Igor Rostislavovich Shafarevich: in Memoriam

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May 12, 2021

The prominent Russian mathematician Igor Rostislavovich Shafarevich passed away on February 19, 2017, leaving behind outstanding contributions in number theory, algebra, and algebraic geometry. The influence of his work on the development of these fields in the second half of the 20th century is hard to overestimate. Besides the fundamental results authored by him and his collaborators he single-handedly created a school of Russian algebraic geometers and number theorists. Many of his numerous students consider their time spent under his guidance as the happiest time in their life as mathematicians. In 1959 he was awarded the Lenin prize. A year earlier he was elected as a correspondent member of the Russian Academy of Sciences and became a full member in 1991. He was also a foreign member of the Italian Academy dei Lincei, the German Academy Leopoldina, the National Academy of Science the USA (from which he resigned in 2003 as a protest against the Iraq War), a member of the Royal Society of London, and received an honorary doctorate from the University of Paris-XI. Shafarevich was an invited speaker at the International Congresses of Mathematicians in Stockholm (1962) and Nice (1970). His name is associated with such fundamental concepts and results in mathematics as the Shafarevich-Tate group, Ogg-Shafarevich theory, Shafarevich map, Golod-Shafarevich Theorem, Golod-Shafarevich groups and algebras, Deuring-Shafarevich formula and several influential Shafarevich Conjectures. His influential textbooks in algebraic geometry and number theory (jointly with Zinovy Borevich) have been translated into English and have served as an introduction to these subjects for several generations of mathematicians. His book “Basic Notions of Algebra” [42] is a bird’s-eye view on algebra that reveals its vast connections with many other fields of mathematics and has become a favorite book in the subject for many mathematicians. Quoting from the preface to a collection of papers “Arithmetic and Geometry” published in two volumes by Birkhäuser in 1983 and edited by M. Artin and J. Tate [2]: ‘Igor Rostislavovich Shafarevich has made outstanding contributions in number theory, algebra, and algebraic geometry. The flourishing of these fields in Moscow since World War II owes much to his influence. We hope these papers, collected for his sixtieth birthday, will indicate to him the great respect and admiration which mathematicians throughout the world have for him.’

In the Preface to [45] Shafarevich writes “At the end of the nineteen sixties the perception of life began to change. The passiveness of thinking and muteness began to feel like irresponsibility. This new feeling seemed to turn me to another road. Otherwise, I would stay until the end of my life in my profession as a mathematician, and my interest in history would remain as a hobby. Instead of this, I had acquired a second working profession to which I became devoted with more and more energy.” It is this subsequent non-mathematical activity that led to his numerous publications on social issues which at the same time both tarnished and magnified his reputation among different
layers of society in Russia and the West.

Biography

Igor Rostislavovich Shafarevich was born in 1923 in the Ukrainian city of Zhitomir. The town name originated in the old Russian word “zhito” which means “rye”. Zhitomir was also the birthplace of many other famous Russians, including the pianist Svyatoslav Richter whose mother was a friend of Shafarevich’s mother and aunt (see [4]).

Shafarevich’s father Rostislav Stepanovich graduated from the mathematical department of Moscow State University (MGU) and, after moving to Moscow, lectured in theoretical mechanics at one of the Institutes of Higher Learning (it happened that the author’s father attended his lectures). His mother Yulia Yakovlevna was a philologist and a gifted pianist, from whom probably her son inherited a lifelong passion for classical music and Russian literature.

Shafarevich’s defining interest was history, a subject to which he was devoted until the end of his life, while Mathematics was his other love. He read his first mathematical books in German in which he was fluent all his life. While at school, he took exams in mathematics at MGU, from which he graduated in 1940 at the age of 17. Although he did not have a formal thesis advisor, he was supported by Boris Nikolaevich Delone. He acknowledged both Israel Moiseevich Gelfand and Alexander Gennadievich Kurosh as his mentors. Three years later, at the age of 20, he completed his graduate studies at MGU with a Ph.D. dissertation on the topic of topological norm-fields. During World War II, he was evacuated together with the university to Ashkhabad, and later to Kazan. After returning to Moscow, he defended his second (doctoral) thesis (a Russian version of the German Habilitation) in 1946. In this work he studied $p$-extensions of the field of $p$-adic numbers and non-ramified $p$-extensions of the fields of algebraic numbers. His doctoral committee included such prominent Russian mathematicians as Dmitry Konstantinovich Faddeev, Anatoly Ivanovich Maltsev and Nikolai Grigorievich Chebotarev. After the defense of his thesis and until his death he worked at the Steklov Mathematical Institute of the Academy of Sciences (MIAN). Starting in 1944 he also taught at MGU, which is where in the nineteen sixties he founded his famous Seminar in algebraic geometry. In 1975 he was fired from MGU because of his participation in dissident movement, and the seminar moved to the Steklov Institute where it continued to meet weekly on Tuesdays. Having directed the Algebra Section of the Institute for many years he was credited with transforming it into a worldwide renowned center of scientific activity in algebra, algebraic geometry, and number theory. Although he was sometimes addressed as “chief” because of his administrative position and also because the word resembles his name, there was never anything bossy in his relationship with his students, colleagues and ordinary Russian people who later came to him for advice on social issues. He always respected his numerous students and colleagues, treated them as equals, and was ready to help them in their mathematical careers and during challenging periods in their lives. His closest friends were the ones with whom he shared his passion for mountain hikes (inherited from Delone) and some of whom even collaborated with him in his dissident activity.

Shafarevich clearly demonstrated his scientific honesty in his mathematical writings. His attribution of known results and historical references should serve as instructive examples for mathematicians of later generations. On several occasions, he stood up to express critical opposition of the mathematical content of some of the theses defended in the Mathematics department at MGU including
the Habilitation dissertation of his former student A. Zhizhchenko that was based on erroneous papers [56].

