$B \to X_sl_i^+l_j^-$ Decays with R-parity Violation

Ji-Ho Jang *, Yeong Gyun Kim † and Jae Sik Lee ‡

Department of Physics, Korea Advanced Institute of Science and Technology
Taejon 305-701, Korea

Abstract

We derive the upper bounds on certain products of R-parity- and lepton-flavor-violating couplings from $B \to X_sl_i^+l_j^-$ decays. These modes of $B$-meson decays can constrain the product combinations of the couplings with one or more heavy generation indices which are comparable with or stronger than the present bounds. From the studies of the invariant dilepton mass spectrum and the forward backward asymmetry of the emitted leptons we note the possibility of detecting R-parity-violating signals even when the total decay rate due to R-parity violating couplings is comparable with that in the standard model and discriminating two types of R-parity-violating signals. The general expectation of the enhancement of the forward backward asymmetry of the emitted leptons in the minimal supersymmetric standard model with R-parity may be corrupted by R-parity violation.

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*E-mail: jhjang@chep6.kaist.ac.kr
†E-mail: ygkim@chep6.kaist.ac.kr
‡E-mail: jslee@chep6.kaist.ac.kr
1. In supersymmetric extensions of the standard model, there are gauge invariant interactions which violate the baryon number \( (B) \) and the lepton number \( (L) \) in general. To prevent occurrences of these \( B \)- and \( L \)-violating interactions in supersymmetric extensions of the standard model, the additional global symmetry is required. This requirement leads to the consideration of the so called R-parity\( (R_p) \). Even though the requirement of \( R_p \) conservation makes a theory consistent with present experimental searches, there is no good theoretical justification for this requirement. Therefore the models with explicit \( R_p \)-violation have been considered by many authors \[1\].

To discover the \( R_p \) violation in future experiments, we need to know what kinds of couplings are severely constrained by present experimental data. Therefore it is important to constrain the \( R_p \) violating couplings from the present data, especially data on the processes forbidden or highly suppressed in the SM. Usually the bounds on the \( R_p \) violating couplings with at least two heavy fields are not stronger than those with at most one heavy field.

In this paper, we derive the upper bounds on certain products of \( R_p \) and lepton flavor violating couplings from \( b \to s l_i^+l_j^- \) decays in the minimal supersymmetric standard model (MSSM) with explicit \( R_p \) violation. These modes of \( B \)-meson decays can constrain the product combinations of the couplings with one or more heavy generation indices.

In the MSSM, the most general \( R_p \)-violating superpotential is given by

\[
W_{R_p} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ij} U_i^c D_j^c D_k^c.
\]

(1)

Here \( i, j, k \) are generation indices and we assume that possible bilinear terms \( \mu_i L_i H_2 \) can be rotated away. \( L_i \) and \( Q_i \) are the \( SU(2) \)-doublet lepton and quark superfields and \( E_i^c, U_i^c, D_i^c \) are the singlet superfields respectively. \( \lambda_{ijk} \) and \( \lambda''_{ij} \) are antisymmetric under the interchange of the first two and the last two generation indices respectively; \( \lambda_{ijk} = -\lambda_{jik} \) and \( \lambda''_{ij} = -\lambda''_{ji} \). So the number of couplings is 45 (9 of the \( \lambda \) type, 27 of the \( \lambda' \) type and 9 of the \( \lambda'' \) type). Among these 45 couplings, 36 couplings are related with the lepton flavor violation.

There are upper bounds on a \textit{single} \( R_p \) violating coupling from several different sources \[2-5\]. Among these, upper bounds from neutrinoless double beta decay \[3\], \( \nu \) mass \[4\] and \( K^+, t- \) quark decays \[4\] are strong. Neutrinoless double beta decay gives \( \lambda'_{111} < 3.5 \times 10^{-4} \). The bounds from \( \nu \) mass are \( \lambda'_{133} < 3 \times 10^{-3} \) and \( \lambda''_{133} < 7 \times 10^{-4} \). From \( K^+ \)-meson decays one obtain \( \lambda''_{ij} < 0.012 \) for \( j = 1 \) and 2. These bounds from \( K^+ \)-meson decays are basis-dependent \[3\]. Here all masses of scalar partners which mediate the processes are assumed to be 100 GeV. Extensive reviews of the updated limits on a single \( R_p \)-violating coupling can be found in \[3\].

