.

We see in Figure 4 that the beam–plasma instability suppression limit (the purple line) is three orders of magnitude lower than the lower limit on the IGMF strength needed to impose a significant time delay of the cascade emission flux due to the magnetic deflection (the green line; Taylor et al. 2011; Finke et al. 2015; Ackermann et al. 2018). We follow Ackermann et al. (2018) in assuming a time period of 10 yr as sufficient for the suppression of the cascade signal. The actual deflection angle would be well below 1°.
Finally, to account for the uncertainty of the nonlinear saturation level of the waves, we consider also the beam–plasma instability model presented in Broderick et al. (2012) with $\delta = 0.2$ and check how the plasma instability suppression limit changes for a certain correlation length. For $\lambda_B = 10^{-11}$ Mpc, the instability limit would shift for Broderick et al. (2012) to $B_{IGM} = 10^{-12.5}$ G, which is two orders of magnitude higher than that based on Vafin et al. (2018) but still below the Fermi
time-delay lower limit. This shift applies also to all the
instability suppression limit points in Figure 4, since the
correlation length and the IGMF strength determine together
the angular spread (Equation (13)) that plays the key role in
determining the linear growth rate.

Although the nonlinear saturation level had changed by four
orders of magnitude between the models of Vafin et al. (2018)
and Broderick et al. (2012), the magnetic field limit had
changed by only two orders of magnitude. This is due to the
dependence of the energy-loss time on the angular spread as
\( \tau_{\text{loss}} \propto (\Delta \theta)^2 \), which is a result of the maximum linear growth rate
dependence on the angular spread as \( \omega_{\text{max}} \propto (\Delta \theta)^{-2} \)
(Vafin et al. 2019) and the energy-loss time relation with the
linear growth rate as \( \tau_{\text{loss}} \propto \omega_{\text{max}}^{-1} \) (Equation (14)).

For a generic beam–plasma instability model with plasma
instability energy-loss time \( \tau_{\text{loss},0} \) in the absence of the IGMF,
the energy-loss time increases as \( \tau_{\text{loss}} \propto (\Delta \theta)^2 \) when the angular spread increases with the IGMF strength and correlation
length as in Equation (13), reaching the inverse Compton scattering energy-loss time at the following IGMF strength:

\[
\log \left( \frac{B_{\text{IGM,lim}}}{\text{Gauss}} \right) = -17.92 - \frac{1}{2} \log \left( \frac{\lambda_B}{\text{pe}} \right) + \log \left( \frac{\text{Myr}}{\tau_{\text{loss},0}} - \frac{\text{Myr}}{\tau_{IC}} \right). \tag{17}
\]

Equation (17) provides the IGMF strength that is sufficient to
suppress a general plasma instability, with energy-loss time
\( \tau_{\text{loss},0} \) in the absence of the IGMF, against the inverse Compton scattering of the blazar-induced pair beam on the CMB. For Vafin’s model, the last logarithmic term on the right-hand side of Equation (17) has a value very close to unity.

5. Conclusion

We investigated the effects of tangled weak IGMFs with small correlation lengths on the electrostatic instability driven
by blazar-induced pair beams. The weak fields increase the angular spread of the pair beam, which decreases the linear growth rate of the electrostatic beam–plasma instability, which in turn reduces the associated energy-loss rate.

In a certain region in the \( B_{\text{IGM}} - \lambda_B \) parameter space, neither the beam–plasma instability nor the IGMF deflection can explain the absence of cascade emission in the spectra of some TeV blazars, and so this parameter space region can be excluded, unless there is a third mechanism that suppresses the GeV-band cascade.

Considering the beam–plasma instability model of Vafin
et al. (2018), we can exclude an IGMF strength within the three
orders of magnitude below the limit above which magnetic deflection imposes a significant time delay of the cascade (10 yr). Even for the nonlinear evolution model of Broderick et al. (2012), we can exclude a range of values that is one order of magnitude wide.

Although the parameter space below the beam–plasma
instability suppression limit is not excluded by the cascade
observations, part of this region (at \( \lambda_B \lesssim 10^{-8} \) Mpc and shaded
in gray in Figure 4) is constrained by MHD turbulent decay
(Banerjee & Jedamzik 2004; Durrer & Neronov 2013). In conclusion, the allowed region for the IGMF lies below \( 10^{-16} \) G at \( \lambda_B \approx 10^{-8} \) Mpc, and the constraint on the IGMF strength is tighter than that for both larger correlation lengths (due to the
instability suppression) and smaller correlation lengths (due to
the MHD turbulent decay).

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Appendix A

Beam Distribution Function with IGMF

Consider a magnetic field with a constant magnitude that
arbitrarily and abruptly changes its direction every correlation length, \( \lambda_B \), along the beam propagation line. In Cartesian coordinates with the \( z \)-axis aligned with the beam, the magnetic field component in the \( x-y \) plane deflects the beam every \( \lambda_B \) interval in a different direction. We will include first the deflection due to the magnetic field component along the \( x \)-axis, and then we will generalize to the \( x-y \) plane. At the end we find that the angular distribution function is a Gaussian with azimuthal symmetry; hence, the electrons and positrons distribution functions are equivalent, and it is sufficient to consider only one species.

