The transport properties of dense stellar electron-proton plasma is studied following an exact relativistic formalism in presence of strong quantizing magnetic field. The variation of transport coefficients with magnetic fields are found to be insensitive for the field strengths $\leq 10^{17}$G, beyond which all of them abruptly go to zero. As a consequence, the electron-proton plasma behaves like a superfluid insulator in presence of ultra-strong magnetic field.

1. INTRODUCTION

The study of the effect of strong magnetic field on dense stellar plasma is one of the oldest branches of physics. It has gotten a new life after the discovery of a few magnetars- the strange stellar objects, with unusually high surface magnetic fields [1–4]. These stellar objects are believed to be strongly magnetized young neutron stars. The surface magnetic fields are observed to be $\sim 10^{15}$G. Then it is quite possible that the field strength at the core region may go up to $10^{18}$G. The exact source of this strong magnetic field is of course yet to be known. These objects are also supposed to be the possible sources of anomalous X-ray and soft gamma emissions (AXP and SGR). If the magnetic fields are really so strong, in particular at the core region, they must affect significantly most of the important physical properties of such stellar objects and the physical processes, e.g., weak and electromagnetic interactions taking place at the core region. Which means, the presence of strong quantizing magnetic field at the core region should modify, both qualitatively and quantitatively the equation of state of dense neutron star matter, and as a consequence the gross-properties of neutron stars [5–8], e.g., mass-radius relation, moment of inertia, rotational frequency etc. should also change significantly. In the case of compact neutron stars, the phase transition from neutron matter to quark matter at the core region, if any, will also be affected by strong quantizing magnetic field. It has been shown that a first order phase transition initiated by the nucleation of quark matter droplets is absolutely forbidden if the magnetic field $\sim 10^{15}$G at the core region [9,10]. However a second order phase transition is allowed, provided the magnetic field strength $< 10^{20}$G. This is of course too high to achieve at the core region.

The elementary processes, in particular, the weak and the electromagnetic processes taking place at the core region of a neutron star are strongly affected by such ultra-strong magnetic field [11,12]. Since the cooling of neutron stars are mainly controlled by neutrino/anti-neutrino emissions, the presence of strong quantizing magnetic field should affect the thermal history of strongly magnetized neutron stars. Further, the electrical conductivity of neutron star matter which directly controls the evolution of neutron star magnetic field will also change significantly.

In another kind of work, the stability of such strongly magnetized rotating objects are studied. It has been observed from the detailed general relativistic calculation that there are possibility of some form of geometrical deformation of these objects from their usual spherical shapes [13–15]. In the extreme case such objects may either become black strings or black disks. In the non-extreme case, however, it is also possible to detect gravity waves from these deformed rotating objects.

In a recent study on microscopic model of dense neutron star matter, we have observed that if most of the electrons occupy the zeroth Landau level, with spin anti-parallel to the direction of magnetic field and very few of them are with spin along the direction of magnetic field and Landau quantum number $> 0$, then either such strongly magnetized system can not exist or such a strong magnetic field is just impossible at the core region of a neutron star [16].

Motivated by the problems as mentioned in preceding two sections, in this paper we shall study the effect of strong quantizing magnetic field on the transport coefficients of dense stellar electron-proton plasma. We shall follow an exact formalism [17] which is applicable for a wide range of magnetic field strengths and obtain the transport coefficients from the relativistic version of Boltzmann kinetic equation by linearizing the distribution function and using relaxation time approximation. We shall obtain the relaxation time from the rates of standard electromagnetic processes taking

*E-Mail: somenath@klyuniv.ernet.in
place inside electron-proton plasma and make necessary modification in the rate calculation due to the presence of strong quantizing magnetic field. We have noticed that the electrical conductivity of the medium becomes extremely small in presence of ultra-strong magnetic field ($\geq 10^{17}$G). The magnetic field at the core region of a magnetar must therefore decay very rapidly (time scale $\sim$ a few mins.) and becomes moderate or low enough. As a consequence there will be in principle no problem on the existence of magnetars (with very low or moderate core magnetic field). The formalism we have developed to obtain rates of electromagnetic processes or the relaxation time is also applicable to evaluate neutrino emissivity and mean free path in presence of strong quantizing magnetic field.

In presence of strong quantizing magnetic field, since the momentum component in the transverse plane with respect to the external magnetic field gets quantized, whereas the component along the field direction varies continuously (from $-\infty$ to $+\infty$), the momentum space volume element becomes

$$\frac{d^3p}{(2\pi)^3} = \frac{dp_x dp_y dp_z}{(2\pi)^3} = \frac{eB_m}{4\pi^2} \sum_{\nu=0}^{\infty} (2 - \delta_{\nu0}) dp_z$$  \hspace{1cm} (1)

(we have assumed $\hbar = c = k_b = 1$) where we have chosen the gauge $A^\mu \equiv (0, 0, xB_m, 0)$, so that the constant magnetic field $B_m$ is along z-direction. We have considered the simplest possible picture of neutron star matter with $n - p - e$ out of thermodynamic equilibrium and the neutrinos are assumed to be non-degenerate. The baryonic components are interacting via $\sigma - \omega - \rho$ meson exchange type mean field and the electrons are assumed to be freely moving particles.

