CMB, Quantum Fluctuations and the Predictive Power of Inflation

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Abstract

I comment on the predictive power of inflation and recent CMB measurements.

When more than twenty years ago G. Chibisov and I were fortunate to discover that quantum fluctuations could be responsible for the large scale structure of the universe, we hardly thought it would be possible one day to verify this prediction experimentally. We wrote a paper [1], published in JETP Lett, Vol. 33, No.10, p. 532, 20 May 1981 (see Appendix), where we derived the spectrum of cosmological metric perturbations generated in a de Sitter stage of accelerated expansion (the word "inflation" had not been invented at this time) from quantum fluctuations. The spectrum came out to be logarithmically dependent on the scale. Our results were obtained for the first particular working model of inflation based on $R^2$—gravity, which is conformally equivalent to a model with a scalar field$^1$.

More than a year later I was very happy to learn that, after some debates, four groups of people, working on perturbations in the other particular scenario of inflation, "new inflation", estimated the perturbations [2],[3],[4],[5] and came to the same conclusion confirming our earlier result [1].

Nowadays, even though there is no reliable particle theory beyond the standard model, there are many successful inflationary scenarios, most of which are based on the idea of [6]. The general theory of the generation of inflationary perturbations which allows us to calculate the perturbations in these scenarios, where the Hubble constant can change by orders of magnitude during inflation, was developed in [7],[8] and its current status is described in the review paper [9], which, to a large extent, is based on [7],[8].

$^1$For this reason the calculations in this model are very similar to the calculations in scalar field models.
The main lesson we learned from the general theory of inflationary perturbations is that despite a multitude of various inflationary scenarios, the spectra of perturbations in the simple models of inflation, that is, models with minimal number of free parameters, are very similar (namely, logarithmically dependent on scale), and in complete agreement with earlier calculations [1]. Contrary to an erroneous belief inflation does not predict the scale-invariant, Harrison-Zel’dovich spectrum. The spectral index should be in the range of $0.92 < n_s < 0.97$. The physical reason for the deviation of the spectrum from the scale invariance is the necessity of having a graceful exit from inflation.

Can one avoid this prediction of inflation? In principle yes, but only by spoiling the most predictive model, namely, simple inflationary model (not be be confused with any one particular scenario). To avoid the main prediction of inflation, i.e. a logarithmically dependent spectrum, one has to introduce unjustified extra parameters and perform some fine tuning. Of course, this diminishes to a large extent the predictivity of inflation and, taken to extremes, one ”can explain” nearly any outcome of any measurement in this way.

The recent precise measurements of the CMB fluctuations, especially by WMAP, seem to be in a very good agreement with the predictions of simple inflationary theory [10]. It is still too early to talk about a reliable detection of logarithmic deviations of the spectrum from the flat one because the measurements are not yet sufficiently accurate. However, the progress in this direction is very impressive and future experiments should be able to verify this very robust prediction of inflation.

Does inflation have any competitor in explaining the outcome of the CMB measurements? Some authors claim that the cyclic universe scenario predicts the same spectrum as inflation. I believe that comparison of these models is misleading. The cyclic model does not predict anything for an expanding universe. In this scenario the spectrum comes to be nearly flat only at the last stage of collapse of the universe, before the singularity. The main obstacle in transferring the spectrum to an expanding branch is the singularity, which makes most dubious not only the conclusions about generated perturbations, but also the whole cyclic scenario. At present, there is not slightest hint how the singularity problem, which seems much more difficult than all other cosmological problems, could be solved. Therefore statements like “the cyclic universe predicts the perturbations spectrum” are completely unjustified and can be considered only as a wishful thinking.

Returning to inflationary cosmology, it is still too early to say that simple inflation has been proved by observations. However, it seems that we are on the right track to confirm the most dramatic prediction of inflationary cosmology that the structure of the universe and hence our lives are due to quantum fluctuations.

The first paper where the spectrum of inflationary perturbations was calculated [1] is not readily available. In order to make this paper easily accessible to those who want to study the inflationary cosmology, I attach it below.

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Quantum fluctuations and a nonsingular Universe

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Abstract. Over a finite time, quantum fluctuations of the curvature disrupt the nonsingular cosmological solution corresponding to a universe with a polarized vacuum. If this solution held as an intermediate stage in the evolution of the universe, then the spectrum of produced fluctuations could have lead to formation of galaxies and galactic clusters.

