Quantum Stability of Generalized Proca Theories

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\textbf{Abstract:} We establish radiative stability of all generalized Proca theories. While standard powercounting arguments would conclude otherwise, we find non-trivial cancellations of leading order corrections by explicit computation of divergent one-loop diagrams up to four-point. These results are crosschecked against an effective action based generalized Schwinger-DeWitt method. Further, the cancellations are understood as coming from the specific structure of the theory through a decoupling limit analysis which at the same time allows for an extension of the results to all orders.

\textbf{Keywords:} vector Galileons, generalized Proca, heat kernel, quantum field theory, effective field theory, dark energy
1 Introduction

Over the last century the theory of General Relativity accumulated a rock solid empirical foundation on a broad band of scales with tests ranging from high precision laboratory experiments to the observation of the predicted gravitational waves traveling through the fabric of space and time [1, 2]. However, almost from beginning the beauty of the theory was smudged by the apparent absence of gravitating vacuum energy, the so called cosmological constant problem [3, 4], a strong indication that Einsteins theory might not be the end of the story on IR gravity. On top of this, the evidence for the current accelerating expansion of the universe [5, 6] additionally drives the search for a plausible generalization of the theory of gravity on cosmological scales with the hope that dynamical dark energy could perhaps at the same time provide a mechanism which screens the cosmological constant.

There exist a multitude of ideas for consistent extensions of GR [7–9]. As it is the unique EFT of a massless spin 2 degree of freedom in four dimensions1, extending it almost inevitably introduces additional degrees of freedom. In a field theory framework, the new degrees of freedom typically manifest themselves as additional scalar, vector or tensor fields. While it appears as a rather easy task to just throw in new degrees of freedom in order to

1Up to reasonable assumptions.
modify gravity on large scales, the challenge is to simultaneously do justice to the unques-
tioned success of GR on smaller scales and denser regions. Hence, the newly introduced 
fields must effectively decouple from matter in these ranges.

In that respect, theories which contain higher order derivative self-interactions become 
interesting, as they naturally incorporate a Vainshtein screening mechanism [10–15]. This 
mechanism essentially relies on the non-linearities becoming large near a massive source, 
such that the kinetic term of perturbations gets enhanced significantly, which in turn weak-
ens their interaction with matter.

In general, theories with derivative self-interactions suffer from Ostrogradsky instabili-
ties [16, 17], propagating a ghost degree of freedom. However, in certain cases it is possible 
to construct theories which evade this rule. A prominent example are the scalar Galileon 
theories in flat spacetime [18], whose finite amount of non-linear derivative interaction 
terms are composed in such a way that they nevertheless lead to second order equations 
of motion and thus still only propagate the desired degree of freedom. Asking theoretical 
consistency, this immediately leads to the question whether these classical interactions are 
stable under quantum corrections. Naively, as the Vainshtein mechanism relies on scales 
for which non-linear interactions are large compared to the kinetic term, one could expect 
that the EFT is not protected against equally non-renormalizable quantum corrections. At 
a closer look, however, the EFT is organized in such a way that there exist a regime for 
which classical non-linearities dominate, while quantum effects are still under control [19–
23]: All terms generated by quantum loops have more derivatives per fields compared to 
the nonlinear galileon interactions. This provides the EFT Lagrangian with two distinct 
expansion parameters, which allow for regions below the UV cutoff scale, notably dense 
regions with non-negligible curvature, where classical non-linearities become important and 
the Vainshtein mechanism screens the coupling of the scalar field to matter, while quantum 
corrections are still under control. On large scales, both classical and quantum derivative 
self interactions become negligible, such that the scalar degree of freedom can be used as 
an extension of classical gravity.

Various counterterms of the galileon EFT have been calculated explicitly [24–27] and 
the theory has been generalized to arbitrary spacetimes [28, 29] which lead to a rediscovery 
of the most general scalar-tensor theory with second order equations of motion [30]. These 
Horndeski theories and associated generalizations have found various applications in cos-
mology [31–44], in particular, galileon theories naturally arise as the zero-helicity part of 
the graviton in higher dimensional models [45] and massive gravity theories (see [46, 47] for 
reviews).

In a cosmological context, scalar fields are by far the most popular choice when it comes 
to adding new degrees of freedom, as they naturally go along with the basic assumptions 
of homogeneity and isotropy. At the same time, this means that throwing in any desired
amount of new scalar dofs is very cheap, in the sense that there are a priori not many restrictions on how to introduce them and the space of possibilities seems endless. It could very well be, that todays inconsistencies in the theory of cosmology require a light departure from the convenient simplifying basic assumptions. This should serve as a motivation to consider the other possibilities at hand.

For instance, when endowing an abelian spin one field with a mass, it’s temporal component can readily serve as an isotropic starting point, with non-abelian cases allowing for even richer structures. Interestingly, a massive vector field\(^3\) also admits a galileon-like ghost free structure of higher order derivative interactions usually referred as generalized Proca theory [49–51], which inherits the benefits of a naturally incorporated Vainshtein screening [52]. Indeed, Proca theories and their various generalizations have already appeared in a cosmological context on various promising occasions [53–77].

The generalized Proca theories, also known as vector Galileons, possess an intimate relation to scalar Galileons. At high energies way above the vector mass the longitudinal polarization dominates, such that the theory acquires a Galileon symmetry and half of the generalized Proca interactions reduce to pure scalar Galileon terms. In particular, in parallel to it’s scalar counterpart the organization of the generalized Proca EFT is highly non-trivial. A crucial step in the analysis of the theoretical viability of any EFT is it’s quantum stability. Yet, a thorough analysis of the behavior of generalized Proca theories under loop corrections is in large parts still missing. The absence of a particular symmetry of the interactions makes it unlikely that the classical structure is protected from quantum detuning, as also indicated by an earlier result [78]. Filling this gap is the goal of the present work.

In §2 we first introduce the particular generalized Proca model we chose for the analysis and reformulate the theory by introducing a scalar Stückelberg field. Section 3.1 is then devoted to the explicit calculation of one-loop UV divergences of Feynman diagrams up to four external legs. In doing so, we correct results obtained in [78] and generalize the analysis to a more complete picture. These results are consolidated by means of re-obtaining them through an effective action based generalized Schwinger-DeWitt method in §3.2. Decoupling limit arguments in §4 allow us to interpret the obtained results and to go one step further by finding strong indications for quantum stability of the vector Galileon theory in its full generality.

2 Generalized Proca Model and Stückelberg Formulation

The most general Lagrangian of a local massive vector field theory with second order equations of motion and three propagating degrees of freedom is restricted to the following structure [49, 51]:

\(^3\)In contrast to the gauge symmetric case, where a no-go theorem for consistent derivative self-interactions has been proven [48].
where the two classical scales of the theory are the mass $m$ and the interaction scale $\Lambda_2$ which controls the interactions expanded in the number of fields $n$ through factors of $\frac{1}{\Lambda_2^n}$.

The dimensionless combination $m/\Lambda_2$ can be viewed in some sense as a coupling constant, generally assumed to be small. The numerical factors in the definitions of $\mathcal{L}_3$ and $\mathcal{L}_4$ are pure convenience.

