Semi-infinite throats at finite temperature and static solutions in exactly solvable models of 2d dilaton gravity

O. B. Zaslavskii

Department of Physics, Kharkov State University, Svobody Sq.4, Kharkov
310077, Ukraine
E-mail: aptm@kharkov.ua

Found is a general form of static solutions in exactly solvable models of 2d dilaton gravity at finite temperature. We reveal a possibility for the existence of everywhere regular solutions including black holes, semi-infinite throats and star-like configurations. In particular, we consider the Bose-Parker-Peleg (BPP) model which possesses a semi-infinite throat and analyze it at finite temperature. We also suggest generalization of the BPP model in which the appearance of semi-infinite throat has a generic character and does not need special fine tuning between parameters of the solution.

PACS number(s): 04.60.Kz, 04.70.Dy

I. INTRODUCTION

Hawking’s discovery of black hole evaporation posed the question about the final fate of a black hole. This issue is far from being fully understood but exploiting two-dimensional (2d) models gained insight into this intriguing puzzle. In particular, Bose, Parker and Peleg (BPP) suggested the exactly solvable modification of the Callan-Giddings-Harvey-Strominger (CGHS) model [1] in which a black hole evaporates leaving, as an end state, a regular static semi-infinite throat [2]. The existence of such a type of solution motivates an interest to finding regular static solutions in 2d dilaton gravity in a more general setting.

Apart from the possible role of such solutions as ”remnants” after the evaporation of black hole, this task is also motivated by the necessity to elucidate the structure of exactly solvable solutions (of any types) in dilaton gravity. In the paper [3] the approach was
suggested which was based on a treatment of non linear \( \sigma \) model. A more simple approach appealing directly to the properties of the action coefficient as functions of a dilaton field was proposed in [4] where it was also shown that the general structure of found exactly solvable models encompasses all known particular ones. The paper [4] deals only with black holes with a regular horizon. Correspondingly, the temperature is put equal to the Hawking one. In the present paper we consider another types of a metric and find the general form of static solutions in exactly solvable models at an arbitrary temperature.

We carry out the analysis of the obtained general solutions for the BPP model and generalizations of it which, as well as BPP ones, do not contain singularities at finite values of a dilaton field. It turns out that in these models there exists a rather rich family of solutions regular everywhere. In spite of the term "throat" in 1+1 dimensional world is somewhat conditional, we will use it, following [2], to denote any regular geodesically complete geometry extending to infinity (which for definiteness is supposed to be a left one) where its curvature is nonzero. If a metric is flat there, we will speak about a soliton-like configuration. It is supposed that at the right infinity a spacetime is flat in any case. We will see that, depending on temperature, the found family of exact solutions contains regular spacetimes of all three possible types - black holes, throats or soliton-like configurations.

II. GENERAL FORM OF STATIC SOLUTIONS

Consider the action

\[ I = I_0 + I_{PL}, \]  

where

\[ I_0 = \frac{1}{2\pi} \int_M d^2x \sqrt{-g} [F(\phi)R + V(\phi)(\nabla\phi)^2 + U(\phi)] \] (2)

and the Polyakov-Liouville action [6] incorporating effects of Hawking radiation and its back reaction on the black hole metric can be written as
\[ I_{PL} = -\frac{\kappa}{2\pi} \int_M d^2x \sqrt{-g} \left[ \frac{(\nabla \psi)^2}{2} + \psi R \right] \] (3)

The function \( \psi \) obeys the equation

\[ \Box \psi = R \] (4)

where \( \Box = \nabla_\mu \nabla^\mu \), \( \kappa = N/24 \) is the quantum coupling parameter, \( N \) is number of scalar massless fields, \( R \) is a Riemann curvature. We omit the boundary terms in the action as we are interested only in field equations and their solutions.

