Thermorheological and magnetorheological effects on Marangoni-Ferroconvection with internal heat generation

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Abstract. Marangoni convective instability in a ferromagnetic fluid layer in the presence of a spatial heat source and viscosity variation is examined by means of the classical linear stability analysis. The higher order Rayleigh-Ritz technique is used to compute the critical Marangoni number. The effective viscosity of the ferromagnetic liquid is taken to be a quadratic function of both the temperature and magnetic field strength. It is shown that the ferromagnetic fluid is significantly influenced by the effect of viscosity variation and is more prone to instability in the presence of heat source compared to that when viscosity is constant. On comparing the corresponding results of heat source and heat sink it is found that heat sink works in tandem with the effect of viscosity variation if magnetic field dependence of viscosity dominates over temperature dependence. If the temperature dependence of viscosity dominates, the effects of viscosity variation and heat sink are mutually antagonistic.

1. Introduction
The manifestation of cellular convective instability in non-magnetic fluid layers heated from below is generally credited to the buoyancy and surface tension mechanisms. The buoyancy driven convection (usually known as Rayleigh-Bénard Convection (RBC)) preponderates the surface tension driven convection (typically referred to as Marangoni Convection (MC)) in the case of not-so-thin fluid layers under usual gravity conditions and it is the other way round for thin fluid layers under microgravity situations (Pearson [1]). Thermally and magnetically induced gradients of magnetization are also responsible for the convective motion transpiring in magnetic fluids besides the buoyancy and surface tension candidates. The idea of regulating the properties of magnetic fluids through a magnetic field has led to numerous fascinating applications (Popplewell[2], Berkovskii et al[3] and Hornet al[4]).

RBC in constant viscosity ferromagnetic fluids, Newtonian as well as non-Newtonian, is fairly well studied (Finlayson [14], Stiles and Kagan [15], Maruthamanikandan [16], Soya Mathew et al [17], Nisha Mary and Maruthamanikandan [18] and Vatani et al [19]). It has been corroborated by Finlayson...
that in very thin layers of magnetic liquids only magnetic forces contribute to convection and that the effect of buoyancy forces could be ignored in such layers. Schwab [20], Qin and Kaloni [21], Odenbach [22], Weilepp and Brand [23] and Geetha and Nanjundappa [24] have studied the surface tension effect on thermomagnetic convection in ferromagnetic fluid.

Technological and biomedical applications of magnetic liquids indicate that these liquids depend greatly on their rheological properties. Several studies such as those of Rosensweig et al [25], Shliomis [26], Kamiyama et al [27], Kobori and Yamaguchi [28] and Chen et al [29] specify that the effective viscosity of a ferromagnetic liquid is enhanced by the application of a magnetic field. This reversible effect, known as magnetorheological effect, is a consequence of the fact that the particles magnetize in the presence of a magnetic field and form chain-like clusters that align with the applied field. These chain-like alignments of the dispersed solid particles impede the motion of the liquid, thereby increasing the viscous characteristics of the suspension. The contemporary applications of the magnetorheological effect include dampers, brakes, pumps, clutches, valves, robotic control systems and the like (Carlsson et al [30]). Balauet et al [31] have pointed out through their experiments that magnetorheological effect is of importance significantly in water-based and kerosene-based solutions, and in physiological-solution-based magnetic liquids even for moderate strengths of applied magnetic field. This is more so in the extraterrestrial context. Prakash [32] studied the effects of magnetic field dependent viscosity and non-uniform basic temperature profiles on thermomagnetic convection in a horizontal layer of ferrofluid.

Another fact about the viscosity of any carrier liquid decreasing with temperature is also well known (Geetha and Nanjundappa [24], Platten and Legros [33], Severin and Herwig [34], Ramanathan and Muchikel [35] and Nanjundappa et al [36]) and is referred to as thermorheological effect. It is imperative therefore to envisage the importance of the MC problem in ferromagnetic liquids involving both magnetic field and temperature dependent effective viscosity. Apart from the rheological effects discussed earlier, the effect of volumetric internal heat source is also important in ferromagnetic liquids from the viewpoint of magnetocaloric pumping. In this paper we aim at studying the effect of internal heat generation on the threshold of MC in a variable viscosity ferromagnetic liquid with a vertical temperature gradient and a vertical magnetic field. The assumed strength of the magnetic field is such that the liquid does not exhibit any non-Newtonian characteristics. The report on the study culminates with an important exploration of the dissimilarity, for Marangoni convection, between heat source and heat sink problems.

