Effects of Compton scattering on the neutron star radius constraints in rotation-powered millisecond pulsars

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ABSTRACT

The aim of this work is to study the possible effects and biases on the radius constraints for rotation-powered millisecond pulsars when using Thomson approximation to describe electron scattering in the atmosphere models, instead of using exact formulation for Compton scattering. We compare the differences between the two models in the energy spectrum and angular distribution of the emitted radiation. We also analyse a self-generated synthetic phase-resolved energy spectrum, based on Compton atmosphere and the most X-ray luminous rotation-powered millisecond pulsars observed by the Neutron star Interior Composition Explorer (NICER).

We derive constraints for the neutron star parameters using both the Compton and Thomson models. The results show that the method works by reproducing the correct parameters with the Compton model. However, biases are found in size and the temperature of the emitting hot spot, when using the Thomson model. The constraints on the radius are still not significantly changed, and therefore the Thomson model seems to be adequate if we are interested only in the radius measurements using NICER.

Key words. stars: atmosphere – stars: neutron – X-rays: binaries – X-rays: stars

1. Introduction

The equation of state of cold matter beyond nuclear densities can be constrained using astronomical observations of masses and radii of neutron stars (NSs) (Steiner et al. 2010, Lattimer 2012, Nättilä et al. 2016, Ozel & Freire 2016, Suleimanov et al. 2016, Watts et al. 2016, Degenaar & Suleimanov 2018, Watts et al. 2019). In case of rapidly rotating NSs having radiating "hotspots" around their magnetic poles, we can model the observed pulses using general relativity, and get thus constraints for their mass and the radius (Poutanen & Gierliński 2003, Miller & Lamb 2015). However, detailed modelling requires knowledge of the spectral energy distribution as well as of the angular emission pattern of radiation emitted by the hotspots. The radiation escaping the hotspots is affected by energy-dependent absorption as well as by the anisotropic and energy-dependent scattering of photons by electrons in the atmosphere of the NS.

There has been several studies trying to constrain NS masses and radii using pulse profiles of accreting millisecond pulsars (AMPs), in which the matter from a low-mass companion star accretes onto the magnetic poles of the NS (see e.g. Poutanen & Gierliński 2003, Leahy et al. 2008, Poutanen 2008, Morsink & Leahy 2011, Salmi et al. 2018). However, these approaches suffer from a relatively high number of unknown NS parameters and from the uncertainties in the atmospheric structure and therefore also in the angular and energy distribution of the emitted radiation.

In case of rotation-powered millisecond pulsars (RMPs), more independent information of the model parameters (e.g. mass and inclination) is often attained from radio data and the existing NS atmospheric models without effects of accretion may be used. In many RMPs the bulk of X-ray radiation is thermal emission coming from the polar caps that are heated by a return flow of relativistic electrons and positrons in the open field line region (see e.g. Harding & Muslimov 2002, Bogdanov 2018). Although, few RMPs exhibit almost pure non-thermal emission generated, most probably, by synchrotron emission from pulsar magnetospheres (Zavlin 2007). We focus on the thermally emitting RMPs, where the composition of the atmosphere is more confidently known than in AMPs (pure hydrogen instead of a mixture with heavier elements), and the temperature is low enough that the electron scattering presumably can be described using Thomson scattering approximation. The angular and energy distribution of the escaping photons can be described by using, for example, a plane-parallel atmosphere model in local thermodynamic equilibrium.

Previously this type of model for RMPs, assuming Thomson scattering, has been implemented in the McGill Planar Hydrogen Atmosphere Code (McPHAC) as described by Hankaßen et al. (2012) (see also e.g. Zavlin et al. 1996, Heinke et al. 2006). This code was also used by Miller (2016) to simulate the data for RMP PSR J1614−2230 that can be provided by the Neutron star Interior Composition Explorer (NICER) and to study the constraints on the NS mass and radius that can be obtained with those data. The question we ask in this paper is how an approximate treatment of Compton scattering affects the radiation spectra escaping from NS atmosphere and how this in turn affects the constraints on NS mass and radius from the NICER data. We note that exact treatment of Compton scattering is very important when considering NS atmospheres heated by accretion.
Suleimanov et al. (2018) or by magnetospheric return currents as, for example, recently was discussed by Bauböck et al. (2019) using a very simplified atmosphere model. Modelling the heated RMP atmospheres is, however, beyond the scope of this work and will be discussed elsewhere.

