Neural networks based post-equalization in coherent optical systems: regression versus classification

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Abstract—In this paper, we address the question of which type of predictive modeling, classification, or regression, fits better the task of equalization using neural networks (NN) based post-processing in coherent optical communication, where the transmission channel is nonlinear and dispersive. For the first time, we presented some possible drawbacks in using each type of predictive task in a machine learning context for the nonlinear channel equalization problem. We studied two types of equalizers based on the feed-forward and recurrent neural networks over several different transmission scenarios, in linear and nonlinear regimes of the optical channel. We observed in all those cases that the training based on regression results in faster convergence and finally a superior performance, in terms of Q-factor and achievable information rate.

Index Terms—Neural networks, nonlinear equalizer, classification, regression, coherent detection, digital signal processing, optical communications.

I. INTRODUCTION

TO improve the performance of optical fiber systems it is important to mitigate the detrimental impact of linear and, most importantly, nonlinear transmission impairments thatcape the systems’ throughput [1]–[3]. Numerous digital signal processing (DSP) algorithms have been proposed and studied [3] for optical fiber channel equalization. One of the most intuitive and simple concepts in this direction is the receiver-based equalizer – a special-purpose DSP device that can (partially) reverse the distortions incurred by a signal when passing over the optical channel. A widespread approach in the channel post-equalization deals with the low complexity devices based on the minimal-mean-squared-error (MMSE) criteria. The usefulness of MMSE equalizers is stipulated by several reasons, including (i) the mean squared error minimization is an optimal condition for the transmission over the additive white Gaussian noise (AWGN) channel; (ii) the MMSE is quite convenient for mathematical optimization because of convexity and differentiability; and (iii) the MMSE equalizer is independent of the underlying waveform or modulation format.

Turning to the problem of coherent discrete constellations (quadrature amplitude modulation, QAM) equalization, it is quite popular there to design the equalizer as a classifier [4], [5]. The advantages attributed to the classification choice are: (i) the classifier is optimized for the specific modulation format used; (ii) it directly maximizes the information rate, the main objective of the channel equalization, and outputs the likelihoods for each received symbol; (iii) even more importantly, it can adapt itself to the correct statistical channel characteristics.

Over the several past years, the “conventional equalizers” have started to evolve toward the designs incorporating machine learning techniques [6]–[9]. More specifically, the artificial neural network (NN) based channel equalization has recently become a topic of intensive research, due to NNs capability to render a high performance in improving the quality of transmission in optical communication systems being, simultaneously, a flexible channel-agnostic technique [5], [9]–[18]. However, at this point, we note that the transfer of the methods developed in the field of machine learning to optical communications should take into account the underlying peculiarities and challenges of NN algorithms themselves, while those are often overlooked when designing the equalizers. In particular, we emphasize that the question of whether to utilize regression or classification for the design of optical channel NN-based post-equalizers has not been addressed whatsoever, being rather a matter of improvised choice or intuition. At the same time, we underline that this choice can be quite important for the derivation of the most efficient equalizers’ design.

Both the classification and regression can be used in data post-processing to improve the system’s performance. However, a fair comparison between these two seems to be somewhat problematic as they produce the different output variable types: discrete versus continuous, respectively. The discussion related to the classification vs. regression comparison, exists in the machine learning research where the logistic regression accuracy is compared to that rendered by the classification [19], [20], or the output of the classification with the different loss functions is analyzed [21]–[23]. However, to our knowledge, only in Ref. [24] the motivation of solving problems with regression instead of classification was directly explained for specific problems. In Refs. [25]–[30] it was recognized that both regression and classification tasks have potential downsides: the regression model cannot utilize the flexibility of discriminative NN models; the classification model, in turn, is not able to capture the value of difference when we misinterpret the classes, which can degrade the modeling quality. Thus, in these works, the combination of the regression and classification was proposed as the best problem fit. This approach was coined the joint classification-regression learning.
The interest in finding the optimum solution between the regression- and classification-based equalization-type algorithms, frequently arising in different areas, prompts us to conduct the investigation of this dilemma for the development of NN-based equalizers in coherent optical systems. We stress that even in the fields where machine learning applications are well established, no general conclusion on the classification versus regression issue has been reached. For the optical channel equalization, this comparison may be made more evident by contrasting the classification output to that obtained with the regression in terms of bit error rate (BER) (i.e. using a hard decision metric). To get even a fairer comparison, we also check the regression and classification results using the value of achieved mutual information (MI), which is a metric that is independent of the decision method at the receiver DSP. In this paper, we compare the performance of classification and regression predictive models and expose the potential drawbacks of each task for the NN-based optical channel equalization that can explain our findings quantitatively. As an additional motivation of our work, we mention that in the end-to-end learning that deals with coherent constellation optimization, the auto-encoder is traditionally trained as a classifier since the bits there are used as inputs/outputs. However, if the training of a classification-based system turns out to have some deficiencies when compared to the regression-based one, it indicates that we may have to reconsider the respective auto-encoder training procedure.

