A Study of P-wave Heavy Meson Interactions in A Chiral Quark Model

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The analytical forms of the interaction potentials between one S-wave and one P-wave heavy mesons as well as the potentials between two P-wave heavy mesons are deduced based on a chiral quark model. Our results explicitly show the attractive property between two heavy mesons. Consequently, a series of possible molecular states are obtained. It is expected that our study might shed some light on the popular discussions of the newly observed XYZ states.

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I. INTRODUCTION

The newly discovered XYZ states, such as new charmonium-like states of \( X(3872) \), \( X(3940) \), \( X(4050) \), \( X(4140) \), \( X(4160) \), \( X(4250) \), \( X(4260) \), \( X(4350) \), \( X(4360) \), \( X(4250) \), \( X(4350) \), \( X(4360) \), \( X(4260) \), \( X(4430) \), and bottomonium-like states of \( Z_b(10610) \), \( Z_b(10650) \), are of great interests, since they cannot be simply considered as normal quarkonium of \( c\bar{c} \) or \( b\bar{b} \), especially for the charged ones. For these XYZ states, many different explanations have been taken into account, including tetraquark, molecular states, hybrid charmonia, cusps, and threshold effects.

In our previous works [22, 23], the possible two S-wave heavy meson molecular explanations for the new resonances of \( X(3872) \), \( X(3940) \), \( Z_b(10610) \) and \( Z_b(10650) \) are obtained. In this paper we extend our study of the S-wave heavy meson interactions to the P-wave heavy meson interactions, and we try to explain the structures of the other XYZ states, particularly for \( X(4250) \), \( X(4350) \), \( X(4360) \), \( X(4260) \), \( X(4430) \). As far as \( X(4250) \) is concerned, it was explained as a \( f^G(J^{PC}) = 0^+(1^-) \) \( D^*\bar{D}_1 \) molecule by Close [24], however Nielsen et al. [25–27] considered it as a \( 1^+(1^-) \) \( D\bar{D}_1 \) molecule with QCD sum rules, while Ding [28] found that the \( D\bar{D}_1 \) and \( D_0^*\bar{D}_1 \) molecules couldn’t be the explanation for \( X(4250) \), but for \( X(4260) \). For the latter resonance, Close [24] and Nielsen et al. [25–27] both agreed with Ding’s conclusion. Moreover, Nielsen et al. [25–27] concluded that \( X(4430) \) could be regarded as a tetraquark or a \( D^*\bar{D}_1 \) molecule. Close [24] and Ding et al. [29] confirmed this molecular explanation, however, the calculation of Liu et al. [30] disfavored the \( D^*\bar{D}_1 \) molecular explanation using the resonating group method in our chiral quark model.

In this paper, our chiral quark model will be employed to study the heavy meson interactions. The model has been explained, in detail, in Refs. [31–38]. In this chiral SU(3) quark model, one-gluon-exchange (OGE), confinement potential, scalar meson exchange as well as pseudoscalar meson exchange are taken into account. Moreover, this chiral SU(3) quark model was developed into the extended chiral SU(3) quark model by introducing the vector meson exchanges. It has been proved that both models are quite successful in reproducing the spectra of the baryon ground states, the binding energy of deuteron, the nucleon-nucleon (NN), kaon-nucleon (KN) scattering phase shifts, and the hyperon-nucleon (YN) cross sections [31–34]. During the past years, by using the Resonating Group Method (RGM), Liu [30, 32, 37] and Wang [38] have studied the interactions and structures of the heavy-quark systems in both the chiral SU(3) quark model and the extended one.
Different from the RGM method, we have deduced analytical effective interaction potentials between the two S-wave mesons in our chiral quark models \[22, 23, 39\]. In this paper, we try to go further to deduce the analytical effective interaction potentials between one S-wave and one P-wave heavy mesons as well as between two P-wave heavy mesons. Then, we’ll systematically calculate the possible bound states associated with $D_0^{*}$, $D_1$, $D_2^{*}$ and their bottom partners $B_1$, $B_2^{*}$, by solving the Schrödinger equation with our analytical potentials, and try to explain the structures for some newly observed XYZ states, particularly for $\psi(4160)$, $X(4250)^{\pm}$, $X(4350)$, $X(4356)$, $X(4260)$, and $X(4430)^{\pm}$. We expect our approach could give a more accurate description of the short-range heavy meson-meson interactions than the RGM does, especially comparing with Ref. \[30\].

The paper is organized as follows. In section II the framework of our chiral quark models is briefly introduced, and the analytical forms of the effective interaction potentials between one S-wave meson and one P-wave meson (S-P) and between two P-wave heavy mesons (P-P) in our chiral quark models are given. The bound state solutions are shown and discussed in Sec. III Finally, a short summary is given in Sec. IV.

II. THE CHIRAL QUARK MODEL

The framework of our models has been discussed extensively in the literature \[31–38, 40–43\]. As mentioned in our previous papers \[22, 23, 39\], the internal kinetic energies and the internal interactions of each meson are not necessary to be calculated. The Hamiltonian of the relative motion, in our approach, takes the form of

$$H = T_{rel} + V_{eff},$$ \hspace{1cm} (1)

where $T_{rel}$ is the kinetic energy operator of the relative motion between the two mesons, and $V_{eff}$ is the effective interaction potential derived from the quark-quark (quark-antiquark) interaction between two mesons by integrating out the internal coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$ of the two mesons:

$$V_{eff} = \sum_{ij} \int \varphi_1^*(\vec{\xi}_1) \varphi_2^*(\vec{\xi}_2) V(\vec{r}_{ij}) \varphi_1(\vec{\xi}_1) \varphi_2(\vec{\xi}_2) d\vec{\xi}_1 d\vec{\xi}_2,$$ \hspace{1cm} (2)

while $\varphi_1(\vec{\xi}_1)$ and $\varphi_2(\vec{\xi}_2)$ are the intrinsic wavefunctions of the two mesons. If a meson is in S-wave, its wave function is taken as one-Gaussian form:

$$\varphi(\vec{\xi}) = \left(\frac{\mu \omega}{\pi}\right)^{3/4} e^{-\mu \omega \xi^2},$$ \hspace{1cm} (3)

and if in P-wave, it is

$$\varphi_{1m}(\vec{\xi}) = \frac{2\sqrt{7}}{3 \pi^{7/4}} (\mu \omega)^{5/4} e^{-\mu \omega \xi^2} Y_{1m}(\theta, \varphi).$$ \hspace{1cm} (4)

Here, $\mu$ is the reduced mass of the two quarks inside each meson cluster and $\omega$ is the harmonic-oscillator frequency of the meson intrinsic wavefunction. $V(\vec{r}_{ij})$ in eq. \[2\] represents the interactions between the $i$-th light quark or antiquark in the first meson and the $j$-th light quark or antiquark in another.

