Low densities in nuclear and neutron matters and in the nuclear surface

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Abstract. Nuclear and neutron matters are investigated in the low density region, well below the nuclear saturation density. Microscopic calculations, based on the Bethe-Brueckner approach with a few realistic nucleon-nucleon potentials, are compared with the predictions of a set of phenomenological effective interactions, mostly employed in nuclear structure studies. An energy functional is constructed on the basis of the microscopic bulk EoS and applied to a selection of nuclei throughout the mass table. The results provide a microscopic basis for a link between nuclear surface behaviour and neutron EoS previously observed with phenomenological effective forces. Possible effects of pairing on asymmetric nuclear matter are also analyzed in detail. The results are expected to illuminate the physical mechanisms which determine the behaviour of the surface density tail in exotic nuclei.
Introduction

Most of the studies of symmetric and asymmetric nuclear matter have been restricted to densities from about saturation ($\sim 0.15 \text{ fm}^{-3}$) to few times this value. This density interval is the most important for the physics of neutron stars and of heavy ion collisions, both at intermediate and relativistic energies.

In finite nuclei, the surface region is characterized by a density profile the detailed shape of which is determined in nuclear structure calculations by the gradient term in phenomenological effective nucleon-nucleon (NN) forces such as the Skyrme forces [1, 2], or by a finite range interaction as in the case with the Gogny force [3]. In exotic nuclei, with large asymmetry, the density drops at the surface with a relatively smooth profile. The neutron to proton ratio can be quite different from the bulk one, which gives rise to phenomena like the appearance of a neutron skin or of a neutron halo. But even for normal nuclei the asymmetry in the far tail may vary substantially from the one in the bulk. In a Local Density Approximation (LDA) picture the detailed asymmetry structure, and thus also the surface profile, should be related to the properties of asymmetric nuclear matter at density much lower than the saturation one. In particular, the symmetry energy $a_y$ at low densities, which is embodied in the effective force, should play a major role in fixing the neutron to proton ratio throughout the surface region. These aspects may also be of importance in other low density nuclear systems such as the expansion phase after central heavy ion collisions at intermediate energies or during the collapse of massive stars.

It therefore appears of great interest to investigate in detail the asymmetry properties of low density nuclear matter. This shall be done in this work using firstly a microscopic Brueckner Hartree-Fock approach starting from the bare NN force and secondly with current phenomenological forces of the Skyrme and Gogny type. The similarities and differences between both approaches shall be analyzed.

A semi-empirical connection between low density nuclear matter Equation of State (EoS) and finite nuclei surface properties has been presented in ref. [4]. The difference between neutron and proton root mean square radii in $^{208}\text{Pb}$ was calculated for an extended set of Skyrme interactions. This difference $r_n - r_p$ turns out to be linearly correlated with the slope $\chi$ of the corresponding neutron matter EoS at the chosen density $\rho = 0.1 \text{ fm}^{-3}$. This slope, i.e. the derivative of the energy per particle with respect to
density, can largely differ from one Skyrme force to another, since no direct phenomenological constraint exists for neutron matter. This study has been extended to relativistic mean field functionals in ref. [5], and the same linear correlation has been found in ref. [6], where the analysis comprises a large set of energy functionals. A partial explanation of the correlation was presented in ref. [7] on the basis of Landau Fermi liquid theory. However, this linear correlation still appears surprising, since it is well known that in order to properly describe the surface properties of finite nuclei the bulk term of the energy functional must be implemented with gradient terms, not included in ref [7], which should also play a role in determining the value of \( r_n - r_p \). One of the main aims of the paper is to establish to what extent the bulk part of the functional, which is directly related to nuclear matter EoS, dominates indeed the nuclear surface behaviour. To this purpose, we shall construct an energy functional based on many-body calculations, in particular for neutron matter, for which the EoS seems to be well established on a microscopical basis. This will be used to restrict the range of possible values of \( r_n - r_p \).

