Spectral response of Bragg gratings in multimode polymer waveguides

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1. INTRODUCTION

Fiber Bragg gratings (FBGs) are extensively used for distributed measurement of physical parameters such as temperature [1] or strain [2] and as a result have been extensively studied in both the single-mode and multi-mode regimes. Kogelnik [3] used coupled mode theory (CMT) to derive the spectral response of an electric field interacting with a refractive index perturbation; Erdogan [4] extended this approach and presented coupled first order differential equations describing the interaction of multiple modes in circular waveguides. Uniform and non-uniform gratings can be modeled using the transfer matrix method [5] (TMM). Using these techniques, it is thus possible to obtain analytic expressions for the Bragg wavelength \(\lambda_b\) as a function of temperature [6] for circular waveguides and thus determine the temperature sensitivity of a FBG sensor.

In the past decade, planar waveguides with Bragg gratings have become of increasing interest, since they can be fabricated using planar manufacturing techniques such as hot embossing [7] and nano imprinting [6], hence avoiding the necessity of expensive techniques such as UV interference lithography. As seen in Figure 1, in contrast to fibers, the core cross section of planar waveguides is typically non-circular, such as the rectangular Bragg grating (RBG) based on an inverted rib waveguide shown in the figure.

Understanding the effects of non-circular cross sections and index perturbations in the multimode domain on the spectral response helps to provide insight into optical behavior as well as manufacturing requirements for planar waveguides used as Bragg sensors. However, four approximations used in the derivation of analytical expressions for the spectral response of FBGs need to be re-examined to compute the spectral response of RBGs:

1. A 3D model of FBGs can be simplified using cylindrical symmetry to a 1D problem [8]; however, RBGs can be reduced to at-most a 2D model.

2. In the case of 2D models, the intensity profile of the fun-
We begin by proposing a general 3D expression describing the intensity guides sensors, and their intensity profiles can be assumed to a fundamental mode propagating in a FBG can be well approximated by a Gaussian function [9]; in RBGs, the higher modes have to be described with Bessel functions [10].

3. The refractive index profile of FBGs is found over the complete cross section of the core, as seen in Figure 1; for RBGs, it changes only in a portion of the core section and, in addition, the index perturbation can be in either the core, the cap or the substrate.

4. The magnitude of energy exchange between forward and backward traveling modes in a fiber is represented by the coupling coefficient $\kappa$. Given assumptions two and three above, and a grating with index perturbation $\delta n_{eff}$, $\kappa$ is often approximated as $2\pi \delta n_{eff}$ [4] for single mode FBGs for light with wavelength $\lambda$. In RBGs, $\kappa$ has multiple values for every possible mode combination and thus must be numerically computed.

We thus propose here modifications to these approximations to derive the multimodal spectral response of a grating placed on top of the core of an inverted rib waveguide; such a grating is referred to as a surface Bragg grating. A commercial computation tool is used to perform simulations and simulated spectral responses are compared with actual measurements. We thus derive a means to easily compute the spectral response of a multimode RBG and use this to derive its response to temperature changes when used as a sensor.

2. THEORY

We begin by proposing a general 3D expression describing the electric field, $\vec{E}_{\text{inc}}$, in a RBG supporting multiple modes, whose unit mode profiles $\vec{c}_j$ are 2D non-Gaussian functions. The interaction of this field with the grating is analysed and coupled first-order differential equations are formulated, overcoming assumptions 3 and 4 above. These equations are numerically solved and an equation describing the multimodal spectral response of a surface Bragg grating, as shown in Figure 2, is derived.

