Periodic Trajectory of Relative Motion Controlled by Constant Thrust

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Abstract. Formation flying has received widespread attention in recent years. However, the conventional control approaches require high demand for engine. To deal with this problem, the relative periodic trajectory can be achieved by multiple two-section constant thrust control proposed in this study. Firstly, the concept of the constant thrust control is presented. Furthermore, the strategy is presented to guarantees periodic trajectory considering J2 perturbation. Simulations are conducted to demonstrate the efficacy of the method. The proposed control approach is applied for engineering applications.

1. Introduction
In recent years, formation flying technology has received wide attention missions [1–2]. In order to obtain precise relative motion models, lots of researches have been conducted [3–7]. Spacecraft formation flying refers to one or more deputy spacecraft flying around a chief spacecraft within a bounded distance. Scheeres [8] proposed the hovering formation for inertial-frame system. Wang et al. [9] developed continuous-low-thrust control to maintain fast fly-around formation. Furthermore, Zhu et al. [10] presented optimal finite-thrust control approach. However, these methods require a time-varying continuous-thrust engine. To overcome these difficulties, Rao et al. [11] proposed a 3-D “droplet” hovering formation. However, these hovering formations are only suitable for two-body case, which limits application for engineering.

The main objective of this study is to design control approach to keep the chasing spacecraft along the periodic trajectory. In this paper, we assume that the chief spacecraft is not affected by J2 perturbation. Firstly, the concept of the two-section thrust control is presented, which can be transformed into an optimization problem to solve. Then, the control strategy based on two-section thrust control is proposed for circular reference orbit. Finally, the effectiveness of the proposed method is verified with numerical simulations and the results indicate that the presented approach can provide a practical choice for engineering applications.

The outline of this paper is as follows. The concept and mathematical model of two-section thrust control is introduced in Sec. II, and the control strategy based on two-section thrust control is given in Sec. III. The proposed method is verified with numerical simulations in Sec. IV. Finally, Sec. V summarizes the paper.

2. The conception and mathematical model of two-section constant thrust control equations
The classical perturbation equations can be expressed as:

\[
\frac{da}{dt} = 2 \frac{a^3 v}{\mu} f_i, \\
\frac{de}{dt} = \frac{1}{v} \left[ 2 (e + \cos \theta) f_i - \frac{r}{a} \sin \theta f_n \right], \\
\frac{di}{dt} = \frac{r}{h} \cos (\omega + \theta) f_h, \\
\frac{d\Omega}{dt} = \frac{r}{h \sin i} \sin (\omega + \theta) f_h, \\
\frac{d\omega}{dt} = \frac{1}{e} \left[ 2 \sin \theta f_i + \left( \frac{2e}{v} + \frac{r \cos \theta}{av} \right) f_n - \frac{r \sin (\omega + \theta) \cos i}{h \sin i} f_h \right], \\
\frac{dM}{dt} = n - \frac{dM_f}{dE} = n - \frac{b}{eav} \left[ 2 (1 + \frac{e^2 r}{p}) \sin \theta f_i + \frac{r}{a} \cos \theta f_n \right].
\]

(1)

Where \(f_i, f_e, f_h\) denote the components of the perturbation acceleration in the TNH frame [12].

The so-called constant thrust means that the magnitude and direction of the applied thrust remain unchanged in the TNH frame. Furthermore, the two-section constant thrust control refers to two constant thrust applied respectively for the first and second half of the control time. In addition, the similar results considering J2 perturbation can be obtained when equations (1) adding J2 term.

As shown in Fig. 1, the blue curve and the yellow curve represent the trajectory of the spacecraft when the thrust acceleration are \((f_{i1}, f_{e1}, f_{h1})\) and \((f_{i2}, f_{e2}, f_{h2})\) respectively. With the two-section control, the spacecraft arrives \(X_{i_f}\) exactly at \(t_f\).

![Figure 1. Two-section constant thrust control.](image)

However, these constant thrust forces cannot be solved by analytic formulas because of the strong nonlinearity of the equations (1). Therefore, the thrust solution must be solved accurately by the numerical method. Let consider a nonlinear least-squares optimization problem:

\[
\min_{f_{i0}, f_{e0}, f_{h0}} F = \min_{f_{i1}, f_{e1}, f_{h1}} \left\| X_e - X \left( \vec{f}, t_f \right) \right\|^2
\]

(2)

Where \(f_{i0}, f_{e0}, f_{h0}\) are optimization variables. If \(\min F = 0\), (2) is equal to equations (1). Thus, the optimum solution for (2) can be regard as the solutions for (1).

3. Periodic trajectory for circular reference orbit on considering J2 perturbation

Based on ROEs, Rao et al. [11] proposed a 3-D “droplet” hovering formation with impulsive control strategies. The formation can be expressed by a complete set of parameters \((x_h, y_h, z_h, \Delta u)\).
($x_h, y_h, z_h$) denotes the relative position at the vertex of the “droplet”, and $\Delta u$ represents the period of the hovering formation.

