Estimation of turbulent diffusion transport in the boundary layer by the SIV method

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Abstract. The study aims at an experimental estimation of turbulent diffusion transport of Reynolds stresses in a developed turbulent zero-gradient boundary layer. Estimates were derived from dynamics of two-component instantaneous velocity vector fields measured by an Smoke Image Velocimetry (SIV) optical method. The obtained profiles were compared with DNS results at a similar value of Reₜ.

1. Introduction
To obtain a closed system of Reynolds-averaged Navier-Stokes equations (RANS), an approximation of the unknown terms of the conservation equations is necessary. Most of the approximation models are derived from the Navier-Stokes equations and contain the empirical parameters. One can say that these models are heuristic and not universal, and they do not obey the stringent requirements of Galilean and tensor invariance [1, 2]. The reason for heuristic models is the lack of information on the structure and kinematics of small-scale turbulent structures and the energy balance of turbulent pulsations.

The indirect methods to test the various closure models were used to calibrate them. This was due to the difficulty in measuring pressure and velocity with sufficient spatial resolution. To verify the correctness of the model and select the empirical parameter, the proposed model was used in the transport equations of turbulent fluctuations together with analogous models for other terms of the equation. The adequacy of the model was estimated by computing a flow with the model and by comparing the predicted mean velocity, Reynolds stresses and pressure coefficient with available experimental data [3-5]. Obviously, the probability of choosing the formulation of the model for a given term of the equation substantially depended on the correctness of the modeling of the remaining terms under this approach.

This paper is a follow-up of [6], where the planar field optical method of Smoke Image Velocimetry (SIV) [6, 7] has been employed to make an attempted estimation of the terms of the turbulent diffusion transport of the Reynolds stress equations in the boundary layer. The profiles of the statistical turbulent characteristics based on the results of experimental measurements have been obtained for the first time compared to the results obtained by just direct numerical simulation (DNS).
2. Experimental setup

The experimental setup and optical measurement method are described in [6], therefore this paper covers only the main details.

The air flow in a smooth plane channel has been studied. The test section was a rectangular 75x150 mm\(^2\) channel with the length of 1 m and a smooth inlet with 6:1 contraction. A turbulence generating grid with 5 mm cell size, 1.2 mm steel wire diameter and 36% solidity was mounted downstream of the smooth inlet. A 50-mm long strip of abrasive P24 [8] was glued onto the channel perimeter. This provided fully developed turbulent boundary layer in the measurement area during the experiments. Table 1 gives the main characteristics of the turbulent boundary layer calculated using SIV measurements. In the considered turbulent boundary layer, Kolmogorov length scale ranged from about 0.1 mm near the wall to 0.18 mm at the outer edge of the boundary layer, i.e. not more than 3 pixels in the image. These length scales did not exceed 1.6 of Kolmogorov length scale for the external boundary and 2.9 of Kolmogorov length scale near the wall.

| Table 1. Characteristics of turbulent boundary layer measured. |
|---------------------------------------------------------------|
| Value                                                         |
| Freestream velocity, \(U_\infty\) (m/s)                        | 4.06 |
| Turbulent boundary layer thickness, \(\delta_{99\%}\) (mm)     | 15.88|
| Displacement thickness TBL, \(\delta^*\) (mm)                | 2.44 |
| Momentum thickness TBL, \(\theta\) (mm)                      | 1.58 |
| Friction factor, \(C_\tau\)                                 | 0.005|
| Dynamic velocity, \(u_t\) (m/s)                              | 0.204|
| \(R_0\)                                                      | 425  |
| \(R_1\)                                                      | 214  |

3. Results

Since the dynamics of instantaneous velocity fields was measured only in the \(x_1x_2\) plane, there was not possibility to estimate the triple correlations gradient in the transverse direction \(x_3\) when estimating the terms of turbulent diffusion transport \(T_{ij}^a\) (1). Therefore, the latter were expressed assuming that the triple correlations gradients along the \(x_1\) and \(x_2\) directions are small compared to \(x_3\). When estimating the terms of turbulent diffusion transport, we neglected the terms describing the diffusion generated by the pressure pulsations. In addition to the difficulties in experimental estimation, these terms almost do not contribute to the total energy balance for the case of a steady turbulent boundary layer.

\[
T_{ij}^{a} = \left( -\frac{\partial u'_i u'_j}{\partial x_k} - \frac{\partial p}{\partial x_k} \left( \delta_{ij} u'_t + \delta_{ik} u'_t \right) \right) \frac{\nu}{u'_t^2},
\]

where \(\delta_{ij}\) is the Kronecker delta, \(\nu\) is the kinematic viscosity.

