CORRELATIONS BETWEEN $<p_T>$ AND MULTIPLICITY IN A SINGLE BFKL POMERON *
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Strong correlations are obtained between the number and the average transverse momentum of jets emitted by the exchange of a single BFKL Pomeron.

1. Introduction

Strong correlations are observed experimentally between the average $p_T$ and multiplicities of particles produced in high-energy hadronic collisions [1]. Average $p_T$ grows with multiplicity. To interpret this fact it is tacitly assumed that with only one hard collision there are no correlations between $<p_T>$ and multiplicity. Theoretically this assumption can only be tested within the Balitskii-Fadin-Kuraev-Lipatov (BFKL) dynamics, which presents a detailed description of particle (actually jet) production at high energies under certain simplifying assumptions (a fixed small coupling constant). The present calculation is aimed to see if there exist correlations between $<p_T>$ and the number of produced jets in the hard Pomeron

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described by the BFKL chain of interacting reggeized gluons [2]. We limit ourselves to the leading order BFKL model.

2. The Formalism

The BFKL equation for the amputated BFKL amplitude, \( f(y, k) \), when \( y \) is the rapidity and \( k \) is the two-dimensional transverse momentum of the virtual (Reggeized) gluon, may be written in the form

\[
f(y, k) = f^{(0)}(y, k) + \bar{\alpha}_s \int_0^y dy_1 \int \frac{d^2 k_1}{\pi q^2} \left( \frac{k^2}{k_1^2} f(y, k) - f(y, k_1) + f(y_1, k_1) \theta(k_1^2 - q^2) \right),
\]

where \( \bar{\alpha}_s = 3 \alpha_s / \pi \) and \( q = k - k_1 \) is the transverse momentum of the emitted (real) gluon.

Defining as an observable jet a real gluon with \( q^2 \geq \mu^2 \), one splits the integration over momenta and thus the integration kernel in (1) into two parts, a resolved one, \( K_R \), corresponding to emitted gluons with \( q^2 > \mu^2 \), and an unresolved one, \( K_{UV} \), which combines emission of gluons with \( q^2 < \mu^2 \) and the subtraction term in (1). Exclusive probabilities to produce \( n \) jets are obtained by introducing \( n \) operators \( K_R \) between the Green functions of the BFKL equations with kernel \( K_{UV} \) [3]. If one presents the full gluon distribution \( f \) as a sum of contributions \( f_n \) from the production of \( n \) jets then one gets a recursive relation

\[
f_n(y) = \int_0^y dy_1 K(y - y_1) f_{n-1}(y_1),
\]

where \( K(y) \) is an \( y \)-dependent operator in the transverse momentum space

\[
K(y) = e^{y K_{UV}} K_R.
\]

Eq. (2) allows one to successively calculate the relative probabilities to produce \( n = 0, 1, 2, ... \) jets starting from the no-jet contribution.

The exclusive physical probabilities to observe \( n \) jets are obtained by convoluting \( f_n \) with the gluon distribution in the projectile (the projectile impact factor). Both the impact factors of the target and of the projectile should vanish as \( k \to 0 \).

3. The Calculation

We are interested in the average values of \( \langle q \rangle_n \) in the observed jets, provided their number \( n \) is fixed. The momentum \( k \) which serves as an argument of \( f(y, k) \) refers to the virtual gluon, and not to the emitted one,
whose momentum $q$ is hidden inside the kernel $K_R$. Therefore to find an average of any quantity $\phi(q)$ depending on the emitted real jet momentum, one has to introduce the function $\phi(q)$ into the integral defining $K_R$, thus changing the kernel $K_R$ to the kernel $K_{av}$:

\[
(K_{av}f)(k) = \bar{\alpha}_s k^2 \int \frac{d^2k_1}{\pi q^2 k_1^2} \theta(q^2 - \mu^2) \phi(q)f(k_1).
\] (4)

