Effect of $D^0$-$\overline{D}^0$ Mixing upon Cabibbo-favored $D^0$ Decays

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Abstract

The parameter $y_{CP}$ is used to characterize mixing in the $D^0$-$\overline{D}^0$ meson system. To determine $y_{CP}$, one measures the effective decay width of a $D^0$ or $\overline{D}^0$ decaying to a $CP$ eigenstate, relative to the effective width for a decay to a Cabibbo-favored, flavor-specific final state. When using $y_{CP}$ to extract information about $D^0$-$\overline{D}^0$ mixing and $CP$ violation, the decay width of the Cabibbo-favored decay is usually assumed to equal $1/\tau$, the reciprocal of the $D^0$ lifetime. However, there is a small correction to this that should be taken into account when $y_{CP}$ is measured with sufficiently high precision. We calculate this correction in terms of charm mixing and $CP$ violation parameters $x$, $y$, $|q/p|$, and $\phi$.

1 Introduction

An important parameter in the phenomenology of charm mixing is $y_{CP}$. It is defined as the difference from unity of the effective lifetime of $(D^0 + \overline{D}^0)$ decays to a $CP$ eigenstate, relative to the effective lifetime of $D^0$ or $\overline{D}^0$ Cabibbo-favored decay to a flavor eigenstate. Specifically [1, 2],

$$y_{CP} = \frac{\hat{\Gamma}(D^0, \overline{D}^0 \rightarrow K^+K^-)}{\Gamma[D^0 \rightarrow K^-\pi^+] - 1}. \quad (1)$$

The effective lifetimes are measured by fitting the corresponding decay time distributions to exponential functions. When taking the ratio of effective lifetimes in Eq. (1), many systematic uncertainties cancel out. Theoretically, for equal numbers of $D^0$ and $\overline{D}^0$ decays (as is the case, e.g., at an $e^+e^-$ collider), one finds [2]

$$y_{CP} = \frac{1}{2} \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) y \cos \phi - \frac{1}{2} \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) x \sin \phi, \quad (2)$$

where $q$ and $p$ are complex coefficients relating the flavor eigenstates $D^0$ and $\overline{D}^0$ to the two mass eigenstates of the $D^0, \overline{D}^0$ system, and $\phi = \text{Arg}(q/p)$. Equation (2) shows that $y_{CP}$ is a combination of the mixing parameters $x = \Delta M/\Gamma$ and $y = \Delta \Gamma/(2\Gamma)$, where $\Delta M$ and $\Delta \Gamma$ are the differences in masses and decay widths, respectively, between the mass eigenstates, and $\Gamma$ is their mean decay width.

In calculating Eq. (2), it is assumed that $\hat{\Gamma}(D^0 \rightarrow K^-\pi^+) = \Gamma$, the mean decay width. In the past, this assumption was sufficiently accurate given the measured precision of $\hat{\Gamma}(D^0 \rightarrow K^+K^-)$. However, a recent measurement of $\hat{\Gamma}(D^0 \rightarrow K^+K^-)$ by the LHCb Collaboration [3] has significantly greater precision than previous measurements [4] and thus requires more careful consideration of $\hat{\Gamma}(D^0 \rightarrow K^-\pi^+)$. In particular, the effective decay width deviates from $\Gamma$ due to $D^0$ mesons that oscillate to $\overline{D}^0$ and subsequently decay to $K^-\pi^+$ via a doubly Cabibbo-suppressed amplitude. As the
$D^0\overline{D}^0$ mixing rate is very small, and the branching fraction for $D^0 \to K^-\pi^+$ is only $1.4 \times 10^{-4}$ \cite{1}, the effect of this process upon the decay time distribution is tiny and typically neglected. However, in Eq. (1), $\hat{\Gamma}(D^0 \to K^-\pi^+)$ is compared to $\hat{\Gamma}(D^0 \to K^+K^-)$, whose deviation from $\Gamma$ is also very small; thus the tiny effect in $\hat{\Gamma}(D^0 \to K^-\pi^+)$ can have an appreciable effect upon $y_{CP}$.

