Piezoelectric mirror shifter transfer function measurement, modelling, and analysis using feedback based synthetic-heterodyne Michelson interferometry

MICHAEL J. CONNELLY,1,* JOSÉ HENRIQUE GALETI,2 AND CLÁUDIO KITANO3

1Optical Communications Research Group, Department of Electronic and Computer Engineering, University of Limerick, Limerick, V94 T9PX, Ireland
2Federal Institute of Mato Grosso do Sul, 79641-162, Três Lagoas-MS, Brazil
3Department of Electrical Engineering, São Paulo State University (UNESP), 15385-000, Ilha Solteira-São Paulo, Brazil

* michael.connelly@ul.ie

Abstract: Laser vibrometry has many applications in non-contact dynamic displacement and vibration measurement. A test beam reflected from a target and a reference beam are combined and detected by a photodiode; the photodetected signal is then processed to determine the target displacement and vibration. This paper describes the use of a 9 kHz measurement bandwidth system, consisting of a Michelson interferometer and self-correcting feedback synthetic-heterodyne signal processing technique, to measure the displacement impulse response of a commercial piezoelectric mirror shifter (PMS), consisting of a mirror mounted on a Piezoelectric transducer and a connecting 50 Ω electrical coaxial cable. The actual non-ideal applied impulse and measured impulse response data were used in conjunction with the instrument variable method to determine a Laplace domain linear transfer function approximation to the actual PMS transfer function. The best transfer function fitting, having a 84% normalized root mean square goodness of fit, was obtained using a 5-th order transfer function having two complex conjugate pole pairs, with associated natural frequencies of 6.29 and 6.79 kHz, and a single real pole. The transfer function zeros consisted of a single complex conjugate zero pair, having an antiresonance frequency of 6.38 kHz and a single real zero. Knowing the analytic transfer function of PMS based nanopositioners is useful for example in the design of closed-loop phase-locked interferometers for wideband sensing.

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1. Introduction

Laser interferometry has many applications in the measurement of dynamic displacement and velocity of vibrating objects [1–4]. In the case of the Michelson interferometer based vibrometer considered in this paper, a laser source is split into two lightwaves. One lightwave acts as a reference; the other is reflected from a vibrating object under test (OUT). The reference and test lightwaves are combined and the resultant total optical field is detected using a photodetector. There is a nonlinear relationship between the photocurrent and the relative phase shift between the lightwaves, the dynamic component of which is proportional to the OUT dynamic displacement. Many different signal processing methods have been investigated to recover the dynamic phase shift; however the detection sensitivity can be adversely affected by environmentally induced optical path variations. Synthetic-heterodyne (SH) processing is particularly useful as it eliminates slowly varying changes in sensitivity due to environmentally induced optical path variations such as those caused by temperature fluctuation induced refractive index changes [5,6]. The basic principle of the basic SH technique is to modulate the optical source frequency with a
high frequency sinusoid that is much higher than the highest vibration signal of interest. The resulting photodetected signal is a frequency modulated type signal, the phase of which is the sum of two components: a slowly varying component due to environmental fluctuations and a component proportional to the vibration signal. Theoretically the photodetected signal can be expanded as a series of bands centered at integer multiples of the carrier frequency each of which is multiplied by either the sine or cosine of the photodetected signal phase. It is then relatively simple to determine the dynamic displacement by a processing the two multiplier outputs using filtering, differentiation, and cross-multiplication and integration. A significant disadvantage of classic SH interferometry is that system parameters, which can significantly drift with time, such as the mean optical path difference, laser source wavelength sensitivity with respect to drive current, received optical power and interferometer visibility must be known. Our recently developed self-correcting SH methods obviate the need for prior calibration [7] and, by the use of phase and gain feedback loops, the requirement for a-priori knowledge of system parameters [8]. In this paper we describe the application of the latter system, having a measurement bandwidth extending from approximately 200 Hz to 9 kHz, to measure the impulse response of a Piezoelectric Mirror Shifter (PMS). The impulse response was used to determine an expression for the PMS transfer function in the s-domain, from which the PMS resonances and characteristic modes and frequency domain transfer function can be determined. Knowing the transfer function of PMS based nanopositioners is required for example in the design of closed-loop phase-locked interferometers for wideband sensing [9].

