A note on brane boxes at finite string coupling

Eric G. Gimon* and Martin Gremm†

California Institute of Technology, Pasadena, CA 91125

Abstract

We consider $\mathcal{N} = 1$ supersymmetric $SU(N_c)$ gauge theories, using the type IIB brane construction recently proposed by Hanany and Zaffaroni. At non-zero string coupling, we find that the bending of branes imposes consistency conditions that allow only non-anomalous gauge theories with stable vacua, i.e., $N_f \geq N_c$, to be constructed. We find qualitative differences between the brane configurations for $N_f \leq 3N_c$ and $N_f > 3N_c$, corresponding to asymptotically free and infrared free theories respectively. We also discuss some properties of the brane configurations that may be relevant to constructing Seiberg’s duality in this framework.

*email: egimon@theory.caltech.edu

†email: gremm@theory.caltech.edu
I. INTRODUCTION

Many field theories with varying numbers of supersymmetries and in various dimensions can be realized on the common directions of type II branes. For a comprehensive review and references see [1]. In four-dimensional $\mathcal{N} = 1$ theories the problem arises that not all global symmetries are fully realized. Specifically, for an $SU(N_c)$ theory with $N_f$ massless flavors one expects $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry. However, in the IIA brane construction of this theory only the diagonal $SU(N_f)$ is visible. This problem was first addressed in [2] where it was pointed out that the full chiral symmetry can be restored under certain conditions. Subsequently, many theories with full chiral symmetry and chiral matter content were constructed [3–7].

More recently Hanany and Zaffaroni [8] proposed a construction of chiral gauge theories using grids of NS fivebranes as well as D5 branes in type IIB string theory. These constructions have the advantage that the full chiral symmetry is manifest. They also provide a method of constructing genuinely chiral theories, i.e., theories for which no mass term can be added to the superpotential. In Ref. [8] the authors consider these brane constructions at zero string coupling. In this case all branes meet at right angles or are parallel. At zero coupling there does not seem to be any obstacle in constructing anomalous gauge theories, such as an $SU(N_c)$ with only anti-fundamentals.

In this note, we take a first step towards an analysis of these brane configurations at finite string coupling. Once the string coupling is switched on, there are certain restrictions on the angles at which NS fivebranes and D5 branes can meet. This is quite familiar from the study of $(p, q)$-webs of fivebranes [9]. Past experience has shown that consistency conditions in string theory usually enforce such field theory requirements as the vanishing of anomalies. In this note we analyze the simplest case discussed in [8], an $SU(N_c)$ gauge theory with fundamental flavors, at finite string coupling. We propose consistency conditions based on the bending of the branes and show that they guarantee that the gauge theories are non-anomalous. They also restrict the number of flavors to $N_f \geq N_c$, which excludes theories
with instable vacua [10,11]. There are qualitative differences between the brane constructions of $SU(N_c)$ theories with $N_f \leq 3N_c$, which are asymptotically free, and the theories with $N_f > 3N_c$, which are free in the infrared. The consistency conditions also help to explain the origin of some puzzling properties of the brane configurations at zero string coupling. These problems appear when we try to find Seiberg’s duality [10,11] by moving the branes in the same way as in the type IIA construction of these $SU(N_c)$ theories [12]. We show two examples of these difficulties and demonstrate that the consistency conditions may offer a way to resolve them.

II. BOXES AT FINITE COUPLING

Following [8], we construct a 3+1 dimensional $SU(N_c)$ gauge theory using type IIB five-branes with world volumes: NS(012345), NS'(012367), D5(012346). The NS and NS' branes bound the D5 branes in the 4 and 6 direction as shown in Fig. 1. If we stack $N_c$ D5 branes on top of each other in the finite box we get the desired $SU(N_c)$ gauge theory. Matter
in the fundamental representation can be included by placing additional D5 branes in the semi-infinite boxes surrounding the box with the $N_c$ finite branes. As indicated in the figure, we place an arbitrary number of semi-infinite D5 branes in each of these boxes. At finite string coupling there will be consistency conditions that restrict these numbers. We can determine the (chiral) matter content of this theory using the rules listed in [8].

The angle at which the D5 and NS branes intersect is given by

$$\Delta x : \Delta y = p + \tau q,$$

(2.1)

