Vocal imprecision as a universal constraint on the structure of musical scales

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Theories of the origin of musical scales from the ancient Greeks to the present day have assumed that the intervals comprising scales are defined by specific mathematical ratios. Such theories are predicated on pre-tunable instruments, and yet the voice is almost certainly the original musical instrument. Therefore, the analysis of vocal scales offers a more naturalistic approach to understanding the origin of musical scales. In the present study, we conducted a large-scale computational analysis of vocal pitch-class properties and their implications for scale structure. We analyzed 418 field recordings of solo, unaccompanied songs from across 10 principal musical-style regions of the world. The results revealed a mean vocal pitch-class imprecision of approximately 1.5 semitones, and comparable results were obtained across all regions. These results suggest that vocal imprecision is universal and is mainly derived from the physiological limitations of the voice. Such vocal imprecision fundamentally constrains the formation of musical scale structure: it provides a lower limit on the spacing between adjacent scale tones and thus an upper limit on the number of scale tones that an octave can contain. We discuss these results in terms of an Interval Spacing model of the evolution of musical scales.

One of the oldest ideas about the origins of music in Western culture is that musical intervals can be mathematically described as simple integer ratios between the lengths of vibrating strings. This idea has served as the foundation for theoretical accounts not just of musical intervals, but of musical scales more generally. A dominant theory of the origin of diatonic Western scales argues that they are entirely derived from simple integer ratios between the scale degrees’ acoustic frequencies (e.g., 2:1 for the octave, 3:2 for the perfect fifth, 4:3 for the perfect fourth, and so on)1. In 1722, Rameau affirmed that harmonics were the ultimate basis of scale structure: “a knowledge of harmony is sufficient for a complete understanding of all of the properties of music” (p. 3)2. We will consider this viewpoint as the Harmonicity theory of the origin of scales, and it has become the established model within music psychology and psychoacoustics. As Kreitler and Kreitler describe it, scales are derived from the “overheard overtones” (p. 129)3 assumed to be perceivable in the harmonic series of single pitched sounds, most especially speech sounds4. The idealized scales of Harmonicity theory use tuning systems of “just intonation,” in which all of the scale degrees are defined as simple integer ratios of one another.

Whereas Harmonicity theory is a top-down theory of scales based on optimizing mathematical simplicity, an alternative bottom-up model is grounded in the physiology of vocal pitch production5, 6. The voice is the oldest and most universal musical instrument, and thus vocal constraints on the structure of musical scales are of utmost importance in thinking about the evolution of music. In fact, vocal music has been a highly neglected source of information in the study of musical scales. Perhaps the most agreed-upon feature of vocal music is the imprecision in pitch production. Numerous studies have found that the voice is highly imprecise as a pitch-generating instrument, especially compared to tunable instruments like violins and flutes5, 7. While synthesizers can produce the exact frequency dictated, acoustic instruments produce pitch- and interval-classes with a distribution of frequencies surrounding the target pitch8–10. In the case of the voice, such a pitch distribution often spans at least a semitone (100 cents) on either side of the target frequency9. Wide distributions such as these have been described as “interval islands”, analogous to the “vowel ellipses” of vowel sounds in phonetics11–14. This observation has led to the formulation of a voice-based model of the origin of musical scales—called the Interval Spacing theory5—that posits that the intrinsic imprecision of vocal production constrains the spacing of pitch-classes in musical scales. Adjacent pitch-classes must be far enough apart to be distinguishable from one another, but not so distant as to hinder accurate singing9.

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Evaluating whether Harmonicity theory or Interval Spacing theory better accounts for the nature of musical scales requires the analysis of a culturally diverse sample of music and the development of an empirically descriptive model thereof. Most theories of musical scales have been predicated on the prescriptive tuning regimens of various art and classical musics, including systems of theoretical modes and instrumental tunings. Such an approach allows scales to be predefined in an abstract sense, but without any reference to actual melody production. The approach thus hinders an investigation into fundamental issues surrounding the origin of musical scales. Does melody preclude scale or vice versa? How do scales arise in vocal music given that the voice cannot be pre-tuned?

