Dynamic spherical cavity expansion analysis of concrete/rock based on Hoek-Brown criterion

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Abstract. Cavity expansion theory has been widely applied to predict the penetration depth of projectiles into geological materials, in which the yield criterion and the constitutive model are combined to describe the shear and compaction of the material, separately. For a precise description of the material behaviors in plastic for geological materials with varied compressive strength, in this present manuscript, the Hoek-Brown yield criterion and the Dilatant-Kinematic equation were introduced into the plastic region. A response region called elastic-cracking-dilatant-compaction region for concrete and rock was established. Then the effects of various parameters, including brittleness coefficient, material integrity coefficient and dilatant coefficient, on the relationship between cavity pressure and normal expansion velocity were discussed. Finally, a penetration depth formula of rigid projectiles into semi-infinite concrete and rock targets was established. It showed that, compared with the classical Forrestal model, the proposed model had a well-being as well as higher accuracy prediction of the penetration depth of concrete and rock materials with various strengths.

1. Introduction
The research of resistance models for projectiles penetrating into concrete or rock targets has great significance in the field of impact and defense. In the past 200 years, numerous empirical and theoretical resistance models [1-4] have been developed, and detailed information can be found in the review papers [5-7]. Among the numbers of theoretical models, the dynamic cavity expansion model has been widely applied in impact engineering for its simplicity, as the normal stress on the surface of a cavity is expressed as a quadratic function of the particle velocity. The pioneering work can be dating back to 1945, Bishop [8] firstly calculated the static pressure required to enlarge a spherical hole in a plastic-incompressible material. Subsequently, this theory was developed as dynamic cavity expansion theory(DCE) by applying the Hugoniot jump conditions and the conservation equations [9].

Since the constitutive model and yield criterion are combined to describe the material response in the plastic region in DCE, selecting a precise yield criterion and a constitutive model to capture the material response in penetration case is vital. In terms of the constitutive or EOS models, the
traditional HJC [10, 11] and P-alpha [12] model were selected to describe the compressive behavior of concrete materials. Recently, some new models have been proposed to meet the demand of the development of high-speed projectiles penetration. Wang [13] used the Bingham liquid constitutive model to characterize incompressibility for concrete material suffering ultra-high pressures, such as in the case of long rod hypervelocity penetration. Wu [14] introduced the Murnaghan equation to describe the plastic behavior of concrete material within the pressure of 50GPa. As for the yield criterion, the linear yield criteria were adopted in early stage to describe the material shear behaviors, such as Tresca [15], Mohr-Coulomb [16] and Griffith [17] yield criterion. Lately, Bavdekar [18] developed a generalized non-linear for brittle ceramic, named Extended-Mohr-Coulomb model. Wu [14] improved the dynamic cavity expansion model by using a hyperbolic yield criterion for concrete material. Those all above-mentioned constitutive models and yield criteria provide a good description of the brittle material mechanical characteristics in impacting conditions. However, some shortcomings, as shown below, should be considered when using the DCE theory.

(1) The linear yield criteria, as depicted in [26], are not sufficient to capture the response of the brittle materials. Although some non-linear yield criteria have been developed, the physical meaning of the parameters in those models are not explicit, such as the parameter ‘k’ in Extended-Mohr-Coulomb model [26]. This makes it difficult to be used in other brittle materials.

(2) The constitutive model and yield criterion are combined to describe the compaction and shear behaviors of concrete material, separately. It means that the geological materials were treated as the metals (no dilatancy flow in plastic region) referencing the methods in classical elastic-plastic mechanics. However, As the material under high pressure has become granular or pulverized, the concrete and the rock will undergo dilation and compaction.

(3) The existing models rarely verify its accuracy by the penetration experiments using geological materials with varied compressive strengths. On the other hand, the concrete and rock with different compressive strength have been widely used in practice, especially the development of the ultra-high performance concrete [19].