In this article, we shall first calculate the transport coefficients of electron gas. Then it is very easy to obtain the transport coefficients for proton matter just by replacing mass, chemical potential etc. of electrons by protons and taking into account the proper modification in presence of $\sigma - \omega - \rho$ meson exchange type mean field [6]. Spinor solutions for electrons in presence of strong quantizing magnetic fields are then given by

$$\Psi^{(\uparrow)}(x) = \frac{1}{\sqrt{L_y L_z}} \exp(-i\varepsilon^{(c)}_\nu t + ip_y y + ip_z z) \times \begin{pmatrix} (\varepsilon^{(c)}_\nu + m_e) I_{\nu-1; p_y}(x) \\ 0 \\ p_z I_{\nu; p_y}(x) \\ -i(2veB_m)^{1/2} I_{\nu-1; p_y}(x) \end{pmatrix}$$  \hspace{1cm} (2)

and

$$\Psi^{(\downarrow)}(x) = \frac{1}{\sqrt{L_y L_z}} \exp(-i\varepsilon^{(c)}_\nu t + ip_y y + ip_z z) \times \begin{pmatrix} 0 \\ (\varepsilon^{(c)}_\nu + m_e) I_{\nu-1; p_y}(x) \\ i(2veB_m)^{1/2} I_{\nu; p_y}(x) \\ -p_z I_{\nu-1; p_y}(x) \end{pmatrix}$$  \hspace{1cm} (3)

where $\uparrow$ and $\downarrow$ represent up and down spin states respectively,

$$I_{\nu; p_y}(x) = \left(\frac{eB_m}{\pi}\right)^{1/4} \frac{1}{\sqrt{2^{\nu/2}}} \times \exp\left[-\frac{1}{2}eB_m \left(x - \frac{p_y}{eB_m}\right)^2\right] H_{\nu} \left[\sqrt{eB_m} \left(x - \frac{p_y}{eB_m}\right)\right],$$  \hspace{1cm} (4)

with $H_{\nu}$ the Hermite polynomial of order $\nu$ and

$$\varepsilon^{(c)}_\nu = (p_z^2 + m_e^2 + 2veB_m)^{1/2}$$  \hspace{1cm} (5)

the energy eigen value with $\nu = 0, 1, 2, \ldots$, the Landau quantum numbers. For neutron we consider the usual spinor solutions. Since the temperature of the system is $\ll$ electron chemical potential, we have not considered the negative energy spinor solutions. In the energy eigen value $p_\perp = (2veB_m)^{1/2}$ is the transverse component of electron momentum. To overcome the serious problem on the mechanical stability and hence the existence of magnetars, we have studied the variation of transport coefficients, in particular the electrical conductivity of electron gas / proton matter in presence of strong quantizing magnetic field and try to show whether the electrical conductivity which is solely responsible for the evolution of neutron star magnetic field, becomes sufficiently small in presence of ultra strong magnetic field.

The paper is organized in the following manner. In section 2, we have developed the relativistic version of Boltzmann kinetic equation for fermions in presence of strong quantizing magnetic field and obtain the expressions for transport coefficients following de Groot [18]. We shall use the relaxation time approximation for the collision term. In section 3 we shall evaluate the the relaxation time from the rates of standard electromagnetic processes taking place at the core region. We shall incorporate the necessary changes in the rate calculation due to the presence of strong quantizing magnetic field (or the effect of quantized Landau levels). In the last section we shall discuss importance of the results.
2. BOLTZMANN EQUATION

The relativistic version of Boltzmann transport equation is given by [18]

\[ p_\mu \partial^\mu f + e F^{\mu \nu} p_\nu \partial f \partial \mu + \Gamma^\rho_{\nu \lambda} p^\nu p^\lambda \partial f \partial p^\rho = C \]  

(6)

where the second and the third terms are coming from electromagnetic and gravitational (general relativistic) interactions respectively and \( C \) is the collision term. Since we have considered the flat space-time geometry and noticed that there is no contribution of external magnetic field, only the induced electric field arising from local charge non-neutrality, causes an induced current in the system will contribute in the electromagnetic interaction term and the curvature term is neglected.

