Oscillons in $\phi^6$-theories: Possible occurrence in MHD

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In this work, we report on the possibility of occurrence of oscillon configurations in the fourth state of matter. Oscillons are extremely long-lived, time-periodic, spatially-localised scalar field structures. Starting from a scalar field theory in 1+1 space-time dimensions, we find out that small-amplitude oscillons can be obtained in the framework of a $\phi^6$ self-interacting potential. A connection between our results and ideal MHD theory is established. Perspectives for a development of the present work are pointed out.

Keywords: nonlinear, oscillons, plasma

INTRODUCTION

It is widely acknowledged that, under certain circumstances, a large number of natural systems can exhibit a nonlinear behavior. For instance, we may observe nonlinearities in the condensed state of matter [1], elementary particle physics [2], cosmological scenarios [3], biological systems [4], but also in plasma physics [5, 6].

In classical field theories, there is an important class of configurations, termed solitons [7], whose existence is entirely due to the nonlinearity of the field equations. Such solutions are well known in Lorentz and CPT breaking systems [8], modified theories of gravitation [9], two-dimensional N=1 supersymmetric quantum field theories [10], non-integrable quantum field theories [11], fibre optics [12], as well as in plasma physics [13, 14]. A soliton is a static field configuration whose energy density profile is localised in space. In particular, it exhibits the distinctive property of to retain its shape after collision with another soliton.

Quite interestingly, in the 1970s, a new class of localised nonperturbative solutions, which may be derived in the realm of nonlinear theories, was reported [15–18]. Their most noticeable features were the time-dependence, extreme long-life, and spatial localisation. In spite of such an intriguing behavior, only in the 1990s their analytical properties were firstly explored by Gleiser [19], who coined them oscillons [20].

Oscillons have attracted attention from several areas of research, namely standard model-extensions [21], supersymmetry theories [22], gravitational waves [23, 24], high energy systems in presence of external potentials [25], the Abelian-Higgs model [26, 27], certain Lorentz violating scenarios [28], spontaneous symmetry breaking phenomena [29], and cosmological background investigations [30]. Such an interest may be attributed to the unexpected longevity combined with nearly periodic oscillation in time, as exhibited by those structures.

Nonlinearities may be also observed in the fourth state of matter. Actually, solitons are ubiquitous in investigations of nonlinear dispersive media [31], electron-beam plasmas [32], weakly relativistic plasmas containing electrons, positrons, and ions [33], fermionic quantum plasmas [34], dusty plasmas [35, 36], plasma slabs [37], and cold plasma columns bounded by deformable dielectrics [38]. Recently, a novel regime of soliton-plasma interaction has been found in a gas-filled hollow-core photonic crystal fiber [39]. Despite the impressive list, as mentioned above, matching solitons to plasmas, investigations linking oscillons to the latter from first principles still lack.

Given that oscillons are known to play a central role in the nonlinear dynamics of a wide number of physical systems, in this work, we propose to explore their possible emergence on a plasma background. Thus, we shall consider a scalar field theory with nonlinear interactions responsible to preserve the localisation of energy for a remarkably long time. In particular, we shall regard only small amplitude oscillons living in 1+1 space-time dimensions [40, 41]. The reason to confine ourselves to this special regime stems from the flexibility it offers to apply our results to a definite plasma system. However, as we will show, our approach opens up a new window to explore a large number of nonlinear plasma scenarios.

SCALAR FIELD DYNAMICS

Let us start by considering a real scalar field theory (SFT) in 1+1 space-time dimensions, described by the action

