Dispersion Dynamics of Clustered Particles by Centrifugal Force

Tiancong Feng
Shanghai Starriver Bilingual School, 2588 Jindu Road, Minhang, Shanghai, China
Email: alexfengtc@sina.com

Abstract. In this paper an approach is proposed to solve the problem of aggregation in nanomaterials through the mean of rotational separation aiming to quickly disperse clustered nanoparticles while not affecting their purity. If it is possible, this approach may replace the current mean of mechanical mixing, which may cause impurities issues. The hypothesis is that the centrifugal force due to rotational velocity acting on the nanoparticles can overcome the cohesive force between the nanoparticles, therefore dispersing the clustered nanoparticles. The experimental mean is to put different spheres connected by different types of glues imitating different nanoparticle clusters into centrifuges imitating the swivel plate. The results from both the theoretical model and the experiment show that for a cluster with a cohesive force of 1.75N, a rotational velocity of about 800 rad./s is required to disperse the cluster. While for a cluster with a cohesive force of 0.25N and the same mass and position, a rotational velocity of about 150 rad./s is required to disperse the cluster. Except for the cohesive force, the mass and position of the nanoparticle on the swivel plate also have a large effect on the required rotational velocity. The observation of the physical mechanism of the dispersion has also shown that while using this way, the cluster is dispersed slowly with small parts separated from it. Therefore, this way can also eliminate re-clustering problems of nanoparticles.

1. Introduction

With the improvement of various properties such as mechanical strength, heat resistance and permeability, composite nanomaterials have been emerged as a new alternative to conventional nanomaterials. More and more kinds of nanoparticles are synthetized and mixed in order to tailor properties of the materials and to bring some specific properties to nanocomposites. However, the dispersing and homogenous mixing of fine particles into other matrix remains a challenge. Thus, dispersion of nanoparticles and powders has been a important requirement in various applications.

In all these industrial processes, high intensity of energy is required to overcome the adhesion forces of clustered nanoparticles or to increase interfacial surface area per unit volume, which was achieved by high-pressure homogenizers, ultrasonic-assisted devices, and high shear mixers [1]. Ultrasonic-assisted devices need longer time to make the clustered materials disperse, but the longer-time using of this device may make material become heat and new clustering could occur. High shear mixers, which are comprised of rotational and stationary parts, were found that the rotational part could produce friction with the materials to be mixed and thus some impurity may be brought into the mixed materials, leading to the deviation of properties of the material.

In this paper, a new approach is proposed to solve the problem of clustering and aggregation of nanomaterials by using rotational separation. It may replace the current mean of mechanical mixing, which may cause impurities issues. The approach is based on fact that the centrifugal force acting on
the nanoparticles can overcome the adhesive force of the nanoparticles. The paper aims to quickly disperse the nanoparticles while not affecting the purity.

2. Theoretical Framework and Analysis

In this paper, small rigid metal spheres with mass and volume connected by a contact force $F$ are adapted as the main subject of theoretical model and experimentation in replacement of nanoparticles. Such small rigid spheres have more controllable properties such as their mass, volume, cohesive force, and the number of spheres in a cluster. All of the following theoretical models are created based on these small rigid spheres, and most of the experiments are conducted based on them.

2.1. Clustered Double Spheres

To begin from the simplest form of cluster, given the drawing below, two rigid spheres with mass $m_1$, $m_2$ and radius $r_1$, $r_2$ have distance $R_1$, $R_2$ to the centre of a boundless swivel plate. There is a cohesive force $F$ between the two spheres. The coefficient of friction ($\mu$) between each sphere and plate is constant. The critical rotational velocity at which the two balls lose contact can be found.

In order to have the two spheres lose contact, there must be inadequate net force on the spheres to support the centripetal acceleration required for the spheres to reach the angular velocity. First, while the angular velocity has only a small value, the friction of both spheres is able to fully support the centripetal force. But as the velocity increases, the centrifugal force will exceed the friction.

Let the centripetal direction be the positive x direction. While the cohesive force $F$ is not put into consideration,

\[ \Sigma F_x = f = mR\omega^2 \]  
\[ \mu mg = mR\omega^2 \]  
\[ \mu g = R\omega^2 \]  

Therefore, the greater the distance, the quicker the frictional force would not be able to support the centrifugal acceleration as the angular velocity increases. Thus, a force analysis on the sphere farther to the centre of the swivel plate can be done to derive the critical angular velocity required for the two spheres to lose contact.