A more careful treatment of $\hat{\Gamma}(D^0 \to K^-\pi^+)$ adds a correction term to Eq. (2). Alternatively, one can define $y_{CP}^{KK} \equiv [\hat{\Gamma}(D^0 \to K^+K^-)/\Gamma] - 1$ and $y_{CP}^{\pi\pi} \equiv [\hat{\Gamma}(D^0 \to K^-\pi^+)/\Gamma] - 1$, and then it is $y_{CP}^{KK}$ that equals the right-hand side of Eq. (2). The right-hand side of Eq. (1), which is what experiments measure, would essentially equal $y_{CP}^{KK} - y_{CP}^{\pi\pi}$. To use Eqs. (11) and (2) to extract information from $y_{CP}$ about $x$, $y$, $|q/p|$, and $\phi$ (as done, e.g., by the Heavy Flavor Averaging Group \cite{6}), one must correct the measured value of $y_{CP}^{KK} - y_{CP}^{\pi\pi}$ for $y_{CP}^{\pi\pi}$.

This paper presents a calculation of $y_{CP}^{\pi\pi}$, or equivalently $\hat{\Gamma}(D^0 \to K^-\pi^+)$, the effective decay width of a Cabibbo-favored decay. This correction was first pointed out in Ref. \cite{7}. In that paper, $y_{CP}^{\pi\pi}$ is calculated in terms of parameters $x_{12}$, $y_{12}$, $\phi_f^M$, and $\phi_f^G$, the magnitudes and phases of the off-diagonal elements of the $D^0\overline{D}^0$ dispersive mass matrix and absorptive decay matrix \cite{8}. In this paper, we calculate $y_{CP}^{\pi\pi}$ in terms of the more common mixing and $CP$ violation parameters $x$, $y$, $|q/p|$, and $\phi$ \cite{4}.

## 2 Calculation

Starting from a pure $|D^0\rangle$ at $t = 0$, the state found at a later time $t$ is

$$|D^0(t)\rangle = g_+(t)|D^0\rangle + \left(\frac{q}{p}\right)g_-(t)|\overline{D}^0\rangle,$$

where

$$g_\pm(t) = \frac{1}{2}(e^{-i\omega_1 t} \pm e^{-i\omega_2 t})$$

(4)

and

$$\omega_{1,2} = m_{1,2} - \frac{i}{2}\Gamma_{1,2}$$

(5)

are the eigenvalues of the two mass eigenstates $|D_1\rangle$ and $|D_2\rangle$. The parameters $m_{1,2}$ and $\Gamma_{1,2}$ are the eigenvalues of the Hermitian $2 \times 2$ mass and decay matrices, respectively, and are real. The decay amplitude to a final state $f$ is

$$\mathcal{A}(D^0 \to f) = \langle f|H|D^0(t)\rangle = g_+(t)\mathcal{A}_f + \left(\frac{q}{p}\right)g_-(t)\overline{\mathcal{A}}_f,$$

(6)

where $\mathcal{A}_f \equiv \langle f|H|D^0\rangle$ and $\overline{\mathcal{A}}_f \equiv \langle f|H|\overline{D}^0\rangle$ are decay amplitudes for pure flavor eigenstates $D^0$ and $\overline{D}^0$. The decay rate is thus

$$r(t) = \left|\langle f|H|D^0(t)\rangle\right|^2 = \left|g_+(t)\mathcal{A}_f + \left(\frac{q}{p}\right)g_-(t)\overline{\mathcal{A}}_f\right|^2$$

(7)

$$= \left|\mathcal{A}_f\right|^2\left|g_+(t)\right|^2 + \left(\frac{q}{p}\right)^2\overline{\mathcal{A}}_f g_-(t)\right|^2$$

