Neutron–Antineutron Oscillations: Discrete Symmetries and Quark Operators

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We analyze status of C, P and T discrete symmetries in application to neutron-antineutron transitions breaking conservation of baryon charge \( B \) by two units. At the level of free particles all these symmetries are preserved. This includes P reflection in spite of the opposite internal parities usually ascribed to neutron and antineutron. Explanation, which goes back to the 1937 papers by E. Majorana and by G. Racah, is based on a definition of parity satisfying \( P^2 = -1 \), instead of \( P^2 = 1 \), and ascribing \( P = i \) to both, neutron and antineutron. We apply this to C, P and T classification of six-quark operators with \( \Delta B = 2 \). It allows to specify operators contributing to neutron-antineutron oscillations. Remaining operators contribute to other \( \Delta B = 2 \) processes and, in particular, to nuclei instability. We also show that presence of external magnetic field does not induce any new operator mixing the neutron and antineutron provided that rotational invariance is not broken.

1. A phenomenon of neutron-antineutron oscillation was suggested by Kuzmin \cite{1} in 1970, and the first theoretical model – by Mohapatra and Marshak in 1980 \cite{2}. It is now under active discussion (for a review, see \cite{3}). A discovery of this oscillations would be a clear evidence of baryon charge nonconservation, \( |\Delta B| = 2 \). In this note we discuss the issue of C, P and T symmetries in the \( \Delta B = 2 \) transitions, applying this to analysis of six-quark operators. We also analyze effects of external magnetic field and show that it does not add any new \( \Delta B = 2 \) operator if the rotational invariance is not broken.

Essentially the same issues were addressed in our previous note \cite{4}. There we emphasize the point that parity P, defined in such a way that \( P^2 = 1 \), is broken, as well as CP, in the neutron-antineutron transition. This is an immediate consequence of the opposite parities of neutron and antineutron when \( P^2 = 1 \). Indeed, we deal then with mixing of the states with different parities. Although we also noted that in the absence of interaction it does not automatically imply an existence of CP breaking physics we did not present a detailed analysis of the problem. We have corrected this at the INT workshop in September 2015, defining \( P_z \) such that \( P_z^2 = -1 \).

Following our note \cite{4} the issue of parity definition in the \( \Delta B = 2 \) transitions was addressed in a number of related publications \cite{5,6,7}. Unfortunately, together with correct statements some of these analyses are clearly erroneous. For instance, McKeen and Nelson in their interesting paper \cite{5} about CP violation due to baryon oscillations wrongly insisted that one can keep \( P^2 = 1 \) for the parity definition. It shows that the subject deserves a further discussion. Actually, the issue of parity definition for fermions was resolved long ago. Below we present more details of parity definition story which has been started in 1937 by Ettore Majorana in his famous paper \cite{8} where he introduced a notion of Majorana fermions. In the same journal issue the parity definition was discussed in more details by Giulio Racah \cite{9}.

2. Let us start with the Dirac Lagrangian

\[ \mathcal{L}_D = i \gamma^\mu \partial_\mu n - m \tau n \]  

with the four-component spinor \( n_\alpha \), \( (\alpha = 1, ..., 4) \) and the mass parameter \( m \) which is real and positive. The Lagrangian gives the Lorentz-invariant description of free neutron and antineutron states and preserves the baryon charge, \( B = 1 \) for \( n \) and \( B = -1 \) for \( \bar{n} \) Its conservation is associated with the continuous U(1)_B symmetry

\[ n \rightarrow e^{i\alpha} n, \quad \bar{n} \rightarrow e^{-i\alpha} \bar{n} \]  

of Lagrangian \cite{1}. Corresondingly, at each spatial momentum there are four degenerate states, the spin doublet of the neutron states with the baryon charge \( B = 1 \), and the spin doublet of the antineutron states with \( B = -1 \), i.e., two spin doublets which differ by the baryon charge \( B \). Note that another bilinear mass term,

\[ -im \, \tau_{15} n, \]  

consistent with the baryon charge conservation, can be rotated away by chiral U(1) transformation \( n \rightarrow e^{i\gamma_5 \gamma^5} n \).

How the baryon number non-conservation shows up at the level of free one-particle states? In Lagrangian description it could be only modification of the bilinear mass terms. Generically, there are four such Lorentz invariant bilinear terms:

\[ n \tau C n, \quad n \tau C \gamma_5 n, \quad \pi \tau n^T, \quad \pi \tau C \gamma_5 \bar{n}^T. \]  

Here \( C = i\gamma^2\gamma^0 \) is the charge conjugation matrix in the Dirac (standard) representation of gamma matrices. It has the same form in the Weyl (chiral) representation. In the Majorana representation \( C = -\gamma^0 \).

Using the chiral basis we show in the part 4 that all these modifications \cite{3} are reduced by field redefinitions...
to just one possibility for the baryon charge breaking by two units,\[\Delta \mathcal{L}_g = -\frac{1}{2} \, \epsilon \left[ n^T C n + \pi C \pi^T \right], \tag{5}\]
where \(\epsilon\) is a real positive parameter. The possibility of such redefinitions is based on \(U(2)\) symmetry of the kinetic term \(i \pi \gamma^\mu \partial_\mu n\). Four-parametric \(U(2)\) transformations allow to exclude the term \(3\) and to reduce four terms \(4\) to just one structure \(5\).

