Imaging the hybridized eigenmodes of a confined magnetic vortex resulting from propagating spiral spin waves from the core

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The hybridized spin wave modes of a ferromagnetic vortex confined to a microscale disc have been directly observed in response to a microwave field excitation using time-resolved scanning Kerr microscopy. Micromagnetic simulations demonstrate that the observed curling nature of confined spin waves in the region of circulating in-plane magnetization is a result of the hybridization of different gyrotropic eigenmodes of the vortex core with the azimuthal (4 - 9 GHz) and radial (~10 GHz) eigenmodes of the in-plane magnetization. Hybridization with the fundamental gyrotropic mode leads to splitting of azimuthal modes with counter propagating wavevector, while hybridization of an azimuthal mode with the first-order gyrotropic mode allows the direction of the core gyration to be determined through hybridization rules. A higher frequency radial mode reveals evidence of excitation at the disc perimeter, but also evidence of hybridization with the first higher order gyrotropic mode. These experimental observations confirm the recent theoretical predictions of such hybridization. The measured spatio-temporal character of the hybridized modes is accurately reproduced by the simulations, which demonstrate that the mechanism for the hybridization is the emission of propagating short-wavelength spiral spin waves from the core. These results will have importance in the field of magnonics and spintronics that aim to utilize spin wave emission from highly localised, nanoscale regions of non-uniform magnetization, and their subsequent interaction with modes that may be supported nearby.

I. INTRODUCTION

A magnetic vortex consists of a flux-closure equilibrium state of circulating in-plane magnetization that surrounds an out-of-plane vortex core singularity with diameter of only a few tens of nanometers [1]. Vortex states confined to thin film ferromagnetic discs generate negligible stray field at the edge of the disc, exhibit stability without the need for a biasing magnetic field, and can support a rich spectrum of spin waves making them attractive elements for high density, low energy, microwave frequency magnetic devices. In particular, spin waves are eagerly anticipated as an energy efficient carrier of information in tunable micron and sub-micron scale magnonic circuits for magnetic logic, memory, and oscillator applications [2–9]. For this purpose, the magnetization dynamics of circular ferromagnetic discs with a vortex equilibrium state have been studied intensively, from works to acquire a greater understanding of the gyrotropic core dynamics [10–12] to emerging signal processing applications, such as tuneable microwave emission of a spin torque vortex oscillator (STVO) [13]. For applications, phase locking of multiple oscillators is required in order to enhance microwave emission to a technologically useful level [13–16]. One approach is to simply couple isolated oscillators in an array by dipolar interaction [16]. However, an understanding of the magnetization dynamics of the individual oscillator elements is an essential prerequisite to understand the collective modes.

When an in-plane pulsed magnetic field is applied to a vortex, the lowest energy mode that can be excited is the gyration of the vortex core about an equilibrium position with uniform displacement across the magnetic film thickness, where the gyration frequency depends upon the aspect ratio of the disc [17, 18]. Higher order gyrotropic modes may also be excited with nodal points in the core displacement across the film thickness [19, 20]. In addition, a complete set of modes related to azimuthal and radial spin waves appear [21–23]. Azimuthal modes exhibit a wavevector around the disc azimuth and corresponding nodal lines along its radius. Conversely, radial modes exhibit wavevectors along the disc radius and nodal lines of constant radius. Radial spin waves are related to Damon-Eshbach modes where their wavevector \( \mathbf{k} \) is perpendicular to the equilibrium magnetization \( \mathbf{M} \), since in a vortex configuration the magnetization is circulating in-plane around the core [24].

The spin wave spectrum of a vortex can be significantly different depending on the disc thickness and more generally, its aspect ratio. Spiralling spin waves found in vortex configurations have been previously explained as the hybridization of a stationary azimuthal mode and a higher order gyrotropic mode that shows no radial propagation, or alternatively as a burst of incoherent spin wave emission during a vortex core reversal [25, 27]. It is well known that microscale confinement can lead to a non-uniform magnetization such as the vortex state [1], or S and C single domain states [25]. Related inhomogeneity of the internal magnetic field in the region of the vortex core, or in the vicinity of edges perpendicular to an applied magnetic field, have been recently shown to be sources of spin waves due to a gradient in the magnonic...
refractive index [29, 30]. Such spin wave emission from these regions has been demonstrated using micromagnetic simulations and direct imaging techniques [31, 32].

In the frequency domain, techniques such as Brillouin light scattering (BLS) and vector network analyzer ferromagnetic resonance (VNA-FMR) can be used to acquire the spin wave spectra of confined nanostructures. In spatially resolved BLS microscopy (micro-BLS), the amplitude of excited spin waves can be directly imaged in individual discs with <300 nm resolution, but with limited phase information [33-35]. On the other hand VNA-FMR can provide amplitude and phase, but no spatial information and typically averages the response of an array of discs [36, 37]. In the time domain, time-resolved scanning transmission x-ray microscopy (TR-STXM) has been used to directly image spiral spin waves in circular discs [38, 39]. In the latter references, the authors explain the spiral nature as the hybridization of azimuthal spin waves with first order perpendicular standing spin waves across the thickness of the disc. At lower frequency, time-resolved scanning Kerr microscopy (TRSKM) can be used to image the spatial character of spin waves with wavelength larger than the diffraction limited optical spatial resolution [32, 41-44].