Considering the deficiencies of the existing models and the realistic demand, a new DCE model was established in this paper based on the Hoek-Brown yield criterion [20] and the Dilatant-Kinematic [21] equation in the plastic region in DCE. A numerical solution of cavity wall pressure is obtained by solving the dynamic spherical cavity expansion resistance. Then, a resistance model and a calculation formula of penetration depth are established. The reliability of the model is verified by comparison with test data.

2. Basis equations and division of the response regions

2.1. Description of Hoek-Brown yield criterion and Dilatant-Kinematic equation

In traditional dynamic cavity expansion theory, the Mohr-Coulomb yield criterion and Tresca-limit shear condition were combined to describe the shear behaviors of geological materials [9], as shown in figure 1(a). However, It is not sufficient to capture the non-linear relationship between \( P \) and \( \tau \) of material as observed in experimental [22]. To overcome this deficiency, the Hoek-Brown [20] model, a hyperbolic yield criterion, was introduced in DCE. It was a statistic results based on a significant
quantity of laboratory triaxial test data for various rock types, as depicted in figure 1(b). The equations
selected to describe the yield criterion of the concrete/rock is

\[
\sigma_1 = \sigma_3 + Y \sqrt[m]{\frac{\sigma_3}{Y} + s}
\]

where \(\sigma_1\) and \(\sigma_3\) are respectively the first and third principle stresses of concrete or rock materials;
Y is the compressive strength; \(m\) and \(s\) are material constants.

In [23], it has been derived theoretically from fracture mechanics that the constant \(m\) is not simply an empirical constant but represents the brittleness of rock-like materials; \(s=1\) for intact rock.

The relationship between \(Y / f_t\) and \(m\) is

\[
\frac{Y}{f_t} = 8.62 + 0.7m
\]

where \(f_t\) is the uniaxial tensile strength of the rock material.

However, note that equation (2) can only be used for brittle rocks and common strength concretes, as the statistic results did not include the experimental data of high strength concrete under high hydrostatic pressure. Thus, for the high strength concrete, the value of \(m\) should be modified according to the triaxial test results. In figure 2, the yield envelopes of Hoek-Brown model shows great agreement with the test data [22], in which \(m\) are separately 2.5 and 5.5 for the 140 MPa and 48 MPa concrete. Then adding the test data from Li [24], a quadratic function relation between concrete compressive strength and \(m\) is obtained by fitting, as shown in figure 3. It suggested that the value of \(m\) for common strength concrete is in the range of 5-10, which is consistent with the conclusions in [20]. While, for the high-strength concrete, the value of \(m\) dramatically decreases with the increasing of the concrete compressive strength, this result will be used in section 4.3.
In addition, as mentioned in section 1, geological materials will undergo dilatancy in compression due to the effect of nucleation and growth of micro-cracks. Thus the volumetric compression and expansion of geological materials in compressive tests should be reflected in constitution model to make a precise resistance prediction. Based on a large number of explosion experiments in rocks, a relationship between volumetric strain rate and shear strain rate of geological materials was proposed [25],

\[
\frac{\dot{\varepsilon}_v}{\dot{\gamma}} + \frac{2\varepsilon}{r} = \frac{2-k}{k+1} \left(\varepsilon - \frac{\dot{\varepsilon}_v}{\dot{\gamma}}\right)
\]  

(3)

where \(\varepsilon\) is particle velocity, \(\dot{\varepsilon}_v = \frac{\partial \varepsilon}{\partial r} + 2\varepsilon/r\) is volumetric strain rate, \(\dot{\gamma} = \varepsilon/r - \frac{\partial \varepsilon}{\partial r}\) is shear strain rate; \(k\) is dilatant coefficient.
It can be seen from equation (3) that $k$ determines the compression or dilation of material volume: if $k > 2$, the material is compressible; if $k = 2$, the material is incompressible; and if $k > 2$, the material is dilated[26]. Moreover, equation (3) suggested that the effects of shear dilatancy and compressibility are combined together, which is the typical property of geological materials.