3. What is the status of discrete \(C\), \(P\) and \(T\) symmetries under the baryon charge breaking modification \(\Delta B\)? Let us first consider the charge conjugation \(C\), which can be viewed as a plain exchange symmetry between \(n\) and \(n^c\) fields,\[C : \quad n \leftrightarrow n^c = C \pi^T. \tag{6}\]
This is a sort of discrete \(Z_2\) symmetry, \(C^2 = 1\). The most simple it looks in the Majorana representation where\[n^c = n^*. \tag{7}\]
It is straightforward to verify that both Lagrangians above, \(1\) and \(5\), are \(C\) invariant. Indeed, they could be rewritten in the form\[\mathcal{L}_D = -\frac{i}{2} \left[ \bar{\pi}_n \gamma^\mu \partial_\mu n + \bar{n}^c \gamma^\mu \partial_\mu n^c \right] - \frac{m}{2} \left[ \pi n + \bar{n} n^c \right], \tag{8}\]
\[\Delta \mathcal{L}_g = -\frac{\epsilon}{2} \left[ \bar{n}^c n + \bar{n} n^c \right], \tag{8}\]
which makes their \(C\) invariance explicit.

The Lagrangians are diagonalized in terms of Majorana fields \(n_{1,2}\),\[n_{1,2} = \frac{n \pm n^c}{\sqrt{2}}, \tag{9}\]
which are even and odd under the charge conjugation \(C\), \(n_{1,2} = \pm n_{1,2}\). Namely,\[\mathcal{L}_D = \frac{1}{2} \sum_{k=1,2} \left[ \bar{\pi}_k \gamma^\mu \partial_\mu n_k - m \bar{n}_k n_k \right], \tag{10}\]
\[\Delta \mathcal{L}_g = -\frac{1}{2} \epsilon \left[ \bar{n}_1 n_1 - \bar{n}_2 n_2 \right]. \tag{10}\]
It demonstrates that the baryon charge breaking leads to splitting into two Majorana spin doublets. The \(C\)-even \(n_1\) field gets the mass \(M_1 = m + \epsilon\) while the mass of the \(C\)-odd \(n_2\) is \(M_2 = m - \epsilon\).

Turn now to the parity transformation \(P\). It involves (besides reflection of the space coordinates) the substitution\[P : \quad n \rightarrow \gamma^0 n, \quad n^c \rightarrow -\gamma^0 n^c, \tag{11}\]
where \(\gamma^0 C \gamma^0 = -C\) is used. The opposite signs in transformations for \(n\) and \(n^c\) reflect the well-known theorem \(10\) on the opposite parities of fermion and antifermion. The definition \(11\) satisfies \(P^2 = 1\), so the eigenvalues of \(P\) are \(\pm 1\) and opposite for fermion and antifermion states.

Different parities of neutron and antineutron imply that their mixing breaks \(P\) parity, and, indeed, the substitution \(11\) changes \(\Delta \mathcal{L}_g\) to \((- \Delta \mathcal{L}_g)\). Together with \(C\) invariance it implies then that \(\Delta \mathcal{L}_g\) is also \(CP\) odd. However, this \(CP\) oddness does not translate immediately into observable \(CP\) breaking effects. To get them one needs an interference of amplitudes and this is provided only when interaction is present.

It shows a subtlety in the definition of parity transformation \(P\), see textbook discussions, e.g., in Refs. \(11, 12\). Let us remind it.

When baryon charge is conserved there is no transition between sectors with different \(B\), and one can combine \(P\) with a baryonic \(U(1)_B\) phase rotation \(\alpha\) and define \(P_\alpha\),\[P_\alpha = P e^{iB\alpha} : \quad n \rightarrow e^{i\alpha} \gamma^0 n, \quad n^c \rightarrow -e^{-i\alpha} \gamma^0 n^c. \tag{12}\]
Of course, then \(P_\alpha^2 = e^{2iB\alpha} \neq 1\) but the phase is unobservable when \(B\) is conserved.

When baryon charge is not conserved the only remnant of baryonic \(U(1)_B\) rotations is \(Z_2\) symmetry associated with changing sign of the fermion field, \(n \rightarrow -n\). This symmetry is protected: unphysical \(2\pi\) space rotation changes the sign of the fermion field. It means that besides the original \(P^2 = 1\) we can consider a different parity definition \(P_\pi\), such that \(P_\pi^2 = -1\).

Thus, choosing \(\alpha = \pi/2\) in Eq. \(12\) we come to a new parity \(P_\pi\),\[P_\pi = P e^{iB\pi/2} : \quad n \rightarrow i\gamma^0 n, \quad n^c \rightarrow i\gamma^0 n^c \tag{13}\]
with \(P_\pi^2 = -1\). Now \(P_\pi\) parities of \(n\) and \(n^c\) states are the same and equal to \(i\), so their mixing does not break \(P_\pi\) parity. It means that all discrete symmetries, \(C\), \(P\), and \(T\) are preserved by the baryon breaking term \(\Delta \mathcal{L}_g\).

Couple of related comments. First, one can choose \(\alpha = -\pi/2\) and have parities of fermion and antifermion both equal to \((-i)\) instead of \(i\). The absolute sign has no physical meaning – it could be changed by a \(2\pi\) space rotation – but relative parity between two different fermions does make sense. Second, it is amusing that the same \(P_\pi\) parity for \(n\) and \(n^c\) equal to \(i\) is still consistent with the notion of opposite parities of fermion and antifermion, having in mind that that for the complex value of parity we should compare \(P_\pi(n)\) with \([P_\pi(n^c)]^*\). Also for a fermion-antifermion pair the product \(P_\pi(n)P_\pi(n^c) = -1\). One more comment is to notice that \(P_\pi\) commutes with \(C\), i.e., \(CP_\pi = P_\pi C\), in contrast with \(P\) which instead anticommutes with \(C\), i.e., \(CP = -PC\). For Majorana fermion both charge and parity conjugations are diagonal in the Hilbert space: their actions (in the rest frame) do not lead to a different physical state. It means that only the commuting case, i.e., \(P_\pi\) not \(P\), is allowed.

