Recent results on $V_{us}$ from KLOE, KTeV and NA48

Contribution to the proceedings of HQL06, Munich, October 16th-20th 2006

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1 Introduction

The flavour structure in the quark sector of the Standard Model is described by the CKM matrix \[1,2\]. Its unitarity leads to a number of relations for its elements and in particular for the first row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$  \hspace{1cm} (1)

Since $V_{ub} \cong 4 \times 10^{-3}$ the contribution of the last term could be neglected at the current level of uncertainty in $V_{ud}$ and $V_{us}$. This approximation gives $V_{us} = \sin \theta_c$ as originally suggested by Cabibbo.

The most precise value of $V_{ud}$ comes from the super-allowed $0^+ \rightarrow 0^+$ beta transitions between nuclei and $V_{us}$ is usually calculated from the branchings of the kaon semileptonic decays. Going back to PDG 2004 \[3\] $V_{us} = 0.2195 \pm 0.0025$ and $V_{ud} = 0.9738 \pm 0.0005$ giving a deviation from unitarity at the level of $2.3\sigma$ where the contribution from the uncertainties of $V_{ud}$ and $V_{us}$ in the final error are almost equal.

In the last few years a significant progress in the kaon physics has been made by three experiments - KLOE, KTeV and NA48. The reflection of their results to the extraction of $V_{us}$ is subject of this review.

KTeV at the Main Injector (Fermilab) \[5\] and NA48 at SPS (CERN) \[6\] are fixed target experiments and exploit similar techniques of kaon decays in flight. Both consist of a spectrometer system measuring the charged particles momentum and a calorimetry system used for measurement of the energy of photons and electrons. The calorimetry system also provides a way to distinguish between the different type of charged particles through their interactions with matter. A muon veto system is

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placed at the end of each detector complex. The primary purpose of both experiments was to measure the direct CP violation parameter $\epsilon'/\epsilon$ in the neutral kaon system. In 2003 NA48 modified its setup in order to study charged kaon decays.

KLOE experiment is situated at $D\Phi NE$, the Frascati $\phi$ factory, where $e^+e^-$ beams collide with a center of mass energy at the $\phi$ meson mass (1020 MeV). With a probability of $\approx 83\%$ $\phi$ decays into neutral or charged kaons, anticoincident in the $\phi$ center of mass (almost true also in the laboratory system). The presence of $K_L/S$ ($K^\pm$) tags $K_{S/L}$ ($K^\mp$). KLOE detector has $2\pi$ symmetry, the momentum of the decay products is measured by a magnetic spectrometer which is followed by an electromagnetic calorimeter.

## 2 Kaon semileptonic decays

Within the Standard Model the kaon semileptonic decays $K \to \pi l\nu$ (so called Kl3) decay appear as a tree level process of $s \to u$ transition. The inclusive branching ratios of all four modes ($K^0 e^3$, $K^0 \mu^3$, $K^\pm e^3$ and $K^\pm \mu^3$) could be written conveniently in the form

$$Br(K_{l3}(\gamma)) = \frac{G_F^2 M_K^2 S_{EW} I_K C^2 (1 + \delta_{EM}^I)}{128 \pi^3 \tau_K} \times |V_{us} f_{K\pi}^{+\gamma}(0)|^2$$  \hspace{1cm} (2)

where $G_F$ is the Fermi constant, $M_K$ and $\tau_K$ are the corresponding kaon mass and lifetime, $S_{EW}$ is the the short distance electroweak enhancement factor, $S_{EW} \approx 1 + \frac{2\pi(1 - \frac{\alpha}{4\pi})}{\pi} \times \log \frac{M_Z}{M_H} = 1.023$ [8], $C$ is the Klebsh-Gordon coefficient, $C = 1$ for $K^0$ and $C = \sqrt{\frac{1}{2}}$ for $K^\pm$, $\delta_{EM}^I$ represents the long-distance electromagnetic correction [9][10], $f_{K\pi}^{+\gamma}(0)$ is the value of the vector form-factor at zero transferred momentum and $I_K$ is the phase space integral dependent on the mode and the shape of the form-factor.

