Design of Adaptive Distributed Secondary Control Using Double-Hidden-Layer Recurrent-Neural-Network-Inherited Total-Sliding-Mode Scheme for Islanded Micro-Grid

QUAN-QUAN ZHANG and RONG-JONG WAI, (Senior Member, IEEE)

1Department of Electronic and Computer Engineering, National Taiwan University of Science and Technology, Taipei 106, Taiwan
2Department of Electrical Engineering, Yuan Ze University, Taoyuan 320, Taiwan

Corresponding author: Rong-Jong Wai (rjwai@mail.ntust.edu.tw)

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ABSTRACT This study proposes an adaptive double-hidden-layer recurrent-neural-network (DRNN)-based distributed secondary control (ADRNN-SC) scheme for the voltage restoration and the optimal active power sharing in an islanded micro-grid (MG). Based on the dynamic model composed of MG network model and primary control, a total-sliding-mode (TSMC)-based distributed secondary control (TSMC-SC) scheme is firstly developed for the properties of fast convergence and overall robustness during the control process, where the issues of voltage restoration and optimal active power sharing are converted to local-neighborhood synchronization and tracking problems. Meanwhile, focused on the problems of the control chattering phenomenon and the model dependence, a model-free DRNN structure is used to mimic the designed TSMC-SC law and to inherit its robust performance. The double-hidden-layer neural network (NN) in the DRNN structure needs less neuron nodes than the one with a single hidden layer at the same control performance because of its strong presentation ability. Thus, the computational complexity of the proposed ADRNN-SC scheme can be reduced. Moreover, the recurrent loop in the DRNN structure delivers the feedback signals of the output layer to the input layer, which possesses associative memory and accelerates the convergence process. Therefore, the DRNN structure can engage with a strong approximation ability and superior dynamic performance. In addition, the network parameters are online tuned adaptively to enhance the network learning ability. Furthermore, based on the small-signal model of the proposed control method embedded with communication delays, the delay margin and the influence of control parameters are also investigated. The effectiveness of the proposed control method is verified by numerical simulations.

INDEX TERMS Double-hidden-layer recurrent-neural-network (DRNN), islanded micro-grid (MG), distributed secondary control, small-signal analysis, total sliding-mode control (TSMC).

I. INTRODUCTION

As an efficient way to deal with the rapidly growing distributed generation of renewable energy, the micro-grid (MG) can be employed in either a grid-connected mode or an islanded mode [1], [2]. The droop control can provide the islanded-operated MG system with stable voltage and frequency support, and is implemented as a local controller at each distributed generation (DG) unit [3]. However, the droop-control-based primary controller will lead to unwanted frequency and voltage deviations and poor voltage-droop-corresponding power sharing [4]. To cope with these problems, secondary control methods with the requirement of information exchange have been widely studied [5]–[8]. The traditional secondary control is implemented in a centralized control structure, and may suffer from single-point failures.
Therefore, as an alternative, the sparse-communication-based distributed secondary control (DSC) scheme has been studied extensively due to its merits of high flexibility, re-configurability and economic efficiency [7]–[9]. Since the sliding-mode control (SMC) is a well-known method due to its strong robustness to uncertainties, it has become an effective scheme for the synthesis of the networked control systems (NCSs), and has been effectively applied for the NCSs [10]–[14]. A comprehensive literature review on recent advances of SMC for NCSs has been presented in [10]. Recently, a great number of SMC techniques have been given for solving the problems of systems with time-delays, e.g. the prediction-based method in [11], the time-shift approach in [12], and the delay-fractioning method in [13]. Niu and Ho [14] firstly discussed the issue of packet losses of the SMC-based NCSs. Then, Jia et al. [15] and Chen et al. [16] investigated the SMC for a general stochastic system and a Markovian jump system with packet losses, respectively. In [15] and [16], the sliding functions based on the occurrence probability of packet dropout occurred in state feedback side have been constructed, where the reachability conditions have been given by adopting a novel Lyapunov function with respect to system states and sliding function variables. By considering the effects from the matched and unmatched uncertainties (MUU), Jouini et al. [17] proposed a novel super-twisting algorithm based on a second-order SMC. Moreover, Karami-Mollaee et al. [18] introduced multiple sliding surfaces with virtual inputs to overcome the mismatched uncertainty, and utilized an integrator to solve the chattering issue. In addition, Jiao et al. [19] developed a super-twisting sliding-mode-based anti-disturbance control method by providing an extended sliding-mode observer (SMO). Furthermore, Cao et al. [20] and Cui et al. [13] investigated finite-time SMC methods for uncertain networked Markovian jump systems with randomly occurring injection attacks and packet losses, respectively. Furthermore, the SMC scheme deserves further attention in applications of fault estimation (FE) and handing the cyber-attacks because of its attractive properties. Li et al. [21] addressed the fault-tolerant SMC problem for a T-S fuzzy stochastic delayed system with augmented variables including system states, actuator faults and sensor faults. In [21], two sliding functions have been constructed for the discontinuous input design and the sliding-mode fault-tolerant-control design, respectively. In order to reduce the influence of actuator attacks, the SMO technique has been introduced in [22] for a type-2 fuzzy system. Moreover, Cao et al. [23] used a neural network (NN) structure to approximate the attack signal, and presented a novel adaptive SMC approach to achieve the security-guaranteed requirement.

For better system performance, some control algorithms, such as input-output feedback linearization method [8], [9], SMC [24], [25], and model predictive control (MPC) [6], have been applied for distributed secondary control structures. Since control schemes based on the traditional SMC always face the control chattering phenomenon problem caused by the sign function in the SMC law, Shen et al. [25] used a radial-basis-function neural-network (RBF-NN) structure to approximate the designed SMC-based distributed secondary control algorithm. As a result, the chattering phenomenon can be alleviated, and the proposed RBF-NN-based distributed secondary controller is endowed with properties of fast convergence and high robustness to uncertainties. However, these methods in [6], [8], [9], [24], and [25] are built on the entire knowledge of MG models and primary control. Unfortunately, the detailed MG information, including network topologies, line impedances and loads, may be fully or partially unavailable to establish accurate models in some cases. Moreover, this model-dependent feature might contradict the concept of hierarchical control and restrict their generalization. Any changes of system structures or parameters could affect the control performance and even result in instability. Besides, it is very hard to precisely capture such dynamics [26] because there are uncertain dynamics and disturbances in distributed-energy-resource (DER)-rich MGs. Obviously, models with poor accuracy can significantly deteriorate the control performance. Recently, model-free control has attracted considerable attention due to its advantages of robustness and flexibility [27]. For the existing model-free control methods, they mostly resort to proportional-integral (PI) control frameworks, which often suffer from high starting overshoot, high sensitivity to controller gains and sluggish response to disturbances [28]. To alleviate the dependence on accurate dynamic models, Pilloni et al. [29] presented an ad-hoc-sliding-mode-based DSC method; Dekordi et al. [30] proposed a super-twisting-sliding-mode-based DSC method, and Mehmood et al. [31] investigated a distributed SMC-based control method. Although these methods in [29]–[31] do enhance robustness, partial model parameters as well as chattering phenomenon cannot be fully avoided.

In order to relax the requirement of system dynamics in controller designs, Amoateng et al. [32] conducted a secondary control framework that incorporates the distributed cooperative control and an adaptive neural network (ANN), where the system dynamics were approximated by the ANN to achieve a model-free controller. However, the separate design of secondary control and primary control might contradict the concept of hierarchical control, which restricts its generalization. Neves et al. [33] designed a multi-task fuzzy secondary controller for an AC MG to regulate the voltage, frequency, and the active and reactive powers, simultaneously. Although the stability analysis of the fuzzy secondary controller is performed in [33], appropriate fuzzy rules are difficult to be defined. Ma et al. [27] proposed a model-free secondary voltage control (SVC), where a nonlinear adaptive controller using ANNs is designed to improve the voltage tracking performance. But, the ANNs in [27] are trained offline, and the design and implementation of the overall secondary control algorithm seem to be complicated. Similarly, Yu et al. [34] presented
an NN-based adaptive secondary controller to deal with the issues of the voltage restoration and the reactive power allocation. Owing to the strong online learning ability of NN, unknown/uncertain dynamics can be compensated, and improved robustness against load changes and external disturbances can be obtained. Unfortunately, detailed stability analysis of the controlled system with communication delays has been rarely involved for these intelligent-control-based researches in [27] and [32]–[34].

In general, the adaptive control is a popular control strategy with advantages to deal with system structure uncertainties, and it can achieve optimized performance by adaptive adjustment of inner parameters [35]. On the other hand, the SMC is a well-known method due to its superior performance of fast convergence, strong robustness to uncertainties and efficient for nonlinear systems [24], [25]. However, uncertain parameter deviations or disturbances in practical applications will lead to an undesired chattering phenomenon in the control effort of SMC [36]–[38]. The chattering problem may degrade the system performance, e.g., the occurrence of actuator saturation [37], the increase of power losses, the production of severe electromagnetic compatibility noise [39], and even leading to system instability [40]. Therefore, the adaptive control is more suitable for improving the aforementioned deficiency of the SMC.

As an effective intelligent control algorithm, the NN control has a powerful approximation ability to the nonlinear function and unknown components, which can be used to alleviate the chattering phenomenon in SMC and relax the requirement of detail information of controlled plants [25], [32]. Therefore, the NN-based adaptive control can be used to inherit the fast dynamic response and robust properties of the SMC, while solving the problems of chattering phenomenon and relaxing the requirement of precise information of the MG models and primary control. However, high computational burden is a key barrier to the application of NN in practical engineering, and it is also hard for the normal NN with a single hidden layer to approximate some quite complex functions [41]–[43]. In the previous researches, a double-hidden-layer NN (DNN) only needs less neuron nodes than the one with a single hidden layer at the same control performance because of its strong presentation ability [41], [42]. Therefore, there are far less parameters in the DNN such that it can also obtain satisfactory control precision, and accelerate the network training speed. Moreover, by a recurrent neural network (RNN) structure, the neurons of the input layer can receive signals from the neurons of the output layer. It is generally known that RNNs with feedback loops are adept at accepting past memory elements. That is, more information will be transmitted to the output node, and the system can achieve better approximation performance and superior dynamic capability compared with the conventional feedforward NNs [41], [42]. As a result, the double-hidden-layer RNN (DHLRNN) structure possesses the advantages of the DNN and the RNN, and it can engage with a strong approximation ability and superior dynamic performance. Chu et al. [41] proposed a novel DHLRNN-based global sliding-mode controller for a three-phase active power filter (APF). The novel DHLRNN structure could combine the advantages of a deep NN and a feedback NN, thereby achieving better approximation performance and showing wonderful static and dynamic properties. In order to realize the high-performance control for an APF, Hou et al. [43] developed an intelligent global sliding-mode control (IGSMC) via a recurrent feature selection NN (RFSNN), where the RFSNN was added to the Quasi-GSMC to replace the required system information. The RFSNN not only could realize a higher mimicking accuracy with recurrent layers, but also could reduce the computational burden of a NN carried with the full parameter adjustment. Moreover, the utilization of the feature selection mechanism avoids redundant or inefficient computation time for the real-time control. To further enhance the performance of power quality improvement, Hou et al. [44] proposed an adaptive type-2 fuzzy neural network (T2RFSFNN) control system. The combination of terminal sliding-mode control (TSMC), type-2 fuzzy neural networks (T2FNN), feature selection approach and recurrent mechanism provides the system with superior robustness, finite-time convergence, and model-free property, while maintaining an acceptable computational complexity for mitigating harmonics. Fei and Chu [42] presented a self-regulated double-hidden-layer output-feedback NN (DHLFNN) for an APF to improve the response characteristic and power quality. The DRNN framework indeed can be applied for the adaptive control to obtain a stronger function fitting capacity and a high learning accuracy. Therefore, the investigations of using the DRNN for the adaptive control in MG applications are carried out in this study, where the DRNN-based adaptive control scheme acts as a distributed secondary controller (DSC) to achieve the aims of voltage restoration and optimal power sharing. Moreover, the online-trained DRNN structure can overcome the drawback in the TSMC about the dependency of detail information of MG models and the primary control. Thus, the designed cooperation-based DSC is engaged with a model-free control design feature without the chattering phenomenon, and is robust to unpredictable disturbances.

This study develops an adaptive double-hidden-layer recurrent-neural-network (ADRRNN)-based distributed secondary control (ADRRNN-SC) scheme for an islanded MG. The proposed ADRNN-SC scheme is possessed with the advantages of a DNN, an RNN, a TSMC, and the distributed secondary control manner, which endows the system with a model-free control structure, high control precision, fast dynamic performance, superior robustness to unpredictable perturbation, and no chattering phenomenon in control efforts. The main contributions of the proposed method can be summarized as follows:

1) In order to enhance the system robustness during the whole control process, a TSMC-based distributed secondary control (TSMC-SC) scheme is designed for dealing with the issues of voltage restoration and optimal active power allocation.
sharing, which can be converted to local-neighborhood synchronization and tracking problems. Then, a DRNN structure is used to mimic the designed TSMC-SC law to eliminate its chattering phenomenon, and to relax the knowledge requirements of the specifications of the MG network topology, the primary controller, line impedances and loads. Therefore, the proposed ADRNN-SC method presents strong robust to unpredictable disturbances, MG structure, and parameter changes. Since the proposed ADRNN-SC structure combines an ADRNN-inherited TSMC algorithm and a distributed cooperative control manner, information exchanges are only required between each DG unit and its neighbors, and the system reliability can be improved.

