Abstract

We obtain the electromagnetic form factors of the $\gamma N\Delta$ transition by analyzing recent pion-electroproduction data using a fully relativistic dynamical model. Special care is taken to satisfy Ward-Takahashi identities for the Born term in the presence of form factors thereby allowing the use of realistic electromagnetic form factors of the nucleon and pion. We parametrize the $Q^2$ dependence of the bare $\gamma N\Delta$ form factors by a three-parameter form which is consistent with the asymptotic behavior inferred from QCD. The parameters of the bare $\gamma N\Delta$ form factors are the only free parameters of the model and are fitted to the differential cross-section and multipole-analysis data up to $Q^2 = 4$ (GeV/c)$^2$ in the $\Delta(1232)$-resonance region. This analysis emphasizes the significance of the pion-cloud effects in the extraction of the resonance parameters.

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Recently a covariant quasipotential description of pion-nucleon (\(\pi N\)) interaction [1] has been extended to the process of pion photo-production [2]. Here we further extended this description to the process of pion electro-production and use the resulting model to extract the electromagnetic \(N\) to \(\Delta\) transition form factors. In doing so we exploit the results of the recent pion production experiments at LEGS [3], MAMI [4], JLab [5, 6], and MIT-Bates [7], as well as the multipole analysis MAID [8].

One of the aims of our investigation is to disentangle the effect of the resonance excitation from the competing background mechanisms, such as pion rescattering in the final state. Admittedly, any such separation of resonance and background is model dependent. We have attempted to constrain this model-dependence by adhering to general principles, such as relativistic covariance, current conservation, unitarity, chiral symmetry. The model is based on a \(\pi N-\gamma N\) coupled-channel equation which when solved to the first order in the electromagnetic coupling \(e\) leads to the electroproduction amplitude, \(T_{\pi\gamma}\) = \(V_{\pi\gamma} + T_{\pi\pi}G_{\pi}V_{\pi\gamma}\), where \(V_{\pi\gamma}\) is an basic electroproduction potential, \(G_{\pi}\) is the pion-nucleon propagator and \(T_{\pi\pi}\) is the full \(\pi N\) amplitude. Thus, the pion rescattering effects are included as the final state interaction. The \(\pi N\) amplitude satisfies an integral equation on its own. The details on reconstructing this amplitude and its fit to the \(\pi N\) elastic scattering are presented in [1].

Our model potential for the pion electro-production is shown in Fig. 1. It includes the Born term (we use the pseudo-vector \(\pi NN\) coupling required by chiral symmetry considerations), the \(t\)-channel exchange of \(\rho\) and \(\omega\) mesons, and the \(\Delta\)-isobar exchange.

The \(\pi N\) final state interaction dresses the \(s\)-channel nucleon and resonance contributions, leading in particular to the mass, field and coupling constant renormalizations. Therefore, both \(N\)- and \(\Delta\)-pole contributions in \(V_{\pi\gamma}\) are included using the bare mass and coupling parameters obtained from the equation for the \(\pi N\) amplitude. The renormalization conditions together with unitarity demand that the same propagators and \(\pi N\) vertices, including the cutoff functions, appear in both the \(\pi N\) and \(\gamma N\) potentials. Thus, all these ingredients are fixed by the analysis of \(\pi N\) scattering [1].

On the other hand, the electromagnetic interaction is constrained by the electromagnetic gauge invariance. At this point one is often concerned with the problem of how to introduce the electromagnetic form factors for nucleon and pion in a way consistent with gauge invariance. A common, but not viable, solution to this problem, implemented for instance in Refs. [8, 9, 10], is to choose all of the electromagnetic form factors that go into the Born term (i.e., nucleon, pion and axial form factors) to be the same. This prescription does enforce the current conservation, however the Ward-Takahashi (WT) identities cannot be satisfied in this way. Furthermore, it is clear that the requirement of gauge invariance should not be able to restrict the \(Q^2\) behavior of the electromagnetic interaction (see, e.g., Ref. [11]). Finally, it is well known, especially in view of the new JLab experiments [13], that these form factors are not the same.

