An Algorithm for Selecting QGP Candidate Events from Relativistic Heavy Ion Collision Data Sample*

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Abstract

The formation of quark-gluon plasma (QGP) in relativistic heavy ion collision, is expected to be accompanied by a background of ordinary collision events without phase transition. In this short note an algorithm is proposed to select the QGP candidate events from the whole event sample. This algorithm is based on a simple geometrical consideration together with some ordinary QGP signal, e.g. the increasing of $K/\pi$ ratio. The efficiency of this algorithm in raising the 'signal/noise ratio' of QGP events in the selected sub-sample is shown explicitly by using Monte-Carlo simulation.

Key words: Relativistic heavy ion collision Signal of QGP formation Factorial moments

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Recently the lead-lead collision at incident energy as high as 160 A GeV has been performed at CERN-SPS. Within a few years gold-gold and lead-lead collisions at even higher energies will be realized in Brookhaven (RHIC) and CERN (LHC). At such ultra-relativistic energies the condition for the formation of a new state of matter — quark-gluon plasma (QGP) will most probably be achieved. In this case, our main concern will be turned from the question: “Whether QGP is formed” to the question: “How to identify the QGP events from the background”.

Owing to the highly complexity of ultra-relativistic heavy ion collision, it could be expected that even when the energy threshold for QGP phase transition is reached in an experiment, the thermalization and equilibrium phase transition may not be realized in every event. Complicated nuclear collision events without phase transition may still be present, forming a background for QGP formation. Any physical characteristic quantity, that may serve as QGP signal, after averaging over the whole event sample, will at least partly be smeared out. Even if we measure this quantity event by event and study their distribution, the peak from background events may disturb the observation of the peak from QGP events. Therefore, it is worthwhile to find some effective way for selecting the QGP events from the whole ultra-relativistic heavy ion collision data sample. The aim of this letter is to propose a possible algorithm for this purpose.

For concreteness, let us take the increasing of $K/\pi$ ratio as an example of QGP signal. As a crude estimate\cite{1} the QGP events might exhibit an almost twofold enhancement of the average $K/\pi$ ratio $\langle R_{K/\pi}\rangle$ at mid-rapidity from the value of 0.14 for the non-QGP events. The relative width $\sigma(R_{K/\pi})/\langle R_{K/\pi}\rangle$ of $K/\pi$ ratio distribution is estimated to be about 0.3 for CERN-SPS energy and higher. Let us assume the $R_{K/\pi}$ distributions for both 'non-QGP' and 'QGP' events be Gaussian with parameters:

$$\langle R_{K/\pi}\rangle^{\text{non-QGP}} = 0.14, \quad \langle R_{K/\pi}\rangle^{\text{QGP}} = 0.25, \quad \sigma(R_{K/\pi})/\langle R_{K/\pi}\rangle = 0.3.$$  \hspace{1cm} (1)

The corresponding distributions of 'non-QGP', 'QGP' and mixed events for three typical cases, in which the percentage of QGP events $\rho_{\text{QGP}}$ equal to 20%, 33% and 50% respectively, are shown in Fig.1.

It can be seen from the figure that the shape of $K/\pi$-ratio distribution for mixed events is mainly determined by the non-QGP events. Even when $\rho_{\text{QGP}} = 50\%$ the QGP events only appear as a shoulder in the distribution. This shoulder can be utilized in constructing a 'QGP event sub-sample'. Introducing a cut at $R_{\text{cut}} = 0.22$ and picking out the events with $R_{K/\pi} > R_{\text{cut}}$, a 'QGP event sub-sample' is obtained. It raises the percentage of QGP events from 50% to 95.5%, with the cost of losing
33.6% of QGP events.

However, when the percentage of QGP events $\rho_{QGP}$ is as low as 33% or 20%, no evident shoulder can be seen and the method of using $K/\pi$ ratio to select QGP events failed.

In order to increase the efficiency of QGP event selection, we make use of a simple geometrical consideration. As is well known, heavy ion collision at conventional energies can be regarded as the superposition of a large number of elementary collision processes. A necessary condition for the formation of QGP is the thermalization of the produced particles in individual elementary collision processes, forming a unified system. Therefore, if we could get a criterion to judge whether the final state particles are coming from a unique system or from a large number of sub-systems, we would be able to make use of this criterion to increase the purity of 'QGP event sub-sample'.