The lagrangian term $\mathcal{L}_2$ contains all possible potential contributions including the mass term, as well as kinetic and interaction terms constructed out of the building blocks $A_\mu$, its field strength $F_{\mu \nu}$ and the dual $\tilde{F}^{\mu \nu} \equiv \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} F_{\rho \sigma}$, which by construction do not give rise to any dynamics of the temporal component $A_0$. On the other hand, $\mathcal{L}_{3, \ldots, 6}$ represent derivative self-interactions which nevertheless remain ghost-free and thus only propagate the required three degrees of freedom [49]. This is ensured by their construction via two Levi-Civita tensors which at the level of the equations of motion only allows for at most second order terms restricted to the very specific gauge invariant form $\tilde{\partial} F^\mu$.4

Being interested in quantum corrections which potentially renormalize the given classical structure we will choose a minimal model with standard canonically normalized kinetic and mass term and where

$$f_{3, 4}(x) = c_{3, 4} x, \quad \tilde{f}_4(x) = \tilde{c}_4 x, \quad f_{5, 6}(x) = c_{5, 6}, \quad \tilde{f}_{5, 6}(x) = \tilde{c}_{5, 6}.$$ (2.2)

With this choice the terms proportional to $c_5$ and $c_6$ are total derivatives and effectively drop out of the analysis, while the other terms up to total derivatives take on the form5

$$\mathcal{L}_2 = -\frac{1}{4} F^{\mu \nu} F_{\mu \nu} + \frac{1}{2} m^2 A^2,$$

$$\mathcal{L}_3 = \frac{m^2}{\Lambda_2^2} c_3 A^2 \partial \cdot A,$$

$$\mathcal{L}_4 = \frac{m^2}{\Lambda_2^2} A^2 \left( c_4 \left[ (\partial \cdot A)^2 - \partial_\mu A_\nu \partial^\nu A^\mu \right] + \tilde{c}_4 F^2 \right),$$ (2.3)

$$\mathcal{L}_5 = -\frac{1}{\Lambda_2^2} \tilde{c}_5 \epsilon^{\mu \nu \rho \sigma} \partial_\mu A_\nu \partial_\alpha A_\beta \partial_\rho A_\gamma,$$

$$\mathcal{L}_6 = -\frac{1}{\Lambda_2^2} \tilde{c}_6 \epsilon^{\mu \nu \rho \sigma} \partial_\mu A_\nu \partial_\alpha A_\beta \partial_\rho A_\gamma \partial_\sigma A_\delta.$$
Note that the operator proportional to $\tilde{c}_4$ is actually a higher order $\mathcal{L}_2$ term. We will nevertheless keep it in order to explicitly see what happens with this class of terms.

It will be useful in the following to rewrite this theory of a self-interacting massive vector field by introducing a redundancy in the form of an additional scalar field $\phi$ through the replacement\footnote{It is important to note that this replacement is not a change of field variables and neither a decomposition of $A_\mu$ into transverse and longitudinal degrees of freedom. It merely introduces redundancy in the description.}

$$A_\mu \rightarrow A_\mu + \frac{1}{m} \partial_\mu \phi.$$  \hspace{1cm} (2.4)

where the mass scale is fixed by canonically normalizing the kinetic term of the scalar field. This formulation goes back to the work of Stückelberg [79, 80] and can be viewed as an explicit reintroduction of the eaten Goldstone boson. The specific form of the replacement (2.4) is such that gauge invariant terms remain untouched and moreover suggests the definition of a covariant derivative $D_\mu \phi \equiv \partial_\mu \phi + mA_\mu$. The new theory is thus effectively obtained by making the replacements

$$A_\mu \rightarrow \frac{1}{m} D_\mu \phi, \quad F \rightarrow F \quad \text{and} \quad \tilde{F} \rightarrow \tilde{F}$$  \hspace{1cm} (2.5)

in (2.3). This renders the theory invariant under the simultaneous gauge transformation

$$\phi \rightarrow \phi + m \alpha, \quad A_\mu \rightarrow A_\mu - \partial_\mu \alpha.$$  \hspace{1cm} (2.6)

Note that the unitary gauge choice $\alpha = -\frac{2}{m}$ sets $\phi = 0$, which shows that the new theory is indeed equivalent to (2.3) and only propagates three degrees of freedom. Through a different gauge choice $\partial_\mu A^\mu + m \phi = 0$ implemented in a Fadeev-Popov procedure one obtains the propagators of $A_\mu$ and $\phi$ [46]

$$\frac{-i \eta_{\mu\nu}}{p^2 + m^2} \quad \text{and} \quad \frac{-i}{p^2 + m^2},$$  \hspace{1cm} (2.7)

which at high energies behave as $\sim \frac{1}{p^2}$ compared to $\sim \frac{1}{m^2}$ in the old formulation.

The lowest strong coupling scale of the theory is found by looking at the pure scalar sector. For instance, the $2 \rightarrow 2$ tree-level amplitude coming from the operator of the schematic form $\sim \frac{m^2}{\Lambda^2} \frac{1}{m^2} (\partial \phi)^2 (\partial^2 \phi)^2$ in $\mathcal{L}_4$ goes like $\mathcal{M}_{2 \rightarrow 2} \sim \frac{F_6}{\Lambda^4 m^2}$, such that at energies above the scale

$$\Lambda_3 \equiv (\Lambda^3 m)^{\frac{1}{3}},$$  \hspace{1cm} (2.8)

the theory becomes strongly interacting. Note that as long as $m^2 \ll \Lambda^2$ (small classical coupling constant) this new scale is separated from the vector mass $m$ by a parametrically large gap, which is essential for the healthiness of the EFT. Moreover, it is a requirement for the decoupling limit to be valid. In this limit, one zooms into the cutoff $\Lambda_3$ of the theory by keeping it fixed, while sending the smaller and higher scales away to zero and infinity respectively

$$m \rightarrow 0 \quad \text{and} \quad \Lambda_2 \rightarrow \infty, \quad \text{while} \quad \Lambda_3 \equiv (\Lambda^3 m)^{\frac{1}{3}} = \text{const.}$$  \hspace{1cm} (2.9)
This decouples in large parts the vector from the Goldstone boson, as the vector field only survives in the gauge invariant combinations \( F \) and \( \tilde{F} \). In particular, the coupled gauge symmetry (2.6) is broken apart and only the one of \( A_\mu \) prevails, while the scalar field merely retains an independent global shift symmetry

\[
\phi \rightarrow \phi + c, \quad A_\mu \rightarrow A_\mu - \partial_\mu \alpha. \tag{2.10}
\]

The surviving terms can directly be obtained from the original theory (2.3) by replacing

\[
A_\mu \rightarrow \frac{1}{m} \partial_\mu \phi, \quad F \rightarrow F \quad \text{and} \quad \tilde{F} \rightarrow \tilde{F}, \tag{2.11}
\]

and applying the limit (2.9) which yields

\[
L_2 = -\frac{1}{4} F^2 + \frac{1}{2} (\partial \phi)^2, \\
L_3 = \frac{1}{\Lambda^3} c_3 (\partial \phi)^2 \Box \phi, \\
L_4 = \frac{1}{\Lambda^6} (\partial \phi)^2 c_4 \left[ \left( \Box \phi \right)^2 - (\partial_\mu \partial_\nu \phi)^2 \right], \\
L_5 = -\frac{1}{\Lambda^3} \tilde{c}_5 \tilde{F}^{\mu \alpha} \tilde{F}_{\alpha \beta} \partial_\mu \partial_\nu \phi, \\
L_6 = -\frac{1}{\Lambda^6} \tilde{c}_6 \tilde{F}^{\mu \alpha} \tilde{F}^{\nu \beta} \partial_\mu \partial_\nu \phi \partial_\alpha \partial_\beta \phi. \tag{2.12}
\]

Hence, the mass term of the vector field and the term proportional to \( \tilde{c}_4 \) vanish, while the terms proportional to \( c_3 \) and \( c_4 \) reduce to pure scalar Galileon interactions [18].

From (2.11) it follows that the decoupling limit can alternatively be viewed as a high energy limit in the original theory, in the sense that scaling down \( m \) is the same as scaling up the energy in the factor \( \frac{\partial}{m} \). This makes contact with the Goldstone boson equivalence theorem: At rest, all three polarizations are equivalent, but at higher energies, the transverse polarizations and the rapidly moving longitudinal polarization are clearly distinguished.

The decoupling limit is thus particularly useful when analyzing the quantum stability of an EFT, as it focuses on the high energy behavior right at the relevant scale, while ignoring all others. Moreover, in this limit \( A_\mu \) exclusively propagates the transverse modes with the gauge symmetry (2.10) ensuring the absence of ghost instabilities and quantum detuning. In the decoupling limit, the analysis of the radiative stability of the EFT is thus reduced to an analysis of the behavior of the Goldstone. However, before being able to perform a thorough hierarchy classification of terms in the EFT in §4, an explicit calculation of the most important counterterms at one-loop in section 3 is in order.

