CP Violation, Higgs Couplings, and Supersymmetry

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Abstract

Supersymmetric extensions of the standard model generically contain additional sources of CP violation. We discuss how at one loop a potentially large CP violating coupling of the lightest Higgs, $h^0$, to leptons is induced in the minimal supersymmetric standard model (MSSM). The CP violating couplings of $h^0$ in extensions of the MSSM, such as the next-to-minimal supersymmetric standard model (NMSSM) are also considered. We indicate how this CP violation might be observed; in particular a polarization-dependent production asymmetry, in the context of a muon collider, provides a means to access this coupling cleanly. In the MSSM, existing limits on the electric dipole moment (EDM) of the electron, coupled with standard universality assumptions, severely constrains any such signal. Nevertheless, extensions of the MSSM, such as the NMSSM, allow CP-violating signals as large as 100%.

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1 Introduction

Supersymmetric (SUSY) extensions of the Standard Model generically contain several additional CP-violating phases beyond the usual Cabbibo-Kobayashi-Maskawa phase. Elucidating their magnitude and structure is important if we are properly to understand the origin of CP violation, or the closely related question of the origin of the cosmic matter-antimatter asymmetry. In this paper we will be concerned with new sources of CP violation in the Higgs sector of SUSY models.

Despite the large number of new phases in the model as a whole, it is well known that in the minimal SUSY extension of the standard model (MSSM) the tree level Higgs potential contains just one complex parameter, the term $B \mu H_u H_d$. Even this phase can be removed by redefinitions of the Higgs fields. Then the vacuum expectation values of the Higgs scalars will be real, with no mixing between the physical scalar and pseudoscalar Higgs fields.

At one loop the situation is different. In Section 2 we will demonstrate that sizable Higgs sector CP violation can be induced at one-loop, even within the MSSM, and especially so at large $\tan \beta$. One place where CP-violating effects can manifest themselves is in the couplings of the lightest neutral Higgs boson to Standard Model fermions. In fact, we will see that these are in principle accessible at a suggested muon collider operating on the Higgs resonance, at least in some regions of the SUSY parameter space. Alternatively, in variations of the MSSM with extended Higgs sectors (such as the so-called NMSSM, defined below) the tree-level Higgs potential contains irremovable CP violating phases. Then substantial CP violation is possible even for small $\tan \beta$.

The possible magnitude of CP violation in the Higgs sector is severely constrained by experimental limits on electric dipole moments (EDMs) of fermions. This is because the same diagrams that contribute to the CP-violating Higgs couplings also contribute to the EDMs of fermions. As we discuss in Section 3, current bounds on the $\mu$EDM do little to constrain our scenario, but within the MSSM, bounds on the $e$EDM are highly constraining given minimal theoretical prejudices. In Section 3 we therefore also discuss how observation of CP violation, or lack thereof, in the Higgs-lepton coupling fits into the broader picture painted by flavor-changing and CP-violation constraints on SUSY extensions of the Standard Model. We emphasize that CP violation in the Higgs-fermion couplings is a way of discriminating the MSSM from more elaborate extensions, such as the NMSSM. Our conclusions are summarized in Section 4.

Though most of our discussion is phrased in terms of the coupling of the lightest neutral Higgs, $h^0$, to charged leptons, much of our analysis applies more generally with only slight alterations to Higgs-quark couplings. There are also potentially interesting effects of CP violation in the heavy Higgs sector, which are under study.
2 CP Violation in the Higgs Sector

2.1 The MSSM

Let us first discuss the situation in the minimal supersymmetric extension of the standard model (MSSM).

In the absence of SUSY breaking, the charged leptons couple to the Higgs field $H_d^0$, but not $H_u^0$. After SUSY breaking, a coupling of the leptons to $H_u^0$ is generated at one loop, and in general it will not be real, due to phases in the soft-breaking parameters and the $\mu$ term. The resulting coupling of the lepton to the Higgs fields is of the form

$$\mathcal{L} = a \overline{\ell} L H_d^0 + b \overline{\ell} L H_u^0 + h.c.$$  \hspace{1cm} (1)

By redefining lepton fields while keeping the vacuum expectation values of the Higgs fields real, we may choose $a$ real but must then allow $b$ to be complex. The resulting lepton mass term is then:

$$\mathcal{L}_{\text{mass}} = \overline{\ell} L (av_d + bv_u) \ell R + h.c.$$  \hspace{1cm} (2)

where $v_{u,d} = \langle H_{u,d}^0 \rangle$. In the Standard Model with only one Higgs doublet, no physical CP violation can arise from the Higgs coupling, because the same rotations that make the lepton masses real also make their couplings to the neutral Higgs particle real. But in this two-doublet extension the fermion masses do not correspond directly to the couplings to the physical Higgs states. Specifically, we can write

$$\text{Re} \left( H_u^0 \right) = \frac{1}{\sqrt{2}} \left( \cos \alpha h^0 + \sin \alpha H^0 \right)$$  \hspace{1cm} (3)

$$\text{Re} \left( H_d^0 \right) = \frac{1}{\sqrt{2}} \left( -\sin \alpha h^0 + \cos \alpha H^0 \right)$$

where $m_{h^0} < m_{H^0}$.