To obtain the shear and bulk viscosity coefficients, heat conductivity and electrical conductivity of dense electron gas we make the relaxation time approximation, given by

\[ C = -\frac{p_\mu}{\tau} (f(x, p) - f^0(p)) \]  

(7)

where

\[ f(x, p) = f^0(p) (1 + \chi(x, p)) \]  

(8)

i.e., the system is assumed to be very close to the local equilibrium configuration. Here \( \tau \) is the relaxation time and \( f^0 \) is the (local) equilibrium distribution (Fermi distribution) function, given by.

\[ f^0(p) = \frac{1}{\exp \beta(\epsilon_p - \mu) + 1} \]  

(9)

We shall now follow the general technique to obtain the perturbed part \( \chi \) as a linear sum of driving forces. Following the formalism developed in the book by de Groot [18], we express the four derivative as the sum of a space like part and a time like part, given by \( \partial^\mu = u^\mu D + \nabla^\mu \) with \( u^\mu \) the hydrodynamic velocity, \( D = u^\mu \partial_\mu \) is the convective time derivative and \( \nabla^\mu = \Delta^\mu_\nu \partial_\nu \) is the gradient operator, with \( \Delta^\mu_\nu = g^\mu_\nu - u^\mu u^\nu \) some kind of projection operator, \( g^\mu_\nu = \text{diag}(1, -1, -1, -1) \) the metric tensor. Then using the eqns.(6)-(9), the decomposition of \( \partial^\mu \) and the equations of motion, given by

\[ Dn = -n \nabla_\mu u^\mu, \quad Dw^\mu = \frac{1}{n \hbar} \nabla^\mu P, \quad C_v DT = -F(T) \nabla_\mu u^\mu \]

we obtain the perturbative part \( \chi \), given by

\[ \frac{T p^0}{\tau} \chi = Q X \left(1 - f^0\right) - (p^\mu u_\mu - \hbar) p_\nu X^\nu \left(1 - f^0\right) + p^\mu p^\nu X^0_{\mu \nu} \left(1 - f^0\right) + e \left[ \frac{p_\mu u^\mu}{\hbar} + 1 \right] p_\nu E^\nu \left(1 - f^0\right) \]  

(10)

where

\[ X = -\nabla^\mu u_\mu \]  

(11)

is the driving force for bulk-viscosity,

\[ X^\mu_q = \nabla^\mu T - \frac{T}{n \hbar} \nabla^\mu P \]  

(12)

is the driving force for heat conduction,

\[ X^0_{\mu \nu} = \nabla^\mu u_\nu - \frac{1}{3} \Delta^\mu_\nu \nabla_\sigma u^\sigma \]  

(13)

is the driving force for shear viscosity and \( E^\nu \) is the driving force for electric current (\( E^\nu \) for \( \nu = i = 1, 2, 3 \) are the components of electric field vector). The quantity \( Q \) is given by
\[ Q = -\frac{1}{3} \Delta^{\mu\nu} p_\mu p_\nu + (p^\mu u_\mu)^2 \frac{F(T)}{T} (1 - \gamma) + T^2 (1 - \gamma) \frac{F(T)}{T} \frac{\partial}{\partial T} \left( \frac{\mu}{T} - n \frac{\partial \mu}{\partial n} \right) p^\mu u_\mu \]  
(14)

with \( F(T) = P(T)/n(T) \), \( P(T) \) and \( n(T) \) are the equilibrium local kinetic pressure and number density respectively. For a non-relativistic Boltzmann gas \( F(T) = T \), the local temperature of the system and finally \( h = (\epsilon + P)/n \), the enthalpy per particle with \( \epsilon \) the local energy density and \( \gamma = C_p/C_v \) the ratio of specific heats at constant pressure and constant volume respectively.

Now from the definition, the heat flow four current is given by

\[ I_q = \frac{eB_m}{4\pi^2} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} (p^\nu u_\nu - h) p^\mu f(x,p) \]
(15)

where the first term is the equilibrium contribution, which is identically zero. Then omitting the symbol (1), we have the irreversible term

\[ I_q^{(1)\mu} = I_q = \frac{eB_m}{4\pi^2} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} p^\nu (p^\nu u_\nu - h) f^{(0)}(p) \chi(x,p) \]
(16)

Again using the definition

\[ I_q^{\mu} = \lambda^{\mu\nu} X_{\rho\nu}, \]
(17)

we have the heat conductivity coefficient

\[ \lambda = \frac{1}{3} \Delta^{\mu\nu} \lambda_{\mu\nu} = -\frac{1}{3T^2} \frac{eB_m}{4\pi^2} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} (p^\nu u_\nu - h)^2 f^{(0)}(1 - f^{(0)}) \Delta^{\mu\nu} p_\mu p_\nu \]
(18)

Now from the definition, the energy-momentum tensor is given by

\[ T^{\mu\nu} = \frac{eB_m}{4\pi^2} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} p^\mu p^\nu f(x,p) \]
(19)

which can also be written as \( T^{\mu\nu} = T^{(0)\mu\nu} + T^{(1)\mu\nu} \), with the equilibrium value

\[ T^{(0)\mu\nu} = \frac{eB_m}{4\pi^2} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} p^\mu p^\nu f^{(0)}(p) \]
(20)

and the non-equilibrium part

\[ T^{(1)\mu\nu} = T^{\mu\nu} = \frac{eB_m}{4\pi^2} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} p^\mu p^\nu \chi(x,p) f^{(0)}(p) \]
(21)

which is a symmetric second rank tensor. Now we consider a model in which a flow of electron gas with cylindrical symmetry is assumed. Then considering \( \mu = r, \nu = z \), we have from the definition

\[ T^{(1)rz} = -\eta \frac{du_z}{dr} \]
(22)

