Influences of Elastic Parameters on Drag-Reduction Effect in Turbulent Flow of Viscoelastic Fluid

Chen Yang\textsuperscript{1,2,*}, Wei Feng\textsuperscript{1,2}, Jianzeng Liu\textsuperscript{1,2}, Xuan Gao\textsuperscript{1,2}, Zhida Tian\textsuperscript{3} and Caizhong Wang\textsuperscript{1,2}

\textsuperscript{1} College of Petroleum Engineering, China University of Petroleum at Beijing, Beijing, China;
\textsuperscript{2} State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum at Beijing, Beijing, China
\textsuperscript{3} Drilling & Production Research Institute, Liaohe Oilfield Company, PetroChina, Panjin 124010, China

Corresponding author’s e-mail: Yangchen@cup.edu.cn

Abstract. A 3D Reynolds-averaged Navier–Stokes (RANS) model was refined in the paper based on the \( k - \varepsilon - v^{2/3} - f \) turbulence model to simulate the drag reduction (DR) flow in the tube. The influence of elastic parameters on the drag reduction effect was studied using this numerical model and validated by the DNS data. The results showed that DR effect increased with both the relaxation time \( \lambda \) and the maximum extensibility of molecule \( L \). DR effect had an upper limit with the increasing of the maximum extensibility of molecule \( L \), and the growth of relaxation time \( \lambda \) also could enhance that upper limit.

1. Introduction

Newtonian fluid with polymer additives could exhibit a viscoelastic rheology characteristic. Tom (1948) \cite{1} first found that the turbulent friction drag of viscoelastic fluid can be significantly reduced compared with Newtonian fluid. Because of this feature, the viscoelastic fluid, as a drag reduction (DR) flow carrier, was widely applied in modern industry \cite{2}. In order to figure out the DR mechanism, an ocean of researches were conducted both experimentally and numerically.

Experiment is the most intuitive method to observe the phenomenon of drag reduction. However, the up-to-date experimental techniques cannot yet obtain the influence law of elastic parameters for studying the mechanism of turbulent DR. The development of computer technology prompts researchers to pay more attentions to numerical methods. Direct numerical simulation (DNS) is one of the numerical methods to study turbulent DR flow. However, because DNS results could show all the flow characteristics precisely, it can be used to validate other numerical models such as Reynolds averaged Navier-Stokes (RANS) model.

Leighton et al. (2003) \cite{3} first introduced the RANS method in the DR flow simulation. This method was widely utilized for simulation of industrial applications because of its advantages of less time consumption and computer memory occupation compared with DNS method. In their study, a Reynolds-stress transport equation model was presented. Pinho (2008) \cite{4} provided a new model for DR flow using the FENE-P constitutive equation based on \( k - \varepsilon \) model, and presented closures for...
turbulent correlation. Resende (2011) [5] optimized the Pinho’s model and provided new closures for nonlinear term. Iaccarino et al. (2010) [6] proposed a $k - \epsilon - v'^2 - f$ model based on Durbin’s model (1991) [7] and $k - \epsilon$ model. This model included two additional equations to represent the turbulence anisotropy. But some predictions did not coincide with DNS results well. Masoudian (2013) [8] modified and simplified the $k - \epsilon - v'^2 - f$ model. This model could provide more accurate predictions for DR flow by improving the closures for viscoelastic eddy viscosity. Based on the improved model, Zheng (2014) [9] tried to simulate turbulent DR flow by embedding model into FLUENT software with the functionalization of user-defined function (UDF), and obtained reliable results. At present, due to the complexity of the DR mechanism of viscoelastic fluid, the effect of elastic parameters on DR is still obscure.

In this paper, a 3D RANS model was refined based on the $k - \epsilon - v'^2 - f$ turbulence model and validated using DNS results. Then, the validated model was developed and embedded into the FLUENT to simulate the influence of elastic parameters on the DR.

2. Numerical model and method

2.1 Model
A 3D RANS model is refined in this paper to simulate the DR flow in the tube based on an existing model proposed by Zheng (2014) [9]. In Zheng’s model, $M_{kk}$, introduced by Masoudian (2013) [8], plays a great role in the viscous layer and is controlled by wall-normal direction. Actually, the turbulent DR flow in the tube is controlled by spanwise and wall-normal direction. This is because the regional scale of flow is similar in two directions. So $M_{kk}$ in the present model is modified as

$$M_{ii} = \frac{2\lambda}{f_c\mu_0} (U_i + U_i^*)$$

(1)

where $\lambda$ is the relaxation time of viscoelastic fluid, $f_c$ is the Peterlin function, $\mu_0$ is the viscoelastic eddy viscosity. More detail information about turbulent model is introduced in [9].

2.2 Method
The present model is embedded into “FLUENT” code through UDF based on an original turbulent model provided by FLUENT software. Non-uniform collocated mesh is adopted in the computational domain in each direction, mesh should be denser near the wall in order to get accurate turbulent information. On the other hand, it will not reduce the accuracy that using a sparser mesh in the turbulent core region and in the streamwise direction. In this way, the grids shown in Figure 1 along the spanwise and wall-normal direction are getting denser gradually from central to the wall and the grids in the streamwise direction are sparse. Coordinate of normalized radius ($r_{1i}^* = r_i^* + 2i$) is $R = 10$mm.

