A finite fracture mechanics approach for interlaminar crack initiation using the scaled boundary finite element method

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In the present study, crack initiation induced by the free-edge effect in symmetric angle-ply laminates is investigated. For this purpose, the coupled stress and energy criterion within the framework of finite fracture mechanics is used. In order to obtain the required quantities efficiently the semi-analytical scaled boundary finite element method is implemented. The obtained effective failure stresses are compared to experimental findings from literature.

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1 Introduction

Free edges of composite laminates induce locally high, theoretically infinite interlaminar stresses due to different layer orientations. As a consequence, the so-called free-edge effect, described among others by Pipes and Pagano [1], may lead to interlaminar crack onset between differently oriented laminate plies. Due to the locally inhomogeneous stress field and the lack of a pre-existing crack, neither strength-of-material approaches, nor fracture mechanics approaches allow for a prediction of interlaminar crack onset. To overcome these drawbacks the coupled stress and energy criterion, suggested by Leguillon [2] within the framework of finite fracture mechanics is applied to predict interlaminar failure. The present work employs similar methodology as Martin et al. [3] but the semi-analytical scaled boundary finite element method (SBFEM) is used to provide the required quantities efficiently. The SBFEM was introduced by Song and Wolf [4] and allows for a more efficient solution of the present linear boundary value problem than the classical finite element method. The unknown interface fracture properties are determined by a parameter fit but taking realistic epoxy resin properties into account.

2 Mechanical Model

The mechanical model, introduced by Pipes and Pagano [1], is based on the assumption of a stress field which only depends on two coordinates (Fig. 1). Hence, the considered representative part of the laminate has to be in a sufficient distance to the load introduction, where local effects occur (Saint-Venant’s Principle). Linear elastic material behavior is presumed whereupon an existing symmetric angle-ply laminate from literature [5] (AS1/3501-6) is taken into account. It can be shown that the displacement field is given by

\[ u(x, y, z) = \varepsilon_0 \cdot x + \bar{u}(y, z), \quad v(x, y, z) = \bar{v}(y, z) \quad w(x, y, z) = \bar{w}(y, z). \]  

(1)

Fig. 1: Left: Symmetric angle-ply laminate subjected to a uniaxial extension \( \varepsilon_0 \). Center: Deduction of a quarter model. Right: Two-dimensional quarter model for the SBFEM.

3 Finite fracture mechanics

Strength-of-material approaches as well as linear elastic fracture mechanics are generally not suitable for the prediction of crack initiation. To surpass these limitations Leguillon [2] suggested the sudden formation of a crack of a definite size \( \Delta A_f \) if a stress and an energy subcriterion are fulfilled simultaneously. The so-called coupled criterion is given by

\[ f(\sigma(x, \varepsilon_0)) \geq \sigma_c \forall x \in \Gamma_c \quad \land \quad \varphi(\Delta A, \varepsilon_0) \geq G_c, \quad \text{with} \quad \varphi = -\frac{\Delta \Pi}{\Delta A}, \]  

(2)
where \( \mathcal{G} \) denotes the incremental energy release rate which is defined as change of the potential energy \( \Delta \Pi \) between a cracked and uncracked state with respect to the crack size \( \Delta A = aL \).

### 4 Scaled boundary finite element method

A new scaled \( \xi, \eta \)-coordinate system is introduced as depicted in Fig. 1. Furthermore, a separation of variables ansatz is employed in the governing principle of virtual displacements. After transformation into the scaled boundary coordinate system, introduction of a discretization only in the boundary coordinate \( \eta \) using one-dimensional quadratic shape functions and integration in \( \eta \), the basic SBFEM equations are obtained:

\[
\begin{align*}
\xi^2 K_0 \ddot{u}_\xi(\xi) + \xi \left( K_0 + K_1^T - K_1 \right) \ddot{u}_\xi(\xi) - K_2 \ddot{u}(\xi) + \xi (S_\xi - S_\eta) &= 0, \\
K_0 \ddot{u}_\xi(\xi = 1) + K_1^T \ddot{u}(\xi = 1) + S_\xi &= 0, \\
K_1 \ddot{u}(\xi = 0) &= 0.
\end{align*}
\]

(3)

The system of ordinary differential equations only depends on the scaling coordinate \( \xi \) and can be solved analytically.

### 5 Results

Subsequently, the obtained interlaminar stresses as well as the incremental energy release rates are incorporated into the coupled criterion. The failure related material properties in terms the interlaminar fracture toughness \( G_c \) and interlaminar shear strength \( \tau_c \) are unknown. A parameter fit is performed taking the effective strengths in dependence on the single ply thickness into account. It is postulated that they are related to the epoxy resin. The obtained values are of the same order of magnitude as values of similar laminates found in literature. The predicted effective longitudinal failure stresses are in good agreement to the experiments carried out by Brewer and Lagace [5].

![Fig. 2: Predicted effective longitudinal failure stress (solid line) and corresponding normalized initiated crack length (dashed line) with respect to the normalized individual ply thickness. The experimentally observed longitudinal effective failure stresses (circles), provided by Brewer and Lagace [5] are shown for comparison as well.](image)

### 6 Conclusion

Crack initiation induced by the free-edge effect in symmetric angle-ply laminates as well as the corresponding effective failure stresses have been investigated using the coupled approach. The required quantities have been determined efficiently by the semi-analytical scaled boundary finite element method. The numerical effort is significantly reduced compared to the standard finite element method. The achieved results are in accordance to experimental findings from literature.

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