Decays $Z_H \to Z\gamma$ and $Z_H \to ZZ$ in the littlest Higgs model

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Abstract. The study of the phenomenology of an extra neutral gauge boson, $Z_H$, can help us to unravel the underlying theory from which this particle arises. We study the decay of such a particle into two neutral gauge bosons, $Z_H \to Z\gamma$ and $Z_H \to ZZ$, in the littlest Higgs model. These decays are induced at the one-loop level by a fermion triangle and are interesting as they are strongly dependent on the mechanism of anomaly cancellation of the model. Other relevant tree-level two- and three-body $Z_H$ decays are also calculated. It is found that the branching ratios for the $Z_H \to Z\gamma$ decays can be as large as that of a tree-level three-body decay but the $Z_H \to ZZ$ decay is very suppressed. We also discuss the experimental prospects for detecting these decays at the LHC.

1. Introduction
Little Higgs models [1, 2, 3, 4, 5, 6, 7] have emerged recently as an interesting alternative to solve the little hierarchy problem without the need of fine-tuning. This class of theories are based on the old idea that the Higgs boson is a pseudo-Goldstone boson arising from a global symmetry spontaneously broken at a scale of the order of a few TeVs. In these models there is a set of new particles that play the role of partners of the standard model (SM) gauge bosons and the top quark. These new particles cancel the quadratic divergences to the Higgs boson mass, $m_H$, arising at one-loop from the exchange of SM particles, thereby allowing a naturally light Higgs boson. The most popular realization of this idea is the littlest Higgs model (LHM). Apart from reproducing the SM at the electroweak scale, the LHM predicts heavy partners for the top quark and the SM gauge bosons, which are necessary to cancel the quadratic divergences of the Higgs boson mass at the one-loop level.

An extra neutral gauge boson arises in models in which the SM group is extended with an extra gauge group or if it is embedded into a larger gauge group. The study of the phenomenology of such a particle may be helpful to identify the model from which it arises. In the LHM there are two extra neutral gauge bosons: the $Z$ gauge boson partner, $Z_H$, which is associated with an additional $SU(2)$ gauge group, and the photon partner, $A_H$, which is associated with an extra $U(1)$ gauge group. Although the latter is the lightest new particle and has a great potential to show up at a particle collider, it would not offer a robust signal of the model due to the arbitrariness of the charge assignments of the SM fermions under the extra $U(1)$ gauge group.

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In this work we will present the calculation of the decays $Z_H \rightarrow V_i Z$ ($V_i = \gamma, Z$) in the framework of the LHM. These decays, which are interesting as their rate is dictated by the mechanism of anomaly cancellation, have been already studied in the context of a superstring-inspired $E_6$ model [8], the minimal 331 model [9], and 5D warped-space models [10].

2. The littlest Higgs model

The LHM is a nonlinear sigma model with a global symmetry under the $SU(5)$ group and a gauged subgroup $[SU(2) \otimes U(1)]^2$. The Goldstone bosons are parametrized by the following $\Sigma$ field

$$\Sigma = e^{i\Pi/\xi} \Sigma_0 e^{i\Pi^T/\xi}$$

where $\Pi$ is the pion matrix. The $\Sigma$ field transforms under the gauge group as $\Sigma \rightarrow \Sigma' = U \Sigma U^T$, with $U = L_1 L_2 Y_2$ an element of the gauge group.

The pattern of symmetry breaking is as follows. The $SU(5)$ global symmetry is broken down to $SO(5)$ by the sigma field VEV, $\Sigma_0$, which is of the order of the scale of the symmetry breaking. After the global symmetry is broken, 14 Goldstone bosons arise accommodated in multiplets of the electroweak gauge group: a real singlet, a real triplet, a complex triplet and a complex doublet. The latter will be identified with the SM Higgs doublet. At the same scale, the gauge symmetry is broken down to its diagonal subgroup, $SU(2) \times U(1)$. The real singlet and the real triplet are absorbed by the gauge bosons associated with the broken gauge symmetry, which acquire their masses. A Coleman-Weinberg potential induces a VEV for the complex doublet and masses for the components of the complex triplet. Finally, electroweak symmetry breaking proceeds in the usual way at the Fermi scale.