In this work we report on the direct observation and micromagnetic simulation of the curling nature of magnetization dynamics of the in-plane magnetized region of the disc that result from the hybridization of azimuthal and radial modes with the gyrotropic modes of the core. We identify that the hybridization is mediated by propagating short wavelength spiral spin wave emission from the core. A sufficiently large disc thickness was chosen to yield a rich mode spectrum, while our results are reported over a broad frequency range that captures both azimuthal and radial modes and their hybridization with different orders of the gyrotropic mode. TRSKM with a spatial resolution of ~300 nm was used to image the curling of hybridized curling modes in a NiFe 2 um diameter and 40 nm thickness disc over a frequency range extending from 4 GHz to over 10 GHz. The simulations reveal that at low frequencies, hybridization of azimuthal modes with core dynamics, mediated by short wavelength spiral spin waves emitted by the core, yields a curling nature of the resulting mode. While these latter modes cannot be spatially resolved experimentally, micromagnetic simulations with fixed core spins demonstrate that the short wavelength modes are no longer emitted, and the curling nature vanishes. The experimental observation of the curling using TRSKM thus provides an indirect confirmation of the emission of the short wavelength spiral spin waves from the core since our experimental measurements show very good agreement with the micromagnetic simulations.

We note that the short wavelength spiral spin waves emitted by the core and the curling, spiral nature of the hybridized azimuthal and radial modes may become confused. Here on in, we therefore refer to these modes as ‘propagating spiral spin waves from the core’ and ‘curling azimuthal and radial modes’ respectively.

The disc studied in this work had a thickness that has been predicted to coincide with a transition between two dynamical regimes, with different spin wave character excited in discs with diameter of 0.5 um [45]. In Ref. [45], when the thickness is below the transition thickness, the frequency of the first order gyrotropic mode of the core is greater than both azimuthal modes with node index \( m = \pm 1 \) and forms an anticrossing (hybridization) with the higher frequency azimuthal mode \( m = +1 \). Above the crossover thickness, the frequency of the first order gyromode becomes lower than the azimuthal modes and crosses the lower frequency azimuthal mode \( m = -1 \). While the aspect ratio (thickness to diameter) of the disc studied in this work is one quarter that of the modelled disc in Ref. [45], we can also describe the curling of the azimuthal mode in terms of a hybridization of the azimuthal mode via spatially unresolved core spiral spin waves.

II. METHODS

A. Time-resolved scanning Kerr microscopy

Time-resolved scanning Kerr microscopy (TRSKM) was used to image the spin wave modes within a single NiFe disc with diameter of 2 um and thickness 40 nm. The spin waves were imaged at remanence (~<10 Oe) and in response to a uniform RF magnetic field applied in the plane of the disc.

The RF excitation was generated using a 50 Ω impedance matched coplanar waveguide (CPW). The CPW and NiFe disc were fabricated from a multilayer stack of composition Ta(5)/Cu(25)/[Ta(3)/Cu(25)]_3/Ta(10)/Ru(5)/Ni_{81}Fe_{19}(40)/Al(1.5) (thicknesses in nm) as described in more detail elsewhere [44]. TRSKM was carried out using a Tisapphire mode locked laser to generate ~100 fs pulses with 800 nm wavelength at a repetition rate of 80 MHz. Second harmonic generation was then used to generate pulses with 400 nm wavelength that were passed along a 4 ns optical time delay line, expanded by a factor of 5 and linearly polarized, before being focused to a diffraction limited spot on the surface of the disc using a high numerical aperture (NA) microscope objective lens (NA 0.6, ×50). The beam was filtered to remove a residual 800 nm component and attenuated so that less than 200 uW average power was incident on the disc. The reflected light was collected by the same objective lens so that changes in the polarization resulting from polar magneto-optical Kerr effect could be analyzed using a polarizing balanced photodiode detector.

Two types of measurement were performed. First, a time-resolved (TR) scan was performed. The laser spot was positioned 0.5 um from the centre of the disc along the y-direction parallel to the RF magnetic field. In this
FIG. 1. A schematic of the time-resolved scanning Kerr microscope featuring second harmonic generation (SHG) and an optical time delay for ~300 nm spatial resolution and picosecond temporal resolution. The microscale disc was fabricated on the central conductor of a constricted section of coplanar waveguide (CPW, not to scale) where the RF field (h_{RF}) was enhanced, in-plane, and perpendicular to the conductor. Images of magnetization dynamics (e.g. inset) correspond to the measured TR polar Kerr signal as the disc is scanned beneath the focused laser spot at a fixed time delay (phase) of the RF field excitation. A ~50 Ω NiCr resistor (black rectangle) was incorporated into the end of the CPW to attenuate the time-varying RF current and minimise multiple reflections.

region the RF excitation of the in-plane equilibrium magnetization is expected to be maximum. The polar Kerr signal was recorded while the time delay was scanned yielding a sinusoidal response corresponding to ∆m_z as the magnetization precesses, Fig.2(a). In the second measurement the time delay was fixed at a particular time delay of interest, and then the disc was scanned in the x-y plane beneath the laser spot to acquire a polar Kerr image corresponding to ∆m_z, e.g. see the inset of Fig.2(b) for polar Kerr images of the disc acquired at opposite (+, −) antinodes of the TR signal (red curve and open symbols) in Fig.2(a). For all RF frequencies used, time delays were selected so that images were acquired at similar increments in phase throughout a single RF cycle, e.g. see phases indicated by open symbols overlaid on the red curve in Fig.2(a). The resulting images were then drift-corrected and arranged according to their time delay to construct movies of a particular spin wave.