Hence the shear viscosity coefficient is given by

\[ \eta = \frac{eB_m}{4\pi^2T} \sum_\nu (2 - \delta_{\nu 0}) \int_{-\infty}^{+\infty} \frac{dp_\nu}{p_0} (p^\nu p^r)^2 f^{(0)}(1 - f^{(0)}) \tau \]
(23)

where \( p_r = (2\nu eB_m)^{1/2} \), the transverse component of electron momentum. Then it is quite obvious that in presence of ultra strong magnetic field, for which \( \nu_{\text{max}} = 0 \), the shear viscosity coefficient vanishes. Here \( \nu_{\text{max}} \) is the maximum value of quantum number of the Landau levels occupied by electrons for a given density and temperature.
ν_{max} = \left[ \frac{\mu^2 - m_e^2}{2eB_m} \right],

where [ ] indicates the nearest integer less than the actual value.

To obtain an expression for bulk viscosity coefficient, we next consider the pressure tensor, given by

\[ \Pi^{\mu\nu} = \Delta_{\mu}^\sigma T^{\sigma\tau} \Delta_{\tau}^{\nu} + P \Delta^{\mu\nu} \]  

(24)

Where the reversible part

\[ \Pi^{(0)\mu\nu} = 0 \]  

(25)

and the non-equilibrium part

\[ \Pi^{(1)\mu\nu} = \Delta_{\mu}^\sigma T^{\sigma\tau(1)} \Delta_{\tau}^{\nu} \]

\[ = \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} \frac{dp_z}{p_0} \Delta^{\mu}_{\sigma} \Delta^{\nu}_{\tau} p^\sigma p^\tau \chi(x, p) f^{(0)}(p) \]  

(26)

Hence omitting 1, we have the traceless part of pressure tensor

\[ \Pi = -\frac{1}{3} \Pi^\mu_{\mu} \]

\[ = -\frac{1}{3} \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} \frac{dp_z}{p_0} \Delta_{\sigma\mu} \Delta^{\nu}_{\tau} p^\sigma p^\tau f^{(0)}(p) \chi(x, p) \]

(27)

Hence we have the bulk viscosity coefficient

\[ \eta_v = \frac{1}{3T} \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} \frac{dp_z}{p_0} \Delta_{\sigma\mu} \Delta^{\nu}_{\tau} p^\sigma p^\tau \tau Q(1 - f^{(0)}) f^{(0)} \]

(28)

We shall now calculate the the electrical conductivity for electron gas. The electric four current is given by

\[ j^\mu(x) = \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} \frac{dp_z}{p_0} p^\mu f(x, p) \]

(29)

Now because of local charge neutrality, the reversible part \( j^{(0)\mu} = 0 \) and the non-equilibrium part is given by

\[ j^{(1)\mu} = j^\mu = \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} \frac{dp_z}{p_0} p^\mu f^{(0)}(x, p) \]

(30)

Then using the covariant form of Ohm’s law

\[ j^\mu = \sigma^{\mu\nu} E_\nu \]

(31)

the electrical conductivity tensor is given by

\[ \sigma^{\mu\nu} = \frac{e^2}{T} \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} \frac{dp_z}{p_0} p^\mu p^\nu \left( \frac{p^\sigma u^\sigma}{h} + 1 \right) \]

\[ f^{(0)}(1 - f^{(0)}) \]

(32)

This equation shows that due the presence of strong quantizing magnetic field the electrical conductivity becomes a second rank tensor- even if the space is isotropic in nature. The \( zz \)-component is given by

\[ \sigma_{zz} = \frac{e^2}{T} \frac{eB_m}{4\pi} \sum_{\nu} (2 - \delta_{\sigma 0}) \int_{-\infty}^{+\infty} dp_z \left( \frac{p^\sigma u^\sigma}{h} + 1 \right) \]

\[ f^{(0)}(1 - f^{(0)}) \]

(33)

The \( \perp \perp \)-component is given by
\[ \sigma^{\perp \perp} = \frac{e^2 \epsilon B_m}{T} \sum_{\nu} (2 - \delta_{\nu,0}) (2 \nu eB_m) \int_{-\infty}^{+\infty} dp_z \frac{\tau p_z}{p_0^2} \left( 1 + \frac{p^2 u_\omega}{h} \right) f(0)(1 - f(0)) \]  

(34)

and finally the \( \perp z \)-component is given by

\[ \sigma^{\perp z} = \sigma^{z \perp} = \frac{e^2 \epsilon B_m}{T} \sum_{\nu} (2 - \delta_{\nu,0}) (2 \nu eB_m)^{1/2} \int_{-\infty}^{+\infty} dp_z \frac{\tau p_z}{p_0^2} \left( 1 + \frac{p^2 u_\omega}{h} \right) f(0)(1 - f(0)) \]  

(35)

From eqn.(34) and (35) it is quite obvious that just like the shear viscosity coefficient, both \( \sigma^{\perp \perp} \) and \( \sigma^{\perp z} \) components of electrical conductivity vanish for \( \nu_{\text{max}} = 0 \), i.e., in the ultra strong magnetic field limit. This is of course not at all evident for \( \eta_e \), \( \lambda \) and \( \sigma^{zz} \).