Here, for the sake of simplicity, we have made the good approximation of omitting additional “pseudoscalar” components on the right hand side. In principle one loop contributions to the Higgs effective potential can spontaneously break CP \[\llbracket\], and/or communicate explicit CP-violation in the soft masses and $\mu$-term to the Higgs sector. In either case, a small phase in the Higgs vacuum expectation values is induced which leads to scalar-pseudoscalar mixing, but this effect is \lesssim 1%, too small to affect our conclusions. A relative phase between $\langle H_u \rangle$ and $\langle H_d \rangle$ arises in the MSSM when a Higgs quartic coupling term $(H_u H_d)^2$ is induced, but since supersymmetry is broken\footnote{In general, because of SU(2)$_L$-breaking, one should write one set of terms for the lepton mass generation and another set for their Yukawa couplings to the Higgses. However, if the electroweak symmetry-breaking effects are small ($m_Z$ smaller than $M_{\text{SUSY}}$), both the mass term and the Higgs coupling will to a good approximation arise from the terms in Eq. (\[\rrbracket\]).}.
softly, such quartic terms arise from finite box graphs, which lead to a very small coefficient.

Since the transformations that eliminate phases in the lepton and quark masses do not eliminate phases from their couplings to $h^0$ and $H^0$, there is a residual violation of CP. In order to extract the CP violating portion of the $h^0\ell\ell$ coupling, we must look for some mismatch between its phase and the phase of the $\ell\ell$ mass term. It is convenient to write the Higgs coupling and mass terms in the forms 

$$L_{h\ell\ell} = \frac{1}{\sqrt{2}} \left[ -a \sin \alpha + b \cos \alpha \right] h^0 e^{i(\varphi - \delta)} \gamma^5 \ell.$$ 

The observable CP-violating phase is then $\varphi - \delta$. It is clear that in the limit $\alpha \to \beta - \frac{\pi}{2}$, or equivalently $\cot \alpha \to -\tan \beta$, the phase $\varphi - \delta$ disappears (recall $\tan \beta \equiv v_u/v_d$). This is the well-known Higgs decoupling limit of the MSSM in which the second doublet becomes much heavier than the weak scale ($m_A \gg m_Z$) so that the low-energy Higgs sector closely approximates that of the SM. The Higgs mixing angle aligns itself with that of the fermion mass terms so that the fermions effectively couple to only one scalar Higgs field, allowing all phases to be removed.

We are interested in the CP violation in the Higgs sector arising from the one loop induced parameter $b$. Representatives of the two basic classes of diagrams that contribute to the $\ell\ell H^0_u$ amplitudes are shown in Fig. 1. These are in the same class of diagrams whose real parts have been studied both in certain fermion mass generation scenarios [2] and have been found to significantly shift the $b$-quark mass at large $\tan \beta$ [3]; here we will confine ourselves to the leptonic sector of the theory and study the imaginary parts of the diagrams. We will only work to leading order in the slepton, neutralino and chargino mixing, which is to say $m_Z \ll \mu, M_2, m_\tilde{\mu}$, and in the limit of $\tan \beta \gg 1$. In these limits, all of the contributing diagrams are linearly dependent on $\mu/\cos \beta$, though for different reasons. (Contributions proportional to soft trilinear $A$-terms do not receive this $1/\cos \beta$ enhancement and thus their contributions to Higgs sector CP violation are unobservably small in generic scenarios.) This is also the approximation in which our effective Lagrangian is adequately described by Eq. (1); that is, we ignore terms in $\mathcal{L}$ with multiple insertions of $H^0_{u,d} H^0_{u,d}$ suppressed by the SUSY-breaking scale. This approximation is sufficient for our purposes since in the region where the CP violating Higgs couplings of the MSSM are significant, the corrections due to exact diagonalization of the various mass matrices are small.
Figure 1: Representative diagrams which contribute to the $\bar{t}_L \ell_R H^0_u + h.c.$ coupling.

