Guarcs in the Inside Hadronic Four-Dimensional Euclidean Space with Real Time

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Abstract

The paper represents the results of the study of the four-dimensional Euclidean space with real time (E-space), where $0 \leq ||VE|| \leq \infty$, in sub-hadronic physics. This closed space has a metric that distinguished from the Minkowski space and the results obtained in the model are different from physical law in the Minkowski space. As it follows from the model of Lagrangian Mechanics, quarks in the central-symmetric attractive potential, kinetic energy of quark diminishes while the speed grows as the quarks exchange their energy-mass with gluons possessing a zero rest mass, so that to ensure the permanent proton mass. This dependence describes the dynamical relation of constituent and current quarks masses.

In the quantified motion model it has been stated, that the oscillations of the particles are cyclic, including alternating localization and translation phases, the action per cycle for a free particle equals $\frac{h}{2\pi}$. The calculation of charge distribution density in proton, carried out on the basis of this model, conforms to the results of the experimental research. All relations between physical values in the E-space, mapped in the Minkowski space, correspond to the principles of SR and are Lorentz-covariant and the infinite velocity is equal to the velocity of light in the Minkowski space. These models have a transparent physical sense.

Keywords: Dynamics of quarks in the proton; Euclidean invariants; Motion of quarks and gluons; Quantum cyclic motion; Charge distribution in the proton

Introduction

Non-perturbative effects are of great importance for the theory of space inside hadron. Supposing a sequence of QCD problems are concentrated in the branch of occurrences that can be described through the transition from the Minkowski space $M(x_{0},x_{1},x_{2},x_{3})$ (M-space) into the Euclidean space inside hadron via the analytical extension of the time axis onto the lower semi plane $z=x-iy$. In this case we get the Euclidean space with the imaginary time $E_{im}(x_{0},x_{1},x_{2},x_{3})$ ($E_{im}$ is space), and $X_{E im},X_{i}$ is automatically $V_{E} = V_{i}$, and $0 \leq ||V_{E}|| \leq 1$. The use of such a space has brought to great results: the QCD vacuum. Its estimations by means of QCD in lattice have shown the following range of probable values: $\lambda^{2} = 0.45 \pm 0.17eV^{-1}$ [6,7], $E_{im}$ is homomorphic in respect of the M-space and non-local, in other words, the instantaneous interactions even at some low $\lambda$ value contradict with S principles.

It gives a reason to consider that the use of merely a part of four-dimensional Euclidean space volume in the models with $E_{im}$ does not allow using its potential to the full extent. The article expounds the first steps in the research of the inside hadronic four-dimensional Euclidean space with real time model $E(x_{0},x_{1},x_{2},x_{3})$ (E-space), where $0 \leq ||V_{E}|| \leq \infty$, and its aim is to show the expedience of the studies in the E-space as a probable prospective direction of sub-hadronic physics development. The article contains researches of the E-space properties in protons and it is presupposed that the obtained correlations have a common nature and can cover all the hadrons. Moreover it has been considered been considered that the models in the E-space will not be an alternative for the theoretical developments in $E_{im}$, but will extend their possibilities. The following requirement is the basic condition enabling this model to exist:

**Requirement 1**: Space-time relations and regularities in the E-space model mapped into the M-space must correspond to the principles of SR and be Lorentz-covariant.

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Received February 28, 2015; Accepted June 09, 2015; Published June 16, 2015

Citation: Kreymer E (2015) Guarcs in the Inside Hadronic Four-Dimensional Euclidean Space with Real Time. J Phys Math 6: 140. doi:10.4172/2090-0902.1000140

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Inside Hadronic Euclidean Frames of Reference

In the E-space no frames of reference, which are microscopic in reality, can be physically implemented. To determine the spatial coordinates of the laboratory frame of reference LFR with the coordinates \((x_{M0}, x_{M1}, x_{M2}, x_{M3})\) has been used, where hadron rests, and \(dx_{M0}, dx_{M1}\). The own time of particles in the LFR is admitted to be the temporal coordinate \(x_{E0}\). Thus the E-space is "subsidiary" towards the M-space.

