“Luttinger” and insulating spin liquids in two dimensions

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Abstract

In the present work, we implement an explicit two-loop renormalization of a two-dimensional flat Fermi surface (FS) in the framework of a field theoretical renormalization group (RG) approach. In our scheme, we derive the RG equations for both coupling functions and Fermi energy. In this way, we are able to probe the existence of spin-charge separation by showing that the low-energy sector of the system is in fact a non-Fermi liquid. In addition, associating the true interacting FS to the infrared stable (IR) fixed point of the Fermi energy, we demonstrate here that it either acquires a small curvature and behaves as a “Luttinger liquid” or it suffers a truncation in k-space depicting an insulating spin liquid.

1 - Introduction

A better understanding of the physical properties of highly interacting electrons in two spatial dimensions (2d) is central for high-Tc superconductivity. Soon after the discovery of the high-Tc superconductors, Anderson [1] suggested that a strongly interacting 2d electron gas should resemble a 1d Luttinger liquid state. This question remains unresolved to this date. Thanks to the high precision of the angular resolved photoemission experiments performed in a variety of materials [2], we know, at present, important facts concerning the Fermi surface (FS) of the cuprates. The FS’s for underdoped and optimally doped Bi2212 and YBaCuO compounds contain both flat and curved sectors [3]. As a result, they are nearly perfectly nested along certain k-directions. As is well-known, whenever there is a flat FS, the corresponding one-electron dispersion is 1d-like in momentum space.

Originally, the cuprates are Mott insulators which become metallic at very low doping [4]. At half-filling, Hubbard-like models have a square shape FS imposed
Figure 1: The initial Fermi surface. The corners are rounded to avoid van Hove singularities.

by electron-hole symmetry. The FS changes as we vary the filling factor and, as soon as it is lightly doped, it acquires nonzero curvature sectors in k-space. In the immediate vicinity of half-filling, there are, at most, isolated curved spots in momentum space. Consequently, in a zeroth order approximation, one may neglect their presence altogether. Following this scheme, several workers investigated the properties of a 2d electron gas in the presence of a totally flat FS \[5, 6, 8, 9\]. In their approaches, the FS is always kept fixed and never deviates from its original flat form. Besides that, their results conflict with each other. Conventional perturbation theory calculations \[5\], parquet method results \[6\], as well as one-loop perturbative RG calculations \[7\] indicate that, for repulsive interactions, there is never a Luttinger liquid state in 2d. In contrast, applying bosonization methods, Luther was able to map the square FS onto two sets of perpendicular chains \[8\]. As a result of that, the corresponding electron correlation functions become sums of power law terms with exponents only differing in form from those of a Luttinger liquid \[9\]. We revisit this problem in this letter.

We report a two-loop field theoretical RG calculation for the electron gas in the presence of the same FS model as used by Dzyaloshinskii and co-workers \[6\]. The novel aspect of our work, apart from taking into account important higher order corrections, is the fact that we show explicitly how the FS changes its shape due to interactions. As a result, we are able to determine when the FS may suffer a truncation in k-space. To our knowledge, this was never done before in such a systematic and detailed form. Needless to say, to show how the FS is renormalized by interaction is a very intricate many-body problem \[10\].

To deal with this here, we calculate how the charge renormalization functions \(Z\) and all physical parameter vary along the renormalized FS itself, by means of appropriate RG flow equations for the Fermi energy and coupling functions. We observe that \(Z\) is nullified at FS and, as a result, there exists spin-charge separation in 2d. In addition, we explore the existence of nontrivial fixed points which vary
continuously along the Fermi surface. Accordingly, we show that the renormalized FS naturally develops a nonzero curvature, if one associates its physical nature to the infrared (IR) stable fixed points. It follows from this that its behavior is regulated by the variation of the anomalous dimension exponent $\gamma^*$ with respect to the momentum $p_\parallel$ along FS. When $1/2 \leq \gamma^* \leq 1$ the renormalized FS is truncated and develops a charge pseudogap in this region of k-space. The physical system behaves as an insulating spin liquid in this case. In contrast, when $0 < \gamma^* < 1/2$ the non-Fermi liquid is metallic and resembles a Luttinger liquid state.

