Dynamical Supersymmetry Breaking

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Abstract

We review mechanisms of dynamical supersymmetry breaking. Several observations that narrow the search for possible models of dynamical supersymmetry breaking are summarized. These observations include the necessary and sufficient conditions for supersymmetry breaking. The two conditions are based on non-rigorous arguments, and we show examples where they are too restrictive. Dynamical effects present in models with product gauge groups are given special attention.
1 Introduction

Supersymmetry breaking is an inherent part of any realistic supersymmetric theory. One of the motivations for supersymmetry is that it stabilizes the ratio of the electroweak scale to the Planck scale against large radiative corrections. If supersymmetry is broken dynamically, logarithmic running of a gauge coupling would also provide an explanation for the smallness of the ratio of the electroweak and Planck scales.

Almost all phenomenologically viable models consist of two sectors: the Minimal Supersymmetric Standard Model (MSSM) and the supersymmetry breaking sector. The information about supersymmetry breaking is transmitted to the MSSM either by gravitational or by gauge interactions, or a combination thereof. In the case of gravity mediated supersymmetry breaking, the electroweak scale, $M_{\text{weak}}$, is proportional to $\frac{\Lambda_{SB}^2}{M_{\text{Planck}}}$, where $\Lambda_{SB}$ is the scale of supersymmetry breaking and $M_{\text{Planck}}$ is the Planck scale. When supersymmetry breaking is mediated by gauge interactions, $M_{\text{weak}} \propto \alpha^n \Lambda_{SB}$, where $\alpha$ is the structure constant of the relevant gauge group. In any scenario, the electroweak scale is tied to the supersymmetry breaking scale. If the scale of supersymmetry breaking is naturally small, so is the electroweak scale.

The scale of supersymmetry breaking can be small naturally because of the supersymmetric non-renormalization theorem \[1\]. Since the superpotential receives no radiative corrections, if supersymmetry is unbroken at the tree-level, it remains unbroken at any order of perturbation theory. Therefore, only non-perturbative effects can be responsible for dynamical supersymmetry breaking.

Consequently, analyzing models of dynamical supersymmetry breaking (DSB) requires knowledge of the non-perturbative behavior of supersymmetric gauge theories. Models of DSB have been around for over ten years \[2, 3, 4\]. These models relied on knowledge of only a small number of dynamical phenomena present in supersymmetric theories, or sometimes on plausibility arguments when one did not know the dynamics. After the giant leap in understanding of the low-energy behavior of supersymmetric theories \[5\], several new mechanisms and models of dynamical supersymmetry breaking have been found. The older models of DSB were analyzed again and there is now a lot more evidence for supersymmetry breaking in these theories.

In the next section, we discuss general conditions that help to identify potential candidate theories for DSB. Then, in Section \[3\] we give a summary of the results on the low-energy dynamics of supersymmetric QCD. We will also illustrate mechanisms of DSB in several simple examples in that section. In Section \[4\], we will explain how these mechanisms give rise to supersymmetry breaking in more complicated theories, like the ones with two non-abelian gauge groups. Afterwards in Section \[5\], we show examples when our general conditions for supersymmetry breaking do not need to be satisfied.
2 Some general arguments

In this section, we summarize both the necessary and sufficient conditions for dynamical supersymmetry breaking. It is necessary for the theories to be chiral to break supersymmetry dynamically \[3\]. It is sufficient for supersymmetry breaking that a theory without flat directions has a spontaneously broken global symmetry \[3, 4\]. We will also review an observation by Dine, Nelson, Nir and Shirman \[7\].

The Witten index \(\text{Tr}(-1)^F\) measures the number of bosonic states of zero energy minus the number of fermionic ones. Since unbroken supersymmetry implies that the vacuum energy is zero, the Witten index counts the difference between the number of supersymmetric bosonic and fermionic vacua. If the index is nonzero, then there are certainly supersymmetric vacua, so supersymmetry is preserved in the ground state. It turns out that pure Yang-Mills theories have a non-zero index \[6\].

The index does not change when the parameters of a theory vary continuously. If it is possible to write the mass terms for all matter fields in the theory, then all mass parameters can be adjusted to take large values. Consequently, all matter fields can be decoupled from the theory. The low-energy theory is pure Yang-Mills, which cannot break supersymmetry. We will therefore consider chiral theories as candidates for DSB. However, the index can change discontinuously when a change of parameters alters the asymptotic behavior of the theory. We will return to this possibility in Section 5.

