Research Article

Research on Hyperbola Fitting Algorithm for Turbulence Level Measurement Test Data

Yufeng Du,1 Long Wu,2 Xunnian Wang,3 Jun Lin,1 and Neng Xiong1

1High Speed Aerodynamics Institute, China Aerodynamics Research and Development Center, Mianyang 621000, China
2Science and Technology on Scramjet Laboratory, China Aerodynamics Research and Development Center, Mianyang 621000, China
3State Key Laboratory of Aerodynamics, China Aerodynamics Research and Development Center, Mianyang 621000, China

Correspondence should be addressed to Yufeng Du; 1415776643@qq.com

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1. Introduction

Wind tunnel test is the most effective method for aerodynamic research. Even if the computer-based numerical simulation technology and the model flight test technology are rapidly improving, wind tunnel test is still an indispensable method for the research of complex aerodynamic characteristics during the research and development of aircraft. The precise design of advanced aircraft requires high accuracy of wind tunnel test results. However, there exist many factors and phenomena which finally result in the inaccuracy of measurements in the wind tunnel, such as the influence of flow qualities, Reynolds and Mach numbers, standard model test results, wind tunnel walls, and supporting system interference in 2D and 3D tests [1–7].

As an important flow quality in wind tunnels, turbulence level will affect the accuracy of wind tunnel test results, such as calculation of aircraft force and moment coefficients [1], measurement of test model attitude angle [8], and measurement of transition characteristics of the boundary layer on test model surface [9]. For aircraft design, the errors in wind tunnel tests mean that there will be design errors in aerodynamic parameters such as lift and drag coefficients. The design errors will lead to estimation errors in aircraft weight, which severely restricts the economy and safety of aircraft [10]. Therefore, it is very important to evaluate the turbulence level in wind tunnels accurately and quantitatively.

Hot-wire anemometry (HWA) is currently the most widely used method for turbulence level measurement due to its advantages such as high-frequency response, high sensitivity, and cost effectiveness [11–14]. In the compressible flow, the hot-wire response function derived by the changing overheat ratio method conforms to a hyperbolic relationship. Therefore, the problem of solving turbulence level can be converted into the problem of solving hyperbola fitting parameters of a set of two-dimensional scattered points [15]. However, in the actual measurement and data acquisition process, the two-dimensional scattered points may deviate from hyperbolic distribution due to unsteady
flow characteristics. In some cases, there are even straight lines and other conic distributions, which make it difficult to solve turbulence level precisely.

Many researchers have studied hyperbola fitting, and the methods utilized differ due to different application scenarios. Hough transform (HT) is a method of graph detection and fitting based on pattern recognition. The HT method converts scattered points in the image space to those in the parameter space and converts the problem of scattered points fitting to the problem of solving extreme value in the parameter space, thereby realizing the detection and fitting of the hyperbola. The HT method has the advantages of strong robustness, but its shortcomings of low computational efficiency limit its real-time application [16–20]. The least-squares (LS) method is the most commonly used method for hyperbola fitting. The key idea of the LS method is to establish a suberror function for each scattered point and use the sum of the squares of all suberror functions as the error function of hyperbola fitting. The suberror function can be expressed by residuals or orthogonal distances between scattered points and fitted hyperbolic curves. Hyperbolic curves are obtained by calculating the extreme value of the error function. The LS method has been widely used due to its ease of application and high computational efficiency [21–25]. Both Lebica VA [12, 26, 27] and Radespiel R [28–31] have done a lot of research in the field of turbulence level measurement and obtaining the fitting solution in compressible flow. They conducted a comprehensive and thorough analysis of fluctuation measurement results, which provided data support for the uncertainty analysis of subsequent wind tunnel tests. However, they have not conducted too much research on the hyperbola fitting method. Instead, the solution of the overdetermined equations (SOE) method is used directly to calculate the hyperbolic parameters to complete the hyperbola fitting. The SOE method is based on the LS method, which means it has high computational efficiency, while the SOE method cannot obtain precise turbulence level results due to overfitting if the scattered points deviate from hyperbolic distribution.