Because there is no color-interrelated interaction between the two color-singlet clusters, OGE interaction and confinement potential between two mesons don’t exist, we only consider the meson-exchange interactions between the two meson clusters. In our chiral SU(3) quark model, we have

$$V(\vec{r}_{ij}) = \sum_{\alpha=0}^{8} V^{\sigma=}(\vec{r}_{ij}) + \sum_{\alpha=0}^{8} V^{\pi=}(\vec{r}_{ij}),$$ \hspace{1cm} (5)
and in the extended chiral SU(3) quark model

\[ V(\vec{r}_{ij}) = \sum_{a=0}^{8} V_{\sigma a}(\vec{r}_{ij}) + \sum_{a=0}^{8} V_{\pi a}(\vec{r}_{ij}) + \sum_{a=0}^{8} V_{\rho a}(\vec{r}_{ij}), \]  

(6)

\( V_{\sigma a}(\vec{r}_{ij}) \) and \( V_{\pi a}(\vec{r}_{ij}) \) are the scalar meson exchange and pseudoscalar meson exchange interactions, respectively. \( V_{\rho a}(\vec{r}_{ij}) \) indicates the vector meson exchange interactions. For quark-quark (antiquark-antiquark) interaction, the explicit forms of \( V_{\sigma a}(\vec{r}_{ij}) \), \( V_{\pi a}(\vec{r}_{ij}) \) and \( V_{\rho a}(\vec{r}_{ij}) \) have been described, in detail, in Refs. [22, 31-38, 40-43] as

\[
V_{\sigma a}(\vec{r}_{ij}) = -C(g_{ch}, m_{\sigma a}, \Lambda) X_1(m_{\sigma a}, \Lambda, r_{ij}) (\lambda^a_\sigma \lambda^a_\sigma),
\]

(7)

\[
V_{\pi a}(\vec{r}_{ij}) = C(g_{ch}, m_{\pi a}, \Lambda) \frac{m_{\pi a}^2}{12m_im_j} X_2(m_{\pi a}, \Lambda, r_{ij}) (\sigma_i \cdot \sigma_j) (\lambda^a_\sigma \lambda^a_\sigma),
\]

(8)

\[
V_{\rho a}(\vec{r}_{ij}) = C(g_{chv}, m_{\rho a}, \Lambda) \left[ X_1(m_{\rho a}, \Lambda, r_{ij}) + \frac{m_{\rho a}^2}{6m_im_j} \left( 1 + \frac{f_{chv} m_i + m_j}{g_{chv} M_N} + \frac{f_{chv}^2 m_i m_j}{g_{chv}^2 M_N^2} \right) \right] 
\times X_2(m_{\rho a}, \Lambda, r_{ij}) (\sigma_i \cdot \sigma_j) (\lambda^a_\sigma \lambda^a_\sigma),
\]

(9)

with

\[
C(g_{ch}, m, \Lambda) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2},
\]

(10)

\[
X_1(m, \Lambda, r_{ij}) = Y(m r_{ij}) - \Lambda m Y(\Lambda r_{ij}),
\]

(11)

\[
X_2(m, \Lambda, r_{ij}) = Y(m r_{ij}) - \left( \frac{\Lambda}{m} \right)^3 Y(\Lambda r_{ij}),
\]

(12)

\[
Y(x) = \frac{1}{x} e^{-x},
\]

(13)

where \( \lambda^a \) is the Gell-Mann matrix in flavor space, and \( \Lambda \) is the cutoff mass indicating the chiral symmetry breaking scale. In eqs. (7-13), \( m_i \) and \( m_j \) are the masses of the \( i \)-th light quark or antiquark in the first meson and the \( j \)-th light quark or antiquark in another, respectively, while \( m_{\sigma a}, m_{\pi a} \) and \( m_{\rho a} \) in eqs. [22, 31-38, 40-43] are the masses of the scalar, pseudoscalar and vector nonets, respectively. \( M_N \) in eq. [22, 31-38, 40-43] is a mass scale usually taken as the mass of nucleon \( \frac{m_N}{2} \). \( g_{ch} \), in the above eqs., is the coupling constants for the scalar and pseudoscalar nonets, while \( g_{chv} \) and \( f_{chv} \) are the coupling constants for the vector coupling and tensor coupling of vector nonets, respectively. For the quark-antiquark interactions, the G-parity of the exchanged mesons should also be taken into account.

In the two heavy meson systems, we don’t consider the one-meson exchange interactions between two heavy quarks or between one heavy quark and one light quark, because these interactions are beyond our SU(3) models. By using the method in Refs. [22, 23, 39] and integrating out the internal coordinates of two mesons as described by eq. [24], we can get the total analytical effective interaction potentials between two S-wave heavy mesons. Analogy to the two S-wave systems, we can go further to deduce the effective interaction potentials between one P-wave heavy meson and one S-wave heavy meson and between two P-wave Heavy mesons. These effective interaction potentials are listed as follows.

\[
V_{eff}(\vec{R}) = \sum_{a=0}^{8} V_{\sigma a}(\vec{R}) + \sum_{a=0}^{8} V_{\pi a}(\vec{R}) + \sum_{a=0}^{8} V_{\rho a}(\vec{R}),
\]
with

\[
V_{q\bar{q}}^\sigma (\vec{R}) = -G_{\sigma, a} C(g_{ch}, m_{\sigma a}, \Lambda) X_{1q\bar{q}}(m_{\sigma a}, \Lambda, R) \left( \lambda_1^a \lambda_1^b \right), \tag{14}
\]

\[
V_{q\bar{q}}^\pi (\vec{R}) = G_\pi C(g_{ch}, m_{\pi a}, \Lambda) \frac{m_{\pi a}^2}{12m_q m_{\bar{q}}} X_{2q\bar{q}}(m_{\sigma a}, \Lambda, R) \left( \sigma_q \cdot \sigma_{\bar{q}} \right) \left( \lambda_1^a \lambda_1^b \right), \tag{15}
\]

\[
V_{q\bar{q}}^{\rho_{\bar{c}}} (\vec{R}) = G_{\rho_{\bar{c}}} C(g_{ch}, m_{\rho_{\bar{c}}}, \Lambda) \left[ X_{1q\bar{q}}(m_{\rho_{\bar{c}}}, \Lambda, R) + \frac{m_{\rho_{\bar{c}}}^2}{6m_q m_{\bar{q}}} \left( 1 + \frac{f_{ch} m_q + m_{\bar{q}}}{g_{ch} M_N} + \frac{f_{ch}^2 m_q m_{\bar{q}}}{g_{ch}^2 M_N^2} \right) \right] \times X_{2q\bar{q}}(m_{\rho_{\bar{c}}}, \Lambda, R) \left( \sigma_q \cdot \sigma_{\bar{q}} \right) \left( \lambda_1^a \lambda_1^b \right). \tag{16}
\]

