Scalable and Sparsity-Aware Privacy-Preserving K-means Clustering with Application to Fraud Detection

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ABSTRACT
K-means is one of the most widely used clustering models in practice. Due to the problem of data isolation and the requirement for high model performance, how to jointly build practical and secure K-means for multiple parties has become an important topic for many applications in the industry. Existing work on this is mainly of two types. The first type has advantages in efficiency, but there is information leakage and potential privacy risks. The second type is provable secure but is inefficient and even helpless for the large-scale data sparsity scenario. In this paper, we propose a new framework for efficient sparsity-aware K-means with three characteristics. First, our framework is divided into a data-independent offline phase and a much faster online phase, and the offline phase allows to pre-compute almost all cryptographic operations. Second, we take advantage of the vectorization techniques in both online and offline phases. Third, we adopt a sparse matrix multiplication for the data sparsity scenario to improve efficiency further. We conduct comprehensive experiments on three synthetic datasets and deploy our model in a real-world fraud detection task. Our experimental results show that, compared with the state-of-the-art solution, our model achieves competitive performance in terms of both running time and communication size, especially on sparse datasets.

CCS CONCEPTS
- Security and privacy → Privacy-preserving protocols;  
- Computing methodologies → Machine learning.

KEYWORDS
Privacy-Preserving Machine Learning, Clustering, Secret Sharing, Homomorphic Encryption

1 INTRODUCTION
K-means [31] has been widely used in many applications, such as feature extraction [26], document clustering [39], targeted marketing [23], and outlier detection [32]. Risk control, including fraud detection, is the core for financial companies. Since fraud patterns are diverse and transient and more than 90% of them are unlabeled, traditional financial companies typically use the K-means algorithm for fraud detection, often in partnership with partner companies. For example, they group transactions based on the independent attribute values, such as credit card values from the payment company and buyer values from the merchant, which distinguish outliers when the input size is large enough [9]. However, over the past few years, the situation of isolated data islands [46] has become a severe problem. Privacy laws and regulations are getting stricter, preventing data sharing between multiple entities. In contrast, a single company with limited data can hardly meet the high-precision requirements of modeling.

Application Scenarios. Solving the above dilemma in financial application scenarios needs to meet some requirements. First, financial scenarios are more strictly regulated than other industries, are more sensitive to information leakage, and prefer jointly trained models that reveal nothing but outputs. Secondly, due to the application in production, efficiency is also essential while meeting security. Lastly, sparseness is an inherent dataset property in real-world applications, mainly from incomplete user profiles or feature engineering such as one-hot. It causes a severe efficiency problem for existing privacy-preserving techniques [10].

Existing Work and Unresolved Problems. The existing privacy-preserving K-means models mainly fall in two categories: (1) partial privacy protection based and (2) provable security based. The former attempts to use perturbation, permutation, or other cryptographic protocols to protect some sensitive information, such as labels and attributes. However, there is still a certain amount of information leakage in the model, such as cluster category [45], centroids [43] and cluster to be merged [19]. In contrast, the latter uses cryptographic protocols to design algorithms with provable security. However, their running times are much longer than the former due to the high computation and communication complexity of the cryptographic techniques [2, 7, 21]. Payman Mohassel et al. [33] proposed the state-of-the-art privacy-preserving clustering scheme. They proposed an ingenious distance computation protocol of centroids to all samples and built a customized garbled circuit to compute binary secret sharing of the minimum. Nevertheless, their scheme still does not work efficiently for three reasons. Firstly, they did not take advantage of the pre-computation property of cryptographic operations, which would otherwise result in a very efficient online phase [14]. Secondly, they operated on numerical values, which is not as efficient as vectors in secret shared setting.

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We optimize all of these steps using appropriate cryptographic techniques. The results show that our protocol significantly outperforms the state-of-the-art in the online phase. Moreover, we conduct the experiments for our proposed protocols and show that they perform better than previous approaches.

2 RELATED WORK

2.1 Partial Privacy Protection Based K-means

Algorithms in this category attempt to provide partial privacy protection for certain sensitive information. The methods of Vaidya and Clifton [43], Jha et al. [22], and Jagannathan and Wright [20] provide security guarantees on attributes and labels of data from different parties. However, none of them can protect the intermediate centroids from being leaked. Centroids in each iteration are very sensitive because they reflect much sample information. Unlike these previous methods, the work of Jagannathan et al. [19] does not reveal intermediate candidate cluster centers. However, it reveals the cluster category, which exposes which samples are qualitatively similar. Especially in the horizontal scene that they are working for, there is serious information leakage. In addition, there are other researches [29, 47], in which each iteration of k-means clustering can be performed without revealing most of the intermediate values. Nevertheless, they reveal the number of entities in each cluster, which also contains the general distribution information of private inputs.