Students

Shafarevich began advising Ph.D. dissertations already in the late nineteen forties. The list of students he advised would be even longer were he not been fired from the University. The following is, hopefully, a complete list of his Ph.D. students. Together with their descendants, the list would contain more than 300 names (see http://www.mi.ras.ru/ shafarev/tree.html).

Table 1: Ph.D. Students

| Name          | Institution | Year |
|---------------|-------------|------|
| Pavlov        | MGU         | ?    |
| Milner A.A.   | MGU         | ?    |
| Medvedev P. A. | MGU       | ?    |
| Berman Samuil D. | MGU       | 1952 |
| Demyanov V. V. | MGU       | 1952 |
| Lapin Andrei I. | MGU       | 1952 |
| Zhizhchenko Alexei B. | MGU       | 1958 |
| Demushkin Sergei, | MGU       | 1959 |
| Golod Evgeny S. | MGU       | 1960 |
| Kostrikin Alexsei I. | MIAN    | 1960 |
| Manin Yuri I.  | MGU         | 1961 |
| Vvedenskii Oleg N. | MGU       | 1963 |
| Tyurina Galina N. | MGU       | 1963 |
| Markshaitis Gamlet N. | MGU       | 1963 |
| Averbuch Boris G. | MGU       | 1964 |
| Koch Helmut    | MGU         | 1964 |
| Tyurin Andrei N. | MGU       | 1965 |
| Neumann Olaf   | MGU         | 1966 |
| Parshin Alexei N. | MGU       | 1967 |
| Dolgachev Igor V. | MGU       | 1970 |
| Drozd Yuriii A | MGU         | 1970 |
| Gizatullin Marat H. | MGU       | 1970 |
| Arakelov Suren J. | MGU       | 1974 |
| Kulikov Valentine S. | MGU     | 1975 |
| Rudakov Alexei N. | MGU       | ?    |
| Abrashkin Victor | MGU       | 1976 |
| Shabat George B. | MGU       | 1976 |
| Todorov Andrey N. | MGU       | 1976 |
| Kulikov Viktor S. | MGU       | 1977 |
| Nikulin Vyacheslav V. | MIAN     | 1977 |
| Belyi Gennady V. | MIAN       | 1979 |
| Kolyvagin Victor A. | MGU       | 1981 |
1 Scientific work

Algebraic number theory

In his Habilitation dissertation Shafarevich studied non-abelian $p$-extensions of local and global fields. For example, he proved that given a finite degree $n$ extension $k$ of the field $\mathbb{Q}_p$ of rational $p$-adic numbers that does not contain $p$-roots of unity, the Galois group of a finite $p$-extension of $k$ is a quotient of a free group with $n+1$ generators [20]. For this work Shafarevich was awarded the prize of the Moscow Mathematical Society. In his next work he made a major contribution to number theory by giving an explicit formula for the local symbol $(\alpha,\beta)_p$ [21]. The formula is reminiscent of a familiar formula for the residue of a differential on a Riemann surface. The theory developed in his dissertation gave a new approach to global and local class field theory (see [14]). His next work was even more impressive. In [22] published in 1954, he solves the inverse Galois problem for solvable groups in the case of fields of algebraic numbers. A gap in the proof of this fundamental result, pointed out later by H. Koch, was based on a reference to a mistaken group-theoretical result of A. I. Skopin. It was fixed by Shafarevich in 1989 in one of the footnotes to his Collected Works [44], p. 752 and, in more details, in [33], where he corrected Skopin’s result. The proof was based on his earlier paper on construction of $p$-extensions of algebraic number fields, and it used, then pioneering, methods of homological algebra developed at that time by D. K. Faddeev. A complete proof that uses new modern tools can be found in [19].

The next problem addressed by Shafarevich was the problem of embedding of local and global fields $k$. Given a Galois extension $L/k$ with Galois group $G$ and its Galois subextension $K/k$, the subgroup of $G$ fixing the elements from $K$ is a normal subgroup of $G$ with quotient group $G'$ isomorphic to the Galois group of $K/k$. Giving a surjective homomorphism of groups $G \to G'$, the embedding problem asks whether there exists an embedding of a Galois extension $K/k$ with Galois group $G'$ into a Galois extension $L/k$ with Galois group $G$ that realizes the surjective homomorphism as the quotient map. In the case when $G$ is abelian and the surjection $G \to G'$ with kernel $H$ makes $G$ to be a semi-direct product $H \rtimes G'$, the problem was solved in 1929 by A. Scholz. Shafarevich generalized this result to the case where $H$ is a nilpotent group of a certain class. He returns to the embedding problem later in a joint work with his former student Sergei Demushkin, first considering the case of local fields [9] and later the case of global fields [10].

In 1963 Shafarevich published an important paper in Publ. Math. IHES [26] (after World War II it was an extremely rare event for a Soviet mathematician to publish a paper in a Western journal) on the problem of $p$-extensions of algebraic number fields, by considering finite extensions of these fields with a fixed set $S$ of ramified divisors. In the case when $S$ is the empty set, he shows that the minimal number $d$ of generators of the pro-finite Galois group of the composition of all such extensions and the number $r$ of minimal number of relations between these generators satisfies the inequality $r \leq d + \rho$, where $\rho$ is the number of generators of the group of units of the field.

