Vibration Suppression and Over-Quadrant Error Mitigation Methods for a Ball-Screw Driven Servo System With Dual-Position Feedback

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ABSTRACT

For high-precision control in a ball-screw driven servo system, the full-closed loop position control structure with position measurements from both the drive side and the load side are usually required. However, compared with semi-closed loop position control, if only the load side position feedback is being used, then the inherent elastic link is included in the position loop, and the mechanical vibration will be easily induced. For this situation, the stability of the servo system will be affected. Moreover, in the ball-screw driven servo system, the backlash and friction in the mechanical transmission chain will introduce a typical contour error–over-quadrant error, which cannot be eliminated by adopting full-closed loop control. In order to maintain stability and high precision positioning performance, both vibration suppression and over-quadrant error mitigation methods are proposed in this paper. First, the model of full-closed loop position control system is established, on this basis, the reasons that why the full-closed loop control is more easily to induce the position vibration than the semi-closed loop control is analyzed. Then, a dual-position feedback control method, by introducing the drive side position information with a filter to the position loop feedback channel, is proposed, which combines the advantages of semi-closed loop control and full-closed loop, and features high gain margin and high control precision. Furthermore, based on the analysis of the mechanism of over-quadrant error, an adaptive backlash error compensation method which can reduce the over-quadrant error is proposed. Finally, simulation and experimental results in both single axis and dual axis are provided to demonstrate the feasibility and effectiveness of the proposed methods.

INDEX TERMS

Vibration suppression, position control, ball-screw, over-quadrant error.

I. INTRODUCTION

Positioning systems using rotary permanent magnet synchronous motors (PMSMs) and mechanical power transmission systems with ball-screw have been widely used in various applications such as numerical control (NC) machine, industrial robots and factory automation, for their advantageous features such as increased reliability and reduced cost [1]–[3]. In manufacturing procedures, it is a very important issue to improve the accuracy and dynamic performance of position control thus to achieve high manufacturing quality. Therefore, in a properly designed servo control system, the precision of position control becomes a significant index [4], [5].

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The semi-closed loop (SCL) position control, which feedbacks the rotation angle information of the motor shaft, is the most commonly used control method and its structure is shown in Fig. 1(a). The precision of the ball screw has a significant influence on the performance of the SCL control. If the pitch error and backlash distribute averagely over the entire ball screw lead, the positioning accuracy can be improved though pitch compensation and backlash compensation. However, the size of backlash will be influenced by the weight and the location of the load. Moreover, the temperature of working condition may bring about accumulation pitch error to the ball screw.

In order to improve the precision of position control, the full-closed loop (FCL) control, with its structure being shown in Fig. 1(b) is used to replace the SCL control [6], [7]. In the FCL, the linear encoder is attached to the load side to...
feedback the actual position error to the position controller. FCL control and SCL control have a similar controller structure except for the location of the position feedback sensors, and thus the FCL features high accuracy than that of the SCL. However, structural resonances or nonlinearities such as backlash or stiction can deteriorate the servo performance due to the nonlinear elements that are contained in the closed loop [8]. That is, setting a high controller gain will result in an unstable servo system. If the resonance frequency of the two-mass system is lower than the controller bandwidth, the position control system will lose stability. Therefore, in order to get rid of the system oscillation, it is necessary to increase the rigidity of the two-mass system, decrease the friction coefficient and remove the possibility of lost motion. Obviously, it is not a cost-friendly method in most of the industrial application

In FCL, when it is difficult to modify the rigidity of the system, it is necessary to lower the controller gain. However, the positioning performance in terms of response time and accuracy will be deteriorated with a lower controller gain.

In this case, many methods have been proposed in the literature for the vibration problem of the two-mass system. The researches on mechanical oscillation suppression are mainly divided into the passive suppression and the active suppression method. In [9]–[11], methods such as PI control strategy based on speed differential feedback, multi-degree-of-freedom control strategy, state feedback control, and load disturbance observation feedback have been proposed, which made great contributions to two-mass system vibration suppression control. However, these methods are complicated to implement due to the complex parameter design. In [12], two degrees of freedom robust controller is proposed based on $\mu$-Synthesis structure to cope with the time-varying nature of mechanical parameters, which can obtain a good control effect. However, the genetic algorithm in this method need a long response time, and the parameters of feedforward compensation are difficult to design.

Most directly, filters such as notch filter, low-pass filter, Bi-Quad filter and acceleration feedback method are used to suppress the vibration [13]–[16]. Among these methods, the notch filter and Bi-Quad filter have better suppression effects, while the notch filter cannot adjust the notch width and the notch depth at the same time; Bi-Quad filter has poor system parameter robustness, and cannot adaptively identify the anti-resonant frequency of the system. Moreover, both notch filter and Bi-Quad filter are more suitable for mid and high frequency resonance suppression. When being used for low frequency vibration suppression, the two filters will reduce the system bandwidth. The acceleration feedback method is suitable for systems with relatively high damping factor and low vibration frequency, and the bandwidth of the observer will limit the realization of the algorithm.