2.2 Provable Security Based K-means

In contrast, the second category enjoys provable security using some cryptographic protocols such as oblivious transfer [8, 33], homomorphic cryptosystems [7, 21] and secret sharing [33, 38]. They do not reveal any information such as the private inputs, cluster category, and numbers of entities in a cluster. However, most of them suffer from high communication and computation costs and cannot scale to large datasets. One of the most notable examples in this category is [21]. They use fully homomorphic encryption and fractional encoding to allow addition, multiplication, and division on encrypted data. Unfortunately, it takes almost 1.5 years on a real-world dataset with 400 entries, 2 dimensions, and 3 classes, which is not practical for large data sets. Similar to Payman’s privacy-preserving clustering scheme [33], in this paper, we focus on how to propose new efficient privacy-preserving K-means clustering for multi-parties with full privacy guarantees to address three problems in their work.
• SADD: \( z \leftarrow (x) + (y) \), where \( z = x + y \mod 2^l \). It can be easily done by adding participants’ local shares and can be extended to linear operation that \( z \leftarrow \alpha \cdot (x) + (y) + \beta \), where \( z = \alpha x + \beta y \mod 2^l \).
• SMUL: \( z \leftarrow \text{Mul}(x, y) \), where \( z = x \cdot y \mod 2^l \).
• A2B: \((x_0)^B, \cdots , (x_l-1)^B \leftarrow A2B(\langle x \rangle)\), where \( x = \sum_{i=0}^{l-1} x_i \cdot 2^i \).
• B2A: \((x) \leftarrow B2A((x_0), \cdots , (x_l-1))^B \).
• MSB: \((x_0)^B \leftarrow \text{MSB}(\langle x \rangle)\).
• CMP: \((x) \leftarrow \text{CMP}(\langle x \rangle, \langle y \rangle)\).
• MUX: \((x) \leftarrow \text{MUX}(\langle z \rangle, \langle x \rangle, \langle y \rangle) = \langle z \rangle \cdot \langle x \rangle + (1 - \langle z \rangle) \cdot \langle y \rangle\).

3.2 Additive Homomorphic Encryption

Additive homomorphic encryption, such as Okamoto-Uchiyama (OU) [36] and paillier [37], is a method that supports secure addition when given a ciphertext. It mainly includes the following schema and operations.

Encryption and Decryption. One party generates a public-private key pair \((pk, sk)\) and distributes \(pk\) to the other party. A plaintext \(u\) encrypted by \(pk\) is denoted by \([u]\), e.g., \([u] = \text{Enc}(pk; x, r)\), where \(r\) is a random number that makes sure the ciphertexts are different even when the plaintexts are the same. Given a ciphertext \([v]\), it needs to be decrypted with its corresponding private key, i.e., \(v = \text{Dec}(sk; [v])\).

Homomorphic Operation. We overload ‘+’ as a homomorphic addition operation on ciphertext space. For any plaintext \(u\) and \(v\) encrypted by the same \(pk\), additive homomorphic encryption satisfies \([u] + [v] = [u + v]\). There are two common variants of it, i.e., \([u \cdot v] = u \cdot [v]\) and \([u + v] = u + [v]\).

3.3 Conversion Between SS and HE

Conversions between SS and HE are 2-party functionalities. Here, we just introduce \(\text{HE2SS}\) protocol between parties \(A\) and \(B\), which converts data from HE format to SS format [10]. That is, \((X) \leftarrow \text{HE2SS}(\langle X \rangle_B, \langle pk_B, sk_B \rangle)\), where \(\langle X \rangle_B\) is a ciphertext encrypted by party \(B\) and held by \(A\). During implementation, \(A\) generates \((X)_1\) using Pseudo-Random Generator \((\text{PRG})\) in the same field \(\langle Z_{gB} \rangle\) as HE, and sends \([X]_2 = [X] - (X)_1 \mod \phi\) to \(B\). \(B\) decrypts it to get \((X)_2\), where \(X = (X)_1 + (X)_2 \mod \phi\). The correctness and security is guaranteed by \([u + v] = u + [v]\). It can be applied to the multi-party setting as well.

3.4 K-means Clustering Algorithm

As the most popular algorithm in clustering, K-means was first proposed by MacQueen [31]. It can automatically partition a collection of data sets into separated groups according to the similarity of the objects, where users preset the number of clusters, and each cluster is described by its center. There are many criteria to measure the similarity, e.g., Euclidean distance [15, 35], manhattan distance [15, 42], and cosine similarity [30, 41]. In our case, we assume data objects are elements of \(\mathbb{R}^d\) and adopt the Euclidean distance as our criterion.

The K-means clustering algorithm consists of two phases: cluster centroids initialization and Lloyd’s iteration. Its core step is Lloyd’s iteration, which mainly consists of three parts:

- **Distance Computation**: Computes the Euclidean distance between each data point and each centroid.
- **Assignment**: Assigns each data point to the nearest centroid.
- **Update**: Updates the centroids to be the mean of all the data points assigned to the same centroid.