**Definition 1:** Inner hadronic four-dimensional Euclidean Frame of Reference \((x_{M0}, x_{M1}, x_{M2}, x_{M3})\) EFR, is a system, where the space coordinates are indexed by the coordinates \((x_{Mi}, x_{Mj})\) LFR and the own time of the particles is equal to the own time of the particles in the LFR

\[
dx_{E0} = dx_{M0} \sqrt{1 - \frac{v^2}{c^2}},
\]

(2.1)

Where \(V_{mi}\) is the velocity of the i-number particle in the LFR. The transition to the other IFR is carried out by means of the Lorentz transformation. The E-space of the real particles corresponds to the M-space upper closed cone \(\nabla = [x_{E0} \in M: x_{E0}^2 \geq 0, x_{Ei} \geq 0] \) and \(x_{E0}[0=0]\), that ensures the execution of the causality principle. The EFR has an invariant which taking into consideration the Definition 1 is equal to

\[
dx^2_{E0} + dx^2_{E1} = dx^2_{M0}.
\]

(2.2)

Then there is symmetry between EFR and LFR: the time of one space is the invariant of the other.

From (2.1) and (2.2) it follows that

\[
v_{Ei} = \frac{v_{Mi}}{\sqrt{1 - \frac{v^2}{c^2}}}.
\]

(2.3)

Where \(v_{Ei}\) is the velocity of the particle in the EFR. And, correspondingly

\[
v_{E} = \frac{v_{M}}{\sqrt{1 - \frac{v^2}{c^2}}}.
\]

(2.4)

If \(v_{Ei} \rightarrow \infty\), then \(v_{Mi} \rightarrow 1\). There is also 4-vector of velocity in the EFR

\[
u_{E} = \left(\frac{1}{\sqrt{1 + \frac{v^2}{c^2}}} v_{E}, \frac{v_{E}}{\sqrt{1 + \frac{v^2}{c^2}}} \right).
\]

And its invariant is equal to the invariant of the corresponding relativistic 4-vector.

**SO(2)** Group of the rotation of plane \((x_{M0}, x_{j})\), cannot be applied in the EFR, because the existence of the infinite velocity makes the time absolute, and \(x_{M0}\) can take no negative values. In accordance with (2.2), E-group of position-vector rotations \(E\) which describes the particles moving with different velocity is valid in the EFR. This group does not mix the temporal \(x_{E0}\) and spatial coordinates \(x_{Ej}\). Mapping kinetic parameters of the particle in EFR observed in into LRF putting the fundamental quadratic forms

\[
E = (dx_{E0}, g_{ij}, dx_{Ei}) = dx_{M0}^2, \quad \text{where } g_{ij} = -\text{Kronecker symbol}, \mu = 0,1,2,3
\]

and

\[
M = (dx_{M0}, g_{ij}, dx_{Mj}) = dx_{E0}^2, \quad \text{where } g_{ij} = -\text{metric tensor}.
\]

The translation matrix

\[
\varphi_{ME} : [E] = [M] = (dx_{E0}^0, g_{ij}, dx_{Ei}) \rightarrow (K_{i0}s_{0}^i, G_{i00}, K_{i0}s_{Ei}) = (dx_{M0}^0, g_{ij}, dx_{Mj})
\]

must involve kinematic \(K_{ij}\) and metric \(G_{ij}\) transformations. With (2.2) we obtain the kinematic transformation matrix

\[
[K_{ij}] = \text{diag}[1, \sqrt{1 - \frac{v^2}{c^2}}, 1, 1, 1, 1, 1, 1]
\]

and the metric transformation matrix \([G_{ij}] = \text{diag}[1, 1, 1, -1, -1]\). There is a distinction of properties of the studied space from the Minkowski space that emerges because of different metric: E-group of radius-vector rotations \(x_{E0}\) does not mix the temporal \(x_{E0}\) and spatial coordinates \(r_{i} = \{x_{Ei}, x_{Ej}\}\).

