Explicit Electroweak symmetry breaking?

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Abstract

We hypothesise that all electroweak symmetry breaking terms such as fermion masses and the W and Z gauge boson masses arise radiatively from just one explicit symmetry breaking term in the Lagrangian. Our hypothesis is motivated by the lack of experimental support and the lack of predictivity of the standard symmetry breaking scenario. We construct a simple model which illustrates our ideas. We also discuss the possible importance of scale invariance.

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The standard model (SM) is a very successful model. It answers many questions and explains many experiments. However, in some aspects it seems to be unsatisfactory. In particular, the (SM) does not give any understanding of the fermion mass hierarchy. It can incorporate the empirical masses but there is no explanation of, e.g., why are $m_e, m_u << m_t$? At present there are few fundamental problems in theoretical physics in which experiment is ahead of theory. A quantitative understanding of the fermion and gauge boson masses is, we believe, one such fundamental problem. A quantitative understanding of the fermion and gauge boson masses is a difficult problem and if such an understanding could be achieved it would undoubtedly lead to a deeper insight into nature.

One of the obstacles in understanding the masses of the elementary particles is that the masses are connected with the physics of symmetry breaking (since the known fermion and gauge boson mass terms do not respect the electroweak gauge symmetry). The origin of electroweak symmetry breaking is not known at present. The SM ascribes symmetry breaking to a vacuum expectation value (VEV) of a fundamental scalar field. This scalar couples to fermions and gauge bosons and the VEV consequently gives tree-level masses to the fermions and bosons. This sector of the SM does not have support yet from experiment. The SM is simple but it is not compelling.

In this note we propose a completely different scenario for the form of symmetry breaking. We hypothesise that all gauge symmetry breaking arises radiatively from just one explicit symmetry breaking term in the Lagrangian.\[1\] We have no a priori justification for this hypothesis. We propose to study it for the following reasons. Firstly it is a simple hypothesis. Secondly, it has not been previously studied (as far as we are aware) \[2\] and most important, if the hypothesis is correct then it might be possible to make some progress in understanding the fermion and gauge boson mass hierarchy, since our hypothesis requires all fermion and gauge boson masses to appear ultimately from just one source term in the Lagrangian. Since we consider explicit symmetry breaking rather than spontaneous symmetry breaking (as in the SM) our model will be non-renormalizable. Our point of view is that the divergences will be cancelled by some new and unknown physics, possibly associated with the symmetry breaking source term. Our approach is a bottom up approach. We do not propose ‘a theory of everything’ and we do not claim to know all the answers.

What then is the one electroweak symmetry breaking term? Perhaps the simplest candidate for the symmetry breaking term is for it to be a mass term (rather then some type of interaction term). This term could be either a gauge boson mass term or a fermion mass term. If the symmetry breaking term is assumed to be a gauge boson mass term then it seems to be impossible to radiatively generate the fermion mass terms (due to chiral symmetry). Thus we argue that if there is only one symmetry breaking term in the Lagrangian, then a fermion mass term is a good candidate.

Throughout this paper, we assume that the gauge symmetry of the La-
grangian is the same as in the standard model, i.e. $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. If symmetry breaking arises from just one tree-level fermion mass term, then we expect that this fermion mass, $M$, will satisfy

$$M \gg M_W, M_Z.$$  (1)

Since the $W$ and $Z$ boson masses ($M_W, M_Z$) are by assumption assumed to arise radiatively from the fermion mass $M$. None of the known fermions (or even the top quark) satisfy this condition so we expect that the fermion mass term must be something exotic. Perhaps the simplest (or at least most obvious) possibility is to assume the existence of right-handed gauge singlet Weyl fields and consider a large Dirac tau neutrino mass term:

$$M_{\nu_D} \gg M_W, M_Z.$$  (2)

We assume a see-saw type picture where there is a very large Majorana mass for the right-handed neutrinos (so that the physical left-handed neutrino mass is small enough to be within the experimental bounds). It has already been shown [3], that this case does lead to satisfactory one-loop induced masses for the $W$ and $Z$ gauge bosons (which are proportional to $M_{\nu_D}$) and these masses automatically satisfy the relation

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_w} \simeq 1,$$  (3)

where the weak mixing angle, $\theta_w$ is defined by the ratio of $SU(2)_L$ and $U(1)_Y$ gauge coupling constants i.e. $\tan \theta_w = g/g'$. The one-loop diagrams are logarithmically divergent however, so that a cutoff is required [4]. This, unfortunately leads to a lack of predictivity, but we don’t consider it a problem for the low energy theory since the cutoff is presumed to be associated with the physics of the symmetry breaking term, or some other type of unknown physics. Despite this lack of predictivity, we think that in principle quantitative predictions can still be made since the infinities can cancel if we consider quantities such as mass ratios (for example).