Varying the action with respect to a metric gives us \( (T_{\mu\nu} = 2\frac{\delta I}{\delta g_{\mu\nu}}) \):

\[ T_{\mu\nu} \equiv T_{\mu\nu}^{(0)} + T_{\mu\nu}^{(PL)} = 0 \] (5)

where

\[ T_{\mu\nu}^{(0)} = \frac{1}{2\pi} \left\{ 2(g_{\mu\nu} \Box F - \nabla_\mu \nabla_\nu F) - U g_{\mu\nu} + 2V \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} V (\nabla \phi)^2 \right\}, \] (6)

\[ T_{\mu\nu}^{(PL)} = -\frac{\kappa}{2\pi} \left\{ \nabla_\mu \psi \nabla_\nu \psi - 2\nabla_\mu \nabla_\nu \psi + g_{\mu\nu} \left[ 2R - \frac{1}{2} (\nabla \psi)^2 \right] \right\} \] (7)

Variation of the action with respect to \( \phi \) gives rise to the equation

\[ R \frac{dF}{d\phi} + \frac{dU}{d\phi} = 2V \Box \phi + \frac{dV}{d\phi} (\nabla \phi)^2 \] (8)

Hereafter we assume that a dilaton field is not constant identically (such special kinds of solutions are considered in [7], [8]). Then, as is shown in [3], [4], we can gain a rich set of exactly solvable models if the action coefficients in (2) \( F(\phi), V(\phi) \) and \( U(\phi) \equiv 4\lambda^2 c F_0^\phi \omega(\phi') d\phi' \) are restricted by one constraint equation. This equation can be written as

\[ V = \omega (u - \frac{\kappa \omega}{2}) \] (9)

where \( u \equiv \frac{dF}{d\phi} \). (In fact, eq. (3) admits further generalization due to a possible term \( A(u - \kappa \omega)^2 \) but, as it leads, generally speaking, to metrics which are not asymptotically flat, hereafter we put \( A = 0 \).) Then it turns out that the metric is a pure static. In the present paper we will use the conformal gauge
\[ ds^2 = g(-dt^2 + d\sigma^2) \]  

where \( g = g(\sigma) \). In this gauge the curvature \( R = -g^{-1}(g'/g)' \). Throughout the paper the prime denotes differentiation with respect to a spatial coordinate. Substituting it into (10) we get after simple manipulations

\[ g = e^{-\psi - a\sigma} \]  

where \( a \) is a constant. It is implied that time is normalized in such a way that in (11) the function \( g \to 1 \) at \( \sigma \to \infty \).

After simple rearrangement the (00) and (11) field equations (5) with the metric (11) are reduced to one equation

\[ \xi_1 \phi'' + \xi_2 \phi'^2 - \xi_1 \frac{g'\phi'}{g} = 0 \]  

where \( \xi_1 = \frac{d\tilde{F}}{d\phi}, \xi_2 = \frac{d^2\tilde{F}}{d\phi^2} - \tilde{V}, \tilde{F} = F - \kappa\psi, \tilde{V} = V - \frac{\kappa}{2} \left( \frac{d\psi}{d\phi} \right)^2. \)

In [4] we found that for exactly solvable models (4) the function \( \psi \) is equal to \( \psi = \psi_0(\phi) \) where

\[ \psi_0 = \int \omega d\phi \]  

For a given metric this function is defined up to an arbitrary function \( \chi \) such that \( \Box \chi = 0 \) as it follows from (11). For a static metric (12) this gives us (up to the constant) \( \chi = \gamma \sigma \) where \( \gamma \) is a constant. We will see below that for \( \gamma = 0 \) we return to black hole solutions obtained in [4] whereas for \( \gamma \neq 0 \) we find new types of static solutions which include, in particular, finite temperature generalizations of "semi-infinite throats" [2]. In what follows we will deal with solutions of the form

\[ \psi = \psi_0 + \gamma \sigma \]  

