Flexible FOND Planning with Explicit Fairness Assumptions

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Abstract

We consider the problem of reaching a propositional goal condition in fully-observable non-deterministic (FOND) planning under a general class of fairness assumptions that are given explicitly. The fairness assumptions are of the form $A/B$ and say that state trajectories that contain infinite occurrences of an action $a$ from $A$ in a state $s$ and finite occurrence of actions from $B$, must also contain infinite occurrences of action $a$ in $s$ followed by each one of its possible outcomes. The infinite trajectories that violate this condition are deemed as unfair, and the solutions are policies for which all the fair trajectories reach a goal state. We show that strong and strong-cyclic FOND planning, as well as QNP planning, a planning model introduced recently for generalized planning, are all special cases of FOND planning with fairness assumptions of this form which can also be combined. FOND+ planning, as this form of planning is called, combines the syntax of FOND planning with some of the versatility of LTL for expressing fairness constraints. A new planner is implemented by reducing FOND+ planning to answer set programs, and the performance of the planner is evaluated in comparison with FOND and QNP planners, and LTL synthesis tools.

Introduction

FOND planning has become increasingly important as a way of solving other types of problems such as probabilistic (MDP) planning, where actions have a probabilistic effect on states (Bertsekas and Tsitsiklis 1996; Geffner and Bonet 2013), LTL planning, where goals to be reached are generalized to temporal conditions that must be satisfied possibly by plans with cycles (Calvanese, De Giacomo, and Vardi 2002; Camacho, Bienvenu, and McIlraith 2019; Aminof et al. 2019), and generalized planning, where plans are not for single instances but for collections of instances (Srivastava, Immerman, and Zilberstein 2011; Hu and De Giacomo 2011), and they can be obtained from suitable abstractions encoded as QNP planning problems (Srivastava et al. 2011; Bonet and Geffner 2020).

A critical limitation of strong, strong-cyclic, and QNP planners, is that the fairness assumptions are implicit in their models and solvers, and as a result, cannot be combined. These combinations, however, are often needed (Camacho and McIlraith 2016; Ciolek et al. 2020), and indeed, a recent FOND planner handles combinations of fair and adversarial actions in what is called Dual FOND planning (Geffner and Geffner 2018). In this work, we go beyond this integration by also enabling the representation and combination of the conditional fairness assumptions that underlie QNP planning. This is achieved by extending FOND planning with a general class of fairness assumptions that are given explicitly as part of the problem. The fairness assumptions are pairs $A/B$ of sets of actions $A$ and $B$ that say that state trajectories that contain infinite occurrences of actions $a$ from $A$ in a state $s$, and finite occurrences of actions from $B$, must also contain infinite occurrences of action $a$ in the state $s$ followed by each one of its possible outcomes. The infinite trajectories that violate this condition are regarded as unfair. The solutions of a FOND problem with conditional fairness assumptions of this type, called a FOND+ problem, are the policies for which all fair state trajectories reach the goal.

We show that strong, strong-cyclic, and QNP planning, are all special cases of FOND+ planning where the fairness assumptions underlying these models can be combined. FOND+ planning extends the syntax and semantics of FOND planning with some of the versatility of the LTL language for expressing fairness constraints. The conditional
A state for each action \( \pi \) and \( s \) state in the sequence being a goal state, \( (s_0, s_1, \ldots, s_n) \) \( \in F(\pi(s), s) \), one for each action \( a \in A \), each state \( s \), and each possible outcome \( E_i \) of the action \( a \), where \( s \) stands for the conjunction of literals that \( s \) makes true. However, unlike LTL, synthesis and planning that are 2EXP-Complete (Pnueli and Rosner 1989; Camacho, Bienvenu, and McIlraith 2019; Aminof, De Giacomo, and Rubin 2020), FOND planning is in NEXP (non-deterministic exponential time).

A planner for FOND\(^+\) is obtained by reducing FOND\(^+\) planning over the explicit state space to an elegant answer set program (ASP), a convenient and high-level alternative to SAT (Brewka, Eiter, and Truszczyński 2011; Lifschitz 1998; Gebser et al. 2012), using the facilities provided by the CLINGO ASP solver (Gebser et al. 2019). The performance of this ASP-based planner is evaluated in comparison with FOND and QNP planners, and LTL synthesis tools.

The paper is organized as follows. We review first strong and strong-cyclic FOND planning, and QNP planning. We introduce then FOND\(^+\) planning, where the assumptions underlying these models are stated explicitly and combined, and present a description of the ASP-based FOND\(^+\) planner, an empirical evaluation, and a discussion.

**FOND Planning**

A FOND model is a tuple \( M = (S, s_0, S_G, Act, A, F) \), where \( S \) is a finite set of states, \( s_0 \in S \) is the initial state, \( S_G \subseteq S \) is a non-empty set of goal states, \( Act \) is a set of actions, \( F(a, s) \) is the set of successor states when action \( a \) is executed in state \( s \), and \( A(s) \subseteq Act \) is the set of actions applicable in state \( s \), such that \( a \in A(s) \) iff \( F(a, s) \neq \emptyset \). A FOND problem \( P \) is a compact description of a FOND model \( M(P) \) in terms of a finite set of states, so that the states \( s \) in \( M(P) \) correspond to truth valuations over the atoms, represented by the set of states that are true. The standard syntax for FOND problems is a simple extension of the STRIPS syntax for classical planning. A FOND problem is a tuple \( P = (At, I, Act, G) \) where \( At \) is a set of atoms, \( I \subseteq At \) is the set of atoms true in the initial state \( s_0 \), \( G \) is the set of goal atoms, and \( Act \) is a set of actions with atomic pre-conditions and effects. If \( E_i \) represents the set of positive and negative effects of an action in the classical setting, action effects in FOND planning can be deterministic of the form \( E_i \), or non-deterministic of the form \( oneof(E_1, \ldots, E_n) \).

A policy \( \pi \) for a FOND problem \( P \) is a partial function mapping non-goal states into actions. A policy \( \pi \) for \( P \) defines a set of, possibly infinite, compatible state trajectories \( s_0, s_1, s_2, \ldots \), also called \( \pi \)-trajectories, where \( s_{i+1} \in F(a_i, s_i) \) and \( a_i = \pi(s_i) \) for \( i \geq 0 \). A trajectory \( \tau \) compatible with \( \pi \) is maximal if it is infinite, or is finite of the form \( \tau = s_0, \ldots, s_n \), for some \( n \geq 0 \), and either \( s_n \) is the first state in the sequence being a goal state, \( \pi(s_n) \notin A(s_n) \) (i.e., the action prescribed at \( s_n \) is not applicable), or \( \pi(s_n) = \bot \) (i.e., no action is prescribed). Likewise, the policy \( \pi \) reaches a state \( s \) if there is a \( \pi \)-trajectory \( s_0, \ldots, s_n \), where \( s = s_n \), and \( \pi \) reaches a state \( s' \) from a state \( s \) if there is a \( \pi \)-trajectory \( s_0, \ldots, s_n \), where \( s = s_i \) and \( s' = s_j \) for \( 0 \leq i \leq j \leq n \). A state \( s \) is recurrent in trajectory \( \tau \) if it appears an infinite number of times in \( \tau \). The strong and strong-cyclic solutions or policies are usually defined as follows:

**Definition 1 (Solutions).** A policy \( \pi \) is a strong solution for a FOND problem \( P \) if all the maximal \( \pi \)-trajectories reach a goal state, and it is a strong-cyclic solution if \( \pi \) reaches a goal state from any state reached by \( \pi \).

