Dark Energy and Dark Matter unification
via superfluid Chaplygin gas

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Abstract

A new model describing the dark sector of the universe is established. The model involves Bose-Einstein condensate (BEC) as dark energy (DE) and an excited state above it as dark matter (DM). The condensate is assumed to have a negative pressure and is embodied as an exotic fluid with Chaplygin equation of state. Excitations are described as a quasiparticle gas. It is shown that the model is not in disagreement with the current observations of the cosmic acceleration. The model predicts increase of the effective cosmological constant and a complete disappearance of the matter at the far future.

Keywords: accelerated expansion, Dark Energy, Dark Matter, relativistic superfluid, Chaplygin gas

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1 Introduction

The energy content of the Universe is a fundamental issue in cosmology. Observational data are evidence of accelerating flat Friedmann-Robertson-Walker model, constituted of about 1/4 of baryonic and dark matter and about 3/4 of a dark energy component. The DM content was originally inferred from spiral galactic rotation curves and then was supported by gravitational lensing and cosmic microwave background observations.

The essential feature of DE is that its pressure must be negative to reproduce the present accelerated cosmic expansion. The simplest DE model, the cosmological constant, is indeed the vacuum energy with the equation of state $p = -\rho$. The models for which $p < -\rho$ has been denoted phantom energy, and possesses peculiar properties, such as, an infinitely increasing energy density [1], negative temperatures [2], and the violation of the null energy condition. A number of models, such as quintessence [3] and k-essence [4], are based on scalar field theories. These models are parameterized by an equation of state $p < -\rho/3$. For a recent review of DE models and references see [5].

An alternative model is that of the Chaplygin gas, also denoted as quartessence, based on a negative pressure fluid, which is inversely proportional to the energy density [6]. The equation of state representing the generalized Chaplygin gas (GCG) is given by $p_{\text{Ch}} = -A/\rho_{\text{Ch}}^\alpha$ with positive constants $A$ and $\alpha$ ($0 < \alpha \leq 1$) [7]. An attractive feature of these models, is that at early times, the energy density behaves as a matter, $\rho_{\text{Ch}} \propto a^{-3}$, where $a$ is the scale factor, and as a cosmological constant at a later stage, $\rho_{\text{Ch}} = \text{const}$. It is also suggested that at an intermediate stage the energy density $\rho_{\text{Ch}}$ consists of both vacuum and soft matter (matter with the equation of state $p = \alpha \rho$) contributions. This is favorable to use the GCG model for a DE and DM unification [7, 8, 9].

Some different approach to the same problem is realized in this work. The feature of the scalar field used in the model [8] is spontaneous symmetry breaking that leads to a nonzero expectation value of the field. In other words, Bose-Einstein condensation of the scalar field into the state with zero momentum takes place. Similar effect holds in the theories of superconductivity and superfluidity. At zero temperature superfluid is in its ground state. If $T \neq 0$ particle-like excitations arises above the ground state. In this case the system can be divided into a background superfluid condensate and a quasiparticle gas. This separation naturally leads to two-fluid dynamics in which the condensate is said to be a superfluid component and the quasiparticle gas forms a normal component.

The model proposed in this letter represents the dark sector of the universe as a superfluid where the superfluid condensate is considered as DE and the normal component is interpreted as DM. To provide the accelerated expansion the potential of the scalar field must have specific form and entail a negative pressure of the superfluid background.

We base our analysis on the action

$$S = \int \left(-\frac{R}{16\pi G} + \mathcal{L}\right) \sqrt{-g} \, d^4x,$$  \hspace{1cm} (1)
where Lagrangian $\mathcal{L}$ associated with a generalized hydrodynamic pressure function depends only on one variable if we consider pure condensate, and on three variables when we include the excitation gas.

2 Condensate in WKB-approximation

Consider a system with spontaneously breaking of $U(1)$ symmetry described by a complex scalar field $\hat{\psi}$ with nonzero Gibbs expectation $\phi(x) = \langle \hat{\psi} \rangle$, the condensate wave function. Quantum fluctuations $\hat{\psi}' = \hat{\psi} - \phi$ can be considered in terms of quasiparticles.

