Generalized Yang-Mills theory and the D-brane effective action

Athanasios Chatzistavrakidis\(^1\) and Fridrik Freyr Gautason\(^1\)

\(^1\)Institute for Theoretical Physics, Leibniz University Hannover &
\(^2\)Center for Quantum Engineering and Spacetime Research, Appelstrasse 2, 30169, Hannover, Germany.
(Dated: October 2, 2018)

Non-abelian gauge theories in the context of generalized complex geometry are discussed. The generalized connection naturally contains standard gauge and scalar fields, unified in a purely geometric way. We define the corresponding Yang-Mills theory on particular subbundles of a Courant algebroid, known as Dirac structures, where the generalized curvature is a tensor. Different Dirac structures correspond to different known theories, such as the bosonic sector of maximally supersymmetric Yang-Mills in ten and four dimensions and reduced matrix models. Furthermore, we revisit the non-abelian world volume effective action of D-branes in this formalism, where the gauge field on the brane and the transverse scalars are unified, while the action does not contain pullbacks of fields and its consistency with T-duality is verified at face value.

The D-brane effective action has the remarkable property that it keeps the same form under T-duality [1]. This is true for both the Dirac-Born-Infeld (DBI) sector, as well as the Wess-Zumino (WZ) sector that describes the couplings to the Ramond-Ramond (RR) gauge potentials of the type II superstrings. The T-duality is realized on the world volume degrees of freedom, the gauge field \(A_a\) and the transverse scalars \(\Phi^i\), by exchanging their components in the dualized direction, i.e.

\[
A_p \quad \overset{T_p}{\leftrightarrow} \quad \Phi^p.
\]

On the other hand, the effective action depends on both the gauge and scalar fields, either explicitly or through pullbacks and interior products. Here we show that the two dual sets of degrees of freedom can be treated in a unified way in the context of generalized geometry, as components of a single generalized connection. This was already partially discussed in Ref. [2], for the abelian case of a single D-brane and for the DBI sector. Here we argue that this result extends to the non-abelian case of a stack of D-branes, and we examine the WZ sector.

Our approach will be to start from the basic structures that appear in generalized complex geometry [4] in order to construct a non-abelian generalized gauge theory, where 1-forms and 1-vectors appear as components of a single generalized vector. We show that the corresponding generalized Yang-Mills (YM) action correctly reproduces theories with gauge and scalar fields, such as the bosonic sector of \(N = 4\) supersymmetric YM theory and the D-instanton and D0 reduced matrix models, in a way that differs from standard dimensional reduction. Then we briefly review the proposal that a D-brane with its fluctuations is a twisted Dirac structure [2, 3] (see also the discussion on generalized complex submanifolds in Ref. [4], Refs. [2, 5] for further work on this). This point of view will allow us to prove that the full non-abelian effective action for D-branes can be written solely in terms of a generalized curvature \(\mathcal{F}\) and spacetime background fields. The resulting action is written without any pullbacks or interior products and its consistency with T-duality is verified at face value.

Generalized non-abelian gauge theory

In this letter we will need some basic tools of generalized geometry [4, 5]. Let us consider a manifold \(M\), equipped with local coordinates \(x^a, x^i\), where \(a = 0, \ldots, p\) and \(i = p + 1, \ldots, m = \dim M - 1\), and its generalized tangent bundle \(\mathcal{T}M\), which is locally isomorphic to the sum of the tangent and cotangent bundle \(TM \oplus T^*M\). The space of sections \(\Gamma(\mathcal{T}M)\) of the generalized bundle consists of generalized vectors \(X = x^\alpha \eta + \eta\), where \(X \in \Gamma(TM)\) is a standard 1-vector and \(\eta \in \Gamma(T^*M)\) is a 1-form. The generalized tangent bundle is equipped with a bracket, called the Courant bracket and defined as

\[
[X + \eta, Y + \xi]_C = [X, Y]_L + \mathcal{L}_X \xi - \mathcal{L}_Y \eta - \frac{1}{2} \delta(\iota_X \xi - \iota_Y \eta),
\]

