Efficient implicit schemes for the treatment of the contact between deformable bodies: Application to shock-absorber devices.

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(Received 01 June 1998; and in revised form 25 February 2000)

Abstract - Classical and non-classical implicit time integration schemes are presented for the transient response of highly nonlinear problems of large deformation analysis exhibiting important material energy dissipation. Surprisingly these schemes lead to excellent convergence properties that make them cost efficient alternative to explicit schemes generally advocated as the best choice for these problems. Numerical illustrations presented herein show the crushing of a shock-absorber device, as well as the resolution of the dynamical buckling of a thin-walled steel cylinder. In these examples, it is shown that implicit schemes are more efficient and require less CPU-cost for a given precision of the contact situation and higher precision reachable than the explicit scheme. Both of which are using the Penalty method for the treatment of the contact problem.

NOTATION

- $x$ vector of nodal positions
- $\dot{x}$ vector of nodal velocities
- $\ddot{x}$ vector of nodal accelerations
- $M$ mass matrix
- $F^{\text{int}}$ vector of internal forces
- $F^{\text{ext}}$ vector of external forces
- $R$ residual (out-of-balance) vector
- $\sigma$ Cauchy stress tensor
- $b$ body force per unit mass
- $f$ surface forces
- $\rho$ current mass density of the material
- $\rho_0$ initial mass density of the material
- $N$ matrix of FEM shape functions
- $B$ matrix of the derivatives of the FEM shape functions
- $\gamma_s$ safety parameter in $[0,1]$ allowing for non-linearities
- $L_e$ geometric factor depending on the space discretization ($e$ is the element number)
- $C_p$ pressure wave velocity in the material
- $\beta_1$ first Newmark parameter
- $\gamma$ second Newmark parameter
- $\alpha_1$ first free parameter balancing sampling time around $[t_n, t_{n+1}]$ for averaging inertia terms
- $\alpha_f$ second free parameter balancing sampling time around $[t_n, t_{n+1}]$ for averaging forces
- $K_T$ tangent stiffness matrix
- $C_T$ tangent damping matrix
- $s_n$ normal penetration
- $e_n$ Penalty coefficient for penetration
- $e_f$ Penalty coefficient for friction
- $\mu$ friction coefficient
- $S(t)$ current surface of the body
- $V(t)$ current volume of the body
- $S_0$ initial surface of the body
- $V_0$ initial volume of the body
- $\Delta t$ time increment: $\Delta t = t_{n+1} - t_n$
- $\tau$ time

INTRODUCTION

When dealing with dynamics for highly nonlinear problems, for instance those with coupled material and geometrical non-linearities as encountered in crash analysis or in metal forming processes, one is often faced with the choice of a time-stepping algorithm: explicit or implicit. For those problems, the current trend in Computational Mechanics is as follows [1]: for dynamics where low frequencies dominate the response, implicit algorithms of the Newmark family (i.e. Newmark; Hilber, Hughes
Taylor; ...) are generally felt to be more appropriate. On the contrary, wave propagation problems such as shock response from impact or explosion are commonly supposed to be best solved using explicit techniques that advocate lower CPU-cost.

However these assertions are mainly based on linear analysis. For example, stability and/or accuracy properties are determined from a linearized problem in a spatial neighbourhood of a given configuration and for vanishingly small time steps. If the properties of the time-stepping algorithms predicted by this technique are reliable for explicit problems where time steps are effectively very small, it's not the case for implicit algorithms where the time steps can be much higher and the assertions predicted can be completely false for general nonlinear problems even if they are true for linear problems.