**The Model of E-invariant Lagrange Mechanics Particle**

4-vector energy- momentum

LaGrange function of the free particle

\[
L_{E} = m_{E} \sqrt{1 + \frac{v^{2}}{c^{2}}}.
\]

(3.1)

The momentum of the particle

\[
p_{E} = \frac{mv_{E}}{\sqrt{1 + \frac{v^{2}}{c^{2}}}},
\]

(3.2)

And the kinetic energy

\[
E_{E} = L_{E} - v_{E} \frac{dp_{E}}{dv_{E}} = \frac{m_{E}}{\sqrt{1 + \frac{v^{2}}{c^{2}}}},
\]

(3.3)

This equation is valid under condition that \(E_{E} \geq 0\). At the same time

\[
E^{2}_{E} = p^{2}_{E} = m^{2}_{E}.
\]

(3.4)

From (3.4) we can make a conclusion that there is a 4-vector of energy-momentum in the EFR, and its invariant is equal to the invariant of the corresponding relativistic 4-vector and it is one more symmetry between the LFR and EFR. Translating the 4-vector of the particle in LFR through (2.3), we obtain \(E_{M} = m_{M} \sqrt{1 - \frac{v^{2}}{c^{2}}} \) and \(p_{M} = m_{M} v_{M}\). These values stay \(\mathbb{E}\) -invariant.

Formula (3.3) testifies to an unusual behavior in the \(\mathbb{E}\)-space of the kinetic energy: it diminishes when the speed grows. The next unit will demonstrate that it is so because of the energy-mass exchange between quarks and gluons.

**Mechanics of quark in the proton**

Here we use the model where quarks are considered electrically neutral particles, and we admit that in the center of a proton there is a hypothetical source creating central-symmetrical attractive potential \(V(r)\) of strong interactions. It is considered that this simplified model will provide the possibility to determine some peculiarities of quarks motion in the proton.

The E-invariant Lagrange function of the quark in the potential \(V(r)\)

\[
L^{q}_{E} = \frac{1}{2} m_{E} \sqrt{1 + \frac{v_{E}^{2}}{c^{2}}} - V(r),
\]

(3.5)

where \(m_{E}\) - constituent mass of the quark. E-invariance of this function is ensured by \(dx_{E} = dx_{M}\) potential \(V(r)\) will be identical for each proton in LRF.

On the analogy with (3.3) the energy of the system "quark – potential \(V(r)\)"

\[
E_{E} = E_{q} + V(r) = \text{const}.
\]

(3.6)

If a particle is under the influence of power \(F = -\nabla V(r)\) parallel to the velocity, that it will change the momentum as follows:

\[
\frac{dp_{E}}{dt} = \frac{1}{\left(1 + \frac{v_{E}^{2}}{c^{2}}\right)^{\frac{3}{2}}} \frac{dv_{E}}{dx_{E0}} F - F_{E} v_{E}.
\]

(3.7)

The alteration of the energy

\[
\frac{dE_{E}}{dx_{E0}} = -v_{E} m_{E} \frac{dv_{E}}{dx_{E0}} F - F_{E} v_{E}.
\]

(3.8)
From which
\[
\frac{d\mathbf{p}_q}{dx_q} = -F_{Dq} \mathbf{a}_q + \frac{dE_{qz}}{dx_q} = \mathbf{F}_{v} - \mathbf{F}_{qz} + \mathbf{F}_v = \mathbf{V}(x) \quad (b).
\]

(3.9)

From the (3.9b) and (3.6) it follows that \( E_v = 0 \). The zero-value of \( E_v \) is a result of the fact that gluons have not been taken into account. To ensure the constant proton mass, the alteration of the quark kinetic motion must be compensated by the relevant alteration of gluons energy – mass. Taking gluons into account.

\[
L_{qz} = L_{qz}(x_v) + L_{qz}(v) - V(r), \quad \text{where} \quad L_{qz}(v) \quad \text{is the Lagrange function for gluons.}
\]

The preserved energy of “quark – gluon – potential \( V(r) \) system makes
\[
E_{vz} = L_{qz}(x_v) - v \frac{\partial L_{qz}(x_v)}{\partial x_v} + v \frac{\partial L_{qz}(v)}{\partial v} - L_{qz}(v) - V(r) = m_q.
\]

(3.10)

This equation has a solution, if \( v = v_c \) Then in the potential \( V(r) \)
\[
L_{qz} = m_q \sqrt{1 + v_c^2} - V(r) - m_q.
\]

(3.11)

Gluon momentum is
\[
P_{qz} = \frac{m_q v_c}{\sqrt{1 + v_c^2}},
\]

(3.12)

And the energy
\[
E_{qz} = m_q - \frac{m_q}{\sqrt{1 + v_c^2}}.
\]

(3.13)

From Eqn. (3.3) and (3.13) we can draw a conclusion, that the energy – mass of the quark translates into the energy - mass of the gluon, and their sum makes equals \( m_q \). At the same time \( P_{qz} = P_{qz} \) and gluons are moving along with quarks creating valon. As a result, the constituent mass of quarks includes zero rest mass. This determines the dynamical relation of constituent and current quarks’ masses. The quark mass diminishes as it approaches to the center of a proton. This corresponds to the existing idea that quark has a minimum mass under the condition that \( \frac{q^2}{a} \to 0 \).