The strong solutions correspond also to the strong-cyclic solutions that are acyclic; namely, where the policies \( \pi \) do not give rise to \( \pi \)-trajectories that can visit a state more than once. Alternatively, strong and strong-cyclic solutions can be understood in terms suitable notions of fairness that establish which \( \pi \)-trajectories are deemed possible. If we say that a policy \( \pi \) solves problem \( P \) when all the fair \( \pi \)-trajectories reach the goal, then in strong planning, all \( \pi \)-trajectories are deemed fair; while in strong-cyclic planning, all \( \pi \)-trajectories are deemed fair except those containing a recurrent state \( s \) that is followed a finite number of times by a successor \( s' \in F(\pi(s), s) \).

In order to make this alternative “folk” characterization of strong and strong-cyclic planning explicit, let us say that all the actions in strong FOND planning are adversarial (or “unfair”), and that all the actions in strong-cyclic FOND planning are fair. The state trajectories that are deemed fair in each setting can then be expressed as follows:

**Definition 2.** If all the actions are adversarial, all \( \pi \)-trajectories are fair. If all the actions are fair, a \( \pi \)-trajectory \( \tau \) is fair iff states \( s \) that occur an infinite number of times in \( \tau \), are followed an infinite number of times by each possible successor \( s' \) of \( s \) given \( \pi \), \( s' \in F(\pi(s), s) \).

Provided with these notions of fairness, strong and strong-cyclic solutions can be characterized equivalently as:

**Theorem 3.** A policy \( \pi \) is a strong (resp. strong-cyclic) solution of a FOND problem \( P \) iff all the fair trajectories compatible with \( \pi \) in \( P \) reach the goal, under the assumption that all actions are adversarial (resp. fair).

**Proof.** When all actions are adversarial, all trajectories are fair and the claim follows directly. For strong-cyclic planning, observe that an infinite fair trajectory \( \tau \) visits each state that is reachable from any recurrent state in \( \tau \). We show the contrapositive of the two implications. If there is a maximal fair trajectory that does not reach the goal, by the observation, there is a state \( s \) that is reachable from the initial state and that is not connected to a goal; i.e., \( \pi \) is not strong-cyclic. Conversely, if \( \pi \) is not strong-cyclic, there is a state \( s \) that is reachable from the initial state and that is not connected to the goal, and thus there is a maximal fair trajectory that does not reach a goal; i.e., \( \pi \) does not solve \( P \).

Methods for computing strong and strong-cyclic solutions for FOND problems have been developed based on OBDDs (Cimatti et al. 2003), explicit forms of AND/OR search (Mattmüller et al. 2010), classical planners (Muir, McIlraith, and Beck 2012), and SAT (Chatterjee, Chmelík, and Davies 2016). Some of these planners actually handle a combination of fair and adversarial actions, in what is called Dual FOND planning (Geffner and Geffner 2018).
QNP Planning

Qualitative numerical planning problems (QNPs) were introduced by Srivastava et al. (2011) as a model for generalized planning, that is, planning for multiple classical instances at once. QNPs have been used since in other works (Bonet et al. 2017; Bonet, Frances, and Geffner 2019) and have been analyzed in depth by Bonet and Geffner (2020).

The syntax of QNPs is an extension of STRIPS problems $P = \langle At, I, O, G \rangle$ with negation where $At$ is a set of ground (boolean) atoms, $I$ is a maximal consistent set of literals from $At$ describing the initial situation, $G$ is a set of literals describing the goal situation, and $O$ is a set of (ground) actions with precondition and effect literals. A QNP $Q = \langle At, V, I, O, G \rangle$ extends a STRIPS problem with a set $V$ of numerical variables $X$ that can be decremented or incremented qualitatively; i.e., by indeterminate positive amounts, without making the variables negative. A numerical variable $X$ can appear in action effects as $X^+$ (increments) and $X^-$ (decrements), while literals of the form $X = 0$ or $X > 0$ (an abbreviation of $X \neq 0$) can appear everywhere else (initial situation, preconditions, and goals). The literal $X > 0$ is a precondition of all actions with $X^-$ effects.

A simple example of a QNP is $Q = \langle At, V, I, O, G \rangle$ with $At = \{p\}$, $V = \{n\}$, $I = \{\neg p, n > 0\}$, $G = \{n = 0\}$, and actions $O = \{a, b\}$ given by

$$a = \langle p, n > 0; \neg p, n\rangle$$

$$b = \langle \neg p; p \rangle$$

where $\langle C; E \rangle$ denotes an action with preconditions $C$ and effects $E$. Thus action $a$ decrements $n$ and negates $p$ that is a precondition of $a$, and $b$ restores $p$. This QNP represents an abstraction of the problem of clearing a block $x$ in Blocksworld instances with stack/unstack actions that include a block $x$. The numerical variable $n$ stands for the number of blocks above $x$, and the boolean variable $p$ stands for the robot gripper being empty. A policy $\pi$ that solves $Q$ can be expressed by the rules:

$$\text{if } p \text{ and } n > 0, \text{ do } a$$

$$\text{and } \text{ if } \neg p \land n > 0, \text{ do } b.$$

A key property of QNPs is that while numerical planning is undecidable (Helmert 2002), qualitative numerical planning is not. Indeed, a sound and complete, two-step method for solving QNPs was formulated by Srivastava et al. (2011): the QNP $Q$ is converted into a standard FOND problem $P = T_D(Q)$ and its (strong-cyclic) solution is checked for termination. The QNP solutions are in correspondence with the strong-cyclic plans of the direct translation $P = T_D(Q)$ that terminate. Moreover, since the number of policies that solve $P$ is finite, and the termination of each can be verified in finite time, plan existence for QNPs is decidable. More recent work has shown that the complexity of QNP planning is the same as that of FOND planning by introducing a polynomial reduction from the former into the latter, and another in the opposite direction (Bonet and Geffner 2020).

We do not need to get into the formal details of QNPs but it is useful to review the direct translation $T_D$ of a QNP $Q$ into a FOND problem $P = T_D(Q)$, and the notion of termination (Srivastava et al. 2011). Concretely, the translation $T_D$ replaces each numerical variable $n$ by a boolean atom $p_n$ that stands for the (boolean) expression $n = 0$. Then, occurrences of the literal $n = 0$ in the initial situation, action preconditions, and goals are replaced by $p_n$, while occurrences of the literal $n > 0$ in the same contexts are replaced by $\neg p_n$. Likewise, effects $n \uparrow$ are replaced by effects $\neg p_n$, and effects $n \downarrow$ are replaced by non-deterministic effects $\text{oneof}(p_n, \neg p_n)$. Actions in the FOND problem $P = T_D(Q)$ with effects $\neg p_n$ (i.e., $n > 0$) are said to “increment $n$”, while actions with effects $\text{oneof}(p_n, \neg p_n)$ (i.e., either $n > 0$ or $n = 0$) are said to “decrement $n$”, even if there are no numerical variables in $P$ but just boolean variables. This information needs to be preserved in the translation $P = T_D(Q)$, as the semantics of $P$ is not the semantics of FOND problems as assumed by strong or strong-cyclic planners.

