Mixed Integer Linear Programming Model for Peak Operation of Gas-Fired Generating Units with Disjoint-Prohibited Operating Zones

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Abstract: Due to booming economic development over the past decades, energy demands in most of China’s provincial power grids have increased sharply, and it has become challenging to guarantee the energy balance at peak periods. In many provincial electric systems of China, gas-fired generators are one of the most important peaking power sources to respond the load change at peak periods. To meet this practical necessity, a novel mixed integer linear programming model is proposed in this paper for the peak operation of gas-fired generating units with disjoint-prohibited operating zones. In the developed model, the objective function is chosen to minimize the peak-valley difference of the remaining load series that is obtained by subtracting the total generation of all the gas-fired units from the original load curve. The real-world simulations in several cases show that the developed model is able to generate satisfying scheduling results by reasonably allocating the power outputs of all the gas-fired generators in the scheduling horizon. Then, the management implications obtained lie in the fact that it is necessary to increase the share of peak power sources in the mid- to long-term planning of an electrical power system; and in the daily operation of the power grid, greater flexibility should be given to the gas-fired units to reduce peak pressure.

Keywords: mixed integer linear programming; peak shaving operation; gas-fired generating unit; disjoint prohibited operating zones; provincial electric system

1. Introduction

In the last few decades, electricity consumption and power load in most of China’s power grid has markedly increased due to rapid economic development [1–3]. Figure 1 illustrates the changing process of electricity consumption and maximum load in Zhejiang power grid from 2010 to 2018. It can be seen clearly that within less than 10 years, the electricity consumption grows from 2824 TWh to 3963 TWh while the peak load grows from 42 GW to 73 GW. Meanwhile, renewable energy (like wind farms and photovoltaic power plants) has developed rapidly in recent years, while their randomness and fluctuation further increases the peak pressure of the power system [4–6]. Obviously, it has become a giant challenge to guarantee the dynamic balance of energy production and electricity consumption during peak periods. Figure 2 shows the installed capacity structure of Zhejiang power grid by the...
end of 2015, where the statistical data is taken from the 13th Five-Year Electric Power Development Plan of Zhejiang province. It can be observed that the thermal power plants occupy the dominant position in the energy structure of Zhejiang power grid, while gas energy has the largest share among all the peaking power sources. Under this background, the peak operation of the adjustable gas energy is attracting increasing popularity in China’s provincial grids [7].

The gas-fired generating units emit less pollution in comparison with thermal plants. When the output of the gas-fired unit exceeds half of the total installed capacity, only a small amount of oxynitride and carbon monoxide is produced [8]. In specific terms, the oxynitride and carbon monoxide of the widely-used F-level gas-fired generators are about 50 mg/m³ and 25 mg/m³, far less than 100 mg/m³ and 35 mg/m³ in China’s national standard. Moreover, the gas-fired generating units can modify the operation state in a short time of period as the working conditions change. For the single-cycle gas turbines, it only takes about 15–20 min to turn from the shutdown mode to the full-load operation mode, and about 20–30 min to reduce the output from full-load to zero; for the combined cycle generators, the start-stop time is relatively long but still far less than the thermal plants [9]. Thus, with the merits of high efficiency and low pollution, the gas-fired generators have become one of the most important peaking power sources in the provincial electricity system of China.

In practice, certain loads will lead to operational instabilities of the gas-fired generators. To avoid this problem, the idea of disjoint-prohibited operating zones is used to reduce the adverse effect of the potential instabilities [10]. As shown in Figure 3, the original continuous state space is divided into a sequence of discrete decision sub-spaces due to the disjoint-prohibited operating zones [11]. In other words, the existence of a prohibited operating zone makes the studied problem become a typical combinatorial optimization problem, which has sharply increased the optimization difficulty of the peak operation of the gas-fired generating units. In order to reduce the complexity of the problem effectively, the relatively fixed mode based on historical experience is used to determine the generation process of gas-fired units [12]: the gas-fired units often start before the morning-peak times, and then
increase the output during peak periods based on the necessary security constraints, and gradually stop the service when the load demand is reached at a certain small level. Although the current method can reduce the operators’ workload, unreasonable scheduling results may be produced in some cases because the differences of load demand, power generation and other factors are not given full attention at the same time. Thus, it is necessary to further develop effective methods for the peak operation of gas-fired generating units in China’s provincial electric system.

As mentioned above, the goal of this paper is to determine the optimal unit commitment of all the gas-fired generating units to smooth the residual load series of power system. Mathematically, the problem presented here is classified as a constrained optimization problem subjected to a variety of complicated equality or inequality constraints. Over the past decades, a variety of optimization algorithms has been developed by scholars throughout the world [13–18], like mixed integer linear programming (MILP) [19], non-linear programming [20], dynamic programming [21–23], Lagrangian relaxation [24] and evolutionary algorithms [25–27].

For non-linear programming, it is difficult to guarantee the global optimal solution for the indefinite problem while the costs of execution time and memory usage may be too large [28]. For dynamic programming, the dimensionality problem has limited its widespread application in high-dimensional optimization problems [29]. For Lagrange relaxation, it is often difficult to select the appropriate Lagrange multiplier and conversion strategies between dual and original solutions [30]. For evolutionary algorithms, it is not easy to produce stable solutions in different runs due to the premature convergence problem [31–33]. By contrast with the aforementioned methods, MILP-based approaches can obtain the globally optimal solution within a finite number of iterations when the objective function and physical constraints become the linear functions of decision variables [34–36]. Besides, numerous softwares have been developed to solve large-scale MILP problems and the users can gain access to the information on the proximity to the optimal solution in the search process. Due to the above merits, a great deal of attention has been paid to the popular MILP-based methods in practical problems [34–37]. For instance, the MILP models are developed to determine the generation of head-sensitive reservoir and distribution networks [38–40]; the MILP model is used for the short-term operation of hydro-wind-solar hybrid system [41]; the MILP procedure is developed for the analysis of electric grid security under a disruptive threat [42]; a mixed-integer linear framework is developed for robust hydrothermal unit commitment [43]; the MILP model is developed for day-ahead hydro-thermal self-scheduling considering price uncertainty and forced outage rate [44].