Indirectly, input shaping technology was applied to vibration suppression, expanding from flexible beam structures (such as robotic arms) to hard disk servos with high rigidity resonance [17]–[19]. However, in multi-axis system, the parameters of each axis are different, the adoption of input shaper will make the multi-axis motion trajectory not synchronized.

In order to achieve both high precision positioning and vibration suppression in the ball-screw driven system with both drive side and load side position sensors, considering the characteristics of FCL and SCL, a dual-position feedback control method is studied in this paper, as shown in Fig. 1(c). The dual-position feedback control contains two control loops: a SCL that detects the position from the drive side, and a FCL that is based on a linear encoder from the load side. In SCL, high gain control can be achieved since the nonlinear element is not contained in the closed loop. The FCL improves accuracy by compensating the errors that cannot be controlled by the SCL. Since the FCL is only used to compensate position error, it performs well despite the lower gain. But it should be noticed that the selection rules of the compensation element play an important role in position control performance. So, in order to solve the vibration frequency variation under different servo drive systems, filter is introduced in the feedback channel in this paper, which improves the feasibility and simplicity of dual-position loop control.

Further, the backlash, friction, deformation and other nonlinear factors in the surface of the mechanical transmission structure can result in an unexpected short-term standstill phenomenon when doing a speed reversal movement [20].

![Figure 1](image-url)
The short-term standstill phenomenon means lost motion, which is impossible to be eliminated by choosing a position controller due to the nonlinear factors [21], [22]. The lost motion will introduce a typical contour error-over-quadrant error in the load side. However, studies on over-quadrant error mitigation in full-closed loop control system is rare in the existing reference. Most of the researches on this issue are conducted under semi-closed loop conditions, and most of the proposed methods need to know the backlash width or friction model parameters in advance [23], [24].

In addition, many scholars focus on the design of control law for the nonlinear systems, these methods can achieve good control performance in applications such as robot arm, flexible beam, and automatic welding machine [25]–[27]. These control methods solve the control problems of nonlinear systems from the position command trajectory side, and are usually implemented in the motion control unit. However, in CNC machine tools and similar high-precision feed systems, time lag is unavoidable in the response of the load side position to the position reference and the nonlinear factors exist for a short time. Therefore, this kind of transient nonlinear problems cannot be solved from the trajectory reference side, and needs to be solved from the inner loop control of the system, such as the servo unit.

In this paper, after understanding the mechanism of over-quadrant error, the effective information of each loop of servo drive control are combined, and an adaptive over-quadrant error compensation method based on torque feedforward is proposed. The proposed method does not need to measure the backlash width in advance, and the compensation signal is automatically generated by the proposed algorithm.

This paper is organized as follows. Section II illustrates the characteristics of a ball-screw driven system. Section III discusses the property of FCL, SCL and the proposed position controller in detail. Section IV analyzes the mechanism of over-quadrant error and introduces the proposed compensation method. The experimental results are given in Section IV. Conclusions are summarized in Section VI.

II. CHARACTERISTICS OF BALL-SCREW DRIVEN SERVO SYSTEM

In order to analysis the position control characteristics of the ball-screw driven servo system with dual-position sensor feedback as shown in Fig. 2, the frequency response of a two-mass system in Fig. 3(a) with the parameters in Table. 1 is introduced first. Block diagram of the two-mass model is shown in Fig. 3(b).

According to Fig. 3(b), the differential equations of two-mass model can be expressed as

\[
\begin{align*}
J_m \dot{\omega}_m &= T_e - T_s \\
J_l \dot{\omega}_l &= T_s - T_l \\
T_s &= C_s (\dot{\theta}_m - \dot{\theta}_l) + K_s (\theta_m - \theta_l) \\
\omega_m &= \dot{\theta}_m \\
\omega_l &= \dot{\theta}_l
\end{align*}
\]  

where \( J_m, J_l \) are the motor and load inertia; respectively, \( K_s, C_s \) are the shaft stiffness coefficient and damping factor; respectively; \( \theta_m, \theta_l \) are the position feedbacks from motor side and load side; respectively; \( \omega_m, \omega_l \) are the speed of motor side and load side, respectively; \( T_e \) is the drive shaft torque.

Since the commonly used ball-screw transmission mechanism has a small viscous friction coefficient and weak damping effect in practice, when analyzing the characteristics of the full-closed loop position control system, the damping coefficient can be ignored \((C_s = 0)\). From equation (1), transfer function (t.f.) of motor speed and electromagnetic torque can be expressed as (2), the t.f. of load speed and electromagnetic torque can be expressed as (3), and the t.f. between load speed and motor speed can be expressed as (4)

\[
G_1(s) = \frac{\omega_m}{T_e} = \frac{J_l s^2 + K_s}{J_m J_l s^3 + s (J_m + J_l) K_s}
\]
The frequency corresponding to the conjugate pole in (3) and (4) are the natural torsional frequency \( f_{NTF} \) and anti-resonant frequency \( f_{ARF} \), respectively, which can be given as

\[
\begin{align*}
    f_{ARF} &= \frac{1}{2\pi} \sqrt{\frac{K_s}{J_1}} \text{ Hz} \\
    f_{NTF} &= \frac{1}{2\pi} \sqrt{\frac{K_s}{J_1} \left(\frac{1}{J_m} + \frac{1}{J_l}\right)} \text{ Hz}
\end{align*}
\]

