Resonant tunnelling diode nano-optoelectronic spiking nodes for neuromorphic information processing

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In this work, we introduce an interconnected nano-optoelectronic spiking artificial neuron emitter-receiver system capable of operating at ultrafast rates (∼100 ps/optical spike) and with low energy consumption (< pJ/spike). The proposed system combines an excitable resonant tunnelling diode (RTD) element exhibiting negative differential conductance, coupled to a nanoscale light source (forming a master node) or a photodetector (forming a receiver node). We study numerically the spiking dynamical responses and information propagation functionality of an interconnected master-receiver RTD node system. Using the key functionality of pulse thresholding and integration, we utilize a single node to classify sequential pulse patterns and perform convolutional functionality for image feature (edge) recognition. We also demonstrate an optically-interconnected spiking neural network model for processing of spatiotemporal data at over 10 Gbps with high inference accuracy. Finally, we demonstrate an off-chip supervised learning approach utilizing spike-timing dependent plasticity for the RTD-enabled photonic spiking neural network. These results demonstrate the potential and viability of RTD spiking nodes for low footprint, low energy, high-speed optoelectronic realization of neuromorphic hardware.

I. INTRODUCTION

With the magnitude of data production increasing exponentially, machine learning (ML) approaches and the field of artificial intelligence (AI) have been undergoing a booming development, rapidly becoming ubiquitous in all domains of human endeavour. These methods have allowed machines to gain human-like information processing capabilities (e.g. learning, computer vision, natural language processing (NLP) or complex pattern recognition) and solve significant computational problems [1]. While AI algorithms achieve new breakthroughs, the hardware used to run those receives in turn less attention. Nowadays, large scale ML models are typically trained on cloud-based computing clusters, with some estimates placing the training energy consumption for a state-of-the-art NLP model on par with six years of total power energy consumption of a human brain [2]. Driven by the goal of reducing energy consumption as well as by the plateauing of empirical chip scaling laws, there has recently been significant growth of interest in non-conventional computing approaches. Neuromorphic (brain-like) engineering develops computer hardware architectures inspired by the brain and by the behaviour of biological neurons. Neuromorphic systems can be operated at various degrees of biological plausibility, directly mapping conventional artificial neural network algorithms onto hardware or capitalising on the rich dynamical behaviour of biological neurons for information processing. While there already are powerful neuromorphic systems based on electronics [3][4], the reliance on CMOS technology imposes limits in terms of interconnectivity and component density, with dozens of transistors required per neuron and additional external memories needed for synaptic weights. This results in several μm large neurons. Since dedicated wiring for every synapti-
TABLE I. Comparison of photonic and optoelectronic technologies capable of spike (pulse) based signalling.

| Photonic platform                        | Energy/event | Spike event timescales |
|------------------------------------------|--------------|------------------------|
| superconducting Josephson junctions     | $\sim 10^{-14}$ J | $\sim 10^{-12}$ J      |
| phase change material cells              | $\sim 10^{-14}$ J | $\sim 10^{-12}$ J      |
| micropillars                             | $\sim 10^{-14}$ J | $\sim 10^{-12}$ J      |
| graphene-SA laser                        | $\sim 10^{-14}$ J | $\sim 10^{-12}$ J      |
| RTD optoelectronic node [this work]     | $\sim 10^{-14}$ J | $\sim 10^{-12}$ J      |

II. NEUROMORPHIC RTD-POWERED OPTOELECTRONIC NODES

In this work, we introduce an optoelectronic neuromorphic system utilizing a resonant tunnelling diode (RTD) element based on a double barrier quantum well (DBQW) epi-layer structure. The DBQW consists of a narrow bandgap semiconductor layer embedded between two thin layers with wider bandgap (Fig. 1I, inset), with typical barrier thicknesses ranging from 4 to 8 nm, and 1 nm to 3 nm, respectively. Under applied voltage, the structure works as a filter for the carrier’s energy, leading to high carriers’ transmission when energy of the electrons (Fermi sea) resonates with the confinement energy levels of the DBQW. The voltage-controlled probability for incident electrons to cross the barrier is locally maximized, which results in the typical N-shaped function voltage-current relation $f(V)$ with one or more regions of negative differential conductance (NDC) in between two or more regions of positive differential conductance (PDC) [35] as shown in Fig. 1I. The presence of the nonlinearity and gain in the NDC region, persisting from DC up to THz frequencies [36], makes RTDs particularly suitable for high frequency oscillators [37]. The nonlinearity is key for operation of the proposed neuromorphic RTD node as a fast, excitable spiking nonlinear source with intrinsic electrical gain. Previous works have investigated triggering of stochastic excitatory responses in hybrid integrated optoelectronic RTD circuits [39, 40] and operation of RTDs with delayed feedback [41], addressing only operation of a single (solitary) device. In this work, we investigate for the first time interconnected systems consisting of multiple independent RTD-based monolithic integrated optoelectronic nodes. We employ the nodes as stateless excitatory devices and take advantage of the spike-based signalling to implement information processing tasks and multi-device networks, with prospects for very low footprint, low energy and high-speed operation due to sub-$\lambda$ elements. We utilize two types of nodes: an electronic-optical (E/O) RTD-LD system, realized with a RTD element coupled to a nanoscale laser diode (LD), and a optical-electronic (O/E) RTD-PD system, realized with a photodiode (PD) coupled to a RTD element. In both node types, spiking threshold can be adjusted via bias voltage tuning. An illustration of two nodes with an unidirectional optical weighted link, representing two feedforward linked neurons, is depicted in Fig. 1I.

