Suppression of effects of Doppler shifts of multipath signals in underwater acoustic communication

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Abstract: In underwater acoustic communication, the Doppler shift can severely degrade demodulation performance. In addition, multipath signals can have the Doppler shifts that are different from that of a direct signal. Although conventional signal processing in underwater acoustic communication can deal with multipath signals with the Doppler shifts equal to that of the direct signal, they cannot sufficiently deal with multipath signals with different Doppler shifts. In this paper, we provide a mathematical description of how multipath signals with the different Doppler shifts degrade demodulation performance. Moreover, signal processing, which suppresses the effects of such multipath signals, has been proposed. In addition, to confirm the improvement due to the proposed processing, simulations of communication between a small surface vessel and an underwater vehicle were carried out in this study. The results show that the proposed processing yields a demodulation performance better than that of conventional processing. Furthermore, we investigated how the proposed processing improved the performance under some motional conditions. Finally, through simulations at various symbol rates, the motional and signal conditions under which the proposed processing can improve the performance efficiently are discussed.

Keywords: Underwater acoustic communication, Decision feedback equalizer, Doppler shift, Multipath signals, Single carrier modulation

1. INTRODUCTION

It is well known that Underwater acoustic communication (UWAC) faces three severe challenges: a slow propagation velocity, a rich multipath environment, and a narrow available bandwidth [1–3].

Owing to the slow propagation velocity, slight motions of the sources and receivers lead to severe Doppler effects. In addition, multipath signals in UWAC have large amplitudes and lags in many cases, making it difficult to deal with the effects caused by multipath signals. Moreover, the physical characteristics of seawater limit the available bandwidth depending on the communication range.

To improve the UWAC in an environment with these challenges, many techniques have been actively pursued for decades. A decision feedback equalizer (DFE) combined with a digital phase-locked loop (DPLL) has been proposed to suppress the effects of temporal instabilities and multipath signals [4,5]. Specifically, the DPLL can suppress the effects of a severe Doppler shift of a direct signal, and a DFE with a DPLL was applied for actual use.

For horizontal UWAC with a rich multipath environment, time-reversal communication has been intensively studied [6–8]. Horizontal UWAC has technical difficulties in most cases because it tends to have a nonminimum phase channel. In the fundamental theory of time reversal, the multiplication of the phase conjugate of a channel response leads to the reproduction of a transmitted signal according to spatial reciprocity. In this process, the signal-to-noise ratio (SNR) is improved by the effective usage of the energies of multipath signals [9]. Recently, applications for multiple-input/multiple-output (MIMO) and multiuser communication with time reversal have been studied [10–14].

Recently, several research results have been utilized for actual use. For an example of high-data-rate UWAC, a data rate faster than 70 kbps has been reported for an actual operation of a manned submersible with a large mothership [15,16].

Meanwhile, the demand for multiuser communication with multiple underwater vehicles and small surface vessels has increased and has been studied recently
In those studies, vertical UWAC was assumed where there is a small number of multipath signals in comparison with horizontal UWAC. Furthermore, it is easier to deal with a channel of vertical UWAC than that of horizontal UWAC because vertical UWAC has a minimum phase channel in most cases. However, small surface vessels roll and pitch with maximum angles larger than 10° in short periods of 2 to 4 s, which causes nonuniform and severe Doppler shifts of both direct and multipath signals [17,19].

In previous studies [18,19], multiuser communication in the frequency division multiple access (FDMA) scheme was investigated. In the FDMA scheme, each user can only use the assigned frequency band. Although there is no interference between users in the FDMA scheme in principle, the FDMA scheme has a disadvantage in that the available bandwidth for each user is narrow. In an environment with nonuniform Doppler shifts, a narrow bandwidth reportedly leads to a degradation of demodulation performance [21]. This is because the bandwidth equals the symbol rate and the updating rate in the estimation of the channel response in the DFE. In cases with a narrow bandwidth, the estimation of a channel response cannot follow the temporal changes in the Doppler shifts of both direct and multipath signals.

