Transverse electric current induced by optically injected spin current in cross-shaped InGaAs/InAlAs system

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We examine electric response of a linearly polarized light normally shed on a cross-shaped quasi 2-dimensional InGaAs/InAlAs system with structure inversion asymmetry. The photo-excited conduction electrons carry a pure spin current with in-plane spin polarization due to the Rashba spin-orbit interaction. We use Landauer-Büttiker formalism to show that this spin current induces two inward or outward transverse charge currents, which are observable in experiments. This effect may serve as an experimental probe of certain types of spin current.

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Spin coherent transport of conduction electrons in semiconductor heterostructures is currently an emerging subject due to its possible application in a new generation of electronic devices [1]. There have been considerable efforts to achieve spin polarized current or pure spin current in semiconductors such as injection from ferromagnetic contact [2], quantum spin pump [3], spin Hall effect [4], and spin like Andreev reflection [5]. Optical injection of spin current is largely based on the fact that the spin polarized carriers in conduction band can be injected in semiconductors via absorption of the circularly or linearly polarized light [6, 7]. While there are successful ways to inject or generate spin current, its detection is still a subtle problem. Despite the optical methods which indicated spin accumulation due to the spin current [8], it is tempting to probe spin current by measuring its electric effects [9, 10, 11, 12]. In this Letter, we theoretically study electric transverse current driven by an optically injected spin current in a two-dimensional electron gas of InGaAs/InAlAs with structure inversion asymmetry. A linearly polarized light pumps electrons from valence to conduction bands, which induces a pure spin current with in-plane polarization due to the spin-orbit coupling. The Hall effect related to this spin current in a cross-shaped mesoscopic system is explored, which yields two measurable inward or outward electric transverse currents. By using the Landauer-Büttiker formalism we provide an estimate of the electric transverse current measurable in experiments.

We begin with the optical injection of spin current in quasi-2D InGaAs/InAlAs with structural inversion asymmetry. The low energy band structure is well known and plotted schematically in Fig. 1(a). The conduction electron can be described by an effective Hamiltonian,

$$H_{\text{conduction}} = \frac{p^2}{2m_e} - \frac{\alpha}{\hbar} (p \times \sigma) \cdot \hat{z} + V(z)$$  \hspace{1cm} (1)

where $\sigma$ are the Pauli matrices, $V(z)$ is the asymmetric confining potential perpendicular to the sample ($x$-$y$) plane, and $\alpha$ is the strength of the Rashba spin-orbit coupling. The zero-field splitting of the conduction band arises because of the Rashba coupling, and electrons are spin polarized normal to the momentum in each subband, as shown in Fig. 1(b). The electrons in the valence band near the $\Gamma$ point can be described by the Luttinger Hamiltonian,

$$H_{\text{valence}} = (\gamma_1 + \frac{5}{2} \gamma_2) \frac{p^2}{2m_e} + 2\gamma_2 \frac{(p \cdot S)^2}{2m_e} + V(z)$$  \hspace{1cm} (2)

where $m_e$ is the free electron mass, $\gamma_1$ and $\gamma_2$ are two Kohn-Luttinger parameters, which are taken to be $\gamma_1 = 7.0$, and $\gamma_2 = 1.9$ in this calculation [13]. $S$ represent three $4 \times 4$ spin-3/2 matrices. For simplicity, we approximate $V(z)$ in Eqs. (1) and (2) by an infinite potential wall with a width of $d = 10\text{nm}$. The wave functions for the holes in the valence band can be obtained by a truncation method. We diagonalize $H_{\text{valence}}$ in a truncated Hilbert space only including the lowest $N$ basis states.
In the present paper, we take $N = 80$, which is accurate enough for the lowest 4 hole subbands.

The process of optical excitation is schematically illustrated with the $e1$ and HH1 subbands in Fig. When a linearly polarized light is shed normally onto the sample plane, the electrons are pumped from the HH1 subband of the valence band to the $e1$ subband of the conduction band via direct optical absorption provided that the photon energy is higher than the band gap, i.e. $\hbar \omega > E_g$. Due to the Rashba spin-orbit coupling, the conduction bands are split into two subbands as shown in Fig. 1(a) and (b). A photo-excited electron has an in-plane spin polarization perpendicular to its momentum, which induces a pure spin current. The spin current operators for electrons in the conduction band and holes in the valence band can be expressed in terms of the velocity and spin operators of electrons and holes, respectively, $J_{\nu}^{(e)} = \frac{i}{2} \{ \sigma^\nu, v^{(e)} \}$ and $J_{\nu}^{(h)} = \frac{i}{2} \{ \sigma^\nu, v^{(h)} \}$, where $v^{(e)}(k)$ and $v^{(h)}(k)$ are the velocity operators for the conduction and valence bands, respectively. The total spin current from the photo-excited electrons and holes are,

