Bosonic Higher Spin Gravity in any Dimension with Dynamical Two-Form

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Abstract: We first propose an alternative to Vasiliev’s bosonic higher spin gravities in any dimension by factoring out a modified \textit{sp}(2) gauge algebra. We evidence perturbative equivalence of the two models, which have the same spectrum of Fronsdal fields at the linearized level. We then embed the new model into a flat Quillen superconnection containing two extra master fields in form degrees one and two; more generally, the superconnection contains additional degrees of freedom associated to various deformations of the underlying non-commutative geometry. Finally, we propose that by introducing first-quantized \textit{sp}(2) ghosts and duality extending the field content, the Quillen flatness condition can be unified with the \textit{sp}(2) gauge conditions into a single flatness condition that is variational with a Frobenius–Chern–Simons action functional.
1 Introduction

Higher spin gravity concerns the extension of ordinary gravity by Fronsdal fields so as to facilitate the gauging of nonabelian higher spin symmetries. Fully nonlinear higher spin gravities have been formulated by Vasiliev by extending spacetime by internal non-commutative directions so as to obtain non-commutative geometries described by Cartan integrable systems, first in four and lower spacetime dimensions \[1\ 3\] by means of twistor oscillators, and later in arbitrary spacetime dimensions \[4\] using vector oscillators (for reviews, see \[5\ 7\]). The latter family is a direct generalization\[4\] to any dimension of the four-dimensional Type A model \[8\], which consists perturbatively of one real Fronsdal field for every even spin, including a parity even scalar field.

In this paper, we revisit the family of Type A models in any dimension, first by modifying their internal \(sp(2)\) gauging without affecting the higher spin gauge algebra nor the perturbative spectrum, and then by modifying the field content and the higher spin algebra. The first step yields a model that agrees with Vasiliev’s original model at the linearized level, and we shall argue that the two models are perturbatively equivalent. The latter step yields a distinct model with bi-fundamental higher spin representations containing additional propagating degrees of freedom,

\[4\]Strictly speaking, the equivalence between the twistor and the vector formulations in four dimensions has been established only at the linearized level.

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which is a natural generalization of the four-dimensional Frobenius–Chern-Simons model proposed in \[9\], motivated primarily by the fact that the extended symmetries restrict drastically the class of higher spin invariants, hence the form of a possible effective action, thus improving upon the predictive powers.

The modification is also motivated by the fact that it facilitates an off-shell formulation as a topological field theory directly in terms of differential forms on an extended non-commutative manifold with boundaries containing spacetime manifolds. This formulation is akin to topological open string field theory \[10\,12\], which we consider to be a desirable feature in view of past Vasiliev inspired works \[13\] (see also also \[14\,16\] on the tensionless limit of string theory in anti-de Sitter (for related holography motivated works, see \[13\,17\,19\]) as well as the more recent progress \[20\,22\] on relating the Fronsdal program \[23\] (for a review see \[24\]) to Vasiliev’s formulation.

The perturbative spectrum of the Type A model on five-dimensional anti de Sitter spacetime can be obtained by truncating the supermultiplets of the first Regge trajectory of the Type IIB superstring on its maximally symmetric anti-de Sitter vacuum down to the maximal spin field in each supermultiplet, save the two scalar fields of the Konishi multiplet. The Type A models have also been proposed \[13\] as bosonic truncations of effective descriptions of tensionless strings and membranes on anti-de Sitter backgrounds, as supported by various considerations based on holography \[17\,19\], whereby the natural candidates for holographic duals are free conformal field theories. Thus, the Type A models may open up a new window to holography permitting access to a wide range of physically interesting quantum field theories in four and higher dimensions, including four-dimensional pure Yang–Mills theories.

The symmetries of Vasiliev’s equations, which one may characterize as being star product local on the higher dimensional non-commutative geometries, induce highly non-local symmetries of the effective deformed Fronsdal theory, causing a tension with the standard Noether procedure, used as a tool for obtaining a classical action serving as a path integral measure, as substantiated by the results of \[21\]. This fact, when taken together with the nature of the holographic duals and inspired by the on-shell approach to scattering amplitudes and topological field theory methods, suggests that the intrinsic spacetime formulation of higher spin dynamics as a stand-alone deformed Fronsdal theory without any reference to higher dimensional non-commutative geometries, is to be treated as a quantum effective theory without any classical limit, governed by higher spin gauge symmetry and unitarity. Accordingly, Vasiliev’s equations, once subjected to proper boundary conditions on the extended non-commutative spaces where they are formulated, should be equivalent to quantum effective equations of motion in spacetime for deformed Fronsdal fields.

As for the path integral formulation of higher spin gravity, it has thus been proposed \[25\,26\] (see also \[9\,27\] and the review \[28\]) to use the language of topological quantum field theories on (higher dimensional) non-commutative Poisson manifolds, which naturally describes the Vasiliev’s equations, and provides the aforementioned link to underlying first-quantized topological field theories in two dimensions \[14\,16\]. Thus, the basic rules for constructing the classical action are to work with the basic \(n\)-ary products and trace operations for non-commutative differential graded algebras, resulting in the notion of star-product local non-commutative topological field theories. These theories have been proposed \[9\] to admit boundary states weighted by boundary observ-
ables fixed essentially by the requirements of higher spin symmetry and admissibility as off-shell deformations of Batalin–Vilkovisky master actions; the simplest example of such deformations are off-shell topological invariants, given by generalized Chern classes. Of the latter, a subset do not receive any quantum corrections, mainly due to the conservation of form degrees at the vertices, and they reduce on-shell to classical higher spin invariants that one may propose are equal, once proper boundary conditions are imposed, to the free energy functionals of deformed Fronsdal theories; these ideas are substantiated by properties of higher spin invariants closely related to the Chern classes, known as zero-form charges [29–33] (for recent progress, see [34]). The spectrum of boundary states and deformations is, however, much richer, and may hence open up new bridges between conformal and topological field theories; it would be interesting to compare these to similar correspondences that have already been established using string and M–theory [35,36].

In order to formulate Vasiliev’s, or Vasiliev-like, higher spin gravities as topological field theories, of key importance is the fact that the original Vasiliev system contains closed and central elements in form degree two, which combine with the Weyl zero-form built on-shell from the Weyl tensors of the Fronsdal fields (and the scalar field), into deformations of the non-commutative structure on symplectic leafs of the base manifold. Recently [9], the twistor formulation of four-dimensional higher spin gravity has been modified such that the aforementioned closed and central elements arises as background values of a dynamical two-form master field, suggesting that the new theory possesses a moduli space of non-commutative geometries. A key feature of the new model is thus that it is formulated in terms of only dynamical fields, which in the maximally duality extended case form a gapless spectrum of forms, fitting into a Quillen superconnection [37], as would be expected from a theory with a string-like first-quantized origin [10–12]. More precisely, the dynamical field content can be packaged into a single Quillen superconnection [37] valued in a Frobenius algebra akin to a topological open string field, leading to a renovated version of the proposal of [13]. Indeed, a stringy feature of the model is that its moduli contain various geometric deformations of the base manifold. More precisely, some combinations of zero-form and two-form moduli deform its symplectic structure, while others are transmitted into Weyl tensors for Fronsdal fields.

A simple observation, which will be of importance in what follows, is that the introduction of the dynamical two-form implies that on a general background the equations of motion cannot be rewritten as a Wigner deformed oscillator algebra. In the case of the four-dimensional twistor theory, this implies that the Lorentz covariance can only be made manifest within the Vasiliev-type phase, as here the deformed oscillator algebra is restored. As for the higher-dimensional vectorial models, the consequences reach further, as the deformed oscillator algebra enters the field dependent $sp(2)$ at the core of Vasiliev’s original model. In this paper, we shall instead factor out an $sp(2)$ algebra with field independent generators, which we shall refer to as $sp(2)^{(Y)}$, that does not refer to any underlying Wigner deformed oscillator algebra. At the linearized level, this implies that the classical moduli appearing via vacuum expectation values of the zero- and two-form consists of Fronsdal fields.