For these reasons we sought for a solution that permits arbitrary choices of the electromagnetic form factors, yet is fully consistent with gauge invariance. Following the arguments given in, e.g., [11, 12], we find that an arbitrary form factor \(F(Q^2)\) can be accommodated by the following replacement the current:

\[
J^\mu \rightarrow J^\mu(Q^2) = J^\mu + [F(Q^2) - 1] O^{\mu\nu} J_\nu, \tag{1}
\]

where \(O^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu/q^2\), and \(q\) is the photon 4-momentum, \(Q^2 = -q^2\). It is easy to see that the resulting current \(J^\mu\) obeys exactly the same WT identities as \(J^\mu\). Thus, as long as gauge-invariance is implemented at the real-photon point, the inclusion of the form factors
via Eq. (1) will give the gauge-invariant current for $Q^2 \neq 0$. For example, the bare $NN\gamma$ and $\pi\pi\gamma$ vertex functions and the Kroll-Rudermann term are:

$$
\Gamma^\mu_{NN\gamma} = e \gamma^\mu + e [F_1(Q^2) - 1] O^{\mu\nu} \gamma_\nu + \frac{eK_N}{2m_N} F_2(Q^2) i\sigma^{\mu\nu} q_\nu,
$$

(2a)

$$
\Gamma^\mu_{\pi\pi\gamma} = e(k + k')^\mu + e [F_\pi(Q^2) - 1] O^{\mu\nu}(k + k')_\nu,
$$

(2b)

$$
J^\mu_{KR} = \frac{eg_{\pi N}}{2m_N} \left\{ \gamma^\mu + [F_A(Q^2) - 1] O^{\mu\nu} \gamma_\nu \right\} \gamma_5.
$$

(2c)

The above procedure allows us to use the experimentally determined form factors in the Born terms. Since many of the form factors are of the dipole form we introduce: $F_D(Q^2; \Lambda^2) = (1 + Q^2/\Lambda^2)^{-2}$. The newly measured proton electric form factor [13], we represent by the following form: $G^p_\rho(Q^2) = (1 + Q^2 e^{-Q^2}) F_D(Q^2; 0.4)$; it is in a very good agreement with the more common form [14] in the range of $Q^2$ under consideration. For the neutron electric form factor we use the parametrization of Galster [15]. For the magnetic form factors of both proton and neutron we use the dipole form: $G^p_\rho(Q^2) = \mu_p/n F_D(Q^2, 0.71)$. For the pion form factor we use the monopole form: $F_\pi(Q^2) = (1 + Q^2/0.45)^{-1}$, while, for the axial form factor we use: $F_A(Q^2) = F_D(Q^2, 0.9)$. For the vector mesons $(\rho/\omega)$ we use the prediction of [16]: $F_{\rho/\omega} = (1 + Q^2)/(1 + 3.04Q^2 + 2.42Q^4 + 0.36Q^6)$.

The only undetermined form factors in our model are then the $\gamma N\Delta$ form factors. These we determine within the framework of our model by fitting to the MAID multipole analysis of data in the region of the $\Delta$-resonance.

Let us first consider the $\gamma N\Delta$ vertex function. We write it in the Lorenz-covariant form that obeys both electromagnetic and spin-3/2 gauge symmetries (see [18, 19, 24] for details):

$$
\Gamma^{\alpha\mu}_{\gamma N\Delta}(p,q) = -\frac{3e(m_\Delta + m_N)}{2m_N[(m_\Delta + m_N)^2 - q^2]} \left\{ g_M(Q^2) \varepsilon^{\alpha\mu\upsilon\nu} p_\beta q_\nu 
+ g_E(Q^2) (p \cdot q g^{\alpha\upsilon} - q^\alpha p^\upsilon) i\gamma^5
+ g_C(Q^2) (1/m_N) [q^2(p^\mu \gamma^\alpha - \not{q} g^{\alpha\mu}) + q^\alpha(p \not{q} - \not{p} \cdot q \gamma^\alpha)] i\gamma^5 \right\},
$$