A brief description of K-means implementation and centroids update. A brief description of K-means implementation and centroids update. The K-means clustering algorithm is presented in Algorithm 1.

\begin{algorithm}[H]
\caption{K-means clustering}
\begin{algorithmic}[1]
\Require Data \(X_{(n \times d)}\), Cluster number \(k\)
\Ensure Centroids \(\mu_{(k \times d)}\)
1. Randomly choose \(k\) centroids \(\mu_0\)
2. \textbf{repeat}
3. \hspace{1em} \textbf{[Distance Compute]}
4. \hspace{2em} Compute Euclidean distance \(D_{(n \times k)}\), where \(D_{ij} = \|X_i - \mu_j\|_2\)
5. \hspace{1em} \textbf{[Cluster Assignment]}
6. \hspace{2em} Reassign the sample to their nearest center by comparison, gets binary matrix \(C_{(n \times k)}\), \(C_i = \arg\min_j D_{ij}\)
7. \hspace{1em} \textbf{[Centroids Update]}
8. \hspace{2em} Recompute the cluster centers \(\mu_j = \frac{\sum_{i=1}^{n} I(c_{ij})X_i}{\sum_{i=1}^{n} I(c_{ij})}\)
9. \hspace{1em} \textbf{until} Has repeated for a fix number of times or the improvement in one iteration is below a threshold
10. \hspace{1em} \textbf{return} Centroids \(\mu_{(k \times d)}\)
\end{algorithmic}
\end{algorithm}

4 THE PROPOSED METHOD

4.1 Problem Statement and Setting

**Problem Statement.** Our privacy-preserving K-means follows the framework of the standard K-means algorithm. It must achieve comparable performance with the plaintext K-means while protecting data privacy. In the following, we consider by default the setting of (semi-honest) 2PC with reprocessing. For the convenience of description, in the following, we take two parties as an example, which is easy to apply to the multi-party setting. The input is the data owned by two parties and the number of clusters \(k\). And the output is the share of the final cluster allocation held by each party.

**Data Setting.** Data provided by different parties is partitioned horizontally or vertically in the data space [27]. We use \(X_A / X_B\) to denote the plaintext data of party \(A / B\), where each column represents an attribute and each row represents a sample. The feature dimension and sample size of party \(A / B\) and joint data are denoted as \(d_A / d_B / d\) and \(n_A / n_B / n\) respectively. In horizontally partitioned situation, the joint data can be formalized as \(X = [X_A X_B]^T\) and \(n = n_A + n_B, d = d_A + d_B\). In vertically partitioned situation, the joint data can be formalized as \(X = [X_A, X_B]\), and \(n = n_A = n_B, d = d_A + d_B\).

**Security Setting.** We consider the standard semi-honest model, widely used in many work [3, 6, 11, 28, 48], where a probabilistic polynomial-time adversary with honest-but-curious behaviors is considered. Assume that the adversary engages in protocol strictly while trying to preserve all intermediate outcomes and infer as much as possible. We also assume that parties do not collude with each other.

**Online-Offline Setting.** Similar to [13, 14, 34], we split our protocols into a data-independent offline phase and a much faster online phase. The offline phase consists mainly of cryptographic operations, which can be performed without the presence of data. Meanwhile, the online stage contains the data-dependent steps in the K-means algorithm. Take two-party secret sharing matrix.
multiplication of as an example [5, 34]. To multiply two secretly shared matrices \((A)\) and \((B)\), firstly, we prepare a shared matrix triple (Beaver’s triplet) \((U) (V)\) and \((Z)\), where each element in \(U\) and \(V\) is uniformly random in \(\mathbb{Z}_{2^l}\) and \(Z = UV \mod 2^l\). Secondly, given two shared matrices \((A)\) and \((B)\), two parties can compute \((E) = (A) - (U)\) and \((F) = (B) - (V)\) locally. After one round interaction, both parties can reconstruct \(E\) and \(F\) and get the final multiplication result \((C) = iEF + (A)F + E(B) + (Z)\), where \(i \in \{0, 1\}\). The triplets generation step (the first step) is time-consuming because it needs a large number of cryptographic operations, such as oblivious transfer or homomorphic encryption [14, 34]. Fortunately, this step is data-independent and can be prepared in advance as an offline phase, using either cryptography-based methods or a trusted third party. After it, the online phase (the second step), which depends on the input data, could be done efficiently.

### 4.2 Privacy-Preserving K-means

Recall that the K-means clustering can be divided into cluster centroids initialization and Lloyd’s iteration. In this subsection, we will first introduce some initialization methods, then describe the vector secure Lloyd’s iteration, which helps improve the efficiency of the privacy-preserving K-means algorithm.

**Initialization.** Cluster centroids initialization is an important issue because it determines the convergence rate. It can be done using different strategies. A simple and common strategy is random initialization. All parties can pick random values or jointly negotiate random indexes of all K groups. There is another way to start with better initials. Each party locally runs the plain-text K-means, which can be computed efficiently.

**Secure Lloyd’s Iteration.** After initialization, K-means will iteratively update the centroid until convergence using Lloyd’s iteration. During each iteration, there are three steps i.e., Secure Distance Computation, Secure Cluster Assignment, and Secure Centroid Update. In these steps, we focus on two issues: security and efficiency. For security, once the input data is shared through SS, during the whole process, each calculated intermediate value (e.g., Euclidean squared distance) is secretly shared as two uniformly distributed values held by each party. The final result will be reconstructed only once at the end of the protocol. Meanwhile, we solve the efficiency problem in two ways. Firstly, we divide the secure computation steps into a data-independent offline phase and a data-dependent online phase. Secondly, we use vectorization to speed up the protocols. Compared with privacy-preserving numerical operations, the matrix form significantly reduces computation and communication in both online and offline phases. Below we will present the details of vectorized secure Lloyd’s iteration.

