Cross-domain tolerance design for directional control valves*

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The task of tolerance analysis usually addresses the question of the mechanical mountability of an assembly. We extend this viewpoint when talking about directional control valves in a cross-domain tolerance analysis; an analysis whose task is to determine the possible variation in the key product characteristics such as response dynamics, or flow gain, induced by a specific tolerance concept. On the other hand, tolerance synthesis aims at the determination of an optimal tolerance concept resulting in the compliance of the demanded tolerances for key product characteristics. Both issues require a way to identify the noise factors to be tolerated, a mathematical representation of the tolerances and a method to propagate their impact on the key product characteristics.

Keywords: manufacturing tolerances, analysis, synthesis, multi-physics, uncertainty

1. Introduction

In general, the key product characteristics of a directional control valve such as response dynamics or flow gain vary from piece to piece, e.g. due to: manufacturing variations, variations of material properties, and environmental influences. The conscious exploit of the individual component’s design limits and the use of special material properties generally require tighter tolerances in order to prevent a failure of the required functionality as a result of these variations. Nevertheless, these tolerances cannot be chosen arbitrarily tight in cost-effective production.

The aim of our research activities was therefore to identify suitable methods and tools for cross-domain tolerance analysis and for the systematic and knowledge-based definition of the tolerances allowed during valve design. The chosen approach was exemplified by both pneumatic switching valves and hydraulic proportional valves. To avoid confusion, this paper is restricted to an 5/2 indirect solenoid actuated spool valve for pneumatic applications.

2. Nomenclature

| Symbol | Description |
|--------|-------------|
| $A$    | area        |
| $b$    | critical pressure ratio |
| $C$    | discharge coefficient; cost |
| $F$    | force       |
| $g$    | restriction |
| $i$    | current     |
| $LB/UB$ | lower/upper bound |

3. Handling of noise factors

Tolerances need to be established for all noise factors that occur during production, e.g. variations of geometric features or material properties, taking into consideration noise factors arising from the environment, e.g. temperature (compare Figure 1).

Based on a structural analysis possible noise factors for typical directional control valves can be identified and logged according to different requirements areas along the phases of the product life cycle. Since the mechanical components are function carriers these noise factors can be taken into account during the evaluation of the system’s transfer function via the physical technical operating principle of the elementary functions.
The considered key product characteristics for this investigation are shown on the right side of figure 1. In particular these are the switching times $t_{A2}$ of the armature and $t_{S2}$ of the spool as well as the force reserve $F_{M, \text{endstop}}$ at the endstop of the armature and the current reserve $\Delta I_{M, \text{release}}$ when the armature releases from the valve seat.

![Diagram of Directional control valves](image)

**Fig. 1** System context for tolerancing illustrated by a parameter diagram following Taguchi’s quality engineering method

Unfortunately, the available information about the noise factors is almost always imperfect. This imperfection constitutes of imprecision and/or uncertainty $^{1)}$, e.g. different expert opinions about the possible value range of a parameter or an experimentally determined probability distribution function. Especially in early design phases the available information is limited. Therefore, an appropriate mathematical formulation has to be chosen carefully. While imprecision can be modelled by crisp or fuzzy sets, uncertainty is typically expressed by measures like probability, possibility or plausibility. Probability approaches are most often used in literature as they allow wider tolerance ranges for the individual features to meet the acceptable tolerance ranges of the key product characteristics. This is due to the fact that they do not overestimate the coincidence of extreme values. On the other hand, the knowledge of the actual distribution functions is often not guaranteed; something that calls the trustworthiness of the results in question. The more detailed the knowledge about the noise factors gets during product development cycle, the more reasonable a probability treatment is.

The methods for propagating imperfect information through system models depend on the mathematical representation of these imperfections. There are intrusive and non-intrusive methods. To be able to use simulation tools which have already proven their suitability for the analysis of fluid power components, only non-intrusive methods were further considered. Probability methods in particular are the focus of a lot of research activities, e.g. Lee et al.$^{2)}$ differentiate between simulation based methods, local expansion based methods, most probable points based methods, functional expansion based methods, and numerical integration based methods.

4. **Computational Models**

Directional control valves show strong interactions between the electrical, magnetic, mechanical and fluidic domains as well as non-linear transfer functions. A closed analytical equation for the calculation of the key product characteristics is not available and simulation models are needed to calculate the system’s response.