The half period of quark vibration is \( \theta = 2a_{qz}^{-1} \) and to preserve \( x_{qz} \) in the given range of values we need to put (3.15) it in the following way:
\[
z = \pm \sqrt{Q^2 - x_{qz}^2},
\]

(3.15)

and
\[
v_{qz} = \frac{a_{qz}^2 - z^2}{z}.
\]

(3.16)

The dependence of the quark energy on the radius is \( E_{qz} = m_q a_{qz}^{-1} \).

From the (3.6) we can draw a conclusion that \( V(r_{\text{max}}) = m_q a_{qz} \), where \( r_f \) is the radius of a proton and \( a_{qz} = r_p \). Under the condition that \( -a_{qz}^{-1} \leq x_{qz} \leq a_{qz}^{-1} \) in the coordinates \( (x_{qz}, z) \) quark makes a circumference with a radius \( a_{qz}^{-1} \).

But the allowable values are \( x_{qz} = [0, \infty] \) and this formula must be specified. The half period of quark vibration is \( \theta = 2a_{qz}^{-1} \) and to preserve \( x_{qz} \) in the given range of values we need to put (3.15) it in the following way:
\[
z = \pm \sqrt{Q^2 - (a_{qz}^{-1} - a_{qz}^{-1})^2},
\]

(3.17)

Where \( x_{qz} = x_{qz} - [n]2a_{qz}^{-1} \), \([n]\) is the biggest whole number in \( x_{qz} / 2a_{qz}^{-1} \). The digits before the root take turns depending on the alteration of \([n]\).

Thus, a vibrating quark makes two half circumferences with \( z = 0 \) and \( z = 0 \), moved at \( 2a_{qz}^{-1} \). The figure 1 shows the graph of the quark oscillations. The calculation involves the rms radius of the proton \( r_p = 0.84 \) fm.

Here we can show how the formula (3.6) is functioning. Under \( z = 0 \) and \( V = 0 \) the speed makes \( v_{qz} = 0 \) and \( E_{qz} = 0 \) (points A, C, E). Under \( z = \frac{1}{2} a_{qz}^{-1} \) and \( v_{qz} = 0 \) as well as \( E_{qz} = m_q \) as well as \( V = m_q \) (points B, D). And therefore \( E_{qz} - V = 0 \).

This brings up a question: how do the oscillations of quarks provide total zero momentum in the motionless proton while they are oscillations? Under multi-particle interactions, a symmetric disposition of particles corresponds to the minimum of energy and therefore a proton possesses a spherical symmetry and that means that 3 quarks make diatomic oscillations creating a space angle \( \pi \) and their impulses are getting balanced. This supposition correlates with analytical studies described in [14]; according to them effective fields in baryons has a Y-shaped configuration of quarks’ plane making an equilateral triangle. This conclusion has also been confirmed by calculations in lattice on depending.

Models of E-Invariant Quantized Motion of Massive Particles

A peculiarity of inside hadronic E-space is that its size in the three-
dimensional space is comparable to the Compton quark wave length and the maximum value of quark kinetic energy makes $m_q$. According to the quantum mechanics the minimum quark energy in the limited space must exceed its mass. This is also applicable for oscillators’ energy in the quantum field theory. Thus the wave equations cannot be applied in our case, including probability interpretation. Though the quarks’ behavior in hadrons has a casual nature and the definite metric of the E-space enables to precede straightforward to the probability characteristics.

**Free scalar particle**

The model is oriented towards the inside hadronic space, in which a particle cannot be free, so this part is of a methodic character.