A. Optoelectronic RTD-system architecture

In both the RTD-LD and RTD-PD nodes, the two functional blocks are integrated in a monolithic, metal dielectric cavity micro-pillar with the DBQW regions on GaAs/AlGaAs materials [42] for operation at the wavelength of 850 nm and InP materials [43] for operation at 1550 nm. For simplicity, in this work we focus on one of the two material platforms and investigate InP-based RTD systems throughout our analyses. The micro-pillar is coated by a dielectric cap (typically made of SiO$_2$) and metallic layer (typically Au or Ag), similarly to previously reported waveguide-coupled nanoLEDs [44]. A significant advantage of the semiconductor RTD epi-layer design is that it can be used to realize all the required functional blocks in the proposed neuromorphic optoelectronic nodes, including ultra-sensitive photodetectors [45, 46], high bandwidth nonlinear behaviour (including spiking responses) in the electric domain, and light emission, including both coherent (laser) and non-
coherent (light emitting diodes, LEDs) operation. This brings the possibility of all-in-one monolithic integration of the required functional blocks into singular sub-micron scale devices. Specific epi-layer designs based upon different materials platforms targeting operation at forementioned wavelength ranges, i.e. 1550 nm (InP) and 850 nm (GaAs), are currently being investigated towards the fabrication of the systems proposed in this work. For non-coherent signalling between nodes, the RTD-LD can also be realized using an RTD-LED sub-λ element at high (multi-gigahertz) speeds with very low power consumption (<1 pJ per emitted spike) [42]. It was observed that the light emission efficiency of the pillar design increases with smaller sizes, with sub-lambda pillars yielding very high light-extraction efficiency [47]. RTD-powered nanolasers and light-sources may also benefit from their small size in terms of improved operation speed and reduced lasing threshold [48]. In a review [2], it was stated that a minimum lateral size of hardware neurons is to be expected around 100 µm. RTD components, embedded as singular or monolithic sub-micron structures, have the potential to be significantly smaller, overcoming one of the key expected disadvantages (large footprint) in such systems. Unlike the superconducting and fluxonic solutions, the RTD-based optoelectronic node can be operated at room temperatures.

The synaptic links in this work are required for optical signal propagation between nodes and signal weighting (controllable optical signal attenuation). Recent advances in integrated, tuneable waveguide meshes offer chip-scale solutions for linear matrix transformations [50], which typically underpin the weighting functionality in neural networks. The micropillars can be directly coupled to waveguides by heterogenous integration [51] or coupled together by means of two-photon polymerization [52] guiding structures. Signal attenuation in photonic waveguides can be realized for example by means of balanced Mach-Zehnder interferometers, directional couplers [50] or nano-scale phase change material (PCM) cells [53]. PCM-based synaptic cells also exhibit suitability for all-optical spike-timing based plasticity [54] and all-optical synaptic signal weighting can also be realized using VCSOAs [55]. Synaptic interconnections can also be realized using photorefractive III-V photonic structures on silicon [56].

III. RTD-LD→RTD-PD: THEORY AND DYNAMICS

A. Single optoelectronic node

We consider the monolithic nodes as optoelectronic circuits based on an RTD element connected to electrical
Here, \( V \) is the voltage along the RTD, \( I(t) \) is the circuit’s total current, \( S(t) \) is the photon number and \( N(t) \) is the carrier number. \( R \) is the circuit equivalent resistance and \( L \) is the intrinsic inductance of the circuit while \( C \) is the parasitic capacitance of the RTD. \( V_m(t) \) is the modulation voltage function. We consider two node models: a) an O-E RTD integrated with a photodetector (PD), governed by Eqs. [1-2], which can be driven by external optical pulses (represented as \( S_m(t) \)) where \( \kappa \) is the photodetector conversion factor translating input optical intensity into a photocurrent [11] signal; b) an E-O RTD-LD node, governed by all the shown equations (Eq. [1-4]) with omission of the PD term. For simplicity of analysis, the rate equation model includes only homogeneous broadening effects. \( N_0 \) is the transparency carrier number, \( \tau_p \) is the photon lifetime, \( \gamma_m, \gamma_r, \gamma_n \) are respectively the spontaneous emission rate into the lasing mode (where \( \gamma_m S \) is the stimulated emission rate), radiative decay into the leaky modes and non-radiative spontaneous emission coefficients. \( q_e \) is the electron charge and \( J \) is an input bias current injected into the LD in addition to the RTD current \( I(t) \). The stochastic nature of the system is given in Eq. [3] by the term \( \gamma_m N \) and the multiplicative noise \( \sqrt{\gamma_m N} \xi(t) \), where \( \xi(t) \) is a time-uncorrelated white noise function. The parameters used in this work are available from the Supplementary information.

