Laser cooling of a trapped two-component Fermi gas

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We study the collective Raman cooling of a trapped two-component Fermi gas using quantum master equation in the festina lente regime, where the heating due to photon reabsorption can be neglected. The Monte Carlo simulations show, that 3D temperatures of the order of 0.008 TF can be achieved. We analyze the heating related to background losses, and show that our laser-cooling scheme can maintain the temperature of the gas without significant additional losses.

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The realization of Bose-Einstein condensation (BEC) and degenerated Fermi gases has outburst the interest in the physics of ultracold bosonic and fermionic gases. A Fermi gas with attractive interactions undergoes for temperatures $T$ below a critical one, $T_c$, a transition into the superfluid Bardeen-Cooper-Schrieffer (BCS) phase. The accomplishment of BCS and its possible detection have been considered in detail (see e.g. Refs. [1]). Unfortunately, evaporative cooling currently employed in experiments is based on collisional processes, and has not yet allowed to reach $T \ll T_c$, since $T_c$ is much smaller than the Fermi temperature $T_F$, and for $T < T_F$ the collisions are strongly suppressed due to Pauli blocking. Recently, however, it has been proposed that $T_c$ may be significantly increased by employing Feshbach resonances, or optical lattices. This opens the possibility to achieve BCS at a temperature where the collisions are still efficient enough.

In this Rapid Communication we study the laser cooling of a two-component Fermi gas in the festina lente (FL) regime, where the spontaneous emission rate $\gamma$ is smaller than the trap frequency $\omega$. In this regime the heating due to photon reabsorption is prevented, as recently observed for tightly bound atoms in optical lattices. The laser cooling in the FL regime has been already predicted to work for bosons and for polarized fermions. Using Monte Carlo simulations, we show here that laser cooling allows for bringing a two component Fermi system to $T \ll T_F$.

The laser cooling of fermions toward $T \ll T_F$ is obstructed by several problems. One of them is the inhibition of spontaneous emission, which results in the decrease of the cooling efficiency. In Ref. [11] we have predicted that the inhibition problem can be overcome by either dynamically adjusting the spontaneous emission rate in a Raman cooling process, or by employing specially designed anharmonic traps. In this paper we apply the former solution. Another obstacle is related to the inelastic losses that create holes deeply in the Fermi sea, producing a significant heating. We show that background collisions do not affect significantly the laser cooling. In fact, our cooling method can be employed to maintain a two-component Fermi gas at a fixed $T$, for a relatively long time in a trap. Finally, we derive an analytic formula that describes the heating of the trapped gas due to background collisions. The heating is shown to be smaller than in the homogeneous case.

We consider fermionic atoms with an accessible electronic three-level $\Lambda$ scheme, containing states $|g\rangle$, $|e\rangle$ and $|r\rangle$. The ground state $|g\rangle$ is coupled via a Raman transition to the metastable state $|e\rangle$. The latter state is also coupled by an optical transition to the upper state $|r\rangle$, from which atoms rapidly decay into $|g\rangle$. The adiabatic elimination of $|r\rangle$, leads to an effective two-level system, characterized by tunable parameters: Rabi frequency $\Omega$, and spontaneous emission rate $\gamma$. The value of $\gamma$ can be controlled by modifying the coupling from $|e\rangle$ to $|r\rangle$.

The atoms are placed in a dipole trap characterized by a Lamb-Dicke parameter $\eta = 2\pi a/\lambda$, with $a = \sqrt{\hbar/2m}\omega_e$ being the size of the ground state of the trap, and $\lambda$ the laser wavelength. A dipole trap was recently used for the all-optical production of a degenerate gas of two Li species, and in current experiments in Mg.

We consider a spherically symmetric trap with incommensurable frequencies $\omega^g$ and $\omega^r$, for the ground and the excited state respectively. The latter assumption simplifies the dynamics of the spontaneous emission processes in the FL limit. The cooling process consists of sequences of Raman pulses of frequencies, adjusted in such a way that they induce the transition of atoms to the lower motional states of the trap. We assume that the power of cooling lasers is sufficiently weak, and therefore during each pulse no significant population in $|e\rangle$ is present. This allows to eliminate adiabatically the level $|e\rangle$, and to consider only the density matrix $\rho(t)$ for the atoms in $|g\rangle$, and being diagonal in the Fock representation corresponding to the bare trap levels.

