Structure of Abrikosov Vortices in SU(2) Lattice Gauge Theory

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(Received )

We calculate the electric flux and magnetic monopole current distribution in the presence of a static quark-antiquark pair for SU(2) lattice gauge theory in the maximal Abelian gauge. The current distribution confines the flux in a dual Abrikosov vortex whose core size is comparable to the flux penetration depth. The observed structure is described by a dual Ginzburg-Landau model.
I. INTRODUCTION

One explanation for the absence of free quarks is that the QCD vacuum naturally expels color-electric flux in a manner similar to the expulsion of magnetic flux in a superconductor. In this scenario the vacuum must contain objects that respond to a color-electric field by generating screening currents to confine the electric flux in a narrow tube similar to the Abrikosov flux tube produced in a superconductor. While this can be achieved in Abelian models by adding additional Higgs fields, in the dual superconductor model dynamically generated topological excitations (magnetic monopoles) generate the currents. In 4D U(1) lattice gauge theory there is considerable indirect evidence from “bulk properties” of the vacuum such as the monopole density, monopole susceptibility, and static quark potential that Dirac magnetic monopoles are associated with the phenomenon of confinement.

In a recent paper we reported the first direct evidence that the monopoles in U(1) lattice gauge theory actually react to the electric flux from a static quark-antiquark pair by producing a solenoidal current distribution to confine the flux to a narrow tube. In the confined phase the curl of the monopole current and the local electric field obey a dual version of the London equations for a superconductor. We also found evidence for quantization of the electric fluxoid. The flux tube that formed between a static $q\bar{q}$ pair was structureless with no “normal core” visible on the scale of a lattice spacing. In the deconfined phase none of this behavior was observed.

While the origin of confinement in U(1) lattice gauge theory is fairly clear, progress in understanding confinement in non-Abelian SU(N) theories has been slow. One promising approach is to fix the non-Abelian degrees of freedom in the maximal Abelian gauge, leaving a residual $U(1)^{N-1}$ gauge freedom, with $(N-1)$ species of $U(1)$ Dirac monopoles. The monopoles have been observed to be abundant in the confined phase and dilute in the (finite temperature) unconfined phase. We present here the first direct evidence that a dual Abrikosov vortex also forms in SU(2) lattice gauge theory with static quarks. We
investigate the structure of the Abrikosov vortex and find that the flux tube in the present case has a normal core of size comparable to the flux penetration depth. Our results are consistent with a dual version of the Ginzburg-Landau model of superconductivity [3]. A preliminary report of our work was presented at the LATTICE 92 conference [14].

II. SIMULATIONS

Our simulations were performed on a $13^3 \times 14$ lattice with skew-periodic boundary conditions. Each link from point $s$ in the $\mu$ direction carried an SU(2) element $U(s, \mu)$ and the plaquette operators $P_{\mu\nu}(s)$ were formed in the usual fashion as a directed product of link variables. We used a standard SU(2) Wilson action

$$S = \beta \sum_{s, \mu > \nu} \left( 1 - \frac{1}{2} \text{Re} \, \text{Tr} \, P_{\mu\nu}(s) \right).$$

(1)

We generated configurations distributed according to $\exp(-S)$ using a combination of Monte Carlo [15] and overrelaxation [16]. Simulations were performed for $\beta=2.4$ and 2.5 with an initial 1000 sweeps to thermalize followed by 50 sweeps between measurements.

We converted our configurations to the maximal Abelian gauge [8] by finding the gauge transformation that maximized the quantity [9]

$$R = \sum_{s, \mu} \text{Tr} \left[ \sigma_3 U(s, \mu) \sigma_3 U^\dagger(s, \mu) \right],$$

(2)

which is equivalent to diagonalizing

$$X(s) = \sum_{\mu} \left[ U(s, \mu) \sigma_3 U^\dagger(s, \mu) + U(s - \mu, \mu) \sigma_3 U^\dagger(s - \mu, \mu) \right]$$

