Theoretical studies of the deformability and stress state of sleepers in the subway tunnel

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Abstract. The purpose of this article is to perform the calculation of this design for the strength of the sleepers in compression according to the “The Strength and Stability Calculation Rules for Railway Track”, which has not been used before. Wooden sleepers in the subway tunnel are encased in track concrete with 2/3 of their length resting on the concrete base. Under the action of rolling stock, stresses and deformations occur in the upper track structure elements. Their dependence on the forces affecting the track is complicated and hard to define. To solve the strength calculation problem, some prerequisites were assumed, and a design diagram and model were created. The magnitude of the vertical modulus of elasticity of the rail pad of the non-ballast track on wooden sleepers in the subway tunnel, the horizontal modulus of elasticity of the track rail, the dynamic vertical pressure of the wheel on the top of the rail, as well as estimated normal stress in the sleeper under the pad were calculated to determine the vertical forces acting on the rail pad of the intermediate rail fastening. The results obtained from the calculation of the wooden sleeper using the “The Strength and Stability Calculation Rules for Railway Track” showed that its bearing capacity is ensured.

1. Introduction

In engineering practice, beams lying on a solid elastic basis are commonly used. Such structures include railway sleepers and rails, as well as continuous foundations of building on the ground etc.

Wooden sleepers in the subway tunnel are encased in track concrete and rest on the concrete base for 2/3 of their length [1]. The strength calculation of this design will be performed when the sleepers are in compressive strain according to “The Strength and Stability Calculation Rules for Railway Track” [2].

2. Formatting the title, authors and affiliations

Many scientists have been engaged in theoretical research in this field, among them professors M.F. Verigo, A.Ya. Kogan, E.I. Danilenko, V.V. Rybkin, A.N. Darenkiy, V.N. Danilov, O.P. Ershkov, M.A. Frishman, G.M. Shakhunyants, V.G. Albrekht, S.V. Amelin, T.G. Yakovleva, V.A. Shamaev, L.V. Klimenko, M.P. Sysyn, A.V. Zamuhovskiy [3-10].
Stresses and deformations occur in the upper track structure elements under the action of rolling stock. Their dependence on the forces affecting the track is complicated and hard to define accurately. Therefore, the following prerequisites have been assumed in the strength calculation rules for the railway track in the subway tunnel:

- the rail is regarded as a beam of infinite length with the constant cross-section, which freely rests on a solid equally elastic base or on individual points of equally elastic supports;
- the basic calculation of the rail is performed for the influence of vertical forces; vertical forces are assumed to be applied in the plane of symmetry of the rail, and both rails are equally loaded. Horizontal shear forces and eccentric application of vertical load are taken into account using the empirical coefficient $f$ defined previously for the corresponding rolling stock;
- the calculation is performed using the formulas which are used in static application of loads; variable dynamic forces from the estimated wheel are taken in their maximum probable value, while the variable dynamic forces from the last wheels are taken in their average values;
- contact stresses, stresses under the top of the rail and other local stresses are neglected;
- the track specifications (such as modulus of elasticity, etc.) are assumed as determined;
- it is assumed in the calculations that the wheels do not come off the rails during the movement and do not create shock impacts;
- when a weight system affects the subway track, the superposition law is used – the stresses and strains in the cross-section under consideration are summed up taking into account their magnitude and sign;
- the resultant of all vertical forces transmitted by the estimated wheel to the rail is taken as the most probable value $P_{cal} = P_{dyn}^{\text{max}}$ as a statistical set of random components with a probability level $\phi = 0.994$ (with the normalizing factor $\lambda = 2.5$);
- the impact of all other wheels of the train on the rail is taken into account by loading the influence lines of the moments $M$ and the transverse forces $Q$ with the wheel load system $P_{cal}$ and determining the equivalent loads $P_{eq}^I$ and $P_{eq}^II$;
- it is assumed that the track and rolling stock are in proper operational condition, complying with the requirements of the Railway Operating Rules and maintenance standards.

It is assumed that the strength of the upper track structure is determined primarily by the flexural strength of the rails.

3. The basic part
To solve the calculation problem, a calculation scheme and a model of the subway track should be created as shown in figure 1.