We now turn to the second criterion for DSB. A generic supersymmetric gauge theory without tree-level superpotential has a large set of possible vacuum states. These are the points where the D-terms vanish, the so-called flat directions. The D-terms are

\[D^\alpha = \sum_i \Phi_i^\dagger T^\alpha \Phi_i, \tag{1}\]

where \(T^\alpha\) denotes the gauge generators in the representations under which the chiral superfields \(\Phi_i\) transform. Knowledge of the D-flat directions (or the classical moduli space) is the first step in analyzing any theory. Finding all values of \(\Phi_i\) that satisfy the equation \(D^\alpha \equiv 0\) for all \(\alpha\)'s is generally quite complicated. Some useful techniques were presented in Refs. \[2, 4\]. This difficult algebraic exercise can be circumvented by using a theorem on a one-to-one correspondence between flat directions and vacuum expectation values of gauge-invariant holomorphic polynomials \[8\]. Instead of solving \(D^\alpha \equiv 0\), one can find all independent gauge-invariant polynomials constructed from chiral superfields. Vacuum expectation values of the gauge-invariant polynomials parameterize the classical moduli space. The only difficulty in this approach is ensuring that one has a complete set of gauge invariants.

When the superpotential is added, some flat directions are lifted, meaning that the F-terms are usually non-zero along D-flat directions. It is crucial to know if all flat directions are lifted. The following procedure is useful. First, compute all F-terms by taking the equations of motion. Next, make the F-terms gauge invariant by all
possible contractions with chiral superfields. If it is possible to determine all gauge invariants by setting the F-terms to zero, then the F-terms vanish only at one point of the moduli space (usually the origin) and flat directions are lifted [8].

Let us consider an example: a theory with an $SU(3) \times SU(2)$ gauge group [4]. The field content of the theory is

|     | $SU(3)$ | $SU(2)$ |
|-----|---------|---------|
| $Q$ | □       | □       |
| $\bar{U}$ | □ | 1 |
| $\bar{D}$ | □ | 1 |
| $L$ | 1 | □ |

where $X$, $Y$ and $Z$ form a complete set of holomorphic gauge invariants. Take as superpotential

$$W = Q\bar{U}L.$$  

(3)

By setting the $\bar{U}$ equation of motion to zero and multiplying by $\bar{U}$ and $\bar{D}$ one obtains $X = 0$ and $Y = 0$. Similarly, the F-term for $L$ sets $Z$ to zero, thus the superpotential of Eq. 3 lifts all flat directions. We will return to this theory in the next section and describe its quantum mechanical behavior.

We are now ready to present the sufficient condition for supersymmetry breaking. Suppose a theory does not have flat directions either because it does not have any gauge invariants constructed from chiral superfields or because flat directions are lifted by appropriate choice of tree-level superpotential. If such a theory has a continuous global symmetry which is spontaneously broken, then supersymmetry is also spontaneously broken [3, 4]. A spontaneously broken global symmetry implies the presence of a Goldstone boson. In a supersymmetric theory, this Goldstone boson has to combine with another massless scalar particle to form a supersymmetric multiplet. Since we assume that there are no non-compact flat directions, then there cannot be another massless scalar in the theory. Hence, supersymmetry must be broken. We want to stress that this argument applies to any global symmetry, not necessarily an R-symmetry. R-symmetry is especially useful when the superpotential is a generic function of chiral superfields consistent with symmetries [9].

The program looks quite simple: take a chiral theory, then lift all flat directions without explicitly breaking all global symmetries. This, however, is more complicated than it seems. An interesting observation by Dine, Nelson, Nir and Shirman helps to find such theories. Suppose one knows a theory that breaks supersymmetry dynamically. Instead of the original theory consider a theory with the gauge group reduced to a subgroup. The matter fields are the same, except that the representations they transform in are obtained by decomposing the original representations into those of the subgroup. Such a theory is guaranteed to be anomaly-free just as the the original one was. Moreover, it is frequently possible to lift all flat directions while preserving a global symmetry. We should stress here that we do not imply a physical procedure...
of breaking the gauge group by the Higgs mechanism. Also, the superpotential in the new theory is not derived from the original one. The theory with reduced gauge group has fewer D-terms, while the same number of chiral superfields. Usually, lifting flat directions in the new theory requires additional terms in the tree-level superpotential.

It is easy to understand why this procedure is likely to yield new theories of DSB. Suppose one adds an adjoint chiral superfield to the theory. This field in the vector-like representation does not affect the Witten index. The gauge symmetry can be broken by arranging the superpotential for the adjoint, where gauge bosons corresponding to broken symmetries and uneaten fields from the adjoint become massive. This idea is quite difficult to carry out explicitly in general \[10\]. Depending on the chosen pattern of gauge symmetry breaking, the adjoint may not be sufficient, and one needs other fields in vector-like representations to achieve the breaking.

## 3 Basic mechanisms for DSB

The obvious question to be addressed next is what kind of non-perturbative phenomena can be responsible for spontaneous global symmetry breaking and as a result supersymmetry breaking. We first review the low-energy effects in supersymmetric QCD, and then illustrate how the non-perturbative dynamics can lead to DSB. Refs. \[5, 11\] contain a detailed review about supersymmetric QCD and analysis of other theories, as well as extensive collection of references.

