Time-odd mean fields in covariant density functional theory
I. Non-rotating systems.

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Time-odd mean fields (nuclear magnetism) are analyzed in the framework of covariant density functional theory (CDFT). It is shown that they always provide additional binding to the binding energies of odd-mass nuclei. This additional binding only weakly depends on the RMF parametrization reflecting good localization of the properties of time-odd mean fields in CDFT. The underlying microscopic mechanism is discussed in detail. Time-odd mean fields affect odd-even mass differences. However, our analysis suggests that the modifications of the strength of pairing correlations required to compensate for their effects are modest. In contrast, time-odd mean fields have profound effect on the properties of odd-proton nuclei in the vicinity of proton-drip line. Their presence can modify the half-lives of proton-emitters (by many orders of magnitude in light nuclei) and affect considerably the possibilities of their experimental observation.

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I. INTRODUCTION

The development of self-consistent many-body theories aiming at the description of low-energy nuclear phenomena provides necessary theoretical tools for an exploration of the nuclear chart into known and unknown regions. Theoretical methods (both relativistic and non-relativistic) formulated within the framework of density functional theory (DFT) and effective field theory (EFT) are the most promising tools for global investigation of the properties of atomic nuclei. The DFT and EFT concepts in nuclear structure models have been extensively discussed in a number of recent articles [1, 2, 3]. The power of the models based on these concepts is essentially unchallenged in medium and heavy mass nuclei where 'ab-initio' type few-body calculations are computationally impossible and the applicability of spherical shell model is restricted to a few regions in the vicinity of the doubly shell closures.

The self-consistent mean-field approach to nuclear structure represents an approximate implementation of Kohn-Sham density functional theory (DFT) [4, 5, 6, 7], which is successfully employed in the treatment of the quantum many-body problem in atomic, molecular and condensed matter physics. The DFT enables a description of the nuclear many-body problem in terms of a universal energy density functional, and mean-field models approximate the exact energy functional, which includes all higher-order correlations, with powers and gradients of ground-state nucleon densities. Although it models the effective interaction between nucleons, a general density functional is not necessarily related to any nucleon-nucleon (NN) potential. By employing global effective interactions, adjusted to reproduce empirical properties of symmetric and asymmetric nuclear matter, and bulk properties of some spherical nuclei, the current generation of self-consistent mean-field methods has achieved a high level of accuracy in the description of ground states and properties of excited states in arbitrarily heavy nuclei, exotic nuclei far from β-stability, and in nuclear systems at the nucleon drip-lines (see Refs. [8, 9, 10] and references therein).

The self-consistent methods (such as Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB)) based on zero range Skyrme forces or finite range Gogny forces are frequently used in nuclear structure calculations [8, 9]. These approaches represent non-relativistic energy density functionals based on the Schrödinger equation for many-body nuclear problem [8].

On the other hand, one can formulate the class of relativistic models based on the Dirac formalism, which can generally be defined as covariant density functionals (CDF) [9]. These models, such as quantum hadrodynamics (QHD) [12, 13], are based on concepts of non-renormalizable effective relativistic field theories and DFT, and they provide a very interesting relativistic framework for studies of nuclear structure phenomena at and far from the valley of β-stability [9]. Relativistic mean field (RMF) models [12] are analogs of the Kohn-Sham formalism of DFT [8], with local scalar and vector fields appearing in the role of local relativistic Kohn-Sham potentials [1, 12]. The exact energy density functional is approximated with powers and gradients of auxiliary meson fields or nucleon densities. The EFT building of the energy density functional allows error estimates to be made, provides a power counting scheme which separates long- and short-distance dynamics and, therefore, removes model dependences from the self-consistent mean field approach [14]. In the description of nuclear ground states and properties of excited states the self-consistent mean-field implementations of quantum hadrodynamics, the relativistic Hartree-Bogoliubov model (RHB) and the relativistic (quasiparticle) random phase approximation (RQRPA) and their subversions, are employed [9].

The mean field is a basic concept of every DFT. One can specify time-even and time-odd mean fields [15, 16] dependent on the response of these fields to the action
of time-reversal operator. The properties of time-even mean fields in nuclear density functionals are reasonably well understood and defined \[8, 9\]. This is due to the facts that (i) many physical observables such as binding energies, radii etc. are sensitive only to these fields, and (ii) the model parameters are fitted to such physical observables.

On the other hand, the properties of time-odd mean fields, which appear only in nuclear systems with broken time-reversal symmetry, are still poorly understood. However, it is already known that these fields are important for proper description of rotating nuclei \[15, 16, 17\], band terminations \[18, 19\], magnetic moments \[20\], large amplitude collective dynamics \[21\], fusion process \[22\], the strengths and energies of Gamow-Teller resonances \[23\], binding energies of odd-mass nuclei \[24, 25, 26\] and the additivity of angular momentum alignments \[27\]. They also may play a role in the \( N = Z \) nuclei \[24, 28\] and affect the definition of the strength of pairing correlations \[26, 29\].

There was a dedicated effort to better understand time-odd mean fields in the framework of the Skyrme energy density functional (EDF) theory (see Refs. \[13, 17, 18, 23\] and references therein). On the contrary, much less attention has been paid to these fields in covariant mean fields, which appear only in nuclear systems with broken time-reversal symmetry, are still poorly understood. This is due to the properties of time-even mean fields. The properties of time-odd mean fields modify the properties of odd-odd nuclei. Finally, connections with odd-even mass scatterings are also considered in this work of the RMF realization of the CDFT. The results of the study of these fields in rotating nuclei will be presented in a forthcoming article \[30\] which represents a continuation of the current investigation.

The manuscript is organized as follows. The cranked relativistic mean field theory and its details related to time-odd mean fields are discussed in Sect. \[11\]. Section \[11\] is devoted to the analysis of the impact of time-odd mean fields on binding energies of odd-mass nuclei. The mass and particle number dependences of this impact and their connections with odd-even mass scatterings are also considered. The microscopic mechanism of additional binding in odd-mass nuclei induced by time-odd mean fields is analyzed in Sect. \[14\]. The impact of time-odd mean fields on the properties of proton-unstable nuclei is studied in Sect. \[15\]. Section \[17\] considers how time-odd mean fields modify the properties of odd-odd nuclei. Finally, Sect. \[17\] contains the main conclusions of our work.

## II. THEORETICAL FORMALISM

The results presented in the current manuscript have been obtained using Cranked Relativistic Mean Field (CRMF) theory \[31, 32, 33\]. This theory has been successfully employed for the description of rotating nuclei (see Ref. \[34\] and references therein) in which time-odd mean fields play an important role, but it is also able to describe the nuclear systems with broken time-reversal symmetry in intrinsic frame at no rotation. In this theory the pairing correlations are neglected which allows to better isolate the effects induced by time-odd mean fields. This is because time-odd effects also play a role in pairing channel \[21\]. The CRMF computer code is formulated in the signature basis. As a result, the breaking of Kramer’s degeneracy of the single-particle states is taken into account in a fully self-consistent way. This is important for an accurate description of time-odd mean fields in fermionic channel (see Sect. \[14\]).

Most important features of the CRMF formalism related to time-odd mean fields are outlined below (for more details see Refs. \[31, 33\]) for the case of no rotation (rotational frequency \( \Omega_z = 0 \)).