These questions can only be addressed by taking a descriptive and comparative approach to the study of musical scales, particularly in indigenous and traditional vocal songs. In comparative linguistics, researchers construct vowel inventories by analyzing the way that people actually speak. By analogy, comparative musicologists need to construct models of scale structure by analyzing the way that people actually sing. This is not a new idea. Meyer discussed a similar approach when he stated that "scales are not the raw material of melodies. It is just the other way around: melodies are the materials from which scales are abstracted" (pp. 216–217). The overall approach of the present study is to move beyond the study of abstract instrumental tunings in art music traditions and instead investigate the actual production of melodies in indigenous and traditional vocal music, and do so at a global level that accounts for the diverse panoply of scale types and singing styles worldwide.

A few previous studies have recognized the evolutionary significance of the human voice and have posited that its harmonic features might account for scale structure. However, the explanatory power of such models for the origin of musical scales has been questioned, largely because they fail to account for the spacing between scale degrees and thus melodic intervals. Other recent comparative musicology studies have analyzed cross-cultural features of melodies and scales and some have looked at specific pitch-class features, including how these features relate to scale structure, although this has yet to be a globally-comprehensive, computational survey of the structural features of pitch-classes using a worldwide database of recorded vocal melodies. Such a study is necessary to understand how pitch-production constraints might influence how scales are structured and used, which is itself necessary in order to develop a generalizable, descriptive model of musical scales, and eventually to compare these ties to physiological, cognitive, cultural, and evolutionary mechanisms.

In the present study, we constructed a large, culturally-diverse database of 418 solo, monophonic vocal song recordings, focusing exclusively on indigenous and traditional musics. We attempted to represent music from across 10 principal musical-style regions of the world, as characterized in Alan Lomax’s Cantometrics project and as presented in the book Folk Song Style and Culture. For each song, we extracted the fundamental frequencies of the melody using Tarsos and then measured two dependent variables for each pitch-class: imprecision, corresponding to the range of the pitch-class's frequency distribution in cents, and inaccuracy, corresponding to how well (or poorly) the tuning conformed to the mathematical template given by twelve-tone equal temperament (12-ET), also in cents. We analyzed global trends for both variables and looked for cross-cultural differences. Because indigenous vocal music is not typically studied in Western research, we also wanted to compare these results to those obtained from a more-typical sample of recordings, while controlling for melodic features that may impact pitch production. To do so, we carried out a small experimentally-controlled study in which pitch-class imprecision and inaccuracy in order to determine whether these variables differed across the instruments as a function of the fixedness of their tuning.

We generated three major predictions for the overall study. First, if vocal imprecision is a universal physiological constraint for melody production, then it should occur at a comparably high level in all cultures, with pitch-classes being minimally 100 cents wide. This would be supportive of the Interval Spacing theory. However, if imprecision is low in some or all cultures, or if it is variable across cultures, then this would oppose the theory, in which case the role of enculturation on pitch production and scale structure would need to be further explored. Second, if Harmonicity theory is a valid tuning theory not just for Western instrumental art music but for traditional vocal musics worldwide, then pitch inaccuracy should occur at a comparably low level across all cultures, indicating that people's sung pitches conform well with the analyzed tuning model. However, if inaccuracy is high in all cultures, then this would call into question scale theories based on mathematically-optimized notions of tonality. Third, if pitch-class imprecision and inaccuracy depend on the degree of pre-tuning of an instrument, then they should vary significantly between the organ, flute, trombone, and voice. We again measured pitch-class imprecision and inaccuracy in order to determine whether these variables differed across the instruments as a function of the fixedness of their tuning.

Methods
Study 1: global song analysis. We constructed a global database of 418 solo, unaccompanied, monophonic private recordings of vocal songs. We obtained these recordings from various private ethnographic collections, independent record labels and, most significantly, the Smithsonian Folkways record label. We restricted our sampling to indigenous musical traditions as much as possible, supplementing them with folk traditions where appropriate, but avoiding art traditions. Within this focus, we aimed to maximize the geographic diversity of our database while avoiding overly sparse sampling of any one culture. The geographic sampling aimed to represent songs from each of the 9 musical-style regions characterized by Lomax in his global analysis of song style, except for an isolate called "Tribal India", due to an inability to find recordings from this region.
Because Lomax’s designation of “Old High Culture” covers an extremely large expanse of geography, and primarily characterizes the art music traditions therein, we divided it into three smaller, more traditional regions: Middle East-North Africa, Central Asia, and East Asia, resulting in 10 final regions. In addition, we opted, based on the terminology in Savage et al.\textsuperscript{30}, to use the term “Circumpolar” to describe what Lomax described as “Arctic Asia” since this style-region includes the arctic part of North America as well (e.g., Inuit). Figure 1 presents a geographic map of the song sampling, including a listing of the number of samples analyzed for each stylistic zone.

