Probing D0-D6 Black Hole Configuration:
2-Loop Matrix Theory Correction to D0-Brane
Effective Action

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Abstract

We present a calculation of the matrix theory 2-loop effective action for a D0-brane in the background of the recently discussed D0-D6 bound state configuration. The effective DBI action of a D0-brane probe in the background of the corresponding 4-dimensional non-supersymmetric black hole solution to low-energy type IIA string theory compactified on a 6-torus is known to agree with the matrix theory calculation at 1-loop order, in the limit in which the ratio of the D0-brane to the D6-brane charges carried by the black hole is large. Agreement at 2-loop between the supergravity description and a conjectured nonabelian BDI effective super Yang-Mills description has also been recently reported. However, we find uncanceled ultraviolet divergences in our direct matrix theory calculation of the 2-loop effective action. This is consistent with the expected nondecoupling of massive open string states from the 6-brane.

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1 Introduction and Summary

Recently several authors have discussed a non-supersymmetric configuration of D0- and D6-branes in the context of supergravity/superYang-Mills correspondence conjectured by matrix theory. These studies have uncovered many interesting features of this brane configuration. This configuration appears as a classical solution in (6+1)-dimensional superYang-Mills theory. The energy spectrum of the configuration with arbitrary number of D0- and D6-branes, which is labelled by the corresponding Ramond-Ramond charges, turns out to be identical to the mass spectrum of a non-supersymmetric extremal black hole solution to the classical equations of low-energy type IIA string theory compactified on a six-torus down to 4-dimensions. This is surprising because in the absence of supersymmetry in this system one would have expected the weak coupling gauge theory result for the energy spectrum to get modified in the strong coupling regime corresponding to supergravity. Furthermore, the configuration has been studied with various brane probes and agreement has been found at the 1-loop level between matrix theory and supergravity calculations, in the limit of large zero to six brane charge ratio. In this limit, a D0-brane scattering off this configuration is dominated by D0-D0 scattering and this is what explains the agreement between the two calculations of the $v^4$ terms in the 1-loop effective action, even in the absence of supersymmetry in this system. However, the agreement of the $v^2$ and constant ($v$-independent) terms in the effective action is difficult to understand since these terms manifestly represent the supersymmetry breaking interactions between D0- and D6-brane. Recently, a 2-loop calculation for a D6-brane probe using the effective superYang-Mills action of Chepelev and Tseytlin has also been reported to be in agreement with the corresponding supergravity calculation at 2-loop. These results are surprising not only because the configuration is non-supersymmetric but also because one does not expect the bulk to decouple from the 6-brane. It is, therefore, of interest to ask whether the agreement at 2-loop level persists in a direct matrix theory calculation.

In this paper we present a direct calculation in the matrix theory framework of the 2-loop effective action of a D0-brane probe in the background of the D0-D6 bound state configuration. We find uncanceled divergences in the matrix theory calculation. These divergences seem to be related to the ultraviolet divergences one might expect in a (6 + 1)-dimensional field theory. This result, which is expected, makes the agreement found in all the more intriguing.

The present work draws heavily on and should be regarded as a sequel to that work. We will follow the conventions and notations of this reference. We also refer the reader to this work for all the background material which we will need here. The organization of this paper is as follows. In the next section we first briefly discuss the D0-D6 configuration given in and then give the terms in the matrix theory action, expanded around this configuration, which are relevant to the 2-loop calculation. In section 3 we give the various 2-point functions needed for the calculation. The calculation of the 2-loop effective action is presented in section 4. In section 5 we discuss the origin of the uncanceled divergences in the 2-loop effective action. We end with some concluding remarks in section 6. Expressions for the various terms in the 2-loop effective action are collected together in the Appendix.
2 The Matrix Theory Action

In this section we will expand the matrix theory action around the D0-D6 background configuration discussed in section 5 of [3], and briefly reviewed below, and obtain the terms relevant for a 2-loop calculation. The matrix theory action is

\[
S = \frac{1}{2g} \int dt \, \text{Tr}\left\{ (D_{i}X_{j})^{2} + \frac{1}{2}[X_{i}, X_{j}]^{2} - (\bar{D}_{i}A)^{2} + \theta^{T}D_{i}\theta \\
+ i\theta^{T}\gamma^{i}[X_{i}, \theta] + 2\partial_{i}C^{T}D_{i}C - 2[C^{T}, B_{i}][X_{i}, C] \right\}
\]  

(2.1)

The notations and conventions are the same as in [3].

Background Configuration

The D0-D6 background configuration consists of multiple D6-branes with the D0-branes appearing on them as magnetic fluxes. We present this configuration below:

\[
B_{4,6,8} = \begin{bmatrix}
Q_{1,2,3} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0
\end{bmatrix}, \quad B_{5,7,9} = \begin{bmatrix}
P_{1,2,3} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0
\end{bmatrix}
\]

Here, the entry in the lower right corner is a single-element one. This entry is reserved for the probe D0-brane. For a single D6-brane, \([Q_{a}, P_{a}] = ic_{a}, a = 1, 2, 3\). For multiple parallel D6-branes, the \(Q_{a}\)’s and the \(P_{a}\)’s have a further structure:

\[
Q_{a} = \begin{bmatrix}
Q_{a}^{(1)} & Q_{a}^{(2)} & \cdots & Q_{a}^{(Q_{6})}
\end{bmatrix}, \quad P_{a} = \begin{bmatrix}
P_{a}^{(1)} & P_{a}^{(2)} & \cdots & P_{a}^{(Q_{6})}
\end{bmatrix},
\]

where \([Q_{a}^{(1)}, P_{a}^{(1)}] = ic_{a}^{(1)}, \) etc. The D6-branes are wrapped on \(T^{6}\) with volume \(V_{6}\) which is assumed to be large since we will be neglecting the effect of winding modes.

The \(Q_{6}\) D6-branes are organized into four sets, each consisting of \(n\) D6-branes, where \(4n = Q_{6}\). Each D6-brane in the first set carries magnetic fluxes \((F_{45}, F_{67}, F_{89}) = (f, f, f)\). The D6-branes in the other three sets carry the magnetic fluxes \((f, -f, -f), (-f, f, -f)\) and \((-f, f, f)\). Here \(f\) is the parameter which is related to the ratio of the D0-brane to D6-brane charge in the bound state [3]. In view of the above, a more suitable notation for the \(Q_{a}\)’s and \(P_{a}\)’s is \(Q_{a}^{\alpha}, P_{a}^{\alpha}, \) where \(\ell = 1, 2, 3, 4\) and \(\alpha = 1, 2, \cdots, n\) and \([Q_{a}^{\alpha}, P_{b}^{\mu\beta}] = i\delta_{ab}\delta^{\alpha\beta}\delta^{\mu\nu}c_{a}^{\alpha}\). For the desired configuration we need \(c_{a}^{\alpha} = c_{a}^{\ell}\) to be independent of \(\alpha\). Moreover, the four triplets of numbers \(\{c_{\ell}^{a}\} = (c_{\ell}^{1}, c_{\ell}^{2}, c_{\ell}^{3}) \equiv \bar{c}_{\ell}\) correspond to the four triplets of fluxes listed above and so we may write

\[
\bar{c}_{\ell} = c\bar{c}_{\ell}, \quad \bar{c}_{1} = (1, 1, 1), \quad \bar{c}_{2} = (1, -1, -1), \quad \bar{c}_{3} = (-1, 1, 1), \quad \bar{c}_{4} = (-1, 1, -1).
\]  

(2.2)

Agreement with supergravity at 1-loop requires \(c = f^{-1} \to 0\).
A D0-brane scattering off this background in a direction transverse to the D6-branes is represented by the additional background fields

\[
\begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & vt
\end{bmatrix}, \quad
\begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & b
\end{bmatrix}, \quad B_3 = 0.
\]

The only nonzero entry in the above matrices is in the lower right corner. Here \(v\) is the velocity of the D0-brane, assumed to be moving along the \(x^1\) direction, and \(b\) is the impact parameter.

**Fluctuations**

There are basically two different types of fluctuations that we need to consider. The first type are the ones with nonzero entries only in the last row or column of the various matrix variables. These represent open strings connecting the probe D0-brane to the branes in the background. These type of fluctuations are the only ones that contribute at the 1-loop level. We shall call them “column type” fluctuations. The other type of fluctuations have nonzero entries everywhere other than the last row and the last column. These “matrix type” fluctuations represent open strings connecting the various branes in the background. They do not contribute to the 1-loop effective action, but do contribute at 2-loop level and beyond. We have done the 2-loop computation in the limit of a large number of D6-brane in the background configuration, i.e. for large values of \(Q_6\). The fluctuation in the bottom right corner entry in the various matrix variables can be ignored for the leading contribution in this limit. This is what we have done in the following.