In order to solve the problem mentioned above, this paper proposes using the constrained least-squares (CLS) method to convert the problem of hyperbola fitting to the inequality constrained optimization problem and then using the Lagrange programming neural network (LPNN) method to solve turbulence level iteratively. The results show that the LPNN method is superior to the traditional SOE method, which verify the feasibility of the LPNN method for solving turbulence level in wind tunnels.

2. The SOE Method

According to the literature [26–32], the response function of the constant temperature hot-wire anemometer (CTA) in compressible flow is

\[
\frac{\Delta E}{E} = F_{\text{CTA}} \frac{\Delta m}{m} - G_{\text{CTA}} \frac{\Delta T_0}{T_0}
\]

(1)

where \(E\) is the output voltage of CTA, \(m\) and \(T_0\) are the gas mass flow rate and total temperature of the hot-wire probe measuring point, and \(F_{\text{CTA}}\) and \(G_{\text{CTA}}\) are the mass flow rate and total temperature sensitivity coefficients of CTA, respectively. \(F_{\text{CTA}}\) and \(G_{\text{CTA}}\) are only related to the overheating ratio, namely, the working temperature, of the hot-wire probe under a fixed flow condition. Dividing equation (1) by \(G_{\text{CTA}}\) and defining \(\theta = \frac{1}{G_{\text{CTA}}} \frac{\Delta E}{E}\) as the response function and \(r = F_{\text{CTA}} / G_{\text{CTA}}\) as the independent variable, we can get

\[
\theta = \frac{\Delta m}{m} r - \frac{\Delta T_0}{T_0}
\]

(2)

Take the mean square value of equation (2):

\[
\left( \sqrt{\theta^2} \right)^2 = \left( \frac{\Delta m}{m} \right)^2 r^2 - 2 \left( \frac{\Delta m}{m} \right) \left( \frac{\Delta T_0}{T_0} \right) r + \left( \frac{\Delta T_0}{T_0} \right)^2
\]

(3)

From equation (3), we can know that the response function of CTA conforms to the hyperbolic relationship with \(r\) as the independent variable and \(\sqrt{\theta^2}\) as the dependent variable. The two-dimensional scattered points \((R, \Theta)\) to be fitted can be obtained by continuously changing the overheating ratio \(n\) times and recording the output voltage of CTA at the same time:

\[
R = [ r_1 \ r_2 \ \ldots \ r_n ]^T,
\]

\[
\Theta = [ \sqrt{\theta_1^2} \ \sqrt{\theta_2^2} \ \ldots \ \sqrt{\theta_n^2} ]^T.
\]

(4)

Construct the coefficient matrix \(A\), the variable vector \(X\) to be sought, and the nonhomogeneous term vector \(\Pi\) of overdetermined equations:

\[
A = \begin{bmatrix}
r_1^2 & r_1 & 1 \\
r_2^2 & r_2 & 1 \\
\vdots & \vdots & \vdots \\
r_n^2 & r_n & 1
\end{bmatrix},
\]

\[
X = \begin{bmatrix}
\frac{\Delta m}{m}^2 - 2 \left( \frac{\Delta m}{m} \right) \left( \frac{\Delta T_0}{T_0} \right) \left( \frac{\Delta T_0}{T_0} \right)^2 \\
\left( \sqrt{\theta_1^2} \right)^2 - \left( \sqrt{\theta_2^2} \right)^2 - \left( \sqrt{\theta_n^2} \right)^2
\end{bmatrix}^T,
\]

\[
\Pi = \begin{bmatrix}
\left( \sqrt{\theta_1^2} \right)^2 \\
\left( \sqrt{\theta_2^2} \right)^2 \\
\vdots \\
\left( \sqrt{\theta_n^2} \right)^2
\end{bmatrix}^T.
\]