NUMERICAL INTEGRATION OF TRANSIENT PROBLEMS

Equations of motion

FEM (space) semi-discretization of the equations of motion of a nonlinear structure leads to the coupled set of second order nonlinear differential equations [1 - 5]:

\[ R = M \ddot{x} + F^{\text{int}}(x, \dot{x}) - F^{\text{ext}}(x, \dot{x}) = 0 \]  

This set is completed by two sets of given initial conditions, for both

\[ x_0 = x(t_0) \quad \text{and} \quad \dot{x}_0 = \dot{x}(t_0) \]  

Internal and external forces expressions can be written:

\[ F^{\text{int}}(x, \dot{x}) = \int_{V(t)} [B]^T \{ \sigma \} dV \]  

\[ F^{\text{ext}}(x, \dot{x}) = \int_{V(t)} \rho [N]^T \{ b \} dV + \int_{S(t)} \{ f \} dS \]  

Note that expression in equation (4) collects all types of loading (applied through local or distributed actions, in a follow-up way or not, reactions to imposed displacements and contact situations) and that the consistent mass matrix reads

\[ M = \int_{V(t)} \rho [N]^T [N] dV = \int_{V_0} \rho_0 [N]^T [N] dV_0 \]  

Explicit scheme

This is the most advocated scheme [1,2] for integrating equation (1) in case of wave propagation and impact problems, i.e. high speed dynamics. It reads for a time step \( \Delta t = t_{n+1} - t_n \):

\[ \dot{x}_{n+\frac{1}{2}} = \frac{\dot{x}_n + \dot{x}_{n+1}}{2} + \Delta t \ddot{x}_n \]  

\[ x_{n+1} = x_n + \Delta t \dot{x}_{n+\frac{1}{2}} \]  

\[ \ddot{x}_{n+1} = M^{-1} \left( F_{n+1}^{\text{ext}} - F_{n+1}^{\text{int}} \right) \]
and is subject to unstable behaviour unless [1] time steps are below a critical value (equation (9)) where the minimum is searched for all elements of the mesh.

\[
\Delta t_{\text{crit}} = \gamma \min_{\text{for all } \varepsilon} \left( \frac{L^*}{C^p_\varepsilon} \right)
\]

Implicit schemes: The generalized-\(\alpha\) trapezoidal scheme

Implicit schemes are classically designed for vibrations and low speed dynamics of structures. The most general scheme for implicit integration of equation (1) is a generalized trapezoidal scheme [6] where updating of positions and velocities are based on “averaged” accelerations stemming from associated values between \(t_n\) and \(t_{n+1}\). It reads for instance

\[
\dot{x}_{n+1} = \dot{x}_n + (1 - \gamma) \Delta t \ddot{x}_n + \gamma \Delta t \ddot{x}_{n+1}
\]

(10)

\[
x_{n+1} = x_n + \Delta t \dot{x}_n + \left( \frac{1}{2} - \beta \right) \Delta t^2 \ddot{x}_n + \beta \Delta t^2 \ddot{x}_{n+1}
\]

(11)

or equivalently

\[
\ddot{x}_{n+1} = \frac{1}{\beta \Delta t^2} \left[ x_{n+1} - x_n - \Delta t \dot{x}_n - \left( \frac{1}{2} - \beta \right) \Delta t^2 \ddot{x}_n \right]
\]

(12)

\[
\dot{x}_{n+1} = \frac{\gamma}{\beta \Delta t} \left[ x_{n+1} - x_n + \left( \frac{\beta}{\gamma} - 1 \right) \Delta t \dot{x}_n + \left( \frac{\beta}{\gamma} - \frac{1}{2} \right) \Delta t^2 \ddot{x}_n \right]
\]

(13)

The discretized motion equation (1) becomes:

\[
R_{n,n+1} = \frac{1 - \alpha_M}{1 - \alpha_F} M \ddot{x}_{n+1} + \frac{\alpha_M}{1 - \alpha_F} M \ddot{x}_n + \frac{1}{1 - \alpha_F} \left( F_{n+1}^{\text{int}} - F_{n+1}^{\text{ext}} \right) + \frac{\alpha_F}{1 - \alpha_F} \left( F_n^{\text{int}} - F_n^{\text{ext}} \right) = 0
\]

(14)

This general form was introduced, for linear problems, by Chung and Hulbert [6]. Particular choice of the parameters leads to well known \([1,2,6]\) schemes such as