With $n$ jets, one has to substitute one of the $n$ operators $K_R$ which generate the jets by $K_{av}$, take a sum of all such substitutions, and divide by $n$. One has further to integrate over all momenta of the virtual gluon $k$ multiplied by the projectile impact factor, and normalize the result to the total probability to have $n$ jets. To formalize this recipe we introduce a generalized operator in the virtual gluon momentum space

\[
K_1(y) = e^{y K_{UV}}[K_R + K_{av}].
\] (5)

Let the function $F(y, k)$ obey the equation

\[
F(y) = f_0(y) + \int_y^0 dy_1 K_1(y - y_1) F(y_1).
\] (6)

One can split the function $F$ into a sum of contributions $F_{nm}$ corresponding to the action of $n$ operators $K_1$, out of which $m = 0, 1, ... n$, are operators $K_{av}$ (evidently $F_{n0} = f_n$). We are interested in the contribution $F_{n1} \equiv g_n$ which contains a single operator $K_{av}$. The average value of interest is determined by

\[
< \phi(q) >_n = \frac{1}{n} \int \frac{dk^2/k^4}{h(k) f_n(y, k)}.
\] (7)

In analogy with Eq. (2), one easily sets up a recursion relation for $g_n$:

\[
g_n = \int_0^y dy_1 K(y - y_1) g_{n-1}(y_1) + \int_0^y dy_1 e^{(y-y_1) K_{UV}} K_{av} f_{n-1}(y_1),
\] (8)

with the initial condition $g_0(y) = 0$. Together with (2), this relation allows one to calculate the function $g_n$ for $n = 1, 2, ...$, and then to use (7) to find the desired averages.

The concrete choice of $\phi(q)$ is restricted by the condition of convergence at large $q$: $\phi(q) < q^2$, as $q \to \infty$. To facilitate our calculation we make a natural choice $\phi(q) = q$, which allows the angular integration to be done analytically.
4. The Results

We defined our jets by taking \( \mu = 2 \text{ GeV/c} \). As for the cutoffs, we used

\[
1 \text{ GeV/c} < k_1 < 100 \text{ GeV/c},
\]

and we used a simplified expression for the virtual photon impact factor, independent of rapidity [2].

We have calculated the functions \( f_n \) and \( g_n \) from Eqs. (2) and (8) up to \( n = 5 \) and \( y = 15 \). Following [3] we have used the expansion in \( N \) Chebyshev polynomials to discretize the kernels in a simple way.

In Figure 1 we present the averages \( < q >_n \) for \( n = 1 - 5 \) and \( x = e^{-y} = 3.10^{-7} - 0.1 \), for the \( \gamma^*\)-hadron collisions (DIS) at \( Q^2 = 100 \text{ (GeV/c)}^2 \).

As one observes, \( < q >_n \) strongly grows with \( n \) at all rapidities, being the growth approximately linear.
As an interesting by-product of our study we find that the averages $\langle q \rangle_n$ go down with rapidity for all $n \geq 2$. This is quite unexpected, since in the BFKL approach an overall average $\langle q \rangle$ rapidly grows with $y$.

Similar results are obtained for purely hadronic collisions [2].

5. Discussion

Emissions of high-$p_T$ jets in DIS seem to be a suitable place to see the BFKL signatures. Our results show that in such emissions strong positive correlations are predicted between $\langle p_T \rangle$ and the number of jets, already for a single Pomeron exchange. This indicates that in fact such correlations are already present in the basic mechanism of jet production. The linear growth of $\langle p_T \rangle$ with $n$ that has been obtained could be a random-walk effect, $\langle p_T \rangle$ becoming larger and larger at each step (with each new produced jet) [4]. The extension of our study to the case of the BFKL equation with a running coupling constant would be important in order to establish the stability of our results.

An unexpected result obtained in our calculation is that $\langle q \rangle_n$ at fixed $n \geq 2$ fall with energy. Certainly this phenomenon deserves further investigation including higher $y$ and/or $n$. We hope that it can be tested experimentally as a possible signature of the BFKL Pomeron.

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4. We thank A.B. Kaidalov for enlightening discussions on this point.