The contribution to the Yukawa coupling from diagram (a) contains a factor of $\mu^*/\cos \beta$ coming from the left-right mixing of the sleptons:

$$A^\ell_1 = \frac{3\alpha_1}{20\pi} y_\ell \mu^* M_1 f \left( M_1^2, m_{\tilde{\ell}}^2, m_{\tilde{\ell}_c}^2 \right)$$  \hspace{1cm} (7)

where $y_\ell$ is the lepton Yukawa coupling, $M_1$ is the U(1) gaugino (i.e., bino) mass and

$$f \left( m_{\tilde{\ell}}^2, m_{\tilde{\ell}_c}^2, m_{\tilde{\ell}_c}^2 \right) = \frac{1}{m_3^2} \left[ \frac{x \ln x}{1 - x} - \frac{y \ln y}{1 - y} \right] \frac{1}{x - y}$$  \hspace{1cm} (8)

with $x = m_{\tilde{\ell}}^2/m_{\tilde{\ell}_c}^2$ and $y = m_{\tilde{\ell}}^2/m_{\tilde{\ell}_c}^2$. Diagram (b) picks up a $1/\cos \beta$ from the Yukawa coupling of the external muon to the $\tilde{H}_d$ higgsino and a $\mu^*$ from the mixing of the $\tilde{H}_d$ with $\tilde{H}_u$ on the internal line:

$$A^\ell_2 = \frac{\alpha_2}{8\pi} y_\ell \mu^* M_2 \left[ f \left( \mu^2, m_{\tilde{\ell}}^2, M_2^2 \right) + 2 f \left( \mu^2, m_{\tilde{\ell}_c}^2, M_2^2 \right) \right]$$  \hspace{1cm} (9)

where $M_2$ is the SU(2) gaugino (i.e., wino) mass. The contributions from both the charged and neutral gaugino/higgsino loops are included in $A^\ell_2$. Finally, there are also contributions analogous to those of diagram (b) but including only the (neutral) bino states in the loops. They contribute to the amplitude

$$A^\ell_3 = -\frac{3\alpha_1}{40\pi} y_\ell \mu^* M_1 \left[ f \left( \mu^2, m_{\tilde{\ell}}^2, M_1^2 \right) - 2 f \left( \mu^2, m_{\tilde{\ell}_c}^2, M_1^2 \right) \right].$$  \hspace{1cm} (10)

Note that the function $\mu^* M_2 f(\mu^2, m_{\tilde{\ell}}^2, M_2^2)$ in Eq. (9) (and similarly in Eq. (10)) has a maximum value of 1. In contrast, the function $\mu^* M_1 f(M_1^2, m_{\tilde{\ell}}^2, m_{\tilde{\ell}_c}^2)$ of Eq. (7) has a maximum value of $\mu^*/M_1$ which can be significantly larger than 1.

The parameter $b$ in the effective Lagrangian is then simply the sum of the $A^\ell_i$’s. Numerically, $|b \tan \beta| \ll a$ (since $b$ is loop-suppressed) so that we can approximate

$$\eta_{CP}^\ell \equiv \tan(\varphi - \delta) \simeq -\frac{\text{Im} \left( \frac{b}{a} \right) \cot \alpha + \tan \beta}{\text{Im} \sum A^\ell_i} \min(m_{A}^2, m_{Z}^2)$$  \hspace{1cm} (11)

$$\simeq -\frac{2 \text{Im} \left( \sum A^\ell_i \right) \min(m_{A}^2, m_{Z}^2)}{y_\ell \mu^*} \frac{m_{A}^2 + m_{Z}^2}{m_{A}^2 + m_{Z}^2}$$  \hspace{1cm} (12)

In order to set our sign convention for $\mu$, we take $W = \mu(H_d^+ H_u^0 - H_d^0 H_u^0)$. 

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where \( \eta_{\text{CP}}^\ell \) will be used henceforth to parameterize the amount of CP violation in the Higgs-lepton couplings. To get this last equation, we have used the well-known relation [4] of the MSSM, 
\[
\sin 2\alpha \simeq -\frac{(m_A^2 + m_Z^2)/(|m_A^2 - m_Z^2|)}{\sin 2\beta} \text{ for large } \tan \beta.
\]
Note that Eq. (12) demonstrates explicitly the suppression of the CP violation in the Higgs decoupling (large \( m_A \)) limit.