![Figure 1. The strength calculation: a – model of the subway track; b – calculation scheme.](image-url)
First, the magnitude of the vertical modulus of the rail pad $U$ of the non-ballast track on wooden sleepers in the subway tunnel should be calculated to determine the vertical forces acting on the rail pad of the intermediate rail fastening of the Subway type, for subsequent designing alternative structures of the rail pad, instead of replacing wooden sleepers (rotten and flimsy). Considering the single-layer construction of the rail pad, it can be written on the basis of Hooke's law that the stresses arising in the material $\sigma_i$ are proportional to the magnitude of the relative deformation of the material $\varepsilon_i$:

$$\sigma_i = \varepsilon_i \cdot E_i,$$  \hspace{1cm} (1)

where $E_i$ – the modulus of elasticity.

It is known that the relative deformation can be defined as the ratio of the absolute deformation $y_i$ to the initial linear size of the material $h_i$:

$$\varepsilon_i = \frac{y_i}{h_i}.$$ \hspace{1cm} (2)

If we combine expressions (1) and (2), we get:

$$\sigma_i = \frac{y_i}{h_i} \cdot E_i.$$ \hspace{1cm} (3)

However, ratio $E_i / h_i = C_i$ – module of subgrade reaction of the material. If the material or construction of the considered material does not have certain elastic characteristics $E_i$ under the action of force $D$, which creates its deformation $y = 1$ cm, then we can write:

$$D_i = C_i \cdot \omega_i,$$ \hspace{1cm} (4)

where $\omega_i$ – the bearing surface of the structure.

If there is more than one layer in the rail fastening design – a wooden pad, a rubber pad and the wooden sleeper itself, it can be concluded that it is fair to determine the total magnitude of force $D$ as follows:

$$D = \frac{1}{\sum D_i} = \frac{1}{\sum \frac{h_{wp}}{\omega_{wp} \cdot E_{wp}} + \frac{h_{rp}}{\omega_{rp} \cdot E_{rp}} + \frac{h_s}{\omega_s \cdot E_s}},$$ \hspace{1cm} (5)

where $\omega_{wp}$ – the area of the wooden pad, cm$^2$; $\omega_{rp}$ – the area of the rail pad, cm$^2$; $\omega_s$ – the area of the supporting surface of the part of the sleeper covered with the load, cm$^2$; $E_{wp}$ – the modulus of elasticity of rubber under compression, kg/cm$^2$; $E_{rp}$ – the modulus of elasticity of the plywood pad, kg/cm$^2$; $E_s$ – the modulus of elasticity of wood under compression across the grain, kg/cm$^2$; $h_{wp}$ – the thickness of the rubber pad, cm; $h_{rp}$ – the thickness of the wooden pad, cm; $h_s$ – the thickness of the wooden sleeper, cm.

The magnitude of the vertical modulus of elasticity of the rail pad of the non-ballast track on wooden sleepers in the subway tunnel without taking into account the elastic characteristics of track concrete of grade 150 and is as follows:

$$U = \frac{D}{L},$$ \hspace{1cm} (6)

where $L$ – the distance between the axis of the adjoining sleepers, m.

The magnitude of the horizontal modulus of elasticity of the rail line $U_z$ when the base of the rail and the rail pad are deformed for the magnitude of elastic squeezing-out can be determined by the formula:

$$U_z = \frac{D_s}{L},$$ \hspace{1cm} (7)
where \( D_z \) – the magnitude of the force that causes horizontal deformation of the rail line on the base of the rail due to squeezing-out of track screws, which is \( y_i = 1 \) cm.

The magnitude of force \( D_z \) is:

\[
D_z = \frac{Q_{S,R}}{y_z}.
\]  

The squeezing out resistance to of track screws is known to be 50-60% of the resistance to squeezing out of track spikes. The magnitude of the resistance to squeezing out of track spikes is 3/4 of the pullout force. Thus, the magnitude of force that squeezes out track screws can be determined:

\[
Q_{S,R} = 0.5 \cdot Q_{Sp-R}.
\]  

It is known that when horizontal forces from the subway rolling stock are acting, they are perceived only by externally located track screws.

To determine the estimated vertical load on the half-sleeper from the action of the subway rolling stock, the following is required.

