Efficient multi-mode quantum memory based on photon echo in optimal QED cavity

Sergey A. Moiseev,1,2,3 Sergey N. Andrianov,2,3 and Firdus F. Gubaidullin1,2

1 Kazan Physical-Technical Institute of the Russian Academy of Sciences, 10/7 Sibirsky Trakt, Kazan, 420029, Russia
2 Institute for Informatics of Tatarstan Academy of Sciences, 20 Mushtary, Kazan, 420012, Russia
3 Physical Department of Kazan State University, Kremlevskaya 18, Kazan, 420008, Russia

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Effective multi-mode photon echo based quantum memory on multi-atomic ensemble in the QED cavity is proposed. Analytical solution is obtained for the quantum memory efficiency that can be equal unity when optimal relations for the cavity and atomic parameters are held. Numerical estimation for realistic atomic and cavity parameters demonstrates the high efficiency of the quantum memory for optically thin resonant atomic system.

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Quantum communications and quantum computation require an effective quantum memory (QM) that should possess a multi-mode and high fidelity character. Most well-known QM based on electromagnetically induced transparency effect demonstrates an efficient storage and retrieval only for a specific single temporal mode regime. Photon echo QM offers most promising properties for realization of the multi-mode QM. However, the quantum efficiency of all discussed multi-mode variants of the photon echo QM tends to unity for infinite optical depth αL as $[1 - \exp(-\alpha L)]^2$, where $\alpha$ and $L$ are resonant absorption coefficient and length of the medium along the light field propagation. It imposes a fundamental limit for the QM efficiency so it is necessary to increase either the atomic concentration or the medium length. However, the QM device should be compact and the large increase of the atomic concentration gives rise to atomic decoherence due to the dipole-dipole interactions limiting thereby a storage time. So, using the free space QM scheme is quite problematic for practical devices. Efficient photon echo QM with controlled reverse of inhomogeneous broadening (CRIB) have been studied recently in ideal cavity and in bad cavity where high QM efficiency has been demonstrated only for a specific optimal single mode regime. Here, we propose a general approach for multi-mode photon echo type of QM in QED cavity (single mode resonator). We demonstrate a high efficiency of the QM for the optimized system of atoms and QED cavity at arbitrary temporal shape of the stored field modes. We find a simple analytical solution for QM efficiency and the optimal conditions for matching of the atomic and cavity parameters where the QM efficiency can reach unity even for small optical depth of the medium loaded in the cavity.

Basic equations: We analyze resonant multi-atomic system in a single mode QED cavity coupled with signal and bath fields. By following to the cavity mode formalism, we use a Tavis-Cumming Hamiltonian $\hat{H} = \hat{H}_o + \hat{H}_1$, for $N$ atoms, field modes and their interactions taking into account the inhomogeneous and homogeneous broadenings of the atomic frequencies and continuous spectral distribution of the field modes where

$$\hat{H}_o = \hbar \omega_0 \sum_{j=1}^{N} \hat{S}_j^z + \hat{a}^+ \hat{a} + \sum_{l=1}^{2} \int \hat{b}_l^+ (\omega) \hat{b}_l (\omega) d\omega,$$

are main energies of atoms ($S_j^z$ is a $z$-projection of the spin $1/2$ operator), energy of cavity field ($\hat{a}^+$ and $\hat{a}$ are arising and decreasing operators), energies of signal ($l=1$) and bath ($l=2$) fields ($\hat{b}_l^+$ and $\hat{b}_l$ are arising and decreasing operators of the field modes $[\hat{b}_l^+ (\omega'), \hat{b}_l (\omega)] = \delta_{l,l'} \delta (\omega' - \omega)$),

$$\hat{H}_1 = \hbar \sum_{j=1}^{N} (\Delta_j (t) + \delta \Delta_j (t)) \hat{S}_j^z$$

$$+ \hbar \sum_{l=1}^{2} \int (\omega - \omega_0) \hat{b}_l^+ (\omega) \hat{b}_l (\omega) d\omega$$

$$+ i \hbar \sum_{j=1}^{N} [g_j \hat{S}_j^z \hat{a}^+ - g_j^* \hat{S}_j^z \hat{a}]$$

$$+ i \hbar \sum_{l=1}^{2} \int \kappa_l (\omega) [\hat{b}_l (\omega) \hat{a}^+ - \hat{b}_l^+ (\omega) \hat{a}] d\omega.$$