### 3. RATE OF ELECTROMAGNETIC PROCESSES

Now to obtain the numerical values of all these transport coefficients, or their variations with the strength of magnetic field we have to know the relaxation time \( \tau \), given by

\[ \frac{1}{\tau} = \sum_i W_i \]  

(36)

where \( W_i \) is the rate of \( i^{th} \) electromagnetic process and the sum is over all possible electromagnetic processes taking place involving the electrons.

Therefore to obtain the relaxation time we evaluate the rates of the basic electromagnetic processes, given by \( e + e \rightarrow e + e \) and \( e + p \rightarrow e + p \). Now in the case of \( e - e \)-Scattering it is necessary to consider both direct and exchange processes, whereas in the case of \( e - p \) scattering only the direct term contributes. Let us first consider the \( e - e \)-scattering process, then the direct part is given by

\[ T^{(d)}_n = \frac{i e^2}{L_y^2 L_z^2 Q^2} (2\pi)^3 \delta(E_1 + E_2 - E_3 - E_4) \delta(k_{1y} + k_{2y} - k_{3y} - k_{4y}) \delta(k_{1z} + k_{2z} - k_{3z} - k_{4z}) \int_{-\infty}^{\infty} dx \langle \bar{u}(k_3, x) \gamma_{\mu} u(k_1, x) \rangle \langle \bar{u}(k_4, x) \gamma^{\mu} u(k_2, x) \rangle \]  

(37)

Similarly the exchange term is given by

\[ T^{(ex)}_n = -\frac{i e^2}{L_y^2 L_z^2 Q^2} (2\pi)^3 \delta(E_1 + E_2 - E_3 - E_4) \delta(k_{1y} + k_{2y} - k_{3y} - k_{4y}) \delta(k_{1z} + k_{2z} - k_{3z} - k_{4z}) \int_{-\infty}^{\infty} dx \langle \bar{u}(k_4, x) \gamma_{\mu} u(k_1, x) \rangle \langle \bar{u}(k_3, x) \gamma^{\mu} u(k_2, x) \rangle \]  

(38)

where \( Q \) is the exchanged momentum. The rate element is then given by

\[ dW = \lim_{t \to \infty} \frac{\left| T^{(d)}_n + T^{(ex)}_n \right|^2}{t} \]  

(39)

Since the spinors are functions of \( x \)-coordinate through \( I_{\nu, p_y}(x) \) (see eqns.(2)-(4)), we use the relation

\[ |f|^2 = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' f(x) f(x') \]  

(40)

and we also need the positive energy projection operator, given by

\[ \Lambda^+ = \sum_{\text{spin}} u(k, x) \bar{u}(k, x') \]  

(41)

Substituting the positive energy spinor solutions (eqns.(2)-(3)), we have

\[ \Lambda^+ = \frac{1}{2E_{\nu}} (Ak_{\mu} \gamma^{\mu} (\mu = 0 \text{ and } z) + mA + Bk_{\mu} \gamma^{\mu} (\mu = y \text{ and } p_y = p_\perp) \]  

(42)
The matrices $A$ and $B$ are given by

$$A = \begin{pmatrix}
I_{\nu} I'_{\nu} & 0 & 0 & 0 \\
0 & I_{\nu-1} I'_{\nu-1} & 0 & 0 \\
0 & 0 & I_{\nu} I'_{\nu} & 0 \\
0 & 0 & 0 & I_{\nu-1} I'_{\nu-1}
\end{pmatrix}$$

(43)

$$B = \begin{pmatrix}
I_{\nu-1} I'_{\nu} & 0 & 0 & 0 \\
0 & I_{\nu} I'_{\nu-1} & 0 & 0 \\
0 & 0 & I_{\nu-1} I'_{\nu} & 0 \\
0 & 0 & 0 & I_{\nu} I'_{\nu-1}
\end{pmatrix}$$

(44)

where primes indicate the functions of $x'$. Since the Dirac $\gamma$ matrices are traceless and both $A$ and $B$ matrices are diagonal in nature with identical blocks we have a few interesting relations, e.g.,

$$\text{Tr}(\gamma^\mu \gamma^\nu A_1 A_2 \ldots B_1 B_2 \ldots) = \text{Tr}(A_1 A_2 \ldots B_1 B_2 \ldots) g^{\mu\nu},$$

(45)

$$\text{Tr}(\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma A_1 A_2 \ldots B_1 B_2 \ldots) = \text{Tr}(A_1 A_2 \ldots B_1 B_2 \ldots) (g^{\mu\nu} g^{\sigma\lambda} - g^{\mu\lambda} g^{\nu\sigma} + g^{\mu\lambda} g^{\nu\sigma})$$

