Collaborators:

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Talk based on:

G. Bíró, G.G. Barnaföldi, T.S. Bíró, J. Phys. G, 47.10 (2020), 105002.

Related publications:

G. Bíró, G.G. Barnaföldi, K. Ürmössy, T.S. Bíró, Á. Takács, Entropy, 19(3), (2017), 88
G. Bíró, G.G. Barnaföldi, T.S. Bíró, K. Shen, EPJ Web Conf., 171, (2018), 14008
Ratio of identified hadrons in small to large systems...

...but what is small?

Small systems can have large multiplicities too...

Where does the quark-gluon plasma start in multiplicity?
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Motivation

Ratio of identified hadrons in small to large systems...

...but what is small?

Small systems can have large multiplicities too...

Where does the quark-gluon plasma start in multiplicity?
Non-extensive statistics – summary:

\[ S_q = 1 - \frac{1}{q} \left( 1 - \sum_{i=1}^{N} p_i^q \right) \]

\[ \lim_{q \to 1} S_q = S_{BG} \]

Thermodynamical consistency:

\[ P = T_s + \mu n - \epsilon \]

\[ P = g \int d^3p \left( \frac{8}{\pi^3} \right)^3 T_f \]

\[ N = g V = \int d^3p \left( \frac{8}{\pi^3} \right)^3 f \]

\[ q_s = g \int d^3p \left( 2 \pi \right)^3 \left[ E - \mu T_f + f \right] \]

\[ \epsilon = g \int d^3p \left( 2 \pi \right)^3 E f \]

Final size effects:

\[ T = \frac{E \langle n \rangle}{T} \]

\[ T = E \left[ \delta^2 - \frac{q-1}{q} \langle n \rangle \right] \]

\[ q = 1 - \frac{1}{\langle n \rangle} + \Delta n^2 / \langle n \rangle^2 \]

\[ \Delta n^2 = \frac{\delta^2}{d^2} \frac{N^2}{\pi^2} p_T d_T d_y \]

\[ T = \left[ 1 + \frac{q-1}{T} \langle m_T - m \rangle \right]^{-\frac{q}{q-1}} \]

\[ \text{Entropy 16(12), (2014), 6497-651. Eur.Phys.J.A 55 (2019) 8, 126} \]
Non-extensive statistics – summary:

$q$-entropy:

\[ S_q = \frac{1}{q-1} \left( 1 - \sum_{i=1}^{W} p_i^q \right) \]

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The $q$ and $T$ parameters can track down the size evolution!
**Motivation**

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\[ s = g \int \frac{d^3p}{(2\pi)^3} \left[ \frac{E - \mu}{T} f^q + f \right] \]

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\[ \frac{d^2N}{2\pi p_T dp_T dy} = A m_T \left[ 1 + \frac{q-1}{T} (m_T - m) \right]^{-\frac{q}{q-1}} \]

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\[ T = E \left[ \delta^2 - (q - 1) \right] \]

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\[ \frac{\Delta n^2}{\langle n \rangle^2} \equiv \delta^2 \]

\[ \frac{d^2N}{2\pi p_T dp_T dy} = AmT \left[ 1 + \frac{q - 1}{T} (m_T - m) \right]^{-\frac{q}{q-1}} \]
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**The $q$ and $T$ parameters can track down the size evolution!**
Motivation

Phenomenological approach:
Map the thermodynamically consistent non-extensive parameter space of the available experimental data and compare it with theoretical QCD calculations

- 11 identified hadron species: from $\pi^\pm$ to $\Omega$
- Various collision systems: proton-proton, proton-nucleus, nucleus-nucleus
- Wide range of multiplicities: $2.2 \leq \langle dN_{ch}/d\eta \rangle \leq 2047$
- Wide range of CM energies: $130 \leq \sqrt{s_{NN}} \leq 13000$ GeV
- More than 30 published experimental datasets

Goal: calibrate the Tsallis-thermometer
### Results

**Parametrizations:**

\[
A = A_0 + A_1 \ln \frac{\sqrt{s_{NN}}}{m} + A_2 \langle dN_{ch}/d\eta \rangle
\]

\[
T = T_0 + T_1 \ln \frac{\sqrt{s_{NN}}}{m} + T_2 \ln \ln \langle dN_{ch}/d\eta \rangle
\]

\[
q = q_0 + q_1 \ln \frac{\sqrt{s_{NN}}}{m} + q_2 \ln \ln \langle dN_{ch}/d\eta \rangle
\]

1. The \(A, q\) and \(T\) parameters characterize the collision

2. Strong **grouping:** \(T_{eq} \approx 0.144\) GeV, \(q_{eq} \approx 1.156\)
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\[ q = q_0 + q_1 \ln \frac{\sqrt{S_{NN}}}{m} + q_2 \ln \ln \langle dN_{ch}/d\eta \rangle \]

Radial flow:

\[ T = T_{fr0} + m \langle u_t \rangle^2 \]

\[ \langle v_t \rangle = \frac{\langle u_t \rangle}{\sqrt{1 + \langle u_t \rangle^2}} \]

1. The \( A, q \) and \( T \) parameters characterize the collision
2. Strong grouping: \( T_{eq} \approx 0.144 \text{ GeV}, \ q_{eq} \approx 1.156 \)
3. Test: results are comparable with experiments (Phys. Rev. C 83 (2011), 064903)
**Results**