There are more stringent bounds on some products of the \( R_p \)-violating couplings from the mixings of the neutral \( K \)- and \( B \)-mesons and rare leptonic decays of the \( K_L \)-meson, the muon and the tau \[6\]. \( B^0 \) decays into two charged leptons \[7\], \( b\bar{b} \) productions at LEP \[10\] and muon(ton) conversion, and \( \tau \) and \( \pi^0 \) decays \[11\].

In this paper, we assume that the baryon number violating couplings \( \lambda'' \)'s vanish in order to avoid too fast proton decays. Especially in the models with a very light gravitino \((G)\) or axino \((\tilde{a})\), \( \lambda'' \) have to be very small independently of \( \lambda' \) from the proton decay.
$p \rightarrow K^+ G$ (or $K^+ a$) ; $\lambda''_{112} < 10^{-15}$ \cite{12}. One can construct a grand unified model which has only lepton number non-conserving trilinear operators in the low energy superpotential when $R_p$ is broken only by bilinear terms of the form $L_i H_2$ \cite{13}. And usually it may be very difficult to discern signals of $B$-violating interactions above QCD backgrounds \cite{14}.

2. The exchange of the sleptons or squarks leads to the four-fermion interactions in the effective lagrangian. Among these four-fermion operators, there are terms relevant for $b \rightarrow s l_i^+ l_j^-$. These effective terms have 2 down-type quarks and 2 charged leptons. From Eq. (1) we obtain

$$\mathcal{L}_{R_p}^{\text{eff}} = \frac{3}{2} \sum_{n=1}^{3} \frac{m_l^2}{m_{l_n}^2} \lambda'^*_{njk} \lambda''_{nlm}(\bar{e}_j P R e_k)(\bar{d}_m P L d_l) + \text{h.c.}$$

where $K$ is the CKM matrix and we assume the matrices of the soft mass terms are diagonal and the CKM matrix comes from the mixings between down type quarks. From this Lagrangian, one finds that the various semileptonic $b$ decays could appear at tree level; $b \rightarrow q e^+ e^-, q \mu^+ \mu^-, q \tau^+ \tau^-, q e^\pm \mu^\mp, q \mu^\pm \tau^\mp, q e^\pm \tau^\mp$ ($q = d, s$).

At presents, the measurements of the branching ratios of the $b \rightarrow s l_i^+ l_j^-$ processes give the upper bounds (at 90% C.L.) \cite{14}

$$BR(b \rightarrow s e^+ e^-) < 5.7 \times 10^{-5},$$

$$BR(b \rightarrow s \mu^+ \mu^-) < 5.8 \times 10^{-5},$$

$$BR(b \rightarrow s e^\pm \mu^\mp) < 2.2 \times 10^{-5}.$$

In the SM, the process $b \rightarrow s e^\pm \mu^\mp$ is forbidden due to the conservation of each lepton flavor number. On the other hand, the decay $b \rightarrow s \mu^+ \mu^-$ ($b \rightarrow s e^+ e^-$) is dominated by the electroweak penguin and receives small contributions from box diagrams and magnetic penguins. A recent analysis gives $BR(b \rightarrow s e^+ e^-)_{SM} = (8.4 \pm 2.3) \times 10^{-6}$, $BR(b \rightarrow s \mu^+ \mu^-)_{SM} = (5.7 \pm 1.2) \times 10^{-6}$ \cite{16}. The experimental bounds are almost one order of magnitude larger than the standard model expectations. If we neglect the SM contribution, the decay rate of the processes $b \rightarrow s l_i^+ l_j^-$ reads

$$\Gamma(b \rightarrow s l_i^+ l_j^-) = \frac{m_b^5}{6144\pi^3 m_i^4} \left[ 4(|A_{ij}|^2 + |B_{ij}|^2 + |C_{ij}|^2) \right]$$

The constants $A_{ij}, B_{ij}$ and $C_{ij}$ are given by

$$A_{ij} = \sum_{n=1}^{3} \lambda'^*_{njl} \lambda''_{nl32},$$

$$B_{ij} = \sum_{n=1}^{3} \lambda'^*_{njj} \lambda''_{nl43},$$

$$C_{ij} = \sum_{n=1}^{3} \lambda'^*_{njl} \lambda''_{nl53}.$$
\[ B_{ij} = \sum_{n=1}^{3} \lambda_{nij} \lambda_{n23}^*, \]
\[ C_{ij} = \sum_{n,p,s=1}^{3} K_{np} K_{ns} \lambda_{nij}^* \lambda_{ps2}^* = \sum_{n=1}^{3} \lambda_{n3}^* \lambda_{m2}^*. \]  

