Spin polarization effects in a two dimensional mesoscopic electronic structure with Rashba spin-orbit and lateral confinement

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Because of the peculiar coupling of spatial and spin degrees of freedom effected by the Rashba spin-orbit interaction, lateral confinement of a two dimensional electronic system leads to a finite transverse spin polarization near the longitudinal edges of a current carrying quantum wire. The sign of this component of the polarization is opposite at the two edges and can be reversed upon inversion of the current. Interestingly for small spin orbit coupling this is the largest contribution to the total polarization, its magnitude being of second order in the coupling constant. As a consequence this phenomenon cannot be revealed in lowest order perturbative approaches. An in-plane spin polarization component is also present that is perpendicular to the current. Within the same model this component would be also present in the bulk. On the other hand while in the latter case its magnitude is linear in the coupling constant, we find that it only represents a third order effect in the wire geometry. Our results are consistent with a general rigorous argument on the parity of the components of the spin polarization with respect to the sign of the spin orbit coupling constant.

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We consider in this paper the effects of the lateral confinement on the properties of a two-dimensional (2D) electron liquid in the presence of linear Rashba spin orbit. The basic idea underlying our work stems from the simple observation that in order to construct a localized wavefunction one must superimpose plane waves with opposite components of the wave vector along the direction of localization. Because spin-orbit forces the latter to have opposite spin components along the unrestricted direction, it is clear that in the direction of confinement the electronic states must display a correspondingly inhomogeneous spin polarization whose direction also changes in space. In the particular case in which a 2D electronic paramagnetic system is laterally confined to produce a quantum wire, the superposition involves spins that are parallel to the plane of motion and therefore combine to give an out of plane polarization component. Since the sign of this component of the magnetization is opposite for states with opposite momenta along the wire, it is clear that a net polarization can be established when the occupation of such states is unbalanced as in the case in which a net steady state current is forced through the system. As we will show this net perpendicular static spin polarization is most pronounced near the lateral boundaries and displays an oscillatory behavior. Inverting the direction of the current simply flips the polarization. No spin current is involved in the problem.

In order to demonstrate and exemplify this idea, we present here a complete analysis of the simplest possible model Hamiltonian describing a system of non interacting electrons subject to a homogeneous neutralizing background, a linear Rashba spin orbit coupling and a lateral confining potential. The effect of impurities is neglected although impurity scattering in the channel will not change our main conclusions. Although more realistic model potentials could be readily employed we have moreover chosen to describe the latter by means of infinite potential barriers, an assumption that allows us to present a transparent and explicit discussion while at the same time affording us a full description of the physical phenomenon. A great advantage of our model is that an analytic perturbative expansion in the spin-orbit coupling constant can be developed, a procedure that shows how the effect would not be revealed by lowest order perturbation theories. A generalization to more general smooth gating potentials is readily implemented.

That a steady state current can lead to a spin polarization in a 2D electron liquid subject to Rashba spin orbit coupling is hardly unexpected. It is in fact well known that a current forced along the $x$ direction of a bulk 2D electronic system will be accompanied by an in-plane polarization along $y$. For small spin orbit this polarization is linear in the coupling constant.

As we shall see however, while this component of the polarization is still present, confinement leads to a spin polarization that, for small spin orbit, is mostly out of plane. Interestingly, as already stated, this polarization can be flipped simply by inverting the current.

The magnitude of the effect strongly depends on the strength of the spin orbit coupling. A quick order of magnitude estimate can be obtained by inspecting the caption of our Figure while making reference to our model Hamiltonian (Eq. (1)).

While the problem can be completely solved by numerical means we will show that a careful study of the perturbation expansion in the spin orbit coupling constant is crucial in understanding the interplay of the various components of the polarization.

The paper is organized as follows: Section I defines our model and details our formulation; Section II describes the numerical and perturbative solutions to the single particle problem providing a discussion of various prop-
properties of the eigenfunctions and eigenenergies. Section III is devoted to the study of the total spin polarization in the presence of a steady state current. Many of our most relevant results are presented in this Section. Finally Section IV provides a discussion of the most interesting physics including that of the crossover of the in plane component of the polarization and our conclusions.

