Directional dark matter by polar angle direct detection and application of columnar recombination

Jin Li
Center for Underground Physics, Institute for Basic Science (IBS), Daejeon 305-811, Korea

We report a systematic study on the directional sensitivity of a direct dark matter detector that detects the polar angle of a recoiling nucleus. A WIMP-mass independent method is used to obtain the sensitivity of a general detector in an isothermal galactic dark matter halo. By using two-dimensional distributions of energy and polar angle, a detector without head-tail capability is shown to achieve the same performance level as a full three-dimensional tracking dark matter detector. Optimum orientation geometries are obtained for various experimental configurations, with detectors that are space- or Earth-fixed, have head-tail capability or not, and use energy information or not. Earth-fixed detectors are found to have best sensitivity when the polar axis is aligned with the Earth’s pole. The WIMP-mass dependence of the performance of a detector with a 3 keV energy threshold that uses xenon as target material is reported. We apply realistic experimental resolutions and thresholds for a columnar recombination detector that detects two channel recombination and ionization processes from gaseous xenon. We find that with a $5 \times 10^{-46}$ cm$^2$ spin-independent WIMP-neutron cross-section and a 30 GeV WIMP, a 636 kg year’s exposure with a columnar recombination detector can make a three sigma discovery of directional WIMPs in the isothermal galactic dark matter halo.

PACS numbers: 95.35.+d, 29.40.-n

I. INTRODUCTION

Many experiments have attempted to directly detect Weakly Interacting Massive Particle (WIMP) dark matter candidates via their elastic scattering on target nuclei [1]. The LUX experiment has recently limited the spin-independent cross sections to be under $7.6 \times 10^{-46}$ cm$^2$ for 33 GeV WIMP mass [2]. A method that has commonly been used is to measure signals associated with the deposited energy of the nuclear recoil. However, background processes, such as those induced by neutrons, can mimic WIMP signals [3]. To overcome this problem, a “smoking gun” WIMP signal would be its unique directional event-rate dependence [4]. When viewed from the Earth, the average WIMP velocity in galactic coordinates is from the Cygnus direction, while the directions of background sources are fixed in an Earth-based coordinate system. This effect can be seen by the directions of the recoiling nuclei that are strongly correlated with the directions of the incoming WIMPs [5], and produce a diurnal variation of rates due to the Earth’s rotation [6, 7]. Prospects for a working directional detector have focused on low pressure gas time projection chambers, such as DRIFT-II [8], DMTPC [9], NEWAGE [10], MIMAC [11], and the D$^3$ prototype [12], and emulsion techniques [13]. The sensitivity of those detectors has been studied assuming they are either capable of reconstructing full three-dimensional (3D) tracks [5–14] or only tracks projections on a particular detector-fixed plane (2D detector) [5, 15]. It is found that of order tens of events are required to reject isotropy of recoil angle distributions, and the number of required events is one order of magnitude larger for a detector that can not measure the sense, defined as the absolute sign ($\pm \vec{x}$) of the recoil vector. However, it is still a daunting task to fully reconstruct sub-100 keV nuclear recoil tracks.

In this paper, we consider a detector that can detect the polar angle of the recoil track with respect to a fixed z axis, while the azimuthal angle is not measured. In the following, we refer to this as a polar detector. In general, for a polar detector, since reconstruction of recoiling nuclear tracks is not required, the experimental realization can be more reliable and feasible. One issue for a directional dark matter detector is whether or not it possesses sense recognition capability, where we call the former a head-tail detector and the latter an axial detector. Considerable technological effort is required to provide head-tail detection capability. Throughout paper, the standard isothermal galactic halo model is used to model the WIMP velocity distribution. In this case, a head-tail polar detector with $z$ axis aligned with the WIMP wind direction has the same performance as a full 3D detector, because in this case the azimuthal distribution is completely flat and provides no information at all.

One example of an axial polar detector is a stilbene crystal, which is an organic single crystal whose scintillation efficiency depends on the nuclear recoil direction relative to crystallographic axes [16]. Recently, a new technique of columnar recombination (CR) has been shown to be capable of realizing a working axial polar detector [17]. The directional sensitivity in a CR detector comes from the dependence of electron-ion recombination on the angle between recoil track and electric field. Here the experimental signals are the scintillation light for the recombinations, and drift electrons from the surviving ionizations.

In most previous sensitivity studies such as those reported in Refs. [3, 14, 18, 19], the recoil energy is integrated out and only distributions with respect to angular
variables are studied. When all the energies are integrated out, the shape of angular distribution does not depend on the WIMP mass [20] and, thus, the distribution and the analysis are greatly simplified. However, we note that in a real experiment, all relevant information are used in order to achieve the highest possible sensitivity. Thus, both energy and directional information are used simultaneously in this work. The one previous study that did use the two dimensional energy and angular information [21], treated the WIMP mass as a known parameter that was kept at a fixed value in the fit. While this procedure is simpler to implement for producing plots of cross-section upper limits versus WIMP mass, it is problematic for a general directional sensitivity calculation. In this study, we address this issue by treating the WIMP mass as a nuisance parameter in a statistical framework that uses the well established profile likelihood method.

