Spinning up the Surface: Evidence for Planetary Engulfment or Unexpected Angular Momentum Transport?

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Abstract

In this paper, we report the potential detection of a nonmonotonic radial rotation profile in a low-mass lower-luminosity giant star. For most low- and intermediate-mass stars, the rotation on the main sequence seems to be close to rigid. As these stars evolve into giants, the core contracts and the envelope expands, which should suggest a radial rotation profile with a fast core and a slower envelope and surface. KIC 9267654, however, seems to show a surface rotation rate that is faster than its bulk envelope rotation rate, in conflict with this simple angular momentum conservation argument. We improve the spectroscopic surface constraint, show that the pulsation frequencies are consistent with the previously published core and envelope rotation rates, and demonstrate that the star does not show strong chemical peculiarities. We discuss the evidence against any tidally interacting stellar companion. Finally, we discuss the possible origin of this unusual rotation profile, including the potential ingestion of a giant planet or unusual angular momentum transport by tidal inertial waves triggered by a close substellar companion, and encourage further observational and theoretical efforts.

Unified Astronomy Thesaurus concepts: Red giant branch (1368); Star-planet interactions (2177); Asteroseismology (73); High resolution spectroscopy (2096); Gaia (2360); Stellar rotation (1629)

1. Introduction

Stellar rotation rates are a sensitive tracer of stellar structure, magnetism, and internal transport of angular momentum. As single stars evolve, several physical processes are expected to act, including angular momentum loss via stellar winds, meridional circulation, internal waves, and magnetic instabilities, among others (see e.g., Maeder & Meynet 2000; Mathis 2013; Aerts et al. 2019). Unfortunately, their relative strength, effectiveness, and timescales have been challenging to constrain theoretically. Each mechanism is, however, generally expected to leave behind a characteristic radial rotation profile. For example, in radiative regions purely hydrodynamic mechanisms such as shear instabilities can act to reduce shear while advective currents can create it (e.g., Zahn 1992; Talon et al. 1997; Meynet & Maeder 2000; Palacios et al. 2003; Decressin et al. 2009; Mathis et al. 2018). Internal magnetic fields are often expected to lead to uniform rotation profiles in large stellar regions (e.g., Mestel & Weiss 1987; Spruit 2002; Eggenberger et al. 2005; Maeder & Meynet 2005; Cantiello et al. 2014; Fuller et al. 2019), while internal gravity waves can create complex and even counter-rotating zones (Kumar et al. 1999; Talon & Charbonnel 2005; Rogers 2015).

The existence of large databases of rotation rates, including spot modulations (e.g. McQuillan et al. 2014; Ceillier et al. 2017;

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 Stars enter the pre-main sequence with a range of rotation rates inherited from their birth clouds. Interactions with their circumstellar disks as well as winds and outflows serve to regulate that angular momentum as they contract (e.g., Cieza & Baliber 2007), and leave behind a distribution of rotation rates that is likely correlated with the properties of these disks (Mamajek 2009; Rebull et al. 2018). At first, the core of these stars is decoupled from the angular momentum loss happening from the wind at the surface (Denissenkov et al. 2010), but the two zones recouple on mass-dependent timescales of order tens to hundreds of millions of years as these stars approach the main sequence (MacGregor & Brenner 1991; Gallet & Bouvier 2013; Lanzafame & Spada 2015; Somers & Pinsonneault 2016). The complexity of these processes (see also Curtis et al. 2020; Godoy-Rivera et al. 2021) makes it challenging to predict a priori what the rate of rotation or its radial or latitudinal profile should be for stars on the main sequence.

Empirically, it has been found that stars rotate approximately as solid bodies on the main sequence (Van Reeth et al. 2016, 2018; Li et al. 2019; Jermy et al. 2020) and that seismic estimates of the envelope rotation rates in these stars are consistent with surface rotation measurements from spots or \( v \sin i \) (Benomar et al. 2015; Nielsen et al. 2015; Beck et al. 2018; Hall et al. 2021). Once stars leave the main sequence, they seem to rotate close to a solid body for a short evolutionary interval (Deheuvels et al. 2020), then the core contracts and the envelope expands, leading to a rapidly rotating core and a slowly rotating envelope.

However, it has been noted by many authors that the cores do not spin up to the level expected of angular momentum conservation (see e.g., Tayar & Pinsonneault 2013), and so some mechanism must transport angular momentum from the core to the surface. Explorations of various hydrodynamic or magnetic mechanisms have thus far failed to match the necessary amount of angular momentum transport for all the observed stars (Eggenberger et al. 2012; Ceillier et al. 2013; Marques et al. 2013; Cantiello et al. 2014).

The identity of this mechanism has remained unclear, as no current theories perfectly match all the observations (e.g., Eggenberger et al. 2019; den Hartogh et al. 2020). In addition, the radial location of the differential rotation is unclear, and various authors have argued that it could be in the surface convection zone (Kissin & Thompson 2015; Tayar & Pinsonneault 2018; Takahashi & Langer 2021) although others expect it to be concentrated in radiative regions (Fuller et al. 2019; Fellay et al. 2021). For most stars, getting measurements of the rotation rate at more than two locations—usually a “core” and a “surface”—has been challenging because of the limitations of the measurable oscillation modes, and thus this conflict has remained unresolved.

In addition, it is known that orbital angular momentum can be exchanged with rotational angular momentum through tidal interactions (Zahn 1977). Work has suggested that binary synchronization (Song et al. 2013), mass transfer (Packet 1981; Renzo & Götberg 2021), and planet engulfment (Privitera et al. 2016a, 2016b; Bolmont & Mathis 2016; Ahuir et al. 2021) can all affect the rotation of a star. However, the impact of these sorts of interactions on the stellar rotation profile, including the rate at which various regions of the star are spun up, is poorly constrained.

