DILUTING SOLUTIONS OF THE COSMOLOGICAL CONSTANT PROBLEM

Olindo Corradini
Dipartimento di Fisica, Università di Bologna and INFN, Sezione di Bologna via Irnerio 46, I-40126 Bologna, Italy
E-mail: corradini@bo.infn.it

Alberto Iglesias
C.N. Yang Institute for Theoretical Physics
State University of New York, Stony Brook, NY 11794
E-mail: iglesias@insti.physics.sunysb.edu

Zurab Kakushadze
Royal Bank of Canada Dominion Securities Corporation†
1 Liberty Plaza, 165 Broadway, New York, NY 10006
E-mail: zurab.kakushadze@rbccm.com

Abstract

We discuss the cosmological constant problem in the context of higher codimension brane world scenarios with infinite-volume extra dimensions.

I. INTRODUCTION

The cosmological constant problem is an outstanding obstacle in the quantum field theory description of gravity interacting with matter. The problem is to reconcile the expected

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contribution of the zero point energy of matter and gauge fields to the vacuum energy density with the observations (coming from Supernova and Cosmic Microwave Background data), which imply that the universe is flat with a positive cosmological constant. The former (assuming supersymmetry breaking around TeV) is at least of order $10^{14}$ TeV, while the latter is at most of order $(10^{-15}$ TeV)$^4$.

We will present a possible way to circumvent the problem by allowing gravity to propagate in infinite volume extra dimensions while the Standard Model fields are localized on a lower dimensional hypersurface (brane).

In order to introduce some general features of brane world scenarios with infinite volume extra dimensions and summarize some results we can consider the following action

$$S = \bar{M}_P^{D-d-2} \int_{\Sigma} d^{D-d}x \sqrt{-\bar{G}} \left( \bar{R} - \bar{\Lambda} \right) + M_P^{D-2} \int_{\mu_D} d^Dx \sqrt{-G} R .$$  \hspace{1cm} (1)

Here $\mu_D$ is the D-dimensional bulk in which gravity propagates, and the second term is a D-dimensional Einstein-Hilbert term with a fundamental gravitational scale $M_P$, while $\Sigma$ is a lower dimensional hypersurface, a brane, with tension $\bar{M}_P^{D-d-2}\bar{\Lambda}$ (which includes contributions to the vacuum energy density from the Standard Model fields whose dynamics is otherwise neglected). The brane Einstein-Hilbert term containing $\bar{R}$ is induced via loops of gauge and matter fields interacting with bulk gravity as long as the world volume theory is non-conformal. $\bar{M}_P$ is then related to a world volume theory scale and is identified with $1/\sqrt{G_N}$. Lower dimensional gravity is reproduced at distances ranging from $1/M_P$ to $\bar{M}_P/M_P^2$ becoming higher dimensional at larger distances, while below the $1/M_P$ scale the effective field theory description breaks down. Gravity is localized on the brane with only ultralight modes leaking into the bulk. This was shown for the case of a tensionless brane case [1] \textit{i.e.}, when $\bar{\Lambda} = 0$, and for the case of a non-zero tension 3-brane in 6 dimensional bulk (and, more generally, for the case of a non-zero tension codimension-2 brane) [2].

Within this context, a step toward solving the cosmological constant problem would then be to find solutions where the brane is flat without having to finetune the brane tension \textit{i.e.}, for a continuous range of values of $\bar{\Lambda}$ that would account for the zero point energies of the brane fields. We refer to such solutions as “diluting”. Such solution do not seem to exist when only one extra dimension is considered, therefore, we will focus on higher codimension setups.

Considering a higher codimension brane with no thickness (a $\delta$-function like brane) leads to a singular graviton propagator at the location of the brane. Taking $r$ to be the radial direction in the extra space, it grows like $\log(r)$ for the case of a brane with two extra dimensions, and as $r^{2-d}$ for branes of codimension $d > 2$.

