How inclusive school students understand fractions

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Abstract. This research aims to analyze students’ ways of understanding when they learn fractions. The respondents were 23 students of the 7th graders in the inclusive school. The research approach was qualitative, with case study design. The data were collected through paper and pencil measure, observation, and interview. The data were analyzed by grounded theory. The results showed four mental acts; those are interpreting, problem-solving, explaining, and inferring as well as ways of understanding and ways of thinking that are relevant to those mental acts.

1. Introduction
Inclusive education is an education service system that includes children with special needs to learn together with their peers in the regular school. The spirit of implementing inclusive education is to provide the broadest possible opportunity or access to all children to obtain a good quality of education and to meet the individual needs of learners without discrimination.

Students in the inclusive school struggle with mathematics because of different reasons. Some of the students have mathematics learning disabilities (MLD) symptoms; 6% of students with MLD have difficulties that came from a cognitive origin [1]. The other research findings concluded that students with MLD have different error pattern [2], often making an error in processing numerical representation [3], misunderstand in comparing and estimating numbers [4], inaccurate in performing arithmetic calculations [5], get wrong in solving elementary numbers [6].

In mathematics education, several communities had inclusive spirit in their principle. For example, Principles and Standards for School Mathematics [7], there is the Equity Principle which stated that all students must have opportunities to study and support to learn mathematics; regardless of their characteristics, backgrounds, or physical challenges. Furthermore, the Equity Principle stated that equity does not mean that each student should receive the same instruction; however, it depends on the need of all students.

All of us agreed about the equity principle in the teaching and learning mathematics. However, studies that examine the learning of mathematics in the inclusive setting are still not widely practiced. A survey of the practice of inclusive education identified only five studies published between 2000 and 2013 relating to mathematics classes [8].

Based on the background of the problem, the problem of this research is how students in the inclusive school understand fraction. In this study, we aim to examine students’ ways of understanding when they learn fractions.
Mathematics is the set of ways of understanding (WoU) and ways of thinking (WoT). WoU is the product of mental acts; it consists of axioms, definitions, theorems, proofs, problems, and solutions. WoT is the characteristics of mental acts; it includes all the ways of thinking that used to produce WoU [9].

The definition implies an excellent pedagogical implementation. Mental action is a characteristic of thinking in line with the problems encountered. The flow of thinking formed from the mental act become WoT. When the current of thinking proceeds and comes into contact with specific contexts to form an understanding, the path leads to WoU. The reasoning of human involves many mental acts such as interpreting, guessing, concluding, proving, explaining, composing, generalizing, applying, predicting, classifying, seeking and problem-solving.

2. Experimental method
This research used a qualitative approach. We want to assess the way of students thinking in understanding fractions in the inclusive schools. This study used case study with the single-case (holistic) designs [10]. The case study method is used to describe field findings related to the research problem formulation, i.e., how students in the inclusive school understand fraction.

This research was conducted in a Junior High Inclusion School in Cimahi, West Java, Indonesia, with 23 students in the grade 7 as respondents. Students were given research instruments, then the students’ work was analyzed using Harel's theory. The data collection techniques used in this study are paper and pencil measure (test), observation and interview. The data were analyzed by grounded theory, with coding and constant comparison [11].

3. Result and discussion
From the analysis of data, students did mental acts as follows:

3.1. Explaining
Explaining is one form of human reasoning. Explaining means making ideas, situations, or issues clear to someone, by explaining them in more detail or by disclosing relevant facts or opinions. From the test result, students who perform mental act explaining were found, as follows:

Figure 1 shows a student who answered the question with a mental act explaining. Students are asked about which fraction representation image is larger. There are three pairs of images.

In the student answer above, it appears that students understand the problem and answer it by using mental act explaining. The student's solution contains a very coherent explanation with good reason too. The explanation from the student is crystal clear that make researcher surprise.

From the student answer, we can also identify the student's way of understanding that is an explanation. The explanation is a cognitive product of mental act explaining. Another example of explanation from student's answer as follow:
In figure 2, students are asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. This explanation is a description of the ways of understanding students, as a result of mental act explaining.