Thus, we demonstrated that neutron-antineutron mixing by \(\Delta \mathcal{L} = \pm 2\) Majorana term in the mass matrix leads to a specific definition of the conserved parity \(P_\pi\), making
it complex and satisfying $P^2_z = -1$ instead of $+1$. It is this definition which should be used in analyzing $CP_z$ violating interactions.

Having in mind that invariance under the charge conjugation was already checked, preservation of $T$ invariance follows from the $CPT$ theorem provided by Lorentz invariance and locality. A specific $P_z$ definition of parity transformation defines a specific $T$ transformation.

A few words about the history of the parity definition. As we mentioned earlier Ettore Majorana and Giulio Racah were the first to realize a necessity of $P$-parity breaking in $\Delta B = \pm 2$ processes. Similar to our initial claim in [4], it is due $P^2 = 1$ for the parity definition what leads to the opposite parities $\pm 1$ for neutron and antineutron. McKeeen and Nelson in [6] also missed this point, and, as we mentioned at the beginning, incorrectly insisted that one can stay with $P^2 = 1$. Technically, the origin of the mistake is that their $\Delta B = \pm 2$ Lagrangian, given by Eq. (A6) in [6], becomes a total derivative when $C$ and $P$ are conserved with $P^2 = 1$. Then, all its matrix elements vanish - no oscillations. Gardner and Yan in [7] followed Majorana neutrino case [13] and correctly defined the parity inversion with $P^2_z = -1$ and the same $P_z = i$ for both, neutron and antineutron.

4. To show that the above consideration covers a generic case it is convenient to introduce two left-handed Weyl spinors, forming a flavor doublet:

$$\psi^\alpha = \begin{pmatrix} \psi^{1\alpha} \\ \psi^{2\alpha} \end{pmatrix}, \quad i = 1, 2, \quad \alpha = 1, 2,$$  \hspace{1cm} (14)

together with their complex conjugates,

$$\psi^\dagger = \begin{pmatrix} (\psi^{1\dagger})^* \\ (\psi^{2\dagger})^* \end{pmatrix}, \quad i = 1, 2, \quad \alpha = 1, 2,$$ \hspace{1cm} (15)

representing the right-handed spinors. One can raise and lower spinor $\alpha, \dot{\alpha}$ and flavor $i$ indices using $\epsilon_{\alpha\beta}, \epsilon_{\dot{\alpha}\dot{\beta}}$ and $\epsilon_{ik}$ with $\epsilon^{12} = 1$.

In terms of Dirac spinor $n$ two left-handed Weyl spinors [13] are associated with $n_L$ and $n_L^* = i\gamma^2(n_R)^*$. In particular, in the chiral (Weyl) basis of gamma-matrices we have:

$$n = \begin{pmatrix} \psi^1 \\ -i\sigma^2(\psi^2)^* \end{pmatrix} = \begin{pmatrix} \psi^{1\alpha} \\ \psi^{2\alpha} \end{pmatrix}, \quad n^\dagger = \begin{pmatrix} \psi^{2\alpha} \\ \psi^{1\alpha} \end{pmatrix}. \hspace{1cm} (16)$$

The generic Lorentz invariant Lagrangian quadratic in fermionic fields $\psi^\alpha$ and $\psi^\dagger$ is

$$L = \frac{i}{2} \left( \bar{\psi}^\dagger \gamma^\alpha \psi^\alpha \mp \bar{\psi} \psi \gamma^\alpha \right) - \frac{1}{2} \left( m_{ik} \psi_i \psi_k + m_{ik} \bar{\psi}_i \bar{\psi}_k \right),$$ \hspace{1cm} (17)

where $\mp = (\sigma^\mu)^{\alpha\dot{\alpha}} \partial_\mu$, $\sigma^\mu = \{1, \sigma\}$, and $m_{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} \partial_\mu$, $\bar{\sigma}^\mu = \{-1, -\sigma\}$, $m_{ik}$ is the symmetric mass matrix, $m_{ik} = m_{ki}$ and $m_{ik}$ is its conjugate.

In the above equation we are implying a standard diagonal form for kinetic terms. These terms in [17] are $U(2)$ symmetric: besides flavor $SU(2)$ rotations it includes also $U(1)$ associated with the overall phase rotation of the flavor doublet [14] which in terms of Dirac spinors [16] is just a chiral transformation. The $U(2)$ symmetry of kinetic terms it clearly generic: starting with $i\psi^\dagger C_{\alpha\dot{\alpha}} \gamma^\mu \psi^\alpha$ where $C^\dagger_{\alpha}$ is an arbitrary Hermitian flavor matrix, one can always diagonalize and normalize these terms.

