Effects of Electrode Configuration for Performances of Voltage-Induction-Type Electrostatic Motors*

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Abstract
This paper describes a general theory for the characteristics of voltage-induction-type electrostatic motors (VITEMs). VITEMs are one of macro-scale electrostatic film motors with multi-phase electrodes on both the sliders and the stators. The motor drives synchronously without voltage feeding wires to its slider due to the indirect power supply using electrostatic induction. Although the numbers of electrode phases can be chosen arbitrary to realize the motor, the characteristics of the motors were analyzed only for two specific cases in the previous studies. This work extends the previously proposed theories so that they can handle a general case, in which the slider and the stator have \( m \)- and \( n \)-phase electrodes with arbitrary numbers for \( m \) and \( n \). The theory can estimate the optimized thrust force performance for any electrode configurations, when given fundamental parameters regarding the capacitance coefficients of the motor, which can be obtained either experimentally or numerically. Using the proposed theory, several different configurations of VITEMs are analyzed and compared, in terms of thrust force performance. The results indicate that electrode thickness considerably affects the thrust force performance of VITEMs, which is not the case for other synchronous electrostatic film motors, and thus suggest that the design of VITEMs should more carefully consider the three-dimensional structure of the electrodes.

Key words: Electrostatic Actuator, Voltage Induction, Induction Motor, Finite Element Method, Design Optimization

1. Introduction
In many industrial applications, motors have significant roles to determine their performances. Among various motors, most applications utilize electromagnetic motors due to their high performances. However, electromagnetic motors sometimes cannot fulfill new requirements raised in several fields, such as home robotics and human interactive devices. Such newly emerging fields require, for example, lightweight, thin, and even sometimes transparent motors for better interaction with humans. Electrostatic film motors are one of the promising motors for such new fields, since they possess the above-mentioned requirements.

Several types of electrostatic film motors have been so far reported. Among them, dual excitation multiphase electrostatic drive (DEMED) has realized superior output performance, which is even comparable to that of electromagnetic motors. The motor is composed of two thin films, a slider and a stator, both of which possess embedded parallel multi-phase electrodes. Supplying multi-phase ac power to the both electrodes can achieve high power output. In addition, its synchronous characteristics facilitate easy feedback control.

One drawback of DEMED is that it requires voltage-feeding wires to its slider, since the wiring can cause mechanical disturbance or it limits the use of the motor in several applications, which do not prefer the existence of feeding wires to their moving elements.
other hand, electrostatic induction motors, which indirectly induces electric potential in its slider by electrostatic induction, do not have voltage feeding wires to their sliders,\(^{5)-(14)}\) however their output performances are often limited due to absence of high electric potential in their sliders. Recently, new types of induction motors have been proposed to solve this problem.\(^{15),(16)}\) In these new motors, the slider voltage is indirectly fed as same as the conventional induction motors, but the voltage on the slider is much enhanced with a help of dedicated induction capacitors\(^{15)}\) or embedded LC resonance circuit.\(^{16)}\) These new motors can achieve high output performance thanks to the high electric potential induced on their sliders, without wiring to the sliders. This paper focuses on one of these new motors that utilize dedicated induction capacitors.\(^{15)}\)

The motor, voltage-induction-type electrostatic motors (VITEMs),\(^{17)}\) realizes high voltage on its slider using dedicated induction capacitors. In conventional electrostatic induction motors, electrostatic induction occurs within the driving area; the voltages on the stator driving electrodes induce potential on their facing slider area\(^{6)-(12)}\) (or facing slider electrodes\(^{13),(14)}\). On the other hand, VITEM utilizes external capacitors for electrostatic induction. Since the external capacitors can be design to have much larger capacitance than those of the driving area, the resultant induction voltages can be much larger. In addition, the ”external” capacitors can be formed on the same films for actuation, the resultant form of the motor is not so much different from DEMED; only electrode patterns on the films are different.

The operation principle of VITEM was analyzed in,\(^{17),(18)}\) which provided theoretical model to calculate slider voltages and output forces. The analyses, however, were only conducted for specific electrode designs, such as two-to-four phase combination (which means the slider has two-phase electrode and the stator has four-phase) or three-to-three phase combination (which means both the stator and the slider have three-phase), even though VITEMs can be, in principle, realized with many different phase combinations. Thus, it has not been clear which electrode design can perform better.