where the first term on the right hand side is the Lie bracket between two vectors, the middle terms are Lie derivatives of 1-forms along vectors and the last term is the exterior derivative of interior products between vectors and 1-forms. This structure can be promoted to a Courant algebroid [10], provided that we introduce an anchor map \(\rho : \mathcal{T}M \to TM\), and a bilinear operation

\[
\langle X + \eta, Y + \xi \rangle = \frac{1}{2} (\iota_X \xi + \iota_Y \eta),
\]

as well as certain axioms that provide compatibility conditions among them. The Courant bracket is skew-symmetric but it does not satisfy the Jacobi identity at the level of the Courant algebroid. However, one can find subbundles with the property that the restricted Courant bracket is closed and satisfies the Jacobi identity, and where the bilinear operation gives zero for any two elements of the subbundle. Such subbundles are known as

---

\(^*\)ITP-UH-09/14
Dirac structures \[\text{[11].} \] Dirac structures are Lie algebroids and moreover the Courant algebroid can be identified with a Lie bialgebroid of the form \(L \oplus L^*\), where \(L\) and \(L^*\) are two dual Dirac structures \[\text{[14].}\]

According to Ref. \[\text{[4].}\], a generalized connection (also called a Lie algebroid connection) is an operator \(D\) that maps the sections \(\Gamma(E)\) of a vector bundle \(E\) to the sections \(\Gamma(L^* \otimes E)\), where \(L\) is a Lie algebroid, which we will consider here to be a Dirac structure. It satisfies a Leibniz rule,

\[
D(fs) = (d_L f) \otimes s + f Ds , \tag{3}
\]

where \(f\) is an arbitrary smooth function on \(M, s\) is a section of the vector bundle \(E\), and \(d_L f = \partial_a f \, dx^a\). The generalized connection of interest for our purposes is

\[
D = d_L + A , \tag{4}
\]

where \(A = A + \Phi\) is a generalized vector, which consists of a 1-form \(A = A_a \, dx^a\) (a standard gauge field) and a vector \(\Phi = \Phi_i \partial_i\) (a multiplet of scalars fields). Moreover we consider the following dual pair of Dirac structures; one is identified with the subbundle \(L = \text{span}\{\partial_a, dx^i\} \subset TM\), and the dual one is \(L^* = \text{span}\{\partial_i, dx^a\} \subset TM\). Obviously, \(L \oplus L^* \cong TM\). Additionally, we assume that the gauge field takes values in the Lie algebra of a non-abelian gauge group, thus implementing the non-abelian case in the present formulation. However, we suppress the corresponding gauge indices.

The curvature \(F\) of a generalized connection \(D\) is defined as

\[
F(X, Y) = D_X D_Y - D_Y D_X - D_{[X,Y]} , \tag{5}
\]

where \(X, Y \in \Gamma(L)\). At the level of the Dirac structure, this curvature operator is tensorial in nature as an element of \(\Gamma(\wedge^2 L^*)\), which is not generally true for the full Courant algebroid \[\text{[12].}\]

For this reason it makes sense to write a physical theory on a Dirac structure. Using the definition \[\text{[4].}\], it is simple to compute the curvature of the particular generalized connection \[\text{[1].}\]. We find:

\[
F = \frac{1}{2} F_{ab} \, dx^a \wedge dx^b + (\partial_i \Phi^i + [A_a, \Phi^i]) \, dx^a \wedge \partial_i + \frac{1}{2}[\Phi^i, \Phi^j] \partial_i \wedge \partial_j , \tag{6}
\]

where \(F = dA + A \wedge A\) is the non-abelian curvature of the standard connection 1-form \(A\) \[\text{[21].}\].