### 2. KIC 9267654: Previous Analyses

In this paper, we present observations of KIC 9267654 that suggest it has a nonmonotonic radial rotation profile (see Figure 1). Asteroseismology, spectroscopy, and photometry agree that the star is a lower red giant branch \((T_{\text{eff}} = 4792 \pm 75 \, \text{K}, \, \log g = 2.98 \pm 0.007)\) of 1.138 \pm 0.039 \,(random) \pm 0.035 \,(systematic) \,M_\odot \,(Pinsonneault et al. 2018). It was observed by the Kepler mission (Borucki et al. 2010) for its full duration. Its seismic oscillations were first characterized by Mosser et al. (2012) and detailed modeling of its oscillation frequencies and rotational splittings were carried out byCorsaro et al. (2015); Pérez Hernández et al. (2016), and Triana et al. (2017). In particular, these works noted a well-constrained core rotation rate of \( \Omega_c = 908–939 \, \text{nHz} \) \((P_{\text{core}} \sim 12.6 \, \text{days})\), slightly higher than average for stars in this regime (Gehan et al. 2018). Several methods of estimation suggested an envelope rotation rate of \( \Omega_{\text{env}} = 20.8–65.0 \, \text{nHz} \) \((P_{\text{env}} \sim 178–556 \, \text{days})\), with most closer to the faster end of that estimate, although a linear fit to the approximate trapping parameter suggested a potentially counter-rotating envelope (see Triana et al. 2017, for more discussion of the various methods used to estimate the envelope rotation and their relative precision). We
note that these results are consistent with the ensemble of core rotation rates available for similar stars (Gehan et al. 2018), other estimates of the core rotation rate of this star ($\Omega_c = 920 \pm 10.06$ nHz; Gehan et al. 2018), and also simple theoretical models of the surface rotational evolution. The oscillation pattern was used to estimate an inclination angle of $72.4^{+7.0}_{-6.0}$ degrees for this star (Gehan et al. 2021), its modes, including the $\ell = 1$ modes, looked to have normal visibility, and there was no evidence of surface spots or activity (Ceillier et al. 2017; Gaulme et al. 2020). This star was observed for two sectors by the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015), and designated TIC 164832220, but it is too faint for oscillation modes to be easily detected in those data (see Stello et al. 2022).

Very recently, Daher et al. (2022) estimated surface rotation velocities for a large number of stars with spectroscopic data from the Sloan Digital Sky Survey (Blanton et al. 2017) Apache Point Galactic Evolution Experiment (APOGEE; Majewski et al. 2017). These stars have moderate resolution ($R \sim 22,000$) spectra taken in the $\text{H-band}$ on the du Pont and Sloan Foundation (Gunn et al. 2006) telescopes and reduced automatically using the APOGEE Stellar Parameters and Chemical Abundances Pipeline (ASPCAP; Holtzman et al. 2015; Nidever et al. 2015; Zamora et al. 2015; Garcia-Perez et al. 2016; Holtzman et al. 2018; Jonsson et al. 2020) to obtain spectroscopic parameters. For red giant stars, the Daher et al. (2022) analysis ran an additional search for rotational broadening using cross-correlation methods to artificially broadened template spectra ( Tayar et al. 2015; Dixon et al. 2020). In their analysis, they suggested a relatively rapid rotation velocity of $6.25 \text{ km s}^{-1}$ for this star, much faster than the the $\langle <1 \text{ km s}^{-1} \rangle$ rotation velocity suggested by the seismic envelope rotation and inclination constraints. They also found no strong evidence of a close binary companion in the APOGEE radial velocity measurements ($\Delta \text{RV}_{\text{max}} = 0.3 \text{ km s}^{-1}$, $v_{\text{scatter}} = 0.18 \text{ km s}^{-1}$), consistent with previous work by Price-Whelan et al. (2020).

The APOGEE survey also provided chemical abundance information for this star (see Section 6), and characterized it as a relatively common evolved solar-like object with a metallicity [Fe/H] = $-0.02 \pm 0.03$ dex and an alpha-element enhancement [$\alpha$/Fe] = $0.00 \pm 0.01$, suggesting that it should be comparable to the ensemble of well-studied stars in the Kepler field. This star has also been observed by Gaia. Identified by the Gaia DR3 ID 2107170676942345088, its photometry is consistent with being a red giant ($G = 11.97$ and BP-RP = 1.193; Gaia Collaboration et al. 2021) relatively nearby (parallax = $0.8444$ mas; distance 1151 pc; Bailler-Jones et al. 2021). It follows a disk-like orbit with a low eccentricity (0.18), and it is currently about one thin disk scale height above the Galactic plane ($z_{\text{max}} = 408$ pc). The Renormalized Unit Weight Error for this star is 0.969, which indicates that it is well fit as a single point source.

3. Binary Analysis

The Gaia radial velocity analysis published as part of the Gaia DR3 data release (Gaia Collaboration et al. 2022) indicates a linear trend in the 24 radial velocities of KIC 9267654 of $-7.5 \pm 3.8 \text{ m s}^{-1} \text{ day}^{-1}$, significant at the 2$\sigma$ level. The star is denoted as an SB1, meaning there are no indications for a second set of lines and implying that the secondary must be fainter than the primary. It need not be much less massive than the primary though since the primary is evolved already, while the secondary might still be on the main sequence. In the following, we derive limits on the mass of the secondary given the radial velocity trend and the primary mass. The 24 radial velocities measurements, which are not available yet individually, have been taken over a time interval of 931.915 days. The mean radial velocity of all measurements taken in that time frame is $-28.8 \pm 1.1 \text{ km s}^{-1}$.