Such a criterion can be provided by the higher-dimensional factorial moment (FM) analysis$^{[2,3]}$. The $q$th order factorial moment $F_q$ is defined as$^{[4]}$

$$F_q(M) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)\cdots(n_m-q+1) \rangle}{\langle n_m \rangle^q}, \quad (2)$$

where a region $\Delta$ of phase space is divided into $M$ cells, $n_m$ is the multiplicity in the $m$th cell, and $\langle \cdots \rangle$ denotes vertically averaging over many events. It has been shown$^{[2]}$ that in the 2-D($\eta$, $\varphi$ or $y$, $\varphi$) and 3-D ($\eta$, $p_t$, $\varphi$ or $y$, $p_t$, $\varphi$) space, due to the superposition of particles coming from a large number of sub-systems, the log-log plot of FM versus $M$ for nucleus-nucleus collisions will be strongly bending upwards. On the other hand, if there is only one unique system the upward-bending of ln-FM versus ln-$M$ will disappear.

Basing on the above observation, we propose the following algorithm for selecting QGP events from the whole data sample.

1. Do the $K/\pi$ ratio $R_{K/\pi}$ and higher-dimensional second order factorial moment $F_2(M)$ analysis event by event simultaneously.

2. In order to get rid of the influence of transverse momentum conservation$^{[5]}$, throw away the point(s) in ln-$F_2$ versus ln-$M$ plot with $M_y < 2$. Denote the first of the remaining points as $M_0$. Change the origin of coordinate system to this point:

$$x = \ln M - \ln M_0, \quad y = \ln F_2(M) - \ln F_2(M_0). \quad (3)$$

Do a quadratic fit:

$$y = ax^2 + bx \quad (4)$$
and use the fitting parameter $a$ as a characteristic of the degree of upward-bending of \( \ln F_2 \) versus \( \ln M \).

(3) Draw the frequency of events as a function of their respective $R_{K/\pi}$ along with their upward-bending parameter $a$. Plot the projection distributions of $R_{K/\pi}$ and $a$.

Then the 'QGP event sub-sample' can be selected by the following ways.

Method A: If the distribution of $R_{K/\pi}$ has double peak or a peak with a shoulder, then take the boundary of the two peaks or that of the peak and the shoulder as $R_{K/\pi}^{\text{cut}}$. Form the 'QGP event sub-sample' from the events with $R_{K/\pi} > R_{K/\pi}^{\text{cut}}$.

Method B: If the distribution of bending parameter $a$ has double peak or a peak with a shoulder, then take the boundary of the two peaks or that of the peak and the shoulder as $a^{\text{cut}}$. Form the 'QGP event sub-sample' from the events with $a < a^{\text{cut}}$.

Method C: In the scattering plot and contour plot of event frequency as function of $R_{K/\pi}$ and $a$ try to find out the dense event region at small $R_{K/\pi}$ and large $a$. This dense region should mainly consist of non-QGP events. Identify the center of this region and draw an ellips with suitable long and short axes to cover this region as precisely as possible. Throwing away the events falling inside this ellips, a 'QGP event sub-sample' is obtained.

For illustration we have made a simple model\cite{ref2} to do Monte Carlo simulation. In this model a nucleus-nucleus collision event with $N$ elementary collisions is simulated by $N$ two-dimensional random cascading $\alpha$ models\footnote{The details of $\alpha$ model used here can be found in Ref.[7].}. Each $\alpha$ model corresponds to one of the $N$ elementary collisions. The investigation region of the nuclear collision is taken to be $(0, 1) \times (0, 1)$. The regions of the $N$ elementary collisions in direction $b$ ('transverse' direction) are all $(0, 1)$, coinciding with that of the nuclear collision, while the region in direction $a$ ('longitudinal' direction) for the $i$th elementary collision ($i = 1, \ldots, N$) is taken to be of width $1 + \omega$ with the center placed randomly at $( (1 - \omega)/2, (1 + \omega)/2 )$, where $0 < \omega \leq 1$ is a fixed parameter. The resulting particles from all the $N$ elementary $\alpha$ models falling in the region $(0, 1) \times (0, 1)$ are then superposed together to form the “nucleus-nucleus collision event”. The model parameters are taken to be $\alpha = 0.5$ and $\omega = 1$.

For a non-QGP nucleus-nucleus collision event the number $N$ of elementary collisions is taken to be 100, while for a QGP candidate event $N = 1$. The number of charged particles in a single event is $n_{\text{ch}} = 1000$.

