Analytic estimates of quenched penguins

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When we embed the strong penguin operator \(Q_6\) in the quenched theory, it does not remain a singlet under right-handed chiral transformations. As a consequence, more low-energy constants associated with this operator appear than in the unquenched theory. We give analytic estimates of the leading constants. The results suggest that the effects of quenching on this operator are large.

The gluonic penguin operator

\[
Q_6 = \frac{1}{2} Q_6^{QS} + Q_6^{QNS},
\]

\[
\begin{align*}
Q_6^{QS} & = 2(\bar{q}_L \gamma_\mu d_R^k) (\bar{q}_L^\alpha q_R^\beta), \\
Q_6^{QNS} & = 2(\bar{q}_L^\beta q_R^\alpha) (\bar{q}_L^\beta q_R^\alpha - \bar{q}_L^\alpha q_R^\beta),
\end{align*}
\]

where \(q\) is summed over \(u, d, s\), and \(\bar{q} = \bar{u}, \bar{d}, \bar{s}\) are the bosonic (ghost) quarks used to define the quenched theory. We see that while \(Q_6^{QS}\) is a singlet under the quenched flavor group SU(3)\(_R\), \(Q_6^{QNS}\) is not.

To leading order in chiral perturbation theory (ChPT), these operators maybe represented by

\[
\begin{align*}
Q_6^{QS} & \rightarrow -\alpha_{q_1}^{(8.1)} \text{str} (\Lambda L_\mu L_\mu) \\
& \quad + \alpha_{q_2}^{(8.1)} \text{str} (2 B_0 \Lambda (\Sigma M + M \Sigma^\dagger)), \\
Q_6^{QNS} & \rightarrow f^2 \alpha_{q_3}^{NS} \text{str} (\hat{N} \Sigma \hat{N} \Sigma^\dagger),
\end{align*}
\]

\(\hat{N} = \frac{1}{2} \text{diag}(1, 1, 1, -1, -1, -1)\),

with \(\bar{q}\Lambda q = \bar{q}d\), \(M\) the quark-mass matrix, and \(\text{str}\) is the supertrace. \(\hat{N}\) represents the non-singlet structure of \(Q_6^{QNS}\). The low-energy constants (LECs) \(\alpha_{q_1,2}^{(8.1)}\) correspond to LECs also appearing in the unquenched theory, whereas the new LEC \(\alpha_{q_3}^{NS}\) is a quenched artifact. Note that \(\alpha_{q}^{NS}\) is of order \(p_0^2\), while the others are of order \(p_0^3\).

The new LEC \(\alpha_{q}^{NS}\) shows up for instance in

\[
\langle 0| Q_6^{QNS} | K^0 \rangle = \frac{4i}{f} \left\{ \frac{1}{2} \alpha_{q_1}^{(8.1)} + \beta_{q}^{NS} \right\} (M_K^2 - M_\pi^2) + \frac{\alpha_{q_3}^{NS}}{16 \pi^2} \sum_{q=u,d,s} \left( M_{sq}^2 \log M_{sq}^2 - M_{dq}^2 \log M_{dq}^2 \right),
\]

\[
M_{qq'}^2 = B_0 (m_q + m_{q'}),
\]

where \(\beta_{q}^{NS}\) is a higher-order LEC associated with \(Q_6^{QNS}\). The appearance of \(\alpha_{q}^{NS}\) is important because it influences the determination of \(\alpha_{q_2}^{(8.1)}\) from a simulation, but it is hard to determine \(\alpha_{q}^{NS}\) this way in practice.

However, there is a trick which makes \(\alpha_{q}^{NS}\) more easily accessible. One rotates \(d_L \rightarrow \tilde{d}_L\) in \(Q_6^{QNS}\) (calling this \(\tilde{Q}_6^{QNS}\)), and considers \(K^0 \rightarrow 0\) with \(\tilde{K}^0\) made out of an anti-s quark and a ghost-

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\(\text{arXiv:hep-lat/0309101v1 16 Sep 2003} \)

\(\text{[1]} \) presenter at conference

\(\text{[2]} \) See Ref. \([1]\) for more discussion of this observation.
\(d\) quark. One finds that
\[
\langle 0 | \hat{Q}_6^{QNS} | \tilde{K}^0 \rangle = 2i f \alpha_q^{NS} + O(p^2). \tag{5}
\]

