Freely Falling Observer and Black Hole Radiation

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Abstract

We find radiation in an infalling frame and present an explicit analytic evidence of the failure of no drama condition by showing that an infalling observer finds an infinite negative energy density at the event horizon. The negative and positive energy density regions are divided by the newly defined zero-energy curve. The evaporating black hole is surrounded by the negative energy which can also be observed in the infalling frame.

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Since the discovery of Hawking’s thermal radiation from a black hole has raised an intriguing information loss problem \([1,3]\), there have been intensive studies in the quantum-gravity arena. In particular, it has been proposed that the information can be significantly released after the Page time \([4,5]\). On the other hand, the information cloning problem can also be overcome by black hole complementarity (BHC) \([6,8]\), which claims that there is no contradictory physical process between the freely falling observer and the distant observer. Recently, Almheiri, Marolf, Polchinski, and Sully (AMPS) \([9]\) have suggested an amazing puzzle referred to as the firewall paradox of quantum black holes (for a similar prediction from different assumptions, see \([10]\)). They argued that a freely falling observer experiences something special near the horizon and burns up because of high energy quanta. Subsequently, much attention has been paid to resolve this problem along with the information loss problem from various viewpoints \([11,27]\).

Now, we are going to investigate whether radiation can be found in an infalling frame or not by using the amenable setting called Callan-Giddings-Harvey-Strominger (CGHS) model \([28]\), which is consistent, renormalizable, and exactly soluble classically. Moreover, the Hawking flux can be exactly calculated semiclassically in an evaporating black hole. First of all, we assume simply two things: one is that energy-momentum tensors transform as true tensors without any anomalies, and the other is that the semiclassical equations of motion are valid. We do not have to postulate the complete evaporation of the black hole which plays an important role in the firewall argument \([9]\). It implies that the present argument has nothing to do with the remnant issue in \([26,29]\). On the other hand, it has also been claimed that there is no apparent need for firewalls because unitary evolution of black hole entangles a late mode located outside the horizon with a combination of early radiation and black hole states, instead of either of them separately \([30]\). The aim of the present work will be to show that there can exist non-trivial effect at the horizon based on the conventional quantum field theory without resort to the firewall argument.

In this work, we will show that there exists radiation in the infalling frame, in particular, the infinite negative energy density at the horizon, which is related to the failure of no drama condition at the event horizon. Moreover, we introduce a newly defined zero-energy curve (ZEC) dividing spacetime into the negative energy region and the positive energy region.

Let us start with the two-dimensional dilaton gravity coupled to massless scalar fields
given by CGHS \[28\]

\[
S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4(\nabla\phi)^2 + 4\lambda^2 \right) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right], \tag{1}
\]

where \(\phi\) is a dilaton field, \(f_i\) are scalar fields, and \(N\) is the number of scalar fields. For the conformal gauge of \(ds^2 = -e^{2\rho} dx^+ dx^-\) in the light-cone coordinates, the equations of motion and constraints can be solved for the shock wave described by the energy-momentum tensor \((1/2) \sum_i \partial_+ f_i \partial_+ f_i - M \delta(x^+ - x^+_0)\). Thus, one can find the solution \(e^{-2\rho} = e^{-2\phi} = -M(x^+ - x^+_0)\Theta(x^+ - x^+_0) - \lambda^2 x^+ x^-\) in the Kruskal coordinates. Next, one can take the coordinate transformation as \(e^{\lambda \sigma} = \lambda x^+\) and \(e^{-\lambda \sigma} = -\lambda x^- - M/\lambda\). Then, the metric can be written in the form of

\[
e^{2\rho} = \begin{cases} 
1 + \frac{M}{\lambda} e^{\lambda \sigma^-} & \text{for } \sigma^+ < \sigma^+_0, \\
1 + \frac{M}{\lambda} e^{-\lambda (\sigma^+ - \sigma^- - \sigma^-_0)} & \text{for } \sigma^+ > \sigma^+_0.
\end{cases} \tag{2}
\]

As for the Hawking radiation \[28\], one can use the one-loop trace anomaly of \(\langle T^f_{++} \rangle = -\kappa \partial_+ \partial_- \rho\) with \(\kappa = N/12\), while the covariant conservation of the energy-momentum tensors is maintained, which yields the energy-momentum tensors as

\[
\langle T^f_{\pm\pm} \rangle = -\kappa \left[ \left( \partial_{\pm} \rho \right)^2 - \partial_{\pm}^2 \rho + t_\pm(\sigma^\pm) \right], \tag{3}
\]

where the functions \(t_\pm\) reflect the nonlocality of the trace anomaly.