Individual spin wave modes of the vortex state were excited using RF frequencies ranging from 4.24 GHz to 10.24 GHz. The frequencies were selected from the Fourier spectrum of a TR scan acquired from the same point in the disc, but in response to a broadband pulse excitation, Fig.3(a). A pulse generator with ~30 ps rise time and ~70 ps duration was used to excite all modes that would couple to a uniform in-plane excitation field on picosecond timescales. The frequency and power of the excited modes was then identified from the spectrum. To excite these modes individually for TRSKM imaging the RF excitation (previously described) was applied with frequency equal to an integer multiple of the laser repetition rate, and with RF power adjusted to compensate for the lower power of some of the modes observed in the spectrum, e.g. particularly at higher frequency, see Fig.3(b). Such modes were excited with an RF power that was enhanced by the approximate difference in power with respect to that of the highest power mode at 5.2 GHz.

To confirm that all modes were imaged within the linear response regime, the RF power dependence was explored for the largest amplitude mode at 5.2 GHz in the

FIG. 2. (a) TR polar Kerr signals for an RF frequency of 5.2 GHz and RF power (P_{RF}) ranging from −10 dBm (light grey trace) to 10 dBm (black dashed trace). (b) The squared Kerr amplitude as a function of P_{RF} (in mW) showing a linear dependence up to 3.3 mW (5 dBm). Inset in (b) are polar Kerr images corresponding to the antinodes (+, −) of the mode excited by a RF field h_{RF} with frequency 5.2 GHz and power P_{RF} = 0 dBm (1 mW, red trace and symbols in (a)). The TR signals in (a) were acquired from the right hand side of the 2 μm disc (large dashed circle overlaid on inset of (b)) from a small circular region corresponding to the optical spatial resolution (solid circle in inset of (b)). For all modes imaged, the symbols on the red trace in (a) indicate the relative phase at which Kerr images were acquired.
FFT spectrum, Fig.3(b). The RF power dependence on the TR polar Kerr signal is shown in Fig.2(b). At 1 mW (0 dBm) the mode excitation was within the linear regime. This suggests that the excitation of modes with lower amplitude identified in the FFT spectrum of Fig.3(b) will also be excited within the linear regime, even when the excitation power is increased to compensate for the reduced mode amplitude in the spectrum.

Slow phase drift of the microwave synthesiser waveform on timescales similar to that required for image acquisition can lead to mismatched spatial character of the spin wave at subsequent phases. To minimize this, repeated TR signals were acquired from the same position in the right hand side of the disc (∼ +0.5 μm from center of disc) to ensure the phase (time delay) was correctly set. Multiple images were acquired to ensure repeatability, and TR images were acquired in a non-sequential order to avoid a systematic accumulation of phase drift (nodal points imaged first, antinodes next, then intermediate phases to complete the series of images). Furthermore, the magneto-optical Kerr effect probes only the average response of the magnetization within the optical skin depth (∼20 nm). In the simulations, variation in the phase across the thickness of the disc to a depth of 20 nm (top 5 layers of cells) was explored. It was confirmed that the phase was approximately uniform across the disc thickness far from the vortex core.

**B. Micromagnetic simulations**

To understand the dynamics of the observed curling, spiral nature of azimuthal and radial modes, we performed a set of micromagnetic simulations using Mumax3 [46]. We simulated a disc with diameter of 2 μm and thickness 40 nm with the typical material parameters of Permalloy at room temperature with saturation magnetization $M_S = 8 \times 10^5$ Am$^{-1}$, exchange constant $A_{ex} = 1.1 \times 10^{-11}$ Jm$^{-1}$, Curie temperature from a weighted average of iron and nickel $T_C = 710$ K and Gilbert damping constant $\alpha = 0.008$ [46, 47]. With these parameters, the single circular disc was simulated in a hexahedral grid. The grid was discretized in the x, y, z-space into $512 \times 512 \times 10$ cells with a cell size of 3.9 nm along x and y, and 4 nm along z such that the cell size along all dimensions was smaller than the exchange length of permalloy (5.3 nm) [48]. The number of cells along x and y were chosen to be powers of $2^n$ (where $n = 8$) for computational efficiency. The edges of the disc were smoothed to reduce staircase effects from hexahedral cells. The smoothed edge volume is found by averaging $p^3$ samples per cell, where $p$ is the parameter input to the function. Since the geometry is a circular disc, the ‘SmoothEdges’ function was set to its maximum value ($p = 8$) [49].

