Influence Factors of Gravitational Acceleration near the Earth

Xuefeng Shi, Xingsheng Wang, Ling Zhang and Kaimin Guo*

School of Physical Science and Technology, Baotou Teachers’ College, Baotou, China

*Corresponding author: kaiminguo@bttc.cn

Abstract. Objects on the earth are subject to gravity due to universal gravitation. This paper systematically studies the influence factors, including latitude, altitude, depth inside the earth, surrounding stars, on gravitational acceleration of object near the earth. The results show that latitude has a relatively large impact on gravitational acceleration; depth and altitude have the greatest impact on gravitational acceleration. The orbit and the position of the object (dayside or nightside of the earth) have little effect on the gravitational acceleration of the object. Our work can offer reference for accurate calculation of gravity acceleration.

Keywords: Gravitational acceleration, universal gravitation, latitude, height, depth.

1. Introduction

The most important force the object near the earth has been exerted is gravity due to universal gravitation. Therefore, when studying mechanics-related issues near the earth, gravity is the basic force need to be taken into account. The gravitational acceleration $g$ is the acceleration of an object caused by the force of gravitation. Normally, $g$ is 9.8 m/s$^2$. In fact, the gravitational acceleration near the earth is not a fixed value. The gravitational acceleration is varies when latitude changes. Previous studies have found that when the latitude increases while the object keep at the same altitude, $g$ gradually increases, $g$ is also changing with the increasing of altitude while the object keep at the same latitude. Although lots of investigations have studied the influence factors such as altitude, latitude, and the shape of the earth on gravitational acceleration, however, there is no systematic analysis of these factors on $g$. This paper will give a systematic and detailed explanation of the influence factors of $g$ and analyze the influence of surrounding stars on $g$.

2. Gravity and universal gravitation

2.1. The origin of gravity

Newton discovered the law of universal gravitation, i.e. there is an attractive force between any two objects. The interaction force between two objects is proportional to the product of the two objects’ mass and inversely proportional to the second power of the distance between the two objects, which can be expressed as [1].
\[ F = G \frac{m_1 m_2}{r^2} \]  

Among them, \( G \) is the gravitational constant, which is measured by Cavendish using the torsion balance experiment, \( m_1 \) and \( m_2 \) is mass of the two objects, \( r \) is the distance of the two objects.

Universal gravitation provides gravity for the object near the earth. Because of the rotation of the earth and the irregularity of the earth sphere, the objects in different places on the earth gain different gravity.

2.2. The direction of gravity

Use \( F_1 \) to denote universal gravitation between the earth and the object on the earth, and \( F_g \) to denote the gravity that the object receives. Gravity on the earth provides the centripetal force required for gravity and the earth's rotation. It can be seen from Figure 1 that the direction of \( F_1 \) points to the center of the earth, the centripetal force when the earth rotates is perpendicular to the axis of the earth, and the direction of gravity is not pointing to the center of the earth, that is to say, it is not vertical down. In fact, the direction of gravity is perpendicular to the horizontal plane, that is to say, the direction of gravity is vertically downward.

![Figure 1. The force analysis of universal gravitation and gravity.](image_url)

3. Factors affecting the gravitational acceleration

3.1. The influence of latitude

Considering this problem, we should approximate the earth as a regular circular sphere [2]. The force analysis of gravity and universal gravitation of object near the earth’s surface is shown in Figure 1.

The radius of the earth is \( R \), and \( \theta \) represents the angle between the equatorial plane of the earth and the position of the object, which is what we usually call latitude. From Figure 1, the forces of universal gravitation \( F_1 \), gravity \( (F_g) \), centripetal force \( (F_n) \) satisfy the vector superposition principle.

\[ F_1 = F_g + F_n \]  

\[ F_n = m\omega^2 R \cos \theta \]  

\[ F_g = mg \]  

\[ F_1 = G \frac{Mm}{R^2} \]
It can be seen qualitatively from the above formula that as the latitude rises, the cosine value of the latitude decreases continuously and decreases to 0 at the poles. At this time, the gravitational acceleration at the poles is 

\[ g_0 = \frac{GM}{R^2} = \frac{g_0}{\omega^2 R^2} \]

where the mass of earth \( M = 5.98 \times 10^{24} \) kg, the gravitational constant \( G = 6.6799 \times 10^{-11} \) N\( \cdot \)m/kg\(^2\), and the average rotation angular velocity of the earth \( \omega = 2\pi/T \), the radius of the earth [3] is \( R = 6371.03 \) km.

Analyse and fit the function (6) quantitatively, the result is shown in Fig.2.

