Performance analysis of secondary networks with multiple primary users for spectrum database system

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**Abstract:** This paper presents the performance evaluation for secondary networks with multiple primary users (PU) with a stochastic geometry approach. On the assumption that the secondary users (SUs) follow a Poisson hole process, we derive a PU’s outage probability (OP) and SU’s area spectral efficiency (ASE), where the PU’s OP and SU’s ASE are defined as the probability that the aggregate interference at a PU from transmitting SUs exceeds a threshold and the product of the transmitting SU’s density and the SU’s average throughput, respectively. The theoretical results agree with the simulation results, and these results show that both the PU’s OP and SU’s ASE decrease along with the primary exclusive region (PER) size, i.e., we quantitatively evaluate the trade-off between the PU’s OP and SU’s ASE.

**Keywords:** cognitive radio, spectrum sharing, spectrum database, exclusive region, stochastic geometry

**Classification:** Terrestrial Wireless Communication/Broadcasting Technologies

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1 Introduction

Toward 5G mobile network, database-driven spectrum sharing, which is one of cognitive radio techniques, is a promising system [1]. In this system, a database collects primary user’s (PU’s) usage information of licensed spectrum bands at each location and establishes a spectrum sharing policy for a secondary user (SU) such that the SU’s operation will not interfere with the PU’s operation [2, 3]. A secondary transmitter (ST) queries the database with its location, and then it operates in the licensed spectrum bands based on the spectrum sharing policy.

Even when the SU follows the spectrum sharing policy, the PU can experience harmful interference. This occurs owing to unpredictable propagation paths from changes in the geological and building information. To reduce the probability of PU interference, the spectrum sharing policy should be adjusted to reduce the aggregate interference at the PU; however, an efficient spatial reuse of the licensed bands cannot be achieved.

In [4], the authors have proposed a framework that the database updates the spectrum sharing policy on the basis of an emergency message from the PU. The authors have considered a circular primary exclusive region (PER) [5] centered at a single PR as the spectrum sharing policy, i.e., STs outside the PER transmits simultaneously. The authors assume a single monostatic radar system, i.e., the transmitter and receiver are co-located, but a practical radar system also contains a bistatic radar, i.e., the transmitter and receiver are separated, and multiple receivers are located [6].

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In this paper, we consider a system that has multiple STs and PRs, and define a PER as the union of disks centered at the PRs. For this system, we derive the PU’s outage probability (OP) and SU’s area spectral efficiency (ASE) using stochastic geometry [7], where the PU’s OP and SU’s ASE are defined as the probability that the aggregate interference at a PR from transmitting STs exceeds a threshold and the product of the transmitting ST’s density and the SU’s average throughput, respectively. We evaluate the impact of both the PER size on the PU’s OP and SU’s ASE.

The rest of this paper is organized as follows. Section 2 presents the system model. In Section 3, we derive the PU’s OP and SU’s ASE with a stochastic geometry approach. Section 4 describes the numerical evaluation. Finally, we conclude this paper in Section 5.

2 System model

We consider the bipolar network model on \( \mathbb{R}^2 \) as the secondary network. Fig. 1 shows an example of the system. PRs are randomly distributed following a homogeneous Poisson point process (PPP) \( \Phi_{PR} \) with density \( \lambda_{PR} \). Similarly, STs are randomly distributed following a homogeneous PPP \( \Phi_{ST} \) with density \( \lambda_{ST} \). The STs attempt to transmit with random power levels distributed with a PDF \( f_p \). A database is connected with all PRs and STs via control channel, but the database is not shown in Fig. 1.

The database establishes a circular region of radius \( r_{PER} \) centered at each PR. The region \( S_{PER} = \bigcup_{x \in \Phi_{PR}} b(x, r_{PER}) \) is called “PER”, where \( b(x, r) \) is a closed disc of radius \( r \) centered at \( x \). The database forbids STs on \( S_{PER} \) to transmit. The database permits STs outside \( S_{PER} \) to transmit, and then the STs transmit simultaneously. The transmitting STs are called “active STs.”

3 Analytical framework

In this section, we derive two performance metrics. One is the probability that harmful interference to the PU occurs. We define that the harmful interference occurs when the aggregate interference at PR from all active STs exceeds a certain threshold \( I_0 \). We call the probability that harmful interference to the PU occurs “PU’s OP.” Let the aggregate interference at PR from all active STs be denoted by
where \( I_{S2P} \), and then the PU’s OP can be expressed as \( \mathbb{P}(I_{S2P} > I_{th}) \). The other is a SU’s ASE. We define the ASE as \( \lambda_{ST,act} \mathbb{E}_{SINR} \ln(1 + SINR) \), where \( SINR \) denotes the signal-to-noise-plus-interference ratio (SINR) of a secondary receiver (SR). \( \mathbb{E}_{SINR} \ln(1 + SINR) \) indicates the Shannon throughput of the SU [8].