Note that the CGHS model has some important properties of the event horizon and the curvature singularity like the four-dimensional back holes, and especially the conformal anomaly for the scalar fields can be employed to calculate the energy-momentum tensors. On general grounds, however, one may consider a realistic scalar field on the Schwarzschild black hole background, then it will be a non-trivial task to realize the conformal anomaly or the effective action for the matter field directly from dimensional reduction at the quantum level. If we consider a four-dimensional scalar field on the background of the Schwarzschild black hole, the original four-dimensional action for the scalar field can be represented as a sum over modes of two-dimensional effective action before renormalization; however, this is not the case generically after renormalization because of dimensional-reduction anomaly \[31\]. This comes from the fact that the four-dimensional renormalization is not equivalent to renormalization of the two-dimensional effective theory since the number of divergent terms
depends on the number of dimensions. So the modified effective action may give a different type of energy-momentum tensors which are related to Hawking radiation. In the above, we simply took the conventional trace anomaly for two-dimensional scalar fields without taking into account the origin of the scalar fields for simple argument. Before we get down to the calculations of the radiations, let us define a coordinate transformation from a fixed observer ($\sigma^\pm$) to a freely falling observer ($\tilde{\sigma}^\pm$) satisfying the geodesic equations given by

$$0 = \left(\frac{d^2\tilde{\sigma}^\pm}{d\tau^2}\right)_P = \left(\frac{\partial\tilde{\sigma}^\pm}{\partial\sigma^\pm}\right) \left[\frac{d^2\sigma^\pm}{d\tau^2} + \Gamma_{\pm\pm}^\pm \frac{d\sigma^\pm}{d\tau} \frac{d\sigma^\pm}{d\tau}\right]_P,$$

at a particular point $P$, where the affine connections are defined by $\Gamma_{\pm\pm}^\pm |_P \equiv (\partial_{\pm\pm}^\pm)^{-1}(\partial_{\pm\pm}^\pm)^|_P$. Then, the transformation from the fixed coordinates to the locally flat coordinates in the vicinity of the point $P$ up to second order is implemented by

$$\tilde{\sigma}^\pm = \tilde{\sigma}_P^\pm + b^\pm_\pm (\sigma^\pm - \sigma_P^\pm) + \frac{1}{2}b^\pm_\pm \Gamma_{\pm\pm}^\pm |_P (\sigma^\pm - \sigma_P^\pm)^2,$$

where $\tilde{\sigma}_P^\pm = \tilde{\sigma}^\pm (\sigma_P^\pm)$ are arbitrary constants. Actually, the point $P$ can be located either in the linear dilaton-vacuum region or in the black hole region as seen in Fig. 1. From the metric (2), the explicit form of $b^\pm_\pm$ can be calculated as

$$b^\pm_\pm = \left[1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^\pm - \sigma_0^\pm)}\Theta(\sigma_P^\pm - \sigma_0^\pm)\right]^{-1/2},$$

which makes the metric become the local Minkowski spacetime at the point $P$, i.e., $e^{-2\tilde{\sigma}(\tilde{\sigma}_P^\pm, \tilde{\sigma}_P^-)} = 1$. According to the transformation (5), the metric in the freely falling frame around the point $P$ can also be written as

$$ds^2 = -e^{2\tilde{\sigma}(\tilde{\sigma}^+, \tilde{\sigma}^-)} d\tilde{\sigma}^+ d\tilde{\sigma}^-$$

where

$$e^{-2\tilde{\sigma}(\tilde{\sigma}^+, \tilde{\sigma}^-)} = b^+_\pm b^-_\pm e^{-2\rho(\sigma^+, \sigma^-)} \left[1 + \Gamma_{++}^+ |_P (\sigma^+ - \sigma_P^+) + \Gamma_{--}^- |_P (\sigma^- - \sigma_P^-) + \Gamma_{++}^+ |_P \Gamma_{--}^- |_P (\sigma^+ - \sigma_P^+) (\sigma^- - \sigma_P^-)\right].$$

Note that the freely falling frame was defined at each point so that the frame is dropped from the point $P$ with zero velocity. For a nonvanishing velocity, Eq. (6) should be modified in such a way that it contains a velocity-parameter as shown in Ref. [32]. Now, we are in a position to study what happens when the infalling observer sees nothing special.