A further important aspect of low density asymmetric nuclear matter may be its pairing properties. Indeed we will find that low density nuclear matter with asymmetry such that it is slightly unbound can become bound solely through the action of neutron-neutron pairing which is quite strong at low densities. This phenomenon may have important consequences in the formation of neutron-skins and -haloes in very neutron rich nuclei.

In detail the paper is organized as follows. In Sec. 2 we give a brief introduction to the microscopic many-body theory and present the results for different nucleon-nucleon realistic interactions, for symmetric nuclear matter, pure neutron matter and for asymmetric nuclear matter in general. In Sec. 3 the results for a few phenomenological NN effective forces are presented, and their comparison with the microscopic calculations is discussed. In Sec. 4 we construct a microscopically based density functional and we study the relevance of the gradient terms in determining the surface properties of different nuclei. Sec. 5 is devoted to the possible effects of the pairing correlations on the equation of state of very asymmetric nuclear matter at low density. Conclusions are drawn in Sec. 6.

2. The BHF approach to low density asymmetric matter

Let us consider homogeneous nuclear matter at low enough density so that two-body correlations dominate. Of course, at very low density, clustering phenomena can occur, like deuteron, triton, helium and alpha particle
formation. Therefore, we are going to consider densities which are low with respect to saturation, but still larger than the ones for the onset of clustering. Typically, we are considering densities between about one tenth to one half of the saturation density. In this density region, microscopic calculations based on Bethe-Brueckner approach are expected to be quite accurate, and they will be, therefore, the starting point for our analysis. The Bethe-Brueckner theory is well documented in the literature (see e.g. ref. [8]) and we will not repeat any details here. We simply want to mention that we will use the continuous choice of the single particle spectrum, as described in ref. [9], since it was demonstrated there that the convergence properties are better than with the gap choice. Three bare forces, the Argonne v$_{14}$ [10], the Argonne v$_{18}$ [11] and the Paris force [12] will be used. As it is well known, these two-body forces have to be supplemented by three-body forces in order to obtain a saturation point close to the empirical one. The semi-empirical force of ref. [13, 14] is used, with parameters that turn out to be the same for all the considered two-body forces. It has to be stressed that the relevance of the three-body force is restricted to densities around and above saturation. In particular, the three-body contribution around the density $\rho = 0.1 \text{ fm}^{-3}$ is less than 0.2 MeV and, therefore, it plays no role in our considerations. The introduction of three-body force is demanded here only to the aim of constructing a realistic energy functional, as it will be discussed in the sequel. Similar considerations apply to neutron EoS.

In Fig. 1 we show the results for the energy per particle as a function of density for symmetric nuclear matter and for pure neutron matter. As a reference we take the Argonne v$_{18}$ interaction, for which we did systematic calculations with a small density grid. The corresponding curves (small circles) have been obtained by a polynomial fitting to the calculated points. Its explicit form is reported in the Appendix. This gives the detailed trend of the neutron matter and symmetric nuclear matter equation of state at low density. At density lower than $0.02 \text{ fm}^{-3}$ the equations of state have been extrapolated imposing that the polynomial crosses the origin.

We have then calculated few representative points for the Argonne v$_{14}$ and Paris potentials. We see that the results of the three forces are practically identical in this density region, both in the case of symmetric nuclear matter and pure neutron matter. Deviations are known to be present [15] among the different forces at saturation density and above. This close agreement at low density shows that the equation of state is determined mainly by the
nucleon-nucleon phase shifts, namely by the on-shell G-matrix.

Another way to see this agreement among the different forces is shown in Fig. 2, where we display the symmetry energy as a function of density, a quantity which should be relevant for the properties of nuclear matter at low densities, in particular in the surface region of nuclei. Here the symmetry energy is just the difference between the neutron and symmetric matter EoS (at the same density). In view of the agreement among the different forces, one can conclude that the equation of state of symmetric nuclear matter and pure neutron matter at low density is well determined from microscopic calculations and they can be considered as established.

