Study on reflection spherical wave at the near surface

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Abstract. In real seismic surveys, seismic wave propagates in the form of spherical wave and plane wave is the approximations of spherical wave in the far-field and for high frequencies. The traditional seismic processing and interpretation methods based on the plane wave theory are not exact. Furthermore the difference between the plane wave and spherical wave is strong, especially in the near surface. Studying the characteristics of reflection spherical wave near the surface can develop more accurate wave theory. In this paper, a two-layer model with a high-velocity upper layer is built to analyze the spherical reflection wave characteristics. Tests show when the incident angle reaches a certain level, there will be an irregular wave, the pseudo-spherical wave, following the main reflection spherical wave, which has a great influence on the main reflection wave. This discovery has enriched our understanding of wave propagation theory and it has important guiding significance for accurate study on wave phenomena.

1. Introduction

Spherical wave reflection coefficient (SRC) is frequency-dependent and the spherical wave theory can give more precise description of real seismic reflection than plane wave [1] According to Li et al. [2], the frequency-dependence of the SRC is strong especially at the near surface. Thus studying the wave theory based on the spherical wave will give a more precise guide for the near surface wave field research. As we know, for two layers of semi-infinite homogeneous near-surface medium, when the velocity of the upper medium is lower than the velocity of the lower medium, and when the incident angle reaches a certain angle, the seismic wave is totally reflected. At this time, the reflected spherical wave and the refracted wave are coupled together and the reflected wave will be distorted. Otherwise, when the velocity of the upper medium is higher than the velocity of the lower medium, the transmitted wave will also be coupled with an irregular wave, which has been studied for a long term. Ott [3] and Brekhovskik [4] both analyzed the characteristics of irregular wave groups transmitted to low-velocity media. Kozak [5] observed the existence of this wave for the first time through experiments but didn’t give an explanation by ray theory. After that, detailed experiments have studied for the transmitted irregular wave in two layers of media. Červený[6] verified the existence of pseudo-spherical wave transmitted to low-speed media through physical simulations and analyzed the characteristics of the waves. The existence of this wave will be coupled with the transmitted wave, so it will have a greater impact on the characteristics of the transmitted wave.

The previous researches mainly focused on the transmission wave characteristics. In actual seismic exploration, the reflection wave characteristics has more important significance for seismic data processing and interpretation. In this paper, a two-layer model with a shallow high-velocity upper layer is built to study the characteristics of spherical wave. According to our conventional knowledge, totally reflection will not occur and there are no other waves will be coupled into the reflected wave in the...
model. In fact, Sommerfeld integral and finite difference modeling results both verify that when the incident angle reaches a certain level, an irregular wave field with a low frequency, the pseudo-spherical wave, is found. The influence of the irregular wave on the reflection coefficient is analyzed through the amplitude spectrum, the result on the complex plane and the time-domain reflection waveform. This discovery has enriched our understanding of wave propagation theory. It has important guiding significance for accurate wave field characteristics of reflected waves.

2. **Algorithm**

For the two layers of semi-infinite homogeneous medium, as shown in Figure 1, the source is located in the upper layer of the medium. The reflection coefficient can be obtained by numerical simulation based on wave equations in a 2D cylindrical coordinate system. The acoustic wave equation in a two-dimensional cylindrical coordinate system is expressed as follows:

\[
\frac{\partial p}{\partial t} = -\rho c^2 \left( \frac{\partial v_r}{\partial r} + \frac{1}{r} v_r + \frac{\partial v_z}{\partial z} \right)
\]

\[
\frac{\partial v_r}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial r}
\]

\[
\frac{\partial v_z}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z}
\]

Where \(v_r\) and \(v_z\) are particle speed in the radial and vertical directions in the cylindrical coordinate system, respectively. \(p\) is the sound pressure, \(\rho\) and \(c\) are the density and velocity of the medium, respectively.

In addition, the spherical acoustic wave reflectivity can also be computed by Sommerfeld integral as follows:

\[
R_{\text{sph}} = \frac{\int J_0(\omega r) x^2 \rho_1 v_1^2 / \rho_2 v_2^2 \left( 1 - \frac{v_1^2}{v_2^2} \right) e^{-i \omega h} dx + \int J_0(\omega r) x^2 \rho_1 v_1^2 / \rho_2 v_2^2 \left( 1 + \frac{v_1^2}{v_2^2} \right) e^{-i \omega z} dx}{\int J_0(\omega r) x^2 \rho_1 v_1^2 / \rho_2 v_2^2 \left( 1 - \frac{v_1^2}{v_2^2} \right) e^{-i \omega h} dx + \int J_0(\omega r) x^2 \rho_1 v_1^2 / \rho_2 v_2^2 \left( 1 + \frac{v_1^2}{v_2^2} \right) e^{-i \omega z} dx}
\]

Where \(\rho_1\) and \(\rho_2\) are the densities of upper and lower medium respectively, \(v_1\) and \(v_2\) are the velocities of upper and lower medium respectively, \(i\) is the imaginary unit, \(\omega\) is angular frequency, \(J_0\) is the zero-order Bessel function, \(r\) is the source-receiver offset, \(h\) and \(z\) are the vertical distances from the receiver and the source to the reflector, respectively.