Here, \( m_q \) and \( m_{\bar{q}} \) are masses of the light quark and antiquark, respectively, \( G_{\sigma, \pi, \rho_{\bar{c}}} \) is the \( G \)-parity of the exchanged meson, and \( \vec{R} \) is the relative coordinate between the two different mesons, namely, the relative coordinate between the centers-of-mass coordinates of the two mesons. Moreover in eqs. (14-16)

\[
X_{1q\bar{q}}(m, \Lambda, R) = Y_{q\bar{q}}(mR) - \frac{\Lambda}{m} Y_{q\bar{q}}(\Lambda R), \tag{17}
\]

\[
X_{2q\bar{q}}(m, \Lambda, R) = Y_{q\bar{q}}(mR) - \left( \frac{\Lambda}{m} \right)^3 Y_{q\bar{q}}(\Lambda R), \tag{18}
\]

where the \( Y_{q\bar{q}}(mR) \) is the modified Yukawa term.

For the two S-wave heavy meson interactions, as mentioned in [22, 23, 39], the modified Yukawa term in eqs. (17,18) reads

\[
Y_{q\bar{q}}(mR) = \frac{1}{2mR} e^{\frac{m^2}{2}} \left\{ e^{-mR} \left[ 1 - erf \left( -\sqrt{\beta} (R - \frac{m}{2\beta}) \right) \right] - e^{mR} \left[ 1 - erf \left( \sqrt{\beta} (R + \frac{m}{2\beta}) \right) \right] \right\}. \tag{19}
\]

For the one S-wave heavy meson and one P-wave heavy meson interactions, they receive the contributions from direct and exchange terms. The modified Yukawa term of the direct term reads

\[
Y_{q\bar{q}}(mR) = \frac{1}{3mR} e^{\frac{m^2}{2}} \left\{ e^{-mR} \left( \frac{m^2 q^2 m^2}{4\omega (m_Q + m_{\bar{Q}}) q_{\bar{Q}}} + \frac{3}{2} \right) \left[ 1 - erf \left( -\sqrt{\beta} (R - \frac{m}{2\beta}) \right) \right] \right. \\
\left. -e^{mR} \left[ 1 - erf \left( \sqrt{\beta} (R + \frac{m}{2\beta}) \right) \right] \right\} - \beta^2 R e^{\frac{-m^2 R^2}{2\omega (m_Q + m_{\bar{Q}}) q_{\bar{Q}}}} \tag{20}
\]

and the one of the exchange term, associated with the charge-parity \( C \) in flavor wave functions (see eqs. [27, 28] in section III), reads

\[
Y_{q\bar{q}}(mR) = \frac{1}{3mR} \left( m_{q\bar{Q}} \omega \right)^2 \left( m_{q\bar{Q}} \omega \right)^2 \left( - \frac{m_{\bar{Q}}}{m_{q} + m_{\bar{Q}}} \frac{m_{Q}}{m_{\bar{Q}}} \partial_a + \frac{m_{q} + m_{\bar{Q}}}{4m_{\bar{Q}}} \frac{m_{q} + m_{\bar{Q}}}{m_{\bar{Q}}} \partial_b \right) \\
\times \left\{ a\beta \frac{1}{2} e^{\frac{m^2}{2}} \left[ e^{-mR} \left[ 1 - erf \left( -\sqrt{\beta} (R - \frac{m}{2\beta}) \right) \right] - e^{mR} \left[ 1 - erf \left( \sqrt{\beta} (R + \frac{m}{2\beta}) \right) \right] \right] \right\}. \tag{21}
\]
Moreover, for the two P-wave heavy meson interaction, the modified Yukawa term is

\[ Y_{qq}(mR) = \frac{2}{9mR} \left\{ \frac{e^{mR}}{4\omega(m_Q + m_q)^2\mu_{qQ}} \left( \frac{m_Q^2m_q^2}{4\omega(m_Q + m_q)^2\mu_{qQ}} + \frac{3}{2} \right) \right. \]

\[ \times \left\{ e^{-mR} \left[ 1 - erf \left( - \sqrt{\beta(R - \frac{m}{2\beta})} \right) \right] - e^{mR} \left[ 1 - erf \left( \sqrt{\beta(R + \frac{m}{2\beta})} \right) \right] \right. \]

\[ + \frac{m_Q^2}{(m_Q + m_q)^2\mu_{qQ}} \left( \frac{m_Q^2}{(m_Q + m_q)^2\mu_{qQ}} \right) Re^{-\beta R^2} \left( - \frac{m^2}{4} \beta \frac{1}{2} + \frac{3}{2} \beta \frac{1}{2} + R^2 \beta \frac{1}{2} \right) \]

\[ \left. - \frac{3\beta^2}{2\omega} Re^{-\beta R^2} \left[ \frac{m_Q^2}{(m_Q + m_q)^2\mu_{qQ}} + \frac{m_Q^2}{(m_Q + m_q)^2\mu_{qQ}} \right] \right\}, \quad (22) \]

In the above equations,

\[ \beta = b - \frac{c^2}{4a}, \quad (23) \]

where

\[ a = \mu_{qQ}\omega \left( \frac{m_Q}{m_q + m_Q} \right)^2 + \mu_{qQ}\omega \left( \frac{m_Q}{m_q + m_Q} \right)^2, \quad (24) \]

\[ b = \mu_{qQ}\omega \left( \frac{m_q + m_Q}{m_q} \right)^2 + \mu_{qQ}\omega \left( \frac{m_q + m_Q}{m_q} \right)^2, \quad (25) \]

\[ c = \mu_{qQ}\omega \frac{m_q + m_Q}{m_q + m_Q} \frac{m_Q}{m_q + m_Q} - \mu_{qQ}\omega \frac{m_q + m_Q}{m_q + m_Q} \frac{m_Q}{m_q + m_Q}, \quad (26) \]

and \( m_Q \) and \( m_q \) are the masses of the heavy quark and antiquark, respectively, and \( \mu_{qQ} = \frac{m_qm_Q}{m_q + m_Q} \).

