The Evolution and Fate of Super-Chandrasekhar Mass White Dwarf Merger Remnants

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ABSTRACT

We present stellar evolution calculations of the remnant of the merger of two carbon-oxygen white dwarfs (CO WDs). We focus on cases that have a total mass in excess of the Chandrasekhar mass. After the merger, the remnant manifests as an $L\sim3\times10^4 L_\odot$ source for $\sim10^4$ yr. A dusty wind may develop, leading these sources to be self-obscured and to appear similar to extreme AGB stars. Roughly $\sim10$ such objects should exist in the Milky Way and M31 at any time. As found in previous work, off-center carbon fusion is ignited within the merger remnant and propagates inward via a carbon flame, converting the WD to an oxygen-neon (ONe) composition. By following the evolution for longer than previous calculations, we demonstrate that after carbon-burning reaches the center, neutrino-cooled Kelvin-Helmholtz contraction leads to off-center neon ignition in remnants with masses $\geq 1.35 M_\odot$. The resulting neon-oxygen flame converts the core to a silicon WD. Thus, super-Chandrasekhar WD merger remnants do not undergo electron-capture induced collapse as traditionally assumed. Instead, if the remnant mass remains above the Chandrasekhar mass, we expect that it will form a low-mass iron core and collapse to form a neutron star. Remnants that lose sufficient mass will end up as massive, isolated ONe or Si WDs.

Key words: white dwarfs – supernovae: general – neutron stars

1 INTRODUCTION

In double WD systems with sufficiently short initial orbital periods, angular momentum losses from gravitational wave radiation shrink the orbit and can lead to a merger of the WDs. The outcome of such a merger is strongly dependent on the mass of the individual WDs and their mass ratio. For recent summaries of the many possible outcomes, see fig. 1 in Dan et al. (2014) or fig. 3 in Shen (2015).

Iben & Tutukov (1984) and Webbink (1984) proposed that mergers of two carbon-oxygen (CO) WDs whose total mass was in excess of the Chandrasekhar mass would lead to the central ignition of carbon fusion and hence to a Type Ia supernova. It was quickly pointed out by Saio & Nomoto (1985) and by Nomoto & Iben (1985) that the rapid mass transfer in such an event would lead to the off-center ignition of carbon, triggering the quiescent conversion of the remnant to an oxygen-neon (ONe) composition. Standard models conclude that subsequently, electron-capture reactions cause the ONe core to collapse and form a neutron star (Miyaji et al. 1980; Schwab et al. 2015).

Saio & Nomoto (1985) and Nomoto & Iben (1985) approximated the effects of the WD merger as the accretion of material on to the more massive WD. They studied constant accretion rates $\dot{M} \lesssim 10^{-5} M_\odot$ yr$^{-1}$, motivated by the assumption that the material is accreting from a disc at a rate bounded by the Eddington limit. Since this early work, smoothed-particle hydrodynamics (SPH) simulations (e.g., Benz et al. 1990; Dan et al. 2011) of double WD systems have been used to investigate the dynamics of the merger. Schematically, the primary (more massive) WD remains relatively undisturbed and the secondary (less massive) WD is tidally disrupted. Some material from the disrupted WD is shock-heated, forming a thermally-supported layer at the surface of the primary WD; the rest of the material is rotationally-supported, forming a thick disc at larger radii.$^1$

$^1$ Exceptions to this picture include WD collisions (e.g., Raskin et al. 2009) or cases where the mass ratio is nearly unity (e.g., van Kerkwijk et al. 2010).
Yoon et al. (2007) took an important step forward by constructing 1D models that mimicked the results of SPH simulations of WD mergers. They then used a hydrodynamic stellar evolution code to evolve these models, allowing them to follow the secular evolution of the remnants. However, Shen et al. (2012) and Schwab et al. (2012) studied the post-merger evolution of these systems in more detail and showed that the transport of angular momentum via magnetic stresses occurs on a time-scale (∼ 10^4 s) far shorter than the time-scale on which the remnant cools (∼ 10^7 yr). Material initially in the disc quickly became thermally supported and spherical. In the models of Yoon et al. (2007), who neglected the effects of magnetic fields, the time-scale for angular momentum redistribution was often comparable to or longer than the cooling time. The disc was assumed to be long-lived and modeled as a low accretion rate on to the central remnant. One of the key conclusions of Shen et al. (2012) and Schwab et al. (2012) was that the rapid viscous evolution should be taken into account before exploring the long-term thermal evolution of the merger remnant.

In this work, we follow the long-term evolution of the merger of two CO WDs. The initial conditions for these calculations are generated self-consistently, beginning from SPH simulations of the dynamical merger of the two WDs (Dan et al. 2011; Raskin et al. 2014) and explicitly modeling the subsequent phase of viscous evolution as in Schwab et al. (2012). We then use the MESA stellar evolution code ( Paxton et al. 2011, 2013, 2015) to follow the evolution of these merger remnants over thermal and nuclear time-scales.

In Section 2 we discuss our initial models along with the key options and input physics that enable our MESA calculations. In Section 3 we discuss the ignition and propagation of a carbon flame. In Section 4 we discuss the ignition of a neon-oxygen flame and then in Section 5 outline how subsequent evolution may lead to the formation of a neutron star. In Section 6 we discuss the observational properties of the remnant. In Section 7 we conclude, presenting a schematic overview of our results and suggesting avenues for future work. Fig. 12 summarizes the possible end states of super-Chandrasekhar WD mergers.

### 2 SETUP OF MESA CALCULATIONS

#### 2.1 Initial Models

As discussed in the introduction, SPH simulations of WD mergers show that at the end of the dynamical phase of the merger, the primary white dwarf remains mostly undisturbed. It is now surrounded by the tidally disrupted secondary, which includes a significant amount of material in a rotationally-supported disc. In Schwab et al. (2012), we took the output of SPH merger calculations, mapped them into a grid-based hydrodynamics code (ZEUS-MP2; Hayes et al. 2006), and followed the evolution of these remnants under the action of an α-viscosity. We found that the viscous stresses transformed the disc material into a spherical, thermally-supported envelope on a time-scale of hours.

In this work, we focus on two super-Chandrasekhar WD merger remnants, one with a total mass of approximately 1.5 M⊙ (model M15) and another with total mass of approximately 1.6 M⊙ (model M16). Both remnants are the result of a merger with mass ratio q = 2/3 and have a composition of 50 per cent carbon and 50 per cent oxygen by mass. Table 1 contains a summary of these models, including references to the papers in which the simulations of the dynamical and viscous phases were first reported.

The initial conditions for the thermal evolution calculations in this work are the end states of our viscous evolution calculations. In Appendix A, we discuss the details of how we transfer the output of our hydrodynamic simulations into MESA. In Fig. 1, we show the density-temperature profiles of our initial MESA models. We define the burning time as \( t_{\text{burn}} = c_p T / \epsilon_{\text{nuc}} \), where \( c_p \) is the specific heat at constant pressure, \( T \) is the temperature and \( \epsilon_{\text{nuc}} \) is the specific energy generation rate from nuclear reactions. At the temperature peaks of both models, \( t_{\text{burn}} > 10^4 \) s, the approximate duration of the viscous evolution. This demonstrates that substantial nuclear energy was not liberated during the viscous phase.

#### 2.2 MESA Version and Important Options

The calculations performed in this paper use MESA version r6596 (released 2014-06-08). As required by the MESA manifesto, the inlists necessary to reproduce our calculations will be posted on http://mesastar.org.

