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Simulation of Semi-Autogenous Grinding (SAG) Mill using Circular-Disks-based Model

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Abstract. System of semi-autogenous grinding (SAG) mill is modeled in this work based on circular-disk objects, which represents not only the ground materials but also the mill liners. Interaction between these objects is through linear spring dash-pot force, which is solved numerically in soft-sphere scheme using molecular dynamics (MD) method. There is also force due to earth gravity. For ground materials a grain consists of several smaller grains, which are attached through some adhesive force. As the ground materials are smashed to the mill liners, when they reach maximum compression, they will be shattered into the smaller grains. Shattering process happens in a special materials motion known as cataracting mode. Friction is not considered in this work due to its complexity.

1. Introduction

The application of semi-autogeneous grinding (SAG) mill is depend on the type of motion. Cascading and cataracting mode are the optimal type of motion to crush the particles into smaller ones.

Figure 1. Motion of particles and zones in cataracting mode.
There are 4 zones in cataracting mode, consist of dead zone, abrasion zone, impact zone, and empty zone (Wills, 2015). As the mill was rotated in clockwise direction, the particle will move up on the left-bottom side of the mill and free fall in parabolic path once they reach the certain height. The smaller grains will be generated in impact zone.

The experimental research regarding two-dimensional SAG mill had been done using serrated cylindrical wall in order to increase milling efficiency, by optimizing the cataracting mode in grinding process (Faizin et al., 2016). In this paper, we conducted a simulation and analysis of SAG with circular liner using event-driven simulation.

2. Model

In this work a SAG (semi-autogenous grinding) mill is simplified into a two-dimensional system and modeled as a compound of circular disks. Then, the whole system consists only of two-dimensional circular objects, i.e. the ground materials and also the mill liners.

2.1. Mill model

The mill with radius $R$ will have a set of liners represented by $N_L$ circular disks as given in the following Figure 1.

![Figure 2. Two-dimensional model of SAG mill with radius R and liners in the form of N circular disks is rotated with angular velocity $\omega_M$.](image)

Consider two adjacent liner of diameter $D$ which is separated in angular position by $\Delta \theta$ as shown in Figure 2 (left). From Figure 2 (right) it can be obtained that

$$\sin\left(\frac{1}{2} \Delta \theta\right) = \frac{1}{2} \frac{D_L}{R_M},$$

(1)

where

$$N_L = \frac{2\pi}{\Delta \theta}.$$  

(2)

Substitution of Equation (2) into Equation (1) will give
\[ D_L = 2R_M \sin \left( \frac{\pi}{N_L} \right). \]  

(3)

\[ N_L = 2 \Rightarrow D_L = 2R_M \sin \left( \frac{\pi}{2} \right) = 2R_M, \]  

(4a)

\[ N_L = 3 \Rightarrow D_L = 2R_M \sin \left( \frac{\pi}{3} \right) = \sqrt{3}R_M, \]  

(4b)

\[ N_L = 4 \Rightarrow D_L = 2R_M \sin \left( \frac{\pi}{4} \right) = \sqrt{2}R_M, \]  

(4c)

\[ N_L = 6 \Rightarrow D_L = 2R_M \sin \left( \frac{\pi}{6} \right) = R_M. \]  

(4d)

**Figure 3.** Geometry relation among \( R_M, D_L, \) and \( \Delta \theta. \)

Equation (3) can be easily proven for \( N_L = 2, 3, 4, \) and 6 as shown in Figure 3, which are

\[ \theta_i = \theta_0 + (i - 1)\Delta \theta. \]  

(5)

Then, it is defined that
\[ \theta_0 = \theta_0 + \omega_M t, \quad (6) \]

where \( \theta_0 \) is initial angular position of the first liner. The angular position is measured to the x axis in counter-clockwise (CCW) direction the same as direction of \( \omega_M \), as it is shown previously in Figure 1. With Equations (2), (5), and (6) following relation can be obtained

\[ \theta_i = \theta_0 + (i-1) \frac{2\pi}{N_L} + \omega_M t, \quad i = 1, 2, 3, ..., N_L, \quad (7) \]

which gives angular position of liner \( i \) at every time \( t \). Position of every liner is then

\[ x_i = x_M + R_M \cos \theta_i \]

and

\[ y_i = y_M + R_M \sin \theta_i. \]

The kinematics of every liner is defined sufficiently using Equation (7)-(9) with input parameters \( N_L, R_M, \) and \( \omega_M \).