To suppress the effects of the nonuniform Doppler shifts of the direct signal, a new process for adjusting the down-converted time was proposed [22]. In the processing, the down-converted time in the digital downconverter is adjusted adaptively in accordance with the phase compensation in the DFE. In a previous study, it was confirmed that processing improves the demodulation performance that is affected by the nonuniform temporal changes of the direct signal. However, it cannot deal with the Doppler shifts of multipath signals because the Doppler shifts of multipath signals differ from those of the direct signal owing to differences in the direction of arrival.

In this paper, under the condition of vertical UWAC with a minimum phase channel, an improved DFE for suppressing the effects of the Doppler shifts of multipath signals is proposed. In this proposal, additional DPLLS are combined with the conventional DFE to compensate for the phase shifts of multipath signals. In addition, the improvement was investigated through simulation. In this study, the demodulation performance is evaluated under many motion conditions because communication between a small surface vessel and an underwater vehicle is assumed.

This paper comprises five sections. In Sect. 2, we describe how the nonuniform Doppler shifts of multipath signals affect demodulation performance. Furthermore, a new processing method to suppress these effects is proposed in this section. Next, the simulation and evaluation index used in this study are described in Sect. 3. In addition, the simulation results and discussion are presented in Sect. 4. Finally, conclusions are presented in Sect. 5.

2. DPLLS FOR MULTIPATH SIGNALS

2.1. Doppler Shifts of Multipath Signals

In this study, a set of impulse responses for direct and other signals received through the corresponding path, including signals reflected at the sea surface and sea bottom, was assumed for simplicity. Each signal with its corresponding impulse response is called a multipath signal in this paper. Furthermore, $m$ is defined as an index to identify a multipath signal. The index $m$ is set as a nonnegative integer, and $m = 0$ is defined as the direct signal. Moreover, the channel response is assumed to be a minimum phase system in this paper. In addition, communication with single-carrier modulation was applied. The signal configuration is depicted in Fig. 1. As shown in Fig. 1, synchronization signals with a common signal length $T_{sync}$ are located at the beginning and end of the signal. The spacing between the synchronization signals is denoted by $T_{sync}$. In addition, the spacing between the synchronization signals of the received signals is denoted by $T_{sync}$.

In this section, the number of multipath signals is assumed to be arbitrary. The transmitted signal is denoted by $s(t) = \Re\{x(t)\exp[i2\pi f_c(t - t_0^s)]\}$, where $x(t)$, $f_c$ and $t_0^s$ denote the baseband signal, a carrier frequency and time when the source begins to transmit the signal, respectively. In addition, the propagation times and amplitudes for the $m$th multipath signals are denoted by $\tau_m(t)$ and $A_m$, respectively, where $A_m$ is assumed to be constant for simplicity. Furthermore, $t_0^s$ represents the time when the beginning part of the direct signal is received. Using $t_0^s$, $t_0^s$ can be rewritten as $t_0^s - \tau_0(t_0^s)$. The relationships among $t_0^s$, $\tau_m(t)$ and $t_0^s$ are shown in Fig. 2.

The received signal $r(t)$ can be written as

$$r(t) = \sum_{m} A_m s(t - \tau_m(t))$$

$$= \sum_{m} A_m \Re\{x(t - \tau_m(t)) \times \exp[i2\pi f_c(t - \tau_m(t) - t_0^s + \tau_0(t_0^s))])\}. \quad (1)$$

Furthermore, the propagation time of the $0$th multipath signal $\tau_0(t)$ can be written as

$$\tau_0(t) = \tau_0(t_0^s) + \beta(t - t_0^s) + \tau_0^d(t). \quad (2)$$

**Fig. 1** Signal configuration.
where $\tau_0(t_0^0)$, $\beta(t - t_0^0)$ and $\tau_0^n(t)$ are constant, linear and nonlinear components, respectively. The relationships between them are depicted in Fig. 3. The variable $\beta$ was calculated as $1 - T_{rx}/T_{tx}$ and follows [21]. Moreover, both $\tau_0^n(t_0^0)$ and $\tau_0^n(T_{rx} + t_0^0)$ are equal to zero, as shown in Fig. 2. Note that the terms $\beta(t - t_0^0)$ and $\tau_0^n(t)$ correspond to the uniform and nonuniform Doppler shifts of the 0th multipath signal, respectively.