We use approx21.net, a 21-isotope α-chain nuclear network. In order to avoid spurious flashes, it is important to resolve the burning fronts in the convectively-bounded deflagrations that develop in our models (e.g., Saio & Nomoto 1998). We use the options selected by Farmer et al. (2015) in their study of carbon flames in super-AGB stars. In particular, we use the controls

\[
\text{mesh_dlog_burn_c_dlogP_extra} = 0.10 \\
\text{mesh_dlog_cc_dlogP_extra} = 0.10 \\
\text{mesh_dlog_co_dlogP_extra} = 0.10
\]

which add additional spatial resolution in regions where carbon burning is occurring. This ensures that that flame is well-resolved. We use analogous controls to ensure that the neon-oxygen-burning flames in our models are also well-resolved.

| ID  | SPH Ref. | M2  | M1  | Viscous Ref. | M_{\text{tot}} |
|-----|----------|-----|-----|--------------|---------------|
| M15 | Dan11    | 0.60| 0.90| Schwab12     | 1.486         |
| M16 | Raskin14 | 0.64| 0.96| Raskin14     | 1.586         |

Table 1. A summary of the two merger systems studied in this work. The ID reflects the total mass of the system. “SPH ref.” indicates the primary reference containing the details of the SPH calculation of the merger (Dan11: Dan et al. 2011, Raskin14: Raskin et al. 2014). \( M_2 \) is the mass (in M⊙) of the secondary, the less massive of the two WDs; \( M_1 \) is the mass of the more massive primary. “Viscous Ref.” refers to the primary reference containing the details of the subsequent viscous evolution (Schwab12: Schwab et al. 2012, Raskin14: Raskin et al. 2014). \( M_{\text{tot}} \) is the total mass of the bound remnant (in M⊙) at the end of the viscous phase simulation, and hence the initial mass of the MESA model.
3 CARBON FLAME

In each of the initial models shown in Fig. 1, the rate of energy release from nuclear reactions exceeds the rate of energy loss from thermal neutrino cooling at the initial temperature peak. As described in Schwab et al. (2012), this is because heating by viscous dissipation leads to off-center ignition of self-sustaining carbon fusion within hours of the merger. When we begin our MESA calculation, there is thus immediately off-center carbon burning in the remnant. The energy release in this carbon-burning shell quickly leads to the formation of a convection zone. Heat from the burning region is conducted into the degenerate interior, giving rise to a deflagration wave that begins propagate towards the center of the remnant. We refer to this deflagration as the “carbon flame”. Fig. 2 shows the evolution of this flame in our MESA calculations of model M15. After an initial transient phase with a duration of $\lesssim 100$ years ($t = 0$ to time 1), the deflagration forms and propagates to the center over $\approx 20$ kyr (time 1 to time 4). Because the structure of the 1.6 M$_{\odot}$ model is so similar to that of the 1.5 M$_{\odot}$ model (see Fig. 1) its evolution and outcomes are similar; for simplicity, we will primarily discuss model M15.

It is important to note that the carbon flame, while off-center, is still deep in the interior. The convective zone outside the burning region satisfies a “balanced power” condition, where the total luminosity of thermal neutrino emission from the zone is approximately equal to the rate of energy release from fusion at its base (Timmes et al. 1994). This neutrino-cooled convective zone has a radial extent of order the pressure scale height, and its upper boundary is sufficiently deep that the time-scale for radiative diffusion to transport the energy to the surface is longer than the evolutionary time-scale of the remnant. Thus, while key to the final fate of the remnant, the energy release of the carbon flame is not coupled to the surface, and does not power the

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2 Here, $dX_C$ and $dX_O$ indicate the mass fraction enhancements above the (relatively small) amount of carbon and oxygen already included in $Z$. Also, the stellar evolution preceding the formation of the individual WDs would have modified some of the abundances contained in the standard solar $Z$ (e.g., $^{14}$N would have been processed to $^{22}$Ne during helium burning) but we neglect those changes.

3 This is in contrast to calculations that model the merger as Eddington-limited accretion, in which carbon ignition does not occur for $\gtrsim 10^4$ yr, until sufficient material has accreted to adiabatically compress the base of the shell to higher temperatures. In a lower mass merger, say $M_{\text{mer}} \approx 1.4 M_{\odot}$, we find that carbon fusion does not ignite during the viscous evolution. In this case, there is a similar time delay, as carbon ignition must wait for the cooling envelope to compress material at its base. An example of such evolution is shown in Shen et al. (2012).
The propagation of the carbon flame in model M15 with no convective boundary mixing. The x-axis shows time, as measured from the beginning of the MESA calculation; effectively this is the time since merger. The y-axis shows the Lagrangian mass coordinate. The extent of the convective region associated with the flame is shaded. The location of maximum nuclear energy generation corresponds to the location of maximum temperature. The thickness of the flame is \( \sim 10^7 \) cm. This profile from our MESA calculation is from a time slightly before time 2, as marked in Fig. 2.

Figure 3. Structure of a carbon-burning flame. The shaded grey region marks the convection zone. The temperature \( (T/10^9 \text{ K}) \), density \( (\rho/10^8 \text{ g cm}^{-3}) \), energy generation rate \( (\epsilon_{\text{nuc}}/10^7 \text{ erg s}^{-1} \text{ g}^{-1}) \), and \(^{12}\text{C} \) mass fraction are shown as function of radius. The thickness of the flame is \( \sim 10^7 \) cm. This profile from our MESA calculation is from a time slightly before time 2, as marked in Fig. 2.

Figure 4. The evolution of the core of model M15 in temperature-density space as the carbon flame propagates inward. Each line corresponds to one of the times indicated in Fig. 2; time 3 is omitted because it appears extremely similar to time 2. The location of the flame corresponds to the location of maximum temperature in each line. The dashed grey line marks the degeneracy condition (above/left is non-degenerate; below/right is degenerate). Early in the evolution (time 1) the center has \( \log(\rho/\text{g cm}^{-3}) > 7 \) and is degenerate. As the flame propagates inward, it lifts the degeneracy of additional material, and the central density decreases (time 2). Once the flame reaches the center (time 4), it has lifted the degeneracy throughout the star.
4 KELVIN-HELMHOLTZ CONTRACTION AND NEON IGNITION

Once the carbon flame reaches the center, it has lifted the degeneracy of the material throughout the star. This is illustrated in Fig. 4, where the dash-dot line (labeled time 4) shows the temperature-density profile of the model at the time of central carbon exhaustion. The newly non-degenerate core will now Kelvin-Helmholtz (KH) contract. The core is sufficiently hot and dense that it cools through thermal neutrino losses. As a result, it will develop an off-center temperature peak. We note that the evolution of the central density and temperature is similar to that seen in models of intermediate mass stars ($\approx 8 - 10 \, M_\odot$), as these objects also develop degenerate cores with similar masses (e.g., Jones et al. 2013, 2014).

As discussed by Nomoto (1984), there is a critical core mass for off-center neon ignition. Nomoto demonstrated this by means of a simple calculation in which stellar models of pure neon were constructed and allowed to contract. Neon ignition occurred only in models with a mass above 1.37$M_\odot$. We repeat this calculation and find a slightly lower critical mass of 1.35$M_\odot$ for pure $^{20}$Ne models (see Appendix C). We use the results of these pure neon models to guide our interpretation of the central evolution of the WD merger remnants.