For each song in the database, we recorded the following metadata: source collection; copyright owner; cultural group; recording year (if known); album title; song title; and song length. Some samples originally included in the database were excluded from analysis if the recordings were too noisy or had audio issues; if the pieces contained significant accompaniment, had no solo sections of sufficient length, or were sung by a child; or if the pitch-tracking algorithm could not produce sufficient fundamental-frequency annotations for quantitative analysis. We aimed to analyze 50 samples per region, although the limited availability of songs from certain regions led the final number of analyzed samples to be 418, rather than 500, for the 10 musical-style regions (see Fig. 1).

**Scale analysis.** We identified by ear major structural sections of the melodies, including repeated verses and points of considerable pitch shift or modulation, such as a sudden change in the set of pitch-classes used. We then chose a short excerpt (mean: 20.5 s; SD: 8.52) that contained at least one complete melodic idea for further analysis, but that did not contain significant pitch drift, such as the singer becoming gradually sharp or flat. For each melodic excerpt, we identified the scale series by ear. Scale series were notated using the closest possible Western alphabetical notes, from the lowest to highest pitch. The perceived tonic was indicated with a star. When the sung notes occurred in between keyboard notes, this was indicated with a slash (e.g., Eb/E). Non-discrete, unstable, and/or uncertain notes were indicated using parentheses. Secondary tonics, if present, were indicated using a double star. Figure 2 presents an example of such an analysis.

**Quantitative analysis.** We analyzed each song using Tarsos\textsuperscript{26} and its built-in YIN pitch-tracking algorithm\textsuperscript{31}. After a pitch-trace was created, we used the “annotations visualizer” to select a melodic excerpt (time) and the salient range of annotations (pitch). These raw data were used for analysis in Jupyter Notebook\textsuperscript{32}. All of our pitch analyses were carried out in cents, where 100 cents = 1 equal-tempered semitone. The pitch annotations for an excerpt were ordered from the lowest to highest value in cents, and the first derivative was calculated. The peaks of this function were then superimposed onto a melograph of the annotations to visualize initial estimations of the pitch-class boundaries. The boundaries were then adjusted by hand using the melograph as a visual reference. An example of a segmented melograph is shown on the left side of Fig. 2. When the final pitch-class boundaries were selected, they were used to bin the cent annotations into discrete pitch-classes. The tonic pitch was identified by ear and validated through inter-rater review (see below). The other pitch-classes were set relative to the mean of the tonic pitch-class, which was assigned a value of 0, where negative cent values referred to pitches below the tonic pitch. Non-discrete pitch-classes and noise were removed.
Two dependent variables were derived from these data, relating to the imprecision and tuning inaccuracy of the vocal production, respectively. For imprecision, we calculated the range of each pitch-class in cents by subtracting the minimum cent value from the maximum value. The range therefore corresponded to the width of the pitch-class (in cents). In order to constrain this variable, we considered values greater than 400 cents (+/− a whole tone [200 cents]) as outliers and removed them from the analysis. Outliers were uncommon (n = 13), and we confirmed that a less-sensitive measure of variability (inter-quartile range) showed a similar patterns of results. To measure how accurately the pitch-classes conformed to a mathematical model of tuning (12-ET in this case), we calculated the absolute distance of each pitch-class’s mean cent value from the nearest 100-cent interval from the tonic, where the tonic marks 0 cents. This variable was constrained to be a positive value less than 50 cents, since a value of 50 cents marks the halfway point between any two adjacent pitch-classes in 12-ET. Values near 0 would indicate a close adherence to 12-ET, whereas larger values would indicate greater levels of tuning inaccuracy relative to this mathematical template. The right side of Fig. 2 presents values for both of the dependent variables for a sample song.

