Creep salt array with a cavity

V I Andreev
NR MGSU, 26 Yaroslavskoye sh., Moscow, 129337, Russia

E-mail: asv@mgsu.ru

Abstract. The problem under consideration is connected with a technological problem creating cavities with camouflage explosions. Often these cavities are created in an array of rock salt [1, 2]. In rock salt even under light loads pronounced rheological properties appear, solution of the problem is of interest about creep of salt massif with cavity under the action of both power loads (soil pressure) and temperatures. As a result of the calculation, stress plots for various points in time are obtained, as well as displacements.

Introduction
The study of the mechanical properties of rock salt has been the subject of numerous papers [3, 4] elastic modulus and the creep process of rock salt at normal and elevated temperatures. Since significant creep deformations develop in salt rocks even with a small load, according to the test results it is difficult to determine the proportionality module between stresses and strains. According to recommendations [5], the value $E$ should be determined at the point of the diagram corresponding to $\sigma = 0.4 R_{pr}$, where $R_{pr}$ – prism strength. Fig. 1 shows the experimental points of the dependence of the modulus of elasticity of rock salt on temperature, obtained in the interval $20^\circ C \leq T \leq 100^\circ C$ and approximating curve $E(T)$. This dependence is quite close to linear, but in the temperature range under consideration it can also be described by formula (1) with $E_0 = 77\,200$ MPa and $\delta = 0.0064$.

\[
E(T) = E_0 \exp(-\delta T)
\]  

(1)

Figure 1. The dependence of the modulus of elasticity of rock salt on temperature:

State of the problem
Figure 2 shows the design scheme of the problem. The spherical cavity of radius $a$ is located at a depth $H$. Assuming that the radius of the cut-out array is $b >> a$, and $H >> b$, we can assume that the
pressure of the soil on the external surface of the array is constant and equal \( p = \gamma H \) [6]. Thus, we solve a centrally symmetric problem in which stresses \( \sigma_r \) and \( \sigma_\theta = \sigma_\phi \), as well as strains \( \varepsilon_r \) and \( \varepsilon_\theta = \varepsilon_\phi \) are non-zero.

![Figure 2. Design scheme](image)

To describe the creep process of rock salt at elevated temperatures, one can use the dependence given in [7], which has the following form under uniaxial loading:

\[
\varepsilon^* = A \exp\left(-\frac{Q}{RT}\right) \left|\sigma^*_r\right|^m
\]

where \( Q \) – creep activation energy, \( R \) – characteristic gas constant, \( T \) – temperature, \( K \), \( A \), \( n \) and \( m \) – empirical coefficients. Based on the above experimental creep studies in the range of \( 20^\circ C \leq T \leq 100^\circ C \) the following values of the parameters of relation (2) were obtained:

\[
A = 7.55 \cdot 10^{-8} \frac{1}{(sec)^n (MPa)^m}; n = 2.7; m = 0.328.
\]

For a complex stress state, for creep strains rates, we obtain, taking into account (2), we obtain:

\[
\dot{\varepsilon}_r^* = \frac{3}{2} \frac{\sigma_r - \sigma_{\text{mid}}}{\sigma_r} Am |\sigma_r|^{n-1} \cdot \exp\left(\frac{Q}{RT}\right) \cdot \dot{\varepsilon}_r^*; \quad \dot{\varepsilon}_\theta^* = \frac{3}{2} \frac{\sigma_\theta - \sigma_{\text{mid}}}{\sigma_\theta} Am |\sigma_\theta|^{n-1} \cdot \exp\left(\frac{Q}{RT}\right) \cdot \dot{\varepsilon}_\theta^*
\]

where \( \sigma_i \) – stress intensity.

**Conclusion resolving equations**

The problem is solved in voltages. As the resolving function is stress \( \sigma_r \). The equilibrium equation (in the absence of bulk forces) and the condition of compatibility of deformations in this problem are:

\[
\frac{\partial \sigma_r}{\partial r} + 2 \frac{\sigma_r - \sigma_0}{r} = 0
\]

\[
\frac{\partial \varepsilon_r}{\partial r} + \frac{\varepsilon_r - \varepsilon_0}{r} = 0
\]

In these equations, partial derivatives are used, since all functions depend on the radius and time. The Cauchy relations in spherical coordinates are represented by the equalities:

\[
\varepsilon_r = \frac{\partial u}{\partial r} \; , \; \varepsilon_\theta = \frac{1}{r} \frac{\partial u}{\partial \theta}
\]

Total deformations are represented by the sum of the three components:

\[
\varepsilon_r = \varepsilon_r^0 + \varepsilon_r^* + \alpha T \; , \; \varepsilon_\theta = \varepsilon_\theta^0 + \varepsilon_\theta^* + \alpha T
\]
We assume that the linear thermal expansion coefficient $\alpha$ is constant, $\varepsilon_r^0$, $\varepsilon_\theta^0$ are the elastic deformations for which Hooke's law is valid:

$$\varepsilon_r^0 = \frac{1}{E} \left( \sigma_r - 2\nu \sigma_\theta \right) \quad \varepsilon_\theta^0 = \frac{1}{E} \left[ \sigma_\theta \left( 1 - \nu \right) - \nu \sigma_r \right]$$

(8)

$\varepsilon_r^*$ and $\varepsilon_\theta^*$ - creep deformations for which equality (2) is true.