(5)

Therefore, the flow fluctuations can be obtained by solving the following overdetermined equations:

\[
A \cdot X = \Pi.
\]

(6)

Since \(A\) is not a square matrix, it is impossible to solve the overdetermined equations by solving the inverse matrix of \(A\). Instead, the generalized inverse matrix of \(A\) can be solved by the singular value decomposition (SVD) method to solve the overdetermined equations. Any real matrix \(A\) \((n \times q)\) can be decomposed:
\[
A = U \cdot \begin{bmatrix} W_z & 0 \\ 0 & 0 \end{bmatrix} \cdot V^T,
\]

where \( U(n \times n) \) and \( V(q \times q) \) are orthogonal matrices, \( W_z = [\text{diag}(w_1, w_2, \ldots, w_z)] \), \( w_1, w_2, \ldots, w_z \) are the non-zero singular values of matrix \( A \), and \( w_1 \geq w_2 \geq \cdots \geq w_z > 0 \), \( z \) is the rank of \( A \). When the rank of \( A \) is full, that is, \( z = q \), the SVD form of \( A \) is

\[
A = U \cdot \begin{bmatrix} W_z & 0 \\ 0 & 0 \end{bmatrix} \cdot V^T.
\]

According to equation (8), the generalized left inverse matrix of \( A \) is

\[
A^+ = V \cdot \begin{bmatrix} W_z^{-1} & 0 \end{bmatrix} \cdot U^T,
\]

which satisfies \( A^+ \cdot A = I \). Therefore, the solution of the overdetermined equations (6) using the SVD method is

\[
X = A^+ \cdot \Pi.
\]

After obtaining the flow fluctuations, the following equations can be used to solve the turbulence level \( T_u \) of the flow:

\[
\frac{\Delta m}{m} = \frac{\Delta (\rho u)}{\rho u} = \frac{\Delta \rho}{\rho} + \frac{\Delta u}{u},
\]

\[
\frac{\Delta T_0}{T_0} = \alpha \frac{\Delta T}{T} + \beta \frac{\Delta u}{u},
\]

where \( \rho, u, T, \) and \( p \) are the gas density, velocity, static temperature, and static pressure of the hot-wire probe measuring point, respectively, and \( M \) is the Mach number. Turbulence level \( T_u \) can be solved by the following equations:

\[
T_u = \left( \frac{\Delta u}{\rho u} \right)^{1/2} = \left( \left[ H^2 \ H \ G^2 \right] \cdot X \right)^{1/2},
\]

\[
H = \frac{1}{1 - M^2},
\]

\[
G = \frac{M^2}{1 - M^2} \beta.
\]

In the actual measurement process, conventional wind tunnels cannot guarantee the strict steadiness of flow due to the long duration of the turbulence level measurement test by using the changing overheat ratio method. Therefore, there will be cases where the two-dimensional scattered points do not strictly obey hyperbolic distribution, and even the distribution is similar to straight lines and other conic curves. The schematic diagram is shown in Figure 1.

The \( O \) points and the solid blue line in Figure 1 are the scattered points under ideal conditions and the hyperbola obtained by the SOE method, and the \( + \) points and the dotted red line are the scattered points with flow disturbances and the curve obtained by the SOE method. As a result of overfitting, the fitting curve is approximately a straight line, which has deviated from the ideal hyperbola. In this situation, the SOE method will not accurately solve the turbulence level.

3. Proposed Algorithm for Solving Turbulence Level

In order to solve the problem mentioned above, the CLS method is used to convert the problem of hyperbola fitting to the inequality constrained optimization problem, and then the LPNN method is used to obtain turbulence level iteratively.