As for the mass terms they generically break both, $U(1)$ and $SU(2)$ flavor symmetries, so no continuous symmetry remains. To see how the $U(1)_B$ symmetry [2] associated with the baryonic charge could survive note that one can interpret $U(2)$ transformations as acting on the external mass matrix $m_{ik}$. This matrix is charged under $U(1)$, the overall phase rotation, so this $U(1)$ symmetry is always broken by nonvanishing mass. In respect to $SU(2)$ transformations the symmetric tensor $m_{ik}$ is the adjoint representation, i.e., can be viewed as an isovector $\mu^a$, $a = 1, 2, 3$,

$$m^a_k = \varepsilon^{ijk} m_{jk} = \mu^a (r^a)_k, \quad a = 1, 2, 3.$$ \hspace{1cm} (18)

Because $\mu^a$ is complex, we are actually dealing with two real isovectors, $Re \mu^a$ and $Im \mu^a$. The $SU(2)$ transformations are equivalent to simultaneous rotation of both vectors, while $U(1)$ changes phases of all $\mu^a$ simultaneously, which is equivalent to $SO(2)$ rotation inside each couple $\{Re \mu^a, Im \mu^a\}$. Only in case when these vectors are parallel we have an invariance of the mass matrix which is just a rotation around this common direction. This symmetry is the one identified with the baryonic $U(1)_B$ in Eq. [2]. When it happens all $Im \mu^a$ can be absorbed in $Re \mu^a$ by $U(1)$ transformation.

Let us show now that in the absence of the common direction we get two spin 1/2 Majorana fermions with different masses. From equations of motion

$$i \partial_\alpha \psi^\alpha = -m_{ik} \psi^k = 0, \quad i \partial_{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} = -m_{ik} \bar{\psi}_{\dot{\alpha}} = 0,$$ \hspace{1cm} (19)

come to the eigenvalue problem for $M^2 = p_\mu p^\mu$,

$$M^2 \psi^\alpha = -m_{ik} m_{il} \psi^i = 0.$$ \hspace{1cm} (20)
Using definition $\mu^a$ of the squared mass matrix can be presented as a combination of isoscalar and isovector pieces:

$$m^\mu m^\nu = \mu^a \bar{p}^a \delta_n^\mu + i e^{abc} \mu^a \bar{p}^b (\tau^c)^a_n.$$  \hspace{1cm} (21)

Correspondingly, there are two invariants defining $M^2$. The isoscalar part gives the sum of eigenvalues,

$$M_2^2 + M_3^2 = \mu^a \bar{p}^a = (\text{Re} \mu^a)^2 + (\text{Im} \mu^a)^2$$  \hspace{1cm} (22)

while the length of the isovector part defines the splitting of the eigenvalues,

$$\frac{M_2^2 - M_3^2}{2} = 2 \sqrt{|e^{abc} \text{Re} \mu^a \text{Im} \mu^b|^2}.$$  \hspace{1cm} (23)

This shows how the splitting is associated with the breaking of the baryon charge.

To follow the discrete symmetries we can orient the mass matrix $m_{ik}$ in a convenient way. In terms of $\mu^a$ the mass matrix $\hat{m}$ has the form

$$\hat{m} = m_{ik} = \begin{pmatrix} -\mu^1 - i\mu^2 \\ \mu^3 \\ \mu^1 + i\mu^2 \end{pmatrix}.$$  \hspace{1cm} (24)

Without lost of generality one can render the vectors $\text{Re} \mu^a$ and $\text{Im} \mu^a$ orthogonal using the overall $U(1)$ phase transformation. Then by remaining $SU(2)$ rotations we can put both of them onto the $23$ plane, i.e., put $\mu^1 = 0$. and choose the direction of $\text{Im} \mu^a$ as the $2$-nd axis. So, only two non-vanishing parameters, $\text{Re} \mu^3$ and $\text{Im} \mu^2$, remain and the mass matrix takes the form

$$\hat{m}_0 = \begin{pmatrix} \text{Re} \mu^3 \\ \text{Im} \mu^2 \\ \text{Re} \mu^3 \\ \text{Im} \mu^2 \end{pmatrix} = \begin{pmatrix} \epsilon & m & \epsilon \\ m & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix},$$  \hspace{1cm} (25)

where correspondence, $m = \text{Re} \mu^3$, $\epsilon = \text{Im} \mu^2$, with parameters introduced earlier in four-component spinor notations is also shown. Then $M_{2,3}^2 = (m \pm \epsilon)^2$ as in the previous Section.

In other words, an arbitrary mass matrix $\hat{m}$, as in Eq. (24), can be brought to quasi-Dirac form $\hat{m}_0$, given by Eq. (25) with real parameters $m$ and $\epsilon$, by a certain $U(2)$ transformation $V$,

$$\hat{m}_0 = V^T \hat{m} V.$$  \hspace{1cm} (26)

Indeed, 6 real parameters in the matrix $\hat{m}$ are diminished to 2 in $\hat{m}_0$ by 4 parameters of $U(2)$ rotations.

In the limit $\epsilon = 0$ the neutron becomes a Dirac particle, and the baryon symmetry $U(1)_B$ associated with $SU(2)$ rotations around $3$-rd axis with a diagonal generator $\tau^3/2$ arises. Non-zero Majorana mass $\epsilon$ breaks this symmetry but in real situation $\epsilon \ll m$, the neutron behaves practically as Dirac particle, and $U(1)_B$ remains an approximate symmetry.\footnote{Present experimental limits on $n-\bar{n}$ oscillation\cite{3} yield the upper bound $\epsilon < 2.5 \times 10^{-33}$ GeV.}

It is convenient to discuss discrete symmetries in this basis.

