Quark Confinement in QCD and New Bosons

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Abstract

If the dual Meissner effect due to abelian monopole condensation is the quark confinement mechanism of QCD as suggested in recent Monte-Carlo simulations of lattice QCD, new axial-vector and scalar bosons with the mass of O(1GeV) would appear as physical states which are different from ordinary hadrons and glueballs. The axial-vector boson can not decay into ordinary color-singlet hadrons and glueballs owing to a remaining global discrete permutation symmetry with respect to colors (Weyl symmetry) if the vacuum respects the symmetry as suggested from lattice MC simulations.

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I. INTRODUCTION

The 'tHooft idea on quark confinement mechanism in QCD starts with partially
gauge-fixing color $SU(3)$ in such a way that the maximal torus group $U(1) \times U(1)$ remains
unbroken. This is called abelian projection. After the abelian projection, monopoles appear
and then QCD can be regarded as abelian $U(1) \times U(1)$ theory with electric charges (quarks
and gluons) and magnetic charges (monopoles). 'tHooft conjectured, if the monopoles make
condensation, electric charges and then quarks are confined due to a mechanism dual to the
Meissner effect.

Suppose the 'tHooft confinement mechanism is actually realized in QCD. Then after
abelian projection, abelian components of gluons and abelian monopoles are expected to be
essential dynamical quantities governing quark confinement mechanism. Numerical studies
have been done by many groups in order to test the confinement mechanism in the framework
of lattice QCD. The present results are summarized as follows:

1. (Abelian dominance) Essential features of confinement such as the string tension seem
to be explained in terms of $U(1) \times U(1)$ operators composed of abelian link fields
alone \cite{2,10} in the maximally abelian (MA) gauge \cite{11,12} and in some cases also in
the Polyakov gauge.

2. (Monopole dominance) The $U(1) \times U(1)$ operators are written by a product of
two parts, a monopole current or Dirac string part and a photon part. The
confinement phenomena seem to be reproduced well by the monopole part alone
\cite{13,3,4,14–17,5–9,18}.

3. (Scaling of monopole density and monopole dynamics) Monopoles in QCD seem to
remain important in the continuum limit as seen from scaling behaviors \cite{19,3,20–23,10}.
Monopoles are jammed \cite{11} and make a long connected loop in the confinement phase.
The long loop seems to be responsible for confinement \cite{17,22,24}.
4. (The dual Meissner effect and flux squeezing) Abelian electric (color) flux is seen to be squeezed and the QCD vacuum seems to be near the border between type 1 and type 2 magnetic superconductor [25–27].

5. (Order parameters of confinement) It is found a candidate of the order parameter of confinement which transforms under the dual $U(1)$ symmetry nontrivially and which vanishes in the confinement phase [28].

6. (Monopole action and monopole condensation) The effective monopole action can be derived in $SU(2)$ and $SU(3)$ QCD. The block-spin transformation on the dual lattice strongly suggests that $SU(2)$ QCD is always in the monopole condensed phase (and so in the confinement phase) for all $\beta$ in the infinite volume limit [22,23,10].

7. (Gauge (in)dependence) Gauge independence of the mechanism is the biggest problem to be proved.

   It is the aim of this note to show that, if the 'tHooft idea is realized in nature, there must appear new axial-vector and scalar bosons with the mass of $O(1\text{GeV})$ as physical states which are not confined. The field operators of the bosons are not invariant under a global discrete Weyl transformation except their special combinations. If the vacuum also respects the Weyl symmetry, the states which are globally color non-singlet are predicted to exist. The new states are seen to have characteristic decay modes into ordinary hadrons and glueballs which are trivial under the Weyl transformation. Experimental tests of such bosons are essential to prove the 'tHooft mechanism in addition to lattice Monte-Carlo simulations.

II. ABELIAN PROJECTION

Abelian projection of QCD is a partial gauge fixing leaving the maximal torus group unbroken. For example, it is done as follows. Choose an operator $X(x)$ which transforms non-trivially under $SU(3)$ transformation:
\[ A_{\mu 0}(x) \rightarrow A_{\mu}(x) = V(x)A_{\mu 0}(x)V^{\dagger}(x) - \frac{i}{g} \partial_{\mu}V(x)V^{\dagger}(x) \]  

(1)

\[ \psi_{0}(x) \rightarrow \psi(x) = V(x)\psi_{0}(x). \]  

(2)

Then abelian projection is to choose \( V(x) \) so that \( X(x) \) is diagonalized:

\[ X(x) \rightarrow \tilde{X}(x) = \text{diagonal}. \]  

(3)

It is known that, once an ordering of the diagonal elements of \( \tilde{X}(x) \) is chosen, the nonabelian part of the gauge is fixed uniquely \[1\]. The diagonal element \( d(x) \) of \( SU(3) \) is not fixed. \( \{d(x)\} \) is the maximum torus group of \( SU(3) \), which is the residual \( U(1) \times U(1) \) gauge symmetry.

