An improved analysis of neutrino-nucleon interaction cross-section at ultra high energy

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Abstract. We determine the Ultra High Energy (UHE) neutrino-nucleon interaction cross-sections numerically both for the charged and the neutral current in the leading order (LO) and find our numerically determined results to be in sufficient agreement with the results obtained by other authors. For this, we exclusively carry out extrapolation at ultra low $x$ and find our method to be satisfactory.

1. Introduction
In this paper, cross-sections of UHE neutrino-nucleon interaction are determined by carrying out numerical integration of the double differential cross-section at one-loop level. For this purpose, we take the help of GRV94 parton distributions [1]. We then compare our predictions with the results of several other authors [2–4] and our previously done analytical results [5] obtained by using the solutions of the non-singlet and singlet Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations respectively [6].

2. Formalism
2.1. Charged current-numerical solution
The differential cross-section for the charged current (CC) UHE interaction is given by [7],

$$
\frac{d^2 \sigma^{\nu(p)}_{CC}}{dxdQ^2} = \frac{G_F^2}{2\pi} \left( 1 + \frac{Q^2}{M_W^2} \right)^{-2} \frac{1}{x} \left[ \left( 1 - y + \frac{y^2}{2} \right) F_2^{\nu(p)_{CC}}(x,t) \pm i \frac{y}{2} x F_3^{\nu(p)_{CC}}(x,t) \right].
$$

(1)

where Fermi coupling constant $G_F = 1.1663 \times 10^{-5}$ GeV$^{-2}$, $Q^2 = xy s = 2ME_{\nu(p)}$ is the centre-of-mass energy squared, $x$ is the usual Bjorken variable, $y$ is the inelasticity parameter. The total cross-section for UHE CC interaction in an isoscaler target $N[\equiv \frac{(p+n)}{2}]$ is given by,

$$
\sigma^{\nu(p)}_{CC} = \int \frac{dx}{x} \int \frac{dQ^2}{Q_0^2} \int \frac{dQ^2}{Q_0^2} \frac{d^2 \sigma^{\nu(p)}_{CC}}{dxdQ^2}.
$$

(2)
We only consider all light \((u,d,s)\) quarks. We have the following LO expressions in QCD for charged current (CC) interactions \([7]\),

\[
F_1^{\nu,\text{light}} = \frac{1}{2} (\bar{u} \hat{d}) + \frac{1}{2} (d + u)|V_{ud}|^2 + s|V_{us}|^2.
\]

\[
F_2^{\nu,\text{light}} = 2x F_1^{\nu,\text{light}}
\]

\[
F_3^{\nu,\text{light}} = - (\bar{u} \hat{d}) + (d + u)|V_{ud}|^2 + 2s|V_{us}|^2.
\]

where \(u(x,t), d(x,t)\) etc. represent parton distributions at various values of \(x\) and \(t\) and \(V_{ud}, \ V_{us}\) etc. are the relevant CKM matrix elements \([8]\). For obtaining the expression of total cross-section, we first put the explicit parton distributions equations (3-5) in equation (1) and then with the help of equation (2), we finally get \([9]\)

\[
\sigma^{\nu(\pi)N}_{\text{CC}} = \frac{G_F^2}{2\pi} \int_{Q_0^2}^s dQ^2 \left( 1 + \frac{Q^2}{M_W^2} \right)^{-2} \int_{Q_0^2}^s \frac{dx}{x} \left[ 1 - \frac{Q^2}{xs} \pm \frac{Q^4}{2x^2s^2} \right] x \left\{ \begin{array}{l}
\frac{1}{2} \left[ \bar{u}(x,t) + \bar{d}(x,t) \right] \\
+ \frac{1}{2} \left[ d(x,t) + u(x,t) \right] |V_{ud}|^2 + s|V_{us}|^2 \nonumber
\end{array} \right\}.
\]

\[
\sigma^{\nu(\pi)N}_{\text{NC}} = \int_{Q_0^2}^s dx \int_{Q_0^2}^s dQ^2 \frac{d^2 \sigma^{\nu(\pi)N}_{\text{NC}}}{dx dQ^2}.
\]

where

\[
\frac{d^2 \sigma^{\nu(\pi)N}_{\text{NC}}}{dx dQ^2} = \frac{G_F^2}{2\pi} \left( 1 + \frac{Q^2}{M_Z^2} \right)^{-2} \int_{Q_0^2}^s \frac{dx}{x} \left[ 1 - y + \frac{y^2}{2} \right] F_2^{\nu(\pi)N_{\text{NC}}}(x,t)
\]

\[
\pm y \left( 1 - \frac{y}{2} \right) x F_3^{\nu(\pi)N_{\text{NC}}}(x,t).
\]