We would like to stress that the new model differs from Vasiliev’s original family of Type
A models in two ways, as the latter does not contain any dynamical two-form and is based on representations obtained by factoring out an $sp(2)$ algebra with field dependent generators, constructed using deformed Wigner oscillators as well as undeformed oscillators, which we shall refer to as $sp(2)^{(\text{diag})}$ as it is the manifest $sp(2)$ symmetry acting by rotating all doublet indices simultaneously. However, despite this apparent advantage, to our best understanding, the $sp(2)^{(\text{diag})}$ gauged model does not admit any bi-fundamental extension nor can it be coupled to a dynamical two-form.

We emphasize that the existence of two possible $sp(2)$ gaugings stems from the fact that both meet the basic criteria for choosing the $sp(2)$ gauge algebra, namely Cartan integrability of the full nonlinear system, and the Central On Mass Shell Theorem [5], i.e. consistency of the linearized system, as we shall spell out in detail in Section 3. Thus, starting at the linearized level, where the two theories are clearly equivalent, the old gauging is possible only on special non-commutative base manifolds while the new gauging, which is thus more akin to topological open string theory, is distinguished by its potential extension to general non-commutative base manifolds.

The paper is organized as follows: In Section 2, we review selected features of Vasiliev’s original formulation of higher spin gravities in arbitrary dimensions. In Section 3, we proceed with the formulation of the new model based on a modified $sp(2)$ gauging. We compare the resulting new model with the original Vasiliev’s Type A model at the (full) perturbative level as well as at the level of higher spin invariants, highlighting the crucial rôle played by the duality extension in the new model. In Section 4, we couple the new model to a dynamical two-form and further extend the system to a flat superconnection. Introducing $sp(2)$ ghosts we construct a BRST operator and propose an action principle that encodes the flatness condition and $sp(2)$ invariance of the system. We conclude in Section 5 pointing to a number of future directions.

2 Vasiliev’s Type A model

In what follows, we outline Vasiliev’s original formulation of self-interacting totally symmetric higher spin gauge fields in arbitrary spacetime dimensions.

2.1 Master field equations

A basic feature of Vasiliev’s original theory, that will remain essentially intact in the new theory, is the formulation of higher spin gravity in terms of horizontal forms on non-commutative fibered spaces, which we refer to as correspondence spaces. The space of horizontal forms is a differential graded associative algebra, whose differential and binary product we shall denote by $d(\cdot)$ and $(\cdot) \ast (\cdot)$, respectively. Locally, these spaces are direct products of a base manifold with coordinates $(X^M, Z_A^i)$ and line elements $(dX^M, dZ_A^i)$, and a fiber space with coordinates $Y_i^A$. The horizontal differential on the correspondence spaces is thus given by

$$d = dX^M \partial_M + dZ_A^i \frac{\partial}{\partial Z_A^i}. \quad (2.1)$$
Here $X^M$ coordinatize a commutative manifold, containing spacetime, whereas $Z^A_i$ and $Y^A_i$ are non-commutative coordinates, with non-trivial commutation relations

$$[Y^A_i, Y^B_j]_* = 2i \epsilon_{ij} \eta^{AB}, \quad [Z^A_i, Z^B_j]_* = -2i \epsilon_{ij} \eta^{AB},$$

(2.2)

where $\eta^{AB}$ is the $\text{so}(2, D - 1)$ invariant symmetric tensor and $\epsilon_{ij}$ is the $\text{sp}(2)$ invariant antisymmetric tensor. In order to define Lorentz tensors, one introduces a constant frame field $(V^A_A, V^a_A)$ obeying $\eta^{AB} V^A_A V^B_B = -1$, $\eta^{AB} V^a_A V^b_B = \eta^{ab}$, and defines $Y_i := V^A_A Y^A_i$ and $Y^a_i = V^a_A Y^A_i$ idem $Z_i$ and $Z^a_i$.

The dynamical fields, all of which are horizontal, are a twisted-adjoint zero-form $\Phi(X, Z; Y)$ and an adjoint one-form $W = dX^M W_M(X, Z; Y) + dZ^A_i W_A(X, Z; Y)$, which we shall refer to as master fields as they comprise infinite towers of tensor fields on the commuting manifold. The system is put on-shell by imposing the constraints

$$F + \Phi \star J = 0, \quad D\Phi = 0,$$

(2.3)

$$DK_{ij} = 0, \quad [K_{ij}, \Phi]_\pi = 0,$$

(2.4)

where $K_{ij}$ generate an $\text{sp}(2)$ algebra, viz.

$$[K_{ij}, K_{kl}]_* = 4i \epsilon_{(ij}(k K_{l)i)} ,$$

(2.5)

which together form a quasi-free differential algebra, and factoring out the orbits generated by the shift symmetries

$$\delta W = K_{ij} \star \alpha^{ij}, \quad \delta \Phi = K_{ij} \star \beta^{ij},$$

(2.6)

where $\alpha^{ij}$ and $\beta^{ij}$ are triplets under the adjoint and twisted-adjoint action of $\text{sp}(2)$, respectively, viz.

$$[K_{ij}, \alpha^{kl}]_* = 4i \delta^{(k}_{(i} \alpha^{l)}_{j)}, \quad [K_{ij}, \beta^{kl}]_\pi = 4i \delta^{(k}_{(i} \beta^{l)}_{j)}. $$

(2.7)

The equations of motion transform covariantly under gauge transformations

$$\delta \epsilon W = D\epsilon, \quad \delta \epsilon \Phi = -[\epsilon, \Phi]_\pi, \quad \delta \epsilon K_{ij} = -[\epsilon, K_{ij}]_*.$$

(2.8)

In the above, the following definitions have been used: The curvature and covariant derivatives

$$F := dW + W \star W,$$

(2.9)

$$D\Phi := d\Phi + [W, \Phi]_\pi,$$

(2.10)

$$DK_{ij} := dK_{ij} + [W, K_{ij}]_*,$$

(2.11)

where the $\pi$-twisted commutator

$$[f, g]_\pi := f \star g - (-1)^{\deg(f)\deg(g)} g \star \pi(f),$$

(2.12)

using the automorphism $\pi$ of the star product algebra defined by

$$\pi(X^M, Z^a_i, Z_i; Y^a_i, Y^i_i) := (X^M, Z^a_i, -Z_i; Y^a_i, -Y^i_i), \quad \pi d = d\pi.$$

(2.13)
The element $J$ is a closed and central two-form
\[ J = -\frac{i}{4} dZ^i dZ_i \kappa , \] (2.14)
where $\kappa$ is an inner Klein operator obeying
\[ dZ^i dZ_i (\kappa \ast f - \pi(f) \ast \kappa) = 0 , \quad \kappa \ast \kappa = 1 , \] (2.15)
for general horizontal forms $f$. It follows that
\[ \kappa = \kappa_Y \ast \kappa_Z , \] (2.16)
where
\[ dZ^i dZ_i (\kappa_Z \ast f - \pi_Z(f) \ast \kappa_Z) = 0 , \quad \kappa_Y \ast f - \pi_Y(f) \ast \kappa_Y = 0 \] (2.17)
for general horizontal forms, and
\[ \pi_Z(X^M, Z^a_i, Z_i; Y^a_i, Y_i) := (X^M, Z^a_i, -Z_i; Y^a_i, Y_i) , \quad \pi_Z d = d\pi_Z , \] \[ \pi_Y(X^M, Z^a_i, Z_i; Y^a_i, Y_i) := (X^M, Z^a_i, Z_i; Y^a_i, -Y_i) . \] (2.18)
Finally, the master fields obey the reality conditions
\[ W^\dagger = -W , \quad \Phi^\dagger = \pi(\Phi) , \quad J^\dagger = -J , \] (2.19)
where the hermitian conjugation operation is defined by
\[ (df)^\dagger = d(f^\dagger) , \quad (f \ast g)^\dagger = (-1)^{\deg(f)\deg(g)} g^\dagger \ast f^\dagger , \] \[ (X^M, Y^A_i, Z^A_i)^\dagger = (X^M, Y^A_i, -Z^A_i) . \] (2.20)