**Secure Distance Computation \(F_{\text{ESD}}\).** Secure distance computation aims to measure the similarity between all samples and the centroids using Euclidean Square Distance (ESD). In order to simplify the calculation, we use Euclidean Squared Distance (ESD) as the substitute for the Euclidean Distance in the SS state. Formally, take a sample \(X_i\) and a centroid \(\mu_j\) of \(j\)-cluster at the round \(t\) as an example. Both parties can calculate their A-share of ESD \(D_i^{(t)}\) with \(X_i\) and A-share \(| \mu_j^{(t)}|\) using \(F_{\text{ESD}}\):

\[
D_i^{(t)} = ESD(X_i, \mu_j^{(t)}) = ||X_i - \mu_j^{(t)}||_2^2
\]

It is well-known that K-means only needs to compare the distance between all the samples and the different centers. Since \(\sum_{i=1}^{n} X_i^2\) remains unchanged with a fixed \(i\), it can be omitted when calculating Equation (1). That is, only \(D_i^{(t)}\) needs to be calculated:

\[
(D_i^{(t)}) = F_{\text{ESD}}(X_i, \mu_j^{(t)}) = d \sum_{l=1}^{d} X_{il}^2 + d \sum_{l=1}^{d} |\mu_{jl}^{(t)}|^2 - 2 \sum_{l=1}^{d} X_{il} |\mu_{jl}^{(t)}|. \tag{2}
\]

Vectorizing it with matrix form to get:

\[
(D_i^{(t)}) = F_{\text{ESD}}(X, \mu_j^{(t)}) = \langle U \rangle - 2X\mu_j^{(t)\text{T}}, \tag{3}
\]

where \(\langle U \rangle = 1_{n \times 1} |\|\mu_j^{(t)}\|_2^2, |\|\mu_j^{(t)}\|_2^2, \ldots, |\|\mu_j^{(t)}\|_2^2\|_{n \times k}\) with the dimension of \(n \times k\). The traditional method needs to calculate the distance from each sample to each center point one by one. The total number of interactions in each iteration is \(nk\). After vectorizing in Equation (3), the first item \(\langle U \rangle\) only needs to be calculated once before the iteration starts, and the second item \(X\mu_j^{(t)\text{T}}\) only needs to be interacted with once per iteration due to the matrix operation. Thus, it is constructive for improving efficiency in high-latency scenarios.

Below we will separately describe the details under two data distribution scenarios. Under vertically partitioned scenario, \(X(\alpha \times d) = [X_A, X_B]\) and we can get:

\[
(D_i^{(t)}) = F_{\text{ESD}}(X, \mu_j^{(t)}) = \langle U \rangle - 2X\mu_j^{(t)\text{T}} - 2[X_A(\mu_j^{(t)\text{T}})_A + (\mu_j^{(t)})_B] + X_B(\mu_j^{(t)})_B, \tag{4}
\]

where \((\mu_j^{(t)})_A\) with dimension \((k \times d_A)\) and \((\mu_j^{(t)})_B\) with dimension \((k \times d_B)\) are held by party A \(][(\mu_j^{(t)})_A(\mu_j^{(t)})_A = \mu_j^{(t)}_A]\). Similarly, \((\mu_j^{(t)})_B\) with dimension \((k \times d_B)\) and \((\mu_j^{(t)})_B\) with dimension \((k \times d_B)\) are held by party B \(([(\mu_j^{(t)})_B, (\mu_j^{(t)})_B] = \mu_j^{(t)}_B\). Note that \(X_A(\mu_j^{(t)})_A\) and \(X_B(\mu_j^{(t)})_B\) can be computed locally by A and B respectively. While, \(X_B(\mu_j^{(t)})_A\) and \(X_A(\mu_j^{(t)})_B\) should be jointly computed using vectorized secret sharing multiplication. Under horizontally partitioned data setting, \(X(\alpha \times d) = [X_A(\alpha \times d), X_B(\alpha \times d)]\) and we can get:

\[
(D_i^{(t)}) = F_{\text{ESD}}(X, \mu_j^{(t)}) = \langle U \rangle - 2X\mu_j^{(t)\text{T}} - 2[X_A(\mu_j^{(t)\text{T}})_A + (\mu_j^{(t)})_B] + X_B(\mu_j^{(t)})_B, \tag{5}
\]

where \(X_A(\mu_j^{(t)})_A\) and \(X_B(\mu_j^{(t)})_B\) should be jointly computed using vectorized secret sharing multiplication.

**Secure Cluster Assignment \(F_{\text{ESD}}^k\).** This step aims to find the clusters to which each sample belongs. For distances from one sample to all current centroids, we perform secure comparisons to find the minimum among them and securely mark its position as
the clustered index. The output of $F_{\text{min}}^k$ is an A-share of the clustered index to which this sample belongs. Formally, two parties run $(C_i^{(1)}) \leftarrow F_{\text{min}}^k\big((D_i^{(1)})\big)$, where $i \in [0, n]$ to reassign sample $i$ to its nearest center $c_i^{(1)} = (0, \ldots, 1, 0, \ldots, 0)$, where 1 appears at the $j$-th element if cluster center $j$ is the closest to sample $i$. To accelerate function $F_{\text{min}}^k$, we utilize the classical binary tree reduction method, which consists of $k - 1$ CoMParison Module (CMPM).