Fig. 5 shows a Kippenhahn diagram of model M15 from the time the carbon flame reaches the center until neon ignition. Initially carbon is being burned in the core, but over the first $\approx 1$ kyr, the central convection zone shrinks and vanishes as the carbon in the center is exhausted. As the star KH contracts, a series of off-center carbon flashes occur. Additionally, an off-center temperature peak develops. Its mass coordinate is indicated by the black dotted line.

Fig. 6 shows the evolution of the temperature and density at both the center (solid black line) and the off-center temperature peak (dashed black line) in model M15 during the KH contraction phase. The “wiggles” in the evolution of the center are manifestations of changes to the stellar structure due to the off-center carbon flashes shown in Fig. 5. The grey lines show the analogous temperature and density evolution of a 1.385$M_\odot$ pure neon model. The qualitative agreement between the two models is good.

The agreement between our full remnant models and our simple pure neon calculations demonstrates that the off-center neon ignition is a simple consequence of the mass of the remnant. Once ignited, the neon burning will propagate to the center in a manner similar to the carbon burning, converting the object to silicon-group elements. (See Section 5 for more discussion of this process and the subsequent evolution.) Because the critical mass for neon ignition (1.35$M_\odot$) is less than the critical mass needed to trigger the collapse of an ONe core (1.38$M_\odot$; Schwab et al. 2015), we conclude that is is difficult to produce an ONe core with a sufficient mass to undergo AIC in a WD merger.

Off-center neon ignition does not appear to have been observed in previous studies of CO WD merger remnants. The salient difference between this work and previous work appears to be that we evolved the remnants for longer. Saio & Nomoto (1985) halted their calculation when the carbon flame was at a mass coordinate of $M_\odot = 0.005$M$_\odot$ because it had become too computationally costly to continue. Later work by the same authors (Saio & Nomoto 1998) allowed the flame to reach the center, but did not continue the evolution beyond this point. The work by Yoon et al. (2007), which this work is most similar to, focused on avoiding off-center carbon ignition; in cases where off-center carbon ignition did occur, the authors did not continue their evolutionary calculations.

As an illustration of the effects of mass loss on the core, we run a version of model M15 with a mass loss rates drawn from Blocker (1995). That is, we use the MESA options

\begin{verbatim}
AGB_wind_scheme = 'Blocker'
Blocker_wind_eta = 0.1d0
\end{verbatim}

The Blocker (1995) mass loss rates were motivated by atmospheric calculations of Mira-like stars and thus they are not directly applicable to this problem. However, we simply want to remove some mass when the object is cool and luminous, and so this is a suitable heuristic. We discuss mass loss in more detail in Section 6.

Fig. 7 shows the same quantities as Fig. 6, but for a model in which the remnant shed approximately 0.2M$_\odot$ of material during the phase while the carbon flame was propagating to the center. As the object contracts, an off-center temperature peak develops, but it fails to reach temperatures where neon burning exceeds thermal neutrino losses. Instead, the core becomes supported by electron degeneracy pressure, halting the KH contraction. As a result, the peak temperature reaches a maximum and then begins to
385 M⊙, and now has a total mass of ≈ 0.6 M⊙. For visual clarity, the line for the temperature peak is shown only after the central density exceeds log(ρ/g cm−3) ≈ 7.3; the first plotted point of the dashed (temperature peak) line is marked by a solid black circle and the contemporaneous point on the solid (center) line is marked by an open circle. The grey lines show the evolution of a contracting 1.385 M⊙ pure neon model; the solid grey line shows conditions at the center and the dashed grey line shows conditions at the temperature peak, which due to neutrino cooling, develops off-center. The black dotted line shows approximately where the energy release from neon burning is equal to the energy loss rate from thermal neutrinos. Off-center neon ignition occurs in this model.

Figure 6. The evolution of temperature and density at the center of model M15 (solid black line) and at the temperature peak (dashed black line) during the KH contraction shown in Fig. 5. The total mass of the remnant is ≈ 1.5 M⊙. For visual clarity, the line for the temperature peak is shown only after the central density exceeds log(ρ/g cm−3) ≈ 7.3; the first plotted point of the dashed (temperature peak) line is marked by a solid black circle and the contemporaneous point on the solid (center) line is marked by an open circle. The grey lines show the evolution of a contracting 1.385 M⊙ pure neon model; the solid grey line shows conditions at the center and the dashed grey line shows conditions at the temperature peak, which due to neutrino cooling, develops off-center. The black dotted line shows approximately where the energy release from neon burning is equal to the energy loss rate from thermal neutrinos. Off-center neon ignition occurs in this model.

5 NEON-OXYGEN FLAME AND SUBSEQUENT EVOLUTION

The phase beginning with off-center neon ignition is relatively unexplored. Although, we are unable to self-consistently evolve the remnants all the way to their final fate (as discussed in Section 5.2), the likely outcome for sufficiently massive remnants is the formation of a neutron star via an iron core collapse. Analogous phases of evolution in single intermediate mass stars (≈ 8 – 10 M⊙; Jones et al. 2014; Woosley & Heger 2015) and in ultra-stripped binary systems (i.e., ones that form helium cores 2.5-3.5 M⊙; Tauris et al. 2013, 2015) continue to be active areas of research. Future results in these areas can be applied to the evolution of super-Chandrasekhar mass WD merger remnants.

5.1 Neon-Oxygen Flame

In model M15, off-center neon ignition occurs at a mass coordinate Mr ≈ 0.6 M⊙. This quickly forms a convectively-bounded neon-oxygen-burning deflagration front which begins to propagate inwards towards the center of the star. Fig. 8 shows the location of the flame and its accompanying convection zone during this phase. In our MESA calculations, we assume that there is no mixing (i.e., convective overshoot or thermohaline mixing) beyond the convective boundary.

As can be shown from the analytic estimates given in Timmes et al. (1994) and as discussed in Woosley & Heger (2015), neon-oxygen-burning flames are much faster (though still extremely subsonic) and thinner than carbon-burning flames. This is due to the higher temperatures and energy generation rates associated with neon and oxygen fusion. Neon-burning consists of photo-disintegration 20Ne + γ → 16O + α followed by α-captures, with the net result being the rearrangement 20Ne → 16O + 24Mg (Woosley et al. 2002). Subsequently, additional α-captures and oxygen-burning produce silicon-group elements (here, 28Si and 26S). In our calculations, the oxygen-burning occurs not in the flame itself, but in the hottest parts of the bounding convection zone. Fig. 9 shows the structure of the neon-oxygen-burning flame in our MESA model; note that the flame thickness is ≈ 103 cm and the flame velocity is ≈ 0.1 cm s−1.

In our MESA calculation we are directly resolving the flame, which is extremely computationally inefficient. Future work will benefit from a sub-grid model such as that used in Woosley & Heger (2015), where the flame is not resolved in the full-star simulation, but tabulated velocities from resolved, micro-zoned flame calculations are used to propagate a model for the flame.

5.2 Silicon Burning and Core Collapse

As neon-oxygen-burning migrates to the center, it leaves behind hot silicon-group ashes. Unlike the carbon flame, the neon-oxygen flame does not fully lift the degeneracy of the...
material; in model M15, \( \eta_{\text{center}} \equiv E_F/(k_B T) \approx 7 \) immediately after the neon-oxygen flame propogates to the center. Still, some KH contraction occurs, leading to a series of off-center flashes that consume the remaining oxygen and neon, converting almost the entire remnant to silicon-group elements. (Fig. 10 summarizes the evolution of the composition of the remnant.) In Appendix C we find a critical mass for off-center silicon ignition of \( \approx 1.41 M_\odot \). This suggests that in a narrow range of WD mergers (remnants with masses \( 1.35 M_\odot \lesssim M \lesssim 1.41 M_\odot \), accounting for mass loss), the final merger remnant could be a WD with a silicon-group composition.

