Angular dependence of the radiation power of a Josephson STAR-emitter

Richard A. Klemm$^{1,\dagger}$ and Kazuo Kadowaki$^{2,\dagger}$

$^1$Department of Physics, University of Central Florida, Orlando, FL 32816, USA
$^2$Graduate School of Pure & Applied Sciences, University of Tsukuba, 1-1-1, Tennodai, Tsukuba, Ibaraki 305-8573, Japan

(Dated: August 19, 2009)

We calculate the angular dependence of the power of stimulated terahertz amplified radiation (STAR) emitted from a dc voltage applied across a stack of intrinsic Josephson junctions. During coherent emission, we assume a spatially uniform $ac$ Josephson current density in the stack acts as a surface electric current density antenna source, and the cavity features of the stack are contained in a magnetic surface current density source. A superconducting substrate acts as a perfect magnetic conductor with $H_{J,ac} = 0$ on its surface. The combined results agree very well with recent experimental observations. Existing Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystals atop perfect electric conductors could have Josephson STAR-emitter power in excess of 5 mW, acceptable for many device applications.

PACS numbers: 07.57.Hm, 74.50.+r, 85.25.Cp

At present, broad-band terahertz (THz) electromagnetic (EM) waves generated from femtosecond laser pulses and monochromatic THz waves generated by laser mixing, parametric resonance techniques, and quantum cascade lasers, etc., are the most common THz sources. But these techniques are not cost effective in the “THz gap” region 0.1-10 THz required for many important applications. After many years of effort, by application of a series of techniques, THz radiation sources, which use lasers to stimulate radiation, have Josephson ST AR-emitter power in excess of 5 mW, acceptable for many device applications.

We first assume the mesa is suspended in vacuum. In Lorentz gauge, the vector potentials from the respective radiation sources are $\mathbf{A}(\mathbf{x}, t) = \sum_{n=1}^{\infty} e^{-i\omega_nt} \frac{\mu_0}{4\pi} \int \frac{d^3\mathbf{r}' J_{Sn}(\mathbf{x}')}{R} e^{i\mathbf{k_n} \cdot \mathbf{r}}$, (1)

where $\mathbf{A}(\mathbf{x}, t)$ is the vector potential at the point $\mathbf{x}$ due to the radiation source at the point $\mathbf{x}'$, $\omega_n$ is the $n$th harmonic of the Josephson angular frequency $\omega_J$, $\mu_0$ is the magnetic permeability, $\epsilon$ is the electric constant, $\mu_0$ is the vacuum magnetic permeability, and $\epsilon_n$ is the relative amplitude of the $n$th harmonic of the Josephson current.

We first consider cylindrical mesas of radius $a \approx 50 \mu m$ and height $h \approx 1 \mu m$. In cylindrical $(\rho', \phi', z')$ coordinates, the sources $J_{Sn}(\mathbf{x}')$ and $M_{Sn}(\mathbf{x}')$ are

$$J_{Sn}(\mathbf{x}') = 2\delta\eta \tilde{E}_0 \cos(\phi' - \phi_0) \eta_M(z') \delta(\rho' - a),$$

where $\tilde{E}_0 = E_0 J_1(k_1 a)$ is the effective electric field amplitude.
tude at the mesa edge, $k_1' a = 1.8412$ satisfies $J_1'(k_1' a) = 0$ in order that $H_a(p' = a) = 0$ for the lowest transverse magnetic (TM) cylindrical cavity mode. TM$_{110}[^3]$, $J_n(x)$ is a standard Bessel function, and for no substrate, $\eta J(z') = \eta M(z') = \eta_0 = \Theta(z') \Theta(h - z')$, where $\Theta(x)$ is the Heaviside step function. The higher energy cavity mode energies are not integral multiples of one another. Assuming the fundamental frequency of a non-uniform $J_{ac}(x', t)$ excites a cavity mode, higher harmonics of it will not, so we take $M_{Sn} \propto \delta_n h$. Since $h << a, r$ is the smallest length in the problem, $\eta(z') \to h \delta(z')$ for no substrate. Outside the mesa, we write $A(x, t)$ and $F(x, t)$ in spherical coordinates $(x, y, z) = r(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. After averaging over $\phi_0$ and $t$, in the radiation zone $r >> a$ far from the mesa, the emitted power per unit solid angle is

$$\frac{dP}{d\Omega} \to \frac{Z_0 J_z V k_0^2}{32\pi^2} \left[ \sin^2 \theta \sum_{n=1}^\infty n \alpha_n S_n(\theta) J_0(n k_0) \right]$$

$$+ \alpha(\theta) \left( \cos^2 \theta J_n^2(k_0) + J_n^2(k_0) \right), \quad (4)$$

where $\alpha(\theta) = \frac{1}{2} \left[ 1 + \frac{E_0 S_n^2(\theta) / (Z_0 J_z)^2}{J_z^2(z) + J_z^2(z)} \right]$, $J_z(z) = (J_2(z) \pm J_0(z))/2$, $k_0 = k_1 a \sin \theta$, $Z_0 = \sqrt{\epsilon_0 / \mu_0}$ is the vacuum impedance, $S_n^M(\theta) = S_n^J(\theta) = 1$ for no substrate, and the details are presented elsewhere.[12] In Fig. 2, plots for no substrate of the intensity $I(\theta) \propto dP(\theta)/d\Omega$ versus $\theta$ at the fundamental with $k_1 a = 1.8412/\sqrt{3}$ from separate $M_{S1}$ and $J_S$ sources are shown by curves (A) and (B), respectively, and the combined fundamental output with $\alpha(0) = 0.6$ is shown by curve (C). $I(\theta)$ at the second $ac$ Josephson current harmonic with $k_2 = 2k_1$ is shown by curve (E).

