Some questions of the contact interaction theory in two-roll modules

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Abstract. The study is devoted to the analysis of contact stresses distribution in a two-roll module. The results of the study of the theory of contact interaction in two-roll modules are presented: an analytical description of the curve shape of the roll contact and determination of the relationship between tangential and normal stresses. Mathematical models of the roll contact curves and the friction stress models in a generalized two-roll module are determined. The obtained models are general in the sense that they are applicable for partial cases of interaction in two-roll modules. Formulas are found for calculating the neutral angles of two-roll modules. Dependencies between the forces acting in the rolls and the stresses distributed under the influence of these forces are stated. It is revealed that these dependences do not change with a change in the angle of supply of the material layer to the line of centers and the angle of inclination of the upper roll relative to the vertical.

1. Introduction

Roller machines are widely used in many branches of industry. The main operating element of roll machines is a roll pair. The roll pair and the processed material together compose a two-roll module.

The main task of the theory of contact interaction of two-roll modules is the modeling of contact (normal and tangential) forces, that is, obtaining analytical dependencies that describe the patterns of distribution of these forces. When modeling the contact forces, the main factors are the models of friction stresses that take into account the effect of friction in the contact zone of the rolls and link tangential and normal stresses, as well as the models of the rolls contact curves that describe the shapes of these curves.

In two-roll modules, the forces applied to the roll axis are transmitted to the material being processed along the contact curves of the rolls. In two-roll modules with rolls having elastic coatings, mutual strain of the contacting bodies occurs. In this case, the contact curves have complex configurations and depend mainly on the geometrical and strain characteristics of the roll surface layers and the material being processed. The exact shapes of the roll contact curves in this case have not yet been clarified. In this regard, the problem of contact interaction (for example, analysis of contact stresses distribution) is solved by preliminary selecting the formula for the roll contact curve.

Usually an arc of a circle, an ellipse and a parabola is applied, described by formula \( r = \frac{a}{1 + e \cos \theta} \) [1], or simple mechanical models defined by equations \( r = R \) and \( r = \frac{R \cos \phi}{\cos \theta} \) [2].

There are many mathematical models of friction stresses obtained by theoretical, experimental or experimental-theoretical methods. In the study of contact interaction in two-roll modules, the dry friction model (Amonton-Coulomb law) [3,4,5] is mainly used. From the point of view of the theory
of interaction, this is equivalent to considering the slip zones only on the contact surface. In fact, as experiments have shown, the slip zones are initiated at the entrance and exit of the contact area, and in the central part the surface of the contacting bodies fully adhere. This part of the contact area is called the adhesion zone. However, a theoretical model of the friction stress for the adhesion zone during rolling does not exist. In [6,7,8], empirical dependencies \( t_s = \varphi(x) \) are usually used.

To carry out calculations with these dependences, data are required that can only be obtained during complex experimental studies. For example, in dependence \( t_s = t_0 \left( \frac{h_r - h_0}{l} \right) \) [7], experimental data are needed on the extent of the adhesion zone \( l \), on the friction stress at the beginning of the adhesion zone \( t_0 \), and the strip thickness in the neutral section \( h_0 \). Therefore, the models of friction stresses currently used in the theory of contact interaction of two-roll modules are considered approximate. For this reason, the theoretical curves of contact stresses distribution obtained are considered approximate, and they do not correspond to the experimental distribution diagrams [8].

It follows from the foregoing that obtaining theoretical curves of contact stress distribution corresponding to experimental diagrams is currently impossible due to the lack of correct models of roll contact curves and friction stress models.

Two-roll modules belong to the main operating elements of the roll machine or perform auxiliary functions. In this regard, many of them are asymmetric. Moreover, quite often several types of asymmetry are realized simultaneously, for example, two types of geometrical asymmetry - different diameters and the tilt of the material layer relative to the horizontal.

In this regard, to systematize the studies of contact interaction, first of all, a generalized scheme for the interaction of a roll pair with the processed material was selected, that is, a generalized model of two-roll modules was developed [10] based on the analysis of functional structures and classifications of roll modules [9], it served as the object of study of contact phenomena.