5. In the Weyl description with the mass matrix $\hat{m}_0$ given by (25) the charge conjugation $C$,

$$C : \begin{array}{c} \psi^1 \alpha \leftrightarrow \psi^2 \alpha, \\ \bar{\psi}_{1\alpha} \leftrightarrow \bar{\psi}_{2\alpha}, \end{array}$$  \hspace{1cm} (27)

is just interchanging fields of the same chirality but with the opposite baryon charges. In terms of $U(2)$ transformations it is the $SU(2)$ rotation by angle $\pi$ around the first axis up to the factor $(-i)$ which is the $U(1)$ rotation:

$$C : \psi \rightarrow U_C \psi, \quad U_C = e^{-i\pi/2} e^{i\pi \tau^3/2} = \tau^1.$$  \hspace{1cm} (28)

This is in the basis where the mass matrix has the quasi-Dirac form (25)\footnote{In the limit $\epsilon = 0$, when only $3$-rd axis is fixed, $m = \text{Re} \mu^3$, one can consider any combination of rotations around $1$-st and $2$-nd axes, $U_C = \cos \omega \tau^1 + \sin \omega \tau^3$. This is the origin of the well-known phase freedom in the definition of $C$ transformation for Dirac fermion, $n \rightarrow e^{-i\omega} n$, $n' \rightarrow e^{i\omega} n$. Non-zero $\epsilon$ removes the phase freedom and leaves only the possibility $\sin \omega = 0.$.} For generic form of the mass matrix we can use Eq. (26) to get

$$U_C = V \tau^1 V^T.$$  \hspace{1cm} (29)

Moreover, we can write the matrix $U_C$ in an arbitrary basis,

$$U_C = \exp(-i\pi/2) \exp(i\pi \tau^a n^a/2) = n^a \tau^a,$$

$$n^a = \frac{\epsilon^{abc} \text{Re} \mu^b \text{Im} \mu^c}{|\epsilon^{abc} \text{Re} \mu^b \text{Im} \mu^c|},$$  \hspace{1cm} (30)

with a straightforward geometrical interpretation. Indeed, it is just a combination of the $SU(2)$ rotation around the normal $n^a$ to the plane of $\text{Re} \mu^a$ and $\text{Im} \mu^a$ by angle $\pi$ with the chiral $U(1)$ rotation by angle $(-\pi/2)$. Evidently, this is a discrete symmetry of the mass matrix, $U_C^\dagger \hat{m} U_C = \hat{m}$. The $SU(2)$ rotation by $\pi$ changes the sign of $\hat{m}$ and the $U(1)$ rotation, $\exp(-i\pi/2) = -i$, compensates this sign.

The transformation $C$ together with $C^2 = I$ composes the discrete $Z_2$ subgroup that survives from $U(2)$ for a generic mass term. The only other discrete symmetry is $Z_2$ associated with changing sign for all fermion fields.

Let us turn now to the parity transformation $P_z$ defined by by Eq. (13) in terms of Dirac spinors. In terms of Weyl spinors\cite{14} and\cite{15} the inversion of space coordinates then implies

$$P_z : \begin{array}{c} \psi^{1\alpha} \rightarrow \bar{\psi}_{2\alpha}, \\ \bar{\psi}_{1\alpha} \rightarrow i \psi^{2\alpha}, \end{array}, \quad \psi^{2\alpha} \rightarrow i \bar{\psi}_{1\alpha};$$

$$\bar{\psi}_{1\alpha} \rightarrow i \psi^{2\alpha}, \quad \bar{\psi}_{2\alpha} \rightarrow i \psi^{1\alpha}.$$  \hspace{1cm} (31)

This is in the basis where the mass matrix has the form (24). Again, similar to $C$, it can be written in the form:

$$P_z : \psi \rightarrow i \bar{\psi} U_p, \quad \bar{\psi} \rightarrow i U_p^\dagger \psi.$$  \hspace{1cm} (32)
where \( U_P = \tau^1 \) in the basis \( \{ \mathbf{2} \} \) and \( U_P = V \tau^1 V^T \) in an arbitrary basis. The transformation \( \{ \mathbf{2} \} \) clearly demonstrates \( P_2^2 = -1 \).

The operation \( \mathbf{C_P} \) which changes both, charge and chirality, has the form,

\[
\mathbf{C_P} : \psi \rightarrow i \bar{\psi} U_C U_P = i \bar{\psi} V V^T, \quad \bar{\psi} \rightarrow U_P^T U_C \bar{\psi} = i V^* V^T \psi .
\]

It is just \( \psi^\alpha \rightarrow i \bar{\psi}_\dot{\alpha} \) and \( \bar{\psi}_\dot{\alpha} \rightarrow i \psi^\alpha \) in the basis \( \{ \mathbf{2} \} \). Note, that \( \mathbf{C} \) and \( P_2 \) commute and \( (\mathbf{CP}_2)^2 = -1 \).

Finally, one can define \( T \) transformation which besides the time inversion and reordering operators in the Lagrangian \( \{ \mathbf{17} \} \) implies

\[
T : \psi^\alpha \rightarrow \bar{\psi}_{\dot{\alpha}} , \quad \bar{\psi}_{\dot{\alpha}} \rightarrow -\psi^\alpha,
\]

in the basis \( \{ \mathbf{2} \} \) and

\[
T : \psi \rightarrow \bar{\psi} V V^T, \quad \bar{\psi} \rightarrow -V^* V^T \psi^\alpha ,
\]

in the arbitrary basis. Clearly, \( T \) anti-commutes with \( \mathbf{CP}_2 \) and \( T^2 = -1 \).

Combining, we get \( \mathbf{CP}_2 T \) transformation, which acts as \( n \rightarrow i \gamma^5 \bar{n} \) on the Dirac spinor together with inversion of all space-time coordinates and reordering of operators in the Lagrangian,

\[
\mathbf{CP}_2 T : \psi^\alpha \rightarrow i \psi^\alpha , \quad \bar{\psi}_{\dot{\alpha}} \rightarrow -i \bar{\psi}_{\dot{\alpha}} .
\]

It satisfies \( (\mathbf{CP}_2 T)^2 = -1 \) and presents an invariance of any local and Lorentz-invariant Lagrangian.

