IMPACTS OF COLLECTIVE NEUTRINO OSCILLATIONS ON CORE-COLLAPSE SUPERNOVA EXPLOSIONS

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ABSTRACT

By performing a series of one- and two-dimensional (1D and 2D) hydrodynamic simulations with spectral neutrino transport, we study possible impacts of collective neutrino oscillations on the dynamics of core-collapse supernovae. To model the spectral swapping, which is one of the possible outcome of the collective neutrino oscillations, we parameterize the onset time when the spectral swap begins, the radius where the spectral swap occurs, and the threshold energy above which the spectral interchange between heavy-lepton neutrinos and electron/anti-electron neutrinos takes place. By doing so, we systematically study how the neutrino heating enhanced by the spectral swapping could affect the shock evolution as well as the matter ejection. We also investigate the progenitor dependence using a suite of progenitor models (13, 15, 20, and 25 $M_\odot$). We find that there is a critical heating rate induced by the spectral swapping that triggers explosions, and which significantly differs between the progenitors. The critical heating rate is generally smaller for 2D than for 1D due to the multidimensionality that enhances the neutrino heating efficiency. For the progenitors employed in this paper, the final remnant masses are estimated to range between $1.1-1.5 \, M_\odot$. For our 2D model of the 15 $M_\odot$ progenitor, we find a set of oscillation parameters that could account for strong supernova explosions ($\sim 10^{51}$ erg), simultaneously leaving behind a remnant mass close to $\sim 1.4 \, M_\odot$.

Key words: hydrodynamics – neutrinos – radiative transfer – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Although the explosion mechanism of core-collapse supernovae is not yet completely understood, current multidimensional (multi-D) simulations based on refined numerical models show several promising scenarios. Among the candidates are the neutrino heating mechanism aided by convection and standing accretion shock instability (SASI; e.g., Marek & Janka 2009; Bruenn et al. 2009; Suwa et al. 2010), the acoustic mechanism (Burrows et al. 2007b), and the magnetohydrodynamic mechanism (e.g., Kotake et al. 2004, 2006; Obergaulinger et al. 2006; Burrows et al. 2007a; Takiwaki et al. 2009). Probably the best-studied one is the neutrino heating mechanism, whose basic concept was first proposed by Colgate & White (1966) and later reinforced by Bethe & Wilson (1985) to take the currently prevailing delayed form.

An important lesson from the multi-D simulations mentioned above is that hydrodynamic motions associated with convective overturn (Herant et al. 1994; Burrows et al. 1995; Janka & Mueller 1996; Fryer & Warren 2002, 2004) as well as the SASI (e.g., Blondin et al. 2003; Scheck et al. 2006; Ohnishi et al. 2006; Foglizzo et al. 2007; Murphy & Burrows 2008; Iwakami et al. 2008; Guilet et al. 2010; Fernández 2010) can help the onset of the neutrino-driven explosion, which otherwise generally fails in spherically symmetric one-dimensional (1D) simulations (Liebendörfer et al. 2001; Rampp & Janka 2002; Thompson et al. 2003; Sumiyoshi et al. 2005). This is mainly because the accretion timescale of matter in the gain region can be longer than in the 1D case, thereby enhancing the strength of neutrino–matter coupling there.

Neutrino-driven explosions have been obtained in the following state-of-the-art two-dimensional (2D) simulations. Using the MuDBaTH code, which includes one of the best available neutrino transfer approximations, Buras et al. (2006) first reported explosions for a non-rotating low-mass ($11.2 \, M_\odot$) progenitor from Woosley et al. (2002), and then for a $15 \, M_\odot$ progenitor from Woosley & Weaver (1995) with a moderately rapid rotation imposed (Marek & Janka 2009). By implementing a multi-group flux-limited diffusion algorithm in the CHIMERA code (e.g., Bruenn et al., 2009), Yakunin et al. (2010) obtained explosions for non-rotating $12 \, M_\odot$ and $25 \, M_\odot$ progenitors from Woosley et al. (2002). More recently, Suwa et al. (2010) pointed out that a stronger explosion is obtained for a rapidly rotating $13 \, M_\odot$ progenitor from Nomoto & Hashimoto (1988) compared to the corresponding non-rotating model, when the isotropic diffusion source approximation (IDSA) for spectral neutrino transport (Liebendörfer et al. 2009) is implemented in the ZEUS code.

However, this success further begs new questions. First of all, the explosion energies obtained in these simulations are typically underpowered by one or two orders of magnitudes to explain the canonical supernova kinetic energy ($\sim 10^{51}$ erg). Moreover, the softer nuclear equation of state (EOS) such as the Lattimer & Swesty (1991, hereafter LS) EOS with an incompressibility $K = 180$ MeV at nuclear densities is employed in those simulations. On top of evidence that favors a stiffer EOS based on nuclear experimental data (Shlomo et al. 2006), the soft EOS may not account for the recently observed massive neutron star of $\sim 2 \, M_\odot$ (Demorest et al. 2010; see the maximum mass for the LS180 EOS in O’Connor & Ott 2011). With a stiffer EOS, the explosion energy may be even lower than inferred from Marek & Janka (2009), who did not obtain the neutrino-driven explosion for their model with $K = 263$ MeV. What is then still missing? We may obtain the answer by using three-dimensional (3D) simulations (Nordhaus et al. 2010) or...
by taking into account new concepts, such as exotic physics in the core of the protoneutron star (Sagert et al. 2009), viscous heating by the magnetorotational instability (Thompson et al. 2005; Masada et al. 2011), or energy dissipation via Alfvén waves (Suzuki et al. 2008).

Joining in these efforts, we explore in this study the possible impacts of collective neutrino oscillations on energizing the neutrino-driven explosions. Collective neutrino oscillations, i.e., neutrinos of all energies that oscillate almost in phase, are attracting great attention, because they can induce dramatic observable effects such as a spectral split or swap (e.g., Raffelt & Smirnov 2007; Duan et al. 2008; Dasgupta et al. 2008; and references therein). These effects are predicted to emerge as distinct features in the energy spectra (see Duan et al. 2010; Dasgupta 2010; and references therein, for reviews of the rapidly growing research field). Among a number of important effects possibly created by self-interaction, we choose to consider the effect of spectral splits between electron- ($\nu_e$) anti-electron neutrinos ($\bar{\nu}_e$), and heavy-lepton neutrinos ($\nu_x$, i.e., $\nu_{\mu}$, $\nu_{\tau}$, and their anti-particles) above a threshold energy (e.g., Fogli et al. 2007). Since $\nu_x$ have higher average energies than the other species in the postbounce phase, the neutrino flavor mixing would increase the effective energies of $\nu_e$ and $\bar{\nu}_e$, and hence increase the neutrino heating rates in the gain region. A formalism to treat the neutrino oscillation using the Boltzmann neutrino transport is given in Yamada (2000) and Strack & Burrows (2005), but it is difficult to implement. To mimic the effects in this study, we perform the spectral swap by hand as a first step. By changing the average neutrino energy, ($\langle \epsilon_{\nu_x} \rangle$, as well as the position of the neutrino spheres ($R_{\nu_x}$) in a parametric manner, we hope to constrain the parameter regions spanned by ($\langle \epsilon_{\nu_x} \rangle$) and $R_{\nu_x}$ wherein the additional heating from collective neutrino oscillations could have impact on the explosion dynamics. Our strategy is as follows. We will first constrain the parameter regions to some extent by performing a number of 1D simulations. Here we also investigate the progenitor dependence using a suite of progenitor models (13, 15, 20, and 25 $M_\odot$). After squeezing the condition in the 1D computations, we include the flavor conversions in 2D simulations to see their impact on the dynamics and also discuss how the critical condition for the collective effects in 1D can be subject to change in 2D.