(8)

$$= \left|\mathcal{A}_f\right|^2\left|g_+(t)\right|^2 + \left(\frac{q}{p}\right)^2\overline{\mathcal{A}}_f g_-(t)\right|^2$$

$$= \left|\mathcal{A}_f\right|^2\left[\left|g_+(t)\right|^2 + \left|g_-(t)\right|^2 + 2\Re[\lambda g^*_+(t)g_-(t)]\right],$$

(10)
where (to reduce clutter) we have defined the parameter \( \lambda \equiv (q/p)(\mathcal{A}_f/\mathcal{A}_f) \). Similarly, starting from a pure \( |\mathcal{D}^0\rangle \) state at \( t = 0 \), the decay rate to a final state \( \tilde{f} \) is

\[
\tau(t) = \left| \langle \tilde{f} | H | \mathcal{D}^0(t) \rangle \right|^2 = \left| g_+(t) \mathcal{A}_f + \left( \frac{p}{q} \right) g_-(t) \mathcal{A}_f \right|^2
\]

\[
= \left| \mathcal{A}_f \right|^2 \left| g_+(t) \right|^2 + \left( \frac{p}{q} \right) \left( \frac{\mathcal{A}_f}{\mathcal{A}_f} \right) \left| g_-(t) \right|^2
\]

\[
= \left| \mathcal{A}_f \right|^2 \left| g_+(t) \right|^2 + \left| \tilde{\lambda} \left| g_-(t) \right|^2
\]

\[
= \left| \mathcal{A}_f \right|^2 \left\{ \left| g_+(t) \right|^2 + \left| \tilde{\lambda} \right| \left| g_-(t) \right|^2 + 2 \Re \{ \tilde{\lambda} \left| g_+(t) \right| \left| g_-(t) \right| \} \right\},
\]

where \( \tilde{\lambda} \equiv (p/q)(\mathcal{A}_f/\mathcal{A}_f) \).

We calculate the following:

\[
\left| g_+(t) \right|^2 = \frac{1}{4} \left| e^{-i\omega_1 t} + e^{-i\omega_2 t} \right|^2
\]

\[
= \frac{1}{4} \left\{ \left| e^{-i\omega_1 t} \right|^2 + \left| e^{-i\omega_2 t} \right|^2 + 2 \Re \{ e^{i\omega_1 t} e^{-i\omega_2 t} \} \right\}
\]

\[
= \frac{1}{4} \left\{ \left| e^{-i(m_1 - i\Gamma_1)/2} \right|^2 + \left| e^{-i(m_2 - i\Gamma_2)/2} \right|^2 + 2 \Re \{ e^{i(m_1 + i\Gamma_1)/2} t e^{-i(m_2 - i\Gamma_2)/2} t \} \right\}
\]

\[
= \frac{1}{4} \left\{ \left| e^{-\Gamma t} t + e^{-\Gamma_2 t} \right|^2 + 2 \Re \{ e^{i\Delta m t} e^{-\Delta t} \} \right\}
\]

\[
= \frac{1}{4} \left\{ \left| e^{-\Gamma t} - e^{-\Gamma_2 t}/2 + e^{(-\Gamma_2 + \Gamma_1)/2} \right|^2 + e^{-\Delta t \cos(\Delta m t)} \right\}
\]

\[
= \frac{e^{-\Delta t \cos(\frac{\Delta \Gamma}{2})}}{2} \left\{ \cosh \left( \frac{\Delta \Gamma}{2} \right) \right. + \cos(\Delta m t) \right\},
\]

where \( \Delta m \equiv m_2 - m_1, \Delta \Gamma \equiv \Gamma_2 - \Gamma_1, \) and \( \tilde{\Gamma} \equiv (\Gamma_1 + \Gamma_2)/2 \). Similarly,

\[
\left| g_-(t) \right|^2 = \frac{1}{4} \left| e^{-i\omega_1 t} - e^{-i\omega_2 t} \right|^2
\]