(46)

$$\text{Tr}(\text{product of odd no of } \gamma \text{s with any number of } A \text{ and/or } B \text{ matrices}) = 0$$

(47)

etc. The beautiful form of $A$ and $B$ matrices allow to multiply $\gamma$-matrices in the above relations from any side or in any order. The other interesting aspects of $A$ and $B$ matrices are

i) $k_{1\mu} k_{2\nu} \text{Tr}(A_1 A_2) = (E_1 E_2 - k_{1z} k_{2z}) \text{Tr}(A_1 A_2)$

ii) $k_{1\mu} k_{2\nu} \text{Tr}(B_1 B_2) = k_{1z} k_{2z} \text{Tr}(B_1 B_2)$

iii) $k_{1\mu} k_{2\nu} \text{Tr}(A_1 B_2) = k_{1z} k_{2z} \text{Tr}(A_1 B_2) = 0$

iv) $p_{1\mu} k_{1\nu} p_{2\nu} k_{3\nu} \text{Tr}(A_1 B_2) \neq 0 = (E_{1\nu} E_{2\nu} - p_{1z} k_{1z}) p_{2z} k_{2z} \text{Tr}(A_1 B_2)$

All these results are entirely new and to our knowledge not reported before in the literature. We do believe that these new results can have interesting applications in various studies of properties of strongly magnetized dense stellar matter. With all these new formulae, the transition matrix element for $e e$ scattering direct term is given by

$$W_{ee:cd} = \sum_{v_2} \sum_{v_3} \sum_{v_4} \int dxdx' \frac{e^4}{(2\pi)^3} \frac{1}{4} \delta(E_{v_1} + E_{v_2} - E_{v_3} - E_{v_4}) \delta(p_{1y} + p_{2y} - p_{3y} - p_{4y}) \delta(p_{1z} + p_{2z} - p_{3z} - p_{4z})$$

$$\int dxdx' \frac{1}{(p_1 - p_4)^2} \times \frac{1}{8E_{v_1} E_{v_2} E_{v_3} E_{v_4}} [\delta(k_3 k_4) l_{k_1 k_2}]$$

(48)

$$\{ \text{Tr}(A_3 A_4) \text{Tr}(A_4 A_2) + \text{Tr}(A_3 B_1) \text{Tr}(B_1 A_4) + \text{Tr}(B_3 A_1) \text{Tr}(B_3 A_2) + \text{Tr}(B_3 B_1) \text{Tr}(B_3 B_2) \}$$

$$+ (k_3 k_4) l_{k_1 k_2} \{ \text{Tr}(A_4 A_1) \text{Tr}(A_4 A_2) + \text{Tr}(A_4 B_1) \text{Tr}(A_4 B_2) + \text{Tr}(B_4 A_1) \text{Tr}(B_4 A_2) + \text{Tr}(B_4 B_1) \text{Tr}(B_4 B_2) \}$$

$$- m_c^2 (k_3 k_4) \{ \text{Tr}(A_3 A_1) \text{Tr}(A_4 A_2) + \text{Tr}(B_3 A_1) \text{Tr}(B_4 A_2) \} - m_c^2 (k_4 k_3) \{ \text{Tr}(A_3 A_1) \text{Tr}(B_4 A_2) \}$$

$$+ \text{Tr}(A_3 A_1) \text{Tr}(A_4 A_2) \} + 2 m_c^2 \{ \text{Tr}(A_3 A_1) \text{Tr}(A_4 A_2) \} \text{dp}_{2y} \text{dp}_{2z} \text{dp}_{3y} \text{dp}_{3z} \text{dp}_{4y} \text{dp}_{4z}$$

$$f_0(p_{2z}) (1 - f_0(p_{3z})) (1 - f_0(p_{4z}))$$

(48)

where $\gamma_{v_i} = (2 - \delta_{v_i 0})$.

The exchange term is given by

$$W_{ee:ex} = \sum_{v_2} \sum_{v_3} \sum_{v_4} \int dxdx' \frac{e^4}{(2\pi)^3} \frac{1}{4} \delta(E_{v_1} + E_{v_2} - E_{v_3} - E_{v_4}) \delta(p_{1y} + p_{2y} - p_{3y} - p_{4y}) \delta(p_{1z} + p_{2z} - p_{3z} - p_{4z})$$

$$\frac{1}{(p_1 - p_4)^2} \times \frac{1}{8E_{v_1} E_{v_2} E_{v_3} E_{v_4}} [\delta(k_3 k_4) l_{k_1 k_2}]$$

