Generation of Sources of Light with Well Defined Orbital Angular Momentum

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Abstract. In this work, a technique to produce spatial electromagnetic modes with definite orbital angular momentum is presented. The method is based in the construction of binary diffractive gratings generated by computer. In the classical regime the gratings produce the well known Laguerre-Gaussian modes distributions when illuminated by a plane wave. In the quantum regime the grating is placed in the signal path of a spontaneous parametric down conversion layout and the diffraction pattern, observed in the coincidence count rate, shows that the single photons are projected onto spatial states consistent with a Laguerre-Gaussian modes distribution.

1. Introduction

Quantum engineering is a branch of science that emerged recently. Its objective is to understand how to improve the manipulation and control of physical systems for the development of new technologies approaching the quantum regime. Of the infinity of processes that occur at the quantum level, the interaction of radiation with matter is of fundamental importance. This is why the interest in understanding the properties of radiation and how to apply them in handling quantum systems has been growing in recent years. One of such properties is the ability to transmit energy and momentum to material systems at a microscopic scale. The energy and linear momentum of radiation are well studied concepts. However, a complete analysis of the optical orbital angular momentum and its potential applications is still under development. The concept itself has gained much interest since its discovery in 1992 [1–3] due to the fact that beams with well defined orbital angular momentum can be easily produced in practice by means of different methods [4–8]. At the same time, the extraordinary perspectives of application of their properties in the manipulation and trapping of matter, quantum information and photonics were immediately evident [9–12]. Experimental studies involving electromagnetic orbital angular momentum in nonlinear optics began shortly in the generation of frequency doubling [13, 14], where the phase matching conditions for helical beams proven to be the same as for plane waves. A similar phenomenon occurs in more general cases of frequency mixing [15, 16]. Since then, too much interest has been paid to the study of the conservation of the orbital angular momentum, spatial correlation and entanglement, of single photons produced by spontaneous parametric
down conversion [17–24]. Studies of these phenomena intends to give a better understanding of the transfer of angular momentum between matter and light at the quantum level. The purpose of this work is to present a procedure to generate sources of light in definite orbital angular momentum states in the classical as well as in the quantum regime. In section 2, the Laguerre-Gaussian modes are considered, from the theoretical point of view, as solutions of the paraxial Helmholtz equation with definite orbital angular momentum. In section 3, the procedure used to generate Laguerre-Gaussian modes distributions is presented. Section 4 contains the experimental setup used to generate single photons in definite spatial states consistent with a Laguerre-Gaussian modes distribution. A spontaneous parametric down conversion (SPDC) layout is used as the source of single photons. The article closes with some conclusions.

2. Orbital angular momentum of light

The fundamental Gaussian beams of light are characterized by their plane wavefronts and Gaussian transverse intensity distributions (see Figure 1). For those modes the linear momentum density $\epsilon_0\mathbf{E} \times \mathbf{B}$ points along the direction of propagation, meaning that there is no component of the angular momentum in that direction.

![Figure 1](image1.png)

**Figure 1.** The Gaussian transverse intensity distribution (a), and the plane wavefront (b) for a fundamental Gaussian mode.

For higher-order modes, as e.g. Laguerre-Gaussian beams, the points of constant phase are delayed with respect to each other and the wavefronts are no longer plane surfaces [25]. The transverse intensity distributions in those cases have also complex structures (see Figure 2).

![Figure 2](image2.png)

**Figure 2.** The “annular” profile of the transverse intensity distribution (a) and the helical wavefront (b) for a Laguerre-Gaussian mode with orbital angular momentum $\ell = 1$.

These spatial modes are predicted by the Maxwell wave description of light as solutions of the wave equation in the paraxial approximation [26]. In particular, the cylindrically symmetric solutions known as the Laguerre-Gaussian modes have the form [27]

$$E(r) = \frac{E_0}{\sqrt{z^2 + z_R^2}} \left( \frac{2r^2}{\omega^2(z)} \right)^{\ell/2} L_n^{(|\ell|)} \left( \frac{2r^2}{\omega^2(z)} \right) e^{-i \frac{\mu r^2}{2 R(z)} e^{-\frac{z^2}{\omega^2(z)}}} e^{i(2n+|\ell|+1) \arctan \frac{z}{z_R}} e^{-i(kz+\ell \theta)}, \quad (1)$$

where $E_0$ is a constant vector, $\ell$ is an integer number, $z_R$ is the Rayleigh range, $\omega(z)$ is the transverse width of the beam (assuming that the waist is placed at $z = 0$), $R(z)$ is the radius of...
curvature of the wavefront and $L_n^{(|\ell|)}$ is the associated Laguerre polynomial [27]. These modes form a complete set of orthogonal solutions to the paraxial Helmholtz equation. The phase $\ell \theta$ is responsible for the helical shape of the wavefronts (see Figure 2) and is directly related to the orbital angular momentum of the beam.