Let us look at QCD at this stage without further fixing the gauge of the residual symmetry. First, we explore how the fields after the abelian projection transform under an arbitrary \( SU(3) \) gauge transformation \( S(x) \). Since \( V(x) \) is a functional of (gauge) fields and so it transforms non-trivially under \( S(x) \). Let us fix the form of \( V(x) \) such that all diagonal components of the exponent of \( V(x) \) are zero. This is always possible if one uses the residual symmetry. Then \( V(x) \) is found to transform under \( S(x) \) as

\[ V(x) \xrightarrow{S} V^{\text{S}}(x) = d^{\text{S}}(x)V(x)S^{\dagger}(x). \]  

(4)

\( V^{\text{S}}(x) \) diagonalizes an operator which is transformed from \( X(x) \) under \( S(x) \). \( d^{\text{S}}(x) \) is necessary for \( V^{\text{S}}(x) \) to take the same form as \( V(x) \) fixing the arbitrariness due to the remaining \( U(1) \times U(1) \).

The gauge field after the abelian projection, \( A_{\mu}(x) \), transforms under \( S(x) \) as

\[ A_{\mu}(x) \xrightarrow{S} A_{\mu}^{\text{S}}(x) = d^{\text{S}}(x)A_{\mu}(x)d^{\dagger S}(x) - \frac{i}{g} \partial_{\mu}d^{\text{S}}(x)d^{\dagger S}(x). \]  

(5)

After the abelian projection, \( A_{\mu}(x) \) transforms only under the diagonal matrix \( d^{\text{S}}(x) \). Since the last term of \( (5) \) is composed of the diagonal part alone, the diagonal part of \( A_{\mu}(x) \) transforms like a photon. The off diagonal part of \( A_{\mu}(x) \) transforms like a charged matter. The quark field transforms under \( S(x) \) as
It is important that, after abelian projection, \( \bar{\psi}^i(x) \psi^i(x) \) and \( \psi^1(x) \psi^2(x) \psi^3(x) \) are neutral and at the same time invariant under any \( SU(3) \) transformation \( S(x) \).

The most interesting fact of abelian projection is that monopoles appear in the residual abelian channel. We treat \( SU(2) \) QCD for simplicity. After abelian projection, we define an abelian field strength as

\[
f_{\mu\nu}(x) = \partial_\mu A_\nu^3(x) - \partial_\nu A^3_\mu(x). \tag{7}
\]

\( f_{\mu\nu}(x) \) can be rewritten in terms of the original field as

\[
f_{\mu\nu}(x) = \partial_\mu (\hat{Y}^a(x) A^a_{\nu0}(x)) - \partial_\nu (\hat{Y}^a(x) A^a_{\mu0}(x)) - \frac{1}{g} \varepsilon_{abc} \hat{Y}^a(x) \partial_\mu \hat{Y}^b(x) \partial_\nu \hat{Y}^c(x) \tag{8}
\]

where \( \hat{Y}(x) = V^\dagger(x) \sigma_3 V(x) = \hat{Y}^a(x) \sigma^a \). \( \hat{Y}^a(x) \) obeys

\[
\hat{Y}^a(x) \hat{Y}^a(x) = 1. \tag{9}
\]

A current

\[
k_\mu(x) = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial^\nu f^{\rho\sigma}(x) \tag{10}
\]

\[
= \frac{1}{2g} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{abc} \partial^\nu \hat{Y}^a(x) \partial^\rho \hat{Y}^b(x) \partial^\sigma \hat{Y}^c(x) \tag{11}
\]

is always zero if \( V(x) \) is fixed. However, at a point \( x \) where the eigenvalue of the diagonalized operator \( X(x) \) is degenerate, \( V(x) \) is not well defined and \( k_\mu(x) \) does not vanish there. We calculate the charge in the three dimensional volume \( \Omega \) around \( x \):

\[
g_m = \int_\Omega k_0(x) d^3x = \frac{1}{2g} \int_\Omega \varepsilon_{0\nu\rho\sigma} \varepsilon_{abc} \partial^\nu \hat{Y}^a(x) \partial^\rho \hat{Y}^b(x) \partial^\sigma \hat{Y}^c(x) d^3x \tag{12}
\]

\[
= \frac{1}{2g} \int_{\partial\Omega} \varepsilon_{ijk} \varepsilon_{abc} \hat{Y}^a(x) \partial_j \hat{Y}^b(x) \partial_k \hat{Y}^c(x) d^2\sigma_i \tag{13}
\]

\[
= \frac{4\pi n}{g}, \tag{14}
\]

where \( n \) is an integer. \( n \) is a topological number corresponding to a mapping between the sphere \( \{ \} \) in the parameter space and the sphere \( \partial\Omega \) of \( \Omega \). Because this equation represents
the Dirac quantization condition, $g_m$ can be interpreted as a magnetic charge. The monopole current $k_\mu(x)$ is a topologically conserved current $\partial_\mu k^\mu(x) = 0$. Abelian projected QCD can be regarded as an abelian theory with electric charges and monopoles. 'tHooft conjectured if the monopoles condense, abelian charges are confined due to the dual Meissner effect. This means quark confinement.

III. THE WEYL SYMMETRY

Once an abelian projection is done with a choice of a certain gauge-fixing matrix, abelian charge neutrals are invariant also under color $SU(3)$ as proved above. However, since such a proof is done on a fixed gauge orbit, it does not mean always that all abelian neutrals are also $SU(3)$ color singlets literally.