For the case of the MSSM, lower \( \tan \beta \) means proportionally smaller \( \eta_{\text{CP}}^\ell \). Therefore we will restrict our attention to the large \( \tan \beta \) regime, in which \( \eta_{\text{CP}}^\ell \) is maximized, when discussing the MSSM. Of course, even if the underlying CP violating phase \( \arg(\mu) \sim O(1) \), the effects in the Higgs-lepton couplings will always be suppressed by a loop factor. Thus even at \( \tan \beta \sim 50 \) one does not expect more than a 10% effect in the Higgs couplings, i.e., \( \eta_{\text{CP}}^\ell \lesssim 0.1 \).

On the other hand, for the down-type quarks in the MSSM, the one-loop induced CP violation in the Higgs-quark couplings can be substantially bigger (of order 100%) owing to the \( \tan \beta \)-enhanced contribution from the gluino.

In Figure 2 we have shown a contour plot of the value of \( \eta_{\text{CP}}^\mu \) for \( \tan \beta = 50 \) and \( \arg(\mu) = \pi/4 \), with \(|\mu|\) along the x-axis. For simplicity we parameterize the masses of the gauginos along the y-axis using the usual supergravity-type parameter, \( M_{1/2} \), where gaugino mass unification is assumed, i.e., \( M_i = \alpha_i M_{1/2}/\alpha_{\text{unif}} \). We also assume scalar mass unification, defining a “common” scalar mass \( M_0 \); however this last assumption is only used to define \( m_{\tilde{\ell}_L}^2 = M_0^2 + 0.5M_{1/2}^2 \) and \( m_{\tilde{\ell}_R}^2 = M_0^2 + 0.15M_{1/2}^2 \) which are the formulas which follow from a renormalization-group analysis in supergravity models. For this figure we make the further illustrative choice that \( M_0 = 2M_{1/2} \), though the qualitative features of the figures are independent of this, or any other, simplification. Both \(|\mu|\) and \( M_{1/2} \) are allowed to vary from 50 GeV to 10 TeV logarithmically.

Note that the biggest effects occur when the SUSY masses are large compared to the weak scale, with both Higgs doublets remaining light. This is not necessarily an unnatural scenario. Indeed, within supergravity-mediated models of SUSY-breaking, one expects the Higgs potential at large \( \tan \beta \) to be extremely flat. In this case the second derivative along the imaginary direction (i.e., \( m_A^2 \)) will be small, ensuring light Higgs doublets.

### 2.2 The NMSSM

The next-to-minimal supersymmetric standard model (NMSSM) is the simplest extension of the MSSM. In this model, the \( \mu H_d H_u \) term is replaced by the superpotential interaction \( \Delta W = \lambda N H_d H_u + \frac{k}{3} N^3 \), where \( N \) is a gauge singlet [5]. This has the advantage that the explicitly dimensionful parameter \( \mu \) of the MSSM is replaced by \( \lambda \langle N \rangle \), with \( \lambda \) dimensionless. For our present purposes, the most important difference from the MSSM is that now there is an irremovable phase in the Higgs sector, which can generate large CP violation even if all the soft supersymmetry breaking parameters are approximately real. Explicitly, the Higgs potential now has three terms with
Figure 2: Plot of $\eta_{\mu}^{CP}$ in the MSSM as a function of $|\mu|$ and the gaugino mass parameter $M_{1/2}$, both varying on a log scale from 50 GeV to 10 TeV. We have chosen $\tan \beta = 50$, $\arg(\mu) = \pi/4$ and the smuon soft mass parameter $M_0 = 2M_{1/2}$. The shaded regions correspond to $\eta_{\mu}^{CP} > 10\%, 5\%, 2.5\%, 1\%, 0.5\%, 0.25\%$, and 0.125% going from darkest to lightest.

non-trivial phase structure: $V \supset \lambda k^* H_u H_d N^* + A_\lambda \lambda H_u H_d N + A_k k N^3 + h.c.$ Even if the soft terms $A_\lambda \lambda$ and $A_k k$ are real, the phase of $\lambda k^*$ cannot be removed. This generally leads to complex vev’s for $H_u$, $H_d$ and $N$. The physical Higgs particles will then be admixtures of scalars and pseudoscalars.

Thus in the NMSSM there are, already at tree-level, CP-violating couplings of the mass eigenstate Higgs particles to fermions. This follows simply from

$$\mathcal{L} = a \bar{\ell} L \ell R H^0_d + h.c.$$  
$$= \frac{a}{\sqrt{2}} \left( O_{11} \bar{\ell} \ell h^0 - i O_{21} \bar{\ell} \gamma^5 \ell h^0 \right) , \quad (13)$$

where $O_{ij}$ is the matrix diagonalizing the Higgs sector mass eigenstates in the basis $(\text{Re}\, H^0_d, \text{Im}\, H^0_d, \text{Re}\, H^0_u, ...)^T = O(h^0, ...)^T$. We can again define $\eta_{\mu}^{CP}$ for the NMSSM (or any larger extension of the MSSM) as the amount of CP violation present in the Higgs-lepton couplings: $\eta_{\mu}^{CP} \equiv -O_{21}/O_{11}$, assuming $\eta_{\mu}^{CP} \ll 1$. (The above arguments follow equally well for Higgs couplings to quarks in the NMSSM.)