The cluster assignment in Figure 1 as an example, where we assume the number of clusters $k$ is 6. All the nodes (shown in yellow rectangles) in this inverted tree structure consist of two parts. The left part of a node (shown in red font) is the record of the smallest distance among the leaf nodes of its subtree. In Figure 1, (1) in the right node of layer 2 is the smallest distance in (7), (2), (1), (3). The right part of a node (shown in black font) is the relative position of the smallest distance. In particular, the relative positions of all the leaf nodes in layer 4 are initialized to (1). The key to $F_{\text{min}}^k$ is CMPM, and we describe the main idea of which in a dotted box in Figure 1. The input of CMPM are two distances to be compared (e.g., $D_i^{(1)} = (2), D_j^{(1)} = (1)$) and their relative positions (e.g., $c_i^{(1)} = (01), c_j^{(1)} = (10)$). The CMPM mainly contains two primitive operations: CMP and MUX, as described in Section 3.1. First, we use CMP to compare “less than”, which judges the shared most significant bit of the subtraction using the A2B, B2A, and MSB. Take the details of a CMPM in Figure 1 for example. When we need to compare distances (2) and (1), we extract and judge the shared sign bit of $(2-1)$ to get the result $z = (0)$. Second, we use MUX to pick out the smaller one from the distances to be compared. In Figure 1, we get the smaller distance ((1)) from two distances of child nodes ((2), (1)). Then the relative position of the smaller distance can be securely marked by concatenation of $b(c_i^{(0)})$ and $((1) \leftarrow \langle b \rangle(c_j^{(0)}), e.g., (0010) \leftarrow F_{\text{min}}^k((7), (2), (1), (3))$ in Figure 1. Finally, after the reduction of the whole tree, we can get secretly shared index matrix $(C_i^{(1)})$ of sample $i$ as the output of $F_{\text{min}}^k\big((D_i^{(1)})\big), i \in [0, n]$. 

**Secure Centroids Update $F_{\text{SCU}}$.** After reassigning the sample to their nearest center, the cluster centers $\mu^{(t+1)}$ should be recomputed by $(C_i^{(t)})$ and $X_i$ below:

$$\mu^{(t+1)} = F_{\text{SCU}}\left((C_i^{(t)}), X\right) = \frac{\sum_{i=1}^{n}(C_i^{(t)})e_i = j)X_i}{\sum_{i=1}^{n}(C_i^{(t)})e_i = j)} = (C_i^{(t)})^TX_{1 \times n}(C_i^{(t)})^T$$

where $1 \leq j \leq k, 1 \leq i \leq n$, and $I(e_i = j)$ is a indicator function that equals 1 if $e_i = j$ and 0 otherwise. To compute the secret sharing of the updated cluster $(\mu^{(t+1)})$, parties separately compute the numerator and denominator. For vertically partitioned scenario, $(C_i^{(t)})^TX = ((C)_A + (C)_B)^T[X_A(n \times d_A), X_B(n \times d_B)]$. And for horizontally partitioned data setting, $(C_i^{(t)})^TX = ((C)_A + (C)_B)^T[X_A(n \times d), X_B(n \times d)]$. Their processing is the same as the distance computation. If the inputs are local, the multiplication can also be calculated locally. Other multiplications are computed jointly. Then the reminders are calculated using the broadcasting secret division operation, which is converted to secret multiplication and addition.

**Checking the Stopping Criterion $F_{\text{CSC}}$**. The iteration terminates when it converges or reaches a certain iteration. For convergence, parties can jointly invoke the secure comparison protocol $F_{\text{CSC}}(\mu^1, \mu^{(t+1)}, e) \leftarrow \text{CMP}(\mathcal{F}_{\text{MCD}}(\mu^1, \mu^{(t+1)})), e)$ to check the stop threshold $\epsilon$.

### 4.3 Optimization for Sparse Scenario

**Motivation.** Feature sparsity is usually caused by missing feature values or feature engineering such as one-hot. Such data has two characteristics. First, it will become dense after using secret sharing. For example, there is a 4-dimensional sparse vector $(0, 0, 1, 0)$. After secret sharing, it will be randomly split into two shares uniformly distributed over a finite field, e.g., $(13, 1, 7, 8)$ and $(3, 15, 10, 8)$ in $\mathbb{Z}_{2^4}$. This will cause much unnecessary computation since we cannot determine the position of $0$ in the state of sharing. Especially in the high-dimensional sparse feature situation, communication becomes the bottleneck of the SS-based model. It can even make the model unusable in practice. Second, it is usually not all $0$s for the entire row or column. Therefore, our algorithm should be optimized for the scenario where any element in the data may be $0$.