Let us introduce the probabilistic space indexed by $E$-elements and defined by three quantities $(\Omega, \Sigma, \mu)$ where $\Omega$ is a multitude of eve, $\Sigma$ is algebra of $\Omega$ subsets and $\mu$ is a positive measure normalized, and $\mu(\Omega) \leq 1$. If $X_E$ is the real random variable and $X_E \in \Omega$, then the distribution of $X_E$ is the probabilistic measure on $\Omega$ $\mu = P(X_E \in X_E < X_E)$.

**Definition 2**: The state of the particle is described by the function $\Omega(x_E, x_0) = 1 - \mu = \bar{\mu}$, belonging to $E$ and selected for $\Phi(x_E, x_0, x_0, x_0, x_0) = 0$.

If the functions $\Phi(x_E, x_0, x_0, x_0, x_0)$ describe a scalar particle then its Lagrangian will equal $L = \frac{1}{2}(\partial_t \Phi)^2 + \frac{m^2}{2} x_0^2$. (4.1)

From which in a usual way we can get a Klein-Gordon-Fock equation in the $E$-space

$$(\partial_t^2 + \Delta) \Phi(x_E, x_0, x_0) = m^2 \Phi(x_E, x_0, x_0),$$

(4.2)

Correspondingly to the (3.4)

The obvious function $\Phi(x_E, x_0, x_0) = \exp[-(p_{E0} x_E + p_{x_0} x_0)]$ may not be the solution of the (4.2), as it will give the conditional expectation value $E(x_E, x_0, x_0) = 0$. Under $\Phi = 0$ we obtain the nonphysical value $E(x_E, x_0, x_0) = 0, x_0 = 0 = \infty$. It is also impossible to use the transition to $E$-representation through a Fourier transformation, as the frequency $K_{E0}$ and the wave vector $K_x$ not satisfy the (3.4).

Despite the time coordinate, (4.2) describes the static state. However the infinite velocity in $E$-space makes it possible to transform the equation for the description of dynamic systems. Now represent $\Phi(x_E, x_0, x_0, x_0)$ as the product of two functions $\Phi(x_E, x_0) = \phi_0(x_E) \phi_0(x_0) \phi_0(x_0)$, each depending on just one variable. Such separation has the following physical meaning. For $x_0 = 0$ the function $\phi_0(x_E) = \phi_0(x_E) \phi_0(x_0), x_0 = 0$ will describe the localization phase, and the function $\phi_0(x_0) = \phi_0(x_E) \phi_0(x_0), x_0 = 0$ the translation phase for $x_0 = \infty$. These phases cannot exist simultaneously, and supposing the average duration of the localization phase is $\tau_L$, and that of the translation phase is $\tau_T = x_E / \tau_L$, in the average phase change occurring with $\tau_L$ and $\tau_T$, then after every cycle of phase change we get the motion of the particle at the average velocity of $v_{x_0} = \frac{\tau_T}{\tau_L} \tau_L$. Such a separation is due to the infinite velocity.

Probabilistic approach in compliance with definition 2, consider $\Omega(x_E, x_0, x_0) = P(X_E, x_0, x_0, x_0, x_0)$ being the multidimensional random vector. The random projection of this vector on, for example, axis $x_{E0}$ defines the probability of event $P(x_{E0} > x_{E0})$ and requires the condition $P(x_{E0} \leq x_{E0})$ to be commonly met. The latter condition is met for $x_{E0} = 0$, that is $\Phi(x_{E0}) = P(x_{E0} > x_{E0})$. Accordingly, $\Phi(x_{E0}) = P(x_{E0} > x_{E0})$ for $P(x_{E0} > x_{E0}) = 1$, i.e. $x_{E0} = 0$. As a result, we arrive at (4.2).

Separating the variables it is necessary to take into account that $\tau_L$ and $\tau_T$ must be a 4-vectors: $\tau_L + \tau_T = \tau_T^\prime$, and $\tau_T^\prime - E$-invariant and $\Theta = e(x_{E0}) \tau_T = e(x_{E0}) \tau_T$, where

$$c(x_{E0}) = (1 + v_{x_0}^2)^{-1/2}; c(x_{E0}) = v_{x_0}(1 + v_{x_0}^2)^{-1/2}.$$ (4.3)