However, in the traditional mathematical models, the goal is often set to minimize the variance of the residual load series, making it become a typical quadratic programming (QP) based problem. Obviously, all the variables in the scheduling horizon are tightly coupled and then the Hessian matrix involved in the objective function may become indefinite in practice [45]. By this time, it will be NP-hard to resolve the peak operation problem of gas-fired generators. Thus, there is a visible gap between the existing research results and the problem presented here, and it is necessary to develop...
more effective optimization models for the peak operation of gas-fired units. Based on the depth analysis, it is found that the widely used inseparable non-linear objective can be replaced by a linear objective minimizing the peak-valley difference of the residual load curve, while all the considered physical constraints can be linearly expressed by the decision variables. On this basis, a novel MILP model is developed for the peak operation of gas-fired generating units with disjoint-prohibited operating zones. The practicability and feasibility of the MILP model is successfully proved in the real-world simulations.

Finally, to better understand our work, the novel contributions are given as below: one is that a practical mixed integer linear programming model is developed for the peak operation of gas-fired generators with disjoint prohibited operating zones, which may be the first research report by far; the other is that the developed model achieves satisfying performance in reducing the peak loads of the power system, demonstrating its practicability and reliability in different cases. Hence, an effective model is presented for the optimal operation of China’s provincial power system where the gas-fired generators play an important role in responding to the load change at peak periods.

The rest of this research is organized as below. Section 2 develops the MILP model for the peak operation of the gas-fired generating units with disjoint-prohibited operating zones. Section 3 tests the performances of the MILP model in different cases. Section 4 gives the conclusions.

2. Mixed Integer Linear Programming Model for the Peak Operation of Gas-Fired Generating Units

2.1. Objective Function

In the traditional modeling, the generation cost is one of the most important issues in the daily generation scheduling of a power system. The widely-used economic goal holds in the case where the installed electricity capacity is large enough to satisfy the load balance of a power system. However, in numerous Chinese provincial power grids, it is often difficult to satisfy the huge load change at peak periods for various reasons like insufficient flexible power sources, equipment maintenance or energy supply deficit. Then, safety accidents will occur when the peak loads are not met. As a result, the peak operation, rather than economic benefit has become the top priority in practice. Generally, the residual load curve is obtained by subtracting the total outputs of all the working generators from the original load series. A smoother residual load curve is beneficial to the steady operation of other energies with poor efficiency (like a thermal system) and the total operation cost of the power system (like startup and shutdown costs) in the scheduling horizon will be reduced.

As mentioned above, the gas-fired generating units are often in charge of responding to the abrupt load change of an electric power system so that other power plants operate in high-efficiency zones. In the existing literature, the goal in Equation (1) is often chosen to minimize the variance of the residual load series, which can be classified as a typical QP-based problem. However, it is often difficult to guarantee theoretically the global optima of the QP problem with an indefinite Hessian matrix in polynomial time. Thus, operators in the dispatching center of China’s provincial power grids are using the experience-based method to determine the scheduling scheme of the gas-fired generating units. This method is relatively simple but may fail to achieve the desired result. Taking the specified generated energy as an example, a larger value may make the small-capacity gas-fired generators operate in the base-load zones, while a smaller value may limit the peak regulation capability of the high-capacity generators. In other words, the existing method cannot give full play to the peak regulation ability of gas-fired generators, and it is of great necessity to further develop a practical optimization model considering all the complicated factors (like load demand, monthly generation plan and schedule attainment).

$$\text{minimize } F = \left\{ \frac{1}{2} \sum_{j=1}^{I} \left[ C_j - \sum_{i=1}^{I} P_{i,j} \right]^2 \right\}$$  

(1)
where \( F \) is the standard deviation of the optimized residual load series. \( P_{ij} \) is the output of the gas-fired generator \( i \) at period \( j \). \( C_{j} \) denotes the values of the original load curve at period \( j \). \( I \) and \( J \) are the number of gas-fired units and periods, respectively.

For Equation (1), the best objective value is equal to zero in the ideal case where the residual load is a horizontal line and there are no differences in the load values of any two periods. Unfortunately, it is usually difficult to find such an ideal solution in practice. Hence, it is necessary to develop an alternative solution that can provide feasible scheduling results. From Figure 4, it can be found that the remaining load curve will gradually become smoother when all the gas-fired generating units increase the power generation at peak periods. In other words, a smaller value of the peak-valley difference has better performance in smoothing the residual load series. Then, this paper aims at developing a practical linear objective function minimizing the valley-to-peak of the residual load series, which can be expressed as below:

\[
\begin{align*}
\text{minimize} & \quad F^1 = \left\{ \max_{1 \leq j \leq J} \{ C_{j} \} - \min_{1 \leq j \leq J} \{ C_{j} \} \right\} \\
C_{j}^1 & = C_{j} - \sum_{i=1}^{I} P_{ij}
\end{align*}
\]  

(2)

(3)

where \( F^1 \) is the peak-valley difference of the optimized residual load series. \( C_{j}^1 \) is the values of the residual load at period \( j \).

![Figure 4. Sketch map of the peak operation of the gas-fired generators.](image)

2.2. Operation Constraints

Generally, the gas-fired generating units should satisfy the equality and inequality constraints in the modeling process.

1. Generation balance equation, which is employed to ensure the smooth implementation of the power agreement between generation enterprises and power grid:

\[
\sum_{j=1}^{J} P_{ij} \Delta_j = E_i, \; i \in [1, I]
\]  

(4)

where \( E_i \) is the generation of the gas-fired generator \( i \). \( \Delta_j \) is the total hours at period \( j \).

2. Output limit constraint, which is used to ensure that the produced output of the generator vary in the safe operation zone:

\[
g_{i,j} \cdot P_{i,j}^{\min} \leq P_{i,j} \leq g_{i,j} \cdot P_{i,j}^{\max}, \; i \in [1, I], \; j \in [1, J]
\]  

(5)
where \( P_{\text{max}}^{i, j} \) and \( P_{\text{min}}^{i, j} \) are the maximum and minimum output of the gas-fired generator \( i \) at period \( j \), respectively. \( g_{i,j} \in \{0, 1\} \) is the binary variable indicating the operating status of the gas-fired generator \( i \) at period \( j \). If \( g_{i,j} = 1 \), the generator is online; otherwise, it is offline.