Supersymmetric QCD is an \(SU(N_c)\) theory with \(N_f\) fields \(Q^i\) in the fundamental representation and \(N_f\) fields \(\bar{Q}_j\) in the antifundamental. The indices \(i, j = 1, \ldots, N_f\) denote flavor degrees of freedom, while the color indices are suppressed. Classically, the flat directions are parameterized by “meson” fields \(M^i_{j1} = Q_i Q^j\), and “baryon” fields \(B^i_{j1 \ldots i N_f} = \epsilon_i^{i1 \ldots i_{N_f}} Q^{i1} \ldots Q^{i_{N_f}}\). Baryon fields exist only when \(N_f \geq N_c\). The mesons and baryons are not independent. They obey constraints

\[
B^i_{j1 \ldots i_{N_f} - N_c} M^i_{j1} = 0, \quad \bar{B}^i_{j1 \ldots i_{N_f} - N_c} M^i_{j1} = 0, \\
M^i_{j1 \ldots i_{N_f}} \epsilon_{i1 \ldots i_{N_f}} \epsilon^{j1 \ldots j_{N_f}} = B^i_{j1 \ldots i_{N_f}} \bar{B}^i_{j1 \ldots i_{N_f}}.
\]

These constraints are easy to verify if we express them in terms of the underlying fields \(Q^i\) and \(\bar{Q}_i\). Vacuum expectation values (VEVs) of these gauge invariant polynomials, subject to constraints, describe the classical moduli space. The classical theory has a large set of degenerate vacuum states. For many of these states, the degeneracy is accidental—not protected by symmetries—and can be removed by quantum effects. The quantum-mechanical picture depends on the number of flavors \(N_f\) \[5\]. We explain briefly each interesting case.

When \(N_f < N_c\), non-perturbative effects generate a superpotential of the form

\[
W_{\text{dyn}} = \left( \frac{\Lambda^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)}.
\]

4
The scalar potential corresponding to this superpotential is inversely proportional to the VEVs of chiral superfields. Therefore, supersymmetric QCD has no stable vacuum state for $0 < N_f < N_c$. It is possible to lift flat directions by adding small mass terms and have a stable vacuum state. In the ground state all fields have VEVs, and the gauge group is broken to $SU(N_c - N_f)$. Even though flat directions are lifted by adding mass terms, supersymmetric QCD does not break supersymmetry since it is non-chiral.

The low-energy theory with $N_f = N_c$ confines. The physical degrees of freedom are $N_f^2$ mesons $M^i_j$ and baryons $B$, $\bar{B}$. Classically, these fields obey the constraint $\det M - B\bar{B} = 0$. In the quantum regime, the constraint is modified: $\det M - B\bar{B} = \Lambda^{2N_f}$, so the classical and quantum moduli spaces are different. It is important that the origin of the moduli space—where all fields have zero expectation values—does not belong to the quantum moduli space. Quantum mechanically, some of the fields necessarily have VEVs; consequently some global symmetries are always broken. The constraint can be implemented by including it with a Lagrange multiplier $\mu$ in the superpotential:

$$W = \mu(\det M - B\bar{B} - \Lambda^{2N_f}).$$

(6)

A theory with one more flavor $N_f = N_c + 1$ is also confining. In this case, the classical and quantum moduli spaces are identical. The gauge invariant operators $M$’s, $B$’s and $\bar{B}$’s correspond to physical degrees of freedom describing the theory at the origin. The mesons and baryons interact via a “confining superpotential”

$$W = \frac{1}{\Lambda^{2N_c-1}} \left( \bar{B}^i M^j_i B_j - \det M \right).$$

(7)

One of the consistency checks on the confining picture is that ’t Hooft anomaly matching conditions between the high and low-energy degrees of freedom are satisfied. In the previous confining case, $N_f = N_c$, anomalies do not match at the origin, which indicates that the origin is not part of the moduli space. However, anomalies match on the points that belong to the quantum moduli space.

For a larger number of flavors $N_c < N_f < 3N_c$, supersymmetric QCD is either in the free-magnetic or conformal phase. Its infrared fixed point can be described equivalently in terms of another theory—supersymmetric QCD with $N_f - N_c$ colors. The “dual” theory has also $N_f$ flavors of “magnetic quarks” $q_i$, $\bar{q}^i; N_f^2$ elementary gauge singlet “mesons” $\tilde{M}^i_j$ and the following superpotential

$$W = \tilde{M}^i_j \bar{q}^i q_i.$$

(8)

Gauge invariant operators of the original $SU(N_c)$ correspond to the gauge invariants of the dual $SU(N_f - N_c)$. The mesons $Q\bar{Q}$ are mapped into gauge singlet fields $\tilde{M}$, while baryon operators $Q^{N_c}$ are mapped into baryons $q^{N_f - N_c}$.

\footnote{The $SU(N_f) \times U(1)_B$ global symmetry preserved by the mass terms is not broken in the ground state.}
Supersymmetric QCD is infrared free for $N_f \geq 3N_c$ and has no interesting non-perturbative dynamics. We again refer the reader to the original papers [5] and the lecture notes [11] for more details.