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A nonsingular cosmological model with a polarized vacuum has been attracting particular interest recently [1]. It has been pointed out elsewhere that quantum fluctuations may prove important in cosmology at energy density comparable to the Planck value [2]. Since this are in fact the energy densities characteristic of the nonsingular polarized-vacuum model [1], we believe it is worthwhile to study the role of quantum fluctuations in order to determine whether there is a singularity in this model.

For an isotropic metric, the single-loop corrections describing the polarization of vacuum of physical fields in a strong gravitational field lead to the following Einstein equations [3]:

\[ R^i_k - \frac{1}{2} \delta^i_k R = \frac{1}{H^2} \left( R^i_l R^l_k - \frac{2}{3} RR^i_k - \frac{1}{2} \delta^i_k R^m_l R^l_m + \frac{1}{4} \delta^i_k R^2 \right) - \frac{1}{6M^2} \left( 2R^i_{,ik} - 2\delta^i_k R^l_{,l} - 2RR^i_k + \frac{1}{2} \delta^i_k R^2 \right), \]  

(1)

where the coefficients \( M^2 \) and \( H^2 \) result from the sum over the effects of all the fields. For stability of Minkowski space, \( M^2 \) must be positive. For \( H^2 > 0 \), Eqs. (1) have a particular solution of the de Sitter type [3],

\[ ds^2 = g_{ik} dx^i dx^k = a^2 (\eta) \left( d\eta^2 - \sum_{\alpha=1}^{3} (dx^\alpha)^2 \right), \]  

(2)
\[ a(\eta) = -\frac{1}{H\eta}, \quad -\infty < \eta < 0, \quad R = -12H^2 = \text{const.} \]

The curvature invariants, in particular \( R \), have no singularities in the limit \( \eta \to -\infty \), showing that there is no actual singularity in the universe described by metric (2).

Let us consider, against the background of this metric, some small perturbations that satisfy Eqs. (1). We restrict the discussion to scalar perturbations. In a synchronous frame of reference \( \delta g_{\alpha 0} = 0 \) these perturbations have the following tensor structure:

\[ h^\alpha_\beta = -\frac{1}{a^2} \delta g_{\alpha \beta} = \left( \nabla^\alpha \nabla_\beta - \frac{1}{3} \delta^\alpha_\beta \Delta \right) \lambda - \frac{1}{3} \delta^\alpha_\beta \Delta \mu, \quad (3) \]

where \( \nabla^\alpha = \frac{\partial}{\partial x^\alpha} \), and \( \Delta \) is the Laplacian.

The convolution of Eqs. (1), linearized with respect to \( h^\alpha_\beta \), yields a second order equation for perturbations of the curvature scalar: \( \delta R \):

\[ \delta R'' + 2 \frac{a'}{a} \delta R' - \Delta \delta R - M^2 a^2 \delta R = 0, \quad (4) \]

where the prime denotes differentiation with respect to \( \eta \), and \( a(\eta) = -1/H\eta \).

Also using Eqs. (1), we can show that all the quantities in which we are interested (in particular, the \( h^\alpha_\beta \)), can be expressed in terms of \( \delta R \). This quantity plays a special role because of its invariance with respect to so called fictitious perturbations \[ \] which stem from transformations of the coordinate system which does not disrupt the synchronism. Fictitious perturbations of scalar functions result from the change in the origin for the time scale \[ \] since \( R \) is independent of the time \( (R = \text{const}) \) in the de Sitter model, with which we are concerned here, we have \( \delta R = 0 \) for these fictitious modes.

Adopting a perturbation of the curvature scalar as a physical variable, we find the corresponding action in the form \[ \]

\[ \delta S_b = \frac{1}{2} \int d^4 x \left[ \phi'^2 - \nabla^\alpha \phi \nabla_\alpha \phi + \left( \frac{a''}{a} + M^2 a^2 \right) \phi^2 \right], \quad (5) \]

where \( \phi = 1/\sqrt{18(4H^2 - M^2)} \ a \delta R/M\ell \), and \( \ell = (8\pi G/3)^{1/2} = 4.37 \times 10^{-33} \text{ cm} \) is the Planck length.