2. Mathematical Formulation
Consider an infinite horizontal layer of a thin ferromagnetic liquid (with a free upper surface) that maintains a temperature gradient and a magnetic field $\vec{H}_0$ in the vertical direction. The gradient in temperature is by virtue of a prescribed temperature difference $\Delta T$ (> 0 for fluid heated from below) across the layer and a uniform distribution of heat source/sink of intensity $\mathcal{S}$ in the liquid. The liquid is assumed to have an effective variable viscosity $\mu$ that depends on the magnitude of the magnetic field and the temperature. The upper boundary interface has a temperature and magnetic field dependent surface tension $\sigma (H,T) = \sigma_o + \sigma_H (H - H_o) - \sigma_T (T - T_a)$, (2.1)

where $\sigma$ is the surface tension, $\sigma_o = \sigma (H_o, T_o)$, $\sigma_H = (\partial \sigma / \partial H)_{H_o, T_a}$, $\sigma_T = - (\partial \sigma / \partial T)_{H_o, T_a}$ and $T_a$ is the constant average temperature. The system of equations associated with the Marangoni instability situation in a variable viscosity ferromagnetic liquid with uniform heat source is (Wilson [8], Finlayson [14] and Severin and Herwig [34])

$$\nabla \cdot \vec{q} = 0, \quad (2.2)$$
\[\rho_o \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \mu_o (\vec{M} \cdot \nabla) \vec{H} + \nabla \cdot \left[ \mu(H,T) \left( \nabla \vec{q} + \nabla \vec{q}^T \right) \right], \tag{2.3}\]

\[\rho_o C_{V,H} H - \mu_o \frac{\partial \vec{M}}{\partial T}, \quad \frac{dT}{dt} + \mu_o T \left( \frac{\partial \vec{M}}{\partial T} \right), \quad \frac{d\vec{H}}{dt} = k_i \nabla^2 T + S, \tag{2.4}\]

\[\vec{M} = \frac{\vec{H}}{H} M(H,T), \tag{2.5}\]

\[M = M_o + x_m(H - H_o) - K_1(T - T_a), \tag{2.6}\]

\[\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = 0, \quad \vec{B} = \mu_o \left( \vec{H} + \vec{M} \right), \tag{2.7}\]

where \(\vec{q} = (u,v,w)\) is the fluid velocity, \(\rho_o\) is the density, \(t\) is the time, \(p\) is the pressure, \(\mu\) is the effective viscosity, \(\vec{H}\) is the magnetic field, \(\vec{B}\) is the magnetic induction, \(T\) is the temperature, \(\mu_o\) is the magnetic permeability, \(\vec{M}\) is the magnetization, \(k_i\) is the thermal conductivity, \(V\) is the vector differential operator, \(C_{V,H}\) is the specific heat at constant volume and magnetic field, \(M_o\) is the reference magnetization, \(x_m\) is the magnetic susceptibility, \(K_1\) is the pyromagnetic coefficient and \(\text{Tr}\) designates the transpose.

It is of interest to note that the well-known viscosity variation with temperature is a non-Boussinesq effect (Selak and Lebon[37]). Further, for a ferromagnetic liquid, we have one more non-Boussinesq influencing factor for the viscosity that arises only when the magnetic field is present. The effective viscosity is known to escalate due to the magnetic field in the case of these synthetic liquids (Chen et al[38]) owing to the reorientation of magnetic particles. There exist a number of correlations of viscosity-temperature and viscosity-magnetic field strength including linear, quadratic and exponential proportionalities. The quadratic and exponential viscosity variations have been brought in owing to the fact that the linear viscosity variation is inadequate in showing the destabilizing nature of temperature dependence of viscosity and stabilizing nature of magnetic field dependence of viscosity. The problem under consideration also necessitates a nonlinear viscosity variation rather than the linear one. In view of this we assume the effective viscosity \(\mu(H,T)\) to be a quadratic function of \(H\) and \(T\) in the form

\[\mu(H,T) = \mu_1 \left[ 1 + \delta_1 (H - H_o) + \delta_2 (H - H_o)^2 - \delta_3 (T - T_a) - \delta_4 (T - T_a)^2 \right], \tag{2.8}\]

where \(\delta_i\) \((i = 1, 2, 3, 4)\) are small positive quantities. We note that it suffices to take other quantities such as surface tension and magnetization as linear functions of temperature and magnetic field. Taking the basic state components of magnetization and magnetic field to be \([0, 0, M_b(z)]\) and \([0, 0, H_b(z)]\), the solution pertaining to the quiescent basic state reads

