Coherent QCD multiple scattering in proton-nucleus collisions

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We argue that high energy proton-nucleus (p+A) collisions provide an excellent laboratory for studying nuclear size enhanced parton multiple scattering where power corrections to the leading twist perturbative QCD factorization approach can be systematically computed. We identify and resum these corrections and calculate the centrality and rapidity dependent nuclear suppression of single and double inclusive hadron production at moderate transverse momenta. We demonstrate that both spectra and dihadron correlations in p+A reactions are sensitive measures of such dynamical nuclear attenuation effects.

PACS numbers: 12.38.Cy; 12.39.St; 24.85.+p

Copious experimental data from central Au+Au reactions at the Relativistic Heavy Ion Collider (RHIC) has generated tremendous excitement by pointing to the possible creation of a deconfined state of QCD with energy density as high as 100 times normal nuclear matter density. In order to diagnose its properties, we first need to quantitatively understand the multiple scattering between a partonic probe and the partons of the medium in simpler strongly interacting systems, for example p+A. The importance of such theoretical investigations has been recently stressed by the measured in-medium nuclear size enhancement in p+A collisions, which has so far been considered a challenge for the perturbative QCD factorization approach. Coherent multiple scattering and the well-studied elastic multiple scattering co-exist in nuclear collisions and their relative role depends on the probes (or observables) and the underlying dynamics. Elastic scattering has played an important role in understanding the k_T-broadening and the Cronin effect. It dominates when the probe is so localized that the quantum correlation between the different scattering centers can be neglected. In terms of the factorization approach, contributions from coherent multiple scattering to a physical cross section are power suppressed in comparison to the leading hard partonic processes. However, as we demonstrate below, when enhanced by the large nuclear size these may become important at small and moderate transverse momenta at RHIC.

Hard scattering in nuclear collisions requires one large momentum transfer Q ≈ xP ≫ Q_{QCD} with parton momentum fraction x and beam momentum P. A simple example, shown in Fig. (a), is the lepton-nucleus deeply inelastic scattering (DIS). The effective longitudinal interaction length probed by the virtual photon of momentum q^L is characterized by 1/xP. If the momentum fraction of an active initial-state parton x ≪ x_c = 1/2m_Nr_0 ≈ 0.1 with nucleon mass m_N and radius r_0, it could cover several Lorentz contracted nucleons of longitudinal size ∼ 2r_0(2P/Q^2) in a large nucleus. In the photon-nucleus frame, Fig. (a), the scattered parton of momentum l will interact coherently with the partons from different nucleons at the same impact parameter. The on-shell condition for l fixes the initial-state parton’s momentum fraction to the Bjorken variable x = x_B ≈ Q^2/(2P · q) but additional final state scattering forces the rescaling x = x_B(1 + ξ^2(A^{1/3} − 1)/Q^2) in p+A reactions. Here, A is the nuclear atomic weight and the scale of power corrections

\[ ξ^2 \approx \frac{3πα_s(Q)}{8r_0^2} \langle p|\tilde{F}^2|p⟩, \]

FIG. 1: Coherent multiple scattering of the struck parton in deeply inelastic scattering (a) and in proton-nucleus collisions (b). A set of resummed two gluon exchange diagrams (c) in p+A reactions.
with the matrix element \( \langle p | \hat{F}^2 | p \rangle = \frac{1}{2} \lim_{x \to 0} x G(x, Q^2) \).
In Ref. [3], we demonstrated that the hard parton interactions for any number of coherent multiple scattering in DIS are infrared safe, which explicitly verifies the factorization theorem. For \( \xi^2 = 0.09 - 0.12 \text{ GeV}^2 \) the calculated reduction in the DIS structure functions, known as nuclear shadowing, is consistent with the isolated reduction in the DIS structure functions, known as factorization theorem. For DIS are infrared safe, which explicitly verifies the factorization in hadronic collisions was proved to be valid at the ruin the factorization [10, 11]. Nevertheless, factorization in hadronic collisions was proved to be valid at the leading power [3] as well as the leading power corrections \( O(1/Q^2) \) [12]. It has been argued [13] that factorization can be extended to the calculation of \( (A^{1/3}/Q^2)^N \)-type nuclear size enhanced power corrections in p+A collisions at all powers of \( N \).

