Laboratory Geometric Calibration of Digital Aerial Camera based on Focused Collimator-Array

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Abstract: The geometric calibration of digital aerial camera, including the calibration of inner orientation elements and distortion model parameters, is an indispensable technical of aerial photogrammetry. This paper proposes the use of a focused collimator-array (FCA) device to achieve a stable, repeatable laboratory geometric calibration of digital aerial camera. This device uses multiple collimators and star testers as target generators. After measuring the coordinates of targets on the CCD plane and corresponding standard angles between the collimator optical axis and the principal optical axis, the high-precision geometric calibration of digital aerial camera can be realized according to the geometric calibration model of this paper. Experiment results show that the FCA device can calibrate various types of area digital aerial cameras at different focal lengths and prove the correctness and the reliability of this calibration method using a typical digital aerial camera - Cannon EOS 5D Mark II. This device has good repeatability and high stability, which can satisfy the requirements of digital aerial camera calibration standards.

1. Introduction
In recent years, aerial photogrammetry technology based on digital aerial camera is widely used in the field of natural resources geographic information production as the main of extracting surface features. Due to the accuracy requirements of geographic information products, the aerial camera requires high precision. However, the distorted of aerial camera is existed, especially for non-metric digital cameras used in low altitude large-scale mapping, which have larger distortion values [1]. In order to ensure the quality of geographic information products, the state and industry have adopted relevant laws and regulations, before carrying out photogrammetry work, accurate geometric calibration of digital aerial camera must be performed to ensure compliance with the relevant technical specifications.

Geometric calibration methods of digital aerial cameras mainly include test field calibration method and laboratory calibration method.

The test field calibration method includes three-dimensional (3D) control field indoor [2], 3D control field outdoor [3-5], or even flight self-test field method [6-7]. This method is usually based on the collinear equation and consists of a certain number of 3D control points with known precise coordinates. After obtaining the test field image, which can be based on direct linear transformation, single-space resection, beam adjustment, regional network aerial triangulation or self-calibration method to solve the inner orientation elements and distortion model parameters. However, this method exhibits the disadvantages of computational complexity, large environmental factors, poor
reproducibility, uncontrollable calibration conditions, and inability to traceability. So far, there are no mature measurement and calibration techniques, methods and technical standards.

In order to solve the traceability problem, the fundamental road is to carry out laboratory geometry calibration method, which is reliable, condition controllable, traceable, high precision, and calibration standards. In recent years, there have been many reports on the calibration of laboratory measuring angle method [8-11] in ISPRS academic circles. This method is not suitable for large area digital aerial camera calibration. However, the laboratory collimator-array method can make up for the lack of measuring angle method. By reasonably setting multiple collimators, the distribution pitch and the standard angle value, which can satisfy the requirements of various types of digital aerial cameras calibration with different focal lengths. In this paper, based on the principle of collimator-array method and the characteristics of digital aerial camera, the focused collimator-array (FCA) device and its geometric calibration algorithm are developed to form scientific calibration technology and objective unified quality evaluation standard, which provide technical standard for aerial camera quality evaluation.

2. The principle of Focused Collimator-Array
The optical principle of collimator-array method is shown in Figure1. A multiple of collimators are placed on the object side based on the design angle value, and a digital aerial camera is fixed on the image side for imaging the infinity points target projected by the parallel light. The inner orientation elements and distortion model parameters of the digital aerial camera are calculated by image measurement and the angle between the multiple collimators and the principal optical axis.

![Figure 1. The optical principle of collimator-array method.](image)

Using the characteristics of the parallel lights of collimators to focus on the convex lens, the number of collimators, the distribution pitch and the standard angle value can be reasonably set to satisfy the requirements of digital aerial cameras of different focal lengths. As the number of collimators increases, multiple collimators can satisfy higher geometric calibration accuracy. Based on the geometric calibration principle of the collimator-array method. The focused collimator-array (FCA) device is proposed in this paper. The FCA device consists of multiple collimators and a digital aerial camera for imaging the infinity targets of the parallel light. This structural principle is that several collimators and a central collimator are evenly distributed on the same rotating paraboloid. A multiple of collimators and star testers as target generators to simulate infinity targets, the optical axis of collimator is passed through rotating parabolic focus, focusing on the optical entrance of the aerial camera. The optical axis of a central collimator serves as the principal optical axis, and the principal optical axis is vertically aligned with the center of the image as much as possible, and several collimators are symmetric about the central collimator to obtain a regular target image. The FCA device is shown in Figure 2, and the FCA structural principle is shown in Figure 3.

