An Insight into Adaptive Elliptical and Hexagonal Arrays Synthesis using Normalized Fractional Least Mean Squares Algorithm

G. Viswanadh Raviteja¹*, K. Sridevi¹ and A. Jhansi Rani²

¹Department of ECE, GITAM University, Visakhapatnam - 530045, Andhra Pradesh, India; tejar512@gmail.com, kadiyamsridevi1980@gmail.com
²Department of ECE, VR Siddhartha Engineering College, Vijayawada - 520007, Andhra Pradesh, India; jhansirani@gmail.com

Abstract

Background: Over the last two decades the use of adaptive arrays in wireless industry has seen a rapid growth. Smart antennas have the capability of ability to direct its main beam towards the desired user and suppress nulls at the same time.

Methods: In this paper, two different arrays namely elliptical and hexagonal arrays are considered. Four stochastic gradient based algorithms; Least Mean Squares (LMS), Fractional Least Mean Squares (FLMS), Normalized Least Mean Squares (NLMS) and Normalized Fractional Least Mean Squares (NFLMS) are evaluated.

Findings: From the results conducted it is shown that hexagonal array using Normalized Fractional Least Mean squares algorithm as the weighting scheme provide optimum results in terms of side lobe reduction and more precise main beam direction when compared to the other array configuration taken into consideration. Applications: Smart antennas increase signal range, suppress the interfering signals to a minimum range which provide greater advantage in wireless systems.

Keywords: Antenna Arrays, Array Factor, NFLMS, Side Lobe Reduction, Smart Antennas

1. Introduction

The demand for services related to voice, data and video is rapidly growing year by year. Despite the huge amount spent to meet the required necessities, vast majority of people around the world do not have quality services¹. Fiber optics on the other hand is very much expensive to establish at home and business. Therefore this made wireless connection a quick alternative and also a cost effective proposal.

The universal spread of mobile phone technology show the public's acceptance of wireless technology. It is seen that more than 25 million users utilized this technology over the last few years in United States alone. At present the number of users using cellular services is growing at rapid rates at all parts around the world². This increase in wireless networks and the bandwidth being used led to the scarcity of the available resources. This made the product to be sold at a higher price³. To compensate all the problems related to the wireless technology smart antennas came into existence.

An antenna is a device which radiates and/or receives electromagnetic waves. RF energy is coupled from transmitter into the open space and in reverse, it coupled form free space to the receiver for receiving purpose⁴. In the last few decades for far field radiation pattern synthesis, the antenna arrays applications are very useful⁵. A group of similar antenna elements are together known to be as arrays. Much attention was gained in communication technology for smart antennas because of their adaptive features⁶. Depending on the geometrical arrangement of array taken into consideration the radiation pattern is always determined⁷. Linear arrays have the disadvantage that the omnidirectional pattern is disturbed by the cor-

*Author for correspondence
ners and also it is very difficult the steer the beam 360° making it less efficient. Circular arrays on the other hand can be easily electronically steered since they do not have edge elements. For some applications to improve the results the planer circular arrangement is considered. But the problem with this is the mutual coupling effect that arises as a result of the planer arrangement. To overcome this problem hexagonal arrays are preferred. Hexagonal arrays are basically two concentric N-elements circular arrays having different radii. Several weighting schemes are proposed over the years namely Recursive Least Squares (RLS) and Constant Modulus Algorithm (CMA), etc. In this paper four adaptive algorithms are taken into consideration. Least Mean Squares (LMS) is the very basic algorithm to be discussed. Fractional Least Mean Squares (FLMS) and Normalized Least Mean Squares (NFLMS) are derived later to provide better convergence. Lastly, the modified form of LMS namely Normalized Fractional Least Mean Squares (NFLMS) is discussed. The implementation of smart antennas using elliptical array is studied in. The use of adaptive array using SMI algorithm is given in. And the nature inspired Meta heuristic algorithms are observed in.

2. Array Geometry

The array configurations for elliptical and hexagonal arrays are given in the following way. (Figure 1). The array factor for elliptical array is given in the following way:

\[ AF(\theta, \phi) = \sum_{n} I_n \exp\left(jk \sin(\theta_n \cos(\phi_n) \cos(\phi) + \sin(\theta_n \sin(\phi)) + \alpha_n \right) \]  \( (1) \)

Where \( I_n \) = amplitude of excitation, \( \alpha_n \) = phase of \( n^{th} \) element, \( \theta \) = elevation angle from z axis, \( e \) = eccentricity of elliptical array i.e., 0.5.

