Prediction of the subjective impression of passenger car roll dynamics on the driver based on frequency-domain characteristic values

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ABSTRACT
Characteristic values are essential for the design and assessment of driving dynamics during the early stages of the development process of passenger cars. Compared to other aspects of vehicle dynamics however, the relationship between measurable parameters and the subjective perception of vehicle roll dynamics has not been researched extensively. In this paper, a study is presented in which several variants of a vehicle with an electronically controlled suspension were rated by test subjects regarding its roll dynamics and measured in a standardised driving manoeuvre. The subjective ratings and objective characteristic values were then used to derive models to predict the subjective liking of several roll dynamics aspects based on objective frequency-domain parameters which could be achieved with relatively high accuracy. The resulting prediction models were validated using measurements of additional vehicles even though this only allowed for a rough validation as no subjective ratings were available for the validation set. Nevertheless, the predictions for the validation variants seemed plausible and reflected the basic expectations regarding the vehicle characters of the validation set. The results of this investigation thus laid a good foundation for follow-up studies to further increase the reliability and robustness of the prediction models.

Introduction
In the early stages of the development process of a new vehicle, simulation often is the only development tool available to engineers as actual prototype vehicles do not exist yet. Furthermore, with the increasing performance and accuracy of simulation methods and models as well as the decreasing number of physical prototype vehicles built during the development of a new car, simulations will be used even more in the future.

In this simulation driven phase, prediction models are essential for the assessment and goal-oriented design of the driving dynamics of a vehicle in order to make the driving characteristics of the virtual prototype converge as much as possible towards the desired...
driving characteristics of the final series production vehicle, see [1]. Additionally, those predictors help in assessing the current development stage as well as changes between iterative development steps. The predictors can be derived based on experience, by using predecessor vehicles or by employing a methodical approach. The latter is when methods of the objective evaluation of vehicle dynamics are used.

By now, there have been several attempts to describe the individual aspects of vehicle dynamics using characteristic values derived from standardised driving manoeuvres. The focus of those studies was often laid on lateral dynamics or steering feel (e.g. [2–9]) as these aspects are intuitively relevant for a driver’s subjective impression of vehicle dynamics.

Despite the large influence of the roll dynamics of a vehicle on the subjective driving impression of its passengers, to this day it was only addressed in a small number of publications with merely a few studies explicitly picking the roll dynamics as a more central theme [10–16]. However, in most of these publications only some partial aspects were broached without focusing too much on how the driver’s perception of roll dynamics can be predicted quantitatively for several different vehicles. In short, there are still no reliable models for the assessment of a vehicle’s roll dynamics based on objective characteristic values as of yet, which is why at this point no reliant objective assessment of roll dynamics is possible, therefore motivating the research presented in the following.

This paper starts with the description of a study which was conducted to collect subjective ratings of several different roll dynamics variants of a BMW X5 which were also measured in a standardised driving manoeuvre to obtain objective measurement data. After pre-processing both subjective ratings and objective measurement data and selecting characteristic values from the latter, correlation analyses were performed to validate the data sets and to obtain initial findings. Subsequently, regression analyses were used to derive more complex models for the prediction of subjective ratings of roll dynamics aspects. Finally, the models were validated using measurements of additional vehicles.

**Objective evaluation and subjective assessment of vehicle roll dynamics**

The general approach regarding the objective evaluation of vehicle dynamics is fairly well established by now. Basically, several vehicle variants which differ in the research aspects are measured using standardised driving manoeuvres and rated by several test subjects. The resulting objective and subjective data is analysed using descriptive and analytical statistics in order to establish reliable relations between the subjective ratings and the characteristic values (CVs) extracted from the objective measurement data. Furthermore, regression analyses are often used to derive more detailed but also more complex models for the prediction of the liking of certain vehicle aspects, e.g. [3,8].

**Study design**

To obtain the subjective ratings of the roll dynamics variants, a single-blind study with 16 test subjects was conducted on the BMW proving grounds in Aschheim near Munich. The tests subjects were asked to rate several variants of a BMW X5 with electronically controlled dampers, electronically controlled anti-roll bars and electric power steering. The roll dynamics of the car was modified by changing the calibration of the above-mentioned control systems. By using only one vehicle in the study instead of several different ones,
the risk of the test subjects subconsciously including additional effects while rating the roll dynamics could be reduced.

Overall, the 16 test subjects assessed seven criteria of six vehicle variants which in total amounted to 1824 individual ratings, where they rated intensity, direction of improvement and liking for six roll dynamics criteria and additionally rated the overall impression of the roll dynamics of the vehicle variant. Specifically, the following variants were used, each variation described in relation to the reference variant:

- A reference variant similar to the production vehicle version of the BMW X5 (variant RV).
- Two variants with increased and decreased roll damping (variants RD↑ and RD↓, respectively).
- Two variants with different levels of steering support resulting in an increased and decreased steering wheel torque (variants ST↑ and ST↓, respectively).
- A variant with decreased eigenfrequency of the transfer function from lateral acceleration to roll angle (EF↓) which essentially increased the roll angle at lower excitation frequencies.

These vehicle variants were selected because they were considered to represent some of the most dominant effects on the perception of vehicle roll dynamics. Additionally, the number of variants rated during the study was set to six to limit its duration to not make it too exhaustive for the test subjects. It also made it easier for the subjects to compare the variants over the course of the study. All test subjects could be considered expert drivers who were working in the development department of driving dynamics at BMW Group in Munich and had extensive experience in rating vehicle dynamics. Each test subject started with the reference variant followed by an individually rearranged order of the remaining variants to avoid familiarisation effects over the course of the assessments, the only exception being the reference variant which was placed at the beginning and the end of each subject’s assessment run in order to be able to check their repetition accuracy. The test subjects were asked to evaluate the following three aspects for each criterion and variant:

- Perceived intensity of the criterion.
- Subjective liking.
- Direction of improvement.

The intensity rating (also called quasi-objective rating, ranging from ‘very small’ to ‘very large’) is used to verify the assessment quality of the test subjects, i.e. if the perceived subjective change of the vehicle variant is consistent with the actual change in vehicle dynamics, and also to select more robust CVs from the correlation analysis (cf. [3]).