(3)

where $q$ ($\mu$) and $p$ ($\alpha$) are the four-momenta (vector-indexes) of the photon and $\Delta$ respectively. This vertex function has the advantage of decoupling the unphysical spin-1/2 sector of the spin-3/2 field, which can be viewed as the result of the transversality property [19]: $p_\mu \Gamma^{\alpha\mu}_{\gamma N\Delta} = 0$.

At the $\Delta$-pole (the mass shell of the $\Delta$), we can relate these couplings to the more conventional decomposition of Jones and Scadron [17] which is done in terms of $G_M, G_E,$ and $G_C$ where $M, E,$ and $C$ refer to magnetic, electric and Coulomb $\gamma N\Delta$ form factors, similar to Sachs form factors of the nucleon. Defining, $D(Q^2) = Q^2 + (m_\Delta - m_N)^2$ and $P(Q^2) = m_\Delta^2 - m_N^2 - Q^2$, we find the following relation between the two sets of form factors:

$$
g_M = G_M - G_E
$$

(4a)

$$
g_E = \frac{2}{D}[P G_E + Q^2 G_C]
$$

(4b)

$$
g_C = \frac{m_N}{m_\Delta D}[4m_\Delta^2 G_E - P G_C]
$$

(4c)
In the actual calculations we use the vertex function \( \gamma N \Delta \) with form factors expressed in terms of \( G_M, G_E \) and \( G_C \) via Eq. (4). The \( \Delta \) contribution to the resonant multipoles \( M_{1+}^{3/2}, E_{1+}^{3/2} \) and \( S_{1+}^{3/2} \) are directly proportional to \( G_M, G_E \) and \( G_C \), respectively.

To parametrize the \( \gamma N \Delta \) form factors we universally use the form:

\[
G_I(Q^2) = G_I \frac{1 + (Q^2/A_I) e^{-Q^2/B_I}}{(1 + Q^2/A_I^2)^2}, \quad I = M, E, C.
\]  

Here we have built in a constraint from perturbative QCD (pQCD) such that these form factors fall as \( Q^{-4} \) (modulo logs) for asymptotically large \( Q \), see e.g. (21). This is the principal difference with parametrizations of Refs. (9, 10) which fall off exponentially and hence do not satisfy the pQCD constraint.

The photo-couplings, i.e., \( G_M \) and \( G_E \), are determined by the photoproduction multipoles \( M_{1+}^{3/2}, E_{1+}^{3/2} \) at the resonance position \( W \approx 1232 \) MeV. The strength of \( G_C \) is determined by \( S_{1+}^{3/2} \) at low \( Q^2 \). We then have determined the \( Q^2 \) dependence of \( G_M \) by comparing our calculations to the experimentally extracted \( M_{1+} \) multipole at an invariant energy \( W = 1232 \) MeV, see Fig. 2. The solid line here represents our full model calculation of this multipole. The dotted line represents the result obtained by including only the \( \Delta \) s-channel exchange in the electroproduction potential. In doing so we exclude mechanisms of resonance electroexcitation through the pion cloud, see, e.g., Fig. 3. In our model these mechanisms result from the \( \pi N \) rescattering through the resonant channel. Such mechanisms apparently account for about 50\% of the \( M_{1+} \) strength near the photon point and about 25\% near \( Q^2 = 4 \) GeV\(^2\). This is in a qualitative agreement with the findings of Refs. (9, 10).