In this way 2000 non-QGP events and the same number of QGP candidate events are mixed to form a sample with $\rho_{\text{QGP}} = 50\%$. Taking randomly 1000 and 500 QGP
candidate events from the 2000 events obtained above, mixing them with the 2000 non-QGP events, two samples with $\rho_{\text{QGP}} = 33\%$ and 20\% respectively are obtained.

The procedure 1–3 mentioned above are then done for the two samples with $\rho_{\text{QGP}} = 50\%$ and 20\% and the Methods A, B and C are used to get the 'QGP sub-samples'. The results are shown in Fig.s 1–3 and Table I.

**Table I** $\rho_{\text{QGP}}$ and percentage of lost QGP events in the 'QGP event sub-sample' obtained through three different methods.

| $\rho_{\text{QGP}}$ (original) = 50\% | $\rho_{\text{QGP}}$ (original) = 20\% |
|-------------------------------------|-------------------------------------|
| Method A | Method B | Method C | Method B | Method C |
| $\rho_{\text{QGP}}$ (sub-sample) | 95.5\% | 98.2\% | 98.6\% | 98.3\% | 94.7\% |
| Lost QGP events | 33.6\% | 3.8\% | 2.5\% | 7.4\% | 2.8\% |

The following observations can be made from the figures and Table I.

(1) As has been pointed out above, no evident shoulder can be seen in the $R_{K/\pi}$ distribution when the percentage of 'QGP' events $\rho_{\text{QGP}} \leq 30\%$ and the method (Method A) using $K/\pi$ ratio to select 'QGP' events failed in these cases. On the other hand, the method using the upward-bending parameter $a$ of $\ln-FM$ versus $\ln M$ (Method B) and the method using $R_{K/\pi}$ and $a$ simultaneously (Method C) can be applied to very low $\rho_{\text{QGP}}$ provided the number $N$ of elementary nucleus-nucleus collision is high enough (greater than 100 for example\[1\]).

(2) The percentage $\rho_{\text{QGP}}$ of QGP events in the 'QGP candidate sub-sample' is higher in Methods B and C than in Method A and the percentage of lost QGP events in the sub-sample is lower in Methods B and C than in Method A. This shows that Methods B and C are more efficient in selecting QGP events than Method A.

(3) The percentage $\rho_{\text{QGP}}$ of QGP events in the 'QGP candidate sub-sample' obtained by using 2-dimensional elliptical cut (Method C) is almost equal to that using 1-dimensional $a$ cut (Method B), but less 'QGP' events are lost.

In this letter the method for selecting QGP candidate events from the whole relativistic heavy ion collision event sample has been discussed in some detail. Basing on a simple geometrical consideration, the upward-bending of higher-dimensional factorial moment has been utilized to select QGP candidate events from the whole event sample. The algorithm using the upward-bending parameter $a$ and the one using $a$

\[2\]We have done simulation with different number $N$ of elementary collisions. As $N$ increases, the $a$ distribution moves to larger $a$ and becomes more and more sharp. The Methods B and C can safely be applied when $N \geq 100$. 

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and $R_{K/\pi}$ simultaneously have been shown to be more powerful and more efficient than the one using $R_{K/\pi}$ alone.

The algorithm proposed in this letter is primarily for the analysis of heavy ion collision data at CERN-SPS energy. It can also be applied to the analysis of the coming ultra-relativistic heavy ion collision data from Brookhaven-RHIC and CERN-LHC experiments. The comparison of the scattering plots of event frequency as function of $R_{K/\pi}$ and $a$ for different energy and/or different centrality of heavy ion collisions might provide useful information about the thermalization and phase transition of the system.

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Figure Captions

Fig. 1 Results of model calculation of $K/\pi$ ratio’ distribution for ‘non-QGP’, ‘QGP’ and mixed event sample. Dashed line is the $R(K/\pi)$ cut for selecting ‘QGP’ events.

Fig. 2 Simulation results of the distribution of upward-bending parameter $a$ of ln-FM versus ln$M$ for ‘non-QGP’, ‘QGP’ and mixed event sample. Dashed line is the $a$ cut for selecting ‘QGP’ events.

Fig. 3 The scattering plots and contour plots of event frequency as function of $R_{K/\pi}$ and $a$ for mixed event sample from model simulation. The dashed ellips is the cut for selecting ‘QGP’ events.
$K/\pi$ ratio

Fig. 1
Fig. 3