**Thermodynamical consistency:**

\[ P = Ts + \mu n - \varepsilon \]

Comparison of the thermodynamical variables with theoretical calculations

Interpretation of the grouping phenomenon in the \( T - (q - 1) \) parameter space:
**Thermodynamical consistency:** ✓

\[ P = T s + \mu n - \varepsilon \]

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Interpretation of the grouping phenomenon in the \( T - (q - 1) \) parameter space:

1. Overlapping region with theoretical QCD calculations → **presence of hot QCD matter** just before the hadronization?
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3. This QGP does certainly **not** follow an equilibrium Boltzmann – Gibbs statistics
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With the **parametrizations**: \( \sqrt{s} \) and \( \langle dN_{ch}/d\eta \rangle \) regions:

- \( \sqrt{s} \gtrsim 7000 \) GeV: \( \langle dN_{ch}/d\eta \rangle \gtrsim 130 \)
- \( \sqrt{s} \gtrsim 13000 \) GeV: \( \langle dN_{ch}/d\eta \rangle \gtrsim 90 \)
Summary

- Consistent non-extensive analysis of a very large set of experimental data
- $q \neq 1$ for all hadron spectra: dependency on the size of the collisional system through multiplicity fluctuations
- Various checks of the non-extensive framework
- Grouping of the $T$ and $q$ parameters, comparison with theoretical QCD calculations
- Tsallis-thermometer: final state hadrons may originate from a previously present strongly interacting QCD matter at event multiplicities as low as $\langle dN_{ch}/d\eta \rangle \sim 100$

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Thank you for your attention!
| System, $\sqrt{s_{NN}}$ (GeV) | $\eta$ or $y$ | Hadron Mult. classes | $p_T$ range (GeV/\(c\)) |
|-------------------------------|---------------|----------------------|--------------------------|
| AuAu, 130                     | $|\eta| < 0.35$ | $\pi^\pm$            | [0.2; 2.2]               |
|                               |               | $K^\pm$              | [0.45; 1.65]             |
| CuCu, 200                     | $|y| < 0.5$   | $K^0$                | [0.5; 9.0]               |
|                               |               | $\Xi^\pm$            | [0.7; 6.0]               |
|                               |               | $\Omega^\pm$         | [1.0; 4.5]               |
|                               |               | $\Phi$               | [0.45; 4.5]              |
| AuAu, 200                     | $|y| < 0.2$   | $\pi^\pm$            | [0.2; 2.0]               |
|                               |               | $K^\pm$              | [0.4; 3.0]               |
|                               |               | $p(\bar{p})$         | [0.3; 3.0]               |
|                               |               | $K^0$                | [0.5; 9.0]               |
|                               |               | $\Lambda^0$          | [0.5; 8.0]               |
| PbPb, 2760                    | $|y| < 0.5$   | $\pi^\pm$            | [0.1; 3.0]               |
|                               |               | $K^\pm$              | [0.2; 3.0]               |
|                               |               | $K^0$                | [0.4; 12.0]              |
|                               |               | $K^{*0}$             | [0.3; 20.0]              |
|                               |               | $p(\bar{p})$         | [0.3; 4.6]               |
|                               |               | $\Lambda^0$          | [0.6; 12.0]              |
|                               |               | $\Phi$               | [0.5; 21.0]              |
|                               |               | $\Xi^\pm$            | [0.6; 8.0]               |
|                               |               | $\Omega^\pm$         | [1.2; 7.0]               |
| pPb, 5020                     | $-0.5 < |y| < 0.0$ | $\pi^\pm$            | [0.1; 20.0]              |
|                               |               | $K^\pm$              | [0.2; 20.0]              |
|                               |               | $K^{*0}$             | [0.0; 16.0]              |
|                               |               | $p(\bar{p})$         | [0.35; 20.0]             |
|                               |               | $\Phi$               | [0.4; 20.0]              |
| pp, 7000                      | $|y| < 0.5$   | $\pi^\pm$            | [0.1; 10.0]              |
|                               |               | $K^\pm$              | [0.1; 10.0]              |
|                               |               | $K^0$                | [0.1; 10.0]              |
|                               |               | $p(\bar{p})$         | [0.3; 20.0]              |
| pbPb, 5020                    | $|y| < 0.5$   | $\pi^\pm$            | [0.1; 10.0]              |
|                               |               | $K^\pm$              | [0.2; 20.0]              |
|                               |               | $K^0$                | [0.0; 12.0]              |
|                               |               | $K^{*0}$             | [0.0; 10.0]              |
|                               |               | $p(\bar{p})$         | [0.3; 20.0]              |
|                               |               | $\Phi$               | [0.4; 10.0]              |
|                               |               | $\Lambda^0$          | [0.6; 6.5]               |
|                               |               | $\Xi^\pm$            | [0.9; 5.5]               |
| pp, 13000                     | $|y| < 0.5$   | $K^0$                | [0.0; 12.0]              |
|                               |               | $\Lambda^0$          | [0.4; 8.0]               |
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|                               |               | $\Omega^\pm$         | [0.9; 5.5]               |