2.2. Interaction forces

Each of the disk-formed ground particles \( i \) could have specific value of diameter \( D_i \) and density \( \rho_i \) (or mass \( m_i \)). These particles interact with each other and also with particles representing mill liners through normal force \( \hat{N} \). Then, the two types of particles are also suffering from earth gravitation force \( \hat{G} \) and air drag force \( \hat{D} \). Only these three types of forces are considered in this work. According to coordinate system used in Figure 1 then gravitation force on particle \( i \) is simply

\[ \hat{G}_i = -m_i \hat{g} \hat{e}_j, \]

where \( \hat{g} \) is earth gravity with value about 9.807 m/s\(^2\). Assuming that particles are moving in still air, then

\[ \hat{D}_i = -c_p v_i \hat{v}_i. \]

is the formulation for drag force. Relative position of particle \( i \) related to particle \( j \) is

\[ \hat{r}_{ij} = \hat{r}_i - \hat{r}_j \]

and its relative velocity is

\[ \hat{v}_{ij} = \hat{v}_i - \hat{v}_j. \]

From Equation (12) unit vector of relative position can be obtained

\[ \hat{r}_{ij} \]

Overlap between particles \( i \) and \( j \) is defined as

\[ \xi_{ij} = \max(0, \frac{1}{2}D_i + \frac{1}{2}D_j - r_{ij}) \]

and it derivative with respect to time \( t \) is

\[ \frac{d\xi_{ij}}{dt} = -v_{ij} \text{ sign}(\xi_{ij}). \]

Normal force on particle \( i \) due to interaction with particle \( j \) is formulated as (Schäfer et al., 1996)
\[ N_y = \left( k_N \xi_y - \gamma_N \frac{d \xi_y}{dt} \right) \dot{r}_y . \]  

(17)

2.3. Newton’s second law of motion

Acceleration of particle \( i \) can be obtained from

\[ \ddot{a}_i = \frac{1}{m_i} \sum \vec{F} , \]

(18)

where \( \sum \vec{F} \) is sum of all forces worked on the particle. Supposed that if there are \( N_L \) liner particles and \( N_G \) ground material particles then from Equations (10), (11), and (17)

\[ \ddot{a}_i = \frac{1}{m_i} \left( \dot{G}_i + \dot{D}_i - \sum_{j \neq i} N_y + \sum_{j \neq i} N_y \right) \]

or

\[ \frac{d^2 \vec{r}_i}{dt^2} = -\frac{1}{m_i} \sum_{j \neq i} \left( k_N \xi_j - \gamma_N \frac{d \xi_j}{dt} \right) \left( \ddot{r}_i - \ddot{r}_j \right) + c_L \frac{d \vec{r}_i}{dt} + g \hat{e}_j = 0 \]  

(20)

is the equation of motion of particle \( i \), that should be solved. It is actually \( N_G \times (N_G - 1 + N_L) \) coupled differential equations. Due to the complexity they cannot be so solved simultaneously, but it can be approached using numerical method, which will be described in Simulation section.

3. Simulation

Equation (20) will be solved using Euler method (Seinfeld et al. 1970), that relates motion variable at time \( t + \Delta t \) with its value at time \( t \) and also its time derivative

\[ \frac{d^n f}{dt^n} \bigg|_{t+\Delta t} = \frac{d^n f}{dt^n} \bigg|_{t} + \frac{d^{n+1} f}{dt^{n+1}} \bigg|_{t} \Delta t , \]

(21)

where \( f \) stands for \( \ddot{r} \) if \( n = 0 \) and for \( \ddot{v} \) if \( n = 1 \). Acceleration \( \ddot{a} \) is obtained from Newton’s second law of motion. After one time step \( \Delta t \) of calculation all values of motion variables from all particles have been changed, then Equations (19) and (21) are repeated until the desired simulation repetition. Normally a final stopping criterion is set, e.g. \( t = t_{\text{end}} \), where initial time is set to 0 or \( t_{\text{beg}} \). In this research, the simulation run at \( 10^{-3} \) s time step interval, \( \Delta t \).
Figure 5. Initial condition of simulation with 20 circular disks as liner and 25 grains inside the mill. The liner was rotated in clockwise direction with angular velocity of 0.8 rad/s.