A. Multimode propagation

Laser diodes; light emitting diodes; and super-luminescent diodes are commonly used as light sources for fiber and waveguide sensors, and their intensity profiles can be assumed to a good approximation to be a Gaussian function [9] (with peak intensity $I_o$, peak wavelength $\lambda_o$ and FWHM $\sigma$). The incoupled light $I(\lambda)$ experiences coupling losses $\xi_0$ due to Fresnel reflections when it is incident on the waveguide facet as shown in Figure 3. The electric field $\vec{E}_t$ at the facet then excites a wave in the waveguide $\vec{E}_{\text{inc}}$ given by

$$|\vec{E}_{\text{inc}}(\lambda)|^2 = \xi_0 |\vec{E}_t(\lambda)|^2 = \xi_0 \xi_c \cdot I(\lambda) = \xi_0 \xi_c \cdot \frac{I_0}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(\lambda - \lambda_o)^2}{2\sigma^2} \right)$$

where $\xi_c$ represents the excitation loss.

The parameters of Equation 1 are shown graphically in Figure 3 for a surface Bragg grating with $\rho$ modes $\vec{A}_j$ propagating toward the grating and $p$ modes $\vec{B}_j$ reflected from the grating. The forward propagating electric field $\vec{E}_{\text{inc}}$ is the superposition of $p$ modes and is formulated as

$$\vec{E}_{\text{inc}}(\lambda, z) = \xi_0 \sum_{j=0}^{p-1} |\vec{E}_t(\lambda)| \xi_j \vec{A}_j(z).$$

In this expression, $\vec{E}_{\text{inc}}$ is a function of wavelength $\lambda$ and distance $z$ in the longitudinal direction. The origin is placed at the start of the grating. Each mode $j$ is a distinct wave, represented by $\vec{A}_j$, namely

$$\vec{A}_j(z) = e^{i(\beta_x z + \omega y)} \cdot \vec{c}_j(x, y) \cdot e^{-az}$$

$$|\vec{A}_j(z = 0)| = 1$$

and each mode is defined as having unit power. $\vec{c}_j(x, y)$ is the transverse mode profile of the wave as shown in Figure 2.

The distribution of energy among modes [11] [12] is described by the modal transfer function [13], where $\xi_j$ is the fraction of power in mode $j$. For a wave with unit power, the modal transfer function is defined as

$$\sum_{j=0}^{p-1} \xi_j = 1.$$
Interaction of coupling dominates (Figure 4), so that

\[
E_j(z) = (A_j(z)e^{ih_jz} + B_j(z)e^{-i\beta_jz}) \cdot e^{-i\omega t} \cdot \delta(x, y).
\]  

Using Equation 11a, 8a and 8b then take the form

\[
\frac{\partial A_j}{\partial z} = i\kappa_i A_i e^{-2i\beta_i z} + i\kappa_j B_j
\]

\[
\frac{\partial B_j}{\partial z} = -i\kappa_i A_i - i\kappa_j B_j e^{2i\beta_j z}.
\]

The transfer matrix method [5] can now be used to solve these coupled differential equations numerically [16]. Equations 12a and 12b can be expressed in matrices as functions of distance \(z\) along the grating as

\[
\frac{\partial}{\partial z} \begin{pmatrix} A_j \\ B_j \end{pmatrix} = C(z) \begin{pmatrix} A_j \\ B_j \end{pmatrix}.
\]

The coupling matrix \(C(z)\) is composed of coupling coefficients, namely

\[
C(z) = \begin{bmatrix} ik_j e^{-2\beta_j z} & i\kappa_j \\ -i\kappa_j e^{2\beta_j z} & -ik_j \end{bmatrix}.
\]

The transfer matrix \(F\) models the energy exchange between modes and is calculated from the coupling matrix \(C\) using the TMM. Numerical computation of \(F\) can be significantly accelerated using eigen-decomposition [17]. The solution to Equation 13 can be written in terms of the transfer matrix \(F\) as

\[
\begin{pmatrix} A_j(z = 0) \\ B_j(z = 0) \end{pmatrix} = F \begin{pmatrix} A_j(z = L) \\ B_j(z = L) \end{pmatrix}.
\]

Equation 15 has four unknowns and two equations. By definition, the magnitude of \(A_j\) at the start of the grating \((z = 0)\) is unity and the magnitude of \(B_j\) at the end of the grating \((z = L)\) is zero,

\[
|A_j(z = 0)| = 1
\]

\[
|B_j(z = L)| = 0.
\]
so that substituting these boundary conditions in Equation 15, yields

\[
\begin{bmatrix}
1 \\
B_j(0)
\end{bmatrix} = F \begin{bmatrix}
A_j(L) \\
0
\end{bmatrix}.
\] (17)

