Evaluation of wall sand roughness influence of friction in boundary layer

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Evaluation of wall sand roughness influence of friction in boundary layer

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Abstract. In this work an universal transition function is obtained for logarithmic profile constant. This dependence is obtained according to the experimental data of Nikuradze for the coefficient of hydraulic resistance of a pipe with sand roughness. The refined velocity profile was used for assessment calculation of the boundary layer on a flat rough plate with adjusted free-stream velocity. The conclusion is made on the influence of roughness on the resistance of the plate with distance from the leading edge and a comparison with experimental data is performed.

1. Introduction

An empirical model of interaction of a turbulent flow with a rough wall is proposed in the paper [1]. According to this model, the distribution of the flow velocity $u_y(y)$ in the zone of developed near-wall turbulence is taken in the form of the function

$$\frac{u_y(y)}{V_s} = f(\eta) = 2.5 \ln \frac{yV_s}{\nu} + 5.5 - (\Phi_g - \Phi_{sh})\Phi\left(\frac{\delta V_s}{\nu}\right)$$

(1)

where $y$ is the distance from the wall, $V_s = \sqrt{\tau_{en}/\rho}$ - dynamic velocity, $\nu$ - kinematic viscosity of a liquid, $\delta$ - characteristic height of roughness, $\Phi_g$ is the function

$$\Phi_g = 2.5 \ln \frac{\delta V_s}{\nu} + 5.5$$

(2)

In formula (1), $\Phi$ is a universal transition function that varies from 0 to 1, depending on the ratio between the roughness height and the thickness of the viscous sublayer,
In (1) $\Phi_{sh}$ is the scalar factor of the roughness shape, a number that does not depend on the height of the roughness, but is determined by its relative dimensions, that is, the shape. The numerical value of the $\Phi_{sh}$ is defined as the limiting, viscosity-independent value of the constant in the logarithmic profile written in the form

$$\frac{\mu}{V_x} = 2.5 \ln \frac{y}{\delta_y} + \Phi$$

and numerically equal to the speed in $V_x$ units at the boundary of the roughness zone at $y = \delta_y$. For sand roughness $\Phi_{sh} = 8.48$.

Strictly relation (1) should be valid in the near-wall region of the flow. Approximation of the function (1) to the entire cross section of a circular tube with subsequent integration, allowed the calculation of resistance to fluid motion in round rough pipes. In Fig. 1 the results of the calculations are compared with the classical Nikuradze measurements of resistance of pipes with sandy roughness [2]. This comparison illustrates the convincing agreement of the calculation with experiment. In [3,4] the empirical relations for calculating the coefficient of hydraulic resistance of pipes with sand roughness are also obtained. However, according to the authors, the proposed model is more universal.

![Figure 1. Hydraulic characteristic of a round pipe with sand roughness.](image)

1 – laminar flow, 2 – calculation according to Blausius formula, 3 – calculation according to Nikuradze's formula, 4-11 – calculation by the proposed method.
2. Plate resistance with sand roughness

According to (1) the interaction of a turbulent flow with rough walls according to (1) is determined by the local wall parameters. These are the height of the roughness protrusions $\delta$, the shape factor of the roughness $\Phi$, the dynamic velocity $V_*$. Involving turbulence models in the computation and using (1) to formulate the boundary conditions of the body surface for the turbulence model can be considered as a common universal approach for calculating the hydrodynamics of complex channels, including the case of a surface roughness variable on the surface. For channels of simple shape (a round tube, a flat channel, an annular layer, etc.) application of (1) to all cross sections of the channels gives good results. In the present study, profile (1) is used to calculate the "simple" flow-the boundary layer on a flat rough plate.

The purpose of the hydrodynamic calculation of the boundary layer is to determine friction on the surface at a given velocity of the oncoming flow

$$\tau_{cm} = C_f (Re_x) \frac{\rho u_0^2}{2}$$

In a boundary layer flow is laminar up to $Re_x = \frac{u_0 x}{v} \approx 3 \cdot 10^5$. Wherein $C_f (Re_x) = \frac{0.664}{Re_x^{0.5}}$. A turbulent boundary layer is established when $Re_x > 5 \cdot 10^5$.