(5)

Note that we assume the universal soft mass \( \bar{m} \) and ignore the lepton mass. To remove the large uncertainty in the total decay rate associated with the \( m_0^5 \) factor, it is convenient to normalize \( \text{BR}(b \rightarrow s l_i^+ l_j^-) \) to the semileptonic rate \( \text{BR}(b \rightarrow c e \bar{\nu}) \). We then obtain

\[ 4(|A_{ij}|^2 + |B_{ij}|^2) + |C_{ij}|^2 = \frac{6144 \bar{m}^4 G_F^2 |V_{cb}|^2 f_{ps}(m_e^2/m_b^2) \text{BR}(b \rightarrow s l_i^+ l_j^-)}{192 \text{BR}(b \rightarrow c e \bar{\nu})}, \]  

(6)

where \( f_{ps}(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \) in \( x \) is the usual phase-space factor. Inserting into Eq. (6) the semileptonic rate \( \text{BR}(b \rightarrow c e \bar{\nu}) \approx 10.5\% \), \( f_{ps}(m_e^2/m_b^2) \approx 0.5 \), \( |V_{cb}| \approx 0.04 \), we obtain

\[ 4(|A_{ij}|^2 + |B_{ij}|^2) + |C_{ij}|^2 = 3.3 \times 10^{-3} \left( \frac{\bar{m}}{100 \text{ GeV}} \right)^4 \text{Br}(b \rightarrow s l_i^+ l_j^-). \]

(7)

In the case of \( b \rightarrow s e^+ e^- \) decay \((i = 1, j = 1)\), we obtain

\[ 4(|A_{11}|^2 + |B_{11}|^2) + |C_{11}|^2 < 1.9 \times 10^{-7}, \]

(8)

from the Eq. (7) and the upper limit on the branching ratio, Eq. (3). Under the assumption that only one product combination is not zero, we can get the upper bounds on the following combinations of the \( \lambda \lambda' \)- and \( \lambda' \lambda' \)-type;

\[ |\lambda_{n11} \lambda_{n32}^*| < 2.2 \times 10^{-4}, \]
\[ |\lambda_{n11} \lambda_{n23}^*| < 2.2 \times 10^{-4}, \]
\[ |\lambda_{n3}^* \lambda_{m2}^*| < 4.3 \times 10^{-4}. \]

(9)

For the product combinations of \( \lambda \lambda' \) type, we observe that the bounds on \( \lambda_{121} \lambda_{232}^*, \lambda_{131} \lambda_{323}^*, \lambda_{121} \lambda_{232}^*, \lambda_{131} \lambda_{323}^* \) are stronger than the previous bounds (see Table I).

In a similar way, another upper bounds can be obtained in the case of \( b \rightarrow s \mu^+ \mu^- \) and \( b \rightarrow s e^\pm \mu^\mp \) decays. For the \( b \rightarrow s \mu^+ \mu^- \) decay, the bound on \( \lambda_{233} \lambda_{232}^* \) is stronger than the previous bounds. And for the \( b \rightarrow s e^\pm \mu^\mp \) decay, the bounds on \( \lambda_{233} \lambda_{232}^* \) and all product combinations of \( \lambda \lambda' \) type are stronger than the previous bounds.

In fact, the \( B_s \rightarrow l_i^+ l_j^- \) process is described with the same parameters \( A, B, C \) and in some cases gives slightly more stringent bounds [1]. But, two things make the decay \( b \rightarrow s l_i^+ l_j^- \) more useful than the \( B_s \rightarrow l_i^+ l_j^- \) process. One thing is that the contribution of the product combinations of \( \lambda \lambda' \) type is vanishing in the limit of zero lepton mass and so these parameters cannot be constrained in the case of the \( B_s \rightarrow l_i^+ l_j^- \) process. The other thing is that the experimental upper limit on the branching ratios exist only in the \( B_s \rightarrow \mu^+ \mu^- \) process at present.
defined by the SM \([16,18]\) and the minimal MSSM with \(R\)

between contributions of the SM and the \(R\)

type measured with respect to the \(b\)-quark direction in the dilepton center of mass system.