These results can be used as a reference for more phenomenological approaches, which are expected to reproduce the microscopic equation of state if they have to be applied in the low density region.

In principle calculations can be extended to asymmetric nuclear matter with an arbitrary value of the asymmetry parameter. However, it is well known that the dependence of the equation of state on the asymmetry parameter $\beta = (\rho_n - \rho_p)/\rho$ can be approximated by a quadratic form with very good accuracy [17], i.e. the energy per particle $e = E/A$ at a given total density $\rho = \rho_n + \rho_p$ and asymmetry $\beta$ can be written as $e = e_n \cdot \beta^2 + (1 - \beta^2) \cdot e_s$, being $e_n$ and $e_s$ the energy per particle for neutron and symmetric matter, respectively, at the same density. Once the symmetric matter and pure neutron matter equations of state are determined, it is then possible to calculate the equation of state for a generic asymmetry. In Fig. 3 we show a quantitative interpolation for various asymmetries of the energy per particle for the Argonne $v_{14}$ case. The striking point is that for the asymmetry parameter $\beta \approx 0.75$ the Equation of State is practically flat with $E/A \sim -400$ keV in the density range considered. No such behaviour is found for the phenomenological forces. The qualitative explanation for this behaviour of the microscopic calculation can be as follows. The average kinetic energy per particle increases as $\rho^{\frac{4}{3}}$. If one neglects the density and momentum dependence of the G-matrix, the potential part at low density increases (in absolute value) linearly. However the Pauli operator quenches the scattering processes, and therefore the absolute value of the G-matrix is slowly decreasing with density. This means that the potential energy has an increasing trend that is slightly less rapid than linear. It is conceivable that for a suitable asymmetry the trend of the potential energy can be mimicked by a density dependence close to $\rho^{\frac{4}{3}}$, which can compensate the increase of the kinetic energy in a relatively
wide density range.

3. Phenomenological forces

Results for the different phenomenological forces are also reported in Fig. 1 and Fig. 2. We have considered two Skyrme forces, Sly4 [1] and SkM* [2] and the finite range Gogny force [3]. They are among the most used ones in nuclear structure calculations. In particular, the Sly4 force has been adjusted also in order to reproduce the microscopic results of ref. [19] for pure neutron matter at high density. One can see that they are quite close to each other in the low density symmetric nuclear matter case, with the Gogny force slightly less binding. The situation is quite different in the pure neutron matter case, where a large spread of values for the EoS is apparent. This could be of course expected, since the Skyrme and Gogny forces are mainly adjusted to reproduce properties of finite nuclei, where the asymmetry is quite small. These discrepancies reflect into the symmetry energy displayed in Fig. 2. It has to be stressed that at saturation the symmetry energy trend for the different forces shows a substantial agreement. Indeed, at $\rho = 0.16 \text{ fm}^{-3}$ for Gogny force $a_y = 31.62 \text{ MeV}$, for SKM* $a_y = 31.35 \text{ MeV}$, while for the Sly4 force $a_y = 32.72 \text{ MeV}$. Thus the trend at saturation is not preserved at lower densities, i.e. the value of the symmetry energy at saturation does not fix the behaviour of the symmetry energy at low density.

As already mentioned, for microscopic calculations the trend of the symmetry energy at low density can be considered independent of a realistic NN interaction and therefore we think that these results provide accurate values for the nuclear matter symmetry energy at low density.

Comparison of the results with the microscopic calculations indicate that additional constraints on the phenomenological forces can be introduced if pure neutron matter has to be well described. These constraints can be of great relevance for the description of the surface region of exotic nuclei with a large neutron excess. The slope of the neutron matter EoS, taken at a reference value of the density, characterizes the behaviour of the EoS in the low density region, in particular the amount of binding for a given EoS.

A comparison similar to the one of Fig. 1, between microscopic calculations and phenomenological ones for pure neutron matter only, can be found in ref. [18]. The microscopic results for pure neutron matter, both from variational and Dirac-Brueckner, show a close agreement with the ones reported in Fig. 1.