The two frequencies can be found in the bode diagram of transfer t.f. \( G_1(s) \) and \( G_2(s) \), as shown in Fig. 4. As can be seen from Fig. 4, the phase delay of the t.f. \( G_1(s) \) does not exceed 90 degrees, and the phase of t.f. \( G_2(s) \) is delayed by more than 90 degrees in a frequency band higher than the resonance frequency because there is no anti-resonance.

\( G_2(s) \) shows the relationship between the load speed and the electromagnetic torque. When the frequency reaches the anti-resonant frequency, the energy input to the motor immediately flows into the load. And from \( G_1(s) \), it can be seen that the motor does not obtain energy from the electromagnetic torque at this frequency, so that the kinetic energy of the load side and the elastic potential energy between the shafts are mutually converted, which results in shaft torsional vibration and end oscillation. Especially in the system with large motor inertia and small elastic coefficient, which means the anti-resonance frequency is low. With the increase of controller gain, this frequency band can be easily covered within the system control bandwidth, which is also the main reason why the flexible system is difficult to control.

### III. POSITION CONTROLLER DESIGN

#### A. CHARACTERISTICS OF FCL AND SCL

To better understand the position control performance of the introduced ball-screw driven system. The following contents give discussion on the design procedure and characteristics of the semi-closed loop and full-closed loop position controller. Fig. 5 shows the position controller with cascade P-PI (Proportional - Proportional Integral, P-PI) controller. The semi-closed loop control as shown in Fig. 5(a) realizes position control with the position feedback from the motor shaft. Therefore, a stable servo drive can be obtained with SCL control, but with large position error. In order to achieve high precision position control, the feedback information should be directly measured from the mechanical load as shown in Fig. 5(b), which is called full-closed loop control. Moreover, the SCL and the FCL have the same inner velocity loop.

Through the Mason formula, the closed loop transfer function of SCL control can be expressed as

\[
\theta_{ref} = \frac{b_1 s + b_0}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \theta_{ref} - \frac{T_L(s)}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} T_L(s)
\]

where

\[
\begin{align*}
    a_5 &= J_m J_l, \quad a_4 = J_l k_m k_{vp}, \\
    a_3 &= J_l k_m k_{vi} + J_l k_m k_{vp} k_p + J_l k_s + J_m k_s, \\
    a_2 &= J_l k_m k_{vi} k_p + k_m k_s k_{vp} k_p, \quad a_1 = k_m k_s k_{vi} + k_m k_s k_{vp} k_p, \\
    a_0 &= k_m k_s k_{vi} k_p,
\end{align*}
\]
\[ b_1 = k_m k_s k_p, \quad b_0 = k_m k_s k_p, \]
\[ c_3 = J_m, \quad c_2 = k_m k_p, \quad c_1 = k_m k_v + k_m k_p k_p, \quad c_0 = k_m k_v k_p \]

The closed loop transfer function of FCL control can be expressed as
\[
\theta_i(s) = \frac{b_3 s + b_2}{a_{12} s^5 + a_{10} s^4 + a_9 s^3 + a_8 s^2 + a_7 s + a_6} + \frac{\theta_{\text{ref}}}{c_6 s^3 + c_5 s^2 + c_4 s + \varepsilon_3} T_i(s)
\]
(8)

where
\[ a_{12} = J_m I_1, \quad a_{10} = J_I k_m k_p, \quad a_9 = J_I k_m k_v + (J_m + J_I) k_s, \]
\[ a_8 = k_m k_s k_v, \quad a_7 = k_m k_s k_v + k_m k_p k_p, \quad a_6 = k_m k_v k_p, \]
\[ b_3 = k_m k_s k_p k_p, \quad b_2 = k_m k_v k_p k_s, \quad c_6 = J_m, \]
\[ c_5 = k_m k_p, \quad c_4 = k_m k_v + k_s \]

In (7) and (8), \( k_p \) is the position controller gain, \( k_{vp}, k_v \) are the proportional and integral gains of speed loop controller; respectively, \( k_m \) is the torque coefficient of the motor.

Based on the final value theorem, the steady-state error of SCL and FCL control under unit step disturbance can be expressed as
\[
e_{\text{semi}} = \lim_{t \to \infty} e_{\text{semi}}(t) = \lim_{s \to 0} s E_{\text{semi}}(s)
\]
\[
= \lim_{s \to 0} s \left( -\frac{c_3 s^3 + c_2 s^2 + c_1 s + c_0}{a_5 s^4 + a_4 s^3 + a_3 s^2 + a_2 s + a_1} \right) s
\]
\[
= -\frac{1}{k_p} (9)
\]
\[
e_{\text{full}} = \lim_{t \to \infty} e_{\text{full}}(t) = \lim_{s \to 0} s E_{\text{full}}(s)
\]
\[
= \lim_{s \to 0} s \left( -\frac{c_6 s^3 + c_5 s^2 + c_4 s}{a_{12} s^5 + a_{10} s^4 + a_9 s^3 + a_8 s^2 + a_7 s + a_6} \right)
\]
\[
= 0 (10)
\]

It can be found that under unit step disturbance, the SCL control has a steady-state error, which is inversely proportional to the shaft stiffness coefficient. When adopting FCL control, step disturbance will not produce steady-state error, thus higher control accuracy can be achieved.