Let us now make use of these results and observations from Section 2. The first mechanism which can break supersymmetry is the dynamically generated superpotential. We will analyze in detail the well-known 3-2 model [4], whose field content we already presented in the previous section. The model has two gauge groups: $SU(3)$ and $SU(2)$. Let us first assume that the dynamical scale of the $SU(3)$ interactions, $\Lambda_3$, is much larger than that of $SU(2)$. In this limit, non-perturbative effects of the $SU(2)$ dynamics can be neglected. $SU(2)$ gauge group makes the theory chiral and also lifts some flat directions. The $SU(3)$ theory is an example of supersymmetric QCD with $N_f < N_c$. It generates a dynamical superpotential $W_{\text{dyn}} = \frac{\Lambda^7_3}{QUQD}$. We have seen that the tree-level superpotential $W = QUL$ lifts all flat directions, it also preserves a $U(1)_R$ symmetry. Because of the dynamically generated superpotential, some fields get VEVs which break the R-symmetry. Thus, supersymmetry must be broken as well. One can check that the vacuum energy is non-zero when the F-terms are computed from the full superpotential

$$W = \frac{\Lambda^7_3}{QUQD} + QUL.$$  \hfill (9)

What happens in the 3-2 model when the $SU(2)$ dynamics is more important, that is when $\Lambda_2 \gg \Lambda_3$? The $SU(2)$ has four doublets, so it has a quantum modified constraint. We also expect supersymmetry to be broken. All global symmetries are preserved only at the origin of the moduli space. Since in the quantum theory the origin does not belong to the moduli, some global symmetries and also supersymmetry can be broken [12]. Again, one can explicitly check that the full superpotential

$$W = \mu \left[ \text{Pf}(QL)(QQ) - \Lambda^4_2 \right] + (QUL)$$  \hfill (10)

breaks supersymmetry. We indicated degrees of freedom confined by the $SU(2)$ dynamics in parenthesis, they are the physical fields. The description of the 3-2 model can be found for arbitrary ratio $\Lambda_3/\Lambda_2$ [12]. The model breaks supersymmetry in the whole region of parameter space, as expected from arguments about spontaneously broken global symmetries in the absence of flat directions.

Not only the dynamically generated superpotential or the quantum modified constraint that can lead to DSB. Confinement can also cause supersymmetry breaking. When a theory confines the physical degrees of freedom are gauge-invariant fields. The low-energy description of the theory has to be written in terms of the confined fields. If the theory has a tree-level superpotential added to lift flat directions, after confinement the superpotential has to be re-expressed in terms of the new physical fields. Because of that, the low-energy theory can have the form of an O’Raifeartaigh model.
Let us consider an $SU(2)$ theory with one field $Q$ in the three-index symmetric representation of $SU(2)$ [13]. The authors of Ref. [13] argued that this theory confines at low energies and there is one light confined field $T = Q^4$. Global symmetries of the theory do not allow a dynamically generated superpotential. The tree-level superpotential $W = Q^4$ lifts the flat directions. In the infrared, this superpotential should be written as $W = T$, which breaks supersymmetry. It is important that the Kähler potential for the $T$ field is not singular. Classically, the Kähler potential expressed as a function of the invariant $Q^4$ has a singularity at the origin.

Another example is perhaps more illustrative. We consider an $SU(7)$ theory with two antisymmetric tensors $A^i$ and six antifundamentals $\bar{Q}_a$ [14]. Both $i = 1, 2$ and $a = 1, \ldots, 6$ are flavor indices. This theory behaves like supersymmetric QCD with $N_f = N_c + 1$; it confines and has a confining superpotential. There are two kinds of confined degrees of freedom: $H = A\bar{Q}^2$ and $N = A^4\bar{Q}$, in terms of which the confining superpotential is $W = \frac{1}{\Lambda^2}(N)^2(H)^2$. This theory is chiral and it is possible to lift all the flat directions with the following superpotential

$$W = A^1\bar{Q}_1\bar{Q}_2 + A^1\bar{Q}_3\bar{Q}_4 + A^1\bar{Q}_5\bar{Q}_6 + A^2\bar{Q}_2\bar{Q}_3 + A^2\bar{Q}_4\bar{Q}_5 + A^2\bar{Q}_6\bar{Q}_1.$$  \hspace{1cm} (11)

After confinement, the tree-level terms turn into linear terms, and the full superpotential is

$$W = H_{12}^1 + H_{34}^1 + H_{56}^1 + H_{23}^2 + H_{45}^2 + H_{61}^2 + \frac{1}{\Lambda^3}(N)^2(H)^2.$$  \hspace{1cm} (12)

Since at low energies fields $H$ and $N$ are to be interpreted as elementary degrees of freedom, this theory takes the form of an O’Raifeartaigh model. The tree-level linear terms force VEVs for some fields. These VEVs break global symmetries and supersymmetry turns out to be broken as well [14].