In neutron stars different spatial configurations of almost pure neutron
matter have been conjectured to occur just below the solid crust. These configurations are characterized by non-uniform density profiles, like rods, “lasagne”, and so on. In particular, neutron bubbles should be quite sensitive to the detailed trend of the pure neutron matter EoS in the considered density range, and therefore an accurate tuning of the phenomenological forces, aimed to reproduce the microscopic neutron EoS, is probably appropriate.

4. From nuclear matter to finite nuclei

In order to study the possible link between low density neutron and nuclear matter EoS and the surface properties of nuclei, we have constructed a simplified energy functional based on the microscopic calculations of Sec. 2. This will enable also a more stringent comparison with the finite nuclei calculations based on Skyrme forces.

In the BHF approach the nuclear matter energy density is just the sum of the free kinetic energy density and the two-body correlation energy

$$\frac{E}{\hbar^2} = \frac{1}{V} \sum_k \frac{h^2 k^2}{2M} n(k) + \frac{1}{V} \sum_{k,k'} \langle kk'|G|kk'\rangle n(k)n(k')$$

$$= \tau(\rho) + U(\rho)$$

where $n(k) = \Theta(k_F - k)$ is the free Fermion gas occupation number, $M$ the nucleon mass, $G$ the Brueckner $G$-matrix, and the variables $k, k'$ include momentum, spin and isospin quantum numbers. Both kinetic energy term and correlation energy term are well defined functions of the density $\rho$. This sum will be identified with the bulk part of the energy functional. Notice that no effective mass is here explicitly introduced, at variance with the Skyrme functionals we are considering (Sec. 3). All the correlation effects are embodied in the interaction term $U(\rho)$.

The full functional appropriate to finite nuclei can be obtained by keeping the same correlation energy $U(\rho)$ and including a possible gradient term of the form $\eta(\nabla \rho)^2$. Of course, this term cannot be obtained from nuclear matter calculations, and the parameter $\eta$ has to be considered as a phenomenological one. This energy density functional could be understood within the Kohn-Sham (KS) theory [24] where the finite range Hartree term has been expanded in terms of distributions [25] providing the gradient term and where the bulk BHF energy density basically corresponds to the exchange-correlation part. In a KS approach based on this functional the hamiltonian
density will be

$$\mathcal{H}(\rho) = \frac{\hbar^2}{2M} \sum_i (\nabla \psi_i)^2 + \left[ U(\rho) + \eta(\nabla \rho)^2 \right]$$ (2)

where $\psi_i$ are the single particle orbital wave functions, $\rho(x) = \sum |\psi_i|^2$.

The wave functions $\psi_i$ are determined by minimizing the total energy with the constraint of particle number conservation. As it is well known, this gives the KS equations for $\psi_i$ and the total energy of the nucleus

$$E = \int d^3 x \mathcal{H}(\rho(x))$$ (3)

To properly describe finite nuclei, the asymmetry and Coulomb energies as well the spin-orbit term have to be included. For the density-dependent symmetry energy we take the difference between neutron and symmetric matter energy densities locally. Also the asymmetry parameter $\beta = (\rho_n - \rho_p)/\rho$ is calculated at each point. For asymmetric nuclei we will keep, for simplicity, the scalar gradient term of Eq. (2) and neglect the possible isovector contributions, which have a small influence and which can be included in the coefficient of the isoscalar term. The Coulomb energy is obtained from the proton density and its exchange term is evaluated in the Slater approximation. The spin-orbit contribution is equal to the one used in the Skyrme or Gogny forces. With this complete functional, which can be included within the quasi-local density functional theory [26] due to the spin-orbit term, an explicit link is established between the microscopic nuclear and neutron matter EoS and finite nuclei. A similar procedure was followed in ref. [20] for a relativistic extended Thomas-Fermi calculation of finite nuclei based on a Dirac-Brueckner-Hartree-Fock EoS.