Inter-rater review. Three raters independently analyzed each song for its time-segments, scale series, tonic pitch, and pitch-class boundaries. The amount and type of disagreement were recorded. Consensus was reached by listening to the piece together and discussing each point of disagreement. Occasionally, if two analysts failed to come to an agreement, the third analyst would act as the tie-breaker. The results of the final agreed-upon analysis are presented here.

Study 2: instrumental pitch production. Samples. This study was an exploratory experimental analysis that aimed to compare the voice with three Western aerophones (i.e., instruments for which air is the primary sound source). In particular, we wanted to investigate how the relative tuning fixedness of the various instruments would influence their pitch-class imprecision and inaccuracy. The instruments were selected to create a gradient of fixedness: organ > flute > trombone > voice. We predicted that pitch-class imprecision would negatively correlate to the degree of fixedness, with the voice being the least fixed and thus least precise. Eight short melodies were randomly chosen from our database (each from a different region) and transcribed into Western musical staff notation by the first author, with review by the other analysts. These melodies reflected a diverse sample of global musical styles, tonal structures, and melodic structures. The musical scores are available at the OSF repository for this study (see “Ethical approval”).

Participants. Three semi-professional musicians—a flutist, a trombonist, and a vocalist—were recruited via personal connections; the first author served as the organizer. The mean age of the participants was 25 years old (SD = 1), and the mean experience at their instrument was 17 years (SD = 3). Participants were remunerated $100 (Canadian), with the exception of the first author. After consultation with the Research Ethics Board (REB), it was determined that the musicians were only being hired to assist with sample preparation, and as such were not considered as participants under the REB’s protocol. We followed pre-approved Covid safety measures during the on-campus and in-person recording session.
Methods. The musicians were sent the sheet music for the eight transcribed vocal songs several months in advance of the experiment. They reported spending an average of 6 h (SD = 5.3) practicing the music before the recording session. They were told to transpose each piece up or down in octaves as needed to place the melody within their comfortable register. They were instructed to perform all pieces with neutral musical articulation, a minimum of expression and vibrato, and to pay particular attention to tuning and pitch accuracy. The singer was instructed to sing the melodies using a single neutral vowel. Before each piece, the initial note was played on the organ, and the starting tempo was played on a metronome.

The musicians were recorded individually in a university concert hall, all on the same day. They performed all eight pieces in a different semi-randomized order. They were allowed to do a second recording of any piece if they desired and then inform us of which version to analyze. Recordings took approximately 30 min per musician. The recordings were made on an Audio Technica 4040 Cardioid Large Diaphragm Condenser Microphone placed 2 m in front of the performer’s sound source, using a Focusrite Clarett 2Pre USB audio interface at 24-bit depth and 96 kHz sampling rate. After the recording session, participants filled out a brief questionnaire in which they rated the perceived difficulty of the melodies, as well as an assessment of their own pitch imprecision and inaccuracy. Quantitative analysis of the recordings was carried out exactly as in Study 1 to obtain measures of pitch-class imprecision and inaccuracy.

Ethical approval. No human participants were involved in this study. All protocols were carried out in accordance to Canada's Tri-Council Policy Statement.

Results

Study 1: global song analysis. Imprecision. The mean global pitch-class imprecision, defined as the mean pitch-class range across the 418 songs, was 155 cents (SD = 62.2, slight right skew = 0.59). The mean values regionally spanned from 121 cents in the Middle East-North Africa to 170 cents in the Insular Pacific and Circumpolar regions. Hence, no world region showed an imprecision in vocal production less than 1 semitone, with the global mean being just beyond 1.5 semitones. The distributions of the pitch-class ranges, as shown in Fig. 3a, differed across regions. Central Asia, the Middle East-North Africa, and the Insular Pacific had seemingly bimodal distributions, whereas the Circumpolar region, Insular Pacific and South America were the least precise.

