Quantitative ultrasound (QUS) allows estimating the intrinsic tissue properties. Speckle statistics are the QUS parameters that describe the first order statistics of ultrasound (US) envelope data. The parameters of Homodyned K-distribution (HK-distribution) are the speckle statistics that can model the envelope data in diverse scattering conditions. However, they require a large amount of data to be estimated reliably. Consequently, finding out the intrinsic uncertainty of the estimated parameters can help us to have a better understanding of the estimated parameters. In this paper, we propose a Bayesian Neural Network (BNN) to estimate the parameters of HK-distribution and quantify the uncertainty of the estimator.

**Index Terms**— Quantitative ultrasound, Tissue characterization, Homodyned K-distribution, Uncertainty, Bayesian Neural Network

1. INTRODUCTION

Quantitative ultrasound (QUS) aims to characterize the tissue by revealing information about the scatterers. These microstructures are smaller than the wavelength and scatter the ultrasound wave. Speckle statistics provide insight about the number and coherency of the scatterers which are correlated with the tissue properties [1, 2]. The scatterer density is an important property of the tissue which is defined as the number of scatterers per resolution cell (an ellipsoidal volume defined by -6 dB point of the beam profile [1]). Coherency of the scatterers is also another parameter that is related to spatial organization of the scatterers. Homodyned K-distribution can comprehensively model the envelope data under diverse number of scatterers (from low to high) and coherency levels.

The HK-distribution does not have closed form solution and conventional methods of estimating the values of the parameters of the HK distribution such as the method based on moments [3] and log compressed moments (we refer to it as XU) [4], rely on iterative optimization methods. In our previous works, we employed Convolutional Neural Networks (CNN) to classify and segment the US data into fully developed (high scatterer number density) and underdeveloped (low scatterer number density) speckle [5, 6]. Recently, an Artificial Neural Network (ANN) was introduced by Zhou et al. [7]. The proposed method was a Multi Layered Perceptron (MLP) that employed speckle statistics to estimate the parameters of HK-distribution.

The ANN estimator employs MLP layers which are prone to overfitting. In addition to this, there is no metric to investigate the reliability of the estimated value. In this paper, we address these two issues and aim to improve the estimation of the HK-distribution parameters and quantify the uncertainty using Bayesian Neural Networks (BNN). The proposed method can also be used to extract QUS parametric images, and detect the regions with high uncertainty.

2. MATERIAL AND METHOD

2.1. Homodyned K-distribution parameters

The Homodyned K-distribution (HK-distribution) is defined as [4]:

\[
P_{HK}(A|\epsilon, \sigma^2, \alpha) = A \int_0^\infty u J_0(\epsilon u) J_0(uA)(1+\frac{u^2\sigma^2}{2})^{-\alpha} du
\]

(1)

where \(\alpha\) is the scatterer clustering parameter that depends on the scatterer number density, \(A\) is the envelope of the backscattered echo signal, and \(J_0(.)\) denotes the zero-order Bessel function. The coherent signal power is \(\epsilon^2\), and the diffuse signal power can be obtained by \(2\sigma^2\alpha\) [4]. The parameter \(k\) is defined as the ratio of coherent to diffuse signal power \((\frac{\epsilon^2}{2\sigma^2\alpha})\) and along with \(\alpha\) has been employed widely for tissue characterization and we refer to them as HK-distribution parameters. The main purpose of this paper is to estimate \(k\) and \(\log_{10}(\alpha)\) (similar to [7]) and quantify the uncertainty of their estimation.

In order to generate training data for both ANN and BNN, sampling from HK-distribution is required. Similar to [3, 7], we employed the following equation produce synthetic sam-
where $X$ and $Y$ are independent and identically distributed (i.i.d) samples from unit Normal distribution, $a_i$ is the generated sample from HK-distribution, and $Z$ is sampled from the Gamma distribution with shape parameter $\alpha$ and scale parameter of 1. To generate training data, $\log_{10}(\alpha)$ is randomly selected from values ranging -0.3 to 1.4 which corresponds to $\alpha$ of 0.5 to 25. $k$ is also randomly selected from values ranging 0 to 1.

Different sizes of data, results in different values for the calculated feature. We generated different sizes of data (we refer to it as $N_x$) to train the networks (similar to [7]). The network is trained for each size separately using 10000 generated training data. The test data is generated with the same network is trained for each size separately using 10000 generated sample from HK-distribution, and $Z$ is also randomly selected from values ranging 0.5 to 25.

The accuracy of the estimators of HK-distribution parameters heavily depends on the number of available i.i.d samples. We evaluated the methods using different number of samples ($N_x$). The Relative Root Mean Square Error (RRMSE) and MAE are employed as the metrics which can be defined as [3, 7]:

$$RRMSE = \sqrt{\frac{\langle (y - \bar{y})^2 \rangle}{|y| + \epsilon}},$$

where $< \cdot >$ denote averaging operation and $\epsilon$ is a small number (here 0.001) to avoid division by zero. The simulation results for $\log_{10}(\alpha)$ and $k$ are given in Tables 1 and 2, respectively. According to the tables, the proposed BNN has lower
Table 1. RRMSE and MAE of $\log_{10}(\alpha)$ using different numbers of HK-distribution samples ($N_s$).

