Top Physics in WHIZARD

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ABSTRACT

In this talk we summarize the top physics setup in the event generator *Whizard* with a main focus on lepton colliders. This includes full six-, eight- and ten-fermion processes, factorized processes and spin correlations. For lepton colliders, QCD NLO processes for top quark physics are available and will be discussed. A special focus is on the top-quark pair threshold, where a special implementation combines a non-relativistic effective field theory calculation augmented by a next-to-leading threshold logarithm resummation with a continuum relativistic fixed-order QCD NLO simulation.
1 Top Physics in WHIZARD

In this talk, the capabilities of the multi-purpose event generator WHIZARD focusing on top physics for lepton colliders are covered. Though WHIZARD is used by both LHC collaborations, ATLAS and CMS, for top physics simulations, we concentrate here exclusively on general issues and topics special for lepton colliders.

WHIZARD \cite{1} is a modular package, that contains its own (tree-level) matrix element generator O’MEGA \cite{2}. It uses recursive algorithms to generate code avoiding all kinds of redundancies in the amplitudes from the very beginning, for the Standard Model (SM) and in principle arbitrary generalizations thereof. WHIZARD contains a special module, CIRCE1/2 \cite{3} that allows to simulate lepton collider beam spectra including beam energy spectra, and also photon collider options of linear lepton colliders. VAMP \cite{4} is WHIZARD’s adaptive multi-channel Monte Carlo integrator. The elaborate phase space parametrization inside WHIZARD is particularly suited for electroweak productions at (but not only) lepton colliders. A massive support for beyond the SM physics (supersymmetry, composite models, extra dimensions, effective field theories) is available \cite{5}. Beyond this, automatically generated BSM models can be included via interfaces to external tools \cite{6}. WHIZARD can handle (tree-level) processes with six, eight or even ten fermions in the final state that are needed to compute processes like \(e^+e^- \rightarrow tt, tth\) completely off-shell. Support for multi-threading with OpenMP and more elaborate techniques to parallelize (SM) processes are provided \cite{7}. Besides full matrix elements, WHIZARD allows to treat processes factorized including full spin correlations, which for testing purposes, however, can be switched off, or reduced to the classical (diagonal) correlations. It is also possible to specify a particular helicity of a decaying resonance.

Top quarks are an important tool for searches for new physics. Anomalous top quark

![Figure 1](image-url)
couplings are implemented in Whizard in an effective-field theory setup including dimension-6 operators (this also includes 4-fermion operators) \cite{8,9}. Furthermore, flavor-violating top quark couplings have been implemented to allow for the simulation of $t \rightarrow c$ transitions.

To produce full events, Whizard contains its own QCD parton shower implementation, supporting both an analytical as well as a $p_T$ ordered shower \cite{10}. To connect hard matrix elements to the shower, Whizard uses the color flow formalism \cite{11}. Hadronization, however, has to be done using external tools. Precision state-of-art predictions for SM processes should be at next-to-leading order (NLO) in the strong coupling constant. After early attempts for the inclusion of QED NLO corrections \cite{12,13} and QCD NLO corrections \cite{14,15}, Whizard now allows for the automatic generation of QCD NLO events using external virtual one-loop matrix elements from either Gosam \cite{16} or OpenLoops \cite{17}. For a finite integration, soft- and collinear regions in an NLO calculation have to be treated by a subtraction formalism. Whizard automatically generates FKS \cite{18,19} subtraction terms, i.e. the corresponding phase space mappings for the soft- and collinear regions. As an example for top physics we show in Figure 1 differential distributions for the off-shell top-quark production $e^+e^- \rightarrow W^+W^-b\bar{b}$ at NLO QCD for $\sqrt{s} = 500$ GeV. The left-hand side shows the $bW$ invariant mass which is smeared out from the top peak mainly by the real radiation at NLO, while the right-hand side shows the energy distribution of hardest jet in the events. Whizard also allows for a proper matching of the fixed-order NLO calculation to the parton shower using the POWHEG scheme, again for arbitrary (lepton-collider) processes.

