Catalysis of partial chiral symmetry restoration by Delta matter

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We study the phase structure of dense hadronic matter including \( \Delta \) as well as \( N \) on the parity partner structure, where the baryons have their chiral partners with a certain amount of chiral invariant masses. We show that, in symmetric matter, \( \Delta \) enters into matter in the density region of about one to four times of normal nuclear matter density, \( \rho_B \sim 1 - 4 \rho_0 \). The onset density of \( \Delta \) matter depends on the chiral invariant mass of \( \Delta \), \( m_{\Delta} \). The larger \( m_{\Delta} \), the bigger the onset density. The \( \Delta \) matter of \( \rho_B \sim 1 - 4 \rho_0 \) is unstable due to the existence of \( \Delta \), and the stable \( \Delta \)-nuclear matter is realized at about \( \rho_B \sim 4 \rho_0 \), i.e., the phase transition from nuclear matter to \( \Delta \)-nucleon matter is of first order for small \( m_{\Delta} \), and it is of second order for large \( m_{\Delta} \).

We find that, associated with the phase transition, the chiral condensate changes very rapidly, i.e., the chiral symmetry restoration is accelerated by \( \Delta \) matter. As a result of the accelerations, there appear \( \Sigma^* \) (1535) and \( \Delta^* \) (1700), which are the chiral partners to \( N \) (939) and \( \Delta \) (1232), in high density matter, signaling the partial chiral symmetry restoration. Furthermore, we find that complete chiral symmetry restoration itself is delayed by \( \Delta \) matter. We also calculate the effective masses, pressure and symmetry energy to study how the transition to \( \Delta \) matter affects such physical quantities. We observe that the physical quantities change drastically at the transition density.

I. INTRODUCTION

Studying the origin of hadron masses is one of interesting problems in hadron physics. As it is well known, the mass of current quarks could explain roughly 2% of the nucleon mass. A standard folklore in nuclear physics states that the nucleon mass other than that from the current quarks could be explained in terms of spontaneous chiral symmetry breaking. However, in the parity doublet model it becomes even more interesting as the nucleon mass gets an additional contribution from a so-called chiral invariant mass. This structure can be formulated in models with parity doublet structure \(^2\) \[2\].

It is interesting to investigate the chiral invariant mass portion of the nucleon mass compared to that from chiral symmetry breaking. Since the chiral symmetry is expected to be partially restored in dense matter, a part of hadron masses originated from the spontaneous chiral symmetry breaking is changed. This change will provide a clue for elucidating the origin of hadron masses.

Dense nuclear matter, which is intimately related to heavy ion collisions, nuclear structure, and neutron stars, has served as a testing ground for our understanding of non-perturbative QCD in hadron and nuclear physics. Dense matter with large neutron-proton number asymmetry is important to understand neutron star properties. Isospin asymmetric dense matter has been even more highlighted thanks to existing and forthcoming rare isotope accelerator facilities.

There are many extensive studies of dense matter in the parity doublet models \(^3\) \[11\]. It was shown in Ref. \[11\] that the phase structure depends on the value of the chiral invariant mass. The density dependence of the effective nucleon mass was also presented, and it was pointed that the masses change, reflecting the partial chiral symmetry restoration in nuclear matter. It will be also interesting to ask What happens to the masses of other hadrons in matter associated with the partial restoration of chiral symmetry.

In the present study, we focus on the role of the \( \Delta \) baryon in terrestrial dense matter that could be created in heavy ion collisions at a few hundreds MeV \[12\] \[13\]. In Ref. \[12\], it was shown that, in symmetric matter, the \( \Delta \) matter appears when the coupling of \( \sigma \) mean field to \( \Delta \) is larger than that to nucleon, and that the on-set density strongly depends on the saturation properties. In Ref. \[13\], the \( \Delta \) matter in asymmetric matter was studied with focusing on the stability of the system. It was shown that there can be instabilities when \( \Delta \) matter appears.

There are also many works on the \( \Delta \) baryons in neutron stars \[14\] \[15\]. It was pointed that the \( \Delta \) baryon enters into the matter around two to three times of the normal nuclear matter density \( \rho_0 \).