Concluding this section, let us emphasize that we have shown that for any pattern of the neutron mass terms, including the Dirac mass respecting the baryon number conservation, as well as the Majorana ones violating it by two units, one can always consistently define the operations of parity transformation \( \mathbf{P}_2 \) and charge conjugation \( \mathbf{C} \) as preserved symmetries in spite of breaking of the baryon charge conservation. In fact, a generic mass matrix \( \tilde{m} \) in \( \{ \mathbf{23} \} \) can be always rotated by flavor transformation \( V V^T \tilde{m} V \) to a pseudo-Dirac form \( \{ \mathbf{25} \} \) where these symmetries are defined in an unique way.

Thus, the neutron-antineutron oscillation in itself does not violate discrete symmetries. However, \( \mathbf{C} \), \( \mathbf{P}_2 \) and also \( \mathbf{CP}_2 \) (which is an equivalent of \( \mathbf{T} \)), generically will not be respected by the interaction terms. Consider, e.g., the neutron \( \beta \)-decay \( n \rightarrow p e^+ \bar{\nu} \), implying that interaction has the standard, baryon charge preserving, form. Then the presence of \( L_B \) terms would induce also the “wrong” decays \( n \rightarrow p e^+ \bar{\nu} \) (though extremely suppressed). Furthermore, \( \mathbf{CP}_2 \) violation could be manifested in difference of branching ratios of “wrong” decays between the neutron and antineutron, \( \text{Br}(n \rightarrow p e^+ \bar{\nu}) \neq \text{Br}(\bar{n} \rightarrow p e^+ \bar{\nu}) \), even if observation of these decays is only a gedanken possibility. However, some \( \mathbf{CP}_2 \) violating processes related to new \( B \)-violating physics that induces the \( n - \bar{n} \) oscillation could be at the origin of the baryon asymmetry of the Universe.

6. In the Standard Model (SM) conservations of baryon \( B \) and lepton \( L \) numbers are related to accidental global symmetries of the SM Lagrangian.\(^5\) The violation of \( B \) by two units can originate only from new physics beyond SM which would induce the effective six-quark interaction

\[
\mathcal{L}(\Delta B = -2) = \frac{1}{M^5} \sum c_i \mathcal{O}^i ,
\]

\[
\mathcal{O}^i = T^i_{\chi_1 \chi_2 \chi_3} q^{A_1} q^{A_2} q^{A_3} q^{A_4} q^{A_5} q^{A_6} ,
\]

where coefficients \( T^i \) account for different flavor, color and spinor structures and the large mass scale \( M \) coming from new physics leads to the smallness of baryon violation.

In particular, the \( n \bar{n} \) mixing term \( \chi \) emerges as a matrix element between \( n \) and \( \bar{n} \) states of the operator \( \mathcal{O} \), see diagram in Fig.\( \text{I} \)

\[
\langle \bar{n} | \mathcal{L} (\Delta B = -2) | n \rangle = -\frac{1}{2} \epsilon v_n^* C u_n ,
\]

where \( u_n, v_n \) are Dirac spinors for \( n, \bar{n} \). Generically, it gives a complex value for \( \epsilon \) but by a phase redefinition of \( n, \bar{n} \) states we always can make it real and positive. Thus, an estimate of the parameter \( \epsilon \), which is inverse of the oscillation time \( \tau_{n\bar{n}} \), is

\[
\epsilon = \frac{1}{\tau_{n\bar{n}}} \sim \frac{\lambda_0^6}{M^5} .
\]

For \( u \) and \( d \) quarks of the first generation the full list of \( \Delta B = -2 \) six-quark operators was determined in Refs.\( \{15, 16\} \),

\[
\mathcal{O}^1_{\chi_1 \chi_2 \chi_3} = u_{\chi_1}^T C u_{\chi_1}^* d_{\chi_2}^T C d_{\chi_2}^* C d_{\chi_3}^T C d_{\chi_3}^* \left[ \epsilon_{i k m} \epsilon_{j l n} + \epsilon_{i k n} \epsilon_{j l m} + \epsilon_{i j m} \epsilon_{k l n} + \epsilon_{i j n} \epsilon_{k l m} \right] ,
\]

\[
\mathcal{O}^2_{\chi_1 \chi_2 \chi_3} = u_{\chi_1}^T C d_{\chi_1}^* u_{\chi_2}^T C d_{\chi_2}^* C d_{\chi_3}^T C d_{\chi_3}^* \left[ \epsilon_{i k m} \epsilon_{j l n} + \epsilon_{i k n} \epsilon_{j l m} + \epsilon_{i j m} \epsilon_{k l n} + \epsilon_{i j n} \epsilon_{k l m} \right] .
\]

Here \( \chi_i \) stands for \( L \) or \( R \) quark chirality. Accounting for relations

\[
\mathcal{O}^1_{\chi L R} = \mathcal{O}^2_{\chi R L} , \quad \mathcal{O}^2_{\chi \chi \chi} = \mathcal{O}^3_{\chi \chi \chi} ,
\]

\[
\mathcal{O}^2_{\chi \chi \chi} - \mathcal{O}^1_{\chi \chi \chi} = 3 \mathcal{O}^3_{\chi \chi \chi} ,
\]

\( ^5 \) Nonperturbative breaking of \( B \) and \( L \), preserving \( B - L \), is extremely small.
we deal with 14 operators for $\Delta B = -2$ transitions and 14 Hermitian conjugated ones for $\Delta B = +2$.

