Application of Algorithms for High Precision Metrology

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ABSTRACT. This paper evaluates the performance of algorithms suitable for processing the measurements from two laser beam metrology systems, with particular reference to the Gaia Basic Angle Monitoring device. The system and signal characteristics are reviewed in order to define the key operating features. The low-level algorithms are defined according to different approaches, starting with a simple, model free method, and progressing to a strategy based on the signal template and variance. The signal model is derived from measured data sets. The performance at microarcsec level is verified by simulation in conditions ranging from noiseless to large perturbations.

Online material: color figures

1. INTRODUCTION

High precision astrometry is often supported by metrology, either on ground or in space. An example of the former case is the control of atmospheric disturbances in long baseline interferometry, typically in the near-infrared range (Schmid et al. 2012; Robertson et al. 2012), and of the overall instrument geometry (Woillez & Lacour 2013) for narrow- to medium-angle astrometry. The latter case is mostly aimed at measurement or modeling of the instrumental parameter secular evolution and long term variations over time scales longer than a few revolutions periods, i.e., above about one day.

Metrology usually employs interferometric combination of laser beams to meet the precision goal thanks to the high photon flux. As opto-electronic systems become more and more complex, the metrology signal may be affected by a number of implementation aspects, potentially introducing errors. The data processing algorithms then become a performance factor, as their limitations might compromise part of the potential precision associated with the hardware design.

We address a set of algorithms suited to the relative phase of multiple fringe patterns from two-beam laser interferometry, in particular the low-level, short range automated processing which provides the relevant information to higher level diagnostics software and to the scientists evaluating the system response. The current work is focused mainly on applications to the Gaia metrology subsystem, although the concept is quite general and could be applied, in the future, to other high precision astrometry experiments (Gai et al. 2012a).

The Gaia mission (Perryman 2005; de Bruijne 2012) is aimed at global astrometry at the few microarcsec (hereafter, μas) level, producing an all-sky catalogue of position, proper motion, and parallax, complete to the limiting magnitude V ≃ 20 mag. The Gaia concept relies on self-consistency of the astrometric information of celestial objects throughout operation, factoring out the instrument parameters and their evolution by calibration of the overall data set. The Hipparcos experience suggests that this approach is viable with respect to detection and modeling of the instrumental parameter secular evolution and long term variations over time scales longer than a few revolution periods, i.e., above about one day.

Gaia takes advantage of the simultaneous observations of two telescopes mounted on a common high-stability torus to precisely measure the relative positions of stars in regions of the sky separated by a wide basic angle BA = 106.5°. Perturbations to the instrument geometry induce displacements of each telescope line of sight (LOS); the common mode displacement of the two telescopes does not affect the measured angular separation between stars, and appears as a disturbance of the overall instrument, with an effect similar to attitude jitter, i.e., a small degradation of image sharpness. However, the differential LOS perturbation appears as a BA variation (BAV), which directly affects the astrometric measurement of star separation.

The design specifications are derived from the short term astrometric performance requirements, corresponding to a BA stability σ(BA) ≃ 7 μas. In order to ensure that the short term stability requirement is actually met throughout operation, a custom on-board metrology system is also integrated in the payload design, the Basic Angle Monitoring (BAM) device (Gielesen et al. 2012).
The metrology concept is based on pairs of collimated beams from a laser source, split by a dedicated distribution system in front of the *Gaia* astronomical telescopes, and following an optical path very close to that of the stellar photons to generate an interferometric pattern on a dedicated section of the focal plane. The rationale is that perturbations to the optical configuration, affecting the position of the stellar images on the focal plane, i.e., the basic ingredient of the astrometric measurement, would also affect the BAM beam path, and therefore the interferogram phase. The match between astronomical and metrology photons is not complete, e.g., because the laser beam hits a much smaller region of the optical system, but it is assumed that the most relevant variations of the telescope geometry will also induce an effect on the interferogram phase, which is monitored, allowing for estimation of astrometric corrections to be applied in the data reduction, if required.

