Does the Superluminal Neutrino Uncover Torsion?

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Abstract

I investigate the possibility of the propagation of neutrino with superluminal speed through matter in the context of the relation between gravity, spin and torsion. Using a lemma of Penrose and earlier works on the relation between spin, torsion and gravity I glimpse on a framework in which superluminal speed of the neutrinos moving through matter become possible. In presence of torsion neutrinos are found to follow spacelike geodesics by tunneling through the light cone into the spacelike region and consequently appear to have superluminal speed in our timelike world, whereas photons always follow null geodesics. This framework may set new frontiers for spacetime physics.

1 Introduction

Measuring the flight time of the muon neutrinos, with average energy of 17.5 GeV, the OPERA collaboration reported that they have measured a superluminal speed for these neutrinos [1]. They claims that their experiment gives a travel time for the ultrarelativistic neutrinos which is about 60 ns less than expected when compared to the speed of light. This means that neutrinos propagate with superluminal speed with \( \delta v = (v - c)/c \sim 2.5 \times 10^{-5} \)
where \( c \) is the speed of light in vacuum. Earlier experiments on high energy neutrinos like the MINOS project [2] have shown that \( \delta v \sim 5.1 \times 10^{-5} \).
Astronomical detection of neutrinos from the supernova SN1987a has given much less figure of about \( \delta v \sim 2 \times 10^{-9} \).

Several interpretations for the OPERA claim has been published in preprints recently, most of them are ad hoc suggestions that lacks rigor and firm foundations. In fact, a more profound explanation is needed to explain
the Opera, MINOS and the SN1987a results, since if true this result will have immense implications on the understanding of spacetime and particles, including the question of general covariance. In this letter I am not going to present any full fledged theory on the subject but will hint on some views that might enable tackle the subject from another point of view toward an approach that might explain these results.

2 Torsion, Spin and Gravity

Symmetry is one of the most beautiful aspects of nature that our minds may envision. However, in a deeper scientific prospect symmetry stands as the unifying pedestal on which laws of nature rests; behind every symmetry there hides a conservation law that keeps the balance. Forces and potentials may break the symmetry but then other forces and potentials come into play to restore symmetry in a wider scope and keep the balance in nature.

Several alternatives to the theory of general relativity has been suggested since it was proposed in 1915. Perhaps the most interesting of these was Cartan’s introduction of torsion as the antisymmetric part of an asymmetric affine connections \[3\]. Cartan recognized the tensor character of torsion and developed a differential geometric formulation and he had some ideas about torsion of the spacetime being connected with the intrinsic angular momentum of matter and later Schrödinger tried to develop a unified theory of gravity and electromagnetism in 1943 where torsion was related to electromagnetic potential \[4\], consequently Schrödinger found that photons acquire a non-zero rest mass (for an overview of the literature see \[5\] and for more recent review see \[6\]). This culminated later into the formulation of what is called the $U_4$ theory by Kibble \[8\] and \[9\] which came in the context of the gauge approach to gravity. Both Kibble and Sciama arrived at a set of field equations and laid down the basic structure of $U_4$ theory. Further development of this approach was taken by Hehl, Von der Heyde and Kerlick \[10\], and Trautman \[11\] and others. The birth of local gauge theory in the 1950s breathed new life into torsion. With the first attempts by Utiyama \[7\] paving some ground, and Sciama \[9\] emphasizing torsion as being related to spin. Kibble \[8\] showed how to describe gravity with torsion as a local gauge theory of the Poincare’ group, and by 1976 Hehl et al \[5\] formulated a gravitation theory with torsion as resulting from local Poincare’ gauge invariance. The beauty and success of global Lorentz invariance was generalized with modern gauge principles to form a compelling new picture. The two Casimir invariants of the Poincare’ group, the square of the translation operator $P^2$ and Pauli–Lubanski spin operator $L^2$ found perfect interpretations
in a theory of gravity with torsion: generalizing the notion that mass curves space, now we also have spin giving rise to torsion. It was anticipated that having formulated gravitation as a gauge theory the charm and success of the quantization of the $SU(n)$ theories might rub off on gravity, yielding a quantizable theory. However, it was disappointing that the early predictions indicated that torsion forces were too weak to measure, and, in some formulations, torsion did not even propagate into vacuum. However if the torsion tensor is to be taken as being a gradient of a scalar potential then it could propagate in vacuum as shown by Hammond [6].

2.1 Gravity with Torsion

Gravity, according to Einstein, is a curvature of the spacetime, and matter would tell spacetime how to curve. This was the view according to Wheeler. However, matter seem to have other effects on spacetime other than curvature. If we would understand curvature in terms of the length of trajectories and durations of time the we can only do that in a comparative context. But if we have to understand curvature in its intrinsic character then we have to look for an intrinsic context within which we can appreciate the meaning of curvature. This is usually done through what we call transporting a vector parallel to itself a long a closed trajectory or a surface in the spacetime. If the vector is preserved throughout the displacement precisely then the curvature of the spacetime is zero and the spacetime is called flat. Otherwise, if there would be a difference in the magnitude or the direction of the vector throughout the trip then in this case we say that the spacetime is curved. In order to preserve the symmetry in curved space a covariant derivative should replace the ordinary derivative to express the infinitesimal translation. Curvature is the property of the spacetime endowed with gravity, such a spacetime can be available outside the matter source. However, once matter (in the form of a test particle for example) is introduced in this spacetime new effects may appear and among these is torsion. The spacetime get rapped in a chiral manner causing light cones to shrink in presence of torsion. This will change the teleparallel behavior of the spacetime, and accordingly the metric and the covariant derivative all will change.