The LHM effective Lagrangian is composed of the kinetic energy Lagrangian of the $\Sigma$ field, $L_K$, the Yukawa Lagrangian, $L_Y$, and the kinetic terms of the gauge and fermion sectors. The sigma field kinetic Lagrangian is given by

$$L_K = \frac{f^2}{8} \text{Tr}[D_\mu \Sigma]^2,$$

with the $[SU(2) \times U(1)]^2$ covariant derivative defined by

$$D_\mu \Sigma = \partial_\mu \Sigma - i \sum_{j=1}^2 \left[ g_j W^a_{\mu j} (Q_j^a \Sigma + \Sigma Q_j^a T) + g'_j B_{\mu j} (Y_j \Sigma + \Sigma Y_j^T) \right].$$

The heavy $SU(2)$ and $U(1)$ gauge bosons are $W^\mu_j = \sum_{a=1}^3 W^a_{\mu j} Q_j^a$ and $B^\mu_j = B_{\mu j} Y_j$, with $Q_j^a$ and $Y_j$ the gauge generators, while $g_j$ and $g'_j$ are the respective gauge couplings. The VEV $\Sigma_0$ generates masses for the gauge bosons and mixing between them. The heavy gauge boson mass eigenstates are given

$$W'^a = -c W_1^a + s W_2^a,$$
$$B' = -c' B_1 + s' B_2,$$

with masses $m_{W'} = \frac{f}{2} \sqrt{g_1^2 + g_2^2}$ and $m_{B'} = \frac{f}{\sqrt{20}} \sqrt{g_1^2 + g_2^2}$.

The orthogonal combinations of gauge bosons are identified with the SM gauge bosons:

$$W^a = s W_1^a + c W_2^a,$$
$$B = s' B_1 + c' B_2,$$

which remain massless at this stage.
The gauge and Yukawa interactions that break the global $SO(5)$ symmetry induce radiatively a Coleman-Weinberg potential for the complex doublet, $h$, and triplet, $\phi$, whose explicit form can be obtained after expanding the $\Sigma$ field:

$$V_{\text{CW}} = \lambda_{\phi^2} f^2 \text{Tr} |\phi|^2 + i\lambda_{h\phi h} f \left(h\phi^+ h^T - h^* \phi^+ h\right) - \mu^2 |h|^2 + \lambda_{h^4} |h|^4,$$

where $\lambda_{\phi^2}$, $\lambda_{h\phi h}$, and $\lambda_{h^4}$ depend on the fundamental parameters of the model, whereas $\mu^2$ is treated as a free parameter of the order of $f^2/16\pi^2$. The Coleman-Weinberg potential induces a mass term for the complex triplet, whose components acquire a mass of the order of $f$. The neutral component of the complex doublet develops a VEV, $v$, of the order of the electroweak scale, which is responsible for EWSB.

At the electroweak scale, EWSB proceeds as usual, yielding the final mass eigenstates: the three SM gauge bosons are accompanied by three heavy gauge bosons which are their counterpart, $A_H$, $W_H$, and $Z_H$. In this stage the masses of the heavy gauge bosons get corrected by terms of the order of $(v/f)^2$ and so are the masses of the weak gauge bosons. The heavy gauge boson masses are given by

$$m_{Z_H}^2 \approx m_{W_H}^2 \approx f^2 \left(\frac{5}{8 s_w^2 c_w^2} - 1\right),$$

$$m_{A_H}^2 \approx m_{W_W}^2 \approx f^2 \left(\frac{5}{8 s_w^2 c_w^2} - 1\right),$$

with $t_W = s_W/c_W$, being $s_W$ and $c_W$ the sine and cosine of the Weinberg angle $\theta_W$.

The fermion sector of the LHM is identical to the SM one except in the top quark, which requires a new vector-like top quark $T$, which is known as the top partner. The $T$ loops cancel the quadratically divergent contribution to the Higgs mass arising from the top quark loops. This fixes the Yukawa interactions, given by [2]

$$\mathcal{L}_Y = \frac{1}{2} \lambda_1 f \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} u_{3i} + \lambda_2 f \tilde{H} \tilde{H} + \text{H.c.},$$

