Three-body couplings in RMF and its effects on hyperonic star equation of state✩

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Abstract
We develop a relativistic mean field (RMF) model with explicit three-body couplings and apply it to hyperonic systems and neutron star matter. Three-baryon repulsion is a promising ingredient to answer the massive neutron star puzzle; when strange hadrons such as hyperons are taken into account, the equation of state (EOS) becomes too soft to support the observed two-solar-mass neutron star. We demonstrate that it is possible to consistently explain the massive neutron star and hypernuclear data when we include three-body couplings and modify the hyperon-vector meson couplings from the flavor SU(3) value.

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Keywords: Nuclear matter, Hypernuclei

PACS: 21.65.+f, 21.80.+a

1. Introduction

In constructing dense matter equations of state (EOSs), it is strongly desired to respect hypernuclear data; hyperons are expected to emerge in the neutron star core, and they drastically soften the dense matter EOS. While nucleonic EOSs without hyperons predict the maximum mass of neutron stars as (1.5 − 2.7)M⊙, hyperons are expected to appear and reduce the maximum mass to (1.3 − 1.6)M⊙. Contrary to these understandings, a two-solar-mass neutron star (M = 1.97 ± 0.04M⊙) is discovered recently by using the Shapiro delay, and it is claimed that most of the EOSs with the appearance of strange hadrons are ruled out [2]. It is also questionable even for surviving nucleonic EOSs to support the massive neutron star, when hyperons are introduced. Thus we need to find either the reason why hyperons do not appear in dense neutron star matter or the mechanism how EOSs can be stiff enough even with hyperons.

One of the possible mechanisms to make the EOS stiffer is the three-baryon repulsion. In microscopic G-matrix calculations, the three-baryon repulsion is found to be necessary to support the 1.44 M⊙ neutron star when hyperons are included [3]. In relativistic mean field (RMF) models, attractive contribution from the scalar potential grows more slowly than the baryon density. In terms of non-relativistic languages, this behavior can be interpreted as the implicit three-body repulsion caused by relativistic effects. This three-body repulsion had been considered to be enough to support neutron stars even if hyperons are taken into account, until the two-solar-mass neutron star was discovered. When hyperon-meson couplings are chosen away from the SU(6) values and the ω meson self-energy is ignored, the calculated neutron star maximum mass can be compatible with the observed massive neutron star [4].

✩ Report number: YITP-12-97
However, these hyperon-meson couplings have not been seriously verified in finite hypernuclear systems, and the density dependence of the vector potential in these treatments would not be compatible with the relativistic Brückner-Hartree-Fock (RBHF) calculation [5], whose results ensure that RMF models are reasonable. Thus the most natural way to make the EOS with hyperons stiff enough would be to introduce explicit three-body repulsion.

In this work, we examine how three-body couplings affect the neutron star matter EOS in the framework of RMF. We include the interaction terms of baryon-meson-meson (BMM) and three-meson (MMM) couplings. Two mesons in the BMM term couple with two other baryons, and three mesons in the MMM term couple with three baryons. Thus BMM and MMM couplings correspond to the explicit three-body forces. Each term in the RMF Lagrangian can be characterized by the number \( n = B/2 + M + D \) in the Furnstahl-Serot-Tang (FST) expansion [6], where \( B \) is the number of baryon fields, \( M \) is the number of non-Nambu-Goldstone boson fields, and \( D \) is the number of derivatives. The baryon-meson coupling terms BMM belong to \( n = 2 \), and the present three-body coupling terms correspond to \( n = 3 \) — the next-to-leading order interactions in the FST expansion. While some of the \( n = 3 \) terms are considered to be absorbed by field redefinitions [6], we need to modify higher order \((n \geq 4)\) terms to compensate the redefinitions. We demonstrate the importance of \( n = 3 \) terms, especially the BMM terms, on the dense matter EOS.