I. FORMULATION OF THE PROBLEM AND BOUNDARY CONDITIONS

The model Hamiltonian we consider is the following:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \alpha (\hat{\sigma}_x \hat{p}_y - \hat{\sigma}_y \hat{p}_x) + V(y),$$

where the motion is restricted to the $x, y$ plane and the lateral confining potential is given by:

$$V(y) = \begin{cases} 0 & \text{for } |y| < W/2 \\ \infty & \text{for } |y| \geq W/2. \end{cases}$$

Since the system is translationally invariant along the $x$ direction, the eigenfunctions of the problem can be constructed by linear superposition of free space ($V(y) = 0$) eigenstates with definite values of $k_x$. Furthermore the reflection symmetry $x \rightarrow -x$ allows one to generally assume $k_x > 0$.

For reference, we recall here that the free space electron eigenfunctions and eigenenergies are given by

$$\varphi_{k,\pm}^{(0)}(x, y) = \frac{e^{i(k_x x + k_y y)}}{\sqrt{2L^2}} \left( \pm \frac{1}{\hbar}\right)^\frac{1}{2} e^{i\phi_k}$$

$$\alpha_{k,\pm} = \frac{\hbar^2 (k_x^2 + k_y^2)}{2m} \mp \alpha \hbar \sqrt{k_x^2 + k_y^2},$$

where $L$ is the linear size of the system and $\phi_k$ is the angle between the direction of the wave vector and the $x$-axis. Interestingly the Rashba spin-orbit coupling forces each plane wave state to have its own distinct orientation of the spin quantization axis. This direction lies in the $x - y$ plane and makes an angle of $\frac{\pi}{2}$ with $\mathbf{k}$. These states form two split bands which are characterized by opposite chirality in the sense that states with the same wave vector have opposite spin directions in the two bands.

Moreover, because of time reversal, in the presence of Rashba spin orbit, states within the same band with opposite wave vectors have opposite spins. The corresponding surfaces (lines) at constant energy $\epsilon$ are concentric circles with radii $K_\pm$. These are given as the positive solutions of the equation

$$\epsilon = \frac{\hbar^2 K_\pm^2}{2m} \mp \hbar \alpha K_\pm.$$  

Constant energy lines are schematically displayed in Figure 1.

At a given energy $\epsilon$, there are only four free space eigenstates that can be combined to form solutions satisfying the zero boundary condition of the inhomogeneous problem. This is depicted in Figure 1 for each spin subband $\pm$ we have two states with opposite value of $k_y$. The four possible values of $k_y$ are $\pm k_y^\pm$, with

$$k_y^\pm = \sqrt{K_\pm^2 - k_x^2},$$

where $K_\pm$ are given in Eq. (5) above.

Since we are interested in spatially undamped solutions, we always take $k_x < k_+ \leq k_- < k_x$. However, it will necessary to include the case $K_- < k_x < K_+$, where $k_y = -|k_y^-|$ is purely imaginary. We also define the angles $\phi_\pm$ by means of:

$$\phi_\pm = \arctan \frac{k_y^\pm}{k_x}.\hspace{1cm}(7)$$

If $k_y = -|k_y^-|$ we have that also $\phi_-$ is purely imaginary and from the above formula we get $\phi_- = \arctan|k_y^-/k_x^-|$.

It is convenient to consider the following two couples of (unnormalized) states, which are also eigenstates of the reflection symmetry $y \rightarrow -y$:

$$\psi_{\pm\pm}(\epsilon, k_x, y) = \left(\begin{array}{c} \cos(\frac{k_y^\pm y}{2}) \\ \pm i \sin(\frac{k_y^\pm y}{2}) \end{array}\right) e^{i k_x x},$$

$$\psi_{\pm\mp}(\epsilon, k_x, y) = \left(\begin{array}{c} \pm i \cos(\frac{k_y^\mp y}{2}) \\ \sin(\frac{k_y^\mp y}{2}) \end{array}\right) e^{i k_x x}.$$

The first $\pm$ subscript refers to the chirality of the spin subband while the second $\pm$ subscript refers to the reflection operation $y \rightarrow -y$. The eigenstates in the confined system are then expressed in the following way:

$$\varphi_n^+(k_x, y) = c_n^+ \psi_{++} + c_n^- \psi_{--},$$

$$\varphi_n^-(k_x, y) = c_n^- \psi_{--} + c_n^+ \psi_{++},$$

where the superscript in $\varphi_n^\pm$ refers to the reflection parity, and not to the chirality. As one would expect, the solutions of the confined system are an admixture of states.
with different chirality. A spin subband index will be introduced as an approximate quantum number in the case of small spin orbit in a following Section where we present a perturbative treatment of the problem.