This paper provides an extended theory to describe the characteristics of VITEM for a general case, which allows us to calculate the performance for any combinations of electrode phases. As well as the previous theory, the new theoretical formulae require several capacitance parameters that need to be determined experimentally. This paper demonstrates several analyses by obtaining the capacitance parameters using finite element method (FEM) simulation to exemplify performance comparison among different electrode designs of VITEM. The comparison reveals that the electrode thickness can considerably affect the motor performance, in contrast to DEMED, the performance of which is hardly affected by the electrode thickness. In previous studies on the film type electrostatic motors, such as DEMED, electrode design was mainly discussed in two-dimensional patterns. This study clearly points out the third dimension, which is thickness, should be carefully designed in case of VITEM.

The rest of the paper is structured as follows. The next section reviews the basic structure of VITEM. In §3, the paper proposes a new force equation that can handle a general case, in other words \(m\)-to-\(n\) phase combination with arbitrary numbers for \(m\) and \(n\). The proposed equations are verified with the test using experimental prototypes. Section 4 extends the formulae to facilitate calculation of optimal forces that are achieved by optimizing the area ratio between induction electrodes and driving electrodes. This calculation is imperative to compare performances of different electrode designs, since the fair comparison can be achieved only when their performances are optimized. Using the optimizing theory, the section compares different designs of VITEM, with a help of FEM analyses. Final section concludes this work with a future remark.

2. Structure of VITEM

2.1. Structure of 2-4 phase combination; an example

The overview of VITEM are shown in Fig. 1, using a two-to-four (2-4) phase combination,\(^{15),(17)}\) as an example. In this paper, \(m\)-to-\(n\) \((m-n)\) phase combination means that the
slider and the stator have \(m\)-phase and \(n\)-phase electrodes, respectively. The motor generally consists of two thin films, typically made using flexible printed circuit (FPC) technology, both of which have driving and induction electrodes. One of the films functions as the stator, and the other as the slider. The surfaces of all the electrodes are covered with insulating films except for connecting pads to the voltage sources. Driving electrodes on the two films are fine parallel electrodes that are connected to bus lines at their ends to comprise multiphase structures, which are, in this particular case, two-phase in the slider and four-phase in the stator. The structural period of the multiphase electrodes are the same for both the stator and the slider.

Larger solid electrodes formed on both sides of the films are induction electrodes. The numbers of induction electrodes formed on the films are same as the number of slider phases, which is two in this example. Induction electrodes on the stator and the slider are facing each other to comprise parallel plate capacitors. The induction electrodes on the stator are connected to power sources, whereas those on the slider are connected to the driving electrodes of the slider.

By applying multi-phase ac high voltages to the stator’s driving electrodes and to the induction electrodes (which are, in this example, four-phase and two-phase, respectively), sinusoidal voltage waves are formed on both driving electrodes of the stator and the slider, as shown in Fig. 2. Since the structural period of the driving electrodes are the same for the stator and the slider, the resulting voltage waves also have the same wavelength. On the stator, the voltage wave travels along the lateral direction, with a speed proportional to the frequency of the four-phase voltage, whereas on the slider, the voltage wave is a standing wave (due to the two-phase structure). Since the electrostatic attraction force arises to keep the relative position between the two voltage waves, the slider is electrostatically actuated along the lateral direction to keep a constant relative position of the two waves. Therefore, the motor operates as a synchronous motor, whose speed is determined by the frequency difference of the two applied multiphase voltages.

2.2. Structure of \(m\)-to-\(n\) phase combination; the general case

Figure 3 shows a generalized model of the motor, which has \(m\)-phase slider electrodes and \(n\)-phase stator electrodes. The number of phases, \(m\) and \(n\), are arbitrary numbers greater than one; however, both of \(m\) and \(n\) cannot be two at the same time since 2-2 phase combination does not work as a motor. Pitches of the electrodes are \(\frac{n}{m}\) and \(p\), respectively. The resultant structural periods are the same for both films and are \(np\).

Both films have \(m\) induction electrodes. In the slider, each phase of the driving electrodes is connected to the corresponding induction electrode. In the stator, voltage supplies of \(m\)-phase and \(n\)-phase are connected to the induction electrodes and the driving electrodes, respectively.

The voltage applications excite voltage waves on both the stator and the slider. Their
wavelengths are the same as the structural periods of the electrodes and are $np$. For $m$ and $n$ larger than two, the resultant voltage wave becomes a traveling wave, whereas $m$ or $n$ being two makes a standing wave. The interaction between the two voltage waves give rise to the lateral thrust force, which actuates the slider so as to keep the relative speed of the two waves to zero.