(3) Total output limit constraint, which is adopted to guarantee that the total outputs of all the generators are smaller than the power load:

\[
\sum_{i=1}^{I} P_{i,j} \leq C_j, \quad j \in [1, J] \tag{6}
\]

(4) Ramping limit constraint, which is used to avoid the rapid change of the produced output:

\[-D_i \leq P_{i,j} - P_{i,j-1} \leq U_i, \quad i \in [1, I], \quad j \in [1, J] \tag{7}\]

where \( U_i \) and \( D_i \) denote the upper and lower ramping rates of the gas-fired generating unit \( i \) at period \( j \), respectively.

(5) Generator maintenance limit, which is adopted to guarantee that the operating status of the gas-fired generator do not conflict with the repair schedule.

\[g_{i,m} = 0, \quad i \in [1, I], \quad m \in [\max\{1, a_i\}, \min\{b_i, J\}] \tag{8}\]

where \( a_i \) and \( b_i \) are the initial and final periods determined in the repair schedule, respectively.

(6) Operation status constraints, which is adopted to ensure the logical relationship of all the binary variables, including start-up status, shut-down status and operation status.

\[
\begin{align*}
y_{i,j} - \bar{y}_{i,j} &= S_{i,j} - \bar{S}_{i,j-1} \\
y_{i,j} + \bar{y}_{i,j} &\leq 1
\end{align*}, \quad i \in [1, I], \quad j \in [1, J] \tag{9}\]

where \( y_{i,j} \in \{0, 1\} \) and \( \bar{y}_{i,j} \in \{0, 1\} \) are the binary variable indicating the start and stop status of the gas-fired generating unit \( i \) at period \( j \). If \( y_{i,j} = 1 \), it is started; if \( \bar{y}_{i,j} = 1 \), it is stopped.

(7) Minimum online limits, which is used to guarantee the safe operation of power system:

\[
y_{i,j} + \sum_{h=j+1}^{j+TA_i-1} \bar{y}_{i,h} \leq 1, \quad i \in [1, I], \quad j \in [1, J] \tag{10}\]

where \( TA_i \) is the minimum online periods of the gas-fired generating unit \( i \).

(8) Minimum offline limits, which is used to avoid the frequent downtime of the generator:

\[
\bar{y}_{i,j} + \sum_{h=j+1}^{j+TB_i-1} y_{i,h} \leq 1, \quad i \in [1, I], \quad j \in [1, J] \tag{11}\]

where \( TB_i \) is the minimum offline periods of the gas-fired generating unit \( i \).

(9) Maximum start-up times limit, which is used to avoid the frequent start-up of the unit:

\[
\sum_{j=1}^{I} y_{i,j} \leq Y_i, \quad i \in [1, I], \quad j \in [1, J] \tag{12}\]

where \( Y_i \) is the maximum start-up times of gas-fired generating unit \( i \).
(10) Maximum stop times limit, which is used to avoid the frequent shut-down of the unit:

\[
\sum_{j=1}^{J} y_{i,j} \leq Y_i, \quad i \in [1, I], \quad j \in [1, J]
\] (13)

where \(Y_i\) is the maximum stop times of the gas-fired generating unit \(i\).

(11) Disjoint prohibited operating zones limit, which is used to avoid a possible accident of the gas-fired generator in unsafe operation zones:

\[
\left( p_{i,j} - p_{i,j}^{\min} \right) \times \left( p_{i,j} - p_{i,j}^{\max} \right) \leq 0, \quad i \in [1, I], \quad j \in [1, J], \quad k \in [1, K_{i,j}]
\] (14)

where \(K_{i,j}\) is the number of feasible operating zones of the gas-fired generator \(i\) at period \(j\). \(p_{i,j}^{\max}\) and \(p_{i,j}^{\min}\) represent the maximum and minimum of the \(k\)th feasible operating zones of the gas-fired generator \(i\) at period \(j\), respectively.

2.3. Optimization Model

Obviously, Equation (2) is a complex non-linear min–max function, and it is difficult to use the existing mathematical solver to solve this problem directly. To reduce the problem complexity, two auxiliary variables \((\overline{C}_j^1, \underline{C}_j^1)\) are introduced, and then the original objective function can be rewritten as:

\[
F = \min \{ \overline{C}_j^1 - \underline{C}_j^1 \} \tag{15}
\]

\[
\overline{C}_j^1 = \max_{1 \leq j \leq J} \left( C_j - \sum_{i=1}^{I} P_{i,j} \right) \tag{16}
\]

\[
\underline{C}_j^1 = \min_{1 \leq j \leq J} \left( C_j - \sum_{i=1}^{I} P_{i,j} \right) \tag{17}
\]

where \(\overline{C}_j^1\) and \(\underline{C}_j^1\) are the possible peak and valley values of the residual load series.

Besides, to ensure the physical significance of two auxiliary variables, any value in the residual load series \(\overline{C}_j^1\) should be larger than the valley load \(\underline{C}_j^1\) but smaller than the peak load \(\overline{C}_j^1\) at the same time, which can be described as below:

\[
\begin{align*}
\overline{C}_j^1 &= C_j - \sum_{i=1}^{I} P_{i,j} \leq \overline{C}_j^1, \quad j \in [1, J] \\
\underline{C}_j^1 &= C_j - \sum_{i=1}^{I} P_{i,j} \geq \underline{C}_j^1
\end{align*}
\] (18)

Due to the disjoint-prohibited operating zones in Equation (14), the original state space is divided into a sequence of sub-spaces, making it become a discrete combinatorial optimization problem. To reduce the problem complexity, the additional binary variable \(z_{i,j,k} \in \{0, 1\}\) is used to capture the disjoint operating zones: if \(z_{i,j,k} = 1\), the gas-fired generating unit \(i\) at period \(j\) falls into the \(k\)th feasible zone; \(0\) otherwise. Besides, for a gas-fired generating unit with disjoint-prohibited operating zones, when it is online \((g_{i,j} = 1)\), only one sub-region is chosen and there will be \(\sum_{k=1}^{K_{i,j}} z_{i,j,k} = 1\); when it is offline \((g_{i,j} = 0)\), all the binary variables \(z_{i,j,k}\) are equal to \(0\) and there will be \(\sum_{k=1}^{K_{i,j}} z_{i,j,k} = 0\). Then, the aforementioned analysis can be mathematically expressed by Equation (19). Supposing that the \(i\)th generator at the \(j\)th period has two feasible operating zones \((K_{i,j} = 2)\), and there are only three cases
that will not appear at the same: offline, or work in the first zone, or work in the second zone. Then, the relationship between two binary variables \((z_{i,j,1} \text{ and } z_{i,j,2})\) and operating status \((g_{i,j})\) is given in Table 1. It can be seen that Equation (19) always holds for the above three cases; otherwise, the illogical relationship violates common sense.