In this work we study the chiral phase structure of dense matter that could be created during heavy ion collisions at a few hundreds A MeV. First we construct a parity doublet model extended from the model in Ref. \[11\] to include the \( \Delta \) baryon following Ref. \[2\] \[19\]. In the present model, \( N \) (1535) and \( \Delta \) (1700) are included as chiral partners to \( N \) (939) and \( \Delta \) (1232), respectively, having certain amounts of chiral invariant masses. We first investigate the dependence of matter constituents on the chiral invariant mass of \( \Delta \), \( m_{\Delta} \), and show that the onset density of the appearance of \( \Delta \) matter strongly depends on \( m_{\Delta} \). Then, we study the chiral phase structure and show that the partial chiral symmetry restoration is accelerated by \( \Delta \) matter, while the chiral symmetry restoration itself is delayed. In addition, we also calculate the effective masses, pressure and symmetry energy to study how the transition to \( \Delta \) matter affects such physical quantities.

This paper is organized as follows:

In Sec. \[11\] we introduce the parity doublet model with the \( \Delta \) baryon, and in Sec. \[11\] we evaluate the effective \( \Delta \) and nucleon masses in cold dense matter. In Sec. \[14\] we...
determine the model parameters and present our results focusing on the possibility of having the \( \Delta \) baryons in dense matter, i.e., a phase transition from nuclear matter to \( \Delta \) matter. Then, in Sec. V, we investigate how the chiral structure is affected by \( \Delta \) matter. Finally, we summarize the present study with a brief discussion on the issues related to \( \Delta \) matter in Sec. VI.

II. LAGRANGIAN FOR \( \Delta \) BARYON

In the present analysis, nucleons are included using a hadronic model in Ref. [11]. We further include \( \Delta \) baryon and its chiral partner based on the parity doublet structure following Ref. [2, 19].

For expressing \( \Delta \) baryon and its chiral partner, we introduce two fields of spin \( 3/2 \), \( \psi^\mu_{1,2} \), to which we impose the constraints \( \gamma^\mu \psi^\mu = 0 \) and \( \partial_\mu \psi^\mu = 0 \), to reduce extra degrees of freedom. To define the chiral transformation of the fields, we introduce

\[
\psi^\mu_{1,2} = \frac{1 + \gamma_5}{2} \psi^\mu_{1,2} \; , \quad \psi^{\mu\nu}_{1,2} = \frac{1 - \gamma_5}{2} \psi^{\mu\nu}_{1,2} \; .
\]

Their representations under the chiral SU(2)_R \times SU(2)_L are given as

\[
(\psi^\mu_{1,2})^{\gamma\beta}_{\alpha\beta} \in \left( \begin{array}{c} 1 \\ 2 \end{array} \right) , \quad (\psi^{\mu\nu}_{1,2})^{\gamma\delta}_{\alpha\beta} \in \left( \begin{array}{c} 1 \\ 2 \\ 1 \end{array} \right) ,
\]

where \( \alpha, \beta \) are indices for SU(2)_L and \( \gamma, \delta \) for SU(2)_R.

We introduce the iso-singlet scalar meson \( \sigma \) and the iso-triplet pion \( \pi^A \ (A = 1, 2, 3) \) through the matrix field \( M \) as

\[
M = \sigma + if \sum_{A=1}^3 \pi^A \tau^A ,
\]

where \( \tau^A \) are the Pauli matrices. The chiral representation of this \( M \) is

\[
M \in \left( \begin{array}{c} 1 \\ 2 \\ 1 \end{array} \right) .
\]

The Lagrangian for \( \Delta \) baryon and its chiral partner is expressed as

\[
\mathcal{L}_\Delta = \left( \psi^\mu_{1}\right)^{\alpha\beta}_{\gamma\beta} \left\{ \sigma_{\mu\nu}, i \right\} D_{\beta\gamma}^{\emptyset} \left( \psi^\nu_{1}\right)^{\gamma\beta}_{\alpha\beta} + (\psi^{\mu\nu}_{1})^{\gamma\delta}_{\alpha\beta} \left\{ \sigma_{\mu\nu}, i \right\} D_{\xi\gamma}^{\emptyset} \left( \psi_{1}\right)^{\gamma\delta}_{\xi\gamma} + \cdots
\]