The questionnaire used in the study is shown in Figure 1. To rate the subjective liking of a variant, an absolute scale ranging from 1 to 10 using a finer resolution in the interval which most of the ratings were expected to end up in was chosen (5.5–9.5). This scale is fairly common in the automotive industry, which is why the test subjects were already familiar with it (see [17]). Values below 6 (‘Barely satisfactory’) are usually considered unacceptable, a value of 8 (‘Very good’) indicates that expectations were fully met. Values 9 (‘Excellent’) and 10 (‘Outstanding’) are used if expectations were surpassed. Additionally, by using an
absolute scale in contrast to a relative one, the results of the study are more universally applicable because they can be used for the estimation of actual subjective ratings instead of only a relative order of variants. The following seven criteria were rated by the test subjects:

- Absolute roll angle at lower excitation frequencies below 1 Hz (RAL).
- Absolute roll angle at higher excitation frequencies above 1 Hz (RAH).
- Time delay between driver input and vehicle roll motion at lower excitation frequencies below 1 Hz (TDL).
- Time delay between driver input and vehicle roll motion at higher excitation frequencies above 1 Hz (TDH).
- Initial roll motion of the vehicle body at the beginning of the driver input (IRM).
- Overshoot of the body roll motion at the end of the driver input and its attenuation (ROS).
- Overall rating for the roll dynamics of the variant (OR).

A two-lane straight was selected as test track which the subjects could perform their driving manoeuvres on and which the standardised manoeuvres were recorded on as well. The test subjects were instructed to evaluate the vehicle from a customer’s perspective (i.e. up to a maximum lateral acceleration of 4 m/s²) at a velocity of about 100 km/h because the objective measurements were performed at the same vehicle speed. The study conductor was not present in the car during the evaluation of each variant so as not to influence the assessor’s driving behaviour. There was no predetermined time span during which the subjects had to assess a variant. Instead, they could decide on their own how much time they needed for their assessment. After each evaluation round they left the test track to fill out
the questionnaire while the study conductor set up the next variant by changing the calibration of the car’s control systems. This procedure was repeated until all variants had been rated.

**Statistical analysis**

Prior to the actual analysis of the subjective data, the ratings of the test subjects were standardised to consider inter-subject differences in using the rating scales and then transformed back onto a rating scale common to all test subjects. For that, each test subject’s ratings $x_{ij}$ were standardised using the mean value $\mu_i$ and the standard deviation $\sigma_i$ of the test subject’s ratings, where $i$ indicates the test subject and $j$ each individual rating by the test subject:

$$z_{ij} = \frac{x_{ij} - \mu_i}{\sigma_i}$$

The standardised data set was then inversely transformed onto the mean value $\mu_{tot}$ and the standard deviation $\sigma_{tot}$ of the data set of all 16 test subjects, mapping the ratings measured on each test subject’s individual scale onto a common rating scale:

$$x_{ij} = \sigma_{tot} \cdot z_{ij} + \mu_{tot}$$

Outliers were detected using Tukey’s fences where a value is treated as an outlier if its distance to the quartiles Q1 and Q3 exceeds 1.5 times the interquartile range (IQR) where Q1 is 25th percentile and Q3 is the 75th percentile. Instead of just trimming the outlier ratings from the data set, they were winsorized, limiting each value to the Tukey fence value it was closest to in order to not completely disregard the information contained within those values.

Correlations were assumed to be statistically significant if the corresponding $p$-values were less than the significance level $\alpha = 0.05$. In this case, correlation coefficients listed in any subsequent table are printed in boldface. Finally, regression analyses were performed to identify equations which describe statistically significant (nontrivial) relationships between the measured objective data and the subjective ratings. During the stepwise linear regressions, terms were added and removed iteratively to the model equations in order to increase the value of the adjusted R-squared $R^2_{adj}$.

**Objective measurement data and characteristic values**

To obtain reliant objective data, an inertial measurement unit and a dedicated measurement steering wheel were used to record the vehicle data during the standardised driving manoeuvres. The manoeuvre used in this analysis is the continuous sine steering (CSST) which provides information about the frequency-domain behaviour of a vehicle. For the CSST manoeuvre the steering frequency is continuously increased while vehicle speed and steering wheel angle amplitude are kept constant, starting with a predetermined amplitude for a given stationary value of the lateral acceleration. In this case a vehicle speed of 100 km/h and a lateral acceleration of 4 m/s$^2$ were used. This driving manoeuvre was chosen because it is commonly used as a standard driving maneuver in the automotive industry which can also be used to analyse a lot of other aspects of vehicle dynamics.
whereas the steering step in comparison only represents a small frequency range. Moreover, this analysis focussed on the transient roll dynamics behaviour because it allows for a more detailed description of vehicle roll dynamics than quasi-stationary vehicle roll which has been identified as insufficient for the description of vehicle roll dynamics in previous research [12]. Therefore, quasi-steady-state cornering as driving manoeuvre and the roll angle gradient derived from it were also not considered.

The transfer function plots can then be estimated from the measurement data using Welch’s averaged modified periodogram which yields the quotient of the cross power spectral density (CPSD) of the signals used. The magnitude of the CPSD represents the Bode magnitude plot and the phase angles of the CPSD represent the Bode phase plot of the transfer function \( G(s) = Y(s)/U(s) = G_{y/u}(s) \), where \( U(s) \) is the system input and \( Y(s) \) is the system output.

A simplified chain of effects for the roll dynamics of a vehicle is shown in equation (3)

\[
M_H \rightarrow \delta_H \rightarrow (\dot{\psi}, \dot{\beta}) \rightarrow a_y \rightarrow \ddot{\varphi} \rightarrow \varphi
\]

where \( M_H \) is the driver steering wheel torque, \( \delta_H \) is the steering wheel angle, \( \psi \) is the yaw angle, \( \beta \) is the slip angle, \( a_y \) is the lateral acceleration and \( \varphi \) is the vehicle roll angle. In contrast to previous research (cf. [12,14,16]), the steering wheel torque was also included, so that no potentially influencing variable between driver input and vehicle output is neglected.