Precisely measuring the angle between the optical axis of the collimators and the principal optical axis, and set the precision theodolite at the focus of all the collimators to measure the standard angle between the optical axis of the collimators and the principal optical axis.
3. Calibration Calculation Based on Collimator-Array Method

The geometric calibration of the digital aerial camera is to determine inner orientation elements and distortion model parameters of the digital aerial camera.

3.1 Calculation of internal orientation elements

The inner orientation elements of a camera that describe the correlated position between the projection centre and the image plane, including the principal distance and the position of principal point.

Based on the principle of the collimator-array method, the calculation of the inner orientation elements is to search for a method of calculating the principal point and principal distance according to the coordinate of image points and the corresponding standard angles, so that the distortions of the camera can satisfy certain constraints.

For the FCA device, combined with the distortion least squares algorithm [12-14], this paper proposes geometric calibration of an aerial camera based on the collimator-array method, from two directions collimator-array (TDCA) calibration to derive of multiple directions collimator-array (MDCA) calibration method. This method can be selected the distributed measured points to establish the distortion model. The principal point and principal distance are calculated by the distortion least squares algorithm which can be beneficial to the accuracy of aerial photogrammetry.

3.1.1 Two directions collimator-array calibration method

The two rows of collimators are distributed in the space, and the collimator-array is divided into two regions. A multiple of collimators are vertically arranged in a cross shape, as shown in figure 4. The principle of geometric calibration of the collimator-array method is shown in figure 5, taking the X-axis of the image plane as an example, where P is the position of the principal point, the $\Delta W$ is the angle corresponding to the point P, and O is the center of the CCD, $S_i$ is a measured point, $W_i$ is the angle corresponding to point $S_i$, $x_P$ is the distance from P to O in the X-direction, and $f_x$ is the X-direction principal distance.
According to the definition of distortion of camera, the distortion corresponding to point \( S_i \) can be expressed as:

\[
\Delta x_i = f_x \times \tan (W_i - \Delta W) + x_p - x_i
\]  

(1)

\( \Delta W \) is the angle between the principal point and the centre of image plane, the offset of principal point is small, \( \Delta W \) can be expressed as:

\[
\Delta W = \frac{x_p}{f_x}
\]  

(2)

Therefore, the item \( \tan (W_i - \Delta W) \) can be approximated by Taylor’s formula:

\[
\tan (W_i - \Delta W) \approx \tan W_i - \sec^2 W_i \times \frac{x_p}{f_x}
\]  

(3)

And camera distortion expression can be expressed as:

\[
\Delta x_i = f_x \times \tan W_i - x_p \times \sec^2 W_i - x_i
\]  

(4)

In equation (5) that \( x_i \) and \( W_i \) are obtained by measuring, the matrix \( V \), \( X \), \( B \), and \( N \) are respectively defined, so there should be:

\[
V = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} \tan W_1 - \tan^2 W_1 \\ \tan W_2 - \tan^2 W_2 \\ \vdots \\ \tan W_n - \tan^2 W_n \end{bmatrix}, \quad N = \begin{bmatrix} f_x \\ x_p \end{bmatrix}
\]  

(6)

Then \( V = X - BN \), the matrix can be solved by the least squares, and the expression of the principal point and principal distance can be expressed as:

\[
x_p = \frac{\left( \sum x_i \tan W_i \times \sum \tan^2 W_i \right) - \left( \sum x_i \tan^2 W_i \times \sum \tan^3 W_i \right)}{\left( \sum \tan^2 W_i \times \sum \tan^4 W_i \right) - \left( \sum \tan^3 W_i \right)^2}
\]  

(7)

\[
f_x = \frac{\left( \sum x_i \tan W_i \times \sum \tan^3 W_i \right) - \left( \sum x_i \tan^3 W_i \times \sum \tan^2 W_i \right)}{\left( \sum \tan^2 W_i \times \sum \tan^4 W_i \right) - \left( \sum \tan^3 W_i \right)^2}
\]  

(8)

According to the horizontal collimator measured data, \( x_p \) and \( f_x \) can be calculated. Similarly, according to the vertical direction collimators measured data, \( y_p \) and \( f_y \) can be calculated. The principal distance of the camera is the average of two directions. Since the measured points are distributed on the X-axis and the Y-axis of the photo in the two directions, the measured points of the entire image cannot be selected to establish the distortion model, the calculation of principal point and principal distance reliability is poor. Therefore, the MDCA calibration method is proposed based on the TDCA calibration method.