with an arbitrary \( \gamma \neq 0 \). At the right infinity where spacetime is supposed to be flat the function \( \psi \sim \sigma \to \infty \) for a generic \( g \) [4] irrespectively of whether or not \( \gamma = 0 \). Meanwhile, at the left coordinate infinity \( \sigma \to -\infty \) (a reader should bear in mind that if a spacetime is not
geodesically complete, the coordinate $\sigma$ does not cover the whole manifold) the admittance of $\gamma \neq 0$ can qualitatively change the character of solution. For instance, instead of a black hole for which at a horizon $g \to 0$ and $\psi$ is finite \cite{7} one can get at a left infinity a ”semi-infinite throat” \cite{2} for which both $g$ and $\psi$ diverge there to ensure, by definition of the ”semi-infinite throat”, the divergencies of a proper distance at a finite value of a coordinate $\sigma$ (see below for details).

The structure of the action coefficients in our case is the same as in \cite{4}. Namely, the relationship between them is represented by eq. (9). The new feature introduced in our treatment as compared to \cite{4} is connected with another character of boundary conditions for the function $\psi$. Whereas in \cite{7}, \cite{4} one deals with a black hole in the Hartle-Hawking state for which $\psi = \psi_0$ is regular on a horizon, for solutions discussed below $\psi$ may diverge at $\sigma \to -\infty$ due to the term $\gamma \sigma$ in (14).

It is convenient to split coefficients in eq. (12) into two parts singling out the term which is built up with the help of $\psi_0$: $\xi_1 = \xi_1^{(0)} - \kappa \gamma \frac{d \sigma}{d \phi}$, $\xi_2 = \xi_2^{(0)} - \kappa \gamma \frac{d^2 \sigma}{d \phi^2} + \kappa \left[ \frac{d \psi_0}{d \phi} \gamma \frac{d \sigma}{d \phi} + \frac{1}{2} (\gamma \frac{d \sigma}{d \phi})^2 \right]$, $\xi_1^{(0)} = \frac{d \tilde{F}(0)}{d \phi}$, $\xi_2^{(0)} = \frac{d^2 \tilde{F}(0)}{d \phi^2} - \tilde{V}(0)$, $\tilde{F}(0) = F - \kappa \psi_0$, $\tilde{V}(0) = V - \frac{\kappa}{2} \left( \frac{d \psi_0}{d \phi} \right)^2$. (15)

Then eq. (12) takes the form

$$\xi_1^{(0)} \phi'' + \xi_2^{(0)} \phi'^2 + \xi_1^{(0)} \phi' (\psi^{(0)'} + \delta) = \kappa \gamma (\delta - \frac{\gamma}{2})$$

(16)

where $\delta = \gamma + a$. Let us multiply this equation by the factor $\eta$ such that $\xi_2^{(0)} \eta = \frac{d \xi_1^{(0)} \eta}{d \phi}$. Then eq. (12) can be cast into the form

$$z' + z(\psi^{(0)'} + \delta) = \kappa \gamma (\delta - \frac{\gamma}{2}) \eta$$

(17)

where $z = \eta \xi_1^{(0)} \phi' = \eta \frac{d \tilde{\psi}(0)}{d \sigma}$. It follows from (3), (13) and (15) that $\eta = e^{-\psi_0}$. Then after integration we get from (17)

$$\tilde{F}(0) = C + D e^{-\sigma \delta} + \kappa \gamma (1 - \frac{\gamma}{2 \delta}) \sigma$$

(18)