We study signals from spin-independent WIMP-nucleus interactions, and assume zero background to provide benchmark results, since this is expected for the next generation of directional WIMP detectors. In case of non-zero background experiments, the background contribution will dilute the directionality of the total distribution. Nevertheless, the analysis procedure described in this paper can be directly applied, where an observation of directionality still unambiguously leads to an observation of WIMPs.

In sections that follow, we will start by calculating the distribution of observables, and then study the sensitivity for various detector configurations using the standard profile likelihood method. Finally, we give an estimate of target mass times measurement time for a realistic columnar recombination detector that would be required to see a three sigma directional signal.

II. DISTRIBUTIONS OF OBSERVABLES

The general double-differential derivative of the recoil rate per unit target mass, with respect to nuclear recoil energy \( E \) and solid angle \( \Omega \) in the nuclear recoil direction \( \hat{q} \), can be expressed as [22]:

\[
\frac{d^2R}{dE d\Omega} = \frac{n \sigma_0}{4 \pi \mu^2} F(E) f(v_{\min}, \hat{q}).
\]

(1)

Here \( \mu = m M / (m + M) \) is the reduced mass of the WIMP-nucleus system, where \( m \) and \( M \) are the WIMP and target nucleus mass, respectively. \( F \) is the form factor, and \( n = \rho \rho^0 / m \) is the number density of the WIMP, where we use \( \rho^0 = 0.3 \text{ GeV/cm}^3 \). The minimum WIMP speed required for a recoil of energy \( E \) is \( v_{\min} \). The Radon transform, \( f(v_{\min}, \hat{q}) \), of the WIMP velocity distribution \( f(v) \) represents the sum of the probability densities where the velocity projection on direction \( \hat{q} \) is equal to \( v_{\min} \):

\[
f(v_{\min}, \hat{q}) = \int \delta(v \cdot \hat{q} - v_{\min}) f(v) d^3 v.
\]

(2)

In the standard isothermal galactic halo model, the distribution of WIMP velocities \( v \) relative to the target is Maxwellian, with an average value of \( v_E \):

\[
f(v) = \frac{1}{(\sqrt{2 \pi} v_0)^3} e^{-\frac{(v - v_E)^2}{2 v_0^2}}.
\]

(3)

In this case the Radon transform is:

\[
f(v_{\min}, \hat{q}) = \int e^{-\frac{(v_{\min} - q \cdot v_E)^2}{2 v_0^2}} d^3 v.
\]

(4)

The maximum recoil energy for a WIMP with velocity \( v \) is \( E_{\text{max}}(v) = 2 \mu n \sigma_0 / M \), and by reversing the formula, we obtain \( v_{\min} = \sqrt{2 \mu n \sigma_0 / M - \int v f(v) | v_E = 0 | d^3 v} \), with the definition \( E_0 = E_{\text{max}}(v_0) \). We also calculate the standard total rate \( R_0 \) when \( v_E = 0 = v_0 \):

\[
\frac{d^2R}{dE d\Omega} = \frac{1}{4 \pi E_0} F(E) e^{-\frac{(v_{\min} - q \cdot v_E)^2}{2 v_0^2}}.
\]

(5)

It is convenient to define two variables:

\[
x \equiv \frac{v_{\min}}{v_0} = \sqrt{\frac{E}{E_0}}; \quad x_E \equiv \frac{v_E}{v_0}.
\]

(6)

In a reference frame where the velocity \( z \) axis is parallel to the WIMP wind direction, or the Cygnus direction, the solid angle dependence reduces to polar angle \( \theta \) dependence, and the differential rate can be expressed as:

\[
\frac{d^2R}{dE d\cos \theta} = \frac{R_0}{2 E_0} F(E) e^{-\frac{(x_E \cos \theta - x)^2}{2 T E_0}}
\]

(7)

For simplicity, the peculiar velocity of the Sun and the Earth’s orbital velocity about the Sun are ignored and the velocity of Earth \( v_E \) is taken to be equal to the velocity of the Local Standard of Rest \( \vec{v}_{\text{LSR}} \). In this case the magnitude of \( v_E \) and \( v_0 \) is same, which is usually given the value of 220 km/s. Hence in this paper we use \( x_E = 1 \) to represent the standard halo model.

It is feasible to build a detector that is oriented at a fixed direction in space, where the WIMP wind direction and the \( z \) axis of detector form a fixed angle \( \theta_0 \). In the frame of a detector system, the polar and azimuth angles of recoil \( \theta_L \) and \( \phi_L \) provide the directional information and the \( \cos \theta \) term in Eqn. 7 can be expressed in terms of \( \theta_L \) and \( \phi_L \) as \( \cos \theta = \cos \theta_0 \cos \theta_L + \sin \theta_0 \sin \theta_L \cos \phi_L \). Here the WIMP wind direction is assumed to lie in the \( x - z \) plane, without any loss in generality. The resulting differential rate in lab frame with respect to \( E \), \( \cos \theta_L \) and \( \phi_L \) is then:

\[
\frac{d^3R}{dE d\cos \theta_L d\phi_L} = \frac{R_0}{4 \pi E_0} F(E) e^{-\frac{(x_E \cos \theta_0 \cos \theta_L + x_E \sin \theta_0 \sin \theta_L \cos \phi_L - x)^2}{2 T E_0}}.
\]

(8)
Here, a solid angle transform \( \frac{1}{2} \frac{d^2 R}{dE d \cos \theta} \rightarrow \frac{d^2 R}{dE d \cos \theta_L d\phi_L} \) is applied from Eqn. 7.