In the first stage of the micromagnetic simulations the stable equilibrium magnetization state was simulated. A vortex state with counter-clockwise circulation (chirality index 1) and core polarization towards the substrate (polarization index -1) was manually set as the initial state and then allowed to relax in a simulation with a high damping parameter ($\alpha = 1$). This particular configuration of core polarization and chirality reproduced the experimental findings as discussed in Section III. The magnetization continued to relax until the maximum change of core polarization and chirality reproduced the experimental findings as discussed in Section III. The magnetization continued to relax until the maximum change in induction (defined as ‘MaxTorque’ parameter in Mumax3, which describes the maximum torque/γ over all cells, where γ is the gyromagnetic ratio of the material) reached $10^{-7}$ T indicating convergence to the equilibrium vortex state of magnetization. The typical time taken to achieve the equilibrium state was 100 ns. Once the equilibrium state was obtained the spin configuration of the disc was recorded and then used as the initial state for simulations with a pulsed magnetic field excitation. To generate a uniform excitation across a desired frequency range in the dispersion diagram, a sinc-shaped magnetic pulse was used $B_1(t)$,

$$B_1(t) = A_1 \text{sinc}(2\pi f_c(t - t_d)),$$

(1)

where $f_c$ is the microwave excitation cut-off frequency, set to be 30 GHz, $t_d = 5$ ns is a pulse delay and $A_1 = 10$ mT is the pulse amplitude. The excitation power was uniformly distributed over the selected frequency range.
of the pulse, so each mode up to the cut-off frequency of 30 GHz is excited with an in-plane magnetic field with amplitude of 0.3 mT. This was chosen to be sufficiently small to ensure that all modes were excited within the linear regime and to avoid any changes to the equilibrium state. To simulate the time evolution of the spatial character of an individual spin wave mode with frequency $f_0$, a small amplitude continuous wave excitation $B_2(t)$ was applied.

$$B_2(t) = A_2 \sin(2\pi f_0 t).$$  \hspace{1cm} (2)

The mode frequency $f_0$ was identified from the first Fourier transform (FFT) spectrum of the simulated temporal response of the spatially averaged out-of-plane component of the magnetization $\langle m_z(t) \rangle$, in response to the pulsed field $B_1(t)$. A sufficiently small amplitude of $A_2 = 0.3$ mT was chosen to ensure that each mode remains in the linear regime while driven at its resonance frequency. A sampling period of $T_s = 25$ ps was used to record 1024 simulated snap-shots of the mode spatial character, but only after the steady state was reached some time after the onset of the excitation. With these parameter values, the Nyquist criterion \cite{49} was satisfied for the whole range of excitation frequencies covered in this work, since the sampling frequency $f_S = 1/T_s = 40 \text{ GHz}$ is almost $\times 4$ larger than the highest excitation frequency used (largest $f_{RF} = 10.24 \text{ GHz}$).

**III. RESULTS AND DISCUSSION**

The TR Kerr signals acquired in response to a pulsed magnetic field, exhibit an almost identical response 0.5 $\mu$m to the left and to the right of the center of the disc, but have opposite sign, Fig.3(a). This is the expected dynamic response of regions of circulating in-plane equilibrium magnetization that lie perpendicular to the pulsed magnetic field. Since these regions to either side of the vortex core have antiparallel magnetization, the initial torque exerted by the pulsed magnetic field will have opposite sign, leading to the observed signals in Fig. 3(a). Clear beating of the TR signals indicates a multi-mode excitation in these regions. The average response of the core probed by the same $\sim 300 \text{ nm}$ focused laser spot positioned at the center shows a response that has reduced amplitude and more complicated beating. This suggests that magnetization dynamics are detected with spatial features smaller than the focused laser spot leading to a diminution of the net response at the centre of the disc compared to that observed 0.5 $\mu$m to the left and right of the center.

Comparison of the FFT spectra of the response detected at the centre of the disc (black curve) and in the right hand side of the disc (grey shaded) are shown in Figure 3(b). Simulated spectra (red curves) extracted from similar regions of the simulated disc in response to the sinc pulse $B_1(t)$ have been overlaid with the measured spectra. While the complete spectral response of measured and simulated spectra show differences, it is clear that there is good agreement of some of the spectral peaks identified from the experimental spectra and the simulated spectra. The differences in the magnitude of the simulated and measured spectral response is due to the uniform power delivered at all frequencies in the simulations, while the power dependence is known to be non-uniform in such experiments, e.g. see reference \cite{50}. In particular the shoulder below 5.2 GHz in the measured spectra is supported by the simulations which reveal at least one additional mode between 4 and 5 GHz. At around 7 and 9 GHz the simulated spectral response from the center (solid red curve) exhibits peaks that coincide with those of the measured spectrum from the center. On the other hand a mode at $\sim 10 \text{ GHz}$ may be seen in the simulated spectral response away from the center (dashed red curve). The simulated mode at around 10 GHz is not well resolved from neighboring peaks, but does support the identification of a mode at 10.24 GHz in the measured spectrum away from the center. The frequencies identified from the experimental spectra alone are indicated by the vertical dashed lines in Fig. 3(b).

It will be shown that the hybridized mode character observed at different frequencies is well reproduced by the micromagnetic simulations. The simulations confirm that the character of the observed dynamics changes with frequency since gyrotropic modes may only hybridize with azimuthal modes at low frequency, while at higher frequency, hybridization may instead take place between a higher order gyrotropic mode and a radial mode. The discussion that follows in this section is split into two parts which address (A) the low frequency regime below 9 GHz, and (B) the high frequency regime above 9 GHz.