Let us start with the usual $SU(3)$ QCD Lagrangian after abelian projection:

$$L = \frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + i \sum_f \bar{\psi}_f^i \gamma^\mu (D_\mu)_{ij} \psi_j^i - \sum_f m_f \bar{\psi}_f^i \psi_f^i + L_{GF+FP},$$

(15)

where

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c,$$

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu - ig \sum_\alpha \Lambda_{ij}^\alpha A_\mu^\alpha,$$

and $L_{GF+FP}$ is a gauge-fixing term. For example, in the MA gauge,

$$L_{GF+FP} = \delta_B \{ \sum_{i \neq j} \bar{c}^j i (\partial_\mu + ig (A_\mu^i - A_\mu^j)) A_\mu^{ij} \},$$

(16)

where $\delta_B$ is the BRS transformation, $\bar{c}$ is the Faddev-Popov ghost and the gluon field $3 \times 3$ matrix is

$$A = \left( A^{ij} \right) = \sum_{\alpha=1}^8 \frac{1}{2} A^a \lambda_\alpha,$$

(17)

with the GellMann matrices $\lambda_\alpha$. In a unitary gauge where an adjoint operator $X$ is diagonalized,
\[ L_{GF+FP} = \delta_B \{ \sum_{i \neq j} \bar{c}^{ij} X^{ij} \}. \]  

Note that one has to further fix the gauge of the remaining \( U(1) \times U(1) \) in the continuum to get the Fadeev-Popov determinant. Also the monopole contribution to the functional measure should be taken into account.

What symmetries are left unbroken after abelian projection? It is well known that maximally abelian torus group \( U(1) \times U(1) \) is unbroken as a local symmetry. In addition, any global discrete permutation with respect to three colors makes the Lagrangian (13) and (16) or (18) unchanged. The discrete permutations compose the permutation group which is the Weyl group of \( SU(3) \). The discrete symmetry corresponds to the fact that one can choose any ordering of the diagonal elements of \( \bar{X}(x) \) in (3) in the case discussed above. In \( SU(3) \), one can choose any one of six \( SU(3)/U(1)^2 \) gauge-fixing matrices corresponding to six different orderings of three eigenvalues. The Weyl transformation interchanges the different gauge-fixing matrices. Hence the Weyl group is a subgroup of the original \( SU(3) \). Monopole physics are unchanged in any choice, since space-time points where the two eigenvalues are degenerate are the same and the topology is unchanged.

Consider for example a permutation (12) which is, in the matrix representation, expressed by

\[
V_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in SU(3). \]  

From the transformation properties

\[ \psi \rightarrow V_3 \psi , \]  

\[ A \rightarrow V_3 A V_3^T , \]  

we get

\[ \bar{\psi} \lambda_3 \psi \rightarrow -\bar{\psi} \lambda_3 \psi , \]
\[ \bar{\psi} \lambda_8 \psi \rightarrow \bar{\psi} \lambda_8 \psi , \]  
(23)

\[ A^3 \rightarrow -A^3 , \quad A^8 \rightarrow A^8 , \]  
(24)

\[ A^{12} \rightarrow A^{21} , \quad A^{13} \rightarrow A^{23} , \quad \text{etc.} , \]  
(25)

Similarly, under a permutation (31), \( A^3 + \sqrt{3} A^8 \) and \( \bar{\psi} (\lambda_3 + \sqrt{3} \lambda_8) \psi \) change their signs.

Considering also the \( U(1) \times U(1) \) property, one can see, for example, \( \bar{\psi} \lambda_i \psi \ (i = 3, 8) \) and the physical parts of \( A^i \ (i = 3, 8) \) are \( U(1) \times U(1) \) neutral but not invariant under the Weyl transformation. Namely these are not global \( SU(3) \) color-singlets. However, since

\[ \sum_{i=1}^{3} \bar{\psi}^i \psi^i = \sum_{i=1}^{3} \bar{\psi}^i \psi^i , \]  
(26)

\( \bar{\psi} \psi \) is global color-singlet.

Such a state exists also in the case of baryons. There are six \( U(1) \times U(1) \) neutral baryons \( \psi_{f_1}^i \psi_{f_2}^j \psi_{f_3}^k \) where \( f_i \) denotes the set of quantum numbers like flavor other than color. It is possible to prove

\[ \sum_{i,j,k=1}^{3} \epsilon_{ijk} \psi_{0f_1}^i \psi_{0f_2}^j \psi_{0f_3}^k = \sum_{i,j,k=1}^{3} \epsilon_{ijk} \psi_{f_1}^i \psi_{f_2}^j \psi_{f_3}^k . \]  
(27)

Hence the antisymmetric combination is equal to the original color singlet baryon. However, other five combinations are \( U(1) \times U(1) \) neutral, but Weyl variant.