The function \( f(V) \) accounts for the nonlinear relation between the voltage applied across the RTD and the current passing through it. We use an analytical expression for \( f(V) \) derived in [62] and detailed in the Supplementary Information. The device operates at room temperature (300°K). Fig. 1B shows the experimentally fitted I-V characteristic (parameters available from Supplementary information) with a relatively narrow region of negative differential conductance embedded in between two regions of positive differential conductance, labelled as NDC, PDC I and PDC II, respectively. The curve peak is located at \( V = 609.6 \text{ mV} \), with a local maximal current of 338.6 \( \mu \text{A} \). At the right of the peak, the current abruptly drops from 340 \( \mu \text{A} \) to 80 \( \mu \text{A} \) in a span of less than 1 \( \text{mV} \). Further rightwards, \( f(V) \) continues to decrease although with a much more moderate rate, until it reaches a valley at \( V = 720.7 \text{ mV} \) and a local minimal current of 73.6 \( \mu \text{A} \). Beyond this point, \( f(V) \) increases again following a diode-like behaviour.

### B. Dynamical behaviour

When the system (Eqs. [1-2]) is biased in the proximity of the peak or valley of its I-V curve and injected with positive or negative voltage pulses respectively, it behaves as an excitable system able to respond with electronic spikes. Using this functionality, numerical simulations of Eqs. [1-4] are run, where a train of square negative voltage pulses \( V_m \) is used to trigger a spike in the RTD-LD optoelectronic master node (Fig. 2b). The period of the train is 2 ns and each pulse is 50 ps long and 100 mV deep. No optical modulation is used (i.e., \( S_m(t) = 0 \)). The RTD is biased close to the valley point at 750 mV. 50 simulations are run over 10 periods (thus, a total of 500 pulses are injected). The RTD responds with upwards current pulses (Fig. 2b), each about 275 ps long and reaching a peak of 342 \( \mu \text{A} \). The response delay is roughly 25 ps and the rest value of the signal is 74 \( \mu \text{A} \). Such a pulse elicits a weak response when injected into the LD because its peak value it slightly surpasses the LD threshold current for a very brief time. This is why the additional input bias current \( J \) is necessary. When the LD is biased at \( J = 210 \mu \text{A} \), the total current injected \( J + I_{\text{master}}(t) \) has a rest value of 284 \( \mu \text{A} \) and a peak value of 552 \( \mu \text{A} \), well below and well above the threshold current, respectively. In consequence, the LD remains inactive most of the time and emits a pulse in response to each current pulse (Fig. 2c). The optical pulse is shorter, with a duration of 40-60 ps (with temporal fluctuations due to the white noise term in Eq. [3]) because the LD takes a relatively long time to respond to an above-threshold current, while it quickly stops emitting as the current descends under the threshold. This results in the optical pulse being shortened and the response latency increased up to about 75 ps. This phenomenon is typical in systems that exhibit transcritical bifurcations and is known as critical slowing down [63–66]. The estimate for RTD-LD power consumption is based on the idle state current (284 \( \mu \text{A} \)), multiplied by the idle voltage bias (750 mV for valley point), resulting in 213 \( \mu \text{W} \). This is inclusive of the additional \( J \) term that sets the sub-threshold operation current of the laser. The spiking itself, due to its very short temporal timescales, will require small amount of additional power. Assuming for the spiking event a peak current of 552 \( \mu \text{A} \) and same voltage value of 750 mV gives a power of 414 \( \mu \text{W} \) during an approximate time of 100 ps (based on pulse shape from Fig. 2b) with maximum spiking repetition rate interval of approx. 420 ps.
further increasing the prospects of larger fan-in/fan-outs in networks.

IV. INFORMATION PROCESSING WITH RTD-BASED OPTOELECTRONIC NODES

A. Single node 8-bit pattern recognition task

Neurons have the ability to integrate a series of input stimuli and elicit a single spike firing response. This happens due to the cumulative effect of separate input perturbations which, when combined, can exceed the neuron firing threshold intensity. A similar integrate and fire (I&F) behaviour can be replicated with RTD devices.