Using Eqs. (1) and (2), the time term of the exponent in Eq. (1) is given by

$$t - \tau_m(t) - t_0^m + \tau_0^n(t_0^m) = (1 - \beta)(t - t_0^m) - [\tau_m(t) - \tau_0^n(t)] - \tau_0^n(t). \quad (3)$$

Let us denote a nonlinear component of the phase shift of the direct signal as $\phi_0^n(t) = -2\pi f_c \tau_0^n(t)$. Additionally, the phase shift of the $m$th multipath signal from the phase of the direct signal is denoted by $\phi_m(t) = -2\pi f_c [\tau_m(t) - \tau_0^n(t)]$. With the terms $\phi_0^n(t)$ and $\phi_m(t)$, Eq. (1) can be written as

$$r(t) = \sum_m A_m \text{Re} \{x(t - \tau_m(t)) \times \exp[i2\pi f_c (1 - \beta)(t - t_0^m) + i\phi_m(t) + i\phi_0^n(t)]\}. \quad (4)$$

The term $\phi_m(t)$ represents the difference between the phase shifts of the direct and $m$th multipath signals, and $\phi_0^n(t)$ is the phase shift caused by only the nonuniform Doppler shift of the direct signal.

Next, we focus on demodulation processing. Figure 4 shows a block diagram of the demodulation with a DFE. During demodulation, the received signal $r(t)$ is down-converted by the replica frequency $f_r = (1 - \beta)f_c$. The baseband version of the received signal $y(t)$ is written as

$$y(t) = \sum_m A_m x(t - \tau_m(t)) \times \exp[i\phi_m(t) + i\phi_0^n(t)]. \quad (5)$$

Equation (5) shows that the $m$th multipath signal is affected by the phase shift $\phi_m(t) + \phi_0^n(t)$ in the baseband signal. Note that the 0th multipath signal is affected only by the phase shift of $\phi_0^n(t)$ because $\phi_0^n(t)$ always equals 0 by definition. In addition, in the digitization process in Fig. 4, the signal $y(t)$ is digitized at $t = t_0^n + [T_{sync} + n/f_{bw}]/(1 - \beta)$, where $n$ and $f_{bw}$ denote an integer and the bandwidth, respectively. The digitized form of the signal yielded by the digitization is denoted as $y_d(n)$.

In conventional demodulation processing, the DFE is utilized to suppress these effects. A block diagram of a conventional DFE is shown in Fig. 5. In Fig. 5, the term $y_m(l)$ means the $l$th input of the DFE and is defined as

$$y_m(l) = [y_d(l - N_1), \ldots, y_d(l + N_2)]^T, \quad (6)$$

where $N_1$ and $N_2$ are set as non-negative integers and $N_{ff} = N_1 + N_2 + 1$ means the length of the feedforward filter in Fig. 5. Moreover, $[\ldots]^T$ represents the transpose.

The DFE consists of feedforward and feedback filters and a DPLL. The feedforward filter compensates for the
channel response of the 0th multipath signal, and the DPLL compensates for the phase shift of the 0th multipath signal \( \phi_0(n) \). The feedback filter suppresses the effects of multipath signals. If the phase shifts \( \phi_m(t) \) vary slowly, the taps of the feedback filter can be adaptively updated to follow the temporal change in \( \phi_m(t) \). However, it is difficult for taps to follow fast changes. Therefore, the phase shifts of multipath signals can degrade the demodulation performance when \( \phi_m(t) \) changes rapidly.

### 2.2. DFE Combined with DPLLs for Multipath Signals

As stated in Sect. 2.1, the DPLL in the conventional DFE compensates for the phase shift caused by the Doppler shift of the direct signal. In this paper, an improved DFE that suppresses the effects of the phase shifts of multipath signals is proposed. \([\ldots]^\text{H}\) represents the Hermitian conjugate.

