Order parameter scaling in fluctuation dominated phase ordering

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In systems exhibiting fluctuation-dominated phase ordering, a single order parameter does not suffice to characterize the order, and it is necessary to monitor a larger set. For hard-core sliding particles (SP) on a fluctuating surface and the related coarse-grained depth (CD) models, this set comprises the long-wavelength Fourier components of the density profile. We study both static and dynamic scaling laws obeyed by the Fourier modes \(Q_m\) and find that the mean value obeys the static scaling law \(\langle Q_m \rangle \sim L^{-\phi} f(m/L)\) with \(\phi \approx 2/3\) and \(\phi \approx 3/5\) with Edwards-Wilkinson (EW) and Kardar-Parisi-Zhang (KPZ) surface evolution respectively. The full probability distribution \(P(Q_m)\) exhibits scaling as well. Further, time-dependent correlation functions such as the steady state auto-correlation and cross-correlations of order parameter components are scaling functions of \(t/L^z\), where \(L\) is the system size and \(z\) is the dynamic exponent with \(z = 2\) for EW and \(z = 3/2\) for KPZ surface evolution. In addition we find that the CD model shows temporal intermittency, manifested in the dynamical structure functions of the density and a weak divergence of the flatness as the scaled time approaches zero.

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I. INTRODUCTION

In equilibrium statistical mechanics, the notion of an ordered state is well understood and the degree of order is characterized with the aid of two-point and one-point functions. Thus, the long-range order (LRO) \(m_c^z\) is defined through the asymptotic behaviour of a two-point correlation function, while the order parameter \(m_c\), the spontaneous value in a vanishing field, is a one-point function. In the thermodynamic limit, fluctuations become insignificant and \(m_c\) and \(m_c^z\) have well defined values \([1]\). Further, for systems with short-ranged interactions, conditions under which ordering is possible are known. Ordering occurs only if the spatial dimension exceeds one for a scalar order parameter, or exceeds two if ordering breaks a continuous symmetry \([2]\).

In nonequilibrium steady states, other types of behaviour are possible and characterization of order needs to be addressed afresh. Of particular interest is a class of systems in which fluctuations are anomalously strong, but which nevertheless have a propensity to order, leading to fluctuation-dominated phase ordering (FDPO)\([3-5]\). FDPO arises in several types of systems. These include passive scalar systems, in which a passive species is driven by an autonomously evolving field, e.g., particles driven by a fluctuating surface or a noisy Burgers fluid in one and two dimensions \([3, 5]\), or, in the context of active systems, by a 2D nematic field \([6]\). FDPO also arises in a model 1D granular gas in which the coefficient of restitution depends on the velocity of approach \([7]\). The signature of FDPO is a cusp singularity at small argument in the scaled two-point correlation function, signifying a breakdown of the Porod law. The relation of this singularity to giant number fluctuations has been discussed in Ref. \([8]\) in the context of active systems. Stronger singularities of the scaling function arise if the passive particles do not have hard core interactions \([3]\).

Finally, there seems to be an intriguing connection between FDPO and systems with quenched disorder, and cusp singularities have been observed in a number of systems with rough surfaces and interfaces \([10, 11]\).

This paper deals with the characterization of order in a passive scalar system which exhibits FDPO. The system consists of hard-core particles sliding locally downwards, along the local slopes of a one-dimensional fluctuating surface with overall slope zero \([3, 4]\). The surface evolves through its own dynamics whereas the particle movement is guided by local surface gradients. As time passes, particles are driven towards each other and the spatial extent of particle-rich regions increases as a function of time, until it is of the order of system size in steady state.

In earlier work, this tendency was quantified by studying coarsening properties, as the system evolves in time starting from a completely disordered state. As usual
for coarsening systems, the equal time correlation function $C(r, t)$ obeys scaling, with a growing length scale $L(t) \sim t^{1/z}$. The unusual point here is that the scaling function shows a cusp singularity as the argument $r/L(t)$ approaches zero. In momentum space, this translates into the scaled structure factor varying as $[kL(t)]^{-(1+\alpha)}$ where $k$ is the wave vector. The exponent $\alpha$ is less than 1 signifying a cusp singularity, which represents a marked deviation from the linear behaviour characteristic of the Porod law ($\alpha = 1$) which holds normally for scalar field coarsening [13, 16]. Further, cusp singularities were also observed in the correlation function in steady state [3, 4], and also in the decay of the temporal auto-correlation function [17].

