Note on the reheating temperature in Starobinsky-type potentials

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The relation between the reheating temperature, the number of e-folds and the spectral index is shown for the Starobinsky model and some of its descendants through a very detailed calculation of these three quantities. The conclusion is that for viable temperatures between 1 MeV and $10^9$ GeV the corresponding values of the spectral index enter perfectly in its $2\sigma$ C.L., which shows the viability of this kind of models.

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1. INTRODUCTION

The Starobinsky model based on $R^2$-gravity in the Jordan frame \cite{1}, which was extensively studied in the literature (see for instance \cite{2,3} and \cite{4} for a detailed dynamical analysis), is one of the most promising scenarios to explain the inflationary paradigm proposed by A. Guth in \cite{5} because it provides theoretical data about the power spectrum of perturbations, which matches very well with the recent observational data obtained by the Planck team \cite{6}. In addition, contrary to the Guth’s paper, in \cite{1} the author briefly details a successfully reheating mechanism based on the production of particles named scalarons whose decay products reheat the universe (see \cite{3,7,8} for a detailed discussion of this mechanism), obtaining a reheating temperature around $10^9$ GeV \cite{9} (see also \cite{2} for the derivation of this reheating temperature when the decay products are massless and minimally coupled with gravity).

Working in the Einstein frame, $R^2$-gravity leads to the well-known Starobinsky potential \cite{2}, which has been recently studied as an inflationary potential and the reheating temperature provided by the model is related to its corresponding spectral index \cite{10,11,12}. However, contrary to \cite{13,14} where the authors consider the gravitational production of superheavy particles, in those papers the reheating mechanism is not taken into account; instead of it, it is assumed that during the oscillations of the inflaton field the effective Equation of State (EoS) parameter is constant. From our viewpoint, it is difficult to understand how it is possible to make any meaningful statements about reheating temperature without consideration of its concrete mechanisms, apart from the hypothesis of instant thermalization, which has to be still justified \cite{15}.

Anyway, although we do not discuss any reheating mechanism, the main goal of this note is to review these papers and find a very precise relation between the reheating temperature and the number of e-folds as a function of the spectral index of scalar perturbations, especially for the Starobinsky-type potentials.

The work is organized as follows: In Section II we perform a very accurate calculation of the number of e-folds from the moment in which the pivot scale leaves the Hubble horizon to the end of inflation, which will be used in Section III to relate the spectral index provided by the Starobinsky-type potentials with its reheating temperature. And we show numerically that for temperatures between 1 MeV and $10^9$ GeV the spectral index ranges in its $2\sigma$ Confidence Level, which means that these reheating temperatures are compatible with the model. Section IV is devoted to the study of the particular case where the effective EoS parameter during the oscillations of the inflaton field is equal to 1/3. This is a very particular case where it is impossible to define exactly when the radiation starts and, thus, it is impossible to obtain the value of the reheating temperature. What we show is that this case is physically unacceptable and all its consequences derived from it must be disregarded. Finally, in the last section we discuss the obtained results.

The units used throughout the paper are $\hbar = c = 1$ and the reduced Planck’s mass is denoted by $M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \approx 2.44 \times 10^{18}$ GeV.

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2. THE NUMBER OF E-FOLDS

First of all, we will assume that from the end of inflation to the beginning of radiation era the effective Equation of State (EoS) parameter, namely $w_{re}$ following the notation of [11], is constant. In this situation, when $w_{re} \neq 1/3$, the number of e-folds from the moment in which the pivot scale crosses the Hubble horizon to the end of inflation, namely $N_k$, is given by (see formula (2.4) of [12])

$$N_k = \ln\left(\frac{a_{eq}}{a_k}\right) + \frac{\ln(\rho_{eq}/\rho_{end})}{3(1 + w_{re})} + \frac{3w_{re} - 1}{12(w_{re} + 1)} \ln(\rho_{eq}/\rho_{re}),$$

(1)

where “eq” means the matter-radiation equality and “end” the end of the inflationary period.