2.2. Response regions

According to the given Hoek-Brown yield criterion and Dilatant-Kinematic equation, the material outside the cavity is divided into two parts, i.e., an elastic part (elastic, cracked) and a comminuted part (dilatant and compacted), as shown in figure4, where $V$, $C_3$, $C_2$, $C_1$, $C_4$ and $t$ are the cavity expansion velocity, compacted-dilatant boundary velocity, dilatant-cracked boundary velocity, cracked-elastic boundary velocity and time.

![Figure 4. Response regions.](image)

Equations of mass conservation and momentum of material particles in Eulerian coordinate system with spherical symmetry are as follows[13]

$$\rho \left( \frac{\partial \rho}{\partial r} + \frac{2 \rho v}{r} \right) = -\left( \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} \right)$$

(4)

$$\frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\theta)}{r} = -\rho \left( \frac{\partial v}{\partial t} + \frac{\partial v r}{\partial r} \right)$$

(5)

where $\rho$ is the material density, $v$ is the particle velocity of the material, and $r$ is the radical coordinate; $\sigma_r$ and $\sigma_\theta$ are respectively the radical stress and circumferential stress in material.

With cavity expansion caused by the radical pressure on the cavity surface, the stress wave will transfer in each response region. Thus, the following Hugoniot jump conditions must be satisfied on the interface of each response area[13],

$$\rho_+ (v_+ - c_n) = \rho_- (v_- - c_n)$$

(6)

$$\sigma_+ + \rho_+ v_+ (v_+ - c_n) = \sigma_- + \rho_- v_- (v_- - c_n)$$

(7)
where $\rho^+, v^+$ and $\sigma^+$ are density, particle velocity and stress of the material before the wave; $\rho^-, v^-$ and $\sigma^-$ are density, particle velocity and stress of the material after the wave, and $c_n$ is boundary velocity.

In section 3, the radial stress and particle velocity in different regions is solved with the similarity transformation method and a fitting quadratic function relation of cavity expansion velocity and expansion stress on the surface of the cavity is obtained.

3. Dynamic spherical cavity expansion analysis

3.1. elastic region

In this region, generalized Hooke’s rule is applied to describe the radial stress and circumferential stress of the material in spherical coordinates,

\begin{equation}
\sigma_r = -\frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \frac{\partial u}{\partial r} + 2\nu \frac{u}{r} \right] \tag{8}
\end{equation}

\begin{equation}
\sigma_\theta = -\frac{E}{(1+\nu)(1-2\nu)} \left[ \nu \frac{\partial u}{\partial r} + \frac{u}{r} \right] \tag{9}
\end{equation}

where $E$ and $\nu$ are separately the Young’s model and Poisson’s ratio of the material, $u$ is the radical displacement.

The displacement of the material in elastic is obtained by using Eqs. (8, 9, 5), in which the convective term is neglected as the particle velocity is extremely small.

\begin{equation}
\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} - \frac{2u}{r^2} = \frac{1}{c_d^2} \frac{\partial^2 u}{\partial t^2} \tag{10}
\end{equation}

\begin{equation}
c_d = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho_0}} \tag{11}
\end{equation}

By using $\xi_1 = r / c_d t$, $\bar{u}_1 = u / c_d t$ in equation (10) and similarity transformations, the equation (10) is transformed into non-dimensionless forms,

\begin{equation}
\frac{d^2 \bar{u}_1}{d\xi_1^2} + \frac{2}{\xi_1} \frac{d\bar{u}_1}{d\xi_1} - \frac{2\bar{u}_1}{\xi_1^2} = \frac{1}{\alpha^2} \frac{d^2 \bar{u}_1}{d\xi_1^2} \tag{12}
\end{equation}

where $\alpha = c_d / c_1$, and the non-displacement in the elastic region is

\begin{equation}
\bar{u}_1 = A\xi_1^\alpha - B \frac{\alpha^2 - 3\xi_1^2}{3\xi_1^2}, 1 \leq \xi_1 \leq \alpha \tag{13}
\end{equation}
in which, the constants $A$ and $B$ are determined by using the boundary conditions at the inner surface of the elastic region and the elastic-cracked interface.