2 Top-antitop threshold

The c.m. energy region around the top-antitop threshold, i.e. where $\sqrt{s} \sim 2m_t$, is of particular interest for experiments at a future lepton collider like the ILC or CLIC. The measurement of the (total) cross section near threshold will allow for a determination of the top mass in a theoretically well-defined (short-distance) scheme and with unprecedented accuracy ($\delta m_t \lesssim 100$ MeV) by fitting the theoretical prediction to the resonance lineshape. Also other important (SM) parameters like the top decay width (i.e. $V_{tb}$), the strong coupling $\alpha_s$ or the top Yukawa coupling can be determined very precisely from the cross section close to threshold \cite{20,21}. Such measurements can in principle be fully inclusive, but in practice due to experimental cuts and tagging in the final state, more exclusive/differential observables might help to improve the precision of the extracted top mass or increase the sensitivity to the parameter of interest.

Our aim is therefore to provide fully differential Monte Carlo predictions for the top threshold region. Crucially, this requires the resummation of Coulomb singular terms $\propto (\alpha_s/v)^n$ to all orders in perturbation theory, which reflect the bound-state nature of the non-relativistic top-antitop system. Here $v \sim \alpha_s \sim 0.1$ represents the relative velocity of the top quarks close to threshold. In addition to the Coulomb singularities, large logarithms $\propto \ln^n v$ can spoil the perturbative series and should be resummed. The resummation of all threshold enhanced terms can be systematically carried out using a Schrödinger equation and the velocity renormalization group within the non-relativistic effective field theory vNRQCD \cite{22}. In vNRQCD the relevant dynamical scales are the soft scale, given by the top momentum $\sim m_t v$, and the ultrasoft scale,
given by the kinetic energies of the tops $\sim m_t v^2$. Particle modes with momenta of the order of the hard scale $m_t$ (or bigger) have been integrated out.

The decay of the top quarks – predominantly into $Wb$ – plays a key role for the prediction of top-antitop threshold production. The large decay width $\Gamma_t \gg \Lambda_{\text{QCD}}$ effectively serves as an infrared cut-off in the vNRQCD calculation and thus allows for a perturbative description of the process. Upon resummation of the singular terms the normalized cross section (R-ratio) close to threshold schematically takes the form

$$ R = \frac{\sigma_{\bar{t}t}}{\sigma_{\mu^+\mu^-}} \sim v \sum_k (\frac{\alpha_s}{v})^k \sum_i (\alpha_s \ln v)^i \times \left\{ 1 \text{ (LL)}; \alpha_s, v \text{ (NLL)}; \alpha_s^2, \alpha_s v, v^2 \text{ (NNLL)}; \ldots \right\}, $$

(1)

where we have indicated the terms at leading-logarithmic (LL) order, next-to-leading logarithmic (NLL) order, etc. The most up-to-date vNRQCD prediction of the total cross section has reached NNLL [23,24,25] precision.

### 2.1 Treatment inside the generator

For the implementation in WHIZARD, which was first discussed in [26], we supplement the SM $tt\gamma$ and $ttZ$ (vector and axial-vector) vertices in the LO $e^+e^- \rightarrow W^+W^-b\bar{b}$ Monte Carlo process with NLL (S- and P-wave) non-relativistic form factors. As this is a modification of the plain standard model, a special model, $\text{SM}_{tt}\text{threshold}$ is available for that purpose in WHIZARD. The mentioned $tt\gamma$ and $ttZ$ form factors consist of a vertex (Green) function and a Wilson coefficient, which is subject to (velocity) RG running at NLL [27,28]. The Green function is computed with the TOPPIK code [29], which numerically performs the Coulomb resummation, and for the purpose of the threshold resummation is shipped together with the WHIZARD distribution. For numerical stability and sufficient speed during integration and event generation, we create interpolation grids for the form factor in $\sqrt{s}$ and the square of the top three-momentum prior to the MC integration. As we are aiming for differential cross sections, it is important to also include the P-wave contributions, which only represent a minor effect in the total cross section but are crucial to describe e.g. the forward-backward asymmetry correctly. Besides providing fully differential predictions, the embedding in WHIZARD and O’MEGA also has the advantage that the leading effects from the interference with the non-resonant $W^+W^-b\bar{b}$ background can be taken into account.