The $P_z$ reflection interchanges $L$ and $R$ chirality $\chi_i$ in the operators $O_{\chi_1 \chi_2 \chi_3}^i$. Note, that the $P_z$ reflection for $u$ and $d$ quarks is defined similar to the neutron by Eq. (13). This is consistent with the $udd$ wave function of neutron. Thus, we can divide operators into $P_z$ even and $P_z$ odd ones,

$$O_{\chi_1 \chi_2 \chi_3}^i \pm L \leftrightarrow R.$$  \hspace{1cm} (42)

The charge conjugation $C$ transforms operators $O_{\chi_1 \chi_2 \chi_3}^i$ into the Hermitian conjugated $[O_{\chi_1 \chi_2 \chi_3}^i]^\dagger$.

Again, our phase definitions for quarks are consistent with those for neutron. So, combinations

$$O_{\chi_1 \chi_2 \chi_3}^i \pm H.c.$$  \hspace{1cm} (43)

represent $C$ even and $C$ odd operators. In total, we break all 28 operators into four groups with different $P_z$, $C$ and $CP$ features, each group contains seven operators,

$$[O_{\chi_1 \chi_2 \chi_3}^i + L \leftrightarrow R]^H.c., \quad P_z = +, \ C = +, \ CP_z = +;$$

$$[O_{\chi_1 \chi_2 \chi_3}^i + L \leftrightarrow R]^H.c., \quad P_z = +, \ C = -, \ CP_z = -;$$

$$[O_{\chi_1 \chi_2 \chi_3}^i - L \leftrightarrow R]^H.c., \quad P_z = -, \ C = +, \ CP_z = -;$$

$$[O_{\chi_1 \chi_2 \chi_3}^i - L \leftrightarrow R]^H.c., \quad P_z = -, \ C = -, \ CP_z = +. \hspace{1cm} (44)$$

Only the first seven operators, which are both $P_z$ and $C$ even, contribute to $n\bar{n}$ oscillations. It is, of course, up to small corrections due to electroweak interactions where the discrete symmetries are broken.

What about the remaining 21 combinations which are odd either under $P_z$ or $C$ transformations? Although they do not contribute to the $n\bar{n}$ transition, their effect show up in instability of nuclei. This source of instability in this case is not due to neutron-antineutron oscillations but due to processes of annihilation of two nucleons inside nucleus like $N + N \rightarrow \pi + \pi$, and, in particular, two proton annihilation, $pp \rightarrow \pi^+\pi^+$, shown on Fig. 2. This could be particularly interesting in case of suppressed $n\bar{n}$ oscillations.

The operators of the type of involving strange quark, like $udsuds$, could induce $\Lambda - \bar{\Lambda}$ mixing. However, such operators would also lead to nuclear instability via nucleon annihilation into kaons $N + N \rightarrow K + K$, see the diagram in Fig. 2 where in upper lines $d$ quark is substituted by $s$ quark $(\pi^+ \leftrightarrow K^+)$. In fact, nuclear instability bounds on $\Lambda - \bar{\Lambda}$ mixing are only mildly, within an order of magnitude, weaker than with respect to $n - \bar{n}$ mixing which makes hopeless the possibility to detect $\Lambda - \bar{\Lambda}$ oscillation in the hyperon beam. (Instead, it can be of interest to search for the nuclear decays into kaons in the large volume detectors.) The nuclear instability limits on $\Lambda - \bar{\Lambda}$ mixing are about 15 orders of magnitude stronger than the sensitivity $\delta_{\Lambda\bar{\Lambda}} \sim 10^{-6}$ eV which can be achieved in the laboratory conditions $^{17}$. The nuclear stability limits make hopeless also the laboratory search of $bus$-like baryon oscillation due to operator $usbusb$ suggested in Ref. $^{18}$.

7. Our above consideration refers to the neutron-antineutron oscillation in vacuum. Now we show that even in the presence of magnetic field no new $|\Delta B| = 2$ operator appears. A similar consideration was done in Ref. $^{19}$ in application to a possible magnetic moment of neutrino.

In the Weyl formalism the field strengths tensor $F_{\mu\nu}$ is substituted by the symmetric tensor $F_{\alpha\beta}$ and its complex conjugate $F_{\bar{\alpha}\bar{\beta}}$. They correspond to $\tilde{E} \pm i\tilde{B}$ combinations of electric and magnetic fields. Then Lorentz
invariance allows only two structures involving electromagnetic fields,

\[ F_{\alpha\beta} \psi^{\alpha} \bar{\psi}^{\beta} \epsilon_{ik}, \quad \bar{F}_{\alpha\beta} \bar{\psi}^{\alpha} \gamma^{\beta} \psi_{ik} \]  

(45)

Antisymmetry in flavor indices implies that spinors with the opposite baryon charges enter. So both operators preserve the baryon charge, and in fact they describe interactions with the magnetic and electric dipole moments of the neutron. In terms of Majorana mass eigenstates \( m \), these are transitional moments between \( n_1 \) and \( n_2 \). However, no transitional moment can exist between \( n \) and \( n' \).

The authors of Ref. [20] realize that the operator \( n^T \sigma^{\mu\nu} C n F_{\mu\nu} \) with \( \Delta B = -2 \) is vanishing due to Fermi statistics. They believe, however, that a composite nature of neutron changes the situation and a new type of magnetic moment in \( \Delta B = \pm 2 \) transitions may present. In other words, they think that the effective Lagrangian description is broken for composite particles.