where $\epsilon_{ijk}$ and $\epsilon_{xy}$ are antisymmetric tensors. The subscripts $i, j$ (x, y) are summed over 1..3 (4,5). In addition, $t_3$ is the SM top quark, $u'_3$ is a new vector-like top quark, and $\chi = (b_3, t_3, \tilde{t})$. The first term of $\mathcal{L}_Y$ induces the couplings of the Higgs boson to the fermions such that the quadratic divergences from the top quark loop are canceled by the top partner loop. The expansion of the $\Sigma$ field leads to the physical states, $H, H^*$, and $\tilde{H}$, whose explicit form is given in terms of two free parameters, $\lambda_1$ and $\lambda_2$. The masses of the SM top quark and the new top quark $T$ are given by [11]

$$m_t = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} v, \quad m_T = f \sqrt{\lambda_1^2 + \lambda_2^2}.$$
We now turn to the one-loop level two-body decays $H \rightarrow V_i V_j (V_i = \gamma, Z)$. For more details, the interested reader is referred to \[13\].

3. Decays of the extra neutral $Z_H$ gauge boson

We will now present the decay widths of the most relevant tree-level decay channels of the $Z_H$ gauge boson together with those of the one-loop decays $Z_H \rightarrow V_i Z (V_i = \gamma, Z)$. For more details, the interested reader is referred to \[13\].

3.1. Tree-level decays

The dominant decays of the extra neutral gauge boson $Z_H$ are the tree-level induced two-body decays $Z_H \rightarrow f \bar{f}$, $Z_H \rightarrow W^+W^-$, $Z_H \rightarrow ZH$, and $Z_H \rightarrow A_H H$. The latter is the only kinematically allowed tree-level two-body decay involving a new particle as a final state.

The decay width into the fermion pair $f \bar{f}$ can be written as:

$$
\Gamma(Z_H \rightarrow f \bar{f}) = \frac{g^2 m_{Z_H} N^f_{c}}{24\pi g_W^2} \sqrt{1 - 4y_f (1 - y_f)} g_L^2, \tag{13}
$$

where we introduced the notation $y_a = \left(m_a/m_{Z_H}\right)^2$, $g_L^f = g_{V}^{\gamma} T_3^f$, with $T_3^f = 1$ ($-1$) for up (down) type fermions, and $N^f_{c}$ is the fermion color number.

On the other hand, the $Z_H \rightarrow W^+W^-$ and $Z_H \rightarrow ZH$ decay widths are given by

$$
\Gamma(Z_H \rightarrow WW) = \frac{g^2 Z_H W W m_{Z_H}}{192\pi g_W^2} (1 - 4y_W)^{3/2} \left(1 + 20y_W + 12y_W^2\right), \tag{14}
$$

and

$$
\Gamma(Z_H \rightarrow ZH) = \frac{g^2 Z_H Z H}{192\pi m_{Z_H} y_Z} \sqrt{(1 - (\sqrt{y_H} - \sqrt{y_Z})^2) (1 - (\sqrt{y_H} + \sqrt{y_Z})^2)} \\
\times \left(1 + (y_H - y_Z)^2 + y_Z^2 - 2(y_H - 5y_Z)\right), \tag{15}
$$

where $g_{Z_H WW} = \frac{g c_s \left(c^2 - s^2\right)^2}{2t^2}$ and $g_{Z_H ZH} = -\frac{g^2 v \left(c^2 - s^2\right)}{2c_s}$. The $Z_H \rightarrow A_H H$ decay width can be obtained from $\Gamma(Z_H \rightarrow ZH)$ after the replacements $y_Z \rightarrow y_{A_H}$ and $g_{Z_H ZH} \rightarrow g_{Z_H A_H H} = -\frac{g^2 v \left(c^2 + s^2\right)}{4c_s c^2 s^2}$ are done.

For completeness we will also present numerical results for the following tree-level three-body decays into SM particles: $Z_H \rightarrow f f \gamma$, $Z_H \rightarrow f f Z$, $Z_H \rightarrow t \bar{t} H$, $Z_H \rightarrow ZH H$, $Z_H \rightarrow ZW^+ W^-$, $Z_H \rightarrow \gamma W^- W^+$, and $Z_H \rightarrow ZZZ$. Other kinematically allowed three-body decays involve a heavy photon: $Z_H \rightarrow A_H H H$, $Z_H \rightarrow A_H WW$, $Z_H \rightarrow A_H ZZ$, and $Z_H \rightarrow A_H A_H A_H$. To obtain the decay widths, we squared the decay amplitude for each set of Feynman diagrams with the aid of the FeynCalc package \[14\], and the integration over the three-body phase space was performed numerically. We would like to note that the decay widths for $Z_H \rightarrow f f \gamma$ and $Z_H \rightarrow \gamma W^- W^+$ were obtained with the assumption of $E_\gamma \geq 10$ GeV to avoid infrared divergences.