where $\tau = i/g_s + \chi/(2\pi)$. (See [3,8] for details.) This condition ensures that no additional supersymmetries are broken. For $g_s = 0$, $\chi = 0$ all branes meet at right angles, but for finite string coupling the angles are determined by the $(p, q)$ charges of the branes involved. The arguments in this section do not depend on the value of the string coupling as long as it is not zero. Fig. 2 shows three slices in the $x_4 - x_7$ plane through the configuration in Fig. 1. Figures 2a,b,c show $x_6$ positions to the left of the two NS branes, between the NS branes, and to the right of the NS branes respectively. The angles are determined by the number of D5 branes ending on each NS' brane from above and below. For an arbitrary number of D5 branes in each of the semi-infinite boxes, the six angles in Fig. 2 will in general be different. This requires that far out in the $x_7$ direction the NS' brane is either discontinuous in the $x_6$ direction or, if the discontinuity is smoothed out, violates Eq. (2.1) at least for some $x_6$ positions. There is an alternative way of looking at this. To the right of the D5 branes in Fig. 4, the NS' branes look like $(p, 1)$ branes with $p$ depending on the $x_6$ position. There are smooth paths along the NS' branes to the right of the D5 branes in Fig. 2, that connect patches with different $p$'s. Since the $(p, q)$ charges are globally defined quantities, this is not necessarily inconsistent. However, far away from the finite D5 branes, we expect the angle of the $(p, q)$ brane to be given by its charges. For arbitrary charges, the $(p, q)$ brane would have different $p$ charges far out in the $\pm x_6$ direction. We take these considerations to indicate that the three angles involving each of the NS' branes have to be identical. This imposes the following restrictions on the numbers of branes we can put in the semi-infinite
FIG. 2. Three slices showing the $x_4 - x_7$ plane: a) to the left of the NS branes, b) between, c) to the right of the NS branes. The lines labeled $(0, 1)$ are the NS’ branes and the vertical lines are the stacks of D5 branes.

We can make an identical argument for the bending of the NS branes by considering slices in the $x_6 - x_5$ plane at different $x_4$ positions. In order for the NS branes to bend in a consistent way, the numbers of semi-infinite D5 branes must satisfy

\begin{align}
  n_1 - m_1 &= n_2 - N_c = n_3 - m_3, \\
  l_1 - m_1 &= l_2 - N_c = l_3 - m_3.
\end{align}

(2.2)

It is interesting to note that this second set of conditions is linearly dependent on the first and does not impose any new constraints. Solving these consistency conditions gives

\begin{align}
  m_1 &= n_1 - n_2 + N_c, \\
  m_3 &= -n_2 + n_3 + N_c.
\end{align}

(2.4)
\[ l_1 = l_3 + n_1 - n_3, \]
\[ l_2 = l_3 + n_2 - n_3, \]

with \( n_1, n_2, n_3, l_3 \) arbitrary, and \( N_c \) the number of colors. The rules given in [8] can be used to find the matter content of this theory. The consistency conditions guarantee that the \( SU(N_c) \) theory is anomaly free. There are \( l_3 + n_1 + N_c \) flavors of fundamentals and anti-fundamentals. Note that the minimum number of flavors in a consistent theory is \( N_c \).

This agrees with the field theory analysis which shows that there is no stable vacuum for \( N_f < N_c \) [10,11]. There is also a a qualitative change in the brane construction as the number of flavors exceeds \( 3N_c \). To see this, consider a theory with \( m_1 = m_3 = N_c \) and \( n_1 = n_2 = n_3 = N_f \) and the other semi-infinite boxes empty. This configuration satisfies our consistency conditions. It gives rise to an \( SU(N_c) \) gauge theory with \( N_f + N_c \) flavors. For \( N_f < N_c \) the \((p, q)\) branes bend away from the stacks of \( N_c \) D5 branes as shown in Fig. [2], but for \( N_f > N_c \) both \((p, q)\) branes bend in the same directions in the \( x_4 - x_7 \) plane. If \( N_f > 2N_c \), the two \((p, q)\) branes meet somewhere to the right of the D5 branes in Fig. [2].

There are two inequivalent choices for the brane configurations to the right of the D5 branes. Either the two \((p, q)\) branes merge to form a \((p', q')\) brane, or they intersect. Both of these configurations differ from \((p, q)\)-webs of fivebranes studied in [1] by the presence of additional branes. An analysis of these configurations is beyond the scope of this paper. Here we simply note that no such configurations arise in brane constructions that give rise to asymptotically free theories with stable vacua. Thus, the brane configuration in Fig. [1] gives rise to the expected gauge theories for a total number of flavors between \( N_c \) and \( 3N_c \).

Repeating this analysis for a general brane configuration that satisfies our consistency conditions yields a set of inequalities that give the constraint \( n_1 + l_3 \leq 2N_c \). This implies that in the general case, the \((p, q)\) branes also do not intersect unless the total number of flavors exceeds \( 3N_c \). Thus the consistency conditions pick out those theories that have a stable vacuum and are asymptotically free.

The consistency conditions help resolve some puzzling aspects of the theories one can
construct at zero coupling. For instance, consider the theory obtained by setting $m_1 = l_1 = N_f > N_c$ and all other boxes empty. This theory is forbidden by our consistency conditions, but at zero coupling we expect to get an anomaly-free $SU(N_c)$ with $N_f$ flavors. For $N_f > N_c$ there should be a brane motion that yields Seiberg’s duality. In order to move the two NS branes past each other, we connect $N_c$ D5 branes across the left NS brane so that they extend from the right NS brane to infinity in the $-x_6$ direction. Then we move the two NS branes to different $x_7$ positions, move them past each other in the $x_6$ direction and bring them back to the same $x_7$ location. The result of this operation is a brane configuration of the kind shown in Fig. 1 with $m_1 = l_1 = l_2 = N_f$ and gauge group $SU(N_f - N_c)$. According to the rules in [8] this theory is anomalous, since it has $2N_f$ fundamentals and $N_f$ anti-fundamentals. Instead of the expected Seiberg-duality $SU(N_c)$, we found a brane motion that transforms an anomaly-free theory into an anomalous one. What went wrong? At finite coupling we see that this brane configuration is inconsistent but even if it were consistent, the motion of the two NS branes past each other is possible only if the NS’ branes are planes. This requires that the same number of D5 branes end on each NS’ from above and below. In other words, we must require that $n_1 = m_1 = l_1 = N_f$ and gauge group $SU(N_f - N_c)$. The original theory had $N_f + N_c$ flavors while the ‘dual’ has $2N_f - N_c$ flavors. Also, instead of the expected gauge group $SU(N_f)$ the brane motion yields $SU(N_f - N_c)$.