\[
= \frac{1}{4} \left\{ \left| e^{-i\omega_1 t} \right|^2 + \left| e^{-i\omega_2 t} \right|^2 - 2 \Re \{ e^{i\omega_1 t} e^{-i\omega_2 t} \} \right\}
\]

\[
= \frac{1}{4} \left\{ \left| e^{-i(m_1 + i\Gamma_1)/2} \right|^2 + \left| e^{-i(m_2 - i\Gamma_2)/2} \right|^2 - 2 \Re \{ e^{i(m_1 - i\Gamma_1)/2} t e^{-i(m_2 - i\Gamma_2)/2} t \} \right\}
\]

\[
= \frac{1}{4} \left\{ \left| e^{-\Gamma t} \right|^2 - e^{(-\Gamma_1 + \Gamma_2)/2} t e^{(-\Gamma_2 + \Gamma_1)/2} t \right|^2 - 2 \Re \{ e^{-\Delta t \cos(\Delta m t)} \} \right\}
\]

\[
= \frac{e^{-\Delta t \cos(\frac{\Delta \Gamma}{2})}}{2} \left\{ \cosh \left( \frac{\Delta \Gamma}{2} \right) \right. - \cos(\Delta m t) \right\}.
\]

Finally,

\[
g_+^*(t) g_-(t) = \frac{1}{4} \left( e^{i\omega_1 t} + e^{i\omega_2 t} \right) \left( e^{-i\omega_1 t} - e^{-i\omega_2 t} \right)
\]

\[
= \frac{1}{4} \left( e^{i(\omega_1 - \omega_1) t} - e^{i(\omega_2 - \omega_2) t} + e^{i(\omega_2 - \omega_1) t} - e^{i(\omega_1 - \omega_2) t} \right)
\]

\[
= \frac{1}{4} \left( e^{-\Gamma_1 t} - e^{-\Gamma_2 t} + e^{-i\Delta m t} e^{-\Delta t} - e^{i\Delta m t} e^{-\Delta t} \right)
\]

\[
= \frac{1}{4} e^{-\Delta t} \left( e^{(-\Gamma_1 + \Gamma_2)/2} t - e^{(-\Gamma_2 + \Gamma_1)/2} t + e^{-i\Delta m t} - e^{i\Delta m t} \right)
\]

\[
= \frac{e^{-\Delta t \cos(\frac{\Delta \Gamma}{2})}}{2} \left\{ \sinh \left( \frac{\Delta \Gamma}{2} \right) \right. + i \sin(\Delta m t) \right\}.
\]
Thus,
\[
2 \text{Re}\left[ \lambda^2 g_+^2(t) g_-^2(t) \right] = e^{-\Gamma t} \left\{ \text{Re}(\lambda) \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \text{Im}(\lambda) \sin (\Delta m t) \right\}.
\]  
(17)

Inserting Eqs. (15), (16), and (17) into Eq. (10) gives
\[
r(t) = \frac{|A_f|^2}{2} e^{-\Gamma t} \left\{ (1 + |\lambda|^2) \cosh \left( \frac{\Delta \Gamma}{2} t \right) + (1 - |\lambda|^2) \cos (\Delta m t) \right\}
+ 2 \text{Re}(\lambda) \sinh \left( \frac{\Delta \Gamma}{2} t \right) - 2 \text{Im}(\lambda) \sin (\Delta m t) \right\}.
\]  
(18)

Notating \( \Gamma \) as simply \( \Gamma \) and defining mixing parameters \( x \equiv \Delta m/\Gamma \) and \( y \equiv \Delta \Gamma/(2\Gamma) \), Eq. (13) becomes
\[
r(t) = \frac{|A_f|^2}{2} e^{-\Gamma t} \left\{ (1 + |\lambda|^2) \cosh(y \Gamma t) + (1 - |\lambda|^2) \cos(x \Gamma t) \right\}
+ 2 \text{Re}(\lambda) \sin(y \Gamma t) - 2 \text{Im}(\lambda) \sin(x \Gamma t) \right\}.
\]  
(19)