### 3 One-loop Corrections

For now we will stick to the original formulation of the theory (2.3) which corresponds to a unitary gauge choice and analyze its radiative stability. After discussing a general powercounting, explicit calculations of the logarithmic divergent part of the 1PI Feynman...
diagrams up to four external legs are presented, crosschecked with a perturbative calculation based on the generalized Schwinger-DeWitt method developed in [81]. In doing so, we for instance correct and extend the previous work of Charmchi et al. [78]. This will set the ground for a complete analysis of the quantum stability of the generalized Proca theory offered in §4.

3.1 Feynman Diagram Calculation

In this section we compute the quantum behavior of the generalized Proca model in the unitary gauge at the one-loop level using standard Feynman diagram techniques. Each diagram represents a contribution to the reduced matrix element $M_{1PI}$ in the perturbative expansion of the S-matrix:

$$
\langle k_{\text{out}} | S | k_{\text{in}} \rangle_{1PI} = 1 + (2\pi)^4 \delta^4(k_{\text{out}} - k_{\text{in}}) i M_{1PI}.
$$

(3.1)

The reduced matrix element is calculated by summing over all possible Wick contractions of the form:

$$
\begin{align*}
  A_{\mu}(x)A_{\nu}(y) &= D_{\mu\nu}(x-y) \\
  A_{\mu}(x) |k,\epsilon\rangle &= \epsilon_{\mu}^k e^{-ikx} \\
  \langle k,\epsilon | A_{\mu}(x) &= \epsilon^{*\mu}_k e^{ikx},
\end{align*}
$$

(3.2)

where

$$
D_{\mu\nu}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} i \frac{-\eta_{\mu\nu} + p_{\mu}p_{\nu}}{p^2 - m^2}
$$

(3.3)

is the propagator of the massive vector field with implicit Feynman-prescription and $\epsilon^{\mu}_k$ denotes the associated polarization vector.

Our minimal choices (2.2) only allow for vertices with up to four legs

whose value depend on the derivative structure of the insertion that translates in fourier space into a dependence on the momenta which run on each leg.

Following the $\overline{\text{MS}}$-scheme, the one-loop counterterms can then be inferred from the UV divergence of the 1PI diagrams which we will extract using dimensional regularization. We are thus after the log-divergent part of the one-loop 1PI diagrams with $N$ external legs $M_{N}^{1\text{PI}}$ which will be a function of the external momenta $k_i$, $i=1,...,N-1$, since the overall delta-function $\delta^4(k_{\text{out}} - k_{\text{in}})$ always allows to express one momentum $k_N$ in terms of the others. Throughout this work we will treat all momenta as incomming such that the overall delta-function translates to $\sum_{i=1}^{N} k_i = 0.$

In the following we will calculate explicit divergent off-shell contributions up to four external legs and comment on their implications.
3.1.1 Two-point

Within our minimal generalized Proca model characterized by the choices (2.2), the perturbative renormalization procedure of the two-point function at one-loop requires the calculation of only two distinct 1PI diagram structures depicted in Fig.1 coming from $\mathcal{L}_{3,5}$ and $\mathcal{L}_{4,6}$ respectively.

![Figure 1: Two distinct one-loop 1PI diagrams giving rise to corrections of the two point function.](image)

The first diagram represents contributions from the three possible combinations out of $\mathcal{L}_3$ and $\mathcal{L}_5$ and the second one separate contributions from $\mathcal{L}_4$ and $\mathcal{L}_6$. Each diagram comes with a symmetry factor of 2.

At the level of the propagator, there are essentially two ways in which quantum corrections could lead to a breakdown of the given EFT structure. First of all, we should check that the mass only receives $\mathcal{O}(1)$ corrections, such that the hierarchy between scales $m \ll \Lambda_3 \ll \Lambda_2$ remains intact. Secondly, the classical, gauge invariant form of kinetic term $F^2$ prevents the propagation of the ghostly temporal component of the vector field. If quantum corrections should interfere with this specific form and generate an operator $\sim (\partial_{\mu} A^\mu)^2$ they should better be suppressed at least by $m^2/\Lambda_3^2$. In other words, the mass of a dynamical temporal degree of freedom should at least be of the order of the cutoff $m_t^2 \sim \Lambda_3^2$. Order one corrections to the kinetic term must therefore preserve the gauge invariant structure.

Before doing any explicit calculations, we can already estimate what to expect by power-counting the presumably required counterterms. For this, bare in mind, that the mass term of the theory modifies the propagator (3.3) in such a way that it’s high energy behavior is $\sim 1/m^2$, as already mentioned. As a first example, consider the quantum corrections to the propagator induced by the term $\mathcal{L}_3 \sim \frac{m^2}{\Lambda_3^4} \partial A^3$. Cutting off the internal momentum loop at the scale $\Lambda_3$ and following dimensional arguments one can expect a mass counterterm of the order

$$\mathcal{L}_{\mathcal{L}_3\mathcal{L}_3}^c \sim \frac{m^4}{\Lambda_3^4} \frac{1}{m^4} A_3^6 A^2 = m^2 A^2 \subset \delta m^2 A^2,$$

(3.4)

which is of $\mathcal{O}(1)$ and thus preserves the parametrically large gap between the mass and the cutoff as required. The same goes for $\mathcal{L}_4$. However, note that for instance with two insertions of $\mathcal{L}_5 \sim \frac{1}{\Lambda_2^4} \partial \bar{\partial} A^3$ naive powercounting allows for two potentially worrisome operators

$$\mathcal{L}_{\mathcal{L}_5\mathcal{L}_5}^c \sim \left( m^2 A_3^6 + \Lambda_3^8 \right) \frac{1}{\Lambda_2^4} \frac{1}{m^4} (\partial A)^2 = \left( 1 + \frac{\Lambda_3^2}{m^2} \right) (\partial A)^2 \subset \delta \partial_2 (\partial A)^2.$$

(3.5)

The first one would lead to order one corrections to the propagator which is problematic as long as it messes with the gauge invariant structure of the kinetic term. The latter has

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to vanish completely in order not to destroy the hierarchy between classical and quantum terms. This means that for the EFT to remain healthy at this stage highly non-trivial cancellations are required in order to cure the above leading orders.

Let’s investigate this by an explicit calculation. First of all, note that in dimensional regularization only the logarithmically divergent terms are picked up. Hence, the power-counting in dimensional analysis together with Lorentz invariance leads to the following expectation for the results

\[
\mathcal{M}_{\mathcal{L}_5, \mathcal{L}_5} \sim \frac{m^4}{\Lambda_2^4} \left( m^2 + k^2 + \frac{k^4}{m^2} + \frac{k^6}{m^4} \right),
\]

\[
\mathcal{M}_{\mathcal{L}_5, \mathcal{L}_5, \mathcal{L}_5} \sim \frac{m^2}{\Lambda_2^2} \left( m^4 + m^2 k^2 + k^4 + \frac{k^6}{m^2} + \frac{k^8}{m^4} \right),
\]

\[
\mathcal{M}_{\mathcal{L}_5, \mathcal{L}_5, \mathcal{L}_6} \sim \frac{1}{\Lambda_2^2} \left( m^6 + m^4 k^2 + m^2 k^4 + k^6 + \frac{k^8}{m^2} + \frac{k^{10}}{m^4} \right),
\]

where \( k \) stands for external momenta and the series presumably stops at \( 1/m^2 \) or \( 1/m^4 \), depending how many propagators are involved in the loop. The above arguments are thus shifted towards the generation of second order operators in the fields, but higher order in derivatives again potentially leading to ghosts. Note that from an EFT point of view, these terms should be treated as additional vertices of the theory rather than including them in the propagator. Here, the potentially worrisome terms are the ones involving eight or higher powers of external momenta. To see this consider a counterterm induced by the \( \sim k^8 \) contribution and focus on the relevant scale by taking the decoupling limit

\[
\sim \frac{\partial^8}{\Lambda_2^8 m^8} A^2 \frac{\partial^6}{\Lambda_2^6 m^6} (\partial \phi)^2.
\]

In the decoupling limit, this term technically blows up and the quantum correction is out of control compared to the classical kinetic term.