We may now estimate \(\alpha_q^{NS}\) analytically in the following way \[8\]. First we fierz
\[
\hat{Q}_6^{QNS} = -4 \left( \bar{\pi} P_R q \bar{\pi} P_L \tilde{d} + \bar{\pi} P_R \tilde{q} \bar{\pi} P_L \tilde{d} \right) \tag{6}
\]
(taking into account that the ghost fields commute), and then Wick contract to find
\[
\langle 0 | \hat{Q}_6^{QNS} | \tilde{K}^0 \rangle = (\bar{s}s - \bar{d}d) \langle 0 | \bar{\gamma}_5 \tilde{d} | 0 \rangle \tag{7}
\]
\[-4 \langle 0 | (\bar{\pi} P_R \bar{q} \bar{q} \bar{P}_L \tilde{d} + \bar{\pi} P_R \bar{q} \bar{q} \bar{P}_L \tilde{d}) | \tilde{K}^0 \rangle,
\]
correct to order \(1/N_c^2\). It can be shown \[8\] that the unfactorized term (2nd line) is order \(p^2\) in ChPT, and thus it does not contribute to \(\alpha_q^{NS}\). Using that
\[
\bar{d}d = -\bar{s}s = \frac{1}{2} f^2 B_0
\]
(note the minus sign: ghost quarks commute!), one obtains
\[
\alpha_q^{NS} = -\frac{1}{2} f^2 B_0^2 \left( 1 + O \left( \frac{1}{N_c^2} \right) \right) \tag{8}
\]

We now consider the singlet operator \(Q_6^{QS}\), which is interesting because it will yield a result for \(\alpha_q^{(8,1)}\), which we may then compare to its unquenched value, as well as to our estimate of \(\alpha_q^{NS}\). \(Q_6^{QS}\) can be fierzed into
\[
Q_6^{QS} = -8 \left( \bar{\pi} P_R q \bar{\pi} P_L d - \bar{\pi} P_R \tilde{q} \bar{\pi} P_L \tilde{d} \right). \tag{9}
\]

Since the quark and ghost-quark propagators are equal (by construction!), contributions in which \(q\) and \(\bar{q}\) or \(\tilde{q}\) and \(\bar{\pi}\) are contracted cancel. Thus, to order \(1/N_c^2\) only the factorized contribution survives, and leads to the long-known result (with operator mixing taken into account to leading-log order)
\[
\alpha_q^{(8,1)} = -8L_5 f^2 B_0^2 \left( 1 + O \left( \frac{1}{N_c^2} \right) \right) \tag{10}
\]
where \(L_5\) is one of the Gasser–Leutwyler \(O(p^4)\) LECs of the strong effective lagrangian \[7\].

From these results, we draw the following two conclusions. First, \(\alpha_q^{NS}\) is not small compared to \(\alpha_q^{(8,1)}\):
\[
\frac{\alpha_q^{NS}}{\alpha_q^{(8,1)}} = \frac{1}{16L_5} \sim 60, \tag{11}
\]
using that \(L_5 \sim 10^{-3}\) (in quenched \[9\] and unquenched \[7\] QCD).

- \(\alpha_q^{NS}\) can thus not be ignored in any quenched computation which involves \(Q_6^{QNS}\).

In the unquenched case, it was found that the unfactorized contribution vanishes in the quenched case,

- the quenched value of \(\alpha_q^{(8,1)}\) maybe substantially smaller than the unquenched one.

We close with a few remarks.

First, these estimates maybe extended to the partially quenched case, in which \(N\) sea quarks are added to the quenched theory, see Ref. \[8\].

Second, to extract leading order LECs, one works in the chiral limit. While the quenched theory is probably singular in the chiral limit, we believe that this is not a problem for the calculation of \(\alpha_q^{NS}\) and \(\alpha_q^{(8,1)}\).

Finally, we believe that it should be instructive to adapt analytic estimates of other hadronic quantities to the quenched and partially quenched cases. This will give quantitative information about the effects of quenching, which ChPT by itself cannot provide.

Acknowledgements

MG was supported in part by the US Dept. of Energy, and SP was supported by CICYT-FEDER-FPA2002-00748, 2001 SGR00188 and by TMR EC-Contracts HPRN-CT-2002-00311 (EU-RIDICE).

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