Termination and SIEVE

A policy $\pi$ for the FOND problem $P = T_D(Q)$ is said to terminate if all the state trajectories in $P$ that are compatible with the policy $\pi$ and with the fairness assumptions underlying the QNP $Q$ are finite. Termination is the result of the absence of cycles in the policy that could be traversed forever. The latter arises when a cycle includes an action that decrements a numerical variable and none that increments it. Since numerical variables cannot become negative such cycles eventually terminate.

The procedure called SIEVE (Srivastava et al. 2011) provides a sound and complete termination test that runs in time that is polynomial in the number of states reached by the policy. SIEVE can be understood as an efficient implementation of the following procedure that operates on a policy graph $G(\pi)$ induced by the FOND problem $P$ and the policy $\pi$, where the nodes are the states $s$ that can be reached in $P$ via the policy $\pi$, and the edges correspond to the state transitions $(s, s')$ that are possible given the policy $\pi$ (i.e., $s' \in F^{G}(\pi(s), s)$).

Starting with the graph $G = G(\pi)$, SIEVE iteratively removes edges from $G$ until $G$ becomes acyclic or does not admit further removals. In each iteration, an edge $(s, s')$ is removed from $G$ if $\pi(s)$ is an action that decrements a variable $x$ that is not incremented along any path in $G$ from $s'$ back to $s$. SIEVE accepts the policy $\pi$ iff SIEVE renders the resulting graph $G$ acyclic. It can be shown that the resulting graph $G$ is well defined (i.e., it is the same independently of the order in which edges are removed), and that SIEVE removes an edge $(s, s')$ when it cannot be traversed by the policy an infinite number of times.

It is useful to capture the logic of SIEVE in terms of an inductive definition that considers states instead of edges:

Definition 4 (QNP Termination). Let $\pi$ be a policy for the FOND problem $P = T_D(Q)$ associated with the QNP $Q$. The policy $\pi$ terminates in $P$ iff every state $s$ that is reachable by $\pi$ in $P$ terminates, where a state $s$ terminates if:

1. there is no cycle on node $s$ (i.e., no path from $s$ to itself),
2. every cycle on $s$ contains a state $s'$ that terminates, or

\[\text{this inductive definition and the ones below imply that there is a unique sequence of state subsets } S_0, S_1, \ldots, S_n \text{ such that } S_{n+1} \text{ is } S_n \text{ augmented with all the states that can be added to } S_n \text{ when assuming that the only terminating states are those in } S_n.\]
3. $\pi(s)$ decrements a variable $x$, and every cycle on $s$ containing a state $s'$ for which $\pi(s')$ increments $x$, also contains a state $s''$ that terminates.

**Theorem 5.** Let $Q$ be a QNPs and $\pi$ a policy. Then, SIEVE accepts the policy graph $G(P, \pi)$ iff policy $\pi$ terminates in $P$, where $P = T_D(Q)$.

**Proof.** Since the resulting graph of executing SIEVE does not depend on the order in which edges are removed, we can consider any execution of SIEVE.

**Forward implication.** Let $e_0, e_1, \ldots, e_m$ be the edge removed by SIEVE along some execution. We show by induction that the state $s_i$ in the edge $S e_i = (s_i, s'_i)$ removed by SIEVE is terminating. SIEVE removes $e_0$ because $\pi(s_0)$ decrements a variable $x$ and there is no cycle involving $s_0$ with actions that increment $x$. If so, condition 3 applies, and $s$ must terminate. Let us assume that the claim holds for the first $k$ iterations of SIEVE, and let us consider the $k + 1$-st iteration. Edge $e_{k+1}$ is removed because $\pi(s_{k+1})$ decrements a variable $x$ and there is no cycle, in the current graph, involving $s_{k+1}$ with actions that increment $x$. That is, either there is no such cycle in the original graph, or all such cycles have been “broken” by the removal of previous edges. Hence, by inductive hypothesis, condition 3 applies again and $s$ must terminate. Now, if the graph resulting from SIEVE is acyclic, all states that have not yet been labeled as terminating can be labeled as such using conditions 1 and 2. On the other hand, if the resulting graph is not acyclic, then it is easy to show that the states in any such cycle cannot be labeled as terminating.

**Backward implication.** Let $S_0, S_1, \ldots, S_k$ be the state subsets associated with the inductive definition (cf. footnote 1). We construct an execution $e_0, e_1, \ldots, e_m$ of SIEVE. If $s$ is $S_0$, either there is no cycle involving $s$ or $\pi(s)$ decrements a variable that is not incremented along any cycle that involves $s$. In the latter case, SIEVE removes all edge $(s, s')$. Let us assume that we have constructed an execution of SIEVE that removes all edges $(s, s')$ for $s$ in $S_0 \cup S_1 \cup \cdots \cup S_k$. If $s$ is $S_{k+1}$, either there is no cycle involving $s$ or $\pi(s)$ decrements a variable $x$, and any any cycle that involves $s$ and decrements $x$, has a state $s''$ that has been labeled as terminating. Then, using the inductive hypothesis, the current execution of SIEVE can be extended by the removal of all edge $(s, s')$. If all reachable states are labeled as terminating, the constructed execution of SIEVE renders an acyclic graph. On the other hand, if some reachable state $s$ is not terminating, it can be shown that the resulting graph has a cycle that involves $s$; i.e., it is not acyclic.

Since solutions to QNPs $Q$ are known to be the strong-cyclic policies of the FOND problem $P = T_D(Q)$ that are accepted by SIEVE (Srivastava et al. 2011; Bonet and Geffner 2020), the solutions for $Q$ can also be expressed as:

**Theorem 6.** A policy $\pi$ is a solution to a QNP $P$ iff $\pi$ is a strong-cyclic solution of $P = T_D(Q)$ that terminates.

**Proof.** The result is direct given that $\pi$ is a solution of $Q$ iff $\pi$ is a strong-cyclic solution for $P$ accepted by SIEVE (Srivastava et al. 2011; Bonet and Geffner 2020), and the equivalence between SIEVE acceptance and the notion of policy termination in Theorem 5.

The characterization that results from this theorem has been used to verify QNP solutions but not for computing them. Indeed, the only available complete QNP planner is based on a polynomial reduction of QNP planning into strong-cyclic FOND planning that avoids the termination test (Bonet and Geffner 2020).

**FOND$^+$ Planning**

In this section, we move from strong, strong-cyclic, and QNP planning to the FOND$^+$ setting where the fairness assumptions underlying these models can be explicitly stated and combined. A **FOND$^+$ planning problem** $P_c = (P, C)$ is a FOND problem $P$ extended with a set $C$ of fairness assumptions:

**Definition 7.** A FOND$^+$ problem $P_c = (P, C)$ is a FOND problem $P$ extended with a set $C$ of (conditional) fairness assumptions of the form $A_i/B_i, i = 1, \ldots, n$ and where each $A_i$ is a set of non-deterministic actions in $P$, and each $B_i$ is a set of actions in $P$ disjoint from $A_i$.

The fairness assumptions play no role in constraining the state trajectories that are possible by following a policy $\pi$, the so-called $\pi$-trajectories:

**Definition 8.** A state trajectory compatible with a policy $\pi$ for the FOND$^+$ problem $P_c = (P, C)$ is a state trajectory that is compatible with $\pi$ in the FOND problem $P$.