2 - Renormalized hamiltonian and electron self-energy

Our starting point is a strongly interacting 2d electron gas in the presence of the flat FS shown schematically in fig. 1. In order to keep a closer contact with well-known works in one-dimensional physics [11], we split the FS in four patches. However, we expand the bare single-particle energy dispersion in the vicinity of the renormalized (i.e physical) FS. The parametrization of the corresponding interactions is shown schematically in fig. 2. If we use this model to calculate physical quantities using a naive perturbation theory, we find divergent results for particular values of the external momenta [12, 13]. We circumvent this problem following the standard field theory procedure of introducing appropriate counterterms in the hamiltonian to render the physical parameters finite in all scattering channels [14]. In this way, the original hamiltonian is rewritten in a more convenient form in terms of the low-energy parameters, which are in turn physically measurable. Thus, we have $H = H_R + H_C$, where

$$H_R = \sum_{p\sigma} v_{F_R}(|p_\perp| - k_{F_R})\psi_{R\sigma}^\dagger(p)\psi_{R\sigma}(p)$$

$$+ \sum_{pqk\sigma} (U_{1R} + U_{2R} + U_{3R} + U_{4R})\psi_{R\sigma}^\dagger(p + q - k)\psi_{R,-\sigma}(k)\psi_{R,-\sigma}(q)\psi_{R,\sigma}(p)$$

(1)

and

$$H_C = \sum_{p\sigma} \frac{Z(p)}{2m_B}(k_{F_R}^2 - k_{F_B}^2) + (Z(p)\frac{m_R}{m_B} - 1)v_{F_R}(|p_\perp| - k_{F_R})\psi_{R\sigma}^\dagger(p)\psi_{R\sigma}(p)$$

$$+ \sum_{pqk\sigma\sigma'} (\Delta U_{1R} + \Delta U_{2R} + \Delta U_{3R} + \Delta U_{4R})\psi_{R\sigma}^\dagger(p + q - k)\psi_{R,\sigma'}(k)\psi_{R,\sigma'}(q)\psi_{R,\sigma}(p)$$

(2)
Figure 2: The parametrization of the interactions in the present model: (a) $U_{1R}$-processes, (b) $U_{2R}$-processes, (c) $U_{3R}$-processes, and (d) $U_{4R}$-processes.

Here the subscripts “R” and “B” stand for renormalized and bare respectively. Besides, $Z(p)$ is the charge renormalization function, which is well-defined only in the following regions of k-space depicted in fig.1: $-\lambda \leq p_{\perp} \pm k_{FR} \leq \lambda$ and $-\Delta \leq p_{\parallel} \leq \Delta$. Finally, the renormalized and bare fields are related to each other by $\psi_{B}(p) = Z^{1/2}(p)\psi_{R}(p)$, whereas the renormalized and bare couplings are connected by $\prod_{i=1}^{4} Z^{1/2}(p_{i})U_{iB} = U_{iR} + \Delta U_{iR}$. Even if, initially, the couplings are taken to be constants, the renormalization process necessarily forces $U_{iR}$ to be momenta dependent all along $FS$. The counterterms, for this reason, form a continuum set in momenta space.

Since our model is renormalizable, the counterterms originate naturally from the initial form of the Hamiltonian. Here we neglect umklapp processes to start with. Then, the only divergent terms that arise in perturbation theory come from the interaction processes described by the $U_{1R}$ and $U_{2R}$ couplings as one can most easily verify. In view of this, in a first order approximation, we can disregard the perturbative terms coming from the other two couplings, namely $U_{3R}$ and $U_{4R}$ and their corresponding counterterms. As a result, the two sets of parallel patches of $FS$ decouple from each other, and there is no ambiguity in locating a particle either at a solid line or at a dashed line patch instead.

