Hybrid quantum system of a nanofiber mode coupled to two chains of optically trapped atoms

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New Journal of Physics 12 (2010) 103014 (18pp)
Received 10 June 2010
Published 8 October 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/10/103014

Abstract. A tapered optical nanofiber is simultaneously used to trap and optically interface cold atoms through evanescent fields and constitutes a new and well controllable hybrid quantum system. The atoms are trapped in two parallel one dimensional (1D) optical lattices generated by suitable far blue and red detuned evanescent field modes very close to opposite sides of the nanofiber surface. Collective electronic excitations (excitons) of each of the optical lattices are resonantly coupled to the second lattice forming symmetric and antisymmetric common excitons. In contrast to the inverse cube dependence of the individual atomic dipole–dipole interaction, we analytically find an exponentially decaying coupling strength with distance between the lattices. The resulting symmetric (bright) excitons strongly interact with the resonant nanofiber photons to form fiber polaritons, which can be observed through linear optical spectra. For large enough wave vectors, the polariton decay rate to free space is strongly reduced, which should render this system ideal for the realization of long-range quantum communication between atomic ensembles.

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1. Introduction

The effects related to light–matter interactions continue to be of great interest in the main research today, due to their importance for fundamental physics and applications. Manipulation and trapping of atoms are some of the consequences of such interactions [1] and serve as a framework for the development of quantum information technologies, e.g. the implementation of quantum memories, optical communications and quantum computations [2]. Furthermore, trapping and guiding of atoms are useful tools for atom optics and atom interferometry [3]. Historically, the system that has drawn most attention so far is cavity quantum electrodynamics, which offers a powerful tool for controlling atom–photon interactions [4] and even for achieving the trapping of a single atom in a cavity [5, 6]. Recently, a Bose–Einstein condensate of an ultracold atomic gas was produced between optical cavity mirrors, where the strong electric dipole coupling regime was achieved [7, 8]. Moreover, strong magnetic coupling of an ultracold gas in a superconducting coplanar waveguide resonator can be achieved [9, 10].

Different directions of research, over the last few years, have been opened for the light–matter coupling which rests on optical fibers. In a hollow core fiber, the atoms are funneled into a capillary in the center of the fiber and couple to the guided fiber photons, where this system can guide and confine both atoms and photons [11]–[13]. Another setup that is of importance for the present work is tapered optical fibers with a nanofiber waist, where the atoms are outside the fiber and couple to the evanescent field surrounding the fiber [14, 15]. A method for trapping and guiding neutral atoms outside a thin optical fiber has been proposed in [16] and studied earlier for a large fiber [17]. This system combining both atomic and solid state devices and can be considered as a hybrid quantum system, where trapping and optical interfacing of atoms can be simultaneously achieved. Moreover, the strong evanescent field around the nanofiber is efficiently used for trapping, detecting and manipulating of a single atom [18]. Spontaneous emission of an atom is significantly affected in the vicinity of an optical nanofiber [19]. Also, the case of a pair of atoms near a nanofiber was studied in [20], where a substantial radiative exchange between distant atoms mediated by the nanofiber modes was
demonstrated. Furthermore, the possibility of directional guided superradiance from an array of distant atoms parallel to a nanofiber was shown in [21].

The guided modes of ultrathin optical fibers, with diameters smaller than the wavelength of the guided light, exhibit strong transverse confinement and a pronounced evanescent field. Interference of two-color evanescent fields surrounding an optical nanofiber gives rise to an array of optical microtraps [22]. The system was realized recently for cesium atoms interacting with a multicolor evanescent field surrounding an optical nanofiber [23], where the atoms are localized in a one-dimensional (1D) optical lattice parallel to the nanofiber, which is shown to be efficiently interrogated with a resonant light field sent through the nanofiber. Moreover, conventional optical lattices are formed by counterpropagating laser beams to get a standing wave in free space, and then the ultracold atoms are loaded [24]. This system is well described by the Bose–Hubbard model, which predicts the quantum phase transition from the superfluid to the Mott insulator phase with a fixed number of atoms per site [25].

In the present paper, we investigate a simplified model related to the setup presented in [23]. The hybrid quantum system is made of a tapered optical nanofiber and two optical lattices. The 1D optical lattices formed by off-resonance nanofiber modes appear parallel to each other and are localized at two opposite sides of the nanofiber. We consider the case of a single two-level atom per lattice site. The system is excited and probed by photons in a fiber mode resonant with the atomic transition. Due to resonant dipole–dipole interactions the electronic excitation dynamics for each chain is dominated by the formation of collective electronic excitations (excitons) incorporating the optical lattice translational symmetry [26]–[28]. Naturally, excitons in the two chains interact directly and via the resonant fiber mode. In the limit of long-wavelength excitons and for a distance of the two chains larger than the lattice constant, the interaction between the two optical lattices can be calculated explicitly and strongly deviates from single particle dipole–dipole coupling. Eventually, interlattice interactions give rise to dark and bright excitons, which mix with the nanofiber photons in the strong coupling regime to form fiber polaritons [29]. In the experiment [23], cold thermal atoms are used, and due to the small trapping volumes, the average occupancy is about half. In the present paper we consider the ideal case of one atom per site, which allows the formation of propagating exitons.

