Pulsar slow glitches in a solid quark star model

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ABSTRACT
A series of five unusual slow glitches of the radio pulsar B1822-09 (PSR J1825-0935) were observed over the 1995-2005 interval. This phenomenon is understood in a solid quark star model, where the reasonable parameters for slow glitches are presented in the paper. It is proposed that, because of increasing shear stress as a pulsar spins down, a slow glitch may occur, beginning with a collapse of a superficial layer of the quark star. This layer of material turns equivalently to viscous fluid at first, the viscosity of which helps deplete the energy released from both the accumulated elastic energy and the gravitation potential. This performs then a process of slow glitch. Numerical calculations show that the observed slow glitches could be reproduced if the effective coefficient of viscosity is \( \sim 10^2 \text{ cm}^2\text{s}^{-1} \) and the initial velocity of the superficial layer is order of \( 10^{-10} \text{ cm}\text{s}^{-1} \) in the coordinate rotating frame of the star.

Key words: dense matter - stars: neutron - pulsars: general - pulsars: individual: PSR B1822-09

1 INTRODUCTION

It is a pity that we are now not able to determine confidently which state of matter really exists in pulsar-like stars due to the difficulty of calculation from the first principles of the elementary strong interaction, even 40 years after the discovery of pulsars. Nuclear matter (related to neutron stars) is one of the speculations even from the Landau’s time, while quark matter (related to quark stars) is an alternative due to the fact of the asymptotic freedom of strong interaction between quarks (see reviews, e.g., Madsen 1999; Glendenning 2003; Lattimer & Prakash 2004; Weber 2003). We then have to focus on astrophysical observations in order to solve this important and fundamental question.

Actually, pulsars are ideal astro-laboratories for physics of cold matter at supranuclear density. Based on the Planck-like spectrum without atomic features and the precession properties of pulsar-like stars, a solid quark matter conjecture was suggested (Xu 2003). Additionally, other features naturally explained within this model could possibly include sub-pulse drifting in radio emission (Xu et al. 1999), glitches (Zhou et al. 2004), strong magnetic fields, birth after a successful core-collapse supernova, and detection of small bolometric radii (for a short review, see, e.g., Xu 2006). Besides, the observational features of anomalous X-ray pulsars/soft gamma-ray repeaters may also reflect the nature of solid quark stars (Owen 2003; Horvath 2005; Xu et al. 2006).

The slow glitches recently observed in PSR B1822-09 (Shabanova 1998; Shabanova & Urama 2003; Zou et al. 2004; Shabanova 2003, 2007) could be a new probe into cold solid quark matter. The pulsar has a period of 0.769 s and a relative young age of 230 kyr. What is interesting in this pulsar is the change in its period known as slow glitches. Different from the typical feature of glitch phenomenon of other pulsars, this pulsar shows glitches that have rather slow increase in spin frequency, with a time scale of 200-300 days (while the normal glitches experience this process much swifter, maybe less than a spin period), and moreover, this pulsar didn’t experience any relaxation progress during the post slow glitch days.

A series of five slow glitches are shown in Fig. 1. Fig. 1(a) shows the frequency derivative \( \dot{\nu} \). The peaks of \( \dot{\nu} \) are enveloped in a parabolic curve, that may indicate that all the glitches could be the components of one process. This process could be triggered by the small glitch (\( \Delta\nu/\nu \sim 8 \times 10^{-10} \)) occurred in 1994 September. Both Fig. 1(b) and (c) show the frequency residual \( \Delta\nu \), but relative to fits to data for different intervals (1991-1994 for the former, and 1995-2004 for the latter). In this observational result, it is evident that the typical character of slow glitch in Fig. 1(b): the spin frequency experiences a process that increases gradually but never decreases (i.e. no post-glitch relaxation there).

We are trying to explain this phenomenon in the regime of solid quark stars. Since the pulsar is supposed to be mostly built up of solid quark matter and the typical density there is order of \( 10^{14} \text{ g/cm}^3 \), soon after a quake, the motion of only a thin layer of the quark matter may cause significant change in the moment of inertia (and thus the spin frequency) of the star owing to the extremely high density. Numerical calculation is carried out to simulate the observational results of both the slow increase in the rotating frequency and the lack of the relaxation of the post-glitch progress.
ν² results in an eccentricity of Figure 1. The observational data of frequency residual ∆ν² Peng, Xu, 2004, during which both elastic and gravitational energies are released. In the model of Zhou et al. (2004), the moment of inertia, I, decreases suddenly (to result in a sharp jump of ∆ν), and then increases gradually (to result in a following post-glitch relaxation). However, as elastic force develops to a critical value, a solid rotator with smaller shear modulus might not be so violent (i.e., I does not decrease suddenly) but in a gentle way for I to decrease, and could thus reproduce the observational feature of slow glitches. No I-increment occurs if no significant elastic energy is converted to kinetic energy (and thus no post-glitch). The small glitch at 1994 of PSR B1822-09 may lead to a small effective shear modulus (e.g., significant part of the quark matter would be shiver-like), and thus trigger the slow glitches. A pre-glitch could then be expected for slow glitches in the star-quake model of quark stars, in order to have an effective small shear modulus.