In the tensionless case the extra space can be taken to be Minkowski space. However, when $\bar{\Lambda}$ is non-vanishing the extra space is no longer flat and a singularity is already found in the background solution. In the codimension-2 case it is a wedge with a deficit angle and the singularity is a conical one at the location of the brane. Here, a range of tensions is consistent with a flat brane\textsuperscript{2}, namely [2],

\begin{enumerate}
    \item Along with higher derivative terms as well as nonlocal terms.
    \item A scale as low as $10^{-15}$ TeV $\sim 0.1 mm$ is required in these models, in such a way that the
\[ 0 \leq T \lesssim 4\pi M_P^4 \sim (10^{-15} \text{ TeV})^4, \]  

which does not improve the observed bound. In this light we are led to consider branes of codimension 3 or higher. The singularity of the background is still an issue in these cases and, furthermore, another type of singularity appears at a finite distance from the brane [6]. The topic of the next section is how to cure this singular behavior of such solutions.

**II. SINGULARITIES**

In order to cure the singular behavior at the origin we give some structure to the brane in the extra space. Thus, instead of taking the brane to be just a point in the extra dimensions we could grow this point into a \( d \)-dimensional ball \( B_d \) of non-zero radius \( \epsilon \). This type of resolution raises a problem, the gravitational modes on the brane contain an infinite tower of tachyonic modes (already for the tensionless case [4]). This can be circumvented by considering a partial smoothing where one replaces the \( d \)-dimensional ball by a \((d-1)\)-dimensional sphere \( S^{d-1} \) of radius \( \epsilon \). This suffices to smooth out higher codimension singularities in the propagator because close to the brane effectively we now have a codimension-1 brane for which the propagator is non-singular. Moreover, in the tensionless case as well as in the case of a non-zero tension codimension-2 brane we will then have only one tachyonic mode\(^3\) (with ultra-low \(-m^2\)) which is expected to be an artifact of not including non-local operators on the brane. As to the background itself this procedure does smooth out \( r = 0 \) singularities. Thus, now we are considering the following action

\[
S = \tilde{M}_P^{D-3} \int_{\Sigma} d^{D-1} x \sqrt{-\tilde{G}} \left( \tilde{R} - \tilde{\Lambda} \right) + M_P^{D-2} \int_{\mu D} d^D x \sqrt{-G} R ,
\]

where \( \Sigma = R^{D-d-1,1} \times S^{d-1} \). However, not surprisingly, this procedure does not suffice for smoothing out the aforementioned finite-distance singularities that appear in the codimension-3 and higher cases [2]. It was suggested that higher curvature terms in the bulk might be responsible for curing such singularities [8]. Studying such backgrounds in the presence of higher curvature terms in the bulk is rather non-trivial and in order to make the problem more tractable, we focus on a special kind of higher curvature terms in the bulk. In particular, we consider adding the second-order (in curvature) Gauss-Bonnet combination in the bulk. This combination has some remarkable properties. Thus, it is an Euler invariant in four dimensions (but it contributes nontrivially to the equations of motion in higher dimensions). It appears in Supergravity theories as the necessary combination to supersymmetrize the Chern-Simons term, and is also expected to appear in a low energy effective action derived from string theory as it is the only combination that does not introduce propagating ghosts. But for our purposes the most important property is that it

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\(^3\)In the case of solitonic branes one does not expect tachyons at all [12]. However, we are not considering here a solitonic brane, rather the original setup of [1].
does not introduce terms with third and fourth derivatives in the background equations of motions, but rather makes them even more non-linear. Albeit non-trivial, these equations can in certain cases be analyzed analytically, so we can get some insight into the effect of higher curvature terms on the finite distance singularities. Our action now reads:

\[ S = \tilde{M}_P^{D-3} \int_{\Sigma} d^{D-1}x \sqrt{-\tilde{G}} \left[ \tilde{R} - \tilde{\Lambda} \right] + \]

\[ M_P^{D-2} \int d^Dx \sqrt{-G} \left[ R + \xi \left( R^2 - 4R^2_{MN} + R^2_{MNP} \right) \right], \tag{4} \]

where \( \xi \) is the Gauss-Bonnet coupling.