The paper is organized as follows. In section 2, we derive the exciton dispersions in two parallel optical lattices, and the exciton damping rate is presented. The excitons coupling to nanofiber photons is obtained in section 3. The strong coupling regime is treated in section 4 to give fiber polaritons. The transmission spectrum is calculated in section 5, and a summary appears in section 6. The appendix includes rigorous calculations of the intra- and inter-lattice exciton dynamical matrices.

2. Excitons in two parallel 1D optical lattices

We consider two identical parallel 1D optical lattices with one atom per site, where the lattice constant is \(a\), and they are separated by a distance \(d\). The optical lattice is located at a distance \(b\) from the fiber surface, where the two optical lattices are at opposite sides of the fiber, as is seen in figure 1, which is related to the setup in [23]. The total Hamiltonian of the hybrid quantum system is \(H = H_{\text{ex}} + H_{\text{ph}} + H_{\text{in}}\), where \(H_{\text{ex}}\) is the electronic excitation Hamiltonian in the two optical lattices, \(H_{\text{ph}}\) is the fiber photon Hamiltonian and \(H_{\text{in}}\) is the coupling Hamiltonian between the fiber photons and electronic excitations in the two optical lattices. First, we concentrate on the dynamics of electronic excitations in the two optical lattices. We examine the formation

\[\text{New Journal of Physics 12 (2010) 103014 (http://www.njp.org/)}\]
Figure 1. Two 1D optical lattices with one atom per site, which are located parallel to a fiber at opposite sides. An incident field is sent from the left side and the transmitted field is observed on the right side. The lattice constant is $a$ and the distance between the lattices is $d$. The transition dipole $\mu$, which makes an angle $\theta$ with the lattice direction, is also seen.

of hybrid collective electronic excitations (excitons) among the two lattices. An electronic excitation delocalizes in a lattice due to resonant dipole–dipole interactions and forms an exciton, which is a coherent state with wave number $k$ \cite{26,27}. Then, we check whether such an exciton can transfer between the two lattices due to resonant dipole–dipole interactions, and whether it can coherently oscillate between the two lattices to form a collective electronic excitation in the whole system of the two optical lattices.

The electronic excitation Hamiltonian in the two optical lattices is

$$H_{\text{ex}} = \sum_{n,\alpha} E_{\alpha} B_{n\alpha} \dagger B_{n\alpha} + \sum_{nm,\alpha\beta} J_{nm}^{\alpha\beta} B_{n\alpha} \dagger B_{m\beta}. \quad (1)$$

Here $(n, m)$ run over all the lattice sites, and $(\alpha, \beta)$ stands for the two lattices $(1, 2)$. The atomic transition energy is $E_{\alpha} = \hbar \omega_{\alpha}$ for two-level atoms, which is the same at the two lattices. $B_{n\alpha} \dagger$ and $B_{n\alpha}$ are the creation and annihilation operators of an electronic excitation at site $n$ in lattice $\alpha$, respectively. We consider the case of a single electronic excitation or low excitation density, then the operators obey bosonic commutation relations. The coupling parameter $J_{nm}^{\alpha\beta}$ is for resonant dipole–dipole interactions, and gives rise to excitation transfer between two sites. The transfer term includes transfer between atoms in the same lattice, $J_{11}^{nm}$ and $J_{22}^{nm}$, and transfer between atoms at different lattices, $J_{12}^{nm}$ and $J_{21}^{nm}$.

The Hamiltonian can be diagonalized in the lattice sites using the transformation

$$B_{n\alpha} = \frac{1}{\sqrt{N}} \sum_{k} e^{ikx_n^\alpha} B_{k\alpha}, \quad (2)$$

where $N$ is the number of lattice sites, which is taken to be a large number, and $k$ is the wave number, which takes the discrete values $k = \frac{2\pi}{Np}$, with $(p = 0, \pm 1, \pm 2, \ldots, \pm N/2)$, by using the periodic boundary condition. Here $x_n^\alpha$ is the position of site $n$ in lattice $\alpha$. The new states represent collective electronic excitations, which are called excitons \cite{26}, and they are waves that propagate to the left and the right of the lattice with wave number $k$, which is a good quantum number. Now the Hamiltonian reads

$$H_{\text{ex}} = \sum_{k,\alpha} E_{\alpha} B_{k\alpha} \dagger B_{k\alpha} + \sum_{k,\alpha\beta} J_{k\alpha}^{\alpha\beta}(k) B_{k\alpha} \dagger B_{k\beta}, \quad (3)$$
which is diagonal in $k$. We defined the exciton dynamical matrix by

$$J^{\alpha\beta}(k) = \sum_L e^{i k L} J^{\alpha\beta}(L),$$

(4)

and we used $J^{\alpha\beta}_{nm} = J^{\alpha\beta}(L)$, with $L = x_m^{\beta} - x_n^{\alpha}$, where the dipole–dipole interaction is a function of the distance between the two atoms.