If the quasiparticle gas is dilute or interaction between the quasiparticles and the condensate is weak then influence of the elementary excitations on the ground state can be neglected. In this case the condensate is described by the Lagrangian

$$\mathcal{L} = \partial_\nu \phi^* \partial^\nu \phi - V(\phi^* \phi).$$

(2)

It is useful for further study to represent the Lagrangian (2) in an equivalent hydrodynamic form. For this purpose we write the condensate wave function in terms of modulus and phase:

$$\phi = \frac{\sigma}{\sqrt{2}} e^{-i\chi}.$$  

(3)

Substituting (3) into the field equation, the real and imaginary parts yield respectively

$$\nabla_\nu \nabla^\nu \sigma + \sigma \left( 2 \frac{dV}{d\sigma^2} - \partial_\nu \chi \partial^\nu \chi \right) = 0,$$

(4)

and

$$\nabla_\nu (\sigma^2 \partial^\nu \chi) = 0.$$  

(5)

Equation (5) is a conservation law for the 4-current

$$j_\nu = i (\phi^* \partial_\nu \phi - \partial_\nu \phi^* \phi) = \sigma^2 \partial_\nu \chi.$$  

(6)

The gradient of the condensate phase is a superfluid momentum which can be written in terms of a unit 4-vector $V_\nu$:

$$\partial_\nu \chi = \mu_\nu = \mu V_\nu,$$

(7)

where $\mu$ is a chemical potential. In the present context $j_\nu$ is the particle current, therefore

$$\sigma^2 = \frac{n_c}{\mu},$$

(8)

where $n_c$ is a particle density of BEC.

If the modulus $\sigma$ varies slower than the phase $\chi$, $\partial_\nu \sigma \leq \sigma \partial_\nu \chi$, then we can neglect derivatives $\partial_\nu \sigma$ that corresponds to the WKB expansion of the condensate wave function up to the first order.
In this approximation equation (4) takes the form
\[ \sigma \mu^2 - \frac{dV}{d\sigma} = 0, \] (9)
and the particle density is
\[ n_c^2 = \sigma^3 \frac{dV}{d\sigma}. \] (10)

The energy-momentum tensor is found as variation of the Lagrangian with respect to the metric:
\[ T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} (\sqrt{-g} L) = \sigma^2 \partial_\mu \chi \partial_\nu \chi - g_{\mu\nu} \left( \frac{\sigma^2}{2} \partial_\lambda \chi \partial^\lambda \chi + V(\sigma^2) \right) = (\rho_c + p_c) V_\mu V_\nu - p_c g_{\mu\nu}, \] (11)
where the pressure and the energy density
\[ p_c = \frac{1}{2} \frac{dV}{d\sigma} \rho - V, \quad \rho_c = \frac{1}{2} \frac{dV}{d\sigma} \rho + V \] (12)
depend on only one variable \( \mu \).

For various contexts we can apply a rather different kind of potential. In our case the condensate is assumed to possess a negative pressure. For this reason let us take the potential
\[ V(\phi^* \phi) = M \left( \frac{\phi^* \phi}{\lambda} + \frac{\lambda}{\phi^* \phi} \right). \] (13)

In accordance with equations (10) and (12) one finds that
\[ n_c = \frac{2 \lambda \mu}{\sqrt{1 - \lambda \mu^2/M}}, \quad \rho_c = \frac{2 M}{\sqrt{1 - \lambda \mu^2/M}}, \] (14)
and the function of the generalized pressure has the form
\[ P(\mu) = p_c = -2 M \sqrt{1 - \lambda \mu^2/M} \] (15)
For further purpose it is more rational to eliminate the chemical potential \( \mu \) and to express the hydrodynamic quantities via the energy density:
\[ n_c = \sqrt{\frac{\lambda}{M} \rho_c^2 - 4 M^2}, \quad p_c = -\frac{4 M^2}{\rho_c}, \] (16)
and the adiabatic speed of sound is
\[ c_s^2 = \frac{dp_c}{d\rho_c} = \frac{4 M^2}{\rho_c^2}. \] (17)

The equation of state (16) is uniquely proper to Chaplygin gas suggested recently by Kameshchik et al. [6] as an alternative to quintessence and developed by a number of authors for description of the dark sector of the universe [7, 8].

