CBR ANISOTROPY IN AN OPEN INFLATION, CDM COSMOGONY

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ABSTRACT

We compute the cosmic background radiation anisotropy, produced by energy-density fluctuations generated during an early epoch of inflation, in an open cosmological model based on the cold dark matter scenario. At $\Omega_0 \sim 0.3 - 0.4$, the COBE normalized open model appears to be consistent with most observations.

Subject headings: cosmic microwave background — large-scale structure of the universe — galaxies: formation

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1. INTRODUCTION

Observational evidence (summarized in Peebles 1993 and Ratra & Peebles 1994b, hereafter RPb) suggests that the cosmological clustered mass density parameter, $\Omega_0$, is significantly smaller than the Einstein-de Sitter value of unity, but possibly somewhat larger than the baryon density value predicted from the standard nucleosynthesis consideration (Walker et al. 1991). Among the low-density cold dark matter (CDM) cosmogonies now under discussion, a model consistent with the familiar version of the inflation picture (Guth 1981; Kazanas 1980; Sato 1981a,b) is a low-density flat universe dominated by a cosmological constant $\Lambda$ (Efstathiou, Sutherland, & Maddox 1990; Kofman, Gnedin, & Bahcall 1993; Stompor & Górski 1994), while a low-density model with open spatial hypersurfaces and $\Lambda = 0$ (Ratra & Peebles 1994a, hereafter RPa; RPb, and references therein)
could perhaps be accommodated in a variant of the inflation picture in which a single-bubble open inflation model is created by tunnelling in a spatially-flat de Sitter spacetime which also inflates (Gott 1982; Guth & Weinberg 1983). In the open case, the first epoch of inflation smooths away initial inhomogeneities, which, if significant on the scale set by space curvature in the second epoch of inflation, would result in an unacceptable large-scale CBR anisotropy (Kashlinsky, Tkachev, & Frieman 1994).

In a model with open spatial sections, the radius of curvature of the space sections introduces a new global length scale (in addition to that set by the Hubble parameter, $H$), and one can either assume a simple functional form for the spectrum of energy density perturbations (Wilson 1983; Sugiyama & Gouda 1992; Kamionkowski & Spergel 1993, hereafter KS; Sugiyama & Silk 1994, hereafter SS), or compute the spectrum that arises from quantum-mechanical zero-point fluctuations during an early epoch of inflation in an open model (Lyth & Stewart 1990; Ratra 1994; RPa).

In RPb the spectrum that results from such a computation, and a generalization to the open model of the Sachs-Wolfe relation between the cosmic background radiation (CBR) anisotropy and the mass distribution (Anile & Motta 1976; RPa), were used to determine the CBR quadrupole anisotropy, $Q$. To fix the inflation-epoch parameters of the model $Q = 10^{e \pm 1} \mu K$ was taken as the range allowed by the measurements (Bennett et al. 1994, hereafter B94; Wright et al. 1994b; Ganga et al. 1994; Górski et al. 1994). A number of statistics of cosmological interest were then estimated, with results that were observationally encouraging, but with large uncertainty because of the relatively large range of $Q$ allowed by the observations and by theoretical cosmic variance. SS have recently studied large-scale CBR anisotropies in this and other low-density models; they, however, did not examine large-scale structure.

Here we summarize a computation of the lowest two thousand CBR multipoles in this model, use the result to normalize the model to the anisotropy at $10^\circ$ (which is observationally better determined than $Q$, and has smaller cosmic variance), and tabulate statistics of cosmological interest. In agreement with earlier conclusions, depending on the $10^\circ$ CBR anisotropy, when $\Omega_0 \sim 0.3$ or maybe somewhat larger, but still significantly below the Einstein-de Sitter value, the open model does fairly well at fitting most observations. In addition, the shape of the large-scale CBR anisotropy multipole spectrum differs from that in the $\Omega_0 = 1$ CDM model, and may allow an observational test of this open model.
The open inflation model is discussed in §2. In §3 we summarize the CBR anisotropy computation, and in §4 we consider the predictions of the model.

2. MODEL AND POWER SPECTRUM

The inflation epoch of the open cosmological model of RP\textsubscript{a,b} is characterized by the potential for the inflaton scalar field \( \Phi \),

\[
V(\Phi) = 12\hbar^2 [1 - \epsilon \Phi],
\]

where the first term, \( 12\hbar^2 \), is responsible for the expansion during inflation, and the second term, with \( \epsilon \) small, forces the mean value of \( \Phi \) ‘down the hill’. At reheating \( V(\Phi) \) vanishes and the \( \Phi \) energy density is converted to radiation energy density (Ratra 1992).