$$u_{r=\alpha} = 0$$  \hspace{1cm} (14)

$$\sigma_{r=\alpha} = -f_t$$  \hspace{1cm} (15)

where $f_t$ is the tensile strength of the material

$$B = \frac{-3}{2} A$$  \hspace{1cm} (16)

$$A = \left( \frac{f_t}{\rho_0 c^2} \right) \frac{2\alpha(1-\nu)}{2\nu(1-\alpha^2) + (2-3\alpha + \alpha^2)}$$  \hspace{1cm} (17)

3.2. Cracked region

For geological materials, its tensile strength is lower than compressive strength, and materials are easy to produce tensile cracks under compression. In this region, the circumferential stress of materials is set as zero and the momentum conservation equation and radial stress are

$$\frac{\partial \sigma_r}{\partial r} + \frac{2\sigma_r}{r} = -\rho_0 \frac{\partial^2 u}{\partial t^2}$$  \hspace{1cm} (18)

$$\sigma_r = -E \frac{\partial u}{\partial r}$$  \hspace{1cm} (19)

Substituting equation (18) into equation (19) and using the transformation in equation (20). Thus

$$\xi_2 = \frac{r}{c_2 t}, \hspace{1cm} \bar{u}_2 = \frac{u}{c_2 t}, \hspace{1cm} \beta = \frac{c_2}{c_p}, \hspace{1cm} \gamma = \frac{c_1}{c_2}$$  \hspace{1cm} (20)

$$\frac{d^2 \bar{u}_2}{d\xi_2^2} + \frac{2}{\xi_2} \frac{d\bar{u}_2}{d\xi_2} = \beta^2 \xi_2 \frac{d^2 \bar{u}_2}{d\xi_2^2}$$  \hspace{1cm} (21)

where $c_p = \sqrt{E/\rho_0}$. The solution of equation (20) is

$$\bar{u}_2 = C - D \frac{1 + \beta^2 \xi_2^2}{\xi_2^2}, \hspace{1cm} 1 \leq \xi_2 \leq \gamma$$  \hspace{1cm} (22)

in which, the parameters of $C$ and $D$ are evaluated from the continuity conditions of displacement at the elastic-cracked interface and the yield criterion at the cracked-dilatant interface.

$$\bar{u}_{r=\gamma} = \gamma \bar{u}_{r=\alpha}$$  \hspace{1cm} (23)

$$\sigma_{r=\alpha} = -E \frac{d\bar{u}_2}{d\xi_2} = Y$$  \hspace{1cm} (24)
\[ D = \frac{Y}{E(\beta^2 - 1)} \]  
\[ C = Y \left[ \alpha^2 - \frac{3}{2} \left( \frac{\alpha^2}{3} - 1 \right) \right] + D \left( \frac{1 + \beta^2 \gamma^2}{\gamma} \right) \] 

With equation (19, 22, 25, 26) and Hook’s law, the radical stress and volumetric strain in the cracked region at the elastic-cracked interface are derived as

\[ \sigma_{2, \xi, \gamma} = -ED \left( \frac{1}{\gamma^2} - \beta^2 \right) \]  
\[ \eta_{2, \xi, \gamma} = -D \left( \frac{1}{\gamma^2} - \beta^2 \right) (1 - 2\nu) \] 

Similarly, the particle velocity, radical stress, volumetric strain and density of the material in the cracked region at the cracked-dilatant interface are

\[ v_{2, \xi, \nu} = -c_2 \frac{d\xi_2}{d\tau} + c_2 \pi_2 = c_2 (C - 2D) \]  
\[ \sigma_{2, \xi, \nu} = Y \]  
\[ \eta_{2, \xi, \nu} = \frac{Y (1 - 2\nu)}{E} \]  
\[ \rho_2 = \frac{\rho_0}{1 - \eta_{2, \xi, \nu}} \] 