To demonstrate this, we modelled the dynamical response of a single RTD-LD node when driven by an AC signal \( V_{DC} \) consisting of short negative sub-threshold square pulses. In this case, the RTD was biased at a voltage \( V_{DC} = 730 \text{mV} \) \( (I_{DC} = 73 \mu\text{A}) \), which positions the device’s operation point in the valley slightly to the right of the NDC region. The LD was biased at \( J = 210 \mu\text{A} \), thus the total current injected \( (J + I_{DC}) \) has a rest value of 283 \( \mu\text{A} \) (below the firing threshold current). For simplicity, we do not include a receiver RTD-PD circuit, but it is assumed that a perturbation \( S_{master} \) can be propagated to a receiver node in the form of an excitatory current pulse. To show the circuit’s I&F functionality, the RTD element was driven by an AC signal consisting of 50 ps pulses of amplitude \( V_{ac} = -15 \text{mV} \), separated by 50 ps. Thus, the resulting modulation signal is \( V_{m} = V_{DC} + V_{ac} \). Figure 3(a) shows the input signal \( V_{m} \) (top), which consists of three pulse trains with ×6, ×7 and ×8 pulses respectively, and the resulting RTD-LD output trace \( S_{master} \) (bottom) as a function of time. For modulation signals containing <8 pulses the output remains unperturbed. However, as the number of pulses is increased to 8, their combined intensities trigger a firing event in the RTD element, eliciting in turn a spike in the LD output. This I&F behaviour can be exploited to perform 8-bit pattern recognition tasks by a single RTD-LD node at very high speed rates, as it is demonstrated in Fig. 3(b)-d.

In this example, seven different 8-bit patterns, representing Tetris-like blocks, are mapped into a 4 by 2 grid with individual values of 1 and -1 (representing black and white colour pixels respectively). The corresponding pattern is described by \( V_{m} \) as a serialized 8-bit signal (top of Figure 3(b)). Subsequently, each pattern is multiplied offline by an array of weights \( W \) associated to a target Tetris piece. In the example shown in Figure 3(b), the element-wise multiplication between the \( J \)-block pattern and \( W = [-1, -1, -1, -1, 1, 1, 1] \) converts the input to a serialized all-negative 8-bit signal \( V_{weight} \). For the simulation, \( V_{weight} \) included 7 patterns temporally separated by 1 ns. Each bit had an activation time of 50 ps with an amplitude \( V_{ac} = \pm 15 \text{mV} \), separated by 50 ps. Two examples of a weighted modulation signal, used to recognise
b) Example of a Tetris J-block represented by a 4 × 2 grid and corresponding serialized signal $V_m$ (top). The J-block is weighted offline by element-wise multiplication with a matrix $W$ converting $V_m$ to $V_{\text{weight}}$ (bottom). c)-d) Simulation of pattern recognition tasks, where $W$ was chosen to target the J c) and S-block d) respectively. The corresponding driving signal $V_{\text{weight}}$ is shown (top) accompanied by the LD output trace (bottom). The LD outputs have been smoothed by taking a moving average $t_{MA} = 0.1$ ns to approximate the effect of the response time of the photodetector and to ease the visualization.

FIG. 3. a) RTD-LD response to an AC modulation signal: containing three sets of negative square signals with 6, 7 and 8 negative pulses respectively (top) and corresponding LD output trace (bottom). b) Example of a Tetris J-block represented by a 4 × 2 grid and corresponding serialized signal $V_m$ (top). The J-block is weighted offline by element-wise multiplication with a matrix $W$ converting $V_m$ to $V_{\text{weight}}$ (bottom). c)-d) Simulation of pattern recognition tasks, where $W$ was chosen to target the J c) and S-block d) respectively. The corresponding driving signal $V_{\text{weight}}$ is shown (top) accompanied by the LD output trace (bottom). The LD outputs have been smoothed by taking a moving average $t_{MA} = 0.1$ ns to approximate the effect of the response time of the photodetector and to ease the visualization.

B. Image edge detection task using sub-threshold pulse integration

We further demonstrate the possibility of using a single RTD-LD node to perform image edge-detection operation. For this task we utilized a binary image $M$ of size $n \times n$ (Figure 4a), where black and white pixels are assigned values of 1 or -1 respectively. On the preprocessing phase, an element-wise product between a 3 × 3 matrix kernel $K$ and sections of the binary image $M_h$ is performed offline such that:

$$P = K \circ M_h = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} M_{i,j} & M_{i,j+1} & M_{i,j+2} \\ M_{i+1,j} & M_{i+1,j+1} & M_{i+1,j+2} \\ M_{i+2,j} & M_{i+2,j+1} & M_{i+2,j+2} \end{bmatrix}$$

where $i$ and $j$ are the indices of the individual pixels.
FIG. 4. a) Steps followed to perform an edge detection task by a RTD-LD device. The process consists of four main phases: offline multiplication of a binary image $M$ and kernel $K$, serialization and sorting of the 9-bit pattern to generate a modulation signal $V_m$, simulation of RTD-LD response to $V_m$, and reconstruction of the LD output to a binary image $Q$. b) 11×11 pixels binary image used for edge detection task, where pixels are assigned values of 1 (black) or −1 (white). c) Example of modulation signal used as input to drive the RTD (top) and corresponding LD output trace (bottom). d) Colour plot showing the complete LD output series used for edge detection of $M$ (The red box corresponds to the $S_{master}$ output plot shown in c). e) Reconstruction of the LD output trace into a binary image $Q$. f) 50×50 binary image of Strathclyde’s Institute of Photonics (IoP) logo. g) Reconstructed image after single RTD-LD edge detection task.