Assuming that a linear trend without notable deviations can be maintained over at most half of the period of an orbit, the minimum period of the system would be twice as long, i.e., 1864 days or about 5.1 yr. For periods upwards of this limit, we have derived the constraint on the minimum companion mass (companion mass times the sine of the unknown orbital inclination) that would be compatible with a radial velocity semi-amplitude consistent with a linear trend of the given value over half of the assumed period under the assumption of a circular orbit. The blue line in Figure 2 illustrates this constraint, while the blue area indicates its 1$\sigma$ confidence region. The red line indicates the primary mass; the secondary, assuming it is not only less massive but also less luminous than the primary, should have a somewhat smaller mass. As can be seen, orbital periods between 5.1 yr and about 12 yr are all compatible with the nominal observed radial velocity trend, and would generate minimum secondary masses between about 0.3 and 1.0 $M_\odot$. If the error on the radial velocity trend is taken into account, the minimum masses could be even smaller (down to about 0.1 $M_\odot$) and the orbital periods somewhat larger (up to about 21 yr). Ground-based radial velocity measurements from APOGEE and the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST; Cui et al. 2012) are sparse and uncertain enough that while they are consistent with the presence of such a trend at about the same order of magnitude, they do not add significant additional constraints. The minimum astrometric signature of all of these orbits is too small (at maximum of the order of 20 $\mu$as for the widest orbits) to be detected by Gaia, so this is consistent with the star not having been identified as an astrometric binary already. Depending on

Figure 2. Illustration of the periods and (minimum) secondary masses compatible with the radial velocity trend derived from Gaia DR3 radial velocities. The blue line indicates the minimum secondary mass derived from the radial velocity trend for each period, while the blue shaded region indicates the 1$\sigma$ confidence region. The red line denotes the primary mass, and the red shaded region shows the limit on the secondary mass based on the fact that the star is an SB1 only. The purple shaded overlap region indicates the combinations of period and secondary mass compatible with both constraints. Based on the nominal value for the radial velocity trend, periods roughly between 5 and 12 yr and companion masses between 0.3 and 1.0 $M_\odot$ are compatible with all constraints.
the inclination, the astrometric signature could in theory be larger than this limit, and the astrometric excess noise of 63 μas observed for this star could in principle be due to the spectroscopic companion. However, since the Gaia astrometric residuals also suffer from systematics, we refrain from modeling the orbit further based on the astrometric jitter.

We conclude that it is highly likely that KIC 9267654 is orbited by a low-mass stellar companion with a period of the order of 5 to 20 yr. However, it is complicated to use such a companion to directly explain the surface rotation period. In general, tidal forces (see Section 7.1 for more discussion) work toward synchronizing the orbital and rotational motion, so in the case of a circular orbit, a companion with a 5 yr period would be both too far away to be strongly interacting and would be forcing the surface toward a 5 yr rotation period, not the 42 day period observed. In eccentric orbits, the rotation rate is spun up toward the pseudosynchronous rotation rate, although it does not always reach that value on short timescales (Zimmerman et al. 2017). In order for the likely companion inferred from Gaia to be pseudosynchronized with the observed surface rotation would require an orbital eccentricity greater than 0.98. While highly eccentric binaries including giants do exist (e.g., Heeren et al. 2021), the system properties require careful tuning to survive the main-sequence phase, so we work under the assumption that the wide companion is not directly responsible for the observed rotation. There are also orbital configurations that lead to more complex dynamics where a wide companion can tighten the orbit of an inner pair (e.g., the eccentric Koçak–Lidov effect; Naoz 2016), and it is the unidentified inner body that is causing the anomalous surface rotation. Such a scenario is not inconsistent with our observations, but we do not have sufficient data to confirm the presence of any inner companion directly.

We also note that given we do not detect a second set of oscillations or an anomalous convective background (see Section 5) it is almost impossible for any additional evolved giant stars to be present in the system. Since our primary star is a giant, and therefore quite bright, any main-sequence companion would be significantly fainter. If we assume single star evolution, the less-evolved secondary would be less massive than the primary and therefore at brightest a G star, contributing at most a few percent of the light of the system and therefore unlikely to affect any spectroscopic results derived from deep lines or many lines (Halbwachs et al. 2014), although there is a possibility that some of the abundances derived only from weak lines could be affected. If we allow for the possibility of mass transfer during the main sequence, following the work of Sun et al. (2021), we expect the star we currently see as the primary to have gained mass during its main sequence. This would have reset the rotation rate, but it would likely have spun down to a normal rotation rate relatively quickly (Leiner et al. 2018), and thus would still require an additional interaction much closer to the current time.

Given this evidence, we therefore conclude that the companion identified in the Gaia data should be well separated from and significantly fainter than the star of interest, so it should not substantially impact the detailed seismic or spectroscopic analyses discussed in the following sections. We also do not expect it to be directly related to the unusual rotation profile we observe, but suggest that further study could prove interesting in a dynamical sense.