The combination \( \Gamma t = t/\tau_{D^0} \equiv \tilde{t} \), the decay time in units of \( D^0 \) lifetime. For the range of \( \tilde{t} \) measured by experiments, \( x \tilde{t} \ll 1 \), \( y \tilde{t} \ll 1 \), and we can use the following approximations: \( \cos(x \tilde{t}) \approx 1 - (x \tilde{t})^2/2 \), \( \cosh(y \tilde{t}) \approx 1 + (y \tilde{t})^2/2 \), \( \sinh(x \tilde{t}) \approx x \tilde{t} \), and \( \sin(y \tilde{t}) \approx y \tilde{t} \). With these approximations, Eq. (19) becomes
\[
r(t) = \frac{|A_f|^2}{2} e^{-\Gamma t} \left\{ (1 + |\lambda|^2) \left( 1 + \frac{y^2}{2} \right) + (1 - |\lambda|^2) \left( 1 - \frac{x^2}{2} \right) + 2 \text{Re}(\lambda) y \tilde{t} - 2 \text{Im}(\lambda) x \tilde{t} \right\}
= |A_f|^2 e^{-\Gamma t} \left\{ 1 + \frac{y^2 - x^2}{4} + |\lambda|^2 \frac{x^2 + y^2}{4} + \text{Re}(\lambda) y \tilde{t} - \text{Im}(\lambda) x \tilde{t} \right\}
= |A_f|^2 e^{-\Gamma t} \left\{ 1 + \frac{y^2 - x^2}{4} + \frac{|q/p|^2}{|\overline{A}_f/A_f|^2} \left( x^2 + y^2 \right) + \left| \frac{q}{p} \right| \left( y \cos(\phi - \delta) - x \sin(\phi - \delta) \right) \right\} \Gamma \tilde{t},
\]  
(20)

where \( \phi \equiv \text{Arg}(q/p) \) and \( \delta \equiv \text{Arg}(A_f/\overline{A}_f) \). While \( \phi \) is a purely weak phase difference, \( \delta \) is almost purely a strong phase difference: the weak phase difference between \( D^0 \) decay amplitudes is tiny due to charm decays proceeding almost exclusively via the first two flavor generations.

The various terms in Eq. (20) have very different magnitudes. The term \( (y^2 - x^2)/4 \) is quadratic in the small (\( < 1\% \)) mixing parameters \( x \) and \( y \) and thus is negligible relative to the leading term. The term \( (x^2 + y^2)/4 \) is also quadratic in mixing parameters; however, if the amplitude \( \overline{A}_f \) is Cabibbo-favored and \( A_f \) is doubly Cabibbo-suppressed, e.g., \( f = K^+\pi^- \), then the term is greatly enhanced by the factor \( |\overline{A}_f/A_f|^2 \) and cannot be neglected. The last two terms, which are linear in mixing parameters, would be enhanced by the factor \( |\overline{A}_f/A_f| \) and thus should also be kept. These three terms yield the usual formula for the decay-time dependence of “wrong-sign” \( D^0 \to K^+\pi^- \) decays; see Refs. [9][10][11][12][1].

