Time dependence of adhesion contact between a rigid sphere and an elastic body

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Abstract. Adhesion contact between a rigid sphere and an elastic body with a flat surface is experimentally investigated. The time dependence of the contact radius on loading and unloading is observed. Based on the total energy of the JKR (Johnson, Kendall, and Roberts) contact, the hysteresis of the contact radius in loading-unloading trajectories is discussed. The correlation between the energy gradient and the velocity or rate of change in contact radius is introduced to discuss the time dependence of the adhesion qualitatively.

1. Introduction

Understanding thermodynamically reversible adhesion contact is one of the keys to saving the Earth's limited resources and to contributing to sustainable development. Some solutions to this issue can be found by carefully studying the adhesion phenomena between solids from the functions of living creatures. Recently, reversible adhesion using polymers is attracting researchers' interests, e.g. in the field of biomimicry technology, and is giving rise to some reports on mimicking gecko foot-hair structure for wall-climbing robots [1,2].

Adhesion contact behavior between elastic solids has been studied for several decades because of its significance in the areas of tribology, joining technology, and micro-manipulating technology [3]. One of the most popular adhesion tests involving the contact between a sphere and a flat surface is the theory of Johnson, Kendall, and Roberts (JKR theory) [4]. Based on the JKR theory, experimental studies on interfacial adhesion between polymers were carried out, which led to the observation of adhesion hysteresis [5,6]. Johnson later introduced the assumption of the energy release rate to characterize adhesion hysteresis [7]. Since the JKR theory deals with adhesion contact of elastic solids based on linear fracture mechanics, the time dependence of the adhesion contact is not taken into account.

The time dependence of adhesion contact is essential and practically important in order to utilize and control reversible adhesion. However, time dependent adhesion of elastic bodies has not been extensively studied.
In this study, we report an observation of JKR adhesion contact between a rigid sphere and an elastic body with a flat surface, and discuss the evolution of the contact radius during adhesion contact using the total energy of the JKR theory.

2. Adhesion contact model and theoretical background

The JKR contact between a rigid sphere and an elastic body with a flat surface used in this study is shown schematically in Figure 1. A displacement $\delta$ is externally given to press the rigid sphere of radius $R$ against the surface of the elastic body, which gives rise to a circular contact area of radius $a$ at the interface of the two bodies. $\delta = 0$ is defined at a virtual point contact, i.e. at the origin of the $yr$-coordinate system shown in Figure 1. For an appropriately small contact radius, i.e. the two bodies are assumed to contact under a light load, the spherical surface of the rigid body is approximated to be a paraboloid as

$$y = \frac{r^2}{2R},$$

and is shown by a dashed line in Figure 1.

![Figure 1. Schematic illustration of JKR contact between a rigid sphere and an elastic body with a flat surface. Displacement $\delta$ applied to the sphere by external load $F$ gives rise to deformation of the elastic surface and contact radius $a$.](image)

According to a previous work by Takahashi, one of the authors of this paper, the total energy $E_{\text{total}}$ of the system is given as a function of the displacement and contact radius [1], expressed as

$$E_{\text{total}} = \frac{4E}{45(1-\nu^2)R^2}a^5 + \frac{Ea}{(1-\nu^2)}\left(\frac{a^2}{3R} - \delta\right)^2 - \pi a^2 \Delta \gamma,$$

where $E$ and $\nu$ are the Young’s modulus and Poisson’s ratio of the elastic body, respectively, and $\Delta \gamma$ is the work of adhesion required to separate a unit area of adhered interface. Under constant displacement, the two bodies come into contact at equilibrium, from which the relation between the displacement and equilibrium contact radius can be obtained as

$$\delta = a^2 - \sqrt{\frac{2\pi(1-\nu^2)\Delta \gamma a}{E}}.$$

Equation (3) can be used to determine the work of adhesion from measurements of the contact radius and applied displacement. However, in practical adhesion contacts, due to surface roughness, the viscoelastic behavior of elastomeric materials and so on, the work of adhesion appears different between loading and unloading. Following the assumption introduced by Johnson [5] that, during loading and unloading, an energy release rate $G$ has to overcome the work of adhesion as well as some
additional energy denoted by a fraction $\alpha$ of the elastic energy dissipation. Therefore, the work of adhesion $\Delta \gamma$ in Equation (3) should be replaced by

$$ G = \frac{\Delta \gamma}{(1 + \alpha)} \quad \text{on loading}, \quad (4) $$

$$ G = \frac{\Delta \gamma}{(1 - \alpha)} \quad \text{on unloading}. \quad (5) $$