Imposing the boundary condition at \( y = W/2 \) for the \( \psi_n^\pm \) leads to the following conditions:

\[
\begin{pmatrix}
\cos \left( \frac{k_y^+ W - \phi_+}{2} \right) & \sin \left( \frac{k_y^- W - \phi_-}{2} \right) \\
i \cos \left( \frac{k_y^- W + \phi_-}{2} \right) & -i \sin \left( \frac{k_y^- W + \phi_-}{2} \right)
\end{pmatrix}
\begin{pmatrix}
\epsilon^{++} \\
\epsilon^{--}
\end{pmatrix} = 0 , \quad (12)
\]

\[
\begin{pmatrix}
\sin \left( \frac{k_y^+ W - \phi_+}{2} \right) & \cos \left( \frac{k_y^- W - \phi_-}{2} \right) \\
i \sin \left( \frac{k_y^- W + \phi_-}{2} \right) & -i \cos \left( \frac{k_y^- W + \phi_-}{2} \right)
\end{pmatrix}
\begin{pmatrix}
\epsilon^{+-} \\
\epsilon^{--}
\end{pmatrix} = 0 . \quad (13)
\]

At fixed \( k_x \), the the determinants of (12) and (13) are functions of \( \epsilon \), through the implicit dependence of \( k_y^\pm \) and \( \phi_\pm \). The zeros of these determinants provide two sets of discrete energies \( \epsilon_n^\pm(k_x) \), corresponding to states of opposite parity.

If one is not interested in separating states with different reflection parity, one can combine the conditions ensuing from (12) and (13) into the compact equation:

\[
(1 - \cos k_y^+ W \cos k_y^- W) \sin \phi_+ \sin \phi_- + (1 - \cos \phi_+ \cos \phi_-) \sin k_y^+ W \sin k_y^- W = 0 ,
\]

which gives the whole spectrum for a given \( k_x \). The eigenfunctions, dropping the reflection parity index, are henceforth quite generally denoted as \( \psi_\epsilon(k_x, y) \).

We finally notice that for values of \( \epsilon \) and \( k_x \) such that \( k_y^- \) is purely imaginary, the structure of the above formulas still holds, and one need only substitute hyperbolic for trigonometric functions.

II. SOLUTION OF THE PROBLEM

A. Exact solution at \( k_x = 0 \)

A case in which the solutions can be expressed in simple closed form is that of \( k_x = 0 \). For this particular value of \( k_x \) it is immediate to see that Eq. (11) admits solutions which are eigenstates of \( \hat{\sigma}_x \). Assuming the following form for the wavefunctions

\[
e^{\mp i \epsilon n W \phi}(y)|\pm\rangle_z ,
\]

\( \Phi(y) \) is found to satisfy:

\[
\left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + V(y) - \frac{1}{2} m\alpha^2 \right] \Phi(y) = \epsilon \Phi(y) . \quad (16)
\]

This result is quite general holding for an arbitrary form of the confining potential \( V(y) \). For the case of an infinite well we obtain:

\[
\epsilon_n = \frac{\hbar^2}{2m} \left( \frac{\pi n}{W} \right)^2 - \frac{1}{2} m\alpha^2 , \quad (17)
\]

where \( n \) is a positive integer. For every eigenvalue there is an additional twofold spin degeneracy.

B. Numerical results

We reproduce in Figures 2-6 numerical results, calculated with the methods of the previous Section. In Figure 2 and 3 we present two examples of the energy spectrum as function of \( k_x \) for different values of the spin-orbit coupling \( \alpha \).

The properties of the eigenfunctions are perhaps more interesting and can be best exemplified by the corresponding number and spin polarization densities. Figures 2, 5 and 6 refer to these properties of the eigenfunctions,
and have been obtained with the same parameters of Figure 2 and an occupation characterized by a Fermi energy $\epsilon_F = 6$ meV. In particular, we show there for each state the relative number density $N_\nu(k_x, y)$ and spin polarization density $\mathcal{P}_\nu(k_x, y)$. These quantities are defined as follows:

$$N_\nu(k_x, y) = \langle \varphi_\nu(k_x, y')|\delta(y - y')|\varphi_\nu(k_x, y') \rangle, \quad (18)$$

$$\mathcal{P}_\nu(k_x, y) = \langle \varphi_\nu(k_x, y')|\hat{\sigma}_y(y - y')|\varphi_\nu(k_x, y') \rangle. \quad (19)$$

From simple symmetry considerations, it follows that the polarization is vanishing in the $x$ direction, along the wire, and states with the same energy and opposite values of $k_x$ have the same density but opposite spin polarization. As it will be discussed in a following Section, this fact allows in principle to produce a net spin polarization along the $y$ and $z$ direction, by driving an electrical current along the wire.

