A POSSIBLE SOLUTION FOR THE $M/L$–[$Fe/H$] RELATION OF GLOBULAR CLUSTERS IN M31. I. A METALLICITY- AND DENSITY-DEPENDENT TOP-HEAVY IMF

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ABSTRACT

The observed mass-to-light ($M/L$) ratios of a large sample of globular clusters (GCs) in M31 show an inverse trend with metallicity compared to what is expected from simple stellar population (SSP) models with an invariant canonical stellar initial mass function (IMF), in the sense that the observed $M/L$ ratios decrease with increasing metallicity. We show that when incorporating the effect of dynamical evolution the SSP models with a canonical IMF cannot explain the decreasing $M/L$ ratios with increasing metallicity for the M31 GCs. The recently derived top-heavy IMF as a function of metallicity and embedded cluster density is proposed to explain the lower-than-expected $M/L$ ratios of metal-rich GCs. We find that the SSP models with a top-heavy IMF, retaining a metallicity- and cluster-mass-dependent fraction of the remnants within the clusters, and taking standard dynamical evolution into account, can successfully explain the observed $M/L$–[$Fe/H$] relation of M31 GCs. Thus we propose that the kinematic data of GCs can be used to constrain the top-heaviness of the IMF in GCs.

Key words: galaxies: individual (M31) – galaxies: star clusters: general – methods: numerical

1. INTRODUCTION

Globular cluster (GC) systems are major tracers for studying the formation and early evolution of their host galaxies. They are also useful for studying the stellar initial mass function (IMF) because they provide us with large, nearly single-aged and mono-metallicity populations of stars located at the same distance, which means that the only significant difference between the stellar populations in different clusters is the mass and metallicity of the clusters.

The stellar mass-to-light ($M/L$) ratio is an important parameter of a GC because it establishes a connection between the luminosity and the gravitating mass of a population. The bolometric luminosity of a GC is mostly due to the small number of evolved stars, while the mass is dominated by the more numerous unobserved low-mass stars and possibly stellar remnants. This implies that the $M/L$ ratio of a GC reflects its present-day mass function (MF), and hence, through its evolution, also the IMF.

For a simple stellar population (SSP), in which all stars are formed in an instantaneous burst with single metallicity, the stellar $M/L$ ratio depends on the age and the IMF. In the beginning, a large fraction of the mass in the cluster can be found in high-mass stars (i.e., $M \geq M_\odot$). As a result of stellar evolution, the fraction drops and the high-mass end of the MF turns into compact remnants. The total luminosity of a star cluster that is due to the high-mass stars drops by an order of magnitude within the first 2 Gyr and by roughly another order of magnitude within the next 10 Gyr of evolution (Baumgardt & Makino 2003). Therefore, the $M/L$ ratio increases constantly as the most luminous stars disappear.

While stellar evolution only raises the $M/L$ ratio of a stellar population with time, dynamical evolution, on the other hand, leads to a decrease in the $M/L$ ratio because of the preferential evaporation of faint, low-mass stars (i.e., high $M/L$ stars) driven by two-body relaxation for star clusters evolving in the tidal field of a host galaxy.

Within the last decade the structural parameters and kinematic properties of extragalactic GCs with distances up to a few tens of megaparsecs have been well measured. For example, Strader et al. (2009, 2011) presented the observed velocity dispersions for 200 GCs in M31 using new high-resolution MMT/Hectochelle spectra and covering a wide range of cluster masses and metallicities. For 163 GCs of this sample they have derived the $M/L$ values in both the optical (V-band) and near-infrared (K-band). Since line blanketing increases with metallicity, stellar population synthesis predicts fainter optical luminosities for more metal-rich clusters, while there is no similar dependence expected in the K-band. They found that the $M/L$ ratios are lower than what is expected from the predictions of stellar population models with a canonical stellar IMF. The discrepancy between the observed $M/L$ and the SSP model is larger at high metallicities, where the $M/L$ values fall below the SSP values by a factor of more than three. Moreover, this discrepancy is more pronounced for clusters with a lower mass and for clusters with a shorter two-body relaxation timescale (Strader et al. 2009).