This expression could be written as

$$N_k = -\ln(1 + z_{eq}) + \ln\left(\frac{H_k}{k_{phys}}\right) + \frac{\ln(\rho_{eq}/\rho_{end})}{3(1 + w_{re})} + \frac{3w_{re} - 1}{12(w_{re} + 1)} \ln(\rho_{eq}/\rho_{re}),$$

(2)

where $z$ denotes the red-shift and $k_{phys}$ is the physical value of the pivot scale.

We choose for example $k_{phys} = \frac{k}{a_0} = 0.05\text{Mpc}^{-1} \cong 1.31 \times 10^{-58}M_{pl}$, $z_{eq} = 3365$, $\rho_{eq} = \frac{\pi^2}{30}g_{eq}T_{eq}^4$ with $g_{eq} = 3.36$ and, from the adiabatic evolution of the universe after reheating, we have that $a_{eq}T_{eq} = a_0T_0 \Rightarrow T_{eq} = (1 + z_{eq})T_0$, where the present CMB temperature is $T_0 = 2.725 \text{K} \cong 2.35 \times 10^{-4} \text{eV}$.

We also write $\rho_{re} = 2\rho_{r, re}$ where $\rho_{r, re} = \frac{\pi^2}{30}g_{re}T_{re}^4$ is the energy density of the relativistic plasma at the reheating time and $g_{re} = g(T_{re})$ is the effective number of degrees of freedom at the beginning of the radiation epoch.

We use as well that $\rho_{end} = \frac{3}{2}V_{end}$ and, in order to get the value of $H_k$, we need the spectrum of scalar perturbations when the pivot scale crosses the Hubble horizon [16], namely

$$P_\zeta = \frac{H_k^2}{8\pi^2M_{pl}^2\epsilon_k} \cong 2 \times 10^{-9},$$

(3)

where

$$\epsilon_k = \frac{M_{pl}^2}{2} \left(\frac{V_\phi(\phi_k)}{V(\phi)}\right)^2$$

(4)

is the main slow-roll parameter at the crossing time.

Then, we can write the number of e-folds as follows:

$$N_k = -\ln(1 + z_{eq}) + \ln\left(\frac{H_k}{k_{phys}}\right) + \frac{1}{4} \ln(\rho_{eq}/\text{GeV}^4) + \frac{\ln(\text{GeV}^4/\rho_{end})}{3(1 + w_{re})} + \frac{3w_{re} - 1}{12(w_{re} + 1)} \ln(\text{GeV}^4/\rho_{re})$$

$$\cong 96.5684 + \frac{1}{2} \ln \epsilon_k + \frac{\ln(\text{GeV}^4/\rho_{end})}{3(1 + w_{re})} + \frac{3w_{re} - 1}{12(w_{re} + 1)} \ln(\text{GeV}^4/\rho_{re}).$$

(5)

3. DIFFERENT MODELS

We will consider the following kind of Starobinsky-like potentials, depicted in Figure [1]

$$V_n(\phi) = \lambda_n M_{pl}^4(1 - e^{-\sqrt{\frac{2}{3}}\phi/\phi_0})^2,$$

(6)

where $\lambda_n$ is a dimensionless parameter.
In Figure 1 we see that for \( n \) even (the odd case is clear) the inflaton field oscillates in the deep well potential after inflation, thus leaving its energy in order to produce enough particles to reheat the universe.

This kind of potentials, contrary to the power law ones, are allowed by the observational Planck results because the values of the spectral index \( n_s \) and the ratio of tensor to scalar perturbations \( r \) enter perfectly in the marginalized confidence contour in the plane \((n_s, r)\) at 1\( \sigma \) and 2\( \sigma \) Confidence Level.

In addition, near the origin the potential is like \( \phi^{2n} \), that is, the shape of the well of the Starobinsky-type potential is the same as for a power law potential. Then, during the oscillations of the inflaton field, for a potential \( V(\phi) = V_0 \phi^{2n} \) and using the virial theorem, we get that the effective EoS parameter is given by [17][18]

\[
    w_{re} = \frac{n - 1}{n + 1},
\]

meaning that this also holds for the potentials [6].

