$|\Delta S| = 1$ HADRONIC WEAK DECAYS OF HYPERONS IN A SOLITON MODEL

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ABSTRACT

We study the parity violating hyperon non-leptonic weak decays in the three flavor Skyrme model. We follow the approach in which the symmetry breaking terms in the action are diagonalized exactly within the collective coordinate approximation. We show that although this method introduces some configuration mixing, the $\Delta I = 1/2$ rule is numerically well satisfied. In addition, and in contrast to previous calculations, we show that not only the relative amplitudes are in good agreement with the empirical values but also their absolute values. The issue of whether the strong interaction enhancement factors should be included in soliton calculations is also addressed.
Nonleptonic weak decays are still one of the least understood aspects of low energy weak interactions. The main difficulty is related with the evaluation of hadronic matrix elements of the weak hamiltonian. In the absence of good hadronic wave functions obtained directly from QCD one has to resort to effective low energy models. In this sense, quark models with QCD enhancement factors have been quite successful in predicting hyperon S-wave decay amplitudes (see Ref.[1] and references therein). The situation in soliton models seemed to be rather different, however. Calculations performed in the mid-eighties showed[2, 3] that although octet dominance was present in such models (that is, $\Delta I = 1/2$ rule was well satisfied) and predicted relative amplitudes were in good agreement with the empirical values, their absolute values turned out to be far too small. Such calculations have been done using the so-called “perturbative” approach to the $SU(3)$ Skyrme model. In such an approach, $SU(3)$ collective coordinates are introduced to quantize the soliton and symmetry breaking terms are treated in first order perturbation theory. It is well-known by now that this naive approach leads to very poor predictions even for the hyperon spectra[4]. Moreover, in those calculations the pion decay constant $f_\pi$ (taken as a free parameter) has to be adjusted to less than one half of its empirical value in order to reproduce some of the observed mass splittings. This small value of $f_\pi$ was believed to be at the origin of the failure in reproducing the absolute weak decay amplitudes. With the introduction of more refined methods to treat chiral symmetry breaking terms the situation was somewhat improved. In Ref.[5] it was shown that within a framework in which hyperons are treated as soliton-kaon bound systems[6] the calculated matrix elements are indeed larger than those obtained in the perturbative approach. However, they still fall quite below the empirical ones. In this paper we will show that the correct absolute values can be naturally obtained within a scheme in which $SU(3)$ collective coordinates are used but symmetry breaking terms are diagonalized exactly. This approach was pioneered by Yabu and Ando[7] and improved by several authors (for a review see Ref.[8]). As a result of this diagonalization process, configuration mixing appears. One might wonder whether this fact, together with the inclusion of kinematic symmetry breaking terms (needed to obtain good predictions for different observables) will not induce deviations from the empirical well satisfied $\Delta = I = 1/2$ rule. As we will see this is not the case for a reasonable parameterization of the model.

As well-known[9], using PCAC and isospin symmetry the seven different hyperon non-leptonic amplitudes can be expressed in terms of five independent ones. They are related to the parity conserving weak hamiltonian according to

$$A(\Lambda^0) = -\frac{1}{\sqrt{2}f_\pi} < n|H_{pc}^{\Lambda^0,\Delta S=1}|\Lambda > , \quad A(\Sigma^+_0) = \frac{1}{2f_\pi} < p|H_{pc}^{\Sigma^+_0,\Delta S=1}|\Sigma^+ >$$

\(^1\)Here and in what follows we use the phase convention given in Ref.[1]. Note also that $f_\pi$ is defined in such a way that empirically $f_\pi = 93 \text{ MeV}$. 

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where $L$ is the Fermi coupling constant and $\theta$ is the symmetry breaking terms:

$$W = \text{non-linear realization of the pseudoscalar octet.} \quad \Gamma$$

$J_{\mu}^{L,a}$ are the left hadronic currents and $\tilde{G} = G_F \sin \theta_c \cos \theta_c / \sqrt{2}$, where $G_F$ is the Fermi coupling constant and $\theta_c$ is the Cabbibo angle. Moreover, we have used the shorthand notation $\pi^- = 1 - i2$ and $K^+ = 4 + i5$. Within the Skyrme model the currents $J_{\mu,a}^{L}$ can be obtained as Noether currents of the effective chiral action supplemented with appropriate symmetry breaking terms. We use the form

$$\Gamma = \Gamma_{SK} + \Gamma_{WZ} + \Gamma_{SB}, \quad \text{(3)}$$

where $\Gamma_{SK}$ is the Skyrme action

$$\Gamma_{SK} = \int d^4x \left\{ f_\pi^2 \frac{m_\pi^2 + 2f_K^2 m_K^2}{12} \text{Tr} \left[ \partial_\mu U (\partial^\mu U)^\dagger \right] + \frac{1}{32\epsilon^2} \text{Tr} \left[ (U^\dagger \partial_\mu U) U^\dagger \partial_\nu U \right]^2 \right\}. \quad \text{(4)}$$

