Unpacking Pre-service Physics Teachers’ Understanding of the $P$-$V$-$T$ Diagram and the Associated Mathematics

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Abstract. This study was conducted to assess the ability to apply an appropriate concept of state function and its graphical and mathematical representation, including to draw, to translate and to interpret the $P$-$V$-$T$ diagram for several thermal processes and thermal cycles. Pre-service physics teachers, respectively 42 students of academic year 2014/2015 and 49 students of academic year 2015/2016, were involved in this study. The findings showed as follows: first, students encountered most difficult in drawing $PV$, $PT$, and $VT$ diagram of adiabatic process; second, students who correctly answer $PV$ diagram were greater than those who correctly answer $PT$ and $VT$ diagram; third, students prefer to understand thermal processes with its characteristics, such as isobaric with constant pressure, isothermal with constant temperature, and isochoric with constant volume. Lack of understanding in drawing the diagram of thermal process also causes lack of ability to draw thermal cycles. One of the promised solutions to address these learning difficulties is by implementing pre-class tutorial.

1. Introduction

Physics closely related to either the relation among variables or among quantities, such as in the learning of thermodynamics. Physics quantities in thermodynamics include the macroscopic quantities of pressure, temperature, volume, entropy, internal energy, enthalpy, Helmholtz free energy and Gibbs free energy. The experiments that have previously been done, which involve those eight of state variables of the system, generated a lot of relationships and formulas describing the functional relationships between those quantities.

Especially for a closed system, i.e. a system of gas whose number of particle unchanged, there are three important conclusions as follows. First, all experiments show that if the system is in equilibrium then each coordinate is expressed as a function of two other coordinates. Secondly, only two of those eight coordinates are independent variables of the system. Third, in the equilibrium state of thermodynamics (mechanical, chemical, thermal and phase equilibrium) applies the relationship: $f (x, y, z) = 0$, with $x$, $y$, and $z$ representing thermodynamic macroscopic coordinates.

Suppose we review the system of gas in a closed sealed (a closed system) whose composition is unchanged, no chemical reactions can alter the number of particles in the system, and no diffusion occurs, the system of gas is at a certain volume ($V$), and temperature ($T$) can be arranged either at any temperature or at a certain temperature, and can be given any volume ($V$). This is most likely because there are 3 adjusting thermodynamic coordinates, such as pressure ($P$), so that the system of gases can be illustrated by a pair of free coordinates ($P$, $V$, $T$) = 0.
Besides expressed in the form of mathematical models, the relationship between variables in thermodynamics is often in the form of graphs / diagrams. An ability to create and interpret data in multiple representatives is needed by a pre-service physics teacher. Interpretation and construction in the form of graphs in reporting a relationship is very important as it is part of the experiment [1].

Previous research at the university level clearly showed that students demonstrated significant difficulties when learning thermodynamics, including conceptual difficulties with heat, temperature, the ideal gas law, and the first law of thermodynamics (Rozier and Viennot, 1991[2]; Yeo and Zadnik, 2001[3]; Loverude et al., 2002[4]; Meltzer, 2004[5]; Kautz et al., 2005[6]; Sari, 2017[7]), as well as learning difficulties with the interpretation of ideal gas processes on PV diagrams [8]. In line with the results of previous research, this study focused on investigating the difficulties of pre-service physics teachers in drawing and interpreting PV, PT, and VT diagrams from a thermal equilibrium of several thermal processes (isobaric, isothermal, isochoric and adiabatic) and thermal cycles (Carnot cycle, Otto cycle, and Diesel cycle).

2. Method
The descriptive method was used to describe the understanding of pre-service physics teachers in drawing and interpreting the PV, PT, and VT diagrams from a thermal equilibrium of various thermal processes (isobaric, isothermal, isochoric and adiabatic) and thermal cycle (Carnot cycle, Otto cycle, and Diesel cycle). Participants involved in this study were 42 students of physics education program in academic year 2014/2015 and 49 students of academic year 2015/2016. The written test instruments for half and final semester program were used to collect data and then the analysis was done through student answer sheets on written tests. We probe student understanding through their response which is provided in their exam sheet. A percentage of students who correctly answer the written problem were analyzed quantitatively to show how many students understood the concept after instruction, while a degree of understanding was determined by analyzing their response that was confirmed through the necessary interview. In assessing the degree of student understanding, we have used the criterion as a measure of understanding of the PVT diagram and the associated mathematics the degree to which an individual successfully applies that concept to the interpretation of simple state of a closed system of gas. We adopted the same criterion that is used by Trownbridge and McDermott (1980) to assess student understanding of kinematics [9].

3. Result And Discussion
In the written test about thermal diagram, students were asked to represent various thermal processes in the form of PV, VT and PT diagrams (see Fig. 1). The findings in this study were obtained through widely administered assignments and course examinations, which provides a student understanding as the subject to descriptive analysis. Our findings are sequentially outlined in this section.