3.2. One-loop decays

We now turn to the one-loop level two-body decays $Z_H \rightarrow \gamma Z$ and $Z_H \rightarrow ZZ$, which are induced by the fermion triangle shown in Fig. 1. It is worth mentioning that the $Z_H$ decay into a photon pair is forbidden by the Landau-Yang theorem. The $Z_H \rightarrow \gamma Z$ decay amplitude can be written as

$$
\ldots
$$
It is evident that the \( C \) scalar functions \( B \) symmetry and can be written as

\[
A^{\gamma Z}_k \propto C \] (16)

respectively. This amplitude displays explicitly electromagnetic gauge invariance. The \( A^{\gamma Z}_k \) coefficients can be written in terms of Passarino-Veltman scalar functions as follows

\[
A^{\gamma Z}_1 = \left( \frac{g}{8\pi c W} \right)^2 \frac{2}{(1 - y_Z)^2} \sum_f \left( \xi^{\gamma Z}_f \left( y^2_Z (2 + B_c - B_a + 2(2y_f + 1)C_a) + B_a - B_c \right) - 2y_Z (1 + 3(B_b - B_c) + (2y_f + 1)C_a) \right),
\]

\[
A^{\gamma Z}_2 = \left( \frac{g}{8\pi c W} \right)^2 \frac{2}{(1 - y_Z)^3} \sum_f \left( \xi^{\gamma Z}_f \left( y^2_Z (B_a - B_c - 2(1 + C_a)) + B_c - B_a - 4y_f C_a \right) + 2y_Z (1 + B_b - B_c + (2y_f + 1)C_a) \right) + 4\lambda^{\gamma Z}_f (1 - y_Z)^2 y_f C_a,
\]

where \( \xi^{\gamma Z}_f = N^f_{\gamma} Q^f_{L} g^f_{L} y^L_{f} \) and \( \lambda^{\gamma Z}_f = N^f_{\gamma} Q^f_{L} g^f_{L} y^L_{f} \), with \( Q^f \) the fermion electric charge, and \( g^f_{L,R} \) the couplings of the \( Z \) gauge boson to the fermions. The sum is over all the charged fermions. Anomaly cancellation requires that \( \sum_f \xi^{\gamma Z}_f = 0 \). Bi stands for the following two-point scalar functions \( B_a = B_0(0, m^2_0, m^2_f) \), \( B_b = B_0(m^2_0, m^2_f, m^2_f) \), and \( B_c = B_0(m^2_0, m^2_f, m^2_f) \), while \( C_a = m^2_0 C_0(0, m^2_0, m^2_f, m^2_f, m^2_f) \) and \( C_b = m^2_0 C_0(0, m^2_0, m^2_f, m^2_f, m^2_f) \) is a three-point scalar function scaled by the \( m^2_0 \) mass.

It is evident that the \( A^{\gamma Z}_k \) coefficients are free of ultraviolet divergences.

The \( Z_H \rightarrow \gamma Z \) decay width follows easily and it is given by

\[
\Gamma(Z_H \rightarrow \gamma Z) = \frac{1}{3} \frac{(1 - y_Z)^5 (1 + y_Z) m_{ZH}}{2\pi y_Z} |A^{\gamma Z}_1 - A^{\gamma Z}_2|^2.
\]

(19)

We now concentrate on the \( Z_H \rightarrow ZZ \) decay. The respective amplitude must obey Bose symmetry and can be written as

\[
\mathcal{M}(Z_H \rightarrow ZZ) = \frac{i A^{ZZ}_{1/\mu}}{m_{ZH}} (k^\nu_{1} \epsilon^{\nu\alpha\lambda\rho} + k^\mu_{2} \epsilon^{\mu\alpha\lambda\rho}) k_{1\lambda} k_{2\rho} \epsilon_a(k) \epsilon_\mu(k_1) \epsilon_\nu(k_2),
\]

