Lepton Flavor Violation within a realistic SO(10)/G(224) Framework

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Abstract

Lepton flavor violation (LFV) is studied within a realistic unified framework, based on supersymmetric SO(10) or an effective G(224) = SU(2)\textsubscript{L} × SU(2)\textsubscript{R} × SU(4)\textsuperscript{c} symmetry, that successfully describes (i) fermion masses and mixings, (ii) neutrino oscillations, as well as (iii) CP violation. LFV emerges as an important prediction of this framework, bringing no new parameters, barring the few SUSY parameters, which are assumed to be flavor-universal at \( M^* \gtrsim M_{GUT} \). We study LFV (i.e. \( \mu \to e\gamma \), \( \tau \to \mu\gamma \), \( \tau \to e\gamma \) and \( \mu N \to eN \)) within this framework by including contributions both from the presence of the right handed neutrinos as well as those arising from renormalization group running in the post-GUT regime (\( M^* \to M_{GUT} \)). Typically the latter, though commonly omitted in the literature, is found to dominate. Our predicted rates for \( \mu \to e\gamma \) show that while some choices of \( (m_0, m_{1/2}) \) are clearly excluded by the current empirical limit, this decay should be seen with an improvement of the current sensitivity by a factor of 10–100, even if sleptons are moderately heavy (\( \lesssim 800 \) GeV, say). For the same reason, \( \mu - e \) conversion (\( \mu N \to eN \)) should show in the planned MECO experiment. Implications of WMAP and \( (g - 2)_\mu \)-measurements are noted, as also the significance of the measurement of parity-odd asymmetry in the decay of polarized \( \mu^+ \) into \( e^+\gamma \).
1 Introduction

Individual lepton numbers ($L_e$, $L_\mu$ and $L_\tau$) being symmetries of the standard model (SM) (with $m_i^L = 0$), processes like $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are forbidden within this model. Even within simple extensions of the SM (that permit $m_i^L \neq 0$), they are too strongly suppressed to be observable. Experimental searches have put upper limits on the branching ratios of these processes: $\text{Br}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [1], $\text{Br}(\tau \to \mu\gamma) \leq 3.1 \times 10^{-7}$ [2] and $\text{Br}(\tau \to e\gamma) \leq 3.7 \times 10^{-7}$ [3]. The extreme smallness of these branching ratios poses a challenge for physics beyond the standard model, especially for supersymmetric grand unified (SUSY GUT) models, as these generically possess new sources of lepton flavor violation that could easily lead to rates even surpassing the current limits.

In this paper, we study how lepton flavor violation (LFV) gets linked with fermion masses, neutrino oscillations and CP violation within a predictive SUSY grand unified framework, based on either SO(10) [4], or an effective (presumably string derived) symmetry $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ [5]. The desirability of having an effective symmetry that possesses SU(4)-color [5], in view of the observed neutrino oscillations and the likely need for leptogenesis as a means for baryogenesis [6, 7], has been stressed elsewhere (see e.g. [8]).

A predictive framework based on supersymmetric SO(10) or G(224)-symmetry has been proposed by Babu, Pati and Wilczek (BPW) in [9], which successfully describes the masses and mixings of all fermions including neutrinos. In particular it makes seven predictions, all in good accord with observations. This framework was recently extended by us in Ref. [10] to describe the observed CP and flavor violations by allowing for phases in the fermion mass matrices. Remarkably enough, this extension could successfully describe the masses of all the quarks and leptons (especially of the two heavier families), the CKM elements, the observed CP and flavor violations in the $K^0 - \bar{K}^0$ system (yielding correctly $\Delta m_K$ and $\epsilon_K$) and the $B_d^0 - \bar{B}_d^0$ system (yielding the correct values of $\Delta m_{B_d}$ and $S_{\psi K_S}$).

In this paper, we study lepton flavor violating processes, i.e. $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ and $\mu N \to eN$, within the same framework [9,10]. The subject of LFV has been discussed widely in the literature within supersymmetric extensions of the standard model. (For earlier works see Ref. [11–13]). Our work based on SUSY SO(10) or G(224) differs from those based on either MSSM with right-handed neutrinos (RHN’s) [11,13,14] or SUSY SU(5) [15], because
for these latter cases the RHN’s are singlets and thereby their Yukawa couplings are \textit{a priori} arbitrary. By contrast, for G(224) or SO(10) the corresponding Yukawa couplings are determined in terms of those of the quarks at the GUT-scale (such as $h(\nu^\tau)_{Dirac} \approx h_{top}$) (see Ref. [9]). Thus the SUSY G(224)/SO(10)-framework is naturally more predictive than the MSSM or SUSY SU(5)-framework.

In addition, our work differs from all others, including those based on SUSY SO(10) [16] as well, in two other important respects: First, we work within a predictive and realistic framework [9, 10] which (as mentioned above) successfully describes a set of phenomena – i.e. (a) fermion masses, (b) CKM mixings, (c) neutrino oscillations, (d) observed CP and flavor violations in the K and B systems, as well as (e) baryogenesis via leptogenesis [8]. As we will see, lepton flavor violation emerges as an important prediction of this framework, bringing no new parameters (barring the few flavor-universal SUSY-parameters).

