Analysis of a hybrid fractal curve antenna using the segmentation method

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Abstract

A microstrip-fed hybrid Fractal antenna for WLAN application is presented in this article. The proposed antenna is composed of meander and Koch elements combined to form a longer curve that extends the electrical length. This approach simultaneously reduces the operating frequency and antenna size. The Green’s functions and segmentation method are used to calculate the input impedance of the hybrid Fractal antenna. For analysis, the amalgamated antenna shape is decomposed into simple regular segments. The impedance parameters of the newly arranged shapes are calculated by the corresponding Green’s functions. The impedance matrix equations of all the segments are presented so as to display in a single matrix whose solution determines the input impedance of the complete antenna geometry. To validate the theory, a prototype of the proposed antenna has been numerically simulated and experimentally measured.

KEYWORDS
fractal antenna, iteration function system, segmentation method

1  |  INTRODUCTION

The microstrip monopole antennas are seen as the preferred candidates for devices with slim profiles and limited real estate for their placement. Methods to iterate the straight radiating wires to form the Fractal curves have proven to be advantageous in improving the antenna’s performance further.1 The Fractal geometries have been extensively studied for their multi-band or wide-band behavior, which are due to the self-similarity of their shapes.2-5 These geometries are known to increase the current path, which results in the reduced size of the antennas. This concept has been reinforced by a relatively recent trend of combining several Fractal geometries, to form hybrid Fractal antennas, has proven effective in further reducing the antenna size.6 Many efforts have been devoted to linking the mathematical properties of Fractals with antenna performance.7-10 However, there are not many reports on the calculation of the impedance of Fractal antennas; this is mainly because the Fractal antennas do not have a regular shape. Accurate analytical models exist for regular and simple antenna shapes.11-13 However, the analysis of the Fractal and other irregular antennas cannot be performed directly using the two commonly used analytical techniques—the transmission line model and the cavity model.
Due to the unavailability of closed-form formulations, the parameters of irregular antenna shapes are mostly calculated using the numerical techniques such as the method of moments (MoM), finite difference time domain method, and finite element method. Although they have the ability to predict the antenna parameters accurately, the in-house or commercially available numerical techniques provide little insight into the physics of an antenna design that limits the understanding and optimization of antenna characteristics.

As for the use of analytical techniques, only a few microstrip Fractal antennas have been reported for the calculation of their input impedance. These include microstrip Sierpinski Fractal carpet antennas, and Minkowski Fractal ring antennas. However, the analyses for these antennas cannot be applied to Fractal curves as they are only suitable for the microstrip ring structures. Besides, an analysis method based on the inductor circuit model in Reference 17 is also not ideal for the microstrip Fractal curve antennas because its application is confined to the wire dipole antennas. In addition to analytical and numerical methods, there are reports on the calculation of Fractal antenna parameters using soft computing techniques such as artificial neural networks. The parameters, crucial to making an antenna resonant at the desired frequency, are predicted and then simulated to determine the complete antenna behavior. Although the soft computing techniques could be useful in reliably calculating design parameters, such methods do not provide an insight into the behavior of complex-shaped Fractal antennas.

In this article, the design and analysis of a hybrid Fractal antenna have been proposed. The meander and Koch segments are merged to form an intricate curve that extends the current path to reduce the operating frequency. The shape of the antenna is generated using the iteration function system (IFS). The additional qualities possessed by the presented Fractal antenna are reduced cost of production due to its fabrication on the inexpensive FR-4 material and acceptable radiation performance.

After the design, the primary intent of this work is to calculate the input impedance of the proposed Fractal antenna. However, as mentioned earlier, due to tortuous Fractal shape the transmission line and cavity models cannot be used for its analysis. Moreover, in the absence of Green’s functions for irregular shapes, the coupling impedance formulas cannot be used for analysis as well. Nevertheless, the segmentation method, which is based on the generalized cavity model can be used in such situations. In this method, irregular antenna shapes are decomposed into various regular shapes and their corresponding Green’s functions are used to calculate their impedances independently. The individual contributions of all the units are then combined using the method of segmentation to calculate the input impedance of the complete structure. Since its development, this technique has been successfully used for the analysis of several irregular microstrip antennas including a Minkowski Fractal ring antenna. However, the analysis for both shape and bends of the Minkowski Fractal ring antenna in Reference 16 was carried out using the impedance Green’s functions of rectangular segments. Therefore, the proposed method in Reference 16 is only suitable to analyze the Fractal ring antennas with fixed indentation angles of 90°. Furthermore, the initial feed impedance in Reference 16 was calculated assuming a probe feed being physically attached to the antenna and the effects of the actual electromagnetically coupled feed were included later to obtain the actual input impedance. While this method is effective to analyze the port impedances of probe fed antennas, it cannot be used to calculate the input impedance of a Fractal antenna that is directly fed at the edge or through a microstrip feed line.