In this paper, our focus is on the study of both static and dynamic aspects of the order parameter or one point function $Q_m(k, t)$. We argue that a single scalar order parameter does not suffice to properly characterize the order and we need to monitor a larger set. This set is built from the long-wavelength Fourier components of the density profile. We present evidence that these Fourier components have probability distributions which remain broad in the thermodynamic limit; that they are anticorrelated with each other; and that the probability distributions of the components for different sizes are described by simple scaling laws. We supplement our studies of the sliding particle system by studying a related coarse-grained depth model, and show that this model exhibits broadly similar behaviour. Further, the temporal behaviours (of autocorrelation and cross-correlation of the Fourier modes) are studied and scaling properties are investigated. This includes a study of dynamical structure function and the flatness, which is found to exhibit a divergence at small argument, indicating that the behaviour in time is intermittent [18, 19].

The paper is organized as follows: In Sec. II we introduce the sliding particle (SP) and coarse-grained depth (CD) models of fluctuating surfaces and review the behaviour of the two-point correlation function. In Sec. III we discuss the static properties of one point function, arguing that long-wavelength Fourier transforms of the density profile constitute an appropriate set of order parameters. We discuss their probability distributions, the correlations between them, and scaling properties. In Sec. IV we discuss the corresponding dynamical properties including cross correlations between modes. We conclude with a discussion of our results in Sec. V.

II. THE MODEL AND TWO-POINT CORRELATION FUNCTIONS

In this section, we discuss some simple models that show FDPO. The sliding particle (SP) model involves a system of hard core particles sliding under gravity on a stochastically evolving surface [3, 4] through either Edwards-Wilkinson (EW) or Kardar-Parisi-Zhang (KPZ) dynamics [20, 21]. We also define the coarse grained depth (CD) model which only involves the height field of the fluctuating surface, and which provides considerable insight. We then summarize the results of [3, 4] on the scaling behaviour of two-point correlation functions in these models.

A. Sliding particle (SP) model

This is a lattice model of particles moving on a 1D fluctuating surface [3, 4]. The particles and links are represented respectively by $\{\sigma_i\}$ and $\{\tau_{i, i-1/2}\}$. Both $\sigma$’s and $\tau$’s are Ising like variables that can take values $\pm 1$ on a one dimensional lattice with periodic boundary conditions. The occupation of any site $i$ can be given in terms of $\sigma_i$’s, as $n_i = \frac{1}{2}(1 + \sigma_i)$ ($n_i = 0, 1$), and the local slope of the surface connecting site $i$ is given by the variable $\tau_{i, i-1/2} = +1$ or $-1$ and is denoted by $\lor / \land$ respectively.

The evolution of the surface can be modelled by either the EW or the KPZ dynamics. This is incorporated in our model by the stochastic corner flips involving exchange of adjacent $\tau$’s: thus $\land \rightarrow \lor$ occurs with rate $p_1$, while $\lor \rightarrow \land$ changes with rate $q_1$. The symmetric surface fluctuations, i.e. $p_1 = q_1$, belong to the EW class whereas $p_1 \neq q_1$ belongs to the KPZ class. The dynamics of the particles can be modelled by exchanging the position of a particle and a vacuum (hole) pair at adjacent sites $(i, i+1)$ with rates that depend on the intermediate local slope: thus the moves $\lor \lor \rightarrow \lor \lor$ and $\lor \lor \rightarrow \lor \lor$ occur at a rate $p_2$, while the reverse moves occur at a rate $q_2 < p_2$. This asymmetry reflects the fact that it is easier to move downwards under the influence of a gravitational field. In our study we consider the strong field limit i.e., $q_2 = 0$ for the particle system and set $p_2 = p_1$. The dynamics conserves both $\sum \sigma$ and $\sum \tau$, and we study that sector where $\sum \sigma$ and $\sum \tau$ vanish corresponding to the half-filled system of particles on a surface with zero average tilt.