We begin by calculating first the electron self-energy in the vicinity of $p_{\perp} = k_{FR}$. Using conventional Feynman rules, the diagrams shown in fig.3 produce

$$\Sigma_{R}(p_{\perp}, p_{\parallel}) = \frac{\lambda U_{1R}}{4\pi^{2}} - (Z(p) - 1)p_{0} + \frac{Z(p)}{2m_{B}}(k_{FR}^{2} - k_{FB}^{2})$$

$$+ (Z(p)\frac{m_{R}}{m_{B}} - 1)v_{FR}(p_{\perp} - k_{FR}) - \frac{(3\Delta^{2} - p_{\parallel}^{2})}{256\pi^{4}4\Delta v_{FR}^{2}}(\tilde{U}_{1R}^{2} + \tilde{U}_{2R}^{2})(p_{0} - v_{FR}(p_{\perp} - k_{FR}))$$

$$\times \left[ \ln \left( \frac{\Omega - v_{FR}(p_{\perp} - k_{FR}) - p_{0} - i\delta}{v_{FR}(p_{\perp} - k_{FR}) - p_{0} - i\delta} \right) + \ln \left( \frac{\Omega - v_{FR}(p_{\perp} - k_{FR}) + p_{0} - i\delta}{v_{FR}(p_{\perp} - k_{FR}) + p_{0} - i\delta} \right) \right]$$

(3)
where $\tilde{U}_{iR}(p_\parallel) = \int dk_\parallel U_{iR}(p_\parallel, k_\parallel)$. The parameter $\lambda$ is the ultraviolet momentum cutoff with the corresponding energy cutoff given by $\Omega = 2v_{FR}\lambda$, and $2\Delta$ is the length of each path along $FS$. In general, the renormalized coupling functions depend on three distinct momenta components parallel to the Fermi surface. However, in calculating the Hartree diagram, shown in fig.3(a), the vertex depend only on two different momenta and we naturally arrive at the definition of $\tilde{U}_{iR}$. In contrast, in the sunset diagrams of figs.3(b) and 3(c), the renormalized vertices depend explicitly on three different momenta components along $FS$. In whatever way, in two-loop order, the tadpole diagram is in fact the only contribution from the self-energy which produce the renormalization of the Fermi surface. As a result, for simplicity, in a zeroth order approximation, we take $\tilde{U}_{iR}(p_\parallel) = (2\Delta)U_{iR}$, neglect the momenta dependence of the vertices and rewrite the renormalized coupling in terms of the corresponding $\tilde{U}_{iR}$’s. For comparison, we show in Appendix 1, how the diagrams $\Sigma_{R}^{(3b)}$ and $\Sigma_{R}^{(3c)}$ are modified if the full momenta dependence of the renormalized coupling functions are taken into account at all steps.

In principle, since this a nonrelativistic system, there should be two scaling parameters in the problem: one for the energy ($\omega$) and another one for the momentum ($\Lambda$) [15]. By making an appropriate choice of the renormalization prescription, these two parameters do not mix with each other. Here we choose to work with the energy scale $\omega$ only. In doing this we implicitly assume that $\Lambda \to 0$ much faster than $\omega \to 0$. Defining the renormalized one-particle irreducible function $\Gamma_{R}^{(2)}(p_\perp, p_\parallel, p_0; \omega)$, which is nothing but the inverse of the full single-particle Green’s function, such that $Re\Gamma_{R}^{(2)}(p_\perp = k_{FR}, p_\parallel, p_0 = \omega; \omega \approx 0) = \omega$, we find that

$$Z(p_\perp = k_{FR}, p_\parallel; \omega \approx 0) = 1 - \frac{(3\Delta^2 - p_\parallel^2)}{128\pi^4 (2\Delta^2 v_{FR}^2)(\tilde{U}_{1R}^2 + \tilde{U}_{2R}^2) \ln(\frac{\Omega}{\omega})} + ..., \quad (4)$$

and

$$\mu_B = \frac{k_{FR}^2}{2m_B} = Z^{-1}(p_\perp = k_{FR}, p_\parallel; \omega \approx 0) \left( \frac{\mu_R}{\Delta} + \frac{\lambda \tilde{U}_{1R}}{4\pi^2} \right) + ..., \quad (5)$$
where we assumed, for simplicity, that the bare and renormalized masses differ only by a multiplicative factor, namely the charge renormalization function, i.e., 
\[ m_B = Z(p_\perp = k_{FR}; p_\parallel; \omega \approx 0) m_R. \]

