Evolution of Quasar Stochastic Variability along Its Main Sequence

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Abstract

We explore the evolution of the time variability (in the optical g-band and on timescales of weeks to years) of Sloan Digital Sky Survey Stripe 82 quasars along the quasar main sequence. A parent sample of 1004 quasars within $0.5 \leq z \leq 0.89$ was used for our statistical studies; we then made subsamples from our parent sample: a subsample of 246 quasars with similar luminosities, and a subsample of 399 quasars with similar $R_{\text{Fe II}}$ (i.e., the ratio of the equivalent width of Fe II within 4435–4685 Å to that of H$\beta$). We find the variability amplitude decreases with luminosity ($L_{\text{bol}}$). The anticorrelation between the variability amplitude and $R_{\text{Fe II}}$ is weak but statistically significant. The characteristic timescale, $\tau$, correlates mostly with quasar luminosity; its dependence on $R_{\text{Fe II}}$ is statistically insignificant. After controlling luminosity and $R_{\text{Fe II}}$, the high- and low-FWHM samples have similar structure functions. These results support the framework that $R_{\text{Fe II}}$ is governed by Eddington ratio and the FWHM of H$\beta$ is mostly determined by orientation. We then provide new empirical relations between variability parameters and quasar properties (i.e., luminosity and $R_{\text{Fe II}}$). Our new relations are consistent with the scenario that quasar variability is driven by thermal fluctuations in the accretion disk; $\tau$ seems to correspond to the thermal timescale. From our new relations, we find that the short-term variability is mostly sensitive to $L_{\text{bol}}$. Based on this we propose that quasar short-term (a few months) variability might be a new type of “Standard Candle” and can be adopted to probe cosmology.

Key words: galaxies: general – quasars: emission lines – quasars: supermassive black holes

1. Introduction

Quasars\(^5\) show aperiodic luminosity variations across the electromagnetic spectrum (for a review, see Ulrich et al. 1997). The physical nature of quasar variability remains unclear although a number of theoretical scenarios have been proposed. For instance, the local (Lyubarskii 1997) or the global accretion rate (Li & Cao 2008) fluctuations can induce variations in quasar luminosity and have the potential to explain the power spectral density (PSD) and the amplitude of quasar variability. It is also speculated that quasar variability is driven by thermal fluctuations in the accretion disk (e.g., Czerny et al. 1999; Kelly et al. 2013). Moreover, the ultraviolet (UV) or optical variations on short timescales might also be induced by X-ray reprocessing (Czerny et al. 1999; Kubota & Done 2018). X-ray reprocessing could also be responsible for the inter-band time lags (Krolik et al. 1991; Edelson et al. 1996, 2015, 2017; Wanders et al. 1997; Collier et al. 1998; Sergeev et al. 2005; McHardy et al. 2014, 2016, 2017; Cackett et al. 2018; Sun et al. 2018, but see Shappee et al. 2014; Fausnaugh et al. 2016; Starkey et al. 2016, 2017; Gardner & Done 2017; Zhu et al. 2017).

Different physical scenarios manifest as various correlations between the variability parameters and quasar properties. Indeed, previous works on both individual and ensemble quasar variability have revealed that the amplitude and the PSD shape depend on quasar luminosity ($L_{\text{bol}}$), the mass ($M_{\text{BH}}$) of the supermassive black hole (SMBH), and wavelength (see e.g., Uomoto et al. 1976; Hook et al. 1994; Givone et al. 1999; Hawkins 2002; Vanden Berk et al. 2004; de Vries et al. 2005; Wilhite et al. 2008; Bauer et al. 2009; Kelly et al. 2009, 2013; MacLeod et al. 2010, 2012; Zuo et al. 2012; Sun et al. 2015; Kołowski 2016; Guo et al. 2017). Roughly speaking, these correlations are not entirely consistent with theoretical expectations. For instance, according to the classical thin-disk theory (Shakura & Sunyaev 1973), the thermal timescale ($\tau_{\text{th}}$) for a fixed wavelength depends only on quasar bolometric luminosity, i.e., $\tau_{\text{th}} \propto L_{\text{bol}}^{1/2}$. However, MacLeod et al. (2010) constrained the characteristic timescale ($\tau$) of quasar variability by fitting the continuous time first-order autoregressive process (i.e., CAR(1), whose PSD has the following shape, PSD($f$) $\propto 1/(f_0^2 + f^2)$, where $f_0 = 1/\tau$; see e.g., Kelly et al. 2009; Kołowski et al. 2010 and Section 3) to the light curves of the Sloan Digital Sky Survey (SDSS) Stripe 82 (S82) quasars and investigated the scaling relation between $\tau$ and $L_{\text{bol}}$ and $M_{\text{BH}}$: they found that the best-fitting scaling relation is incompatible with the expected scaling relations for the thermal or the viscous timescales. It is unclear whether the discrepancy is real or is simply caused by some systematic biases in estimating the variability parameters and quasar properties.

MacLeod et al. (2012) and Guo et al. (2017) argued that the PSD of the observed light curves on long timescales (i.e., $\gg \tau$) should be steeper than that of the CAR(1) process. A deviation from the CAR(1) process on short timescales (i.e., sub-month) has also been proposed (e.g., Mushotzky et al. 2011; Kasliwal et al. 2015; Simm et al. 2016; Caplar et al. 2017; Smith et al. 2018).

Recently, Kołowski (2017) explored the biases on the estimation of $\tau$ via fitting the CAR(1) process to individual light curves. They concluded that $\tau$ and other variability...
parameters are incorrectly determined if the baseline is too short, and the reported scaling relations between the variability parameters and quasar properties are unlikely to be robust. Instead, the ensemble structure function (which measures the variability amplitude as a function of timescale; see Section 3) is found to be less biased (Kozłowski 2016).