The known propagation methods for imperfect information usually require numerous model evaluations. Therefore, the computational costs are critical and the simulation with distributed parameters, e.g. FEM for magnetic fields, CFD for fluid flows, is not or only to a limited extent permissible. On the other hand, simulations with concentrated parameters allow for the consideration of cross-domain interactions and provide a sufficiently fast model computation, e.g. it is possible to model an electromagnetic actuator as a reluctance network. This allows for the calculation of the armature movement in a fraction of the time that an FEM calculation would need. If only static characteristics are required (e.g. for the analysis of a proportional valve’s flow gain) the phenomenological representation of these three dimensional and non-linear relationships by means of surrogate models has proven to be advantageous. Such a decrease in model complexity is often accompanied by an increase of the model uncertainty and requires a careful model validation. One must ensure to cover all relevant noise factors and their impact on the key product characteristics despite the simplification.

**Table 1** Consideration of noise factors for the pneumatic sample valve

| sub-function         | geometry based implementation | phenomenological implementation |
|----------------------|-------------------------------|-------------------------------|
| pilot stage:         |                               |                               |
| convert $E_{1\text{coil}} \rightarrow E_{\text{mech}}$ | iron core reluctances, coil (temperature-dependent) | number of windings, magnetic permeability |
| pilot stage:         |                               |                               |
| adjust shift element | stroke range, spring pre-stress, mass of armature | spring constant (temperature-dependent), damping |
| pilot stage:         |                               |                               |
| change resistance    | discharge rate C, critical pressure ratio, $b$ (stroke-dependent) |                               |
| main stage:          |                               |                               |
| convert $E_{\text{fluid}} \rightarrow E_{\text{mech}}$ and | stroke range, piston force, mass of spool | friction force (pressure- and velocity-dependent), spring constant (temperature-dependent), damping |
| main stage:          | adjust shift element           |                               |

To investigate the tolerance concepts all identified noise factors need to be related to the key product characteristics within the simulation model. Unfortunately, some of the underlying physics cannot be formulated with respect to geometric tolerances or other noise factors. A good example is the resulting friction force at a piston sealing. It is a com-
plex time/geometry/material/load - depending effect which is currently neither completely understood nor are there model approaches available which can be reduced to ordinary differential equations for implementation in concentrated parameter simulations. In such cases it is again possible to describe these physics phenomenologically with different kinds of surrogate models that take into account noise factors as multipliers estimated from limiting samples. These surrogate models can be empirically motivated algebraic expressions or families of characteristics gained from measurements. When there is no data available, these multipliers can be used to develop limits that must be ensured during product development.

Table 1 summarizes the chosen implementation of the identified noise factors for the analyzed pneumatic directional control valve in a simulation model with concentrated parameters. As shown on the left side in figure 2 all relevant domains are considered within the same simulation model. Furthermore, the simulation results from the whole system’s model and the corresponding measurements are in alignment as depicted on the right side of figure 2.

In some cases (e.g., the calculation of variance based sensitivity indices or the iterative evaluation of dispersion measures within the numerical optimization process for tolerance synthesis) the computational cost for simulations with concentrated parameters are still too high. A common approach is to use surrogate models for estimating the whole system’s behaviour with less computational effort. According to Simpson et al., artificial neural networks are especially suitable for systems with much more than ten noise factors. It is important to check the quality of the surrogate models with respect to the resulting error and against overfitting. The latter is done by using an additional test set; one that is not used for the generation of the surrogate model. When there is only a small error for these additional samples, the model is said to have a good generalization: something that is essential for utilization in tolerance analysis.

5. Tolerance Analysis

The task of tolerance analysis usually addresses the question of the mechanical mountability of an assembly. We extend this viewpoint when talking about directional control valves in a cross-domain tolerance analysis; an analysis whose task is to determine the possible variation in the key product characteristics such as response dynamics, or flow gain, induced by a specific tolerance concept. Different methods for the propagation of imperfect information through a simulation model are described in literature. Monte-Carlo-Simulation is one example which is straightforward and easy to implement. In this case (quasi-)random samples are generated within the tolerance range for each noise factor and the corresponding key product characteristics are determined by iterative model evaluations, as illustrated in figure 3. Latin-Hypercube-Sampling with iteratively reduced correlation has proven an efficient sampling method capable of accounting for different distribution functions. The following discussion explains how descriptive statistics can be used for the characterisation of the resulting variations and the determination of failure rates. For demonstration purposes, an initial tolerance concept was chosen based on the tolerance class “fine” according to the DIN ISO 2768-1:1991 norm for all features.

Histograms like shown in figure 4 can be used for a first visualisation of the calculated deviations. More detailed statistical parameters are depicted in the box plots aligned above these histograms. This type of diagram contains the following information:

- The red line shows the median of the data set.
- The limits of the box mark the first and the third quartile of the data set.
- The whiskers are used to determine outliers which themselves are labelled by plus-signs.

