God (≡ Elohim),
the first small world network

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Abstract

In this paper, the approach of network mapping of words in literary texts is extended to "textual factors": the network nodes are defined as "concepts"; the links are "community connexions". Thereafter, the text network properties are investigated along modern statistical physics approaches of networks, thereby relating network topology and algebraic properties, to literary texts contents. As a practical illustration, the first chapter of the Genesis in the Bible is mapped into a 10 node network, as in the Kabbalah approach, mentioning God (≡ Elohim). The characteristics of the network are studied starting from its adjacency matrix, and the corresponding Laplacian matrix. Triplets of nodes are particularly examined in order to emphasize the "textual (community) connexions" of each agent "emanation", through the so called clustering coefficients and the overlap index, whence measuring the "semantic flow" between the different nodes. It is concluded that this graph is a small-world network, weakly disassortative, because its average local clustering coefficient is significantly higher than a random graph constructed on the same vertex set.

Keywords: textual factors, clustering coefficients, semantic flow, Genesis, overlap index, Kabbalah,

1 Introduction

"Good Lord, it's a small world, isn't it?" [1]

An answer is intended here below:

"Yes, it is: the Good Lord is a small world network".

... It's even the first one. [2]

In modern statistical physics [3], networks [4], underlying opinion formation of agents located at nodes [5], with links defined from data pertaining to social aspects [6], have gathered much interest. Many cases can be found in the literature [7]. Among particularly interesting topics, one encounters the case of finite size networks in which agents have small connectivity values; such cases are known to be "sociologically more realistic" [1, 8].

On the other hand, texts carry messages; they are statistically studied much since Shannon’s introduction of the information entropy definition [9].
More recently, it has been discussed that texts can be transformed into trees \cite{10,11} or better into networks in order to study their structure beside finding word and idea correlations \cite{12}.

Thereafter, one may point to interesting quantitative considerations about network related analyses of the characteristics of literary texts; for example, see \cite{13} about the morphological complexity of a language, \cite{14} about word length frequencies, or about sequences in Ukrainian texts \cite{15,16,17}, and still more recently, enjoyable texts analyses of fables in Slovene as in \cite{18,19}.

There are many other papers reporting studies of word and sequences frequencies, or different language connections as on networks. However to quote all such papers would lead to a useless digression so far, but see the recent \cite{18} which can serve as a recent review, beside these papers: \cite{20}-\cite{28}.

In brief, the present study pertains to applications of statistical physics measures and models like those studied in language evolution and in linguistics \cite{20}-\cite{28}.

In all cases, relevant scientific questions pertain to the dynamics of collective properties, not only of agents on the network, but also by the network structure itself \cite{20}. An interesting structure is the "small world network" (SWN), introduced by Watts and Strogatz \cite{30}. In a SWN, the neighbors of any given node are both likely to be neighbors of each other and also be reached from every other node by a small number of linking "steps" \cite{31,32}.

I propose to discuss a 10 node network, as obtained from the first chapter of Genesis \cite{33}, the so called "Tree of Life", through the kabbalistic (yosher) tradition \cite{34}.

Notice that due to its size, this Genesis network might be also expected to become as useful as the karate club data (which has 44 nodes) \cite{35} or the acquaintance network of Mormons (which has 43 nodes) \cite{36}, both previously known in the literature for paradigmatic studies of SWNs \footnote{Other small networks, recently studied, are the Intelligent design-Darwin evolution controversy, or financial and geopolitical networks.}.

One might wonder why as "serious scientists", interested in social networks for describing communications between agents, we should care about the structure of such an \textit{a priori} "mystic network". Such a network is based on information flow between concepts, - not between words, as it should be emphasized. The matter seems not to have been studied from statistical physics points of view. Nevertheless, one may sort out \cite{37} for a thermodynamic approach. Thus, I hope to connect the network analysis methodology
with that followed in kabbalistic studies, - which are much tied to numerology. Moreover, the present work aims to contribute at introducing a quantitative approach to the analysis of the interaction between "agents", - here being called sephirot [38, 39, 40], in small networks.