$M_{\text{BH}}$, one of the key parameters of SMBHs, is difficult to measure robustly for quasars. The most widely adopted approach is via the single-epoch virial black hole mass estimators (e.g., Vestergaard 2002; Vestergaard & Peterson 2006; Shen et al. 2011; for a recent review, see Shen 2013). These estimators are based on two assumptions: first, the broad emission line region (BLR) radius-quasar luminosity relation is valid for the full quasar population; and second, the line widths of the broad emission lines (BELs) trace the virial motions of the BLR gas. The empirical BLR radius-quasar luminosity relation (e.g., Bentz et al. 2013) is derived from a small sample of sources. There is new evidence that this empirical relation is invalid for high Eddington-ratio sources (Du et al. 2015).

Quasar spectra show diverse features in terms of emission lines. It is shown that the diversity can be well represented by several eigenvectors. It is widely speculated that the Eigenvector 1 (hereafter, EV1), which is the main variance of the diversity, is driven by Eddington ratio (Boroson & Green 1992; Sulentic et al. 2000a, 2000b; Boroson 2002; Runnoe et al. 2014). Shen & Ho (2014) and Sun & Shen (2015) adopted the orientation-independent $M_{\text{BH}}$ indicators and found that, after controlling for quasar luminosity, the Fe II strength, $R_{\text{Fe II}}$ (i.e., the ratio of the equivalent width of Fe II within 4345–4685 Å to that of H/β), anticorrelates with $M_{\text{BH}}$. However, controlling quasi variability and $R_{\text{Fe II}}$, the correlation between FWHM and $M_{\text{BH}}$ is rather weak or absent; it is likely that the line widths of BELs are sensitive to inclination (see also Collin et al. 2006; Runnoe et al. 2013; Pancost et al. 2014; Bisogni et al. 2017; Grier et al. 2017a; Storchi-Bergmann et al. 2017). Therefore, quasars can be unified by Eddington ratio (or $R_{\text{Fe II}}$) and orientation (or the line width of H/β; i.e., the quasar main sequence; see, e.g., Shen & Ho 2014).

It is interesting to investigate the evolution of quasar variability on the main sequence plane. There are only a few published studies on this topic. For instance, Ai et al. (2010) focused on the tight correlation between the long-term variability amplitude and $R_{\text{Fe II}}$.

In order to better understand the relationship between quasar variability and the main sequence, and to test the physical scenarios, we study the g-band light curves of spectrascopically confirmed SDSS S82 quasars by calculating the ensemble structure functions along the main sequence. We choose g-band for two reasons: first, compared with r-band, g-band is less contaminated by galaxy emission; and second, the noise level of g-band is smaller than that of u-band.

This paper is formatted as follows. In Section 2, we introduce our sample selection. In Section 3, we describe the structure function and the CAR(1) process. In Section 4, we present quasar variability along the main sequence. In Section 5, we discuss the implications of our results. We summarize our main conclusions in Section 6. In this work, we adopt a flat ΛCDM cosmology with $h_0 = 0.7$ and $\Omega_m = 0.3$ unless otherwise specified.

### 2. Sample Selection

Our initial parent sample consists of the SDSS S82 quasars considered by MacLeod et al. (2010). The S82 quasars have on average ~60 epochs of accurate photometry in five bands (i.e., ugriz; see Gunn et al. 2006); these light curves can effectively probe rest-frame timescales from weeks to years. The light curve data are produced with improved calibration techniques (Ivezić et al. 2007; Sesar et al. 2007). We then cross match this parent sample with the catalog of quasar properties from SDSS DR7 (Shen et al. 2011) and obtain the emission line properties and quasar parameters (e.g., the bolometric luminosity, $L_{\text{bol}}$). As a second step, we only select quasars with available properties of H/β and Fe within 4435 Å–4685 Å. Radio-loud (i.e., radio loudness $R = f_j (6 \text{ cm}) / f_j (2500 \text{ Å}) > 10$) quasars are also rejected. The resulting parent sample that will be used for our subsequent studies has 1004 quasars within $0.5 < z < 0.89$. We only consider sources in such a narrow range of redshift to eliminate the rest-frame wavelength dependence.

The distribution of our parent sample in the $R_{\text{Fe II}}-L_{\text{bol}}$ plane is shown in Figure 1. To explore the relationship between quasar variability and the main sequence, we made subsamples from our parent sample: a subsample of quasars with similar luminosity and redshift (i.e., the luminosity-matched sample), and a subsample of quasars with similar $R_{\text{Fe II}}$ and redshift (i.e., the $R_{\text{Fe II}}$-matched sample).

The luminosity-matched sample: this sample initially consists of 246 quasars with $10^{45.4} \text{ erg s}^{-1} \leq L_{\text{bol}} \leq 10^{45.6} \text{ erg s}^{-1}$ and $0.5 < z < 0.89$ (i.e., the region defined by two solid lines in Figure 1). We choose such a narrow luminosity range for two reasons. First, the distribution of $L_{\text{bol}}$ peaks at this luminosity range (see Figure 1). Second, the variability amplitude depends critically upon $L_{\text{bol}}$ but weakly on $R_{\text{Fe II}}$ (see Section 4.1). We verified that our conclusions would not change if we, for instance, considered quasars in other
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Table 1

Properties of the Luminosity- and R_{Fe II}-matched Samples

|                        | Number | \( \log L_{bol} \) (erg s\(^{-1}\)) | FWHM (H\(\beta\)) (km s\(^{-1}\)) | \( R_{Fe II} \) |
|------------------------|--------|-------------------------------------|------------------------------------|-----------------|
| The \( L_{bol} \)-matched sample |        |                                     |                                    |                 |
| High-\( L_{bol} \) bin | 82     | 45.50 ± 0.01                        | 3170 ± 220                         | 1.83 ± 0.07     |
| Middle-\( L_{bol} \) bin | 82     | 45.50 ± 0.01                        | 3410 ± 260                         | 0.89 ± 0.03     |
| Low-\( L_{bol} \) bin  | 82     | 45.49 ± 0.02                        | 4590 ± 300                         | 0.40 ± 0.02     |
| The \( R_{Fe II} \)-matched sample |        |                                     |                                    |                 |
| High-\( R_{Fe II} \) bin | 132    | 45.87 ± 0.02                        | 4440 ± 220                         | 0.70 ± 0.02     |
| Middle-\( R_{Fe II} \) bin | 134    | 45.50 ± 0.01                        | 4210 ± 320                         | 0.66 ± 0.04     |
| Low-\( R_{Fe II} \) bin  | 133    | 45.25 ± 0.01                        | 4550 ± 330                         | 0.70 ± 0.02     |

Note. The quoted value is the median of each bin. The 1σ error bar is calculated via bootstrapping.