(48)

$$\{ \text{Tr}(A_1 A_2) \text{Tr}(A_2 A_1) + \text{Tr}(B_1 A_4) \text{Tr}(B_2 B_3) + \text{Tr}(A_1 B_4) \text{Tr}(A_2 B_3) + \text{Tr}(B_1 B_4) \text{Tr}(B_2 B_3) \}$$

$$+ (k_1 k_3) (k_2 k_4) \{ \text{Tr}(A_1 A_4) \text{Tr}(A_2 A_3) + \text{Tr}(B_1 A_4) \text{Tr}(A_2 B_3) + \text{Tr}(A_1 B_4) \text{Tr}(B_2 A_3) + \text{Tr}(B_1 B_4) \text{Tr}(B_2 B_3) \}$$

$$- m_c^2 (k_1 k_3) \{ \text{Tr}(A_1 A_4) \text{Tr}(A_2 A_3) + \text{Tr}(B_1 B_4) \text{Tr}(A_2 A_3) \}$$

$$- m_c^2 (k_2 k_4) \{ \text{Tr}(A_1 A_4) \text{Tr}(B_2 A_3) + \text{Tr}(A_2 A_4) \text{Tr}(B_2 B_3) \}$$

$$+ 2 m_c^2 \text{Tr}(A_1 A_4) \text{Tr}(A_2 A_3) \text{dp}_{2y} \text{dp}_{2z} \text{dp}_{3y} \text{dp}_{3z} \text{dp}_{4y} \text{dp}_{4z}$$

$$f_0(p_{2z}) (1 - f_0(p_{3z})) (1 - f_0(p_{4z}))$$

(49)
and similarly the $e e$-scattering mixed term is given by

$$W_{ee,mix} = \sum_{\nu_2} \sum_{\nu_3} \sum_{\nu_4} \int dx \, \frac{e^4}{(2\pi)^2} \frac{1}{4} \delta(E_{\nu_1} + E_{\nu_2} - E_{\nu_3} - E_{\nu_4}) \delta(p_{1y} + p_{2y} - p_{3y} - p_{4y}) \delta(p_{1z} + p_{2z} - p_{3z} - p_{4z})$$

\[
\gamma_{\nu_2} \gamma_{\nu_3} \gamma_{\nu_4} \frac{1}{(p_1 - p_4)^2 (p_1 - p_3)^2} \times \frac{1}{8E_{\nu_2} E_{\nu_3} E_{\nu_4}} \left[-4(k_3,k_4)k_1 k_2\right] \times \{Tr(A_1 A_2 A_3 A_4) + Tr(A_1 A_2 B_3 B_4) + Tr(B_1 B_2 A_3 A_4) + Tr(B_1 B_2 B_3 B_4)\}
\]

where $\{A_1 A_2 A_3 A_4\}$ etc represent the various 3, 5, etc dimensional integrals.

and finally the rate of $e$-p-scattering is given by

$$W_{ep} = \sum_{\nu_2} \sum_{\nu_3} \sum_{\nu_4} \int dx \, \frac{e^4}{(2\pi)^2} \frac{1}{4} \delta(E_1 + E_2 - E_3 - E_4) \delta(p_{1y} + p_{2y} - p_{3y} - p_{4y}) \delta(p_{1z} + p_{2z} - p_{3z} - p_{4z})$$

\[
\gamma_{\nu_2} \gamma_{\nu_3} \gamma_{\nu_4} \frac{1}{(p_1 - p_4)^2 (p_1 - p_3)^2} \times \frac{1}{8E_{\nu_2} E_{\nu_3} E_{\nu_4}} \left[(k_3,k_4)(k_1,k_2)\right] \times \{Tr(A_1 A_1 A_3 A_4)Tr(A_2 A_2) + Tr(A_3 A_3 B_3 B_4) + Tr(B_3 A_3 A_4)Tr(B_4 A_4) + Tr(B_3 B_3 B_4 B_4)\}
\]

\[
+ \left\{(k_3,k_4)(k_1,k_2)\left\{Tr(A_1 A_1 A_3 A_4)Tr(A_2 A_2) + Tr(A_3 A_3 B_3 B_4) + Tr(B_3 A_3 A_4)Tr(B_4 A_4) + Tr(B_3 B_3 B_4 B_4)\right\}
\]

\[
+ (k_3,k_4)(k_1,k_2)\left\{Tr(B_3 B_3 A_3 A_4)Tr(B_4 B_4) + Tr(B_3 A_3 B_3 B_4) + Tr(B_3 B_3 B_4 B_4)\right\}
\]

\[
- m_0^2 \left\{(k_3,k_4)(k_1,k_2)\left\{Tr(A_1 A_1 A_3 A_4)Tr(A_2 A_2) + Tr(A_3 A_3 B_3 B_4) + Tr(B_3 A_3 A_4)Tr(B_4 A_4) + Tr(B_3 B_3 B_4 B_4)\right\}
\]

\[
dp_{2y} dp_{2z} dp_{3y} dp_{3z} dp_{4y} dp_{4z} f_0(p_{2z})(1 - f_0(p_{3z}))(1 - f_0(p_{4z}))(1 - f_0(p_{4z}))
\]

To evaluate the multidimensional (eight dimensional) integrals, we have used three $\delta$-functions and finally evaluate numerically using multi-dimensional Monte-Carlo integration technique and obtain the results from the eqns.(48)-(51).