The paper opens with descriptions of the initial models and the numerical methods, focusing on how to model the collective neutrino oscillations (Section 2). The main results are shown in Section 3. We summarize our results and discuss their implications in Section 4.

2. NUMERICAL METHODS

2.1. Hydrodynamics

The employed numerical methods are essentially the same as those in our previous paper (Suwa et al. 2010). For convenience, we briefly summarize them in the following. The basic evolution equations are written as

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0,$$

$$\frac{d\mathbf{v}}{dt} = -\nabla P - \rho \nabla \Phi,$$

$$\frac{d\epsilon_{\nu^+}}{dt} + \nabla \cdot [(\epsilon_{\nu^+} + P) \mathbf{v}] = -\rho \mathbf{v} \cdot \nabla \Phi + Q_{\nu^+},$$

where $\rho$, $\mathbf{v}$, $P$, $\mathbf{e}^+$, and $\Phi$ are density, fluid velocity, gas pressure including the radiation pressure of neutrinos, total energy density, and gravitational potential, respectively. The time derivatives are Lagrangian. As for the hydro solver, we employ the ZEUS-2D code (Stone & Norman 1992) which has been modified for core-collapse simulations (e.g., Suwa et al. 2007a, 2007b, 2009; Takiwaki et al. 2009). $Q_{\nu^+}$ and $Q_{\nu^+}$ (in Equations (3) and (4)) represent the change of energy and electron fraction ($Y_e$) due to interactions with neutrinos. To estimate these quantities, we implement spectral neutrino transport using the IDSA scheme (Liebendorfer et al. 2009). The IDSA scheme splits the neutrino distribution into two components, both of which are solving using separate numerical techniques. We apply the so-called ray-by-ray approach in which the neutrino transport is solved along a given radial direction assuming that the hydrodynamic medium for the direction is spherically symmetric. Although the current IDSA scheme does not yet include $\nu_e$ and the inelastic neutrino scattering with electrons, these simplifications save a significant amount of computational time compared to the canonical Boltzmann solvers (see Liebendorfer et al. 2009 for more details). Following the prescription in M"uller et al. (2010), we improve the accuracy of the total energy conservation by using a conservation form in Equation (3), instead of solving the evolution of internal energy as originally designed in the ZEUS code. Numerical tests are presented in the Appendix.

The simulations are performed on a grid of 300 logarithmically spaced radial zones from the center up to 5000 km and 128 equidistant angular zones covering $0 \leq \theta \leq \pi$ for 2D simulations. For the spectral transport, we use 20 logarithmically spaced energy bins ranging from 3 to 300 MeV.

2.2. Spectral Swapping

As mentioned in Section 1, we introduce a spectral interchange from heavy-lepton neutrinos ($\nu_x$, $\bar{\nu}_x$, and their antineutrinos, collectively referred as $\nu_x$ hereafter) to electron-type neutrinos and antineutrinos, namely $\nu_x \rightarrow \nu_e$ and $\bar{\nu}_x \rightarrow \bar{\nu}_e$. Instead of solving the transport equations for $\nu_x$, we employ the so-called light-bulb approximation and focus on the optically thin region outside the neutrinosphere (e.g., Janka & Mueller 1996; Ohnishi et al. 2006).

According to Duan et al. (2010), the threshold energy, $\epsilon_{\text{th}}$, is set to be 9 MeV, above which spectral swap takes place. Below the threshold, neutrino heating is estimated from the spectral transport via the IDSA scheme. Above the threshold, the heating rate is replaced by

$$Q_{E} \propto \int_{\epsilon_{\text{th}}}^{\infty} d\epsilon_{\nu^+} \epsilon_{\nu^+}^{3} [j(\epsilon_{\nu^+}) + \chi(\epsilon_{\nu^+})] f_{\nu^+}(r, \epsilon_{\nu^+}),$$

where $j$ and $\chi$ are the neutrino emissivity and absorptivity, respectively, and $f_{\nu^+}(r, \epsilon_{\nu^+})$ corresponds to the neutrino distribution function for $\nu_{\nu}$ with $\epsilon_{\nu}$ being the energies of the electron neutrinos and antineutrinos. In the light-bulb approach, this is often approximated by the Fermi–Dirac distribution with a vanishing chemical potential (e.g., Ohnishi et al. 2006) as

$$f_{\nu^+}(r, \epsilon_{\nu^+}) = \frac{1}{\epsilon_{\nu^+} / kT_{\nu^+} + 1} g(r),$$
where $k$ and $T_{\nu}$ are the Boltzmann constant and the neutrino temperature, respectively. $g(r)$ is the geometric factor, $g(r) = 1 - \left[1 - (R_{\nu}/r)^2\right]^{1/2}$, which is taken into account for the normalization, with $R_{\nu}$ the radius of the neutrinosphere. The neutrino luminosity of $\nu_{\ell}$ at infinity is given as

$$L_{\nu_{\ell}} = 2.62 \times 10^{52} \left(\frac{\langle \epsilon_{\nu_{\ell}} \rangle}{15 \text{ MeV}}\right)^{4} \left(\frac{R_{\nu}}{30 \text{ km}}\right)^{2} \text{ erg s}^{-1},$$

(8)

where $\langle \epsilon_{\nu_{\ell}} \rangle = \int_{0}^{\infty} d\epsilon_{\nu_{\ell}} \epsilon_{\nu_{\ell}}^{2} f_{\nu_{\ell}}(\epsilon_{\nu_{\ell}}) / \int_{0}^{\infty} d\epsilon_{\nu_{\ell}} \epsilon_{\nu_{\ell}}^{2} f_{\nu_{\ell}}(\epsilon_{\nu_{\ell}})$ is the average energy of the emitted neutrinos. The position where spectral swapping sets in is fixed at 100 km (around the gain radius) and the onset time $t_s$ is varied as a parameter: $t_s = 100, 200, \text{and } 300 \text{ ms after bounce}$. In fact, the threshold energy depends on the neutrino luminosities, spectra, and oscillation parameters (see, e.g., Duan et al. 2010 and references therein) with conserved net $\nu_x$ flux (i.e., the lepton number conservation). However, the conservation of the lepton number is too complicated to satisfy in the dynamical simulation because the neutrino spectrum and the luminosity evolve with time. In order to focus on the hydrodynamic features affected by the spectral modulation induced by the swapping, we simplify just a single threshold energy in this work.

To summarize, the parameters that we use to mimic spectral swapping are the following: (1) $R_{\nu_x}$, the radius of the neutrinosphere of $\nu_x$, (2) $\langle \epsilon_{\nu_x} \rangle$, the average energy of $\nu_x$, and (3) $t_s$, the time when spectral swapping sets in.