The use of transfer function is a popular approach to analyze the properties of position control. Equation (11) and (12) are the open-loop transfer functions of SCL control in Fig. 5(a) and FCL control in Fig. 5(b).

\[
G_{\text{semi--open}} = \begin{bmatrix} k_{\text{pos}} (J_2 k_s k_{sp} s^3 + J_2 k_s k_{si} s^2 + k_s k_{sp} s + k_s k_{si}) \cr J_1 J_2 s^5 + J_2 k_s k_{sp} s^4 + [J_2 k_s k_{si} + (J_2 + J_1) k_s] s^3 \cr + k_s k_{sp} s^2 + k_s k_{si} s \end{bmatrix}
\]
(11)

\[
G_{\text{full--open}} = \frac{k_{\text{pos}} (k_s k_{sp} s + k_s k_{si})}{J_1 J_2 s^5 + J_2 k_s k_{sp} s^4 + [J_2 k_s k_{si} + (J_2 + J_1) k_s] s^3 + k_s k_{sp} s^2 + k_s k_{si} s}
\]
(12)

In order to compare the performance of SCL and FCL control, the Bode diagrams of the transfer function (11) and (12) are shown in Fig. 6. It can be seen that the SCL has a higher position gain \( k_{\text{pos}} \) margin than that of FCL. The \( G_{\text{full--open}} \) margin of t.f. \( G_{\text{semi--open}} \) is increased rapidly because there is no anti-resonant frequency in the control loop. Therefore, when the FCL control bandwidth is higher than the resonance frequency, it will cause system unstable. Thus, in a ball-screw driven system, it is difficult to maintain the stability of the system under high gain feedback using the FCL control.

![Bode diagram for SCL/FCL control. (a) Bode diagram for SCL control. (b) Bode diagram for FCL control.](image)

According to the aforementioned analysis, we can conclude that in a ball-screw driven system, the semi-closed loop control method can obtain a stable servo drive, but with large position error. Obviously, the full-closed loop control method can mitigate the position errors in the controlled load side due to the use of position information of the load side, but the machine resonances are easily stimulated when setting a relatively high position loop gain. Therefore, it is important to choose a proper control method that can provide a high precision and stable position loop in industrial applications.

### B. DUAL-POSITION FEEDBACK CONTROLLER DESIGN

As mentioned above, the use of load-side position sensor feedback can improve position accuracy. In this section, a position control method named frequency selective feedback control (FSFC) uses both drive-side and load-side position feedback is introduced, the block diagram of the control

![Block diagram of dual-position feedback control](image)
structure is shown in Fig. 7. The drive is assumed to be a typical commercial electrical servo drive with a current loop and velocity loop inside a position loop. The FSFC contains two control loops: a semi-closed loop that detects the position from the drive side which can eliminate the mechanical resonance from the position servo loop, and a full-closed loop based on a linear encoder from the load side which is used to compensate for the position errors. The compensation errors are measured from the difference between the drive side and load side position feedbacks. Therefore, FSFC can naturally address both the drive side and load side position information and improve the tracking performance. The closed loop transfer function of FSFC control can be expressed as

$$\theta_1(s) = \frac{b_{5s} + b_4}{a_{17}s^5 + a_{16}s^4 + a_{15}s^3 + a_{14}s^2 + a_{13}s + a_{12}} \theta_{ref}$$

(13)

where

$$a_{17} = J_m l_1, a_{16} = J_1 k_m k_{vp},$$
$$a_{15} = J_1 k_m k_{vi} + J_1 k_m k_{vp} k_p (1 - Q) + (J_m + J_1) k_s,$$
$$a_{14} = J_1 k_m k_{vi} k_p (1 - Q) + k_m k_s k_{vp} k_p,$$
$$a_{13} = k_mk_v k_{vi} + k_m k_s k_{vp} k_p, a_{12} = k_m k_s k_{vi}, k_p,$$
$$b_5 = k_m k_{vi} k_{vp} k_p, b_4 = k_m k_s k_{vi} k_p,$$
$$c_{10} = J_1, c_9 = k_m k_{vp},$$
$$c_8 = k_m k_v k_{vi} + k_m k_{vp} k_p (1 - Q) + k_s, c_7 = k_m k_v k_{vi} (1 - Q)$$