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where we abbreviate $\partial_t = \partial / \partial t$ and $\partial_x = \partial / \partial x$. The quantity $U = U(\phi; \partial_t \phi, \partial_x \phi)$ is a potential functional, with $\phi = \phi(t; x)$ denoting the scalar field. We remark that $U$ depends on both $\phi$ and its derivatives. Our motivation to consider that instance stems from the observation that nonlinear plasma dynamics usually exhibits such a behavior. Actually, the natural framework at which oscillon configurations are most likely to be analytically described in plasmas is the so-called reduced magnetohydrodynamics (RMHD). That formulation allows the identification of three independent time scales, which may be associated with (1) MHD equilibrium, (2) perturbations normal to the externally applied magnetic field $\vec{B}$, and (3) those parallel to $\vec{B}$ [42]. A suitable account of (1), by an appropriate choice of geometry, and elimination of (2), by an adequate requirement of constraints, enables a satisfactory description of (3). In fact, such an approach has been proved consistent with the usual assumption of a perfectly conducting fluid (the so-called ideal limit) in magnetohydrodynamics, even at sufficiently high perturbative frequencies (higher than the inverse Alfvén transit time scale), by robust numerical treatments [43]. In 2+1 dimensions, RMHD may be derived from a variational principle, whose action depends on both the fields and their derivatives [44]. A somewhat more general derivation is possible in the realm of kinetic theory [45]. In this work, we confine ourselves to the consideration of the potential $\phi^6$, which plays a central role in the description of first-order phase transitions [46] and particle physics phenomenology [2]. Such a symmetric model has been also a popular point of departure for several investigations concerning oscillon dynamics [47–49]. In particular, we consider the $\phi^6$ potential, motivated by investigations on oscillons, in an expanding universe [50]. We write

$$V(\phi) = \frac{\omega^2 \phi^2}{2} \left( 1 - \frac{2\phi^2}{3\phi_0^2} \right)^2,$$

where $\omega$ and $\phi_0$ are real, positive valued parameters, and the assigned prefactors have been suitably chosen. The quantity $\omega$ has dimension of mass. In the context of the standard model, one may add the $\phi^6$ interaction to the Higgs potential, in order to describe the strong first-order electroweak phase transition for masses, thereby leading to scales above 100 GeV [51].

The profile of $V(\phi)$ is illustrated in Fig. 1. It shows that the potential exhibits three degenerate minima (the so-called vacua of the model), localized at $\phi(0) = 0$ (the central vacuum) and $\phi(\pm) = \pm(3/2)^{1/2} \phi_0$.

In the next Section, we follow the usual approach to derive oscillons in 1+1 space-time dimensions.

**Oscillon Configurations**

First of all, oscillons are time-periodic structures. Thus, for each fixed point $(T, X)$, we consider solutions of Eq. (4) on the (Euclidean) ellipse

$$\frac{\tau^2}{T^2} + \frac{\chi^2}{X^2} = 1.$$
Next, oscillons are also localised in space. Hence, we regard the scale mapping $\chi = X \epsilon$, where $\epsilon$ is a small, positive parameter, $0 < \epsilon << 1$. Therefore, Eq. (6) shows that the (of course, positive) time coordinate might transform as $0 < \tau = T(1 - \epsilon^2)^{1/2}$. We notice that those transformations naturally appear in the context of perturbation theory, through use of so-called multiscale expansion method [52]. Within that approach, one can introduce several time and space variables, which may be scaled differently and regarded as independent. Thus, it is possible to identify such scalings from the equations of motion.

Given the above considerations, Eq. (4) becomes

$$\omega^2 \phi \left( 1 - \frac{8\phi^2}{3\phi_0^2} + \frac{4\phi^4}{3\phi_0^4} \right) + (1 - \epsilon^2) \partial_{\tau \tau} \phi - \epsilon^2 \partial_{\chi \chi} \phi$$

where use has been made of Eq. (5). Obviously, now $\phi = \phi(\tau; \chi)$.

Departing from Eq. (7), we can derive small amplitude oscillons which are localised in the central vacuum of the model described by Eq. (5). The usual approach to accomplish this task in 1 + 1 dimensions is to consider the expansion of the field $\phi$ about $\phi(0) = 0$ in a power series of the small parameter $\epsilon$. However, it is worth to emphasize that, some time ago, a new kind of approach to nonlinear solutions, which introduces a class of configurations now known as flat-top oscillons, has been developed by Amin and Shirokoff [50].