Second, we do a comprehensive study of LFV processes by including contributions from three different sources: (i) the sfermion mass-insertions, $\hat{\delta}^{ij}_{LL,RR}$, arising from renormalization group (RG) running from $M^*$ to $M_{GUT} \sim 2 \times 10^{16}$ GeV (where $M^*$ denotes the presumed messenger-scale, with $M_{GUT} < M^* \leq M_{string}$, at which flavor-universal soft SUSY breaking is transmitted to the squarks and sleptons, like in a mSUGRA model [17]), (ii) the mass-insertions $\left(\delta^{ij}_{LL}\right)_{RHN}$ arising from RG running from $M_{GUT}$ to the right handed neutrino mass scales $M_{R_i}$, and (iii) the chirality-flipping mass-insertions $\delta^{ij}_{LR,RL}$ arising from $A$–terms that are induced solely through RG running from $M^*$ to $M_{GUT}$ involving gauginos in the loop. All the three types of mass-insertions: $\hat{\delta}^{ij}_{LL,RR}$, $\left(\delta^{ij}_{LL}\right)_{RHN}$ and $\delta^{ij}_{LR,RL}$ are in fact fully determined in our model. (See [10] for details). Most previous works in this regard have included only the second contribution associated with the RH neutrinos in their analysis.\footnote{Barbieri, Hall and Strumia (in Ref. [12]) have discussed the relevance of the contributions from the mass-insertions $\hat{\delta}^{ij}_{LL,RR}$ and those from the induced $A$–terms, but without a realistic framework for light fermion masses and neutrino oscillations.} We find, however, that it is the first and the third contributions associated with post-GUT physics that typically dominate over the second in a SUSY unified framework.

A brief review of our previous work and our results are presented in the following sections.
2 A brief review of the BPW framework and its extension

The Dirac mass matrices of the sectors \( u, d, l \) and \( \nu \) proposed in Ref. [9] in the context of SO(10) or G(224)-symmetry have the following structure:

\[
M_u = \begin{bmatrix}
0 & \epsilon' & 0 \\
-\epsilon' & \zeta_2^u & \sigma + \epsilon \\
0 & \sigma - \epsilon & 1
\end{bmatrix} \mathcal{M}_u^0; \quad M_d = \begin{bmatrix}
0 & \eta' + \epsilon' & 0 \\
\eta' - \epsilon' & \zeta_2^d & \eta + \epsilon \\
0 & \eta - \epsilon & 1
\end{bmatrix} \mathcal{M}_d^0
\]

\[
M_v^D = \begin{bmatrix}
0 & -3\epsilon' & 0 \\
3\epsilon' & \zeta_2^u & \sigma - 3\epsilon \\
0 & \sigma + 3\epsilon & 1
\end{bmatrix} \mathcal{M}_v^0; \quad M_l = \begin{bmatrix}
0 & \eta' - 3\epsilon' & 0 \\
\eta' + 3\epsilon' & \zeta_2^d & \eta - 3\epsilon \\
0 & \eta + 3\epsilon & 1
\end{bmatrix} \mathcal{M}_l^0
\]

These matrices are defined in the gauge basis and are multiplied by \( \Psi_L \) on left and \( \Psi_R \) on right. For instance, the row and column indices of \( M_u \) are given by \((\bar{u}_L, \bar{c}_L, \bar{t}_L)\) and \((u_R, c_R, t_R)\) respectively. These matrices have a hierarchical structure which can be attributed to a presumed U(1)-flavor symmetry (see e.g. [8,10]), so that in magnitudes \( 1 \gg \sigma \sim \eta \sim \epsilon \gg \zeta_2^u \sim \zeta_2^d \gg \eta' > \epsilon' \). The entries \( \epsilon \) and \( \epsilon' \) are proportional to \( B - L \) and are antisymmetric in family space (see below). Thus \((\epsilon, \epsilon') \rightarrow -3(\epsilon, \epsilon') \) as \( q \rightarrow l \). Following the constraints of SO(10) and the U(1)-flavor symmetry, such a pattern of mass-matrices can be obtained using a minimal Higgs system consisting of \( 45_H, 16_H, \bar{16}_H, 10_H \) and a singlet \( S \) of SO(10), which lead to effective couplings of the form [8,10]:

\[
\mathcal{L}_{Yuk} = h_{33} 16_3 16_3 10_H + [h_{23} 16_2 16_3 10_H (S/M) \\
+ a_{23} 16_2 16_3 10_H (45_H/M') (S/M)^p \ + g_{23} 16_2 16_3 16^d_H (16_H/M'')(S/M)^q] \\
+ [h_{22} 16_2 16_2 10_H (S/M)^2 \ + g_{22} 16_2 16_2 16^d_H (16_H/M'') (S/M)^q+1] \\
+ [g_{12} 16_1 16_2 16^d_H (16_H/M'') (S/M)^{q+2} + a_{12} 16_1 16_2 10_H (45_H/M'') (S/M)^{p+2}] .
\]

The mass scales \( M', M'' \) and \( M \) are of order \( M_{\text{string}} \) or (possibly) of order \( M_{\text{GUT}} \) [18]. Depending on whether \( M'(M'') \sim M_{\text{GUT}} \) or \( M_{\text{string}} \) (see [18]), the exponent \( p(q) \) is either one or zero [19]. The VEVs of \( \langle 45_H \rangle \) (which is along \( B - L \)), \( \langle 16_H \rangle = \langle \bar{16}_H \rangle \) (along \( \langle \bar{v}_{RH} \rangle \)) and \( \langle S \rangle \) are of the GUT-scale, while those of \( \langle 10_H \rangle \) and \( \langle 16_H^d \rangle \) are of the electroweak scale [9,20]. The combination \( 10_H . 45_H \) effectively acts like a \( 120 \) which is antisymmetric in family space...
and is along $B-L$. The hierarchical pattern is determined by the suppression of the couplings by appropriate powers of $M_{\text{GUT}}/(M, M'or M'')$. For details on how Eq. (2) emerges from Eq. (1) see Refs. [8–10].