From this it follows that equations for each phase of the i-cycle are to be solutions of (4.1)

$$\delta_0 \phi_0(x_{E0}) = k_0^2 \phi(x_{E0}); \partial_{E_{E0}} \phi_0(x_{E0}) = k_0^2 \phi_0(x_{E0})$$ (4.4)

where

$$k_{E0} = m_{E0}[1 + v_{x_0}^2].$$ (4.5)

Equation (4.4a) has the following solution $\phi_0(x_{E0}) = e(x_{E0}) \exp[-k_{E0} x_{E0}]$. As attractive potential $V(x_{E0}) \geq 0$ equally affects the particle as well as the antiparticle, according to (3.6) $A_{E0} = \sqrt{\mu - v_{x_0}^2} \geq 0$ and correspondingly, $K_{E0} \geq 0$. Considering $x_{E0} \geq 0$ from Definition 2 it follows that $C_{E0} = 1, C_0 = 0$, and there remains the decreasing exponent. The probability density $f(x_{E0}) = d\phi_0(x_{E0}) / dx_{E0} = K_{E0} \exp[-k_{E0} x_{E0}]$ possesses necessary properties: the densities are not negative and the integral of the densities over all values of $x_{E0}$ equals unity. The mathematical expectation of the localization phase duration

$$\tau_L = 1 / K_{E0} = \frac{1}{m_{E0}[1 + v_{x_0}^2]}.$$ (4.7)

For several cycles, segments $X_{E0}$ form the simplest stream with no aftereffect. For the free particle in the translation phase the displacement vector of the particle $x_{E0}$ and vector $k_{E0}$ are co-directed and (4.4b) has the following solution

$$u_{E0}(x_{E0}) = x_{E0} \exp[-k_{E0} x_{E0}],$$ (4.8)

Where $x_{E0} = |x_{E0}|$ and $x_{E0}$ is the unit vector. Probability density $f_{E0}(x_{E0}) = k_{E0} \exp[-k_{E0} x_{E0}]$, i.e. probability density is also positive and the mathematical expectation of the particle displacement in the translation phase is

$$x_{E0} = k_{E0} = m_{E0}[1 + v_{x_0}^2].$$ (4.9)

As has been assumed, $\tau_L$ and $k_{E0}$ are the components of E-vector and with (2.2)

$$\tau_L^2 + k_{E0}^2 = m_{E0}^2 = 

$$ (4.10)

where

$\tau_{E0}$ is the average cycle duration in LRF, $E$-invariant is a value

$$\tau_{E0} P_{E0} \phi_0(x_{E0}) \phi_0(x_{E0})$$ (4.11)

which equals quantum of action.

On the grounds of (4.10) we consider the cycle duration in LRF $x_{E0}$ to be the two-dimensional random vector with random coordinates $x_{E0}$ and $x_{E0}$, distributed by the exponential law. Then $x_{E0}$ is also distributed.
by the exponential law \( u_n(x_{nm}) = \exp(-mx_{nm}) \) with the average value \( \langle x_{nm} \rangle = m^{-1} \). From equality \( dx dx d = dx d x_{nm} \) it follows that \( x_{nm} \) is also distributed by the exponential law \( u(x_{nm}) = x_{nm} \exp(-k x_{nm}) \) and \( \langle x \rangle = \langle x \rangle \). Using equations (2.3b) and (4.9) we obtain

\[
\varepsilon(x) = v x^{-1}, \quad (4.12)
\]

and the average velocity of \( \langle v \rangle = \langle x \rangle / \langle x \rangle \leq 1 \). The value \( \tau_{\alpha} p_{\alpha} = \chi \varepsilon \), where \( h \hbar -1 \varepsilon \) is also relativistically covariant, and equals to relativistic Lagrangian accurate to a coefficient and changes from \( h \) to 0.

Thus the motion of the particle in E-space is discreet, consists of alternating translation and localization phases and the resultant action for every cycle equals a quantum of motion. The averaged graph of free particle motion in E is a random step function with the average step length \( \tau \) and the average height \( \varepsilon \).

The average duration of free particle cycle in LRF quantizes time \( x_{nm} \) into intervals with the average value \( \tau_{\alpha} = m^{-1} \) dependent only on particle mass. And homogeneity is not violated.