In this two-roll module, the rolls are positioned relative to the vertical by tilting to the right at an angle \( \beta \), have unequal diameters \( (R_1 \neq R_2) \) and elastic coatings from the materials of different stiffness and friction coefficients \( (f_1 \neq f_2) \), the lower shaft is a driving one and the upper one is free. The material layer has uniform thickness \( \delta \), and is fed with a tilt downward relative to the line of centers at an angle \( \gamma \) (figure 1).

The aim of the study is a mathematical modeling of the roll contact curves and friction stresses, i.e. an analytical description of the curve shape of the roll contact and determination of the relationship between shear and normal stresses in the generalized two-roll module under consideration.

2. Analytical solutions to the tasks

2.1. Modeling the curve shape of the roll contact

We study the first problem, i.e. modeling the contact curves of the rolls.

First, consider the interaction of the material layer with the lower roll. The shape of the contact curve is analyzed in polar coordinates. This curve (curve \( A_1A_2 \)) consists of two sections \( A_1K \) and \( KA_2 \).
(figure 1). On the $A_1 K$ section, the interacting bodies are compressed, and on $K A_2$ the recovery occurs, where $K$ is the point of the contact curve of the lower roll on the line of centers of the rolls. Any point $B_1$ of section $A_1 K$ is determined by polar coordinates $r_1$ and $\theta_{11}$, and point $B_2$ of section $K A_2$ by $r_2$ and $\theta_{12}$. According to figure 1 $-\phi_{11} \leq \theta_{11} \leq 0$, $0 \leq \theta_{12} \leq \phi_{12}$, where $\phi_{11}, \phi_{12}$ are the contact angles of the lower roll.

In the process of interaction with the rolls, the angle of inclination of the processed material changes: in section $A_1 K$ it decreases from $\gamma_1$ to zero, and on $K A_2$ it increases from zero to $\gamma_2 = m \gamma_1$, where $m$ is the proportionality coefficient.

Assume that

$$\gamma = \gamma(\theta_{11}) = a_1 \theta_{11} + b_1, \quad -\phi_{11} \leq \theta_{11} \leq 0, \quad \gamma = \gamma(\theta_{12}) = a_2 \theta_{12} + b_2, \quad 0 \leq \theta_{12} \leq \phi_{12}.$$  

Coefficients $a_1$, $b_1$, $a_2$, and $b_2$ are found from initial and boundary conditions: at $\theta_{11} = -\phi_{11}$, $\gamma = -\gamma_1$; at $\theta_{12} = \phi_{12}$, $\gamma = 0$; at $\theta_{12} = \phi_{12}$, $\gamma = m \gamma_1$.

Then we have

$$\gamma = \frac{\gamma_1 \theta_{11}}{\phi_{11}}, \quad -\phi_{11} \leq \theta_{11} \leq 0, \quad \gamma = \frac{m \gamma_1 \theta_{12}}{\phi_{12}}, \quad 0 \leq \theta_{12} \leq \phi_{12}.$$  

Contact interaction in two-roll modules with rolls having elastic coatings can be considered by analogy with the rolling of an elastic wheel over deformable soil [11]. In the theory of wheel rolling, the analytical definition of the contact line is associated with the analysis of the ratio of the strain rates of interacting bodies [12]. In various studies, this ratio is considered constant [12,13]. Then the ratio of wheel and soil strains is equal to the ratio of their strain rates [13].

To describe the curve shape of the roll contact, similar to the theory of wheel rolling, the ratio of strains of surface layers of the rolls and the material layer is considered to be equal to the ratio of their strain rates.

At each point of the roll contact curve, the strain in contacting bodies occurs along the $n$ - $n$ line perpendicular to the roll contact curve. Then at point $B_1$ of section $A_1 K$ we have (figure 1)

$$\frac{D_i B_i}{B_i C_i} \frac{d \theta_{11}}{d \theta_{12}} = \frac{D_i B_i}{B_i C_i} = \lambda_{11},$$  

where $\lambda_{11} = \frac{d \theta_{11}}{d \theta_{12}}$ is the ratio of the strain rates of the surface layer of the lower roll and the layer of material under compression; $\psi$ is the angle between the radius $r_1$ and the line $n - n$.