![Figure 1](image1.png)

**Figure 1.** The schematic of observation system for the two-layer model.
3. Algorithm instance
As Figure 1 shows, for the two-layer model, the velocity of upper layer is \( c_1 = 4000 \text{m/s} \), the density is \( \rho_1 = 2.5 \text{g/cm}^3 \). And the velocity of lower layer is \( c_2 = 1000 \text{m/s} \), the density is \( \rho_2 = 2.0 \text{g/cm}^3 \). The distance from the source and receiver to the interface is \( h = 100 \text{m} \) and \( z=100 \text{m} \). For the numerical modeling, a ricker wavelet with a domain frequency of 20Hz is selected and the space interval is \( dr = dz = 1 \text{m} \), and the time interval is \( dt = 0.5 \text{ms} \). Then we use the sommerfeld integration method and the numerical modeling method based on the cylindrical coordinate system to compute the SRC.

Taking the angle of incidence of 10°, the SRC on the complex plane is shown in Figure 2a. It can be seen that the integration result and the finite difference result maintain a good consistency even the finite difference simulation dispersion may cause slight errors. As the Figure 2b shows, the amplitude spectrum results from these two method still maintain a good consistency. The frequency dependence of the spherical wave are more obvious at low frequencies. Then from the reflected waveform in Figure 3, we can see that the spherical reflection wave no longer keeps symmetry because of the frequency dependency, which gives a more accurate description for the near-surface reflection wave.

![Figure 2. The SRC on complex plane (a) and Amplitude of SRC (b) variation for different frequencies at 10°](image)

![Figure 3. The reflection spherical wave](image)

In order to further study the characteristics of SRC, the incident angle was taken to be 80°. The SRC result on the complex plane was obtained. Compared to the Figure 2a, the SRC shown in Figure 4a no longer maintains elliptic characteristics [7], which indicates that the reflection coefficient is distorted. Figure 4b shows the spectrum of the spherical wave, we can see the reflection coefficient can be greater than 1 at the low frequencies. This phenomenon is quite different from conventional knowledge for the reflected wave for the two-layer model with a higher velocity upper layer.
Figure 4. The SRC on complex plane (a) and Amplitude of SRC (b) variation for different frequencies at 80°.

Figure 5. The reflection spherical wave (a) and the zoom of the pseudo-spherical wave (b)

In order to further assure the influence of the irregular wave on the spherical wave, we separate these two waves and Figure 6a shows the separation normalized waveform results. Then the result of these waves on the complex plane is given, from the Figure 6b, the green line is the full reflected wave which is spiraling variation for different frequencies, and the SRC tends to the real reflection coefficient with the increasing frequency, while the blue line is the pseudo spherical wave which is also spiraling and the reflection coefficient of it tends to zeros with the increasing frequency. But for the pure spherical wave as the red line shows, the reflection coefficient will not keep spiraling and will also tend to the real reflection coefficient, which indicates the pseudo-spherical wave mainly causes spiraling phenomenon.

Figure 6. The separation normalized waveforms (a) and reflection coefficient (b)
4. Conclusions
In this paper, the reflection coefficients obtained by numerical simulation results and Sommerfeld results are consistent, and both have verified the existence of pseudo-spherical wave. The existence of spherical waves has a greater impact on spherical reflected waves, especially the far-offset reflected waves. The study of reflected wave characteristics has important guiding significance for the near surface wave field study. This is a new discovery for the characteristics of the spherical wave and is first proposed by our work, but the detailed property of the pseudo-spherical wave needs further research.

References
[1] Tao Y H, Wang S X and Li J N 2016 The Effect of Frequency-dependent Reflection Coefficient on Seismic Attenuation Estimation 78th EAGE Conference and Exhibition.
[2] Li J N, Wang S X and Yuan, S Y 2015 Frequency-dependent spherical wave reflectivity 77th EAGE Conference and Exhibition.
[3] OTT H Reflexion und Brechung von Kugelwelle Effekte 2 Ordnung Ann d Phys 41 (1942) 443-66.
[4] Brekhovskik L M 1960 Waves in Layered Media Academic Press, New York
[5] Kozak J 1970 Kinematic and Dynamic Properties of Elastic Waves Investigated on Seismic Models by Means of the Schlieren Method PhD Thesis Geophysical Institute of Czechoslovak Acad of Sci.
[6] Červený V Kozák I Pšenčík 1971 Refraction of elastic waves into a medium of lower velocity — Pseudospherical waves Pure and Applied Geophysics Volume 92 Issue 1 pp 115–132.
[7] Li J , Wang S and Yin H 2015 Acoustic Wave Equation Modeling in Cylindrical Coordinates with Convolutional PML 77th EAGE Conference and Exhibition.