Soon after a quake, we assume that a superficial layer moves in order to set a new equilibrium (see Fig. 2), while most of the inner part of star keeps almost the same. Part of the energy released turns into heat and melts the debris, while the other part turns to the kinetic energy of the fluid. It is suggested in this model that the broken material will turn into viscous fluid and is therefore able to flow slowly and change the shape of the star towards the equilibrium shape. This progress would surely cause slow decreasing of the moment of inertia of the star. Since this process happens in a pretty short time comparing to the typical age of the pulsar, it is reasonable to suppose that the angular momentum of the star keeps invariant during the whole progress of glitch. As a result, the frequency of the star might increase slowly during the same time. When the fluid is flowing, the viscous interaction among the elements of the fluid may exhaust the kinetic energy of the fluid. The shape of the star stop changing finally, and the material then cools down and is solidified. This is why slow glitch appears no relaxation feature in the post-glitch progress. Calculational results (see §3) explain why this pulsar experiences a serious of five slow glitches rather than a single one.

To assure that model, we are doing numerical calculation to simulate this process. First, the Navier-Stokes equation is known as the basic equations to describe the motion of viscous fluid. In the frame which is fixed on the inner solid part of the star, the general form reads,

\[
\frac{\partial}{\partial t} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \mu \Delta \mathbf{v} + \mathbf{g} - \frac{d\omega}{dt} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) - 2\mathbf{v} \times \omega, \tag{3}
\]

where \( \mathbf{v} \) is the velocity of the superficial matter in the rotating frame, \( \rho \) is the pressure in the star, \( \mathbf{g} \) is the acceleration of gravity on the surface of the star, \( \mu \) denotes the viscosity of the fluid after the phase transformation (from solid to fluid-like matter), and the star is assumed to rotate constantly. The three terms in the bracket of right hand side arises from the inertial acceleration due to the non-inertial frame we choose.

Let’s consider briefly Eq.(3). For a zero order approximation, a solid star could be modelled by a rigidity body,
A slow glitch is supposed to occur at time $t = t_0$. In the left, (1) an imagined ellipsoidal figure (determined by Eq.(1)) at $t = t_0$ if the star is in a fluid state, with spin frequency $\nu = \nu_{\text{before}}$; (2) the real figure of a solid star at $t = t_0$ ($\nu = \nu_{\text{before}}$), with stress energy high enough to quake; (3) the figure after slow glitch at time $t = t_0$, with $\nu = \nu_{\text{after}}$, which might be close to the imagined shape (1); (4) the inner solid part which keeps almost the same during a slow glitch; (5) the superficial crust layer which is transmuted during the slow glitch progress.

with velocity $v = 0$, i.e.,

$$0 = -\frac{1}{\rho} \nabla p + g - \left( \frac{\partial v}{\partial t} + \nabla \times (\omega \times r) \right).$$  \hspace{1cm} (4)

The above equation could at least be adaptable for the case of $v \ll r \omega$. Observationally, the timescale of slow glitches is much longer than the spin period. We thus think that Eq. (4) could be applied in our following simulation. When a glitch happens, the elastic energy is released suddenly, the solid superficial layer turns to a fluid-like state with strong viscosity, but the pressure gradient $\nabla p$ may keep to be almost invariable. It is a key point that $\nabla p$ remains approximately the same in spite of stress release in the model, which means that this is the case in which most of the elastic energy released is changed into heat, rather than into the kinetic energy. The heating may melt down solid quark matter to become viscous fluid. Soon after the starquake, the matter could be re-solidified finally due to cooling. Contrastively, in case that the star breaks globally, most of the elastic energy may turns into the form of the kinetic energy, and the pressure gradient would change significantly during starquake; the star would experience then a normal glitch with sharp frequency increase. Therefore, during a glitch, only $\omega$ and $v$ in Eq. (4) change, with Eq. (5) to be well approximated since $\Delta \omega / \omega \sim 10^{-9}$ and $\Delta \omega / \omega \sim 10^{-2}$ are very small. Then Eq. (5) of the viscous fluid can be reduced to be

$$\frac{\partial v}{\partial t} + (v \cdot \nabla) v = \mu \Delta v - (-2v \times \omega).$$  \hspace{1cm} (5)