(3)

at each site $s$. To measure the gauge fixing in the simulations we used the lattice sum of the magnitude of the offdiagonal component of $X(s)$, $|Z|^2 = \sum_s |X(s)_{12}|^2$. Typically we needed about 650 gauge fixing sweeps to attain $|Z|^2 \approx 10^{-5}$/site for $\beta = 2.4$. Three gauge fixing methods were used: (1) generating and accepting random local changes only if $R$ increased, (2) locally maximizing $R$ exactly at alternate sites, (3) applying overrelaxation using the square of the gauge transformation of method (2) to sample configurations better.
III. MEASUREMENTS

After gauge fixing, the Abelian U(1) link variable is given \([9]\) by the phase of the diagonal component of the SU(2) link variable, \(u(s, \mu) = U(s, \mu)_{11}/|U(s, \mu)_{11}|\). We construct the Abelian plaquette variables \(p_{\mu \nu}(s)\) from the \(u(s, \mu)\) in the usual fashion. The one important difference from our U(1) study is that we have no local U(1) action here, so we cannot identify the magnitude of the Abelian charge \(e\). In the U(1) case we divided the plaquette by \(e\) in order to get the electric field \(\vec{E}\), and we multiplied the integer valued lattice operator for \(\vec{\nabla} \times \vec{J}_M\) by \(e_M = 2\pi/e\) to normalize the magnetic current. In the present case we determine instead the electric “force” \(e\vec{E}\) and the magnetic number current \(2\pi\vec{J}_M/e_M = e\vec{J}_M\) as variables here, so both quantities are known only up to an overall common factor. We shall continue to refer to them as the electric field and magnetic current in keeping with our previous analysis.

Since the electric field and monopole current operators are vectors, they will average to zero unless they are correlated with a Wilson loop \(W\) representing the current from a \(q\bar{q}\) pair; a \(3 \times 3\) loop in the \(z-t\) plane was used in the simulations. Therefore, averages of observables \(\Theta\) are computed as \(\langle \Theta \rangle \equiv \text{Tr}\{\exp(-S)W\Theta\}/\text{Tr}\{\exp(-S)W\}\).

The electric field operator is given by \(a^2\mathcal{E}_\mu = \text{Im } p_{\mu \nu}\), where \(a\) is the lattice spacing. With our choice for the orientation of the Wilson loop only the \(z\)-component of the electric field has a nonzero average. In Fig. 1(a) we show the operator for the electric field \(\mathcal{E}_z\), given by a \(z-t\) plaquette, as a bold line for fixed time.

The magnetic monopoles are identified using the DeGrand-Toussaint \([4]\) construction. It is convenient to associate the monopole current density in each three-volume with a link on the dual lattice, making world lines that define a conserved current density \(\vec{J}_M\). In Fig. 1(b) we show the three-volumes as squares since the time dimension is suppressed, and through the center of each square is the dual link associated with \(\vec{J}_M\). In order to isolate the solenoidal monopole currents, we construct the operator for the line integral of \(\vec{J}_M\) around a dual plaquette, \(\vec{\nabla} \times \vec{J}_M\), from the four three-volumes (squares) shown in Fig. 1(b). Notice
from this construction that $\vec{E}$ and $\nabla \times \vec{J}_M$ take values at the same location within the unit cell of the lattice, indicated by the bold face line in Fig. 1(b).

IV. ANALYSIS

In our previous work [7], we found that the confined phase of 4D $U(1)$ lattice gauge theory exhibited a response to a static $q\bar{q}$ pair that could be described in terms of a dual version of the London theory [3]. In this model, one combines the dual version of Ampere’s law

$$-c\nabla \times \vec{E} = \vec{J}_M,$$

(4)

with the dual London equation governing the generation of the persistent currents in the monopole condensate

$$\nabla \times \vec{J}_M = \frac{c}{\lambda^2} \vec{E},$$

(5)

where $\lambda = (mc^2/n_s e^2)^{1/2}$ is the London penetration depth, with $e$, $m$ and $n_s$ the charge, mass and number density of the monopole condensate. The dual London theory also predicts, based on the single-valuedness of the condensate order parameter, the fluxoid quantization relation

$$\int \vec{E} \cdot d\vec{S} - \frac{\lambda^2}{c} \oint \vec{J}_M \cdot d\vec{S} = n\Phi_e,$$

(6)

where $n$ is an integer and $\Phi_e = e/\sqrt{\hbar c}$ is the quantum of electric flux.

