On UV/IR mixing in noncommutative
gauge field theories

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ABSTRACT: In formulating gauge field theories on noncommutative (NC) spaces it is suggested that particles carrying gauge invariant quantities should not be viewed as pointlike, but rather as extended objects whose sizes grow linearly with their momenta. This and other generic properties deriving from the nonlocal character of interactions (showing thus unambiguously their quantum-gravity origin) lead to a specific form of UV/IR mixing as well as to a pathological behavior at the quantum level when the noncommutativity parameter $\theta$ is set to be arbitrarily small. In spite of previous suggestions that in a NC gauge theory based on the $\theta$-expanded Seiberg-Witten (SW) maps UV/IR mixing effects may be under control, a fairly recent study of photon self-energy within a SW $\theta$-exact approach has shown that UV/IR mixing is still present. We study the self-energy contribution for neutral massless fermions in the $\theta$-exact approach of NC QED, and show by explicit calculation that all but one divergence can be eliminated for a generic choice of the noncommutativity parameter $\theta$. The remaining divergence is linked to the pointlike limit of an extended object.

KEYWORDS: noncommutative quantum field theory, neutrino self-energy
A reasonable expectation about noncommutative (NC) field theories is that they should reduce to their commutative relatives whenever the momenta of the field quanta are well below $|\theta|^{-1/2}$ (or more exactly when the limit $\theta \to 0$ is undertaken). In fact that was one of the main reasons why NC field theories based on star products are so popular. This naive expectation is badly violated at the loop level where the inherent nonlocality of the full theory shows up in the UV/IR mixing phenomenon [1]. The effect is characterized by the appearance of new infrared divergences at the IR-limit of the external momentum and is accompanied by a nonanalytic behavior in the noncommutativity parameter $\theta$. In the combined effect, the theory also shows pathological behavior when the spatial extension of size $|\theta P|$, for a particle moving with momentum $P$ along the region affected by spacetime noncommutativity, gets reduced to a point.

Another naive expectation concerns studies of NC gauge theories where one anticipates the absence of quadratic and linear IR divergences as the corresponding commutative-theory counterparts deal at most with logarithmic divergences. That this second expectation does not hold was found for the first time in [3] for the photon self-energy correction in NC QED, and reconfirmed later on in [4, 5]. In these papers the $\theta$-dependent infrared divergent terms (quadratic poles) were found in the real part of the photon two-point function at one-loop order. It should be however stressed that such an inappropriate behavior was not found in the imaginary part of the self-energy, nor in the one-loop correction to the electron self-energy [4, 5, 8], where, due to miraculous cancellation of phase factors accompanied with the two vertices, the contribution boils down to the one found in ordinary QED. We note that this second expectation does hold for supersymmetric NC gauge theories, where, because of cancellation between fermion and boson loops, the UV/IR-mixing problem is softened in such a way that only logarithmic divergences appear at small values of NC momenta [4].

The next stage in the development of NC gauge theories occurred with the seminal paper [9], where NC fields and gauge transformation parameters were interpreted as nonlocal, enveloping algebra-valued functions of their commutative counterparts and of the noncommutativity parameter $\theta$. Such a connection, known as the Seiberg-Witten map, has virtues compared to the early attempts based solely on star products in that now gauge covariance [10] and gauge fixing are more easily understood. This also enables one to deform commutative gauge theories with essentially arbitrary gauge group and representation [11, 12, 13, 14, 15, 16]. Such a procedure furthermore allows the construction of NC extensions of important particle physic models like the NC standard model and grand unified theory models [17, 18, 19, 20, 21, 22]. Moreover, upon expanding up to first order in $\theta$, those models were shown to develop trouble-free one-loop quantum corrections [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. Still, the Seiberg-Witten map is also known not to be free of freedom/ambiguities [34, 35].

It has been argued [24] that due to the prodigious freedom in the Seiberg-Witten map, the photon self-energy in NC QED can be made free from UV/IR mixing when expansion in the NC parameter $\theta$ is accomplished. However, it has been demonstrated recently that in the $\theta$-exact Seiberg-Witten map UV/IR mixing yet reappears [36].

