A six-quark dressed-bag description of $np \rightarrow d\gamma$ radiative capture

M. M. Kaskulov

Physikalisches Institut, Universität Tübingen, D-72076 Tübingen, Germany and
Institute of Nuclear Physics, Moscow State University, 119899 Moscow, Russia

V. I. Kukulin

Institut für Theoretische Physik, Universität Tübingen, D-72076 Tübingen, Germany and
Institute of Nuclear Physics, Moscow State University, 119899 Moscow, Russia

P. Grabmayr

Physikalisches Institut, Universität Tübingen, D-72076 Tübingen, Germany

Abstract

The radiative capture process $np \rightarrow d\gamma$ is considered within the framework of a recently developed six-quark dressed-bag model for the nucleon-nucleon interaction. The calculations presented here include both the nucleon current and the meson-exchange current contributions. The latter uses short-range hadronic form factors for the pion exchange currents consistent with the soft cut-off parameter $\Lambda_{\pi NN}$ from the $NN$-potential. Contributions of the pion exchange current and $\Delta$-isobar current to the total cross section still cannot explain the discrepancy between the theoretical and experimental cross sections. Possibilities for new types of meson exchange currents associated with chiral fields inside multi-quark dressed-bag states in nuclei are discussed.
INTRODUCTION AND MOTIVATION

Radiative capture of thermal neutrons by hydrogen, $^{1}\text{H}(n, \gamma)^{2}\text{H}$, is now a well-studied reaction which is of prime importance in both astrophysical applications and in fundamental nuclear physics. This process is one of the simplest classical nuclear interactions involving an electromagnetic (e.-m.) “probe”, and thus it is doubly interesting due to its close relationship to $180^\circ$ inelastic electron-deuteron scattering near threshold, as well as to deuteron photodisintegration $\gamma d \rightarrow np$. The primary interaction is of e.-m. nature, and thus it is well understood; the leading effects of strong interaction occurring between two nucleons are also well characterised. Therefore, observables for this reaction, if measured with sufficient precision, can provide a sensitive test to phenomena associated with, for example, the role of sub-nucleonic degrees of freedom.

The numerous measurements done in previous years have demonstrated that the accepted values for $\sigma_{np}(1\text{H})$ are free of systematic errors and without masking due to competing processes. The modern accepted value of the $np \rightarrow d\gamma$ cross section for thermal neutron capture $^{1}\text{H}$ at a neutron velocity of 2200 m/sec is

$$\sigma_{np}(1\text{H}) = 334.2 \pm 0.5 \text{ mb}. \quad (1)$$

We will compare this experimental result with theory.

For many years a 10% discrepancy has existed between experiment and theoretical calculations for the total radiative thermal $np$ capture cross section, $np \rightarrow d\gamma$. The discrepancy was first pointed out by Austern $^{2}$ as early as in 1953 and has been repeatedly verified in the literature since then. The successful explanation of the cross section in terms of meson exchange currents (MEC) given three decades ago by Riska and Brown $^{3}$ was considered as one of the cornerstones of mesonic degrees of freedom in nuclear physics. Using a realistic hard-core wave function for the deuteron, these authors computed the two diagrams with one-pion exchange (OPE) initially suggested in 1947 by Villars $^{4}$ plus $\omega$ and $\Delta$ resonance diagrams. Although suspected since the Yukawa force was introduced, the work of Riska and Brown was the first evidence for the explicit role of mesons, in particular that of pions, in nuclear interactions. In recent years the status of meson-exchange currents in the deuteron, including the $np \rightarrow d\gamma$ reaction, has been discussed exhaustively by Arenhövel $^{5}$, Mathiot $^{6}$ and Riska $^{7}$.

However, apart from quantitative calculations showing the sensitivity of the cross section to the $NN$ wave function (at a level of 1.5%) $^{8}$, short-range contributions to exchange currents (π$NN$ form factor, ρ-exchange currents) have been studied only recently. These contributions appear to be of some importance, especially in the light of current pictures of the quark substructure of the nucleon.

In the framework of chiral perturbation theory ($\chi$PT) it has been shown that the short-range interaction is suppressed in the exchange current contributions $^{9}$. From this the conclusion was reached $^{10}$, that only soft one-pion exchange terms contribute to the two-body current. This means that all short-range hadronic processes and higher order corrections are cancelled out to rather high accuracy for the magnetic transitions leaving only the pion degree of freedom — “a chiral filter”. This important conclusion shifted the problem of sensitivity of all the MEC-contributions to the values of cut-off parameters $\Lambda_{\pi NN}, \Lambda_{\rho NN}, \Lambda_{\pi N\Delta}$ etc. used in current theories for MEC. In fact, all modern one-boson exchange (OBE)-models suggested up to date for the description of the $NN$-interaction employ rather high cut-off values $\Lambda_{\pi NN} \simeq 1.5$ GeV $^{11}$ and thus the currently accepted approaches for MEC also
include similarly high values of cut-off parameters. Meanwhile, all consistent theories for \( \pi - N \) interactions and also numerous experimental data result unambiguously in soft cut-off parameters around 600 MeV/c [12]. However, these soft cut-off values cannot be matched with the quantitative description of the \( NN \)-interaction within the OBE-model [13, 14]. Moreover these soft cut-offs, being in agreement with the above chiral filter ideas, lead un-avoidably to a significant reduction of all the MEC-contributions to the \( np \rightarrow d\gamma \) process, and thus there appears again some visible disagreement between the experimental data and current theoretical estimations for MEC-contributions.

In the present work we try to solve this puzzle by use of a new \( NN \) microscopic force model developed recently jointly by the Moscow-Tübingen (MT) group [15, 16]. Contrary to meson-exchange models, the new model includes soft cut-off parameter values only, and thus it is in agreement with the above “chiral filter” ideas while providing a quantitative and accurate description of \( NN \)-data [16].

Another specific feature of the new force model is the fact that mesonic and quark degrees of freedom are included into the model in a consistent and explicit form, so that the total wave function of the \( NN \) system is described as a Fock row:

\[
\Psi(NN) = \begin{pmatrix}
\Psi_{NN} \\
\Psi_{6q+\sigma} \\
\Psi_{6q+2\pi} \\
\Psi_{6q+\rho} \\
\vdots
\end{pmatrix}
\]  

(2)

here the second and higher components are of quark-mesonic nature and are essential at short and intermediate \( (r < 1 \text{ fm}) \) ranges. On the other hand, it would be very interesting to check whether there is a non-negligible contribution to the \( np \rightarrow d\gamma \) reaction at low energies from the region \( r < 1 \text{ fm} \), especially for meson exchange-current contributions. Another interesting problem to study here is the role of “inner” tensor force introduced within the new \( NN \)-model [15, 16]. This specific short-range tensor force is based on the symmetry of the six-quark wave functions and leads also to some enhancement of the \( D \)-wave function of the deuteron in the asymptotic region. This modification should affect the radiative capture cross section.

To answer the above key questions, we present in this paper a consistent \textit{ab initio} calculation for the cross section of the \( np \rightarrow d\gamma \) reaction with thermal neutrons using the new \( NN \)-force model.

The structure of the article is the following: section 2 is dedicated to the general features of \( np \)-radiative capture and its relationship to the underlying \( NN \)-force model. In section 3 we present a brief description of the new force model. The deuteron structure emerging from the new \( NN \)-model is discussed in section 4. The central section 5 is devoted to the formalism for e.-m. currents within the quark-meson force model while the calculational results are presented in section 6. In the concluding section 7 we discuss the results obtained and outline general perspectives of our new approach.

**GENERAL FEATURES OF \( np \) CAPTURE**

The main features of low energy \( np \) capture are generally supposed to be well understood [17]. We list here the basic points specific to this process.
At threshold the neutron is captured predominantly from an $l = 0$ state. This is to be expected due to dominant $S$-wave component of the deuteron wave function. This statement was verified by explicit calculations reported by Adler [18] and Jankus [19]. Moreover, it was found that the dominant transition proceeds from the $np \, ^1S_0$ initial state, while the $^3S_1$ state is assumed to be of little importance. Since the deuteron ground state is dominantly $^3S_1$, the transition involves a change in total angular momentum of $\Delta J = 1$ and thus it is a $M1$ transition.

The transition from the $^1S_0$ to the $^3S_1$ component of the deuteron wave function dominates the impulse approximation [20]. Since the continuum $^1S_0$ state is of isovector while the deuteron is of isoscalar nature, the dominant $^1S_0 \rightarrow d$ transition has an isovector character and thus the transition involves an isospin-flip amplitude; the emitted photon has $I = 1$. The isovector magnetic moment for the two-nucleon system, $(\mu_p - \mu_n)$, can be used to simplify the calculations. Similarly the $^3S_1 \rightarrow d$ transition is isoscalar and involves the isoscalar magnetic moment, $(\mu_p + \mu_n)$, and isoscalar charge.

The problem under question is closely related to both deuteron photo- and electro-disintegration. Indeed, the magnetic photo-disintegration is merely the time-reversed process to the $np \rightarrow d\gamma$ reaction and thus involves precisely the same matrix elements and transition currents as the latter reaction. The deuteron electro-disintegration requires a broader knowledge of the transition current process; more precisely, the relevant magnetic transition current involves a value of momentum-transfer squared, $q^2 = 0$. The radiative capture process is accompanied by emission of a $\gamma$-quant with $q^2 = \omega^2 - q^2 = 0$ where $|q|$ equals the deuteron binding energy, $|q| = |q|$. Thus, the current we are dealing with here is merely a special case of the currents involved in electro-disintegration.

The expression for the cross section can be divided into the phase space and kinematical factors and into the current transition matrix element which describes the dynamics:

$$d\sigma_{fi} = \frac{\delta(P_f - P_i)}{|v_n - v_p|} \frac{1}{(2\pi)^2 2\omega \gamma} \frac{1}{E_p E_n E_d} |M_{fi}|^2 d^3p_d d^3q_\gamma$$

where the transition amplitude takes the form:

$$M_{fi} = \langle \psi_d | \vec{j}(q) \cdot \vec{\epsilon}_\lambda | \psi_{pn} \rangle$$

with $\vec{j}(q)$ for the hadronic e.-m. current and $\vec{\epsilon}_\lambda$ stands for the photon polarisation vector. $\psi_d$ and $\psi_{pn}$ are the initial and final state wave functions.