And

\[ \alpha_n = -k \sin(\theta_n)(a \cos(\phi_n) \cos(\phi) + b \sin(\phi_n) \sin(\phi)) \]  \( (2) \)

Where \( \theta_n = 90^\circ \), \( \phi_0 = 0^\circ \); \( N \) = number of elements.

Area (A) = \( \pi ab \)

Circumference (c) = \( \pi(3a + b) - \sqrt{(3a + b)(a + 3b)} \)

The array factor for the hexagonal array is given by:

\[ AF(\theta, \phi) = \sum_{n} [A_n e^{j\theta_n \cos(\phi_n) + \phi} + B_n e^{j\theta_n \sin(\phi_n) + \phi}] \]  \( (3) \)

3. Adaptive Algorithms

The LMS algorithm is basically a simplification of steepest-descent method, in which the gradient vector is estimated from available data when we operate in an unknown environment. The fractional LMS in addition to the first derivative has a fractional derivative.

The LMS has a disadvantage of gradient noise amplification problem. Normalized LMS is used in order to overcome the problem. The main difference between the latter and former is the way in which weights are update.

The normalized version of the FLMS algorithm has better performance than the conventional LMS filters. Also the NFLMS has faster convergence than the LMS algorithm. The weight update formula for this algorithm is given by:

\[ w(n+1) = w(n) + \frac{\mu}{\delta + \|u(n)\|^2} e(n)u(n) \]

\[ + \frac{\mu f}{\delta + \|u(n)\|^2} e(n)u(n) \frac{\Gamma(2 - v)}{\Gamma(2 - v)} \]  \( (4) \)

4. Simulations and Results

The parameters for the Normalized Fractional Least Mean Squares algorithm are taken as \( \mu = 0.6 \) for step unit and \( \mu_c = 0.055 \) for the fractional step input. The fractional derivative order \( v = 0.2 \). Number of elements considered are \( N = 12 \). The simulations are carried using MATLAB.

The results are simulated using the four algorithms taken, namely Least Mean Squares (LMS), Fractional Least Mean Squares (FLMS), Normalized Least Mean Squares (NLMS) and Normalized Fractional Least Mean Squares (NFLMS) algorithm.

The obtained results are compared in terms of the radiation pattern characteristics and side lobe levels.
The radiation plot for elliptical array using the four algorithms with 2 interferences (Figure 2).

Figure 2. Desired angle at 10°; 1st interference at 20°; 2nd interference at -40°.

The radiation plot for hexagonal array using the four algorithms with 2 interferences (Figure 3).

Figure 3. Desired angle at 30°; 1st interference at 40°; 2nd interference at -50°.

The radiation plot for uniform circular array, planar uniform circular array, elliptical array and hexagonal array using the NFLMS algorithm with 2 interferences (Figure 4).

Figure 4. Using NFLMS only. Desired angle at 0°; 1st interference at -45°; 2nd interference at 30°.

The radiation plot for uniform circular array, planar uniform circular array, elliptical array and hexagonal array using the NFLMS algorithm with 2 interferences (Figure 5).

Figure 5. Using NFLMS only. Desired angle at 0°; 1st interference at -15°; 2nd interference at 50°.
Table 1 represents the HPBW and SLL values for the configurations taken using the four algorithms.

### Table 1. Half power beam width and side lobe levels are tabulated

| Array configuration | Algorithm | HPBW [Deg] | SLL [dB] |
|---------------------|-----------|------------|----------|
| Elliptical array    | LMS       | 38.41°     | -6.37    |
|                     | FLMS      | 39.12°     | -7.13    |
|                     | NLMS      | 39.45°     | -7.74    |
|                     | NFLMS     | 40.12°     | -9.37    |
| Hexagonal array     | LMS       | 42.21°     | -5.19    |
|                     | FLMS      | 42.96°     | -6.93    |
|                     | NLMS      | 43.64°     | -7.97    |
|                     | NFLMS     | 43.71°     | -9.92    |

Table 2 represents the HPBW and SLL values for the configurations taken using the NFLMS algorithm only.

### Table 2. Using NFLMS algorithm, the values for half power beam width and side lobe levels

| Algorithm                    | Array configuration | HPBW [Deg] | SLL [dB] |
|------------------------------|---------------------|------------|----------|
| Normalized Fractional Least Mean Squares | UCA                 | 33.65°     | -7.35    |
|                              | PUCU                | 34.16°     | -8.40    |
|                              | Elliptical          | 35.61°     | -9.82    |
|                              | Hexagonal           | 37.43°     | -12.04   |

Table 3 represents the distributions of elements of the elliptical and hexagonal arrays using the Normalized Fractional Least Mean Squares algorithm.