We can also recognize student's way of thinking which corresponds to mental act explaining, that is the way of explaining (how to explain). How to explain is the cognitive characteristic of the mental act explaining. How to explain closely related to students' mathematical communication skills. Indicator of students' mathematical communication ability can be seen from: (1) Ability to express mathematical ideas through oral, written, and demonstrate and visualize it; (2) Ability to understand, interpret, and evaluate mathematical ideas both orally and in other visual forms; (3) Ability to use terms, mathematical notations and structures to present ideas, describe relationships and situational models [12].

By paying attention to students' answers above, it can be seen that student’s way of explaining can be categorized on how to explain/express mathematical ideas through writing.

3.2. Interpreting
The second mental act found in the analysis of students’ answer is interpreting. Interpreting means explaining the meaning of information, a word, or an action. From the study of the student solution, we found some students who do mental act interpreting, as follows:

In figure 3, students are asked to describe fractions $\frac{1}{2}$ and $\frac{2}{5}$ in two different ways. In the student's answer, it appears that students perform mental act interpreting by interpreting fractions $\frac{1}{2}$ and $\frac{2}{5}$ with shaded circle and rectangles.

From the student answer, we can also identify the student's way of understanding which corresponds to mental act interpreting, that is interpretation. Interpretation is the cognitive product of mental act interpreting. From the analysis of student answers, we also obtained another example of the way of understanding interpretation, as follows:
In figure 4, students are asked to describe fractions $\frac{1}{2}$ and $\frac{2}{5}$ in two different ways. Students' solutions are obtained by using mental act interpreting that produces a rectangular and a heart picture with a half shaded as an interpretation of $\frac{1}{2}$. While the $\frac{2}{5}$ was interpreted as square and cycle pictures whose two parts are shaded, while the other three parts are not shaded.

We can also recognize student's way of thinking which corresponds to mental act interpreting, that is multiple interpretations of a fraction. Multiple interpretations of fraction symbols are cognitive characteristic of mental act interpreting. In the above answer, we can see that students have multiple interpretations of $\frac{1}{2}$; there are rectangular, heart, and cycle picture. The other examples of students' way of thinking (multiple interpretations) are as follow:

In figure 5, students are asked to explain which fractions are larger between $\frac{1}{2}$ and $\frac{2}{3}$. Students interpreted the fraction as a decimal to answer the question. He wrote $\frac{1}{2}$ as 0.5 and $\frac{2}{3}$ as 0.6.

3.3. Inferring
The third mental act that found from the analysis of student answers is inferring. Inferring means to deduce (information) from evidence and reasoning, not from explicit statements. From the analysis of student answers, we found a student who performs the mental act inferring, as follows:

In figure 6, students are asked which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. To answer this problem, students do the reasoning by making the same denominator for the two fractions. Fractions $\frac{2}{3}$ changed to $\frac{8}{12}$, and $\frac{3}{4}$ changed to $\frac{9}{12}$. Then the students conclude that $\frac{2}{3} < \frac{3}{4}$ from the inferring.
From the student answer, we can also identify the student's way of understanding which corresponds to mental act inferring, that is inference. The inference is the cognitive product of mental act inferring. From the analysis of student answers, we obtained another example of the way of understanding inference, as follows:

![Figure 7. The second example of student answer on inferring.](image1)

In figure 7, students are asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. Students' answers are obtained by performing a mental act inferring so that the product is inference (conclusion). The conclusion is $\frac{2}{3} < \frac{3}{4}$. This conclusion is derived from the following reasoning: $\frac{2}{3} = 66\%$ and $\frac{3}{4} = 75\%$.

We can also recognize student's way of thinking which corresponds to mental act inferring, that is how to draw an inference (conclusion). How to draw an inference is a cognitive characteristic of mental act inferring. How to conclude can be done with deductive and inductive reasoning [4]. From the previous picture, here is one example of student work that does deductive reasoning in answering test questions:

![Figure 8. The third example of student answer on inferring.](image2)

In figure 8, students are asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. Students make deductive reasoning to answer the question. He replaced $\frac{2}{3}$ with $\frac{8}{12}$ and replaced $\frac{3}{4}$ with $\frac{9}{12}$. Students then compare that $\frac{8}{12} < \frac{9}{12}$, so it can be deduced that $\frac{2}{3} < \frac{3}{4}$.