Considering the local angular momentum density of the electromagnetic field [28]

$$ l = \epsilon_0 r \times (E \times B), \quad (2) $$

and using the expression (1), it is possible to show that the ratio of orbital angular momentum to energy in the direction of propagation is $\ell/\omega$ [29], suggesting that the Laguerre-Gaussian modes posses definite orbital angular momentum proportional to $\ell$. In this way, the integer number $\ell$ determines the number of sheets in the helical surface of the wavefront and the amount of orbital angular momentum of each axial mode of the electromagnetic field.

![Figure 3. Geometry of two beams interference.](image)

### 3. Production of light with orbital angular momentum

Different techniques have been developed in order to generate structured beams in practice. For example, it is possible to transform Hermite-Gaussian beams into Laguerre-Gaussian ones using a set of two cylindrical lenses called mode converter [1, 7, 8]. The converting process is based on the decomposition of a Laguerre-Gaussian and a diagonally oriented Hermite-Gaussian modes into Hermite-Gaussian ones. By changing the separation of the set of lenses it is possible to revert the helicity of the resulting beam. However, the most common technique in the production of helical wavefronts with any value of orbital angular momentum is the use of diffractive gratings generated by computer [4–6]. In this case the interference pattern of a plane wave and the mode to be generated is recorded in a photographic film. The resulting grid has an $\ell$–fold dislocation (a fork) at the center of the pattern. When a plane wave illuminates this pattern, the diffracted beam acquire the desired distributions of phase and intensity. Spatial light modulators (SLMs) have also been used to generate beams of a wide range of phase and intensity distributions [10, 23]. They consist in pixelated liquid crystal devices instead of the photographic film, in which different holographic patterns can be displayed.

In this work the Laguerre-Gaussian beams were produced by means of binary diffraction gratings. In order to make the design of these holograms, the interference process of a plane wave and a Laguerre-Gaussian beam of the same frequency and amplitude is considered. The total amplitude of the electric field at a point $P$ in the far field is (see Figure 3)

$$ E'(r, t) = E_0 e^{i(kr_1 - \omega t)} + E_0 e^{i(kr_2 + \ell \theta - \omega t)}, \quad (3) $$

where the first term is the plane wave and the second one the Laguerre-Gaussian mode. Note the additional term in the phase of this field corresponding to the phase angle that characterizes
the helical beams. The total irradiance of the electromagnetic field at point $P$ is given by

$$I(\mathbf{r}) = |E(\mathbf{r})|^2 = 2|E_0|^2(1 + \cos[k(r_1 - r_2) - \ell\theta]). \quad (4)$$

Assuming that $r_1$ and $r_2$ are much greater than the separation $a$ between the sources one obtains (Figure 3)

$$I(\mathbf{r}) = 2|E_0|^2 \left(1 + \cos \left[ \frac{ka}{z_0} r \cos \theta - \ell\theta \right] \right). \quad (5)$$

Next, it is assumed that the boundaries between the clear and dark fringes of the interference pattern are found whenever the argument of the periodic term of (5) crosses integer multiples of $\pi$, i.e.

$$\ell\theta - \frac{ka}{z_0} r \cos \theta = m\pi, \quad m = 0, \pm 1, \pm 2, \ldots \quad (6)$$