**Privacy-Preserving Sparse K-means.** After vectorization, we can see that the cluster update involves many matrix multiplications. There is a secure sparse matrix multiplication protocol combining the advantages of HE and SS [10]. Given a sparse matrix $X$ held by $\mathcal{A}$ and a dense matrix $Y$ held by $B$, the protocol can efficiently calculate $XY$ without revealing the value of $X$ and $Y$, which multiplies sparse data under HE space and uses HE2SS to convert the output to the secret-shared state for facilitating the following calculations under SS space. As shown in Protocol 1, Line 2 is the ciphertext multiplication using additive HE. It can significantly speed up sparse matrix multiplication by eliminating calculations involving zeros because the sparse matrix is $X$ held by $\mathcal{A}$. Lines 3-5 show how to generate secret shares under homomorphically encrypted field. Compared with SS, this protocol does not need to transmit $X$-sized matrices. Therefore, the communication cost of it will be much cheaper when the shape of $Y$ is much smaller than $X$. We show the overall framework of our privacy-preserving K-means algorithm with sparse optimization in Algorithm 3, where we take the vertical distribution of data as an example. For sparse
We present the security proof of our privacy-preserving K-means clustering algorithm in this subsection. First, we introduce the security definition of our algorithm against semi-honest adversaries. We adopt the security definition from [18], where the security is defined over statistical security parameter $\kappa$ and computational security parameter $\lambda$. We also use $\xi$ to denote computationally indistinguishability by the parameter $\lambda$.

Definition 4.1. We say a protocol $\pi$ is a secure instantiation of $f = (f_0, f_1)$ against semi-honest adversaries, if for all sufficiently large $\lambda \in \mathbb{N}^+$, $x, y \in \{0, 1\}^*$, there exists two probabilistic polynomial-time simulators $(\text{Sim}_0, \text{Sim}_1)$, where the following holds,

\[
\begin{align*}
\{\text{Sim}_0(1^\lambda, x, f_0(x, y), f(x, y))\}_{x,y,\lambda} &\subseteq \{\text{view}^\pi_{\lambda}(x, y, \text{output}^\pi(x, y))\}_{x,y,\lambda} \quad (7) \\
\{\text{Sim}_1(1^\lambda, y, f_1(x, y), f(x, y))\}_{x,y,\lambda} &\subseteq \{\text{view}^\pi_{\lambda}(x, y, \text{output}^\pi(x, y))\}_{x,y,\lambda}. \quad (8)
\end{align*}
\]

Lemma 4.2. Our privacy-preserving (sparse-aware) k-means protocol is a secure instantiation of k-means algorithm against semi-honest adversaries with the existence of ideal MPC functionalities $\mathcal{F}_{\text{ESD}}$, $\mathcal{F}_{\text{SCU}}^k$, and $\mathcal{F}_{\text{CSC}}^k$.

Proof. Since the executions of both parties are identical, we only show the proof that $P_0$ is corrupted. With the existence of the above ideal functionalities, the message received by $P_0$ is:

For each iteration $i$:

1. The output of $\mathcal{F}_{\text{ESD}}$ denoted as $\{f_{\text{ESD}}\}^{(i)}_0$.

Algorithm 3 Privacy-preserving (sparse) K-means for vertically partitioned data

Input: sparse matrix $X_A, X_B$ hold by A, B respectively (the joint data can be formalized as $X = [X_A, X_B]$) and $n = n_A + n_B, d = d_A = d_B$; cluster number $k$

Output: Centroids $\{\mu^{(i)}\}$

1. Randomly choose $k$ centroids $\mu^{(0)}$ or each of them locally run the K-means for initialization.
2. repeat
3. [Distance Compute]
4. A locally calculates $X_A(\mu^{(i)})^T_{A1(k \times d_A)}$, $(U)^A_{\lambda}$.
5. B locally calculates $X_B(\mu^{(i)})^T_{B1(k \times d_B)}$, $(U)^B_{\lambda}$.
6. A and B securely calculate $X_B(\mu^{(i)})^T_{A2(k \times d_A)}$ and $X_A(\mu^{(i)})^T_{B1(k \times d_B)}$ using secure sparse matrix multiplication to get $(D)\wedge{1} = \mathcal{F}_{\text{ESD}}^k \langle D \rangle_{\lambda, \nu}$. Use $(D)\wedge{1}$ to denote the cluster centers updated for the next iteration.
7. [Centroids Update]
8. Reassign the sample to their nearest center by binary trees compare recursively, gets binary matrix $(C^{(i)})$, where $(C^{(i)}_{\wedge{1}})_{\lambda} \leftarrow \mathcal{F}_{\text{CSC}}^k \langle (D)^{\wedge{1}} \rangle_{\lambda}(i \in [n, n])$.
9. [Centroids Update]
10. A locally calculates $(C^{(i)})^T_{A1(k \times d_A)}$.
11. B locally calculates $(C^{(i)})^T_{B1(k \times d_B)}$.
12. A and B securely recompute the cluster centers using $\mu^{(i+1)} = (C^{(i)})^T_{A1(k \times d_A)} \mathcal{F}_{\text{CSC}}^k \langle 1 \rangle_{\lambda, \nu} \mathcal{F}_{\text{ESD}}^k \langle C \rangle_{\lambda, \nu}$ by secure sparse matrix multiplication and secret sharing division which is converted to SADD & SMUL operations.
13. until Repeated for a fix number of times or the improvement in one iteration is below a threshold.
14. return Centroids $\{\mu^{(i)}\}$.
Also, given that we instantiate all of our functionalities using arithmetic MPC, all constructions of our functionalities \(F_{\text{ESD}}, F_{\text{min}}^{\text{K}}, F_{\text{CU}}\) satisfy the definition of MPC gates with semi-honest security.