Due to the non-normality (Shapiro–Wilk’s test, W = 0.98, p < 0.001) and non-homogeneity of variance (Levene’s test, F = 6.78, df = 9, p < 0.01) in the data, we tested for differences in imprecision between world regions using a multiple-comparisons method that is robust to non-heteroscedasticity and that controls the family-wise error rate49. The results revealed several significant regional differences. Pitch-class imprecision was (a) lower in the Middle East-North Africa region than all other regions (p < 0.001), except Central Asia; (b) lower in Central Asia than the Circumpolar and Insular Pacific regions (p < 0.001), as well as East Asia and South America (p < 0.01); (c) higher in the Insular Pacific region than Europe (p < 0.01) and North America (p < 0.05); and (d) higher in the Circumpolar region than Europe (p < 0.05). No other comparisons were significant. To summarize, songs from the Middle East-North Africa region, and to a lesser extent Central Asia, demonstrated lower-than-average pitch-class imprecision, whereas songs from the Insular Pacific, and to a lesser extent Circumpolar region, demonstrated higher-than-average pitch-class imprecision. Similar results for pitch-class imprecision were found using the inter-quartile range: songs from the Middle East-North Africa region were the most precise, whereas songs from the Circumpolar region, Insular Pacific and South America were the least precise.
Inaccuracy. Figure 3b demonstrates the global pattern of pitch-class inaccuracy. The average global pitch-class inaccuracy was 24 cents (SD = 14.5), which is indicative of a generally random pattern of tuning compared to the mathematical template (i.e., 12-ET). This pattern was consistent across regions. An analysis using the same multiple-comparisons method as above revealed that pitch-class inaccuracy was significantly lower in Europe than in North America (p < 0.001), Africa, and East Asia (p < 0.01), and the Middle East-North Africa (p < 0.05). However, no other comparisons were significant. Another way to evaluate inaccuracy is to compare the variance of the inaccuracy values, not just the values themselves. Songs that are well-tuned to 12-ET, such as European art songs, should include pitch-classes with consistently low inaccuracy values, demonstrating low inaccuracy and low inaccuracy variance. Songs that are not tuned to 12-ET should produce pitch-classes with a random distribution (from 0 to 50 cents) of inaccuracy values, leading to a high variance for inaccuracy, though not necessarily a high average inaccuracy. The variance of inaccuracy values across regions was compared using multiple pairwise Levene tests with Bonferroni correction. The results showed that the variance of pitch-class inaccuracy was not significantly different among regions. A general Levene test also showed no heterogeneity (F = 0.47, df = 9, p = 0.89). The distributions of pitch-class inaccuracy, as seen in Fig. 3b, are also of note. The bimodal distribution of Africa is particularly interesting, as is the nearly reciprocal relationship between the distributions of North and South America. To summarize, these results suggest that a cappella traditional songs are generally not sung in 12-ET, although European folk songs are more accurate to that tuning system than are songs from other global regions.

Study 2: instrumental pitch production. Imprecision. Figure 4a presents the distributions of pitch-class imprecision across the instruments studied. The expected trend of organ < flute < trombone < voice was generally observed. Due to the non-homogeneity of variance in the data (Levene's test, p < 0.01), we tested for differences between instruments using the same multiple-comparisons method as above. The pitch-class imprecision for each instrument differed significantly from every other instrument (p < 0.001), except for the organ and flute, which did not differ (p = 0.5).

Inaccuracy. Figure 4b shows the distributions of pitch-class inaccuracy across the instruments studied. The expected trend for pitch-class inaccuracy (organ < flute < trombone < voice) was not observed, mainly because the voice was more accurate than predicted. Due to the non-normality (Shapiro–Wilk's test, p < 0.01) and non-homogeneity of variance (Levene's test, p < 0.01) in the data, we tested for differences between instruments using the same multiple-comparisons method as above. The pitch-class inaccuracy of the organ was significantly lower than every other instrument (p < 0.001). The pitch-class inaccuracy of the trombone was significantly higher than that of the flute (p < 0.005) and voice (p < 0.05), but the inaccuracy of the flute and voice did not differ significantly (p > 0.5). As above, we compared the variance of the inaccuracy values across instruments using multiple pairwise Levene tests with Bonferroni correction. The organ had significantly lower variance in pitch-class inaccuracy than all of the other instruments. The flute did not differ significantly from either the voice or the trombone. However, the voice showed a significantly lower inaccuracy variance than the trombone. These results generally support the previous conclusion: the organ was the most accurately tuned, whereas the trombone was the least accurately tuned. In summary, the results for both pitch-class imprecision and inaccuracy mostly followed the expected trends, except that the singer was more accurate than expected compared to the trombone.