\[
\begin{align*}
K_{i,j} \sum_{k=1}^{z_{i,j,k}} &= g_{i,j} \\
K_{i,j} \sum_{k=1}^{z_{i,j,k}} P_{i,j,k} &\leq P_{i,j} \leq K_{i,j} \sum_{k=1}^{z_{i,j,k}} P_{i,j,k} \\
&\forall i \in [1,I], \; j \in [1,J]
\end{align*}
\]

(19)

Table 1. Explanation for Equation (19) where the generator has two feasible zones.

| No. | \(z_{i,j,1}\) | \(z_{i,j,2}\) | \(z_{i,j,1}+z_{i,j,2}\) | \(P_{i,j}\) | \(g_{i,j}\) | Description |
|-----|----------------|----------------|--------------------------|----------------|----------------|-------------|
| 1   | 0              | 0              | 0                        | \(P_{i,j} = 0\) | 0              | Offline     |
| 2   | 1              | 0              | 1                        | \(P_{i,j} \leq P_{i,j,1} \leq \bar{P}_{i,j,1}\) | 1              | Online, and work in the first feasible zone |
| 3   | 0              | 1              | 1                        | \(P_{i,j} \leq P_{i,j,2} \leq \bar{P}_{i,j,2}\) | 1              | Online, and work in the second feasible zone |
| 4   | 1              | 1              | 2                        | -               | -              | Impossible because the output only appears in a single zone |

By this time, a mixed integer linear programming model is developed for the peak operation of the gas-fired generator with disjoint-prohibited operating zones, where the objective function is in Equation (15), while the physical constraints are in Equations (4)–(13) and (18)–(19). At each period, the decision variables are composed of the output \(P_{i,j}\), operation status \(g_{i,j}\), start-up status \(y_{i,j}\) and shut-down status \(\bar{y}_{i,j}\) and operating zone status \(z_{i,j,k}\).

It should be mentioned that, even though the MILP model has been developed in Reference [46] for the conventional unit commitment problem, there are still obvious differences between two models: (1) Research object. the thermal unit commitment and the gas-fired generating units are the focuses of Reference [46] and this paper, respectively; (2) Objective function. our paper aims at minimizing the peak-valley difference of the residual load curve, while the goal in Reference [46] is set to minimize the total operational cost of all the thermal units; (3) Constraint set. Some physical constraints (like prohibited operating and generator maintenance limits) are not considered in Reference [46], while the load balance equation is not mandatory in our model; (4) Application scenario. Reference [46] works for power systems where the installed capacity is large enough to satisfy the energy demand, while our model is suitable for power systems lack of flexible peaking power resources. To sum up, this paper tries to address a brand-new practical problem in China and can make certain useful contributions to the operation optimization of an electrical power system with similar engineering requirements.

3. Case Studies

3.1. Engineering Background

In this section, the gas-fired generators in Zhejiang province are used to testify to the feasibility of the proposed model. Table 2 lists the basic information of the gas-fired generators that shoulder the responsibility of meeting the huge load change of the Zhejiang provincial electric system. At the present stage, the dispatching center of Zhejiang power grid uses the empirical method to determine the dispatching plan of the gas-fired generating units, and there are certain improvement spaces to enhance the dispatch schedule. In the following sections, several typical load curves are chosen to test the performance of the MILP model.
Table 2. Basic information of the gas-fired generating units in Zhejiang provincial electric system.

| Item (MW)          | Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Unit 9 | Unit 10 | Unit 11 | Unit 12 | Unit 13 |
|--------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Maximum output     | 904    | 435    | 1195   | 1226.2 | 789.2  | 787.6  | 681    | 257.5  | 344    | 225    | 112    | 115    | 56     |
| Minimum output     | 45     | 22     | 60     | 61     | 39     | 39     | 34     | 13     | 17     | 11     | 6      | 6      | 3      |

3.2. Parameter Setting of the Developed Model

Currently, many mature solvers can find the global optimal solution for the standard MILP model. Here, the famous MATLAB is used to resolve this problem presented in this paper. Then, the defaulted computational parameters are used in the model. For instance, the absolute and relative gap tolerance are set as 0 and $1 \times 10^{-4}$; the constraint tolerance and integer are set as $1 \times 10^{-4}$ and $1 \times 10^{-5}$; the cut max iterations and heuristics max nodes are set as 10 and 50. Besides, the experimental results are executed on a personal computer with 3.30 GHz processor and 8 GB RAM.

3.3. Case 1: Validation of the Proposed Model in Summer

Here, the summer load curve is adopted to verify the feasibility of the developed model. Table 3 shows the statistical indicators of the original load and the optimized residual load. From Figure 5, it can be clearly found that compared with the original load curve, the developed model can produce a smooth scheduling result where all the gas-fired generating units quickly increase the electric energy production at peak periods, which is in line with the statistical value listed in Table 3. Besides, the developed model can effectively reduce the peak-valley difference as well as increase the load rate of the power system. For instance, the peak load and the peak-valley difference are reduced by approximately 6.28% and 17.92% while the load rate index is improved by about 4.94%. Based on the famous statistical theory, the MILP model can achieve the goal of reducing the peak loads and the power system has a stronger capability to respond the load change. Table 4 shows the operation status of all the gas-fired generating units. It can be found that the minimum offline and online duration periods are well satisfied, while no illogical “flip-flop” occurs in the scheduling horizon. Thus, it can be concluded that the MILP method can produce feasible solutions when employed to address the peak operation of the gas-fired generators with disjoint-prohibited operating zones.