We are interested in solutions with vanishing \((D-d)\)-dimensional cosmological constant, which, at the same time, are radially symmetric in the extra \( d \) dimensions. The corresponding ansatz for the background metric reads:

\[ ds^2 = \exp(2A) \eta_{\mu\nu} dx^\mu dx^\nu + \exp(2B) \delta_{ij} dx^i dx^j, \tag{5} \]

where \( A \) and \( B \) are functions of \( r \) but are independent of \( x^\mu \) (the \((D-d)\) coordinates on the original brane) and \( x^\alpha \) (the angular coordinates on the extra space over which the brane was smoothed). The metric for the coordinates \( x^i = (x^\alpha, r) \) is conformally flat.

The equations of motion are highly non-linear and difficult to solve in the general case. However, in the \( d = 4 \) and especially \( d = 3 \) cases various (but not all) terms proportional to \( \xi \) vanish. This is due to the fact that the Gauss-Bonnet combination is an Euler invariant in four dimensions. To make our task more tractable, from now on we will focus on the codimension-3 case \((d = 3)\). We do not expect higher codimension cases to be qualitatively different.

Not only is the complexity of the equations of motion sensitive to the value of \( d \), but also to the value of \( \overline{d} \equiv D - d \). In particular, we have substantial simplifications in the cases of \( \overline{d} = 2 \) and \( \overline{d} = 3 \) corresponding to the non-compact part of the brane \( \Sigma \) being a string respectively a membrane. Note that these simplifications are specific to the Gauss-Bonnet combination. In particular, if we set the Gauss-Bonnet coupling \( \xi \) to zero (that is, if we keep only the Einstein-Hilbert term in the bulk), there is nothing special about the \( \overline{d} = 2, 3 \) cases. This suggest that the conclusions derived from explicit analytical computations for the \( \overline{d} = 2, 3 \) cases can be expected to hold in \( \overline{d} \geq 4 \) cases as well (in particular, in the case of a 3-brane in 7D and, as we mentioned above, even higher dimensional bulk). In fact, the analytical and numerical results in the case of a 3-brane in 7D bulk confirm this expectation.

What follows is a review of the results of a recent collaboration [3].

**III. NO EINSTEIN-HILBERT BULK TERM**

In order to test the hypothesis of smoothing out by the presence of higher curvature terms we first studied a more of a toy model type of action in which the bulk Einstein-Hilbert term was neglected. Let us first consider a string \((\overline{d} = 2)\) propagating in 5D bulk \((d = 3)\). The equations of motion then read:

\[ (A')^2 \left[ 3(B')^2 + 6\frac{B'}{r} + \frac{2}{r^2} \right] = 0, \tag{6} \]
\[
\left[ A'' + (A')^2 - A'B' \right] \left[ (B')^2 + 2 \frac{B'}{r} \right] + 2A' \left( B' + \frac{1}{r} \right) \left( B'' + \frac{B'}{r} \right) = \\
\frac{1}{8\xi} e^{3B} (\kappa - 2) \lambda \tilde{L} \delta (r - \epsilon) ,
\]
(7)

\[
(\kappa')^2 (B'' + \frac{B'}{r}) + 2 \left[ A'' + (A')^2 - A'B' \right] A' \left( B' + \frac{1}{r} \right) = \\
\frac{\kappa}{8\xi} e^{3B} \lambda \tilde{L} \delta (r - \epsilon) ,
\]
(8)

where we have introduced the notation

\[
\tilde{L} = \frac{\tilde{M}_P^{D-3}}{M_P^{D-2}} , \quad \lambda = \frac{d - 2}{\epsilon^2} e^{-2B} , \quad \kappa \equiv \tilde{\Lambda}/\lambda .
\]
(9)

We solve these equations for \( r \neq \epsilon \) and then impose the two matching conditions for the \( r < \epsilon \) and \( r > \epsilon \) parts of the solution that produce the jump in the first derivatives of the warp factors implied by the \( \delta \) functions on the r.h.s. of (7) and (8).

For \( r \neq \epsilon \) flat space is a solution, i.e., we can simultaneously set \( A \) and \( B \) to be constant. A nontrivial solution is also easily found, namely,

\[
A(r) = \ln \left( \frac{r_A}{r} \right)^{\alpha - 1} , \quad B(r) = \ln \left( \frac{r_B}{r} \right)^{\alpha} ,
\]
(10)

where

\[
\alpha = \alpha_\pm = 1 \pm \frac{1}{\sqrt{3}} .
\]
(11)

Thus we can consider two possibilities: Either we take the solution to be nontrivial inside the spherical shell at \( r = \epsilon \), while outside we take constant warp factors; or we consider flat space inside the sphere taking the nontrivial part of the solution on the outside. We will refer to the former solutions as \textit{interior} and call the latter solutions \textit{exterior}.