2.1 Form factors

The kaon form factors are defined as [11]

$$\langle \pi(q)| s\gamma\mu u |K(p)\rangle = f_+^{K\pi}(t) \times (p + q) + f_-^{K\pi}(t) \times (p - q)$$  \hspace{1cm} (3)

where $t = (p - q)^2$ is the transferred momentum. Instead of the couple $f_+, f_-$ usually another set of form-factors is used $f_+(t)$ and $f_0(t) = f_+(t) + \frac{\tau}{M_K - M_\pi} f_-(t)$ inspired by the VMD model. The dependence of the transferred momentum could be written as

$$f_{+,0}^{K\pi}(t) = f_{+,0}^{K\pi}(0)(1 + \delta f(t))$$  \hspace{1cm} (4)
It is convenient to express the charged kaon form factor by the neutral one \( |f_+^{K^+\pi^0}(0)|^2 = (1 + \delta_{SU2}) \times |f_+^{K^0\pi^-}(0)|^2 \). The SU2 breaking parameter is obtained within the Chiral Perturbation Theory, \( \delta_{SU2} = 0.046 \pm 0.004 \) [9, 12]. \( f_+^{K^0\pi^-}(0) \) was calculated for the first time in the 80s [12]

\[
f_+(0) = 0.961 \pm 0.008.
\] (5)

However more recent analysis give higher values \( f_+(0) = (0.981 \pm 0.012) \) [10]. Another result \( f_+(0) = (0.960 \pm 0.009) \) comes from lattice QCD [13] which is consistent with [11]. Since \( f_+(0) \) enters directly in the calculation of \( V_{us} \) a clarification of this problem is highly desirable. In this review (5) is used.

The term \( \delta f_{+,0}(t) \) enters in the phase space integral calculation and is subject to different parametrization. The Taylor expansion gives

\[
\delta f_{+,0}(t) = \lambda'_{+,0} \frac{t}{M^2_{\pi}} + \frac{1}{2} \lambda''_{+,0} \frac{t^2}{M^4_{\pi}}.
\] (6)

while within the VMD model \( f_{+,0} \) correspond to vector or scalar meson exchange and are parametrized by the mass of the pole:

\[
\delta f_{+,0}(t) = \frac{M^2_{V,S}}{M^2_{V,S} - t} - 1
\] (7)

In both cases the unknown parameters are determined experimentally. If in equation (6) the quadratic term is neglected then the shape of the form factor is given only by its slope \( \lambda_+ \). The three collaborations have studied the form factors in the case of \( K_L \rightarrow \pi^0 e\nu \) decays and the results can be summarized in the following table:

|        | \( \lambda'_{+} \)         | \( \lambda''_{+} \)     | \( \lambda_+ \)      | Pole mass |
|--------|--------------------------|------------------------|----------------------|-----------|
| NA48   | 0.0280 ± 0.0024          | 0.0004 ± 0.0009        | 0.0288 ± 0.0012      | 859 ± 18  |
| KTeV   | 0.0217 ± 0.0020          | 0.0029 ± 0.0008        | 0.0283 ± 0.0006      | 881 ± 7.1 |
| KLOE   | 0.0255 ± 0.0018          | 0.0014 ± 0.0008        | 0.0286 ± 0.0006      | 870 ± 9.2 |

The values agree in the case of linear and pole parametrization but there is a discrepancy for the necessity of a quadratic term in (6). Recently the KTeV collaboration has performed a new calculation of the phase space integral with a reduced model uncertainty, \( I_{K_{0e}} = 0.10262 \pm 0.00032 \) [17]. For the rest of the phase-space integrals we use \( I_{K_{0\mu}} = 0.06777 \pm 0.00053 \) with the KTeV quadratic form factor parametrization, \( I_{K_{e3}} = 0.1060 \pm 0.0008 \) and \( I_{K_{\mu3}} = 0.0702 \pm 0.0005 \) with the IS-TRA+ measurement of the form factors [18]. A 0.7% error is added to account for the difference between the quadratic and the pole parametrization of the form-factors.
2.2 Kaon lifetime