2. Model description

We adopt here an RMF Lagrangian, \( \mathcal{L}_{\text{RMF}} = \mathcal{L}_{\text{eff}} + V_{\text{eq}} + \mathcal{L}_{\text{3b}} \), where \( \mathcal{L}_{\text{3b}} \) corresponds to the ordinary RMF Lagrangian including \( n = 2 \) two-body couplings, and \( V_{\text{eq}} \) represents the meson self-energies, which include an \( \omega^4 \) potential and a logarithmic potential of scalar-isoscalar mesons [7]. Here we ignore the nucleon coupling with scalar and vector \( \bar{s}s \) mesons (\( \zeta \) and \( \phi \)), as usually assumed. For the three-body coupling terms, \( \mathcal{L}_{\text{3b}} \), we first consider symmetric nuclear matter case, where \( \mathcal{L}_{\text{3b}} \) is taken to be

\[
\mathcal{L}_{\text{3b}}^{n=3} = -\frac{1}{f_2} \sum_B \sum_B \bar{\psi}_B \left[ g_{\sigma\alpha\beta} \sigma^\alpha \omega^\beta \right] \psi_B + c_{\text{3b}} f_3 \sigma \omega \phi^\mu \phi^\nu \left[ \right].
\]

The first and second terms modify the effective mass of nucleons, \( M_\nu^e = M_N - \frac{f_2}{g_{\omega N}} \sigma - \frac{f_2}{g_{\omega N} \sigma^2} \), where \( \sigma \) represents the temporal component. These \( n = 3 \) terms modify the effective mass from the \( n = 2 \) coupling with \( \omega^2 \). The third term modifies the vector potential of nucleons at high density, \( U_v = \frac{f_2}{g_{\omega N}} \sigma \omega \), and the fourth term represents the \( \omega \) meson mass shift at finite density. In RBHF calculations, the vector potential at low densities is almost proportional to the baryon density, but the vector potential to baryon density ratio \( U_v/\rho_b \) is suppressed around \( \rho_b \) or at higher densities. This suppression in RBHF is sometimes simulated by the \( \omega^2 \) term [8] or by the density dependent coupling [9]. When we simulate the suppression only with the \( \omega^3 \) term, the ratio \( U_v/\rho_b \) is monotonically decreasing with increasing \( \rho_b \). With a large coefficient of \( \omega^2 \), EOS at high density is thus too softened to support the massive neutron star [7].

For the massive neutron star puzzle, hyperon-meson coupling is another key. In many of the RMF parameter sets, the hyperon- and nucleon-vector meson coupling ratio is chosen to be \( R \equiv g_{\omega \Lambda}/g_{\omega N} = 2/3 \) based on the spin-flavor SU(6) symmetry or the quark counting arguments. This choice is the main reason why we cannot support the massive neutron star with hyperons. Mesons in RMF models describe scalar and vector potentials from various origins; two pion exchange, correlation from two-baryon short range repulsion, meson pair exchanges, and so on, in addition to the meson fields consisting of \( \bar{q}q \). Thus it is not mandatory to impose the spin-flavor SU(6) or flavor SU(3) relations among the coupling constants. In our previous work [7], we have adopted a more phenomenological prescription; while the hyperon-isoscalar vector meson couplings \( (g_{\sigma \Lambda}, g_{\omega \Lambda}) \) have been chosen to be the flavor SU(3) values, other couplings in the \( S = -1 \) hyperon sector \( (g_{\pi \gamma}, g_{\pi \gamma}, g_{\pi \gamma}) \) have been fitted to the hypernuclear data, including \( \Lambda \) separation energies \( (S_\Lambda) \), the bond energy of the double \( \Lambda \) hypernucleus \( (\Delta B_{\Lambda \Lambda}) \), and the \( \Sigma^- \) atomic shift data. It should be noted that we need to adopt the \( \Sigma^- \) coupling much smaller than the flavor SU(3) value in order to fit the \( \Sigma^- \) atomic shift data [2,3]. We here explore the results using the hyperon-\( \omega \) coupling value other than the SU(3) value. Specifically, we examine the results with \( R = 0.8 \) in the later discussion.
We prepare two three-body coupling parameter sets, TB-a and TB-b. The scalar meson part of $V_{\text{GB}}$ is taken from our previous work (SCL3 RMF model) \cite{7}, which describes known properties of finite and infinite nuclear systems such as binding energies per nucleon ($B/A$). Three-body couplings in TB-a are determined so as to reproduce the density dependence of the vector potential in RHBF at high densities. In TB-b, the repulsion from three-body couplings on the vector potential is chosen to be about twice of that in TB-a as shown in the left panel of Fig. 1. The density dependence of the vector potential in RHBF at high densities. In TB-b, the repulsion from three-body couplings on the vector potential is chosen to be about twice of that in TB-a as shown in the left panel of Fig. 1.