A covariant $\theta$-exact approach, inspired by exact formulas for the SW map [15, 37, 38]
and developed for the first time in [36], was used later to study photon-neutrino phenomenology, namely, photon-neutrino interactions in various astrophysical/cosmological environments [39, 40]. The central result in those papers was a derivation of the tree-level coupling of neutrinos with photons, absent in the standard model settings. There, a gauge field couples to a spinor field in the adjoint representation of U⋆(1), which enables particles not charged under the gauge group to have an electric dipole moment proportional to θ. More generally, electrically neutral matter fields will be promoted via (hybrid) Seiberg-Witten map to NC fields that couple via star commutator to photons and transform in the adjoint representation - this is also the case for phenomenologically promising NC grand unified theories [19, 22]. Since such an interaction has no its commutative counterpart, it switches off when θ → 0. Such interactions have already been studied in the framework of NC gauge theories, defined by the θ-expanded Seiberg-Witten map [41, 42]. There, like in almost all other studies of covariant NC field theory an expansion in θ ∼ E/ΛNC was used, where ΛNC is the NC scale and E is the characteristic energy for a given process. There exists, however, exotic processes like scattering of ultra high energy neutrinos from extraterrestrial sources, in which the interacting energy scale runs higher than the current experimental bound on ΛNC [39]. In this case the aforementioned expansion is no longer applicable, so one is forced to resort to NC field theoretical frameworks involving the full θ resummation.

The θ-exact SW map expansion employed in [36] could be derived from direct recursive computation using consistency conditions [40]. Instead of expanding in momenta, NC fields are expanded here in powers of the commutative gauge-field aμ and hence in the powers of the coupling constant. At each order in aμ, however, θ-exact expressions can be determined. The motivation behind this procedure is to sort the interaction vertices by the number of field operators. In tree-level neutrino-photon coupling, an expansion to the lowest nontrivial order in aμ (but all orders in θ) will suffice.

In the present paper, we have undertaken an explicit and exact calculation of the neutral (massless) fermion self-energy at one loop in NC QED using the θ-exact Seiberg-Witten map of [36]. In spite of pretty involved and tedious computation, we have managed to express the final result in an analytic form, so that all pathological behavior originating from UV/IR mixing can be studied in an unambiguous way \(^1\). The structure of the final result encountered in our calculation is quite a unique one, not seen in any of the previous approaches. The crucial novelty is due to θ resummation. Namely, although the structure of the self-energy can be guessed by noting that in addition to the momentum of the neutral fermion, two extra 4-vectors (\(\tilde{p}^μ\) and \(\tilde{p}'^μ\), to be defined below) can be constructed in NC spacetimes, what we have found here is that the coefficients in front of this new spinor structures are not constant, but rather functions of the momentum and of the NC parameter θ (we give explicit expressions for them). This brings in consequences for the UV/IR mixing as well. First of all, the second naive expectation does hold here (without supersymmetry) as UV/IR mixing is represented by a logarithmic divergence. We show that if (without loss of generality) θ is taken to lie in the (1, 2) plane, the UV/IR mixing

\(^1\)We stress a difference with regard to the previous paper [24] on this topic, in which only a partial structure of the photon self-energy was explicitly evaluated.
term disappears and the rest of the contribution is well behaved in the UV and IR region, having also a continuous commutative limit. However, we show that a divergence linked to the spatial extension of the particle, in a direction of space affected by \( \theta \), still persists. These are the central results of our paper.

We start with the following model of a Seiberg-Witten type NC U\(_s\)(1) gauge theory:

\[
S = \int \frac{-1}{4} F^{\mu\nu} \star F_{\mu\nu} + i \bar{\Psi} \star \partial \Psi ,
\]

with definitions of the nonabelian NC covariant derivative and the field strength, respectively:

\[
D_{\mu} \Psi = \partial_{\mu} \Psi - i A_{\mu} \Psi \quad \text{and} \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - i [A_{\mu} \Psi, A_{\nu}] .
\]

All the fields in this action are images under (hybrid) Seiberg-Witten maps of the corresponding commutative fields \( a_{\mu} \) and \( \psi \). In the original work of Seiberg and Witten and in virtually all subsequent applications, these maps are understood as (formal) series in powers of the noncommutativity parameter \( \theta^{\mu\nu} \). Physically, this corresponds to an expansion in momenta and is valid only for low energy phenomena. Here we shall not subscribe to this point of view and instead interpret the noncommutative fields as valued in the enveloping algebra of the underlying gauge group. This naturally corresponds to an expansion in powers of the gauge field \( a_{\mu} \) and hence in powers of the coupling constant \( e \). At each order in \( a_{\mu} \) we shall determine \( \theta \)-exact expressions. In the following we discuss the model construction for the massless fermion case (we set \( e = 1 \) throughout the paper).