Based on the final value theorem, the steady-state error of FSFC under unit step disturbance can be expressed as

$$e_{dual} = \lim_{t \to \infty} e_{dual}(t) = \lim_{s \to 0} s E_{dual}(s)$$

$$= \lim_{s \to 0} s \left( -\frac{c_{10}s^3 + c_9s^2 + c_8s + c_7}{a_{17}s^5 + a_{16}s^4 + a_{15}s^3 + a_{14}s^2 + a_{13}s + a_{12}} \right)$$

(14)

Here, the transfer function of FSFC from reference position $\theta_{ref}$ to $\theta_1$ is given as (15), as shown at the bottom of the page, where Q represents the low-pass filter (LPF), and the transfer function of Q gives as:

$$Q = \frac{1}{1 + \tau s}$$

(16)

where $\tau$ is the time constant of the low-pass filter. The difference between the drive side position and the load side position is used as an input to a LPF. The bandwidth of the low-pass filter is determined by setting the parameter $\tau$. When $\tau = 0$, then FSFC is same as full-closed loop control; when $\tau = \infty$, then FSFC is same as semi-closed loop control. In practice, the filter is setting at a very low bandwidth for the reason to remove structural dynamic frequencies from the correction process.

The Bode diagram of $G_{FSFC-open}$ when setting $\tau = 0.1s$ and $\tau = 0.001s$ are shown in Fig. 8.

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**FIGURE 7.** Bode diagram of frequency selective feedback control.

**FIGURE 8.** Bode diagram for FSFC (a) Bode diagram for $\tau = 0.1s$, (b) Bode diagram for $\tau = 0.001s$. It can be seen from the Bode diagram that the FSFC is equal to semi-closed loop control under transient behavior, and is equal to full-closed loop control under steady state behavior.
IV. OVER-QUADRANT ERROR COPENSATION METHOD

The position loop controller plays a key role in the servo drive system, especially in the non-direct drive mechanical transmission system. Its performance can directly determine the quality of the position control. The FSFC can provide a good position loop with high gain margin and high control precision. However, backlash, friction, deformation and other nonlinear factors in the surface of the mechanical transmission structure of ball-screw stage can result in an unexpected short-term standstill (lost motion) phenomenon when doing speed reversal movement, which will directly affect the machining accuracy of the worktable. In SCL system, the lost motion phenomenon causes the positioning error, which is mainly affected by the width of the backlash. In FCL system, the positioning error can be completely eliminated due to the use of load side position information. However, there will still be a position error over the quadrant, which is mainly composed of the bandwidth of servo axis and friction, and is restricted by the size of the backlash. The over-quadrant error is impossible to eliminate by the position controller itself. Hence, an adaptive compensation method is proposed in this section.

A. ANALYSIS OF THE MECHANISM OF OVER-QUADRANT ERROR

The friction-drive hysteresis backlash model can describe the reverse lost motion phenomenon by including the characteristics of friction in the reverse process, as shown in Fig. 9. The structure of the hysteresis backlash model is shown in Fig. 10.

In Fig. 10, \( k_l x_m \) and \( x_i \) are the input and output of the backlash model, respectively, which can be expressed as

\[
\dot{x}_i = \begin{cases} 
  k_l \dot{x}_m(t), & \dot{x}_m(t) > 0 \text{ and } k_l x_m(t) - b/2 = x_i(t) \\
  k_l \dot{x}_m(t), & \dot{x}_m(t) < 0 \text{ and } k_l x_m(t) + b/2 = x_i(t) \\
  0 & \text{others}
\end{cases}
\]  

(17)

To facilitate analysis of over-quadrant error, on the basis of hysteresis backlash model, the FCL system model can be simplified as Fig. 11. In which \( B_l \) is the backlash model,

\[
M(\omega) = |G_c(j\omega)| = \frac{k_a k_p}{\sqrt{(k_a k_p)^2 + \omega^2}}
\]

(20)
To facilitate the analysis and display the over-quadrant error phenomena, the position error $\varepsilon_{h,circular}$ at the over-quadrant can be divided into two parts, radial error $\varepsilon_r$ and transient backlash error $\varepsilon_b$, as follows

$$\varepsilon_r = r \cos \phi - r$$
$$\varepsilon_b = r \cos \phi \left( \sqrt{1 + \sin^2(\omega t_b)} - 1 \right) \quad (21)$$

It can be seen from (21) that when the dynamic parameters of the XY axis are the same, the circular contour error consists of two components: radial error and circular transient backlash error. Among them, the amplitude of the radial error always remains unchanged, and its amplitude is determined by the radius of the circle $r$ and the amplitude-frequency response of the servo axis; the transient backlash error only exists for a short time at the over-quadrant, and the amplitude of the transient backlash error is related to the backlash width $\beta$, the feed angular rate $\omega$ and the circle radius $r$.

On the other hand, if only considering the friction characteristics of the backlash meshing period, the over-quadrant error can be analyzed as follows.