\[
+ b \left( \psi^{\mu}_{1\beta\gamma} \sigma_{\mu\nu} (\psi^\nu_{2\beta})^{\delta\gamma}_{\alpha\beta} + (\psi^{\mu\nu}_{1})^{\gamma\delta}_{\alpha\beta} (M^1)^{\delta\gamma}_{\beta\alpha} \right) \; ,
\]

where

\[
(D_{\mu})^{\gamma\beta\alpha}_{\alpha\beta} = \partial_{\mu} \gamma_{\beta\beta} \delta_{\beta\gamma} - i(\mathcal{L})^{\gamma\beta} \delta_{\beta\gamma} - i \delta_{\gamma\beta} \delta_{\beta\gamma} (R_{\mu})^{\alpha\beta} ,
\]

\[
(D_{\mu})^{\gamma\delta\alpha}_{\alpha\beta} = \partial_{\beta} \gamma_{\gamma\beta} \delta_{\beta\gamma} - i(\mathcal{L})^{\gamma\beta} \delta_{\beta\gamma} - i \delta_{\gamma\beta} \delta_{\beta\gamma} (R_{\mu})^{\alpha\beta} .
\]

III. EFFECTIVE MASSES

We first construct a thermodynamic potential of nuclear matter at mean field level including nucleons of positive and negative parities in the model given in Ref. [11], with the baryon number chemical potential \( \mu_B \) and the isospin chemical potential \( \mu_I \). Here, we assume that \( \mu_I \) is small and there is no pion condensation, and that the rotational invariance is not spontaneously broken. Then, the relevant mean fields are \( \bar{\sigma} \) for the scalar meson, \( \bar{\omega} \) and \( \bar{\rho} \) for the time components of the vector mesons, which are determined by minimizing the thermodynamic potential.
In nuclear matter, the existence of the mean fields of $\omega$ and $\rho$ changes the dispersion relation of the nucleon as

$$E_n = \sqrt{m_N^2 + k^2 + g_{\omega NN}\omega - g_{\rho NN}\rho}$$

$$E_p = \sqrt{m_N^2 + k^2 + g_{\omega NN}\omega + g_{\rho NN}\rho},$$

where $E_{n,p}$ are the energies of a neutron and a proton measured from their respective chemical potentials given by

$$\mu_p = \mu_B + \frac{1}{2}\mu_I,$$

$$\mu_n = \mu_B - \frac{1}{2}\mu_I,$$

(10)

$g_{\omega NN}$ and $g_{\rho NN}$ are the couplings of $\omega$ and $\rho$ mesons to the nucleon. Here, the nucleon mass $m_N$ is expressed by $\bar{\sigma}$ as

$$m_N = \frac{1}{2}\left(\sqrt{(g_1 + g_2)^2\bar{\sigma}^2 + 4m_{N0}^2} + (g_1 - g_2)\bar{\sigma}\right),$$

(11)

where $m_{N0}$ is the chiral invariant mass of the nucleon, $g_1$ and $g_2$ are Yukawa couplings of the scalar meson to the nucleon. The mass of its partner is given by

$$m_{N^*} = \frac{1}{2}\left(\sqrt{(g_1 + g_2)^2\bar{\sigma}^2 + 4m_{N0}^2} - (g_1 - g_2)\bar{\sigma}\right),$$

(12)

which is the mass of $N^*(1535)$ as in Ref. [11]. Then, using the masses of $N(939)$ and $N^*(1535)$ as inputs, we determine the values of the $\sigma NN$ couplings for given value of the chiral invariant mass $m_{N0}$. In Table I, we list the values of $g_1$ and $g_2$ for several choices of $m_{N0}$.

| $m_{N0}$ (MeV) | 500 | 700 | 900 |
|---------------|-----|-----|-----|
| $g_1$         | 9.03| 7.82| 5.97|
| $g_2$         | 15.49| 14.28| 12.43|

TABLE I. Values of $\sigma NN$ coupling constants for several choices of the chiral invariant mass $m_{N0}$.