We now consider a rectangular mesa of width $w$, length $\ell$, and height $h$ with no substrate. To fit experiment, we assume TM$_{500}$ modes, which oscillate in position with integral multiples of half-wavelengths along the mesa widths $\bar{2}$ $\bar{3}$ $\bar{4}$ $\bar{10}$. In rectangular source coordinates $(x', y', z')$,

$$J_{Sn}(x') = \bar{z} \frac{J_{tan}}{4} \eta J(z') \sum_{\sigma = \pm} \left[ f_{\sigma}(x', y') + g_{\sigma}(x', y') \right], \quad (5)$$

$$M_{Sn}(x') = \bar{E}_{0n} \eta M(z') \sin[\pi(x' - x_n) / w]$$

$$\times \sum_{\sigma = \pm} \sigma \left[ y' f_{\sigma}(x', y') - x' g_{\sigma}(x', y') \right], \quad (6)$$

$$f_{\sigma}(x', y') = w \delta(x' + \sigma w/2) \Theta[(\ell/2)^2 - (y')^2], \quad (7)$$

$$g_{\sigma}(x', y') = \ell \delta(\sigma w/2) \Theta[(w/2)^2 - (x')^2], \quad (8)$$

where the TM$_{500}$ cavity mode energy is degenerate for $-w/2 \leq x_n \leq w / n$.

We treat $M_{Sn}$ in two models. In Model I, the average output power is taken to be $\langle P(x_n) \rangle_I = \frac{1}{2}P(0) + P(w/n)[3]$. In Model II, we let $\langle P(x_n) \rangle_{II} = J_{\omega n/n} dx_{n} P(x_{n}) n / 2w$. Then, in spherical coordinates, the time-averaged power per unit solid angle in the radiation zone is

$$\frac{dP}{d\Omega} \to \frac{Z_0 J_z V k_0^2}{128\pi^2} \sum_{n=1}^\infty n^2 \left[ \sin^2 \alpha_n \chi_n S_n^J(\theta) \right]$$

$$+ \alpha_n(\theta) \left( C_n^i + D_n^i - \sin^2 \theta (C_n^i \cos^2 \phi \right)$$

$$+ D_n^i \sin^2 \phi - E_n^i \sin \phi \cos \phi \right), \quad (9)$$

$$\chi_n = \bar{X}_n \sin Y_n / Y_n + \cos Y_n \bar{X}_n / X_n, \quad (10)$$

$X_n = (k_n w / 2) \sin \theta \cos \phi$, $Y_n = (k_n \ell / 2) \sin \theta \sin \phi$, where $i = I, \bar{I}$, $\alpha_n(\theta) = |E_{0n} S_n^M(\theta) / (2Z_0 J_z)|^2 / V^2$, $\bar{V} = w \ell \epsilon$, and the $C_n^i(\phi, \theta)$, $D_n^i(\phi, \theta)$, and $E_n^i(\phi, \theta)$ are given elsewhere.[12].

Plots of $I(\theta, 0) \propto dP(\theta, 0)/d\Omega$ in arbitrary units versus $\theta$ in degrees at $\phi = 0$ from Eq. (9) with $S_n^M(\theta) = S_n^J(\theta) = 1$, $\ell = 20w/3$, $k_1' = \pi / w = k_1 \sqrt{\epsilon}$, $k_2 = 2k_1$ are given by curves (A) and (B) in Fig. 3.[12]. In experiment, the maximum fundamental $I(\theta, 0)$ is generally at $\theta_{\text{max}} \approx 30 - 40^\circ$, and $I(90^\circ, 0) = 0$. Although curve (A), obtained from $M_{S1}$ alone, yields $I(90^\circ, 0) = 0$, it has $\theta_{\text{max}} = 0$. Hence, it is necessary to add the $J_S$ source. Curve (B) is obtained for $\alpha(0) = 0.20, 0.40$ for $i = I$ and $\bar{I}$, respectively. It has $\theta_{\text{max}} \approx 40^\circ$, nearly as observed, but also yields a large $I(90^\circ, 0)$ value, unlike the observations. The corresponding second harmonic intensity is shown in curve (C), which also shows $I(90^\circ, 0) \neq 0$, unlike present experiments.[2]. This unexpected combination of $I(90^\circ, 0) = 0$ and $\theta_{\text{max}} \approx 30^\circ$ led us to consider the effects of existing BSCCO substrates.[2] As
sketched in Fig. 1 (b), during coherent Josephson radiation, the ac Josephson current is essentially confined to the mesa. With only a dc surface current density $\propto I_0$ and $B_{\perp,ac}(t) = 0$ beyond the skin depth ($\approx 0.15 \mu m$) inside the BSCCO substrate due to the ac Meissner effect, the Ampère-Maxwell boundary condition forces $H_{\perp,ac}(t) = 0$ just above the BSCCO substrate. This corresponds to a perfect magnetic conductor (PMC) substrate, with the effective image sources opposite to those of a PEC substrate. Thus, for a BSCCO substrate, we replace $\eta_1(z')$ and $\eta_M(z')$ in Eqs. (2) and (3) by