Writing \(X_i = B_i + \sqrt{g} Y_i\), we have

\[
Y_i = \begin{bmatrix}
Z_i & \cdots & \phi_i \\
\vdots & \ddots & \vdots \\
\phi_i^\dagger & \cdots & 0
\end{bmatrix}
\]

Similarly,

\[
A = \sqrt{g} \begin{bmatrix}
Z_A & \cdots & \phi_A \\
\vdots & \ddots & \vdots \\
\phi_A^\dagger & \cdots & 0
\end{bmatrix}, \quad \Theta = \sqrt{g} \begin{bmatrix}
Z_\theta & \cdots & \chi_\theta \\
\vdots & \ddots & \vdots \\
\chi_\theta^\dagger & \cdots & 0
\end{bmatrix}, \quad C = \sqrt{g} \begin{bmatrix}
Z_c & \cdots & \phi_c \\
\vdots & \ddots & \vdots \\
\phi_c^T & \cdots & 0
\end{bmatrix}
\]

Note that because \(C\) is not hermitian, \(\bar{\chi}_c \neq \chi_c^*\). Also, in the notation that we have used for the background, the fluctuations have the index structure \(\phi^{\ell \alpha}, \chi^{\ell \alpha}, Z^{\ell \alpha, m \beta}\). Finally, it is useful to parameterize the background as

\[
B_i = \begin{bmatrix}
D_i & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & d_i
\end{bmatrix}.
\]
Action for Fluctuations

The action (2.1) may now be expanded around the background $B_i$. It is convenient to write the result as follows:

$$S = S_{Bgd} + S_2 + S_{\text{Int}}.$$  \hfill (2.3)

Here $S_2$ is the part of the action quadratic in fluctuations and $S_{\text{Int}}$ is the part containing interactions. Furthermore, it is convenient to write

$$S_2 = S_2^{X+A} + S_2^\theta + S_2^C.$$  \hfill (2.4)

We find,

$$S_2^{X+A} = \int d\tau \left\{ \frac{1}{2} \text{Tr} \left( \dot{Z}_i^2 - [D_i, Z]^2 - 2[D_i, D_j][Z_i, Z_j] \right) + \phi_i^\dagger \dot{\phi}_i + \phi_j^\dagger D_i^2 \phi_j + 2\phi_i^\dagger [D_i, D_j] \phi_j + \frac{1}{2} \text{Tr} \left( \dot{Z}_i^2 - [D_i, Z_A]^2 \right) + \dot{\phi}_A^\dagger \phi_A + \phi_A^\dagger D_i^2 \phi_A + 2i\dot{\phi}_i \left( \phi_i^\dagger \phi_A - \phi_A^\dagger \phi_i \right) \right\}.$$  \hfill (2.5)

$$S_2^\theta = \int d\tau \left\{ \frac{i}{2} \text{Tr} \left( Z_\theta^T \dot{Z}_\theta - Z_\theta^T Z_\theta \right) + i\chi_\theta \dot{\chi}_\theta - i\chi_\theta \gamma^T D_i \chi_\theta \right\},$$  \hfill (2.6)

$$S_2^C = \int d\tau \left\{ \text{Tr} \left( \dot{Z}_i^T \dot{Z}_c - [D_i, Z_c^T][D_i, Z_c] \right) + \dot{\phi}_c \phi_c + \phi_c^\dagger D_i^2 \phi_c + \dot{\phi}_c^\dagger D_i^2 \phi_c \right\}.$$  \hfill (2.7)

In writing the above, we have already made a Wick rotation to Euclidean time, $t \rightarrow i\tau$, $A \rightarrow -iA$ and $vt \rightarrow v_E \tau$.\footnote{Here $v_E = iv$ is the Euclidean velocity, to be taken real during the course of this calculation. For convenience of notation, we will drop the subscript ‘E’ in the following.} Also, a dot represents derivative with respect $\tau$, and we have used the notation

$$D_i \equiv D_i - d_i 1.$$ \hfill (2.8)

Similarly to the quadratic piece, it is convenient to write the interaction piece of the action for fluctuations as

$$S_{\text{Int}} = S_{\text{Int}}^{X+A} + S_{\text{Int}}^\theta + S_{\text{Int}}^C.$$ \hfill (2.9)

We find

$$S_{\text{Int}}^{X+A} = \sqrt{g} \int d\tau \left\{ \left( \phi_i^\dagger D_i Z_j \phi_j + \phi_j^\dagger D_i Z_i \phi_i - 2\phi_j^\dagger D_i Z_j \phi_i + h.c. \right) + \left( \phi_i^\dagger D_i Z_A \phi_A + \phi_A^\dagger D_i Z_i \phi_A - 2\phi_A^\dagger D_i Z_A \phi_i + h.c. \right) + \left( i\phi_i^\dagger Z_i \phi_A + i\phi_i^\dagger Z_A \phi_i + 2i\phi_i^\dagger Z_i \phi_A + h.c. \right) \right\} + g \int d\tau \left\{ \phi_i^\dagger Z_i Z_j \phi_j + \phi_j^\dagger Z_i^2 \phi_j - 2\phi_i^\dagger Z_i Z_j \phi_j \right\}.$$
\[ S_{\text{int}}^\theta = \sqrt{g} \int d\tau \left\{ -i\chi_0^\dagger \chi_0 iZ_0^i\phi_i + i\phi_i^\dagger Z_0^i\gamma_i\chi_0 - \chi_0^\dagger Z_0^i\phi_A - \phi_A^\dagger Z_0^i\chi_0 + \chi_0^\dagger Z_A^i\phi_A \right\}. \]  

(2.10)

\[ S_{\text{int}}^C = \sqrt{g} \int d\tau \left\{ \phi_c^T \left[ D_1, Z_c^\dagger \right] \phi_c + \phi_i^T \left[ D_i, Z_i^\dagger \right] \phi_i - \phi_c^T Z_i \phi_c + \phi_i^T Z_c \phi_i + \phi_c^T Z_c \phi_c \right. \]

\[ \left. + \phi_i^T D_i Z_i \phi_i - \phi_i^T D_i Z_c \phi_i - i \left( \phi_c^T Z_i^\dagger \phi_A + \phi_i^T Z_c \phi_A + \phi_A^T Z_i^\dagger \phi_c \right) \right\}. \]

(2.12)

In the above, ‘h.c.’ stands, as usual, for hermitian conjugate. Also, we have dropped all terms that either do not contribute to the 2-loop effective action of the probe D0-brane in the leading large \( Q_0 \) limit or contribute terms that do not depend on the impact parameter \( b \) and the velocity \( v \) of the probe.

Note that in the notation we have used for the background, a term like, for example, \( \text{Tr} \bar{Z}_i^2 \) reads \( \sum_{i,m=1} \sum_{\alpha,\beta=1} \text{tr}(\bar{Z}_i^{\alpha\beta} \bar{Z}_i^{\eta\alpha\beta}) \). The remaining trace ‘tr’ is over the space in which the harmonic oscillator variables \( Q_\alpha^a, P_\alpha^{a0} \) operate.

## 3 Two-Point Functions

There are two different kinds of 2-point functions corresponding to the two different kinds of fluctuations, namely “column type” fluctuations \( \phi_i, \phi_A, \chi_0, \phi_c \) and \( \phi_i \) and “matrix type” fluctuations \( Z_i, Z_A, Z_\theta, Z_c \). One minor complication in both the sectors is that there is mixing at the quadratic level, so the propagators cannot be immediately read-off from the quadratic action, \( S_2 \), and a rediagonalization is needed. Furthermore, to obtain explicit expressions for the 2-point functions, it is necessary to use an explicit representation for the Heisenberg algebra, \( \left[ Q_\alpha^a, P_\beta^{a0} \right] = i\epsilon^{\alpha\beta} \delta^{\alpha\beta} \epsilon_\ell^a \). In the limit \( c \to 0 \), we may use the representation in terms of functions of 3 real variables, one for each of the three values of the index ‘\( \eta \)’. We will denote these three real variables by the triplet \( \bar{x} \equiv (x_1, x_2, x_3) \).

Note that in terms of there variables the explicit form for a term like, for example, \( \text{Tr} \bar{Z}_i^2 \) is \( \sum_{i,m=1} \sum_{\alpha,\beta=1} \int d^3\bar{x} \int d^3\bar{y} |\bar{Z}_i^{\alpha\beta}(\bar{x}, \bar{y}, \tau)|^2 \). We will now list below all the nonzero 2-point functions. It is convenient to use the proper time representation and this is what we do below.

**Column Type Fluctuations**

Let us first consider the “column type” fluctuations, \( \phi_i, \phi_A, \) etc. Introducing the notation

\[ \langle \phi_i^{(\alpha)}(\bar{x}, \tau) \phi_j^{(\beta)}(\bar{x}', \tau') \rangle \equiv \delta^{\alpha\beta} \Delta^{(\alpha\beta)}(\bar{x}; \bar{x}', \tau, \tau'), \]

(3.1)
where \( I, J \epsilon(i, A, c, \tilde{c}, \theta) \) we have

\[
\Delta_{\alpha}^{\alpha} = \Delta_{11}^{\alpha} = \int_{0}^{\infty} ds \, \cosh 2vs \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (3.2)
\]

\[
\Delta_{\alpha}^{\alpha} = \Delta_{1A}^{\alpha} = i \int_{0}^{\infty} ds \, \sinh 2vs \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (3.3)
\]

\[
\Delta_{\alpha}^{22} = \Delta_{33}^{\alpha} = \Delta_{66}^{\alpha} = \Delta_{cc}^{\alpha} = \int_{0}^{\infty} ds \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (3.4)
\]

\[
\Delta_{ii}^{\alpha} = \int_{0}^{\infty} ds \, \cosh 2cs \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (i = 4, 5, \cdots, 9) \quad (3.5)
\]

\[
\Delta_{45}^{\alpha} = \Delta_{54}^{\alpha} = \int_{0}^{\infty} ds \, \sinh 2cs \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (3.6)
\]

\[
\Delta_{67}^{\alpha} = \Delta_{76}^{\alpha} = \int_{0}^{\infty} ds \, \sinh 2cs \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (3.7)
\]

\[
\Delta_{89}^{\alpha} = \Delta_{98}^{\alpha} = \int_{0}^{\infty} ds \, \sinh 2cs \, K_{s}(\vec{x}, \tau; \vec{x}' \tau'), \quad (3.8)
\]

\[
\Delta_{\theta \theta}^{\alpha} = i \left( \partial_{\tau} + \gamma^{j} \mathcal{D}_{jx}^{\alpha} \right) \int_{0}^{\infty} ds \, \exp \left[ -s(v\gamma^{1} + c\vec{\sigma} \cdot \vec{\epsilon}_{\ell}) \right] K_{s}(\vec{x}, \tau; \vec{x}' \tau'). \quad (3.9)
\]

In the above

\[
K_{s}(\vec{x}, \tau; \vec{x}' \tau') = \frac{1}{4\pi^{2}}(v/c^{3} \sinh^{3} 2cs \sinh 2vs)^{1/2} e^{-sb^{2}} \times \exp \left[ -\frac{v}{4} \left\{ (\tau + \tau ')^{2} \tanh vs + (\tau - \tau ')^{2} \coth vs \right\} \right. \\
\left. -\frac{1}{4c} \left\{ (\vec{x} + \vec{x}')^{2} \tanh cs + (\vec{x} - \vec{x}')^{2} \coth cs \right\} \right]. \quad (3.10)
\]