The first term in comprises the perturbation energies of atoms where $\Delta_j (t)$ is a controlled frequency detuning of $j$-th atom $\Delta_j (t < \tau) = \Delta_j$ and $\Delta_j (t > \tau) = -\Delta_j$, $\delta \Delta_j (t)$ is its fluctuating frequency detuning determined by local stochastic fields, $g_j$ is a photon-atom coupling constant in the QED cavity, $S_j^z$ and $S_j^z$ are the transition spin operators. Ensemble distributions over the detunings $\Delta_j$ and $\delta \Delta_j (t)$ determine inhomogeneous $\Delta_{in}$ and homogeneous $\gamma_{21}$ broadenings of the resonant atomic line. In the following we use Lorentzian shape for inhomogeneous broadening (IB) and typical anzatz for ensemble average over the fluctuating detunings $\delta \Delta_j (t)$:
\[ \sum_{j=1}^{N} |g_j|^2 \exp{-i\Delta_j(t-t')} \Phi_j(t,t') \equiv \nonumber \\
N|g|^2 \exp{-i(\Delta_{in} + \gamma_2)}t - t'|, \]

where \( \Phi_j(t,t') = \exp{-i\varphi_j(t,t')} \), \( \delta \varphi_j(t,t') = \int_t^{t'} dt'' \Delta_j(t'') \), \(|g|^2\) is quantity averaged over the atoms. Second term in (2) contains frequency detunings of the field operators and for the atomic operators in the rotating frame representation:

\[ \text{for the field operators and for the atomic operators in the} \]

\[ \text{the following linearized system of Heisenberg equations} \]

\[ \text{Neglecting a population of excited atomic state and} \]

\[ \text{we use the Laplace transformation for} \]

\[ \hat{a}_L(p) = \int_{t_0}^{\infty} e^{-p(t-t_0)} \hat{a}(t) \]

\[ \text{and similarly for} \]

\[ \hat{b}_L(p) = \int_{t_0}^{\infty} e^{-p(t-t_0)} \hat{b}(t) \]

\[ \text{leads} \]

\[ \hat{a}_L(p) = \sum_{n=1}^{N} \hat{a}_{n,L}(p), \]

\[ \text{where} \]

\[ \hat{a}_{1,L}(p) = f(p)\hat{a}(t_0), \]

\[ \hat{a}_{2,L}(p) = f(p)\sum_{j=1}^{N} g_j \hat{S}_{j}^L(t_0)\Phi_j^{(s)}(p), \]

\[ \hat{a}_{3,L}(p) = f(p)\sqrt{\gamma_2}\hat{b}_{2,L}(p), \]

\[ \hat{a}_{4,L}(p) = f(p)\sqrt{\gamma_1}\hat{b}_{1,L}(p), \]

\[ f(p) = \left( p + \frac{(\gamma_1 + \gamma_2) + (N_1|g|^2)}{2} \right)^{-1}. \]

After inverse Laplace transformation, we find a solution \( \hat{a}(t) = \sum_{n=1}^{N} \hat{a}_n(t) \), where four terms of the cavity field \( \hat{a}_n(t) = \frac{1}{\sqrt{2\pi}} \int_{t_0}^{\infty} dp e^{i\omega(p-t_0)} \hat{a}_{n,L}(p) \) have different temporal and physical properties. The first field \( \hat{a}_1(t) \) is determined by the initial field \( \sim \hat{a}_1(t_0) \) that disappears rapidly in the cavity on time interval \( t - t_0 > 0 \). Second field component \( \hat{a}_2(t) \) is excited due to the interaction with atomic coherence at \( t = t_0 \). Third \( \hat{a}_3(t) \) and fourth \( \hat{a}_4(t) \) field components are excited by the bath modes \( \hat{b}_2(\nu) \) and by the signal field \( \hat{b}_1(\nu) \). Due to the initial state \( |\Psi_{in}(t_0)\rangle \), the field components \( \hat{a}_2(t) \) and \( \hat{a}_3(t) \) redetermine only the QED cavity vacuum without excitation of real photons. By taking into account the expectation values \( \langle \hat{b}_2^\dagger(\nu)\hat{b}_2(\nu) \rangle = \langle \hat{S}_2^L(t_0)\hat{S}_2^L(t_0) \rangle = 0 \) for initial state, we leave only the nonvanishing term for the atomic coherence at \( t = \tau \) determined by the signal field