In thermodynamics, we discuss many thermal processes, i.e. the change of macroscopic coordinates of a system due to interacting with the environment; include a isochoric, isobaric, adiabatic and isotherm. Explain those processes qualitatively for ideal gases. Represent each process by using P-V, P-T, and V-T diagram, and provide an interpretation (mathematically or physically) based on the diagrams you make.

Figure 1. A written problem posed on the examinations

Based on student’s response on written tests, it can be determined the percentage of students who are able to correctly create and interpret PV, PT, and VT diagrams of a thermal equilibrium condition of various thermodynamic processes. A detailed data can be found in Table 1.
Table 1. Results from a written problem in which students were asked to draw and interpret the $P\text{-}V\text{-}T$ diagram. Question was administered to two different groups of pre-service teacher.

| Diagram | Thermal Process | Number of students who answer correctly |
|---------|-----------------|----------------------------------------|
|         |                 | 2014/2015 (N=42) | 2015/2016 (N=49) |
| $P\text{-}V$ | Isobaric | 36 | 85.71 | 41 | 83.67 |
|         | Isotherm | 29 | 69.05 | 28 | 57.14 |
|         | Isochoric | 34 | 80.95 | 39 | 79.59 |
|         | Adiabatic | 19 | 45.24 | 20 | 40.82 |
| $P\text{-}T$ | Isotherm | 28 | 66.67 | 28 | 57.14 |
|         | Isochoric | 24 | 57.14 | 21 | 51.02 |
|         | Adiabatic | 18 | 42.86 | 23 | 46.94 |
| $V\text{-}T$ | Isotherm | 33 | 78.57 | 37 | 75.51 |
|         | Isochoric | 32 | 76.19 | 33 | 67.35 |
|         | Adiabatic | 13 | 30.95 | 11 | 22.45 |

The student difficulties that we identified from analyzing student’s response to the written question indicated some points as follows: (a) the biggest difficulty in each $P\text{-}V$, $P\text{-}T$, and $V\text{-}T$ diagram is to draw an adiabatic process. It is allegedly due to this process has a unique characteristic “no heat, or $Q=0$”, none quantity that can be represented directly in the $P\text{-}V$, $P\text{-}T$, and $V\text{-}T$ diagram. It is in line with the previous study reported by Meltzer (2004) that students tend to have persistence belief that no heat transferred in adiabatic process; (b) the percentage of students who correctly answer the $P\text{-}V$ diagram is higher than those who correctly answer the $P\text{-}T$ and $V\text{-}T$ diagrams. Students get used to finding a thermodynamic equilibrium for various thermal processes which are presented using the $P\text{-}V$ diagram; and (c) students prefer understanding thermal processes with their specific characteristic, such as isobaric with constant pressure, isotherm with constant temperature, and isochoric with volume constant. It means that the diagram that involves these constant quantities is easy to draw because it only requires drawing a straight line showing the constant value.

A detailed analysis on student’s response gives a clear explanation about a degree of student understanding on each thermal process. First, almost half the students answered correctly the $PV$ and $PT$ diagrams of adiabatic process, but still answered incorrectly the $VT$ diagram. In addition, most of the students did not answer the mathematical or physical reasoning related to the diagram they created. Allegedly some students just memorized the form of the diagram from the textbook. It was unconfirmed through interview but clearly shown in their response on the exam sheet as presented in Figure 2(a).
Figure 2. (a) Sample of student’s response on the question of adiabatic process diagram; (b) the correct VT diagram of adiabatic process.

Student response indicated that the form of VT diagram is made nearly resembles the PT diagram. In interview session, a student stated that “for ideal gases PV=nRT; we have P and V in the left side, and T in the right side, so that P and V are proportional to T, it means that P-T and V-T diagram tends to have similar curve.” Students have an incorrect understanding, because the adiabatic condition applies $PV^n = K$, if substituting $P = \left(\frac{nRT}{V}\right)$, then it will be obtained $TV^{n-1} = C$, in which $V = \left(\frac{C}{T}\right)^{\frac{1}{n-1}}$ and $K, C$ are constant. For $\frac{n}{n-1} = \frac{3}{2}$, the VT diagram is shown in Fig. 2(b).

Second, most students answered correctly the PV diagram of other thermal processes, but not for VT and PT diagrams as shown in Fig. 3. Student responses illustrated that PT and VT diagrams of isothermal and adiabatic processes are made in the same curve as if both processes have the same characteristics. Further analysis of student interviews obtained some findings as follows: students believe that (a) the adiabatic process is similar to the isothermal process. PV diagrams of the two processes differ only in steepness, isothermal processes are steeper than adiabatic; (b) in the adiabatic process, there is no heat transfer, $Q = 0$, based on the equation $Q = mc\Delta T$, which also means no change in temperature. Therefore, in the adiabatic process, "there is no temperature change", which means constant temperature.