Existence of the remaining Weyl symmetry in generic abelian projection is proved as follows. Note that one can always find an adjoint operator which is diagonalized under any abelian projection whenever the gauge-fixing matrix is well-defined. Define a gauge-fixing \( SU(3) \) matrix \( V(x) \) of an abelian projection. Then \( \hat{Y}(x) = V^\dagger(x) \lambda_0 V(x) \) where \( \lambda_0 \) is any linear combination of \( \lambda_3 \) and \( \lambda_8 \) is a functional of gluon (quark) fields and transforms like an adjoint operator as seen from (4). An abelian projection can be characterized as the diagonalization of the matrix \( \hat{Y}(x) \), i.e., \( \hat{Y}^i \neq j = 0 \) at any space-time point where monopoles do not exist. These conditions are trivially Weyl symmetric. Monopoles exist where \( V(x) \) and \( \hat{Y}(x) \) are ill-defined. A Weyl transformation changes any \( \hat{Y}(x) \) among the set of the matrices \( V^\dagger(x) \lambda_0 V(x) \). Hence monopole physics remain unchanged.
IV. DUAL MEISSNER EFFECT AS THE DUAL HIGGS MECHANISM

The monopole condensation causes the dual Meissner effect and the quark confinement. The effect is a kind of the Higgs mechanism just as the usual Meissner effect in superconductivity. Here the theory is well described in terms of the dual variables after a dual transformation. The spontaneously broken symmetry is magnetic $U(1) \times U(1)$ which is dual to the remaining electric $U(1) \times U(1)$ maximal torus group \[30-32\]. Hence axial vector massive gauge bosons and scalar (dual Higgs) bosons are predicted to exist just as in the usual Higgs mechanism.

The situations can be seen more clearly when one constructs a dual abelian $U(1) \times U(1)$ Higgs model composed of two dual photons $(1^+)$ (which are dual to two abelian gluons $A^3$ and $A^8$ after abelian projection) and scalar $(0^+)$ bosons coupled to the dual photons. Actually the present author and his collaborators have derived such a model called dual Ginzburg-Landau (DGL) model starting from QCD \[30-33\]. After summing up all contributions from closed loops of monopole currents appearing after abelian projection, the model is composed of two degenerate dual photons $C^3_\mu(x)$ and $C^8_\mu(x)$ $(1^+)$ and three degenerate complex scalar $(0^+)$ fields $\chi_i(x) \ (i = 1 \sim 3)$ with magnetic charges as well as quarks and gluons which play the role of simple charged particles. Let me call the former dual photons as strong bosons and the latter magnetically charged scalar as monopole particles. In the confinement phase, strong bosons become massive due to the Higgs mechanism and massive monopole particles appear in addition to usual hadrons composed of quarks and glueballs. When we neglect dynamical quarks and charged gluons for the moment and consider only an external electric current $\vec{j}_{ext}^3 = (j_{ext}^3, j_{ext}^8)$, the model is written as \[30-33\]

$$
\mathcal{L}_{\text{eff}} = -\frac{1}{4} F^2_{\mu \nu} + \sum_{\alpha=1}^{3} \{ |(\partial_{\mu} + ig_{m} \vec{e}_\alpha \cdot \vec{C}_\mu) \chi_\alpha|^2 - \lambda (|\chi_\alpha|^2 - v^2)^2 \} - \lambda (\sum_{\alpha=1}^{3} |\chi_\alpha|^2 - 3v^2)^2 + \kappa \chi_1 \chi_2 \chi_3, \tag{28}
$$

where

$$
\vec{e}_1 = (1, 0), \quad \vec{e}_2 = (- \frac{1}{2}, -\frac{3}{2}), \quad \vec{e}_2 = (- \frac{1}{2}, \frac{3}{2}), \quad \Im(\chi_1 \chi_2 \chi_3) = 0,
$$
\[ \vec{H}_{\mu\nu} = \partial_\mu \vec{C}_\nu - \partial_\nu \vec{C}_\mu + \epsilon_{\mu\nu\alpha\beta} n^\alpha (n \cdot \partial)^{-1} \vec{\gamma}^\beta J_{ext} \cdot \vec{C}_\mu = (C_3^3, C_8^8). \]

In unitary gauge \(Im \chi_\alpha = 0\), the classical field equations

\[ \begin{align*}
\partial_\mu \vec{H}^{\mu\nu} + 2g_m^2 \sum_{\alpha=1}^{3} \vec{e}_\alpha \cdot (\vec{e}_\alpha \cdot \vec{C}_\nu)^2 \chi_\alpha^2 &= 0 \\
\partial_\mu \partial^\mu \chi_\alpha - g_m^2 (\vec{e}_\alpha \cdot \vec{C}_\mu)^2 \chi_\alpha + 2\lambda (\chi_\alpha^2 - v^2) \chi_\alpha &= 0.
\end{align*} \tag{29, 30} \]

where \(\lambda' = 0\) and \(\kappa = 0\) are assumed for simplicity. In case of static hadrons we set static charge configurations in \(\vec{J}_{ext}\).

The model can reproduce analytically the linear potentials between static quark-antiquark (meson) \cite{30,32,34} and also between three quarks (baryon) \cite{34}. It can also explain the characteristic features of finite-temperature transition of pure QCD found by Monte-Carlo simulations, that is, the first (second) order phase transition in \(SU(3)\) (\(SU(2)\)) QCD \cite{33}. A long-range Van der Waals force is shown not to appear between meson-meson interactions \cite{35}. Monopole condensation seems to enhance chiral symmetry breaking \cite{36,35}.

Both strong bosons and monopole particles are neutral with respect to electric \(U(1) \times U(1)\) and so are proved to be physical which are composed of gluons (glueball-like states). To search for such bosons and to establish them experimentally are therefore very crucial in order to prove the correctness of the dual Meissner effect. Also, numerical Monte-Carlo measurements of such particles on large enough lattices in the framework of lattice QCD is very important in order to prepare for real experiments.