There are two ways to reduce $M/L$: adding stars with low $M/L$ ratios (i.e., RGB/AGB stars) or removing stars with high $M/L$ ratios (low-mass dwarfs). Different IMFs for metal-poor and metal-rich clusters are proposed by Strader et al. (2009) to explain the $M/L$–[$Fe/H$] relation of M31 GCs.

Most recently, Shanahan & Gieles (2015) showed that the mass segregation bias in $(M/L)_V$ as a function of [Fe/H] can explain the observed discrepancy in $M/L$ between the dynamical and SSP models, without invoking the depletion of low-mass stars or variations in the IMF. This scenario is supported by the discrepancy between the observed $(M/L)_V$ and the SSP models being more pronounced for low-mass clusters with a shorter two-body relaxation timescale (Strader et al. 2011), because dynamical mass segregation is a more efficient process in clusters with lower mass. However, ignoring the effect of evaporation of low-mass stars driven by two-body relaxation is nearly valid only in clusters with a long two-body relaxation timescale; for GCs that are older than their half-mass relaxation timescale the depletion of low-mass stars should be included.
Despite all the evidence for a universal IMF (e.g., Kroupa 2001, 2002; Bastian et al. 2010), several indications have recently begun emerging for a possible environmental dependence of the shape of the IMF. Several observational and theoretical indications suggest that the IMF becomes top-heavy under extreme starburst conditions (Dabringhausen et al. 2009, 2010, 2012; Marks et al. 2012; see Kroupa et al. 2013 for a review of the evidence for the top-heavy IMF). The data suggest that the IMF becomes less top-heavy with increasing cluster metallicity and decreasing density. The aim of the present paper is to assess the different scenarios for explaining the $M/L$-[Fe/H] discrepancy and to constrain the most reliable of them. The paper is organized as follows: in Section 2 we investigate whether only dynamical evolution is able to explain this discrepancy. In Section 3 we assess whether SSP models using the top-heavy IMF can explain the $M/L$-[Fe/H] correlation. Our conclusion and discussion are presented in Section 4.

2. THE CANONICAL IMF: DYNAMICAL EVOLUTION AND THE $M/L$ RATIO

Figure 1 shows the observed $M/L$ ratio of M31 GCs versus their metallicity [Fe/H] in the V- and K-bands (Strader et al. 2011). In order to compare with the theoretical expectation from SSP models, we overplot the SSP models for both optical and near-infrared $M/L$ ratios assuming an age of 12.5 Gyr and the canonical IMF using the flexible stellar population synthesis code (FSPS, Marigo et al. 2008; Conroy et al. 2009; Conroy & Gunn 2010), in which the Padova isochrones and the Basel stellar library are utilized. The canonical ($\alpha_2 = \alpha_3$) IMF, $\xi(m)$, can be conveniently written as a two-part power-law function, $\xi(m) \propto m^{-\alpha}$, where $\alpha_1 = 1.3$ for stars with mass $0.08 \leq m/\odot \leq 0.5$, $\alpha_2 = 2.3$ for $0.5 \leq m/\odot \leq 1$, and $\alpha_3$ for $m > 1 \odot$ (Kroupa 2001; Kroupa et al. 2013) with the upper mass limit of 100 $\odot$. As can be seen in Figure 1 (solid line), no strong dependence of $M/L_K$ on metallicity is expected for SSP models, while $M/L_V$ increases significantly with metallicity. An observed trend of strongly decreasing $M/L_K$ values with increasing metallicity for M31 GCs can be seen, while only a minimal dependence of $M/L_K$ on metallicity is expected from SSP models (e.g., Bruzual & Charlot 2003; Maraston 2005; Conroy & Gunn 2010). The observed $M/L_V$ ratios of metal-rich GCs deviate strongly from the SSP models, which predict fainter luminosities and larger $M/L_V$ ratios for more metal-rich systems.