3.1. The CLS Method

Compared with the traditional LS method, the CLS method can limit the parameters to be fitted within a certain range to prevent overfitting. Suppose that the fitted hyperbolic equation of the two-dimensional scattered points \( (R, \Theta) \) is

\[
f^2(r) = c_1 r^2 + c_2 r + c_3,
\]

where \( r \) is the independent variable, \( f(r) \) is the dependent variable, and \( c_1, c_2, \) and \( c_3 \) are the hyperbola fitting coefficients. According to equation (14), the dependent variable fitting values \( F(R) \) and residuals \( E \) are

\[
F(R) = [f(r_1), f(r_2), \ldots, f(r_n)]^T,
\]

\[
E = \Theta - F(R) = [e_1, e_2, \ldots, e_n]^T,
\]

where \( e_i = \sqrt{e_i^2} - f(r_i), \quad (i = 1, 2, \ldots, n). \)

Select the error function \( S \) of the CLS method as the sum of squares of the residuals of each scatter:

\[
S = \sum_{i=1}^{n} e_i^2.
\]

Comparing equations (3) and (14), it can be seen that the hyperbola fitting coefficients \( c_1, c_2, \) and \( c_3 \) in the CLS method are the flow fluctuations to be sought; namely,

\[
C = [c_1, c_2, c_3]^T = \hat{X}.
\]

According to the physical meanings of the variables in equations (5) and (17), the constraints are established as follows:

(1) Since the mean square values of \( m \) and \( T_0 \), fluctuations are always positive, we can say \( c_1 > 0 \) and \( c_3 > 0 \).
(2) The variances of \( m \) and \( T_0 \), their covariance, and correlation coefficient are defined, respectively, as follows:

\[
\sigma_m^2 = (\Delta m)^2, \\
\sigma_{T_0}^2 = (\Delta T_0)^2, \\
\sigma_{mT_0} = (\Delta m)(\Delta T_0), \\
\rho_{mT_0} = \frac{\sigma_{mT_0}}{\sigma_m \sigma_{T_0}}, \quad -1 \leq \rho_{mT_0} \leq 1. 
\]  

(18)

From equations (18) and (19), the correlation coefficient \( \rho_{mT_0} \) can be rewritten as

\[
\rho_{mT_0} = \frac{\sigma_{mT_0}}{\sigma_m \sigma_{T_0}} = \frac{1}{2} \frac{-2(\Delta m)(\Delta T_0)}{\sqrt{(\Delta m)^2 (\Delta T_0)^2}}. 
\]  

(20)

Constraints can be established based on the range of the correlation coefficient \( \rho_{mT_0} \):

\[
c_2^2 - 4c_1c_3 \leq 0. 
\]  

(21)

In summary, the problem of hyperbola fitting of two-dimensional scattered points can be converted to the following optimization problem:

\[
\arg \min_C S(C), \\
\text{s.t.} \quad c_1 > 0 \ & \ c_3 > 0 \ & \ c_2^2 - 4c_1c_3 \leq 0. 
\]  

(22)

3.2. The LPNN Method. To solve the nonlinear inequality constrained optimization problem in equation (22), the LPNN method is applied [33]. The inequality constraints can be rewritten as follows:

\[
g_1(C) = -c_1 + \tau \leq 0, \\
g_2(C) = -c_3 + \tau \leq 0, \\
g_3(C) = c_2^2 - 4c_1c_3 \leq 0, 
\]  

(23)

where \( \tau \) is the correction value of coefficients \( c_1 \) and \( c_3 \) and \( \tau = 10^{-10} \). Its value is determined according to the flow quality values of the conventional wind tunnels. The Lagrange function \( L \) with inequality constraints is defined as

\[
L(C, \nu, \lambda) = S(C) + \sum_{i=1}^{3} \lambda_i [g_i(C) + \nu_i], 
\]  