In the Hartree approximation, the stationary Dirac equation for the nucleons in the intrinsic frame is given by

\[
\hat{h}_D \psi_i = \varepsilon_i \psi_i \tag{1}
\]

where \( \hat{h}_D \) is the Dirac Hamiltonian for the nucleon with mass \( m \)

\[
\hat{h}_D = \alpha (-i \nabla - V(r)) + V_0(r) + \beta (m + S(r)). \tag{2}
\]

It contains the average fields determined by the mesons, i.e. the attractive scalar field \( S(r) \)

\[
S(r) = g_\sigma \sigma(r), \tag{3}
\]

and the repulsive time-like component of the vector field \( V_0(r) \)

\[
V_0(r) = g_\omega \omega_0(r) + g_\rho \tau_3 \rho_0(r) + \frac{1 - \tau_3}{2} A_0(r), \tag{4}
\]

A magnetic potential \( V(r) \)

\[
V(r) = g_\omega \omega(r) + g_\rho \tau_3 \rho(r) + \frac{1 - \tau_3}{2} A(r), \tag{5}
\]

originates from the space-like components of the vector mesons. Note that in these equations, the four-vector components of the vector fields \( \omega^\mu, \rho^\mu, \) and \( A^\mu \) are separated into the time-like \( (\omega_0, \rho_0, \tau_3) \) and space-like \( (\omega^\tau, \omega^\rho, \omega) \), \( \rho = (\rho^\tau, \rho^\rho, \rho) \), and \( A = (A^\tau, A^\rho, A) \) components. In the Dirac equation the magnetic potential has the structure of a magnetic field. Therefore the effect produced by it is called nuclear magnetism (NM) \[31\].

The corresponding meson fields and the electromagnetic potential are determined by the Klein-Gordon equations

\[
\begin{align*}
\{- \Delta + m_\omega^2\} \omega_0(r) & = g_\omega [\rho^\tau_0(r) + \rho^\rho_0(r), \\
\{- \Delta + m_\rho^2\} \rho_0(r) & = g_\rho [\rho^\tau_0(r) + j^\rho_0(r), \\
\{- \Delta + m_A^2\} A_0(r) & = e \rho^\rho_0(r), \\
\end{align*}
\]

\[
\begin{align*}
\{- \Delta + m_\omega^2\} \omega(r) & = g_\omega [\rho^\tau(r) + \rho^\rho(r), \\
\{- \Delta + m_\rho^2\} \rho(r) & = g_\rho [\rho^\tau(r) + j^\rho(r), \\
\{- \Delta + m_A^2\} A(r) & = e \rho^\rho(r).
\end{align*}
\]
with source terms involving the various nucleonic densities and currents

\[ \rho^{n,p}_i(r) = \sum_{i=1}^{N,Z} (\psi_i(r))^{\dagger} \beta \psi_i(r), \quad (12) \]

\[ \rho^\omega_{n,p}(r) = \sum_{i=1}^{N,Z} (\psi_i(r))^{\dagger} \lambda \psi_i(r), \quad (13) \]

\[ j^{n,p}(r) = \sum_{i=1}^{N,Z} (\psi_i(r))^{\dagger} \alpha \psi_i(r), \quad (14) \]

where the labels \( n \) and \( p \) are used for neutrons and protons, respectively. In the equations above, the sums run over the occupied positive energy shell model states only (no-sea approximation) \[33\]. Note that the spatial components of the vector potential \( A(r) \) are neglected in the calculations since the coupling constant of the electromagnetic interaction is small compared with the coupling constants of the meson fields.

The magnetic potential \( V(r) \) in the Dirac equation as well as the currents \( j^{n,p}(r) \) in the Klein-Gordon equations do not appear in the RMF equations for time-reversal systems \[12\]. Similar to the nonrelativistic case, their presence leads to the appearance of time-odd mean fields. Thus, we will use the terms nuclear magnetism and time-odd mean fields interchangeably throughout this manuscript. The magnetic potential is the contribution to the mean field that breaks time-reversal symmetry in the intrinsic frame and induces non-vanishing currents \( j^{n,p} \) (Eq. \(14\)) in the Klein-Gordon equations (Eqs. \(8\) \[10\]), which are related to the space-like components of the vector mesons. In turn, the space-like components of the vector \( \omega \) and \( \rho \) fields form the magnetic potential \[15\] in the Dirac equation. Note that the current \( j^{n,p} \) change the sign upon the action of time-reversal operator \[30\]. The currents \( j^{n,p} \) have relativistic meaning since they are the products of the large (F) and small (G) components of the Dirac spinors \( \psi_i(r) \).

The spatial components of the vector \( \omega \) and \( \rho \) mesons lead to the interactions between possible currents. For the \( \omega \)-meson this interaction is attractive for all combinations (\( pp, nn \) and \( pn \)-currents), and for the \( \rho \)-meson it is attractive for \( pp \) and \( nn \)-currents but repulsive for \( pn \)-currents. Within mean field theory such currents occur only in the situations of broken time-reversal symmetry.

Note that time-odd mean fields related to NM are defined through the Lorentz invariance \[3\] and thus they do not require additional coupling constants: the coupling constants of time-even mean fields are used also for time-odd mean fields.

The currents are isoscalar and isovector in nature for the \( \omega \) and \( \rho \) mesons (Eqs. \(8\) \[10\]), respectively. As a consequence, the contribution of the \( \rho \)-meson to magnetic potential is marginal in the majority of the cases. Thus, time-odd mean fields in the RMF framework depend predominantly on the spatial components of the \( \omega \) meson. Neglecting the contribution of the \( \rho \) meson, one can see than only two parameters, namely, the mass \( m_\omega \) and coupling constant \( g_\omega \) of the \( \omega \) meson define the properties of time-odd mean fields (Eqs. \(33\) \[3\] and \(10\)). Table 1 clearly indicates that these parameters are well localized in the parameter space for all modern non-linear parametrizations of the RMF Lagrangian. This suggests that the parameter dependence of the impact of time-odd mean fields on the physical observables should be quite weak. Indeed, the analysis of terminating states in Ref. \[19\] showed that time-odd mean fields are defined with an accuracy around 15% in the RMF framework.

| Parametrization | \( m_\omega \) | \( g_\omega \) |
|-----------------|----------------|-----------------|
| NL1 (Ref. [35]) | 795.359        | 13.2846         |
| NL3 (Ref. [37]) | 782.501        | 12.8675         |
| NL3* (Ref. [38]) | 782.60         | 12.8065         |
| NL-Z (Ref. [39]) | 780.00         | 12.9086         |
| NL-Z2 (Ref. [40]) | 780.00        | 12.9084         |
| NL-RA1 (Ref. [41]) | 783.00        | 12.9212         |
| NLSH (Ref. [42]) | 783.00         | 12.9451         |
| TMA (Ref. [43]) | 781.95         | 12.8424+3.191A^{-0.4} |

The total energy of the system is given in Refs. \[31\] \[33\]. In order to facilitate the discussion we split it into different terms as \[62\]

\[ E_{\text{tot}} = E_{\text{part}} + E_{\text{cm}} - E_\sigma - E_{\sigma\text{NL}} - E_\omega^T - E_\rho^T \]
\[ - E_\omega^S - E_\rho^S - E_{\text{Coul}}, \quad (15) \]

where \( E_{\text{part}} \) and \( E_{\text{cm}} \) represent the contributions from fermionic degrees of freedom, while other terms are related to mesonic (bosonic) degrees of freedom. In Eq. \(15\)

\[ E_{\text{part}} = \sum_i A_i \varepsilon_i, \quad (16) \]

is the energy of the particles moving in the field created by the mesons (\( \varepsilon_i \) is the energy of \( i \)-th particle and the sum runs over all occupied proton and neutron states)

\[ E_\sigma = \frac{1}{2} g_\sigma \int d^3r \sigma(r) \left[ \rho^{\omega}_n(r) + \rho^{\omega}_p(r) \right], \quad (17) \]

is the linear contribution to the energy of isoscalar-scalar \( \sigma \)-field

\[ E_{\sigma\text{NL}} = \frac{1}{2} \int d^3r \left[ \frac{1}{3} g_2 \sigma^3(r) + \frac{1}{2} g_3 \sigma^4(r) \right], \quad (18) \]

is the non-linear contribution to the energy of isoscalar-scalar \( \sigma \)-field

\[ E_\omega^T = \frac{1}{2} g_\omega \int d^3r \omega_0(r) \left[ \rho^{\omega}_n(r) + \rho^{\omega}_p(r) \right], \quad (19) \]
is the energy of time-like component of isoscalar-vector $\omega$-field

$$E^T_\omega = \frac{1}{2} g_\omega \int d^3 r \rho_0 (r) [j^\rho_0 (r) - j^\rho_0 (r)],$$