The stellar population models only account for the effects of stellar evolution, whereas dynamical evolution and preferential depletion of low-mass (i.e., high $M/L$) stars through two-body relaxation, mass segregation, and evaporation from evolving GCs change the shape of the stellar MF within a cluster (Vesperini & Heggie 1997; Baumgardt & Makino 2003), causing the $M/L$ ratio to decrease.

In order to take the dynamical evolution into account we follow the N-body results of Baumgardt & Makino (2003) and estimate the effect of dynamical evolution on the slope of the stellar MF of M31 GCs. The dissolution time of star clusters in circular orbits can be expressed as (Baumgardt & Makino 2003)

$$T_{\text{diss}} = \beta \left( \frac{N}{\ln(\gamma N)} \right)^x \frac{R_c}{[\operatorname{kpc}]} \left( \frac{V_c}{200 \operatorname{km} s^{-1}} \right)^{-1},$$

where $\gamma = 0.02$ (Giersz & Heggie 1996), $x = 0.75$, and $\beta = 1.91$ for King $W_0 = 5.0$ clusters (Baumgardt & Makino 2003). The initial number of stars, $N$, is calculated from the initial mass of clusters assuming a mean stellar mass of 0.55$M_\odot$. It should be noted that the exact value of the mean mass depends on the assumed IMF. In addition, it varies in the simulated clusters because the MF changes during the evolution. However, the change is only marginal and does not affect the nature of the conclusions.

In order to iteratively calculate the initial mass of individual clusters (using Equations (10) and (12) of Baumgardt & Makino 2003) to produce a cluster with the present-day mass after 12.5 Gyr, one needs the orbital parameters of each cluster,
which are not yet available. To estimate the the initial masses of the clusters we refer to Table 2 of Baumgardt & Makino (2003), where 10 clusters are listed for which orbital information and determinations of the slope of the MF are simultaneously available. According to their Table 2, the mean value of the initial mass over the current mass is $\langle M_0/M \rangle \simeq 3 \pm 1.5$, where $M$ is the present-day mass (as given in Table 4 of Strader et al. 2011) and $M_0$ is the initial mass of the cluster. We assume a mean value independent of $M_0$ because, although more massive clusters evaporate their stars on a longer timescale than low-mass clusters, if it were the case that the initially more massive clusters were more centrally concentrated in the protogalaxy and thus subject to a stronger tidal field, this bias would be reduced. For example, Arp 220 has a central starburst in very massive forming clusters with an abnormally high supernova rate there (Dabringhausen et al. 2012 and references therein); our own Milky Way has very massive and very young central clusters (Arches and Quintuplet, e.g., Portegies Zwart et al. 2010); the nearby disk galaxy M33 has been suggested to have a systematically radially decreasing population of most massive young star clusters (Pflamm-Altenburg et al. 2013); and many galaxies have central nuclear clusters. Therefore we assume for this analysis that the above mean value is independent of $M_0$. For each individual observed cluster we assume that $M_0/M$ is a random number that is generated from a Gaussian distribution with a mean value of 3 and standard deviation of 1.5.

Since only the projected galactocentric radius of each M31 GC is available (Caldwell et al. 2009), calculating the dissolution time and consequently the dynamical evolution of a cluster in a realistic tidal field is uncertain. Assuming that clusters move on eccentric orbits with an eccentricity of $e$, the timescales of clusters, $T_{\text{diss}}(e)$, decrease as a function of eccentricity compared to clusters moving on circular orbits with radius equal to the apogalactic distance of the eccentric orbit as