![Figure 1. Division of the grids](image-url)

3. Validation
In order to validate the present model, the results of the numerical simulation were compared with DNS results. Masoudian (2013) gave the DNS results of LDR (Low DR) and HDR (High DR) on a fully developed channel flow research with computational domain size of $16h*2h*8h$. Half channel height
is $h=0.1$ m and Reynold number $Re_\tau$ is $= 395$. Other parameters are shown in Table 1 ($Wi_\tau$ is Weissenberg number, $\beta$ is solver viscosity ratio, $L$ is maximum extensibility of polymer molecule).

### Table 1. DNS Parameters

|       | $Wi_\tau$ | $\beta$ | $L$ |
|-------|-----------|---------|-----|
| Low DR| 25        | 0.9     | 30  |
| High DR| 100      | 0.9     | 120 |

The present model in this paper is adopted to simulate the molecule extension ($C_{kk}$) and velocity ($U^+$) with the same parameters provided by Masoudian. The results by the present model and DNS are shown in follow Figures.

To compare with DNS results provided by Masoudian, all the results are given in the dimensionless form. The polymer extension simulated by the present model is shown and compared with DNS data in Figure 3, the results of cases (LDR and HDR) are both in qualitative agreement with DNS results. In Figure 4, the simulated velocities under the cases of different DR and the DNS results are presented. The results of LDR case are in better agreement with DNS data than those of HDR case. The maximum deviation of HDR case is about 15.2% and the maximum deviation of LDR case is less
than 4.5%.
Therefore, the present model is validated to be able to simulate the viscoelastic turbulent drag-reduction flow and can be utilized for further research.

4. Parameters analysis
In this section, we apply the validated model to investigate the influence of elastic parameters on the drag reduction effect in the tube.

4.1 Influence of $\lambda$ on DR
To make study convenient, the effect of Relaxation Time $\lambda$ on DR can be converted to the effect of Weissenberg Numbers $Wi_r$ based on equation $Wi_r = \lambda U_r^2 / (\mu_p + \mu_s)$.

In this research, five cases are placed with different $Wi_r$ which are 5, 10, 25, 50 and 100 separately, while other parameters remain the same. Dynamic viscosity of the viscoelastic polymer is set as $\mu_p = 0.00011 \text{ Pa} \cdot \text{s}$, so the solvent viscosity ratio is $\beta = \frac{\mu_s}{(\mu_p + \mu_s)} = 0.9$, and the Reynolds Number is $Re = \frac{\rho U_* R}{(\mu_p + \mu_s)} = 300$. The maximum extensibility of polymer molecule is set as $L = 100$.

Computational results of DR by the present model are shown in Table 2.

Table 2. Computational drag-reduction with different Weissenberg number

| Case | $Wi$ | DR |
|------|------|----|
| Case | 5    | -4%|
| Case | 10   | 5% |
| Case | 25   | 22%|
| Case | 50   | 38%|
| Case | 100  | 61%|

Table 2 shows that DR grows with the increasing of $Wi_r$, indicating that DR increases with the relaxation time $\lambda$. The DR effect will not occur when $Wi_r$ is less than 8. Since $\lambda$ is presented as a coefficient in FENE-P constitutive equation, the change of $\lambda$ will influence every quantity in turbulence equations. Therefore, the turbulent boundary layer theory is applied to further illustrate the influence of $\lambda$ on DR.

Figure 4. Computational results of normalized mean velocity

By comparison shown in Figure 4, the coincident segment between computational results and $U^+ = r^+$ gets longer, which means the viscous sublayer will get thicker. The length of the linear segment shortens, indicating that the size of log-law region gets smaller. The slope of the linear segment becomes larger, which implies the velocity gradient of the log-law region increases. Because
the viscous stress is dominant in viscous sublayer and the Reynolds stress is dominant in the log-law region, the thickening of viscous sublayer leads to the increase of viscous stress and the decrease of Reynolds stress in the whole flow region. Therefore, the turbulent dissipation of energy reduces. In other words, the mean velocity increases in whole flow region. So, DR effect increases when $\lambda$ gets larger.

### 4.2 Influence of $L$ on DR

The influence of the maximum extensibility $L$ of polymer molecule on DR is also investigated by the present model. In this study, five cases are set with different $L$ (30, 60, 90, 120 and 150), while other parameters remain the same. Weissenberg Numbers is set as $Wi_{\tau} = 30$; Simulation results of DR are shown in Table 3.

| Case | $L$ | DR |
|------|-----|----|
| Case F | 30 | 18% |
| Case G | 50 | 27% |
| Case H | 100 | 32% |
| Case I | 120 | 35% |
| Case J | 140 | 35% |

Table 3 shows that DR grows with the increasing of $L$ when $L$ is less than 120 and keeps constant after $L$ larger than 120. But this trend cannot be well explained only by the turbulence boundary layer theory. According to FENE-P constitutive equation, it can be found that $C_{kk}$ (deformability tensor) will be influenced by $L$. The elastic deformation of the molecule stores the energy of turbulent eddy. The increase of $C_{kk}$ indicates that the molecule is stretched along the flow direction and the storage of elastic energy increases. The DR grows with the increasing of $C_{kk}$. The change of $C_{kk}$ of these cases are shown in Figure 5 I to better understand this law.

