Analysis of inverter circuit of Sinusoidal Pulse Width Modulation with keystone waveform  
Fangjian Huang  
School of Energy Science and Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China  
Email: huangfangjian@hotmail.com

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Abstract. Third harmonic is usually combined with the standard sinusoidal signal to increase the DC voltage utilization within the Sinusoidal Pulse Width Modulation (SPWM) based inverters. The inverter of SPWM with 3rd keystone waveform was analyzed and the one with which to get the best DC voltage utilization ratio was presented. Two inverters of SPWM with these two different waveforms were simulated in Matlab. And the conclusion is that the Total Harmonic Distortion (THD) of line voltage from the inverter of SPWM with keystone waveform is smaller than the one from the inverter of SPWM with 3rd harmonic.

1 Introduction

Pulse Width Modulation (PWM) is one of the most popular control methods in the full-controlled device circuits[1] and is widely used in lots of engineering areas [2, 3]. There are many feedback schemes [4] and kinds of specific optimized circuits for PWM method. Different source signals are chosen according to the different requirements when PWM method is employed. When inverter concerned, the source signal is usually sinusoidal waveform. The PWM waveform whose pulse width varies by sine law is named Sinusoidal Pulse Width Modulation (SPWM) waveform. And the technology used to acquire the SPWM waveform is called SPWM technology. Its application in inverter circuit is well known [7]. Till now, it is the control scheme employed in vast majority of inverter circuits of small and medium power.

DC voltage utilization ratio denotes the ration between the output baseband amplitude of the inverter circuit and the input voltage across the collector and the emitter of IGBT. In high power circuits, higher DC voltage utilization ratio presents better economic value. For the normal three-phase SPWM inverter circuit, the DC voltage utilization radio is just $\sqrt{3}/2 \approx 0.866$. This is mainly caused by the fact that the amplitude of sinusoidal modulation signal cannot exceed the one of the carrier.

In order to get the highest DC voltage utilization ratio and lower total harmonic distortion (THD) than those the normal modulation method can provide, we add a keystone waveform which triples the frequency of the baseband sinusoidal signal into it. Simulation shows that the design meets the requirement expected.

The rest part is arranged as following, in Section 2 we find the maximum DC voltage utilization ratio is 1 when triple frequency signal is add into the baseband modulation signal. The parameters of keystone signal which can acquire the DC voltage utilization ratio 1 when added into the baseband modulation signal are presented in Section 3. In the last section, we compare the THDs of the line voltages of inverter circuits based on the 3rd harmonic and triple keystone signal.

2 Analysis of sinusoidal waveform combined with triple frequency waveforms

In order to increase the utilization ratio of DC voltage, 3rd harmonic is often added when the base sine waveform is modulated. And there exists 3rd harmonic in the phase voltage of the output of the PWM inverter. When we combine the phase voltage into line voltage, the 3rd harmonic will be eliminated and there is only sine waveform in the line voltage theoretically. Holmes [9] indicates
that the modulation ratio of 3 \text{rd} harmonic added into the base sine signal is 1/6, if DC voltage utilization ratio 1 is needed.

Actually, the maximum DC voltage ratio is same for all the continuous waveforms which are symmetrical about the origin and triple frequency of baseband sine waveform. Let \( f \) denote the frequency of the base sine waveform and \( \omega = 2\pi f \). Assume that \( g(t) \) is some periodic continuous function of variable \( t \) and \( g(t) \) is symmetrical about the origin and the frequency of \( g(t) \) is 3f. Let \( h(x) = h(\omega t) = g(t) \), then we have \( h\left(x + \frac{2}{3}\pi\right) = h\left(\omega \left(t + \frac{1}{3t}\right)\right) = g\left(t + \frac{1}{3t}\right) = g(t) = h(x) \).

This means that the period of \( h(x) \) is 2\pi/3. From \( h\left(\frac{\pi}{3} - x\right) = -h\left(x - \frac{\pi}{3}\right) = -h\left(\frac{\pi}{3} + x\right) \), we know that \( h(x) \) is symmetrical about \( \pi/3 \) and \( h(\pi/3) = 0 \) since \( h(x) \) is continuous. So the point \( (\pi/3, \sqrt{3}/2) \) is on the combined waveform of base sine and \( h(\omega t) \). Then the utilization ratio of DC voltage of the output line voltage is no more than 1 and can reach 1 when the parameters set properly.