In his talk at the ICM in Stockholm, he remarked that if one proves that $r(G) - d(G) \to \infty$, where the limit is taken over the set of all $p$-groups, then unramified infinite extensions do exist. In a joint work with his former student Evgeny Golod [11] he proved that the limit does, in fact, goes to infinity thereby solving the class field tower problem in number theory that have been around for more than 40 years.
Elliptic curves

The transition of Shafarevich’s interests from number theory to algebraic geometry was rather smooth and was based on his, now famous, work on elliptic curves. Already in 1956 in his talk at the Third Congress of Soviet Mathematicians he pointed out to analogy between the problem of embedding of fields of algebraic numbers and the problem of classification of elliptic curves over such fields. Both problems use the local-to-global approach: find a solution for all completions of the field and decide whether it leads to a solution over a global field. In the case of elliptic curves, this leads to the question whether the set of elliptic curves with a fixed absolute invariant isomorphic to a fixed curve over all completions of the field is finite. In a short announcement note [23] published in Doklady AN SSSR he shows that the set of elliptic curves isomorphic to a fixed curve over some extension of the ground field form a group that admits a cohomological interpretation as the first Galois cohomology group with coefficients in the group of points of the Jacobian curve. The fact that such a set forms an abelian group was not new; in the case when the ground field is the field of real numbers it was discovered by François Châtelet in 1947, whose construction used the same cocycles. In 1955 A. Weil extended this result to the case of abelian varieties of arbitrary dimension although he did not give a cohomological interpretation of the group. The paper of S. Lang and J. Tate of 1958 gives a foundation of the theory of principal homogeneous spaces over an abelian variety based on its cohomological interpretation (with no reference to Shafarevich’s paper). The authors called the group the Châtelet group, later known under the name the Weil-Châtelet group. In the same paper Shafarevich proved that the subgroup of the Weil-Châtelet group of elements that admit a point of degree $n$ over the ground field (they represent elliptic curves of degree $n+1$ in $\mathbb{P}^n$) and become trivial over all completions of the field is a finite group. In the subsequent paper [24] in Doklady Shafarevich proved the existence of elliptic curves of arbitrary degree $n$ not isomorphic to any curve of smaller degree, thus giving a solution of an old problem in the theory of Diophantine equations.

In 1961 Shafarevich published a paper devoted to a systematic study of the Weil-Châtelet group $H^1(K, A)$ of an abelian variety $A$ over a field $K$ of algebraic functions in one variable over an algebraically closed field $k$ [25]. He divided this study into three parts by determining the structure of three groups: the local group $H^1(K_p, A_p)$, where the field $K$ is replaced by one of its completions, the kernel and the cokernel of the restriction homomorphisms to the product of these groups with respect to the set of all completions of $K$. The kernel group (where $K$ is replaced by a number field or a more general function field) was later named the Shafarevich-Tate group and Shafarevich’s contribution to the theory of elliptic curves was honored by a common acceptance of using the Russian letter Sha for its notation $Ш(A)$. A similar theory was independently developed about the same time by Andrew Ogg in Berkeley. Later on, the theory was given a more modern approach by Grothendieck who gave a cohomological interpretation of $Ш(A)$ as the first étale cohomology group with coefficients in an étale sheaf on a nonsingular projective model of the field $K$ which is represented by the Néron model of $A$. The main result of Ogg and Shafarevich on the structure of $Ш(A)$ is now known as the Grothendieck-Ogg-Shafarevich formula. In their work, Ogg and Shafarevich dealt only with the part of the Weil-Châtelet group prime to the characteristic of
The subsequent work of several people including Oleg Vvedensky, a student of Shafarevich, completed the work by settling the $p$-part [5].

It is remarkable that the last mathematical published work of Shafarevich, who was by then 90 years old, was in number theory. In [43] he provided a new proof, using the theory of modular forms, of Stark’s theorem that there are only nine imaginary quadratic fields with class number one.

**Algebraic Geometry**

It would appear that Shafarevich was always interested in algebraic geometry; for example, in 1950 he authored an article on algebraic geometry in the Russian Encyclopedia. In his paper [24] he gave a reference to a paper of Enriques of 1899 that contains geometric analogs of some of his results. It should be said that algebraic geometry and the theory of algebraic functions in one variable were always peripheral to the interests of the Russian mathematics (although some important work in real algebraic geometry have been done by I. G. Petrovsky and his students). The only textbook in this field was Chebotarev’s book [7] published in 1948 that gives an exposition of algebraic theory of algebraic curves. In 1961-1963 Shafarevich and a group of his students ran a seminar on algebraic surfaces whose goal was to understand some of the classical works of the Italian algebraic geometers from a modern point of view. The new techniques, based on topological methods and the use of the new theory of cohomology of algebraic coherent sheaves developed earlier by Jean-Pierre Serre, served as common tools in their work. The same activity was undertaken about the same time by Oscar Zariski and David Mumford at Harvard and, from more analytical point of view, by Kunihiko Kodaira. A book 'Algebraic surfaces' [1] had appeared in Russian in 1965 and was translated into English the same year. For many years this book was the only source for learning the classification of algebraic surfaces from a modern point of view. Shafarevich himself contributed two chapters to the book. In one of them, he translated his previous work on principal homogeneous spaces of elliptic curves into geometric language, in particular, reconstructing Enriques’ work on elliptic surfaces. In another, he gave a modern proof of Enriques’ criterion of ruledness of algebraic surfaces. As his students will acknowledge, his influence on the book as a whole was much more than the contribution of the two chapters and the preface. Shafarevich was always interested in and had a broad knowledge of the history of mathematics and its classical sources. He had passed this passion to some of his students.

In 1971 Shafarevich turned his attention to the study of complex K3 surfaces, which together with abelian surfaces represent two-dimensional analogs of elliptic curves. Their occurrence in many areas of mathematics and even mathematical physics is truly remarkable. K3 surfaces share one common property with elliptic curves, namely the existence of a unique, up to proportionality, holomorphic differential form of top degree. However, they differ from elliptic curves by the property that they are simply-connected. It is a simple fact that the complex structure of an elliptic curve is determined by its periods, i.e. the values of integrals of its holomorphic form on a basis of 1-homology of the curve. Considered as a vector $(\int_{\gamma_1} \omega, \int_{\gamma_2} \omega)$ modulo proportionality and modulo of the group $\text{SL}_2(\mathbb{Z})$ acting via basis changes, it represents a point in $\mathbb{C}$ that determines the curve up to isomorphism. The proof of this fact follows easily from representing an elliptic curve as the quotient of $\mathbb{C}$ by the lattice spanned by the periods. The absence of this representation for K3 surfaces made André Weil’s guess that the periods of K3 surfaces should also determine its holomorphic structure seem too daring to attempt to prove. Weil himself recognized this by giving the naming K3 surfaces [54]:
“il s'agit des variétés kähhleriennes dites K3, ainsi nommées en l'honneur de Kummer, Kähler, Kodaira et de la belle montagne K2 au Cachemire.”