\[\vec{q}_b = \vec{0}, \quad T_b(z) = T_a - f(z), \quad p = p_b(z), \quad \mu_b(z) = \mu_1 \left[ 1 + V_1 f(z) - V_2 \{ f(z) \}^2 \right], \tag{2.9}\]

\[\vec{H}_b = \left[ \begin{array}{c} H_o - \frac{K_1}{1 + x_m} f(z) \\ \hat{k} \end{array} \right], \quad \vec{M}_b = \left[ \begin{array}{c} M_o + \frac{K_1}{1 + x_m} f(z) \\ \hat{k} \end{array} \right].\]
where \( V_1 = \delta_3 - \frac{\delta_1 K_1}{1 + \chi_m} \), \( V_2 = \delta_4 - \frac{\delta_2 K_1}{(1 + \chi_m)^2} \) and \( f(z) = (S z^2/2k_1) + (\Delta T z/d) - (S d^2/8k_1) \). In arriving at the above solution it has been assumed that \( T = T_o \) at \( z = d/2 \) and \( T = T_i (= T_o + \Delta T) \) at \( z = -d/2 \) where \( d \) is the thickness of the liquid layer. The dominance of magnetic dependency over temperature dependency of viscosity is signified by the condition \( V_1 < 0 \) and \( V_2 < 0 \), while \( V_1 > 0 \) and \( V_2 > 0 \) signifies dominance of temperature dependency.

We next study the stability of the system by resorting to the method of small perturbation (Finlayson [14]). Introducing the magnetic potential \( \Phi' \), eliminating the pressure \( p \) and incorporating the solution in equation (2.9), we obtain the following equations pertaining to the perturbed state

\[
\rho_o \frac{\partial}{\partial t} (\nabla^2 w') - \mu_b(z) \nabla^4 w' + D^2 \mu_b(z) \left( \nabla_1^2 - D^2 \right) w' - 2D \mu_b(z) \nabla^2 (Dw')
\]

\[
- \frac{\mu_o K_1^2}{1 + \chi_m} Df(z) \nabla_1^2 T' + \mu_o K_1 Df(z) \nabla_1^2 (D\Phi') = 0,
\]

\[
\left[ \rho_o c - \frac{\mu_o K_1^2}{1 + \chi_m} f(z) \right] \frac{\partial T'}{\partial t} - \mu_o K_1 \left[ T_o - f(z) \right] \frac{\partial}{\partial t} (D\Phi')
\]

\[
+ \left[ \frac{\mu_o T_o K_1^2}{1 + \chi_m} - \rho_o c \right] Df(z) w' = k_1 \nabla_1^2 T',
\]

\[
(1 + \chi_m) D^2 \Phi' + \left[ 1 + \frac{M_o}{H_o} \right] \nabla_1^2 \Phi' - K_1 D T' = 0,
\]

where \( \rho_o c = \rho_o C_{V,H} + \mu_o K_1 H_o \) and \( D = \partial/\partial z \). A separable solution to equations (2.10) – (2.12) in the form of periodic waves reads

\[
\begin{bmatrix}
    w' \\
    T' \\
    \Phi'
\end{bmatrix}
= \exp \left[ i \left( k_x x + k_y y \right) \right].
\]

Use of equation (2.13) in equations (2.10) – (2.12), we obtain

\[
\rho_o \left( D^2 - k^2 \right) \frac{\partial w}{\partial t} - \mu_b(z) \left( D^2 - k^2 \right)^2 w - D^2 \mu_b(z) \left( D^2 + k^2 \right) w
\]

\[
- 2D \mu_b(z) \left( D^2 - k^2 \right) Dw - \frac{\mu_o K_1}{1 + \chi_m} k^2 Df(z) \left[ (1 + \chi_m) DT - K_1 T \right] = 0
\]
\[
\left[ \rho_o c - \frac{\mu_o K^2_1}{1 + \chi_m} f(z) \right] \frac{\partial T}{\partial t} - \mu_o K_1 \left[ T_a - f(z) \right] \frac{\partial (D\Phi)}{\partial t} + \left( \frac{\mu_o T_a K^2_1}{1 + \chi_m} - \rho_o c \right) Df(z)w - k_1 \left( D^2 - k^2 \right) T = 0
\]

\[
(1 + \chi_m) D^2 \Phi - \left( \frac{1 + M_o}{H_o} \right) k^2 \Phi - K_1 DT = 0,
\]