3. Basic analysis for $m$-to-$n$ phase combination

In previous literatures, the performance of the motor has been analyzed for two special cases: two-to-four phase$^{17}$ and three-to-three phase$^{18}$. In this section, generalized theory is provided so that we can handle any phase combinations

3.1. Induced voltages and forces of $m$-to-$n$ phase combination

First, induced voltages and thrust force of $m$-to-$n$ phase combination are analyzed. The analysis starts from a capacitance network model to represent relationships among electrodes as similar as previous analyses$^{17}$–$^{19}$. The generalized model shown in Fig. 3, which has $m$-phase for the slider and $n$-phase for the stator, can be modeled using the capacitance network shown in Fig. 4. The model has $2m + n$ terminals that correspond to the stator’s driving electrodes and both induction electrodes on the stator and the slider. They are numbered from 1 to $n$ for the stator’s driving electrodes, $n + 1$ to $n + m$ for the slider electrodes, and $n + m + 1$ to $n + 2m$ for the stator’s induction electrodes. Since the driving electrodes on the slider are connected with the induction electrodes, each pair of driving and induction electrodes is regarded as virtually one electrode and is given the same number. The capacitive relationship among the electrodes is expressed mathematically using $(2m + n) 	imes (2m + n)$ matrix, which is called capacitance coefficients matrix (CCM). In the matrix, the capacitance coefficient at $i$-th row and $j$-th column ($C_{i,j}$) represents the capacitance between $i$-th and $j$-th electrodes if they are multiplied by $−1$. (By definition, non-diagonal elements of a capacitance matrix are always negative.)

By assuming no capacitive coupling between induction electrodes and driving electrodes, we can divide the whole CCM into two sub-matrices.

$$\begin{align*}
C(\theta) &= \begin{bmatrix}
C_{\text{drv}}(\theta) & 0_{m+n,m} & 0_{m,n} & 0_{2m,n} \\
0_{m,n+m} & O_{m,n} & O_{m,n} & O_{2m,n} \\
O_{m,n} & O_{m,n} & C_{\text{ind}} & 0_{2m,n}
\end{bmatrix}
\end{align*} \quad (1)
$$
where $C_{drv}$ and $C_{ind}$ independently represent capacitive relationship within driving electrodes and induction electrodes, respectively. The detailed descriptions of $C_{drv}$ and $C_{ind}$ are shown in the Appendix. It should be noted that the electrodes from $n + 1$ to $n + m$, which are the slider electrodes, have capacitances both in driving and induction electrodes.

Each capacitance coefficient is defined as follows. First, as same as the previous analyses, this work assumes that the capacitance between a slider electrode and a stator electrode (i.e. $C_{i,j+n}$, $1 \leq i \leq n$, $1 \leq j \leq m$) changes sinusoidally in terms of the slider position as:

$$C_{i,j+n} = -C_{m0} - C_{m1} \cos \left\{ \theta_x - \frac{2(i-1)\pi}{n} + \frac{2(j-1)\pi}{m} \right\}$$

where $\theta_x = \frac{2\pi x}{np}$ is an electrical angle representation of the slider position $x$. The other capacitances are regarded constant, which include capacitances between induction electrodes on the stator and the slider, capacitances within stator, and those within slider. It is also assumed that each induction electrode has a capacitance only against its facing electrode (i.e. $C_{(n+i),(n+m+j)}$, $1 \leq i \leq m$); the other capacitances such as capacitances among neighboring induction electrodes are assumed zero.

The capacitance matrix relates charges on the electrode with the voltages. Using a voltage vector $V$ and a charge vector $q$, which represent the voltages and charges on the electrodes, the following equation holds:

$$q = C(\theta_x)V$$

Voltage vector $V$ can be written as:

$$V = (v_t(1), \ldots, v_t(n), V_{ind}(1), \ldots, V_{ind}(m), v_l(1), \ldots, v_l(m))$$

where the applied voltages to the stator’s $i$-th driving electrode and the $j$-th induction electrode are respectively written as:

$$v_t(i) = V_t \sin \left\{ \omega t - \frac{2(i-1)\pi}{n} \right\}$$

$$v_l(j) = V_l \sin \left\{ \omega t - \frac{2(j-1)\pi}{m} \right\}.$$  

The slider induction voltages, $V_{ind}(i)$, are unknown. The charge vector can be written as:

$$q = (q_1, \ldots, q_n, 0, \ldots, 0, q(1+m+n), \ldots, q(2m+n))$$

Since the slider’s electrodes are electrically floating, their charges are found as zero; the others are unknown.

By solving Eq. (3), the unknown voltages and charges can be obtained. Practically, however, it is not easy to solve the equation for the general case, this work introduces the following
assumption to ease the calculation. Since the model comprises a linear circuit, the induced voltage should have two frequency components that correspond to \( v_t(i) \) and \( v_l(j) \). Considering the geometric relation between the stator and slider driving electrodes, the following form is assumed for the induced voltage on the \( j \)-th slider’s electrode:

\[
V_{\text{ind}}(j) = A \sin \left\{ \omega t + \frac{2(j-1)\pi}{m} - \theta_x \right\} + B \sin \left\{ \omega t + \frac{2(j-1)\pi}{m} \right\} \tag{8}
\]

This assumption does not conflict with the results of the previous studies, which are done for two-to-four phase and three-to-three phase combinations.