Based on equation 3, the transfer functions from the excitation states \( M_H, \delta_H \) and \( a_y \) to the roll angle \( \varphi \) and its time derivatives were selected and the values of the amplitude gains \( A \) and phase angles \( \phi \) at the frequencies 0.6, 0.9, 1.2 and 1.5 Hz determined. Additionally, the following parameters were used in the analysis:

- Quasi-stationary value of the amplitude gain at a frequency \( f = 0.3Hz \):
  \[
  A_{y/u}^{0.3} = |G_{y/u}(0.3)|
  \]

- Maximum amplitude gain of the frequency response:
  \[
  A_{y/u}^{\text{max}} = \max |G_{y/u}(j\omega)|
  \]

- Magnification factor \( \beta \):
  \[
  \beta_{y/u} = \frac{\max |G_{y/u}(j\omega)|}{A_{y/u}^{0.3}}
  \]

- Position of the maximum magnification of the frequency response of \( G_{y/u}(j\omega) \):
  \[
  \omega_{y/u}^{0}
  \]

Because the phase angles of the frequency response are shifted by +90° for each differentiation of the output signal with respect to time, no further information is contained in additional phase response plots. Thus, only the phase plot of the transfer function \( G_{y/u}(s) \) is shown in Figure 2 and only the phase angle values from the transfer functions to the roll angle \( \varphi \) are used. Since the modification of steering wheel torque did affect neither the
yaw dynamics nor the roll dynamics but instead only the relation between steering wheel torque and steering wheel angle, the transfer behaviour of the reference variant (solid blue line) and the variant with increased steering wheel torque (dotted black line) are hidden behind the solid pink line of the variant with decreased steering wheel torque in the second and third column of Figure 2 and thus not directly visible.

**Results**

In the following, the data obtained via the before-mentioned methods is presented. The averaged ratings of the variants are discussed and the effects of the roll dynamics variations on the subjective ratings are analysed. The measurement data is used to determine the frequency response of the roll dynamics system represented by the Bode plots which was already shown in Figure 2. Characteristic values are then selected and along with the subjective ratings analysed for correlations. Finally, regression models are derived to predict the subjective ratings based on these CVs.
Figure 3. Spider plot of the subjective liking ratings.

Table 1. Averaged standardised liking ratings of each variant.

| Variant       | RAL | RAH | TDL | TDH | IRM | ROS | OR |
|---------------|-----|-----|-----|-----|-----|-----|----|
| Reference     | 7.8 | 6.9 | 7.4 | 7.0 | 7.6 | 7.7 | 7.3|
| Roll damping  | ↑   |     |     |     |     |     |    |
| Roll damping  | ↓   |     |     |     |     |     |    |
| Steering torque| ↑   |     |     |     |     |     |    |
| Steering torque| ↓   |     |     |     |     |     |    |
| Eigenfrequency|     |     |     |     |     |     |    |
| Δ_{min/max}   | 0.5 | 1.1 | 0.7 | 0.6 | 0.7 | 0.5 | 0.7|

Notes: RAL = roll angle at lower frequencies; RAH = roll angle at higher frequencies; TDL = time delay at lower frequencies; TDH = time delay at higher frequencies; IRM = initial roll motion; ROS = roll overshoot; OR = overall rating.

Subjective ratings

The individual subjective ratings of each test subject were standardised and then transformed onto the mean and the variance of the ratings of the whole group of test subjects as described in the previous section. The resulting averaged standardised ratings of each variant and criterion are shown in Figure 3 and listed in Table 1 and described in the following.

Additionally, the difference between highest and lowest rating is listed in the last row of the table as an indicator of how large the differences between the individual variants
Figure 4. Changes of the subjective liking ratings for each vehicle variant relative to the ratings of the reference variant.

perceived by the test subjects were. The rating change for each criterion relative to the reference variant rating is shown in Figure 4. The increase of roll damping resulted in a rating improvement of both the higher frequency roll angle and time delay. This could be expected as it primarily reduced the absolute roll angle at the roll eigen frequency which was located at about 1.4 Hz (see Figure 2). Every other criterion was rated worse compared to the reference variant. This can probably be attributed to the fact that the relative distance between the roll motion at lower and higher frequencies was reduced by increasing the roll damping. As a result, the roll motion at lower frequencies seemed larger when being compared to the (now smaller) roll motion at higher frequencies. Because even expert drivers rarely use steering wheel frequencies above 1.2 Hz [16,p.91], it is not surprising that all but the higher frequency criteria were rated worse compared to the reference variant. The biggest rating improvement of an individual criterion could be observed for the higher frequency roll angle of this variant (by 0.6). Still, the increased ratings for the higher frequency criteria did not improve the overall impression of the roll dynamics of that variant. The reduction
Figure 5. Subjective intensity ratings of the vehicle variants.

of roll damping on the other hand lowered the ratings for all criteria except for the time delay at lower excitation frequencies which remained on the same level. Overall, this variant also received the worst ratings. These results are consistent with the findings in [12, p.47].

The increase of steering wheel torque improved the ratings of all criteria except for the roll gain at lower frequencies. This variant also turned out to receive the highest ratings overall with five out of the seven criteria rated higher than the reference variant. This emphasises the importance of steering feel for the perception of roll dynamics and highlights an important aspect which – apart from the yaw gain of a vehicle – has been neglected in essentially every research that focussed on vehicle roll dynamics. This can be attributed to all these studies either using only one vehicle [10,16] or not including the steering wheel torque in the analysis of roll dynamics when different vehicles were used [12–15]. In contrast, the lowered steering wheel torque reduced the ratings of all criteria except for the higher frequency roll angle. Both effects of the steering wheel torque manipulation can be attributed to its influence on how the driver stimulates the vehicle system, lower steering wheel torques leading to steering wheel angles which are too large for the desired vehicle trajectory and vice versa. In essence, a higher steering wheel torque results in a smaller vehicle excitation and thus less roll motion, which improved the ratings in this study.