Combining Fig. 2 and Fig. 11, if ignoring the mechanical damping and inductance of the motor, the current differential equation of the motor can be obtained as

$$[(x_{cmd} - x_l)K_p - \dot{\theta}]K_{vp} - i \right) K_c - \dot{\theta}K_e = Ri \quad (22)$$

where, $\theta$ is the motor shaft position, $K_c$ is the gain of current loop, $K_e$ and $R$ are the back EMF coefficient and resistance of the motor, respectively.

Assuming an elastic connection exists between the ball screw and the worktable, the partial torque balance equation of the motor and the ball screw is

$$iK_m = J\ddot{\theta} + (x_m - x_l)K_cK_a \quad (23)$$

where $x_m$ is the position of ball-screw nut, $k_a$ is the transmission coefficient.

In addition, during the backlash meshing, the differential equation of the force balance of the table part can be expressed as

$$(x_m - x_l)K = M\ddot{y} + B\ddot{y} + F_f \text{sgn}(y) \quad (24)$$

where $M$ is table quality, $B$ is the friction damping coefficient, $F_f$ is the friction, and $\text{sgn}()$ is sign function.

Since the bandwidth of the current loop is much larger than that of the position loop, the current and the motor output torque can be regarded as a linear relationship. From equation (22) to (24), the system differential equation can be obtained as

$$[(x_{cmd} - x_l)K_p - \dot{\theta}]K_{vp}K_cK_m/R = J\ddot{\theta} + K_a(M\ddot{y} + B\ddot{y} + F_f \text{sgn}(y)) \quad (25)$$

For the circular trajectory, suppose the servo axis moves in the X-Y plane, passing through the X-axis reversal position with radius $r$, at angular velocity $\omega$, near the quadrant position $(x_{cmd}(t) = r \sin(\omega t), t_0 = \pi/2\omega)$, it has

$$\dot{\theta}(t_0) \approx 0, \quad \dot{x}_l(t_0) \approx 0$$

$$\ddot{\theta} \approx K_a r \omega^2, \quad \ddot{x}_l(t_0) \approx r \omega^2 \quad (26)$$

In a FCL system, the lost motion phenomenon is mainly affected by the friction during the over-quadrant period, but the duration of the lost motion phenomenon is affected by the width of the backlash, which is also the main factor affecting the amplitude of the backlash error. At the over-quadrant, the frictional force is close to the maximum static friction $F_m$, $F_f = F_m$, substituting into equation (25), the position error at the over-quadrant is given by

$$x_{cmd} - x_l = \frac{K_a R}{K_p K_{vp} K_c K_m} [(M + J) r \omega^2 + F_m]$$
$$\varepsilon_f = K [(M + J) r^2/r + F_f \text{sgn}(\omega)] \quad (27)$$

where $K = K_a R/(K_p K_{vp} K_c K_m)$.

It can be seen from equation (27) that the position error at the over-quadrant is composed of the frequency response characteristics of the position loop and the friction force. For the friction part, because the system state is in zero-speed zone and the low-speed zone, even if the LuGre dynamic friction model can describe many friction phenomena, the effect of the duration of the lost motion phenomenon that appears under the actual FCL condition cannot be obtained. Therefore, it is very difficult to determine the motion characteristics under the influence of nonlinear friction and establish a friction model.

In fact, (27) and (19) have similar structures, and both show that the quadrant error is composed of two components: radial error and circular transient error. Since in actual processing, when the over-quadrant error occurs, the phenomenon of lost motion on load side will occur, while the motor side will not be affected. Therefore, it can be concluded that the transient error is caused by the existence of friction factors, but at the over-quadrant, the duration of lost motion of the load side position directly determines the amplitude of transient error, which is mainly related with the width of the backlash.

It can be seen from the above discussion that, compared with SCL, the positioning error can be completely eliminated in FCL due to the introduction of the position information from the end. However, due to the lost motion phenomenon, there will still be a position error at the over-quadrant, which is mainly caused by inertia and friction. But the magnitude of the over-quadrant error is constrained by the size of the backlash.

B. ADAPTIVE OVER-QUADRANT ERROR COMPENSATION METHOD BASED ON TORQUE FEEDFORWARD

Noticing the fact that the backlash meshing process can be regarded as an integral process, the conventional method realizes error compensation by combining the amplitude and duration of the speed compensation signal with the backlash width. This method can reduce the error to a certain extent, but it needs to know the width of backlash in advance. Moreover, the choice of the amplitude and the duration time of the applied compensation speed signal is not a simple matter in actual application.
Different from the conventional backlash compensation method, the theoretical analysis of over-quadrant error provides a compensation idea from the perspective of torque balance. The block diagram of adaptive over-quadrant error compensation method is shown in Fig. 12. The proposed compensation method is divided into two parts, the first part is the automatic acquisition of the compensation signal, and the second part is the function of the compensation signal. In the case that the backlash width is unknown, through step 1, in pre-setting stage, given the circular track command signal, complete the automatic acquisition of the backlash compensation signal. Under normal working conditions, step 2 realizes real-time over-quadrant error compensation.

\[
\text{backlash forward meshing} \begin{cases} 
\text{start} & \text{if } \dot{\theta} > 0 \text{ and } \dot{x}_l = 0 \\
\text{end} & \text{if } \dot{\theta} > 0 \text{ and } \dot{x}_l > 0 
\end{cases}
\]