The number of circular disks and radius of mill are required to be set before the simulation begins. Circular disks and grains are numbered in order to analyse the behaviour of each grains. Numbering on grains is depend on the number of circular disks and the number of the whole particles in the system. The configuration of simulation which was shown in Fig.4 used 45 particles with 20 of them were circular disks. In initial condition, the grains were positioned at the centre of the mill.

Figure 6. Initial condition of SAG simulation with 10 ball mills at the top of grains configuration.

The ball mills, which were marked with black line, are placed at the top two rows of grains configuration in order to push the grains to reach the bottom of liner wall as the mass was 5 times greater than the particles and speed up the grinding process. The number of ball mills was set lesser than the particles to be milled. In this simulation, we did the same characteristic of rotation with the previous one.
4. Results and discussion

Simulation was conducted using JavaScript program (Viridi and Kurniadi, 2017).

![Simulation Display](image)

**Figure 7.** Display of simulation with $N_L = 20$, $N = 45$, $R_M = 38$ at (a) $t = 0$ s, (b) $t = 2$ s, (c) $t = 3$ s, (d) $t = 4$ s, (e) $t = 6$ s, (f) $t = 7$ s, (g) $t = 8$ s, (h) $t = 10$ and (i) $t = 12$ s.

As the consequence of the gravitational attraction, the grains move in negative direction of $y$-axis, which was shown in Fig.7b. The grains started to reach the bottom of the mill at $t = 2$ s and interacted with the rotated liner. Interaction between grains and the circular disks generates the normal force where the direction of movement is affected by the properties of interacting particles such as vector and magnitude of velocity. When the value of normal force takes greater contribution than
gravitational force, the movement of particle will obey the normal force direction as shown particle numbered 42 in Fig. 7c.

![Image](https://via.placeholder.com/150)

Figure 8. Movement of particle numbered (a) 21, (b) 22, and (c) 42.

Particle numbered 21 and 22 were moving in cascading mode, then become cataracting mode at $t = 7\, s$ forward which were described in Fig. 8a and 8b. While particle numbered 42 was moving in cataracting mode by $t = 3\, s$.

The particles were moving in cascading and cataracting mode according to the rotation of mill which was in clockwise direction. This condition was described in Fig. 7e where the particles assembled at the left-bottom side which is convenient with the characteristic of cataracting mode described in Fig. 1.
Figure 9. Display of simulation with $N_L = 20$, $N = 45$, $R_m = 38$ at (a) $t = 0$ s, (b) $t = 2$ s, (c) $t = 3$ s, (d) $t = 4$ s, (e) $t = 6$ s, (f) $t = 7$ s, (g) $t = 8$ s, (h) $t = 10$ s, and (i) $t = 12$ s.

In SAG simulation, the particles and the grinding balls were moving down due to gravitational force in first two seconds. The motion of particles which affected by normal force were retained by the falling ball mills, therefore the particles distributed at the lower region of mill if compare to the distribution in Fig.7.

Grinder ball numbered 40 hit the particle numbered 35 in impact zone, allows the particle to be smaller parts. The cataracting mode in this simulation are shown in Fig.9d until Fig.9i. Particle numbered 21, 22, and 26 were moving up due to the rotation of the wall at $t = 6$ s, afterwards they jump to the impact zone. This condition is similar to Fig.8.
5. Conclusion
The simulation program used in this paper offer the adjustable configuration of mill, such as size and number of circular liner and also the radius of mill. The simulation run results an acceptable distribution of grains when the grinding occurs, especially for cascading and cataracting mode. The existence of ball mills can accelerate the grinding process.

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