The reduction of roll eigenfrequency resulted in reduced ratings of every criterion but the higher frequency roll angle and time delay. This can be attributed to the slight decrease of roll motion at higher frequencies while at the same time increasing the lower frequency roll motion. It was also the vehicle variant which received the second-worst ratings, especially with respect to the initial roll motion (by −0.6). This reveals the importance of the lower frequency domain for the subjective impression of roll dynamics aspects and is consistent with the conclusions from the roll damping variation.
Table 2. Subjective intensity ratings of each criterion.

| Variant                  | RAL | RAH | TDL | TDH | IRM | ROS |
|--------------------------|-----|-----|-----|-----|-----|-----|
| Reference                | 2.4 | 3.2 | 2.5 | 3.1 | 2.4 | 2.5 |
| Roll damping ↑           | 2.7 | 2.6 | 2.7 | 3.1 | 2.4 | 2.7 |
| Roll damping ↓           | 2.5 | 3.9 | 2.6 | 3.5 | 3.0 | 3.0 |
| Steering torque ↑        | 1.7 | 3.1 | 2.2 | 3.0 | 2.2 | 2.5 |
| Steering torque ↓        | 2.4 | 3.6 | 2.5 | 2.9 | 2.6 | 2.8 |
| Eigenfrequency ↓         | 2.7 | 3.4 | 2.7 | 2.8 | 2.9 | 2.7 |
| Δ_{min/\max}             | 1.0 | 1.3 | 0.5 | 0.7 | 0.8 | 0.5 |

Table 3. Liking ratings distinguished with statistical significance.

| Variant                  | RV | RD↑ | RD↓ | ST↑ | ST↓ | EF↓ |
|--------------------------|----|-----|-----|-----|-----|-----|
| Reference (RV)           | –  | –   | –   | –   | –   | –   |
| Roll damping (RD) ↑      | –  | –   | –   | –   | –   | –   |
| Roll damping (RD) ↓      | –  | –   | –   | –   | –   | –   |
| Steering torque (ST) ↑   | –  | –   | –   | RAH | –   | –   |
| Steering torque (ST) ↓   | –  | –   | RAH | –   | –   | –   |
| Eigenfrequency (EF) ↓    | –  | –   | RAH | RAL | –   | –   |

The intensity ratings of the six variants are shown in Figure 5 and in Table 2. They are, for the most part, inversely proportional to the liking ratings except for the time delay at higher frequencies (see subsequent correlation analysis in section ‘Results’ for detailed information about the correlation coefficients). Also, compared to the other criteria, the rating differences between the variants of both the time delay criteria and of the roll overshoot criterion are small.

The effect of a higher steering wheel torque (less excitation of the vehicle lateral and thus roll dynamics by the driver) can be clearly observed from the intensity ratings of the lower frequency roll angle and time delay as well as the initial roll motion, which were perceived smaller than the reference variant’s.

Based on the dataset of the liking ratings shown in Table 1, a two-pooled two-sample t-test for unequal means and unequal variances at a significance level α = 0.05 was used to examine which vehicle variants could be distinguished with statistical significance by the test subjects. As could be expected from the artificial manipulation of individual vehicle dynamics properties, the test indicated that only a few criteria of the different variants could be distinguished with statistical significance regarding the subjective ratings in Table 2, which is summarised in Table 3.

The results obtained for both time delay criteria, the initial roll motion, the roll overshoot, and the overall rating thus need to be interpreted with care as further analysis might be required. As to both the criteria regarding the absolute roll angle, the test subjects distinguished the combinations shown in Table 3 with statistical significance.

Regarding the actual rating changes, the modification of roll damping primarily affected the rating of the higher frequency roll angle. This is consistent with expectation because the variation of roll damping has its greatest effect on the roll gain at the roll eigenfrequency of a vehicle which is usually located at higher excitation frequencies. Typically, a driver excites
the vehicle body in the lower frequency area well below 0.6 Hz, which significantly reduces
the effect of any variation of roll damping.

The modification of steering wheel torque affected all rating criteria approximately
equally, an increase of steering wheel torque leading to better ratings of the criteria and
vice versa. Because the driver needs less force to excite the vehicle with increasing steering
support, he in this case also tends to use greater steering wheel angles while controlling
the vehicle compared to a vehicle with less steering support. This in turn leads to more roll
motion which is interpreted as a less supported vehicle body and ultimately reflected in the
ratings.

The eigenfrequency modification had a primary effect on the lower frequency roll angle
and time delay, the initial roll motion, the roll overshoot and the overall rating, i.e. on every
criterion but the higher-frequency ones. Compared to the other variants, the ratings of this
variant deviated the most from the reference variant, which again shows the importance of
lower excitation frequencies for the perception of roll dynamics. No statistically significant
differences were found between the ratings of the reference variant and its repetition, which
confirms the assessment qualification of the test subjects.

**Objective parameters**

The numerical values of the previously presented CVs are listed in Table 4. The individual
amplitude gain and phase angle values were omitted for the sake of clarity. Overall, 126
CVs have been selected for each vehicle variant from the measurement data.

**Correlation analysis**

Both auto- and cross-correlations were evaluated for the three data sets obtained through
the objective measurements and the subjective assessments (subjective ratings of both lik-
ing and intensity, and objective parameters). The resulting auto-correlation coefficients of
the quasi-objective ratings are listed in Table 5, where a blue background colour equals a
correlation coefficient of +1, a white background colour equals a correlation coefficient
of 0 and a correlation coefficient of -1 equals a red background colour. The same colour
scheme was also used in Tables 6–8. Low correlation coefficients are an indication of the
test subjects perceiving the variation of the assessment criteria as independent which is
consistent with the artificial creation of the variants.

Overall only two statistically significant correlation coefficients were found, one
between the lower frequency roll angle and the lower frequency time delay and the other
between the initial roll motion and the roll overshoot. Because time delays often are harder
to perceive and compare than other criteria (which was also stated by several test subjects
during the study) and due to the small number of distinguished variants regarding the time
delay at lower excitation frequencies, it is not surprising the test subjects unconsciously
based their rating of the lower frequency time delay on the lower frequency roll angle with
the latter being the only other lower frequency criterion.