where the horizontal wavenumber \( k \) is defined as \( k^2 = k_x^2 + k_y^2 \). We next make equations (2.14)–(2.16) dimensionless by introducing the following definitions
\[
t^* = \frac{\kappa}{d^2} t, \quad z^* = \frac{z}{d}, \quad w^* = \frac{d}{\kappa} w, \quad T^* = \frac{T}{\Delta T}, \quad \Phi^* = \frac{(1 + \chi_m)}{K_1 \Delta T d} \Phi, \quad a^* = kd, \quad (2.17)
\]

where the quantities with asterisk are dimensionless. Equations (2.14)–(2.16), upon using equation (2.17), read
\[
\frac{1}{Pr} \left( D^2 - a^2 \right) \frac{\partial w}{\partial t} - \left[ 1 + \Gamma_1 g(z) - \Gamma_2 \left( g(z) \right)^2 \right] \left( D^2 - a^2 \right)^2 w
\]

\[
- 2 \left[ \Gamma_1 Dg(z) - 2\Gamma_2 g(z)Dg(z) \right] \left( D^2 - a^2 \right) Dw - R_M a^2 Dg(z) \left[ D\Phi - T \right] = 0,
\]

\[
\left[ 1 - M_2 \frac{\Delta T}{T_a} g(z) \right] \frac{\partial T}{\partial t} - M_2 \left[ 1 - \frac{\Delta T}{T_a} g(z) \right] \frac{\partial (D\Phi)}{\partial t}
\]

\[
= \left( D^2 - a^2 \right) T + \left( 1 - M_2 \right) Dg(z)w,
\]

\[
(D^2 - M_3 a^2) \Phi - DT = 0,
\]

where \( g(z) = N_S z^2 + z - (N_S / 4) \) and the asterisks have been removed for simplicity. Thenon-dimensional parameters appearing in equations (2.18)–(2.20) are the Prandtl number \( Pr = \frac{H_1}{\rho_o \kappa} \), linear variable viscosity parameter \( \Gamma_1 = V_1 \Delta T \), quadratic variable viscosity parameter \( \Gamma_2 = V_2 (\Delta T)^2 \), the heat source (sink) parameter \( N_S = \frac{S d^2}{2 k_1 \Delta T} \), magnetic Rayleigh number \( R_M = \frac{\mu_o (K_1 \Delta T d)^2}{(1 + \chi_m) \mu_1 \kappa} \), the parameter defining ratio of thermal flux (due to magnetization) to magnetic flux \( M_2 = \frac{\mu_o K^2_1 T_a}{(1 + \chi_m) H_o} \), and the parameter measuring nonlinearity in magnetization \( M_3 = \frac{M_o + H_o}{(1 + \chi_m) H_o} \).

The Prandtl number, \( Pr \), is the ratio of the speed of propagation of momentum to that of heat transport. The heat source (sink) parameter \( N_S \) is the ratio of strength of the internal heat source to
external heating. The magnetic Rayleigh number $R_M$ is the ratio of magnetic force to viscous dissipation. The parameter $M_2$, being equal to $10^{-6}$ (Finlayson[14]), shall be discarded in the subsequent analysis. Equations (2.18) – (2.20) are solved together with the following boundary conditions

$$w = \left[ 1 + I_1 \left\{ \frac{1}{2} - \right\} I_2 \left\{ g \left( \frac{1}{2} \right) \right\} \right] D^2 w + a^2 M \alpha T - a^2 M \alpha H D \Phi = DT = 0,$$

and

$$D \Phi + \frac{d \Phi}{dz} + T = 0 \text{ at } z = \frac{1}{2},$$

$$w = D w = T = D \Phi - \frac{d \Phi}{dz} = 0 \text{ at } z = -\frac{1}{2},$$

(2.21)

where $M = \frac{\tau_T \Delta T d}{\mu_1 \kappa}$ and $M_M = \frac{\tau_H K_1 \Delta T d}{(1 + \chi_m) \mu_1 \kappa}$ are the thermal and magnetic Marangoni numbers respectively. $M_H$ is the ratio of thermorheological factors favouring fluid motion to forces opposing motion. Likewise $M_M$ is the ratio of magnetorheological factors supporting fluid motion to forces opposing motion (which is assumed negligible in the further analysis). Since the occurrence of oscillatory instability is ruled out for the problem at hand (Lam and Bayazitoglu [6], Finlayson [14] and Weilepp and Brand [23]), the stability equations associated with the stationary instability therefore read

