Robust Control of High-Speed Elevator Transverse Vibration Based on LMI Optimization

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Abstract. Aiming at the problem of transverse vibration of high-speed elevator caused by surface roughness excitation of guide rail and relative motion between car and car frame, firstly, this paper establishes an eight-degree-of-freedom active control model for the transverse vibration of the high-speed elevator separated of car from the car frame. Secondly, the transverse vibration state equation of the high-speed elevator is derived. The \( H_2 \) norm is used to minimize the vibration acceleration of the car system. Based on Linear matrix inequality (LMI) optimization technology, a robust controller for the high-speed elevator is designed. Finally, the robust controller is solved by MATLAB and its simulation analysis is carried out. The results show that the vibration of the car and the car frame is inconsistent in actual operation. At the same time, the simulation results also show that the root mean square value, mean value and maximum value of the acceleration of the transverse vibration of the car and the car frame are reduced by more than 19% after using the robust controller. Therefore, the robust controller designed in this paper can effectively suppress the transverse vibration of the vibration car system.

1. Introduction

Nowadays, with the aggravation of social and economic development, population growth, land resource shortage and other problems, the number of high-rise and super-high-rise buildings is increasing, and the speed of elevator as a vertical transportation tool is also increasing. The results show that the transverse vibration acceleration of the car of a high-speed elevator running at a speed of more than 3m/s is significantly greater than that of a low-speed elevator [1]. The traditional method of vibration reduction is to install a spring between the car frame and the guide shoes [2]. This passive vibration reduction method is simple in structure, easy to realize, economical and reliable, and has satisfactory effect when the speed of the elevator is low, because it does not need external energy. However, with the increase of elevator speed, the limitation of passive vibration reduction is shown. For example, it is only suitable for external disturbance with constant frequency or small change, and it can only provide small vibration reduction force, so it is difficult to meet people's requirements. Active vibration control technology has become a new way to solve elevator vibration because of its good effect and adaptability. Many scholars have made a lot of research on it and achieved a lot of useful results [3-4]. In the research process, these scholars often regard the car and the frame as a whole of rigid connection, and then control the whole, but in the actual operation process of the elevator, the car and the frame are connected by the damping rubber block. With the increase of the
elevator speed, the transverse relative displacement between the car and the car frame increases, at this
time, the car and the car frame cannot be treated as a rigid body.
In view of the above shortcomings, this paper establishes the dynamic model of active transverse
vibration control of high-speed elevator with car and car frame as two rigid bodies, and derives its
state equation. The $H_2$ norm is used to minimize the vibration acceleration of the output car system.
Based on LMI optimization technology, a robust controller for transverse vibration of high-speed
elevator is designed, and the simulation analysis is carried out by using MATLAB.

2. Active Control Model for Transverse Vibration of High-Speed Elevator
This paper only considers the transverse vibration between the elevator car and the guide rail, that is,
the vibration parallel to the direction of the car door. In order to simplify the model and conform to the
actual operation conditions of the elevator as far as possible, the following reasonable assumptions are
made according to the structure and movement law of the elevator.
(1) In order to simplify the calculation, it is assumed that the centroid of the car and the frame does
not shift in the vertical direction during the discussion of the relative motion of the car and the
frame, and the car and the frame are referred to as the car system.
(2) Because the roller is close to the guide rail and the relative displacement between them is small in
the course of vibration, the guide shoe can be simplified as a mass spring damping system.
(3) The car is connected with the car frame by the damping rubber block. The damping rubber block
is simplified as a spring damping system, and the structure and parameters of each damping
rubber block are identical.
(4) The structure and parameters of each roller are identical.
(5) Consider only the incentive of guide rail roughness.