$$
\langle J^\nu \rangle = \sum_k Tr \{ J_{\nu}^{(e)} \rho^{(e)}(k) - J_{\nu}^{(h)} \rho^{(h)}(k) \}
$$

where $\rho^{(e)}(k)$ and $\rho^{(h)}(k)$ are the density matrices for the conduction and valence bands, respectively. The density matrices that appear in Eq. (3) can be obtained in a relaxation time approximation. In the present study only the diagonal components of the density matrices are kept and the results can be written as

$$
\rho^{(e,h)}_{nn}(k) = \frac{\pi^{\tau_{e,h}}}{2\hbar} \sum_{m} M^*_{nn}(k) M_{nm}(k) \times \delta \{ E^{e,h}_n(k) + E^{e,h}_m(k) - \hbar \omega_{ph} \}
$$

where $\tau_{e,h}$ is the relaxation time for electrons (holes) and $M$ is the $2 \times 4$ transition matrix describing the direct optical transition between the conduction and valence subbands, which is caused by the external light. Under the dipole approximation, $M$ can be expressed by $M = D_{c1} MD_{c2}$, where $D_{c}$ and $D_{v}$ are the $2 \times 2$ and $4 \times 4$ transformation matrices which diagonalize the Hamiltonian in Eqs. and respectively, and the matrix $\tilde{M}$ is the coupling matrix in the original basis of the row $\{|S_z = \pm 1/2, -1/2\rangle\}$ and column $\{|S_z = \pm 3/2, 1/2, -1/2, -3/2\rangle\}$ as

$$
\tilde{M} = \left( \begin{array}{cc} g e^{i\phi} & 0 \\ 0 & g e^{-i\phi} / \sqrt{3} \end{array} \right)
$$

where $\phi$ is the polarization angle of the linearly polarized light and the factor $g$ is determined by the Bloch functions of electron and hole at the $\Gamma$ point. In this way the pure spin current is obtained as a function of the frequency and polarization of the light. The dominant component of the spin current flowing in the $x$ direction is $J^x_2$, which is similar to the equilibrium current proposed by Rashba. To calculate the spin current we adopt the parameters from a sample of In$_x$Ga$_{1-x}$As/In$_{0.52}$Al$_{0.48}$As, with the Rashba coupling strength $\alpha = 6.1 \times 10^{-12}$eVnm, $(\alpha/h \approx 3 \times 10^{-4} c)$, where $c$ is the speed of light, the effective mass $m_e = 0.05 m_e$, the incident light power is $100\mu$W with the wavelength $\lambda = 880\mu m$. Also we extract from the experimental data that $\lambda = 0.78 \times 10^{-3}$, which is used in calculating the density matrices. Given a quantum well with the size to be $L \times L$, the induced current $I_\nu = J_{\nu} L$ varies approximately with $\cos 2\phi$, as shown in Fig. (c) with $L = 100\mu m$, the function of $I^\nu_\nu$ is fitted to be $I^\nu_\nu \approx I_0 + I_1 \cos 2\phi$ with $I_0 = 12.32\mu A$ and $I_1 = 0.75\mu A$. And it is noticeable that there is also a non-vanishing $I^\nu_\nu$ component of the spin current, although it’s comparatively negligible.

We now turn to investigate the consequence of applying this in-plane polarized spin current to a cross-shaped mesoscopic system with the Rashba spin-orbit coupling, and it turns out that electric transverse currents will be induced in this case. Inevitably this is a situation reminiscent of the reciprocal version of the spin Hall effect proposed by Hirsch and Zhang et al, in which a transverse current was predicted to arise when a spin current polarized perpendicular to the plane flows through the scattering region. And a general Onsager relation between the spin Hall conductance and the electric Hall conductance has been found in a similar condition. While the spin current is indeed a tensor, not a conventional vector, its spin polarization as well as the symmerties of the scattering spin-orbit coupling play key roles in generating the electric transverse current, which implies that for in-plane polarized spin current, the induced transverse currents may have a quite different character. Here we show this difference from the symmetry point of view, estimate the amplitude of transverse current numerically, and then propose an experiment to observe the transverse electric currents induced by the in-plane polarized spin current in our case.