Thus we argue that physics below 100 GeV can be satisfactorily explained by assuming that electroweak symmetry breaking is explicit (rather than spontaneous). At high energies of the order of a TeV, the tree-level elastic scattering cross section for longitudinal $W$ and $Z$ bosons will violate the partial wave unitarity bound. Thus, we expect the model to break down at the TeV scale.

We have not said anything yet about how the fermion masses can arise from the symmetry breaking source term. In the model as it stands, there is no mechanism for the fermions to gain masses. Of course this is a crucial issue. In this paper we focus on illustrating our idea, and (like many other authors [2]) assume the existence of elementary scalars (which do not get any VEVs) to communicate the symmetry breaking to the fermions. Once we introduce
elementary scalars, we also introduce arbitrariness as in the case of the SM. This is clearly unsatisfactory but we do not have many better ideas at the moment. Perhaps a better alternative might be to consider the possibility of a larger gauge symmetry than the SM, and to try and generate the fermion masses with loops involving gauge bosons. This possibility is interesting, but not easy since we will need to extend our hypothesis to include the additional symmetry breaking of the larger gauge group. We hope to study this possibility sometime in the future but for the present, we assume only the SM gauge symmetry, and allow for the possibility of scalars (with no VEVs) to generate the fermion masses radiatively. This will allow us to illustrate our idea.

Since the top quark mass is expected to be about the same as the $W$ and $Z$ gauge boson masses (i.e. within a factor of 2), we would also like to have this mass generated at one-loop. However assuming the SM gauge symmetry, it is impossible to generate the top quark mass from $M_{\nu_D}$ at one-loop using elementary scalars (with no VEVs) to connect these two fermions together. To see this, note that the Feynman diagram should have the form shown in figures 1a or 1b. It is straightforward to show that such diagrams could not exist. To see this note that the first vertex in figure 1a requires a $\bar{t}_L \chi \nu_{DR}$ interaction term and the second vertex in figure 1a requires a $\nu_{DL} \chi \bar{t}_R$ interaction term, but $\bar{t}_L \chi \nu_{DR}$ does not transform like $\bar{t}_R \nu_{DL}$ under the SM gauge symmetry (and for the alternative possibility of the top quark being generated from figure 1b, a similar argument holds). One can of course construct a diagram with two different $\chi$'s which are mixed together by a symmetry breaking mass term. However in this case the Lagrangian would contain two symmetry breaking terms (and would consequently violate our hypothesis of only one symmetry breaking term in the Lagrangian).

Thus we are led to consider a completely new type of fermion with a large mass $M$ which is presumed to be the origin of symmetry breaking. If we add a doublet of fermions and give one of them a large mass, then the induced $W$ and $Z$ gauge boson masses do not automatically satisfy eq. (3). However as already mentioned, if (for example) the doublet of fermions contains a neutral fermion, and if one of the Weyl components of the neutral fermion is a $SU(2)_L \otimes U(1)_Y$ singlet and has a large Majorana mass (which is just the see-saw type picture), then the induced $W$ and $Z$ gauge boson masses do automatically satisfy eq.(3). The Majorana mass term of the gauge singlet does not violate our hypothesis since it is not a gauge symmetry breaking term.