It follows from the trace of eq. (3), (7) that
As for any function \( f(\sigma) \) we have in the metric (11) \( \Box f = g^{-1} f'' \), we get from (19), (11) and (13) the relationship between constants \( D \delta^2 = 4 \lambda^2 \). In what follows we assume that the function \( \tilde{F} \) as well as \( F \) has the asymptotic behavior \( \tilde{F} \sim e^{\omega_0 \phi} \) (\( \omega_0 = \text{const} \)) at the right infinity \( \sigma \to \infty \) where \( \phi \to -\infty \), \( \psi_0 \sim \omega_0 \phi \) and the metric is flat, \( g \sim e^{-\omega_0 \phi - \delta \sigma} \to 1 \). This asymptotic condition is satisfied, in particular, by CGHS [1], RST [9] and BPP models with \( \omega_0 = -2 \). Thus, we get \( D = 1, \delta = -2 \lambda \) (it is assumed for definiteness that \( \omega_0 < 0 \)) and the relationship between a dilaton field and coordinate reads \( \tilde{F}(0) = f(y) \equiv e^{2y} - By + C \) where \( y \equiv \lambda \sigma, B = -\kappa^2 (1 + \frac{\gamma}{\lambda}) \). The value of \( \gamma \) can be found from asymptotical conditions at right infinity where spacetime is supposed to be flat. Let us impose such a condition which describes quantum fields at finite temperature \( T \). It means that at right infinity the quantum stress-energy tensor should have the form

\[
T^{\mu(P/L)} = \frac{\pi^2 N}{6} T^2 (1, -1)
\]  

(20a)

Comparing it with the asymptotic form of eq. (7), we find that \( T^{\mu(P/L)} = \frac{\pi}{4\pi} \psi'_\infty^2 (1, -1) \) where \( \psi'_\infty = \lim_{\sigma \to \infty} \frac{d\psi}{d\sigma} \). Hence, \( \psi'_\infty^2 = \frac{2\pi^2 T^2 N}{3\kappa} = 16\pi^2 T^2 \). Remembering that for exactly solvable models under consideration the Hawking temperature is equal to \( T_0 = \lambda/2\pi \) [4] we obtain \( \psi'_\infty^2 = 4\lambda^2 T^2 / T_0^2 \). On the other hand, we have from (11) and the condition \( g \to 1 \) at infinity that \( \psi'_\infty = -a \). Then \( \gamma = \delta - a = -2\lambda + \psi'_\infty = 2\lambda (T/T_0 - 1) \) whence \( B = \kappa (1 - T^2/T_0^2) \) (the sign of \( \psi'_\infty \) is chosen to ensure \( \gamma = 0 \) for \( T = T_0 \) when \( \psi = \psi_0 \) and we return to the situation described in [4]). Collecting all basic formulas, we obtain

\[
ds^2 = g(-dt^2 + d\sigma^2), \quad g = \exp(-\psi_0 + 2y), \quad y = \lambda \sigma, \quad \psi_0 = \int \omega d\phi.
\]

(21)

\[
\tilde{F}(0)(\phi) = f(y) \equiv e^{2y} - By + C, \quad B = \kappa (1 - T^2/T_0^2), \quad T_0 = \lambda/2\pi.
\]

If \( T > T_0 \), the coefficient \( B > 0 \) and the function takes its minimum at the point \( y_0 = \frac{1}{2} \ln B \) where \( f(y_0) = C - C^* \),

\[
C^* = -\frac{B}{2} (1 + \ln \frac{2}{B})
\]

(22)
Let $T = T_0$, then the coefficients $\gamma = B = 0$ whence we obtain from (21) $g = \exp(-\psi_0)(\tilde{F}(0) - C)$. If we identify $C = \tilde{F}(\phi_h)$ we obtain black hole solutions with the horizon at $\phi = \phi_h$ analyzed in [4]. Then the equality of temperatures simply means that the state of the Euclidean metric we deal with is the Hartle-Hawking one. To make the comparison more clear, let us pass from the conformal gauge (11) to the Schwarzschild one $ds^2 = -gdt^2 + g^{-1}dx^2$ used in [4]. This can be achieved by the coordinate transformation $x = \int d\sigma g$. It follows from (21) that $x = (2\lambda)^{-1} \int d\phi e^{-\psi F'}$ that agrees with eq. (27) of [4].

