A computer based classification of caps in 
\( PG(5,2) \)

Daniele Bartoli, Stefano Marcugini, Fernanda Pambianco

Abstract

In this paper we present the complete classification of caps in \( PG(5,2) \). These results have been obtained using a computer based exhaustive search that exploits projective equivalence.

1 Introduction

In the projective space \( PG(r, q) \) over the Galois Field \( GF(q) \), a \( n \)-cap is a set of \( n \) points no 3 of which are collinear. A \( n \)-cap is called complete if it is not contained in a \( (n + 1) \)-cap. For a detailed description of the most important properties of these geometric structures, we refer the reader to [4]. In the last decades the problem of determining the spectrum of the sizes of complete caps has been the subject of a lot of researches. For a survey see [1]. In this work we search for the classification of complete and incomplete caps in \( PG(5,2) \), using an exhaustive search algorithm. In Section 2 the algorithm utilized is illustrated; in Section 3 the complete list of non equivalent complete and incomplete caps is presented.

2 The searching algorithm

In this section the algorithm utilized is presented. Our goal is to obtain the classification of complete and incomplete caps in \( PG(5,2) \). It is not restrictive to suppose that a cap in \( PG(5,2) \) contains this six points:

\[
\mathcal{R} = \{(1 : 0 : 0 : 0 : 0 : 0); (0 : 1 : 0 : 0 : 0 : 0); (0 : 0 : 1 : 0 : 0 : 0); \\
(0 : 0 : 0 : 1 : 0 : 0); (0 : 0 : 0 : 0 : 1 : 0); (0 : 0 : 0 : 0 : 0 : 1)\}.
\]
Then we define the set $Cand$ of all the points lying no 2-secant of $R$. We introduce in $Cand$ the following equivalence relationship:

$$P \sim Q \iff C \cup \{P\} \cong C \cup \{Q\},$$

where $\cong$ means that the two sets are projectively equivalent. This relationship spreads the candidates in equivalent classes $C_1, \ldots, C_k$.

The choice of the next point to add to the building cap can be made only among the representatives of the equivalent classes, in fact two caps one containing $C \cup \{P\}$ and the other one $C \cup \{Q\}$, with $P$ and $Q$ in $C_i$, are equivalent by definition of orbit.

Suppose now that we have construct all the caps containing $C \cup \{P\}_i$, with $i \leq j$. Considering the caps containing $C \cup \{P\}_j$ with $j < i$, all the points of the classes $C_k$ with $k < j$ can be avoided. In fact a cap containing $C \cup \{P\}_j \cup \{P^\ast\}_k$, with $P^\ast_k \in C_k$ and $k < j$, is projectively equivalent to a cap containing $C \cup \{P_k\} \cup \{P\}_j$, already studied.

When we add a new point to the cap, we can divide all the remaining candidates in equivalence classes, as above. Two points $P$ and $Q$ are in relationship with the $j$-th class $C_j$, i.e. $P \sim Q$, if $C \cup \{P\}_j \cup \{P\}$ and $C \cup \{P\}_j \cup \{Q\}$ are projectively equivalent.

At the $m$-th step of the extension process if the cap $C \cup \{P_{i_m}\} \cup \ldots \cup \{P_{i_{m-1}}\} \cup \{P\}$ is projectively equivalent to the cap $C \cup \{P_{i_m}\} \cup \ldots \cup \{P_{i_{m-1}}\} \cup \{Q\}$ with $P_{s_{i_1 \ldots i_m}} \in C_{s_{i_1 \ldots i_m}}$, then $P$ and $Q$ are in relationship ($P \sim_{i_1 \ldots i_m} Q$) and they belong to the same class $C_{s_{i_1 \ldots i_m}}^{i_1 \ldots i_m}$.

Iterating the process we can build a tree similar to the following:

The tree is important to restrict the number of candidates in the extension process. Suppose that we have generated a $n$-cap containing the cap $C \cup \{P_{i_1}\} \cup \{P_{i_2}\} \cup \ldots \cup \{P_{i_{m-1}}\} \cup \{P\}$, after having generated $n$-caps containing $C \cup \{P\}_j$ with $j < i_1$, $C \cup \{P_{i_1}\} \cup \{P_{i_2}\} \cup \ldots \cup \{P_{i_{m-1}}\} \cup \{P\}_j$ with $j < i_2, \ldots, C \cup \{P_{i_1}\} \cup \{P_{i_2}\} \cup \ldots \cup \{P_{i_{m-1}}\} \cup \{P_{i_j}\}$ with $j < i_m$, with $P_{s_{i_1 \ldots i_m}} \in C_{s_{i_1 \ldots i_m}}$. Then the
points belonging to $C_1 \cup \ldots \cup C_{i-1} \cup C_{i1}^{i1} \cup \ldots \cup C_{i2}^{i1} \cup \ldots \cup C_{i1}^{i1} \cup \ldots \cup C_{im-1}^{i1}$ can be avoided, because a cap containing one of them is equivalent to one already found. For example a $n$-cap containing $C \cup \{P_{i1}\} \cup \ldots \cup \{P_{im-1}\} \cup \{P\} \cup \{Q\}$ with $Q \in C_h$ for some $h < i_1$ is equivalent to a $n$-cap containing $C \cup \{P_h\}$, which is already found.

3 Results

In this Section all non equivalent caps, complete and incomplete, in $PG(5, 2)$ are presented.

3.1 Non-equivalent caps $K$ in $PG(5, 2)$

This table shows the number and the type of the non equivalent examples of all the caps.