Forward backward asymmetry of the emitted leptons in the presence of \(R\) would be interesting to compare the invariant dilepton mass spectrum and the dilepton mass spectrum as follows

\[
\frac{dA(\hat{s})}{d\hat{s}} = \int_0^1 \frac{d^2B}{d\hat{s}dz} - \int_{-1}^0 \frac{d^2B}{d\hat{s}dz},
\]

where \(\hat{s} \equiv s/m_b^2\), \(s\) is the invariant mass of the lepton pair, and \(z \equiv \cos \theta\) is the angle of \(l^+\) measured with respect to the \(b\)-quark.

Neglecting the masses of the strange quark and leptons, we obtain the \(R\) invariant dilepton mass spectrum as follows

\[
\frac{dB^{R_\nu}(b \rightarrow sl^+_l l^-_j)}{d\hat{s}} = \frac{B_{s\ell}(1 - \hat{s})^2 [24(1 - 2 + |C_{ij}|^2 + |B_{ij}|^2)\hat{s} + |C_{ij}|^2(1 + 2\hat{s})]}{16f_{ps}(m_t^2/m_b^2)G_F^2\bar{m}_l^4|V_{cb}|^2},
\]

where \(B_{s\ell}\) is the semileptonic rate \(BR(b \rightarrow ce\bar{\nu})\). We find that there are no interferences between contributions of the SM and the \(R\)-violating model under consideration. So, the total invariant dilepton mass spectrum is given by the direct sum of the SM contributions and \(R\)-violating contributions. In Fig. 1 (a), we show the normalized invariant dilepton mass spectrum \(\frac{d^2B}{d\hat{s}dz} \equiv \frac{1}{\hat{s}} \frac{d\hat{s}}{dz}\). The solid line denotes the typical SM prediction \([16,18]\) and dashed and dotted lines the \(R\)-violating contributions to the invariant dilepton mass spectrum. Since the structure of the effective lagrangian corresponding to \(\lambda\lambda'\)-type \(R\) violation is different from that of \(\lambda\lambda'\)-type one, the \(\hat{s}\) dependences of the invariant dilepton mass spectrum due to these two types of \(R\) violation are quite different from each other. The \(\hat{s}\) dependence of \(\lambda\lambda'\)-type \(R\) violation is nearly the same as that of the SM and this type of \(R\) violation enhances the decay rate on the low \(\hat{s}\) region. But the behavior of the contribution from \(\lambda\lambda'\)-type \(R\) violation is different from the SM and this type of \(R\) violation enhances the invariant dilepton mass spectrum in the region around \(\hat{s} = 1/3\). Therefore, from the invariant dilepton mass spectrum one can detect the \(R\) violating signals even if the magnitude of \(R\) violating coupling is comparable with that of the SM and it is possible to discriminate two types of \(R\) violating signals, \(\lambda\lambda'\)- and \(\lambda\lambda'\)-type.

We also obtain \(R\) forward backward asymmetry of the emitted leptons

\[
\frac{dA^{R_\nu}(b \rightarrow sl^+_l l^-_j)}{d\hat{s}} = \frac{-3B_{s\ell}|C_{ij}|^2(1 - \hat{s})^2\hat{s}}{32f_{ps}(m_t^2/m_b^2)G_F^2\bar{m}_l^4|V_{cb}|^2}.
\]

Let us note that sign of this asymmetry is negative. In Fig. 1 (b), we show the normalized forward backward asymmetry of the emitted leptons \(\frac{dA}{d\hat{s}} \equiv \frac{1}{B} \frac{d\hat{s}}{d\hat{s}}\). There is no contribution to the asymmetry from the \(\lambda\lambda'\)-type \(R\) violation. But there is negative contribution from \(\lambda\lambda'\)-type \(R\) violation which can compensate the SM asymmetry. Since there are no interferences between contributions of the SM and the \(R\)-violating model under consideration, the total asymmetry \(\frac{dA}{d\hat{s}}\) is given by
\[
\frac{d\tilde{A}}{d\tilde{s}} = \left( \frac{dA}{ds} \right)_{\text{SM}} + \frac{dA}{ds} \bigg|_{R_p} \equiv \left( \frac{dA}{ds} \right)_{\text{SM}} + \frac{dA}{ds} \bigg|_{R_p} + \xi_{\lambda\lambda'} \left( \frac{dA}{ds} \right)_{\lambda\lambda'}^{R_p} + \xi_{\lambda\lambda'} \left( \frac{dA}{ds} \right)_{\lambda\lambda'}^{R_p},
\]