### 2.2 Diagonal $sp(2)$ generators

In Vasiliev’s Type A model, the $sp(2)$ gauge algebra is taken to be generated by
\[ K_{ij} = K_{ij}^{(\text{diag})} := K_{ij}^{(0)} - K_{ij}^{(S)} , \quad K_{ij}^{(0)} := K_{ij}^{(Y)} + K_{ij}^{(Z)} , \] (2.22)
where the two first generators are field independent, viz.
\[ K_{ij}^{(Y)} := \frac{1}{2} Y^A(i) \ast Y^A j A \equiv K_{ij} , \quad K_{ij}^{(Z)} := -\frac{1}{2} Z^A(i) \ast Z^A j A , \] (2.23)
and $K_{ij}^{(S)}$ is the field dependent generator
\[ K_{ij}^{(S)} := -\frac{1}{2} S^A(i) \ast S^A j A , \] (2.24)
built from the generalized Wigner deformed oscillator
\[ S_{Ai} := Z_{Ai} - 2i W_{Ai} , \quad (S_{Ai})^\dagger = -S_{Ai} , \] (2.25)
which is an adjoint element in the sense that

$$
\delta_\epsilon S_{Ai} = -\{\epsilon, S_{Ai}\}.
$$

The $sp(2)$ generators defined above form three copies of $sp(2)$, viz.

$$
[K^{(Y)}_{ij}, K^{(Y)}_{kl}] = 4i\epsilon_{(j(i(k) K^{(Y)}_{lj})i)} , \quad [K^{(Z)}_{ij}, K^{(Z)}_{kl}] = 4i\epsilon_{(j(i(k) K^{(Z)}_{lj})i)},
$$

$$
[K^{(S)}_{ij}, K^{(S)}_{kl}] = 4i\epsilon_{(j(i(k) K^{(S)}_{lj})i)},
$$

of which the latter follows from

$$
[S_{Ai}, S_{Bj}] = -2i\epsilon_{ij}(\eta_{AB} - V_A V_B \Phi \kappa),
$$

$$
S_{ai} \Phi - \Phi \pi(S_{ai}) = 0, \quad S_i \Phi + \Phi \pi(S_i) = 0,
$$

which is an equivalent way of writing $F_{Ai,Bj} = -\frac{i}{2}\epsilon_{ij} V_A V_B \Phi \kappa$ and $D_{Ai}\Phi = 0$ as a direct sum of an undeformed oscillator $S_{ai}$ and a Wigner deformed oscillator $S_i := V_A S_{Ai}$, with $\Phi$ playing the role of deformation parameter.

As for the $sp(2)$ invariance conditions, it follows from $D_M S_{Ai} = 0$ and $[S_{Ai}, \Phi] = 0$ that

$$
D_M K_{ij}^{(diag)} = 0 \iff [K_{ij}^{(0)}, W_M] = 0,
$$

$$
[K_{ij}^{(diag)}, \Phi] = 0 \iff [K_{ij}^{(0)}, \Phi] = 0,
$$

while

$$
D_{Ai} K_{jk}^{(diag)} = 0 \iff [S_{Ai}, K_{jk}^{(0)} - K_{jk}^{(S)}] = 0 \iff [K_{ij}^{(0)}, S_{Ak}] = 2iS_{A(\epsilon_j)k},
$$

from which it follows that

$$
[K_{ij}^{(diag)}, K_{kl}^{(diag)}] = 4i\epsilon_{(j(i(k) (K_{lj}^{(0)} + K_{lj}^{(S)})) - [K_{ij}^{(0)}, K_{kl}^{(S)}] - [K_{ij}^{(S)}, K_{kl}^{(0)}]},
$$

$$
= 4i\epsilon_{(j(i(k) (K_{lj}^{(0)} - K_{lj}^{(S)})]} = 4i\epsilon_{(j(i(k) K_{lj}^{(diag)}),
$$

i.e. the desired $sp(2)$ commutation rules [2.5]. Under a gauge transformation, one has

$$
\delta_\epsilon K_{ij}^{(diag)} = -\delta_\epsilon K_{ij}^{(S)} = -[\epsilon, K_{ij}^{(S)}],
$$

and hence $\delta_\epsilon K_{ij}^{(diag)} = -[\epsilon, K_{ij}^{(diag)}]$ holds true provided that

$$
[K_{ij}^{(0)}, \epsilon] = 0,
$$

which is indeed compatible with [2.29].

2.3 Symbol calculus, gauge conditions and $sp(2)$ symmetry

Having specified the basic ingredients, the following observations are in order:

Although there is no canonical way to realize the star product as a convolution formula, there are two choices that are particularly convenient for the most basic purposes.
As far as finding (perturbatively) exact solutions is concerned, which shall be a topic below, it is convenient to separate completely the $Y$ and $Z$ variables by representing horizontal forms $f$ by their Weyl ordered symbols $f_W = [f]_W$, where $[\cdot]_W$ thus denotes the map sending an operator to its Weyl ordered symbol, sometimes referred to as the Wigner map. Conversely, we write $f = [f_W]_W$, where thus $[\cdot]_W$ is the inverse Wigner map sending classical functions to operators. One way of defining the Wigner map, is to convert the operator product $f_W \star g_W$ to a corresponding non-local composition rule

$$f_W \star g_W = [[f_W]_W \star [f_W]_W]_W,$$

for symbols, which is given by the twisted convolution formula

$$(f_W \star g_W)(Y,Z) = \int \! d\mu \, d\tilde{\mu} \, e^{iV_i^A U_A^i + \tilde{V}_i^A \tilde{U}_A^i} f_W(Y + U, Z + \tilde{U}) \, g_W(Y + V, Z - \tilde{V}),$$

where $d\mu = (2\pi)^{-2(D+1)} d^2 U d^2 V$, idem $d\tilde{\mu}$. It follows that

$$(f(Y) \star g(Z))_W = f_W(Y) \, g_W(Z).$$

In particular, in the case of the inner Klein operator $$(2.16),$$ one finds

$$\kappa_Y = [2\pi \delta^2(Y_i)]^W, \quad \kappa_Z = [2\pi \delta^2(Z_i)]^W, \quad \kappa = [(2\pi)^2 \delta^2(Y_i) \delta^2(Z_i)]^W.$$

On the other hand, in order to describe asymptotically anti-de Sitter regions using perturbatively defined Fronsdal tensors, one needs to use another ordering scheme in which all master fields are real analytic at $Y = 0 = Z$. To this end, one may choose to work with normal ordered symbols $f_N = [f]_N$ in terms of which the star product reads

$$(f_N \star g_N)(Y,Z) = \int \! d\mu \, e^{iV_i^A U_A^i} f_N(Y + U, Z + U) \, g_N(Y + V, Z - V).$$

Consequently,

$$\kappa_Y = [2\pi \delta^2(Y_i)]^N, \quad \kappa_Z = [2\pi \delta^2(Z_i)]^N, \quad \kappa = [\exp(iY_i Z_i)]^N.$$

It also follows that if $f = f(Y)$ and $g = g(Z)$ then

$$f_W(Y) = f_N(Y), \quad g_W(Z) = g_N(Z).$$

Working in normal order, one can show that $[4]$ the unfolded description of free Fronsdal fields, as spelled out by the Central On Mass Shell Theorem $[5]$, is contained in the equations

$$[F_{MN}]_N|_{Z=0} = 0, \quad [D_M \Phi]_N|_{Z=0} = 0$$

in their free limit, obtained by expanding perturbatively around the anti-de Sitter background for $W$, provided that $i)$ all linearized symbols are real analytic at $Y = 0 = Z$; and $ii)$ the gauge condition

$$W_{ai} = 0, \quad Z^i [W_i]_N = 0.$$
which we shall refer to as the Vasiliev-Fronsdal gauge, holds in the linearized approximation. More generally, we shall argue that in order to describe deformed Fronsdal fields in asymptotically anti-de Sitter spacetimes, conditions (i) and (ii) must be imposed in the leading order of the generalized Fefferman–Graham expansion to all orders in classical perturbation theory, together with boundary conditions at infinity of \( Z \)-space in addition, essentially as boundary conditions on a gauge function and Weyl zero-form.