- \(\alpha_M = \alpha_F = 0\) for Newmark scheme
- \(\alpha_M = 0\) for Hilber-Hughes-Taylor scheme
- \(\alpha_F = 0\) for Wood-Bossak-Zienkiewicz scheme

Iterative solution of the nonlinear system in equation (14) requires first the elimination of acceleration and velocity at time \(t_{n+1}\) with the help of equations (12 & 13) and, secondly, the writing of the Hessian matrix of the system, i.e.

\[
S = \frac{1}{\beta \Delta t^2} \left( \frac{1 - \alpha_M}{1 - \alpha_F} \right) M + \frac{\gamma}{\beta \Delta t} C_\gamma + K_\gamma
\]

(15)
where $K_T, C_T$ are respectively the tangent stiffness and damping matrices defined by

$$K_T = \frac{\partial}{\partial x} (F^{\text{int}} - F^{\text{ext}})$$  \[16\]

$$C_T = \frac{\partial}{\partial x} (F^{\text{int}} - F^{\text{ext}})$$  \[17\]

Unconditional stability and second order accuracy of the scheme for linear problems \([1,2,6]\) require that

$$\gamma \geq 1/2 - \alpha_M + \alpha_F$$  \[18\]

$$\alpha_M \leq \alpha_F \leq 1/2$$

$$\beta \geq 1/4 + (\alpha_F - \alpha_M)/2$$

Associated rules for Newmark scheme are

$$\gamma \geq 1/2$$  \[19\]

$$\beta \geq \frac{1}{4} (y + 1/2)^2$$

It is worth pointing out that, though classical schemes require $0 < \alpha_F < 1/2$ (i.e. sampling the force in the second half of $[t_n, t_{n+1}]$), here no such rule is followed for $\alpha_M$, the sampling parameter for the inertia terms: it might be negative for instance, thus leading to an interpolation at $t_{n+1}$ instead of an extrapolation.

**Implicit schemes: The generalized-\(\theta\) mid-point scheme (GMP)**

An alternative to the preceding is a generalized midpoint scheme with constant acceleration over the time step \([3,5]\). In this case, the equation of motion (1) is solved at the sampling time:

$$t_{n+\theta} = t_n + \theta(t_{n+1} - t_n)$$

with $\theta > 0$, i.e.

$$R_\theta = M \ddot{x}_{n+\theta} + F^{\text{int}}(x_{n+\theta}, \dot{x}_{n+\theta}) - F^{\text{ext}}(x_{n+\theta}, \dot{x}_{n+\theta}) = 0$$  \[20\]

where

$$\ddot{x}_{n+\theta} = \frac{2}{(\theta \Delta t)^2} [x_{n+\theta} - x_n - \theta \Delta t \dot{x}_n]$$  \[21\]

$$\dot{x}_{n+\theta} = \frac{2}{\theta \Delta t} [x_{n+\theta} - x_n - \frac{\theta \Delta t}{2} \dot{x}_n]$$  \[22\]

Iterative solution of the nonlinear system, equation (20), requires the evaluation of the Hessian matrix of the system that is
Let the $\theta$-GMP scheme be stressed with respect to $\alpha$-family:

- all forces, even contact ones, are exactly estimated at time $t_{n+1}$ instead of being averaged between the values at $t_n$ and $t_{n+1}$ as in equation (14).

- the present scheme is $\alpha_n$-independent, yielding thus the final acceleration, as a post-treatment result given by equation (21)

$$\ddot{x}_{n+1} = \frac{\ddot{x}_{n+1} - \ddot{x}_n}{\Delta t} = \ddot{x}_{n+1}$$  \hspace{1cm} [24]

- since, for $\theta = 1$, it corresponds to the Newmark scheme with $\gamma = 1$ and $\beta = 0.5$, and can be stated that it is conditionally stable and exhibits strong numerical damping. However, this scheme has proved to be very efficient provided $\theta$ is taken bigger than unity, i.e. choosing a sampling point out of the range $[t_n, t_{n+1}]$, thus producing some backward evaluation of the final status of the system at the end of the time step, with respect to the sampling point for residual evaluation.