We assume that the laser acts only on one component, whereas the other one is cooled sympathetically, which is sufficient to reach $T \ll T_F$. Using the standard theory of quantum-stochastic processes [12, 13] we derive the quantum master equation (ME) for the density matrix $\rho(t)$ in a similar way as that for bosons:

$$\dot{\rho}(t) = L_0 \rho + L_1 \rho + L_2 \rho, \quad \text{(1)}$$

where $L_0 \rho = -iH_{\text{eff}}\rho(t) + i\rho(t)H_{\text{eff}}^\dagger + J\rho(t)$, $L_1 \rho = \ldots$. 
The standard fermionic anticommutation relations:

\[ \{ n, b^\dagger \} = \{ m, b^\dagger \} = \delta_{mn}. \]

Here \( g_m \) (\( e_1 \)) is the annihilation operator of atoms of the first component, and in the trap level \( n \), is denoted by \( b_n \). These operators fulfill the standard fermionic anticommutation relations: \( \{ g_m, g_n^\dagger \} = \{ e_1, e_1^\dagger \} = \{ b_m, b_n^\dagger \} = \delta_{mn}. \) In Eq. (4) \( \gamma \) denotes the single-atom effective spontaneous emission rate, \( \omega_m^g, \omega_m^b \) are the energies of state \( m \) of the trap: \( |g\rangle \), \( |e_1\rangle \), \( |b\rangle \) respectively, and \( \delta \) is the laser detuning from the atomic transition. The coefficients \( \xi_{lm} \) are defined as follows:

\[ \xi_{lm} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \langle \mathcal{W}(\theta, \phi)|\eta_{lm}(\mathcal{k}_L)|^2 \].

The interaction of the laser light with the atoms is governed by the Hamiltonian \( \hat{H}_{\text{las}} \):

\[ \hat{H}_{\text{las}} = \frac{\Omega}{2} \sum_{l,m} \eta_{lm}(\mathcal{k}_L) e_1^\dagger g_m + \text{H.c.}, \]

where \( \Omega \) is the Rabi frequency and \( \mathcal{k}_L \) is the wavevector of the cooling laser. Binary collisions are described by

\[ \hat{H}_{\text{coll}} = \sum_{m,n,q,p} U_{m,n,q,p} g_m^\dagger g_n^\dagger b_q^\dagger b_p, \]

where due to Fermi statistics only collisions between different species are allowed. We neglect the collisions with the atoms in \( |e\rangle \), since only a small fraction of atoms is excited in each pulse. The collisional amplitudes are defined as \( U_{m,n,q,p} = (4\pi e_{\text{osc}} / m) \int_{R^3} d^3 x \phi_m^*(x) \phi_n(x) \beta_q^*(x) \beta_p(x) \), where \( \phi_\alpha \) (\( \beta_\alpha \)) denotes the wavefunction of the state \( \alpha \) in the \( |g\rangle \), \( |b\rangle \) trap, and \( e_{\text{osc}} \) is the scattering length.

We require the FL condition \( \gamma < \omega \) at least in the initial phase of the cooling process, and also assume that \( \Omega < \gamma \) and \( \Omega^2 / \gamma \ll \omega \), which allows for the adiabatic elimination of \( |e\rangle \) performed by means of projection operator techniques [14]. For weak interactions between the two species, the dynamics due to the collisions and the dynamics due to the laser-cooling are independent [16]. The laser cooling may be described by the ME without the collisional part, whereas the collisional part takes the form of a quantum Boltzmann master equation (QBME) [17]. The probability of a laser-induced equation transition from state \( m \) to state \( n \) of the trap \( |g\rangle \) is given by

\[ P_{m\to n}^{\text{opt}} = \frac{\Omega_n^2 N_n^1 (1 - N_n^1) \times}{4 \left[ \sum_{l,m} \xi_{lm} \eta_{lm}(\mathcal{k}_L)^2 \right] + \gamma^2 / (\omega_n^g - \omega_n^q)^2 + \gamma^2 R_{ml}^2}, \]