3. RESULT

3.1. One-dimensional Models

3.1.1. 1D without Spectral Swapping

In this subsection, we first outline the 1D collapse dynamics without spectral swapping. We take a 13 $M_\odot$ progenitor (Nomoto & Hashimoto 1988) as a reference.

At around 112 ms after the onset of gravitational collapse, the bounce shock forms at a radius of $\sim$10 km with an enclosed mass of $\sim$0.7 $M_\odot$. The central density at this time is $\rho_c = 3.6 \times 10^{14} \text{ g cm}^{-3}$. The shock propagates outward but finally stalls at a radius of $\sim$100 km. Due to the decreasing accretion rate through the stalled shock, the shock can be still pushed outward. However, after some time, the shock radius begins to shrink. The ratio of the advection timescale, $\tau_{\text{adv}}$, and the heating timescale, $\tau_{\text{heat}}$, is an important indicator for the criteria of neutrino-driven explosion (Buras et al. 2006; Marek & Janka 2009; Suwa et al. 2010). In our 1D simulations, $\tau_{\text{adv}}/\tau_{\text{heat}}$ is generally smaller than unity in the postbounce phase. This is the reason why our 1D simulations do not yield a delayed explosion. This is also the case for the other progenitors (15, 20, and 25 $M_\odot$) investigated in this study. As for the accretion phase (later than ~50 ms after the bounce), the typical neutrino luminosity at $r = 5000 \text{ km}$ is $3 \times 10^{52} \text{ erg s}^{-1}$ for both $\nu_e$ and $\bar{\nu}_e$, and the typical average energy is $\langle \epsilon_{\nu_{\ell}} \rangle \approx 9 \text{ MeV}$ and $\langle \epsilon_{\bar{\nu}_{\ell}} \rangle \approx 12 \text{ MeV}$ as shown in Figure 1. Figure 2 indicates the resultant neutrino luminosity spectrum 100 ms after the bounce.

3.1.2. 1D with Spectral Swapping

The investigated models with spectral swapping are summarized in Table 1. As already mentioned, the model parameters are the neutrinosphere radius ($R_{\nu_x}$), the average energy of neutrinos ($\langle \epsilon_{\nu_{\ell}} \rangle$), and the onset time of spectral swapping ($t_s$). The model names include these parameters: “NH13” represents the progenitor model, “R..” represents $R_{\nu_x}$ in units of km, “E..” represents $\langle \epsilon_{\nu_{\ell}} \rangle$ in MeV, “T..” represents $t_s$ in ms, and “S” represents 1D (spherical symmetry). Figure 3 presents the time evolution of the mass shells for models NH13R30E12T100S and NH13R30E13T100S. The difference between these panels is the average energies of neutrinos ($\langle \epsilon_{\nu_{\ell}} \rangle = 12 \text{ MeV}$ for the top panel and 13 MeV for the bottom panel. The thick solid lines represent the radial positions of the shock waves. Regardless of a small difference in $\langle \epsilon_{\nu_{\ell}} \rangle$, model NH13R30E13T100S shows a shock expansion after the manual spectral swapping is switched on (see the thick line in the bottom panel of Figure 3), while the stalled shock does not revive for model NH13R30E12T100S (top panel). This suggests that there is a critical condition for successful explosion induced by spectral swapping. In the bottom panel, the regions enclosing the mass of $M_c \sim 1.2 \ M_\odot$ (thin black line) correspond to the so-called mass cut, which could be interpreted as the final mass of the remnant. The fact that a clear mass cut emerges

![Figure 1. Time evolution of the neutrino luminosity (top panel) and average energy (bottom panel) for $\nu_e$ (red solid line) and $\bar{\nu}_e$ (blue dashed line). (A color version of this figure is available in the online journal.)](image1)

![Figure 2. Neutrino luminosity spectrum of $\nu_e$ (red solid line) and $\bar{\nu}_e$ (blue dashed line) without spectral swapping 100 ms after the bounce. For comparison, we show the injected luminosity spectrum of $\nu_e$ with $\langle \epsilon_{\nu_{\ell}} \rangle = 15 \text{ MeV}$, which will be swapped with the original spectrum of $\nu_e$ and $\bar{\nu}_e$ at $\epsilon_{\nu} > 9 \text{ MeV}$ for models including spectral swapping. (A color version of this figure is available in the online journal.)](image2)

![Figure 3. The investigated models with spectral swapping are summarized in Table 1. As already mentioned, the model parameters are the neutrinosphere radius ($R_{\nu_x}$), the average energy of neutrinos ($\langle \epsilon_{\nu_{\ell}} \rangle$), and the onset time of spectral swapping ($t_s$). The model names include these parameters: “NH13” represents the progenitor model, “R..” represents $R_{\nu_x}$ in units of km, “E..” represents $\langle \epsilon_{\nu_{\ell}} \rangle$ in MeV, “T..” represents $t_s$ in ms, and “S” represents 1D (spherical symmetry). Figure 3 presents the time evolution of the mass shells for models NH13R30E12T100S and NH13R30E13T100S. The difference between these panels is the average energies of neutrinos, $\langle \epsilon_{\nu_{\ell}} \rangle = 12 \text{ MeV}$ for the top panel and 13 MeV for the bottom panel. The thick solid lines represent the radial positions of the shock waves. Regardless of a small difference in $\langle \epsilon_{\nu_{\ell}} \rangle$, model NH13R30E13T100S shows a shock expansion after the manual spectral swapping is switched on (see the thick line in the bottom panel of Figure 3), while the stalled shock does not revive for model NH13R30E12T100S (top panel). This suggests that there is a critical condition for successful explosion induced by spectral swapping. In the bottom panel, the regions enclosing the mass of $M_c \sim 1.2 \ M_\odot$ (thin black line) correspond to the so-called mass cut, which could be interpreted as the final mass of the remnant. The fact that a clear mass cut emerges](image3)
Figure 3. Time evolution of mass shells for NH13R30E12T100S (top) and NH13R30E13T100S (bottom). The black thin line corresponds to 1.2 \( M_\odot \) and the black thick line represents the shock wave position, respectively. The difference between these panels is the average energies of neutrinos, \( \langle \epsilon_{\nu_x} \rangle = 12 \text{ MeV} \) for the top panel and 13 MeV for the bottom panel.