In contrast to these works where pressure of Chaplygin gas is formed by both DE and DM, this model implies that the equation of state (16) concerns with only BEC which is interpreted as DE.
3 Relativistic superfluid dynamics

An efficient approach to description of the excited state is two-fluid hydrodynamics. This theory does not depend on details of microscopic structure of the quantum liquid and exploits effective macroscopic quantities. In the theory there exist two independent flows, the coherent motion of the ground state named a superfluid component, and a normal component produced by the quasiparticle gas. For this reason it is necessary to increase the number of independent variables in the generalized pressure (15) from one to three \[10, 11\]. They correspond to three scalar invariants which can be constructed from the pair of independent vectors, namely superfluid \(\mu_\alpha\) and thermal \(\theta_\alpha\) momentum covectors so that the general variation of the generalized pressure in a fixed background is

\[
\delta P = n^\alpha \delta \mu_\alpha + s^\alpha \delta \theta_\alpha. \tag{18}
\]

The coefficients \(n^\alpha\) and \(s^\alpha\) are to be interpreted as particle number and entropy currents correspondingly. By virtue of its invariance the pressure is given as a function of three independent variables, \(I_1 = \frac{1}{2} \mu_\alpha \mu^\alpha, I_2 = \mu_\alpha \theta^\alpha, I_3 = \frac{1}{2} \theta_\alpha \theta^\alpha\). Taking the derivatives of the pressure, one finds

\[
n^\alpha = \frac{\partial P}{\partial I_1} \mu^\alpha + \frac{\partial P}{\partial I_2} \theta^\alpha, \quad s^\alpha = \frac{\partial P}{\partial I_2} \mu^\alpha + \frac{\partial P}{\partial I_3} \theta^\alpha. \tag{19}\]

As soon as the generalized pressure is the Lagrangian density in the action (1) its variation with respect to the metric gives the energy-momentum tensor

\[
T_{\alpha\beta} = \frac{\partial P}{\partial I_1} \mu_\alpha \mu_\beta + \frac{\partial P}{\partial I_2} (\mu_\alpha \theta_\beta + \theta_\alpha \mu_\beta) + \frac{\partial P}{\partial I_3} \theta_\alpha \theta_\beta - P g_{\alpha\beta}. \tag{20}\]

Instead of the thermal momentum \(\theta_\alpha\) let us introduce an inverse temperature vector \(\beta^\alpha = s^\alpha/(s^\beta \theta_\beta)\) which we use as the independent vector together with the superfluid momentum \(\mu_\alpha\) since they are comoving to the excitation gas and the condensate respectively. Corresponding unit 4-velocities are

\[
U^\alpha = \frac{\beta^\alpha}{\sqrt{\beta^\beta \beta_\beta}}, \quad V^\alpha = \frac{\mu^\alpha}{\sqrt{\mu^\beta \mu_\beta}}. \tag{21}\]

In place of the scalars \(I_1, I_2, I_3\) we use new three invariants, a chemical potential \(\mu = \sqrt{\mu^\beta \mu_\beta}\), scalar \(\gamma = V_\alpha U^\alpha\) associated with the relative motion of the components, and inverse temperature with respect to the reference frame comoving to the excitation gas \(\beta = \sqrt{\beta^\beta \beta_\beta}\).

Using (19) and (21) the energy-momentum tensor and the particle number current are readily represented as

\[
n^\alpha = n_c V^\alpha + n_n U^\alpha \tag{22}\]

\[
T_{\alpha\beta} = \mu n_c V_\alpha V_\beta + n_n U_\alpha U_\beta - P g_{\alpha\beta}, \tag{23}\]

Relations between the macroscopic quantities involved in equations (22) and (23) are of a quite general form. More detail information about them can be obtained from statistical description of the elementary excitations. The quasiparticle energy spectrum has a significant
nonlinear dispersion at high energy, and therefore completely relativistic description has been carried out only for a low energy excitations, phonons [12, 13]. Based on the relativistic kinetic theory of the phonon gas [13] we in particular can obtain

\[ \frac{\mu_n}{\gamma} = (1 - c_s^2)W_n, \] (24)

when phonons prevail over another sorts of quasiparticles.

Let us assume that the generalized pressure function is separated as follows:

\[ P(\mu, \beta, \gamma) = p_c(\mu) + p_n(\mu, \beta, \gamma). \] (25)

Equation (25) is best suited for the phonon gas since it describes neglect of excitation influence on the ground state. This ansatz retains equations (16) and (17) valid for the condensate in the framework of two-fluid dynamics.

