Determination of the mixing between active neutrinos and sterile neutrino through the quark-lepton complementarity and self-complementarity

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Abstract

It is suggested that there is an underlying symmetry which relates the quark and lepton sectors. Namely, among the mixing matrix elements of CKM for quarks and PMNS for leptons there exist complementarity relations at a high energy scale (such as the see-saw or even the GUT scales). We assume that the relations would remain during the matrix elements running down to the electroweak scale. Observable breaking of the rational relation is attributed to existence of sterile neutrinos which mix with the active neutrino to result in the observable PMNS matrix. We show that involvement of a sterile in the (3+1) model, induces that \( |U_{e4}|^2 = 0.040, |U_{\mu 4}|^2 = 0.009 \) and \( \sin^2 2\alpha = 0.067 \). We also find a new self-complementarity \( \vartheta_{12} + \vartheta_{23} + \vartheta_{13} + \alpha \approx 90^\circ \). The numbers are generally consistent with those obtained by fitting recent measurements, especially in this scenario, the existence of a sterile neutrino does not upset the LEP data i.e. the number of neutrino types is very close to 3.

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I. INTRODUCTION

By the recent observation, the neutrino masses are much lighter than the corresponding leptons, but the origin of the neutrino masses remains as a mystery. Even so, in analog to the quark sector, the neutrino flavor eigenstates are different from their mass eigenstates, so the Pontecorvo-Maki-Nakawaga-Sakata (PMNS) matrix appears [1, 2]. On the aspect, the quark sector has been treated separately and a corresponding mixing matrix Cabibbo-Kobayashi-Maskawa (CKM) [3–5] plays the role. It is well known that to cancel the gauge anomaly, quark and lepton sectors must exist simultaneously and have the same generations. Therefore, one is tempted by the one-to-one correspondence of quark and lepton to conjecture there might be some intimate relations between the two sectors. Even though at the practical world, the two sectors look quite differently, one may consider that at very high scales such as the see-saw, Grand Unification or even Planck scales, they originate from the same or at least related sources. Therefore there might exist a large symmetry which relates the two sectors. Indeed, at the high energy scale of about $10^{15}$ GeV, where the strong, electromagnetic (EM) and weak interactions are unified into a large symmetry such as SU(5), SO(10), E6 etc. [6], quarks and leptons may reside in the same representations of the large group, therefore by the Grand Unification Theories (GUTs) it is natural to expect that such complementary relations may exist. Earlier in 2004, Raidal [7] suggested that GUTs may relate the quark and lepton sectors and predicted some phenomenological consequences, and Ma et al. [8] considered it as the theoretical base of complementarity.

If the two sectors indeed originate from a large symmetry, even though during the process of running down from higher energy scale to our practical electro-weak scale many quantities look different, some of the relations may remain.

With a certain parametrization the quark-lepton complementarity [8] and self-complementarity [10–13] are noticed. Indeed such relations are approximate. Motivated by the picture described above, we assume that the complementarity and self-complementarity are exact and guaranteed by the residual symmetry which even though is not clear yet. On other aspect, the experimental measurements show that such relations are only approximate. One may think that such deviations are due to measurement errors, or there exists new physics whose existence results in the declination from exact complementarity and self-complementarity. The goal of this work is to search for a possible new physics scenario which may cause such a declination.

The short-baseline neutrino oscillations indicates there may exist light sterile neutrinos if CPT-invariance is conserved [14–16]. The sterile neutrinos do not directly participate in weak interaction, but may mix with the active neutrinos of three generations. Therefore they would make substantial contributions to the observable physical quantities via the mixing.

The mixing among active neutrinos and sterile neutrinos produces an extended $4 \times 4$ PMNS matrix [17, 18] with certain parameters. In the previous $3 \times 3$ PMNS matrix [3, 11, 12] the mixing with sterile neutrinos was not included, so that the quark-lepton complementarity and self-complementarity are approximate. Now we assume the quark-lepton complementarity and self-complementarity to be exact, whereas the mixing among active and sterile neutrinos
causes the apparent declination. The starting point of this work: the mixing angles for quarks and leptons possess an exact complementarity, but contaminated by existence of sterile neutrinos.