II. NEURAL NETWORK-BASED EQUALIZERS

A. Designing Regression-based and Classification-based Equalizers

Classification predictive modeling is the task of approximating a data mapping function, $f$, from input variables, $Y$, to discrete output variables, $X$. In the case considered, $f$ is the inverse of the transfer function of a coherent optical channel. Taking the probability of $X$ being the output of the NN, this probability can be interpreted as the posterior of a given example $Y$ belonging to each class in a discrete space of $X: P(X|Y)$. The predicted probability can be converted into a class value by selecting the class label that has the highest probability. The approximation of a mapping function $f$ from input variables $Y$ to a continuous output variable $X$ is known as regression predictive modeling. From the optical communication perspective, the classification output is measured in terms of accuracy, which is similar to the symbol error rate (SER) metric, while the regression evaluates the efficiency using the mean squared error (MSE), which is similar to the concept of signal-to-noise ratio (SNR) or, even more precisely, to the error vector magnitude (EVM) metric.

Another important and more universal (decision unrelated) metric qualifying the transmission system performance is the input-output MI, $I(X;Y)$. For the classification task, the MI is directly proportional to the loss function used in the training, called the cross-entropy loss (CEL): these two quantities both measure the entropy of the transmission. For the classification, since it is the case of jointly discrete random variables, the MI metric can be explicitly written through the continuous input $Y$ and discrete output $X$ as

$$I(X;Y) = H(X) - H(X|Y) = \log_2(MF) - CEL(Y, X),$$

where $H(X)$ is the marginal entropy, $H(X|Y)$ is the conditional entropy, and $MF$ is the modulation format order (constellation cardinality) of the transmission, defining how many distinct symbol classes we transmit.

In the case of regression, the MI cannot be expressed as directly as in the previous case because now we deal with the soft symbols themselves but not with their distributions. We can, however, estimate the approximate value of $I(X;Y)$ by assuming a single-input single-output AWGN channel, which gives us a sub-optimal value and indicates the MI’s lower bound. So the lower bounds to $I(X;Y)$ can be expressed as [37]–[39]:

$$I(X;Y) = \mathbb{E} \left[ \log_2 \left( \frac{p(y|x_k)}{\sum_{i=0}^{MF-1} p(i)p(y|x_k)} \right) \right],$$

where $p(i)$ represents the probability distribution of each $k$-th QAM alphabet symbol, and $p(y|x_k)$ defines the conditional probability of the received constellations given the $k$-th QAM input symbol. We used a multivariate Gaussian distribution estimator [40], [41] to estimate $p(y|x_k)$ with the transmitted-received complex symbols, which will result in some underestimation of the final MI values. The summary of this section’s content is given in Table I.

| NN Task | Input | Output | Loss | Metric | NN | MI |
|---------|-------|--------|------|--------|----|----|
| Regres. | Received symbols | Recovered symbol | MSE | SNR | Under Estimated |
| Class.  | Received symbols | Probab. of Classes | CEL | SER | Measured |

B. Drawbacks of Regression and Classification in the Optical Channel Nonlinearity Mitigation Task

We notice that any empirical dataset’s posterior probability can be learned by a perfectly trained NN classifier with a sufficient number of degrees of freedom [42]. This explains the choice of the classifier loss function, the CEL, that is used to train the NN: when the NN output layer contains a “sigmoid” or “softmax” nonlinearity, and the goal is to maximize the likelihood of properly classifying the input data, the CEL is the most appropriate choice from a probabilistic standpoint [42]. On the other hand, because the regression model training procedure employs the MSE loss function with a linear output layer, in the regression task we assume that the target is continuous and normally distributed. In other words, the MSE can be seen as the cross-entropy between the distribution of the model’s prediction and the distribution of the target variables within the framework of a maximum likelihood estimation and under the assumption of Gaussian target variable distribution.