(28)

\[
\text{backlash reverse meshing} \begin{cases} 
\text{start} & \text{if } \dot{\theta} < 0 \text{ and } \dot{x}_l = 0 \\
\text{end} & \text{if } \dot{\theta} < 0 \text{ and } \dot{x}_l < 0 
\end{cases}
\]

(29)

While obtaining the backlash meshing interval sign signal by equation (28) and (29), the feedback current values corresponding to point a, b, c, and d in Fig. 12 can be obtained from the servo drive. The current distortion that occurs during the backlash meshing interval is mainly affected by the maximum static friction force $F_m$. If the response time from $i_a$ to $i_b$ or $i_c$ to $i_d$ is reduced, the duration of the lost motion phenomenon can be shortened, thereby reducing the over-quadrant error. So, the compensation signal $i_{cp}$ can be expressed as

\[
i_{cp} = \begin{cases} 
i_b - i_a & \text{if } \dot{\theta} > 0 \text{ and } \dot{x}_l = 0 \\
i_d - i_c & \text{if } \dot{\theta} < 0 \text{ and } \dot{x}_l = 0 \\
0 & \text{others} 
\end{cases}
\]

(30)

V. EXPERIMENTAL RESULTS

In order to test the presented method for a ball-screw driven system, the experiments are carried out on CNC machine tools, which is depicted in Fig. 14, the main parameters of the CNC are list in Table 2. The actual resonant frequency of the linear system is 36.02Hz, which is obtained from Chirp signal injection test, as shown in Fig. 15.
TABLE 2. Main parameters of CNC machine tools

| Parameters          | Value    | Unit     |
|---------------------|----------|----------|
| Screw lead          | 16       | mm       |
| Scale accuracy      | 50       | nm       |
| Load inertia        | 8.67×10⁻³ | kg·m²    |
| Motor rated power   | 3.9      | kW       |
| Motor inertia       | 2.89×10⁻³ | kg·m²    |
| Motor rated torque  | 15       | N·m      |
| Motor rated speed   | 2500     | r/min    |

included in the loop control, it is more likely to cause position loop oscillation problems than semi-closed-loop control. To verify the correctness of the theoretical analysis, the experiments were carried out under the position command as shown in Fig. 16.

As shown in Fig. 17, when the gain of the position loop controller is low ($K_p = 750\text{rad/s}$), the overall trend of the position tracking error of the two controls is the same regardless of the acceleration phase or the deceleration phase, and the tracking error is between $−0.2\text{mm}$ to $0.2\text{mm}$. As shown in Fig. 18, when the gain of the position loop is increased ($K_p = 1500\text{rad/s}$), the tracking error of the two types of controls is significantly reduced, and it drops to between $−0.15\text{mm}$ and $0.15\text{mm}$. But unlike the low position gain, Fig. 18 shows that the tracking error of the conventional full-closed-loop control gradually diverges and oscillates, accompanied by the low-frequency vibration of the machine tool, while the semi-closed-loop control remains stable overall. It can be seen from the partially enlarged view in Fig. 18 that the difference between the two is more obvious. The bump in the figure at the zero-crossing point originates from the lag when the speed crosses zero.

Perform FFT analysis on the acceleration value of the position signal from the linear encoder under two kinds of controls, as shown in Fig. 19. It can be found that the oscillation frequency of the conventional FCL control system is $32.74\text{Hz}$. With the same position gain, only the FCL control stimulates the oscillation at this frequency point. Compared with the Chirp sweep result, $f_{ARF} = 36.02\text{Hz}$, there
exist slight deviation between the two frequencies, which is caused by the digital discrete system and the frequency sweep method.

Therefore, the system is controlled stably under semi-closed loop control, and the mechanism of oscillation under full closed loop control is consistent with the previous analysis, that is, caused by ARF.

It can be seen from the above experimental results that although reducing the position gain can suppress the conventional FCL control oscillation problem, but the high position gain means the reduction of the tracking error. Thus, the SCL control can achieve smaller tracking error with higher position gain. On the contrary, the conventional FCL control cannot achieve the position performance of the SCL control, so the advantages of positioning accuracy cannot be brought in with a low position gain.

In FSFC, the filter time constant is selected as 100ms, and the position loop gain coefficient is $K_p = 1500\text{rad/s}$. The experimental results are shown in Fig. 20. From the experimental waveforms, it can be found that from the overall trend, under the same parameters, compared with the conventional FCL control, the FSFC can ensure the stability of the system. In the partially enlarged view, during the whole acceleration and deceleration process, the FSFC is equivalent to the SCL control. In addition, it can be seen from the figure that, due to the error between the two position feedback, the position tracking error of the SCL control shifts downward as a whole, but the FSFC can correct the tracking error back to near zero.

The analysis of acceleration value of the position signal of the linear encoder under the three kinds of control, as shown in Fig. 21. It can be seen that the FFT of the SCL control basically coincides with the result of the FSFC. It means that FSFC can avoid triggering the oscillation of the resonance point with a high position loop gain.