(20)

is the energy of the time-like component of isovector-vector $\rho$-field

$$E^S_\rho = -\frac{1}{2} g_\rho \int d^3 r \omega (r) [j^\rho (r) + j^n (r)],$$

(21)

is the energy of the space-like component of the isoscalar-vector $\omega$-field

$$E^S_\omega = -\frac{1}{2} g_\omega \int d^3 r \omega (r) [j^n (r) - j^\rho (r)],$$

(22)

is the energy of the space-like component of isovector-vector $\rho$-field

$$E_{\text{Cont}} = \frac{1}{2} e \int d^3 r A_0 (r) \rho^a_0 (r),$$

(23)

is the Coulomb energy

$$E_{\text{cm}} = -\frac{3}{4} 41 A^{-1/3} \text{MeV},$$

(24)

is the correction for the spurious center-of-mass motion approximated by its value in a non-relativistic harmonic oscillator potential.

The CRMF equations are solved in the basis of an anisotropic three-dimensional harmonic oscillator in Cartesian coordinates characterized by the deformation parameters $\beta_0 = 0.3$ ($\beta_0 = 0.5$ in the case of superdeformed states) and $\gamma = 0^\circ$ as well as oscillator frequency $h\omega_0 = 41 A^{-1/3}$ MeV. The truncation of basis is performed in such a way that all states belonging to the shells up to fermionic $N_F=12$ and bosonic $N_B=16$ are taken into account; numerical analysis indicates that this truncation scheme provides sufficient numerical accuracy. Single-particle orbitals are labeled by $[Nn_\pi A] \Omega^{\pm \text{sign}}$, [$Nn_\sigma A] \Omega$ are the asymptotic quantum numbers (Nilsson quantum numbers) of the dominant component of the wave function. The superscripts sign to the orbital labels are used sometimes to indicate the sign of the signature $\pi$ for that orbital ($\pi = \pm i$). The majority of the calculations are performed with the NL3 parametrization \cite{37} of the RMF Lagrangian.

In order to investigate the impact of NM (time-odd mean fields) on physical observables, the CRMF calculations are performed with and without NM (the later will be further denoted as WNM). This is only the way in which the effects of time-odd mean fields can be studied and as such it is frequently used in the DFT studies, both in relativistic and non-relativistic frameworks \cite{15, 16, 19, 20, 21, 22, 23, 32, 40}. One should however keep in mind that if time-odd fields are neglected, the local Lorentz invariance (Galilean invariance in non-relativistic framework \cite{15, 47}) is violated. The inclusion of time-odd mean fields restores the Lorentz invariance.

The calculations of odd and odd-odd nuclei require that only one state among two time-reversal counterpart states is occupied in proton or/neutron subsystem(s). This is achieved in the CRMF code by occupying one signature ($r = + i$ or $r = - i$) of the single-particle state, and keeping opposite signature empty. We will call the occupied signature as blocked state in order to simplify the discussion.

### III. BINDING ENERGIES IN ODD MASS NUCLEI

The time-reversal invariance is conserved in the ground states of even-even nuclei. The nucleon states are then pairwise degenerated, and the contribution of the state to the currents cancels with the contribution of its time-reversed partner. Time-odd mean fields reveal themselves in odd- and odd-odd mass nuclei and in two-(multi-)quasiparticle states of even-even nuclei. This is because unpaired (odd) nucleon breaks time-reversal invariance in intrinsic frame and produces the contribution to the currents and spin. In this case, the Kramer’s degeneracy of time-reversal partner orbitals is also broken.

The modifications of the binding energies and quasiparticle spectra are most important issues when considering time-odd mean fields in non-rotating systems. The binding energies are important in nuclear astrophysics applications \cite{48}, and their modifications due to time-odd mean fields may have considerable consequences for the $r$- and $rp$-process abundances. They are also one of the most valuable input data for the parameter fit of the nuclear structure models. Thus, it is important to understand the influence of time-odd mean fields on binding energies of odd- and odd-odd mass nuclei especially in the context of mass table fits \cite{49}. With current focus on spectroscopic quality DFT \cite{50}, a knowledge on how the time-odd mean fields influence the relative energies
of different (quasi)particle states in model calculations is also needed.

While there was a considerable interest in the study of time-odd mean fields in odd- and odd-odd mass nuclei at no rotation within the Skyrme EDF \cite{24, 25}, relatively little is known about their role in the framework of the CDFT. So far the impact of time-odd mean fields on binding energies has been studied in the CDFT framework only in odd-mass nuclei around doubly magic spherical nuclei in Ref. \cite{51}, and in few deformed nuclei around \( ^{32}\text{S} \) \cite{52} and \( ^{254}\text{No} \) \cite{20}. Only last two publications treat time-odd mean fields in a fully self-consistent manner; the breaking of Kramer’s degeneracy is neglected in Ref. \cite{51}.

### A. Binding energies in light nuclei

The impact of NM on the binding energies of light odd-mass nuclei is shown in Fig. 1. One can see that in all cases the presence of NM leads to additional binding, the magnitude of which is nucleus and state dependent. The absolute value of this additional binding is typically below 200 keV and only in some lower mass nuclei it reaches 300 keV. On the average, the magnitude of additional binding due to NM correlates with the mass of nucleus; it is largest in lightest nuclei and smallest in the heaviest nuclei. For each isotope chain, it is largest in the vicinity of the proton-drip line and smallest in the vicinity of neutron-drip line. Fig. 2 shows that it only weakly depends on the RMF parametrization; this is also seen in the analysis of terminating states in Ref. \cite{19}. In both cases, the biggest deviation from the NL3 results is observed in the case of the NLSH parametrization.

It is interesting to compare these results with the ones obtained in the Skyrme EDF (see Fig. 4 in Ref. \cite{24}). The modifications of total binding energy due to time-odd mean fields are given by the \( E^{\text{to}} \) quantity in Ref. \cite{24}, which is an analog of the \( E^{\text{NM}} - E^{\text{WNM}} \) quantity.

The general dependence of both quantities on \( N - Z \) is similar in odd-mass nuclei apart of few cases such as \( ^{43}\text{Ti} \) and \( ^{48}\text{Sc} \) in SLy4 Skyrme EDF (Fig. 4 in Ref. \cite{24}). Neither of them indicates the enhancement of time-odd mean fields in the vicinity of the \( N = Z \) line. In addition, the absolute values of \( E^{\text{to}} \) and \( E^{\text{NM}} - E^{\text{WNM}} \) quantities are similar being below 300 keV in the majority of the cases. The principal difference between the RMF and the Skyrme EDF lies in the fact that time-odd mean fields are always attractive and show very small dependence on the parametrization in the RMF calculations (this is also supported by the analysis of terminating states, see Ref. \cite{19}), while they can be both attractive (SLy4 force) or repulsive (SIII force) and show considerable dependence on the parametrization in Skyrme EDF (Ref. \cite{24}).

### B. Binding energies in the Ce (\( Z = 58 \)) isotopes

The role of time-odd mean fields is studied here in medium mass Ce isotopes in order to facilitate the comparison with the results obtained within the Skyrme EDF with the SLy4 force in Ref. \cite{23}. This reference represents the most detailed study of time-odd mean fields in odd-mass nuclei within the Skyrme EDF. We consider lowest one-particle states of positive and negative parities, while Ref. \cite{23} studies only lowest one-particle state in each nucleus.