**A. Low frequency regime**

In Fig.3(b) the most prominent spectral peaks are found in the low frequency regime for which simulations predict that spiral spin waves emitted from the core have the largest amplitude. In Fig. 4(d), 4(e), and 5(a) and 5(b), the TR spatial character of the modes is shown for frequencies of 5.2 GHz, 4.24 GHz, 6.8 GHz and 8.96 GHz, respectively. Notably, Fig. 4(c) and 4(d) reveal good agreement of the measured and simulated spatial character of the curling azimuthal mode as a function of time.

In the low frequency regime, a whole set of azimuthal spin wave modes can arise \cite{12}. At an appropriate thickness \cite{45}, core dynamics may hybridize with the standing azimuthal modes leading to the observed curling, spiral nature of the hybridized mode about the core. Such hybridization may only be observed above a threshold thickness \cite{27}. To demonstrate that the curling effect is a result of the hybridization, an equivalent disc was simulated with the spins of the core region fixed using the
FIG. 4. Simulated (a, b and c) and measured (d) TR images corresponding to the out-of-plane component of the dynamic magnetization in response to an in-plane excitation of 5.2 GHz frequency. In (e) measured images are also shown for an excitation frequency of 4.24 GHz. In (a, b and c) the $m_z$ component is extracted from the second layer of cells from the top surface of the disc. The spins in the vicinity of the core are fixed in (a) and in a ring around it in (b) and are free to precess in (c). In (d and e) the disc perimeter is indicated by the overlaid yellow circle.

‘frozenspins’ function in Mumax3 [46]. Fig.4(a) reveals that, when the spins are fixed, short wavelength spiral spin waves are no longer emitted by the core. The spatial character of the azimuthal mode then remains static and does not exhibit the curling, spiral nature observed in the experiments and in the simulations where the core is free to gyrate (Fig.4(c) and 4(d)). Additional simulations (Fig.4(b)) in which spins are fixed in a concentric ring-shaped region around the core, show that the spiral spin waves are emitted by the core but do not propagate through the ring to reach the standing azimuthal mode and so the curling of the azimuthal mode does not occur. This demonstrates the curling is a consequence of hybridization of the azimuthal mode with the gyrotropic core dynamics via the exchange (rather than dipolar) interaction mediated by the spiral spin waves emitted by the core.

At 5.2 GHz (Fig. 4(d)) and 4.24 GHz (Fig. 4(e)) the curling of the azimuthal mode is found to be in the opposite sense about the core. This is a direct observation of the hybridization of the fundamental mode of vortex core gyration with azimuthal modes with opposite index of propagation, which leads to frequency splitting of the azimuthal mode. This idea is supported by additional simulations which show evidence of the fundamental gyromode profile at 4.24 GHz (Fig. 6(a)). Movies of the hybridized modes observed in the experiments can be found in the Supplemental Material (1).

At higher frequency, hybridization of the azimuthal mode and first order gyrotropic mode may only take place when their azimuthal motion is of the same sense [45]. The sense of gyration, and therefore polarization, may then be inferred from TR images of azimuthal modes hybridized at higher frequency, such as those identified in Fig. 3(b). Ref. [45] suggests that a significant frequency gap can be expected in the spectrum where the first order gyrotropic mode is hybridized with the higher frequency azimuthal mode, which means that the hybridized azimuthal mode may be observed at a frequency lower than that predicted for the first-order gyrotropic mode.

The simulated images of Figures 4(c) (5.2 GHz), 5(a) (6.8 GHz) and 5(b) (8.96 GHz) clearly show the emission of a shorter wavelength spiral spin wave from the core. In Fig.5(a) a snapshot of the simulated core region (dashed red square) at 0.809 ns is shown in more detail (inset right). Spiral spin wave emission from the core was also observed in simulations at 4.24 GHz (not shown). The spatial resolution of the experimental technique prevents the direct visualisation of these spin waves. To
demonstrate this, the top row of simulated images in Fig. 5(a) have been spatially down-sampled using Gaussian smoothing with a width corresponding to the optical spatial resolution of \( \sim 300 \) nm. The smoothed simulated images (smoothed sim) are shown in the center row in Fig. 5(a) and reveal greater similarity with the measured images where the short wavelength spiral spin waves emitted from the core are not resolved and lead to a reduction of the net signal in the core region.

The TR polar Kerr images of Figures 4(e) (4.24 GHz), 5(a) (6.8 GHz), and 5(b) (8.96 GHz) all exhibit a curling motion with the same counter-clockwise sense, but with varying degrees of the spiral nature. Ref. [45] suggests that the spiral character of the hybridized mode is more marked when the azimuthal mode is hybridized with the first-order gyrotropic mode at higher frequency. In this work at 8.96 GHz, the hybridized azimuthal mode exhibits the strongest spiral spatial character observed at any of the studied frequencies (Fig. 5(b)). Furthermore, the hybridized mode at 8.96 GHz is observed at a frequency within 2 GHz of the first-order gyrotropic mode frequency. For the dimensions (2000 nm \( \times \) 40 nm) and material parameters of the disc, the analytical dispersion relation from [19] yields an eigenfrequency of 10.69 GHz for the first order gyrotropic mode \((n = 1)\). When it is considered that the linewidth of the mode at 8.96 GHz is \( \sim 1 \) GHz, that a sizable frequency gap \((> 1 \) GHz\) opens in the spectrum as a result of hybridization [45], and that the frequencies of the azimuthal and gyrotropic modes only need be close to hybridize, the TR spatial character of the observed dynamics in Fig. 5(b) provides compelling evidence that the mode at 8.96 GHz is a hybridization of the azimuthal mode and the first-order gyrotropic mode. This idea is also supported by simulations which show evidence of the excitation of the first order gyromode profile at 8.96 GHz (Fig. 6(b)). Previous work have also shown the first order gyrotropic mode and related antisymmetric dynamical magnetization combined with a uniform profile in the core region at frequencies as low as 6.8 GHz in discs of the same thickness (40 nm) [27]. Since the motion of the azimuthal and first order gyrotropic mode must be of the same sense for hybridization to take place, their counter-clockwise curling direction (see red arrow in Fig. 5(b)) also indicates the direction of the core gyration.