Table 3. Comparison of the original load and the optimized load obtained by mixed integer linear programming (MILP) in Case 1.

| Item          | Peak Load (MW) | Peak-Valley Difference (MW) | Standard Deviation (MW) | Load Rate |
|---------------|----------------|-----------------------------|-------------------------|-----------|
| Original      | 42,204.00      | 14,800.00                   | 5035.77                 | 0.835     |
| Optimized by MILP | 39,551.64     | 12,147.64                   | 4490.09                 | 0.877     |
| Reduction     | 2652.4         | 2652.4                      | 545.7                   | 0.041     |
| Improve (%)   | 6.28           | 17.92                       | 10.84                   | 4.94      |
In the second case, the typical load in winter is adopted to test the feasibility of the MILP model. There are significant differences in the load features of both winter and summer, like peak load, peak appearance time, and valley-to-peak. Table 5 shows the results of the optimized and original load curves, in which Case 1 means that no prohibited operating zones are considered for all the gas-fired generators, while Cases 2–4 means that there is one, two and three prohibited operating zones for all the generators per period. Figure 6 draws the detailed computational results of the MILP model. From Table 5 and Figure 6, it can be clearly observed that the performances of the MILP model in different cases are satisfying since the statistical indexes of the residual load curves are obviously improved as compared with the original load. On the other hand, there are obvious differences in the scheduling processes of all the gas-fired generators as well as the standard deviations of the residual loads in four cases, while the best performance is achieved in the case without prohibited operating zones. Thus, this case indicates that the MILP model can gather sufficient generation from gas-fired generators to respond to the peak loads of the electric power system, while the optimized result is affected by the prohibited operating zones. Thus, the above two cases with different load curves indicate that the MILP model is an effective tool for the peak operation of the gas-fired generating units with disjoint-prohibited operating zones in a provincial electric system.
while the average load rates in Cases 3 and 4 are increased by about 6.0%. On the whole, the average improvements of the load rates are up to about 5.02%, while the peak load, peak-valley difference and standard deviation are improved by 6.14%, 19.71% and 12.93% on average, respectively. To sum up, as compared with the original load in various cases, the developed model is able to reduce the peak-valley difference and peak values of the residual loads.

Table 5. Comparison of the original load and the optimized load obtained by MILP in Case 2.

| Case | Item | Peak Load (MW) | Peak-Valley Difference (MW) | Standard Deviation (MW) | Load Rate |
|------|------|----------------|----------------------------|------------------------|-----------|
|      | Original | 38,380  | 12,182  | 3929.26  | 0.830  |
| Case 1 | MILP | 35,758.62  | 9560.62  | 3447.31  | 0.878  |
|      | Improve (%) | 6.83  | 21.52  | 12.27  | 5.73  |
| Case 2 | MILP | 37,346.85  | 11,148.85  | 3651.94  | 0.840  |
|      | Improve (%) | 2.69  | 8.48  | 7.06  | 1.23  |
| Case 3 | MILP | 37,346.85  | 11,148.85  | 3655.20  | 0.840  |
|      | Improve (%) | 2.69  | 8.48  | 6.97  | 1.23  |
| Case 4 | MILP | 37,346.85  | 11,148.85  | 3879.94  | 0.840  |
|      | Improve (%) | 2.69  | 8.48  | 1.26  | 1.23  |

3.5. Case 3: Validation of the Proposed Model in the Long Run

In this section, 12 cases based on the real operation data are adopted to further test the feasibility of the developed MILP model. Table 6 shows the detailed results of the MILP model in the long run. Then, the following interesting conclusions can be obtained from Table 6: ① the optimization difficulty of the peak operation is rather large due to the complicated and changeable features of the load curves in different cases. For instance, the maximum load in Case 1 is reduced by about 29.5% as compared with Case 2, while the peak-valley differences in Case 9 is reduced by about 27.1% as compared with Case 12; ② there is positive correlation between the peak load and the peak-valley difference while the value of the standard deviation changes slightly, which means that the whole variation trend of the load curves are basically identical while the maximum loads at peak periods are increased rapidly; ③ the peak-valley differences, standard deviations and load rates of the residual loads are sharply improved by the MILP model. For instance, the peak-valley differences in Cases 7 and 8 are reduced by about 21.1% and 22.4%, the standard deviation in Cases 5 and 6 are reduced by 12.8% and 12.9%, while the average load rates in Cases 3 and 4 are increased by about 6.0%. On the whole, the average improvements of the load rates are up to about 5.02%, while the peak load, peak-valley difference and standard deviation are improved by 6.14%, 19.71% and 12.93% on average, respectively. To sum up, as compared with the original load in various cases, the developed model is able to reduce the peak-valley difference and peak values of the residual loads.
Table 6. Comparison of the original load and the optimized load obtained by MILP in Case 3.

| Case | Item     | Peak Load (MW) | Peak-Valley Difference (MW) | Standard Deviation (MW) | Load Rate |
|------|----------|----------------|-----------------------------|-------------------------|-----------|
|      |          |                |                             |                         |           |
| Case 1 | Original  | 50,043.2       | 12,809.4                    | 4104.0                  | 0.877     |
|       | MILP     | 47,577.6       | 10,343.8                    | 3531.2                  | 0.912     |
|      | Improve (%) | 4.93       | 19.25                       | 13.96                   | 4.00      |
| Case 2 | Original  | 38,642.0       | 12,364.0                    | 4006.1                  | 0.827     |
|       | MILP     | 35,968.9       | 9690.9                      | 3507.8                  | 0.875     |
|      | Improve (%) | 6.92       | 21.62                       | 12.44                   | 5.81      |
| Case 3 | Original  | 38,147.0       | 12,598.0                    | 3976.7                  | 0.826     |
|       | MILP     | 35,488.4       | 9939.4                      | 3461.6                  | 0.874     |
|      | Improve (%) | 6.97       | 21.10                       | 12.95                   | 5.83      |
| Case 4 | Original  | 38,341.0       | 13,152.0                    | 4073.5                  | 0.823     |
|       | MILP     | 35,594.4       | 10,405.4                    | 3528.8                  | 0.872     |
|      | Improve (%) | 7.16       | 20.88                       | 13.37                   | 5.99      |
| Case 5 | Original  | 38,342.0       | 13,008.0                    | 4085.0                  | 0.821     |
|       | MILP     | 35,609.6       | 10,275.6                    | 3561.9                  | 0.870     |
|      | Improve (%) | 7.13       | 21.01                       | 12.81                   | 5.96      |
| Case 6 | Original  | 38,903.0       | 13,513.0                    | 4194.6                  | 0.820     |
|       | MILP     | 36,045.2       | 10,655.2                    | 3652.9                  | 0.870     |
|      | Improve (%) | 7.35       | 21.15                       | 12.91                   | 6.08      |
| Case 7 | Original  | 40,296.0       | 12,804.0                    | 3940.5                  | 0.833     |
|       | MILP     | 37,592.5       | 10,100.5                    | 3439.2                  | 0.880     |
|      | Improve (%) | 6.71       | 21.11                       | 12.72                   | 5.61      |
| Case 8 | Original  | 38,443.0       | 11,326.0                    | 3464.0                  | 0.832     |
|       | MILP     | 35,900.9       | 8783.9                      | 2993.8                  | 0.879     |
|      | Improve (%) | 6.61       | 22.44                       | 13.57                   | 5.62      |
| Case 9 | Original  | 43,087.6       | 16,179.5                    | 5734.7                  | 0.833     |
|       | MILP     | 40,718.9       | 13,810.8                    | 5126.3                  | 0.866     |
|      | Improve (%) | 5.50       | 14.64                       | 10.61                   | 3.98      |
| Case 10 | Original | 51,697.7       | 11,778.0                    | 3734.2                  | 0.909     |
|        | MILP     | 49,745.0       | 9825.3                      | 3296.0                  | 0.935     |
|        | Improve (%) | 3.76       | 16.58                       | 11.73                   | 2.89      |
| Case 11 | Original | 47,332.0       | 12,924.9                    | 4225.3                  | 0.861     |
|        | MILP     | 45,354.5       | 10,947.5                    | 3572.7                  | 0.887     |
|        | Improve (%) | 4.18       | 15.30                       | 15.45                   | 3.03      |
| Case 12 | Original | 41,974.9       | 12,727.6                    | 4065.2                  | 0.831     |
|        | MILP     | 39,254.5       | 10,007.3                    | 3551.4                  | 0.876     |
|        | Improve (%) | 6.48       | 21.37                       | 12.64                   | 5.41      |