Figs. 3 and 4 show the additional binding due to NM. The comparison with the Skyrme EDF results of Ref. \cite{23} reveals the number of important differences. First, similar to the results in light nuclei (Sect. VI A) and in actinide region (Sect. VI H in Ref. \cite{23}), time-odd mean fields are attractive in the RMF calculations for the Ce isotopes. On the contrary, they are repulsive in the SLy4 parametrization of the Skyrme EDF \cite{23}. Note that the SLy4 force produces attractive time-odd mean fields in light nuclei (Ref. \cite{24}). This mass dependence of the effects of time-odd mean fields in the Skyrme EDF may be due to the competition between isovector and isoscalar effects \cite{23}. The average absolute magnitude of the change of binding due to time-odd mean fields in the RMF calculations is only half of the one seen in the Skyrme calculations with the SLy4 parametrization. It was also checked on some examples that additional binding due to NM only weakly depends on the parametrization of the RMF Lagrangian.

Second, the results of calculations do not reveal strong dependence of additional binding due to NM on deformation. For example, the deformation of the \( \nu(615)^{11/2} \) state in the \( ^{173-181}\text{Ce} \) chain changes drastically from \( \beta_2 \sim 0.23 \) down to \( \beta_2 \sim 0.06 \) (Fig. 3 bottom panel), but

\[
\begin{align*}
E^{\text{NM}} - E^{\text{WNM}} & = \ldots \\
\text{or} \quad E^{\text{to}} & = \ldots \\
E^{\text{NM}} & = \ldots \\
E^{\text{WNM}} & = \ldots \\
E^{\text{total}} & = \ldots 
\end{align*}
\]
The impact of NM on binding energies of positive parity states in odd mass Ce (Z = 58) nuclei. Upper panel shows the additional binding $E^{NM} - E^{WNM}$ due to NM and its configuration dependence. The structure of the states is shown by the Nilsson labels; the states at and to the right of the Nilsson label up to the next Nilsson label have the same structure. The bottom panel shows the corresponding deformations of the single-particle configurations. The calculations have been performed with the NL3 parametrization of the RMF Lagrangian. They cover the nuclei from proton-drip line up to neutron-drip line.

FIG. 3: The impact of NM on binding energies of positive parity states in odd mass Ce (Z = 58) nuclei. Upper panel shows the additional binding $E^{NM} - E^{WNM}$ due to NM and its configuration dependence. The structure of the states is shown by the Nilsson labels; the states at and to the right of the Nilsson label up to the next Nilsson label have the same structure. The bottom panel shows the corresponding deformations of the single-particle configurations. The calculations have been performed with the NL3 parametrization of the RMF Lagrangian. They cover the nuclei from proton-drip line up to neutron-drip line.

The additional binding due to NM remains almost the same (Fig. 3 top panel). The $\nu[523]7/2$ and $\nu[505]11/2$ states are another examples of this feature (Fig. 3).

Third, the binding energy modifications due time-odd mean fields are completely different in the RMF and Skyrme EDF calculations. In the Skyrme EDF calculations, the magnitude of these modifications correlates with three properties of the blocked orbital, namely, in decreasing order of importance they are: a small $\Omega$ quantum number, a down-sloping behavior of the energy of the single-particle state with mass number $A$, and a large total angular momentum $j$ for the spherical shell from which the single-particle state originates [25]. On the contrary, the RMF calculations do not reveal this type of correlations. Indeed, the single-particle states which have the largest changes in binding energies due to NM ($|E^{NM} - E^{WNM}| \geq 0.1$ MeV) are $[413]5/2$, $[404]7/2$, $[640]1/2$, $[631]3/2$, $[505]9/2$ and $[501]1/2$.

Neutron current distributions in one-neutron configurations of selected nuclei having similar quadrupole deformations are shown in Figs. 5 and 6. They are predominantly defined by the currents generated by the blocked orbitals and are characterized by the complicated patterns in different cross-sections of nucleus. However, there are clear correlations between the patterns of the currents in the $y - z$ plane ($z$ is the symmetry axis and $x$ is the rotation axis in the CRMF) and the projection $\Omega_{bl}$ of the angular momentum of the blocked orbital on the axis of symmetry (Fig. 5). At $\Omega_{bl} = 1/2$, the currents show circulations (vortices) which are concentrated in the central region of nucleus. However, with increasing $\Omega_{bl}$, the currents are pushed away from the axis of symmetry of the nucleus towards the surface area. In addition, the strength of currents correlates with $\Omega_{bl}$: as follows from the values of factor $F$ the strongest currents appear for low $\Omega_{bl}$, while the configurations with large $\Omega_{bl}$ are characterized by weak currents. In the $x - y$ plane, the majority of the configurations show the current pattern (although with different strengths of the currents) visible on Fig. 6, while in the $x - z$ plane the currents are typically directed towards (away) from the $z = 0$ axis (see, for example, Fig. 6a). Figs. 5d,e show that the change of the signature of the blocked orbital leads to a change in the direction of the currents.
C. Mass and particle number dependences of additional binding due to NM

Mass, neutron and proton number dependences of additional binding due to NM (the $|E_{NM} - E_{WNM}|$ quantity) are presented in Figs. 7 and 8. These figures are based on the results obtained in Sects. III A and III B and on some extra calculations for odd-mass nuclei around $^{249}$Cf. The latter calculations were performed in order to check the impact of pairing on the $|E_{NM} - E_{WNM}|$ quantity and to add some high-$A, Z, N$ data to better define particle number dependences of this quantity. These calculations without pairing were compared with the Relativistic Hartree-Bogoliubov results for some one-quasiparticle states shown in Table IV of Ref. 28. Although the pairing decreases additional binding due to NM in most of the cases, there are still one-(quasi)particle configurations in which the $|E_{NM} - E_{WNM}|$ quantity is smaller in the calculations without pairing. This is a consequence of the complicated nature of the $E_{NM} - E_{WNM}$ quantity defined by (i) the interplay of time-odd mean fields and the polarization effects (Sect. IV) and by (ii) the differences in the impact of pairing on different terms of total energy.

The calculated $|E_{NM} - E_{WNM}|$ quantities were fitted
both be positive. An analogous proton OES indicator modified in the presence of time-odd mean fields as obtained in the calculations with and without time-odd mean fields, and $\delta E_{TO}$ is the contribution coming from time-odd mean fields. If the $\Delta^{(3)}(N)$ quantity is centered at odd-$N$ nucleus, the $\delta E_{TO}$ quantity represents the change of binding energy of this odd-mass nucleus induced by time-odd mean fields. This is because the time-odd mean fields have no effect on binding energies of the ground states of even-even nuclei. Note that with such selection $\delta E_{TO}$ is negative if time-odd mean fields provide additional binding in odd-mass nucleus.

In the CDF theory, the $\delta E_{TO}$ quantity is equal to $E^{NM} - E^{WMN}$ and thus it is always negative: this result does not depend on the RMF parametrization (see Sect. IIIA for the dependence of the $E^{NM} - E^{WMN}$ quantity on the RMF parametrization). In addition, the magnitude of the $\delta E_{TO}$ quantity depends only weakly on the RMF parametrization. On the contrary, the sign and the magnitude of $\delta E_{TO}$ depends strongly on the parametrization in the Skyrme EDF calculations. For example, in the calculations with the SLy4 force the $\delta E_{TO}$ quantity

$\Delta E = \frac{c}{Q^\alpha}$

where $Q$ is equal either to proton $Z$, neutron $N$ or mass $A$ numbers. Note that the $|E^{NM} - E^{WMN}|$ values from odd-proton (odd-neutron) nuclei were used in the fit of $Z - (N-)$dependence of $\Delta E$, while all available values were used in the fit of $A-$dependence. The results of the fits are shown by solid lines in Figs. 6 and 7. One can see that the powers $\alpha$ are similar for different fits (proton, neutron or mass) and it is likely that some differences between them are due to limited set of data used in the fit. On the other hand, the magnitudes $c$ differ considerably between proton and neutron quantities. More extensive calculated data for $|E^{NM} - E^{WMN}|$ will definitely improve the accuracy of such fits and may allow to define the sources of observed difference in the magnitudes $c$ between proton and neutron quantities. However, such study is beyond the scope of the current manuscript.