For a detector where the only directional measurable is the polar angle \( \theta_L \), the azimuthal angle \( \phi_L \) should be integrated out:

\[
\frac{d^2 R}{dE d \cos \theta_L} = \int_0^{2\pi} \frac{d^3 R}{dE d \cos \theta_L d\phi_L}. \tag{9}
\]

For an axial detector where the sense of the recoil direction cannot be distinguished, the distributions should be folded, as

\[
\frac{d^2 R}{dE d \cos \theta_L | d(\phi_L + \pi)}, \quad \text{and} \quad \frac{d^2 R}{dE d \cos \theta_L} = \frac{d^2 R}{dE d \cos \theta_L} + \frac{d^2 R}{dE d \cos \theta_L | d(\phi_L + \pi)}.
\]

Now we consider a common case where a detector is fixed in the Earth’s coordinate frame. Since the Earth itself is rotating, the angle \( \theta_0 \) modulates with a period of one sidereal day. The WIMP wind from Cygnus is directed to the Earth at a constant angle \( \epsilon = 42^\circ \) to the Earth’s polar axis, as shown in Fig. 1. If the \( z \) axis of the detector is oriented relative to Earth’s polar axis at a fixed angle \( \theta_D \), then the dependence of the angle \( \theta_0 \) as a function of time \( t \) in units of sidereal days is:

\[
\cos \theta_0 = -\cos \epsilon \cos \theta_D - \sin \epsilon \sin \theta_D \cos(2\pi t). \tag{10}
\]

Here \( t \) is zero when the WIMP velocity lies in the plane made by the detector \( z \) axis and Earth’s polar axis. Since \( t \) distributes uniformly within 0 and 1 when all energies and angles are considered, the appropriate distribution for the Earth-fixed case, now in terms of \( (E, \cos \theta_L, t) \), is determined by replacing \( \theta_0 \) in Eqn. 8 with Eqn. 10 and integrating out \( \phi_L \).

Note that here, instead of using angular information only as in \[6\ [14\ [18\ [19\], we use two and three dimensional distributions that include energy and sidereal time as additional variables.

### III. STATISTICAL TEST FOR DIRECTIONAL SIGNAL

The most common method that is used in particle physics to determine significance of an observation is the profile likelihood ratio test statistic method. It uses a hypothesis test against a null or trivial hypothesis

where the data is assumed to correspond to the distribution of an alternative or interesting hypothesis \( H_1 \). Usually, a set of parameters of interest \( \nu \) of \( H_1 \) are fixed to a specific value \( \nu_0 \) to obtain \( H_0 \). A likelihood ratio is calculated as the maximum likelihood of a null hypothesis divided by the maximum likelihood of an alternative hypothesis, with nuisance parameters \( \theta \) floating:

\[
\lambda(\nu_0) = \frac{L(\nu = \nu_0, \hat{\theta})}{L(\hat{\nu}, \hat{\theta})}. \tag{11}
\]

The test statistic \( q \) is defined as \(-2 \ln \lambda(\nu_0)\). The \( p \) value is the probability to have a discrepancy larger than the observed one \( q^{\text{obs}} \), as

\[
p = \int_{q^{\text{obs}}}^\infty f(q|H_0) dq_0. \tag{12}
\]

The smaller the \( p \) value, the more credible that hypothesis \( H_0 \) is not correct. For example, a \( p \) value of 0.00135 corresponds to a 3\( \sigma \) signal significance. In principle, the distribution \( f(q|H_0) \) needs to be obtained using Monte Carlo methods with high statistics. However, according to Wilk’s theorem, \( q \) follows a \( \chi^2 \) distribution \[23\]. Then the \( p \) value can be directly calculated as the probability above \( q^{\text{obs}} \) for a \( \chi^2 \) distribution \( P \) with degrees of freedom equal to the number of fixed parameters in the numerator of Eqn. 11:

\[
p = \int_{q^{\text{obs}}}^\infty f(q|H_0) dq_0.
\]

The likelihood function \( L \) used in this paper is the unbinned product of normalized probability density function (PDF) of observed quantities for all events. The PDF is normalized over two or three dimensional variables, such as \((E, \cos \theta_L)\) or with the inclusion of \( t \). Later, for a columnar recombination detector, the recombination and ionization signals will replace \( E \) and \( \cos \theta_L \). A non-zero \( x_E \) indicates a finite average WIMP speed from the Cygnus direction. Thus, for studies in this paper, \( x_E \) as \( \nu \) is the sole parameter of interest, with the null hypothesis being \( x_E = 0 \). The WIMP mass, since it is unknown, is a nuisance parameter and floated in the fits for both the numerator and denominator of Eqn. 11.