In Fig. 1, we plot the density dependence of the mean field $\bar{\sigma}$ in symmetric nuclear matter with including only nucleons in matter. This $\bar{\sigma}$ is interpreted as the pion decay constant in nuclear matter. An experiment [21] shows that the value of the pion decay constant at the normal nuclear matter density $\rho_0$ is about 0.8 times of that in vacuum [22]. Figure 1 indicates that $m_{N0} = 900$ MeV leads to a too small value for $\bar{\sigma}$. Then, in the following analysis, we use $m_{N0} = 500$-700 MeV.

As in Ref. [23], we define the effective nucleon masses as the energies at $k = 0$:

$$m_n^{(eff)} = m_N + g_{\omega NN}\omega - g_{\rho NN}\rho$$

![FIG. 1. Density dependence of $\bar{\sigma}$ in symmetric nuclear matter for $m_{N0} = 500$ MeV (red curve), 700 MeV (green curve) and 900 MeV (blue curve).](image1)

In Fig. 2, we plot the density dependence of these effective masses in symmetric matter, which are given by taking $\bar{\rho} = 0$ in Eq. (13), for $m_{N0} = 500$ and 700 MeV. This shows that the effective mass $m_n^{(eff)}$ for $m_{N0} = 500$ MeV is quite similar to the one for $m_{N0} = 700$ MeV for $\rho_B \lesssim \rho_0$. A reason is as follows: As in Ref. [11], we use the saturation density $\rho_0 = 0.16$ fm$^{-3}$ and $\mu_B = 923$ MeV at saturation density as inputs. The former leads to the Fermi momentum of nucleons as $k_F \simeq 270$ MeV. Then, from Eqs. (9) and (13), we obtain

$$\mu_B - m_n^{(eff)} = \sqrt{k_F^2 + m_N^2} - m_N \simeq \frac{k_F^2}{2m_N}$$

$$\simeq \frac{k_F^2}{2\mu_B} \simeq 40 \text{ MeV},$$

(14)

independently of the choice of the chiral invariant mass $m_{N0}$.

In the density region higher than $\rho_0$, Fig. 2 shows that $m_n^{(eff)}$ decreases against increasing density for $m_{N0} = 500$ MeV since $m_N$ in Eq. (11) decreases rapidly. On the other hand, $m_n^{(eff)}$ for $m_{N0} = 700$ MeV increases since
\( m_N \) in Eq. (11) decreases very slowly and the large \( \omega \) contribution pushes up the effective mass.

The mass of the \( \Delta \) baryon with positive parity is obtained as

\[
m_{\Delta}^+ = \frac{1}{2} \left( \sqrt{(a + b)^2 + 4m_{\Delta 0}^2} - (a - b)\tilde{\sigma} \right),
\]

(15)

where \( m_{\Delta 0} \) is the chiral invariant mass of \( \Delta \). The mass of the \( \Delta \) baryon with negative parity is given by

\[
m_{\Delta}^- = \frac{1}{2} \left( \sqrt{(a + b)^2 + 4m_{\Delta 0}^2} + (a - b)\tilde{\sigma} \right).
\]

(16)

Using the masses of \( \Delta(1232) \) and \( \Delta(1700) \) in free space as inputs, we determine the values of the couplings \( g_\omega \) and \( g_\rho \) for fixed \( m_{\Delta 0} \), which is summarized in Table II for typical values of \( m_{\Delta 0} \).

| \( m_{\Delta 0} \) | 500 | 700 | 1000 | 1300 | 1400 |
|---|---|---|---|---|---|
| \( a \) | 17.5 | 16.5 | 14.1 | 9.87 | 7.25 |
| \( b \) | 12.2 | 11.4 | 9.08 | 4.87 | 2.18 |

The dispersion relations for the \( \Delta \) baryons with positive parity in nuclear matter are given by

\[
E_{\Delta^{++}} = \sqrt{m_{\Delta}^2 + k^2} + g_\omega \Delta \Delta \tilde{\omega} + 3g_\rho \Delta \Delta \tilde{\rho},
\]

\[
E_{\Delta^+} = \sqrt{m_{\Delta}^2 + k^2} + g_\omega \Delta \Delta \tilde{\omega} + g_\rho \Delta \Delta \tilde{\rho},
\]

\[
E_{\Delta^0} = \sqrt{m_{\Delta}^2 + k^2} + g_\omega \Delta \Delta \tilde{\omega} - g_\rho \Delta \Delta \tilde{\rho},
\]

\[
E_{\Delta^-} = \sqrt{m_{\Delta}^2 + k^2} + g_\omega \Delta \Delta \tilde{\omega} - 3g_\rho \Delta \Delta \tilde{\rho},
\]