The three-point indicator \[54\]

$$\Delta^{(3)}(N) = \frac{\pi_N}{2} [B(N-1) + B(N+1) - 2B(N)]$$

is frequently used to quantify the odd-even staggering (OES) of binding energies. Here $\pi_N = (-1)^N$ is the number parity and $B(N)$ is the (negative) binding energy of a system with $N$ particles. In Eq. (26), the number of protons $Z$ is fixed, and $N$ denotes the number of neutrons, i.e. this indicator gives the neutron OES. The factor depending on the number parity $\pi_N$ is chosen so that the OES centered on even and odd neutron number $N$ will both be positive. An analogous proton OES indicator $\Delta^{(3)}(Z)$ is obtained by fixing the neutron number $N$ and replacing $N$ by $Z$ in Eq. (26).

The $\Delta^{(3)}(N)$ (and similarly $\Delta^{(3)}(Z)$) quantity will be modified in the presence of time-odd mean fields as

$$\Delta^{(3)}_{TO}(N) = \Delta^{(3)}_{WTO}(N) + \delta E_{TO}$$

where the subscripts 'TO' and 'WTO' indicate the values obtained in the calculations with and without time-odd mean fields, and $\delta E_{TO}$ is the contribution coming from time-odd mean fields. If the $\Delta^{(3)}(N)$ quantity is centered at odd-$N$ nucleus, the $\delta E_{TO}$ quantity represents the change of binding energy of this odd-mass nucleus induced by time-odd mean fields. This is because the time-odd mean fields have no effect on binding energies of the ground states of even-even nuclei. Note that with such selection $\delta E_{TO}$ is negative if time-odd mean fields provide additional binding in odd-mass nucleus.

In the CDF theory, the $\delta E_{TO}$ quantity is equal to $E^{NM} - E^{WMN}$ and thus it is always negative: this result does not depend on the RMF parametrization (see Sect. IIIA for the dependence of the $E^{NM} - E^{WMN}$ quantity on the RMF parametrization). In addition, the magnitude of the $\delta E_{TO}$ quantity depends only weakly on the RMF parametrization. On the contrary, the sign and the magnitude of $\delta E_{TO}$ depends strongly on the parametrization in the Skyrme EDF calculations. For example, in the calculations with the SLy4 force the $\delta E_{TO}$ quantity

FIG. 6: (Color online) The same as in Fig. 6 but for neutron current distributions $j^n(r)$ in the $z - x$ plane (at $y = 0.48$ fm) and in the $y - x$ plane (at $z = 0.53$ fm) for the $\nu[413]5/2^+$ configuration of $^{119}$Ce.

FIG. 7: (Color online) Mass dependence of additional binding due to NM. Open and solid circles are used for odd-proton and odd-neutron nuclei, respectively.
is positive for medium mass nuclei (Refs. 24, 53, 56) but negative in light nuclei (Ref. 24). On the other hand, the $\delta E_{\text{TO}}$ quantity will be positive in light nuclei in the calculations with the SIII parametrizations 24.

It is interesting to compare the averaged effects of time-odd mean fields as given by the $\Delta E$ quantity with the experimental global trends for OES as shown by dashed lines in Fig. 2 in Ref. 52. The latter trends were obtained using phenomenological parametrization with the same functional dependence as in Eq. (26) with $c = 4.66$ MeV (4.31 MeV) and $\alpha = 0.31$ for neutron (proton) data sets. The comparison of theory and experiment suggests that time-odd mean field contributions into OES can be as large as 10% in light systems and around 5-6% in heavy systems. These are non-negligible contributions which have to be taken into account when the strength of pairing interaction is defined from fits to experimental OES. The analysis of the Sn isotopes in Ref. 29 showed that the time-even and time-odd polarization effects induced by the odd nucleon produce about 30% reduced OES as compared to the standard spherical calculations. As a consequence, an enhancement of pairing strength by about 20% is required to compensate for that effect. Our calculations show much smaller reduction of OES in part because the polarization effects in time-even channel are already taken into account in the calculations without NM. In addition, the polarization effects in time-odd channel (in particular, the breaking of Kramer’s degeneracy which is neglected in Ref. 29) are more correctly treated in the current manuscript (see Sect. IV). Thus, the current calculations suggest that much smaller increase of the strength of pairing (by approximately 5%) would be required to compensate the reduction of OES due to time-odd mean fields.

IV. THE MECHANISM OF ADDITIONAL BINDING DUE TO NM IN ODD-MASS NUCLEI

In the current section a detailed analysis of the impact of NM on the energies of the single-particle states and on different terms in the total energy expression (Eq. (15)) is performed in order to better understand the microscopic mechanism of additional binding due to NM. We use the $\nu5/2[413]$ configuration of $^{119}$Ce as an example in this analysis.

A. Energy splittings of time-reversal counterpart single-particle states in the presence of NM

Fig. 9 shows that the presence of time-odd mean fields leads to the energy splitting $\Delta E_{\text{split}}(i)$ of the single-particle states which are time-reversal counterparts. This corresponds to the removal of the Kramer’s degeneracy of these states. One of these states moves up by $\approx \Delta E_{\text{split}}/2$ as compared with its position in the absence of NM, while another down by $\approx \Delta E_{\text{split}}/2$.

Detailed analysis of the single-particle spectra in $^{119}$Ce and $^{123}$Xe reveals general features which are also found in other nuclei. The $^{119}$Ce nucleus is axially symmetric ($\gamma = 0^\circ$) while $^{123}$Xe is triaxial with $\gamma = -26^\circ$. This difference in the symmetry of nucleus results in important consequences: the energy splittings appear in all single-particle states in triaxial nuclei, while only the states with $\Omega = \Omega_{\text{bl}}$ ($\Omega$ is the projection of the total angular momentum of the particle on the axis of symmetry and the subscript ‘bl’ indicates the blocked state) experience such splittings in axially symmetric nuclei. The former feature is due to the fact that $\Omega$ is not good quantum number in triaxial nuclei and each single-particle state represents a mixture of the basic states with different values of $\Omega$.

It is important to mention that the occupied and unoccupied states as well as proton and neutron states show energy splittings (Fig. 9). The splittings of the proton and neutron states of the same structure are similar. This is because the largest contribution to magnetic potential (Eq. (21)) is due to space-like components of the $\omega$-meson fields which do not depend on the isospin. In addition, the occupied state is always more bound than its unoccupied time-reversal counterpart.

The change of the signature of the blocked state leads to the inversion of the signatures in other pairs of time-reversal orbitals (compare columns (1) and (3) in Fig. 9). This is due to the change of the direction of the currents caused by the change of the signature of the blocked state (compare Fig. 5d with Fig. 5e). However, the additional binding due to NM (the $E_{\text{NM}} - E_{\text{WNM}}$ quantity) does not depend on the signature of the blocked state in odd-mass nuclei.

B. Polarization effects induced by NM

The polarization effects induced by NM are investigated by considering its impact on different terms of the total energy (Eq. (13)). The results of this study are shown in Table I. One can see that the total energy terms can be split into two groups dependent on how they are affected by NM. The first group includes the $E_{\omega N L}, E_{\omega}^T, E_{\rho}^T$ and $E_{\text{Coul}}$ terms which are only weakly influenced by NM, and thus, will not be discussed in detail.