The resulting matrix $P$ is serialized as a 9-bit pattern, where each bit is assigned a 50 ps activation pulse and a 50 ps separation for a total of 100 ps per bit. Each pulse is assigned an amplitude $V_{ac} = \pm 16$ mV * $P_{k,l}$, where $k$ and $l$ are the indices of individual matrix elements in $P$. The serialized bits are sorted such that their amplitude is rearranged in descending order. This ensures all negative pulses are integrated consecutively to elicit a firing response. The resulting 9-bit modulation signal ($V_m$) is used as the electrical input for the RTD-LD node. The process described above is repeated for each row of $M$ taking steps of 1 pixel. Finally, the output of the RTD-LD device is used to reconstruct a binary image $Q$, where pixels are assigned a value of 1 if the laser output trace exhibits a spike and -1 otherwise.

An example of an 11×11 binary image, used to demonstrate edge detection operation, is shown in Figure 4b. Each row of $M$ is described by a modulation signal $V_m$, like shown in the top of Figure 4c, consisting of 9 patterns with a duration of 100 × 9 ps each and temporally separated by 750 ps to account for the time required for the LD output to return to zero. For the simulation the RTD was biased at the valley $V_{DC} = 730$ mV. The corresponding $S_{master}$ time trace, displayed in the bottom of Figure 4c, shows two spikes of the LD output (pixels 4 and 6) as a result of the I&F response of the RTD (red box in Figure 4b,d). Figure 4e shows a colour plot of the LD output traces for each row of $M$, where the high values of
$S_{\text{master}}$ correspond to a detected edge. A binary image $Q$, reconstructed from the RTD-LD output, is shown in Figure 4a. It can be observed that, following an offline element-wise multiplication operation with a single 3 by 3 kernel, the RTD-LD node is able to consistently detect all edges of $M$, regardless of their orientation. We further show the capability of an RTD-LD to consistently detect all edge features, by using a 50 × 50 pixels binary image of the logo of the Institute of Photonics (IoP) at the University of Strathclyde (Fig. 4). The reconstructed image in Figure 4 shows the RTD-LD node is able to detect all edges with a 99.7% accuracy. This results are a good example of functional tasks which can be performed by exploiting the I&F response of an RTD-LD node.

C. Feedforward network of optoelectronic nodes

Since the information processing capability of an artificial neural network (ANN) typically grows with increasing network complexity, demonstrating networking performance with multiple optoelectronic spiking nodes is of key importance. Here, we numerically investigate the operation of a spiking variation on the single layer, feedforward perceptron model with all-to-one layout. Such network processes input spike-represented data by weighting the signal from each upstream node and summing up all the weighted inputs on the the downstream node, which fires a spike if the weighted input sum exceeds the firing threshold. In the demonstrated model, the spatiotemporal patterns of input superthreshold stimuli are injected into the first layer of neurons (PREs), where each stimulus results in a guaranteed optical spiking outcome from the corresponding PRE node. The optical spiking signals from the PREs are weighted by attenuating them (multiplying their intensity by a given factor $w_n$ in the numerical model). During the network learning phase, a guiding signal carries the data labels alongside each pattern, marking it as wanted (True) or unwanted (False) via change in amplitude. The downstream POST neuron performs the temporal integration of the upstream inputs and fires a spike if the voltage of spiking threshold is surpassed (integrate & fire operation). A diagram of the network is depicted in Fig. 5, showing how different patterns (consisting of spikes, in blue) may result in activation of spikes and illustrating the dependence of weights on the output of the downstream node.

In particular, the investigated network consists of five layer 1 RTD-LD nodes (PREs, biased in the valley (in PDR II), $V_{DC} = 770$ mV), whose output optical signals are propagated through unidirectional, feed-forward links (each with weight $w_l$) to a single, layer 2 PD-RTD (POST) node biased in the valley (in PDR II). In the PD-RTD, the PD is current-coupled into the spiking RTD element (with the PD conversion factor $\kappa$), directly converting the incoming optical intensity into the electrical domain and resulting in activation of an electronic spiking signal. In the PREs, we utilize superthreshold input trigger pulses of length $t_{\text{pulse}} = 80$ ps, resulting in excitatory (increasing intensity) optical pulses. Since the output current of a PD at a given time directly depends on the input light intensity and temporal distance from previous optical spikes, only certain weighted pulse patterns may result in sufficiently strong current modulation, activating a spike in the downstream node. That is the working principle of the network model for input spatiotemporal spike-pattern recognition. Visualization of the pattern recognition in the network is shown in Fig. 6. In this network, the temporal separation between each 5-bit input pattern is set to 420 ps, corresponding to full network processing capacity of 11.9 Gbps.