The form of the generalized field strength \[\text{[6].}\] is very suggestive. Based on it, it is possible to write down a generalized YM action on the Dirac structure

\[
S_{GYM} = \int \text{Tr} \, F \wedge *L, F , \tag{7}
\]

where the trace is taken over the non-abelian gauge group. The volume form and the Hodge duality operator refer to the Dirac structure. More precisely, the volume on the Dirac structure \(L^*\) is

\[
\begin{align*}
\text{vol}_{L^*} &= \text{vol}_{L} \wedge \text{vol}_{L^*} \\
&= \sqrt{-\det g_{ab}} \, dz^0 \wedge \ldots \wedge dz^p \wedge \\
&\wedge \sqrt{-\det g^{ij}} \, \partial_{p+1} \wedge \ldots \wedge \partial_m \\
&:= \sqrt{-\det g_{ab}} \sqrt{\det g^{ij}} \, \text{d}^{m+1}x , \tag{8}
\end{align*}
\]

and it is part of the full generalized volume of \(\mathcal{V}M\). In Eq. \[\text{[8].}\] we considered the splitting \(L^* = L_{\|}^* \oplus L_{\perp}^*\), with \(L_{\|}^* \subset T^*M\) and \(L_{\perp}^* \subset TM\). The Hodge operator takes a generalized \(q\)-vector of \(\wedge^pL_{\|}^* \wedge^r L_{\perp}^*\) with \(q = r + s\) and returns a dual generalized \((\dim M - q)\)-vector of \(\wedge^{p+1-r}L_{\|}^* \wedge^m L_{\perp}^*\), such that \(*L \cdot 1 = \text{vol}_{L^*}\). In order to make sense of the integration in Eq. \[\text{[7].}\], we use a prescription similar to that of Ref. \[\text{[13].}\] (see also Ref. \[\text{[14].}\]). In particular, we identify the measure on \(L_{\perp}^*\) with the measure \(\text{d}^{m-s}p = \text{d}p_{s+1} \wedge \ldots \wedge \text{d}p_m\) on a momentum space that is associated with the transverse coordinates \(x^i\). This prescription makes sense only for fields that do not depend on \(p_i\), which we assume from now on. This was already pointed out in Ref. \[\text{[13].}\] and it makes perfect sense e.g. for the case of D-branes, where the degrees of freedom depend only on the world volume coordinates.

After this assumption is made, we are able to integrate out the \(p_i\)’s when necessary. This essentially corresponds to dimensional reduction, albeit in an unusual guise. For the following we normalize this integrated measure to unity, namely \(\int \sqrt{\det g^{ij}} \, \text{d}p_{s+1} \wedge \ldots \wedge \text{d}p_m = 1\).

Using the expression \[\text{[10].}\], the action may be rewritten as

\[
S_{GYM} = \int \text{Tr} \left( \frac{1}{2} F \wedge *F + \frac{1}{2} \sum_i D \Phi^i \wedge * D \Phi^i + \right. \\
\left. \frac{1}{2} |\Phi^i, \Phi^j|^2 \text{vol}_{L_{\|}^*} \wedge \text{vol}_{L_{\perp}^*} \right) , \tag{9}
\]

where the gauge covariant derivative is

\[
D_a = \partial_a + [A_a, _i] , \tag{10}
\]

and the Hodge duality operator \(*\) is now the standard one on \(L_{\perp}^*\). Then, if we start with \(\dim M = 10\), the action \[\text{[7].}\] neatly reproduces some theories that are well-known. For \(p = 9\) it reduces to the bosonic sector of the maximally supersymmetric YM theory in ten dimensions, while for \(p = 3\) one gets the bosonic sector of the \(\mathcal{N} = 4\) supersymmetric YM theory, after integrating out the \(\text{vol}_{L_{\perp}^*}\). We recall that in the conventional formulation the \(\mathcal{N} = 4\) supersymmetric YM theory in four dimensions is obtained by dimensional reduction of the \(\mathcal{N} = 1\) theory in ten dimensions \[\text{[12].}\]. Here the action arises from generalized gauge theory on a specific Dirac structure and dimensional reduction is implemented by projecting the theory on \(L_{\|}^*\). Furthermore, for \(p = -1\) the Dirac structure is
the same as TM and we obtain an action that contains only the square of the scalar commutator, which is identical to the D-instanton matrix model [16]. Similarly, the D0 matrix model [17] is obtained for $p = 0$.