2) The DRNN structure is a combination of a DNN and a RNN, which endows the system with better approximation performance. Since the DRNN structure with a double-hidden-layer NN is more expressive than the conventional NN with a single hidden layer, less neuron nodes are required in the DRNN control structure, and the computational complexity can be reduced. Moreover, by the RNN, the signals from the output layer are given back to the input layer, which means more information to be delivered to the input layer. Therefore, the approximation ability can be improved and superior dynamic performance can be achieved. In addition, network parameters are online tuned adaptively to enhance the network learning ability. As a result, the proposed ADRNN-SC method can inherit the merits of fast convergence and robustness during the whole control process with a model-free control structure and without control chattering phenomenon.

3) The small-signal model of the proposed control method embedded with communication delays is established, and the delay margin and the influence of control parameters are investigated by the eigenvalue analysis, which can serve as a parameter design indicator.

The rest of this study is organized as follows. Section II presents the design process of a TSMC-SC structure. In Section III, the TSMC-SC law is imitated by a proposed ADRNN-SC method, where a DNN structure and a recurrent loop of the output signal are introduced to enhance the approximation performance. In Section IV, the small-signal model of an islanded MG embedded with communication delays is established, and the delay margin and the influence of control parameters are investigated. Section V conducts numerical simulation studies and Section VI concludes the work.

II. PROBLEM FORMULATION

As an efficient nonlinear control method, the total sliding-mode control (TSMC) has been extensively investigated in various fields for its merits of fast dynamic response and strong robustness to uncertainties [45], [46]. In this study, the TSMC technique is integrated into the distributed cooperative secondary control framework to achieve the properties of fast convergence and the strong robustness against uncertainties for the voltage restoration and the optimal active power sharing, as shown in Fig. 1. Because of the space limitation, this paper only focuses on the active power and voltage droop control loop, the reactive power and frequency responses are not shown here. Note that, the proposed distributed secondary control also can be effectively applied for reactive power sharing and the frequency restoration to be investigated in the future research.

Firstly, the system dynamic model consisting of the primary control, an MG network model and an optimal active power sharing scheme are derived. Since the inner voltage and current controllers operate much faster than the power controller, the dynamics of the primary control can be modelled by only considering the power controller.

For a resistive-impedance-dominated system, the $P\cdot V/Q-f$ droop control is generally used to achieve a desirable power sharing ratio by regulating the voltage amplitude ($V_i$) and the angular frequency ($\omega_i$) around their nominal values. By considering a low-pass filter between the measured power values ($P_i^m$ and $Q_i^m$) and the output power values ($P_i$ and $Q_i$), the simplified $i$-th distributed generation (DG) model based on the primary control can be derived as [4]

$$
\begin{align}
\tau_i \dot{V}_i + V_i - V^* + m_i P_i &= 0 \\
\tau_i \dot{\omega}_i + \omega_i - \omega^* - n_i Q_i &= 0
\end{align}
$$

where $V_i$ and $\omega_i$ represent the output voltage amplitude and the angular frequency of DG$_i$, respectively; $i$ denotes the $i$-th DG; $V^*$ and $\omega^*$ are the nominal voltage amplitude and the nominal angular frequency, respectively; $m_i$ and $n_i$ denote droop gains; $\tau_i$ represents the time constant of a first-order low-pass filter.

For an MG with multiple DGs, both the communication network and physical architecture can be modeled as undirected graphs [24]. According to the graph theory in [24], the $i$-th node in the graph denotes the $i$-th DG (bus) and the edges represent the communication links or the line impedances among DGs; i.e., the line admittance $Y_{ij} = G_{ij} + jB_{ij}$, where $G_{ij}$ represents the conductance, and $B_{ij}$ represents the susceptance. The set of neighbors of the $i$-th DG is denoted as $N_i$. If $j \notin N_i$, then $Y_{ij} = 0$, which means that there is no connection between $i$-th DG and $j$-th DG. Otherwise, if $j \in N_i$ and $j \neq i$, then $Y_{ij} \neq 0$. $Y_{ij}$ serves as the connection weight between $i$-th DG and $j$-th DG. According to the power transmission characteristic analyzed in [4], the active power $P_{li}$ injected to the public network by the $i$-th three-phase inverter can be derived as

$$
P_{li} = \sum_{j \in N_i} 3(V_i^2 G_{ij} - V_i V_j |Y_{ij}| \cos(\delta_i - \delta_j - \phi_{ij}))/2
$$

where $V_i$ and $\delta_i$ represent the voltage amplitude and the phase angle of the $i$-th DG; $V_j$ and $\delta_j$ represent the voltage amplitude and the phase angle of the $j$-th DG; $|Y_{ij}| = \sqrt{G_{ij}^2 + B_{ij}^2}$ is the magnitude of the admittance $Y_{ij}$; $\phi_{ij} = \arctan(B_{ij}/G_{ij})$ is the admittance angle of $Y_{ij}$.

By the power balance theorem [47], the output active power $P_i$ of the $i$-th inverter is the sum of the power consumed by
the local load $P_{L_i}$ and the power injected to the public network $P_I$. Therefore, the architecture model in (3) can be obtained.

$$P_I = P_{L_i} + \sum_{j \in N_i} 3(V_i^2 G_{ij} - V_i V_j |Y_{ij}| \cos(\delta_i - \delta_j - \phi_{ij}))/2$$  \hspace{2cm} (3)

where a ZIP load consisting of a constant impedance load (Z), a constant current load (I), and a constant power load (P) is employed, and the local load $P_{L_i}$ can be expressed as [48]

$$P_{L_i} = P_{1i} V_i^2 + P_{2i} V_i + P_{3i}$$  \hspace{2cm} (4)

in which $P_{1i}$, $P_{2i}$, and $P_{3i}$ denote the nominal constant impedance load, the nominal constant current load, and the nominal constant power load, respectively. The indexes 1, 2 and 3 are the coefficient subscripts of these loads.

Since the conventional P-V droop control method in (1) is designed to achieve the proportional active power sharing among DGs according to their capacities ratios, the economic system operation is not considered. Therefore, an optimal active power sharing method is adopted in this study to minimize the generation cost. Generally, the generation cost for the $i$-th DG is assumed to be a quadratic function [49], which can be expressed as

$$GC_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i$$  \hspace{2cm} (5)

where $\alpha_i$, $\beta_i$, and $\gamma_i$ are given cost parameters.

By the equal increment principle (EIP) [50]–[52], the optimal dispatch can be described as

$$\eta_1(P_1) = \eta_2(P_2) = \cdots = \eta_n(P_n) = \lambda$$  \hspace{2cm} (6)

where $\eta_i(P_i) = 2\alpha_i P_i + \beta_i$ is defined as the incremental cost of the $i$-th DG.

In order to solve the problem of voltage and frequency deviations caused by the primary control, and optimize the power sharing performance, a novel distributed secondary control method is proposed in this study. Typically, the secondary control inputs ($u_i$) are added to the nominal voltage amplitude ($V^*$) of the primary control model. Thus, the following formula can be obtained:

$$\tau_0\dot{V}_i + V_i - V^* + k_{ed}\eta_i(P_i) + u_i = 0$$  \hspace{2cm} (7)

where $k_{ed}$ is the droop coefficient for the active power sharing optimization. By substituting (3) into (7), and choosing $x_i = V_i$ as the system state variable, the system dynamic model based on the primary control, the MG network model, and the optimal active power sharing scheme can be expressed as

$$\dot{x}_i = f_i - g_iu_i$$  \hspace{2cm} (8)

where $g_i = 1/\tau_0$; $u_i$ denotes the secondary control input; $f_i = f_0i + \Delta f_i$, in which $f_0i$ and $\Delta f_i$ are the nominal and the uncertain terms of $f_i$, respectively. The value of the uncertain terms ($\Delta f_i$) is assumed to be bounded as $|\Delta f_i| \leq f_b$, where $f_b$ is a given positive number. The nominal term ($f_0i$) and the uncertain term ($\Delta f_i$) can be expressed as

$$f_{0i} = -\frac{k_{ed}\alpha_i(2P_{01i} + \sum_{j \in N_i} 3G_{0ij})}{\tau_0} V_i^2 + \frac{3k_{ed}\alpha_i}{\tau_0} \sum_{j \in N_i} G_{0ij}V_i V_j$$

$$+ \frac{1 + 2k_{ed}\alpha_i P_{02i}}{\tau_0} V_i + \frac{V^* - 2k_{ed}\alpha_i P_{03i} - k_{ed}\beta_i}{\tau_0}$$

$$\Delta f_i = -\frac{k_{ed}\alpha_i(2\Delta P_{1i} + \sum_{j \in N_i} 3\Delta G_{ij})}{\tau_0} V_i^2 + \frac{3k_{ed}\alpha_i}{\tau_0} \sum_{j \in N_i} \Delta G_{ij} V_i V_j$$

$$- \frac{2k_{ed}\alpha_i \Delta P_{3i}}{\tau_0} V_i$$  \hspace{2cm} (9)

According to the distributed cooperative control theory [9], [25], the objectives of the voltage recovery and
the optimal power sharing can be converted to an average consensus problem together with a synchronization-tracking problem. Because each DG only requires information of its own and its neighbors, the communication network can be simplified, and the system reliability can be improved. Define a local-neighborhood-synchronization tracking error as

\[ e_i = c_p e_{ip} + c_v e_{iv} \]  
(11)

where \( c_p \) and \( c_v \) represent the switch gains for the optimal active power sharing and the voltage restoration, respectively; \( e_{ip} \) and \( e_{iv} \) represent the distributed control protocols of the optimal active power sharing and the voltage restoration, respectively, which can be expressed as

\[ e_{ip} = k_e \sum_{j \in N_i} a_{ij}(\eta(P_j) - \eta(P_i)) \]  
(12)

\[ e_{iv} = \sum_{j \in N_i} a_{ij}(\bar{V}_j - \bar{V}_i) + h_i(V_{ref} - \bar{V}_i) \]  
(13)

where \( P_i^m \) is the output measured active power; \( \eta(P_i) \) represents the incremental cost of DG \( i \) and is defined as \( \eta(P_i) = 2\alpha_i P_i + \beta_i \); \( a_{ij} \) represents the communication weight between DG \( i \) and DG \( j \); \( h_i \) is a pinning gain, and \( h_i \neq 0 \) for DGs that have communication access to the terminal voltage reference \( V_{ref} \). Assume that at least one DG in the MG has access to \( V_{ref} \). \( N_i \) represents the communication neighborhood set of DG \( i \), \( \bar{V}_i \) and \( \bar{V}_j \) are the estimated average voltages of DG \( i \) and DG \( j \), respectively, which can be steered to the practical MG average voltage by a dynamic-consensus-rule-based estimator [53] as

\[ \bar{V}_i = V_i + c_E \int \sum_{j \in N_i} a_{ij}(\bar{V}_j - \bar{V}_i) dt \]  
(14)

where \( c_E \) denotes an estimator parameter. By using (12), the local-neighbor optimal-active-power-sharing error can be corrected. Based on (13) and (14), the estimated average voltages of all DGs can be synchronized to the practical MG average voltage, resulting in a unified estimated average voltage. By a consensus algorithm with the tracking error in (11), the pinned DG regulates its estimated average voltage. By using (12), the consensus problem together with a synchronization-tracking problem can be achieved [54], [55].

Then, a TSMC law will be designed in the distributed cooperative secondary control framework for the synchronization and tracking problems in (11). Firstly, a total sliding surface is defined as

\[ S_i(t) = e_i(t) + k_1 \int_0^t e_i(\tau) d\tau - e_i(0) \]  
(15)

where \( k_1 \) is a given positive constant value; \( e_i(0) \) represents the initial value of \( e_i(t) \), which endows the system with the strong robustness during a whole sliding motion and the fast convergence.

According to the traditional TSMC design steps in [46], the TSMC-based distributed secondary control (TSMC-SC) law can be derived as

\[ u_{TSMC_i} = \frac{1}{g_i} [f_{0i} - \frac{1}{M} d_i - f_2 \text{sgn}(S_i) - k_2 S_i] \]  
(16)

where \( k_2 \) is given positive constant value; \( \text{sgn}(\cdot) \) is the sign function; \( M = c_v \sum_{j \in N_i} a_{ij} + h_i \) is always larger than zero; \( d_i \) can be expressed as

\[ d_i = c_v \sum_{j \in N_i} a_{ij} \dot{V}_j - (c_v \sum_{j \in N_i} a_{ij} + h_i)c_E \sum_{j \in N_i} a_{ij}(\tilde{V}_j - \bar{V}_i) \\
+ c_p k_{ed} \sum_{j \in N_i} a_{ij}[\hat{\eta}(P_j^m) - \hat{\eta}(P_j^m)] + k_1 e_i \]  
(17)

As can be seen in (16), the term \( f_{0i} \) in the TSMC-SC law requires the MG parameter information, e.g., the prior MG network structures, the line impedance, the loads, and the primary control parameters. For a practical application, these parameters are uncertain due to the aging component and the system perturbation, which may lead to system instability or large tracking errors. Therefore, a model-free double-hidden-layer recurrent-neural-network (DRNN)-based secondary control (DRNN-SC) is developed in the next section to mimic the TSMC-SC law in (16) without any knowledge of the specifications of the primary control layer and the MG network model.

