Short-time scaling behavior of growing interfaces

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Abstract

The short-time evolution of a growing interface is studied analytically and numerically for the Kadar-Parisi-Zhang (KPZ) universality class. The scaling behavior of response and correlation functions is reminiscent of the “initial slip” behavior found in purely dissipative critical relaxation (model A). Unlike model A the initial slip exponent for the KPZ equation can be expressed by the dynamical exponent \(z\). In 2+1 dimensions \(z\) is estimated from the short-time evolution of the correlation function for ballistic deposition and for the RSOS model.

1. Introduction

Interface formation and growth are typical processes in nonequilibrium systems. Two important examples are fluid flow in porous media \cite{1} and deposition of atoms during molecular beam epitaxy (MBE) \cite{1,2}. It is expected that at times much later than typical aggregation times and on macroscopic length scales these interfaces develop a characteristic scaling behavior, where the scaling exponents fall into certain dynamic universality classes \cite{1,3,4}. In certain cases, however, interfaces can also show turbulent, i.e., spatial multiscale behavior \cite{4}. Usually a \(d\)-dimensional interface is embedded in \(d+1\)-dimensional space such that the interface position at time \(t\) can be described by a height function \(h(x,t)\), where \(x\) denotes the lateral position in a \(d\)-dimensional reference plane given by the surface of a substrate. Complete information about the scaling behavior is contained in the dynamic structure factor, which is related to the time displaced height-height correlation function \(C(x - x', t, t') = \langle h(x, t)h(x', t') \rangle - \langle h(x, t) \rangle \langle h(x', t') \rangle\), where a laterally translational invariant system is assumed. For \(t, t' \to \infty\) and finite \(|t - t'|\) the correlation function displays the asymptotic scaling behavior

\[
C(x - x', t, t') = |x - x'|^{2\alpha} F_C(|t - t'|/|x - x'|^z),
\]

where \(\alpha\) denotes the roughness exponent and \(z\) is the dynamic exponent \cite{1,2}. For a laterally translational invariant system the interfacial width \(w(t) \equiv \langle h^2(x, t) \rangle - \langle h(x, t) \rangle^2\) is only a function of \(t\) and displays the scaling behavior \(w(t) \sim t^\beta\) for late times, where \(\beta = \alpha/z\) is the growth exponent. For MBE as an example the scaling behavior displayed in Eq.(1.1) gives access to the exponents \(\alpha\) and \(z\) both experimentally by reflection high energy electron diffraction (RHEED) (see, e.g., chapter 16 of Ref.\cite{1}) and by direct imaging using a surface tunneling microscope \cite{6} and theoretically by continuum models \cite{1,3} and Monte-Carlo simulations \cite{2,5}.

Continuum descriptions of interfacial growth processes can be obtained from general symmetry principles and conservation laws obeyed by the growth process \cite{1}. For a wide class of growth processes the resulting continuum model is given by the well-known Kadar-Parisi-Zhang (KPZ)
equation [7], which reads
\[
\frac{\partial}{\partial t} h(x, t) = \nu \nabla^2 h(x, t) + \frac{\lambda}{2} (\nabla h(x, t))^2 + \eta(x, t).
\] (1.2)

The noise \(\eta(x, t)\) has a Gaussian distribution with \(\langle \eta(x, t) \rangle = 0\) and
\[
\langle \eta(x, t)\eta(x', t') \rangle = 2D\delta(x - x')\delta(t - t').
\] (1.3)

The parameters \(\nu, D,\) and \(\lambda\) are assumed to be constants and averages \(\langle \ldots \rangle\) are taken over the noise distribution. In the long time limit Eq. (1.2) has a global symmetry which is commonly denoted as Galileian invariance \([1, 7]\). An important consequence is that the exponents \(z\) and \(\alpha\) of the KPZ equation fulfill the scaling relation \(\alpha + z = 2\). The exponents of the KPZ equation are exactly known only in \(d = 1\), where \(z = 3/2\) and \(\alpha = 1/2\) due to the existence of a dissipation fluctuation theorem \([8, 11]\). In \(d = 2\) numerical investigations indicate \(z \simeq 1.6\) and \(\alpha \simeq 0.4\) \([9]\). For \(d > 2\) the asymptotic scaling behavior is either governed by linear theory \((\lambda = 0, \) weak coupling regime) or by another set of exponents inaccessible by analytical methods \((\lambda \neq 0, \) strong coupling regime) depending on the value of the effective coupling constant \(g \equiv D\lambda^2/(4\nu^3)\) \([8, 9]\). Furthermore, it is interesting to note that the nonlinearity in Eq. (1.2) renders all other possible nonlinearities irrelevant in the renormalization group sense in the long-time limit. For intermediate times, however, the presence of other nonlinearities in the growth equation gives rise to various crossover phenomena \([1, 12]\).