In our proposal the whole setup is carved on an InGaAs/InAlAs heterojunction with a central scattering region and four leads for measurements, as shown in the inset of Fig. 2. The transverse leads (leads $y+$ and $y-$) and the scattering region should be masked to prevent from the light shining explicitly, while the longitudinal leads (leads $x+$ and $x-$) are opened to accept the linearly polarized light to generate the incident in-plane polarized spin current. By avoiding any possible interfaces of current injection, the spin current is expected to circulate in the $x$ direction without conventional spin injection problem. Under the Landauer-Büttiker formalism, which has been extensively used in the study of quantum transport in mesoscopic systems, and will be applied to analyze the transverse currents in our case, we
simulate the generation of the spin current and the measurement of the transverse currents by assuming a proper setting of the spin-dependent chemical potential related to each lead. In detail, if we denote the effective voltage related to spin polarization $\mu$ ($\mu = \uparrow, \downarrow$) at lead $p$ by $V^\mu_p$, then we assume $V^\uparrow_p = -V^\downarrow_p = V^x_p = V^y_p = 0/2$ and $V^\downarrow_p = V^\uparrow_p = V^+ + V^y_p = 0$. It should noted that $\mu$ can be oriented in $x$, $y$ or $z$ direction (to consider either in-plane or perpendicular-to-plane polarized spin current), which will be denoted by $\mu \sim x, y$ or $z$. Given this voltage setting, the symmetry properties of the currents will be fully presented by the spin-dependent transmission functions between the leads \ref{Rashba spin-orbit term}, as we will see.

In each lead (assumed to be ideal and semi-infinite by convention) attached to the central shaded region in the inset of Fig. 2, the wave function can be expanded in terms of separate propagating modes, which are the eigenstates in the lead. In $x$, for example, the eigenstates are $\psi_{m\mu}^i(x, y) = C e^{\pm i k_m x} \phi_m(y) \otimes \chi_\mu$, where $C$ is the normalization constant, $\phi_m(y)$ is the $m$th eigenstate in the transverse dimension, $\chi_\mu$ is the spin eigenstate with $\mu = \uparrow$ or $\downarrow$ and $\pm$ denotes the mode is incoming or outgoing. The expansion coefficients of the actual wave functions in terms of these eigenstates are known as the wave amplitudes $a^a_{m\mu}^{in/out}$, which are related by an unitary matrix $a^{out}_{m\mu} = \sum_{n,\nu} s^a_{mn} a^{in}_{n\nu}$ and all the symmetries of the transport properties in the system are embedded in this S-matrix. For simplicity we assume that the quantum modes in opposite leads are symmetrical, which means that for each mode with wave vector $k_m$ and spin polarization $\mu$ in lead $x$ ($y$) there is a mode with the same wave vector and spin polarization in lead $x$ ($y$), and vice versa. In the clean limit, the Hamiltonian with the Rashba spin-orbit coupling term is obviously invariant under three unitary transformations, i.e. $H$ commutes with the time-reversal operator $T = -i\sigma_y K$, where $K$ is the complex-conjugate operator, and two combined operators $\sigma_x R_x$ and $\sigma_y R_y$, where $R_x (R_y)$ denotes the mirror reflection operator transforming $x \rightarrow -x$ ($y \rightarrow -y$). While the transformed eigenstates remain as eigenstates of the original Hamiltonian, some of the amplitudes are transformed in the following ways:

under the transformation of $T$,

$$a^{in}_{m\mu} \rightarrow (a^{out}_{m\mu})^*; \quad a^{out}_{m\mu} \rightarrow (a^{in}_{m\mu})^*;$$

under the transformation of $\sigma_i R_i$,

$$a^{in(out)}_{m\mu} \rightarrow a^{in(out)}_{m\mu}(m \in i, i; \mu \sim i)$$

$$a^{in(out)}_{m\mu} \rightarrow a^{in(out)}_{m\mu}(m \in i, i; \mu \sim j \neq i);$$

where $i = x$ or $y$, $\mu = -\mu$ and $\bar{m}$ is the counterpart of $m$ in its opposite lead. The phase factors are neglected in the above transformations because they will not be manifested in the following calculations of the transmission probabilities. The symmetries as well as the unitary condition impose constraints on the S-matrix, and thus on the transmission probability from mode $\{n, \nu\}$ to mode $\{m, \mu\}$, which is defined as $T_{mn}^{\mu\nu} \equiv \left| s^{\nu}_{mn} \right|^2$. For the time-reversal symmetry $T$, this yields