(20)
where $k_1 \mu$ and $k_2 \nu$ are the four-momenta of the outgoing $Z$ gauge bosons. The coefficient $A^{ZZ}$ is

$$A^{ZZ} = \frac{g^3}{8\pi^2 c_W} \left( \frac{1}{(1 - 4y_Z)^2} \sum_f \left( (1 + 4y_Z^2) (B_b - B_c + (1 + 2y_f) C_b + 2) - 2y_Z (3 - (1 - 4y_f) (B_b - B_c) + 5y_f C_b) + 2y_f ((B_b - B_c) + C_b) - 4y_Z^2 C_b) \xi^f_{ZZ} \
+ 2y_f (1 - 4y_Z) (B_b - B_c + y_Z C_b) \lambda^f_{ZZ} \right) - 2y_f (1 - 4y_Z) (2(B_b - B_c) + (1 - 2y_Z) C_b) \rho^f_{ZZ} \right),$$

(21)

with $\xi_{ZZ}^f = N_{c}^f g_{L_L}^f g_{L_R}^f$, $\lambda_{ZZ}^f = N_{c}^f g_{L_L}^f g_{R_R}^f$, and $\rho_{ZZ}^f = N_{c}^f g_{L_L}^f g_{R_L}^f$. The two-point scalar functions $B_b$ and $B_c$ were given above, while the scaled three-point scalar function is $C_b = m_Z^2 C_0 (m_Z^2, m_W^2, m_W^2, m_T^2, m_T^2, m_T^2)$. The sum is now over all fermions. Anomaly cancellation requires that $\sum_f \xi_{ZZ}^f = 0$.

The $Z_H \rightarrow ZZ$ decay width is given by

$$\Gamma(Z_H \rightarrow ZZ) = \frac{1}{3} \frac{m_{Z_H}}{2 \pi y_Z} (1 - 4y_Z)^{5/2} |A^{ZZ}|^2.$$  

(22)

We will examine below the behavior of the branching ratios of all the above decays as functions of the symmetry breaking scale $f$ and the mixing angle $c$.

4. Numerical Results

In Fig. 2 we show the corresponding results for the $Z_H$ branching ratios discussed above as a function of the mixing angle $c$ and for $f = 4$ TeV, which corresponds to the strongest constraint on the scale $f$ [15]. For simplicity we used $\tan \theta = c'/s' = 1$, although there is little dependence on this parameter. Also, the value 120 GeV is used for the mass of the Higgs boson. We observe that around $c = 1/\sqrt{2}$, several couplings of the $Z_H$ gauge boson vanish and so it decays mainly into fermions. Apart from the color factor, the branching ratios for the light fermions are almost identical since the fermion mass is negligible. The subdominant decays are $Z_H \rightarrow t\bar{t}H$, $Z_H \rightarrow f\bar{f} \gamma$, $f\bar{f}Z$, $Z_H \rightarrow A_H H$, $Z_H \rightarrow A_H WW$, $Z_H \rightarrow A_H ZZ$, and $Z_H \rightarrow A_H HH$. The branching ratios for the last two decays are of similar magnitude and the latter was not included in the plot. Other tree-level decays such as $Z_H \rightarrow WW$, $Z_H \rightarrow ZH$, $Z_H \rightarrow ZHH$, $Z_H \rightarrow ZWW$, and $Z_H \rightarrow \gamma WW$ exactly vanish when $c = 1/\sqrt{2}$. On the other hand, when $c$ is not close to $1/\sqrt{2}$, the decay $Z_H \rightarrow WW$ width can be as dominant as the fermion decays. As far as the one-loop decays are concerned, the $Z_H \rightarrow \gamma Z$ branching ratio can be as high as the tree-level decays $Z_H \rightarrow A_H H$ or $Z_H \rightarrow t\bar{t}Z$, but the $Z_H \rightarrow ZZ$ decay has a very small branching ratio. The former has a branching ratio of the order of $10^{-3}$ while the latter has a rate of about $10^{-5}$.

We now show the $Z_H$ branching ratios as functions of the scale $f$ and for $c = 1/\sqrt{2}$ in Fig. 3. In this case, several decay channels vanish due to the vanishing of the $Z_H$ couplings. It is interesting to note that the $Z_H \rightarrow \gamma Z$ branching ratio is even larger than the ones for the tree-level decays involving a heavy photon. The latter are suppressed by phase space. However, the decay $Z_H \rightarrow ZZ$ has a negligible branching ratio and it would hardly have the chance of being detected.