\[ \hat{S}_2^L(\tau) = -g_j^* \int_{t_0}^{\tau} dt' \Phi_j(\tau, t') \exp{-i\Delta_j(\tau-t')} \hat{a}_4(t'), \]
By using \( \text{(13)} \) and \( \text{(3)} \), we calculate a storage efficiency of the signal field \( Q_{ST}(\tau) = \bar{P}_{ee}(\tau)/\bar{n}_1 \) where \( \bar{P}_{ee}(\tau) = \sum_{j=1}^N (S_j^+(\tau)S_j^-(\tau)) \) is an excited level population of atoms after the interaction with last M-th signal field mode for \( \tau > \tau_M + \delta t_M \) (we assume usual relation for temporal duration and spectral width of k-th mode \( \delta t_k \approx \delta \omega_k^{-1} \)). Total number of photons in the input signal field is \( \bar{n}_1 = \sum_{k=1}^M \bar{n}_{1,k} \), \( \bar{n}_{1,k} = \int dt \langle \hat{b}_{1,k}(t)\rangle \) is initial number of photon in k-th temporal mode, (\( \langle \cdot \rangle \)) is a quantum averaging over the initial state |\( \Psi_{in}(t) \rangle \). Performing the algebraic calculations of \( \bar{P}_{ee}(\tau) \), we find the quantum efficiency of storage \( Q_{ST} = (1/\bar{n}_1) \sum_{k=1}^n Q_{ST,k} \) where the storage efficiency of k-th mode is

\[
Q_{ST,k} = \int_{-\infty}^{\infty} dv Z(\nu, \Delta_{tot}, \Gamma_{tot}) \frac{\langle \hat{n}_{1,k}(\nu) \rangle}{\bar{n}_{1,k}},
\]

(14)

where spectral function

\[
Z(\nu, \Delta_{tot}, \Gamma_{tot}) = \frac{\Delta_{tot}^2}{1 - \nu^2/\Delta_{tot}} \left( \frac{\nu}{\Delta_{tot} + \nu^2} \right)^{\gamma_1} + \frac{4\gamma_1 \Gamma_{tot}}{\left[1 - i\nu / \Delta_{tot}\right] + 2i\nu^2}.
\]

(15)

\( \Gamma_{tot} = 2N \bar{\gamma}_0^2 / \Delta_{tot} \) is a photon absorption rate by N-atomic ensemble in unit spectral domain within the IB line, \( \Delta_{tot} = \Delta_{in} + \gamma_2 \) is a total line width. For relatively narrow spectral width \( \delta \omega_k \) of the k-th signal field and weak atomic decoherence rate in comparison with IB (\( \delta \omega_k, \gamma_2 \ll \Delta_{tot} \)), we get from Eqs. (14) and (15):

\[
Q_{ST,k} = \frac{\gamma_1}{(\gamma_1 + \gamma_2)} \frac{4\Gamma_{tot} / (\gamma_1 + \gamma_2)}{1 + \Gamma_{tot} / (\gamma_1 + \gamma_2)}.
\]

(16)

Quantum efficiency \( Q_{ST,k} \) reaches unity at optimal matching conditions \( \Gamma_{tot}/\gamma_1 = 1 \) and \( \gamma_2/\gamma_1 \ll 1 \) that provide a perfect storage of each k-th signal mode (maximum number of the modes is limited by \( M_{\text{max}} \sim \Delta_{in} / \gamma_2 \gg 1 \)). At the optimal matching, the each input k-th mode entering in the QED cavity will completely transfer to the IB atomic system since the IB atomic system absorbs each spectral component of the input signal field \( \bar{n}_k(\omega_k) \) with the same optimal rate \( \Gamma_{tot} = \gamma_1 \) leading to relation \( \bar{a}(t) = \sqrt{\gamma_1} \bar{b}_1(t) \) for arbitrary temporal mode shape that means an absence of the reflection of any input signal field from the QED cavity. So the storage of the multi-mode field in IB atomic system will occur by one step procedure.