A subtle but pivotal aspect of this computation is the top decay. Per default, O’MEGA attaches the tree-level decay of the top to the resummed production graph. This implies that the LO width should be used in all top propagators such that upon integration the total on-shell $tt$ cross section is approximately reproduced as $\Gamma_t \ll m_t$. On the other hand, to reconcile the prediction for the threshold with the continuum in Section 2.2 we have to use the NLO width everywhere. This in turn requires the NLO decay of the tops, also in the non-relativistic computation. Furthermore, in combination with the LL resummed production, the NLO correction to the decay can be an important NLL effect in exclusive cross sections near threshold and should be taken into account. It is particularly important when an additional
hard gluon is resolved. We are currently working on a consistent implementation of the NLO top decay using factorization. In this approach, we refrain from applying an on-shell projection, but evaluate production and decay matrix elements with the mass set to the top invariant mass. This allows to apply the factorization also below threshold and is similar to the procedure in Ref. [30], but without the boost to equal invariant masses. An additional benefit of this implementation of the decay in the threshold resummed process is that gauge invariance is guaranteed by working with on-shell decays instead of a restricted set of Feynman diagrams, i.e. signal diagrams. As in every resummed computation used for event generation, this setup has full NLL+NLO accuracy only for sufficiently inclusive observables. For arbitrarily exclusive observables the precision is formally limited to LL+NLO, where the LL is with respect to threshold resummation.

We have performed several cross checks of the numerical implementation of the non-relativistic resummation in the Whizard code. For example, we have verified that the analytic results, augmented with relativistic corrections, are precisely reproduced for $\alpha_s \rightarrow 0$ when using moderate cuts. We can also use analytic results as crosscheck in the threshold region, i.e. where $v \rightarrow 0$ or $\sqrt{s} \rightarrow 2m_t$: For the on-shell process, we have verified that Whizard perfectly reproduces the analytic result using on-shell form factors expanded to $O(\alpha_s)$ in the threshold region. Another important consistency check is the agreement of the prediction using the expanded off-shell form factor with the full QCD NLO result in the threshold region as shown in Figure 2.

2.2 Matching to the relativistic continuum

Future lepton colliders might run at energies close to, but a bit off threshold (cf. e.g. the 380 GeV staging from the CLIC study group). Therefore a smooth transition (matching), between the resummed threshold prediction and the fixed order prediction at large energies is required. The necessary ingredients for this are: the full $e^+e^- \rightarrow W^+bW^-\bar{b}$ NLO result, the threshold resummed vNRQCD prediction, its expansion up to $O(\alpha_s)$ (with relativistic scale setting) and a switch-off function that turns off the unphysical resummation effects away from threshold. For the NLO result, we can obtain the virtual amplitude conveniently via the BLHA interface from OpenLoops [17]. The FKS subtraction in Whizard [31] automatically identifies the singular regions and adds and subtracts the necessary terms in the real and virtual components. We then add the non-relativistic resummed process and, to avoid double counting, subtract the non-relativistic NLO terms with the hard scale, cf. the orange curve in Figure 2. As noted already above, this expanded form factor gives a very good approximation of the full process as it contains the dominant terms close to threshold.

Finally, we have to cure the problem that the resummed prediction keeps growing arbitrarily with $\sqrt{s}$, which indicates the break-down of the non-relativistic approximation and is seen by the rise of the green curve in Figure 2. This is done by multiplying the relevant couplings with a switch-off function that smoothly approaches zero as one moves away from threshold. One has some freedom in the definition of this function as well as the decision where to switch off the resummation, i.e. at which energies one does not trust the resummed results anymore. This freedom has to be treated as an uncertainty that should eventually be combined with the scale uncertainties (and other potential error sources) for a reliable theory error estimate in the
Figure 2: Matching the NLL resummed threshold prediction to the fixed-order NLO QCD continuum for the total cross section of the process $e^+e^- \rightarrow W^+bW^-b$. The blue dots show our (preliminary) result of the matching between the non-relativistic threshold and relativistic continuum region. The solid green line corresponds to the LO process with insertions of the NLL resummed form factors. The orange curve shows the same result with the form factors expanded to first order in $\alpha_s$ with relativistic (hard) scales, i.e. $\alpha_H \equiv \alpha(m_t)$. The red crosses represent the full relativistic fixed-order NLO result.

intermediate region. Given the rather smooth transition between the threshold and continuum region in the matched prediction of Figure 2, we however expect this error to be relatively small for the total cross section. In summary, the procedure described above allows us to consistently add the terms beyond NLO from the NLL threshold resummation to the full NLO QCD result in Whizard.

3 Summary

In this talk, we presented the current status of top quark physics inside the event generator Whizard with a special emphasis on lepton colliders. Two main ongoing projects are the general automation of (QCD) fixed-order NLO corrections for SM processes and a proper matching of the continuum off-shell top pair production with the non-relativistic resummed corrections at the top threshold.
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References

[1] W. Kilian, T. Ohl and J. Reuter, Eur. Phys. J. C 71, 1742 (2011) doi:10.1140/epjc/s10052-011-1742-y [arXiv:0708.4233 [hep-ph]].