To test the sensitivity of a measurement, the median (not mean) value of the test statistic \( q_{\text{med}} \) of a large ensemble of hypothesis tests is used as a measurement quantity of sensitivity for the work reported here.
ensemble size $N$ is 1000 for all of calculations discussed below. The error of $q_{\text{med}}$ is $\frac{1}{\sqrt{N f(q_{\text{med}})}}$, where $f(q)$ is the normalized probability density function that is obtained from the raw distribution of $q$ for the ensemble by a kernel density estimation method [24]. Here, for each hypothesis test, a toy Monte Carlo dataset with a certain number of events is generated and fitted to obtain the $q$ value for this dataset.

With only one parameter of interest, the significance in units of $\sigma$ is calculated as the square root of the test statistic: $Z = \sqrt{q}$. So a $Z = 3\sigma$ significance corresponds to $q = 9$. In all the studies of this work, $q_{\text{med}}$ is found to be proportional to the total event number for cases where $q_{\text{med}} \geq 5$, as will be shown in Fig. 5. So we use a linear proportional function to fit $q_{\text{med}}$, and find the number of events corresponding to $q_{\text{med}} = 9$ as the required size for a $3\sigma$ discovery.

IV. GENERAL POLAR DETECTOR

To study the general performance of a polar detector, a general detector with zero energy threshold and unit form factor is considered. In the absence of the form factor term in Eqn. 5, the energy dependence of the rate is expressed through the ratio $E/E_0$, i.e., in units of $E_0$.

Since the WIMP mass dependence of the rate function only enters through $E_0$, a change of WIMP mass just changes $E_0$, the overall scale of the energy $E$. Thus the sensitivity for the general polar detector has no dependence on the WIMP mass. In this section, general statistical analysis results applicable to all WIMP masses are shown.

Figure 2 shows the median value of $q$ for space-fixed detectors, with polar and axial configurations, for different detector orientation angles. We can see that the optimum orientation for a space-fixed case is always at $\cos \theta_0 = 1$, where the detector’s $z$ axis points toward the...
Cygnus. This is expected since in this case the detector can gain maximum information from the measured \( \cos \theta_L \). Non-trivial curves for axial detectors are discovered with a minimum at around \( \cos \theta_0 = 0.56 \), where the distribution of \( \cos \theta_L \) is close to flat due to the folding of positive and negative \( \cos \theta_L \) values.

In the Earth-fixed case, the quantity \( q_{\text{med}} \) is averaged over many directions, and depends on the orientation \( \theta_D \). Figure 2 shows the dependence of \( q_{\text{med}} \) on the angle \( \theta_D \). A general head-tail detector is shown to have the most sensitivity when the \( z \) axis is oriented parallel to the Earth’s polar axis (\( \theta_D = 0^\circ \)). Interestingly, the optimal \( \theta_D \) value for a general axial detector is near \( \theta_D = 45^\circ \). A fit using an empirical function \( q_{\text{med}} = Q \cos(\alpha(\theta_D - \theta_D^0)) \) gives the optimum value \( \theta_D^0 = 44.48 \pm 0.33 \), with the fitted curve shown in Fig. 5 as a dashed red line.

Placing four types of detector in their optimal orientation, i.e., \( \cos \theta_0 = 1 \) for space-fixed detectors, and \( \theta_D = 45(0)^\circ \) for Earth-fixed axial (head-tail) detectors, the required number of events for 3\( \sigma \) discovery are displayed as red dots in row one of Table I. Of Order 10 to 20 event are required for axial detectors. We can also see that a change from space to Earth-fixed basis worsens the sensitivity by factors of 3.1 and 1.9 for axial and head-tail configurations.

As discussed in the introduction section, the optimal space-fixed head-tail detector behaves the same as a full 3D detector with its \( z \) axis aligned along the the WIMP wind. By dividing the two numbers for space-fixed case, we expect that an axial detector would have same sensitivity as a full 3D detector, provided it accumulated 6.3 times as many events.

V. ANGULAR-ONLY DETECTOR

Most previous estimates of the sensitivity of a directional dark matter detector have only considered angular information only. To study detectors with polar angle information only, the rate as a function of \( \cos \theta_L \) is obtained from Eqn. 8 by integrating out the energy part:

\[
\frac{dR}{d \cos \theta_L} = \int_0^\infty dE \frac{d^2 R}{dEd \cos \theta_L}.
\]

The same statistical procedure is performed as in the general polar detector case, for space- or Earth-fixed, axial or head-tail configurations. The \( q_{\text{med}} \) dependence on detector orientation is shown in Figs. 2 and 5. In all four detector configurations, the shape of the \( \cos \theta_0 \) and \( \theta_D \) dependence is similar to that for the general polar detector case. For a space-fixed axial detector, the dependence on \( \cos \theta_0 \) is also not monotonic. In Earth-fixed case, the red dotted curve in Fig. 5 has a maximum at \( \theta_D = 43.68 \pm 0.28^\circ \). Results for the optimal values \( \cos \theta_0 = 1 \) for space-fixed, \( \theta_D = 45^\circ \) and \( 0^\circ \) for Earth-fixed axial and head-tail detectors, are reported in row 2 of Table I. For head-tail detectors, of order of 10 (20) events are required to see directionality in space(Earth)-fixed basis, similar to results from previous studies [13, 15, 19] for 3D and 2D detectors. The sensitivity worsens by factors of 3.3 and 1.9 for axial and head-tail configurations, respectively, when changing a space-fixed to an Earth-fixed basis, similar to the general polar detector case.

