Inhomogeneous phase of a gluon plasma at finite temperature and density

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Abstract – By considering the nonperturbative effects associated with the fundamental modular region, a new phase of a Gluon Plasma at finite density is proposed. It corresponds to the transition from glueballs to nonperturbative gluons which condense at a nonvanishing momentum. In this respect the proposed phase is analogous to the color superconducting LOFF phase for fermionic systems.

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Quantum Chromodynamics (QCD) under extreme conditions has been intensively studied and a rich phase diagram in the $T$-$\mu$ plane is now well established.

At small chemical potential, there is a critical temperature, $T_c$, where the string tension goes to zero and there is a crossover to the deconfined quark-gluon plasma (QGP). Moreover the lattice simulations of the QGP phase transition clearly indicate nonperturbative effects above $T_c$,[1–10], that is: a) the Stefan-Boltzman limits for the pressure and the energy density of the system are not yet reached at $T_c \simeq 4 T_c$; b) correlated $\bar{q}q$ bosonic pairs survive up to $T_c \simeq 3 T_c$; c) for $T < T_c$ the gluon condensate is temperature independent whereas for $T > T_c$ its chromoelectric part rapidly decreases and, up to $2 T_c$, the chromomagnetic contribution remains almost constant.

On the other side of the phase diagram, i.e. at small temperature and above a critical quark chemical potential, $\mu_c$, there is a transition to a color superconducting phase, which turns out to be stable in the so-called LOFF phase, where the fermion pairs condense in a state with total momentum $\vec{q} \neq 0$ (see footnote 1) [11–17].

Phenomenological quasi-particle models have been applied to fit lattice results [18–21] and, in particular, for a gluon plasma (GP), i.e. a pure $SU(3)$ gauge theory at finite temperature, lattice data on pressure and energy density can be fitted [22], for $T > 2 T_c$, by a gluon gas with the following gluon dispersion relation:

$$E(k) = \sqrt{k^2 + \frac{M^4}{k^2}}, \quad (1)$$

where $M \simeq 0.7$ GeV [22]. Equation (1) has been derived in [24, 25] after an analysis of the physical configurations of a non-Abelian gauge theory, once the Coulomb gauge condition, $\partial_i A_i = 0$, is fixed. This condition still leaves the freedom of having gauge equivalent configurations (or Gribov copies) which should not be counted when enumerating the physical states. The corresponding reduction to the physical states only, the so-called Fundamental Modular Region (FMR), has the effect of changing the massless dispersion relation of the gluon, $E(k) = |k|$, into eq. (1).

The dispersion relation in eq. (1) is extremely interesting not only for the theoretical reasons associated with confinement [26] but also because the energy has a minimum at a finite value of the momentum, $|k|_g = M$, and increases both in the infrared and in the ultraviolet regions of the momentum. These are exactly the general conditions discussed by Brazovskii [27] which imply a stripe phase, i.e. a boson condensate in a state with $k \neq 0$. This approach has been formulated for phase transitions in condensed matter [28], which have been experimentally tested [29].

Moreover, infrared and ultraviolet behaviors analogous to eq. (1) are typical in noncommutative self-interacting

1This has been shown by using Nambu - Jona Lasinio model with color charge, see [11–17].
scalar field theory [30–32] with the consequence that spontaneous symmetry breaking cannot occur for a homogeneous background but only for an inhomogeneous phase, where bosons condense at nonzero momentum, analogously to the LOFF phase for fermion pairs.

Therefore, a nonvanishing minimum gluon energy, as predicted from eq. (1), opens the possibility of studying nontrivial dynamical effects also at finite density. In particular, in the analysis of a pure SU(3) gauge theory at finite temperature and density, the natural question arises if the dispersion relation in eq. (1) implies that at large density there is a gluon stripe phase. In this letter we shall address this issue.