To analyze the aggregate interference from active STs, we approximate that the locations of the active STs follow a PPP. The active STs follow a Poisson hole process \( \Phi_{ST,active} = \Phi_{ST} \setminus \Phi_{PER} \), and its first order density \( \lambda_{ST,act} \) can be expressed as \( \lambda_{ST,act} = \lambda_{ST} \exp(-\lambda_{PR}\pi r_{PER}^2) \) [7]. For using Campbell’s theorem for the PPP [7], we approximate that the active STs follow a PPP \( \Phi_{ST,act} \) with density \( \lambda_{ST,act} \).

### 3.1 Probability of harmful interference to primary user

We focus on a PR, and then analyze the aggregate interference at the PR. The PR is called “typical PR.” Without loss of generality, we assume that the typical PR is located at the origin. We define the aggregate interference at the typical PR as \( I_{S2P} \approx \sum_{x \in \Phi_{ST,act}} k(x, y) p_x h_{x,y} ||x||^{-\alpha} \), where \( \alpha \) is the path loss exponent, \( ||x|| \) is the Euclidean distance between \( x \in \mathbb{R}^2 \) and the origin, \( p_x \) is the transmission power level of the active ST located at \( x \in \Phi_{ST,act} \), and \( h_{x,y} \) is the fading coefficient between \( x \in \mathbb{R}^2 \) and \( y \in \mathbb{R}^2 \) with unit mean and is distributed with a PDF \( f_h \).

Deriving cumulants of \( I_{S2P} \), \( \kappa_n(I_{S2P}) \), with Campbell’s theorem [7] and matching the cumulants to that of a shifted log-normal distribution [9, 10], the PU’s OP \( \mathbb{P}(I_{S2P} > I_{th}) \) can be expressed as follows:

\[
\mathbb{P}(I_{S2P} > I_{th}) \approx Q \left( \frac{\ln(I_{th} - c_{SLN}) - \mu_{SLN}}{\sigma_{SLN}} \right)
\]

(1)

\[
c_{SLN} = \kappa_1(I_{S2P}) - \exp(\mu_{SLN} + \sigma_{SLN}^2/2)
\]

(2)

\[
\mu_{SLN} = \frac{1}{2} \left[ \ln \left( \frac{\kappa_2(I_{S2P})}{\exp(\sigma_{SLN}^2)} - 1 \right) - \sigma_{SLN}^2 \right]
\]

(3)

\[
\sigma_{SLN}^2 = \ln \left( \sqrt{W + \sqrt{W^2 - 1}} + \sqrt{W - \sqrt{W^2 - 1}} \right)
\]

(4)

\[
W = 1 + \kappa_3(I_{S2P})^3/(2\kappa_2(I_{S2P})^3)
\]

(5)

\[
\kappa_n(I_{S2P}) \approx \frac{2\pi \lambda_{ST,act} E_p(p^n) E_h(h^n)}{(na - 2)r_{PER}^{na - 2}}, \quad n = 1, 2, \ldots,
\]

(6)

where \( Q(\cdot) \) denotes the Q-function defined as \( Q(x) = (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2) \, dt \).

### 3.2 Area spectral efficiency of secondary user

We derive the SU’s ASE. We focus on a SR receiving data from an active ST, and then derive the SINR of the SR. The SR is called “typical SR”, and the active ST transmitting data to the typical SR is called “typical ST.” Without loss of generality, we assume that the typical SR is located at the origin. Let the location of the typical ST be denoted by \( x_o \), and let the distance between \( x_o \) and the origin be denoted by \( d_{link} \). The transmit power of the typical ST is assumed to be \( E_p(p) \). The SINR of the typical SR \( SINR \) is defined as \( SINR = S_{2S}/(I_{S2S} + \sigma_N^2) \), where \( S_{2S} \) denotes the signal power at the typical SR from the typical ST, \( I_{S2S} \) denotes the aggregate interference power at the typical SR from active STs except the typical ST, and \( \sigma_N^2 \) is the noise power.
In an interference limited environment ($I_{S2S} \gg \sigma_N^2$), $\text{SINR}$ can be approximated as $\text{SINR} \approx S_{S2S}/I_{S2S}$. We define $S_{S2S}$ and $I_{S2S}$. $S_{S2S}$ is defined as $S_{S2S} = E_p(p)h_{\text{link}} d_{\text{link}}^{-\alpha}$, where $h_{\text{link}}$ denotes the fading coefficient between the typical ST and SR with unit mean and follows a exponential distribution with unit mean because the channel between the typical ST and SR is assumed to be Rayleigh fading channel. $I_{S2S}$ is defined as $I_{S2S} \approx \sum_{x \in \Phi_{\text{ST,act}}} p(h_{x,\text{ref}}) ||x||^{-\alpha}$.