At a given correlation length interval denoted by \( i \), the component of the magnetic field in the \( x \)-direction \( (B_{x,i} = B_{\text{IGM}} \sin \theta' \cos \varphi') \) deflects the beam positrons along the \( y \)-direction with a deflection angle

\[
\Delta \theta_i(\theta', \varphi') = \frac{\lambda_B e B_{\text{IGM}} \sin \theta' \cos \varphi'}{p}, \tag{A1}
\]

where \( p \) is the momentum of the beam particle and \( e \) is the
elementary electric charge. \( \Delta \theta \) is a random variable that depends on the random variables \( \theta' \) and \( \varphi' \). Since all the possible magnetic field orientations have the same probability, the mean deflection is

\[
\mu = \frac{1}{4\pi} \int_0^\pi \sin \theta' d\theta' \int_0^{2\pi} d\varphi' \Delta \theta_i(\theta', \varphi') P(\Delta \theta_i(\theta', \varphi')) = \frac{\lambda_B e B_{\text{IGM}}}{4\pi p} \int_0^\pi d\theta' \int_0^{2\pi} d\varphi' \sin^2 \theta' \cos \varphi' = 0, \tag{A2}
\]

and the variance is

\[
\sigma^2 = \frac{1}{4\pi} \left( \frac{\lambda_B e B_{\text{IGM}}}{p} \right)^2 \int_0^\pi d\theta' \int_0^{2\pi} d\varphi' \sin^3 \theta' \cos^2 \varphi' = \frac{1}{3} \left( \frac{\lambda_B e B_{\text{IGM}}}{p} \right)^2. \tag{A3}
\]

The total deflection of the beam is computed as

\[
\Delta \theta = \sum_{i=0}^n \Delta \theta_i, \tag{A4}
\]

where \( n = \lambda_{IC}/\lambda_B \) is the total number of the correlation lengths crossed by the beam during its energy-loss length (substituted here by the inverse Compton scattering length). Since \( n \) is very large in our case, \( \lambda_{IC} \gg \lambda_B \), we can use the central limit theorem with \( n \to \infty \) to find the distribution function of the
Note that by definition \( n = \lambda_C/\lambda_B \). Combining the two distributions in Equations (A5) and (A6) using the result in Appendix B, we get the full angular distribution of the pair beam

\[
f_{b,\theta}(\theta, p) = \frac{1}{2\pi n \sigma} \exp \left\{ -\frac{1}{2} \left( \frac{\theta}{\sqrt{n} \sigma} \right)^2 \right\}, \quad -\pi \leq \theta \leq \pi, \quad 0 \leq \theta_x \leq \pi, \quad 0 \leq \varphi \leq 2\pi, \tag{A7}
\]

where

\[
\Delta \theta_{\text{IGMF}} = \frac{eB_{\text{IGMF}}}{p} \sqrt{\frac{2}{3}} \frac{\lambda_C}{\lambda_B}. \tag{A8}
\]

What we have considered here is a fixed IGMF amplitude. Considering an IGMF with different amplitudes leads to the same result in terms of the rms IGMF with a numerical factor.

**Appendix B**

**Transformation of** \( f_2(\theta_x,f_2(\theta_y)) \) **to** \( f(\theta, \varphi) \)

Combining the distributions \( f_2(\theta_x) \) and \( f_2(\theta_y) \) gives the distribution \( f(\theta, \varphi) \)

\[
f(\theta, \varphi) = f_2(\theta_x)f_2(\theta_y) \frac{1}{\sin \theta} \frac{1}{\sin \varphi} \cdot \frac{1}{\sin \theta} \frac{1}{\sin \varphi} \cdot \frac{1}{2\pi n \sigma} \exp \left\{ -\frac{1}{2} \left( \frac{\theta}{\sqrt{n} \sigma} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left( \frac{\varphi}{\sqrt{n} \sigma} \right)^2 \right\},
\]

where \( \theta \) and \( \varphi \) are the spherical coordinates and \( \theta_x \) and \( \theta_y \) are defined in Figure 5. We rewrite Equation (B1) using the Jacobian determinant

\[
f(\theta, \varphi) = f_2(\theta_x(\theta, \varphi))f_2(\theta_y(\theta, \varphi)) \frac{1}{\sin \theta} \frac{1}{\sin \varphi} \cdot \frac{1}{2\pi n \sigma} \exp \left\{ -\frac{1}{2} \left( \frac{\theta}{\sqrt{n} \sigma} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left( \frac{\varphi}{\sqrt{n} \sigma} \right)^2 \right\},
\]

where \( \theta \) and \( \varphi \) are the spherical coordinates and \( \theta_x \) and \( \theta_y \) are defined in Figure 5. We rewrite Equation (B1) using the Jacobian determinant

\[
f(\theta, \varphi) = f_2(\theta_x(\theta, \varphi))f_2(\theta_y(\theta, \varphi)) \frac{1}{\sin \theta} \frac{1}{\sin \varphi} \cdot \frac{1}{2\pi n \sigma} \exp \left\{ -\frac{1}{2} \left( \frac{\theta}{\sqrt{n} \sigma} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left( \frac{\varphi}{\sqrt{n} \sigma} \right)^2 \right\},
\]

for a small \( \theta \). Putting this in Equation (B2) gives

\[
f(\theta, \varphi) \approx f_2(\theta_x(\theta, \varphi))f_2(\theta_y(\theta, \varphi)) \frac{\tan \theta}{\sin \theta},
\]

for a small \( \theta \).

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