### 3 - Curvature and truncation in the renormalized FS

Using perturbation theory, we calculate next the one-particle irreducible functions \( \Gamma^{(4)}_{ir}(p, k, q; \omega) \), which are essentially the renormalized two-particle interaction. Since our main intention in this work is to analyze the nature of the resulting renormalized FS, we restrict ourselves to scattering processes with two independent external momenta only. Employing appropriate renormalization group prescriptions such as

\[ \Gamma^{(4)}_{1R\uparrow\downarrow}(p_\parallel, k_\parallel, p_0 + k_0 = \omega, k_0 - p_0 = \omega; \omega \approx 0) = -i U_{1R}(p_\parallel, k_\parallel; \omega), \]

\[ \Gamma^{(4)}_{2R\uparrow\downarrow}(p_\parallel, k_\parallel, p_0 + k_0 = \omega; \omega \approx 0) = -i U_{2R}(p_\parallel, k_\parallel; \omega), \]

and

\[ \Gamma^{(4)}_{2R\uparrow\uparrow(\downarrow\downarrow)}(p_\parallel, k_\parallel, p_0 + k_0 = \omega; \omega \approx 0) = 0, \]

for \( p_\perp = -k_\perp = k_{FR} \), together with the perturbative expansions for the one-particle irreducible functions, we can relate the corresponding renormalized and bare coupling functions to each other. Following the same approximating scheme as before, with respect the dependence on the momentum component parallel to FS in the vertices of the various Feynman diagrams, and taking into account the RG conditions \( \omega \partial \tilde{U}_{iB}/\partial \omega = 0 \), we find the resulting RG equations

\[ \beta \tilde{U}_{1R} = \frac{1}{v_{FR}} \frac{\partial \tilde{U}_{1R}}{\partial \omega} = \left( \frac{3 \Delta^2 - p_\parallel^2}{16 \pi^2 v_{FR}^2 \Delta^2} \right) \tilde{U}_{2R} + \left( \frac{17 \Delta^2 - 3 p_\parallel^2}{384 \pi^4 v_{FR}^4 \Delta^2} \right) \tilde{U}_{1R} \left( \tilde{U}_{2R}^2 + \tilde{U}_{1R}^2 \right) + ..., \]

\[ \beta \tilde{U}_{2R} = \frac{1}{v_{FR}} \frac{\partial \tilde{U}_{2R}}{\partial \omega} = \left( \frac{3 \Delta^2 - p_\parallel^2}{8 \pi^2 v_{FR}^2 \Delta^2} \right) \tilde{U}_{2R} \tilde{U}_{1R} + \left( \frac{17 \Delta^2 - 3 p_\parallel^2}{384 \pi^4 v_{FR}^4 \Delta^2} \right) \tilde{U}_{2R} \left( \tilde{U}_{1R}^2 + \tilde{U}_{2R}^2 \right) + .... \]

For comparison, we show in Appendix 2 how the RG equations for the corresponding renormalized coupling functions look like if we take into account the full
momenta dependence at all stages. Finally, using eq.(5), and following a similar procedure with the renormalized Fermi energy $\mu_R(p_\parallel, \omega)$, we get the last RG equation of our interest

$$
\beta_{\mu_R} = \frac{1}{\Omega} \frac{\omega}{\omega} \frac{\partial \mu_R}{\partial \omega} = \frac{3\Delta^2 - p_\parallel^2}{128\pi^4 v_{FR}^2 \Delta^2 \Omega} (\tilde{U}_{1R}^2 + \tilde{U}_{2R}^2) \left( \mu_R + \frac{\Omega \tilde{U}_{1R}}{8\pi^2 v_{FR}} \right) - \frac{1}{8\pi^2} \omega \frac{\partial}{\partial \omega} \left( \frac{\tilde{U}_{1R}}{v_{FR}} \right) + \ldots
$$