As follows from (4), to determine the friction coefficient $C_f (Re_x)$ for turbulent flow, it is sufficient to find the ratio of the velocity of the oncoming flow and the dynamic velocity

$$\sqrt{\frac{2}{C_f}} = \frac{u_0}{V_*}$$

The velocity distribution along the thickness of the boundary layer differs from the universal profile due to the phenomenon of intermittency in the outer part of the layer. According to [3], taking into account the Cowels correction,

$$\frac{u_y(y)}{V_*} = f \left( \frac{y V_*}{v} \right) + \Pi f_1 \left( \frac{y}{\delta} \right)$$

where $f(\eta)$ in the case of a rough wall should be taken equal to (1), $f_1 \left( y/\delta \right)$ - a universal function of the ratio of the distance from the wall to the thickness of the boundary layer

$$f_1 \left( \frac{y}{\delta} \right) = 1 + \sin \left[ \frac{\pi}{2} \left( \frac{2 y}{\delta} - 1 \right) \right]$$

$\Pi$ – coefficient depending on the pressure gradient and equal to $\Pi = 1.375$ for $\frac{dP}{dx} = 0$. 


Substituting in (6) \( y \) equal to the thickness of the boundary layer \( \delta \), we obtain a velocity equal to the velocity of the oncoming flow. Taking (1) into account, we have

\[
\frac{u_0}{V_c} = 2.5 \ln \frac{\partial V_x}{v} + 8.25 - (2.5 \ln \frac{\partial V_x}{v} + 5.5 - \Phi_{\delta h}) \phi \left( \frac{\partial V_x}{v} \right)
\]  

(7)

If the velocity distribution over the thickness of the layer (6) is known, the unknown thickness of the turbulent boundary layer \( \delta \) contained in (7) can be obtained from the integral momentum equation

\[
\frac{d}{dx} \int_0^\delta u_x(y)(u_0 - u_x(y))dy = \frac{\tau_{cm}}{\rho} = V_c^2
\]  

(8)

In the first approximation neglecting by the influence of the wall roughness on the thickness of the turbulent boundary layer, we use the result obtained for flow past a smooth wall [5].

\[
\delta(x) = b \frac{V_c}{u_0} x
\]  

(9)

where \( b=0.3 \).

Taking into account (5) and (9)

\[
\frac{\partial V_x}{v} = b \left( \frac{V_c}{u_0} \right)^2 \frac{u_0 x}{v} = \frac{b}{2} C_f \text{Re}_x
\]

(10)

For sandy roughness, the size of the sand particles \( \delta_f \) is used as the grain size \( \Delta \). Wherein

\[
\frac{\partial V_x}{v} = \Delta V_x \frac{\Delta u_0}{v} \frac{V_x}{u_0} = \frac{C_f}{2} \text{Re}_\Delta
\]

(11)

where \( \text{Re}_\Delta = \frac{\Delta u_0}{v} \) is the dimensionless parameter determining the size of the roughness.

Taking into account (10), (11), the relation (7) is transformed into a transcendental equation for determining the coefficient of friction \( C_f \). The coefficient of friction is the function of the distance from the leading edge of the plane \( \text{Re}_x \) and the dimensionless dimension of the roughness \( \text{Re}_\Delta \)

\[
\sqrt{\frac{2}{C_f}} = 2.5 \ln(0.15C_f \text{Re}_x) + 8.25 - (2.5 \ln \left( \frac{C_f}{2} \text{Re}_\Delta + 5.5 - \Phi_{\delta h} \right)) \phi \left( \frac{\sqrt{C_f}}{2} \text{Re}_\Delta \right)
\]

(12)

where the form parameter is equal to \( \Phi_{\delta h} = 8.48 \).

If \( \phi = 0 \), from (12) follows the friction equation for a turbulent flow on a smooth wall. The results of the calculations are shown in Fig. 2 for the dimensions of the roughness of the corresponding \( \text{Re}_\Delta=250,500 \).
Figure 2. Hydraulic characteristic of a flat plate: 1 – laminar flow, 2 – turbulent regime of a smooth plate, 3-4 – calculated according to (12) values for Re$_\Lambda$=250,500, 5 – experimental data of [6].

3. Conclusions
The main conclusion following from the calculation results – the influence of the roughness on the resistance decreases with distance from the leading edge. The roughness has the strongest effect at the beginning of the zone of turbulent flow and in the zone of transition to turbulent regime.

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