Another interesting example of theory that breaks supersymmetry because of strong dynamics is an $SU(5)$ theory with one antisymmetric tensor and one antifundamental [3]. This theory has no flat directions since there are no invariants that can be constructed out of single $10$ and single $\bar{5}$ of $SU(5)$. This theory has two $U(1)$ symmetries. The authors of Ref. [3] argued that one of these symmetries must be broken in the ground state. Thus, supersymmetry is broken as well since the theory has no flat directions. The theory has been analyzed recently by adding fields in vector-like representations. It is then possible to find low-energy description of these $SU(5)$ theories with extra fields. Theories with mass terms for the additional fields break supersymmetry [15]. A similar theory is $SO(10)$ with one spinor field. It also does not have flat directions and breaks supersymmetry [3]. This model has also been studied with larger matter content and appropriate mass terms [16].

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2In fact, since $T$ is a free field this model has an accidental $U(1)$ symmetry. A linear combination of this accidental symmetry and the R-symmetry is preserved by superpotential. If $T$ has non-zero VEVs at the ground state, the $U(1)$ symmetry is spontaneously broken.
Note that the 3-2 model can be obtained from the $SU(5)$ theory with $10$ and $\bar{5}$ by decomposing $SU(5)$ into its $SU(3) \times SU(2)$ subgroup. This is an illustration of the observation by Dine, Nelson, Nir and Shirman explained in the previous section. Other possible decompositions of the $SU(5)$ theory like $SU(4) \times U(1)$ or $Sp(4) \times U(1)$ also break supersymmetry. It is interesting that these theories are chiral only because of the $U(1)$ charge assignment.

In the next section we will analyze theories whose field content is obtained by decomposing an $SU(N)$ theory with an antisymmetric tensor $A$ and $N - 4$ antifundamentals $\bar{F}_i$. Let us outline the mechanism of supersymmetry breaking in the $SU(N)$ theory with an antisymmetric tensor and $N - 4$ antifundamentals. Without tree-level superpotential this theory has flat directions described by the gauge invariants $A\bar{F}_i\bar{F}_j$. Along a generic flat direction, the $SU(N)$ gauge group is broken to $SU(5)$. The uneaten fields are $10$ and $\bar{5}$ of $SU(5)$. The vacuum energy in the $SU(5)$ theory is proportional to the dynamical scale of $SU(5)$: $E_{\text{vac}} \propto \Lambda_5$.

When $SU(N)$ is broken to $SU(5)$ by VEVs of order $\langle \phi \rangle$ the scales of $SU(N)$ and $SU(5)$ are related by matching:

$$\Lambda_5 = \Lambda_N^{(2N+3)/15} \langle \phi \rangle^{-(2N-10)/13}. \tag{13}$$

Here, $\langle \phi \rangle$ indicates a generic value of a VEV for either $A$ or $\bar{F}$. Therefore, the vacuum energy as a function of the VEVs is $E_{\text{vac}} \propto \langle \phi \rangle^{-(2N-10)/13}$. This resembles the situation in models with a dynamically generated superpotentials. The low-energy $SU(5)$ generates a potential which decreases to zero at large VEVs. When flat directions are lifted by the tree-level superpotential $W = \lambda^{ij}A\bar{F}_i\bar{F}_j$, the theory breaks supersymmetry. Here, $\lambda^{ij}$ is a matrix of rank $N - 5$.

## 4 Product groups

In this section we describe models obtained by decomposing the field content of the $SU(N)$ theory with an antisymmetric tensor and $N - 4$ antifundamentals into $SU(N - M) \times SU(M) \times U(1)$ subgroup. Depending on $N$ and $M$, the two gauge groups have different number of flavors. By adjusting $N$ and $M$ we can analyze theories in different phases.

$SU(N)$ theories with an antisymmetric tensor, $N - 4$ antifundamentals and some number of fundamental-antifundamental pairs behave very much like supersymmetric QCD. When $N_f < 3$, the theory generates a dynamical superpotential. (A superpotential is not generated when $N_f = 0$ and $N$ is odd, since all holomorphic gauge invariants vanish classically.) For $N_f = 3, 4$ the theory confines respectively with a quantum deformed moduli space or with a confining superpotential. When the number of flavors is larger than four the theory admits a dual description.

\footnote{These VEVs are related because of the D-flatness conditions.}
The fields of $SU(N)$ decompose under $SU(N - M) \times SU(M) \times U(1)$ as follows
\begin{align*}
A & \rightarrow A(\overline{\mathbb{1}}, 1)_{2M} + a(1, \overline{\mathbb{1}})_{2M-2N} + T(\mathbb{1} \overline{\mathbb{1}})_{2M-N}, \\
\bar{F}_i & \rightarrow \bar{F}_i(\overline{\mathbb{1}}, 1)_{-M} + \bar{Q}_i(1, \overline{\mathbb{1}})_{N-M},
\end{align*}
where the subscripts indicate the $U(1)$ charge. It is quite tedious to show that the following superpotential lifts all flat directions and preserves an R-symmetry
\begin{align*}
W_{\text{tree}} = A F_1 F_2 + \ldots + A F_{N-6} F_{N-5} + a Q_2 \bar{Q}_3 + \ldots + a Q_{N-5} F_1 + \\
T \bar{F}_1 \bar{Q}_1 + \ldots + T \bar{F}_{N-4} \bar{Q}_{N-4}.
\end{align*}
The field $T$ is the only field, which transforms under both gauge groups. Through that field the two groups can affect each other's dynamics.