The other significant correlation was found between initial roll motion and the roll over-
shoot which could either indicate that the intensity of both initial and final body roll motion
was perceived similarly for the assessed variants or that one criterion was rated according
to the other. Given the above-mentioned small difference between the variants with respect
Table 4. Selected frequency-domain CVs of the six vehicle variants.

| Transfer function $G_{y/u}$ | CV | Variant |
|-----------------------------|----|---------|
| $G_{\psi/Mh}$               | $A_{\psi/u}^{0.3}$ | 0.18 0.18 0.17 0.18 0.18 0.25 $^\circ$/Nm |
| $A_{\psi/u}^{\max}$         | $y/u$ | 0.43 0.32 0.52 0.43 0.43 0.39 $^\circ$/Nm |
| $y_0$                       | 1.33 1.32 1.37 1.33 1.33 1.26 Hz |
| $\beta_{\psi/y}$           | 2.36 1.77 3.10 2.36 2.36 1.59 $-^\circ$ |
| $G_{\psi/bh}$               | $A_{\psi/u}^{0.3}$ | 0.36 0.38 0.34 0.36 0.36 0.47 $^\circ$/(sNm) |
| $A_{\psi/u}^{\max}$         | $y/u$ | 3.09 2.35 3.73 3.09 3.09 2.72 $^\circ$/(sNm) |
| $y_0$                       | 1.46 1.51 1.46 1.46 1.46 1.43 Hz |
| $\beta_{\psi/y}$           | 8.63 6.21 11.06 8.63 8.63 5.75 $-^\circ$ |
| $G_{\psi/bh}$               | $A_{\psi/u}^{0.3}$ | 0.64 0.73 0.62 0.64 0.64 0.82 $^\circ$/(s^2Nm) |
| $A_{\psi/u}^{\max}$         | $y/u$ | 23.3 18.0 28.0 23.3 23.3 20.3 $^\circ$/(s^2Nm) |
| $y_0$                       | 1.57 1.60 1.54 1.57 1.57 1.55 Hz |
| $\beta_{\psi/y}$           | 36.4 24.6 45.4 36.4 36.4 24.9 $-^\circ$ |
| $G_{\psi/ah}$               | $A_{\psi/u}^{0.3}$ | 0.025 0.025 0.023 0.025 0.025 0.033 $-^\circ$ |
| $A_{\psi/u}^{\max}$         | $y/u$ | 0.049 0.039 0.058 0.049 0.049 0.043 $-^\circ$ |
| $y_0$                       | 1.21 0.99 1.28 1.21 1.21 1.05 Hz |
| $\beta_{\psi/y}$           | 2.48 2.45 2.60 2.48 2.48 2.79 $-^\circ$ |
| $G_{\psi/ah}$               | $A_{\psi/u}^{0.3}$ | 0.050 0.052 0.046 0.050 0.050 0.064 $1/s$ |
| $A_{\psi/u}^{\max}$         | $y/u$ | 0.33 0.25 0.40 0.33 0.33 0.27 $1/s$ |
| $y_0$                       | 1.33 1.34 1.36 1.33 1.33 1.28 Hz |
| $\beta_{\psi/y}$           | 6.72 4.69 6.86 6.72 6.72 4.29 $-^\circ$ |
| $G_{\psi/ah}$               | $A_{\psi/u}^{0.3}$ | 0.090 0.102 0.086 0.090 0.090 0.111 $1/s^2$ |
| $A_{\psi/u}^{\max}$         | $y/u$ | 2.37 1.78 2.88 2.37 2.37 1.91 $1/s^2$ |
| $y_0$                       | 1.41 1.46 1.42 1.41 1.41 1.39 Hz |
| $\beta_{\psi/y}$           | 26.4 17.4 33.7 26.4 26.4 17.1 $-^\circ$ |
| $G_{\psi/ay}$               | $A_{\psi/u}^{0.3}$ | 0.19 0.18 0.14 0.16 0.22 0.20 $^\circ$/m |
| $A_{\psi/u}^{\max}$         | $y/u$ | 0.39 0.30 0.38 0.34 0.46 0.26 $^\circ$/m |
| $y_0$                       | 1.36 0.92 1.40 1.36 1.36 1.22 Hz |
| $\beta_{\psi/y}$           | 2.24 2.26 2.70 2.24 2.24 2.57 $-^\circ$ |
| $G_{\psi/ay}$               | $A_{\psi/u}^{0.3}$ | 0.37 0.38 0.29 0.32 0.43 0.38 $^\circ$/m |
| $A_{\psi/u}^{\max}$         | $y/u$ | 2.82 1.98 2.80 2.45 3.32 1.76 $^\circ$/m |
| $y_0$                       | 1.45 1.44 1.47 1.45 1.45 1.33 Hz |
| $\beta_{\psi/y}$           | 7.71 5.18 9.71 7.71 7.71 4.68 $-^\circ$ |
| $G_{\psi/ay}$               | $A_{\psi/u}^{0.3}$ | 0.64 0.73 0.53 0.56 0.76 0.65 $1/s$ |
| $A_{\psi/u}^{\max}$         | $y/u$ | 21.0 14.8 21.0 18.2 24.7 12.4 $1/s$ |
| $y_0$                       | 1.50 1.52 1.53 1.50 1.50 1.40 Hz |
| $\beta_{\psi/y}$           | 32.5 20.4 40.0 32.5 32.5 19.0 $-^\circ$ |

To the roll overshoot and partly for the initial roll motion as well, the first assumption seems more probable.