The methodology in this article, which can be generalized for the analysis of microstrip-fed Fractal curve antennas, is described by applying the segmentation method to the proposed meander-Koch curve Fractal antenna. The Fractal generator of this antenna is constructed with the indentation angles of 90° and 45°. To develop the network models for these bent shapes, the whole geometry is divided into units of rectangular and right-angled isosceles triangular segments. The distribution of the continuous current sheet is replaced by the discretized ports. These ports, assigned electric currents, and voltages, are placed along the peripheries of the segments representing the continuous currents and voltages in the compound structure. The relationship between the voltages-currents and impedances at the connection ports are used to determine the input impedance of the entire meander-Koch curve Fractal element. For validation, the calculated input reflection coefficient of the meander-Koch curve Fractal antenna is numerically and experimentally verified.

2 PROPOSED ANTENNA GEOMETRY

The main aim of the proposed antenna geometry is to extend the convoluted length of the curve to lower the resonance frequency. This is achieved by merging the meander line and Koch curve elements. The iterative approach to generate the desired antenna shape is shown in Figure 1. The construction of the hybrid meander-Koch curve Fractal antenna begins with a linear strip shown in Figure 1A. It is divided into five equal parts, and then a rectangular element is created in the
FIGURE 1 Transformations of the hybrid meander-Koch curve Fractal antenna (A) 0th transformation (initiator); (B) first transformation (generator); (C) second transformation; (D) Addition of meander and Koch curve geometries to construct a generator.

Center. In this way, a meander element is formed to occupy the 3/5 region of the total strip length. As shown in Figure 1B, it consists of two vertical lines and a horizontal line, the Koch curve element is created over the latter. It is obtained by dividing the horizontal line equally into three segments and the middle part is replaced by two lines with indentation angles of 45° and −45° respectively. In the next step of the geometric transformation, as shown in Figure 1C, the same two-level procedures are repeated on every linear line until the geometry of the meander-Koch curve antenna is obtained. In simple terms, this iterative process involves an initiator (a straight line) that is transformed using a generator created by combining the meander and Koch curve geometries.

Figure 1D shows the details of the first transformation that resulted in the merger of the meander and the Koch curve shapes. It could be seen that the number of segments in the resultant geometry has increased to eight, whereas individually the meander and the Koch curve elements contain five and four segments, respectively. The IFS is used to construct the Fractal generator. This method is useful to create a number of Fractal geometries.1,26-29 Using IFS, the meander-Koch curve Fractal antenna is formed after a series of geometric operations–which comprises scaling, rotation, and translation–known as transformation. The Fractal transformations are given by the following expression:

$$W \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{r} \cos \theta & -\frac{1}{s} \sin \theta \\ \frac{1}{r} \sin \theta & \frac{1}{s} \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$  \hspace{1cm} (1)

where, $r$ and $s$ scale the segments in $x$- and $y$- directions, respectively; $\theta$ is the angle of rotation and the column vector $(e, f)$ translates the segments respectively in $x$ and $y$ directions. The shape of the first transformation of the meander-Koch curve element as shown in Figure 1B is generated by the following matrix equations:
where, the generator $W(A)$ is obtained by combining $W_1$ to $W_8$ which are a set of linear affine transformations and $A$ is the initial geometry. The Fractal dimension $D$ for the first transformation of the meander-Koch curve Fractal antenna is calculated as 

$$D = \frac{\log(N)}{\log(r)} = \frac{\log 8}{\log 5} = 1.29$$

where, $N$ is the total number of segments at each iteration and $r$ is number by which the initiator is divided at each iteration.

The antenna size is related to the wavelength in the dielectric medium, which is also responsible for the operation of an antenna at any particular frequency. The meander-Koch curve Fractal antenna has a longer curve which increases the wavelength. At every iteration, the effective length $l$ of a meander-Koch curve Fractal antenna is calculated as $l = L(N/r)^n = L(8/5)^n$ where $L$ is the length of the initiator, $n$ is the number of iterations, and $N/r = 1.6$ is the factor by which the initial length is increased at the first iteration. The generation of the longer curve is due to the two characteristic properties of the Fractals: the self-affine and space-filling properties. Due to the self-affine property, the structure shown in Figure 1C resembles its parts in some way, albeit at different scales. From Figure 1C, it can also be noticed that the shape of the meander-Koch curve Fractal antenna has occupied more volume compared to a linear strip shown in Figure 1A. The space-filling property of the Fractal antennas is related to their iterative expansion that extends the current path while maintaining the same height as the linear strip radiator shown in Figure 1A. The elongated current path lowers the fundamental resonant frequency that eliminates the need to increase the antenna size.

In theory, a perfect Fractal shape is based on the application of infinite iterations. However, in realistic situations, there are limitations related to the practical designs such that an increase in the iterations causes the Fractal shape to convolve and the adjacent microstrip lines intersect each other if their width is not narrow enough. Furthermore, most
of the physical volume increase between the first and second iterations and after the first few iterations, the electrical length of the Fractal antennas does not increase noticeably.