3.1.2 Multiple directions collimator-array calibration method

The multi-row collimators are distributed in the space, and the collimator-array is divided into six regions, and a multiple of collimators are arranged in a vertical shape at an angle \( \theta_k \), which is the angle between the L radial line and the horizontal X-ray, as shown in figure 6.

The principle of geometry calibration of multiple directions collimator-array (MDCA) is shown in Figure 7. Taking the L-axis of the image plane as an example, where \( H' \) is the back node and \( O \) is the center of CCD. \( k=0,1,2 \ldots, 5 \), \( P_k \) is the principal point position of the radial line of angle \( \theta_k \), the \( W_i \) is
the angle corresponding to the point $P_k$, $P_k$ is the angle $\theta_k$ radial line the principal point distance, $S_i$ is a measured point, $L_i$ is the distance from $S_i$ to $O$, $W_i$ is the angle between the collimators and the central collimator, the $L_i$ direction corresponds to the positive and negative of the $W_i$, and $f_i$ is the principal point distance of the radial line of the angle $\theta_i$.

According to the formula of the principal point and principal distance based on the TDCA calibration method, the MDCA calibration formula is as follows:

$$f_k = \frac{\left( \sum L_i\tan W_i \times \sum \tan^4 W_i \right) - \left( \sum L_i \tan^2 W_i \times \sum \tan^3 W_i \right)}{\left( \sum \tan^2 W_i \times \sum \tan^4 W_i \right) - \left( \sum \tan^3 W_i \right)^2}$$  \hspace{1cm} (9)

$$p_k = \frac{\left( \sum L_i\tan W_i \times \sum \tan^3 W_i \right) - \left( \sum L_i \tan^2 W_i \times \sum \tan^2 W_i \right)}{\left( \sum \tan^2 W_i \times \sum \tan^4 W_i \right) - \left( \sum \tan^3 W_i \right)^2}$$  \hspace{1cm} (10)

Where $p_k$ is the principal point distance of the angle $\theta_i$ radial line, and $f_i$ is the principal point distance of the angle $\theta_i$ radial line. According to the equation (11), the principal point $(x_p, y_p)$ are solved according to the least squares, and the principal distance is the average value of the principal direction values in six directions.

$$(x_p - x_0) \times \cos \theta_i + (y_p - y_0) \times \sin \theta_i = p_k$$  \hspace{1cm} (11)

### 3.2 Calculation of distortion model parameters

The distortion model parameters mainly include radial distortion $k_0$, $k_1$, $k_2$, $k_3$, eccentric distortion $p_1$, $p_2$, deformation coefficients in CCD plane $b_1$, $b_2$.

After the inner orientation elements are calculated, distortion of each measured point which the $k$ paraboloid and the $i$ angle can be calculated by the equation (12):

$$\Delta r_{ik} = f \times \tan(W_i - \frac{P_k}{f}) - (L_i - p_k)$$  \hspace{1cm} (12)

To achieve the purpose of distortion correction, the distortion model is used to fit the distortion. Firstly, the radial distortion correction is performed. The radial distortion is a distortion symmetric with the optical axis, which is only related to the optical structure of the objective lens. The radial distortion accounts for the majority of the distortion. According to the principle of geometrical optics, the radial distortion can be expressed as the following polynomial:

$$\Delta r_{ik} = k_0 r_{ik}^0 + k_1 r_{ik}^1 + k_2 r_{ik}^2 + k_3 r_{ik}^3 + k_4 r_{ik}^4$$  \hspace{1cm} (13)

In equation (13), according to the radial distortion of a series of measured points, based on the principle of least squares, radial distortion parameters $k_0$, $k_1$, $k_2$, $k_3$, $k_4$ are calculated. Where $r$ is the radial distance and $r^2=x^2+y^2$. 