With this functional we performed Kohn-Sham calculations for the magic nuclei $^{16}\text{O}$, $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{90}\text{Zr}$ and $^{208}\text{Pb}$. The free parameters for our functional i.e. the parameter $\eta$ and the strength of the spin-orbit force $W_0$, are fitted to obtain a good reproduction of the experimental binding energy, charge radius and spin-orbit splittings of some levels near the Fermi surface of the previously mentioned magic nuclei.

At the practical level, the correlation energy density $U(\rho)$ of the $v_{18}$ EoS was fitted with a simple polynomial form, to be used in the actual KS calculations, as already mentioned in Sec. 2. For densities close to the saturation
one we allowed some small deviations of few hundreds keV from the calculated energy values, so that the saturation point turns out very close to the phenomenological one. This deviations are within the numerical accuracy of the microscopic calculations and are introduced to make the functional as realistic as possible. The main region of interest for the nuclear surface is of course the low density region, where the microscopic calculation is accurately reproduced. We got for the saturation density $\rho_0 = 0.155 \text{ fm}^{-3}$ and the corresponding energy $E/A = -15.4 \text{ MeV}$. The incompressibility and symmetry energy at saturation are $K = 247 \text{ MeV}$ and $a_y = 31.34 \text{ MeV}$, respectively.

With this functional we performed KS calculations for a set of nuclei and varied the parameter $\eta$ in order to get a good reproduction of the data throughout the periodic table. As a reference we took the value from the SkM* force, $\eta = -68 \text{ MeV fm}^5$. The optimal value turns out to be $\eta \approx -53$. The spin-orbit strength is fixed to $130 \text{ MeV fm}^5$. Some results are reported in Table 1, in comparison with SkM*, which gives excellent reproduction of the phenomenological data. Of course, this microscopically based functional cannot compete with more phenomenological ones, like the SkM* functional, since in this case only the parameters $\eta$ and $W_0$ are adjusted. However, this procedure allows to obtain a realistic enough functional as basis for our considerations on the link between surface properties and nuclear matter EoS.

The results reported in Table 2 for the neutron $r_n$ and proton $r_p$ root mean square radii show the possible influence of the gradient term on the value of its difference $r_n - r_p$. One can see that this quantity is, to a large extent, insensitive to the precise value of $\eta$, and even for $\eta = 0$ its value is not dramatically changed. In other words, the value of $r_n - r_p$ is mainly determined by the bulk part of the functional. This result provides an explanation, or at least a justification, of the existing link between the value of $r_n - r_p$ and nuclear matter EoS.

Also the absolute values of the neutron and proton radii seem to be only marginally affected by the value of $\eta$. We have also checked that a possible isovector gradient term (i.e. the gradient of the difference between neutron and proton densities) does not affect appreciably the results on radii. On the contrary, as expected, the binding energy is quite sensitive to the strength of the gradient term, which has a large effect on the surface energy as it can be seen from Table 2. An estimate of the mass formula surface energy coefficient $a_s$ can be obtained by subtracting from the total energy the bulk energy.
in asymmetric nuclear matter as well as the Coulomb and spin-orbit contributions, which can be all easily extracted from the KS calculations. This coefficient estimated for several values of the $\eta$ parameter are also reported in Table 2. In the case of $^{208}$Pb, one indeed realizes that the surface energy reduces by about a factor 1.7 going from the value $\eta = -53$ to $\eta = 0$, pointing out the key role of the gradient term in the energy density to properly describe the nuclear surface.