At this point, it may seem intuitive to argue that the classification is better than regression for the optical channel equalization because the regression task can be reformulated

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as a classification task by discretizing the continuous output space to obtain class labels, and in the resulting classification task, we use the multi-class loss (the categorical CEL) with no prior Gaussian assumptions on the outputs [24]. However, as it was mentioned previously, learning the posterior probabilities requires the “perfect training”. As a result, if we have difficulties in achieving that “perfect training” for our NN-based equalization, the classification strategy can have severe drawbacks. Thus, both regression and classification algorithms have some flaws, and we need to figure out which particular problem impacts most of the performance of NN-based channel equalizers.

For the regression, in terms of optical channel equalization, the MSE loss function can be treated as a simplification of the true likelihood measurement: it fails to capture all the information in the distorted sequences if the noise distribution is signal-dependent or non-Gaussian [43]. Furthermore, from a qualitative point of view, consider the following first-order perturbative solution of the channel model [44]:

\[ y_k = x_k + \varepsilon \sum_{m,n=-M}^{M} C_{m,n} x_{k+m} x_{k+n} + \eta(\vec{x}), \quad (3) \]

where \( y_k \) is the received symbol in the \( k \)-th time slot, \( x_k \) is the transmitted symbol in the \( k \)-th time slot; \( \varepsilon \) is equal to the ratio of the dispersion length to the nonlinearity length; \( C \) is the coupling matrix that governs the signal-signal interactions, particularly those which are responsible for the non-circular distribution of the distortion; “\( * \)” means the complex conjugate; \( M \) is the channel memory size, \( \eta(\vec{x}) \) is a random function that models the noise-signal interaction and another additional signal-independent noise. It is clear that we can divide the expression in Eq. (3) into deterministic and stochastic parts. With this in mind, the assumption that makes the MSE equivalent to the CEL means that \( \eta(\vec{x}) \) is a normal Gaussian distribution with the variance independent of \( \vec{x} \). In practical terms, by considering the MSE, we try to recover the deterministic part assuming that the stochastic part is a Gaussian distribution with signal-independent variance, which is reasonable since the noise-signal interaction is generally much smaller than the transmitter-induced and additive optical amplifier signal-independent noises that normally can be approximated as a Gaussian process [45].

For the classification, we can highlight two main drawbacks that come from machine learning-related issues. First, in the CEL, the confusion between the two classes (i.e. between the two constellation points, for our problem) has the same “cost” value regardless of the corresponding inaccuracy in the target space: the penalization ignores the spatial proximity of the labels, reflecting just the very fact that the constellation point has been misclassified. In the traditional classification task, each class represents a distinct concept that can be identified using a NN, e.g. we classify different types of coordinate objects. However, when it comes to the optical equalization problem, every class bears more information than just a label. As shown in Fig. 1, different classes, which correspond to distinct point positions in the constellation (in this case – in the 16-QAM), have individual nonlinear distortion levels. These levels provide additional information about the nature of each label. From the machine learning perspective, we can say that the labels of the target variable exhibit a natural ordering [46]. To better understand this question, consider an explanatory representative in which the class labels are imbued with order information, such as four classes: “healthy”, “slightly ill”, “moderately ill”, and “very ill”, that state the conditions of patients in a specific database. It is important to remember that misclassifying a “very ill” patient as “healthy” must cost more than misclassifying a “very ill” patient as “moderately ill”, because the impact of the former confusion is considerably more dangerous for the patient. Turning now to our subject, i.e. to classifying the symbols in Fig. 1 the misclassification between “A” and “B” should cost less than the misclassification between “A” and “C”: for the first case, the two constellation clouds would naturally share some symbols due to the noise-induced clouds’ spreading and overlap as “A” and “B” share the same decision boundary, which, obviously, is not true for the “A” and “C” classes.

The value of the gradients arising in the training process with the CEL is the second classification disadvantage. According to Ref. [23], the CEL surfaces have less local minima than the square error-based losses (SEL), to which the MSE loss belongs. However, the CEL exhibits stronger gradients than the SEL, which leads to overfitting in the systems trained with the CEL, resulting in SEL’s having a better generalization property in almost all the cases tested in [23]. This behavior was explained there assuming that the CEL loss surface is more prone to sharp minima (narrow valleys) than the SEL’s surface; thus, the CEL-based systems experience overfitting much easier. For our problem, we have observed the same pattern, when the MSE-based system generalized better compared to the case of using the categorical CEL, the
phenomenon occurring due to the overfitting. 
In this paper, for the first time, we demonstrate how the aforementioned two classification drawbacks impact the performance of classification equalizers when compared with regression-based ones in the coherent optical channel equalization. We show that the machine learning-related deficiencies overpower the other classification benefits.