Since from eq. (1) the minimum energy of the gluon is 

\[ E_g^* = \sqrt{2} M, \]

then at \( T = 0 \) and \( \mu = 0 \) we can derive a rough estimate of glueball mass, where two valence gluons are bound by a one-gluon effective exchange interaction [33]. Thus the corresponding glueball mass is

\[ M_G \simeq 2E_g^* - \frac{\alpha_s(r)}{r}, \]

(2)

where the typical scale of the bound state is \( r \simeq 1/M \) and the one loop expression for \( \alpha_s(r) \) has been used, with \( \Lambda_{QCD} \simeq 200 \) MeV. For the fitted value of \( M \) in [22], \( M = 0.7 \) GeV, \( M_G \simeq 1.5 \) GeV. This suggests that the zero-momentum glueball condensate, associated with \( \langle \alpha_s/\pi G_{\mu\nu} G^{\mu\nu} \rangle \neq 0 \), corresponds to confined gluons of energy \( E_g^* \) or minimum momentum \( |\vec{k}|_g = M \) and that, by increasing the density, one can expect (see for example ref. [34]) a transition from a glueball condensate to a deconfined, but still nonperturbative, gluonic phase with condensation in the mode \( |\vec{k}|_g \).

We are mainly interested in understanding the qualitative behavior of the phase diagram of the system and therefore the approximated values of the critical temperature and of the critical density can be evaluated by comparing the pressure in the two phases, and determining the transition line at \( p_g - B = 0 \), where

\[ p_g = -TD_g \int \frac{d^3k}{(2\pi)^3} \ln[1 - e^{-\beta(E(k) - \mu)}] \]

(3)

is the gluon pressure, \( \mu \) is the gluon (color independent) chemical potential, \( D_g = 16 \) is the gluon degeneracy factor and \( B \) is the bag constant that in the pure gauge theory one identifies with \( \langle \alpha_s/\pi G_{\mu\nu} G^{\mu\nu} \rangle \). At \( \mu = 0 \), the critical temperature turns out to be \( T_c \simeq 0.29 \) GeV for \( \langle \alpha_s/\pi G^2 \rangle_0 \simeq 0.005 \) GeV\(^4 \), which is smaller than the average value obtained by QCD sum rules but still within the phenomenological uncertainty [35].

By increasing the chemical potential one obtains the critical lines in the \( T-\mu \) plane, depicted in fig. 1. At a certain value of the temperature, \( T^*_c \), the critical line reaches the maximum value of the chemical potential allowed by eq. (3), \( \mu_c = E_g^* \). In fact for \( \mu > \mu_c \), the integral in eq. (3) is ill-defined. Then, according to the standard picture of the Bose-Einstein condensation (see, e.g., [36]), for \( T < T^*_c \) and \( \mu = \mu_c \), which corresponds to the vertical ending piece of the critical line in fig. 1, gluons progressively condense in the state of minimum energy \( E_g^* \). When the temperature reaches the point \( T = 0 \), the totality of the gluons is in this state. Unlike the standard condensation where the condensed bosons carry no momentum, in this framework the gluons, after condensation, have nonvanishing momentum \( |\vec{k}|_g = M \).

This would correspond to an inhomogeneous phase and therefore one can wonder about the application of a quasi-particle approach suitable for an homogeneous system, as implicitly assumed in eq. (3). In this sense, one is considering a slowly varying inhomogeneous background which introduces small deviations from translational invariance. Indeed, in a noncommutative field theory approach to nonuniform symmetry breaking, for fermionic and bosonic systems [31], it has been shown that the corrections to the translational invariant case, due to a smooth and slowly varying background, are of the order \( P^2 \), where \( P \) is the dimensionless parameter which describes the inhomogeneity. In our case, one can show that for the simplest strip background \( \phi(x) = A \cos(k_y x) \), which reduces to the homogeneous case for \( k_y \rightarrow 0 \), the corrections are of order the ratio of the typical scales of the problem, \( O(k^2_y/M^2_G) \), which is less than 25%.

Therefore, let us consider the energy density of the system given by

\[ \epsilon_g = D_g \int \frac{d^3k}{(2\pi)^3} \frac{1}{[e^{\beta(E(k) - \mu)} - 1]} + B. \]

(4)

The “interaction measure” \( (\epsilon_g - 3p_g)/T^4 \) can be evaluated and compared with lattice data for \( \mu = 0 \).