Now, we notice that if $H$ is an integer which satisfies the condition $H \geq 2$, then Eq. (7) just exhibits odd powers of $\phi$. Since only terms up to $\epsilon^3$-order will be fundamental to determine the oscillon profile, we choose $H = 2$ for the simplest case,

$$\omega^2 \phi \left( 1 - \frac{8\phi^2}{3\phi_0^2} + \frac{4\phi^4}{3\phi_0^4} \right) + (1 - \epsilon^2) \partial_{\tau \tau} \phi - \epsilon^2 \partial_{\chi \chi} \phi$$

where $\phi = \phi(\tau; \chi)$.

Substituting Eq. (9) in Eq. (8), the resulting series expansion yields

$$\epsilon(\partial_{\tau \tau} \phi_1 + \omega^2 \phi_1) +$$

$$\epsilon^3 \left[ (\partial_{\tau \tau} \phi_3 + \omega^2 \phi_3) - \left( \partial_{\tau \tau} \phi_1 + \partial_{\chi \chi} \phi_1 + \frac{8\omega^2 \phi_1^3}{3 \phi_0^2} \right) \right] +$$

$$\mathcal{O}(\epsilon^5) = 0.$$ (10)

From Eq. (10), the coefficients of $\epsilon$ and $\epsilon^3$ lead to

$$\partial_{\tau \tau} \phi_1 + \omega^2 \phi_1 = 0,$$ (11)

$$\partial_{\tau \tau} \phi_3 + \omega^2 \phi_3 = \partial_{\tau \tau} \phi_1 + \partial_{\chi \chi} \phi_1 + \frac{8\omega^2 \phi_1^3}{3 \phi_0^2},$$ (12)

respectively. Now, since the functions $\phi_n$ are even in both $\tau$ and $\chi$, it follows that an also even function $\varphi(\chi)$ exists. The amplitude of the latter shall decay to zero as $|\chi| \to \infty$, and the solution of Eq. (11) may be promptly deduced,

$$\phi_1(\tau; \chi) = \varphi(\chi) \cos(\omega \tau).$$ (13)

Substituting Eq. (13) in Eq. (12), we obtain

$$\partial_{\tau \tau} \phi_3 + \omega^2 \left[ \phi_3 - \frac{2 \varphi^3}{3 \phi_0^2} \cos(3\omega \tau) \right]$$

$$= \left[ \varphi_{\chi \chi} - \omega^2 \varphi \left( 1 - \frac{2 \varphi^2}{\phi_0^2} \right) \right] \cos(\omega \tau),$$ (14)

where $\varphi_{\chi \chi} \equiv d^2 \varphi/d\chi^2$.

It is clear that the general solution of Eq. (14) is neither periodic in time nor localised in space. However, the periodicity of $\phi_3$ requires that $\partial_{\tau \tau} \phi_3 + \omega^2 \phi_3$ be orthogonal to $\cos(\omega \tau)$, which imposes the condition
\[
\varphi_{xx} = \omega^2 \varphi \left( 1 - \frac{2\varphi^2}{\phi_0^2} \right)
\]

(15)

to be satisfied. As one may easily check, both \(\phi_1\) and \(\phi_3\) (thus, also \(\phi\)) qualify to describe oscillon configurations. Up to a translation, Eq. (15) exhibits a nontrivial solution, which exactly tends to zero as \(|\chi| \to \infty\). Thus, after a straightforward calculation, the solution of Eq. (15) can be written in the form

\[
\varphi (\chi) = \phi_0 \sech (\omega \chi),
\]

(16)

and the small amplitude oscillon is determined by (see Eqs. (9) and (13))

\[
\phi (\tau; \chi) = \epsilon \phi_0 \sech (\omega \chi) \cos (\omega \tau) + \mathcal{O} (\epsilon^3).
\]

(17)

By plugging back the original variables, we finally define the oscillon field as

\[
\phi^{(e)}(t; x) \equiv \epsilon \phi_0 \sech (\epsilon \omega_d x) \cos \left[ \left( 1 - \frac{\epsilon^2}{2} \right) \omega_c t \right] + \mathcal{O} (\epsilon^3),
\]

(18)

where use has been made of the binomial approximation \((1 - \epsilon^2)^{1/2} \approx (1 - \epsilon^2/2)\) for \(0 < \epsilon \ll 1\), and we have defined \(\omega_c \equiv \omega (1 + 2c)^{-1/2}\) and \(\omega_d \equiv \omega (1 - 2d)^{-1/2}\).