Thus, in this paper, the previous approaches on text structure studies through word correlations is extended to "textual factors". Indeed, the network nodes are defined as "concepts"; the links are "community connexions". The characteristics of the network are studied starting from its adjacency matrix, - its eigenvalues, whence providing a measure of the "semantic flows" between the different nodes. The network Laplacian matrix is also studied along the same lines. Together with Kirchhoff’s theorem, and Cheeger’s inequality, the "spectrum gap" (between the two smallest eigenvalues) can be used to calculate the number of spanning trees for a given graph [41]. Indeed, the sparest cut of a graph can be approximated through the second smallest eigenvalue of its Laplacian by Cheeger’s inequality. Furthermore, the spectral decomposition of the Laplacian matrix allows constructing low dimensional embeddings that appear in many machine learning applications and determines a spectral layout in graph drawing, as claimed in https://en.wikipedia.org/wiki/Laplacian_matrix (accessed on March 26, 2022).

In so doing, one adds a quantitative set of values for an answer to a question raised in [42] on the classes of SWN examined in the literature [43].

Besides, the present numerical approach might be in line with modern studies in Kabbalah research about numbering [44, 45], and quantitative studies on religious adhesion or religiosity aspects [46]-[53], as recently used in socio-physics for examining growth processes, opinion formation, and related topics. This paper is in line with such a frontiers in physics approach.

After introducing the data set, its origin, in Sect. 2 it seems rather appropriate to provide the whole adjacency matrix (10 x 10). Its construction goes in lines with studies on large-scale networks, like co-authorship networks [43, 54]. The present network structural aspects are first outlined, before searching for subsequent numerical and statistical aspects, through a few usual network characteristics in Sect. 3. A similar study is performed for the network Laplacian matrix.

Nevertheless, let it be here pointed out that triplets of nodes are particularly examined in order to emphasize the agent ("emanation") community connexions through the so called clustering coefficient [30] and the overlap index [55], in Sect. 4 and Sect. 5 respectively. The results prove the SWN
nature of the 10 sephirots network. For completeness, some other network characteristics, like the assortativity coefficient [56], are calculated and reported. A kabbalistic ”generalized point of view” is provided.

2 The data set

Let us consider as the demonstration of the approach a text in which concepts are somewhat hidden, - in the present case within some mystical concept, but without any loss of generality from a theoretical point of view. The data, downloaded from [34], emerges from the kabbalistic interpretation of the occurrence of ”spiritual principles” at the universe creation. In brief, the Kabbalah [38, 39, 40] seems to infer, from the Genesis first chapter, that ”The Infinite” (God) has ”emanations” which form a network of ten nodes, like on Fig. 1; the ”node names” are given for further reference in Table 1. The network so symmetrically displayed is made of 3 ”columns”. (Alternative configurations are given by different schools in the historical development of Kabbalah, with each articulating different spiritual aspects; to distinguish the variants is not very relevant for the present investigation [39]. The enumeration of the 10 nodes, as on Fig. 1 is stated in the Sefer Yetzirah [38, 39, 40] .) Notice that the Tree of Life nodes are arranged onto seven planes; 7 being a mystic (or sacred) number.

Between the 10 sephirots, run 22 channels, or paths [59]. These links are interpreted as the specific connections of (”spiritual”) information flow. In the present case, the flow of information goes according to the node number hierarchy; such a type of directed flow consideration has been recently studied in [60] in a different context.

In so doing, an adjacency matrix \( G = (g_{ij}) \in \mathbb{R}^{N \times N} \) can be built, with \( g_{ij} = 1 \) for an existing link between 2 connected nodes, \( \nu_i \) and \( \nu_j \), selected among the \( N = 10 \) nodes here, and \( g_{ij} = 0 \) otherwise, i.e.,

\[g_{ij} = \begin{cases} 1 & \text{if } (\nu_i, \nu_j) \text{ is an edge} \\ 0 & \text{otherwise} \end{cases}\]

This paper is not intended to justify of infirm studies of the Bible through Kabbalah methods [57, 58]. However, it can be pointed out that the interaction of Kabbalah with modern physics has generated its own literature, up to including renaming the elementary particles with kabbalistic (Hebrew) names or developing kabbalistic approaches to debates on evolution.

For example, instead of a ”tree” with 3 ”columns”, the iggulim representation depicts the sefirot as a succession of concentric circles [34].
Figure 1: The network of 10 sephirots; notations of node labels are found in the main text.
\[ g_{ij} = \begin{cases} 
1 & \text{if } \nu_i \text{ and } \nu_j \text{ are connected nodes} \\
0 & \text{otherwise.} 
\end{cases} \quad (1) \]

Thus, all diagonal terms are 0; the matrix is symmetric; it has 44 finite elements, i.e. \(2L\), the number of links. In this study, the links are neither directional nor weighted; the nodes have also no "strength".