5. Experimental perspectives

At the LHC, the production of an extra neutral gauge boson would proceed mainly via the Drell-Yan process [12, 11]. We have calculated the number of $Z_H \rightarrow V_i Z$ ($V_i = \gamma, Z$) events
Figure 2. Branching ratios for the one-loop decays $Z_H \to \gamma Z$ and $Z_H \to ZZ$ in the LHM as a function of the mixing angle $c$. We also include the main tree-level two- and three-body decays. We used the value $m_H = 120$ GeV for the Higgs boson mass. The branching ratios for the decays $Z_H \to ZZH$, $Z_H \to \gamma WW$, and $Z_H \to A_HHH$ are not shown in the plot but $Br(Z_H \to ZZH) \sim Br(Z_H \to 3Z)$, $Br(Z_H \to \gamma WW) \sim Br(Z_H \to ZWW)$ and $Br(Z_H \to A_HHH) \sim Br(Z_H \to A_H ZZ)$. For the one-loop decays we used the package LoopTools [16, 17] to numerically evaluate the Passarino-Veltman scalar functions.

Figure 3. Branching ratios for the one-loop decays $Z_H \to \gamma Z$ and $Z_H \to ZZ$ in the LHM as a function of the scale of symmetry breaking $f$ and for $c = 1/\sqrt{2}$, as well as for $c = 1$. We also include the main tree-level two- and three-body decays. We used the value $m_H = 120$ GeV for the Higgs boson mass. The branching ratios not shown vanish exactly for this value of $c$.

at the LHC and the results are presented in [13]. For comparison purpose, we also calculated the number of $Z_H \to \ell\ell$ events. There are promising expectations for the discovery of an extra neutral gauge boson decaying into a lepton pair. As soon as the LHC reaches the nominal $\sqrt{s} = 14$ TeV energy, it will take a few years to collect 100 fb$^{-1}$ of integrated luminosity. For $f = 4$ TeV and $c = 1/\sqrt{2}$, which corresponds to the value $m_{Z_H} = 2.6$ GeV, there would be a few hundreds of $Z_H \to \ell\ell$ events. These results are similar to those obtained in other models [18, 19, 20], though they are one or two orders of magnitude smaller than those reported by ATLAS and CMS, using the SSM model. As far as the potential observation of the $\gamma Z$ and $ZZ$ decay channels is concerned, LHC experiments have studied the sensitivity to the production of SM diboson events, including the $\gamma Z$ and $ZZ$ signals with the $Z$ gauge boson decaying into a highly energetic lepton pair ($e\ell$ and $\mu\ell$) accompanied by an isolated high $p_T$ photon [21, 22, 23, 24].
The s-channel $\gamma Z$ production is of special interest due to its sensitivity to the $Z\gamma V_i$ ($V_i = Z, \gamma$) vertex [25], which is forbidden at tree-level in the SM. In Ref. [13] it was shown that there would be about one dozen of $Z_H \rightarrow Z\gamma$ events at the LHC. For a heavier $Z_H$, an increase of about one order of magnitude in the integrated luminosity would be necessary to look for this decay process. As for the $Z_H \rightarrow ZZ$ decay channel, where the final state includes four leptons, it is clear that an experimental signature is not favorable. However, if we consider that the total LHC integrated luminosity will be of the order of 3000 fb$^{-1}$, there is still some chance to observe the $Z_H \rightarrow ZZ$ decay channel.

6. Conclusions
We have calculated the one-loop decays $Z_H \rightarrow V_i Z$ ($V_i = \gamma, Z$) in the framework of the LHM. While the branching ratio for the $Z_H \rightarrow \gamma Z$ decay can be as large as $10^{-3}$, the $Z_H \rightarrow ZZ$ decay has a branching ratio of the order of $10^{-5}$. We have also discussed the prospects for the experimental observation of these decays at the LHC. We conclude that the detection of the $Z_H \rightarrow V_i Z$ decays would not be favorable. In fact, it would be necessary to collect more than 1000 fb$^{-1}$ of integrated luminosity in order to have a few handful of $Z_H \rightarrow \gamma Z$ candidate events. The situation of the $Z_H \rightarrow ZZ$ decay is less favorable.

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