Nevertheless, a joint work with Ilya Iosifovich Piatetski-Shapiro went on to do exactly that. They proved that a projective complex algebraic K3 surface is uniquely determined by its vector of periods modulo proportionality and changes of a basis in the subgroup of 2-homology group orthogonal to the class of its hyperplane section. This result became known as the Global Torelli Theorem for algebraic K3 surfaces named after an Italian algebraic geometer Ruggiero Torelli who proved a similar result for algebraic curves [30]. A corollary of this theorem allows them to reduce the study of the automorphism group of a K3 surface to some arithmetical property of an integral quadratic form of algebraic cycles on the surface. This became an essential tool in the subsequent extensive study of automorphism groups of K3.

The absence of topological and analytical methods in the study of K3 surfaces defined over fields of positive characteristic seemed to be an unsurpassable obstacle for the extension of study of K3 surfaces. A paper of Michael Artin [3] (as Shafarevich told me, it was as one of the most beautiful papers he had read in his life) was a breakthrough in this direction. In it, he introduced the periods of supersingular K3 surfaces that are distinguished by the property that they have maximal possible number of linearly independent algebraic cycles. In a long series of influential papers with his former student, Alexei Rudakov, Shafarevich undertook a comprehensive study of K3 surfaces over fields of positive characteristic. Thus, they proved the unirationality of K3 surfaces, in this case, proved the non-degeneracy of supersingular K3 surfaces, the absence of non-trivial regular vector fields on K3 surfaces and laid the foundations for the theory of inseparable morphisms of algebraic varieties. Using the non-degeneracy results of Shafarevich and Rudakov, Arthur Ogus was able to prove a Global Torelli Theorem for supersingular K3 surfaces over fields of odd characteristic.

The Global Torelli Theorem for K3 surfaces together with the Surjectivity Theorem for for K3 surfaces, proven by his former students Victor Kulikov allows one to construct a coarse moduli space for polarized algebraic K3 surfaces as an arithmetic quotient of a Hermitian symmetric domain of orthogonal type. A subsequent work of his former student Andrey Todorov extended the Surjectivity Theorem to the case of Kähler K3 surfaces. Apparently, Shafarevich was interested in the theory of arithmetic groups and automorphic functions for a long time. In 1954 he wrote a preface and edited the Russian translation of Siegel’s book [51]. In his paper with Piatetski-Shapiro [27] he studies a pro-algebraic variety with the field of rational functions equal to the limit of the fields of automorphic functions of subgroups of finite index of a discrete arithmetic group of automorphisms of a bounded symmetric domain. The second volume of his ‘Basic Algebraic Geometry’ ends with a discussion of a problem of uniformization of algebraic varieties and makes his famous Shafarevich Conjecture that states that the universal cover of a complex projective variety $X$ is holomorphically convex, or, equivalently, the universal cover of $X$ admits a proper map to a Stein manifold with connected fibers. Another reformulation, due to Janos Kollár, is that there is a proper map $\text{sh}_X : X \to \text{III}(X)$ onto a normal variety $\text{III}(X)$ with connected fibers that contracts all closed subvarieties $Y$ of $X$ such that the natural homomorphism of the fundamental group $\pi_1(Y')$ of a resolution of singularities of $Y$ to the fundamental group $\pi_1(X)$ has finite image. Kollár named a map with this property the Shafarevich map. His monograph [15] contains an extensive study of the Shafarevich Conjecture and culminates with a proof of an existence of a birational map $\text{sh}_X$ with the similar properties. The Shafarevich conjecture is closely related to the group-theoretical properties of $\pi_1(X)$, for example, with the existence of a faithful representation of this group in a simple compact Lie group whose
The Shafarevich map $\text{sh}_X$ should be considered as a non-abelian generalization of the Albanese map $a_X : X \to \text{Alb}(X)$ that has the same property with respect to abelian unramified covers of $X$. In his popular article [35] published in Mathematical Intelligencer Shafarevich proposed that the major challenge of modern mathematics can be summed up as a “non-abelianization of mathematics”. He acknowledges that the “non-abelian mathematics of the future” philosophy also inspired him when he was starting his work in mathematics.

The joint interest of Shafarevich in number theory and algebraic geometry is explained by the many close analogies between the two theories going back to Leopold Kronecker and David Hilbert. Shafarevich’s talk at the ICM in Stockholm in 1962 is entirely devoted to the connections between the two fields. In particular, he stated two very influential conjectures in his talk. The analog of the Hermite conjecture about the finiteness of the number of finite extensions of an algebraic number field with the fixed discriminant becomes his conjecture about the finiteness of the set of algebraic curves of fixed genus $g > 0$ over a number field $k$ with fixed discriminant and an analog of Minkowski’s theorem that there are no unramified extensions of $\mathbb{Q}$ that now states that there are no smooth families of curves of positive genus over $\text{Spec}(\mathbb{Z})$. The attempts to prove the first conjecture played a crucial role in Falting’s proof of the Mordell Conjecture.

The beginning of the nineteen sixties was the time when many algebraic geometers of the present and earlier generations had to re-educate themselves in learning the new language of algebraic geometry developed in the fundamental work of Alexander Grothendieck. Shafarevich insisted on the importance of this new development in algebraic geometry. His lectures on the $\zeta$-functions at MGU in 1969 and Bombay Lectures on minimal models of two-dimensional schemes [28] together with Manin’s lectures at MGU on affine schemes and Mumford’s Lectures of curves on algebraic surfaces [17] became instrumental tools for Russian algebraic geometers for accomplishing this goal.