Imprecision/inaccuracy correlations. Figure 5 examines correlations between pitch-class imprecision (on the x axis) and pitch-class inaccuracy (on the y axis) for both studies. Figure 5a demonstrates that vocal pitch-
class imprecision and inaccuracy were relatively uncorrelated in the global ethnographic sample from Study 1, and that there were no clear regional clusters. The global Pearson's correlation between these two variables was 0.06 (t = 1.20, df = 415, p = 0.23, dashed line in Fig. 5a). The correlation was strongest in East Asia (r = 0.31), Africa (r = 0.30), and Central Asia (r = 0.30), and weakest in the Middle East-North Africa (r = 0.08).

Figure 5b shows the correlations for Study 2, and provides further support for the observation that vocal pitch-class imprecision and inaccuracy are only weakly correlated, even for the trained Western singer (Pearson's correlation = 0.25, t = 2.29, df = 6, p = 0.55), and especially compared to the fixed-pitch instruments. The overall correlation between pitch-class imprecision and inaccuracy in Study 2 was 0.39 (t = 2.29, df = 30, p = 0.03). But when the voice data was removed, the correlation between instrumental pitch-class imprecision and inaccuracy was 0.67 (t = 4.29, df = 22, p < 0.001). Unlike in Study 1, relatively clear clusters are present in the data from Study 2, suggesting that the relationship between pitch-class imprecision and inaccuracy can be used as an instrumental profile. Fixed-pitch instruments (including the organ and flute) are both accurate and precise, whereas variable-pitch instruments (including the trombone) are both less precise and less accurate. In contrast, the voice is relatively imprecise, regardless of the singer's accuracy.

Discussion
In the present study, we carried out the first analysis of vocal imprecision and inaccuracy in a global corpus of indigenous vocal songs, covering the principal musical-style regions of the world, as based on Lomax's seminal work28. The study moves beyond the circularity of tunable instruments being engineered to produce music according to mathematically-specified templates, and instead investigates the actual pitch properties of indigenous vocal songs. The main take-aways of our analyses are clear. Study 1 indicated that vocal imprecision in sung pitch production is universal, with the range of vocal pitch-classes averaging just beyond 1.5 semitones worldwide. In addition, it demonstrated that our sample of indigenous and traditional music did not conform to the mathematical predictions of 12-ET. Study 2 supported the observation that vocal imprecision is universal, to their degree of pre-tuning, where the voice is both the least tunable and the least precise instrument studied here. As such, these results provide strong evidence for universal vocal imprecision and a dearth of evidence for mathematically-based vocal tuning worldwide, which collectively support physiology-based models of music evolution like the Interval Spacing theory4,6. The results also indicate that future models of scale structure, beyond proposing universal physiological mechanisms, must also account for the enculturating effects that long-term musical training may exert on vocal physiology, especially with regard to tuning accuracy.

The results of the present study point to a fundamental and universal constraint that has shaped the nature of musical scales from their probable origin in vocal music. Harmonicity theory cannot account for either the number of tones in a scale or the spacing between successive scale tones. Previous cross-cultural analyses have demonstrated a preference for scales having 5–7 tones per octave and a predominant spacing of a whole tone between adjacent scale tones24,30. In Study 1, we found a global pitch-class imprecision of 155 cents (just greater than 1.5 semitones). Scale-steps smaller than this would cause adjacent pitch-class distributions to overlap, leading to a reduction in pitch distinguishability and musical communication. Therefore, if scale tones have a tendency to be at least 1.5 semitones apart on average, then this constrains scales to have fewer than 8 pitch-classes per octave (1200 cents in an octave / 155 cents of vocal imprecision = 7.8 pitch-classes). Likewise, a pitch-class
imprecision of 155 cents creates a driving force to distance successive scale pitches by steps closer to 200 cents (a whole tone) than 100 cents (a semitone) or less, which jibes with the observed cross-cultural trends. In other words, our results provide a physiological rationale for why semitones are considered to be the smallest scale interval in world musics, a phenomenon that is in no way explainable by Harmonicity theory.