Table 1: Number and type of non equivalent examples

| $|K|$ | # COMPLETE CAPS | # INCOMPLETE CAPS | $|K|$ | # COMPLETE CAPS | # INCOMPLETE CAPS |
|-----|----------------|-------------------|-----|----------------|-------------------|
| 7   | 0              | 4                 | 20  | 1              | 23                |
| 8   | 0              | 7                 | 21  | 0              | 16                |
| 9   | 0              | 12                | 22  | 0              | 15                |
| 10  | 0              | 24                | 23  | 0              | 9                 |
| 11  | 0              | 34                | 24  | 0              | 8                 |
| 12  | 0              | 43                | 25  | 0              | 5                 |
| 13  | 1              | 46                | 26  | 0              | 4                 |
| 14  | 0              | 49                | 27  | 0              | 2                 |
| 15  | 0              | 44                | 28  | 0              | 2                 |
| 16  | 0              | 48                | 29  | 0              | 1                 |
| 17  | 5              | 35                | 30  | 0              | 1                 |
| 18  | 1              | 32                | 31  | 0              | 1                 |
| 19  | 0              | 25                | 32  | 1              | 0                 |

3.2 Description of the caps

In this section we describe each cap of size $k + 1$, with $7 \geq k \geq 31$, as union of a cap of size $k$ and a point in $PG(5, 2)$. We start from these non equivalent examples of 7-incomplete caps.
We will call these caps $C_1^7, C_2^7, C_3^7, C_4^7$ respectively. They have stabilizer ($O$) of size

| $K$ | STABILIZER  |
|-----|-------------|
| $C_1^7$ | $|O| = 5040$ |
| $C_2^7$ | $|O| = 144$ |
| $C_3^7$ | $|O| = 240$ |
| $C_4^7$ | $|O| = 720$ |

In the following tables the symbol $C_j^i$ indicates the $j$-th cap of size $i$ and $C_j^i$ means that the cap is complete. If it is possible to write a cap in two different ways, we choose that one with the lowest value of $j$.

| $K$ | CORRESPONDS TO | STABILIZER  |
|-----|----------------|-------------|
| $C_1^7$ | $C_2^7 \cup \{(1, 1, 0, 1, 1, 0)\}$ | $|O| = 48$ |
| $C_3^7$ | $C_2^7 \cup \{(0, 0, 1, 0, 1, 0)\}$ | $|O| = 96$ |
| $C_5^7$ | $C_2^7 \cup \{(1, 1, 1, 0, 1, 0)\}$ | $|O| = 1152$ |
| $C_7^7$ | $C_1^7 \cup \{(1, 1, 0, 1, 1, 0)\}$ | $|O| = 144$ |

| $K$ | CORRESPONDS TO | STABILIZER  |
|-----|----------------|-------------|
| $C_8^7$ | $C_2^7 \cup \{(0, 1, 1, 0, 1, 1)\}$ | $|O| = 144$ |
| $C_6^7$ | $C_2^7 \cup \{(1, 1, 1, 1, 1, 0)\}$ | $|O| = 144$ |
| $C_4^7$ | $C_2^7 \cup \{(1, 1, 0, 1, 1, 1)\}$ | $|O| = 144$ |
| $C_5^7$ | $C_2^7 \cup \{(1, 1, 1, 1, 1, 1)\}$ | $|O| = 144$ |

4
| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|-----------|--------------|------------|-----------|--------------|------------|
| $C_0^6$   | $C_0^6 \cup \{(0, 0, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 48$ | $C_0^6$   | $C_0^6 \cup \{(0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ |
| $C_0^{10}$ | $C_0^{10} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 48$ | $C_0^{10}$ | $C_0^{10} \cup \{(0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ |
| $C_0^{12}$ | $C_0^{12} \cup \{(0, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ | $C_0^{12}$ | $C_0^{12} \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 48$ |

| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|-----------|--------------|------------|-----------|--------------|------------|
| $C_0^6$   | $C_0^6 \cup \{(0, 0, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 48$ | $C_0^6$   | $C_0^6 \cup \{(0, 0, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ |
| $C_0^{10}$ | $C_0^{10} \cup \{(0, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ | $C_0^{10}$ | $C_0^{10} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 48$ |
| $C_0^{12}$ | $C_0^{12} \cup \{(0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ | $C_0^{12}$ | $C_0^{12} \cup \{(0, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 48$ |