(17)

where \( g_\omega \Delta \Delta \) and \( g_\rho \Delta \Delta \) are the couplings of the \( \omega \) and \( \rho \) mesons to the \( \Delta \) baryon, and the energies are measured from their respective chemical potentials given by

\[
\mu_{\Delta^{++}} = \mu_B + \frac{3}{2} \mu_I,
\]

\[
\mu_{\Delta^+} = \mu_B + \frac{1}{2} \mu_I,
\]

\[
\mu_{\Delta^0} = \mu_B - \frac{1}{2} \mu_I,
\]

\[
\mu_{\Delta^-} = \mu_B - \frac{3}{2} \mu_I.
\]

(18)

We define the effective masses as

\[
m_{\Delta^{++}}^{(\text{eff})} = m_{\Delta} + g_\omega \Delta \Delta \tilde{\omega} + 3g_\rho \Delta \Delta \tilde{\rho},
\]

\[
m_{\Delta^+}^{(\text{eff})} = m_{\Delta} + g_\omega \Delta \Delta \tilde{\omega} + g_\rho \Delta \Delta \tilde{\rho},
\]

\[
m_{\Delta^0}^{(\text{eff})} = m_{\Delta} + g_\omega \Delta \Delta \tilde{\omega} - g_\rho \Delta \Delta \tilde{\rho},
\]

\[
m_{\Delta^-}^{(\text{eff})} = m_{\Delta} + g_\omega \Delta \Delta \tilde{\omega} - 3g_\rho \Delta \Delta \tilde{\rho}.
\]

(19)

We plot the density dependence of the effective mass in symmetric matter (\( \bar{\rho} = 0 \)) for \( m_{\Delta 0} = 500 \) and 700 MeV together with the baryon chemical potential \( \mu_B \) in Figs. 3 and 4. These figures show that the effective mass of \( \Delta \) becomes smaller than the baryon chemical potential around \( p_B / p_0 \sim 2-3 \), which indicates that the \( \Delta \) baryon is populated in the ground state, forming \( \Delta \) fermi sea.

**IV. DELTA MATTER**

We now construct a thermodynamical potential with the \( \Delta \) baryons following Ref. [11]:

\[
\Omega = -\frac{1}{2} m_\rho^2 \bar{\rho}^2 - \frac{1}{2} m_\omega^2 \bar{\omega}^2 + V_\sigma - \sum_{\alpha=p,n} \int \frac{dk}{\pi^2} k^2 (\mu_\alpha - E_\alpha) \theta(\mu_\alpha - E_\alpha) - 2 \sum_{\alpha=++,-0,-} \int \frac{dk}{\pi^2} k^2 (\mu_{\Delta \alpha} - E_{\Delta \alpha}) \theta(\mu_{\Delta \alpha} - E_{\Delta \alpha})
\]
where \( E_{\rho,n} \) and \( E_{\Delta^{++},\Delta^{+},\Delta^{0},\Delta^{-}} \) are the effective energies given in Eqs. (9) and (17), \( \mu_{\rho,n} \) and \( \mu_{\Delta^{++},\Delta^{+},\Delta^{0},\Delta^{-}} \) are the chemical potentials given in Eqs. (10) and (18). In this expression, we drop the antiparticles because they do not contribute at zero temperature.