However, while in strong and strong-cyclic FOND planning all actions are considered as adversarial and fair, respectively, in the FOND$^+$ setting, each action is labeled fair or unfair depending on the assumptions in $C$ and the trajectory where the action occurs. We define what it means for an action $a = \pi(s)$ to behave “fairly” in a recurrent state $s$ of an infinite $\pi$-trajectory as follows:

**Definition 9.** The occurrence of the action $\pi(s)$ in a recurrent state $s$ of a $\pi$-trajectory $\tau$ associated with the FOND$^+$ problem $P_c = (P, C)$ is **fair** if for some fairness assumption $A/B \in C$, it is the case that $\pi(s) \in A$ and all the actions in $B$ occur infinitely often in $\tau$.

The meaning of a conditional fairness assumption $A/B$ is that the actions $a \in A$ can be assumed to be fair in any recurrent state $s$ of a $\pi$-trajectory $\tau$, provided that the condition on $B$ holds in $\tau$; namely, that actions in $B$ do not occur infinitely often in $\tau$. Otherwise, if any action in $B$ occurs infinitely often in $\tau$, then $a$ is said to be **unfair** or adversarial. Once actions $\pi(s)$ occurring in recurrent states $s$ are “labeled” in this way, the standard notion of fair trajectories (Definition 2) extends naturally to FOND$^+$ problems:

**Definition 10.** A $\pi$-trajectory $\tau$ for a FOND$^+$ problem $P_c = (P, C)$ is **fair** if for every recurrent state $s$ in $\tau$ where the action $\pi(s)$ is fair and every possible successor $s'$ of $s$ due to action $\pi(s)$ (i.e., $s' \in F(\pi(s), s)$), state $s$ is immediately followed by state $s'$ in $\tau$ an infinite number of times.

The solution of FOND$^+$ problems can then be expressed in a standard way as follows:
Definition 11 (Solutions). A policy $\pi$ solves the FOND$^+$ problem $P_c = (P, C)$ if the maximal $\pi$-trajectories that are fair reach the goal.

A number of observations can be drawn from these definitions. Let us say that one wants to model a non-deterministic action $a$ whose behavior is fair in that it always displays all its possible effects infinitely often in every recurrent state $s$ such that $\pi(s) = a$. To so, we consider a fairness constraint $A/B$ in $C$ such that $a \in A$ and $B$ is empty. On the other hand, to model an adversarial action $b$, whose behavior is not fair (may not yield all its effects infinitely often in a recurrent state $s$ with $\pi(s) = b$), we do not include $b$ in any set $A$. This immediately suggests the way to capture standard strong and strong-cyclic planning as special forms of FOND$^+$ planning:

Theorem 12. The strong solutions of a FOND problem $P$ are the solutions of the FOND$^+$ problem $P_c = (P, \emptyset)$.

Proof. Notice that policies are not defined on goal states, and thus there are no infinite $\pi$-trajectories that reach a goal, for any policy $\pi$. If $\pi$ is a strong solution for $P$, there are no infinite $\pi$-trajectories and thus all maximal $\pi$-trajectories are finite and goal-reaching; i.e., $\pi$ solves $P_c$.

On the other hand, if $\pi$ solves $P_c$, by definition, the maximal $\pi$-trajectories that are fair reach the goal. Since there are no constraints, any infinite $\pi$-trajectory is fair and non-goal reaching, and thus, if there is any such trajectory, $\pi$ cannot solve $P_c$. Therefore, all maximal $\pi$-trajectories are finite and goal reaching; i.e., $\pi$ is a strong solution for $P$.

Theorem 13. The strong-cyclic solutions of a FOND problem $P$ are the solutions of the FOND$^+$ problem $P_c = (P, \{A/0\})$, where $A$ is the set of all the non-deterministic actions in $P$.

Proof. Let $\pi$ be a policy for $P$ and let $\tau$ be an infinite $\pi$-trajectory in $P$. Since $B$ is empty, any action $\pi(s)$ in $A$ for a recurrent state $s$ in $\tau$ is fair (cf. Definition 9). Thus, by Definition 10 and that $A$ contains all the non-deterministic actions, $\tau$ is fair in $P$ iff $\tau$ is fair in $P_c$, and by Theorem 3, $\pi$ is a strong-cyclic solution for $P$ iff $\pi$ solves $P_c$.

Similarly, QNP problems are reduced to FOND$^+$ problems in a direct way, in this case, making use of both the head $A$ and the condition $B$ in the fairness assumptions $A/B$ in $C$:

Theorem 14. The solutions of a QNP problem $Q$ are the solutions of the FOND$^+$ problem $P_c = (P, C)$ where $P = T_D(Q)$ and $C$ is the set of fairness assumptions $A_i/B_i$ one for each numerical variable $x_i$ in $Q$, such that $A_i$ contains all the actions in $P$ that decrement $x_i$, and $B_i$ contains all the actions in $P$ that increment $x_i$.

Proof. Let $Q$ be a QNP problem, let $P_c = (P, C)$ be the FOND$^+$ problem associated with $Q$ where $P = T_D(Q)$, and let $\pi$ be a policy for $Q$ (and $P$).

Forward direction. Let us assume that $\pi$ solves $Q$ and suppose it does not solve $P_c$. Then, there is a maximal fair (cf. Definition 10) $\pi$-trajectory $\tau$ that does not reach the goal.

The role of the fairness assumptions underlying strong, strong-cyclic, and QNP planning, FOND$^+$ planning integrates these planning models as well. We illustrate the new possibilities with an example.

Let $P$ be a FOND problem with state set $\{s_0, s_1, s_2, g\}$, two non-deterministic actions $a$ and $b$, initial and goal states being $s_0$ and $g$, respectively. Action $a$ can only be applied in state $s_0$, leading to either $s_1$ or $s_2$, whereas action $b$ can be applied only in $s_1$ and $s_2$, leading, in both cases, to either $s_0$ or $g$; see Figure 1. The FOND problem $P$ admits a single policy, namely, $\pi(s_0) = a$ and $\pi(s_1) = \pi(s_2) = b$, which we analyze in the context of different FOND$^+$ problems $P_i = (P, C_i)$ that can be built on top of $P$ using different sets of fairness assumptions $C_i$. For convenience, in the sets $C_i$, we use $a/b$ to denote the fairness assumption $\{a\} \cup \{b\}$.
and $a$ to denote the assumption $\{a\}/\emptyset$. The marks ‘✓’ and ‘✗’ express that the policy $\pi$ solves or does not solve, resp., the FOND$^+$ problem $P_i$, where $C_i$ is:

- $C_1 = \{\}$; $a$ and $b$ are adversarial.
- $C_2 = \{a, b\}$; $a$ and $b$ are fair.
- $C_3 = \{a\}$; $a$ is fair and $b$ is adversarial.
- $C_4 = \{b\}$; $b$ is fair and $a$ is adversarial.
- $C_5 = \{a/b\}$; $a$ is conditionally fair on $b$; $b$ adversarial.
- $C_6 = \{a/b, a\}$; QNP like: $a : x_1 \downarrow, x_2 \uparrow$ and $b : x_2 \downarrow$.
- $C_7 = \{b, a/b\}$; QNP like: $b : x_1 \downarrow, x_2 \uparrow$ and $a : x_2 \downarrow$.
- $C_8 = \{a/b, b/a\}$; QNP like: $a : x_1 \downarrow, x_2 \uparrow$ and $b : x_2 \downarrow, x_1 \uparrow$.