where \( R_{ml} = \sum_{n} \xi_{ln} (1 - N_n^1 + \delta_{n,m}) \) are the factors modifying the spontaneous emission rate in a Fermi gas, and \( N_n^1 \) and \( N_n^2 \) denotes the number of atoms occupying the state \( m \) of the trap \( |g\rangle \) and \( |b\rangle \), respectively. In the regime of quantum degeneracy, coefficients \( R_{ml} \) vanish, inhibiting the spontaneous emission and forcing the atoms to remain excited for a long time. This prolongs the cooling process, and can result in excited-ground collisions, which lead to heating and losses. In addition, the adiabatic elimination used in the derivation of Eq. (4) ceases to be valid. The cooling efficiency is also decreased due to the fermionic inhibition factor \( (1 - N_n) \) in the numerator of (4). The negative influence of the statistics can be overcome by dynamically increasing \( \gamma \) during the Raman cooling [11], in order to avoid the inhibition effects, but still remaining in the FL regime: \( \gamma R_{ml} < \omega \). Still, a small fraction of the atoms will remain in the excited state after the cooling pulses, and has to be removed from the trap in order to avoid inelastic collisions. The latter aim can be achieved by optically pumping the excited atoms to a third non-trapped level.

The probability of a collision between two fermions of different species, from the states \( n \) and \( p \) to the states \( m \) and \( q \), respectively, is given by

\[ P_{n,p\to m,q}^{\text{coll}} = \frac{\pi}{\omega} N_n^1 N_p^2 (1 - N_m^1) (1 - N_q^2) \times \left[ \langle U_{m,n,q,p} \rangle^2 \delta_{E_n + E_p, E_m + E_q} \right]. \]

The fermionic inhibition factors \( (1 - N_n^1) \) also slow the collisional processes.

In our simulations we assumed the atoms as confined in an isotropic, 3D harmonic trap with frequency \( \omega = \omega^g = \omega^e = \omega^b \). Due to the limitations when simulating relatively large systems \((N \sim 10^4)\), we employ ergodic approximation, i.e. we assume that the populations of the states with the same energy are equal. This approximation relies on the fact that thermalization inside the same energy shell is much faster than between different energy shells. In addition we assume that the collisional processes is much faster than the laser cooling. The detailed calculation of the transition probabilities within ergodic approximation is presented in Ref. [18]. Due to numerical limitations we assumed \( \eta = 2 \), which could be e.g. the case of potassium atoms in a dipole trap with \( \omega = 2 \pi \times 2.4 \text{ kHz} \) (employed in Li experiments [12]), and a laser wavelength \( \lambda \sim 720 \text{ nm} \). We consider a scattering length for the interactions between the two species \( a_{\text{sc}} = 157 a_0 \), where \( a_0 \) is the Bohr
radius. This value corresponds to the interactions between $F = 9/2, m_F = 9/2$ and $|F = 9/2, m_F = 7/2|$ of $^{40}$K [21]. The trap is assumed to have the same depth for both species, and contains 81 energy levels (91881 states). We assume both components to have equal initial number of atoms, $N = 10660$, corresponding to a Fermi energy $E_F = 38\hbar\omega$. For this relatively large number of atoms, the dynamics generated by the collisional part of the ME equation leads to an equilibrium distribution, which agrees very well with the one calculated from the grand canonical ensemble [18]. Hence we start the simulations from a thermal distribution.

In a Fermi gas the main loss sources are provided by background collisions, resulting from non ideal vacuum conditions, and photoassociation. The photoassociation losses (when the laser is tuned between molecular resonances) are typically of the order of $10^{-14}\text{cm}^3\text{s}^{-1}$ for laser intensities of $1\text{mW/cm}^2$ [27]. In our case the applied laser intensities are typically 1000 times smaller and we estimate that for $N = 10660$ atoms, the atomic density is smaller than $3.5 \times 10^{13}\text{cm}^{-3}$. Therefore the photoassociation losses can be safely neglected. On the contrary, the background collision apart from decreasing the number of atoms, generate holes deep within the Fermi sea. Those holes, after subsequent thermalization, may lead to a significant heating [11]. We have assumed that background losses depopulate each state of the trap with the rate $\gamma_{\text{bg}}$, which is independent of the energy of the state

$$\dot{N}_j = -\gamma_{\text{bg}} N_j^2,$$

where $j = 1, 2$ enumerates the components. In the simulations we assume $\gamma_{\text{bg}} = 1/350\text{Hz}$ [8]. We include also in the simulations the losses produced by the removal of long-living excited atoms at the end of each cooling pulse.

