Excitation of plasma oscillations by moving Josephson vortices in layered superconductors

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Abstract

Electric and magnetic fields created by moving uniform lattice of Josephson vortices in the magnetic field parallel to the layers are calculated in the frame of exactly solvable model. At large velocities of the vortex lattice the plasma oscillations of superconducting electrons are excited by vortex motion, this results in interference features in I-V curves at low temperatures. The spectrum of electromagnetic radiation by moving vortices contains peaks related to the excitation of plasmons and to the Cherenkov radiation.

It is known that in high-$T_c$ superconductors there exist low-damping plasma oscillations [1]-[7] in the range of hundreds megahertz - terahertz. The presence of an underdamped eigenmode leads to a resonance behavior of the driven modes when frequency and wave-vector of the driven force are close to those of the eigenmode. In particular, frequencies of the electromagnetic field of sliding lattice of Josephson vortices created by magnetic field parallel to the layers can enter the range of plasma modes if the lattice is moving fast enough. The velocity of such vortices driven by transport current perpendicular to the layers may be very large, because the amplitude of the order parameter $\Delta$ inside Josephson vortices is perturbed very slightly. Thus, at large enough voltages across the superconductor the resonance conditions can be satisfied. This results in the generation of plasma oscillations and influences the I-V curves. Below we study the motion of uniform lattice of Josephson vortices in an infinite crystal in magnetic field which is small enough, so that non-linear cores of the vortices do not overlap. We assume that the characteristic frequencies of the problem are small in comparison to $\Delta$.

In the presence of vortices the spectrum of the eigenmodes is different from the zero-field spectrum due to oscillations of vortex displacements leading to the sound-like [1],[7],[8] spectrum of the modes. In contrast, the response of a superconductor to the motion of the vortex lattice driven by the transport current calculated in our study, is not related to the oscillations of the vortices, driven modes being related to the plasmons with the plasma edge.

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The current distribution in the vortices is influenced by their motion, this makes calculations of the vortex dynamics difficult since usual perturbational approach is not valid at large velocities of the lattice. To solve the problem strictly one must find solution of Maxwell’s equations with expressions for charge and current densities in layered superconductors. Having in mind high-$T_c$ superconductors we shall consider the order parameter having the symmetry close to d-pairing. Expressions for current and charge densities are non-linear functions of the phase differences between adjacent layers, which are rather complicated functions in general case. Quasi-particle current densities depend on phase differences, on the superconducting momentum $P_n = (1/2)i\mathbf{q}\chi_n - (1/c)\mathbf{A}_n$ and, in addition, on the gauge-invariant scalar potential $\mu_n = (1/2)(\partial\chi_n/\partial t) + \Phi_n$, where $\mathbf{A}_n$ is vector potential, $\Phi_n$ is electric potential, and $\chi_n$ is the phase of the order parameter in layer $n$. (We assume here $\hbar = 1, e = 1$). In general case potential $\mu_n$ to be found from the Poisson’s equation, however in the vortex problem at low temperatures considered here, $T \ll \Delta$, the quasi-particle branch imbalance, and, hence, potential $\mu_n$ can be neglected provided $(r_0/d)^2 \ll 1$, where $r_0$ is Thomas-Fermi screening radius, and $d$ is the period of the crystal in the direction perpendicular to the layers. Current density along the layers can be described by expression for linear response since the characteristic value of the current is determined by the critical current density $j_c$ in the perpendicular direction, which is small in comparison to the critical current in the parallel direction. Then the expression for current density in layer $n$, presented by means of Fourier transformation has the form

$$j_n = \frac{e^2}{4\pi\lambda^2}P_n - i\omega\sigma_{\parallel}(\omega)\mathbf{P}_n,$$

The first term here describes the superconducting current, and the second is related to the quasi-particle current, $-i\omega$ and $i\mathbf{q}$ correspond to the time derivative and to the gradient in the direction parallel to the layers. The frequency dependence of conductivity $\sigma_{\parallel}(\omega)$ is determined by symmetry of the order parameter and depends on the momentum scattering time. We shall use the expressions from Ref.[10] to describe $\sigma_{\parallel}(\omega)$.