For further reference, let us here introduce an alternative to the adjacency matrix, i.e. the so called Laplacian matrix of the network: \( \Lambda = (\lambda_{ij}) \in \mathbb{R}^{10 \times 10} \), with

\[ \lambda_{ij} = \begin{cases} 
-1 & \text{if } \nu_i \text{ and } \nu_j \text{ are connected nodes} \\
d_{\nu_i} & \text{if } \nu_i \equiv \nu_j \\
0 & \text{otherwise,} 
\end{cases} \quad (2) \]

where \(d_{\nu_i}\) is the degree of the node \(\nu_i\), i.e. the number of links at the node.
In brief, the Laplacian matrix \(\Lambda\) is the difference between a diagonal matrix \(\Delta\) reporting the degree of the node and the adjacency matrix of the graph.

For completeness, let us mention the finite elements of \(\Delta = (d_{ij}) \in \mathbb{R}^{N \times N}\) through the degree list defined as \(D = (d_{\nu_1}, d_{\nu_2}, ..., d_{\nu_N})\), which reads here \(D = (3, 5, 5, 5, 8, 4, 4, 4, 1)\)

Thus, the adjacency matrix reads

\[
G = \begin{pmatrix}
-1 & 1 & 1 & - & - & 1 & - & - & - & - \\
1 & -1 & 1 & 1 & 1 & - & - & - & - & - \\
1 & 1 & - & 1 & 1 & 1 & - & - & - & - \\
1 & 1 & - & - & 1 & 1 & 1 & - & - & - \\
-1 & 1 & - & 1 & 1 & 1 & - & - & - & - \\
-1 & 1 & 1 & - & 1 & 1 & 1 & - & - & - \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & - \\
-1 & - & - & 1 & 1 & -1 & 1 & 1 & - & - \\
-1 & - & - & - & 1 & 1 & 1 & -1 & 1 & - \\
-1 & - & - & - & - & 1 & 1 & 1 & -1 & - \\
-1 & - & - & - & - & - & - & 1 & - & - 
\end{pmatrix} \quad (3)
\]

in which each 0 is replaced by a - for better readability. The \(\Lambda\) matrix is written and analyzed below.

Anyone knows that when there is a matrix, one looks for eigenvalues and eigenvectors: the (necessarily real) eigenvalues are found to be equal to:

\[5.02314, 2.21045, 0.61803, 0.13191, 0.00000, -1.00000, -1.36550, -1.61803, -2.00000, -2.00000,\]
Table 1: Characteristics of the network matrix $G$, with hereby defined node $(i)$ notations, (Hokm. = Hokmah; Geb. = Gebourah; Tiph. = Tiphereth; Malk. = Malkouth) in the first and second columns, and their corresponding number of links ($d_{\nu_i}$). The values of usual structural information for networks are given: the probability $p_i$ that a vertex $\nu_i$ has a degree $d_{\nu_i}$; $q_i$ is fully defined in Eq. (10) in terms of $p_i$ and the $i$ vertex degree $d_{\nu_i}$; the possible maximum number of different wedges, $(d_{\nu_i}(d_{\nu_i} - 1)/2)$; the number of triads ($e_i$) associated to a given node $(i)$ in $G$; VCC, the corresponding clustering coefficient ($c_i$) of a vertex $i$, - from which one deduces the global clustering coefficient (GCC) of the network; and $\Gamma_i$, the local clustering coefficient (LCC) of a vertex $i$, - from which one deduces the average local clustering coefficient for the network, see Sect. 4.2.