Figure 6 shows a block diagram of the proposed method. In comparison with Fig. 5, additional DPLLs are inserted into the feedback filter in Fig. 6. The additional DPLLs compensate for \( \phi_m(t) \) in Sect. 2.1, which is caused by the phase shift of the multipath signals. In addition, each DPLL is combined with the corresponding tap in the feedback filter and deals with only the phase shift of the tap; therefore, the proposed DFE can simultaneously compensate for phase shifts of multiple multipath signals that the feedback filter can detect.

In this study, taps of the feedforward and feedback filters are denoted by \( a_{ff}^n \) and \( b_{fb}^n \), respectively. Furthermore, the length of the feedforward filter \( N_f \) is set to be a value large enough for the feedforward filter to deal with the 0th multipath signal with varying \( \tau_m(t) \). The length of the feedback filter \( N_b \) is set to be a value large enough for the feedback filter to cover multipath signals other than the 0th multipath signal. First, the output of a set of the feedforward filter and the DPLL can be written as

\[
p(l) = a_{ff}^H y_m(l) \exp[-i\phi_0]. \tag{7}
\]

The input of the feedback filter equals \( \{\hat{d}(l-1), \hat{d}(l-2), \ldots, \hat{d}(l-N_b)\}^T \), where \( \hat{d}(n) \) stands for the \( n \)th decision data symbol. Then, the corresponding additional DPLL, which is represented as \( \exp[-i\phi_k] \) in Fig. 6, is applied to output of each tap of the feedback filter. The summation of the output of the additional DPLLs can be written as

\[
q(l) = \sum_{k=0}^{N_b} b_{fb}^k(\hat{d}(l-k) \exp[-i\phi_k]). \tag{8}
\]

where \( b_{fb}^k \) denotes the \( k \)th tap of the feedback filter. Furthermore, \( \exp[-i\phi_k] \) in Eq. (8) means the additional DPLL that applies to the \( k \)th tap of the feedback filter. In addition, the compensation phases of the additional DPLLs \( \phi_k \) are variables independent of each other; hence, each additional DPLL works independently. As shown in Fig. 6, the estimate of the \( l \)th data symbol in the DFE can be written as

\[
\hat{d}(l) = p(l) - q(l). \tag{9}
\]

The compensation phases of the additional DPLL, \( \phi_k \), are updated using the least mean squares (LMS) algorithm in the same way as the update of the DPLL in the conventional DFE. The derivative of the square error between both cases of LMS and recursive least squares (RLS) algorithms [4,5].
As shown in Figs. 5 and 6, the proposed DFE coincides with the conventional DFE when \( \theta_b \) is fixed at 0. This means that the proposed DFE is an extended version of the DFE in order to deal with the Doppler shifts of multipath signals.

3. MODELS OF SIMULATION

3.1. Propagation Simulation

In this study, simulations were carried out to investigate the improvement of the demodulation performance by the proposed DFE combined with the additional DPLLs, as stated in Sect. 2.2.

In the simulations, communication between the receiver of a surface vessel and a source on an underwater vehicle was assumed. The source was located at a fixed point, and the receiver moved in accordance with the roll motion of the vessel. The roll motion was modeled as a simple harmonic motion (SHM) with the roll angle \( \theta \) and is written as

\[
\theta_i(t) = \theta_{\text{max}} \sin(2\pi t / T_{\text{SHM}} + \psi_0),
\]

where \( \theta_{\text{max}} \) and \( T_{\text{SHM}} \) denote the maximum roll angle and roll period, respectively. Furthermore, \( \psi_0 \) represents an SHM phase at the time when the first synchronization signal is detected on the receiver. Furthermore, the motion phase is defined as \( \psi(t) = 2\pi t / T_{\text{SHM}} + \psi_0 \); hence, with the phase \( \psi \), the roll angle in Eq. (13) can be expressed as

\[
\theta_i(t) = \theta_{\text{max}} \sin(\psi(t)).
\]

To focus on the Doppler shifts caused by the motion of the receiver, the sea surface was assumed to be flat and stable. Furthermore, the center of the roll motion was fixed at the sea surface for simplicity.