From figure 1 we find

$$D_i B_i = O_i D_i - O_i B_i = R_i - r_{11}, \quad B_i C_i = O_i B_i - O_i C_i = r_{11} - R_i \frac{\cos(\phi_{11} + \gamma)}{\cos(\theta_{11} + \gamma)}.$$  

In view of expressions (2), equalities (1) take the form

$$\frac{R_i - r_{11}}{r_{11} - R_i \frac{\cos(\phi_{11} + \gamma)}{\cos(\theta_{11} + \gamma)}} = \lambda_{11},$$  

Having solved equalities (3) with respect to $r_{11}$, we find the equation of the contact curve of section $A_1 K$

$$r_{11} = \frac{R_i}{1 + \lambda_{11}} \left(1 + \lambda_{11} \frac{\cos(\phi_{11} + \gamma)}{\cos(\theta_{11} + \gamma)}\right).$$  

To simplify research, expression $\cos(\phi_{11} + \gamma)$, where $0 \leq \gamma \leq \gamma_1$, in formula (4), is replaced with
expression \( \cos(\phi_1 + \gamma_1) \).

Then we have

\[
r_{11} = \frac{R_1}{1 + \lambda_{11}} \left[ 1 + \lambda_{11} \frac{\cos(\phi_1 + \gamma_1)}{\cos(\theta_1 + \gamma)} \right], \quad \gamma = \frac{\gamma_1 \theta_{11}}{\phi_{11}}, \quad -\phi_1 \leq \theta_1 \leq 0. \tag{5}
\]

Geometrical analysis shows that the transition from formula (4) to formula (5) means replacing the straight line \( A_2 E_1 \) with the straight line \( A_1 M_1 \) (figure 1). Since the angle of inclination of the material layer relative to the line of centers is usually small, such a replacement is acceptable (the calculation error does not exceed 1.5% - 3.2%).

By analogy with formula (5), the equation of the contact curve of section \( K A_2 \) is written

\[
r_{12} = \frac{R_2}{1 + \lambda_{12}} \left[ 1 + \lambda_{12} \frac{\cos(\phi_2 + \gamma_1)}{\cos(\theta_2 + \gamma)} \right], \quad \gamma = \frac{\gamma_1 \theta_{12}}{\phi_{12}}, \quad 0 \leq \theta_2 \leq \phi_2, \tag{6}
\]

where \( \lambda_{12} = \frac{dr_{12}}{dh} \) —is the ratio of the strain rates of the surface layer of the lower roll and the material layer during recovery.

Summarizing equations (5) and (6), we find the equations of the lower roll contact curve

\[
\begin{align*}
\frac{r_1}{R_1} &= \frac{1}{1 + \lambda_{11}} \left[ 1 + \lambda_{11} \frac{\cos(\phi_1 + \gamma_1)}{\cos(\theta_1 + \gamma)} \right], \quad \gamma = \frac{\gamma_1 \theta_{11}}{\phi_{11}}, \quad -\phi_1 \leq \theta_1 \leq 0, \\
\frac{r_2}{R_2} &= \frac{1}{1 + \lambda_{12}} \left[ 1 + \lambda_{12} \frac{\cos(\phi_2 + \gamma_1)}{\cos(\theta_2 + \gamma)} \right], \quad \gamma = \frac{\gamma_1 \theta_{12}}{\phi_{12}}, \quad 0 \leq \theta_2 \leq \phi_2.
\end{align*}
\tag{7}
\]