The transfer matrix contains numerical values from which reflectivity \( r_j \) and transmissivity \( t_j \) can be expressed as

\[
r_j = B_j(0) \tag{18a}
\]
\[
t_j = A_j(L). \tag{18b}
\]

D. Multimode Spectrum

To determine the multimode spectrum, each reflected mode \( B_j(z) \) is calculated for each incident mode \( A_j(z) \). The multimode reflectivity \( r_m \) can be defined as the ratio of reflected intensity to incident intensity of the wave as

\[
r_m(z, \lambda) = \left| \frac{E_r(\lambda)}{E_i(\lambda)} \right|^2 = \left( \sum_{i=0}^{p-1} |\tilde{A}_i(0)| \cdot r_i(L, \lambda) \right)^2. \tag{19}
\]

Since \( |A_i(0)| = 1 \), we have

\[
r_m(0, \lambda) = \left( \sum_{i=0}^{p-1} r_i(\lambda) \xi_i \right)^2. \tag{20}
\]

Similarly, an equation for \( z = L \) can be written for Equation 19 and we can obtain overall transmissivity as

\[
t_m(z, \lambda) = \left| \frac{E_r(\lambda)}{E_i(\lambda)} \right|^2 = \left( \sum_{i=0}^{p-1} |\tilde{A}_i(0)| \cdot t_i(L, \lambda) \right)^2 \tag{21}
\]

such that

\[
t_m(L, \lambda) = \left( \sum_{i=0}^{p-1} t_i(z, \lambda) \xi_i \right)^2. \tag{22}
\]

Thus, the spectra of individual modes are combined to obtain Equations 20 and 22 which the describe the spectral response of a RBG.

E. Temperature dependence

Since one important application of Bragg gratings is for temperature sensing, and that application will be used as an example here, we consider the expected change in the spectrum as a function of temperature. To that end, the physical length of the waveguide varies as a function of temperature (T) as

\[
L(\Delta T) = L_0(1 + \alpha \Delta T) \tag{23}
\]

where \( \alpha \) is the linear thermal expansion coefficient and \( \Delta T = T_0 - T \) and \( T_0 = 273K \). The material refractive index of the cap, core and substrate layers, \( n_c, n_p, n_s \), vary as

\[
n(\Delta T) = n_0 + N' \Delta T \tag{24}
\]

where \( N' \) is the thermo-optic coefficient of the material.

Equations 23 and 24 are substituted into Equation 14 \[18\]. Thus peak reflectivity \( r_m \) is then found as a function of \( T \).

The sensitivity of FBGs to changes in temperature, \( S_{\lambda_0} \), can be expressed using the analytic expression \[1\]

\[
S_{\lambda_0} = \frac{\partial \lambda_0}{\partial T}, \tag{25}
\]

\[
= 2 \frac{\partial n_{\text{eff}}}{\partial T} \lambda + 2 n_{\text{eff}} \frac{\partial \lambda}{\partial T} \tag{26}
\]

\[
= 2 \left( N' \lambda + a n_{\text{eff}} \right). \tag{27}
\]

As an alternative, we may employ the multimode reflectivity \( r_m(\lambda) \) for planar surface Bragg gratings, which is a numerically computed value and is calculated as a function of \( \lambda \); the wavelength with the highest reflectivity is denoted as the Bragg wavelength, \( \lambda_b \). Even though \( \lambda_b \) is not an analytic expression, a more general relationship,

\[
S_m = \frac{\partial \lambda_b}{\partial T}, \tag{28}
\]

may also be used to define sensitivity.

3. FABRICATION

To define the structures which will be subject to analysis by simulation in Section 4, we briefly consider the fabrication techniques used for the rectangular Bragg grating structures employing a planar surface Bragg gratings which will then provide the experimental results of Section 6.