The auto-correlation results of the liking ratings are presented in Table 6. There were six significant linear correlations between the liking ratings of the assessment criteria. The large majority of ratings did not correlate, which could be expected from the isolated variation of the roll dynamics aspects. The correlation of the lower frequency roll angle and
Table 5. Auto-correlation coefficients of the quasi-objective ratings.

| Criterion (Intensity) | RAL  | RAH  | TDL  | TDH  | IRM  | ROS  |
|-----------------------|------|------|------|------|------|------|
| RAL                   | 1.00 |      |      |      |      |      |
| RAH                   | 0.00 | 1.00 |      |      |      |      |
| TDL                   | 0.98 | -0.09| 1.00 |      |      |      |
| TDH                   | -0.01| 0.27 | 0.12 | 1.00 |      |      |
| IRM                   | 0.60 | 0.73 | 0.56 | 0.30 | 1.00 |      |
| ROS                   | 0.52 | 0.66 | 0.54 | 0.47 | 0.86 | 1.00 |

Table 6. Auto-correlation coefficients of the liking ratings.

| Criterion (Liking)     | RAL  | RAH  | TDL  | TDH  | IRM  | ROS  | OR  |
|------------------------|------|------|------|------|------|------|-----|
| RAL                    | 1.00 |      |      |      |      |      |     |
| RAH                    | -0.19| 1.00 |      |      |      |      |     |
| TDL                    | 0.87 | -0.48| 1.00 |      |      |      |     |
| TDH                    | -0.01| 0.95 | -0.24| 1.00 |      |      |     |
| IRM                    | 0.89 | 0.15 | 0.70 | 0.31 | 1.00 |      |     |
| ROS                    | 0.58 | 0.43 | 0.38 | 0.60 | 0.83 | 1.00 |     |
| OR                     | 0.70 | 0.39 | 0.53 | 0.57 | 0.93 | 0.94 | 1.00|

time delay is not surprising given the correlation of the corresponding quasi-objective ratings. Because the quasi-objective ratings of the higher frequency roll angle and time delay do not correlate, the correlation of their subjective ratings suggests that the time delay criterion was rated according to the higher frequency roll angle. Due to the correlation of the intensity ratings of the initial roll motion and the rollover overshoot, the auto-correlation of the corresponding liking ratings could be expected. The correlation of the overall rating with both the criteria initial roll motion and rollover overshoot could be explained either by one of the individual aspects dominating the subjective impression of the variants or by the individual ratings being more difficult to assess so the test subjects rated them according to the overall impression of the vehicle variant.

Based on the results of the auto-correlation analyses in Tables 5 and 6 an elimination of some rating criteria could be considered. In case of the lower frequency criteria that would be two of the three criteria absolute roll angle, time delay or initial roll motion, and in case of the higher frequency criteria it would be either the absolute roll angle or the time delay. There are two main reasons why at this point all criteria were kept:

- Future studies using the same criteria were planned to enlarge the set of subjective ratings and objective measurements to iteratively improve the prediction models with the added data.
- Because of the limited variation of some of the rating aspects of the variants in this study, the direction of causality cannot be identified reliably, i.e. it cannot be said with certainty if the roll angle at lower frequencies or the time delay or the initial roll motion are sufficient if additional vehicles are considered.
Table 7. Cross-correlation coefficients of the subjective and the quasi-objective ratings.

| Criterion (Liking) | RAL     | RAH     | TDL     | TDH     | IRM     | ROS     |
|-------------------|---------|---------|---------|---------|---------|---------|
| RAL               | -0.79   |         |         |         |         |         |
| RAH               | 0.11    | -0.88   |         |         |         |         |
| TDL               | -0.90   | 0.27    | 0.90    |         |         |         |
| TDH               | -0.14   | -0.89   | -0.10   | -0.56   | 0.88    |         |
| IRM               | -0.74   | -0.35   | -0.75   | -0.23   | 0.83    | 0.97    |
| ROS               | -0.51   | -0.55   | -0.55   | -0.45   | -0.83   | -0.97   |
| OR                | -0.69   | -0.48   | -0.73   | -0.49   | 0.91    | -0.92   |

Still, a consolidation of the roll dynamics aspects investigated in this study will be considered in the future once the models have been validated with additional data.

Table 7 shows the coefficients of the correlation of the subjective ratings with the quasi-objective ratings. Ideally, there should be significant correlations on the main diagonal, which would indicate consistency between the ratings of liking and intensity for each criterion and confirm the assessment quality of the test subjects. The only criterion for which this is not true is the higher frequency time delay which might again point towards the difficulty of perceiving and comparing small differences between time delays. The correlation of the quasi-objective ratings of the initial roll motion and the roll overshoot with the subjective ratings of the roll overshoot and the initial roll motion can be explained from the corresponding auto-correlation results of both the subjective and the quasi-objective ratings shown in Tables 5 and 6, respectively.

Furthermore, based on the correlation results of the subjective ratings and the objective CVs, individual CVs for each criterion for an approximate estimation of its subjective impression can be picked. During this selection it is important to consider the correlation coefficients of both liking and intensity ratings of each CV because the selection of a CV does not make sense if the correlation between CV and liking rating is strong while the correlation between CV and intensity rating is not and vice versa. Any of these two cases would be a strong indication of a random correlation. Basically, the intensity ratings are used to increase the reliability of the selected CVs.

The best individual correlation pair for each criterion from the study data is shown in Table 8, i.e. strong correlations which were consistent with the expectation of how the rating criterion could be predicted. In Table 8, $r_{s/o}$ denotes the correlation coefficients between subjective liking ratings and objective CVs, $r_{qo/o}$ the correlation coefficients between quasi-objective intensity ratings and CVs.

Overall, CVs correlating highly with both liking and intensity ratings could be found for every criterion except for lower frequency roll angle and time delay. Because the perceptible variation of the lower frequency roll angle and time delay between the variants was not very high, it does not surprise that the correlation coefficients of the intensity ratings and the CVs $r_{qo/o}$ for the lower frequency roll angle and time delay are smaller than for the other criteria. The reasoning behind the selection of the individual CVs in Table 8 will be explained in a bit more detail in the following.