III. ADAPTIVE DISTRIBUTED SECONDARY CONTROL BASED ON DRNN SCHEME

The control structure of the proposed adaptive double-hidden-layer recurrent-neural-network (DRNN)-based distributed secondary control (ADRNN-SC) scheme and the DRNN framework are depicted in Fig. 2(a) and 2(b), respectively. As can be seen from Fig. 2(b), the four-layer DRNN designed in this study is embedded with double hidden layers and a recurrent loop of the output signal. The functional structure and the signal propagation of the DRNN are explained as follows:

1) The input layer receives the input signal vector \( \mathbf{x} = [x_1, \ldots, x_m, \ldots, x_N]^T \) and the previous-step output signal \( (Z^{-1}y) \) of the output layer. The output vector of this layer can be expressed as \( \mathbf{\theta} = [\theta_1, \ldots, \theta_m, \ldots, \theta_N]^T \), where

\[ \theta_m = x_m w_{rom}(Z^{-1}y) \]  
(18)

where \( w_{rom} \) denotes the element of the external recurrent weight vector, \( w_{ro} = [w_{ro1}, \ldots, w_{rom}, \ldots, w_{ron}]^T \); \( Z^{-1} \) is the one-step delay operator. In this study, the inputs of the input layer are the sliding surface \( (x_1 = S_i) \) and its derivative \( (x_2 = \dot{S}_i) \).

2) The first hidden layer receives the signals from the input layer and maps them to a higher dimensional hidden space using a Gaussian membership function, which is

\[ \mu_i^j = \exp[-\sum_{m=1}^{N} (\theta_m - c_i^j)^2/(b_i^j)^2] \]  
(19)
convergence and robust performance without the requirement.

designed to mimic the TSMC-SC law to inherit its fast convergence.

\( b_1, \ldots, b_{N_h} \) represent the optimal weight vectors, the optimal mean vectors, and the optimal standard deviation vectors, respectively. The output of the proposed ADRNN-SC can be presented as

\[
u_i = \hat{u}_{DFNNSC}(x, \hat{w}, \hat{b}_1, \hat{c}_1, \hat{b}_2, \hat{c}_2, \hat{w}_m) = \hat{w}^T \hat{\mu}_2 + \zeta\]  

where \( \zeta \) denotes a mapping error; \( (w^*, w_m^*) \), \( (c_1^*, c_2^*) \), and \( (b_1^*, b_2^*) \) denote the optimal weight vectors, the optimal mean vectors, and the optimal standard deviation vectors, respectively. The output of the proposed ADRNN-SC can be presented as

\[
u_i = u_{DFNNSC}(x, \hat{w}, \hat{b}_1, \hat{c}_1, \hat{b}_2, \hat{c}_2, \hat{w}_m) = \hat{w}^T \hat{\mu}_2\]  

where \( \hat{w}, \hat{w}_m, \hat{c}_1, \hat{c}_2, \hat{b}_1 \) and \( \hat{b}_2 \) are the corresponding real estimations, which are updated online by the designed adaptation laws in (24)-(29) to be introduced later.

**Theorem 1:** As for the system dynamics based on the primary control and the MG network model in (8), if the proposed ADRNN-SC law in (23) and the parameter updating rules in (24)-(29) are adopted, the parameter convergence and the stability of the proposed ADRNN-SC system can be guaranteed.

If \(|\hat{w} - B_w| < B_w \) or \(|\hat{w} - B_w^T \hat{\mu}_2| < 0\)

\[
\dot{\hat{w}} = -\varepsilon_1 \hat{S}_i \hat{\mu}_2
\]

(24a)

If \(|\hat{w} - B_w| = B_w \) and \(S_i \hat{w}^T \hat{\mu}_2 < 0\)

\[
\dot{\hat{w}} = -\varepsilon_1 \hat{S}_i \hat{\mu}_2 + \varepsilon_1 S_i \hat{w}^T \hat{\mu}_2 \hat{w} / \|\hat{w}\|^2
\]

(24b)

If \(|\hat{c}_1| < B_{c_1} \) or \(|\hat{c}_1| = B_{c_1} \) and \(S_i \hat{w}^T \mu_{2c_1} \hat{c}_1 \geq 0\)

\[
\dot{\hat{c}}_1 = -\varepsilon_2 S_i (\hat{w}^T \mu_{2c_1})^T
\]

(25a)

If \(|\hat{c}_1| < B_{c_1} \) and \(S_i \hat{w}^T \mu_{2c_1} \hat{c}_1 < 0\)

\[
\dot{\hat{c}}_1 = -\varepsilon_2 S_i (\hat{w}^T \mu_{2c_1})^T + \varepsilon_2 (S_i \hat{w}^T \mu_{2c_1} (\hat{c}_1 \hat{c}_1^T / \|\hat{c}_1\|^2))^T
\]

(25b)

If \(|\hat{c}_2| < B_{c_2} \) or \(|\hat{c}_2| = B_{c_2} \) and \(S_i \hat{w}^T \mu_{2c_2} \hat{c}_2 \geq 0\)

\[
\dot{\hat{c}}_2 = -\varepsilon_3 S_i (\hat{w}^T \mu_{2c_2})^T
\]

(26a)

If \(|\hat{c}_2| < B_{c_2} \) and \(S_i \hat{w}^T \mu_{2c_2} \hat{c}_2 < 0\)

\[
\dot{\hat{c}}_2 = -\varepsilon_3 S_i (\hat{w}^T \mu_{2c_2})^T + \varepsilon_3 (S_i \hat{w}^T \mu_{2c_2} (\hat{c}_2 \hat{c}_2^T / \|\hat{c}_2\|^2))^T
\]

(26b)

If \(|\hat{b}_1| < B_{b_1} \) or \(|\hat{b}_1| = B_{b_1} \) and \(S_i \hat{w}^T \mu_{2b_1} \hat{b}_1 \geq 0\)

\[
\dot{\hat{b}}_1 = -\varepsilon_4 S_i (\hat{w}^T \mu_{2b_1})^T
\]

(27a)

If \(|\hat{b}_1| = B_{b_1} \) and \(S_i \hat{w}^T \mu_{2b_1} \hat{b}_1 < 0\)

\[
\dot{\hat{b}}_1 = -\varepsilon_4 S_i (\hat{w}^T \mu_{2b_1})^T + \varepsilon_4 (S_i \hat{w}^T \mu_{2b_1} (\hat{b}_1 \hat{b}_1^T / \|\hat{b}_1\|^2))^T
\]

(27b)

If \(|\hat{b}_2| < B_{b_2} \) or \(|\hat{b}_2| = B_{b_2} \) and \(S_i \hat{w}^T \mu_{2b_2} \hat{b}_2 \geq 0\)

\[
\dot{\hat{b}}_2 = -\varepsilon_5 S_i (\hat{w}^T \mu_{2b_2})^T
\]

(28a)

If \(|\hat{b}_2| = B_{b_2} \) and \(S_i \hat{w}^T \mu_{2b_2} \hat{b}_2 < 0\)

\[
\dot{\hat{b}}_2 = -\varepsilon_5 S_i (\hat{w}^T \mu_{2b_2})^T + \varepsilon_5 (S_i \hat{w}^T \mu_{2b_2} (\hat{b}_2 \hat{b}_2^T / \|\hat{b}_2\|^2))^T
\]

(28b)
where \( \mu_1, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \) and \( \varepsilon_6 \) are positive learning rates; \( B_{iv}, B_{1i}, B_{2i}, B_{i1}, B_{i2}, \) and \( B_{wvo} \) represent parameter bounds; \( \| \cdot \| \) is the Euclidean norm.

**Proof:** The approximation error between the real estimation and the TSMSC-SC law can be expressed as

\[
\tilde{u}_i = u_{TSMSCI} - u_i = w^*T \mu_2^2 + \zeta \tilde{w}_i \mu_2 = \tilde{w}_i \mu_2^2 + \zeta
\]

where \( \tilde{w} = w^* - \tilde{w} ; \mu_2 = \mu_2^2 - \mu_2 \). By Taylor series expansion [37], \( \mu_2 \) can be linearized as

\[
\mu_2 = \frac{\partial \mu_2}{\partial c_1} |_{c_1 = \hat{c}_1}(c_2^* - \hat{c}_1) + \frac{\partial \mu_2}{\partial c_2} |_{c_2 = \hat{c}_2}(c_2^* - \hat{c}_2) + \frac{\partial \mu_2}{\partial b_1} |_{b_1 = \hat{b}_1}(b_1^* - \hat{b}_1)
\]

\[
+ \frac{\partial \mu_2}{\partial b_2} |_{b_2 = \hat{b}_2}(b_2^* - \hat{b}_2)
\]

\[
+ \frac{\partial \mu_2}{\partial w_{ro}} |_{w_{ro} = \hat{w}_{ro}}(w_{ro}^* - \hat{w}_{ro}) + O_H
\]

\[
= \mu_2c_1 + \mu_2c_2 + \mu_2b_1 + \mu_2b_2 + \mu_2w_{ro} + O_H
\]

where \( \mu_2c_1 = c_1^* - c_1 ; \mu_2c_2 = c_2^* - c_2 ; \mu_2b_1 = b_1^* - b_1 ; \mu_2b_2 = b_2^* - b_2 ; \mu_2w_{ro} = w_{ro}^* - \hat{w}_{ro} ; O_H \) represents a high-order-term vector, and

\[
\mu_2c_1 = [\partial \mu_2/\partial c_1, \partial \mu_2/\partial c_1, \ldots, \partial \mu_2/\partial c_1]^T |_{c_1 = \hat{c}_1} ;
\]

\[
\mu_2c_2 = [\partial \mu_2/\partial c_2, \partial \mu_2/\partial c_2, \ldots, \partial \mu_2/\partial c_2]^T |_{c_2 = \hat{c}_2} ;
\]

\[
\mu_2b_1 = [\partial \mu_2/\partial b_1, \partial \mu_2/\partial b_1, \ldots, \partial \mu_2/\partial b_1]^T |_{b_1 = \hat{b}_1} ;
\]

\[
\mu_2b_2 = [\partial \mu_2/\partial b_2, \partial \mu_2/\partial b_2, \ldots, \partial \mu_2/\partial b_2]^T |_{b_2 = \hat{b}_2} ;
\]

\[
\mu_2w_{ro} = [\partial \mu_2/\partial w_{ro}, \partial \mu_2/\partial w_{ro}, \ldots, \partial \mu_2/\partial w_{ro}]^T |_{w_{ro} = \hat{w}_{ro}} .
\]

Then, the approximation error in (30) can be rewritten as

\[
\tilde{u}_i = w^*T \mu_2 + \zeta - \tilde{w}_i \mu_2
\]

\[
\tilde{u}_i = w^*T \mu_2 + \tilde{w}_i \mu_2 + \mu_2c_1 + \mu_2c_2 + \mu_2b_1 + \mu_2b_2 + \mu_2w_{ro} + O_H + \zeta
\]

\[
\tilde{u}_i = w^*T \mu_2 + \tilde{w}_i \mu_2 + \mu_2c_1 + \mu_2c_2 + \mu_2b_1 + \mu_2b_2 + \mu_2w_{ro} + O_H + \zeta
\]

where \( \tilde{v} = \tilde{w}_i \mu_2 + \mu_2c_1 + \mu_2c_2 + \mu_2b_1 + \mu_2b_2 + \mu_2w_{ro} + O_H + \zeta \). Then, the derivative of the sliding surface in (15) can be rewritten as

\[
\dot{S}_i = -M_{fi} - M \Delta f_i + d_i + g_i M (u_{TSMSCI} - \tilde{u}_i)
\]

\[
\dot{S}_i = -M_{fi} + m(S_i) - M_{c2} \tilde{S}_i - g_i M (\tilde{w}_i \mu_2 + \mu_2c_1 + \mu_2c_2 + \mu_2b_1 + \mu_2b_2 + \mu_2w_{ro} + O_H + \zeta)
\]

\[
\dot{S}_i = -M_{fi} - M \Delta f_i + d_i + g_i M (u_{TSMSCI} - \tilde{u}_i)
\]

\[
\dot{S}_i = -M_{fi} + m(S_i) - M_{c2} \tilde{S}_i - g_i M (\tilde{w}_i \mu_2 + \mu_2c_1 + \mu_2c_2 + \mu_2b_1 + \mu_2b_2 + \mu_2w_{ro} + O_H + \zeta)
\]

where \( r_i = \Delta f_i / g_i E \) represents the uncertainty term, and is also assumed to be bounded by \(|r_i| \leq b_i\). Consider a Lyapunov function candidate as

\[
V = \tilde{w}_i^T (\tilde{v}_1 \mu_2 + \tilde{v}_2 \tilde{v}_2 + \tilde{v}_3 \tilde{v}_3 + \tilde{v}_4 \tilde{v}_4)
\]

\[
\dot{V} = \tilde{w}_i^T \dot{\tilde{v}}_1 \mu_2 + \tilde{v}_1 \dot{\tilde{w}}_1 \tilde{w}_1 + \tilde{v}_2 \dot{\tilde{v}}_2 + \tilde{v}_3 \dot{\tilde{v}}_3 + \tilde{v}_4 \dot{\tilde{v}}_4
\]

\[
\dot{V} = \tilde{w}_i^T \dot{\tilde{v}}_1 \mu_2 + \tilde{v}_1 \dot{\tilde{w}}_1 \tilde{w}_1 + \tilde{v}_2 \dot{\tilde{v}}_2 + \tilde{v}_3 \dot{\tilde{v}}_3 + \tilde{v}_4 \dot{\tilde{v}}_4
\]

\[
\dot{V} = \tilde{w}_i^T \dot{\tilde{v}}_1 \mu_2 + \tilde{v}_1 \dot{\tilde{w}}_1 \tilde{w}_1 + \tilde{v}_2 \dot{\tilde{v}}_2 + \tilde{v}_3 \dot{\tilde{v}}_3 + \tilde{v}_4 \dot{\tilde{v}}_4
\]

By differentiating (34) and substituting (33), it yields

\[
\dot{V} \leq -k_2 \tilde{v}_1 \tilde{v}_1 - k_2 \tilde{v}_2 \tilde{v}_2 - k_2 \tilde{v}_3 \tilde{v}_3 - k_2 \tilde{v}_4 \tilde{v}_4
\]

As can be seen from (36), the function of \( V \) is negative semi-definite, which guarantees \( V_i \) and parameter approximation errors \( (\tilde{w}_i, \tilde{c}_1, \tilde{c}_2, \tilde{b}_1, \tilde{b}_2) \), and \( \tilde{w}_{ro} \) to be bounded. Let function \( F(t) \equiv k_2 S_i \) and integrate \( F(t) \) with respect to time

\[
\int_0^t F(t) dt \leq V(0) - V(t)
\]

Because \( V(0) \) is a bounded function, and \( V(t) \) is also a non-increasing and bounded function, it can conclude that

\[
\lim_{t \to \infty} \int_0^t F(t) dt \leq \infty
\]

Since \( \int_0^t F(t) dt \) and \( F(t) \) are bounded for all time, one can obtain the relation of \( \lim_{t \to \infty} F(t) = 0 \) by the Barbalat’s lemma [56]. It implies that \( S_i(t) = 0 \) as \( t \to \infty \). As a result, the stability of the proposed ADRNN-SC system can be guaranteed.