In DIS, as shown in Fig. 1(a), only diagrams with multiple final-state interactions to the scattered parton lead to medium size enhanced power corrections [3]. On the other hand, in p+A collisions, shown in Fig. 1(b), all diagrams with either final state and/or initial-state multiple interactions could in principle contribute. However, the coherent multiple scattering is a result of an extended probe size, which is determined by the momentum exchange of the hard collisions. As shown in Fig. 1(b), once we fix the momentum fractions \( x_a \) and \( z_1 \), the effective interaction region is determined by the momentum exchange \( q^0 = (x_a P_a - P_c/z_1)^0 \). In the head-on frame of \( q - P_b \), the scattered parton of momentum \( \ell \) interacts coherently with partons from different nucleons at the same impact parameter. Interactions that have taken place between the partons from the nucleus and the incoming parton of momentum \( x_a P_a \) and/or the outgoing parton of momentum \( P_c/z_1 \) at a different impact parameter are much less coherent and actually dominated by the independent elastic scattering [14].

We pursue the analogy with the DIS results of Ref. [3] and express the lowest order single and double inclusive hadron production cross sections as follows:

\[
\frac{d\sigma_{h_1/N}}{dy_1 d^2p_{T_1}} = \sum_{abcd} \int \frac{dz_1}{z_1} D_{h_1/c}(z_1) \int dx_a \phi_{a/N}(x_a) \left[ \frac{1}{x_a S + U/z_1} \right] \frac{\alpha^2}{S} \int dx_b \delta(x_b - \bar{x}_b) F_{ab \to cd}(x_b),
\]

\[
\frac{d\sigma_{h_1h_2}}{dy_1 d^2p_{T_1} d^2p_{T_2}} = \delta(\Delta \varphi - \pi) \sum_{abcd} \int \frac{dz_1}{z_1} D_{h_1/c}(z_1) D_{h_2/d}(z_2) \phi_{a/N}(\bar{x}_a) \frac{\alpha^2}{S^2} \int dx_b \delta(x_b - \bar{x}_b) F_{ab \to cd}(x_b),
\]

where \( \sum_{abcd} \) runs over all parton flavors, \( \phi_{a/N}(x_a) \) are the parton distribution functions (PDFs) [16] and \( D_{h_1/c}(z_1) \), \( D_{h_2/d}(z_2) \) are the fragmentation functions (FFs) [16]. The dependence on the small momentum fraction \( x_b \) is isolated in the function

\[
F_{ab \to cd}(x_b) \equiv \frac{\phi_{b/N}(x_b)}{x_b} |M_{ab \to cd}|^2,
\]

where the \( 2 \to 2 \) on shell squared partonic matrix elements \( |M_{ab \to cd}|^2 \) are given in [17]. In Eq. 2 the invariants \( T = -p_{T_1} \sqrt{S} \epsilon^{-y_1}, U = -p_{T_1} \sqrt{S} \epsilon^{y_1} \) and the momentum fraction from the nucleus \( \bar{x}_b = -T/(z_1 S + U/x_a) \). In Eq. 3 \( \Delta \varphi = \varphi_2 - \varphi_1, z_2 = z_1 p_{T_2}/p_{T_1} \), and the solution for the momentum fractions carried by the partons from the nucleus and the nucleon are \( \bar{x}_b = p_{T_1} (\epsilon^{y_1} + \epsilon^{y_2})/z_1 \sqrt{S} \) and \( \bar{x}_a = p_{T_1} (\epsilon^{y_1} + \epsilon^{y_2})/z_1 \sqrt{S} \), respectively. In both Eqs. 2 and 3 the possible K-factor for the lowest order formula and all renormalization and factorization scale dependences are suppressed. Ratios of cross sections in p+A and p+p reactions are insensitive to their choice.

As in [3], the leading \( A^{1/3} \)-enhanced contribution to the cross sections comes from the coherent scatterings with N nucleons shown in Figs. 1(b) and 1(c). The two gluon exchange with an individual nucleon gives rise to the characteristic scale of high twist corrections \( \xi^2 = 0.09 - 0.12 \text{ GeV}^2 \), defined in Eq. 1. For fixed \( N \), the final state interactions of parton \( d \), illustrated in Fig 1(c), replace the integral over the small \( x_b \) dependent function \( F_{ab \to cd}(x_b) \) in Eqs. 2 and 3 by

\[
\int dx_b \left( \frac{-1}{N!} \right)^N C_d \bar{x}_b \frac{\xi^2}{1 - T} (A^{1/3} - 1)^N \times \frac{d^N \delta(x_b - \bar{x}_b)}{dx_b^N} F_{ab \to cd}(x_b).
\]