3 | ANALYSIS OF MEANDER-KOCH CURVE FRACTAL ANTENNA

The analysis of the meander-Koch curve Fractal antenna is performed using the segmentation technique and coupling impedance formulas.

3.1 | Segmentation method

The composite shape of the meander-Koch curve Fractal antenna is generated using the IFS. Meander and Koch curve geometries are merged to scale down the antenna size. The electrical length of the meander-Koch element is 0.19\(\lambda\) that corresponds to its length (\(L_{M-K} = 23\) mm). This configuration resulted in a 20% antenna size reduction compared to the 0.25\(\lambda = 120\) mm (free space wavelength) monopole operating at the 2.45 GHz. In the segmentation method, the length of the Fractal curve is distributed over its 59 segments. However, the width of these segments is varied to find the best match at the 2.45 GHz. The width was not extended beyond 1 mm to avoid the overlapping of sharp corners of the Fractal curve. Therefore, width constraints have been imposed in such a way that \(W_{M-K} < 1\).

As shown in Figure 1C, the shape of the proposed meander-Koch curve Fractal antenna is fairly tortuous with many bends. Therefore, the input impedance \(Z_{in}\) cannot be calculated directly using the coupling impedance formulas. To apply the segmentation technique for the analysis, this antenna shape is divided into rectangular and triangular (45°, 45°, 90°) segments. In general, regardless of antenna shapes, the complete geometry is divided into 0, 1, 2, … \(d - 1\) segments, where \(d\) is the last segment of the geometry. The geometry of the meander-Koch curve element contains a total of \(d = 59\) (0-58) segments.

The decomposition of the antenna shape into various segments disrupts the continuous current distribution. For the interconnection between the segments, discrete ports are assigned at certain points along the edges to replace the continuous connection between the segments. Each port has the corresponding voltage-current matrix circuit equations that are based on the coupling impedances. In general, for any \(x\) number of segments, two sets of ports are required to be attached to both connected sides where the impedance parameters are calculated. The number of ports is dependent on the width of the segment and for our application, two ports are used for the internal connection of the segments. In curve-type geometries, the last segment is an exception because it has only one set of interconnection ports that connect it to the previous segment. Therefore, for the input impedance calculation of meander-Koch curve Fractal antenna, the total number of required matrix circuit equations to be solved is 117 for combining the coupling impedances of all the regular shaped elements. In this manner, the impedance of an irregular antenna (constructed using many regular shapes) can be analyzed. With the new formation of the interconnections using the discrete ports, the meander-Koch curve Fractal antenna is modeled as the multiport network, and it is now possible to compute the self and mutual impedance matrix elements between the segment interconnections. To determine the overall impedance of the amalgamated geometry, all the matrix elements are combined through the segmentation method. The generalized steps for the analysis of the curve-type antennas are given below:

1. The Fractal antennas are modeled as the regular multiport segments. Each segment is connected to the adjacent segment via the internal virtual ports. There is one external feed port (port 1 located at segment 0).
2. The impedance matrix of the whole geometry is given by combining the self and the coupling impedance contributions from all the segments.
3. Equations are deduced from the impedance matrix. For the continuity of the electric and magnetic fields across the interfaces of the interconnecting segments, the constraints at the boundaries are imposed, which equates port voltages-currents of the two interconnected segments. This process is performed for all the segments in the geometry.
4. The coupling impedance formulas for rectangular and triangular (45°, 45°, 90°) segments are used to determine the matrix elements (self and mutual impedances). The selection of the appropriate formulas is dependent on the shape of the segment and the location of their respective ports.
5. All the matrix equations are solved until the feed port impedance \(Z_{01}\) is obtained, which is a single complex number representing the overall impedance of the complete geometry.
FIGURE 2  Segmentation of the meander-Koch curve Fractal antenna. (A) Second iteration (B) detailed and enlarged view of the decomposition of the generator into rectangles and $45^\circ$, $45^\circ$, $90^\circ$ triangles

The impedance matrix for the boundary connection between all the segments in Figure 2 is given below:

\[
\begin{bmatrix}
v_{01} \\ v_{02} \\ v_{11} \\ v_{12} \\ v_{21} \\ v_{22} \\ v_{31} \\ \vdots \\ v_{581}
\end{bmatrix} =
\begin{bmatrix}
Z_{011} & Z_{012} & \cdots & \cdots & \cdots & & 0 \\
Z_{021} & Z_{022} & \cdots & \cdots & \cdots & & 0 \\
0 & 0 & Z_{111} & Z_{112} & \cdots & \cdots & 0 \\
0 & 0 & Z_{121} & Z_{122} & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & Z_{211} & Z_{212} & \cdots & 0 \\
0 & 0 & 0 & 0 & Z_{221} & Z_{222} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & Z_{5811}
\end{bmatrix}
\begin{bmatrix}
\vdots \\
i_{581}
\end{bmatrix}
\]

