STRONGLY DISTORTED BARYON WAVE–FUNCTIONS: HYPERON BETA–DECAY AND THE SPIN OF THE Λ AND THE NUCLEON

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Within the collective coordinate approach to chiral soliton models we suggest that breaking of $SU(3)$ flavor symmetry mainly resides in the baryon wave–functions while the charge operators have no (or only small) symmetry breaking components. In this framework we study the $g_A/g_V$ ratios for hyperon beta–decay as well as the various quark flavor components of the axial charge of the nucleon and the Λ–hyperon.

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1 Introduction and Motivation

Using results on the axial current matrix elements from deep–inelastic scattering as well as hyperon beta–decay data together with flavor covariance results in sizable polarizations for the non–strange quarks, $\Delta U_\Lambda = \Delta D_\Lambda \approx -0.20$ together with $\Delta S_\Lambda \approx 0.60$ for the strange quark inside the Λ–hyperon $[1]$. The assumption of flavor covariance is motivated by the feature that the Cabibbo scheme $[2]$ utilizing the $F&D$ parameterization for the flavor changing axial charges works unexpectedly well $[3]$ as the comparison in table 1 exemplifies.

Table 1. The empirical values for the $g_A/g_V$ ratios of hyperon beta–decays $[4]$, see also $[3]$. For the process $\Sigma \rightarrow \Lambda$ only $g_A$ is given. Also the flavor symmetric predictions are presented using the values for $F&D$ which are mentioned in section III. Analytic expressions which relate these parameters to the $g_A/g_V$ ratios may e.g. be found in table I of $[6]$.

| Process     | $\Lambda \rightarrow p$ | $\Sigma \rightarrow n$ | $\Xi \rightarrow \Lambda$ | $\Xi \rightarrow \Sigma$ | $\Sigma \rightarrow \Lambda$ |
|-------------|----------------------------|-------------------------|----------------------------|-----------------------------|----------------------------|
| emp.        | 0.718 ± 0.015              | 0.340 ± 0.017           | 0.25 ± 0.05                | 1.287 ± 0.158               | 0.61 ± 0.02                |
| $F&D$       | 0.725 ± 0.009              | 0.339 ± 0.026           | 0.19 ± 0.02                | 1.258 = $g_A$               | 0.65 ± 0.01                |

To account for flavor symmetry breaking effects we consider the Skyrme model approach in which baryons emerge as solitons in an effective meson theory. In such models baryon states are obtained by quantizing the large amplitude fluctuations (zero modes) about the soliton. Exact eigenstates are obtained for any strength of symmetry breaking $[6]$. We focus on a picture with the symmetry breaking mainly residing in the baryon wave–functions, including important contributions which would be missed in a first order treatment. In contrast, we assume that the current operators, from which the charges are computed, are dominated by their flavor

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covariant components. This approach approximately reproduces the data with no (or only minor) explicit symmetry breaking in the charge operators. In addition we present the results obtained from a realistic vector meson soliton model that supports the suggested picture. Details omitted here may be traced from ref \[7\].

2 Symmetry Breaking in the Baryon Wave–Functions

Here we review the collective coordinate quantization for the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in soliton models. The collective coordinates $A$ are introduced via

$$U(\vec{r}, t) = A(t)U_0(\vec{r})A^\dagger(t), \quad A(t) \in SU(3).$$  \hspace{1cm} (1)

$U_0(\vec{r})$ describes the soliton embedded in the isospin subgroup. A prototype model Lagrangian for the chiral field $U(\vec{r}, t)$ would consist of the Skyrme model supplemented by the Wess–Zumino–Witten term as well as suitable symmetry breaking pieces. We parameterize the collective coordinates by eight “Euler–angles”

$$A = D_2(\hat{I})e^{-\nu\lambda_4}D_2(\hat{R})e^{-i\rho/\sqrt{3}\lambda_8},$$  \hspace{1cm} (2)

where $D_2$ denote rotation matrices of three Euler–angles for each, rotations in isospace ($\hat{I}$) and coordinate–space ($\hat{R}$). Substituting (1) into the model Lagrangian yields upon canonical quantization the Hamiltonian for the collective coordinates $A$:

$$H = H_s + \frac{1}{4}\gamma\sin^2\nu.$$  \hspace{1cm} (3)

The symmetric piece of this collective Hamiltonian only contains Casimir operators and may be expressed in terms of the $SU(3)$–right generators $R_a \ (a = 1, \ldots, 8)$:

$$H_s = M_{cl} + \frac{1}{2\alpha^2}\sum_{i=1}^{3} R_i^2 + \frac{1}{2\beta^2}\sum_{\alpha=4}^{7} R_{\alpha}^2.$$  \hspace{1cm} (4)

$M_{cl}, \alpha^2, \beta^2$ and $\gamma$ are functionals of the soliton, $U_0(\vec{r})$. The generators $R_a$ can be expressed in terms of derivatives with respect to the ‘Euler–angles’. The eigenvalue problem $H\Psi = \epsilon\Psi$ reduces to sets of ordinary second order differential equations for isoscalar functions which only depend on the strangeness changing angle $\nu$ [6]. Only the product $\omega^2 = \frac{3}{2}\gamma\beta^2$ appears in these differential equations which is thus interpreted as the effective strength of the flavor symmetry breaking. A value in the range $5 \lesssim \omega^2 \lesssim 8$ is required to obtain reasonable agreement with the empirical mass differences for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons [8].

