New method for a continuous determination of the spin tune in storage rings and implications for precision experiments

D. Eversmann, V. Hejny, F. Hinder, A. Kacharava, J. Pretz, F. Rathmann, M. Rosenthal, F. Trinkel, S. Andrianov, W. Augustyniak, Z. Bagdasarian, M. Bai, W. Bernreuther, S. Bertelli, M. Berz, J. Bsaïsou, S. Chekmenev, D. Chiladze, G. Ciullo, M. Contalbrigo, J. de Vries, S. Dymov, R. Engels, S. Dymov, F. Trinkel, K. Grigoryev, D. Grzonka, G. Guidoboni, C. Hanhart, D. Heberling, N. Hempelmann, J. Hetzel, R. Hipple, D. Höltscher, A. Ivanov, V. Kamerdzhiev, B. Kamys, I. Keshelashvili, A. Khoutaz, I. Koop, H.-J. Krause, I. Keshelashvili, R. Engels, S. Andrianov, G. Macharashvili, H. Stockhorst, H. Stockhorst, H. Stockhorst, M. Tabidze, R. Talman, P. Thöring, G. Ciullo, H. Glückler, M. Gaisser, H. Soltner, M. Nioradze, M. Bai, W. Bernreuther, S. Mey, Yu. Uzikov, M. Zakrzewska, E. Vassiliev, and D. Zyuzin (JEDI collaboration)

1 III. Physikalisches Institut B, RWTH Aachen University, 52056 Aachen, Germany
2 Institut für Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany
3 JARA–FAME (Forces and Matter Experiments), Forschungszentrum Jülich and RWTH Aachen University, Germany
4 Faculty of Applied Mathematics and Control Processes, St. Petersburg State University, 198504 Petersburg, Russia
5 Department of Nuclear Physics, National Centre for Nuclear Research, 00681 Warsaw, Poland
6 High Energy Physics Institute, Tbilisi State University, 0186 Tbilisi, Georgia
7 Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, 52056 Aachen, Germany
8 University of Ferrara and INFN, 44100 Ferrara, Italy
9 Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
10 Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany
11 Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, 141380 Dubna, Russia
12 Zentralinstitut für Engineering, Elektronik und Analytik, Forschungszentrum Jülich, 52425 Jülich, Germany
13 Center for Axon and Precision Physics Research, Institute for Basic Science, 291 Daehak-ro, Yuseong-gu, Daejeon 305-701, Republic of Korea
14 Institut für Hochfrequenztechnik, RWTH Aachen University, 52056 Aachen, Germany
15 Institute of Physics, Jagiellonian University, 30348 Cracow, Poland
16 Institut für Kernphysik, Universität Münster, 48149 Münster, Germany
17 Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia
18 Peter Grüning Institut, Forschungszentrum Jülich, 52425 Jülich, Germany
19 Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn, 53115 Bonn, Germany
20 L.D. Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia
21 Research Institute for Nuclear Problems, Belarusian State University, 220030 Minsk, Belarus
22 Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia
23 Indiana University Center for Spacetime Symmetries, Bloomington, Indiana 47405, USA
24 Cornell University, Ithaca, New York 14850, USA
25 Department of Physics, KTH Royal Institute of Technology, SE-10691 Stockholm, Sweden
26 Petersburg Nuclear Physics Institute, 188300 Gatchina, Russia
27 Physics and Astronomy Department, UCL, London, WC1E 6BT, UK

A new method to determine the spin tune is described and tested. In an ideal planar magnetic ring, the spin tune – defined as the number of spin precessions per turn – is given by $\nu = \gamma \cdot G$ ($\gamma$ is the Lorentz factor, $G$ the magnetic anomaly). For 970 MeV/c deuterons coherently precessing with a frequency of $\approx 120$ kHz in the Cooler Synchrotron COSY, the spin tune is deduced from the up-down asymmetry of deuteron carbon scattering. In a time interval of $2.6\,\text{s}$, the spin tune was determined with a precision of the order $10^{-6}$, and to $1 \cdot 10^{-10}$ for a continuous $100\,\text{s}$ accelerator cycle. This renders the presented method a new precision tool for accelerator physics: controlling
The spin motion of particles to high precision is mandatory, in particular, for the measurement of electric dipole moments of charged particles in a storage ring.