Substituting Eqs. (1), (4), (7), and (8) into Eq. (3) gives the following solutions for the coefficients \( A \) and \( B \) of (8).

\[
A = -2C_i V_l \csc (\theta_x + \omega_l t - \omega_t t) \sin(\phi) + nC_{m0} V_l \tag{9}
\]

\[
B = \frac{C_i V_l \csc (\theta_x + \omega_l t - \omega_t t) \sin(\theta_x + \phi + \omega_t t) - 2nC_{m0} V_i}{C_i + C_{m0} + 2C_l \cos \frac{2\pi}{m}}.
\]

Finally, substituting Eq. (9) into Eq. (8) gives the induction voltages as:

\[
V_{\text{ind}}(j) = \frac{2C_i V_l \sin \left\{ \omega t + \frac{2n(j-1)\pi}{m} + \phi \right\} + \frac{2nC_{m0} V_l}{C_i + nC_{m0} + \delta C_l (1 - \cos \frac{2\pi}{m})}}{2 \left( C_i + nC_{m0} + \delta C_l (1 - \cos \frac{2\pi}{m}) \right)} \tag{10}
\]

where the parameter \( \delta \) represents the number of adjacent electrodes in different phases counted for each driving electrode in the slider, as shown in Fig. 5; \( \delta = 1 \) for \( m = 2 \) and \( \delta = 2 \) for \( m > 2 \). In addition, the relationship of \( C_{m0} = nC_{m0} + \delta C_l \) was used for this solution. (This relationship comes from the intrinsic feature of the CCM; if all the electrostatic interactions are closed within the system, the sum of each row or each column of the CCM always becomes zero.)

Using the obtained voltages, thrust force can be calculated using the virtual work principle as:

\[
f_m = \sum_{j=1}^{m} \sum_{i=1}^{n} v_l(i) \frac{\partial C_{i,j,n}}{\partial x} V_{\text{ind}}(j) \tag{11}
\]

The force is obtained as:

[In the case of \( m = 2, n > 2 \),]

\[
f_{m=2,n>2} = -\frac{\pi C_i C_{m0} V_l}{C_i + nC_{m0} + 2C_l} \sin \left\{ (\omega_n + \omega_l) t - \frac{2\pi x}{n p} + \phi \right\} + \frac{n C_{m0} V_i}{2 C_i V_l} \sin \left\{ 2\omega_l t - \frac{4\pi x}{n p} \right\} \tag{12}
\]

[In the case of \( m > 2, n > 2 \),]
\[ f_{m>2,n>2} = \frac{m}{2} \pi C_i C_{m1} V_i V_t \sin \left( (\omega_l - \omega_t) t + \frac{2 \pi s}{n p} + \phi \right) \]

Note that formula in the condition of \( n = 2 \) is skipped but can be easily derived as same as \( m = 2 \) condition.

Reviewing the obtained thrust force found that the thrust force has three frequency components when \( m = 2 \). On the other hand, for the case both \( m \) and \( n \) is larger than 2, the force has only one frequency component. For the latter case, the behavior of the motor can be easily understood. Under a constant load, the thrust force is always kept constant so that it balances with the load. In other words, the argument for the sin function in Eq. (13) becomes constant. This condition provides the synchronous speed as:

\[ \frac{d}{dt} x = \frac{np}{2 \pi} (\omega_l - \omega_t) \]  

When \( m = 2 \), the synchronous speed cannot be obtained for a general case, due to the existence of three different frequency components. In previous studies, \( 15, 17 \) VITEM with a two-phase slider was driven using two high frequency signals with a small frequency difference. In that case, only \( \omega_l - \omega_t \) has a low frequency, whereas the other two, \( \omega_l + \omega_t \) and \( 2 \omega_t \), have high frequencies, which are almost double the applied frequencies. Since the slider cannot mechanically respond to high frequencies, the high frequency components attenuate mechanically, and thus can be ignored. Ignoring the two high frequency components, we obtain the same synchronous speed as in Eq. (14) for the \( m = 2 \) case.

For both cases, \( m = 2 \) with high frequency voltages and \( m,n > 2 \), the maximum thrust force is reached when the argument for the sin function becomes \( \pi/2 \), which is given as:

\[ f_{Amp} = \frac{m C_i C_{m1} V_i V_t}{2p \left( C_i + n C_{m0} + 2 C_l (1 - \cos \frac{2 \pi s}{np}) \right)} \]  

These analytical results indicate that capacitances within the slider, \( C_l \), and those between slider and stator, \( C_{m0}, C_{m1} \) have important roles for the performance, which is not the case for DEMED in which only \( C_{m1} \) affects on the motor performance.