D. Networks: supervised learning method for spatiotemporal pattern recognition

Training algorithms are fundamental for useful utilization of artificial neural networks (ANNs). However, training methods for spiking neural networks (SNNs) differ from those used for conventional ANNs, which are typically based on backpropagation [6]. SNNs can be trained using either biologically-plausible local learning rules (e.g. spike-timing dependent plasticity (STDP), long-term potentiation) or using other specially designed algorithms such as ResSuMe [68]. Resilient-Backpropagation (RProp) inspired supervised learning [69] and SuperSpike [70], among others. In this work, we introduce an offline supervised learning rule, following the approach introduced in [71] for training memristor-based neural networks. However, in contrast to [71], our system propagates information using optical spike trains, allowing us to fully benefit from the advantages of optical signaling (e.g. high-bandwidth, low loss waveguiding, non-interacting signals etc). Data processing in our network follows the two typical phases: a) training phase and b) inference phase. During the training phase, labelled patterns are processed by the network. By comparing the output state of the network with the label, according adjustments are made to the network weight matrix. The learning phase consists of multiple epochs, and progresses until the weights stabilize. During a single epoch, the dynamical evolution of all the RTD-based nodes in the network is numerically evaluated. The use of teacher signals (which carry the label of the pattern) allows for processing of multiple patterns in a single epoch. In the learning phase, three independent patterns are processed per single epoch of $t = 5$ ns.

Fig. 6 shows the learning process. The target input is a 5-bit spatial pattern, either [0 1 0 1 0] (Fig. 6a) or [1 0 1 0 1] (Fig. 6b), and the network is initiated with all weights set to an initial value $w = 0.4$. We want to note here that the weights depend on the current conversion factor $\kappa$ of the PD, which was selected in this demonstration to bound the weights in the usual interval [0,1]. During each learning epoch, three random patterns are picked, with a probability $P_l = 0.25$ of picking the
FIG. 5. a) Network architecture diagram, illustrating how patterns of input electronic pulses (in blue) enter the RTD-LD nodes and are propagated as optical signals to the downstream node using weighted connection. The output state of the downstream node is compared to the label, and if there is a mismatch between label and output state, the weights are updated. Desired pattern is highlighted with the target icon. b) Visualization of inference in 5-to-1 feedforward network numerical model. The guiding signals representing pattern labels are visualized as background shading (green for 'True', grey for 'False'). Only a particular spatial pattern \([1 \ 0 \ 1 \ 0 \ 1]\), green results in firing of electric spike of the downstream RTD-PD node (green current trace). The red timetrace represents a simple moving average of the LD output optical signal over \(t_{MA} = 0.1\) ns.

FIG. 6. Demonstration of the supervised learning process for two different spatial patterns with varying number of active bits: a) \([0 \ 1 \ 0 \ 1 \ 0]\) and b) \([1 \ 0 \ 1 \ 0 \ 1]\). As the system is used to process to labelled patterns in each epoch, the weights are adjusted using the local learning rule, strengthening connections which produced false negative results and weakening links which produced false positive results. The background colour shows network state (True/False) during each step. Target and \(P_f = 0.75\) of picking any other pattern. Fig. 6 shows the evolution of the weights during each learning step. Green background represents True outcomes (True positive, True negative) while red represents False outcomes (False positive, False negative). For either True output state, no weights are adjusted during the learning step. For the False positive output state, the weights that contributed to the firing are weakened, with \(\Delta w\) being a function of PRE-POST spike separation. The closer the PRE node’s spike was to activation of a False positive POST spike, the higher is the depotentiation (weakening) effect. This is a supervised variation on the STDP learning protocol, a specific kind of Hebbian learning approach which is believed to constitute part of the learning process in biological neural networks. A simple rational function was selected for the weight adjustment:

\[
\Delta w_n = \frac{a}{b \cdot |\Delta T_n| + c} + d \quad (6)
\]

where

\[
\Delta T_n = T_{POST} - T_{PRE,n} \quad (7)
\]

represents the time interval between the spikes from the POST and the PRE neuron \(n\), \(a = 9.35 \cdot 10^{-3}\), \(b = 5 \cdot 10^3\), \(c = 0.8\), \(d = 1.5 \cdot 10^{-3}\). The numerical coefficients in the rational function were selected based on observed distances between spikes in PRE-POST neurons and the corresponding desired weight adjustments. Weight adjustment factors as a function of spike separation time can be seen in Fig. 7.

As the training process proceeds, the occurrence of false outcomes gets more and more rare. For both tested patterns, the system reaches a stable weight setting in approximately 300 patterns (100 epochs). This network
implementation utilizes only positive weight values, making the solution physically feasible. After the training phase, the network can perform inference for recognition of the selected spatiotemporal 5-bit pattern. We tested all patterns with equal number of active bits against a single desired target pattern: [0 1 0 1 0] in one measurement, and [1 0 1 0 1] in the other. When testing inference accuracy for [0 1 0 1 0] against all patterns with $n_{ON} = 2$ active bits, the total True response accuracy (with 540 inferred patterns) was 97.4%. Inferring the pattern [1 0 1 0 1] against all patterns with same number of ON bits ($n_{ON} = 3$) in 540 inference steps yields total True response accuracy of 94.8%. The confusion matrices for both of these inference procedures are shown in Fig. 8.