During the last year two new measurements of the $K_L$ lifetime have been published by KLOE. One of them is obtained from the the proper time distribution of $K_L \to 3\pi^0$ decays [19], giving $\tau_{K_L} = (50.92 \pm 0.30)\text{ns}$. The second method produces a result for the lifetime as a byproduct of the measurement of the major $K_L$ branching fraction imposing the condition that their sum should be unity [20]. The result is $\tau_{K_L} = (50.72 \pm 0.37)\text{ns}$, independent of the previous measurement. The combined value including also the only previous measurement in the 70s is $\tau_{K_L} = (51.01 \pm 0.20)\text{ns}$. For the $K_S$ lifetime the PDG [22] average is used.

Concerning the charged kaons a new preliminary result for the $K^\pm$ lifetime has been presented by KLOE $\tau_{K^\pm} = (1.2367 \pm 0.0078) \times 10^{-8}\text{s}$ [21]. For the moment the PDG average $\tau_{K^\pm} = (1.2385 \pm 0.0025) \times 10^{-8}\text{s}$ is used and we are waiting for the final result.

2.3 Branching ratios

For a long time the branching ratios of the kaon semileptonic decays were fixed in the PDG due to the lack of new measurements. The BNL result for $Br(K^+e3) = (5.13 \pm 0.10)\%$ [23] published in 2003 was in disagreement with the PDG 2002 value $(Br(K^+e3) = (4.87 \pm 0.06)\%)$ [24] and initiated a lot of experimental activity.

All six major $K_L$ branching fractions have been measured by KTeV determining their ratios of decay rates [25]. The results for $Br(K^+_L e3)$ and $Br(K^+_L \mu3)$ are

$$Br(K_L \to \pi^\pm e^\mp \nu) = (40.67 \pm 0.11)\% \quad (8)$$
$$Br(K_L \to \pi^\pm \mu^\mp \nu) = (27.01 \pm 0.09)\% \quad (9)$$

KLOE has also measured the dominant $K_L$ branchings [20] as mentioned above obtaining for the semileptonic decays

$$Br(K_L \to \pi^\pm e^\mp \nu) = (40.07 \pm 0.15)\% \quad (10)$$
$$Br(K_L \to \pi^\pm \mu^\mp \nu) = (26.98 \pm 0.15)\% \quad (11)$$

Apart from the $K_L$ KLOE has studied $K_S e3$ decays [26]. Using $K_S \to \pi^+\pi^-$ for normalization channel the result is four times more precise than the previous value:

$$Br(K_S \to \pi^\pm e^\mp \nu) = (7.046 \pm 0.091)\% \quad (12)$$

The NA48 experiment has measured the ratio of the branching ratios of $K_L e3$ and all two track events [27]. In this way $Br(K_L e3) = R_e(1.0048 - Br(K_L 3\pi^0))$, where $Br(K_L 3\pi^0)$ is the external input. Using the measured $R_e = 0.4978 \pm 0.0035$ and the current PDG value for $Br(K_L 3\pi^0) = (19.69 \pm 0.26)\%$ the result for the $K_L e3$ branching is

$$Br(K_L \to \pi^\pm e^\mp \nu) = (40.22 \pm 0.31)\% \quad (13)$$
Figure 1: Recent measurements of the kaon semileptonic branching ratios. $Br(K_S \to \pi^\pm e^\mp \nu) = (7.046 \pm 0.091)\%$

Preliminary results for the charged semileptonic decays have also been presented by NA48 [28,29] and KLOE [21].