where \(\xi_{(\lambda\lambda',\lambda\lambda')} \equiv B_{(\lambda\lambda',\lambda\lambda')}^{R_p}/B_{\lambda\lambda'}^{\text{SM}}\) and \(\left( \frac{dA}{ds} \right)_{\lambda\lambda'}^{R_p} = 0\). In the case \(\xi_{\lambda\lambda'} \gg \xi_{\lambda\lambda'}\), the SM asymmetry will be diluted by the factor of \(\sim 1/(1 + \xi_{\lambda\lambda'})\). In the case \(\xi_{\lambda\lambda'} \ll \xi_{\lambda\lambda'}\), even the sign of the asymmetry could be different from the prediction of the SM depending on the size of \(\xi_{\lambda\lambda'}\). For illustration, we show the above forward backward asymmetry with \(\lambda\lambda' = 1\) or \(\lambda\lambda' = 1\) in Fig. 2. So, also from the forward backward asymmetry of the emitted leptons one can detect the \(R_p\) violating signals even if the magnitude of \(R_p\) violating coupling is comparable with that of the SM and it is possible to discriminate two types of \(R_p\) violating signals, \(\lambda\lambda'\)- and \(\lambda\lambda'\)-type.

From the studies of the process \(b \rightarrow s l^+ l^-\) in the minimal MSSM with \(R_p\) \textsuperscript{[19]}, the SUSY signal may appear as the enhancement of this asymmetry by more than 100% relative to SM expectations. But, in the presence of \(R_p\) violation this minimal MSSM enhancement may be corrupted \textsuperscript{[1]}.

4. To conclude, we have derived the more stringent upper bounds on certain products of \(R_p\)- and lepton-flavor-violating couplings from the recent measurements of \(b \rightarrow s l^+ l^-\) decay rates at CLEO. From the studies of the invariant dilepton mass spectrum and the forward backward asymmetry of the emitted leptons, we note the possibility of detecting \(R_p\)-violating signals even when the total decay rate due to \(R_p\)-violating couplings is comparable with that in the SM and discriminating two types of \(R_p\)-violating signals, \(\lambda\lambda'\)- and \(\lambda\lambda'\)-type. The general expectation of the enhancement of the forward backward asymmetry of the emitted leptons in the minimal MSSM with \(R_p\) may be corrupted by R-parity violation.

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\textsuperscript{1}In fact, the precise definition of the asymmetry in the first reference of \textsuperscript{[19]} is different from ours. But, our conclusion is not changed by this difference.
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TABLE I. Upper bounds on the magnitudes of products of couplings derived from $b \to s l_i^+ l_j^-$.