### 2. Analytic Theory

In Fourier space the KPZ equation Eqs. (1.2) and (1.3) is equivalent to the dynamic functional \(\mathcal{J} = \mathcal{J}_0 + \mathcal{J}_1\) \([6, 11, 13]\) which consists of the Gaussian part
\[
\mathcal{J}_0[\tilde{h}, h] = \int \frac{d^d q}{(2\pi)^d} \int_0^\infty dt \left\{ D\tilde{h}(q, t)\tilde{h}(-q, t) - \tilde{h}(q, t) \left( \frac{\partial}{\partial t}\tilde{h}(-q, t) + \nu q^2\tilde{h}(-q, t) \right) \right\}
\] (2.1)

and the interaction part
\[
\mathcal{J}_1[\tilde{h}, h] = -\frac{\lambda}{2} \int \frac{d^d q_1}{(2\pi)^d} \int \frac{d^d q_2}{(2\pi)^d} \int_0^\infty dt \, q_1 \cdot q_2 \tilde{h}(-q_1 - q_2, t) h(q_1, t) h(q_2, t),
\] (2.2)

where \(\tilde{h}(q, t)\) is the Fourier transform of the response field \([12]\). The initial condition \(h(q, 0) = 0\), which is implicitly assumed in Eqs. (2.1) and (2.2), breaks the temporal translational invariance of the KPZ dynamics. The analytic treatment of Eqs. (2.1) and (2.2) is based on the identities
\[
\frac{\partial}{\partial t} h(q, t = 0) = 2D\tilde{h}(q, t = 0) \quad \text{and} \quad G(q, 0, t, t' < t) = 1,
\] (2.3)

where the response function \(G\) is given by the formal average \(\langle h(-q, t)\tilde{h}(q, t') \rangle\) with respect to the dynamic functional \(\mathcal{J}[\tilde{h}, h]\) \([15]\). For details of the field-theoretic treatment of Eqs. (2.1) and (2.2) we refer to Refs. \([11, 15]\) and only quote the main results for later reference.

The correlation function \(C\) and the response function \(G\) are found to obey the scaling relations
\[
C(q, t, t' \ll t) = (t'/t)^\theta |q|^{-d-2\alpha} f_C(|q|^2 t) \quad \text{and} \quad G(q, t, t' \ll t) = (t'/t)^{\tilde{\theta}} |q|^{-d} f_G(|q|^2 t),
\] (2.4)

respectively, where in contrast to model A \([11]\) the short-time exponents \(\theta\) and \(\tilde{\theta}\) are given by
\[
\theta = (d + 4)/z - 2 = (d + 2\alpha)/z \quad \text{and} \quad \tilde{\theta} = 0
\] (2.5)
at the nontrivial fixed point \((\lambda \neq 0)\) of Eq.\((1.2)\). In \(d = 1\) the exact value \(\theta = 4/3\) can be obtained from the exact value \(z = 3/2\) and is confirmed by a Monte-Carlo simulation of ballistic deposition \([15]\). From numerical estimates for \(z\) in \(d = 2\) one obtains \(\theta \simeq 1.7\). The exponent relation given by Eq.\((2.3)\) simply means that the short-time and the long-time scaling behavior of the correlation function are identical, i.e., the short-time scaling behavior can be obtained by extrapolating the \(t'\)-dependence of \(C(q,t',t')\) from \(t' \sim t\) to \(t' = 0\). In fact, the scaling relation given by Eq.\((2.3)\) can be derived independently by analyzing the fluctuation spectrum of the interface displacement velocity averaged over a macroscopic portion of the interfacial area \([16]\). It should be noted, however, that the perturbative analysis of Ref.\([15]\) only constitutes a rigorous proof of Eq.\((2.5)\) for \(d = 1\). For \(d \geq 2\) one encounters the strong coupling regime of Eq.\((1.2)\) which can no longer be treated analytically.

Finally, we remark that an alternative scaling form for \(C\) can be obtained from the definition of the growth exponent \(\beta\) which leads to \(\theta = d/z + 2\beta\). The scaling behavior displayed in Eq.\((2.4)\) can then be written in the simplified form \(C(q,t,t' \ll t) = t'^{\beta}g_C(|q|^2t')\), where \(g_C(y) = y^{-\beta}f_C(y)\).