Figure 3. Sample of student’s response on the diagram of other thermal processes
Both of these conceptions are obviously incorrect. First, adiabatic is similar to isotherms, but the $PV$ diagram of the adiabatic process should be steeper than the isothermal process because there are factors $\gamma$ (Laplace constant, $\gamma > 1$) or $PV = K$ in the adiabatic process, whereas the isotherm only satisfies $PV = K$. Second, the conception of adiabatic $Q = 0$, based on the equation $Q = mc\Delta T$, means that there is no change in temperature only applies to systems that transfer the energy through heat mechanism only (release or absorption). However, for the system of gas, heat is not the only way to transfer the energy into or out of the system. There has heat ($Q$), work ($W$) or both as the mechanism of energy transfer. In addition, it is not generally accepted in thermodynamics that every net work done by or to the system will then be converted into releasing or absorbing energy through heat mechanism, but it may also be converted in the form of change in internal energy of the system ($U$).

![Figure 4](image)

**Figure 4.** (a) Student’s response on the $PV$ diagram of isothermal process (the bottom left diagram). (b) The correct $PV$ diagram of isothermal process

Even though students answered correctly the $PV$ diagram, but some are still mistaken. One interesting thing is their false reasoning about the $PV$ diagram of isothermal process. Some students referred to the ideal gas equation $PV = nRT = K$, that is because $PV$ is constant, so the pressure of $P$ is inversely proportional to volume $V$. Some students made the $P$ and $V$ relationship in the $PV$ diagram as a straight line as shown in Fig. 4(a) (the bottom left diagram). Although the relationship of $P$ and $V$ is inversely proportional, it does not mean that this relationship if made in the $PV$ diagram will be a straight line. In this case, the students hold persistence belief that "on isothermal process, if $P$ increases then $V$ decreases, and vice versa". This relationship is generally true, but if it refers to the mathematical equation it can be derived $PV = K \rightarrow P = \frac{K}{V}$ or if made in a PV diagram will be as presented in Fig. 4(b).
Table 2. Results from a written problem in which students were asked to draw and interpret the diagram of various thermal cycles. Question was administered to two different groups of pre-service teacher.

| Diagram | Thermal Cycles | Number of students who answer correctly |
|---------|----------------|----------------------------------------|
|         |                | 2014/2015 (N=42) | 2015/2016 (N=49) |
|         |                | n  | %  | n  | %  |
| P-V     | Carnot         | 27 | 64.29 | 29 | 59.18 |
|         | Otto           | 18 | 42.86 | 21 | 42.86 |
|         | Diesel         | 19 | 45.24 | 19 | 38.78 |
| P-T     | Carnot         | 22 | 52.38 | 24 | 48.98 |
|         | Otto           | 17 | 40.48 | 19 | 38.78 |
|         | Diesel         | 16 | 38.10 | 19 | 38.78 |
| V-T     | Carnot         | 20 | 47.62 | 25 | 51.02 |
|         | Otto           | 19 | 45.24 | 22 | 44.90 |
|         | Diesel         | 20 | 47.62 | 20 | 40.82 |

Lack of understanding in creating and interpreting various diagrams of thermal processes in the thermal equilibrium condition also impacted student understanding on $PV$, $PT$, and $VT$ diagrams of thermodynamic cycles, such as Carnot cycle, Otto cycle, and Diesel cycle. This is evident from the student responses on the written question. The percentage of student who answered correctly the thermal cycles test is presented in Table 2.

The findings of this study have implications for the instruction of the thermal diagrams of either various thermal processes or cycles for it can be used to guide the design of the learning resources that address problems. One of the efforts we developed to improve student understanding is a pre-class tutorial design. Implementation of pre-class tutorials on mechanics course has shown the potential of this curriculum to improve student understanding [10][11]. Sample of pre-class tutorial design on thermodynamics course that are being designed under our on going research are presented in Fig. 5.

Some researchers have developed several strategies to pre service physics teacher. P sinaga and s feranie (2017) has implemented writing task strategy to enhance critical thinking of third year pre-service physics teacher student in modern physics course [12]. Correction some other researchers have implemented other strategy in their learning design such as literacy strategies [13][14].
4. Conclusion

We have conducted a study that investigated student understanding of $PV$, $PT$ and $VT$ diagrams for various thermal processes and cycles. The results showed some of the findings as follows. First, the greatest difficulty is in describing PV, PT and VT are adiabatic processes. Second, the percentage of students who answered correctly on the PV diagram is higher than for PT and VT diagrams. Third, students better understand the thermodynamic processes in their typical characteristics, such as isobaric process with constant pressure, isotherm with constant temperature, and isochoric with constant volume. Lack of student understanding in drawing the diagram of various thermal processes also affects their ability in drawing diagrams of various thermal cycles. These findings are in line with previous study, and some are the first report in student understanding of $PVT$ diagram. One alternative solution that can be implemented is a pre-class tutorial, the modified tutorial design which is pioneered by Physics Education Group of University of Washington.

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