3.4. Problem-Solving
The fourth mental act that found from the analysis of student answers is problem-solving. Problem-solving means the process of finding solutions for difficult or complex problems. From the analysis of student answers, we found some students who do mental act problem-solving, as follows:

![Figure 9. The example of student answer on problem-solving.](image3)

In figure 9, to answer this word problem, students do mental act problem-solving by modeling the problem into a sum of fractions. Then, the student solved the fraction sum problem with common
denominator approach. Fractions $\frac{2}{5}$ changed to $\frac{14}{35}$ and $\frac{3}{7}$ changed to $\frac{15}{35}$. Then the students add up the two fractions and get the result $\frac{29}{35}$.

From the student answer, we can also identify the student's way of understanding which corresponds to mental act problem-solving that is the solution. The solution is the cognitive product of mental act problem-solving.

We can also recognize student’s way of thinking which corresponds to mental act problem-solving that is problem-solving approach. Problem-solving approach is a cognitive characteristic of mental act problem-solving. From the results of student tests, we found some students who perform various problem-solving approach, as follows:

3.4.1. **Common denominator approach.** Here is one example of student work that uses common denominator approach:

![Figure 10](image1.png)

*Figure 10.* The example of student answer on common denominator approach.

In figure 10, the student solved the fraction sum problem with common denominator approach. Fractions $\frac{2}{5}$ changed to $\frac{14}{35}$ and $\frac{3}{7}$ changed to $\frac{15}{35}$. Then, the student added up the two fractions and get the result $\frac{29}{35}$.

3.4.2. **Change to decimal approach.** Here is one example of student work using change to a decimal approach:

![Figure 11](image2.png)

*Figure 11.* The example of student answer on change to decimal approach.

In figure 11, students are asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. Students used the change to decimal approach to answer the question. He changed $\frac{2}{3}$ to 0.66 and $\frac{3}{4}$ to 0.75, then found the solution is $\frac{2}{3} < \frac{3}{4}$.

3.4.3. **Change to percent approach.** Here is one example of student’s work using change to a percent approach:

![Figure 12](image3.png)

*Figure 12.* The example of student answer on change to percent approach.
In figure 12, students are asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. Students used the change-to-percent approach to answer the question. He changed $\frac{2}{3}$ to 66% and $\frac{3}{4}$ to 75%.

3.4.4. Draw a picture approach. Here is one example of student work that uses draw a picture approach:

![Image of student work]

**Figure 13.** The example of student answer on the draw a picture approach.

In figure 13, students are asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. Students used the draw a picture approach to answering the problem. He describes $\frac{2}{3}$ as a circle divided into three parts with two shaded and $\frac{3}{4}$ as a circle divided into four parts with three shaded. Then, the students found the solution that is $\frac{2}{3} < \frac{3}{4}$.

We summarize the findings of the mental act, way of understanding, and way of thinking in the table 1 below:

| Mental Acts     | Ways of Understanding | Ways of Thinking                                      |
|-----------------|-----------------------|-------------------------------------------------------|
| Interpreting    | Interpretation        | Multiple interpretations                               |
| Problem-solving | Solution              | Problem-solving approach: common denominator approach, change to a decimal approach, change to a percent approach, draw a picture approach |
| Explaining      | Explanation           | how to explain/express mathematical ideas through writing |
| Inferring       | Inference             | how to draw inference: deductive reasoning           |

In the table 1, the findings of the mental acts, WoU, and WoT are based on general observable features, although in reality, they can intersect each other.

The findings are in line with Harel's opinion that when analyzing students' thinking, the researchers begin with a mental act, look at the type of cognitive product (way of understanding associated with it), and tries to find common cognitive properties between ways understanding, every characteristic that noticed is the way of thinking related to the mental act. The acquisition of meaning and concepts from WoU in one case will have an impact on the improvement of subsequent mental action [9].

4. Conclusion
In the inclusive class, there are four mental acts: those are interpreting, problem-solving, explaining, and inferring with corresponding WoU and WoT. The inclusive school system is our future, the study of students’ ways of thinking when understanding mathematics in the inclusive setting is critical. The results of this study can be made as one of didactic anticipation when teachers teach the concept of fractions in the inclusive school.

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