In [37] Shafarevich stated the following conjecture: the set of Picard lattices of K3 surfaces defined over a fixed field of algebraic numbers of degree $n$ over $\mathbb{Q}$ is finite. He proved this conjecture for K3 surfaces with maximal Picard number equal to 20. He also proved its geometric analog for one-dimensional families of Kummer surfaces. In a paper [40] that was published in the same year he studied the Shimura variety of abelian surfaces with quaternionic multiplication (fake elliptic curves) and proved that the number of isomorphism classes of non-constant fake elliptic curves defined over an extension $K/\mathbb{C}(t)$ of degree $\leq n$ is finite.

Algebra

Shafarevich’s work in number theory led him to some fundamental problems in group theory. Thus, the solution of the problem of the existence infinite tower of class field towers resulted in him and Golod proving that $r > (\frac{d-1}{2})^2$, where $d$ is the smallest number of generators of a $p$-group $G$ and $r$ is the smallest number of relations between $d$ generators. It is known that the numbers $r, d$ and $t = r - d$ can be interpreted in terms of the group cohomology $r = \dim H^2(G, \mathbb{F}_p)$, $d = \dim H^1(G, \mathbb{F}_p)$ and $t = \dim H^2(G, \mathbb{Z})_p$. Thus the Golod-Shafarevich inequality becomes an equality of the Betti numbers $b_i$ of the graded algebra of cohomology $H^*(G, \mathbb{Z})$. The main implication of the Golod-Shafarevich inequality (later improved by E. Vinberg and P. Roquette to the form $r \leq d^2/4$) is that
the small number of relations compared to the number of generators implies that the group must be infinite. In this way, a statement of this sort in different categories can be proved by similar methods and is referred as the Golod-Shafarevich Theorem. This led also to the definition of the Golod-Shafarevich group as a $p$-group with certain properties of its presentation that implies that the group is infinite. There is an extensive study of the Golod-Shafarevich groups and their analogs in other categories, as well as new applications of the Golod-Shafarevich theory. For example, Alexander Lubotzky proved that the fundamental group of a hyperbolic 3-manifold of finite volume contains a Golod-Shafarevich subgroup of finite index.

In 1964-66 Shafarevich run a seminar at the Steklov Insitute on Cartan’s classification of simple transitive transformation Lie pseudogroups. A result of this seminar is a joint paper of Shafarevich and his former student Alexei Kostrikin [16] in which they make a very important observation that Cartan’s classification is closely related with the classification of restricted Lie algebras over a field of characteristic $p > 0$. A transitive Lie algebra of a Lie pseudogroup admits a natural filtration defined by transformations that preserve $k$-jets of function at a fixed point that becomes an infinite-dimensional graded Lie algebra, sometimes infinite-dimensional Lie algebra. An important role in Cartan’s classification is played by four algebras realized as subalgebras of the algebra of derivation of the algebra of formal power series $k[[t_1,\ldots,t_n]]$ over a field $k$ of characteristic 0: algebra of all derivations $D_n$; all derivations $\partial$ that preserve the volume form $\omega = dt_1 \wedge \cdots \wedge dt_n$; all derivations that preserve a symplectic form; all derivations $\partial$ such that $\partial(\omega) = f \omega$ for some $f \in k[[t_1,\ldots,t_n]]$. The algebras have ideals of finite codimension that consist of derivations $\partial = \sum f_i \frac{\partial}{\partial t_i}$ with $f_i \in (t_1^p,\ldots,t_n^p)$. In characteristic $p > 0$ they represent new so-called nonclassical restricted Lie algebras. They further made a bold conjecture that the class of restricted Lie algebras consists of classical ones and the four algebras as above. In 1988 Richard Block and Robert Wilson proved this conjecture for $p > 7$ [6] (in the case $p < 7$ there are other infinite families of restricted Lie algebras).

The study of Cartan pseudogroups led Shafarevich to study of infinite-dimensional groups of biregular transformations of affine algebraic varieties. In his brief note [29] (named the “Italian paper”) Shafarevich announced some fundamental results about the structure of the group of automorphisms of the ring of polynomials in $n$ variables based on his theory of infinite-dimensional algebraic groups. Answering some criticism of the lengthy review of the paper by T. Kambayashi, he returns to this topic 15 years later by giving in [32] some detailed proofs of the announced results and laying a foundation for the concept of an infinite-dimensional algebraic group. He proved that in the case when the characteristic is zero, it is a nonsingular infinite-dimensional algebraic variety. Another important result is that the group of automorphisms $\text{Aut}(k[x_1,\ldots,x_n])$ is generated as an algebraic group by affine transformations and de Jonquières transformations and its subgroup $\text{Aut}(k[x_1,\ldots,x_n])^0$ of automorphisms with trivial Jacobian is simple as an algebraic group. Note that both results are not true for the group of abstract automorphisms of the ring. According to I. Shestakov and U. Umirbaev [50] the group generated by affine and de Jonquières transformation is a proper subgroup of $\text{Aut}(k[x_1,x_2,x_3])$ and according to a result of Vladimir Danilov [8] the group $\text{Aut}(k[x_1,x_2])^0$ is not simple as an abstract group. In 2004 Shafarevich returns to his theory of infinite-dimensional groups by studying the group $\text{GL}(2,K[t])$ [41]. He defines two different structures of an infinite-dimensional algebraic group on $\text{GL}(2,K[t])$ and studies singular points of their finite-dimensional closed subschemes.

In a paper [34] Shafarevich studied the algebraic variety $A_n$ parameterizing finite-dimensional nilpotent commutative algebras of dimension $n$ over a field. For example, in [34], he considers such
algebras $N$ of nilpotent class 2, i.e. satisfying $N^3 = 0$. In the case when the ground field is algebraically closed and of characteristic zero he proves that the irreducible components of $A_n$ coincide with its subvarieties $A_{n,r}$ parameterizing algebras $N$ satisfying $\dim N^2 = r$ assuming that $1 \leq r \leq (n-r)(n-r+1)/2$. He reveals an interesting behavior of the number of the irreducible components of $A_n$.