Let us also suppose that the pressure of the normal component depends on temperature by the power law, \( p_n \propto T^{\nu+1} \). This assumption is a generalization of the dependence \( p_n \propto T^4 \) obtained by Carter and Langlois [12] for the equilibrium distribution of the phonon gas followed by the relation \( p_n = c_s^2 W_n / 4 \). In our case it transforms to

\[ p_n = \frac{c_s^2}{1 + \nu} W_n. \] (26)

Equation (26) is a kind of barotropic equation of state and \( \nu \) is a properly polytropic index. Its value governs so-called second sound speed \( c_2 \), the sound speed in the excitation gas. In the limit of low temperatures when the quasiparticle contribution becomes small \( c_2 \to c_s / \sqrt{\nu} \), and in the case of the equilibrium phonon gas it coincides with the result obtained in [12].

We restrict our consideration to the equation of state (26) for the normal component situated between the dust one and the stiff one. It is evident from equation (26) that this constraint implies \( \nu \geq 1 \).

4 Universe with BEC

4.1 Equations of motion

The cosmic medium is now regarded as a matter which particularly is in the BEC state and its particle number current and energy-momentum tensor have the form (22) and (23). We assume that the self-dependent condensate ansatz (25) holds and the superfluid background obeys the equation of state (16) and the excited state is described by the relations (24) and (26). This means that we ignore nonlinear high-energy part of the quasiparticle spectrum. Although these restrictions render the model incomplete they essentially simplify the evolution equations retained simultaneously the key features of the model.
Let us consider a homogeneous and isotropic spatially flat universe. In this case the superfluid and normal velocities are equal and thus $\gamma = 1$. Einstein equations then reduce to

$$3\frac{\dot{a}^2}{a^2} = 8\pi G \rho_{\text{tot}}, \quad -6\frac{\ddot{a}}{a} = 8\pi G(3p_{\text{tot}} + \rho_{\text{tot}}),$$

(27)

where $\rho_{\text{tot}}$ consists of the condensate density $\rho_c$ and the normal one $\rho_n = W_n - p_n$ that are interpretable as DE and DM densities respectively, and $p_{\text{tot}} = p_c + p_n$. In accordance with the integrability conditions of Einstein equations we require local energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$ that yields

$$\dot{\rho}_{\text{tot}} + 3\frac{\dot{a}}{a}(p_{\text{tot}} + \rho_{\text{tot}}) = 0.$$  

(28)

The interaction between DE and DM is implicitly included in equation (28) and also in particle number conservation $\nabla_\mu n^\mu = 0$ that leads to

$$\dot{n}_{\text{tot}} + 3\frac{\dot{a}}{a}n_{\text{tot}} = 0 \quad \implies \quad n_c + n_n = \frac{n_0}{a^3}, \quad n_0 = \text{const.}$$

(29)

This approach distinguishes the present model from [15] where the rate of the transition between ground and excited states $\Gamma$ is explicitly used as an interaction factor and equation (28) breaks down into separated balance equations for DE and DM. In [16] the similar splitting is applied for interaction between Chaplygin gas (it is regarded as DE) and CDM.

Taking into account the expressions (24)–(26) and (29) we reduce equations (27) and (28) to following two dimensionless equations:

$$3(1 + \nu)\frac{\dot{a}^2}{a^2} = \frac{1}{\rho} + \frac{k}{a^3} \left( \frac{\nu \rho}{\sqrt{\rho^2 - 1}} + \frac{\sqrt{\rho^2 - 1}}{\rho} \right),$$

(30)

$$3\frac{\dot{a}}{a} \left( 1 + \nu - \frac{k}{a^3} \frac{1}{\sqrt{\rho^2 - 1}} \right) + \frac{\dot{\rho}}{\rho} \left( 1 - \frac{k}{a^3} \left( \frac{1}{\sqrt{\rho^2 - 1}} - \frac{\nu \rho^2}{(\rho^2 - 1)^{3/2}} \right) \right) = 0,$$

(31)

where $\rho = \rho_c/2M$, and $k = n_0/2\sqrt{\lambda M}$. The dimensionless time variable $t'$ is connected with real time $t$ as $t' = \sqrt{16\pi GMt}$.