In the multipath environment, it is assumed that two reflected signals can arrive at the receiver, one reflected from the surface and another reflected from the bottom. In this paper, the signals reflected from the surface and the bottom are defined as the 1st and 2nd multipath signals, respectively. The simulation conditions are shown in Fig. 7, and the parameters are listed in Table 1.

In the geometry shown in Fig. 7, the 1st and 2nd multipath signals arrive at the receiver approximately 2 ms and 5.8 ms later than the 0th multipath signal, respectively. The differences in propagation time between the 0th and other multipath signals are depicted in Fig. 8. The difference in the case of the 1st multipath signal varies largely whereas that of the 2nd multipath signal is almost constant. The result indicates that the Doppler shift of the 1st multipath signal differs from that of the 0th multipath signal and the Doppler shift of the 2nd multipath signal is almost the same as that of the 0th multipath signal.

In the simulations, the received signal \( r(t) \) was calculated at a sampling rate \( f_s \) of 400 kHz using Eq. (1). The propagation time for the multipath signals, \( \tau_\text{m}(t) \), was derived at each discrete time in accordance with ray theory. In addition, it was assumed that a signal reflected on the surface had an amplitude \( A_1 = 0.3A_0 \) impinged on the receiver, whereas a signal reflected on the bottom had \( A_2 = 0.1A_0 \). The speed \( c \) of sound was set as a constant of 1,500 m/s.

In the simulations, \( \theta_{\text{max}} \) was set to be \( 2^\circ \) to \( 20^\circ \) in steps of \( 2^\circ \). \( \theta_{\text{max}} \) larger than \( 10^\circ \) corresponds to the case of a small vessel with a size of several meters. \( T_{\text{SHM}} \) was set...
to be 1 to 10 s in steps of 1 s. The measured periods ranged from approximately 2 to 4 s in the case of a small vessel, while $T_{\text{SHM}} = 10$ s corresponds to that of the large vessel with a size of several tens of meters.

### 3.2. Communication Condition

In this study, the carrier frequency $f_c$ was set to 20 kHz, and the symbol rate was set to be 1 to 10 kHz in steps of 1 kHz. In all cases, the transmitted signal had the configuration shown in Fig. 1 and a length $T_s$ equal to 1 s because it is close to the parameter for actual use [19]. As synchronization signals, Zadoff–Chu signals with a length of 127 symbols were utilized. In addition, it was assumed that the arrival times of the synchronization signals of the 0th multipath signals were known to focus on the difference in the demodulation performance of DFEs.

In this work, the parameters of the proposed DFE, except for the additional DPLLs, were set to be the same as those of the conventional DFE. A training signal of 500 symbols was utilized for the initial estimation of the channel response in all simulations. Furthermore, the taps of the feedforward and feedback filters in both DFEs were updated using the RLS algorithm. The length of feedforward filters $N_f$ was set to 21 with $N_1 = 10$ and $N_2 = 10$. Meanwhile, the number of feedback filters was set to 10 ms for each symbol rate. Furthermore, quadrature phase shift keying (QPSK) was utilized in all simulations. In addition, no error correction code was utilized in this study to focus on the improvement due to channel response estimation using the proposed DFE.

### 3.3. Evaluation

In this paper, the proposed process improves demodulation performance through a physical layer in communication; hence, demodulation performance is evaluated using the output SNR. The output SNR is the energy ratio of the transmitted data symbol to the error of the estimate after the equalizer and is defined as

$$\text{output SNR} = 10 \log_{10}\left[\frac{\sum_l |d(l)|^2}{\sum_l |d(l) - \hat{d}(l)|^2}\right],$$

(14)

where $d(l)$ is the $l$th transmitted data symbol and $\hat{d}(l)$ stands for the estimate defined in Eq. (9). The output SNR can indicate the performance in a physical layer, even in the case without any bit error. To investigate the improvement due to the channel response estimation by the proposed DFE without degradation caused by the wrong symbol decision, the output SNR is adopted as an evaluation indicator in this study.