$$\left[ 1 + I_1 \left\{ \frac{1}{2} - \right\} I_2 \left\{ g(z) \right\} \right] (D^2 - a^2)^2 w$$

$$+ \left[ I_1 D^2 g(z) - 2 I_2 \left\{ g(z) D^2 g(z) + \Phi g(z) \right\} \left( D^2 - a^2 \right) w \right]$$

$$+ 2 \left[ I_1 D g(z) - 2 I_2 g(z) \Phi D g(z) \right] \left( D^2 - a^2 \right) D w + R_M a^2 D \Phi \left[ D \Phi - T \right] = 0,$$

$$\left( D^2 - a^2 \right) T + D g(z) w = 0,$$

$$\left( D^2 - M^2 a^2 \right) \Phi - DT = 0.$$  

(2.23)

(2.24)

3. Method of Solution

The system of equations (2.22) – (2.24) together with the conditions in equation (2.21) poses an eigenvalue problem for $M$ with $I_1$, $I_2$, $N_S$, $R_M$, $M_3$ and $\chi_m$ as parameters. A closed form solution of the problem at hand is unlikely on account of the presence of variable coefficients in equations (2.22) and (2.23). We therefore employ the Rayleigh-Ritz technique to obtain the critical eigenvalue $M_Tc$ and the corresponding critical wavenumber $a_c$. Accordingly $w(z)$, $T(z)$ and $\Phi(z)$ have expansions in the form $w(z) = \sum \alpha_i w_i(z)$, $T(z) = \sum \beta_i T_i(z)$ and $\Phi(z) = \sum \gamma_i \Phi_i(z)$ where $\alpha_i$, $\beta_i$ and $\gamma_i$ are constants, and $w_i(z)$, $T_i(z)$ and $\Phi_i(z)$ are trial functions. We choose the trial functions $w_i = \left( z - \frac{1}{2} \right)^{i+1}$, $T_i = \left( z(z-1) - \frac{3}{4} \right)^{i+1}$ and $\Phi_i = z^i$ guided by the chosen boundary conditions and variational considerations.
4. Results and discussion

External regulation of rheological properties and thereby the control of surface tension driven instability in a variable viscosity ferromagnetic liquid in the presence of internal heat generation and vertical uniform magnetic field is studied. The critical values pertaining to stationary convection have been computed using the Rayleigh-Ritz technique. The results arrived at in the problem could be understood better if we observe the profile of the basic state temperature distribution which sheds light on the effect of heat source/sink on the stability of the system.

Figure 1. Plot of dimensionless basic state temperature profile $\theta(z)$ for different values of heat source/sink parameter $N_S$.

Figure 1 is a plot of $z$ versus the basic state temperature distribution $\theta(z)$. We note that the curves are asymmetric about the lines $\theta=0$ and $z=0$ when $N_S \neq 0$. The asymmetry is obviously due to the variation in the parameter $N_S$. From Figure 1 it is clear that when $N_S = 1$, the highest temperature in the liquid layer occurs at the lower bounding surface, that is, at $z=-1/2$. As $N_S$ increases beyond the value of 1, the location of the point of extrema approaches closer to $z=0$. Thus, when $N_S \geq 1$, the point of highest temperature always lies in the lower half of the layer, that is, in $-1/2 \leq z < 0$. It is therefore clear that the effect of increasing $N_S$ is to hasten instability. On the other hand, when $N_S \leq -1$, the point of highest temperature always manifests in the upper half of the layer, that is, in the part $0 < z \leq 1/2$. Hence the effect of a decrease in $N_S$ is to impede the Marangoni instability.

In arriving at the documented critical values, we have made use of a four-term Rayleigh-Ritz technique that ensures satisfactory convergence. As to the accuracy of the Rayleigh-Ritz technique, attention is paid to the case of $MC$ in a constant viscosity non-magnetic liquid without internal heating. In this case, the Rayleigh-Ritz technique yields the critical values of $Ma_{tc} = 79.9$ and $ca = 2.0$, that are in excellent agreement with the existing values (Pearson [1]). It is also found, in the absence of viscosity variation and for $N_S \neq 0$, that the results compare very well with those obtained by Char and Chiang [7] for a non-magnetic liquid.

Figures 2 and 3 show the variation of $Ma_{tc}$ with $N_S$, $\Gamma_2$ and $R_M$ for magnetic field dominance of viscosity and temperature dominance of viscosity respectively. The destabilizing effect of increasing the magnetic Rayleigh number $R_M$ is obvious from the figures. A striking result from
Figures 2 and 3 is that the thermorheological and magnetorheological effects are more pronounced for a uniform heat sink than for a uniform heat source.