![Confusion Matrix](image)

**FIG. 7.** Weight adjustment factor as a function of POST-PRE spike temporal distance $|\Delta T_n|$. For false negative (FN, in green) cases, the weight adjustment is a constant fixed value. For false positive, the magnitude of weight adjustment is a function of $|\Delta T_n|$, with close spikes being depotentiated more strongly.

**FIG. 8.** *(Left)* Confusion matrix for inference of [0 1 0 1 0] against all other pattern with two ON bits ($n = 10$ different patterns, 540 total inference steps). *(Right)* Confusion matrix for inference of pattern [1 0 1 0 1] against all other pattern with three ON bits ($n = 10$ different patterns, 540 total inference steps).

V. CONCLUSIONS

In this work, we introduce a nano-optoelectronic neuromorphic node based on the DBQW-based resonant tunnelling diode exhibiting regions of negative differential conductance (NDC), enabling neuron-like electronic spiking responses at over GHz rates. The nodes consist of this highly nonlinear, high bandwidth RTD element coupled to either a high-sensitivity photodetector or a nanoscale laser to enable the reception and transmission of optical spikes, respectively. These devices offer multiple desirable properties such as suitability for either hybrid or monolithic integration, highly reduced footprint, high-speed operation (<100 ps long input signals) and low energy requirements (operation with mV trigger pulse amplitudes and energies of <pJ/spike). All these aspects are crucial for the future development of on-chip integrated neuromorphic photonic computing platforms. We investigate and analyze the dynamical behaviour of the proposed neuromorphic optoelectronic system and discuss feasible hardware implementations of individual nodes as well as nodes in interconnected network architectures. We also numerically demonstrate functional information processing tasks, including 8-bit pattern recognition and image edge-feature detection at over 10 Gbps rates (using 50 ps ps long input signals). Finally, we demonstrate network operation, investigating a 5-to-1 feedforward (two-layer) spiking neural network architecture. Using physical models for each node, we demonstrate that the numerically implemented network can be used to classify spatial 5-bit pulse patterns encoded in time, and we propose a supervised learning scheme that employs a spike-timing dependent learning rule. During the inference phase, we demonstrate 94%+ accuracy for spatial pulse pattern recognition. To the best of our knowledge, the reported results are the first theoretical demonstration of RTD-based neuromorphic optoelectronic spike information processing, delivering successful operation in key tasks (e.g. pattern recognition, image edge-detection) by utilizing either implementation-friendly single node solution, or using multiple interconnected devices in the form of a photonic feed-forward spiking neural network.

VI. ACKNOWLEDGEMENTS

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SUPPLEMENTARY INFORMATION

I. RTD I-V CHARACTERISTICS AND SIMULATION PARAMETERS

The RTD I-V characteristics used in this work has an analytical form of [62]:

\[
f(V) = a \ln \left( \frac{1 + e^{\frac{q_e}{k_B T} (b-c+n_1 V)}}{1 + e^{\frac{q_e}{k_B T} (b-c-n_1 V)}} \right) \times \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{c-n_1 V}{d} \right) \right) + h \left( e^{\frac{q_e}{k_B T} n_2 V} - 1 \right).
\]

(S1)

Here, \( T \) is the temperature, \( q_e \) is the electron charge and \( k_B \) is the Boltzmann’s constant. The parameters \( a, b, c, d, n_1, n_2, h \) depend on the geometry of the barrier and the confinement energy levels. The following parameter values were found after fitting experimental data:

| Parameter | Value |
|-----------|-------|
| \( a \)  | \(-5.5 \times 10^{-5}\) A |
| \( b \)  | 0.033 V |
| \( c \)  | 0.113 V |
| \( d \)  | \(-2.8 \times 10^{-6}\) V |
| \( n_1 \) | 0.185 |
| \( n_2 \) | 0.045 |
| \( h \)  | \(18 \times 10^{-5}\) A |

The use of negative parameter values is justified as Eq. (S1) is derived from an idealised system that has been fitted to a realistic one adjusting the parameter values.

