NEW BOUNDS FOR THE RATIO OF POWER MEANS

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Abstract. We show that for real numbers $p, q$ with $q < p$, and the related power means $P_p$, $P_q$, the inequality

$$\frac{P_p(x)}{P_q(x)} \leq \exp\left(\frac{p - q}{8} \left(\ln\left(\frac{\max x}{\min x}\right)\right)^2\right)$$

holds for every vector $x$ of positive reals. Moreover we prove that, for all such pairs $(p, q)$, the constant $\frac{p - q}{8}$ is sharp.

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