The ratios of the resonant multipoles: \( R_{EM} = \text{Im} E_{1+}/\text{Im} M_{1+} \) and \( R_{SM} = \text{Im} S_{1+}/\text{Im} M_{1+} \) carry important information about the \( N \) to \( \Delta \) transition and about the admixture of the \( D \)-wave component in the nucleon wave function in particular. The focus of several recent theoretical (3, 10, 23, 24) and experimental (3, 6) studies has been the extraction of the \( Q^2 \)-dependence of these ratios. This dependence can potentially tell us about the range of the momentum transfer where pQCD becomes applicable (pQCD predicts \( R_{EM} = 100\% \) and \( R_{SM} = \text{const} \)).

In Fig. 4 we display the \( Q^2 \)-dependence of \( R_{EM} \) and \( R_{SM} \) obtained in our model and compared with a number of results of recent measurements and calculations. In particular, at the real photon point we find \( R_{EM} \cong -2.7\% \) and \( R_{SM} \cong -2.3\% \). As for the \( Q^2 \) dependence, we can see that \( R_{EM} \) shows a systematic tendency to cross zero in the region between 3 and 4 (GeV/c\(^2\)) . This is in contrast to the recent data analysis (3) or Sato and Lee model (2) which conclude that \( R_{EM} \) stays negative and is virtually flat in this domain of \( Q^2 \). Our results are in a better agreement with the data analysis of Kamalov et al. (23) and Aznauryan (24), which indicate the sign change of \( R_{EM} \) below \( Q^2 = 4 \) (GeV/c\(^2\)). Additional experiments and extractions of multipoles at higher \( Q^2 \) would be desirable to investigate this point. Our result for \( R_{SM} \) is in agreement with most of the mentioned analyses. Again the dotted curves represent the result of resonance dominance, when \( R_{EM} = -G_E/G_M \) and \( R_{SM} = G_C/G_M \).

After determining the \( Q^2 \)-dependence of the \( \gamma N \Delta \) form factors, we calculated the virtual-photon differential cross sections, spanning a wide range of \( Q^2 \) values as well as various energies \( W \). In Fig. 5 we show several of these calculations. We find the agreement with the
recent JLab experiments quite pleasing, especially in view of the fact that the model has very few adjustable parameters, all given in Table I.

In summary, we have extended the relativistic dynamical model of Refs. [1, 2] to calculate the pion electro-production reactions. We were able to use realistic electromagnetic form factors at each photon vertex by carefully treating the problem of gauge invariance for the Born terms. For the resonance terms we parameterized thebare \( \gamma N\Delta \) form factors such that the pQCD asymptotic \( Q^2 \) constraint is fulfilled, and then fitted them to the cross-section data and resonant multipoles of MAID up to \( 4 \) (GeV/c\(^2\)). Our result for \( R_{EM} \) in this \( Q^2 \) domain is still very far from the pQCD prediction of 100\%. Although, we do find that \( R_{EM} \) shows a strong tendency to cross zero and change sign in the region between 3 and 4 (GeV/c\(^2\)). Our model calculation of the virtual photon cross section of \( p(e, e'p)\pi^0 \) shows a very good agreement with recent data up to \( 4 \) (GeV/c\(^2\)) in the \( \Delta \) resonance energy region. A thorough study of the existing data base of pion electroproduction reactions using this model is underway [25].