(12)

Except $v_{01}$ which is the feed port voltage, for the continuity of current between the peripheries of the segments, it is required that $v_{x_i} = v_{(x+1)i}$ and $i_{x_i} = -i_{(x+1)i}$. Using these relations (12) is modified to:

\[
\begin{bmatrix}
v_{01} \\ v_{02} \\ v_{02} \\ v_{01} \\ v_{12} \\ v_{12} \\ v_{22} \\ \vdots \\ v_{581}
\end{bmatrix} =
\begin{bmatrix}
Z_{011} & Z_{012} & \cdots & \cdots & \cdots & & 0 \\
Z_{021} & Z_{022} & \cdots & \cdots & \cdots & & 0 \\
0 & 0 & Z_{111} & Z_{112} & \cdots & \cdots & 0 \\
0 & 0 & Z_{121} & Z_{122} & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & Z_{211} & Z_{212} & \cdots & 0 \\
0 & 0 & 0 & 0 & Z_{221} & Z_{222} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & Z_{5811}
\end{bmatrix}
\begin{bmatrix}
\vdots \\
i_{581}
\end{bmatrix}
\]

(13)
The matrix circuit equations for (13) are given as

\[ \begin{align*}
    v_0 &= Z_{01}i_0 + Z_{02}i_2 \\
    v_1 &= Z_{01}i_0 + Z_{02}i_0 \\
    v_2 &= -Z_{11}i_0 + Z_{12}i_2 \\
    v_3 &= -Z_{11}i_0 + Z_{12}i_1 \\
    v_4 &= -Z_{21}i_1 + Z_{22}i_2 \\
    v_5 &= -Z_{21}i_2 + Z_{2}i_3 \\
    \vdots \\
    v_{57} &= -Z_{57}i_5 + Z_{57}i_7 \\
    v_{57} &= -Z_{58}i_8 + Z_{58}i_8 \\
\end{align*} \]

(14)

This system of matrix circuit equations in (14) is solved until the equations are reduced to the following equation:

\[ v_0 = i_0 \left[ Z_{01} + Z_{02}(-Z_{11} - Z_{02} + Z_{12}N_{57}Z_{12})^{-1}Z_{02} \right] \]

(15)

Therefore, the overall impedance of the meander-Koch Fractal antenna is given as

\[ Z_{in} = \frac{v_0}{i_0} = [Z_{01} + Z_{02}(-Z_{11} - Z_{02} + Z_{12}N_{57}Z_{12})^{-1}Z_{02}] \]

(16)

where,

\[ \begin{align*}
    N_{57} &= (Z_{21} + Z_{12} - Z_{22}N_{56}Z_{22})^{-1} \\
    N_{56} &= (Z_{31} + Z_{22} - Z_{32}N_{55}Z_{32})^{-1} \\
    N_{55} &= (Z_{41} + Z_{32} - Z_{42}N_{54}Z_{42})^{-1} \\
    N_{54} &= (Z_{51} + Z_{42} - Z_{52}N_{55}Z_{52})^{-1} \\
    \vdots \\
    N_1 &= (Z_{58} + Z_{57})^{-1} \\
\end{align*} \]

where, \( Z_{01} \) is a sub-matrix having a single element. The sub-matrix \( Z_{02} \) is a vector matrix of dimensions \( 1 \times N \) and \( Z_{02} = Z_{02}^T \). All the other sub-matrices are square matrices with equal dimension \( N \times N \).

### 3.2 Coupling impedance formulas

For the calculation of the coupling impedances, the generalized coupling impedance formula for any microstrip shape has been given by Okoshi.\(^{32}\)

\[ Z_{yx} = \frac{1}{W_p W_c} \int \int G(x_p, y_p | x_c, y_c) \, dr_p \, dr_c \]

(17)
where, \( x \) is the segment number, the ports \( p \) and \( c \) have associated widths \( W_p \) and \( W_c \), respectively. The co-ordinate location for the port \( p \) is \((x_p, y_p)\) and similarly, the port \( c \) is located at \((x_c, y_c)\). \( dr_p \) and \( dr_c \) are the incremental port distances over the widths \( W_p \) and \( W_c \), respectively. The port widths \( W_p \) and \( W_c \) should be minuscule so that the current density remains uniform across the port widths. The port widths should satisfy \( W_p, W_c \ll \lambda \). \(^{33}\)

The segmentation method is applied by decomposing the meander-Koch curve element into a number of rectangular and triangular \((45^\circ, 45^\circ, 90^\circ)\) segments. The coupling impedances of these shapes are given by their respective Green’s functions. To calculate the mutual impedance between the various placements of the ports on the edges of the rectangular segments, the following three cases have been considered\(^{34}\):