4. Spectroscopic Analysis

Given the highly unusual detection of a surface rotation faster than the bulk envelope rate, it seemed necessary to confirm the surface rotation velocity. In particular, the APOGEE resolution is only $R \sim 22000$, which has a formal $v \sin i$ detection limit of $13.1 \text{ km s}^{-1}$. While cross-correlation analysis has suggested that it is possible to distinguish slower rotation velocities, values below 10 km s$^{-1}$ are still difficult, and most authors are highly skeptical of constraints below $\sim 5–8 \text{ km s}^{-1}$ (Tayar et al. 2015; Dixon et al. 2020; Simonian et al. 2020).

We therefore obtained a higher resolution spectrum using the High-Efficiency and high-Resolution Mercator Echelle Spectrograph (HERMES; Raskin et al. 2011) instrument on the 1.2 m Mercator telescope on La Palma. The star was observed on 2021 October 28 for two observations of 45 minutes, each with a signal-to-noise ratio $(S/N)$ of $\approx 20$ at 618 nm and with a resolution of $R \approx 85,000$. Data were reduced using the dedicated HERMES reduction pipeline, which includes flat-fielding and wavelength calibration (Raskin et al. 2011). In order to infer the spectroscopic parameters, the HERMES spectra were first normalized by fitting a second-order spline to the continuum following the method described in Abdul-Masih et al. (2021). Then the two spectra were co-added and compared to a large grid of model atmospheres using the spectral analysis code Grid Search in Stellar Parameters (GSSP; Tkachenko 2015). GSSP uses a grid of atmosphere models precomputed with LLMODELS (Shulyak et al. 2004) and computes synthetic spectra on the fly using the SYNThV (Tsymbal 1996) spectrum synthesis code. The HERMES spectral range covers from 400 to 900 nm, but only the region from 450 to 700 was used for the stellar parameter search. This analysis was done independently of the APOGEE parameters.

The resulting fits (see Figure 3) were very similar to the APOGEE values with $T_{\text{eff}} = 4750^{+220}_{-210}$ K, $\log g = 3.16 \pm 0.46$ dex, and $[\text{M/H}] = -0.05^{+0.18}_{-0.20}$ dex, suggesting that the published spectroscopic parameters are likely to be reliable. In addition, this analysis gave an estimate for the surface rotation velocity of $v \sin i = 5.96_{-0.91}^{+0.91}$ km s$^{-1}$, consistent with the APOGEE constraint, and significantly inconsistent with the seismic envelope rotation rate.

In addition, the cross-correlation function of the individual HERMES spectra show no evidence for a second set of lines that would indicate a spectroscopic binary companion, nor do they show any strong radial velocity shifts that may have been expected in a tidally locked system with a stellar mass secondary component, although we cannot confirm or further constrain the sort of wide companion suggested by the Gaia data. We also report the nondetection of lithium features at around 670 nm in our optical spectrum. Given the measurement constraints, this rules out a lithium abundance at or above solar ([Li/Fe] $< 0$, $A(\text{Li}) < 1.1$). Such abundances would be consistent with previous observations of the lithium abundance for a star of this mass, metallicity, and surface gravity, as for most giants the lithium line is not detectable, although the detectability does scale with temperature (Martell et al. 2021). However, given that we only measure an upper limit, we cannot rule out this star being depleted in lithium, which has been suggested to occur in some cases where planetary engulfment drives mixing through the thermohaline instability (Deal et al. 2015).

Barium is an element often used to identify stars that have been polluted by AGB companions. We have examined our
core rotation estimate was later shown to be consistent with an entirely independent analysis published in Gehan et al. (2018), emphasizing that this result, which is much simpler to measure, is likely to be robust. However, the envelope rotation rate can be more challenging to estimate, and so they applied six different methods including models, inversions, inferences, and fits (see Triana et al. 2017, for a more detailed discussion of each method). For KIC 9267654, four of the rotation rates measured by these methods agreed to within the uncertainties.

For this paper, we have taken a careful look at the available Kepler data, using the KEPSEISMIC light curves (García et al. 2011, 2014; Pires et al. 2015), where the data systematics, gaps, and instrumental effects have been carefully treated, and compare the results to the ensemble of Kepler data that have been homogeneously analyzed in the time since the original papers on this star were published. In our analysis, we have searched for any indication that would suggest that any of the properties might be unusual or mismeasured in a way that could impact the conclusions of this paper. We have found no such evidence.

Specifically, we find that the convective background for this star (see the top panel in Figure 4) is entirely consistent with the relations found for the Kepler red giants (Mathur et al. 2011; Kallinger et al. 2014), in agreement with what was found for this star in Sayeed et al. (2021). This also makes it hard to imagine that the spectroscopic broadening we are interpreting as rotation is caused by unusual atmospheric macroturbulence, which should also correlate with the stellar surface convection and thus the granulation. We also find that the mode amplitudes and widths for this star are fully consistent with the expectations for a star of this type (e.g., Mosser et al. 2011; Corsaro et al. 2017), with no obvious indications of e.g., strong magnetic fields (Fuller et al. 2015; Loi 2020; Bugnet et al. 2021). All of these properties are measured at extremely high S/N (see the bottom panel in Figure 4) using the Markov Chain Monte Carlo (MCMC) Bayesian fitting code apollinaire (Breton et al. 2022). By doing so, we ensured that the new results were not dependent on the Bayesian fitting scheme used (i.e., MCMC with apollinaire versus a nested sampling tool in DIAMONDS; Corsaro & De Ridder 2014).

Because the signal-to-noise in the seismic modes is so high, estimating rotational splittings for this star is much less challenging than usual. The g-mode dominated $\ell = 1$ mixed modes are significantly narrower and offset from the usual $\ell = 1$ ridge; their splitting is clear and consistent with previous estimates of the core rotation rate.