The estimation of the \(T_{ij}^a\) terms was performed in two ways. The first way was differentiating the measured discrete values of the triple correlations using a 3-point approximation scheme with a 50% window overlap. The second way was differentiating the functions (2) obtained from the approximation results of the triple correlation profiles by polynomials of the fifth degree from \(\log(x_i^*)\) within the interval \(x_3^* \in [7; 170].\)

\[
\begin{align*}
\bar{u}'_i u'_j x^*_i & = 7.99 \left( \log x_i^* \right)^3 - 68.38 \left( \log x_i^* \right)^2 + 226.8 \left( \log x_i^* \right) - 361.6 \left( \log x_i^* \right)^{-1} + 274.1 \left( \log x_i^* \right) - 78.72, \\
\bar{u}'_i u'_j x^*_j & = -3.31 \left( \log x_i^* \right)^3 + 24.90 \left( \log x_i^* \right)^2 - 71.72 \left( \log x_i^* \right) + 98.66 \left( \log x_i^* \right)^{-1} - 65.22 \left( \log x_i^* \right) + 16.70, \\
\bar{u}'_i u'_j x^*_k & = -1.39 \left( \log x_i^* \right)^3 + 11.20 \left( \log x_i^* \right)^2 - 34.78 \left( \log x_i^* \right) + 51.89 \left( \log x_i^* \right)^{-1} - 37.29 \left( \log x_i^* \right) + 10.35.
\end{align*}
\]

Estimation of the derivatives of polynomial functions has three undeniable advantages. First, the approximation procedure is a kind of filter against the random error of the experimental measurements. Secondly, the issues concerning the choice of the approximation scheme order for the derivatives and interrogation window overlap are not considered already. Thirdly, this allows extension the range of estimation of the characteristics, because this range is substantially reduced due to impossibility of estimation the derivatives with high-order difference schemes at the boundary.
points of the measurement area. It was shown in [6] that when differentiating the results of SIV-measurements with respect to the spatial coordinate, the degree of window overlap using to estimate the instantaneous velocity vector fields at two neighboring points should not exceed 50%. In our experiments, the minimal distance between the points was 2 pixels, it corresponds to 88% overlap with interrogation window size of 16 pixel. To ensure a 50% overlap, we had to increase the differentiation interval to 8 pixels. With respect to the 3-point approximation scheme, we failed to estimate $T_{ij}$ in the first three points, which is clearly visible in Figure 1 (dashed lines).

When the profiles of triple correlations are approximated by polynomials in $\lg(x^*_2)$, the transition to a logarithmic coordinate system leads to a nonuniform distribution of the profile points along the ordinate axis, namely, to a concentration at a distance from 0. Approximation of such data will obviously result in the graph of the polynomial function so that it is good to describe the area far from the wall. In order to achieve uniform distribution of the triple-correlation profiles along the ordinate axis in the logarithmic coordinate system, we have thinned out the profiles before the approximation procedure.

![Figure 1](image-url)

**Figure 1.** Turbulent transport rate terms of the second-moment transport equation in wall coordinates. Lines DNS ($Re_τ = 180$; Mansour 1987); circle markers SIV ($Re_τ = 214$); dashed lines SIV (differentiation of functions (2), $Re_τ = 214$).
Figure 1 shows the profiles of turbulent diffusion transport terms obtained from the SIV measurements, which are compared with the results of DNS [4] at the close Re. Good agreement with the DNS results for T_{11} and T_{12} is obtained starting with the coordinate x_2^+=20…25, and for profile T_{22} starting with the coordinate x_2^+=50. The T_{22} profiles obtained by differentiating the discrete values and the polynomial function of lg(x_2^+) do not agree very well, obviously because of the insufficient degree of the approximating polynomial. When considering the T_{22} profile form from the DNS results, the approximation by the fifth-degree polynomial here served as a filter from the random error of the experimental estimation of v'v'. Taking into account the order of T_{22} value in comparison with T_{11} and T_{12}, the use of a polynomial with a higher degree is not justified. Using the approximation of the triple correlation profiles by polynomial functions for subsequent differentiation yielded minimal deviation of the T_{ij} profiles from the DNS results.

4. Conclusions
The profiles of turbulent diffusion transport of Reynolds stresses in a developed turbulent zero-gradient boundary layer obtained from the SIV measurement agree well with DNS results at a similar value of Re.

Deviation between the profiles estimated from SIV measurements of turbulent diffusion transport terms and DNS results has been reduced by polynomial approximation of profiles. The approximation procedure served as a filter cutting off the random measurement uncertainty.

To demonstrate the capabilities of the SIV technique for estimating small-scale turbulence characteristics in this paper, a flow regime with a relatively small Reynolds number was chosen, for which reliable DNS results are available. However, the estimation of small-scale characteristics based on SIV measurements can be performed at higher Re numbers for which DNS capabilities are limited.

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