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**Figure 6.** Distribution of the MDCA.  
**Figure 7.** The principle of the MDCA calibration.
Eccentric distortion and deformation coefficients in the CCD plane can be expressed as the following polynomial:

\[
\Delta_{ix} = P_i [r_i^2 + 2(x_i - p_1^x)^2] + 2P_2 (x_i - x_0) (y_i - y_0) + B_1 (x_i - x_0) + B_2 (y_i - y_0) \\
\Delta_{iy} = 2P_1 (x_i - x_0) (y_i - y_0) + P_2 [r_i^2 + 2(y_i - y_0)^2]
\]  \text{(14)}

In equation (14), according to the residual distortion of the corrected radial distortion, the eccentric distortion parameters \(p_1, p_2\), the scale parameter \(b_1\), and the trim parameter \(b_2\) are calculated by least squares.

4. Experiments and analysis

In this paper, based on the principle of collimator-array calibration, the digital aerial camera laboratory geometry calibration software is developed. Its main functions include the high-precision position of the target image and calculation of inner orientation elements and distortion model parameters. To verify the correctness of the method, the mainstream digital aerial cameras, Canon EOS 5D Mark II, are used to take experiments, and the results are analyzed.

4.1 Experiments and analysis of geometric calibration

4.1.1 Experiments of geometric calibration

Experiment device is focused collimator-array (FCA), experiment camera is a Canon EOS 5D Mark II. The nominal focal length 24mm, pixel size 0.0064mm, the resolution is 5616 × 3744 pixels.

The specific calibration process is as follows, as shown in Figure 8: ① The camera is fixed on the two-dimensional turntable, and the position of the camera is precisely adjusted so that the focus of the paraboloid of rotation coincides with the entrance point of the aerial camera. The target image is acquired by the camera to accurately measure the position of the target image points on the CCD image plane. The positional accuracy is generally within 0.1 pixels, and the target image is shown in Figure 9. ② The geometric calibration results are solved by the image points coordinate of the target point and corresponding to standard angles between the collimator optical axis and the principal optical axis. The inner orientation elements and distortion model parameters of the camera are calculated and listed according to the method of this paper in Table 1.

The distortion correction requirement is to establish a mathematical model as close as possible to the actual measurement results. Figure 10 is original distortion of the camera and residual distortion after distortion fitting of the camera. The six directions include 0-0, 1-1, 2-2, 3-3, 4-4, 5-5. Before the distortion is fitted, the radial distortion value of the measured points is the original distortion. After the distortion is fitted, the residual distortion is the difference between the distortion model fitting and the original distortion.

| parameter     | value    |
|---------------|----------|
| principal point \(x_p\) (mm) | -0.02442 |
| principal point \(y_p\) (mm) | 0.14556  |
According to the least squares accuracy estimation method, the standard deviation of the distortion fitting can be expressed as:

$$\sigma = \left( \frac{\sum_{i=1}^{n} v_i^2}{n-t} \right)^{1/2}$$

(15)

According to this formula, $v_i$ is the residual distortion after distortion fitting, $t$ is the number of unknowns, $t=8$; $n$ is the times of measured independently with equal precisions. The standard deviation of camera is 0.211 pixels in X direction, 0.225 pixels in Y direction, and 0.308 pixels in radial direction.
Figure 10. The SDCA of original distortion and original distortion: (a) 0-0 distortion; (b) 3-3 distortion; (c) 1-1 distortion; (d) 4-4 distortion; (e) 2-2 distortion; (f) 5-5 distortion.

4.1.2 Results analysis

(1) Table 1 shows that the principal point and principal distance are calculated by geometric calibration. The principal point have obvious offset and the principal distance are different from their nominal focal length. It shows the necessity of geometric calibration of digital aerial cameras.

(2) In Figure 10, there are obvious distortion in the six directions of the camera, and the distortion in each direction has obvious symmetry with the distance from the image point to the principal point. And the laws of distortion in the six directions are basically the same. This is a kind of typical radial distortion.

(3) In Figure 10, the residual distortion in the six directions is significantly reduced after distortion fitting, which shows the correctness of the distortion model of this paper.