We have also looked at the available $\ell = 2$ and $\ell = 3$ modes for potential detection of rotational splittings from the envelope region. Using apollinaire, we fit five orders of $\ell = 0$, 2 modes and three $\ell = 3$ modes between 95 and 136 $\mu$Hz. We allow the splitting, $\nu_s$, to vary between 0.01 and 0.6 $\mu$Hz, and allow the amplitude ratios between the different degree modes to vary freely. Our MCMC chain had 500 walkers and between 20,000 and 55,000 steps. We tested a variety of different constraints on the inclination angle. We concluded that it is not possible with the current data to effectively constrain the inclination angle of the star using only the p modes. It was only possible to constrain a combination of the projected splitting and the inclination angle (Ballot et al. 2006). For this reason,
we used the inclination angle derived from the mixed modes by Gehan et al. (2018) to run a final fit of the $p$ modes; the result is shown in Figure 5. This requires assuming that the inclination of the envelope is the same as the inclination of the core, which may or may not be true depending on what has happened to this star. Nevertheless, this seems to be the most reasonable starting assumption, and it produces a posterior probability of the inclination angle that is rather flat with a median value of $78.1\pm8.1$ degrees and a rather Gaussian posterior probability for the rotational splitting centered at $0.021\pm0.006$ $\mu$Hz (corresponding to rotation periods between 445 and 825 days). It is worth noting that for all of the fits using only $p$ modes that we have tried, the envelope rotation periods obtained were longer than 200 days, regardless of the chosen inclination constraint. They were also generally slower than the rotation constraints published in Triana et al. (2017), but for our purposes that only makes the detected rapid rotation of the surface more significant.

Further explorations of this star might consider imposing a nonmonotonic rotation profile ad hoc on top of a stellar structure model whose fundamental parameters reproduce those of KIC 9267654. The imposed rotation profile should match the inferred estimates at the core-envelope boundary, subsurface convection zone, and surface, respectively. From here, one could determine that model’s theoretical pulsation spectrum using the GYRE stellar oscillation program (Townsend & Teitler 2013), including the rotation kernels of each oscillation mode observed in the spectrum, and compare this to the observations to determine whether additional constraints can be placed on the stellar latitudinal or radial rotation profile (Garcia et al. 2008; Davies et al. 2015). However, given the challenges of such an exercise (Gizon & Solanki 2003), we consider it beyond the scope of this study.

6. Chemical Offsets

It has been shown that much of the variance in the elemental abundances of dwarf stars in the Galactic disk is confined to a small number of independent chemical vectors that can be mapped to various Galactic enrichment processes (Price-Jones & Bovy 2018; Weinberg et al. 2019; Griffith et al. 2021; Joyce et al. 2022). This provides us with fairly strong expectations for the abundance patterns of stars as a function of age, mass, and metallicity, making it possible to identify outliers from the expected pattern in a fairly straightforward manner. If the unusual rotation profile of KIC 9267654 is due to some kind of mass transfer or collision with a companion or a planet, enrichment by an AGB star, or the result of formation in an unusual environment, it is conceivable that such an event could have resulted in a deviation of its surface abundances from this expected pattern. Again, we find no strong evidence of such a deviation.

To explore this possibility, we selected a comparison sample from the APOKASC-2 catalog of stars with measured asteroseismic gravities, ages, and masses (Pinsonneault et al. 2018) and many elemental abundances determined in APOGEE DR16 (Ahumada et al. 2020). We kept all stars with valid results for seismic $\log g$, age, and mass, and with $T_{\text{eff}}$ within 50 K, $\log g$ within 0.3 dex, and mass within 0.1 $M_\odot$ of KIC
are strongly correlated with their inferred ages. Masses that correlate slightly with metallicity, and masses that look at a restricted metallicity range in the Kepler sample, and they are all quite similar in these fundamental quantities. As one might expect when moving from a stellar or substellar companion. Using the Galactic Archaeology with HERMES DR2 data set to the comparison set, mildly enhanced in carbon, manganese, and cerium, and depleted in aluminum and enhanced in copper relative to the comparison set, mildly enhanced in carbon, manganese, and cerium, and depleted in silicon and potassium. These are not known signals of any common internal mixing, binary mass transfer, or planetary engulfment process. We are aware that the abundances presented in this Figure could shift with the inclusion of NLTE effects. However, since the comparison stars are of similar temperatures, surface gravities, and metallicities, we expect the whole sample to be shifted in similar ways and thus we do not expect NLTE effects to significantly change which abundances in our star are anomalous relative to the comparison sample.

We also explore whether the lack of detected lithium enhancement can place constraints on the accretion of material from a stellar or substellar companion. Using the Galactic Archaeology with HERMES DR2 data set (GALAH; Buder et al. 2018), merged with effective temperature and luminosity measurements from Gaia DR2, we investigated a population of 26,183 stars with lithium abundance signatures and

Figure 6. Top: metallicity vs. $\alpha$-enhancement for KIC 9267654 (green circle) and the comparison stars from APOKASC (smaller black circles). Bottom: mass vs. age for KIC 9267654 and the comparison stars from APOKASC. The blue cross in the lower left corner of each panel shows the median uncertainty for the comparison stars. The reported APOGEE uncertainties in the fundamental stellar properties $[M/H]$ and $[\alpha/M]$ are expanded by a factor of 5 for visibility.

