A Fundamental of the LPM showers in water up to $10^{21}$eV

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Abstract. In order to pursue the highest energy of neutrino events in lepton physics, say, $\sim 10^{21}$ eV, as attained in extensive air showers, in hadron physics, the fundamental understanding towards electron-initiated cascade showers, namely, the LPM showers at extremely high energies, is strongly requested, because the LPM showers at these energies behave in completely different manner against those of the conventional BH showers. For the purpose, we need, at first of all, start from the morphological study of the LPM showers from $10^{16}$ eV to $10^{21}$ eV in water and $10^{21}$ eV in the atmosphere. The different characteristics of the LPM shower are clarified in different energy region.

1. Introduction

The final goal of the present paper, not at present, but in future, is to offer the fundamental materials for the design study for the electron neutrino astrophysics above $10^{21}$ eV, which is attained already by extensive air showers. For the purpose, we should carefully investigate big diversity of the LPM showers at extremely high energies, which should be regarded the strongest characteristics of the LPM showers [1].

In figure 1, the overall average pictures for the LPM showers over wider primary energies are shown. To make it possible to understand characteristics of the LPM showers over wider energy ranges, we show the cascade curves for shower particles with different primary energies, keep $E_{\text{prim}}/E_{\text{min}}=10^5$, for the LPM showers, where $E_{\text{prim}}$ and $E_{\text{min}}$ denote the primary energy and the minimum energy for the LPM showers, respectively. Primary energies are covered with $10^{15}$ eV to $10^{21}$ eV and the primary particles are electrons. If we assume the absence of the LPM effect, we expect all the cascade showers show the same structures as the BH shower shown in the figure, irrespective of primary energies as the results of under Approximation A [2].

2. Morphology of the LPM showers from mono-peak to multi-peak structure

As emphasized in Introduction, the characteristics of the LPM showers at extremely high energy lies in the existence of the extraordinary diversities among the LPM showers. Our final aim is the establishment of the reliable energy determination of LPM showers at extremely high energies which behave in very complicated manner and nobody could understand their characteristics in detail. In such the situations, the first of all, we must get accurate information around the LPM showers as much as possible. This is the reason why we need morphological studies around the LPM showers.
2.1. The LPM effect begins to be effective \( \sim 10^{15} \text{ eV} \) in water and become clear \( \sim 10^{16} \text{ eV} \)

The LPM effect begins to influence over the development of cascade showers in water \( \sim 10^{15} \text{ eV} \), as shown in figure 1. However, the LPM effect becomes effective essentially at \( \sim 10^{16} \text{ eV} \). In figure 2, we show clear indication of the LPM effect in both aspects, namely, one is the prolongation of the cascade shower compared with the BH shower and the other is the clear fluctuations around their average values. Here, [r.l] (radiation length) is the same as [c.u.] (cascade unit). 1 r.l=36cm in water.

2.2. A LPM shower with primary energies above \( 10^{17} \text{ eV} \). The appearance of multi-peak structure

The most distinguished character of the LPM showers which we never expect in the usual cascade showers (BH showers) lies in the existence of the multi-peak structure [1]. It is the direct reflection of the strong deviations among shower particles produced in both bremsstrahlung and pair production with LPM effect. This characteristic is firstly found by Konishi et al in the LPM shower with \( 10^{17} \text{ eV} \) in lead [1]. Compared the LPM shower in water shown in figure 3 with the corresponding one in lead with the same primary energy [1], “the degree” of multi-peak structure in figure 3 is rather weaker than that in lead, because the LPM effect is a kind of density effect which is stronger in higher density.

2.3. The appearance of multi-peak structure ( \( > 4 \) \( \sim 10^{18} \text{ eV} \))

The characteristic of the LPM showers \( \sim 10^{18} \text{ eV} \) is the appearance of clear multi-peak (\( > 4 \)) in addition to large prolongation of the cascade curve as shown in figure 4.

2.4. Appearance of the sub-showers with multi-peak structure \( \sim 10^{19} \text{ eV} \)

In these energy regions, as the energy distribution among shower particles may lean on one particle in both bremsstrahlung and pair production, some of shower particles penetrate into deeper depth without energy loss. In figure 5, it is understood that the first sub-shower is initiated at 20 r.l. (10 meter) by the

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**Figure 1.** Cascade curves for electrons by the averaged LPM showers with different primary energies from \( 10^{15} \text{ eV} \) to \( 10^{23} \text{ eV} \) for keeping of \( E_{\text{prim}}/E_{\text{min}}=10^5 \). The average BH shower with \( 10^{15} \text{ eV} \) is attached for readers’ reference.
primary electron penetrate there. The second sub-shower is generated at the point ~70 r.l. (25 meter) after the first sub-shower terminates. Such phenomena are never imagined in the BH showers.

**Figure 2.** In two reasons, LPM effect appear clear, (1) average picture is clearly different from that of the BH shower(see, Figure 1,also), (2) the real one shows clear fluctuation compared with the average picture.