In order to explicitly evaluate the contributions one has to perform the usual sum over all possible Wick-contractions which a priory gives 18 and 12 possible contractions for each diagram respectively without counting the vertex exchange factor which as usual cancels the prefactor of the exponential expansion. Alternatively and more conveniently, one can also directly compute the sum of the \( 3!^2 \) respectively \( 4! \) possible combinations divided by the corresponding symmetry factor of two of each diagram. Considering all possible combinations of vertices, summing up all diagrams and following a standard dimensional regularization procedure with \( d = 4 + 2\epsilon \), the divergent part of the reduced matrix element up to two powers of momenta reads

\[
\mathcal{M}_{\mathcal{L}_2}^{\text{div}} = \frac{\epsilon_{\alpha}^\beta \epsilon_{\beta}^{-k}}{16\pi^2 \epsilon \Lambda_2^2} \left( k^2 \eta_{\alpha\beta} m^4 \left( -3 c_3^2 + 6 \hat{c}_4 - 4 c_3 \hat{c}_5 + 2 \hat{c}_3^2 \right) + \eta_{\alpha\beta} m^6 \left( -3 c_3^2 + 6 \hat{c}_4 \right) \right)
\]

\(^7\)Note that at one loop the divergent part is blind to the extra factors of \( d \) in the Levi-Civita contractions, such that we will disregard them.

\(^8\)Note the different relative factors compared to equation (3.13) in [78]. In particular, when trying to reproduce the results of [78] we already obtain discrepancies in earlier steps, for instance eq. (3.9). Given that our computation (3.8) is confirmed by an entirely independent method (3.21) we are very confident about our results.
\[ +k_\alpha k_\beta m^4 \left( 12 c_3^2 - 6 \tilde{c}_4 + 16 c_3 \tilde{c}_5 + \frac{11}{2} \tilde{c}_5^2 \right) \]
\[ +k^2 k_\alpha k_\beta m^2 \left( -3 c_3^2 - 2 c_3 \tilde{c}_5 + \frac{2}{3} \tilde{c}_5^2 \right) - k^4 \eta_{\alpha\beta} \frac{m^2}{\Lambda^2} \frac{19}{6} \tilde{c}_5^2 \]
\[ +k^4 k_\alpha k_\beta \left( \frac{1}{2} c_3^2 - \frac{13}{12} \tilde{c}_5^2 \right) + k^6 \eta_{\alpha\beta} \frac{4}{3} \tilde{c}_5^2 \]
\[ +k^6 \frac{1}{m^2} \frac{1}{6} \tilde{c}_5^2 \left( k_\alpha k_\beta - k^2 \eta_{\alpha\beta} \right) \]

(3.8)

where \( k_1 = -k_2 = k \).

Observe that \( \mathcal{L}_6 \) and the term in \( \mathcal{L}_4 \) proportional to \( c_4 \) does not contribute at all, while the counterterm induced by \( \tilde{c}_4 \) preserves the ghost free structure of the kinetic and mass term \( (\Box + m^2) \eta_{\alpha\beta} - \partial_\alpha \partial_\beta \) as could have been expected by the structure of the operator. However, the contributions from \( \mathcal{L}_3 \) and \( \mathcal{L}_5 \) introduce in general a detuning of the gauge invariant kinetic combination by introducing a counterterm of the form

\[ \sim \frac{m^4}{\Lambda^2} (\partial_\mu A^\mu)^2. \]

(3.9)

This term is however heavily suppressed such that the mass of the associated ghost \( m_t^2 \sim \Lambda^6_3/m^4 \) does not come close to the cutoff. As discussed above, only the terms involving a power of external momenta equal or higher than eight are troublesome. Let’s thus focus on the last line in (3.8): The terms with momentas to a power of ten are absent, even though they would technically be allowed. Hence, the structure of the generalized Proca model is precisely such, that these dangerous corrections are canceled. However, there is a contribution \( \sim k^8 \). Yet again, the theory remarkably only allows for this contribution to induce a counterterm with the specific gauge preserving combination \( \Box \eta_{\alpha\beta} - \partial_\alpha \partial_\beta \). This cures the EFT structure as can be seen in the decoupling limit, where compared to (3.7) we now have

\[ \sim \frac{\partial^6}{\Lambda_3^6 m^2} F^2 \xrightarrow{DL} \frac{\partial^6}{\Lambda_3^6} F^2, \]

(3.10)

perfectly fitting into the hierarchy between classical and quantum terms.

At this point, the cancellations observed above magically seem to rescue the EFT. In order to better understand these nice properties of the EFT we will change gears in the next section 4 and perform a thorough decoupling limit analysis which will allow us to prove the quantum stability of the generalized Proca theory in its full generality.

But first, let’s also explicitly calculate the higher point one-loop contributions.

### 3.1.2 Three-point

With three external legs there exist as well two distinct 1PI one-loop diagram structures represented in Fig.2. The first diagram receives contributions from combinations out of \( \mathcal{L}_3 \) and \( \mathcal{L}_5 \) and the second one pairs \( \mathcal{L}_{4,6} \) with \( \mathcal{L}_{3,5} \).
Two distinct one-loop 1PI diagrams giving rise to corrections of the three point function. The diagrams represent contributions from the four possible combinations out of $\mathcal{L}_3$ and $\mathcal{L}_5$ and contributions from the mixing of even and odd numbered interaction terms respectively.

Again, we can have a look at what awaits us by invoking dimensional analysis together with Lorentz invariance. Note that there are now three indices of external polarization vectors to be contracted.

\begin{align}
M_{\mathcal{L}_3^3} & \sim \frac{m^6}{\Lambda_2^6} \left( k + \frac{k^3}{m^2} + \frac{k^5}{m^4} + \frac{k^7}{m^6} \right), \\
M_{\mathcal{L}_3^2 \mathcal{L}_5, \mathcal{L}_2 \mathcal{L}_4} & \sim \frac{m^4}{\Lambda_2^4} \left( m^2 k + k^3 + \frac{k^5}{m^2} + \frac{k^7}{m^4} + \frac{k^9}{m^6} \right), \\
M_{\mathcal{L}_3^2 \mathcal{L}_5, \mathcal{L}_3 \mathcal{L}_6, \mathcal{L}_5 \mathcal{L}_4} & \sim \frac{m^2}{\Lambda_2^2} \left( m^4 k + m^3 \partial^3 + k^5 + \frac{k^7}{m^2} + \frac{k^9}{m^4} + \frac{k^{11}}{m^6} \right), \\
M_{\mathcal{L}_3^2 \mathcal{L}_5, \mathcal{L}_3 \mathcal{L}_6} & \sim \frac{1}{\Lambda_2^2} \left( m^6 k + m^4 k^3 + m^3 k^5 + k^7 + \frac{k^9}{m^2} + \frac{k^{11}}{m^4} + \frac{k^{13}}{m^6} \right),
\end{align}

where again $k$ denote external momenta and the series stops at $1/m^4$ or $1/m^6$ depending on how many propagators are involved. Hence, by the same arguments as above, we should give special attention to the $\mathcal{L}_3^2 \mathcal{L}_5$, $\mathcal{L}_3^3$ and $\mathcal{L}_5 \mathcal{L}_6$ contributions containing external momenta to the power 11 or higher, as they potentially destabilize the EFT structure.

In order to calculate the diagrams explicitly, let’s quickly go through the combinatorics. The first diagram gets four different contributions from combinations out of $\mathcal{L}_3$ and $\mathcal{L}_5$. For each of these, there are $3!$ possible ways of exchanging the vertices, which for the $\mathcal{L}_3^3$ contribution for example simply cancels the prefactor of the expansion of the exponential. But for the combinations $\mathcal{L}_3^2 \mathcal{L}_5$ and $\mathcal{L}_3 \mathcal{L}_6$, a redistribution of vertices leads to three distinct results, hence each of these come only with a vertex exchange factor $2!$. After that, there remains $3!^3$ possible Wick-contractations for each diagram, as the symmetry factor is 1. Note that not all of these contractions are independent of course.

For the second diagram, there are three distinct channels which need to be considered. For each of these channels at fixed vertices, there are a priori 72 different ways of contracting in the S-matrix expansion (3.1) or in other words $3!^4$! different ways of distributing the insertions over the legs divided by the symmetry factor of two. Note that for vertices with a different number of legs there is no additional vertex exchange factor which could cancel the $1/2!$ in the exponential expansion in (3.1).