THE EFFECTIVE $NN$-INTERACTION IN THE DRESSED-BAG STATE MODEL

The long-range part of the $NN$-potential is certainly the most accurately known and it is generated from the exchange of $\pi$-mesons. This part of the potential provides a rather good description of the static properties of the deuteron, and gives rise to a noticeable $D$-wave component in the deuteron wave function. The structure of the $NN$-interaction at intermediate and especially at short distances is still not well understood. In conventional models it is assumed to be generated from two-and three-pion exchange, although such a construction is very complicated. Moreover, this part is fundamental in the treatment of the nucleon structure since the $\pi$-mesons are regarded as the Goldstone bosons associated with spontaneous breaking of chiral symmetry. A complete account of the physics related to the pion degrees of freedom can be found in ref. [21].
The most natural way to take into account the multiple \( \pi \)-exchange is to use dispersion relation techniques. The important ingredients are the correlated and uncorrelated \( 2\pi \)-exchange contributions. The part which comes from uncorrelated \( 2\pi \)-exchange involves isobar components in the intermediate states; both crossed and uncrossed diagrams contribute. Another part comes from pion rescattering and is included through the \( \pi\pi \) scattering amplitude to the \( t \)-channel. It was assumed in previous years [13], that the \( 2\pi \)-exchange amplitude with \( \pi - \pi \) resonance-like interaction in the \( s \)-channel and \( \Delta \)-isobar intermediate states can be parametrised, in first order, by the exchange of heavier scalar-isoscalar mesons. This gave rise to the concept of the scalar-isoscalar \( \sigma \) meson exchange. The \( NN \)-potential constructed in this way is momentum- (and/or energy-) dependent. This is certainly not very surprising since at very short distances the potential has no reason to be local. In the literature we can find several modern parametrisations of the \( NN \)-potential (see e.g. refs. [11, 12, 13, 22]); they may be classified according to their description of the medium and short-range part. However, it has been demonstrated independently by a few groups that the exchange of a correlated \( 2\pi \) pair in the scalar-isoscalar channel cannot give any significant attraction between two nucleons sufficient to couple them to a deuteron or to produce nuclear binding (see e.g. ref. [23]). This failure has perhaps a fundamental character for our understanding of the nuclear force, and thus could mean some deep revision of the force at intermediate distances.

Another fundamental problem with the current status of the force is the short-range behaviour of the \( NN \)-interaction in terms of meson-nucleon form factors. In fact, at short distances (\( r < 0.8 \) fm), all these potentials are regularised in either a completely phenomenological way (e.g. as in the case of the Paris \( NN \)-potential) or by phenomenological form factors at each meson-nucleon vertex with cut-off parameters fitted to \( NN \)-phase shifts [13]. This short-range part of the \( NN \)-potential is indeed very crucial in nuclear physics since it gives rise to a strong repulsion between the nucleons and leads in particular to the saturation of nuclear matter in the usual non-relativistic many-body formalism. However the cut-off parameter values \( \Lambda_{\pi NN}, \Lambda_{\rho NN}, \Lambda_{\pi N\Delta} \) etc., accepted in current OBE-models for \( NN \)-potentials, e.g. \( \Lambda_{\pi NN} \approx 1.5-1.7 \) GeV, exceed the respective cut-off parameter values derived independently from all microscopic and dynamical models for \( \pi N \) interaction, as well as the values inferred from experiment, \( \Lambda_{\pi NN}^{\text{mono}} \approx 0.5 - 0.7 \) GeV.

There is another serious problem with the short-range \( NN \)-interaction which is attributed mainly to \( \omega \)-meson exchange. The \( \omega NN \) coupling constant \( g_{\omega NN}^2/4\pi \), providing a good fit to \( NN \)-scattering data, i.e. \( g_{\omega NN}^2/4\pi = 13/15 \), exceeds three times the value dictated by \( SU(3) \)-symmetry [12], while all remaining couplings are still in agreement with this symmetry. Moreover, Feshbach has demonstrated that the \( SU(3) \) symmetry should be a rather good one in this region (see the detailed discussion of the above mentioned disagreements and inconsistencies in OBE-models in ref. [12]). These strong disagreements peculiar to the current OBE-models require a search for another model for the short-range behaviour of \( NN \)-potentials. Such an alternative model for the \( NN \)-force has been developed recently [13, 16] by the Moscow-Tübingen group.

In conventional models the \( NN \)-potential appears when the mesonic degrees of freedom are eliminated explicitly from the nuclear wave function. Contrary to this, in the new model the main part of the interaction appears when mesonic degrees of freedom are included explicitly. The key element of the new approach is a dense six-quark droplet surrounded by strong meson-fields, Fig. 1. Naturally, such a bag has been called as Dressed Bag while the respective driving mechanism for the \( NN \)-force in the model is the Dressed-Bag-State (DBS).
FIG. 1: The graph illustrates two sequential \( \pi \)-meson emission and absorptions via an intermediate \( \sigma \)-meson and the generation of a six-quark bag.

Contrary to the conventional OBE-models, here the mesonic degrees of freedom are included in an explicit form. Thus the total wave function of the \( NN \) system should be presented in form of eq. (2). The \( NN \) wave function can be generalised further with respect to the first version of the model of ref. [16] by the important configurations which incorporate charged \( 2\pi \)-clouds around the six-quark core (see Figs. 2a and 2b).

The channel couplings between different components are calculated from a quark-meson microscopic model [16]. Typical diagrams representing the above couplings have a structure shown in Figs. 2a and 2b, where the \( \sigma \)-field represents the scalar-isoscalar channel, while the \( s \)-channel \( 2\pi \)-exchange (shown in Fig. 2b) is dominated by the isovector combination of two \( \pi \)s.

The \( t \)-channel (external) \( \pi \)- and \( 2\pi \)-exchanges are assumed to be usual Yukawa-type interactions with soft \( \Lambda_{\pi NN} \) cut-offs, which represent the peripheral parts of the \( NN \)-force. These soft cut-off form factors provide very natural separation between \( t \)-channel Yukawa-like (i.e. peripheral) \( \pi \)- and \( 2\pi \)-exchanges and \( s \)-channel \( \sigma \)-, \( \rho \)- and \( 2\pi \)-exchanges localised in the bag region, i.e. at \( r < 1 \) fm. The attractive interaction in the \( NN \) channel in the DBS approach comes mainly from the \( \sigma \)-field (or \( \sigma \)-exchange in the \( s \)-channel) generated in the intermediate dense six-quark bag. The dominating dressing in even \( NN \)-partial waves (\( S, D, \cdots \)) is the \( \sigma \)-meson field while in odd partial waves this will come from the \( \rho \)-meson field. It is also important that the initial and final \( NN \)-components (in \( S \)- and \( D \)-waves) are described by mixed symmetry six-quark states \( |s^2p^2[42]_x; L = 0, 2, ST \rangle \) whereas the intermediate bag has a completely symmetric structure \( |s^6[6]_x; L = 0, ST \rangle \); both six-quark components being orthogonal to each other. Thus, the effective interaction in the \( NN \) channel will result after elimination of other Fock-components in the row (eq. 2) except \( \Psi_{NN} \). The effective one-channel Hamiltonian for the \( NN \)-component takes the form [15, 16]:

\[
\mathcal{H}_{eff} = H_0 + V_{OPE} + V_{TPE} + V_{NqN} + \lambda_{ort} \Gamma.
\]  

(5)

Here \( V_{OPE} \) and \( V_{TPE} \) are the \( \pi \)- and \( 2\pi \)-exchange potentials in the \( t \)-channel taken with the soft cut-off parameter \( \Lambda_{\pi NN} \simeq 0.5 \) GeV. \( V_{NqN} \) is the main part of the interaction.
at intermediate ranges, which is generated due to the intermediate DBS-production with dominating $\sigma$-meson dressing of the bag and includes inevitably an energy dependence which is the consequence of the elimination of some channels from the many-channel problem. The orthogonality pseudo-potential $\lambda_{\text{ort}} \Gamma$ with $\lambda_{\text{ort}} \to \infty$ (in practice the value of $\lambda$ is usually taken about $10^6$ MeV$\cdot$ fm$^{-2}$) and

$$\Gamma = |0s\rangle\langle 0s|$$

is the projection operator onto the $|0s\rangle$-state in the $NN$ channel, where the $\Gamma$ operator excludes the $|s^6\rangle$ six-quark component from the initial $NN$ channel. As form factors $|0s\rangle$ we use the h.o. function with the common radius $r_0$:

$$|0s\rangle \equiv \frac{2}{\sqrt{\pi}} r_0^{1/2} r_0^{3/2} e^{-r_0^2/2r_0^2}.$$  (7)

The $NN$-interaction in the fully symmetric six-quark state $|s^6\rangle$ (which is undressed here) is strongly repulsive $^{24}$ both in One-Goldstone-Boson (OGB)-exchange $^{25, 26}$ and in One-Gluon-Exchange (OGE)-models $^{27, 28, 29}$ (for $q-q$ interaction), and thus we replace this highly non-local $NN$-repulsion by the repulsive pseudo-potential $\lambda_{\text{ort}} \Gamma$ in a separable form. The forms for $V_{\text{OPE}}$ and $V_{\text{TPE}}$ adopted in this work are given in the Appendix to this paper.