2. Theory and implementation

A schematic of the experimental setup and the optical subsystem are shown in Fig. 1 and Fig. 2(a) respectively. The light source used as the input to the Michelson interferometer is a 658 nm single-mode laser diode (Ondax TO-658-PLR35) having an output power of 20 mW at a bias current of 60 mA. The OUT is a PMS (Piezomechanik STr-35 having an unloaded resonance frequency of 10 kHz) consisting of a mirror mounted (Piezomechanik ST35 1” mirror support, which is a flat screw on cap, to which the mirror is inserted) on a Piezoelectric Transducer (PZT). The PZT is itself affixed to a mounting structure to absorb shocks [Fig. 2(a)]. The actuator is driven, via a 1 m long connecting 50 Ω coaxial cable, by a High-Voltage Amplifier (HVA, Thorlabs HVA200) that has a flat voltage gain response from DC to 1 MHz. The reflected reference and test lightwaves are combined and photodetected (Thorlabs photodiode SM05PD2A). The resulting photocurrent is amplified by a Transimpedance Amplifier (TA, Stanford SR570), which includes a 100 kHz 3 dB bandwidth anti-aliasing filter. Within this bandwidth the optical receiver (photodiode-TA) transfer function has constant magnitude and linear phase (constant group delay). The amplifier output voltage is acquired by a 16-bit, 250 kHz sampling rate data acquisition (DAQ) card (National Instruments USB-6211) and processed in real-time on a computer using LabView based software; the interface of which is shown in Fig. 2(b). The software also implements the real-time control systems described below. The DAQ is also used to apply the laser bias and modulation currents via a DC-50 kHz laser diode controller (Thorlabs IP500) and to provide test signals to the PMS HVA.

Small-signal modulation of the laser current leads to a linearly proportional modulation of the optical frequency, which can produce a significant interferometric phase modulation, depending on the difference in the optical paths between the interferometer arms [6]. The phase and gain feedback SH system uses a modulation current consisting of the sum of high frequency \( \frac{\omega_0}{2\pi} = 20 \text{ kHz} \) and low frequency \( \frac{\omega_1}{2\pi} = 20 \text{ Hz} \) sine waves. The former limits the theoretical maximum detectable frequency to 10 kHz although the digital filters used in the signal processing system limits it to approximately 9 kHz. The theoretical voltage signal \( v(t) = A + AV \cos \left[ C_0 \cos(2\pi f_0 t) + \varphi(t) + \theta(t) \right] \) at the DAQ input, ignoring ambient phase shifts,
Fig. 1. Schematic of the experimental setup.

Fig. 2. (a) Optical configuration. (b) LabView based control and signal processing software, which includes allows adjustment of laser modulation and control algorithm settings as well as carrying out signal monitoring and processing.

can be expanded in terms of Bessel functions as [8],

\[
v(t) = A \left\{ 1 + V J_0(C_0) \cos[C_1 \cos(\omega_1 t) + \phi(t)] + 2V \sum_{k=1}^{\infty} (-1)^k G_{2k}(C_0) \cos[2k \omega_0 t + \psi_{2k}] \\
\times \cos[C_1 \cos(\omega_1 t) + \phi(t)] - 2V \sum_{k=0}^{\infty} (-1)^k G_{2k+1}(C_0) \cos[(2k + 1) \omega_0 t + \psi_{2k+1}] \right\} \\
\times \sin[C_1 \cos(\omega_1 t) + \phi(t)]
\]