Books

The name of Shafarevich should be familiar to any mathematician, particularly a student, who is interested in the background on algebraic geometry. His textbook ‘Basic Algebraic Geometry’ was first published in Russian in 1968, republished in 1972, available in a vastly extended edition in 1988, followed by the final edition in 2007. The 1972 edition was translated into English by K.A. Hirsch in 1974 and into German by Rudolf Fragel. The 1988 and 2007 editions were translated in English by Miles Reid in 1994 and in 2007.

Another well-used textbook written jointly with Zinovy I. Borevich is ‘The Theory of Numbers’. Its first edition was published in Russian in 1964 and republished in 1972. It was translated into German by Helmut Koch, into English by Newcomb Greenleaf in 1966, and into French by Myriam and Jean-Luc Verley in 1967.

Shafarevich published also several books for a broad audience. A book ‘Geometry and Groups’ written jointly with his former student Vyacheslav Nikulin and published in Russian in 1983 deals with 2- and 3-dimensional locally Euclidean geometries and their transformation groups. It was translated into English by Miles Reid in 1987.

A book ‘Discourses on algebra’, translated into English from Russian by William Everett in 2003, was addressed to high school students and teachers. In words of the author, the task of the book is to show that algebra is just as fundamental, profound, and equally beautiful as geometry.

For many years Shafarevich was one of the editors of several volumes of “Encyclopedia of Mathematical Sciences” published by Springer as translations from Russian originals published in Itogi Nauki i Tekniki. Sovremennye problemy v matematiki. Fundamental’nye napravleniya. He contributed to the volumes himself writing jointly with Vassily A. Iskovskikh an article about algebraic surfaces in ‘Algebraic geometry’, vol. 3. His other contribution to the series is his book ‘Algebra I’ published in 1990 and reprinted in 1997. This masterpiece gives a beautiful exposition of main concepts and ideas of algebra from a broader perspective of a mathematician working in other areas of mathematics, confirming Shafarevich’s lifeview of mathematics as a whole body with ideas freely circulating from one field to another.
2 Non-mathematical activity

Dissident movement

We refer to Krista Berglund’s [4] for a meticulously researched comprehensive study of this part of Shafarevich’s life. Another rather detailed account of Shafarevich’s activity as a dissident can be found in the book of Robert Horvath [13]. Here we limit ourselves to only a brief summary of Shafarevich’s public life outside of mathematics.

Already in 1955, Shafarevich was courageous enough to sign a letter, along with other 300 scientists, denouncing the works of the Soviet biologist Trofim Lysenko who, using his power under Stalin, opposed and prosecuted scientists working in genetics theory. In 1968, he was one of the 99 cosigners of a letter in defense of a mathematical logician Aleksander Esenin-Volpin who was forcibly taken to a psychiatric hospital. The result was, however, that many of those who signed were deprived of any possibility to travel abroad. Since 1971 he took part in a Moscow Human Rights Committee organized by Andrei Sakharov. In September 1973 Shafarevich wrote an open letter in defense of Sakharov. Such activities led to him being fired from a teaching position at the University in 1975 (a position he was also briefly fired from for unknown reasons back in 1949). His dismissal deprived the university of a brilliant mathematician, a popular lecturer and a mentor of graduate students. As in the case of Sakharov, his membership in the Soviet Academy of Sciences and his worldwide fame as a scientist prevented the authorities to impose a harsher punishment.

In 1974 Shafarevich left the Sakharov Human Rights Committee and began to collaborate with Alexander Solzhenitsyn in publishing an anthology ‘Iz pod glyb’ (‘From under the rubble’) [52]). First published in Russian by YMCA-Press in 1974, it was translated the following year in France, United States, England, and Germany. In this collection of articles the authors living in Russia at the time discuss the present and the possible future of Russia. Consequently, condemned by official Soviet propaganda as expressing hatred of socialistic ideas, it was also condemned by many left-inclining Russian dissidents as expressing Russian nationalism, chauvinism and an attempt to replace a democratic society with an autocratic one. Shafarevich contributed three essays, on ethics, on the national problem, and on socialism. The latter one was the synopsis of his book [47] which he had already written a year earlier but would publish only later in 1977 by YMCA Press with a foreword by Solzhenitsyn. The book was translated into French the same year. Before the book ‘Iz pod Glyb’ had been released in the West, Solzhenitsyn was forcefully deported from Russia, leaving to Shafarevich the responsibility to discuss the book at several press-conferences for foreign journalists (New-York Times, Frankfurter Allgemeine, BBC). On many occasions, Solzhenitsyn expressed his respect of Shafarevich. Thus he writes in his essay “Bodalsia telenok s dubom” of 1975: “We have two thousand people in Russia with world-wide fame, for many of them, it was much louder than for Shafarevich (mathematicians exist on Earth in weak minority), however as citizens they are zeros because of their cowardice; and from this zero only a dozen took over and have grown to a tree, and among them is Shafarevich.” On another occasion he wrote: “The depth, the solidity of this man, not only in his figure, but in all his live-image, were immediately noticed.”

In 1973 Shafarevich was amongst the few members of the Academy of Sciences who protested against the malicious campaign in Soviet Press directed at Andrey Sakharov and he even wrote an Open Letter distributed in Samizdat and abroad. The following year he wrote two letters protesting
against the deportation of Alexander Solzhenitsyn with a bitter reproach to the Russian people for the passive silence and even acquiescence to this action. On many other occasions, Shafarevich’s name could be found on various petitions in defense of unlawfully prosecuted human right activists, including mathematicians Leonid Plusch, Yuri Gastev and a physicist Yuri Osipov. Along with Sakharov, he attended their trial proceedings.

After 1979 Shafarevich stepped aside from the dissident movement. Although some of the dissidents attributed this to a crackdown on the dissident movement that started the same year, this action cannot be explained merely by cowardice on his part and would be quite contrary to his actions during his whole life. As Shafarevich writes himself, he got disappointed with the movement causes - such as the preoccupation with the right to the Jewish emigration - that he considered minor compared to the real problems of the Russian people.