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The matter-antimatter asymmetry that emerges from the Standard Model (SM) of particle physics falls short by many orders of magnitude compared to the observed value \[1\]. Physics beyond the SM is thus required and is sought at high energies and by high-precision measurements at lower energies, for instance in the search for electric dipole moments (EDMs). A permanent EDM of a particle or composite non-degenerate system would violate both parity \((P)\) and time reversal \((T)\), and via the CPT theorem, also CP symmetry. A non-zero EDM measurement would be a telltale sign of physics beyond the SM \[2\]. In addition, EDM measurements of various systems would point towards the underlying extension of the SM \[3, 4\].

Up to now, upper limits of hadronic EDMs have been determined for the neutron \[5\] and the proton, but the latter only indirectly from a measurement on \(^{199}\)Hg \[6\]. EDMs of charged hadrons are proposed to be measured in storage rings with a precision of \(10^{-29}\) e·cm by observing the influence of the EDM on the spin motion \[7–9\]. The high level of sensitivity is maintained only when the event arrival times with respect to the beginning of the experiment were assigned to each recorded event \[13\].

Another limiting factor of the storage ring approach to EDM searches, however, is controlling the spin motion of particles to high precision is mandatory, in particular, for the measurement of electric dipole moments of charged particles in a storage ring. The spin motion of a particle in the electric and magnetic fields in order to unambiguously determine the EDM signal. Consequently, the measurement described in this paper constitutes one cornerstone of storage ring EDM searches: the first precise measurement of the spin tune during a complete accelerator cycle.

The spin motion of a particle in the electric and magnetic fields of a machine is governed by the Thomas-BMT equation \[10, 20\],

\[
\frac{d\mathbf{s}}{dt} = \mathbf{s} \times \left( \Omega_{\text{MDM}} + \Omega_{\text{EDM}} \right).
\]

Here, \(\mathbf{s}\) denotes the spin vector in the particle rest frame in units of \(\hbar\), \(t\) the time in the laboratory system, and \(\Omega_{\text{MDM}}\) and \(\Omega_{\text{EDM}}\) the angular frequencies due to magnetic dipole (MDM) and electric dipole moments (EDM).

In a real machine, field imperfections, magnet misalignments, and the finite emittance of the beam lead to spin rotations around non-vertical axes and the spin tune deviates from the ideal one given in Eq. \[3\].

In the following, spin rotations due to EDMs, being many orders of magnitude smaller than those produced by MDMs, are neglected. In this case, the spins of particles that orbit in an ideal planar machine precess about the vertical magnetic field \(\vec{B}\) with the angular frequency

\[
\Omega_{\text{MDM}} = \frac{q}{m} G \vec{B},
\]

where \(q\) and \(m\) denote particle charge and mass, and \(G\) the magnetic anomaly. Dividing \(|\Omega_{\text{MDM}}|\) by the cyclotron angular frequency \(|\Omega_{\text{cyc}}| = qB/(m\gamma)\) yields the number of spin revolutions per turn, called spin tune \(\nu_s\) \[21, 22\]. For a particle on the closed orbit in an ideal magnetic ring, the spin tune is thus given by

\[
\nu_s = \gamma G.
\]

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The experiment was performed at COSY. A polarized deuteron beam of \(10^9\) particles was accumulated, accelerated to the final momentum of \(970\) MeV/c, and electron-cooled to reduce the equilibrium beam emittance. The beam polarization, perpendicular to the ring plane, was alternated from cycle to cycle using two vector-polarized states, \(p_\zeta^+ = 0.57 \pm 0.01\) and \(p_\zeta^- = -0.49 \pm 0.01\), and an unpolarized state. The tensor polarization \(p_\xi\) of the beam was smaller than 0.02. An rf cavity was used to bunch the beam during the \(140\) s long cycle. After the beam was prepared, the electron cooler was turned off for the remaining measurement of period \(100\) s.