3.2. Experimental verification

This subsection verifies the above theoretical model using two prototype models, one has 2-4 phase combination and the other has 3-3. Figure 6 shows the films used for the two prototypes. Figure 6 (a) shows the stator and the slider of the 2-4 phase motor. In this motor, the slider measures 95 mm by 120 mm, whereas the stator measures 160 mm by 100 mm. The sizes of the driving electrodes are 86.4 mm by 25 mm in the slider, and 132.8 mm by 25 mm in the stator. The size of each induction electrode measures 86.4 mm by 32.4 mm in the slider, and 132.8 mm by 32.4 mm in the stator. The pitches of the driving electrodes are 0.4 mm for the slider and 0.2 mm for the stator. The thickness of the electrodes is 18 \( \mu m \).

Figure 6 (b) shows the film used for the 3-3 phase motor. For this motor, the same film design is used for both the stator and the slider. The film measures 95 mm by 120 mm. It has three induction electrodes, each of which are further divided into two electrodes on both sides, for a symmetric design of the film. The areas of the whole driving electrode and that of the induction electrode for one phase are 86.4 mm by 25 mm and 86.4 mm by 20 mm, respectively. The driving electrode has a pitch of 0.2 mm and is skewed with 0.75 pitches for better accordance with the theoretical model, as described in \( 19 \). (In 3-3 phase motor, electrode skew is essential to obtain sinusoidal capacitance variation, whereas 2-4 phase motor naturally possesses sinusoidal variation.) The thickness of the electrodes is 35 \( \mu m \), which is almost twice as that for the 2-4 phase motor. Capacitances for both motors were measured at 1 kHz by the method described in \( 19 \) and were found as in Table 1.

Induced voltages and forces were also measured for both motors. For the measurements, the 2-4 phase motor was operated at \( \omega_l/2\pi = 15 Hz \) and \( \omega_t/2\pi = 10 Hz \), whereas the 3-3
Table 1: Measured capacitances from prototypes of VITEM

| Capacitances | 2-4 VITEM (pF) | 3-3 VITEM (pF) |
|--------------|----------------|----------------|
| $C_{sl}$     | 200.4          | 467.5          |
| $C_{l}$      | 85.6           | 167.1          |
| $C_{m0}$     | 36.8           | 46.4           |
| $C_{m1}$     | 27.5           | 20.4           |
| $C_{i}$      | 464.5          | 225.4          |

Table 2: Induced voltages and forces in two-to-four phase motor and three-phase motor (voltage amplitude: 2 kV, voltage frequency: $\omega_{l}/2\pi = 15$ Hz and $\omega_{l}/2\pi = 10$ Hz for 2-4 phase motor, $\omega_{l}/2\pi = 2$ Hz and $\omega_{l}/2\pi = -2$ Hz for 3-3 motor.)

| Comp. | Induced Voltage | Thrust Force | 2-4 VITEM | 3-3 VITEM |
|-------|----------------|--------------|-----------|-----------|
|       | $\omega_{l}$   | $\omega_{l}$| experimental | theoretical | experimental | theoretical |
|       | 824 V          | 139 V        | 1238 V     | 147 V     | 523 V        | 63 V        |
| $\omega_{l} - \omega_{l}$ | 828 mN        | 592 mN       | 1070 mN    | 1070 mN   | 374 mN       | 374 mN      |
| $\omega_{l} + \omega_{l}$ | 76.6 mN       | 127 mN       | -          | -         | -            | -           |

Vorlage measurement for the slider is quite difficult in case of VITEM. Since the slider is electrically floated, non-contact measurement is imperative; if the slider electrodes are connected to an oscilloscope for voltage measurement, the finite impedances of the voltage probes change the voltages on the slider. For better estimation of the slider voltages, this work utilized a non-contact electrostatic surface voltmeter (Model 344, probe:6000B-8, TReK Inc.). Since the surface voltmeter requires $\phi 15$ mm target area, it was impossible to directly measure the potential of the fine slider electrodes. Therefore, an isolated large conductive plate was connected to a target electrode to facilitate the measurement by the surface voltmeter. Forces were measured using a load cell, which was fixed to the slider. The measured results for both voltage and force were then analyzed by fast Fourier transform (FFT) program to find frequency components.

The measured values are shown in Table 2, together with the theoretical values calculated...
using the capacitances in Table 1. Generally, the experimental results underperform the theoretical estimations. The difference possibly came from the impedance effect of the conductive plate for the non-contact voltage measurement and frictional loss in the measurement setup. However, the overall tendency agrees well, which verifies the theoretical analysis.