Table 1
1D Simulations

| Model            | Dimension | \( R_\nu \) (km) | \( \langle \epsilon_{\nu_x} \rangle \) (MeV) | \( L_\nu \) \( (10^{53} \text{ erg s}^{-1}) \) | \( t_s \) (ms) | Explosion | \( E_\nu^{\text{diag}} \) \( (10^{51} \text{ erg}) \) | \( M_{10}^{\text{mej}} \) \( (M_\odot) \) | \( M_{10}^{\infty} \) \( (M_\odot) \) |
|------------------|-----------|------------------|---------------------------------|------------------|---------------|-----------|-----------------|-----------------|-----------------|
| NH13R10E1T100S   | 1D        | 10               | 15 MeV                          | 0.29             | 100           | No        | ...             | 1.18            | ...             |
| NH13R10E1T100S   | 1D        | 10               | 17 MeV                          | 0.48             | 100           | No        | ...             | 1.18            | ...             |
| NH13R10E1T100S   | 1D        | 10               | 18 MeV                          | 0.60             | 100           | No        | ...             | 1.18            | ...             |
| NH13R10E1T100S   | 1D        | 10               | 19 MeV                          | 0.75             | 100           | Yes       | 1.00            | 1.18            | 1.14            |
| NH13R10E20T100S  | 1D        | 10               | 20 MeV                          | 0.92             | 100           | Yes       | 1.49            | 1.18            | 1.12            |
| NH13R20E1T100S   | 1D        | 20               | 13 MeV                          | 0.66             | 100           | No        | ...             | 1.18            | ...             |
| NH13R20E1T100S   | 1D        | 20               | 13 MeV                          | 0.66             | 150           | No        | ...             | 1.21            | ...             |
| NH13R20E1T200S   | 1D        | 20               | 13 MeV                          | 0.66             | 200           | No        | ...             | 1.25            | ...             |
| NH13R20E1T100S   | 1D        | 20               | 14 MeV                          | 0.88             | 100           | No        | ...             | 1.18            | ...             |
| NH13R20E1T100S   | 1D        | 20               | 14 MeV                          | 0.88             | 150           | No        | ...             | 1.21            | ...             |
| NH13R20E1T200S   | 1D        | 20               | 14 MeV                          | 0.88             | 200           | No        | ...             | 1.25            | ...             |
| NH13R20E1T100S   | 1D        | 20               | 15 MeV                          | 1.16             | 100           | Yes       | 0.97            | 1.18            | 1.15            |
| NH13R20E1T100S   | 1D        | 20               | 15 MeV                          | 1.16             | 150           | Yes       | 0.54            | 1.21            | <1.24           |
| NH13R20E1T200S   | 1D        | 20               | 15 MeV                          | 1.16             | 200           | Yes       | 0.47            | 1.25            | <1.26           |
| NH13R20E21T100S  | 1D        | 20               | 16 MeV                          | 4.47             | 100           | Yes       | 5.56            | 1.18            | 1.07            |
| NH13R20E21T100S  | 1D        | 20               | 16 MeV                          | 4.47             | 200           | Yes       | 5.56            | 1.18            | 1.07            |
| NH13R20E21T100S  | 1D        | 20               | 17 MeV                          | 4.47             | 300           | Yes       | 5.56            | 1.18            | 1.07            |
| NH13R20E21T100S  | 1D        | 20               | 17 MeV                          | 4.47             | 400           | Yes       | 5.56            | 1.18            | 1.07            |
in model NH13R30E13T100S indicates that a neutron star will be left behind in this model. Such a definite mass cut has been observed by Kitaura et al. (2006) who reported a successful neutrino-driven explosion (in 1D) for a lighter progenitor star, which is, however, difficult to realize for more massive stars in 2D (e.g., Figure 2 in Marek & Janka 2009 and Figure 1 in Suwa et al. 2010).

As a tool to measure the strength of an explosion, we define a diagnostic energy that refers to

$$E_{\text{diag}} = \int_D dV \left( \frac{1}{2} \rho |v|^2 + e - \rho \Phi \right),$$  \hspace{1cm} (9)

where $e$ is the internal energy, and $D$ represents the domain in which the integrand is positive. Figure 4 shows the time evolution of $E_{\text{diag}}$ for some selected models. The diagnostic energy increases with time for the green dotted line, but decreases for the red line, noting that the difference between the pair of models is $\Delta(\epsilon_{\nu_x}) = 1$ MeV. The blue dashed line (model NH13R30E15T100S) has $\langle \epsilon_{\nu_x} \rangle = 15$ MeV and reaches larger $E_{\text{diag}}$ than the green line (NH13R30E13T100S; $\langle \epsilon_{\nu_x} \rangle = 13$ MeV). On the other hand, the later injection of spectral swapping leads to smaller $E_{\text{diag}}$, i.e., the brown dot-dashed line ($t_s = 200$ ms) shows smaller $E_{\text{diag}}$ than the blue dashed line ($t_s = 100$ ms). For models that experience earlier spectral swapping with higher neutrino energy, the diagnostic energy becomes higher in an earlier stage, as expected.

Looking at Figure 4 again, $E_{\text{diag}}$ for the exploding models seems to saturate with time. These curves can be fitted by the following function,

$$E_{\text{diag}}(t) = E_{\text{diag}}^{\infty} (1 - e^{-at+bt}),$$  \hspace{1cm} (10)

where $E_{\text{diag}}^{\infty}$ is a converging value of $E_{\text{diag}}$, and $a$ and $b$ are the fitting parameters. As for NH13R30E13T100S, $E_{\text{diag}}^{\infty} = 8.5 \times 10^{50}$ erg. This fitting formula allows us to estimate the final diagnostic energy especially for the strongly exploding models whose diagnostic energy we cannot estimate in principle because the shock goes beyond the computational domains ($r < 5000$ km) before saturation.

Figure 5 shows the summary of 1D models for a given neutrino luminosity determined by $R_{\nu_x}$ and $\langle \epsilon_{\nu_x} \rangle$ (Equation (8)). The gray lines correspond to the neutrino luminosities determined by the pairs of $R_{\nu_x}$ and $\langle \epsilon_{\nu_x} \rangle$ which is $(1-5) \times 10^{52}$ erg s$^{-1}$ from bottom to top. Circles and crosses correspond to the exploding and non-exploding models, respectively. Not surprisingly, explosions are more easily obtained for higher neutrino luminosity.

As is well known, the combination of $\langle \epsilon_{\nu_x} \rangle$ and $R_{\nu_x}$ is an important quantity in diagnosing the success or failure of explosions, because the neutrino heating rate in the so-called gain region, $Q^*_\nu$, is proportional to $\langle \epsilon_{\nu_x} \rangle^2 R_{\nu_x}$ (e.g., Equation (23) in Janka 2001).

Figure 6 shows $E_{\text{diag}}^{\infty}$ as a function of $\langle \epsilon_{\nu_x} \rangle^2 R_{\nu_x}$. Note in the plot that we set the horizontal axis not as $\langle \epsilon_{\nu_x} \rangle^2 R_{\nu_x}$ but as $\langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x}$, so that we can deduce the following dependence more clearly and easily. In this figure, let us first focus on the red pluses, green crosses, and blue squares whose difference is characterized by $t_s$ (2D results (filled circles) will be mentioned in a later section). The red ($t_s = 100$ ms), green ($t_s = 150$ ms), and blue ($t_s = 200$ ms) points have a clear correlation with $\langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x}$. The orange and light-blue regions represent the

$$\langle \epsilon_{\nu_x} \rangle = \int_0^\infty d\epsilon_{\nu_x} \epsilon_{\nu_x} f_{\epsilon_{\nu_x}} / \int_0^\infty d\epsilon_{\nu_x} \epsilon_{\nu_x}^2 f_{\epsilon_{\nu_x}} (\epsilon_{\nu_x})$$

and $\langle \epsilon_{\nu_x} \rangle$ can be simply connected as $\langle \epsilon_{\nu_x} \rangle = 2.1 (\epsilon_{\nu_x})^2$ for the neutrino spectrum of Equation (7).
non-exploding regions for the red and blue points, respectively. Both of them show that the minimum $E_{\text{diag}}$ decreases with $t_\nu$, indicating that the critical values of $(\epsilon_\nu)^2 L_{\nu}$ for explosion sharply depend on $t_\nu$. This is because the mass outside the shock wave gets smaller with time so that the minimum energy to blow up the star gets smaller, too. By the same reason, $E_{\text{diag}}$ becomes larger as $t_\nu$ becomes smaller given the same $(\epsilon_\nu)^2 L_{\nu}$. To obtain a larger $E_{\text{diag}}$, earlier spectral swapping is preferred.