The planar waveguide (Figure 1) was realized using UGS45E, a mixture of 45% ethylene glycol dimethyl acrylate and 55% Syntholux, a commercially available UV-curable mixture based on an epoxy acrylate, by weight and poly(methyl meth-acrylate) (PMMA) as the substrate \( (n_s) \) \[19\]. A hot embossed grating was defined using PMMA \[20\] and allowed realization of two grating configurations, which we refer to as the cap sensor (Figure 5) and the core sensor (Figure 6). The cap sensor is made by placing a grating on top of polymerised core layer whereas a core sensor is made by pressing the grating stamp into the unpolymerised (soft) core layer, followed by UV polymerisation.

4. DESIGN & SIMULATION

Based on the theoretical considerations above and the structure of the Bragg gratings which can be fabricated, CAD models of the cap and core sensors were defined using the commercial simulation software Rsoft, in particular the module BeamPROP which uses the beam propagation method \[21\] \[22\] (BPM) to estimate mode profiles \( \tilde{e}_i \). The module GratingMOD was used to solve Equations 12a and 12b numerically.
A. Assumptions

A number of simplifying assumptions for the simulations were made at the outset:

1. Modal power redistribution due to scattering is neglected;
2. All features have zero deviations from the ideal;
3. All modes are uniformly TE polarised;
4. The polarisation is constant as the wave propagates;
5. Dispersion is assumed to be zero;
6. Scattering & absorption losses are zero;
7. All thermal coefficients are linear; and
8. Loss coefficients \( \zeta_c, \zeta_o, \zeta_d \) and \( \zeta_s \) are assumed to be independent of \( \lambda \).

We discuss the implications of some of these assumptions on the results in Section 6E.

B. Model input parameters

Common design variables of both (cap and core) sensors are illustrated in Figure 7 and tabulated in Table 1. The two sensors differ in the location of the Bragg index modulation and its magnitude. In both sensors, the index modulation has a square wave pattern and is not across the entire cross section of the waveguide. Structural details are illustrated in Appendix B.

For the cap sensor, the propagating wave encounters two alternating materials (PMMA and air) in the cap, with an index modulation of \( \delta n_{\text{cap}} = 1.488 - 1 = 0.488 \). For the core sensor, the index perturbation is in the core and has a modulation of \( \delta n_{\text{core}} = 1.543 - 1.488 = 0.055 \). The actual dimensions of the fabricated sensors (Figure 8) were measured after fabrication and used to improve the accuracy of the simulations. The most relevant parameters are listed in Table 2. In addition, the material properties of the core and cladding layers were characterized experimentally and the measured values are given in Table 3.

For readers interested in replicating the simulations, parameters affecting numerical accuracy with which CMT & BPM are computed using the finite element method (FEM) are listed in Table 4 of Appendix C.

| Design parameters | | |
|---|---|---|
| Name | Symbol | Design |
| Core height | \( H_c \) | 2 \( \mu \)m |
| Rib height | \( H_r \) | 0.8 \( \mu \)m |
| Rib width | \( W_r \) | 20 \( \mu \)m |
| Etch depth | ED | 110 nm |
| Duty cycle | DC | 50% |
| Core Width | \( C_w \) | 60 \( \mu \)m |
| Cap Height | \( H_c^2 \) | 3 \( \mu \)m |
| Substrate Height | \( H_s \) | 4 \( \mu \)m |
| Grating period | \( \Lambda \) | 278 nm |
| Surrounding index | \( n_o \) | 1 (Air) |
| Core index | \( n_g \) | 1.52 |
| Substrate index | \( n_s \) | 1.488 |
| Thermal expansion | \( \alpha \) | 77 ppm K\(^{-1}\) |

Table 2. Measured dimensions

| Name | Design | Measured |
|---|---|---|
| Core height | 2 \( \mu \)m | 1.61 \( \mu \)m |
| Rib height | 0.8 \( \mu \)m | 0.79 \( \mu \)m |
| Rib width | 20 \( \mu \)m | 20.23 \( \mu \)m |
| Grating period | 278 nm | 277.88 nm |
| Grating length | - | 12.5 mm |

C. Mode excitation

A final consideration is the excitation of the multiple modes in a multimode waveguide and the possible transfer of energy between them. As we discussed in Section 2A, the maximum number of modes supported in a multimode waveguide is given by \( p \) and the energy distribution of the modes by \( \Psi_j \) characterise \( \bar{E}_w \). Estimating how many modes are actually excited is crucial in order to compare simulations and measurements.