To show that is not the case let us consider the process of annihilation of two neutrons into virtual photon,

\[ n(p_1) + n(p_2) \rightarrow \gamma^n(k), \]  

(46)

which is the crossing channel to \( n - \bar{n} \gamma^n \) transition. The number of invariant amplitudes for the process (46) which is \( 1/2^+ + 1/2^+ \rightarrow 1^- \) transition is equal to one. Only orbital momentum \( L = 1 \) and total spin \( S = 1 \) in the two neutron system are allowed by angular momentum conservation and Fermi statistics. The gauge-invariant form of the amplitude is

\[ u^T(p_1) C \gamma^{\gamma} \gamma_5 u(p_2) k^\nu (k_\mu \epsilon_{\nu} - k_{\nu} \epsilon_{\mu}), \]  

(47)

where \( u_{1,2} \) are Dirac spinors describing neutrons and \( \epsilon_{\mu} \) refers to the gauge potential. In space representation we deal with \( \partial^\nu F_{\mu\nu} \) the quantity which vanishes outside of the source of the electromagnetic field, and, in particular, for the distributed magnetic field. It proves that there is no place for magnetic moment of \( n - \bar{n} \) transition, and effective Lagrangian description does work. Let us also remark that \( n \rightarrow \bar{n} \gamma^n \) transition with a virtual photon connected to the proton, as well as \( nn \rightarrow \gamma^n \) annihilation, would destabilize the nuclei even in the absence of \( n - \bar{n} \) mass mixing.

Even in the absence of new \( n - \bar{n} \) magnetic moment the authors of [20] claim that suppression of \( n - \bar{n} \) oscillations by external magnetic field can be overcome by applying the magnetic field transversal to quantization axis. Following our criticism [4] Gardner and Yan recognized in [7] that it would break the rotational invariance. As a consequence the magnetic field suppression does present indeed.

The situation is different if one considers oscillation \( n - n' \) where \( n' \) is a mirror neutron, twin of the neutron from hidden mirror sector [21]. In this case one deals with the mass mixing between two Dirac fermions, \( \epsilon \bar{m} + \text{h.c.} \) forming a combination of baryon numbers \( B + B' \). Hence, also operators \( \pi \sigma^{\mu\nu} n' F_{\mu\nu} \) and \( \pi \sigma^{\mu\nu} \gamma_5 n' F_{\mu\nu} \) are allowed which describe respectively the transitional magnetic and electric dipole moments between \( n \) and \( n' \) states (and the similar operators with mirror electromagnetic field, \( F_{\mu\nu} \rightarrow F'_{\mu\nu} \)). Since underlying new physics generating \( n - n' \) mixings generically should violate CP-invariance, both transitional magnetic and electric dipole moments can be of the same order. In large enough magnetic (or electric) field \( n \rightarrow n' \) transition probabilities should not depend on the field value, with possible implications for the search of neutron–mirror neutron transitions, and in particular for testing experimentally solution of the neutron lifetime puzzle via \( n - n' \) transitions [22].

8. Our use of the effective Lagrangian for the proof means that the Lorentz invariance and CPT are crucial inputs. Once constraints of Lorentz invariance are lifted new \( |\Delta B| = 2 \) operators could show up.

Such operators were analyzed in Ref. [23] for putting limits on the Lorentz invariance breaking. In particular, the authors suggested the operator \( n^T C \gamma^3 \gamma^2 n \) as an example which involves spin flip and, correspondingly, less dependent on magnetic field surrounding.

Note, however, that besides breaking of Lorentz invariance this operator breaks also 3d rotational invariance, i.e., isotropy of space. Such anisotropy could be studied by measuring spin effects in neutron-antineutron transitions.

9. The construction we used for neutron-antineutron transition could be applied to mixing of massive neutrinos. As an example, let us take the system of left-handed \( \nu_e \) and \( \nu_\mu \) and their conjugated partners, right-handed \( \bar{\nu}_e \) and \( \bar{\nu}_\mu \). One can ascribe them [24] a flavor charge \( F = L_e - L_\mu \) (analogy of \( \mathcal{B} \)), to be \( +1 \) for \( \nu_e \) and \( -1 \) for \( \nu_\mu \). Then, \( C \) conjugation is interchange of \( \bar{\nu}_e \) and \( \nu_\mu \). Again, \( F \) breaking mass term would be \( C \) and \( P \), even but odd for \( P \).

A similar scenario can be staged in case of Dirac massive neutrino.

10. In summary, we show that the Lorentz and CPT invariance lead to the unique \( |\Delta B| = 2 \) operator in the effective Lagrangian for the neutron-antineutron mixing. This mixing is even under the charge conjugation \( C \) as well as under the modified parity \( P \), which takes the same value \( i \) for both, neutron and antineutron in contrast with standard \((+1)\) and \((-1)\) values. It means that observation of the neutron-antineutron mixing per se does not give a signal of CP violation. It could be compared with the \( K^0 - \bar{K}^0 \) transition amplitude with \( |\Delta S| = 2 \) where to separate \( CP \) conserving and \( CP \) breaking parts one needs to relate it to \( |\Delta S| = 1 \) decay amplitudes.

We applied the discrete symmetries to classification of possible \(|\Delta B| = 2 \) six-quark operators, separating those which contributes to the neutron-antineutron mixing. Other \(|\Delta B| = 2 \) operators contribute to instability of
We also showed that switching on external magnetic field influences the level splitting, what suppresses $n - \bar{n}$ oscillations, but does not add any new $|\Delta B| = 2$ operator in contradistinction with recent claims in literature.

Our classification of $|\Delta B| = 2$ operators coming from new physics, could be useful in association with Sakharov conditions for baryogenesis which involves both, non-conservation of baryon charge and CP-violation.

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