Figure 7. Top panels: abundance vs. metallicity for the light elements C, N, and O in KIC 9267654 (green circle) and the comparison stars from APOKASC (smaller black circles). Bottom: the carbon-to-nitrogen ratio correlates well with stellar mass in the sample. The comparison of KIC 9267654 to the trend suggests it could not have gained more than a few hundredths of a solar mass since its first dredge-up. The blue cross in the lower left corner of each panel shows the median uncertainty for the comparison stars. As in Figure 6, the uncertainty in $[M/H]$ is expanded by a factor of 5 for visibility.

9267654. Figure 6 compares these 176 RGB stars to KIC 9267654 in $\alpha$-enhancement versus metallicity and mass versus age, to give a sense of the properties of this star as well as the range of the comparison sample, and they are all quite similar within the distribution of the APOKASC comparison sample, which similarly does not seem to require significant abundance changes or mass gain from a companion after the first dredge-up.

In Figure 8 elements from the different nucleosynthetic groups are labeled in red (light odd-Z), green (alpha), magenta (iron peak), and purple (neutron capture). KIC 9267654 is clearly depleted in aluminum and enhanced in copper relative to the comparison set, mildly enhanced in carbon, manganese, and cerium, and depleted in silicon and potassium. These are not known signals of any common internal mixing, binary mass transfer, or planetary engulfment process. We are aware that some of the abundances presented in this Figure could shift with the inclusion of NLTE effects. However, since the comparison stars are of similar temperatures, surface gravities, and metallicities, we expect the whole sample to be shifted in similar ways and thus we do not expect NLTE effects to significantly change which abundances in our star are anomalous relative to the comparison sample.
[Fe/H] = 0 ± 1 dex. Following the procedure outlined in Soares-Furtado et al. (2021), we binned these signatures by the stellar effective temperature and luminosity, calculating the median lithium baseline for each bin, as well as the median absolute deviation of the median lithium baseline. Using a postprocessing approach that is also outlined in Soares-Furtado et al. (2021), we then determined the lithium enrichment signature accompanying the ingestion of a 1 M\textsubscript{Jup} substellar companion.

For a star of 1.1 M\textsubscript{E}, T\textsubscript{eff} = 4792 K, and L = 15.4 L\textsubscript{E}, we calculated that a lithium baseline of A(Li) = 1.03 ± 0.18 dex would be expected on average, although a large range of lithium abundances is observed in the population. For such a host, the ingestion of a 1 M\textsubscript{Jup} companion would result in a lithium abundance signature of 1.1 dex, which would only represent a statistically insignificant 0.5\textsigma increase in the expected lithium abundance.

To produce a statistically significant (5\textsigma) lithium enrichment signature, a star of this mass and evolutionary state would be required to consume a companion of 33 M\textsubscript{Jup}—a massive brown dwarf or tens of gas giants. We illustrate the minimum-mass companion required to produce an ingestion-derived lithium enrichment signature with a 5\textsigma statistical significance in Figure 9, where KIC 9267654 is illustrated in the H-R diagram as a red star. Also depicted are the location of the main-sequence turnoff (gray-hued cross) and luminosity bump (salmon-hued diamond) corresponding to this stellar track. Given the large mass that would need to be ingested to create a significant lithium enhancement, we determine that an observable lithium enrichment signature is extremely unlikely to be present in a star of this mass and evolutionary state, and therefore the lack of a lithium detection in this star is not sufficient evidence to rule out a significant interaction event. Similarly, the upper limit we are able to place on the lithium abundance is not low enough to argue for mixing-driven lithium depletion (Deal et al. 2015) and so the observed lithium constraint argues neither for nor against any particular formation scenario. For consistency with Soares-Furtado et al. (2021), we have used the GALAH DR2 data set for this analysis and comparison. While the GALAH DR3 data set is now available (Buder et al. 2021), we do not expect that changing to the newer measurements would substantially change our conclusion that significant mass would have to be accreted to create a noticeable change in the lithium abundance for a star with this deep of a surface convection zone.

7. Theoretical Explanations

Given that our further investigations of the rotation of KIC 9267654 seem to indicate that the inferred nonmonotonic
rotation profile is robust, it is interesting to consider how the star ended up in this unusual configuration. There are several potential explanations that must be considered in our endeavor to understand the unusual rotation profile of KIC 9267654.

### 7.1. Tidal Waves

One exciting possibility would be the ingestion or interaction with a substellar body, perhaps an exoplanet. The Kepler mission in particular has emphasized that a large fraction of stars are orbited by at least one planet that is close enough in that it should be engulfed as the star expands on the red giant branch. Such an event may impact the surface rotation (e.g., Prvitera et al. 2016; Bolmont & Mathis 2016; Qureshi et al. 2018; Stephan et al. 2020; Ahuir et al. 2021), and the lithium abundance (Aguilera-Gómez et al. 2016) of some stars, but is unlikely to cause other changes to the abundances in the surface convection zone. These predictions are consistent with our observations of KIC 9267654. If this is indeed a case of planetary ingestion, the existence of a rapidly rotating surface layer would seem to suggest that the angular momentum gained from the tidal interaction is happening from the outside in layer by layer, rather than treating the whole convection zone as a single piece (Stephan et al. 2020). Our detection of a star in this presumably transient nonmonotonic state would also put significant constraints on the timescales and efficiencies of internal angular momentum transport mechanisms acting in this phase.