$$T_{mn}^{\mu\nu} = T_{\bar{n}\bar{m}}^{\bar{\nu}\bar{\mu}};$$

for the symmetry under $\sigma_i P_i$,

$$T_{mn}^{\mu\nu} = T_{mn'}^{\mu\nu} \quad (\mu, \nu \sim i)$$

$$T_{mn}^{\mu\nu} = T_{m'n'}^{\mu\nu} \quad (\mu, \nu \sim j \neq i);$$

where $i = x$ or $y$, $m' = \bar{m}$ if $m \in i, i$, or $m$ otherwise. By summing up all the transmission probabilities between two leads with specific spin polarizations, the transmission functions $T_{pq}^{\mu\nu} = \sum_{m,\mu,\nu} T_{mn}^{\mu\nu} T_{nm'}^{\nu\mu}$, and the currents are obtained using the extended Büttiker formula

$$I^c_p = \frac{e}{\hbar} \sum_{q,\nu} \sum_{q,\nu} (T_{pq}^{\mu\nu} V^\nu_q - T_{pq}^{\nu\mu} V^\mu_p)$$

with the electric and spin current defined as $I^c_p = e(I^x_p + I^y_p)$ and $I^c_p = \frac{e}{2}(I^x_p - I^y_p)$, respectively. Combining the symmetry-derived Eqs.\ref{symmetry under T} and \ref{symmetry under sigma P}, with the preceding voltage configuration, we find

$I^x_p = -I^x_p = 0, \quad (\mu \sim x)$

$I^y_p = I^y_p, \quad (\mu \sim y)$

$I^z_p = -I^z_p, \quad (\mu \sim z)$

It is clear now to see the difference between the transverse electric currents induced by a $z$-direction polarized spin current and an in-plane polarized spin current, that is in the former case the transverse current is a truly circulating one, which can be naturally regarded as a reversed effect of the spin Hall effect $\ref{spin Hall effect}$, whereas in
our case of in-plane polarization, the transverse currents are flowing both inward or outward instead of circulating through the two transverse leads. Consequently this will make an essential difference in the measurements of the currents in such systems. To present a quantitative estimate of the induced currents, we make numerical calculations with the tight-binding approximation in this cross-shaped mesoscopic system, and the ratio of the induced electric transverse current $I^c_y$ to the spin current $I^s_x$ are plotted in Fig. 2. Combined with the calculated value of the injected spin current, we notice that the induced current is about $0.1 \sim 0.2 \text{nA}$, which is large enough to be measured experimentally, while the small part of the spin current with spin polarization along $x$ axis does not contribute to the transverse currents.

In practice, there are two major factors which will affect our conclusions, the disorder effect and the absence of the symmetry of leads we’ve assumed previously. The disorder breaks the symmetries of the system in a way as adding some random on-site potentials. Our numerical calculation with on-site potentials varying within $[-W/2, W/2]$, illustrated in Fig. 3, shows that despite the current approaching zero with increasing $W$, the symmetry relations shown in Eq. (10a) and Eq. (10b) are well preserved in terms of the average values provided $W$ is not too large. This is reasonable and consistent with previous works [19]. As for the absence of the symmetry of the quantum channels in the leads, we mimic its effect by modifying the width of each lead, and the calculations show that the profile of each current is retained with its value changed according to the modification of the channel number, which implies that within a limited range of asymmetry, our conclusions are still applicable.

In conclusion, a linearly polarized light may pump electrons from the valence band to the conduction band in InGaAs/InAlAs heterojunction, and the zero-field splitting of the conduction band with structural inversion asymmetry drives the photon-excited electrons and holes to form the spin current with in-plane spin polarization. The spin current was estimated by means of the electric-dipole approximation and relaxation time approximation. This mechanism provides a source of spin current to explore spin-dependent transport in mesoscopic systems. Furthermore, the Landauer-Büttiker formula is used to calculate the electric Hall currents while optical injection of spin current is empirically regarded as a source of spin-dependent potentials. As a result two inward or out electric transverse currents are observed. All parameters in our calculation are adopted from a realistic sample of InGaAs/InAlAs heterojunction. Thus the effect may serve as an experimental probe of in-plane polarized spin currents.

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