Entering expression (7) for the components of the total strain into the compatibility equation (5), using Hooke's law (8) and eliminating the stress $\sigma_\theta$ with the help of the static equation (4), we obtain the finally resolving equation for $\sigma_r$:

$$\frac{\partial^2 \sigma_r}{\partial r^2} + \varphi(r,t) \frac{\partial \sigma_r}{\partial r} + \psi(r,t) \sigma_r = f(r,t)$$

(9)

where

$$\varphi(r,t) = \frac{4}{r} - \frac{1}{E} \frac{\partial E}{\partial r} \quad \psi(r,t) = -\frac{1}{r} \cdot \frac{2(1-2\nu)}{(1-\nu)} \frac{\partial \sigma_r}{\partial r} \quad \frac{f(r,t)}{r} = -\frac{2E}{r(1-\nu)} \left( \alpha \frac{\partial T}{\partial r} + \frac{\partial \sigma_\theta^0}{\partial r} + \varepsilon_\theta^* \right)$$

(10)

The boundary and initial conditions of the problem are presented in the form:

$$r = a, \quad b \gg a, \quad \sigma_r = -p_a, p_b, \quad t = 0, \varepsilon_j = 0, (j = r, \theta, \varphi)$$

(11)

Here $p_a$ is the possible internal pressure in the cavity, $p_b = \gamma H$ is the pressure of the medium under the assumption $H \gg b$.

Solution Method

The resolving equation (9) is a second-order partial differential equation with variable coefficients. Due to the nonlinearity and complexity of the coefficients, an analytical solution of these equations cannot be found even with substantial simplifications.

If we assume that the temperature field and power loads slowly change in time, then the creep problem can be considered as quasistationary. One of the first papers in which the “layered” method of solving quasistationary creep problems was proposed was [8]$.^1$ The essence of the “layer-by-layer integration” method is as follows. At the zero stage (at $t = 0$), the problem of determining the temperature field is first solved and the dependences of the elastic and relaxation parameters of the material on temperature and coordinates are found.

If the loading is considered instantaneous, then at the time $t = 0$ the initial condition (11) will be valid. Thus, at the zero stage we arrive at an elastic problem. In this case, equation (10) becomes an ordinary differential equation, which, with the boundary conditions (11), is a two-point boundary value problem, which, because of the complexity of the coefficients of the equation, must be solved numerically. One of the effective methods for solving such boundary-value problems is the sweep method [10 and others].

Having determined all necessary quantities at the zero stage (displacements, deformations and stresses), from equations (3) one can find the creep strain rates $\left( \frac{\partial \varepsilon_r^*}{\partial t} \right)_0$, $\left( \frac{\partial \varepsilon_\theta^*}{\partial t} \right)_0$. Assuming that the time step $\Delta t$ can be arbitrarily small, one can make a linear approximation in time and calculate the creep deformations on the following “time layer” $t = \Delta t$:

1 Such methods for solving creep problems are also called stepwise, incremental, etc.
By numerical differentiation, the derivatives of creep deformations along the radius that are included in the right-hand side of equation (9) are also found. Having thus formed the right side of equation (9), we again arrive at an elastic problem with a new function \( f(r) \) and, in the general case, with new coefficients \( \varphi(r) \) and \( \psi(r) \). Solving this problem numerically, we obtain the solution at the first stage. Continuing the process to an arbitrary point in time, one can determine stresses, strains and displacements at any point of the body.

**Results**

Below are the results of the calculation obtained by the described method with a variable step in time and radius with the following initial data: \( a = 2 \) m; \( b = 10 \) m; \( p_b = 21.5 \) MPa; \( \nu = 0.275; \) \( T_a = 100^\circ \text{C}; \) \( T_b = 20^\circ \text{C}. \)

Fig. 3 shows the stress diagrams for different points in time. It should be noted that in the process of creep, the stresses near the contour of the cavity are significantly reduced, and increase as the distance from the cavity increases, which is due to the need to perform the integral equilibrium equation

\[
\int_{a}^{b} \sigma_\theta r \, dr = \frac{1}{2} p_b b^2 \varphi
\]