B. Coarse-grained depth (CD) model

In order to describe the dynamics of the hills and valleys of the surface, let us consider the height profile $\{h_i\}$ with $h_i = \sum_{1 \leq j \leq i} \tau_{j-1/2}$. We wish to define a model involving only $\{h_i\}$, to mimic the SP model. Since SP particles slide down and occupy the lower portion of the height profile, we define an analogous variable $\sigma_i$ in the CD model as taking on the values $+1$, $-1$, or 0 depending on whether the surface profile at site $i$ is below, above, or exactly coincident with a reference level: $\sigma_i = -sgn[h_i - \langle h(t) \rangle]$, where $\langle h(t) \rangle = \frac{1}{L} \sum_{i=1}^{L} h_i(t)$ is the instantaneous average height which fluctuates with time. The nomenclature Coarse-grained Depth model derives from the fact that the mapping $h_i \rightarrow \sigma_i$ may be viewed as a coarse graining which eliminates all fluctuations of
concerned with the static and dynamic properties of \(\{t_i\}^2\), which mimics the particle configuration in the SP model.

C. Two-point correlation function: overview

The two-point correlation function characterizes the steady state approached by the system of particles driven by a fluctuating surface that exhibits LRO. We monitor the evolution of the local density using Monte Carlo simulations. For both the EW and KPZ surfaces, no equilibration is needed because every configuration of hills and valleys with periodic boundary condition carries equal weight and can be chosen at random as a valid surface configuration in the steady state. The particles are distributed randomly on sites. In every Monte Carlo step, we performed 2\(L\) updates (\(L\) each for sites and bond variables) at random. The density distribution is guided by the evolution of the surface profile.

To quantify the tendency towards clustering, one can define the equal time two point correlation function as \(C(r,t) = \langle \sigma_i(t)\sigma_{i+r}(t)\rangle\) for the SP model, and \(C(r,t) = \langle s_i(t)s_{i+r}(t)\rangle\) for the CD model. The correlation functions have the scaling form

\[
C(r,t) = Y \left( \frac{r}{L(t)} \right),
\]

where \(L(t)\) is a time-dependent length scale which describes the typical linear size of a cluster at time \(t\), typically growing as a power law in time i.e. \(L(t) \sim t^{\frac{z}{2}}\). Here, \(z\) is the dynamical exponent characteristic of the surface fluctuations. This length scale \(L(t)\) for density fluctuations is set by the base lengths of typical coarse-grained hills which have overturned in time \(t\).

In 1D, we have \(z = 2\) for EW and \(z = 3/2\) for KPZ surface evolution. The existence of such a single growing length scale is a signature of coarsening towards a phase-ordered state. The coarsening is driven by surface fluctuations, rather than temperature quenching. As previously studied in Refs. 3, 4, the scaling curves have cusps at small values of \(r/L(t)\) with a nonzero intercept. That is, \(C(y) = C_0 - C_1 y^{\alpha}\). The intercept \(C_0\) is equal to the LRO measure \(m_0^2 = \lim_{r \to \infty} \langle \sigma_i \sigma_{i+r} \rangle\). This is because an arbitrary large but fixed distance \(r\) corresponds to \(y = r/L(t) \to 0\) in a coarsening system, since \(L(t)\) diverges as \(t \to \infty\). The cusp exponent \(\alpha\) is found numerically to be \(0.5\) for EW evolution and \(0.25\) with KPZ dynamics of the interface. In fact, the cusp exponent \(\alpha = 1/2\) can be found analytically for the CD model. The cusp implies a tail of scaled structure factor \(S(k) \sim (kL)^{-1+\alpha}\) for large \(kL\). This shows a deviation from the Porod law \(S(k) \sim (kL)^{-2}\), characteristic of customary coarsening.

If instead of considering coarsening in an infinite system, we consider steady state in a finite system of size \(L\), we may replace \(L(t)\) by \(L\) to conclude that the scaled two-point correlation function in the steady state can be written as

\[
C_s \left( \frac{r}{L} \right) = m_0^2 - c_1 \left( \frac{r}{L} \right)^{\alpha},
\]

with \(|r/L| \ll 1\). Here, as with coarsening, the cusp exponent \(\alpha = 0.5\) for EW and \(\alpha = 0.25\) for KPZ surface evolution.

III. ORDER PARAMETER STATICS

The long-wavelength Fourier transforms of the density profile (in the SP model) or the coarse-grained depth (in the CD model) constitute a set of order parameters appropriate to characterize the type of large-scale clustering that sets in. In this section, we study the probability distributions of these parameters and their scaling properties as the system size is changed.
A. The order parameter set

Let us define Fourier components of the density

$$Q(k) = \frac{1}{L} \sum_{j=1}^{L} n_j \exp \left( i \frac{2\pi mj}{L} \right) ,$$

where, \( n_j = \frac{1}{2} (1 + \sigma_j) \) and \( m = 1, 2, \ldots, L - 1 \). As we will see below, the lowest nonzero Fourier components \( Q_m^* = \langle Q(2\pi m/L) \rangle \) can be used as a measure of the phase separation in our system with conserved dynamics.