We start by allowing for the most general $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ transformation properties of the new fermion $F_L, F_R$. Since $SU(3)_c$ is presumed unbroken, $F_L, F_R$ must have identical $SU(3)_c$ transformation properties (otherwise the $\tilde{F}_L F_R$ mass term would break $SU(3)_c$). For simplicity, we assume that they are color singlets. The simplest possibility is then:

$$F_L \sim (1, M_1, y_L), \ F_1R \sim (1, 1, 0), \ F_2R \sim (1, M_1 - 1, y_R).$$  

The one tree-level $SU(2)_L \otimes U(1)_Y$ symmetry breaking mass term will be a
$M\bar{F}_1F_{1R}$ term in the Lagrangian (where $F_{1L}$ is one component of the multiplet $F_L$) i.e.

$$\mathcal{L}_{SB} = M\bar{F}_1F_{1R} + H.c.$$ (5)

As discussed earlier, we assume a large Majorana mass term $M\bar{F}F_{1R}(F_{1R})^c$ (with $M \gg \mathcal{M}$) for the gauge singlet. This leads to acceptable $W$ and $Z$ gauge boson masses which are proportional to the fermion mass $M$ [3]. We also require that the top quark gains mass at one-loop order. The required Feynman diagram is shown in figure 2. In order to connect the top quark with the fermion $F$ we assume the existence of a scalar $\chi$ (with no VEV). For the top quark to gain mass from this diagram we must have the Lagrangian terms:

$$\mathcal{L}_\chi = \lambda_1\bar{t}_LF_1R\chi + \lambda_2\bar{F}_Lt_R\chi^\dagger + H.c.$$ (6)

The first term in the Lagrangian implies that

$$\chi \sim t_L\bar{F}_1R \sim (3, 2, 1/3).$$ (7a)

The second term implies that

$$\chi \sim \bar{F}_Lt_R \sim (3, M_1, 4/3 - y_L).$$ (7b)

So we see that $M_1 = 2$ and $y_L = 1$. Our notation is as follows:

$$F_L = \begin{pmatrix} E \\ V \end{pmatrix}_L \sim (1, 2, 1), \quad F_{1R} = V_R \sim (1, 1, 0), \quad F_{2R} = E_R \sim (1, 1, 2).$$ (8)

Thus assuming there exists a scalar $\chi$ with interactions described by the Lagrangian terms $\mathcal{L}_\chi$ (eq.(6)), the top quark (as well as the $W$ and $Z$ gauge bosons) gain masses at one-loop order. All the other fermions remain massless so we need to add more scalars. The exotic fermion $E$ must have mass greater than about 45 GeV, otherwise it would have been detected in the LEP measurements of the $Z$ boson width. Thus it is quite heavy like the $W, Z$ gauge bosons and the top quark and we would therefore expect its mass to be generated at the same order in perturbation theory as the $W, Z$ gauge bosons and the top quark. Thus the $E$ fermion should have its mass generated at one-loop, and this requires the introduction of a new scalar $\phi$, with interactions:

$$\mathcal{L}_{\phi} = \lambda_3\bar{F}_L\phi V_R + \lambda_4\bar{F}_L\phi^c E_R + H.c.$$ (9)

Note that, like the scalar $\chi$, the scalar $\phi$ is assumed to have no VEV. It is straightforward to show that this Lagrangian is consistent with $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge invariance provided

$$\phi \sim (1, 2, 1).$$ (10)

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The Lagrangian terms $L_{\phi_1}$ imply that the new fermion $E$ gets its mass at one-loop, as shown in figure 3. Given that $\phi \sim (1, 2, 1)$, it can also couple to the SM quarks and leptons through the Lagrangian terms:

$$L_{\phi_2} = \lambda_{3j} f_{1L} \phi \bar{e}_{jR} + \lambda_{3j} f_{1L} \phi \bar{\nu}_{jR} + \lambda_{4j} Q_{IL} \phi D_{jR}$$

$$+ \lambda_{4j} Q_{ij} \phi \bar{e}_{jR} + \lambda_{5j} f_{1L} \phi \bar{\nu}_{jR} + \lambda_{6j} f_{1L} \phi \bar{\nu}_{jR} + H.c.$$  \hspace{1cm} (11)

with $i, j = 1, \ldots, 3$ representing the generations, i.e.