By comparing the event numbers in columns 1 and 3 of Table I, i.e., for the axial configurations of general and angular only detectors, we can see that 3.7 and 4.0 larger statistical samples are needed when the energy information is not used. However, for head-tail detectors (columns 2 and 4), only 1.3 and 1.2 times the statistics are required. Thus, the importance of combining energy and angular observables is established, at least for axial detectors. We note that the performance may be improved over that reported in Refs. [15, 19], if the observed energy is simultaneously used.

VI. STANDARD XENON DETECTOR

Now we consider a detector with a specific material and a minimum energy threshold. Since the proposed columnar recombination detector uses gaseous xenon as target, a xenon Helmh form factor [25] \( F(E) \) is computed and inserted into Eqn. 8. For the minimum energy threshold, we refer to the LUX experiment [2], where the trigger efficiency reaches 50% for 4 keV nuclear recoil events. In gaseous xenon the 50% efficiency point can be achieved at a lower energy, because the efficiency from a single scatter will increase due to smaller multiple scattering per unit track length. In this study, a minimum energy threshold 3 keV is used, since it is also the minimum recoil energy that is considered in the LUX final analysis [2]. From the shape of the form factor, the event rate will be suppressed by a factor less than \( 10^{-6} \) if the recoil is greater than 100 keV. Thus, a detection energy range from 3 to 100 keV is chosen.

Because of the detection threshold, the required number of observed events for detectors with different thresholds cannot be directly translated to detector performance, since in this case the total rate depends on the thresholds. To make fair comparisons of the sensitivities, we introduce a quantity called the number of point interactions (\( N_{\text{pint}} \)), which is the number of WIMP-nucleus interactions with no threshold and no form factor effects for a given total target mass \( M_T \) and measurement time \( T \): \( N_{\text{pint}} = R \sigma_M T = \int_0^\infty dE \int d\Omega \frac{d\sigma_R}{d\Omega} |F(E)| = \int_0^\infty dE \int d\Omega \frac{d\sigma_R}{d\Omega} |F(E)| = 1.304 R_0 M_T T = 2.608 \sigma_0 \sigma_T M_T T / \sqrt{\pi} \). The number of observed events, \( N_{\text{obs}} \), is the integral of the differential rate within the detection range including the form factor: \( N_{\text{obs}} = \int_{\text{det. range}} d^2 R / d\Omega dE / (1.304 R_0) \). For a xenon target, the spin-independent WIMP-nucleus cross-section \( \sigma_0 \) is related to the WIMP-nucleon cross-section \( \sigma_n \) as \( \sigma_0 = \mu / \mu_{W-n} L^2 \sigma_n \), where \( \mu_{W-n} \) is the WIMP-nucleon reduced mass. Inputting the Xenon atomic num-
number $A = 131.3$, and the nucleon mass 0.938 GeV, we obtain $N_{\text{pin}} = 1.70 \times 10^{16} \text{cGy} / \text{m} \cdot \text{GeV} \cdot (\text{barn} \cdot \text{kg} \cdot \text{year})^{-1}$.

This standard xenon detector can be viewed as a columnar recombination detector with perfect resolution. So we only study the axial configuration. For this there will be some WIMP-mass dependence to the sensitivity since the form factor and the threshold depends on absolute energy and do not scale with $E_D$. The same statistical analyses are done for different WIMP masses. Figure 4 shows the raw sensitivity as a function of WIMP mass, for space-fixed detectors with the optimal orientation $\cos \theta_0 = 1$ and with 200 observed events within the detection energy range. The curves monotonically decreases with increasing WIMP mass. This is because for light WIMP masses, the high energy recoils are less suppressed by the form factor. The decrease of the scale factor $f_{\text{obs}}$ in the low WIMP mass region is due to the effect of the threshold that eliminates more events for the low WIMP mass case. The decrease of $f_{\text{obs}}$ for high WIMP masses is due to the form factor.

In the Earth-fixed case, the sensitivity dependence on $\theta_D$ is studied for four representative WIMP mass values of 20, 30, 50 and 100 GeV. Figure 5 shows the results, with fitted curves superimposed. The fitted optimal values for $\theta_D$ are $45.08 \pm 0.22$, $45.21 \pm 0.30$, $44.42 \pm 0.49$, $44.30 \pm 0.79$ degree for 20, 30, 50 and 100 GeV WIMP mass. These are all nearly equal; a weighted average value is $44.97 \pm 0.17$. In the following, $\theta_D = 45^\circ$ is used as the optimal detector orientation for Earth-fixed standard xenon and columnar recombination detectors.