**NA48**

\[ Br(K^\pm \to \pi^0 e^\mp \nu) = (5.14 \pm 0.06)\% \]  
\[ Br(K^\pm \to \pi^0 \mu^\mp \nu) = (3.46 \pm 0.07)\% \]  

**KLOE**

\[ Br(K^\pm \to \pi^0 e^\mp \nu) = (5.047 \pm 0.043)\% \]  
\[ Br(K^\pm \to \pi^0 \mu^\mp \nu) = (3.310 \pm 0.048)\% \]  

which confirm the discrepancy with the PDG observed by BNL.
This ten new measurements of the kaon semileptonic branching ratios together with the BNL result for $\text{Br}(K^\pm e3)$ are averaged depending on the decay mode and are shown on Figure 1 (apart from $\text{Br}(K_S e3)$, measured only by KLOE). As can be seen they show very good consistency.

2.4 $V_{us}$ from kaon semileptonic decays

Combining all the inputs mentioned above the values for $V_{us} \times f_+ (0)$ from the different modes together with the average are shown on Figure 2.

![Figure 2](image_url)

Figure 2: The experimentally measured quantity $V_{us} \times f_+ (0)$ from kaon semileptonic decays

The precision on the combined measurement of $V_{us} \times f_+ (0)$ is approximately 0.16%. Using for $f_+ (0)$ the value obtained by Leutwyler and Roos the result for $V_{us}$ is

$$V_{us} = 0.2251 \pm 0.0019$$

where the dominant contribution to the error comes from the uncertainty of $f_+ (0)$. 

6
3 \( V_{us} \) from \( Kl2 \) decays

A complementary way to extract \( V_{us} \) is to use the ratio of the branching ratios of the pion and the kaon leptonic decays \([30]\). It can be written as

\[
\frac{Br(K^\pm \to \mu^\pm \nu(\gamma))}{Br(\pi^\pm \to \mu^\pm \nu(\gamma))} = \frac{|V_{us}|^2 f_K^2}{|V_{ud}|^2 f_{K}^2} \times \frac{\tau_K}{\tau_{\pi}} \frac{M_K(1 - \frac{M_{\mu}^2}{M_K^2})^2}{M_{\pi}(1 - \frac{M_{\mu}^2}{M_{\pi}^2})^2} \times \frac{1 + \frac{\alpha}{\pi} C_{K}}{1 + \frac{\alpha}{\pi} C_{\pi}}
\]

(19)

where \( \tau_{K,\pi} \) and \( f_{K,\pi} \) are the meson lifetimes and decay constants correspondingly and \( C_{K,\pi} \) parametrize the electroweak correction. Using the new measurement of \( Br(K^\pm \to \mu^\pm \nu(\gamma)) = (63.66 \pm 0.17)\% \) from KLOE \([31]\) and the lattice QCD calculation of \( f_K/f_{\pi} \) \([32]\) we get \( |V_{us}|/|V_{ud}| = 0.2286^{+0.0026}_{-0.0014} \) which together with the measurement of \( V_{ud} \) \([33]\) gives

\[
V_{us} = 0.2223^{+0.0026}_{-0.0014}
\]

(20)

The accuracy of the result is comparable to (18). The dominant error comes from the uncertainty on the ratio \( f_K/f_{\pi} \).

4 Conclusions

The values of \( V_{us} \) extracted from kaon semileptonic decays and from \( K\mu2 \) decay agree. The average is

\[
V_{us} = 0.2241 \pm 0.0015.
\]

(21)

Using \( V_{ud} = 0.97377(27) \) we have

\[
|V_{ud}|^2 + |V_{us}|^2 = 0.9985 \pm 0.0009.
\]

(22)

This result is compatible with the Standard Model and the unitarity of the CKM matrix.

Acknowledgements

I would like to thank prof. dr. Leandar Litov for the valuable help during the preparation of this review and the Joint Institute for Nuclear Research - Dubna for the sponsorship.
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