Let us analyze the singularity structure in the above solutions. Singularities can potentially occur at \( r \to \infty \) in the exterior solutions and at \( r \to 0 \) in the interior ones. Thus, the line element in the non-trivially warped part of the space-time is given by:

\[
ds^2 = \left( \frac{r_A}{r} \right)^{2(\alpha - 1)} \eta_{\mu\nu} dx^\mu dx^\nu + \left( \frac{r_B}{r} \right)^{2\alpha} \delta_{ij} dx^i dx^j ,
\]
(12)

which is singular at \( r = 0 \) for both roots \( \alpha = \alpha_\pm \). However, only for \( \alpha = \alpha_- \) is the space truly singular, whereas for \( \alpha = \alpha_+ \) we merely have a coordinate singularity. Thus, consider a \( 2n \)-derivative scalar. Such an object - let us call it \( \chi^{(n)} \) - has the following expression in terms of the extra-space radius

\[
\chi^{(n)} \sim e^{-2nB} \frac{1}{r^{2n}} \sim r^{2n(\alpha - 1)} .
\]
(13)

The latter blows up for \( \alpha = \alpha_- \) as \( r \) approaches zero; this singularity is a naked singularity. One can indeed consider radial null geodesics with affine parameter \( \sigma \) and use that \( G_{tt} dt/d\sigma \) is constant along geodesics to obtain
\[ \frac{dr}{d\sigma} \sim r^{2\alpha-1}. \] (14)

For \( \alpha = \alpha_- \) these geodesics terminate with finite affine parameter as \( r \) approaches zero:

\[ \sigma \sim r^{2(1-\alpha)} + \text{constant}. \] (15)

Thus, we have incomplete geodesics reaching a point of divergent curvature. On the other hand, for \( \alpha = \alpha_+ \) the expression (13) vanishes as \( r \) approaches zero, the aforementioned geodesics are complete (i.e., \( \sigma \to \infty \)), and it is not difficult to see, by doing a similar calculation, that radial time-like geodesics extend to infinite proper time in this limit. Therefore, \( r = 0 \) is a coordinate singularity in this case. This then implies that for the interior solutions we must choose \( \alpha = \alpha_+ \). It then follows that the Gauss-Bonnet coupling \( \xi \) is positive in this case.

Similar considerations apply to the \( r \to \infty \) singularity. In this case we have a naked singularity for \( \alpha = \alpha_+ \), while for \( \alpha = \alpha_- \) we merely have a coordinate singularity. This implies that for the exterior solutions we must choose \( \alpha = \alpha_- \). Note that in this case the Gauss-Bonnet coupling \( \xi \) is also positive. Moreover, it is also not difficult to check that the volume of extra space is infinite.

Thus, as we see, we have sensible infinite-volume non-singular solutions if we take the bulk action to be given by the Gauss-Bonnet combination. However, these solutions exist only for a fine-tuned value of the brane tension. Indeed, the matching conditions at \( r = \epsilon \) for the interior (+) and exterior (−) solutions reduce to:

\[ \pm \frac{\alpha(\alpha - 1)(\alpha - 2)}{\epsilon} = -\frac{\kappa - 2}{8\xi} \tilde{L} \left( \frac{r_B}{\epsilon} \right)^\alpha, \] (16)

\[ \pm \frac{(\alpha - 1)^3}{\epsilon} = -\frac{\kappa}{8\xi} \tilde{L} \left( \frac{r_B}{\epsilon} \right)^\alpha, \] (17)

Note that the brane tension is always positive (assuming that the “brane-width” \( \epsilon \) is non-zero). An unwelcome feature of these solutions is that the tension is determined by the other parameters (the integration constant \( r_B \) drops out): which implies that

\[ \tilde{\Lambda} = \frac{8\tilde{L}^2}{\xi^2}. \] (18)

That is, these solutions are not “diluting”, rather they exist only if we fine-tune the brane tension and the Gauss-Bonnet coupling. In the following we will see that this is specific to the case at hand, and diluting solutions do exist in other cases. In all the other cases we studied, for which the equations are not so simple, a dependence on one of the integration constants from the nontrivial part of the solution appears in the l.h.s. of the matching conditions. This extra freedom translates into a brane tension being unfixed by the other parameters of the theory.