Figure 7 shows the neutrino heating rate and the density distribution of NH13R30E13T100S for 10 ms and 250 ms after $t_\nu$ (= 100 ms after the bounce). As the shock wave propagates outward, the density in the gain region drops sharply (e.g., 100–200 km, dashed blue line), leading to the suppression of the heating rate (dashed red line). This is the reason for the saturation in $E_{\text{diag}}$ as shown in Figure 4.

The remnant mass is an important indicator to diagnose the consequences of the explosion in producing either a neutron star or a black hole. The last two lines in Table 1 show the integrated masses in the regions of $\rho \geq 10^{19}$ g cm$^{-3}$ at $t = t_\nu$ and $t = \infty$. The latter one is estimated by fitting as

$$M_{10}(t) = M_{10}^{\infty}(1 + e^{-ct+td})$$

(11)

where $c$ and $d$ are the fitting parameters. For the exploding models, $M_{10}^{\infty}$ becomes generally smaller than $M_{10}^{\text{init}}$, because of the mass ejection. Exceptions are weakly exploding models (NH13R20E15T150S, NH13R20E15T200S, NH13R30E13T100S, and NH13R50E11T100S), in which the mass accretion continues after $t_\nu$ and eventually stops at late time (maximum masses are presented in Table 1). For the non-exploding models, the remnant mass simply increases with time. Regarding the 13 $M_\odot$ progenitor investigated in this section, the remnant masses in models that produce strong explosion ($E_{\text{diag}} \gtrsim 10^{51}$ erg) are considerably smaller (1.1–1.2 $M_\odot$) if compared to the typical mass of observed neutron stars $\sim 1.4 M_\odot$ (Lattimer & Prakash 2007). This may simply reflect the light iron core ($\sim 1.26 M_\odot$) inherent to the progenitor or the existence of mass accretion induced by the matter fallback after the explosion. We now move on to investigate the progenitor dependence in the next section.

3.1.3. The Progenitor Dependence

In addition to the 13 $M_\odot$ progenitor by Nomoto & Hashimoto (1988), we are going to investigate the progenitor dependence in 1D simulations. The computed models are NH15 (15 $M_\odot$; Nomoto & Hashimoto 1988), s15s7b2 (15 $M_\odot$; Woosley & Weaver 1995), s15.0 (15 $M_\odot$), s20.0 (20 $M_\odot$), and s25.0 (25 $M_\odot$; Woosley et al. 2002), which are all listed in Table 2. The first set of characters for these models indicate the density profiles of investigated progenitors 100 ms after the bounce as functions of the enclosed mass.

Figure 8 depicts density profiles of these progenitors 100 ms after the bounce as a function of the progenitor mass, $M$. The circles are progenitors from Nomoto & Hashimoto (1988), the square is from Woosley & Weaver (1995), and the crosses are from Woosley et al. (2002), respectively. The error bars represent the distance between the last failing and the first exploding model in our grid of models. The symbols are located at the centers of error bars. The error bar is small for model NH13 because we calculated a more refined grid of models for the 13 $M_\odot$ progenitor (Table 1) than for the 15–25 $M_\odot$ progenitors (Table 2).

Figure 9 shows the critical heating rates, $(\epsilon_\nu)^2 L_{\nu}$, as a function of the progenitor mass, $M$. The circles are progenitors from Nomoto & Hashimoto (1988), the square is from Woosley & Weaver (1995), and the crosses are from Woosley et al. (2002), respectively. The error bars represent the distance between the last failing and the first exploding model in our grid of models. The symbols are located at the centers of error bars. The error bar is small for model NH13 because we calculated a more refined grid of models for the 13 $M_\odot$ progenitor (Table 1) than for the 15–25 $M_\odot$ progenitors (Table 2).
heating rates for models WHW25 and NH13 are in the high and low ends, respectively. However, the critical heating rate for model WHW20 is almost the same as the one for model NH13 although the envelope of model WHW20 is much thicker than that of model NH13 (see Figure 8). Our results show that the critical heating rate is indeed affected by the envelope mass; however, the relation is not one-to-one. It is also interesting to note that the critical heating rates for the 15 $M_{\odot}$ progenitors of WW15, WHW15, and NH15 are different by a factor of $\sim 3$, which may send a clear message that an accurate knowledge of supernova progenitors is also pivotal to pin down the supernova mechanism.

The integrated masses with $\rho \gtrsim 10^{10}$ g cm$^{-3}$ for $t = t_s$ and $t = \infty$ are listed in the last two lines in Table 2 and Figure 10. The tendencies are the same as those found with NH13. As for model WHW25, we obtain results with $E_{\text{diag}}^{\infty} > 10^{51}$ erg and $M_{10}^{\infty} > 1.4 M_{\odot}$, simultaneously.

### 3.2. Two-dimensional Models

Here we discuss the effects of spectral swapping in 2D (axisymmetric) simulations. Since our 2D simulations, albeit utilizing the IDSA scheme, are still computationally expensive, it is not practical to perform a systematic survey in 2D as we have done in 1D simulations. Looking at Figure 9 again, we choose models WHW15 (Woosley et al. 2002) and NH13 (Nomoto & Hashimoto 1988), whose critical heating rates are in the high and low ends, respectively.

#### 3.2.1. 2D without Spectral Swapping

The basic hydrodynamic picture is the same with 1D before the shock stall (e.g., until $\lesssim 10$ ms after bounce). After that, convection as well as SASI sets in between the stalled shock and the gain radius, which leads to the neutrino-heated shock revival for model NH13 (e.g., Suwa et al. 2010). For model WHW15, the position of the stalled shock, following several oscillations begins to shrink at $\gtrsim 400$ ms after bounce.

Even after the shock revival, it should be emphasized that the shock propagation for model NH13 is the so-called passive one (Buras et al. 2006). This means that the amount of mass ejection is smaller than the accretion in the post-shock region of the expanding shock (see the motions of mass shells in the post-shock region of Figure 1 in Suwa et al. 2010). Some regions have a positive local energy (Equation (9)), but the volume integrated value is quite small, $\lesssim 10^{50}$ erg at the maximum. In