Also, we have used the following notation in (3.9):

\[
\mathcal{D}_{1x}^{\alpha} = -v \tau, \quad \mathcal{D}_{2x}^{\alpha} = -b, \quad \mathcal{D}_{3x}^{\alpha} = 0,
\]

\[
\left( \mathcal{D}_{4x}^{\alpha}, \mathcal{D}_{6x}^{\alpha}, \mathcal{D}_{8x}^{\alpha} \right) = (x_{1}, x_{2}, x_{3}),
\]

\[
\left( \mathcal{D}_{5x}^{\alpha}, \mathcal{D}_{7x}^{\alpha}, \mathcal{D}_{9x}^{\alpha} \right) = -ic \left( \epsilon_{1} \frac{\partial}{\partial x_{1}}, \epsilon_{2} \frac{\partial}{\partial x_{2}}, \epsilon_{3} \frac{\partial}{\partial x_{3}} \right), \quad (3.11)
\]

and

\[
\vec{\sigma} = (\sigma^{1}, \sigma^{2}, \sigma^{3}) \equiv (i\gamma^{A} \gamma^{5}, i\gamma^{6} \gamma^{7}, i\gamma^{8} \gamma^{9}). \quad (3.12)
\]

\[\text{Here the subscript } \tilde{c} \text{ refers to } \tilde{\phi}_{c}.\]
Matrix Type Fluctuations

We now consider the matrix type fluctuations, \(Z_i, Z_A\), etc. We introduce a notation similar to (3.1),

\[
\langle Z_i^{f_a,m^β}(\vec{x}, \vec{y}, τ) Z_j^{c_α,m^β'}(\vec{x'}, \vec{y'}, τ') \rangle \equiv \delta^{ij} \delta^{mm'} \delta^{αα'} \delta^{ββ'} \Lambda_{ij}^{f_a,m^β}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

where \(I, J\ell(= a, c, θ)\). A complication here is that the cases (i) \(ℓ = m\) and (ii) \(ℓ ≠ m\) need to be considered separately since the 2-point functions are different in the two cases. This is because there is mixing at the quadratic level in case (ii). The 2-point functions, however, depend only on the combination \(\{ε^a_{\ell m}\} \equiv \{ε^a_{\ell m}\}\) and therefore only four independent cases need to be considered, instead of a possible sixteen. We now list all the nonvanishing 2-point functions in their proper time representation.

(i) \(ℓ = m\)

\[
\Lambda_{\thetaθ}^{f_a,ℓβ} = i \{ \partial_τ + γ^i (D_{f_x}^{f_a} - D_{f_y}^{ℓβ}) \} \tilde{Λ}_{θθ}^{f_a,ℓβ},
\]

\[
\Lambda_{ii}^{f_a,ℓβ} = \Lambda_{AA}^{f_a,ℓβ} = \Lambda_{cc}^{f_a,ℓβ} = \tilde{Λ}_{θθ}^{f_a,ℓβ}
\]

\[
= \frac{1}{16π^2c^4} \int_0^{∞} \frac{ds}{s^2} \exp \left[ -s(\vec{x} - \vec{y})^2 - \frac{(\vec{x} - \vec{x'})^2}{4c^2s} - \frac{(τ - τ')^2}{4s} \right] δ^{(3)}((\vec{x} - \vec{x'}) - (\vec{y} - \vec{y}')).
\]

(ii) \(ℓ ≠ m\)

\[
\Lambda_{ii}^{f_a,m^β}(i = 1, 2, 3) = \Lambda_{AA}^{f_a,m^β} = \Lambda_{cc}^{f_a,m^β} = \int_0^{∞} ds \ G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{44}^{f_a,m^β} = \Lambda_{55}^{f_a,m^β} = \int_0^{∞} ds \ \cosh \left[ 2cs(1 - ε^a_{\ell m}) \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{45}^{f_a,m^β} = -\Lambda_{54}^{f_a,m^β} = -iε^a_{\ell m} \int_0^{∞} ds \ \sinh \left[ 2cs(1 - ε^a_{\ell m}) \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{66}^{f_a,m^β} = \Lambda_{77}^{f_a,m^β} = \int_0^{∞} ds \ \cosh \left[ 2cs(1 - ε^a_{\ell m}) \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{67}^{f_a,m^β} = \Lambda_{76}^{f_a,m^β} = -iε^a_{\ell m} \int_0^{∞} ds \ \sinh \left[ 2cs(1 - ε^a_{\ell m}) \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{88}^{f_a,m^β} = \Lambda_{99}^{f_a,m^β} = \int_0^{∞} ds \ \cosh \left[ 2cs(1 - ε^a_{\ell m}) \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{89}^{f_a,m^β} = \Lambda_{98}^{f_a,m^β} = -iε^a_{\ell m} \int_0^{∞} ds \ \sinh \left[ 2cs(1 - ε^a_{\ell m}) \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ'),
\]

\[
\Lambda_{θθ}^{f_a,m^β} = i \{ \partial_τ + γ^i (D_{f_x}^{f_a} - D_{f_y}^{mβ}) \} \int_0^{∞} ds \ \exp \left[ -cs(\vec{ε}_ε - ε^a_{\ell m}) \cdot \vec{σ} \right] G_s^{(ε^a_{\ell m})}(\vec{x}, \vec{y}, τ; \vec{x'}, \vec{y'}, τ').
\]

Note that for the choice of \(\vec{ε}_ε\)’s given in (2.2), \(\{ε^a_{\ell m}\}, ℓ ≠ m\), always has one +ve and two -ve signs. For \(\{ε^a_{\ell m}\} = (+, -, -)\) we have
\[ G_s^{(+,-,-)} = (4\pi^2 c^2 s \, \sinh 4cs)^{-1} \exp \left[ -s(x_1 - y_1)^2 - \frac{(x_1 - x'_1)^2}{4c^2 s} - \frac{(\tau - \tau')^2}{4s} \right] \]
\[
\times \exp \left[ -\frac{1}{8c} \left\{ ((x_2 + x'_2) - (y_2 + y'_2))^2 + ((x_3 + x'_3) - (y_3 + y'_3))^2 \right\} \tanh 2cs \right. \\
- \frac{1}{8c} \left\{ ((x_2 - x'_2) - (y_2 - y'_2))^2 + ((x_3 - x'_3) - (y_3 - y'_3))^2 \right\} \coth 2cs \right] \\
\times \delta ((x_1 - x'_1) - (y_1 - y'_1)) \delta ((x_2 - x'_2) + (y_2 - y'_2)) \delta ((x_3 - x'_3) + (y_3 - y'_3)). \\
\]

\[ G_s^{(-,+,-)} \] is obtained from the above by the substitutions \( x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2 \) and \( x'_1 \leftrightarrow x'_2, y'_1 \leftrightarrow y'_2 \). Similarly, \( G_s^{(-,-,+)} \) is obtained by the substitutions \( x_1 \leftrightarrow x_3, y_1 \leftrightarrow y_3 \) and \( x'_1 \leftrightarrow x'_3, y'_1 \leftrightarrow y'_3 \).

Using the 2-point functions listed above and the triple and quartic interaction vertices given in (2.10) – (2.12), standard rules of perturbation theory may be used to evaluate the 2-loop diagrams.

### 4 The Two-Loop Effective Action

There are basically two types of 1PI diagrams that contribute to the effective action at the 2-loop level. Diagrams of the type shown in Fig. 1 involve two 3-point vertices and diagrams of the type shown in Figs. 2 and 3 involve a single 4-point vertex. In these diagrams the thin lines represent “column type” propagators and the thick lines represent “matrix type” propagators. Diagrams of type Figs. 2 and 3 are easier to evaluate and so we compute these first.

**Diagrams Involving a 4-Point Vertex**

(i) Diagrams involving a “matrix type” propagator, Fig. 2, come from terms of order \( g \) in
$S_{\text{int}}^{X+A}$, (2.10), containing two $Z$'s. There are five such terms. Their contribution is

$$
\Gamma_Z = g \sum_{\ell,m=1}^4 \sum_{\alpha,\beta=1}^n \int d\tau \int d^3\vec{x} \int d^3\vec{y} \int d^3\vec{z} \left[ -2\Delta_{ij}^{\ell_{ij}} \Lambda_{ij}^{\ell_{ij},m_{ij}} + \Delta_{ij}^{\ell_{ij}} \Lambda_{ij}^{\ell_{ij},m_{ij}} + \Delta_{ii}^{\ell_{ii}} \Lambda_{ii}^{\ell_{ii},m_{ii}} + \Delta_{AA}^{\ell_{AA}} \Lambda_{AA}^{\ell_{AA},m_{AA}} \right],
$$

(4.1)

where $\Delta_{ij}^{\ell_{ij}} \equiv \Delta_{ij}^{\ell_{ij}}(\vec{z}, \tau; \vec{x}, \tau)$ and $\Lambda_{ij}^{\ell_{ij},m_{ij}} \equiv \Lambda_{ij}^{\ell_{ij},m_{ij}}(\vec{x}, \vec{y}, \tau; \vec{z}, \vec{y}, \tau)$. We have evaluated this using the proper time representations for the 2-point functions involved. The result is given in the Appendix in (A6).

(ii) Diagrams involving only “column type” propagators, Fig. 3, come from the rest of the order $g$ terms in (2.10). There are seven such terms. Their contribution is

$$
\Gamma_\phi = g \sum_{\ell,m=1}^4 \sum_{\alpha,\beta=1}^n \int d\tau \int d^3\vec{x} \int d^3\vec{y} \left[ -\Delta_{ij}^{\ell_{ij}} \Delta_{ij}^{m_{ij}} + \frac{1}{2} \Delta_{ii}^{\ell_{ii}} \Delta_{ii}^{m_{ii}} \right],
$$

(4.2)

where $\Delta_{ij}^{\ell_{ij}} \equiv \Delta_{ij}^{\ell_{ij}}(\vec{x}, \tau; \vec{x}, \tau)$ and $\Delta_{ij}^{m_{ij}} \equiv \Delta_{ij}^{m_{ij}}(\vec{y}, \tau; \vec{y}, \tau)$. This has also been evaluated and is given in the Appendix in (A7).