| $N_s$   | ANN RRMSE | ANN MAE  | BNN RRMSE | BNN MAE |
|---------|-----------|----------|-----------|---------|
| 65536   | 0.054     | 0.048    | 0.012     | 0.035   |
| 16384   | 0.052     | 0.061    | 0.029     | 0.054   |
| 4096    | 0.125     | 0.091    | 0.090     | 0.083   |
| 1024    | 0.393     | 0.129    | 0.388     | 0.123   |

Table 2. RRMSE and MAE of $k$ using different numbers of HK-distribution samples ($N_s$).

| $N_s$   | ANN RRMSE | ANN MAE  | BNN RRMSE | BNN MAE |
|---------|-----------|----------|-----------|---------|
| 65536   | 0.143     | 0.074    | 0.122     | 0.053   |
| 16384   | 0.218     | 0.084    | 0.235     | 0.073   |
| 4096    | 0.359     | 0.118    | 0.291     | 0.103   |
| 1024    | 0.538     | 0.153    | 0.460     | 0.139   |

error compared to ANN for estimation of both $\log_{10}(\alpha)$ and $k$ in the most of sample sizes.

The RRMSE and MAE error maps are shown for $N_s = 16384$ and different ground truth values of $\log_{10}(\alpha)$ and $k$. RRMSEs high values around the ground truth zero are due to the division by the small number. For better visualization, RRMSEs are plotted in log scale. Fig. 1 shows that the proposed BNN method has lower error than ANN (notice the blue regions in RRMSEs).

The proposed method can also provide uncertainty of the prediction (Eq 7). Fig. 2 shows the uncertainty of the estimation of the parameters. It can be seen that areas in Fig. 1 that high error is presents, the uncertainty is high which can provide an insight about the reliability of the estimation.

3.2. Experimental Phantom Results

A two layered phantom was constructed from an emulsion of ultrafiltered milk and water-based gelatin having 5–43 $\mu$m diameter glass beads (3000E, Potters Industries, Valley Forge, PA, USA) as the source of scattering. Data was collected by a 18L6 probe, linear array transducer, using a Siemens Acuson S2000 scanner (Siemens Medical Solutions USA, Inc.) with operating center frequency of 8.9 MHz. The middle layer was made to have a higher backscattering coefficient than the other two layers by increasing the concentration of scatterers (higher $\alpha$). The backscattering coefficient of top and bottom layers is $3.52 \times 10^{-3} \text{ cm}^{-1} \text{ sr}^{-1}$ and it is $0.37 \times 10^{-3} \text{ cm}^{-1} \text{ sr}^{-1}$ for the middle layer at the center frequency. Data from this phantom has been reported in the previous publication [10].

The B-mode image of the phantom is shown in Fig. 3 (top). Two large patches of size $14.40 \times 13.6 \text{ mm}$ (patch 1) and $12.68 \times 13.6 \text{ mm}$ are extracted from low and high scatterer concentration layers, respectively. In order to avoid introducing bias, neighbor samples (14 samples in axial and 3 in lateral) are skipped to reduce the correlation between samples before computing the features. The obtained features are averaged over 12 frames and then given to the networks. The features were passed to the BNN multiple times to acquire different samples of the predicted distribution. The results are shown in Fig. 3.

The patch 2 has higher $\alpha$ than the patch 1 which is expected since patch 2 has a higher scatterer concentration. Although the phantom has very low coherent components, the predicted $k$ parameter is discernible. One possible explanation could be the false coherency due to low number of samples [11]. Comparing the two methods, ANN only provides a single estimate of the parameters while, BNN offers the distribution of the parameters which can be sampled multiple times. By looking closely at the BNN results, it can be observed that the network has a higher uncertainty for patch 2. This has physical interpretation that by increasing the scatterer number density, the estimation would be more difficult and a higher uncertainty is obtained.

The exact value of $\alpha$ is not known for the phantom but the ratio of high to low scatterer density is close to the ratio of their corresponding backscattering coefficients which is known. The mean value $\pm$ standard deviation of the BNN prediction of $\log_{10}(\alpha)$ for the patch 1 is $0.749 \pm 0.0206$, and it is $0.967 \pm 0.0273$ for the patch 2. The ratio of backscattering coefficients of patch 2 to patch 1 is $6.37 \pm 1.81$. The ratio of the predicted $\alpha$ values is $\frac{10^{0.967\pm0.0273}}{10^{0.749\pm0.0206}} = 1.65 \pm 0.128$. It can be observed that the ratio of the estimated $\alpha$ values is very

Fig. 1. The RRMSE and MAE error maps of BNN (top) and ANN (bottom) for $N_s = 16384$. The RRMSEs are shown in log scale for better visualization.

Fig. 2. The estimated uncertainty (standard deviation of predictions) of $\log_{10}(\alpha)$ and $k$ for $N_s = 16384$ using BNN. The areas with high uncertainty correspond to areas with high error in Fig. 1.
Fig. 3. B-mode image of the layered phantom (top) and predictions for the patches specified in the b-mode image (bottom) for BNN and ANN trained on $N_s = 16384$. The shaded areas show the 2 times of the standard deviation of the predictions.

close to the ground truth ratio of backscattering coefficients.

4. CONCLUSION

In this paper, a Bayesian Neural Network (BNN) is proposed to estimate HK-distribution parameters. The method provides the distribution of estimated parameters which can be sampled multiple times to acquire the mean prediction and uncertainty. It is compared with a recent neural network approach using simulation and experimental phantom data.

5. COMPLIANCE WITH ETHICAL STANDARDS

This is a numerical and experimental phantom study for which no ethical approval was required.

6. ACKNOWLEDGMENTS

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