Turning to the \( sp(2) \) gauging, the choice of \( sp(2) \) generators made in (2.22) amounts to gauging the rigid transformations that act by simultaneous rotation of the doublets \( (Y^A_i, Z^A_i, dZ^A_i, W^i_A) \), which is a manifest symmetry in normal order, due to the particular form of \( \kappa_N \) given in (2.40). This property of \( sp(2)^{(\text{diag})} \) together with the fact that its generators reduce to those of \( sp(2)^{(Y)} \) in the free limit was the rationale behind Vasiliev’s original construction. More precisely, factoring out \( sp(2)^{(Y)} \) from the free theory yields linearized fluctuations in \( W_M \) and \( \Phi \) consisting of unfolded Fronsdal tensors and corresponding Weyl tensors on-shell, respectively.

3 New Type A model

Examining Vasiliev’s original formulation, one notes that its consistency relies on the facts that

1) The \( sp(2) \) generators form a star product Lie algebra.

2) The element \( J \) is closed and central.

3) The \( sp(2) \) gauge conditions have the desired free limit (in perturbative expansion around the AdS vacuum).

The key observation of this paper is that all of these conditions hold true as well if one instead of \( K_{ij}^{(\text{diag})} \) uses the undeformed \( sp(2) \) generators\footnote{The undeformed \( sp(2) \) generators \( K_{ij}^{(0)} \) or \( K_{ij}^{(Z)} \) obey conditions (1) and (2) but not (3).}

\[
K_{ij} = K_{ij}^{(Y)},
\]

which thus yields an alternative Type A model that is distinct from the original one, as we shall demonstrate explicitly in the next section by solving the two models perturbatively and comparing the results.

Clearly, the two alternative Vasiliev-type models agree at the linearized level in a perturbative expansion around the standard anti-de Sitter vacuum, since \( K_{ij}^{(\text{diag})} - K_{ij} \) are given by nonlinear corrections in such an expansion.

At the non-linear level, the key feature of the \( sp(2) \) gauge conditions is that the \( sp(2) \) generators form an algebra, as this assures that in applying classical perturbation theory to solve the \( Z \)-space constraints there is no risk of encountering any inconsistency in the form of additional algebraic constraints in the remaining \( X \)-space constraints at \( Z = 0 \). In this sense, both \( sp(2)^{(\text{diag})} \) and \( sp(2)^{(Y)} \) gaugings are admissible, even though the former is based on a symmetry that is manifest in any order (acting as rotations of the doublets \( (Y^A_i, Z^A_i, dZ^A_i, S^A_i) \)), while the
latter is based on a symmetry that is manifest in Weyl order, and hence in any ordering scheme related to Weyl order by means of re-orderings and gauge transformations.\footnote{Formally, a star product algebra is defined up to re-orderings generated by totally symmetric poly-vector fields, which form symmetries of trace operations given by integrals with suitable defined measures; for details, see \cite{42,43}.

As we shall see below, for both models, the differential constraints can formally be solved perturbatively for general zero-form initial data and gauge functions by working in a convenient gauge in Weyl order, that we shall refer to as the integrable gauge. Based on existing results for similar perturbative expansions in the four-dimensional twistor version of the Type A (and B) model, we shall propose that for suitable initial data and gauge functions, the resulting field configurations can be mapped to the Vasiliev–Fronsdal gauge (in which the normal ordered symbols of the master fields have perturbative expansions in terms of Fronsdal tensors that are weakly coupled at weak curvatures, such as in asymptotically anti-de Sitter regions).

The aforementioned map is given by a similarity transformation that does not leave the $sp(2)^{(Y)}$ generators invariant. Consequently, in the old model, the $sp(2)^{(\text{diag})}$ generators are field dependent in both the integrable and Vasiliev–Fronsdal gauges, while in the new model, the $sp(2)^{(Y)}$ gauge condition is imposed using field independent generators in the integrable gauge and field dependent similarity transformed $sp(2)$ generators in the Vasiliev–Fronsdal gauge. Hence, strictly speaking, in the new model, we shall refer to (2.43) as the the Vasiliev–Fronsdal basis (rather than gauge).

Below, we shall also propose to construct higher spin invariants, referred to as zero-form charges \cite{32}, using trace operations and quasi-projectors that annihilate the two-sided ideals generated by the $sp(2)$-generators. As the zero-form initial data in the integrable gauge is related to that in the Vasiliev–Fronsdal gauge by means of a nonlinear map, the zero-form charges have non-trivial perturbative expansions in the Vasiliev–Fronsdal gauge (which thus provides observables in the asymptotic weak coupling region of spacetime \cite{9}). Whether these two sets of observables can be used to map the two type A models into each other remains an open problem.

### 3.1 Manifest $sp(2)^{(Y)} \times sp(2)^{(Z)}$ symmetry

We would like to stress that the $sp(2)^{(Y)}$ transformations can be made into a manifest symmetry of the equations of motion. In fact, these equations can be rewritten as to exhibit an even larger symmetry, generated by $sp(2)^{(Y)} \times sp(2)^{(Z)}$. To this end, one first goes to Weyl order, in which the symbol calculus takes the form

\[ Y^A_i \star Y^B_j := [Y^A_i Y^B_j]_W + i\eta^{AB} \epsilon_{ij}, \quad Y^A_i \star Z^B_j := [Y^A_i Z^B_j]_W, \quad (3.2) \]

\[ Z^A_i \star Y^B_j := [Z^A_i Y^B_j]_W, \quad Z^A_i \star Z^B_j := [Z^A_i Z^B_j]_W - i\eta^{AB} \epsilon_{ij}, \quad (3.3) \]

which indeed has manifest $sp(2)^{(Y)} \times sp(2)^{(Z)}$ symmetry. Likewise, we recall that the inner Kleinian $\kappa$ can be rewritten as to make the $sp(2)^{(Y)} \times sp(2)^{(Z)}$ symmetry manifest, viz.

\[ \kappa = \kappa_Y \star \kappa_Z, \quad \kappa_Y = [2\pi \delta^2(Y^i)]^W, \quad \kappa_Z = [2\pi \delta^2(Z^i)]^W. \quad (3.4) \]
Thus, in Weyl order, both the \( \star \) product and the central element \( J \) are manifestly \( sp(2)^{(Y)} \times sp(2)^{(Z)} \) invariant, and hence they are in particular invariant under the \( sp(2)^{(Y)} \) symmetry used to gauge the new model.

### 3.2 Perturbative solution in integrable gauge

The differential equations in \( X \)-space can be solved using a gauge function, \textit{viz.}

\[
W = L^{-1} \star (W' + d) \star L \quad \Phi = L^{-1} \star \Phi' \star \pi(L) \quad W_M^2 = 0 \quad \tag{3.5}
\]

The primed fields, which are thus \( X \)-independent, obey the reduced equations

\[
d'W' + W' \star W' + \Phi' \star J = 0 \quad d'\Phi' + W' \star \Phi' - \Phi' \star \pi(W') = 0 \quad d' = dZ_i^A \frac{\partial}{\partial Z_i^A} \quad \tag{3.6}
\]

Imposing an initial condition on the zero-form in Weyl order, \textit{viz.}

\[
[\Phi']_W |_{Z=0} = [C']_W \quad \tag{3.7}
\]

and imposing the gauge condition

\[
Z^i [W'_i]_W = 0 \quad W'_{ai} = 0 \quad \tag{3.8}
\]

the resulting solution space can be written as

\[
\Phi' = C' \quad W' = \sum_{n \geq 1} w^{(n)} (C' \star \kappa_y)^n \quad \pi_Z (w^{(n)}) = w^{(n)} \quad \tag{3.9}
\]

where the perturbative corrections can be grouped into a generating element

\[
w' := \sum_{n \geq 1} w^{(n)} \nu^n \quad w^{(n)} = dZ^i w^{(n)}_i (Z^j) \quad \nu \in \mathbb{C} \quad \tag{3.10}
\]

obeying the deformed oscillator problem \textsuperscript{3}

\[
d'w' + w' \star w' + \nu j' = 0 \quad j' := -i^4 dZ^i dZ_i \kappa_z \quad \tag{3.11}
\]

Its solutions\textsuperscript{7} can be obtained by adapting the method for the four-dimensional twistor formulation of the Type A model spelled out in \textsuperscript{29}, by introducing an auxiliary frame \( U_i^{\pm} \) in \( Z \)-space defining creation and annihilation operators \( Z^\pm \), and representing the dependence of \( w'_i \) on \( Z^j \) as an inverse Laplace transform in the variable \( Z^+ Z^- \), or equivalently, solving the problem using a basis for symbols in \( Z \)-space defined using normal order, followed by mapping back to Weyl order; for details on the latter approach, see \textsuperscript{38}.