3. Definition of Structure Function and the CAR(1) Process

3.1. Structure Function

The structure function,\(^9\) \( SF(\Delta t) \), measures the statistical dispersion of two random variables (i.e., a magnitude pair) separated by time intervals, \( \Delta t \). The structure function can be used to characterize the statistical dispersion of \( \Delta m \) for a sample of many similar quasars with the same (or close) \( \Delta t \), where \( \Delta m \) is the magnitude difference between two observations. We adopted the interquartile range (i.e., IQR) to measure the statistical dispersion as it is robust against outliers or tails in the distribution. Therefore, we calculate the statistical dispersion as follows (MacLeod et al. 2010; Sun et al. 2015),

\[
SF_{IQR}(\Delta t) = 0.74 \text{IQR}(\Delta m),
\]

where IQR(\(\Delta m\)) is the 25%–75% interquartile range of \(\Delta m\). The constant 0.74 normalizes the IQR to be equivalent to the standard deviation of a Gaussian distribution. Therefore, 0.74 IQR is known as the normalized IQR (hereafter NIQR).

It should be noted that the measured statistical dispersion (i.e., Equation (1)) is a superposition of measurement errors and quasar variability. On very short timescales (e.g., days), the amplitude of quasar variability is small and the statistical dispersion is dominated by measurement errors. Therefore, we can estimate measurement errors from the statistical dispersion on timescales of a few days. On timescales of months to years, the contribution of measurement errors becomes negligible.

3.2. The CAR(1) Process

The CAR(1) process is often referred to as the damped random walk or the Ornstein–Uhlenbeck (OU) process; this process has proven to be effective in describing the light curves of quasar continuum emission (e.g., Kelly et al. 2009; Kozlowski et al. 2010; MacLeod et al. 2010, 2012; Zu et al. 2013). The structure function of the CAR(1) process is given by

\[
SF(\Delta t|\tau, \delta) = \delta \sqrt{\tau (1 - \exp(-\Delta t/\tau))},
\]

where \( \Delta t = |t_i - t_j| \) is the separation time between two observations. That is, the CAR(1) process is characterized by two parameters, \( \delta \) and \( \tau \). The former, \( \delta \), determines the short-term variability amplitude while the latter, \( \tau \), is the characteristic timescale.

It should be noted that quasar variability might be more complex than the CAR(1) process. Therefore, Kelly et al. (2014) proposed more flexible continuous-time autoregressive moving-average (i.e., CARMA(p, q)) models to describe quasar light curves; the CAR(1) process corresponds to the CARMA(1, 0) process. For each source in our parent sample, we use the Python CARMA package\(^10\) and adopt the Akaike information criterion (AIC; Akaike 1974) to choose the order of the CARMA(p, q) models (i.e., determining \( p \) and \( q \) that minimize AIC; see Section 3.5 of Kelly et al. 2014); we also calculated AIC for the CAR(1) process (hereafter AIC(1)). We found that, for most of our light curves (~90%), the differences between the minimum AIC and AIC(1) is less than 10. Therefore, it seems that the data quality of our sample is insufficient to distinguish between the CAR(1) process and other more complex models. In Section 5.2, we model the structure functions as the CAR(1) process (i.e., Equation (2)); however, more complex models (i.e., Equation (3)) are also discussed. If quasar variability is indeed not driven by the stochastic models we assumed or the light curve is a nonstationary process, the uncertainties of our model parameters in Section 5.2 and Tables 3 and 4 might be inaccurate (or even underestimated; see, e.g., White 1982).

\(^9\) For a general discussion, see, e.g., Emmanoulopoulos et al. (2010) and Kozlowski (2016).

\(^10\) This package can be downloaded from https://github.com/brandonkelly/carma_pack.
Figure 2. $g$-band ensemble structure functions for the three bins, controlling $L_{\text{bol}}$ and $z$. Low-$R_{\text{Fe II}}$ quasars are more variable. The solid lines represent our best-fitting models (see Section 5.2).

4. The Ensemble Structure Function and Quasar Main Sequence

4.1. The Ensemble Structure Function and $R_{\text{Fe II}}$

We aim to explore the ensemble variability of quasar continuum as a function of $R_{\text{Fe II}}$. The ensemble structure functions for the three bins of the luminosity-matched sample are presented in Figure 2. Low-$R_{\text{Fe II}}$ quasars tend to be more variable (for a statistical description of our conclusion, see Section 5.2). Our result is well expected if: (1) EV1 is indeed driven by Eddington ratio and (2) for fixed luminosity, high Eddington-ratio quasars are more stable. The former assumption is supported by independent tests (e.g., Shen & Ho 2014; Sun & Shen 2015). We discuss possible explanations of the second requirement in Section 5.3.