**Remark 1:** In general, the NN has been widely applied in identification and control for its strong capacity in approximating to arbitrary unknown continuous functions of dynamic systems [57]–[59]. The detailed verification of this conclusion can be referred to [60, Th. 2.1], which is omitted here. However, the TSMSC-SC law to be approximated by the DRNN contains a discontinuous term \(-f_i sgn(S_i) g_i \) in (16). If a further discussion on the sufficient condition of the DRNN as an approximator to the TSMSC-SC law is required, one should smooth the TSMSC-SC law by introducing the concept of a boundary layer (i.e., \( u_{TSMSCI} = f_0 - d_i / M - f_i sgn(S_i) / g_i \), which is a saturation function and \( r_i \) is the thickness of the boundary layer) to form a continuous function. After this modification, the approximation accuracy of the designed DRNN to this smooth TSMSC-SC law can be verified by referring to the proof of [61, Th. 19.2]. As can be seen from these theorems in [57]–[61], the approximation errors can be as small as
possible (i.e., the minimum reconstructed-error vector $\xi$ in (22) is bounded) if one sufficiently samples the domains of interests and has a large number of nodes in the network structure. Note that, the design of the DRNN to approximate the TSMC-SC law in this study is just to use the intelligent ability of the DRNN without the requirement of system information for maintaining the robust characteristic of the TSMC-SC law, and not really to make the DRNN output as the same as the output of the TSMC-SC law possessed with a chattering phenomenon.

### IV. SMALL-SIGNAL MODEL ANALYSIS OF PROPOSED DISTRIBUTED SECONDARY CONTROLLER

#### A. SMALL-SIGNAL MODEL OF PROPOSED DISTRIBUTED SECONDARY CONTROLLER FOR ISLANDED MICRO-GRID WITH CONSIDERATION OF TIME DELAYS

In order to investigate the influence of control parameters and time delay on the system stability, the eigenvalue analysis based on the small-signal model of a 4-bus islanded microgrid (MG) in Fig. 3 is conducted.

According to the low-pass filter between the measured power values ($P_{m}^{i}$ and $Q_{m}^{i}$) and the output power values ($P_{i}$ and $Q_{i}$), the small-signal model of the measured active power can be derived as

$$\Delta P_{m}^{i} = -g_i \Delta P_{i}^{m} + g_i \Delta P_{i}$$  \hspace{1cm} (39)$$

Moreover, the primary droop control function with the optimal active power sharing scheme can be reformulated as

$$\Delta \dot{V}_i = -g_i \Delta V_i - g_k \Delta \eta_i(P_i) - g_i \Delta u_i$$  \hspace{1cm} (40)$$

where $\Delta \eta_i(P_i) = 2\alpha_i \Delta P_i$. In addition, the small signal dynamics of the network model can be derived as

$$\Delta P_i = 2P_{i0} V_{i0} \Delta V_i + P_{2i} \Delta V_i + \sum_{j \in N_i} 3G_{ij}(V_{j0} \Delta V_j - (V_{j0} \Delta V_i + V_{i0} \Delta V_j)/2)$$  \hspace{1cm} (41)$$

where $V_{i0}$ and $V_{j0}$ represent the initial values of voltage amplitude of the $i$-th DG and $j$-th DG, respectively.

The small-signal model of the estimated average voltage of the $i$-th DG in (14) can be presented as

$$\hat{\Delta} \dot{V}_i(t) = \Delta \dot{V}_i(t) + c_E \sum_{j \in N_i} a_{ij}(\Delta \dot{V}_j(t - \tau_d) - \Delta \dot{V}_i(t))$$  \hspace{1cm} (42)$$

where $\tau_d$ represents a constant communication delay. By combining (15)-(17), the time-delayed control laws can be reformulated as

$$\Delta u_i = (k_1 k_2/g_i)(c_i \sum_{j \in N_i} a_{ij} + h_i) \Delta \dot{V}_i - c_v \sum_{j \in N_i} a_{ij} \Delta \dot{V}_j(t - \tau_d))$$
$$+ (k_1 k_2 c_k \bar{c}_d/g_i)(\sum_{j \in N_i} a_{ij}(\Delta \eta_j(P_{m}^{j}) - \Delta \eta_j(P_{m}^{m})(t - \tau_d)))$$
$$+ (-2k_1 c_k \bar{c}_d/2)(\sum_{j \in N_i} 3G_{ij}(V_{j0} + \tau_d) \Delta V_j)$$
$$- (1 + 2k_1 c_k \bar{c}_d/2) \Delta \dot{V}_i + 3k_1 c_k \bar{c}_d \sum_{j \in N_i} G_{ij} V_{j0} \Delta \dot{V}_j$$
$$+ ((k_1 + k_2(c_v \sum_{j \in N_i} a_{ij} + h_i)) - c_E \sum_{j \in N_i} a_{ij}/g_i) \Delta \dot{V}_i$$
$$- (k_1 c_v/(g_i(c_v \sum_{j \in N_i} a_{ij} + h_i)) + (k_2 c_v - c_E)/g_i)$$
$$\times \sum_{j \in N_i} a_{ij} \Delta \dot{V}_j(t - \tau_d))(k_1 k_2 c_k \bar{c}_d/g_i)(\sum_{j \in N_i} a_{ij}(\Delta \eta_j(P_{m}^{j}) - \Delta \eta_j(P_{m}^{m})(t - \tau_d)))$$
$$- (c_v \sum_{j \in N_i} a_{ij} \Delta \dot{V}_j(t - \tau_d) + k_2 c_k \bar{c}_d \sum_{j \in N_i} a_{ij}(\Delta \eta_j(P_{m}^{j}) - \Delta \eta_j(P_{m}^{m})(t - \tau_d)))$$
$$- (c_v \sum_{j \in N_i} a_{ij} \Delta \dot{V}_j(t - \tau_d) + k_2 c_k \bar{c}_d \sum_{j \in N_i} a_{ij}(\Delta \eta_j(P_{m}^{j}) - \Delta \eta_j(P_{m}^{m})(t - \tau_d)))$$  \hspace{1cm} (43)$$

Thus, the small-signal model of the complete MG system with consideration of a consistent time delay ($\tau_d$) can be derived from (39)-(43) as

$$\Delta \dot{x}_{MG} = A_{MG}\Delta x_{MG} + A_{d1}\Delta x_{MG}(t - \tau_d)$$
$$+ A_{d2}\Delta x_{MG}(t - 2\tau_d) + A_{d3}\Delta x_{MG}(t - 3\tau_d)$$  \hspace{1cm} (44)$$

where $\Delta x_{MG} = [\Delta x_{DG1} \Delta x_{DG2} \ldots \Delta x_{DGn}]^T$, with $\Delta x_{DGi} = [\Delta V_i \Delta \dot{V}_i \Delta \eta(P_i) \Delta P_i^{m}/\Delta P_i]$, $A_{MG}$ and ($A_{d1}$, $A_{d2}$, $A_{d3}$) represent the system state matrices of ordinary state and delayed state, respectively, which are given in the Appendix.

#### B. INFLUENCES OF CONTROL PARAMETERS AND TIME DELAYS ON SYSTEM PERFORMANCE

The stability and dynamic performance of the distributed control system in (44) can be analyzed by observing the roots of its characteristic equation in (45).

$$\text{det}(sI - A_{MG} - A_{d1}e^{-\tau_d s} - A_{d2}e^{-2\tau_d s} - A_{d3}e^{-3\tau_d s}) = 0$$  \hspace{1cm} (45)$$

When all the characteristic roots of (45) lie in the left half of the $s$-domain, the system could be considered to be stable. However, the time delay introduces transcendental terms, resulting in an infinite dimensional system, which makes it extraordinarily complex to process the delay differential equation. In this study, the method of tracing critical eigenvalue [62] is introduced to analyze the stability of the system with time delays. The key point is to acquire
all the possible critical stability points, where the purely imaginary roots are spontaneously produced due to the continuity property of root locus with time delays. For a stable system with \( \tau_d = 0 \), the value of \( \tau_d \) is then increased gradually until the purely imaginary roots appear at \( \tau_d = \tau_m \). The term of \( \tau_m \) is defined as the delay margin, which means the system is stable with \( \tau_d < \tau_m \), and unstable with \( \tau_d > \tau_m \).

As for the conjugate imaginary eigenvalues (\( \lambda = \pm iw \)), the following equation should be satisfied:

\[
iw = e^{i\xi} = \varepsilon (A_{MG} + A_{d1} e^{-i\tau_d w} + A_{d2} e^{-i2\tau_d w} + A_{d3} e^{-i3\tau_d w})
\]

where

\[
\Delta (w, \tau_d) = A_{MG} + A_{d1} e^{-i\tau_d w} + A_{d2} e^{-i2\tau_d w} + A_{d3} e^{-i3\tau_d w}
\]

Let \( \xi = \tau_d w \), and then (47) can be reformulated as

\[
\Delta (w, \tau_d) = A_{MG} + A_{d1} e^{-i\xi} + A_{d2} e^{-i2\xi} + A_{d3} e^{-i3\xi}
\]

Note that, these terms of \( e^{-i\xi} \), \( e^{-i2\xi} \), and \( e^{-i3\xi} \) vary periodically when the value of \( \xi \) changes at the periods of \( 2\pi, \pi \) and \( 2/3\pi \), respectively. Therefore, according to (48), the transformation matrix \( \Delta (w, \tau_d) \) and its eigenvalues both vary periodically as the value of \( \xi \) changes at a period of \( 2\pi \). By regulating the value of \( \xi \) from 0 to \( 2\pi \), then all the purely imaginary eigenvalues can be captured. If a pair of purely imaginary eigenvalues (\( \pm iw_1 \)) exist at \( \xi = c \), the corresponding critical time delay can be derived as \( \tau_c = \xi/c |w_1| \), where \( \cdot | \cdot \) is the operator of an absolute value. There may have plural critical time delays \( \{ \tau_{c1}, \tau_{c2}, \ldots, \tau_{co} \} \) corresponding to multiple pairs of imaginary eigenvalues \( \{ \pm iw_{c1}, \pm iw_{c2}, \ldots, \pm iw_{co} \} \), and the delay margin is set as \( \tau_m = \min \{ \tau_{c1}, \tau_{c2}, \ldots, \tau_{co} \} \). Based on electric parameters and control coefficients in Tables 1 and 2, the root locus of the transformation matrix \( \Delta (\xi) \) with the value of \( \xi \) varying in the range of \([0, 2\pi]\) is depicted in Fig. 4. As can be seen from Fig. 4, two pairs of purely imaginary eigenvalues exist, which corresponding to critical delays of \( \tau_{c1} \) and \( \tau_{c2} \). Therefore, the delay margin can be derived as \( \tau_m = 0.289s \) by the relation of \( \tau_m = \min \{ \tau_{c1}, \tau_{c2} \} \) in this study.

By taking the time derivative of (15) and considering the relation of \( \dot{S}_1(t) = 0 \), the error dynamic response \( \dot{e}_1(t) + k_1 e_1(t) = 0 \) for the distributed cooperative secondary control loop can be obtained. The parameter \( k_1 \) in the sliding surface can be roughly selected by considering the dynamic response of the first-order system \( \dot{e}_1(t) + k_1 e_1(t) = 0 \). Moreover, the small-signal stability analysis can provide the theoretical basis for the parameter design. Based on the small-signal model of the complete MG system in (44), the influence of \( k_1 \) on the system stability and dynamic performance can be analyzed by observing the roots of its characteristic equation in (45). The dominant root locus with the value of \( \tau_d = 0s \) and the sliding-surface parameter \( k_1 \) varying from 50 to 10000 is depicted in Fig. 5. As can be seen from Fig. 5, the system dynamics are mainly influenced by three pairs of conjugate eigenvalues denoted by \( \lambda_1 \rightarrow \lambda_0 \). When a relatively small value of \( k_1 \) is adopted, the dominant eigenvalue \( \lambda_1 \) is located on the negative real axis, which endows the system with poor dynamic response. As the value of \( k_1 \) increases, the dominant eigenvalues shift away from the imaginary axis, resulting in a faster dynamic response. Then, the conjugate complex roots \( \lambda_1 \) and \( \lambda_2 \) gradually move towards the imaginary axis. Therefore, by increasing the value of \( k_1 \), the system dynamic response can be improved. However, a too big value of \( k_1 \) may lead to oscillations. Therefore, \( k_1 = 300 \) is selected in this study. Note that a bigger value of \( k_1 \) also can be chosen to achieve faster dynamic response.