Here, the hard scale \( t = q^2 = (x_a P_a - P_c/z_1)^2 \) and \( C_d \) is a color factor depending on the flavor of parton \( d \). \( C_{q(\bar{q})} = 1 \) and \( C_q = C_A/C_F = 9/4 \) for quark (antiquark) and gluon, respectively. By resumming all contributions in Fig. 1(c), \( N = 0, \ldots, \infty \) in Eq. 5, we derive the nuclear modified cross sections for single and double inclusive hadron production in p+A collisions. These have the same form as Eqs. (2) and (3) with the following
Centraly dependence is implicit in Eq. (7) and the attenuation increases in the forward rapidity region. The attenuation is equivalent to a shift of rapidities

\[ y^\text{min.bias} = 5.6 \text{ fm} \]

Similarly to the DIS case, resumming the coherent scattering with multiple nucleons is equivalent to a shift of the momentum fraction of the active parton from the nucleus and leads to a net suppression of the cross sections. The \( t \)-dependence of this shift in Eq. (8) indicates that the attenuation increases in the forward rapidity region.

We consider d+Au reactions at RHIC at \( \sqrt{S} = 200 \text{ GeV} \) with the deuteron and the nucleus moving with rapidities \( y_{\text{max}} = \pm 5.4 \), respectively. Small \( p_T = 1 \text{ GeV} \) and forward \( y_1 = 4 \) hadron production corresponds to nuclear \( x_0 \geq 1.3 \times 10^{-4} \) where coherence and our calculated power corrections will certainly become important. Dynamical nuclear effects in multi-particle production can be studied through the ratio

\[ R_{AB}^{n} = \frac{d\sigma_{d^hA^h}/dy_1 \cdots dy_n/d^2p_T_1 \cdots d^2p_T_n}{(N_{\text{coll}})^n/d\sigma_{dA^h}/dy_1 \cdots dy_n/d^2p_T_1 \cdots d^2p_T_n}. \]

Centrality dependence is implicit in Eq. (7) and the modified cross section per average collision \( d\sigma_{dA^h}^{n}/(N_{dA}^{\text{coll}}) \) can be calculated from Eqs. (2) and (3) via the substitution Eq. (4). The necessary scaling with impact parameter, \( b \), of the enhanced power corrections can be obtained through the nuclear thickness function, \( A^{1/3} \rightarrow A^{1/3}/T_A(b) \), where \( T_A(b) = \int_{b_{\text{min.bias}}}^{\infty} \rho_A(z,b)dz \).

The top panels of Fig. 2 show the rapidity and transverse momentum dependence of \( R_{dA}^{n} \) for minimum bias collisions. The amplification of the suppression effect at forward \( y_1 \) comes from the build-up of coherence and the decrease of the Mandelstam variable \( (-t) \). At high \( p_T \), the attenuation is found to disappear in accord with the QCD factorization theorems. BRAHMS data on \( h^+ \) production at forward rapidity is shown for comparison. Our result should be contrasted with a recent numerical study of non-linear parton evolution that found a factor of 2 suppression of the hadron production in p+A collisions at \( p_T = 10^{-5} \text{ GeV} \) but negligible rapidity dependence of \( R_{dA}^{n} \) at small \( p_T \). The bottom panels of Fig. 2 show the growth of the nuclear attenuation effect with centrality.

In the double collinear approximation, Eq. (8), the leading hadrons are produced strictly back-to-back. The acoplanarity, \( \Delta \phi \neq \pi \), arises from soft gluon resummation next-to-leading order corrections and in the presence of nuclear matter - transverse momentum diffusion. We consider dihadron correlations associated with hard scattering partonic processes that have the bulk many-body collision background subtracted and can be approximated by near-side and away-side Gaussians:

\[ C_2(\Delta \phi) = \frac{1}{\text{Norm}} \frac{d^2N_{\text{dijet}}}{d\Delta \phi} \approx \frac{A_{\text{Near}}}{\sqrt{2\pi \sigma_{\text{Near}}}} e^{-\frac{\Delta \phi^2}{2\sigma_{\text{Near}}^2}} + \frac{A_{\text{Far}}}{\sqrt{2\pi \sigma_{\text{Far}}}} e^{-\frac{(\Delta \phi-\lambda)^2}{2\sigma_{\text{Far}}^2}}. \]