The subtle cases are the last four. The policy $\pi$ does not solve $P_6$, because there are trajectories like $\tau = s_0, s_1, s_0, s_2, s_0, s_1, s_2, \ldots$ that are fair but do not reach the goal. The reason is that while $a/b \in C_5$, the occurrences of the action $a = \pi(s_0)$ in the recurrent state $s_0$ in $\tau$ are not fair. Thus, both $a$ and $b$ have an adversarial semantics in $\tau$. The policy $\pi$ does not solve $P_6$ either, because in the same trajectory $\tau$, the action $a$ is fair in $s_0$ as $a \in C_6$ but $b$ is not fair in either $s_1$ or $s_2$, as the assumption $b/a$ is in $C_6$ but $a$ occurs infinitely often in $\tau$. As a result, $\tau$ is fair but non-goal reaching in $P_6$.

The situation is different in $P_7$, where $b$ is fair and $a$ is unfair. Here, $\tau$ is unfair, as any other trajectory in which some or all the states $s_0, s_1$, and $s_2$ occur infinitely often. This is because $b$ being fair in $s_1$ and $s_2$ means that the transitions $(s_1, g)$ and $(s_2, g)$ cannot be skipped forever, and the goal must be reached eventually. Finally, in $P_8$, the trajectory $\tau$ becomes fair again, as both $a$ and $b$ are adversarial in $\tau$.

**Termination and SIEVE$^+$ for FOND$^+$**

We now consider the computation of policies for FOND$^+$ problems. Initially, we look for a procedure to verify if a policy $\pi$ solves a problem $P_c = \langle P, C \rangle$, and then transform this verification procedure into a synthesis procedure.

The solutions for FOND$^+$ problems are policies that **terminate in the goal**, a termination condition that combines and goes beyond the solution concept for QNPs that only requires goal reachability (strong-cyclicity) and termination (finite trajectories). The termination condition for FOND$^+$ planning can be expressed as follows:

**Definition 15 (FOND$^+$ termination).** Let $\pi$ be a policy for the FOND$^+$ problem $P_c = \langle P, C \rangle$. State $s$ in $P$ terminates iff

1. $s$ is a goal state,
2. $s$ is fair and some state $s' \in F(\pi(s), s)$ terminates, or
3. $s$ is not fair, all states $s' \in F(\pi(s), s)$ terminate, and $F(\pi(s), s)$ is non-empty.

where $s$ is fair if for some $A_i/B_i$ in $C$, $\pi(s) \in A_i$, and every path that connects $s$ to itself and that contains a state $s'$ with $\pi(s') \in B_i$, also contains a state $s''$ that terminates.

FOND$^+$ termination expresses a procedure similar to SIEVE, that we call SIEVE$^+$, that keeps labeling states $s$ as terminating (the same as removing all edges from $s$ in the policy graph) until no states are left or no more states can be labeled. The key difference with SIEVE is that the removals are done backward from the goal as captured in Definition 15. This is strictly necessary for SIEVE$^+$ to be a sound and complete procedure for FOND$^+$ problems.

**Theorem 16.** A policy $\pi$ solves the FOND$^+$ problem $P_c = \langle P, C \rangle$ iff all the states $s$ that are reachable by $\pi$ terminate according to Definition 15.

The solutions to FOND$^+$ problems cannot be characterized as those of QNPs, as policies that are strong-cyclic and terminating in the sense that the policy cannot traverse edges in the policy graph forever. The policy $\pi$ for the example $P_3$ is indeed strong-cyclic and terminating in this sense, but as shown above, it does not solve $P_3$. The policy terminates because the action $a$ cannot be done forever, but it does not terminate in a goal state. In QNPs, this cannot happen, as strong-cyclic policies that are terminating, always terminate in a goal state.

**Proof.** Let $S_0, S_1, \ldots$ be the chain of state subsets labeled as terminating corresponding to Definition 15 (cf. footnote 1). For a state subset $R$, $\pi(R)$ denotes the set $\{\pi(s) : s \in R\}$.

**Backward implication.** For a proof by contradiction, let us assume that every reachable state terminates, and let us suppose that $\pi$ does not solve $P_c$; i.e., there is a maximal and fair non-goal reaching $\pi$-trajectory $\tau$. We consider two cases.

**Case 1:** $\tau = s_0, s_1, \ldots, s_n$ is finite. Since $\tau$ is maximal and $s_n$ is not a goal, $F(\pi(s_n), s_n)$ is empty. Therefore, $s_n$ cannot terminate contradicting the assumption.

**Case 2:** $\tau$ is infinite. Let $R$ be the set of recurrent states in $\tau$, let $j$ be the minimum index such that $R \cap S_j \neq \emptyset$, and let $s$ be a state in $R \cap S_j$. Clearly, $j > 0$ since $S_0$ is the set of goal states and $\tau$ does not reach such states. We further consider two subcases:

- $s$ is not fair according to Definition 15. Then, every state $s'$ in $F(\pi(s), s)$ must belong to some $S_k$ for $k < j$. This is impossible by the choice of $j$.
- $s$ is fair according to Definition 15. That is, for some constraint $A_i/B_i$, $\pi(s) \in A_i$, and if $\pi(R) \cap B_i = \emptyset, R$ must contain a state $s'' \in S_k$ for $k < j$. Since the latter is impossible by the choice of $j$, $\pi(R) \cap B_i = \emptyset$. This implies that the occurrence of the action $\pi(s)$ in the trajectory $\tau$ (cf. Definition 9), and thus that $s$ is followed in $\tau$ by each of its possible successors $s' \in F(\pi(s), s)$ infinitely often; i.e., $\pi(s), s' \subseteq R$. However, by Definition 15, some such successor $s'$ must terminate (i.e., $s' \in S_k$ for $k < j$), something that is impossible by the choice of $j$.

**Forward implication.** For a proof by contradiction, let us assume that $\pi$ solves $P_c$, and let us suppose that there is a reachable state $s$ that does not terminate. We are going to construct an infinite $\pi$-trajectory $\tau$ that is fair and does not reach a goal, contradicting the assumption.

First observe that $s$ is not a goal state and thus it must have successor states. If $s$ is fair (cf. Definition 15), every state $s' \in F(\pi(s), s)$ does not terminate, whereas if $s$ is not fair, some state $s' \in F(\pi(s), s)$ does not terminate. It is then easy to see that we can construct an infinite $\pi$-trajectory $\tau'$ seeded at $s$ such that for the set $R$ of its recurrent states:
1. no state in \( R \) terminates,
2. if \( s' \) in \( R \) is fair, it is followed infinitely often by each \( s'' \in F(\pi(s'), s') \), and
3. if \( s' \) in \( R \) is not fair, it is followed infinitely often by each \( s'' \in F(\pi(s'), s') \) that does not terminate, but there is \( s'' \in F(\pi(s'), s') \) that does not follow \( s'' \) infinitely often.

Since the state \( s \) is reachable by \( \pi \), we can construct a \( \pi \)-trajectory \( \tau \) that reaches \( s \) from the initial state \( s_0 \) of \( P \) and that then follows \( \tau' \). The set of recurrent states for \( \tau \) is the set \( R \) of recurrent state for \( \tau' \); in particular, \( R \) does not contain a goal state. We finish by showing that \( \tau \) is fair.

We do so using Definitions 9 and 10 for fair actions and fair trajectories respectively. Let \( s' \) be a state in \( R \) such that the action \( \pi(s') \) is fair in \( \tau \); in particular, \( \pi(s') \in A_i \) for some index \( i \). If the state \( s' \) is fair (cf. Definition 15), \( F(\pi(s'), s) \subseteq R \) by construction of \( \tau \). Else, if \( s' \) is not fair, we show below that the occurrence of the action \( \pi(s') \) is not fair in \( \tau \). Hence, in both cases, the trajectory \( \tau \) is fair.