The right-handed neutrino masses arise from the effective coupling of the form [21]:

$$\mathcal{L}_{\text{Maj}} = f_{ij}\overline{16}_j \overline{16}_H \overline{16}_H / M$$

where the $f_{ij}$'s include appropriate powers of $\langle S \rangle / M$. The hierarchical form of the Majorana mass-matrix for the RH neutrinos is [9]:

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

(4)

Following flavor charge assignments (see [8]), we have $1 \gg y \gg z \gg x$. The magnitude of $M_R$ is estimated (with $f_{33} \approx 1$, $\langle \overline{16}_H \rangle \approx 2 \times 10^{16}$ GeV and $M \approx M_{\text{st}} \approx 4 \times 10^{17}$ GeV) as [8, 9]: $M_R = f_{33}(\overline{16}_H)^2 / M \approx (10^{15} \text{ GeV})(1/2–2)$.

Thus the Majorana masses of the RH neutrinos are given by [8, 9]:

$$M_3 \approx M_R \approx 10^{15} \text{ GeV} (1/2–1), \quad M_2 \approx |y|^2 M_3 \approx (2.5 \times 10^{12} \text{ GeV})(1/2–1),$$

$$M_1 \approx |x - z|^2 M_3 \approx 10^{10} \text{ GeV}(1/4–2).$$

(5)

Note that both the RH neutrinos and the light neutrinos have hierarchical masses.

In the BPW model of Ref. [9], the parameters $\sigma, \eta, \epsilon$ etc. were chosen to be real. To allow for CP violation, this framework was extended to include phases for the parameters in Ref. [10]. Remarkably enough, it was found that there exists a class of fits within the SO(10)/G(224) framework, which correctly describes not only (a) fermion masses, (b) CKM mixings and (c) neutrino oscillations [8,9], but also (d) the observed CP and flavor violations in the K and B systems (see Ref. [10] for the predictions in this regard). A representative of this class of fits (to be called fit A) is given by [10]:

$$\sigma = 0.109 - 0.012i, \quad \eta = 0.122 - 0.0464i, \quad \epsilon = -0.103, \quad \eta' = 2.4 \times 10^{-3},$$

$$\epsilon' = 2.35 \times 10^{-4}e^{i(69^\circ)}, \quad \zeta_2^d = 9.8 \times 10^{-3}e^{-i(149^\circ)}, \quad (\mathcal{M}_u^0, \mathcal{M}_d^0) \approx (100, 1.1) \text{ GeV}.$$  

(6)

In this particular fit, $\zeta_2^u$ is set to zero for the sake of economy in parameters. However, allowing for $\zeta_2^u \approx (1/3)(\zeta_2^d)$ would still yield the desired results. Because of the success
of this class of fits in describing correctly all four features (a), (b), (c) and (d)-which is a non-trivial feature by itself - we will use fit A as a representative to obtain the mass-insertion parameters $\hat{\delta}_{LL,RR}^{ij}$, $(\delta_{LL}^{ij})^{RHN}$ and $\delta_{LR,RL}^{ij}$ in the lepton sector and thereby the predictions of our model for lepton flavor violation.

The fermion mass matrices $M_u$, $M_d$ and $M_l$ are diagonalized at the GUT scale $\approx 2 \times 10^{16}$ GeV by bi-unitary transformations:

$$M_{u,d,l}^{diag} = X_{L,R}^{(u,d,l)\dagger}M_{u,d,l}^{(u,d,l)} X_{L,R}^{(u,d,l)}$$

(7)

The analytic expressions for the matrices $X_{L,R}^{d}$ can be found in [10]. The corresponding expressions for $X_{L,R}^{l}$ can be obtained by letting $(\epsilon, \epsilon') \to -3(\epsilon, \epsilon')$.

We now discuss the sources of lepton flavor violation in our model.

3 Lepton Flavor Violation in the SO(10)/G(224) Framework

We assume that flavor-universal soft SUSY-breaking is transmitted to the SM-sector at a messenger scale $M^*$, where $M_{GUT} < M^* \leq M_{string}$. This may naturally be realized e.g. in models of mSUGRA [17], or gaugino-mediation [22]. With the assumption of extreme universality as in CMSSM, supersymmetry introduces five parameters at the scale $M^*$:

$$m_o, m_{1/2}, A_o, \tan \beta \text{ and } sgn(\mu).$$

For most purposes, we will adopt this restricted version of SUSY breaking with the added restriction that $A_o = 0$ at $M^*$ [22]. However, we will not insist on strict Higgs-squark-slepton mass universality. Even though we have flavor preservation at $M^*$, flavor violating scalar (mass)$^2$-transitions arise in the model through RG running from $M^*$ to the EW scale. As described below, we thereby have three sources of lepton flavor violation.

(1) RG Running of Scalar Masses from $M^*$ to $M_{GUT}$.