C. Perturbation theory

Although the numerical analysis provides the complete answer to the problem, useful insight can be gained from an examination of the perturbative approach. We therefore consider next the case of small spin orbit coupling and will assume that the Rashba term can indeed be treated as a perturbation. In order to simplify the notation, in this Section we consider a potential that confines the electrons in $0 < y < W$ rather than $-W/2 < y < W/2$. For simplicity we will label the perturbed eigenstates via the (approximate) quantum numbers of the unperturbed solutions. Accordingly we write

$$\varphi_{n\pm}(k_x, y) = \varphi^0_{n\pm}(k_x, y) + O(\alpha), \quad (20)$$

where

$$\varphi^0_{n\pm}(k_x, y) = \sqrt{\frac{2}{2LW}} e^{ik_x x} \sin \left(\frac{n\pi y}{W}\right) \left(\pm\right), \quad (21)$$

with $n$ a positive integer. We also find convenient to choose the spinors along $y$, in such a way that the states are eigenstates of the $y$ reflection symmetry. The relevant matrix elements of the spin orbit interaction are:

$$\langle \varphi^0_{n'\pm}|\hat{R}_0|\varphi^0_{n\pm}\rangle = \mp \hbar \alpha k_x \delta_{nn'} \delta_{k_x k'_x}, \quad (22)$$

$$\langle \varphi^0_{n'\mp}|\hat{R}_0|\varphi^0_{n\pm}\rangle = \mp \frac{2\hbar \alpha mn'(1 - (-1)^{n + n'})}{W} (n^2 - n'^2) \delta_{k_x k'_x}. \quad (23)$$

We can easily obtain the form of the perturbation expansion, up to second order:

$$\epsilon_{n\pm}(k_x) = \frac{\hbar^2}{2m} \left(\frac{\pi n}{W^2}\right)^2 + \frac{\hbar^2 k_x^2}{2m} + \mp \hbar \alpha k_x - \frac{1}{2} m \alpha^2 + O(\alpha^3). \quad (24)$$

Higher order terms are also readily obtained.

The necessary condition for the validity of the perturbative expansion is that the matrix elements be smaller than the unperturbed energy differences. The latter are of order $\frac{\hbar^2}{mW^2}$. This gives:

$$\alpha \ll \frac{\hbar}{mk_x W^2} \quad \text{and} \quad \alpha \ll \frac{\hbar}{mW}. \quad (25)$$

Clearly the perturbative expansion fails for large values of $k_x$, or for a channel with large $W$.

The perturbation theory allows us to explicitly calculate a number of interesting quantities. In particular the $z$-component of the spin polarization density of the confined states (see Eq. (19)) is given by:

$$\mathcal{P}^z_{n\pm}(k_x, y) = \langle \varphi_{n\pm}(k_x, y')|\hat{\sigma}_z(y - y')|\varphi_{n\pm}(k_x, y') \rangle, \quad (26)$$

$$= \mp \frac{\alpha m}{W} \frac{F_n(y/W)}{\bar{F}_n(y/W)} + \left(\frac{\alpha mW}{\hbar}\right)^2 k_x G_n(y/W) + O(\alpha^3),$$

where the functions $F_n$ and $G_n$ are given by:

$$F_n(y/W) = \frac{64}{\pi^2} \sum_{m=1}' \left[ (n^2 - m^2)^2 \sin \frac{\pi ny}{W} \sin \frac{\pi my}{W} \right],$$

$$G_n(y/W) = \frac{256}{\pi^2} \sum_{m=1}' \left[ (n^2 - m^2)^2 \sin \frac{\pi ny}{W} \sin \frac{\pi my}{W} \right], \quad (27)$$

where the sum extends over positive even (odd) integers $m$ for odd (even) $n$.