(24)

where \( \lambda = [\lambda_1, \lambda_2, \lambda_3]^T \) are the constraint coefficients and \( \lambda_i \geq 0 \) for \( i = 1, 2, 3 \). \( \nu = [\nu_1, \nu_2, \nu_3]^T \) are the relaxation variables. There are three types of neurons in the LPNN method: variable neurons control \( C \), Lagrange neurons control \( \lambda \), and relaxation neurons control \( \nu \). According to the KKT condition, the state equations of neurons can be obtained as follows:

\[
\frac{dC}{dt} = -\frac{\partial L(C, \nu, \lambda)}{\partial C}, \\
\frac{d\nu}{dt} = -\frac{\partial L(C, \nu, \lambda)}{\partial \nu}, \\
\frac{d\lambda}{dt} = -\frac{\partial L(C, \nu, \lambda)}{\partial \lambda}, 
\]  

(25)

where \( t \) is characteristic time. The Euler iteration equation is

\[
C^{k+1} = C^k + \frac{\partial L(C^k, \nu^k, \lambda^k)}{\partial C} \cdot \Delta t, \\
\nu^{k+1} = \nu^k + \frac{\partial L(C^k, \nu^k, \lambda^k)}{\partial \nu} \cdot \Delta t, \\
\lambda^{k+1} = \lambda^k + \frac{\partial L(C^k, \nu^k, \lambda^k)}{\partial \lambda} \cdot \Delta t, 
\]  

(26)

where \( k \) is the number of iterations and \( \Delta t \) is the iteration time step. The component form of equation (26) is

\[
c_i^{k+1} = c_i^k - \left[ \frac{\partial S(C^k)}{\partial c_i^k} + \sum_{j=1}^{3} \frac{\partial g_j(C^k)}{\partial c_i^k} \right] \cdot \Delta t, \quad i = 1, 2, 3, \\
\nu_i^{k+1} = \nu_i^k - 2\lambda_i^k \Delta t, \quad i = 1, 2, 3, \\
\lambda_j^{k+1} = \lambda_j^k + \left[ g_j(C^k) + \nu_j^k \right]^2 \cdot \Delta t, \quad i = 1, 2, 3, 
\]  

(27)

3.3. Stability Analysis of the LPNN Method. According to the literature [33], when performing stability analysis, it is necessary to verify that the iterative equilibrium point
obtained by the LPNN method is the asymptotically stable point of the neural network. Let the iterative equilibrium point be \((C^*, \nu^*, \lambda^*)\), and this point can be obtained by the following equation:

\[
\begin{bmatrix}
\frac{dC}{dt} \\
\frac{d\lambda}{dt}
\end{bmatrix} =
\begin{bmatrix}
\nabla_{CC}^2 L(C^*, \nu^*, \lambda^*) & \nabla_{C\nu} g(C^*, \nu^*) \\
-\nabla_{C\nu} g(C^*, \nu^*) & 0
\end{bmatrix}
\begin{bmatrix}
C - C^* \\
\lambda - \lambda^*
\end{bmatrix}
\]

\[= -G \begin{bmatrix}
C - C^* \\
\lambda - \lambda^*
\end{bmatrix},\]  

(28)

where \(C = [C^T, \nu^T]^T\) and \(g_i(C, \nu) = g_i(C) + \nu_i^2\) for \(i = 1, 2, 3\). Additionally, we have

\[
\nabla_{C\nu} g(C^*, \nu^*) =
\begin{bmatrix}
-1 & 0 & -4c_3^1 \\
0 & 0 & 2c_2^2 \\
0 & -1 & -4c_1^2 \\
2\nu_1^* & 0 & 0 \\
0 & 2\nu_2^* & 0 \\
0 & 0 & 2\nu_3^*
\end{bmatrix}
\]

(29)

\(\nabla_{C\nu} g(C^*, \nu^*)\) can be easily verified to be a full column rank. To be more precise, the gradients above at the equilibrium point are linearly independent, which means that \(C^*\) is a regular point and \((C^*, \nu^*, \lambda^*)\) is a Kuhn–Tucker point of the Lagrange function \(L\) [33]; namely, the following equation holds:

\[
\nabla_{CC}^2 L(C^*, \nu^*, \lambda^*) > 0.
\]  