Next we diagonalize the above Hamiltonian relative to the two lattice indexes, $(\alpha, \beta)$, by applying the transformation

$$B^\dagger_{k\nu} = \frac{B_{k1} \pm B_{k2}}{\sqrt{2}}.$$  

(5)

The new states represent entangled states between excitons from the two lattices, which are symmetric and antisymmetric states. The symmetric state is denoted by $\nu = s$ and takes the plus sign ($+$); the antisymmetric state is denoted by $\nu = a$ and takes the minus sign ($-$). The Hamiltonian is cast into the diagonal form

$$H_{ex} = \sum_{k,\nu} E^{\nu}_{ex}(k) B^\dagger_{k\nu} B_{k\nu},$$

(6)

where the eigenenergies are

$$E^s_{ex}(k) = E_A + J(k) + J'(k),$$

$$E^a_{ex}(k) = E_A + J(k) - J'(k).$$

(7)

For identical lattices, inside the same lattice we have

$$J(k) = J^{11}(k) = J^{22}(k),$$

(8)

and among different lattices we have

$$J'(k) = J^{12}(k) = J^{21}(k).$$

(9)

Our main task now is to calculate the two exciton dynamical matrices, $J(k)$ and $J'(k)$.

2.1. Exciton dispersion

The resonant dipole–dipole interaction between two atoms that are separated by a distance $R$, and of transition dipole $\mu$, is defined by

$$J(R) = \frac{1}{4\pi \varepsilon_0} \frac{|R|^2|\mu|^2 - 3(\mu \cdot R)^2}{|R|^5}.$$  

(10)

Such an interaction holds in the limit of $|R| < \lambda_A$, where $E_A = \hbar c/\lambda_A$; otherwise one needs to include radiative corrections. We neglect here the effect of the nanofiber materials on the dipole–dipole interaction. For the dipole moment we take the general case of $\mu = (\mu_x, \mu_y, \mu_z)$. The two optical lattices are parallel to the $x$-axis, one lattice with $z_1 = 0$ and sites at $(la, 0, 0)$ and the other lattice with $z_2 = d$ and sites at $(la, 0, d)$, where $l$ is an integer that runs over all the lattice sites. See figure 1.

In appendix A.1, we calculate the exciton dynamical matrix for interactions inside the same lattice, $J(k)$. In the limit of long-wavelength excitons, that is, $ka \ll 1$, we obtain

$$J(k) \approx \frac{\zeta(3)}{2\pi \varepsilon_0} \frac{\mu_x^2 + \mu_y^2 - 2\mu_z^2}{a^3}.$$  

(11)
Here the $y$ and $z$ components, which are normal to the lattice direction, are repulsive; and the $x$ component, which is parallel to the lattice direction, is attractive. Note that using the nearest-neighbor interaction in this limit gives $J = \frac{1}{2\pi \epsilon_0} \frac{\mu_x^2 \mu_z^2 - 2\mu_x^2}{a^2}$.

Appendix A.2 includes the calculation for the exciton dynamical matrix for interaction between the two optical lattices, $J'(k)$. The result is obtained for the limit of long-wavelength excitons, that is, $ka \ll 1$. We present here the results for the limit of large distance between the two lattices $kd \gg 1$, namely we have $d \gg a$. Even though the present experiment [23] is far from this limit, future experiments should achieve it. The result is given by

$$J'(k) \approx \frac{\sqrt{2\pi} (kd)^{3/2}}{ad^2} \frac{\mu_x^2}{4\pi \epsilon_0} e^{-kd} \left\{ \mu_x^2 - \mu_z^2 + \frac{4\mu_x^2}{3kd} \right\}. \tag{12}$$

Here the $x$ and $y$ terms are repulsive, as they are parallel for the different lattices, but as the $x$ component is parallel to the lattice direction and the $y$ component is normal to it, the contribution of the $x$ component is much larger than the $y$ one. The $z$ term is attractive; even though these components are normal to the lattice direction, they are parallel for different lattices. Approximately the last term is much smaller than the first two (or we can assume $\mu_y = 0$), so we obtain

$$J'(ka \to 0, kd \to \infty) \approx \frac{\sqrt{2\pi} (kd)^{3/2}}{ad^2} \frac{\mu_x^2}{4\pi \epsilon_0} e^{-kd} \left\{ \mu_x^2 - \mu_z^2 \right\}. \tag{13}$$

The main result of the present derivation is that the interaction decays exponentially with the distance between the two lattices. In place of the inverse cube dependence of the interaction between two dipoles, the collective effect in each lattice gives the coupling between two excitons that decays exponentially with the distance.

The exciton dispersions ($\nu = a, s$), for $\mu_y = 0$, are

$$E_{ex}^a(k) \approx E_A + \frac{\zeta(3)}{2\pi \epsilon_0 a^3} \left\{ \mu_x^2 - 2\mu_x^2 \right\} + \frac{\sqrt{2\pi} (kd)^{3/2}}{4\pi \epsilon_0} e^{-kd} \left\{ \mu_x^2 - \mu_z^2 \right\}. \tag{14}$$

For the case of a dipole in the $(x-z)$ plane, with $\mu = (\mu_x, 0, \mu_z) = \mu (\cos \theta, 0, \sin \theta)$, where $\theta$ is the angle between the dipole and the lattice direction, as seen in figure 1, we obtain

$$E_{ex}^s(k, \theta) \approx E_A + \frac{\xi(3) \mu_x^2}{2\pi \epsilon_0 a^3} \left\{ 1 - 3 \cos^2 \theta \right\} + \frac{\sqrt{2\pi} (kd)^{3/2}}{4\pi \epsilon_0} e^{-kd} \mu^2 \left\{ 2 \cos^2 \theta - 1 \right\}. \tag{15}$$