On the other hand, dealing with the power spectrum of scalar perturbations, we have that

\[
    \epsilon_k \approx \frac{4n^2}{3} \left( \frac{\phi_k}{M_{pl}} \right)^{2(n-1)} e^{-2\sqrt{\frac{2}{3}} \phi_k / M_{pl}^n}.
\]

and

\[
    \eta_k = M_{pl}^2 \frac{V_{\phi\phi}(\phi_k)}{V(\phi_k)} \approx - \frac{4n^2}{3} \left( \frac{\phi_k}{M_{pl}} \right)^{2(n-1)} e^{-\sqrt{\frac{2}{3}} \phi_k / M_{pl}^n}.
\]
and, thus, the spectral index is given by
\[ 1 - n_s \cong 6 \epsilon_k - 2 \eta_k \cong \frac{8 n_s^2}{3} \left( \frac{\phi_k}{M_{pl}} \right)^{2(n-1)} e^{-\sqrt{2} \phi_k^2/M_{pl}^2}. \]

Only for the exact Starobinsky model \((n = 1)\) one can express analytically \(\epsilon_k\) as a function of \(1 - n_s\) obtaining \(\epsilon_k \cong \frac{3}{16} (1 - n_s)^2\). In the other cases \((n \neq 1)\) one has to obtain it numerically.

Note also that inflation ends when
\[ \epsilon_{end} = \frac{4 n_s^2}{3} \left( \frac{\phi_{end}}{M_{pl}} \right)^{2(n-1)} \frac{e^{-2\sqrt{\frac{2}{3}} \phi_{end}^2/M_{pl}^2}}{(1 - e^{-\sqrt{2} \phi_{end}^2/M_{pl}^2})^2} = 1 \]
and the value of \(\phi_{end}\) can only be obtained analytically for the exact Starobinsky model.

### 3.1. Case \(n = 1\): The exact Starobinsky model

As we have already explained in the introduction, this potential comes from \(R^2\)-gravity in the Einstein frame (see for example [13] for a detailed explanation) and, since \(n = 1\), \(w_{re} = 0\). In addition, from [11] one gets
\[ \phi_{end} = -\sqrt{\frac{3}{2}} \ln(\sqrt{3}(2 - \sqrt{3})) M_{pl} \cong 0.9402 M_{pl}, \]

obtaining
\[ V_{end} = 4 \lambda (2 - \sqrt{3})^2 M_{pl}^4 = \rho_{end} = 6 \lambda (2 - \sqrt{3})^2 M_{pl}^4. \]

To calculate the value of the parameter \(\lambda\) we use that \(H_k^2 \cong \frac{V(\phi_k)}{3 M_{pl}^2} \cong \frac{\lambda M_{pl}^2}{3} \). Therefore, from the formula of the power spectrum of scalar perturbations [3] one gets
\[ \lambda \cong 9 \pi^2 (1 - n_s)^2 \times 10^{-9} \]
and the number of e-folds is given by
\[ N_k \cong 95.7314 + \ln(1 - n_s) + \frac{1}{3} \ln \left( \frac{\text{GeV}^3}{\rho_{end}} \right) \cong 44.9381 + \frac{1}{3} \ln(1 - n_s) + \frac{1}{3} \ln \left( \frac{g_{re}^{1/4} T_{re}}{\text{GeV}} \right). \]

On the other hand, the number of e-folds could also be calculated using the formula
\[ N_k = \int_{t_k}^{t_{end}} H dt = \frac{1}{M_{pl}} \int_{\phi_k}^{\phi_{end}} \frac{1}{\sqrt{2 \epsilon}} d\phi. \]

