Single bubble rising dynamics in porous media using lattice Boltzmann method

Xi Jia, Juan Shi *, Zhenqian Chen

School of Energy and Environment, Southeast University, Nanjing, 210096, China
shi_juan@seu.edu.cn

Abstract. The modified pseudopotential Lattice Boltzmann model has been used to study the motion and morphology of a single rising bubble in porous media. Effects of the wettability at solid-fluid interfaces and non-dimensional number Eo on bubble rising dynamics are illustrated in this study. The wettability of the solid surface is modeled by employing an interaction force between the solid wall and the fluid.

1. Introduction
Multiphase flow is frequently observed in many fields of research such as nucleate boiling. Bubble rising dynamics is one of the most fundamental phenomena in gas-liquid two-phase flow, which has been studied through experimentation and numerical simulation for many years. The behavior of bubbles is characterized by non-dimensional numbers such as Reynolds number (Re), Eotvos number (Eo) and Morton number (Mo) [1-2]. Porous media exhibit excellent performance in enhancing boiling heat transfer. Research on the mechanisms of bubble motion in pore structure is helpful to improve boiling heat transfer or gas-liquid separation in porous media. Meanwhile wetting and spreading phenomena of fluids on solid phase have attracted much attention [3].

The continuous deformation of the liquid-vapor interface, the mass transfer at the interface, the complex structure of porous media and so on make it challenging to simulate bubble motion in porous media [4]. The lattice Boltzmann method (LBM) shows superiority to other methods in simulating multiphase flow in complex media due to its mesoscopic characteristics. LBM can easily describe the interaction between different phases with easy-to-implement boundary conditions. Several popular multiphase models in LBM have been proposed during the past twenty years including the Rothman-Keller (RK) model [5-6], the pseudopotential model (SC model) [3,7-9] and the free energy model [10]. The pseudopotential model is widely employed owing to its simplicity and versatility [11]. Huang [3] analyzed the relation between the adhesion parameters and the fluid-solid contact angle in the SC model. Huang [7] applied the SC model to discuss the immiscible two-phase flow in porous media. Liu [8] proposed an improved model by modifying the pseudopotential in the SC model. Gong [9] proposed a new method of calculating the interparticle interaction force on the basis of the modified SC model.

In this paper, single bubble rising dynamics in porous media is studied through the modified pseudopotential LBM model for multiphase flow. Effects of the wettability at solid-fluid interfaces and non-dimensional number Eo on bubble motion and morphology are investigated, which is helpful in enhancing the two-phase flow and heat transfer in porous media.

2. Methodology
2.1. The lattice Boltzmann method

A LBM model contains three parts which are discrete velocity model, equilibrium distribution function and evolution equation. The evolution equation is given by

$$f_i^k(x, t + \Delta t) - f_i^k(x, t) = -\frac{1}{\tau_k} \left( f_i^k(x, t) - f_i^{k(eq)}(x, t) \right) + \Delta f_i^k(x, t)$$ (1)

where $f_i^k(x, t)$ represents the particle distribution function for the $k$th component with velocity $e_i$ at position $x$ and time $t$. The parameter $k$ indicates the phase of fluid that is equal to 1 or 2 for the liquid phase or the gas phase respectively. $\tau_k$ is the relaxation time which is related to the kinematic viscosity as $\tau_k = \frac{3\nu_k}{(c_s^2 \delta t) + 0.5}$. The equilibrium distribution function $f_i^{k(eq)}(x, t)$ and the body force term $\Delta f_i^k(x, t)$ can be calculated as

$$f_i^{k(eq)} = w_i \rho_k \left[ 1 + \frac{e_i \cdot u_k}{(c_s^k)^2} + \frac{(e_i \cdot u_k)^2}{2(c_s^k)^4} - \frac{u_k^2}{2(c_s^i)^2} \right]$$ (2)

$$\Delta f_i^k(x, t) = f_i^{k(eq)}(x, t) \rho_k(x, t), u_k + \Delta u_k - f_i^{k(eq)}(x, t), u_k$$ (3)

D2Q9 model is chosen for 2D simulation in this study. For the D2Q9 scheme, the values of weight coefficients $w_i$ are $4/9, 1/9, 1/36$ respectively for $i = 0, 1 \sim 4$ and $i = 5 \sim 8$. The discrete velocity vectors are governed by

$$e_i = \left\{ \begin{array}{ll}
(0,0) & i = 0 \\
\frac{\sqrt{2}c_c}{c} \left[ \cos \left( (i-1)\frac{\pi}{4} \right), \sin \left( (i-1)\frac{\pi}{4} \right) \right] & i = 1, 2, 3, 4 \\
c \left[ \cos \left( (2i-1)\frac{\pi}{2} \right), \sin \left( (2i-1)\frac{\pi}{2} \right) \right] & i = 5, 6, 7, 8
\end{array} \right.$$ (4)

where $c = \delta x / \delta t$ is the lattice speed, with $\delta x$ being the lattice spacing and $\delta t$ being the time spacing. $\delta x$ and $\delta t$ are always adopted as 1.0 in LBM. $c_c$ is the lattice acoustic velocity and $c = c_c/\sqrt{3}$. In Eq. (3), $\Delta u_k = F_k \delta t / \rho_k$ is the variation in velocity caused by the body force $F_k$ which can be expressed as

$$F_k = F_{int,k}(x) + F_{s,k}(x) + F_{g,k}(x)$$ (5)

where $F_{int,k}$ is the interparticle interaction force, $F_g$ is the gravitational force and $F_s$ is the fluid-solid interaction force. The density and velocity of the fluid are defined as

$$\rho_k = \sum_i f_i^k, \quad \rho_k u_k = \sum_i e_i f_i^k$$ (6)