| G matrix | node name | n.links | $p_i$ (%) | $q_i$ (%)² | $d_{\nu_i}(d_{\nu_i} - 1)/2$ | n.triads | VCC | LCC | $\Gamma_i$ |
|----------|-----------|---------|-----------|-----------|----------------------------|----------|-----|-----|----------|
| 1Kt      | Kether    | 3       | 6.82      | 9.21      | 3                         | 3        | 1   | 6/6 |
| 2Hk      | Hokm.     | 5       | 11.4      | 25.6      | 10                        | 6        | 0.6 | 13/15|
| 3Bn      | Binah     | 5       | 11.4      | 25.6      | 10                        | 7        | 0.7 | 12/15|
| 4Hs      | Hesed     | 5       | 11.4      | 25.6      | 10                        | 7        | 0.7 | 12/15|
| 5Gb      | Geb.      | 5       | 11.4      | 25.6      | 10                        | 7        | 0.7 | 12/15|
| 6Tph     | Tiph.     | 8       | 18.2      | 65.5      | 28                        | 13       | 0.464| 21/36|
| 7Nt      | Netsah    | 4       | 9.09      | 16.4      | 6                         | 3        | 0.5 | 8/10|
| 8Hd      | Hod       | 4       | 9.09      | 16.4      | 6                         | 3        | 0.5 | 8/10|
| 9Ys      | Yesod     | 4       | 9.09      | 16.4      | 6                         | 3        | 0.5 | 7/10|
| 10Mlk     | Malk.    | 1       | 2.27      | 1.02      | 0                         | 0        | 0   | 1   |
Table 2: The \((N_{i,j})\) number of different (undirected information flow) paths between two nearest neighbors \(i\) and \(j\), through a nearest neighbor \(k\).

They are distributed in a (quasi logarithmically) decreasing order: \(y = 4.503 - 6.865 \log(x)\), with \(R^2 = 0.977\).

Thereafter, one can look for the 10 eigenvectors; however, they are not shown for saving space, - their writing being irrelevant within the present aim. Nevertheless, the above suggests that a Principal Component Analysis can be a complementary valuable investigation for "community detection".

The network Laplacian matrix reads

\[
\Lambda = \begin{pmatrix}
3 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 5 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 5 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & 5 & -1 & -1 & 8 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 4 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & 4 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & 4 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & 4 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 4 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1
\end{pmatrix}
\]  

(4)

The eigenvalues are:

9.01939, 6.61803, 6.48072, 6.00000, 5.13659, 4.38197, 3.48940, 2.13004, 0.74387, 0.00000.

They are distributed according to: \(y \simeq 9.478 - 0.923 x\); with \(R^2 = 0.971\).
Together with Kirchhoff’s theorem, the Laplacian matrix eigenvalue spectrum can be used to calculate the number of spanning trees for a given graph, \( \eta \). The sparsest cut of a graph can be approximated through the second smallest eigenvalue of its Laplacian, i.e. \( \lambda_2 = 0.74387 \), here, by Cheeger’s inequality. Since the Laplacian matrix spectral gap is also obviously equal to 0.74387, one finds

\[
\frac{\lambda_2}{2} \leq \eta \sqrt{\lambda_2(2d_{v_j}^{(M)} - \lambda_2)}
\]  

(5)

where \( d_{v_j}^{(M)} (= 8) \) is the largest node degree. Thus, \( 0.372 \leq \eta \leq 3.369 \).

3 Data statistical analysis

Next, one proceeds performing some classical structural analysis as usual on such networks, i.e. an analysis of indicative coefficients: one obtains the network node in- and out-degree distributions, the network assortativity, the (Global and Local) Clustering Coefficients, and the Average Overlap Index.

In the present case, the matrix, or network, is symmetric, whence the number of links exiting from a node \( v_j \), i.e. the out − degree, is equal to the in − degree number. The largest degree (=8) is for node 6; the smallest (=1) is for node 10; the average degree, counting both out − degrees and in − degrees is easily found to be 4.4.

On Table 1, one also gives for each node, the degree, i.e. the number of links \( d_{v_i} \) exiting from or entering into each node \( v_i \). Table 1 also reports the possible maximum number of different wedges, \( (d_{v_i}(d_{v_i} - 1)/2) \), and triads, \( (e_i) \), associated to a given node \( v_i \) in \( G \).

In addition, we report he number \( (N_{i,j}) \) of different paths going through a nearest neighbor \( k \) of two nearest neighbors \( i \) and \( j \) in Table 2. This number is equivalent to the number of triangles sharing the link \( (i,j) \).

4 Clustering

The tendency of the network nodes to form local interconnected groups is a convincing arguments for describing social networks along the statistical physics modern formalism. Such a behavior is usually quantified by a measure referred to as the clustering coefficient [30]. The amount of studies on this
characteristic of networks has led to the particularization of the term in order to focus on different complex features of networks. Here, one considers the global clustering coefficient and the local clustering coefficient, together with the overlapping index, and the assortativity for a text mapped into a network.