Here, $\epsilon$ is the dimensionless Skyrme parameter. Furthermore the chiral field $U$ is the non–linear realization of the pseudoscalar octet. $\Gamma_{WZ}$ is the Wess-Zumino action:

$$\Gamma_{WZ} = -\frac{iN_c}{240\pi^2} \int d^5x \epsilon^{\mu\nu\rho\sigma\tau} \text{Tr} [L_\mu L_\nu L_\rho L_\sigma L_\tau] \quad \text{(5)}$$

where $L_\mu = U^\dagger \partial_\mu U$ and $N_c = 3$ is the number of colors. Finally, $\Gamma_{SB}$ represents the symmetry breaking terms:

$$\Gamma_{SB} = \int d^4x \left\{ \frac{f_\pi^2 m_\pi^2 + 2f_K^2 m_K^2}{12} \text{Tr} \left[ U + U^\dagger - 2 \right] + \sqrt{3} \frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{6} \text{Tr} \left[ \lambda_8 \left( U + U^\dagger \right) \right] \right. \right.
\left. \left. + \frac{f_K^2 - f_\pi^2}{12} \text{Tr} \left[ \left( 1 - \sqrt{3} \lambda_8 \right) \left( U(\partial_\mu U) U^\dagger \partial_\nu U + U^\dagger \partial_\mu U(\partial^\nu U) \right) \right] \right\}. \quad \text{(6)}$$

Here $f_K$ is the kaon decay constant while $m_\pi$ and $m_K$ are the pion and kaon masses, respectively.

A straightforward calculation shows that the corresponding left current can be expressed as

$$J_{\mu,a}^{L} = -\frac{i}{2} f_\pi^2 \text{Tr} \left( \lambda_a R_\mu \right) + \frac{i}{8\epsilon^2} \text{Tr} \left[ \left[ \lambda_a, R_\nu \right] [R_\mu, R_\nu] \right)$$

$$+ \frac{N_c}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( \lambda_a R_\mu R_\alpha R_\beta \right) - i \frac{f_K^2 - f_\pi^2}{12} \text{Tr} \left( 1 - \sqrt{3} \lambda_8 \right) \left[ U, \lambda_a \right] R_\mu, \quad \text{(7)}$$

where $R_\mu = \partial_\mu U U^\dagger$. The contribution of the different terms in Eq. (6) to the left current can be easily recognized.
In the soliton picture we are using the strong interaction properties of the low–lying \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) baryons are computed following the standard \( SU(3) \) collective coordinate approach to the Skyrme model. We introduce the ansatz

\[
U(r, t) = A(t) \begin{pmatrix} c + i \mathbf{r} \cdot \hat{\mathbf{r}} & s & 0 \\ 0 & 1 \end{pmatrix} A^{\dagger}(t) \tag{8}
\]

for the chiral field. Here we have employed the abbreviations \( c = \cos F(r) \) and \( s = \sin F(r) \) where \( F(r) \) is the chiral angle which parameterizes the soliton. The collective rotation matrix \( A(t) \) is \( SU(3) \) valued. Substituting the configuration Eq.(8) into \( \Gamma \) yields (upon canonical quantization of \( A \)) the collective Hamiltonian. Its eigenfunctions and eigenvalues are identified as the baryon wavefunctions \( \Psi_B(A) = \langle B | A \rangle \) and masses \( m_B \). Due the symmetry breaking terms in \( \Gamma_{sb} \) this Hamiltonian is obviously not \( SU(3) \) symmetric. As shown by Yabu and Ando [7] it can, however, be diagonalized exactly. This diagonalization essentially amounts to admixtures of states from higher dimensional \( SU(3) \) representations into the octet \((J = \frac{1}{2})\) and decouplet \((J = \frac{3}{2})\) states. This procedure has proven to be quite successful in describing the hyperon spectrum and static properties [8].