The stationary condition for \( \bar{\sigma} \) is expressed as

\[
\bar{m}e = m_{\pi}^{2} f_{\pi} ,
\]

from the values in Table III. The values of the four couplings \( \sigma \) and \( \Delta \) are fixed as in Tables II and III. The value of \( \bar{\mu} \) is determined once the values of \( \lambda \) and \( \lambda_{0} \) are determined via the stationary condition of \( V_{\sigma} \) in Eq. (21) in vacuum:

\[
\bar{\mu}^{2} = \lambda_{4} f_{3}^{2} - \lambda_{6} f_{5}^{2} - \bar{m}e .
\]

Imposing that the energy density is minimized at \( \rho_{0} \), we obtain the pressure vanishing at \( \rho_{0} \):

\[
P|_{\rho_{B}=\mu_{B}=\mu_{\pi}=0} = -\Omega|_{\rho_{B}=\mu_{B}=\mu_{\pi}=0} = 0 .
\]

Then, using \( E_{\text{bind}} = -16 \text{MeV} \) at \( \rho_{B} = \rho_{0} \) and \( m_{N_{0}} = 939 \text{MeV} \) at vacuum, we get \( \mu_{0} = \mu_{B_{0}} = 923 \text{MeV} \):

\[
\rho_{0} = \rho_{B} |_{\rho_{B}=\mu_{B}=\mu_{\pi}=0} = \left( \frac{\partial \Omega}{\partial \mu_{B}} \right)_{\rho_{B}=\mu_{B}=\mu_{\pi}=0} .
\]

In addition to these, the incompressibility \( K \) and the symmetry energy \( E_{\text{sym}} \) at the normal nuclear matter density are determined by

\[
K = 9 \rho_{0} \frac{\partial \mu_{B}}{\partial \rho_{B}} |_{\rho_{B}=\mu_{B}=\mu_{\pi}=0} ,
\]

\[
E_{\text{sym}} = 4 \rho_{0} \frac{\partial \mu_{1}}{\partial \rho_{1}} |_{\rho_{B}=\mu_{B}=\mu_{\pi}=0} .
\]

First, for given values of \( m_{N_{0}}, m_{\Delta} \) and the ratio \( g_{\omega\Delta}/g_{\omega NN} \), we determine the values of \( \bar{\omega} \) and \( g_{\omega NN} \) using \( \rho_{0} \) in Eq. (31) and the stationary condition for \( \bar{\omega} \) in

\[
\bar{m}e = m_{\pi}^{2} f_{\pi} ,
\]

the empirical values of them, which we use as inputs, in Table IV.

## Table III. Physical inputs in vacuum (MeV).

| \( m_{N_{+}} \) | \( m_{N} \) | \( m_{\Delta_{+}} \) | \( m_{\Delta_{-}} \) | \( m_{\omega} \) | \( m_{\rho} \) | \( f_{\pi} \) | \( m_{\pi} \) |
|----------------|--------|----------------|----------------|--------|--------|--------|--------|
| 939            | 1535   | 1232           | 1700           | 783    | 776    | 92.3   | 140    |

## Table IV. Physical inputs at the normal nuclear matter density. \( \rho_{0} \): saturation density, \( E_{\text{bind}} \): binding energy, \( K \): incompressibility, \( E_{\text{sym}} \): symmetry energy.

| \( \rho_{0} \) | \( E_{\text{bind}} \) | \( K \) | \( E_{\text{sym}} \) |
|----------------|--------------------|-------|----------------|
| 0.16 fm\(^{-3}\) | -16 MeV | 240 MeV | 31 MeV |
Equation (22). Second, we fix the values of $\lambda_4$, $\lambda_6$ and $\bar{\sigma}$ using the pressure $P$ in Equation (30), $K$ in Equation (32) and the stationary condition for $\bar{\sigma}$ in Equation (26). Third, for a given value of $g_{\Delta\Delta}/g_{\rho\rho}$, we determine the values of $\bar{\rho}$ and $g_{\rho\rho}$ from $E_{\text{chem}}$ in Equation (22) and the stationary condition for $\bar{\rho}$ in Equation (25). Once we fix the parameters as above, we can calculate several physical quantities with given chemical potentials $\mu_B$ and $\mu_f$.

Let us first show that the $\Delta$ baryons are unlikely to exist in nuclear matter with $\mu_B \sim \rho_0$ for $m_{N0} = m_{\Delta0} = 700$ MeV and $g_{\omega\omega\Delta} = g_{\omega\rho\rho}$.