Figure 7 shows the detailed scheduling results of the develop model and the original loads. It can be seen that all the gas-fired generating units can increase the produced outputs at peak periods and cut down the power at valley periods to produce a smooth residual load series left for other kinds of energy resources. In such a way, the frequent change of low-efficiency generators is avoided and then the operational cost of the power system will be sharply reduced. Thus, the feasibility of the MILP model in scenically allocating the gas-fired units to respond to the load demand of provincial power system is proved in this section.
As previously mentioned, the peak operation of the gas-fired generating units with disjoint-prohibited operating zones is a changeable task for operators in the dispatching center of a provincial electric system. For the sake of reducing work stress, the operators are using their work experience to determine the generation scheduling plan of the gas-fired generators in the practical engineering. This method is relatively simple but fails to take full advantage of the flexible gas-fired generating units for peak regulation in some scenarios, which will produce unreasonable scheduling results and increase the operational cost. To improve the work efficiency, this paper develops a computationally efficient MILP model for the peak operation of gas-fired generating units with disjoint-prohibited operating zones. The simulations demonstrate the practicability of the developed model.

Generally, the global optimal solution of the developed MILP model can be obtained easily in polynomial time by calling the solvers in mature optimization software. Besides, the MILP model can effectively avoid the possible defects in the traditional methods, like huge computational burden in non-linear programming, premature convergence in evolutionary algorithms, and the dimensionality problem in dynamic programming. With the advantages of easy implementation and a clear principle, the MILP model has been embedded in the decision-making system of Zhejiang power grid. In practice, with a minor adjustment of the basic computational parameters, the operators can quickly obtain feasible scheduling plans for all the gas-fired generators based on the actual situations, which can result in an obvious improvement in the work efficiency.

It is necessary to mention that although the feasibility of the developed model is proved in different cases, there are still some shortcomings deserving careful attention: ① all the elements involved in the studied energy system are assumed to be normal or functional, which is not in line with the reality because the malicious attacks or platform-level failures usually produce faulty elements [47–49]; ② the synergistic effect of multiple energies (like thermal or wind) is ignored to some extent, which may not...
be suitable for an actual power system that is often composed of hybrid power resources [32,50,51]; the developed model may not work in a market environment because the generating costs and operating incomes in various cases (like peak or offpeak periods, wet or dry years) are not given full attention [52–54]. Thus, in the future, research work can be deepened in terms of the following aspects: one is the information diffusion in the presence of noncooperative nodes to guarantee the safety of power system; the other is the cooperative operation of gas-fired generators and other energy sources to reduce the peak regulation pressure of the power system; the third is the market trading rule to balance the revenue of a power enterprise and the safety of a power grid under uncertainty.

4. Conclusions

Due to booming economic development in recent decades, peak operation pressure has become a giant challenge for almost all the power grids in China. In practice, gas-fired units are asked to satisfy the rapid load change at peak periods in many provincial power systems where the installed capacity of adjustable flexible energy is not large enough. Hence, this paper proposes a practical mixed integer linear programming model to optimize the generation scheduling process of the gas-fired generating units with disjoint-prohibited operating zones and peak shaving operation aspects. The gas-fired generating units in the dispatching center of Zhejiang power grid of China are chosen to verify the feasibility of the developed model. The simulations show that with thousands of constraints and decision variables, the proposed model is able to generate satisfying results within reasonable execution time. For instance, the developed model can make about 6.14%, 19.71%, 12.93% and 5.02% improvements on the peak, peak-valley difference and standard deviation of the original loads in the long run, which will effectively alleviate the peak regulation pressure and guarantee the safe operation of the power grid. Thus, the presented model is an alternative method to reduce the peak operation pressure of China’s provincial electric system.

Besides, based on the empirical results, the following management implications can be obtained: a well-designed optimization model is helpful to reduce the solution difficulty and produce a better scheduling result for peak operation; the dynamic adjustment of the essential factors (like generated energy and operation limits) can increase the adaptive ability of gas-fired generators to the actual working conditions; and the share of peak power sources should be increased to improve the energy structure of China in the long run.