### Table 2

| Model          | Dimension | $R_{\nu}$ (km) | $\langle \nu \rangle$ (MeV) | $L_{\nu}$ ($10^{52}$ erg s$^{-1}$) | $t_s$ (ms) | Explosion | $E_{\text{diag}}^{\infty}$ ($10^{51}$ erg) | $M_{10}^{\infty}$ ($M_{\odot}$) | $M_{10}^{\infty}$ ($M_{\odot}$) |
|----------------|-----------|----------------|----------------------------|-----------------------------------|------------|-----------|------------------------------------------|---------------------------------|--------------------------------|
| WHW15R30E11T100S | 1D        | 30             | 11 MeV                    | 0.76                              | 100        | No        | ...                                      | 1.34                            | ...                            |
| NH15R30E12T100S | 1D        | 30             | 12 MeV                    | 1.07                              | 100        | No        | ...                                      | 1.34                            | ...                            |
| WHW15R30E13T100S | 1D        | 30             | 13 MeV                    | 1.48                              | 100        | Yes       | 0.65                                     | 1.34                            | <1.38                          |
| NH15R30E14T100S | 1D        | 30             | 14 MeV                    | 1.99                              | 100        | Yes       | 2.17                                     | 1.34                            | 1.25                           |
| WHW15R30E15T100S | 1D        | 30             | 15 MeV                    | 2.62                              | 100        | Yes       | 3.73                                     | 1.34                            | 1.21                           |
| WW15R30E11T100S | 1D        | 30             | 11 MeV                    | 0.76                              | 100        | No        | ...                                      | 1.40                            | ...                            |
| WW15R30E12T100S | 1D        | 30             | 12 MeV                    | 1.07                              | 100        | No        | ...                                      | 1.40                            | ...                            |
| WW15R30E13T100S | 1D        | 30             | 13 MeV                    | 1.48                              | 100        | No        | ...                                      | 1.40                            | ...                            |
| WW15R30E14T100S | 1D        | 30             | 14 MeV                    | 1.99                              | 100        | Yes       | 1.94                                     | 1.40                            | 1.31                           |
| WW15R30E15T100S | 1D        | 30             | 15 MeV                    | 2.62                              | 100        | Yes       | 3.41                                     | 1.40                            | 1.25                           |
| WHW20R30E11T100S | 1D        | 30             | 11 MeV                    | 0.76                              | 100        | No        | ...                                      | 1.49                            | ...                            |
| WHW20R30E12T100S | 1D        | 30             | 12 MeV                    | 1.07                              | 100        | No        | ...                                      | 1.49                            | ...                            |
| WHW20R30E13T100S | 1D        | 30             | 13 MeV                    | 1.48                              | 100        | No        | ...                                      | 1.49                            | ...                            |
| WHW20R30E14T100S | 1D        | 30             | 14 MeV                    | 1.99                              | 100        | Yes       | 2.20                                     | 1.45                            | 1.34                           |
| WHW20R30E15T100S | 1D        | 30             | 15 MeV                    | 2.62                              | 100        | Yes       | 3.55                                     | 1.49                            | 1.36                           |
| WHW25R30E11T100S | 1D        | 30             | 11 MeV                    | 0.76                              | 100        | No        | ...                                      | 1.45                            | ...                            |
| WHW25R30E12T100S | 1D        | 30             | 12 MeV                    | 1.07                              | 100        | No        | ...                                      | 1.45                            | ...                            |
| WHW25R30E13T100S | 1D        | 30             | 13 MeV                    | 1.48                              | 100        | No        | ...                                      | 1.45                            | ...                            |
| WHW25R30E14T100S | 1D        | 30             | 14 MeV                    | 1.99                              | 100        | Yes       | 0.99                                     | 1.45                            | 1.34                           |
| WHW25R30E15T100S | 1D        | 30             | 15 MeV                    | 2.62                              | 100        | Yes       | 2.17                                     | 1.45                            | 1.29                           |
| WHW25R30E16T100S | 1D        | 30             | 16 MeV                    | 3.39                              | 100        | Yes       | 0.73                                     | 1.69                            | <2.00                          |
| WHW25R30E17T100S | 1D        | 30             | 17 MeV                    | 4.32                              | 100        | Yes       | 5.92                                     | 1.69                            | 1.49                           |
| WHW25R30E18T100S | 1D        | 30             | 18 MeV                    | 5.26                              | 100        | Yes       | 9.21                                     | 1.69                            | 1.41                           |

![Figure 10](https://example.com/figure10.png)  
**Figure 10.** Final NS masses as a function of the progenitor mass, $M$. Circles are progenitors from Nomoto & Hashimoto (1988), squares are from Woosley & Weaver (1995), and crosses are from Woosley et al. (2002), respectively.
order to reverse the passive shock into an active one it is most important to energize the explosion in some way. Using these two progenitors that produce a very weak explosion (model NH13) and do not show even a shock revival (model WHW15), we hope to explore how the dynamics would change when spectral swapping is switched on.

### 3.2.2. 2D with Spectral Swapping

Table 3 shows a summary for our 2D models, in which the last character of each model (A) indicates “Axisymmetric.” Models NH13A and WHW15A are 2D models without spectral swapping for NH13 and WHW15, respectively.

As in 1D, the onset of spectral swapping is taken to be \( t_s = 100 \) ms after bounce. At this time, model NH13 shows the onset of gradual shock expansion with a small diagnostic energy of \( \langle E_{\text{diag}} \rangle \sim 3 \times 10^{50} \) erg; the shock radius is located at \( \sim 300 \) km. As for model WHW15, there is no region with a positive local energy (e.g., Equation (9)) and the shock radius is \( \sim 200 \) km. The density profile for this model is essentially the same as the one in the 1D counterpart (see Figure 8) but with small angular density modulations due to convection.

In Figure 6, red filled circles represent \( E_{\text{diag}}^{2D} \) for model NH13. It can be seen that the critical heating rate to obtain \( E_{\text{diag}}^{2D} \sim 10^{51} \) erg is smaller for 2D than the corresponding 1D counterparts (compare the heating rates for \( \langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x} \sim 2.2 \times 10^{54} \text{MeV}^2 \text{erg s}^{-1} \)). In fact, models with \( \langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x} \lesssim 2.2 \times 10^{54} \text{MeV}^2 \text{erg s}^{-1} \) fail to explode in 1D, but succeed in 2D (albeit with a relatively small \( E_{\text{diag}}^{2D} \): less than \( 10^{51} \) erg). As opposed to 1D, it is rather difficult in 2D to determine a critical heating rate due to the stochastic nature of the explosion triggered by SASI and convection. In our limited set of 2D models, the critical heating rate is expected to be close to \( \langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x} \sim 1.5 \times 10^{54} \text{MeV}^2 \text{erg s}^{-1} \), below which the shock does not revive (e.g., \( \langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x} \lesssim 1 \times 10^{54} \text{MeV}^2 \text{erg s}^{-1} \) is the lowest in the horizontal axis in the figure).

As seen from Figure 6, \( E_{\text{diag}}^{2D} \) becomes visibly larger for 2D than 1D especially for a smaller \( \langle \epsilon_{\nu_x} \rangle^2 L_{\nu_x} \). As the heating rates become larger, the difference between 1D and 2D becomes smaller because the shock revival occurs almost in a spherically symmetric way (before SASI and convection develop nonlinarily). In Table 3, it is interesting to note that model NH13R30E11T100A fails to explode, while we observe shock revival for the corresponding model without spectral swapping (model NH13A). This is because the heating rate of model NH13R30E11T100A is smaller than that of NH13A due to the small \( \langle \epsilon_{\nu_x} \rangle \), which can make it more difficult to trigger \( \nu_x \) explosions. On the other hand, if the energy gain due to the swap is high enough (i.e., for models with greater \( \langle \epsilon_{\nu_x} \rangle \) than E12 in Table 3), the swap can facilitate explosions.

