The Flux Creep Effect in Superconducting Magnetic Bearings
Levitation Force

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Abstract. The main characteristic of superconducting magnetic bearings (SMB’s) is the levitation force between the permanent magnet rotor and the superconducting stator. This force can relax with time due to the flux creep effect in the superconductor. In order to investigate the effect of the flux creep in the levitation force, several dynamic levitation force measurements were made in simple axis-symmetric SMB’s. These measurements were conducted with different magnet approaching speeds and with the magnet stopped after the movement at a fixed position. The SMB’s were composed of superconducting YBCO seeded melt textured cylinders and Nd-Fe-B cylindrical permanent magnets, with different YBCO diameters, heights and pinning characteristics. The measurements were compared with simulations that have taken the flux creep into account. The simulation algorithm finds the current density profile inside the superconductor at each magnet approaching time step. The current density profile was found by solving an integral inside the superconductor and using a power-law for calculating the electric field. This power-law relationship can be derived from the Anderson-Kim model. The time evolution was obtained using time dependent finite difference method and the levitation force was calculated from the Lorentz force. The simulation results show a good agreement with measurements.

1. Introduction
The use of melt textured high Tc superconductors in magnetic levitation has become common recently, especially in superconducting magnetic bearing (SMB) [1]. Several prototypes of SMB were developed to operate in different kind of application [2], such as: generators, motors, high-speed flywheels and satellite positioning. It is usual to find in the literature models to simulate the interaction among magnetic fields and superconductors [3,4], where most of them are based in the critical state model [5,6]. In the results presented by D. Ruiz-Alonso et al the levitation force between a permanent magnet and a superconductor bulk was calculated, showing less than 10% of disagreement with measurements. However, that calculation is not able to predict the flux creep effect. The flux creep is important to determine the final levitation force in the SMB after a transient period, a mechanical disturbance or a load increase.

On the other hand, effects such as the force relaxation can be simulated, as presented by [3,7], to calculate the levitation force in superconductors. In the present paper, the same algorithm used in previous work [7] has been applied to calculate the levitation force between a permanent magnet and an YBCO disk, taking the flux creep into account. One disadvantage of this model is the difficulty to
simulate complex geometries. The comparison between simulated and measured results, show the high potential of this model.

2. Model Implementation

In order to simulate the movement between a permanent magnet (PM) and a superconductor, which arrangement is described in Figure 1, one needs to obtain the total magnetic field of the system. As illustrated in Figure 1, the magnetic field of the PM is modeled by a superficial current density $K$, which is related to a residuary magnetic flux density $B_{res}$, $K = B_{res} / \mu_0$. The details of this modeling can be found in [3]. Using the Biot-Savart’s Law together with the definition of the vector potential, the expression for the vector potential can be written as

$$A_{pm}(r) = \frac{B_{res}}{2\pi} \int_{\sigma_m} r_{pm} \cos(\phi'_{pm}) \ln \left[ \frac{\sqrt{r_{pm}^2 + \rho^2 - 2\rho r_{pm} \cos(\phi'_{pm})} + (z + r_{pm})^2}{\sqrt{r_{pm}^2 + \rho^2 - 2\rho r_{pm} \cos(\phi'_{pm})} + z^2} \right] d\phi'_{pm}, \quad (1)$$

where $r = (\rho, z)$. The integral of equation (1) does not have an analytical solution. This equation has to be solved numerically, and the steps followed can be found in [8].

![Figure 1. Superconductor cylinder in the presence of a cylindrical permanent magnet (PM) together with the mapping of the superconductor section and the superficial current density $K$ of the PM.](image)

The magnetic field produced by the screening current $J$ in the superconductor, is modeled by the magnetic vector potential. This expression can also be evaluated by using the Biot-Savart’s Law:

$$A_J(r, t) = \mu_0 \int_0^\rho \int_{\phi_0}^{\phi_1} d\rho' \int_{\phi_0}^{\phi_1} Q_{cilpm}(r, \rho') J(r, t) d\phi', \quad (2)$$

where the integral Kernel is expressed by

$$Q_{cilpm}(r, \rho') = \frac{\rho' \cos(\rho')}{2\pi \sqrt{\rho'^2 - 2\rho\rho' \cos(\rho') + \rho^2 + (z - \rho')^2}}. \quad (3)$$

Then, using the superposition principle, the total vector potential can be found by adding the permanent magnetic, equation (1), and the screening currents, equation (2), contributions, resulting in

$$A(r, t) = \mu_0 \int_0^\rho \int_{\phi_0}^{\phi_1} d\rho' \int_{\phi_0}^{\phi_1} dz Q_{cilpm}(r, \rho') J(r, t) + A_{pm}(r, t). \quad (4)$$

To simulate the movement between the PM and the superconductor, a stationary reference system was fixed and located in the PM. In such a way, the approach or removal of the PM in relation to the superconductor is given by
\[ z_{\text{mov}}(t) = z_{00} + z_0 - v_s t, \]  \hspace{1cm} (5)

where \( z_{00} + z_0 \) is the maximum distance between the PM and the superconductor, \( z_{00} \) is the minimum distance, and \( v_s \) is the relative velocity between the superconductor and the PM.

The time derivative of equation (4) needs to be evaluated. The flux creep here is implemented taking an exponential law to describe the behavior of the local electric field and the current density [8]. The final expression for the electric field can be written as

\[ E(J) = E_c \left| \frac{J J^*}{J} \right|^n \text{ with } n = \frac{U_0}{kT}, \]  \hspace{1cm} (6)

where \( E_c \) is the critical electric field of the superconductor, \( U_0 \) is the critical value for the activation energy, \( k \ldots \) and \( T \ldots \).

