The conditions for coherent scattering of starlight in the forward direction, and it’s consequences

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Abstract

I have been too conservative in ‘A review of the properties of the scattered starlight which contaminates the spectrum of reddened stars’ (NewA, 2002, 7/4, 191) concerning the conditions implied by an additional component of scattered light in the spectrum of reddened stars. I will review and simplify these conditions. Implications for the column density of the scatterers will then be discussed. Estimates for coherent Rayleigh scattering by hydrogen are in excellent agreement with the observations.

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1 Introduction

The conditions I have set in ‘A review of the properties of the scattered starlight which contaminates the spectrum of reddened stars’ (Zagury, 2002c) for scattered starlight to represent an important proportion of the spectrum of a reddened star are too conservative. In this paper I will review and simplify these conditions (section 2). I will then look further into how the angular distance from the reddened star effects the scatterers contribution. In section 3, the conditions found in section 2 are used to see the implications for the column density and the nature of the scatterers.

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2 Conditions for coherent scattering

Two conditions out of the three set in Zagury (2002c) cannot be avoided. Firstly, the scatterers have to be identical and spherically symmetric so that the amplitudes of the scattered waves will be equal and add positively. Secondly, the phase lag between the scattered waves has to be small, which implies that the scatterers are very close to the reddened star direction: the order of magnitude of the difference between the path followed by the scattered waves and the star-observer distance is given by the UV wavelengths ($\sim 1000\text{Å}$).

Interferometry with a large source of light, a candle for instance or, at a larger scale, a star, requires the product of the dimension of the source by the distance between the receptors to be small compared to the product of the wavelength by the distance between the source and the receptors (Crawford, 1968). This condition, applied to the scattering of starlight by interstellar particles on the line of sight of a star, is a third and major constraint on the conditions for coherent scattering given in Zagury (2002c). It is more restrictive than the condition which was set on the optical paths followed by the scattered waves. In the particular case of the light scattered in the complete forward direction, this third condition can be proved not to be necessary.

Consider the star as the set of the many point sources which altogether give the overall light of the star. For one such source, $s$, the particles (supposed to be identical and spherically symetric) very close to the source-observer axis scatter the source’s light with negligible phase lag. The intensity of the scattered starlight received by the observer is proportional to $N_s^2$, the square of the column density of scatterers on the point-source line of sight: $I_s = i_s * N_s^2$, with $i_s$ the intensity the observer would receive from one particle alone.

Since $N_s = N$ should not differ from one point source to the other, the intensity of the scattered light received by the observer is proportional to $N^2$.

If condition 1 from Zagury (2002c) is released, coherent scattering certainly occurs from the particles within an angle $\theta$ (viewed from the observer) from the star which satisfies equation 2 of Zagury (2002c):

$$\theta \ll 5 \times 10^{-8} \left( \frac{\lambda}{2000 \text{Å}} \right)^{0.5} \left( \frac{100 \text{pc}}{l_0} \right)^{0.5} \left( \frac{d_0}{D} \right)^{0.5}$$

(1)

$l_0$ is the observer-cloud distance, $d_0$ the star-cloud distance, $D$ the star-observer distance ($D = d_0 + l_0$).

A typical order of magnitude for the angular distance from the star within which scattered starlight is coherent is $10^{-8''}$, a larger value than the $10^{-12''}$
found in Zagury (2002c). For a cloud at 100 pc this represents \( \sim 10^{14} \text{ cm}^2 \) (a radius of \( \sim 100 \text{ km} \)). If \( \beta \) is the proportion of scatterers relative to the number of hydrogen atoms, for a typical \( N_H = 10^{21} \text{ H/cm}^2 \) column density, coherent scattered light is, at the first order, \( \beta 10^{35} \) times what it would be if scattering was incoherent.

3 Further contribution of scatterers at larger distances from the star-observer axis

Particles at larger distances from the star-observer axis will give a scattered wave with a larger phase-lag, and can be thought to give a negative contribution to the scattered starlight. In Zagury (2001b), I thought this could explain the 2200 Å bump.

Such contribution will not annihilate the positive ones, but will change the wavelength dependence of the scattered starlight. This is seen by integrating all contributions, in a calculus similar to that given by Van de Hulst (1969), section 4.3.

The \( 1/\lambda^4 \) dependence observationally found for the scattered light (Zagury, 2001a, 2002a) indicates that the contribution of destructive waves is probably negligible.

The most obvious reason is that, when moving away from the line of sight of the star, the spatial distribution of the scatterers becomes an important factor to consider in the addition of the amplitudes of the scattered waves. Coherent scattering at too large distances from the star direction will occur only in specific, ordered configurations, which cannot be the case in interstellar space. Since we compare differences of astronomical distances to wavelengths of less than one micron, deviations from the first Fresnel zones introduce random phase lags which annihilate the contribution of these waves (Bohren & Huffman, 1983).