In this paper we will employ the scenario with three active neutrinos plus one sterile neutrino (3+1). Thus we may fix mixing angles between the light sterile neutrino with the active ones in term of the assumed quark-lepton complementarity and self-complementarity. Though the scenario is simple, its prediction generally coincides with the experimental observation, thus the present data do not suggest us to abandon the simple version[19].

To be explicit, we re-state our strategy as: By supposing the quark-lepton complementarity and self-complementarity to be exact, we calculate the mixing matrix elements $|U_{e1}|$, $|U_{e2}|$, $|U_{e3}|$, $|U_{\mu3}|$ and $|U_{\tau3}|$ of the original matrix PMNS (i.e. without mixing with the sterile neutrino). Then the mixing matrix is extended to a $4 \times 4$ matrix which includes mixing between active neutrinos and a sterile neutrino as suggested in literature, and the $3 \times 3$ submatrix at the left-upper corner of the $4 \times 4$ matrix is the practical PMNS matrix. Comparing the matrix elements with the data one can fix the mixing angles between the sterile neutrino and the active neutrinos, and determine the weak CP phase. (see the later context for details).

Finally, we test the scenario by calculating the number neutrino generations, which is determined by the LEP data very accurately as very close to 3. Our result shows that this number is perfectly respected in the new scenario.

The paper is organized as follows. After the introduction we describe our detailed strategy and derivation of relevant formulas in section II. In section III, we present our numerical results along with all the inputs and discuss both experimental and theoretical errors. In section IV we will make a summary.

II. THE MIXING OF FERMIONS AND QUARK-LEPTON COMPLEMENTARITY AND SELF-COMPLEMENTARITY

In this section we show explicitly how to fix the mixing angles between the active neutrinos and sterile neutrino and the CP-phase under the hypothesis of the exact quark-lepton complementarity and self-complementarity.

A. the mixing of fermions in SM

The mixing among quarks or leptons is described by the CKM and PMNS matrices which appear in the weak charged currents. The quark sector involves the u-type and d-type quarks, whereas the leptonic sector involves neutrinos and charged leptons. The relevant Lagrangian is

$$
\mathcal{L} = \frac{g}{\sqrt{2}} \bar{U}_L \gamma^\mu V_{CKM} D_L W_{\mu}^+ - \frac{g}{\sqrt{2}} \bar{E}_L \gamma^\mu V_{PMNS} N_L W_{\mu}^+ + h.c.,
$$

(1)
where $U_L = (u_L, c_L, t_L)^T$, $D_L = (d_L, s_L, b_L)^T$, $E_L = (e_L, \mu_L, \tau_L)^T$ and $N_L = (\nu_1, \nu_2, \nu_3)^T$. $V_{CKM}$ and $V_{PMNS}$ are the CKM and PMNS matrices respectively. If there were no sterile neutrino, both quark and lepton sectors contain three generations, so their mixing matrices are similar. As is well known that real physics is independent of any parametrization schemes, so it is convenient to set $V_{CMS}$ and $V_{PMNS}$ in the P1 parametrization\cite{8} as

$$V = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\nu1} & U_{\nu2} & U_{\nu3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -c_{12}s_{23} - s_{12}c_{23}e^{i\delta} & -s_{12}s_{23}c_{13} + c_{12}c_{23}e^{i\delta} & s_{23}c_{13} \\ c_{12}s_{23} + s_{12}c_{23}e^{i\delta} & -s_{12}s_{23}c_{13} - c_{12}c_{23}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (2)$$

Here $s_{ij}$ and $c_{ij}$ denote $\sin \theta_{ij}(\sin \vartheta_{ij})$ and $\cos \theta_{ij}(\cos \vartheta_{ij})$ with $i, j = 1, 2, 3$. In this work, we use $\theta_{ij}$ for quark sector and $\vartheta_{ij}$ for lepton sector respectively.