An rf solenoid-induced spin resonance was employed to rotate the spin by \(90^\circ\) from the initial vertical direction into the transverse horizontal direction. Subsequently, the beam was slowly extracted onto an internal carbon target using a white noise electric field applied to a stripline unit. Scattered deuterons were detected in scintillation detectors consisting of rings and bars around the beam pipe \[23\], and their energy deposit was measured by stopping them in the outer scintillator rings. The event arrival times with respect to the beginning of each cycle and the frequency of the COSY rf cavity were recorded in a long-range time-to-digital converter (TDC), i.e., the same reference clock was used for all signals. The number of orbit revolutions could thus be unambiguously assigned to each recorded event \[13\].

In the following, we use a right-handed coordinate system, the \(z\)-axis points in beam direction, \(y\) upwards, and \(x\) sideways. The differential cross section, for scattering of purely vector-polarized deuterons with a vertical polarization component \(p_\xi = 0\) off an unpolarized target,
can be written as \[\sigma(\vartheta, \phi) = \sigma_0(\vartheta) \left[ 1 - \frac{3}{2} p_x(t)A_y^4(\vartheta) \sin(\phi) \right]. \tag{4} \]

Here, \(\sigma_0(\vartheta)\) denotes the differential cross section for unpolarized beam, \(\vartheta\) the polar scattering angle, \(\phi\) the azimuthal scattering angle, and \(A_y^4(\vartheta)\) the deuteron vector analyzing power. According to Eqs. (1) and (2), \(p_x(t)\) in Eq. (4) precesses as

\[p_x(t) = p_\xi \sin(\Omega_s t + \varphi), \tag{5}\]

where \(\Omega_s = 2\pi f_{\text{rev}} \nu_s\) denotes the angular frequency of the horizontal spin precession, \(\varphi\) the phase, and \(p_\xi = \sqrt{p_x^2 + p_z^2}\) the magnitude of the in-plane vector polarization. Because of the COSY straight sections, \(f_{\text{rev}}\) differs from the cyclotron frequency \(f_{\text{cyc}} = \Omega_{\text{cyc}}/(2\pi)\).

In order to determine the spin tune from Eqs. (4–5), and to cancel possible acceptance and flux variations during the measurement, asymmetries are formed using the counts of the Up (U) and Down (D) detector quadrants. The quadrants are centered at \(\varphi_U \approx 90^\circ\) and \(\varphi_D \approx 270^\circ\), covering polar angles from \(\vartheta = 9^\circ\) to \(13^\circ\), and an azimuthal range of \(\Delta \varphi_U \approx \Delta \varphi_D \approx 90^\circ\). An expression for the event rate \(R_X\) of a detector quadrant \(X = (U\text{ or }D)\) is obtained by integration over the solid angle, yielding

\[R_X = I d \int_X a_X(\vartheta, \phi) \sigma(\vartheta, \phi) d\Omega = I d a \sigma_{0,X} \left[ 1 - \frac{3}{2} p_x(t)A_y^4 \right]. \tag{6}\]

Here, \(a_X(\vartheta, \phi)\) denotes the combined detector efficiency and acceptance, \(I \text{ [s}^{-1}]\) the beam intensity, \(d \text{ [cm}^2\text{]}\) the target density, \(a \sigma_{0,X}\) the integrated spin-independent cross section, and \(|A_y^4| \approx 0.4\) the average analyzing power of the respective quadrants.