Finally, replacing the vector potential, using Faraday’s Induction Law, by \( \partial A / \partial t = -E \), and using flux creep consideration, the time derivative of the current density can be written as

\[ \frac{\partial J(r,t)}{\partial t} = -\mu_0^{-1} \int_{V} d\rho \int_{V} d\zeta \mathcal{Q}_{\text{clm}}^{-1}(r,\rho,\zeta) \left[ E \left| \frac{J(r,t)}{J_c} \right|^n + \frac{\partial A_{\text{em}}(r,t)}{\partial t} \right]. \]  \hspace{1cm} (7)

For the numerical implementation, a grid with equidistant points was mapped, as illustrated in figure 1. Equation (6) does not have an analytical solution and must be solved for each point of the mesh using the Method of Moments. These calculations and implementation are more precisely described in [7].

3. Measurements Procedure and Results

To compare the magnetic levitation force behaviour and the flux creep effect with the simulated results one needs to use a measurement system which is able to measure the interaction force between a PM and a superconducting disk. The PM used is a NdFeB magnet disk with 22mm of diameter and 10mm of height. The residual magnetic flux density in the center of its face is 445mT (\( B_{\text{res}} \)). For the superconducting material, two different YBCO disks were used. The first one has 28mm of diameter and 10mm of height (sample A) and the other has 31mm of diameter and 17mm of height (sample B).

![Figure 2. Magnetic levitation force comparison between the measured (filled line) and the simulated (dashed line) results with an approach velocity of 1.25mm/s. These results are for the sample B.](image-url)
The tests were done following the procedure described below. Firstly, the magnet was placed above the superconducting disk with an initial gap of 50mm, where the influence of its magnetic field is negligible. In our measurements we have used two values of the final gap, 5mm and 3mm. The YBCO disk was cooled in LN$_2$ without the presence of magnetic field (ZFC - Zero Field Cooling), and then the magnet is moved towards to the YBCO disk with a constant velocity of 1.25mm/s until it reaches the final position (final gap). After that, the procedure is done in two different ways. In the first one, the PM is elevated and then returned to the initial position and in the second one it stops and is held there. This second procedure is used to observe how the force varies with time when the gap is fixed (flux creep effect) [4].

Figure 2 shows the comparison between the results of the magnetic levitation force measured and that obtained from the simulation taking the flux creep into account. In this simulation, the values used for the YBCO parameters were $n=21$, $E_c=1$ V/cm and $J_c=7 \times 10^7$ A/m$^2$ for sample A, and $n=21$, $E_c=1$ V/cm and $J_c=2.3 \times 10^7$ A/m$^2$ for sample B. The value of the final gap used for this simulation was 5mm.

Figure 3. Flux creep comparison between the measured (filled line) and the simulated results (dashed line) for sample A. The final gap is 5mm and the approach velocity is 1.25mm/s.

Figure 4. Flux creep comparison between the measured (filled line) and the simulated results (dashed line) for sample A. The final gap is 3mm and the approach velocity is 1.25mm/s.

Figure 5. Flux creep comparison between the measured (filled line) and the simulated results (dashed line) for sample B. The final gap is 5mm and the approach velocity is 1.25mm/s.

It can be seen that the model predicts correctly the maximum value of the levitation force, which is in accordance with the experimental results, but has a small difference during the approach and back
processes. This may be due to parameter $J_c$, was a constant in this model (Bean model). If a $J_c$ dependence on the local magnetic flux was included, as in the Kim model [6], the results for the levitation force probably will be better.

On the other hand the goal of this model is to take into account the flux creep effect. Figures 3 and 4 (sample A) and Figure 5 (sample B) show the measured and simulated results for the magnetic levitation force, when the PM is approximated and then it is kept in a final position. For sample A, when the final gap=5mm the force decreases about 3% and for a final gap=3mm the force decreases about 5%. For sample B, when the final gap=5mm the force decreases about 5%. By comparison both samples, it is observed that for the same final gap (5mm) the force decrease is greater for sample B. This occurs due to the smaller value of $J_c$ of sample B.

Once the model can correctly predict the magnetic levitation force and the flux creep effect, it is possible to investigate the effect of different values of approaching velocity. Figure 6 shows the simulated results considering three values of approaching velocity. Due to the $E-J$ constitutive law given by equation (6), as the velocity increases the induced electric field will also increase and the vortices in the superconductor will have less time to relax. Therefore the maximum levitation force will increase as the velocity increases and the hysteretic effect will be less stressed. It can be observed in Figure 7 that the model predicts a logarithmic dependence between the maximum levitation force and the relative velocity of the PM and the HTS.

![Figure 6. Magnetic force simulation for different values of approach velocity. The final gap is 5mm.](image)

![Figure 7. Maximum levitation force for different values of approach velocities.](image)

4. Conclusion
This paper has presented a model to simulate the levitation force between a permanent magnet and a high temperature superconductor. This model is able to take the flux creep effects into account, by applying the $E-J$ power law. The measurements of the levitation force relaxation due to the flux creep effect showed satisfactory agreement with simulated results. The model can also predict changes in the hysteretic behaviour due to different approaching velocity and a logarithmic dependence between the maximum levitation force and the relative velocity of the PM and the HTS.

Acknowledgments
This work was supported partly by CNPq/MCT.

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