The tendency between $R_{\text{Fe II}}$ and the quasar variability amplitude might be induced by FWHM of H$\beta$ since there might be an anticorrelation between FWHM and $R_{\text{Fe II}}$ (see Table 1) and $M_{\text{BH}} \propto$ FWHM$^2$. In order to verify this speculation, we explore quasar variability as a function of FWHM after controlling $R_{\text{Fe II}}$, $L_{\text{bol}}$, and $z$. Therefore, we construct samples as follows. First, we select quasars within $10^{45.3}$ erg s$^{-1} \leq L_{\text{bol}} \leq 10^{45.6}$ erg s$^{-1}$ and $0.4 < R_{\text{Fe II}} < 1$. We now choose a slightly wider luminosity bin to increase the statistic. Second, these sources are divided into two bins according to FWHM, i.e., the low- (high-) FWHM bin with FWHM being smaller (larger) than Median(FWHM). Third, we ensure $L_{\text{bol}}$ and Fe II strength of the two samples are matched via the methodology in Section 2. The number of quasars in the low- (high-) FWHM bin is 75 (74). The median values of FWHM for the two subsamples are 3127 km s$^{-1}$ and 6278 km s$^{-1}$, respectively. As shown in Figure 3, the ensemble structure functions for the two subsamples are quite similar. Therefore, it seems that the relation between quasar variability and the virial $M_{\text{BH}}$ is rather weak or absent since, for fixed quasar luminosity, $M_{\text{BH}} \propto$ FWHM$^2$.

We also control FWHM, $L_{\text{bol}}$, and $z$, and divide sources into two $R_{\text{Fe II}}$ bins following the method mentioned above. That is, we select quasars within $10^{45.3}$ erg s$^{-1} \leq L_{\text{bol}} \leq 10^{45.6}$ erg s$^{-1}$ and 3000 km s$^{-1} < $ FWHM $< 5000$ km s$^{-1}$ and divide them into two bins according to $R_{\text{Fe II}}$. We calculate the structure functions for the two bins. We again find that sources with larger $R_{\text{Fe II}}$ tend to be less variable (see Figure 4). These conclusions provide additional evidence supporting the claim that orientation determines the dispersion of FWHM (e.g., Shen & Ho 2014). We discuss this idea in Section 5.1.

4.2. The Ensemble Structure Function and Quasar Luminosity

In the previous section, we demonstrate the relation between quasar variability and $R_{\text{Fe II}}$. To examine whether there is an additional dependence on $L_{\text{bol}}$, we compare the ensemble structure functions of the $R_{\text{Fe II}}$-matched sample (see Figure 5). On short timescales (i.e., $1 \lesssim \Delta t \lesssim 100$ days), there is a clear anticorrelation between quasar variability and $L_{\text{bol}}$ (for a statistical description of our conclusion, see Section 5.2). This tendency diminishes on long timescales (i.e., $\Delta t \gtrsim 100$ days). Therefore, it seems that: (1) $L_{\text{bol}}$ controls the short-term ($1 \lesssim \Delta t \lesssim 100$ days) quasar variability and (2) $R_{\text{Fe II}}$ drives quasar variability on timescales of $\Delta t \gtrsim 10$ days.

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11 Throughout this work, FWHM refers to H$\beta$, unless otherwise specified.
5. Discussion

5.1. Implications for the Structure of BLR

According to our inspection of the structure function described in Section 4, quasar variability at a given wavelength in the UV/optical bands and on timescales from weeks to years can be characterized by $L_{\text{bol}}$ and $R_{\text{Fe II}}$. There is no additional correlation between quasar variability and FWHM. Our results can be well explained in the framework that the Eddington ratio and orientation govern most of the quasar diversity (Shen & Ho 2014). According to this scenario, the EV1 is driven by the Eddington ratio; high-Fe II-strength sources have high Eddington ratios and are less variable; FWHM is a tracer of orientation and does not correlate with quasar variability.

To test whether FWHM traces orientation, we compare the $r - W1$ color of the low-FWHM sample with that of the high-FWHM sample, where $W1$ refers to the WISE 3.4 $\mu$m band. To obtain $W1$, we cross-match our quasars with the ALLWISE catalog (Wright et al. 2010; Mainzer et al. 2011) with the maximum matching radius of 2". The left panel of Figure 6 presents our results. Indeed, sources in the broad-FWHM bin tend to have redder $r$-band SED than those in the narrow-FWHM sample (the $p$ value of the Anderson–Darling test is <0.01). Similar results have been obtained by Shen & Ho (2014). Therefore, broad- (narrow-) FWHM sources are consistent with being viewed more edge- (face-) on. If so, the geometry of BLR is disk-like rather than spherical, which is consistent with other observations (e.g., Jarvis & McCue 2006; Pancoast et al. 2014; Grier et al. 2017a; Storchi-Bergmann et al. 2017; Xiao et al. 2018). The orientation scenario also naturally explains the lack of correlation between quasar variability and FWHM (Figure 3).

[O III] EW has also been proposed as a tracer of orientation (e.g., Risaliti et al. 2011). We also show the distributions of [O III] EW for the broad and narrow FWHM samples in the right panel of Figure 6. Contrary to our expectation, we cannot reject the null hypothesis that the two distributions of [O III] EW are drawn from the same population (the $p$ value of the Anderson–Darling test is 0.4). Therefore, we conclude that [O III] EW is driven by $R_{\text{Fe II}}$ (i.e., the EV1; Boroson & Green 1992) or the maximum disk temperature (Panda et al. 2017a; Panda et al. 2017b; Panda et al. 2018) rather than orientation.

5.2. Modeling Quasar Variability

Previous works (e.g., MacLeod et al. 2010, 2012; Kozłowski 2016) aimed to find correlation between quasar variability as a function of $L_{\text{bol}}$ and $M_{\text{BH}}$. Often in these works, $M_{\text{BH}}$ is estimated via the single-epoch virial black hole mass estimators, i.e., $\log M_{\text{BH}} = p_0 + p_1 \log L + p_2 \log \text{FWHM}$, where $p_0$, $p_1$, and $p_2$ are constants (e.g., Vestergaard 2002; Vestergaard & Peterson 2006; Shen et al. 2011). However, as we demonstrate in Section 4.1 and Figures 3 and 4, there is no clear relation between quasar variability and FWHM. Therefore, we relate quasar variability to $L_{\text{bol}}$ and $R_{\text{Fe II}}$.