The remainder of this paper is structured as follows. In Sect. 2 we discuss the methods, including modelling the NS atmosphere, ray-tracing, and our method to create and analyse synthetic data. In Sect. 3 we first compare our spectral results to those computed with McPHAC, and then obtain NS parameter constraints fitting the data, that are created with the exact Compton model, with both the exact Compton and approximate Thomson scattering models. We conclude in Sect. 4.

2. Methods

We first construct a model for NS atmosphere consisting of pure hydrogen, which is justified, because without continuing accretion gravitational stratification will leave only the lightest elements in the atmospheric layers, which determine the properties of the escaping radiation. We compute the atmosphere model and the angular distribution of specific intensity of the escaping radiation using three different approaches. In the first one we use our code (Suleimanov et al. 2012), which treats Compton scattering using the exact relativistic Klein-Nishina cross-section and redistribution function derived and presented in details by [Aharonion & Atoyan (1981), Prasad et al. (1986), Naribner & Poutanen (1994), Poutanen & Svensson (1996), and Poutanen & Yurum (2010)]. As a second model, we use the same code but where Compton scattering is treated in Thomson approximation. This simplifies and accelerates the calculations dramatically. The third model is constructed using McPHAC code, which also treats Compton scattering in Thomson approximation (we use their anisotropic version of the model).

The parameters of the model are the effective temperature $T_{\text{eff}}$ (which we will call just $T$ for brevity) and the surface gravity $g$. The solution of the equations that describe the NS atmosphere (see e.g. Suleimanov et al. 2012) provides us with the intensity of the escaping radiation. We tabulate these intensities at 360 photon energies $E$ (keV) (spaced equally in log $E$ from $-3.4$ to $1.3$) and 7 cosines of the zenith angle $\mu$ (in the interval between 0 and 1 using Gaussian nodes) for 11 values of temperature $T$ (K) spaced equally in log $T$ from 5.5 to 6.6 and 10 values of surface gravity $g$ (cm s$^{-2}$) spaced equally in log $g$ from 13.7 to 14.6.

The observed spectra depend on the NS mass, equatorial radius $R_{\text{eq}}$ and NS spin (which determine gravitational acceleration $g$ as a function of co-latitude), the spot local effective temperature $T$, the spot angular radius $\rho$, and the co-latitude $\theta$ of the spot centroid. The spectra also depend on the inclination of the NS and the distance to the source. To compute the observed phase-resolved spectra and pulse profiles, we use “oblate Schwarzschild” approximation (see e.g. Poutanen & Loborodov 2006; Morsink et al. 2007; Miller & Lamb 2015; Salmi et al. 2018) taking into account the deformed shape of the star together with the special and general relativistic corrections to the photon trajectories and angles. For calculations of the total observed flux, integration over the spot surface is needed. However, in order to speed-up the computations, the surface gravity $g$ for the atmospheric model was assumed to be constant within the spot (using the correct value for the spot center). Thus, for each model only one piece-wise 2-dimensional linear interpolation is made from the set of pre-calculated spectral tables to obtain a single 2-dimensional array of intensities as functions of $E$ and $\mu$ only (corresponding to a given temperature $T$ and surface gravity log $g$). In our examples this is justified, because we consider only relatively small spots, where the changes in the NS radius within the spot are small. Then, for each position within the spot we make again a piece-wise 2-dimensional linear interpolation to obtain intensities corresponding to a required photon emission zenith angle and energy.

The synthetic data are created keeping in mind the most promising NICER targets. Instead of PSR J1614−2230 used by Miller (2016) as an example case, we focus on PSR J0437−4715 (the closest known RMP), or similar pulsar with high expected count rate (in order to observe any possible differences in the parameter constraints from the two spectral models). This pulsar has a complicated pulse profile presumably produced by two small high-temperature spots surrounded by a cooler annular region, and also an additional power-law component (Bogdanov 2013). However, because we aim only to compare the Thomson and Compton models, and are not necessarily interested in modelling this particular pulsar, we ignore these complications, and assume two spots with constant temperature and pure thermal spectrum. PSR J0437−4715 is mainly used the obtain typical values for the parameters of the synthetic data. The model parameters are the following: spot temperature $T = 0.27$ keV (3.1 MK), spot angular radius $\rho = 5.0^\circ$, spot co-latitude $\theta = 36^\circ$, equatorial radius of the star $R_{\text{eq}} = 12$ km, and an arbitrary phase shift. These parameters are treated as free when fitting the data. Other model parameters are the NS mass $M = 1.76 M_\odot$, NS spin frequency $\nu = 173.6$ Hz, the distance to the star $D = 156.3$ pc, the inclination $i = 42.4^\circ$, and neutral hydrogen column density for interstellar absorption $N_{\text{H}} = 7 \times 10^{19}$ cm$^{-2}$ (see e.g. Bogdanov 2013; Deller et al. 2008; Verbiest et al. 2008). They are regarded as fixed because they are or can be determined from other (radio) observations with a relatively good accuracy. We assume that the observation of the source is long enough to accumulate the total number of observed counts of $4 \times 10^7$.