The generalized field strength $\mathcal{F}$ is invariant under the generalized gauge transformation

$$\delta \mathcal{A} = D\lambda = d_{L} \lambda + [\mathcal{A}, \lambda],$$

where $\lambda$ is a gauge transformation parameter. This reproduces exactly the expected gauge transformations for the gauge field $\mathcal{A}$ and scalars $\Phi^{i}$ as we know them from the theories mentioned above.

**D-branes as Dirac structures.**

The approach of the previous section exhibits a unified treatment of gauge and scalar fields. Therefore it is the appropriate framework to examine the D-brane effective action. Indeed, the authors of Ref. [2] showed that a Dp-brane can be identified with a particular Dirac structure or, equivalently, with a specific leaf of a foliated structure. In particular, the two dual Dirac structures $L$ and $L^{*}$ that we introduced above were also introduced in Ref. [2] and they play a central role in their argument. The sections of the first one have the form

$$X_{L} = v^{a}(x)\partial_{a} + \xi_{a}(x)dx^{a},$$

and $L$ contains all the necessary information for the geometric description of a D-brane. The second subbundle is the dual of $L$, with sections

$$X_{L^{*}} = v^{i}(x)\partial_{i} + \xi_{i}(x)dx^{i},$$

and it contains the information for the fluctuations of the brane, as well as for the world volume components of the background fields. The fluctuations of the brane world volume include the gauge field $\mathcal{A}$ living on the brane, and the transverse scalars that form a vector $\Phi = \Phi^{i}(x)\partial_{i}$. This is suggestive of the generalized vector $\mathcal{A} = A + \Phi \in \Gamma(L^{*})$, which unifies the longitudinal and transversal degrees of freedom of the Dp-brane in a single mathematical object. As we saw above, it is part of the generalized connection [14] on the Lie algebroid $L$ with differential $d_{L}$ and curvature $\mathcal{F}$, given in Eq. (6).

Using the generalized field strength $\mathcal{F}$, one can determine the fluctuations of the brane just by considering the deformation of the Dirac structure $L$ corresponding to

$$L_{\mathcal{F}} = e^{\mathcal{F}} L .$$

The latter is also a Dirac structure provided that the intergrability condition

$$d_{L}\mathcal{F} + \frac{1}{2}[\mathcal{F}, \mathcal{F}]_{S} = 0$$

holds [10], where the bracket in this equation is the Schouten bracket. The abelian case was considered in Ref. [2]. In the non-abelian case, which corresponds to multiple D-branes, using Eqs. (10) and (12) we find that

$$L_{\mathcal{F}} = \text{span}\{\partial_{a} + D_{a}\Phi^{i}\partial_{i} + F_{ab}dx^{b},$$

$$dx^{i} - D_{a}\Phi^{i}dx^{a} + [\Phi^{i}, \Phi^{j}]\partial_{j}\} ,$$

with $D_{a}$ given in Eq. (10). We observe that the commutator of the scalar fields appears in $L_{\mathcal{F}}$ and this will account for the non-abelian couplings of the D-brane.

**The effective action for D-branes revisited**

In Ref. [2] the generalized metric seen by the D-brane was determined and the DBI Lagrangian was reformulated in terms of it. According to our previous discussion on integration, the reformulated action takes the form

$$- T_{p} \int d^{10}x \ e^{-\phi}(\det g)^{1/4} (\det s_{\mathcal{F}})^{1/4} ,$$

where $g$ is the Riemannian metric on TM and $s_{\mathcal{F}} \in L^{*} \otimes L^{*}$ is the metric seen by the $\mathcal{F}$-twisted Dirac structure $L_{\mathcal{F}}$. More precisely, denoting by $t$ the map from $L$ to $L^{*}$ that implements T-duality geometrically, $s$ refers to its symmetric part and $a$ to its antisymmetric one, seen as an element of $\otimes^{2}L^{*}$. Then the components of $t$,

$$t_{ab} = E_{ab} - E_{ak}E^{kj}E_{lb} , \quad t_{a}^{j} = E_{ak}E^{kj} , \quad t^{ij} = E^{ij} ,$$

are identical to the ones of $E = g + B$ after a number of T-dualities, as they appear in Ref. [1], where $B$ is the usual Kalb-Ramond 2-form. Then $s_{\mathcal{F}}$ is determined to be equal to $s - (a - \mathcal{F})s^{-1}(a - \mathcal{F})$ [2]. Although the action [10] was formulated for a single D-brane, here we show that in the non-abelian case it remains the same, with the difference that the curvature $\mathcal{F}$ is now replaced by the non-abelian expression [14], and that it has to be traced over the gauge group.