In the same way, while $\mathcal{P}^y_{n\pm}(k_x, y)$ vanishes, the corresponding perturbative result for the polarization along $y$ of the $n\pm$ states can be expressed as:

$$\mathcal{P}^y_{n\pm}(k_x, y) = \langle \varphi_{n\pm}(k_x, y')|\hat{\sigma}_y(y - y')|\varphi_{n\pm}(k_x, y') \rangle,$$

$$= \mp \frac{2}{W} \sin^2 \frac{\pi ny}{W} \mp \frac{2\alpha m}{\bar{F}_n(y/W)} W H_n(y/W) + O(\alpha^3),$$

where the function $H_n$ is defined as follows:

$$H_n(y/W) = \frac{128}{\pi^4} \sum_{m=1}' \left[ \frac{m^2 n^2}{(n^2 - m^2)^2} \sin^2 \frac{\pi ny}{W} \sin \frac{\pi my}{W} \right] - 2 \sum_{m,n,m'} \left[ \frac{m n m'}{(n^2 - m^2)(m^2 - n'^2)} \sin \frac{\pi ny}{W} \sin \frac{\pi ny'}{W} \right], \quad (28)$$

where the sums extend over the positive integers. In particular while $n'$ is of the same parity as $n$, both $m$ and $m'$ are of opposite parity. Moreover $n' \neq n$.

Notice that as expected, in all these formulas the substitution $k_x \rightarrow -k_x$ and $\mp \rightarrow \mp$ gives a state with the same energy and opposite spin polarization.

III. EDGE SPIN POLARIZATION

We consider here the spin accumulation at the boundaries of the wire in the presence of an electric current in the $x$ direction.
FIG. 4: Number density \( N_\nu(y) \) (in m\(^{-1}\) units) of individual eigenstates at the Fermi level of a quantum wire. The parameters are the same as in Figure 2. The wave vector values are determined by the crossings of the bands with the horizontal dashed line at 6 meV in Figure 2 and are given at the top (dots on the \( k_x \)) line. The corresponding \( N_\nu(y) \) functions are shown in the panels, where the numbers label the first twelve wave vectors, counted from left to right on the top \( k_x \) line. States with opposite wave vectors have the same density.

FIG. 5: Spin polarization density \( P_z^\nu(y) \) along the \( z \) direction (in m\(^{-1}\) units) of the same eigenstates of Figure 4. States with opposite wave vectors have polarization of opposite sign. Solid lines correspond to the first twelve \( k_x \) values, counted from left to right on the top \( k_x \) line, while dashed lines refer to \(-k_x\).

Although the response of a system of electrons subject to Rashba spin orbit and an applied electric field is quite complicated and requires a careful analysis, for simplicity sake we will model here a current carrying steady state by assuming a modified electron occupation distribution. In particular we will assume that a finite chemical potential difference \( \delta \mu = \mu_+ - \mu_- \) is established between the right moving and left moving states. This is depicted in Figure 2 by the two short horizontal solid lines (whose separation from the equilibrium Fermi level - dashed - is not in scale). The present model is meant to describe ballistic transport with all the chemical potential drop occurring at the contacts at \( x = \pm \infty \).

Within this framework, the total spin polarization density due to the current flux can be obtained in the linear regime from the relation:

\[
\mathcal{M}(y) = \pm \frac{g\mu B}{2} \sum_\nu \frac{\delta \mu}{2\pi \hbar v_{F\nu}^F} P_\nu(k_{F\nu}, y),
\]

where the sum is restricted to the occupied one-dimensional subbands with \( k_{F\nu} \) and \( v_{F\nu} \) being the (positive) Fermi wave vector and Fermi velocity relative to the occupied band as labeled by the index \( \nu \). In this formula the positive sign refers to the case of hole transport.
As a consequence the direction of the magnetization only depends on the current direction and not on the type of carrier involved.

The perturbative result for the $z$ component $\mathcal{M}_z$ is readily obtained from Eqs. (26):

$$\mathcal{M}_z(y) \simeq \mp \delta \mu m g_{\mu B} \left( \frac{m a W}{\hbar} \right)^2 \sum_n G_n(y/W),$$

where the sum is limited to the positive integers corresponding to the occupied bands. The results of the (exact) numerical calculations of $\mathcal{M}_z(y)$ are shown in Figure 7.