(30)

The specific form of \(\nabla_{CC}^2 L(C^*, \nu^*, \lambda^*)\) in matrix \(G\) in equation (28) is

\[
\nabla_{CC}^2 L(C^*, \nu^*, \lambda^*) =
\begin{bmatrix}

0 & 0 & 0 \\
0 & 2\lambda_1^* & 0 \\
0 & 0 & 2\lambda_2^* \\
0 & 0 & 0
\end{bmatrix}
\]

(31)

According to the results of equation (30) and \(\lambda_i \geq 0\) for \(i = 1, 2, 3\), we can say that \(\nabla_{CC}^2 L(C^*, \nu^*, \lambda^*)\) is a strict positive definite matrix and the coefficient matrix \(G\) in equation (28) is a negative semidefinite matrix, which means that the equilibrium point \((C^*, \nu^*, \lambda^*)\) is the asymptotically stable point of the network.

4. Results and Analysis

4.1. Results of the Proposed Algorithm. Three sets of typical turbulence level measurement test results are selected to be processed. The independent variable and dependent variable data in equation (3) are shown in Table 1.

The LPNN method is used to iteratively solve the turbulence level corresponding to the three sets of test data above in Table 1. Considering convergence speed, the initial value of iteration is selected as \(10^{-8}, 10^{-7}, 10^{-6}\), and \(10^{-7}\), respectively, and the iteration time step is \(\Delta t = 10^{-8}\). The iteration results of variable \(C\) are shown in Figure 3. The iteration results of variables \(\nu\) and \(\lambda\) have no concern with solving the turbulence level, so they are not listed here.

From Figure 3, we can easily find that variable \(C\) converges to a fixed value within approximate \(4 \times 10^6\) steps, and consistent results can be obtained with different initial values. The results indicate good and stable convergence of the LPNN method.
Table 1: Data on turbulence level measurement test.

| $M = 0.449$ | $M = 0.516$ | $M = 0.599$ |
|-------------|-------------|-------------|
| $R$ | $\sqrt{R} \%$ | $r$ | $\sqrt{r} \%$ | $r$ | $\sqrt{r} \%$ |
| 0.146 | 0.093 | 0.147 | 0.058 | 0.146 | 0.074 |
| 0.177 | 0.110 | 0.179 | 0.068 | 0.178 | 0.073 |
| 0.214 | 0.112 | 0.217 | 0.080 | 0.215 | 0.079 |
| 0.257 | 0.111 | 0.261 | 0.092 | 0.259 | 0.100 |
| 0.413 | 0.181 | 0.426 | 0.159 | 0.421 | 0.180 |
| 0.742 | 0.314 | 0.788 | 0.273 | 0.773 | 0.279 |

Figure 3: Continued.
Figure 3: Results of variables from the iteration process. (a) $C_1$ for $M = 0.449$. (b) $C_2$ for $M = 0.449$. (c) $C_3$ for $M = 0.449$. (d) $C_1$ for $M = 0.516$. (e) $C_2$ for $M = 0.516$. (f) $C_3$ for $M = 0.516$. (g) $C_1$ for $M = 0.599$. (h) $C_2$ for $M = 0.599$. (i) $C_3$ for $M = 0.599$. 
4.2. Comparison with the SOE Method. The comparison results of the fitted hyperbola obtained by the LPNN method and the traditional SOE method are shown in Figure 4. The turbulence level and other flow qualities by the two methods are shown in Table 2.

For the case $M = 0.449$, Figure 4(a) shows that the scattered points of turbulence level measurement test results basically conform to the hyperbolic distribution, and the fitted hyperbolae obtained by the LPNN method and the traditional SOE method basically coincide. The results in Table 2 show that the turbulence level and other flow qualities are basically equal for $M = 0.449$. Therefore, in the case when the scattered points conform to the hyperbolic distribution, both the LPNN method and the SOE method can be used to solve the turbulence level accurately.