We would like to note that so far we have not imposed any \( sp(2) \) gauge conditions, and consequently we have treated the new and old models in parallel.

\textsuperscript{7} We expect the structure of the resulting moduli space to resemble that of the four-dimensional twistor formulation of the Type A model, which decomposes into discrete branches, each labelled by a flat connection on \( Z \)-space, and coordinatized by (continuous) zero-form initial data in their turn belonging to cells separated by “walls” given by critical deformation parameters; for details, see \textsuperscript{29,38,39}. 


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3.3 Similarity transformation to Vasiliev–Fronsdal basis

Let us proceed, still in parallel between the old and new models, by finding the gauge function $L$ that brings the solution from the integrable gauge to the Vasiliev–Fronsdal basis obeying

$$Z^i [W_i]_N = 0,$$

(3.12)

where thus the gauge fields become Fronsdal tensors in weak coupling regions. To this end, it is useful to introduce the homotopy contractor

$$\rho_{\vec{v}}(f) := [\iota_{\vec{v}}(L_{\vec{v}})^{-1}f]_N, \quad \vec{v} = Z^{A_i} \partial_{A_i},$$

(3.13)

that can be used to invert the action of $d'$ on operators $f$ whose normal ordered symbols obey $\iota_{\vec{v}}f_N = 0$, viz.

$$\rho_{\vec{v}} d' f = f - \delta_0 \deg(f) [f_N|_{Z=0}]^N.$$

(3.14)

For explicit calculations, one can use the integral representation

$$L_{\vec{v}}^{-1} = \int_0^1 \frac{dt}{t} L_{\vec{v}},$$

(3.15)

which has a well-defined action on symbols defined in normal order that are real analytic in $Z$-space at $Z = 0$. Thus $L$ can be obtained in normal ordered form by first expanding

$$L = \sum_{n \geq 0} L^{(n)},$$

(3.16)

and then iterating (3.12), which yields

$$L^{(n)} = -L^{(0)} \ast \left( \sum_{n_1 + n_2 + n_3 = n} \rho_{\vec{v}} \left( (L^{-1})^{(n_1)} \ast W^{(n_2)} \ast L^{(n_3)} \right) + \sum_{n_1 + n_2 = n} \rho_{\vec{v}} \left( (L^{-1} - (L^{(0)})^{-1})^{(n_1)} \ast d'L^{(n_2)} \right) \right),$$

(3.17)

for $n \geq 1$, as can be seen from

$$0 = \rho_{\vec{v}} W = \rho_{\vec{v}} \left( L^{-1} \ast (W' + d') \ast L \right),$$

(3.18)

by using $d'L^{(0)} = 0$ to write

$$L^{-1} \ast d'L = d' \left( (L^{(0)})^{-1} \ast L \right) + \left( L^{-1} - (L^{(0)})^{-1} \right) \ast d'L,$$

(3.19)

and $((L^{(0)})^{-1} \ast L)|_{Z=0} = 1$ to integrate

$$\rho_{\vec{v}} d_z ((L^{(0)})^{-1} \ast L) = (L^{(0)})^{-1} \ast L - 1.$$

(3.20)

The relation now reads

$$-(L^{(0)})^{-1} \ast L - 1 = -\rho_{\vec{v}} \left[ L^{-1} \ast W' \ast L + \left( L^{-1} - (L^{(0)})^{-1} \right) \ast d'L \right],$$

(3.21)
and one recovers the perturbative solution (3.17) by inserting the expansion (3.16), which is thus well-defined provided that the arguments of the homotopy contractors are real analytic in $Z$ space in normal order.

The latter problem is similar to that studied in the case of the four-dimensional twistor formulation of the Type A model, where it was found that $L^{(1)}$ exists if the gauge function $L^{(0)}$ and the zero-form initial data $\Phi'$ are Gaussian elements corresponding, respectively, to the anti-de Sitter vacuum and fluctuations thereabout given by the particle and black-hole-like modes. In what follows, we shall assume that an analogous result holds for the Type A model in any dimension for $\Phi'$ consisting of particle modes, that is, that it is possible to map initial data in lowest weight spaces to linearized Fronsdal fields on-shell.

### 3.4 $sp(2)$ gauging

In order to gauge $sp(2)$, we first impose the $sp(2)$ invariance conditions, which we shall tend to next, after which we shall proceed by factoring out the corresponding ideals at the level of higher spin invariants. As we shall see, the resulting $sp(2)$ gaugings are equivalent at the linearized level.

#### 3.4.1 $sp(2)$ invariance

**Old model ($sp(2)_{(\text{diag})}$).** We recall that, in the old model, the $sp(2)_{(\text{diag})}$ invariance conditions read

\begin{align}
D_M K^{(\text{diag})}_{ij} = 0 & \iff [K^{(Y)}_{ij} + K^{(Z)}_{ij}, W_M]_* = 0 , \\
D_A K^{(\text{diag})}_{ij} = 0 & \iff [K^{(Y)}_{ij} + K^{(Z)}_{ij}, S_A]_* = 4i S_A (\epsilon^j_{ik}) , \\
[\Phi, K^{(\text{diag})}_{ij}]_* = 0 & \iff [K^{(Y)}_{ij} + K^{(Z)}_{ij}, \Phi]_* = 0 .
\end{align}

(3.22) (3.23) (3.24)

In the integrable gauge, these conditions are equivalent to

\begin{equation}
[K^{(Y)}_{ij}, C'_j]_* = 0 .
\end{equation}

(3.25)

In the Vasiliev–Fronsdal gauge, the $sp(2)_{(\text{diag})}$ invariance holds provided that

\begin{equation}
[K^{(Y)}_{ij} + K^{(Z)}_{ij}, L^{(0)}]_* = 0,
\end{equation}

(3.26)
as this condition implies that $[K^{(Y)}_{ij} + K^{(Z)}_{ij}, L]_* = 0$ by virtue of the fact that the homotopy contractor $\rho_v$ is $sp(2)_{\text{diag}}$ invariant.

**New model ($sp(2)^{(Y)}$).** In the integrable gauge, the $sp(2)^{(Y)}$ invariance conditions reads ($K_{ij} \equiv K_{ij}^{(Y)}$)

\begin{equation}
[K_{ij}, W']_* = 0 = [K_{ij}, \Phi']_* ,
\end{equation}

(3.27)

which are equivalent to

\begin{equation}
[K_{ij}, C'_j]_* = 0 .
\end{equation}

(3.28)
In the Vasiliev–Fronsdal basis the fields obey the following similarity transformed $sp(2)^{(Y)}$ invariance conditions:

$$[\Phi, K^{(L)}_{ij}]_* = 0 \, , \quad DK^{(L)}_{ij} \equiv dK^{(L)}_{ij} + [W, K^{(L)}_{ij}]_* = 0 \, ,$$

(3.29)

where

$$K^{(L)}_{ij} := L^{-1} \star K_{ij} \star L = (L^{(0)})^{-1} \star L^{-1} \star K_{ij} \star (L^{(0)})^{-1} \star L \, ,$$

(3.30)

which are field dependent generators such that $(K^{(L)}_{ij})^{(0)} = K_{ij}$.

**Equivalence between old and new model.** In the Vasiliev–Fronsdal gauge, and prior to factoring out the ideal, both models have perturbatively defined solution spaces obeying the same differential equations, gauge conditions, viz.

$$W_{ai} = 0 \, , \quad Z^i W_i = 0 \, ,$$

(3.31)

and $sp(2)$ invariance conditions, viz.

$$D_i K_{jk} = 0 \, , \quad [K_{ij}, \Phi]_x = 0 \, , \quad [K_{ij}, K_{kl}]_* = 4i \epsilon_{jk} K_{il} \, .$$

(3.32)

with $sp(2)$ generators subject to the same functional initial condition, viz.

$$K_{ij}|_{\Phi=0} = K^{(Y)}_{ij} \, .$$

(3.33)

This suggests that the two models are perturbatively equivalent, modulo redefinitions of zero-form initial data and modifications of the Vasiliev–Fronsdal gauge condition away from the asymptotic region. This could be examined by comparing the first order corrections to $K^{(L)}_{ij}$ and $K^{(\text{diag})}_{ij}$, which we leave for a separate work.