**Figure 2. 3-D hovering formation proposed by Rao [11].**

As shown in Fig. 2, the Rao’s hovering configuration composes of trajectory $C_1^k C_2^k$. In each cycle, the chasing spacecraft moves along $C_1^i C_2^k$ and $C_2^k C_1^{i+1}$ without any control, while travels at point $C_2^k$ with an instantaneous impulse. The spacecraft starts from vertex $C_1^i (C_1^i)$ and then moves back to $C_1^{i+1} (C_1^{i+1})$ in a hovering period $\Delta u$.

However, the Rao’s hovering formation is designed only for two-body case, which is not suitable for J2 case. In order to form a completely coincident periodic relative trajectory, the chief spacecraft needs to assume that it is not affected by J2 perturbation. Furthermore, to ensure periodic revisits when considering J2 perturbation, the relative position and velocity vectors of spacecraft should be the same at the vertex $C_1^k$, which can be achieved by two-section constant thrust control. Thus, it is only to solve the two-point boundary value problem regarding the time and states of two adjacent vertices $C_1^k$ and $C_1^{k+1}$.

However, despite the periodic relative trajectory, the J2 perturbation still changes the shape of the trajectory and makes it much larger than Rao’s hovering formation, which causes that the parameters $(x_h, y_h, z_h, \Delta u)$ cannot describe the periodic trajectory. In order to ensure that the shape of the formation is close to Rao’s formation, the multiple two-section constant thrust control is needed in each period. That is, except for the vertex $C_1^k$, some “control points” of two-body periodic trajectory are selected so that the spacecraft can revisits these points periodically. The more control points are selected, the closer the trajectory is to the Rao’s formation. Generally, the hovering periods are divided into equal parts and then the corresponding points at each moment are regarded as control points.

**Figure 3. 3-D periodic trajectory controlled by multiple two-section constant thrust.**
As shown in Fig. 3, the periodic trajectory represented by red curve includes vertex $b_0^i$ and four control points $(B_1^i, B_2^i, B_3^i, B_4^i)$. For each part, a two-section constant thrust control is necessary to maintain the shape of the periodic trajectory. The multiple-two-section constant thrust control for each part can be solved by equations (2).

4. Simulation Results
To demonstrate the advantages of the constant thrust proposed in this paper, two numerical simulations are carried out, and the reference orbit is circular orbit. First, the periodic trajectory controlled by two-section constant thrust is presented, which proves the realizability of periodic trajectory with proposed control method. Then, another simulation is carried out to validate the effectiveness of multiple-two-section constant thrust control strategy maintaining the shape of the Rao’s formation when considering J2 perturbation. Table 1 and Table 2 show the orbit elements of the circular reference satellite (orbit) and parameters of the periodic trajectory.

Table 1. Orbit elements of the circular reference satellite

| reference satellite | $a$, m  | $e$   | $i$, deg | $\Omega$, deg | $\omega$, deg | $M$, deg |
|---------------------|--------|-------|----------|----------------|--------------|----------|
| Satellite A         | 7755000| 0     | 45       | 0              | 0            | 0        |

Table 2 Parameters of the periodic trajectory

|                     | Value |
|---------------------|-------|
| $x_h$, m            | 500   |
| $y_h$, m            | 500   |
| $z_h$, m            | 500   |
| $\Delta\mu$, deg    | 120   |

Fig.4 illustrated the ten-loop of the periodic trajectory controlled by two-section constant thrust. When the reference satellite(orbit) is assumed not subjected by J2 perturbation, periodic trajectories can be formed if $2\pi$ is an integral multiple of $\Delta\mu$. In this simulation, $2\pi = 3\Delta\mu$ and thus the whole relative trajectory takes three loops of the periodic trajectory as a cycle. Therefore, there are only three loops of the periodic trajectory in Fig.4. However, despite the periodic relative trajectory, the J2 perturbation still changes the shape of the trajectory and makes it much larger than Rao’s hovering formation.

Figure 4 Relative trajectory of the periodic trajectory in 3-D space
To ensure that the shape of the periodic trajectory is close to that of Rao’s formation, multiple two-section constant thrust controls are needed in each loop. Fig. 5 illustrated the Rao’s formation controlled by impulse method (blue) and periodic trajectory controlled by multiple two-section constant thrust method (red) for J2 case respectively. The shape of the periodic trajectory under J2 perturbation is close to the Rao’s formation. Thus, multiple two-section constant thrust control method is validated as an effective method for realizing Rao’s formation for J2 case.

5. Conclusions
The control approaches to new periodic relative trajectory are validated by two simulations. To ensure periodic revisits to specific relative position considering J2 perturbation, the multiple two-section constant control strategy is presented to achieve Rao’s hovering formation maintenance. Compared with the conventional control strategy, two-section constant control strategy is much more convenient because in each section the thrust is a constant vector. Thus, the proposed control approach can provide practical choice for engineering applications.

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7. References
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