The density of the superconducting current between layers $n$ and $n+1$ is given by

$$j_{\perp n}^{(s)}(\varphi_n) = j_c \sin \varphi_n,$$

where $\varphi_n$ is the gauge invariant phase difference. We shall adopt an exactly solvable model substituting $\sin \varphi_n$ by the saw-tooth function

$$j_{\perp n}^{(s)}(\varphi_n) = j_c \arcsin \sin \varphi_n,$$

as it was done in [11]. Such a model was used to study the Josephson vortex flow in the linear approximation on velocity in [12]. Following (1) we shall use linear relation between the quasi-particle current and the phase difference

$$j_{\perp n}^{(qp)} = -i\omega\sigma_{\perp}(\omega)\varphi_n/(2d).$$

The expressions for the current densities we insert to the Maxwell equation

$$\text{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c} \frac{\partial\mathbf{D}}{\partial t},$$

(2)
in which we shall use discrete approximation for spatial derivatives in $z$ direction perpendicular to the layers, and express magnetic field in $y$ direction as $\varphi_n$ and $P_n$

$$H_y = \frac{c}{2d} \frac{\partial \varphi_n}{\partial x} - \frac{c}{d}(P_{n+1} - P_n),$$

where $x$ is the coordinate in the direction of the vortex flow.

In the displacement current we take into account only the component along $z$ axis, since due to the strong anisotropy of a layered superconductor the plasma frequency in the direction parallel to the layers, $\Omega_p = c/\lambda$, is much larger, than typical frequencies of the problem, which are of the order of the plasma frequency for perpendicular direction, $\omega_p = c_z/\lambda_c$. Here $c_z = c/\sqrt{\epsilon}$, $\epsilon$ is a dielectric constant in $z$ direction, $\lambda$ and $\lambda_c = c/\sqrt{8\pi d_j c}$ are the screening lengths for the current flowing parallel and perpendicular to the layers, respectively. Then we get the equations

$$j_{\perp n}(\varphi_n) - \frac{c^2}{8\pi d} \frac{\partial^2 \varphi_n}{\partial x^2} + 2\lambda_c^2 \frac{\partial}{\partial x}(P_{n+1} - P_n) = -\frac{\lambda_c^2}{c^2} \frac{\partial}{\partial t} \left( \frac{4\pi \sigma_{\perp}}{\epsilon} + \frac{\partial}{\partial t} \right) \varphi_n$$

$$\frac{c^2}{2d^2} \frac{\partial}{\partial x}(\varphi_n - \varphi_{n-1}) + \Omega_p^2 P_n - \frac{c^2}{d^2}(P_{n+1} + P_{n-1} - 2P_n) = -\frac{\partial}{\partial t} \left( 4\pi \sigma || + \frac{\partial}{\partial t} \right) P_n.$$  

The solution of these equations yields spatial and temporal dependencies of electric and magnetic fields.

Using the model current-phase relation (1) we can find the exact solution of equations (3-4) by means of the Fourier transformation. In principle, we must take into account a bending of the vortex center $x_0(y)$ along $x$ axis is small at distances $y \approx \lambda_c$. Estimation of the vortex bending based on equations of a balance between forces acting to the vortices and created by the transport current and by the vortex currents shows that the deformation of vortices can be neglected in magnetic fields $H \gg H_{c1}$ provided

$$\frac{x_0(\lambda_c)}{\lambda_c} \approx \frac{\lambda}{\pi d \ln(\lambda/d)} \frac{j_{\perp} H_{c1}}{j_c H} \ll 1.$$  

We assume that $H$ is large in comparison to $H_{c1}$ and condition (5) is satisfied.

Then for the triangle lattice sliding with velocity $u$ we get

$$(\tilde{\omega}_p^2 - \omega^2 + i\omega \omega_r)\varphi - 2\epsilon c^2 q K P/\epsilon = \Pi \delta(q - \omega/u)$$

$$-(c^2 q K/2d^2)\varphi + (\tilde{\Omega}_p^2 - \omega^2 + i\omega \omega_r)P = 0.$$  

Here $K = 2 \sin k/2$, $|k| < \pi$ is the wave number obtained from the discrete Fourier transformation with respect to the layer number, $\tilde{\omega}_p^2 = \omega_p^2(1 + \lambda_c^2 q^2)$, $\tilde{\Omega}_p^2 = \Omega_p^2(1 + \lambda^2 K^2/d^2)$, $\omega_r = 4\pi \sigma_{\perp} \epsilon$, $\Omega_r = 4\pi \sigma ||$ and, finally,

$$\Pi = 4\pi^2(\omega_p^2/i\omega) \sum_{l,m} \exp[-i(l + m/2)qX - imkZ/d].$$

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Solution of equations (3-7) for the phase differences takes the form