V. ESTIMATE OF THE MASSES OF NEW BOSONS

The phenomenological analyses of the DGL model lead us to predict existence of strong bosons and monopole particles having masses of the order \(O(1\text{GeV})\), that is, \(0.5\text{GeV} \sim 2.0\text{GeV}\) \cite{32,36}. The value of the mass can not be fixed definitely at present, but it can not be too large, because they are related to the value of the string tension \(\sqrt{\sigma} \sim 450\text{MeV}\) and the QCD gauge coupling constant \(g\) through Dirac’s quantization condition \(gg_m = 4\pi n\) (\(n = \text{integer but } n = 1\) is actually considered).
Introducing a static quark and antiquark source

\[ \vec{j}_{ext}^{\mu} = \vec{Q} g^{\mu\nu} \delta(x) \delta(y) \{ \delta(z - \frac{R}{2}) - \delta(z + \frac{R}{2}) \}, \tag{31} \]

where \( \vec{Q} = (g/2, g/2\sqrt{3}) \) and \( n^\mu = (0, 0, 0, 1) \), we can evaluate the string tension \( \sigma \) by numerically solving the equations of motions (29) and (30) \[30–32,36\].

Especially, exact results can be derived analytically in the extreme type 2 case (where the Ginzburg-Landau parameter \( \kappa = \sqrt{2\chi}/(\sqrt{3}g_m) \gg 1/\sqrt{2} \)) and also at the border between type 1 and type 2 (\( \kappa = 1/\sqrt{2} \)) as is well known in the usual superconductor case.

In the extreme type 2 case, I (partially with Maedan) derived

\[ \sigma = \frac{\vec{Q}^2 m_c^2}{4\pi} K_0 \left( \frac{\sqrt{2} m_c}{m_\chi} \right), \tag{32} \]

where a natural infrared cutoff is introduced and \( K_0 \) is a modified Bessel function. Recently, Suganuma et al. \[30\] have pointed out that in this case we need not introduce the infrared cutoff and have obtained

\[ \sigma = \frac{\vec{Q}^2 m_c^2}{8\pi} \ln \left( \frac{m_c^2 + m_\chi^2}{m_c^2} \right), \tag{33} \]

In the extreme type 2 case \( m_\chi \gg m_c \), both give about the same results

\[ \sigma = 4\pi v^2 \ln \left( \frac{m_\chi}{m_c} \right), \tag{34} \]

where we have used \( <\chi_1> = <\chi_2> = <\chi_3> = v, gg_m = 4\pi \) and \( m_c = \sqrt{3}g_m v \tag{31} \].

Adopting \( \sigma = (450)^2 \text{ MeV}^2 \) from the Cornell potential fit to charmonium spectra, we get, say, for \( m_\chi/m_c = 1.5 \sim 4 \),

\[ v = 198 \sim 108(\text{MeV}). \tag{35} \]

On the other hand, at the border between type 1 and type 2 cases, one can get the first integral of the equations of motions (29-30):

\[ \sqrt{\frac{2}{3}} \frac{1}{\rho \partial \rho} (\rho \tilde{C}) = \sqrt{2\chi}(v^2 - \chi^2), \tag{36} \]
where it is enough to consider only one common $\chi$ and $C$ fields \[32\]. Also $C = C_D + \tilde{C}$ where $C_D$ is the Coulomb part and the cylindrical coordinate $(\rho, \theta, z)$ is adopted. When two sources are far apart, the string tension is expressed by

$$\sigma = | \int \rho d\rho d\theta [2\lambda (v^4 - \chi^4) - 2g_m^2 \chi^2 (C_D + \tilde{C}) \tilde{C}]|,$$

which reduces using (36) to

$$\sigma = 2\pi | \int_0^\infty d\rho \frac{2\sqrt{\lambda}}{\sqrt{3}} \frac{\partial}{\partial \rho} [(\rho \tilde{C})(v^2 + \chi^2)] |$$

$$= 4\pi v^2,$$

where we have used $\tilde{C} \rightarrow -C_D \rightarrow -g/(4\pi \rho)$ and $\chi \rightarrow v$ as $\rho \rightarrow \infty$. Hence we get in this case

$$v = 127\text{(MeV)}.$$  

The mass $m_c$ depends on the value $g$. Non-perturbative effects are not known and $g = 2 \sim 5$ may be possible \[36\]. Then we get

$$m_c = 0.5 \sim 2.0\text{(GeV)} \quad \text{for the extreme type 2 case},$$

$$m_c = m_\chi = 0.6 \sim 1.4\text{(GeV)} \quad \text{for the border case}.$$  

The masses can be determined also from the abelian electric flux distribution and the correlation between the electric flux and the rotation of monopole currents as done similarly in the superconductor. The Monte-Carlo measurements have been done by some groups \[25-27\]. Although the lattices used are not large enough, the $SU(3)$ data suggest both masses are almost equal and of order $1.5 \sim 2.0$ GeV. Namely the QCD vacuum seems near the border between type 1 and type 2 magnetic superconductor. This is consistent with numerical analyses of the DGL model \[32\] and a preliminary Monte-Carlo evaluation of axial-vector and scalar correlations using abelian Wilson loops \[37\].