The switching time \( t_{\text{A2}} \) of the armature shows a small variation of about 3.3% around the mean value. Also, the electrical current reserve \( I_{\text{Mrelease}} \) when the armature releases from the valve seat varies only within 3.8%. Even when the reserve of the magnetic force at the armature end stop \( F_{\text{Mendstop}} \) possesses an estimated standard variation of 12.2% with respect to the mean value it will not become zero and therefore proper...
functioning of the pilot stage is ensured. On the contrary the
time for the spool travel \( t_{S2} \) is divided into two groups. For the
first group of samples the spool reaches the end stop shortly
after the pilot stage pressurizes the piston. For the second
group of samples the spool will not arrive at the end stop until
the end of simulation time and is, therefore, faulty. As shown
on the right side of figure 4 an empirical cumulative distribution
function plot can be used to estimate the corresponding failure rates.

It was possible to identify the failure cause as the interaction
between the spring force, the pressurisation of the piston,
and the friction force as a function of the pilot pressure and
the spool velocity. An increasing spring pre-stress requires a
higher pilot pressure on the piston to move the spool. Due to the
higher pressure, the piston sealing lays tighter to the housing
and, therefore, the friction force increases. This slows down
the spool movement and finally the spool stops at an inclined
position. In figure 5 the resulting threshold level between
operating (labelled “1”) and faulty (labelled “0”) samples is
clearly visible.

Furthermore, the calculated samples can be utilised to perform a sensitivity analysis. Without prior knowledge of the
underlying transfer function characteristics, it is nearly im-
possible to choose an adequate sensitivity measure from the plurality of possible ones (e.g. screening or variance decom-
position, a detailed review is given in literature\(^6\)). As this
holds for the analysed directional control valve different sens-
itivity measures where calculated and compared. The results
are shown in figure 6.

In the direct comparison of figure 6, each of the different sensitivity measures identify the same noise factors as signif-
ificant. One should keep in mind that Sobol’s main effects sum up to one, while every correlation coefficient and every
regression coefficient can take values in the interval \([-1,1]\). Since only variance based sensitivity measures quantify the ratio of the noise factor variation to the variation of the
key product characteristics, it seems to filter the data better
than correlation or regression measures which measure linearity/monotonicity and are proportionality factors, respectively.
This becomes evident by the sensitivities of \( t_{S2} \) where Sobol’s
indices only identify two significant noise factors whereas the other measures identify up to six significant noise factors.

There is nearly no difference between the correlation coef-
ficients and the regression coefficients. That indicates not only
monotonicity, but also a linear relationship within the range
of tolerances. Furthermore, there seems to be no strong in-
teractions between noise factors due to the fact that Sobol’s
main and total effects show nearly no differences. The only
exception is the switching time \( t_{A2} \) whose cause of failure was
already discussed as the interaction between spring stiffness
and friction force.

Variance based sensitivity measures require no restrictions
about the linearity of the system’s transfer function, but their
calculation is far more computationally expansive. This is
especially relevant when a large amount of noise factors are
taken into account for the analysis. In this case surrogate
models were used to gain convergence within a bearable computa-
tional timeframe.

Concerning the valve’s key product characteristics strong
dependencies between the switching time \( t_{A2} \) of the pilot stage
and the noise factors related to the supply voltage, the parasitic
air gaps, and the stiffness of the armature return spring are
visible. Most influence to the switching time of the main stage
\( t_{S2} \) comes from the noise factors for the friction force and the
stiffness of the spool return spring. The variations of parasitic
air gaps, the supply voltage and the magnetic permeability of
the yoke’s material dominate the variations of the force reserves

![Fig. 4 Results from a Monte-Carlo-Simulation for a directional control valve: histograms and box plots of the key product characteristics and empirical cumulative distribution function plot for the determination of failure rates.](image)

![Fig. 5 Analysis of Failure Mode: threshold level between operating (labelled “1”) and faulty (labelled “0”) samples and comparison of switching process for one operating and one faulty sample.](image)

![Fig. 6 Comparison of different sensitivity measures calculated for a directional control valve (\( S_{H1} \) and \( S_{T} \): main and total sensitivity indices from Sobol’s method; SRC: standardized regression coefficient; SRRC: standardized rank regression coefficient; \( r \): Pearson’s correlation coefficient; \( p \): Spearman’s rank correlation).](image)
for release and hold of the armature.