With increasing cavity expansion velocity, the interface velocity of \( C_2 \) will catch up the interface velocity of \( C_1 \), which means the cracked region disappeared [27]. In this case, the elastic region is directly adjacent to the dilatant region, and the boundary conditions in elastic region at the elastic-dilatant interface is equation (31). Thus the constants A and B in equation (13) becomes

\[ B = -\frac{3}{2} A \]  
\[ A = -\left( \frac{Y}{\rho_0 c_d^2} \right) \frac{\alpha (1 - \nu)}{(1 + \nu) - \alpha^2 (1 - 2\nu) - 3\nu \alpha} \] 

3.3. dilatant region

In this region the material satisfies both the Hoek-Brown yield criterion and Dilatant-Kinematic
equation. Applying the boundary condition $v(c_i t, t) = v_3$ and integrating equation (3), the particle velocity can be expressed as

$$v = v_3 \left( \frac{c_i t}{r} \right)^{k_i}$$  \hspace{1cm} (35)

where $v_3$ is the particle velocity in the dilatant region at the cracked–dilatant interface, $k_i$ is the dilatant coefficient in dilatant region.

With equation (35), the mass conservation equation (1) becomes

$$\rho(2 - k_i) v_3 \left( \frac{c_i t}{r} \right)^{k_i} \frac{1}{r^{k_i+1}} = -\left( \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial r} \right)$$  \hspace{1cm} (36)

Using the transformations in equation (37) and an ordinary differential equation form equation (36) is obtained.

$$\varepsilon_3 = \frac{r}{V} \text{ or } \frac{r}{c_i t}, \quad \delta = \frac{c_3}{V} \text{ or } \frac{c_3}{c_5}, \quad S_3 = \frac{\sigma}{Y}$$  \hspace{1cm} (37)

$$\left( 2 - k_i \right) \cdot v_3 \cdot \delta^{k_i} \cdot \frac{1}{c_3} \cdot \varepsilon_3 = \frac{d \rho}{\rho}$$  \hspace{1cm} (38)

Equation (38) integrates to

$$\rho = \rho_i \left( \frac{-N_i}{\varepsilon_3^{k_i+1}} + 1 \right)$$  \hspace{1cm} (39)

$$M_i = (2 - k_i) \cdot v_3 \cdot \delta^{k_i} \cdot \frac{1}{c_3}$$  \hspace{1cm} (40)

$$N_i = \frac{v_3}{c_3} \cdot \delta^{k_i}$$  \hspace{1cm} (41)

To describe the compaction or dilation state of the material, the density of the concrete material in comminuted region must be known. Combining the boundary condition $\rho(\varepsilon_3 = \delta) = \rho_i$, $\rho_i$ is derived from equation (39).

$$\rho_i = \frac{\rho_i \left( \varepsilon_3 = \delta \right)}{\left( \frac{-N_i}{\delta^{k_i+1}} + 1 \right)^{\frac{M_i}{N_i(n+1)}}}$$  \hspace{1cm} (42)

With equation (1, 35), the conservation equation (5) becomes
\[
\frac{\partial \sigma_{r}}{\partial r} + \sqrt{4\sigma_{m}Y + (mY)^2 + 4Y^2s - mY} = -\rho \left( v_3 k_i \frac{c_i}{r_k} + v \frac{\partial v}{\partial r} \right)
\]
(43)

Introducing equation (37) to equation (43), an ordinary differential equation is obtained,

\[
\frac{dS_3}{d\xi_3} = -\rho \left( -\frac{N_1}{\xi_3^{k_i} + 1} \right) \left[ v_3 k_i \left( \frac{\partial}{\partial \xi_3} \right)^{k_i} c_3 - v_3^{2} k_i \left( \frac{\partial}{\partial \xi_3} \right)^{2k_i} - \frac{m}{4} \left( \frac{S_3}{m} + 1 + 4 \frac{s}{m} - m \right) \right]
\]
(44)

The boundary conditions at the cracked-dilatant interface are as follows when the cracked region exists:

\[
v_3 \xi_{3,\infty} = \frac{\rho_3}{\rho_1} \left( v_2 \xi_{3,1} - c_2 \right) + c_2
\]
(45)

\[
\sigma_3 \xi_{3,\infty} = \sigma_2 \xi_{3,1} + \rho_2 v_2 \xi_{3,1} \left( v_2 \xi_{3,1} - c_2 \right) - \rho_3 v_3 \xi_{3,\infty} \left( v_3 \xi_{3,\infty} - c_2 \right)
\]
(46)