The fitting procedure of the data is mostly same as presented in Salmi et al. (2018). We use Bayesian analysis and an affine invariant ensemble sampler (Goodman & Weare 2010) to get posterior probability distributions for the free model parameters. The only exception is the phase shift, for which we find the maximum likelihood solution in each fit. Additionally, we have the intrinsic scatter of the model as a free parameter log $\sigma_r$. It is a measure of the systematic errors coming from the choice of the model (see e.g. Salmi et al. 2018). We assume the prior probability distributions to be uniform in all of the parameters. The limits of the priors are set to (11 km, 13 km) in $R_{\text{eq}}$, (0 $^\circ$, 90$^\circ$) in $\theta$, (1$^\circ$, 40$^\circ$) in $\rho$, (0.08 keV, 0.35 keV) in $T$, and (0.868, 5.212) in log $\sigma_r$. The synthetic pulse-profile data are binned into 16 phase bins and NICER energy channels located between 0.3 and 10 keV. In addition, we require each modelled energy-phase bin to have more than 5 observed counts.

3. Results

3.1. Spectral properties

We begin our calculations by confirming that our code gives similar results with McPHAC when we use Thomson scattering instead of Compton. This is shown in Figs. 1 and 2 with the former showing the emergent spectrum and the latter one the angular dependencies of the emitted radiation. Fig. 1 also shows the results computed with the exact Compton model. The comparison of the angular dependencies from that model to those of McPHAC are shown in Fig. 3. The parameters of the model, tem-
temperature and surface gravity, have been chosen to be $T = 10^{6.5}$ K ($= 3.16$ MK) and $\log g = 14.3856$, reasonable for RMPs.

From the aforementioned figures we see that the calculations with McPHAC agree our Thomson version within a few per cent, although showing a small systematic discrepancy that increases with energy and is probably connected to the increasing error at high zenith angles (i.e. small $\mu$). This should have only a minor effect to the fitted effective temperature. A much larger difference is seen between our Compton and the Thomson models, which also becomes more significant at higher energies (above 3 keV for the chosen temperature) and small $\mu$. This difference in spectrum is similar to that presented in Suleimanov & Werner (2007).

Taking into account the energy response matrix of the NICER instrument, we also show the modelled phase-averaged count spectra in Fig. 4 with $T = 2$ MK and in Fig. 5 with $T = 3.1$ MK (other parameters being the same as in Sect. 2). The Compton version in Fig. 5 shows also our synthetic data used in the following sections. From the figures we see that the discrepancy between the models at the highest energies can be partly hidden.
because of only few detected counts, and therefore large statistical errors. We also assume the calibration error of the instrument to be 1%. In any case, a clearly observable difference above 3 keV remains when \( T = 3.1 \) MK.

3.2. Parameter constraints with the correct model

We now apply the method described in Sect. 2 with exact treatment of Compton scattering in the atmosphere, to fit the synthetic data created using the same model. We confirm the robustness of our method by getting no strong biases in the constraints for radius and other parameters. The fitted pulse profiles are shown in Fig. 6 (integrated to 3 energy bins), and the posterior probability distributions are shown in Fig. 7. The credible limits of all parameters are also listed in Table 1. The best-fit solution presented in Fig. 6 has \( \chi^2/\text{d.o.f.} = 7411/(7649-6) \approx 0.97 \) for 3 free parameters, and the amount of detected counts (4 \( \times \) 10\(^7\)) is sufficient to make a fit. The blue crosses show the correct solution. The model describes the synthetic data well, as seen in the posterior probability distribution for intrinsic scatter \( \sigma_i \). The mean \( \sigma_i < 1 \) of the posterior translates to an error of less than 10 counts in each phase-energy bin, which effectively means zero intrinsic scatter because it is significantly smaller than the Poisson noise of the data, which is 50 counts on average in each fitted phase-energy bin. For the radius we find the 68% (95%) limits of \( R_{eq} = 12.01^{+0.06}_{-0.02} \) km. We note that this, and the other limits presented here and in the following section, are considerably tighter than what is expected, if comparing, for example, to the approximation in Eq. (5) by Psaltis et al. (2014), that has been used to predict 5% accuracy for the NICER targets. With our model parameters and the amount of detected counts (4 \( \times \) 10\(^7\)), we should have about 1% accuracy. Our even tighter limits could be due to the anisotropic effects (ignored in the aforementioned equation), that can strongly increase the second harmonic of the pulse profile signal \( P_{am} \), and thus decrease the uncertainty in its measurement (as we regard the atmospheric effects to be known). In any case, this is not critical, because we