It is worth stressing, however, that the class of solutions (21) is much more general than that found in [4]. It includes not only black holes due to the parameter $B$ which describes a system at arbitrary temperature when black holes with a regular horizon cannot exist. In what follows we will use the notation $T_0 = \lambda/2\pi$ even in cases when $T_0$ does not have the meaning of the Hawking temperature. Equations (21) represent in the closed form the general expression describing static solutions in exactly solvable models of 2d gravity [3], [4]. The function in the right hand side of eq. (21) has the universal form while a particular model is characterized by the choice of $\tilde{F}^{(0)}(\phi)$. It is the analysis of this class of solutions that we now turn to. As we are mainly interested in finding regular solutions, we discuss the BPP model and its generalizations.

### III. BPP MODEL AT FINITE TEMPERATURE

This model is specified by the choice

$$F = e^{-2\phi} - 2\kappa\phi, \quad \tilde{F} = e^{-2\tilde{\phi}}, \quad \omega = -2, \quad \psi_0 = -2\phi, \quad V = 4e^{-2\phi} + 2\kappa \quad (23)$$

If $T = T_0$ we have $B = 0$ and $e^{2y} = e^{-2\phi} - e^{-2\phi_h}$, $g = 1 - e^{2(\phi - \phi_h)}$. As seen from the above formulas, the expression for the metric (either in terms of $\phi$ or in terms of a coordinate) has the same form as in the classical limit ($\kappa = 0$), so in this sense the metric does not acquire quantum corrections [4]. In the paper [4] the authors considered the case $T = 0$. Then asymptotically the energy density of quantum field approaches zero (radiationless solutions).
In general, spacetime contains a time-like singularity or a singular horizon depending on the value of $C$ that determines whether or not the function $f(y)$ has a zero. The special case arises when simultaneously $f(y) = 0 = f'(y)$ at $y = y_0$, $C = C^* = -\frac{\kappa}{2}(1 + \ln \frac{2}{\kappa})$ (a reader should have in mind that we define the quantum parameter $\kappa = N/24$ whereas in $\kappa = N/12$). Then the spacetime does not contain singularities at all extending from the left infinity where $R = -4\lambda^2$ (semi-infinite throat) to the right one where $R = 0$.

In fact, the analysis is tractable to the case of an arbitrary $T$ due to the universality and simplicity of the function $f(y)$ with an arbitrary $B$. Let, first, $T < T_0$, so $B > 0$. For $C < C^*$ there is a time-like naked singularity ($y = y_0$) at a finite proper distance. If $C > C^*$, there exists a singular horizon at $\phi = \infty$. The throat exists provided $C = C^*$ where $C^*$, according to (22), depends on temperature via $B(T)$. The calculation of curvature shows that at the throat $R = -4\lambda^2$ independently of temperature. In the limit $T = 0$ $B = \kappa$ and we return to radiationless solutions described in [3].

If $T > T_0$, $B < 0$ the function $f(y)$ is monotonic and does not turn into a zero. Then for any $C$ there is a time-like singularity at $\phi = \infty$.

In the point $T = T_0$ the coefficients $B = 0 = C^*$. If $C > 0$ we have a black hole in the Hartle-Hawking state with a regular horizon. If $C < 0$ there exists a time-like singularity. For $C = 0$ the spacetime is flat (linear dilaton vacuum $\phi = -y$). The situation can be summarized as follows.

| BPP model | $T < T_0$ | $T = T_0$, $B = 0 = C^*$ | $T > T_0$ |
|-----------|-----------|--------------------------|-----------|
| $C < C^*$ | time-like singularity | time-like singularity | time-like |
| $C = C^*$ | throat | linear dilaton vacuum, flat spacetime | singularity |
| $C > C^*$ | singular horizon | space-like singularity behind a horizon for any $C$ |

The main feature of the model in question is the fact that the semi-infinite throat may exist at any finite temperature $T < T_0$. Meanwhile, this needs the special choice $C = C^*(T)$. It turns out, however, that there are another exactly solvable models for which the existence of a throat is generic and does need fine tuning in constants.
IV. GENERALIZATIONS OF BPP MODEL