2.2. Fluid-fluid and fluid-solid interaction forces

The interparticle interaction force and the gravitational force are computed respectively as

$$F_{int,k}(x) = -\psi_k(x) \sum \sum w_i \psi_k(x + e_i \Delta t) e_i$$ (7)

$$F_{g,k}(x) = \mathbf{g} \cdot (\rho_k(x) - \rho_l)$$ (8)
where $\psi_k(x)$ is related to the density of the $k$th component. $g_{k,k}$ determines the interaction strength between the $k$th and $k'$th components and $g$ is gravitational acceleration.

The wettability of the solid surface can be modeled by employing an interaction force between the solid wall and the fluid that is defined as

$$ F_{s,k}(x) = -g_{\text{ads},k} \psi_k(x) \sum_i w_i s(x + e_i \Delta t) e_i $$

where $s(x + e_i \Delta t)$ is an indicator function that is equal to 1 for solid or 0 for fluid domain. The interaction strength between the fluid and the solid wall is determined by $g_{\text{ads},k}$. In this simulation, $g_{\text{ads},1} = -g_{\text{ads},2}$.

2.3. The physical model

The schematic diagram of the physical model of single bubble motion in porous media is shown in Fig 1. The domain of interest is a two-dimensional rectangular area of height $H$ and width $L$. The initial position of bubble with diameter $D$ is $(L/2, h)$. The porous media is simplified as regular structure.

![Figure 1. Schematic diagram of the physical model.](image)

2.4. Non-dimensional numbers

The transformation between the lattice unit and the actual physical unit can be achieved by introducing the non-dimensional numbers as follows

$$ X = \frac{x}{L}, \quad Y = \frac{y}{H}, \quad \theta = \frac{t \sqrt{H/g}}{E_0}, \quad E_0 = \frac{(\rho_l - \rho_g) g D^2}{\sigma} $$

where $\theta$ is the dimensionless time. $E_0$ is Eotvos number. $\sigma$ is the surface tension coefficient which is taken as 0.4. Subscript $l$ and $g$ stands for liquid and gas respectively.

3. Results and discussion

To record the bubble rising process, the top point of bubble is chosen as the its location in flow field for simplicity. The structure parameters are taken as follows: $L = 140$, $H = 500$, $D = 40$. In this section, the single bubble rising morphology and its dimensionless location with dimensionless time in different cases will be analyzed.

3.1. Effects of wettability
In this part, $Eo$ is set as 0.04. As shown in Fig 2, with the increase of $g_{ads,2}$, the bubble rising process from the bottom to the top boundary consumes less time. When $g_{ads,2} = -3$, the adhesion force between gas phase and solid walls is so big that the bubble cannot escape and adheres to the surface of the porous media, as shown in Fig 3. As a consequence, $Y$ value keeps stable when it reaches a particular value.

**Figure 2.** The effect of wettability on bubble motion.  
**Figure 3.** Viscosity phenomenon of bubble when $g_{ads,2} = -3$.

Comparing Fig 4a and Fig 4b, it can be seen that the interface between the bubble and the solid walls is clearer when $g_{ads,2} = 1$. For example, when $\theta = 0.80$, the bubble deformation in Fig 4a is obviously stronger. On the contrary, the skeletons of porous media are inside the lower surface of bubble in Fig 4b and the bubble deforms slightly.
It can be concluded that the increase of $g_{ads,2}$, which represents the solid surface becomes from hydrophobic to hydrophilic, contributes to the increase of bubble rising velocity and stronger bubble deformation. In other words, the solid wall becomes more and more aerophobic when $g_{ads,2}$ increases so that the interface between the gas phase and the solid walls becomes increasingly clear.

3.2. Effects of Eotvos number

In this part, $g_{ads,2}$ is set as 1. As shown in Fig 5, with the increase of $Eo$ number, that means with the increase of gravity or buoyancy, the dimensionless rising time for bubble to reach the top boundary from the bottom decreases.

When $Eo = 0.006$, bubble rises slowly under the combined influence of gravity, buoyancy, drag force and viscous force. As the ratio of buoyancy is small, the bubble cannot escape from the skeleton of porous media. Therefore, $Y$ value keeps stable when it reaches a certain value. It can be concluded
that the increase of \( E_o \) number, which represents higher ratio of buoyancy, contributes to the increase of bubble rising velocity.

4. Conclusions
In this study, dynamics of a single rising bubble in porous media is investigated using the Lattice Boltzmann method based on the pseudopotential model. Conclusions are summarized as follows in terms of two aspects, which are the effects of the wettability of solid surface and Eotvos number on bubble motion and morphology.

- The increase of \( g_{ads,2} \), which means the solid surface becomes from hydrophobic to hydrophilic, contributes to the increase of bubble rising velocity and clearer interface between the bubble and the solid walls.
- The increase of \( E_o \) number, that represents higher ratio of buoyancy, contributes to the increase of bubble rising velocity.

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