III. SIMULATING SIGNAL PROPAGATION IN COHERENT OPTICAL TRANSMISSION SYSTEMS

To illustrate the effects addressed in our work, we numerically simulated the dual-polarization (DP) transmission of a single-channel signal at a 34.4 Gbd rate. First, a bit sequence was generated using the Mersenne twister generator [59], which has the periodicity equal to \(2^{19937} - 1\). Then, the signal is pre-shaped with a root-raised cosine (RRC) filter with 0.1 roll-off at an upsampling rate of 8 samples per symbol. In addition, the signal could have four possible modulation formats: 16 / 32 / 64-QAM. To cover different physical scenarios, we considered the following two test cases: (i) the transmission over the optical link consisting of 9×50 km true-wave classic (TWC) spans; and (ii) transmission over 5×100 km of standard single-mode fiber (SSMF) spans. The optical signal propagation along the fiber was simulated by solving the Manakov equation via split-step Fourier method [50] with the resolution of 1 km per step. The parameters of the TWC fiber are: the attenuation parameter \(\alpha = 0.23\) dB/km, the dispersion coefficient \(D = 2.8\) ps/(nm·km), and the effective nonlinearity coefficient \(\gamma = 2.5\) (W·km)\(^{-1}\). The SSMF parameters are: \(\alpha = 0.2\) dB/km, \(D = 17\) ps/(nm·km), and \(\gamma = 1.2\) (W·km)\(^{-1}\). The purpose of testing two different fibers is to see if the regression task works better in the SSMF transmission because, due to the dispersion, the constellation points distributions, in that case, would be closer to Gaussian; for the TWC we have 6 times lower dispersion and 2 times higher nonlinearity, such that the non-Gaussianity would be more pronounced.

In our model, every span is followed by an optical amplifier with the noise figure \(NF = 4.5\) dB, which fully compensates for the fiber losses and adds the ASE noise. At the receiver, a standard Rx-DSP was used. It includes the full electronic chromatic dispersion compensation (CDC) using a frequency-domain equalizer, the application of a matched filter, and downsampling to the symbol rate. Finally, the received symbols were normalized (by phase and amplitude) to the transmitted ones. After the Rx-DSP, the output symbols were processed by an NN-based equalizer for further signal enhancement. Fig. 2 shows all the blocks involved in the transmission simulations, where we highlight regression/classification-based NN equalizers with red boxes. Besides the MI, another performance metric used in this paper is the Q-factor expressed through BER after the hard decision as:

\[
Q = 20 \log_{10} \left[ \sqrt{2} \text{erfc}^{-1}(2BER) \right],
\]

where \(\text{erfc}^{-1}\) is the inverse complementary error function. Note that the hard-decision block is optional: it is used for the Q-factor computation, but redundant when we deal with the MI.

The NN input mini-batch shape, for both regression and classification tasks, can be defined by three dimensions [9]: \((B, M, 4)\), where \(B\) is the mini-batch size, \(M\) is the memory size defined through the number of neighbors \(N\) as \(M = 2N + 1\), and 4 is the number of features for each symbol, referring to the real and imaginary parts of two polarization components. For the regression, the output target is to recover the real and imaginary parts of the \(k\)-th symbol in one of the polarization, so the shape of the NN output batch can be expressed as \((B, 2)\). In the case of classification, the output will provide the vector probability of a received symbol to belong to a certain class, and so the output batch shape is equal to \((B, MF)\). Finally, we note that the different random seeds were used to produce both the training and testing datasets to ensure their independence and avoid overestimation, with the cross-correlation not exceeding 0.02.