Following refs. [4, 5, 6] we consider the possible correlation between the difference $r_n - r_p$ and the nuclear matter EoS. Instead of considering the slope of the neutron EoS at the density $\rho = 0.1$ fm$^{-3}$, we prefer to consider the slope $\chi$ of the symmetry energy $a_y$ (at the same density), i.e. $\chi = da_y/d\rho$. As shown in Fig. 4 the approximate linear correlation, found in refs. [4, 5, 6], is also valid with this different choice for the EoS parameter. This is probably due to the fact that the symmetric matter slope at the reference density is quite similar for all forces. The set of relativistic mean field functionals clearly cluster on the upper-right part of Fig. 4, while most of the Gogny and Skyrme forces are concentrated in the lower-left part. The point (the star in Fig. 4) corresponding to the microscopic functional, constructed in the present work, lies within the Gogny and Skyrme region. This indicates that the small discrepancies between the microscopic and Skyrme forces EoS, as displayed in Fig. 1 and Fig. 2, do not modify appreciably the considered linear relationship. An accurate measurement of the neutron radius in $^{208}$Pb, as projected through parity violating electron scattering at Jefferson Laboratory should allow to obtain a value of $r_n - r_p$ precise enough to discriminate between different sets of EoS, i.e. the one based on relativistic mean field and the one on Skyrme forces or microscopic functionals, respectively. The non-relativistic microscopic functional gives a well defined prediction for the position of the point along the linear plot. The uncertainty contained in this analysis, namely the precise value of the parameter $\eta$, is well below the discrepancy between the two sets of functionals.

5. Superfluid properties

It is well known that neutron superfluidity is a pronounced function of density and is strongest for about one fifth of the saturation density, where the gap is about 3 MeV [21]. We therefore can expect that for $\beta \approx 0.75$ where the EoS is completely flat, the additional pairing correlations can form a local pocket in the EoS where the superfluid correlations are strongest. We therefore made a calculation where in addition to the Brueckner approach
we included pairing correlations in the BCS approximation using the bare force in the gap equation. The procedure is the same as the one used in ref. [22]. Many-body correlations can alter substantially the value of the gap [23]. However, the issue is still controversial and we prefer to use the value calculated within the BCS approach with the bare interaction, keeping in mind that the results have to be taken as qualitative. In Fig. 5 we show a blow up of the EoS in the region close to zero binding with inclusion of the superfluid phase, again for the Argonne\textsubscript{14} potential. We see that indeed a broad depression around $\rho = \rho_0/5 - \rho_0/8$ is formed which is about 200 KeV deep. This means that nuclear systems with $\beta \approx 0.75$ at very low density should be stabilized around this range of density values. For example such a scenario may take place in the outer part of the skin of very neutron rich nuclei where indeed the $\beta$-values become very large and the neutron density drops to low values. Of course we are aware that such type of picture based on the LDA idea may have only qualitative value. Nevertheless it may shed light on the results of more quantitative structure calculations for finite nuclei.

6. Conclusions

In this paper we investigated within the Brueckner-Hartree-Fock approach nuclear and neutron matter at low density typically between one tenth and one half of the saturation value. At these densities the various realistic forces employed give practically identical results, and we may say that neutron and symmetric matter EoS are well established in this density domain. We also give the EoS as a function of asymmetry with the help of the well founded quadratic interpolation formula. We are particularly interested in the EoS for large asymmetry as they may be found in the surface tail of neutron rich nuclei but to a less extent also in stable nuclei.