Multi-mode Quantum Memory: In accordance with photon echo QM protocol \( \text{(10)} \), after complete absorption of the signal QM, we recover the rephasing atomic coherence by changing the frequency detunings of atoms at time moment \( t = \tau \). Inversion of the atomic detunings \( \Delta_j \rightarrow -\Delta_j \) can be done by using a Doppler effect \( \text{(10)} \), properties of local fields \( \text{(11)} \), or by direct changing a polarity of the external magnetic or electric fields \( \text{(12)} \). It is possible to recover the atomic coherence with quite large efficiency by exploiting the atomic frequency comb structure of the IB line \( \text{(23)} \). At first, we demonstrate the QM in QED cavity in most general way by using a Schrödinger picture by taking into account the interaction with cavity mode and other field modes.

In spite of huge complexity of the compound light atoms system, here, we show that their quantum dynamics governed by \( H_1 \) in \( \text{(2)} \) can be perfectly reversed in time on our demand in a simple robust way. By transferring to the new field operators \( \hat{a} = -\hat{A} \) and \( \hat{b}_1(\omega_0 + \Delta \omega) = \hat{B}_1(\omega_0 + \Delta \omega) \) (with similar relations for the Hermit conjugated operators), we get a new Hamiltonian with an opposite sign in comparison with initial one

\[
H_1 = -\hat{H}_1 \text{ determining a temporally reversed evolution} \ 
\]

\[
U_2[(t - \tau)] = \exp\{-i\hat{H}_1(t - \tau) / \hbar\} \exp\{i\hat{H}_1(t - \tau) / \hbar\}.
\]

Ignoring weak interaction with the bath modes and slow atomic decoherence, i.e. assuming \( \gamma_2 \approx 0, \gamma_2 \approx 0 \), we find that the quantum evolution \( U_2 \) recovers the initial quantum state of the multi-mode signal field and atoms at \( t = 2\tau \) due to unitary reversibility of the echo signal emission making the echo field spectrum inverted relatively to the central frequency \( \omega_0 \) in comparison with the original one.

Coming back to the real parameters of atomic decoherence rate \( \gamma_2 \) and cavity parameters \( \gamma_1, \gamma_2 \), we analyze below a retrieval of the echo field and QM efficiency for the multi-mode signal field (the field index "e" is introduced to indicate the echo emission stage). By changing \( \Phi_j^{(s)}(p) \) to \( \Phi_j^{(e)}(p) = \int_{-\infty}^{\infty} dt \exp\{-\nu(t - \Delta_j)(t - \tau)\} \Phi_j(t, \tau) \) we find the Laplace image of the quantum echo field irradiated by the atomic coherence \( S_2^j(\tau) \) in accordance with Eq. (9). We find the echo field in time domain picture \( \bar{a}_e(t) \) after inverse Laplace transformation, calculation of all temporal integrals and summation over the atomic responses. By taking into account large IB in comparison with the atomic decoherence rate \( \Delta_{in} \gg \gamma_2 \), we find the echo field irradiated in the QED cavity

\[
\bar{a}_e(t) = -\frac{\exp\{-2\gamma_2(t - \tau)\}}{\sqrt{\pi} \Gamma_{tot}} \int_{-\infty}^{+\infty} dv \overline{Z(\nu, \Delta_{in}, \Gamma_{in}) \int_{-\infty}^{+\infty} dt \frac{\hat{b}_1(k)(\nu) \exp\{i\nu(t + \tau - 2\tau)\}}{\sqrt{2\pi}}},
\]

