Modification of Zenner and Liu Criterion Due to Non-Proportionality of Fatigue Load by Means of MCE Approach

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Keywords: multiaxial fatigue, non-proportional loading, fatigue criteria, fatigue life estimation

Abstract. The results of experimental verification of Zenner and Liu’s criterion confirm its efficiency in the case of estimation of fatigue life for proportional loading. Due to application of general damage parameter, the criterion is useful in case of various types of proportional loading and for multiple materials. However, for non-proportional loadings, of high non-proportionality level, the error of estimation of fatigue lives often exceeds scatter band 3. The hereby work presents a proposal for Zenner and Liu’s criterion modification based upon introduction of loading non-proportionality measure with minimum circumscribed ellipse method (MCE).

Introduction

Significant number of materials shows sensitivity to non-proportionality of loading [1,2], resulting in reduction of life and fatigue strength. It is visible on e.g. Wöhler’s graphs prepared for equivalent stresses calculated acc. to generally used fatigue criteria (Fig. 1).

Characteristic feature of non-proportional loadings is rotation of principal axes during loading cycle. It is assumed that the change of location of primary axes has vast influence on reduction fatigue life and fatigue strength [3]. Loading non-proportionality also results in change of location of shear stress vector on physical plane crossing the analyzed material point (Fig. 2).

The majority of generally used fatigue criteria do not consider changes of direction of shear stress vector solely accounting for the ranges of stresses or strains which occur during loading cycle. It has vast influence on the quality of obtained results [4]. At the same time there is no uniformity on what values should be considered as amplitudes of stresses in case of non-proportional loading [5,6]. Currently any of the proposed models of estimation of life and fatigue strength for non-proportional loadings has not been generally accepted. Their further development and discussion is still needed.
Zenner and Liu’s criterion in non-proportional loadings conditions

One of the most commonly applied fatigue criterion for combined loadings, of integral criteria group, is Zenner and Liu’s criterion [6]. The criterion for loadings without mean values can be expressed by formula:

\[
\sigma_Z = \sqrt{\frac{15}{8\pi} \int_0^{2\pi} \int_0^{\pi} (a\tau_{\gamma\varphi,a}^2 + b\sigma_{\gamma\varphi,\varphi}^2) \sin \gamma \, d\gamma \, d\varphi} \leq Z_{rc},
\]

(1)

where \(a\) and \(b\) are the coefficients calculated on the basis of tension-compression and torsion fatigue limits.

The results obtained with the criterion during estimation of fatigue strength for single-axis and proportional loadings can be deemed as satisfactory [4,7]. However, the results for non-proportional loadings differ from the experimental ones to great extent. This can be caused by the fact that, similarly to many other criteria, for calculation of values of equivalent stress only maximum values of modules of shear stress vector are considered, with omission of changes in its acting direction (Fig. 2).

![Fig. 2. Changes of modules and locations of normal vector \(\sigma_{\gamma\varphi}\) and shear stress vector \(\tau_{\gamma\varphi}\) on physical plane during non-proportional loading cycle.](image)

Modification proposal for Zenner-Liu’s criterion

The first stage of the proposed modification of Zenner-Liu’s criterion is different approach to the history of loading cycle. It is based on calculation of equivalent stress values for each moment of loading cycle, and not, as in its original form, for the whole cycle. Then the maximum value of the course of equivalent stress is calculated (Fig. 3) \(\max_t (\sigma_Z(t))\).

The next modification stage is to specify the degree of loading cycle non-proportionality. The proposed method is based on assumption that for a given material type there is a critical path of shear stress vector, resulting from acting of the most damaging loading.

Fig. 4 presents paths marked by shear stress vector on exemplary sectional plane for loadings of sinusoidal courses of various values of phase shift angle \(\delta\) and various values of coefficient \(\lambda\), at constant value of equivalent stress calculated in the aforementioned manner. It can be noticed that for constant value \(\lambda\), the path describing area of the greatest surface area is for angle \(\delta = 90°\), and for constant value \(\delta\) for coefficient \(\lambda = Z_{so}/Z_{rc}\). Thus, it can be stated that the path describing area of the greatest surface area shall be always in case of loading with value of phase shift angle \(\delta = 90°\) and coefficient \(\lambda = Z_{so}/Z_{rc}\). Big surface area described with hodograph of shear stress vector indicates that on a given plane during the loading cycle, in each direction a vector of significantly large module has operated. The relation between fatigue life and the shape and size of paths of shear stress vector is confirmed by Wöhler’s diagrams (Fig. 5 and 6), prepared for values of equivalent amplitudes calculated as in the first step of the discussed method. Types of loadings on graphs are marked as follows: tension– TC, torsion – T, proportional loading P5, non-proportional loading N. The numbers next to letters notify the value of coefficient \(\lambda\) multiplied by 10. For copper, for which \(Z_{so}/Z_{rc} \approx 0,5\) [8], the most damaging is non-proportional loading of ratio \(\lambda = 0,5\). It also
generates the path of shear stress vector of the greatest surface area. In case of X2CrNiMo17-12-2 steel the most damaging has been non-proportional loading of coefficient $\lambda = 0.8$. Also in case of this material the value is almost identical with quotient $Z_{so}/Z_{rc} \approx 0.8$.