Off-center silicon ignition does occur in model M15, in accordance with our expectation given its mass, at a mass coordinate of \( M_c \approx 0.75 M_\odot \); this occurs about 10 years after the neon-oxygen flame reaches the center. The conditions at this location (at the time shown in the bottom panel of Fig. 10) are \( \rho = 2.7 \times 10^8 \text{ g cm}^{-3} \) and \( T = 3.6 \times 10^8 \text{ K} \). Recently, Woosley & Heger (2015) discussed the presence of Si-flashes in the evolution of 9–11 M_\odot stars. They found silicon deflagrations beginning in models with CO core masses of \( \approx 1.4 M_\odot \), with increasingly intense silicon-burning flashes as the mass increased. Model M15 ignites silicon further off-center than any of the models described in Woosley & Heger (2015), so it is difficult to associate model M15 with a specific model in that work. However, it does appear that we may be in a regime where unstable Si-burning plays a role. Future work will clarify its importance.

Our calculations are not well-suited to study silicon-burning, so we leave this to future work. In particular, we are limited by our use of an \( \alpha \)-chain network. For example, this small network does not include electron-capture reactions that will have occurred during the pre-silicon-burning evolution, reducing the electron fraction and increasing the abundance of the more neutron-rich isotopes \( ^{30}\text{Si} \) and \( ^{34}\text{Si} \). For now, we can envision that silicon-burning in remnants with \( M \gtrsim 1.41 M_\odot \) (accounting for mass loss) quiescently leads to the formation of an Fe-core and that subsequently this low-mass Fe core will collapse to form a neutron star. Thus, while we have revised the evolutionary story for super-Chandrasekhar WD mergers, we think they are still likely to produce a population of single, low-mass neutron stars. More quantitative details about such neutron stars, including the identification of possible distinguishing characteristics, await models that have been evolved up to core infall and through core-collapse.
6 OBSERVATIONAL PROPERTIES OF THE MERGER REMNANT

In this section we describe the observational properties of super-Chandra WD merger remnants, focusing on the time between the merger and the final collapse to form a neutron star. As described in Sections 3 and 5.1, the energy released by fusion during the carbon and neon-oxygen flames does not reach the surface. It is instead lost primarily to thermal neutrino cooling deep in the stellar interior, at sufficiently high optical depths that the existence of the flame does not modify the observational properties of the WD merger remnant. The latter are instead governed by the heat released during the merger.

The outer envelope of the remnant responds to the energy deposited during the merger and begins to radiate away this energy. Fig. 11 shows the location of model M15 in the HR diagram during our MESA calculation, over the phase in which the carbon flame is propagating to the center. The remnant radiates at the Eddington luminosity for a solar mass object with an effective temperature \( \approx 4000 - 5000 \) K for \( \approx 5 \) kyr, and then evolves to the blue, spending \( \approx 10 \) kyr with \( T_{\text{eff}} \gtrsim 10^5 \) K. The track shown in Fig. 11 for the remnant resembles that of a star evolving from the AGB to the planetary nebula stage (e.g., Kwok 1993). In some ways, the merger has formed an object similar to the core of an intermediate mass star. However, the lack of hydrogen- and helium-burning shells means that thermal pulses will be absent. Near the end of the AGB phase, stars are seen to exhibit extreme (\( M \gtrsim 10^{-4} \, M_\odot \, \text{yr}^{-1} \)) mass loss rates (e.g., van Loon et al. 1999). Winds from these cool, luminous stars are thought to be launched by a two stage process in which pulsation-driven shock waves lead to dust formation and then radiation pressure on the dust accelerates the wind (e.g., Höfner 2015). The pulsations in AGB stars are driven by H and/or He ionization (Wood 1979; Ostlie & Cox 1986); such pulsations may not be present in pure carbon-oxygen envelopes. The structural and compositional differences between extreme AGB stars and these remnants mean that proximity in the
HR diagram does not itself directly imply analogous mass loss properties.

The track shown in Fig. 11 does not include the effects of mass loss. The inclusion of mass loss has two primary effects: (1) it alters the observational properties of the remnant, likely obscuring it in a dusty wind and (2) it reduces the mass of the remnant, potentially influencing the final outcome. In particular, with \( \dot{M} \approx 10^{-4} M_\odot \text{yr}^{-1} \), over the duration of the cool giant phase, the remnant would shed \( \Delta M \approx 0.1 M_\odot \), sufficient to change a model with total mass \( \approx 1.5 M_\odot \) from super-to-sub Chandrasekhar (see Section 4).

However, we note that our model M16 displays similar evolution as M15, but with its higher total mass \( \approx 1.6 M_\odot \), it can shed \( \Delta M \approx 0.1 M_\odot \), and remain super-Chandrasekhar.

In Fig. 11 the lowest effective temperature \( (T_{\text{eff}} \approx 4000 \text{ – } 5000 \text{ K}) \) reflects the steep decline in the opacities at these temperatures—see Section 2.3 and Appendix B.\(^5\)

The R Coronae Borealis stars are giants with similar effective temperatures \( (4000 \text{ – } 7000 \text{ K}) \) with He-dominated, C-enhanced atmospheres (e.g., Clayton 1996). These objects exhibit high-amplitude dimming events \( (\approx 10 \text{ mag in the optical}) \) which are understood to be the result of dust formation events outside the photosphere. It is possible that the super-Chandrasekhar WD merger remnants would exhibit similar variability.

Given their cool photospheres between times 1 and 2 in Fig. 11 and their almost pure carbon and oxygen composition, it seems likely that these remnants would form copious amounts of dust. The mass loss rate is then set by the dust formation rate near the stellar photosphere. Reprocessing by this dust makes obscured objects bright infrared (IR) sources, though because the remnants are hydrogen-free, such an object should not exhibit any OH-maser emission, as seen in the OH/IR stars (Wilson & Barrett 1968).

The properties of the dust also depends on the C/O ratio (the number ratio of carbon and oxygen atoms) and as such, the remnants could manifest either as C-rich or O-rich objects. We expect that the surface C/O ratio will be set largely by the C/O ratio of the disrupted WD. This quantity is likely not universal—depending on the mass of the disrupted WD, for example.

The C/O ratio predicted by stellar models depends on still-uncertain input physics such as the \( ^{12}\text{C}(\alpha, \gamma)\) \(^{16}\text{O} \) reaction rate. The recent study of Fields et al. (2016) explored the effects of this and other reaction rate uncertainties using STARLIB (Sallaska et al. 2013). For WDs in the mass range \( 0.55 - 0.65 M_\odot \), their models using the “median” reaction rates had total C/O ratios in the range \( \approx 0.8 - 1.1 \), with this interval expanding to \( \approx 0.7 - 1.3 \) when varying the reaction rates between their “low” and “high” values (C. E. Fields, private communication). Asteroseismic observations of pulsating WDs can provide observational constraints on the core abundances. Unfortunately, asteroseismic fits to grids of stellar evolution models (e.g., Romero et al. 2012) have compositions that are “fixed” by the choice of a single set of standard nuclear reaction rates. Fits using parameterized composition profiles, such as Bischoff-Kim et al. (2014), do not suffer from this limitation. They report core oxygen mass fractions of a 0.55 \( M_\odot \) WD. Their two best fit models (fitting to a grid with a step-size \( \Delta X_O = 0.05 \) have \( X_O = 0.55 \) (which is C/O \( \approx 1.1 \)) and \( X_O = 0.70 \) (which is C/O \( \approx 0.6 \)).\(^6\)

We conclude that it is not yet theoretically or observationally clear whether we expect the material from the disrupted WD to be C-rich or O-rich.