**EXPLICIT VS IMPLICIT SCHEME**

Relative merits of explicit and implicit schemes have long been advocated in a dichotomic way [1]:

| Feature             | Explicit | Implicit                  |
|---------------------|----------|---------------------------|
| Stability ($\Delta t$) | Bounded  | Unbounded                 |
| Cost per Step       | Low      | Large (Iterative)         |
| Memory              | Low      | Large                     |
| Implementation      | Easy     | Less easy                 |
| Numerical Damping   | None     | Variable                  |

More important are the reference problems to which they apply best. Most authors agree to state that:

- high speed dynamics involving high frequency response are best treated through explicit schemes (wave propagation, impact, ...)
- inertia problems involving basic (low) frequency modes are best treated via implicit schemes (vibrations, fluid dynamics except for shock waves, seismic analysis, ...)

The present paper is aimed at stressing that, for particular nonlinear problems (short duration high-frequency wave propagation) unclassical choice, i.e. implicit schemes, may reveal cost efficient.

Though dedicated mathematical studies are lacking at this stage of research to prove the evidence, two types of reasons may be invoked:

- first, high numerical (artificial) damping associated with these implicit schemes is not coercitive with respect to the own damping properties of the material, thus preventing the actual response to be inhibited by the scheme. On the contrary, this added numerical damping proves to stabilize greatly the transient response of spurious components.
second, standard stability and accuracy analysis are based on linear theory applied to ideal oscillators (elastic materials). They might thus lead to inaccurate statements in the nonlinear range.

The key point of the newly interpolated implicit schemes presented is indeed in the way of sampling the \(\alpha\)- or \(\beta\)-family outside the classical range \([t_n, t_{n+1}]\), which could not be inferred by the linear theory. Hence, looking at the solution at times later than \(t_{n+1}\), before stepping backwards for the solution at \(t_{n+1}\) (thus producing an effective interpolation of the solution at \(t_{n+1}\) between those at \(t_n\) and \(t_{n+\theta}\) or \(t_{n+\sigma_M}\)), seems to be the reason of their success as demonstrated in the following examples.

NUMERICAL EXAMPLES

Introduction

As a numerical illustration of these concepts, the behaviour of two types of shock-absorber devices has been investigated. The finite element package METAFOR [7] was used. This finite element package is able to simulate quasi-static and transient large deformation problems with complex material behaviour, as well as frictional contact.

Let it be pointed out that during numerical experiments, time steps size for implicit schemes were not guided by physical concepts related to frequency contents of the response but mainly by convergence purposes linked with the limited radius of convergence of the Newton-Raphson algorithm.

Shock absorber device

This first example deals with the numerical modelling of a shock absorber device. It is based on the turning inside-out of a thin walled ductile metal tube. This is generally called an "invertube" device. In this case, a plain tube is confronted with a hard curved die to produce the inversion, figure 1.
This inversion, in turn, produces very large plastic strains which form an efficient energy absorbing mechanism during impact. In this way, the kinetic energy of the impacting bodies is dissipated through plastic deformation, in a controlled fashion at an acceptable rate. The yield limit of the material keeps the transmitted force below an acceptable upperbound. Hence, the deceleration is lower and less harmful for the people inside the car.

Numerical modelling of the collapse of such energy dissipating structures requires not only to take into account the plastic behaviour of the tube material, as well as inertial forces, but also to consider very large strains and large amplitude rigid body motions that develop and also, in this case, the accurate prediction of frictional forces. Thus a great number of advanced code capabilities are tested by running this kind of problems.