The case of a membrane in 6D bulk can also be solved analytically. We found smooth diluting interior as well as exterior solutions in this case. In the case of a 3-brane in 7D bulk we found smooth (with just a coordinate singularity at \( r = 0 \)) diluting solutions with positive brane tension via numerical methods.

Encouraged by these results we then studied the case in which both the bulk Einstein-Hilbert and Gauss-Bonnet terms are present.
IV. EINSTEIN-HILBERT AND GAUSS-BONNET TERMS IN THE BULK

Let us now concentrate on the case of a 3-brane in 7-dimensional space with both Einstein-Hilbert and Gauss-Bonnet bulk terms, which is the most interesting case from the phenomenological point of view. By introducing the following variables:

\[ V = rA', \quad U = rB' + 1, \quad z = \frac{r^2 e^{2B}}{\xi}, \]

the \((rr)\) equation can be rewritten as

\[ U^2(72V^2 - z) - 2U4V(z - 12V^2) - z(6V^2 - 1) - 12V^2(2 - V^2) = 0, \]

which can be used to express \(U\) as a function of \(V\) and \(z\). And similarly, we can rewrite the \((rr - \alpha\beta)\) equation of motion which gives, away from the brane, the following first order differential equation for \(V\) (treating \(V\) as a function of \(z\)):

\[ 2zU \frac{dV}{dz} = \frac{g_1}{g_2}, \]

where \(g_1\) and \(g_2\) are known polynomials of \(U\), \(V\) and \(z\). And, as in the previous cases, the solution must satisfy two matching conditions at \(r = \epsilon\).

In order to have non-trivial solutions consistent with the matching conditions, \(V(\epsilon+)\) (for exterior solutions), \(V(\epsilon-)\) (for interior solutions) and \(z(\epsilon)\) must be chosen in certain regions of the \(V - z\) plane.

For the positive Gauss-Bonnet coupling case, \(\xi > 0\) (i.e., \(z > 0\)) some of the interior solutions that are consistent with the matching conditions are smooth with just a coordinate singularity \(r \to r_0+\) limit (where \(r_0\) is an integration constant). Both the affine parameter of radial null geodesics and the proper time of time-like geodesics diverge in that limit as \(1/\ln(r/r_0)\). Thus, space is complete if we cut it at \(r = r_0\). The values of \(\kappa\) consistent with the matching conditions range from \(-\infty\) to 2.15. These solutions are of particular interest as some of these solutions have positive 3-brane tension (the ones with \(\kappa > 2\)) and, furthermore, they are diluting.

In order to make contact with the phenomenology mentioned in the introduction we have to give a more precise definition of the brane tension. Up to this point we have been referring to \(\tilde{\Lambda}\) (or, more precisely, \(\tilde{T} \equiv \tilde{M}_P^{D-3}\tilde{\Lambda}\)) as the brane tension. This quantity is indeed the tension of the smoothed brane \(\Sigma\) whose world-volume has the geometry of \(\mathbb{R}^{d-1,1} \times S^{d-1}_{\epsilon}\). Note, however, that a bulk observer at \(r > \epsilon\) does not see this brane tension - to such an observer the brane appears to be tensionless. Indeed, the warp factors are constant in the aforementioned solution at \(r > \epsilon\). The non-vanishing (in fact, positive) tension of the brane \(\Sigma\) does not curve the space outside of the sphere \(S^{d-1}_{\epsilon}\). Instead, it curves the space inside the sphere \(S^{d-1}_{\epsilon}\), that is, at \(r < \epsilon\). And this happens without producing any singularity at \(r < \epsilon\), and with the non-compact part of the world-volume of the brane remaining flat.