According to the literature [5-6] and the high-speed elevator model provided by a company, this paper
establishes an active control model for the transverse vibration of the high-speed elevator car system.
As shown in Fig. 1, with the center of mass $O$ of the car and frame as the origin, and the x axis
transversely, The rotation of the car around its centroid $O$ is $\theta_c$, and the rotation of the car frame
around its centroid $O$ is $\theta_f$. In this paper, the vibrations of the car and the frame in the direction of x
axis and the vibrations in the direction of rotation about its centroid are called transverse vibrations. At
the same time, each guide shoe of the car system has its own degrees of freedom in the horizontal

![Figure 1. Active control model for transverse vibration of high-speed elevator](image-url)
direction, that is, the elevator model has eight degrees of freedom. The mass of the elevator car is \(m_c\), the mass of the elevator car is \(m_e\). The moment of inertia of the car is \(J_c\), the moment of inertia of the car frame is \(J_f\). The horizontal displacement of the centroid of the car is \(x_c\), the horizontal displacement of the centroid of the frame is \(x_r\), the horizontal displacement of the roller is \(x_e\) \((i=1,2,3,4)\). The stiffness coefficient and damping coefficient between roller and guide rail are \(k_e\) and \(c_e\), respectively. The stiffness and damping coefficients between roller and car frame are \(k_r\) and \(c_r\), respectively. The stiffness coefficient and damping coefficient of the rubber block connected between the car and the car frame are \(k_f\) and \(c_f\), respectively. The vertical distances of the centroids of the active guide shoes 1, 3 and 2, 4 to the car system are \(l_1\) and \(l_4\), respectively. The vertical distances of the damping rubber blocks 1, 3 and 2, 4 to the car system center of mass are \(l_1\) and \(l_2\), respectively. The guide rail roughness excitation is \(f_i\) \((i=1,2,3,4)\). The active control force is \(f_i\) \((i=1,2,3,4)\).

According to Lagrange equation, the dynamic equation of active control for transverse vibration of high-speed elevator is established,

\[
\begin{align*}
M_1 \ddot{q} + C_1 \dot{q} + C \dot{x} + K_1 q + K x_e &= GF \\
M_2 \ddot{x} + C'_2 \dot{x} + C \dot{x} + G(C \dot{x} + K \dot{x}) q + (K + K) x_e + G K x_c &= G \dot{F}
\end{align*}
\]

where,

\[
q = \begin{bmatrix} x_c & x_r & \Theta_e & \Theta_f \end{bmatrix}^T, \quad \dot{x} = \begin{bmatrix} x_c & x_r & x_e & x_r \end{bmatrix}^T, \quad \Delta l_i = l_i - l_e,
\]

\[
\Delta l_i = l_3 - l_4, \quad h_i = l_3^2 + l_4^2, \quad M_2 = diag(m_c, m_e, m_e), \quad C_e = diag(c_e, c_e, c_e), \quad C_r = diag(k_r, k_r, k_r, k_r)
\]

\[
C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\
-c_e & -c_e & -c_e & -c_e \\
0 & 0 & 0 & 0 \\
c_i l_3 & c_i l_4 & c_i l_3 & c_i l_4 \end{bmatrix}, \quad K_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\
-k_e & -k_e & -k_e & -k_e \\
0 & 0 & 0 & 0 \\
k_i l_3 & -k_i l_4 & k_i l_3 & -k_i l_4 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
M_1 = \begin{bmatrix} m_c & 0 & 0 & 0 \\
0 & m_e & 0 & 0 \\
0 & 0 & J_c & 0 \\
0 & 0 & 0 & J_f \end{bmatrix}, \quad C = \begin{bmatrix} 4c & -4c & -2c \Delta l_1 & 2c \Delta l_1 \\
-4c & 4(c_e + c) & 2c \Delta l_1 & -2c \Delta l_2 - 2c \Delta l_1 \\
-2c \Delta l_1 & 2c \Delta l_1 & 2ch_i & -2ch_i \\
2c \Delta l_1 & -2c \Delta l_2 - 2c \Delta l_1 & -2ch_i & 2ch_i + 2ch_i \end{bmatrix}
\]

\[
K = \begin{bmatrix} 4k & -4k & -2k \Delta l_1 & 2k \Delta l_1 \\
-4k & 4(k_e + k) & 2k \Delta l_1 & -2k \Delta_l_2 - 2k \Delta l_1 \\
-2k \Delta l_1 & 2k \Delta l_1 & 2kh_i & -2kh_i \\
2k \Delta l_1 & -2k \Delta l_2 - 2k \Delta l_1 & -2kh_i & 2kh_i + 2kh_i \end{bmatrix}
\]