## 5 EXPERIMENTS AND APPLICATIONS

Our experiments intend to answer the following questions: **Q1:** how does our model perform compared with the state-of-the-art privacy-preserving clustering protocol [33]? **Q2:** How does the division of online and offline affect the performance? **Q3:** How does vectorization affect the performance? **Q4:** What is the acceleration of our optimized model in sparse scenarios? **Q5:** How about the effectiveness of our framework for real-world multi-party fraud detection applications?

### 5.1 Experimental Setup

**Implementation and Hardware Details.** We use the C++ programming language to implement our algorithm and run it on a server equipped with a 2.5 GHz Intel Core E5-2682 processor and 188 GB RAM. We consider two network settings. For Q1, we use a Local Area Network (LAN) setting with 0.02ms round-trip latency and 10 Gbps network bandwidth, the same as the comparison model [33]. For Q2 to Q4, we use the Wide Area Network (WAN) with a 20Mbps maximum throughput and 40ms round-trip latency.

**Hyper-Parameters.** For HE method in our experiments, We choose Okamoto-Uchiyama encryption (OU) [36] which outperforms Pailler over all operations [16]. In this experiment, we set key length as 2,048 and \(\psi\) as a large number with longer than 1,365 bits (2/3 of the key length) to meet the requirements of both security and efficiency. For the finite field of SS, we choose integers modulo as \(2^{64} (l = 64)\), which is computationally beneficial [12]. Additionally, we use 20 out of 64 bits to represent the fractional part. For multiplication triples generation, we choose OT-based method to implement it [17] and set the security parameter \(\kappa = 128, \varphi = 3,072\) according to [4]. With these parameters, the security lifetime of the algorithm can be extended after 2030.

**Dataset.** We conduct experiments on 4 datasets, including 1 real-world dataset and 3 synthetic datasets. To compare the online-offline setting (in Q1&Q2) and verify the performance with different parameters (e.g., dimension, sample size, cluster number, and sparse degree in Q3&Q4), we choose to generate the required data. We also take two parties as an example, which is easy to expand to multi-parties. For the vertically and horizontally partitioned setting, the only difference is how Eq. (3) and Eq. (6) are calculated, as we have described in Section 4.2. Since computation and communication costs of both settings are almost equivalent, without loss of generality, we vertically split these datasets during experiments.

**Benchmarks.** The MPC-based K-means protocol in [33] is the most efficient private K-means scheme among the work that is provable secure. They proposed an ingenious distance computation protocol for centroids to all samples and built a customized garbled circuit to compute binary secret sharing of the minimum. In the remainder of this paper, we term it as M-Kmeans and will compare it in terms of efficiency. Specifically, we use the publicly available implementation\(^1\). As suggested in their paper, we set its computational security parameter \(\kappa = 128\), and the bitlength \(l = 32\).

### 5.2 Comparison with M-Kmeans (Q1)

**Methods.** We compare our model with M-Kmeans in terms of the run time and communication. Specifically, we show the results of the online phase, offline phase, and entire process of our proposed model and compare them with the total process of M-Kmeans. For the sake of fairness, we follow the paper of M-Kmeans [33] to run experiments on the synthetic dataset under LAN. The synthetic data is generated from \(k \in \{2, 5\}\) clusters with the sample size \(n \in \{10^4, 10^5\}\) and feature dimension \(d = 2\). Since the running time and communication increase linearly with the number of iterations, we fix iteration \(t = 10\).

**Results.** We report the results of running time and communication cost in Table 1 and 2 respectively. From them, we can see that (1) the overall cost (both running time and communication) of our model and M-Kmeans is of the same order of magnitude, (2) the online phase of our model is extremely efficient. Our model is about 5x - 6x faster than M-Kmeans in terms of running time and communication volume. Note that if there is a trusted third party that does the offline phase (Beaver’s triples generation), the overall efficiency will improve further.

### 5.3 Study of Online-Offline Setting (Q2)

**Methods.** We examine the effect of online-offline framework from two aspects: running time and communication. To do it, we run experiments on a synthetic dataset consisting of 1,000 data points with 4 clusters and 2 dimensions in the WAN setting and iteration \(t = 10\). Recall that secure Lloyd’s iteration consists of three steps, i.e., secure distance computation (S1), secure clusters assignment (S2), and secure centroids update (S3), as described in Section 4.2. To study the performance of each step, we report results in Figure 2.

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\(^1\)https://github.com/osu-crypto/secure-kmean-clustering

| Parameters | \(n\) | \(k\) | Online | Offline | Total Time | M-Kmeans |
|------------|-------|-------|--------|---------|------------|----------|
| \(10^4\)   | 2     | 0.33  | 1.61   | 1.94    | 1.92       |          |
|            | 5     | 0.94  | 4.70   | 5.64    | 5.81       |          |
| \(10^5\)   | 2     | 3.12  | 15.19  | 18.31   | 18.02      |          |
|            | 5     | 9.06  | 48.39  | 57.45   | 58.09      |          |

Table 1: Comparison of running time (in minutes) on synthetic data \((t = 10, l = 64)\).

| Parameters | \(n\) | \(k\) | Online | Offline | Total Time | M-Kmeans |
|------------|-------|-------|--------|---------|------------|----------|
| \(10^4\)   | 2     | 1,084 | 3,660  | 4,744   | 5,118      |          |
|            | 5     | 3,156 | 12,900 | 16,056  | 18,632     |          |
| \(10^5\)   | 2     | 14,147| 32,598 | 46,745  | 47,342     |          |
|            | 5     | 33,572| 131,243| 164,815 | 192,192    |          |

Table 2: Comparison of communication size (in MB) on synthetic data \((t = 10, l = 64)\).
Results. As clearly seen from Figure 2, the data-independent offline phase is time-consuming and communication-consuming, mainly because of the complicated cryptographic operations. We can conclude that after the time-consuming offline phase is ready, the data-dependent online phase is significantly efficient, which is suitable for most practical applications.