By analogy with the definition of the equation of the contact curve of the lower roll, we determine the equation of the contact curve of the upper roll

\[
\begin{align*}
\frac{r_{21}}{R_2} &= \frac{1}{1 + \lambda_{21}} \left[ 1 + \lambda_{21} \frac{\cos(\phi_2 - \gamma_1)}{\cos(\theta_2 - \gamma)} \right], \quad \gamma = \frac{\gamma_1 \theta_{21}}{\phi_{21}}, \quad -\phi_2 \leq \theta_2 \leq 0, \\
\frac{r_{22}}{R_2} &= \frac{1}{1 + \lambda_{22}} \left[ 1 + \lambda_{22} \frac{\cos(\phi_2 - \gamma_1)}{\cos(\theta_2 - \gamma)} \right], \quad \gamma = \frac{\gamma_1 \theta_{22}}{\phi_{22}}, \quad 0 \leq \theta_2 \leq \phi_2.
\end{align*}
\tag{8}
\]

where \( \lambda_{21} = \frac{dr_{21}}{dh}, \lambda_{22} = \frac{dr_{22}}{dh} \) are the ratios of the strain rates of the surface layer of the upper roll and the material layer during compression and recovery, respectively.

The systems of equations (7) and (8) describe the shapes of the roll contact curves in the two-roll module under consideration. In this case, the influence of geometrical characteristics of one of the rolls on the shape of the contact curve of another one determines the contact angles, and the strain characteristics of the material layer and both rolls are completely included into the rate ratio. In the two-roll module under consideration, the upper roll is free. It can be a driving one. In this case, the two-roll module has a transmission mechanism between the rolls. The gripping conditions in a two-roll module with one driving roll differ from the gripping conditions with two driving rolls. The gripping conditions in the two roll module determine the contact angles. Therefore, two-roll modules with one driving roll and with two driving rolls have different contact angles [14]. In this regard, we can say that the influence of the transmission mechanism on the curve shape of contact rolls is also determined by the contact angles.

All partial cases of the two-roll module under consideration and the corresponding partial types of the system of equations (7) and (8) are analyzed. It was revealed that the formulas obtained are general in the sense that they describe all partial cases of the interaction of the processed material with the pairs of rolls in two-roll modules. Therefore, they can be used in modeling friction stresses.
2.2. Friction stress modeling

Now proceed to the study of the second problem, i.e. to friction stresses modeling.

First, we analyze the stress state of the contact interaction of the material layer and the lower roll along the contact curve $A_1$.

In the steady-state interaction process, the lower roll is affected by: pressure force of the clamping devices $Q_1$, horizontal reaction of the roll supports $F_1$, moment of resistance force $M_1$, elementary forces of normal pressure and friction, acting along the entire curve of the contact roll. Elementary forces in the zones of compression $(N_{11}, T_{11})$ and recovery $(N_{12}, T_{12})$ are presented separately.

The friction forces at the beginning and at the end of the contact zone have opposite directions. They change their signs, turning to zero at a neutral point $A_2$.

Let the neutral point $A_3$ correspond to the angle $\theta_{11} = -\theta_{13}$ (figure 2).

Considering the lower roll in equilibrium under applied forces, we obtain

$$
\begin{align*}
\sum X &= N_{1x} + T_{1x} + F_{1x} + Q_{1x} = 0, \\
\sum Y &= N_{1y} + T_{1y} + F_{1y} + Q_{1y} = 0,
\end{align*}
$$

(9)

where $N_{1x}, N_{1y}, T_{1x}, T_{1y}$ - are the projections of the principal normal and tangential forces on the axes $x$ and $y$, equal to the sum of the projections of the principal normal and tangential forces in the zones of compression ($j = 1$) and recovery ($j = 2$):

$$
N_{1x} = \sum_{j=1}^{2} N_{1xj}, \quad N_{1y} = \sum_{j=1}^{2} N_{1yj}, \quad T_{1x} = \sum_{j=1}^{2} T_{1xj}, \quad T_{1y} = \sum_{j=1}^{2} T_{1yj}.
$$

(10)

Assume that

$$
F_{1x} = \sum_{j=1}^{2} F_{1xj}, \quad F_{1y} = \sum_{j=1}^{2} F_{1yj}, \quad Q_{1x} = \sum_{j=1}^{2} Q_{1xj}, \quad Q_{1y} = \sum_{j=1}^{2} Q_{1yj}.
$$

(11)

With expressions (10) and (11) the system of equations (9) has the form

$$
\begin{align*}
N_{1xj} + T_{1xj} + F_{1xj} + Q_{1xj} &= 0, \\
N_{1yj} + T_{1yj} + F_{1yj} + Q_{1yj} &= 0,
\end{align*}
$$