4. Electrode area optimization for the largest force per unit area

4.1. Analytical result for optimized condition

In VITEMs, both induction electrodes and driving electrodes are formed on the same films. Even with the same size of the films, the maximum thrust force changes by the area ratio between the driving and induction electrodes, as previous analysis indicated. This section discusses the optimized area ratio to maximize the thrust force performance.

The optimum area ratio can be found from the ratios of $C_i$, $C_l$, $C_{m0}$, $C_{m1}$, and $C_{sl}$ for their unit areas. This work defines the ratios as

$$\frac{C_i}{C_l} : \frac{C_{m0}}{C_i} : \frac{C_{m1}}{C_l} : \frac{C_{sl}}{C_i} = 1 : k_1 : k_2 : k_3$$

where $\delta$ is the same index as the previous section ($\delta = 1$ when $m = 2$, and $\delta = 2$ when $m > 2$). Here, $C_i$ represents the induction capacitance per its unit area. The other four, $C_{m0}$, $C_{m1}$, and $C_{sl}$ are capacitances of $C_i$, $C_{m0}$, $C_{m1}$, and $C_{sl}$ for unit area of the driving electrodes.

By defining the total film area as $S$ and the driving electrode area $S_{drv}$, the ratio of the driving electrodes, $S_{ratio}$ is defined as $S_{ratio} = S_{drv}/S$. Using these area parameters, capacitance of the motor are represented as:

$$C_i = \frac{(1 - S_{ratio})S}{m} C_{i,unit}$$

$$C_{m0} = k_2 S_{ratio} S_{i,unit}$$

$$C_{m1} = k_3 S_{ratio} S_{i,unit}$$

$$C_{sl} = \frac{k_3 S_{ratio} S_{i,unit}}{\alpha_{mn} m n k_2}$$

Substituting Eqs. (17)-(20) into Eq. (15) derives the thrust force per unit film area as

$$f_{Amp,unit} = \frac{s m n k_3 C_{i,unit}(1 - S_{ratio})S_{ratio} V_i^2}{2 p (1 + (-1 + 2 \delta k_1 \sin^2 \frac{\pi}{m} + m n k_2) S_{ratio})}$$

where $f_{Amp,unit} = f_{Amp}/S$.

Differentiating Eq. (21) with respect to $S_{ratio}$ and finding the extremum ($\frac{\partial f_{Amp,unit}}{\partial S_{ratio}} = 0$) provides optimal $S_{ratio}$ as:

$$S_{ratio, opt} = \frac{1}{1 + \sqrt{\alpha_{mn}}}$$

$$\alpha_{mn} = m \left(2 \delta k_1 \sin^2 \frac{\pi}{m} + n k_2 \right).$$

At this optimal condition, thrust force per unit area is maximized as follows:

$$f_{Amp,unit, opt} = \frac{s m n k_3 C_{i,unit} V_i}{2 p(1 + \sqrt{\alpha_{mn}})^2}$$

It can be found that larger $k_3$ and $C_{i,unit}$, and smaller $k_1$ and $k_2$ result in better performance.

The above analyses indicate that the capacitance ratios ($k_1$, $k_2$, and $k_3$) define the optimal structure of the motor. Since the ratios depends on various practical parameters, such as shapes of the electrodes, permittivities of the materials, and the number of electrode phases, determination of those ratios require either experiments or numerical simulations.
4.2. Comparison between 2-4 phase and 3-3 phase

In §3.2, thrust forces of 2-4 phase combination and 3-3 phase combination were compared. Although their effective film areas were almost the same for both 2-4 and 3-3 phase combinations, the result showed that the thrust force of 2-4 phase combination is more than twice larger than that of 3-3. This result, however, does not correctly compare the real performances of the two different phase combinations. As stated above, each electrode configuration has its optimal electrode ratio, $S_{\text{ratio}}$. Therefore, for fair comparison, it should be checked if the experimental prototypes were designed with their own optimal ratios.

Figure 7 shows the theoretical forces of the two motors in §3.2, which are plotted over $S_{\text{ratio}}$. These theoretical values were calculated using the capacitance ratios of the two prototype motors. It can be found that the $S_{\text{ratio}}$ is almost optimal in the 3-3 phase motor, while that of the 2-4 phase motor did not reach its optimal performance. This analysis seems to imply that 2-4 phase motor is far better than 3-3 phase motor with the same film size.