It is not possible to determine the spin tune \(\nu_s\) from the observed event rates by a simple fit with \(\nu_s\) as a parameter using Eq. (6), because at a detector rate of \(\approx 5000 \text{ s}^{-1}\) and a spin frequency of \(f_s = |\nu_s| f_{\text{rev}} \approx 0.16 - 750 \text{ kHz} = 120 \text{ kHz}\), only about one event is detected per 24 spin revolutions. Therefore, as described below, an algorithm is applied that maps all events into one oscillation period. It generates an asymmetry, largely independent of variations of acceptance, flux, and polarization that oscillates around zero. For each event, the integer turn number \(n\) is calculated, using the event time compared to the time of the COSY rf cavity. Based on the turn number, the 100 s measurement interval is split into 72 turn intervals of width \(\Delta n = 10^6\) turns (each turn lasting \(\approx 1.3 \mu\text{s}\)).

For all events, the spin phase advance \(\varphi_s = 2\pi n \nu_s\) is calculated under the assumption of a certain spin tune \(\nu_s\). Each of the turn intervals is analyzed independently, and the events are mapped into a \(4\pi\) interval, which yields the event counts \(N_U(\varphi_s)\) and \(N_D(\varphi_s)\) shown in Fig. 1(a). In order to obtain from \(N_U(\varphi_s)\) and \(N_D(\varphi_s)\) a sinusoidal wave form that oscillates around zero, four new event counts for the two quadrants \((X = U\text{ or }D)\) are defined, \(N_X^\pm(\varphi_s)\) with \(\varphi_s \in [0, 2\pi]\) using the counts \(N_U(\varphi_s)\) and \(N_D(\varphi_s)\), shown in panel (a). The vertical error bars show the statistical uncertainties, the horizontal bars indicate the bin width.

\[N_X^\pm(\varphi_s) = \begin{cases} N_X(\varphi_s) \pm N_X(\varphi_s + 3\pi) & \text{for } 0 \leq \varphi_s < \pi \\ N_X(\varphi_s) \pm N_X(\varphi_s + \pi) & \text{for } \pi \leq \varphi_s < 2\pi \tag{7} \end{cases} \]

FIG. 1. (a): Counts \(N_U\) and \(N_D\) after mapping the events recorded during a turn interval of \(\Delta n = 10^6\) turns into a spin phase advance interval of \(4\pi\). (b): Count sums \(N_U^\pm(\varphi_s)\) and differences \(N_U^\pm(\varphi_s)\) of Eq. (7) with \(\varphi_s \in [0, 2\pi]\) using the counts \(N_U(\varphi_s)\) and \(N_D(\varphi_s)\), shown in panel (a). The vertical error bars show the statistical uncertainties, the horizontal bars indicate the bin width.

The above equations provide sums, \(N_U^\pm(\varphi_s)\) and \(N_D^\pm(\varphi_s)\), and differences, \(N_U^\pm(\varphi_s)\) and \(N_D^\pm(\varphi_s)\), of counts depicted in Fig. 1(b). While the sums are constant, the differences oscillate around zero, and the asymmetry,

\[\epsilon(\varphi_s) = \frac{N_D^-(\varphi_s) - N_U^+(\varphi_s)}{N_D^-(\varphi_s) + N_U^+(\varphi_s)} = \frac{3}{2} p_\xi \frac{\sigma_D - \sigma_U A_y^4}{\sigma_D + \sigma_U A_y^4} \sin(\varphi_s + \varphi), \tag{8}\]

in the range \(\varphi_s \in [0, 2\pi]\), independent of beam intensity and target density, has the functional form

\[\epsilon(\varphi_s) = \hat{\epsilon} \sin(\varphi_s + \varphi). \tag{9}\]

Since the spin coherence time (SCT) of the in-plane vector polarization \(p_\xi\) is long \((t_{\text{SCT}} \approx 300\text{ s})\), the polariza-
position is assumed to be constant within the duration of the turn interval \( \Delta n \) (1.3 s).