The intralattice exciton dynamical matrix $J(k)$ is much larger than the interlattice one $J'(k)$, in the present limit. The splitting between the two exciton dispersions is

$$\Delta(k, \theta) = 2 \frac{\sqrt{2\pi} (kd)^{3/2}}{4\pi \epsilon_0} e^{-kd} \mu^2 \left\{ 2 \cos^2 \theta - 1 \right\}, \tag{16}$$

which is the coupling parameter for the coherent oscillation of the exciton between the two lattices. Such oscillation is possible only if the splitting is larger than the exciton damping rate. The symmetric–antisymmetric splitting vanishes at $\theta = 45^\circ$, where we obtain the energy $E(\theta = 45^\circ) \approx E_A - \frac{\xi(3) \mu_x^2}{4\pi \epsilon_0 a^3}$. At $\theta \approx 54.7^\circ$ the on-lattice term vanishes, where $1 - 3 \cos^2 \theta = 0$, and we obtain

$$E_{ex}^a(k, \theta \approx 54.7^\circ) \approx E_A + \frac{\sqrt{2\pi} (kd)^{3/2} \mu_x^2}{3ad^2} e^{-kd}. \tag{17}$$
2.2. Exciton damping rate

Spontaneous emission of atoms near a nanofiber is widely studied, for a single atom [19], two atoms [20] and a chain of independent atoms [21]. In the next section, we consider only the strong coupling of excitons to resonant nanofiber photons, and we neglect the weak coupling to off-resonant nanofiber photons. Therefore, in spite of the existence of the nanofiber, we consider here the damping rate of excitons only into free space.

For a single 1D optical lattice with one atom per site, we calculated in [30] the radiative damping rate into free space. The exciton damping rate is given by

\[ \Gamma_{ex}(k, \theta) = \frac{\mu^2 E_{ex}^2(k)}{4 \epsilon_0 \hbar \omega_{ex}(k)} \left\{ 1 + \cos^2 \theta - \frac{\hbar k c}{E_{ex}^2(k)} \left( 2 \cos^2 \theta - \sin^2 \theta \right) \right\}, \]

which is a function of \( k \) and \( \theta \). Here the exciton energy is \( E_{ex}(k) = E_A + J(k) \), which is the same as the above result but neglecting \( J'(k) \). This result needs to be compared with the damping rate of a single atom, which is \( \Gamma_A = \frac{\mu^2 E_A^3}{3 \epsilon_0 \hbar c^3} \). As we have discussed in more detail in [30], the exciton damping rate is found to be much larger than \( \Gamma_A \) for small wave numbers, that is, \( ka \ll 1 \), and the excitons can be considered as superradiant states. For larger wave numbers, the damping rate of excitons with polarizations orthogonal to the lattice increases, while that for polarizations parallel to the lattice decreases and becomes zero at a critical wave number. As the spontaneously emitted photon has a wave vector component parallel to the lattice equal to the exciton wave number, the critical wave number is obtained from the energy conservation condition \( E_{ex}^2(k_c) = \hbar k_c c \). Beyond this critical wave number, excitons become metastable with zero radiative damping rate, and they can propagate in the lattice without radiative decay.

In the present system we have two optical lattices, and we showed above that their mutual coupling gives rise to symmetric and antisymmetric excitons. Direct calculations yield that the antisymmetric excitons are dark with zero damping rate, that is, \( \Gamma_{ex}^a = 0 \), whereas the symmetric excitons are bright with twice the single exciton damping rate, that is, \( \Gamma_{ex}^s = 2 \Gamma_{ex} \). Note that according to the previous discussion, the symmetric exciton damping rate also becomes zero beyond the critical wave number \( k_c \). The dark excitons cannot be excited directly through the coupling to the fiber photons and need other methods to excite them. As we explain in the next section, the symmetric excitons can be excited directly through the electric dipole interaction with the fiber photons, where this coupling appears also for excitons with wave number beyond the critical wave number. This regime is useful for long-range transmission through the system, and the only source of damping will be the fiber photon decay.

3. Fiber photon–exciton interactions

Now we treat the fiber photons and their coupling to the two optical lattices. Rigorous treatment of tapered nanofiber modes is given in [14, 31], but here we use a simplified picture. The 1D optical fiber modes are given by the Hamiltonian

\[ H_{ph} = \sum_k \hbar \omega_{ph}(k) \mathbf{a}_k^\dagger \mathbf{a}_k, \]
where $a_k^\dagger$ and $a_k$ are the fiber photon creation and annihilation operators with wave number $k$, respectively. The photon dispersion is taken to be of the simplified form

$$\omega_{ph}(k) = \frac{c}{\sqrt{\epsilon}} \sqrt{k_0^2 + k^2},$$

(20)

where the wave number $k_0$ results from the fiber transverse confinement, and $\epsilon$ is the fiber average dielectric constant. The electric field operator outside the fiber is

$$\hat{E}(r) = i \sum_k \sqrt{\hbar \omega_{ph}(k) \over 2\epsilon_0 V} \mathbf{e} u(r) [a_k e^{ikz} - a_k^\dagger e^{-ikz}],$$

(21)

where $\mathbf{e}$ is the photon linear polarization unit vector, $V$ is the normalization volume and $u(r)$ is the mode function, which includes the fiber field complexity. The fiber photons can decay into free space with a given damping rate fixed by the nanofiber quality.