5.4 Experiments for Vectorization (Q3)

Methods. To test the effectiveness of vectorization, we compare running time before and after vectorization under the WAN setting. The synthetic data is generated for 4 clusters with 1,000 samples and feature dimension $d \in \{2, 4, 6, 8\}$. Without loss of generality, we chose the distance calculation step, for example.

Results. Figure 3 shows the speedup gained from vectorization. We can conclude from it that there are significant improvements for both online phase and offline phases. Compared with the numerical operation, the running time after vectorization increases slower with the growth of the feature dimension. In other words, the larger feature dimension, the more improvement of vectorization.

5.5 Experiments for Sparse Optimization (Q4)

Methods. (a) To study our model acceleration in sparse scenarios, we first compare the online running time of our model with and without sparse optimization on a synthetic sparse dataset (with sparse degree 0.2, that is 20% of the elements are 0) in WAN setting. As usual, we fix the sample size to $10^6$, cluster number to 2, and iteration to 10. (b) Secondly, we vary the sparsity degree in $[0, 0.5, 0.9, 0.99]$ and sample size from $10^6$ to $5 \times 10^6$ to test the effectiveness of our sparse optimization. We also choose the distance calculation step, for example.

Results. (a) We can find from Figure 4(a) that both models scale linearly with feature dimension, but the slope of our model with sparse optimization is smaller than that without sparse optimization. The result proves the scalability of sparse optimization. (b) The results shown in Figure 4(b) reveal that our spare optimization can significantly improve efficiency, especially when data sparsity is severe. With the increase in sample size, the improvement becomes more evident, which again proves that our model has good scalability in the large-scale data-sparse scenario.

5.6 Deployment in Fraud Detection (Q5)

Scenario. There is a payment company that provides online payment services, whose customers include both individual users and large merchants. Users can make online transactions with merchants through it. Therefore, rich features, including user features and transaction (context) features in the payment company and merchant, are the key to building intelligent fraud detection models. However, due to the data isolation problem, these data cannot be shared directly. To build a more intelligent fraud detection task, we deploy our work to the above two companies to collaboratively build privacy-preserving K-means clustering.

Methods and Data. We examine the effectiveness of our framework from two aspects. First, we compare its accuracy with M-Kmeans to verify its correctness. Second, we compare it with the plaintext K-means using the payment company data only to show the improvements in multi-party modeling. The real-world dataset consists of 10,000 data points and 42 dimensions, where the payment company has 18 transaction features (e.g., transaction amount) and partial user features (e.g., user age). In contrast, the merchant has 24 other partial user behavior features (e.g., visiting count). Since fraud patterns are diverse and transient, the latest data is used in the application, and 90% of them have no label. In order to compare the effect of multi-party modeling on accuracy improvement, the data used in this paper was collected from earlier happened transactions. We can use the Jaccard coefficient to measure the difference between the outliers found by the algorithm and the ground-truth outliers. The Jaccard coefficient ($J$) is defined as $J(R, R^*) = \frac{|R \cap R^*|}{|R \cup R^*|}$, where $R$ is the set of outliers returned by the algorithm, and $R^*$ is the ground-truth. Note that by design, $0 \leq J(R, R^*) \leq 1$. The higher the value, the closer the two sets are.

Results. We performed 10 runs for each experiment and report the average. The Jaccard coefficient of our framework is 0.86, while the result of M-Kmeans has the same performance with 0.83, which proves that our framework can achieve satisfying performance.
The result of the model using payment company data only is \(0\) which is much lower than that of joint modeling. The results are easy to interpret: more valuable features will naturally improve fraud detection ability. Traditionally, the payment company can build the K-means model using its plaintext features only. With our framework, it can build a better model together with the merchant without compromising their private data.

### 6 CONCLUSION

In this paper, to solve the efficiency and security problem of the existing secure and privacy-preserving K-means models, we propose a novel online-offline vectorized framework to build efficient K-means that provide the full privacy guarantee. Especially, for the data-sparse scenario in fraud detection task, we further adopt the large-scale sparse matrix multiplication in K-means model which combines Homomorphic Encryption (HE) and Secret Sharing (SS) to achieve both efficiency and security. We conduct comprehensive experiments with three synthetic datasets and deploy our model in a real-world fraud detection task. Our experimental results show that the execution time of our online phase is 5x faster than the state-of-the-art solution in average while the overall running time is about the same. In practice, the offline process can be completed in advance. For the dataset with a certain sparse degree, the data size increases, the advantages of our solution become more and more obvious. The results show the efficiency and scalability of our proposed privacy-preserving K-means.

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