3. Experimental

3.1. Preparation of samples
A transparent silicon rubber (Fusougomu) with dimensions of $35 \times 40 \times 10$ mm is used for the elastic body, of which the Young’s modulus and Poisson’s ratio are measured using a compact table-top universal tester (Shimadzu, EZ Test) to be $E = 45.5$ kPa, $\nu = 0.5$. For the rigid sphere, two steel balls (Sato Tekko, SUJ-2) of different radii, 12.5 mm and 15.08 mm, are used. Prior to each experiment, the samples are cleaned using an ultrasonic cleaner with ethyl alcohol followed by pure water.

3.2. Experimental setup and procedures
The experimental setup is shown schematically in Figure 2. The rigid sphere is fixed on an electronic balance (Sartorius, TE153S) to enable measurement of the applied load. The elastic body is attached to the bottom of a transparent acrylic beam connected directly to a motorized precision stage (Suruga Seiki, K701-20LMS). A crossed roller and goniometer stage (Suruga Seiki, B43-38N, B54-60UNR) are used to adjust the horizontal position and level of the elastic body. Loading and unloading are precisely controlled by a stepping motor on the motorized precision stage, which is manipulated by a series of digital pulses, with a resolution of $0.05 \mu$m. An optical microscope (Keyence, VH-7000) is used to measure the radius of the contact area through the transparent elastic body.

To measure the work of adhesion, starting from initial displacement, loading/unloading is done by adding/reducing the displacement to the sphere step-by-step with a certain increment, up to a target displacement, in each of which the contact radius is measured as a function of a given displacement after it becomes stable or at equilibrium.

To measure the time dependence of the adhesion, after applying a target displacement with an average rate of loading/unloading of less than 0.5 mm/s, the evolution of the contact radius $a$ is captured over time by a computer with a rate of 30 frames per second until the contact area becomes stable. The displacement, once given, is kept constant during the measurement. The contact radius is then measured manually, frame-by-frame, using a circle-fitting function of the optical microscope.

Note that all experiments are conducted in ambient conditions, and performed in an acrylic box on a vibration isolation table (Sigma Koki, TDI-129LA) to reduce the effects of dust, air currents, moisture and noise.

![Figure 2. Schematic illustration of the experimental setup. The elastic body is brought into contact with the sphere by controlling the stepping motor. The contact radius is measured through the elastic body using an optical microscope.](image-url)
4. Results and discussions

Experimental results of the loading/unloading cycles of the two spheres, \( R = 12.5 \) mm and 15.08 mm, on the elastic body are shown in Figures 3 and 4. Filled symbols (■) depict experimental data for the equilibrium contact radius \( a \) at an applied displacement \( \delta \) on loading, whereas open symbols (○) depict those measured on unloading. As shown in the figures, adhesion hysteresis between loading and unloading can be observed in both experiments.

![Figure 3](image1)

![Figure 4](image2)

**Figure 3.** Experimental results of equilibrium contact radius as a function of displacement, used for estimating the work of adhesion \( \gamma \Delta \) and energy release rate \( \alpha \), for \( R = 12.5 \) mm.

**Figure 4.** Experimental results of equilibrium contact radius as a function of displacement, used for estimating the work of adhesion \( \gamma \Delta \) and energy release rate \( \alpha \), for \( R = 15.08 \) mm.

The work of adhesion \( \gamma \Delta \) and the fraction \( \alpha \) of the elastic energy dissipation can be determined by fitting the experimental data with Equations (3), (4) and (5). Taking into account the condition given in Equation (1), data pairs of \( \delta \) and \( a \) that satisfy \( a/R < 0.18 \) are selected for fitting. Consequently, \( \gamma \Delta \) and \( \alpha \) are estimated as 0.0462 J/m\(^2\) and 0.292 for \( R = 12.5 \) mm, and 0.0824 J/m\(^2\) and 0.174 for \( R = 15.08 \) mm, respectively. To check the estimated results, theoretical curves are plotted as shown in Figures 3 and 4. The dashed curves do not take account of the energy dissipation, while the solid curves, on loading and unloading, fit the measurements very well for small contact radii that satisfy Equation (1). The hysteresis observed in this study is due to dry contact. However, hysteresis is also observed in lubricated contact as reported by Lei et al. (our research group) in a study on area contact using the same elastic material as that used in this study [8].
Figure 5. Measurements of the evolution of contact radius during adhesion. Filled symbols depict loading, and open symbols depict unloading. $R = 12.5$ mm.