The boundary conditions at the elastic-dilatant interface are as follows when the cracked region vanishes:

\[
v_3 \xi_{3,\infty} = \frac{\rho_3}{\rho_3} \left( v_2 \xi_{3,1} - c_2 \right) + c_2
\]
(47)

\[
\sigma_3 \xi_{3,\infty} = \sigma_2 \xi_{3,1} + \rho_2 v_2 \xi_{3,1} \left( v_2 \xi_{3,1} - c_2 \right) - \rho_3 v_3 \xi_{3,\infty} \left( v_3 \xi_{3,\infty} - c_2 \right)
\]
(48)

Here, according to the division the response regions, two different boundary conditions should be considered for the inner surface \((\xi_3 = 1)\) of the dilatant region. If there is no compacted-region, the cavity expansion velocity is equal to the particle velocity as in equation (45) or (46). If the compacted-region exists, the particle velocity and material density in dilatant region at the dilatant-compacted interface are deduced from equation (35) and (39).

\[
v_j \left( \xi_3 = 1 \right) = v_3 \left( \xi_3 = \delta \right) \delta^{j_i}
\]
(49)

\[
\rho_j \left( \xi_3 = 1 \right) = \rho_3 \left( 1 - N_1 \right) \xi_3^{k_i (i+k_i)}
\]
(50)

3.4. Compacted region

In this region, material is also described by both the Hoek-Brown yield criterion and Dilatant-Kinematic equation. Different with the dilatant region, the compacted coefficient \(k_2\) is applied in the Dilatant-Kinematic equation for the compacted region. Following dimensionless transformations are introduced,

\[
\xi_3 = \frac{r}{V_I}, \quad S_4 = \frac{\sigma_{r}}{Y}, \quad \zeta = \frac{c_3}{V}
\]
(51)

Similar to the derivation process of the dilatant region, the dimensionless stress differential
equation of compact region is

\[ \frac{dS_4}{d\xi_4} = -\rho_c \left( -\frac{N_2}{\xi_4^{k_1+1}} + 1 \right) + \rho_c \left[ \frac{M_2}{\xi_4^{k_2+1}} \left( \frac{\delta}{\xi_4} \right)^{k_2} - v_4^2 k_2 \frac{\delta^2 k_2}{\xi_4^{2k_2+1}} \right] - m \sqrt{\frac{4s_4}{m} + 1 + \frac{s}{m} - m} \]

(52)

\[ M_2 = (2 - k_2) \cdot \frac{1}{V} \]

(53)

\[ N_2 = \frac{v_4}{V} \cdot \xi_4^{k_2} \]

(54)

Combining the boundary condition \( \rho = \rho_4 (\xi_4 = \xi) \) and the Hugoniot jump conditions, The boundary conditions at the dilatant-compacted interface are

\[ v_4^{\xi_4 = \xi} = \rho_4 \left( v_4^{\xi_4 = \xi} - c_1 \right) + c_3 \]

(55)

\[ \sigma_4^{\xi_4 = \xi} = \sigma_4^{\xi_4 = \xi} + \rho_4 v_4^{\xi_4 = \xi} \left( v_4^{\xi_4 = \xi} - c_1 \right) - \rho_4 v_4^{\xi_4 = \xi} v_4^{\xi_4 = \xi} \left( v_4^{\xi_4 = \xi} - c_1 \right) \]

(56)

The ordinary differential equations, equation (44) and (52) are solved by using the Runge-Kutta method to get the radical stress of the material in the dilatant and compacted regions. The boundary velocities \( V \) and \( C_1 \) are solved by iteration method until the given boundary conditions are satisfied.