\footnote{\( f = K^+\pi^- \), our sign convention for \( \delta \) is opposite that used for the strong phase \( \delta \) in these papers.}
However, if \( A_f \) is Cabibbo-favored and \( \overline{A}_f \) is doubly Cabibbo-suppressed, e.g., \( f = K^-\pi^+ \), then the \((x^2 + y^2)/4\) term can also be neglected. In this case, Eq. (20) becomes

\[
\begin{align*}
  r_{D^0 \to K^-\pi^+}(t) & \approx |A_f|^2 e^{-\Gamma t} \left\{ 1 + \frac{|A_f|}{|\overline{A}_f|} \left[ y \cos(\phi - \delta) - x \sin(\phi - \delta) \right] \Gamma t \right\} \\
  & \approx |A_f|^2 e^{-\Gamma t} e^{-y_{K\pi} \Gamma t} = |A_f|^2 e^{-(1+y_{K\pi}) \Gamma t}, \quad (21)
\end{align*}
\]

where

\[
y_{K\pi} = \frac{|q/p|}{2} \sqrt{R_f} \left[ x \sin(\phi - \delta) - y \cos(\phi - \delta) \right] \quad (22)
\]

with \( R_f \equiv |\overline{A}_f/A_f|^2 \). Equation (21) implies that the decay-time distribution of Cabibbo-favored \( D^0 \to K^-\pi^+ \) decays is essentially exponential, with a decay constant of \((1 + y_{K\pi}) \times \Gamma\).

For Cabibbo-favored \( \overline{D}^0 \to K^+\pi^- \) decays, the decay rate \( \bar{r}(t) \) is obtained by comparing Eq. (14) with Eq. (10). From this comparison, we conclude that

\[
\begin{align*}
  \bar{r}_{\overline{D}^0 \to K^+\pi^-}(t) & \approx |\overline{A}_f|^2 e^{-(1+y_{K\pi}) \Gamma t}, \quad (23)
\end{align*}
\]

where

\[
\bar{y}_{K\pi} = \frac{|p/q|}{2} \sqrt{R_f} \left[ x \sin(-\phi - \delta) - y \cos(-\phi - \delta) \right] \quad (24)
\]

with \( R_f \equiv |A_f/\overline{A}_f|^2 \). If one selects a combined sample of Cabibbo-favored \( D^0 \to K^-\pi^+ \) and \( \overline{D}^0 \to K^+\pi^- \) decays, with equal numbers of such decays, then the resulting decay-time distribution (assuming \(|A_f|^2 = |\overline{A}_f|^2\)) will have a time dependence of

\[
\begin{align*}
  e^{-\Gamma t} \left( e^{-y_{K\pi} \Gamma t} + e^{-\bar{y}_{K\pi} \Gamma t} \right) & = 2 e^{-\Gamma t} e^{-(y_{K\pi} + \bar{y}_{K\pi}) \Gamma t/2} \cosh \left( \frac{y_{K\pi} + \bar{y}_{K\pi}}{2} \right) \Gamma t \\
  & \approx 2 e^{-\Gamma t} e^{-(y_{K\pi} + \bar{y}_{K\pi}) \Gamma t/2} \\
  & \approx 2 e^{-\Gamma t} e^{-y_{CP} \Gamma t} \\
  & \approx 2 e^{-(1+y_{CP}) \Gamma t}, \quad (25)
\end{align*}
\]

where we have defined \( y_{CP} = (y_{K\pi} + \bar{y}_{K\pi})/2 \). This decay time distribution is also exponential, with a decay constant differing from \( \Gamma \) by the factor

\[
y_{CP} = \frac{y_{K\pi} + \bar{y}_{K\pi}}{2} = \frac{1}{2} \left\{ \frac{|q|}{p} \sqrt{R_f} \left[ x \sin(\phi - \delta) - y \cos(\phi - \delta) \right] \\
+ \frac{|p|}{q} \sqrt{R_f} \left[ x \sin(-\phi - \delta) - y \cos(-\phi - \delta) \right] \right\}
\]