With family universality at the scale $M^*$, all sleptons have the mass $m_o$ at this scale and the scalar (mass)$^2$ matrices are diagonal. Due to flavor dependent Yukawa couplings, with $h_t = h_b = h_\tau (=h_{33})$ being the largest, RG running from $M^*$ to $M_{GUT}$ renders the third family lighter than the first two (see e.g. [12]) by the amount:

$$\Delta \hat{m}_{\tilde{l}}^2 = \Delta \hat{m}_{\tilde{b}}^2 = \Delta \hat{m}_{\tilde{\tau}_L}^2 = \Delta \hat{m}_{\tilde{\tau}_R}^2 \equiv \Delta \approx -\frac{(30m_o^2)}{16\pi^2} h_t^2 \ln(M^*/M_{GUT}) .$$

(8)
The factor 30→12 for the case of G(224). The slepton (mass)^2 matrix thus has the form 
\( \tilde{M}_l^{(o)} = \text{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2 - \Delta) \). As mentioned earlier, the spin-1/2 lepton mass matrix is diagonalized at the GUT scale by the matrices \( X_{L,R} \). Applying the same transformation to the slepton (mass)^2 matrix (which is defined in the gauge basis), i.e. by evaluating \( X_L^{(t)}(\tilde{M}_l^{(o)})_{LL} X_L \) and similarly for L→R, the transformed slepton (mass)^2 matrix is no longer diagonal. The presence of these off-diagonal elements (at the GUT-scale) given by:
\[
(\tilde{\delta}_{LL,RR})_{ij} = \left( X_{L,R}^{(t)}(\tilde{M}_l^{(o)})_{LL} X_{L,R} \right)_{ij} / m_l^2
\]
induces flavor violating transitions \( \tilde{l}_{L,R} \rightarrow \tilde{l}'_{L,R} \). Here \( m_l \) denotes an average slepton mass and the hat signifies GUT-scale values.

(2) RG Running of the A-parameters from \( M^* \) to \( M_{\text{GUT}} \).

Even if \( A_o = 0 \) at the scale \( M^* \) (as we assume for concreteness, see also [22]), RG running from \( M^* \) to \( M_{\text{GUT}} \) induces A-parameters at \( M_{\text{GUT}} \), invoing the SO(10)/G(224) gauginos; these yield chirality flipping transitions \( (\tilde{l}_{L,R} \rightarrow \tilde{l}'_{R,L}) \).

Evaluated at the GUT scale, the A-parameters, induced respectively through the couplings \( h_{ij}, a_{ij} \) and \( g_{ij} \), are given by:
\[
A_{ij}^{(1)} = \frac{63}{2} \frac{1}{8\pi^2} g_1^2 \eta \ln(\frac{M^*}{M_{10H}}) \quad (i,j = 2,3)
\]
\[
A_{ij}^{(2)} = \frac{95}{2} \frac{1}{8\pi^2} a_{ij} (\frac{45H}{M^*})^2 g_1^2 \eta \ln(\frac{M^*}{M_{10H}}) \quad (i,j = 23,12)
\]
\[
A_{ij}^{(3)} = \frac{90}{2} \frac{1}{8\pi^2} g_{ij} (\frac{10H}{M^*})^2 g_1^2 \eta \ln(\frac{M^*}{M_{16H}}) \quad (i,j = 23,22,12)
\]

The coefficients \( (\frac{63}{2}, \frac{95}{2}, \frac{90}{2}) \) are the sums of the Casimirs of the SO(10) representations of the chiral superfields involved in the diagrams. For the case of G(224), \( (\frac{63}{2}, \frac{95}{2}, \frac{90}{2}) \rightarrow (\frac{27}{2}, \frac{45}{2}, \frac{42}{2}) \). Thus, summing \( A^{(1)}, A^{(2)} \) and \( A^{(3)} \), the induced A matrix for the leptons is given by:
\[
A^{l}_{LR} = Z \left\{ K_{10} \left[ \begin{array}{ccc} 0 & -285\epsilon' & 0 \\ 285\epsilon' & 63\zeta_{22} & -285\epsilon + 63\sigma \\ 0 & 285\epsilon + 63\sigma & 63 \end{array} \right] + K_{16} \left[ \begin{array}{ccc} 0 & 90\eta' & 0 \\ 90\eta' & 90(\zeta_{22} - \zeta_{22}) & 90(\eta - \sigma) \\ 0 & 90(\eta - \sigma) & 0 \end{array} \right] \right\} \quad (11)
\]
where \( Z = (\frac{1}{16\pi}) h_4 g_1^2 \eta \ln(\frac{M^*}{M_{10H}}) \), \( K_{10} = \ln(\frac{M^*}{M_{10H}}) \) and \( K_{16} = \ln(\frac{M^*}{M_{16H}}) \). For simplicity if we let \( M_{16H} \approx M_{10H} \approx M_{\text{GUT}} \), we can write the A matrix in the SUSY basis as:
\[
A^{l}_{LR} = Z \ln(\frac{M^*}{M_{\text{GUT}}}) (X_L^t) \left[ \begin{array}{ccc} 0 & -285\epsilon' + 90\eta' & 0 \\ 285\epsilon' + 90\eta' & 90(\zeta_{22} - \zeta_{22}) & -285\epsilon + 90(\eta - \sigma) \\
0 & 285\epsilon + 90(\eta - \sigma) - 27\sigma & 63 \end{array} \right] X_R^t \quad (12)
\]
Approximate analytic forms for $X_{L,R}^d$ are given in Ref. [10], and $X_{L,R}^l$ can be obtained from $X_{L,R}^d$ by the substitutions $(\epsilon, \epsilon') \rightarrow -3(\epsilon, \epsilon')$. The chirality flipping transition angles are defined as:

$$ (\delta_{LR})_{ij} \equiv (A_{LR})_{ij} \left( \frac{v_d}{m_i^2} \right) = (A_{LR})_{ij} \left( \frac{v_u}{\tan \beta m_i^2} \right). \quad (13) $$

(3) RG Running of scalar masses from $M_{GUT}$ to the RH neutrino mass scales:

We work in a basis in which the charged lepton Yukawa matrix $Y_l$ and $M_{\nu R}'$ are diagonal at the GUT scale. The off-diagonal elements in the Dirac neutrino mass matrix $Y_N$ in this basis give rise to lepton flavor violating off-diagonal components in the left handed slepton mass matrix through the RG running of the scalar masses from $M_{GUT}$ to the RH neutrino mass scales $M_{R_i}$. The RH neutrinos decouple below $M_{R_i}$. (For RGEs for MSSM with RH neutrinos see e.g. Refs. [13] and [23].) In the leading log approximation, the off-diagonal elements in the left-handed slepton (mass)$^2$-matrix, thus arising, are given by:

$$ (\delta_{LL}^{l})_{RHN}^{ij} = -\frac{(3m_o^2 + A_o^2)}{8\pi^2} \sum_{k=1}^{3} (Y_N)_{ik} (Y_N^*)_{jk} \ln \left( \frac{M_{GUT}}{M_{R_k}} \right) \cdot \quad (14) $$

The superscript RHN denotes the contribution due to the presence of the RH neutrinos. We remind the reader that the masses $M_{R_i}$ of RH neutrinos are well determined within our framework to within factors of 2 to 4 (see Eq. (5)). The total LL contribution is thus:

$$ (\delta_{LL}^{l})_{ij}^{Tot} = (\delta_{LL}^{l})_{ij} + (\delta_{LL}^{l})_{ij}^{RHN} \quad (15) $$

Now, most authors including those using SUSY SU(5) with RHN’s or SUSY SO(10) [15, 16] have cosidered only the second term $(\delta_{LL}^{l})_{RHN}$ that arises due to the right-handed neutrinos. As mentioned in the introduction, however, the first term $(\delta_{LL}^{l})$ and the contribution of the $A-$term $\delta_{LR,RL}^{l}$ (Eq. (13)) are found to dominate over the second term (as long as $\ln(M^*/M_{GUT}) \sim 1$). We obtain our results by including the contributions from all three sources listed above in Eqs. (9), (13) and (14). They are presented in the following section.

4 Results

The decay rates for the lepton flavor violating processes $l_i \rightarrow l_j \gamma$ ($i > j$) are given by:

$$ \Gamma(l_i^+ \rightarrow l_j^+ \gamma) = \frac{e^2 m_i^3}{16\pi} \left( |A_L^{ji}|^2 + |A_R^{ji}|^2 \right) \quad (16) $$
Here $A_j^{ij}$ is the amplitude for $(l_i)_{L}^{+} \rightarrow (l_j)^{+} \gamma$ decay, while $A_j^{ij} = A((l_i)_{R}^{+} \rightarrow (l_j)^{+} \gamma)$. The amplitudes $A_j^{ij}_{L,R}$ are evaluated in the mass insertion approximation using the $(\delta_{LL}^{ir})^{Tot}$, $\delta_{RR}^{ir}$, $\delta_{LR,RL}^{ir}$ calculated as above. The general expressions for the amplitudes $A_j^{ij}_{L,R}$ in one loop can be found in e.g. Refs. [13] and [23]. We include the contributions from both chargino and neutralino loops with or without the $\mu-$term.

We evaluate the amplitudes by first going to a basis in which the chargino and the neutralino mass matrices are diagonal. Analytic expressions for this diagonalization can be obtained in the approximation $|M_2 \pm \mu| \gg m_Z$ and $|M_2\mu| > m_W^2 \sin 2\beta$ [24]. This approximation holds for all the input values of $(m_o, m_{1/2})$ that we consider.

In Table 1 as well as in Fig. 1, we give the branching ratios of the processes $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $\tau \rightarrow e\gamma$ for the case of SO(10), with some sample choices of $(m_o, m_{1/2})$. For these calculations we set $\ln\left(\frac{M^*}{M_{GUT}}\right) = 1$, i.e. $M^* \approx 3M_{GUT}$, $\tan\beta = 10$, $M_{R_1} = 10^{10}$ GeV, $M_{R_2} = 10^{12}$ GeV and $M_{R_3} = 5 \times 10^{14}$ GeV (see Eq. (5)), and $A_o(\text{at } M^*) = 0$. The corresponding values for G(224) are smaller approximately by a factor of 4 to 6 in the rate, provided $\ln(M^*/M_{GUT})$ is the same in both cases (see comments below Eqs. (8) and (10)).

| $(m_o, m_{1/2})//\tan\beta$ | $\text{Br}(\mu \rightarrow e\gamma)$ | $\text{Br}(\tau \rightarrow \mu\gamma)$ | $\text{Br}(\tau \rightarrow e\gamma)$ |
|-----------------------------|------------------------|------------------------|------------------------|
| $\mu > 0$ | $\mu < 0$ | $\mu > 0$ | $\mu < 0$ | $\mu > 0$ | $\mu < 0$ |
| I (600, 300)//10 | $3.3 \times 10^{-12}$ | $9.8 \times 10^{-12}$ | $2.4 \times 10^{-9}$ | $3.1 \times 10^{-9}$ | $2.4 \times 10^{-12}$ | $3.3 \times 10^{-12}$ |
| II (800, 250)//10 | $2.9 \times 10^{-13}$ | $1.7 \times 10^{-12}$ | $1.9 \times 10^{-9}$ | $1.9 \times 10^{-9}$ | $2.0 \times 10^{-12}$ | $2.0 \times 10^{-12}$ |
| III (450, 300)//10 | $2.7 \times 10^{-11}$ | $4.6 \times 10^{-11}$ | $2.7 \times 10^{-9}$ | $5.6 \times 10^{-9}$ | $2.7 \times 10^{-12}$ | $6.1 \times 10^{-12}$ |
| IV (500, 250)//10 | $5.9 \times 10^{-12}$ | $1.9 \times 10^{-11}$ | $4.8 \times 10^{-9}$ | $6.4 \times 10^{-9}$ | $5.0 \times 10^{-12}$ | $6.9 \times 10^{-12}$ |
| V (100, 440)//10 | $1.02 \times 10^{-8}$ | $1.02 \times 10^{-8}$ | $8.3 \times 10^{-8}$ | $8.4 \times 10^{-8}$ | $1.0 \times 10^{-10}$ | $1.0 \times 10^{-10}$ |
| VI (1000, 250)//10 | $1.6 \times 10^{-13}$ | $5.6 \times 10^{-12}$ | $9.5 \times 10^{-10}$ | $9.0 \times 10^{-10}$ | $1.0 \times 10^{-12}$ | $9.5 \times 10^{-13}$ |
| VII (400, 300)//20 | $9.5 \times 10^{-12}$ | $3.8 \times 10^{-11}$ | $1.4 \times 10^{-8}$ | $1.8 \times 10^{-8}$ | $1.5 \times 10^{-11}$ | $1.9 \times 10^{-11}$ |