The second group is represented by the $E_{\text{part}}, E_{\sigma}, E_{\omega}^S$ and $E_{\omega}^T$ terms which are strongly affected by NM. The $E_{\omega}^T$ term is directly connected with the nucleonic currents (see Eq. (27)). The $E_{\sigma}$ and $E_{\omega}^T$ terms depend only indirectly on the time-odd mean fields: the minimization of the total energy in the presence of time-odd terms leads to a very small change of equilibrium deformation induced by NM. The quadrupole and hexadecapole moments change by $10^{-4}$ of their absolute value when the NM is switched on; a similar magnitude of changes is seen also in $E_{\sigma}$ and $E_{\omega}^T$. One should keep in mind that only the $E_{\sigma} + E_{\omega}^T$ quantity has a deep physical meaning since
FIG. 8: (Color online) The same as in Fig. 7 but for neutron and proton dependences of additional binding due to NM.

\[
\Delta E = 0.788N^{-0.611} \quad \text{and} \quad \Delta E = 0.415Z^{-0.567}
\]

FIG. 9: (Columns (1) and (3)) The energy splittings \( \Delta E_{\text{split}} \) between different signatures of the single-particle states in the presence of NM. The results of the calculations are shown for the configurations of \(^{119}\text{Ce}\) in which either the \( \nu[413]5/2^- \) (column (1)) or \( \nu[413]5/2^+ \) (column (3)) states are blocked. These signatures are degenerated in energy in the calculations without NM (column (2)). Note that the single-particle states of interest are shown at arbitrary absolute energy in the column (2). Solid (open) circles indicate occupied (unoccupied) states. Solid (dotted) lines are used for the \( r = +i \) \( (r = -i) \) states.

TABLE II: The impact of NM on different terms of the total energy (Eq. (15)) in the \([413]5/2\) configuration of \(^{119}\text{Ce}\). Second column shows the absolute energies [in MeV] of different energy terms in the case when NM is neglected. Third column shows the changes [in MeV] in the energies of these terms induced by NM.

| Quantity | \( E_{i}^{WM} \) | \( E_{i}^{NM} - E_{i}^{WM} \) |
|----------|------------------|----------------------|
| 1        | -2849.889        | -0.410               |
| 2        | -17079.532       | -2.231               |
| 3        | 343.341          | -0.017               |
| \( E_{\text{part}} \) | 14356.156        | 2.054                |
| \( E_{\sigma}^{S} \) | 0.0              | -0.124               |
| \( E_{\sigma}^{T} \) | 2.044            | 0.003                |
| \( E_{\rho}^{S} \) | 0.0              | -0.010               |
| \( E_{\rho}^{T} \) | -6.252           | 0.0                  |
| \( E_{\text{Coul}} \) | 481.196          | 0.017                |
| \( E_{\text{cm}} \) | -959.349         | -0.104               |

\( E_{\text{part}} \) is defined by both time-odd mean fields and the polarization effects in time-even mean fields induced by time-odd mean fields. Therefore, the rest of the modification of \( E_{\text{part}} \) is related to small changes in the single-particle energies of occupied states caused by the changes in the equilibrium deformation induced by NM.

This detailed analysis clearly indicates that the \( E_{\text{part}}^{NM} - E_{\text{part}}^{WM} \) quantity is defined by both time-odd mean fields and the polarization effects in time-even mean fields induced by time-odd mean fields. \( E_{\text{part}}^{NM} - E_{\text{part}}^{WM} = -104 \) keV is a result of near cancellation of the contributions from the time-odd mean fields and the polarization effects in time-even mean fields.
due to fermionic \((-410\text{ keV})\) and bosonic \((-306\text{ keV})\) degrees of freedom. Note that the latter appears with negative sign in Eq. \(\text{(15)}\). The fermionic degrees of freedom are represented by the \(E_{\text{part}}\) and \(E_{\text{cm}}\) terms, while other terms of the total energy are related to the bosonic degrees of freedom. The fermionic contribution into \(E^{NM} - E^{WNM}\) is defined by more or less equal contributions from time-odd mean fields and polarization effects in time-even fields. On the contrary, the time-odd mean fields define only \(\approx 1/3\) \((E^{S}_{\text{split}} = -0.124\text{ keV})\) of the bosonic contribution into \(E^{NM} - E^{WNM}\), while the rest is due to polarization effects in time-even mean fields.

It turns out that these contributions are highly correlated as can be seen from the ratio \(\Delta E_{\text{split}}/(E^{NM} - E^{WNM})\) in the Ce isotope chain (Fig. 10). \(\Delta E_{\text{split}}\) depends only on time-odd mean fields in fermionic channel, while \((E^{NM} - E^{WNM})\) depends both on time-odd mean fields and polarization effects in time-even mean fields in fermionic and bosonic channels. One can see that \(\Delta E_{\text{split}}/(E^{NM} - E^{WNM}) \approx 4\) for majority of nuclei. Similar relation exists also in the Skyrme EDF calculations for the Ce isotopes (see Eq. \(\text{(7)}\) in Ref. \[25\]).

V. THE IMPACT OF TIME-ODD MEAN FIELDS ON THE PROPERTIES OF PROTON-UNSTABLE NUCLEI

The blocked state has always lower energy than its unoccupied time-reversal counterpart in the calculations with NM; this fact does not depend on the signature of the blocked state (Sect. \[IV.A\]). The energy of blocked state in the presence of NM is lower by \(\approx \Delta E_{\text{split}}/2\) than the energy of the same state in the absence of NM. This additional binding will affect the properties of the nuclei in the vicinity of the proton-drip line via two mechanisms discussed below. They are schematically illustrated in

![Graph of \(\Delta E_{\text{split}}/(E^{NM} - E^{WNM})\) vs. mass number for Ce isotopes.](image)

**Fig. 10:** The ratio \(\Delta E_{\text{split}}/(E^{NM} - E^{WNM})\) in the Ce isotopes. The structure of the blocked states is shown by the Nilsson labels; the states at and to the right of the Nilsson label up to the next Nilsson label have the same structure.

In the first mechanism, the nucleus, which is proton unbound (state A in Fig. 11) in the calculations without NM, becomes proton bound in the calculations with NM (state A’ in Fig. 11). The necessary condition for this mechanism to be active is the requirement that the energy of the single-particle state in the absence of NM is less than \(\Delta E_{\text{split}}/2\). This mechanism can be active both in the ground and excited states of the nuclei in the vicinity of the proton-drip line.

In the second mechanism, the energy of the single-particle state (state B’ in Fig. 11) is lower in the presence of NM, but the state still remains unbound. This will affect the decay properties of proton emitters and the possibilities of their observation. Indeed, the lowering of the energy of the single-proton state will decrease the probability of emission of the proton through combined Coulomb and centrifugal barrier. Many results of the physics of proton emitters are conventionally expressed in terms of the \(Q_p\) energies which depend on the difference of binding energies of parent (odd-proton) and daughter (even-proton) nuclei. Note that for simplicity we consider here only even-\(N\) nuclei. NM leads to an additional binding in odd-proton nucleus but it does not affect the binding of even-proton nucleus. Thus, the \(Q_p\) values are lower by the value of this additional binding when the NM is taken into account.

Two consequences follow from lower \(Q_p\) values. First, experimental observation of proton emission from the nucleus will become impossible if the \(Q_p\) value moves out-
side the $Q_p$ window favorable for the observation of proton emission or becomes possible if the $Q_p$ value moves into the $Q_p$ window favorable for the observation of proton emission. The size of the $Q_p$ window for rare-earth proton emitters is about $0.8 - 1.7$ MeV, while it is much smaller in lighter nuclei. Large $Q_p$ values outside this window result in extremely short proton-emission half-lives, which are difficult to observe experimentally. On the other hand, the decay width is dominated by $\beta^+$ decay for low $Q_p$ values below the $Q_p$ window. This consequence of the lowering of $Q_p$ due to NM is especially important in light nuclei where the impact of NM on binding energies is especially pronounced and the $Q_p$ window is narrow.