In order to prove the above assertion, we essentially have to show that the product of the fourth roots of the determinants is equal to

$$\sqrt{\det \left( E_{ab} - E_{ax}E^{ij}E_{xb} + E_{ab}E^{kj} + D_{a}\Phi^{j} - E^{ik}E_{kb} - D_{b}\Phi^{i} - E^{ij} + [\Phi^{i}, \Phi^{j}] \right) / \det E^{ij}} .$$

which is the expression for the full DBI Lagrangian [1]. On the other hand the matrix that appears in this expression is equal to the components of the tensor $t_{\mathcal{F}} = t - \mathcal{F}$. Furthermore, the authors of Ref. [2] showed the following two identities in the abelian case,

$$\det t_{\mathcal{F}} = (\det s)^{1/2}(\det s_{\mathcal{F}})^{1/2} ,$$

$$\det s = (\det E^{ij})^{2}\det g .$$
Following the same steps, a direct computation reveals that they also hold in the non-abelian case. The computation is unaltered for Eq. 18, which does not involve \( A \) and \( \Phi \) at all, and it may be found in Appendix A.6 of Ref. 2. For Eq. 17 it is simple to see that the replacement of the abelian generalized curvature by the non-abelian counterpart 0 does not affect the calculation. Then it is simple to see that

\[
\mathcal{L}_{\text{DBI}} = \sqrt{\frac{\det g}{\det s_F}} = \sqrt{\frac{\det g \sqrt{\det s_F}}{\det E^{ij}}},
\]

as required.

The remarkable property of the action 18 is that it obviates the need to check that it is consistent with T-duality. Indeed, first it is important to observe that the factor

\[
e^{-\tilde{\phi}} = e^{-\phi} (\det g)^{1/4}
\]

is automatically invariant under T-duality. According to the usual Buscher rules for multiple T-dualities,

\[
e^{2\phi} \rightarrow e^{2\tilde{\phi}} \det E^{ij},
\]

\[
g_{ab} \rightarrow g_{ab},
\]

\[
g_{kl} \rightarrow \det g_{ij} \det E^{ij} g^{kl},
\]

where we refer to the parametrization of Ref. 18, in which the metric acquires the convenient factorization

\[
det g = \det g_{ab} \det g_{ij}. \]

It is then simple to see that \( e^{-\tilde{\phi}} \) is T-duality invariant. Now let us look at the remaining factor in the action, namely \( (\det s_F)^{1/4} \). First of all, let us discuss the behavior of \( F \) under T-duality, since this will also be useful in the following. Splitting the world volume directions as \( x^a = \{ x^\alpha, x^i \} \) and using the basic duality rule 1 for all directions labelled by \( i \), we obtain:

\[
\frac{1}{2} F_{ab} dx^a \wedge dx^b \rightarrow \frac{1}{2} F_{ab} dx^a \wedge dx^b + D_a \Phi^i dx^\alpha \wedge \partial_i \]

\[+ \frac{1}{2} [\Phi^i, \Phi^j] \partial_i \wedge \partial_j \]

\[D_a \Phi^i dx^\alpha \wedge \partial_i \rightarrow D_a \Phi^i dx^\alpha \wedge \partial_i + [\Phi^i, \Phi^j] \partial_i \wedge \partial_j \]

\[= \frac{1}{2} [\Phi^i, \Phi^j] \partial_i \wedge \partial_j \rightarrow \frac{1}{2} [\Phi^i, \Phi^j] \partial_i \wedge \partial_j \].

Summing up the respective sides of the above relations we directly obtain that the expression 13 for \( F \) does not change under T-duality. The same result holds for the character \( e^F \). On the contrary, recall that the Chern character \( e^F \) does not behave well under T-duality 7.