While our analysis of the typological properties of the scales in this corpus will be presented elsewhere, we report here that the mean intervallic spacing between successive pitches in the 418 scales was 2.2 semitones, even larger than the 1.5-semitone imprecision in pitch-classes. This raises the important point that the true constraint on scale structure might not stem from the imprecision of pitch-classes per se, but instead the imprecision of interval-classes, since intervals define the actual pitch movement of the voice during melody production. As mentioned in the Introduction, scales are an abstraction of melodies, not the other way around. We are currently in the process of developing a computational method for analyzing interval-class imprecision. It is quite likely, as per the results of Pfordresher and Brown, that interval-class imprecision for this sample would exceed 1.5 semitones and be more in line with cross-cultural trends for the spacing between successive scale tones.

We observed some cross-cultural variation in the pitch-class imprecision data (see Fig. 3a), spanning from a lower mean value of 121 cents in cultures that possess a priori scale theories (the Middle East-North Africa and Central Asia) to a higher value of 170 cents in more-indigenous cultures from the Circumpolar region and the Insular Pacific (e.g., the indigenous Taiwanese populations). This suggests that a certain degree of cultural mediation is superimposed upon the presumed physiological constraint on vocal precision, a supposition that is further supported by the results with the trained Western singer in Study 2 (Fig. 4a). This apparent enculturation effect bears further study.

The claim that scales should have 7 or fewer pitches per octave and favor successive intervals of a whole tone or larger does not, by itself, specify the actual sequences of steps that comprise scales worldwide. In fact, we have observed immense scale diversity both within and between cultures. Scales can combinatorially create a near-infinite variety of scale sequences, even when constrained by vocal interval-spacing considerations. From the standpoint of the current discussion, it will be important to examine if differences in pitch-class imprecision between the cultures mentioned above are correlated with differences in scale properties. We address this issue in our forthcoming typological analysis of the scales in the corpus.

Pitch-class inaccuracy demonstrated a much more uniform trend across cultures than imprecision, with a mean value of 24 cents. The plots in Fig. 3b revealed that inaccuracy values spanned the full extent of the 50-cent bin, suggesting a random pattern with regard to the mathematical template. In addition, we found little evidence of cross-cultural variation in inaccuracy, although Europe—the source of 12-ET—showed slightly lower inaccuracy than the rest of the world. This brings into question a key assumption of tuning theories based on a priori mathematical principles. Vocal music does not seem to show a concern for precise mathematical ratios, which is perhaps not surprising given the “island” nature of pitch-classes (the current study) and interval-classes in vocal music. Overall, imprecision seems to be a stronger constraint in the formation of scale structure than inaccuracy. Despite all of the attention given to perfect ratios in academic accounts of scales, mathematical tuning does not seem to be a critical factor in vocal scales cross-culturally. Scales seem to show flexibility with regard to perfect ratios, especially in indigenous cultures and especially in vocal music. It is important to state that, for cultures that lack literary texts about a priori tuning theories, we have no idea what the actual pitches should be in their music. The best we can do is examine the sung pitches against a mathematical template and determine the extent to which the pitches conform with it. The current study, using the tuning protocol of 12-ET as its template, showed a lack of regard for this tuning system in all world regions.

Although this is, to our knowledge, the largest and most diverse analysis of global song thus far researched, it has a number of key limitations. First, very few recordings of solo traditional songs were obtainable for several world regions. For example, it was difficult to find samples from Central Asia, and especially India, since most vocal recordings from this latter region are of the classical tradition. Defining indigenous and folk musical traditions, in contrast to art and classical traditions, as well as delimiting the geographical regions of various cultures, was generally a challenge for this study. This approach cannot account for within-culture diversity, cultural blending due to migration, and other nuances of cultural identity. Other significant stylistic regions that were not sampled in this study included Melanesia and Polynesia, which we aim to sample in future work.