From Fig.2 we can see that the gravitational acceleration is gradually increasing from the equator to the poles, but it is a non-linear change, and the gravitational acceleration changes fast at first and then slowly. Through calculation and fitting, the gravitational acceleration at the equator is \( g_{eq} = 9.7929 \) m/s\(^2\), the gravitational acceleration at the two poles is equal to \( g_{poles} = 9.8266 \) m/s\(^2\). The difference between the gravitational acceleration considering with rotation and without rotation is

\[ \delta g = 0.0336 \text{ m/s}^2 \]

It can be seen that the rotation of the earth has a small effect on the gravitational acceleration.

The measured gravitational acceleration near the equator is \( g_{eq} = 9.78030 \) m/s\(^2\), the calculated equatorial gravitational acceleration is \( g_0 = 9.7929 \) m/s\(^2\), the relative error is 1.3%. The measured gravitational acceleration at the poles is \( g_{poles} = 9.832 \) m/s\(^2\), the calculation of the two-pole gravitational acceleration is \( g_{poles} = 9.8266 \) m/s\(^2\), the relative error is 0.5%. The reason for this is the earth is not an ideal sphere, and the surface is uneven.

3.2. The influence of the earth's altitude

The surface of the earth is not a horizontal plane. The geomorphology of the earth varies tremendously with the change of places. Therefore, the altitude will also affect the gravitational acceleration.

When an object is brought to a certain height near the earth, universal gravitation between the earth and the object still provides gravity. In the 3.1, it can be seen that the rotation of the earth has little effect on the distribution of gravitational acceleration on the earth, so when calculating the influence of the height on g, the influence of the earth's rotation can be ignored.

It can be obtained from equation (1) that the universal gravitation provides gravity for the object at \( h \) above the earth,

\[ mg = G \frac{Mm}{(R + h)^2} \]
\[ g = \frac{G M}{(R + h)^2} \]

**Figure 3.** Graph of gravitational acceleration with altitude.

### 3.3. The depth of the object

When the earth is regarded as a uniformly distributed sphere and the density distribution inside the earth is uniform, the gravitational acceleration of the object will varies with the change of depth inside the earth.

![Schematic diagram of distribution of the mass of the earth.](image)

**Figure 4.** Schematic diagram of distribution of the mass of the earth.

When an object is moved from the core of the earth to the surface, according to Gauss's theorem, the mass surrounded by the Gaussian surface will change, and the part of the earth providing gravity will change, so the gravitational acceleration will also change.

According to equation (1), it can be obtained that the universal gravitation between the earth and the object in the earth.

\[ G \frac{M m}{r^2} = mg \]

Where \( M^* \), represents the mass of the earth contained at \( r \) inside the earth,

\[ M^* = \frac{r^3}{R^3} M \]

The gravitational acceleration for the object at \( r \) inside the earth is,

\[ g = \frac{G M r}{R^3} \]

Through the above analysis, it can be obtained that the gravitational acceleration of the object within the earth changes linearly with the depth.
Figure 5. Gravitational acceleration changes with the depth inside the earth.

When the earth is regarded as a uniformly distributed sphere, according to the Gauss theorem, the change of the earth's gravitational acceleration can be obtained from the core of the earth to the outside as shown in Fig.6.

Figure 6. Gravitational acceleration changes with the depth and height of the object.

3.4. The influence of the sun
The earth is not a isolated system in the universe. There are many stars around the earth. These stars will have effect on the earth. Therefore, in reality, the gravitational acceleration will be affected by surrounding stars. Taking the influence of the sun as an example, the orbit of the earth's revolution is not a circle but an ellipse with a small curvature.

The perihelion is $R_{ph} = 1.47 \times 10^8 \text{km}$, the aphelion is $R_{ah} = 1.52 \times 10^8 \text{km}$. When the earth revolves, the sun’s force on the earth provides the centripetal acceleration of the earth’s rotation around the sun.

At perigee, on the side close to the earth acceleration provided by the sun, 

$$g_p = G \frac{M}{R_{ph}^2} = 0.0061 \text{m/s}^2$$

At apogee, on the side close to the earth acceleration provided by the sun, 

$$g_a = G \frac{M}{R_{ah}^2} = 0.0057 \text{m/s}^2$$

Therefore, when the earth is in different positions in its orbit, it is affected by the sun’s gravity. The difference between perigee and apogee is 0.0004 m/s².

3.5. Dayside and nightside effect
It has been pointed out in 3.4 that the gravitational acceleration on the earth will be affected by the sun. At the same distance (perihelion or aphelion), the object on the dayside and nightside of the earth gain
different forces. On the equator, the resultant force received are the force from the earth, the force from the sun and the rotation of the earth. Since the diameter of the earth is compared small with the radius of the orbit, it can be ignored.