In the simulation, the Doppler shifts of the 0th and other multipath signals varied with changes in the corresponding propagation time. For this reason, simulations with various initial motional phases $\psi_b$ were carried out as described in Sect. 3.1. Furthermore, the demodulation performance for each SHM period $T_{\text{SHM}}$ and maximum roll angle $\theta_{\text{max}}$ was evaluated using the most frequent value of the output SNR distribution dependent on $\psi_b$. The most frequent value was defined as the output SNR at the peak of the histogram [22]. It is the output SNR yielded at the highest probability and is an effective indicator for designing a communication system for actual use under motional conditions.

### 4. RESULTS AND DISCUSSION

#### 4.1. Cases of a Symbol Rate of 3 kHz with a Multipath Signal

Figure 9 shows the simulation results for a symbol rate of 3 kHz with $A_1 = 0.3A_0$ and $A_2 = 0$. Figure 9(a) shows how the difference in propagation time between the 0th and 1st multipath signals depends on the motion phases $\psi = 2\pi t/T_{\text{SHM}} + \psi_b$. Furthermore, in Fig. 9(b), the output SNRs due to the use of two DFEs dependent on $\psi_b$ are shown. As shown in Fig. 9, the difference in propagation time has four extremums at $\psi = 0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$, while the output SNRs of the proposed DFE also have four extremums. Furthermore, almost all output SNRs of the proposed DFE are higher than the corresponding output SNRs of the conventional DFE. To focus on the dependence of the demodulation performance on the motional

![Fig 9](image-url)
condition, the results in the cases of $\psi_b = 45^\circ$, $85^\circ$ and $160^\circ$ are discussed below.

First, Fig. 9(a) shows that the difference in propagation time varied nonuniformly at almost all parts of the received duration of the signal in the case of $\psi_b = 45^\circ$. In particular, the difference around the extremum at $\psi = 90^\circ$, which varies nonlinearly, severely affects the signals and causes the nonuniform Doppler shift of the 1st multipath signal, which differs from that of the 0th multipath signal.

Next, most of the difference in propagation time varied linearly while signals in the cases of $\psi_b = 85^\circ$ and $160^\circ$ were received, as shown in Fig. 9(a). In Fig. 9(b), the proposed DFE marks the output SNRs higher than those of the conventional DFE by more than 3 dB. The difference in propagation time is proportional to the phase shift, which is compensated by the additional DPLL in the proposed DFE. Therefore, as stated in Sect. 2.2, the term $\alpha_2 \sum_{j=1}^l \Phi_b(j)$ in Eq. (11) indicates the contribution to the linear change in the phase shift caused by the difference in propagation time. Consequently, the proposed DFE can effectively suppress the effects of the phase shift of multipath signals in the cases of $\psi_b = 85^\circ$ and $160^\circ$. Meanwhile, the phase shift varies nonlinearly in the case of $\psi_b = 45^\circ$, as stated earlier, which makes it difficult for the proposed DFE to deal with the phase shift of the multipath signal.

It is difficult to individually evaluate functions for filters and DPLLs in DFEs because the taps and compensation phases are cooperatively updated on the basis of estimation errors using the RLS algorithm. However, the better estimation and compensation of a channel response always show a decrease in square errors between the transmitted symbols and estimates by equalization. Therefore, to understand the relationship between improvement due to the proposed DFE and the motional state, the calculated phase shift $\phi_b(l) - \phi_b(l_{0}^b)$ and the square errors of the conventional and proposed DFEs $|\hat{d}(l) - d(l)|^2$ are shown in Fig. 10.