To determine the fixed points of the model, we calculate next the zeros of these RG equations for both renormalized couplings, and the renormalized Fermi energy. It then turns out that, aside from the usual infrared unstable Fermi liquid fixed point, and yet another nontrivial unstable fixed point, we get two infrared stable nontrivial fixed points which are, as we shall see, associated with non-Fermi liquid phases. For conciseness, we will only present the final expression for these IR stable fixed points

$$
\tilde{U}_{1R}^* = -16\pi^2 v_{FR}^* \left( \frac{3\Delta^2 - p_\parallel^2}{17\Delta^2 - 3p_\parallel^2} \right)
$$

$$
\tilde{U}_{2R}^* = \pm \sqrt{2} \tilde{U}_{1R}^*
$$

$$
k_{FR}^* = 8\lambda \left( \frac{3\Delta^2 - p_\parallel^2}{17\Delta^2 - 3p_\parallel^2} \right)
$$

Figure 4: The flow diagram in the $(\tilde{U}_{1R}, \tilde{U}_{2R})$ plane.
Figure 5: The anisotropic suppression of $Z(p_\parallel)$ in the present model as we approach the FS for three values of $(\omega/\Omega)$. The black line is for $(\omega/\Omega) = 10^{-5}$, the light grey line is for $(\omega/\Omega) = 10^{-7}$, and the dotted line is for $(\omega/\Omega) = 10^{-9}$.

We observe that they depend upon $p_\parallel$ in an essential way. In view of that, the Fermi surface of the system also acquires a $p_\parallel$-structure and deviates slightly from its initial flat form. This FS deformation comes out naturally from the renormalization process. This feature of the fixed points appears only in calculations up to two-loop order or beyond. In fig.4, we show schematically the scaling trajectories in the $(\tilde{U}_{1R}, \tilde{U}_{2R})$ plane.

The nature of the electron liquid associated with the nontrivial fixed points can be inferred by the flow of the charge renormalization function $Z(p_\parallel, \omega)$. Since $\gamma = (\omega/Z)(\partial Z/\partial \omega)$, in the vicinity of the fixed point, we have that $Z(p_\parallel, \omega)$ scales as $(\omega/\Omega)^{\gamma^*(p_\parallel)}$ with the anomalous dimension being simply $\gamma$ evaluated at those fixed point values. Note that it is always positive definite along FS. Calculating $\gamma^*(p_\parallel)$ explicitly, we find that, indeed, the charge renormalization function vanishes most rapidly at the center of the FS patch (fig.5). This anisotropic suppression of $Z(p_\parallel, \omega)$ was also emphasized by Kishine and Yonemitsu within a Wilsonian RG approach [16].

Another particularly interesting feature is that, although the FS is IR stable within given boundaries, there is no guarantee that the physical FS is well-defined throughout the original patch. The nature of the resulting fermionic system can be determined by the behavior of the corresponding momentum distribution function $n(p)$. If $n(p)$ has an infinite slope at the renormalized FS, the state is metallic and resembles a Luttinger liquid. If it turns out otherwise that $n(p)$ is perfectly smooth at $k_{FR}$, the renormalized FS is truncated, and there appears a charge pseudogap at those points. This gapped state depicts an insulating spin liquid instead. To make the argument more quantitative, we use the momentum distribution function.
calculated for a similar FS in ref. [13]. In that work, it was shown explicitly that, for $1/2 < \gamma^*(p_\parallel) < 1$, the FS must be truncated at the corresponding $p_\parallel$ values. Here this condition is fulfilled for $|(p_\parallel/\Delta)| \leq 0.41$. After the elimination of the corresponding Fermi surface segments, the remains of the interacting FS are shown in fig.6. This truncation scenario will be discussed in great length elsewhere [17].

4 - Conclusion

In summary, we showed explicitly by a two-loop RG calculation that, even if the initial FS is entirely flat in two spatial dimensions, the true interacting Fermi surface becomes slightly curved as a result of interactions. Using field theoretical methods, we explored the existence of infrared stable nontrivial fixed points, which are associated with the non-Fermi liquid behavior in the low-energy sector. We showed that the fixed points vary continuously with the momentum $p_\parallel$ along the Fermi surface. In addition, we argued that the criterion $Z(\omega) \to 0$ as $\omega \to 0$ does not suffice in determining the nature of the resulting non-Fermi liquid state. We called attention to a possible route of physical characterization of those states in terms of the momentum distribution function. For the $p_\parallel$ values, in which the anomalous dimension $\gamma(p_\parallel)$ is such that $1/2 \leq \gamma^*(p_\parallel) \leq 1$, the renormalized FS suffers a truncation. This takes place, in our case, at the central region of the original FS patches.