$$f_{1L} = \left( \begin{array}{c} \nu_e \\ e \end{array} \right)_L, \; f_{2L} = \left( \begin{array}{c} \nu_\mu \\ \mu \end{array} \right)_L, \; f_{3L} = \left( \begin{array}{c} \nu_\tau \\ \tau \end{array} \right)_L, \; etc.$$ \hspace{1cm} (12)

and our notation for the SM quarks should be self explanatory. Note that we have also added gauge singlet right-handed neutrino fields to the theory. The Lagrangian terms $L_{\phi_2}$ imply that a Dirac mass term $M_{\nu_{\tau}D}$ is also generated at one-loop (and one can check that no other fermion masses are generated at one-loop order). This is shown in figure 4. Thus, with the scalars $\phi, \chi$ with interactions $L_{\phi}, L_{\chi}$, the $W, Z$, the top quark, the exotic $E$ fermion and a Dirac mass term for $\nu_\tau$ are all generated at one-loop order in perturbation theory. Each of these radiatively induced masses are directly proportional to the symmetry breaking fermion mass $M$. Note that we assume a large Majorana mass for the right-handed neutrinos so that the physical neutrino masses are small (which is just the well known seesaw mechanism). The simplest way that we have found to extend the model so that all of the other quarks and leptons get masses is to add another copy of $\phi$ with interactions of the same form as in eqs.(9) and (11).

We denote the two scalars as $\phi_1$ and $\phi_2$. Assuming the most general Lagrangian for the interactions of the scalars $\phi_1$ and $\phi_2$, then in the quark sector, at two-loops two charged $-1/3$ quarks will gain masses (which we take to be the $b$ and $s$ quarks). At three-loops, the $c$ and $u$ quarks will gain mass and the $d$ quark will get mass at four-loop order. In the lepton sector, the $\tau$ and $\mu$ leptons get masses at two-loops. At three-loops, the $\nu_\mu$ and $\nu_e$ get masses and the electron gets mass at four-loop order in perturbation theory [5] [6]. Of course, the model as it stands can only partially explain the fermion mass hierarchy and it certainly does not give any quantitative mass predictions. Also note that the model is not complete since it has triangle anomalies and there is also the requirement of a cutoff. However the point of it is to illustrate our idea [7].

Finally, we would like to make some remarks concerning scale invariance [8]. Perhaps an interesting possibility is that scale invariance might also be explicitly broken at tree-level by just one term in the Lagrangian. In particular, maybe both gauge and scale symmetry breaking arises from just one source term [9]. It is not too difficult to modify our model so that this scenario is realized. In our model, the only terms which break scale invariance at tree-level are the masses for the scalars, the electroweak symmetry breaking mass and the Majorana masses for the exotic $V_R$ as well as the $\nu_R$ fields. The masses for
the scalars will arise at one-loop if they are absent in tree-level. The Majorana mass of $V_R$ may be the source of the majorana masses of the $\nu_R's$ (through one-loop diagrams involving new scalars for example). The $V_R$ Majorana mass can be combined with the electroweak symmetry breaking mass, so that the one symmetry breaking term is then

$$L_{SB} = M\bar{V}_R (\cos \phi (V_R)^c + \sin \phi V_L) + H.c.$$  \hfill (13)

We would also like to stress again that the symmetry breaking term is not only the source of all the gauge and scale symmetry violating terms, it is also the source of the non-renormalizability of the theory. (This is unlike the situation in the SM where symmetry breaking is spontaneous in origin and is renormalizable when terms with (mass) dimension of less than or equal to four are allowed at tree-level). We have two comments on renormalizability. Firstly, the model is not complete and some new physics must exist somewhere which renders the divergent integrals finite. Perhaps the symmetry breaking source term should be viewed as some effective term. Perhaps it is itself of dynamical origin in which case there maybe no gauge or scale symmetry breaking terms in the Lagrangian at all in the tree-level [10]. Secondly, it would be pleasing if renormalizability followed from some principle. This is because, for example, in nature there maybe some effective regulator which renders the divergent integrals finite. In this case there is no reason to exclude non-renormalizable terms from the tree-level Lagrangian. However, if we start adding non-renomalizable terms to the standard model, then some things would start to go wrong. For example, the proton could decay and flavour changing neutral currents could occur. What type of principle could prevent non-renormalizable terms from appearing in the Lagrangian of a gauge theory? Scale invariance seems to be a good candidate for such a principle [11].