Second, the lowering of the $Q_p$ values due to NM will increase the half-lives of proton emitters. For example, the lowering of $Q_p$ due to NM will be around 50 keV in rare-earth region since this is typical value of additional binding due to NM in odd-mass nuclei of this region. This can increase the half-lives of proton emitters in this mass region.

On the other hand, the impact of NM can be dramatic on the half-lives of proton emitters in lighter nuclei. This is due to two factors, namely, (i) the general increase of additional binding due to NM and the magnitude of $\Delta E_{\text{split}}$ with decreasing mass and (ii) the narrowing of the $Q_p$ window with the decrease of mass due to the lowering of the Coulomb barrier. This can be illustrated by several examples. The change in proton energy of around 300 keV in $^{89}\text{Br}$ causes a change in the proton decay lifetime of 11 orders of magnitude. This effect is even more pronounced in lighter systems. The half-life window of 10 to $10^{-4}$ s corresponds to proton energies of 100 - 150 keV in nuclei around $Z = 20$, while the variation of the $Q_p$ value between 3 to 50 keV in $^7\text{B}$ changes the half-lives by 30 orders of magnitude. The energy changes quoted in these examples are either of similar magnitude or even smaller as compared with the changes of the energies of single-proton states and the $Q_p$ values induced by NM. As a result, one can conclude that the effects of time-odd mean fields have to be taken into account when attempting to describe the properties of proton emitters in light nuclei.

VI. ODD-ODD MASS NUCLEI: A MODEL STUDY OF IMPACT OF NUCLEAR MAGNETISM ON BINDING ENERGIES.

The nuclei around $^{32}\text{S}$ in superdeformed minimum are considered in the present Section. Their selection is guided in part by the desire to compare the CRMF results with the ones obtained in the Skyrme EDF in Ref. [51], where the signature separation induced by time-odd mean fields has been found in the excited SD bands of $^{32}\text{S}$. The CRMF calculations have been performed for some SD configurations in $^{32}\text{S}$ and in neighboring nuclei. The starting point is the doubly magic SD configuration $\pi^3\nu^2$ in $^{32}\text{S}$ (further ‘SD core’) (see Ref. [52]) in which all single-particle orbitals below the $N = Z = 16$ SD shell gaps are occupied (Fig. 12). Then the configurations in the nuclei under consideration (Fig. 13) are created by either adding particles into the $[202]5/2^+$ orbital(s) or/and creating holes in the $[330]1/2^+$ orbitals: these are the orbitals active in signature-separated configurations discussed in Ref. [61].

Similar to the results shown in Sects. IIIA and IIIIB the NM leads to additional binding in one-particle configurations of odd mass nuclei (the configurations in $^{33}\text{S}$ and $^{33}\text{Cl}$ created by adding a particle to the SD core or the configurations in $^{31}\text{P}$ and $^{31}\text{S}$ created by removing a particle from the SD core, see Fig. 13). This additional binding does not depend on the signature of the single-particle state.

Fig. 13 shows that additional binding due to NM is smaller for the proton states as compared with the neutron ones. For example, the single-particle configurations in $^{31}\text{P}$ and $^{31}\text{S}$ are built on the same Nilsson state. However, additional binding due to NM is smaller for the proton state ($^{31}\text{P}$) than for the neutron state ($^{31}\text{S}$). Similar
situation exists also in $^{33}$S and $^{33}$Cl. These results are consistent with a general systematics (Sect. III C) which shows that additional binding due to NM is smaller in the proton subsystem than in the neutron one. The redistribution of the proton density due to Coulomb interaction may be one of possible reasons for such difference between proton and neutron states.

The situation is more complicated in odd-odd nuclei ($^{30}$P and $^{34}$Cl) in which considerable energy splitting between the $r = +1$ and $r = −1$ configurations is obtained in the calculations. The microscopic mechanism of binding modifications is illustrated in Table III on the example of configurations A and B in $^{34}$Cl.

NM provides additional binding of around 0.4 MeV in the configuration A which has signature $r = −1$. The proton and neutron currents due to the occupation of proton and neutron $5/2^+$ states are in the same direction which results in appreciable baryonic current. This baryonic current leads to sizeable modifications in the $E_{\text{part}}$, $E_\sigma$, $E_\gamma$, and $E_\beta$ terms (Table III). These are precisely the same terms which are strongly affected by NM in odd-mass nuclei, see Sect. IV B. The fermionic contribution into $E_{WNM}^{NM} − E_{WNM}^{NM}$ (the $E_{\text{part}}$ term) is defined by more or less equal contributions from time-odd mean fields and polarization effects in time-even mean fields. On the contrary, the time-odd mean fields define only approximately 1/3 ($E_\sigma = −0.413$ MeV) of the bosonic contribution into $E_{WNM}^{NM} − E_{WNM}^{NM}$, while the rest is due to polarization effects in time-even mean fields (the $E_\sigma$, $E_\gamma$ terms).

On the contrary, the NM leads to the loss of binding in the configuration B which has $r = +1$. In this configuration, the proton and neutron currents due to $\pi[202]5/2^+$ and $\nu[202]5/2^−$ states are in opposite directions, so the total baryonic current is very close to zero. As a result, the impact of NM is close to zero for the majority of the terms in Eq. (1) (see Table III). The only exception is the $E_\beta$ term which represents space-like component of the isovector-vector $\rho$-field. This term depends on the difference of proton and neutron currents (Eq. (22)), which for the present case of opposite currents gives non-zero result. As follows from Table III, this term is predominantly responsible for the loss of binding due to NM in the configuration B.

Fig. 13 also shows the results for the 4-particle excited SD state $\pi(a+b)+\nu(a+b)$ in $^{32}$S, for which the calculated rotational structures display the signature separation induced by time-odd mean fields [19, 61]. The configurations are formed by exciting proton and neutron from

![FIG. 13: The impact of NM on binding energies of different single-particle configurations calculated under study. The energies of single-particle configurations calculated without NM are normalized to zero. Short lines show the magnitude of additional gain (negative energies) or loss (positive energies) in binding energies in the presence of NM. The configurations are labeled by the particle (p) and/or hole (h) states with respect to the $^{32}$S SD core and the total signature $r$ of the configuration.](image1)

![FIG. 14: (Color online) The impact of NM on binding energies of the lowest two-particle configurations in odd-odd Al nuclei. Upper panel shows the $E_{NM}^{NM} − E_{WNM}^{NM}$ quantity for different signatures. The structure of the states are shown by the Nilsson labels only in the cases when two-particle configurations are near-prolate. The same state is blocked in the proton subsystem of all nuclei. The bottom panel shows the $\beta_2$ and $\gamma$-deformations of nuclei in the configurations under study.](image2)

![FIG. 15: (Color online) The same as in Fig. 13 but for the lowest two-particle configurations in odd-odd Cl nuclei.](image3)
### TABLE III: The changes in different terms of the total energy (Eq. (13)) in the SD configurations $A \equiv \pi[202]5/2^- \otimes \nu[202]5/2^-(r = -1)$ and $B \equiv \pi[202]5/2^+ \otimes \nu[202]5/2^+(r = +1)$ of $^{34}$Cl induced by NM (third and fourth columns). Second column shows the absolute energies [in MeV] of different energy terms in the case when NM is neglected. The configurations are given with respect of doubly magic SD configuration in $^{32}$S.