Not long ago, a new ARPES data for the high-temperature superconductors $Bi_2Sr_{2-x}La_xCuO_{6-\delta}$ was reported showing some indication of spin-charge separation and “Luttinger” liquid like behavior in the normal state of that compound [18]. That data was, most recently, fitted consistently by a “Luttinger” liquid-like phe-
nomenclature \cite{10}. Those workers found that, contrary to what happens in one dimension, the anomalous exponents vary strongly with momentum along the Fermi surface. This is in agreement with the findings of our work. We also note that, in the cuprates, there exists a pseudogap phase with a truncated FS around special k-values in momentum space. In view of that, it is therefore natural to ascribe to those gapped systems an insulating spin liquid nature instead \cite{21}. Again, it is suggestive to associate that result to our work. It is therefore important to explore this scenario further. In a even more general experimental framework, one could also try to apply pressure in a interacting metallic state to move the Fermi surface towards its critical condition inducing, in this way, a new kind of quantum phase transition \cite{20}. This would, certainly, open more possibilities to test the limits of our results.

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Appendix 1

Using conventional Feynman rules, the contributions of the diagrams from figs.3(b) and 3(c) for the self-energy and the non-interacting single-particle Green’s functions associated with our model hamiltonian are

\[
\Sigma^{(3b)}_R (p, p_\perp = k_{FR}) = -\frac{1}{64\pi^4} \int_{D_3} dk_{\parallel} dq_{\parallel} \left( \frac{1}{v_{FR}^2} \right) U_1 R \left( k_{\parallel}, -k_{\parallel} + p_{\parallel} + q_{\parallel}, q_{\parallel} \right)
\]

\[
\times U_1 R \left( p_{\parallel}, -k_{\parallel} + p_{\parallel} + q_{\parallel} \right) \left( p_0 - v_{FR} (p_{\perp} - k_{FR}) \right)
\]

\[
\times \left[ \ln \left( \frac{\Omega - v_{FR} (p_{\perp} - k_{FR}) - p_0 - i\delta}{v_{FR} (p_{\perp} - k_{FR}) - p_0 - i\delta} \right) + \ln \left( \frac{\Omega - v_{FR} (p_{\perp} - k_{FR}) + p_0 - i\delta}{v_{FR} (p_{\perp} - k_{FR}) + p_0 - i\delta} \right) \right]
\]

and

\[
\Sigma^{(3c)}_R (p, p_\perp = k_{FR}) = -\frac{1}{64\pi^4} \int_{D_3} dk_{\parallel} dq_{\parallel} \left( \frac{1}{v_{FR}^2} \right) U_2 R \left( -k_{\parallel} + p_{\parallel} + q_{\parallel}, k_{\parallel}, q_{\parallel} \right)
\]

\[
\times U_2 R \left( p_{\parallel}, q_{\parallel}, k_{\parallel} \right) \left( p_0 - v_{FR} (p_{\perp} - k_{FR}) \right)
\]
\[
\times \left[ \ln \left( \frac{\Omega - v_{FR}(p_\perp - k_{FR}) - p_0 - i\delta}{v_{FR}(p_\perp - k_{FR}) - p_0 - i\delta} \right) + \ln \left( \frac{\Omega - v_{FR}(p_\perp - k_{FR}) + p_0 - i\delta}{v_{FR}(p_\perp - k_{FR}) + p_0 - i\delta} \right) \right] \tag{16}
\]

where the domain of integration \( D_3 \) is \(-\Delta \leq k_\parallel \leq \Delta, -\Delta \leq q_\parallel \leq \Delta, \) and \(-\Delta \leq -k_\parallel + p_\parallel + q_\parallel \leq \Delta. \) If, for simplicity, the coupling functions and the Fermi velocity are considered to be independent of the momenta components parallel to \( FS, \) our results follow immediately.