From the stationary condition for $\bar{\omega}$ at $\mu_B = \rho_0$ with $g_{\Delta\Delta} = g_{\rho\rho}$, we obtain

$$
m_\omega^2 \bar{\omega} = g_{\omega\rho\rho} \rho_0,
$$

which fixes the value of $g_{\omega\rho\rho}$ for given value of $\bar{\omega}$. Then, we determine the the value of $\bar{\sigma}$ using Equation (22), which is shown by the blue curve in Figure 5. On the other hand, for given values of $\bar{\omega}$ and $\bar{\sigma}$ we check if $\mu_\Delta - E_\Delta > 0$ ($\mu_\Delta = \bar{\rho} = 0$) is satisfied. The green-colored area in Figure 5 shows the region where $\mu_\Delta - E_\Delta > 0$ is satisfied. Since the value of $\bar{\sigma}$ cannot be so small, from this figure, we conclude that the $\Delta$ baryon cannot enter the matter at the normal nuclear matter density for this parameter choice. In the following, we will show our results with several parameter choices. For all cases we have checked that the $\Delta$ baryon does not pile up in the ground state, i.e., no transition to $\Delta$ matter, at or near the normal nuclear matter density.

We now show our results for symmetric matter ($\mu_\rho = 0$). In Figure 6 for a parameter choice of $m_{N0} = 550$ MeV, $m_{\Delta0} = 550$ MeV and $g_{\omega\omega\Delta} = g_{\omega\rho\rho}$, we show the resultant relation between the chemical potential $\mu_B$ and the pressure. Since the stationary condition for $\bar{\sigma}$ is nonlinear, we often have a few solutions for a given $\mu_B$ with fixed parameters. Then, there exist a few values of pressure for a given $\mu_B$. On the straight line AB except the point D, we have a solution corresponding to vacuum, $\bar{\sigma} = f_\pi$ and $\bar{\omega} = 0$, so that the pressure and the density $P_B$ is zero, $P = 0$ and $\rho_B = 0$. At the point D, we have two solutions: one corresponds to the vacuum and another to normal nuclear matter. Along the curve from the point B to D through C, there exists a solution corresponding to the negative pressure with the density smaller than the normal nuclear matter density. In Figure 7, we show the densities of $N(939)$, $\Delta(1232)$ and $N^*(1535)$ against the baryon number density $\rho_B$ given as sum of these densities. This figure shows that only $N(939)$ exists for $\rho_B < \rho_0$. Since the pressure is negative, we call this region the “$N$ liquid-gas coexistence”, which is indicated by the red area in Figure 8.

When the pressure increases along the curve from the point D to G in Figure 6, the nucleon density increases from $\rho_B/\rho_0 = 1$ to about 2.7 as shown in Figure 7. Here the ordinary nuclear matter exists and we call this region the “stable $N$ matter” indicated by the green area in Figure 8.
sponds to $\rho_B/\rho_0 \simeq 2.7$ to 3.4 in Fig. 7. In this region we have another solution having larger pressure for the same value of the chemical potential along the curve GH. Then, the region between G and E is not energetically favored. Along the curve from E to G through F, the $\Delta$ baryon appears in matter as in Fig. 7. This region is also unfavored since we have another solution having the larger pressure for the same value of the chemical potential along DG. We call this region $\rho_B/\rho_0 \simeq 2.7$ - 4 the “coexistence of $\Delta$-N matter and nuclear matter” indicated by the blue area in Fig. 8.

At the point G in Fig. 6, there is a solution corresponding to $\rho_B/\rho_0 \simeq 4$, at which both the nucleon and $\Delta$ exist in matter as shown in Fig. 7. At $\rho_B/\rho_0 \simeq 4.2$ which corresponds to the point H in Fig. 6, $N^*(1535)$ also enters into the matter as shown in Fig. 7. Between the points G and H, there is a stable matter including the $\Delta$ baryon and the nucleon, which we call the “stable $N$-$\Delta$ matter” indicated by cyan region in Fig. 8.