Considering a typical case with 30 GeV WIMP mass, rows 3 and 5 of Table I show the required observed event numbers and point interactions for a 3\sigma discovery. Compared to the general axial polar detector case, the required numbers of observed events increases by about 20% when the effects of the 3 keV threshold and form factor for a xenon target are considered.

VII. REALISTIC COLUMNAR RECOMBINATION DETECTOR

A columnar recombination detector using high pressure xenon gas can measure the angle between the nuclear recoil track and electric drift field $[17, 26]$, without sense recognition capability. Thus it is an axial polar detector. It is based on a novel concept of columnar recombination effect in electron-ion pairs, where electrons encounter fewer positive ions if the field lead electrons away is perpendicular to the ion column, and conversely, a higher level of recombination occurs if the electrons' drift parallel to the ion column. By adding a small amount of trimethylamine (TMA) to the xenon gas, the xenon-TMA Penning transfer $[27]$ can convert the primary excitation to ionization, thus increasing the CR effect. TMA can also hasten electron thermalization and act as a wavelength shifter for the near-UV ($\gtrsim 300 \text{ nm}$) scintillation light $[17]$.

In a real columnar recombination experiment, recombination and ionization signals are read out as scintillation and charge for each event. The advantage of not requiring track imaging is obvious. Since the track
FIG. 6: Two dimensional plots of ionization versus recombination signals for a columnar recombination detector as density maps. A xenon target and a 30 GeV WIMP mass are used. From left to right, for case of \( x_E = 0 \) (isotropic), \( x_E = 1, \cos \theta_0 = 1 \) (parallel electric field), and \( x_E = 1, \cos \theta_0 = 0 \) (perpendicular electric field). In each plot, contours where the rate is equal to 0.05, 0.02, and 0.01 \( R_0/\text{keV}^2 \) are drawn as increasing sized solid curves. The threshold of 3 keV is shown as blue dashed lines.

TABLE I: The required number of events for 3\( \sigma \) discovery for all detector types. Numbers are the abscissa value corresponding to \( q_{\text{med}} = 9 \) level in Fig. 5 for the fitted straight lines. \( N_{\text{pint}} \), defined in the text, is the number of point interactions without threshold and form factor effect, which directly relates to the detector performance. The indicated errors are statistical.

| Detector type             | space-fixed | Earth-fixed |
|---------------------------|-------------|-------------|
|                           | Axial       | Head-tail   | Axial       | Head-tail   |
| General                   | 62.30 ± 0.56| 9.90 ± 0.09 | 193.2 ± 2.1 | 19.22 ± 0.19|
| Angular Only              | 231.4 ± 2.7 | 12.70 ± 0.13| 767.0 ± 9.2 | 23.91 ± 0.25|
| Xenon, 30 GeV WIMP \( (N_{\text{obs}}) \) | 72.97 ± 0.68 | 23.4 ± 2.5 |
| Real CR, 30 GeV WIMP \( (N_{\text{obs}}) \) | 78.06 ± 0.78 | 242.4 ± 2.8 |
| Xenon, 30 GeV WIMP \( (N_{\text{pint}}) \) | 168.1 ± 1.56 | 535.6 ± 5.8 |
| Real CR, 30 GeV WIMP \( (N_{\text{pint}}) \) | 180.3 ± 1.8  | 559.0 ± 6.5 |

FIG. 7: Mean value of Recombination versus Ionization in binned profile histograms for middle and left plots of Fig. 6. The solid line is for \( x_E = 1, \cos \theta_0 = 1 \) and the dashed line is for \( x_E = 0 \). This provides a better view of the differences between the two distributions than the density maps in Fig. 6. The points with error bars are for one generated data sample of 500 events, where we can compare the size of errors to the differences between the two histograms.

length is irrelevant, the working density can be as large as 0.5 g/cm\(^3\), just before where the energy resolution starts to worsen [28]. Large volumes can be realized since the limit of drift length due to diffusion for an imaging system does not apply. In addition, the excellent energy resolution of gas xenon can be exploited [29]. We use the quantities \( C \) and \( I \) to denote the corresponding calibrated energy deposits from the recombination (scintillation) and ionization readouts.

For electrons depositing energy in high pressure xenon gas in a low electric drift-field, the average energy loss per primary scintillation \( (W_S) \) is measured to be 76 ± 12 eV [30] at 15 bar, and 72 ± 6 eV [31] for the 1-3 bar pressure range. The corresponding average energy for one electron-ion pair is estimated to be \( W_J = 24.8 \text{ eV} [32] \). According to Ref. [33], the primary scintillation yield due to initial excitation for nuclear recoils is quenched by a factor of roughly 2, and the ionization yield is quenched by a factor of roughly 5. So by applying these quenching factors to \( W_S \) and \( W_J \), we use the numbers \( W_S^{\nu} = 150 \text{ eV} \) and \( W_J^{\nu} = 125 \text{ eV} \) to calculate the signal yields for nuclear recoils in a columnar recombination detector in the following. Consider a nuclear recoil that de-
posits an energy of 30 keV. This will induce, on average, 240 ionizations and 200 initial excitations. Assuming a 10% geminate recombination of free electrons, this corresponds to 216 ionization and 224 excitations. The inclusion of TMA mixture would produce a large Penning transfer effect that would convert 80% of the excitations to free electrons. This will increase the free electron yield to 216 + 0.8 × 224 = 395, corresponding to an effective ionization work function for nuclear recoils \( W_1^{\text{eff}} = 30 \text{ keV}/395 = 75.9 \text{ eV} \). The columnar recombination principle would apply to these 395 electron-ion pairs. The recombination fraction of electrons is at a maximum for recoils aligned with electric field (\( \theta_L = 0 \)), while it is at a minimum for recoil direction perpendicular to the electric field. We use a function \( f_{\text{comb}} = 0.8 - 0.4 \sin^2 \theta_L \) to describe the recombination fraction and, hence, the ionization fraction is \( 1 - f_{\text{comb}} \). Thus \( C \) and \( I \) are then \( C = (0.8 - 0.4 \sin^2 \theta_L)E \) and \( I = (0.2 + 0.4 \sin^2 \theta_L)E \). Here we assume that the maximum and minimum recombination fractions are 80% and 40%.