There is, however, one more important aspect that needs to be examined for fair comparison between the two motors. Since the experimental prototypes have different electrode thicknesses, 18 $\mu$m for 2-4 and 35 $\mu$m for 3-3, the difference of thickness could possibly affect the performance. The effect of the different thickness can be evaluated by considering the parameter set of ($k_1, k_2, k_3, C_{i,\text{unit}}$). These parameters for the two motors are summarized in Table 3. Noticeable differences are found in the values of $k_1$, which is a parameter of capacitances between slider’s driving electrode. The difference of $k_1$ mainly came from the different electrode thicknesses, as well as different electrode pitches. In other words, the large difference in performance of 2-4 and 3-3 phase combinations could be a result of their different thicknesses, and did not probably reflect the intrinsic performances of the two electrode designs. To confirm this, the following evaluates different electrode designs with a help of FEM simulation.

4.3. FEM model for calculating capacitances

In this work, fundamental capacitance parameters for different electrode designs were obtained using FEM simulations, which were then substituted to the proposed theoretical model to analytically estimate the thrust force performances of a given design.

The FEM simulation utilizes 2D models because the length of each electrode is long...
enough compared with the thickness and the gap against its facing electrode. The 2D models are constructed based on the dimensions of cross-sections of commercially available FPC films, as shown in Fig. 8. There are many different structures in commercial FPC products. They differ in thicknesses of base or cover films, thickness of electrode, or existence of adhesive layer between the electrode and the base film. (20)

In this work, three typical structures are modeled. Figure 8(a) shows a single-sided FPC film. The thickness of the laminated copper electrode was set to 20 μm, which represents the typical thickness of 18 μm in real FPCs but was rounded for ease of modeling. This single-sided design can be used only for two-phase film. Hereafter, the structure is called ‘S2 type’.

The four-phase film and three-phase film need to be fabricated using double-sided design. The double-sided FPC film shown in Fig. 8(b) has copper-laminated polyimide film for its base. The laminated copper is patterned and then covered with a cover film using adhesive. In the following, this type is referred to as ‘D2 type’. The electrode thickness of D2 type was set to 20 μm.

The last one is also a double-sided FPC film, but differs from D2 type in their structures of copper foil and the base film. In this type, the copper foil is glued to the base plastic film, which means that there is an adhesive layer between the copper and the base. In the following, this type is referred to as ‘D3 type’. This type of FPC can be often found in old FPC films, together with relatively thicker copper foil. Therefore, the thickness of the copper electrode was set to 35 μm for this type.

First, we modeled the experimental prototype to verify the correctness of the FEM simulation. Figure 9 shows the finite elements models that represent the experimental prototypes of 2-4 and 3-3 phase motor. In these models, 2-4 phase motor utilizes S2 type for its slider and D2 type for its stator, whereas 3-3 phase motor utilizes D3 type for both its stator and
slider. For 3-3 phase motor, the electrode skew, which was employed in the prototype, was not considered in the FEM model; the effect of the skew is considered in the post process. The simulation was done using ANSYS (Ansys Inc.) FEM simulator. Relative permittivity of polyimide films, adhesive, dielectric liquid, and air were set to 3.0, 5.0, 1.8, and 1.0, respectively.

Capacitance coefficients of the motor were calculated by moving the slider model with a step of 20 μm with respect to the stator model. Capacitance coefficients were obtained from the total charges stored in electrodes, when unity volt was given to each electrode. The results are shown in Fig. 10. The result for 3-3 phase motor shown Fig. 10 (b) includes the effect of
electrode skew, which was calculated in the post process. The skew effect was considered by taking moving average of the original results over 1.5 times electrode pitch.\(^{(19)}\)

Comparisons of the results with the measured values in §3.2 are shown in Fig. 11. The calculated values show a good agreement in both models with the experimental values. These results verified the correctness of the FEM models.

### 4.4. Comparison among various electrode designs

The performances of various different electrode designs were estimated using FEM models and the proposed theory. First, 2-4 phase motor with different film types were compared in Fig. 12(a). The three models all have the same planar electrode pattern; they only differ in FPC film types. This result clearly shows that the performance of the motor is greatly affected by FPC structure, even though their planar electrode designs are the same. Next, the performances of different phase combinations were compared for the film type of D3 in Fig. 12(b). This comparison includes 2-3 phase combination, which was additionally modeled for this comparison. All the models were designed to have the same structural periled of the driving electrodes. For 3-3 phase combination, a result calculated for non-skewed electrode is also plotted for reference (but it should be noted, that the non-skewed electrode does not have sinusoidal capacitance variations and therefore does not fit the theory provided in this paper. The result was obtained ignoring the distortion in the capacitance variation and is shown only for reference.) It is found that regardless of the phase combination, the performances of the motor are almost the same, if we focus on non-skewed electrode for 3-3 phase combination. In practice, however, 3-3 phases motor is often configured using the skewed electrode for smooth characteristics, which can result in lower performance than the other phase combinations.