Political activity

Following Perestroika, Shafarevich began taking an active part in Russian political life. First supporting Yeltsyn and Sakharov, in a series of articles in “Nash Sovremennik”, he began to criticize the current regime for the drastic economic changes that cast many ordinary people into poverty. He also criticised the plans for the creation of the Soviet Sovereign Republics which meant the de facto dissolution of the USSR. His main complaint was that such an important issue had to be given a serious public discussion, whereas announcement of the decision had appeared only five days before the date of its signature. The August Putsch of 1991 that followed after this was a tragic event - unfortunately one of many - in Russian history. In articles written after the Putsch, he compared the dissolution of the Soviet Union and the Communist Party with the previous revolution that left large swaths of ordinary people despairing with the new ideological and economic situation.

As a result of this event, Shafarevich decided “to go into politics”. Joining the opposition camp to a regime that was claimed in the Western media as progressive resulted in another blow to his reputation. In December 1991 he joined the Russian All-People’s Union party and spoke at its first congress. The new political body that brought together representatives of many patriotic and democratic movements disillusioned with Yeltsin was claimed in the West as “the new right”, (proto)-fascist and the “red-browns”. The address that Shafarevich gave, however, called for dropping all sectarian interests and work on behalf of the Russian people. In February 1992, he was elected (although he did not stand for this) to the central council of the similar new organization Russian People’s Assembly (Rossiiskoe Narodnoe Sobranie, RNS). The same organization that, according to the official media, was responsible for the assault on the Moscow TV station in Ostankino, leading to more sharp criticism of Shafarevich.

In October 1992 Shafarevich joined the organizing committee of the National Salvation Front representing various ideological doctrines. In a statement signed by Shafarevich, the Front demanded that Yeltsin and the government accept the responsibility for the hardship of ordinary citizens brought about over the past years by the radical economic policy and suggested that the Front was ready to take on new executive power to prevent the country from collapsing. As Krista Berglund suggests “the moderation and sanity penetrating the Front’s statement together with lucid style and many formulations and emphases familiar from Shafarevich’s statements make it plausible that he significantly contributed to it.” In response, Yeltsin promptly banned the Front through a decree.
Of course, this was the period when Yeltsin was trying to consolidate the powers granted to him after the August Putsch while his relationship with the State Duma (the lower house of the Federal Assembly of Russia) had reached its worst. But for Shafarevich, according to his statement at the Front’s press-conference, this was the repeat of his experience as a dissident 20 year ago. The subsequent confrontation between Yeltsin and the State Duma led Yeltsin to the decision to hold a referendum that should confirm his power over the State Duma. Shafarevich vehemently opposed this referendum by demanding general elections of the President and the new Duma instead. As is well-known this confrontation ended in the bloodshed near the building of the Parliament that left hundreds dead. Although the Front did not play any organizational role in this conflict, many of its members participated on the side of the Parliament, subsequently compromising the Front itself. Following an unsuccessful attempt to get elected to the new Parliament as a representative of the Party of the Constitutional Democracy, Shafarevich ended his political activity.

Ten years later, when asked by Krista Berglund ‘whether he had a feeling that this thing [participating in political organizations] was not quite “my own”, his answer was an emphatic Yes, with the the exception being the participation in the National Salvation Front. Having left all the political parties in 1995 he agreed only to be on the editorial board of the journal “Questions of Nationalism” of the new National Democratic Party of Konstantin Krylov in 2012.

Non-mathematical writings

Three volumes of collected works of Shafarevich were published in 1994 [45]. In 2014 the Institute of Russian Civilization published six-volume collected works that contain a lengthy introduction [49]. The last volume is devoted to his mathematical works. From the preface: “Shafarevich is a classic of Russian national thought. His books enter into the golden fund of Russian national heritage. For millions of Russians, the thoughts expressed in them become a guide in their spiritual and social life.”

Many of the non-mathematical works collected in the first five volumes were published abroad in Russian or other languages. The first such essay published by the YMCA Press in 1973 was his report “Zakonodatelstvo o religii v SSSR” (‘The legislation on religion in USSR’) for the Human Rights Committee. The French translation had been published in 1974 by Éditions du Seuil, Paris. His second book “Socialism kak yavlenie mirovoy istorii” (‘Socialism as a phenomenon of world history’) was published by the YMCA Press in Russian in 1977 along with a French translation by the same publisher as the previous book. In 1980 Harper Collins along with Regnery Publishing published the English translation titled “The Socialist Phenomenon”. The first translation contained a preface written by A. Solzhenitsyn.

It was around the same period in the nineteen seventies that Shafarevich began writing his most controversial opus “Russophobia”, which in turn brought him simultaneously both the love and admiration from wide circles in Russia while making him a persona non grata among many Russian and Western intelligentsia. Although hardly invented by Shafarevich, the word “Russophob” became often associated with his book. Being distributed in Samizdat in Russia since 1982, it had been officially published (in abridged version) in Russia in 1988 by a literary magazine “Nash Sovremen-nik”. In the same year the Russian original was published by Munich-based journal Veche. It was followed by translations into Italian (Insigna del Veltro, 1990), French (Edition Chapitre Douze,
1993), Serbian (Pogledi, 1993) and German (Verlag der Freunde, 1995). It is amazing that no commercial English translation has appeared so far. Probably the existence of the book had became known to the mathematical community through Smilka Zdravkovska’s interview with Shafarevich [55]. Soon, a non-commercial translation was made by Joint Publication Research Service of the US Department of Commerce in 1990 and by a mathematician Larry Shepp in 1992 on his own initiative. While Shafarevich never considered the book as one of his most important works, nevertheless, it made his name known for the first time in the West outside the mathematical circles. In this book Shafarevich develops further the theory of French historian Augustin Cochin (1876-1916) who claimed that the French revolution of 1789 has been initiated by a small group of intellectuals constituting Malyi Narod (“Lesser or Small People”) and was opposed to the “Large People” who represent the organic basis of the given society. Although he never claimed that in the modern Russian history the “Small People’ consisted entirely of Jews, he did try to demonstrate that they at least represented a major part of it. As probably happens in many a historical study, he filtered the factual material and citations selectively to prove his point.