Now, we will assume that there exists no radiation in the linear dilaton-vacuum and then investigate whether the radiation exists or not in the infalling frame. For this purpose, it is required that in the linear dilaton region, $\in (T_{\pm\pm}^f (\sigma^+ < \sigma_0^+))_{in} = 0$, where the vacuum is
FIG. 1. An incoming shock wave collapses to a black hole. The spacelike curve represents the curvature singularity and the event horizon is located at $\sigma^- = \infty$. The two lower timelike curves describe the geodesics for infalling observers. The uppermost timelike curve represents ZEC starting from $\sigma_1^+ = \sigma_0^+ + 2\lambda^{-1}\ln 2$.

denoted by the in-vacuum $|0\rangle_{\text{in}}$, which determines the boundary functions $t_+(\sigma^+) = 0$ and $t_-(\sigma^-) = - (\lambda^2/4) [1 - (1 + (M/\lambda)e^{\lambda\sigma^-})^{-2}]$ \cite{33,34}. So the energy-momentum tensor on the line $\sigma^- = \sigma_0^-$ can be calculated as

$$\langle T_{\sigma^-}^f (\sigma^+ > \sigma_0^+, \sigma^-) \rangle_{\text{in}} = \frac{\kappa \lambda^2}{4} \left[ \left( 1 + \frac{M}{\lambda} e^{-\lambda(\sigma^+ - \sigma^- - \sigma_0^+)} \right)^{-2} - \left( 1 + \frac{M}{\lambda} e^{\lambda\sigma^-} \right)^{-2} \right]$$

for $\sigma^+ \to \infty$.

Note that the boundary functions correspond to the unknown constants of the auxiliary field to localize the quantum effective action \cite{35}. These constants are determined by the boundary conditions and related to the divergent structure in the Schwarzschild black hole in two and four dimensions.
Next, the coordinate transformation to the infalling frame is performed as

\[
\begin{align*}
\text{in}(\tilde{T}_-^I(\tilde{\sigma}^+, \tilde{\sigma}^-))_{\text{in}|_{\sigma^+ > \sigma_0^+, \sigma^- = \sigma_P^-}} &= (b_-)^{-2} \text{in}(T_-^I(\sigma^+ > \sigma_0^+, \sigma_P^-))_{\text{in}} \\
&= \frac{\kappa \lambda^2}{4} \left(1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^- - \sigma_0^+ - \sigma_P^+)}\right) \times \\
&\quad \times \left[ \left(1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^- + (b_-)^{-1}(\tilde{\sigma}^+ - \tilde{\sigma}^-)) + O(\tilde{\sigma}^+ - \tilde{\sigma}^-)^2 - \sigma_0^+}ight)^{-2} - \left(1 + \frac{M}{\lambda} e^{\lambda \sigma_P^-}\right)^{-2} \right],
\end{align*}
\]

where the transformation (5) was employed. The nonvanishing outgoing radiation (9) in the infalling frame at the point \(P\) can be found as

\[
\begin{align*}
\text{in}(\tilde{T}_-^I(\tilde{\sigma}_P^+, \tilde{\sigma}_P^-))_{\text{in}|_{\sigma_P^+ > \sigma_0^+}} &= \frac{\kappa \lambda^2}{4} \left[1 - \left(1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^- - \sigma_0^+ - \sigma_P^+)}\right)^{-2} \right] - \left(1 + \frac{M}{\lambda} e^{\lambda \sigma_P^-}\right)^{-2} \right],
\end{align*}
\]

Note that the outgoing radiation is zero at the horizon which is a conventional result. By the way, the radiation (11) for \(\sigma_P^+ \to \infty\) tells us that it is interestingly coincident with the Hawking flux (8), so that the extreme infalling observer at the null infinity \(\mathcal{I}^+_R\) can detect the same amount of radiation with the Hawking radiation. It seems to be plausible in that it is impossible to distinguish physically the two asymptotic observers (for a realistic four-dimensional numerical analysis, see [36].)

Now, the ingoing flux on the line \(\sigma^+ = \sigma_P^+ > \sigma_0^+\) measured by the fixed observer is given by

\[
\begin{align*}
\text{in}(T_+^I(\sigma_P^+, \sigma^-))_{\text{in}|_{\sigma_P^+ > \sigma_0^+}} &= -\frac{\kappa \lambda^2}{4} \left[1 - \left(1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^- - \sigma_0^+ - \sigma_P^+)}\right)^{-2} \right],
\end{align*}
\]

which is negative and finite at the horizon, \(i.e., -\kappa \lambda^2/4\). By using the transformation (5), the ingoing flux in the infalling frame at the point \(P\) is obtained as

\[
\begin{align*}
\text{in}(\tilde{T}_+^I(\tilde{\sigma}_P^+, \tilde{\sigma}_P^-))_{\text{in}} &= -\frac{\kappa \lambda^2}{4} \left[1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^- - \sigma_0^+ - \sigma_P^+)}\right] - \left(1 + \frac{M}{\lambda} e^{-\lambda(\sigma_P^- - \sigma_0^+ - \sigma_P^+)}\right)^{-1} \right],
\end{align*}
\]

where the ingoing flux is negatively divergent at the horizon. It is interesting to note that this divergent flux does not mean that a physical observer will observe all particle states because the detector cannot register whose wavelength is much larger than the size of the detector. Actually, the size of the freely falling detector based on the spirit of the local inertial frame should be taken as very small enough to smooth out the tidal force so that
only the high frequency modes can be detected in the detector and the detector will miss most of the particles from the horizon. In this respect, an infalling observer might not burn at the horizon.