Fig. 1 shows the evolution of the temperature of the laser-cooled two-component Fermi gas. Initially we consider $T_0 = T_F$. The cooling process was divided into four stages, each one consisting of a sequence of two Raman pulses. The employed pulses are characterized by the following parameters: detuning $\delta/\omega = \{-11, -12, -16, -17, -19, -20, -25, -26\}$ respectively, Rabi frequency $\Omega/\gamma = \{(0.113, 0.0113), (0.008, 0.012), (0.0025, 0.004), (0.003, 0.011)\}$ respectively, and length $\Delta t/\omega^{-1} = \{(250, 250), (2000, 2000), (4000, 4000), (4000, 6500)\}$ respectively. For these parameters not more than 10% of the atoms is excited during each pulse, and thus the conditions of the adiabatic elimination are fulfilled. The temperature was determined by fitting the calculated distribution of fermions to a thermal distribution. As one can observe, a final temperature $T \approx 0.008T_F$ may be reached within 8 s. The losses associated with the cooling process do not exceed 2% and are slightly larger for the laser-cooled component, due to the removal of the long-living excited atoms (inset of Fig. 1).

We have analyzed the effect of the losses on the laser-cooled gas. To this aim we have considered an initial gas at $T = 0.008T_F$ (end of the cooling process in Fig. 1), and compared two different cases: (i) the laser is turned off, and the gas is heated due to the background collisions, (ii) the laser is turned on, and the cooling pulses of the last stage are continuously applied. Fig. 2 presents the evolution of the temperature of both species for these two cases. As observed, the laser cooling compensates for both components (darker and lighter curves) during the cooling process from a thermal distribution. As one can observe, a final temperature $T \approx 0.008T_F$ may be reached within 8 s. The losses associated with the cooling process do not exceed 2% and are slightly larger for the laser-cooled component, due to the removal of the long-living excited atoms (inset of Fig. 1).
for the heating induced by the creation of holes in the degenerate distribution. Hence, it helps to maintain the degenerate gas for a relatively long time in the trap. We have verified that the continuous application of the laser cooling does not lead to substantially larger losses.

Finally, we have analyzed the results for the heating due to background collisions with the help of an analytical model. For the background losses that decrease the population of the trap levels in the way described by Eq. 2, one can calculate the evolution of the temperature analytically \([\dot{T} = \dot{T}_0 + \frac{E_F(N_0)}{E_F(N(t))} 3 \frac{\pi^2}{2} (\frac{E_F(N_0)}{E_F(N(t))} - 1) \).

For a large system \(E_F(N) \approx \hbar \omega (6N)^{1/3} \), and \(N(t) \approx N_0 \exp(-\gamma_{bg} t) \), hence

\[
\dot{T}(t) = \sqrt{\frac{E_F(N_0)}{E_F(N(t))}} \dot{T}_0 + 3 \left( \frac{E_F(N_0)}{E_F(N(t))} - 1 \right) / 2 \pi^2. \tag{9}
\]

Fig. 3 presents the evolution of the temperature for a rate \(\gamma_{bg} = 10 \text{ Hz} \), which is much larger than the typical experimental rates, and was employed to reduce the amount of time-consuming numerical calculations. Fig. 3 compares our numerical results, the predictions of the analytical formula of (9) derived for a homogeneous system, and the data calculated from Eq. 9 with \(E_F(N) \approx \hbar \omega ((6N)^{1/3} - 2) \), which is more appropriate for a finite size system. The analytic curve fits very well to the numerical data. The small discrepancy for larger temperatures originate from the fact, that the expression for the mean energy, which we use, is valid up to order \((T/T_F)^2\).

In conclusion, we have studied the laser cooling of trapped two-component Fermi gases. Our method exploits the dynamical adjusting of the effective spontaneous emission rate in Raman cooling. Using Monte Carlo simulations, we have shown that laser cooling is able to cool the two-component Fermi gas below \(0.01 T_F \). We have also discussed the losses which may affect the laser-cooled gas. In this context, we have shown that our laser-cooling scheme can be employed to maintain a two-component Fermi gas at a fixed temperature in the presence of background collisions, without significant additional atom losses. Finally we have derived an analytic formula for the temperature of a trapped Fermi gas heated by background collisions. We have shown that the heating rate is smaller than in the homogeneous case.

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