In every turn interval, the parameters \( \dot{\epsilon} \) and \( \dot{\phi} \) of Eq. (10) are fitted to the measured asymmetry of Eq. (8), and the procedure is repeated for several values of \( \nu_s \) in a certain range around \( \nu_s = \gamma G \) (see e.g., Fig. 5 of [13]). The fits, for which \( \dot{\epsilon} \) becomes maximal (an example is shown in Fig. 2), yield a first approximation of \( \nu_s \) with a precision of about \( 10^{-6} \).

In order to determine the spin tune more accurately, the phase parameter \( \dot{\phi} \) is determined from the fits with Eq. (10) for all turn intervals of a complete cycle. A fixed common spin tune \( \nu_s^{\text{fix}} = -0.160975407 \) is chosen such that the phase variation \( \dot{\phi}(n) \) is minimized, as shown in Fig. 3 (a). The spin tune as a function of turn number is given by

\[
\nu_s(n) = \nu_s^{\text{fix}} + \frac{1}{2\pi} \frac{d\dot{\phi}(n)}{dn} = \nu_s^{\text{fix}} + \Delta \nu_s(n),
\]

independent of the particular choice of \( \nu_s^{\text{fix}} \), because a differently chosen \( \nu_s^{\text{fix}} \) is compensated for by a corresponding change in \( \Delta \nu_s(n) \).

Without any assumption about the functional form of the phase dependence in Fig. 3 (a), one can calculate the spin tune deviation \( \Delta \nu_s(n) \) from \( \nu_s^{\text{fix}} \) by evaluating \( d\dot{\phi}(n)/dn \) using two consecutive phase measurements, corresponding to a measurement time of 2.6 s. In this case, at early times the statistical accuracy of the spin tune reaches \( \sigma_{\nu_s} = 1.3 \cdot 10^{-8} \), and toward the end of the cycle \( \sigma_{\nu_s} = 3 \cdot 10^{-8} \), due to the decreasing event rate.

An even higher precision of the spin tune is obtained by exploiting the observed parabolic phase dependence, fitted to \( \dot{\phi}(n) \) in Fig. 3 (a), which indicates that the actual spin tune changes linearly as a function of turn number. As displayed in Fig. 3 (b), in a single 100 s long measurement, the highest precision is reached at \( t \approx 38 \) s with an error of the interpolated spin tune of \( \sigma_{\nu_s} = 9.7 \cdot 10^{-11} \).

The achieved precision of the spin tune measurements compares well with the statistical expectation. The error of a frequency measurement is approximately given by \( \sigma_f = \sqrt{6/N/(\pi \varepsilon T)} \), where \( N \) is the total number of recorded events, \( \varepsilon \approx 0.27 \) is the oscillation amplitude of Eq. (9), and \( T \) the measurement duration. In a 2.6 s time interval with an initial detector rate of 5000 s\(^{-1} \), one would expect an error of the spin tune of \( \sigma_{\nu_s} = \sigma_f/\nu_{\text{rev}} \approx 1 \cdot 10^{-8} \), and, during a 100 s measurement with \( N \approx 20000 \) recorded events, an error of \( \sigma_{\nu_s} \approx 10^{-10} \).

The new method can be used to monitor the stability of the spin tune in the accelerator for long periods of time. As shown in Fig. 4, the spin tune variations from cycle to cycle are of the same order (\( 10^{-8} \) to \( 10^{-9} \)) as those within a cycle [Fig. 3 (b)], illustrating that the spin tune determination provides a new precision tool for the investigation of systematic effects in a machine. It is remarkable that COSY is stable to such a precision, because it was not designed to provide stability below \( 10^{-9} \) with respect to, e.g., magnetic fields, closed-orbit corrections and power supplies. Presently investigations are underway to locate the origins of the observed variations in order to develop feedback systems and other means to minimize them further.

Several systematic effects that may affect the spin tune...
measurement are briefly discussed below.