Models obtained by decomposing $SU(N)$ to $SU(N - 1) \times U(1)$ and $SU(N - 2) \times SU(2) \times U(1)$ were analyzed in Ref. [7]. In both cases the $SU(N - 1)$ or $SU(N - 2)$ groups generate a dynamical superpotential. Not surprisingly, these theories break supersymmetry when flat directions are lifted [7]. Let us examine in detail the decomposition of $SU(7)$ to $SU(4) \times SU(3) \times U(1)$ [17]. When the gauge groups are analyzed independently, $SU(4)$ has an antisymmetric tensor and three flavors. It confines with a quantum modified constraint. Because the modified constraint breaks the R-symmetry, one expects supersymmetry to be broken in the limit $\Lambda_4 \gg \Lambda_3$.

We will analyze the theory in the opposite limit $\Lambda_4 \ll \Lambda_3$ and show that supersymmetry is broken as well. Above the scale $\Lambda_4$, $SU(4)$ is still weakly gauged and we can neglect its non-perturbative dynamics. Below $\Lambda_3$, the $SU(3)$ confines since it is a supersymmetric QCD with $N_f = N_c + 1$. We have to describe the physics in terms of confined mesons and baryons. These are
\begin{align*}
B = (T^3), \quad 2 \cdot \bar{B} = (a \bar{Q}^2), \quad \bar{B} = (\bar{Q}^3), \quad 3 \cdot M = (T \bar{Q}), \quad M = (Ta)
\end{align*}
Note that in the case of $SU(3)$ the field $a$ transforms as an antifundamental, just like the $\bar{Q}$'s. Since the underlying fields transform under $SU(4)$, some of the composites objects carry $SU(4)$ quantum numbers. In particular, $\bar{B}$ transforms as an antifundamental of $SU(4)$, $M$'s as fundamentals while $\bar{B}$'s are singlets.

After confinement of $SU(3)$, the field content of the $SU(4)$ group has changed. The $SU(4)$ has now one more flavor. Above the $SU(3)$ confining scale, the $SU(4)$ gauge group has three flavors in addition to an antisymmetric tensor. Below $\Lambda_3$, there are four flavors: four fundamentals $M$ and four antifundamentals $\bar{F}_i$ and $B$. The effective $SU(4)$ theory below $\Lambda_3$ is an analog of supersymmetric QCD with $N_f = N_c + 1$ and not $N_f = N_c$. It confines and has a confining superpotential. When we go below the new effective scale $\Lambda_4$, we have to again change the description into gauge invariant

\footnote{This R-symmetry is anomalous with respect to the $U(1)$ gauge group. However, the Goldstone boson resulting from the spontaneous breaking of this symmetry is massless. Therefore, the argument for supersymmetry breaking when a global symmetry is broken still holds.}
fields. It turns out that by simply adding the confining superpotentials obtained from $SU(3)$ and $SU(4)$ gauge dynamics one obtains a correct description of the theory for any ratio $\Lambda_3/\Lambda_4$ \cite{17}. For instance, one of the equations of motion properly reproduces the quantum modified constraint expected in the $\Lambda_4 \gg \Lambda_3$ limit. When the tree-level superpotential of Eq. (13) is added

$$ W = W_{\text{conf}}^{SU(3)} + W_{\text{conf}}^{SU(4)} + W_{\text{tree}}, $$

the low-energy theory is an O’Raifeartaigh model and breaks supersymmetry. Again, the tree-level terms become linear and force VEVs for some fields. The $U(1)$ gauge group does not play any dynamical role, its sole purpose is to lift some flat directions. A model with similar dynamics based on the $SU(3) \times SU(2)$ gauge group was analyzed in Ref. \cite{12}.

In the above example we saw that the dynamics of one gauge group affected dynamics of the other—it changed the effective number of flavors. There is, however, another way to see that supersymmetry is broken in the $\Lambda_4 \ll \Lambda_3$ limit, which will be essential for other examples. So far, we have first analyzed the theory without including the tree-level superpotential. We found the low-energy description without superpotential, then added the superpotential and checked if supersymmetry is broken. The tree-level superpotential may play an important dynamical role, it can alter the infrared behavior of the theory.