Indeed, the most relevant elements of a heterogeneous agent interaction network can be identified by analyzing global and local connectivity properties. In the present case, this can be attempted by analyzing the number of triangles with agent (or "emanation") nodes belonging to the same "community" or not, depending on the type of connexions. The former number gives some hierarchy information; the latter some reciprocity measure, i.e. recognition of leadership or proof of some challenging conflict among the emanations.

4.1 Global Clustering Coefficient

The global clustering coefficient (GCC) of the network is defined as \(< c_i \rangle\), the average of \(c_i\) over all the vertices in the network, \(< c_i \rangle = \sum c_i / N\), where \(N\) is the number of nodes of the network, and where the clustering coefficient \(c_i\) of a vertex \(i\) is given by the ratio between the number \(e_i\) of triangles sharing that specific vertex \(i\), and the maximum number of triangles that the vertex could have. If a node \(i\) has \(d_{\nu_i}\) neighbors, then the so called clique, i.e. a complete graph in fact, would have \(d_{\nu_i}(d_{\nu_i} - 1)/2\) triangles at most, thus one has,

\[
c_i = \frac{2}{d_{\nu_i}(d_{\nu_i} - 1)} e_i
\]

The value of GCC is found to be \(< c_i \rangle = 0.5564\), from the raw data in Table 4.

4.2 Local Clustering Coefficient

In the literature \([43]\), the term 'clustering coefficient' refers to various quantities, relevant to understand the way in which nodes form communities, under some criterion. By definition, the "local clustering coefficient" (LCC) \(\Gamma_i\) for a node \(i\) is the number of links between the vertices within the nearest neighbourhood of \(i\) divided by the maximum number of links that could possibly exist between them. It is relevant to note that the above GCC is not trivially related to the LCC, e.g. the GCC is not the mean of LCC. In the former
case, triangles having common edges are emphasized, in the latter case only the number of links is relevant. This number of links common to triangles sharing the node \(i\) can vary much with the number of connected nearest neighbour nodes indeed. Basically, the GCC value quantifies how much the neighbors of \(i\) are close to being part of a complete graph. In contrast, LCC rather serves to determine whether a network is a SWN \([30]\) or not.

The LCC \((\Gamma_i)\) values are given in Table 1 under a ratio form in order to emphasize that the numerator of the fraction is the sum of \(d_{\nu_i}\) plus the number of links making triangles in the nearest neighborhood of \(i\), while the denominator is obviously \(d_{\nu_i}(d_{\nu_i} + 1)/2\). It is easily deduced that \(< \Gamma_i > = 0.8217\).

There is no drastic conclusion to draw from this specific value, since not many corresponding values are reported in the literature allowing a comparison with other networks \([61]\). Yet, let it be recalled that the lower the \(\Gamma_i\) values, the less "fully connected" appears to be the network. This is not the present case.

However, let it be emphasized that a graph is considered to be small-world, if its average local clustering coefficient is significantly higher than a random graph constructed on the same vertex set, i.e. here with \(N = 10\). Thus, one confirms that the present network looks like a SWN rather than either a random network (RN) or a complete graph (CG).

5 Average Overlap Index

Finally, for characterizing members of communities, in another hierarchical way, let us also calculate the Average Overlap Index (AOI) \(O_{ij}\); its mathematical formulation and its properties are found in \([55]\) in the case of a unweighted network made of \(N\) nodes linked by \((ij)\) edges,

\[
O_{ij} = \frac{N_{ij} (d_{\nu_i} + d_{\nu_j})}{4 (N - 1) (N - 2)}, \quad i \neq j
\]

(7)

where \(N_{ij}\) is the measure of the common number of (connected) neighbors to the \(i\) and \(j\) nodes. In the present case: \(4(N - 1)(N - 2) = 288\) N.B. in a fully connected network, \(N_{ij} = N - 2\). Of course, \(O_{ii} = 0\) by definition.
Table 3: \( O_{i,j} \): the numerator of the overlap index, Eq. (7), of the neighboring \( \nu_i \) and \( \nu_j \) nodes; and \( \langle O_i \rangle \), the average overlap index, Eq. (8).

N.B. \( 288 = 4(N - 1)(N - 2) \), while \( \sum_i \sum_j O_{i,j} = 1228 \).