The main purpose of this section is to provide new empirical relations for future variability modeling. Therefore, for simplicity, we assume quasar variability is a CAR(1) process (which can, in practice, describe the light curves well; see, e.g., Kelly et al. 2009; MacLeod et al. 2010, 2012; Zu et al. 2013).

We aim to explore the correlations between the CAR(1) parameters (i.e., $\tau$ and $\hat{\sigma}$) and quasar properties (i.e., $L_{\text{bol}}$ and $R_{\text{Fe II}}$). Following Kozłowski (2016), we constrain $\hat{\sigma}$ and $\tau$ by modeling the ensemble structure function with

$$\text{SF}_E^2(\Delta t; \tau, \hat{\sigma}) = \hat{\sigma}^2 \tau (1 - \exp(-\Delta t/\tau)) + \sigma_p^2,$$  

where $\sigma_p$ is the uncertainty of the magnitude difference between two observations separated by $\Delta t$. We fix $\beta = 1$ (i.e., the CAR(1) process, see Equation (2)) in our subsequent analysis (we try to set $\beta$ as a free parameter in Section 5.3).

To explore the dependence of $\tau$ on $L_{\text{bol}}$ and $R_{\text{Fe II}}$, we perform the following analysis. For each bin of the $R_{\text{Fe II}}$-matched sample, we assume that the ensemble CAR(1) parameters are determined by

$$\log \tau = c_1 + \kappa_1 \log(L_{\text{bol}}/10^{45.5} \text{ erg s}^{-1}),$$  

and

$$\log \hat{\sigma} = c_2 + \kappa_2 \log(L_{\text{bol}}/10^{45.5} \text{ erg s}^{-1}),$$

where $L_{\text{bol}}$ is the average of $L_{\text{bol}}$ in each bin. We also try to remove galaxy contamination to $L_{\text{bol}}$ by applying the empirical relation of Equation (1) in Shen et al. (2011). We then calculate

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**Figure 5.** $g$-band ensemble structure functions for the three $L_{\text{bol}}$ bins, controlling $R_{\text{Fe II}}$ and redshift. On short timescales (i.e., $1 \lesssim \Delta t \lesssim 100$ days), quasar variability and $L_{\text{bol}}$ are anti-correlated. This tendency diminishes on long timescales (i.e., $\Delta t \gtrsim 100$ days). The solid lines represent our best-fitting models (see Section 5.2).

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**Figure 6.** Left: distributions of $r - W1$ color for the broad- and narrow-FWHM bins, controlling $L_{\text{bol}}$, redshift, and $R_{\text{Fe II}}$. Sources in the broad-FWHM bin tend to have redder $r - W1$ colors. Right: The distributions of EW([O III]) for the broad- and narrow-FWHM bins, controlling $L_{\text{bol}}$, redshift, and $R_{\text{Fe II}}$. The two bins are consistent with being drawn from the same population of EW([O III]).
the theoretical structure function from these two equations and Equation (3).

We fit the theoretical structure functions to the three ensemble structure functions of the $R_{\text{Fe ii}}$-matched sample via a Bayesian approach. The likelihood function is

$$
\ln p(f|x, \text{pms}, \sigma_{\text{int}}) = -\frac{1}{2} \sum_{i=1}^{3} \sum_{n} \left[ \left( f_{\text{in},i} - f_{\text{model},n,i} \right)^2 + \ln(2\pi\sigma_{\text{in},i}^2) \right] + \ln L_{\text{int}},
$$

where $x$ represents a set of quasar parameters (e.g., $L_{\text{bol}}$, $R_{\text{Fe ii}}$); pms is a collection of parameters $c_{11}$, $c_{21}$, $\kappa_{11}$, and $\kappa_{21}$; $f_{\text{model},n,i}$ and $f_{\text{in},i}$ are the theoretical and observational structure functions, respectively; $i = 1, 2, 3$ represents the three bins; and $n$ indicates each $\Delta(n)$. The vector $\sigma_{\text{in},i}$ denotes the summation of the measurement uncertainty of $f$ and the (possible) intrinsic scatter which is assumed to be $\sigma_{\text{int},f_{\text{model},n,i}}$.

Combining the best-fitting relations for the $R_{\text{Fe ii}}$- and luminosity-matched samples, we can derive quasar variability as a function of $L_{\text{bol}}$ and $R_{\text{Fe ii}}$, i.e.,

$$
\log \tau = 2.49 + 0.50(\log L_{\text{bol}} - 45.50),
$$

and

$$
\log \sigma = -1.788 - 0.26(\log L_{\text{bol}} - 45.50) - 0.08R_{\text{Fe ii}}.
$$

Table 2: Priors of the Parameters

| Parameter | Min | Max | Distribution |
|-----------|-----|-----|--------------|
| $c_{11}$  | 0.0 | 4.0 | Uniform      |
| $c_{22}$  | -10.0 | 10.0 | Uniform      |
| $\kappa_{11}$ | -2.0 | 2.0 | Uniform      |
| $\kappa_{22}$ | -2.0 | 2.0 | Uniform      |

The priors are summarized in Table 2. The statistical properties of the distributions are summarized in Table 3. To our surprise, the correlation between $\tau$ and $R_{\text{Fe ii}}$ is statistically insignificant as $\kappa_{12} = 0.001 \pm 0.070$. Therefore, we conclude that $\tau$ depends mostly on $L_{\text{bol}}$.

We then refit the ensemble structure functions of the luminosity-matched sample with Equations (7) and (8) but fix $\kappa_{12} \equiv 0$ (i.e., we assume $\tau$ does not depend on $R_{\text{Fe ii}}$). The statistical properties of the distributions are summarized in Table 3. The best-fitting structure functions are the solid lines in Figure 2. By fixing $\kappa_{12} \equiv 0$, the intrinsic scatter of the fit ($\ln \sigma_{\text{int}} = -2.41$) is similar to that of the previous fit ($\ln \sigma_{\text{int}} = -2.42$). That is, $\tau$ and $R_{\text{Fe ii}}$ are not tightly correlated.