To model the excitation of the modes by illumination of the waveguide facet, the modal transfer function is a useful tool; the modal transfer function of a waveguide illuminated by LEDs and LDs has been studied previously [23]. When these light sources are used with coupling lenses whose NA is larger than the NA of the waveguide, the modal transfer function can be assumed to be uniform [24], so that

\[
\Psi_j = 1. \tag{29}
\]

In general, however, \( \Psi_j \) can take on different values. For low NA, the relationship is exponential [25]

\[
\Psi_j = \mu e^{-\mu j} \tag{30a}
\]

\[
\mu = 1 \tag{30b}
\]
and for intermediate NA we have \([25]\)

\[
\Psi_j = \begin{cases} 
1, & j \leq q \\
0, & j > q 
\end{cases} 
\]  

(31)

As the modal transfer function is defined as relative energy distribution between modes, we formulate it as \([13]\)

\[
\xi_j = \frac{\Psi_j}{\sum_{l}^{j} \Psi_j}.
\]

(32)

namely the portion of energy in a given mode normalized by the total energy of all modes.

Since the NA of the waveguide we fabricated and modeled was 0.31, an in-coupling objective with a higher NA (0.65) was used to assure uniform excitation in the subsequent experiments. We thus assumed that all possible modes may be excited. Analytic expressions for determining the maximum number of modes are available for circular waveguides \([26]\) but not inverted rib waveguides. Based on a modal analysis of the structures, mode profiles of 79 waveguide and 21 substrate modes were subsequently used in simulation.

D. Spectrometer output

The simulations were done with a resolution of 0.1 pm. This resolves each sinusoidal oscillation in the single mode Bragg spectrum into 36 points. The measured spectra of the lightsource and cap sensor have a resolution of 56 pm and 35 pm. The simulated responses were rescaled so that their resolution matches with the spectrometer.

5. EXPERIMENTAL SETUP

For experimental investigation of the fabricated Bragg sensor, a 800-860 nm super luminescent light-emitting diode (SLD-351 from SUPERLUM) is used as the light source, as shown in Figure 9. The incident light passes through a beam splitter and is focused on the input facet through a RMS40X - 40X Olympus Plan Achromat objective. The light emerging from the waveguide is coupled into a spectrometer (Optical Spectrum Analyzer, AQ-6315A/-6315B, Yokogawa) through a long working distance 50x microscope objective from Mitutoyo. This output spectrum is measured from 815 to 855 nm in steps of 40 pm.

6. SIMULATION & MEASUREMENT RESULTS

We compare the simulated and measured characteristics of the different sensor configurations and derive from these an understanding of the nature of the optical transmission behavior.

Fig. 9. Experimental setup to measure the spectral response of the fabricated Bragg sensor

A. Effect of mode number on Bragg wavelength

The measured transmitted spectrum of a cap sensor (recall Figure 5) on a multi-mode waveguide is shown in Figure 10. Multiple dips in the transmission can be seen, in contrast to the characteristic of a single mode Bragg grating, which shows only a single dip \([27]\). The simulated spectrum, also shown in the figure, explains the origin of the multiple minima: they result from the spectral overlap of multiple modes. The simulated results are offset to the right by 5.5 pm on account of manufacturing tolerances.

In contrast with a single mode Bragg grating, where the Bragg wavelength is solely determined by the grating period and core refractive index, the number of excited modes and the modal transfer function also affect the Bragg wavelength in a multimode surface grating. In simulation, when the number of modes considered is increased, the output spectrum significantly changes, as shown in Figure 11. The dips to the right of the spectrum are caused by waveguide modes and dips to the left, by substrate modes. The results show how important it is to estimate the number of propagating modes correctly, if the output spectrum is to be accurately modeled.

