On the Strong Comparison Principle for Degenerate Elliptic Problems with Convection.

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Abstract

We will discuss the weak and strong comparison principles (WCP and SCP, respectively) for quasilinear elliptic boundary value problems with the $p$-Laplacian in one space dimension, $\Delta_p(u) \overset{\text{def}}{=} \frac{d}{dx} \left(|u'|^{p-2}u'\right)$. We treat the “degenerate” case of $2 < p < \infty$ and allow also for the nontrivial convection velocity $b : [-1,1] \to \mathbb{R}$ in the underlying domain $\Omega = (-1,1)$. We establish the WCP under a rather general, “natural sufficient condition” on the convection velocity, $b(x)$, and the reaction function, $\varphi(x,u)$. Furthermore, we establish also the SCP under a number of various additional hypotheses. In contrast, with these hypotheses being violated, we present also a few rather natural counterexamples to the SCP and discuss their applications to an interesting classical problem of fluid flow in porous medium, “seepage flow of fluids in inclined bed”. Our methods are based on a mixture of classical and new techniques.
Keywords: Quasilinear elliptic equation; degenerate $p$-Laplacian; examples and counterexamples to strong comparison principle; Hopf’s boundary point lemma

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