1) Estimated output specifications of rolling stock.

In order to calculate the magnitude of the vertical load on the half-sleeper from the action of the subway rolling stock of Ezh modification (model 81-707), which has a two-level spring suspension, the estimated magnitude of static deflection of spring sets should be determined by the formula:

\[
F_{ct} = \frac{Q_{sys}}{K_{pec}},
\]  

where \( Q_{b} \) – the load from the car body, kN; \( S_{sp} \) – is the stiffness of the spring sets, kN/m, determined using the formula:

\[
S_{sp} = \max \left\{ \frac{8(s_{i\alpha} + s_{e\alpha}) + 4(s_{s\xi} + s_{s\eta})}{8s_{jb} + 4(s_{s\mu} + s_{s\nabla})} \right\},
\]  

where \( s_{i\alpha} \) – the stiffness of the internal spring of the axlebox link, kN/m; \( s_{e\alpha} \) – the stiffness of the external spring of the axlebox link, kN/m; \( s_{s\xi} \) – the stiffness of the external bolster suspension spring, kN/m; \( s_{s\eta} \) – the stiffness of the internal bolster suspension spring, kN/m; \( s_{jb} \) – the stiffness of the spring of the chair of the journal box, kN/m.

Static deflection of springs at static wheel load \( P_a \) is:

\[
f_{sp} = \frac{8(P_a - q)}{S_{sp}},
\]  

where \( q \) – unsprung borne load related to the wheel, kN.

The length of the rigid bogey wheelbase \( L_{sw} \), m.

2) The estimated specifications of the rail track:

- \( I_V \) – moment of inertia of the cross-section of the rail, m^4;
- \( W_B \) – the moment of resistance of the cross-section of the rail relative to the center of gravity of the rail base during bending in a vertical plane, m^3;
- \( \beta \) – the coefficient that takes into account the ratio of moments of inertia,
- \( \varepsilon \) – the coefficient that takes into account the material of the sleepers;
- \( \alpha_w^o \) – is the coefficient taking into account the ratio of the masses of the track and the wheels oscillating during the interaction:

\[
\alpha_w^o = \frac{m_w}{m_i + m_w},
\]
where \( m_w \) – is the mass of an oscillating wheel, kN·sec\(^2\)/m; \( m_t \) – is the mass of an oscillating track, \( m_t = 1.31m_w \), kN·sec\(^2\)/m; \( \alpha_1 \) – is coefficient taking into account the ratio of coefficients \( \alpha_o^{rc} \) and \( \alpha_o^w \):

\[
\alpha_1 = \frac{\alpha_o^{rc}}{\alpha_o^w},
\]

where \( \alpha_o^{rc} \) – is the fraction of the mass of an oscillating wheel during the interaction of the wheel and track on reinforced concrete sleepers (when the track oscillates with a ballast layer under sleepers \( m_t = 0.205m_w \)):

\[
\alpha_o^{rc} = \frac{m_w}{m_w + 0.205m_w};
\]

where \( \gamma \) – the coefficient that takes into account the deflection of the rail, depending on the type of ballast, which in this case is absent.

The coefficient taking into account the relative rigidity of the rail pad and the main rail is:

\[
K_z = \frac{U_z}{4EI_r},
\]

where \( E \) – the modulus of elasticity of the rail steel, kg/cm\(^2\).

To determine the dynamic vertical pressure of the wheel on the top of the rail, we use the known method according to “The Strength and Stability Calculation Rules for Railway Track” [1] with the known specifications of rolling stock and railway track in the subway tunnel.

The estimated normal stresses in the sleeper under the pad are determined using the formula:

\[
\sigma_s = \frac{Q}{\omega_p},
\]

where \( Q \) – the value of the maximum vertical shear load transferred by the rail to the pad, kN; \( \omega_p \) – is the area of the pad, m\(^2\).

\( \sigma_s = 867 \text{ kN/m}^2 < [\sigma_s] = 2200 \text{ kN/m}^2. \)

4. Conclusions
Analysis of the obtained results using the methodology given in “The Strength and Stability Calculation Rules for Railway Track” suggests that the wooden sleepers in the subway tunnel have a large margin of safety under these operating conditions.

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