The resultant force exerted of object on the surface of the earth at the perihelion:

\[ F_1 = F_g - F_{ph} \]  
\[ g_1 = g_0 - g_{ph} \]

It can be obtained that \( g_1 = 9.7868 \text{ m/s}^2 \) Compared with the actual measured value at the equator \( g_r = 9.78030 \text{ m/s}^2 \), the relative error is 0.06%.

The resultant force of the aphelion on the surface of the earth is:

\[ F_2 = F_g - F_{ah} \]  
\[ g_2 = g_0 - g_{ah} \]

It can be obtained that \( g_2 = 9.7872 \text{ m/s}^2 \) Compared with the actual measured value at the equator \( g_r = 9.78030 \text{ m/s}^2 \), the relative error is 0.0705%.

The resultant force of object on the dayside of the earth (perihelion),

\[ F_3 = F_g + F_{ph} \]
\[ g_3 = g_0 + g_{ph} \]

The resultant force exerted by the object on the nightside of the earth,

\[ F_4 = F_g + F_{ah} \]  
\[ g_4 = g_0 + g_{ah} \]

It can be obtained that \( g_3 = 9.799 \text{ m/s}^2 \). Comparing with the actual measured value at the equator the relative error is 0.19%. \( g_4 = 9.7986 \text{ m/s}^2 \) the relative error is 0.187% with the actual measured value at the equator.

It can be seen that at the perihelion of the earth's orbit, the gravitational acceleration measured at dayside and nightside are also different.

3.6. Other factors

The earth is an irregular ellipsoid with a slightly bulged equator. The influence of the distribution of the earth ellipsoid on calculation and measurement also needs to be considered. Miao Yunying [5] concluded that the earth should be regarded as a spheroid, which shows that the shape of the earth is also a factor that affects the gravitational acceleration.

| Earth's internal structure | Depth (km) | Density range (g/m³) |
|---------------------------|-----------|---------------------|
| Crust                     | 0-15      | 2.83                |
| Mantle                    | 15-3000   | 3.31-5.62           |
| Outer core                | 3000-5000 | 9.89-12.7           |
| Kernel                    | 5000-5370 | 12.7-13.0           |

The earth is an ellipsoid with uneven density distribution. The structure of the earth is roughly divided into four layers: inner core, outer core, mantle, and crust. In this way, the earth is a sphere composed of several concentric circles, and the material composition, density and temperature of each layer of the earth are not the same [6]. The density and thickness of each layer of the earth [7] are shown in Table 1. Hu Jing, et al. calculate the gravitational acceleration using the "Gauss theorem of gravitational field" on the earth's surface and give the density distribution of each layer.

The density of each layer inside the earth is different, so the gravitational acceleration inside the earth does not change linearly. The most areas of the earth surface are covered with water. For the surface of the earth, the distribution of matter at various places is uneven. Therefore, when calculating
the gravitational acceleration, only an approximate average density is selected. However, the density distribution of the earth will also affect the distribution of the gravitational acceleration.

4. Conclusion
The earth is an approximate ellipsoid with uneven distribution, irregular shape, slightly flattened poles and slightly bulged middle. Since these factors have little influence on the distribution of gravitational acceleration, the difference between the actual measured value of gravitational acceleration on the equator and the calculated value of the earth as a regularly distributed sphere is 0.0126 m/s², and the difference between the two poles is 0.0054 m/s². A rough comparison of the changes in gravitational acceleration can see the earth as a uniformly distributed regular sphere. According to Gauss's theorem, it can be concluded that as the latitude rises, the gravitational acceleration gradually decreases. At the same latitude, as the altitude increases, the gravitational acceleration gradually decreases, and decreases in inverse proportion. As the depth increases, the earth's gravitational acceleration is gradually decreasing.

The earth is not an isolated system in the solar system. The gravitational force of the sun on the earth will affect the gravitational acceleration on the surface of the earth. At the perigee and apogee positions on the earth's orbit, the gravitational acceleration of the earth is affected by the influence of the sun. However, the influence of the sun on the earth's gravitational acceleration is relatively small. The gravitational acceleration of object on the sunward surface at perihelion is slightly larger than at aphelion. At the same position of the earth's orbit, the gravitational acceleration on the dayside and nightside will also cause different gravitational accelerations due to the different directions of the sun's force. Moreover, the gravitational acceleration is not only affected by the sun but also by other stars.

At the same time, the factors such as the shape of the earth, the density distribution and the altitude will also affect the distribution of gravitational acceleration. Correction needs to be considered these factors in the accurate calculation of gravitational acceleration.

Acknowledgments
This work was financially supported by National Natural Science Foundation of China (11765013) fund.

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