We show here only the schematic form of the result as a sum of contributions $M^{(i)}_3 (c_i)$ where $i$ denotes the power of external momenta involved, while the arguments $(c_j)$ show which diagrams contribute at the given order. In order not to clutter the result we will...
leave the argument in a general form whenever all possible contributions are involved. The
detailed expressions can be found in the appendix A.

\[ M_{3}^{\text{div}} = \frac{m^{6}}{16\pi^{2}\epsilon\Lambda_{2}^{2}} \left[ M_{3}^{(0)}(c_{1},c_{2},c_{3},c_{4}) + \frac{1}{m^{2}}M_{3}^{(2)}(c_{j}) + \frac{1}{m^{4}}M_{3}^{(4)}(c_{j}) + \frac{1}{m^{6}}M_{3}^{(6)}(c_{j}) \right. \\
\left. + \frac{1}{m^{8}}M_{3}^{(8)}(c_{j}) \right]. \quad (3.12) \]

Hence, the calculated series stops at \( \sim k^{9} \) and again even though dangerous contributions
would technically be allowed (3.11) non of the contributions reach the problematic powers
of external momenta.

And this point one could conclude, that all quantum corrections which renormalize
the given classical structure involving gauge breaking operators, although being heavily
suppressed, come from diagrams involving either \( \mathcal{L}_{5} \) of \( \mathcal{L}_{6} \). In other words, upon a restriction
of the generalized Proca model to the even numbered terms \( \mathcal{L}_{2}, \mathcal{L}_{4} \) and \( \mathcal{L}_{6} \) by choosing \( c_{3} = \tilde{c}_{4} = 0 \), the only one-loop correction so far is a gauge preserving operator proportional to
\( \tilde{c}_{4} \). Moreover, this choice is technically natural, since with only \( \mathcal{L}_{4,6} \) insertions no diagrams
with an odd number of external legs can be constructed. However, these properties are lost
as soon as corrections to higher point functions are taken into account as it will become
clear below.

### 3.1.3 Four-point

For completeness, we also calculate corrections to the four point function, but restrict
ourselves by simplicity to the contributions of the diagram in Fig. 3. The symmetry factor
of the diagram is two, such that for each of the three distinct channels there are at first
sight \( 4! \times 4! / 2 = 288 \) possible Wick contractions. Again vertex exchange cancels the \( 2! \) in the
exponential expansion.

![Figure 3](image)

**Figure 3:** A 1PI diagram giving rise to corrections of the four point function. The diagrams
represent contributions from the three possible combinations out of \( \mathcal{L}_{4} \) and \( \mathcal{L}_{6} \).

After adding all the contractions together and going through the dimensional regularization
procedure, the schematic form of the result reads

\[ M_{4}^{\text{div}} = \frac{m^{8}}{16\pi^{2}\epsilon\Lambda_{2}^{2}} \left[ M_{4}^{(0)}(\tilde{c}_{1}) + \frac{1}{m^{2}}M_{4}^{(2)}(c_{1} + \tilde{c}_{1}) + \frac{1}{m^{4}}M_{4}^{(4)}(c_{2}) + \frac{1}{m^{6}}M_{4}^{(6)}(c_{3}) + \frac{1}{m^{8}}M_{4}^{(8)}(\tilde{c}_{4}) \right. \\
\left. + \frac{1}{m^{10}}M_{4}^{(10)}(c_{1} + \tilde{c}_{1}) + \frac{1}{m^{12}}M_{4}^{(12)}(\tilde{c}_{4}) \right], \quad (3.13) \]

with the same notation as above and details in the appendix A.\footnote{Due to the lengthiness of the results we only present the cases \( i = 0, 2 \)}

Thus also the classical terms \( \mathcal{L}_{4} \) and \( \mathcal{L}_{6} \) get renormalized. In particular, \( M_{4}^{(2)} \) generates operators of the form...
\[ A^\mu A^\nu (\partial_\mu A_\alpha \partial_\nu A^\alpha + \partial_\alpha A_\mu \partial^\alpha A_\nu) \] which destroy the classical ghost-free tuning. Yet again these operators come with heavy suppressions such that the associated ghost degree of freedom will have a mass way above the cutoff. Only the contribution involving twelve external momenta is potentially worrisome as it naively diverges in the decoupling limit. But as it will become clear in the next section, the corresponding counterterm again preserves gauge invariance, such that the actual decoupling limit is of the form

\[ \sim \frac{\partial^8}{\Lambda_2^8 m^4} F^4 \frac{\partial^8}{\Lambda_3^8 F^2} F^2. \] (3.14)

### 3.2 Schwinger-DeWitt Technique

As a complementary check, we compute the one-loop counterterms above using an alternative, effective action based method which combines background field and generalized Schwinger-DeWitt techniques. This method has the additional advantage that it naturally generalizes to curved space-time.\(^{10}\)

The starting point is the one-loop effective action given by:

\[ \Gamma^{(1)}_{\text{div}} = \frac{i}{2} \text{Tr} \ln \hat{F}, \] (3.15)

computed after a split of the field \( A_\mu \to \bar{A}_\mu + B_\mu \) into background and quantum parts with \( \hat{F} \) denoting the bilinear form of the action (2.3)

\[ S^{(2)} = -\frac{1}{2} \int d^4 x B \hat{F} B, \quad \hat{F} = \hat{D}_2 + \hat{P}, \] (3.16)

which can be decomposed into its principle part \( [\hat{D}_2]_{\mu \nu} = (\Box + m^2)\eta_{\mu \nu} - \partial_\mu \partial_\nu \) and the sub-leading perturbations \( \hat{P} = \sum_{i=3}^6 \hat{D}_i (\bar{A}) \) depending on the background field which originate from the interaction terms \( \mathcal{L}_{3,..6} \).

The decomposition (3.16) together with an expansion of the logarithm in (3.15) leads to

\[ \text{Tr} \ln \hat{F} = \text{Tr} \ln \hat{D}_2 + \text{Tr} \left[ \hat{P} \hat{D}_2^{-1} \right] - \frac{1}{2} \text{Tr} \left[ \hat{P} \hat{D}_2^{-1} \hat{P} \hat{D}_2^{-1} \right] + \mathcal{O}(\hat{P}^3), \] (3.17)

where the principle operator can be inverted to give

\[ [\hat{D}_2^{-1}]_{\mu \nu} = \frac{1}{\Box + m^2} \left( \eta_{\mu \nu} + \frac{\partial_\mu \partial_\nu}{m^2} \right). \] (3.18)

The trick is now to transform the expansion (3.17) above into a sum of terms proportional to universal functional traces whose divergent part can be evaluated by resorting to Schwinger-DeWitt techniques [81]. In flat spacetime, the only non-vanishing universal functional traces in dimensional regularization with \( d = 4 - 2\epsilon \) are

\[ \text{Tr} \left. \mathcal{P}^{\mu_1 \ldots \mu_N} (\bar{A}) \partial_{\mu_1} \ldots \partial_{\mu_N} \frac{1}{(\Box + m^2)^n} \right|_{\text{div}} = \frac{i}{16\pi^2 \epsilon} \int d^4 x \mathcal{P}^{\mu_1 \ldots \mu_N} (\bar{A}) \frac{(-1)^n m^{2l}}{2^N l! (n-1)!} \eta^{(N)}_{\mu_1 \ldots \mu_N}, \] (3.19)

\(^{10}\)See [82] for a specific application of the method to a similar theory.
where \(2N = 2n - 4 + 2l\), \(N \geq 1\), \(n \geq 1\), \(l = 0, 1, 2, \ldots\) and \(\eta_{\mu_1 \ldots \mu_{n-2+2l}}^{(n-2+l)}\) is the totally symmetrized product of \(n - 2 + l\) metrics. Note that the background field dependent piece \(\mathcal{P}(\bar{A})\) just goes along the ride, regardless of it’s precise structure.