**Figure 3.** The multiple peak structure of the LPM shower in water clearly appears $\sim 10^{17}$ eV.

**Figure 4.** The LPM shower with $10^{18}$ eV. The typical structure of the multi-peak structure.

**Figure 5.** The separated sub-showers appear in $\sim 10^{19}$ eV. A deformed kind of multi-peak structure.
2.5. Appearance of the sub-showers with longer traversed length $\sim 10^{20}$ eV.
It should be noticed that the LPM shower traverses too long distance. In figure 6, we show one example with $10^{20}$ eV which shows the total length $\sim 1790$ r.l. (640 meter). The extension of the first sub-shower is 390 r.l. (140 meter) and that of second is 1250 r.l. (450 meter), which of the separation between two is 150 r.l. (60 meter). The second sub-shower is composed of many peaks.

2.6. Appearance of single cascade shower with $10^{21}$ eV over 6000 r.l. (2100 meter)
We show a single LPM shower with many peaks but continuously developed over 6000 r.l. (2100 meter) in figure 7. Namely, this LPM shower could not be stored within the interior of the KM3 detectors. Nobody has imagined such a characteristics of cascade showers.

![Figure 6](image1.png)
![Figure 7](image2.png)

Figure 6. An example of two separated sub-showers with $10^{20}$ eV.
Figure 7. The LPM shower with $10^{21}$ eV traverses with many multi-peak without producing sub-shower. Notice that shower traverse 6000 c.u. (~2100 meter!!).

3. Track length distributions from the view point of energy estimations of the LPM showers
All reliable energy estimation of cascade showers is finally reduced to the exact estimation of track lengths of electrons in the corresponding cascade showers, irrespective of the different means for the measurements, namely by optical measurements for Cherenkov lights or Radio pulse measurements from cascade showers.

The fluctuation in the track lengths for shower particles as the function of the depth traversed by cascade particles is rather small compared with the corresponding ones due to cascade curves which are shown in figures 2 to 7. Therefore, the track length might be a measure for more reliable estimation.

In figure 8, one example of the cascade curves for track length with $10^{20}$ eV is shown. It is seen from the figure that the track lengths saturate roughly at $\sim 1000$ r.l. From this fact, one may conclude that the length for reliable energy estimation is roughly, 1000 r.l. (360 meters). In similar manner, one may conclude from figure 9 that we need $\sim 3000$ r.l. ($\sim 1$ km) for $10^{21}$ eV. Of course, one may say that the present calculation is limited to $E_{\text{prim}}/E_{\text{min}}=10^5$, however, in the real experiment, $E_{\text{prim}}/E_{\text{min}}=10^4$ for $10^{20}$ eV, where one adopts 1 MeV as the minimum energy. In spite of big difference between $10^5$ and $10^4$, the depth where the saturation is attained is independent on the value of $E_{\text{prim}}/E_{\text{min}}$. (See, the next section)
Could the LPM showers at extremely high energy keep their morphological features down to lower energy, say, 1 MeV? The minimum energies of the LPM showers under consideration are rather high because of $E_{\text{prim}}/E_{\text{min}} = 10^{5}$. On the other hand, really observed LPM showers include many shower particles whose energies are so low (~1 MeV) that the LPM effect can be completely neglected there. Consequently, it is natural for someone to have the same opinion as the conjecture by Kamata and Nishimura [3]. However, this conjecture is finally excluded [3]. Namely, the LPM showers with extremely higher primary energies keep their essential features even if the energy of each shower particle become so low that the LPM effect does not influence over them [3].

In figures 10 and 11, we give two examples among the LPM showers in the atmosphere whose primary energies are $10^{23}$ eV. The showers whose zenith angles are 90 degrees traverse through the deepest atmosphere (on the ground) whose total length is ~7000g/cm$^2$. Here, we take into account of the change of the density in the atmosphere which influence over the LPM effect.

In these calculations, we utilize the hybrid method for calculation of the total number of electrons. Namely, we calculate cascade shower down to $10^{16}$ eV where the LPM effect is still effective in the atmosphere by the exact Monte Carlo method and we utilize the NKG function for the connection with results by the Monte Carlo results. In the energy region below $10^{16}$ eV in the atmosphere, the cascade showers concerned are regarded as BH showers in which fluctuation effect is really can be neglected. In the NKG function, we assume no fluctuation. Therefore, in figures 10 and 11, the fluctuation effect appears only in the LPM shower, namely in the exact Monte Carlo part. The NKG function is an approximation to the BH showers. In real BH shower, of course, the fluctuation effect exists but its effect is very small compared with the case in the LPM shower. Therefore, for the rough estimation of the reliable energy of the LPM shower, it may be allowed to connect NKG function with the exact Monte Carlo method for the LPM shower. It is shown in figure 10 and 11 that green lines represent cascade curves by the exact Monte Carlo method and red lines represent cascade curves by hybrid method. From both figures, we conclude that the features in the green curve keep even red curves.

This can be applied to the discussion around figure 8 and 9 directly. It is clear from those figures that track lengths of shower particles whose minimum energy ~1 MeV saturate at nearly the same points shown as those of figures 8 to 9 in spite of big numerical difference in track lengths themselves.
5. Conclusion
For the moment, the most high energy cosmic ray events have been found in extensive air shower. We should try to find higher events in electron neutrino astrophysics.

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