### TABLE 1. Electrical parameters for MG system.

| Parameter | DG1 | DG2 | DG3 | DG4 |
|-----------|-----|-----|-----|-----|
| DGs       | \( L_1 = 2.4mH \) | \( L_1 = 1.4mH \) | \( L_1 = 2.4mH \) | \( L_1 = 1.4mH \) |
|           | \( C_1 = 20\mu F \) | \( C_1 = 20\mu F \) | \( C_1 = 20\mu F \) | \( C_1 = 20\mu F \) |
| \( r_L \) | \( 0.047\Omega \) | \( 0.047\Omega \) | \( 0.047\Omega \) | \( 0.047\Omega \) |
| \( R_1 \) | \( 0.08\Omega \) | \( 0.18\Omega \) | \( 0.12\Omega \) | \( 0.15\Omega \) |
| \( L_2 \) | \( 0.36mH \) | \( 0.36mH \) | \( 0.36mH \) | \( 0.36mH \) |
| Rated powers | \( P_L^R = 10kW \) | \( P_L^R = 10kW \) | \( P_L^R = 5kW \) | \( P_L^R = 5kW \) |
|           | \( Q_L^R = 6kVar \) | \( Q_L^R = 4.5kVar \) | \( Q_L^R = 4.5kVar \) | \( Q_L^R = 4.5kVar \) |
| Loads     | \( P_1 = 0.01 \) | \( P_2 = 0.01 \) | \( P_2 = 0.01 \) | \( P_2 = 0.01 \) |
|           | \( P_2 = 1 \) | \( P_2 = 2 \) | \( P_2 = 3 \) | \( P_2 = 4 \) |
|           | \( P_1 = 6 \times 10^4 \) | \( P_2 = 6 \times 10^4 \) | \( P_2 = 5 \times 10^4 \) | \( P_2 = 4 \times 10^4 \) |
|           | \( Q_1 = 0.01 \) | \( Q_2 = 0.01 \) | \( Q_2 = 0.01 \) | \( Q_2 = 0.01 \) |
|           | \( Q_1 = 1 \) | \( Q_2 = 2 \) | \( Q_2 = 3 \) | \( Q_2 = 4 \) |
|           | \( Q_2 = 6 \times 10^4 \) | \( Q_2 = 5 \times 10^4 \) | \( Q_2 = 3 \times 10^4 \) | \( Q_2 = 3 \times 10^4 \) |
| Lines     | \( R_{ab} = 0.32\Omega \), \( L_{ab} = 0.26mH \) | \( R_{ab} = 0.26\Omega \), \( L_{ab} = 0.18mH \), | \( R_{ab} = 0.24\Omega \), \( L_{ab} = 0.16mH \), | \( R_{ab} = 0.32\Omega \), \( L_{ab} = 0.26mH \) |

### TABLE 2. Control coefficients.

| Parameter | DG1 | DG2 | DG3 | DG4 |
|-----------|-----|-----|-----|-----|
| Primary droop control gains | \( m_1 = 1 \times 10^3 \) | \( m_1 = 1 \times 10^4 \) | \( m_1 = 2 \times 10^3 \) | \( m_1 = 2 \times 10^3 \) |
|           | \( n_1 = 5 \times 10^4 \) | \( n_1 = 7.5 \times 10^4 \) | \( n_1 = 1 \times 10^4 \) | \( n_1 = 1 \times 10^3 \) |
|           | \( \tau_p = 0.016 \) | \( \tau_p = 0.016 \) | \( \tau_p = 0.016 \) | \( \tau_p = 0.016 \) |
| Cost parameters | \( \alpha = 0.94 \) | \( \beta = 0.078 \) | \( \alpha = 0.105 \) | \( \alpha = 0.082 \) |
|           | \( \beta_1 = 1.22 \) | \( \beta_2 = 3.41 \) | \( \beta_2 = 2.53 \) | \( \beta_2 = 4.02 \) |
|         | \( \gamma = 51 \) | \( \gamma = 31 \) | \( \gamma = 78 \) | \( \gamma = 42 \) |
| Estimator parameter | \( c_{ir} = 100 \) | \( c_{ir} = 100 \) | \( c_{ir} = 100 \) | \( c_{ir} = 100 \) |

Nominal values: \( V_{ref} = 311V \), \( \omega_{ref} = 377rad/s (60Hz) \)
V. CASE STUDIES

The effectiveness of the proposed adaptive double-hidden-layer recurrent-neural-network (DRNN)-based distributed secondary control (ADRRN-SC) scheme is verified by simulating a 4-bus islanded micro-grid (MG) in Fig. 3 via the MATLAB/Simulink environment. The value of the sliding-surface parameter is selected to be \( k_1 = 300 \) by considering the system dynamic performance based on the small-signal model analysis in Section IV. In the DRNN, the learning rates are selected as \( \varepsilon_1 = 0.02, \varepsilon_2 = 0.2, \varepsilon_3 = 0.15, \varepsilon_4 = 0.15, \varepsilon_5 = 0.02, \) and \( \varepsilon_6 = 0.3 \). By considering the on-line training process of the network parameters, the initial values of the Gaussian function parameters can be roughly given as \( c_1 = [73.50 - 3.5 - 7]^T, b_1 = [22222]^T, c_2 = [73.50 - 3.5 - 7]^T, \) and \( b_2 = [22222]^T \). The initial weights are randomly set between -1 and 1. Moreover, the parameter bounds can be determined as \( B_w = 500, B_{c_1} = 100, B_{c_2} = 100, B_{b_1} = 50, B_{b_2} = 50, \) and \( B_{wro} = 500 \), correspondingly. The MG electrical parameters and other control coefficients are listed in Tables 1 and 2, respectively.

In the DRNN, the learning rates (\( \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 \)) are related to the tracking error convergence speed \([44], [63]\). By increasing the values of these parameters, the system dynamic response can be improved. However, too big values of these parameters may result in larger overshooting. According to our knowledge, there is no specific online tuning mechanism for these parameters. In this study, these parameters are chosen by empirical rules as well as trial and error. The predetermined initial values of network coefficients are determined according to the possible variation of its inputs. Moreover, the inaccurate selection of the initialized parameters can be retrieved by the on-line training methodology. By considering the on-line training process of the network parameters, some heuristics or expert knowledge can be used to roughly initialize the parameters of the DRNN for obtaining better transient performance in practical applications. For example, the initial weights are randomly set between -1 and 1. Moreover, the initial values of the means and standard deviations of Gaussian functions can be selected roughly by dividing equally as follows:

\[
\begin{align*}
\varepsilon_1^j &= S_{i_{\text{max}}} - (j - 1) \frac{S_{i_{\text{max}}} - S_{i_{\text{min}}}}{N_{h1} - 1}, \\
\varepsilon_2^j &= \frac{S_{i_{\text{max}}} - (k - 1) S_{i_{\text{max}}} - S_{i_{\text{min}}}}{N_{h2} - 1}, \\
\varepsilon_3^j &= \frac{S_{i_{\text{max}}} - (m - 1) S_{i_{\text{max}}} - S_{i_{\text{min}}}}{N_{h3} - 1},
\end{align*}
\]

where \( S_{i_{\text{max}}} \) and \( S_{i_{\text{min}}} \) are the predetermined maximal and minimal bounds of \( S_i \). Based on these pre-set initial network parameters, a pre-training process can be conducted by running the procedure once in advance and store the final values as the initial values of the next process to enhance the convergent speed. Then, the parameters are online adjusted, and the effect due to the inaccurate selection of the initialized parameters also can be retrieved by the online adaptation laws in (24)-(29). The parameter bounds \( (B_w, B_{c_1}, B_{c_2}, B_{b_1}, B_{b_2}, B_{wro}) \) are selected by considering the variation range of \( S_i \) and the predetermined initial values. As for the elements of the projection algorithm in (24)-(29), the parameter bounds \( (w, c_1, c_2, b_1, b_2, w_{ro}) \) should not be selected too small in case that network parameters \( (w, c_1, c_2, b_1, b_2, w_{ro}) \) may not be tuned to their optimal values. Therefore, the parameter bounds \( (B_w, B_{c_1}, B_{c_2}, B_{b_1}, B_{b_2}, B_{wro}) \) are comprehensively decided as the Euclidian norms of the pre-treatment steady-state values of the corresponding network parameters \( (w, c_1, c_2, b_1, b_2, w_{ro}) \).

Selection of the upper bound of the uncertain term \( (f_b) \) has a significant effect on the control performance of the total-sliding-mode (TSMC)-based distributed secondary control (TSMC-SC) law. If the bound is selected too large, the sign function in the TSMC-SC law will result in serious chattering phenomena in the control efforts. The chattering problem may degrade the system performance and even lead to system instability \([40]\). On the other hand, if the bound is selected too small, the stability conditions may not be satisfied. It will cause the controlled system to be unstable. By considering the values of line impedances and loads deviating 50%, and the voltage amplitude difference between DGs to be 15V, the value of the uncertain term \( |\Delta f_i| \) can be estimated according to (10). By using the parameters in Tables 1 and 2, it can roughly obtain the theoretical bound value of \( |\Delta f_i| \) to be about 100. In order to prove the stability of the closed-loop system integrating the proposed TSMC-SC law in (16), a Lyapunov function candidate is defined as

\[
V_1 = S^2_i(t)/(2M)
\]

where \( M = c_v \sum_{i \notin N_l} a_{ij} + h_i \) is always larger than zero. When \( S_i(t) \neq 0 \), the time derivative of \( V_1 \) can be derived from (51), and expressed as

\[
\dot{V}_1 = S_i \dot{S}_i/M = S_i([-f_{\text{ui}} - \Delta f_i + g_i u_i + d_i/M] = S_i([-\Delta f_i - f_{\text{sp}} \text{sgn}(S_i) - k_2 S_i]) \leq |S_i| |\Delta f_i| - f_{\text{sp}} |S_i| - k_2 S_i^2 = |S_i| (|\Delta f_i| - f_{\text{sp}}) - k_2 S_i^2 \leq -k_2 S_i^2 < 0
\]
result, all voltages in the small-scale MG can be maintained at its rated value (311V), which is obvious in Fig. 6(b). As a whole, the estimated average voltage of each DG are regulated to the value of 311 V, where the largest voltage deviation rate with respect to the voltage amplitudes of DGs are dropped from their rated values, where the largest voltage deviation rate of DGs before and after load changes is only about 0.64%. In addition, Figure 6 shows that the proposed ADRNN-SC scheme indeed provides superior dynamic response and strong robustness during algorithm activation and load changes at \( t = 0.5s \), \( t = 1.5s \), \( t = 2.5s \), respectively.

**B. IMPACTS OF SLIDING-SURFACE PARAMETER AND COMMUNICATION DELAY**

Since a time delay around 10-40 ms usually exists in modern communication technologies [65], a fixed communication delay of 50 ms is considered for testing the performance of the proposed ADRNN-SC scheme. Figure 7 depicts the system performance with the proposed ADRNN-SC scheme under the same conditions as the ones in Section V-A, but the sliding-surface parameter is changed to the value of \( k_1 = 600 \) in Fig. 7(a)-(c), and a communication delay of 50 ms is considered in Fig. 7(d)-(f). By comparing Fig. 7(a)-(c) and Fig. 6(a)-(c), it can be noticed that the increasing of the sliding-surface parameter \( k_1 \) can generate a faster dynamic response after the algorithm activation at \( t = 0.5s \), and load changes at \( t = 1.5s \) and \( t = 2.5s \), which corresponding to the root locus analysis in Fig. 5. By comparing Fig. 7(d)-(f) and Fig. 6(a)-(c), it is obvious that when the communication delay is considered, the convergence time is increased with acceptable oscillatory dynamics, and the output signals return to steady-state values about 1.2 s at load changes. Since communication time delays in the distributed secondary control of islanded MGs have effects on the system performance, the optimized controllers for compensating the influences of communication delays to enhance the stability of the MG system is worthy to investigate in the future research.

**C. PERFORMANCE OF PROPOSED ADRNN-SC METHOD AT COMMUNICATION FAILURE**

This case adopts the same parameter values in Section V-A. In order to preserve the connectivity of the communication network in Fig. 3, a communication link between DG1 and DG4 is established. Prior to the communication failure, the optimal active power sharing and the voltage restoration can be achieved. The communication link between DG3 and DG4 is interrupted from \( t = 1.5s \) to \( t = 4s \), and load variations within an acceptable range after \( t = 0.5s \) in Fig. 6(c), and the largest voltage deviation rate of DGs before and after load change is only about 0.64%. In addition, Figure 6 shows that the proposed ADRNN-SC scheme indeed provides superior dynamic response and strong robustness during algorithm activation and load changes at \( t = 0.5s \), \( t = 1.5s \), \( t = 2.5s \), respectively.