In this work, we take our model parameters, which have been determined in our previous works, as follows \(^{31,38,40,42}\). The up/down quark mass \( m_q \) is fitted as the nucleon mass and taken as \( M_N/3 \sim 313 \text{ MeV} \). The coupling constant for the scalar and pseudoscalar chiral fields \( g_{ch} = 2.621 \) is fixed by the relation of

\[ \frac{g_{ch}^2}{4\pi} = \frac{9}{25} \frac{g_{NN\pi}^2}{M_N^2} \]

with \( g_{NN\pi}^2/4\pi = 13.67 \) determined from experiments.

In our extended chiral SU(3) quark model, the vector coupling constant \( g_{chv} \) and tensor coupling constant \( f_{chv} \) in eqs. \(^9,16\) are fitted by the mass difference between \( N \) and \( \Delta \), under the condition that the strength of the OGE is taken to be almost zero. When the tensor coupling is neglected, \( g_{chv} = 2.351 \) and \( f_{chv} = 0 \); and when the tensor coupling is considered, \( g_{chv} = 1.973 \) and \( f_{chv} = 1.315 \). The harmonic-oscillator frequency \( \omega \) (being as \( 1/(m_u b_u^2) = 1/(m_d b_d^2) \) where \( b_u \) is fitted by the \( N - N \) scattering phase shifts) is taken as \( 2.522 \text{fm}^{-1} \) in the chiral SU(3) quark model and \( 3.113 \text{fm}^{-1} \) in the extended chiral SU(3) quark model, respectively. In our calculation, the masses of the mesons are taken from the PDG \(^{44}\), except for the \( \sigma \) meson, which does not have a well-defined value. Here \( m_{\sigma} \) is obtained by fitting the binding energy of deuteron \(^{33}\). It is \( m_{\sigma} = 595 \text{ MeV} \) in our chiral SU(3) quark model, 535 MeV for neglecting tensor coupling and 547 MeV for considering tensor coupling in our extended chiral SU(3)
quark model. The cutoff mass $\Lambda$ means the chiral symmetry breaking scale and is taken as 1100 MeV as a convention. In addition, we find that the final results are not sensitive to the variation of the heavy quark masses, and we take $m_c = 1430$ MeV \cite{45} and $m_b = 4720$ MeV \cite{46} as the two typical values.

III. RESULTS AND DISCUSSIONS

To make sure the two-heavy meson systems have definite quantum numbers of isospin $I$ and $C$-parity $C$, we follow the definitions of the flavor wavefunctions in Refs. \cite{22, 36, 47, 48}. For the hidden-charm systems constituted by two different mesons $\bar{D}'$ and $\bar{D}$, the flavor wavefunctions are

$$
I = 1:\begin{cases}
\frac{1}{\sqrt{2}}(D'^0 + cD^0) \\
\frac{1}{\sqrt{2}}(D'^0 - cD^0) \\
\frac{1}{2}((D^0\bar{D}^0 - D^+\bar{D}^-) + c(D^0\bar{D}^0 - D^+\bar{D}^-)),
\end{cases}
$$

$$
I = 0: \frac{1}{\sqrt{2}}[(D^0\bar{D}^0 + D'^0\bar{D}^-) + c(D^0\bar{D}^0 + D'^0\bar{D}^-)].
$$

(27)

Here, $c=1$ for $C=+$ and $c=-1$ for $C=-$. As for $\bar{D}$ meson-anti $\bar{D}$ meson systems, the flavor functions are

$$
I = 1:\begin{cases}
D^+\bar{D}^0 \\
D^-\bar{D}^0 \\
\frac{1}{\sqrt{2}}(D^0\bar{D}^0 - D^+\bar{D}^-),
\end{cases}
$$

$$
I = 0: \frac{1}{\sqrt{2}}(D^0\bar{D}^0 + D^+\bar{D}^-).
$$

(29)

For the two $\bar{D}$ systems, the similar expressions can be obtained.

A. Two $D$ mesons

1. S-P: $DD_1$, $(DD'_0, DD'_2, D^*_1)$ and $(D^*_1, D^* D'_2)$

For the two pseudoscalar meson system of $DD_1$, the spin factor $<\sigma_q \cdot \sigma_{\bar{q}}>$ in eqs. \cite{15, 16} is 0. Therefore, the spin-correlated interactions don’t exist. Its interaction is associated with isospin $I$ and $C$-parity $C$. We find two bound states, and their binding energies are shown in Table I. In the case of $I = 0 C = +$, the $I^G(J^{PC}) = 0^+(1^{+-})$ $DD_1$ bound state has a mass of 4253-4285 MeV. In the case of $I = 0 C = -$, the total interaction potential is shown in Fig. 1 and the $0^-(1^{--})$ $DD_1$ bound state has a mass of 4264-4285 MeV, Its mass and quantum umbers match the $X(4260)$. So in our approach, the $0^-(1^{--})$ $DD_1$ molecule might be an explanation to the state of $X(4260)$.

For the systems of $DD'_0$, $DD'_2$ and $D^*_1$, because they share the same interaction potential with the same isospin and $C$-parity, we put them together in discussion. We find the total interaction potentials in all cases are attractive, due to the strong attraction provided by $\sigma$ exchange. In the $I=0 C=+$ case, the spin factor $<\sigma_q \cdot \sigma_{\bar{q}}>$ in eqs. \cite{15, 16} is 1, and the attractive interaction potential, as shown in Fig. 2 is especially strong, because $\pi$ and $\sigma'$ exchanges also provide strong attraction.
FIG. 1: The total interaction potential of $I=0$ and $C=-D\bar{D}_1$ system. The solid, dashed and dotted lines represent the results obtained from the chiral SU(3) quark model, the extended chiral SU(3) quark model neglecting and including the tensor coupling of vector field, respectively.

FIG. 2: The total interaction potential of $I=0$ and $C=+D\bar{D}_0^*, D\bar{D}_2^*$ and $D^*\bar{D}_1$ systems. The solid, dashed and dotted lines represent the same meaning as in Fig. 1.