$$T_{\text{diss}}(e) = T_{\text{diss}}(0)(1 - e).$$

Since the orbital shapes of M31 GCs are unknown, we assume that these clusters are following eccentric orbits with an average eccentricity of $e = 0.5$. We also assume that they are likely near the apocenter to estimate the maximum effect of dynamical evolution. It has been shown that the slope of the stellar MF at time $T$ and in the low mass range, $m \leq 0.5M_\odot$, changes in a universal way, independent of the orbital shape of the cluster as (Baumgardt & Makino 2003)

$$\alpha_1 = 1.3 - 1.51 \left( \frac{T}{T_{\text{diss}}(e)} \right)^2 + 1.69 \left( \frac{T}{T_{\text{diss}}(e)} \right)^3 - 1.50 \left( \frac{T}{T_{\text{diss}}(e)} \right)^4.$$  

(3)

This is used here to estimate the impact of dynamical evolution on the stellar $M/L$ ratio of each individual GC assuming $T = 12.5$ Gyr. Note that the change in the slope of the MF in the mass range $m \geq 0.5M_\odot$ ($\alpha_2$) over time is negligible and is more noticeable at the low-mass part ($\alpha_1$). This is because of the dynamical mass segregation whereby the high-mass stars sink into the inner region of the cluster and experience a weak tidal field, while the low-mass stars that are distributed in the outer regions preferentially escape due to the external tidal interaction (see, e.g., Baumgardt & Makino 2003; Haghhi et al. 2015).

Figure 1 depicts the observed $M/L$ ratios of M31 GCs and the prediction of SSP models with (red plus symbols) and without (solid lines) adding the effect of dynamical evolution. To calculate the position of the red symbols we first assume that the initial (i.e., birth) $M/L$ ratio of an observed cluster (black dots in Figure 1) at its [Fe/H] value is that provided by our SSP model shown by the solid curves in Figure 1. These $M/L$ values correspond to the canonical IMF (Section 2) and are evolved to the $M/L$ values for the present-day MF using Equation (3). The $M/L$ value at $T = 12.5$ Gyr is then calculated anew using the SSP code FSPS for the present-day MF thus obtained. Therefore, if the model captures reality well, then the $M/L$ values thus calculated (the red crosses) ought to lie close to their corresponding black dots.

Basically we expect dynamical evolution to decrease the $M/L$ ratios, but this decline is not dependent on metallicity and does not show any trend of the $M/L$ ratios with [Fe/H], in either the V-band or the K-band. In other words, although the dynamical evolution leads to a decrease in the stellar $M/L$ ratios from what is predicted by SSP models, it does not provide the observed trend between $M/L$ and metallicity. We therefore conclude that standard dynamical evolution alone is not the reason for the observed discrepancy between the observed data and the SSP models, confirming the conclusion by Strader et al. (2011).

3. THE TOP-HEAVY IMF CASE

In a gas cloud of low metallicity the Jeans mass is larger, favoring the formation of more massive stars, and the fraction of high-mass to low-mass stars increases (Larson 1998). Adams & Fatuzzo (1996), on the other hand, discuss the possibility that, because there is no single Jeans mass per cloud, a stellar mass grows to a final mass through a balance between accretion rate and self-generated energy feedback, both of which are dependent on metallicity and density with the same general metallicity dependence of the stellar mass as in the above Jeans-mass argument. In very dense star-forming cores, pre-stellar cores may coalesce before they form protostars, thus leading to a top-heavy IMF (Dib et al. 2007a, 2007b).

The present-day masses of individual clusters are taken from Strader et al. (2011).

These fundamental theoretical arguments lead to the expectation that the IMF ought to become metallicity-dependent by being more top-heavy (i.e., flatter) in metal-poor and denser environments. It was disconcerting that, until recently, observational data did not indicate this long-expected IMF variation (Kroupa et al. 2013).