In § 2 we present the main characteristics of the signal, and in § 3 we define accordingly the algorithm concepts. In § 4 we evaluate the performance achieved with a simple implementation of such algorithms; then, in § 5, we discuss the implications of their usage within the *Gaia* data reduction scheme.

### 2. SIGNAL CHARACTERISTICS

The BAM design provides a fringe pattern with period comparable to the stellar image size in the high resolution direction, detected by a CCD similar to the science ones used for object detection, astrometry, and photometry. The interferogram can be considered as a large set of artificial stars, coherently providing an instrumental phase information. The BAM concept is based on two assumptions: (1) small amplitude of the instrument perturbations to be monitored, and (2) high signal to noise ratio (S/N).

The latter condition leads to a potential measurement precision corresponding to a small fraction of the interferogram period, thus making the device suitable to fulfill the first requirement.

The actual relationship between the interferogram phase and the telescope pointing direction is unknown, depending on the actual optical configuration, but is it assumed that, in the expected small perturbation regime, they will coincide apart from an offset, or “zero point”. The BA variation, i.e., the variation of the angle between the telescope observing directions, is thus connected to variation of the relative phase between the interferograms. In particular, correlated phase variations are associated with common mode telescope pointing errors, and differential phase variations with more critical BA variations affecting the astrometric measurement.

In the medium to long period, the BA variation measured by the BAM device can be calibrated with respect to its corresponding value on the sky, as deduced by a convenient combination of observations; e.g., variation in the angular separation among stars at different epochs. The comparison between BAM measurements and astrometry will therefore provide an assessment of science quality throughout the mission.

The interferogram associated with an ideal BAM, feeding an ideal telescope, is the point spread function of the unobstructed circular aperture related to the individual laser beams, modulated by fringes with the Young period. Practical implementation introduces deviations from such a simple framework because of optical aberrations due to design, manufacturing, and alignment in the metrology distribution system and in the telescope. Also, the laser beam intensity is not uniform over its cross section, but it is expected to be described, e.g., by a truncated Gaussian.

This still generates a useful signal (Riva et al. 2012) with high contrast fringes within a more complex diffraction envelope. The fringe period is basically preserved, but the phase modulation bears contributions from the real system. Also, the envelope has generally larger size, and significant shape variation, with respect to the ideal case. However, it is expected that the crucial information of BA variation is still encoded in the mutual relationship between interferograms. In particular, the small BA variations expected during operation will introduce interferogram displacements on the μas scale, with amplitude much smaller than the fringe period.

The BAM device, by design, generates an interferogram for each telescope, imaged in different positions of a common dedicated CCD. Each signal is affected by its own photon shot noise, background (depending on the current pointing position), and comparable readout noise. Photon noise is expected to be the main contribution, since the laser source intensity is tuned in order to use most of the detector dynamic range.

The BAM is based on a geometry similar to that of Young’s interference experiment, in which the two slits are replaced by small beams with diameter \( D = 9 \) mm, separated by a baseline \( B = 0.54 \) m, generating a modulated signal on the focal plane, i.e., an interferogram, with \( \sim 2.44B/D = 146 \) fringes, limited by the diffraction envelope of the individual aperture. The two telescopes are fed by separate beam pairs, generating similar interferometric spots on the detector; the fringe period is \( \lambda/B = 325 \) mas, corresponding to 55 μm, i.e., 5.5 pixels. The interferogram from each telescope is repeatedly measured, at a rate of an image every \( \sim 20 \) s.

Hereafter, we will refer to the beam pair monitoring one telescope, the related optical system, and the corresponding detected signal as a “BAM channel”. Only a limited region of interest over the central lobe of diffraction is actually acquired for each BAM channel, as shown by the dashed rectangle in Figure 1. The interferogram is integrated across the fringes after readout in order to achieve a more easily tractable one-dimensional signal, shown in Figure 2. Since the fringe patterns are mostly aligned to the CCD array, to within 1.5°, only a marginal smearing is introduced.