So, using the value \(\phi_k = -\sqrt{\frac{3}{2}} \ln(\sqrt{3}(1 - n_s))\), that \(\epsilon \cong \frac{4}{3} \left( \frac{s}{1+n_s} \right)^2\), where \(s = e^{-\sqrt{\frac{2}{3}} \phi_{end}^2/M_{pl}^2}\), and that \(s_{end} \cong -3 + 2\sqrt{3}\) (which corresponds to \(\epsilon_{end} = 1\)), one gets that
\[ N_k \cong \frac{3}{4} \left( \frac{8}{3(1 - n_s)} + \frac{1}{3 - 2\sqrt{3}} + \ln \left( \frac{3}{8} \frac{n_s - 1}{3 - 2\sqrt{3}} \right) \right) \cong -1.7759 + \frac{2}{1 - n_s} + \frac{3}{4} \ln(1 - n_s), \]
which leads to \(41.34 \leq N_k \leq 95.29\) for the values of \(n_s\) given by Planck’s team [6] within its 2\(\sigma\) C.L., namely \(n_s = 0.968 \pm 0.006\). We note that, if we invert this function, the obtained result coincides with a great extent with the relation in equation (32) of [20], reached through a next-to-leading order expansion.

Now, from equations [13] and (17) we get the following relation between the reheating temperature and the spectral index of scalar perturbations,
\[ 46.714 + \frac{1}{3} \ln \left( \frac{g_{re}^{1/4} T_{re}}{\text{GeV}} \right) = \frac{5}{12} \ln(1 - n_s) + \frac{2}{1 - n_s}. \]
Here, it is important to take into account that a lower bound of the reheating temperature is 1 MeV because the Big Bang Nucleosynthesis (BBN) occurs at this scale and the universe needs to be reheated at this epoch. In the same way the upper bound of the reheating temperature could be obtained imposing that relic products such as gravitinos or modulus fields which appear in supergravity or string theories do not affect the BBN success, which happens for reheating temperatures below \(10^6\) TeV (see for instance [20]).

![FIG. 3: The reheating temperature and the number of e-folds for \(n = 1\) as a function of the spectral index, only for temperatures between 1 MeV and \(10^6\) TeV.](image)

In Figure 3 we can see that for reheating temperatures between 1 MeV and \(10^6\) TeV the spectral index satisfies \(0.9564 < n_s < 0.9639\), which enters perfectly in its 2\(\sigma\) C.L., and the number of e-folds ranges between 41.8 and 51.1, which is in agreement with the previous and maybe not so exact calculations made in [11,12], but does not coincide at all with the result obtained in [10] by using a diagrammatic approach, because in Figure 3 of [10] one can see that for \(n_s = 0.964\) the reheating temperature is greater than \(10^{10}\) GeV, which contradicts our Figure 3 where for \(n_s = 0.964\) one has \(T_{re} \approx 10^9\) GeV. Note also that we have used as \(g_{re}\) the function obtained as a linear interpolation of the values in Table 1 of [21].

We end this subsection pointing out that the Starobinsky potential could also be used in quintessential inflation improving the well-known Peebles-Vilenkin model [22]. In that case it was shown in [23] that the reheating temperature depends on the mechanism used to reheat the universe. More precisely, when superheavy particles (whose decay products will reheat the universe) are gravitationally produced, the upper bound of \(T_{re}\) is around 40 TeV and, when the mechanism is the so-called instant preheating [24,25], one gets the following lower bound, \(T_{re} \geq 20\) TeV.

### 3.2. Case \(n \neq 1\)

When \(n \neq 1\) the relation between the reheating temperature and the spectral index has to be calculated numerically. For each value of \(n_s\) in the 2\(\sigma\) C.L. interval, we have numerically solved equations (10) and (11) in order to find the values of \(\phi_k\) and \(\phi_{end}\). Then we have used the value of \(\epsilon_k\) in equation (8) in order to calculate the number of e-folds \(N_k\) as stated in (16). And finally we have obtained the reheating temperature by setting this value equal to the one in equation (5).