In the next step we expand the action in terms of the commutative gauge parameter \( \lambda \) and fields \( a_{\mu} \) and \( \psi \) using the following SW map solution [36]

\[
A_{\mu} = a_{\mu} - \frac{1}{2} \theta^{\rho\sigma} a_{\rho} \star_{2} (\partial_{\rho} a_{\mu} + f_{\rho\mu}) + O(a^{3}) ,
\]

\[
\Psi = \psi - \theta^{\rho\sigma} a_{\rho} \star_{2} \partial_{\rho} \psi + \frac{1}{2} \theta^{\rho\sigma} \theta^{\rho\sigma} \left\{ (a_{\rho} \star_{2} (\partial_{\rho} a_{\mu} + f_{\rho\mu})) \star_{2} \partial_{\rho} \psi + 2 a_{\rho} \star_{2} (\partial_{\rho} a_{\mu} + f_{\rho\mu}) \right\} + O(a^{3}) \psi
\]

\[
\Lambda = \lambda - \frac{1}{2} \theta^{ij} a_{i} \star_{2} \partial_{j} \lambda + O(a^{2}) \lambda ,
\]

with \( \Lambda \) being the NC gauge parameter and \( f_{\mu\nu} \) is the abelian commutative field strength \( f_{\mu\nu} = \partial_{\mu} a_{\nu} - \partial_{\nu} a_{\mu} \).

The Mojal-Weyl star product \( \star \), and its two generalizations, \( \star_{2} \) and \( \star_{3} \), appearing in [3], are defined, respectively, as

\[
(f \star g)(x) = f(x) e^{\star \theta^{\rho\sigma} \partial_{\rho} \partial_{\sigma}} g(x) ,
\]

\[
f(x) \star_{2} g(x) = [f(x) \star g(x)] = \frac{\sin \left( \frac{\partial_{\rho} \theta \partial_{\rho} \partial_{\sigma}}{2} + f(x_{1})g(x_{2}) \right)}{\left. f(x_{1})g(x_{2}) \right|_{x_{1} = x_{2} = x}} ,
\]

\[
(f(x)g(x)h(x))_{\star_{3}} = \left( \sin \left( \frac{\partial_{\rho} \theta \partial_{\rho}}{2} + f(x_{1})g(x_{2})h(x_{3}) \right) \right)_{\star_{3}}
\]

\[
(f(x)g(x))_{\star_{3}} = \left( \sin \left( \frac{\partial_{\rho} \theta \partial_{\rho}}{2} + f(x_{1})g(x_{2})h(x_{3}) \right) \right)_{\star_{3}} + \{1 \leftrightarrow 2 \} .
\]
Figure 1: One-loop self-energy of a neutral (massless) fermion

where $\star$-product is associative but noncommutative, while $\star_2$ and $\star_3$ are both commutative but nonassociative. The resulting expansion defines the one-photon-two-fermion, the two-photon-two-fermion and the three-photon vertices, $\theta$-exactly.

The expansion of action is straightforward using the SW map expansion (3). In this way, the photon self-interaction terms up to the lowest nontrivial order are obtained as

$$S_g = \int i\partial_\mu a_\nu \star [a^\mu \star a^\nu] + \frac{1}{2} \partial_\mu (\theta^{\rho\sigma} a_\rho \star_2 (\partial_\sigma a_\nu + f_{\sigma\nu})) \star f^{\mu\nu} + \mathcal{O}(a^4).$$

The photon-fermion interaction up to 2-photon-2-fermion terms can derived by using the first order gauge field and the second order fermion field expansion,

$$S_f = \int \bar{\psi} \gamma^\mu [a_\mu, \psi] + (\theta^{ij} \partial_i \bar{\psi} \star_2 a_j \gamma^\mu [a_\mu, \psi] + i(\theta^{ij} \partial_i \bar{\psi} \star_2 a_j \partial_j \psi - i\bar{\psi} \star \phi(\theta^{ij} a_i \star_2 \partial_j \psi)
- \bar{\psi} \gamma^\mu [a_\mu, \theta^{ij} a_i \star_2 \partial_j \psi] - \bar{\psi} \gamma^\mu [\frac{1}{2} \theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{\mu j})) \star_2 \partial_j \bar{\psi} - i(\partial^i \bar{\psi} \star_2 a_j) \bar{\psi} - \bar{\psi}(\theta^{kl} a_k \star_2 \partial_l \bar{\psi})
+ \frac{i}{2} \theta^{ij} \theta^{kl} [(a_k \star_2 (\partial_l a_i + f_{ili})) \star_2 \partial_j \bar{\psi} + 2a_i \star_2 (\partial_j (a_k \star_2 \partial_l \psi)) - a_i \star_2 (\partial_k a_j \star_2 \partial_l \psi)
+ (a_i \partial_k \bar{\psi}(\partial_j a_l + f_{jli})) - \partial_k \partial_i \bar{\psi} a_j a_l) \star_3 \partial_j \psi + \frac{i}{2} \theta^{ij} \theta^{kl} \bar{\psi} \phi[(a_k \star_2 (\partial_l a_i + f_{ili}) \star_2 \partial_j a_l + f_{jli}) \star_2 \partial_j \bar{\psi}
+ 2a_i \star_2 (\partial_j (a_k \star_2 \partial_l \psi)) - a_i \star_2 (\partial_k a_j \star_2 \partial_l \psi) + (a_i \partial_k \bar{\psi}(\partial_j a_l + f_{jli}) - \partial_k \partial_i \bar{\psi} a_j a_l) \star_3 \partial_j \psi + \bar{\psi}(a^3) \psi].$$