1. The self-impedances (eg, \( Z_{0_{11}}, Z_{0_{22}}, Z_{1_{11}}, Z_{1_{22}} \), and so on) of the rectangular segments when both the ports are placed on the same side are calculated as

\[
Z_{xp} = jo\mu h \left[ -\cot \frac{a_k}{b_k} + \frac{2b^2}{W_p^2 \pi^3} \sum_{n=1}^{\infty} \frac{(\sin n\theta_1 - \sin n\theta_2)^2}{n^2 \sqrt{n^2 - B^2}} \coth \frac{a\pi}{b} \sqrt{n^2 - B^2} \right] \tag{18}
\]

where,

\[
\theta_1 = \frac{\pi}{b} \left( y_p + \frac{W_p}{2} \right); \quad \theta_2 = \frac{\pi}{b} \left( y_p - \frac{W_p}{2} \right); \quad \theta_3 = \theta_1; \quad \text{and} \quad \theta_4 = \theta_2
\]

2. If ports are located on the opposite sides of the rectangular segments, the coupling impedance formula for the calculation of port impedances (eg, \( Z_{0_{12}}, Z_{1_{12}}, Z_{2_{12}}, \) and so on) is given as

\[
Z_{xp} = jo\mu h \left[ \frac{1}{bk \sin ak} - \frac{2b^2}{W_p W_c \pi^3} \sum_{n=1}^{\infty} \frac{(\sin n\theta_1 - \sin n\theta_2)(\sin n\theta_3 - \sin n\theta_4)}{n^2 \sqrt{n^2 - B^2} \sinh \frac{a\pi}{b} \sqrt{n^2 - B^2}} \right] \tag{19}
\]

where

\[
\theta_1 = \frac{\pi}{b} \left( y_p + \frac{W_p}{2} \right); \quad \theta_2 = \frac{\pi}{b} \left( y_p - \frac{W_p}{2} \right); \quad \theta_3 = \frac{\pi}{a} \left( x_c + \frac{W_c}{2} \right); \quad \text{and} \quad \theta_4 = \frac{\pi}{a} \left( x_c - \frac{W_c}{2} \right)
\]

3. For the adjacent placement of ports on the edges of the rectangular segments, the coupling impedances (eg, \( Z_{5_{12}}, Z_{17_{12}}, Z_{29_{12}}, \) and so on) are calculated using the following formula

\[
Z_{xp} = jo\mu h \left[ \frac{\sin A(\pi - \theta_4) - \sin A(\pi - \theta_3)}{bk^2 W_c \sin ak} \right]
+ \frac{2b^2}{W_p W_c \pi^3} \sum_{n=1}^{\infty} \frac{(\sin n\theta_1 - \sin n\theta_2) \left[ \sinh \frac{a(\pi - \theta_4)}{b} \sqrt{n^2 - B^2} - \sinh \frac{a(\pi - \theta_3)}{b} \sqrt{n^2 - B^2} \right]}{n(n^2 - B^2) \sinh \frac{a\pi}{b} \sqrt{n^2 - B^2}} \right] \tag{20}
\]

where

\[
\theta_1 = \frac{\pi}{b} \left( y_p + \frac{W_p}{2} \right); \quad \theta_2 = \frac{\pi}{b} \left( y_p - \frac{W_p}{2} \right); \quad \theta_3 = \frac{\pi}{a} \left( x_c + \frac{W_c}{2} \right); \quad \text{and} \quad \theta_4 = \frac{\pi}{a} \left( x_c - \frac{W_c}{2} \right)
\]

For all the rectangular segments, the lengths of the two sides along \( x \)- and \( y \)-axes are given by \( a \) and \( b \), respectively. Port location for port \( p \) is \((x_p, y_p)\), and similarly for port \( c \) is \((x_c, y_c)\). \( W_p \) and \( W_c \) are the widths of the two ports \( p \) and \( c \), respectively, \( \omega = 2\pi f, A = ak/\pi, B = bk/\pi, h \) is the height of the dielectric substrate and \( k = \sqrt{\omega^2\mu_0\epsilon_\perp(1 - j/Q)} \).

Similarly, to calculate the self and mutual impedances of the triangular \((45^\circ, 45^\circ, 90^\circ)\) segments, the following three cases are considered\(^{35}\):
1. When both the ports are located on the perpendicular side of the 45°, 45°, 90° triangle, the self-impedances (eg, \(Z_{3,1}\), \(Z_{7,1}\), \(Z_{11,1}\), and so on) are calculated using the following formula:

\[
Z_{x_p} = j \omega \mu \left[ -\frac{\cot ak}{ak} + \frac{\sin A\theta_2 - \sin A\theta_1}{ak^2 W_p \sin ak} + \frac{2a^2}{W_p^2 \pi^3} \sum_{m=1}^{\infty} \left( \frac{\sin m\theta_1 - \sin m\theta_2)^2 \coth \pi \sqrt{m^2 - A^2}}{m^2 \sqrt{m^2 - A^2}} \right) \right. \\
\left. + \frac{2a^2}{W_p^2 \pi^3} \sum_{m=1}^{\infty} \left( \frac{(-1)^m \sin m\theta_1 - \sin m\theta_2)(\sin _{\theta_1} \sqrt{m^2 - A^2} - \sin _{\theta_1} \sqrt{m^2 - A^2})}{m(m^2 - A^2) \sinh \pi \sqrt{m^2 - A^2}} \right) \right]
\]

where,

\[
\theta_1 = \frac{\pi}{a} \left( y_p + \frac{W_p}{2} \right); \theta_2 = \frac{\pi}{a} \left( y_p - \frac{W_p}{2} \right); \theta_3 = \theta_1; \text{and} \theta_4 = \theta_2
\]

2. The coupling impedances (eg, \(Z_{3,1}\), \(Z_{7,1}\), \(Z_{11,1}\), and so on) of the 45°, 45°, 90° triangles whose ports are placed along the perpendicular and hypotenuse sides are calculated by:

\[
Z_{x_p} = j \omega \mu \left\{ \frac{2}{a^2 k^2} + \frac{2\sqrt{2}[\sin(A\pi - \theta_1) - \sin(A\pi - \theta_4)]}{ak^2 W_c \sin ak} - \frac{2[\sin(A\theta_1/\sqrt{2}) - \sin(A\theta_2/\sqrt{2})]}{ak^2 W_p \sin(ak/\sqrt{2})} \right\}
\]

where,

\[
\theta_1 = \frac{\pi}{a} \left( y_p + \frac{W_p}{2} \right); \theta_2 = \frac{\pi}{a} \left( y_p - \frac{W_p}{2} \right); \theta_3 = \frac{\pi}{a} \left( x_c + \frac{W_c}{2\sqrt{2}} \right); \theta_4 = \frac{\pi}{a} \left( x_c - \frac{W_c}{2\sqrt{2}} \right)
\]

3. When the ports are only located on the hypotenuse side, the coupling impedances (eg, \(Z_{3,1}\), \(Z_{7,1}\), \(Z_{11,1}\), and so on) are calculated using the following formula:

\[
Z_{n_x} = j \omega \mu \left\{ \frac{1}{a^2 k^2} - \frac{\cot(\sqrt{2}k)}{\sqrt{2}ak} + \frac{\sin \left[ A(\pi^2 - 2\theta_1)/\sqrt{2} \right] - \sin \left[ A(\pi^2 - 2\theta_2)/\sqrt{2} \right]}{\sqrt{2}ak^2 W_p \sin \left( \frac{ak}{\sqrt{2}} \right)} + \frac{\sin \left[ A(\pi^2 - 2\theta_3)/\sqrt{2} \right] - \sin \left[ A(\pi^2 - 2\theta_4)/\sqrt{2} \right]}{\sqrt{2}ak^2 W_c \sin \left( \frac{ak}{\sqrt{2}} \right)} \right\}
\]

where,

\[
\theta_1 = \frac{\pi}{a} \left( x_p + \frac{W_p}{2\sqrt{2}} \right); \theta_2 = \frac{\pi}{a} \left( x_p - \frac{W_p}{2\sqrt{2}} \right); \theta_3 = \frac{\pi}{a} \left( x_c + \frac{W_c}{2\sqrt{2}} \right); \theta_4 = \frac{\pi}{a} \left( x_c - \frac{W_c}{2\sqrt{2}} \right)
\]
For all the triangular \((45^\circ, 45^\circ, 90^\circ)\) segments, \(a\) is the length of the perpendicular side. The co-ordinate locations for ports \(p\) and \(c\) are \((x_p, y_p)\) and \((x_c, y_c)\), respectively. \(W_p\) is the width of the port \(p\), while \(W_c\) is for port \(c\); \(\omega = 2\pi f\), \(\Lambda = a k / \pi\), \(h\) is the height of the dielectric substrate and \(k = \sqrt{\omega^2 \mu \varepsilon_0 \varepsilon_r (1 - j/Q)}\).

### 4 | EXPERIMENT RESULTS

A prototype of the meander-Koch curve Fractal antenna was developed to validate the theory for its analysis in Section 3. The photograph of the meander-Koch curve Fractal antenna prototype is shown Figure 3 and the details of the antenna dimensions are given in Table 1. The antenna was fabricated on the FR-4 substrate with a dielectric constant \(\varepsilon_r = 4.3\) and thickness \(h = 1.6\) mm. The reflection coefficient measurement was carried out using the Agilent PNA E8363C.