While this model explained our $U(1)$ results, our results for $SU(2)$ do not obey Eqn. (5). Figure 2 shows the result of 480 measurements at $\beta=2.4$ of $E_z$ and $(\nabla \times \vec{J}_M)_z$ measured midway between the $q\bar{q}$ pair. Since $-(\nabla \times \vec{J}_M)_z$ has a positive value at one point off axis, there is clearly no linear combination of the off axis data for $E_z$ and $(\nabla \times \vec{J}_M)_z$ that will satisfy Eqn. (6).

Therefore, we have adopted a dual form of Ginzburg-Landau (G-L) theory [3], which generalizes the London theory to allow the magnitude of the condensate density to vary in
space. As before, the superconducting order parameter is a complex function $\psi(\vec{x})$, where $|\psi(\vec{x})|^2$ is the condensate density. We define $\psi(\vec{x}) = \sqrt{n_s} f(\vec{x}) \exp(i\alpha(\vec{x}))$, where $n_s$ is the London (bulk) condensate density, and $f$ and $\alpha$ are real functions describing the spatial variation of the condensate; the London model presumes $f \equiv 1$. The characteristic scale over which the condensate density varies is $\xi$, the G-L coherence length.

The dual London Eqn. (5) is replaced with a dual version of the G-L equations

\begin{align}
\vec{E} &= \vec{\nabla} \times \vec{A}_E \\
\vec{J}_M &= \frac{c}{\lambda^2} f^2 \left( \vec{A}_E - \frac{\Phi_e}{2\pi} \vec{\nabla} \alpha \right)
\end{align}

and the condensate density obeys

\begin{equation}
0 = -\xi^2 \nabla^2 f + \xi^2 \left( \vec{\nabla} \alpha - \frac{2\pi \Phi_e}{\Phi} \vec{A}_E \right)^2 f - f + f^3,
\end{equation}

while the fluxoid quantization relation of Eqn. (6) is generalized to

\begin{equation}
\int \vec{E} \cdot d\vec{S} - \frac{\lambda^2}{c} \oint \frac{1}{f^2} \vec{J}_M \cdot d\vec{\ell} = n\Phi_e
\end{equation}

The dual Abrikosov vortex for a flux tube along the $z$ axis is a solution to Eqns. (4), (7) and (8) where the GL order parameter in polar coordinates varies as $\psi = f(r) e^{i\theta}$ with $f(r)$ given approximately by

\begin{equation}
f(r) = \tanh(0.9r/\xi).
\end{equation}

To fit our data, we chose an analytic function for the azimuthal component of the monopole current of the form

\begin{equation}
J_M(r) = \frac{a_1}{r} \left[ (1 + a_2 r) e^{-a_2 r} - (1 + a_3 r) e^{-a_3 r} \right]
\end{equation}

with $a_1$, $a_2$, and $a_3$ as fitting parameters. The form of Eqn. (11) was simply chosen to ensure that $J_M(r)$ vanished linearly at $r = 0$ and faster than $1/r$ as $r \to \infty$. The curl of the monopole current is then found from $(\vec{\nabla} \times \vec{J}_M(r))_z = (1/r)d(rJ_M)/dr$ and the parameters where fixed by fitting the data for the curl of the monopole current. The electric field data
was then fit using Eqns. (10) and (11) in Eqn. (7). We excluded the point at $r = 0$ in the fit to $E_z(r)$ since we are using a continuum approximation for the electric field to fit lattice data for the electric flux through a plaquette, and at the origin our continuum approximation to $E_z(r)$ diverges while the electric flux though the plaquette is still finite.