The effective $NN$-interaction $V_{NqN}$ induced by the intermediate DBS generation has the following form:

$$V_{NqN} = \lambda_{000}^0(E)|2s\rangle\langle 2s|,$$  (8)

for the singlet $^1S_0$ channel. For the triplet $^3S_1 - ^3D_1$ coupled channels we have

$$V_{NqN} = \begin{pmatrix} \lambda_{100}^1(E)|2s\rangle\langle 2s| & \lambda_{101}^1(E)|2s\rangle\langle 2d| \\ \lambda_{120}^1(E)|2d\rangle\langle 2s| & \lambda_{122}^1(E)|2d\rangle\langle 2d| \end{pmatrix}.$$  (9)

The bag radius is taken as $b = 0.5$ fm and relates to h.o. radius $r_0 = \sqrt{2/3} b$. The standard normalised h.o. wave functions in the $NN$ relative motion $|2s\rangle$ and $|2d\rangle$ are formulated as follows:

$$|2s\rangle \equiv \sqrt{4\pi}(\pi r_0^2)^{-3/4} \sqrt{\frac{3}{2}} \left(1 - \frac{2r_0^2}{3r_0^2}\right) e^{-r^2/2r_0^2},$$  (10)

$$|2d\rangle \equiv \sqrt{4\pi}(\pi r_0^2)^{-3/4} \sqrt{\frac{4}{15}} \left(\frac{r^2}{r_0^2}\right) e^{-r^2/2r_0^2}.$$  (11)
The energy dependent coupling constants $\lambda^J_{SLL'}(E)$ in eqs. (8) and (9) are calculated from microscopic $6q$- and $3P_0$-quark pion coupling models [16] and then are compared with those found from a fit to $NN$ phase shifts, Fig. 3 from zero energy up to 1 GeV. An impressive agreement for all relative values $\lambda^{J}_{SLL'}(0)$ was found. The energy dependence for $\lambda^J_{SLL'}(E)$ has been parameterised in ref. [16] in the Padé-approximant form

$$\lambda^J_{SLL'}(E) = \lambda^J_{SLL'}(0) \frac{E_0 - aE}{E_0 - E}. \quad (12)$$

However, the energy dependence $\lambda^J_{SLL'}(E)$ derived from the fit occurred to be somewhat stronger compared to the theoretical predictions. In the present work we use the values and the energy dependencies for $\lambda^J_{SLL'}(E)$ as tuned from the fit.

The parameters for $H_{eff}$ in eq. (5), which provide a very good fit to $NN$-phase shifts from zero energy until as high as 1 GeV for $3S_1 - 3D_1$ and $1S_0$-channels, are given in Table II.

It should be stressed that the range and strength parameter values displayed in Table II are not only in good agreement with the values derived from the quark-meson microscopical calculations and empirical high-energy $NN$ phase shifts, but the same values of the parameters result in a perfect agreement for effective range parameters both in triplet and singlet channels; they also yield very good predictions for the deuteron structure (see below).
TABLE I: The potential parameters found by fitting the $NN$ scattering phase shifts up to 1 GeV (Ref. [16]).

| $r_0$ | $r_2$ | $\lambda_{00}$ | $\lambda_{122}^1$ | $\lambda_{102}^1$ | $E_0$ | $a$ | $V_{TPE}^0$ | $\beta$ |
|-------|-------|-----------------|-------------------|-------------------|------|----|-----------|------|
| fm    | fm    | MeV            | MeV              | MeV              | MeV  | MeV| MeV      | fm   |

$^3S_1 - ^3D_1$

| $0.41356$ | $0.59423$ | $-328.55$ | $-15.65$ | $-44.06$ | $693$ | $-0.05$ | $-4.0573$ | $0.5$ |

$^1S_0$

| $0.430$ | - | $-328.9$ | - | $693$ | $-0.05$ | $-8.803$ | $0.6441$ |

THE DEUTERON STRUCTURE IN THE DBS MODEL

For this work on the deuteron we restrict ourselves to two components of the full Fock column eq. (2), which can be represented by:

$$
\Psi_d = \begin{pmatrix}
\Psi_{NN} \\
\Psi_{6q+\sigma}
\end{pmatrix}
$$

(13)

The additional configurations generate new currents, which will be presented in another paper, however they are of no relevance here. According to eq. (13), the deuteron can be found in two different phases:

1) in the $NN$ cluster-like phase, in which its state is described in terms of the $NN$-relative motion variable $r$, total spin $S_{NN} = S = 1$, isospin $T_{NN} = T = 0$, and the total angular momentum $J_{NN} = J = 1$;

2) in the $6q + \sigma$ quark-meson phase, where one must use the dressed bag variables, i.e. the quark coordinates $r_1$, $r_2$, ... $r_6$ together with their spins, colours, isospins, the total spin of the six-quark system $S_{6q}$, and also the momentum of the $\sigma$-meson $k$ and the total angular momentum of the bag $J_B = J = 1$.

The first component describes the arrangement of the six quarks into nucleon clusters with the wave function given by the resonating group method (RGM) [55]:

$$
\Psi_{NN} = A \left[ \psi_N(123) \psi_N(456) \chi_{NN}(r) \right]
$$

(14)

where $r \equiv (r_1 + r_2 + r_3)/3 - (r_4 + r_5 + r_6)/3$. The $\psi_N(i,j,k)$ is the nucleon wave function:

$$
\psi(1,2,3) = \varphi(\xi_1, \xi_2) \left[ [1^3]C S_{3q}, ([21]CS) T_{3q} : [1^3]CST \right]
$$

(15)

and the relative motion function $\chi_{NN}(r)$ has the following angular decomposition:

$$
\chi_{NN}(r) = \frac{u(r)}{r} Y_{1100}^{M_0}(\hat{r}) + \frac{w(r)}{r} Y_{1120}^{M_0}(\hat{r})
$$

(16)
with

$$J_{JSLT}^{M_J,M_T}(\hat{r}) = \sum_{M_L,M_S} (LM_LSM_S|JM_J)Y_{LM_L}(\hat{r})|SM_S|TM_T)$$ (17)

and $|SM_S\rangle$ and $|TM_T\rangle$ are the spin and isospin functions for the two-nucleon deuteron state.

The DBS-component in simplest approximation (i.e. no additional pion loops are considered) can be described by the Yukawa-like wave function

$$\Psi_{\varphi_6} = N \varphi_6 \cdot \chi(r_\sigma)$$ (18)

where $N$ is the normalising factor. We use the abbreviations $|\varphi_6\rangle = |s[6],L = 0, ST\rangle$ and $\chi(r_\sigma) \sim \exp(-\mu_\sigma r_\sigma)/\mu_\sigma r_\sigma$, with $r_\sigma$ being the distance of the $\sigma$-meson from the six-quark core. The $\sigma$ mass is not fixed strictly in this model because it represents a quasi-particle defining the quantum numbers of the scalar-isoscalar channel. It depends slightly upon the particular system and the type of the process under investigation. Thus, $\mu_\sigma \simeq 400$ MeV [30] is a sensible choice.

**TABLE II: Deuteron properties in different models.**

| Model | $E_d$(MeV) | $P_d$(%) | $r_m$(fm) | $Q_d$(fm$^2$) | $\mu_d$(µN) | $A_S$(fm$^{-1/2}$) | D/S |
|-------|------------|----------|-----------|--------------|-------------|----------------|-----|
| RSC   | 2.22461    | 6.47     | 1.957     | 0.280        | 0.8429      | 0.8776         | 0.0262 |
| AV18  | 2.2245     | 5.76     | 1.967     | 0.270        | 0.8521      | 0.8850         | 0.0256 |
| Moscow 99$^a$ | 2.22452 | 5.52     | 1.966     | 0.272        | 0.8483      | 0.8844         | 0.0255 |
| Bonn 2001 | 2.22458 | 4.85     | 1.966     | 0.270        | 0.8521      | 0.8846         | 0.0256 |
| DBS model (NN component only) | 2.22454 | 5.42     | 2.004     | 0.286        | 0.8489      | 0.9031         | 0.0259 |
| DBS model (NN + 6q(3.66%)) | 2.22454 | 5.22     | 1.972     | 0.275        | 0.8548      | 0.8864         | 0.0264 |
| Experiment | 2.2245475(9)$^b$ | 1.971(3)$^c$ | 0.2859(3)$^d$ | 0.857406(1)$^e$ | 0.8846(44)$^f$ | 0.0264$^g$ |

$^a$) Ref. [31]; $^b$) Ref. [32]; $^c$) Ref. [33]; $^d$) Refs. [34, 35]; $^e$) Ref. [36]; $^f$) Ref. [37]; $^g$) Ref. [38]

Having chosen all the parameters of the effective Hamiltonian $\mathcal{H}_{eff}$ in the $NN$ channel, the deuteron two-phase structure can be derived with no additional parameters. In Table II the main static properties of the deuteron as emerging from the above $NN$-model are compared to experiment and to predictions of other modern $NN$-force models.

The weight of the DBS-component in the deuteron wave function, as calculated from the above model, is 3.66% [56]. We emphasise here that the six-quark component weight in this model is a direct consequence of the six-quark model parameters established either
FIG. 4: Deuteron wave functions $u(r)$ and $w(r)$ calculated in the six-quark dressed bag model.

theoretically or from the fit to $NN$ phase shifts. And thus this weight cannot be varied or fitted to other data. This weight looks to be quite significant and it is within the limits found with other hybrid models. A reasonable variation of the radius parameter $r_0$ for the 6q-core in connection with the readjustment of the strength $\lambda$ does not result in any significant change of the weight for the DBS component in the deuteron. This holds also for other observables. The level of agreement between theoretical predictions of this model and the most accurate experimental data for deuteron observables is not worse than for the best modern $NN$-potentials of conventional type. Our results for the $np \rightarrow d\gamma$ process presented in the paper can be fully compared with those of other modern $NN$-models.

It is also of interest to compare the form of the deuteron wave functions predicted by our model and traditional ones; see Fig. 4 for both the $S$- and the $D$-wave components. It is evident from Fig. 4 that the DBS model generates a deuteron $S$-wave function with a node at short distances ($r \sim 0.5 \div 0.6$ fm). This node is a consequence of the requirement of orthogonality between all $S$- (and $P$-) wave functions in the $NN$ channel and the quark part of the DBS-state, projected onto the $NN$ channel. There is also a rather small short-range loop in the deuteron $D$-wave component of DBS-wave function.