Referring to [7], we derive $E_{\text{SINR}}(\ln(1 + \text{SINR}))$ as the following steps. Firstly, on the assumption that $h_{\text{link}}$ follows an exponential distribution with unit mean, $E_{\text{SINR}}(\ln(1 + \text{SINR}))$ can be modified as follows:

$$E_{\text{SINR}}(\ln(1 + \text{SINR})) = \int_0^\infty \mathcal{L}_{I_{S2S}}(s) \exp(-sI_{S2S}) \frac{e^{-sI_{S2S}}}{s} \, ds,$$

where $\mathcal{L}_{I_{S2S}}(s)$ denote the Laplace transform (LT) of $I_{S2S}$. Then, using Campbell’s theorem [7] and Slivnyak’s theorem [11], $\mathcal{L}_{I_{S2S}}(s)$ can be expressed as follows:

$$\mathcal{L}_{I_{S2S}}(s) = E_{I_{S2S}}(e^{-sI_{S2S}}) \approx \exp(-\pi \lambda_{\text{ST,act}} \delta^2 E_p(p) E_h(h) \Gamma(1 - \delta)), \quad (8)$$

where $\delta = 2/\alpha$ and $\Gamma(\cdot)$ denotes the gamma function. Finally, $E_{\text{SINR}}(\ln(1 + \text{SINR}))$ can be expressed as follows:

$$E_{\text{SINR}}(\ln(1 + \text{SINR})) \approx \int_0^\infty \exp\left(-\pi \lambda_{\text{ST,act}} \delta^2 E_p(p) \frac{e^{u}}{E_p(p) E_h(h)} (e^{u} - 1)^{\delta}\right) du. \quad (9)$$

In the special case $\alpha = 4$, $E_{\text{SINR}}(\ln(1 + \text{SINR}))$ can be simplified as follows:

$$E_{\text{SINR}}(\ln(1 + \text{SINR})) \approx e^{iA} E_1(iA) + e^{-iA} E_1(-iA), \quad (10)$$

where $A = \pi^{1/2} \lambda_{\text{ST,act}} \delta^2 E_p(p) E_h(h)^{1/2}$, $i$ denotes the imaginary unit, and $E_1(z)$ denotes the first order exponential integral function defined as $E_1(z) = \int_1^\infty e^{-zk} k^{-1} \, dk$.

### 4 Numerical Evaluation

We numerically evaluate the PU’s OP and SU’s ASE for the PER radius using the derived theoretical expressions.

Table I shows the parameter values, where the active ST’s power level $p$ is assumed to be a constant $P$. We use Nakagami-$m$ fading superimposed on log-normal shadowing and can write $E_h(h)\alpha$ as

$$E_h(h) = \frac{\Gamma(m_N + n)\Gamma(m_N)}{\Gamma(m_N)m_N^\alpha} \exp\left[\frac{n(n - 1)\sigma_{S_{\text{dB}}}}{2\sigma_s^2}\right], \quad (11)$$

where $m_N$ and $\sigma_{S_{\text{dB}}}$ denote the Nakagami-$m$ fading parameter and the log-normal shadowing parameter with the dB-spread, respectively, and $\xi = 10/\ln(10)$ [12]. Note that the channel between the typical ST and the typical SR is assumed to be Rayleigh fading channel for using (9).

Figs. 2(a) and 2(b) show the PU’s OP $P(I_{S2P} > I_h)$ and SU’s ASE $\lambda_{\text{ST,act}}E_{\text{SINR}}(\ln(1 + \text{SINR}))$ for the PER radius $r_{\text{PER}}$, respectively. In each figure, the red line is the theoretical result, and the blue crosses are the simulation results.
obtained from a Monte Carlo simulation with $2 \times 10^4$ runs. The theoretical results agree with the simulation results, and both the PU’s OP and SU’s ASE decreases with the PER radius.

### Conclusion

In this paper, we derived the PU’s OP and SU’s ASE by using stochastic geometry, assuming that the active STs follow a Poisson hole process. We evaluated the PU’s OP and SU’s ASE for the PER radius, and we showed the trade-off between the PU’s OP and SU’s ASE and the agreement of the theoretical results with the simulation results.

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**Table 1.** Parameter values.

| Parameters | Values |
|------------|--------|
| $\alpha$   | 4      |
| $\lambda_{PR}$ | $10^{-7}$ m$^{-2}$ |
| $\lambda_{ST}$ | $10^{-6}$ m$^{-2}$ |
| $d_{link}$ | 1 m    |
| $m_N$      | 1      |
| $\sigma_{S,\mathrm{dB}}$ | 6 dB  |
| $P$        | 1 W    |
| $I_{th}$   | $10^{-9}$ W |

**Fig. 2.** Trade-off between PU’s OP and SU’s ASE.

(a) PU’s OP.

(b) SU’s ASE (1 nat = 1/$(\ln 2)$ bit).