3 Charge Operators

In the soliton description the effect of the derivative type symmetry breaking terms is mainly indirect. They provide the splitting between the various decay constants and thus increase $\gamma$ which is proportional to $f_K^2m_K^2 - f_{\pi}^2m_{\pi}^2 \approx 1.5f_{\pi}^2(m_K^2 -$
Otherwise the derivative type symmetry breaking terms may be omitted. Whence there are no symmetry breaking terms in current operators and the non–singlet axial charge operator is parameterized as \((a = 1, \ldots, 8, i = 1, 2, 3)\)

\[
\int d^3 r A_i^{(a)} = c_1 D_{a1} - c_2 D_{a8} R_i + c_3 \sum_{\alpha, \beta = 4}^7 d_{i\alpha\beta} D_{a\alpha} R_{\beta},
\]

(5)

where \(D_{ab} = \frac{1}{2} \text{tr} \left( \lambda_a A \lambda_b A^\dagger \right)\). In the limit \(\omega^2 \to \infty\) (integrating out strange degrees of freedom) the strangeness contribution to the axial charge of the nucleon should vanish. Noting that \(\langle N | D_{83} | N \rangle \to 0\) and \(\langle N | \sum_{\alpha, \beta = 4}^7 d_{i\alpha\beta} D_{a8} R_{\beta} | N \rangle \to 0\) while \(\langle N | D_{88} | N \rangle \to 1\) for \(\omega^2 \to \infty\), we demand

\[
\int d^3 r A_i^{(0)} = -2 \sqrt{3} c_2 R_i \quad i = 1, 2, 3.
\]

(6)

for the axial singlet current because it leads to the strangeness projection, \(A_i^{(s)} = (A_i^{(0)} - 2 \sqrt{3} A_i^{(8)})/3\) that vanishes for \(\omega^2 \to \infty\). Actually all model calculations in the literature \([9, 10]\) are consistent with this requirement. In order to completely describe the hyperon beta–decays we also demand matrix elements of the vector charges. These are obtained from the operator

\[
\int d^3 r V_0^{(a)} = \sum_{b=1}^8 D_{ab} R_b = L_a,
\]

(7)

which introduces the \(SU(3)\)–left generators \(L_a\).

The values for \(g_A\) and \(g_V\) (only \(g_A\) for \(\Sigma^+ \to \Lambda e^+ \nu_e\)) are obtained from the matrix elements of respectively the operators in eqs (5) and (7), sandwiched between the eigenstates of the full Hamiltonian \([3]\). We choose \(c_2\) according the proton spin puzzle and subsequently determine \(c_1\) and \(c_3\) at \(\omega_{\text{fix}}^2 = 6.0\) such that the nucleon axial charge, \(g_A\) and the \(g_A/g_V\) ratio for \(\Lambda \to p e^- \bar{\nu}_e\) are reproduced\([4]\). We are not only left with predictions for the other decay parameters but we can also study the variation with symmetry breaking. This is shown in figure \([4]\). The dependence on flavor symmetry breaking is very moderate\([5]\) and the results can be viewed as reasonably agreeing with the empirical data, cf. table \([4]\). The observed independence of \(\omega^2\) shows that these predictions are not sensitive to the choice of \(\omega_{\text{fix}}^2\). We therefore have a two parameter \((c_1\) and \(c_3, c_2\) is fixed from \(\Delta \Sigma_N\) fit of the hyperon beta–decays. The two transitions, \(n \to p\) and \(\Lambda \to p\), which are not shown in figure \([4]\), exhibit a similar negligable dependence on \(\omega^2\). Comparing the results in figure \([4]\) with the data in table \([4]\) we see that the calculation using the strongly distorted wave–functions agrees equally well with the empirical data as the flavor symmetric \(F&D\) fit. We also observe that the singlet current does not get modified. Hence we have the simple relation \(\Delta \Sigma_N = \Delta \Sigma_A\) for all values of \(\omega^2\).

\[1\) In this section we will not address the problem of the too small model prediction for \(g_A\) but rather use the empirical value \(g_A = 1.258\) as an input to determine the \(c_n\).

\[2\) However, the individual matrix elements entering the ratios \(g_A/g_V\) vary strongly with \(\omega^2\).\]
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Fig. 1. The predicted decay parameters for the hyperon beta–decays using $\omega^2_{\text{fix}} = 6.0$. The errors originating from those in $\Delta \Sigma$ are indicated.

Fig. 2. The contributions of the non–strange (left panel) and strange (right panel) degrees of freedom to the axial charge of the $\Lambda$. Again we used $\omega^2_{\text{fix}} = 6.0$.