Consider the model

\[ F^{(0)} = \alpha e^{-2b\phi} + e^{-2\phi} - 2\kappa\phi, \quad \alpha > 0, \ 0 < b \leq 1/2, \ \omega = -2, \ V = 4(e^{-2\phi} + b\alpha e^{-\phi}) + 2\kappa, \quad (24) \]

\[ \tilde{F}^{(0)} = \alpha e^{-2b\phi} + e^{-2\phi} \]

which is, according to (9), exactly solvable and the Hawking temperature for black hole solutions is equal to \( T_0 \) \([4]\). Let first \( B > 0 \) (\( T < T_0 \)). Then near the point \( y_0 \) where \( f(y_0) = 0 \) we have for any \( C < C^* \) \( f \sim y - y_0, \ \tilde{F}^{(0)} \sim e^{-2b\phi} \rightarrow 0, \ \phi \rightarrow \infty \), so \( e^{2\phi} \sim (y - y_0)^{-1/b} \) and, according to (21), \( g = e^{2\phi + 2y} \sim (y - y_0)^{-1/b} \). Then the curvature \( R \sim (y - y_0)^{1/b - 2} \). If \( b < 1/2 \), the curvature \( R = 0 \) at left infinity, the proper distance \( l \) between \( y_0 \) and any other point \( y > y_0 \) diverges like \( l \sim (y - y_0)^{1-1/2b} \). Thus, we obtain a soliton-like configuration.

If \( C = C^* \), \( f \sim (y - y_0)^2, \ g \sim (y - y_0)^{-2/b}, \ R \sim (y - y_0)^{2/b - 2} \rightarrow 0 \), so the configuration is again soliton-like for any \( 0 < b < 1 \). For \( C > C^* \) spacetime cannot be regular everywhere, it contains the singular horizon where \( y \rightarrow -\infty, \ g \sim e^{2y |y|^{-1/b}}, \ R \sim -e^{-2y |y|^{1/b - 2}} \), a proper distance \( l \) is finite.

The rest of cases can be treated in a similar manner. We only dwell on the fact that for \( T = T_0 \) a spacetime represent a black hole whose curvature is finite everywhere. In particular, at left infinity where \( \phi \rightarrow \infty \) the curvature \( R \rightarrow 0 \), so spacetime is asymptotically flat at both infinities. The variety of cases is tabulated as follows.

| \( \tilde{F}^{(0)} = e^{-2\phi} + \alpha e^{-2b\phi} \), \( 0 < b < 1/2 \) | \( T < T_0 \) | \( T = T_0, \ B = 0 = C^* \) | \( T > T_0 \) |
|---|---|---|
| \( C < C^* \) | soliton-like | soliton-like | soliton-like |
| \( C = C^* \) | soliton-like for \( 0 < b < 1 \) | soliton-like | for |
| \( C > C^* \) | singular horizon | black hole regular everywhere | any \( C \) |

Let now \( b = 1/2 \). The results of consideration, details of which we omit, can be summarized in the table below.

\[ \tilde{F}^{(0)} = e^{-2\phi} + \alpha e^{-\phi} \]
The most interesting feature of this model is the existence of a throat at any temperature (for $C < C^*$, if $T \leq T_0$ and for an arbitrary $C$ if $T > T_0$).