The Average Overlap Index for the node \( i \) is defined as:

\[
\langle O_i \rangle = \frac{1}{N-1} \sum_{j=1}^{N} O_{ij}.
\]  

(8)

The values are given in Table 3.

This measure, \( \langle O_i \rangle \), can be interpreted indeed as an other form of clustering attachment measure: the higher the number of nearest neighbors, the higher the \( \langle O_i \rangle \), the more so if the \( i \) node has a high degree \( d_{\nu_i} \). Since the summation is made over all possible \( j \) sites connected to \( i \) (over all sites in a fully connected graph), \( \langle O_i \rangle \) expresses a measure of the local (node) density near the \( i \) site.

Recall also that in magnetic networks, the links are like exchange integrals between spins located at \( i \) and \( j \). An average over the exchange integrals provides an estimate of the critical (Curie) temperature at which a spin system undergoes an order-disorder transition, and conversely. Therefore \( \langle O_i \rangle \) can also be interpreted, in a physics sense, as a measure of the stability of the node versus perturbations due to an external (for example, thermal) cause. In other words, in the present context, a high \( \langle O_i \rangle \) value reflects the \( i \) node strong attachment to its community: the main "textual factor" is
thereby emphasized. Here, for our illustrative example, the highest value ($\approx 0.074$) correspond to the 6th node ("emanation"), Tiphereth, - as should have been also visually expected from Fig. 1.

The average overlap index of each node, obtained according to Eq. (8), are given in Table 3. The order of magnitude of the $\langle O \rangle$ values are $\sim 0.05$, much smaller than in other investigated cases, like in [55] (or [62]). This is due to the low value of $N_{ij}$, in the present case.

For completeness, observe that $\sum_i \sum_j O_{ij} = 1228$, whence $1288/288 = 4.472$, which divided by $N$ leads to $\sim 0.4472$, as another characteristic of the average overlap number of triangles throughout the network.

In order to indicate some aspect of the attachment process in a network, one can calculate its so called "assortativity" [56]. The term refers to a preference for a network node to be attached to others, depending on one out of many node properties [56]. Assortativity is most often measured after a (Pearson) node degree correlation coefficient $r$

$$r = \frac{\sum_{i,j=1}^{N} q_i q_j g_{ij} - [\sum_{i,j=1}^{N}(q_i + q_j)g_{ij}]^2 / L}{\sum_{i,j=1}^{N}((q_i^2 + q_j^2)g_{ij}) - [\sum_{i,j=1}^{N}(q_i + q_j)g_{ij}]^2 / L}$$

(9)

where

$$q_i = \frac{k_i p_i}{\sum_i k_i p_i},$$

(10)

where $k_i$ is the $i$ vertex (total) degree $d_{i\nu}$, in which $p_i$ is the probability that a vertex $i$ has a degree $d_{i\nu}$ (this can be here obtained/read from Fig. 1 or Table 1): $L$ is the number of connecting channels ($= 22$, here); $r = 1$ indicates perfect assortativity, $r = -1$ indicates perfect "dis-assortativity", i.e. a perfectly negative correlation.

For the (text based) network of interest here, we have found, $r = -0.229$, a quite negative value for the assortativity notion, in most networks. The present finding is somewhat surprising, since according to [56], almost all "non-social networks" [56] seem to be quite dis-assortative, even though the "social networks" usually present significantly assortative mixing. However, the technological and biological networks usually are all dis-assortative: the hubs are (primarily) connected to less connected nodes, dixit Newman [56]. The present case is a weakly dis-assortative network.

In order to show a positive value of $r$, a network must have some specific additional structure that favors assortative mixing, i.e. a division into communities or groups; a contrario, to see significant dis-assortativity, the
highest degree vertices in the network need to have degree on the order of \( \sqrt{N} \), where \( N \) is the total number of vertices, so that there is a substantial probability of some vertex pairs sharing two or more edges. Here \( \sqrt{22} \approx 4.69 \); the highest degree which is for \( Tiphereth = 8 \) is (at once visually found from Fig. 1) the “knowledge transfer hub”, - the most important emanation.

One may consider the practical aspects resulting from the node characteristics, next those from links. In relation to a “generalized kabbalistic point of view”, one may make the following comments.