(17)

where the total photon number operator of the echo field signal irradiated at time \( t \gg 2\tau \) is \( \hat{n}_e = \int_{-\infty}^{\infty} dv \hat{b}_e(k)(\nu) \exp\{i\nu(t + \tau - 2\tau)\} \hat{b}_e(k)(\nu) \), where \( \hat{n}_{e,k} = \gamma_1 \int_{-\infty}^{\infty} dt \hat{a}_e(t)(\tau) \hat{a}_e(t)(\tau) \) relates to the k-th field mode with average photon number \( \langle \hat{n}_{e,k} \rangle = \exp\{-4\gamma_2(t - \tau)\} \} Q_{ME,k} \bar{n}_{1,k} \) and

\[
Q_{ME,k} = \int_{-\infty}^{+\infty} dv Z(\nu, \Delta_{in}, \Gamma_{in}) \frac{\langle \hat{n}_{1,k}(\nu) \rangle}{\bar{n}_{1,k}}.
\]

(18)
We have assumed in (18) that $\gamma_2 \delta t_k \ll 1$. A spectral function $[Z(\nu, \Delta_{in}, \Gamma_{in})]^2$ filtering the echo spectrum demonstrates an influence of two similar steps of the light-atoms interaction in accordance with their temporal reversion, factor $\exp\{-4\gamma_2 (\tau - \tau_k)\}$ is a result of the atomic decoherence on the QM efficiency during the storage time $2(\tau - \tau_k)$ of the $k$-th mode. Total quantum efficiency $Q_{ME}$ of the multi-mode field retrieval is

$$Q_{ME} = \frac{1}{\bar{n}_1} \sum_{k=1}^{M} \exp\{-4\gamma_2 (\tau - \tau_k)\} \hat{Q}_{ME,k} \bar{n}_{1,k}. \quad (19)$$

The quantum efficiency of the multi-mode field retrieval is depicted in Fig. 1 in accordance with Eqs. (18)-(19) for broad range of ratio $\Gamma_{in}/\gamma_1$ and reasonable parameters of the IB resonant atoms for Lorentzian spectral shape of each $k$-th temporal field mode $\langle \hat{n}_{1,k}(\nu) \rangle = \frac{1}{\hat{b}_1^\dagger(\nu) \hat{b}_1(\nu)} = \frac{1}{\bar{n}_1, k \delta \omega_k / (\delta \omega_k^2 + \nu^2)}$

As seen in Fig. 1, the QM efficiency for multi-mode field is close to unity at optimal atomic and cavity parameters $\Gamma_{in}/\gamma_1 = 1$. The QM efficiency is equal unity in the theoretical limit $\gamma_1 > \Delta_{in} \gg \delta \omega_k$ and $\gamma_2 \ll \gamma_1$ if $\Gamma_{in} = \gamma_1$ leading to the following relation for the optimal optical depth $\alpha L \sim \gamma_1 L/c$ (where $c$ is a speed of light) that yields 100% QM efficiency even for thin optical depth. To give an example, we assume $L = 1 \text{ mm}$ and $\gamma_1 = 10^8 \text{ s}^{-1}$ that leads to the optimal optical depth $\alpha L \approx 3 \cdot 10^{-4}$. Such small but optimal optical depth can be prepared by spectral tailoring of the original IB resonant line of rare-earth ions in the dielectric crystals.

**Conclusion:** We have found that an efficient multi-mode photon echo QM in QED cavity can be constructed at the optimal choice of the atomic and cavity parameters. Here, high QM efficiency can be realized even for the atomic system with thin optical depth determined by the matching condition for the atoms and cavity depending only on the spectral characteristics of the signal field (not on its temporal shape). We stress a principle advantage of the proposed multi-mode photon echo QM in QED cavity with respect to the QMs based on well-known EIT or early numerous variants of photon echo QMs [5] where 100 % efficiency occurred only for infinite optical depth of the coherent resonant atomic system ($\alpha L \gg 1$). So, using the QED cavity not only increases the optical depth via the well-known Purcell factor [25], but makes its possible to realize an optimal condition for interaction between the external field and atoms. Thus, the predicted here possibility to get a highest efficiency for the QM at finite values of the parameters easily achievable in experiment and its multi-mode character opens a door for practical applications.

* Electronic address: samoi@yandex.ru

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