Observe that the distance between two clear (or dark) fringes becomes $\Lambda = 2z_0/ka$ as $r \to \infty$. Thus, the algebraic relation giving the desired grating turn out to be

$$\frac{s\theta}{\pi} = m + \frac{2r}{\Lambda} \cos \theta, \quad s = 1, 2, \ldots \quad (7)$$

where $(r, \theta)$ are the polar coordinates, $s$ is the number of dislocations in the grid, $m$ is an integer number and $\Lambda$ is the period of the grid at large distances from the dislocation [6, 9]. By assigning different values to these parameters one may generate diverse grids that produce particular spacial light distributions. In Figure 4 we present two diffractive gratings constructed with $s = 2$ (a) and $s = 6$ (b). As they were illuminated by a plane wave, the well known distributions of Laguerre-Gaussian modes were observed. In order to evaluate the vorticity of these modes we use the self-interference method [1], which consists in superposing two modes of the same charge $\ell$ and opposite handedness. Then, an interference pattern with $N = 2\ell$ regions of constructive interference can be observed. In Figure 5 we show the interference pattern of the Laguerre-Gaussian mode distribution generated by a grating with charge $s = 2$ superposed with itself but rotated $\pi$rad. We can observe a central mode of order 0, a second order mode composed by a distribution of four maxima, a third order one with eight maxima and so on. This means that the corresponding Laguerre-Gaussian modes are associated to $\ell = 0, \pm 2, \pm 4, \ldots$. Similar experiments with different spatial distributions allow to conclude that Laguerre-Gaussian modes associated to $\ell = sn$, $n = 0, 1, 2, \ldots$ can be generated by means of grids with $s$–fold dislocations.
Interference pattern of the Laguerre-Gaussian mode distribution, generated by a grating with charge $s = 2$, superposed with itself but rotated $\pi$ rad. The regions of constructive interference lead to the conclusion that this grids produce Laguerre-Gaussian modes with $\ell = 0, \pm 2, \pm 4, \ldots$.

General outline of the setup used to observe the spatial correlation of down converted photons with orbital angular momentum.

4. Spatial correlation of light with orbital angular momentum

In this section the experimental setup used to test the generation of light with definite orbital angular momentum in the quantum regime is presented. A SPDC layout is used as a source of single photons. In our setup the pump beam is generated by a violet diode laser, of wavelength $\lambda = 405 \text{nm}$ and a bandwidth of 0.78 $\text{nm}$, with horizontal polarization. A type I BBO crystal cut at 30° with respect to the optical axis and a semi-cone angle of 5° is placed to 13 cm from the laser output as shown in Figure 6. A beam with a spot of transversal diameter 1.5 mm and power of 100 mW is incident on the BBO crystal. Two light collectors are placed at a distance of 80 cm from the crystal, on the signal and idler arms of the SPDC setup. The collectors consist on coupling lenses connected to optical fiber to send the down converted light to the detection system. For the measurement of count rates, two avalanche photodiodes (APD), with a detection efficiency for 810 nm single photons of about 60%, are used. When the APDs detect a photon, a 3.5 V TTL pulse of 15 ns is triggered. Then the coincidence system detects either individual or coincidence counts.

A diffractive grating to produce Laguerre-Gaussian modes with $\ell = 0, \pm 2, \pm 4, \ldots$ (see Figure 4(a)) is set on the signal arm of the SPDC source at a distance of 27 cm from the BBO crystal. An He-Ne laser of wavelength 633 nm is used to track the course of signal photons in order to align the grid. The first collector is then placed at the position of maximum counts of idler photons while the second collector make an horizontal scanning on the signal arm searching for coincidences. The aim of each idler photon is to localize in both, time and position, the corresponding signal photon. In Figure 7 it is shown the diffraction pattern observed in coincidence count rate.
Figure 7. Coincidence count rate between the idler and signal photons as a function of the signal detector horizontal position.

Observe that this distribution shows that the signal photons are projected onto spacial states consistent with the three lowest orders of a Laguerre-Gaussian modes distribution, suggesting that they posses definite orbital angular momentum.

5. Concluding remarks

A technique for the generation of Laguerre-Gaussian modes with different values of angular momentum was established. This technique consist in the design of diverse types of diffractive gratings by means of the interference pattern of a plane wave and different Laguerre-Gaussian modes. The angular momentum of the resulting modes was determined by observing the interference pattern of two modes with opposite helicity. The designed holograms shown to produce helical modes with orbital angular momentum proportional to the number of dislocations in the grids. A spontaneous parametric down conversion source was used to produce single photons and the coincidence count rate diffraction pattern, when a fork grid is placed in the signal path was observed. The experiment allowed to conclude that the parametric down converted photons were projected onto spacial states consistent a Laguerre-Gaussian modes distribution.