Indeed, the inferred IMF slope at the high-mass end, $\alpha_3$, for a sample of MW GCs suggests that the high-mass IMF was more top-heavy (flatter) in more massive and denser environments (see Figures 2 and 3 of Marks et al. 2012). This implies that denser and, to a lesser extent, metal-poorer systems form more massive stars compared to the canonical IMF (see also Weidner et al. 2013 and Kroupa et al. 2013 for reviews of the evidence for the top-heavy IMF). The independent data analyzed by Dabringhausen et al. (2009, 2012) and Marks et al. (2012) on ultra-compact dwarf galaxies and globular clusters, respectively, suggest that the slope of the IMF for stellar masses larger than $1M_\odot$, $\alpha_3$, and its variation can be described as
The birth half-mass radius, $r_h$, can be calculated as follows:

$$r_h (\text{pc}) = 0.1 \times \left( \frac{M_{\text{cl}}}{M_\odot} \right)^{0.13}.$$  

This birth half-mass radius is used only to calculate $\rho_{\text{cl}}$, and we assume that the young GCs thereafter evolve through stellar-evolution mass loss and expulsion of residual gas to the present-day radii. The $K$- and $V$-band $M/L$ ratios from the SSP models, calculated as in Section 2 but here assuming a top-heavy IMF and corrected for the effect of dynamical evolution but retaining all stellar remnants in the $T = 12.5$ Gyr cluster model, are compared to the observed values in Figure 2. Although the top-heavy IMF now leads to a $M/L$–[Fe/H] trend (especially in the $K$-band, compare with Figure 1), the predicted $M/L$ values (red crosses) are still larger than the observed values (black dots). As can be seen, even the effect of dynamical evolution cannot sufficiently reduce the $M/L$ ratios to make them consistent with observations, and there remain a large number of GCs with observed $M/L$ ratios that are lower than those predicted by SSP models with a top-heavy IMF.

It should be noted that so far we have kept 100% of the stellar remnants within the clusters, which leads to higher $M/L$ ratios because the remnants contribute to the mass but not to the luminosity. However, it is still unclear how many remnants receive a velocity kick at formation and get ejected immediately. Some recent studies have shown that, even if kicks from black hole (BH) formation are not sufficiently strong to eject BHs from young GCs, a significant fraction of the formed BHs are expelled through stellar dynamical evolution up to the typical ages (12 Gyr) of the GCs (Mackey et al. 2008; Banerjee et al. 2010; Banerjee & Kroupa 2011; Breen & Heggie 2013; Sippel & Hurley 2013; Heggie & Giersz 2014; Morsch et al. 2015). This implies that the assumption of a 100% retention fraction is not reasonable for GCs.

If BHs receive similar velocity kicks to neutron stars (NSs) upon formation, and if not all NSs receive a strong kick, a retention fraction of 30% for all remnants may be assumed. Keeping 30% of all BHs and NSs within the clusters, we recalculate the $M/L$ ratios from SSP models with a top-heavy IMF. As shown in Figure 3, the improvement with respect to the panels in Figure 1 is considerable. The use of the top-heavy IMF in SSP models, and the dynamical evolution and partially retention of the remnants in GCs contribute to this improvement.

If the dependence of the IMF’s top-heaviness on the metallicity of the progenitor molecular cloud is enhanced, e.g., by changing the $x$ parameter in Equation (4) to

$$x = -0.70 \log_{10} \left( \frac{\rho_\odot}{10^6 M_\odot \text{pc}^{-3}} \right),$$

a remarkable agreement between the observed $M/L$–[Fe/H] trends and that expected from theoretical models can be achieved, as we show in Figure 4. Such an enhanced metallicity-dependent top-heavy IMF may be argued for, given that the original formulation (Equation (4)) can at present only be seen as a first approximation, as derived from GCs by Marks et al. (2012).
According to Figure 4, metal-rich GCs still typically show lower $M/L_V$ values than expected from SSP values. This discrepancy could be explained by the bias in the inferred mass from the integrated light properties (as a result of the assumption that mass traces light) that underestimates the true mass, especially at high metallicities as recently discussed by Shanahan & Gieles (2015).