The terms appearing in the expansion (3.17) are cast into the specific form appearing on the left hand side of (3.19) by commuting all the operators \(1/(\Box + m^2)\) to the right. This procedure is efficient, as each commutation decreases the number of partial derivatives in the numerator of (3.19) compared to the factors of \(1/(\Box + m^2)\) and increases the number of derivatives on the background operator \(\hat{P}\):

\[
\left[ \frac{1}{\Box + m^2}, \hat{P} \right] = - \frac{1}{\Box + m^2} [\Box, \hat{P}] \frac{1}{\Box + m^2}, \text{ where } [\Box, \hat{P}] = (\Box \hat{P}) + 2(\partial^\alpha \hat{P}) \partial_\alpha \tag{3.20}
\]

This means that the while the log expansion (3.17) will be cut off by the maximum number of background fields one is interested in, the iterative commutation of operators (3.20) will constantly increase the number of derivatives applied on the background fields, which thus allows for the computation of counterterms up to any desired but fixed order in derivatives as well as in the fields.

Using this method we have verified the Feynman diagram calculations of §3.1 up to four background fields, but restricted ourselves to a maximum of four derivatives acting on them, which translates into a limitation to four powers of external momentas. This nevertheless represents a very powerful check of the results, since in the diagramatic momentum space computations all powers of external momentas are treated at the same time.

We will now present the most important results. First of all, note that in contrast to the massless case, the expansion in factors of \(m^2\) measured by the integer \(l\) allows for divergent contributions of the linear terms in (3.17) with \(n = 1\). However, tadpole contributions arising from the interaction terms \(\mathcal{L}_3\) and \(\mathcal{L}_5\) are immediately ruled out by the odd number of derivative factors. Thus, the linear terms will only provide potential corrections to the two point function via contributions from \(\mathcal{L}_4\) and \(\mathcal{L}_6\).

The next terms in the log expansion (3.17) \(\sim \hat{P}^2\) give rise to contribution to the 2-point function originating in the interactions \(\mathcal{L}_3\) and \(\mathcal{L}_5\) and contributions to the 3- and 4-point functions by a suitable mixing of all interaction terms. We explicitly show here the full logarithmic divergent contribution at one loop to the 2-point effective action up to the given order in derivatives

\[
\Gamma_{1,2}^{\text{div}} = \frac{m^4}{16\pi^2\epsilon \Lambda_2^2} \int d^4x \left[ \left( -\frac{3}{2} c_3^2 + 3 \tilde{c}_4 - 2 c_3 \tilde{c}_5 + \tilde{c}_5^2 \right) \partial_\mu \bar{A}_\nu \partial^\mu \bar{A}^\nu \\
+ \left( \frac{3}{2} c_3^2 + 3 \tilde{c}_4 \right) m^2 \bar{A}_\mu \bar{A}^\mu \\
+ \left( 6 c_3^2 - 3 \tilde{c}_4 + 8 c_3 \tilde{c}_5 + \frac{11}{4} \tilde{c}_5^2 \right) (\partial_\mu \bar{A}^\mu)^2 \\
- \left( \frac{3}{2} c_3^2 + c_3 \tilde{c}_5 - \frac{1}{3} \tilde{c}_5^2 \right) \frac{1}{m^2} \partial_\mu \partial_\nu \bar{A}^\nu \Box \bar{A}^\mu \\
- \frac{19}{12} \tilde{c}_5^2 \frac{1}{m^2} \Box \bar{A}_\mu \Box \bar{A}^\mu \right]. \tag{3.21}
\]
In order to relate this result with the Feynman diagram calculation (3.8) recall that the effective action is a generating functional of 1PI correlation functions

\[
\frac{\delta^n \Gamma [\pi]}{\delta A^{\mu_1} (x_1) ... \delta A^{\mu_n} (x_n)} \bigg|_{A = \langle A \rangle} = \langle A^{\mu_1} (x_1) ... A^{\mu_n} (x_n) \rangle_{\text{1PI}}. \tag{3.22}
\]

The 1PI correlation functions in turn are given by the sum of all 1PI diagrams with \( n \) external points. Thus, fourier transformed functional derivatives of divergent one-loop effective action results at vanishing mean field should coincide with the corresponding divergent off-shell results of the 1PI diagrams calculated in §3.1. We explicitly checked this for all calculations. For instance for the 2-point result (3.21) it can be seen by eye that it precisely matches the momentum space calculation (3.8) as the conversion essentially merely introduces a factor of 1/2.

As concerns higher point results, we won’t need terms in the log expansion (3.17) higher than \( \sim \hat{P}^3 \), as these cover all cases considered through Feynman calculations in §3.1. We explicitly present here a selection of the most relevant 3 and 4-point leading order results up to three powers of external momenta which serve as highly non trivial checks of the Feynman diagram based momentum space calculations.

\[
\Gamma_{1,3}^{\text{div}} \supset \frac{1}{16\pi^2} \frac{m^6}{\Lambda_2^2} \int \! d^4 x \left[ 9 \tilde{c}_6 \tilde{c}_5 \tilde{A}^2 \bar{\partial}_\mu \tilde{A}^\nu + \frac{1}{12} \bar{c}_4 \frac{1}{m^2} \left\{ 11 ( \partial_\mu \tilde{A}^\nu )^3 + 51 \partial_\mu \tilde{A}^\nu \partial_\nu \tilde{A}^\rho + ( \partial_\mu \tilde{A}^\nu )^2 \right\} - 6 \partial^\mu \tilde{A}^\nu \partial_\alpha \tilde{A}^\rho \tilde{A}^\sigma - 2 \partial^\mu \tilde{A}^\nu \partial_\alpha \tilde{A}^\rho \tilde{A}^\sigma \right] \right] \tag{3.23}
\]

\[
\Gamma_{1,4}^{\text{div}} \supset \frac{1}{16\pi^2} \frac{m^6}{\Lambda_2^2} \int \! d^4 x \left[ 9 \tilde{c}_1^2 m^2 (\tilde{A}^2)^2 - (2 \tilde{c}_1^2 + 16 \tilde{c}_4 \tilde{c}_6 + 20 \tilde{c}_4^2 + 3 \tilde{c}_4 \tilde{c}_6) \tilde{A}^2 \bar{\partial}_\mu \tilde{A}^\nu \partial^\rho \tilde{A}^\sigma - (2 \tilde{c}_1^2 - 10 \tilde{c}_4 \tilde{c}_6 - 2 \tilde{c}_4^2 - 3 \tilde{c}_6 \tilde{c}_6) \tilde{A}^2 ( \partial_\mu \tilde{A}^\nu )^2 + 2 ( \tilde{c}_1^2 + 3 \tilde{c}_4 \tilde{c}_6 - 9 \tilde{c}_4^2 ) \tilde{A}^2 \partial_\mu \tilde{A}_\rho \tilde{A}^\nu \partial^\sigma \tilde{A}^\rho \right. \\
- 2 ( \tilde{c}_1^2 + 3 \tilde{c}_4 \tilde{c}_6 - 3 \tilde{c}_6 \tilde{c}_6 - 5 \tilde{c}_6^2 ) \tilde{A}^2 \bar{\partial}_\mu \tilde{A}^\nu \partial_\rho \tilde{A}^\sigma + 4 ( 4 \tilde{c}_1^2 + \tilde{c}_6^2 ) \tilde{A}^2 \bar{\partial}_\mu \tilde{A}^\nu \partial_\rho \tilde{A}^\sigma \bar{\partial}_\sigma \tilde{A}^\rho \right] \right]. \tag{3.24}
\]

4 Decoupling Limit Analysis

Based on the above results we will now intent a complete radiative stability analysis of the Generalized Proca EFT. To this end we will leave the unitary gauge employed in the previous section and instead take the decoupling limit. This will at a first step shed light on many important aspects already discussed in §3 and allow a generalization of the results to any order even beyond one-loop. It turns out that the structure of the generalized Proca EFT bares many similarities with equally self-interacting non-abelian \( SU(2) \) spin 1 fields endowed with a mass term and thus the hierarchy structure of the weak sector of the Standard Model EFT.