To study the feasibility of a realizable columnar recombination experiment, a realistic detector response should be considered. The measured quanta for the recombination channel is scintillation photons, where the detection efficiency and quantum efficiency of PMTs. For the ionization channel is scintillation photons, where the detection efficiency and quantum efficiency of PMTs. For the ionization channel, the thresholds are 1.52 keV for scintillation photoelectrons or ionization electrons are observed in each channel, the thresholds are 1.52 keV for \( C \) and 0.30 keV for \( I \).

For gaseous xenon, the energy resolution can be expressed using Fano’s theory, with Fano factor \( F = 0.14 \) \cite{25, 32}, such that \( \sigma_C = \sqrt{FWI_1^{\text{eff}}C/e_C} \), and \( \sigma_I = \sqrt{FWI_1^{\text{eff}}I/e_I} \). At present, no result on the correlation between scintillation and ionization yields from nuclear recoil events in gaseous xenon is available. On the other hand, the correlation between the two for alpha particle energy deposits \( \sigma_{\text{nuc}} \) has been measured to be \(-0.2\), a negative number whose absolute value is much less than one. This anti-correlation may be explained by the fact that no quenching for alpha particles in either ionization or scintillation occurs \( \sigma_{\text{nuc}} \), and the total energy is mostly split between the scintillation and ionization, similar to the electron energy deposit case. However, for nuclear recoils, both signals are quenched, and the majority of energy goes to heat, which means that the correlation will be much less than that for the alpha case, which is already quite mild. Thus it is safe and reasonable to assume no correlation in this study.

Combining all information together, for a space-fixed CR detector, the distribution of the observables \( C \) and \( I \) is expressed as:

\[
\frac{d^2R}{dCIdI} = \frac{d^2R}{dEd\cos\theta_L} J(E, |\cos\theta_L|; C, I) \times G(C; \sigma_C)G(I; \sigma_I)
\]

Here \( J \) is the Jacobian that transforms \( (E, \cos\theta_L) \) to \( (C, I) \), the symbol \( \otimes \) denotes a two dimensional convolution over \( C \) and \( I \), and \( G(x; \sigma) \) denotes a Gaussian function of \( x \) with zero mean and width \( \sigma \). For an Earth-fixed CR detector with sidereal time variable \( t \), the distribution \( \frac{d^2R}{dCIdt} \) is obtained using the same method from the distribution \( \frac{d^2R}{dEd\cos\theta_L} \) for an Earth-fixed general detector. A two-dimensional rectangular fit range of 1.52-64 keV for \( C \), and 0.30-48 keV for \( I \) is used.

Figure 6 shows the two dimensional distributions for \( (C, I) \) for the standard isothermal galactic halo model with a typical 30 GeV WIMP mass. The isotropic case with \( x_E = 0 \) is shown in a reduced region for better view, together with standard halo cases with parallel and perpendicular oriented electric field with respect to the WIMP wind. The differences in three distributions are clearly visible, which suggests that no huge amount of statistics is needed to distinguish between them. The 3 keV threshold used in the standard xenon detector is shown as a dashed line, which correspond to the formula \( C + I = 3 \text{ keV} \). It roughly corresponds to the two independent thresholds in \( C \) and \( I \) shown in the figure.

To see the discrimination power visually, profile histograms as mean value of recombination versus Ionization are plotted in Fig. 7 for isotropic \( (x_E = 0) \) and directional \( (x_E = 1, \cos\theta_E = 1) \) WIMP halos, on space-fixed basis. A clear difference in two histograms is seen, with the mean value of \( C \) always higher in directional halo. For 500 observed events in the standard WIMP halo with fixed WIMP mass, the error size shown in error bars is small enough to have a statistical significant shift from the isotropic histogram. Quantitative numbers for a 3\( \sigma \) discovery from the likelihood analysis procedure using the distribution in Eqn. 14 are displayed in fourth row of Table I. Compared to numbers in the third row, we can see that the resolution of a real CR detector has a minor effect. The required \( N_{\text{pint}} \) value of 180.3 and 559.0 from the last row of Table I translate to 636 and 1973 kg-year measurement, using a \( 5 \times 10^{-46} \text{ cm}^2 \) spin-independent WIMP-nucleon cross-section and a 30 GeV WIMP mass.