Having proven the consistency of \( F \) with T-duality, it is easy to see that \( s_F \), and therefore the full action, is also consistent.

A final essential remark has to do with the implementation of the gauge trace. In the conventional formulation one has to choose a prescription for this. In the present formalism, where all the expressions \( F_{ab}, D_a \Phi^i \) and \( [\Phi^i, \Phi^j] \) are components of the curvature of a single generalized connection, the symmetrized trace prescription, adopted in Ref. 1, is automatic.

### WZ Couplings

We now turn our attention to the gauge invariant WZ couplings of D-branes to the RR fields. First we note that the generalized tangent bundle \( TM \) extends naturally to a Clifford bundle \( \text{Cl} \) using the non-degenerate bilinear operation in Eq. 23 4. The Clifford bundle has a natural representation on \( \wedge^* T^* M \), given by the extension of the action

\[
\mathcal{X} \cdot \omega = (t_X + \eta \wedge) \omega,
\]

where \( \mathcal{X} = X + \eta \in \Gamma(TM) \) and \( \omega \in \Gamma(\wedge^* T^* M) \). Then the generalized curvature \( F \) can be used to construct gauge invariant couplings to any section of \( \wedge^* T^* M \). Let for example \( C \) be a collection of odd p-forms, given as the formal sum \( C = \sum_{i=1}^p C_i \), as in type IIA superstring theory. Similar considerations hold in type IIB for even p-forms. Consider the coupling

\[
\int \text{Tr} e^F \cdot (C \wedge e^B) \wedge \text{vol}_{L^*} \big|_{L^*},
\]

where the exponential is defined through its power series. Here we have introduced the restriction to \( L^* \) since otherwise the integrand will be a general section of \( \wedge^* TM \). The spinorial actions of each term of \( F \) commute, which means that we can expand Eq. 23 and obtain

\[
\int \text{Tr} e^F \wedge \left[ e^{D_a \Phi^i dx^\alpha \wedge \partial_i} \cdot e^{\omega_{ab} + \omega_{ij} D_a \Phi^i D_b \Phi^j} \right] \big|_{L^*},
\]

where the action has been reduced to the world volume of the brane. This is precisely the Wess-Zumino coupling of a stack of D-branes to the RR sector gauge potentials of type IIA string theory, as explained in Ref. 1 22. This becomes obvious using the fact that for any spacetime p-form \( \omega \),

\[
P[\omega] = e^{D_a \Phi^i dx^\alpha \wedge \partial_i} \cdot \omega \big|_{L^*},
\]

where on the left hand side we encounter the pullback of \( \omega \) on the D-brane world volume. This assertion is easily proven by expanding the exponential and choosing a convenient gauge so that

\[
P[\omega]_{ab} = \omega_{ab} + 2 \omega_{ij} [\Phi^i, \Phi^j] + \omega_{ab} D_a \Phi^i D_b \Phi^j,
\]

where we have taken \( \omega \) to be a 2-form for simplicity, but the result evidently holds for any p-form. Then using the relation 25, it is easily shown that the action 23, and therefore the action 26 too, becomes

\[
S_{\text{WZ}} = \int \text{Tr} e^F \wedge P[e^{\omega_{ab} \cdot C \wedge e^B}],
\]

where the
as required.

A final remark about the action (23) has to do with its behavior under T-duality. We already showed that $e^x$ is consistent with T-duality. Moreover, it is well-known that although the RR gauge potentials $C_i$, as well as the sum $C$, are not well-behaved under T-duality, the improved ones $C'_i = C_i \wedge e^B$, and the corresponding sum, keep their form under it. Therefore we conclude that the action (23) is consistent with T-duality.

Discussion.