Terms with a vertical vector and a tensor polarization have been omitted in the derivation of $\epsilon(\varphi_s)$ [Eq. (5)]. A detailed analysis taking these terms into account shows that $p_y$ has no influence on the spin tune at all, because the particle ensemble precesses about the y-axis; $p_y$ thus merely dilutes the asymmetry $\epsilon(\varphi_s)$. Although a small tensor polarization of up to $\pm 0.02$ leads to higher harmonics in the oscillation pattern from which the spin tune is derived (Fig. 2), these contributions alter neither the location of the zero crossings nor that of the extrema, and thus have no bearing on the extracted spin tune.

Effects of time-dependent variations of the in-plane polarization, acceptance, and flux were studied using a Monte Carlo simulation, for which detector rates were generated using Eq. (4). The analysis, carried out assuming these quantities to be constant, showed that even extreme variations such as a complete loss of polarization or acceptance during a 100 s measurement, does not affect the spin tune determination down to a level of $10^{-11}$.

The work presented here can be compared to the measurement of the muon precession frequency $|\tilde{\Omega}_{\text{MDM}}|$, which was determined in the muon $(g - 2)$ experiment with a relative precision of $\approx 10^{-6}$ per year [26]. This corresponds to an absolute precision of the spin tune of $\sigma_{\nu_s} \approx 3 \cdot 10^{-8}$ per year. The much higher precision achieved here is mainly attributed to the much longer measurement time of 100 s compared to the measurement time of 600 µs in the muon $(g - 2)$ experiment.

Ring imperfections introducing MDM rotations about non-vertical axes make it impossible at this stage to use the new technique to directly determine the magnetic anomaly $G$ with high precision from the measured spin tune. One application of the described technique would provide a high-precision CPT test in the hadronic sector, similar to a measurement performed with electrons and positrons [27]. Once polarized antiprotons are available [28, 29], with a clockwise stored beam of polarized protons in a conventional magnetic storage ring, and a simultaneously stored counter-clockwise beam of polarized antiprotons, a measurement of the ratio of the proton and antiproton $G$ factors would become possible because, when the beams are kept on the same orbit, ring imperfections are largely cancelled. A precision of $10^{-9}$ or better could be reached which has to be compared with the present precision of CPT tests based on the comparison of the magnetic moments, $\sigma(p_d/\bar{p}_d) = 5 \cdot 10^{-6}$ [30].

The method to determine the spin tune, described in this paper, can be readily extended to protons. In addition, increasing the measurement period by a factor of ten with $T_{SCT}$ of a few hundred seconds further increases the precision of $\nu_s$ by about the same factor. Undoubtedly, already the presently achieved precision will have a substantial impact on future precision measurements. The new technique has the potential, for instance, to determine the energy of a stored beam with unprecedented precision (using Eq. (6), including higher-order corrections), where the fundamental limit is set by the precision of proton and deuteron $G$ factors themselves ($\sigma_{G_d}/G_d = 8.4 \cdot 10^{-6}$, $\sigma_{G_p}/G_p = 8.2 \cdot 10^{-9}$ [31]). To put this into perspective, in a recent state-of-the-art measurement of the $\eta$ mass, already a moderate relative momentum uncertainty of the used deuteron beam of $\sigma_{p_d}/p_d = 3 \cdot 10^{-5}$ allowed a competitive mass determination [32].

Future charged particle EDM searches with an anticipated precision of $10^{-29} e \cdot cm$ shall be carried out in frozen-spin mode, i.e., with the spins aligned along the momentum of the particles, and clockwise and counter-clockwise beams to cancel out systematic effects [9]. These investigations demand a new class of primarily electrostatic storage rings. Using an existing magnetic machine, we could perform a first direct measurement of the proton or deuteron EDM using an rf Wien filter, providing partially frozen spins [32, 34]. In this case, however, one has to cope with the fast spin precession due to the deflection and focusing in the magnetic elements. The new precision determination of the spin tune will pave the way for the development of feedback systems to lock the phase of the spin precession to the rf phase of the Wien filter.