Considering the vacuum seems near the border between type 1 and type 2 magnetic superconductor, we could guess both masses are between $0.5$ GeV and $2.0$ GeV. In this rough sense, the new bosons are predicted to have the mass of $O(1\text{GeV})$. 

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VI. SELECTION RULES FROM THE WEYL SYMMETRY

The above Weyl symmetry is expected to lead us to interesting selection rules with respect to transition matrix elements of the strong bosons and the monopole particles.

A. Weyl transformation properties of new boson operators

Ordinary color singlets mesons $\sum_i \bar{\psi}_i \psi_i$ and baryons $\sum_{i,j,k=1}^3 \epsilon_{ijk} \psi_{f_1} \psi_{f_2} \psi_{f_3}$ are naturally invariant under the Weyl group. Other $U(1) \times U(1)$ neutral hadrons composed of quarks and gluons such as $\bar{\psi} \lambda_3 \psi$ are Weyl nontrivial.

What about the new bosons? The strong bosons $C^3$ and $C^8$ have the same transformation property as those of the abelian gauge fields $A^3$ and $A^8$ after abelian projection, since they are canonical conjugates and are not independent. Hence the strong bosons are Weyl nontrivial. The Weyl symmetry is common in the original and in the dual expressions of the abelian projected QCD. However, it is easy to prove that the followings are Weyl invariant:

\[(C^3)^2 + (C^8)^2 = C^+ C^- , \quad (43)\]

\[\bar{\psi}(C^3 \lambda_3 + C^8 \lambda_8) \psi = \bar{\psi}(C^+ \lambda_- + C^- \lambda_+) \psi, \quad (44)\]

where $C^\pm \equiv (C^3 \pm iC^8)/\sqrt{2}$ and $\lambda_\pm \equiv (\lambda_3 \pm i\lambda_8)/\sqrt{2}$. Here we have not written the space-time dependence explicitly.

The monopole fields $\chi_\alpha$ have the coupling with the strong bosons as follows:

\[\sum_{\alpha=1}^3 |(\partial_\mu + ig_m \bar{\epsilon}_\alpha \cdot \vec{C}_\mu) \chi_\alpha|^2. \quad (45)\]

Hence each $\chi_\alpha$ is Weyl nontrivial. Actually, it is easy to see the strong boson triplet $\bar{\epsilon}_\alpha \cdot \vec{C}$ ($\alpha = 1 \sim 3$) and the monopole triplet $\chi_\alpha$ ($\alpha = 1 \sim 3$) changes each other under any Weyl transformation. For example, under the permutation (31),

\[\frac{-C^3 - \sqrt{3}C^8}{2} \rightarrow \frac{C^3 + \sqrt{3}C^8}{2}, \quad (46)\]

\[\frac{-C^3 + \sqrt{3}C^8}{2} \rightarrow -C^3, \quad (47)\]

\[\chi_2 \rightarrow \chi_2^*, \quad \chi_3 \rightarrow \chi_1^*. \quad (48)\]
Also under the cyclic permutation (123),
\[-\frac{C^3 - \sqrt{3}C^8}{2} \to C^3, \quad C^3 \to -\frac{C^3 + \sqrt{3}C^8}{2},\]
\[-\frac{C^3 + \sqrt{3}C^8}{2} \to -\frac{C^3 - \sqrt{3}C^8}{2},\]
\[\chi_1 \to \chi_2, \quad \chi_2 \to \chi_3, \quad \chi_3 \to \chi_1.\]  

(49) \hspace{1cm} (50) \hspace{1cm} (51)

However the mixed state
\[\chi^0 \equiv |\chi_1 + \chi_2 + \chi_3|/\sqrt{3}\]

is Weyl trivial.

**B. The Weyl property of the vacuum**

To fix the transformation properties of the new states, one has to study the vacuum. Does the vacuum respect the Weyl symmetry?

In the framework of the DGL theory, the vacuum can be fixed by the self-interaction terms of monopole fields:
\[V = \lambda \left( \sum_{\alpha=1}^{3} |\chi_{\alpha}|^2 \right)^2 + \lambda' \left( \sum_{\alpha=1}^{3} |\chi_{\alpha}|^2 \right)^2 + \kappa \chi_1 \chi_2 \chi_3 - \mu^2 \sum_{\alpha=1}^{3} |\chi_{\alpha}|^2.\]

When magnetic $U(1) \times U(1)$ is spontaneously broken ($\mu^2 > 0$), both vacuum states with spontaneous broken and unbroken Weyl symmetry are possible, depending on the choice of the parameters $\lambda, \lambda'$ and $\kappa$. On the other hand, only the symmetric vacuum is chosen in the case of $SU(2)$ vacuum.

However, MC simulations of abelian projection of lattice QCD strongly suggest that QCD vacuum also respects the Weyl symmetry. After abelian projection in lattice $SU(3)$ QCD, there are two independent abelian link variables corresponding to $A^3$ and $A^8$. The value of the string tension, the Polyakov loops and the masses of the strong bosons are seen to be the same when we evaluate them in terms of each abelian link variable, although the fact is not explicitly written in the published papers [27,5,8]. This suggests that the $SU(3)$ QCD vacuum respects the Weyl symmetry.
In the previous section, we have taken $\lambda' = 0$ in which the Weyl symmetry is not broken spontaneously. Hence we have tacitly assumed the invariance of the vacuum as suggested in the MC simulation.