To see this in some more detail, consider the theory with $n_1 = m_1 = l_1 = N_f$ and $n_2 = l_2 = N_c$. In this case the two NS’ branes are planes, so there is no impediment to moving the NS branes in the $x_7$ direction. If we repeat the brane motion described above, we get a theory with $n_1 = m_1 = l_1 = N_f$, $n_2 = l_2 = N_f - N_c$ and gauge group $SU(N_f - N_c)$. The original theory had $N_f + N_c$ flavors while the ‘dual’ has $2N_f - N_c$ flavors. Also, instead of the expected gauge group $SU(N_f)$ the brane motion yields $SU(N_f - N_c)$.

In this analysis we did not take the bending of the NS branes into account. Fig. 3 shows a section in the $x_5 - x_6$ plane of the initial brane configuration. The brane motion described above corresponds to exchanging the $x_6$ positions of the two NS branes. The solid lines in Fig. 4 show the final configuration. In order to obtain brane configurations that give rise to
FIG. 3. A section in the $x_5 - x_6$ plane showing the $(p, q)$ charges of the branes. The (0, 1) branes are the two NS branes and the vertical line represents the D5 branes. The duality motion corresponds to exchanging the $x_6$ positions of the two NS branes.

... asymptotically free field theories, we require that the NS branes do not intersect before the brane motion. In the initial configuration this restricts $N_f$ to the range $0 \leq N_f \leq 2N_c$. As shown in Fig. 4, exchanging the $x_6$ positions of the NS branes requires that the $(p, q)$ branes intersect when they coincide in the $x_7$ direction. This is true for all values of $N_f$ except $N_f = 2N_c$, which corresponds to parallel NS branes to the left and parallel $(p, q)$ branes to the right of the D5 branes.

It was shown in Ref. 9 that an intersection of $(p, q)$ branes can be viewed as a special case of a $(p, q)$ web theory with four external branes. We can replace the intersection point by a finite size face, because changing the size of a face in a $(p, q)$-web is a flat direction corresponding to a modulus in the five-dimensional theory on the intersection of the fivebranes. The resulting brane configuration is shown in Fig. 4, where the dashed lines show one possible face that could replace the intersection of the $(p, q)$ branes. We can increase the size of the face until one edge of it touches the D5 branes. The massless states from strings stretched between the edge and the stacks of D5 branes are degrees of freedom.
FIG. 4. The brane configuration after the duality motion. The solid lines to the right of the D5 branes are the intersecting \((p, q)\) branes and the dashed lines show one possible face.

in the four-dimensional field theory. An analysis of these effects is beyond the scope of this note, but these considerations suggest that we may need to modify the counting rules proposed in Ref. [8], if there are extra intersections of \((p, q)\) branes. One possibility is, that the configurations before and after the brane motion give rise to the same field theory after the states from the intersection are included.

III. CONCLUSIONS

We analyzed the type IIB construction of four-dimensional \(\mathcal{N} = 1\) \(SU(N_c)\) gauge theories at non-zero string coupling. The condition that no further supersymmetries be broken determines the angles at which NS and D5 branes can intersect. Based on these conditions we obtained consistency requirements that constrain which gauge theories can be constructed in this approach. We found that the gauge theories are guaranteed to be non-anomalous and that the total number of flavors is restricted to the range \([N_c, 3N_c]\). If the total number of flavors exceeds \(3N_c\), extra intersections between \((p, q)\) branes appear and it is not clear
what effect this has on the field theory. The IIB construction without the extra intersections automatically gives rise only to asymptotically free theories with stable vacua.

We also demonstrated that brane motions analogous to the IIA brane motions that provide a string theory realization of Seiberg’s duality, naively give results that contradict the field theory expectations. At finite string coupling, these brane motions require that new intersections of \((p, q)\) branes appear. We argued that there may be additional massless states in the four-dimensional field theory that arise from these intersections.

The results presented here are only a first step toward a more complete analysis. In particular, it would be very interesting to analyze theories with intersecting \((p, q)\) branes. This may shed some light on how to construct Seiberg’s duality in the IIB framework. An analysis of the flat directions of these brane configurations would be useful. The zero coupling results may get modified when the bending of branes and the possibility of replacing intersections with faces is taken into account. Another interesting question is, how to modify the consistency conditions to incorporate orientifolds. We hope to address some of these questions in the future.

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