Frequency Evolution of the Gravitational Waves for Compact Binaries

B. Mikócsi

Departments of Theoretical and Experimental Physics, University of Szeged, Szeged 6720, Hungary

Abstract. We present here a complete list of contributions to the gravitational wave frequency evolution from compact binaries on circular orbits up to the second post-Newtonian order by including the interaction of magnetic dipole moments, quadrupole-monopole interactions together with the spin-orbit, the spin-spin and the self-interaction spin contributions. We apply our results to the Manchester et al. model of the J0737-3039A-B double pulsar.

1. Introduction

Inspiralling compact binaries (like double-pulsar systems) emit gravitational radiation. These binary systems are the most promising source of the gravitational waves for the Earth-based interferometric detectors (LIGO, VIRGO, GEO, TAMA and AIGO). The coalescence of such compact binaries is preceded by a milder inspiral phase, for which the post-Newtonian approach provides a reliable description. This description is generally considered valid until the system reaches the innermost stable circular orbit (ISCO). The ISCO is not even precisely defined. For the special case of a non-rotating black hole the ISCO is at \( r=6M \) and at \( r=M \) for a maximally rotating black hole (throughout the paper we use \( G = c = 1 \)). For neutron stars the gravitational wave frequency is 800-1230 Hz \( \text{Oechslin et al. 2004} \) at ISCO. The LIGO sensitivity is 1000 Hz.

The description of motion was given to 3.5 post-Newtonian (3.5 PN) order accuracy, with the inclusion of spin-orbit effects (SO) \( \text{Wex 1995, Rieth \& Schafer 1997, Gergely et al. 1998} \) and their first PN correction \( \text{Tagoshi et al. 2001} \), spin-spin (SS) \( \text{Kidder 1995, Gergely 2000} \), quadrupole-monopole (QM) \( \text{Poisson 1998, Gergely \& Keresztes 2003} \) and magnetic dipole-magnetic dipole (DD) \( \text{Ioka \& Taniguchi 2000, Vasúth et al. 2003} \) and self-interaction spin term (SS-self) \( \text{Gergely 2000, Mikócsi et al. 2005} \) contributions to the accelerations were also discussed. On the long run due to the emission of gravitational waves the orbit tends to circularize. Therefore we consider circular orbits, for which the gravitational wave frequency is twice the orbital frequency. We evaluate the rate of increase of \( f \). This is given by the rate of change of the orbital angular frequency \( \omega = \pi f \) under radiation reaction. We present the accumulated number of cycles left until the ISCO and the spin parameters in the Manchester et al. model of the J0737-3039A-B double pulsar.
In this section we summarize some of the results of (Mikóczy et al. 2005) on the frequency evolution and accumulated number of cycles for compact binaries on circular orbit. The radial projection of the acceleration defines the orbital angular velocity $\omega$. We expressed both the energy and the energy loss with $\omega$.

Then the evolution of the radiative orbital angular frequency is:

$$\left\langle \frac{d\omega}{dt} \right\rangle_{\text{circ}} = \frac{96\eta m_1^{5/3} \omega^{11/3}}{5} \left[ 1 - \left( \frac{743}{336} + \frac{11}{4}\eta \right) (m\omega)^{2/3} + (4\pi - \beta) m\omega \right. \left. + \left( \frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 + \sigma \right) (m\omega)^{4/3} \right], \quad (1)$$

where $\eta = m_1 m_2 / m^2$, $m = m_1 + m_2$ and

$$\sigma = \sigma_{S_1S_2} + \sigma_{SS-self} + \sigma_{QM} + \sigma_{DD}. \quad (2)$$