As shown in Fig. 10, the square errors in both DFEs become the minimum around the extremums of the phase shifts. This means that both DFEs successfully suppress the effects of the multipath signal around the extremums of the phase shift, where it varies only slightly. Meanwhile, the phase shift changes greatly when the phase shift varies linearly. Because it is difficult for both DFEs to deal with the large variation in phase shift, the square errors grow significantly during the linear variation. In addition, the proposed DFE shows square error smaller than that of the conventional DFE during the linear variation, except at the beginning part of the variation. In general, second-order DPLLs require steps to estimate the linear variation and adjust the term $\alpha_2 \sum_{j=1}^l \Phi_b(j)$ in Eq. (11); thus, the improvement due to the proposed DFE appears after the estimation of the linear variation.

As discussed above, the simulation results show that a large linear variation of the phase shift severely degrades the demodulation performance and that the proposed DFE can reduce the square errors after the estimation of the linear phase variation by the additional DPLLs.

### 4.2. Dependence on Motional Conditions and Symbol Rates

In this study, simulations were carried out under many motional conditions. As described in Sect. 3.1, the period $T_{\text{SHM}}$ was set from 1 to 10 s, and the maximum roll angle $\theta_{\text{max}}$ was set from $2^\circ$ to $20^\circ$. The symbol rate was set from 1 to 10 kHz. In this section, results are shown and discussed in three cases: no multipath signal, only the 1st multipath signal and with 1st and 2nd multipath signals.

Figure 11 shows the most frequent values of the output SNRs of the conventional and proposed DFEs and the improvement due to the proposed DFE when only the 0th multipath signal is received, in other words, $A_1 = 0$ and $A_2 = 0$. Furthermore, the most frequent values were evaluated, as described in Sect. 3.3. For the evaluation, histograms of the output SNRs with bins of width equal to 1 dB were utilized.

Figure 11 shows that the demodulation performance of the proposed DFE is almost the same as that of the conventional DFE. In principle, the proposed DFE cannot improve the demodulation performance when no multipath signals are received because all taps of the feedback filter, specifically, $b_{R_{b}}$ in Eq. (8), become 0. Therefore, the result that both DFEs yield almost the same output SNRs coincides with the theoretical characteristics stated earlier.
Note that the most frequent value of the proposed DFE is 6 dB better than that of the conventional DFE at a symbol rate of 7 kHz, $T_{\text{SHM}} = 2$ s and $\theta_{\text{max}} = 4^\circ$. Figure 12 shows how the output SNRs of both DFEs depend on the initial motion phases $\psi_b$ under the condition with a symbol rate of 7 kHz, $T_{\text{SHM}} = 2$ s and $\theta_{\text{max}} = 4^\circ$. In Fig. 12, the difference in output SNRs at the same initial motion phase $\psi_b$ is less than 1 dB in most cases, which is different from the results in Fig. 9(b). The results in Fig. 12 are consistent with the theoretical prospect that the proposed DFE cannot improve the demodulation performance when only the 0th multipath signal is received. The most frequent value of the proposed DFE is higher than that of the conventional DFE because the probability density distribution functions of output SNRs of the two DFEs have two peaks at the minimum and maximum values of output SNRs like the distribution of a sinusoidal function. Therefore, Fig. 12 shows that the difference in the most frequent values does not mean the improvement is due to the proposed DFE under the simulation condition.
The signal lengths $T_{\text{tx}}$ were commonly set to 1 s in this study; hence, a large $T_{\text{SHM}}$ indicates a motional condition where both phase shifts of the 0th and 1st multipath signals vary almost linearly while receiving the signal. Because both phase shifts vary linearly, the difference also varies linearly. As discussed in Sect. 4.1, the proposed DFE can effectively improve the demodulation performance affected by the linear variation of the difference in the phase shift. If $T_{\text{tx}}$ is smaller and $T_{\text{SHM}}$ is fixed, the phase shifts of the 0th and 1st multipath signals and their difference vary more linearly. Therefore, a smaller $T_{\text{tx}}$ yields a better demodulation performance, whereas it decreases the data rate.