(1)

where \(A\) is the average voltage and \(V\) the interferometer visibility. \(\phi(t) = 4\pi x(t)/\lambda\) is the dynamic phase difference between the reference and test lightwaves, where \(x(t)\) is the OUT dynamic displacement in the same plane as the test beam and \(\lambda\) the source wavelength. In this paper \(x(t)\) increases in the positive sense such as to decrease the interferometer path difference. \(J_m(C_0)\) is a Bessel function of the first kind of order \(m\) with argument \(C_0\). \(C_0\) and \(C_1\) are equal to the amplitudes of the high and low frequency modulation currents multiplied by \((4\pi L/c)dv/di\), where \(dv/di\) is the laser optical frequency sensitivity with respect to current (\(\sim 1.5\) GHz/mA as measured using a 0.06 nm resolution optical spectrum analyzer) and \(L\) the mean path difference.
GM and ψm are the optical receiver voltage gain relative to DC and the induced phase shift at integer multiples m of ω0, respectively. The low frequency modulation current is set to a value such that C1 is ≥ π, so the peak-to-peak value of v(t) is always equal to 2AV, which can then be measured from its long-term average. The modulation current amplitudes to reach the necessary C0 and C1 are small (typically < 5 mA) compared to the bias current. The associated amplitude modulation induced variations in laser power did not have an appreciable effect on the performance of the demodulation scheme.

The objective of the improved SH method is to process v(t) so as to obtain a vibration signal that does not require a-priori knowledge of system parameters and which, is independent of variations in the optical receiver frequency response. A flow-chart of the signal processing algorithm, including the gain and phase control loops is shown in Fig. 3.

The terms in (1) centered at ω0 and 2ω0 are selected by multiplying v(t) by in-phase and quadrature local oscillators at the same frequencies but having controllable phases Γ1 and Γ2 respectively, and low-pass filtering (linear phase finite impulse response filters), to give signals,

\[
y_1(t) = -AVG_1J_1(C_0) \sin[\phi(t) + \Psi(t)] \cos(\psi_1 - \Gamma_1)
\]

\[
y_2(t) = -AVG_2J_2(C_0) \cos[\phi(t) + \Psi(t)] \cos(\psi_2 - \Gamma_2)
\]

\[
y_3(t) = AVG_1J_1(C_0) \sin[\phi(t) + \Psi(t)] \sin(\psi_1 - \Gamma_1)
\]

\[
y_4(t) = AVG_2J_2(C_0) \cos[\phi(t) + \Psi(t)] \sin(\psi_2 - \Gamma_2)
\]

, where \(Ψ(t) = C_1 \cos(ω_1 t)\). The phase error signal \(e_P(t) = (ψ_1 - Γ_1) - (ψ_2 - Γ_2)\) is equal to \(\tan^{-1}\left(\frac{y_2y_3 - y_4^2}{y_1y_2 + y_3y_4}\right)\). Because the optical receiver has a linear phase response the
ratio $K$ defined as $K = \psi_2/\psi_1$ is equal to 2. By setting $\Gamma_2 = K\Gamma_1$, then $e_p(t) = (K - 1)(\Gamma_1 - \psi_1)$. A Proportional-Integral (PI) controller was used to adjust $\Gamma_1$ such that $e_p = 0$; then calculating $y_3(t) = (y_1y_2 - y_2y_1)/(y_1^2 + y_2^2)$ gives,

$$y_3(t) = \frac{G_1G_2J_1(C_0)J_2(C_0)\left[\phi(t) + \Psi(t)\right]}{G_1^2J_1^2(C_0)\sin^2[\phi(t) + \Psi(t)] + G_2^2J_2^2(C_0)\cos^2[\phi(t) + \Psi(t)]}$$  \hspace{1cm} (6)

$y_3(t)$ is equal to $\dot{\phi}(t) + \dot{\Psi}(t)$ at a particular value of $C_0$ such that $G_1J_1(C_0) = G_2J_2(C_0)$. When observed over a time span exceeding $1/(2\pi \omega_1)$, $y_1(t)$ and $y_2(t)$ have maximum values of $AVG_1J_1(C_0)$ and $AVG_2J_2(C_0)$ respectively. The gain error $e_G(t) = y_{1\text{max}} - y_{2\text{max}}$ is minimized by adjusting the amplitude of the modulation current at $\omega_0$ using a PI loop. When $e_G(t) = 0$, $y_3(t) = \dot{\psi}(t) + \dot{\phi}(t)$, from which $\dot{\phi}(t)$ and thereby $\dot{x}(t)$ are obtained by highpass filtering $y_3(t)$ to eliminate the slowly varying $\dot{\Psi}(t)$. $x(t)$ is calculated by integrating $\dot{x}(t)$ and then multiplying by $\lambda/4\pi$. The accuracy and stability of the system for sinusoidal vibration signals was demonstrated in our previous work; in particular, the experimental results were in excellent agreement with measurements taken using two independent industry standard calibration methods [10,11].