The comparison is shown in fig. 2 where our results, the green points, are plotted together with those obtained in [22] and the lattice findings. Our points are evaluated for \( T > 1.2T_c \) since a quasi-particle approach is unreliable near the critical point. The introduction of the gluon condensate clearly improves the agreement with
Inhomogeneous phase of a gluon plasma at finite temperature and density

![Graph showing comparison of \((c - 3p)/T^4\) evaluated from eqs. (1), (2) (green points) with lattice data, for \(T > 1.2T_c\). The red points correspond to the calculations with \(B = 0\) [22].](image)

respect to the case \(B = 0\) [22] and we stress that, independently of the improvement of the quantitative agreement with lattice data, fig. 2 shows that the nonperturbative effects associated with the fundamental modular region are the whole story only for large temperature. For \(T \approx T_c\) there are other nonperturbative effects that we describe by gluon condensation. In other words, above \(T_c\) and for \(T \leq 2T_c\) there are still glueballs. This result is analogous to the survival of quark-antiquark correlated pairs above the deconfinement temperature in the full QCD theory [6].

The results in fig. 1 have been obtained by considering a \(\mu\) independent condensate \(\langle \alpha_s/\pi G^2 \rangle_0\). In fact the effect of the finite density on the value of the condensate should be taken into account but there is no model or lattice simulation available in the literature. A rough indication of the influence of a finite density can be obtained by following the analysis performed in [37], where \(\langle \alpha_s/\pi G^2 \rangle_\mu\) is estimated for a system with finite baryon number, by using the quark/gluon color factor which gives the correct result for \(\mu = 0\) [23] \((11/9 = 11(N_c)/(11N_c - 2N_f))\), and the initial value at \(\mu = 0\) corresponding to the previously used value \(B = \langle \alpha_s/\pi G^2 \rangle_0 \approx 0.005\) aG. This analysis gives no qualitative difference with respect to the results in fig. 1, but since \(B(\mu)\) decreases by increasing the density, the transition line is obviously characterized by a smaller critical density.

In our opinion, the previous considerations give qualitative but clear indications that there is a critical line in the \(T-\mu\) plane where eq. (1) leads to a phase transition to an inhomogeneous condensate. As firstly realized by Brazovskii, for systems in which the fluctuation spectrum has a minimum at a nonzero momentum, \(p_c\), there is a first-order transition to a stripe phase, i.e. a periodic ordered state with spatial period \(2\pi/p_c\). In Brazovskii’s approach the minimum in the inverse propagator at nonzero momentum is determined by a self-consistent Hartree approximation and by expanding \(E(k)\) around its minimum one obtains an effective Lagrangian. For example, the Brazovskii-like 4-dimensional effective Lagrangian for a complex scalar field can be written as [28,30]

\[
S_{eff} = \int d^4x \left[ \alpha \left( \partial^2 + p^2 \right) \phi^2 + \beta \phi^4 + \gamma |\phi|^4 \right]
\]

that for \(\beta < 0\) has a classical minimum at \(\phi = A \exp(ipx)\) with \(|A|^2 = -\beta/\gamma\) and \(|p| = p_c\). In the present analysis, the minimum in \(E(k)\) is due to the QCD infrared dynamics [22,38].

On the other hand, the use of a quasiparticle approach has strong limitations and a definite answer on the existence of a stripe phase for a gluon plasma can be addressed only by lattice simulations. The correlation method [39] has the correct signal for this transition could be that, at finite density, some gauge invariant QCD correlator, \(C\), shows an oscillating behaviour \(C = a cos(p_c x)\) corresponding to a macroscopic occupation of the modes \(|\vec{k}| \approx p_c\). Alternatively one can study a momentum-dependent order parameter as in the case of lattice simulations of noncommutative field theories [32].

There is also the question about the survival of this phase when quarks are taken into account. At large quark chemical potential there is the transition to a color crystalline phase with a nonuniform colored condensate. Therefore if the gluon condensation occurs at zero momentum one should obtain a LOFF fermionic phase with a superimposed gluon uniform phase. This seems unlikely, as shown in ref. [40], where a gluon inhomogeneous phase combines with the quark LOFF phase. From this point of view, future lattice results which support the stripe phase for pure gauge theory can give an indirect indication of the existence of the color crystalline phase at large quark chemical potential where lattice calculations are still unreliable.

In conclusion, by considering the nonperturbative effects associated with the fundamental modular region, we propose a new phase of a Gluon Plasma at finite density which corresponds to the transition from glueballs to nonperturbative gluons which condense at a nonvanishing momentum. It will be interesting to verify if even this phenomenon is common to field theory and condensed matter as it happens for Bose-Einstein condensation and spontaneous symmetry breaking, superconductivity and chiral symmetry, LOFF phase and quarks color crystalline phase.

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