[2] M. Moretti, T. Ohl and J. Reuter, hep-ph/0102195.

[3] T. Ohl, Comput. Phys. Commun. 101, 269 (1997) doi:10.1016/S0010-4655(96)00167-1 [hep-ph/9607454].

[4] T. Ohl, Comput. Phys. Commun. 120, 13 (1999) doi:10.1016/S0010-4655(99)00209-X [hep-ph/9806432].

[5] J. S. Kim, J. Reuter, K. Rolbiecki and R. R. de Austri, arXiv:1512.06083 [hep-ph]; W. Kilian, T. Ohl, J. Reuter and M. Sekulla, Phys. Rev. D 93, no. 3, 036004 (2016) doi:10.1103/PhysRevD.93.036004 [arXiv:1511.00022 [hep-ph]]; J. Reuter and M. Tonini, JHEP 1501, 088 (2015) doi:10.1007/JHEP01(2015)088 [arXiv:1409.6962 [hep-ph]]; W. Kilian, T. Ohl, J. Reuter and M. Sekulla, Phys. Rev. D 91, 096007 (2015) doi:10.1103/PhysRevD.91.096007 [arXiv:1408.6207 [hep-ph]]; J. Reuter, M. Tonini and M. de Vries, JHEP 1402, 053 (2014) doi:10.1007/JHEP02(2014)053 [arXiv:1310.2918 [hep-ph]]; J. Reuter and D. Wiesler, Eur. Phys. J. C 73, no. 3, 2355 (2013) doi:10.1140/epjc/s10052-013-2355-4 [arXiv:1212.5559 [hep-ph]]; N. Pietsch, J. Reuter, K. Sakurai and D. Wiesler, JHEP 1207, 148 (2012) doi:10.1007/JHEP07(2012)148 [arXiv:1206.2146 [hep-ph]]; J. Reuter and D. Wiesler, Phys. Rev. D 84, 015012 (2011) doi:10.1103/PhysRevD.84.015012 [arXiv:1010.4215 [hep-ph]]; J. Kalinowski, W. Kilian, J. Reuter, T. Robens and K. Rolbiecki, JHEP 0810, 090 (2008) doi:10.1088/1126-6708/2008/10/090 [arXiv:0809.3997 [hep-ph]]; A. Alboteanu, W. Kilian and J. Reuter, JHEP 0811, 010 (2008) doi:10.1088/1126-6708/2008/11/010 [arXiv:0807.3914 [hep-ph]]; W. Kilian, D. Rainwater and J. Reuter, Phys. Rev. D 74, 095003 (2006) [Phys. Rev. D 74, 099905 (2006)] doi:10.1103/PhysRevD.74.095003, 10.1103/PhysRevD.74.099905 [hep-ph/0609119]; M. Beyer, W. Kilian, P. Krstonosic, K. Monig, J. Reuter, E. Schmidt and H. Schroder, Eur. Phys. J. C 48, 353 (2006) doi:10.1140/epjc/s10052-006-0038-0 [hep-ph/0604048]; K. Hagiwara, W. Kilian, F. Krauss, T. Ohl, T. Plehn, D. Rainwater, J. Reuter and S. Schumann, Phys. Rev. D 73, 055005 (2006) doi:10.1103/PhysRevD.73.055005 [hep-ph/0512260]; W. Kilian, D. Rainwater and J. Reuter, Phys. Rev. D 71, 015008 (2005) doi:10.1103/PhysRevD.71.015008 [hep-ph/0411213]; W. Kilian and J. Reuter, Phys. Rev. D 70, 015004 (2004) doi:10.1103/PhysRevD.70.015004 [hep-ph/0311095].

[6] N. D. Christensen, C. Duhr, B. Fuks, J. Reuter and C. Speckner, Eur. Phys. J. C 72, 1990 (2012) doi:10.1140/epjc/s10052-012-1990-5 [arXiv:1010.3251 [hep-ph]].
[7] B. Chokoufe Nejad, T. Ohl and J. Reuter, Comput. Phys. Commun. 196, 58 (2015) doi:10.1016/j.cpc.2015.05.015 [arXiv:1411.3834 [physics.comp-ph]].

[8] F. Bach and T. Ohl, Phys. Rev. D 86, 114026 (2012) doi:10.1103/PhysRevD.86.114026 [arXiv:1209.4564 [hep-ph]].