Now, however, in contrast to the MSSM, $\eta_{\mu}^{CP}$ has no strong dependence on $\tan \beta$, and no loop suppression, so that there can be large CP-violating effects over a wide range of $\tan \beta$. 

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2.3 Collider Searches

In order to measure directly the amount of CP violation in the Higgs-lepton couplings, one will undoubtedly require a very large number of well-tagged Higgs bosons. To our knowledge, the most promising scheme for producing such a large sample is to operate a muon collider on the Higgs resonance. There are in principle several CP-violating observables which are accessible in a muon facility. The most straightforward analysis is for the left-right polarization production asymmetry:

\[ A = \frac{\sigma(\mu_L^+ \mu_R^- \rightarrow h^0 \rightarrow b\bar{b}) - \sigma(\mu_R^+ \mu_R^- \rightarrow h^0 \rightarrow b\bar{b})}{\sigma(\mu_L^+ \mu_L^- \rightarrow h^0 \rightarrow b\bar{b}) + \sigma(\mu_R^+ \mu_R^- \rightarrow h^0 \rightarrow b\bar{b})} \equiv \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{RR}} \quad (14) \]

which is zero in the absence of CP violation. It is simple to show that

\[ A \approx \frac{4\eta_{\mu CP}^\mu P}{1 + P^2} \quad (15) \]

for beam polarizations \( P \), assuming the same polarization for both beams and \( \eta_{\mu CP}^\mu \ll 1 \). This is to be compared to a background from \( \mu^+ \mu^- \rightarrow (\gamma, Z) \rightarrow b\bar{b} \), which has a cross-section of the same order as that of the Higgs-mediated process, but suppressed by \( (1 - P^2) \). A simple estimate can be made of the integrated luminosity, \( \int L \), that will be necessary in order to make a 3σ discovery of non-zero \( A \) (without considering losses due to acceptances and efficiencies):

\[ \int L = \frac{(3/\sqrt{A})^2}{\sigma_S + \sigma_B} \sigma_S^2 \]

where \( \sigma_S(B) \) is the signal (background) cross-section \( \sigma_{LL} + \sigma_{RR} \).

In Fig. 3 we plot the luminosity needed for a 3σ measurement of non-zero \( A \) in one year (10\(^7\) s) against our CP-violating parameter \( \eta_{\mu CP}^\mu \). (The figure assumes \( m_{h^0} = 100 \) GeV, but varying \( m_{h^0} \) changes the figure little.) The plotted lines represent the limit for different beam polarizations: from top to bottom, \( P = 0.2, 0.5, 0.8 \) and 1.0. For current collider design parameters of \( \mathcal{L} = 5 \times 10^{30} \) cm\(^2\)s\(^{-1}\) and “natural” beam polarization \( (P = 0.2) \), one sees from the figure that \( \eta_{\mu CP}^\mu > 8\% \) is accessible. To probe down to \( \eta_{\mu CP}^\mu = 2\% \) would require a 16-fold increase in the luminosity or beam polarizations better than 75%.

As we will discuss in the next section, measurements of the electron EDM, combined with well motivated assumptions about the slepton mass spectrum, constrain the CP violation in the muon coupling to be very small within the MSSM, usually <1%. However, the constraints on the \( \tau \) coupling are much weaker. So if the MSSM model is correct a more theoretically promising, though experimentally demanding, window for viewing CP violation in the Higgs-lepton couplings is in the final state \( \tau \) polarization asymmetry from \( h^0 \rightarrow \tau^+\tau^- \). (See Ref. [8] for a related discussion.)

Within the NMSSM (or other extensions of the MSSM Higgs sector), much larger CP-violating effects are possible in the Higgs-lepton couplings. Though the constraints on the mass spectrum and CP-violating phases arising from the \( e \)EDM and \( \mu \)EDM, reviewed in the following section, are essentially identical to those in the

\[ \dagger \text{For a related discussion on CP violation in two photon coupling of the Higgs, see Ref. [9].} \]
Figure 3: Luminosity needed for a $3\sigma$ measurement of CP violation in the polarized production asymmetry $\mu^+_{L,R}\mu^-_{L,R} \rightarrow h^0 \rightarrow b\bar{b}$ in one year, as a function of the amount of CP violation in the Higgs-muon coupling ($\eta^\mu_{CP}$). The four lines correspond (in descending order) to beam polarizations $P = 0.2, 0.5, 0.8$ and $1.0$.

