Entropic dark energy and sourced Friedmann equations

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Abstract

In this paper we show that a recent attempt to derive dark energy as an entropic force suffers from the same problems as earlier attempts motivated by holography. A possible remedy is again the introduction of source terms.
1 Introduction

The discovery by Bekenstein, [1], that black hole physics lead to thermodynamics, have by many been seen as an important clue to the nature of gravity and its quantization. An especially intriguing step forward in this context was Jacobson’s realization, [2], that the argument also goes in the other direction: you can start with thermodynamics and derive the Einstein equations from there. Many have tried to extend these ideas using holography, and apply them to, among other things, cosmology. The progress has been meager.

In particular, there has been attempts to derive dark energy using this kind of reasoning. Some examples can be found in [5] and [6]. The shortest line of reasoning goes like this. The number of degrees of freedom in a Hubble volume is limited by holography to be given by the area of the horizon. That is, $R^2$ in planckian units where $R \sim 1/H$. To each of these degrees of freedom you may associate a zero point energy of the order $1/R$. The total energy is then $\sim R^2 \times 1/R \sim R$. Spreading this out over the volume $R^3$ gives an energy density $1/R^2 \sim H^2$. Hence this is of the same order as the cosmological constant.

This approach suffers from many problems when you look at the details. The most obvious one is that the mere addition of a component with energy given by $H^2$, leads to the wrong equation of state: the universe will not accelerate. As we will review below, the situation can be improved if we introduce source terms, [8], [7].

In [3] an attempt was made to further develope the ideas of Jacobson. Whether this succeeds or not, depends on whether new conclusions can be reached concerning, e.g., cosmology. This is the subject of a recent paper, [4], where it is claimed that the results of [3] automatically leads to an explanation of dark energy. In this note I will put this claim in the context of previous research, and argue that the status of the subject is unchanged.

2 Modifying the Friedmann equations

In [4] it was proposed that one of the Friedmann equations should be corrected by an extra term, $s$, according to

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_p^2} (\rho + 3p) + s. \tag{1}$$

It was argued that such a term can be motivated from the possible presence of usually discarded boundary terms in the Einstein equations. Alternatively, it can be argued to arise due to the presence of the entropic force discussed in [3]. If this argument is correct, does this mean that physicists over the years have missed an obvious term in the Friedmann equations? Is the claim in [3] that the entropic formulation is equivalent to ordinary gravity, or does it give new and different predicitions in the context of cosmology? What I want to argue for below is that a modification of this
kind does not represent something fundamentally different compared to what has already been considered.

Since we want to approach gravity and cosmology from a thermodynamical point of view, we should first make sure that the equations we write down are manifestly in this spirit. Following [7] and [8], one can apply the reasoning of Jacobson in a simplified cosmological setting. The starting point is the relation between the area of the horizon, and entropy for a black hole given by

$$S = \frac{M^2}{4} A.$$  \hfill (2)

We assume, like in [9], that the cosmological horizon determined by the Hubble constant plays a similar role as the horizon of a black hole, and assign an entropy to the horizon according to

$$S = \frac{\pi M^2}{H^2}.$$  \hfill (3)

From this we can now derive the Friedmann equations using the standard relation between flow of heat and entropy, $dQ = T dS$. The flow of heat out through the horizon is then related to a flow of entropy given by

$$\dot{Q} = \dot{S} T = A (\rho + p),$$  \hfill (4)

where

$$T = \frac{H}{2\pi}.$$  \hfill (5)

Expressing the entropy in terms of the horizon area and the Hubble constant, we find

$$\dot{H} = -\frac{4\pi}{M_p^2} (\rho + p),$$  \hfill (6)

which, indeed, is one of the Friedmann equations.

To completely specify the time evolution we also need the continuity equation

$$\dot{\rho} + 3H (\rho + p) = 0,$$  \hfill (7)

which, combined with (6), give rise to another of the Friedmann equations, i.e.,

$$H^2 = \frac{8\pi}{3M_p^2} \rho + \text{const.}$$  \hfill (8)

Usually, the two Friedmann equations together with the continuum equation are viewed on an equal footing keeping in mind that only two of them are independent. However, from our thermodynamical point of view, there is an important difference between the various choices. [8] is obtained from [9] using integration and there is, therefore, a corresponding constant of integration, the cosmological constant, which
does not appear in the basic equations (6). The usual interpretation is that the cosmological constant corresponds to matter with $p = -\rho$. However, from our thermodynamical point of view, it might be more natural to view the cosmological constant as part of the initial conditions.

Let us now come back to the modified equation (11). It is easy to see that unless $s$ is a constant, (11) is inconsistent with (10) and (7). (6) lies at the heart of thermodynamics and holography and should not be modified. (7), on the other hand, can be modified in a simple way by adding a source term:

$$\dot{\rho} + 3H (\rho + p) = q,$$

which is easily seen to lead to

$$H^2 = \frac{8\pi}{3M_p^2} \rho - \frac{8\pi}{3M_p^2} \int^t q dt. \quad (10)$$

Furthermore, we find that the source term is related to the extra term in (7) through

$$q = -\frac{3M_p^2}{8\pi} \dot{s}. \quad (11)$$

One can also, directly from (11) using (6), arrive at

$$H^2 = \frac{8\pi}{3M_p^2} \rho + s. \quad (12)$$

We conclude that the correcting terms discussed in (11), which are not constant in time, necessarily imply the inclusion of a source term for matter if we remain true to the thermodynamic view on gravity. The interpretation is that the would be cosmological constant, arising as a constant of integration, is promoted into a running dark energy. It is depleted as time goes by and the energy is converted into ordinary matter or radiation through the source term.