If these objects are obscured by a dusty wind, then they may appear in the same luminosity-color cuts used to identify extreme AGB stars (see e.g., Thompson et al. 2009; approximately 9 of their sources are coincident with the luminosities of our objects). In the wind, \( \tau(\nu) \approx M_\kappa/(4\pi\nu \kappa) \), where \( \tau \) is the optical depth, \( \kappa \) is the opacity, and \( \nu \) is the wind velocity. Since \( \tau(\nu) \approx 1 \),

\[
\tau_{\text{ph}} \sim \frac{M_\kappa}{4\pi\nu} \approx 1.5 \times 10^{15} \text{cm} \left( \frac{\dot{M}}{10^{-10} M_\odot \text{yr}^{-1}} \right) \left( \frac{\kappa}{10 \text{ cm}^2 \text{g}^{-1}} \right) \left( \frac{\nu}{30 \text{ km s}^{-1}} \right) \tag{1}
\]

Given the luminosity of our sources, this radius would imply an effective temperature

\[
T_{\text{eff}} = 500 \text{K} \left( \frac{L}{10^{34} L_\odot} \right)^{1/4} \left( \frac{\tau_{\text{ph}}}{100 \text{AU}} \right)^{-1/2} \tag{2}
\]

which is consistent with extreme AGB stars.

We can make a rough estimate of the number of merger remnants in this cool giant or self-obscured phase at a given time. The specific rate of super-Chandrasekhar double WD mergers estimated by Badenes & Maoz (2012) is \( 1.6 \times 10^{-14} \text{yr}^{-1} M_\odot^{-1} \). This implies that the number of sources active in a galaxy is

\[
N_{\text{active}} \approx 1 \times \left( \frac{\text{merger rate}}{10^{-14} \text{yr}^{-1} M_\odot^{-1}} \right) \left( \frac{\text{stellar mass}}{10^{10} M_\odot} \right) \left( \frac{\text{lifetime}}{10^4 \text{yr}} \right) \tag{3}
\]

meaning that roughly one of these remnants will be currently active in M33 and 20 in M31.\(^7\) However, we have no reason to expect sub- and super-Chandrasekhar models to be observationally distinguishable during this phase.\(^8\) Thus, the number of objects in this phase may be a factor of few higher, since sub-Chandrasekhar total mass mergers include lower mass CO WDs closer to the peak of the individual WD mass distribution (Kepler et al. 2007).

Around time 2, in Fig. 11, we expect the extreme mass loss to end and the dusty envelope to detach from the photosphere. The evolution from time 2 to time 3, taking \( \approx 1 \text{ kyr} \), is similar to a proto-planetary nebula phase (e.g., Kwok 2003).

\(^\text{5}\) Our MESA models with cool photospheres also have density inversions in the outer layers (these are visible in Fig. B2). In such cases, the structure of the 3D models may be different than the solution realized in nature, where the energy transport is influenced by inherently multi-dimensional effects (Joss et al. 1973; Jiang et al. 2015). This introduces an additional uncertainty in our models during this phase.

\(^\text{6}\) Note that treating these core values as estimates of the total fractions overestimates the total amount of oxygen, as the models transition from an O-dominated interior to a C-dominated exterior.

\(^\text{7}\) The stellar mass of M33 is \( M_* \approx 3 - 6 \times 10^9 M_\odot \) (Corbelli 2003); the stellar mass of M31 is \( M_* \approx 10 - 15 \times 10^9 M_\odot \) (Tamm et al. 2012).

\(^\text{8}\) A lower total mass merger will not yet have ignited carbon, but recall that the luminous giant phase is powered by thermal energy release during the merger, not carbon burning.
This transitions into a planetary nebula phase as the temperature of the central object rises to $T_{\text{eff}} \gtrsim 10^5$ K, making it a bright extreme ultraviolet/soft X-ray source. This emission will begin to ionize material shed during the earlier phase, forming an ionization nebula. An evolutionary timescale of $\sim 10^5$ yr suggests a size $\lesssim 0.1$ pc for this nebula. The ionized material, which could have a mass $\approx 0.1 \, M_\odot$, will be composed primarily of carbon and oxygen and should be hydrogen and helium free. Eq. (3) suggests that there should currently be tens of these nebulae in the Milky Way.

Depending on the amount of dust formed, the remnant could continue to remain obscured through these later phases. The dust surface density is roughly

$$\Sigma_0 \sim 10^{-5} \, \text{g} \, \text{cm}^{-2} \left( \frac{f_{\text{dust}}}{1.0} \right) \left( \frac{M_{\text{wind}}}{0.1 \, M_\odot} \right) \left( \frac{v}{30 \, \text{km} \, \text{s}^{-1}} \right)^{-2} \left( \frac{\Delta t}{10^4 \, \text{yr}} \right)$$

(4)

where $M_{\text{wind}}$ is the total amount of material ejected in the wind and $f_{\text{dust}}$ is the fraction of this material that forms dust. Approximating the cross-section for UV photons ($\lambda \approx 10 - 100$ nm) as geometric, the dust opacity for a grain size $a$ is $\kappa_{\text{dust}} \approx 10^5 \, \text{cm}^2 \, \text{g}^{-1} (a/10^{-5} \, \text{cm})^{-1}$, which is roughly appropriate when $a > \lambda$. Given Eq. (4), the dust optical depth could be $\sim 1$ even $10^4$ yr after merger. However, the hard radiation from the central object may also destroy some of the dust; the efficiency of this process will depend on both the composition and grain size (e.g., Waxman & Draine 2000). Whether or not they remain completely obscured, it seems likely that these objects would continue to display significant infrared excesses.

During the neon-oxygen flame and later phases of evolution (see Sections 4 and 5), the remnant remains in the vicinity of time 4 (in Fig. 11). This evolution is relatively rapid ($\approx 5$ kyr) and so the object does not yet begin to move down the WD cooling track if the mass remains above the Chandrasekhar mass. However, if the object has shed enough mass that it is below the mass that will lead to neon ignition, then it will begin to cool. Continuing the analogy to existing systems, these objects may then resemble helium-free supernovae of GW Vir. Such objects would likely also exhibit g-mode pulsations, as these are driven by ionization of carbon and oxygen (Quirion et al. 2007). As the objects continue to cool, merger remnants that remain below the Chandrasekhar mass may later appear as carbon (DQ) WDs (Dufour et al. 2007, 2008).