One of the main purposes of the paper is to elucidate the microscopic origin of the recently discovered strong correlation between neutron rms radii of heavy nuclei and the EoS of neutron matter [4] and to what extent this correlation persists within a microscopic scheme other than semi phenomenological energy functionals. We first compared our microscopic EoS in the low density regime with the ones of various modern Skyrme and Gogny type phenomenological forces. Though there is qualitative agreement in the overall behaviour of the EoS there exist important local deviations. Our microscopic results may therefore serve to fine-tune future effective forces in the low density regime possibly improving in this way the surface properties of finite nuclei.
Then we constructed a realistic density functional based on the microscopic bulk EoS and applied it to a set of symmetric and non-symmetric nuclei. The outcome of this analysis can be summarized as follows. i) Some differences between the microscopic and the Skyrme or Gogny EoS appear at low density. In particular, for symmetric nuclear matter around $\rho \approx 0.02\text{ fm}^{-3}$ the microscopic EoS appears to have more binding than all the considered phenomenological ones. ii) It is possible to construct an energy functional from the microscopic EoS which is able to reproduce phenomenological data on nuclei at a level of precision comparable with the best Skyrme or Gogny forces. Only two parameters, the strength $\eta$ of the density gradient term and $W_0$ of the spin-orbit force, are adjusted. iii) It is found that the difference $r_n - r_p$ between the neutron and proton rms radii is only marginally sensitive to the parameter $\eta$. This justifies the possibility of a direct link between this radii difference and the bulk nuclear matter EoS. iv) For the microscopic functional, it turns out that the value of the radii difference (in the lead nucleus) and the slope of the symmetry energy in nuclear matter at the reference density $\rho = 0.1\text{ fm}^{-3}$ are in agreement with the linear correlation found previously for the phenomenological effective forces. The microscopic prediction is close to the Skyrme or Gogny cases, while it differs substantially from the predictions based on relativistic mean field approximation. Therefore, an accurate enough measurement of the neutron radius in $^{208}\text{Pb}$ could give direct informations on the bulk nuclear matter EoS via the neutron skin $r_n - r_p$.

We have also considered the nuclear matter EoS at different asymmetry values. For the strong asymmetry around $\beta \approx 0.75$ we found as a particular feature that the microscopic EoS is practically constant over a quite large range of low density values. Switching on neutron pairing known to have a strong maximum at low density, we find that a local minimum in the EoS at $\beta \approx 0.75$ develops for densities of $\rho_0/5$ to $\rho_0/8$. This local pocket, about 200 keV deep, may have an important bearing for the extra binding of neutrons in the far tail of neutron rich nuclei and eventually in the formation of neutron halos.

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Appendix
In this Appendix we give details on the explicit form of the microscopic energy functional discussed in the paper. The bulk term of the energy functional is the sum of the free kinetic energy and of the correlation energy part \( U(\rho) \). The latter, if we put \( x = \rho/\rho_0 \), with \( \rho_0 = 0.155 \text{ fm}^{-3} \) (see Sec. 4), is given by (energy per particle in MeV)

\[
U(\rho) = \begin{cases} 
\sum_{n=1}^{6} b_n x^n & x < 1 \\
 a_1 \cdot (x - 1) + a_2 \cdot (x - 1)^2 & x > 1
\end{cases} 
\]  

(1)

with the coefficients \( b_1 = -116.734211421291, b_2 = 310.796396961038, b_3 = -584.213036193276, b_4 = 587.659458730963, b_5 = -290.107854880778, b_6 = 55.5404612395750 \); and \( a_1 = -14.4391981216314, a_2 = 16.1424105528328 \).

The two forms match at \( x = 1 \) (\( \rho = \rho_0 \)) up to the second derivative. This functional form must be used up to \( \rho = 0.2 \text{ fm}^{-3} \), which is the interval where the fit has been performed.

For the pure neutron matter case we have a similar expression, just a polynomial in \( x \),

\[
U(\rho) = \sum_{n=1}^{5} b_n x^n
\]

(2)

with \( b_1 = -40.1046417029240, b_2 = 41.3865460399823, b_3 = -32.1381502519477, b_4 = 15.1308678586138, b_5 = -2.71394288779458 \), which is valid in the same density interval. The symmetry energy can be taken as the difference between the two functionals, including the kinetic energy parts (see the text).
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Table 1: Binding energy per particle and charge root mean square radius of some magic nuclei obtained with several values of the parameter $\eta$ (see text). The same values computed with the $SkM^*$ force and the experimental ones are also given.