Figure 11 depicts the entropy distributions for models NH13A (top panel) and NH13R30E13T100A (bottom panel). It can be seen that model NH13A shows a unipolar-like explosion (see also Suwa et al. 2010), while model NH13R30E13A explodes in a rather spherical manner as mentioned above. Model NH13A experiences several oscillations aided by SASI and convection before explosion, while the stalled shock for model NH13R30E13T100A turns into expansion shortly after the onset of spectral swapping. In fact, the shapes of hot bubbles behind the expanding shock are shown to be barely changing with time (bottom panel), which indicates a quasi-homologous expansion of material behind the revived shock.

Figure 12 shows the time evolution of mass shells for models NH13A (thin-gray lines) and NH13R30E13T100A (thin-orange lines). Black and red thick lines represent the shock position at the north pole for each model. The mass shells for model NH13A continue to accrete to the PNS, since the shock passively expands outward as previously mentioned. Due to this continuing mass accretion, the remnant for this model would be a black hole instead of a neutron star. On the other hand, model NH13R30E13T100A shows a mass ejection with a definite outgoing momentum in the post-shock region so that the remnant could be a neutron star. Unfortunately, however, we cannot predict the final outcome due to the limited simulation time. A long-term simulation recently done in 1D (e.g., Fischer et al. 2010) should be indispensable also for our 2D case. This is, however, beyond the scope of this paper.

Here let us discuss the validity of the parameters for the spectral swap that we have assumed so far. For example, the criteria of explosion for model NH13R30E12T100A were \( L_{\nu_x} \approx 1.07 \times 10^{51} \) erg s\(^{-1}\) and \( \langle \epsilon_{\nu_x} \rangle \approx 12 \) MeV. These values are even smaller than the typical values obtained in 1D Boltzmann simulations (e.g., Liebendörfer et al. 2004), which show \( L_{\nu_x} \approx 2 \times 10^{52} \) erg s\(^{-1}\) and \( \sqrt{\langle \epsilon_{\nu_x} \rangle} \approx 20 \) MeV (i.e., \( \langle \epsilon_{\nu_x} \rangle \approx 14 \) MeV with a vanishing chemical potential) earlier in the postbounce phase. Therefore, spectral swapping, if it would work as we have assumed, may potentially assist explosions. It should be noted that the critical heating rate in this study might be too small due to the approximation of the light-bulb scheme. In this scheme, we can include the geometrical effect of the finite size of the neutrinosphere as in Equation (7), but cannot include the back reaction by the matter, i.e., the absorption of neutrino. Some fraction of neutrinos, in fact, are absorbed in the gain region and the neutrino luminosity decreases with the

### Table 3

#### 2D Simulations

| Model       | Dimension | \( R_v \) (km) | \( \langle \epsilon_{\nu_x} \rangle \) (MeV) | \( L_{\nu_x} \) \( (10^{52} \text{erg s}^{-1}) \) | \( t_s \) (ms) | Explosion | \( E_{\text{diag}}^{2D} \) \( (10^{51} \text{erg}) \) | \( M_{\text{diag}}^{\text{crit}} \) \( (M_\odot) \) | \( M_{\text{diag}} \) \( (M_\odot) \) |
|-------------|-----------|----------------|----------------------------------|----------------------------------|-------------|-----------|----------------|----------------|----------------|-----------|
| NH13A       | 2D        | ...            | ...                              | ...                              | ...         | Yes       | ...            | ...            | ...            | ...       |
| NH13R30E11T100A | 2D       | 30             | 11 MeV                           | 0.76                             | 100         | No        | ...            | ...            | ...            | ...       |
| NH13R30E12T100A | 2D       | 30             | 12 MeV                           | 1.07                             | 100         | Yes       | 0.45           | 1.18           | <1.23         | ...       |
| NH13R30E13T100A | 2D       | 30             | 13 MeV                           | 1.48                             | 100         | Yes       | 1.03           | 1.18           | <1.18         | ...       |
| NH13R30E13T100A | 2D       | 30             | 15 MeV                           | 2.62                             | 100         | Yes       | 2.33           | 1.18           | 1.10          | ...       |
| WHW15A      | 2D        | ...            | ...                              | ...                              | ...         | No        | ...            | ...            | ...            | ...       |
| WHW15R30E13T100A | 2D       | 30             | 13 MeV                           | 1.48                             | 100         | No        | ...            | ...            | ...            | ...       |
| WHW15R30E14T100A | 2D       | 30             | 14 MeV                           | 1.99                             | 100         | Yes       | 1.96           | 1.48           | <1.52         | ...       |
| WHW15R30E15T100A | 2D       | 30             | 15 MeV                           | 2.62                             | 100         | Yes       | 3.79           | 1.48           | 1.34          | ...       |
radius. We omit this effect in this study so that the heating rate might be overestimated in the simulation with spectral swapping. Thus, a fully consistent simulation including spectral swapping is necessary for a more realistic critical heating rate, which is beyond the scope of this study.

Finally, we discuss the $15 \, M_\odot$ progenitor labeled WHW15. As mentioned, this progenitor fails to explode without spectral swapping even in 2D. Figure 13 shows the entropy distributions of WHW15A (left; non-exploding) and WHW15R30E15T100A (right; exploding) for 220 ms after the bounce (corresponding to 120 ms after $t_*$ for model WHW15R30E15T100A). The model with $R_\nu = 30$ km and $\langle \epsilon_\nu \rangle = 14$ MeV does not explode in 1D but explodes in 2D (compare Tables 2 and 3). Again, the multidimensionality helps the onset of explosion. The critical heating rate in 2D is in the range of $2.5 \lesssim (\langle \epsilon_\nu \rangle^2 L_\nu)/(10^{54} \text{ MeV}^2 \text{ erg s}^{-1}) \lesssim 3.9$, while it is $3.9 \lesssim (\langle \epsilon_\nu \rangle^2 L_\nu)/(10^{54} \text{ MeV}^2 \text{ erg s}^{-1}) \lesssim 5.9$ in 1D. Therefore, the critical heating rate in 2D can be by a factor of $\sim 2$ smaller than in 1D. In 2D, the critical $\nu_\ell$ luminosity and average energy to obtain explosion are $L_{\nu_\ell} \sim 2 \times 10^{52} \text{ erg s}^{-1}$ and $\langle \epsilon_{\nu_\ell} \rangle \sim 14$ MeV (corresponding to $\sqrt{\langle \epsilon_{\nu_\ell}^2 \rangle} \sim 20$ MeV), which are close to the results obtained in a 1D Boltzmann simulation (Sumiyoshi et al. 2005) for a $15 \, M_\odot$ progenitor. The diagnostic energy as well as the estimated remnant masses are listed in the last three columns in Table 3. $E_{\text{diag}}^\infty$ (as well as $M_{\text{diag}}^\infty$) for exploding models is shown to be larger than the model series of NH13. As a result, some of the 2D models for WHW15 produce strong explosions ($E_{\text{diag}}^\infty \sim 10^{51}$ erg) while simultaneously leaving behind a remnant of $1.34$–$1.52 \, M_\odot$. We think that it is only a solution accidentally found by our parametric explosion models. However, again, the critical heating rates that require the

9 This is consistent with a very recent result by Obergaulinger & Janka (2011), who performed 2D simulations of model WHW15 with spectral neutrino transport.