3. PMS impulse response and transfer function

In order to measure the PMS impulse response, a 200 Hz repetition rate low amplitude 50 $\mu$s full-width at half-maximum (FWHM) low-amplitude voltage pulse $p(t)$ stream was applied to the PMS. Normalized plots of $p(t)$ and the resulting dynamic displacement impulse response $h_p(t)$ and their amplitude spectrums, calculated using the fast Fourier transform, are shown in Fig. 4. The amplitude spectrum has peaks at 6.24 and approximately 6.8 kHz. The presence of two peaks indicates that the PMS dynamics are similar to a two-mass underdamped spring system, the masses being the piezoelectric actuator and the mounted mirror since the latter is not perfectly attached to the former. Typically, the associated mechanical resonances are due to longitudinal compression waves of the mounting structure and shearing modes that cause drumhead-like vibrations of the mounting face [12]. In [13] the mounted mirror dynamics are attributed to small fluctuations in the reflected lightwave caused by small rotations of the mirror.

![Fig. 4.](image_url) (a) Applied impulse (inset) and response waveforms. (b) Amplitude spectrums.

The PMS transfer function $H(s)$, defined as the ratio of the Laplace transforms of the displacement to applied voltage, was determined using the simplified refined instrument variable method (MATLAB function `tfest`) [14]. The method estimates, for a specified number of zeros and poles, the time-continuous transfer function required to minimize the normalized mean squared error between the predicted and measured $h_p(t)$ time-domain responses of $H(s)$ to the
input stimulus \( p(t) \). It was found that the 5-th order linear transfer function,

\[
H(s) = \frac{1037}{s^5 + 21s^4 + 3.5 \times 10^3s^3 + 6 \times 10^4s^2 + 3 \times 10^6s + 4.2 \times 10^7} \text{ nm/V,}
\]  

(7)

where \( s \) has units of ms\(^{-1}\), gave good agreement (84% normalized root mean square goodness of fit), as shown in Fig. 5, between \( h_x(t) \) and the theoretical response \( L^{-1}[H(s)P(s)] \) to \( p(t) \), where \( L^{-1} \) denotes the inverse Laplace transform. Increasing the order of \( H(s) \) did not result in a significant improvement in the goodness of fit. The divergence between the theoretical and experimental responses is particularly noticeable for a short duration around 0.9 ms. This may be an indication that the PMS has some non-linearity, attributed in [15] to hysteresis effects, for which it can be challenging to develop an accurate model. \( h_x(t) \) is not exactly the same as the actual impulse response \( h(t) = L^{-1}[H(s)] \). This is because \( p(t) \) does not have a flat amplitude spectrum over the vibration frequency range of interest as shown in Fig. 4(b), having the effect of emphasizing lower frequencies.

![Fig. 5. Comparison between measured and transfer function model responses to the experimental impulse.](image)

The pole-zero plot of \( H(s) \) is shown in Fig. 6, from which it can be seen that \( H(s) \) has two pairs of complex conjugate poles and a single real pole. The complex pole pairs have natural (resonant) frequencies of 6.29 kHz and 6.79 kHz and damping coefficients of 0.02 and 0.055 respectively, with associated damped frequencies of 6.22 kHz and 6.60 kHz and decay time constants of 1.26 ms and 0.43 ms respectively. The single real pole has a decay time constant of 0.067 ms. There is a single real zero and a pair of complex conjugate zeros having an anti-resonant frequency of 6.38 kHz, between the two resonances. The presence of a single anti-resonance between two adjacent resonances is typically characteristic of a two degree-of-freedom system. \( H(s) \) can be expressed, in terms of the linear and quadratic factors corresponding to the real and complex conjugate poles and zeros, as

\[
H(s) = \frac{(s + 130)(s^2 + 1.6s + 1560)}{(s + 14.8)(s^2 + 1.6s + 1560)(s^2 + 4.6s + 1820)} \text{ nm/V,}
\]  