Let sphere 2 to have greater distance from the centre of mass of the sphere to the centre of the disk. At the moment when the two blocks lose contact, it must be at the point when the net force is just about to exceed the centrifugal force required. For Sphere 2

\[ \Sigma F_x = F_x + f = m_2R_2\omega_{critical}^2 \]
The expression of the \( \cos \alpha \) can be found by solving the triangle \( OO_1O_2 \) in figure 1 using the law of cosines

\[
R_1^2 = R_2^2 + (r_1 + r_2)^2 - 2(r_1 + r_2)R_2 \cos \alpha
\]

(6)

\[
2(r_1 + r_2)R_2 \cos \alpha = R_2^2 + (r_1 + r_2)^2 - R_1^2
\]

(7)

\[
\cos \alpha = \frac{R_2^2 + (r_1 + r_2)^2 - R_1^2}{2(r_1 + r_2)R_2}
\]

(8)

Then substituting the expression back to the equation

\[
F \frac{R_2^2 + (r_1 + r_2)^2 - R_1^2}{2(r_1 + r_2)R_2} + \mu m_2 g = m_2 R_2 \omega_{\text{critical}}^2
\]

(9)

Yielding the critical rotational velocity to be

\[
\omega_{\text{critical}} = \sqrt{F \frac{R_2^2 + (r_1 + r_2)^2 - R_1^2}{2(r_1 + r_2)R_2^3 m_2} + \frac{\mu g}{R_2}}
\]

(10)

2.2. Clustered Triple Spheres

To complicate the situation by a little, one more sphere is added, creating a triple sphere cluster problem. Similarly, three rigid spheres with mass \( m_1, m_2, m_3 \) and radius \( r_1, r_2, r_3 \) have distances \( R_1, R_2, R_3 \) to the centre of a boundless swivel plate. There is a cohesive force \( F \) between the three spheres, the coefficient of friction \( \mu \) between the spheres and the swivel plate. The critical angular velocity at which the sphere farthest to the centre of the swivel plate disperses from the cluster is to be found.

Using the same conclusion derived in the two-spheres cluster model, when the farthest sphere lose contact from the other two, its centrifugal force is exactly equal to the maximum net force. The expression of the net force when the sphere is at its critical velocity can be found.

\[
F \cos \alpha + F \cos \beta + \mu m_2 g = m_2 R_2 \omega_{\text{critical}}^2
\]

(11)

For \( \cos \alpha \) and \( \cos \beta \), they can be derived through the law of cosines

\[
\cos \alpha = \frac{(r_1 + r_2)^2 + R_2^2 - R_1^2}{2(r_1 + r_2)R_2}, \quad \cos \beta = \frac{(r_3 + r_2)^2 + R_3^2 - R_1^2}{2(r_3 + r_2)R_2}
\]

(12)

Then substituting the expressions back to the equation

\[
F \frac{(r_1 + r_2)^2 + R_2^2 - R_1^2}{2(r_1 + r_2)R_2} + F \frac{(r_3 + r_2)^2 + R_3^2 - R_1^2}{2(r_3 + r_2)R_2} + \mu m_2 g = m_2 R_2 \omega_{\text{critical}}^2
\]

(13)

Yielding the critical angular velocity

\[
\omega_{\text{critical}} = \sqrt{F \frac{(r_1 + r_2)^2 + R_2^2 - R_1^2}{2(r_1 + r_2)R_2^3 m_2} + F \frac{(r_3 + r_2)^2 + R_3^2 - R_1^2}{2(r_3 + r_2)R_2^3 m_2} + \frac{\mu g}{R_2}}
\]

(14)
2.3. Theoretical Results

2.3.1. Effect of Positions of Spheres on Critical Rotation Velocity. For double-sphere case, with the parameters: \( m_1 = m_2 = 1.0 \text{mg} \), \( r_1 = r_2 = 1.0 \mu\text{m} \), \( \mu = 0.1 \), and \( F = 0.1 \text{N} \), the equation (10) is used to calculate the critical rotational velocity of swivel plate when the farthest sphere lose contact.

\[
\frac{m_1 + m_2}{2} \omega^2 R = m_1 g \cos \theta - m_2 g \cos \theta - \frac{1}{2} \mu m_1 R \omega^2 - \frac{1}{2} \mu m_2 R \omega^2 + F \cos \theta
\]

Figure 2. The variation of critical velocity of the swivel plate with the distance of the spheres from the centre of swivel plate.

Figure 2 shows the effect of the distance of the spheres from the centre of swivel plate on the critical velocity of the swivel plate. It is known from the figure that with the distance increasing from 0.01m to 0.1m, the critical velocity dramatically decreases from 2200 to 700 rad./s. Thus, it can be concluded that the initial position of the cluster has a large effect on the critical rotational velocity.