This is precisely what is expected if progressive (traveling) tidal waves are excited by a spiraling companion (Goldreich & Nicholson 1989). These waves deposit their angular momentum at so-called critical layers, which correspond to corotation resonances between the local rotation frequency of the stellar fluid and the orbital frequency in the simplest case of a coplanar and circular orbit. This leads to a synchronization of the interior of the star from its surface to the core. In the case of the convective envelope of KIC 9267654, the best candidate to explain such a phenomenon are tidal inertial waves (Ogilvie & Lin 2007), whose restoring force is the Coriolis acceleration. Therefore, when they meet such a corotation layer, they are able to transfer the momentum they carry to the stellar (differential) rotation (Favier et al. 2014; Astoul et al. 2021). This would have significant implications for the estimation of tidal dissipation quality factors and the mechanisms of angular momentum transport in the convective zones of stars hosting companions (Astoul & Barker 2021).

Since planetary engulfment and associated tidal flows are potentially the best candidates to explain the observed subsurface acceleration, each category of tidal flows should be examined. First, the gravitational perturbation by a potential companion triggers large-scale hydrostatic/equilibrium flows (Zahn 1966; Terquem et al. 1998; Remus et al. 2012; Ogilvie 2013). Considering the mathematical expressions derived in Remus et al. (2012) and Terquem (2021), one can demonstrate that the latitudinally averaged flux of angular momentum carried by Reynolds stresses along the vertical direction of these equilibrium flows vanishes. Therefore, these flows cannot explain the needed transport of angular momentum and one should consider the complementary tidal flows: the dynamical tide. The dynamical tide is constituted of waves that propagate in stars (and planets) that can be excited by tidal interactions with a companion (Zahn 1975; Ogilvie & Lin 2004). In a stellar convective envelope, the dynamical tide is constituted by tidal inertial waves excited by the Coriolis acceleration of the equilibrium tide (Ogilvie & Lin 2007; Ogilvie 2013). As has been pointed out at the beginning of this discussion, they are able to transport angular momentum thanks to their Reynolds stresses and critical layers, leading to a synchronization (i.e., an acceleration) of the convective envelope from the surface to the interior. For a companion spiraling toward the star when the orbital mean motion $n_\text{orb}$ is greater than the stellar surface rotation frequency, $\Omega_*$, these waves are excited when $P_\text{orb} > P_*/2$ (e.g., Bolmont & Mathis 2016 with $P_\text{orb} = 2\pi/n_\text{orb}$ the orbital period and $P_*/2 = 2\pi/\Omega_*$ the rotation period of the surface of the star). Using scaling laws for the amplitude of the equilibrium tide that scales as $1/a^3$ (Remus et al. 2012; Ogilvie 2013), where $a$ is the semimajor axis, one can demonstrate that the Reynolds stresses of tidal inertial waves, which are excited by the Coriolis acceleration of the equilibrium tide (Ogilvie 2013), scale as $1/a^5$. They thus strongly increase with the approach of the companion. One challenging remaining question is to quantify the nonlinear interactions between the flows of the turbulent convection, the hydrostatic equilibrium tide, and the dynamical tides (e.g., Duguid et al. 2020; Barker & Astoul 2021; Terquem 2021) and the resulting angular momentum transport (Astoul et al. 2021), another is to compare the normal oscillations we see in this system to the close binary stars with suppressed oscillation amplitudes from Gualme et al. (2014). We do not attempt to study the details of such an interaction here, but suggest that this system might represent an interesting test case for future theoretical models and numerical simulations.

### 7.2. Turbulent Convection

To explain the observed subsurface acceleration, one can also examine the mechanisms that transport angular momentum in single stars. The first one is turbulent convection. Depending on the so-called convective Rossby number, different regimes...
of differential rotation can be triggered (Brun et al. 2017). On the one hand, this dimensionless number can be defined as the ratio of the rotation period to the convective overturn time in stellar evolution theory. This is the stellar convective Rossby number $Ro_\text{c}$ (e.g., Landin et al. 2010). On the other hand, it can be defined as the ratio between the vorticity of convective flows and twice the rotation angular frequency of the star in (magneto-)hydrodynamical numerical simulations of stellar convection zones. This is the so-called fluid convective Rossby number $Ro_f$ (Brun et al. 2017). These Rossby numbers compare the inertia of convective flows to their Coriolis acceleration; their relationships have been discussed in Brun et al. (2017), who showed that they are of the same order of magnitude (we refer the reader to Table 4 and Appendix B in that article) and who identified the different regimes of differential rotation as a function of $Ro_f$. First, for small convective Rossby numbers (fast rotation; $Ro_f \lesssim 0.3$), the Coriolis acceleration constrains convective flows through the so-called Taylor–Proudman constraint (e.g., Rieutord 2015). This leads to cylindrical differential rotation profiles like those observed at the surface of Jupiter and Saturn. Next, for moderate convective fluid Rossby numbers of the order of unity, conical differential rotation profiles are established with a solar-like equatorial acceleration if $0.3 < Ro_f \lesssim 1$ and an antisolar polar acceleration if $Ro_f \gtrsim 1$. Finally, when $Ro_f$ becomes large, the action of the Coriolis acceleration becomes negligible and shellular-like differential rotation profiles depending mainly on the radius can be obtained (Brun & Palacios 2009). Note that this classification has been established in a hydrodynamical framework and that Brun et al. (2022) showed how it could be modified in presence of a dynamo-generated magnetic field. In particular, the feedback of the Lorentz force weakens the absolute value of the differential rotation, particularly in the case of fast-rotating stars. In addition, the transition between the conical solar-like differential rotation and the antisolar differential rotation state is particularly sensitive to the complex properties of turbulent convective flows (Gastine et al. 2014; Käpylä et al. 2014). We compute the value of $Ro_f$ in KIC 9267654 and we obtain that $Ro_f > 7$ in the bulk of the envelope while $Ro_f \sim 1$ at the surface. Therefore, KIC 9267654 seems to be in the latter regime. In this context, the 3D nonlinear global spherical simulations of the dynamics of the convective envelope of a red giant star computed by Brun & Palacios (2009) with the Anelastic Spherical Harmonics (ASH) code (Clune et al. 1999; Brun et al. 2004) is interesting. Indeed, it shows how a shellular differential rotation can be established when $Ro_f$ is large. In that case, the obtained latitudinally averaged differential rotation radial profile is decreasing toward the surface (we refer the reader to the right column of Figure 10 in Brun & Palacios 2009), a trend that is conserved in Brun & Palacios (2009) when increasing the rotation rate that leads to antisolar differential rotation profiles. We are not aware of evidence that the transition in convective Rossby number close to the surface can lead to any observed acceleration of the rotation rate consistent with what we observe for KIC 9267654. Therefore, we consider it more likely that the rotation profile is explained by a planetary engulfment event.