### 3. ALGORITHM DEFINITION

The laser source is monochromatic, but the interferometric signal, due to the limited beam diameter and associated fringe
envelope size, has a spatial frequency distribution spread over
the approximate range \([\frac{1}{2}, \frac{1}{2}]\). A detailed mathematical
description of the fringe pattern is not easily formulated except in
the simplest geometric cases, due to optical aberrations.

However, most phase estimation algorithms are based either
on a template, or on assumptions on the mathematical charac-
teristics of the signal; i.e., they are model dependent. Given the
BAM readout frequency and high S/N, it is possible to define a
numerical “signal template”, as well as its empirical variance,
both deduced from a convenient set of measured data, rather
than analytical formulation of a signal model whose parameters
have to be estimated, e.g., by best fit to the measurements.

It is possible to define algorithms aimed at providing a phase
estimate for each interferogram, related to the average LOS
position of the individual telescope, from which the BAV is simply
deduced from the difference of the phase values at each time.

We focus on a set of simple concepts leading to convenient al-
gorithm implementation, and evaluate their main impact on the
measurements.

We also address a model independent measurement concept,
providing a direct estimate of the BAV as phase difference be-
tween the fringe patterns, without individual phase computation.

A model dependent approach is expected to be, in principle,
more precise, because the expected characteristics of the signal
are taken into account. However, this requires that additional in-
formation be available to define at least the signal template and
variance, as in § 3.1. Hereafter we expand on some of the possible
approaches, which will then be tested on simulated data.

3.1. Template Definition from the Data

From a sufficient number \(N\) of measurement of the interfer-
ometric signals \(S_1, S_2\) related to each telescope, it is possible to
define the sample approximations \(T_1, T_2\) of the corresponding
templates, or noise-free reference functions, from the signal av-
eraged over the data set:

\[
T_{1,2}(x_k) = \frac{1}{N} \sum_{n=1}^{N} S_{1,2}(x_k; t_n). \tag{1}
\]

The dependence on the pixel position \(x_k\) and on the current
exposure time \(t_n\) has been shown explicitly, but it will be omit-
ted below where not necessary for the sake of simplicity.

We define in a similar way the empirical variance \(V_1, V_2\) of
each interferogram pixel as

\[
V_{1,2}(x_k) = \frac{1}{N-1} \sum_{n=1}^{N} \left[ S_{1,2} - T_{1,2} \right]^2. \tag{2}
\]

This approach is based on the assumption that the measure-
ment conditions are stationary, so that the average can be con-
sidered as a reasonable approximation of the template.

The phase noise on the fringes is assumed to be much smaller
than the fringe period, by a factor of \(\sim 10^{-5}\), in normal con-
ditions. However, phase noise will induce errors on the template
and variance estimate, in addition to the amplitude fluctuations
due to photon noise. It can be shown that photon noise, in the
expected conditions, is dominant; thus, the small fringe dis-
placement among subsequent instances does not introduce a sig-
nificant blurring of the mean fringe, which can be considered a
reasonable approximation to the desired signal template.

In the numerical experiments below, the template is built by
averaging over 1,000 frames, so that the expected reduction of
individual measurement noise is of order of \(\sqrt{1000} \approx 31\).

The data set also provides an estimate of the signal dispersion
through the sample variance, so that subsequent measurements
can then be checked for consistency with the template and the
initial data (e.g., in terms of confidence levels). For each BAM

![FIG. 1.—Gaia BAM interferogram. The dashed rectangle represents the read-out area. See the electronic edition of the PASP for a color version of this figure.](image1)

![FIG. 2.—Section of two one-dimensional fringe patterns, integrated across fringe. See the electronic edition of the PASP for a color version of this figure.](image2)
channel, a specific template is defined by averaging the corresponding data set from a suitable measurement sequence.

### 3.2. Model Free Fringe Location: Mutual Correlation (MC)

The signals from the two BAM channels are not exactly sinusoidal, but they have a very narrow spectrum, resulting in a significant modulation at the same nominal spatial frequency. Their envelopes are not equally shaped, but over the readout region their amplitude is quite high.