Not only in this $I=0$ $C=+$ case but also in the case of $I=0$ $C=-$ can we find the bound states after solving the Schrödinger equation. We list the obtained binding energies in Table I and one may find the binding energies, for the systems of $D\bar{D}_0^*$, $D\bar{D}_2^*$ and $D^*\bar{D}_1$, are almost the same, and the small differences are due to the small differences among their reduced masses.

We find the $0^+(0^{++})$ $D\bar{D}_0^*$ molecule has a mass of 4154-4181 MeV, and it could be a plausible explanation for the resonance of $X(4160)$. The mass of $0^-(0^{--})$ $D\bar{D}_0^*$ molecule, in our calculation, is about 4160-4183 MeV. Moreover, we get the $0^+(2^{++})$ $D\bar{D}_2^*$ molecule with the mass of 4297-4325 MeV, and it could be an explanation for $X(4350)$. The $0^-(2^{--})$ $D\bar{D}_2^*$ molecule has a mass of 4303-4326 MeV, and the $0^+(J^{++})$ $D^*\bar{D}_1$ or $0^-(J^{--})$ $D^*\bar{D}_1$ molecules has a mass of 4400-4429 MeV or 4406-4430 MeV, respectively. However, no $I=1$ $D^*\bar{D}_1$ molecule could be found in our model, because $\pi$ and $\sigma$' exchanges are both repulsive. Therefore, our result doesn’t favor the $D^*\bar{D}_1$ molecular explanation of the isovector
TABLE I: Binding energies (MeV) of the bound states of S-P D meson systems. I, II and III refer to the chiral SU(3) quark model, the extended chiral SU(3) quark model neglecting and considering tensor coupling of vector field, respectively.

|               | $I^G(J^{PC})$ | I   | II  | III |
|---------------|----------------|-----|-----|-----|
| $D\bar{D}_1$ | $0^+(1^{-+})$  | 0.4 | 32.4| 22.6|
|               | $0^-(1^{--})$  | 0.3 | 21.0| 12.0|
| $D\bar{D}_0$ | $0^+(0^{++})$  | 1.7 | 29.3| 19.6|
|               | $0^-(0^{--})$  | 0.4 | 23.2| 15  |
| $D\bar{D}_2$ | $0^+(2^{++})$  | 1.8 | 29.7| 20.0|
|               | $0^-(2^{--})$  | 0.5 | 23.6| 15.4|
| $D^*\bar{D}_1$| $0^+(J^{-+})$  | 2.2 | 31.2| 21.2|
|               | $0^-(J^{--})$  | 0.7 | 24.7| 16.3|
| $D^*\bar{D}_0$| $0^+(1^{++})(S=2)$ | 21.8| 30.7| 30.3|
|               | $0^-(1^{--})(S=2)$ | 5.9 | 12.6| 12.1|
| $D^*\bar{D}_2$| $0^+(J^{-+})(S=2)$ | 22.3| 31.3| 30.9|
|               | $0^-(J^{--})(S=2)$ | 6.1 | 12.9| 12.4|

$X(4430)^\pm$. This result is consistent with the conclusion of Liu [30], and contrary to the results of Close [24] and Ding [29].

For the two different vector meson systems, such as $D^*\bar{D}_0$, $D^*\bar{D}_2$, their interaction potentials are simultaneously associated with the isospin $I$, the $C$-parity $C$ and the total spin $S$. In the $I=0$, $C=+$, $S=2$ case, the spin factor $<\sigma_q \cdot \sigma_{\bar{q}}>$, in eqs. (15-16), is 2, which is different from that of $I=0$ $C=+$ of the $D\bar{D}_0$, $D\bar{D}_2$ and $D^*\bar{D}_1$ systems. We find that the $D^*\bar{D}_0$ and $D^*\bar{D}_2$ systems share the same interaction potential shown in Fig. 3 and correspondingly we get the bound states with the binding energies listed in the Table II. In this case the effects of the vector meson exchanges are small in the extended chiral SU(3) quark model. In the case of $I=1$, the bound states also exist.

In our calculation, the mass of $0^+(1^{++})(S=2)$ $D^*\bar{D}_0$ molecule is of 4297-4306 MeV and we find the $0^-(1^{--})(S=2)$ $D^*\bar{D}_0$ molecule has a mass of 4315-4322 MeV, and it might be explained as $X(4360)$. No bound states could be found in the $I=1$ cases in our approach. Our result agrees with Ding [28] that $X(4250)^\pm$ might not be explained as a isovector $D^*\bar{D}_0^*$ molecule. Our results also disfavor the $D^*\bar{D}_0^*$ molecular explanation of $X(4260)$ [25, 27]. Since we ignore the spin-orbital coupling interaction in our calculation, the $0^+(J^{-+})(S=2)$ $D^*\bar{D}_2$ molecule with mass of 4441-4452 MeV and the $0^-(J^{--})(S=2)$ $D^*\bar{D}_2$ molecule with mass of 4459-4466 MeV are degenerate states for the total angular momentum $J$.

2. P-P: $D_1\bar{D}_1$, ($D_1\bar{D}_0$, $D_1\bar{D}_2$) and ($D_0^*\bar{D}_0^*$, $D_2^*\bar{D}_2^*$, $D_0^*\bar{D}_2$)

For the system of $D_1\bar{D}_1$, its interaction is only associated with isospin $I$. It could form a $0^+(J^{++})$ weakly bound state with binding energy listed in Table III. This bound state has a mass of 4824-4842 MeV in our approach. The interaction potential is shown in Fig. 4.
FIG. 3: The total interaction potential of $I=0 \ C=+ \ S=2 \ D^*\bar{D}^*_0$ and $D^*\bar{D}^*_2$ systems. The solid, dashed and dotted lines represent the same meaning as in Fig. 1.

For the $D_1\bar{D}_0$ and $D_1\bar{D}_2$ systems with the same quantum numbers, they share the same interaction potential. Also in the $I=0 \ C=+$ case the interaction potential becomes strong enough to form bound states. This interaction potential is shown in Fig. 2. We see that the interaction potential in the extended chiral SU(3) quark model is weaker than that in the chiral SU(3) quark model, because the vector meson exchanges, in this case, provide repulsive interaction, however the range of the total potential is longer than that of the chiral SU(3) quark model. As a result, the vector meson exchanges don’t obviously change the binding energies as listed in Table II. In our approach, the $0^+(1^{++}) \ D_1\bar{D}_0^*$ and $0^+(J^{++}) \ D_1\bar{D}_2^*$ molecules have the masses of 4719-4727 MeV and 4863-4870 MeV, respectively.