The electronic parameters of the RTD circuit are:

| Parameter | Value |
|-----------|-------|
| \( R \)  | 10 Ω |
| \( L \)  | \(126 \times 10^{-9}\) H |
| \( C \)  | \(2 \times 10^{-15}\) F |

A. Self-oscillations and slow-fast dynamics

It is well-known that an RTD driven by an electrical DC source exhibits self-oscillations when biased in the NDC region, and a steady response (given by the intersection between the load line and the RTD I-V curve) when biased in either PDC region [35, 72, 73]. From the analytical point of view, this notion is valid (although not exact) provided that the load line is "almost vertical" and the coefficient \( \mu = \sqrt{C/L} \) is much smaller than the absolute value of the minimal differential conductance in the NDC region \([74][76]\). The first condition can be assumed in our study, since the slope of the load line (i.e., \(-1/R = -0.1 \Omega^{-1}\)) is much bigger in magnitude than that of the segment that connects the peak and the valley (\(\Delta I/\Delta V = -2.39 \times 10^{-3}\) Ω\(^{-1}\)). The second condition applies as well as the minimal value of \( f'(V) \) in the NDC region is \(-5.5 \Omega^{-1}\) while \(\mu = 1.26 \times 10^{-4} \Omega^{-1}\).

The coefficient \( \mu \) also accounts for the stiffness in the dynamics. Provided that \( \mu \) is much smaller in magnitude than the minimal conductance, the self-oscillations have sharp-cornered orbits and each period has two stages of slow and two stages of fast dynamics [35], as illustrated in Fig. S1. Each slow stage takes place at each PDC region, where the orbit remains close to the I-V curve until reaching either the peak or the valley. Here, the orbit makes a fast jump into the other PDC region, the voltage \(V\) changes abruptly in a very short time, with a slight change in the current \(I\), which does not exhibit the fast stages. Consequently, the RTD generates current and voltage spikes periodically with a frequency that depends on the bias voltage \([76]\) but remains at about 3 GHz.

B. Excitability

The concept of excitability was originally coined to refer to the capacity of living organisms to respond to an external stimulus, provided that such stimulus is above a certain threshold \([77][78]\), but it is also applicable in other areas such as image processing \([79]\), neuron-like semiconductors \([80]\) and signal generation \([81][83]\). A physical
system is referred to as excitable if, after a sufficiently strong (super-threshold) perturbation, it responds with a long, complex trajectory before returning to its natural state of equilibrium. While this response takes place, the system is unable to respond to any other perturbation, strong or weak. The duration of the response is thus known as refractory (lethargic) time. For a perturbation below the threshold, the response is a fast, exponential decay, typical of linear systems. Excitable systems are particularly interesting in the context of spiking because the response to a super-threshold perturbations can be interpreted as a spike or pulse, while unresponsiveness to weak perturbations confers them a robust character.

When the circuit is biased at \( V_0 = 750 \text{ mV} \) (i.e., in the second PDC region but close to the NDC region), it remains steady at about \( I = 74 \mu\text{A} \). Fig. S2 shows the response of the circuit to inhibitory (i.e., downward) square voltage pulses of 50 ps long and depths ranging from 10 to 100 mV. If the pulse depth is 50 mV or less, the circuit responds with low-amplitude relaxation oscillations and decays back into the steady state in a relatively short time. However, if the pulse depth is larger than 50 mV (excitability threshold), the circuit responds with a large current pulse (about 280 \( \mu\text{A} \) in amplitude and 250 ps long) before returning to the steady state. The voltage across the RTD is also affected and the circuit’s response follows a phase-space orbit reminiscent of that of the self-oscillations that arise when the circuit is biased in the NDC. The slow and fast stages of the periodic self-oscillations are also present here.

### II. LASER DIODE SIMULATION PARAMETERS

Given the dimensions of the device under study, the nanoscale LD model utilized in this work uses the following set of parameters:

| Parameter | Value |
|-----------|-------|
| \( N_0 \) | \( 5 \times 10^5 \) |
| \( \gamma_m \) | \( 10^7 \text{ s}^{-1} \) |
| \( \gamma_l \) | \( 10^9 \text{ s}^{-1} \) |
| \( \gamma_{nr} \) | \( 2 \times 10^9 \text{ s}^{-1} \) |
| \( \tau_p \) | \( 5 \times 10^{-13} \text{ s} \) |
| \( \kappa \) | \( 1.1 \times 10^{-7} \text{ A} \) |
| \( \eta \) | 1 |
| \( J \) | 210 \( \mu\text{A} \) |

Thus, the Purcell enhanced factor is \( \beta = \frac{\gamma_m}{(\gamma_m + \gamma_l)} = 0.099 \), proper of nanoscale lasers. When injected with a constant current, laser diodes emit with an intensity that increases in proportion to such current, provided that the current is above the lasing threshold. Indeed, the stable equilibrium photon number is a piecewise linear function of the input bias current,

\[
S = \begin{cases} 
0 & J \leq J_{tr}, \\
\tau_p \left( \frac{\eta}{q_e} - (\gamma_l + \gamma_m + \gamma_{nr}) \left( N_0 + \frac{1}{\gamma_m \tau_p} \right) \right) & J \geq J_{tr},
\end{cases}
\]

where

\[
J_{tr} = \frac{q_e (\gamma_l + \gamma_m + \gamma_{nr}) (N_0 + 1/\gamma_m \tau_p)}{\eta} = 337.54 \mu\text{A}
\]

for the parameters chosen in this study.