Putting the LO expressions for structure functions involving Neutral currents (NC) \([7]\) for light quarks in equation (8) and then with the help of equation (7), we get the expression of total cross-section for neutral current as \([9]\)

\[
\sigma^{\nu(\pi)N}_{\text{NC}} = \frac{G_F^2}{2\pi} \int_{Q_0^2}^s dQ^2 \left( 1 + \frac{Q^2}{M_Z^2} \right)^{-2} \int_{Q_0^2}^s \frac{dx}{x} \left[ 1 - \frac{Q^2}{xs} + \frac{Q^4}{2x^2s^2} \right] \left\{ 0.1432x \right\}
\]

\[
\times \left[ u(x,t) + \bar{u}(x,t) + d(x,t) + \bar{d}(x,t) \right] + 0.1849x
\]

\[
\times \left[ u(x,t) + \bar{u}(x,t) + d(x,t) + \bar{d}(x,t) \right] + 4s \right\}
\]

\[
\pm \frac{Q^2}{xs} \left( 1 - \frac{Q^2}{2xs} \right) \left\{ 0.268x \left[ u(x,t) - \bar{u}(x,t) + d(x,t) - \bar{d}(x,t) \right] \right\}
\]
You can refer to the figure for a visual representation of the data.

2.3. Extrapolation at ultra low $x$

Since the UHE neutrinos coming from various sources have energy more than $10^5$ GeV, so perturbative calculations should be carried out using parton distribution functions at Ultra-low Bjorken $x$ ($x < 10^{-5}$). But high energy measurements of deep inelastic scattering (DIS) of lepton-nucleon at the collider HERA [10] at DESY provide data for ($x \geq 10^{-5}$). So we carry out extensive extrapolation in the region of $x$ still unexplored by terrestrial experiments. This is done by multiplying $F_2(x,t)$ by log($\frac{1}{x}t$) in equation (1) and equation (8), consequently modifying equation (6) and equation (9) for charged and neutral currents respectively. It is worthwhile to mention that the contribution from $xF_3$ to cross-section is negligible at Ultra High Energy of neutrino.

3. Results and discussions

We use GRV94 generalised parton distributions [1] for determining the cross-section numerically. Putting these distributions in equation (6) and equation (9), we obtain the values of non-extrapolated cross-sections for $\nu N$ interactions both for the charged current and the neutral current respectively. For getting extrapolated numerical results, we follow the techniques as mentioned in Subsection 2.3.

In Figure 1, we plot our numerical LO results (with and without extrapolation) for $\sigma^{\nu N}_{CC}$ ($cm^2$) versus neutrino energy. In the same Figure, we also plot next-to-leading order (NLO) results of various other authors [2–4]. Similarly in Figure 2, we compare our LO predictions for $\sigma^{\nu N}_{NC}$ ($cm^2$) (with and without extrapolation) with the NLO results of various other authors [2–4]. In both the cases, we find that our numerically-extrapolated results match better with the result of other authors than our numerically non-extrapolated results. Due to the absence of propagator effect in low energy regime, our numerical result indicates a sharp rise for $E_\nu \leq 10^3$ GeV. For $E_\nu \geq 10^5$ GeV, propagator effect creates the dampening. But there is a severe dampening in our previously calculated non-extrapolated analytical results [5] for $E_\nu \geq 10^5$ GeV, much more than our non-extrapolated numerical results. Of course, our extrapolated analytical results [5] agree more or less well with our extrapolated numerical results and the results of various other authors [2–4] as clearly seen from Figure 1 and Figure 2.
4. Comments and conclusions
To conclude, we have shown that our numerically-extrapolated result agree quite well with the results of other authors [2–4] both at the lower as well as higher ends of the neutrino energy spectrum. This reflects that the exclusive technique adopted by us in this paper for carrying out extrapolation at ultra low $x$ is quite acceptable and satisfactory. Taking into consideration the contribution due to heavy quarks might improve this result marginally.

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