Diagrams Involving Two 3-Point Vertices

It is convenient to group these diagrams, Fig. 1, into sets such that each set has only one type of matrix propagator joining the two 3-point vertices. Thus, we have diagrams with only matrix ghost propagator, etc. We now give below the contributions of the different sets. Note that to arrive at the expressions listed below we have made extensive use of the symmetry properties of the various 2-point functions with respect to their arguments as well as their indices. Also, throughout the following $\Delta_{ij}^{\ell_{ij}} \equiv \Delta_{ij}^{\ell_{ij}}(\vec{x}', \tau'; \vec{x}, \tau)$, $\Delta_{ij}^{m_{ij}} \equiv \Delta_{ij}^{m_{ij}}(\vec{y}, \tau; \vec{y}', \tau')$ and $\Lambda_{ij}^{\ell_{ij},m_{ij}} \equiv \Lambda_{ij}^{\ell_{ij},m_{ij}}(\vec{x}', \vec{y}', \tau; \vec{x}, \vec{y}, \tau)$.

(i) Diagrams with a matrix ghost propagator give the following contribution to the effective action.

$$
\Gamma_C = -g \sum_{\ell,m=1}^4 \sum_{\alpha,\beta=1}^n \int d\tau \int d^3\vec{x} \int d^3\vec{y} \int d\tau' \int d^3\vec{x}' \int d^3\vec{y}' \left[ 2 \left( \partial_\tau \Delta_{cc}^{\ell_{cc}} \Delta_{AA}^{m_{cc}} - iv_\tau \Delta_{cc}^{\ell_{cc}} \Delta_{AA}^{m_{cc}} \right) \partial_\tau \Lambda_{cc}^{\ell_{cc},m_{cc}} + \left\{ \Delta_{cc}^{\ell_{cc},m_{cc}} \left( D_{jx'}^{\ell_{cc}} - D_{jy'}^{m_{cc}^*} \right) D_{iy}^{m_{cc}} \Delta_{ji}^{\ell_{cc}} \Delta_{cc}^{m_{cc}} + \text{c.c.} \right\} \right].
$$

(4.3)

The corresponding proper time expression is given in (A8) of the Appendix. Note that ‘c.c.’ stands for complex conjugate, as usual.
(ii) Diagrams with a matrix fermion propagator give the contribution

\[
\Gamma_{\theta} = -g \sum \cdots \int \cdots \left[ -\text{tr}_{\text{dirac}} \left( \Delta_{\theta\theta}^{\alpha,\beta} \Lambda_{\theta\theta}^{\alpha,\beta} \right) \Delta_{AA}^{m\beta} \right.
\]

\[
+ i \text{tr}_{\text{dirac}} \left( \left[ \Delta_{\theta\theta}^{\alpha,\beta}, \gamma^i \right] \Lambda_{\theta\theta}^{\alpha,\beta} \right) \Delta_{AA}^{m\beta} + \text{tr}_{\text{dirac}} \left( \gamma^k \Delta_{\theta\theta}^{\alpha,\beta} \Lambda_{\theta\theta}^{\alpha,\beta} \right) \Delta_{ik}^{m\beta} \right] \quad (4.4)
\]

In the above, we have used an obvious short-hand notation for the various sums and integrals which are identical to those in (4.3). We will use this notation in the following also. The Dirac trace in the last term is rather tedious to evaluate, but finally one gets a moderately simple expression for \(\Gamma_{\theta}\) in the proper time representation. This is given in the Appendix in (A9).

(iii) Diagrams with a matrix gauge propagator give the contribution

\[
\Gamma_A = \Gamma_{1A} + \Gamma_{2A} + \Gamma_{3A},
\]

where

\[
\Gamma_{1A} = -g \sum \cdots \int \cdots \left[ -\partial_{\tau'} \Delta_{ij}^{\alpha,\beta} \partial_{\tau} \Delta_{ij}^{m\beta} - 2i\tau' \Delta_{AA}^{m\beta} \partial_{\tau} \Delta_{1A}^{m\beta} + 2i\tau \Delta_{AA}^{m\beta} \partial_{\tau'} \Delta_{1A}^{m\beta} + \partial_{\tau} \Delta_{AA} \partial_{\tau'} \Delta_{cc} \Delta_{cc}^{m\beta} \right] \Lambda_{ij}^{\alpha,\beta}, \quad (4.5)
\]

\[
\Gamma_{2A} = -g \sum \cdots \int \cdots \Lambda_{ij}^{\alpha,\beta} \left( 2D_{ix}^{\alpha,\beta} - D_{iy}^{m\beta} \right) \left( 2D_{jx}^{\alpha,\beta} - D_{jy}^{m\beta} \right) \Delta_{AA}^{m\beta} \Delta_{ij}^{m\beta}, \quad (4.6)
\]

\[
\Gamma_{3A} = \frac{1}{2} g \sum \cdots \int \cdots \Lambda_{ij}^{\alpha,\beta} \text{tr}_{\text{dirac}} \left( \Delta_{\theta\theta}^{\alpha,\beta} \Delta_{\theta\theta}^{m\beta} \right). \quad (4.7)
\]

These contributions to the 2-loop effective action have been evaluated and are listed in (A10)–(A12) in the Appendix.

(iv) Finally, diagrams with a matrix bosonic propagator give the contribution

\[
\Gamma_X = \sum_{q=1}^{8} \Gamma_{qX},
\]

where

\[
\Gamma_{1X} = -\frac{1}{2} g \sum \cdots \int \cdots \text{tr}_{\text{dirac}} \left( \Delta_{\theta\theta}^{\alpha,\beta} \gamma^i \Delta_{\theta\theta}^{m\beta} \gamma^j \right) \Lambda_{ij}^{\alpha,\beta}, \quad (4.8)
\]

\[
\Gamma_{2X} = \frac{1}{2} g \sum \cdots \int \cdots \Lambda_{ij}^{\alpha,\beta} \left( D_{ix}^{\alpha,\beta} D_{iy}^{m\beta} \Delta_{cc} \Delta_{cc}^{m\beta} + \text{c.c.} \right), \quad (4.9)
\]

\[
\Gamma_{3X} = -\frac{1}{2} g \sum \cdots \int \cdots \Lambda_{ij}^{\alpha,\beta} \left( D_{ix}^{\alpha,\beta} D_{iy}^{m\beta} + D_{jx}^{m\beta} + D_{jy}^{m\beta} \right) \times \left( \Delta_{jk}^{\alpha,\beta} + \Delta_{AA}^{m\beta} \Delta_{AA}^{m\beta} - 2\Delta_{1A}^{m\beta} \Delta_{1A}^{m\beta} \right), \quad (4.10)
\]
\[ \Gamma_{4X} = -g \sum \ldots \Lambda_{ij}^{\alpha,\beta} \left( 2\mathcal{D}_{kj}^{\alpha} - \mathcal{D}_{kj}^{\beta} \right) \left( 2\mathcal{D}_{fj}^{\alpha} - \mathcal{D}_{fj}^{\beta} \right) \Delta_{ji}^{\alpha} \Delta_{jk}^{\beta}, \quad (4.11) \]

\[ \Gamma_{5X} = -\frac{1}{2}g \sum \ldots \left\{ \Lambda_{ij}^{\alpha,\beta} \left( 2\mathcal{D}_{kj}^{\alpha} - \mathcal{D}_{kj}^{\beta} \right) \left( 2\mathcal{D}_{fj}^{\alpha} - \mathcal{D}_{fj}^{\beta} \right) \Delta_{ji}^{\alpha} \Delta_{jk}^{\beta} + c.c. \right\}, \quad (4.12) \]

\[ \Gamma_{6X} = g \sum \ldots \left\{ \Lambda_{ij}^{\alpha,\beta} \left( \mathcal{D}_{kj}^{\alpha} + \mathcal{D}_{kj}^{\beta} \right) \left( 2\mathcal{D}_{fj}^{\alpha} - \mathcal{D}_{fj}^{\beta} \right) \Delta_{ji}^{\alpha} \Delta_{jk}^{\beta} + c.c. \right\}, \quad (4.13) \]

\[ \Gamma_{7X} = -g \sum \ldots \Lambda_{ij}^{\alpha,\beta} \left\{ -4iv\tau \Delta_{1A}^{\alpha} \partial_{1A} \Delta_{ij}^{\beta} - 2iv\tau \partial_{1A} \Delta_{1A}^{\alpha} \Delta_{ij}^{\beta} + 4\Delta_{AA}^{\alpha} \partial_{1A} \Delta_{ij}^{\beta} + 4\partial_{1A} \Delta_{AA}^{\alpha} \Delta_{ij}^{\beta} \right\}, \quad (4.14) \]

\[ \Gamma_{8X} = -g \sum \ldots \Lambda_{11}^{\alpha,\beta} \left\{ 10iv\tau \Delta_{1A}^{\alpha} \partial_{1A} \Delta_{1A}^{\beta} + 8iv\tau \partial_{1A} \Delta_{1A}^{\alpha} \Delta_{1A}^{\beta} - 5\partial_{1A} \Delta_{1A}^{\alpha} \partial_{1A} \Delta_{1A}^{\beta} + 4v^2\tau^2 \Delta_{1A}^{\alpha} \Delta_{1A}^{\beta} - 4\partial_{1A} \partial_{1A} \Delta_{1A}^{\alpha} \Delta_{1A}^{\beta} \right\}, \quad (4.15) \]

Expressions for these contributions in the proper time representation are given in (A13)–(A20) in the Appendix.