C. Selection rules

In the following also, we assume that the Weyl symmetry is not spontaneously broken in the $SU(3)$ QCD vacuum. Since the strong bosons have the mass of $O(1\text{GeV})$, it is natural to suppose that they are the lightest Weyl nontrivial states which can couple directly to quarks and gluons. Considering that the QCD Hamiltonian is invariant under the Weyl symmetry, we can prove that any matrix element between $C^3_\mu$ ($C^8_\mu$) and ordinary hadron states denoted by $|h\rangle$ vanishes. Applying the (12) permutation, we get

$$\langle C^3_\mu | H | h \rangle = -\langle C^3_\mu | H | h \rangle = 0.$$  \hspace{1cm} (54)

Also under the (31) permutation, we have

$$\langle C^3_\mu + \sqrt{3} C^8_\mu | H | h \rangle = -\langle C^3_\mu + \sqrt{3} C^8_\mu | H | h \rangle = 0.$$  \hspace{1cm} (55)

Hence

$$\langle C^8_\mu | H | h \rangle = 0.$$  \hspace{1cm} (56)

The Weyl trivial states can couple to ordinary hadrons. Hence such a state as $C^+ C^-$ can decay into or produced by ordinary color singlet hadrons. Also Weyl trivial $\chi^0$ can couple to ordinary hadrons through $C^+ C^-$. 

Now one can understand why such a light axial vector state has not been found in the usual lattice search of glueballs. Usually, Wilson loops composed of a full $SU(3)$ link field are used to search for glueball states. But such Wilson loops correspond to totally color singlet Weyl trivial states. Hence one can not get any information of such Weyl nontrivial states like the strong boson. On the otherhand, abelian Wilson loops composed of abelian link fields alone after abelian projection are in general Weyl nontrivial. Actually, Monte-Carlo
simulations using such abelian Wilson loops give the mass of $O(1\text{GeV})$ \cite{25–27}, although the lattice size is not large enough.

**VII. PRODUCTION AND ANNIHILATION OF NEW BOSONS AND EXPERIMENTS**

Is it possible to evaluate matrix elements of (pair) production or pair annihilation of the new bosons analytically? It is very interesting and challenging, but there are some severe problems:

1. If we introduce dynamical charged quarks into the DGL model (still neglecting dynamical charged gluons), we get the following Lagrangian:

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} H_{\mu\nu}^2 + \sum_{\alpha=1}^{3} \{ (|\partial_{\mu} + ig_{m} C_{\alpha} \cdot C_{\mu}|) \chi_{\alpha}|^2 - \lambda (|\chi_{\alpha}|^2 - v^2)^2 \}
+ \lambda' (\sum_{\alpha=1}^{3} |\chi_{\alpha}|^2 - 3v^2)^2 + \kappa \chi_1 \chi_2 \chi_3 + \bar{\psi}(i\gamma^\mu - m)\psi ,
\]  

(57)

where

\[
\begin{align*}
H_{\mu\nu} &= \partial_{\mu} C_{\nu} - \partial_{\nu} C_{\mu} + \epsilon_{\mu\nu\alpha\beta} n^\alpha (n \cdot \partial)^{-1} j^\beta , \\
j^\mu(x) &= -g \bar{\psi}(x) \gamma^\mu \frac{\lambda}{2} \psi(x) , \quad \vec{\lambda} = (\lambda^3, \lambda^8) .
\end{align*}
\]

Since the theory contains two coupling constants $g$ and $g_m$ satisfying the Dirac quantization condition $gg_m = 4\pi$, a perturbative treatment is impossible. We have to resort to some nonperturbative method.

2. Moreover, in the DGL model, electrically charged quarks and gluons are topological quantities just as monopoles in the original QCD. There must arise inevitably non-local interactions between dynamical and topological quantities. This reflects the necessity of the Dirac string and actually there are non-local terms containing $n_\lambda (n \cdot \partial)^{-1}$ in (57).
Maybe, the most reliable method is Monte-Carlo simulations of lattice QCD. The new boson state with non-trivial Weyl property can be constructed in terms of abelian Wilson loops after abelian projection. If we evaluate correlations of such operators and ordinary hadron operators composed of full Wilson loops, we would get information of the matrix elements of, say, pair annihilation of the strong bosons into ordinary hadrons, although we need large lattices and very long CPU time. But this is worth while to be challenged.

Experimentally, there may be severe constraints with respect to such matrix elements [38]. They could be used to test the correctness of the ’tHooft mechanism. Here I only list up some possible examples:

• $e^+ + e^- \rightarrow \gamma + X^0$, where $X^0$ is a pair of the strong bosons or $\chi^0$.

• $J/\psi$ (and $\Upsilon$) $\rightarrow \gamma + X^0$. In this case, the $\gamma + \chi^0$ decay seems severely restricted.

• $\bar{p} + p \rightarrow C^+ + C^-.$

• $\pi^- + p \rightarrow n + X^0.$

If the couplings of $C^+ + C^- \rightarrow$ ordinary hadrons happen to be unexpectedly small due to some unknown mechanism, the new bosons might be a new candidate of the dark matter. Such new bosons are produced much through the transition from quark-gluon phase to hadron phase.

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REFERENCES

[1] G. ’tHooft, Nucl. Phys. B190 (1981) 455.