Figures 5 and 6 show the change in contact radius against time, measured at $\delta = 0.130$ mm for $R = 12.5$ mm, and at $\delta = 0.147$ mm for $R = 15.08$ mm, respectively. Filled symbols depict loading, and open symbols depict unloading. Error bars in the measurements of contact radius are of the same order as the symbol size. The two different values of $\delta$ selected here are intended to obtain (almost) the same ratio $a/R$ for each individual sphere. Although each target displacement is completely given by $t_{\text{stop}} = 0.5$ s from an initial contact, the contact radius, afterward, varies constantly up to 30 s to become stable. It is observed from both experiments that the velocity or rate of change in contact radius on loading is faster comparing to that on unloading.

Figure 7 shows the experimental results for the change in contact radius with time, measured at three different applied displacements, $\delta = 0.047$, 0.072 and 0.147 mm, for the sphere of $R = 15.08$ mm. Each target displacement is completely given by 0.5 s from initial contact. It is observed that the contact radius at each displacement changes in the same manner, but finally converges to a different equilibrium value.

Figure 7. Measurements of the evolution of contact radius during adhesion, measured at different applied displacements, $\delta = 0.047$, 0.072 and 0.147 mm, for $R = 15.08$ mm.

Figure 8. Relation between theoretical total energy and contact radius. Measured contact radii vary along each curve on loading and unloading; however, they cannot reach a stable equilibrium.

Figure 8. Relation between theoretical total energy and contact radius. Measured contact radii vary along each curve on loading and unloading; however, they cannot reach a stable equilibrium.
To give further consideration to these experimental results from the viewpoint of the total energy of the system, we plot the theoretical total energy of the system, Equation (2), for the three displacements. As shown in Figure 8, filled symbols depict starting or initial contact radius \(a_{s,l}\), \(a_{s,u}\), and open symbols depict equilibrium contact radius \(a_{e,l}\), \(a_{e,u}\). Rectangles represent loading, and triangles represent unloading. The cross depicts the thermodynamic equilibrium value of the contact radius \(a_{th,e}\). Precise values of the above-mentioned data are summarized in Table 1.

**Table 1.** Measured data for the contact radius at three different applied displacements, for the sphere of \(R = 15.08\) mm. (Unit: mm)

| \(R\)   | \(\delta\) | \(a_{s,l}\) | \(a_{s,u}\) | \(a_{th,e}\) | \(a_{s,u}\) | \(a_{e,u}\) |
|---------|------------|-------------|-------------|--------------|--------------|-------------|
| 0.047   | 1.413      | 1.522       | 1.566       | 1.764        | 1.653        |
| 15.08   | 0.072      | 1.564       | 1.672       | 1.704        | 1.982        | 1.770       |
| 0.147   | 1.823      | 2.007       | 2.052       | 2.510        | 2.103        |

It is observed from Figure 8 that the three theoretical energy curves are smooth and have a single thermodynamic equilibrium, i.e. stable equilibrium. Therefore, the contact areas at each applied displacement should attain the stable equilibrium. But in actuality, the contact radii from measurements on both loading and unloading cannot attain a stable equilibrium. There is one possibility that the existence of relatively small sub-roughness on the surface of the two bodies gives rise to the existence of an energy-gap on the interface that the contact area cannot overcome.

From the experimental data, we investigated the relation between the energy gradient \(\partial E_{\text{total}} / \partial a\) and the velocity of change in contact radius \(\partial a / \partial t\) for both spheres, as shown in Figures 9 and 10. It is observed that the energy gradient has a negative correlation with the velocity of change in contact radius, on both loading and unloading. As shown in Figure 10, the correlation is seen not to be influenced by the variation of applied displacement.

**Figure 9.** Negative correlation between energy gradient and velocity of change in contact radius can be observed.

**Figure 10.** Negative correlation between energy gradient and velocity of change in contact radius can be observed. The correlation is seen to be independent of the applied displacement.
5. Conclusions
We experimentally investigated the adhesion contact between a rigid sphere and an elastic body with a flat surface. Hysteresis of the contact radius was observed between loading and unloading as a function of the applied displacement, from which the work of adhesion and the energy release rate could be estimated accurately. In the measurement of time dependence of the contact radius, it was observed that the contact radius on loading converges faster than that on unloading. Based on the total energy of the JKR theory, it was observed that the energy gradient has a negative correlation with the velocity of change in contact radius, on both loading and unloading.

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