The quantities $\beta$, $\sigma_{S_1S_2}$, $\sigma_{SS-self}$, $\sigma_{QM}$ and $\sigma_{DD}$ are the spin-orbit, spin-spin, self-interaction spin, quadrupole-monopole and magnetic dipole-dipole parameters, respectively:

$$\beta = \frac{1}{12} \sum_{i=1}^{2} \frac{S_i}{m_i^2} \left( \frac{113}{m_i^2} + 75\eta \right) \cos \kappa_i,$$

$$\sigma_{S_1S_2} = \frac{S_1 S_2}{48\eta m^4} (-247 \cos \gamma + 721 \cos \kappa_1 \cos \kappa_2),$$

$$\sigma_{SS-self} = \frac{1}{96m^2} \sum_{i=1}^{2} \left( \frac{S_i}{m_i} \right)^2 \left( 6 + \sin^2 \kappa_i \right),$$

$$\sigma_{QM} = -\frac{5}{2} \sum_{i=1}^{2} p_i \left( 3 \cos^2 \kappa_i - 1 \right),$$

$$\sigma_{DD} = -\frac{5}{\eta m^4} d_1 d_2 A_0. \quad (3)$$

The N, PN, SO, SS, 2PN and tail contributions were verified to agree with those given in (Poisson & Will 1995), the QM with those given in (Poisson 1998), the DD with those given in (Ioka & Taniguchi 2000) and the SS-self with those given in (Mikóczy et al. 2005), respectively. The constant $A_0$ is also given in (Vasúth et al. 2003).

From here the accumulated number of gravitational wave cycles emerges as:

$$N = \frac{1}{\pi \eta} \left\{ \tau^{5/8} + \left( \frac{3715}{8064} + \frac{55}{96}\eta \right) \tau^{3/8} + \frac{3}{4} \left( \frac{\beta}{4} - \pi \right) \tau^{1/4} \right. \left. + \left( \frac{9275495}{14450688} + \frac{284875}{258048}\eta + \frac{1855}{2048}\eta^2 - \frac{15\sigma}{64} \right) \tau^{1/8} \right\}. \quad (4)$$
where the dimensionless time variable $\tau = \eta(t_c - t)/5m$ is related to the time $(t_c - t)$ left until the final coalescence. The tail and 2PN contributions agree with those given in [Blanchet et al. 2002].

Table 1. Accumulated number of gravitational wave cycles.

| PN Order | $J0737 - 3039$ | $B1913 + 16$ | $BH - BH$ |
|----------|---------------|--------------|------------|
| $f_{in}$ (Hz) | 10            | 10           | $4.199 \times 10^{-4}$ |
| $f_{fin}$ (Hz) | 1000          | 1000         | $3.997 \times 10^{-2}$ |

| $N$            | 18310          | 15772.1      | 21058      |
| $PN$           | 475.8          | 435          | 677        |
| $SO$           | 17.5$\beta$    | 16.5$\beta$ | 36$\beta$ |
| $SS_{self}, SS, QM, DD$ | $-2.1\sigma$ | $-2.1\sigma$ | $-5\sigma$ |
| $Tail$         | $-208$         | $-206$       | $-450$     |
| $2PN$          | 9.8            | 9.5          | 18         |

Table 2. The spin parameters for the two solutions (JR1 & JR2) of the Jenet-Ransom model and for two solutions (M1 and M2) of the model of the Manchester et al. representing the binary pulsar J0737-3039.

| Spin parameters | JR1 $[\kappa_1 = 167^\circ]$ | JR2 $[\kappa_1 = 90^\circ]$ | M1 $[\kappa_1 = 14^\circ]$ | M2 $[\kappa_1 = 60^\circ]$ |
|-----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $\beta$         | $-0.166$                   | 0.001                       | 0.168                       | 0.087                       |
| $\sigma_{S_1S_2}$ (10$^{-4}$) | $-0.372$                 | 0                           | 0.371                       | 0.191                       |
| $\sigma_{SS_{self}}$ (10$^{-4}$) | 0.298                    | 0.345                       | 0.298                       | 0.333                       |

In Table 1 we present numerical results for two well-known binary neutron star systems, the double pulsar J0737-3039 [Burgay et al. 2003, Lyne et al. 2004], Hulse-Taylor pulsar B1913+16 and one example of galactic black hole-galactic black hole binary.