Fig. 7 presents the graph illustrating the difference between values of equivalent stress for tension-compression and non-proportional loadings $\sigma_R$ for various values of coefficient $\lambda$, prepared for copper Cu-ETP. On the basis of it the relation between reduction of fatigue strength and values of coefficient $\lambda$ and existence of critical value of coefficient $\lambda$, for which fatigue strength reduction is the greatest can be stated.

Upon observing the critical path of loading and paths of single-axis loadings of the same equivalent values, one can see that they always have common points PTC and PT (Fig. 8). Paths occurring for non-proportional loadings of coefficient values $\lambda < \lambda_{kr}$ are contact to critical path at the points specified by the path for tension-compression PTC, and of values $\lambda > \lambda_{kr}$ at point specified by the path for torsion PT (fig. 8). On the basis of such information and the fact that for loadings of sinusoidal courses with phase shift, the path of shear stress vector is always an ellipse, it is possible to specify critical path of the same equivalent stress value as the discussed loading case. Construction of the proposed measure of loading non-proportionality is based firstly upon specification of distance, expressed as $\psi$, between current and critical path. For paths of coefficient values $\lambda \leq \lambda_{kr}$ the distance is measured towards minor axis of critical path and for paths of coefficient values $\lambda > \lambda_{kr}$ towards major axis (Fig. 9).
Ultimately, the measure has the form of:

\[
\alpha = \frac{f_{\text{crit}}}{4} - \psi,
\]

where \(f_{\text{crit}}\) means the square root of the sum of distance of semi-axis of critical path of shear stress vector and is the measure proposed by Freitas in [9]. Modified Zenner-Liu’s criterion can thus be expressed by formula:

\[
\sigma_{z, \text{mod}} = \max_t (\sigma_z(t)) + \alpha = \max_t (\sigma_z(t)) + \frac{f_{\text{crit}}}{4} - \psi.
\]

Fig. 7. Difference \(\sigma_R\) between equivalent stress for tension-compression and non-proportional loading of various coefficient values \(\lambda\), obtained for copper Cu-ETP.

As the plane on which \(\alpha\) should be specified for tension-compression with torsion of sinusoidal courses offset in phase normal plane was assumed for vector which location is expressed by angles of spherical system of coordinates of values \(\gamma = 45^\circ\) and \(\varphi = 45^\circ\). It has been assumed that the plane is representative for the discussed loading case as the path marked by the vector of shear stress describes the area of relatively big surface area (Fig. 10).

Comparison of the proposed \(\alpha\) coefficient with values of \(\sigma_R\) are presented for Cu-ETP copper on Fig. 11 and for X2CrNiMo17-12-2 steel on Fig. 12. Significant compatibility of \(\alpha\) coefficient values with values of differences between equivalent stress for tension-compression and non-proportional loadings \(\sigma_R\) can be observed.

Verification of the proposed method

Fig. 13 and 14 present Wöhler’s curves prepared for equivalent stresses calculated acc. to the proposed method of Zenner-Liu’s criterion modification for steel X2CrNiMo17-12-2 and copper Cu-ETP. Figures 15 and 16 present comparison graphs of experimental fatigue lives \(N_{\text{exp}}\) with calculated
Fig. 11. Comparison of values of $\alpha$ coefficient and $\sigma_R$ for Cu-ETP copper.

Fig. 12. Comparison of values of $\alpha$ coefficient and $\sigma_R$ for X2CrNiMo17-12-2 steel.

ones $N_{ca\ell}$. Both in the case of tested steel and copper, due to the proposed method, the results within scatter band $3\sigma$ during estimation of resistance and fatigue strength have been obtained.

Conclusions and discussion on results

The tested materials have shown high sensitivity to loading non-proportionality. The sensitivity is in particular visible on fatigue graphs prepared for equivalent stresses calculated in manner not considering the influence of loading non-proportionality (Fig. 5 and 6). Estimation of fatigue life and fatigue strength for such materials is particularly difficult.

Fig. 13. Wöhler’s diagram prepared for equivalent stresses calculated acc. to proposed method of modification Zenner and Liu’s criterion modification for X2CrNiMo17-12-2 steel.

Fig. 14. Wöhler’s diagram prepared for equivalent stresses calculated acc. to proposed method of modification Zenner and Liu’s criterion modification for Cu-ETP copper.
The proposed method of modification Zenner-Liu’s criterion allows for consideration of influence of loading non-proportionality with new measure. Values of equivalent stresses and fatigue life obtained with the method cover with experimental values to an acceptable degree.

The proposed method, based on critical path idea, should be verified for greater number of materials and other cases of non-proportional loadings.

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