(12)
or

\[ \begin{align*}
    &dN_{1x} + dT_{1x} + dF_{1x} + dQ_{1x} = 0, \\
    &dN_{1y} + dT_{1y} + dF_{1y} + dQ_{1y} = 0.
\end{align*} \]  

(12)

From the force scheme of the compression zone (figure 2) we find

\[ \begin{align*}
    &dN_{1x} = dN_{11} \sin(\theta_1 - \psi_{11}), \quad dN_{1y} = -dN_{11} \cos(\theta_1 - \psi_{11}), \quad dT_{1x} = -dT_{11} \cos(\theta_1 - \psi_{11}), \\
    &dT_{1y} = -dT_{11} \cos(\theta_1 - \psi_{11}), \quad dF_{1x} = dF_{11}, \quad dF_{1y} = 0, \quad dQ_{1x} = 0, \quad dQ_{1y} = dQ_{11},
\end{align*} \]

where \( \psi_{11} \) is the angle between force \( dN_{11} \) and radius \( r_{11} \).

With the expressions from system (12) for the compression zone we have

\[ \begin{align*}
    &dT_{11} \cos(\theta_1 - \psi_{11}) - dN_{11} \sin(\theta_1 - \psi_{11}) = dF_{11}, \\
    &dT_{11} \sin(\theta_1 - \psi_{11}) + dN_{11} \cos(\theta_1 - \psi_{11}) = dQ_{11}.
\end{align*} \]

or

\[ \frac{dF_{11}}{dQ_{11}} = \frac{dT_{11} \cos(\theta_1 - \psi_{11}) - dN_{11} \sin(\theta_1 - \psi_{11})}{dT_{11} \sin(\theta_1 - \psi_{11}) + dN_{11} \cos(\theta_1 - \psi_{11})}. \]  

(13)

Since the considered process is a steady-state process, assume that

\[ \frac{F_1}{Q_1} = C_1, \]  

(14)

where \( C_1 \) is a constant quantity.

Hence we have

\[ d \left( \frac{F_1}{Q_1} \right) = \frac{Q_1 dF_1 - F_1 dQ_1}{Q_1^2} = 0 \quad \text{or} \quad \frac{dF_1}{dQ_1} = C_1. \]

Considering

\[ \frac{dF_{11}}{dQ_{11}} = C_{11}, \]  

(15)

from equality (13) we obtain

\[ \frac{dT_{11}}{dN_{11}} = \frac{\sin(\theta_1 - \psi_{11}) + C_{11} \cos(\theta_1 - \varphi_{11})}{\cos(\theta_1 - \psi_{11}) - C_{11} \sin(\theta_1 - \psi_{11})}. \]  

(16)

Elementary forces are associated with contact stresses by the following relations [2]

\[ dN_{11} = n_1 \sqrt{r_{11}^2 + r_{11}^2} d\theta_{11}, \quad dT_{11} = t_{11} \sqrt{r_{11}^2 + r_{11}^2} d\theta_{11}, \]

where \( n_{11} = n_{11} (\theta_{11}), \quad t_{11} = t_{11} (\theta_{11}) \) are the normal and tangential forces (stresses), respectively, distributed over the compression zone of the roll contact curve.

We substitute expressions \( dN_{11} \) and \( dT_{11} \) in equality (16), then transform according to expressions \( \cos(\theta_{11}) = \frac{r_{11}}{\sqrt{r_{11}^2 + r_{11}^2}}, \quad \sin(\theta_{11}) = \frac{r_{11}}{\sqrt{r_{11}^2 + r_{11}^2}}. \) Then the dependencies connecting the tangential and normal stresses at the points of the compression zone of the lower roll are obtained

\[ t_{11} = \frac{(\sin(\theta_{11}) + C_{11} \cos(\theta_{11})) r_{11} - (\cos(\theta_{11}) - C_{11} \sin(\theta_{11})) r_{11}'}{(\cos(\theta_{11}) - C_{11} \sin(\theta_{11})) r_{11} + (\sin(\theta_{11}) + C_{11} \cos(\theta_{11})) r_{11}'}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \]  