Another reason might come from Section 2 if interferences of the scattered waves implies that relation 1 of Zagury (2002c) be respected. Since this relation is e.g. much more restrictive than relation 1 of section 2, interferences will concern particles which, anyway, do not introduce phase lags between the scattered waves.
4 The column density of the scatterers

In section 6.1 of Zagury (2002c), I have distinguished two possible kinds of scatterers. The scatterers can be atoms or molecules from the gas, the most natural and likely candidate being hydrogen. Small dust particles (the VSG introduced by Sellgren (1983)) will also scatter as $1/\lambda^4$. The scattering cross section of each of these two possible candidates have different expressions, which I will use to examine how likely each carrier can be responsible for the coherent scattering.

The maximum ratio of scattered light to direct starlight corrected for extinction and measured on earth, is of order 15%. It is reached for $\lambda \sim 1500$ Å (Zagury, 2000b).

From equation 7 of Zagury (2001b) the order of magnitude of the ratio (scattered light)/(direct starlight corrected for extinction) can be estimated to be:

$$\frac{\sigma(\mathcal{N}S)^2}{4\pi l_0^2} \left(\frac{D}{d_0}\right)^2 \sim \sigma N^2 \theta^4 l_0^2 \left(\frac{D}{d_0}\right)^2$$

(2)

with $S$ the surface of the cloud around the star-observer axis within which coherent scattering occurs, $\sigma = \sigma_0(2000$ Å$/\lambda)^4$ ($\sigma$ is given in cm$^2$ in the following) the optical depth of the scatterers for Rayleigh scattering.

A ratio of 15% between the scattered light and the direct starlight corrected for extinction is obtained for:

$$N = 4 \times 10^{-10} \left(\frac{1''}{\theta}\right)^2 \left(\frac{d_0}{D}\right) \left(\frac{100\text{ pc}}{l_0}\right) \left(\frac{\lambda}{2000\text{ Å}}\right)^2 \sigma_0^{-0.5}\text{ cm}^{-2}$$

(3)

With $\lambda = 1500$ Å and condition 1 we find a simple condition on $N$:

$$N \gg 10^7\sigma_0^{-0.5}\text{ cm}^{-2}$$

(4)

$\sigma_0$ is given for hydrogen and for the VSG in Zagury (2002c).

For Rayleigh scattering by hydrogen: $\sigma_0 = 2.3 \times 10^{-28}$.

For small spherical grains: $\sigma_0 = 3.2 \times 10^{-8}(a/100\text{ nm})^6$ (Van de Hulst (1969), section 6.4). $a$ is the size of the particles.

Thus, depending on whether the scatterers are hydrogen or small grains, they must satisfy:
\[ N_H \gg 6 \times 10^{20} \text{ cm}^{-2} \]  
\[ N_{grains} \gg 10^7 \left( \frac{100 \text{ nm}}{a} \right)^3 \text{ cm}^{-2} \]  

Condition 5 is in remarkable agreement with the column densities found for hydrogen in interstellar clouds.

From the Bolhin et al. (1978) relation \( A_V/N_H \sim 5 \times 10^{-22} \text{ mag/cm}^2 \), an hydrogen column density \( N_H \sim 6 \times 10^{20} \text{ cm}^{-2} \) corresponds to \( A_V \sim 0.3 \), and \( E(B-V) \sim 0.1 \) (assuming \( R_V \sim 3 \)). \( E(B-V) \sim 0.1 \) is also the average observed value above which scattered starlight starts to represent an appreciable part of the spectrum of reddened stars (Zagury, 2001c, 2002b). The hypothesis, suggested by T. Lehner (Observatoire de Meudon), that hydrogen is the carrier responsible for the contamination of starlight by scattered light, leads to an agreement between theory and observations.

Condition 6 is flexible, if identical and symetric VSG do exist, they can also be the agents of the coherent scattering.

5 Conclusion

I have reviewed conditions for coherent scattering of starlight by interstellar particles. This review leads to substantial corrections of sections 5 and 6.2 of Zagury (2002c).

The conditions specified in this paper concern the nature of the scatterers and the trajectories followed by the scattered waves. The length of these optical paths must differ by less than a wavelength, leading to one mathematic condition, equation 1 of section 2.

The order of magnitude of the angular angle from the reddened star within which starlight is coherently scattered is \( \sim 10^{-8}'' \). For a cloud at 100 pc this represents a surface of radius \( \sim 100 \text{ km} \).

We have observational indications that only the scatterers very close to the star direction participate to the coherent scattering, since, if this was not the case, the \( \lambda \) dependence of the scattered light will have a different power law than \( 1/\lambda^4 \). This rules out the possibility of interferences due to waves scattered at larger distances from the star-observer axis, conforming to the idea that the random distribution of the scatterers eliminates coherent scattering when moving away from the star direction (Bohren & Huffman, 1983).

The scatterers must be small particles, atoms or molecules, or small grains. Conditions on the column density of the scatterers have also been set for each
of these types of particles.

Atomic hydrogen can seriously be considered to be the carrier of the scattered light which contaminates the spectrum of reddened stars. The column densities of hydrogen necessary to provide the amount of scattered starlight are typical of the interstellar medium. Furthermore, the lower limit found for the hydrogen column density agrees with the value of $E(B - V)$, given by observation (Zagury, 2001c, 2002b), above which the scattered light component becomes noticeable in the spectrum of reddened stars.

Note that, reciprocally, section 4 indicates that coherent Rayleigh scattering by hydrogen must give an appreciable contribution to the spectrum of reddened stars.

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