Mass number and isospin dependence of symmetry energy coefficients for finite nuclei

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Abstract

Mass number and isospin dependence of symmetry energy coefficients for isobaric chains with $A = 8, 12, 16, 20, \cdots, 100$ have been globally studied in relativistic mean-field (RMF) theory with the PK1 effective interaction and compared with the corresponding pseudo data from the experimental binding energy and semi-empirical mass formula. The mass dependence of symmetry energy coefficient both from the RMF calculation and pseudo data for different isospin values has been fitted with and without the surface term $b_v/A^{4/3}$. The results confirm that only if with the surface term in the fitting of symmetry energy coefficient in finite nuclei can one get the consistent value as that in infinite nuclear matter.

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I. INTRODUCTION

In the past decades, many exotic nuclei far from the valley of \( \beta \)-stability line with extreme \( N/Z \) ratio have been produced with the new generation of radioactive-ion beam facilities [1]. These exotic nuclei provide us useful information on the equation of state (EOS) of asymmetric nuclear matter, which has not been well determined and is important for understanding both the structure of unstable nuclei and the properties of neutron stars. One of the most important quantities is the nuclear symmetry energy, which affects significantly the binding energy and rms radii of neutron-rich nuclei [2]. Furthermore, the chemical composition, the evolution of lepton profiles, cooling process and the neutrino fluxes in neutron stars depend strongly on the nuclear symmetry energy [3, 4]. Therefore, more and more attention has been paid to study the symmetry energy in both infinite nuclear matter and finite nuclei [2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

In infinite asymmetric nuclear matter, the energy density is usually expanded in terms of isospin asymmetry \( \beta = (\rho_n - \rho_p)/\rho \) as \( E(\rho, \beta) = E(\rho, \beta = 0) + a'_\text{sym}(\rho) \beta^2 + O(\beta^4) \), where the nuclear symmetry energy coefficient is given by \( a'_\text{sym}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \beta)}{\partial \beta^2} \bigg|_{\beta=0} \), and \( E(\rho, \beta = 0) \) is the energy density in symmetric nuclear matter. Many investigations have already been carried out to study the density and isospin dependence of symmetry energy coefficient \( a'_\text{sym} \) [12].

In finite nuclei, nuclear symmetry energy has been studied in the finite-range droplet model (FRDM) [5], Random Phase Approximation (RPA) based on the Hartree-Bogoliubov approach [13], and energy density functionals of Skyrme force [9] as well as relativistic meson-exchange force [10]. It has been found that the nuclear symmetry energy coefficient \( a_{\text{sym}} \) extracted from \( E_{\text{sym}} = a_{\text{sym}} T(T + 1) \) is in better agreement with the empirical data than that from \( E_{\text{sym}} = a_{\text{sym}} T^2 \).

In this paper, the mass number and isospin dependence of symmetry energy coefficients for isobaric chains with \( A = 8, 12, 16, 20, \ldots, 100 \) will be globally studied in a covariant density function theory and compared with the corresponding pseudo data from the experimental binding energy and semi-empirical mass formula. The paper is organized as follows. In Sec. II we briefly describe the method used to obtain the nuclear symmetry coefficient in finite nuclei. The results and discussion will be given in Sec. III. Finally, a short summary is presented in Sec. IV.
II. THE METHOD

The conventional semi-empirical mass formula of Bethe and Weizsäcker in liquid droplet model (LDM) for the binding energy of a nucleus is as follows \[14\],

\[ B(N, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_I \frac{(N - Z)^2}{A} + a_p \frac{\delta}{A^{1/2}}, \]  \hspace{1cm} (1)

with \( N \) and \( Z \) the number of neutrons and protons, \( A \) the total number of nucleons, \( A = N + Z \). \( a_v, a_s, a_c, \) and \( a_I \) are positive and represent the coefficients of bulk or volume, surface, Coulomb, and symmetry energies respectively, and \( a_p \) is a simple parametrization of pairing with \( \delta = \frac{(-1)^N + (-1)^Z}{2} \).

The research of interest in this work is the symmetry energy term in Eq. (1), which can be rewritten in terms of the isospin value \( T \),

\[ E_{\text{sym}}(A, T) = a_I \frac{(N - Z)^2}{A} \equiv a_{\text{sym}}(A, T)T^2, \]  \hspace{1cm} (2)

where the isospin value of a nucleus is given by \( T = |T_z| = |N - Z|/2 \). However, recent studies \[5, 9, 10, 13, 15\] showed that it is more proper to parameterize the nuclear symmetry energy into following form,

\[ E_{\text{sym}}(A, T) = a_{\text{sym}}(A, T)T(T + 1), \]  \hspace{1cm} (3)

which will be adopted to extract the symmetry energy coefficient subsequently.