Thanks to hard experimental measurements on the weak processes where the CKM matrix is involved, the mixing parameters for the quark sector are more precisely fixed and their central values\cite{12} are

$$\theta_{12} = 13.023^\circ, \theta_{23} = 2.360^\circ, \theta_{13} = 0.201^\circ, \delta = 69.10^\circ. \quad (3)$$

Definitely, certain experimental errors still exist and they would cause theoretical uncertainties in our predictions on the PMNS parameters. We will discuss that issue later.

The parameters in the $3 \times 3$ PMNS matrix which are determined by the measured data\cite{12} are

$$\vartheta_{12} = 33.65^\circ, \vartheta_{23} = 38.41^\circ, \vartheta_{13} = 8.93^\circ, \quad (4)$$

which are directly measured by the neutrino-involved experiments, especially the neutrino oscillations.

As we discussed above, among the CKM and PMNS matrix elements, there are complementarity and self-complementarity relations. In the P1 parametrization, the relations reduce to some direct relations among the mixing angles. The quark-lepton complementarity suggests $\theta_{12} + \vartheta_{12} \approx 45^\circ, \theta_{23} + \vartheta_{23} \approx 45^\circ$ and the self-complementarity requires $\vartheta_{12} + \vartheta_{13} \approx \vartheta_{23}$ to be held.

Comparing with data, one immediately notices that even though those relations are in a good approximation, obvious deviation of the obtained mixing matrix from the data,

$$V' = \begin{pmatrix} U'_{e1} & U'_{e2} & U'_{e3} \\ U'_{\mu1} & U'_{\mu2} & U'_{\mu3} \\ U'_{\nu1} & U'_{\nu2} & U'_{\nu3} \end{pmatrix}, \quad (5)$$

demands an explanation. That deviation happens in the scenario with only three types of neutrinos as required by the standard model, so when the theory is extended to involve new components, the problem would be easily solved.
B. the mixing of neutrinos beyond SM

In some previous works, the authors introduced one or more sterile neutrinos to explain the data of short-baseline neutrino oscillation\[14–16\]. In this work we consider the model of three active neutrinos mixing with one sterile neutrino (\(\nu_s\)). The sterile neutrino does not directly participate the weak interaction, so before taking into account its mixing with active neutrinos, the weak interaction Lagrangian for leptonic sector is

\[
- \frac{g}{\sqrt{2}} \left( \bar{e}_L \bar{\mu}_L \bar{\tau}_L \right) \gamma^\mu \left( U'_{e1} U'_{\mu1} U'_{\tau1} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_s \end{pmatrix} \right) W^+_\mu + h.c. .
\] (6)

Apparently as the active neutrinos mix with the sterile neutrino, the values of the mixing matrix elements in eqs.(5) and (6) are definitely affected. Once appropriate mixing parameters are chosen, these modified mixing matrix elements may coincide with the available data. By contrary, if one cannot fix a set of such mixing parameters to make the new matrix elements to meet the data, the model would fail. Later we will show that the adopted scenario succeeds, i.e. the newly obtained PMNS matrix elements are generally consistent with the data and the theoretical uncertainties are smaller than the experimental errors.

To account for the possible mixing between the sterile neutrino and the active ones, we introduce a \(4 \times 4\) matrix. In a complete picture, the mixing of neutrinos (3 active neutrinos+1 sterile neutrino) could be

\[
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = V_{4 \times 4} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix},
\] (7)

where \(\nu_s\) is the sterile neutrino and the extended \(4 \times 4\) matrix is written as