(4) The standard deviation after distortion fitting is about 0.3 pixels, and the distortion correction accuracy is better than one-third pixel. It can be seen from the results of the accuracy analysis that the accuracy of the focused collimator-array device satisfy the standard requirements.

4.2 Comparison experiments of collimator-array calibration in different directions

Based on the FCA device, the principal point and principal distance of the camera are calculated using the TDCA calibration method and the MDCA calibration method respectively. Figure 11(a) shows the 0-0 direction, and the 3-3 direction, Figure 11(b) shows the 1-1 direction and the 4-4 direction, and Figure 11(c) shows the 2-2 direction and the 5-5 direction, Figure 11(d) shows the six directions of the collimator-array. The two directions measured points and the six directions measured points are collected, which are calculated the principal point and principal distance of the camera are listed in Table 2; The results of the six directions measured points are calculated as standard values, Table 3 shows the error of principal point and principal distance between the TDCA calibration method and the MDCA calibration method.

Figure 11. The MDCA target image: (a) direction of 0-0 and 3-3; (b) direction of 1-1 and 4-4; (c) direction of 2-2 and 5-5; (d) six distortions
Table 2. Results of principal point and principal distance.

| direction | xp (mm) | yp (mm) | f (mm) |
|-----------|---------|---------|--------|
| 0,3       | -0.025  | 0.142   | 23.942 |
| 1,4       | -0.023  | 0.146   | 23.938 |
| 2,5       | -0.025  | 0.148   | 23.938 |
| 6 directions | -0.024 | 0.145   | 23.939 |

Table 3. Errors of principal point and principal distance.

| direction | xp (mm) | yp (mm) | f (mm) |
|-----------|---------|---------|--------|
| 0,3       | -0.001  | -0.003  | 0.003  |
| 1,4       | 0.001   | 0.001   | -0.001 |
| 2,5       | -0.001  | 0.003   | -0.001 |

(1) Table 2 and 3 show that the error of principal point and principal distance between the two directions measured points and six directions measured points. Therefore, the result of the principal point and the principal distance are calculated based on the six directions measured point least squares adjustment, which illustrates the necessity of the MDCA calibration method.

(2) Comparing to the calibration results from every two directions measured points to six directions measured points, which shows that the maximum error of the principal point xp is 0.001 mm, the maximum error of the principal point yp is 0.003 mm, and the maximum error of the principal distance is 0.003 mm. It shows that the each two directions measured points are calculated as principal point and the principal distance error is relatively small, which has strong reliability. It verifies the correctness of the FCA device and its geometric calibration adjustment.

4.3 Repeatability experiments of collimator-array calibration

Based on the FCA device, the camera is in the same position for five consecutive geometric calibrations to verify the repeatability of the FCA device. Table 4 shows the continuous calibration results of the principal point and principal distance based on the collimator-array method.

Table 4. Repeatability results of principal point and principal distance

|       | 1     | 2     | 3     | 4     | 5     | max error | medium error |
|-------|-------|-------|-------|-------|-------|-----------|--------------|
| xp (mm) | -0.024 | -0.024 | -0.024 | -0.024 | -0.024 | 0.0000    | 0.0000       |
| yp (mm) | 0.145  | 0.146  | 0.145  | 0.145  | 0.145  | 0.0010    | 0.0004       |
| f (mm)  | 23.939 | 23.939 | 23.938 | 23.938 | 23.938 | 0.0010    | 0.0005       |

The experimental results show that the camera of geometry calibration is continuously performed based on collimator-array method. The repeatability and stability of the calibration results are high. It is proved that the digital aerial camera of the principal point and the principal distance are uniquely for multiple calibrations. The experimental results show that the FCA device has high repeatability and stability, which can satisfy the requirements of digital aerial camera calibration standards.

5. Conclusion

The geometric calibration of digital aerial camera is an indispensable technology in aerial photogrammetry. In this paper, a FCA device is presented. The principle of this device and key techniques of geometric calibration based on the collimator-array method are introduced in detail. A typical digital aerial camera is used to carry out experiments, and the results are analyzed. Experiments results show that the FCA device can realize high-precision geometric calibration of multiple types of areal digital aerial cameras with different focal lengths. This device has the characteristics of good repeatability, strong stability and high calibration precision, which can satisfy the requirements of the digital aerial camera calibration standard.
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