Similar problems were investigated by Beltran and Goicolea [8], by Garcia-Garino [9] with an explicit scheme and by Ponthot & Hogge [4,5] who compared the performances of explicit and implicit algorithms for impact problems. However, all the previous references dealt with frictionless contact. In the present paper, both explicit and implicit schemes have been used to integrate the equations of motion in time. The material consists of an aluminum tube of 50.8 mm outside diameter times 63.5 mm length times 1.63 mm wall thickness. The material is supposed to behave like a J2 elastic-plastic material with linear isotropic hardening. The material parameters are given in table 2 and numerical parameters for the time marching algorithms are given in table 3. For this simulation, the following penalty parameters have been used: \( \nu_N = 10^7 \text{ N/mm} \) & \( \nu_I = 10^8 \text{ N/mm} \)

| Table 2. Material properties for Aluminum |
|----------------------------------------|
| Young Modulus | \( E = 67,000 \text{ N/mm}^2 \) |
| Poisson Ratio | \( \nu = 0.33 \) |
| Density | \( \rho = 2,700 \text{ kg/m}^3 \) |
| Yield Stress | \( \sigma_y = 150 \text{ N/mm}^2 \) |
| Hardening parameter | \( h = 44.7 \text{ N/mm}^2 \) |

| Table 3. Numerical parameters for time marching algorithms |
|-----------------------------------------------------------|
| Explicit scheme: safety parameter | \( \gamma = 0.8 \) |
| Implicit scheme: Newmark parameters | \( \gamma = 0.5 \) and \( \beta = 0.25 \) |

The tube has been modelled using 300 quadrilateral elements (3 x 100) with 4 Gauss points and constant pressure to avoid locking. It is driven against a 3.97 mm radius die made of mild steel at a velocity of 44 m/s (144 Km/h). Thus a 50 mm prescribed vertical displacement over a time period of 0.00125 seconds is imposed on the upper nodes of the tube.
Table 4 displays the performances of the explicit and various implicit algorithms. From this table, it is obvious that if classical implicit scheme (i.e. Newmark), is much slower than the explicit scheme, all alternative implicit schemes, i.e. Hilber-Hughes-Taylor (HHT), Chung-Hulbert (CH) and Generalized Midpoint (GMP) exhibit much better performances.

The history of the deformation (frictionless case) is given in figure 2 and a comparison of the final configurations for the different friction coefficients (μ=0.0 ; 0.15 & 0.30) is given in figure 3. In figure 4 are displayed the time/load curves obtained for the three coefficients of friction and table 5 shows the computational effort for the different frictional case (GMP with θ=1.1).

Table 4. Performances of the explicit and various implicit schemes (frictionless case)

| Scheme | Steps | Iterations | CPU | CPU-ratio |
|--------|-------|------------|-----|-----------|
| Explicit | 32687 | - | 25 min 19 sec | 18.8 |
| Implicit | | | |
| NEWMARK | | | |
| αm = αf = 0 | 4448 | 14334 | 43 min 30 sec | 32.2 |
| HHT(αm = 0) | | | |
| αf = 0.01 | 812 | 2472 | 7 min 36 sec | 5.6 |
| αf = 0.05 | 328 | 966 | 3 min 04 sec | 2.3 |
| αf = 0.1 | 266 | 778 | 2 min 27 sec | 1.8 |
| CH | | | |
| αf = 0.01 αm = -0.97 | 174 | 508 | 1 min 38 sec | 1.2 |
| αf = 0.05 αm = -0.85 | 174 | 514 | 1 min 37 sec | 1.2 |
| αf = 0.10 αm = -0.70 | 160 | 466 | 1 min 30 sec | 1.1 |
| GMP | | | |
| θ = 1.00 | 449 | 1236 | 4 min 06 sec | 3.0 |
| θ = 1.01 | 198 | 575 | 1 min 58 sec | 1.4 |
| θ = 1.05 | 143 | 412 | 1 min 27 sec | 1.1 |
| θ = 1.10 | 142 | 404 | 1 min 22 sec | 1.0 |
| θ = 1.15 | 149 | 431 | 1 min 28 sec | 1.1 |
Figure 3. Comparison of the final configurations as a function of friction coefficient.
(μ=0.0: upper configuration; μ=0.15: intermediate configuration; μ=0.3: lower configuration)

Table 5. Performances of GMP implicit scheme for various friction coefficients

| Friction | Steps | Iterations | CPU           |
|----------|-------|------------|---------------|
| μ = 0.00 | 142   | 404        | 1 min 22 sec  |
| μ = 0.15 | 143   | 415        | 1 min 24 sec  |
| μ = 0.30 | 147   | 432        | 1 min 32 sec  |

Figure 4. Applied load as a function of time for different friction coefficients
Dynamic buckling of a cylinder

This second example deals with the buckling of a thin steel cylinder by axial compression into a rigid matrix. This problem has been first mentioned by Tod A. Laursen [10] who compares the results with and without friction in a quasi-static situation.