Another possibility could be that the higher main-sequence turn-off mass of metal-rich GCs (for clusters of the same age) leads to less mass in remnants (Sippel et al. 2012). For instance, for an initial stellar mass of 50 $M_\odot$, the maximum BH mass is about 28 $M_\odot$ for a metal-poor ([Fe/H] = -2) progenitor and it is about 12 $M_\odot$ for a metal-rich ([Fe/H] = 0) star (Belczynski et al. 2006; Sippel et al. 2012). This suggests that the lower observed $M/L_V$ ratios of metal-rich GCs in M31 may be a result of the smaller contribution of remnants in high-metallicity clusters, and also a spread in the retention fraction of NSs and BHs.

So far, we assumed that the retention fraction is the same for all clusters, while this fraction could vary in principle with the cluster escape velocity. In fact, for a given kick velocity dispersion of remnants, the retention fraction of each remnant type depends on the local escape velocity, which is related to the cluster mass and radius as $V_{\text{esc}} = \sqrt{2GM/r_h}$. In order to estimate the dependence of the retention fraction on the cluster mass the mass-radius relation from Larsen (2004, $r_h \propto M^{0.1}$) or Marks & Kroupa (2012, Equation (5) here) can be used. Regardless of the exact form of the mass-radius relation, it can be easily shown that $V_{\text{esc}} \propto M^{\alpha}$, where $\alpha \approx 0.4$. Therefore, for a given kick velocity dispersion, the remnant retention fraction is set by the cluster mass, in such a way that stellar remnants can be retained in massive clusters and therefore can have a significant impact on the cluster evolution affecting the $M/L$ ratios. Since the exact form of the kick velocity distribution is still a matter of debate, in order to show the influence of the cluster-mass dependence of the retention fraction on the $M/L$–[Fe/H] curve we assume the retention fraction to vary linearly from zero (for the lightest massive cluster in our sample) to 0.7 (for the most massive cluster in our sample) as a function of the cluster initial mass. Figure 5 shows that the change is only marginal and does not affect the nature of the conclusions.

As a final and perhaps most realistic model we adopt Equation (4) for the variation of the IMF and additionally we assume the retention fraction of remnants ($r_f$) to be dependent on the metallicity,

$$r_f([\text{Fe/H}]) = -0.16[\text{Fe/H}] + 0.08,$$  \hspace{1cm} (7)
such that the retention fraction of stellar remnants runs linearly from 0.4 at [Fe/H] = −2 to zero at [Fe/H] = 0.5. We furthermore assumed that 50% of all white dwarfs (WDs) leave their star clusters due to dynamical evolution and stellar-astrophysical processes (Fellhauer et al. 2003). For each [Fe/H] we calculate the corresponding retention fraction of BHs and NSs (Equation (7)) and derive the inferred $M/L$ value of each GC in M31. As shown in Figure 6 such a model well reproduces the observed distribution of $M/L$ values of M31 GCs in both $V$- and $K$-bands. Note that the agreement between
our models and the data can be improved further if the WDs are also assumed to follow the same metallicity-dependent retention fraction as BHs and NSs (Figure 7).

4. CONCLUSION

Observations show a shallow decline of $M/L$ ratios in the $V$- and $K$-bands with increasing metallicity for M31 GCs, while higher $M/L_V$ ratios are expected from SSP models due to the evolution of both mass and luminosity. Also, a minimal dependence of $M/L_K$ on metallicity is expected from SSP models. In this paper we have presented a possible scenario to explain the discrepancy between the observed $M/L$ ratios of a large sample of GCs in M31 and those predicted by SSP models as found by Strader et al. (2011). The main conclusions of these calculations can be summarized as follows.