3. Application for the J0737-3039A-B Double Pulsar

The double-pulsar J0737-3039 has become an important astrophysical laboratory for testing gravitational physics. The measurements on this system where done by the Parkes 64-m radio-telescope at three frequencies: 680, 1390 and 3030 MHz [Manchester et al. 2004].

We evaluate the spin parameters $\sigma_{S_1S_2}$ and $\sigma_{SS_{self}}$ for the case of the double pulsar J0737-3039. The neutron stars in this double pulsar have average radii 15 km, pulse periods of 22.7 ms and 2773.5 ms. The $\kappa_1$ angle is between the spin axis and angular momentum. The Jenet-Ransom model [Jenet & Ransom 2004] has two possible values: $\kappa_1 = 167^\circ \pm 10^\circ$ (JR1), and $\kappa_1 = 90^\circ \pm 10^\circ$ for pulsar A.
The other angle $\kappa_2$ for pulsar B in principle can be determined by solving numerically the spin precession equations. According to [Kaspi et al. 2004] it is likely that wind-torques from the energetically dominant component have driven the spin axis of the other component to align with the direction of $L$, causing $\kappa_2 = 0$. Therefore $\gamma = \kappa_1$. We give the estimates for the spin parameters in Table 2.

Besides the JR1 and the JR2 solutions we present here for the first time an model [Manchester et al. 2004]. According to this model the measurements favorize maximally allowed the angle $\kappa_1 = 60^{\circ}$. The best-fit solution (M1) is located at $\kappa_1 = 14^{\circ}$. We also present results for the maximally allowed value of $\kappa_1$ (M2). As it can be seen in this model the proper spin-spin contribution is comparable with the self-interaction spin contribution.

4. Conclusions

We have presented the complete set of contributions up to second post-Newtonian order (PN, SO, SS, QM, DD, tail and 2PN) to the evolution of gravitational wave frequency and to the accumulated number of gravitational wave cycles left until the final coalescence, with the inclusion the self-interaction spin terms (SS-self).

We have shown that on the Manchester et al. model of the J0737-30309A-B double pulsar the self-interaction spin contributions are comparable with spin-spin contributions.

This work was supported by OTKA no. TS044665 and T046939 grants.

References

Blanchet, L., Faye G., Iyer, B. R., Jiguet, B. 2002, Phys. Rev. D65, 061501
Burgay, M., et al. 2003, Nature 426, 531
Gergely, L. Á. 2000, Phys. Rev. D62 024007; ibid. D61, 024035
Gergely, L. & Á., Keresztes, Z. 2003, Phys. Rev. D67, 024020
Gergely, L. Á., Perjés, Z. I., Vasúth, M. 1998 Phys. Rev. D57, 876; ibid. D57, 3423; ibid. D58, 124001
Ioka, K. & Taniguchi, T. 2000, Phys. Rev. 537, 327
Jenet, F. A. & Ransom, S. M. 2004, Nature 428, 919
Kaspi, V.M., et al. 2004, ApJ. 613, 137
Kidder, L. 1995, Phys. Rev. D52, 821
Lyne, A. G. et al. 2004, Science 303, 1153
Manchester, R. N., et al. 2005, ApJ. 621 L49
Mikózi, B., Vasúth, M., Gergely, L. Á. 2005, Phys. Rev. D71 124043
Oechslin, R., et al. 2004, Mon. Not. Roy. Astron Soc. 349, 1469
Poisson, E. 1998, Phys. Rev. D57, 5287
Poisson, E., Will, C. M. 1995, Phys. Rev. D52, 848
Rieth, R. & Schafer, G. 1997, Class. Quantum Grav. 14, 2357
Tagshi, H., Ohashi, A., Owen, B. J. 2001, Phys. Rev. D63 044006
Vasúth, M., Keresztes, Z., Mihály, A., Gergely, L. Á. 2003, Phys. Rev. D68, 124006
Wex, N. 1995, Class. Quantum Grav. 12, 983