We are concerning the initial and boundary conditions for our simulations. First we assume that only part of a solid quark star breaks under the stellar surface (the shaded region in Fig. 2), with a thickness of $H$. We note that this assumption is for simplifying the calculation, but is not a key for general results in the starquake model for slow glitches (see §3). We denote $h$ to measure the height of matter in the beaked part, i.e., $h = 0$ in the bottom while $h = H$ on the surface. Two cases are considered in the simulated result of Fig. 3, $H = 10\% R$ and $20\% R$, where $R$ is the stellar radius. We suppose that the velocity of matter increases linearly from the bottom of collapsed layer to the top, with a boundary of zero velocity at the bottom, and that the velocity increases gradually from polar to equator. The tangential velocity could then be formulated as

$$\begin{align*}
    v_{\text{max}}(\theta) &= v_i \sin \theta, \\
    v(\theta, r) &= v_{\text{max}}(\theta) \frac{r}{R},
\end{align*}$$  \hspace{1cm} (6)

where $v_i$ is the maximum of the initial velocity soon after a quake. A starquake is assumed to be a trigger for the initial movement with maximum velocity $v_i$. The continues condition of matter (i.e. $\nabla \cdot v = 0$) determines the radial velocity, given that the viscous fluid is incompressible.

Solving Eq. (5) with boundary Eq. (6), one can obtain a 3-dimensional vector field of velocity all over the star. To achieve the changing of the shape and rotating frequency, we adopt the incompressible hypothesis and the conservation law of angular momentum. The movement of incompressible matter in the layer changes gradually the shape of star, and thus the angular frequency because of conservation of angular momentum. In fact the conservation law of angular momentum is not obeyed precisely due to the spin-down torque. However, in light of the rather short duration of the glitch progress, the conservation could be well approximated during a relatively short time.

After a series of slow glitches, we may suppose that the star stays at an equilibrium state which is described by Eq. (2). It is worth clarifying that, as noted in Fig. 2 (note the difference between states (1) and (3)), after each slow glitch (except the last one), the star did not turn to the real equilibrium state, but suffer actually a series of glitches, not only a single one. We can then obtain the eccentricity of the star with the angular frequency at the end of the five slow glitches through Eq. (2). After doing that, we may calculate the star’s spin frequency in any time via the conservation law of angular momentum.

Getting all above, we can carry out numerical calculus after setting the value of initial velocity $v_i$ and viscosity $\mu$. Our goal is to find whether there is reasonable parameter space (of $v_i$ and $\mu$) where the general features of the slow glitches observed could be reproduced. In the calculation, the “heur” method is adopted in order to obtain more accurate values from numerical calculations.

### 3 THE RESULTS

We do the simulation with typical density of $\rho = 10^{14} \text{ g/cm}^3$ and stellar radius of $R = 10$ km for indications. However, according to various simulations, the general conclusions would not change significantly if these two parameters are in the same orders. Observational data of slow glitches and the simulated result in the solid quark star model are shown in Fig. 1(b) and Fig. 3, respectively. The parameters of $\mu$ and $v_i$ for the five slow glitches are also shown in table 1.

In Fig. 3, two simulated curves are shown with different thickness of the surface fluid layer, $H = 10\% R$ and $H = 20\% R$, respectively. From Fig. 3, it is obvious that the
thickness indeed affect the result of $\Delta \nu$. Though the general behavior keeps, the amplitude and the gradient of $\Delta \nu$ increase as percentage of the thickness to the stellar radius increases.

What if the values $v_i$ of and $\mu$ in Table 1 change? In case of $H = 10\% R$, the influence of these parameters on the simulated result of $\Delta \nu$ are shown in Fig. 4 and Fig. 5, respectively.

As shown in Fig. 4, where values of $\mu$ are the same listed in Table 1, $\Delta \nu$-curves for various $v_i$ are illustrated. It is evident that a larger value of $v_i$ could result in a larger glitch amplitude of $\Delta \nu$. It is worth noting, however, that the time durations in which $\Delta \nu$ increases to the maximum are almost the same. This could be easily understood since a larger $v_i$ implies a larger stress and energy released in the starquake. The effect of changing $\mu$-values on the amplitude of $\Delta \nu$ are demonstrated in Fig. 5, where the values of $v_i$ are the same listed in Table 1. As shown in Fig. 5, smaller $\mu$-values could not only result in higher amplitudes of $\Delta \nu$, but also in longer durations for $\Delta \nu$ to increase to maximum. This result is also understandable. In the model, the fluid moves under coriolis’ force and viscous resistance. Smaller $\mu$ could lead to a relative smaller resistance and thus a longer time to reach its equilibrium. In summary, one needs to have a high $v_i$ or a low $\mu$ in order to reproduce a large amplitude of $\Delta \nu$, while a small $\nu$-value could effectively delay the increase of $\Delta \nu$.