In this letter, we studied non-abelian gauge theory in the framework of generalized complex geometry. We showed that this is the appropriate framework to discuss theories that involve standard gauge and scalar fields. Indeed, generalized gauge theory treats them on equal footing and unifies them in a single mathematical object, the generalized connection. The curvature of this connection can be used to define a generalized Yang-Mills theory on a Dirac structure. It is remarkable that by changing Dirac structure this generalized YM theory scans a set of other standard theories, such as the $\mathcal{N} = 1$ YM theory in 10D, the $\mathcal{N} = 4$ YM theory in 4D, the IKKT and the BFSS matrix models. Moreover, the framework is also appropriate to discuss the world volume action of D-branes. Here we extended previous work that appeared in Ref. [2]. In particular we argued that the abelian DBI action of Ref. [2] is appropriate to describe the non-abelian case too, by promoting the generalized connection to its non-abelian counterpart. Furthermore, we examined the WZ term and the non-abelian brane couplings and we showed that it acquires a very simple form in terms of the generalized curvature, while its consistency with T-duality is evident. Although these expressions do not add anything substantially new from the physical point of view yet, the advantages of this rewriting are the following.

First, the consistency of the action with T-duality can be checked with remarkable ease, essentially by simple inspection, since all the quantities that appear in it keep the same form under T-duality. Second, pullbacks and interior products do not appear at all. The fields that enter the action are defined directly on spacetime and not on the world volume of the brane. In this sense, the action has more direct connection to the 10D supergravity theory. Finally, the unification of gauge and scalar fields in a single generalized connection may turn out to be useful in physical contexts, such as in a unified description of dark energy and dark matter along the lines of Ref. [20] and in gaining new geometric insights for symmetry breaking in gauge theories.

Acknowledgments. The authors are grateful to Lars-isa Jonke and Satoshi Watamura for discussions and for reading the manuscript, to Olaf Lechtenfeld for helpful remarks and feedback, and to Ivonne Zavala for discussions on a physical problem that inspired this work.

1 Electronic address: thanasis@itp.uni-hannover.de
2 Electronic address: fridrik.gautason@itp.uni-hannover.de
3 R. C. Myers, JHEP 9912 (1999) 022 [hep-th/9910053].
4 T. Asakawa, S. Sasa and S. Watamura, JHEP 1210 (2012) 064 [arXiv:1206.6964 [hep-th]].
5 T. Asakawa, H. Muraki, S. Watamura, Int. J. Mod. Phys. A 29 (2014) 1545008 [arXiv:1402.0942 [hep-th]].
6 M. Gualtieri, Oxford University DPhil Thesis, math/0401221 [math-dg].
7 P. Koerber, JHEP 0508 (2005) 099 [hep-th/0506154].
8 L. Martucci and P. Smyth, JHEP 0511 (2005) 048 [hep-th/0507099].
9 P. Grange and R. Minasian, Nucl. Phys. B 741 (2006) 199 [hep-th/0512185].
10 P. Grange and S. Schafer-Nameki, Nucl. Phys. B 770 (2007) 123 [hep-th/0609084].
11 N. Hitchin, Quart. J. Math. Oxford Ser. 54 (2003) 281 [math/0209099 [math-dg]].
12 Z.-J. Liu, A. Weinstein, and P. Xu, J. Diff. Geom. 45 (1997) 547 [arXiv:dg-ga/9508013].
13 T. Courant, Trans. Amer. Math. Soc. 319 (1990) 631.
14 M. Gualtieri, arXiv:0710.2713 [math.DG].
15 I. T. Ellwood, JHEP 0712 (2007) 084 [hep-th/0612100].
16 D. Mylonas, P. Schupp and R. J. Szabo, JHEP 1209 (2012) 012 [arXiv:1207.0926 [hep-th]].
17 F. Gliozzi, J. Scherk and D. I. Olive, Nucl. Phys. B 122 (1977) 253.
18 N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B 498 (1997) 467 [arXiv:hep-th/9612115].
19 T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112-5128 [hep-th/9610043].
20 J. Maharana and J. H. Schwarz, Nucl. Phys. B 390 (1993) 3 [hep-th/9207016].
21 B. Jurco, P. Schupp, J. Vysoky, arXiv:1404.2795 [hep-th].
22 T. Koivisto, D. Wills, I. Zavala, arXiv:1312.2597 [hep-th].
23 We use anti-hermitian generators of the gauge group, therefore no factors of i appear in our expressions.
24 It would be very interesting to include also the RR sector in the geometry (see e.g. Ref. [19]) and further simplify the WZ coupling.