6. Tolerance Synthesis

Tolerance synthesis, on the other hand, aims at the determination of an optimal tolerance concept resulting in the compliance of the demanded tolerances for the key product characteristics. It is possible to determine tolerances for the individual part and material properties based on the calculated sensitivity measures. But in doing so, there is no guaranty of finding the most cost-efficient tolerance concept. Considering tolerance synthesis as an optimization problem allows for the utilization of a broad variety of established numerical optimization tools which results in a substitution of heuristics by algorithms7).

**Figure 7** Chosen procedure for tolerance synthesis: numerical optimization of tolerance concept $T_B$ by iterative calculation of dispersion measures and comparative costs

**Figure 7** depicts the chosen approach for the tolerance synthesis of directional control valves. For realizing this concept, one needs to pay attention to the following three aspects:

**Tolerance-Cost-Function** In order to determine the total costs associated with a tolerance concept, the specific manufacturing costs of each feature within the given individual tolerance range need to be known. Tolerance-cost functions typically increase strongly the smaller the tolerance range is. Many authors use reciprocal potential functions like the one shown in the middle of figure 7 as a mathematical description for that relationship. The main difficulty lies in determining suitable regression parameters $a$, $b$ and $k$ for the specific manufacturing processes for the considered feature. There are almost no publicly available data sets as the data is company specific and usually underlying an obligation of confidentiality. An exception is a data set for metal working reported by Chase et al.8). It is based on a study of the U.S. Army from the 1940s which seems quite outdated. But by being formulated as relative costs, this data set allows for the testing of the chosen approach for tolerance analysis. In addition, the tolerance-cost-function for standard parts can be reconstructed from available grades and the corresponding purchasing price.

**Adapted tolerance analysis** During the optimization process the resulting deviations of the key product characteristics associated to each specific tolerance concept need to be determined. Almost all available methods for propagating imperfect information about the noise factors through system models share the necessity for running a huge number of model evaluations. Surrogate model can reduce the calculation time strongly but compared to their use in sensitivity analysis an extended range of validity is required in the case of the algorithm is trying to expand the tolerances beyond their initial range. By doing so usually a significant reduction of the goodness of fit can be observed. Two possible countermeasures have proven themselves in practice: a) the restriction to the operative valve samples during surrogate model generation by utilising an additional pattern recognition neural network and b) a sensitivity based selection of the noise factors whose range of validity is extended for surrogate model generation.

**Numerical optimization** Furthermore an suitable optimization method needs to be selected with regard to the number of noise factors and the characteristics of the objective function. The restriction expressing the compliance to the tolerable deviations of the key product characteristics can be transformed into an additional objective function. This could be the minimization of a weighted sum of a measure of dispersion for each key product characteristic, e.g. the quartile dispersion coefficients which are robust against outliers. This is particularly advantageous because one solution can then be chosen according to the desired cost or precision demands out of a set of so called pareto optimal solutions afterwards. Within this special solution set an objective function cannot be increased without decreasing another objective function. Unfortunately, there is no gradient information available for this multicriteria objective function which results in the necessity to calculate them numerically. This is prohibitive, especially in the case of many noise factors. To avoid this, a stochastic optimization algorithms called “Non-dominated Sorting Genetic Algorithm” (NSGA-II) was chosen for the cross-domain tolerance synthesis of directional control valves which is also able to identify not only local minima but also global ones.

**Fig. 8** Comparison of resulting tolerance concepts with shown reference to sensitivity measure $\rho$. The compared concepts correspond to the coloured dots in the pareto front of figure 7
Three different resulting tolerance concepts are compared to the initial design in figure 8. They are highlighted in figure 7 with the same colours. The pink dot marks the initial tolerance concept. Compared to the concept marked in red, only a small reduction of costs can be achieved at the expense of the resulting deviations for the key product characteristics. On the other hand, the concept marked in blue reduces the deviations by half, but for double the price. A compromise solution is marked by the green colour. Compared to the initial design, costs, as well as the resulting deviations, can be significantly reduced. In this case it is possible to extend the allowed tolerances for most of the system’s parameters. Only a couple of parameters with especially high sensitivity measures are tightened to ensure the precision of the valve characteristics. The tolerance restriction is strongly coupled to the tolerance-cost-function which is extremely important for the results.

7. Summary and Outlook

This paper demonstrates a method for computational tolerance analysis and tolerance synthesis for a directional control valve. Questions like identifying noise factors to be tolerated, a mathematical representation of these tolerances, and a way to propagate their impact on the key product characteristics through simulation models, were all addressed. A comparison of different approaches to describe the noise factors, as well as further application to other valve types (or even to complex fluid power systems) could be the focus of future research.

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