### VIII. Conclusion

The performance of a directional dark matter detector with polar angle detection is studied for various configurations. A WIMP-mass independent method is used to obtain the sensitivity of a general detector. In addition, a realistic columnar recombination detector is modeled. We infer that:

- Both axial and head-tail polar detectors have the highest sensitivity when the \( z \) axis is aligned with
WIMP wind. However, the dependence of sensitivity to detector orientation is not monotonic for an axial detector. To obtain optimal performance when rotating with the Earth, the z axis should be oriented at 45 degree to Earth’s pole for an axial detector, while it should be aligned with Earth’s polar axis for a head-tail detector.

- A head-tail polar detector can detect directionality with of order 10 or 20 events on a space- or Earth-fixed basis. In the absence of sense detection capability, an order of magnitude more statistics is needed.

- Without using energy information simultaneously, the required statistics would be a factor of 3.7(4.0) times higher for a space (Earth)-fixed axial detector. This conclusion will be useful for detector types in which partial directional information is available, such as a 2D plane detector.

- A general axial polar detector with 6.3 times the statistics has the same performance as a general full 3D tracking detector. However, in experimental practice, the target mass for a full 3D detector is limited because of diffusion effects, and accomplishing millimeter tracking is extremely challenging. On the other hand, a detector with polar angle sensitivity without head-tail discrimination requirement can use straightforward experimental techniques. In addition, it can be made in large volume with high-density gas, with a target mass that can be orders of magnitude larger than a conventional full 3D detector. We conclude that it is of great advantage to explore the directional dark matter detection technique using a senseless polar angle detection apparatus.

- A space-fixed detector is generally found to be 3 and 2 times more sensitive than an Earth-fixed detector, in axial and head-tail configurations. This ratio is an important factor when comparing the additional cost for a space fixed detector, since it has to rotate all the time with respect to the Earth. In a space-fixed detector, to distinguish between the WIMP signal from the galaxy coordinate and possible anisotropic background originating from detector frame, manually reversing the detector z axis direction for half of the measurement period can help.

- For a realistic xenon columnar recombination detector, the resolution effect changes the required statistics slightly, and a 636 or 1973 kg-year measurement can reach a 3σ directional signal for a space- or Earth-fixed case, using a $5 \times 10^{-46} \text{cm}^2$ WIMP-nucleon cross-section and a 30 GeV WIMP mass.

Acknowledgments

The author would like to thank Adam Para, Jonghee Yoo, Yeongdok Kim and Stephen Olsen for useful discussions.
[21] J. Billard, F. Mayet and D. Santos, Phys. Rev. D 85, 035006 (2012) [arXiv:1110.6079 [astro-ph.CO]].
[22] P. Gondolo, Phys. Rev. D 66, 103513 (2002) [hep-ph/0209110].
[23] G. Cowan, K. Cranmer, E. Gross and O. Vitells, Eur. Phys. J. C 71, 1554 (2011) [arXiv:1007.1727 [physics.data-an]].
[24] K. S. Cranmer, Comput. Phys. Commun. 136, 198 (2001) [hep-ex/0011057].
[25] J. D. Lewin and P. F. Smith, Astropart. Phys. 6, 87 (1996).
[26] V. M. Gehman, A. Goldschmidt, D. Nygren, C. A. B. Oliveira and J. Renner, JINST 8, C10001 (2013).
[27] S. Cebrian, T. Dafni, E. Ferrer-Ribas, I. Giomataris, D. Gonzalez-Diaz, H. Gomez, D. C. Herrera and F. J. Iguaz et al., JINST 8, P01012 (2013) [arXiv:1210.3287 [physics.ins-det]].
[28] A. Bolotnikov and B. Ramsey, Nucl. Instrum. Meth. A 396 (1997) 360.
[29] V. Alvarez et al. [NEXT Collaboration], Nucl. Instrum. Meth. A 708, 101 (2013) [arXiv:1211.4474 [physics.ins-det]].
[30] A. Parsons, T. K. Edberg, B. Sadoulet, S. Weiss, J. F. Wilkerson, K. Hurley, R. P. Lin and G. Smith, IEEE Trans. Nucl. Sci. 37, 541 (1990) [Proc. SPIE Int. Soc. Opt. Eng. 1159, 45 (1989)].
[31] L. M. P. Fernandes, E. D. C. Freitas, M. Ball, J. J. Gomez-Cadenas, C. M. B. Monteiro, N. Yahlali, D. Nygren and J. M. F. da Santos, JINST 5, P09006 (2010) [Erratum-ibid. 5, A12001 (2010)] [arXiv:1009.2719 [astro-ph.IM]].
[32] D. Nygren, Nucl. Instrum. Meth. A 603, 337 (2009).
[33] J. Renner et al. [NEXT Collaboration], [arXiv:1409.2853 [physics.ins-det]].
[34] V. Alvarez et al. [NEXT Collaboration], JINST 8, P05025 (2013) [arXiv:1211.4508 [physics.ins-det]].