MATHEMATICAL ANALYSIS OF ALTERNATIVE POWER SYSTEMS IN A NORTHERN WISCONSIN HOME

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ABSTRACT
The effects of global warming have been causing irreversible damage to the Earth over the past decades. One way we can reduce our individual impact on the environment is to focus on the production and consumption of eco-friendly sources of electricity such as solar, wind, or bioelectric energy. Based on the current available and suitable energy options, we have proposed to modify a house from Northern Wisconsin by using a hybrid system including solar energy and wind turbine energy to replace the traditional power grid. Based on the sunlight-time, wind speed in Northern Wisconsin, we have projected the immediate and long-term fiscal and environmental impacts. From the cost-benefit analysis mathematical model, we have found that the best hybrid option is a combination of 95% solar power and 5% wind energy, which would save about $43,000 over the next 30 years.

INTRODUCTION
Within the past century, the global temperature has increased by 1.33 °Fahrenheit (0.74 °Celsius) (NASA, 2018). The effects of global warming have been devastating these days. To minimize the damage of climate change, our research team focus on studying the feasibility, immediate and long-term fiscal and environmental impacts of using the renewable energy to replace a traditional grid to power a house located in the Northern Wisconsin.

METHODS

◇ 30-year Electric Bill Estimation
First, we have used the inflation rate to project the electricity price. The USDA predicts that average annual inflation will remain stable at 2.3% for the next 30 years (USDA Economic Research Service, 2018).

The projected the electricity price ($/kWh) over the next 30 years is

\[ y = 0.1325 \times (1 + 2.3\%)^{x-2019} \]  

(1)

The estimated total electric bill over the next 30 years

\[ \sum_{x=2019}^{2048} \text{Annual Electric Usage} \times y \]  

(2)

\[ = 11148(0.1325 + 0.1325 \times 1.023 \times \cdots + 0.1325 \times 1.023^{27}) \]  

\[ = 62821.78 \]

Then, we calculated the cost of installing the solar energy system and wind energy system.

◇ Solar Energy System
The amount of electricity generated by the solar system is determined by the power and peak sun-hour.

The whole peak sun hours

\[ \times \text{the number of days in each month} \]  

\[ = 1596.296 \text{ hours} \]

The output of mechanical power captured from wind by a wind turbine (Pragya Nema 2009) can be formulated as

\[ P_1 = \frac{C_P A V^2}{2} \]  

(3)

The torque developed by a wind turbine can be expressed as

\[ T_1 = \frac{P_1}{\eta} \]  

Where \( P_1 \) is the output power, \( T_1 \) is the torque developed by wind turbine, \( C_P \) is the power coefficient, \( \eta \) is the tip speed ratio, and \( p \) is the air density in kg/m³. \( A \) is the frontal area of wind turbine and \( V \) is the wind speed. The standard power of each wind turbine is 0.4kW when the wind speed is 12.5 m/s (Phil Taylor-Parker, 2018). Thus, the relationship between power and time is,

\[ P(t) = 0.0002048 \times (0.2359375 + 0.88291)^3 \]  

(5)

Finally, we have the amount of electricity generated in the whole year is

\[ E = 0.39818 \times 365 \text{ days} = 145.3441 \text{ kWh} \]  

(6)

RESULTS
We see that there is a huge cost increase when the energy from solar power usage increases from 90% to 100%. Therefore, we proposed to use the wind power system to generate the 90% of the energy for the chosen household. We combine two systems and get the chart of the total cost of energy system when 90%~100% of electricity is provided by solar power.

DISCUSSION
Based on the research we have done, we concluded that the most effective way of producing energy for our chosen residence is a combination of solar and wind energy. When comparing this system to the environmental impact of the current energy system, the benefits become very obvious. Solar and wind energy produces a negligible amount of greenhouse gases compared to coal and other energy sources currently being used at this residence. Wisconsin gets 49.4% of its electricity from fossil fuels, which produce harmful gases like carbon dioxide and sulfur dioxide (Kevin, Klien & Reindl, 2010).

Based on both the price and the environmental impact, it is much more effective to install a combination of solar and wind power instead of staying with our current system. This may not be the most effective system for every house, but it does work for this specific residence. If more resources were used to improve the effectiveness of different forms of clean energy, the amount of carbon emissions would be greatly reduced.

As more research goes into alternate energy sources, this may or may not remain constant. If more clean energy sources are explored, we may discover that another way of generating power is better—either geothermal, hydropower, or biogas. With the resources available now, solar and wind is most effective, but we believe the future will surpass our current knowledge.

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Figure 1 Power of solar energy needed based on the percentage of the total consumption of energy

Figure 2 Total cost of solar panels and inverters

Figure 3 Wind Energy system

Figure 4 Total cost of energy system when 90%~100% of electricity is provided by solar power

Figure 5 Hybrid Power Systems (Energy Saver, 2015)