\[
\frac{\text{Counts}}{\text{Photon flux} EFE/ \langle EFE \rangle} \text{data}
\]
are only interested in the differences between the two spectral models. Similarly, tight constraints are found for other parameters so that the correct point locates inside their 68% limits, except for the temperature where the correct point is slightly offset towards smaller values, but is still inside the 95% limits.

### 3.3 Parameter constraints with the incorrect model

We apply now the method described in Sect. 2 using NS atmosphere model PHAC to fit the synthetic data that are created using the exact Compton scattering model. The fitted pulse profiles, integrated to 3 energy bins, are now shown in Fig. 8 for illustration. We see that the fits are worst at the highest energies due to a large difference in the spectral shapes. The posterior probability distributions are shown in Fig. 9 and the credible limits are again also listed in Table 1.

We find that the constraints for radius are still not biased, but very close to those obtained in the previous section, since $R_{eq} = 12.00^{+0.01}_{-0.01}$ km. However, the credible limits for the temperature and especially for the size of the spot are clearly different. Temperature is larger and the spot size smaller than the correct values. The latter is not coinciding even the 95% limits. The best-fit solution presented in Fig. 8 has $T = 0.27$ keV.

### Table 1. Most probable values and 68% and 95% credible limits for Compton and Thomson models applied to the synthetic data.

| Quantity               | 95% lower limit | 68% lower limit | Most probable value | 68% upper limit | 95% upper limit |
|------------------------|-----------------|-----------------|---------------------|-----------------|-----------------|
| $R_{eq}$ (km)          | 11.99           | 12.00           | 12.01               | 12.02           | 12.03           |
| $\theta$ (deg)         | 35.6            | 35.8            | 35.9                | 36.0            | 36.2            |
| $\rho$ (deg)           | 4.99            | 5.00            | 5.00                | 5.01            | 5.01            |
| $T$ (keV)              | 0.2690          | 0.2693          | 0.2696              | 0.2699          | 0.2701          |
| $\log(\sigma_i)$      | 0.868           | 0.869           | 0.869               | 0.879           | 0.887           |

**Notes.** The quantities shown in the Table are equatorial radius $R_{eq}$, spot co-latitude $\theta$, spot angular radius $\rho$, hotspot temperature $T$, and intrinsic scatter $\log \sigma_i$. The correct values for the model parameters are $R_{eq} = 12$ km, $\theta = 36^\circ$, $\rho = 5^\circ$, and $T = 0.27$ keV.
4. Conclusions

We studied the possible outcomes of using Thomson scattering approximation in the atmosphere calculation instead of an exact Compton scattering model when trying to constrain NS parameters from RMP pulse profile observations of NICER. Our spectral comparisons showed that the difference in the observed spectrum may not be detected, due to the low count rate at highest energies, if the temperature of the emitting hot spot is $T = 2$ MK. However, in case of $T \approx 3$ MK, a significant discrepancy can be observed.

We simulated and fitted synthetic data, based on the Compton atmosphere with $T = 3.1$ MK and the NICER target PSR J0437$-$4715 which is expected to give some of the most constraining limits to NS radius. Fitting with the same Compton model, we got indeed very tight limits for the NS parameters without strong biases, which demonstrates the robustness of our method. Also, fitting with the Thomson model resulted in very similar constraints on the radius. However, the obtained size and the temperature of the hotspot were significantly different. The exact credible limits should not be taken too seriously, as we have exaggerated the predicted count rate in order to emphasize the differences between the two spectral models.