In conclusion, we have considered the hypothesis that all electroweak symmetry breaking is due to a single term in the Lagrangian. All other symmetry breaking terms may arise radiatively from this symmetry breaking term. From this assumption we hope to make progress in understanding the fermion and gauge boson masses. We constructed a simple model illustrating the idea. However our work is at present incomplete since the model contained triangle anomalies and our Feynman diagrams were logarithmically divergent. More importantly, we failed to make any mass predictions, so we have not yet improved substantially upon the SM. Nevertheless, we feel that in principle our idea could be successful. For example, with some additional assumptions, the Yukawa couplings of the scalars in our model can be restricted so that quantitative predictions can be made. Alternatively the use of scalars could be abandoned and the gauge symmetry enlarged. Despite the many shortcomings of our model, we think our idea is interesting. It is important to explore new ideas in relation to the fermion mass problem and we will continue to study this problem. We also pointed out that the single symmetry breaking term could be the only tree-level source of scale symmetry breaking. We believe that scale invariance may

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be an important concept since it is a symmetry which forbids terms of (mass) dimension greater than four in the Lagrangian (in the tree-level).

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Figure Captions:

Figure 1a: Form of the Feynman diagram for radiatively generating the top quark mass from the neutrino mass at one loop.

Figure 1b: Another possible form for a Feynman diagram generating the top quark mass from the neutrino mass at one loop.

Figure 2: Feynman diagram for the radiative generation of the top quark mass from the $V$ tree-level fermion mass term.

Figure 3: Feynman diagram for the radiative generation of the $E$ fermion mass at one loop from the $V$ tree-level fermion mass term.

Figure 4: Feynman diagram for the radiative generation of a Dirac tau neutrino mass term at one loop from the $V$ tree-level fermion mass term.

References:

[1] Note however that there may also be some symmetry breaking due to dynamical effects such as the formation of light quark condensates which are believed to form in QCD. We assume that such effects are negligible compared with the symmetry breaking terms which are radiatively induced from the tree-level source term.

[2] There has been a lot of studies discussing the possibility of generating some fermion masses from others radiatively. However as far as we are aware, no-
one has considered our hypothesis before. For some earlier work on radiative fermion mass generation see for example, B. S. Balakrishna, Phys. Rev. Lett. 60, 1602 (1988); X-G. He, R. R. Volkas, and D.-D. Wu, Phys. Rev. D41, 1630 (1990).

[3] R. Foot and S. Titard, Mod. Phys. Lett. A7, 1991 (1992).

[4] Note that we assume that the cutoff is assumed to preserve electroweak gauge symmetry. This assumption is necessary, and it is consistent (at least in spirit) with our hypothesis that there is only one Lagrangian term which violates electroweak gauge symmetry.

[5] Note that the scalar masses can be arbitrarily large. The model has no naturalness problem and no problem with flavour changing neutral currents from scalar exchange (which can be made arbitrarily small since the scalar masses can be made arbitrarily large).

[6] The details of the mass generation of the quarks and the leptons from the one-loop top quark and Dirac tau neutrino masses, will be presented in more detail elsewhere [see R. Foot, H. N. Long and T. A. Tuan, to be published.]. For this reason we have only sketched the details here.

[7] While our hypothesis is simple, there is of course no unique implementation of it. In our example, we assumed that the symmetry breaking term was a fermion mass term (eq.(5)). Our model has 2 arbitrary mass scales (actually 5 when we include the Majorana masses for the right-handed neutrinos). It might be interesting to assume only one mass scale, say the Majorana mass scale $M$. We could then try to derive all the other masses in the theory from this electroweak symmetric term by using a symmetry breaking interaction term. The one mass scale could be very large (e.g. the Planck mass) and the symmetry breaking term very small.

[8] Scale invariance can be defined mathematically as the symmetry of the action under the scale transformations:

$$x^\mu \to \lambda x^\mu, \quad \{\phi, W^\mu, Z^\mu, A^\mu\} \to \{\phi/\lambda, W^{\mu}/\lambda, Z^{\mu}/\lambda, A^{\mu}/\lambda\},$$

$$f \to f/\lambda^{3/2}$$

where $f$ stands for the set of fermion fields.

[9] There may also be scale symmetry breaking due to dynamical effects discussed in footnote [1].
[10] A. Zee [Phys. Rev. Lett. 44, 703 (1980)] has also speculated that the "theory of the world" contains no dimensional parameter, i.e. scale invariant. Zee’s motivation for scale invariance seems to be different from ours though.

[11] It is possible that non-renormalizable terms are present at the tree-level. However, we expect that they must be heavily suppressed due to the observed stability of the proton. There are interesting exceptions to this however. For example, there are some gauge models in which the gauge symmetry is sufficient to forbid proton decay. See R. Foot, Southampton University Preprint 1991 (unpublished) for details.
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