For the last bit, by Definition 15, if \( s' \) is not fair, there is a cycle \( \tau'' \) that a) passes over \( s' \), b) contains a state \( s'' \) with \( \pi(s'') \in B_i \), and c) does not contain a terminating state. Hence, by construction of \( \tau \), \( R \) contains all the states in \( \tau'' \). However, since there is some state in \( F(\pi(s'), s') \) that is not in \( R \), the occurrence of \( \pi(s') \) in \( \tau \) is not fair.

**FOND+ and Dual FOND Planning**

FOND+ planning subsumes Dual FOND planning where fair and adversarial actions can be combined. In order to show that, let us first recall the latter:

**Definition 17** (Geffner and Geffner, 2018). A Dual FOND problem is a FOND problem where the non-deterministic actions are labeled as either fair or adversarial. A policy \( \pi \) solves a Dual FOND problem \( P \) iff for all reachable state \( s \), \( \langle \pi(s), s \rangle \in A(s) \), and there is a function \( d \) from reachable states into \( \{0, \ldots, |S|\} \) such that 1) \( d(s)=0 \) for goal states, 2) \( d(s')<d(s) \) for some \( s' \in F(\pi(s), s) \) if \( \pi(s) \) is fair, and 3) \( d(s')<d(s) \) for all \( s' \in F(\pi(s), s) \) if \( \pi(s) \) is adversarial.

For showing that this semantics coincides with the semantics of a suitable fragment of FOND+ planning, let us recast this definition as a termination procedure:

**Definition 18** (Dual FOND termination). Let \( \pi \) be a policy for the Dual FOND problem \( P \). A state \( s \) in \( P \) terminates iff
1. \( s \) is a goal state,
2. \( \pi(s) \) is fair and some \( s' \in F(\pi(s), s) \) terminates, or
3. \( \pi(s) \) is adversarial, all states \( s' \in F(\pi(s), s) \) terminate, and \( F(\pi(s), s) \) is non-empty.

**Theorem 19.** \( \pi \) is a solution to a Dual FOND problem \( P \) iff for every non-goal state \( s \) reachable by \( \pi \), \( \pi(s) \in A(s) \) and \( s \) terminates according to Definition 18.

**Proof.** If \( \pi \) is a solution for \( P \), there is a minimal function \( d \) that satisfies Definition 17, and that can be used to construct a chain \( S_0, S_1, \ldots \) of state subsets by \( S_i = \{ s : s \text{ is reachable and } d(s) = i \} \). It is not difficult to check that such a chain corresponds to the unique chain of subsets entailed by Definition 18 (cf. footnote 1).

On the other hand, for a policy \( \pi \) that entails such a chain \( S_0, S_1, \ldots \) of terminating states that cover all reachable states, the function \( d \) on states, defined by \( d(s) = i \) for the minimum \( i \) such that \( s \in S_i \), satisfies Definition 17.

The only difference between the termination for Dual FOND and the one for FOND+ (Def. 15) is that in the former the fair and adversarial labels are given, while in the latter they are a function of the explicit fairness assumptions and policy. It is easy to show however that Dual FOND problems correspond to the class of FOND+ problems with conditional fairness assumptions \( A/B \) with empty \( B \):

**Theorem 20.** A policy \( \pi \) solves a Dual FOND problem \( P' \) iff \( \pi \) solves the FOND+ problem \( P_c = (P, C) \) where \( P \) is like \( P' \) without the action labels, and \( C = \{ A/B \} \) where \( A \) contains all the actions labeled as fair in \( P' \), and \( B \) is empty.

**Proof.** By Theorems 19 and 16, it is enough to show that for every \( \pi \)-reachable and non-goal state \( s \) in \( P' \), \( s \) terminates according to Definition 18 iff \( s \) terminates according to Definition 15. Yet, this is direct since for the unique constraint \( A/B \) in \( C \), it is easy to see that a state \( s \) is fair according to Definition 15 iff \( \pi(s) \) is a fair action in \( P' \). With this observation, Definition 18 becomes exactly Definition 15.

**FOND-ASP: An ASP-based FOND+ Planner**

The characterization of FOND+ planning given in Theorem 16 allows for a transparent and direct implementation of a sound and complete FOND+ planner. For this, the planner hints a policy \( \pi \) and then each state reachable by \( \pi \) is checked for termination using Definition 15. The problem of looking for a policy that satisfies this restriction can be expressed in SAT, although we have found it more convenient to express it as an answer set program, a convenient and high-level alternative to SAT (Brewka, Eiter, and Truszczyński 2011; Lifschitz 2019; Gebser et al. 2012), using the facilities provided by CLINGO (Gebser et al. 2019).

The code for the back-end of the ASP-based FOND+ planner is shown in Figure 2. The front-end of the planner, not shown, parses an input problem \( P_c = (P, C) \) and builds a flat representation of \( P_c \) in terms of a number of ground atoms that are shown in capitalized predicates in the figure. The code in the figure and the facts representing the problem are fed to the ASP solver CLINGO, which either returns a (stable) model for the program or reports that no such model exists. In the former case, a policy that solves \( P_c \) is obtained from the atoms \( \pi(s,A) \) made true by the model.

The set of ground atoms providing a flat representation of the problem \( P_c \) contains the atoms \( \text{STATE}(s), \text{ACTION}(s), \) and \( \text{TRANSITION}(s,a,s') \) for each (reachable) state \( s \), ground action \( a \) and transition \( s' \in F(a, s) \) found in a reachability analysis from the initial state \( s_0 \). In addition, the set includes the atoms \( \text{INITIAL}(s_0), \text{GOAL}(s) \) for goal states \( s \), and \( \text{ASET}(i,a) \) and \( \text{BSET}(i,b) \) for a fairness assumption \( A_i/B_i \) in \( C \) if \( a \in A_i \) and \( b \in B_i \) respectively.

The program for the FOND+ problem \( P_c \) is denoted as \( T(P_c) \), while \( T(P_c, \pi) \) is used to refer to the program \( T(P_c) \) but with the line 2 in Figure 2 replaced by facts \( \pi(s,a) \).
when \( \pi(s) = a \) for a given policy \( \pi \), and the integrity constraint in line 23 removed. A model \( M \) for \( T(P_c) \) encodes a policy \( \pi_M \) where \( \pi_M(s) = a \) iff \( \pi_1(s, a) \) holds in \( M \). The formal properties of the FOND-ASP planner are as follows:

**Theorem 21.** Let \( P_c = \langle P, C \rangle \) be a FOND+ problem, and let \( \pi \) be a policy for \( P \). Then,

1. There is a unique stable model \( M \) of \( T(P_c, \pi) \), and \( \text{terminate}(s) \in M \) iff \( s \) terminates (Definition 15).
2. The policy \( \pi \) solves \( P_c \) iff the model \( M \) for \( T(P_c, \pi) \) satisfies the integrity constraint in line 23 in Figure 2.
3. \( M \) is a model of \( T(P_c) \) iff \( M \) is the model of \( T(P_c, \pi_M) \) and \( M \) satisfies the integrity constraints. Thus, FOND-ASP is a sound and complete planner for FOND+.
4. Deciding if \( T(P_c, \pi) \) has a model is in P; i.e., FOND-ASP runs in non-deterministic exponential time.