### 5. Conclusions

This paper analyzed voltage induction type electrostatic motor, VITEM, for a general case and provided theoretical solutions of induced voltage and thrust force. While the previous studies only provided solutions for some specific cases, the solutions in this work can handle any arbitrary combinations of stator and slider phases. This general solution facilitates comparison among different electrode designs of the motor.

As an example of comparisons that can be achieved with the given theory, the work demonstrated comparison of several different electrode designs. Since the theory requires some fundamental capacitance parameters that need to be obtained experimentally, the work utilized FEM simulation. The FEM simulation was used only for obtaining fundamental parameters. The final results were obtained analytically and thus can be utilized for any further analyses or can help better understanding of the motor behavior. In that sense, the analytical scheme is much useful than full FEM simulation, in which the final results, such as induction voltage and thrust force, are directly obtained by FEM.

The comparison indicated that, in this type of induction motor, the FPC structure can affect the motor performance. In previous studies on film-type electrostatic motors, FPC structures were not paid so much attention; the main design effort was made for determining planar electrode design, such as electrode pitch, since previous motor, such as DEMED, was not greatly affected by FPC structures. Our result, however, indicated that FPC design should be carefully considered when designing this type of motors. Although the result is limited for VITEM, similar result is expected for the other high-power induction motor that utilizes LC resonance circuit,\(^{(16)}\) which needs to be clarified in the future work.

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Appendix

The two capacitances matrices, $C_{\text{drv}}$, $C_{\text{ind}}$ were represented as follows:

$$C_{\text{drv}}(\theta_i) = \begin{bmatrix} C_{\text{drv}\text{-st}} & C_{\text{drv}\text{-sl-st}} \\ C_{\text{drv}\text{-sl}} & C_{\text{drv\text{-sl}}} \end{bmatrix} \quad (m+n) \times (m+n) \text{ matrix}$$

$$C_{\text{ind}} = \begin{bmatrix} C_{\text{ind-sub}} & -C_{\text{ind-sub}} \\ -C_{\text{ind-sub}} & C_{\text{ind-sub}} \end{bmatrix} \quad (2m \times 2m \text{ matrix})$$

where the submatrices $C_{\text{drv\text{-st}}}$, $C_{\text{drv\text{-sl-st}}}$, $C_{\text{drv\text{-sl}}}$, and $C_{\text{ind-sub}}$ are represented as follows:

$$C_{\text{drv\text{-st}}} = \begin{bmatrix} C_{\text{st}} & -C_{\text{t}} & 0 & \cdots & 0 & -C_{\text{t}} \\ -C_{\text{t}} & C_{\text{st}} & -C_{\text{t}} & \ddots & 0 & 0 \\ 0 & -C_{\text{t}} & C_{\text{st}} & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & C_{\text{st}} & -C_{\text{t}} \\ -C_{\text{t}} & 0 & 0 & \cdots & -C_{\text{t}} & C_{\text{st}} \end{bmatrix} \quad (n \times n \text{ matrix})$$

$$C_{\text{drv\text{-sl-st}}} = \begin{bmatrix} C_{1,1+n} & C_{1,2+n} & \cdots & C_{1,m+n} \\ C_{2,1+n} & C_{2,2+n} & \cdots & C_{2,m+n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n,1+n} & C_{n,2+n} & \cdots & C_{n,m+n} \end{bmatrix} \quad (n \times m \text{ matrix})$$

$$C_{\text{drv\text{-sl}}} = \begin{bmatrix} C_{\text{sl}} & -C_{\text{i}} & 0 & \cdots & 0 & -C_{\text{i}} \\ -C_{\text{i}} & C_{\text{sl}} & -C_{\text{i}} & \ddots & 0 & 0 \\ 0 & -C_{\text{i}} & C_{\text{sl}} & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & C_{\text{sl}} & -C_{\text{i}} \\ -C_{\text{i}} & 0 & 0 & \cdots & -C_{\text{i}} & C_{\text{sl}} \end{bmatrix} \quad (m \times m \text{ matrix})$$

$$C_{\text{ind-sub}} = \begin{bmatrix} C_{\text{i}} & 0 & \cdots & 0 \\ 0 & C_{\text{i}} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & C_{\text{i}} \end{bmatrix} \quad (m \times m \text{ matrix})$$

where $C_{\text{st}}$ and $C_{\text{sl}}$ are self capacitance of a stator’s driving electrode and that of a slider’s driving electrode, respectively, and $C_{\text{i}}$ and $C_{\text{i}}$ are mutual capacitances between adjacent electrode in each of them, and $C_{\text{i}}$ is a capacitance of induction electrodes.