### 7 CONCLUSIONS

We have presented stellar evolution calculations that follow the evolution of the remnant of the merger of two CO WDs. We focused on systems with a total mass in excess of the Chandrasekhar mass; our fiducial system is the merger of an $0.6 \, M_\odot$ WD and a $0.9 \, M_\odot$ WD (model M15 in Table 1), but we find similar results for a system with a total mass of $1.6 \, M_\odot$ (model M16). Our calculations use the results of SPH simulations of the merger (Dan et al. 2011; Raskin et al. 2014) to set the initial properties of the remnant. Its post-merger viscous evolution was then followed as in Schwab et al. (2012) and the results of these simulations form the initial conditions for our MESA calculations. Thus, our results do not apply to the case in which a merger leads to the detonation of the primary WD, as may occur for particularly massive CO+CO WD mergers (Pakmor et al. 2012).

The flowchart shown in Fig. 12 summarizes the potential final fates of these systems, indicating relevant uncertainties in our models.

Our study has focused on two major questions. First, what is the post-merger observational appearance of super-Chandrasekhar WD merger remnants? Second, what is the fate of such objects? The viscous evolution converts the thick, rotationally-supported disc into a spherical, thermally-supported envelope. Our calculations self-consistently include this material, as opposed to approximating its effects by imposing an Eddington-limited accretion rate at the outer boundary. We can thus better address the emission from WD merger remnants. We find a post-merger phase, powered by thermal energy deposited during the merger, in which the remnant manifests as an Eddington-luminosity ($L \sim 3 \times 10^4 \, L_\odot$) source for $\sim 10^4$ yr. This is consistent, in luminosity and lifetime, with the signature suggested by Shen et al. (2012) who also constructed related stellar evolution models. We improve upon these models by drawing our initial conditions from multidimensional viscous simulations and by incorporating more realistic opacities.

The effective temperature in our models during the giant phase is $\sim 4000 - 5000$ K. However, the mass loss properties of these carbon and oxygen dominated envelopes are uncertain. It seems likely that a dusty wind could develop, leading these sources to be self-obscured and to appear similar to extremely AGB stars. The mass loss during the post-merger phase can also influence the later stages of evolution, leaving an initially super-Chandrasekhar remnant with a sub-Chandrasekhar mass (see Figs. 7 and 12). Based on the observed WD merger rates, we estimate that tens (few) of remnants are presently in this giant phase in the nearby galaxies M31 (M33). As the giant phase ends, the central source evolves, likely producing proto-planetary nebula and planetary nebula phases in which the surrounding nebula would lack hydrogen or helium. These sources might remain dust-obscured throughout, depending on the amount of dust produced and its properties. The planetary nebula phase also lasts for $\sim 10^4$ yr, suggesting that there should currently be tens of such sources in the Milky Way and M31.

In the massive mergers we consider, off-center carbon fusion is robustly ignited within the remnant. Most of the fusion energy released is lost to neutrinos, and that which is not, fails to reach the surface in the evolutionary timescale. Thus, the surface emission is powered solely by the thermal energy release during the merger. As found previously (Saio & Nomoto 1985; Nomoto & Iben 1985; Saio & Nomoto 1998), the carbon-burning quiescently propagates inward, converting the WD to an oxygen-neon composition. However, we follow the evolution of these remnants for longer than previous calculations and demonstrate that when carbon-burning reaches the center it lifts the degeneracy of the remnant.

As the non-degenerate oxygen-neon core undergoes a phase of neutrino-cooled KH contraction, the remnant can ignite off-center neon burning. There is a critical mass for a “hot” (i.e., non-degenerate) ONe core, $M_{\text{Ne,hot}} \approx 1.35 \, M_\odot$, above which off-center neon ignition will occur (Nomoto 1984; see also Appendix C); this is set by the rate of neon
Super-Chandrasekhar WD Merger Remnants

Merger of two CO WDs with super-Chandrasekhar total mass

Does the merger lead to a detonation?

- yes → Type Ia
- no → Remnant evolves viscously

Remnant expands and enters cool, giant phase

- Remnant envelope
- Off-center carbon-burning ignited

Does mass loss make the remnant sub-Chandra?

- yes → Massive WD
- no → Does the carbon flame reach the center?
  - yes → Hybrid Ia
  - no → Neon-oxygen flame propagates to center
    - Subsequent silicon burning leads to formation of an iron core
      - Low-mass NS

Figure 12. A summary of the final fates of super-Chandrasekhar WD merger remnants. Dashed lines indicate phases not explored in this work. Detonations of CO WDs during merger likely requires total masses $\geq 2.0 \, M_\odot$ (Dan et al. 2014; Sato et al. 2015), so this is a rare channel. Hybrid Ias are unlikely according to Lecoanet et al. (2016). Thus, most super-Chandrasekhar mergers will produce a single massive WD or a low mass NS, with the relative rates depending on the uncertain role of mass loss.

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fusion and the rate of thermal neutrino losses. There is also a critical mass for electron-capture-initiated central ignition in a “cold” (i.e., degenerate) ONe core, $M_{\text{Ne,cold}} \approx 1.38 M_\odot$; this is set by the electron Fermi energy needed to favor electron captures on $^{20}\text{Ne}$ (Schwab et al. 2015). The off-center carbon burning ignited in the merger remnant leads to the production of a “hot” ONe core. Because $M_{\text{Ne,hot}} < M_{\text{Ne,cold}}$, we therefore conclude that lower mass merger remnants ($M < M_{\text{Ne,hot}}$) will form massive ONe WDs which will then quiescently cool, while higher mass objects ($M > M_{\text{Ne,hot}}$) will be processed to compositions beyond neon. Thus, contrary to standard models, it is difficult to produce an oxygen-neon core with a sufficient mass to undergo accretion-induced collapse in a WD merger. We emphasize that our conclusions apply to WD mergers and do not affect standard single degenerate AIC scenarios. In the single degenerate case, an ONe core below $M_{\text{Ne,hot}}$ forms in an intermediate mass star and cools. Subsequently, it accretes material and grows to a mass in excess of $M_{\text{Ne,cold}}$ at which point it collapses.

One of the major uncertainties in our calculation is the extent of mass loss during the $\sim 10^4$ yr post-merger thermal evolution. In addition to our fiducial model, we also evolved a remnant with a total mass $\approx 1.6 M_\odot$. Both models had similar initial profiles (Fig. 1) and underwent a qualitatively similar course of evolution. We suspect that yet higher masses would behave similarly, up to the point where carbon detonations are likely to occur during the merger. The results of both Dan et al. (2014) and Sato et al. (2015) suggest that the threshold for carbon detonations is around a total mass $\gtrsim 2 M_\odot$ for CO+CO WD mergers. Since this threshold is significantly super-Chandrasekhar, this suggests that even in the presence of significant mass loss ($\approx 0.2 M_\odot$) both the massive WD and the low mass NS fates in Fig. 12 are realized. However, the exact mapping of merger masses to outcome is uncertain.

If the remnant mass remains super-Chandrasekhar and experiences off-center neon ignition, we expect that it will ultimately collapse to form a neutron star. The neon-oxygen flame propagates to the center converting the remnant to silicon-group elements (Figs. 8 and 9). Computational limitations prevent us from continuing our calculations much beyond the point at which the neon-oxygen flame reaches the center. However, we then argue, based on the work of Jones et al. (2013, 2014) and Woosley & Heger (2015) that Si-burning leads to the formation of an iron core, which will then collapse. Future work in this area will be important in understanding the role of violent Si-burning and examining the structure of the remnant at the time of core infall. The low mass core, with steep density gradients near its edge, means that this progenitor will likely produce a neutrino-driven explosion a short time after core bounce, as is seen in core-collapse calculations involving ONe cores (Kitaura et al. 2006; Janka et al. 2008). A shorter time between core bounce and explosion also implies that there is less time for instabilities to grow, leading to less asymmetry and lower neutron star kicks (e.g., Wongwathanarat et al. 2013). With pre-collapse structures in-hand, future explosion calculations can determine the amount of radioactive material produced and thus the brightness of the accompanying supernova—simulations of ONe cores find only $10^{-3} - 10^{-2} M_\odot$ of $^{56}\text{Ni}$ (Dessart et al. 2006; Kitaura et al. 2006)—but the amount of $^{56}\text{Ni}$ synthesized in remnants that remain significantly super-Chandrasekhar at the time of collapse remains to be determined.