The computation of the fluctuations produced during inflation is described in RP\textsubscript{a} and the results are summarized in RP\textsubscript{b}. We work to linear order in the matter and metric perturbations about a spatially homogeneous open cosmological model, and in the inflation epoch we also work to lowest nontrivial order in an expansion in \( \epsilon \) (Ratra 1989).

One approach to computing the CBR anisotropy makes use of the gauge-invariant fractional energy-density perturbation (\( \Delta \)) power spectrum,

\[
P_\Delta(A, t) = |\Delta(A, t)|^2,
\]

where the radial coordinate wavenumber \( A(0 < A < \infty) \) is related to the eigenvalue of the spatial scalar Laplacian, \( -(A^2 + 1) \). (\( P_\Delta \) should not be confused with the instantaneously Newtonian synchronous hypersurface power spectrum used in RP\textsubscript{b}.) The present linear-theory power spectrum is (RP\textsubscript{a})

\[
\frac{\epsilon^2}{(1 + z_{re})^4} P_\Delta(A) = 2\pi \left( \frac{H_0}{m_p} \right)^2 \frac{\Omega_0}{1 + z_{eq}} \left( \frac{W_1}{c_1} \right)^2 \frac{4 + A^2}{A(1 + A^2)},
\]

where the Planck mass \( m_p = G^{-1/2} \), \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\) is the present Hubble parameter, \( z_{eq} \) is the redshift of equality of radiation and matter mass densities, \( z_{re} \) that of reheating, and

\[
\frac{W_1}{c_1} = \frac{2(1 - \Omega_0)}{45\Omega_0^{1.5}(1 + z_{eq})^{2.5}} + \frac{1 + 2\Omega_0}{1 - \Omega_0} + \frac{3\Omega_0}{(1 - \Omega_0)^{1.5}} \ln \left\{ \frac{1}{\sqrt{\Omega_0}} - \sqrt{\frac{1 - \Omega_0}{\Omega_0}} \right\}.
\]

The first term on the right hand side is a non-power-law correction to the adiabatic solution; it is subdominant unless \( \Omega_0 \) is very small. The wavenumber dependence of equation (2) is consistent with that of the energy-density perturbation power spectrum which may be
derived from the expressions of Lyth & Stewart (1990) (Lyth 1994). As noted in RPa,b, at short wavelengths $|\Delta|^2 \propto A$ (the usual $n = 1$ scale-invariant form), while at long wavelengths $|\Delta|^2 \propto 1/A$ (this growth at small $A$ is not disturbing, since on large scales the spatial harmonics are strongly damped). Finally, we note that $P_\Delta \propto 1/\epsilon^2$, so an observational upper bound on $P_\Delta$ results in a lower limit on $\epsilon$, the slope of the inflaton potential (Ratra 1992, 1989, 1990).

3. CBR ANISOTROPY

The computation of the CBR anisotropy multipole moments $C_l = \langle |a^m_l|^2 \rangle$ (where the temperature anisotropy in comoving coordinates is $\delta T/T = \sum_{l,m} a^m_l Y^m_l$) makes use of the gauge-invariant formalism of Gouda, Sugiyama, & Sasaki (1991). (It can be shown that this is identical to the synchronous-gauge formalism of RPa.) In this preliminary computation of the $C_l$ we take $h = 0.5$ and the present baryon density $\Omega_B = 0.03$, use the wavenumber dependence of the power spectrum in equation (2), numerically integrate the perturbation equations starting from well before $z_{eq}$ (as a result we may ignore the subdominant non-power-law term in eq. [3]), and account for the fuzziness of the last-scattering surface (the CBR anisotropy on this surface is negligible). We find that the scaled quadrupole ($l = 2$) moment ($Q^2$ multiplied by $\epsilon^2/(1 + z_{re})^4$, as in eq. [2]), agrees with that found in RPa to better than 1% (we compared the two computations of the ratios of $Q$ at total $\Omega_0 = 0.1, 0.2$, and 0.2, 0.3, · · · , and 0.5, 0.6; since we use the numerical values from the $h = 0.8$ and $\Omega_B = 0.0$ run of the RPa computations, this result shows that for all practical purposes $Q$ is independent of $h$ and $\Omega_B$, Bond et al. 1994).