**Free spinor particle**

Spinor function \( \psi(x, x) \) also should be a solution to the Dirac equation in E and describe two phases of motion. To derive the Dirac equation model in E we need to take into account that

\[
E = R(x_{mn}) \otimes R(x_{mn}, x_{mn}, x_{mn}),
\]

the sense of such E-space partition is that the rotation is only possible in \( E = R(x_{mn}) \otimes R(x_{mn}, x_{mn}, x_{mn}) \) and consequently only bispinors have effect. Let us factorize (4.2)

\[
\gamma_{\alpha} \gamma_{\alpha} \psi(x, x) = -m \psi(x, x),
\]

Matrices \( \gamma_{\alpha} \) satisfy the relation \( \gamma_{\alpha} \gamma_{\alpha} + \gamma_{\alpha} \gamma_{\alpha} = 2g_{\alpha\beta} \), where \( g_{\alpha\beta} \) - the Kronecker symbol, and equal

\[
\gamma_{\alpha} = \hat{a} = \text{diag}(1, 1, 1), \quad \gamma_{\alpha} = \sigma_{i}.
\]

Function \( \psi(x, x) \) should describe two phases of motion

\[
\psi(x, x) = \psi_{t}(x, x) \chi_{t}(x, x).
\]

For the localization phase together with (4.5) we obtain

\[
\gamma_{\alpha} \gamma_{\alpha} \psi(x, x) = -k_{x} \chi_{t}(x, x),
\]

where \( \chi_{t}(x, x) = \beta \exp(-k_{x} x_{mn}) \) - bispinor with \( k_{x} \neq 0 \) and \( x_{mn} \geq 0 \).

Equation for the translation phase is

\[
\gamma_{\alpha} \gamma_{\alpha} \chi_{t}(x, x) = -k_{x} \chi_{t}(x, x),
\]

and in compliance with (4.8) the solution is \( \chi_{t}(x, x) = \beta k_{x} \exp(-k_{x} x_{mn}) \).

Then

\[
\chi_{t}(x, x) = k_{x} \chi_{t}(x, x),
\]

and the space of bispinor \( \chi_{t}(x, x) \) is a proper space of the diagonal matrix \( \sigma_{i} \) with positive and negative helicity and there may be only a discrete transition between these subspaces. The duration of localization phases and the extent of translation phases are defined by formulae (4.7) and (4.11).

All the features of the quantum theory of the scalar particle are valid for spinors as well. But in the latter case we have a new detail of helicity. In E, the helicity of massive fermions is only observed in the translation phase, and it is a “good” quantum number, whereas in M the helicity of massive fermions with a nonzero mass can’t be a quantum number characterizing the particle, since it can be inverted by appropriate Lorentz transformations. Nevertheless, in nature, there exist left and right fermions that are quite different particles and this is seen in E.

**Neutral spin or particle in the strong potential**

If the particle is affected by the attractive potential which in the general case equals \( V(x_{nm}) \), then (4.13) will take the form

\[
y_{\alpha} \gamma_{\alpha} \psi(x, x) = -(m + V(x_{mn})) \chi_{t}(x, x) \psi_{t}(x, x) \psi_{t}(x, x) V(x_{mn}),
\]

and potential \( V(x_{nm}) \) works then in the localization phase

\[
y_{\alpha} \gamma_{\alpha} \psi_{t}(x, x) = -k_{x} \left(1 + V(x_{mn}) / m \right) \psi_{t}(x, x).
\]

The solution to this equation is

\[
\psi_{t}(x, x) = \beta \exp(-k_{x} (1 + V(x_{mn}) / m) x_{mn}) \chi_{t}(x, x).
\]

The average duration of the localization phase is

\[
\tau_{\alpha} = \frac{1}{k_{x} (1 + V(x_{mn}) / m)}.
\]

The equation for the translation form will take the form

\[
y_{\alpha} \gamma_{\alpha} \psi_{t}(x, x) = -k_{x} (1 + V(x_{mn}) / m) \psi_{t}(x, x) \psi_{t}(x, x) V(x_{mn}),
\]

and the solution

\[
\psi_{t}(x, x) = \beta x_{mn} \exp(-k_{x} (1 + V(x_{mn}) / m) k_{x} x_{mn}).
\]

The average extent of the translation phase is

\[
X_{t} - \frac{1}{k_{x} (1 + V(x_{mn}) / m)}.
\]

With the quantized motion for \( V(r) = cr \) (3.9) takes the following form

\[
E(\delta \psi_{t}) = F \chi_{t}, \quad E(\delta \psi_{t}) = -F \chi_{t}.
\]

Equation (4.27) proves that while the translation phase is on when \( x_{mn} = \text{const} \), there are instant nonlocal interactions in E. However, when mapped in M they take place with speed c.