Note that actions for gauge and matter fields obtained above, (7) and (8) respectively, are nonlocal objects due to the presence of the (generalized) star products: $\star$, $\star_2$ and $\star_3$.

As depicted in Fig. 1, there are four Feynman diagrams contributing to the neutral massless fermion self-energy at one-loop: the bubble diagram ($\Sigma_1$), the 3-fields tadpole with fermion/photon loop ($\Sigma_3$ and $\Sigma_4$, respectively), and the fourth one is the 4-field (2-fermion-2-photon) tadpole ($\Sigma_2$). Only the $\Sigma_1$ and $\Sigma_3$ contributions have their commutative-theory analogs. With the aid of (8), we have verified by explicit calculation that the 4-field tadpole
(Σ₂) does vanish. The 3-fields tadpoles (Σ₃ and Σ₄) can be ruled out by invoking the NC charge conjugation symmetry [19]², so we do not take them into account here. Thus only the bubble diagram needs to be evaluated. By extracting the relevant Feynman rule from (8), one obtains in spacetime of the dimensionality D,

\[ \Sigma₁ = μ⁴−D \int \frac{d^{D}q}{(2\pi)^{D}} \left( \sin \frac{qθ}{2} \right) \left[ \frac{1}{q^2 (p + q)^2} \right] \left( \begin{array}{c} 1 \\ p^2 (p + q)^2 \end{array} \right) + \gamma \left( (qθp)^2 (4 - D) (\hat{p} + \hat{q}) \right) + (qθp) \left[ \hat{p}(2p^2 + 2p \cdot q) - \hat{q}(2q^2 + 2p \cdot q) \right] + \left[ \hat{p}(q^2 (p^2 + 2p \cdot q) - q^2 (\hat{p}^2 + 2\hat{p} \cdot \hat{q})) + \hat{q}(\hat{p}^2 (q^2 + 2p \cdot q) - p^2 (q^2 + 2p \cdot q)) \right], \]

where \( (\tilde{p}^\mu = (θp)^\mu = θ^{μν}p_ν) \), and in addition \( (\tilde{p}^\mu = (θp)^\mu = θ^{μν}θ_{νρ}p^ρ) \).

To perform computations of those integrals using the dimensional regularization method, we first use the Feynman parametrization on the quadratic denominators, then the Heavy Quark Effective theory (HQET) parametrization [43] is used to combine the quadratic and linear denominators. In the next stage we use the Schwinger parametrization to turn the denominators into Gaussian integrals. The outcome is then combined with different phase factors emerging from the sine term in (3).

Evaluating the relevant integrals for \( D = 4 - \epsilon \) in the limit \( \epsilon \to 0 \), we obtain the final expression for the self-energy as

\[ \Sigma₁ = γµ \left[ p^µ A + (θθp)^µ \frac{p^2}{(θp)^2} B \right], \]

where

\[ A = -\frac{1}{(4π)^2} \left[ p^2 \left( \frac{trθθ}{(θp)^2} + 2 \frac{(θθp)^2}{(θp)^4} \right) A_1 + \left( 1 + p^2 \left( \frac{trθθ}{(θp)^2} + 2 \frac{(θθp)^2}{(θp)^4} \right) \right) A_2 \right] \]

\[ A₁ = \frac{2}{\epsilon} + \ln(µ^2(θp)^2) + \ln(πe^{cE}) + \frac{1}{2} \left( \ln \frac{p^2(θp)^2}{4} + 2ψ₀(2k + 2) \right) \],

\[ A₂ = -\frac{(4π)^2}{2} B = -2 \]

\[ + \sum_{k=0}^{∞} \frac{(p^2(θp)^2/4)^{k+1}}{(2k + 1)(2k + 3) \Gamma(2k + 2)} \left( \ln \frac{p^2(θp)^2}{4} - 2ψ₀(2k + 2) - \frac{8(k + 1)}{(2k + 1)(2k + 3)} \right), \]

with \( γ_E \simeq 0.577216 \) being Euler’s constant. It is to be noted here that the spinor structure proportional to \( \hat{p} \) is missing in the final result. This conforms with the calculation of the neutral fermion self-energy in the \( θ \)-expanded SW map approach [44].