The atomic transition dipole operator is

$$\hat{\mu} = \mu \sum_{n,\alpha} (B_{n\alpha} + B_{n\alpha}^\dagger).$$

The matter–light coupling is given by the electric dipole interaction

$$H_{in} = -i \sum_{k,l,n,\alpha} \frac{\hbar \omega_{ph}(k) \mu^2}{2\epsilon_0 VN} \{a_k B_{n\alpha}^\dagger e^{i(k-l)x_n} - a_k^\dagger B_{n\alpha} e^{-i(k-l)x_n}\},$$

(22)

where by using the inverse transformation of equation (2), we have

$$H_{in} = -i \sum_{k,\alpha} \frac{\hbar \omega_{ph}(k) \mu^2}{2\epsilon_0 VN} \{a_k B_{k\alpha}^\dagger - a_k^\dagger B_{k\alpha}\}. $$

(23)

Using $\frac{1}{N} \sum_n e^{i(k-l)x_n} = \delta_{kl}$, we obtain

$$H_{in} = -i \sum_{k,\alpha} \frac{\hbar \omega_{ph}(k) \mu^2 N}{\epsilon_0 VN} \{a_k B_{k\alpha}^\dagger - a_k^\dagger B_{k\alpha}\}. $$

(24)

We represent the exciton operators in terms of symmetric and antisymmetric operators by using the inverse of the transformation (5), to obtain

$$H_{in} = -i \sum_{k} \frac{\hbar \omega_{ph}(k) \mu^2 N}{\epsilon_0 V} \{a_k^\dagger B_{ks} - a_k B_{ks}^\dagger\}. $$

(25)

It is clear that only the symmetric excitons are coupled to the fiber photons, whereas the antisymmetric ones are dark and decouple from the photons.

Defining the coupling parameter

$$\hbar f_k = -iu(b) \frac{\hbar \omega_{ph}(k) \mu^2}{\epsilon_0 S a},$$

(26)

where $S$ is the mode cross area, we can write

$$H_{in} = \sum_k [\hbar f_k a_k B_{ks}^\dagger + \hbar f_k^\dagger a_k^\dagger B_{ks}]. $$

(27)

The photon dispersion and the exciton–photon coupling are taken to be $\theta$ independent.
4. Fiber polaritons

The total Hamiltonian is given by

\[ H = \sum_k \hbar \left( \omega_{\text{ex}}^e(k, \theta) B_{ka}^\dagger B_{ka} + \omega_{\text{ex}}^s(k, \theta) B_{ka}^\dagger B_{ks} + \omega_{\text{ph}}(k) a_k^\dagger a_k + f_k a_k B_{ks}^\dagger + f_k^* a_k^\dagger B_{ks} \right), \]  

where, due to translational symmetry along the lattice and the fiber axis, the Hamiltonian is separated for each \( k \).

In the strong coupling regime, where the coupling is larger than both the exciton and photon linewidths, we define polaritons by diagonalizing the Hamiltonian \cite{26, 29}, to obtain the total Hamiltonian as

\[ H = \sum_k \hbar \omega_{\text{pol}}^e(k, \theta) A_k^\dagger A_k + \sum_k \hbar \omega_{\text{ex}}^e(k, \theta) B_{ka}^\dagger B_{ka}, \]  

with the polariton dispersions

\[ \omega_{\text{pol}}^\pm(k, \theta) = \frac{\omega_{\text{ph}}(k) + \omega_{\text{ex}}^e(k, \theta)}{2} \pm \Delta(k, \theta), \]  

where

\[ \Delta(k, \theta) = \sqrt{\delta^2(k, \theta) + |f_k|^2}, \]  

with the detuning

\[ \delta(k, \theta) = \frac{\omega_{\text{ph}}(k) - \omega_{\text{ex}}^e(k, \theta)}{2}. \]  

The polariton operators are a coherent superposition of symmetric excitons and photons, where

\[ A_k^\pm = X^\pm(k, \theta) B_{ka} + Y^\pm(k, \theta) a_k, \]  

and where the amplitudes are given by

\[ X^\pm(k, \theta) = \pm \sqrt{\frac{\Delta(k, \theta) \mp \delta(k, \theta)}{2\Delta(k, \theta)}}, \]

\[ Y^\pm(k, \theta) = \frac{f_k}{\sqrt{2\Delta(k, \theta)\Delta(k, \theta) \mp \delta(k, \theta)}}. \]