In Eq. (8) the near-side width \( \sigma_{\text{Near}} \) and away-side width \( \sigma_{\text{Far}} \) are independent by jet fragmentation. If the strength of the away-side correlation function in elementary N+N collisions is normalized to unity, dynamical quark and gluon shadowing in p+A reactions will manifest in the attenuation of the area \( A_{\text{Far}} = R_{pA}^{h_1h_2}(b) \). The away-side width \( \sigma_{\text{Far}} \) is related to the accumulated dijet transverse momentum squared in a plane perpendicular to the collision axis:

\[ \langle k_T^2 \rangle_{\text{dijet}} = 2 \langle k_T^2 \rangle_{\text{vac}} + \sum_{i,a,c,d} \frac{\mu^2 L_{\text{eff}}}{\lambda} (1 + \tanh^2 y_i). \]

In Eq. (9) \( y_i \) are the interacting parton rapidities and \( \mu^2 L_{\text{eff}}/\lambda = 0.72 \text{ GeV}^2 \) characterizes the elastic scattering power of cold nuclear matter in minimum bias d+Au collisions and varies with centrality as \( T_{A(b)}(b_{\text{min.bias}}) \). The vacuum broadening \( \langle k_T^2 \rangle_{\text{vac}} \) is taken from the data.

In triggering on a large transverse momentum hadron, the nuclear modification from enhanced power corrections and transverse momentum diffusion is reflected in
the away-side correlation function. For large vacuum broadening, \( k_f^2 \nu \) = 2.5 – 3.5 GeV\(^2\), elastic scattering may lead to only a moderate additional growth of \( \sigma_{\text{FAR}} \). The left panels of Fig. 3 show that for \( p_T = 4 \) GeV, \( p_T = 2 \) GeV the dominant effect in \( C_2(\Delta \varphi) \) is a small increase of the broadening with centrality, compatible with the PHENIX \([21]\) and STAR \([22]\) measurements. Even at forward rapidity, such as \( y_1 = 2 \), the effect of power corrections in this transverse momentum range is not very significant. At small \( p_T = 1.5 \) GeV, \( p_T = 1 \) GeV, shown in the right hand side of Fig. 3, the apparent width of the away-side \( C_2(\Delta \varphi) \) is larger. In going from midrapidity, \( y_1 = y_2 = 0 \), to forward rapidity, \( y_1 = 4, y_2 = 0 \), we find a significant reduction by a factor of 3 - 4 in the strength of dihadron correlations from the nuclear enhanced power corrections. We emphasize again that this result depends sensitively on centrality and the choice of transverse momenta and disappears at high \( p_T \).

The dynamical cross section attenuation calculated here does not contradict, instead, complements the effects from a possible modification of the nuclear parton distribution functions (nPDFs), known as leading twist shadowing \([23]\). In our formalism, one can include both effects by replacing the PDFs in Eqs. \([2]\) and \([3]\) with the corresponding nPDFs and applying Eq. \([6]\). The weaker scale dependence of nPDFs will thus slow down the disappearance of the nuclear suppression at high \( p_T \) in Fig. 3. A combined global QCD analysis would, however, be required and is beyond the scope of this Letter.

In conclusion, we presented a systematic approach to the calculation of coherent QCD multiple scattering and resummed the nuclear enhanced power correction to the single and double inclusive hadron production cross sections in p+A reactions. At low \( p_T \) we find a sizable suppression, which grows with rapidity and centrality. At high \( p_T \) the nuclear modification disappears in accord with the QCD factorization theorems. We demonstrated that both particle spectra and dihadron correlations are sensitive measures of such dynamical attenuation of the parton interaction rates.

Our approach, with its intuitive and transparent results, can be easily applied to study the nuclear modification of other physical observables in p+A reactions and its predictions can be readily tested against RHIC data. The systematic incorporation of coherent power corrections provides a tool to address the most interesting transition region between “hard” and “soft” physics in hadron-nucleus collisions. It allows for the extension of perturbative QCD calculations to relatively small transverse momenta and for bridging the gap between the parton model picture and the possible onset of gluon saturation \([2, 24]\) at very large collider energies and very small values of nuclear \( x \).

This work is supported in part by the US Department of Energy under Grant No. DE-FG02-87ER40371 and by the J. Robert Oppenheimer fellowship of the Los Alamos National Laboratory.

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