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References

1. Shen, J.; Cheng, C.; Cheng, X.; Lund, J.R. Coordinated operations of large-scale UHVDC hydropower and conventional hydro energies about regional power grid. *Energy* 2016, 95, 433–446. [CrossRef]

2. Feng, Z.K.; Niu, W.J.; Cheng, C.T.; Zhou, J.Z. Peak shaving operation of hydro-thermal-nuclear plants serving multiple power grids by linear programming. *Energy* 2017, 135, 210–219. [CrossRef]

3. Yu, L.; Li, Y.P.; Huang, G.H. A fuzzy-stochastic simulation-optimization model for planning electric power systems with considering peak-electricity demand: A case study of Qingdao, China. *Energy* 2016, 98, 190–203. [CrossRef]

4. Catalão, J.P.S.; Pousinho, H.M.I.; Mendes, V.M.F. Short-term wind power forecasting in Portugal by neural networks and wavelet transform. *Renew. Energy* 2011, 36, 1245–1251. [CrossRef]
5. Chen, D.; Liu, S.; Ma, X. Modeling, nonlinear dynamical analysis of a novel power system with random wind power and it's control. *Energy* 2013, 53, 139–146. [CrossRef]
6. Chen, F.; Zhou, J.; Wang, C.; Li, C.; Lu, P. A modified gravitational search algorithm based on a non-dominated sorting genetic approach for hydro-thermal-wind economic emission dispatching. *Energy* 2017, 121, 276–291. [CrossRef]
7. Feng, Z.K.; Niu, W.J.; Cheng, C.T. China’s large-scale hydropower system: Operation characteristics, modeling challenge and dimensionality reduction possibilities. *Renew. Energy* 2019, 136, 805–818. [CrossRef]
8. Lin, L.; Tian, X.; Cai, X. Gas unit deep peak regulation and power system energy efficiency in consideration of conditional cost. *Autom. Electr. Power Syst.* 2018, 42, 16–23.
9. Wang, J.; Lü, Q.; Li, W.; Zhao, W. Peak load regulation transaction mode for gas turbine in electricity market. *Electr. Power Autom. Equip.* 2014, 34, 48–54.
10. Ding, T.; Bo, R.; Li, F.; Sun, H. A Bi-Level branch and bound method for economic dispatch with disjoint prohibited zones considering network losses. *IEEE Trans. Power Syst.* 2015, 30, 2841–2855. [CrossRef]
11. Ding, T.; Bo, R.; Gu, W.; Sun, H. Big-M based MIQP method for economic dispatch with disjoint prohibited zones. *IEEE Trans. Power Syst.* 2014, 29, 976–977. [CrossRef]
12. Zhou, L.; Shen, J.; Li, J.; Zhang, J.; Cheng, C.; Wu, H. Peak regulation method for daily start-stop gas-fired units. *Proc. Chin. Soc. Electr. Eng.* 2017, 37, 5913–5921.
13. Liu, P.; Cai, X.; Guo, S. Deriving multiple near-optimal solutions to deterministic reservoir operation problems. *Water Resour. Res.* 2011, 47. [CrossRef]
14. Ma, C.; Wang, H.; Lian, J. Short-term electricity dispatch optimization of Ertan hydropower plant based on data by field tests. *J. Renew. Sustain. Energy* 2011, 3, 063109. [CrossRef]
15. Madani, K. Game theory and water resources. *J. Hydrol.* 2010, 381, 225–238. [CrossRef]
16. Yin, X.A.; Yang, Z.F.; Petts, G.E. Reservoir operating rules to sustain environmental flows in regulated rivers. *Water Resour. Res.* 2011, 47. [CrossRef]
17. Zhao, T.; Zhao, J. Improved multiple-objective dynamic programming model for reservoir operation optimization. *J. Hydroinf.* 2014, 16, 1142–1157. [CrossRef]
18. Niu, W.J.; Feng, Z.K.; Feng, B.F.; Min, Y.W.; Cheng, C.T.; Zhou, J.Z. Comparison of multiple linear regression, artificial neural network, extreme learning machine, and support vector machine in deriving operation rule of hydropower reservoir. *Water* 2019, 11, 88. [CrossRef]
19. Feng, Z.K.; Niu, W.J.; Wang, S.; Cheng, C.T.; Jiang, Z.Q.; Qin, H.; Yi, L. Developing a successive linear programming model for head-sensitive hydropower system operation considering power shortage aspect. *Energy* 2018, 155, 252–261. [CrossRef]
20. Cai, X.; McKinney, D.C.; Lasdon, L.S. Solving nonlinear water management models using a combined genetic algorithm and linear programming approach. *Adv. Water Resour.* 2001, 24, 667–676. [CrossRef]
21. Zhang, Y.; Jiang, Z.; Ji, C.; Sun, P. Contrastive analysis of three parallel modes in multi-dimensional dynamic programming and its application in cascade reservoirs operation. *J. Hydrol.* 2015, 529, 22–34. [CrossRef]
22. Feng, Z.K.; Niu, W.J.; Cheng, C.T. Optimizing electrical power production of hydropower system by uniform progressive optimality algorithm based on two-stage search mechanism and uniform design. *J. Clean. Prod.* 2018, 190, 432–442. [CrossRef]
23. Feng, Z.K.; Niu, W.J.; Cheng, C.T.; Liao, S.L. Hydropower system operation optimization by discrete differential dynamic programming based on orthogonal experiment design. *Energy* 2017, 126, 720–732. [CrossRef]
24. Liu, B.; Cheng, C.; Wang, S.; Liao, S.; Chau, K.W.; Wu, X.; Li, W. Parallel chance-constrained dynamic programming for cascade hydropower system operation. *Energy* 2018, 165, 752–767. [CrossRef]
25. Feng, Z.K.; Niu, W.J.; Cheng, C.T. Optimization of hydropower reservoirs operation balancing generation benefit and ecological requirement with parallel multi-objective genetic algorithm. *Energy* 2018, 153, 706–718. [CrossRef]
26. Yang, T.; Gao, X.; Sellars, S.L.; Sorooshian, S. Improving the multi-objective evolutionary optimization algorithm for hydropower reservoir operations in the California Oroville-Thermalito complex. *Environ. Modell. Softw.* 2015, 69, 262–279. [CrossRef]
27. Ming, B.; Chang, J.X.; Huang, Q.; Wang, Y.M.; Huang, S.Z. Optimal operation of Multi-Reservoir system Based-On cuckoo search algorithm. *Water Resour. Manag.* 2015, 29, 5671–5687. [CrossRef]
28. Catalão, J.P.S.; Pousinho, H.M.I.; Mendes, V.M.F. Hydro energy systems management in Portugal: Profit-based evaluation of a mixed-integer nonlinear approach. *Energy* 2011, 36, 500–507. [CrossRef]