Thus, two-component wave functions are present in our approach, both in the initial and in the final (i.e. deuteron) channel. It should be stressed, that both components have a different structure and both will contribute to the $np \rightarrow d\gamma$ transition amplitude, and the contribution of the DBS-component will by no means be negligible. Our main concern in this paper is the comparison between predictions of our model and the conventional $NN$-potential models, with the inclusion of both the nucleon currents and MEC-contributions. Because the conventional models have no other contributions we postpone the study of the new currents inside the DBS-model to future work. We present in this paper the comparison with traditional OBE-models in the $NN$-sector and the conventional MEC-operators.

The wave functions of the $np$ continuum and the deuteron are obtained by solving the Schrödinger equation with $H_{eff}$ from eq. 3. In the present calculation the same potential model is used to determine both the initial scattering state as well as the deuteron wave function.
GENERAL TRANSITION ELECTROMAGNETIC CURRENTS

From a quantitative point of view, the relationship between the nuclear wave function, the NN-potential and e.-m. operators is expressed via a continuity equation, which ensures that the total current is conserved (global gauge invariance). The e.-m. current operator \( \mathbf{j} \) of a nuclear many-body system is related to the Hamiltonian of the system by the continuity equation

\[
\nabla \cdot \mathbf{j} + i [H, \rho] = 0,
\]

where \( \rho \) is the charge operator of the system. So that once the total system Hamiltonian is known, eq. (19) gives the current operator \( \mathbf{j} \) for the meson-exchange-current (MEC) calculations.

The contribution of MEC to e.-m. processes on nuclei constitutes an important manifestation of meson degrees of freedom for charged mesons mediating the strong interactions between nucleons in nuclei. They participate in the interaction of an external e.-m. field with a nucleus and result formally in the appearance of nuclear currents in the form of two- or many-body current operators. So they are intimately linked to the underlying NN-interaction. However, for a given NN-potential there does not exists an a-priori way of constructing the appropriate MEC-operator, unless the NN-potential is derived from an underlying more fundamental field-theoretical framework with explicit incorporation of sub-nuclear degrees of freedom. Although the existence of exchange currents associated with an NN-potential has been acknowledged for a long time, the early realistic potentials, being phenomenological to a large extent, prevented thus the consistent consideration of such exchange currents [57].

The situation is complicated by the fact that the e.-m. current operator contains irreducible two-body exchange current components, which through the continuity equation depend on the potential model. Therefore, consistent gauge invariant calculations require in general, that the same potential model must be used to generate both the wave functions and the exchange current operators.

The full hadronic current operator has the following matrix form in the DBS model:

\[
\hat{\mathbf{j}}_\mu(x) = \begin{pmatrix}
\hat{j}^{NN}_\mu(x) & \hat{j}^{NN\rightarrow DBS}_\mu(x) \\
\hat{j}^{DBS\rightarrow NN}_\mu(x) & \hat{j}^{DBS}_\mu(x)
\end{pmatrix}
\]

where the diagonal currents

\[
\begin{align*}
\hat{j}^{NN}_\mu(x) &= \hat{j}^{IA}_\mu(x) + \hat{j}^{MEC}_\mu(x) + \hat{j}^{IC}_\mu(x) \\
\hat{j}^{DBS}_\mu(x) &= \hat{j}^{cloud}_\mu(x) + \hat{j}^{6q}_\mu(x)
\end{align*}
\]

(20)

(21)

(22)

(23)

(24)

correspond to the interaction of an external e.-m. field with the \( NN \) and the Dressed Bag, respectively. The third term in eq. (21) corresponds to the isobar current (IC) associated with the intermediate \( \Delta(1232) \) excitation. The transition (non-diagonal) currents \( \hat{j}^{NN\rightarrow DBS}_\mu \) or \( \hat{j}^{DBS\rightarrow NN}_\mu \) describe the interphase transitions induced by the external e.-m. field.

We consider here only that part of the hadronic e.-m. current operator \( \hat{j}^{NN}_\mu \) given by eq. (21), which is associated with cluster-like nucleon-nucleon configurations.
FIG. 5: One-body e.-m. current in IA which is constructed in the DBS model on the basis of the diagrams presented in Fig. 2. Region A corresponds to the generation of the initial $NN$ scattering wave function: in the region B, the emission of a $\gamma$-quantum by a single nucleon takes place and leads to the $M1$ transition; region C generates the final deuteron wave function.

Impulse approximation

The simplest description of nuclei is based on a nonrelativistic many-body theory of interacting nucleons. Within this framework, the nuclear e.-m. operators are expressed in terms of those associated with the individual protons and neutrons — the so-called impulse approximation (IA). In this approximation the one-body nuclear e.-m. $\rho(\vec{q})$ and $\vec{j}(\vec{q})$ operators are obtained from the covariant single-nucleon current [39]:

$$j^\mu = \bar{u}(\vec{p}′) \left[ F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} \right] u(\vec{p}), \quad (23)$$

where $\vec{p}$ and $\vec{p}'$ are the initial and final momenta, respectively, of a nucleon of mass $m_N$, and $F_1(Q^2)$ and $F_2(Q^2)$ are its Dirac and Pauli form factors taken as a function of the four-momentum transfer:

$$Q^2 = -q_\mu q^\mu, \quad q_\mu = p'_\mu - p_\mu. \quad (24)$$

The current $j_\mu$ is expanded in powers of $1/m_N$ and, including terms up to an order of $1/m_N$, the nuclear current operator can be written as a sum of one-body current operators which takes the form [40]:

$$\vec{j}(\vec{q}) = \sum_i \frac{1}{4M_N} \left[ G_{E}^S(Q^2) + G_{E}^V(Q^2)\tau_{z,i} \right] (\vec{p}_i + \vec{p}_i)e^{i\vec{q}\vec{r}_i}$$

$$+ \frac{i}{4M_N} \left[ G_{M}^S(Q^2) + G_{M}^V(Q^2)\tau_{z,i} \right] [\vec{\sigma}_i \times \vec{q}] e^{i\vec{q}\vec{r}_i} \quad (25)$$

where the first term is the so-called convection current and the second is the magnetisation current.

The Sachs form factors $G_E$ and $G_M$ are related to the Dirac and Pauli form factors in eq. (23) via [39]:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2), \quad (26)$$
\[ G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \]  

(27)

and are normalised so that

\[ G^S_E(Q^2 = 0) = G^V_E(Q^2 = 0) = 1, \quad G^S_M(Q^2 = 0) = \mu_p + \mu_n = 0.880 \mu_N, \]  

(28)

\[ G^V_M(Q^2 = 0) = \mu_p - \mu_n = 4.706 \mu_N \]  

(29)

where \( \mu_p \) and \( \mu_n \) are the magnetic moments of the proton and neutron in terms of the nuclear magneton \( \mu_N \). The superscripts \( S \) and \( V \) of the Sachs form factors \( G_E \) and \( G_M \) denote isoscalar and isovector combinations of the proton and neutron electric and magnetic form factors, respectively. Then the transition matrix elements take the form

\[ M = \langle \chi^{3S_1\rightarrow 3D_1}_d | \vec{j}(\vec{q}) \cdot \vec{e}_\lambda | \chi^{1S_0}_{np} \rangle \]  

(30)

where

\[ |\chi^{3S_1}_d(\vec{r})\rangle = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r} |1M_S \rangle_S |00\rangle_T \]  

(31)

and

\[ |\chi^{3D_1}_d(\vec{r})\rangle = \frac{w(r)}{r} \sum_{M_L M_S} (2M_L 1M_S |1M_J \rangle Y_{2M_L}(\hat{\vec{r}}) |1M_S \rangle_S |00\rangle_T \]  

(32)

are \( ^3S_1 \) and \( ^3D_1 \)-deuteron wave function components, respectively.

For the initial \( np \, ^1S_0 \) continuum state we use a similar form

\[ |\chi^{1S_0}_{np}\rangle = \frac{1}{\sqrt{4\pi}} \frac{z(r)}{r} |00\rangle_S |10\rangle_T \]  

(33)

The direct calculation for the impulse approximation (IA) results in the following expressions for the amplitudes \( M \)

\[ M_{1, S_0 \rightarrow 3S_1} = G_{1, S_0 \rightarrow 3S_1}^{IA} \sqrt{\frac{8\pi}{3}} \sum_k (1M_J 1k 1\lambda) Y_{1k}(\hat{\vec{q}}) \]  

(34)

\[ M_{1, S_0 \rightarrow 3D_1} = -G_{1, S_0 \rightarrow 3D_1}^{IA} \sqrt{\frac{8\pi}{3}} \sum_k (1M_J 1k 1\lambda) Y_{1k}(\hat{\vec{q}}) \]  

(35)

where the functions \( G_{1, S_0 \rightarrow 3S_1}^{IA} \) and \( G_{1, S_0 \rightarrow 3D_1}^{IA} \) are taken as follows:

\[ G_{1, S_0 \rightarrow 3S_1}^{IA} = G_M(Q^2 = 0) \frac{q}{2M_N} \left[ \int_0^\infty u(r)z(r)j_0(\frac{qr}{2})d\ r \right] \]  

(36)

\[ G_{1, S_0 \rightarrow 3D_1}^{IA} = G_M(Q^2 = 0) \frac{q}{2M_N} \left[ \frac{1}{\sqrt{2}} \int_0^\infty w(r)z(r)j_2(\frac{qr}{2})d\ r \right] \]  

(37)

and with

\[ G_{1, S_0 \rightarrow d}^{IA} = [G_{1, S_0 \rightarrow 3S_1}^{IA} - G_{1, S_0 \rightarrow 3D_1}^{IA}] \]  

(38)
\[ \mathcal{M}_{IA} = G_{IA}^{1S_0 \to d} \left( \frac{8\pi}{3} \sum_k (1M_J1k1\lambda)Y_{1k}(\mathbf{q}) \right) \] (39)

The IA cross section is thus proportional to the quantity \( \left[ G_{IA}^{1S_0 \to d} \right]^2 \) as one can neglect the states other than \(^1S_0\) in the entrance channel because of its low energy:

\[ G_{IA}^{1S_0 \to d} = G_M^{ij}(Q^2 = 0) \frac{q}{2M_N} \int_0^\infty \left[ u(r)z(r)j_0(qr) - \frac{1}{\sqrt{2}} w(r)z(r)j_2(qr) \right] dr \] (40)

The result of the destructive interference between \( G_{IA}^{1S_0 \to ^3S_1} \) and \( G_{IA}^{1S_0 \to ^3D_1} \) is a sharp minimum in the cross section for the \( e^+d \to e^-n+p \) reaction which has not been seen experimentally. Notice that at high momenta the transition to the \( D\)-state of the deuteron is as important as the transition to the \( S\)-state. The total matrix element changes its sign at about \( q^2 \approx 12 fm^{-2} \).