**Application of the Model of Quantized Motion of Quarks to Determine Some Properties of Quarks in Protons**

**Quantized motion of quarks**

The calculation of the quantized motion of quarks has been done on the basis of the IVC (Figure 1) on the assumption that the quark moves along the axis z which passes through the centre of the proton, parameters of motion \( r_{0} \) and \( X_{0z} \) being of average value. The following data are used in the calculation: root-mean-square radius of the proton \( r_{0} = 0.84 \text{ fm} \) and \( a_{0} = 0.84 \text{ fm} \), averaged constituent mass \( u \) and \( d \) of quarks 0.33Gev. This mass is included into the calculation as Compton wave-length of a quark \( \lambda_{0} = 0.66 \text{ fm} \). The motion of a quark is divided into deceleration and acceleration portions. The
initial point for the calculation (point A) is chosen at the beginning of the deceleration portion when a quark has passed through the centre of a proton and at point $x_0 = 0, z = \lambda / 2 = 0.3\text{fm}$ the quark localization phase starts. The acceleration portion starts with the translation phase at point B when $v_z = 0$ and the end of the translation phase coordinate $x_{\text{end}}$ has become more than 0.84fm.

The quark deceleration in the second half-period of oscillation starts also with the localization phase at point C for $v_z = \infty$ and $z < 0$ and the calculation is done in the way similar to the first half-period. Here the following peculiarity is disclosed: the coordinates of the beginning of the second oscillation (0.08fm 0.32fm) are close to the accepted coordinates of the beginning of the first oscillation (0.0, 0.3fm).

**Charge distribution in the proton**

Central-symmetric motion of quarks (Section 3.1) makes it possible to confine to the calculation of the charge distribution for one quark considering that its charge equals the charge of a proton. The calculation is done on the assumption that $V(r) = cr$ and the charge density is defined by the probability of the quark being at a given point of radius $r=|Z|$. After the approximation by the exponential function the equation for the charge density calculation is obtained as $\rho = -\frac{e}{r} \exp(-3.9\text{fm})/\text{fm}^2$ for validity factor $R^2=0.85$. The calculated charge density along the radius is $j_r = 4\pi r^2 \rho r / \text{fm}$ (Figure 2).

For the comparison the experimental data for the electric form-factor of the proton have been used which are usually described by dipole approximation $G = (1 + q^2/0.71)^{-2}$ [16] for the preset square of 4-momentum $q$. This dependence gives the experimental value of charge density $\rho(r) = 3.0 \exp(-4.35r) / \text{fm}^3$ and that of the distribution of a charge along the radius $j_r(r) = 4\pi r^2 \rho(r) / \text{fm}$ (Figure 2).

Graph $j_r(r)$ systematically exceeds $j_c(r)$. It is connected with the fact that definition domain $j(r)$ equals $0 < r < 0.85$ fm and the box under $j(r)$ equals $d$. Definition domain $j(r)$ equals $0 < r < \infty$ and the box under this curve on the section $0 < r < 0.9\text{fm}$ equals 0.6.

**Conclusion**

It is stated that in the E-space model, Radius-vector rotations group does not mix temporal and spatial coordinates; kinetic energy diminishes when the speed grows. This determines the existence of constituent and current quarks and describes the dynamic relation of their masses; to describe quantum movement in the E-space, wave equations cannot be applied. The application of the random function theory has shown that the quarks’ movement consists of localization and translation phases; helicity of massive fermions can be observed only during translation phase and is a “good” quantum number: an infinite velocity and non-local interactions connected with it while mapping in the M-space does not upset the RS-principles: the maximum interaction transmission velocity and the maintenance of causality principle; the proton charge calculation result plausibly agrees with the experimental data; the four-dimensional values in the E-space are the 4-vector with scalar invariants which have analogies in the M-space; E-invariant models have a transparent physical content and are no alternative for the existing QCD methods, but expand their possibilities.

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