COSMIC CONCORDANCE

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Abstract

It is interesting, and perhaps surprising, that despite a growing diversity of independent astronomical and cosmological observations, there remains a substantial range of cosmological models consistent with all important observational constraints. The constraints guide one forcefully to examine models in which the matter density is substantially less than critical density. Particularly noteworthy are those which are consistent with inflation. For these models, microwave background anisotropy, large-scale structure measurements, direct measurements of the Hubble constant, $H_0$, and the closure parameter, $\Omega_{\text{Matter}}$, ages of stars and a host of more minor facts are all consistent with a spatially flat model having significant cosmological constant $\Omega_\Lambda = 0.65 \pm 0.1$, $\Omega_{\text{Matter}} = 1 - \Omega_\Lambda$ (in the form of “cold dark matter”) and a small tilt: $0.8 < n < 1.2$. 
Our approach has been to objectively apply the best known observational constraints to classes of theoretical models and map out any permitted range of parameters. In addition to the astronomical measurements that have been standardly considered, we incorporate recent measurements of the cosmic background radiation (CBR) anisotropy at large, intermediate and small angular scales. We present the results of a most promising case, spatially flat models with cold dark matter and cosmological constant. These models are consistent with a primary prediction of the inflationary paradigm\(^1\) (for a recent review, see Ref. 2); that is, to high precision, \(\Omega_\Lambda + \Omega_m + \Omega_{\text{rad}} = 1\) today. Although the common theoretical prejudice is that \(\Omega_\Lambda = 0\), we consider it plausible that there is a new energy scale (few meV) being unveiled, associated with weakly coupled fields (e.g. neutrinos or dark matter).

We first consider astronomical measurements that delimit the \((\Omega_0, \text{Matter}, h)\) plane (Fig. 1). For the Hubble parameter, most recent observations\(^3\) are in the range \(h = 0.70 \pm 0.15\), including the major recent study using the Hubble Space Telescope and the classical Cepheid variables\(^4\) \((h = 0.82 \pm 0.17)\) and studies using Type I supernovae\(^5\) \((h = 0.67 \pm 0.07)\). We will take as a lower bound on the age of the universe the ages of the oldest globular clusters: \(t_0 = 15.8 \pm 2.1\) based on the main sequence turnoff\(^6\) and \(t_0 = 13.5 \pm 2.0\) using giant branch fitting\(^7\) (The 11.5 Gyr lower bound is illustrated in Fig. 1; using the 13.7 Gyr value would reduce the concordance region by roughly half, still maintaining a substantial area.)

The cosmological constant, \(\Omega_\Lambda\), is constrained by many tests\(^8\) but most directly by gravitational lensing which gives \(\Omega_{0,\Lambda} < 0.75\). This is consistent with the lower bound \(0.2 < \Omega_{0,\text{Matter}}\) based on observed light density and cluster mass-to-light ratios\(^9\) or by utilization of large-scale structure measurements\(^1\).

The X-ray measured gas masses and the total virial masses\(^2\) imply \(\Omega_B h^{3/2}/\Omega_{0,\text{Matter}} = 0.07 \pm 0.03\), and light element nucleosynthesis\(^13\) constrains \(\Omega_B h^2 = 0.015 \pm 0.005\), where \(\Omega_B\) is the baryonic contribution to the critical density. These combine into a constraint on \(1 - \Omega_{0,\Lambda} = \Omega_{0,\text{Matter}} = (0.21 \pm 0.12)h^{-1/2}\). Finally, the growth of large scale structure in CDM models\(^14\) requires \(\Gamma \equiv \Omega_{0,\text{Matter}} h = 0.25 \pm 0.05\).
FIGURES

FIG. 1. The range of models (hatched area) in concordance with the best known astronomical observations of the Hubble constant \( (H_0) \), age (allowed region is above the dashed curve), large scale structure \( (\Gamma \equiv \Omega_{\text{Matter}}h) \), baryons in galaxy clusters (dot-dashed curves), and gravitational lenses (allowed models are below the shaded region). Dot indicates a representative model with \( h = 0.65 \) and \( \Omega_\Lambda = 0.65 \).