5
| $\kappa$ | CORRESPONDS TO | STABILIZER | $|\mathcal{O}|$ | $\kappa$ | CORRESPONDS TO | STABILIZER | $|\mathcal{O}|$ |
|--------|----------------|------------|-------------|--------|----------------|------------|-------------|
| $C^{11}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 144$ | $D_6 \times C_2$ | $C^{12}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 1384$ |
| $C^{13}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 1152$ | $D_6 \times C_2$ | $C^{14}_{11}$ | $C^{6}_{10} \cup \{(1, 0, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 96$ |
| $C^{11}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 48$ | $D_4 \times C_2$ | $C^{16}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 120$ |
| $C^{17}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 1008$ | $D_4 \times C_2$ | $C^{18}_{11}$ | $C^{6}_{10} \cup \{(0, 0, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 32$ |
| $C^{21}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 0, 0, 0, 0)\}$ | $|\mathcal{O}| = 1384$ | $D_4 \times C_2$ | $C^{22}_{11}$ | $C^{6}_{10} \cup \{(0, 0, 1, 1, 0, 1)\}$ | $|\mathcal{O}| = 48$ |
| $C^{23}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 384$ | $D_4 \times C_2$ | $C^{24}_{11}$ | $C^{6}_{10} \cup \{(1, 0, 0, 0, 0, 0)\}$ | $|\mathcal{O}| = 1920$ |
| $C^{25}_{11}$ | $C^{6}_{10} \cup \{(1, 0, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 720$ | $D_4 \times C_2$ | $C^{26}_{11}$ | $C^{6}_{10} \cup \{(0, 0, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 1920$ |
| $C^{27}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 32$ | $D_4 \times C_2$ | $C^{28}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 1334$ |
| $C^{29}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 1, 1, 0, 1)\}$ | $|\mathcal{O}| = 192$ | $D_4 \times C_2$ | $C^{30}_{11}$ | $C^{6}_{10} \cup \{(1, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 48$ |
| $C^{31}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 0, 1, 1, 0)\}$ | $|\mathcal{O}| = 384$ | $D_4 \times C_2$ | $C^{32}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 1, 1, 1, 0)\}$ | $|\mathcal{O}| = 48$ |
| $C^{33}_{11}$ | $C^{6}_{10} \cup \{(0, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 48$ | $D_4 \times C_2$ | $C^{34}_{11}$ | $C^{6}_{10} \cup \{(1, 1, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 1920$ |
| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|---|---|---|---|---|---|
| $C_{13}^7$ | $C_{12}^6 \cup \{(0,0,0,1,1,1,1)\}$ | $|\mathcal{O}| = 1152$ | $C_{13}^7$ | $C_{12}^6 \cup \{(1,0,1,0,1,0,0)\}$ | $|\mathcal{O}| = 48$ |
| $C_{13}^3$ | $C_{12}^4 \cup \{(1,1,1,0,0,0,0)\}$ | $|\mathcal{O}| = 32$ | $C_{13}^4$ | $C_{12}^4 \cup \{(1,1,1,1,0,0,0)\}$ | $|\mathcal{O}| = 32$ |
| $C_{13}^5$ | $C_{12}^2 \cup \{(1,1,1,0,0,0,0)\}$ | $|\mathcal{O}| = 256$ | $C_{13}^6$ | $C_{12}^2 \cup \{(1,0,1,0,1,1)\}$ | $|\mathcal{O}| = 256$ |
| $C_{13}^7$ | $C_{12}^1 \cup \{(1,1,1,0,0,0,0)\}$ | $|\mathcal{O}| = 2304$ | $C_{13}^8$ | $C_{12}^1 \cup \{(1,0,1,1,0,0,0)\}$ | $|\mathcal{O}| = 2304$ |
| $C_{13}^9$ | $C_{12}^5 \cup \{(0,1,0,0,1,1,1)\}$ | $|\mathcal{O}| = 384$ | $C_{13}^9$ | $C_{12}^5 \cup \{(0,1,0,1,0,1,0)\}$ | $|\mathcal{O}| = 384$ |
| $C_{13}^{10}$ | $C_{12}^3 \cup \{(0,1,1,1,1,1,1)\}$ | $|\mathcal{O}| = 48$ | $C_{13}^{10}$ | $C_{12}^3 \cup \{(0,0,1,1,1,1,1)\}$ | $|\mathcal{O}| = 48$ |
| $C_{13}^{11}$ | $C_{12}^2 \cup \{(0,0,1,1,1,1,1)\}$ | $|\mathcal{O}| = 48$ | $C_{13}^{11}$ | $C_{12}^2 \cup \{(0,0,1,0,0,0,0)\}$ | $|\mathcal{O}| = 48$ |
| $C_{13}^{12}$ | $C_{12}^1 \cup \{(0,0,1,1,1,1,1)\}$ | $|\mathcal{O}| = 48$ | $C_{13}^{12}$ | $C_{12}^1 \cup \{(0,0,1,0,1,1,1)\}$ | $|\mathcal{O}| = 48$ |
| $C_{13}^{13}$ | $C_{12}^0 \cup \{(0,0,0,1,1,1)\}$ | $|\mathcal{O}| = 48$ | $C_{13}^{13}$ | $C_{12}^0 \cup \{(0,0,0,1,0,1)\}$ | $|\mathcal{O}| = 48$ |