\[
V_{4 \times 4} = \begin{pmatrix} U''_{e1} & U''_{e2} & U''_{e3} & U''_{e4} \\ U''_{\mu1} & U''_{\mu2} & U''_{\mu3} & U''_{\mu4} \\ U''_{\tau1} & U''_{\tau2} & U''_{\tau3} & U''_{\tau4} \\ U''_{s1} & U''_{s2} & U''_{s3} & U''_{s4} \end{pmatrix},
\] (8)

which can be realized in a rotation\[17, 18\]

\[
V_{4 \times 4} = R_{23} \phi R_{13} R_{12} R_{14} R_{24} R_{34},
\] (9)

and the relevant matrices \(R_{23}, R_{13}, R_{12}, R_{14}, R_{24}, R_{34}\) and \(\phi\) are simple and straightforward as done in literature, however for readers’ convenience we present them in the appendix. It is noted that the left upper \(3 \times 3\) sub-matrix corresponds to the measured PMNS mixing matrix whose elements are fixed by the neutrino oscillation experiments.
C. the strategy to fix the mixing parameters

As a sterile neutrino is introduced into the model, there would be more free parameters. We have obtained the modules of $U'_{e1}$, $U'_{e2}$, $U'_{e3}$, $U'_{\mu 3}$ and $U'_{\nu 3}$ in eq.(5). Taking into account the mixing with the sterile neutrino, the elements $U'_{e1}$, $U'_{e2}$, $U'_{e3}$, $U'_{\mu 3}$ and $U'_{\nu 3}$ are modified to $U''_{e1}$, $U''_{e2}$, $U''_{e3}$, $U''_{\mu 3}$ and $U''_{\nu 3}$ in eq.(8). By adjusting the mixing parameter, we can make those elements to eventually coincide with the measured values.

Supposing that the quark-lepton complementarity and self-complementarity hold for the 3-generation neutrino structure the central values of the measured CKM matrix elements for quarks would fully determine $\theta_{12} = (13.023 \pm 0.038)^\circ$, $\theta_{23} = (2.360 \pm 0.052)^\circ$ and $\theta_{13} = (0.201 \pm 0.009)^\circ$, then we can obtain $\vartheta'_{12} = (31.977 \pm 0.038)^\circ$, $\vartheta'_{23} = (42.640 \pm 0.052)^\circ$, $\vartheta'_{13} = (10.663 \pm 0.014)^\circ$ which are deviate from the values given in Eq.(4). Let us re-write the PMNS matrix in terms of the obtained angles as

$$|V'| = \begin{pmatrix} 0.834\pm0.001 & 0.520\pm0.001 & 0.185\pm0.001 \\ - & - & 0.666\pm0.001 \\ - & - & 0.723\pm0.001 \end{pmatrix}. \quad (10)$$

The corresponding experimental values in the $3 \times 3 \ V_{PMNS}$ is [12]

$$|V_{PMNS}| = \begin{pmatrix} 0.822\pm0.011 & 0.547\pm0.016 & 0.155\pm0.008 \\ - & - & 0.614\pm0.018 \\ - & - & 0.777\pm0.014 \end{pmatrix}. \quad (11)$$

One can notice the deviation.

Then we introduce the mixing with the sterile neutrino and re-calculate the modules of $U''_{e1}$, $U''_{e2}$, $U''_{e3}$, $U''_{\mu 3}$ and $U''_{\nu 3}$ in the $V_{1\times4}$ matrix. Now the numbers can be compared with the measured values of the $V_{PMNS}$ elements. Here let us explicitly show the expression of $|U_{e1}|$ as an example

$$|U''_{e1}| = \cos \vartheta'_{12} \cos \vartheta'_{13} \cos \alpha = 0.834 \cos \alpha. \quad (12)$$

Comparing with the data,

$$|U''_{e1}| = |U_{e1}|, \quad (13)$$

we fix the mixing parameters. The other elements and CP phase $\delta'$ are simultaneously fixed, when the $\chi^2$ methods is employed [20, 21].

At last, using these parameters we complete the generalized and practical $4 \times 4$ matrix and its left-upper $3 \times 3$ sub-matrix is just the practical matrix $|V_{PMNS}|$.