Our calculations assume that the bounding convection zone does not induce significant mixing in either the carbon flame or the neon flame. The results of Lecoanet et al. (2016), which we invoked to justify this assumption in the case of the carbon flame do not strictly apply to the neon-oxygen flame. The neon-oxygen flame self-crossing time (which is equivalent to the burning time-scale) is much shorter, $\sim 10^4$ yr, on the order of the convective turnover time in the region behind the flame. This invalidates the assumption made by Lecoanet et al. (2016) that the flame is effectively stationary over a convective turnover time. Nevertheless, it seems likely that fewer convective turnover times make it even less likely that convection will disrupt the flame, as convective perturbations will not be able to accumulate because the flame moves away.

Because the duration of the giant phase is shorter than the time for the carbon flame to reach the center, our results indicate that at the time of neutron star formation the remnant is likely in a compact configuration (Fig. 11), implying minimal signatures due to interaction or shock breakout. However, if violent Si-flashes unbind some mass, as Woosley & Heger (2015) found in some of their analogous intermediate mass star models, this could produce strong circumstellar interaction in the resulting supernovae. At late times after the supernova explosion ($\sim 10^4$ yr), the ejecta from the explosion (with $v \sim 10^4$ km s$^{-1}$) will catch up to the wind generated during the giant phase (with $v \sim 10$ km s$^{-1}$); however this interaction is radiatively inefficient.

Future work should continue to explore the variety of final outcomes shown in Fig. 12 and work to clarify further differences between neutron stars formed via single-degenerate accretion-induced collapse and those formed in WD mergers.

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Each approach is problem-specific: one must identify the important parts of the solution and design a mapping that preserves them. Our goal is to use the output of a 2D Eulerian code (ZEUS-MP2, which solves the viscous fluid equations) as input to a 1D Lagrangian code (MESA, which solves the stellar structure equations).

At the end of our hydrodynamic simulations of the viscous phase of evolution, most of the WD merger remnant is slowly rotating and in hydrostatic equilibrium and thus quite spherical (e.g., Fig. 5 in Schwab et al. 2012). We make the choice to study the non-rotating case and thus do not initialize or evolve a rotational velocity variable. Under the assumption of spherical hydrostatic equilibrium, the complete structure of an object can be specified by its specific entropy, \( s(M_r) \), and composition, \( X_i(M_r) \), as a function of Lagrangian mass, \( M_r \). Therefore, the first step is to spherically average our multi-D simulations and calculate these quantities.

We perform volume averages, \( \rho(r) = \frac{1}{2} \int_0^\pi d\theta \sin(\theta) \rho(r,\theta) \) (A1), \( c(r) = \frac{1}{2} \int_0^\pi d\theta \sin(\theta) c(r,\theta) \) (A2), and \( \rho(r) X_i(r) = \frac{1}{2} \int_0^\pi d\theta \sin(\theta) \rho(r) X_i(r,\theta) \) (A3) so that the appropriate quantities (e.g., mass, energy) are conserved. Assuming full ionization, it is simple to calculate \( \bar{A} \) (the average mass per ion) and \( \bar{Z} \) (the average charge per ion) from the mass fractions \( X_i \). Given \( \rho, c, \bar{A}, \bar{Z} \), we use the Helmholtz (Timmes & Swesty 2000) equation of state, which is used by both our ZEUS-MP2 simulations and MESA (for \( \rho \gtrsim 3 \times 10^3 \text{g cm}^{-3} \)) to calculate the specific entropy \( s(r) \). The Lagrangian mass is

\[ M_r = \int_0^r dr' 4\pi r'^2 \rho(r') \] (A4)

and we record a 1D approximation to our 2D simulation consisting of the values of \( M_r, s(r) \), and \( X_i(r) \) for each radial grid point in the computational domain.

We want to create a MESA model which matches this profile. Instead of trying to create a MESA model file and then reading it in, we begin with a model unlike what we want, but slowly reshape it into our desired profile. The steps in this procedure were arrived at by trial-and-error. There is nothing inherently correct about many of the particulars of this approach; we demonstrate at the end that this stellar engineering gives us the desired result.

First, we create a pre-main sequence model with the mass of the remnant. We evolve this model, with nuclear reactions turned off, until the central density is equal to \( 10^9 \text{ g cm}^{-3} \). Then we resume the evolution, using the built-in capability of MESA to relax our model to a specified composition. With the composition relaxation complete, we evolve this model, with nuclear reactions and mixing turned off, until the central density reaches \( 10^7 \text{ g cm}^{-3} \).

Again, we resume the evolution, this time making use of a custom routine in run_starExtras.f which relaxes the model to the desired thermodynamic profile. We take advantage of the other_energy routine, which allows us to add an additional term to the energy equation, and set

\[ Q_{\text{extra},k} = -c_v \frac{\delta T}{t_0} \alpha_k \] (A5)

where \( c_v \) is the specific heat at constant volume and \( T \) is the temperature. The subscripts \( k \) indicate that these quantities are evaluated on a per-cell basis. The values of \( \alpha_k \) were chosen to drive the model towards the desired profile. We chose a heating/cooling time-scale, \( t_0 \), which is short compared to the thermal time of the star and is comparable to the total duration of our viscous evolution calculations, typically \( t_0 = 10^8 \text{s} \).

To determine \( \alpha_k \), we read in the spherically-averaged entropy profile from ZEUS and use built-in MESA interpolation routines to sample it such that we have a target entropy function, \( \delta(q) \), where \( q = M_r/M_{\text{tot}} \). In the inner part of the model, which corresponds to the cold, undisturbed primary WD, we will ignore the entropy profile and instead relax the profile to an isothermal one. This approach requires three additional parameters: the isothermal temperature, \( T_c \), the region which should be isothermal, \( q_c \), and the width of a region in which we blend between these, \( \Delta \). At each point in the MESA model, we evaluate

\[ \delta_1 = 1 - \delta(q_c) / s_k \] (A6)
\[ \delta_2 = 1 - T/T_k \] (A7)
\[ \delta_k = f_k \delta_1 + (1 - f_k) \delta_2 \] (A8)

where

\[ f_k = \frac{1}{2} \left( \tanh \left( \frac{q_k - q_c}{\Delta} \right) + 1 \right) \] (A9)

From these quantities, we locally calculate

\[ \alpha_k = \delta_k \tanh \left( \delta_k \right) \] (A10)

and in order to check whether the profile matches, calculate a global quantity

\[ \epsilon = \sum_{k=1}^{n_k} |\delta_k| dq_k \] (A11)

We consider our relaxation complete when \( \epsilon < 3 \times 10^{-4} \). Empirically, this approach works quickly and robustly.