### 3. Monte-Carlo Results

The scaling behavior of \(C(q,t,t' \ll t)\) according to Eq.\((2.4)\) can be tested numerically by a Monte-Carlo simulation of simple deposition models on lattices \([1]\). The continuum description underlying Eqs.\((1.2)\) and \((1.3)\) is replaced by a discretized description according to

\[
h(x,t) = h(x = a\bar{j}, t = n/(FL^d)) \equiv a h_j(n),
\]

where the lattice constant \(a\) is assumed to be the same both in the plane of the substrate and perpendicular to it and \(\bar{j} = (j_1, \ldots, j_d)\). The lattice has \(L^d\) sites, \(F\) is the incoming particle flux, and \(n\) is the number of deposited particles. Furthermore, the incoming particle flux \(F\) has been normalized to unity, so that \(t\) in Eq.\((3.1)\) is dimensionless and given by the number of deposited layers. Finally, \(h_j(n)\) defined by Eq.\((3.1)\) is also dimensionless and denotes the number of particles deposited at lattice site \(j\) after \(n\) particles have been deposited on the lattice. Ballistic deposition on a two-dimensional substrate is defined by the deterministic growth rule

\[
h_{j,k}(n + 1) = \max(h_{j-1,k}(n), h_{j,k-1}(n), h_{j,k}(n) + 1, h_{j+1,k}(n), h_{j,k+1}(n)),
\]

(see, e.g., Ref.\([1]\)), where the site \((j, k)\) in Eq.\((3.2)\) has been selected randomly from the \(L \times L\) sites of the lattice and periodic boundary conditions are applied.

In order to measure the short-time exponent given by Eq.\((2.3)\) it is sufficient to probe the integrated time displaced correlation function, i.e., one probes \(C(q = 0,t,t')\) as described in Ref.\([15]\). Like a real deposition process the simulation is characterized by an a priori unknown microscopic aggregation time \(t_a\). The scaling behavior of \(C\) according to Eq.\((2.4)\) can only be observed for \(t' \gg t_a\). On the other hand \(t' \ll t\) is required for Eq.\((2.4)\) to hold, so that short-time scaling is restricted to the time window \(t_a \ll t' \ll t\). Furthermore, the lattice size \(L\) must be chosen sufficiently large in order to avoid the onset of finite-size crossover effects if \(t^{1/2} \sim L\) when \(t'\) is still much smaller than \(t\). For the simulation described here \(t = 1000\) and \(L \geq 200\) fulfill the above requirements. In order to cope with the very small signal to noise ratio in each measurement of \(C(0, t, t')\) for \(t' \ll t\) averages are taken over \(2 \times 10^4\) realizations. These are distributed over 20 individual runs at every point in time so that the jackknife method can be applied for the data.
Fig. 1: Correlation function $C(0, t, t')$ in $d = 2$ for ballistic deposition (top) and the RSOS model (bottom) as a function of $t'/t$ for $0.005 \leq t'/t \leq 1$ and $L \times L = 300 \times 300$ (solid line). Error bars are shown only at a few selected points in time and represent one standard deviation. The dashed lines display power laws with the measured short-time exponents $\theta = 1.729 \pm 0.042$ and $\theta = 1.733 \pm 0.041$, respectively. The data follow this power law rather accurately in the interval $0.02 \leq t'/t \leq 0.2$ (bold arrows).

analysis. The result for ballistic deposition according to Eq. (3.2) and for the RSOS model in $d = 2$ according to Ref. [17] is displayed in Fig. 1, where $C(0, t, t')$ is shown as a function of $t'/t$ for
\[ t = 1000 \text{ and } L = 300. \] According to Fig. 1, the interval \( 0.02 \leq t'/t \leq 0.2 \) is available to determine the short-time exponent \( \theta \). From Eq.(2.5) we obtain the estimate \( z = 1.608 \pm 0.013 \) by averaging over the two values for \( \theta \) given in Fig. 1. Finally, we note that according to the scaling relation \( \alpha + z = 2 \) one has \( \alpha = 0.392 \pm 0.013 \) for the roughness exponent. These values are in agreement with other numerical data for \( z \) and \( \alpha \) in \( d = 2 \) (see chapter 8 of Ref. [1] for a collection of recent estimates) and they therefore provide some support for the general validity of Eq.(2.5).

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