### 3.4.2 Factoring out the $sp(2)$ ideal

Thus, the perturbatively defined configurations [3.9] with $sp(2)$-invariant zero-form initial data obey the differential equations of motion as well as the $sp(2)$ invariance conditions in the old as well as new models. In both models, the problem of factoring out the $sp(2)$ orbits from these solution spaces combines naturally with the problem of constructing higher spin invariants.

The (two-sided) ideal $I$ in the algebra $\mathcal{A}_0$ of $sp(2)$ invariant master fields generated by the $sp(2)$ gauge algebra can be factored out from invariants by using the trace operation

$$\text{Tr}_M [f] := \text{Tr} M \star f \, ,$$

(3.34)

where $[f] \in \mathcal{A}_0/I$ is the equivalence class of $f \in \mathcal{A}_0$; $\text{Tr}$ is the trace operation on $Y$- and $Z$-space, and $M$ obeys

$$K_{ij} \star M = 0 = M \star K_{ij} \, ,$$

(3.35)

the covariant constancy condition

$$D_M M = 0 \, , \quad [B, M]_* = 0 \, , \quad B := \Phi \star \kappa \, ;$$

(3.36)
and is a quasi-projector in the sense that \( M \star A_0 \) exists (but not \( M \star M \star A_0 \)). In the new model, we have

\[
M = M^{(Y)} = F(K_{ij}^{(Y)}K_{ij}^{(Y)}) ,
\]

(3.37)

where \( F \) is real analytic and nonvanishing at the origin, and

\[
M^{(L)} = L^{-1} \star M^{(Y)} \star L = M^{(Y)} + \text{h.o.t.,}
\]

(3.38)
in the Vasiliev–Fronsdal basis; in the old model, we have

\[
M = M^{(\text{diag})} = M^{(Y)} + \text{h.o.t.}
\]

(3.39)

where the higher order terms can be found by solving \( K^{(\text{diag})}_{ij} \star M^{(\text{diag})} = 0 \) perturbatively \cite{40}.

It follows that

\[
D_M (M \star B) = 0 , \quad D_M (M \star S_{Ai}) = 0 ,
\]

(3.40)

of which the first equation indeed contains the correct linearized mass-shell conditions for generalized Weyl tensors (including the dynamical scalar field) \cite{40}.

The simplest invariants are the zero-form charges \cite{29,31} given by

\[
\mathcal{O}_C := \text{Tr}_M W_C(S) ,
\]

(3.41)

where \( W_C \) are twisted (open) Wilson lines along curves \( C \) from \( Z = 0 \) to \( Z = \Lambda(C) \), which can be straightened out into star products of vertex-like operators \cite{34,41}, viz.

\[
W_C = f_C(B) \star V_\Lambda , \quad V_\Lambda := \exp_{\ast} (iA_i S_{Ai}) ,
\]

(3.42)

where \( f_C \) is a star function (i.e. its dependence on \( B \) is in terms of monomials \( B^*n \) for \( n = 0, 1, 2, \ldots \)) depending on the shape of \( C \). The zero-form charges are de Rham closed by virtue of

\[
\partial_M \mathcal{O} = \text{Tr} D_M (M \star W_C) = 0 ,
\]

(3.43)

and hence higher spin invariant.

More general invariants \cite{5,31}, that can be evaluated on non-trivial elements [\( \Sigma \)] in the singular homology of \( X \)-space, can be constructed by choosing a structure group \( G \) with connection \( \Omega_M \) and splitting

\[
W_M = \Omega_M + E_M ,
\]

(3.44)

where \( E_M \) is a soldering one-form, that is, a generalized frame field, whose gauge parameters belong to sections that can be converted to globally defined vector fields on \( X \) (modulo a \( G \) gauge transformation with composite parameter). This facilitates the definition of \( G \)-invariant tensors on \( X \)-space, which induce top forms on representatives \( \Sigma' \in [\Sigma] \) whose integrals over \( \Sigma' \) define generalized volumes whose extrema (as one varies \( \Sigma' \)) are diffeomorphism invariants, and hence higher spin gauge invariant by the soldering mechanism. These geometries also support closed abelian even forms

\[
H_{[2p]} = \text{Tr}_M (E \star E)^* p ,
\]

(3.45)
on $X$-space, whose charges $f_{2p}H_{2p}$ are higher spin gauge invariant.

As first suggested in \cite{13}, the zero-form charges have perturbative expansions over asymptotically anti-de Sitter solutions in terms of boundary correlation function, as has been verified and
developed further in the context of four-dimensional twistor oscillator models \cite{30,32,33}, where it has also been proposed \cite{38} that they can be interpreted as extensive charges for families of
localizable black-hole like solutions. Thus, zero-form charges together with other invariants could serve as tools for establishing the perturbative equivalence between the old and new Type A
models.\footnote{They could also be useful in establishing the equivalence between the vector and twistor oscillator formulations of the Type A model in four dimensions.}

4 Coupling of the new Type A model to a dynamical two-form

The new Type A model can be coupled to a dynamical two-form, leading to an extended higher
spin gravity model of Frobenius–Chern-Simons type based on a superconnection suitable for an
off-shell formulation and possibly also for making contact with topological open strings.

4.1 Master field equations

We introduce two separate connections $A$ and $\tilde{A}$, with curvatures

$$F := dA + A \ast A, \quad \tilde{F} := d\tilde{A} + \tilde{A} \ast \tilde{A}, \quad (4.1)$$

and a two-form $\tilde{\Phi}$, and take $(\Phi, \tilde{\Phi})$ to transform in opposite twisted bi-fundamental representa-
tions, with covariant derivatives

$$D\Phi := d\Phi + A \ast \Phi - \Phi \ast \pi(\tilde{A}), \quad D\tilde{\Phi} := d\tilde{\Phi} + \pi(\tilde{A}) \ast \tilde{\Phi} - \tilde{\Phi} \ast A, \quad (4.2)$$

such that $\Phi \ast \tilde{\Phi}$ and $\pi(\tilde{\Phi} \ast \Phi)$ can be used to source $F$ and $\tilde{F}$, respectively. The resulting Cartan integrable equations of motion read

$$F + \Phi \ast \tilde{\Phi} = 0, \quad 0 = \tilde{F} + \pi(\tilde{\Phi} \ast \Phi), \quad (4.3)$$

$$D\Phi = 0, \quad 0 = \tilde{D}\tilde{\Phi}, \quad (4.4)$$

$$DK_{ij} = 0, \quad 0 = \tilde{D}K_{ij}, \quad (4.5)$$

$$[K_{ij}, \Phi]_{\pi} = 0, \quad 0 = [K_{ij}, \pi(\tilde{\Phi})]_{\pi}, \quad (4.6)$$

where $K_{ij}$ form a star product $sp(2)$ algebra that reduce to $K_{ij}^{(Y)}$ in the free limit, and field
configurations are considered to be equivalent if they belong to the same orbit generated by the
shift symmetries

$$\delta\tilde{\Phi} = K_{ij} \ast \tilde{\beta}^{ij}, \quad \delta\tilde{A} = K_{ij} \ast \tilde{\alpha}^{ij}, \quad \delta\Phi = K_{ij} \ast \alpha^{ij}, \quad \delta\tilde{\Phi} = K_{ij} \ast \beta^{ij} \quad (4.7)$$

for general undeformed $sp(2)$-triplets $(\tilde{\beta}^{ij}, \tilde{\alpha}^{ij}, \alpha^{ij}, \beta^{ij})$. Finally, its reality conditions are