Select the state vector as \(X = [q \quad \dot{q} \quad x_c \quad \dot{x}_e]^T \in \mathbb{R}^{16 \times 1}\). Select the control input vector as \(U = [f_1 \quad f_2 \quad f_3 \quad f_4]^T \in \mathbb{R}^{4 \times 1}\). Select the interference input vector as \(W = [x_e \quad \dot{x}_e]^T \in \mathbb{R}^{8 \times 1}\). Therefore, the state space equation of the transverse vibration model of an eight-degree-of-freedom active control elevator car system is expressed as follows,

\[
\dot{X} = AX + BU + B_2 W
\]

where,
The working principle of the active guide shoe in the current active control technology: the vibration of the car system can be suppressed by the force opposite to the roller displacement generated by the actuator on the active guide boot. The evaluation index of elevator ride comfort is the vibration acceleration of car system. Therefore, the transverse vibration acceleration of car system is chosen as the evaluation index of active control effect. So, select the output vector as

$$
T_{c} = \begin{bmatrix} C_x & X & B & U \\ \end{bmatrix},
$$

that is,

$$
y = C_{a}X + DU
$$

where,

$$
C_{a} = \begin{bmatrix} -M_{1}^{-1}K & -M_{1}^{-1}C & -M_{1}^{-1}K_{1} & -M_{1}^{-1}C_{1} \end{bmatrix}, D = -M_{1}^{-1}G.
$$

3. Robust Controller Design

The root mean square value of vibration acceleration is often used in engineering as an index to evaluate the vibration acceleration of car system. According to the physical meaning of $H_2$ norm, when the input signal of the system is independent of each other, the $H_2$ norm is the root mean square (RMS) value of the system output [7]. In this paper, the excitation disturbance input is random excitation and independent of each other. For the performance output $y_1$, the $H_2$ norm is used to describe, and the output is the root mean square value of the output $y_1$. When $G_1$ is used to represent the transfer function from $W(t)$ to $y_1(t)$, the active control problem in this paper can be described as follows,

$$
\min \left\| G_1 \right\|_2
$$

If the state feedback gain of the system is $K_2$, then the control input is $U=K_2X$. Therefore, the elevator system in this paper can be expressed as follows,

$$
\begin{cases}
\dot{X}(t) = AX(t) + BU(t) + BW(t) = (A + B_1K_2)X(t) + B_2W(t) \\
Y_1(t) = C_{a}X(t) + DU(t) = (C_{a} + DK_2)X(t)
\end{cases}
$$

that is,

$$
\begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl1} & D_{cl1} \end{bmatrix} = \begin{bmatrix} A + B_1K_2 & B_2 \\ C_{a} + DK_2 & 0 \end{bmatrix}
$$

In the case of multi-objective control, the linear time invariant system has a conclusion [8]: If the system state matrix $A_{cl}$ is stable, and $\left\| G_1 \right\|_2 < v$, if and only if there exists $P=PT>0, S>0$, such that,

$$
\begin{bmatrix} A_{cl}^TP + PA_{cl} & PB_2 \\ B_2^TP & -I \end{bmatrix} < 0, Trace(S) < v, D_{cl1} = D_{cl2} = 0, \begin{bmatrix} P & C_{cl1}^T \\ C_{cl1} & S \end{bmatrix} > 0
$$

Let $Q=P^{-1}$ and $Y=KQ$, using $S$-Procedure and Schur complement formula to get LMI of $v, Q, Y, S$.
\[
\begin{bmatrix}
Q A^T + A Q + Y^T B_1^T + B_2 Y + B_2^T \\
B_2
\end{bmatrix} < 0, \quad \text{Trace}(S) < v, \quad D_{21} = D_{22} = 0, \quad \begin{bmatrix}
Q C_{Q}^T + Y^T D^T \\
S
\end{bmatrix} > 0
\]