\[
= \frac{1}{2} \left\{ \frac{|q|}{p} \sqrt{R_f} \left[ x \sin(\phi \cos \delta - \cos \phi \sin \delta) - y (\cos \phi \cos \delta + \sin \phi \sin \delta) \right] \\
+ \frac{1}{2} \left\{ \frac{|p|}{q} \sqrt{R_f} \left[ x \cos \phi \sin \delta + y \cos \phi \cos \delta \right] \\
+ \frac{1}{2} \left\{ \frac{|q|}{p} \sqrt{R_f} - \frac{|p|}{q} \sqrt{R_f} \right\} \right\} \left( x \sin \phi \cos \delta - y \sin \phi \sin \delta \right). \quad (26)
\]
This expression is complicated. However, we can also apply this formula to \( f = K^+K^- \) or \( \pi^+\pi^- \); for these self-conjugate final states, \( |\mathcal{A}_f/A_f| = 1 \) and the \((x^2+y^2)/4\) term in Eq. (20) can be neglected as done for Cabibbo-favored decays. For \( f = K^+K^- \), \( R_f = R_f = 1 \), \( \delta = \pi \) (due to our phase convention \( CP|D^0| = -|\bar{D}^0| \), \( CP|\bar{D}^0| = -|D^0| \)), and Eq. (26) simplifies to

\[
y_{CP}^{K\pi} = \frac{1}{2} \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) y \cos \phi - \frac{1}{2} \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) x \sin \phi.
\]  

(27)

This quantity is the well-known parameter \( y_{CP} \). [4, 6], i.e., we obtain Eq. (2).

As a final step, we utilize the fitted parameters of the Heavy Flavor Averaging Group (HFLAV) [6]

\[
R_D \equiv \frac{|A_f|^2 + |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2}, \quad A_D \equiv \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2},
\]

and note that \( R_f = R_D(1 - A_D) \) and \( R_f = R_D(1 + A_D) \). Inserting these expressions into Eq. (26) gives

\[
y_{CP}^{K\pi} = -\sqrt{R_D} \left( \left| \frac{q}{p} \right| \sqrt{1-A_D} + \left| \frac{p}{q} \right| \sqrt{1+A_D} \right) (x \cos \phi \sin \delta + y \cos \phi \cos \delta) \\
+ \sqrt{R_D} \left( \left| \frac{q}{p} \right| \sqrt{1-A_D} - \left| \frac{p}{q} \right| \sqrt{1+A_D} \right) (x \sin \phi \cos \delta - y \sin \phi \sin \delta).
\]

(29)

To estimate how large \( y_{CP}^{K\pi} \) is, we insert values obtained from the most recent HFLAV global fit [13]: \( x = 0.407\% \), \( y = 0.647\% \), \( R_D = 0.344\% \), \( A_D = -0.76\% \), \( |q/p| = 0.994 \), \( \delta = 11.7^\circ \), and \( \phi = -2.6^\circ \). The result is \( y_{CP}^{K\pi} = -4.19 \times 10^{-4} \), which has a magnitude \( \ll 1 \) and thus is typically neglected in Eq. (25). However, to measure \( y_{CP} \), one compares the effective decay constant \( \hat{\Gamma} \) of \( D^0 \to K^+K^- \) or \( D^0 \to \pi^+\pi^- \) decays with the effective decay constant of \( D^0 \to K^-\pi^+ \) or \( \bar{D}^0 \to K^+\pi^- \) decays (or a combined sample), i.e., Eq. (11). The right-hand side of Eq. (11) equals

\[
\frac{\hat{\Gamma}_{K^+K^-}}{\hat{\Gamma}_{K\pi}} - 1 \approx \frac{1 + y_{CP}^{K\pi}}{1 + y_{CP}^{K\pi}} - 1 \approx (1 + y_{CP}^{K\pi})(1 - y_{CP}^{K\pi}) - 1 \\
\approx y_{CP}^{K\pi} - y_{CP}^{K\pi}.
\]

(30)

As the left-hand side of Eq. (30) has recently been measured with an uncertainty of \( 2.9 \times 10^{-4} \) [3], it is necessary to account for \( y_{CP}^{K\pi} \) – which is larger than this uncertainty – to determine \( y_{CP}^{KK} \) (usually referred to as simply \( y_{CP} \)).

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