For the cases $M = 0.516$ and $M = 0.599$, Figures 4(b) and 4(c) show that the scattered points deviate from the hyperbolic distribution due to the unsteadiness of the flow, and there are even straight lines and other conic curves for the scattered points distribution. The fitting curve obtained by the traditional SOE method is an ellipse, which is inconsistent with the theoretical results from equation (3). Table 2 shows the high goodness of fitting values and negative $C_3$ values for the SOE results of $M = 0.516$ and $M = 0.599$. $C_3$ represents the mean square value of $T_{0i}$; thus, $C_3$ should be positive for all situations according to its physical meaning. However, the results of $C_3$ obtained by the SOE method are negative, which indicate that overfitting has occurred for $M = 0.516$ and $M = 0.599$. The scattered points are overfitted into ellipses by the SOE method, which results in negative $C_3$.  

Figure 4: Comparison of hyperbola fitting from LPNN and SOE results. (a) $M = 0.449$. (b) $M = 0.516$. (c) $M = 0.599$. 

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values. In this situation, although equations (12) and (13) can still be used to solve turbulence levels, the results are not true values.

The fitting curve for the cases $M = 0.516$ and $M = 0.599$ obtained by the LPNN method is still hyperbola with the influence of the unsteadiness of the flow, and the goodness of fitting values are high, which is consistent with the theoretical results. From the results, we can easily find that the LPNN method is better than the SOE method, and the results verify the feasibility of applying the proposed algorithm to solve the turbulence level in wind tunnels.

5. Conclusions

In this paper, we propose a new hyperbola fitting algorithm for turbulence level measurement test data, based on the Lagrange programming neural network method, to solve the turbulence level in wind tunnels iteratively. The LPNN method is stable and the equilibrium point is proved to be asymptotically stable. The results show that, better than the SOE method, the LPNN method can be utilized to solve turbulence level, and the results will not be affected by the unsteadiness of the flow. The results of this paper will be mainly used to solve the turbulence level more precisely to ensure the high accuracy of wind tunnel test results.

Future research will continue to further develop the hyperbola fitting algorithm on the basis of the LPNN method to improve the accuracy of turbulence level results. In addition, the LPNN method will also be improved to be applied in data processing for wind tunnel test results.

Data Availability

The data used to support the findings of this study are included within the tables of this article. More turbulence level measurement test data are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Table 2: Comparison of flow quality from LPNN and SOE results.

| Method | $(\Delta m/\bar{m})^2$ ($C_1$) | $-2(\Delta m/\bar{m})(\Delta T_0/T_0)$ ($C_2$) | $(\Delta T_0/T_0)^2$ ($C_3$) | $Tu$ (%) | Goodness of fitting |
|--------|-------------------------------|--------------------------------|----------------------------|----------|-------------------|
| $M = 0.449$ | LPNN $2.03 \times 10^{-5}$ | $-2.97 \times 10^{-6}$ | $9.27 \times 10^{-7}$ | 0.54 | 0.9981 |
| SOE | $2.02 \times 10^{-5}$ | $-2.90 \times 10^{-6}$ | $9.11 \times 10^{-7}$ | 0.54 | 0.9981 |
| $M = 0.516$ | LPNN $1.26 \times 10^{-5}$ | $-4.51 \times 10^{-7}$ | $1.54 \times 10^{-7}$ | 0.44 | 0.9974 |
| SOE | $9.40 \times 10^{-6}$ | $2.41 \times 10^{-6}$ | $-2.93 \times 10^{-7}$ | 0.46 | 0.9992 |
| $M = 0.599$ | LPNN $1.33 \times 10^{-5}$ | $9.36 \times 10^{-8}$ | $1.61 \times 10^{-7}$ | 0.48 | 0.9864 |
| SOE | $6.28 \times 10^{-6}$ | $6.29 \times 10^{-6}$ | $-7.92 \times 10^{-7}$ | 0.52 | 0.9936 |

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