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$$
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\log \sigma = -1.788 - 0.26(\log L_{\text{bol}} - 45.50) - 0.08R_{\text{Fe ii}}.
$$

For each S82 quasar with “good” data (e.g., at least 10 epochs and small measurement errors), MacLeod et al. (2010) fit the CAR(1) process to the light curve and constrained $\delta$ and $\tau$. In principle, we can adopt their data and fit the best-fitting parameters as a function of quasar properties. However, Kozłowski (2017) recently demonstrated that if the baseline is not ~5–10 times larger than $\tau$, the best-fitting CAR(1) parameters are biased. The biases are negligible for $\delta$ but are rather strong for $\tau$. Therefore, we should only focus on $\delta$.

The function we use to relate $\delta$, $L_{\text{bol}}$, and $R_{\text{Fe ii}}$ is

$$
\log \delta = a + b(\log L_{\text{bol}} - 45) + c \log \text{FWHM}.
$$

For comparison, we also try to fit the following function:

$$
\log \delta = a + b(\log L_{\text{bol}} - 45) + c \log \text{FWHM}.
$$

We fit the functions of Equations (11) and (12) to quasars in the range $0.5 < z < 0.89$ (i.e., a narrow range of redshift) via a Bayesian approach. The likelihood function is

$$
\ln p(\log \delta|x, \sigma_{\text{in}}, \text{pms}, \sigma_{\text{int}}) = -\frac{1}{2} \sum_{i=1}^{3} \sum_{n} \left[ \left( \log \sigma_{\text{in}} - \log \sigma_{\text{model},n,i} \right)^2 + \ln(2\pi\sigma_{\text{in},i}^2) \right] + \ln L_{\text{int}},
$$

where $x$ is $[\log(L_{\text{bol}}), R_{\text{Fe ii}}]$ (or $[\log(L_{\text{bol}}), \text{FWHM}]$); $\sigma_{\text{in}}$ is the uncertainty of $x$; pms represents parameters $a, b$, and $c$; $\bar{\sigma}_{\text{model}}$ is given by Equation (11) or (12); and $\sigma_{\text{int}}$ is a summation of the measurement uncertainty of $\delta$ and the intrinsic scatter. Furthermore, $\bar{\sigma}^2 = \sigma_{\text{int}}^2 + (b\sigma_{\text{in}})^2 + (c\sigma_{\text{in}})^2$, where $\sigma_{\text{in}}$, $\sigma_{\text{in}}$, and $\sigma_{\text{in}}$ are the measurement errors of $L_{\text{bol}}$, $R_{\text{Fe ii}}$ (or FWHM), respectively, and $\sigma_{\text{int}}$ represents the statistical dispersion due to either measurement errors of $\log \delta$ or the intrinsic scatter. The priors are summarized in Table 2.

The statistical properties of the parameters $a, b, c$, and $\sigma_{\text{int}}$ for $\delta$ as a function of $L_{\text{bol}}$ and $R_{\text{Fe ii}}$ (i.e., Equation (11)) are presented in Table 4. Our results indicate that while the short-term variability is mainly driven by $L_{\text{bol}}$, an additional dependence on $R_{\text{Fe ii}}$ (or Eddington ratio) is also statistically significant.

In the works of MacLeod et al. (2010) and Kozłowski (2016), they explored the dependencies of $S_{\text{Fe i}}$ and $\tau$ on $L_{\text{bol}}$ and $M_{\text{BH}}$. Using their best-fitting relations, we can also obtain

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The intrinsic scatter is considered during the fit since the bootstrap method might significantly underestimate the errors of the ensemble structure functions (Emmanoulopoulos et al. 2010).
the relation between $\dot{\sigma}$, $L_{\text{bol}}$, and Eddington ratio. In both works, the dependence of $\dot{\sigma}$ on $L_{\text{bol}}$ is close to our result. Kelly et al. (2009, 2013) also obtained a similar relation using light curves of the international AGN Watch projects. However, the correlation between $\dot{\sigma}$ and Eddington ratio is statistically insignificant in these works.

It is quite possible that in previous works the correlation between $\dot{\sigma}$ and Eddington ratio was diluted by the large uncertainty in $M_{\text{BH}}$ due to orientation. Indeed, after controlling $L_{\text{bol}}$ and $R_{\text{Fe II}}$, the ensemble structure function does not depend on FWHM (see Figure 3). To confirm our guess, we explore the dependence of $\dot{\sigma}$ on $L_{\text{bol}}$ and FWHM by fitting Equation (12). The priors are summarized in Table 2. The statistical properties of the distributions are summarized in Table 4. As we expected, there is indeed no correlation between $\dot{\sigma}$ and FWHM (the slope, $c_2$, is statistically consistent with 0). Therefore, the additional dependence of $\dot{\sigma}$ on Eddington ratio is missed in previous works.

5.3. Implications for Accretion Physics

In this work, we find a dependency of the variability parameters on $L_{\text{bol}}$ and $R_{\text{Fe II}}$. Therefore, it is likely that the optical/UV variability is produced in the quasar central engine. Several models have been proposed to explain the connection between the optical/UV variability and quasar properties. For

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is that Eddington ratio might correlate with gas metallicity (e.g., Matsuoka et al. 2011). If so, high-$R_{\text{Fe} \ \text{II}}$ quasars are iron-overabundant, and their accretion disks are more stable (Jiang et al. 2016).