The total 2-loop effective action, \( \Gamma_{\text{2-loop}} \), is given by the sum of the contributions in (4.1)–(4.15).

### 5 Ultraviolet Divergences in the 2-loop Action

At low velocities and for small values of the parameter \( c \), on general dimensional grounds we expect the 2-loop effective action to be a double series of the following form:

\[ \Gamma_{\text{2-loop}} = \frac{\xi}{c^6} \left( a_1 \frac{b^1}{v} + a_2 b^7 v + a_3 b^3 v^3 + d_1 v^5 + \cdots \right) \]

\[ + \frac{\xi}{c^4} \left( a_4 \frac{b^5}{v} + a_5 b^3 v + d_2 v^3 + \cdots \right) \]

\[ + \frac{\xi}{c^2} \left( a_6 \frac{b^3}{v} + d_3 v + \cdots \right) \]

\[ + d_4 \frac{\xi}{bv} + \cdots \quad (5.1) \]

Here the quantities \( a_1, \ldots, a_6 \) and \( d_1, \ldots, d_4 \) are pure numbers and \( \xi \) is defined in the Appendix in (A1). The structure of (5.1) follows from the observation that, as defined in (2.1), the parameter \( g \) has dimensions of [length]\(^{-3}\). Moreover, \( b^2, v \) and \( c \) all have the dimensions of [length]\(^{-2}\) and the leading term in the 2-loop calculation goes as \( 1/vc^6 \).
The terms in (5.1) with coefficients $a_1, \cdots, a_6$ all have positive powers of $b$ with them. On physical grounds all these coefficients should vanish since they would otherwise give rise to a potential that diverges at large distances. The terms with coefficients $d_1, \cdots, d_4$ all give rise to a potential that goes as $\rho^{-2}$, where $\rho^2 = b^2 + v^2 + \tau^2$. This is precisely the structure of the leading terms, in the limit $c = f^{-1} \to 0$, of the 2-loop effective potential calculated using the DBI action, given in (6.11) of [6]. For agreement with matrix theory, the coefficients of these terms must match between these two calculations.

We have calculated the various terms in (5.1) using the results presented in the Appendix. We find that while indeed $a_1 = 0$, all the other coefficients have uncanceled divergences. For example, for the term proportional to $\xi v/c$, we get

$$3 \int_0^\infty ds_1 \int_0^\infty ds_2 e^{-b^2(s_1+s_2)} \left\{ \frac{(s_1 + s_2)^{3/2}}{s_1^{7/2} s_2^{5/2}} - \frac{16}{5} \frac{(s_1 + s_2)^{-1/2}}{s_1^{5/2} s_2^{5/2}} \right\}$$

$$-2 \int_0^\infty ds_1 \frac{e^{-b^2 s_1}}{s_1^2} \int_0^\infty ds_3 \frac{ds_3}{s_3^{7/2}}.$$  \hfill (5.2)

Similar ill-defined expressions are obtained for all the other terms. Since the general structure of all the terms is similar to (5.2), let us see in some detail where the two type of terms in (5.2) originate.

Consider the second term in (5.2) first. The structure of this term suggests that it originates from $\Gamma_Z$, (A6). There is actually another contribution to this term which comes from those terms in the sum of (4.3)–(4.15) which are proportional to $b^2$. To see this, let us write the total effective action at 2-loops as

$$\Gamma_{\text{2-loop}} = \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 e^{-b^2(s_1+s_2)} W(s_1, s_2, s_3)$$

$$+ \xi \int_0^\infty ds_1 \int_0^\infty ds_2 e^{-b^2(s_1+s_2)} X(s_1, s_2)$$

$$+ \xi \int_0^\infty ds_1 \int_0^\infty ds_3 e^{-b^2(s_1+s_2)} Y(s_1, s_3),$$  \hfill (5.3)

$W, X$ and $Y$ can be read-off from the results given in the Appendix. Let us now explicitly separate out the $b^2$ piece in $W$:

$$W(s_1, s_2, s_3) = \omega(s_1, s_2, s_3) + b^2 \mathcal{U}(s_1, s_2, s_3).$$  \hfill (5.4)

The $b^2$ terms come only from diagrams like Fig. 1 containing a matrix gauge or bosonic propagator. So the quantity $\mathcal{U}(s_1, s_2, s_3)$ can be read-off from the sum of (4.5)–(4.15). An explicit expression for it is given in the Appendix in (A21). Now, using the identity

$$b^2 e^{-b^2(s_1+s_2)} = -\frac{1}{2} (\partial_{s_1} + \partial_{s_2}) e^{-b^2(s_1+s_2)},$$

3Extensive use of Mathematica was made in these calculations.
we may rewrite this piece as follows:

\[
\frac{1}{2} \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \; e^{-b^2(s_1+s_2)} U(s_1, s_2, s_3)
\]

Thus, we have

\[
\Gamma_{2\text{-loop}} = \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \; e^{-b^2(s_1+s_2)} \tilde{W}(s_1, s_2, s_3)
\]

where

\[
\tilde{W}(s_1, s_2, s_3) = \omega(s_1, s_2, s_3) + \frac{1}{2}(\partial_{s_1} + \partial_{s_2}) U(s_1, s_2, s_3),
\]

and

\[
\tilde{Y}(s_1, s_3) = Y(s_1, s_3) + \frac{1}{2}[U(s_1, 0, s_3) + U(0, s_1, s_3)].
\]

Using (A6) and (A21) we find that

\[
\tilde{Y}(s_1, s_3) = \frac{1}{16} \left( c^3 s_3^{7/2} \sinh^3 c_1 \sinh v s_1 \right)^{-1} \left( 12 + 13 \cosh 2 v s_1 + 39 \cosh 2 c_1 
\]

Expanding in powers of \(1/c\), to leading order we get

\[
\tilde{Y}(s_1, s_3) = (4c^6 s_1^3 s_3^{7/2} \sinh v s_1)^{-1}(51 + 13 \cosh 2 v s_1 - 64 \cosh v s_1) + 0(1/c^4). \quad (5.10)
\]

Now, further expanding this in powers of \(v\), we see that the order \(v^{-1}\) term vanishes, but all the higher powers of \(v\) have nonvanishing coefficients. In particular, the order \(v/c^6\) term exactly reproduces the second term in (5.2).

The trick employed in (5.5) has a diagramatic representation. In terms of diagrams like Fig. 1, the second term in (5.5) results from shrinking one of the “column type” propagators
to a point (large $b^2$ or small $s$ expansion). Doing this to Fig. 1 produces diagrams of the type shown in Fig. 2, and hence there is mixing of these two types of contributions, as we have seen above.

The divergence in the second term in (5.2) comes from small values of the proper time variable, and this suggest that they are ultraviolet in nature. This can be seen more directly as follows. We have seen that this divergence has basically to do with diagrams of type Fig. 2. A typical contribution of such a diagram is

$$\int d\tau \int d^3\vec{x} \int d^3\vec{y} \int d^3\vec{z} \Delta(\vec{z}, \tau; \vec{x}, \tau) \Lambda(\vec{x}, \vec{y}, \tau; \vec{z}, \vec{y}, \tau).$$

(5.11)

In writing (5.11) we have ignored the various indices since they are not relevant to the present discussion. Now, introducing the momentum space representation for the matrix propagator,

$$\Lambda(\vec{x}, \vec{y}, \tau; \vec{x}', \vec{y}', \tau') = \int \frac{d^3\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \int d^3\vec{x} \int d^3\vec{y} \tilde{\Lambda}(\omega, \vec{k}, \vec{x} - \vec{y}) \delta^3((\vec{x} - \vec{y}) - (\vec{x}' - \vec{y}')),$$

where

$$\tilde{\Lambda}(\omega, \vec{k}, \vec{x}) \equiv [\omega^2 + \vec{k}^2 + \vec{x}^2]^{-1},$$

which can be obtained from (3.15), we may rewrite (5.11) as

$$\int \frac{d^3\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \int d^3\vec{x} \int d^3\vec{y} \tilde{\Lambda}(\omega, \vec{k}, \vec{x} - \vec{y}) \Delta(\vec{x}, \tau; \vec{x}, \tau) \Delta(\vec{y}, \tau; \vec{y}, \tau)$$

$$= \left[ \int \frac{d^3\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \int d^3\vec{y} \tilde{\Lambda}(\omega, \vec{k}, \vec{y}) \right] \left[ \int d\tau \int d^3\vec{x} \Delta(\vec{x}, \tau; \vec{x}, \tau) \right].$$

The first factor looks like a massless bosonic loop in 7-dimensional space-time, which is ultraviolet divergent. The divergence in the second factor can also be seen to be coming from the high frequency oscillators in the energy representation of this integral.

In a similar fashion one can see that the divergences in the first term in (5.2) are ultraviolet in nature. This term receives contributions from diagrams of the type shown in Fig. 1 as well as those in Fig. 3. The general structure of the contribution of the former type of diagrams, after integrating over the proper time variable $s_3$ associated with the matrix propagators, is exactly like those of the latter type of diagrams. In fact, integrating over $s_3$ effectively shrinks the matrix propagator in Fig. 1, thus producing the diagram in Fig. 3. A typical contribution of these latter type of diagrams is

$$\int d\tau \left[ \int d^3\vec{x} \Delta(\vec{x}, \tau; \vec{x}, \tau) \right] \left[ \int d^3\vec{y} \Delta(\vec{y}, \tau; \vec{y}, \tau) \right].$$

It is clear from this expression that high frequency fluctuations produce the divergences seen in the first term in (5.2).

We have discussed above in detail the origin and the structure of the divergences in the coefficient $a_2$. Similar general discussion applies to the coefficients $a_3, \cdots, a_6$ and one finds that these coefficients are also ultraviolet divergent. Somewhat more work is required for
the coefficients $d_1, \ldots, d_4$. This is because in a naive double expansion in powers of $1/c$ and $v$, the integrals over the proper time variable $s_3$ in the different contributions to the 2-loop effective action diverge at the large $s_3$ end. This happens essentially because the matrix propagators are “massless”, i.e. there is no factor of $e^{-b^2s_3}$ in the integrand. The correct procedure is to first integrate over $s_3$ and then perform the double expansion. Once this is done, one finds that the coefficients $d_1, \ldots, d_4$ are also ultraviolet divergent.