IV. COMPARISON OF EQUALIZERS BASED ON REGRESSION OR CLASSIFICATION

A. Fairness of Comparison

Before moving on to the results section, we discuss how we make the comparison of regression and classification as fair as possible. First, we use two types of equalizers: the first is based on the feed-forward multi-layer perceptron (MLP) with three hidden layers, while the second one is the recurrent

![Fig. 2: The schematic of the setup used in our simulations. The two available equalization types (classification and regression) are inserted at the receiver side after the matching filter and the DSP blocks: CDC and Phase/Amplitude Normalization. The equalizers are highlighted with a red box.](image)

1Yet another drawback of classification-based systems is that the discretization method must be tailored to each task, i.e., it must have the specified fixed number of outputs corresponding to the constellation’s cardinality. This indicates that the classifier model’s operation is specific to the modulation format on which it was trained. In other words, the practical (say, hardware) classifier implementation would limit the respective device’s application to just a specific format. But in the case of regression, as demonstrated in Refs. [17], [38], we do not need to retrain the model at all for it to work on other modulation formats. Because of this, the regression model is far more versatile than the classification model, allowing us to use it in situations other than those in which it was trained.

2The fine step resolution guarantees that we truly model the optical channel properties captured by the NN and not address some by-side simulation effects.
structure, consisting of one layer of bidirectional Long short-term memory (biLSTM). The comparison of these equalizers’ complexity and functioning is given in Ref. [9]. These two cases are taken to demonstrate that our outcomes are true for different NN architectures, but we note that the biLSTM layer has demonstrated better performance in previous studies [9]. The only difference between using each architecture for regression or classification tasks is the structure of the output layer, as shown in Fig 3 and the loss function type. In the case of regression, the output layer has two linear neurons referring to the real and imaginary parts of the recovered symbol, and the loss function used is the MSE. For the classification, the number of neurons in the output layer is determined by the modulation format cardinality, and the NN structure ends with the softmax layer, while the loss function is the categorical CEL. We point out once more that besides these two differences, the regression and classifier models that we compare, share the same number of inputs, hidden layers, neurons in each layer, and hyperparameter values; the training/test datasets are also the same. As for memory size, for both prediction types, we used the same memory: \( M = 51 \) for the SSMF case and \( M = 41 \) for the TWC case. The summary of the NN parameters used in this study is presented in Table II.

The comparison of the regression- and classification-based (but otherwise equivalent) systems’ performance for different modulation format orders is depicted in Fig. 4. From the results of Fig. 4(a) and (c), one can see the impact on the classifier’s performance when increasing the number of classes in the problem (i.e. increasing the modulation format order), in terms of the Q-factor for 6 and 10 dBm respectively.

For the biLSTM equalizer, the percentage of how greater the Q-factor is after the regression equalization compared to the classification one was roughly 8%, 31%, and 31% for the 16-QAM, 32-QAM, and 64-QAM scenarios, respectively, for the 6 dBm test case, Fig. 4(a), and 22%, 29%, and 50% for the 16-QAM, 32-QAM, and 64-QAM scenarios, respectively, for the 10 dBm test case, Fig. 4(c). For the MLP equalizer, as compared to the classification output in 16-QAM, 32-QAM, and 64-QAM scenarios, the regression equalization always delivered better results, yielding 14%, 19%, and 15% Q-factor improvement, respectively for the 6 dBm test case, Fig. 4(a), and 7%, 11%, and 19% Q-factor improvement, respectively, for the 10 dBm test case, Fig. 4(c). When using the biLSTM equalizers, we can observe a greater difference between the regression and classification for different modulation formats, because the biLSTM equalizers, on average, perform much better than the MLP equalizers [9, 51]. So, in the biLSTM case, we can see better how much the classification loss function gets degraded by ignoring the difference between distinct miss-classification occurrences.

| Equalizer | Mini-Batch | \( N_h \) | \( N_1 / N_2 / N_3 \) | Training / Testing Dataset size |
|-----------|------------|----------|-----------------|-------------------------------|
| Recurrent | 4331       | 226      | -               | \( 10^{18} / 10^{18} \)         |
| MLP       | 4331       | -        | 481 / 31 / 263  | \( 10^{18} / 10^{18} \)         |

Fig. 3: Schemes of different NN architectures considered in this paper. At the top, we show the regression and classification systems based on the recurrent equalizer with \( N_h \) hidden units. At the bottom, we also show both tasks implemented with the MLP equalizer having three hidden layers, with \( N_1 \), \( N_2 \), and \( N_3 \) neurons in each consecutive layer, respectively. In all cases, the output is marked with red to highlight the difference in the regression- and classification-based approaches. For our case, the activation function \( \varphi \) is “tanh”.