Fig. A1 compares the 1D-averaged ZEUS-MP2 profiles and the initial MESA profiles for our model M15. The density, entropy, and temperature are in good agreement for \( 0.5 \lesssim q \lesssim 0.9 \). This is the critical region, because it is where carbon ignition will occur and where the thermal energy that will power the giant phase is located. In the deep interior (\( q \lesssim 0.5 \)), our assumed isothermality means the temperature and entropy of the material has been altered, but because the material is degenerate, this has little effect on the structure (i.e., the density agrees well). In the outer regions (\( q \gtrsim 0.9 \)), modest differences are introduced by the fact that the material was not spherical, but is now assumed to be.

APPENDIX B: OPACITIES

As discussed in Section 2.3, we generate and use a set of low temperature opacities for material with a carbon-oxygen
transitioning into a region in which the opacities are tabulated only as a function of $X$, $Y$, and $Z$. Thus, the compositions assumed in the calculation of the low and high temperature opacities are necessarily different. The carbon and oxygen composition of our WD model nominally corresponds to $Z = 1$, but since the assumed abundance distribution within $Z$ is based on solar abundances, a choice of $Z = 1$ effectively assumes the material is dominated by $^{12}\text{N}$ and $^{56}\text{Fe}$. One of the ways MESA can handle this is to assume a user-specified value of $Z$ ($Z_{\text{base}}$), use the value of $X$, and put the rest of the abundance in $Y$. The line for the low temperature FA05 opacities in Fig. B1 makes this choice, assuming $Z = 0.02$ and thus $X = 0, Y = 0.98$. In addition to being physically inconsistent, these choices are numerically unsatisfactory because they result in large change in the opacity and its derivatives over a small temperature range. Our use of the Kasen opacities avoids both of these issues.

The opacities used in the MESA calculations do not include the effects of molecules. If we were to include these effects, the opacity at $\log(T/K) \lesssim 3.5$ would be extremely sensitive to the C/O ratio of the material (e.g., Ferguson & Dotter 2008). This quantity was not, however, self-consistently determined in our initial WD models, which have equal mass fractions of $^{12}\text{C}$ and $^{16}\text{O}$.

**APPENDIX C: CRITICAL MASSES**

For a contracting stellar model of a given composition, there is a critical mass above which the temperature will become sufficiently high to ignite nuclear fusion. This is familiar for typical hydrogen-rich compositions, where this critical mass marks the boundary between a brown dwarf and a low mass star. Nomoto et al. (1984) calculated this critical mass for pure neon stars and used this simple model to gain insight into the evolution of contracting ONe cores. For pure neon models, the higher density and temperatures mean the KH contraction is driven by neutrino cooling from the interior as opposed to photon cooling from the surface. The density dependence of the neutrino cooling rates lead to the formation of an off-center temperature peak, and when ignition does occur, it occurs off center.

In the spirit of Nomoto et al. (1984), we run a series of MESA models with masses in the range $1-30\,M_\odot$ to $1.44\,M_\odot$, using a stride of $0.005\,M_\odot$, and with neon, oxygen-neon, oxygen, and silicon compositions. We create a pre-main sequence model with the desired mass and then relax the model to the desired composition. With nuclear reactions off, we evolve the model until $\log(p_0/g\,\text{cm}^{-3}) = 4$. At this point, we load the model using the approx21 nuclear network. We then evolve each model until either (1) the rate of energy release from nuclear reactions exceeds the rate of energy loss from neutrinos anywhere in the star and subsequently exceeds $10^{14}\,\text{erg}\,\text{s}^{-1}\,\text{g}^{-1}$, in which case we classify the model as having ignited or (2) the peak temperature reaches a maximum value and subsequently declines, in which case we know the model will never ignite.

In Fig. C1, we show the evolution of three of our pure neon models. We find the lowest mass model that ignites has $M = 1.35\,M_\odot$. This is slightly lower than the value of $1.37\,M_\odot$ reported by Nomoto et al. (1984). We specu-
Figure B1. The solid black line (labeled OPAL Type 2) is the carbon- and oxygen-enhanced radiative opacities from Iglesias & Rogers (1996). The calculations presented in this paper use low-temperature opacities calculated with a code developed by one of us (Kasen; dashed black line). The thin grey line (labeled FA05) is the low-temperature opacities from Ferguson et al. (2005); these are the opacities that MESA would use if we did not provide our own. The difference between the OPAL/Kasen and FA05 curves are because MESA has assumed a helium-dominated composition (see text for discussion). In all cases, the relative metal abundances are drawn from Grevesse & Sauval (1998). In our MESA calculations, we blend the OPAL and Kasen opacities between log($T/K$) = 4.1 and 4.2 (see Fig. B2). The left panel shows the opacity as a function of temperature at fixed log($R$) $\equiv$ log$\rho - 3\log T + 18$ (cgs); the right panel shows it at fixed $\rho$.

late that this minor difference is due to differences in the input microphysics. Curves analogous to these, but for different masses, are shown as the grey lines in Figs. 6 and 7. The qualitative agreement between our WD merger remnant models and these simple homogeneous models demonstrates that the off-center neon ignition found in our WD merger remnants is a simple consequence of their mass.

The mass coordinate at which ignition occurs is shown in Fig. C2. We show 4 different compositions: pure $^{20}$Ne (Ne), an equal mixture by mass of $^{20}$Ne and $^{16}$O (Ne/O), pure $^{28}$Si (Si), and an equal mixture by mass of $^{28}$Si and $^{32}$S (Si/S). The large dots indicate the lowest mass models in which off-center ignition was observed; these are the critical ignition masses quoted in the text. Caveats apply to our treatment of silicon-burning: we have used a small $\alpha$-network and neglected neutron-rich isotopes such as $^{30}$Si and $^{34}$S. However, since the same caveats apply in our calculation of the remnant evolution, these simple models are well-suited for use in interpreting our results.

This paper has been typeset from a TeX/\LaTeX\ file prepared by the author.
Figure B2. Coverage of temperature-density space by the opacity tables used in our MESA calculation. Solid grey lines show the boundaries of the two types of opacity tables: OPAL Type 2 (Iglesias & Rogers 1996) and Kasen (Appendix B). “Blend” indicates a smooth blend between these two values. Recall \( \log R \equiv \log \rho - 3 \log T + 18 \) (cgs). The other lines show the temperature-density profiles of our model at the set of times also marked in Figs. 2, 4, and 11. We observe density inversions at both the Fe opacity bump (\( \log(T/K) \approx 5.3 \)) and in bumps associated with carbon and oxygen.

Figure C1. The evolution of temperature and density at the center (solid lines) and at the temperature peak (dashed lines) during the KH contraction of pure neon models. The dotted line shows approximately where the rate of energy generation from neon burning is equal to the energy loss rate from thermal neutrinos. The energy generation rate for neon-burning is that given in Woosley et al. (2002). This figure is a reproduction using MESA of the results presented in fig. 1 of Nomoto et al. (1984). We find a slightly lower critical mass for neon ignition of 1.35 \( M_\odot \).

Figure C2. The mass coordinate of off-center ignition for different assumed compositions: pure \( ^{20}\text{Ne} \) (Ne), an equal mixture by mass of \( ^{20}\text{Ne} \) and \( ^{16}\text{O} \) (Ne/O), pure \( ^{28}\text{Si} \) (Si), and an equal mixture by mass of \( ^{28}\text{Si} \) and \( ^{32}\text{S} \) (Si/S). Each model is indicated with a dot; the lowest mass model that ignited is indicated with a large dot. The dots are connected by lines to guide the eye.