6 Concluding Remarks

In this paper we have presented the results of a calculation of the 2-loop effective action for a D0-brane in the background of the recently discussed D0-D6 bound state configuration. As we have seen, the 2-loop effective action has uncanceled ultraviolet divergences. Similar ultraviolet divergences are potentially present in the calculation of the 1-loop effective action also. However, in that calculation, done in the proper time representation like the present calculation, the divergent parts precisely cancel in the sum of contributions from different sectors (boson, fermion, gauge and ghost), leaving behind a finite answer for the 1-loop effective action. In general, one expects matrix theory loop calculations to be divergent for the present background because of nonrenormalizability of 7-dimensional gauge theory. The ultraviolet divergent 2-loop result that we have got is consistent with this expectation. Since a full string theory calculation is expected to be ultraviolet finite, this means that massive open string states do not decouple from the 6-brane, as expected from general considerations 18, 19.

One of our motivations for the present calculation was the surprising supergravity/superYang-Mills agreement at 2-loop found in 17. In view of our result and the above considerations, it now seems even more surprising that a simple 2-loop effective superYang-Mills action should reproduce the supergravity result. Presumably the effective superYang-Mills action can be obtained from a full string theory calculation. It would be nice to have a better understanding of this.

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APPENDIX

In this appendix we list the contributions of various diagrams in the proper time representation. For this purpose it is useful to define

$$\frac{g Q_6^2}{64 \sqrt{\pi}} \equiv \xi, \quad (A1)$$

$$\frac{\sinh \nu s_1 \sinh \nu s_2}{\nu \sinh \nu (s_1 + s_2)} \equiv h_\nu, \quad (s_3 + h_\nu)^{-1} \equiv k_\nu, \quad (A2)$$

$$\frac{\sinh c s_1 \sinh c s_2}{c \sinh c (s_1 + s_2)} \equiv h_c, \quad (s_3 + h_c)^{-1} \equiv k_c, \quad \left(\frac{\tanh 2c s_3}{2c} + h_c\right)^{-1} \equiv \tilde{k}_c, \quad (A3)$$

$$\frac{\sqrt{k_\nu}}{\sinh \nu (s_1 + s_2)} \frac{k_c^3}{c^3 \sinh^3 c (s_1 + s_2)} \equiv \Omega(s_1, s_2, s_3), \quad (A4)$$

$$\frac{\sqrt{k_\nu}}{\sinh \nu (s_1 + s_2)} \frac{k_c}{c^2 \sinh^3 c (s_1 + s_2) \cosh^2 2c s_3} \equiv \tilde{\Omega}(s_1, s_2, s_3). \quad (A5)$$

We now list the various contributions.

$$\Gamma_\nu = \frac{9}{8} \xi \int_0^\infty ds_1 \int_0^\infty ds_3 \frac{e^{-s_1 b^2}}{s_3^{7/2} c^3 \sinh^3 c s_1 \sinh \nu s_1} (2 \cosh^2 \nu s_1 + 3 \cosh 2c s_1)$$

$$+ \frac{3}{4} \xi \int_0^\infty ds_1 \int_0^\infty \frac{ds_3}{s_3^{3/2} \sinh^2 2c s_3} \frac{e^{-s_1 b^2}}{c \sinh^3 c s_1 \sinh \nu s_1} \left[4 \cosh^2 \nu s_1 (1 + 8 \cosh^2 2c s_3) + 2 \cosh 2c s_1 (7 + 20 \cosh^2 2c s_3) + 12 \sinh 2c s_1 \sinh 4c s_3 \right]. \quad (A6)$$

We have used $4n = Q_6$ in arriving at this expression. Also, here and in the following the first term comes from the $\ell = m$ case and the second term from $\ell \neq m$.

$$\Gamma_\phi = \frac{1}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{e^{-b^2(s_1 + s_2)}}{(v \sinh \nu s_1 \sinh \nu s_2 \sinh \nu (s_1 + s_2))^3/2} (\sinh c s_1 \sinh c s_2)^{-3}$$

$$\times \left[\frac{1}{2} + 2 \cosh 2\nu s_1 + 6 \cosh 2c s_1 + \frac{1}{2} \cosh 2\nu s_1 \cosh 2\nu s_2 + \frac{3}{2} \sinh 2\nu s_1 \sinh 2\nu s_2ight.$$ 

$$+ 6 \cosh 2c s_1 \cosh 2c s_2 + \frac{15}{2} \cosh 2c s_1 \cosh 2c s_2 + \frac{9}{2} \sinh 2c s_1 \sinh 2c s_2 + s_1 \leftrightarrow s_2$$

$$+ \frac{3}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{e^{-b^2(s_1 + s_2)}}{(v \sinh \nu s_1 \sinh \nu s_2 \sinh \nu (s_1 + s_2))^3/2} (\sinh c s_1 \sinh c s_2)^{-3}$$

$$\times \left[\frac{1}{2} + 2 \cosh 2\nu s_1 + 6 \cosh 2c s_1 + \frac{1}{2} \cosh 2\nu s_1 \cosh 2\nu s_2 + \frac{3}{2} \sinh 2\nu s_1 \sinh 2\nu s_2ight.$$ 

$$+ 6 \cosh 2c s_1 \cosh 2c s_2 + \frac{15}{2} \cosh 2c s_1 \cosh 2c s_2 - \frac{3}{2} \sinh 2c s_1 \sinh 2c s_2 + s_1 \leftrightarrow s_2 \right]. \quad (A7)$$
In the above, as well as in the following, ‘$s_1 \leftrightarrow s_2$’ stands for the entire preceding expression rewritten with this substitution.

\[
\Gamma_C = \frac{1}{32} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1+s_2)} \Omega(s_1, s_2, s_3) \left[ v h_v k_v \frac{\cosh c(s_1 + 2s_2)}{\sinh c s_1} + s_1 \leftrightarrow s_2 \right] \\
+ \frac{3}{32} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1+s_2)} \tilde{\Omega}(s_1, s_2, s_3) \left[ v h_v k_v (\sinh 2v s_1 + \coth v s_1 \cosh 2v s_2) \right. \\
+ 2ch_c k_c \frac{\cosh c(s_1 + 2s_2)}{\sinh c s_1} + 4ch_c \tilde{k}_c \left( \cosh c(s_1 - 2s_2) \\
- \sinh c(s_1 - 2s_2) \tanh 2cs_3 \right) \left/ \sinh c s_1 + s_1 \leftrightarrow s_2 \right] \].

(A8)

\[
\Gamma_\theta = -\xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1+s_2)} \Omega(s_1, s_2, s_3) \left[ 3ch_c k_c \coth c s_1 \cosh c s_1 (\cosh v s_1 \\
+ \cosh v(s_1 + 2s_2)) + \frac{1}{2} h_v k_v v \frac{\cosh^2 c s_1}{\sinh v s_1} (\cosh c s_1 + 3 \cosh c(s_1 + 2s_2)) \\
+ 6ch_c k_c \cosh c(s_1 + 2s_2) \cosh v s_1 + s_1 \leftrightarrow s_2 \right] \\
- 3\xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1+s_2)} \tilde{\Omega}(s_1, s_2, s_3) \left[ \left( ch_c k_c \frac{\cosh^2 c(s_1 + 2s_3)}{\sinh c s_1} \right) \\
+ 2ch_c \tilde{k}_c \frac{\coth c s_1 \cosh c(s_1 + 2s_3)}{\cosh 2cs_3} \right) (\cosh v s_1 + \cosh v(s_1 + 2s_2)) \\
+ \frac{1}{2} h_v k_v \frac{v}{\sinh v s_1} \left( \cosh c s_1 \cosh^2 c(s_1 + 2s_3) + \cosh c(s_1 + 2s_2) \cosh^2 c(s_1 + 2s_3) \\
+ 2 \cosh c s_1 \cosh c(s_1 + 2s_3) \cosh c(s_1 - 2s_2 - 2s_3) \right) \\
+ 2 \cosh v s_1 \left( ch_c k_c \frac{c}{\sinh c s_1} \cosh c(s_1 + 2s_3) \cosh c(s_1 - 2s_2 - 2s_3) + h_v k_v \frac{c}{\sinh c s_1 \cosh 2cs_3} \\
\times (\cosh c(s_1 + 2s_2) \cosh c(s_1 + 2s_3) + \cosh c s_1 \cosh c(s_1 - 2s_2 - 2s_3)) \right) \left/ \sinh c s_1 + s_1 \leftrightarrow s_2 \right] \].