Using the numerical values for the $C_l$, one could fix the model normalization by requiring that the rms temperature anisotropy at 10° angular resolution agree with the two year COBE value $\delta T = 30.5(1 \pm 0.16)\mu K$ (B94), where the range is that allowed at one standard deviation from the measurement errors and model-dependent cosmic variance added in quadrature. This $\delta T$ is determined from the data after a monopole and dipole is subtracted (which affects the value of the quadrupole and octupole); as a result the cosmic $\delta T$ is likely to be somewhat larger than 30.5$\mu K$ (Wright et al. 1994a). Also, if one uses, as we do, a 10° FWHM gaussian approximation to the DMR beam shape, one must increase $\delta T$ (Wright et al. 1994a). It does not yet seem possible to account for these adjustments in a (theoretical) model independent manner, but for the purpose of this
preliminary comparison it suffices to adopt $\delta T(10^\circ) = 35(1 \pm 0.3)\mu K$, where we have taken the precaution of increasing the range to account for possible model-dependent effects. On this large a scale ($10^\circ$) the dependence of $C_l$ on $h$ and $\Omega_B$ is very weak, and so the normalization is almost independent of the value of $h$ and $\Omega_B$. The numbers in column (2) of the table is the value of $Q$ predicted with this normalization. Comparing to the result of RPb, we see that the parameters of the inflation epoch model must obey $1 + z_{re} \sim 10^{29}\sqrt{\epsilon}$.

The numbers in columns (3) and (4) of the table are the present rms linear fluctuation $\delta M/M$ in the mass averaged over a sphere of radius $8h^{-1}$ Mpc and the present rms value $v_p$ of the line-of-sight peculiar velocity in a window of radius $50h^{-1}$ Mpc. They are scaled by the ratio of $Q$ estimated here to that of RPb, from the numerical values computed in RPb. (RPb took $\Omega_B = 0$, so the $\delta M/M$ numbers in the table are fair for $\Omega_0 \sim 0.3$ and larger, while the rough estimate of $v_p$ in RPb assumed that on $50h^{-1}$ Mpc the transfer function could be ignored.) The range in each entry only accounts for the uncertainty in the $10^\circ$ normalization.

In Figures 1 and 2 we show the $C_l$, to $l = 50$ and $l = 2000$, as a function of $l$. The highest curve in the figures, at $l \sim 50$, is the flat model, with $\Omega_0 = 1$, and the other curves are the open models, all with $h = 0.5$ and $\Omega_B = 0.03$. For $l < 50$ the dependence on $h$, $\Omega_B$, and ionization history is very weak, but for larger $l$ the dependence is significant, and so the $l > 100$ part of the curves in Figure 2 are only meant to be illustrative. Finally, we show in Figure 3 the scaled Newtonian hypersurface physical linear energy density perturbation power spectrum, $(a_0h)^3 \tilde{P}(A)T^2(A)$ (RPb, eq. [2], where $a_0$ is the present cosmological scale factor, and $T^2(A)$ is the transfer function with $\Omega_B = 0$), as a function of the scaled proper wavenumber, $A/(a_0h)$. The highest curve is the Einstein-de Sitter model, with $\Omega_0 = 1$ and $h = 0.5$; the other curves are the open models with $h = 0.65$.

4. DISCUSSION

Dynamical mass estimates on length scales $\lesssim 10h^{-1}$Mpc consistently suggest $\Omega_0 = 0.2 \pm 0.1$ (Peebles 1993), while the preliminary dynamical evidence, on scales $\gtrsim 20h^{-1}$Mpc, from the IRAS/POTENT analysis of large-scale flows is that $\Omega_0 \approx 1$ (Dekel et al. 1993). Large-scale estimates based purely on redshift surveys, however, are consistent with lower $\Omega_0$ (Hamilton 1993; Fisher et al. 1994). Also, as summarized in RPb, most of the rest of the observational evidence is consistent with a low-density open or $\Lambda$-dominated flat
model. In our computation of $\delta M/M$ and $v_p$, we adopt $h = 0.65$ (when $\Omega_0 < 1$), which is in the range of most recent estimates (Jacoby et al. 1992; van den Bergh 1992; Fukugita, Hogan, & Peebles 1993; Birkinshaw & Hughes 1994; Sandage et al. 1994; Schmidt et al. 1994). For $\Omega_0 = 0.4$ this implies an expansion time $\sim 12$ Gyr, which is consistent with, but near the low end of, recent estimates (van den Bergh 1992).