MSSM, the effective amount of CP violation in the Higgs-lepton couplings, $\eta^\ell_{CP}$, is much larger for two reasons. First, $\eta^\ell_{CP}$ in the NMSSM is unsuppressed by loop factors. Second, we can have a significant effect even for small $\tan\beta$. In the next section, we will see that these properties permit much larger values of $\eta^\ell_{CP} \sim 10\%$ to $100\%$ in the NMSSM. Conservative muon collider design parameters are already sufficient to see such large CP violation.

3 Electric and magnetic dipole moments

The fundamental CP-violating phase of the $\mu$-term which is responsible for the existence of a CP-violating Higgs-fermion coupling also contributes to the electric dipole moments (EDMs) of the electron, muon and neutron. In fact, the diagrams of Fig. 1 contribute directly to the EDM of an electron or muon when the external Higgs is replaced by its vacuum expectation value and a photon is attached to any charged line. Because the experimental constraints on the muon and electron couplings are so different, we will consider them each in turn.

When $\ell = e$ in Figure 1, a non-zero $e$EDM is generated. Current experimental constraints on the $e$EDM are extremely strong. Specifically, $d_e = (1.8 \pm 1.6) \times$
Figure 4: EDM constraints for (a) \( \tan \beta = 50 \) and (b) \( \tan \beta = 2 \), as a function of \(|\mu|\) and the gaugino mass parameter, \( M_{1/2} \), both varying on a log scale from 50 GeV to 10 TeV. For illustrative purposes, the soft scalar mass parameter is taken to satisfy \( M_0 = 2M_{1/2} \). Solid, long-dashed, and short-dashed curves correspond to \( \text{arg}(\mu) = \pi/4, 0.1, 0.01 \) respectively. Regions below the curves are excluded by the current eEDM bound.

The effect of this constraint on the SUSY parameter space is astonishing. In Figure 4(a) we have shown the region of \( \mu - M_{1/2} \) parameter space excluded (at 90% C.L.) for \( \tan \beta = 50 \) by the \( e \)EDM bound, following the calculation of Ref. [11]. Different contours correspond to differing sizes of \( \text{arg}(\mu) \): \( \pi/4, 0.1, \) and 0.01. Again, gaugino mass unification is assumed, \( M_i = \alpha_i M_{1/2}/\alpha_{\text{unif}} \), and the soft selectron mass parameter is taken to satisfy \( M_0 = 2M_{1/2} \). Figure 4(b) shows the excluded regions for \( \tan \beta = 2 \).

Plainly for \( O(1) \) phases and \( \tan \beta = 50 \), the masses of the SUSY particles must be so heavy as to approach being unnatural. The end result, in any case, is that one expects very little observable CP violation in the coupling of the Higgs to electrons. (Of course, simply observing the CP-conserving Higgs-electron coupling is so challenging that this is probably a moot point.)

For \( \ell = \mu \), the situation is not as clear. Current experimental bounds on the \( \mu \)EDM, \( d_\mu = 3.7 \pm 3.4 \times 10^{-19} \) cm [12], are not particularly constraining. Taken alone, the current bound on the \( \mu \)EDM does little to limit the SUSY parameter space or the size of \( \eta^\mu_{\text{CP}} \) predicted in the MSSM. Slightly more constraining is the muon magnetic dipole moment which is already measured to an accuracy of approximately \( 10^{-8} \) [12]. It is well-known that \( g - 2 \) of the muon already excludes the MSSM with very large \( \tan \beta \) and very light sleptons: for universal masses and \( \tan \beta = 50 \), the sleptons must lie above 250 GeV at 90% C.L. [13]. In fact, it is the real part of the

\(^3\)In our evaluation of the l-loop contributions to the \( \ell \)EDMs we have numerically diagonalized the full complex neutralino and slepton mass matrices. No mass-insertion approximation has been performed.
diagrams of Fig. 1 that give such large contributions to $g - 2$.

There are plans to improve both the $\mu$EDM measurement as well as that of $(g - 2)_\mu$ in the near future [14]. Over the next few years, direct limits on the $\mu$EDM should dramatically improve, reaching a limit near $10^{-23}$ cm. Scaled by the appropriate $m_e/m_\mu$, this limit approaches within a factor of 10 that already obtained for the electron. If a non-zero $\mu$EDM were to be measured, this would strongly bolster the case for observable CP violation in the direct Higgs-muon coupling. Conversely, non-observation would produce a strong bound, essentially equivalent to the arg($\mu$) = 0.1 $e$EDM bound shown in Figure 4(a), making it unlikely that CP-violation in the MSSM Higgs-muon coupling could be observed.