The behavior of sticks and clocks in torsion free curved spacetime is well developed in general relativity which considers a Riemannian spacetime with all the symmetries enjoyed within like metricity, covariance and conservations of energy and momentum. In such a spacetime the covariant derivative

$$\nabla_\mu = \partial_\mu + \Gamma^\sigma_{\mu\nu}$$

is introduced in order to account for the locality of the gravitational potentials
and to safeguard the general covariance of the translational symmetry. The affine connections $\Gamma^\sigma_{\mu\nu}$ are assumed to be symmetric in $\mu$ and $\nu$ and the metric tensor is taken to be divergenceless expressing the conservation of spacetime. Such assumptions have constrained the spacetime to be torsion free. Now, if we have to take care of the chiral symmetry of the spacetime and look for the introduction of spinning matter we have to introduce torsion; for torsion is the object that is related in essence to chirality. For this goal we should expect the Riemannian affine connections to be modified. Indeed these are given by

$$\hat{\Gamma}^\sigma_{\mu\nu} = \Gamma^\sigma_{\mu\nu} - K^\sigma_{\mu\nu}$$

(2)

where $\Gamma^\sigma_{\mu\nu}$ are the usual Riemann-Christoffel symbols and $K^\sigma_{\mu\nu}$ is the cotorsion tensor related to the torsion by

$$S^\sigma_{\mu\nu} = \Gamma^\sigma_{[\mu\nu]} = -K^\sigma_{[\mu\nu]}$$

(3)

The Covariant derivative in the Riemann-Cartan space takes the form

$$\hat{\nabla}_\mu = \partial_\mu + \Gamma^\sigma_{\mu\nu}$$

(4)

The fact that torsion is basically antisymmetric motivates one to foresee some connection with the rotational properties of the spacetime, and perhaps this what Cartan had in mind originally. This what would introduce the Poincare group into the picture (for more details see sec.IV of Ref [5]).

From the definition of the covariant derivative in (2) and the properties of $K^\sigma_{\mu\nu}$ it is easy to see that $\nabla_\mu g_{\rho\sigma} = 0$, a property by which the metricity of the Riemann-Cartan spacetime is maintained.

### 2.2 The variational consideration

The total action of gravity with matter and torsion is taken to be extremal according to

$$\delta (I_G + I_m + I_s) = 0$$

(5)

where $I_G$ is the geometrical action given by

$$I_G = \frac{c^4}{16\pi G} \int \sqrt{-g} R d^4x$$

(6)

where $R$ is the scalar curvature $g$ is the determinant of the metric tensor. $I_m$ is the matter action given by

$$I_m = \frac{1}{2} \int \sqrt{-g} T^{\mu\nu} \delta g_{\mu\nu} dx^4$$

(7)
with $T^{\mu\nu}$ being the energy-momentum tensor. $I_s$ is the torsion action given by

$$I_s = \frac{1}{2} \int \sqrt{-g} \mu^\mu_\sigma \delta S^\sigma_{\mu\nu} dx^4$$

(8)

where $\mu^\mu_\sigma$ is the spin-energy potential of matter. From (5) the field equations can be obtained as

$$G^{\mu\nu} - \hat{\nabla}_\beta (T^{\mu\nu\beta} + T^{\beta\mu\nu} + T^{\beta\nu\mu}) = \frac{16\pi G}{c^4} T^{\mu\nu}$$

(9)

where

$$T^{\alpha\beta\sigma} = S^{\alpha\beta\sigma} + S^\beta g^{\alpha\sigma} - S^{\beta} g_{\beta\sigma}$$

(10)

is the modified torsion tensor, $S^{\alpha\beta\sigma}$ is the torsion tensor and $S^\beta = S^\beta_{\sigma}$ is the torsion trace or the torsion vector. The torsion tensor is related to matter potentials by the equations

$$S_{\alpha\beta\gamma} = \frac{16\pi G}{c^4} (\mu_{[\alpha\beta]} + \mu_{[\alpha} g_{\beta\gamma]})$$

(11)

where $\mu_{\alpha} = \mu^\sigma_{\alpha\sigma}$. Clearly torsion vanishes in vacuum, therefore exterior to matter source the field equations (9) reduces to the standard Einstein field equations and their solutions will produce the same geodesics that are produced in Riemannian space.