The signal similarity leads to the possibility of processing them with a simple approach based on the correlation technique. The two quasi sinusoidal signals will have a maximum correlation value for a given displacement with respect to each other, which in the example shown in Figure 2 is of order of two pixels. A perturbation to either telescope, inducing a phase variation on the corresponding interferogram, will modify accordingly the position of the signal correlation maximum. Therefore, this algorithm defines a linear variable strictly related to the relative position of the interferograms, and as a consequence of the basic phase variation between telescope lines of sight.

We adopt the estimate of this relative interferogram phase as an operational definition of the BA estimate in order to monitor its variation around an irrelevant zero point to be calibrated on the sky from science data. This approach, by construction, gives a position estimate modulo the fringe period, which is not an “absolute” datum.

The relative displacement of the two fringe patterns can be estimated directly by computation of the maximum correlation position between them, e.g., from parabolic fit of the three correlation values achieved for lag zero and ±1 pixel. This approach completely avoids the issue of template definition; broadly speaking, the interferogram from each channel acts as a template for the other. In practice, the method does not require that the two signals be very similar to each other (which would improve on the correlation value), but only that their shape remains approximately stable over the time frame required for calibration on the sky through the science data.

In more detail, the two signals are arrays of 500 values; given their offset of about 2 pixels, shown in Figure 2, they are roughly “aligned” by removing the first value on the former and the last one on the latter, i.e., reducing the data to \( K = 499 \) values. The offset is due to the simulated placement of readout windows; it is constant over the simulation, due to the small phase noise.

The lag zero signal is then selected as the central part of the array with index \( k = 2, \ldots, K - 1 \), i.e., excluding the end values. The lag ±1 signals include, respectively, either of the end values, discarding the opposite ending value, i.e., with indices \( k = 1, \ldots, K - 2 \), and \( k = 3, \ldots, K \). The correlation is then computed over \( K - 1 = 498 \) values.

This windowing approach sacrifices a small part of the signal, but it avoids data interpolation or extrapolation at each end of the arrays. Processing is fast, as only array indexing is used.

The mismatch between signals from each BAM channel suggests that the noise performance of this algorithm may not be optimal, but its advantages are simplicity and robustness, so that it is applicable also in a context of limited knowledge of the instrument parameters, e.g., at the beginning of operation or after unexpected events inducing large perturbations. It also provides a simple sanity check for other algorithms.

### 3.3. Model Dependent Fringe Location: Correlation with Template (CT)

In the expected operating conditions, the interferometric signal is quite stable, and therefore we assume that it is possible to apply the approach of § 3.1 to define its model (or template) by averaging the measurements according to equation (1). As long as only small perturbations are present, the individual signal instances will be consistent with such signal templates, apart from fluctuations in amplitude (e.g., from photon noise) and phase (related to the BA variation).

For any fringe pattern instance, we compute the correlation with the template, in its nominal position, and with ±1 pixel offset. The indexing of § 3.2 is used for definition of the lag zero and ±1 signals. Also, parabolic fit of the correlation values provides the “exact” location of the current interferogram with respect to its template. We adopt this parameter as an estimate of the relative LOS position, independently for each BAM channel. The difference between LOS values is considered the operational definition of the BA, so that its variation can be used for monitoring.

In stable conditions, each LOS is expected to fluctuate around zero, since each signal instance is close to its template within the noise; similarly, the BA estimate is expected to be centered in zero.

### 3.4. Model Dependent Fringe Location: Maximum Likelihood Estimator (ML)

Correlation is a robust and proven technique, but it is known to sometimes provide less than optimal noise performance. We define an estimator derived from the maximum likelihood framework, which also requires knowledge of the current noise statistics, in particular the signal variance, estimated from the data as from equation (2).

Given two similar functions, hereafter labeled “signal” \( S \) and “template” \( T \), the maximum likelihood approach may be used to identify the best matching position, minimizing a functional inspired to the classical \( \chi^2 \) and defined as

\[
\chi^2 = \sum_{k=1}^{K} \frac{[S_k - T(x_k - \tau)]^2}{\sigma_k^2}, \tag{3}
\]

where \( S_k \) and \( T_k \) are the signal and template values at the \( k \)th sample, \( \tau \) is the assumed offset in sample number, \( \sigma_k \) is the estimated variance at the \( k \)th sample, and \( K \) is the total number of samples. The problem is ill-posed, as the parameters \( \chi^2 \) and \( \tau \) are not uniquely determined.