Therefore, the robust controller of the high-speed elevator described in this paper can be described as,
\[
\text{Solving } \min_{v, Q, Y, S} v \text{ satisfies the above LMI.}
\]

(9)

4. Solution and Simulation of Robust Controller for High-Speed Elevator

4.1. Solution of State Feedback Matrix

The parameters of 4m/s high-speed elevator are shown in Table 1. The LMI solution toolbox (feasp) in MATLAB is used to solve the system state feedback matrix \( K_2 \).

\[
K_2 = \begin{bmatrix}
K_{21} & K_{22} & K_{23} & K_{24}
\end{bmatrix}
\]

where,

\[
K_{21} = \begin{bmatrix}
4.7382e5 & -5.1007e5 & 1.2773e5 & -1.3013e5 \\
4.7382e5 & -5.1007e5 & -1.2773e5 & 1.3013e5 \\
4.7378e5 & -5.1003e5 & 1.0379e6 & -1.0943e6 \\
4.7378e5 & -5.1003e5 & -1.0379e6 & 1.0943e6
\end{bmatrix}
\]

\[
K_{22} = \begin{bmatrix}
2.8781e2 & -4.3483e2 & -6.9734e2 & 7.5525e2 \\
2.8781e2 & -4.3483e2 & 6.9734e2 & -7.5525e2 \\
2.8780e2 & -4.3480e2 & 1.4360e3 & -1.7854e3 \\
2.8780e2 & -4.3480e2 & -1.4360e3 & 1.7854e3
\end{bmatrix}
\]

\[
K_{23} = \begin{bmatrix}
1.4181e4 & 8.2959e3 & 2.1706e4 & 7.9235e3 \\
8.2959e3 & 1.4181e4 & 7.9235e3 & 2.1706e4 \\
1.2867e4 & 1.6761e4 & 2.1485e4 & 9.8989e2 \\
1.6761e4 & 1.2867e4 & 9.8989e2 & 2.1485e4
\end{bmatrix}
\]

\[
K_{24} = \begin{bmatrix}
1.0595 & -9.4490e-1 & -2.2619e-1 & 6.2148e-1 \\
-9.4490e-1 & 1.0595 & 6.2148e-1 & -2.2619e-1 \\
2.4508 & -2.0556 & 1.6155 & -1.5008 \\
-2.0556 & 2.4508 & 1.6155 & 1.5008
\end{bmatrix}
\]

Table 1. Simulation parameters of elevator car system.

| Parameters             | Numerical value | Parameters             | Numerical value |
|------------------------|-----------------|------------------------|-----------------|
| Car mass \( m_c \)     | 1.2e3 kg        | Damping coefficient \( c_r \) | 1.2e2 N·s/m     |
| Car moment of inertia \( J_c \) | 1.3e3 kg·m² | Stiffness coefficient \( k \) | 5.0e5 N·m⁻¹     |
| Car frame mass \( m_f \) | 7.5e2 kg       | Damping coefficient \( c \) | 3.2e2 N·s/m     |
| Car frame moment of inertia \( J_f \) | 3.0e3 kg·m² | Distance \( l_1 \) | 1.2 m           |
| Roller mass \( m_r \)   | 3.1 kg          | Distance \( l_2 \) | 1.2 m           |
| Stiffness coefficient \( k_r \) | 7.0e5 N·m⁻¹ | Distance \( l_3 \) | 1.6 m           |
| Damping coefficient \( c_r \) | 1.34e2 N·s/m | Distance \( l_4 \) | 1.6 m           |
| Stiffness coefficient \( k_e \) | 5.0e4 N·m⁻¹ | Lifting speed \( V \) | 4 m·s⁻¹         |

4.2. Simulation Experiment

The disturbance excitation involved in this paper is the guide rail unevenness excitation. The research shows that the surface unevenness of the guide rail obeys the normal distribution with zero expectation...
The simulation experiment is carried out by using MATLAB. The vibration acceleration response of the car system under the action of uncontrolled and robust controller designed in this section is compared and analyzed, and the validity of the robust controller designed in this paper is verified. 