For the two vector systems of $D_0^*\bar{D}_0^*, \ D_2^*\bar{D}_2^*$ and $D_0^*\bar{D}_2^*$, they share the same interaction corresponding to the same isospin $I$ and total spin $S$, and the possible bound states are also listed in Table II. In the case of $I=0 \ S=2 \ D_0^*\bar{D}_0^*, \ D_2^*\bar{D}_2^*$ and $D_0^*\bar{D}_2^*$ systems, the spin factor $<\sigma_q \cdot \sigma_{\bar{q}}>$ in eqs. (15, 16) is 1, the same as that of the $I=0 \ C=+ \ D_1\bar{D}_0^*$ and $D_1\bar{D}_2^*$ systems. Here the interaction potential is shown.
FIG. 5: The total interaction potential of $I=0$ and $C=\pm$ $D_1\bar{D}_0$ and $D_1\bar{D}_2$ systems and $I=0$ $S=2$ $D_0\bar{D}_0$, $D_2\bar{D}_2$ and $D_0\bar{D}_2$ systems. The solid, dashed and dotted lines represent the same meaning as in Fig. 1.

TABLE II: Binding energies (MeV) of the bound states of P-P D meson systems. I, II and III refer to the same meaning as in Table I.

| $I^G(J^{PC})$   | I    | II   | III  |
|-----------------|------|------|------|
| $D_1\bar{D}_1$  | $0^+(J^{++})$ | 0.4  | 18.2 | 12.0 |
| $D_1\bar{D}_0$  | $0^+(1^{++})$  | 12.2 | 19.5 | 12.4 |
| $D_1\bar{D}_2$  | $0^+(J^{++})$  | 12.5 | 19.8 | 12.4 |
| $D_0\bar{D}_0$  | $0^+(0^{++})(S=2)$ | 12.1 | 19.3 | 12.4 |
|                 | $0^-(0^{+-})(S=1)$ | –    | 16.7 | 11.4 |
|                 | $0^+(0^{++})(S=0)$ | –    | 15.6 | 11.3 |
| $D_2\bar{D}_2$  | $0^+(J^{++})(S=2)$ | 12.7 | 20.0 | 12.9 |
|                 | $0^-(J^{+-})(S=1)$ | –    | 17.5 | 12.1 |
|                 | $0^+(J^{++})(S=0)$ | –    | 16.3 | 11.9 |
| $D_0\bar{D}_2$  | $0^+(2^{++})(S=2)$ | 12.4 | 19.7 | 12.6 |
|                 | $0^+(2^{++})(S=1)$ | –    | 17.1 | 11.7 |
|                 | $0^+(2^{++})(S=0)$ | –    | 16.0 | 11.6 |

in Fig. 5, which is strong enough to form the bound states in our models. Our results tell the possible $0^+(0^{++})$ $D_0\bar{D}_0$ molecule with the mass of 4617-4624 MeV, the $0^+(J^{++})$ $D_2\bar{D}_2$ molecule with the mass of 4904-4911 MeV and the possible $0^+(0^{++})$ $D_0\bar{D}_2$ molecule has a mass of 4760-4768 MeV.

For the systems of $I=0$ $S=0,1$ $D_0\bar{D}_0$, $D_2\bar{D}_2$ and $D_0\bar{D}_2$, we get the bound states in our extended chiral SU(3) quark model, but no bound states in our chiral SU(3) quark model. This is because, in our extended chiral SU(3) quark model, vector meson exchanges provide additional strong attractive interaction. Even though we can’t currently draw a definite conclusion whether or not the molecular
bound states could exist, those states provide a test for our two models.

As a short summary, we have studied twelve sets of the heavy meson interactions associated with P-wave D mesons. They are, S-P: $D\bar{D}_1^0$, $(D\bar{D}_1^0, D\bar{D}_2^0, D^*\bar{D}_1)$ and $(D^*\bar{D}_1^0, D^*\bar{D}_2^0)$; and P-P: $D_1\bar{D}_1$, $(D_1\bar{D}_1^0, D_1\bar{D}_2^0)$ and $(D_1\bar{D}_1^0, D_1\bar{D}_2^0, D_0\bar{D}_2^0)$. We’ve found eighteen possible bound states, and they are all isoscalar. Only the four possible molecules of $0^+(0^+) D\bar{D}_1^0$, $0^-(1^-) D^*\bar{D}_1^0$, $0^-(1^-) D\bar{D}_1$, and $0^+(2^+) D\bar{D}_2^0$ could be explained as $X(4160)$, $X(4360)$, $X(4260)$, $X(4350)$, respectively. Other bound states don’t match any observed XYZ states.

B. Two B masons

Considering the resemblance between B mesons and D mesons, the corresponding binding systems should have the similar properties, while the P-wave B meson bound states should have larger binding energies because of their larger reduced masses.

1. S-P: $B\bar{B}_1$, $(B\bar{B}_1^*, B_1\bar{B}^*)$ and $B^*\bar{B}_2$

As for the $B\bar{B}_1$ system, similar to the $D\bar{D}_1$ system, it becomes bound in the $I = 0$ $C = \pm$ cases and the obtained binding energies are listed in the Table III. In the $I = 0$ $C = -$ case, the $0^+(1^-) BB_1$ molecule has a mass of 10932-10980 MeV, and in the $I = 0$ $C = +$ case, the $0^-(1^-) BB_1$ molecule has a mass of 10955-10993 MeV.