The rms fluctuation in the number of galaxies in a randomly placed sphere of radius $8h^{-1}$Mpc is observed to be $\delta N/N = 0.79$ to 1.1 (Peebles 1993, eqs. [7.33, 7.73]). From Table 1 we see that, depending on the $10^\circ$ CBR anisotropy, when $\Omega_0 = 0.3$ the open model could be consistent with a bias factor of about two. An $\Omega_0 = 0.1$ model would require unreasonably high bias, while $\Omega_0 > 0.4$ could be consistent with no bias. Given the uncertainties, these values of the bias factor should also suffice to make the spectra of Figure 3 consistent with the data. (The $P_\Delta(A) \propto A$ spectrum considered by KS has less large-scale power than the one considered here, and when normalized to COBE it results in a $\delta M/M(8h^{-1}$Mpc) that is $\sim 25\%$ larger at $\Omega_0 = 0.3$.) The observed cluster mass and correlation functions provide another test (Lilje 1992; Kauffmann & White 1992; Weinberg & Cole 1992; Oukbir & Blanchard 1992; Bahcall & Cen 1992; White, Efstathiou, & Frenk 1993). For instance, Cen, Gnedin, & Ostriker (1993) find that an $\Omega_0 h = 0.2$, $\Lambda$-dominated $\Omega_0 = 0.3$ model, with $\delta M/M(8h^{-1}$Mpc) = 0.67, is a reasonable fit to the data. From Table 1 we see that in the open model at $\Omega_0 = 0.4$, $\Omega_0 h = 0.26$ and, depending on the $10^\circ$ CBR anisotropy, $\delta M/M(8h^{-1}$Mpc) $\sim 0.6$.

It is interesting that the values predicted for $Q$ after normalizing at $10^\circ$, column (2) of the table, are larger than the COBE CBR measurement $6(1 \pm 0.5)\mu K$ (B94). This is also the case in the Einstein-de Sitter model, but not for topological defects in an open universe (Spergel 1993). Since the total quadrupole is significantly affected by emission from our galaxy, and cosmic variance is non-negligible, it would be premature to conclude that this rules out the model. (B94 note that the probability of finding the one-standard-deviation measured COBE range from a flat model with $Q = 17\mu K$ is 10%.) The shape of the low-order CBR multipoles (Fig. 1) is quite insensitive to the value of $h$ and $\Omega_B$, but does depend on the value of $\Omega_0$. The shape of the low-$\Omega_0$ open inflation model spectrum is somewhat reminiscent of that in the scale-invariant $\Lambda$-dominated flat model and in the tilted CDM model, but differs from that of the scale-invariant Einstein-de Sitter case (SS). The shape at low $l$ is mostly determined by two effects: the strong damping of the open
model spatial harmonics on scales comparable to that of space curvature; and the long wavelength $1/A$ form of the power spectrum (eq. [2]). Relative to the $P_{\Delta}(A) \propto A$ model (where the shape of the low-$l$ $C_l$ is determined by the damping, KS), we see that here the asymptotic $1/A$ behaviour opposes the damping and raises the low-$l$ $C_l$, as long as $\Omega_0$ is not too small (the $\Omega_0 = 0.1$ multipoles at $l = 3 - 5$ are larger than at $l = 2$; this might be because the present Hubble scale is closer to the space curvature scale and so the damping is more significant for $l = 2$, which goes out to larger scales.) It would be useful to more carefully compare the low-$l$ $C_l$ to the data (and thereby more accurately fix the model normalization). The large $l$ part of the spectrum (Fig. 2) is much more sensitive to the ionization history and the values of $h$ and $\Omega_B$. We see, as noted by Kamionkowski et al. (1994), that in the open case the position of the peak in the spectrum is sensitive to the value of $\Omega_0$, but insensitive to the large wavenumber form of $P_{\Delta}(A)$, and depends weakly on $\Lambda$, $\Omega_B$, $h$, and ionization history. Observations of small-scale CBR anisotropies thus might allow for a discrimination between $\Lambda$-dominated and open models.

Finally, we emphasize that structure formation occurs earlier in the low-density open and $\Lambda$-dominated flat CDM models, compared to the $\Omega_0 = 1$ tilted CDM and mixed dark matter cases (RPb). It would be of some interest to more carefully quantify the differences, since with moderately high redshift data one should be able to see the significant evolution of large-scale structure predicted in those models in which structure forms late.