Figure 1 shows the mean values \( \langle Q(k) \rangle \) as a function of \( k \) for various values of \( L \) for SP and CD models. Two points should be noted. (a) First, for any fixed non zero value of wave-vector \( k \), one observes that \( \langle Q(k) \rangle \to 0 \) as \( L \to \infty \). (b) In order to study the \( k \to 0 \) limit, it is instructive to observe the behaviour of the low-\( m \) modes \( Q_m = Q(2\pi m/L) \) with \( m = 1, 2, 3, 4, \ldots \) and monitor \( Q_m^* = \langle Q_m \rangle = \langle Q(2\pi m/L) \rangle \). It is seen from Fig. 1 that \( Q_1^*, Q_2^*, \ldots \), each approach a finite limit as \( L \to \infty \).

The values of first four Fourier modes for various models are tabulated in Table I for both the EW and KPZ surface evolutions. From the table, we observe that \( Q_1^* \) is the largest, and it thus provides a first characterization of order.

The values of \( Q_1^*, Q_2^*, Q_3^*, \ldots \) characterize the gross form of the macroscopic cluster that occurs in the system. For example, \( Q_1^* \) is largest in configurations with a single dense cluster of particles, \( Q_2^* \) is largest in configurations with two well separated dense clusters and so on. The mean values of the Fourier components \( Q_m^* \) decrease with increasing \( m \), providing strong evidence that macroscopic clustering occurs in the system.

Figure 2 shows the time series for the long-wavelength Fourier modes \( Q_m \). The strong excursions about their average values lead to broad probability distributions. In fact these distributions remain broad in the thermodynamic limit. This is contrary to the situation in normal equilibrium phase transitions where the order parameter has a well defined value in the thermodynamic limit and its distribution consists of delta functions at values that characterize the phase.

In Fig. 3, we have shown the probability distribution functions of the first eight modes \( Q_m \) for both the SP and CD models with EW and KPZ surface evolutions. These distributions are obtained numerically for system sizes \( L = 64, 128, \) and 192 for EW, and \( L = 128, 256, \) and 512 for KPZ surface evolutions. It is

| Model   | Evolution | \( Q_1^* \) | \( Q_2^* \) | \( Q_3^* \) | \( Q_4^* \) |
|---------|-----------|-------------|-------------|-------------|-------------|
| SP      | EW        | 0.18        | 0.09        | 0.07        | 0.06        |
|         | KPZ       | 0.16        | 0.08        | 0.06        | 0.04        |
| CD      | EW/KPZ    | 0.22        | 0.11        | 0.08        | 0.055       |

TABLE I: \( Q_m^* \) values for SP and CD models.
We now turn to a discussion of the scaling properties of time dependent correlation functions in the models under study.

IV. ORDER PARAMETER DYNAMICS

The fact that \( k = 2\pi m / L \) is a good variable in the large \( L \) limit is already incorporated in writing the argument \( y = m / L \). Evidently the scaling function \( Y(y) \to Y_0 \), a constant value, as \( y \to \infty \) in order to be consistent with (a) above; \( Q^* \) falls to zero as \( Y_0 L^{-\phi} \). Turning to (b), the small-\( m \) end of Fig. 4 corresponds to \( y << 1 \) where we expect \( Y(y) \sim y^{-\gamma} \), which would imply an \( L \) dependence of the form \( m^{-\gamma} L^{\phi-\gamma} \) for \( Q^*(m, L) \). Since the data in Fig. 4 suggests finite limiting values as \( L \to \infty \) as noted in (b), we conclude that \( \gamma = \phi \) and the limiting values \( Q^*_m \) fall as \( m^{-\phi} \). This is verified in Fig. 5 which shows the data for the SP model for both EW and KPZ surface dynamics, and for the CD model. As noted earlier, the static properties of the CD model are identical for both the EW and KPZ cases.