To resolve this ambiguity, we assume that the signal \( S \) and template \( T \) are of the same shape, and therefore we restrict the search for the best matching position to a range of \( \tau \) where the signal is expected to deviate significantly from the template. This approach sacrifices a small part of the signal, but it avoids data interpolation or extrapolation at each end of the arrays. Processing is fast, as only array indexing is used.

The mismatch between signals from each BAM channel suggests that the noise performance of this algorithm may not be optimal, but its advantages are simplicity and robustness, so that it is applicable also in a context of limited knowledge of the instrument parameters, e.g., at the beginning of operation or after unexpected events inducing large perturbations. It also provides a simple sanity check for other algorithms.
where the dependence on the coordinates has been shown explicitly only for the template. The matching position has true value \( \tau_0 \), and its estimate \( \tau \) is derived from the measured data. The derivation follows that in Gai et al. (1998), in which the signal \( S_k = S(x_k) \), measured over the \( K \) pixel positions \( x_k \), was considered to be equal to the template discrepancy an evolving signal shape (nonstationary conditions), the signal different functions, e.g., a template not matching the signal, or "labelled measurement error and a shift in the reference position, therein labelled “photo-centre”. It may be noted that, in case of usage of different functions, e.g., a template not matching the signal, or an evolving signal shape (nonstationary conditions), the signal discrepancy

\[
h_k = S_k - T_k
\]

is no longer just due to noise, so that it cannot be expected in general to be uncorrelated and to have zero mean.

The photo-centre estimate must be a stationary point of the \( \chi^2 \) in equation (3), i.e., a solution of the equation

\[
\sum_k \left[ S_k - T_k(\tau_0) \right] \cdot T'_k(\tau_0) / \sigma_k^2(\tau_0) = 0.
\]

Therefore, assuming small errors (\( \tau \approx \tau_0 \)), the square bracket can be simplified, taking advantage of Taylor’s expansion of the template in the current approximate position, to provide

\[
\tau - \tau_0 = - \sum_k \frac{[S_k - T_k(\tau)] \cdot T'_k(\tau)}{\sum_k [T'_k(\tau)]^2 / \sigma_k^2(\tau)}.
\]

The solution is unbiased, i.e., \( \langle \tau - \tau_0 \rangle = 0 \), and has variance equal to

\[
\langle (\tau - \tau_0)^2 \rangle = \sum_k \frac{[T'_k(\tau)]^2}{\sigma_k^2(\tau)}
\]

It may be noted that at increasing values of location error due, e.g., to lower S/N, the validity of the above expressions degrades progressively. The above equation (6) is adopted as the operational definition of the ML estimate of LOS for each BAM channel, and the BA is defined as the LOS difference.

3.5. Residual Diagnostics

In order to assess the consistency of individual interferogram \( S_k \) with the current estimate of signal template \( T \) and of its variance \( \sigma \), from equations (1) and (2), we define a quantity related to the reduced \( \chi^2 \) concept according to equation (3):

\[
\chi^2_R = \frac{1}{K-1} \sum_{k=1}^{K} \left[ S_k - T(x_k - \tau) \right]^2 / \sigma_k^2.
\]

When all residuals are comparable with the expected noise, corresponding to the signal variance, the reduced \( \chi^2 \) is expected to be of order of unity. The normalisation factor \( K - 1 \) is used under the assumption that no model parameter is derived from the current measurement, apart an overall photometric level estimate. The issue of defining the actual number of degrees of freedom is quite sensitive, in general, and may require some care in practical implementation.

However, we may expect that the value of \( \chi^2_R \) as defined in equation (8) might at least help in identifying significant perturbations when it becomes significantly larger than unity.