Although the IA has been utilised frequently in investigations of weak and e.-m. interactions in nuclei, it does not preserve all the salient features characterising the “elementary” interaction with free nucleons. For instance, even though the e.-m. current for on-shell nucleons is conserved as implied by gauge invariance, the nuclear or hadronic e.-m. currents generated by the conventional IA are not conserved in general and are inconsistent with gauge invariance [41]. To restore current conservation in conventional IA, we should incorporate also MEC and take seriously the argument that only the sum of all the contributions shall be conserved. In other words, we have to take seriously the two basic ingredients of the IA, viz.: (1) The interaction with the whole nucleus can be approximated by a simple sum of the “elementary” interactions with the constituent nucleons, and (2) each elementary interaction with the constituent nucleon takes place \textit{instantaneously}. By “instantaneously” it is understood, that the elementary interaction takes place in region \( B \) of Fig. 5 and thus is not affected by the presence of the initial and final state interactions. Even if the off-shell effects arising from the binding of constituent nucleons by the nuclear potential can be neglected, the masses for the initial and final states of the nucleon cannot always be treated as the same because of the instantaneous absorption of the four-momentum transfer. Current conservation is already violated in this elementary interaction if, in spite of such difference in the effective masses, we adopt as our basic input the form factors for free nucleons. To guarantee that the resultant impulse approximation is in accord with current conservation, we have to attempt to restore current conservation for each elementary interaction by generalising the on-shell form factors to include the off-shell effects due to, in particular, the instantaneous absorption of the four-momentum transfer [42].

**Pion-exchange currents with the soft cut-off parameters**

It is important to note here that both the pion-exchange interaction and the model independent pion-exchange current operator may be derived as nonrelativistic limits of the corresponding operators obtained from a relativistic chiral symmetric Lagrangian model for the meson-nucleon system. The two-body currents generated from the meson degrees of freedom can be classified according to their time (charge) and space (current) components, and also to their isospin structure (isoscalar or isovector). These contributions are of different...
nature, the isovector current is of the same $1/M$ order as the one-body current, while all other transitions appear at the relativistic level and should compete with relativistic corrections to the impulse approximation.

The diagrams for MEC of longest range are presented in Figs. 6 and 7. All transition operators representing these exchange-current processes have isovector character, and do therefore lead to transition from the deuteron state to the $^1S_0$ state and not to the $^3\alpha$-state ($^3S_1 + ^3D_1$). There is, of course, no limit to the number of possible contributing exchange-current processes, but only those of pionic range are really significant, since short-range heavy-meson exchange processes are suppressed by the strong short-range repulsion in the nucleon-nucleon interaction [43]. It means that a isoscalar counterpart of the isovector currents, due to $\rho\pi\gamma$ and $\omega\pi\gamma$ contributions contribute only a little to the isovector transition under consideration and thus have been neglected in the present study. Besides that, the class of model-dependent currents between different mesons, i.e. $\rho\pi\gamma$ ($\omega\pi\gamma$), are purely transverse and are of short range, which is due to the large $\rho$- ($\omega$) mass. Therefore, their contribution at low momentum transfer should be very small [39]. It should be emphasized also that the contribution related to these operators are rather sensitive to the values of cut-off parameters. For the soft cut-off parameters used in the DBS model ($\Lambda_\pi \sim 0.7$ GeV) these contributions will be even less important than in the traditional OBEP models where $\Lambda_\pi \sim 1.2$ GeV is usually employed.

The Seagull term

The most important MEC process is due to the seagull current operator $J^{\text{sea}}_\pi$ which is a two-body current operator (see Fig. 6):

$$J^{\text{sea}}_\pi(\vec{k}_1,\vec{k}_2) = i \frac{f_{\pi NN}^2}{m_\pi^2} (\vec{\tau}_1 \times \vec{\tau}_2)^z \left[ \frac{\vec{\sigma}_2(\vec{\sigma}_1 \cdot \vec{k}_1)}{m_\pi^2 + \vec{k}_1^2} - \frac{\vec{\sigma}_1(\vec{\sigma}_2 \cdot \vec{k}_2)}{m_\pi^2 + \vec{k}_2^2} \right]$$

where

$$(\vec{\tau}_1 \times \vec{\tau}_2)^z = \frac{1}{2} \left( \vec{\tau}_1 \times (\vec{\tau}_1 \times \vec{\tau}_2) \right)$$

The configuration-space expression [39] may be obtained from the respective Fourier
Thus the transition matrix element is of the form:

\[ J_\pi^\text{sea}(\vec{q}) = \int d^3 \vec{x} \ e^{i\vec{q} \cdot \vec{x}} \int d^3 \vec{k}_1 \ d^3 \vec{k}_2 \ e^{i\vec{k}_1 \cdot (\vec{r}_1 - \vec{x})} \ e^{i\vec{k}_2 \cdot (\vec{r}_2 - \vec{x})} \ J_\pi^\text{sea}(\vec{k}_1, \vec{k}_2) \]  

which results in:

\[ J_\pi^\text{sea}(\vec{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2} (\vec{r}_1 \times \vec{r}_2)^z (m_\pi + \frac{1}{r}) e^{-m_\pi r} \left\{ \vec{\sigma}_2 (\vec{r} \cdot \vec{r}) e^{i\vec{q} \cdot \vec{r}_2} + \vec{\sigma}_1 (\vec{r} \cdot \vec{r}) e^{i\vec{q} \cdot \vec{r}_1} \right\} \]  

Thus the transition matrix element is of the form:

\[ \mathcal{M}_{1S_0 \rightarrow d} = \langle \psi_d | J_\pi^\text{sea}(\vec{q}) \cdot \vec{e}_\lambda | \psi_{pn} \rangle \]  

It can be naturally decomposed in two terms:

\[ \mathcal{M}_{1S_0 \rightarrow d}^\text{sea} = \left[G_{1S_0 \rightarrow S_1}^\text{sea} + G_{1S_0 \rightarrow D_1}^\text{sea} \right] \times \sqrt{\frac{8\pi}{3}} \sum_i (1M_{J1}i|1\lambda) Y_{1i}(\vec{q}) \]  

where the functions \( G_{1S_0 \rightarrow S_1}^\text{sea} \) and \( G_{1S_0 \rightarrow D_1}^\text{sea} \) are defined as:

\[ G_{1S_0 \rightarrow S_1}^\text{sea} = \left(\frac{f_{\pi NN}^2}{4\pi}\right) \frac{4}{m_\pi^2} \left[ \int_0^\infty u(r)z(r) j_1(r/2) (1 + m_\pi r) \frac{e^{-m_\pi r}}{r^2} dr \right] \]  

\[ G_{1S_0 \rightarrow D_1}^\text{sea} = \left(\frac{f_{\pi NN}^2}{4\pi}\right) \frac{4}{m_\pi^2} \left[ \frac{1}{\sqrt{2}} \int_0^\infty u(r)z(r) j_1(r/2) (1 + m_\pi r) \frac{e^{-m_\pi r}}{r^2} dr \right] \]  

The Pion-in-flight term

Among all the conventional diagrams, the pion-in-flight term is the most difficult one to calculate, since it does involve two-pion propagators (see Fig. 17). This diagram represents the direct coupling of the photon to the exchanged pion. It corresponds to virtual pion photoproduction on a single nucleon by the pion-pole amplitude, followed by absorption on the second one. The result of the calculation for this pion-in-flight (pif) term is:

\[ J_{\pi}^{\text{pif}}(\vec{k}_1, \vec{k}_2) = \frac{1}{m_\pi^2} \left(\vec{r}_1 \times \vec{r}_2 \right)^z \left( \vec{k}_1 - \vec{k}_2 \right) \left( \vec{\sigma}_1 \cdot \vec{k}_1 \right) \left( \vec{\sigma}_2 \cdot \vec{k}_2 \right) \left[ m_\pi^2 + \vec{k}_1^2 \right] \left[ m_\pi^2 + \vec{k}_2^2 \right] \]  

And the transition matrix elements can be written as follows:

\[ \mathcal{M}_{1S_0 \rightarrow d}^{\text{pif}} = \langle \psi_d | J_{\pi}^{\text{pif}}(\vec{q}) \cdot \vec{e}_\lambda | \psi_{pn} \rangle \]  

consisting of two components:

\[ \mathcal{M}_{1S_0 \rightarrow d}^{\text{pif}} = \left[G_{1S_0 \rightarrow S_1}^{\text{pif}} + G_{1S_0 \rightarrow D_1}^{\text{pif}} \right] \sqrt{\frac{8\pi}{3}} \sum_i (1M_{J1}i|1\lambda) Y_{1i}(\vec{q}) \]
FIG. 7: The pion-in-flight term contribution in the DBS model. Notations for the regions are the same as in Fig. 5 for the one-body e.-m. current.

where the functions $G_{1S_0 \rightarrow 3S_1}^{pif}$ and $G_{1S_0 \rightarrow 3D_1}^{pif}$ take the forms:

$$G_{1S_0 \rightarrow 3S_1}^{pif} = \left( \frac{f_{\pi NN}^2}{4\pi} \right) \frac{2}{3} \frac{q}{m_\pi^2} \left[ \int_0^\infty \left\{ \left( \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right) t_0 + \left( \frac{d^2}{dr^2} + 5 \frac{d}{r} \frac{d}{dr} + \frac{3}{r^2} \right) t_2 \right\} u(r)z(r)dr \right]$$