| Quantity          | $E^{NM}_i(A,B)$ | $E^{NM}_i(A) - E^{NM}_i(A)$ | $E^{NM}_i(B) - E^{NM}_i(A)$ |
|-------------------|-----------------|-------------------------------|-------------------------------|
| $E_{part}$        | -836.044        | -1.47                         | -0.004                        |
| $E_s$             | -4416.351       | -7.824                        | +0.004                        |
| $E_{NM}$          | 84.864          | -0.036                        | -0.001                        |
| $E_s^P$           | 3698.757        | 7.126                         | -0.003                        |
| $E_s^N$           | 0.0             | -0.413                        | 0.0                           |
| $E_s^H$           | 0.061           | 0.0                           | 0.0                           |
| $E_s^S$           | 0.0             | 0.0                           | -0.043                        |
| $E_{Coul}$        | 59.839          | 0.051                         | -0.002                        |
| $E_{em}$          | -9.492          | 0.0                           | 0.0                           |
| $E_{tot}$         | -272.705        | -0.374                        | 0.041                         |

FIG. 16: (Color online) The impact of NM on binding energies of two-particle states in odd-odd nuclei in the vicinity of the $N = Z$ line as a function of proton number $Z$. Panel (a) shows the results for two-particle configurations with different proton and neutron states, while panel (b) shows the results for two-particle configurations with the same proton and neutron states. The structure of the states are shown by the Nilsson labels only in the cases when two-particle configurations are near-prolate. Note that in panel (b) only one Nilsson label is shown since proton and neutron states have the same structure. The results are shown only in the cases when the convergence has been achieved for both $N = Z$ and $N = Z - 2$ (or $N = Z + 2$) nuclei.

is broken and additional binding, which depends on the total signature of the configuration (0.907 MeV for the $r = +1$ configurations and 0.468 MeV for the $r = -1$ configurations in the calculations with the NL3 parametrization), is obtained when NM is taken into account. The NL1 and NLSH parametrizations of the RMF Lagrangian give very similar values of additional binding due to NM. The essential difference between the relativistic and non-relativistic calculations lies in (i) the size of the energy gap between the $r = +1$ and $r = -1$ configurations and (ii) the impact of time-odd mean fields on the energy of the $r = -1$ states. This energy gap is about 2 MeV in the Skyrme EDF calculations with the SLy4 force [61], while it is much smaller being around 0.45 MeV in the CRMF calculations with the NL1, NL3 and NLSH parametrizations. The energies of the $r = -1$ states are not affected by time-odd mean fields in the Skyrme EDF calculations [61], while appreciable additional binding is generated by NM for these states in the CRMF calculations (Fig. 13).

Figs. 14 and 15 show the results of the calculations for the ground state two-particle configurations in odd-odd Al and Cl nuclei. The calculations suggest that signature separation due to time-odd mean fields is expected also in two-particle configurations at zero or low excitation energies in odd-odd nuclei. The signature separation is especially pronounced in the $N = Z$ $^{26}$Al (the $\pi[202]5/2^{-} \otimes \nu[202]5/2^{-}$ configuration) and $^{34}$Cl nuclei. This is because the proton and neutron currents in these configurations are almost the same both in strength and in spatial distribution. As a result, their contribution to total energy is large in the case when these currents are in the same direction (the $r = -1$ configurations) and close to zero when these currents are in opposite directions (the $r = +1$ configurations). Note that $^{26}$Al is axially deformed while the $^{34}$Cl is triaxially deformed with the $[330]1/2^{-}$ orbitals below the $N = 16$ and $Z = 16$ SD shell gaps into the $[202]5/2^{-}$ orbitals located above these gaps. They have the $\pi 311/2\nu 311/2$ structure in terms of intruder orbitals. When NM is neglected these four configurations are degenerated in energy. This degeneracy
\( \gamma \sim 30^\circ \). However, both of them show the enhancement of signature separation at \( N = Z \).

The signature separation is rather small for the majority of nuclei away from the \( N = Z \) line. This is a consequence of the fact that the strength of the currents in one subsystem (and thus the impact of NM on binding energies) is much stronger that in another subsystem. As a result, there is no big difference (large signature separation) between the cases in which the proton and neutron currents are in the same and opposite directions. However, some nuclei away from the \( N = Z \) line also show appreciable signature separation. These are \(^{38}\text{Al}\) and \(^{38,48,50}\text{Cl}\) nuclei (Figs. 14, 15) for which the strengths of proton and neutron currents (but not necessarily the spatial distribution of the currents) are of the same order of magnitude.

It was suggested in Ref. [24] that the effects of time-odd mean fields are enhanced at the \( N = Z \) line. However, Fig. 16 clearly shows that the enhancement of signature separation is not restricted to the \( N = Z \) line. Indeed, signature separation of the same two-particle configurations are very similar in the \( N = Z \) and \( N = Z \pm 2 \) nuclei despite the fact that sometimes the deformations of compared nuclei differ appreciably. There are considerable signature separation in two-particle configurations based on the same proton and neutron states in the \( N = Z \) and \( N = Z - 2 \) nuclei (Fig. 16a). On the other hand, almost no signature splitting is observed in the \( N = Z \) and \( N = Z + 2 \) nuclei when two-particle configurations are based on different proton and neutron states (Fig. 16b). This suggests that the enhancement of signature splitting is due to similar proton and neutron current distributions (see discussion in previous paragraph).

When considering odd-odd nuclei one has to keep in mind that the present approach takes into account only the part of correlations between blocked proton and neutron and neglects the pairing. In particular, the residual interaction of unpaired proton and neutron leading to the Gallagher-Moshkowski doublets of two-quasiparticle states with \( K_+ = \Omega_p + \Omega_n \) and \( K_- = |\Omega_p - \Omega_n| \) [62, 63] is not taken into account. Thus, future development of the model is required in order to compare directly the experimental data on odd-odd nuclei with calculations. This question will be discussed in more details in a forthcoming manuscript [30].

VII. CONCLUSIONS

Time-odd mean fields (nuclear magnetism) have been studied at no rotation in a systematic way within the framework of covariant density functional theory. The main results can be summarized as follows:

- In odd-mass nuclei, nuclear magnetism always leads to an additional binding indicating its attractive nature in the CDFT. This additional binding only weakly depends on the parametrization of the RMF Lagrangian. On the contrary, time-odd mean fields in Skyrme EDF can be attractive and repulsive and show considerable dependence on the parametrization of density functional. This additional binding is larger in odd-neutron states than in odd-proton ones in the CDFT framework. The underlying microscopic mechanism of additional binding due to NM has been studied in detail.

- Additional binding due to NM can have a profound effect on the properties of odd-proton nuclei in the ground and excited states in the vicinity of the proton-drip line. In some cases it can transform the nucleus which is proton unbound (in the calculations without NM) into the nucleus which is proton bound. This additional binding can significantly affect the decay properties of proton unbound nuclei by (i) increasing the half-lives of proton emitters (by many orders of magnitude in light nuclei) or (ii) moving the \( Q_p \) value inside or outside the \( Q_p \) window favorable for experimental observation of proton emission.

- Relative energies of different (quasi)particle states in medium and heavy mass nuclei are only weakly affected by time-odd mean fields. This is because additional bindings due to NM are small. In some cases it can transform the nucleus which is proton unbound (in the calculations without NM) into the nucleus which is proton bound. This additional binding can significantly affect the decay properties of proton unbound nuclei.

- The phenomenon of signature separation [61] and its microscopic mechanism have been investigated in detail. It was shown that this phenomenon is active also in two-particle configurations of odd-odd nuclei. It is enhanced for configurations having the same structure of proton and neutron states; this takes place either at ground state or at low excitation energy in the nuclei at or close to the \( N = Z \) line. Some configurations away from the \( N = Z \) line also show this effect but signature separation is appreciably smaller.

The present investigation has been focused on the study of time-odd mean fields in the CDFT with nonlinear parametrizations of the Lagrangian. Point coupling [65] and density dependent meson-nucleon coupling [64] models are other classes of the CDF theories. It is important to compare them in order to make significant progress toward a better understanding of time-odd mean fields. The work in this direction is in progress, and the results will be presented in a forthcoming manuscript [30].
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We follow Refs. 44, 45 in the selection of the signs of the energy terms.