The observed scaling properties of the average value \( Q^*(m, L) \equiv \langle Q(2\pi m / L) \rangle \) suggest that the full probability distributions \( P(2\pi m / L) \) in Fig. 3 may exhibit a scaling collapse as well. This is indeed the case (except for first few modes), as seen in Fig. 5 where we have plotted the scaled distribution \( P(Q_m) / m^\phi \) versus \( Q_m / m^\phi \).

We conclude by commenting on the values of the exponent \( \phi \). The values used in the plots in Fig. 4 and Fig. 5 are \( \phi = 2/3 \) and \( \phi = 3/5 \) (for the SP model with EW and KPZ surface dynamics, respectively) and \( \phi = 3/4 \) (for the CD model). An analytic understanding of these values would be very desirable.

B. Scaling properties

Let us turn to a discussion of the scaling behaviour of \( Q^*(m, L) = \langle Q(2\pi m / L) \rangle \) as a function of mode number \( m \) and system size \( L \). As noted earlier, Fig. 4 shows that: (a) For fixed value of \( k = 2\pi m / L \) away from zero, \( \langle Q(k) \rangle \to 0 \) as \( L \to \infty \). (b) For a fixed (typically small) value of \( m \), the mean value \( Q^*_m \) approaches a finite \( m \) dependent value as \( L \to \infty \). Thus, in Fig. 4 the limit \( k \to 0 \) exposes the set of order parameters we have been discussing. The limiting values (as \( L \to \infty \)) of \( Q^*_m \) fall with increasing \( m \). In order to connect (a) and (b), we postulate the scaling form:

\[
Q^*(m, L) \sim L^{-\phi} Y \left( \frac{m}{L} \right) .
\] (4)
A. Single site autocorrelation: earlier results

An earlier study [17] has shown that the single site occupancy autocorrelation function $A(t, L) \equiv \langle \sigma_i(0)\sigma_i(t) \rangle_L$ exhibits the scaling form

\[ A(t, L) \approx f \left( \frac{t}{L^z} \right), \tag{5} \]

for both SP and CD models, with $z = 2$ and $3/2$ for surface evolutions obeying EW and KPZ dynamics respectively. The scaling function $f(y)$ was found to show a cusp singularity as $y \to 0$

\[ f(y) \approx m_0^2 \left[ 1 - by^\beta \right] \quad \text{for} \quad y << 1, \tag{6} \]

where $m_0$ is a measure of LRO. For the SP model, the exponent $\beta$ was found to be approximately 0.22 for EW, and 0.18 for KPZ dynamics. For the CD model, the value $\beta = 1/4$ was derived analytically for EW dynamics, while the value $\beta \simeq 0.31$ was found numerically for KPZ case [17].

B. Order parameter set: auto-correlation and cross-correlation

Let us define time-dependent correlation functions involving the order parameter set

\[ C_{mn}(t) = \frac{\langle Q_m(0)Q_n(t) \rangle - \langle Q_m \rangle \langle Q_n \rangle}{\langle Q_m \rangle \langle Q_n \rangle}. \tag{7} \]

In Fig. 6 we have plotted $C_{mn}$ against the scaled argument $y = t/L^z$. The collapse, evident in the figure, provides clear evidence of scaling. Moreover, the cross-correlation function $C_{12}(t)$ is negative, implying that the
C. Dynamical structure functions and intermittency

We now turn to the characterization of the dynamical structure function of the first Fourier mode $Q_1$, whose variation in time is shown in Fig. 2. The question arises whether the time series $Q_1(t)$ is self-similar, or whether it displays intermittency. In this subsection, we study the dynamical structure factors, and an associated measure, namely the flatness, to answer this question. For the CD models, we find a weak but definite divergence of the flatness in the limit of small argument, implying that the series is weakly intermittent.

The structure functions are defined as moments of the distributions of temporal variations of $Q_1$:

$$S_n(t, L) = \langle [Q_1(t) - Q_1(0)]^n \rangle_L. \quad (8)$$

In general, $S_n(t, L)$ grows as a power of $t$, but saturates at a value which depends on system size $L$. Figure 4 shows that $S_2(L, t)$ is a scaling function of the variable $t/L^z$ for both $n = 2$ and $4$, where $z$ is the dynamical exponent. For large $t$, $Q(t)$ and $Q(0)$ are independent, and thus the saturation values of $S_2$ and $S_4$ are given by the static quantities $2\langle Q_2^2 \rangle - \langle Q_1^2 \rangle^2$ and $2\langle Q_4^2 \rangle - 8\langle Q_3^2 \rangle\langle Q_1 \rangle + 6\langle Q_3^2 \rangle^2$ respectively. These values can be determined accurately from steady state data and are shown as dashed lines in Fig. 7.