Using the ansatz Eq.(8) in the expression of the left current we obtain that, to leading order in \( N_c \), the weak hamiltonian can be written as

\[
H^{pc}_{\omega, S=1} = -\phi_{SK} R_{\pi^{-}, a} R_{K^{+}, a} + \phi_{WZ} R_{\pi^{-}, 8} R_{K^{+}, 8} - \phi_{SB} \left[ \left( \frac{1 + 2R_{8,8}}{2} \right) R_{\pi^{-}, a} R_{K^{+}, a} + \left( \frac{2 + R_{8,8}}{3} \right) R_{\pi^{-}, 8} R_{K^{+}, 8} \right], \tag{9}
\]

where

\[
\phi_{SK} = \frac{G f_{\pi}^4}{3} \int d^3 r \left( F'^2 + 2 \frac{s^2}{r^2} \right) + \frac{4}{e^2 f_{\pi}^2} \frac{s^2}{r^2} \left( 2F'^2 + \frac{s^2}{r^2} \right) \left( F'^4 + 4F'^2 \frac{s^2}{r^2} + \frac{s^4}{r^4} \right), \tag{10}
\]

\[
\phi_{WZ} = \frac{G N_c^2}{48 \pi^4} \int d^3 r \ F'^2 \frac{s^4}{r^4}, \tag{11}
\]

\[
\phi_{SB} = \frac{G f_{\pi}^2}{9} \left( f_{K}^2 - f_{\pi}^2 \right) \int d^3 r \ (1 - c) \left( F'^2 + \frac{s^2}{r^2} \right) + \frac{4}{e^2 f_{\pi}^2} \frac{s^2}{r^2} \left( 2F'^2 + \frac{s^2}{r^2} \right)). \tag{12}
\]

The \( SU(3) \) rotation matrices are defined by

\[
R_{a,b} = \frac{1}{2} Tr \left( \lambda_a A^\dagger \lambda_b A \right). \tag{13}
\]

For simplicity, in Eq.(13) we have not written the contribution quadratic in \( (f_{K}^2 - f_{\pi}^2) \) since for empirical values of the decay constants it turns out to be numerically completely negligible.
The present model the hyperon decay amplitudes can be computed by taking the matrix elements of the hamiltonian Eq.(9) between the hadronic states expressed as linear combinations of SU(3) D-functions. For this purpose it is convenient to use the Clebsch-Gordan decomposition of the collective operators appearing in the weak hamiltonian. One obtains

\[
R_{\pi^-,a} R_{K^+,a} = -\frac{3\sqrt{6}}{5} D_{\frac{3}{2},0}^8 - \frac{1}{10} D_{\frac{7}{2},0}^{27} - \frac{1}{\sqrt{20}} D_{\frac{7}{2},0}^{27}
\]

and similar relations for those containing \( R_{\pi^-,8} R_{K^+,8} \). Here, the left lower index of the SU(3) D-functions \( I = \frac{1}{2}, \frac{3}{2} \) stands for \( (Y, I, I_3) = (1, I, -\frac{1}{2}) \) while the right lower index for \( (0, 0, 0) \).

At this stage we note the potential advantages and drawbacks of the present approach with respect to the perturbative calculations of Refs.\[2,\ 3\]. On one hand the use of an exact diagonalization allows for the use of empirical meson decay constants \[8\]. This will certainly lead to an improvement of the decay amplitudes absolute values. On the other hand, since as a consequence of this diagonalization baryon wavefunctions contain higher SU(3) representations the relevant matrix elements of the collective operators Eqs.(14,15) will not be, in general, “octet dominated”. In this sense, it is not clear whether the \( \Delta I = 1/2 \) rule will be well satisfied as it was the case in the perturbative calculation.

We turn now to the numerical calculations. We take the meson masses to their empirical values \( m_\pi = 138 \text{ MeV} \) and \( m_K = 495 \text{ MeV} \). Moreover, we use the empirical value \( f_\pi = 93 \text{ MeV} \). As already stressed several times in the literature the use of \( f_\pi \neq f_K \) is essential to reproduce the observed mass differences of the low lying octet and decouplet baryons. Therefore, we take \( f_K = 120 \text{ MeV} \) which together with \( \epsilon = 4.10 \) gives a very good overall description of various hyperon properties \[8\]. As well-known with these parameters the soliton mass turns out to be, at tree level, quite large as compared to the value needed to reproduce the empirical nucleon mass. However, in the last few years it was shown \[10\] that, within the SU(2) soliton model, the inclusion of one-loop meson corrections reduces that value significantly. Very recently \[11\] this same conclusion was extended to the SU(3) models. Therefore, at present time, the parameter set above can be considered as the optimal one within the approach adopted here.

Our results for the decay amplitudes are given in Tables 1 and 2. In Table 1 we show the decay amplitude taken with respect to \( A(\Lambda^0) \) while in Table 2 we give the absolute value of this particular amplitude. The results are presented in this way to make easier the comparison with the values obtained in other models. In fact, also shown in Table 1 are those of the perturbative approach (PTA) to the SU(3) soliton model \[4\], the bound state soliton model (BSA) \[5\] and the empirical values taken from Ref.\[1\]. The value for
the quark model (QM) that appears in Table 2 has been taken from Ref.[12]. Note that in Table 1 only the values of the independent amplitudes Eq.(2) are given. The reason is that all the corresponding model calculations (and the QM as well) are based on the use of PCAC and isospin symmetry which implies

\[ \frac{A(\Lambda^0_0)}{A(\Lambda^0_0)} = \frac{A(\Xi^0_0)}{A(\Xi^-)} = -\frac{1}{\sqrt{2}}. \] (16)

Although the \(\Lambda\)-ratio is not known empirically, the \(\Xi\)-ratio is \(\frac{A(\Xi^0_0)}{A(\Xi^-)}^{emp} = -0.75\). This is usually taken as an indication that PCAC and isospin symmetry can be used in this framework.