We evaluate the fluxoid $\Phi_e$ by two distinct methods. The first identifies $\Phi_e$ with $\Phi_{tot}$, the net flux through the entire lattice. The second method comes from evaluating the fluxoid relation on the central plaquette ($r = 0$). If we interpret the data point $E_z(0)$ as actually representing the flux through a circle of unit area (radius $1/\sqrt{\pi}$), Eqn. (??) gives

$$\Phi_e \approx \left[ E_z(0) + \frac{\lambda^2}{c} \frac{2\pi r J_M(r)}{f^2(r)} \right]_{r=1/\sqrt{\pi}}$$  

(12)

The curves in Figs. 2(a) and 2(b) represent our fit to the data for $\beta=2.4$. The resulting current distribution as a function of the distance from the $q\bar{q}$ axis is shown in Fig. 3. The fitted values we obtained are $\lambda/a = 1.05 \pm 0.12$ and $\xi/a = 1.35 \pm 0.11$. This latter result agrees with the naive estimate that $\xi$ is the distance where $\vec{\nabla} \times \vec{J}_M$ changes sign, i.e., $\approx 1.5a$. The fluxoid found from Eqn. (12) was $0.16 \pm 0.04$, while that found from the total flux was $\Phi_{tot} = 0.176 \pm 0.003$. The errors quoted are the statistical variance in the fitting parameters when the 480 measurements were divided into 4 sets of 120 points and fitting each set independently. For $\beta = 2.5$ we accumulated 208 measurements, which were grouped into 4 independent sets of 52 measurements for analysis. We found a penetration depth $\lambda/a = 1.59 \pm 0.31$, a coherence length $\xi/a = 1.16 \pm 0.08$, and two flux quantum estimates as $\Phi_e = 0.18 \pm 0.03$ and $\Phi_{tot} = 0.253 \pm 0.005$.

V. CONCLUSIONS

From this study we see that the dual superconductivity of the vacuum in SU(2) has a richer spatial structure than that seen in U(1), with a clearly apparent normal core region in the flux tube. The value of $\xi$ measures in effect the thickness of the boundary between the external, confining vacuum and the deconfined interior of a hadron. It will be interesting to investigate the structure of the flux tube in SU(3) lattice gauge theory.
The values we find for $\xi$ and $\lambda$ explain results obtained by Ivanenko et al. [13] on the existence of monopoles in SU(2) lattice gauge theory. They found the string tension between a $q\bar{q}$ pair correlated not with monopoles found by the deGrand-Toussaint construction applied to elementary three-volumes, but rather to “extended” monopoles defined over several three-volumes. Since the correlation length $\xi$ measures the minimum length scale over which a well-defined monopole condensate exists, $\xi \approx a$ means the global monopole condensate is best measured over a larger region than an elementary three-volume.

The gross spatial structure of the flux tube is fixed by the dimensionless Ginzburg-Landau parameter $\kappa \equiv \lambda/\xi$. For a type II superconductor ($\kappa > 1/\sqrt{2}$) the flux tube is compact, while a type I superconductor ($\kappa < 1/\sqrt{2}$) has a highly ramified flux structure [7] in the form of corrugated cylinders or highly reticulated walls. Our result ($\kappa = 1.4 \pm 0.2$) is close to the borderline. We should point out that an analysis by Maedan et al. [18] of an effective Lagrangian for confinement yields a value of $\kappa \gg 1$ for SU(2). However, since our value for $\kappa$ depends on the size of the Wilson loop and on the scaling behavior of $\kappa(\beta)$, we cannot yet reliably compare our results to a continuum model. Further analysis using larger Wilson loops will be needed to definitively establish the magnitudes of $\lambda$ and $\xi$. Rather, the goal of this work was to find an operator $(\vec{\nabla} \times \vec{J}_M)$ that gave a clear signal of the nature of the confinement and showed the structure of the flux tube; an analysis of the electric flux alone does not elucidate this structure [19].

VI. ACKNOWLEDGMENTS

We thank A. Kronfeld, M. Polikarpov, G. Schierholz, T. Suzuki, R. Wensley, J. Wosiek and K. Yee for many fruitful discussions. R.W.H. and V.S. are supported in part by the U. S. Department of Energy under grant DE-FG05-91ER40617 and D.A.B. is supported in part by the National Science Foundation under grant No. NSF-DMR-9020310.
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FIGURES

FIG. 1. Operators for (a) electric field and (b) $\vec{\nabla} \times \vec{J}_M$ on a fixed time slice.

FIG. 2. Profile of (a) $E_z$ and (b) $\vec{\nabla} \times \vec{J}_M$ as a function of distance from the $q\bar{q}$ axis for $\beta = 2.4$, together with the best fit.

FIG. 3. Profile of the current distribution $\vec{J}_M$ derived from the fit to the data of Fig. 2.