The property of intermittency has to do with the behaviour of $S_n(y)$ at small values of $y \equiv t/L^z$, i.e., $S_n \sim y^{\beta_n}$. For self-similar series, the exponents $\beta_n$ themselves grow linearly with $n$, i.e., $\beta_n/n$ is a constant. By contrast, for an intermittent time series, this property is no longer true. A good diagnostic is the flatness $\kappa(t/L^z) = S_2/S_4^2$. For a self-similar system, $\kappa(y)$ approaches a constant value as $y \to 0$, while for an intermittent system, $\kappa(y)$ diverges as $y \to 0$, due to the preponderance of sharp rises and falls of $Q_1$ over relatively short times. The flatness as a function of scaled variable $t/L^z$ for both SP and CD models with EW and KPZ dynamics is plotted in Fig. 8.

Let us first consider the CD models, for both the EW and the KPZ dynamics. The data in Fig. 7 indicate $S_2(y) \sim y^{\beta_2}$ and $S_4(y) \sim y^{\beta_4}$ with $\beta_2 = 0.709 \pm 0.002$ (EW) and $0.907 \pm 0.003$ (KPZ), and $\beta_4 = 1.365 \pm 0.005$ (EW) and $1.74 \pm 0.01$ (KPZ). This would imply $\kappa(y)$ diverging as $y^{-\gamma}$ with $\gamma = 2\beta_2 - \beta_4$, leading to $\gamma = 0.05 \pm 0.01$ (EW) and $\gamma = 0.07 \pm 0.02$ (KPZ). The exponent $\gamma$ can also be extracted directly from the plot for the flatness (Fig. 8). For the CD models we obtain $\gamma = 0.040 \pm 0.001$ (EW) and $0.049 \pm 0.001$ (KPZ). There is a weak but definite divergence. Hence we conclude that the time series $Q_1(t)$ displays weak intermittency for the CD models, in both the EW and KPZ cases. For the SP models in the intermediate $y$ regime, we estimate $\beta_2 = 1.639 \pm 0.007$ (EW) and $1.700 \pm 0.006$ (KPZ), and $\beta_4 = 3.21 \pm 0.02$ (EW) and $3.33 \pm 0.02$ (KPZ). Correspondingly we find $\gamma = 0.07 \pm 0.03$ for both EW and KPZ dynamics. At smaller values of $y$, the curves seem to show a flattening, but we are not able to reach a firm conclusion owing to problems with the precision of the data in this case.

V. CONCLUSIONS

In this paper, we have examined the characterization of order in fluctuation-dominated phase ordering (FDPO), by studying the static and dynamic properties of the order parameter set for a system of sliding hard-core particles driven by a 1D fluctuating surface. In addition to the sliding particle (SP) model, we studied the CD model of surface dynamics which mimics the properties of the SP model. The necessity for having a set of order parameters – here the long-wavelength Fourier modes of the particle density – arises from the fact that the macroscopic cluster of particles has a finite probability of breaking into a small number $m$ of clusters ($m = 1, 2, 3, \ldots$); the occurrence of these clusters is tracked by the $m$th Fourier mode, $Q_m$.

The characteristic of FDPO is that fluctuations remain large, and do not damp down in the thermodyn-
namic limit. Correspondingly, the probability distribution $P(Q_m)$ approaches a broad limiting form as $L \to \infty$. The mean value $Q^* (m, L)$ is found to be a scaling function of mode number $m$ and system size $L$, as is the full probability distribution. The temporal properties of FDPO are influenced by the existence of an $L$ dependent time scale $\sim L^z$ where $z$ is the dynamical exponent of the driving interface ($z = 2$ for EW, and $3/2$ for KPZ case). The dynamical properties are a function of $t/L^z$. The most interesting property is that the time series for the primary order parameter, $Q_1(t)$, displays intermittency in the case of the CD models for both EW and KPZ evolutions for the driving interface. The signature of this is a weak but definite divergence of the flatness, which involves the ratio of the fourth and second moments of the variations of the order parameter. It would be interesting to be able to spell out the conditions for intermittency to arise in systems displaying FDPO, and phase ordering in general.

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