Interestingly the expression given in (27) is not sufficient to obtain the leading corresponding perturbative expression for $\mathcal{M}_y(y)$. The reason is simply that in the confined geometry the first finite contribution to this quantity is clearly of order $\alpha^3$ (see also below). This term can be of course readily calculated. The results for the exact calculation of $\mathcal{M}_y(y)$ are plotted in Figure 8 alongside the perturbative expression.

It is remarkable that the $z$ component of the magnetization is larger for small values of the spin-orbit coupling alongside the perturbative expression. The results for $\mathcal{M}_z(y)$ remain of the same sign and is instead symmetric.

As expected, in all cases, there is good agreement with the approximate analytical formula (30) for small values of $\alpha$.

Our theory can be extended to the case of non-ballistic transport. A simple treatment of such a situation can be obtained by setting $\delta \mu \rightarrow e E v_F \tau_\nu$ in Eq. (29). Here $E$ is the magnitude of the driving electric field and $v_F \tau_\nu$ is the elastic mean free path appropriate for the states of the one-dimensional subband $\nu$, a length scale that we assume much larger than the width $W$ of the wire. By taking for simplicity the same scattering time for all subbands $\tau_\nu = \tau$ the ensuing magnetization can be obtained. The results for $\mathcal{M}_z(y)$ are plotted in Figure 9.

We notice that the oscillations of the magnetization are somewhat damped as compared to the ballistic case. On the other hand, the overall sign of the effect is unchanged, the behavior being in qualitative agreement with the experimental results of Ref. 9. We haste to state however that our model is rather different from the situation of a dirty three-dimensional sample dealt with in the experiment.

IV. DISCUSSION

As we have noted, while for small spin orbit the out of plane (z-axis) polarization $\mathcal{M}_z(y)$ induced by an x-axis current is quadratic in the coupling constant, the in plane (y-axis) polarization $\mathcal{M}_y(y)$ is much smaller, first appearing in third order. This result is only in apparent contradiction with the well known corresponding bulk result in two dimensions which entails a linear dependence of $\mathcal{M}_y(y)$ on the coupling constant. Within the simplified model used in Section III such a behavior can be readily derived by assuming the distorted momentum space electron distribution function of Figure 10. The final expression is given by

$$\frac{1}{L^2} \langle \delta_y \rangle = \frac{\alpha m^2 \delta \mu}{\pi^2 \hbar^2 \sqrt{2E_F + m a^2}},$$

where $\delta \mu = 2\hbar v_F \delta k$. As expected for small $\alpha \mathcal{M}_y(y)$ is linear in the coupling constant.

The difference lies with a quenching of the y-axis polarization due to the confining potential. As it can be

FIG. 6: Spin polarization density $P^y(y)$ along the y direction (in $\text{m}^{-1}$ units) of the same eigenstates of Figure 4. States with opposite wave vectors have polarization of opposite sign. Solid lines correspond to the first twelve $k_z$ values, counted from left to right on the top $k_z$ line, while dashed lines refer to $-k_z$. 

FIG. 8: Spin polarization density $P^y(y)$ along the y direction (in $\text{m}^{-1}$ units) of the same eigenstates of Figure 4. States with opposite wave vectors have polarization of opposite sign. Solid lines correspond to the first twelve $k_z$ values, counted from left to right on the top $k_z$ line, while dashed lines refer to $-k_z$. 

FIG. 10: Electron distribution function of Figure 10. The final expression is given by

$$\frac{1}{L^2} \langle \delta_y \rangle = \frac{\alpha m^2 \delta \mu}{\pi^2 \hbar^2 \sqrt{2E_F + m a^2}},$$

where $\delta \mu = 2\hbar v_F \delta k$. As expected for small $\alpha \mathcal{M}_y(y)$ is linear in the coupling constant. 

The difference lies with a quenching of the y-axis polarization due to the confining potential. As it can be
surmised by inspecting Eq. (27), this effect can be readily seen to originate from the lowest order cancellation of the contributions to $M(y)$ stemming from states within the same spin-split one dimensional band. The cancellation only occurs when the spin-splitting is smaller than the energy quantization associated with the one-dimensional confining potential. This is of course not operational in the bulk since in such a situation the energy spectrum is continuum.