To understand how the hybridization between the azimuthal mode and the gyrotropic core dynamics takes place, radial profiles of the simulated out-of-plane component of the magnetization dynamics at 6.8 GHz are shown in Fig. 7 as a function of time. The profiles in Fig. 7(a) and 7(b) were extracted from the middle layer of cells along the y- and x-direction respectively. The temporal evolution of the radial profiles reveal the onset of the spiral spin wave emission from the core, followed by the subsequent curling of the azimuthal mode. In the first 300 ps, the azimuthal mode remains spatially stationary and exhibits the expected maximum amplitude along the y-direction where the in-plane equilibrium magnetization is perpendicular to the RF field. The initial standing nature of the azimuthal mode is confirmed by the absence of its oscillation along the orthogonal x-direction at time delays \(<300\) ps, and beyond a radius of 300 nm from the core where the azimuthal mode is expected. This standing azimuthal mode exhibits almost constant amplitude across most of the disc radius in the y-direction where the equilibrium magnetization lies in-plane and orthogonal to the RF field. At the same time, and within 50 nm of the center of the disc, core dynamics with a phase difference of approximately \(\pi/2\) with respect to the azimuthal mode can be seen. These core dynamics act as the source of the radially propagating short wavelength spiral spin wave.

FIG. 6. Simulated TR images of a cross section across the thickness of the disc passing through the core region. The contrast corresponds to the normalised out-of-plane component of the dynamic magnetization in response to an in-plane excitation of 4.24 GHz (a), 8.96 GHz (b) and 10.24 GHz (c). The characteristic profile of the fundamental gyromode and that of the first higher order gyromode can be easily identified in (a), and (b) and (c), respectively. The vortex core equilibrium position is centered at x = 0 nm. The dynamic core profile of a x-z cross section obtained at 5.2 GHz is similar to that shown in (a) while the profile at 6.8 GHz is also similar to that shown in (b).
thought of as a time-delayed dragging of the azimuthal wavefront by the exchange interaction with the propagating spiral spin wave from the core. Micromagnetic simulations of a nominally identical disc, but with fixed core spins, strongly support this interpretation. When the core spins are fixed, the azimuthal mode remains as a standing mode in the absence of the core dynamics.

The emission of a short wavelength spiral spin wave from the vortex core is predicted by micromagnetic simulations over the frequency range explored experimentally. The spiral spin wave propagates away from the core as time progresses, and appears as a diagonal propagation wavefront in the space-time plots of Fig. 7 (see yellow dashed line in Fig. 7(a) and 7(b)). Marked changes in the contrast as a function of x can be observed for x between 100 nm and the spiral wave propagating wavefront (dashed yellow line). Beyond this wavefront the contrast is almost constant, which demonstrates how the spiral spin wave, emitted from the core, and interacts with the azimuthal mode and modifies its spatial character. The wavelength of the emitted spiral spin wave at 6.8 GHz is \( \sim 100 \text{ nm} \) (see Fig. 5(a)), corresponding to a k-vector of 0.06 rad nm\(^{-1}\). Experimentally it is not possible to resolve spin waves with a half-wavelength shorter than the spatial resolution of 300 nm. Furthermore, as this spin wave propagates between 0.2 \( \mu \text{m} \) and 0.5 \( \mu \text{m} \) from the core, a phase mismatch occurs between the emitted spin wave and the resulting curling motion of the azimuthal mode, see Fig. 7(b). This leads to an apparent reduction in the amplitude of the hybridized dynamics within \( \sim 300 \text{ nm} \) of the core where the spiral spin waves exhibit their largest amplitude, but are averaged to a weak net signal by the limited spatial resolution of the laser spot. Consequently, these dynamics are not spatially resolved in the experiments, which can be confirmed by applying Gaussian smoothing with a full-width-half-maximum of 300 nm to the simulated images, Fig. 5(a).

The measured TR images of the curling, spiral nature of the hybridized azimuthal mode, and the understanding of its origin from micromagnetic simulations, allows us to unambiguously confirm that short wavelength spiral spin waves are emitted from the core, propagate radially outwards, and being correlated with the curling (statistically necessary condition but not sufficient for the curling), play the role of mediators in the hybridization of the azimuthal mode with core dynamics, despite the insufficient spatial resolution for their direct observation. It should be noted that the onset of hybridization discussed with reference to Fig. 7 can only be observed in the simulations since the TR images are acquired by integrating the polar Kerr signal at each pixel for at least 1 s while the dynamics in the disc are driven through more than \( 10^9 \) cycles in that time, which averages out transient dynamics. However, the observation of the subsequent steady state dynamics, i.e. the curling of the azimuthal mode, confirms that this hybridization process has taken place.