The second volume of the collected works reprints “Russophobia” together with other important articles written in the nineteen nineties. One of the most important articles amongst them is ‘Dve dorogi k odnomu obryvu’ (‘Two roads to the same abyss’). In this article written for the collection ‘Iz pod Glyb’ which we mentioned earlier, Shafarevich rejects both the Socialistic and the Western Democratic style for the future development of Russia and searches for a middle way via the spiritual rebirth of the nation.

Volume 4 of the collected works reprints another book of Shafarevich “Three thousand years of mystery. History of the Jews from perspectives of modern Russia” published in Russia in 2002. Volume 5 contains many articles on historical and current political issues that appeared in the Russian Press, including three articles about Shostakovich and his music. According to [4], p. 160, Solomon Volkov, the professional musicologist who published Shostakovich’s Testimony, stated in 2004 ‘For the last 50 years the person who has written better than anybody about Shostakovich is Igor Rostislavovich Shafarevich.’

Many articles of Shafarevich were not political by nature but more philosophical, historical and religious. The leading thread of his thinking was the eternal fight between the Good and the Evil. From this viewpoint, he discussed the work of Plato as well as the music of Shostakovich. In his lecture delivered on the occasion of the official presentation of the Dannie-Heinemann Award of the Göttingen Academy of Sciences in 1973 he searched for the spiritual sources and purpose of Mathematics [31].

**Accusation in anti-semitism**

The accusations are based on the Shafarevich’s attempt to defend Russia from Russophobia by expressing Judeophobia in his works. According to some of the definitions, antisemitism is based on religious, economic, social, racist, ideological, cultural prejudices toward Jews. Only the last one may apply to Shafarevich. It is defined on the basis of accusing Jews of corrupting a given culture by supplanting it with a uniform, crude ”Jewish culture”. The main purpose of his book, as well of many of his other writings, and of his whole life outside mathematics, was not to express hatred of Jewish people and Jewish culture, as it was claimed, but rather to defend Russian people, Russian
culture and Russian history from varied accusations. Among them are bending under different political regimes, incapacity to grow a democratic society, poor cultural traditions (sic!), racism towards other ethnicities and hostility to the Western social ideas. All of this is especially familiar in our days of the growth of nationalism in many countries, where the accusation of “being for our own people” versus “being for all people” is condemned as racist and chauvinistic.

The reaction of the mathematical community to publishing “Russophobia” is well known and widely available on the Internet. Unfortunately, the reason for the negative reaction of many mathematicians in the West, some of whom probably did not bother or were not able to read Shafarevich’s writings, was not the understandable concern about the fate of Russia in its turbulent time of the nineteen nineties but the outrage at what Shafarevich wrote about Russian-Jewish relations. However, some of the mathematicians (including J.-P. Serre) compared the campaign against Shafarevich with a witch-hunt. Citing from a recent letter of David Mumford [18] “I did not believe then and do not believe now that he was anti-semitic, but rather that he was a fervent believer in his country, its people, its traditions -perhaps one should say its soul.” For most people the love of their country, its history and its traditions and less interest or indifference to other countries and its traditions is natural. Unfortunately, Russia in modern time was exceptional in this way. The assault to the nationalistic feeling of Russian people came from many sides: political, cultural, religious, intellectual, foreign and domestic. Shafarevich and Solzhenitsyn were among a few people who dedicated their lives to defending the right of the Russian people to the respect they deserve among other nations.

Shafarevich expressed his own creed in the following words: "A possibility to influence the future depends on the capability to evaluate and comprehend the past. Indeed, we belong to the species of Homo Sapiens, and the mind is one of the most powerful tools that allows us to find our own path in life. For this reason, it seems to me, this is now one of the most important for Russia concrete questions: stand up for the right to comprehend your own history without any taboos and forbidden topics."

One may disagree with many of Shafarevich’s views, some of them unwillingly historically distorted, but there is a good reason to remind Voltaire’s principle “I wholly disapprove of what you say, but I will defend to the death your right to say it.”

Many accusations of Shafarevich being hostile to individual Jews and, especially doing harm to Mathematics, have not been supported by any evidence. Thus the foreign secretary of the NAS accused Shafarevich of interfering in the careers of young Jewish mathematicians and preventing them from publishing their papers. He had never apologized for this blatant lie. One in four of Shafarevich’s Ph.D. students were of Jewish or partly Jewish origin. Among his non-Jewish students were students of Armenian, Bulgarian, German, Lithuanian, Tartar and Ukrainian origin. His close associate and one of the contributors to “Algebraic Surfaces” was Boris Moishezon, one of the pioneers of the Jewish emigration movement. The coauthor of one of his most influential papers on the Torelli Theorem for K3 surfaces was Ilya Iosifovich Piatetski-Shapiro. One of his friends (to whom he wrote a memorial article) was the famous topologist Vladimir Rokhlin. Shafarevich had taken a lot of efforts and troubles to secure jobs for his students, Jewish or not, for example, arguing before Vinogradov for the merit of giving a position at the Steklov Institute to Yuri Ivanovich Manin. Since 1950 until his death, Shafarevich served on the editorial board of the most important and prestigious Russian mathematical journal “Izvestia”. In the period of 10 years 1967-1977, he was the
associate editor of the journal. The chief editor Ivan Matveevich Vinogradov played only a nominal role in editorial decisions. During this period many Jewish mathematicians (e.g. Victor Kac or Boris Weisfeiler who later emigrated to the USA) were able to publish their important papers in this journal.

Igor Shafarevich had lived a long and productive life as a mathematician, as a philosophical thinker, publicist, a historian and a Russian patriot. His mathematical heritage will certainly last forever, only the future will tell whether his other contributions to intellectual life will be of equal value.

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