2.3.2. Effect of Radius of Spheres on Critical Rotation Velocity. With the parameters: \( m_1 = m_2 = 1 \text{mg} \), \( R_1 = 10 \text{mm} \), \( R_2 = R_1 + r_1 \), \( \mu = 0.1 \), and \( F = 0.1 \text{N} \), the equation (10) is used to calculate the critical rotational velocity of swivel plate when the farthest sphere lose contact.

\[
\frac{m_1 + m_2}{2} \omega^2 R = m_1 g \cos \theta - m_2 g \cos \theta - \frac{1}{2} \mu m_1 R \omega^2 - \frac{1}{2} \mu m_2 R \omega^2 + F \cos \theta
\]

Figure 3. The variation of critical velocity of the swivel plate with the spheres radius.

Figure 3 shows the effect of the sphere radius on the critical velocity of the swivel plate. It is shown from the figure that with the radius increasing from 1\( \mu\text{m} \) to 10\( \mu\text{m} \), the critical velocity slowly decreases
from 2235.55 to 2232 rad./s. Thus, it can be concluded that the sphere radius has only a minor effect on the critical velocity.

2.3.3. Effect of Mass of Spheres on Critical Rotation Velocity. With the parameters: \( r_1 = r_2 = 1.0\mu m \), \( R_1 = 10.0\) mm, \( R_2 = 10.001\) mm, \( \mu = 0.1 \), and \( F = 0.1\) N, the equation (10) is used to calculate the critical rotational velocity of swivel plate when the farthest sphere lose contact.

![Figure 4](image)

**Figure 4.** The variation of critical velocity of the swivel plate with the spheres mass.

Figure 4 shows the effect of the sphere mass on the critical velocity of the swivel plate. It is shown from the figure that with the mass increasing from 1.0mg to 10.0mg, the critical velocity dramatically decreases from 2200 to 700 rad./s. Thus, it can be concluded that the spheres mass has a large effect on the critical rotational velocity.

2.3.4. Effect of Cohesive Force of Spheres on Critical Rotation Velocity. With the parameters: \( m_1 = m_2 = 1.0\) mg, \( r_1 = r_2 = 1.0\mu m \), \( R_1 = 10.0\) mm, \( R_2 = 10.001\) mm, and \( \mu = 0.1 \), the equation (10) is used to calculate the critical rotational velocity of swivel plate when the farthest sphere lose contact.

![Figure 5](image)

**Figure 5.** The variation of critical velocity of the swivel plate with the cohesive force of spheres.

Figure 5 shows the effect of the cohesive force of spheres on the critical velocity of the swivel plate. It is illustrated from the figure that with the force increasing from 0.1 to 1N, the critical velocity increases
from 2000 to 7000 rad./s. Thus, it can be concluded that the cohesive force between the spheres has a deciding effect on the critical rotational velocity.

The analytical results show that for the clustered two-sphere case, with the increasing distance or/and particle mass, the critical velocity dramatically decreases. With the increasing radius of particles or/and increasing friction coefficient, the critical velocity nearly keep constant. With the increasing cohesive force, the critical velocity dramatically increases.

3. Experimental Results and Discussion

After the derivation and analysis of theoretical model, the experiment is conducted to verify and compare the results of the theoretical model with the experimental results.

3.1. Experiment Instruments

The key instrument is a small high-speed centrifuge JOANLAB type MS-12 Pro with rotational velocity of 1000-12000 rpm. In order to measure the cohesive force, a push-pull force probe is used. In order to imitate the cohesive force between each particle, three different types of glues are used: Liquid glue, PVA Solid glue, and 502 Cyanoacrylate Instant Adhesive Super Glue. To imitate the particles, two types of metal spheres with different mass and radius are used. The larger spheres have a diameter of 6mm and are made of 304 stainless steel. The smaller spheres have a diameter of 2.5mm and are made of 316 stainless steel. Besides the small metal spheres, since the ultimate goal of this work is to disperse the nanoparticles with the centrifugal force, SiO$_2$ nanoparticles are also prepared for observation on how the actual nanoparticles will disperse in the centrifuge. While using the SiO$_2$ nanoparticles, they are wrapped together with the weakest liquid glue imitating the cohesive force inside them while in use.

3.2. Experiment Parameters

Before finding the rotational velocity, the strength of the cohesive force from different glues is to be calculated. Through applying adequate amounts of different types of glue at the pull end of the force probe to connect to a 6mm sphere and then pulling the sphere, the value of the cohesive force that each glue provides is derived. The cohesive force of Liquid Glue, PVA Solid Glue and 502 Super Glue are <0.25, 0.27, 1.75N.