**Figure 3.** Redistribution in time stresses \( \sigma_\theta \) in the array with a spherical cavity:

- - - homogeneous array \((T = 20^\circ \text{C})\); ——— inhomogeneous array \((T_a = 20^\circ \text{C}, T_b = 20^\circ \text{C})\)

The latter formula can be obtained from consideration of an element of an array with a spherical cavity (Fig. 4). We make a projection of all forces acting on a given element onto the axis \( 0z \). The integral of the stress \( \sigma_\theta \) is

\[
P_z (\sigma_\theta) = \int_{a}^{b} \sigma_\theta \, dF_1 = \int_{a}^{b} \sigma_\theta \, d\varphi \, dr
\]
The scheme of checking the balance of an array with a spherical cavity.

The projection on the axis $z$ of the integral force from the pressure $p_b$ can be determined by the formula

$$P_z(p_b) = -\int_0^{\pi/2} p_b \cos \theta \, dF = -\int_0^{\pi/2} p_b \cos \theta \cdot b \sin \theta \, d\phi \, d\theta$$

Calculating the last integral, summing expressions (13) and (14) and reducing everything by $d\phi$, we arrive at equality (12). It should be noted that a noticeable increase in stresses $\sigma_\theta$ in the process of creep near the outer contour of the cut out array ($r = b$) is due to the conventionality of the model, i.e. the final value of $b$ (in the calculation was taken $b = 5a$). At $b \to \infty$, the influence of the hole and inhomogeneity does not affect the stress values at a large distance from the hole.

Figure 5 shows diagrams of displacements in an array with a cavity for some points in time. In this case, the actual displacements should be counted from the straight line (the dotted line is shown), corresponding to the compression of the elastic mass without a cavity. In the case of creep, as in the theory of plasticity, the hypothesis of incompressibility of the material is accepted, and therefore a further reduction in the volume of the solid massif as compared with the elastic work stage does not occur.

Elastic displacements of an array without cavity are defined as follows:

$$u = r \cdot \varepsilon_0 = \frac{r}{E} \left[ \sigma_0 - \nu (\sigma_\theta + \sigma_r) \right]$$

Given that in the presence of central symmetry
\[ \sigma_r = \sigma_\theta = \sigma_\phi = -p_b \]

we come to linear dependence:

\[ u = -\frac{(1-2v)p_b r}{E} \]

Note that the calculation of displacements in the problem under consideration is highly relevant, since in the creep process the cavity is swimming, which can lead to a significant decrease in its volume, which allows us to predict.

Note that the calculation of displacements in the problem under consideration is highly relevant, since in the creeping process the cavity is swimming, which can lead to a significant decrease in its volume. Carrying out appropriate calculations allows to predict the service life of created objects.

Summary

The problems of the theory of creep of inhomogeneous bodies represent a rather complex class of problems related to nonlinearity, as well as the need to conduct experimental studies to determine the physical parameters of the material. The results obtained in the article make it possible not only to analyze the stress state of a salt array with a cavity, but also to predict the life cycle of the structure. Similar studies are reflected in the above-mentioned work [8], as well as in recently published papers [10, 11], in which the creep process for metals and polymers is considered.

References

[1] Myasnikov K V, Leonov E A, Romadin N M 1974 Development of the scientific and technical basis for the creation of underground repositories using nuclear explosions in a rock salt massif (Peaceful Nuclear Explosions) III 179-191.
[2] Rawson D, Randolf P, Boardman C, Wheeler V 1966 Post explosion environment resulting from the Salmon event (22.X.1964) (J. of Geophysical Research) 71 14 3415-3426.
[3] Erzhanov Zh S, Bergman E I 1978 Salt Rock Creep (Science, Alma-Ata).
[4] Konstantinova S A, Spirkov V L, Kartashov Yu M 1979 Creep of rock salt samples under uniaxial compression (FTPrpi) 5 43-46.
[5] Proskuryakov N M 1969 Physicomechanical properties of salt rocks (Nedra, Moscow).
[6] Andreev V I 2015 Mechanics of inhomogeneous bodies (Jurait, Moscow).
[7] Boyle J, Spence J 1986 Analysis of stress in creep structures (Moscow).
[8] Andreev V I 1968 On the stability of polymer rods in creep (Mechanics of polymers) 1 22-28.
[9] Demidovich B P, Maron I A, Shuvalova E Z 2006 Numerical Analysis Methods (Lan, Moscow, St. Petersburg).
[10] Andreev V I, Chepurnenko A S, Yazyeva S B 2016 Calculation of creep of circular cylindrical shell by bending theory (Procedia Engineering) 165 1141–1146.
[11] Andreev V, Urumov G 2018 Modeling of Plasticity and Creep under Variable Stresses (MATEC Web of Conferences) 251 04004.