The \( 1/\epsilon \) UV divergence could in principle be removed by a properly chosen counterterm. However (as already mentioned) due to the specific momentum-dependent coefficient in

²Here we take the charge conjugation transformation to be the same as equation (64) to (66) in [19], i.e.
\[ θ^{Cνµ} = -θ^{νµ}. \]
front of it, a nonlocal form for it is required. It is important to stress here that amongst
other terms contained in both coefficients $A_1$ and $A_2$, there are structures proportional to

$$(p^2(\theta p)^2)^{n+1}(\ln(p^2(\theta p)^2))^m, \forall n \text{ and } m = 0, 1. \quad (14)$$

The numerical factors in front of the above structures are rapidly-decaying, thus series are
always convergent for finite argument, as we demonstrate in [15].

Turning to the UV/IR mixing problem, we recognize a soft UV/IR mixing term rep-
resented by a logarithm,

$$\Sigma_{UV/IR} = -\hat{\theta} p^2 \left( \frac{\text{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta p)^2}{(\theta p)^4} \right) \cdot \frac{2}{(4\pi)^2} \ln |\mu(\theta p)|. \quad (15)$$

Thus, we see that the second naive expectation about NC gauge field theories does hold
here even without invoking supersymmetry.

Instead of dealing with nonlocal counterterms, we take a different route here to cope
with various divergences besetting (10). Since $\theta^0 \neq 0$ makes a NC theory nonunitary [16],
we can, without loss of generality, chose $\theta$ to lie in the $(1, 2)$ plane

$$\theta^{\mu\nu} = \frac{1}{\Lambda_{NC}^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (16)$$

Automatically, this produces

$$\frac{\text{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta p)^2}{(\theta p)^4} = 0, \forall p. \quad (17)$$

With (17), $\Sigma_1$, in terms of Euclidean momenta, receives the following form:

$$\Sigma_1 = \frac{-1}{(4\pi)^2} \gamma_\mu \left[ p^\mu \left( 1 + \frac{\text{tr}\theta\theta}{2\theta p^2} \right) - 2(\theta p)^\mu \frac{p^2}{(\theta p)^2} \right] A_2. \quad (18)$$

By inspecting (13) one can be easily convinced that $A_2$ is free from the $1/\epsilon$ divergence and
the UV/IR mixing term, being also well-behaved in the infrared, in the $\theta \to 0$ as well as
$\theta p \to 0$ limit. We see, however, that the two terms in (18), one being proportional to $\hat{\theta}$
and the other proportional to $\tilde{\theta}$, are still ill-behaved in the $\theta p \to 0$ limit. If, for the choice
(16), $P$ denotes the momentum in the $(1, 2)$ plane, then $\theta p = \theta P$. For instance, a particle
moving inside the noncommutative plane with momentum $P$ along the one axis, has a
spatial extension of size $|\theta P|$ along the other. For the choice (16), $\theta p \to 0$ corresponds
to a zero momentum projection onto the $(1, 2)$ plane. Thus, albeit in our approach the
commutative limit ($\theta \to 0$) is smooth at the quantum level, the limit when an extended
object (arising due to the fuzziness of space) shrinks to zero, is not. We could surely claim
that in our approach the UV/IR mixing problem is considerably softened; on the other
hand, we have witnessed how the problem strikes back in an unexpected way. This is, at
the same time, the first example where this two limits are not degenerate.
Summing up, we have demonstrated how quantum effects in the $\theta$-exact Seiberg-Witten map approach to NC gauge field theory reveal a much richer structure for the one-loop quantum correction to the fermion two point function (and accordingly for the UV/IR mixing problem), than observed previously in approximate models restricting to low-energy phenomena. In our approach, UV/IR mixing assumes a new form where the commutative limit and the limit of zero size of the extended object are fully disentangled. Our analysis can be considered trustworthy since we have obtained the final result in an analytic, closed-form manner. We believe that a promising avenue of research would be to use the enormous freedom in the Seiberg-Witten map to look for other forms which UV/IR mixing may assume. One alternative form has been already found \cite{45}. Finally, we mention that our approach to UV/IR mixing should not be confused with the one based on a theory with UV completion ($\Lambda_{UV} < \infty$), where a theory becomes an effective QFT, and the UV/IR mixing manifests itself via a specific relationship between the UV and the IR cutoffs \cite{17, 18, 19, 50}.

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