A typical profile of \(\phi^{(e)}(t; x)\) is illustrated in Fig. 2. It shows that the fields indeed oscillate about their corresponding effective mean value.

To show the variation of the field energy density with respect to the potential coefficients \(c\) and \(d\). As we have shown, those quantities imply dilations on the frequency \(\omega\).

In the next section, we discuss the outgoing radiation of the presently deduced oscillon configuration.

**OUTGOING RADIATION**

In this section, we follow the steps of Hertzberg [53], who has proposed a method to compute the classical radiation in 1+1 dimensional Minkowski space-time. The approach assumes that one can write the solution of the field equations as

\[
\phi^{(e)}(t; x) = \phi^{(e)}_{osc}(t; x) + \zeta (t; x),
\]

(19)

where \(\phi^{(e)}_{osc}(t; x)\) is the oscillon solution and \(\zeta (t; x)\), a small correction. Thus, by substituting Eq. (19) into Eq. (3), we obtain the linearized formula

\[
\partial_{TT} \zeta - \partial_{XX} \zeta + \omega^2 \zeta (T; X) = -\Delta (T; X),
\]

(20)

where use has been made of the space-time dilations and

\[
\Delta (T; X) = \partial_{TT} \phi^{(e)} - \partial_{XX} \phi^{(e)} + \omega^2 \phi^{(e)} (T; X)
\]

(21)

may be interpreted as an external source term, which enables us to write the correction field as

\[
\zeta (T; X) = -\frac{1}{(2\pi)^2} \lim_{\eta \to 0^+} \int d\Omega \int dK \frac{\Delta (\Omega, K) e^{i(KX - \Omega T)}}{K^2 - \Omega^2 + i\eta}
\]

(22)

with the Fourier component

\[
\Delta (\Omega, K) = \int dT \int dX \Delta (T; X) e^{-i(KX - \Omega T)}.
\]

(23)

Eq. (23) shows that the amplitude of the radiation field \(\zeta (t; x)\) is much smaller than \(\epsilon \phi_0\), the oscillon amplitude (see Eq. (18)). This means that the presently given oscillon solutions are actually stable configurations.

In the next section, we establish a possible connection between oscillon configurations and plasma physics.
CONNECTION WITH PLASMA PHYSICS

Ideal magnetohydrodynamics (MHD) is the theory that describes the time and space evolution of a perfectly conducting fluid subjected to a strong external magnetic field. Action principles for MHD are usually constructed to reflect the interaction of both linear and nonlinear waves with a non-uniform background plasma flow [54]. A simple MHD action may be written as [55]

\[ S_{\text{MHD}} = \int dt \int d^3x \left( \frac{\rho u^2}{2} - \rho \Phi - \rho e - \frac{B^2}{2\mu_0} \right), \]  

(24)

where \( \Phi \) is the gravitational potential (for a non-self-gravitating fluid), \( u \) and \( B \) are the strengths of the flow and magnetic fields, respectively, and \( \rho \) and \( e \) are the mass density (per unit volume) and specific energy (per unit mass), respectively, and \( \mu_0 \) is the vacuum magnetic permeability (we use MKS units to comply with the plasma physics community practice).

An arbitrary variation of the MHD action (24) leads to the equation of motion

\[ \rho D_i u_i = -\rho \partial_t \Phi + \partial_j W_{ij}, \]  

(25)

where the differential operator

\[ D_i = \partial_i + u_i \partial_t \]  

(26)

is the material, or convective, derivative (with the abbreviation \( \partial_i = \partial / \partial x_i \)), a repeated index denotes a summation over \( i = 1, 2, 3 \), and the stress tensor (including thermal and magnetic pressures) may be read as

\[ W_{ij} = -\left( p + \frac{B^2}{2\mu_0} \right) \delta_{ij} + \frac{1}{\mu_0} B_i B_j, \]  

(27)

with \( p = -\langle \partial e / \partial v \rangle \), where \( s \) and \( v = \rho^{-1} \) denote the fluid specific entropy and volume, respectively (we regard an equation of state in the form \( e = e(s, v) \), thus \( p \) may be readily computed from the well-known thermodynamic relation \( de = T ds - p dv \), with \( T = \langle \partial e / \partial s \rangle \), denoting the fluid temperature).