4. Simulation

The algorithm performance is evaluated, first in terms of noise sensitivity with respect to signal amplitude, and of linearity with respect to signal phase; then, on externally generated data, with respect to random noise and systematic error performance. The processing is performed on a desktop PC, endowed with an Intel Xeon 3.33 GHz CPU and 8 GB RAM, using the Windows 7 (64 bit) operating system, and the Matlab package.

4.1 Performance as a Function of Amplitude Noise

The phase noise performance is simulated for the three algorithms defined above, using signal profiles considered as realistic for an optical configuration affected by a plausible amount of manufacturing, mounting, and alignment errors.

Over a range of intensity corresponding to increasing S/N, from a few hundreds to about one million, we estimate the theoretical location precision for the ML algorithm according to equation (7). For each algorithm, we generate a noisy signal sample of \( N = 10,000 \) instances, corresponding to constant interferogram phase and random amplitude fluctuations related to the photon noise level associated to the current intensity level. The fringe position, or the corresponding interferogram separation, is estimated according to a simple implementation of each algorithm (MC, CT, ML). The corresponding BA noise due to amplitude fluctuations is then evaluated as the RMS dispersion over the sample. The result is shown in Figure 3. Over most of the S/N range, the trend is basically photon limited, as evidenced by the constant slope in logarithmic units.

The noise decreases with increasing S/N, as expected, and the algorithm performance difference cannot be easily distinguished on this scale. In order to ease the comparison of algorithm results, the relative precision is shown, referred to the estimated precision from equation (7), which can be considered as a conservative estimate of the limiting performance, providing a good match with the Maximum Likelihood location estimator defined in the same framework. The result is shown in Figure 4.

In the high S/N regime, the noise performance of both correlation methods is comparable, and about 10% (CT) and 12.5% (MC) worse than the limiting error defined in the Maximum
Likelihood framework. The fluctuations of the relative performance are compatible with the statistics related to the sample size, i.e., of order of 1%.

4.2. Linearity Performance

The algorithm linearity is evaluated in a noiseless case by simulation, injecting a known LOS displacement on the input signals and verifying the LOS estimate in output (for CT and ML). For the MC algorithm, the BAV estimate is averaged for input displacements applied to either LOS1 or LOS2 (both giving quite similar, but not exactly equal, results). The test range is ±1 mas, i.e., quite large with respect to the small perturbations expected in the normal operating regime. A linearity correction was required for both CT and MC. The input/output discrepancy of ML, and the corresponding residual discrepancy after linearity correction for CT and MC, are shown in Figure 5. The linearity of all algorithms, after correction in the case of MC and CT, is quite good, providing a residual discrepancy of a few 0.01 μas over the ±1000 μas range considered.

4.3. Performance of the Algorithms on Externally Simulated Data

Two data sets including a number of realistic effects have been generated by the Coordination Unit 2 (CU2) of the Gaia Data Processing and Analysis Consortium (DPAC), on the supercomputer Mare Nostrum at the Barcelona Supercomputing Center (Centro Nacional de Supercomputación), including contributions from different real world sources. They are processed according to the algorithms in § 3. The data sets include, respectively, 5,181 and 3,638 interferogram pairs representing sequences of BAM exposures.

4.3.1. Overall Features of Data Set 1

The S/N of this set of fringe pattern pairs is extremely high, of order of $6 \times 10^5$ and $10^6$ respectively for the whole of the LOS 1 and LOS 2 signals. From § 4.1, this corresponds to astrometric noise levels of order of 0.2 μas and 0.1 μas, respectively.

At an early analysis, a discontinuity in the data set was evidenced, as can be seen in Figure 6, showing the average intensity over each interferogram (i.e., the mean level of the fringe pattern). This intensity variation might correspond, e.g., to a variation of the laser source intensity, although the amplitude of the variation (~20%) and its suddenness may not be expected as a realistic common event. However, it is interesting to check the robustness of our diagnostics and measurement algorithms against such variations.

