Dynamical Domain Wall Fermions

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We report on an exploratory study of $N_f = 2$ dynamical domain wall fermions and the DBW2 gauge action at weak coupling. Details of improved simulation algorithms and preliminary results for the hadron spectrum and renormalised light and strange quark masses will be presented.

1. INTRODUCTION

In the quenched approximation Domain Wall Fermions (DWF) have been found to be an extremely successful approach to simulating QCD on the lattice. Crucial to this success is the fact that, when working at weak couplings ($a^{-1} \approx 2$GeV) and using improved gauge actions such as the DBW2 action, the degree of explicit chiral symmetry breaking is very small for practically useful sizes of the fifth dimension ($O(10)$).

Early simulations of dynamical DWF, performed at relatively coarse couplings, suggested that $L_5 \approx O(100)$ would be needed before the degree of chiral symmetry breaking was small enough to be acceptable. Here we will report on the progress of a preliminary study of $N_f = 2$ dynamical DWF in which we both adopt the DBW2 gauge action, and move to weaker coupling, in an attempt to find a region of parameter space where dynamical DWF simulations are practical.

2. SIMULATION PARAMETERS

All the results that will be presented were generated using the DWF action with $L_5 = 12$ and $M_5 = 1.8$, and the DBW2 gauge action with $\beta = 0.80$ on $16^3 \times 32$ lattices. Using the HMC algorithm we have generated three separate evolutions for bare masses of $m_f = 0.02$, $m_f = 0.03$, and $m_f = 0.04$. Table 1 summarises the total number of trajectories collected so far, together with the acceptance. Each HMC trajectory is of length 0.5 in HMC time and is split up into 50 leapfrog integration steps for mass of 0.02 and 0.03, and 40 integration steps for 0.04.

| $am_f$ | trajectories | acceptance |
|--------|--------------|-------------|
| 0.02   | 4716         | 78 \%       |
| 0.03   | 4785         | 78 \%       |
| 0.04   | 3445         | 68 \%       |

3. ALGORITHMIC DETAILS

The number of degrees of freedom of DWF grows with $L_5$, but the number of physical degrees of freedom does not. To cancel off this bulk divergence a set of Pauli-Villars fields,

$$\Phi \dagger D \dagger (m_f = 1)D(m_f = 1) \Phi,$$

(1)

is added to the DWF Lagrangian for dynamical simulations. Previous work has used two sets of pseudo-fermion fields to represent the DWF action: one for the fermion piece of the action and one for Pauli-Villars. The cancellation between these two terms is therefore only apparent after the average over the pseudo-fermion fields. Here we have used the fact that

$$\frac{\det(D \dagger (1)D(1))}{\det(D \dagger (m_f)D(m_f))}$$

(2)

is equal to

$$\det(D(1)[D \dagger D(m_f)]^{-1}D \dagger (1))$$

(3)
to represent the fermion and Pauli-Villars pieces of the action with a single pseudo-fermion field. With this approach the cancellation happens step by step in the leapfrog integration. We find the acceptance of the algorithm is increased by 10\%–20\% while the inversion costs are reduced by 20\%–30\% when using this modified force term.

We have also implemented the chronological inverter technique of [4], leading to a performance improvement of a factor of $\approx 1.7$. After this improvement calculating a single trajectory takes approximately $1.6 \times 10^4$, $9 \times 10^3$ and $6 \times 10^3$ dirac matrix applications for $m_f = 0.02$, 0.03 and 0.04 respectively.

4. RESULTS

While the lattices collected represent part of a larger RBC collaboration project to calculate many hadronic quantities of phenomenological interest, here we will concentrate on a few mesonic observables to determine the basic properties of our simulations such as scale and quark mass. To calculate these quantities we have used every 50th trajectory, leaving out the first $\approx 600$ trajectories to allow the evolutions to thermalise. All quoted errors will be from a jackknife estimate of the statistical error.

To quantify the chiral symmetry breaking from finite $L_s$, we have measured the residual mass, $m_{\text{res}}$ as defined from the breaking term in the Ward-Takahashi identity [5]. To extract this we look at

$$ R(t) = \frac{\sum_{x,y} \langle J_{\mu}^a(y, t) J_5^a(x, 0) \rangle}{\sum_{x,y} \langle J_5^a(y, t) J_5^a(x, 0) \rangle}, $$

which for time greater than some $t_{\text{min}}$ should be time independent and equal to $a m_{\text{res}}$ [6]. Figure 1 shows this for the $m_f = 0.02$ evolution at the dynamical point. As can be seen, a plateau is evident for $t \geq 2$, with an error weighted average between timeslice 6 and the end giving a value of 0.00137(2). This number is relatively insensitive to the quark mass with linear extrapolation to $m_f = 0$ giving $m_{\text{res}} = 0.00136(5)$.

While this value for the residual mass is relatively small compared to the input quark mass, to properly interpret the value we must know the lattice spacing and mass renormalisation. For the purpose of this preliminary analysis we will assign a single lattice spacing for all three evolutions based on a linear extrapolation in the dynamical quark mass of the $\rho$ meson mass. This is shown in Fig 2 and leads to an inverse lattice spacing of 1.806(60)GeV.

Figure 3 shows $M_\pi^2$ for degenerate quark masses versus dynamical mass. Fitting to the naive, first order chiral perturbation theory, expectation that

$$ M_\pi^2 = B_\pi (m_1 + m_2), $$

Figure 1. residual mass extraction for $m_f = 0.02$ evolution

Figure 2. $M_\rho$ - dynamical extrapolation

\[ m_1, m_2 \]
where $B_\pi$ is a constant and $m_1$ and $m_2$ are the quark masses, gives a $\chi^2$ per degree of freedom of 0.2 and results that are consistent with the pion mass vanishing at $m_f = -m_{res}$ with $M^2_\pi(m_f = -m_{res}) = 1.6(11) \times 10^4 \text{MeV}^2$. The experimental value of $M^2_K$ is shown on the figure as a dotted line. Together with Eq 5 this suggests our lightest quark mass is a little above half the strange quark mass.

Going further, following the same approach as [7], we extract preliminary values of the renormalised light and strange quark masses of $3.94(31) \text{MeV}$ and $103(8) \text{MeV}$ in the $\overline{\text{MS}}$-scheme at 2 GeV. An alternative way to get a rough idea of the size our input quark mass in physical terms, which is independent of the way we are setting our scale, is to calculate the ratio $M_\pi/M_\rho$. This is 0.541(8), 0.598(8) and 0.637(8) for $m_f = 0.02, 0.03$ and 0.04 respectively.

In quenched simulations using the DBW2 action it was noticed that tunneling between different topological sectors is suppressed with respect to more standard gauge actions [6]. As such, it is important to study how the topological charge varies with trajectory number in our dynamical simulations. Fig 4 shows this for the $m_f = 0.02$ evolution, determined using a classically $O(a^4)$ improved definition of the topological charge calculated on each lattice after applying 20 steps of APE smearing with a coefficient of 0.45. While it is encouraging that the value of the topological charge is changing, it is clear that, with a separation of 50 trajectories between lattices, strong correlations are present.

5. CONCLUSIONS

A preliminary study of $N_f = 2$ dynamical DWF QCD, using the DBW2 gauge action at an inverse lattice spacing of $\approx 1.8$ GeV, shows that a regime exists for which the explicit chiral symmetry breaking is small for a computationally practical extent of the fifth dimension.

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