**Appendix 2**

Using perturbation theory together with our renormalization group prescriptions, and taking full account of the coupling functions dependence on the momenta components parallel to \( FS \) the RG equations for \( U_{1R} \) and \( U_{2R}, \) for general scattering processes, become

\[
\omega \frac{\partial U_{1R} (p_1||, p_2||, p_3||)}{\partial \omega} = \frac{1}{4\pi^2} \left\{ \int_{D_1} dk_\parallel \left( \frac{1}{v_{FR}} \right) \left[ U_{1R} (p_1||, p_2||, k_\parallel) U_{1R} (p_1|| + p_2|| - k_\parallel, k_\parallel, p_3||) 
\right.
\]

\[
+ U_{2R} (p_1||, p_2||, k_\parallel) U_{2R} (p_1|| + p_2|| - k_\parallel, k_\parallel, p_3||) \left. \right] - \int_{D_2} dk_\parallel \left( \frac{1}{v_{FR}} \right) \left[ U_{1R} (p_1||, p_3|| - p_1|| + k_\parallel, p_3||) \times U_{1R} (k_\parallel, p_2||, p_3|| - p_1|| + k_\parallel) \right) \right\} + \frac{1}{64\pi^4} U_{1R} (p_1||, p_2||, p_3||)
\]

\[
\times \sum_{i=1}^{4} \delta_{p_1|| + p_2|| + p_3|| + p_4||} \int_{D_3} dk_\parallel dq_\parallel \left( \frac{1}{v_{FR}^2} \right) \left[ U_{1R} (p_i||, q_\parallel, -k_\parallel + p_\parallel + q_\parallel) \times U_{1R} (k_\parallel, -k_\parallel + p_\parallel + q_\parallel, q_\parallel) + U_{2R} (p_i||, q_\parallel, k_\parallel) U_{2R} (-k_\parallel + p_\parallel + q_\parallel, k_\parallel, q_\parallel) \right] \tag{17}
\]

and

\[
\omega \frac{\partial U_{2R} (p_1||, p_2||, p_3||)}{\partial \omega} = \frac{1}{4\pi^2} \left\{ \int_{D_1} dk_\parallel \left( \frac{1}{v_{FR}} \right) \left[ U_{1R} (p_1||, p_2||, k_\parallel) U_{2R} (p_1|| + p_2|| - k_\parallel, k_\parallel, p_3||) \right.
\]

\[
+ U_{1R} (p_1|| + p_2|| - k_\parallel, k_\parallel, p_3||) U_{2R} (p_1||, p_2||, k_\parallel) \right) \right\} + \frac{1}{64\pi^4} U_{2R} (p_1||, p_2||, p_3||)
\]

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\[ \times \sum_{i=1}^{4} \delta_{p_1||+p_2||+p_3||+p_4||} \int_{D_3} dk_|| dq_|| \left( \frac{1}{v_{FR}^2} \right) \left[ U_{1R} \left( p_3||, q_||, -k_|| + p_1|| + q_|| \right) \right. \\
\times U_{1R} \left( k_||, -k_|| + p_3|| + q_||, q_|| \right) + U_{2R} \left( p_3||, q_||, k_|| \right) U_{2R} \left( -k_|| + p_3|| + q_||, k_||, q_|| \right) \right] (18) \]

where the domains \( D_1, D_2 \) and \( D_3 \) are given by

\[
D_1 = \begin{cases} 
-\Delta \leq k_|| \leq \Delta \\
-\Delta \leq p_1|| + p_2|| - k_|| \leq \Delta 
\end{cases}
\]

\[
D_2 = \begin{cases} 
-\Delta \leq k_|| \leq \Delta \\
-\Delta \leq p_3|| - p_4|| + k_|| \leq \Delta 
\end{cases}
\]

\[
D_3 = \begin{cases} 
-\Delta \leq k_|| \leq \Delta \\
-\Delta \leq q_|| \leq \Delta \\
-\Delta \leq p_3|| + q_|| - k_|| \leq \Delta 
\end{cases}
\]

Once again if, for simplicity, we neglect the dependence of the coupling functions with respect to the components of the momenta along \( FS \), consider scattering processes associated with only two independent external momenta and integrate over one of them, our simplified RG equations for the \( \tilde{U}_{1R} \)'s are readily reproduced.

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