Here it is important to note that the effective tension of the fat 3-brane (in the present case) whose world-volume is \(\mathbb{R}^{3,1}\) (this 3-brane is fat as it is extended in the extra 2 angular dimensions) is also positive. It is not difficult to see that this brane tension is given by

\[ \tilde{T} = (\tilde{\Lambda} - 2\lambda)v_2\tilde{M}_P^4, \]
where \( v_2 = 4\pi\epsilon^2\epsilon^{2B(\epsilon)} \) is the volume of the sphere \( S^2_\epsilon \) whose radius is not \( \epsilon \) but rather \( R \equiv \epsilon\epsilon^{B(\epsilon)} \).

We therefore have:

\[
\hat{T} = 4\pi(\kappa - 2)\tilde{M}_P^4.
\]

(24)

Thus, for \( \kappa > 2 \) (which is part of the parameter space for the aforementioned solution) this effective fat brane tension is positive.

Next, the 4-dimensional Planck scale \( \tilde{M}_P \) is given by

\[
\tilde{M}_P^2 = v_2\tilde{M}_P^4 = 4\pi R^2\tilde{M}_P^4.
\]

(25)

where \( \tilde{M}_P \) is the 4-dimensional Planck scale, \( \tilde{M}_P \) is the 6-dimensional Planck scale.

A priori we can reproduce the 4-dimensional Planck scale \( \tilde{M}_P \sim 10^{18} \) GeV by choosing \( R \) between \( R \sim \text{millimeter} \) and \( R \sim 1/\tilde{M}_P \). The 6-dimensional Planck scale then ranges between \( \tilde{M}_P \sim \text{TeV} \) and \( \tilde{M}_P \sim \tilde{M}_P \). On the other hand, it is reasonable to assume a “see-saw” modification of gravity [5] in the present case, in particular, once we take into account higher curvature terms on the brane, to obtain 4-dimensional gravity up to the distance scales of order of the present Hubble size, we must assume that the “fundamental” 7-dimensional Planck scale

\[
\tilde{M}_P \sim (\text{millimeter})^{-1}.
\]

(26)

Let us see what range of values we can expect for the 5-brane tension \( \tilde{T} \).

Thus, the 5-brane tension

\[
\tilde{T} = \Lambda\tilde{M}_P^4 = \kappa R^{-2}\tilde{M}_P^4 \sim R^{-4}\tilde{M}_P^2.
\]

(27)

If we assume that the Standard Model fields come from a 6-dimensional 5-brane theory compactified on the 2-sphere, then we might need to require that \( R \lesssim (\text{TeV})^{-1} \). Then \( \tilde{M}_P \) ranges between \( 10^7 - 10^8 \) TeV and \( \tilde{M}_P \), while the 5-brane tension ranges between \( (10^5 \text{ TeV})^6 \) and \( \tilde{M}_P^6 \). Note that a priori this is not in conflict with having the supersymmetry breaking scale in the TeV range.

In principle, the above scenario a priori does not seem to be inconsistent modulo the fact that we still need to explain why the 6-dimensional Plank scale \( \tilde{M}_P \) is many orders of magnitude (30 in the extreme case where \( R^{-1} \sim \tilde{M}_P \sim \tilde{M}_P \)) higher than the seven-dimensional Planck scale \( M_P \). Note, however, that the same issue is present in any theory with infinite-volume extra dimensions.

Let us see what kind of values of \( V(\epsilon-\epsilon) \) we would need to have in order to obtain a solution satisfying the above phenomenological considerations. First, we will assume that the Gauss-Bonnet parameter \( \xi \sim M_P^{-2} \) (its “natural” value).