(51)

$$G_{1S_0 \rightarrow 3D_1}^{pif} = \left( \frac{f_{\pi NN}^2}{4\pi} \right) \frac{2}{3} \frac{q}{m_\pi^2} \left[ \int_0^\infty \left\{ \left( \frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} \right) t_0 + \left( \frac{d^2}{dr^2} + 2 \frac{d}{r} \frac{d}{dr} - \frac{6}{r^2} \right) t_2 \right\} \frac{1}{\sqrt{2}} w(r)z(r)dr \right]$$

(52)

where

$$t_l = \int_0^1 \frac{1}{\alpha} e^{-\alpha r} j_l \left( \frac{\eta qr}{2} \right) d\eta, \quad l = 0, 2$$

(53)

and

$$\alpha = \sqrt{m_\pi^2 + \frac{1}{4} q^2 (1 - \eta^2)}.$$  (54)

In the long-wave limit (limit for small $qr$) the Bessel functions $j_l \left( \frac{\eta qr}{2} \right)$ of the eq. (53) can be reduced according to:

$$j_l(z) \xrightarrow{z \to 0} \frac{z^l}{(2l+1)!!}, \quad (2l + 1)!! = 1 \cdot 3 \cdot 5 \cdots (2l + 1),$$

(55)

so that the eqs. (51) and (52) result in:

$$G_{1S_0 \rightarrow 3S_1}^{pif} = \left( \frac{f_{\pi NN}^2}{4\pi} \right) \frac{2}{3} \frac{q}{m_\pi^2} \left[ \int_0^\infty \left\{ (m_\pi r - 2) \frac{e^{-m_\pi r}}{r} \right\} u(r)z(r)dr \right]$$

(56)

$$G_{1S_0 \rightarrow 3D_1}^{pif} = \left( \frac{f_{\pi NN}^2}{4\pi} \right) \frac{2}{3} \frac{q}{m_\pi^2} \left[ \frac{1}{\sqrt{2}} \int_0^\infty \left\{ (m_\pi r + 1) \frac{e^{-m_\pi r}}{r} \right\} w(r)z(r)dr \right]$$

(57)

18
The $\Delta(1232)$-isobar current term

The process corresponding to the virtual excitation of a $\Delta(1232)$ followed by the $\Delta N \rightarrow NN$ transition interaction (Fig. 8) is described by the $\Delta$-current $J_\Delta^\pi$. There are many approaches to evaluate this effect. We assume $\Delta$-dominance, and we shall use here the static quark model and the sharp resonance approximation.

In the static quark model the exchange current due to the $\Delta$ resonance is \[ J_\Delta^\pi = i \frac{g_{\pi NN}^2}{25M_N^2(M_\Delta - M_N)} \left\{ \frac{4\tau_3}{m_\pi^2 + k_1^2} (\vec{k}_1 \times \vec{q}) (\vec{\sigma}^1 \cdot \vec{k}_1) + \frac{4\tau_3}{m_\pi^2 + k_2^2} (\vec{k}_2 \times \vec{q}) (\vec{\sigma}^2 \cdot \vec{k}_2) \right\} \]

- $(\vec{r}^1 \times \vec{r}^2)_3 \left[ \frac{(\vec{\sigma}^1 \times \vec{k}_2) \cdot \vec{q} (\vec{\sigma}^2 \cdot \vec{k}_2)}{m_\pi^2 + k_2^2} - \frac{(\vec{\sigma}^2 \times \vec{k}_1) \cdot \vec{q} (\vec{\sigma}^1 \times \vec{k}_2)}{m_\pi^2 + k_1^2} \right]$ \[ (58) \]

Here $k_1$ and $k_2$ are the fractional momentum transfers imparted to the first and second nucleons, and $g_{\pi NN}$ is the pseudoscalar $\pi NN$ coupling constant, which is related to the pseudovector $\pi NN$ coupling constant $f_{\pi NN}$ as:

\[ \frac{g_{\pi NN}}{2M_N} = \frac{f_{\pi NN}}{m_\pi} \quad \text{and} \quad \frac{f_{\pi NN}^2}{4\pi} = 0.075. \]

The transition matrix element for the $\Delta$-exchange current is:

\[ M^\Delta_{I S_0 \rightarrow d} = \langle \psi_d | J_\Delta^\pi (\vec{q}) \cdot \hat{\epsilon}_\lambda | \psi_{pn} \rangle. \]

consisting of two contributions:

\[ M^\Delta_{I S_0 \rightarrow S_1} = 0 \]

and

\[ M^\Delta_{I S_0 \rightarrow D_1} = G^\Delta_{I S_0 \rightarrow D_1} \sqrt{\frac{8\pi}{3}} \sum_i (1M_f 1i | 1\lambda) Y_{1i}(\hat{q}), \]

\[ (61) \]
FIG. 9: The one-pion-exchange potential (OPEP) modified by the hadronic form factor of monopole type at each vertex.

where $G_{1S_0 \to 3D_1}^{\Delta}$ in the long-wave limit has the form:

$$
G_{1S_0 \to 3D_1}^{\Delta} = G_M^V(Q^2 = 0) \frac{g_{\pi NN}^2}{4\pi} \frac{q}{M_\Delta - M_N} \left( \frac{m_\pi}{M_N} \right)^3 
\times \frac{4}{25} \left[ \frac{2}{3} \int_0^\infty w(r) z(r) \left( 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right) e^{-m_\pi r} \frac{r^2}{m_\pi} dr \right].
$$

Effect of short range correlations

One important feature of the $\pi$-exchange currents which has not been much emphasised in previous studies of the radiative neutron capture is the sensitivity of these exchange-current effects to the $\pi NN$ form factor. From a general point of view, the $\pi NN$ form factor originates due to the modification of the bare $\pi$-exchange current at short distances [44], where the finite size of the nucleon (and the pion) together with the vertex renormalisation associated with very short-range multi-pion exchange processes (the coupling of the pion to heavier systems such as $\rho$ or $3\pi$) can be of some importance [45]. The former corrections are roughly related to the confinement radius of the quarks in the nucleon.

It is rather difficult to calculate the form factors exactly, as they involve many non-perturbative contributions [53]. For practical applications they are usually parametrised in momentum space by the monopole form, the mass scale $\Lambda$ being a parameter chosen in order to be consistent with the experiment. All the consistent estimates given up to date lead to a cut-off parameter $\Lambda_{\pi NN}$ lying around $0.5 \div 0.8$ GeV/c. In the same way as the e.m. form factor in MEC operators has been related to the longitudinal part of the charge operator from the continuity equation, the hadronic form factors in these operators should be connected to the choice of a particular $NN$-potential.

One takes

$$
F_{\pi NN}(q) = \frac{\Lambda^2_{\pi} - m_\pi^2}{\Lambda^2_{\pi} + q^2}
$$

at each $\pi NN$ vertex. The values of the cut-off masses are, of course, to be related to the cut-offs used in the construction of the $NN$-potential.

One important point in deriving the expressions for MEC with the hadronic form factors is to insure gauge invariance, or in other words, the current conservation. It has been shown a long time ago that current conservation is insured, if one takes a $\pi$-exchange model for the $NN$-potential together with exchange currents (seagull and mesonic) including an appropriate e.m. form factor of nucleons and pions but without any meson-nucleon form factor. The next step is the verification whether insertion of the hadronic form factor at each
operators in eqs. (46) and (51) are modified in the following way:

Apart from the normalisation factor \( \Lambda_\pi^2 \), the exchange-current iteration is the appearance of the two new contributions, shown in Fig. 10a and Fig. 10c, which arise from the diagram shown in Fig. 9, where \( \Pi \) represents a particle of mass \( \Lambda_\pi \).

The leading isovector \( \pi \)-exchange potential is modified by a monopole form factor at each vertex in the following way:

\[
V_{ij}^{\text{OPE}}(\vec{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2} \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + q^2} \right)^2 \frac{(\vec{\sigma}_i \cdot \vec{q})(\vec{\sigma}_j \cdot \vec{q})}{q^2 + m_\pi^2} (\vec{r}_i \cdot \vec{r}_j),
\]

(64)

Apart from the normalisation factor \( (\Lambda_\pi^2 - m_\pi^2)^2 \) this modified potential can be visualised by the diagram shown in Fig. 9 where \( \Pi \) represents a particle of mass \( \Lambda_\pi \) with the same quantum numbers as the \( \pi \)-meson (the short-range vertex renormalisation is represented by an exchange of a virtual heavy particle, of mass \( \Lambda_\pi \)). By minimal substitution on this diagram, it is then straightforward to construct the corresponding exchange currents, modified by hadronic form factors and satisfying gauge invariance. These are depicted in Fig. 10a (modified seagull term) and Fig. 10b, c, d (modified mesonic term).