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References

[1] Allen L, Beijersbergen M W, Spreeuw R J C and Woerdman J P 1992 Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes Phys. Rev. A 45 8185-8189
[2] van Enk S J and Nienhuis G 1992 Eigenfunction description of laser beams and orbital angular momentum of light Opt. Commun. 94 147-158
[3] Beijersbergen M W, Allen L, van der Veen H and Woerdman J P 1993 Asigmatic laser mode converters and transfer of orbital angular momentum Opt. Commun. 96 123-132
[4] Bazhenov V Yu, Vasnetsov M V and Soskin M S 1990 Laser beams with screw dislocations in their wavefronts JETP Lett. 52 429-431
[5] Heckenberg N R, McDuff R, Smith C P, Rubinsztein-Dunlop R and Wegener M J 1992 Laser beams with phase singularities, Opt. Quant. Electronics 24 S951-S962
[6] Arlt J, Dholakia K, Allen L and Padgett M J 1998 The production of multiringed Laguerre-Gaussian modes by computer-generated holograms J. Mod. Opt. 45 1231-1237.
[7] Padgett M, Courtial J and Allen L 2004 Light’s orbital angular momentum Phys. Today 57 35-40
[8] Yao A M and Padgett M 2011 Orbital angular momentum: origins behavior and applications Adv. Opt. Photonics 3 161-204
[9] Andrews L D 2008 Structured light and its applications: an introduction to phase-structured beams and nanoscale optical forces (USA: Academic Press)
[10] Grier D G 2003 A revolution in optical manipulation Nature 424 810-816
[11] Leach J, Dennis M, Courtial J and Padgett M 2004 Laser beams: knotted threads of darkness Nature 432 165
[12] Gibson G, Courtial J and Padgett M J 2004 Free-space information transfer using light beams carrying orbital angular momentum Optics Express 12 5448-5456
[13] Dholakia K, Simson N, Padgett M and Allen L 1996 Second-harmonic generation and the orbital angular momentum of light Phys. Rev. A 54 R3742-R3745
[14] Courtial J, Dholakia K, Allen L and Padgett M 1997 Second-harmonic generation and the conservation of orbital angular momentum with high-order Laguerre-Gaussian modes Phys. Rev. A 56 41934196
[15] Grice W P and Walsley I A 1997 Spectral information and distinguishing in type-II down-conversion with a broadband pump Phys. Rev. A 56 1627-634
[16] Beranskis A, Matijou A, Piskarskas A, Smilgevjei V and Stabinis A 1997 Conversion of topological charge of optical vortices in a parametric frequency converter Opt. Commun. 140 273276
[17] Arlt J, Dholakia K, Allen L and Padgett M J 1999 Parametric down-conversion for light beams possessing orbital angular momentum Phys. Rev. A 59 3950-3952
[18] Chunqing G A O, Guanghi W E I and Weber H 2000 Orbital angular momentum of the laser beam and the second order intensity moments Science in China 43 1306-1311
[19] Maier A, Vaziri A, Weihs G and Zeilinger A 2001 Entanglement of the orbital angular momentum states of photons Nature, 412 3123-316
[20] Franke-Arnold S, Barnett S M, Padgett M J and Allen L 2002 Two-photon entanglement of orbital angular momentum states Phys. Rev. A 65 033823 5 pages
[21] Barbosa G A and Arnaut H H 2002 Twin photons with angular momentum entanglement: phase matching Phys. Rev. A 65 053801 6 pages
[22] Huguenin J A O, Martinelli M, Caetanos D P, Coutinho dos Santos B, Almeidas M P, Souto Ribeiros P H, Nussenzveig P and Khoury A Z 2006 Orbital angular momentum exchange in parametric down conversion J. of Mod. Opt. 53 647-658
[23] Yao E, Franke-Arnold S, Courtial J and Padgett M J 2006 Observation of quantum entanglement using spatial light modulators Optics Express 14 1389-13904
[24] Osorio C I, Molina-Terriza C and Torres J P 2008 Correlations in orbital angular momentum of spatially entangled paired photons generated parametric down-conversion Phys. Rev. A 77 015810 4 pages
[25] Galvez E J 2006 Gaussian beams in the optics course Am. J. Phys. 74 355-361
[26] Bandres M A and Gutierrez-Vega J C 2008 Circular beams Opt. Let. 33 177-179
[27] Siegman A E 1986 Lasers (USA: University Science Books)
[28] Jackson J D 1962 Classical electrodynamics (USA: Wiley)
[29] Allen L, Padgett J M and Babiker M 1999 The orbital angular momentum of light in E. Wolf, Progress in Optics XXXIX Elsevier Science B. V. 291-372