|        | $\eta = -50$ | $\eta = -53$ | $\eta = -55$ | $SkM^*$ | $Exp$ |
|--------|--------------|--------------|--------------|---------|-------|
|        | $E/A$ | $r_{ch}$ | $E/A$ | $r_{ch}$ | $E/A$ | $r_{ch}$ | $E/A$ | $r_{ch}$ |
| $^{16}\text{O}$ | -8.25 | 2.82 | -8.10 | 2.83 | -8.01 | 2.84 | -7.98 | 2.81 | -7.98 | 2.73 |
| $^{40}\text{Ca}$ | -8.63 | 3.52 | -8.52 | 3.54 | -8.45 | 3.55 | -8.53 | 3.52 | -8.55 | 3.49 |
| $^{48}\text{Ca}$ | -8.87 | 3.55 | -8.76 | 3.56 | -8.68 | 3.57 | -8.75 | 3.54 | -8.67 | 3.48 |
| $^{90}\text{Zr}$ | -8.73 | 4.31 | -8.64 | 4.32 | -8.58 | 4.33 | -8.70 | 4.30 | -8.71 | 4.27 |
| $^{208}\text{Pb}$ | -7.79 | 5.53 | -7.73 | 5.54 | -7.69 | 5.55 | -7.87 | 5.51 | -7.87 | 5.50 |
Table 2: Neutron $r_n$ and proton $r_p$ root mean square radii (in fm), total binding energy $E$ (in MeV) and the estimate of the mass formula surface energy coefficient $a_s$ (in MeV) of $^{208}$Pb obtained using several values of the $\eta$ parameter (in MeV $\cdot$ fm$^5$).

|          | $\eta = 0$ | $\eta = -53$ | $\eta = -68$ |
|----------|------------|--------------|--------------|
| $r_n$    | 5.454      | 5.648        | 5.696        |
| $r_p$    | 5.329      | 5.485        | 5.524        |
| $r_n - r_p$ | 0.125      | 0.163        | 0.172        |
| $E$      | -1891.14   | -1608.46     | -1547.64     |
| $a_s$    | 10.55      | 17.66        | 19.31        |

Figure captions

Fig. 1.- The energy per particle $E/A$ is plotted as a function of the density, both for pure neutron matter (upper part) and for symmetric nuclear matter (lower part). The symbols correspond to the Brueckner calculations with realistic forces, Argonne $v_{14}$ (diamonds), Argonne $v_{18}$ (small open circles which correspond to the polynomial fit) and Paris potentials (open squares). The lines correspond to phenomenological nucleon-nucleon forces, the SkM* (solid line) and the Sly4 Skyrme forces (short dashed line), and the Gogny force (long dashed line).

Fig. 2.- The symmetry energy at low density calculated in the Brueckner scheme and with phenomenological nucleon-nucleon forces. Same notation as in Fig. 1.

Fig. 3.- Equation of state at different asymmetries in the parabolic approximation for the symmetry energy and for the Argonne $v_{14}$ potential. The lowest and higher curves correspond to the nuclear and neutron matter EoS of Fig. 1 respectively. The other lines correspond to the EoS of asymmetric nuclear matter from asymmetry $\beta = 0.1$ to asymmetry $\beta = 0.9$ in steps of 0.05. Some values of the asymmetry are reported as label of the corresponding EoS.
Fig. 4.- Linear correlations between the neutron and proton radii and the slope (Mev fm$^{-3}$) of the symmetry energy at the density $\rho = 0.1$fm$^{-3}$. Squares from the top correspond to $NL1$ [27], $NL3$ [28], $G1$, $G2$ [29] and $Z271$ [30]. Triangles to Gogny forces $D280$, $D300$, $D250$, $D260$ $D1$ and $D1S$ [31]. Circles correspond to the Skyrme forces $SV$ [32], $SII$ [33], $SIV$ [32], $SkM$, $SkM^*$ [2], $SLy4$, $SLy5$ [1], $T6$ [34], $SGII$ [35], $SI$ [33], $SIII$ and $SVI$ [32].

Fig. 5.- Equation of state of nuclear matter at asymmetry $\beta = 0.75$ and with the inclusion of pairing energy.