Based on the analysis in above, the relationships between the interface velocities and the expansion velocity of the cavity for the concrete with a uniaxial compressive strength of 48MPa are shown in figure 5. It can be seen that the compacted region appears when \( v \) is above of 350 m/s. The response region is compacted-dilatant-cracked-elastic for the material at this point. With increasing \( V \), the cracked region diminishes and is eliminated eventually, and the response region is compacted-dilatant-elastic. figure 6 presents a dimensionless quadratic curve of the normal stress on the cavity surface versus the cavity expansion velocity.

\[ \frac{\sigma}{Y} = a_1 \left( \frac{V}{\sqrt{V/\rho_0}} \right)^2 + a_2 \left( \frac{V}{\sqrt{V/\rho_0}} \right) + a_3 \]

(57)

where, \( a_1, a_2, \) and \( a_3 \) are inertial, viscosity and strength coefficients \([2]\) with corresponding values 1.23, 3.06 and 9.32, separately. Dilatant coefficients \( k_1 = 1.8 \) and \( k_2 = 2.1 \) are adopted in this model \([26]\).
Figure 5. The propagation of region interfaces while the cavity expansion velocity increased.

Figure 6. Dimensionless radial stress at the cavity surface vs. cavity expansion velocity.

4. Results and validation

4.1. The influences of material parameters

The influences of different material parameters on the relationships of the radial stress $\sigma_r$ and the cavity expansion velocity $V$ are discussed. In figure 7, it suggested that $\sigma_r$ decreases with increasing $k_1$ and $k_2$, which means that dilatancy of the material in comminuted region will increase the cavity expansion resistance, and the compressibility of the material will inversely decrease the cavity expansion resistance. Under the same cavity expansion velocity, radial stress $\sigma_r$ increases as brittleness coefficient $m$ increases, as shown in figure 8 (a). It can be seen that $m$ is a sensitive parameter for radial stress $\sigma_r$, and the conclusions in section 2 will be used to determine the value of $m$ for each geological material. Figure 8 (b) shows that the radial stress $\sigma_r$ increases with increasing intact coefficient $s$. When the dimensionless cavity expansion velocity on the cavity
surface is lower than 2.5, the distinction of the radical stresses under different intact coefficients are obvious; While, the influence of intact coefficient on radical stress can be neglected when the dimensionless expansion velocity is larger than 2.5. The existence of initial flaws reduces the strength of materials: With low cavity expansion velocity, the radial stress is mainly determined by the static term in equation (57); As cavity expansion velocity increases, the percentage of resistance induced by material inertial increases, thus in this condition the influence of the intact coefficient on radical stress is not obvious.

Figure 7. The influence of dilatant coefficient.

Figure 8. Dimensionless radial stress vs. cavity expansion velocity for different coefficients \( m \) and \( s \).

4.2. Rigid projectile penetration model
The projectile penetration process in geological target is described by the two-stage model[27]: cavity stage and tunnel stage. The penetration resistance at the cavity stage is

\[
F_z = cz, \quad 0 \leq z \leq 4R, \quad (58)
\]

where \( c \) is the parameter in [27], \( z \) is the penetration depth, \( R \) is the radius of the projectile. At the tunnel stage the resistance force is determined by assuming that the normal resistance on the surface of the projectile is equal to the radical stress in cavity expansion theory.
\[ F_z = \int \sigma(v_z) \cos \theta d\Gamma \quad 4R < z, \]
\[ v_n = v \cos \theta, \]
\[ \theta = \arccos(e_z, n), \]

where \( e_z \) is the unit vector along the z-axis, \( n \) is the unit vector of interior normal on the projectile surface, \( (e_z, n) \) is the inner product of vectors, \( \theta \) is the angle between \( e_z \) and \( n \), \( v \) is the projectile velocity along the z-axis, and \( \Gamma \) is the lateral surface of the projectile. Here, the friction on projectile surface is neglected.