Then finding the mass of the two different spheres. Since the metal spheres are perfectly spherical, for the 6mm diameter spheres, the density for its 304 stainless steel material is 7930 kg/m$^3$, while for the 2.5mm diameter spheres, the density for its 316 stainless steel material is 7980 kg/m$^3$. Therefore, their masses are found to be $m_{6.5} = 0.0653 \text{ g}$, $m_{6} = 0.897 \text{ g}$.

With a ruler, the distance between the centre and the edge of the centrifuge’s plate is measured to be 25.0mm. The coefficient of friction, since the swivel plate in the centrifuge is made of graphite, is 0.1.

![Figure 6](image.png)

**Figure 6.** The comparison of theoretical and experimental results for the double sphere case.
3.3. Comparison of Experimental and Theoretical Results

Except the actual SiO$_2$ nanoparticles that are tested, all of the experiments are conducted in same situation as the theoretical model. For better comparison, the quantified recorded experiment results will be put into graphs along with the theoretical value.

For this section, the comparison will be done in the same order as the experiments are conducted. As shown in figure 6, the theoretical results and the actual results for the double sphere cluster are analysed below.

The theoretical and the experimental values deviate a little. The bars are the experimental results, while the lines are the theoretical results. The analysis on cohesive force here appears to be applicable, as the critical rotational velocity is dramatically increased while increasing the cohesive force with the 502 Super Glue. However, comparing the experimental values for each graph with the theoretical values, it can be determined that the effect of the mass and radius are different for the two. In the theoretical analysis, it has been expected that the mass of the spheres has a deciding effect on the critical rotational velocity while the radius has only a little deciding effect. Therefore, it is expected that the critical rotational velocity for 2.5mm spheres will exceed the critical rotational velocity for 6mm spheres.

However, the experiment results show that the critical rotational velocity for 2.5mm spheres is actually less than the critical rotational velocity for 6mm spheres. The reason for this deviation is possibly due to the slight difference in the cohesive force between the spheres. As mentioned before, the cohesive force has a deciding effect on the critical rotational velocity. In this case, the larger spheres will form a larger contact surface with each other, therefore creating the larger cohesive force, which impacted on the results of experimentation. If this cohesive force is reasonably considered, it can be found that the results of the experiment does still fit the theoretical model.

4. Conclusion

As a summary, in this paper, the physical and mathematical models for clustered two-spheres, three-spheres, are deduced to model the relationship between the geometry, mass, mechanical parameters of particles as well as the angular velocity of a swivel plate for dispersing the clustered particles. The analytical results showed that for the clustered two spheres and three-sphere models, with the increasing distance from the centre of swivel plate, the critical velocity dramatically decreases. With the increasing radius of particles and friction coefficient, the critical velocity nearly keep constant. With the increasing mass of particles, the critical velocity dramatically decreases. With the increasing cohesive force, the critical velocity dramatically increase. Thus, the distance from the centre of disk, the mass of particles, cohesive force of the spheres have larger on the critical velocity of the disk, and the radius of particles, friction coefficient have smaller effect. The theoretical results are basically in agreement with the experiments results, and thus the approach proposed in this paper may separate the clustered particles and thus quickly disperse the nanoparticles while not affecting the purity.

References

[1] Nguyen S, Didier R, Brice V. 2014 Dispersion of nanoparticles: From organic solvents to polymer solutions [J] Ultrasonics Sonochemistry 21 149–153.
[2] Xie L, Rielly C, Eagles W, Zcan-Tas G. 2007 Dispersion of NanoParticle Clusters Using Mixed Flow and High Shear Impellers in Stirred Tanks [J] Chemical Engineering Research and Design 85 (A5) 676-684.
[3] Adam J, Stuart W. 2008 Dispersion of Nanoparticle Clusters by Ball Milling [J] Journal of Dispersion Science and Technology 29 (4) 600–604.
[4] Chia-Jung C, Lijuan X , Qiang H, Jianjun S. 2012 Quantitative characterization and modeling strategy of nanoparticle dispersion in polymer composites [J] IIE Transactions 44 523–533.
[5] Chenghu K, Jianlin X, Lei N, Chengsi L, Jiliang F. 2021 Effect of thermoplastic polyurethane elastomer on the properties of polybutylene terephthalate matrix flame retardant composites[J] Polymer Composites 42 3098–3113.
[6] Halen T, Laurence S. 2021 Combined combustion and heat-flux measurements on a supersonic jet-in-crossflow configuration using luminescent paint [J] Exp Fluids 62 70.