Here we present the results for a system with the following parameters: the lattice constant is \( a = 1000 \text{Å} \), the transition energy is \( E_A = 1 \text{eV} \), and the transition dipole is \( \mu = 1 \text{eÅ} \) of angle \( \theta = 90^\circ \) with the lattice axis. The distance between the two optical lattices is taken to be \( d = 10a \). The fiber dielectric constant is \( \epsilon = 3 \), with the mode cross area of \( S = 4\pi a^2 \), and the mode function at the lattice position is taken to be \( u(b) = 0.1 \). The cavity mode energy at \( k = 0 \) is taken to be in resonance with the free atom transition energy, that is, \( E_{\text{ph}}(k = 0) = E_A \), then \( k_0 = E_A \sqrt{\epsilon}/hc \). The polariton dispersions are plotted in figure 2 relative to the free atom transition, and the symmetric exciton and photon weights are plotted in figure 3. The exciton–photon intersection point is taken here to be at \( k = 0 \), where the polaritons split by the Rabi splitting, and they are half exciton and half photon. For large \( k \), the upper branch becomes photonic, and the lower one becomes excitonic.
Figure 2. The full lines are for the upper and lower polariton angular frequencies relative to the free atom transition, \( \omega_{\text{pol}} - \omega_A \) versus wave number \( k \). The dashed line is for the symmetric exciton dispersion and the dashed parabola is for the fiber photon dispersion.

Figure 3. The excitonic and photonic weights versus \( k \). For the lower polariton branch the dashed line is for the excitonic weight and the full line for the photonic weight. For the upper polariton branch the dashed line is for the photonic weight and the full line for the excitonic weight.

5. Fiber transmission spectrum

To obtain the linear optical spectra [26], we consider an incident field from the far left side of the fiber, and we calculate the transmission \( T(k, \theta) \) and reflection \( R(k, \theta) \) spectra, with the absorption spectrum \( A(k, \theta) \). The coupling of the fiber field to the external field at the two far edges of the fiber is included in the parameter \( \gamma \), and the fiber photon damping rate is taken to be \( \Gamma_{\text{ph}} \), which is assumed to be \( k \) independent. The symmetric exciton damping rate is included here phenomenologically, and is taken to be \( \Gamma_{\text{ex}}^\pm(k, \theta) \) as discussed in section 2.1. The polariton damping rate is taken to be

\[
\Gamma_{\text{pol}}^\pm(k, \theta) = \Gamma_{\text{ex}}^\pm(k, \theta) \left| X^\pm(k, \theta) \right|^2 + \Gamma_{\text{ph}} \left| Y^\pm(k, \theta) \right|^2.
\] (35)
The linear optical spectra are calculated and given by

\[ R(\omega, k, \theta) = \frac{1}{|1 + i\gamma \Lambda(\omega, k, \theta)|^2} \]  

and

\[ T(\omega, k, \theta) = \frac{\gamma^2 |\Lambda(\omega, k, \theta)|^2}{|1 + i\gamma \Lambda(\omega, k, \theta)|^2}, \]

with

\[ R(\omega, k, \theta) + T(\omega, k, \theta) + A(\omega, k, \theta) = 1, \]

where

\[ \Lambda(\omega, k, \theta) = \sum_r \frac{|Y_r(k, \theta)|^2}{\omega - \omega_{\text{pol}}^r(k, \theta) - i\Gamma_{\text{pol}}^r(k, \theta)}. \]

In the linear spectra plots we use for the photon a small damping rate of \( \hbar \gamma_{\text{ph}} = 10^{-10} \text{ eV} \), and the fiber edge coupling parameter is taken to have a bandwidth of \( \hbar \gamma = 10^{-6} \text{ eV} \). In figure 4, we plot the transmission spectra for a wave number of \( k = 10^{-6} \text{ Å}^{-1} \), where we get two transmission peaks corresponding to the two polariton branches, which are separated by the Rabi splitting. Now we take an incident field with a wide bandwidth of \( \hbar \gamma = 10^{-4} \text{ eV} \). In figure 5, we plot the transmission spectra for a wave number of \( k = 10^{-6} \text{ Å}^{-1} \), where the band gap between the two peaks becomes a dip in the transmission spectrum. Figure 6 is a close look at the minimum that shows a small shift relative to the free atom transition and that results from the dipole–dipole interactions. Here the symmetric exciton linewidth is \( \hbar \Gamma_{\text{ex}} = 2.32 \times 10^{-8} \text{ eV} \), which is larger than the free atom linewidth of \( \hbar \Gamma_A = 2.5 \times 10^{-9} \text{ eV} \). In figure 7 we plot the transmission spectra for a wave number of \( k = 10^{-5} \text{ Å}^{-1} \) and in figures 8 and 9 for \( k = 5 \times 10^{-5} \text{ Å}^{-1} \); in figure 8 the plot is around the upper polariton branch, and in figure 9, it is around the lower polariton branch, which is also seen in figure 8 as a small peak around zero. It is clear that for large wave numbers at the lower branch we get small transmission, whereas for large \( k \) the excitons and photons with the same wave number have large detuning, namely they are off resonance. At the upper branch we get large transmission. As we mentioned previously, the upper branch becomes photonic and the lower one excitonic.
Figure 5. The transmission spectrum for $k = 10^{-6} \text{Å}^{-1}$ and $\hbar\gamma = 10^{-4} \text{eV}$.

Figure 6. The transmission spectrum for $k = 10^{-6} \text{Å}^{-1}$ and $\hbar\gamma = 10^{-4} \text{eV}$. A close look at the minimum.