Combining the Newton’s law, the kinematic equation of the projectile with a mass \( m \) is
\[ m \frac{d^2 z}{dt^2} = -F_z = \begin{cases} cz, & 0 \leq z \leq 4R \\ A_1 v^2 + A_2 v + A_3, & z > 4R \end{cases}. \]

in which, \( A_1, A_2, A_3 \) are the integral coefficients from equation (59). Then, the expression for penetration depth \( P \) is
\[ P = \frac{m}{2A_1} \ln \left[ \frac{Q_1}{(v_1 + Q_1)^2 + Q_3} - \frac{2Q_2}{Q_3} \left( \arctan \frac{Q_2}{\sqrt{Q_3}} - \arctan \frac{v_1 + Q_2}{\sqrt{Q_3}} \right) \right] + 4M_0 \]
\[ v_1 = \frac{-A_2 + \sqrt{A_2^2 - 4(A_1 + \frac{m}{4M_0})(A_1 - \frac{mv_1^2}{4M_0})}}{2(A_1 + \frac{m}{4M_0})} \]
\[ Q = \frac{A_1}{A_2}, \quad Q_2 = \frac{A_2}{2A_1}, \quad Q_3 = Q_1 - Q_2^2 \]

where \( v_s \) and \( v_1 \) are the initial velocity at the first stage and the initial velocity at the second stage separately, \( V \) is the volume of projectile, and \( m \) is the projectile mass.

### 4.3. Model validation

In this section, based on the tests of rigid projectiles penetration into concrete and rock targets, the penetration depths predicted by equation (61) as well as the classical model [28] suggested by Forrestal are compared, respectively.

The penetration depths of rigid projectile penetrating into 36 MPa, 51 MPa and 62.5 MPa common strength concrete [28, 29] are depicted in figure 9 (a). Predictions from the proposed model and the classical model show good agreement with the test data for those three kinds of concrete. When the initial impact velocity is lower than 800 m/s, the proposed model is more accurate than the classical model. When the initial impact velocity is higher than 800 m/s, the predictions form both models are distinct: the prediction results from the classical model are more accurate for the 36 MPa concrete.
while the proposed model gives a better prediction for 51 MPa and 61 MPa concretes. As for the high-strength compressive concrete (87 MPa and 112.5 MPa) [30], the predictions from both model integrating the experiments data are shown in figure 9(b). It can be observed that the present model based on the Hoek-Brown yield criterion and Dilatant-Kinematic equation gives better predictions than the classical model.

The penetration data of limestone with compressive strength of 60 MPa and granite with compressive strength of 154 MPa are selected as typical data of rigid projectiles penetration into low strength rock targets and high strength rock targets respectively [31, 32], as shown in figure 10 (a and b). It can be seen that the prediction results of the proposed model are consistent with the experimental data than that of Forrestal model, especially for the granite targets. It should be point that as the rocks tensile strength and Poisson’s ratio are unavailable in primary literature, $Y = 12f$, and $\nu = 0.26$ are adopted in our model.

**Figure 9.** Depth of penetration vs. initial impact velocity for concrete target.

**Figure 10.** Depth of penetration vs. initial impact velocity for rock target.
5. Conclusion
In this paper, the dynamic cavity expansion theory is applied for the depth prediction of rigid projectiles penetrating into concrete and rock targets, the main works and conclusion are as follows:

(1) The Hoek-Brown yield criterion and Dilatant-Kinematic equation are introduced to describe the plastic behavior of concrete and rock materials, and a precise dynamic cavity expansion model is proposed.

(2) A depth of penetration equation for rigid projectile is proposed combing the Newton’s law, the present model with higher accuracy has a well-being prediction of the penetration depth of concrete/rock materials with different strength.

(3) The dilatancy of the material in comminuted region will increase the cavity expansion resistance, and compressibility of the material will inversely decrease the cavity expansion resistance.

(4) The radical stress $\sigma_r$ increases as brittleness coefficient $m$ increases, and the value of $m$ decreases with increasing uniaxial compressive strength for concrete material.

(5) When cavity expansion velocity is low, the radical stress $\sigma_r$ increases with increasing intact coefficient $s$. However, when the dimensionless cavity expansion velocity on the cavity surface is larger than 2.5, the influence of intact coefficient on radical stress can be neglected.

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