This paper presents the most precise measurement to date of the spin tune in a storage ring. The current precision reaches a level of $\sigma_{\nu_s} = 10^{-10}$ for a 100 s measurement, and in the near future improvements of at least one order in magnitude will become possible. The new method will have a huge impact on future precision measurements in storage rings, such as testing the CPT symmetry and the determination of electric dipole moments of charged particles.

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[1] A. Riotto and M. Trodden, Annual Review of Nuclear and Particle Science 49, 35 (1999)
[2] J. Engel, M. J. Ramsey-Musolf, and U. van Kolck, Progress in Particle and Nuclear Physics 71, 21 (2013)
[3] W. Dekens et al., Journal of High Energy Physics 07, 069 (2007)
[4] J. Bsaisou et al., Journal of High Energy Physics 2015, 104 (2016)
[5] C. A. Baker et al., Phys. Rev. Lett. 97, 131801 (2006)
[6] W. C. Griffith et al., Phys. Rev. Lett. 102, 101601 (2009)
[7] JEDI Collaboration, proposal available from http://collaborations.fz-juelich.de/ikp/jedi/
[8] srEDM Collaboration, proposal available from http://www.bnl.gov/edm/files/pdf/proton_EDM_proposal_20111027_final.pdf
[9] V. Anastassopoulos et al., arXiv:1502.04317 [physics.acc-ph]
[10] R. Maier, Nucl. Instrum. Meth. A390, 1 (1997)
[11] C. Weidemann et al., Phys. Rev. ST Accel. Beams 18, 020101 (2015)
[12] G. Guidoboni, Nuovo Cimento C 36, 29 (2013)
[13] Z. Bagdasarian et al., Phys. Rev. ST Accel. Beams 17, 062803 (2014)
[14] P. Benati et al., Phys. Rev. ST Accel. Beams 16, 049901 (2013)
[15] P. Benati et al., Phys. Rev. ST Accel. Beams 15, 124202 (2012)
[16] L. Thomas, Nature 117, 514 (1926)
[17] L. Thomas, Phil. Mag. 3, 1 (1927).
[18] V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Lett. 2, 435 (1959)
[19] D. F. Nelson, A. A. Schupp, R. W. Pidd, and H. R. Crane, Phys. Rev. Lett. 2, 492 (1959)
[20] T. Fukuyama and A. J. Silenko, Int. J. Mod. Phys. A28, 1350147 (2013)
[21] S. Lee, Spin Dynamics and Snakes in Synchrotrons (World Scientific, 1997).
[22] D. Barber, M. Vogt, and G. Hofstätter (1998) Proceedings of the 6th European Particle Accelerator Conference (EPAC 98), Stockholm, Sweden, pp. 1362-1364, available from http://accelconf.web.cern.ch/AccelConf/e98/contents.html
[23] D. Albers et al., Eur. Phys. J. A22, 125 (2004)
[24] B. v. Przewoski et al., Phys. Rev. C 74, 064003 (2006)
[25] D. Chiladze et al., Phys. Rev. ST Accel. Beams 9, 050101 (2006)
[26] G. W. Bennett et al., Phys. Rev. D 73, 072003 (2006)
[27] I. Vasserman et al., Physics Letters B 187, 172 (1987)
[28] P. Lenisa and F. Rathmann, Nuclear Physics News 23, 27 (2013)
[29] W. Augustyniak et al., Eur. Phys. J. A22, 64 (2012)
[30] J. DiSciacca et al., Phys. Rev. Lett. 110, 130801 (2013)
[31] NIST database, available from http://physics.nist.gov/cuu/Constants/index.html
[32] P. Goslawski et al., Phys. Rev. D 85, 112011 (2012)
[33] Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. 96, 214802 (2006)
[34] W. M. Morse, Y. F. Orlov, and Y. K. Semertzidis, Phys. Rev. ST Accel. Beams 16, 114001 (2013)