On the other hand, using Eqs. (8) and (11), the energy density \( \epsilon \) in the fixed coordinates is calculated as

\[
\epsilon = e^{-4\rho} \left[ \langle T_{++}^f \rangle + \langle T_{+-}^f \rangle + 2\langle T_{+-}^f \rangle \right],
\]

with the help of the trace anomaly. Note that it vanishes in the linear dilaton-vacuum region as expected, while in the black hole region \( \sigma^+ > \sigma_0^+ \), it is explicitly written as

\[
\epsilon = \frac{\kappa \lambda^2}{4} \left\{ 2 - \frac{4M}{\lambda} e^{-\lambda(\sigma^+ - \sigma^- - \sigma_0^+)} \left[ 1 + \left( 1 + \frac{M}{\lambda} e^{\lambda\sigma^-} \right)^{-2} \right] \right\}.
\]

At the horizon, the energy density is negatively divergent, whereas it is positive finite far from the horizon. The energy-momentum tensors of \( \langle T_{\pm\pm}^f \rangle \) and \( \langle T_{++}^f \rangle \) are regular everywhere; however, in our case the divergence comes from the energy density \( \epsilon = \langle T_{00}^f \rangle \). So, we can naturally define ZEC to distinguish two regions by imposing condition of \( \epsilon = 0 \), which gives a curve starting from \( \sigma_1^+ \) shown in Fig. 1:

\[
e^{-\lambda(\sigma^+ - \sigma_0^+)} = \frac{1 + \zeta}{1 + 3\zeta^2 + \sqrt{2\zeta^2(3 + 5\zeta^2)}},
\]

where \( \zeta = 1 + (M/\lambda)e^{\lambda\sigma^-} \). If one considers a spacelike curve from the event horizon to the asymptotic future null infinity, the energy density increases from the negative state at the event horizon to the positive Hawking radiation region across ZEC. At last, the energy density in the infalling frame in the black hole region of \( \sigma^+ > \sigma_0^+ \) is calculated as

\[
\tilde{\epsilon} = \frac{\kappa \lambda^2}{4} \left\{ 2 \left( 1 + \frac{M}{\lambda} e^{-\lambda(\sigma^+ - \sigma^- - \sigma_0^+)} \right)^{-1} - \left( 1 + \frac{M}{\lambda} e^{-\lambda(\sigma^+ - \sigma^- - \sigma_0^+)} \right) \left[ 1 + \left( 1 + \frac{M}{\lambda} e^{\lambda\sigma^-} \right)^{-2} \right] \right\}
\]

\[
- \frac{4M}{\lambda} e^{-\lambda(\sigma^+ - \sigma^- - \sigma_0^+)} \left( 1 + \frac{M}{\lambda} e^{-\lambda(\sigma^+ - \sigma^- - \sigma_0^+)} \right)^{-1} \}
\]

The well-known Hawking radiation flux is exactly recovered at the future null infinity and the energy density also vanishes on ZEC [15]. At the horizon, the infinite negative energy density appears in the infalling frame, which implies the failure of the no drama condition.

In conclusion, we have shown that the infalling observer can find the negative energy zone around the black hole, especially an infinite negative energy density at the event horizon. We also discussed Hawking radiation between the infalling observer and the distant observer.
Furthermore, it will be interesting to extend this work by taking into account the back reaction of the geometry, since there is no divergence at the horizon in the back reacted model in Ref. [38].

The final comment to be mentioned is that the energy density at the horizon in the freely falling frame is divergent; however, it has been expected to be finite [39], since the energy density due to radiation can be cancelled by the negative energy density of the vacuum polarization near the horizon. In our calculations, the divergent energy density was measured by the freely falling observer who starts to move just at the horizon without the long-term journey. If the free fall happens at a far distance from the horizon, then the energy density measured by the freely falling observer will be finite because the energy flux due to the positive Hawking radiation can be cancelled out by sweeping out through the cloud of the vacuum polarization as was claimed in the standard argument [39]. If the back reaction of the geometry were taken into account together, the more rigorous investigation was possible. We hope this issue will be discussed elsewhere in more detail.

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