1. First, we added the effect of standard dynamical evolution to the result of SSP models calculated with a canonical IMF in order to investigate whether such an effect can explain the growing difference between the $M/L$ ratios in the $V$- and $K$-bands with increasing metallicity for M31 GCs. The evolution of the stellar MF is computed by considering the fitting functions derived by Baumgardt & Makino (2003) based on comprehensive direct $N$-body experiments. We found that, although the dynamical evolution leads to a decrease in the $M/L$ ratio as a result of dynamical mass segregation and evaporation of low-mass stars from the star clusters in the tidal field of a host galaxy, this effect alone cannot describe the observed anticorrelation between the $M/L$ ratios and metallicity for the M31 GCs.

2. We next investigated the impact of a top-heavy IMF on the results. We used the recently derived top-heavy IMF as a function of the density and metallicity of embedded clusters, in which star formation leads to a more top-heavy IMF in denser and metal-poorer pre-GC cloud cores (Equation (4)), and showed that by keeping 30% of the remnants (i.e., BHs and NSs) within the clusters, the standard dynamical evolution can significantly reduce the discrepancy between the observed $M/L–[Fe/H]$ relation of M31 GCs and those predicted by such SSP models. We furthermore showed that assuming the retention fraction of remnants to be correlated with the cluster initial mass can remarkably improve the agreement of the calculated $M/L$ values in the $V$- and $K$-bands with observation by decreasing the predicted $M/L$ ratios.

3. We showed that a stronger dependence of the top-heaviness on the metallicity (Equation (5)) can also improve the consistency between the theoretical $M/L$ values and the observed ones. This conclusion can be looked at from another perspective: if the observed $M/L–[Fe/H]$ anticorrelation of M31 GCs is a result of a top-heavy IMF, then this may be interpreted as a new constraint on the strength of the dependence of the top-heaviness of the IMF on the metallicity of the progenitor giant molecular clouds.

4. Finally and perhaps most realistically, we assumed the variation of the IMF as under point 2 above, and in addition we assumed that the retention fraction of stellar remnants depends on metallicity (Equation (6)). Such models reproduce the observed distribution of $M/L$ values of M31 GCs in the $V$- and $K$-bands best.

It is worth pointing out that, although the lower than expected $M/L$ values of the Milky Way GCs can be explained by the depletion of low-mass stars as a result of dynamical evaporation (Kruisjes & Mieske 2009), here we show that this effect alone cannot be the only reason for the observed $M/L–[Fe/H]$ anticorrelation of the M31 GCs, and our calculations suggest that a metallicity-dependent top-heavy IMF might be necessary for the initial conditions of very massive star clusters. Such a dependence has been constrained by Marks et al. (2012).

We remind the reader that, with our approach, we are making some simplifying assumptions. For instance, our computations of the effect of dynamical evolution are based on some simplifying assumption on the average orbital parameters of the M31 GCs, and some rough estimations for the initial masses of the clusters from their present-day masses. Investigating this problem for each individual GC by direct $N$-body simulations or any other faster $N$-body methods (e.g., the Monte Carlo method) can provide an improved estimation of the effect of dynamical evolution on the $M/L$ ratios of GCs, assuming a varying IMF and a possibly metallicity-dependent retention fraction of stellar remnants. It should be noted that in Baumgardt & Makino (2003) the clusters moved through an external galaxy that followed a logarithmic potential. However, this potential is a good assumption for clusters moving in the outer part of the host galaxy ($R_g \geq 10$ kpc); for clusters orbiting in the inner part the effect of disk and bulge should be taken into account. Moreover, in Baumgardt & Makino (2003) all remnants that in principle have a significant impact on the evolution and dissolution rate of star clusters are assumed to escape. Further simulations would be required in the future to modify the results of Baumgardt & Makino (2003) to incorporate the dynamical effects of retained remnants and a more realistic model for the galactic halo on the dynamical evolution.

As we mentioned above, since only the projected distance from the galactic center is available for each M31 GC, it is challenging to calculate their expected evolution due to the degeneracy in orbits of GCs and the resulting differences in tidal forces. Future kinematic data will enable us to provide more detailed estimates of the expected dynamical evolution of many of these objects by improving their structural parameters.

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