29. Feng, Z.K.; Niu, W.J.; Cheng, C.T.; Wu, X.Y. Optimization of hydropower system operation by uniform dynamic programming for dimensionality reduction. *Energy* 2017, 134, 718–730. [CrossRef]

30. Jia, B.; Zhong, P.; Wan, X.; Xu, B.; Chen, J. Decomposition-coordination model of reservoir group and flood storage basin for real-time flood control operation. *Hydrol. Res.* 2015, 46, 11–25. [CrossRef]

31. Zheng, F.; Zecchin, A.C.; Simpson, A.R. Investigating the run-time searching behavior of the different evolution algorithm applied to water distribution system optimization. *Environ. Modell. Softw.* 2015, 69, 292–307. [CrossRef]

32. Niu, W.J.; Feng, Z.K.; Cheng, C.T.; Wu, X.Y. A parallel multi-objective particle swarm optimization for cascade hydropower reservoir operation in southwest China. *Appl. Soft Comput.* 2018, 70, 562–575. [CrossRef]

33. Niu, W.; Feng, Z.; Cheng, C.; Zhou, J. Forecasting daily runoff by extreme learning machine based on quantum-behaved particle swarm optimization. *J. Hydrol. Eng.* 2018, 23, 04018002. [CrossRef]

34. You, L.; Li, Y.P.; Huang, G.H.; Zhang, J.L. Modeling regional ecosystem development under uncertainty—A case study for New Binhai District of Tianjin. *Ecol. Modell.* 2014, 288, 127–142. [CrossRef]

35. Liu, Y.; Huang, G.; Cai, Y.; Dong, C. An inexact mix-integer two-stage linear programming model for supporting the management of a low-carbon energy system in China. *Energies* 2011, 4, 1657–1686. [CrossRef]

36. Li, Y.P.; Huang, G.H.; Chen, X. An interval-valued minimax-regret analysis approach for the identification of optimal greenhouse-gas abatement strategies under uncertainty. *Energy Policy* 2011, 39, 4313–4324. [CrossRef]

37. De La Torre, S.; Arroyo, J.M.; Conejo, A.J.; Contreras, J. Price maker self-scheduling in a pool-based electricity market: A mixed-integer LP approach. *IEEE Trans. Power Syst.* 2002, 17, 1037–1042. [CrossRef]

38. Cheng, C.; Wang, J.; Wu, X. Hydro unit commitment with a Head-Sensitive reservoir and multiple vibration zones using MILP. *IEEE Trans. Power Syst.* 2016, 31, 4842–4852. [CrossRef]

39. Li, X.; Li, T.; Wei, J.; Wang, G.; Yeh, W.W.G. Hydro unit commitment via mixed integer linear programming: A case study of the three gorges project, China. *IEEE Trans. Power Syst.* 2014, 29, 1232–1241. [CrossRef]

40. Munoz-Delgado, G.; Contreras, J.; Arroyo, J.M. Joint expansion planning of distributed generation and distribution networks. *IEEE Trans. Power Syst.* 2015, 30, 2579–2590. [CrossRef]

41. Wei, H.; Hongxuan, Z.; Yu, D.; Yiting, W.; Ling, D.; Ming, X. Short-term optimal operation of hydro-wind-solar hybrid system with improved generative adversarial networks. *Appl. Energy.* 2019, 250, 389–403. [CrossRef]

42. Motto, A.L.; Arroyo, J.M.; Galiana, F.D. A mixed-integer LP procedure for the analysis of electric grid security under disruptive threat. *IEEE Trans. Power Syst.* 2005, 20, 1357–1365. [CrossRef]

43. Razavi, S.E.; Esmaeel Nezhad, A.; Mavalizadeh, H.; Raeisi, F.; Ahmadi, A. Robust hydrothermal unit commitment: A mixed-integer linear framework. *Energy* 2018, 165, 593–602. [CrossRef]

44. Esmaeily, A.; Ahmadi, A.; Raeisi, F.; Ahmadi, M.R.; Esmaeel Nezhad, A.; Janghorbani, M. Evaluating the effectiveness of mixed-integer linear programming for day-ahead hydro-thermal self-scheduling considering price uncertainty and forced outage rate. *Energy* 2017, 122, 182–193. [CrossRef]

45. Feng, Z.; Niu, W.; Wang, W.; Zhou, J.; Cheng, C. A mixed integer linear programming model for unit commitment of thermal plants with peak shaving operation aspect in regional power grid lack of flexible hydropower energy. *Energy* 2019, 175, 618–629. [CrossRef]

46. Carrión, M.; Arroyo, J.M. A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. *IEEE Trans. Power Syst.* 2006, 21, 1371–1378. [CrossRef]

47. Shang, Y. Resilient consensus of switched multi-agent systems. *Syst. Control Lett.* 2018, 122, 12–18. [CrossRef]

48. Shang, Y. Resilient multiscale coordination control against adversarial nodes. *Energies* 2018, 11, 1844. [CrossRef]

49. Shang, Y. Finite-Time weighted average consensus and generalized consensus over a subset. *IEEE Access.* 2016, 4, 2615–2620. [CrossRef]

50. Feng, Z.K.; Niu, W.J.; Cheng, C.T. Multi-objective quantum-behaved particle swarm optimization for economic environmental hydrothermal energy system scheduling. *Energy* 2017, 131, 165–178. [CrossRef]

51. Domínguez, R.; Conejo, A.J.; Carrión, M. Toward fully renewable electric energy systems. *IEEE Trans. Power Syst.* 2015, 30, 316–326. [CrossRef]

52. Carrión, M.; Arroyo, J.M.; Conejo, A.J. A bilevel stochastic programming approach for retailer futures market trading. *IEEE Trans. Power Syst.* 2009, 24, 1446–1456. [CrossRef]
53. Arroyo, J.M.; Alguacil, N.; Carrión, M. A risk-based approach for transmission network expansion planning under deliberate outages. *IEEE Trans. Power Syst.* **2010**, *25*, 1759–1766. [CrossRef]

54. Conejo, A.J.; Nogales, F.J.; Carrión, M.; Morales, J.M. Electricity pool prices: Long-term uncertainty characterization for futures-market trading and risk management. *J. Oper. Res. Soc.* **2010**, *61*, 235–245. [CrossRef]

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