**A. PERFORMANCE OF PROPOSED ADRNN-SC METHOD UNDER ACTIVATION AND LOAD CHANGES**

In this case, the parameters in Tables 1 and 2 are adopted, and the system behavior of the proposed ADRNN-SC scheme over different stages is depicted in Fig. 6. Prior to \( t = 0.5s \), the MG is operated under the droop control method, and the proposed ADRNN-SC scheme is activated at \( t = 0.5s \). A load rated at 6 kW and 4 kVar is then connected to Load 3 at \( t = 1.5s \) and removed at \( t = 2.5s \). Figure 6(a) shows that when only the droop controller is adopted before \( t = 0.5s \), the optimal active power sharing is not covered, thus the equal incremental cost cannot be achieved. Moreover, as can be seen from Fig. 6(c), due to the inherent regulation characteristic of a droop controller, the voltage amplitudes of DGs are dropped from their rated values, where the largest voltage deviation rate with respect to the nominal value (311 V) reaches 2.3%. However, as can be seen from Fig. 6(a), by activating the proposed ADRNN-SC at \( t = 0.5s \), the optimal active power sharing can be achieved because the equal incremental cost among DGs can be reached. In general, due to the inherent contradiction between the voltage restoration and the active power sharing of a P-V/Q-f droop controlled system with the consensus regulation, voltage deviations cannot be eliminated once the active power sharing is achieved [50]. Therefore, this study adopts a dynamic-consensus-rule-based estimator [53], and the estimated average voltage of each DG are regulated to its rated value (311 V), which is obvious in Fig. 6(b). As a result, all voltages in the small-scale MG can be maintained within an acceptable range after \( t = 0.5s \) in Fig. 6(c), and the largest voltage deviation rate of DGs before and after load change is only about 0.64%. In addition, Figure 6 shows that the proposed ADRNN-SC scheme indeed provides superior dynamic response and strong robustness during algorithm activation and load changes at \( t = 0.5s \), \( t = 1.5s \), \( t = 2.5s \), respectively.

![Figure 6](image-url) Performance of proposed ADRNN-SC method. (a) Incremental cost values. (b) Estimated average voltages. (c) Terminal voltages.
FIGURE 7. Performance of proposed ADRNN-SC method under (a)-(c) $k_1 = 600$ and $\tau_d = 0s$; (d)-(f) $k_1 = 300$ and $\tau_d = 50 ms$. (a) Incremental cost values. (b) Estimated average voltages. (c) Terminal voltages.

FIGURE 8. Link 3-4 failure and plug-and-play capability with 4th DG. (a) Incremental cost values. (b) Estimated average voltages. (c) Terminal voltages.

are proceed at $t = 2s$ and $t = 3s$. Moreover, DG4 is cut off from the MG system with its associated communication links also interrupted at $t = 4s$. Figure 8 shows the system performance with the proposed ADRNN-SC scheme for the scenario of communication failure. As can be seen from Fig. 8, the proposed ADRNN-SC scheme is resilient to the communication failure. The optimal active power sharing and voltage restoration also can be achieved before and after load changes in this scenario. Moreover, as can be seen from Fig. 8(a), when DG4 is removed from the MG system, other DG units increase their supplies according to the optimal cost sharing, and the equal increment principle between the remained DGs also can be reached. In addition, Figure 8(b) shows that the estimated average voltages of the remained DGs are regulated to their rated value (311V), and all voltages in the small-scale MG also can be maintained within an acceptable range in Fig. 8(c).

D. COMPARISONS WITH EXISTING METHODS

In this case, the performance of the proposed ADRNN-SC scheme is compared with the ones of the nonsingular terminal-sliding-mode-based distributed secondary control (NTSM-SC) method in [24] and the adaptive fuzzy-neural-network-based distributed secondary control (AFNN-DSC) method in [38] to highlight its superior properties, which are depicted in Figs. 9-11, respectively. Since the NTSM-SC method in [24] focused on the voltage restoration, the voltage response is compared here for fairness, where $c_p$ and $c_E$ are given $0$ for the proposed ADRNN-SC scheme, and other parameters and conditions are set as the same as the ones in Section V-A. In order to evaluate the system performance with different control methods, the records of the average root-mean-squared error (ARMSE) in (53) is adopted, where a smaller ARMSE value is expected.

$$\text{ARMSE}(e) = \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{1}{T} \sum_{t=1}^{T} e_i^2(t)}$$ (53)

where $T$ is the total sampling instants; $e_i$ represents the local-neighborhood-synchronization tracking error of DG$_i$ $e_i^1$; $n$ represents the total amount of DGs.

As can be seen from Fig. 9(a) and (b), the NTSM-SC method in [24] can restore the DG terminal voltages to their nominal values with the convergence time of about 0.33s, and a small ARMSE value of 0.283 for the local-neighborhood-synchronization tracking error can be achieved. However, as can be seen from Fig. 9(c), serious chattering phenomena exists in the control efforts of the NTSM-DSC method, which makes it difficult to be directly implemented in practical
applications. Besides, the undesired chattering phenomena in the control efforts may excite unstable system dynamics. As can be seen from Fig. 10(a)-(c), the AFNN-DSC scheme endows the system with the properties of a smaller ARMSE value of 0.203, faster dynamic response with the convergence time of about 0.12s. Moreover, smooth control efforts in Fig. 10(c) can be achieved. Figure 11 indicates that the proposed ADRNN-SC scheme also can alleviate the chattering phenomenon of the NTSM-DSC method, and demonstrate even better transient response and smaller tracking errors than the NTSM-DSC method [24] in Fig. 9 and the AFNN-DSC method [38] in Fig. 10. Due to the strong approximation ability and superior dynamic performance of the DRNN structure, the convergence speed with the proposed ADRNN-SC method can be improved by 84.8% in comparison with the NTSM-SC method in [24], and by 58.3% in comparison with the AFNN-DSC method in [38]. Moreover, the proposed ADRNN-SC method exhibits an even smaller tracking error with improvements of 70.0% and 58.1% in comparison with the methods in [24] and [38], respectively.

The comparative results of the proposed ADRNN-SC method with previous works [8], [24], [38] are summarized in Table 3. The NTSM-SC method in [24] performs better dynamic response and stronger robustness at local-neighborhood-synchronization tracking than the distributed-cooperative-control-based secondary control (DCSC) method in [8]. However, the practical application of the NTSM-DSC method is restricted by its chattering phenomena and system-dynamic dependence, whereas it is not the constraint for both the AFNN-DSC method in [38] and the proposed ADRNN-SC scheme. Compared with the methods in [8], [24], and [38], the proposed ADRNN-SC method exhibits an even better dynamic behavior and smaller tracking errors due to its enhanced approximation performance by the deep learning neural network structure and the output recurrent loop. Moreover, the proposed ADRNN-SC scheme considers the optimal power sharing to minimize the generation cost, and all voltages in the small-scale MG can be maintained within an acceptable range, simultaneously. As a result, the contradiction between
The voltage recovery and the power sharing in conventional frameworks can be relaxed.

### E. TEST OF ROBUSTNESS AGAINST UNCERTAINTIES AND DISTURBANCES

In order to test the robustness of the proposed ADRNN-SC scheme against uncertainties and disturbances inheriting the merits of SMC, line impedances in Table 1 are all increased by 50% to test for verification. Other conditions are the same as the ones in Section V-D. By comparing Fig. 12 (a)-(b) and Fig. 11 (a)-(b), it is obvious that the model-free ADRNN-SC scheme still can perform a superior synchronization and tracking performance during both dynamic and steady state at the line impedance increasing by 50%, and thus presents strong robustness against parameter variations. Moreover, smooth control efforts also can be achieved, as can be seen from Fig. 12(c).

### F. INFLUENCE INVESTIGATION OF DOUBLE-HIDDEN-LAYER STRUCTURE AND RECURRENT LOOP

In order to verify the strong presentation ability of the double-hidden-layer recurrent-neural-network (DRNN), the performance of the proposed ADRNN-SC scheme is compared with a single-hidden-layer recurrent-neural-network (SRNN)-based distributed secondary control (SRNN-SC) in Fig. 13, where the SRNN-SC method adopts the same number of neuron nodes as in the ADRNN-SC scheme. Moreover, a double-hidden-layer neural-network (DNN) without the recurrent loop is applied for the distributed secondary control to demonstrate the effectiveness of recurrent loop on accelerating the convergence process, which is depicted in Fig. 14.

Because it is hard for the normal NN with a single hidden layer to approximate some quite complex functions [41]–[43], the SRNN-SC method in Fig. 13 shows poor dynamic response with large fluctuations. Note that, a DNN needs less neuron nodes than the one with a single hidden layer at the same control performance because of its strong presentation ability. Therefore, there are far less parameters in the DNN, which also can obtain satisfactory control precision, and accelerate the network training speed. By comparing the system performance of the proposed ADRNN-SC scheme in Fig. 11 with the SRNN-SC method in Fig. 13, the network with a double-hidden-layer structure can achieve better control performance than the one with a single hidden layer structure at the same number of neuron nodes because of its strong presentation ability. Moreover, by a recurrent neural network (RNN) structure, the neurons of the input layer can receive signals from the neurons of the output layer. It is generally known that RNNs with feedback loops are adept at accepting past memory elements. That is, more information will be transmitted to the output node, and the system can achieve better approximation performance and superior dynamic capability compared with the conventional feedforward NNs [41], [42]. As can be seen from Figs. 11 and 14, the proposed ADRNN-SC scheme with a recurrent loop presents better dynamic performance than the DNN-SC method without the recurrent loop. Therefore, due to the combination of the DNN structure and the recurrent loop in the proposed ADRNN-SC scheme, the system is endowed with better approximation performance and superior dynamic performance.

**Remark 2:** The sliding-mode control (SMC) is a well-known method due to its superior performance of fast convergence, strong robustness to uncertainties and efficient for nonlinear systems [24], [25]. However, SMC-based secondary control methods are built on the entire knowledge of MG models and primary control. Unfortunately, the system modeling is an approximation process, where the detailed MG information (including network topology, line impedances and loads) and the uncertain dynamics of power electronics and disturbances may be fully or partially unavailable to establish accurate models. Any changes of system
structures or parameters could affect the control performance and even result in instability. Besides, it is very hard to precisely capture such dynamics [26] because there are uncertain dynamics and disturbances in distributed-energy-resource (DER)-rich MGs. In addition, uncertain parameter deviations or disturbances in practical applications will lead to an undesired chattering phenomenon in the control effort of SMC [36]–[38]. Obviously, models with poor accuracy and the chattering phenomenon can significantly deteriorate the control performance.

In general, the adaptive control is a popular control strategy with advantages to deal with system structure uncertainties, and it can achieve optimized performance by adaptive adjustment of inner parameters [35]. The DRNN-based adaptive control is designed in this study to mimic the ideal TSMC-SC law, which makes sense a clear operating rule and specific control law of the proposed method, and it also can inherit the advantages of the ideal TSMC. Moreover, the DRNN-based adaptive control is investigated to construct the model-free control design feature in the proposed strategy by its powerful approximation ability to a nonlinear function without a prior knowledge of system models. Therefore, the online-tuned DRNN structure can overcome the drawbacks in the TSMC about the dependency of detail system dynamic information and the chattering phenomenon, and thus presenting a better control performance.

Remark 3: The major difficulty in this study is to determine the delay stability margin of the system. First, a small-signal model of the proposed distributed secondary controller for an islanded MG with the consideration of time delays is established as (44). Then, the characteristic equation in (45) can be obtained, and the stability of the distributed control system in (44) can be analyzed by eigenvalue analysis of (45). However, as can be seen in (45), transcendental terms introduced by the time delay result in an infinite dimensional system, which makes it extraordinarily complex to process the delay differential equation. In previous researches, some approaches ranging from the frequency-domain direct methods [62], [66] to the time-domain indirect methods [67] have been proposed successively. By considering the conservativeness of indirect time-domain methods, the direct approach based on tracing critical eigenvalues [62] is introduced in this study to analyze the stability of MG with the consideration of time delays. The key point is to acquire all the possible critical stability points, where the purely imaginary roots are spontaneously produced due to the continuity property of root locus with time delays. For a stable system with \( \tau_d = 0 \), the value of \( \tau_d \) is then increased gradually until the purely imaginary roots appear at \( \tau_d = \tau_m \). The term of \( \tau_m \) is defined as the delay margin, which means the system is stable with \( \tau_d < \tau_m \), and unstable with \( \tau_d > \tau_m \). There may have plural critical time delays \( (\tau_{c1}, \tau_{c2}, \ldots, \tau_{co}) \) corresponding to multiple pairs of imaginary eigenvalues \( (\pm iw_{c1}, \pm iw_{c2}, \ldots, \pm iw_{co}) \) of (46), and the delay margin is set as \( \tau_m = \min\{\tau_{c1}, \tau_{c2}, \ldots, \tau_{co}\} \). Finally, based on the root locus in Fig. 4, the delay margin can be derived.

VI. CONCLUSION

This study has successfully designed an adaptive distributed secondary control using double-hidden-layer recurrent-neural-network-inherited total-sliding-mode scheme for improving the local-neighborhood synchronization and the tracking performance of voltage restoration, and achieving the objective of the optimal active power sharing in an islanded micro-grid (MG). The double-hidden-layer neural network (DNN) possesses superior approximation performance due to its strong presentation ability, and can
achieve the properties of high precision, rapid learning speed and efficient training. Moreover, the recurrent neural network (RNN) also can improve the approximation capability to unknown functions by associative memory, which further accelerates the convergence process. Owing to the strong approximation ability, the proposed adaptive double-hidden-layer RNN (DRNN)-based distributed secondary control (ADRRNN-SC) method can inherit the merits of fast convergence and robustness during the whole control process with a model-free control structure and without control chattering phenomenon.