The input impedance of the antenna was calculated using the segmentation method and then transformed into the reflection coefficient \((S_{11})\) to compare the calculated result with the simulated and measured results. The meander-Koch curve Fractal antenna was simulated using the CST Microwave Studio. The comparison plot between the measured, simulated, and calculated reflection coefficients is shown in Figure 4. The measured, simulated, and calculated resonance frequencies are 2.4 GHz, 2.26 GHz, and 2.46 GHz respectively. The calculated minimum input reflection coefficient is \(-22\) dB. Further, the result shows that the calculated and measured resonant frequencies are in good agreement with a percentage error of 2.85%. Moreover, the measured and simulated minimum reflection coefficients are \(-19\) dB and \(-17.5\) dB, respectively. The predicted resonant frequency \((\min|S_{11}|)\) matches the simulated one with the percentage error of about 8.37%. The reason for the higher calculated resonant frequency is the assumption made by the segmentation method (which is based on the generalized cavity model) that the ground plane is similar in dimensions to the top of the radiating element. However, the virtual prototype for simulation and the actual measurement prototype of the meander-Koch curve Fractal antenna were developed with a reduced ground plane to transform the microstrip antenna into a printed monopole antenna. Therefore, as seen in Figure 4, the calculated resonant frequency corresponding to meander-Koch curve Fractal antenna backed by a full rectangular ground plane underneath is higher compared to the simulated and measured results that refer to a modified meander-Koch curve Fractal antenna backed by a reduced rectangular ground plane.

![Figure 3](image)

**Figure 3** Photograph of the meander-Koch Fractal antenna

| \(L\) | \(W\) | \(L_{M,K}\) | \(W_{M,K}\) | \(L_f\) | \(W_f\) |
|------|------|----------|---------|------|------|
| 40   | 25   | 23       | 0.66    | 16   | 1.8  |

**Table 1** Dimensions (mm) of the proposed meander-Koch curve Fractal antenna
The measured and simulated 10-dB impedance bandwidths ($|S_{11}|<-10\, \text{dB}$) are 22.58% (2.03-2.55 GHz), and 14.5% (2.65-2.29 GHz), respectively. Noticeably, the segmentation method has performed inadequately in predicting the impedance bandwidth. The 10-dB calculated impedance bandwidth ranges from 2.43 to 2.51 GHz. The reason for this wide discrepancy is that the segmentation method only considers the model of a microstrip antenna for the calculation of the input impedance over a frequency range. However, the actual antenna is a variation of the microstrip antenna model that has been realized with a reduced ground plane which remains beneath the meander-Koch curve strip. In cases where the geometric size of the small antenna is not similar to the size of the ground plane, the currents on a reduced ground plane become a source of radiation and are also influential in the calculation of impedance and bandwidth. The printed monopole antenna is thought to contain a large air dielectric substrate with permittivity ($\varepsilon_r \cong 1$), which extends beyond the physical substrate. This thick air substrate is assumed to enhance the bandwidth of the printed monopole antennas. Another reason for the wide bandwidth in the simulated and measured results is that the printed monopole antenna is analogous to a vertical monopole antenna that comprises a cylindrical wire mounted over a ground plane. An increase in the diameter of the wire increases the bandwidth. Hence, the printed monopole antenna is considered similar to a cylindrical monopole antenna that has a large effective diameter. These effects are not taken into account for the calculation of the input reflection characteristics using the segmentation method. Here, it is assumed that the substrate has a very small thickness in comparison to the wavelength ($h \ll \lambda$); therefore, the magnetic field has the components along the $x$- and $y$- axes, whereas the electric field is oriented along the $z$- axis only. Moreover, there is no field variation along the $z$- axis (perpendicular to the ground plane).

In general, the cavity model-based methods have high accuracy when they are applied to the wide microstrip patches. However, all the segments of the meander-Koch curve Fractal antennas have narrow widths. Due to the reasons mentioned above, the analysis using the segmentation method was unable to accurately predict the impedance matching over a frequency range (bandwidth). However, it has performed reasonably to predict the reflection characteristics and the resonant frequency.

The simulated and measured radiation patterns of the meander-Koch Fractal antenna are presented in Figure 5. From the results, it can be seen that the meander-Koch curve Fractal antenna has stable patterns. The Y-Z plane pattern is omnidirectional at 2.4 GHz. Although the measured E-plane pattern, as shown in Figure 5B, has the same “8” shape pattern as the simulated one. However, the agreement is not as close as it looks in the case of the H-plane patterns. The simulated and measured gains are 2 dBi and 1.9 dBi respectively at 2.4 GHz.

### 5 | PARAMETRIC STUDY

In order to characterize the design of the meander-Koch curve Fractal antenna, its certain parameters are studied for their effects on the resonance behavior of the antenna. The study on the effects of the design parameters is carried out using CST MWS. The length and width of the Fractal strip are taken as variables and these parameters are varied one by one.
The simulated reflection characteristics for different values of length $L_{M,K}$ and width $W_{M,K}$ of the meander-Koch curve Fractal antenna are shown in Figures 6 and 7, respectively.