Table 1. Branching ratios of $l_i \rightarrow l_j\gamma$ for the SO(10) framework with $\kappa \equiv \ln(M^*/M_{GUT}) = 1$; $(m_o, m_{1/2})$ are given in GeV, which determine $\mu$ through radiative electroweak symmetry breaking conditions. The entries for $\text{Br}(\mu \rightarrow e\gamma)$ for the case of G(224) would be reduced by a factor $\approx 4 – 6$ compared to that of SO(10) (see text).

To give the reader an idea of the magnitudes of the various contributions, we exhibit in table 2 the amplitudes for the process $\mu \rightarrow e\gamma$ calculated individually from the four sources
\( \delta_{LL}^{ij}, \delta_{LR,RL}^{ij} \) and \( (\delta_{LL}^{ij})^{RHN} \) (see Eqs. (9), (13) and (14)), for a few cases of table 1.

| \((m_0, m_{1/2})(\text{GeV})\) | \(A_L^{(1)}(\delta_{LL})\) | \(A_L^{(2)}(\delta_{LR})\) | \(A_R(\delta_{RL})\) | \(A_L^{(3)}((\delta_{LL})^{RHN})\) |
|-----------------|----------------|----------------|----------------|----------------|
| I, (600, 300)   | \(3.3 \times 10^{-13}\) | \(-6.7 \times 10^{-13}\) | \(-5.9 \times 10^{-13}\) | \(2.4 \times 10^{-14}\) |
| II, (800, 250)  | \(2.9 \times 10^{-13}\) | \(-1.8 \times 10^{-13}\) | \(-1.6 \times 10^{-13}\) | \(2.0 \times 10^{-14}\) |
| IV, (500, 250)  | \(4.8 \times 10^{-13}\) | \(-9.7 \times 10^{-13}\) | \(-8.5 \times 10^{-13}\) | \(3.4 \times 10^{-14}\) |

Table 2. Comparison of the various contributions to the amplitude for \(\mu \to e\gamma\) for cases I, II and IV, with \(\mu > 0\). Each entry should be multiplied by a common factor \(a_0\). Imaginary parts being small are not shown. Note that columns 2, 3 and 4 arising from RG running from \(M^* \to M_{GUT}\) (see text) dominate over the RHN contribution.

Glancing at tables 1 and 2, the following features of our results are worth noting:

1. It is apparent from table 2 that the contribution due to the presence of the RH neutrinos\(^2\) (fifth column) is about an order of magnitude smaller, in the amplitude, than those of the others (proportional to \(\delta_{LL}^{ij}, \delta_{LR}^{ij}\) and \(\delta_{RL}^{ij}\)), listed in columns 2, 3 and 4. The latter arise from RG running of the scalar masses and the \(A\)-parameters in the context of SO(10) or G(224) from \(M^* \to M_{GUT}\). It seems to us that the latter, which have commonly been omitted in the literature, should exist in any SUSY GUT model for which the messenger scale for SUSY-breaking is high \((M^* > M_{GUT})\), as in a mSUGRA model. The inclusion of these new contributions to LFV processes arising from post-GUT physics, that too in the context of a predictive and realistic framework, is the distinguishing feature of the present work.

2. Again from table 2 we see that the two dominant contributions to \(A_L = A(\mu_L^- \to e^+\gamma)\), arising from \(\delta_{LL}\) and \(\delta_{LR}\)-insertions, partially cancel each other if \(\mu > 0\); they would however add if \(\mu < 0\). By contrast, \(A_R\) gets contribution dominantly only from \(\delta_{RL}\) (column 4).\(^3\) As a result we find that in our model, typically, \(|A_R| > |A_L|\) if \(\mu > 0\) and \(|A_L| > |A_R|\) if \(\mu < 0\).

3. Owing to the general prominence of the new contributions from post-GUT physics, we

\(^2\)In the context of contributions due to the RH neutrinos alone, there exists an important distinction (partially observed by Barr, see Ref. [16]) between the hierarchical BPW form [9] and the lop-sided Albright-Barr (AB) form [25] of the mass-matrices. The amplitude for \(\mu \to e\gamma\) from this source turns out to be proportional to the difference between the (23)-elements of the Dirac mass-matrices of the charged leptons and the neutrinos, with (33)-element being 1. This difference is (see Eq. 1) is \(\eta - \sigma \approx 0.041\), which is naturally small for the hierarchical BPW model (incidentally it is also \(V_{\alpha\beta}\), while it is order one for the lop-sided AB model. This means that the rate for \(\mu \to e\gamma\) due to RH neutrinos would be about 600 times larger in the AB model than the BPW model (for the same input SUSY parameters).