B. High frequency regime

Previous studies have demonstrated that in addition to the core, other nanoscale regions of inhomogeneity of the equilibrium magnetic state can act as sources of high frequency spin waves \[31\]. In this work, a higher frequency excitation of 10.24 GHz, revealed magnetization dynamics that extend to the very edge of the disc. In contrast, at the lower frequencies already discussed, there is a diminution of the Kerr signal in the vicinity of the disc perimeter. This suggests that at the higher frequency of 10.24 GHz, the edge of the disc is also a source of spin waves, in addition to the core.

Micromagnetic simulations shown in Fig. 8 predict that at 10.24 GHz an antisymmetric radial mode is excited. The asymmetry is due to the opposite torque acting on the antiparallel in-plane equilibrium magnetization to either side of the core. The spatial character therefore appears as the superposition of a high order radial mode with 3 nodes (excluding the core and the perimeter of the disc) and an azimuthal mode with nodal line perpendicular to the RF field and passing through the centre of the disc \[32\]. This character is most clearly seen in Fig. 8(a) at 1.1712 ns and 1.22 ns when the spins of the core are fixed. When the spins of the core are free to precess, the radial-azimuthal mode appears to propagate from the edge of the disc \[31\] towards the center exhibiting a spiral character that curls about the centre of the disc. The TR simulated images reveal an apparent reverse in chirality (chirality indexes +1 or \(-1\)) of the spiral pattern due to the asymmetry of the radial mode, e.g. compare simulated images in Fig. 8(b) at 1.1468 ns and 1.1712 ns.

In a similar mechanism that led to the curling, spiral nature of the hybridised azimuthal mode at lower frequency, the excitation of the first order gyrotropic mode leads to a curling, spiral nature of the radial mode. This TR spatial character is due to a time-delayed dragging of the radial mode wavefronts as short wavelength, spiral spin waves propagate away from the core and towards the edge of the disc. The propagating spin wave interacts with subsequent wavefronts at increasing time delay, and with reduced coupling, as the propagating spin wave amplitude decays. The hybridized radial mode exhibits additional complexity whereby the tight spiral structure close to the core appears to separate to allow for the propagation of the next radial wavefront with opposite sign of \( m_z \), but then merges with the subsequent wavefront with the same sign of \( m_z \). The result is an apparent alternating chirality of the hybridized curling radial mode.

Fig. 9 shows the simulated amplitude of the out-of-plane component of the dynamic magnetization as a function of time extracted from three positions: approximately 70 nm (in the core region), 140 nm (at the perimeter of the core) and 210 nm (within the in-plane magnetised region) from the vortex core. The core width is approximately 300 nm, and was extracted from the simulated equilibrium vortex state (see inset of Fig. 9).
traces show that the core gyration is delayed by $\pi/4$ radians with respect to the radial wave at the perimeter of the vortex core. After $\sim 0.175$ ns, the radial wave interacts with the core and the phase delay is introduced (see black dashed line in Fig. 9).

The dynamic out-of-plane component of magnetization of the radial spin wave ($m_r$) and of the core counterclockwise gyration ($m_{\Delta g}$) at high frequencies can be naively described in polar coordinates,

$$m_r(\rho, \theta, t) = m_0 \sin(\theta) \sin(k_\rho \rho + \omega_0 t),$$  \hspace{1cm} (3) $$m_{\Delta g}(\rho, \theta, t) = m_0 \sin(\theta - \omega_0 t - \phi_0 - \phi_1) e^{-\rho/\delta_0} - 2m_0 \sin(\theta - \omega_0 t - \phi_0) e^{-\rho/\delta_1},$$  \hspace{1cm} (4)

where $\rho$ and $\theta$ are the radial and azimuthal coordinates respectively, $t$ is time, and $\omega_0$ is the angular frequency of the microwave excitation. The phase difference assumed between the core and the radial mode wavevector is $\phi_0 = \pi/4$ (identified from Fig. 9) and $k_\rho$ is the radial mode wavevector. The amplitude $m_0$ is assumed to be identical in both expressions while the core dynamics are modelled as the gyration of a ‘double-dip’ bipolar profile in $m_z$. The double dip profile has oppositely polarized regions that exist close to the moving vortex core with position described by $\delta_0 (+m_z)$ and $\delta_1 (-m_z)$, a phase difference $\phi_1$ between them [18, 25], and with a radially decaying function since it is limited to the core region. From micromagnetic simulations of the equilibrium vortex state, the core region is estimated to be approximately $|\rho| < \delta_0 = 0.15 \, \mu m$ (see inset in Fig. 9). The outermost part of the ‘double-dip’ is delayed $\phi_1 = \pi/8$ with respect to the innermost part of the profile ($|\rho| < \delta_1 = 0.06 \, \mu m$) to mimic a dragging effect around the core. Assuming superposition of both waves inside the core region ($|\rho| << \delta_0$), the final pattern exhibits a spiral-like profile that appears to change chirality with time at the centre. Fig. 8(c) shows results from this pseudo-analytical model at time frames separated by $T/4$ ns, where $T$ is the period of the microwave excitation ($T = 1/f_0$). While this analytical model does not account for the dragging effect due to hybridization, it does provide insight into the change in chirality of the spiral character of the hybridized mode (Fig. 8(b)) as it curls around the core due to the significant phase difference between the core dynamics and the radial mode.