**Proof.** (Sketch.) For 1. A model \( M \) is stable iff it can be constructed in a “bottom-up manner”. This can be done by constructing a sequence of atom sets \( M_0, M_1, \ldots \) that define the atoms connected/2, blocked/2, fair/1, terminate/1, and reachable. Using induction, it can be shown that any such sequence of atom sets lead to the same stable model \( M \). This shows uniqueness. The existence of \( M \) follows since for given \( \pi \), there is always a stable model \( M \) for \( T(P_c, \pi) \). For 2. By Theorem 16, \( \pi \) solves \( P_c \) iff \( \pi \) is the unique model of \( T(P_c, \pi) \) and \( M \) satisfies the integrity constraint. For 3. Straightforward by 2. For 4. A model for \( T(P_c, \pi) \) can be constructed by calculating the terminating states for the policy graph for \( \pi \). This can be done in time that is polynomial in the size of the policy graph which lower bounds the size of \( T(P_c) \). FOND-ASP builds \( T(P_c) \) in exponential time, then guesses a policy \( \pi \) is linear time in the size of \( T(P_c) \), and finally checks if \( T(P_c, \pi) \) has a model in time that is polynomial in the size of \( T(P_c) \).

**Complexity**

A direct consequence of Theorem 21 is that the plan-existence decision problem for FOND+ is in NEXP (i.e., non-deterministic exponential time). Since FOND problems are easily reduced to FOND+ problems (Theorem 13) and the plan-existence for FOND is EXP-Hard (Littman, Goldsmith, and Mundhenk 1998; Rintanen 2004), plan-existence for FOND+ is EXP-Hard as well. We conjecture that the NEXP bound is loose and that plan-existence for FOND+ is EXP-Complete. In contrast, LTL planning and synthesis is 2EXP-Complete (Pnueli and Rosner 1989).

**Theorem 22.** The plan-existence problem for FOND+ problems is in NEXP and it is EXP-Hard.

**Proof.** **Inclusion.** On input problem \( P_c = \langle P, C \rangle \), the state space for \( P \) is constructed explicitly and a policy \( \pi \) for \( P_c \) is guessed in non-deterministic exponential time. Then, by Theorem 21, the logic program \( T(P_c, \pi) \) has a model iff \( \pi \) solves \( P_c \). Deciding whether \( T(P_c, \pi) \) has a model can be done in time polynomial in the size of the program.

**Hardness.** By Theorem 13, A FOND problem can be reduced in polynomial time to a FOND+ problem. The claim follows by the EXP-Hardness of the plan-existence problem for FOND.

**Experiments**

We tested FOND-ASP on three classes of problems: FOND problems, QNPs, and more expressive FOND+ problems that do not fit in either class and that can only be addressed using LTL engines. On each class, we compare FOND-ASP with the FOND solvers FOND-SAT (Geffner and Geffner 2018) and PRP (Muise, McIlraith, and Beck 2012), the QNP solver QNP2FOND (Bonet and Geffner 2020) using FOND-SAT and PRP as the underlying FOND solver, and the LTL-synthesis tool STRIX (Luttenberger, Meyer, and Sickert 2020). The pure (strong and strong-cyclic) FOND problems are those in the FOND-SAT distribution, the QNPs are those by (Bonet and Geffner 2020) and two new families of instances that grow in size with a parameter. For more expressive FOND+ planning problems, four new families of problems are introduced that extend the new QNPs with fair and adversarial actions, with only some being solvable. The domain and goals of these problems are encoded in LTL in the usual way, while the fairness assumptions \( A/B \) are encoded as described in the introduction. In all the experiments, time and memory bounds of 1/2 hour and 8GB are enforced.

The results are detailed below. In summary, we observe the following. For pure FOND benchmarks, FOND-ASP does not compete with specialized planners like PRP or FOND-SAT as these problems span (reachable) state spaces that are just too large. For QNPs, on the other hand, FOND-ASP does better than FOND-SAT but worse than PRP on the FOND translations. For expressive FOND+ problems, where these planners cannot be used at all, FOND-ASP performs much better than STRIX on both solvable and unsolvable problems.

**FOND Benchmarks**

FOND-ASP managed to solve a tiny fraction of the benchmarks used for strong and strong-cyclic planning in the FOND-SAT distribution. The number of reachable states in these problems is large (tens of thousands or more) and the size of the grounded ASP program is quadratic in that number. In general, this seems to limit the scope of FOND-ASP to problems with no more than one thousand states approximately, as suggested by the results in Table 1. We have observed however that sometimes FOND-ASP manages to solve strong planning problems with more than 100,000 states. This may have to do with CLINGO’s grounder or with the state space topology; we do not know the exact reason yet.

**QNPs**

The two families of QNPs involve the numerical variables \( \{x_i\}_{i=1}^n \) that have all positive values in the initial state. The goal is to achieve \( x_n = 0 \). Problems in the QNP1 family are solved by means of \( n \) sequential simple loops, while problems in the QNP2 family are solved using \( n \) nested loops. The actions for problems in QNP1 are \( b = (\neg p; p) \), \( a_1 = (p; \neg p, x_1) \), and \( a_i = (p, x_{i-1} = 0; \neg p, x_i) \) for \( 1 < i < n \),

\[\text{Planner available at https://github.com/idrave/aspplaner}\]
while those for QNP2 are \( b = \langle -p; p \rangle, a_1 = \langle p; -p, x_{i1} \rangle \), and \( a_i = \langle p, x_{i-1} = 0; -p, x_{i-1}, x_{i1} \rangle, 1 < i \leq n \).

Table 1 shows the results for values of \( n \) in \{2, 3, …, 10\} and different planners, along with the number of reachable states in each problem. As can be seen, QNP2FOND/PRP is the planner that scales best, followed by FOND-ASP, QNP2FOND/FOND-SAT, and STRIX at the end. As mentioned, the performance of FOND-ASP is harmed by a large number of reachable states. While the number of states for the FOND translation produced by QNP2FOND is much larger, as the translation involves extra propositions, this number does not necessarily affect the performance of FOND planners like FOND-SAT and PRP that can compute compact policies. It is also interesting to see how quickly the performance of the LTL engine STRIX degrades; it cannot even solve qnp1-06 which has 14 states. The table also shows results for QNP problems that capture abstractions for four generalized planning problems, all of which involve small state spaces (Bonet and Geffner 2020).

More Expressive FOND\(^+\) Problems

The third class of instances consists of four families of problems obtained from the two QNP families above. The new problems are not “pure” QNPs, as they also involve actions with non-deterministic effects on boolean variables that can be adversarial or fair. Thus, these problems cannot be translated into FOND problems for the use of planners such as PRP or FOND-SAT. For each family QNP1 and QNP2, two new families \( f01 \) and \( f11 \) of problems are obtained by re-placing the action \( b = \langle -p; p \rangle \) by the non-deterministic action \( b' = \langle -p; oneof \{ p, -p \} \rangle \), leaving the actions \( a_i \) untouched. Since the action \( b' \) does not appear in any fairness assumption, it is adversarial and thus no problem in the class \( f01 \) has a solution as the “adversary” may always choose to leave \( p \) false. The family \( f11 \) is obtained on top of \( f01 \) by adding two additional booleans \( q \) and \( r \), and two actions \( c = \langle q; r, oneof \{ q, -q \} \rangle \) and \( d = \langle r, q, -r \rangle \) such that: 1) the actions \( a_i \) are modified by adding \( q \) as precondition and \( -q \) as effect, and 2) the fairness assumption \( A/B \) with \( A = \{ b' \} \) and \( B \) empty is added. The problems in \( f11 \) thus involve the QNP-like actions \( a_i \), the fair action \( b' \), and the adversarial action \( c \), and they all have a solution.