(17)

Formula connecting the tangential and normal stresses at the points of recovery zone of the lower roll is obtained similarly. It has the form

\[ t_{12} = \frac{(\sin(\theta_{12}) + C_{12} \cos(\theta_{12})) r_{12} - (\cos(\theta_{12}) - C_{12} \sin(\theta_{12})) r_{12}'}{(\cos(\theta_{12}) - C_{12} \sin(\theta_{12})) r_{12} + (\sin(\theta_{12}) + C_{12} \cos(\theta_{12})) r_{12}'}, \quad 0 \leq \theta_{12} \leq \varphi_{12}. \]  

(18)

Note that at the point of the contact curve on the center line the following boundary conditions are satisfied

\[ t_{11}(0) = t_{12}(0), \quad n_{11}(0) = n_{12}(0), \quad r_{11}(0) = r_{12}(0) = R_0, \quad r_{11}'(0) = r_{12}'(0) = 0. \]
These conditions lead to equality \( C_{11} = C_{12} \). Then from expressions (14) and (15) we have

\[
C_1 = C_{11} = C_{12} = \frac{F_1}{Q_1}.
\]

So, from equation (17) and (18) we obtain the model of friction stresses for the lower driving roll

\[
\begin{align*}
    t_{11} &= \frac{(Q_1 \sin \theta_{11} + F_1 \cos \theta_{11})r_1}{(Q_1 \cos \theta_{11} - F_1 \sin \theta_{11})r_1} - (Q_1 \cos \theta_{11} - F_1 \sin \theta_{11})r_1' n_1, & -\phi_{11} \leq \theta_{11} \leq 0, \\
    t_{12} &= \frac{(Q_1 \sin \theta_{12} + F_1 \cos \theta_{12})n_2}{(Q_1 \cos \theta_{12} - F_1 \sin \theta_{12})n_1} + (Q_1 \sin \theta_{12} - F_1 \cos \theta_{12})r_1' n_2, & 0 \leq \theta_{12} \leq \phi_{12}.
\end{align*}
\]  

(19)

In the two-roll module under consideration, the upper roll is free. In this case, the forces \( \bar{T}_2 \) and \( \bar{T}_2' \) acting on the upper roll change direction [9]. Therefore, the quantities \( t_{2j} \) \((j = 1, 2)\) and \( F_2 \) have opposite signs in the formulas of system (19). In this regard, the model of friction stresses for the upper roll has the form

\[
\begin{align*}
    t_{21} &= -\frac{(Q_2 \sin \theta_{21} - F_2 \cos \theta_{21})r_2}{(Q_2 \cos \theta_{21} + F_2 \sin \theta_{21})r_1} + (Q_2 \sin \theta_{21} + F_2 \cos \theta_{21})r_2' n_1, & -\phi_{21} \leq \theta_{21} \leq 0, \\
    t_{22} &= -\frac{(Q_2 \sin \theta_{22} - F_2 \cos \theta_{22})r_2}{(Q_2 \cos \theta_{22} + F_2 \sin \theta_{22})r_2} - (Q_2 \sin \theta_{22} + F_2 \cos \theta_{22})r_2' n_2, & 0 \leq \theta_{22} \leq \phi_{22}.
\end{align*}
\]  

(20)

The systems of equations (19) and (20) determine the models of friction stresses in the considered two-roll module. They show that the models of friction stresses in two-roll modules are independent of the feed inclination of the material layer to the center line and of the upper roll inclination relative to the vertical. An analysis of these models shows that they describe stress models for all partial cases of the two-roll module under consideration.