As discussed in §2, rewriting the generalized Proca model (2.3) by introducing a Stückelberg field \( \phi \) allows one to take a smooth \( m \to 0 \) limit, without loosing any degrees of freedom. In the present interacting theory, this decoupling limit works actually as a high energy limit way above the vector mass and right at the lowest cutoff \( \Lambda_3 \equiv (\Lambda_2^2 m)^{1/3} \). The only operators which survive this high energy limit are the least suppressed ones and thus the decoupling limit puts focus on the operators with the poorest behavior. The resulting theory is described through (2.12), where the transverse modes, in this limit described by a
massless and gauge invariant vector field $A_\mu$, are decoupled in a symmetry sense from the longitudinal Goldstone mode $\phi$. This directly implies that potential ghost-like interactions for the temporal component of the vector field in the original theory gets mapped on to the ghostly interactions of the Goldstone field and quantum stability in the decoupling limit implies quantum stability of the whole theory. Essentially, healthiness of generalized Proca EFTs can be inferred simply from the fact that these theories admit a well defined decoupling limit, together with the regular high energy behavior $\sim 1/p^2$ of propagators in the Stückelberg formulation as discussed below §4.2. Moreover, the decoupling limit offers a deeper understanding of the crucial cancellations observed in the unitary gauge calculations in §3.

### 4.1 Reinterpretation of the Results

As $L_3$ and part of $L_4$ reduce to scalar Galileon terms, it is useful to quickly remind ourselves how the Galileon EFT is structured. It is known that the classical terms are not renormalized, since quantum corrections always come with more derivatives per field. Employing dimensional regularization the full scalar Galileon EFT lagrangian schematically goes like

$$L'_{\text{Gal}} \sim (\partial^2 \pi) \left( \frac{\partial^2 \pi}{\Lambda_3^3} \right)^i + \left( \frac{\partial^2 \pi}{\Lambda_3^3} \right)^{3+n} (\partial^2 \pi)^2 \left( \frac{\partial^2 \pi}{\Lambda_3^3} \right)^{m-2}, \quad 3 \geq i \geq 0, \ n \geq 0, \ m \geq 2. \quad (4.1)$$

This follows again from Lorentz invariance, the massless propagator and the fact that only the log divergent piece enters in the construction of counterterms. This defines the two expansion parameters

$$\alpha_{\text{cl}} = \frac{\partial^2 \pi}{\Lambda_3^3} \quad \text{and} \quad \alpha_{\text{q}} = \frac{\partial^2 \pi}{\Lambda_3^3}, \quad (4.2)$$

and there exists a regime below the UV cutoff, where the a priori irrelevant classical non-linear galileon operators become important compared to the kinetic term $\alpha_{\text{cl}} \sim \mathcal{O}(1)$, while quantum corrections are still under control $\alpha_{\text{q}} \ll 1$. The Galileon EFT is even well defined in regimes where $\alpha_{\text{cl}} \gg 1$ as the enhanced kinetic term suppresses quantum fluctuations further, which cures the potentially dangerous expansion in external legs in (4.1) [20].

Coming back to the generalized Proca theory in the decoupling limit, the considerations above directly imply that in the full theory, for instance also in the unitary gauge, all quantum corrections generated exclusively through $L_3$ and the $c_4$ term in $L_4$ are safe. Since taking the decoupling limit and computing quantum corrections are two operations which should commute, from (4.1) one can as well directly infer the corresponding highest order quantum correction in the unitary gauge

$$L_{L_3,L_4(c_4)}' \sim m^m \frac{\partial^{4+2n+m}}{\Lambda_3^{2n+3m}} A^m, \quad n \geq 0, \ m \geq 2, \quad (4.3)$$

where for example the case $m = 2, n = 0$ corresponds to the two-point result §3.1.1. Moreover, we could also have directly inferred the absence of high momenta power contributions proportional to $c_4$ in (3.8), because $L_{4\text{Gal}}$ does not contribute to the one-loop correction with two external legs.
Up to now, these considerations parallel the powercounting arguments in the unitary gauge. However, quantum corrections involving $L_5$ and $L_6$ required non-trivial cancellations of the leading order dimensional estimations to remain healthy. These cancellations can readily be explained from the point of view of the decoupling limit. Let’s first focus at the one loop corrections to the propagator. $L_5$ in the decoupling limit is an interaction term between the massless vector and the Goldstone (2.12). This directly implies that in the high energy limit no one-loop diagram can be formed between the two terms $L_3 L_5$ and hence, terms proportional to $c_3 \tilde{c}_5$ have no impact close to the cutoff scale $\Lambda_3$ and remain highly suppressed. This is in perfect agreement with the obtained results (3.8). However, two distinct diagrams can be formed with two $L_5$ insertions depicted in Fig.4, where straight lines denote scalar legs $\sim \partial^2 \phi$ or scalar propagators $\sim 1/p^2$ and the wiggled lines massless vector legs and propagators $\sim F$ and $\sim 1/p^2$ respectively.

**Figure 4:** Two distinct one-loop $L_5$ diagram contributions in the decoupling limit giving rise to corrections of the two point function. Solid lines represent scalar legs or propagators. Each external leg comes with two derivatives applied on the field $\sim \partial^2 \phi$. Wiggled lines correspond to gauge preserving vector legs $\sim F$ or corresponding propagators. In the decoupling limit, propagators have a good $\sim 1/p^2$ high energy behavior.

The first diagram induces a counterterm proportional to $(\partial^2 \phi)^2$ and thus leads to a contribution of the same order as (4.1), since the external legs also carry two derivatives per scalar field, while second one is bound to be gauge invariant

$$L^c_{L_5 L_5}^{\text{DL}} \sim \tilde{c}_5^2 \left[ \frac{\partial^4}{\Lambda_3^4} (\partial^2 \phi)^2 + \frac{\partial^6}{\Lambda_3^6} (F)^2 \right]. \tag{4.4}$$

This explains the cancelation of the $\sim k^{10}$ contributions in the one-loop two-point corrections in the original theory as terms of this order cannot be formed in the decoupling limit. Translating back these decoupling limit results to the unitary gauge is done via a replacement $\partial \phi \rightarrow m A$ and gives

$$L^c_{L_5 L_5} \sim \tilde{c}_5^2 \left[ \frac{m^2 \partial^4}{\Lambda_3^4} (\partial A)^2 + \frac{\partial^6}{\Lambda_3^6} (F)^2 \right], \quad (4.5)$$

which is in perfect agreement with the $F^2$ structure obtained in the last line of (3.8).

Similar for higher point functions. For example $\tilde{c}_3^2$ diagrams at most generate counterterms of the form $\sim \frac{1}{\Lambda_3^4} \partial^4 (\partial^2 \phi) F^2$, while it is not possible to form contributions going like $\sim \frac{1}{\Lambda_3^3} \partial^3 F^3$. This is explicitly confirmed by the 3-point calculation (3.12). In a similar manner, the 4-point results can be understood. For instance, the $\tilde{c}_6^2$ high energy contributions come from the three diagrams 5 in the decoupling limit. The schematic form of the
corresponding counterterms is

\[ \mathcal{L}_{\mathcal{L}_6 \mathcal{L}_6}^{DL} \sim \tilde{c}_0^2 \left[ \frac{\partial^4}{\Lambda_3^2} \left( \partial^2 \phi \right)^4 + \frac{\partial^6}{\Lambda_3^2} \left( \partial^2 \phi \right)^2 (F)^2 + \frac{\partial^8}{\Lambda_3^2} (F)^4 \right], \] (4.6)

which in unitary gauge corresponds to

\[ \mathcal{L}_{\mathcal{L}_6 \mathcal{L}_6}^{DL} \sim \tilde{c}_0^2 \left[ \frac{m^4}{\Lambda_3^2} \left( \partial^4 A \right)^4 + \frac{m^2}{\Lambda_3^2} \left( \partial^6 A \right)^2 (F)^2 + \frac{\partial^8}{\Lambda_3^2} (F)^4 \right], \] (4.7)

in perfect agreement with (3.13).

Figure 5: Three distinct one-loop $\mathcal{L}_6$ diagram contributions in the decoupling limit giving rise to corrections of the four point function. Solid lines represent scalar legs or propagators. Each external leg comes with two derivatives applied on the external field $\sim \partial^2 \phi$. Wiggled lines correspond to gauge preserving vector legs $\sim F$ or corresponding propagators. In the decoupling limit, propagators have a good $\sim 1/p^2$ high energy behavior.