10 Note that the progenitor employed in Sumiyoshi et al. (2005) is WW95, so that the direct comparison may not be fair. However, the critical heating rate in 1D for WW15 is smaller than WHW15 (Figure 9) and the mass of the envelope is thicker for WHW15 than for WW15 (Figure 8). This indicates that our discussion above seems to be quite valid, although we really need 1D results for WHW15 to draw a more solid conclusion.
neutrinos and electron energy above which the spectral interchange between heavy-lepton
of the possible outcomes of collective neutrino oscillations, we
port via the IDSA scheme. To model spectral swapping, one
otions of core-collapse supernovae with spectral neutrino trans-
We also investigated the progenitor dependence using a suite of
ale of the multidimensionality that enhances the neu-
transport will give us a more correct answer (see Ott et al.2008;
1D due to the multidimensionality that enhances the neu-
Transport can still be marginally satisfied.

4. SUMMARY AND DISCUSSION

We performed a series of 1D and 2D hydrodynamic simula-
tions of core-collapse supernovae with spectral neutrino trans-
port via the IDSA scheme. To model spectral swapping, one
of the possible outcomes of collective neutrino oscillations, we
parameterized the onset time when the spectral swap begins, the
radius where the spectral swap takes place, and the threshold en-
ergy above which the spectral interchange between heavy-lepton
neutrinos and electron/anti-electron neutrinos occurs. By doing
so, we systematically studied the shock evolution and matter
ejection due to neutrino heating enhanced by spectral swapping.
We also investigated the progenitor dependence using a suite of
progenitor models (13, 15, 20, and 25 M⊙). With these compu-
tations, we found that there is a critical heating rate induced by
spectral swapping to trigger explosions that differs between the
progenitors. The critical heating rate is generally smaller for 2D
than 1D due to the multidimensionality that enhances the neu-
trino heating efficiency (see also Janka & Mueller1996). The
remnant masses which range from 1.1–1.5 M⊙ depending on the
progenitors, can be determined by the mass ejection driven by
the neutrino heating. For our 2D model of the 15 M⊙ pro-
genitor, we found a set of parameters that produces an explosion
with a canonical supernova energy close to 1051 erg and at the
same time leaves behind a remnant mass close to ∼1.4 M⊙.
Our results suggest that collective neutrino oscillations have the
potential to solve the supernova problem if they occur. These
effects should be explored in a more self-consistent manner in
hydrodynamic simulations.

Here it should be noted that the simulations in this paper are
only a first step toward more realistic supernova modeling. For
the neutrino transfer, we omitted the cooling of heavy-lepton
neutrinos and the inelastic neutrino scattering by electrons.
These omissions lead to an overestimation of the diagnostic
energy and they also should relax the criteria for explosion. The
ray-by-ray approximation may lead to an overestimation of the
directional dependence of the neutrino anisotropies. A full-angle
transport will give us a more correct answer (see Ott et al. 2008;
Brandt et al. 2011). Moreover, due to the coordinate symmetry
axis, the SASI develops preferentially along the axis; it could
thus provide a more favorable condition for the explosion. As
several exploratory simulations have been done recently (e.g.,
Iwakami et al. 2008; Scheidegger et al. 2008; Nordhaus et al.
2010), 3D supernova models are indeed necessary also to pin
down the outcomes of spectral swapping.

Finally, we briefly discuss whether the oscillation parameters
in this paper are really valid in view of recent work that
focus on clarifying the still-veiled nature of collective neutrino
oscillations. Following Duan et al. (2010), there are at least two
conditions for the onset of collective neutrino oscillations in the
case of inverted neutrino mass hierarchy.

The first criterion should be satisfied in the so-called bipolar
regime of the collective oscillation. In this regime, the neutrino
number density should exceed the critical value,

\[ n_{\nu_e, crit} \sim \frac{1}{(\sqrt{1 + \chi} - 1)^2} \frac{\Delta m^2}{\sqrt{2} G_F (\epsilon_{\nu_e})} \]

\[ \sim 1.4 \times 10^{29} \text{ cm}^{-3} \left( \frac{0.2}{\chi} \right)^2 \left( \frac{15 \text{ MeV}}{\epsilon_{\nu_e}} \right), \]  \[ (12) \]

where \( \chi \) is the fractional excess of neutrinos over antineutrinos,
\( \Delta m^2 \) is the characteristic mass-squared splitting (a typical value of ∼2.4 \times 10^{-3} \text{ eV}^2 is employed here), and \( G_F \) is the Fermi
coupling constant. By using our simulation results, we can
estimate \( \chi \), which is often treated as a parameter (typically
∼0.01–0.25). The following estimation is given in Esteban-
Pretel et al. (2007): \( \chi \sim F_{\nu_e} / F_{\bar{\nu}_e} - 1 \) in the case of vanishing \( F_{\nu_e} \),
where \( F_{\nu_e} \) is the number flux of \( \nu_e \). From Figure 14, it can be seen that \( \chi \sim 0.2–0.3 \) for 100–400 ms after bounce. Since the typical number density in the post-shock region (\( r \sim 200–300 \text{ km} \) can be
estimated as

\[ n_{\nu_e} = \frac{L_{\nu_e}}{4 \pi r^2 c (\epsilon_{\nu_e})} \sim 1.1 \times 10^{31} \text{ cm}^{-3} \left( \frac{L_{\nu_e}}{10^{52} \text{ erg s}^{-1}} \right) \]

\[ \times \left( \frac{100 \text{ km}}{r} \right)^2 \left( \frac{15 \text{ MeV}}{\epsilon_{\nu_e}} \right), \] \[ (13) \]

the first condition is satisfied.\[11\]
The second criterion is related to the decoherence of collective oscillations by matter. In order to overwhelm the suppression by decoherence, the following condition should be satisfied:

\[ n_{\nu_e, \text{crit}} \sim n_e, \]  

(14)

where \( n_e \) is the number density of electrons where decoherence takes place. This is equivalent to

\[ Y_{\nu_e, \text{crit}} \sim Y_e, \]  

(15)

In our 1D simulation, \( Y_{\nu_e} \sim (0.1–0.2) \times Y_e \) for 100 km \( \lesssim r \lesssim r_{sh} \), where \( r_{sh} \) is the shock radius.\(^{12}\) Since this condition is barely satisfied, the collective oscillations in reality could modify the spectrum to some extent between heavy-lepton neutrinos and electron/anti-electron neutrinos; however, the full swapping assumed in this study may be exaggerated. Very recently,\(^{13}\) Chakraborty et al. (2011a, 2011b) pointed out that the matter effect could fully suppress spectral swapping in the accretion phase using the 1D neutrino-radiation hydrodynamic simulation data of Fischer et al. (2010). However, the current understanding of the collective oscillation is not completed and calculations in this field employ several assumptions (e.g., single angle approximation; but see also Dasgupta et al. 2011 for more recent work). To draw a robust conclusion, one needs a more detailed study that includes the collective neutrino flavor oscillation in the hydrodynamic simulations in a more self-consistent manner, which we are going to undertake as a sequel of this study.

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\(^{12}\) Outside the shock, \( Y_{\nu_e} \gg Y_e \) is achieved due to rapid density decrease.

\(^{13}\) In fact, they posted their papers on astro-ph after our submission.
early postbounce phase when the neutrino burst is launched and the accretion shock expands to its maximum radius. The hydrodynamic quantities are shown in Figures 16, 17, and 18.

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