III. NUMERICAL RESULTS

There are two possible schemes for the 3+1 mixing.

1. The first scheme: the sterile neutrino mixes with the three active neutrino by different mixing parameters, namely the there are three free parameters $\alpha$, $\beta$ and $\gamma$. 


The resultant in the 3+1 neutrino mixing scenario. Their results are presented in table I. 

To fit the data, we set the values: \( \alpha = (0.00 \pm 0.02)^\circ, \beta = (14.19 \pm 0.18)^\circ, \gamma = (12.46 \pm 0.19)^\circ \) and CP phase \( \delta' = (0.00 \pm 0.01)^\circ \). The module of the PMNS matrix reads

\[
|V_{4\times 4}| = \begin{pmatrix}
0.834 \pm 0.001 & 0.505 \pm 0.001 & 0.153 \pm 0.002 & 0.165 \pm 0.002 \\
0.496 \pm 0.001 & 0.541 \pm 0.001 & 0.621 \pm 0.002 & 0.277 \pm 0.003 \\
0.243 \pm 0.001 & 0.627 \pm 0.001 & 0.740 \pm 0.001 & 0.001 \pm 0.004 \\
0 \pm 0.001 & 0.245 \pm 0.004 & 0.209 \pm 0.004 & 0.947 \pm 0.002
\end{pmatrix}.
\]

(14)

The resultant \( |U_{e3}''|, |U_{\mu 4}''|, \) and \( |U_{\tau 3}''| \) are close to data. Based on our calculations we have \( |U_{e4}''|^2 = 0.027 \pm 0.004, |U_{\mu 4}''|^2 = 0.077 \pm 0.006 \) and \( \sin^2 2\alpha = 0 \pm 0.002 \). In the earlier works \[14, 19, 22, 23\] the authors carried out an analysis of short-baseline neutrino oscillations in the 3+1 neutrino mixing scenario. Their results are presented in table I.

2. The second scheme: That is a simplified version of the first scheme, we let \( \alpha = \beta = \gamma \) as discussed in Ref. \[18\]. And then we carry out the same process to determine the single parameter \( \alpha \). The parameters \( \alpha = (7.51 \pm 0.04)^\circ \) and \( \delta' = (0.00 \pm 0.01)^\circ \) are obtained. The modulus of corresponding PMNS matrix is

\[
|V_{4\times 4}| = \begin{pmatrix}
0.826 \pm 0.001 & 0.502 \pm 0.001 & 0.161 \pm 0.001 & 0.199 \pm 0.002 \\
0.492 \pm 0.001 & 0.561 \pm 0.001 & 0.659 \pm 0.001 & 0.096 \pm 0.001 \\
0.241 \pm 0.001 & 0.645 \pm 0.001 & 0.724 \pm 0.001 & 0.042 \pm 0.001 \\
0.131 \pm 0.001 & 0.130 \pm 0.001 & 0.128 \pm 0.001 & 0.974 \pm 0.001
\end{pmatrix}.
\]

(15)

In this scenario, which assumes the mixing between the sterile neutrino and the different active neutrinos is nondistinctive. Our estimates are presented in table I.

Moreover, we find a new self-complementarity \( \psi'_{12} + \psi'_{23} + \psi'_{34} + \alpha \approx 90^\circ \) which is a bit different from that self-complementarity relation given in Ref. \[13\].