$$A^\dagger = -\tilde{A}, \quad \Phi^\dagger = \pi(\Phi), \quad \tilde{\Phi}^\dagger = -\pi(\tilde{\Phi}). \quad (4.8)$$
The equations can be re-written by introducing an outer Klein operator \( k \) that obeys \( k^2 = 1 \) along with
\[
[k, Y^a_i] = 0, \quad \{k, Y_i\} = 0, \quad [k, Z^a_i] = 0, \quad \{k, Z_i\} = 0, \quad dk = kd, \quad (4.9)
\]
and defining
\[
B = \Phi \kappa, \quad \tilde{B} = \kappa \tilde{\Phi}, \quad (4.10)
\]
after which the equations read
\[
F + B \ast \tilde{B} = 0, \quad 0 = \tilde{F} + B \ast B, \quad (4.11)
\]
\[
DB = 0, \quad 0 = \tilde{D} \tilde{B}, \quad (4.12)
\]
\[
DK_{ij} = 0, \quad 0 = \tilde{D} K_{ij}, \quad (4.13)
\]
\[
\{K_{ij}, B\} = 0, \quad 0 = \{K_{ij}, \tilde{B}\}, \quad (4.14)
\]
where now
\[
DB := dB + A \ast B - B \ast \tilde{A}, \quad \tilde{D} \tilde{B} := d \tilde{B} + \tilde{A} \ast \tilde{B} - \tilde{B} \ast A, \quad (4.15)
\]
and the \( \text{sp}(2)^{(Y)} \) gauge symmetries read
\[
\delta \tilde{B} = K_{ij} \ast \tilde{\beta}^{ij}, \quad \delta \tilde{A} = K_{ij} \ast \tilde{\alpha}^{ij}, \quad \delta A = K_{ij} \ast \alpha^{ij}, \quad \delta B = K_{ij} \ast \beta^{ij} \quad (4.16)
\]
for general undeformed \( \text{sp}(2)^{(Y)} \)-triplets \((\tilde{\beta}^{ij}, \tilde{\alpha}^{ij}, \alpha^{ij}, \beta^{ij})\). The reality conditions are
\[
A^\dagger = -\tilde{A}, \quad B^\dagger = B, \quad \tilde{B}^\dagger = -\tilde{B}. \quad (4.17)
\]

The system can be extended further in two independent ways, by allowing general dependence on \( k \), and by duality extension, whereby \((A, \tilde{A}, B, \tilde{B})\) are forms of degrees \((1, 1, 0, 2)\) mod 2, respectively. Reducing the \( k \)-dependence by taking \( B = \Phi k \) and \( \tilde{B} = \kappa \tilde{\Phi} \) and \((A, \tilde{A}, \Phi, \tilde{\Phi})\) to be \( k \)-independent forms of degrees \((1, 1, 0, 2)\) mod 2, respectively, yields the duality extension of the original system with twisted bi-fundamental zero- and two-form.

Prior to eliminating \( k \), the one-form \( S := dZ^A_i S_{Ai} \) with \( S_{Ai} := Z_{Ai} - 2i A_{Ai} \) obeys
\[
[S_{ai}, S_{bj}] = 2 \epsilon_{bij} \tau_{ai} (S \ast S), \quad (4.18)
\]
\[
\pi_k (S_{Ai}) \ast (S \ast S) = 2 \chi_{\tau_Ai} (S \ast S), \quad (4.19)
\]
where
\[
S \ast S = i dZ^A dZ_{Ai} + 4B \ast \tilde{B}, \quad (4.20)
\]
and the inner derivatives \( \tau_{Ai} \equiv \tau_{\partial_{Ai}} \) act from the left, using the rule \([k, \tau_{ai}] = 0 \) and \( \{k, \tau_i\} = 0 \). In deriving \((4.18)\) we have used \( dZ^A dZ_{Ai}, A = -2i dZ^A \partial_{Ai} A \) and \( F = -B \ast \tilde{B} \). Thus, after eliminating \( k \), we have
\[
[S_{Ai}, S_{Bj}] = 2 \epsilon_{Bj\tau_Ai} (S \ast S), \quad S \ast S = i dZ^A dZ_{Ai} + 4\Phi \ast \tilde{\Phi}, \quad (4.21)
\]
that is, the presence of the dynamical two-form implies that \( S_{Ai} \) is no longer a deformed oscillator on-shell. The one-form \( \tilde{S} := dZ^A_i S_{Ai} \) with \( S_{Ai} := Z_{Ai} - 2i \tilde{A}_{Ai} \) obeys similar constraints, and we note that there is no constraint on mutual star products between \( S_{Ai} \) and \( \tilde{S}_{Ai} \) master fields.
As for the choice of \( sp(2) \) gauge algebra generators, the introduction of the dynamical two-form obstructs the Wigner deformed oscillator algebra, and hence the definition of a diagonal \( sp(2) \) algebra. On the other hand, the choice
\[
K_{ij} = K_{ij}^{(Y)},
\]
remains consistent for general two-form backgrounds. With this choice, and assuming that \( Z \) contains an \( S^2 \) on which \( \tilde{B} \) can be wrapped as to produce \( J \) as a vacuum expectation value, the consistent truncation
\[
\tilde{\Phi} = J, \quad \tilde{A} = A = W,
\]
gives back the new Type A model. The non-trivial two-cycle implies, however, that the dynamical two-form contains additional degrees of freedom, that we plan to examine elsewhere; for a related feature in the case of four-dimensional higher spin gravity, see \cite{9,27}.

### 4.2 Frobenius algebra and superconnection

As topological open strings set the paradigm for deforming differential form algebras on Poisson manifolds \cite{14,16,42,46}, this raises the question of whether the field equations admit a format more akin to that expected from a topological open string field theory, namely that of a flatness condition on a graded odd superconnection valued in the direct product of the higher spin algebra and a suitable graded Frobenius algebra \( \mathcal{F} \) \cite{12}.

To this end we take \( \mathcal{F} \equiv \text{Mat}_2(\mathbb{C}) \) to be spanned by \( (I, J = 1, 2) \) \cite{9,27}
\[
e_{IJ} = \begin{bmatrix} e & f \\ \tilde{f} & \tilde{e} \end{bmatrix}, \quad e_{IJ}e_{KL} = \delta_{JK}e_{IL}.
\]
We then define the superconnection \( X \), \( sp(2) \) gauge generators \( K_{ij} \), and nilpotent differential \( q \), respectively, by
\[
X := A e + \tilde{A} \tilde{e} + B f - \tilde{B} \tilde{f}, \quad K_{ij} := (e + \tilde{e})K_{ij}^{(Y)}, \quad q := (e + \tilde{e})d,
\]
introduce the 3-grading \( \deg_\mathcal{F}(\tilde{f}, e, \tilde{e}, f) = (-1, 0, 0, 1) \), and use Koszul signs governed by the total degree given by the sum of form degree and \( \deg_\mathcal{F} \); we note that \( q \) has total degree given by 1, while \( X \) has total degree given by 1 prior to duality extension, and in \( \{1, 3, \ldots\} \) after duality extension. In terms of these requisites, the equations of motion and gauge conditions can be written on the desired format as
\[
qX + X \star X = 0, \quad [K_{ij}, X]_\star = 0,
\]
and the factorization of the \( sp(2) \) ideal amounts to the shift symmetries
\[
\delta X = K_{ij} \star \alpha^{ij}.
\]
4.3 Comments on action and quantum corrections

We propose to make the equations of motion (4.26) (including the $sp(2)$ gauge condition) variational by taking the spacetime manifold to be part of the boundary of an open manifold $\mathcal{X}$, extending $\mathcal{X}$ to a master field $\hat{\mathbf{X}}$ that depends on a set of ghost $(B^{ij}, C_{ij})$ variables obeying

$$\{B^{ij}, C_{kl}\} = \delta_{[k}^{[i} \delta_{l]}^{j]} \; , \tag{4.28}$$

and introducing a master field $\hat{\mathbf{P}}$ that vanishes at $\partial \mathcal{X} \times Z$. The Koszul signs are governed by the total degree given by the sum of the form degree, degree in $\mathcal{F}$ and ghost number. The total degree of $\hat{\mathbf{X}}$ lies in \{1, 3, ..., $2^p - 1$\}, where $\dim(\mathcal{X}) = 2^p - 3$ or $2^p - 4$, subject to the condition that the sum of form degree and degree on $\mathcal{F}$ is non-negative. The total degree of $\hat{\mathbf{P}}$ lies in \{1, 3, ..., $2^p - 1$\} if $\dim(\mathcal{X}) = 2^p - 3$ or in \{0, 3, ..., $2^p - 2$\} if $\dim(\mathcal{X}) = 2^p - 4$, again subject to the condition that the sum of form degree and degree on $\mathcal{F}$ is non-negative.