(A9)
\[ \Gamma_{1A} = -\frac{1}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \frac{v}{\sinh v(s_1 + s_2)} \times \\
(2 + \cosh 2v s_1 \cosh 2v s_2 + 6 \cosh 2c(s_1 + s_2)) \left( \cosh v(s_1 - s_2) + \frac{k_v \sinh^2 v(s_1 - s_2)}{4v \sinh v(s_1 + s_2)} \right) \\
\frac{v}{2 \sinh v(s_1 + s_2)}(1 + \sinh 2v s_1 \sinh 2v s_2) + \frac{1}{2} k_v \sinh 2v s_1 \sinh 2v s_2 \\
\times \left( \frac{1}{4 \sinh^2 v(s_1 + s_2)} - v^2 h_v^2 \right) - v(1 - \frac{1}{2} h_v k_v) \sinh 2v(s_1 - s_2) \sinh v(s_1 - s_2) \left/ \sinh v(s_1 + s_2) \right. \right] \\
-\frac{3}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \frac{v}{\sinh v(s_1 + s_2)} \times \\
(2 + \cosh 2v s_1 \cosh 2v s_2 + 2 \cosh 2c(s_1 + s_2) \\
+4 \cosh 2c(s_1 - s_2)) \left( \cosh v(s_1 - s_2) + \frac{k_v \sinh^2 v(s_1 - s_2)}{4v \sinh v(s_1 + s_2)} \right) \\
\frac{v}{2 \sinh v(s_1 + s_2)}(1 + \sinh 2v s_1 \sinh 2v s_2) + \frac{1}{2} k_v \sinh 2v s_1 \sinh 2v s_2 \times \\
\left( \frac{1}{4 \sinh^2 v(s_1 + s_2)} - v^2 h_v^2 \right) - v(1 - \frac{1}{2} h_v k_v) \sinh 2v(s_1 - s_2) \sinh v(s_1 - s_2) \left/ \sinh v(s_1 + s_2) \right. \right]. \quad (A10) \]

\[ \Gamma_{2A} = -\frac{1}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \frac{b^2}{2} \cosh 2v s_1 \\
+\frac{v}{4} \cosh 2v s_1 \cosh 2v s_2 \left( \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + v h_v^2 k_v \right) + \frac{3}{2} k_v \cosh 2v s_1 \left\{ \frac{5}{2} \cosh 2c s_2 \\
+cs_3 \coth c(s_1 + s_2) + 3ch_c \sinh 2c s_2 - \frac{3 \sinh c(s_1 - s_2)}{2 \sinh c(s_1 + s_2)} \cosh 2c s_2 \right\} + s_1 \leftrightarrow s_2 \right] \\
-\frac{3}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \frac{b^2}{2} \cosh 2v s_1 \\
+\frac{v}{4} \cosh 2v s_1 \cosh 2v s_2 \left( \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + v h_v^2 k_v \right) + \cosh 2v s_1 \left\{ \frac{5}{4} k_v \cosh 2c s_2 \\
+k_v \cosh 2c s_2 \left( \frac{5}{2} + \frac{1}{2} \tanh 2c s_3 \frac{\cosh c(s_1 - s_2)}{\sinh c(s_1 + s_2)} - 4ch_c \tanh 2c s_3 \right) \\
+\frac{1}{2} cs_3 k_v \coth c(s_1 + s_2) + (2 \tanh 2c s_3 - 5ch_c) \tilde{k}_v \sinh 2c s_2 + \frac{3}{2} ch_c k_v \sinh 2c s_2 \\
-\frac{3}{4} \sinh c(s_1 - s_2) \left( (k_v + 2 \tilde{k}_v) \cosh 2c s_2 + 2 \tilde{k}_v \tanh 2c s_3 \sinh 2c s_2 \right) \right\} + s_1 \leftrightarrow s_2 \right]. \quad (A11) \]
\[
\Gamma_{3A} = -\frac{1}{2} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \, e^{-b^2(s_1-s_2)} \Omega(s_1, s_2, s_3) \left[ b^2 \cosh v(s_1-s_2) \cosh^3 c(s_1+s_2) \right. \\
+ \frac{v h_v k_v}{2 \sinh v(s_1+s_2)} \cosh^3 c(s_1+s_2) + 3 c s_3 k_c \cosh v(s_1-s_2) \cosh c(s_1+s_2) \coth c(s_1+s_2) \bigg] \\
- \frac{3}{2} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \, e^{-b^2(s_1-s_2)} \tilde{\Omega}(s_1, s_2, s_3) \left[ b^2 \cosh v(s_1-s_2) \cosh c(s_1+s_2) \times \\
\cosh^2 c(s_1-s_2) + \frac{v h_v k_v}{2 \sinh v(s_1+s_2)} \cosh c(s_1+s_2) \cosh^2 c(s_1-s_2) + c s_3 k_c \cosh v(s_1-s_2) \times \\
\left. \frac{\cosh^2 c(s_1-s_2)}{\sinh c(s_1+s_2)} + \tilde{k_c} \tanh 2 c s_3 \cosh v(s_1-s_2) \cosh c(s_1-s_2) \coth c(s_1+s_2) \right] \right]. \tag{A12}
\]

\[
\Gamma_{1X} = -\frac{1}{2} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \, e^{-b^2(s_1+s_2)} \Omega(s_1, s_2, s_3) \left[ b^2 \left\{ \cosh v(s_1-s_2) \cosh c(s_1+s_2) \\
+ 6 \cosh v(s_1+s_2) \cosh c(s_1-s_2) \right\} \cosh^2 c(s_1+s_2) + \frac{2 v}{\sinh v(s_1+s_2)} \{ \cosh c(s_1+s_2) \\
+ 3 \cosh c(s_1-s_2) \} \cosh^2 c(s_1+s_2) - \frac{3}{4} h_v k_v \left( \cosh c(s_1+s_2) + 2 \cosh c(s_1-s_2) \right) \times \\
\cosh^2 c(s_1+s_2) \bigg] + 3 c s_3 k_c \coth c(s_1+s_2) \left\{ 4 \cosh v(s_1+s_2) \cosh c(s_1-s_2) \\
+ (\cosh v(s_1-s_2) + 2 \cosh v(s_1+s_2)) \cosh c(s_1+s_2) \right\} \right] \\
- \frac{3}{2} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \, e^{-b^2(s_1+s_2)} \tilde{\Omega}(s_1, s_2, s_3) \left[ b^2 \left\{ \cosh v(s_1-s_2) \cosh c(s_1+s_2) \times \\
\cosh^2 c(s_1-s_2) + 2 \cosh v(s_1+s_2) \cosh c(s_1-s_2) \left( \cosh^2 c(s_1-s_2) \\
+ 2 \cosh c(s_1+s_2) \cosh c(s_1+s_2+4 s_3) \right) \right\} \right] \\
+ \frac{v}{\sinh v(s_1+s_2)} \left\{ 4 \left( \cosh c s_1 \cosh c s_2 \cosh^2 c(s_1-s_2) + \cosh c(s_1+s_2) \cosh c(s_1-s_2) \cosh c(s_1+s_2+4 s_3) \right) - h_v k_v \left( \frac{3}{2} \cosh c(s_1+s_2) \times \\
\cosh^2 c(s_1-s_2) + \cosh^3 c(s_1-s_2) + 2 \cosh c(s_1+s_2) \cosh c(s_1-s_2) \cosh c(s_1+s_2+4 s_3) \right) \right\} \\
+ \frac{c s_3 k_c}{\sinh c(s_1+s_2)} \left\{ (\cosh v(s_1-s_2) + 2 \cosh v(s_1+s_2)) \cosh^2 c(s_1-s_2) \\
+ 4 \cosh v(s_1+s_2) \cosh c(s_1-s_2) \cosh c(s_1+s_2+4 s_3) \right\} \right\}.
\]
\[ + \frac{k_c \tanh 2cs_3}{\sinh c(s_1 + s_2)} \left( \cosh (v(s_1 - s_2) + 2 \cosh (s_1 + s_2)) \cosh c(s_1 + s_2) \cosh c(s_1 - s_2) + 2 \cosh v(s_1 + s_2) \left( \cosh c(s_1 + s_2) \cosh c(s_1 + s_2 + 4s_3) + \cosh^2 c(s_1 - s_2) \right) \right) \right] \right). \quad (A13)

\[
\Gamma_{2X} = \frac{1}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1 + s_2)} \Omega(s_1, s_2, s_3) \left[ b^2 + \frac{v}{2} \left( \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + vh^2 b_v \right) \right. \\
+ 3cs_3k_c \frac{\cosh c(s_1 - s_2)}{\sinh c(s_1 + s_2)} \right]
+ \frac{3}{16} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-\bar{b}^2(s_1 + s_2)} \bar{\Omega}(s_1, s_2, s_3) \left[ b^2 + \frac{v}{2} \left( \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + vh^2 b_v \right) \right. \\
+ \frac{3}{8} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-\bar{b}^2(s_1 + s_2)} \bar{\Omega}(s_1, s_2, s_3) \left[ b^2 + \frac{v}{2} \left( \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + vh^2 b_v \right) \right. \\
+ \frac{1}{2} k_c + 2cs_3k_c \frac{\cosh c(s_1 - s_2)}{\sinh c(s_1 + s_2)} + \bar{k}_c \cosh 4cs_3 + 2c \sinh 4cs_3 \right]
\times \left( 1 + \cosh 2v(s_1 + s_2) + \cosh 2c(s_1 + s_2) + 2 \cosh 2c(s_1 - s_2) \right) \quad (A14)

\[
\Gamma_{3X} = -\frac{1}{8} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1 + s_2)} \Omega(s_1, s_2, s_3) \left[ 2b^2 + \frac{v}{2} \left( \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + vh^2 b_v \right) \right. \\
+ \frac{3}{2} k_c + 6cs_3k_c \frac{\cosh c(s_1 - s_2)}{\sinh c(s_1 + s_2)} \right] \right]
+ \frac{1}{2} k_c + 2cs_3k_c \frac{\cosh c(s_1 - s_2)}{\sinh c(s_1 + s_2)} + \bar{k}_c \cosh 4cs_3 + 2c \sinh 4cs_3 \right]
\times \left( 1 + \cosh 2v(s_1 + s_2) + \cosh 2c(s_1 + s_2) + 2 \cosh 2c(s_1 - s_2) \right) \quad (A15)

\[
\Gamma_{4X} = -\frac{1}{32} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-b^2(s_1 + s_2)} \Omega(s_1, s_2, s_3) \left[ 2 + \cosh 2vs_1 + 6 \cosh 2cs_1 \right]
\times \left( b^2 + \frac{v}{2} \cosh 2vs_2 \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + vh^2 b_v \right) \right] + 3cs_3k_c \coth c(s_1 + s_2) \\
+ \frac{3}{2} k_c \cosh 2cs_2 \left( 5 - \frac{\sinh c(s_1 - s_2)}{\sinh c(s_1 + s_2)} \right) + 9cs_3k_c \sinh 2cs_2 + s_1 \leftrightarrow s_2 \right]
\right]
- \frac{3}{32} \xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 \ e^{-\bar{b}^2(s_1 + s_2)} \bar{\Omega}(s_1, s_2, s_3) \left[ 2 + \cosh 2vs_1 + 2 \cosh 2cs_1 \\
+ 4 \cosh 2(c(s_1 + 2s_3)) \right] \left( b^2 + \frac{v}{2} \cosh 2vs_2 \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + vh^2 b_v \right) \right)
\right]
\right]
\right) \quad (A15)
\[ +c s _3 k _c \coth c(s _1 + s _2) + \frac{1}{2} k _c \cosh 2 c s _2 \left( 5 - 3 \frac{\sinh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} \right) + 3 c h _c k _c \sinh 2 c s _2 \]