As a test one would calculate the neutrino flavor number which is determined to be 3 by the LEP data. Ignoring the neutrino masses, the neutrino number is

\[
N_\nu = \sum_{\rho,\sigma=1}^4 \Gamma(Z \to \bar{\nu}_\rho \nu_\sigma)/\Gamma(Z \to \bar{\nu} \nu) = \sum_{\rho,\sigma=1}^4 |\sum_{i=1}^3 (V_\rho^\dagger)_{\rho i} V_{i\sigma}|^2,
\]

(16)

where \( V_{i\sigma} \) is the generalized PMNS matrix which is a \( 3 \times 4 \) matrix and not unitary. Our numerical result shows that in this scenario, \( N_\nu \) is 3, which is fully consistent with the LEP measurement within a reasonable error tolerance. The denominator of the above equation \( \Gamma(Z \to \bar{\nu} \nu) \) stands for the partial decay width of \( Z \) boson into a neutrino pair calculated in the SM.
IV. SUMMARY

In this work we adopt the two quark-lepton complementarity relations and a self-complementarity relation proposed in literatures [7, 9, 11] which is supposed to originate from a higher symmetry and maintain when energy scale runs down to the electroweak scale.

Then the deviation of the determined values from the measured PMNS matrix elements is attributed to the involvement of a sterile neutrino. The mixing of the sterile neutrino with the active ones results in the practical values of the PMNS matrix. Comparing with data, we are able to determine the mixing parameters.

In this work, we choose two schemes, in the first scheme, the sterile neutrino mixes with three different active neutrino by different parameters (i.e. $\alpha$, $\beta$ and $\gamma$ are independent parameters which are determined by fitting data; whereas in the second scheme, we let $\alpha = \beta = \gamma$, so that there is only one parameter to describe the mixing. The numerical values are listed in Tab.1.

It is noted that the previous estimates on the mixing between sterile neutrino and the active ones were obtained by fitting the data, instead, by our strategy, we start with the theoretical assumption: the complementarity and self-complementarity. The relevant mixing elements obtained in previous literatures are quite disperse and the only common point is that the sterile-active mixing is small, no matter how to obtain them.

By the first scheme, our prediction on $|U'_{e4}|^2$ is generally consistent with the results given by the authors of Ref.[20,22] (see table I), but the value of $|U'_{\mu4}|^2$ is slightly bigger. The compatibility of reactor antineutrino anomaly was discussed in Ref. [24] and the mixing parameter $\sin^2 2\alpha = 0.14 \pm 0.08$ was fixed when $\Delta m_{41}^2 > 1.5 eV^2$. Our estimation on $|U'_{e4}|^2$ is consistent also with it within a $2\sigma$ range.

For the second scheme, the numbers look differently, but the trend and consistency degree with those given in literatures are all within the present experimental error tolerance.

The theoretical uncertainties of our predictions originate from the measurement errors of the CKM matrix elements which are relatively small thanks to many years of hard work. On the contrary the experimental errors for measuring the PMNS matrix elements are larger. Thus, our predictions on the mixing between sterile and active neutrinos and that obtained by others are still consistent with each others within 1-2 $\sigma$ ranges.

Recently the Daya Bay collaboration reports their new data[25] on the mixing between the sterile neutrino and active neutrinos, but the errors are still too large to make a conclusive judgement on the validity of our theory yet. The future improved measurement may further narrow down the data ranges, so that we can testify any theoretical ansatz and get a better understanding on neutrinos.

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Appendix A

\[
R_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{23} & S_{23} & 0 \\ 0 & -S_{23} & C_{23} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, R_{13} = \begin{pmatrix} C_{13} & 0 & S_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -S_{13} & 0 & C_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, R_{12} = \begin{pmatrix} C_{12} & S_{12} & 0 & 0 \\ -S_{12} & C_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (A1)
\]

\[
R_{14} = \begin{pmatrix} C_{\alpha} & 0 & 0 & S_{\alpha} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_{\alpha} & 0 & 0 & C_{\alpha} \end{pmatrix}, R_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\beta} & 0 & S_{\beta} \\ 0 & 0 & 1 & 0 \\ 0 & -S_{\beta} & 0 & C_{\beta} \end{pmatrix}, R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & C_{\gamma} & S_{\gamma} \\ 0 & 0 & -S_{\gamma} & C_{\gamma} \end{pmatrix}, \quad (A2)
\]

\[
\phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\delta} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (A3)
\]

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