### V. DISCUSSION AND CONCLUSION

We have found a general form of static solutions in 2d exactly solvable dilaton gravity theories accounting for quantum effects at finite temperature. The criteria of solvability are the same as derived in [3], [4] for eternal black holes and represent one constraint on the action coefficient (9). Thus, every model with black hole solutions considered in [4] generates a family of solutions of another kind which generalize [4] and reduce to it in the particular case $T = T_0$ when the solution describes a black hole with a Hawking temperature $T_0$ in thermal equilibrium with its Hawking radiation. It was indicated in [4] that exactly solvable models in question exhibits a series of universal properties. In particular, it is remarkable that a Hawking temperature is constant for all such models, $T_0 = \lambda/2\pi$. The present consideration extended this universality to a much more wide class solution and enabled us to classified them in an unified manner due to the universal structure of (21) independently of the choice of the model. In fact, the particular properties of the model are encoded in the only function $\tilde{F}^{(0)}(\phi)$. If $\tilde{F}^{(0)}(\phi)$ does not have zeros at finite $\phi$, as it takes place for the BPP model [2] and its generalization considered in the present paper, the only relevant information is contained in the asymptotic behavior of $\tilde{F}^{(0)}$ at $\phi \to \infty$.

We analyzed two concrete examples and found that there exist three types of solutions regular everywhere. First, it is a black hole, if $T = T_0$ (the possibility of solutions of such a kind was pointed out in [4]). Second, this a semi-infinite throat generalizing the observation made in [2]. The most interesting feature of throats found in the model (24)
with \( b = 1/2 \) is that such types of solutions, unlike the BPP case, do not need a special choice of parameters of the solution: semi-infinite throats may exist in the whole range of parameters and temperatures. Third case is a soliton-like configuration extending in both directions to flat infinities without horizons. Such a type of solution is absent in the BPP model but may occur in the generalizations of it.

The conditions which make the existence of semi-infinite throats possible are essentially quantum. Indeed, as seen from (21), such a kind of configuration arises only if the function \( f(y) \) is not monotonic due to the coefficient \( B \) and takes its minimum value at some finite \( y_0 = \frac{1}{2} \ln B \). However, in the classical limit \( \kappa \to 0 \) the coefficient \( B \to 0 \) and we get \( g = e^{-\psi_0}(F^{(0)} - C) \). Choosing \( C = F(\phi_h) \) we can get a classical counterpart of a quantum black hole but not a semi-infinite throat. For example, with \( \omega = -2 \), \( F^{(0)} = e^{-2\phi} \) we have the familiar expression for a static dilaton black hole \( g = 1 - Ce^{2\phi} \) where \( C/2 \) plays the role of a classical ADM mass.

In the present paper we determined possible types of static solutions but did not consider dynamical scenarios in the process of which they could arise. Here we only restrict ourselves by a remark that finite temperature solutions allow us to consider more complicated processes of interaction between a black hole and its quantum surrounding than a evaporation into vacuum. In particular, this enables us to include into consideration directly the role of thermal bath [10]. Another interested problem for further researches is looking for possibilities to gain new exactly solvable models not only due to relaxing the regularity condition for the function \( \psi \) at the left infinity (as was made in the present paper) but also to the generalization of the action structure itself. All this needs a special treatment.

I am grateful to Sergey Solodukhin for careful reading the manuscript and valuable comments. This work is supported by International Science Education Program (ISEP), grant No. QSU082068.
[1] C. G. Callan, S. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D 45 (1992) R1005.

[2] S. Bose, L. Parker, and Y. Peleg, Phys. Rev. D 52 (1995) 3512.

[3] Y. Kazama, Y. Satoh, and A. Tsuichiya, Phys. Rev. D 51 (1995) 4265.

[4] O. B. Zaslavskii, Exactly solvable models of two-dimensional dilaton gravity and quantum eternal black holes, hep-th/9804089 (Phys. Rev. D, in press).

[5] S. N. Solodukhin, Phys. Rev. D 51 (1995) 609.

[6] A. M. Polyakov, Phys. Lett. B 103 (1981) 207.

[7] S. N. Solodukhin, Phys. Rev. D 53 (1996) 824 .

[8] O. B. Zaslavskii, Phys. Lett. B 424 (1998) 271 [hep-th/9802117].

[9] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. D 46 (1992) 3444; Phys. Rev. D 47 (1992) 533.

[10] J. Cruz and J. Navarro-Salas, Phys. Lett. B 387 (1996) 51.