### Table 1

| Number | $\mu$ (cm$^2$·s$^{-1}$) | $v_i$ (cm·s$^{-1}$) |
|--------|----------------|-------------------|
| 1      | $10^{2.4}$     | $10^{-10.61}$     |
| 2      | $10^{2.6}$     | $10^{-9.88}$      |
| 3      | $10^{2.5}$     | $10^{-10.17}$     |
| 4      | $10^{2.4}$     | $10^{-10.00}$     |
| 5      | $10^{2.6}$     | $10^{-11.10}$     |

### Figure 3

The simulation of slow glitches. The vertical axis is the frequency change, $\Delta \nu$ (in unit of $10^{-7}$ Hz), and the horizontal axis measures the time (in days). The two curvatures in this figure are two simulations, given that the thickness of the surface layer to be 10% and 20% $R$, respectively. The starting point of time could be at MJD 49857 in Fig. 1.

### Figure 4

Different curves of $\Delta \nu$ with different parameters of $v_i$. The curve labelled as “V1” is that same one labelled as “10%R” in Fig. 3, with $v_i$-values listed in Table 1. The curves labelled as “2V1” and “0.5V1” are with $v_i$-values double and half the ones listed in Table 1, respectively.

### Figure 5

Different curves of $\Delta \nu$ with different parameters of $\mu$. The curve labelled as “$\mu$” is that same one labelled as “10%R” in Fig. 3, with $\mu$-values listed in Table 1. The curves labelled as “2$\mu$” and “0.5$\mu$” are with $\mu$-values double and half the ones listed in Table 1, respectively.

### 4 CONCLUSIONS AND DISCUSSIONS

A solid quark star model, in which the surface matter breaks during an initial small glitch, is suggested for understanding the general features of slow glitches observed recently. Though this is a real problem of rheology in principle, we deal with the matter as viscous fluid for simplicity. Numerical calculations show that the observed slow glitches could be reproduced if the effective coefficient of viscosity is $\sim 10^2$ cm$^2$·s$^{-1}$ and the initial velocity of the superficial layer is order of $10^{-10}$ cm·s$^{-1}$ in the coordinate rotating frame of the star. We simulated with typical density of $\rho = 10^{14}$ g/cm$^3$ and stellar radius of $R = 10$ km for indications, but the general conclusions would not change significantly if these two parameters are in the same orders.
The pieces of blocks should be re-correlated through solidifying melted surfaces of the pieces, into which the super-\text{ficial layer of a solid quark star breaks soon after a quake. The released energy (both gravitational and elastic) during a slow glitch may result in melting the conjunctural parts of segments (i.e., the surfaces of blocks) at first. The temperature then increases significantly at only a small fraction ($\eta \ll 1$) of the segments. For an order-of-magnitude demonstration, we apply the Debye model to normal solid quark matter and obtain a heat capacity of

$$C_v = 0.15 \frac{k_B^2T^3}{c^5\hbar^2} \rho_0 \eta.$$

(7)

The matter absorbs an energy of $E = \int_0^T C_vdT$ from an initial temperature of $T_0 \ll T$ to $T$. In case of released energy of $E \sim 10^{39}$ erg and a melting temperature of $T \sim 10$ MeV ($\gg$ the star’s global temperature $T_0 \sim$ keV), we have,

$$\eta = 2.7 \times 10^{-22} \frac{c^5\hbar^2}{k_B^2R_{\Omega T_1}} \frac{\rho}{\rho_0} \approx 10^{-8} \frac{1}{R_T^2} \frac{\rho}{R_{\Omega T_1}} \frac{\rho}{\rho_0}$$

(8)

for stars with radius of $R_b \times 10^6$ cm and density of $\rho$, where $T_{11} \times 10^{13}$ K is the melting temperature and $\rho_0$ is the nuclear density. For blocks with length scale of 1 m, this calculation shows a heating surface part with thickness of about $10^7$ fm for each of the blocks. We note the timescale for keeping such a high temperature gradient should be very small, so that the contacting parts cool rapidly to solidify. More and more small blocks become correlated (i.e., a big solid-like bulk matter forms) as a slow glitch evolves, and strain energy develops again then. All the heated points are within the star and the cold surface keeps, and then no significant heating feature could be observed after a glitch.