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[1] V. Pascalutsa and J. A. Tjon, Phys. Rev. C 54, 054003 (2000); Phys. Lett. B 435, 245 (1998).
[2] V. Pascalutsa and J. A. Tjon, arXiv:nucl-th/0407068.
[3] G. Blanpied et al., Phys. Rev. Lett. 79, 4337 (1997).
[4] R. Beck et al., Phys. Rev. C 61, 035204 (2000).
[5] V. V. Frolov et al., Phys. Rev. Lett. 82, 45 (1999).
[6] K. Joo et al., Phys. Rev. Lett. 88, 12201 (2002).
[7] C. Mertz et al., Phys. Rev. Lett. 86, 2963 (2001).
[8] D. Drechsel, O. Hanstein, S. S. Kamalov and L. Tiator, Nucl. Phys. A 645 (1999) 145.
[9] T. Sato and T. -S. H. Lee, Phys. Rev. C 54, 2660 (1996); ibid. 63, 055201 (2001).
[10] S. S. Kamalov and S. N. Yang, Phys. Rev. Lett. 83, 4494 (1999).
[11] J. H. Koch, V. Pascalutsa and S. Scherer, Phys. Rev. C 65, 045202 (2002).
[12] F. Gross and D. O. Riska, Phys. Rev. C 36, 1928 (1987).
[13] M. K. Jones et al., Phys. Rev. Lett. 84, 1398 (2000).
[14] P. A. M. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. 91, 142303 (2003).
[15] S. Galster et al., Nucl. Phys. 32B, 221 (1971).
[16] P. Maris and P. C. Tandy, Phys. Rev. C 65, 045211 (2002).
[17] H. F. Jones and M. D. Scadron, Annals Phys. 81, 1 (1973).
[18] V. Pascalutsa, Phys. Rev. D 58, 096002 (1998); arXiv:nucl-th/0303005.
[19] V. Pascalutsa and R.G.E. Timmermans, Phys. Rev. C 60, 042201(R) (1999).
[20] V. Pascalutsa, Phys. Lett. B 503, 85 (2001).
[21] C. E. Carlson and N. C. Mukhopadhyay, Phys. Rev. Lett. 81, 2646 (1998).
[22] W. Bartel et al., Phys. Lett. 28B, 148 (1968); K. Bätzner et al., ibid. 39B, 575 (1972); J. C. Alder et al., ibid. 46B, 573 (1972); S. Stein et al., Phys. Rev. D12, 1884 (1975); P. Stoler, ibid. 44, 73 (1991); L. M. Stuart et al., ibid. 58, 032003 (1998).
[23] S. S. Kamalov et al., Phys. Rev. C 64, 032001(R) (2001).
[24] I. G. Aznauryan, arXiv:nucl-th/0207087.
[25] G. Caia, Ph.D. Thesis, Ohio University, 2004 (unpublished).
| $G_M(Q^2)$ | $G_E(Q^2)$ | $G_C(Q^2)$ |
|------------|------------|------------|
| 3.10       | 0.05       | -0.18      |
| $\Lambda^2$ [GeV$^2$] | 0.6 | 0.5 | 0.8 |
| $A$ [GeV$^2$] | 1.0 | -1.1 | -0.9 |
| $B$ [GeV$^2$] | 1.2 | 2.0 | 1.0 |

**TABLE I:** Parameters of the $\gamma N\Delta$ form factors extracted in our model.

**FIG. 1:** The Born, vector-meson, and $\Delta$-isobar contributions included in the electroproduction potential.
FIG. 2: Model description of $\text{Im}M_{1+}^{3/2}/F_D$, with $F_D = (1 + Q^2/0.71)^{-2}$. The data at $Q^2 = 2.8$ and 4.0 (GeV/c)$^2$ are taken from [3], other data are from [22]. The dotted line is the calculation including only the direct $\Delta$ exchange in $V_{\pi\gamma}$, hence leaving out resonant mechanisms due to the $\pi N$ rescattering.

FIG. 3: Examples of the dynamical mechanisms of $\Delta$-resonance electroexcitation.

FIG. 4: Results for the ratios of the resonant multipoles $R_{EM}$ and $R_{SM}$. The data points are from CLAS [5, 6], LEGS [3], MAMI [4], MIT-Bates [7], and the MAID reanalysis of JLab data by Kamalov et al. [23]. The asterisks represent predictions of the Sato and Lee model [9].
FIG. 5: Results for the virtual-photon differential cross sections on $p(e, e'p)\pi^0$. The data points are from Refs. [5, 6].