\(^3\)Although \(\delta_{RR}\) is comparable to \(\delta_{LL}\), its contribution to \(A_R\) (via the bino loop) is typically suppressed compared to that of \(\delta_{LL}\) to \(A_L\) (in part by the factor \((\alpha_1/\alpha_2)(M_1/M_2))\) in most of the parameter space.
see from table 1 that case V, (with low $m_o$ and high $m_{1/2}$) is clearly excluded by the empirical limit on $\mu \rightarrow e\gamma$-rate (see Sec. 1). Case III is also excluded, for the case of SO(10), yielding a rate that exceeds the limit by a factor of about 2 (for $\kappa = \ln(M^*/M_{GUT}) \gtrsim 1$), though we note that for the case of G(224), Case III is still perfectly compatible with the observed limit (see remark below table 1). All the other cases (I, II, IV, VI, and VII), with medium heavy ($\sim 500$ GeV) to moderately heavy sleptons (800-1000 GeV), are compatible with the empirical limit, even for the case of SO(10). The interesting point about these predictions of our model, however, is that $\mu \rightarrow e\gamma$ should be discovered, even with moderately heavy sleptons, both for SO(10) and G(224), with improvement in the current limit by a factor of 10–100. Such an improvement is being planned at the forthcoming MEG experiment at PSI.

(4) We see from table 1 that $\tau \rightarrow \mu\gamma$ (leaving aside case V, which is excluded by the limit on $\mu \rightarrow e\gamma$), is expected to have a branching ratio in the range of $2 \times 10^{-8}$ (Case VII) to about $(1 \text{ or } 2) \times 10^{-9}$ (Case VI or II). The former may be probed at BABAR and BELLE, while the latter can be reached at the LHC or a super B factory. The process $\tau \rightarrow e\gamma$ would, however, be inaccessible in the foreseeable future (in the context of our model).

(5) The WMAP-Constraint: Of the cases exhibited in table 1, Case V ($m_o = 100$ GeV, $m_{1/2} = 440$ GeV) would be compatible with the WMAP-constraint on relic dark matter density, in the context of CMSSM, assuming that the lightest neutralino is the LSP and represents cold dark matter (CDM), accompanying co-annihilation mechanism. (See e.g. [26]). As mentioned above (see table 1), a spectrum like Case V, with low $m_o$ and higher $m_{1/2}$, is however excluded in our model by the empirical limit on $\mu \rightarrow e\gamma$. Thus we infer that in the context of our model CDM cannot be associated with the co-annihilation mechanism.

Several authors (see e.g. Refs. [27] and [28]), have, however considered the possibility that Higgs-squark-slepton mass universality need not hold even if family universality does. In the context of such non-universal Higgs mass (NUHM) models, the authors of Ref. [28] show that agreement with the WMAP data can be obtained over a wide range of mSUGRA parameters. In particular, such agreement is obtained for $(m_\phi/m_o)$ of order unity (with either sign) for almost all the cases (I, II, III, IV, VI and VII)$^4$, with the LSP (neutralino) representing CDM.$^5$ (Here $m_\phi \equiv \text{sign}(m^2_{H_u,d})\sqrt{|m^2_{H_u,d}|}$, see [28]). All these cases (including

$^4$We thank A. Mustafayev and H. Baer for private communications in this regard.

$^5$We mention in passing that there may also be other possibilities for the CDM if we allow for either non-universal gaugino
Case III for $G(224)$ are of course compatible with the limit on $\mu \to e\gamma$.

(6) **Coherent $\mu - e$ conversion in nuclei:** In our framework, $\mu - e$ conversion (i.e. $\mu^- + N \to e^- + N$) will occur when the photon emitted in the virtual decay $\mu \to e\gamma^*$ is absorbed by the nucleus (see e.g. [29]). In such situations, there is a rather simple relation connecting the $\mu - e$ conversion rate with $B(\mu \to e\gamma) : B(\mu \to e\gamma)/(\omega_{\text{conversion}}/\omega_{\text{capture}}) = R \simeq (230 - 400)$, depending on the nucleus. For example, $R$ has been calculated to be $R \simeq 389$ for $^{27}Al$, $238$ for $^{48}Ti$ and $342$ for $^{208}Pb$ in this type of models. (These numbers were computed in [29] for the specific model of [12], but they should approximately hold for our model as well.) With the branching ratios listed in Table 1 ($\sim 10^{-11}$ to $10^{-13}$) for our model, $\omega_{\text{conversion}}/\omega_{\text{capture}} \simeq (40 - 1) \times 10^{-15}$. The MECO experiment at Brookhaven is expected to have a sensitivity of $10^{-16}$ for this process, and thus will test our model.

(7) **Parity odd asymmetry in $\mu^+ \to e^+\gamma$ decay:** Parity violation can be observed by studying the correlation between the momentum $\vec{p}_e$ of $e^+$ in $\mu^+ \to e^+\gamma$ decay and the polarization vector $\vec{P}$ of positive muons (from $\pi^+$ decays). The distribution of $e^+$ is proportional to $(1 + A \, \hat{p}_e \cdot \vec{P})$ where $A$ is the $P$-odd asymmetry parameter given by $A(\mu^+ \to e^+\gamma) = (|A_L|^2 - |A_R|^2)/(|A_L|^2 + |A_R|^2)$. Here $A_L$ is the amplitude for $\mu^+_L \to e^+\gamma$ decay, while $A_R = A(\mu^+_R \to e^+\gamma)$. In our model, as noted in (2), we typically have $|A_{\mu}^L| > |A_{\mu}^R|$ and thus $A(\mu^+ \to e^+\gamma) < 0$ if $\mu > 0$, and $|A_{\mu}^L| > |A_{\mu}^R|$ and thus $A > 0$ if $\mu < 0$. For example, with $(m_\mu, m_{1/2}) = (800, 250)$ GeV, $\mu > 0$ and $\tan \beta = 10$, we obtain $|A_L| = |A^{(1)}_L(\hat{\delta}_{LL}) + A^{(2)}_L(\delta_{LR}) + A^{(3)}_L| = 1.3 \times 10^{-13}$ (see table 2) while $|A_R| \simeq 1.6 \times 10^{-13}$, and thus $A \simeq -0.25$, while for $(m_\mu, m_{1/2}) = (500, 250)$ GeV and $\tan \beta = 10$ we get, $|A_L| \simeq 4.7 \times 10^{-13}$ and $|A_R| \simeq 8.6 \times 10^{-13}$, yielding $A \simeq -0.54$. The precise prediction of our model for $A$ would thus be definitive once the SUSY spectrum is known.