4.2 Exact solution

In the formal limit $\nu \to \infty$ equations (30) and (31) are solved analytically. As obvious from (26) the quasiparticle pressure is neglected and DM behaves as dust-like matter. In this case Eq. (31) yields

$$\rho_c = \sqrt{\frac{k^2}{(a^3 + b^3)^2} + 1},$$

(32)

and the scale factor varies in according to the integral

$$t' = \int \sqrt{3a \, da} \sqrt{\frac{1}{a^3 + b^3} \sqrt{k^2(a^3 + b^3)^{-2} + 1}}.$$
The parameter $k$ gives an initial normalized total particle number density and $b$ associates with an initial particle number density for the normal component. More precisely, $b^3 = (n_0/n_n(0) - 1)^{-1}$. If $b = 0$, the normal component is unavailable and the evolution follows the scenario proposed in [6] since the ground state obeys the equation of state (16). In the case of $b \neq 0$ DM is governed by the law

$$\rho_n = \frac{b^3}{a^3} \sqrt{\frac{k^2}{(a^3 + b^3)^2} + 1}. \quad (34)$$

At the beginning stage (i.e. for small $a$) the total energy density is approximated by $\rho_{tot} \propto a^{-3}$ that corresponds to a universe dominated by dust-like matter. The same behavior is a feature of Chaplygin gas [6] but even though in this model the condensate has the same equation of state, such dependence is due to the normal component.

At the late stage (i.e. for large $a$) $\rho_{tot} \to 1$. Separating now DE and DM contributions one finds the subleading terms are

$$\rho_c \sim 1 + \frac{k^2}{2} a^{-6}, \quad (35)$$

$$\rho_n \sim \frac{b^3}{a^3}, \quad (36)$$

whereas the scale factor time evolution corresponds to de Sitter spacetime, namely, $a \propto e^{\dot{a}/\sqrt{3}}$. At this stage the fluid is almost in the ground state so that the condensate wave function is close to the potential minimum. The behavior is similar to GCG [7]: expressions (35) and (36) imply that the system evolves as a mixture of the cosmological constant and the dust-like matter. Note, the asymptotic formula (35) is valid for any value of $\nu$.

### 4.3 Numerical simulation

When $\nu$ has a finite value equations (30) and (31) are solved numerically. It emerges that in the context of the concerned scenario the universe expansion may be decelerated or accelerated from the start. Nevertheless once the universe starts accelerating it cannot decelerate anymore and eventually falls within de Sitter phase. We regard only the solutions with initial deceleration. Photometric observations of apparent Type Ia supernovae attests that the recent cosmological acceleration commenced at $0.3 < z < 0.9$ [14]. To fix time scaling, $\dot{a}$ is assumed to be in the minimum when the redshift $z = 0.5$.

Equations (30) and (31) are solved with initial conditions $a(0) = 1$ and $\rho(0)$ is a constant greater than 1. The coefficient $k$ is imposed to be more than $\sqrt{\rho(0)^2 - 1}$ to ensure that both DE and DM contents are available. At the final stage DM dies out gradually. This means that all particles pass into the ground state and the normalized condensate number of particles in a comoving volume $a^3$, namely $N_c = n_c a^3$, goes to 1 while the number density scalar $n_c$ decays to zero (see Fig. 1). If we start with a small value of the background number density then the
beginning condensate production rate is so high that not only the condensate particle number \( N_c \) increases but the number density \( n_c \) as well. To the contrary, an excess of the condensate particles involves monotone behavior of the condensate number density and the particle number in the comoving volume.

Fig. 2 depicts an evolution of the normalized energy densities \( \Omega_c \) and \( \Omega_n \) of DE and DM respectively. The curves are plotted for different values of \( \nu \) and demonstrate increasing of the DM content when \( \nu \) increases. Correspondence with the current observational value of the DE fraction \( \Omega_c \approx 0.72 \) falls on \( \nu \approx 25 \). This implies a high-degree temperature dependence of the phonon energy and the second sound speed with a value much less than it would be expected for real phonons. Note, that in superfluid helium a lower second sound speed is provided by quasiparticles from the nonlinear part of the energy spectrum (such as rotons). In the context of the pure phonon consideration they are not taken into account and their influence is simulated with a large value of \( \nu \).

Plots of equation-of-state parameters \( w_A = p_A/\rho_A \) (A=c, n, tot) as a function of the redshift \( z \) is shown on Fig. 3. The DE equation-of-state \( w_c \) occurs close to \(-1\) extremely fast after the universe starts to accelerate. This is accompanied by vanishing of the excitations and the total equation-of-state \( w_{\text{tot}} \) is found to be the same at the same time. Simultaneously the DM one \( w_n \) approaches the value of \( 1/\nu \) that is the asymptotic value for the second sound speed as the sound speed \( c_s \rightarrow 1 \). If \( \nu \) is large then DM is perceived as a nonrelativistic one. This CDM-like behavior is observed within the whole range of the redshift under consideration. It is expected that the excitations with a nonlinear dispersion will promote decreasing the equation-of-state \( w_n \) when the complete quasiparticle spectrum is taken into account.