### 4.2 Quantum Stability of Generalized Proca Theories

These decoupling limit arguments can be generalized to all higher point functions, showing that the EFT structure is stable under all possible quantum corrections. The following argument will even go beyond the specific model chosen in (2.2) and include all possible generalized Proca EFTs described by (2.1). First of all note that up to total derivatives, a generic generalized Proca theory has a classical action with the schematic form

\[ \mathcal{L} \sim (F^2 + m^2 A^2) \left( \frac{mA}{\Lambda_2^2} \right)^{2a_1} \left( \frac{F}{\Lambda_2^2} \right)^{a_2} \left( \frac{\partial A}{\Lambda_2^2} \right)^{a_3}, \quad a_{1,2,3} \geq 0, \quad a_3 \leq 4 \] (4.8)

where of course all suppressed Lorentz indices need to be contracted and the classical structure is such that the theory only propagates the required three degrees of freedom. In the language of (2.3), higher order $\mathcal{L}_2$ terms and for instance also the $\tilde{c}_4$ term defined above are build out of powers of $a_1$ and $a_2$, Galileon like contributions out of $a_1$ and $a_3$ but proportional to $m^2 A^2$ only, while the last class of terms to which $\tilde{c}_5$ and $\tilde{c}_6$ again only involve powers of $a_1$ and $a_3$ but are always proportional to $F^2$. This shows that the decoupling limit of all generalized Proca theories is well defined and the classical Lagrangian reduces to

\[ \mathcal{L}_{\text{DL}} \sim (F^2 + (\partial \phi)^2) \left( \frac{\partial^2 \phi}{\Lambda_3^2} \right)^{a_3}, \quad 4 \geq a_3 \geq 0 \] (4.9)

This justifies the specific choice of our model (2.2) in retrospective, since we cover all interesting cases.\footnote{The cases with $a_3 > 2$ only appear for pure Galileon terms proportional to $(\partial \phi)^2$ which yield known contributions.}
Including now loop contributions, this further implies, that each vertex in the decoupling limit comes with a factor of $\frac{1}{\Lambda^3}$. This means that in dimensional regularization at one loop where each vertex at least includes one external leg there are only two distinct schematic building blocks for quantum induced operators $\frac{\partial F}{\Lambda^3}$ and $\frac{\partial^2 \phi}{\Lambda^3}$, and a general one loop counterterm in the decoupling limit therefore has the generic form

$$\mathcal{L}_{DL} \sim \partial^4 \left( \frac{\partial F}{\Lambda^3} \right)^{2b_2} \left( \frac{\partial^2 \phi}{\Lambda^3} \right)^{b_3} \sim \left\{ \begin{array}{ll}
F^2 \left( \frac{\partial^2}{\Lambda^3} \right)^{2+b_2} \left( \frac{\partial F}{\Lambda^3} \right)^{b_2-1} \left( \frac{\partial^2 \phi}{\Lambda^3} \right)^{b_3}, & b_2 \geq 1 \\
(\partial \phi)^2 \left( \frac{\partial^2}{\Lambda^3} \right)^{2+b_2} \left( \frac{F^2}{\Lambda^3} \right)^{b_2} \left( \frac{\partial^2 \phi}{\Lambda^3} \right)^{b_3-2}, & b_3 \geq 2
\end{array} \right.\quad (4.10)$$

where $2b_2 + b_3 = N \geq 2$ with $N$ the number of external fields. Thus, on top of the two expansion parameters $\alpha_{cl}$ and $\alpha_q$ defined in the pure scalar Galileon context (4.2), we can identify a second classical expansion parameter

$$\alpha_{\tilde{c}l} = \frac{F^2}{\Lambda^3}.\quad (4.11)$$

But the analysis remains the same: All operators generated at one loop involve additional factors of $\alpha_q$, such that there exists a parametrically large regime in which quantum contributions are suppressed $\alpha_q \ll 1$, while classical, non-linear terms are still important $\alpha_{cl,\tilde{c}l} \sim O(1)$. Exactly as for the pure scalar Galileon case, in the regime $\alpha_{cl,\tilde{c}l} \gg 1$ the expansion in external legs, hence large $b_i$, is not well defined at this point. However, splitting the scalar field into its background and fluctuation contribution leads to an enhanced tree level kinetic term of quantum fluctuations whenever classical-non-linearities are large, thus effectively pushing the cutoff to higher values. For this to work, it is crucial that the classical contributions do not lead to ghost instabilities which is exactly why generalized Proca theories are so interesting. Even when including higher loop contributions the picture will not change as higher loops will merely introduce additional factors of $\partial^2/\Lambda^3$.

One can thus conclude, that the generalized Proca EFT (2.3) does not loose it’s key properties when including quantum corrections in their full generality and the effective description is theoretically viable.

The expansion (4.10) translated back to the unitary gauge gives access to the least suppressed quantum corrections in the original formulation

$$\mathcal{L}^c \sim \partial^4 \left( \frac{\partial F}{\Lambda^3} \right)^{2b_2} \left( \frac{m \partial A}{\Lambda^3} \right)^{b_3},\quad (4.12)$$

which are the ones with $b_3 = 0$, hence the ones which preserve gauge invariance. Other contributions with non-zero $b_3$ and operators which do not survive the decoupling limit are further suppressed by factors of $m/\Lambda_3$.

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12Once again this schematic form is fixed through Lorentz invariance, powercounting and the well behaved propagators in the decoupling limit.
5 Conclusion

The search for viable extensions to general relativity is guided on the one hand by observational constraints and the requirement of theoretical consistency on the other. As concerns the latter, the modern understanding of renormalization views in particular gravity theories as effective, such that a quantum stability check is indispensable for every proposed model. The sole classical description of an effective field theory does not make sense on a fundamental level. This is especially true for Galileon type models involving irrelevant derivative self-interactions which gain importance only in regimes where loop corrections might harm the classical EFT structure.

In this work we have investigated the stability of generalized Proca theories under quantum corrections and explicitly calculated all one-loop counterterms up to the three-point function with a glimpse towards four-point results. Doing so revealed a generic neutralization of dangerous leading order corrections, preserving the hierarchy between scales and the specific structure of classical operators. These results were confirmed by the use of an entirely independent Heat Kernel method with only the input of the Lagrangian as a common ground. More than a thorough check of the results, this method paves the way to a covariant generalization. Beyond that, a reformulation of the theory in terms of the Stückelberg method permitted an interpretation of the results from a different angle heavily relying on the existence and consistency of the decoupling limit in generalized Proca theories. This empowered us to an extension of the results to arbitrary orders without any restrictions regarding specific model choices. In summary, we have thus shown that the attractive properties of generalized Proca models withstand the quantum check in its full generality.

These results are especially noteworthy with possible cosmological applications in mind, as the hierarchy between classical and quantum non-linearities allow for regimes in which the former dominate while EFT description is still protected against quantum detuning. Including gravity and matter fields, this endows the theory with a natural Vainshtein screening in dense regions whereas the additional vector field serves as a generalization of gravity on cosmological scales. While we leave the explicit coupling of the full EFT to gravity for future work, we should expect a smooth inclusion of graviton loops as each mixed vertex comes with a heavy plank mass suppression.

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\[ M^{(1)} = -2(2c_0 + c_3(c_4 - 2c_4) + 9c_5c_7 - 9c_5c_9)(e_{22} + e_{22} - e_{12}(e_{22} + e_{32})) \] (A.1)

\[ M^{(3)} = -2(3c_3(c_4 - 2c_4) + 9c_5c_7 - 9c_5c_9)(e_{22} + e_{22} - e_{12}(e_{22} + e_{32})) \] (A.2)
\[ + \epsilon_{k1}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k2}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k3}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k4}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k5}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k6}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k7}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k8}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) \]

\[ + \epsilon_{k1}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k2}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k3}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k4}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k5}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k6}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k7}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k8}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k9}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) + \epsilon_{k10}(19k_{1}^{2} + 20k_{1}k_{2} + 78k_{1}k_{2}^{2}) ]}
$$M_1^{(A)} = 72c_6(1 + 32c_{12} + c_{14} + 2c_{16} + c_{22})$$

$$M_2^{(A)} = 12c_6(c_{23} + c_{24} + c_{26} + c_{28} + c_{30} + c_{32} + c_{34} + c_{36})$$

where $c_{ij} = c_{ij} - c_{ij}$, $k_{ij} = k_{ij} - k_{ij}$ and $e_{ij} = e_{ij} - e_{ij}$.
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