3 Propagation of Matter Fields in $U_4$

The propagation of matter fields in the background of Riemann-Car tan spacetime has been studied by few authors and its is shown that scalar field, which has no spin, neither feel nor produce torsion. Photons in $U_4$ are unaffected by the presence of torsion and consequently the causal structure of a $U_4$ spacetime is determined completely by the conformal metric structure of the spacetime [5]. The propagation of massive Dirac field in $U_4$ was studied by Jurgen [12] and it was shown that particles follow non-geodesic trajectories. The force equation is given as

$$m v^\nu \hat{\nabla}_\nu v_\mu = \frac{1}{2} \left( \frac{\hbar}{2} \right) \hat{R}_{\mu\sigma\rho\lambda} b_0 \sigma^{\rho\lambda} b_0 \sigma^\sigma$$

(12)

where $\sigma^{\rho\lambda} = i [\gamma^\rho, \gamma^\lambda]$, $b_0$ is a spinor column matrix, and $\bar{b}_0$ is its conjugate. Clearly the force in Eqn. (12) is taken to the order of $\hbar$, if this order is dropped then the trajectories of Dirac particles will follow geodesics in the
Riemann-Cartan spacetime. However, this force may also hint a verification of torsion through a suitable experiments.

If Neutrinos are to fly with superluminal speed then they have to follow a spacelike trajectories [13], and according to Penrose such trajectories are possible if torsion exist [14]. In this case, as remarked by Penrose, the light cone will be a timelike surface with respect to $\nabla$, but would curl into the inside of the light cone with respect to $\nabla$. Therefore, the trajectory of the neutrino will escape from inside to outside the light cone. The reason why the observed velocity of the neutrinos gained marginal velocity above $c$ is that their torsion potential is low.

The supernova SN1987a neutrinos did not show any significant superluminal speed because it was propagated mostly in vacuum. This is where torsion would not play any significant role. In order to testify our suggestion presented here and other suggestions other experiments with longer base-lines are needed in order to check the path dependence of this effect.

The situation suggested here is geometrically similar to that which is suggested by the bimetric relativity. Recently Moffat [15] suggested that a bimetric structure of the spacetime might offer an explanation for the superluminal neutrino. In fact it is clear that the underlaying gauge $\psi_\mu$ variance is playing the underlaying connecting substratum in both approaches. This might be shown in more details if we consider the calculation of $\hat{\Gamma}_{\mu\nu}^\sigma$ from the given bimetrical form

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \beta \psi_\mu \psi_\nu$$

and calculate $\hat{\Gamma}_{\mu\nu}^\sigma$ where we find a similar relation to (2). In this case the torsion tensor will get its definition in terms of the bivector $\psi_\mu \psi_\nu$. This analogy will help investigate the qualitative and quantitative differences in the physical effects that both theories may predict.

The issue of how neutrinos or other particles can cross the light barrier into the spacelike region is something that has to be investigated more profoundly. However, such tunneling into the spacelike region will no doubt assert the massiveness of the neutrino no matter how small is its mass, otherwise. The very recent suggestion of Ahluwalia, Horvath and Schritt [16] for the mass of the neutrino spacies in the context of discussing a possible limit on the neutrino masses based on the measured speed of the neutrino is consistent with this view.

The point which concerns theoretical physicists is that crossing the light barrier for a superluminal speed would break Lorentz covariance, and in fact this will not be a problem. We are accustomed to view physics in the timelike regions of the spacetime. This is motivated by the wish to maintain causality in the relativistic meaning. But if we take into consideration all the
 spacetime with its timelike and spacelike regions to be under a more general transformation law, then the physics might get easier and better understood. Indeed timelike and spacelike regions can be interchanged using the duality transformation in order to get the full picture of the physical phenomena. For example if we consider the duality transformation of the electromagnetic field we find that on a special choice of the $\theta$ the electric monopole is replaced by the magnetic monopole and vise versa. If the duality transformations are generalized to cover spacetime coordinates then it would imply interchanging spacelike and timelike regions. Consequently, I would suggest here that magnetic monopoles does exist, but only in the spacelike region of the spacetime. Beside this and since magnetic monopoles are necessary for the quantization of charge as shown by Dirac [17] then it would be of interest to understand the action of these monopoles from under the carpet of our timelike world instead of looking for them in labs. In this context comes the necessity to complement the timelike regions of the light cone with the spacelike regions to form a complete manifold for physics. Within this scope the role of exotic objects like magnetic monopoles, tachyons and supersymmetric particles may be better understood, and the coupling of mass to curvature, electromagnetic field to null geodesics, torsion and spin to gravity get clearer to complete the picture of the world.

4 Conclusions

Spinning neutrinos couples to torsion in spacetime. Torsion is a spatial property that is reflected in the translational structure of the spacetime. If superluminal speed of neutrinos proves to be true, then one might conjecture that they follow spacelike geodesics and consequently their superluminal speed is understood. In this context it might be quite possible to use neutrino-experiment as an effective tool for probing the properties of spacetime, specifically this can be used to investigate the torsion effects (if any) at different energies of the probing particles. A long base-line experiments are needed now for two reasons: (i) to make sure that neutrinos are propagated with a speed faster than light and (ii) to see if that effect is distance dependent. Such investigations will enable physics get new frontiers that might help bring gravity and quantum mechanics closer to meet on the bridge of the intrinsic spin of elementary particles.
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