Given the relative stability of the signal level on each side of the discontinuity, we compute the signal template and variance

![Fig. 3.—Algorithm precision as a function of S/N. See the electronic edition of the PASP for a color version of this figure.](image1)

![Fig. 4.—Noise with respect to the ML case. See the electronic edition of the PASP for a color version of this figure.](image2)

![Fig. 5.—Input/output discrepancy for ML, CT, and MC. See the electronic edition of the PASP for a color version of this figure.](image3)
separately for each subset, according to equations (1) and (2); they are labeled respectively “Pre” and “Post”, with reference to the discontinuity. According to § 3.5, we evaluate the reduced $\chi^2$ for each interferogram using both templates. The results are shown in Figure 7.

We remark that the large signal amplitude variation in Figure 6 is evidenced also by a huge $\chi^2$ variation, by about seven orders of magnitude, when processing the data on either side of the discontinuity with the mismatched template. The $\chi^2$ increase is consistent with the intensity variation and the S/N level: the signal change is order of $10^4$ times larger than its variance. Besides, when using matched data and templates, i.e., “Pre” template with “Pre” data and “Post” template with “Post” data, the reduced $\chi^2$ value remains of order of unity, as shown in Figure 7.

### 4.3.2. LOS and BAV of Data Set 1

The LOS evolution over the data set 1 are shown in Figure 8, estimated using the CT algorithm. The linearity correction from § 4.2 is applied. In Figure 9 the LOS estimate from the ML algorithm is shown. A crude background subtraction is applied by removing the fringe mean value. The zero points have been set according to display convenience; also, each display point replaces 10 initial points with their average, also reducing the noise, in order to improve on plot readability.

The ML result is quite similar to that from CT for both LOS 1 and LOS 2. The two algorithms appear therefore to be quite consistent with each other. The signal discontinuity appears to induce a marginal effect on both LOS 1 and LOS 2, corresponding to an offset of a few 0.1 $\mu$as. Both LOSs feature an approximately sinusoidal behaviour, with different phase and period corresponding to the satellite revolution (about 6 hr).
The oscillations represent simulation of the thermo-elastic evolution of the instrument during the spin, i.e., a physical phenomenon; the two telescopes are mounted in different positions and therefore respond independently to a given external perturbation (e.g., residual effects of Sun irradiation). Besides, the LOS and BAV “jump” is due to the change in the signal profile between either side of the discontinuity, filtered by the truncation due to readout windowing and the algorithm response.

The sequence of BAV estimates from each algorithm is shown in Figure 10, also including the nominal BAV values used in input to the simulation (solid line). It may be noted that all algorithms reproduce a large part of the input BA oscillations, to a few 0.1 μas.

The BAV discrepancy from the input value is shown in Figure 11 for each algorithm. The effect of signal discontinuity is a small BAV “jump” (by a few 0.1 μas) on CT and MC estimates (circles and crosses, respectively), and hardly perceivable (≈0.1 μas) on the ML results (triangles).

The mean and RMS discrepancy of the results from each algorithm with respect to the “true” input BAV, and of the result discrepancy between algorithms, are listed in Table 1 separately for the two region. The input/output BAV discrepancy is the algorithm error (a few 0.1 μas for MC and CT, a few 0.01 μ as for ML), whereas the output difference (a few 0.1 μas) is related to the mutual algorithm consistency. The RMS values can be considered to be the random noise of the measurement, whereas the offsets represent a systematic difference.

The noise is of order of 0.1 μas, as expected, within each data region (“Pre” or “Post”); the average value is quite close to zero for ML, evidencing a very small systematic error, consistent with the very small “jump” (Fig. 11). The results from the different algorithms are consistent to within a few 0.1 μas.

The MC and CT results are grouped along straight lines separated by a vertical offset, corresponding to a “jump” (associated to the signal discontinuity) of ≈0.5 μas, significantly larger than the intrinsic dispersion in either “Pre” or “Post” region.

The algorithm linearity can be verified by direct comparison of input and output values; the ideal case of perfect matching between input and output would correspond to zero discrepancy. The BAV discrepancy is shown in Figure 12 for all three algorithms. Their responses are quite linear, since the slope of the output discrepancy versus input BAV is very small; the off-sets are, again, due to the signal discontinuity. The analysis of linear correlation between input and output provides a correlation coefficient in all cases above 99.9%, and a slope very close to unity, for all algorithms.