$\mathcal{K}$ is a set of stabilizers for a code, and $|\mathcal{O}|$ represents the size of the orbit of a particular stabilizer action.
| $\kappa$ | CORRESPONDS TO | STABILIZER | $\kappa$ | CORRESPONDS TO | STABILIZER |
|--------|----------------|------------|--------|----------------|------------|
| $C_{14}^3$ | $C_{13}^4 \cup \{(1, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 48$ | $C_{14}^2$ | $C_{13}^4 \cup \{(0, 1, 1, 0, 1, 1)\}$ | $D_4 \times C_2$ |
| $C_{14}^2$ | $C_{13}^6 \cup \{(0, 0, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 128$ | $C_{14}^4$ | $C_{13}^4 \cup \{(0, 0, 1, 1, 0, 0)\}$ | $D_4 \times C_2$ |
| $C_{14}^1$ | $C_{13}^7 \cup \{(1, 0, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 96$ | $C_{14}^6$ | $C_{13}^6 \cup \{(0, 0, 0, 0, 0, 0)\}$ | $D_4 \times C_2$ |
| $C_{14}^7$ | $C_{13}^5 \cup \{(0, 0, 1, 0, 0, 0)\}$ | $C_2 \times C_2$ | $C_{14}^8$ | $C_{13}^5 \cup \{(0, 1, 0, 0, 0, 1)\}$ | $D_6 \times C_2$ |
| $C_{14}^9$ | $C_{13}^4 \cup \{(0, 1, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 128$ | $C_{14}^9$ | $C_{13}^5 \cup \{(1, 0, 0, 1, 0, 0)\}$ | $D_6 \times C_2$ |
| $C_{14}^{11}$ | $C_{13}^3 \cup \{(0, 1, 0, 1, 0, 1)\}$ | $|\mathcal{O}| = 48$ | $C_{14}^{12}$ | $C_{13}^4 \cup \{(1, 0, 0, 0, 0, 0)\}$ | $D_6 \times C_2$ |
| $C_{14}^{13}$ | $C_{13}^2 \cup \{(1, 0, 0, 1, 0, 1)\}$ | $D_6$ | $C_{14}^{13}$ | $C_{13}^4 \cup \{(0, 0, 1, 0, 1, 1)\}$ | $D_6$ |
| $C_{14}^{14}$ | $C_{13}^2 \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 1536$ | $C_{14}^{15}$ | $C_{13}^4 \cup \{(0, 1, 0, 1, 0, 1)\}$ | $D_6$ |
| $C_{14}^{17}$ | $C_{13}^2 \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 2304$ | $C_{14}^{16}$ | $C_{13}^4 \cup \{(0, 1, 1, 1, 1, 1)\}$ | $D_6$ |
| $C_{14}^{18}$ | $C_{13}^6 \cup \{(0, 1, 0, 0, 0, 0)\}$ | $|\mathcal{O}| = 64$ | $C_{14}^{19}$ | $C_{13}^4 \cup \{(0, 0, 1, 0, 0, 0)\}$ | $D_6$ |
| $C_{14}^{23}$ | $C_{13}^3 \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 48$ | $C_{14}^{20}$ | $C_{13}^4 \cup \{(0, 0, 0, 0, 0, 0)\}$ | $D_6$ |
| $C_{14}^{25}$ | $C_{13}^2 \cup \{(0, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 48$ | $C_{14}^{21}$ | $C_{13}^4 \cup \{(0, 0, 0, 1, 0, 1)\}$ | $D_6$ |
| $C_{14}^{27}$ | $C_{13}^2 \cup \{(1, 0, 0, 0, 0, 0)\}$ | $|\mathcal{O}| = 96$ | $C_{14}^{22}$ | $C_{13}^4 \cup \{(1, 1, 1, 0, 0, 0)\}$ | $D_6$ |
| $C_{14}^{29}$ | $C_{13}^2 \cup \{(1, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 48$ | $C_{14}^{23}$ | $C_{13}^4 \cup \{(0, 0, 0, 1, 0, 0)\}$ | $D_6$ |
| $C_{14}^{33}$ | $C_{13}^4 \cup \{(0, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 64$ | $C_{14}^{34}$ | $C_{13}^4 \cup \{(1, 0, 0, 0, 1, 0)\}$ | $D_6$ |
| $C_{14}^{35}$ | $C_{13}^4 \cup \{(1, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 128$ | $C_{14}^{36}$ | $C_{13}^4 \cup \{(1, 0, 0, 1, 0, 0)\}$ | $D_6$ |
| $C_{14}^{37}$ | $C_{13}^4 \cup \{(1, 0, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 128$ | $C_{14}^{38}$ | $C_{13}^4 \cup \{(1, 0, 0, 0, 0, 1)\}$ | $D_6$ |
| $C_{14}^{39}$ | $C_{13}^4 \cup \{(1, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 1536$ | $C_{14}^{40}$ | $C_{13}^4 \cup \{(1, 0, 0, 0, 1, 0)\}$ | $D_6$ |