[2] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42 (1990) 4257; Nucl.Phys.B(Proc.Suppl.)20 (1991) 236.

[3] S. Hioki et al., Phys. Lett. 272B (1991) 326; errata, Phys. Lett. B281 (1992) 416; Nucl.Phys. B(Proc.Suppl.) 26 (1992) 441.

[4] T. Suzuki, Nucl. Phys. B(Proc. Suppl.) 30 (1993) 176.

[5] T. Suzuki et al., Phys. Lett. B347 (1995) 375; Nucl. Phys. B(Proc. Suppl.) 42 (1995) 529.

[6] O. Miyamura, Phys.Lett. B353 (1995) 91.

[7] O. Miyamura and S. Origuchi, Hiroshima Univ. Report hep-lat 9508015.

[8] T. Suzuki et al., Talk at ’Lattice 95’. To appear in Nucl. Phys. B(Proc. Suppl.). hep-lat 9509010.

[9] S. Ejiri et al., Talk at ’Lattice 95’. To appear in Nucl. Phys. B(Proc. Suppl.). hep-lat 9509013.

[10] T. Suzuki, See the reviews in conferences and workshops. Int. School-Seminar ’93 - Hadrons and Nuclei from QCD - (World Scientific, 1994) 325; YITP Workshop ”From Hadronic Matter to Quark Matter” to appear in Prog. Theor. Phys. Suppl.; German-Japan Seminar on Massively Parallel Computers (World Scientific, 1995); RCNP Workshop on Color Confinement and Hadrons (Osaka 1995 ’Confinement 95’ to appear in Prog. Theor. Phys. Suppl.; ECT Workshop ’Nonperturbative Approaches to QCD’ (Trento 1995).

[11] A.S. Kronfeld et al., Phys. Lett. 198B (1987) 516, A.S. Kronfeld et al., Nucl.Phys. B293 (1987) 461.
[12] S. Hioki et al., Phys. Lett. 271B (1991) 201.

[13] F. Brandstaeter et al., Phys. Lett. 272B (1991) 319.

[14] S. Kitahara et al., Nucl Phys. B(proc. Suppl.) 30 (1993) 557.

[15] J.D. Stack and R.J. Wensley, Nucl.Phys. B371 (1992) 597; Talk at Lattice 95. To appear in Nucl. Phys. B(Proc. Suppl.).

[16] H. Shiba and T. Suzuki, Phys. Lett. B333 (1994) 461.

[17] S.Ejiri et al., Phys. Lett. B343 (1995) 315; Nucl. Phys. B(Proc. Suppl.) 42 (1995) 481.

[18] S. Ejiri, Talk at ‘Lattice 95’. To appear in Nucl. Phys. B(Proc. Suppl.). hep-lat 9509014.

[19] V.G. Bornyakov et al., Phys. Lett. 284B (1992) 99.

[20] S. Hioki et al., Phys. Lett. 285B (1992) 100; Nucl.Phys. B(Proc.Suppl.) 26 (1992) 450.

[21] T.L. Ivanenko et al., Phys. Lett. 252B (1990) 631.

[22] H. Shiba and T. Suzuki, Nucl.Phys. B(Proc.Suppl.) 34 (1994) 182; Nucl.Phys. B(Proc.Suppl.) 42 (1995) 282; Phys. Lett. B351 (1995) 519.

[23] T. Suzuki et al., Talk at ‘Lattice 95’. To appear in Nucl. Phys. B(Proc. Suppl.). hep-lat 9509015.

[24] S. Kitahara et al., Prog. Theor. Phys. 93 (1995) 1; Nucl.Phys. B(Proc.Suppl.) 42 (1995) 511.

[25] V. Singh et al., LSU preprint LSUHEP-1-92 (1992); Nucl. Phys. B(Proc. Suppl.) 30 (1993) 568.

[26] P. Cea and L. Cosmai, Nucl. Phys. B(Proc. Suppl.) 30 (1993) 572.

[27] Y. Matsubara et al., Nucl. Phys. B(Proc. Suppl.) 34 (1994) 176.

[28] L. Del Debbio et al., To appear in Phys. Lett.B; Talk at ‘Lattice 95’. To appear in Nucl.
[29] J. Arafune et al., Jour. Math. Phys. 16 (1975) 433.

[30] T. Suzuki, Prog. Theor. Phys. 81 (1988) 929; 81 (1989) 752.

[31] S. Maedan and T. Suzuki, Prog. Theor. Phys. 81 (1989) 229.

[32] S. Maedan et al., Prog. Theor. Phys. 84 (1990) 130.

[33] H. Monden et al., Phys. Lett. B294 (1992) 100.

[34] S. Kamizawa et al., Nucl Phys. B389 (1993) 563.

[35] Y. Matsubara, Review talk in ECT Workshop ’Nonperturbative Approaches to QCD’ (Trento 1995).

[36] H. Suganuma et al., Nucl. Phys. B435 (1995) 207; Talks in RCNP Workshop on Color Confinement and Hadrons (Osaka 1995); ECT Workshop ’Nonperturbative Approaches to QCD’(Trento 1995).

[37] Y. Tezuka, Master thesis submitted to Kanazawa Univ. (unpublished March, 1995).

[38] Particle Data Group, Phys. Rev. D50 (1994) 1173 and references therein.