| Decay Mode | Combinations Constrained | Upper bound | Previous bound |
|------------|--------------------------|-------------|----------------|
| $b \to se^+e^-$ | $\lambda_{121}\lambda_{232}$ | $2.2 \times 10^{-4}$ | $8.0 \times 10^{-3}$ $^a$ |
| | $\lambda_{131}\lambda_{332}$ | $2.2 \times 10^{-4}$ | $2.9 \times 10^{-2}$ |
| | $\lambda_{121}\lambda_{223}'$ | $2.2 \times 10^{-4}$ | $6.0 \times 10^{-4}$ $^b$, ($9.0 \times 10^{-3}$ $^c$) |
| | $\lambda_{131}\lambda_{323}'$ | $2.2 \times 10^{-4}$ | $7.2 \times 10^{-4}$ $^b$, ($1.2 \times 10^{-2}$ $^c$) |
| | $\lambda_{113}'\lambda_{112}'$ | $4.3 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($4.0 \times 10^{-4}$ $^c$) |
| | $\lambda_{123}'\lambda_{122}'$ | $4.3 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($4.0 \times 10^{-3}$ $^c$) |
| | $\lambda_{133}'\lambda_{132}'$ | $4.3 \times 10^{-4}$ | $1.1 \times 10^{-4}$ $^a$ |
| $b \to s\mu^+\mu^-$ | $\lambda_{122}\lambda_{132}$ | $2.2 \times 10^{-4}$ | $5.5 \times 10^{-5}$ $^d$ |
| | $\lambda_{232}\lambda_{332}'$ | $2.2 \times 10^{-4}$ | $5.5 \times 10^{-5}$ $^d$ |
| | $\lambda_{122}\lambda_{123}'$ | $2.2 \times 10^{-4}$ | $5.5 \times 10^{-5}$ $^d$ |
| | $\lambda_{232}\lambda_{323}'$ | $2.2 \times 10^{-4}$ | $5.5 \times 10^{-5}$ $^d$ |
| | $\lambda_{213}'\lambda_{212}'$ | $4.4 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($8.1 \times 10^{-3}$ $^c$) |
| | $\lambda_{223}'\lambda_{222}'$ | $4.4 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($3.2 \times 10^{-2}$ $^c$) |
| | $\lambda_{233}'\lambda_{232}'$ | $4.4 \times 10^{-4}$ | $2.5 \times 10^{-2}$ $^a$ |
| $b \to se^+\mu^+$ | $\lambda_{122}\lambda_{232}'$ | $1.4 \times 10^{-4}$ | $1.8 \times 10^{-2}$ $^a$ |
| | $\lambda_{132}\lambda_{332}'$ | $1.4 \times 10^{-4}$ | $2.9 \times 10^{-2}$ |
| | $\lambda_{121}\lambda_{132}'$ | $1.4 \times 10^{-4}$ | $8.0 \times 10^{-3}$ $^a$ |
| | $\lambda_{231}\lambda_{332}'$ | $1.4 \times 10^{-4}$ | $2.9 \times 10^{-2}$ |
| | $\lambda_{122}\lambda_{223}'$ | $1.4 \times 10^{-4}$ | $6.0 \times 10^{-4}$ $^b$, ($9.0 \times 10^{-3}$ $^c$) |
| | $\lambda_{132}\lambda_{323}'$ | $1.4 \times 10^{-4}$ | $7.2 \times 10^{-4}$ $^b$, ($1.2 \times 10^{-2}$ $^c$) |
| | $\lambda_{121}\lambda_{123}'$ | $1.4 \times 10^{-4}$ | $6.0 \times 10^{-4}$ $^b$, ($1.0 \times 10^{-2}$ $^c$) |
| | $\lambda_{231}\lambda_{323}'$ | $1.4 \times 10^{-4}$ | $7.2 \times 10^{-4}$ $^b$, ($1.2 \times 10^{-3}$ $^c$) |
| | $\lambda_{113}'\lambda_{212}'$ | $2.7 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($1.8 \times 10^{-3}$ $^c$) |
| | $\lambda_{123}'\lambda_{222}'$ | $2.7 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($3.6 \times 10^{-2}$ $^c$) |
| | $\lambda_{133}'\lambda_{232}'$ | $2.7 \times 10^{-4}$ | $1.1 \times 10^{-4}$ $^a$ |
| | $\lambda_{213}'\lambda_{112}'$ | $2.7 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($1.8 \times 10^{-3}$ $^c$) |
| | $\lambda_{223}'\lambda_{122}'$ | $2.7 \times 10^{-4}$ | $1.4 \times 10^{-4}$ $^b$, ($3.6 \times 10^{-3}$ $^c$) |
| | $\lambda_{233}'\lambda_{132}'$ | $2.7 \times 10^{-4}$ | $2.5 \times 10^{-2}$ $^a$ |

a: Bounds from $B \to X_u l \bar{\nu}$ $^{[17]}$. b: Considering the bounds from $K \to \pi \nu \bar{\nu}$ $^{[5]}$. c: Ignoring the bounds from $K \to \pi \nu \bar{\nu}$. d: Bounds from $B_s \to \mu^+ \mu^-$ $^{[4]}$. Others: See the second reference of $^{[4]}$.
FIG. 1. Plots of normalized invariant dilepton mass spectrum (a) and forward backward asymmetry of the emitted leptons (b). The solid line denotes the typical SM prediction [16,18] and dashed and dotted lines the $R_p$-violating contributions, Eq.(10) and Eq. (11).
FIG. 2. Plots of total forward backward asymmetry of the emitted leptons, \( \frac{\tilde{d}A}{d\hat{s}} \). The solid line denotes the typical SM prediction [16,18] and dashed and dotted lines the \( R_\mu \)-violating contributions, Eq.(12).