\[ \varphi = \Pi(1 + \lambda^2 K^2/d^2)\delta(q - \omega/u)/D, \]

where \( D \) is the determinant of equations (3-7). Zeros of \( D \) yield the spectrum of the eigenmodes, i.e. of the plasmons. At high frequencies and low temperatures, when frequencies of dielectric relaxation \( Re \omega_r \) are small enough, we get

\[ D = [(\omega_p^2 - \omega^2)(1 + \lambda^2 K^2/d^2) + \omega_p^2 \lambda^2 q^2 - i\omega [\omega_r(1 + \lambda^2 K^2/d^2) + \Omega_r(\omega_p^2 - \omega^2 + \omega_p^2 \lambda^2 q^2)/\Omega_p^2]]. \]

Variation of the solution with frequency increasing can be illustrated by the example of the component of the electric field \( E_z = -i(\omega/2d)\varphi \). At frequency \( \omega = qu \) \( (q = 2\pi/X) \) the slowly varying along \( z \) part of the field \( E_z \) created by a single vortex has the form

\[ E_z = \frac{\pi u \omega_p^2 \exp(-z/\Lambda)}{2d\sqrt{(\omega_p^2 - \omega^2 - i\omega\omega_r)(\omega_p^2 - \omega^2 + \omega_p^2 \lambda^2 q^2 - i\omega\omega_r)}}, \quad \Lambda = \lambda \sqrt{\frac{\omega_p^2 - \omega^2 - i\omega\omega_r}{\omega_p^2 - \omega^2 + \omega_p^2 \lambda^2 q^2 - i\omega\omega_r}}. \]

The total field is described by the sum of the fields created by individual vortices. One can see that at small frequencies the decaying length is of the order of \( \lambda \). When frequency becomes larger, than the plasma frequency, the real part of the exponent in (3) becomes small. This means that in the range of plasma oscillations the decaying length for the electric field is large and is determined by the damping of plasmons.

In order to find the relation between transport current \( j_{tr} \) and the velocity of the vortex lattice \( u \), we multiply equation (3) by \( (c^2/\lambda^2)\partial \varphi_n/\partial x \) and (4) by \( 4\partial P_n/\partial x \), then we add these equation, integrate them by \( x \) and sum up over all \( n \). Then in the left-hand part of the resulting equation we get an expression which is a full derivative over \( x \), the integral of which can be reduced to \(-2d^2 \sum_n H_y^2|_{z=\infty} \). This expression is related to the force acting on the vortex lattice and is proportional to the half sum of the magnetic fields at the opposite sides of the sample (i.e. to the externally applied magnetic field), and to the difference of the magnetic fields, which by means of equation (2) can be expressed in terms of the transport current. The right-hand side we express by means of the Fourier transformation using solutions of equations (6-7). Finally, we get

\[ 16\pi^2 d j_{tr} = \int dqdku\omega_p^4[\varepsilon\omega_r(1 + \lambda^2 K^2/d^2)^2 + \Omega_r\lambda^4 K^2 q^2/d^2)]/|D|^2 \sum_{l,m} e^{i(l+m/2)\pi + imkZ/d}. \]

Integrating (10) we find a relation of the transport current to the velocity of the vortex lattice, and, hence, to the average electric field \( \bar{E} = 2\pi n_L u \) (where \( n_L \) is the vortex density).

In the limit of small velocities, \( u \ll (d/\lambda)c \), one can neglect the effect of velocity \( u \) on the shape of vortices. In this limit I-V curve is described by the Ohm’s law with the resistivity

\[ \rho = \frac{1}{\sigma_0 + \sigma_0(\lambda/\lambda_c)^2}\frac{2n_L d\lambda J}{\pi}. \]

where \( \lambda J = d\lambda_c/\lambda \). The last factor in (11) describes the relative volume of the superconductor occupied by the non-linear region. One can see an analogy to the Bardeen-Stephen law with
Figure 1: Typical relation between normalized transport current and vortex velocity in magnetic field $H = 20H_{c1}$.

the area of the Abrikosov vortex core $\xi^2$ substituted by the area of the non-linear part of the Josephson vortex $d\lambda_J$.