\[ + \tilde{k} _c \cosh 2 c s _2 \tanh 2 c s _3 \left( \frac{\cosh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} - 8 c h _c \right) + \tilde{k} _c \tanh 2 c s _3 \sinh 2 c s _2 \times \]

\[ \left( 4 - 3 \frac{\sinh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} \right) + \tilde{k} _c \cosh 2 c s _2 \left( 5 - 3 \frac{\sinh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} \right) - 10 c h _c \tilde{k} _c \sinh 2 c s _2 \]

\[ + s _1 \leftrightarrow s _2 \]. \quad (A16) \]

\[ \Gamma _{5X} = -\frac{1}{16} \xi \int _0 ^\infty d s _1 \int _0 ^\infty d s _2 \int _0 ^\infty d s _3 \ e^{-b^2(s _1 + s _2)} \Omega(s _1, s _2, s _3) \left[ b^2 + \nu \left( \frac{\cosh v(s _1 - s _2)}{\sinh v(s _1 + s _2)} + v h_v ^2 k_v \right) \right] \]

\[ \times \cosh 2 v s _1 \cosh 2 v s _2 + 3 c s _3 k _c \frac{\cosh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} - 6 k _c \cosh 2 c(s _1 - s _2) \]

\[ -\frac{3}{16} \xi \int _0 ^\infty d s _1 \int _0 ^\infty d s _2 \int _0 ^\infty d s _3 \ e^{-b^2(s _1 + s _2)} \tilde{\Omega}(s _1, s _2, s _3) \left[ b^2 + \nu \left( \frac{\cosh v(s _1 - s _2)}{\sinh v(s _1 + s _2)} + v h_v ^2 k_v \right) \right] \]

\[ \times \cosh 2 v s _1 \cosh 2 v s _2 + c s _3 k _c \frac{\cosh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} - 2 k _c \cosh 2 c(s _1 - s _2) \]

\[ + \tilde{k} _c \tanh 2 c s _3 \left( \frac{\cosh c(s _1 - s _2)}{\sinh c(s _1 + s _2)} + 10 c h _c \right) \cosh 2 c(s _1 + s _2 + 2 s _3) \]

\[ -4 \tilde{k} _c \cosh 2 c(s _1 + s _2 + 2 s _3) - 2 c(5 - 9 h_v \tilde{k} _c) \sinh 2 c(s _1 + s _2 + 2 s _3) \]. \quad (A17) \]

\[ \Gamma _{6X} = \frac{1}{8} \xi \int _0 ^\infty d s _1 \int _0 ^\infty d s _2 \int _0 ^\infty d s _3 \ e^{-b^2(s _1 + s _2)} \Omega(s _1, s _2, s _3) \left[ 2 b^2 + \nu \cosh 2 v s _1 \cosh 2 v s _2 \left( v h_v ^2 k_v \right. \right. \right. \]

\[ + \frac{\cosh v(s _1 - s _2)}{\sinh v(s _1 + s _2)} + 6 c s _3 k _c \frac{\cosh c(s _1 - s _2) \cosh 2 c(s _1 + s _2)}{\sinh c(s _1 + s _2)} \]

\[ + \frac{3}{2} k _c \cosh 2 c(s _1 + s _2) + 9 c h _c k _c \sinh 2 c(s _1 + s _2) \] \]

\[ + \frac{3}{8} \xi \int _0 ^\infty d s _1 \int _0 ^\infty d s _2 \int _0 ^\infty d s _3 \ e^{-b^2(s _1 + s _2)} \tilde{\Omega}(s _1, s _2, s _3) \left[ 2 b^2 + \nu \cosh 2 v s _1 \cosh 2 v s _2 \left( v h_v ^2 k_v \right. \right. \right. \]

\[ + \frac{\cosh v(s _1 - s _2)}{\sinh v(s _1 + s _2)} + 2 c s _3 k _c \frac{\cosh c(s _1 - s _2) \cosh 2 c(s _1 + s _2)}{\sinh c(s _1 + s _2)} \]

\[ + \frac{1}{2} k _c \cosh 2 c(s _1 + s _2) + 3 c h _c k _c \sinh 2 c(s _1 + s _2) + \tilde{k} _c \cosh 4 c s _3 \cosh 2 c(s _1 - s _2) \]

\[ - 3 \tilde{k} _c \sinh 4 c s _3 \frac{\sinh c(s _1 - s _2) \sinh 2 c(s _1 - s _2)}{\sinh c(s _1 + s _2)} \right] + 2 c h _c \tilde{k} _c \sinh 4 c s _3 \cosh 2 c(s _1 - s _2) \right] \]
\[
\Gamma_{7X} = -\frac{1}{64}\xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 e^{-\nu^2(s_1+s_2)} \Omega(s_1, s_2, s_3) \left[ v(2 + \cosh 2v s_2 + 6 \cosh 2v s_2) + 4 \coth v s_1 \csc \left( \frac{v(s_1 - s_2)}{\sinh v(s_1 + s_2)} + \cosh v s_1 \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} \right) \right] + s_1 \leftrightarrow s_2 \right]
\]

\[
\Gamma_{8X} = -\frac{1}{64}\xi \int_0^\infty ds_1 \int_0^\infty ds_2 \int_0^\infty ds_3 e^{-\nu^2(s_1+s_2)} \left[ \Omega(s_1, s_2, s_3) + 3\tilde{\Omega}(s_1, s_2, s_3) \right] \times v \left[ 2 \cosh 2v s_1 \sinh 2v s_2 \frac{\sinh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} - 5 \sinh 2v s_1 \sinh 2v s_2 \frac{\cosh v(s_1 - s_2)}{\sinh v(s_1 + s_2)} \right] - \nu h^2 \kappa_v \left[ 2 \cosh 2v s_1 \sinh 2v s_2 (4 \coth v s_1 + 5 \coth v s_2) - \sinh 2v s_1 \sinh 2v s_2 (4 \coth^2 v s_1 + 5 \coth v s_2) \right] + s_1 \leftrightarrow s_2 \right].
\]

\[
U(s_1, s_2, s_3) = -\frac{1}{16} \Omega(s_1, s_2, s_3) \left[ 1 + \cosh 2v s_1 + 2 \cosh 2v(s_1 + s_2) + 3 \cosh 2cs_1 + 6 \cosh 2(s_1 + s_2) + 8 \cosh v(s_1 - s_2) \cosh^3 c(s_1 + s_2) + 24 \cosh v(s_1 + s_2) \cosh c(s_1 - s_2) \cosh^3 c(s_1 + s_2) + s_1 \leftrightarrow s_2 \right]
\]

\[
-\frac{3}{16} \tilde{\Omega}(s_1, s_2, s_3) \left[ 1 + \cosh 2v s_1 + 2 \cosh 2v(s_1 + s_2) + \cosh 2cs_1 \right]
\]
\[+2 \cosh 2c(s_1 + 2s_3) + 2 \cosh 2c(s_1 + s_2) + 4 \cosh 2c(s_1 - s_2) + 8 \cosh v(s_1 - s_2) \cosh c(s_1 + s_2) \cosh^2 c(s_1 - s_2) + 16 \cosh v(s_1 + s_2) \cosh c(s_1 + s_2 + 4s_3) \cosh c(s_1 + s_2) \cosh c(s_1 - s_2) + 8 \cosh v(s_1 + s_2) \cosh^3 c(s_1 - s_2) + s_1 \leftrightarrow s_2 \]. \quad (A21)
References

[1] H.J. Sheinblatt, *Phys. Rev.* **D57** (1998) 2421, hep-th/9705054.

[2] W. Taylor, *Nucl. Phys.* **B508** (1997) 122, hep-th/9705110.

[3] E. Keski-Vakkuri and P. Kraus, *Nucl. Phys.* **B510** (1998) 199, hep-th/9706196.

[4] J.M. Pierre, *Phys. Rev.* **D56** (1997) 6710, hep-th/9707102.

[5] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, *Phys. Lett.* **B423** (1998) 238, hep-th/9711010.

[6] A. Dhar and G. Mandal, *Nucl. Phys.* **B531** (1998), hep-th/9803004.

[7] N. Itzhaki, hep-th/9809063.

[8] T. Banks, W. Fischler, S. Shenker and L. Susskind, *Phys. Rev.* **D55** (1997) 5112, hep-th/9610043.

[9] G. Gibbons and R. Kallosh, *Phys. Rev.* **D51** (1995) 2839.

[10] H. Lu and C. N. Pope, hep-th/9606047.

[11] P. Dobiasch and D. Maison, *Gen. Relativ. Gravitation* **14** (1982) 231.

[12] A. Chodos and S. Detweiler, *Gen. Relativ. Gravitation* **14** (1982) 879.

[13] D. Pollard, *J. Phys.* **A16** (1983) 565.

[14] G. Gibbons and D.L. Wiltshire, *Ann. Phys.* **167** (1986) 201.

[15] G. Lifschytz, *Nucl. Phys.* **B520** (1998) 105, hep-th/9612223.

[16] I. Chepelev and A.A. Tseytlin, *Nucl. Phys.* **B515** (1998) 73, hep-th/9709087.

[17] J. Branco, *Class. Quant. Grav.* **15** (1998) 3739, hep-th/9806180.

[18] A. Sen, *Adv. Theor. Math. Phys.* **2** (1998) 51, hep-th/9709220.

[19] N. Seiberg, *Phys. Rev. Lett.* **79** (1997) 3577, hep-th/9710009.