[9] F. Bach and T. Ohl, Phys. Rev. D 90, no. 7, 074022 (2014) doi:10.1103/PhysRevD.90.074022 [arXiv:1407.2546 [hep-ph]].

[10] W. Kilian, J. Reuter, S. Schmidt and D. Wiesler, JHEP 1204, 013 (2012) doi:10.1007/JHEP04(2012)013 [arXiv:1112.1039 [hep-ph]].

[11] W. Kilian, T. Ohl, J. Reuter and C. Speckner, JHEP 1210, 022 (2012) doi:10.1007/JHEP10(2012)022 [arXiv:1206.3700 [hep-ph]].

[12] W. Kilian, J. Reuter and T. Robens, Eur. Phys. J. C 48, 389 (2006) doi:10.1140/epjc/s10052-006-0048-y [hep-ph/0607127].

[13] T. Robens, J. Kalinowski, K. Rolbiecki, W. Kilian and J. Reuter, Acta Phys. Polon. B 39, 1705 (2008) [arXiv:0803.4161 [hep-ph]].

[14] T. Binoth, N. Greiner, A. Guffanti, J. Reuter, J.-P. Guillet and T. Reiter, Phys. Lett. B 685, 293 (2010) doi:10.1016/j.physletb.2010.02.010 [arXiv:0910.4379 [hep-ph]].

[15] N. Greiner, A. Guffanti, T. Reiter and J. Reuter, Phys. Rev. Lett. 107, 102002 (2011) doi:10.1103/PhysRevLett.107.102002 [arXiv:1105.3624 [hep-ph]].

[16] G. Cullen et al., Eur. Phys. J. C 74, no. 8, 3001 (2014) doi:10.1140/epjc/s10052-014-3001-5 [arXiv:1404.7096 [hep-ph]].

[17] F. Cascioli, P. Maierhofer and S. Pozzorini, Phys. Rev. Lett. 108 (2012) 111601 doi:10.1103/PhysRevLett.108.111601 [arXiv:1111.5206 [hep-ph]].

[18] S. Frixione, Z. Kunszt and A. Signer, Nucl. Phys. B 467, 399 (1996) [hep-ph/9512328].

[19] R. Frederix, S. Frixione, F. Maltoni and T. Stelzer, JHEP 0910, 003 (2009) [arXiv:0908.4272 [hep-ph]].

[20] K. Seidel, F. Simon, M. Tesar and S. Poss, Eur. Phys. J. C 73, no. 8, 2530 (2013) doi:10.1140/epjc/s10052-013-2530-7 [arXiv:1303.3758 [hep-ex]].

[21] H. Baer et al., arXiv:1306.6352 [hep-ph].

[22] M. E. Luke, A. V. Manohar and I. Z. Rothstein, Phys. Rev. D 61, 074025 (2000) doi:10.1103/PhysRevD.61.074025 [hep-ph/9910209].

[23] A. H. Hoang, C. J. Reisser and P. Ruiz-Femenia, Phys. Rev. D 82, 014005 (2010) doi:10.1103/PhysRevD.82.014005 [arXiv:1002.3223 [hep-ph]].
[24] A. H. Hoang and M. Stahlhofen, JHEP 1106, 088 (2011) doi:10.1007/JHEP06(2011)088 [arXiv:1102.0269 [hep-ph]].

[25] A. H. Hoang and M. Stahlhofen, JHEP 1405, 121 (2014) doi:10.1007/JHEP05(2014)121 [arXiv:1309.6323 [hep-ph]].

[26] F. Bach and M. Stahlhofen, arXiv:1411.7318 [hep-ph].

[27] A. Pineda, Phys. Rev. D 66, 054022 (2002) doi:10.1103/PhysRevD.66.054022 [hep-ph/0110216].

[28] A. H. Hoang and P. Ruiz-Femenia, Phys. Rev. D 74, 114016 (2006) doi:10.1103/PhysRevD.74.114016 [hep-ph/0609151].

[29] A. H. Hoang and T. Teubner, Phys. Rev. D 60, 114027 (1999) doi:10.1103/PhysRevD.60.114027 [hep-ph/9904468].

[30] J. M. Campbell, R. K. Ellis, P. Nason and E. Re, JHEP 1504 (2015) 114 doi:10.1007/JHEP04(2015)114 [arXiv:1412.1828 [hep-ph]].

[31] C. Weiss, B. C. Nejad, W. Kilian and J. Reuter, PoS EPS-HEP2015 (2015) 466 [arXiv:1510.02066 [hep-ph]].