Several different types of CV could have been selected for the description of the lower frequency roll angle as they all had similar correlation coefficients (e.g. the quasi-stationary
Table 8. Strongest individual correlations for each criterion.

| Criterion | CV                         | Correlation coefficient |
|-----------|----------------------------|-------------------------|
|           | Liking $r_s/o$ | Intensity $r_qo/o$ |
| RAL       | $V^{0.6}_{\dot{\phi}/ay}$ | -0.82 | 0.39 |
| RAH       | $V^{1.2}_{\dot{\phi}/\delta_H}$ | -0.96 | 0.88 |
| TDL       | $\omega^0_\phi/\delta_H$ | 0.81 | -0.56 |
| TDH       | $\phi^{1.2}_{\phi/M_H}$ | -0.82 | 0.81 |
| IRM       | $\beta_{\phi}/M_H$ | -0.87 | 0.91 |
| ROS       | $\beta_{\phi}/M_H$ | -0.80 | 0.76 |
| OR        | $\beta_{\phi}/M_H$ | -0.87 | — |

The CVs with the strongest correlations identified for the description of the higher frequency roll angle were primarily variations of the gains $V_{y/u}$ from the inputs steering wheel angle $\delta_H$ and lateral acceleration $a_y$ to the outputs roll angle $\varphi$, roll rate $\dot{\varphi}$ and roll acceleration $\ddot{\varphi}$, respectively, at frequencies $f = 0.9$ and $f = 1.2$ Hz. The CV $A^{1.2}_{\phi/\delta_H}$ ($r_s/o = -0.96, r_qo/o = 0.88$) was selected, because it included the vehicle yaw dynamics as the ratio from yaw to roll dynamics was identified as important for the subjective impression of vehicle roll dynamics in [12]. The frequency 1.2 Hz was chosen because Wenzelis [16] observed that even expert drivers did not exceed 1.2 Hz steering wheel frequency while tuning the chassis for a sporty vehicle.

To describe the lower frequency time delay, no strongly-correlating CVs which directly use the phase angle $\phi$ could be identified. The highest correlations were found for $A^{0.3}_{\dot{\phi}/a_y}$ and $A^{3}_{\dot{\phi}/a_y}$, both of which were not used because roll rate $\dot{\varphi}$ and roll acceleration $\ddot{\varphi}$ should be barely noticeably to the test subjects at low excitation frequencies and thus did not seem adequate to describe the lower frequency time delay. The third-strongest correlation was identified for the CV $\omega^0_\phi/\delta_H$ ($r_s/o = +0.81, r_qo/o = -0.56$). As can be observed in the Bode plot (see Figure 2), the variant with reduced eigenfrequency was the only one that significantly changed the phase angle at lower frequencies while at the same time increasing the absolute roll angle, resulting in this correlation. Thus, in context of the vehicle variants used in the study, the selection of the eigenfrequency as CV appears plausible, even though it cannot be ruled out that the test subjects perceived and rated a combination of both effects. In consequence, additional vehicle variants are required to obtain a more robust and reliable CV to describe the subjective impression of this criterion.

For the higher frequency time delay several strongly-correlating CVs could be identified, five of which provided correlation coefficients above 0.75 for both $r_s/o$ and $r_q/o/o$. All of them were combinations of steering wheel torque $M_H$ and roll angle $\varphi$ or roll rate $\dot{\varphi}$ at $f = 0.9$ or $f = 1.2$. The best fit seemed to be $\phi^{1.2}_{\phi/M_H}$ ($r_s/o = -0.82, r_qo/o = 0.81$) because the visually perceptible change of the vehicle body relative to the horizon could be included by using the roll angle which might have had an additional effect during the
assessments. Because phase angle differences usually become more pronounced and thus easier discernible for the driver at higher excitation frequencies, the CV at 1.2 Hz was selected.

The highest correlation for the initial roll motion could be found for CV $\beta_{\varphi/M_H} (rs/o = -0.87, r_{qo/o} = 0.91)$, followed by $\beta_{\varphi/\delta_H} (rs/o = -0.78, r_{qo/o} = 0.75)$. The next-closest CVs were those describing the gain between the system inputs lateral acceleration $a_y$ or steering wheel angle $\delta_H$ and the system outputs roll angle $\varphi$ or its time derivatives $\dot{\varphi}$ and $\ddot{\varphi}$ at a frequency $f = 0.6$ Hz, e.g. $A_{\varphi/a_y}^{0.6} (rs/o = -0.62, r_{qo/o} = 0.56)$ and $A_{\dot{\varphi}/a_y}^{0.6} (rs/o = -0.63, r_{qo/o} = 0.53)$. Their correlation coefficients $r$ were almost identical due to the high inter-correlation of these system states. $\beta_{\varphi/M_H}$ was selected out of these CVs because it reflects the change of roll angle for different excitation frequencies which is generally smaller for well-stabilized vehicle bodies. It also considers the steering wheel torque $M_H$ which strongly influences the initial vehicle stimulation through the driver, and implicitly includes the yaw gain which also influences the perception of the roll dynamics [12,14,16]. Due to the high correlation of the ratings of the initial roll motion and the roll overshoot ($r = 0.80$), the CV $\beta_{\varphi/M_H}$ also correlated strongly with the roll overshoot ratings $(rs/o = -0.80, r_{qo/o} = 0.76)$ which is why it was selected for the description of the roll overshoot. A more appropriate CV for this criterion can presumably be identified in follow-up studies when vehicles with larger differences w.r.t. individual roll dynamics aspects are used.

For the overall impression of vehicle roll dynamics the strongest correlation could be found for the CV $\beta_{\varphi/M_H}$ with $r = -0.87$, followed by CV $\beta_{\varphi/\delta_H}$ with $r = -0.57$. The first CV $\beta_{\varphi/M_H}$ seemed like a reasonable pick as it

- considers the steering wheel torque $M_H$, thus incorporating the whole chain of effects,
- uses the roll angle $\varphi$ which – in contrast to roll rate $\dot{\varphi}$ or roll acceleration $\ddot{\varphi}$ – also reflects quasi-stationary vehicle roll,
- reflects the change of roll stabilisation for different excitation frequencies, and
- has a correlation coefficient $r$ that is significantly higher than the second-strongest correlation.