\[\text{Note a similarity with [10].}\]
We can for instance take the $z \ll |V| \ (|V| \gg 1)$ case in which the matching conditions become,

$$R\bar{L}/\xi \approx -11.77V^3(\epsilon^-) \; ,$$

$$\kappa R\bar{L}/\xi \approx -25.3V^3(\epsilon^-) \; ,$$

which gives $\kappa \approx 2.15$, and $|V(\epsilon^-)| \approx 10^{30}$, and $\epsilon$ is very close to the would-be singularity $r_*$ (here for definiteness we have assumed the extreme case $R^{-1} \sim \bar{M}_P \sim \bar{M}_P$, where $\bar{L} = \bar{M}_P^4/M_P^{12} \sim 10^{120}$ mm):

$$r_*/\epsilon - 1 \approx 10^{-30} \; .$$

Note that the singularity at $r = r_*$ would be there if we took the interior solution for $r < \epsilon$ and continued it for values $r > \epsilon$. However, in our solution (just as in the case without the Einstein-Hilbert term) there is no singularity as for $r > \epsilon$ the warp factors are actually constant, and this is consistent with the matching conditions at $r = \epsilon$. Thus, as we see, in the presence of both the Einstein-Hilbert and Gauss-Bonnet terms in the bulk action we have sensible smooth solutions with positive brane tension. Moreover, these solutions are *diluting*, that is, they exist for a range of values of the brane tension (note that the Gauss-Bonnet coupling in these solutions is *positive*). In particular, no fine tuning appears to be required in our solution.

As we mentioned above, the singularity at finite $r = r_0 < \epsilon$ is harmless as the corresponding geodesics are complete. Note that in the case without the Einstein-Hilbert term the corresponding coordinate singularity is at $r = 0$. The reason why is the following. If we start with a solution corresponding to both the Einstein-Hilbert and Gauss-Bonnet terms present in the bulk, to arrive at the solution corresponding to only the Gauss-Bonnet term present in the bulk we must take the limit $\xi \to \infty$, $M_P^{D-2}\xi$ = fixed. It is then not difficult to check that in this limit the coordinate singularity at $r = r_0$ continuously moves to $r = 0$. Also note that, since the singularity at $r = r_0$ in the full solution is a coordinate singularity, we can consistently cut the space at $r = r_0$. The geometry of the resulting solution then is as follows. In the extra three dimensions we have a radially symmetric solution where a 2-sphere is fibered over a semi-infinite line $[r_0, \infty)$. The space is curved for $r_0 \leq r < \epsilon$, at $r = \epsilon$ we have a jump in the radial derivatives of the warp factors (because $r = \epsilon$ is where the brane tension is localized), and for $r > \epsilon$ the space is actually flat. So an outside observer located at $r > \epsilon$ thinks that the brane is tensionless, while an observer inside of the sphere, that is, at $r < \epsilon$ sees the space highly curved (and it would take this observer infinite time to reach the coordinate singularity at $r = r_0$). This is an important point, in particular, note that we did not find smooth exterior solutions where the space would be curved outside but flat inside. And this is just as well. Indeed, if we have only the Einstein-Hilbert term in the bulk, then we have no smooth solutions whatsoever [8] (that is, smoothing out the 3-brane by making it into a 5-brane with two dimensions curled up into a 2-sphere does not suffice). What is different in our solutions is that we have added higher curvature terms in the bulk, which we expected to smooth out some singularities. But serving as an ultra-violet cut-off higher curvature terms could only possibly smooth out a real singularity at $r < \epsilon$, not at $r > \epsilon$. And this is precisely what happens in our solution - the presence of higher curvature terms ensures that we have only a coordinate singularity at $r < \epsilon$ instead of a real naked singularity as would be the case had we included only the Einstein-Hilbert term.
V. CONCLUSIONS

In the presence of the Einstein-Hilbert and Gauss-Bonnet terms in the bulk action we have smooth infinite-volume solutions which exist for a range of positive values of the brane tension (the diluting property). These solutions, therefore, provide examples of brane world scenarios where the brane world-volume can be flat without any fine-tuning or presence of singularities.

We suspect (albeit we do not have a proof of this statement) that even for generic higher curvature terms solutions with the aforementioned properties should still exist. In particular, we suspect that the fact that we found non-singular solutions has to do with including higher curvature terms in the bulk rather than with their particular (Gauss-Bonnet) combination, which we have chosen to make computations tractable.

There are many interesting open questions to be addressed in scenarios with infinite-volume extra dimensions. As was originally pointed out in [7,11,1], these scenarios offer a new arena for addressing the cosmological constant problem. And addressing the aforementioned open questions definitely seems to be worthwhile. We hope our results presented in this talk will stimulate further developments in this field.

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