Consider now a small perturbation of the (Lagrangean) coordinate \( x_i \) in the form \( x_i \to x_i + \xi_i \), where we assume that the displacement \( \xi_i \) satisfies the condition \( | \xi_i | \ll | x_i | \). Thus, the MHD action (24) becomes

\[ S_{\text{MHD}} = S_{\text{MHD}}(t, x_i; \xi_i, \partial_i \xi_i, \partial_i \xi_j), \]  

thereby yielding the perturbed equation of motion

\[ \rho D_{ti} \xi_i = -\rho (\partial_i \Phi) \xi_j + \partial_j (W_{ijkl} \partial_k \xi_l) + O(\xi^3), \]  

(28)

where the differential operator

\[ D_{ti} = \partial_{ti} + (\partial_t u_i) \partial_i + 2u_i \partial_i \partial_t + u_i (\partial_t u_j) \partial_j + u_i u_j \partial_{ij} \]  

(29)

and the stress tensor

\[ W_{ijkl} = \left[ (\gamma - 1)p + \frac{B^2}{2\mu_0} \right] \delta_{ij} \delta_{kl} + \left( p + \frac{B^2}{2\mu_0} \right) \delta_{il} \delta_{jk} + \frac{1}{\mu_0} B_j B_l \delta_{ik} - \frac{1}{\mu_0} (B_i B_j \delta_{kl} + B_k B_l \delta_{ij}), \]  

(30)

with \( \gamma \) denoting the ratio of specific heats at constant pressure to volume (actually, \( \gamma = (v/p)(\partial^2 e / \partial v^2) |_s \)).

We notice that Eq. (28) provides the basis for a nonlinear perturbation theory for a given ideal MHD flow, since it may be computed at any order in the Lagrangean displacement \( \xi_i \) [56]. With appropriate modifications, the method may be also extended to turbulent flows [57], and even account for certain dissipative effects [58]. Further, the proper approach may be modified to include inertial [59] and Hall [60] terms, and yet set forward in the framework of relativistic MHD [61].

Although oscillon configurations have been discussed on 1+1 dimensions in the previous Sections, and ideal MHD, on 3+1 dimensions in this Section, a contrast of the relevant equations, (10) and (28), qualitatively suggests an analogy between the scalar field \( \phi \) and Lagrangean displacement \( \xi \). Such an observation indicates a perspective for a development of the present work. On one hand, we shall extend our oscillon solution to 2+1 dimensions, and, on the other, restrict the action principle to 2+1 MHD. Thus, we might be able to formulate a quantitative analysis of oscillon dynamics in the realm of plasma physics. That will be presented in a forthcoming communication.

CONCLUSION

In this work, we have reported on the possibility of occurrence of extremely long-lived, time-periodic, spatially-localised scalar field configurations, the so-called oscillon structures, in the fourth state of matter.

Starting from a scalar theory in 1+1 space-time dimensions, whose potential \( U \) depends on both the field \( \phi \) and its derivatives, we have obtained the Euler-Lagrange equations. In particular, the self-interacting part of \( U \) has been suitably chosen to be a \( \phi^6 \) potential. Small amplitude oscillons have emerged as a power series expansion in the scaling parameter \( \epsilon \) of the solutions of the field equations on an Euclidean ellipse. A perturbative analysis has shown that such configurations are actually long-lived structures.

A connection with plasma physics has been established in the framework of ideal MHD theory in 3+1 dimensions. In that realm, the perturbative Lagrangean displacement
\( \xi \) has been shown to play the role of the scalar field \( \phi \), as qualitatively suggested by the contrast of the relevant equations (28) and (10), respectively.

A quantitative analysis deserves a thorough investigation of both formulations by matching them in 2+1 space-time dimensions. That will be presented in a forthcoming communication.

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