The crossover between these two regimes approximately occurs when the spin-splitting equals the energy spacing between the eigenvalues of the confining potential along the $y$ direction, i.e. for $\frac{m_0 W}{\hbar} \approx 1$. In order to exemplify this phenomenon we have plotted in Figure 11 the spatially averaged magnetization density

$$\bar{M}_y = \frac{1}{W} \int_{-W/2}^{W/2} M_y(y)dy,$$

as a function of the value $\hbar\alpha$ of the spin orbit coupling constant. With the parameters chosen in Figure 11, the crossover occurs approximately for $\hbar\alpha = 1.7 \times 10^{-9}$ meV m. The dashed curve superimposed to the $\epsilon_F = 2.5$ meV dotted line is purely cubic and provides a useful guide to the eye for that case.

Exactly the opposite mechanism is responsible for the case of $M_z(y)$, which, as we have seen, is instead vanishing in the bulk. Interestingly, as one can infer from Figure 11, the expression of Eq. (31), formally valid in the bulk for finite values of $\alpha$ when in principle the perturbative condition is inapplicable, still gives good results for the case of a rather narrow channel with $W = 0.15 \mu m$. 

FIG. 7: Spin polarization density in the $z$ direction of a $W = 0.1 \mu m$ wide quantum wire, for carriers with effective mass $m = 0.3 m_0$ and $g = 2$. The Fermi energy is $\epsilon_F = 6$ meV, which corresponds to an electron density of $0.67 \times 10^{12}$ cm$^{-2}$. 

We used $\delta \mu = 0.1$ meV which gives a current in the wire of 23 nA. In the top panel $\hbar\alpha = 1.5 \times 10^{-9}$ meV m while in the bottom one $\hbar\alpha = 3.5 \times 10^{-9}$ meV m. The perturbative result is also plotted (dashed). As a comparison of the magnitude of the effect, the Pauli susceptibility is $\chi = 72.5 \mu_B/\mu m^2 T$.

FIG. 8: Spin polarization in the $y$ direction corresponding to the same parameters of the previous Figure 7. In the top panel $\hbar\alpha = 1.5 \times 10^{-9}$ meV m while in the bottom one $\hbar\alpha = 3.5 \times 10^{-9}$ meV m. The leading perturbative contribution (cubic in $\alpha$) is also plotted (dashed).

FIG. 9: Magnetization density of a $W = 0.1 \mu m$ wide quantum wire, for carriers with same parameters as the first panel of Figure 7. We used $E = 1$ kV/m and a constant value for $\tau = 7.3 \times 10^{-13}$ s, which gives the same current in the wire of Figure 7. The perturbative result is also plotted (dashed). The magnetization is somewhat different from the previous case, where the oscillations of the magnetization are more pronounced. The overall sign of the effect is the same.
We discuss next how the different behaviors of the two components of the spin polarization we have just discussed are compatible with a quite general and rigorous argument regarding the parity of $M_z(y)$ and $M_y(y)$ with respect to the sign of the spin orbit coupling constant. Consider the effect on the hamiltonian (including contributions from the electron electron interaction and non magnetic impurities) of the application of the operator $\hat{\sigma}_z$. Clearly $\hat{\sigma}_z \hat{H}(\alpha) \hat{\sigma}_z = \hat{H}(\alpha)$. Then it immediately follows that given an eigenstate of the problem for a given value of $\alpha$, the corresponding solution for $-\alpha$ can be obtained simply by application of $\hat{\sigma}_z$. This in turn implies that, quite generally, while $M_z(y)$ is even with respect to $\alpha$, $M_y(y)$ is odd.

We have shown that a net spin polarization is established when a steady state current is run through the wire obtained by laterally confining a two dimensional electron gas in the presence of Rashba spin orbit. Reversing the current inverts the spin polarization. The effect is phenomenologically analogous to, yet distinct from the spin-Hall effect that has recently been the subject of intense discussion (for a useful review, see Ref. 11) and was demonstrated experimentally.\textsuperscript{2,12,13,14} The mechanism relevant to the present discussion simply stems from the structure of the electronic wavefunctions in a confined geometry. Accordingly, there is no need to make appeal to spin currents or impurity scattering. Moreover, its geometric nature should leave the phenomenon mostly unchanged in the presence of a moderate amount of impurities in the wire. A similar effect should also be relevant in the case of electronic states localized by an impurity potential as well as in the transition to a low density Wigner crystal in the presence of Rashba spin orbit.

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1 That the effects of the electronic interactions can be neglected is usually a rather dangerous assumption. A systematic study of the interplay of the Coulomb interaction and Rashba spin-orbit in a two dimensional electron liquid can be found in S. Chesi and G. F. Giuliani, unpublished.
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