| $C_{14}^{41}$ | $C_{13}^4 \cup \{(1, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 1152$ | $C_{14}^{42}$ | $C_{13}^4 \cup \{(0, 0, 1, 0, 0, 0)\}$ | $D_6$ |
| $C_{14}^{43}$ | $C_{13}^4 \cup \{(0, 0, 0, 0, 0, 0)\}$ | $|\mathcal{O}| = 96$ | $C_{14}^{44}$ | $C_{13}^4 \cup \{(0, 0, 1, 1, 0, 0)\}$ | $D_6$ |
| $C_{14}^{45}$ | $C_{13}^4 \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 384$ | $C_{14}^{46}$ | $C_{13}^4 \cup \{(0, 0, 1, 0, 0, 0)\}$ | $D_6$ |
| $C_{14}^{47}$ | $C_{13}^4 \cup \{(0, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 576$ | $C_{14}^{48}$ | $C_{13}^4 \cup \{(0, 0, 0, 1, 0, 1)\}$ | $D_6$ |
| $C_{14}^{49}$ | $C_{13}^4 \cup \{(0, 0, 1, 0, 1, 1)\}$ | $|\mathcal{O}| = 36$ | $C_{14}^{50}$ | $C_{13}^4 \cup \{(0, 0, 1, 0, 0, 1)\}$ | $D_6$ |
| $\kappa$ | CORRESPONDS TO | STABILIZER | $\kappa$ | CORRESPONDS TO | STABILIZER |
|---|---|---|---|---|---|
| $C_{15}^3$ | $C_{14}^3 \cup \{0,1,0,0,1,1\}$ | $S_3$ | $C_{15}^2$ | $C_{14}^1 \cup \{0,1,0,0,0,1\}$ | $\mathcal{D}_4 \times \mathcal{C}_2$ |
| $C_{15}^5$ | $C_{14}^5 \cup \{0,0,1,1,1,0\}$ | $S_3$ | $C_{15}^6$ | $C_{14}^2 \cup \{0,1,1,0,1,0\}$ | $\mathcal{D}_4 \times \mathcal{C}_2$ |
| $C_{15}^8$ | $C_{14}^8 \cup \{0,1,0,1,0,0\}$ | $|O| = 64$ | $C_{15}^8$ | $C_{14}^1 \cup \{0,1,1,0,0,1\}$ | $|O| = 32$ |
| $C_{15}^{11}$ | $C_{14}^{11} \cup \{1,1,1,0,0,0\}$ | $|O| = 48$ | $C_{15}^{11}$ | $C_{14}^1 \cup \{0,1,1,0,0,1\}$ | $|O| = 32$ |
| $C_{15}^{13}$ | $C_{14}^{13} \cup \{1,1,1,0,0,0\}$ | $|O| = 48$ | $C_{15}^{13}$ | $C_{14}^1 \cup \{0,1,1,0,0,1\}$ | $|O| = 32$ |
| $C_{15}^{15}$ | $C_{14}^{15} \cup \{0,1,1,1,0,0\}$ | $|O| = 760$ | $C_{15}^{15}$ | $C_{14}^1 \cup \{0,1,1,1,0,0\}$ | $|O| = 720$ |
| $C_{15}^{23}$ | $C_{14}^{23} \cup \{0,1,0,1,0,1\}$ | $|O| = 384$ | $C_{15}^{23}$ | $C_{14}^1 \cup \{0,1,0,0,1,1\}$ | $|O| = 48$ |
| $C_{15}^{25}$ | $C_{14}^{25} \cup \{1,0,0,1,0,1\}$ | $|O| = 128$ | $C_{15}^{25}$ | $C_{14}^1 \cup \{0,1,0,0,1,1\}$ | $|O| = 384$ |
| $C_{15}^{27}$ | $C_{14}^{27} \cup \{1,0,1,0,0,1\}$ | $|O| = 760$ | $C_{15}^{27}$ | $C_{14}^1 \cup \{0,1,0,0,1,1\}$ | $|O| = 720$ |
| $C_{15}^{29}$ | $C_{14}^{29} \cup \{0,0,1,0,1,1\}$ | $|O| = 64$ | $C_{15}^{29}$ | $C_{14}^1 \cup \{0,1,0,0,1,1\}$ | $|O| = 720$ |
| $C_{15}^{17}$ | $C_{14}^{17} \cup \{1,1,0,0,0,1\}$ | $|O| = 2688$ | $C_{15}^{17}$ | $C_{14}^1 \cup \{0,1,0,0,1,1\}$ | $|O| = 48$ |
| $C_{15}^{19}$ | $C_{14}^{19} \cup \{0,1,0,0,1,1\}$ | $|O| = 384$ | $C_{15}^{19}$ | $C_{14}^1 \cup \{0,1,0,0,0,1\}$ | $|O| = 144$ |
| $C_{15}^{21}$ | $C_{14}^{21} \cup \{1,0,0,1,0,1\}$ | $|O| = 48$ | $C_{15}^{21}$ | $C_{14}^1 \cup \{0,1,0,0,1,0\}$ | $|O| = 48$ |
| $C_{15}^{23}$ | $C_{14}^{23} \cup \{1,1,0,0,1,0\}$ | $|O| = 64$ | $C_{15}^{23}$ | $C_{14}^1 \cup \{0,1,0,1,0,1\}$ | $|O| = 48$ |
| $C_{15}^{25}$ | $C_{14}^{25} \cup \{1,0,1,0,1,0\}$ | $|O| = 48$ | $C_{15}^{25}$ | $C_{14}^1 \cup \{0,1,0,1,0,0\}$ | $|O| = 48$ |
| $C_{15}^{27}$ | $C_{14}^{27} \cup \{0,0,1,1,0,1\}$ | $|O| = 32$ | $C_{15}^{27}$ | $C_{14}^1 \cup \{1,0,1,0,0,1\}$ | $|O| = 48$ |
| $C_{15}^{29}$ | $C_{14}^{29} \cup \{1,0,1,0,0,1\}$ | $|O| = 240$ | $C_{15}^{29}$ | $C_{14}^1 \cup \{1,0,1,1,0,1\}$ | $|O| = 48$ |
| $C_{15}^{31}$ | $C_{14}^{31} \cup \{0,0,1,1,1,0\}$ | $|O| = 192$ | $C_{15}^{31}$ | $C_{14}^1 \cup \{0,0,1,1,1,0\}$ | $|O| = 48$ |