Alternatively, let’s consider a crusted strange quark star with a solid crust and a fluid quark matter core. The iner-\text{tia of the crust could be only $\sim 10^{-5}$ times of that of the quark matter core. According to $\Delta I/I \sim \Delta \Omega/\Omega \sim 10^{-8}$ for slow glitches, the crust ellipticity should change a value of $\sim 10^{-3}$, while the actual ellipticity is $\sim 10^{-3}$ for stars with period of $\sim 1$ s. This means that pulsar slow glitches could not be re-produced in a crusted strange star model.

What is the key ingredient which affects a pulsar to undergo a normal or a slow glitch? We think that the stellar mass of quark star could play an important role in this issue.

Let’s introduce two kinds of stress force inside solid stars at first. As noted by Xu (2006), two kinds of factors could result in the development of stress energy in a solid star, and then in star quakes as glitches. (i) As a quark star cools (even spinning constantly), changing state of matter may cause a development of anisotropic pressure distributed inside a solid matter. Such matter cannot be well approximated by perfect fluid, and the equation governing star’s gravitational equilibrium should then not be the TOV equation. In case of spherical symmetry (the simplest case), one can introduce the difference between radial and tangential pressures, $\Delta$. Change of $\Delta$ would lead to no-conservation of stellar volume. We call the force, which acts primary in this case, as \textit{bulk-variable force}. This kind of force may be the key factor for giant quakes during superflares of soft $\gamma$-ray repeaters (Xu et al. 2006). (ii) An uniform fluid star would keep its eccentricity presented in Eq. (1) or Eq. (2), i.e., the eccentricity decreases as a star spins down. However, for a solid star, the shear stress would prevent the star from decreasing eccentricity during spindown. In this case, even the state of matter does not change, stress energy could still develop as solid star spins down. We call this kind of force as \textit{bulk-invariable force} since the total stellar volume may always keep constantly. Starquakes due to bulk-invariable force, and the consequent glitches, were calculated and discussed in Zhou et al. (2004).

Both bulk-invariable and bulk-variable forces could result in decreases of moment of inertia, and therefore in pulsar glitches. These two kinds of forces could trigger normal glitches if they are relatively stronger than the critical stress, but might only conduce to slow glitches if weaker. The bulk-variable force could be stronger than the bulk-invariable one for massive solid stars, since, in a special case of non-rotating, the gravity $\propto M^2/R^2$ ($M$: mass; $R$: radius) is strong there. If the critical stress of solid quark matter is almost the same (note: the effective critical stress could be much small for a shiver-like surface layer), we may think that normal glitches prefer to occur in massive quark stars with possibly strong bulk-variable forces, while slow glitches are likely to take place in low-mass solid quark stars.

What if PSR B1822-09 is a low-mass quark star? The spindown due to magnetodipole torque for a star with a magnetic dipole moment $\mu$, a moment of inertia $I$, and an angular velocity $\Omega$ reads

$$\dot{\Omega} = -\frac{2}{3} \frac{\mu^2 \Omega^3}{I}.$$

(9)

This rule keeps quantitatively for any oblique rotators, as long as the braking torques due to magnetodipole radiation and the unpolar generator are combined (Xu & Qi 2001). Approximating $I = (2/5)MR^2$, $\mu = (1/2)BR^3$, and $M = (4\pi/3)R^3\rho$ with $\rho$ the average density, we have then

$$R = 2.76 \times 10^{31} \rho_{14} B^{-2} \text{ cm}.$$

(10)

where $\rho = \rho_{14} \times 10^{14}$ g/cm$^3$, and $P = 0.769$ s and $\dot{\Omega} = 5.23 \times 10^{-14}$ for PSR B1822-09. The pulsar could be a few kilometers in radius if the polar magnetic field is $\sim 10^{15}$ G.

The potential drop in the open field line region, $\phi$, should be greater than a critical value of $\sim 10^{13}$ V $\approx 3 \times 10^9$ e.s.u. in order to create secondary $e^\pm$ plasma via gap discharges. The potential drop between the center and the edge of a polar cap is therefore (Ruderman & Sutherland 1975)

$$\phi = \frac{2\pi^2}{e^2} R^3 B^2 P^{-2} = 7.84 \times 10^{74} \rho_{14} B^{-5}.$$

(11)

Note that the potential in above equation would be higher if the effect of inclination angle is included (Yue et al. 2006). The drop could be high enough for pair production if the field $B \approx 10^{13}$ G, and the pulsar should then be radio loud.

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