B. Effect of grating length on transmissivity

The transmissivity vs grating length was analyzed as a function of the number of modes. Whereas the single mode transmissivity for a sufficiently long grating always goes to zero \([4]\), the measured multimode transmissivity does not reach zero, as seen in Figure 12. Although light in the vicinity of Bragg wavelength is reflected completely within each mode, the \(\lambda_b\) of each mode is different. Hence light which is reflected from lower modes propagates unhindered in higher modes and vice versa. Since the spectrometer measures the total intensity at each wavelength irrespective of which mode the light came from, the dips in the simulated and measured spectrum for most multimode Bragg gratings, will not reach zero.

C. Cap vs Core Bragg sensor - spectrum

Finally, we compare the cap and core sensor configurations (recall Figures 5 and 6). The transmissivity of the cap sensor is lower than that of the core sensor, as seen in Figures 13 and 14. The origin of this difference is that the overlap between modes
As we have seen, numerical simulation enables an understanding of the spectral behavior of multimode Bragg sensors. Some differences between the measured and simulated spectrum still remain: for example, the dips in the measured spectrum are wider, possibly due to shifts in polarization during propagation.

The redistribution of power due to scattering causes the multimode transmissivity to decrease more in experiment than in the simulations. The net effect is the coupling of light with a wavelength \( \lambda \) from modes that are not reflected by the grating into modes at wavelengths which are reflected. Thus as more light is removed from the transmitted beam, the corresponding dips in that portion of the spectrum will become wider. An additional experimental effect is that the grating period varies along the \( z \) axis due to manufacturing imperfections, which further augments redistribution of power among modes, making the spectral dips wider.

Upon examining the light source spectrum and the reference waveguide spectrum, we realised that the loss coefficients have a noticeable wavelength-dependent distribution, causing the spectrum to shift.

Finally, in the scope of this paper, apodisation functions were assumed to be zero, which effectively means the grating period is constant. Non uniform Bragg gratings allow the spectral response to be tailored by introduction of phase shifts, side lobe suppression and dispersion compensation. The derivation can be applied for non uniform gratings by modifying the description of coupling coefficient, as proposed by Erdogan [4].
The mode profiles as calculated using Beam Propagation Method generated 2D plots of normalised magnitude of electric field $E$ as functions of $x$ and $y$. In BPM, a 3D infinite wave is modelled as a 2D plane wave with boundary conditions. In such cases, one can estimate the magnitude of the Poynting vector as:

$$S_j = \frac{2}{\eta_o} \int \int |E(x, y)|^2 \cdot dx \cdot dy$$  \hspace{1cm} (34)

$\eta_o$ is the resistance in vacuum as is equal to 377 ohms.

The transverse coupling coefficient $k_{ij}$ can be expressed in terms of permittivity profile $\epsilon$, transverse electric field profile of the mode $\vec{e}_{ij}$ and Poynting vector as:

$$k_{ij} = \frac{\omega}{2 \eta_o} \int \int (\epsilon - \epsilon_o) \vec{e}_{ij}^2 \cdot dx \cdot dy$$  \hspace{1cm} (35)

B. Appendix B - Refractive index side profile

Figure 18 shows the side view of a cap sensor while Figure 16 shows the cross-sectional view of its grating.

Figure 19 shows the side view of a core sensor while Figure 17 shows the cross-sectional view of its grating.
C. Appendix C - FEM parameters

Parameters affecting the numerical accuracy with which CMT & BPM are computed using the finite element method (FEM).

Table 4. FEM settings

| Name              | Settings          |
|-------------------|-------------------|
| Grid size         | 20 nm             |
| Grid type         | Uniform           |
| Slices per grating| 4                 |
| Wavelength        | 852 nm            |
| Resolution        | 0.1 pm            |
| Tolerance         | 10⁻⁴              |
| Number of modes   | 100               |
| Wavelength range  | 782 nm - 882 nm   |
| Precision         | Single float      |

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