Suppose we included the superpotential of Eq. (13) from the beginning. We again study the theory in the limit $\Lambda_4 \ll \Lambda_3$. After confinement in $SU(3)$, the superpotential has to be expressed in terms of $SU(3)$ invariants. In particular, all terms of the form $T\bar{F}_i\bar{Q}_j$ will become mass terms $M\bar{F}$, where $M = (T\bar{Q})$ transforms as a fundamental of $SU(4)$. The mass parameters associated with these terms are $\lambda\Lambda_3$, where $\lambda$ is a Yukawa coupling. For brevity, Yukawa couplings were not explicitly specified in Eq. (13). If $\lambda\Lambda_3 > \Lambda_4$, we need to integrate out the massive fields before we take into account the $SU(4)$ dynamics. In the tree-level superpotential there are mass terms for three flavors of $SU(4)$. After integrating them out we obtain an $SU(4)$ theory with an antisymmetric tensor and one flavor, which generates a dynamical superpotential. The dynamically generated superpotential is the reason for supersymmetry breaking in this regime. Of course, this limit could have been recovered from the exact description of the theory without superpotential. It is sometimes easier to analyze the theory taking the tree-level superpotential into account.

We now consider another example obtained by decomposing $SU(N+4)$ theory with an antisymmetric tensor and $N$ fundamentals into $SU(N) \times SU(4) \times U(1)$ subgroup, $N \geq 5$ \cite{17}. When the two non-abelian gauge groups are examined independently, the $SU(N)$ is an analog of supersymmetric QCD with $N_f = N_c + 1$, it confines without modifying the moduli space. $SU(4)$ has an antisymmetric tensor and $N$ flavors. Depending on $N$, $SU(4)$ can be in the free-magnetic, conformal or even infrared-free phase. Naively, there is no mechanism that would cause supersymmetry breaking. It seems that the theory could have a vacuum state at the origin, where no
symmetries are broken and the tree-level superpotential vanishes.

We will investigate the theory in the limit $\Lambda_N \gg \Lambda_4$, and take the superpotential of Eq. [15] into account from the start. The analysis is in fact quite similar to the one of the $SU(4) \times SU(3) \times U(1)$ example. When $SU(N)$ confines, the $T\bar{F}_i\bar{Q}_i$ terms become mass terms $M_i\bar{Q}_i$. We can integrate out these terms and obtain an $SU(4)$ theory with an antisymmetric tensor and one flavor. Because of the dynamically generated superpotential in $SU(4)$, supersymmetry is broken.

These examples may seem to suggest that it is the confining dynamics that is crucial for supersymmetry breaking. Let us consider an $SU(N) \times SU(5) \times U(1)$ theory obtained by decomposing $SU(N+5)$ [17]. The $SU(N)$ gauge group can be given a dual description. In the dual, meson operators are mapped into elementary singlet fields. If we include the tree-level superpotential of Eq. [14], all $T\bar{F}\bar{Q}$ terms become mass terms $\tilde{M}\bar{Q}$. Here, $\tilde{M}$ singlets of the dual gauge group still carry $SU(5)$ charges, since $SU(5)$ was just a spectator in dualizing $SU(N)$. In the dual description, the effective number of $SU(5)$ flavors changes first as a result of duality transformation and second because of the mass terms from the tree-level superpotential. Effective $SU(5)$ theory again generates a dynamical superpotential which breaks supersymmetry.

These examples show that supersymmetry can be broken in product group theories in many cases where one would not expect appropriate dynamical effects to take place. Dynamics of one gauge group can affect the dynamics of other group. It is interesting that the tree-level superpotential is more important than simply lifting flat directions. It can change the phase of the theory. Many other examples of product group theories and theories utilizing dual gauge group dynamics were presented in Ref. [18, 19].

5 Other possibilities

In this section we point out that there can be non-chiral theories which break supersymmetry, and also that in certain cases it is not necessary to lift all classical flat directions. We first consider an $SU(2)$ theory with four doublets $Q_i$ and six singlets $S^{ij}$ [12, 20]. This theory has a global $SU(4)$ symmetry under which $Q_i$’s transform as a fundamental while $S^{ij}$’s as an antisymmetric tensor. The $SU(2)$ has the same number of flavors as colors, so it has a dynamically modified constraint. The tree-level superpotential, $W_{\text{tree}} = \lambda S^{ij}Q_iQ_j$, preserves the global $SU(4)$. This superpotential lifts flat directions associated with $Q$’s, however the singlet fields remain flat. In the quantum theory, the full superpotential includes the modified constraint in addition to the tree-level term,

$$W = \mu (\text{Pf } M_{ij} - \Lambda_2^4) + \lambda S^{ij}M_{ij},$$

(18)

where $M_{ij} = (Q_iQ_j)$. The equations of motion with respect to $S^{ij}$ set all $M$’s to zero, which is incompatible with the constraint. The sufficient condition for supersymmetry
breaking is not useful in this case. Quantum modified constraint breaks the global $SU(4)$ symmetry, but the theory has classical flat directions.