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| $\Omega_0$ | $Q^b$   | $\delta M / M$ $(8 h^{-1} \text{Mpc})$ | $v_p (50 h^{-1} \text{Mpc})^c$ |
|-------|--------|--------------------------------------|-------------------------------|
| 0.1   | 18(1 ± 0.3) | 0.046 – 0.084                         | 62 – 120                      |
| 0.2   | 20(1 ± 0.3) | 0.13 – 0.25                           | 100 – 190                     |
| 0.3   | 20(1 ± 0.3) | 0.25 – 0.46                           | 140 – 260                     |
| 0.4   | 19(1 ± 0.3) | 0.39 – 0.72                           | 180 – 340                     |
| 0.5   | 17(1 ± 0.3) | 0.54 – 1.0                            | 220 – 410                     |
| 1     | 17(1 ± 0.3)$^d$ | 0.73 – 1.4$^d$                          | 280 – 520$^d$                |

$^a h = 0.65$ unless otherwise indicated
$^b$ unit = $\mu$K
$^c$ unit = km s$^{-1}$
$^d h = 0.5$ flat CDM model
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FIGURE CAPTIONS

Fig. 1.– CBR anisotropy multipole moments \( l(l+1)C_l/(2\pi) \times 10^{10} \) as a function of \( l \), to \( l = 50 \), for the open model with \( \Omega_0 = 0.1, 0.2, 0.3, 0.4 \), and 0.5, and the Einstein-de Sitter model with \( \Omega_0 = 1 \), all for \( h = 0.5 \) and baryon density \( \Omega_B = 0.03 \), normalized to an rms temperature anisotropy of 35\(\mu K \) at 10\(^\circ\). The curves are in descending order of \( \Omega_0 \) as one moves down the right hand side of the figure. From left to right, the data points (courtesy of L. Page, from Bond 1994), with vertical one standard deviation error bars and horizontal bars centered on the relevant window function maxima that give the value of \( l \) at which the window function falls to \( e^{-0.5} \) of the maxima, are the flat-model power-law-spectrum multipole fit from COBE (B94), FIRS (Ganga et al. 1994), and the lower end of the error bar from Tenerife (Hancock et al. 1994). We emphasize that these are preliminary estimates.

Fig. 2.– CBR anisotropy multipoles to \( l = 2000 \). Aside from the different scales on the axes, the notation is the same as in Figure 1. The alternating solid and dashed lines are the spectra of the open model with \( \Omega_0 = 0.1, 0.2, 0.3, 0.4 \), and 0.5, and the Einstein-de Sitter model, as one moves up the figure at \( l \sim 100 \). From left to right, the data points (courtesy of L. Page, from Bond 1994) are the flat-model power-law-spectrum multipole fit from COBE, FIRS, Tenerife, ACME (Schuster et al. 1993), Saskatoon (Wollack et al. 1993), the lower end of the Python error bar (with most likely value \( \sim 6.7 \) and half-power points \( l \sim 52 \) and 200, Dragovan et al. 1994), ARGO (de Bernardis et al. 1994), MSAM2 with and without sources (Cheng et al. 1994), MAX-MuPeg (Meinhold et al. 1993; not shown is MAX-GUM with lower bound \( \sim 6.2 \)), MSAM3 with and without sources (Cheng et al. 1994), and, with no vertical error bars, the two standard deviation upper limits from WD (95\% CL upper limit, Tucker et al. 1993) and OVRO (97.5\% Bayesian probability, Readhead et al. 1989). We emphasize that these are preliminary estimates. For \( l \gtrsim 100 \) the spectra are sensitive to the assumed values of \( h \) and \( \Omega_B \), so this part of the figure is only meant to be illustrative. In particular, increasing \( h \) from 0.5 to 0.65 should allow the low-density open models to comply with the WD and OVRO constraints; this could also be accomplished by early mild reionization (Kamionkowski, Spergel, & Sugiyama 1994).

Fig. 3.– Newtonian hypersurface (scaled) physical linear power spectrum of fractional energy density perturbations, \( P(k) \equiv a_0^2 \tilde{P}(A)T^2(A) \), at the present epoch, as a function of (scaled) proper wavenumber \( k(=A/a_0) \). The highest curve is the flat CDM model with
$\Omega_0 = 1$, $\Omega_B = 0$, and $h = 0.5$; with the $10^\circ$ CBR normalization $\delta M/M(8h^{-1}\text{Mpc}) = 1.0$. The other curves are the open models with $h = 0.65$, $\Omega_B = 0$, and $\Omega_0 = 0.5, 0.4, 0.3, 0.2, 0.1$ as one moves down the right hand side of the figure. With the $10^\circ$ CBR normalization $\delta M/M(8h^{-1}\text{Mpc}) = 0.36$ (when $\Omega_0 = 0.3$), $= 0.56(\Omega_0 = 0.4)$, and $= 0.77(\Omega_0 = 0.5)$, larger than the values used in RPb. The points are the IRAS 1.2Jy redshift data rescaled to real space under the assumptions that $\Omega_0 = 1$ and that IRAS galaxies are unbiased. They are estimated from Figure 10 of Fisher et al. (1993).
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