The scatter of $\delta$ as a function of $L_{\text{bol}}$ and $R_{\text{Fe} \ \text{II}}$ is slightly smaller than that of the relation between $\delta$, $L_{\text{bol}}$, and FW. These scatters are caused by measurement errors (which is 0.088 dex) and intrinsic scatter. Guo et al. (2017) argued that the intrinsic scatter is caused by the deviation from the CAR(1) process on long timescales (see their Figure 9). Based on this hypothesis, they constrained the PSD of quasar variability on long timescales to be steeper than $f^{-1.5}$. According to our best-fitting results, the intrinsic scatter in their work is slightly overestimated since they related $\delta$ to FW. Therefore, the PSD of quasar variability on long timescales approaches the $1/f$ relation. Such a PSD is expected from the local variations of accretion rate (Lyubarskii 1997; Noble & Krolik 2009).

We find that the characteristic timescale, $\tau$, is mostly driven by $L_{\text{bol}}$ (see Section 5.2; Table 3). This sole dependence and the normalization encourage us to link $\tau$ with the thermal timescale ($\tau_{\text{TH}}$). The best-fitting slope (0.50 ± 0.08) is remarkably consistent with the theoretical expectation (i.e., the thermal timescale $\tau_{\text{TH}} \sim L_{\text{bol}}^{-1/2}$). It should be noted that even if $\tau$ is the thermal timescale, there might still be an anticorrelation between $\tau$ and $M_{\text{BH}}$ for fixed $L_{\text{bol}}$. This is because the thermal timescale of an accretion disk depends positively on iron abundance (Jiang et al. 2016); quasars with high Eddington ratios might be more metal-rich than those with low Eddington ratios (Matsuoka et al. 2011). However, such a correlation is not found in our results. It is possible that this correlation is weak and is unable to be revealed in our data.

Recent works suggested that significant deviations occur on very short timescales (i.e., ~days; see, e.g., Mushotzky et al. 2011; Kasliwal et al. 2015; Smith et al. 2018). However, on timescales we consider here (months to years), this deviation should not be very important. Kozlowski (2016) revealed a positive correlation between $\beta$ and $L_{\text{bol}}$ by studying the S82 quasars. We then refit Equation (3) to the $R_{\text{Fe} \ \text{II}}$-matched samples via the same Bayesian approach but set $\beta$ as a free parameter. We do not find a significant correlation between $\beta$ and $L_{\text{bol}}$. The discrepancy might be caused by the following factors. First, our selected S82 quasars have much lower luminosity than those of Kozlowski (2016). Second, we use $R_{\text{Fe} \ \text{II}}$ rather than the ratio of $L_{\text{bol}}$ to the virial $M_{\text{BH}}$ (which is likely biased by orientation) to trace the unknown Eddington ratio.

The strong correlation between $\tau$ and $L_{\text{bol}}$ is also found by Caplar et al. (2017). We note, however, that they adopted a different method to constrain $\tau$. In some other previous works (MacLeod et al. 2010; Kozlowski 2016), $\tau$ is found to be insensitive to $L_{\text{bol}}$ but depends on the virial $M_{\text{BH}}$. The differences between our results and those of Kozlowski (2016) might also be caused by the factors mentioned above.\(^{15}\)

However, it should be noted that quasar variability on long timescales is likely not consistent with the CAR(1) process (MacLeod et al. 2012; Guo et al. 2017). If this is found to be true, it is unclear whether or not we can directly compare $\tau$ with $L_{\text{bol}}$.

\(^{15}\)Kozlowski (2017) argued that $\tau$ can easily be biased toward lower values. The bias anticorrelates with the ratio of the (rest-frame) time interval of a light curve to $\tau$. If our $\tau$-$L_{\text{bol}}$ relation is correct, our best-fitting results are less biased since our selected S82 quasars are less luminous (i.e., smaller $\tau$) and have smaller redshifts (i.e., longer rest-frame time interval).
physical timescales. The forthcoming era of time domain astronomy will be key to understanding the physical nature of $\tau$.

### 5.4. Quasar Variability as a Probe of Cosmology?

Our work and many previous works (e.g., MacLeod et al. 2010; Kozłowski 2016; Caplar et al. 2017) indicate that the short-term UV/optical variability amplitude (or $\delta$) critically depends on $L_{\text{bol}}$. In this work, we also find an additional dependence of $\delta$ on $R_{\text{Fe II}}$. This additional dependence is statistically significant but rather weak since the slope is $-0.05 \pm 0.01$ (see Table 4). In practice, we can ignore this additional dependence and fit $\delta$ as a function of $L_{\text{bol}}$ only (i.e., the parameter $c_1$ in Equation (11) is fixed at zero). The best-fitting parameters are $\delta = (-1.74 \pm 0.012) - (0.30 \pm 0.018)L_{\text{bol}}$; the scatter (i.e., $\sigma_{\text{int}}$ which is a combination of measurement errors and the intrinsic scatter) is $\exp(-1.81 \pm 0.026)$ which is the same as that of Equation (11). We can, in principle, estimate $L_{\text{bol}}$ from the $\delta$--$L_{\text{bol}}$ relation without assuming any cosmological models. Therefore, it is possible to use the short-term UV/optical variability of quasars as a probe of cosmology parameters.

The $\delta$--$L_{\text{bol}}$ relation (i.e., Equation (11) with $c_1$ fixed at zero) can be revised as

$$\log \delta = a + b \log(4\pi f/10^{15} \text{ erg s}^{-1}) + 2b \log(D_L),$$

(14)

where $f$ is the observed flux and $D_L$, the luminosity distance, is a function of the cosmological model and can be independently measured if we know $\delta$, $f$, $a$, and $b$. Given the intrinsic scatter of the $\delta$--$L_{\text{bol}}$ relation, such constraints can only be made with a large sample of quasars that span over a wide range of cosmic history.