Let us observe two new integer numbers appearing through the study: 288 and 1228. Notice that

- 288: this number contains profound significance; in Kabbalah, it refers to the number of “sparks” that God had to remove in order to create the world; see https://www.biblegematria.com/288-holy-sparks.html (Accessed March 01, 2022)
- 1228: the Hebrew name of Simon Peter, Symehon Hacephi, is 1228 in Hebrew name numeration; ([63], p. 54).

Comments and suggestions on such a ”society structure” within formal texts can be thought to arise from similar numerical perspectives.

6 Conclusions

In frontiers science, prior to scientific excitations and paper avalanches, there are modest inter-connections, between authors and between fields. This is one of the underlying ideas for the present problem, not at the level of authors but at the semantic level, - justifying the study. Two apparently unrelated research fields are interconnected. One can study texts through network mappings, - nothing new. I recall that the Ukrainian language network used in the selected fables studied in [15] is a strongly correlated, scale-free, small world network. In the present case, one goes a little bit further: instead of another word correlation study, one examines textual concept distributions. Moreover, picking up a basic text with some mystic ingredient, one covers a wide gap between various disciplines, with a physics support.

\[^{4}\text{It is thought that the earth’s average surface temperature = 288 K, but that might neither be relevant, nor suggests further investigation.}\]
One has proposed to examine a theoretical question on applied linguistics, with a specific illustration, but in so doing also asked: do the sephirot, thus nodes and links of a mystic network, mean something from a statistical physics point of view, knowing their "esteem" or "sense" in kabbalistic work? Thus, in fact, the study has some similarity to other "social network" considerations: mutatis mutandis, in the present problem the agents are the sephirot, while the links carry the information flows between emanations.

Practically, the yosher kabbalistic mapping of a selection of concepts from the Genesis in the Bible produces a network [34]. In order to characterize the necessarily small network, based on its adjacency matrix, one has calculated a few specialisation coefficients. Surely, in future work, one could consider many other quantities of interest for networks [64]; the matter is left for the imagination of concerned researchers.

In particular, assortativity characteristics of the network have been examined, - in so doing searching whether there is a proof of any preference of a sephirot attachment to some sub-networks. Examining the whole network, through their communities and the inter-community links, it is found that the sephirots are neither perfectly assortative nor perfectly dis-assortative. From the values of the Pearson node degree correlation coefficient $r$ it is asserted that the network is rather dis-assortative, - but weakly correlated in contrast to the fables in [15]. This is contrasted to fictional social networks [65, 66] which are found to be small-world, highly clustered, and hierarchical, which typically differ from real ones in connectivity and levels of assortativity [18].

According to [65], a clustering coefficient can (also) be defined by

$$C = \frac{1}{N} \frac{[<\kappa^2>-<\kappa>^2]}{<\kappa>^3},$$  \hspace{1cm} (11)

where $\kappa$ is the excess degree; in the present text case, $<\kappa> = 3.4$. Thus, $C = 0.308$.

In order to characterize in greater detail the intercommunity structure complexity, - its "information flow", one can also consider elementary entities made of a few sephirots. The smallest (geometric) cluster to be examined is the triangle. In this respect the study of the local clustering coefficients indicates a low value for these inter-community sub-networks. The average overlap index (AOI) [55] allows to extract from the clusters those nodes which inside their community and with respect to the others are the centers of more
attention. One may claim that one gives some scientific (statistical physics) emphasis to one kabbalistic emanation.

From a fundamental statistical physics point of view, one may emphasize the "added value" of the present investigation. Return to Amaral et al. [42] who have proposed three classes of SWN: (i) scale-free networks, characterized by a vertex connectivity distribution that decays as a power law; (ii) broad-scale networks, characterized by a connectivity distribution that has a power law regime followed by a sharp cutoff; and (iii) single-scale networks, characterized by a connectivity distribution with a fast decaying tail. The analyses presented in the main text suggest that the network belongs to the third category. It should be of course of interest to find out if this conclusion holds for other "textual factors" in other literary texts.

Finally, let it be recalled that some time ago, "God is a mathematician", was concluded by Newman [67] and questioned by Livio [68]. Elsewhere, one may find the question: "Is God a geometer?" [69].

Apparently, according to the present text analysis of the Genesis, God (≡ Elohim) is also the (chronologically) first small world network, - for the monotheistic religions.

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• **Studies involving human subjects. Generated Statement:** No human studies are presented in this manuscript.

• **Inclusion of identifiable human data. Generated Statement:** No potentially identifiable human images or data is presented in this study.

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