Given a generic function in polar coordinates $f(\rho, \theta)$, the reverse of chirality can be described as the even symmetry $f(\theta) = f(-\theta)$. Through algebraic transformations and assuming that $|\rho| << \delta_0$ and $\phi_1$ is negligible, it can be trivially shown that Eq.(4) satisfies the condition

$$m_{\Delta g}(\theta, t) = m_{\Delta g}(-\theta, t + T/4),$$

only if $\phi_0 = \pi/4$. Together with numerical results from Fig. 8, this reveals that the phase delay between the radial mode and the core dynamics reverses chirality every $T/4$ ns. Fig. 8(d) shows the simulated images of Fig. 8(b) after Gaussian smoothing has been applied to reproduce the 300 nm spatial resolution of the optical measurements. While the high resolution of the core dynamics in the simulated images is lost as a result of the smoothing the smoothed

FIG. 7. The temporal evolution of the magnetization dynamics is shown along the radial (a) y- and (b) x-directions from the centre of the disc to its edge. The contrast corresponds to the out-of-plane component of the magnetization dynamics in response to the in-plane RF magnetic field with frequency of 6.8 GHz applied along the x-direction. The red dashed line highlights the time at which the curling makes one quarter of an azimuthal cycle and the yellow dashed line the wavefront of the propagating spiral spin wave emitted from the core.
FIG. 8. Simulated (a, b, and d) and measured (e) TR images corresponding to the out-of-plane component of the dynamic magnetization in response to an in-plane excitation of 10.24 GHz. In (a, b, and d) the $m_z$ component was extracted from the top layer of simulated cells. The spins in the vicinity of the core are fixed in (a) and are free to precess in (b). The simulated effect of a spatial resolution of 300 nm is shown in (d). In (c) TR images calculated from a pseudo-analytical model from Eq.(3) and Eq.(4) and separated by $T/4$ ns. Images yield an accurate reproduction of the measured images in Fig. 8(e).

The understanding gained from the analytical model and micromagnetic simulations allows us to conclude that the observed TR spatial character is a hybridization of a higher order gyrotropic mode of the core and a higher order radial spin wave in the disc. From the dispersion relation in [19], and the size and material parameters of the disc in this work, the first-order gyrotropic mode has frequency 10.69 GHz, which lies within the line width of the observed mode at 10.24 GHz (Fig. 3(b)). Simulated profiles across the thickness and through the vortex core (Fig. 6(c)) show the characteristic first higher order gyrotropic mode profile exhibiting a single node at the centre of the disc thickness and maximum amplitude of precession close to the surfaces, but with opposite phase at the opposite surfaces at the same polar coordinate. The detailed micromagnetic and analytical analysis provides evidence to support our interpretation that the experimentally imaged mode at 10.24 GHz is a hybridization of a higher order vortex radial mode with the first order gyrotropic mode of the core.

FIG. 9. Simulated TR traces for the dynamic out-of-plane component of magnetization extracted from the selected positions. Inset shows the core profile at $t = 0$ ns and the positions where magnetization is recorded as a function of time. The black dashed line highlights the time when the interaction between the radial mode and core dynamics starts.

IV. SUMMARY

We have used TRSKM to image magnetization dynamics of the vortex state in a microscale disc that exhibit curling, spiral nature as a result of the hybridization of azimuthal and radial modes with gyrotropic modes of the core. Micromagnetic simulations predict the emission of short wavelength spiral spin waves from the core that cannot be observed in the measurements due to the limited spatial resolution. However, micromagnetic simulations with frozen core spins demonstrate that the curling, spiral nature of the azimuthal and radial modes cannot be initiated without the emission of the short wavelength spin waves from the core. Therefore, the experimental observation of the curling, spiraling nature in response to a microwave excitation is indirect evidence of the emission of spiral spin waves from the core with wavelength beyond the experimental resolution. At low frequencies, the clockwise and counter-clockwise curling of the azimuthal mode hybridized with the fundamental gyrotropic mode has been observed, while at high frequency both an azimuthal and a higher-order radial mode showed evidence of hybridization with the first-order gyrotropic mode of the core. Unlike the azimuthal modes, the higher-order radial mode also showed spin wave excitation at the edges of the disc. Micromagnetic simulations reveal that the spin waves can propagate towards the core to establish the standing radial mode and form part of the hybridized radial mode dynamics at the edge of the disc. This work provides detailed insight into the hybridized nature of azimuthal and radial vortex modes with gyrotropic modes of the core, via short wavelength...
spiral wave emission from the core, which cannot be observed using a pulsed excitation alone, but by continuous excitation at microwave frequencies. These results will permit further understanding for the control of spin wave emission from the core of a vortex and their interaction with other modes of confined magnetic elements. Such understanding will be important for the design of magnetic nanotechnologies for high frequency logic and oscillator applications.

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All data created during this research are openly available from the University of Exeter’s institutional repository at https://ore.exeter.ac.uk/repository/
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