An analytic expression for the dependence of $u$ on $j_{tr}$ may be obtained easily also for frequencies of the order of the plasma frequency for the case of small plasmon damping

$$j_{tr} = j_c \frac{\sinh b}{\cosh b - \cos a}, \quad a = \frac{X_{cz}}{\lambda_c u}, \quad b = \frac{\omega_r(\omega_p)}{2\omega_p} a.$$

The last factor in this equation describes the oscillations at the I-V curve appearing as a result of interference of the plasmons created by different vortices. In the limit of large velocities the I-V curve approaches the Ohm’s law again, but with resistivity $\rho = 2\pi \sqrt{n_L d\lambda_j/\sigma(\omega_p)}$. The shape of the I-V curves is sensitive to the magnitude and to the anisotropy of the quasi-particle conductivity. An increase of the conductivity results in an increase of the plasmon damping and in a decrease of the peaks. The regime of very small damping is easily achieved in the case of isotropic pairing when the order parameter has no nodes and the quasi-particle density is exponentially small. In the case of d-pairing the damping is higher and peaks at the I-V curves are smaller. In Fig.1 we present the dependence $j_{tr}(u)$ calculated numerically under assumptions that the real part of the conductivity is described by expressions from [10] and is decreasing at frequencies above the quasi-particle momentum scattering rate as $1/\omega^2$. Note that the condition (5) is difficult to satisfy at large currents, therefore the shape of the I-V curve near the maxima in Fig.1 may need more detailed calculations which take into account the bending of the vortices due to their motion.

Thus, in the range of plasma oscillations the regions of negative differential conductivity
appear in which the uniform flow of the vortex lattice is unstable. These regions are separated by the stability regions. At large voltages the uniform motion is stable.

Excitation of plasma oscillations is related to an emission of electromagnetic radiation. We calculate the energy flow traveling along $x$ axis at frequency $\omega_N = \frac{2\pi uN}{X}$, where $N$ is the harmonic’s number. To do this we find the Pointing vector using expressions for electric and magnetic fields calculated by means of (8).

$$S = \int_{-\pi}^{\pi} \frac{ue^2\omega_p^4(1 + \lambda^2k^2/d^2)}{16\lambda^2dX^2|D(\omega = \omega_N)|^2} \sum_m e^{imZk/d}dk.$$ 

It is evident that the energy flow increases at resonance frequencies, when the denominator $|D|$ is small. In the case of triangle lattice we find that the energy of odd harmonics contains maxima in the region of plasma frequency, while even harmonics has additional sharp peaks at the velocities of the lattice close to the velocity of the light in the superconductor $c_z$ playing a role of the Swihart velocity of the tunnel junction. The latter peaks correspond to the Cherenkov radiation. The energy flow created by the first harmonic, $S(u)$, is shown in Fig.2, where the energy flow is presented in units of $S_0 = \frac{\hbar^2c^2\omega_p}{(16e^2\lambda^2\lambda_c)}$. Using values typical for BSCCO, $\epsilon = 25$, $\lambda = 0.2 \mu m$, $\lambda_c = 60 \mu m$, we find $S_0 \sim 10$ W/cm$^2$. The shape of the curve and the peak’s size strongly depend on the conductivity value and on the anisotropy of the conductivity. The values of the lattice velocity corresponding to the maximums at $S(u)$ do not necessarily correlate with the peaks of the conductivity, thus, the peaks of the radiation in the range of the plasma frequency can be observed in the regions of the stability of the uniform vortex lattice flow.
The peak in the energy flow at \( u > c_z \) is much higher than those near the plasma frequency, it can be approximated by the expression

\[
S = \frac{S_0 h^2}{4\pi^2 \left\{ \left[ 1 + hN^2 (1 - u^2/c_z^2) \right]^2 + hN^2 \omega_r^2 (u = c_z) \right\}}.
\]

where \( h = 4\pi^2 \lambda \lambda_c n_L \).

Note that the frequencies of the radiation at the vortex motion with velocities close to \( c_z \) belong to the frequency range below the superconducting gap value only if the magnetic field is small enough, i.e. when the period of the vortex lattice \( X \) is comparable to \( \lambda_c \). In larger fields the corresponding frequency approaches \( \Delta \). However, this must not affect the results considerably, because at such frequencies the part of the response corresponding to damping is very small in pure, \( \tau T_c \gg 1 \), superconductors.

Note also that the effects considered here are related to the range of higher voltages and frequencies, than those typically used in experimental studies of Josephson vortices, e.g. in [13], in which a non-Josephson emission from the moving vortices was reported in BSCCO. As long as we consider here the uniform motion of the lattice in the infinite crystal, such a radiation in our treatment is absent.

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