From the parametric study on the variation of the overall length of the strip $L_{M,K}$, it is observed that when the length of the radiating element increases, it causes a decrease in the resonance frequency. This behavior is typical to the monopole antennas, where an increase or reduction in the strip length inversely affects the resonant frequency. Hence from the parametric study, it is found that the length of the monopole is determining the resonant frequency of the meander-Koch curve Fractal antenna. Since the available substrate length $L$ is not sufficient to accommodate the increment of the strip beyond 23 mm, the length of the strip could not be varied without increasing the substrate length $L$. Therefore, the length of the substrate has also been increased so that the strip length could be extended to 24 mm and 25 mm.

The modeling of the antenna geometry using IFS describes the variation of the length of the line segments in horizontal and vertical directions such that an ideal Fractal geometry is constructed having no or zero width. To realize such an antenna physically, it is essential to assign a width to the strip trace; however, the width of the microstrip line should be narrow enough so that the intersection of the adjacent lines can be avoided. The proposed antenna has been studied for various widths $W_{M,K}$ of the Fractal strip. The values are varied from 0.594 to 0.726. The variation of the return loss with respect to the $W_{M,K}$ shows that increasing the antenna trace width results in the frequency reduction. This behavior is
FIGURE 7  Simulated reflection coefficient of the meander-Koch curve Fractal for various strip widths

![Graph showing simulated reflection coefficient for various strip widths](image)

FIGURE 8  Simulated reflection coefficient of the meander-Koch curve Fractal for various feed line widths

![Graph showing simulated reflection coefficient for various feed line widths](image)

analogous to the monopole antennas where the frequency is reduced if the diameter of the monopole or the width of the microstrip line increases. Moreover, the observed changes with respect to the $W_{M-K}$ indicates that the width of the strip is an important parameter that causes a change in the effective area of the meander-Koch curve Fractal antenna, which in turn affects the resonance frequency.

Figure 8 shows the effect of the variation of the feed line width $W_f$ on the simulated reflection coefficients. A variation in the transmission line width has a more noticeable impact on the impedance matching of the antenna than the shift in the resonance frequency. The width of the feed line $W_f$ is varied from 1.6 mm to 2 mm. At 1.6 mm, the frequency is slightly higher than the other two frequencies that correspond to 1.8 mm and 2 mm; whereas at $W_f = 2$ mm, the matching is better among all the considered widths of the feed line. Therefore, $W_f = 1.8$ mm is selected as a compromise between the higher frequency and better matching.

6  |  COMPARISON WITH STATE-OF-THE-ART

The performance comparison of the proposed antenna with the relevant reported antennas is given in Table 2. The proposed antenna has the smallest size among all the antennas given in the Table 2.

7  |  CONCLUSION

A microstrip-fed hybrid Fractal curve antenna for WLAN is analyzed in this article. The segmentation method is used to segment the composite shape of the proposed meander-Koch curve Fractal antenna. The impedance matrix elements of the segments are calculated using the coupling impedance formulas of the rectangles and $45^\circ$, $45^\circ$, $90^\circ$ triangles. The
The performance comparison of this antenna with other state-of-the-art antennas is given below.

| References | Dimensions (mm) | Operation (GHz) | Bandwidth (MHz) | Gain (dBi) | Radiation pattern |
|------------|-----------------|-----------------|-----------------|------------|------------------|
| 39         | $102 \times 83$ | 2.433-2.449     | 16              | Not reported | Directional      |
| 40         | $53 \times 46.7$| 2.43-2.48       | 50              | 2.65       | Directional      |
| 41         | $48 \times 48$  | 2.26-2.61       | 350             | 3.5        | Omnidirectional  |
| 42         | $39 \times 39$  | 2.36-2.55       | 190             | 2.06       | Omnidirectional  |
| 43         | $71 \times 49$  | 2.3-2.6         | 300             | 1.44       | Omnidirectional  |
| 13         | $147 \times 112$| 2.4, 2.5        | Not reported    | Not reported| Not reported     |
| This work  | $40 \times 25$  | 2.03-2.55       | 520             | 1.9        | Omnidirectional  |

Impedances of the segments are then combined using the multiport connection method to calculate the impedance of the complete geometry. The calculated result is compared to the CST MWS simulation and prototype measurement results. It is shown that the calculation using the segmentation method was able to reliably predict the resonance behavior of the meander-Koch curve Fractal antenna. However, due to the inherent limitations of the cavity model, a vast discrepancy is observed for the calculated impedance bandwidth when compared to the simulated and measured results. Nevertheless, the segmentation method can be effectively utilized as a first step to analyze and understand the behavior of the Fractal curve antennas.

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CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

AUTHORS CONTRIBUTION

Atif Jamil contributed to the conceptualization, data curation, formal analysis, investigation, methodology, software, validation, writing the original draft. Mohd Zuki Yusoff contributed to the funding acquisition, investigation, project administration, supervision. Noorhana Yahya contributed to the formal analysis, project administration, resources.

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