For $BB_1^*$ and $B_1\bar{B}^*$, the systems of one S-wave B meson and one P-wave B meson, they share the same interaction potential corresponding to the same isospin and C-parity. In the $I=0$ case the interaction potential is strong enough to bind $BB_1^*$ and $B_1\bar{B}^*$. For example, in the $I=0$ and $C=+$ case, the interaction potential is shown in Fig. 6 and the obtained binding energies are listed in Table III. We find that the

![FIG. 6: The total interaction potential of $I=0$ $C=+$ $BB_1^*$ and $B_1\bar{B}^*$ systems. The solid, dashed and dotted lines represent the same meaning as in Fig. 4.](image)

$0^+(2^+) BB_2^*$ and $0^-(2^-) BB_2^*$ molecules have the mass of 10962-11009 MeV and 10977-11018 MeV, respectively. In our calculation the $0^+(J^-) B_1\bar{B}^*$ molecule, without a definite total angular momentum $J$, has a mass of 10970-11017 MeV, and the mass of $0^-(J^-) B_1\bar{B}^*$ molecule is about 10985-11026 MeV.
TABLE III: Binding energies (MeV) of the bound states of S-P $B$ meson systems. I, II and III refer to the same meaning as that in Table I.

| $I^G(J^{PC})$ | I  | II | III  |
|---------------|----|----|------|
| $B\bar{B}_1$ | 0$^+(1^{-+})$ | 20.0 | 69.0 | 56.1 |
|               | 0$^-(1^{--})$ | 7.7  | 46.1 | 34.5 |
| $B\bar{B}_2^*$ | 0$^+(2^{-+})$ | 17.8 | 64.6 | 51.2 |
|               | 0$^-(2^{--})$ | 9.2  | 50.1 | 38.6 |
| $B_1\bar{B}^*$ | 0$^+(J^{+})$ | 17.8 | 64.7 | 51.3 |
|               | 0$^-(J^{--})$ | 9.2  | 50.2 | 38.6 |
| $B^*\bar{B}_2^*$ | 0$^+(J^{++})(S=2)$ | 66.8 | 97.8 | 91.7 |
|               | 0$^+(J^{++})(S=1)$ | 2.2  | 9.4  | 7.7  |
|               | 0$^-(J^{--})(S=2)$ | 24.9 | 40.8 | 40.1 |

For the $B^*\bar{B}_2^*$ system, the total interaction potential is associated with isospin $I$, $C$-parity $C=\pm$ and total spin $S$. The possible bound states are listed in the Table III. The interaction potential between $B^*\bar{B}_2^*$ and in the case of $I=0\ C=\pm\ S=2$ is shown in Fig. 7. This interaction potential is strong enough to bind

![Image of Fig. 7](image)

FIG. 7: The total interaction potential of $I=0\ C=\pm\ S=2$ $B^*\bar{B}_2^*$ system. The solid, dashed and dotted lines represent the same meaning as in Fig. I.

$B^*\bar{B}_2^*$ system, and the $0^+(J^{++})(S=2)$ $B^*\bar{B}_2^*$ molecule has a mass of 10974-11005 MeV in our calculation. In the case of $I=0\ C=\pm\ S=1$, the mass of $0^+(J^{++})(S=1)$ $B^*\bar{B}_2^*$ molecule is about 11063-11070 MeV, while in the case of $I=0\ C=\pm\ S=2$, the $0^-(J^{--})(S=2)$ $B^*\bar{B}_2^*$ molecule has a mass of 11061-11047 MeV.
2. P-P: $B_1 \bar{B}_1$, $B_1 B_2^*$ and $B_2^* \bar{B}_2^*$

For the two pseudoscalar meson system of $B_1 \bar{B}_1$, the total spin $S$ is 0, like $D_1 \bar{D}_1$, the total interaction potential is only associated with the isospin $I$. Like the $D_1 \bar{D}_1$ system, $B_1 \bar{B}_1$ forms a bound state in the $I=0$ case in our models as shown in Table IV. Our $0^+(J^{++})$ $B_1 \bar{B}_1$ molecule has a mass of 11408-11436 MeV.

For the $B_1 \bar{B}_2^*$, like the $D_1 \bar{D}_2^*$ system, the interaction potential shown in Fig. 8, in the case of $I=0$ and $C=+$, is strong to bind the $B_1 \bar{B}_2^*$. The obtained binding energies in this case are listed in the Table IV.

![Fig. 8](image)

We find that the $0^+(J^{++})$ $B_1 \bar{B}_2^*$ molecule has a mass of 11435-11444 MeV.

| $I^G(J^{PC})$ | I | II | III |
|---------------|---|----|----|
| $B_1 \bar{B}_1$ | $0^+(J^{++})$ | 7.1 | 39.5 | 30.5 |
| $B_1 B_2^*$ | $0^+(J^{++})$ | 31.1 | 33.2 | 24.3 |
| | $0^-(J^{+-})$ | – | 20.2 | 14.2 |
| $B_2^* \bar{B}_2^*$ | $0^+(J^{++})(S=2)$ | 31.2 | 32.5 | 23.7 |
| | $0^-(J^{+-})(S=1)$ | – | 36.8 | 30.4 |
| | $0^+(J^{++})(S=0)$ | – | 40.1 | 35.7 |

As for the system of $B_2^* \bar{B}_2^*$, the interaction potential is associated with isospin $I$ and the total spin $S$. Similar to the $D_2^* \bar{D}_2^*$ system, we find a bound state in the $I=0$ $S=2$ case where the interaction potential is just the same as the $I=0$ $C=+$ $B_1 \bar{B}_2^*$ system shown in Fig. 8. The binding energies are also listed in the Table IV. In our approach, the $0^+(J^{++})(S=2)$ molecule has a mass of 11454-11465 MeV.

To summarize this subsection, we have studied seven sets of heavy meson interactions associated with P-wave B mesons. They are S-P: $B B_2^*$, $(B^* B_1, \bar{B} B_1)$, and $B^* B_2^*$ and P-P: $B_1 B_2^*$, $B_2^* \bar{B}_2^*$, and $B_1 \bar{B}_1$. 
We’ve found twelve possible bound states and they are all isoscalar. Unfortunately, none of these bound states could match any newly observed XYZ states.

IV. SUMMARY

In this work, we have employed our SU(3) chiral quark model to systematically studied the heavy meson interactions associated with the P-wave. The parameters of our models have already fixed in our previous calculations from the spectra of baryon ground states, the nucleon-nucleon phase shifts and the deuteron binding energy. We have gotten some possible molecular states. For the systems of P-wave B-mesons, unfortunately, those molecules don’t match to any of the newly observed XYZ hadronic states. For P-wave D meson interactions, we find that the $X(4350)$ could be explained as the $0^+(2^{-+})$ $D\bar{D}_2^*$ molecule, the $X(4360)$ could be explained as the $0^-(1^{--})$ $D^*\bar{D}_0$ molecule, and the $X(4160)$ could be explained as the $0^+(1^{++})$ $D\bar{D}_0$ molecule. However, the $X(4430)^\pm$ could not be explained as a isovector $D_1D^*$ charged molecule. Moreover, neither $X(4250)^\pm$ nor $X(4260)$ could be explained as the $D^*\bar{D}_0^*$ molecule, but $X(4260)$ might be explained as a $0^-(1^{--})$ $D\bar{D}_1$ molecule. Finally, we find that we can get bound states in our extended chiral SU(3) quark model in some cases but not in the chiral SU(3) quark model. This is because the vector meson exchanges in the extended chiral SU(3) quark model provide additional attraction. Future experimental results about these special cases would give a discriminates of our two models.