We can compare the predictions of our model for $A$ with those of other SUSY models. In the MSSM with $\nu_R$, since LFV arises through $\delta_{LL}$ type mixings, $A_L \gg A_R$, and thus $A(\mu^+ \to e^+\gamma) \approx +1$, at least for $\tan \beta \leq 30$ or so, regardless of the choice of $(m_\mu, m_{1/2})$. In SUSY $SU(5)$ GUT, with or without $\nu_R$, the GUT threshold effects realized in the regime $M_{\text{GUT}} \leq \mu \leq M_* \delta_{RR}$ type mixings, and will lead to $A_R \gg A_L$ and thus $A \simeq -1$. In the SUSY $SO(10)$ models with symmetric mass matrices, such as the ones studied in [12,30], $A_L = A_R$ from GUT threshold effects, leading to a vanishing $A$. Thus, we see that a
determination of $\mathcal{A}$ may help sort out the specific type of GUT that is responsible for LFV.

(8) Correlation between muon $g - 2$ and $\mu \to e\gamma$: Currently there exists a discrepancy between theory and experiment in the anomalous magnetic moment of the muon: $\Delta a_\mu = a_\mu^{\text{expt}} - a_\mu^{\text{SM}} = 251(93) \times 10^{-11}$ [31]. This is a 2.7 sigma effect and may be an indication of low energy supersymmetry. In our framework, this discrepancy can be considerably reduced for some, but not all, choices of the SUSY spectrum. When the sleptons are relatively light ($\leq 500 \text{ GeV}$) with $\tan \beta = 10 - 20$, the SUSY contribution to $a_\mu$ is in the range $(50 - 200) \times 10^{-11}$. For example, following a recent numerical analysis (see [32] and references there in), we find $\Delta a_\mu^{\text{SUSY}} \approx 180 \times 10^{-11}$ for the cases of both IV and VII (see table 1). Note that when the SUSY contributions to $\Delta a_\mu$ becomes significant, $B(\mu \to e\gamma)$ is enhanced. Thus, a confirmation of new physics contribution to $a_\mu$, for example by improved precision in the $e^+e^- \to \text{hadron}$ data and in the theoretical analysis, would imply (in the context of a SUSY-explanation) that $\mu \to e\gamma$ is just around the corner, within our framework.

In summary, lepton flavor violation is studied here within a predictive SO(10)/G(224)-framework, possessing supersymmetry, that was proposed in Refs. [9, 10]. The framework seems most realistic in that it successfully describes five phenomena: (i) fermion masses and mixings, (ii) neutrino oscillations, (iii) CP violation, (iv) quark flavor-violations, as well as (v) baryogenesis via leptogenesis [7]. LFV emerges as an important prediction of this framework bringing no new parameters, barring the few flavor-preserving SUSY parameters.

As mentioned before, the inclusion of contributions to LFV arising both from the presence of the RH neutrinos as well as those from the post-GUT regime, that too within a realistic framework, is the distinguishing feature of the present work. Typically, the latter contribution, which is commonly omitted in the literature, is found to dominate. Our results show that – (i) The decay $\mu \to e\gamma$ should be seen with improvement in the current limit by a factor of $10 - 100$, even if sleptons are moderately heavy ($\sim 800 \text{ GeV}$, say); (ii) for the same reason, $\mu - e$ conversion ($\mu N \to e N$) should show in the planned MECO experiment, and (iii) $\tau \to \mu\gamma$ may be accessible at the LHC and a super B-factory. It is noted that the muon ($g - 2$)-anomaly, if confirmed, would strongly suggest, within our model, that the discovery of the $\mu \to e\gamma$ decay is imminent. The significance of a measurement of the parity-odd

\footnote{This analysis is based on theory and data on $e^+e^- \to \text{hadron}$. If $\tau \to \nu\tau$ hadron data is used, this discrepancy reduces to 1.3 sigma; this may however be less reliable [31].}
asymmetry in polarized $\mu^+$ decay into $e^+\gamma$ is also noted. In conclusion, the SO(10)/G(224) framework pursued here seems most successful on several fronts; it can surely meet further stringent tests through a search for lepton flavor violation.

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[20] While $16_H$ has a GUT-scale VEV along the SM singlet, it can also have a VEV of EW scale along the “$\tilde{\nu}_L$” direction due to its mixing with $10^d_H$ (see Ref. [9]).

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Fig. 1. Log of $\text{Br}(\mu \rightarrow e\gamma)$ divided by the experimental bound ($1.2 \times 10^{-11}$) obtained for the SO(10) framework with $\ln(M^*/M_{\text{GUT}}) = 1$, $\tan \beta = 10$ and $\mu > 0$ vs $m_\alpha$ (in GeV) with $m_{1/2} = 200$, 250 and 300 GeV.