State equation of car system:

\[
\begin{cases}
\dot{X} = AX + BU + B_2W \\
y_1 = C_N X
\end{cases}
\]  

(11)

where, \(U=K_2X\), \(C_N=C_a+DK_2\).

The equation (11) can be discretized by MATLAB:

\[
\begin{cases}
X(k+1) = A_dX(k) + B_{d1}U(k) + B_{d2}W(k) \\
y_1 = C_d X(k)
\end{cases}
\]

(12)

where, \(A_d, B_{d1}, B_{d2}, C_d\) are matrices with the same dimension as \(A, B_1, B_2, C_N\).

4.2.1. Analysis of Simulation Results of Transverse Vibration of Car. The transverse vibration acceleration of the car's centroid position under the same random excitation is controlled by No Control and robust controller designed in this paper. The transverse vibration acceleration image of the car is shown in Figure 2. The calculation results of root mean square, mean and maximum transverse vibration acceleration of elevator car are shown in Table 2. The contrast analysis shows that after adopting the robust control, the root-mean-square value, the mean value and the maximum value of the transverse vibration acceleration of the car are 25.8%, 26.9% and 19.0% lower than the root-mean-square value, the mean value and the maximum value of the transverse vibration acceleration of the car by no control, respectively. The results show that the robust controller designed in this paper can effectively control the transverse vibration of the car.

![Figure 2. Comparative diagram of transverse vibration acceleration of elevator car](image)

| Table 2. Acceleration parameters of car transverse vibration. |
|---------------------------------------------------------------|
| Elevator Car | No Control | Robust Control |
|----------------|------------|----------------|
| RMS            | 0.0097     | 0.0072         |
| Mean value     | 0.0078     | 0.0057         |
| Maximum value  | 0.0263     | 0.0213         |
4.2.2. *Analysis of Simulation Results of Transverse Vibration of Car Frame.* No control and robust controllers are used to control the transverse vibration acceleration of the centroid position of the car frame under the same random excitation respectively. The transverse vibration acceleration image of the car frame is shown in Figure 3.

The calculation results of root mean square, mean and maximum transverse vibration acceleration of elevator car frame are shown in Table 3. The contrast analysis shows that after adopting the robust control, the root-mean-square value, the mean value and the maximum value of the transverse vibration acceleration of the car frame are 34.5%, 34.8% and 31.5% lower than the root-mean-square value, the mean value and the maximum value of the transverse vibration acceleration of the car frame by no control, respectively. The results show that the robust controller designed in this paper can effectively control the transverse vibration of the car frame.

![Figure 3. Comparative diagram of transverse vibration acceleration of elevator car frame](image)

**Table 3. Acceleration parameters of car transverse vibration.**

| Elevator Car | No Control | Robust Control |
|--------------|------------|----------------|
| RMS          | 0.0139     | 0.0091         |
| Mean value   | 0.0112     | 0.0073         |
| Maximum value| 0.0378     | 0.0259         |

5. **Conclusion**

(1) In view of the relative displacement between the elevator car and the frame during the actual operation of the elevator, this paper establishes the active control model of the transverse vibration of the high-speed elevator, which is in accordance with the actual operation of the elevator. The $H_2$ norm is used to minimize the output acceleration of the car vibration. Based on the LMI optimization technology, the robust controller of the high-speed elevator is designed and solved. The validity of the designed robust controller is verified by the MATLAB simulation.

(2) Through the simulation results, it can be seen that the root-mean-square value, the mean value and the maximum value of the transverse vibration acceleration of the car are smaller than the root mean value, the mean value and the maximum value of the transverse vibration acceleration of the car frame. The conclusion that the vibration of the car and the frame is inconsistent in the actual process is drawn, which further proves the correctness of the model established in this paper.

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