### Table 3. Amplitude and phase excitations generated by NFLMS

| Element No. | Elliptical | Hexagonal |
|-------------|------------|-----------|
| 1           | 1.20 $\angle$ 48.26° | 1.56 $\angle$ 54.93° |
| 2           | 1.62 $\angle$ 88.81° | 1.65 $\angle$ 73.32° |
| 3           | 1.68 $\angle$ 52.65° | 1.45 $\angle$ 62.67° |
| 4           | 1.59 $\angle$ 76.56° | 1.48 $\angle$ -81.92° |
| 5           | 1.84 $\angle$ 46.19° | 1.32 $\angle$ 66.54° |
| 6           | 0.65 $\angle$ 67.51° | 0.95 $\angle$ 78.94° |

5. Conclusion

Results computed show that out of the four stochastic gradient based algorithms, the Normalized Fractional Least Mean Squares algorithm outperforms LMS, FLMS and NLMS algorithms. The side lobe levels are considerably reduced when NFLMS algorithm is used. Hexagonal array when used with NFLMS algorithm obtained better results in terms of half power beam width and reduced side lobe levels. Better results can be achieved when heptagonal and octagonal arrays are considered which may be opted for future work.

6. References

1. Balanis CA, Ioannides PI. Introduction to smart antennas. Synthesis Lectures on Antennas. 2007 Jan; 2(1):1–75.
2. Tsoulos G, Beach M, McGeohan J. Wireless personal communications for the 21st century: European technological advances in adaptive antennas. IEEE Communications Magazine. 1997 Sep; 35(9):102–29.
3. Roy RH. Application of smart antenna technology in wireless communication systems. Mobile Networking with WAP 2000, Vieweg+ Teubner Verlag; 2000. p. 65–71.
4. Cooper M, Goldberg M. Intelligent antennas: Spatial division multiple access. Annual Review of Communications; 1996 Oct 4. p. 999–1002.
5. Raviteja GV, Sridevi K, Rani AJ, Rao VM. Adaptive uniform circular array synthesis using cuckoo search algorithm. Journal of Electromagnetic Analysis and Applications. 2016 Apr; 8(04):71–8.
6. Haupt RL. Adaptive nulling with weight constraints. Progress in Electromagnetics Research B. 2010; 26:23–38.
7. Noordin NH, Zuniga V, El-Rayis AO, Haridas N, Erdogan AT, Arslan T. Uniform circular arrays for phased array antenna. 2011 Loughborough Antennas and Propagation Conference (LAPC); Loughborough. 2011 Nov. p. 1–4.
8. Rahim T. Directional pattern synthesis in circular arrays of directional antennas. London: University of London; 1980. p. 257–43.
9. Ioannides P, Balanis CA. Uniform circular and rectangular arrays for adaptive beam forming applications. IEEE Antennas and Wireless Propagation Letters; 2005; 4:351–4.
10. Bera R, Lanjewar R, Mandal D, Kar R, Ghoshal SP. Comparative study of circular and hexagonal antenna array synthesis using improved particle swarm optimization. Procedia Computer Science. 2015 Dec; 45:651–60.
11. Geravanchizadeh M, Osgouei SG. Dual-channel speech enhancement using normalized fractional least-mean-squares algorithm. 19th Iranian Conference on Electrical Engineering (ICEE); Tehran. 2011 May. p. 1–5.
12. Raviteja GV, Sridevi K, Rao VM, Rani AJ. Synthesis of adaptive uniform circular array using Normalized Fractional Least Mean Squares algorithm. American Journal of Signal Processing. 2016; 6(1):14–8.
13. Rao AV, Ankaiah NB, Cheruku DR. Antenna performance improvement in elliptical array using RMI method of mutual coupling compensation. Journal of Electromagnetic Analysis and Applications. 2016 Jan; 8(01):8–21.
14. Subba Reddy G, Ayyem Pillai V. A study of sample matrix inversion algorithm for smart antenna applications. Indian Journal of Science and Technology. 2016 May; 9(15):1–5.
15. Pavani T, Das RP, Jyothi AN, Murthy AS. Investigations on array pattern synthesis using nature inspired metaheuristic algorithms. Indian Journal of Science and Technology. 2016 Jan; 9(2):1–11.