In figure 2 we display the flavor components of the axial charge of the $\Lambda$ hyperon. Again, the various contributions to the axial charge of the $\Lambda$ exhibit only a moderate dependence on $\omega^2$. The non–strange component, $\Delta U_{\Lambda} = \Delta D_{\Lambda}$ slightly increases in magnitude. The strange quark piece, $\Delta S_{\Lambda}$ grows with symmetry breaking since we keep $\Delta \Sigma_{\Lambda}$ fixed. Our results agree nicely with an $SU(3)$ analysis applied to the data (see above). The observed independence on the symmetry breaking does not occur for all matrix elements of the axial current. An important counter–example is the strange quark component in the nucleon, $\Delta S_{N}$. For $\Delta \Sigma = 0.2$, say, it is significant at zero symmetry breaking, $\Delta S_{N} = -0.131$ while it decreases (in magnitude) to $\Delta S_{N} = -0.085$ at $\omega^2 = 6.0$. 
4 Spin Content of the $\Lambda$ in a Realistic Model

We consider a realistic soliton model containing pseudoscalar and vector meson fields. It has been established for two flavors in ref [11] and been extended to three flavors in ref [11] where it has been shown to fairly describe the parameters of hyperon beta–decay (cf. table 4 in ref [9]). The model Lagrangian contains terms which involve the Levi–Cevita tensor $\epsilon_{\mu\nu\rho\sigma}$, to accommodate processes like $\omega \rightarrow 3\pi$ [12]. These terms contribute to $c_2$ and $c_3$. A minimal set of symmetry breaking terms is included [13] to account for different masses and decay constants. These terms add symmetry breaking pieces to the axial charge operator,

$$\delta A_1^{(\alpha)} = c_4 D_{a8} D_{8i} + c_5 \sum_{\alpha, \beta = 4}^{7} d_{i\alpha\beta} D_{a\alpha} D_{8\beta} + c_6 D_{a1}(D_{88} - 1) , \quad \delta A_1^{(0)} = 2\sqrt{3} c_4 D_{8i}.$$  

The identical coefficient $c_4$ in the octet and singlet currents arises from the model calculation, it is not demanded by the consistency condition as $\omega^2 \rightarrow \infty$.

Unfortunately the model parameters cannot be completely determined in the meson sector [11]. We use the remaining freedom to accommodate baryon properties in three different ways as shown in table 2. The set denoted by ‘b.f.’ refers to a best fit to the baryon spectrum. It predicts the axial charge somewhat on the low side, $g_A = 0.88$. The set named ‘mag.mom.’ labels a set of parameters yielding magnetic moments close to the respective empirical data (with $g_A = 0.98$) and finally the set labeled ‘$g_A$’ reproduces the axial charge of the nucleon and also reasonably accounts for hyperon beta–decay [9]. We observe that in particular the predictions

Table 2. Spin content of the $\Lambda$ in the realistic vector meson model. For comparison the nucleon results are also given. Three sets of model parameters are considered, see text.

|          | A          |          |          |          |          |          |
|----------|------------|----------|----------|----------|----------|----------|
|          | $\Delta U = \Delta D$ | $\Delta S$ | $\Delta \Sigma$ | $\Delta U$ | $\Delta D$ | $\Delta S$ | $\Delta \Sigma$ |
| b.f.     | -0.155     | 0.567    | 0.256    | 0.603    | -0.279    | -0.034    | 0.291    |
| mag. mom. | -0.166     | 0.570    | 0.238    | 0.636    | -0.341    | -0.030    | 0.265    |
| $g_A$    | -0.164     | 0.562    | 0.233    | 0.748    | -0.476    | -0.016    | 0.256    |

for the axial properties of the $\Lambda$ are quite insensitive to the model parameters. Their variation only influences the isovector part of the axial charge operator. The singlet matrix element of the $\Lambda$ hyperon is smaller than that of the nucleon. The full model calculation predicts sizable polarizations of the up and down quarks in the $\Lambda$ which are slightly smaller in magnitude but nevertheless comparable to those obtained from the $SU(3)$ symmetric analyses.

5 Conclusions

In the collective coordinate approach to chiral solitons large deviations from flavor symmetric (octet) wave–functions are required to accommodate the observed
pattern of the baryon mass–splitting. We have suggested a picture for the axial charges of the low–lying $\frac{2}{3}$ baryons which manages to reasonably reproduce the empirical data without introducing (significant) flavor symmetry breaking components in the corresponding operators. Rather, the sizable symmetry breaking resides almost completely in the baryon wave–functions. The empirical data for the parameters of hyperon beta–decay are as reasonably reproduced as in the Cabibbo scheme. We emphasize that the present picture is not a re–application of the Cabibbo scheme since in the present calculation the ‘octet’ baryon wave–functions have significant admixture of higher dimensional representations.

We may take the symmetry breaking parameter to be infinitely large. For consistency then the two flavor model for the nucleon must be retrieved. This consistency condition relates coefficients in the axial singlet current operator to the respective octet components. Disentangling the quark flavor components yields sizable up and down quark polarizations in the $\Lambda$. We also considered a realistic model, wherein the parameters entering the charge operators are actually predicted. This model calculation confirmed the results obtained in the parametrically treatment.

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