In quantizing we should note that the physical system under consideration (a polarized vacuum in a gravitational field) has a finite energy density, which must undergo fluctuations according to the uncertainty principle. These fluctuations are zero-point oscillations of the field of collective excitations of the ordinary physical fields (“scalarons” with mass \( M \)).
The canonical quantization is carried out by a procedure similar to that of Ref. [7]. As a result we can calculate various correlation functions, e.g.,

\[
\langle 0 \left| \delta \hat{R}(x) \delta \hat{R}(x + r) \right| 0 \rangle = \frac{1}{2\pi^2} \int P^2(k) \frac{\sin kr}{kr} \frac{dk}{k},
\]

where \( k \) and \( r \) are the physical wave vector and the physical length, expressed in units of reciprocal centimeters and centimeters, respectively. The spectrum \( P(k) \) is expressed in terms of Bessel functions of index \( 3\nu/2 = (3/2)\sqrt{1 + (4/9)M^2/H^2} \) and argument \( k/H \), which are the exact solutions of Eq. (11). The spectrum \( P(k) \) is shown in Fig.1. A distinguishing feature of this spectrum is a maximum at \( k \sim H(\eta/\eta_0) \) (\( \eta_0 \) is the initial time at which the vacuum of the scalaron field is given); the perturbation amplitude at this maximum increases over time. Over a finite time, which in the most interesting case \( (M^2 \ll H^2) \) is

\[
\delta t_f = \frac{3}{2H} \frac{H^2}{M^2} \ln \left( \frac{1}{2(\ell M)^2} \right),
\]

the amplitude of the curvature perturbations at the maximum reaches characteristic of the original background model, and the universe enters a stage of Friedmann expansion with ordinary hydrodynamic matter as a result of multiple production of scalarons with finite wave numbers (primarily with \( k = H(\eta/\eta_0) \)). Thus we see that the quantum fluctuations, which are necessary present in the system, cause the universe to spend a finite time in the de Sitter state and thus cast doubt on the possibility of a nonsingular beginning of the universe. Regardless of the nature of singularity (classical or quantum), we believe that this fact significantly detracts from the esthetic value of the original model.

A finite duration of the de Sitter stage does not by itself rule out the possibility that this stage may exist as an intermediate stage in the evolution of the universe. An interesting question arises here: Might not perturbations of the metric, which would be sufficient for the formation of galaxies and galactic clusters, arise in this stage? To answer this question, we need to calculate the correlation function for the fluctuations of the metric after the universe goes from the de Sitter stage to the hydrodynamic stage. By analogy with (8) we find

\[
\langle 0 \left| \hat{h}(x) \hat{h}(x + r) \right| 0 \rangle = \frac{1}{2\pi^2} \int Q^2(k) \frac{\sin kr}{kr} \frac{dk}{k},
\]

where \( h = h_\alpha^\beta \) and where, for the most interesting region, \( H > k > H \exp(-3H^2/M^2) \) \( (M^2 \ll H^2) \),

\[
Q(k) \approx 3\ell M \left( 1 + \frac{1}{2} \ln \frac{H}{k} \right) \cdot
\]

The fluctuation spectrum is thus nearly flat. The quantity \( Q(k) \) is the measure of the amplitude of perturbations with scale dimensions \( 1/k \) at the time the universe begins the ordinary Friedmann expansion. With \( \ell M \sim 10^{-3} - 10^{-5} \) and \( M/H \leq 0.1 \)—these values are consistent with modern theories of elementary particles—the amplitude of the perturbations of the metric on the
Figure 1: Spectrum of relative curvature fluctuations, $P(k)/R^{(0)}$, plotted as a function of the wave number $k$. Dashed lines - spectrum of vacuum fluctuations which is specified at some initial time $\eta_0$; solid line - the spectrum into which the vacuum spectrum transforms at some later time.

scale of galactic clusters turns out to be equal to $10^{-3} - 10^{-5}$, and these perturbations can lead to the observed large scale structure of the universe. The form of spectrum (9) is completely consistent with modern theories for the formation of galaxies [5].

To summarize: Using a de Sitter model as an example, we have shown that quantum fluctuations (zero-point vibrations) cause the universe to spend a finite time in a state with polarized vacuum. This result casts doubt on the possibility of a nonsingular origin for the universe. However, models in which the de Sitter stage exists only as an intermediate stage in the evolution are attractive because fluctuations of the metric sufficient for the galaxy formation can occur. Thus we have one possible approach for solving the problem of the appearance of the original perturbation spectrum.

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