10
| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|------|----------------|------------|------|----------------|------------|
| $C_{16}^3$ | $C_{15}^3 \cup \{0, 1, 1, 0, 0, 1\}$ | $\mathcal{D}_5$ | $C_{16}^2$ | $C_{15}^2 \cup \{0, 1, 1, 0, 0, 0\}$ | $\mathcal{D}_4$ |
| $C_{16}^5$ | $C_{15}^5 \cup \{0, 0, 1, 0, 1, 1\}$ | $|\mathcal{O}| = 32$ | $C_{16}^3$ | $C_{15}^3 \cup \{0, 1, 1, 0, 0, 0\}$ | $C_2 \times C_2 \times C_2$ |
| $C_{16}^7$ | $C_{15}^7 \cup \{0, 1, 1, 0, 0, 0\}$ | $|\mathcal{O}| = 128$ | $C_{16}^5$ | $C_{15}^5 \cup \{0, 1, 0, 0, 0, 1\}$ | $|\mathcal{O}| = 768$ |
| $C_{16}^7$ | $C_{15}^7 \cup \{0, 1, 1, 0, 0, 0\}$ | $|\mathcal{O}| = 576$ | $C_{16}^8$ | $C_{15}^8 \cup \{0, 1, 1, 0, 0, 0, 0\}$ | $|\mathcal{O}| = 384$ |
| $C_{16}^9$ | $C_{15}^9 \cup \{0, 1, 0, 0, 1, 1\}$ | $|\mathcal{O}| = 192$ | $C_{16}^{10}$ | $C_{15}^{10} \cup \{0, 0, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 144$ |
| $C_{16}^{11}$ | $C_{15}^{11} \cup \{0, 0, 0, 0, 0, 1\}$ | $|\mathcal{O}| = 128$ | $C_{16}^{10}$ | $C_{15}^{10} \cup \{0, 0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 36$ |
| $C_{16}^{13}$ | $C_{15}^{13} \cup \{0, 0, 0, 0, 0, 1\}$ | $|\mathcal{O}| = 192$ | $C_{16}^{12}$ | $C_{15}^{12} \cup \{0, 1, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 48$ |
| $C_{16}^{14}$ | $C_{15}^{14} \cup \{0, 1, 1, 0, 0, 0\}$ | $|\mathcal{O}| = 192$ | $C_{16}^{10}$ | $C_{15}^{10} \cup \{0, 0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 192$ |
| $C_{16}^{16}$ | $C_{15}^{16} \cup \{0, 0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 11520$ | $C_{16}^{12}$ | $C_{15}^{12} \cup \{0, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 256$ |
| $C_{16}^{17}$ | $C_{15}^{17} \cup \{0, 0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 192$ | $C_{16}^{14}$ | $C_{15}^{14} \cup \{0, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 1536$ |
| $C_{16}^{20}$ | $C_{15}^{20} \cup \{0, 0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 384$ | $C_{16}^{24}$ | $C_{15}^{24} \cup \{1, 0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 2688$ |
| $C_{16}^{21}$ | $C_{15}^{21} \cup \{0, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 384$ | $C_{16}^{26}$ | $C_{15}^{26} \cup \{1, 0, 1, 0, 0, 1\}$ | $|\mathcal{O}| = 64$ |
| $C_{16}^{23}$ | $C_{15}^{23} \cup \{0, 1, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 576$ | $C_{16}^{28}$ | $C_{15}^{28} \cup \{1, 0, 0, 1, 0, 1\}$ | $C_2 \times C_2 \times C_2$ |
| $C_{16}^{25}$ | $C_{15}^{25} \cup \{0, 1, 0, 1, 1, 0\}$ | $|\mathcal{O}| = 576$ | $C_{16}^{30}$ | $C_{15}^{30} \cup \{1, 0, 1, 0, 1, 0\}$ | $C_2 \times C_2 \times C_2$ |
| $C_{16}^{27}$ | $C_{15}^{27} \cup \{0, 1, 0, 1, 0, 0\}$ | $|\mathcal{O}| = 720$ | $C_{16}^{32}$ | $C_{15}^{32} \cup \{1, 1, 0, 0, 0, 0\}$ | $|\mathcal{O}| = 64$ |
| $C_{16}^{29}$ | $C_{15}^{29} \cup \{0, 1, 0, 1, 0, 0\}$ | $|\mathcal{O}| = 576$ | $C_{16}^{34}$ | $C_{15}^{34} \cup \{0, 1, 1, 1, 0, 1\}$ | $-2C_1 \times C_2$ |
| $C_{16}^{31}$ | $C_{15}^{31} \cup \{0, 1, 0, 1, 0, 0\}$ | $|\mathcal{O}| = 720$ | $C_{16}^{36}$ | $C_{15}^{36} \cup \{0, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 48$ |
| $C_{16}^{33}$ | $C_{15}^{33} \cup \{0, 1, 0, 0, 1, 0\}$ | $|\mathcal{O}| = 720$ | $C_{16}^{38}$ | $C_{15}^{38} \cup \{0, 1, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 32$ |
| $C_{16}^{35}$ | $C_{15}^{35} \cup \{0, 1, 0, 0, 0, 0\}$ | $|\mathcal{O}| = 720$ | $C_{16}^{40}$ | $C_{15}^{40} \cup \{0, 1, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 192$ |
| $C_{16}^{37}$ | $C_{15}^{37} \cup \{0, 1, 0, 0, 0, 0\}$ | $|\mathcal{O}| = 20160$ | $C_{16}^{42}$ | $C_{15}^{42} \cup \{0, 1, 0, 1, 0, 0\}$ | $|\mathcal{O}| = 32$ |
| $C_{16}^{39}$ | $C_{15}^{39} \cup \{0, 1, 0, 0, 0, 0\}$ | $|\mathcal{O}| = 192$ | $C_{16}^{44}$ | $C_{15}^{44} \cup \{1, 0, 1, 1, 0, 0\}$ | $C_2 \times C_2 \times C_2$ |
| $C_{16}^{41}$ | $C_{15}^{41} \cup \{0, 1, 0, 0, 0, 0\}$ | $|\mathcal{O}| = 32$ | $C_{16}^{46}$ | $C_{15}^{46} \cup \{1, 0, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 36864$ |
| $C_{16}^{43}$ | $C_{15}^{43} \cup \{0, 1, 1, 0, 0, 0\}$ | $|\mathcal{O}| = 576$ | $C_{16}^{48}$ | $C_{15}^{48} \cup \{0, 0, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 60$ |
| $C_{16}^{45}$ | $C_{15}^{45} \cup \{0, 1, 1, 0, 0, 0\}$ | $|\mathcal{O}| = 1024$ | $C_{16}^{46}$ | $C_{15}^{46} \cup \{0, 0, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 60$ |
| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $|\mathcal{O}|$ |
|---|---|---|---|
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 1, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 17$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 0, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 28$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 39$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 0, 1, 0, 1)\}$ | $|\mathcal{O}| = 50$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 1, 0, 1)\}$ | $|\mathcal{O}| = 51$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 0, 1, 1, 0)\}$ | $|\mathcal{O}| = 52$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 1, 1, 0)\}$ | $|\mathcal{O}| = 53$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 54$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 55$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 0, 1, 1, 1, 0)\}$ | $|\mathcal{O}| = 56$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 1, 1, 1, 0)\}$ | $|\mathcal{O}| = 57$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 0, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 58$ |
| $C_7^{17}$ | $C_7^{16} \cup \{(0, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 59$ |

| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $|\mathcal{O}|$ |
|---|---|---|---|
| $C_7^{16} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 50$ |
| $C_7^{16} \cup \{(0, 0, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 51$ |
| $C_7^{16} \cup \{(0, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 52$ |
| $C_7^{16} \cup \{(0, 0, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 53$ |
| $C_7^{16} \cup \{(0, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 54$ |
| $C_7^{16} \cup \{(0, 0, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 55$ |
| $C_7^{16} \cup \{(0, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 56$ |
| $C_7^{16} \cup \{(0, 0, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 57$ |
| $C_7^{16} \cup \{(0, 1, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 58$ |
| $C_7^{16} \cup \{(0, 0, 1, 1, 1, 0)\}$ | $|\mathcal{O}| = 59$ |

12
| $\kappa$ | CORRESPONDS TO | STABILIZER | $\kappa$ | CORRESPONDS TO | STABILIZER |
|-------|----------------|------------|-------|----------------|------------|
| $C_{1}^{1}$ | $C_{17} \cup \{(1, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 10752$ | $C_{2}^{1}$ | $C_{17} \cup \{(1, 0, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 128$ |
| $C_{1}^{8}$ | $C_{17} \cup \{(0, 1, 1, 0, 0, 0)\}$ | $|\mathcal{O}| = 64$ | $C_{4}^{1}$ | $C_{17} \cup \{(0, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 192$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(0, 1, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 384$ | $C_{6}^{1}$ | $C_{17} \cup \{(0, 1, 0, 1, 1, 0)\}$ | $|\mathcal{O}| = 48$ |
| $C_{1}^{35}$ | $C_{17} \cup \{(1, 0, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 48$ | $C_{7}^{1}$ | $C_{17} \cup \{(0, 0, 1, 0, 1, 1)\}$ | $C_{2} \times C_{2}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 144$ | $C_{8}^{1}$ | $C_{17} \cup \{(0, 1, 0, 0, 1, 0)\}$ | $D_{4}$ |
| $C_{1}^{1}$ | $C_{17} \cup \{(1, 0, 1, 0, 0, 1)\}$ | $C_{2} \times C_{2} \times C_{2}$ | $C_{10}^{1}$ | $C_{17} \cup \{(1, 0, 1, 0, 0, 0)\}$ | $D_{4} \times C_{2}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 0, 0, 1, 1, 0)\}$ | $|\mathcal{O}| = 64$ | $C_{12}^{1}$ | $C_{17} \cup \{(1, 0, 0, 0, 1, 0)\}$ | $D_{6}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 6144$ | $C_{14}^{1}$ | $C_{17} \cup \{(1, 0, 0, 1, 1, 0)\}$ | $|\mathcal{O}| = 32$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(0, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 1536$ | $C_{16}^{1}$ | $C_{17} \cup \{(1, 0, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 384$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 0, 0, 0, 1, 0)\}$ | $D_{6}$ | $C_{18}^{1}$ | $C_{17} \cup \{(1, 1, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 96$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 576$ | $C_{20}^{1}$ | $C_{17} \cup \{(1, 0, 0, 1, 1, 0)\}$ | $C_{2} \times C_{2}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 0, 0, 1, 1, 0)\}$ | $|\mathcal{O}| = 1356$ | $C_{22}^{1}$ | $C_{17} \cup \{(1, 0, 0, 0, 0, 1)\}$ | $D_{5}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 0, 0, 0, 1, 0)\}$ | $D_{6} \times C_{2}$ | $C_{24}^{1}$ | $C_{17} \cup \{(1, 1, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 1152$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 384$ | $C_{26}^{1}$ | $C_{17} \cup \{(1, 0, 1, 0, 0, 0)\}$ | $D_{4}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 0, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 96$ | $C_{28}^{1}$ | $C_{17} \cup \{(1, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 32$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 43008$ | $C_{30}^{1}$ | $C_{17} \cup \{(1, 0, 0, 1, 0, 0)\}$ | $D_{4}$ |
| $C_{1}^{18}$ | $C_{17} \cup \{(0, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 2304$ | $C_{32}^{1}$ | $C_{17} \cup \{(1, 1, 0, 0, 1, 0)\}$ | $|\mathcal{O}| = 32$ |
| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|---|---|---|---|---|---|
| $C_{19}^2$ | $C_{18}^2 \cup \{0, 1, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 1152$ | $C_{19}^2$ | $C_{18}^9 \cup \{0, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 36$ |
| $C_{19}^3$ | $C_{18}^7 \cup \{0, 0, 1, 0, 1, 1\}$ | $S_3$ | $C_{19}^4$ | $C_{18}^9 \cup \{0, 1, 0, 1, 1, 0\}$ | $|\mathcal{O}| = 48$ |
| $C_{19}^5$ | $C_{18}^3 \cup \{0, 1, 0, 1, 1, 0\}$ | $|\mathcal{O}| = 32$ | $C_{19}^6$ | $C_{18}^{15} \cup \{1, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 9216$ |
| $C_{19}^7$ | $C_{18}^9 \cup \{0, 1, 1, 1, 0, 0\}$ | $|\mathcal{O}| = 144$ | $C_{19}^8$ | $C_{18}^3 \cup \{1, 1, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 192$ |
| $C_{19}^9$ | $C_{18}^9 \cup \{1, 1, 0, 1, 0, 0\}$ | $|\mathcal{O}| = 192$ | $C_{19}^{10}$ | $C_{18}^4 \cup \{1, 0, 1, 0, 1, 0\}$ | $|\mathcal{O}| = 48$ |
| $C_{19}^{11}$ | $C_{18}^4 \cup \{1, 0, 1, 1, 1\}$ | $|\mathcal{O}| = 48$ | $C_{19}^{12}$ | $C_{18}^4 \cup \{0, 0, 1, 0, 1\}$ | $|\mathcal{O}| = 96$ |
| $C_{19}^{13}$ | $C_{18}^2 \cup \{1, 0, 1, 1, 1\}$ | $|\mathcal{O}| = 48$ | $C_{19}^{14}$ | $C_{18}^2 \cup \{1, 0, 0, 1, 0, 0\}$ | $|\mathcal{O}| = 32$ |
| $C_{19}^{15}$ | $C_{18} \cup \{0, 0, 1, 1, 1\}$ | $|\mathcal{O}| = 48$ | $C_{19}^{16}$ | $C_{18} \cup \{0, 1, 0, 1, 0, 0\}$ | $D_8 \times C_2$ |
| $C_{19}^{17}$ | $C_{18}^3 \cup \{0, 1, 0, 1, 1, 1\}$ | $|\mathcal{O}| = 2304$ | $C_{19}^{18}$ | $C_{18}^5 \cup \{1, 0, 0, 1, 1, 0\}$ | $D_4$ |
| $C_{19}^{19}$ | $C_{18}^3 \cup \{0, 0, 0, 1, 1\}$ | $D_4 \times C_2$ | $C_{19}^{20}$ | $C_{18}^2 \cup \{0, 1, 1, 1, 1, 0\}$ | $D_4 \times C_2$ |
| $C_{19}^{21}$ | $C_{18}^2 \cup \{1, 1, 1, 1, 1, 1\}$ | $|\mathcal{O}| = 9216$ | $C_{19}^{22}$ | $C_{18}^3 \cup \{1, 1, 0, 1, 1, 1\}$ | $|\mathcal{O}| = 23040$ |
| $C_{19}^{23}$ | $C_{18} \cup \{1, 1, 0, 1, 1\}$ | $|\mathcal{O}| = 96$ | $C_{19}^{24}$ | $C_{18} \cup \{0, 0, 1, 1, 1, 1\}$ | $|\mathcal{O}| = 256$ |
| $C_{19}^{25}$ | $C_{18} \cup \{1, 1, 0, 1, 1\}$ | $|\mathcal{O}| = 96$ | $C_{19}^{26}$ | $C_{18} \cup \{0, 1, 0, 1, 1, 0\}$ | $D_6$ |
| $C_{19}^{27}$ | $C_{18} \cup \{0, 1, 1, 1, 0\}$ | $|\mathcal{O}| = 36$ | $C_{19}^{28}$ | $C_{18} \cup \{1, 1, 0, 0, 0\}$ | $C_2 \times C_2 \times C_2$ |
| $C_{19}^{29}$ | $C_{18} \cup \{0, 0, 1, 1, 0\}$ | $|\mathcal{O}| = 36$ | $C_{19}^{30}$ | $C_{18} \cup \{1, 0, 0, 1, 1\}$ | $|\mathcal{O}| = 36$ |
| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|---|---|---|---|---|---|
| $C_{21}^2$ | $C_{20} \cup \{(0, 1, 0, 0, 0, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 32$ | $C_{21}^2$ | $C_{20} \cup \{(0, 0, 0, 1, 1, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 192$ |
| $C_{21}^4$ | $C_{20} \cup \{(0, 1, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 768$ | $C_{21}^4$ | $C_{20} \cup \{(0, 0, 1, 1, 1, 0, 0, 0, 0, 0)\}$ | $|\mathcal{O}| = 144$ |
| $C_{21}^6$ | $C_{20} \cup \{(1, 0, 1, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 21504$ | $C_{21}^6$ | $C_{20} \cup \{(0, 0, 1, 0, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 384$ |
| $C_{21}^8$ | $C_{20} \cup \{(0, 0, 1, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 48$ | $C_{21}^8$ | $C_{20} \cup \{(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 192$ |
| $C_{21}^{10}$ | $C_{20} \cup \{(0, 0, 0, 0, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 3072$ | $C_{21}^{10}$ | $C_{20} \cup \{(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 768$ |
| $C_{21}^{12}$ | $C_{20} \cup \{(0, 0, 0, 0, 0, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 11520$ | $C_{21}^{12}$ | $C_{20} \cup \{(0, 0, 0, 0, 0, 0, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 1152$ |
| $C_{21}^{14}$ | $C_{20} \cup \{(1, 0, 1, 1, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 72$ | $C_{21}^{14}$ | $C_{20} \cup \{(1, 0, 1, 1, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 72$ |
| $C_{21}^{16}$ | $C_{20} \cup \{(0, 1, 1, 0, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 1008$ | $C_{21}^{16}$ | $C_{20} \cup \{(0, 1, 1, 0, 1, 1, 1, 1, 1, 1)\}$ | $|\mathcal{O}| = 1008$ |

| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|---|---|---|---|---|---|
| $C_{22}^2$ | $C_{21} \cup \{(0, 1, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 32$ | $C_{22}^2$ | $C_{21} \cup \{(0, 1, 0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 192$ |
| $C_{22}^4$ | $C_{21} \cup \{(0, 1, 0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 120$ | $C_{22}^4$ | $C_{21} \cup \{(0, 0, 1, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 120$ |
| $C_{22}^6$ | $C_{21} \cup \{(1, 0, 0, 0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 2688$ | $C_{22}^6$ | $C_{21} \cup \{(1, 0, 0, 0, 0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 2688$ |
| $C_{22}^8$ | $C_{21} \cup \{(1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 72$ | $C_{22}^8$ | $C_{21} \cup \{(1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1)\}$ | $|\mathcal{O}| = 72$ |

| $\mathcal{K}$ | CORRESPONDS TO | STABILIZER | $\mathcal{K}$ | CORRESPONDS TO | STABILIZER |
|---|---|---|---|---|---|
| $C_{23}^2$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 32$ | $C_{23}^2$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 32$ |
| $C_{23}^4$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 120$ | $C_{23}^4$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 120$ |
| $C_{23}^6$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 72$ | $C_{23}^6$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 72$ |
| $C_{23}^8$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 1008$ | $C_{23}^8$ | $C_{22} \cup \{(1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 1008$ |
| $\mathcal{K}$ | CORRESPONDS TO                     | STABILIZER |
|---------|-----------------------------------|------------|
| $C_{25}^1$ | $C_{24} \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 720$ |
| $C_{25}^2$ | $C_{24} \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 144$ |
| $C_{25}^3$ | $C_{24} \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 1152$ |
| $C_{25}^4$ | $C_{24} \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 768$ |
| $C_{25}^5$ | $C_{24} \cup \{(0, 1, 0, 0, 1, 1)\}$ | $|\mathcal{O}| = 64512$ |
| $C_{26}^1$ | $C_{25} \cup \{(1, 1, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 768$ |
| $C_{26}^2$ | $C_{25} \cup \{(1, 0, 1, 1, 0, 0)\}$ | $|\mathcal{O}| = 720$ |
| $C_{26}^3$ | $C_{25} \cup \{(1, 0, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 11520$ |
| $C_{26}^4$ | $C_{25} \cup \{(0, 0, 1, 1, 0, 1)\}$ | $|\mathcal{O}| = 18432$ |
| $C_{27}^1$ | $C_{26} \cup \{(1, 0, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 1920$ |
| $C_{27}^2$ | $C_{26} \cup \{(1, 1, 0, 1, 0, 0)\}$ | $|\mathcal{O}| = 9216$ |
| $C_{28}^1$ | $C_{27} \cup \{(0, 1, 1, 0, 1, 0)\}$ | $|\mathcal{O}| = 258048$ |
| $C_{28}^2$ | $C_{27} \cup \{(0, 1, 0, 1, 0, 1)\}$ | $|\mathcal{O}| = 9216$ |
| $C_{29}^1$ | $C_{28} \cup \{(1, 0, 1, 0, 0, 1)\}$ | $|\mathcal{O}| = 64512$ |
| $C_{30}^1$ | $C_{29} \cup \{(0, 1, 0, 1, 0, 1)\}$ | $|\mathcal{O}| = 645120$ |
| $C_{31}^1$ | $C_{30} \cup \{(1, 1, 0, 0, 0, 1)\}$ | $|\mathcal{O}| = 9999360$ |
| $C_{32}^1$ | $C_{31} \cup \{(0, 0, 1, 1, 0, 1)\}$ | $|\mathcal{O}| = 319979520$ |
All the complete caps of sizes 13, 17, 18 and 20 have projective frame. The 32- complete cap has no projective frame.

References

[1] A. Davydov, G. Faina, S. Marcugini and F. Pambianco, On size of complete caps in projective spaces $PG(n, q)$ and arcs in planes $PG(2, q)$, Journal of Geometry, published online 18/7/2009.

[2] G. Faina, S. Marcugini, A. Milani and F. Pambianco, The size $k$ of the complete $k$-caps in $PG(n, q)$ for small $q$ and $3 \leq n \leq 5$, Ars Combinatoria 50 (1998), 235-243.

[3] M. Hall and J. K. Senior, The group of order $2^n$ ($n \leq 6$), Macmillan, New York, 1964.

[4] J. W. P. Hirschfeld, Projective geometries over finite fields, Claredon Press, Oxford, 1979.

[5] J. W. P. Hirschfeld, Finite projective spaces of three dimension, Claredon Press, Oxford, 1985.