## 5 Conclusion

In this letter we examine the model of superfluid Chaplygin gas (SCG) describing the dark sector of the universe as a matter that behaves as DE while it is in the ground state and as DM when it is in the excited state. Cosmological dynamics is described in the framework of the two-fluid model therefore the interaction between DE and DM is implicitly involved into the conservation laws (28) and (29). In this approach there is no need to introduce different equations for the description of DE and DM evolution and to use the interaction factor as an additional parameter. Moreover, if we abandon the self-dependent condensate ansatz (25), it will be impossible to uniquely divide the total energy density into the DE and DM fractions.

The SCG model is applied to the universe evolution from the deceleration-acceleration transition epoch. It provides the current mixture of the dark contents and approaches de Sitter phase in the future. The normal component (DM) is formed by a pure phonon gas with the pressure varying as the power \( \nu + 1 \) of temperature. Simple fitting the model parameters to the observational data shows that \( \nu \) must be quite large. In this case DM behaves as CDM. In the standard model CDM consists of nonrelativistic massive particles whereas in this model it
is described as an excited state of an exotic matter with a low second sound speed. To develop a more realistic model, a wide quasiparticle spectrum should be taken into account so that the quasiparticles with a nonlinear dispersion are dominant in the modern epoch while the phonons prevail in the future at the temperatures close to zero. Further all the particles fall into the ground state and the final epoch is de Sitter universe. The model is integrated in more early epochs as a usual matter since it almost fully is in the excited state that time.

It should be emphasized the difference between the GCG and SCG models. In the former the energy density $\rho_{\text{tot}} = \rho_{\text{Ch}}$ consists of both vacuum and matter contributions. The parameter $\alpha$ (see introduction) is assumed to satisfy the constraint $\alpha < 0.4$ [17] to be in agreement with observations. In the latter $\rho_{\text{tot}} = \rho_c + \rho_n$, where the condensate with the density $\rho_c = \rho_{\text{Ch}}$ can be considered as GCG and the previous restriction is unnecessary. Moreover ordinary Chaplygin gas ($\alpha = 1$) is preferred since $\alpha$ has no such a dramatic effect for this model as in [7, 8, 9] and its governing role for the interaction between DE and DM goes to $\nu$. This is not to say that $\alpha$ and $\nu$ have an identical significance. We can see that while the GCG model becomes equivalent to $\Lambda$CDM for $\alpha = 0$, SCG persists as a model unified DE and DM for $\nu \to \infty$. Therefore we can use $\alpha$ as an additional parameter in the SCG model later.

There is another problem in the models of DE and DM unification. It concerns the non-negligible sound speed that produces unphysical oscillations and an exponential blow-up in the DM power spectrum at present [18]. This problem was solved for the GCG model in [9] by the special decomposition of the energy density into DE and DM components. In the SCG model DM is generated by excitations and a role of the sound is played by the second sound. We should expect that a perturbative analysis of the energy density fluctuations of DM will lead to additional restrictions on the parameters. The heuristic estimation of $\alpha$ relying on scales of Galaxy clusters obtained in [18] gives the range $|\alpha| < 10^{-5}$ for GCG that means its indistinguishability from $\Lambda$CDM. The similar estimation for SCG can be given by the formal replacement $\alpha \to 1/(1 + \nu)$. As one would expect pressureless DM ($\nu \to \infty$) is free from the blow-up and the restriction $\nu > 10^5$ is in agreement with the foregoing inference regarding the quasiparticle spectrum.
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Figure 1: The condensate number density in the units of $n_0$ as a function of the redshift $z$ for $k = 15$ and $\nu = 3$. The solid ($\rho(0) = 1.02$) and dashed ($\rho(0) = 5$) curves correspond to deficit and exceed of the particles in the ground state respectively.

Figure 2: The ratio of the energy density to the critical density for the different components as a function of the redshift $z$ for $k = 120$ and $\rho(0) = 1.2$.

Figure 3: The equation-of-state parameters as a function of the redshift $z$ for $k = 120$ and $\rho(0) = 1.2$. 