Figure 7. The transmission spectrum for $k = 10^{-5} \text{Å}^{-1}$.

6. Summary

We studied a recently demonstrated integrated hybrid atom–photon quantum system based on a tapered nanofiber and cold atoms. The nanofiber is used to implement two arrays of nanotraps for the atoms using off-resonant photons as well as to spatially confine resonant photons.
efficient in ensuring strong single photon coupling to the two atomic ensembles. Interestingly, the effective coupling parameter of excitons in the two chains decays exponentially with their distance but it is still sufficient to split the energies of symmetric and antisymmetric excitons. Only the symmetric excitons are bright and coherently mix with the nanofiber photons to form fiber polaritons, which can be seen in the transmission spectrum of the nanofiber.

The present system allows one to investigate and deeply understand 1D collective electronic excitations (excitons), where it is possible to excite, store and read out information in the form of coherent electronic excitations. As was shown in our previous work [30], the damping rate of small-wave-number excitons in a 1D lattice is much larger than a single atom excitation and decays fast into free space. But beyond a critical wave number it is shown that 1D excitons become metastable with infinite radiative lifetime. In this regime, excitons cannot be excited by free space photons, but can be easily excited by coupling to fiber photons. These excitons can serve as information storage devices and for communication through energy transfer. In this paper, the discussion was limited to the linear regime. Nonlinear processes induced by electronic excitation saturation effects give rise to more interesting physics, which is our next task. At this point, we have neglected the interaction of the atoms via light forces induced by multiple scattering of photons [32], which can be significantly enhanced through
transverse mode confinement and the fact that the atoms form Bragg-like structures. This could lead to collective oscillations and instabilities of the lattices but can also be used to engineer long-distance motional couplings between ensembles.

This paper considered the ideal case of exactly one atom per site. In the real experiment [23] typical occupancy of at most one atom per site can be obtained, which gives an average occupancy of about half. Future experiments will approach our model in using a deeper chain of nanotraps. However, the exciton picture still holds for the present experiment, where different effects are expected due to scattering and localization of excitons, which leave space for more such studies in future.

Acknowledgments

The work was supported by the Austrian Science Funds (FWF) through the project P21101 and by the Eranet NanoSci-E+ project NOI (I269-N16).

Appendix. Exciton dynamical matrix calculations

In this appendix, we calculate the exciton dynamical matrix for the two cases of the intralattice interactions, \( J(k) \), and the interactions among interlattices, \( J'(k) \).

A.1. Intralattice interactions

For interactions inside the same lattice, from equation (10), we have the resonant dipole–dipole interaction

\[
J(l) = \frac{1}{4\pi \epsilon_0} \frac{\mu_y^2 + \mu_z^2 - 2\mu_x^2}{a^3 l^3},
\]

and in momentum space it reads

\[
J(k) = \frac{1}{4\pi \epsilon_0} \frac{\mu_y^2 + \mu_z^2 - 2\mu_x^2}{a^3} \sum_{l=-\infty}^{\infty} \frac{e^{ikal}}{l^3}
\]

\[
= \frac{1}{4\pi \epsilon_0} \frac{\mu_y^2 + \mu_z^2 - 2\mu_x^2}{a^3} \sum_{l=1}^{\infty} \frac{2 \cos (kal)}{l^3}.
\]

The prime indicates that \( l \) runs from \(-\infty\) to \(+\infty\) but excluding \( l = 0 \), which is for self-interactions. In the limit of long-wavelength excitons, or small wave number, that is, \( ka \ll 1 \), we obtain

\[
J(ka \rightarrow 0) \simeq \frac{1}{4\pi \epsilon_0} \frac{\mu_y^2 + \mu_z^2 - 2\mu_x^2}{a^3} \sum_{l=1}^{\infty} \frac{2}{l^3}
\]

\[
= \frac{1}{2\pi \epsilon_0} \zeta(3) \frac{\mu_y^2 + \mu_z^2 - 2\mu_x^2}{a^3},
\]

where the Riemann zeta function gives \( \zeta(3) \approx 1.202 \).
A.2. Interlattice interactions

In order to calculate the exciton dynamical matrix for interactions between the two lattices, we use Ewald’s method [33], which is applied here for excitons in 1D optical lattices.

The resonant dipole–dipole interaction reads as

$$J'(l) = \sum_{ij} \frac{\mu_i \mu_j}{4\pi \epsilon_0} D_{ij}(l),$$  \hspace{1cm} (A.4)

where $i, j = x, y, z$. The calculations give

$$D_{xx}(l) = \frac{1}{(a^2l^2 + d^2)^{5/2}} - \frac{3a^2l^2}{(a^2l^2 + d^2)^{5/2}},$$

$$D_{xy}(l) = \frac{1}{(a^2l^2 + d^2)^{3/2}},$$

$$D_{xz}(l) = \frac{1}{(a^2l^2 + d^2)^{3/2}} - \frac{3d^2}{(a^2l^2 + d^2)^{5/2}},$$

$$D_{yx}(l) = D_{xy}(l) = -\frac{3adl}{(a^2l^2 + d^2)^{5/2}},$$

$$D_{zy}(l) = D_{yz}(l) = D_{zx}(l) = D_{zy}(l) = 0.$$  \hspace{1cm} (A.5)