On the other hand, according to the LDM formula \[11\], one notices that the difference in binding energies of isobaric nuclei with same odd-even parity is only related to the Coulomb energy and symmetry energy terms,

\[ B(A, T) - B(A, T = 0) = \left[ E_{\text{Coul.}}(A, T) - E_{\text{Coul.}}(A, T = 0) \right] + \left[ E_{\text{sym}}(A, T) - E_{\text{sym}}(A, T = 0) \right]. \]  \hspace{1cm} (4)

Therefore, the pseudo experimental value of symmetry energy in Eq. (3) is determined by subtracting the Coulomb energy from the experimental data of nuclear total binding energy.
as,

\[ E_{\text{sym}}(A, T) = [B(A, T) - B(A, T = 0)] - [E_{\text{Coul}}(A, T) - E_{\text{Coul}}(A, T = 0)], \]  

(5)

where the Coulomb energy \( E_{\text{Coul}}(A, T) \) of a specific nucleus is calculated by assuming the nucleus as an uniformly charged spherical droplet, i.e.,

\[ E_{\text{Coul}}(A, T) = \frac{3}{5} Z^2 e^2 \frac{1}{R_c} [1 - \frac{0.7636}{Z^{2/3}}]. \]  

(6)

The charge radius is given by \( R_c = 1.2A^{1/3} \). The second term is from the Fock-exchange term of proton. Even if there is quadrupole deformation \( \beta \) (typically about 0.1 – 0.5) deviating from sphere in a nucleus, it just reduces the Coulomb energy by a factor of \( \beta^2/4\pi \), which is negligible. Therefore, it is justified to consider the nucleus as a spherical liquid drop rather than a deformed one. Finally, one obtains the pseudo experimental value of symmetry energy coefficient from Eqs. (3) and (5), which depends on the mass number \( A \) and isospin value \( T \).

The theoretical study of symmetry energy coefficients for finite nuclei is based on a covariant density functional theory with nucleon-nucleon interacting by exchange of various mesons and photon for nuclear ground-state energies. The non-linear version of relativistic mean-field theory is used. The corresponding effective Lagrangian density are composed of the free parts of nucleon field \( (\psi) \), two isoscalar meson fields \( (\sigma \text{ and } \omega_\mu) \), isovector meson field \( (\vec{\rho}_\mu) \), photon field \( (A_\mu) \) as well as their couplings \[16, 17, 18\],

\[ \mathcal{L} = \bar{\psi} [i\gamma^\mu \partial_\mu - m - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \tau \cdot \vec{\rho}_\mu - \frac{1}{2} e (1 - \tau_3) \gamma^\mu A_\mu] \psi \\
- \frac{1}{2} \epsilon(1 - \tau_3) \gamma^\mu A_\mu \\
- \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - U_\sigma(\sigma) - \frac{1}{4} \Omega_{\mu\nu} \Omega_{\mu\nu} + U_\omega(\omega_\mu) \\
- \frac{1}{4} \vec{R}^{\mu\nu} \cdot \vec{R}_{\mu\nu} + U_\rho(\vec{\rho}_\mu) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \]  

(7)

where \( m \) and \( m_i(g_i) \) \( (i = \sigma, \omega_\mu, \vec{\rho}_\mu) \) are masses (coupling constants) of the nucleon and the mesons respectively and

\[ \Omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu, \]  

(8a)

\[ \vec{R}^{\mu\nu} = \partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu, \]  

(8b)

\[ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \]  

(8c)
are field tensors of the vector mesons and the photon field. We adopt the arrows to indicate vectors in isospin space and bold types for the space vectors. Greek indices $\mu$ and $\nu$ run over 0, 1, 2, 3, while Roman indices $i$, $j$, etc. denote the spatial components.