We should take a moment to comment on the heavy mass limit. Figure 4(a) clearly shows that EDM constraints do/will require either very small phases or very heavy sparticles. In the case of small phases, $\eta^\ell_{CP}$ is similarly small. However, heavy masses do not necessarily imply small $\eta^\ell_{CP}$, which is to say that this is an example of a SUSY non-decoupling effect. It is easy to understand why we do not find decoupling. In particular, if one takes $m_A$ large along with the SUSY masses, leaving only a one Higgs doublet Standard Model at low energies, then these effects do decouple by virtue of the fact that in a one Higgs doublet model all CP violation can be eliminated through field redefinitions. The same is not true in a two Higgs doublet version of the Standard Model, which is what one has for light $m_A$. Now when the SUSY states are integrated out, they generate CP-violating $h^0\ell\ell$ terms in the Lagrangian of the two doublet model. Because this Higgs coupling is marginal (renormalizable), those couplings are only logarithmically sensitive to the SUSY scale. This is in contrast to the EDM operator, $\bar{\ell}\sigma^{\mu\nu}\gamma^5\ell F_{\mu\nu}$, which is irrelevant (non-renormalizable) and therefore flows quickly to zero in the infrared as $1/M_{\text{SUSY}}$.

Do such large SUSY masses make sense? Perhaps. The main “esthetic” or “naturality” constraint on supersymmetric partner masses is that they not induce too large a correction to the Higgs (mass)$^2$ parameter through diagrams which grow quadratically with the mass of the superpartner. The one-loop contributions of the selectrons and smuons to the Higgs (mass)$^2$ parameter are proportional to their relatively small Yukawa couplings, allowing one to satisfy the naturality constraint with masses as large as 1000 TeV. (By way of contrast, the third generation squarks and sleptons as well as the gauginos, because of their $\mathcal{O}(1)$ couplings to the Higgs, must have masses $\sim 1$ TeV.) However, there are two-loop contributions involving the light generation sfermions not proportional to their small Yukawa couplings, and naturality [15] and vacuum stability [16] require that these sfermion masses be below about $(5-20)$ TeV. Thus it is not immediately clear whether one should consider smuon masses such as those demanded by Figure 4(a) to be unnatural or not. (Note that if the gaugino masses are $\lesssim 1$ TeV, but 1st and 2nd generation scalars have masses in the $5 \sim 10$ TeV range, then $\eta^\ell_{CP}$ in the MSSM is suppressed by a factor of $(m_2/m_\mu) \sim 1/10$; see Eqs. (9) – (10).)

However, as we now explain, under some reasonable assumptions concerning the
SUSY spectrum, the constraint on the $e$-EDM can be used to constrain the $\mu$-EDM already. This is because the phase that enters the leptonic coupling via $\arg(\mu)$ is universal – all sleptons receive exactly the same phase. There can be non-universal phases coming from the trilinear $A$-terms, but they are not enhanced at large $\tan \beta$, and unless one assumes some kind of cancellation, the size of the CP-violating phases of the various sleptons are correlated. Furthermore, if the smuon and selectron are approximately degenerate, then the constraint on the $e$-EDM translates directly into a constraint on the muon-Higgs coupling. It implies for one thing that $d_\mu < 10^{-24} \text{ e cm}$, which means that the BNL E821 experiment looking for a non-zero $\mu$-EDM should obtain a null result. It also means that the current limit on the $e$-EDM already constrains the CP violation in the MSSM Higgs-muon coupling to be unobservable at a Higgs factory, even without any improvement in the $\mu$-EDM measurement.

Why should we assume that the smuons and sleptons will be degenerate? Non-degenerate sleptons generically lead to large flavor-changing neutral currents (FCNC’s), specifically $\mu \to e\gamma$. Very heavy sleptons could also account for the lack of FCNC’s in leptonic processes, but if the sleptons are non-degenerate then it is natural to expect the same for the squarks, and there the constraints are much stronger. In particular, in the presence of generic CP-violating phases, $d$-squark masses would have to exceed approximately 5000 TeV in order to agree with the measured CP violation in the kaon system, specifically $\epsilon_K \simeq 2 \times 10^{-3}$. This is far above any possible definition of a natural squark mass and we do not consider this possibility further. There is also the possibility that the new contributions to $\epsilon_K$ could be eliminated through alignment of the quark/squark mass matrices, but to our knowledge no very attractive model has been built along these lines. Thus we are left with degeneracy in the squark sector as by far the most attractive solution to the dual problems of FCNC’s and $\epsilon_K$, and by extension degeneracy becomes the most attractive scenario for the sleptons as well.