We need to calculate the interactions in momentum space

$$J'(k) = \sum_{ij} \frac{\mu_i \mu_j}{4\pi \epsilon_0} D_{ij}(k),$$  \hspace{1cm} (A.6)

where

$$D_{ij}(k) = \sum_{l=-\infty}^{+\infty} e^{ikl} D_{ij}(l).$$  \hspace{1cm} (A.7)

We define the function

$$S(k) = \sum_{l=-\infty}^{+\infty} \frac{e^{ikl}}{(a^2l^2 + d^2)^{3/2}},$$  \hspace{1cm} (A.8)

and then the matrix elements of the exciton dynamical matrix can be derived from it by

$$D_{xx}(k) = \left( 2 \frac{\partial^2}{\partial k^2} + d^2 \right) S(k),$$

$$D_{xy}(k) = \left( \frac{\partial^2}{\partial k^2} + d^2 \right) S(k),$$

$$D_{xz}(k) = \left( \frac{\partial^2}{\partial k^2} - 2d^2 \right) S(k),$$

$$D_{zx}(k) = D^*_{zx}(k) = i3d \frac{\partial}{\partial k} S(k).$$  \hspace{1cm} (A.9)
The function $S(k)$ can be written in the form of an integral with exponential decay, in place of the $1/R^5$ dependence, in using the identity

$$\frac{4}{3\sqrt{\pi}} \int_0^\infty dt \ t^{3/2} e^{-\alpha t} = \frac{1}{\alpha^{5/2}}. \quad (A.10)$$

Then we arrive at

$$S(k) = \frac{4}{3\sqrt{\pi}} \int_0^\infty dt \ t^{3/2} e^{-d^2 t} \sum_{l=-\infty}^{+\infty} e^{-\sigma^2 t + ika l}. \quad (A.11)$$

In using the relation

$$\sum_{l=-\infty}^{+\infty} e^{-\sigma^2 t + ika l} = \frac{\sqrt{\pi}}{a^{5/2}} \sum_{n=-\infty}^{+\infty} e^{-(1/a^2)(n\pi + (ka/2))^2} \quad (A.12)$$

we convert one summation into another. The first includes oscillations and then decays slowly for a large summation index $l$, while the second decays faster with increasing summation index $n$. Such a change in the series is found to be useful for long wavelength excitons, as we show in the following. In doing so, we obtain

$$S(k) = \frac{4}{3a} \sum_{n=-\infty}^{+\infty} \int_0^\infty dt \ t e^{-d^2 t} e^{-(1/a^2)(n\pi + (ka/2))^2}. \quad (A.13)$$

The integral gives

$$\int_0^\infty dt \ t e^{-(\beta/t) - \gamma t} = \frac{\beta}{\gamma} K_2(2\sqrt{\beta\gamma}), \quad (A.14)$$

where $K_n(x)$ is the modified Bessel function of the second kind of order $n$. Finally, we have

$$S(k) = \frac{4}{3a} \sum_{n=-\infty}^{+\infty} \frac{2(n\pi + (ka/2))^2}{a^2 d^2} K_2 \left( \frac{2d(n\pi + (ka/2))}{a} \right). \quad (A.15)$$

We are interested in the limit of long-wavelength excitons, that is, $ka \ll 1$. In this limit, the dominant contribution comes from the term with $n = 0$, and the other terms decay faster; hence, we obtain

$$S(ka \to 0) \approx \frac{2}{3} \frac{k^2}{ad^2} K_2(kd). \quad (A.16)$$

We need to use the relation

$$\frac{\partial}{\partial x} K_n(x) = -\frac{1}{2} [K_{n-1}(x) + K_{n+1}(x)]. \quad (A.17)$$

Here we use the second main approximation, which is for large $d$ where $kd \gg 1$, and as $ka \ll 1$, we are in the limit of $d \gg a$. For the function $K_n(kd)$ we use the asymptotic expansion

$$K_n(kd \to \infty) \approx \sqrt{\frac{\pi}{2kd}} e^{-kd}, \quad (A.18)$$
which has the same behavior for different $n$. In this limit, we obtain the matrix elements

\[ D_{xx}(ka \to 0, kd \to \infty) \approx \frac{\sqrt{2\pi}}{ad^2} (kd)^{3/2} e^{-kd}, \]

\[ D_{yy}(ka \to 0, kd \to \infty) \approx \frac{4 \sqrt{2\pi}}{3 ad^2} (kd)^{1/2} e^{-kd}, \]  

\[ D_{zz}(ka \to 0, kd \to \infty) \approx -\frac{\sqrt{2\pi}}{ad^2} (kd)^{3/2} e^{-kd}, \]

\[ D_{xz}(ka \to 0, kd \to \infty) \approx -\frac{2i \sqrt{2\pi}}{ad^2} (kd)^{3/2} e^{-kd}. \]  

(A.19)

Finally, we obtain

\[ J'(ka \to 0, kd \to \infty) \approx \frac{\sqrt{2\pi}}{4\pi \epsilon_0} (kd)^{3/2} e^{-kd} \left\{ \mu_x^2 - \frac{2}{3} \mu_z^2 + \frac{4}{3} \mu_y^2 \right\}. \]  

(A.20)

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