### B. OVER-QUADRANT ERROR MITIGATION PERFORMANCE

In order to verify the effectiveness of the compensation algorithm, experiments were carried out under single-axis and dual-axis conditions. Through the test, the X-axis and Y-axis backlash width values of the experimental platform are $3.5\mu\text{m}$ and $6.5\mu\text{m}$.

In order to verify the effectiveness of the proposed over-quadrant error mitigation method, we need to obtain the compensation signals first. The speed and torque signals during the acquisition of the torque feedforward compensation signal of Y servo axis are shown in Fig. 22. In the compensation signal acquisition process, the position reference is given as $\text{sine}$ wave with amplitude of $10\text{mm}$ and a frequency of $0.5\text{Hz}$. It can be seen from the partial enlarged view in Fig. 22 that when the motor is running during the quadrant, the load speed will appear lost motion phenomenon, which is consistent with the theoretical analysis. The backlash meshing interval flag signal can be obtained through the “lost motion” phenomenon, and the electromagnetic torque values corresponding to the rising and falling edges of the signal are recorded, respectively. In Fig. 22, $T_e(t_1)$ to $T_e(t_2)$ are the electromagnetic torque values at the beginning and end of the backlash reverse meshing, $T_e(t_3)$ to $T_e(t_4)$ are the electromagnetic torque values at the beginning and end of the backlash forward meshing. Therefore, through this identification, during backlash forward meshing and backlash reverse meshing, the amplitude of the compensation signal of the electromagnetic torque is $0.3\text{N\cdot m}$ and $−0.3\text{N\cdot m}$ respectively.
FIGURE 22. Torque compensation signal identification of Y servo axis.

FIGURE 23. Position and speed tracking performance without over-quadrant error compensation.

Fig. 23 shows the position and speed response waveforms of Y servo axis without compensation. It can be seen that when the over-quadrant error compensation method is not implemented, lost motion phenomenon is happened at the end position when the speed reversal, and the duration is about 0.029s. In addition, when the speed crosses zero, the position tracking error suddenly increases, and its value is about 7µm, which is approximately equal to backlash value.

In order to show the effectiveness of the over-quadrant error mitigation method proposed in this paper, Fig. 24 compares the compensation results of the method proposed in this paper with the traditional method used in [28] during the over-quadrant region. The tradition over-quadrant compensation method used in [28] is realized by speed feedforward compensation. The speed feedforward signal \( c(t) \) is a square wave pulse signal, and its amplitude \( c_0 \) and duration time \( t_c \) are calculated by (31).

\[
\int_0^{t_c} c(t)dt = b \\
c_0 t_c = b \tag{31}
\]

FIGURE 24. Position tracking performance with different compensation method. (a). Waveform of position response. (b). Waveform of position tracking error.

The experiment of Fig. 24 is carried out in the Y servo axis. In the traditional method, the amplitude of the speed feedforward signal is 1mm/s, and the duration time is 0.007s and 0.008s. The over-quadrant error without compensation is shown by the blue curve in Fig. 24, and its amplitude is about 0.007mm, which is consistent with the width of the backlash. When using the traditional method, as the duration time \( t_c \) increases, the magnitude of over-quadrant error decreases accordingly.

When \( t_c \) is 0.007s, the magnitude of the over-quadrant error is about 0.0035mm, and when \( t_c \) is 0.008s, the over-quadrant error is about 0.0025mm. However, it is obvious that continuing to increase the duration time of the compensation signal will not lead to greater reduction in the magnitude of the over-quadrant error. On the contrary, over-compensation will occur, which will affect the stability of the system. This is decidedly prohibited in the position servo application. In contrast, the proposed method can compensate for...
the over-quadrant error without setting the backlash value in advance. With compensation, the over-quadrant error can almost be compensated completely.

Fig. 25 shows the results of the roundness test of circular track on the XY dual-axis platform. The radius of the circular reference is 50 mm, the feed direction is clockwise, and the feed rate is 1000 mm/min. The Renishaw QC20W wireless ballbar was used for roundness test.

It can be found from Fig. 25(a) that the X-axis over-quadrant contour error is 3 µm, and Y-axis over-quadrant contour error is 6 µm without compensation. After compensation, the X-axis over-quadrant contour error is about 1.1 µm, and the Y-axis over-quadrant contour error is about 2.4 µm. Therefore, the compensation method can greatly reduce the machining error caused by the backlash in the FCL servo machine tool.

VI. CONCLUSION

In this paper, a FCL position control method based on backlash compensation is adopted. The compensation method can adaptively finish the compensation which does not need to set the backlash width or friction model, and the compensation signal can be automatically generated by the proposed algorithm. The method is simple and effective, and avoids over-compensation or under-compensation. In the future work, the more detailed analysis of different types of position controller and the stability of the compensation method will be studied.

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M. Yang et al.: Vibration Suppression and Over-Quadrant Error Mitigation Methods for a Ball-Screw Driven Servo System

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