One of the main advantages of the current project was not only the global sampling of the vocal songs, but the use of Lomax’s 1968 Cantometric map to define the regions of song style with musicological rigor. However, there were limitations in this approach. For example, Lomax’s “Old High Culture” designation was too broad for our purposes, forcing us to segment it into three sub-regions (the Middle East-North Africa, Central Asia, and East Asia). Another limitation is that Lomax revised his own classification scheme in a 1980 book chapter, creating points of difference from the map that we used. For example, Europe became subdivided into Western Europe and Central Europe. In addition, new designations, such as Proto-Melanesia and Nuclear America, were added to the scheme. But perhaps the most fundamental limitation of our current approach to the musical map is that a 10-region division of world musical styles is very coarse-grained in light of the hundreds if not thousands of musical styles that no doubt exist. Therefore, future research in this area, even that restricted to indigenous samples, will need to work towards finer-grained classification of musical styles than was attempted here. However, such classification must be based on well-researched musicological analyses, such as Lomax’s.

Next, there were limitations inherent to our analytical methods. Any qualitative study is prone to subjectivity, and this is especially true for by-ear analyses of non-Western music by Western musicians. Although we consider our main quantitative results to be objective, we ultimately defined the pitch-class boundaries by hand. Three analysts worked in parallel on a subset of the songs, reviewed them together, and then reanalyzed them before moving on to the next subset, and the results of our analyses were indeed consistent across multiple interim checks over roughly a year’s time-span. Still, our particular analysis of each piece is likely irreproducible, as the precise location of pitch-class boundaries might change depending on the analyst—or even within a single
evolution of music\textsuperscript{41, 42}. optimize their structure according to the physiological limitations of the earliest human music, namely song\textsuperscript{5, 6}. as these underlie the Interval Spacing model of the origin of musical scales, which posits that scales evolved to rationale for why semitones are considered to be the smallest scale interval in world musics. Observations such scale tones and an upper limit on the number of scale tones within an octave. They also provide a physiological on the formation of musical scales by simultaneously providing a lower limit on the spacing between adjacent a pitch range of around 1.5 semitones, regardless of culture. Such imprecision provides a significant constraint at melodies produced by tunable instruments such as chordophones, the study has examined ethnographic field ratios, it has looked to bottom-up physiological mechanisms of vocal production. Likewise, instead of looking starting with the top-down mathematical principle that scales optimize their structure according to harmonic constraints may have offered various affordances for scale structure over the course of scale evolution. For example, the need to entrain with others, and/or to harmonize with them, may have encouraged the use of different musical textures. The harmonies emerging from these textures may have shaped musical scales beyond the simple melody-based forms that we studied here. Hence, we view Interval Spacing theory as a fundamental theory of the origin of musical scales, one that can be used to build models of musical tonality\textsuperscript{40} and the cultural evolution of music\textsuperscript{41, 42}. Overall, the study has provided a novel approach to investigating the evolution of musical scales. Instead of starting with the top-down mathematical principle that scales optimize their structure according to harmonic ratios, it has looked to bottom-up physiological mechanisms of vocal production. Likewise, instead of looking at melodies produced by tunable instruments such as chordophones, the study has examined ethnographic field recordings of vocal music from indigenous cultures, doing so from all of the major musical-style regions of the world. The results revealed a universal propensity for singers to produce pitches in an imprecise manner, with a pitch range of around 1.5 semitones, regardless of culture. Such imprecision provides a significant constraint on the formation of musical scales by simultaneously providing a lower limit on the spacing between adjacent scale tones and an upper limit on the number of scale tones within an octave. They also provide a physiological rationale for why semitones are considered to be the smallest scale interval in world musics. Observations such as these underlie the Interval Spacing model of the origin of musical scales, which posits that scales evolved to optimize their structure according to the physiological limitations of the earliest human music, namely song\textsuperscript{6, 8}. Our companion analysis will look at the intervallic and structural properties of the 418 scales. Future work should aim to identify the physiological basis of pitch-class imprecision and inaccuracy, and posit mechanisms underlying the refinement of vocal control that can occur in various forms of musical training, expertise, and enculturation.

Data availability
All materials for this study are available at the following public Open Science Framework repository: https://osf.io/jn78c/?view_only=89846926c138478dad2990e833fb5e38.

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Author contributions

Both authors contributed to the study design. E.P. wrote the computational code, conducted the study, and analyzed the results. Both authors contributed to data interpretation, preparation of figures, and the final manuscript.

Competing interests

The authors declare no competing interests.