We have generated the Hermite polynomials appearing in the matrices $A$ and $B$ numerically. We obtain the relaxation time from eqn.(36) and finally evaluate the kinetic coefficients from eqns.(18),(23),(28),(33)-(35) for various values of magnetic fields $B_m$, temperature $T$ and matter density $n_B$. In figs.(1)-(6) we have shown the variation of kinetic coefficients with magnetic field strength. Identical results can also be obtained for dense proton matter.

4. DISCUSSIONS

From the figs.(1)-(6) it is quite obvious that the kinetic coefficients are almost independent of magnetic field for moderate strengths ($< 10^{17}G$), but all of them go to almost zero for magnetic field strength beyond $10^{17}G$. The first conclusion is therefore, that at ultra strong magnetic field, the matter (electron or proton matter) behaves like a superfluid insulator. The mechanism of superfluidity is of course completely different from conventional neutron matter.

Now as we know from classical plasma physics in presence of strong magnetic field, that the charged particles can only travel along the lines of force, motions are almost forbidden across the field, as a consequence $\sigma_{\perp\perp}$ or $\sigma_{\perp\perp}$ vanish for a classical plasma in presence of ultra-strong magnetic field, whereas $\sigma_{zz}$ remains non-zero even at very high magnetic field strength. On the other hand, in the case of quantum mechanical plasma as the magnetic field become strong enough, the maximum value of Landau quantum number $n_{max} \rightarrow 0$, which further means the system becomes effectively one dimensional in the ultra-strong magnetic field limit. Then in the collision dominated scenario, since $p_z$ varies continuously from $-\infty$ to $+\infty$ and $p_{\perp} \rightarrow 0$, $\sigma_{zz} \rightarrow 0$. To elaborate this point a bit- since the current is flowing along the field lines only, then from the symmetric nature of $p_z$, we have

$$j_z^{(+)} = -j_z^{(-)} \implies j_z = j_z^{(+)} + j_z^{(-)} = 0$$

This is specially true for an effectively one dimensional collision dominated quantum plasma. In the case one dimensional flow (free streaming) of charged particles along the field lines, this is not true; $\sigma_{zz}$ has non-zero finite value. Since all the three component of electrical conductivity vanish in presence of ultra-relativistic magnetic field, this
must affect significantly the Ohmic decay of strong magnetic field. We know that the Ohmic decay time scale is given by

\[ \tau_d = \frac{4\pi\sigma l^2}{c^2} \]  

(53)

where \( l \) is the dimension of the system. Since \( \sigma \to 0 \) for extremely strong magnetic field, as a consequence, the field should decay quickly. We have noticed that a field of strength \( 10^{18} \text{G} \) become \( \sim 10^{15} \text{G} \) within a minute. This moderate field of course will take longer time to decay further (long decay time scale). Therefore the last conclusion is that stronger the magnetic field, shorter the decay time scale and as a result we do believe that the magnetic field strength at the core region of a magnetar can not be high enough and consequently there will be no problem on the mechanical stability or the existence of magnetars.

FIG. 1. Variation of shear viscosity coefficient with magnetic field \( B \). Curve (a): \( T = 15 \text{MeV} \) and \( n_B = 3n_0 \), Curve (b): \( T = 15 \text{MeV} \) and \( n_B = 5n_0 \), and Curve (c): \( T = 30 \text{MeV} \) and \( n_B = 3n_0 \).
FIG. 2. Variation of bulk viscosity coefficient with magnetic field $B$. Curve (a): $T = 15\text{MeV}$ and $n_B = 3n_0$, Curve (b): $T = 15\text{MeV}$ and $n_B = 5n_0$, and Curve (c): $T = 30\text{MeV}$ and $n_B = 3n_0$.

FIG. 3. Variation of heat conductivity coefficient with magnetic field $B$. Curve (a): $T = 15\text{MeV}$ and $n_B = 3n_0$, Curve (b): $T = 15\text{MeV}$ and $n_B = 5n_0$, and Curve (c): $T = 30\text{MeV}$ and $n_B = 3n_0$. 
FIG. 4. Variation of $zz$ component of electrical conductivity with magnetic field $B$. Curve (a): $T = 15\text{MeV}$ and $n_B = 3n_0$, Curve (b): $T = 15\text{MeV}$ and $n_B = 5n_0$, and Curve (c): $T = 30\text{MeV}$ and $n_B = 3n_0$.

FIG. 5. Variation of $pp$ component of electrical conductivity with magnetic field $B$. Curve (a): $T = 15\text{MeV}$ and $n_B = 3n_0$, Curve (b): $T = 15\text{MeV}$ and $n_B = 5n_0$, and Curve (c): $T = 30\text{MeV}$ and $n_B = 3n_0$, 

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FIG. 6. Variation of \( zp \) component of electrical conductivity with magnetic field \( B \). Curve (a): \( T = 15 \text{MeV} \) and \( n_B = 3n_0 \), Curve (b): \( T = 15 \text{MeV} \) and \( n_B = 5n_0 \), and Curve (c): \( T = 30 \text{MeV} \) and \( n_B = 3n_0 \),

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