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[1] Belle Collaboration, S.K. Choi et al., Phys. Rev. Lett. 91, 262001 (2003).
[2] CDF Collaboration, D. Acosta et al., Phys. Rev. Lett. 93, 072001 (2004).
[3] D0 Collaboration, V.M. Abazov et al., Phys. Rev. Lett. 93, 162002 (2003).
[4] Babar Collaboration, B. Aubert et al., Phys. Rev. D 71, 071103 (2005).
[5] Belle Collaboration, K. Abe et al., Phys. Rev. Lett. 98, 082001 (2007).
[6] Belle Collaboration, S.K. Choi et al., Phys. Rev. Lett. 94, 182002 (2005).
[7] Babar Collaboration, B. Aubert et al., Phys. Rev. Lett. 101, 082001 (2008).
[8] Babar Collaboration, J. P. Lees et al., Phys. Rev. D 85, 052003 (2012).
[9] Belle Collaboration, R. Mizuk et al., Phys. Rev. D 78, 072004 (2008).
[10] Belle Collaboration, C.P. Shen et al., Phys. Rev. Lett. 104, 112004 (2010).
[11] CDF Collaboration, T. Aaltonen et al., Phys. Rev. Lett. 102, 242002 (2009).
[12] Belle Collaboration, P. Pakhlov et al., Phys. Rev. Lett. 100, 202001 (2008).
[13] Babar Collaboration, B. Aubert et al., Phys. Rev. Lett. 95, 142001 (2005).
[14] CLEO Collaboration, T.E. Coan et al., Phys. Rev. Lett. 96, 162003 (2006).
[15] Belle Collaboration, C.Z. Yuan et al., Phys. Rev. Lett. 99, 182004 (2007).
[16] Belle Collaboration, X. L. Wang et al., Phys. Rev. Lett. 99, 142002 (2007).
[17] Babar Collaboration, B. Aubert et al., Phys. Rev. D 98, 212001 (2007).
[18] Belle Collaboration, S.-K. Choi et al., Phys. Rev. Lett. 100, 142001 (2008).
[19] Belle Collaboration, R. Mizuk et al., Phys. Rev. D 80, 031104 (2009).
[20] Babar Collaboration, B. Aubert et al., Phys. Rev. D 79, 112001 (2009).
[21] Belle Collaboration, A. Bondar et al., Phys. Rev. Lett. 108, 122001 (2012).
[22] M.T. Li, W.L. Wang, Y.B. Dong, Z.Y. Zhang, J. Phys. G 40, 015003 (2013).
[23] M.T. Li, W.L. Wang, Y.B. Dong, Z.Y. Zhang, Int. J. Mod. Phys. A 27, 1250161 (2012), arXiv:1206.0523[hep-ph].
[24] F. Close, C. Downum, C. E. Thomas, Phys. Rev. D 81, 074033 (2010).
[25] R.M. Albuquerque, M. Nielsen, Nucl. Phys. A 857, 48-49 (2011).
[26] S.H. Lee, K. Morita, M. Nielsen, Nucl. Phys. A 815, 29-39 (2009).
[27] M. Nielsen, F.S. Navarre, S.H. Lee, Phys. Rep. 497, 41-83 (2010).
[28] G.J. Ding, Phys. Rev. D 79, 014001 (2009).
[29] Y.R. Liu, Z.Y. Zhang, arxiv: 0908.1734[hep-ph].
[30] Z.Y. Zhang, A. Faessler, U. Straub, L.Ya. Glozman, Nucl. Phys. A 578, 573 (1994).
[31] Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A. Faessler, and U. Straub, Nucl. Phys. A 625, 59 (1997).
[32] L.R. Dai, Z.Y. Zhang, Y.W. Yu, P. Wang, Nucl. Phys. A 727, 321 (2003).
[33] F. Huang, Z.Y. Zhang, and Y.W. Yu, Phys. Rev. C 70, 044004 (2004).
[34] F. Huang and Z.Y. Zhang, Phys. Rev. C 70, 064004 (2004).
[35] Y.R. Liu and Z.Y. Zhang, Phys. Rev. C 80, 015208 (2009).
[36] Y.R. Liu and Z.Y. Zhang, Phys. Rev. C 79, 035206 (2009).
[37] W.L. Wang, F. Huang, Z.Y. Zhang, and B.S. Zou, Phys. Rev. C 84, 015203 (2011).
[38] M.T. Li, Y.B. Dong, Z.Y. Zhang, Chin. Phys. C 35, 622-628 (2011), arxiv: 1010.2283[hep-ph].
[39] W.L. Wang, F. Huang, Z.Y. Zhang, Y.W. Yu and F. Liu, Eur. Phys. J. A 32, 293-297 (2007).
[40] W.L. Wang, F. Huang, Z.Y. Zhang and F. Liu, J. Phys. G 35, 085003 (2008).
[41] W.L. Wang, F. Huang, Z.Y. Zhang and F. Liu, Mod. Phys. Lett. A 25, 1325-1332 (2010).
[42] W.L. Wang and Z.Y. Zhang, Phys. Rev. C 84, 054006 (2011).
[43] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).
[44] H.X. Zhang, W.L. Wang, Y.B. Dai, and Z.Y. Zhang, Commun. Theor. Phys. 49, 414 (2008).
[45] H.X. Zhang, M. Zhang, and Z.Y. Zhang, Chin. Phys. Lett. 24, 2533 (2007); M. Zhang, H.X. Zhang, and Z.Y. Zhang, Commun. Theor. Phys. 50, 437 (2008).
[46] X. Liu, Z.G. Luo, Y.R. Liu and S.L. Zhu, Eur. Phys. J. C 61, 411 (2009).
[47] Z.F. Sun, J. He, X. Liu, Z.G. Luo and S.L. Zhu, Phys. Rev. D 84, 054002 (2011).
[48] P. Falkensteiner, H. Grosse, F. Schöberl and P. Hertel, Comput. Phys. Commun. 34, 287-293 (1985).
[49] W. Lucha and F. F. Schöberl, Int. J. Mod. Phys. C 10, 607-619 (1999).