As we are dealing with different loss functions, we must consider that the learning rate and the number of epochs required may differ for the regression compared to the classification. To address this potential issue, we optimized the learning rate from the range \([10^{-3}, 5 \times 10^{-4}, 10^{-4}, 5 \times 10^{-5}]\), and used the early stop to get the architecture that performed the best during the training process. In general, the early stop was used if no improvement was seen after 150 epochs out of the total 5000 training epochs.
When evaluating the performances in terms of MI, almost the same behavior was observed: the results are depicted in Fig. 4(b) and (d) for 6 and 10 dBm, respectively. In the case of the biLSTM equalizer, the difference between the MI obtained through regression and classifier for 16-, 32-, and 64-QAM was approximately 0, 0.0191, and 0.1393, for 6 dBm; and 0.0815, 0.161, and 0.1733, for 10 dBm. Again, by increasing the order of the modulation format, the regression achieved better results than the classifiers.

In the case of 6 dBm with the MLP equalizer, the same tendency appeared again: the MI difference was larger when the modulation order increased. The difference between the MI for regression vs. classification was 0.00445, 0.0449, and 0.1126, for modulation formats 16-, 32-, and 64-QAM, respectively. However, when comparing the MLP equalizers’ outcomes, the classifier MI was somewhat higher than that for the regression case in two scenarios at 10 dBm: for 32- and 64-QAM cases. We believe that this happened because the MI of the regression is lower-bounded but not computed exactly, and so the MI value estimated via Eq. (2) and Gaussian approximation, is lower than the true one. This is also corroborated by the fact that when we look at the MLP Q-factor for those cases, we still observe the higher Q-factors attributed to the regression model.

Now we turn to the issue of overfitting in the classification equalizer, addressing two cases: i) the case of a low dispersion fiber (SC-DP 16QAM 34.4 Gbd over 9x50km TWC fiber), and ii) the case of a conventional SSMF fiber (SC-DP 64QAM 34.4 Gbd over 5x100km SSMF fiber). To reveal the overfitting, for both the biLSTM and MLP equalizers, we present the Q-factor/MI values for the training and test (validation) datasets. The difference between the values obtained in training and testing is the qualitative measure of the overfitting strength. In these words, the comparison of classification and regression training and testing results will reveal which approach has generalized better. Fig. 5 shows the results of our analysis where the solid green line is the Q-factor/MI after only linear equalization (regular DSP), the solid blue and red lines indicate the Q-factor/MI of the classification and regression models evaluated with the testing dataset, respectively, and the dashed blue and red lines depict the Q-factor/MI of the classification and regression models evaluated with the training dataset.

When we use the CEL in our equalizers, we see the same trend of higher overfitting level as it was observed in Ref. [23]. As can be seen in all four panels, the Q-factor curves of training and testing for the classifiers show a significant difference (since this metric gives the logarithmic measure), suggesting the presence of noticeable overfitting in
Fig. 5: Generalization study for regression and classification equalizer showing the impact of overfitting in the NN performance and training process on the following scenarios: (a,b) biLSTM analyses and (c,d) MLP analyses for SC-DP-16-QAM, 9x50km TWC fiber and 34.4GBd; (e,f) biLSTM analyses and (g,h) MLP analyses for SC-DP-64-QAM, 5x100km SSMF fiber and 34.4GBd.

The classification model.

Then, we can see that, in comparison to the classifier's result, the training and testing output curves when using the regression, behave almost identically. It means that the
regression model using MSE generalizes much better for all of our test cases (two different NN equalizers and two different transmission setups), which complies with the conclusions reached in Ref. [23]. Furthermore, we were able to see that by using regression equalizers, the Q-factor level after equalization was still higher than with the classification, due to the better generalization of the regression NNs. When we look at the MI values, we see that the classifier’s training performance was overfitted, yielding virtually the maximum MI attainable for each scenario, but in the case of regression, the training and testing curves followed the same trend, indicating a better generalization of the problem.

Finally, we notice that for high modulation formats, such as those shown in Fig. 1, we have an even more reduced classification performance, with no increase in Q-factor observed for both biLSTM and MLP equalizers in 64-QAM 5x100 SSMF transmission scenario.

C. Training Complexity

In this final subsection, we highlight the training process of all equalizers used in this paper, considering not only the problem of overfitting but also the speed of convergence for such equalizers.