Defining the BRST operator

$$\hat{Q} = C^{ij} K_{ij} - 2 i B_{ij} C_j^k C_k^i \; , \quad \hat{Q}^2 = 0 \; , \tag{4.29}$$

and the covariant derivative

$$\hat{\mathcal{D}} = \mathfrak{q} + \text{ad}_Q \; , \quad \hat{\mathcal{D}}^2 = 0 \; , \quad \mathfrak{q} \hat{Q} = 0 \; , \tag{4.30}$$

the flatness condition

$$\hat{\mathcal{D}} \hat{\mathbf{X}} + \hat{\mathbf{X}} \star \hat{\mathbf{X}} = 0 \; , \quad \text{at } \partial \mathcal{X} \; , \tag{4.31}$$

follows from the variational principle applied to

$$\begin{align*}
\text{dim}(\mathcal{X}) \text{ odd} : & \quad S = \int_{\mathcal{X} \times Z} \text{Tr}_A \text{Tr}_\mathcal{F} \text{Tr}_G \left( \hat{\mathbf{P}} \star (\hat{\mathcal{D}} \hat{\mathbf{X}} + \hat{\mathbf{X}} \star \hat{\mathbf{X}}) + \frac{1}{3} \hat{\mathbf{P}} \star \hat{\mathbf{P}} \star \hat{\mathbf{P}} \right) , \\
\text{dim}(\mathcal{X}) \text{ even} : & \quad S = \int_{\mathcal{X} \times Z} \text{Tr}_A \text{Tr}_\mathcal{F} \text{Tr}_G \left( \hat{\mathbf{P}} \star (\hat{\mathcal{D}} \hat{\mathbf{X}} + \hat{\mathbf{X}} \star \hat{\mathbf{X}}) + \frac{1}{2} \hat{\mathbf{P}} \star \hat{\mathbf{P}} \right) ,
\end{align*} \tag{4.32}$$

treating $Z$ as a closed manifold, and where $\text{Tr}_A$ denotes the (cyclic) trace operation over the extended Weyl algebra $A$ generated by polynomials in $Y$, $\kappa_y$ and $k$ (constructed as in [9, 27]); $\text{Tr}_\mathcal{F}$ is the standard trace operation on $\mathcal{F} \equiv \text{Mat}_2$; and $\text{Tr}_G$ is the standard trace over the Clifford algebra $G$ generated by the ghosts. With these definitions, the kinetic term is based on a non-degenerate bilinear form. Thus, the proposal is that Eqs. (4.26) and (4.27) describe the BRST cohomology contained in (4.31).

As for boundary conditions, we assume that $\mathcal{X} \times Z$ is a compact manifold that contain subregions $\mathcal{X}' \times Z$, with $\mathcal{X}'$ corresponding to conformal boundaries, where a subset of the master field components are allowed to blow up; in particular, treating $Z$ as a compact manifold with non-trivial cycles affects the degrees of freedom that are local on $\partial \mathcal{X}$, as already commented on above. The homogenous Dirichlet boundary condition on $\hat{\mathbf{P}}$ does not follow from the classical variational principle; instead it follows from the requirement that the field theory BRST operator is a smooth functional differential of a topological field theory [26, 47]. The latter property is preserved under the addition of topological invariants to $\partial \mathcal{X} \times Z$. If these contain components
of $\hat{X}$ in sufficiently high form degree, then they may receive quantum corrections from the $\hat{P}^{*2}$ and $\hat{P}^{*3}$ vertices. The topological invariants may thus be non-trivial on-shell, thereby providing boundary micro-state observables appearing in the boundary partition function (as $\hat{X}$ is left free to fluctuate at $\partial \mathcal{X} \times \mathcal{Z}$); in addition, if the expectation values in $\hat{X}$ at $\partial \mathcal{X} \times \mathcal{Z}$ (due to non-trivial cycles and including the zero-form initial data) source forms in $\hat{X}$ in higher degrees, then the resulting boundary partition function may contain non-trivial bulk quantum corrections. This suggests that the standard (duality unextended) Chern classes, which only contain one-forms from $\hat{A}$ and $\tilde{\hat{A}}$, correspond to free conformal theories, while their duality extensions, which contain higher forms from $A$ and $\tilde{A}$, correspond to non-trivial conformal field theories.

5 Conclusions

In this work, we have first presented an alternative to Vasiliev’s on-shell formulation of the Type A model in general spacetime dimensions, using the same field content but a different $sp(2)$ gauge symmetry with field independent generators. We have argued that this model propagates the same degrees of freedom as Vasiliev’s original equations, and we have provided evidence that the two models are perturbatively equivalent. Drawing on the field independence of the $sp(2)$ generators of the new model, we have then extended its equations of motion by a dynamical two-form. This extension requires two connection one-forms, gauging the separate left- and right-actions of a complexified higher spin algebra, and a zero- and two-form in opposite (real) bi-fundamental representations. Finally, we have proposed that the latter set of equations describes the BRST cohomology of a system that descends from a variational principle, that is obtained by further extension by first-quantized ghosts and an internal graded Frobenius algebra. If this proposal holds true, then these extensions permit the packaging of the equations of motion and the $sp(2)$ gauge conditions, respectively, into a flatness condition and a set of gauge transformations for a single odd superconnetion $\hat{X}$. The action also requires the introduction of a supermomentum $\hat{P}$ that may quantum deform certain observables, that may be of importance in taking the correspondence between topological open strings and conformal fields beyond the current agreement at the level of conformal particles and free fields [13,30,32–34].

Although the extension with dynamical two-form does not retain manifest Lorentz covariance, it is nevertheless suitable for potential extensions of higher spin gravity to more general non-commutative manifolds. Indeed, the extension by the two-form provides a link to topological open string fields theory, which is the natural framework for deforming non-commutative geometries.

We have deferred a number of technical aspects for future work: First of all, it remains to map linearized states in lowest weight spaces (particle-like solutions) in $\Phi$ to Fronsdal fields in $W^\mu$ by finding a suitable gauge function; for related supporting results for the four-dimensional twistor formulation, see [48–50]. Furthermore, in order to establish whether the old and the new Type A models are perturbatively equivalent; the first step is to examine whether $K^{(diag)}_{ij}$ and $K^{(L)}_{ij}$ agree in Vasiliev-Fronsdal gauge at first sub-leading order.

As for the formulation in terms of the superconnection $X$, the topology and the boundary conditions of $\mathcal{X} \times \mathcal{Z}$ need to be examined. In particular, $\mathcal{Z}$ needs to contain a non-trivial two-
cycle in order for the dynamical two-form to contain the original closed and central element as a non-trivial vacuum expectation value. In this case the alternative Type A master fields arise as a consistent truncation of \( \mathbf{X} \); if so, however, the dynamical two-form leads to new local degrees of freedom in spacetime, whose holographic interpretation remains to be given; for related issues in the case of the four-dimensional holographic twistor theory, see \([9,27]\).

Our proposal for an action, producing the \(sp(2)\) condition as well from a variational principle, relies on the claim made in Section 4.3 concerning the BRST cohomology contained in the flat superconnection \( \hat{\mathbf{X}} \) (obtained by extension by first-quantized \(sp(2)\) ghosts). In the aforementioned action principle, the \(sp(2)\) generators are fixed given operators. In this context, it would be interesting to treat them as new fluctuating degrees of freedom \([51–54]\) of an enlarged string field.

Concerning the basic physical motivation behind our work, namely that from the recent gathering of results concerning the nature of the Noether procedure, it appears that the formulation of higher spin gravity in terms of Fronsdal fields leads to a perturbatively defined quantum effective action making sense in asymptotically maximally symmetric spacetimes, whereas the topological open string field theory formulation provides perturbative expansions around more general backgrounds. In addition, the latter formulation leads to the notion of star product locality, whereby the classical action is built from data obtained from disc amplitudes, thus replacing the more subtle notion of spacetime quasi-(non)locality that needs to be adopted following the standard Noether approach.

Finally, we remark that the alternative \(sp(2)\) gauging for the Type A model presented in this work has a direct generalization to the Type B model based on \(osp(1|2)\) gauging, whose conformal field theory dual expanded around the anti-de Sitter vacuum consists of free fermions; we hope to present this model in more detail in a forthcoming work.

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