$$\eta_1(z') = \eta(z') - \eta(-z') = \text{sgn}(z')\Theta[h^2 - (z')^2]. \quad (11)$$

In the radiation zone, $h \ll a, r$ and $h \ll w, \ell, r$ for both mesa types, so we assume $h \ll 1/k_n$ for the relevant $n$. Expanding $e^{ik_aR}/R$ in Eq. (1) for small $z'$,

$$S_n^J(\theta) = S_n^M(\theta) \xrightarrow{r \to \infty} -ik_nh \cos \theta \Theta(90^\circ - \theta). \quad (12)$$

Plots of $I(\theta, 0)$ in the radiation zone for a cylindrical mesa with $k_1a = 1.8412/\sqrt{\tau}$ and $k_2 = 2k_1$ atop a superconducting substrate are shown by curves (D) and (F) in Fig. 2. Near $90^\circ$, a superconducting substrate has a drastic effect on the power emitted from the $J_S$ source. Plots of $I(\theta, 0)$ for a rectangular mesa on a superconducting substrate in the radiation zone of the fundamental in both models and of the second harmonic in Model II, are shown respectively by curves (D) and (E) in Fig. 3. Since intense higher harmonics can arise only from the $J_{SN}$ in cylindrical mesas, the study of cylindrical mesas could provide valuable information regarding the primary radiation source.

Preliminary experimental fundamental $I(\theta, 0)$ results for mesas with $w = 60 \mu m, \ell = 400 \mu m,$ and $h = 1 \mu m$ are in good agreement with curve (D) shown in Fig. 3. The effect of a superconducting substrate is crucial for $\theta \approx 90^\circ$ (nearly parallel to the substrate), as shown in Figs. 2 and 3. More importantly, a superconducting substrate has a drastic effect upon the radiation power. Since $k_1 = \pi/w$ for rectangular mesa, at the fundamental $\theta_{\max} = 40^\circ$ from curve (B) in Fig. 3, $|S_n^J(\theta)|^2 \approx 1.6 \times 10^{-3}$. Hence, replacing the superconducting substrate by an insulating one could enhance the power output by 600. Replacing it by a standard PEC could further quadruple it. Since output power of $5 \mu W$ was achieved for rectangular mesas on superconducting substrates, coherent THz radiation power in excess of $5 mW$, acceptable for many applications, might be attainable from existing BSCCO samples. Since the frequency range of the coherent radiation lies between those of a maser and a laser, we hereby declare the device to be a Josephson STAR-emitter, for stimulated terahertz amplified radiation emitter.

We thank X. Hu, S. Lin, B. Markovic, N. F. Pedersen, and M. Tachiki for stimulating discussions. This work was supported in part both by the JST (Japan Science and Technology Agency) CREST project, by the WPI Center for Materials Nanoarchitechtonics (MANA), by the JSPS (Japan Society for the Promotion of Science) CTC program and by the Grant-in Aid for Scientific Research (A) under the Ministry of Education, Culture, Sports, Science and Technology (ME) of Japan. R.A.K. would also like to thank the University of Tsukuba for its kind hospitality.

**References**

1. For a review, M. Tonouchi, Nature Photon. 1, 97 (2007).
2. L. Ozyuzer et al., Science 318, 1291 (2007).
3. K. Kadowaki et al., unpublished.
4. J. Millman and C. C. Halkias, Electronic Devices and Circuits, (McGraw-Hill, Tokyo, 1967).
5. J. D. Jackson, Classical Electrodynamics (Wiley, NY, third edition, 1999).
6. C. A. Balanis, Antenna Theory, Analysis and Design (Wiley, Hoboken, NJ, third edition, 2005). See Fig. 4, 13.
7. H. Matsumoto, T. Koyama, and M. Machida, Physica C (2008). doi:10.1016/j.physc.2007.11.030
8. L. N. Bulaevskii and A. E. Koshelev, J. Supercond. and Novel Magn. 19, 349 (2006).
9. L. N. Bulaevskii and A. E. Koshelev, Phys. Rev. Lett. 99, 057002 (2007).
10. A. E. Koshelev and L. N. Bulaevskii, arXiv:0708.3269v2.
11. S. Lin and X. Hu, Phys. Rev. Lett. 100, 247006 (2008).
12. R. A. Klemm and K. Kadowaki, arXiv:0807.3082.