4.3.3. Results of Data Set 2

The S/N of this data set is still very high with respect to usual astronomical data, but significantly lower than the previous case, of order of 5 × 10^4 for both LOS 1 and LOS 2 signals, corresponding to the nominal BAM operation level. From the results of § 4.1, this corresponds to an expected astrometric noise level of order of 1 μas.

The simulation results evidence a noise level of ≈1.4 μas, consistent with expectations. The algorithm behaviour with respect to signal perturbations, including intensity increase by 14% for channel 1, and 12% for channel 2, is again limited to a BAV “jump” of about 1 μas. The results from MC, CT, and ML remain quite consistent with one another. Also, the signal variation is clearly evidenced by the reduced χ^2.

5. DISCUSSION

The performance of low-level algorithm on BAV noise and linearity has been verified on different data sets at the μas level or better, consistently with the S/N limit. In spite of its simplicity, the model free approach (MC) achieves noise performance within a few percent of the template based CT and within...
However, they do provide a clear diagnostic of its occurrence in terms of estimation of relevant system parameter change. The low-level algorithms do not perform any interpretation of the signal variation simulated in data set 1, in terms of estimation of relevant system parameter change. However, they do provide a clear diagnostic of its occurrence through the \( \chi^2 \), and the BAV estimate error is retained within the specifications.

The reduced \( \chi^2 \) evidences oscillations around unity value in data set 1 (Fig. 7), even using the matching template and variance for each data subset. This is assumed to be related to the extremely high S/N, and correspondingly very low measurement noise. In such circumstances, the simple procedure used to build the signal template and variance seems to be insufficient, e.g., because the input phase variation induces signal variations that are not negligible with respect to those associated to photon noise. The fluctuations on ML estimates (Fig. 11), in phase with the BAV, appear to be consistent with this scenario. Such oscillations are no longer present in the reduced \( \chi^2 \) and BAV computed in data set 2.

The current simple approach appears to be adequate for the Gaia mission requirements, even in spite of the large simulated signal variations, resulting in systematic errors compatible with the microarcsec noise level associated with the S/N range expected for the Gaia BAM operation. It may be noted that, with the Gaia pixel size \( \sim 60 \) mas, the best precision case of data set 1, of order of \( 0.1 \) \( \mu \)as, corresponds to a few micropixels, thus matching well the simulations and experimental results in Zhai et al. (2011).

The minimisation of signal model error, according to § 3.1, requires large amounts of data to average out the noise fluctuations. Besides, this may reduce the sensitivity to disturbances acting on a time scale shorter than the period of data accumulation. A suitable trade-off must therefore be defined.

### 6. CONCLUSIONS

We investigate algorithms aimed at low-level processing of metrology signals from two laser beam, high dilution, imaging metrology systems, with particular reference to the Gaia BAM device. The methods range from a model free approach based on mutual correlation (MC), to others using progressively more detailed information, including the signal template (CT), and the signal variance (ML).

The numerical model of the signal is derived by computing the average fringe pattern and its variance over a convenient data set, which may be easily verified for self-consistency, and has minimal dependence on external parameters: mainly pixel size and optical scale to convert linear results (fractions of pixel) into angular values.

The performance is verified at or below the \( \mu \)as level by simulation on data sets generated either in-house, or by independent groups, ranging from the noiseless case to comparably large perturbations. The model free approach still provides quite appealing performance, while the maximum likelihood method features lowest noise and sensitivity to disturbances. Some signal disturbances, including profile variation and up to 20% intensity variation, induce discontinuities in the measured...
positions of order of 1 μas, within the design specifications. However, the signal variation is clearly identified by χ² diagnostics.

The performance of all low-level algorithms appears, therefore, to be of interest for application to the data reduction of the Gaia Basic Angle Monitoring device, and potentially to future high precision astrometry experiments.

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