The nonlinear self-coupling terms $U_\sigma(\sigma)$, $U_\omega(\omega_\mu)$, and $U_\rho(\vec{\rho}_\mu)$ for the $\sigma$-meson, $\omega$-meson, and $\rho$-meson in the Lagrangian density (7) respectively have the following forms:

$$U_\sigma(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4,$$

$$U_\omega(\omega_\mu) = \frac{1}{2} m_\omega \omega^\mu \omega_\mu + \frac{1}{4} c_3 (\omega^\mu \omega_\mu)^2,$$

$$U_\rho(\vec{\rho}_\mu) = \frac{1}{2} m_\rho \vec{\rho}^\mu \cdot \vec{\rho}_\mu.$$  

From Eq.(7) combined with the mean-field approximation, one obtains the corresponding energy density functional $E_{\text{RMF}} = \langle \Phi | \hat{H} | \Phi \rangle$, where $\hat{H}$ is the corresponding Hamiltonian operator. The equations of motion for the nucleon and the mesons can be obtained by requiring that the energy functional be stationary with respect to the variations of single-particle density $\rho$ and meson fields. Finally, one obtains a Dirac equation for nucleon as well as inhomogeneous Klein-Gordon equations for meson fields and photon field. The source terms for Klein-Gordon equations are sums of bilinear products of Dirac spinors restricted in the Fermi sea.

With the restriction of spherical symmetry, both the Dirac equation for the nucleon and the Klein-Gordon equations for the mesons and the photon become radially dependent only. These coupled equations are solved iteratively using the shooting method combined with Runge-Kutta algorithms and employing appropriate boundary conditions in a spherical box of radius $R = 20$ fm with a step size of 0.1 fm. More details about the numerical techniques can be found in Ref. [18].

Moreover, the proper treatment of center of mass (c.m.) motion has been found very important in the explanation of Coulomb energy displacement in mirror nuclei [19]. We adopt the same c.m. correction to the total energy after variation, as it has been used in adjusting the parameter set PK1 [20],

$$E_{\text{cm}}^\text{mic} = -\frac{1}{2mA} \langle \hat{P}_{\text{cm}}^2 \rangle,$$

where $m$ is the mass of neutron or proton, $\hat{P}_{\text{cm}} = \sum_i^A \hat{P}_i$ is the total momentum in the c.m. frame. The total energy for the nuclear ground state is given by the summation of nuclear
part $E_{\text{RMF}}$ and c.m. correction,

$$E_{\text{tot}} = E_{\text{RMF}} + E_{\text{cm}}^{\text{mic}}.$$  \hspace{1cm} (11)

The nuclear symmetry energy for certain $A$ and $T$ is finally obtained as follows,

$$E_{\text{sym}}^{\text{theo}}(A,T) = E_{\text{tot}}(A,T = 0) - E_{\text{tot}}(A,T)$$  \hspace{1cm} (12)

where $E_{\text{tot}}(A,T = 0)$ is the total energy of $N = Z$ isobaric nucleus with mass number $A$. The theoretical value for the coefficient of symmetry energy $a_{\text{sym}}(A,T)$ is thus determined by $E_{\text{sym}}^{\text{theo}}(A,T)/T(T+1)$.

III. RESULTS AND DISCUSSION

In this section, we perform systematical calculations for $A = 8, 12, 16, 20, \cdots, 100$ isobars with the effective interaction PK1. The main interesting is focused on the coefficient of symmetry energy, which is only associated with symmetry energy. It is justified to switch off the Coulomb interaction in the particle-hole channel of nucleon-nucleon effective interaction in the effective Lagrangian density (7).

![Graph showing the coefficients of symmetry energy for nuclei with $T = 2, 4, \cdots, 12$ as a function of the corresponding mass number $A$. The filled symbols represent the pseudo experimental data and the open symbols are the RMF predictions.](image-url)

**FIG. 1:** The coefficients of symmetry energy for nuclei with $T = 2, 4, \cdots, 12$ as a function of the corresponding mass number $A$. The filled symbols represent the *pseudo* experimental data and the open symbols are the RMF predictions.
Figure 1 shows the predicted nuclear symmetry energy coefficients $a_{\text{sym}}$ in nuclei with $T = 2, 4, \cdots, 12$ as a function of the corresponding mass number $A$ in comparison with the corresponding pseudo data. It shows that good agreement in $a_{\text{sym}}$ has been achieved between RMF calculation and the pseudo data except the very light nuclei with $T = 2$. The observed strong oscillating in $a_{\text{sym}}$ of RMF calculation for small $T$ are associated with shell closures [10]. In contrary, there is no effect of shell structure in the LDM formula, which gives a moderate changing of $a_{\text{sym}}$ as a function of mass number $A$. Furthermore, for larger value of $T$, the symmetry energy becomes less sensitive to the shell structure and its value is stabilized. As the consequence, a better agreement between RMF calculation and the pseudo data is found for large isospin value $T$.