In this axisymmetric problem, a steel cylinder is forced downwards (via displacement control of its top surface) into a rigid fixture. The steel is modelled by an elastoplastic law with linear isotropic hardening and has the material properties shown in table 6. Spatial discretization of the cylinder is achieved using 177 bilinear four-nodes elements (3 x 59) with constant pressure. As the calculation proceeds, the cylinder initially moves down into the die, but when a critical axial load is reached, it begins a series of buckles, as shown in Figure 5. Importantly, this buckling occurs without any initial geometric imperfections. The contact is considered frictionless and is solved using the Penalty method. Information about the initial geometry is given in table 7.

| Table 6. Material properties for Steel |
|---------------------------------------|
| Young Modulus E = 210000 N/mm²         |
| Poisson Ratio v = 0.3                  |
| Density ρ = 7850 kg/m³                |
| Yield Stress σ₀ = 700 N/mm²           |
| Hardening parameter h = 808 N/mm²      |

| Table 7. Geometrical properties of the cylinder |
|-----------------------------------------------|
| Internal Diameter D₉ = 27.00 mm               |
| External Diameter D₅ = 31.75 mm              |
| Thickness e = 4.75 mm                        |
| Height h = 180.00 mm                         |

| Table 8. Numerical parameters for time marching algorithms |
|----------------------------------------------------------|
| Explicit                                                  |
| Safety parameter χ = 0.2                                  |
| Implicit (CH)                                             |
| Inertia parameter α_{M} = -0.97                           |
| Force parameter α_{N} = 0.01                              |
| Newmark parameters χ = 0.5 and β = 0.25                   |

To compare the explicit and the implicit schemes, the problem had to be solved dynamically. So it was decided to crash it down 110 mm in 11 ms (10m/s at constant velocity). The problem was solved using a large set of Penalty parameters ranging from 10⁶ N/mm (which gives a maximum penetration encountered lower than 100 µm) to as large as possible, with the explicit scheme and with the Chung-Hulbert (CH) implicit scheme (see tables 9 and 10). The low value of the safety parameter (χ = 0.2) in the explicit scheme is necessary to obtain the physical solution of the problem: using a larger one results in a solution with the third and last buckle generated on the top of the cylinder as shown in figure 6. The correct solution (figure 5) is only found with a χ lower than 0.2 for the explicit scheme and in any case with the implicit scheme.
Figure 5. Buckling of a cylinder, configurations from 0 to 11 milliseconds

Table 9. Buckling of a cylinder (Explicit scheme)

| Penalty coefficient | CPU         | Steps  | Penetration | m |
|---------------------|-------------|--------|-------------|---|
| $10^6$              | 2 h 38 min 26 sec | 367189 | 7.e-2       |   |
| $5.10^6$            | 2 h 40 min 42 sec | 369095 | 1.e-2       |   |
| $10^7$              | Failed      |        |             |   |