The seagull current operator is just modified by adding the corresponding factor \( F_{\pi NN}^2(q) \) in the numerator of the expression given in eq. 41:

\[
J_{\pi}^{\text{sea}}(\vec{k}_1, \vec{k}_2) = i \frac{f_{\pi NN}^2}{m_\pi^2} (\vec{\tau}_1 \times \vec{\tau}_2) \cdot \frac{\vec{\sigma}_2(\vec{\sigma}_1 \cdot \vec{k}_1) \Lambda_\pi^2 - m_\pi^2}{m_\pi^2 + k_1^2} \frac{\vec{\sigma}_1(\vec{\sigma}_2 \cdot \vec{k}_2) \Lambda_\pi^2 - m_\pi^2}{m_\pi^2 + k_2^2}
\]

(65)

The leading isovector \( \pi \)-exchange pion-in-flight current operator given in eq. 48 is thus modified:

\[
J_{\pi}^{\text{pif}}(\vec{k}_1, \vec{k}_2) = i \frac{f_{\pi NN}^2}{m_\pi^2} (\vec{\tau}_1 \times \vec{\tau}_2) \cdot (\Lambda_\pi^2 - m_\pi^2)(\vec{\sigma}_1 \cdot \vec{k}_1)(\vec{\sigma}_2 \cdot \vec{k}_2)
\]

\[
\times \left\{ \frac{1}{\Lambda_\pi^2 + k_1^2} \left[ \frac{1}{(\Lambda_\pi^2 + k_2^2)(m_\pi^2 + k_2^2)} + \frac{1}{(m_\pi^2 + k_1^2)(m_\pi^2 + k_2^2)} + \frac{1}{(m_\pi^2 + k_1^2)(\Lambda_\pi^2 + k_1^2)} \right] \right\}
\]

(66)

The main difference from the usual derivation of exchange currents without form factors is the appearance of the two new contributions, shown in Fig. 10a and Fig. 10c, which arise in the mesonic term. After the inclusion of the hadronic form factor, the exchange-current operators in eqs. (66) and (67) are modified in the following way:

\[
\frac{e^{-m_\pi r}}{r^2}(1 + m_\pi r) \rightarrow \frac{e^{-m_\pi r}}{r^2}(1 + m_\pi r) - \frac{e^{-\Lambda_\pi r}}{r^2}(1 + \Lambda_\pi r) - \frac{\Lambda_\pi^2 - m_\pi^2}{2} e^{-\Lambda_\pi r},
\]

(67)
\[
\frac{e^{-ar}}{\alpha} \Rightarrow \frac{e^{-ar}}{\alpha} - \frac{e^{-\lambda r}}{\lambda} - \frac{(\Lambda_{\pi}^2 - m_{\pi}^2)}{2\lambda^3} e^{-\lambda r}
\]  

(68)

where

\[
\lambda = \sqrt{\Lambda_{\pi}^2 + \frac{1}{4}q^2(1 - \eta^2)}.
\]  

(69)

Thus, eqs. (46), (47), (51), (52), (67) and (68) define the exchange currents within the conventional model to be used in this work. This model has the attractive feature that it satisfies current conservation consistent with the one-pion exchange potential. The effect of the short-range hadronic form factor to the \(\Delta\)-isobar contribution in the \(NN\) wave function was investigated by transition potential methods [46] with \(\pi\)-exchange contributions. For the calculations we employed a potential proposed in ref. [48].

**CROSS SECTIONS FOR THE THERMAL NEUTRON CAPTURE**

The thermal \(np\) capture is a fundamental process for our purpose because it involves two opposite features, long-distance behaviour and short-range processes, where the latter are not completely understood. Clearly, a better determination of the asymptotic behaviour of the theoretical \(NN\) wave function is needed before a quantitative comparison with experiment can be made for the thermal \(np\) capture cross section.

For thermal neutrons we can use a nonrelativistic kinematic regime, and we may even ignore the deuteron recoil energy. Then the \(\gamma\)-ray emitted in the capture process will have an energy essentially equal to the deuteron binding energy \(\epsilon_d\) [47]. In this kinematic regime the differential cross section has the form

\[
\frac{d\sigma_{fi}}{d\Omega_\gamma} = \frac{\delta(P_f - P_i)}{|\mathbf{v}_n - \mathbf{v}_p|} \frac{e^2}{2\omega_\gamma} |M_{fi}|^2 \frac{d^2p_d d^3q_\gamma}{(2\pi)^2}
\]  

(70)

The integrals over \(p_d\) and \(|q|\) in the laboratory system where \(|v_p| = 0\), are easy to take [47] if we ignore the deuteron recoil and set \(q = \omega = \epsilon_d\), i.e. the deuteron binding energy. So, these integrations results in:

\[
\frac{d\sigma_{fi}}{d\Omega_\gamma} = \frac{\omega_\gamma}{|\mathbf{v}_n|} \frac{e^2}{2(2\pi)^2} |M_{fi}|^2
\]  

(71)

Since we neglect the deuteron recoil momentum, there is no angular dependence in the differential cross section \(d\sigma_{fi}/d\Omega_\gamma\) due to phase space. Therefore, all the angular dependence is due to the dynamics of the capture process.

The total transition amplitude takes the form:

\[
M_{S_0 \to d} = M_{S_0 \to d}^{IA} + M_{S_0 \to d}^{sea} + M_{S_0 \to d}^{pif} + M_{S_0 \to d}^{\Delta}
\]  

(72)

where

\[
M_{S_0 \to d} = [G_{S_0 \to d}^{IA} + G_{S_0 \to d}^{sea} + G_{S_0 \to d}^{pif} + G_{S_0 \to d}^{\Delta}] \sqrt{\frac{8\pi}{3}} \sum_i (1M_i |1\lambda)Y_i(\hat{q})
\]  

(73)
To compute the unpolarised cross section from eq. (71) one has to sum over the final and average over the initial spin directions. Carrying out these summations one gets the expression for the differential cross section for thermal np capture to the $^3S_1 - ^3D_1$ deuteron state:

$$
\frac{d\sigma}{d\Omega} = \frac{\omega_\gamma v_n}{4(2\pi)^2} \left[ G_{IA}^{^3S_0\to d} + G_{sea}^{^3S_0\to d} + G_{pif}^{^3S_0\to d} + G_{\Delta}^{^3S_0\to d} \right]^2
$$

(74)

After angular integration and introducing the relative np momentum in the c.m. system of the incident channel:

$$
p = \frac{1}{2}M_N |v_n| = \left( \frac{1}{2}M_N E_n \right)^{1/2},
$$

(75)

one arrives at:

$$
\sigma(np \to d\gamma) = \frac{\alpha \omega_\gamma M_N}{2p} \left[ G_{IA}^{^3S_0\to d} + G_{sea}^{^3S_0\to d} + G_{pif}^{^3S_0\to d} + G_{\Delta}^{^3S_0\to d} \right]^2
$$

(76)

where $\alpha = e^2/4\pi$ is the fine structure constant.

The relative change of the IA result due to the exchange currents can be expressed in terms of the quantity

$$
\delta(MEC) = \frac{G_{sea}^{^3S_0\to d} + G_{pif}^{^3S_0\to d}}{G_{IA}^{^3S_0\to d}} \equiv \delta_{sea} + \delta_{pif}
$$

(77)

The empirical value for $\delta$ is normally deduced from the relations:

$$
\sigma^{exp}(np \to d\gamma) = (1 + \delta)^2 \cdot \sigma^{IA}(np \to d\gamma)
$$

(78)

where

$$
\sigma^{IA}(np \to d\gamma) = \frac{\alpha \omega_\gamma M_N}{2p} \left[ G_{IA}^{^3S_0\to d} \right]^2
$$

(79)

and $G_{IA}^{^3S_0\to d}$ is determined by eq. (40) with $q = \omega_\gamma$. We have now at our disposal all formulae for the analysis of both the IA- and the MEC-contributions in the DBS model. All the above formulas for transition matrix elements are derived for the general case and can be applied to related processes as well. In the long-wave limit, the equations for the MEC will lead to the forms familiar from literature [6].

**RESULTS AND DISCUSSION**

The most important uncertainty in IA comes from the asymptotic behaviour of the wave function, because the momentum transfer carried by the photon is almost zero. Therefore, it is not surprising that the analysis of the theoretical cross sections within IA [4] has demonstrated a very high stability of the IA-results for different NN-potential models (see Table III).

In Table III the IA results and the normalisation $\delta$ to the experiment are compiled for the potentials of de-Tourreil-Sprung [49], Argonne V14 [11], Paris [50] and Reid Soft-Core [51], respectively. The small variations among different $NN$-model predictions are due to the differences in $D$-wave admixture (which changes the value of parameter $A_S$ of asymptotic behaviour of the deuteron wave function) and due to the different short-range repulsion.
TABLE III: The values of the one-body contribution for the different potentials to the thermal neutron capture reaction as compared with experimental data.

| potential     | Ref. | IA [mb] | δ(%) |
|---------------|------|---------|------|
| TS            | [49] | 305     | 4.68 |
| Argonne V14   | [11] | 304.1   | 4.83 |
| Paris         | [50] | 302     | 5.2  |
| RSC           | [51] | 300     | 5.55 |
| DBS model     | present | 304.3   | 4.79 |
| Experiment    | [1]  | 334.2±0.5 |

The potentials are ordered for increasing repulsion from the TS to the RSC potential. Thus, the 10% discrepancy between the experiment and the one-body contribution is significant, and cannot be accounted for by any change in the nuclear wave function compatible with the fit to the accurate $NN$-data.

It is evident from Table III that the IA-results for all realistic $NN$-potential models and the DBS-model as well agree to each other within some narrow corridor. It should be expected because the long-range behaviour of $NN$ wave functions for all the models is quite similar. However, the close similarity of the IA results for all $NN$-models considered here is still informative because the DBS wave functions in both initial and final channels have a node, and thus the short-distance behaviour is sharply distinct from those for the traditional $NN$-potential models. Thus the 30 mb residual of the cross section should come from MEC-contributions for all $NN$-models, no matter whether they are based on meson-exchange mechanism, hybrid six-quark model or on the dressed-bag intermediate state production. This is our first conclusion.