As can be seen from numerical simulations, the convergence speed with the proposed ADRNN-SC method can be improved by 96.5% in comparison with the distributed-cooperative-control-based secondary control (DCSC) method in [8], by 84.8% in comparison with the nonsingular terminal-sliding-mode-based distributed secondary control (NTSM-SC) method in [24], and by 58.3% in comparison with the adaptive fuzzy-neural-network-based distributed secondary control (AFNN-DSC) method in [38]. Moreover, the proposed ADRNN-SC method exhibits an even smaller tracking error with improvements of 93.6%, 70.0%, and 58.1% in comparison with the methods in [8], [24], and [38], respectively.

By the eigenvalue analysis based on the small-signal model of the proposed control method embedded with communication delays, the delay margin and the influence of control parameters are investigated, which can serve as a parameter design indicator for engineers. Although the system response deteriorates as the communication delay increases, the system can still maintain stability within a large communication delay range. Further studies also show that the proposed structure is resilient to communication failures. Numerical simulation results have verified the effectiveness of the proposed ADRNN-SC strategy.

**APPENDIX**

The system state matrices of ordinary states \((A_{MG})\) can be expressed as

\[
A_{MG} = \begin{bmatrix}
A_{M11} & A_{M12} & \ldots & A_{M1n} \\
A_{M21} & A_{M22} & \ldots & A_{M2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{Mn1} & A_{Mn2} & \ldots & A_{Mnn}
\end{bmatrix}_{6n \times 6n}
\]

(A1)

where

\[
A_{Mii} = \begin{bmatrix}
T_{ii1} & 0_{6 \times 4}
\end{bmatrix}_{6 \times 6}
\]

(A2)

in which

\[
T_{ii1} = \begin{bmatrix}
-g_i & 0 & -k_{ed}g_i & 0 & 0 & -g_i \\
-g_i & -2c_E & -k_{ed}g_i & 0 & 0 & -g_i \\
2\alpha_1t_{ii1} & 0 & 2\alpha_1k_{ed}t_{ii1} & 0 & 0 & 2\alpha_1t_{ii1} \\
t_{ii1} & 0 & k_{ed}t_{ii1} & 0 & 0 & t_{ii1} \\
t_{ii} & t_{ii} & t_{ii0} & t_{ii1} & t_{ii2} & t_{ii}
\end{bmatrix}_{6 \times 6}
\]

(A3)

with

\[
t_{ii1} = -g_i(2P_1V_{i0} + P_2) - 3/2 \sum_{j \in N_i} G_{ij}V_{j0} + 3 \sum_{j \in N_i} G_{ij}V_{j0}
\]

(A4)

\[
t_{ii2} = g_i(2 + 2k_{ed}g_i)
\]

(A5)

\[
t_{ii3} = -k_1 - k_2(c_v \sum_{j \in N_i} a_{ij} + h_i) + c_E \sum_{j \in N_i} a_{ij}
\]

(A6)

\[
t_{ii4} = k_1k_2(c_v \sum_{j \in N_i} a_{ij} + h_i)
\]

(A7)

\[
t_{ii5} = 2\alpha_1(k_1k_2c_v/(c_v \sum_{j \in N_i} a_{ij} + h_i) + k_2c_Pk_{ed}) \sum_{j \in N_i} a_{ij}
\]

(A8)

\[
t_{ii6} = 2\alpha_1c_Pk_{ed}g_i/(c_v \sum_{j \in N_i} a_{ij} + h_i) \sum_{j \in N_i} a_{ij}
\]

(A9)

\[
t_{ii7} = (2c_Pk_{ed} \alpha_1k_1k_2/g_i) \sum_{j \in N_i} a_{ij}
\]

(A10)

\[
t_{ii} = t_{ii} + t_{ii3}
\]

(A11)

\[
t_{ii} = (c_Et_{ii} + t_{ii4})/g_i
\]

(A12)

\[
t_{ii0} = k_{ed}t_{ii1}
\]

(A13)

\[
t_{ii1} = -t_{ii} + t_{ii6} + t_{ii7}
\]

(A14)

\[
t_{ii2} = t_{ii} - t_{ii6}
\]

(A15)

\[
A_{Mij} = \begin{bmatrix}
0_T & T_T & 0_T & T_T & 0_T & T_T \end{bmatrix}_6 \times 6
\]

(A16)

in which

\[
T_{ij1} = \begin{bmatrix}
2\alpha_1t_{ij1} & 0 & 2\alpha_1k_{ed}t_{ij1} & 0 & 2\alpha_1t_{ij1}
\end{bmatrix}_{1 \times 6}
\]

(A17)

\[
T_{ij2} = \begin{bmatrix}
t_{ij1} & 0 & k_{ed}t_{ij1} & 0 & t_{ij1}
\end{bmatrix}_{1 \times 6}
\]

(A18)

with

\[
t_{ij1} = 3g_iG_{ij}V_{j0}/2
\]

(A19)

The system state matrices of delayed states \((A_{d1})\) can be represented as

\[
A_{d1} = \begin{bmatrix}
A_{d1,11} & A_{d1,12} & \ldots & A_{d1,1n} \\
A_{d1,21} & A_{d1,22} & \ldots & A_{d1,2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{d1,n1} & A_{d1,n2} & \ldots & A_{d1,nn}
\end{bmatrix}_{6n \times 6n}
\]

(A20)

where

\[
A_{d1,ii} = 0_{6 \times 6}
\]

(A21)

\[
A_{d1,ij} = \begin{bmatrix}
0_T & T_T & 0_T & T_T \end{bmatrix}_{6 \times 6}
\]

(A22)

in which

\[
T_{d1,ij} = \begin{bmatrix}
t_{d1,ij} & t_{d1,ij} & t_{d1,ij} & t_{d1,ij} & t_{d1,ij}
\end{bmatrix}_{1 \times 6}
\]

(A23)
with
\[t_{d1,j1} = a_j(-ti_{i1} - g_i/(c_v \sum_{j \in N_i} a_j + h_i)) - c_E \sum_{i \in N_j} a_j + (c_v \sum_{j \in N_i} a_j + h_i)) \tag{A24}\]
\[t_{d1,j2} = -c_E(a_j + \sum_{i \in N_j} a_j t_{i1})/g_i - k_1 k_2 c_v a_j/g_i - (c_E \sum_{i \in N_j} a_j)^2/(g_i (c_v \sum_{j \in N_i} a_j + h_i)) \tag{A25}\]
\[t_{d1,j3} = k_{edd} t_{d1,j1} \tag{A26}\]
\[t_{d1,j4} = -2 \alpha_j a_j (k_1 c_p k_{ed} / (c_v \sum_{i \in N_j} a_j + h_i) + k_2 c_p k_{ed}) + 2 a_j c_p k_{ed} g_i / (c_v \sum_{j \in N_i} a_j + h_i) \tag{A27}\]
\[t_{d1,j5} = -t_{d1,j4} - 2 c_p k_{ed} \alpha_j a_j k_2 / g_i \tag{A28}\]
\[T_{d2,1} = \begin{bmatrix} 0 & a_i c_E & 0 \end{bmatrix}_{1 \times 6} \tag{A29}\]

The system state matrices of delayed states \((A_{d2})\) can be expressed as
\[
A_{d2} = \begin{bmatrix} A_{d2,11} & A_{d2,12} & \ldots & A_{d2,1n} \\
A_{d2,21} & A_{d2,22} & \ldots & A_{d2,2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{d2,n1} & A_{d2,n2} & \ldots & A_{d2,nn} \\
\end{bmatrix}_{6n \times 6n} \tag{A30}\]

where
\[
A_{d2,i} = \begin{bmatrix} 0^T_{5 \times 10} & T_{d2,1}^T_{6 \times 6} \end{bmatrix}^T \tag{A31}\]
in which
\[
T_{d2,1} = \begin{bmatrix} t_{d2,11} & t_{d2,12} & t_{d2,13} & 0_{1 \times 2} & t_{d2,1} \end{bmatrix}_{1 \times 6} \tag{A32}\]

with
\[
t_{d2,11} = a_j c_E \sum_{j \in N_i} a_j / (c_v \sum_{j \in N_i} a_j + h_i) \tag{A33}\]
\[
t_{d2,12} = a_j c_E t_{i1} / (g_i \sum_{j \in N_i} a_j + h_i) + a_j (c_p k_{ed}) / (g_i (c_v \sum_{j \in N_i} a_j + h_i)) \tag{A34}\]
\[
t_{d2,13} = k_{edd} t_{d2,1} \tag{A35}\]
\[
A_{d2,j} = \begin{bmatrix} 0^T_{5 \times 10} & T_{d2,1}^T_{6 \times 6} \end{bmatrix}^T \tag{A36}\]
in which
\[
T_{d2,1} = \begin{bmatrix} t_{d2,11} & t_{d2,12} & t_{d2,13} & 0_{1 \times 2} & t_{d2,1} \end{bmatrix}_{1 \times 6} \tag{A37}\]

with
\[
t_{d2,11} = \begin{cases} c_E/(c_v \sum_{j \in N_i} a_j + h_i) & \text{if } (j \in N_k \text{ and } k \in N_i) \\
0 & \text{if } (j \notin N_k \text{ and } k \in N_i \text{ or } j \in N_i) \end{cases} \tag{A38}\]

The system state matrices of delayed states \((A_{d3})\) can be represented as
\[
A_{d3} = \begin{bmatrix} A_{d3,11} & A_{d3,12} & \ldots & A_{d3,1n} \\
A_{d3,21} & A_{d3,22} & \ldots & A_{d3,2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{d3,n1} & A_{d3,n2} & \ldots & A_{d3,nn} \\
\end{bmatrix}_{6n \times 6n} \tag{A41}\]

where
\[
A_{d3,j} = 0_{6 \times 6} \tag{A42}\]
\[
A_{d3,j} = \begin{bmatrix} 0^T_{5 \times 10} & T_{d3,1}^T_{6 \times 6} \end{bmatrix}^T \tag{A43}\]
in which
\[
T_{d3,1} = \begin{bmatrix} 0 & t_{d3,11} & 0_{1 \times 4} \end{bmatrix}_{1 \times 6} \tag{A44}\]

with
\[
t_{d3,11} = -a_j \sum_{j \in N_i} a_j (c_E^2 \sum_{i \in N_j} a_j) / (g_i (c_v \sum_{j \in N_i} a_j + h_i)) \tag{A45}\]

REFERENCES

[1] D. E. Olivares, A. Mehrizi-Sani, A. H. Etemadi, C. A. Cañizares, R. Iravani, M. Kazeneri, A. H. Hajimiragha, O. Gomis-Bellmunt, M. Saeedifard, R. Palma-Behnke, G. A. Jiménez-Estévez, and N. D. Hatziargyriou, “Trends in microgrid control,” IEEE Trans. Smart Grid, vol. 5, no. 4, pp. 1905–1919, Jul. 2014.

[2] P. C. Sekhar and S. Mishra, “Storage free smart energy management for frequency control in a diesel-PV-fuel cell-based hybrid AC microgrid,” IEEE Trans. Neural Netw. Learn. Syst., vol. 27, no. 8, pp. 1657–1671, Aug. 2016.

[3] K. Wang, X. Yuan, Y. Geng, and X. Wu, “A practical structure and control for reactive power sharing in microgrid,” IEEE Trans. Smart Grid, vol. 10, no. 2, pp. 1880–1888, Mar. 2019.

[4] R.-J. Wai, Q.-Q. Zhang, and Y. Wang, “A novel voltage stabilization and power sharing control method based on virtual complex impedance for an off-grid microgrid,” IEEE Trans. Power Electron., vol. 34, no. 2, pp. 1863–1880, Feb. 2019.

[5] Y. Khayat, Q. Shaﬁe, R. Heydari, M. Naderi, T. Dragicevic, J. W. Simpson-Porco, F. Dorﬁer, M. Fathi, F. Blaabjerg, J. M. Guerrero, and H. Bevrani, “On the secondary control architectures of AC microgrids: An overview,” IEEE Trans. Power Electron., vol. 35, no. 6, pp. 6482–6500, Jun. 2020.

[6] G. Lou, W. Gu, Y. Xu, M. Cheng, and W. Liu, “Distributed MPC-based secondary voltage control scheme for autonomous droop-controlled microgrids,” IEEE Trans. Sustain. Energy, vol. 8, no. 2, pp. 792–804, Apr. 2017.

[7] I. Schiffer, T. Seel, J. Raisch, and T. Sezi, “Voltage stability and reactive power sharing in inverter-based microgrids with consensus-based distributed voltage control,” IEEE Trans. Control Syst. Technol., vol. 24, no. 1, pp. 96–109, Jan. 2016.

[8] A. Bidram, A. Davoudi, F. L. Lewis, and Z. Qu, “Secondary control of microgrids based on distributed cooperative control of multi-agent systems,” IET Generat. Transmiss. Distrib., vol. 7, no. 8, pp. 822–831, Aug. 2013.
R. Jiao, W. Chou, Y. Rong, and M. Dong, “Anti-disturbance control of nonlinear discrete-time systems,” in *IEEE Trans. Autom. Control*, vol. 56, no. 5, pp. 1263–2628, Nov. 2010.