The variation of critical wavenumber $a_c$ with $N_S$, $\Gamma_2$ and $R_M$ is shown in Figures 4 and 5. The results for an intermediate value of $R_M$ are provided in Table 1. It is found that the qualitative effect of the magnetization parameter $M_3$ and the magnetic susceptibility $\chi_m$ on the onset of convection is akin to that in a constant viscosity ferromagnetic liquid (Finlayson [14]).
As can be seen from Table 1 the effect of heat sink enhances the destabilizing effect of $M_3$ on the system. Further, the convection cell size is vulnerable to the variations in $N_S$, $2\Gamma$, $R_M$ and $M_3$, and it is insensitive to the changes in $\chi_m$ so long as the magnetization parameter $M_3$ is sufficiently large.
Table 1. Critical values for a variable viscosity ferromagnetic liquid with internal heat source/sink for $R_M = 50$ and $\Gamma_1 = 0.5$.

| $N_S$ | $\chi_m$ | $M_3$ | $\Gamma_2 = 0.5$ | $\Gamma_2 = 1$ | $\Gamma_2 = 1.5$ |
|-------|-----------|-------|------------------|----------------|------------------|
|       |           |       | $M a_{TC}$ | $a_c$ | $M a_{TC}$ | $a_c$ | $M a_{TC}$ | $a_c$ |
| -2.5  | 1         | 25    | 952.78       | 1.18 | 868.15       | 1.14 | 777.47       | 1.08 |
|       |           |       | 927.36       | 1.19 | 842.35       | 1.15 | 751.49       | 1.11 |
|       | 5         | 25    | 956.83       | 1.18 | 872.23       | 1.14 | 781.54       | 1.08 |
|       |           |       | 927.61       | 1.19 | 842.63       | 1.15 | 751.81       | 1.11 |
|       | 1         | 25    | 194.89       | 1.73 | 175.44       | 1.72 | 155.34       | 1.71 |
| -1.5  | 5         | 25    | 190.57       | 1.73 | 171.17       | 1.72 | 151.13       | 1.71 |
|       |           |       | 195.66       | 1.73 | 176.20       | 1.72 | 156.10       | 1.71 |
|       | 1         | 25    | 190.58       | 1.73 | 171.18       | 1.72 | 151.14       | 1.71 |
| 0     | 1         | 25    | 77.15        | 2.00 | 71.43        | 2.02 | 65.33        | 2.04 |
|       |           |       | 74.52        | 2.00 | 68.79        | 2.01 | 62.69        | 2.02 |
|       | 5         | 25    | 77.78        | 2.01 | 72.06        | 2.02 | 65.96        | 2.04 |
|       |           |       | 74.54        | 1.99 | 68.82        | 2.01 | 62.72        | 2.02 |
|       | 1         | 25    | 42.27        | 2.09 | 38.35        | 2.11 | 34.06        | 2.13 |
| 1.5   |           |       | 39.38        | 2.06 | 35.40        | 2.07 | 31.06        | 2.08 |
|       | 5         | 25    | 43.08        | 2.10 | 39.16        | 2.12 | 34.88        | 2.14 |
|       |           |       | 39.42        | 2.06 | 35.45        | 2.07 | 31.11        | 2.08 |
|       | 1         | 25    | 28.77        | 2.10 | 23.56        | 2.12 | 17.19        | 2.14 |
| 2.5   |           |       | 25.40        | 2.07 | 19.96        | 2.08 | 13.22        | 2.09 |
|       | 5         | 25    | 29.75        | 2.11 | 24.58        | 2.13 | 18.26        | 2.15 |
|       |           |       | 25.47        | 2.07 | 20.02        | 2.08 | 13.29        | 2.09 |

5. Conclusions
The influence of temperature and magnetic field dependent effective viscosity of magnetic fluid on Marangoni convection with internal heat generation is studied. The following conclusions are arrived at from the study:

- Heat source and heat sink have reverse influence on magnetic fluid Marangoni instability.
- Thermorheological and magnetorheological effects are markedly pronounced when there is a uniform heat sink in the fluid layer.
- Convection cell size is noticeably sensitive to the effect of variable viscosity, internal heat generation and the fluid magnetization.

The problem is important in energy conversion devices and in microgravity application situations involving ferromagnetic liquids as working media.

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