7.3. Internal Gravity Waves

The last mechanisms that could be considered in single stars are traveling (progressive) internal gravity waves and mixed (gravito-acoustic) modes that are efficiently stochastically excited by turbulent convection in evolved low-mass stars (e.g., Talon & Charbonnel 2008; Dupret et al. 2009; Pinçon et al. 2016, 2017). Indeed Schatzman (1993), Zahn et al. (1997), Talon & Charbonnel (2005), Rogers (2015), and Belkacem et al. (2015a, 2015b) have demonstrated that they are able to efficiently transport angular momentum. However, this statement is only true for stably stratified stellar radiation zones where internal gravity waves propagate and gravity modes form; they are evanescent in stellar convective regions. In addition, acoustic modes are not efficient at redistributing angular momentum in convective regions. As a consequence, these mechanisms should be excluded from further discussion of KIC 9267654 where the unexpected differential rotation is localized within the surface convection zone.

8. Discussion

We have presented evidence to support the existence of a nonmonotonic radial rotation profile for KIC 9267654, with a rapidly rotating core ($P_{\text{core}} = 12.6$ days), a slowly rotating envelope ($P_{\text{env}} \gtrsim 178$ days), and a moderately rotating surface ($P_{\text{surf}} = 42$ days), inferred using a combination of spectroscopic and asteroseismic data. We have collected this evidence from two entirely independent spectroscopic investigations, as well as several asteroseismic analyses. The data seem to consistently indicate that KIC 9267654 is a relatively normal lower red giant branch star, whose only glaringly unusual feature is its odd rotation profile, and possibly a few offset abundances. While Gaia suggests a companion in an orbit much longer than the rotation period, we find no evidence for the existence of a tidally synchronized stellar companion or significant ongoing mass transfer in this system, although we cannot exclude such scenarios entirely.

We note that this star is not the only giant with an unusual surface rotation rate. Other stars have been noted whose claimed surface rotation rates are faster than their core rotation rate (e.g., Kurtz et al. 2014; Ceillier et al. 2017; Aerts et al. 2019; Tayar et al. 2019), or significantly faster than single stellar evolution would predict (e.g., Ceillier et al. 2017; Tayar & Pinsonneault 2018). However, in most of these cases, the unusual surface rotation is presumed to be linked to tidal interactions with another star (Tayar et al. 2015; Li et al. 2020a; Daher et al. 2022) or a much earlier planetary engulfment (Carlberg et al. 2012; Privitera et al. 2016c; Bolmont & Mathis 2016; Ahuir et al. 2021). In other cases, these stars come from ensemble analyses, where a few percent of stars are often affected by measurement errors or target contamination (Aigrain et al. 2015), and envelope rotation rates are rarely available. Finally, in other well-studied red giants including e.g., KIC 3744043, the envelope rotation rate estimated with asteroseismic techniques ($P_{\text{env}} \sim 104–159$ days; Triana et al. 2017) may come out slightly slower than the surface rotation rate measured from the broadening of spectroscopic lines ($\nu \sin i = 2.5 \pm 1.0 $ km s$^{-1}$, $P_{\text{rot}} \sim 94.5 \pm 45.9$ days; Thygesen et al. 2012; Gehan et al. 2021). Nonetheless, the disagreement is within the properly propagated measurement uncertainties and its core is rotating significantly faster than both its envelope and surface, over a period of $P_{\text{core}} \sim 21.6$ days (Triana et al. 2017; Gehan et al. 2018), which makes it hard to draw strong conclusions. What makes KIC 9267654 so interesting is the existence of a well-measured three-point rotation profile where all three measurements have been verified by multiple teams and independent analyses, and the surface and envelope...
rotation rates disagree with each other at a significant level given the small measurement uncertainties.

Given the exciting possible explanations of the unusual rotation profile detected in KIC 9267654, and their impact on our understanding either of planetary engulfment and tidal theory or some unexplained physics in low-mass stellar interiors, we encourage further observational and theoretical work on this exciting system. In particular, a more detailed observational analysis of the surface rotational and chemical properties including r-process and s-process abundances could be interesting, as would longer-term searches for spot modulation as well as searches for surviving planetary companions in the system. Theoretical modeling of both tidal theory in general, as well as specific companions and orbital configurations that could produce the characteristics observed for KIC 9267654 might prove quite illuminating. More generally, the ability to not only identify planets in stable configurations similar to our own solar system, but also to trace out their changes as their stars evolve (Grunblatt et al. 2017; Rizzuto et al. 2017; Chontos et al. 2019; Gänsicke et al. 2019) has opened and is likely to continue to open interesting windows toward understanding planetary formation, evolution, and long-term habitability, while simultaneously constraining the physics of stellar surfaces and interiors.

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