First, we compare how many epochs each task required to reach the best possible result. For some selected cases discussed in our paper, Table III displays the training time for the biLSTM and MLP equalizers. When comparing the regression and classification in terms of the number of epochs required to obtain the highest Q-factor, no significant difference in training time was identified, at first. However, for roughly the same amount of training epochs, we can affirm that the regression was more successful because it delivered greater performance outputs.

Finally, we would like to note that when we lowered the learning rate, we saw a reduction in overfitting in the classification task. Fig. 6 shows the example case of 32-QAM at 10 dBm using the SSMF link, where three training and testing MI curves for the biLSTM equalizer are shown for the three learning rates: $10^{-3}$, $10^{-4}$, and $5 \times 10^{-5}$. As can be seen, for $10^{-3}$ the overfitting is much more intense than when using lower learning rates. However, even with such low learning rates as $5 \times 10^{-5}$, the performance after 5000 epochs of training, was not better than that with either the regression or compared to the best case with the classification with $10^{-3}$ learning rate. Also, the overfitting could be seen as the training ML level grew faster than the testing level. The maximum MI measured with the test dataset is shown by the black dashed line in Fig. 6. It is evident from this figure that lowering down the learning rate did not result in any considerable improvement for our test cases.

V. Conclusion

In this work, we carried out the analysis of the regression and classification predictive models’ performance, addressing specifically the functioning of NN-based post-equalizers in coherent optical lines. Our findings indicate that in all cases considered, the regression outperforms the classification in terms of equalization performance. However, we notice that though we have observed similar regression vs. classification behavior in a number of different scenarios, we would rather restrain from making too general conclusions regarding the interplay between these two predictive models. Since the drawbacks of classification come, basically, from the machine learning-related training deficiencies, some advanced regularization and/or more sophisticated problem-specific loss functions may

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**TABLE III: Summary of the main features in the Regression and Classification tasks.**

| Test Scenario | Epoch biLSTM Class. | Epoch biLSTM Regress. | Epoch MLP Class. | Epoch MLP Regress. |
|---------------|---------------------|-----------------------|-----------------|-------------------|
| SSMF 16-QAM 10dBm | 385 | 277 | 257 | 153 |
| SSMF 32-QAM 10dBm | 147 | 117 | 389 | 398 |
| SSMF 64-QAM 10dBm | 462 | 414 | 399 | 391 |
| SSMF 64-QAM 4dBm | 279 | 302 | 363 | 95 |
| TWC 16-QAM 2dBm | 102 | 87 | 152 | 114 |
potentially mitigate the strong overfitting occurring in the classification case. However, we emphasize that the scope of this work was to evaluate the performance of the conventional classifier architectures with the “traditional” loss function, the CEL.

We demonstrated that both the regression and classification tasks have certain design drawbacks. From the statistical analysis point of view, the regression loss function (the MSE) is a special case of the classification loss function (the CEL), in which the stochastic component of the output variables is assumed to be signal-independent and normally distributed. Therefore, the MSE does not take into account the signal-dependent stochastic contribution, which is, obviously, present in the true nonlinear optical channel. Nonetheless, we underline that from the machine learning methods’ application perspective, the classification loss function (the CEL) is unaware of the difference in misclassifying the labels and does not account for the output location in the target space (i.e. of the location of the equalized point on the constellation plane). As we demonstrated, the labels-aware misclassification occurrence, pertaining to the regression task, contains important information that impacts the equalization quality, while this information is lost in the classification-based equalization. Furthermore, according to recent Ref. [23], the CEL landscape typically involves very sharp local minima, which can cause the NN model to overfit, such that it typically generalizes much worse than the regression model with the loss based on the euclidean distance. To evaluate the drawbacks of the classification approach compared to the regression-based equalization, we considered optical data transmission with various modulation formats: 16-QAM, 32-QAM, and 64-QAM, using two different types of fiber, the TWC, and the SSMF. To address distinct equalization models, we analyzed two conceptually different NN equalizers: the recurrent-type biLSTM, and the feed-forward MLP. In all the cases, the regression-based equalization provided better performance, with the Q-factor gain improvement increasing for higher-order modulation. The difference between regression and classification was more pronounced for the biLSTM equalizer that, generally, renders a better equalization quality than a simple MLP. In addition, we could observe that in all cases studied, the classification training curves were still much higher than the testing curves, indicating an overfitting tendency rather than a generalization propensity. On the other hand, for the regression equalizers, the training and testing curves followed the same tendency, which points out the fact that the machine learning drawbacks related to the classification outweigh the “statistical” flaws of regression.

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