In Table 1 we observe that the relative values of the decay amplitudes are quite well reproduced in our model. Of particular interest is \(A(\Sigma^+_\pm)\). In the limit in which the \(\Delta I = 1/2\) rule is exactly satisfied this amplitude is zero. We see that our value, although small, does not vanish. In fact, it nicely reproduces the small departure from the \(\Delta I = 1/2\) rule verified by the empirical amplitudes. The reason for the smallness of our calculated value even in the presence of configuration mixing is twofold. Firstly, higher order representations although essential to obtain a reasonable hyperon spectrum appear with a quite small weight in the low-lying hyperon wavefunction. Secondly, the collective operators that contain stronger “non-octet” contributions (as i.e. \(R_{8,8}R_{\pi^-,-a}R_{K^+,a}\)) appear in terms proportional to \(\phi_{SB}\) which is, numerically, one order of magnitude smaller than the leading contributions (terms proportional to \(\phi_{SK}\)). Nevertheless, as already mentioned above, these “dynamical” symmetry breaking terms are important to obtain good mass splittings. Also in Table 1 we observe that the other calculated ratios are (in absolute value) somewhat larger than the empirical ones. However, they are basically of the same quality as those of the PTA or the BSA. In any case, the main success of the present model over the other soliton approaches is in the prediction of the absolute values of the decay amplitudes. Since we have seen that the ratios to the \(A(\Lambda^0_0)\) amplitude are reasonably described, it is enough to consider the absolute value of such quantity. From Table 2 we see that our calculated value is in good agreement with the empirical one. The improvement with respect to the PTA and BSA is very significant and shows that the use of empirical values for the model parameters is essential to describe the weak decay amplitudes correctly. In Table 2 we also see that our prediction is somewhat better than that of the QM. However, it should be noticed that in the QM this amplitude is particularly problematic. In general, the QM results are of the same quality than ours.

Finally, we discuss the role of strong interaction enhancement factors used in previous soliton calculations. These factors were introduced in the context of the quark model to account for hard gluon exchanges[13]. It is not clear whether they should also be used in soliton calculations since, in principle, they could be already contained in the non-perturbative soliton currents. This question was already raised in the context of the \(SU(2)\) soliton model (see i.e. Ref.[14]). The results corresponding to PTA and BSA given
in Table 2 do include an enhancement factor \( c_1 \approx 2.6 \). In our calculation we have not used such factor. The good agreement we have found with respect to the empirical value of \( A(A^0) \) seems to clearly indicate that there is no need for the enhancement factors in the soliton models.

In conclusion, we have studied the S-wave non-leptonic weak decay amplitudes of the hyperon in the context of an \( SU(3) \) soliton model in which strangeness degrees of freedom are introduced through collective variables and symmetry breaking terms are diagonalized exactly. We have obtained a very nice agreement of both the relative and absolute amplitudes with the corresponding empirical values. In fact, we have found a substantial improvement in the prediction of the absolute amplitudes with respect to previous soliton calculations. Finally, we have seen that although the present approach includes some configuration mixing the corresponding impact on the “octet dominance” is small enough to guarantee that the empirical “\( \Delta I = 1/2 \)” is still well satisfied.

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Table 1: The nonleptonic hyperon decay amplitudes taken with respect to $A(\Lambda^0)$ amplitude.

|             | This work | PTA [2] | BSA [3] | Empirical [1] |
|-------------|-----------|---------|---------|---------------|
| $A(\Sigma^+_0)$ | 0.05      | 0.00    | 0.00    | 0.04          |
| $A(\Sigma^-_0)$ | –1.26     | –1.00   | –1.00   | –1.00         |
| $A(\Sigma^-)$   | 1.74      | 1.43    | 1.41    | 1.31          |
| $A(\Xi^-)$      | –1.54     | –1.43   | –1.73   | –1.39         |

Table 2: Absolute value of the S-wave $\Lambda \to p\pi^-$ decay amplitude.

|               | $A(\Lambda^0) \times 10^6$ |
|---------------|-----------------------------|
| This work     | 0.35                        |
| PTA [2]       | 0.07                        |
| BSA [3]       | 0.08                        |
| QM [12]       | 0.21                        |
| Empirical [1] | 0.32                        |