It is important to realize that the naive application of (11), assuming $\rho \sim a^{-3(1+w)}$, will not lead to the correct results in this setup. The form of $\rho$ will be different due to the energy creation, and a more careful analysis is necessary. This is what we turn to next.

### 3 Solving the sourced Friedmann equations

To proceed we need to be more specific. Let us assume that there are one matter component that is sourced by the dark energy and, for generality, one that is not. We have

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1In principle one can also modify equation (6) demanding instead that there are no source terms for matter. Since, according to [2], there is also a local version of (11), i.e. the Einstein equations, such a procedure implies that the entropic dark energy turns into a local fluid with very specific properties. It is not clear how to interpret this from a holographic point of view, and this option will not be considered further in this paper.

2See, however, previous footnote.
\[ \rho_s + 3H (\rho_s + p_s) = q \]  
\[ \dot{\rho}_{us} + 3H (\rho_{us} + p_{us}) = 0. \]  

(13)  
(14)

We assume that the equations of state are such that \( p_s = w_s \rho_s \) and \( p_{us} = w_{us} \rho_{us} \). One can then show, slightly generalizing [7], that

\[ a^2 H H'' + a^2 H'^2 + (4 + 3w_s) a H H' = -\frac{4\pi}{M_P^2} (1 + w_s) H^{-1} q + \frac{12\pi}{M_P^2} (1 + w_{us}) (w_{us} - w_s) \rho_{us}. \]  

(15)

This is a second order differential equation for \( H \), yielding two constants of integration. One will generalize the cosmological constant, and the other one will fix \( \rho_s \) at some moment in time.

Below we will focus on the case where the sourced matter is in the form of radiation. We will consider two examples of sources.

### 3.1 Entropic dark energy

To capture the entropic approach to dark energy we consider \( s = kH^2 \) for some numerical constant \( k \). The value of \( k \) does not affect the conclusions in any essential way. We find

\[ a^2 H H'' + a^2 H'^2 + 5a H H' = 4kaH H' + \frac{12\pi}{M_P^2} (1 + w_{us}) (w_{us} - w_s) \rho_{us}. \]  

(16)

The homogenous solution is

\[ H^2 = C_1 + C_2 a^{-4/(1+w)}, \]

where we note the presence of a cosmological constant, and a would be radiation component that is decaying less rapidly than \( a^{-4} \) (if \( k > 0 \)). What happens is that the dark energy proportional to \( H^2 \) decays as \( H \) decreases, and the energy is converted into radiation.

Going back to (12), we see that the cosmological constant necessarily is part of \( \rho \) and \( p \). Such a component is necessary in order for \( H^2 \) to be close to constant. Ironically, we conclude that the cosmological constant is a free parameter, and manifestly not given by the entropic contribution in this setup.

### 3.2 An example related to transplanckian physics

An extra source term can also be connected with a modified vacuum. This has been done in [7]. There it was assumed that some of the matter or radiation fields were
put in an excited state at small scales and high energy. In the case of inflation this would correspond to a deviation from the usual Bunch-Davies vacuum. This requires the introduction of a source term that injects the necessary energy into the system. The amount of modes in this excited state is given by the Bogolubov coefficient $|\beta_k|^2$. The total energy generated in this way is given by

$$\rho_\Lambda (a) = \frac{3}{2\pi} \int_\epsilon^\Lambda d\epsilon p^3 |\beta_p|^2 = \frac{3}{2\pi} \frac{\Lambda^4}{a^4} \int_{a_i}^a dx x^3 |\beta_x|^2,$$

where we have introduced a low energy cutoff corresponding to the present energy of modes that started out at $\Lambda$ at a time when the Hubble constant was as small as $H_i$.

A specific case of vacuum fluctuations, suggested in [10], with the characteristic values of the Bogolubov mixing given by

$$|\beta_k|^2 \sim \frac{H^2}{\Lambda^2},$$

was studied in [7]. If we take a derivative of the energy density with respect to the scale factor and use $\frac{d}{da} = \frac{1}{aH} \frac{d}{dt}$, we find

$$\dot{\rho}_s + 4H \rho_s = \frac{3}{2\pi} \Lambda^2 H^3,$$

and we can conclude that the source term is given by

$$q = \frac{3}{2\pi} \Lambda^2 H^3.$$

If $\Lambda \ll M_p$ then we find (ignoring $\rho_{us}$)

$$H^2 = C_1 a^{-2n_1} + C_2 a^{-2n_2}$$

where $C_{1,2}$ are constants of integration, and

$$n_{1,2} = 1 \pm \sqrt{1 - \frac{4\Lambda^2}{M_p^2}}.$$

That is, one term corresponding to a slowly decaying dark energy, and term corresponding to radiation decaying more slowly than usual due to energy transfer.

For these kind of sources we find that one can not assign an unambiguous value to the cosmological constant. We find, instead, that a fixed dimensionful cosmological constant, is effectively replaced by a dimensionless parameter determining the running, given by the ratio of a fundamental scale and the Planck scale.
4 Discussion

We have seen that the modified Friedmann equation (1) of [4] under some reasonable assumptions is equivalent to the addition of a source term to the continuity equation. The explicit example considered in [4], motivated by the entropic force, has the Friedmann equation modified by $s \sim H^2$. This is very similar to previous attempts to argue for the presence of a holographic energy of order $H^2$. As shown already in [6] and [7], this can be accommodated through the addition of a source term obtainable through (11). The conclusion is that the results of [4] do not go beyond those obtained previously in the literature. The motivation for how the dark energy component arises is subtly different but in essence the same.

The relation between gravity and thermodynamics and the possible consequences for cosmology and dark energy remain as elusive as ever.

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