Formula (17) is transformed taking into account expressions \( \tan \psi_{11} = \frac{r_{11}'}{r_{11}} \) and \( C_{11} = C_1 \)

\[
t_{11} = \frac{(\tan \theta_{11} + C_1) - (1 - C_1 \tan \theta_{11}) \tan \psi_{11}}{(1 - C_1 \tan \theta_{11}) + (\tan \theta_{11} + C_1) \tan \psi_{11}} n_{11}
\]

Assuming that \( C_1 = \tan \xi_{11} \), proceed to

\[
t_{11} = \tan (\theta_{11} - \psi_{11} + \xi_{11}) n_{11}.
\]  

(21)

At the neutral point \( t_{11}(-\varphi_{11}) = 0 \) and \( n_{11}(-\varphi_{13}) \neq 0 \). Therefore, according to expression (21), the condition for determining the neutral angle can be represented as

\[
\varphi_{13} + \psi_{11} = \xi_{11},
\]  

(22)

where \( \psi_{11} = \arctan \frac{r_{11}'(-\varphi_{11})}{r_{11}(-\varphi_{13})} \).

From the first equation of system (7) we have

\[
\begin{align*}
    r_{11}(-\varphi_{13}) &= \frac{R_1}{1 + \lambda_{11}} \left( 1 + \lambda_{11} \frac{\cos(\varphi_{13} + \gamma_{1})}{\cos(1 + a_{1}\varphi_{13})} \right), \\
    r_{11}'(-\varphi_{13}) &= -\frac{R_1 \lambda_{11} \cos(\varphi_{13} + \gamma_{1})}{1 + \lambda_{11}} \cdot \frac{(1 + a_{1}) \sin(1 + a_{1}\varphi_{13})}{\cos^2(1 + a_{1}\varphi_{13})},
\end{align*}
\]

where \( a_{11} = \frac{\gamma_{1}}{\varphi_{13}} \).
Since the neutral point is close to the center line [5], the angle $\varphi_{13}$ is close to zero. Therefore, we can assume that $\operatorname{tg}\psi_{13}^* \approx \psi_{13}^*$ (since $\psi_{13}^* < \varphi_{13}$), $\sin(1 + a_1)\varphi_{13} \approx (1 + a_1)\varphi_{13}$, $\cos(1 + a_1)\varphi_{13} \approx 1$, $(1 + a_1)^2 \approx 1 + 2a_1$.

Then the transform of equality (22) leads to the following formula

$$\varphi_{13} = \frac{(1 + \lambda_{11}\cos(\varphi_{11} + \gamma_1))\varphi_1\xi_1}{\varphi_1 - 2\gamma_1\lambda_{11}\cos(\varphi_{11} + \gamma_1)}. \quad (23)$$

This formula determines the neutral angle of the lower roll of the two-roll module in question. The neutral angle of the upper roll can be determined similarly.

The neutral angle is calculated by formula (23). The calculation results show that with increasing angle $\xi_1$, the angle $\varphi_{13}$ increases linearly. According to expression $\xi_1 = \arctg \frac{F_1}{Q_1}$, an increase in $\xi_1$ means an increase in force $F_1$. In this regard, we can say that the greater the force $F_1$, the more to the left from the line of centers the neutral point is. The angle $\varphi_{11}$ also increases with increasing $\lambda_{11}$ and $\gamma_1$. In the first case, the increase occurs in a parabolic law, and in the second case - linearly. With an increase in angle $\varphi_{11}$, a decrease in the neutral angle is observed.

Conclusions
1. Models of roll contact curves and friction stress in a two-roll module are determined, in which the rolls are located relative to the vertical with a tilt to the right, have unequal diameters and elastic coatings from the materials of different stiffness and friction coefficients, the lower roll is a driving one, the upper one is free, the material layer is fed downward with a tilt relative to the line of centers.

2. It was revealed that the obtained models are general in the sense that they are applicable for partial cases of interaction in a two-roll module.

3. It was stated that the neutral angle increases with increasing horizontal reaction of the driving roll supports, the angle of inclination of the feed of the material layer to the line of centers, and the ratio of the strain rates of the roll and the material layer. In the first cases, the increase occurs according to a linear law, and in the second - according to a parabolic law. With an increase in the contact angle, a decrease in the neutral angle is observed.

4. The dependences between the forces acting in the rolls and the stresses distributed under these forces are established. It was revealed that these dependences do not change with a change in the angle of supply of the material layer to the line of centers and the angle of inclination of the upper roll relative to the vertical.

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