Table 10. Buckling of a cylinder (Implicit scheme CH)

| Penalty coefficient | CPU     | Steps | Iterations | Penetration | m |
|---------------------|---------|-------|------------|-------------|---|
| $10^6$              | 33 sec  | 178   | 423        | 4.e-2       |   |
| $5.10^6$            | 40 sec  | 214   | 494        | 1.e-2       |   |
| $10^7$              | 41 sec  | 129   | 533        | 6.e-3       |   |
| $10^8$              | 48 sec  | 178   | 624        | 6.e-4       |   |
| $10^9$              | 1 min 00 sec | 148  | 771        | 6.e-5       |   |
| $10^{10}$           | 1 min 19 sec | 176  | 1038       | 6.e-6       |   |
| $10^{11}$           | 4 min 32 sec | 1647 | 3503       | 6.e-7       |   |
| $10^{12}$           | 6 min 38 sec | 2275 | 5261       | 7.e-8       |   |
| $10^{13}$           | 17 min 18 sec | 5814 | 14188      | 7.e-9       |   |
The CPU-costs and the numbers of time steps and iterations required to solve the problem are given in Table 9 and 10. An idea of the precision obtained is given by the maximum penetration encountered during the process.

Looking at these results, it is obvious that the precision was strongly influenced by the modelling of the contact situation which changed the penalty coefficient: the larger the penalty coefficient, the smaller was the penetration. The best choice would then be to use infinite or very large penalty coefficient to best fit the impenetrability condition. The problem is that the explicit and implicit integration methods encounter different numerical difficulties when this coefficient becomes too large.

Explicit scheme does not accept too large Penalty coefficients because of the too important numerical perturbations induced by a new contact configuration: using a discrete step by step process to solve the problem, the explicit scheme suddenly encounters, at a new contact node (unconstrained at the latest time step), a large contact force. As the explicit scheme computes positions at a given time from the forces at the preceding step, and remembering that the contact force only appears at the present step, we understand that the penetration is nothing of a physical equilibrated value but it is the distance that the node would have covered if there were no contact. However, the time steps in explicit form are generally rather small, preventing from encountering too large "first penetrations" but, if a large Penalty coefficient is used, the contact force induced could be so large that the node in contact is bounced away from the contacting body in the following step. This phenomenon can produce strong numerical oscillations of the contact nodes or even failure of the solving process if the penalty coefficient is too large.

Implicit schemes do not like too large Penalty coefficient. However, for certain reasons a too high penalty coefficient ill-conditions the tangent stiffness matrix of the system; this matrix is necessary to
lead the iterative process towards an equilibrated solution (Newton-Raphson process). The tangent stiffness matrix has contact terms directly linked to the Penalty coefficient and if this latter is too large, the corresponding terms can become much greater than the other physical terms. It can then swap most of the information necessary to converge towards a solution that the method can no longer except. In table 10, it can be seen that not only the precision increases as the penalty coefficient grows, but also the CPU-cost. This is due to the crescent ill-conditioning of the stiffness matrix that makes the process more demanding with respect to time steps and iterations to solve the problem (with a better accuracy).

![Buckling of a Cylinder](image)

Figure 7. Buckling of a cylinder: variation of the CPU-cost with the precision on a log-log scale

Figure 7 shows the variation of the CPU-cost as a function of the maximum penetration. It can therefore be concluded that, for the explicit scheme, CPU-cost is more or less independent of the Penalty coefficient (then, also of the desired precision, as far as this one can be reached) and that for the implicit scheme, it follows, for this problem, the relation \( CPU-cost = C_1 g_n^{-1/4} = C_2 \varepsilon_n^{-1/4} \) with \( g_n \), the normal penetration and \( \varepsilon_n \), the penalty coefficient (\( C_1 \) and \( C_2 \) are two constants). Figure 7 also shows that even for a very large penalty parameter (that gives a very low penetration) the implicit scheme is cheaper than the explicit one. At the limit of precision that can be reached by the explicit scheme, the implicit one gives the same solution with a CPU 240 times lower.

CONCLUSIONS

In this paper, it has been shown using two examples that, for high speed dynamic problems, the explicit scheme can be much more expensive (200 times in the second problem) than implicit schemes. This is in contradiction with what is generally thought in the computational mechanics community. Moreover, if agreed to “pay the price”, the implicit schemes can lead to a much more precise solution. This is simply done by increasing the penalty parameters which in turn lead to smaller penetrations. It has also been shown herein that the explicit scheme has to be very carefully used (safety parameter \( \gamma \), lower than 0.2 in some case) to get an accurate solution.
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