Table 2 shows the result for FOND-ASP and STRIX for as these are the only solvers able to handle the combination of fairness assumptions. As can be seen, FOND-ASP scales better than STRIX on all of these problems, the solvable ones (families \( f11 \)) and the unsolvable ones (families \( f01 \)).

We finally tested FOND-ASP over the seven problems considered in a recent approach to program synthesis over bounded data structures (Bonet et al. 2020). Although the original specifications are in LTL, these can be all expressed in FOND\(^+\) using different types of fairness assumptions. The problems are solved easily by both FOND-ASP and STRIX as their reachable state spaces have very few states.

Related Work

The work is related to three threads: SAT-based FOND planning, QNPs, and LTL synthesis. The SAT-based FOND plan-
more efficient techniques. While the strong fairness assumption connects also to works proposing an extension of FOND planning (Geffner and Geffner 2018). We have formulated an extension of FOND planning that makes use of explicit fairness assumptions of the form $A/B$ where $A$ and $B$ are disjoint sets of actions.

The use of fairness assumptions connects also to works on LTL planning and synthesis (Camacho, Bienvenu, and McIlraith 2019; Aminof et al. 2019), and to works addressing temporally extended goals (De Giacomo and Vardi 1999; Patrizi, Lipovetzky, and Geffner 2013; Camacho et al. 2017; Camacho, Bienvenu, and McIlraith 2019; Aminof, De Giacomo, and Rubin 2020). Our work can be seen as a special case of planning under LTL assumptions (Aminof et al. 2019) that targets an LTL fragment that is relevant for FOND planning and is computationally simpler. While it is possible to express FOND$^+$ tasks as LTL synthesis problems, and we have shown how to do that, it remains to be seen whether the task can be expressed in a restricted LTL fragment that admits more efficient techniques. While the strong fairness assumption on action effects that is required cannot be directly encoded in GR(1) (Bloem et al. 2012), strong-cyclic FOND planning has been encoded in Büchi Games (D’Ippolito, Rodríguez, and Sardiña 2018), a special case of GR(1). It remains to be investigated whether that encoding can be extended to deal with conditional fairness.

Summary

We have formulated an extension of FOND planning that makes use of explicit fairness assumptions of the form $A/B$ where $A$ and $B$ are disjoint sets of actions. While in Dual FOND planning actions are labeled as fair or unfair, in FOND$^+$ planning these labels are a function of the trajectories and the fairness assumptions: an action $a \in A$ is deemed fair in a recurrent state if a suitable condition on $B$ holds. In this way, FOND$^+$ generalizes strong, strong-cyclic, Dual FOND planning, and also QNP planning, which is actually the only planning setting, excluding LTL planning, that makes use of the conditions $B$. We have implemented an effective FOND$^+$ planner by reducing the problem to answer set programs using CLINGO, and evaluated its performance in relation to FOND and QNP planners, which handle less expressive problems, and LTL synthesis tools, which handle more expressive ones. We have shown that FOND$^+$ is in NEXP but have not shown yet whether it is in EXP, like FOND and QNP planning.

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| problem | FOND-SAT | PRP | STRIX | FOND-ASP |
|---------|----------|-----|-------|----------|
| qnp1-02 | 6        | 0.08 | 0.09  | 3.35     | 0.00     |
| qnp1-03 | 8        | 0.13 | 0.11  | 2.63     | 0.00     |
| qnp1-04 | 10       | 0.28 | 0.14  | 5.21     | 0.00     |
| qnp1-05 | 12       | 0.60 | 0.14  | 98.34    | 0.01     |
| qnp1-06 | 14       | 1.27 | 0.15  | —        | 0.01     |
| qnp1-07 | 16       | 2.54 | 0.15  | —        | 0.01     |
| qnp1-08 | 18       | 5.54 | 0.17  | —        | 0.01     |
| qnp1-09 | 20       | 12.96| 0.21  | —        | 0.02     |
| qnp1-10 | 22       | 26.70| 0.19  | —        | 0.02     |
| qnp2-02 | 8        | 0.20 | 0.18  | 2.33     | 0.00     |
| qnp2-03 | 16       | 1.77 | 0.30  | 2.31     | 0.01     |
| qnp2-04 | 32       | 10.00| 0.58  | 14.25    | 0.04     |
| qnp2-05 | 64       | 50.24| 1.15  | 885.37   | 0.20     |
| qnp2-06 | 128      | 302.80| 2.53  | —        | 1.26     |
| qnp2-07 | 256      | 1,969.35| 4.02  | —        | 7.14     |
| qnp2-08 | 512      | —   | 6.96  | —        | 54.37    |
| qnp2-09 | 1,024    | —   | 13.22 | —        | ***      |
| qnp2-10 | 2,048    | —   | 21.94 | —        | ***      |

Table 1: Results for three families of QNPs for QNP2FOND paired with the FOND solvers FOND-SAT and PRP, STRIX (QNP translated to LTL), and FOND-ASP. Entries ‘—’ and ‘***’ denote solver runs out of time and memory, respectively. Time is in seconds.

| problem | STRIX | FOND-SAT | PRP | STRIX | FOND-ASP |
|---------|-------|----------|-----|-------|----------|
| qnp1-xx-02 | 6    | 3.44     | 0.00 | 24 | 6.07     | 0.03     |
| qnp1-xx-03 | 8    | 2.42     | 0.01 | 32 | 6.20     | 0.04     |
| qnp1-xx-04 | 10   | 4.13     | 0.01 | 40 | 98.58    | 0.07     |
| qnp1-xx-05 | 12   | 93.85    | 0.01 | 48 | —        | 0.11     |
| qnp1-xx-06 | 14   | —        | 0.01 | 56 | —        | 0.14     |
| qnp1-xx-07 | 16   | —        | 0.01 | 64 | —        | 0.19     |
| qnp1-xx-08 | 18   | —        | 0.01 | 72 | —        | 0.25     |
| qnp1-xx-09 | 20   | —        | 0.02 | 80 | —        | 0.39     |
| qnp1-xx-10 | 22   | —        | 0.02 | 88 | —        | 0.34     |
| qnp2-xx-02 | 8    | 3.22     | 0.00 | 32 | 5.85     | 0.04     |
| qnp2-xx-03 | 16   | 2.25     | 0.01 | 64 | 8.16     | 0.21     |
| qnp2-xx-04 | 32   | 11.38    | 0.04 | 128| 236.89   | 1.55     |
| qnp2-xx-05 | 64   | 873.09   | 0.21 | 256| —        | 15.45    |
| qnp2-xx-06 | 128  | —        | 1.25 | 512| —        | 46.67    |
| qnp2-xx-07 | 256  | —        | 12.13|1,024|—        |***       |
| qnp2-xx-08 | 512  | —        | 39.56|2,048|—        |***       |
| qnp2-xx-09 | 1,024| —        | ***  | 4,096| —       |***       |
| qnp2-xx-10 | 2,048| —        | ***  | 8,192| —       |***       |

Table 2: Results for four families of FOND$^+$ problems obtained from the QNPs in Table 1 by playing with the fairness assumptions, some solvable, some unsolvable. These problems are handled only by STRIX and FOND-ASP. Entries ‘—’ and ‘***’ denote solver runs out of time and memory, respectively. Time is in seconds.
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