The preceding discussion of degeneracy really only applies to the first two generations of sparticles. The third generation, thanks to its small quark mixings with the other generations, has suppressed contributions to FCNC’s and $\epsilon_K$. It would not be difficult to imagine, for example, that the first two generations of sleptons are degenerate and heavy, while the third is much lighter. In that case the constraint from the $e$-EDM would not limit CP violation in the $\tau$ sector and so the decay $h^0 \to \tau^+ \tau^-$ may be a likely place to observe violation of CP. (If $m_{\tilde{\tau}} = m_{\tilde{e}}$, then the $e$-EDM constraint does apply, leaving little room for CP violation in the Higgs-$\tau$ coupling.)

The situation for the NMSSM and other extensions is slightly different. First, we emphasize that the EDM constraints on the (tree-level) phase are essentially identical (for a given SUSY spectrum) to the constraint on the size of CP-violating phases in the MSSM. Indeed, after suitable field redefinitions, this phase of the NMSSM can be moved into a phase of the effective $\mu$-term, $\lambda \langle N \rangle$. Consequently, precisely the same diagrams contribute to the EDMs in the NMSSM as in the MSSM. The only difference, a minor one, is that the neutralino mass matrix is now a $5 \times 5$ matrix; in the limit of fixed $\mu \equiv \lambda \langle N \rangle$ and $k \langle N \rangle \gg \lambda v$, the contributions involving the $N$
component of the neutralino to the EDM are suppressed, and one recovers exactly the structure arising in the MSSM.

Nevertheless, as we discussed in Sections 2.2-2.3, there are two important differences between the MSSM and NMSSM. First, in the NMSSM the CP violation in the Higgs-lepton couplings is neither enhanced nor diminished by tan $\beta$ since it is a tree-level effect. Therefore one does not require very large values of tan $\beta$ in order to obtain observable violations of CP. From the point of view of the EDM constraints, however, it is advantageous to have small tan $\beta$. This is clear when comparing Figures 4(a) and (b). For the same phase, the mass bounds at tan $\beta = 2$ are roughly four times weaker than those at tan $\beta = 50$. Thus the constraint coming from naturalness (i.e., demanding light scalar masses) is less restricting at small tan $\beta$, allowing in turn larger underlying phases and thus larger $\eta_{\text{CP}}$.

A second difference is that in the NMSSM CP-violating couplings of the Higgs particles to fermions occur at tree-level. Thus there are no loop suppressions which suppress $\eta_{\text{CP}}$ with respect to the EDM.

The final effect of these differences is that for scalar masses in the range 1 to 3 TeV, the underlying CP-violating phases in the NMSSM can be $\mathcal{O}(1)$ and thus $\eta_{\text{CP}} \sim \mathcal{O}(1)$ as well. Even for sleptons in the 500 GeV mass range, it is quite natural to expect $\eta_{\text{CP}} \sim 10\%$, which can easily be probed in a single year at a muon collider of current conservative design parameters (see Figure 3). This is contrasted with the much smaller value ($\eta_{\text{CP}} \sim 1\%$) expected in the MSSM. Indeed, one can take advantage of this difference and use a combination of EDM and CP-violating muon-Higgs coupling measurements to discriminate between the MSSM and its extensions.

4 Conclusions

In this paper we have shown that at 1-loop a potentially large CP-violating coupling of the Higgs to SM fermions is induced in the MSSM. The CP-violating coupling to muons, $\eta_{\mu_{\text{CP}}}$, could be accessed cleanly through the polarization-dependent production asymmetry at a muon collider operating on the Higgs resonance. However, by imposing reasonable theoretical expectations, the motivations for which were discussed in Section 3, together with the current bounds on the $e$EDM, we found stringent constraints on the size of $\eta_{\mu_{\text{CP}}}$ in the MSSM. We argued that the CP-violating coupling of third generation fermions to the Higgs could be substantial nevertheless. In simple, natural extensions of the MSSM, such as the NMSSM, CP-violating couplings of $h^0$ to SM fermions occur at tree level, and large CP violation is plausible for the Higgs couplings of all three generations of SM fermions, even after imposing the $e$EDM constraints. In particular, CP-violating signals at a muon collider of $\mathcal{O}(100\%)$ are not ruled out.
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