In Fig. 3, we show the neutron star matter EOS (NS EOS) obtained with TB-a and TB-b. By including three-body couplings, we can describe the case of $\Lambda$-shells in heavy hypernuclei. We may introduce the tensor coupling of vector mesons to obtain a larger $ls$ potential, but the tensor coupling does not modify the uniform matter EOS in the mean field treatment. In the $R = 0.8$ case, both scalar and vector potentials become stronger, and we find slightly different trends in $S_{\Lambda}$ as shown in Fig. 2. A single particle excitation energies are calculated to be smaller than those without three-body couplings, since the three-body couplings suppress the scalar potential and $\Lambda$ effective mass is larger in the present parametrization. It is necessary to tune hyperon-meson(-meson) couplings more carefully in order to reproduce $\Lambda$ single particle energies, especially of those in $d$- and $f$-shells in heavy hypernuclei. 

In Fig. 3 we show the neutron star matter EOS (NS EOS) obtained with TB-a and TB-b. By including three-body couplings, we can obtain stiffer NS EOSs without spoiling the nuclear matter saturation and finite nuclear properties. We also plot the mass-central density curve in the right panel of Fig. 3. We find that the calculated maximum mass exceeds 1.97$M_\odot$ by adopting repulsive three-body and hyperon-vector couplings (TB-b and $R = 0.8$), while it seems...
to be difficult for TB-a to support the massive neutron star.

The above conclusion may not be in agreement with recent studies using different type of higher-order terms [12], where the $2M_\odot$ mass neutron star is found to be supported in RMF with the SU(6) $R$ value. One of the differences is the existence of the $\omega \rho$ coupling term. The $\omega \rho$ coupling term $\propto (\omega_\rho \rho^2)(\rho^2 \rho^{\omega \rho})$ is related to the symmetry energy at high densities. We have not included this coupling, since it belongs to $n = 4$ in the FST expansion. We may need to include $n = 4$ terms in addition to other $n = 3$ terms in order for a systematic study including the density dependence of the symmetry energy.

4. Summary and conclusion

We have examined three-body couplings ($n = 3$) in the framework of the relativistic mean field (RMF) model. We can obtain stiffer EOSs at high densities while keeping the nuclear matter properties around $\rho_0$ and finite nuclei. We have also demonstrated that finite hypernuclear properties are reasonably well described with a modified $g_{\omega NN}$ from the flavor SU(3) value. By these two modifications in the RMF Lagrangian, we can obtain the neutron star matter EOSs, which support the recently observed two-solar-mass neutron star. These findings indicate that it would be possible to answer the massive neutron star puzzle; we can explain recently observed massive neutron star even if we respect hypernuclear data. The two ingredients discussed here may afford a key to understand the maximum mass of neutron stars. More careful tuning of parameters and introduction of other interaction terms may be necessary for a more satisfactory description of finite normal nuclei and hypernuclei. Isovector part of $n = 3$ couplings should be also investigated, and will be reported elsewhere.

Acknowledgments

This work is supported in part by the Grants-in-Aid for Scientific Research from JSPS (Nos. (B) 23340067, (B) 24340054, (C) 24540271), by the Grants-in-Aid for Scientific Research on Innovative Areas from MEXT (Nos. 24105001, 24105008), by the Yukawa International Program for Quark-hadron Sciences, and by the Grant-in-Aid for the global COE program “The Next Generation of Physics, Spun from Universality and Emergence” from MEXT.

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