### Regression analysis

Finally, linear regression models were identified using a stepwise regression approach. The normalised objective CVs of the seven variants were used as predictor variables $x_{ij}$ and the subjective ratings as response variables $y_i$:

$$y_i = f(x_{ij}) \quad (4)$$

Additional predictor terms were added to the equations only if the value of $R^2_{adj}$ was increased by at least 0.05 which helped in avoiding overfitting and multicollinearity of the regressors. Afterwards, the CVs in the models using more than one predictor were checked for correlation and their variance inflation factors (VIF) calculated with values above 10 indicating a multicollinearity problem. The normalised equations are shown in Table 9 with the largest VIF of the predictors of each equation and the $R^2_{adj}$ in the last two rows. Columns 3–9 in Table 9 each list the intercept term, the CVs and the coefficients of the
Table 9. Summary of the regression models with normalised predictors.

| CV          | Unit       | RAL | RAH | TDL | TDH | IRM | ROS | OR  |
|-------------|------------|-----|-----|-----|-----|-----|-----|-----|
| Offset      |            | 7.6 | 7.5 | 7.5 | 7.5 | -0.18 | -0.29 | -0.28 |
| $A^{0.6}_{\psi/\alpha_y}$ | °/(s²Nm) | -1.12 | -0.75 | -0.53 | -0.42 | -0.27 | -0.70 | -0.52 | -0.61 |
| $A^{\text{max}}_{\psi/\alpha_y}$ | °/m | -0.18 | -0.29 | -0.28 | -0.42 | -0.27 | -0.70 | -0.52 | -0.61 |
| $A^{0.2}_{\phi/\delta_H}$ | 1/s² | 0.19 | 0.70 | 0.93 | 0.65 | 1.83 | 1.89 | 1.89 | 1.89 |
| $\beta \phi/\alpha_y$ | °/s²/m | 1.00 | 0.65 | 0.93 | 0.70 | 1.83 | 1.89 | 1.89 | 1.89 |

For the lower frequency roll angle. The regression models were limited to predictors that seemed appropriate for the description of the criteria, similar to the reasoning in the correlation analysis section. This resulted in a maximum number of two predictors per regression model.

As mentioned before, some roll dynamics aspects could only be changed slightly across the six vehicle variants, which resulted in small differences among the subjective ratings (see Table 1) and the measurement data (see Figure 2). Because of that, the offsets have a comparably large impact on the predictions while the CVs only act as support. Still, as typically only a range of 6–8 is used during the subjective assessment of development or series-production vehicles, the ratios of the offsets to the coefficient are not as unfavourable as it might initially seem. Nevertheless, additional vehicle variants with more distinct roll dynamics will be used in the follow-up studies to improve these models.

The objective measurement data was then inserted into the equations in Table 9 for back-testing, i.e. to check the consistency of the predicted ratings and the original ratings from the study. The resulting predictions next to the original liking ratings are depicted in Figure 6.

To measure the prediction error for each criterion $i$, the root mean squared error (RMSE) defined by

$$\text{RMSE}: \sqrt{\sum_i (y_{i,\text{orig}} - y_{i,\text{pred}})^2}$$

is used, where $y_{i,\text{orig}}$ is the original rating and $y_{i,\text{pred}}$ is the predicted rating of each criterion $i$. The highest RMSE is 0.32 for the initial roll motion, followed by 0.29 for the lower frequency time delay. The RMSEs for the remaining criteria are all below 0.21. Thus, a very good fit of the original and the predicted ratings could be achieved for the given vehicle data and their corresponding subjective ratings. The only exception is the steering wheel torque influence which is not specifically considered in the models for the absolute roll angle and the time delay.
Figure 6. Validation of the prediction models (a) original subjective ratings (b) ratings predicted by the regression models.

Figure 7. Frequency-domain data used for model validation.
Validation of the prediction models

The results of the correlation and regression analysis were then additionally validated using previously recorded measurement data of several other vehicles. Because no subjective ratings for the validation set using the same exact questionnaire with the same roll dynamics aspects were available at this point, this could only serve as a rough validation that would be extended in the future.

The frequency domain data of the validation vehicles is shown in Figure 7. The dataset comprised both extremes of vehicles available to market, ranging from a sports car to a super luxury sedan, with a sports SUV, a sedan and a coupe in between them. The predicted subjective ratings of the five vehicles are presented in Figure 8.

The relative order of the ratings for each criterion seems plausible for the most part. The sports car with barely any roll motion received the highest ratings whereas the super luxury sedan received the lowest ratings. The coupe and the sedan received pretty similar ratings, with the ratings of most of their criteria alternating between second- and third-best. The sports SUV was predicted to be rated fourth-best overall, even though its criteria higher frequency roll angle and both the lower and higher frequency time delays were calculated to be on par with the sports car. Because some of the objective CVs of the validation vehicles lie outside the range of the objective data of the variants from the original study, the predictions from the regression models do not necessarily end up within the original scale from 1 to 10 which is the case here. Nevertheless, the relative order of the predictions should still hold true.
**Discussion**

The correlation analysis provided a set of parameters with relatively high correlation coefficients, even though the correlations of the quasi-objective ratings of the lower frequency roll angle and time delay could be higher. Most of the CVs match the intuitive understanding of how the individual roll dynamics aspects could be predicted, except for the roll overshoot. The low coefficients of the quasi-objective correlation of the lower frequency roll angle and time delay can be attributed to the small difference of the variants regarding these criteria and should be investigated further.

Based on the available frequency-domain parameters, plausible regression models could be derived which provided good predictions of the original subjective ratings. The influence of the steering wheel torque is taken into account as well with five of the seven models using at least one steering wheel torque CV. This indicates that, when analysing the perception of vehicle roll dynamics, it is important to consider the complete chain of effects and not just the relationship between lateral acceleration and roll angle. The rough validation performed for different vehicle classes at the end of this paper showed that trends are captured by the models for the most part but some effects were not included in this study, mainly because the range of vehicle dynamics covered by the original variants could be larger, which is why there is some potential for improvements. On the one hand, the extrapolation with the validation set presented at the end of this paper pointed out that the range of validity of the models was too limited. The models might only reflect the modified roll dynamics aspects of the original six variants, which could be of less importance compared to other aspects not modified in this study. On the other hand, it is assumed that individual roll dynamics aspects can be predicted more reliably by adding time-domain CVs, especially the initial roll motion and the roll overshoot. This is why additional identically designed studies will be conducted with different vehicle classes to obtain both subjective and objective data which then can be combined to improve the overall quality of the prediction models.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

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