In Figure 4 taking viable reheating temperatures from 1 MeV to \(10^6\) TeV, we have depicted the corresponding values of the spectral index for several models, showing that they enter in its 2\(\sigma\) C.L. We have also represented the corresponding number of e-folds for these values of \(n_s\). The models studied correspond to the values \(n = 3, 4\) and 5 which are respectively equal to the following values of the effective EoS parameter, \(w_{re} = 1/2, 3/5\) and 2/3. Note that in all these cases the reheating temperature decreases as \(n_s\) grows, in opposite to what happens when \(n = 1\). This arises from the fact that the last term in equation (5) vanishes for \(n = 2\). As a consequence, \(T_{re}\) is constant in \(n_s\) for \(n = 2\), thus increasing (resp. decreasing) as a function of \(n_s\) for \(n < 2\) (resp. \(n > 2\)).
FIG. 4: The reheating temperature and the number of e-folds as a function of the spectral index, for \( n = 3, 4 \) and 5 for temperatures between 1 MeV and \( 10^6 \) TeV. Here we have used the Planck2018 data \( n_s = 0.9649 \pm 0.0084 \) at 2\( \sigma \) C.L. [27].

4. THE PARTICULAR CASE \( w_{re} = 1/3 \)

This situation is obtained for our potentials when \( n = 2 \) and it has been already shown that it is impossible to obtain neither the value of the reheating temperature \( T_{re} \), nor the number of e-folds from the end of inflation to the beginning of the radiation era \( N_{re} = \ln \left( \frac{a_0 T_0}{H_{k0}} \right) \). The reason is that, in order to obtain the values of \( T_{re} \) and \( N_{re} \), one needs to know the beginning of the radiation epoch, i.e., when the energy density of the light particles obtained from the decay of the inflaton field starts to dominate, which does not happen in this case because during the oscillations of inflaton the effective EoS parameter is the same as in the radiation era [10–12].

However, in this particular case it is possible to calculate the effective number of degrees of freedom at the beginning of reheating, which is obtained using the formula (2.12) of [11]:

\[
g_{re} = \left( \frac{43}{11} \right)^4 \left( \frac{\pi}{30} \right)^3 \left( \frac{H_{k0} T_0}{e^{N_k} \rho_{end}^{1/4} k} \right)^{12}.
\]  

(19)

Now, taking into account that \( H_k/k = 1/a_k \) and that \( a_k e^{N_k} = a_{end} \), one gets

\[
g_{re} = \left( \frac{43}{11} \right)^4 \left( \frac{\pi}{30} \right)^3 \left( \frac{a_0 T_0}{a_{end} \rho_{end}^{1/4}} \right)^{12}
\]  

(20)

and, using that from the end of inflation to the matter-radiation equality the effective EoS parameter is 1/3, which implies \( a_{end} \rho_{end}^{1/4} = a_{eq} \rho_{eq}^{1/4} \), one finally obtains

\[
g_{re} = \left( \frac{43}{11} \right)^4 \left( \frac{\pi}{30} \right)^3 \left( 1 + z_{eq} \right)^{12} \left( \frac{T_0}{\rho_{eq}^{1/4}} \right)
\]  

(21)

This formula is very interesting because it depends neither on the shape of the potential during inflation, nor on the pivot scale. Instead it only depends on parameters which could be observationally measured. For example, choosing \( z_{eq} = 3365 \) and \( T_0 = 2.725 \) K = \( 2.35 \times 10^{-4} \) eV, we get the following abnormally small number, \( g_{re} \simeq 0.732 \), which is in contradiction with the values of the effective degrees of freedom (see for instance Figure 1 of [21]). In fact its minimum value is approximately 3.36, which is obtained at the matter-radiation equality.
This means that the case \( v = 1/3 \) has to be disregarded, as well as all its consequences. For example, the assumption that the value of \( g \) is approximately 100 (see for instance the Section 2.1 of [11]), and also the consequences derived in Section V of [10].

5. CONCLUSIONS

In this short note we have proved that for Starobinsky-type potentials (which seem to be the best for predicting the values of the power spectrum of perturbations according to the recent observations) the reheating temperature ranges in all its allowed values. In this work we take its maximum value as \( 10^6 \) TeV in order that the production of relics such as gravitinos or modulus fields in supergravity theories do not affect the success of the BBN.

We have also studied the particular case when the effective EoS parameter during the oscillations of the inflaton field is equal to \( 1/3 \) showing that this case leads to an absurd value of the number of degrees of freedom at the reheating time, meaning that this situation and its consequences must be disregarded.

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