In the following, we parameterize the coefficient of symmetry energy into following two forms:

$$a_{\text{sym}} = \begin{cases} 
\frac{4a'_{\text{sym}}}{b_v A} & 
\frac{b_v A}{A - b_s A^{4/3}} 
\end{cases}$$

(13)

where $a'_{\text{sym}}$ in the first form corresponds to the standard definition of coefficient of symmetry energy in infinite nuclear matter, while $b_v$ and $b_s$ are the so-called volume and surface components in symmetry energy of finite nuclei and its physical origin is traditionally explained in terms of the kinetic energy and mean isovector potential contributions [21]. Duflo and Zuker [5] performed a global fit to nuclear masses and found $b_v = 134.4, b_s = 203.6$. Ban et al. [10], made a least-square fit to the calculated nuclear symmetry energy for the $A = 40, 48, 56, 88, 100, 120, 140, 160, 164, \text{and} 180$ isobaric chains and found $b_v = 133.2, b_s = 220.3$.

We carry out a global fit of the obtained coefficients of symmetry energy from pseudo data and RMF calculation via the Levenberg-Marquardt method. Table I presents the parameter $a'_{\text{sym}}$ in coefficients of symmetry energy $a_{\text{sym}}$ for isospin values of $T = 2, 4, 6,$ and 8 respectively. It shows that the parameters $a'_{\text{sym}}$ from both the pseudo data and RMF calculation are more than two times smaller than the infinite symmetric nuclear matter value 37.6 MeV [20] for PK1.

Table II presents the parameters $b_v$ and $b_s$ in coefficients of symmetry energy for isospin values of $T = 2, 4, 6,$ and 8 respectively. It shows that the parameters $b_v$ and $b_s$ are consistent with those given in Refs. [3, 10] and the coefficient of symmetry energy in infinite nuclear matter $a'_{\text{sym}} = b_v/4$ with $A$ going to infinity in Eq. (13). It confirms the conclusion that only
TABLE I: The parameter $a'_{\text{sym}} = 4a'_{\text{sym}}/A$ in coefficients of symmetry energy $a_{\text{sym}}$ for isospin values of $T = 2, 4, 6,$ and $8$ respectively.

| $T$ | Exp. | 2   | 4   | 6   | 8   |
|-----|------|-----|-----|-----|-----|
|     |      | 13.5| 16.8| 18.3| 19.2|
| Theory | 9.2  | 14.6| 16.0| 18.2|

TABLE II: The parameters $b_v$ and $b_s$ in coefficients of symmetry energy $a_{\text{sym}} = b_v/A - b_s/A^{4/3}$ for isospin values of $T = 2, 4, 6,$ and $8$ respectively.

| $T$ | Exp. | Theory |
|-----|------|--------|
|     | $b_v$ | $b_v$ |
|     | $b_s$ | $b_s$ |
| 2   | 131.4 | 134.6 |
|     | 183.9 | 232.5 |
| 4   | 92.2  | 111.3 |
|     | 68.4  | 157.8 |
| 6   | 121.5 | 137.0 |
|     | 175.9 | 244.5 |
| 8   | 128.1 | 138.9 |
|     | 206.1 | 254.8 |

with the surface term $b_v/A^{4/3}$ in the fitting of symmetry energy coefficient can one get the consistent value as that in infinite nuclear matter.

Figure 2 shows the coefficients of symmetry energy for nuclei with $A = 40, 60, 80$ and $100$ as a function of the corresponding isospin value $T$ in comparison with the corresponding pseudo data. It shows that the coefficients of symmetry energy $a_{\text{sym}}$ for fixed mass number decrease gradually with the isospin value $T$.

IV. SUMMARY

The mass number and isospin dependence of symmetry energy coefficient in finite nuclei have been studied in RMF theory with the effective interaction PK1 and compared with the pseudo data from the experimental binding energy and semi-empirical mass formula. It has been found that the nuclear symmetry energy coefficient obtained from the RMF calculations decreases gradually with the mass number and isospin value, which is in good agreement with the pseudo data. The mass dependence of symmetry energy coefficient
FIG. 2: The coefficients of symmetry energy for nuclei with $A = 40, 60, 80$ and 100 as a function of the corresponding isospin value $T = (N - Z)/2$. The filled symbols represent the *pseudo* experimental data and the open symbols are the RMF predictions.

for different isospin values has been fitted with and without the surface term $b_v/A^{4/3}$. The results confirm that only if with the surface term in the fitting of symmetry energy coefficient in finite nuclei can one get the consistent value as that in infinite nuclear matter.

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