3 Analysis of triple frequency keystone waveform

When a keystone waveform is modulated as the source signal in the inverter circuit, the strength is that the DC voltage utilization ratio can be around 1.1 while the weakness is there is many harmonics. In this section, we add triple frequency keystone waveform into the base sine single as modulation signal and present the parameters of keystone waveform which can make the maximum DC voltage utilization ratio.

Let \( h(x) \) denote a triple frequency keystone waveform which is symmetrical about the origin. \( \sigma \) is the triangulationrate and \( k \) is the height of the keystone waveform. Figure 1 shows the waveform of \( h(x) \) when \( x \) is between the origin and 2\pi/3.

![3rd Keystone Waveform](image)

Fig.1 3rd Keystone Waveform

And we can use the following piecewise function to express \( h(x) \) when \( x \in [0, 2\pi/3] \).

\[
h(x) = \begin{cases} 
\frac{2k}{L\sigma}x & 0 \leq x \leq \frac{L\sigma}{2} \\
k & \frac{L\sigma}{2} \leq x \leq L - \frac{L\sigma}{2} \\
\frac{2k}{L\sigma}(L - x) & L - \frac{L\sigma}{2} \leq x \leq L + \frac{L\sigma}{2} \\
-k & L + \frac{L\sigma}{2} \leq x \leq 2L - \frac{L\sigma}{2} \\
\frac{2k}{L\sigma}(x - 2L) & 2L - \frac{L\sigma}{2} \leq x \leq 2L 
\end{cases}
\]

in which \( L = \pi/3 \).

From Section 2, \( h(x) + \sin(x) \) must reach the maximum \( \sqrt{3}/2 \) when \( x = \pi/3 \) or \( \pi/2 \). After some simple calculation, we have parameters of \( h(x) \) to achieve the highest DC voltage utilization ratio,

\[
k = 1 - \frac{\sqrt{3}}{2}, \sigma = \frac{12}{\pi} \left(1 - \frac{\sqrt{3}}{2}\right)
\]

From above, modulation signal \( 2/\sqrt{3}(h(x) + \sin(x)) \) can perform the DC voltage utilization ratio to be 1. It is drawn in Figure 2.
4 Comparison of THDs

Theoretically, there is only sinusoidal waveform in the output line voltage of inverters when the source modulation signal is the summation of the baseband sine and its 3rd harmonic. But the THD is not zero when we consider the hardware factors in the inverter circuits. In this section, we simulate two inverter circuits with Matlab/Simulink. These two circuits are both SPWM inverter circuits. The only difference between them is the modulation signal. The first one is \(2/\sqrt{3}(\sin(x) + 1/6 \sin(3x))\), the other one is \(2/\sqrt{3}(h(x) + \sin(x))\) in which \(h(x)\) is as (1) specified by \(k = 1 - \sqrt{3}/2, \sigma = 12/\pi \ (1 - \sqrt{3}/2)\). Above analysis shows that the DC voltage utilization ratios of both inverter circuits are 1. We show that the THD of latter is smaller than the one of the former. Without loss of generality, a normal three-phase PWM inverter model shown in Figure 3 is used for the simulation.

For the simulation, we set the Matlab/Simulink parameters as following, the frequency of base sine signal is 50Hz, the frequency of signal to be added is 150Hz, the frequency of carrier of PWM generator is 1650Hz, the input DC source is 1000v and the transformer is just an isolation one.

The simulation result is shown in Figure 4.

After the inverter system stabled \((t > 0.1 \text{ sec})\), Figure 5, the enlargement of part of Figure 4, shows that the THD of the line voltage in the former inverter circuit whose modulation signal is \(2/\sqrt{3}(\sin(x) + 1/6 \sin(3x))\) is larger than the one in the latter circuit whose modulation signal is \(2/\sqrt{3}(h(x) + \sin(x))\) in which \(h(x)\) is specified by \(k = 1 - \sqrt{3}/2, \sigma = 12/\pi \ (1 - \sqrt{3}/2)\).
5 Summary

In this paper, we discuss the SPWM inverter circuit in which different triple frequency signal is added into the base sine waveform. We achieve the conclusion that the maximum DC voltage utilization ratio is 1 when the modulation signal is the summation of the base sine waveform and some triple frequency signal symmetrical about the origin. We analyze the SPWM inverter whose base sine waveform is added with triple frequency keystone waveform and get the parameters of the added keystone waveform which can provide the DC voltage utilization ratio 1. With the general PWM inverter model, we simulate two different PWM inverter circuits. The modulation signal in the first one is the summation of base sine and its 3rd harmonic signal, and in the second one is the summation of base sine and keystone signal specified in Section 3. Simulation shows that the THD of line voltage in latter is smaller than the one in former circuit.

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