(8)

which in turn can be expressed as a partial fraction expansion \( H(s) = H_1(s) + H_2(s) + H_3(s) \) where,

\[
H_1(s) = \frac{-14.6(s + 11.6)}{s^2 + 1.6s + 1560}
\]  

(9)

\[
H_2(s) = \frac{-47.6(s - 39.6)}{s^2 + 4.6s + 1820}
\]  

(10)
\[ H_3(s) = \frac{62.2}{s - 14.8}, \]  

(11)
each having units of nm/V. \( H_1(s) \) and \( H_2(s) \) correspond to the 6.29 kHz and 6.79 kHz complex pole pairs respectively and \( H_3(s) \) corresponds to the real pole. The inverse Laplace transforms \( h_1(t) \), \( h_2(t) \) and \( h_3(t) \) (time-domain response to a delta function impulse) of \( H_1(s) \), \( H_2(s) \) and \( H_3(s) \) respectively constitute the modes of the system. The relative importance of these modes are shown in the normalized time-domain plots of Fig. 7. In the initial stages of the impulse response all three constituent responses are significant. The effect of the real pole is negligible after a few 10-ths of milliseconds, after which the response is dominated by the complex pole pairs. The lower frequency pole pair has less effect initially but because of its significantly longer time constant its envelope is equal to that of the higher frequency pole pair after approximately 1 ms and thereafter dominates the overall response. |\( H_3(f) \)|

![Fig. 6. PMS transfer function pole-zero diagram.](image)

The PMS amplitude response |\( H(f) \)| and phase response \( \angle H(f) \), determined by letting \( s = j 2\pi f \) in (7) where \( f \) is in kHz, are shown in Fig. 8 as well as are the relative contributions of the constituent amplitude responses |\( H_1(f) \)|, |\( H_2(f) \)| and |\( H_3(f) \)|. |\( H_1(f) \)| and |\( H_2(f) \)| have FWHMs of 0.44 kHz and 1.28 kHz respectively. In contrast to the measured impulse response amplitude spectrum of Fig. 4(b), the higher frequency peak (6.8 kHz) of |\( H(f) \)| is slightly more pronounced than the lower frequency peak (6.2 kHz). At frequencies less than 4 kHz, the relatively flat PMS amplitude response is dominated by and the lower frequency tail of |\( H_2(f) \)|, which has a significantly larger FWHM compared to |\( H_1(f) \)|. From approximately 4 kHz to 8 kHz, the resonant characteristics of |\( H_1(f) \)| and |\( H_2(f) \)| are more pronounced and have significant overlap, which along with |\( H_3(f) \)| leads to the double peaked structure of |\( H(f) \)|. At frequencies greater than approximately 8 kHz, |\( H(f) \)| monotonically decreases as \( f \) increases. MATLAB code and data files corresponding to Figs. 4 to 8 and the transfer function estimation are given in Code 1 [16] and Code 2 [17].
4. Conclusions

A novel feedback based synthetic-heterodyne Michelson interferometer based system, was used to measure the impulse response of a commercially available PMS. The use of two independent control loops results in significant improvements in the stability and accuracy of the detected dynamic displacement and velocity. The complex control and associated signal processing algorithms were implemented in real-time on a computer. The experimental applied impulse was not an ideal impulse and as such its amplitude spectrum monotonically decreases with frequency; thereby the PMS transfer function cannot be determined by simply taking the inverse Laplace transform of \( H(f) \). However, its amplitude spectrum does contain energy at all frequencies within the of vibration frequency range of interest. Although the system measurement bandwidth is limited to approximately 9 kHz, this could be greatly increased by the use of a higher bandwidth laser diode controller and higher sampling rate DAQ. It should also be possible to implement all the real-time control and signal processing on a dedicated processor, which we may investigate in future work. In a manner similar to that carried out in [6], the effect of noise on system sensitivity may also be investigated in future work; important sources of noise
include the optical receiver, DAQ quantization noise and possibly laser phase noise. Given the complexity of the demodulation process such an analysis would be challenging. It would be of interest to determine how the PMS transfer function is related to the PMS structure and how it could be modeled as a coupled damped spring system.

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