In addition, when the maximum $\theta_{\text{max}}$ is small, the difference in the phase shift changes negligibly. Because even the conventional DFE can suppress the effects in this case, the improvement due to the proposed DFE becomes small. Furthermore, the output SNRs of the proposed DFE are smaller than those of the conventional DFE at symbol rates higher than 5 kHz when $\theta_{\text{max}} < 6^\circ$. One of the possible reasons is that the additional DPLLs cause slight instability of the estimated result. Because the symbol rate equals the updating rate of both DFEs, estimation of the DFEs can follow faster change of a channel response when the symbol rate is higher. Therefore, increasing the symbol rate leads to better demodulation performance in most cases. Meanwhile, compensation phases of additional DPLLs have slight instability in principle, as do other adaptive filters. Therefore, the output SNRs of the proposed DFE are smaller than those of the conventional DFE when output SNRs of the conventional DFE are degraded by errors smaller than the effect of the instability of the additional DPLLs. In Fig. 13, most output SNRs of both DFEs are higher than 42 dB, which is within the range where the output SNRs of the proposed DFE are less than those of the conventional DFE. This indicates that the instability of the additional DPLLs alters the demodulation performance by approximately $-42$ dB in comparison with the estimated symbol energy.

When the maximum $\theta_{\text{max}}$ is large, the proposed DFE can improve the performance considerably. This is because the large difference in the phase shift makes it difficult for the conventional DFE to deal with the phase shift of the multipath signal.

Furthermore, the results at symbol rates higher than 1 kHz show marked improvement in comparison with the results at a symbol rate of 1 kHz. This is because the symbol rate equals the update rate of the channel response estimation for both DFEs. At a symbol rate equal to 1 kHz, even additional DPLLs cannot sufficiently follow the variation of the phase shift of the multipath signal because of the low updating rate. Consequently, the proposed DFE greatly improves the performance at symbol rates higher than 1 kHz.

Finally, Fig. 14 shows demodulation performance with both DFEs and the improvement due to the proposed DFE when $A_1 = 0.3A_0$ and $A_2 = 0$. As shown in Fig. 13, the demodulation performance of both DFEs becomes high in cases with a large period $T_{\text{SHM}}$ and a small maximum roll angle $\theta_{\text{max}}$, that is, when the Doppler shifts of the 0th and 1st multipath signals are small. Meanwhile, the improvement manifests mainly as large $T_{\text{SHM}}$ and large maxima $\theta_{\text{max}}$ at all symbol rates.

Note that the improvement due to the proposed DFE appears under some conditions even when the period $T_{\text{SHM}}$ is small; for example, the improvement is 3 dB at $T_{\text{SHM}} = 3$ s and $\theta_{\text{max}} = 16^\circ$ in Fig. 13, and the symbol rate is 3 kHz. In this case, the most frequent value of the output SNRs of the proposed DFE is 20 dB, which yields a bit error rate of less than $2 \times 10^{-6}$ when using 16 quadrature amplitude
modulation (16QAM) even without error correction [23]. Meanwhile, that of the conventional DFE is approximately 17 dB, which yields a bit error rate of $6 \times 10^{-4}$. The improvement indicates that the proposed DFE can contribute to a high data rate or robust communication even for communication between a small surface vessel and an underwater vehicle at a low symbol rate.

5. CONCLUSION

In general, the Doppler shifts of multipath signals can degrade demodulation performance of even conventional processing when they differ from the Doppler shift of a direct signal. In this paper, each signal received through the corresponding path are defined as a multipath signal, and we mathematically described how the Doppler shifts of multipath signals degrade the demodulation performance. Furthermore, to suppress the effects of the Doppler shifts of multipath signals other than the direct signal, a new equalizing method was proposed.

The proposed processing was realized by installing additional DPLLs for conventional processing to compensate for the phase shifts of the multipath signals. Moreover,
through simulation under many motional conditions at various symbol rates, we investigated how the proposed processing improves the demodulation performance. The simulation results showed that the proposed processing improves the performance when the difference in phase shifts between direct and other multipath signals varies linearly. In addition, the simulation results also indicated that the proposed processing can improve the performance even under some fast motion conditions at some symbol rates.

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