The range of concordance with all the quoted observations is indicated by the hatched area in Fig. 1. The black dot denotes a representative model \( h = 0.65, \Omega_\Lambda = 0.65 \). Two conclusions are worth noting here: (1) a substantial permitted area does exist, and (2) removal of any one of the observational sets of limits does not significantly enlarge the permitted area. Or, put differently, recovering the more theoretically desirable \( \Omega_\Lambda = 0 \) requires a combination of many observations to change in a coherent fashion. Using similar astronomical arguments, others have been led to non-zero \( \Omega_\Lambda \) models independently.

We now add the cosmological constraints derived from CBR anisotropy. The amplitude of the CBR power spectrum can be used to determine the value of \( \sigma_8 \), the fluctuations in mass in an eight \( h^{-1} \) Mpc sphere. The extrapolation from the large angular scales probed by COBE DMR down to 8 Mpc depends on the value of the spectral index \( n \), the fractional contribution of gravitational waves to CBR fluctuations, and the values of \( \Lambda, h, \) and \( \Omega_B \). For any given point in the concordance region of Fig. 1, an allowed range for \( n \) can be determined by extrapolating the COBE DMR amplitude down to 8 Mpc and comparing to the range \( \sigma_8 = (0.56 \pm 0.06)(\Omega_{0,\text{Matter}})^{-0.56} \) derived from the great clusters and from the distribution of large-scale structure and velocities. Here we explicitly assume inflation, which fixes a relation between \( n \) and the gravitational wave contribution. We find agreement for tilts, \(-0.2 < n - 1 < 0.2\), consistent with COBE DMR spectral index measurements and with what is achievable in inflationary models. Hence, we find that cosmic concordance with all observations and with inflationary cosmology can be obtained.

We might have taken as a postulate that \( \Omega_\Lambda = 0 \) and considered open universe models
instead. One problem is that open universes can be accommodated with inflation only by very delicate tuning of parameters. Also, the diagram analogous to Figure 1, would show a much smaller concordance region because the age constraint would have shifted substantially to the left. We do not intend to rule out the possibility that yet other types of models, such as mixed dark matter or baryonic isocurvature, might be constructed to fit the observational constraints.

What future observations could be used to further reduce the concordance range? The most promising are measurements of intermediate- and small-scale CBR anisotropy. Figure 2 shows the predicted CBR anisotropy power spectrum for the standard CDM model and a representative model from the middle of the concordance region ($h = 0.65, \Omega_\Lambda = 0.65$). A characteristic feature of inflationary models and a spatially flat universe is the Doppler peak at $\ell \approx 220$. Hence, simply finding (or not finding) a Doppler peak at $\ell \approx 220$ would rule out either all open (or the flat) models.

FIG. 2. Predicted CBR power spectrum showing the spectrum of multipoles ($C_\ell$) as a function of multipole number ($\ell$) for standard CDM ($n = 1$, $h = 0.5$, $\Omega_\Lambda = 0$; dashed line) and a representative concordance model ($n = 0.96$, 20% gravity wave contribution to CBR quadrupole, $h = 0.65$, $\Omega_\Lambda = 0.65$, $\sigma_8 = 0.87$; solid line). The boxes represent the theoretical predictions for present CBR experiments. The horizontal error bars are present one-sigma detections, and the triangles are 95 percent upper confidence limits. (See Ref. 2 for details.) Note that the power spectrum for the concordance model is remarkably similar to the prediction for CDM, except at smaller angular scales ($\ell > 250$), where the concordance model is marginally more consistent with present observational limits.

Experiments at yet smaller angular scales, down to 10 arcminutes, would provide further constraints on cosmological parameters. An amusing feature, illustrated in Figure 2, is that the predicted CBR power spectrum for $\ell < 250$ in standard ($\Omega_\Lambda = 0$) CDM models and in our concordance models are virtually indistinguishable and in equal agreement with observations, despite their substantially different cosmological parameter values. However,
for $\ell > 250$ (spanning 10'-30'), representative concordance models predict somewhat less power than standard CDM, marginally more consistent with current measurements. This example is strong motivation for improved CBR experiments with 10'-30' angular resolution.

We have presented this analysis, in part, to lay down a challenge to the reader: Can one identify a serious problem with the concordance models illustrated in Figs. 1 and 2? If not, perhaps we have already identified models which, in broad outline, capture the essential properties of the large-scale universe. More generally, we offer this analysis as a forward-looking illustration of how new and improved observations will provide quantitative and redundant tests that can decisively discriminate among competing models.
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