This $SU(2)$ theory is not chiral. The Witten index argument fails in this case because of flat directions associated with the $S$ fields. When a mass term $m \text{Pf} S$ is added, the asymptotic behavior of potential for the $S$ fields changes. The form of the potential at infinity changes discontinuously when $m$ becomes non-zero. In fact, the theory with mass terms has supersymmetric vacua at expectation values of $S$ proportional to $\frac{\Lambda^2_2}{m}$ [12]. These supersymmetry-preserving vacua disappear from the theory when $m$ is zero.

It turns out that classical flat directions associated with $S^{ij}$ are not flat when quantum corrections are taken into account [21]. In the limit where $\lambda \langle S \rangle > \Lambda_2$ one finds that the vacuum energy is proportional to $\sqrt{\lambda} \Lambda_2$. When the one-loop running of the Yukawa coupling $\lambda$ is examined, one finds that $\lambda$ as a function of $S$ has a minimum. Therefore, the full scalar potential for $S$ is not flat when the deviation of the Kähler potential from its canonical form is included.

The $SU(2)$ theory illustrates another important point: supersymmetry can be broken even when the classical theory has flat directions. This can happen if along the flat directions the gauge group is not completely broken. In the $SU(2)$ example, VEVs of the singlet fields obviously do not break the gauge group. One is usually worried that flat directions may lead to runaway theories without vacuum state. If the gauge group is completely broken, the larger the VEVs are, the weaker the dynamical effects become. When there is an unbroken subgroup, its effective scale can grow with the vacuum expectation values and lift the flat direction quantum mechanically [21].

In the $SU(2)$ theory considered above, when $\lambda \langle S \rangle$ is large, one can integrate out the doublets and obtain a pure Yang-Mills theory with a low-energy scale $\Lambda_{2,L} = (\text{Pf} S)^{1/6} \Lambda_2^{2/3}$. The scale increases with the expectation values of $S^{ij}$'s. The same can happen in chiral theories. For example, the superpotential of Eq. [13] lifts all flat directions of $SU(N-M) \times SU(M) \times U(1)$ theories described in the previous section. The theory with all $a\bar{Q}_i \bar{Q}_j$ terms left out has classical flat directions. Along a generic flat direction $SU(N-M)$ is unbroken, while $SU(M)$ is completely broken. When fields $\bar{Q}$ get VEVs, $T F \langle \bar{Q} \rangle$ terms are mass terms for $SU(N-M)$ flavors. There are $M$ such mass terms, since $T$ is at most rank $M$. After integrating these out, the effective $SU(N-M)$ has an antisymmetric tensor and $N-M-4$ flavors. This is exactly of the form of the original theory from which $SU(N-M) \times SU(M) \times U(1)$ models were derived. Supersymmetry breaking for this theory was described at the end of Section [3]. The vacuum energy in the effective $SU(N-M)$ is proportional to $\Lambda_{N-M}$, which grows with expectation values of the $a\bar{Q}Q$ flat directions as a result of scale matching [21]. Therefore, these flat directions are lifted in the quantum theory and supersymmetry is broken.
6 Conclusions

We have described the basic facts about models of dynamical supersymmetry breaking. Chiral theories without flat directions in which global symmetries are spontaneously broken also break supersymmetry. The three dynamical mechanisms that can cause supersymmetry breaking are dynamically generated superpotential, quantum modified constraint and confinement. The picture is much more complicated in product group theories. The dynamics of one gauge group can affect the dynamics of the others, and the interplay among the dynamics of the product groups can be non-trivial. This interplay, together with the tree-level superpotentials, can change the effective number of flavors and yield theories which break supersymmetry, even in cases were one would not expect supersymmetry breaking. There exist examples of non-chiral theories or theories with flat directions, which nevertheless break supersymmetry.

We have focused on the technical aspects of finding theories which break supersymmetry. However, we have not addressed the most important problem, that is how to identify which theory will describe nature most accurately. So far, there is no answer to that question. The solution depends strongly on the mechanism of communicating supersymmetry breaking to the MSSM. Refs. [7, 22, 23] contain several recently proposed scenarios of gauge mediation. One cannot argue for or against a given model on the basis of simplicity. One may want to “unify” the DSB sector with the messenger sector and MSSM [22]. This requires theories with relatively large gauge groups.

We also have not discussed the properties of the supersymmetry-breaking ground state in presented models. We were satisfied with showing that the vacuum energy is not zero in each case. The particle spectrum and the unbroken symmetries at the ground state are important for communicating supersymmetry breaking to the visible sector. There are at least two obstacles in identifying the properties of the ground state. First, in many theories we do not know the Kähler potential. If the theory is weakly coupled, one can calculate the Kähler potential in perturbation theory. In some strongly coupled theories, one expects the Kähler potential to be canonical, up to a constant, near the origin. However, there is no systematic way of computing the corrections. The second difficulty is technical. Many of the analyzed theories contain a large number of fields. Finding the minimum for so many variables, even numerically, is not an easy task.

Hopefully, with so many recently constructed models of DSB, one of the theories is the right one. If not, the tools for building models we have already acquired are perhaps enough to find it.
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