The $\delta$-value of eq. (78) necessary to match the DBS-calculation to the experimental cross section of the $np \to d\gamma$ reaction amounts to

$$\delta = 4.79\%$$

Our results for the MEC-contributions within the DBS-model which include the seagull, meson-in-flight and isobar current terms are presented in Table IV. For clarity of the conclusions we show in this Table the results for both soft ($\Lambda_\pi = 697$ MeV) and hard ($\Lambda_\pi = 1200$ MeV) $\pi NN$ and $\pi N\Delta$ form factors. It is evident that going to hard cut-off parameters increases all the MEC-contributions quite visibly, as was expected before. The MEC and IC contributions in our calculation with the soft $\Lambda_{\pi NN}$ give in sum

$$\delta(\text{MEC + IC}) \simeq 2.8\%$$

and hence there remains some 2% of the total cross section still unexplained with the present $NN$-model. Therefore it is appropriate here to discuss which further contributions should appear in our approach.
TABLE IV: Relative contributions of seagull, mesonic and model dependent resonance terms for pion exchange currents in the DBS model for two cut-off factors $\Lambda_\pi$. The values are given for $S \rightarrow S$ and $S \rightarrow D$ transitions. The $\delta^\pi$ present the values for point-like nucleons.

| Term                  | $\delta^\pi_{SS}(\%)$ | $\delta^\pi_{SD}(\%)$ | $\delta^{\pi+FF\pi}_{SS}(\%)$ | $\delta^{\pi+FF\pi}_{SD}(\%)$ | $\delta^{\pi+FF\pi}_{SS}(\%)$ | $\delta^{\pi+FF\pi}_{SD}(\%)$ |
|-----------------------|------------------------|------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
|                       | $\Lambda_\pi = 697 \text{ MeV}$ | $\Lambda_\pi = 1200 \text{ MeV}$ |
| Seagull               | 2.707                  | 0.459                  | 2.120                         | 0.393                         | 2.422                         | 0.447                         |
| Mesonic               | -1.243                 | 0.459                  | -1.065                        | 0.393                         | -1.117                        | 0.447                         |
| Total MEC             | 1.464                  | 0.918                  | 1.005                         | 0.786                         | 1.305                         | 0.954                         |
| Isobar current        | -                      | 1.151                  | -                             | 0.974                         | -                             | 1.062                         |
| Total MEC+IC          | 1.464                  | 2.069                  | 1.005                         | 1.760                         | 1.305                         | 2.016                         |

The main difference between the DBS-model predictions and the conventional $NN$-potential models comes from the mixed two-component quark-meson structure of DBS-approach. The most evident specific contributions are

i) The first non-vanishing contribution, which is solely due to the two-component structure, are transitions between dressed-bag states themselves (these currents can be identified with the operator $\hat{j}^6 \hat{q}_\mu$ in eq. (22)). These can be illustrated for $^1S_0 \rightarrow (^3S_1 - ^3D_1)$ transitions by the following graphs in Fig. 11. This process corresponds to the $M1$ single-quark transition in the bag. We would like to stress the point that the spin-flip of the quark in the bag is a non-exotic process because in the conventional quark-diquark model for the nucleon, the usual nucleon spin-flip (which is responsible for the conventional $(\vec{\sigma} \cdot \vec{\nabla})$- coupling in $\pi N$- amplitude) originates also from the unpaired quark spin-flip transition.

ii) The dressed six-quark bag in our model has mainly an uncharged $\sigma$-meson cloud with the effective mass of the meson being $m_\sigma \simeq 400 \text{ MeV}$. The charged six-quark core and the chargeless $\sigma$-cloud are oscillating in the intermediate DBS-state around the total center-of-mass. Thus these oscillations of the charged six-quark core will induce some e.m. radiation similar to radiation emitted by a charged oscillator in conventional quantum theory. Therefore this radiation from the DBS-component in the $NN$ system may have monopole and also higher multipole character. A $M1$ transition should be especially appropriate to describe polarisation degrees of freedom in the $\vec{n} \ p \rightarrow d\gamma$ reaction. The point here is, that the scattering and bound state wave functions in the $NN$ channel in our model are non-orthogonal to each other due to the non-hermicity of the underlying $NN$-“potential” $V_{NNN}(E)$. However, the total two- (or many-) component functions $\Psi(NN)$ should still be orthogonal to each other and thus monopole $E0$ transitions are quite possible to occur separately in the $NN$- or in the DBS-channel while the monopole transition between the initial and final state vectors should vanish. The above monopole transition would include the transition
FIG. 11: The 6-quark contributions to the DBS e.-m. current induced by transitions between dressed-bag states.

$^3S_1 \rightarrow ^3S_1$ in the deuteron. However, the isoscalar $M1$-transition from the initial $^3S_1$ channel to the final $^3S_1$ channel inside the bag, which is related to the spin rotation of the bag as the whole from $M_S = +1$ (or -1) to $M'_S = 0$ (or vice versa), is still feasible.

iii) The dressing of the bag includes in our model not only neutral $\sigma$, but also charged $\pi$- and $\rho$-meson clouds. It would be quite possible for incoming $\gamma$-quanta to interact first with these charged clouds of the bag. These specific new meson-exchange currents will contribute to the $np \rightarrow d\gamma$ reaction and also to many other e.m. processes [52, 53, 54]. We postpone the discussion of these new effects to our further publications.

In summary, we presented in this paper the first e.-m. calculations for $np \rightarrow d\gamma$ cross sections on the basis of the new $NN$-interaction model. The model employs the soft meson-nucleon cut-off parameter values so that we still observe some underestimation of the experimental cross section. However, there are a few specific new contributions in the model which are fully absent in conventional $NN$-potential models. Our further studies point in this direction.

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ONE- AND TWO-PION EXCHANGE POTENTIALS IN THE DBS-MODEL

If isospin-breaking terms are ignored and staying within the lowest order of perturbation theory, the OPE $NN$- potential in momentum space takes the form:

$$V^{OPE}_{ij}(\vec{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2} \frac{(\vec{\sigma}_i \cdot \vec{q})(\vec{\sigma}_j \cdot \vec{q})}{\vec{q}^2 + m_\pi^2} (\vec{\tau}_i \cdot \vec{\tau}_j),$$

(82)

where $m_\pi$ is the mass of the exchanged pion and $f_{\pi NN}^2$ the coupling constant.
When the potential is Fourier transformed to configuration space, it is usually regularised to remove the singularities at the origin. This can be achieved by introducing a form factor $F(q^2)$ which has a dipole form:

$$F(q^2) = \left( \frac{\Lambda_{\pi NN}^2 - m^2}{\Lambda_{\pi NN}^2 + q^2} \right)^2$$  \hspace{1cm} (83)

where $\Lambda_{\pi NN}$ is the cut-off parameter.

A typical Fourier transform reads as

$$\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2 + m_{\pi}^2}(q^2)^n F(q) \equiv \frac{m_{\pi}}{4\pi}(-\vec{\nabla})^n f_C(r) .$$  \hspace{1cm} (84)

So, in configuration space the OPE potential takes the form:

$$V_{ij}^{OPE}(\vec{r}) = \frac{f^2_{\pi NN} m_{\pi}}{4\pi} \frac{m_{\pi}}{3} [f_C(r) \vec{\sigma}_i \cdot \vec{\sigma}_j + f_T(r) S_{ij}] \vec{\tau}_i \cdot \vec{\tau}_j ,$$  \hspace{1cm} (85)

where the tensor interaction operator is

$$S_{ij} = 3(\vec{\sigma}_i \cdot \vec{r})(\vec{\sigma}_j \cdot \vec{r}) - (\vec{\sigma}_i \cdot \vec{\sigma}_j)$$  \hspace{1cm} (86)

and the central interaction term is

$$f_C(r) = \left[ \frac{e^{-x}}{x} - \frac{e^{-\alpha x}}{x} \right] - e^{-\alpha x} \left( \frac{\alpha^2 - 1}{2\alpha} \right),$$  \hspace{1cm} (87)

with

$$x = m_{\pi} r \quad \text{and} \quad \alpha = \frac{\Lambda_{\pi NN}}{m_{\pi}} .$$  \hspace{1cm} (88)

Using the definition (84), the Fourier transform for the tensor potential can be expressed simply in terms of derivatives of the central potential, i.e.,

$$f_T(r) = \frac{1}{m_{\pi}} \frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} \right) f^0_C(r) ,$$  \hspace{1cm} (89)

with

$$f^0_C(r) = \left[ \frac{e^{-x}}{x} - \frac{e^{-\alpha x}}{x} \right] - e^{-\alpha x} \left( \frac{\alpha^2 - 1}{2\alpha} \right) ;$$  \hspace{1cm} (90)

and the tensor interaction term can be written in terms of the variables $x$ and $\alpha$ as

$$f_T(r) = \frac{e^{-x}}{x} \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) - \alpha^3 \frac{e^{-\alpha x}}{\alpha x} \left[ 1 + \frac{3}{\alpha x} + \frac{3}{(\alpha x)^2} \right] - \frac{\alpha}{2} \left( \alpha^2 - 1 \right) \left[ 1 + \frac{1}{\alpha x} \right] e^{-\alpha x} .$$  \hspace{1cm} (91)

We use the averaged pion mass in the OPE potential:

$$m_{\pi} = \frac{(m_{\pi^0} + 2m_{\pi^\pm})}{3} = 138.0363 \text{ MeV} ,$$

and the respective “weighted” value for the pion-nucleon coupling constant

$$f^2_{\pi NN}/(4\pi) = 0.075$$
The value of the regularisation parameter $\alpha$ amounts to

$$\alpha = 5.054.$$ 

The inclusion of the peripheral two-pion exchange contribution $V_{TPE}$ in our case can be imitated by the simplified form [16]:

$$V_{TPE} = V_{TPE}^0 (\beta r^2)^2 e^{-\beta r^2} \tag{92}$$

and gives only a quite small contribution (about 2-3 MeV) in the intermediate region $r \sim 1.5 \div 2$ fm which is well beyond the DBS radius. However, this small attractive contribution is quite important for the precise description of effective-range parameters and asymptotic properties of the deuteron.

* Electronic address: kaskulov@pit.physik.uni-tuebingen.de

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The quark antisymmetrization operator $\mathcal{A}$ is fully taken into account in the calculation [16] for transitions between the two phases ($NN$) and ($6q + \sigma$). In the cluster-like $NN$ channel its effect reduces to a renormalisation of the $NN$ channel wave function with a constant factor of $\sqrt{10}$ [31].

The contribution of the DBS to r.m.s. radius of matter in deuteron is proportional to the weight of the DBS-state. However due to much wider radial distribution in nucleon component of the deuteron, as compared to the DBS, the real correction to the r.m.s. radius of matter in the deuteron is rather small ($\sim 0.03$ fm); see Table II, sixth and seventh lines in the third column.

One can remark, however, that the minimal account of mesonic degrees of freedom in e.m. transitions has been taken for a long time through the use of the Siegert theorem.

One rather popular modern approach to the reliable determination of the electroweak and strong $\pi NN$ form factors is based on well known QCD sum rules.