T. Jia, Y. Niu, and Y. Zou, “Sliding mode control for stochastic systems subject to packet losses,” *Inf. Sci.*, vol. 217, pp. 117–126, Dec. 2012.

B. Chen, Y. Niu, and Y. Zou, “Sliding mode control for stochastic Markovian jump systems with random switchings,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 45, no. 5, pp. 822–832, May 2015.

K. Hatipoglu, I. Fidan, and G. Radman, “Investigating effect of voltage unbalance on AC microgrids,” *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 2387–2395, Nov. 2013.

Q. Q. Zhang, R.-J. Wai, “Robust power sharing and voltage stabilization in multi-inverter microgrids,” *IEEE Trans. Power Electron.*, vol. 28, no. 6, pp. 304–312, Jul. 2013.

J. Hu, H. Zhang, H. Liu, and X. Yu, “A survey on sliding mode control for networked control systems,” *Int. J. Syst. Sci.*, vol. 52, no. 6, pp. 1129–1147, Feb. 2021.

Z. Chen, J. Fu, X.-W. Tu, A.-L. Yang, and M.-R. Fei, “Real-time predictive sliding mode control method for AGV with actuator delay,” *Adv. Manuf.*, vol. 7, no. 4, pp. 448–459, Dec. 2019.

H. L. D. C. Pinto, T. R. Oliveira, and L. Hsu, “Sliding mode observer for fault reconstruction of time-delay and sampled-output systems—A time shift approach,” *Automatica*, vol. 106, pp. 390–400, Aug. 2019.

Y. Cui, J. Hu, Z. Wu, and G. Yang, “Finite-time sliding mode control for networked singular Markovian jump systems: A delay-fractioning scheme,” *Neurocomputing*, vol. 385, pp. 48–62, Apr. 2020.

Y. Niu and D. W. C. Ho, “Design of sliding mode control subject to packet losses,” *IEEE Trans. Autom. Control*, vol. 55, no. 11, pp. 2633–2641, Nov. 2010.

T. Sreekumar and K. S. Jiji, “Comparison of proportional-integral (P-I) and sliding mode control for AC microgrids,” *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 4454–4465, Jul. 2018.

R. V. A. Neves, R. Q. Machado, V. A. Oliveira, X. Wang, and F. Blaabjerg, “Multitask fuzzy secondary controller for AC microgrid operating in stand-alone and grid-tied mode,” *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 5640–5649, Sep. 2019.

Y. Yu, G.-P. Liu, and W. Hu, “Online learning based voltage and power regulation for AC microgrids,” *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 68, no. 4, pp. 1318–1322, Apr. 2021.

L. Marton and B. Lantos, “Control of robotic systems with unknown friction and payload,” *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 6, pp. 1534–1539, Nov. 2011.

S. Hou, J. Fei, C. Chen, and Y. Chu, “Finite-time adaptive fuzzy-neural-network control of active power filter,” *IEEE Trans. Power Electron.*, vol. 34, no. 10, pp. 10298–10313, Oct. 2019.

R. J. Wai and R. Muthusamy, “Fuzzy-neural-network inherited sliding-mode control for robot manipulator including actuator dynamics,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 2, pp. 274–287, Feb. 2013.

R.-J. Wai, Y.-F. Lin, and Y.-K. Liu, “Design of adaptive fuzzy-neural-network control for a single-stage boost inverter,” *IEEE Trans. Power Electron.*, vol. 30, no. 12, pp. 7282–7286, Dec. 2015.

X. Liu, X. Yang, C. Liu, L. Huang, and W. Chen, “A sliding-mode controller with multiresonant sliding surface for single-phase grid-connected VSI with an LLC filter,” *IEEE Trans. Power Electron.*, vol. 28, no. 5, pp. 2259–2268, May 2013.

H. Ma, Y. Li, and Z. Xiong, “Discrete-time sliding-mode control with enhanced power reaching law,” *IEEE Trans. Ind. Electron.*, vol. 66, no. 6, pp. 4629–4638, Jun. 2019.

Y. Chu, J. Fei, and S. Hou, “Adaptive global sliding-mode control for dynamic systems using double hidden layer recurrent neural network structure,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 4, pp. 1297–1309, Apr. 2020.

J. Fei and Y. Chen, “Double hidden layer output feedback neural adaptive global sliding mode control of active power filter,” *IEEE Trans. Power Electron.*, vol. 35, no. 3, pp. 3069–3084, Mar. 2020.

S. Hou, Y. Chu, and J. Fei, “Intelligent global sliding mode control using recurrent feature selection neural network for active power filter,” *IEEE Trans. Ind. Electron.*, vol. 68, no. 8, pp. 7320–7329, Aug. 2021.

S. Hou, Y. Chu, and J. Fei, “Adaptive type-2 fuzzy neural network inherited terminal sliding mode control for power quality improvement,” *IEEE Trans. Ind. Informat.*, vol. 17, no. 11, pp. 7564–7574, Nov. 2021.

Q. Q. Zhang and R.-J. Wai, “Robust power sharing and voltage stabilization control structure via sliding-mode technique in islanded micro-grid,” *Electron. Lett.*, vol. 57, no. 4, pp. 383–389, Feb. 2021.

Y. Wang and R.-J. Wai, “Adaptive power decoupling strategy for single-phase grid-connected inverter,” *IEEE Trans. Ind. Appl.*, vol. 55, no. 4, pp. 4275–4285, Jul. 2019.

A. R. Bergen and V. Vittal, *Power System Analysis*. Upper Saddle River, NJ, USA: Prentice-Hall, 2000.

K. Hatipoglu, I. Fidan, and G. Radman, “Investigating effect of voltage changes on static ZIP load model in a microgrid environment,” in *Proc. North Amer. Power Symp.*, Sep. 2011.

A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power Generation, Operation, and Control*. New York, NY, USA: Wiley, 2013.

G. Chen and E. Feng, “Distributed secondary control and optimal power sharing in microgrids,” *IEEE/CIE J. Automat. Sist.,",* vol. 2, no. 3, pp. 304–312, Jul. 2015.

G. Chen, J. Ren, and E. N. Feng, “Distributed finite-time economic dispatch of a network of energy resources,” *IEEE Trans. Smart Grid*, vol. 8, no. 2, pp. 822–832, Mar. 2017.

H. Zhang, S. Kim, Q. Sun, and J. Zhou, “Distributed adaptive virtual impedance control for accurate reactive power sharing based on consensus control in microgrids,” *IEEE Trans. Smart Grid*, vol. 8, no. 4, pp. 1749–1761, Jul. 2017.

F. Mehrmodood, B. Khan, and M. S. Ali, “Renewable generation inter-mittence and economic dispatch control of autonomous microgrid with distributed sliding mode,” *Int. J. Electr. Power Syst.*, vol. 130, pp. 106937–106948, Sep. 2021.

D. O. Amoateng, M. Al Hosami, M. S. Elmoursi, K. Turitsyn, and J. L. Kirtley, “Adaptive voltage and frequency control of islanded multi-microgrids,” *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 4454–4465, Jul. 2018.
L. Jiang, W. Yao, Q. H. Wu, J. Y. Wen, and S. J. Cheng, “Delay-dependent H. Fazelinia, R. Sipahi, and N. Olgac, “Stability robustness analysis of N. Wang and M. J. Er, “Self-constructing adaptive robust fuzzy neural D. Chen, S. Li, and Q. Wu, “A novel superstwisting zeroing neural network with application to mobile robot manipulators,” IEEE Trans. Neural Nat. Wang, “Adaptive neural network learning controller design for a class of nonlinear systems with time-varying state constraints,” IEEE Trans. Neural Netw. Learn. Syst., vol. 31, no. 1, pp. 66–75, Jan. 2020. K. Hornik, M. Stinchcombe, and H. White, “Multilayer feedforward networks are universal approximators,” Neural Netw., vol. 2, no. 5, pp. 359–366, 1989. L. X. Wang, A Course in Fuzzy Systems and Control. Englewood Cliffs, NJ, USA: Prentice-Hall, 1997. J. Hongjie and Y. Xiaodan, “A simple method for power system stability analysis with multiple time delays,” in Proc. IEEE Power Energy Soc. Gen. Meeting Convers. Del. Elect. Energy 21st Century, Jul. 2008, pp. 1–7. D. Chen, S. Li, and Q. Wu, “A novel superstwisting zeroing neural network with application to mobile robot manipulators,” IEEE Trans. Neural Netw. Learn. Syst., vol. 32, no. 4, pp. 1776–1787, Apr. 2021. N. Wang and M. J. Er, “Self-constructing adaptive robust fuzzy neural tracking control of surface vehicles with uncertainties and unknown disturbances,” IEEE Trans. Control Syst. Technol., vol. 23, no. 3, pp. 991–1002, May 2015. M. S. Golsorkhi, M. Savaghebi, D. D.-C. Lu, J. M. Guerrero, and J. C. Vasquez, “A GPS-based control framework for accurate current sharing and power quality improvement in microgrids,” IEEE Trans. Power Electron., vol. 32, no. 7, pp. 5675–5687, Jul. 2017. H. Fazelinia, R. Sipahi, and N. Olgaç, “Stability robustness analysis of multiple time delayed system using ‘building block’ concept,” IEEE Trans. Autom. Control, vol. 52, no. 5, pp. 799–809, May 2007. L. Jiang, W. Yao, Q. H. Wu, J. Y. Wen, and S. J. Cheng, “Delay-dependent stability for load frequency control with constant and time-varying delays,” IEEE Trans. Power Syst., vol. 27, no. 2, pp. 932–941, May 2012.

QUAN-QUAN ZHANG was born in Hubei, China, in 1991. She received the B.S. degree in electric engineering and automation and the M.S. degree in electric engineering from the China University of Mining and Technology, Xuzhou, China, in 2014 and 2017, respectively, and the Ph.D. degree in electronic and computer engineering from the National Taiwan University of Science and Technology, Taiwan. Her research interests include renewable energy generation systems, control of parallel converter, and micro-grid.

RONG-JONG WAI (Senior Member, IEEE) was born in Tainan, Taiwan, in 1974. He received the B.S. degree in electrical engineering and the Ph.D. degree in electronic engineering from Chung Yuan Christian University, Taoyuan, Taiwan, in 1996 and 1999, respectively. From August 1998 to July 2015, he was with Yuan Ze University, Taoyuan, where he was the Dean of General Affairs, from August 2008 to January 2021, and he is currently a Distinguished Professor and the Director of the Energy Technology and Mechatronics Laboratory. He is a chapter-author of Intelligent Adaptive Control: Industrial Applications in the Applied Computational Intelligence Set (CRC Press, 1998) and the coauthor of Drive and Intelligent Control of Ultrasonic Motor (Tsang-Hai, 1999), Electric Control (Tsang-Hai, 2002), and Fuel Cell: New Generation Energy (Tsang-Hai, 2004). He has authored more than 180 conference papers and over 200 international journal articles and holds 61 inventive patents. His research interests include power electronics, motor servo drives, mechatronics, energy technology, and control theory applications. The outstanding achievement of his research is for contributions to real-time intelligent control in practical applications and high-efficiency power converters in energy technology.

Dr. Wai is a fellow of the Institution of Engineering and Technology, U.K. He received the Excellent Research Award, in 2000, and the Wu Ta-You Medal and Young Researcher Award, in 2003, from the National Science Council, Taiwan. In addition, he was a recipient of the Outstanding Research Award from Yuan Ze University, in 2003 and 2007; the Excellent Young Electrical Engineering Award and the Outstanding Electrical Engineering Professor Award from the Chinese Electrical Engineering Society, Taiwan, in 2004 and 2010; the Outstanding Professor Award from the Far Eastern Y. Z. Hsu-Science and Technology Memorial Foundation, Taiwan, in 2004 and 2008; the International Professional of the Year Award from the International Biographical Centre, U.K., in 2005; the Young Automatic Control Engineering Award from the Chinese Automatic Control Society, Taiwan, in 2005; the Yuan-Ze Chair Professor Award from the Far Eastern Y. Z. Hsu-Science and Technology Memorial Foundation, in 2007, 2010, and 2013; the Electric Category-Invent Silver Medal Award, in 2007, the Electronic Category-Invent Gold and Silver Medal Awards, in 2008, the Environmental Protection Category-Invent Gold Medal Award, in 2008, the Most Environmental Friendly Award, in 2008, the Power Category-Invent Bronze Medal Award, in 2012, and the Electronic Category-Invent Gold and Silver Medal Awards, in 2015, from the International Invention Show and Technomart, Taipei; the University Industrial Economic Contribution Award from the Ministry of Economic Affairs, Taiwan, in 2010; the Ten Outstanding Young Award from the Ten Outstanding Young Person’s Foundation, Taiwan, in 2012; the Taiwan Top 100 MVP Managers Award from Manager Today magazine, Taiwan, in 2012; the Outstanding Engineering Professor Award from the Chinese Institute of Engineers, Taiwan, in 2013; the Green Technology Category-Scientific Paper Award from the Far Eastern Y. Z. Hsu-Science and Technology Memorial Foundation, in 2014; the Scopus Young Researcher Lead Award-Computer Science from Taiwan Elsevier, in 2014; the Outstanding Research Award, in 2016, 2018, and 2020; and the Excellent Research Award from the National Taiwan University of Science and Technology, in 2022; the Most Cited Researchers Award, in 2016 (field: electrical and electronics engineering); the World’s Top 2% of Scientists (Rank 17th and 19th, field: electrical and electronics engineering), in 2020 and 2021; and the Teaching Excellence Award from the National Taiwan University of Science and Technology, in 2021.