#### Figure 5. Computational results of molecular extension I and II

According to Figure 5 I, $C_{kk}$ grows with the increasing of $L$ and keeps constant after $L$ larger than 100. This illustrates that the increase of stored energy caused by $L$ has an upper limit. Therefore, the growth of DR effect with the increasing of $L$ also has an upper limit. To study whether the increase of $\lambda$ can change this upper limit, a Case K with $Wi_{\tau} = 120$ is compared with Case I.

| Case | $L$ | $Wi_{\tau}$ | DR |
|------|-----|-------------|----|
| Case I | 120 | 30 | 35% |
| Case K | 120 | 120 | 67% |

According to Table 4, the upper limit of growth of DR effect caused by $L$ is broken when $\lambda$
increases. The computational results of $C_{kk}$ between before and after the growth of $\lambda$ are shown in Figure 5 II. The upper limit of the storage of elastic energy is also broken. It means that $C_{kk}$ will no longer change when $L$ exceeds a certain value, then if $\lambda$ increases, the upper limit of $C_{kk}$ will increase and DR effect will increase simultaneously.

It is worth mentioning that, the effect of temperature on drag reduction flow was be ignored in this paper. This should be studied by heat transfer property which will be built in the further research.

5. Conclusions
A 3-D turbulent drag-reducing flow model is refined and validated by DNS results in this paper. The influence of elastic parameters ($\lambda$ and $L$) on the DR effect was studied using the model that embedded into the FLUENT code through utilizing the functionalization of UDF, the relationship between DR and polymer elastic parameter has been ascertained:

DR effect increases with the growth of both relaxation time $\lambda$ and maximum extensibility of molecule $L$. DR effect had an upper limit with the increasing of $L$, and the growth of relaxation time $\lambda$ can enhance that upper limit by increasing the $C_{kk}$.

The structure of the turbulent boundary layer can be changed with the growth of relaxation time $\lambda$, thus reducing the Reynolds stress distribution space and increasing the flow velocity in the flow channel.

The energy storage capacity of tension can be improved by the molecule stretched along the flow direction with the growth of maximum extensibility of molecule $L$. Therefore, when the DR polymer additive is selected, the material with high maximum extensibility should be preferred.

Acknowledgement
Supported by the Science Foundation of China University of Petroleum, Beijing (No: 2462017BJB02)

References
[1] B.A. Toms. (1948). Some observations on the flow of linear polymer solutions through straight tubes at large Reynolds numbers. In the proceedings of the 1st International Congress on Rheology, pp, 135-141.
[2] Angelis, E. D., Casciola, C. M. and Piva, R. (2002). DNS of wall turbulence: dilute polymers and self-sustaining mechanisms. Computers & Fluids, 31(7), 495-507.
[3] Leighton, R., Walker, D. and Stephens, T. (2003). Reynolds Stress Modeling for Drag Reducing Viscoelastic Flows. ASME/JSME 4th Joint Fluids Summer Engineering Conference. American, 735-744.
[4] Pinho, F. T., Li, C. F. and B.A. Younis. (2008). A low Reynolds number turbulence closure for viscoelastic fluids. Journal of Non-Newtonian Fluid Mechanics, 181(2), 89-108.
[5] Resende, P. R., Kim, K. and B.A. Younis. (2011). A FENE-P $k$-$\varepsilon$ turbulence model for low and intermediate regimes of polymer-induced drag reduction. Journal of Non-Newtonian Fluid Mechanics, 166(12), 639-660.
[6] Iaccarino, G., Shaqfeh, E. S. G. and Dubief, Y. (2010). Reynolds-averaged modeling of polymer drag reduction in turbulent flows. Journal of Non-Newtonian Fluid Mechanics, 165(7), 376-384.
[7] Durbin, P. A. (1991). Near-wall turbulence closure modeling without “damping functions”. Theoretical and Computational Fluid Dynamics, 3(1), 1-13.
[8] Masoudian, M., Kim, K. and Pinho, F. T. (2013). A viscoelastic turbulent flow model valid up to the maximum drag reduction limit. Journal of Non-Newtonian Fluid Mechanics, 202, 99–111.
[9] Zheng, Z. Y., Li, F. C. and Li, Q. (2014). Reynolds-Averaged Simulation on Turbulent Drag-Reducing Flows of Viscoelastic Fluid Based on User-Defined Function in FLUENT Package. 14th Joint Fluids Summer Engineering Conference. American