According to our results, Compton scattering seems not to be important if trying to only get accurate radius constraints for RMPs, at least when having model and data similar to that used here. However, for the interpretation of the data from a mission that is more sensitive at high energies and observes more counts at the energies around and above 3 keV, we need to take into account the effects of Compton scattering exactly. These effects will be even more important for atmospheres heated in the surface layers by bombarding particles.

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References

Aharonian, F. A. & Atoyan, A. M. 1981, Ap&SS, 79, 321
Baumböck, M., Psaltis, D., & Özel, F. 2019, ApJ, 872, 162
Bogdanov, S. 2013, ApJ, 762, 96
Bogdanov, S. 2018, in IAU Symposium, Vol. 337, Pulsar Astrophysics the Next Fifty Years, ed. P. Welti, B. M. Perera, & S. Sanidas, 116–119
Degenaar, N. & Suleimanov, V. F. 2018, arXiv e-prints [arXiv:1806.02833]
Delfer, A. T., Verbiest, J. P. W., Tingay, S. J., & Bailes, M. 2008, ApJ, 685, L67
Goodman, J. & Weare, J. 2010, Communications in Applied Mathematics and Computational Science, Vol. 5, No. 1, p. 65–80, 2010, 5, 65
Haakonsen, C. B., Turner, M. L., Taicik, N. A., & Rutledge, R. E. 2012, ApJ, 749, 52
Harding, A. K. & Muslimov, A. G. 2002, ApJ, 568, 862
Heinke, C. O., Rybicki, G. B., Narayan, R., & Grindlay, J. E. 2006, ApJ, 644, 1090
Lattimer, J. M. 2012, Annual Review of Nuclear and Particle Science, 62, 485
Leahy, D. A., Morsink, S. M., & Cadeau, C. 2008, ApJ, 672, 1119
Miller, M. C. 2016, ApJ, 822, 27
Miller, M. C. & Lamb, F. K. 2015, ApJ, 808, 31
Morsink, S. M. & Leahy, D. A. 2011, ApJ, 726, 56

Morsink, S. M., Leahy, D. A., Cadeau, C., & Braga, J. 2007, ApJ, 663, 1244
Nagami, D. I. & Poutanen, J. 1994, Astrophysics and Space Physics Reviews, 9, 1
Nättäli, J., Steiner, A. W., Kajava, J. J. E., Suleimanov, V. F., & Poutanen, J. 2016, A&A, 591, A25
Özel, F. & Freire, P. 2016, ARA&A, 54, 401
Poutanen, J. 2008, in American Institute of Physics Conference Series, Vol. 1068, A Decade of Accreting Millisecond X-ray Pulsars, ed. R. Wijnands, D. Altamirano, P. Solarz, N. Degenaar, N. Rea, P. Casella, A. Patruno, & M. Linares, 77–86
Poutanen, J. & Beloborodov, A. M. 2006, MNRAS, 373, 836
Poutanen, J. & Gierliński, M. 2003, MNRAS, 343, 1301
Poutanen, J. & Svensson, R. 1996, ApJ, 470, 249
Poutanen, J. & Vurm, I. 2010, ApJS, 189, 286
Prasad, M. K., Kershaw, D. S., & Beason, J. D. 1986, Appl. Phys. Lett., 48, 1193
Psaltis, D., Özel, F., & Chakrabarty, D. 2014, ApJ, 787, 136
Salmi, T., Nättäli, J., & Poutanen, J. 2018, A&A, 618, A161
Steiner, A. W., Lattimer, J. M., & Brown, E. F. 2010, ApJ, 722, 33
Suleimanov, V., Poutanen, J., & Werner, K. 2012, A&A, 545, A120
Suleimanov, V. & Werner, K. 2007, A&A, 466, 661
Suleimanov, V. F., Poutanen, J., Klochkov, D., & Werner, K. 2016, European Physical Journal A, 52, 20
Suleimanov, V. F., Poutanen, J., & Werner, K. 2018, A&A, 619, A114
Verbiest, J. P. W., Bailes, M., van Straten, W., et al. 2008, ApJ, 679, 675
Watts, A. L., Andersson, N., Chakrabarty, D., et al. 2016, Reviews of Modern Physics, 88, 021001
Watts, A. L., Yu, W., Poutanen, J., et al. 2019, Science India Physics, Mechanics, and Astronomy, 62, 2903
Zavlin, V. E. 2007, Ap&SS, 308, 297
Zavlin, V. E., Pavlov, G. G., & Shibanov, Y. A. 1996, A&A, 315, 141

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