To illustrate this idea, we perform the following simulation of $10^5$ quasars. For each quasar, the intrinsic $L_{\text{bol}}$ and $R_{\text{Fe II}}$ and their measurement errors are assigned according to the randomly selected quasar from our parent sample. We then calculate $\delta_{\text{th}}$ from our best-fitting Equation (11); a Gaussian noise with standard deviation of $\exp(-1.81)$ (see Section 5.2) is added to $\delta_{\text{th}}$ to generate the observed $\delta$. We also assign galaxy contamination according to Equation (1) of Shen et al. (2011). The observed $\delta$ is diluted by the nonvariable galaxy emission. The observed $L_{\text{bol}}$ and $R_{\text{Fe II}}$ are generated by perturbing intrinsic $L_{\text{bol}}$ and $R_{\text{Fe II}}$ with their measurement errors. In addition, the galaxy emission is added to the observed $L_{\text{bol}}$. To calculate the observed flux, we assume a flat $\Lambda$CDM cosmology with $h_0 = 0.7$ and $\Omega_m = 0.3$; redshift is randomly assigned from a uniform distribution within [0.1, 0.89]. We then fit Equation (14) to the simulated mock sample by considering the $\Lambda$CDM cosmology with $h_0 = 0.7$ via a Bayesian approach. Both $\Omega_m$ (i.e., the matter density fraction) and $\Omega_\Lambda$ (i.e., the dark energy fraction) are free parameters. The likelihood function is the same as Equation (13) and the priors are summarized in Table 2.

The posterior distributions of the model parameters are presented in Figure 8. Even if $\Omega_\Lambda$ is not constrained, the recovered $\Omega_m = 0.28 \pm 0.03$ is accurate. We note that if the sample size is limited to $10^2$, the recovered $\Omega_m = 0.36 \pm 0.12$. Therefore, the large sample size is one of the key factors.

Our simulated sample might be available in the era of time-domain astronomy (e.g., with the Large Synoptic Survey Telescope). However, it remains unclear whether the scatter of the $\delta$--$L_{\text{bol}}$ relation depends on the sample size/redshift or not.

In order to test this hypothesis, we select sources with $0.96 < z < 1.48$ or $1.48 < z < 2.03$. Their $r$ or $i$ bands correspond to the rest-frame of $0.5 < z < 0.9 \text{ g}$ band. We calculate the differences between their $\delta$ and the expected values from our best-fitting $\delta$--$L_{\text{bol}}$ relation. Some of the differences are due to a combination of the scatter of the relation and the measurement error of $L_{\text{bol}}$. $\delta_{\text{L}}$ (i.e., the total scatter is $\sqrt{\sigma_{\text{int}}^2 + (b\sigma_f)^2}$). We then calculate the ratio of the differences to this total scatter. We find that for 95% of sources the ratio is less than 3. Many of the remaining 5% of sources are highly variable (20% have $\delta > 1$). Such sources might be “changing-look” AGN candidates (MacLeod et al. 2016); the origin of such variability could be different. Therefore, it is unlikely that the scatter of the $\delta$--$L_{\text{bol}}$ relation significantly depends on the sample size/redshift. We could also use high-redshift ($z \sim 2$) quasars to constrain cosmological parameters; the accuracy would be further improved.

In addition to our method, it is also proposed that the BLR and dust reverberation (Watson et al. 2011; Yoshi et al. 2014), the nonlinear relation between the UV and X-ray luminosities (Risaliti & Lusso 2015), the X-ray variability and broad line width (La Franca et al. 2014), and the saturated luminosity of super-Eddington AGNs (Wang et al. 2013) could also be adopted as distance measurements. In conclusion, AGNs will play a more important role in measuring the universe (for a recent review, see Czerny et al. 2018).

### 6. Summary

In this work, we have explored the evolution of the optical $g$-band variability of SDSS S82 quasars along the quasar main sequence. Our study focuses on quasar variability on timescales of weeks to years. Our main results are as follows.

![Figure 8. Posterior distributions of the parameters for $\delta$ as a function of $L_{\text{bol}}$ and cosmology parameters ($\Omega_m$ and $\Omega_\Lambda$) from $10^5$ simulated quasars. The blue lines and dot indicate the input parameters. $\Omega_m$ is well constrained to be 0.28 $\pm$ 0.03. On the other hand, $\Omega_\Lambda$ cannot be constrained.](image-url)
1. The variability amplitude decreases with \( L_{\text{bol}} \) (Section 4.2; Figure 5) and \( R_{\text{Fe II}} \) (Section 4.1; Figure 2). After controlling for luminosity and \( R_{\text{Fe II}} \), high- and low-FWHM sources show similar variability (Figure 3). These results support the scenario where \( R_{\text{Fe II}} \) is governed by Eddington ratio (Shen & Ho 2014); FWHM traces orientation (Section 5.1).

2. We provide new empirical relations between the variability parameters \( L_{\text{bol}} \) and \( R_{\text{Fe II}} \) (Section 5.2; Equations (9) and (10)).

3. Our new empirical relations are consistent with the scenario where quasar variability is driven by thermal fluctuations in the accretion disk; \( \tau \) seems to correspond to the thermal timescale (Section 5.3). X-ray reprocessing and/or gas metallicity might also play a role in determining short-term variability.

4. The short-term variability depends mostly upon \( L_{\text{bol}} \). We therefore propose that short-term (a few months) quasar variability might be regarded as a new type of “Standard Candle.” Our simple simulation suggests that the cosmological parameters can be well constrained with a sample of 10^4 quasars (Section 5.4).

In this work, we only focus on the SDSS S82 quasars. Therefore, we cannot constrain quasar variability on timescales of less than months. On such timescales, it has been shown that the PSD of quasar variability has an additional break to \( f^{-n} \) with \( n > 2 \) (Mushotzky et al. 2011; Kasliwal et al. 2015). It would also be interesting to explore the relation between such variability and \( R_{\text{Fe II}} \). Meanwhile, current and future surveys, e.g., SDSS, PTF (Law et al. 2009), DES (Honscheid et al. 2008), and LSST (Ivezic et al. 2008) can provide much better light curves in terms of cadence and baseline. Our results can be justified and extrapolated in the era of time domain astronomy.

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