In Fig. 8, we provide a summary of matter constituents on $\rho_B/\rho_0$-$m_{\Delta 0}$ plane with $m_{N0} = 500$ MeV and $g_{\omega \Delta \Delta} = g_{\omega N N}$ in symmetric nuclear matter. The horizontal axis shows the baryon number density scaled by the normal nuclear matter density, and the vertical axis shows the value of $m_{\Delta 0}$ in unit of MeV. The red area indicates the “$N$-liquid-gas coexistence”, the green the “stable $N$ matter”, the blue the “$\Delta$-N matter and $N$ matter”, and the cyan the “stable $N$-$\Delta$ matter”. In the black area, $N^*(1535)$ enters into matter, while in the yellow area, both $N^*(1535)$ and $\Delta(1700)$ enter.

For $m_{\Delta 0}$ = 590 MeV, the “stable $N$-$\Delta$ matter” indicated by cyan area in Fig. 8 appears at $\rho_B/\rho_0 \sim 4$, independently of the value of $m_{\Delta 0}$.

This implies that the first order phase transition occurs for $510 \leq m_{\Delta 0} \leq 590$, and that, for example, the density jumps from $\rho_B/\rho_0 \simeq 1.2$ to 4 for $m_{\Delta 0} = 510$ MeV.

For $m_{\Delta 0} \geq 600$ MeV, the “stable $N$ matter” by green area is smoothly connected to the “stable $N$-$\Delta$ matter” indicated by cyan area. Furthermore, for $\rho_B/\rho_0 \sim 4.2$ - 4.5, $N^*(1535)$ enters into matter, which is a reflection of the partial restoration of chiral symmetry accelerated by $\Delta$ matter. We will investigate this point in detail in the next section.

In Fig. 9, we show matter constituents for different choices of $g_{\omega \Delta \Delta}$ with fixed $m_{N0} = 500$ MeV.

This shows that the following qualitative structures are similar: (1) The onset density of the “coexistence of $\Delta$-N matter and nuclear matter” is larger for larger value of $m_{\Delta 0}$. (2) There exists the “stable $N$-$\Delta$ matter”.

There are several points which depend on the value of $g_{\omega \Delta \Delta}$: (1) The minimum value of $m_{\Delta 0}$, for which the onset density of $\Delta$ matter is larger than the normal nuclear matter density, is larger for smaller $g_{\omega \Delta \Delta}$; $m_{\Delta 0}^{(\text{min})} = 510$, 590 and 720 MeV for $g_{\omega \Delta \Delta} = 1, 0.8, 0.5$, respectively. This feature is consistent with the one obtained in Ref. [12]. (2) For some values of $m_{\Delta 0}$, in high density region, there appear $N^*(1535)$ and/or $\Delta(1700)$, which are chiral partners to $N(929)$ and $\Delta(1232)$, reflecting the partial chiral symmetry restoration. The onset density of the appearance is larger for smaller $g_{\omega \Delta \Delta}$. We will investigate this point further in the next section.

In Fig. 10 we show matter constituents for different choices of $g_{\omega \Delta \Delta}$ with fixed $m_{N0} = 700$ MeV. This shows that the phase structures are similar to the ones for $m_{N0} = 500$ MeV, but the critical densities for the change of matter constituents depend on the choice of parameters. Comparing Figs. 9 and 10 we also observe that larger $m_{N0}$ tends to lower the transition density to the “stable $N$-$\Delta$ matter”. In Ref. [12], it was shown that the onset density of $\Delta$ matter is larger for larger $m_N^2/m_N$, where $m_N^2$ is the effective nucleon mass at $\rho_B = \rho_0$ and $m_N$ is the mass at vacuum. In the present analysis, even when we change the value of the chiral invariant mass $m_{N0}$, the effective nucleon mass at $\rho_B = \rho_0$ is intact. On the other hand, the effective mass for $\rho_B > \rho_0$ is larger for larger $m_{N0}$ as shown in Fig. 2. Then, one may say that the change of the onset density against the change of $m_{N0}$ in the present analysis is consistent with the one

1. One may wonder why we have more $\Delta$ baryons than nucleons in stable $N$-$\Delta$ matter in Fig. 7. In this $N$-$\Delta$ matter, the effective masses of $\Delta$ and $N$ are close to each other as in Eq. (17). Since $\Delta$ has sixteen degrees of freedom (spin 3/2 and isospin 3/2) compared with four (spin 1/2 and isospin 1/2) for nucleon, the $\Delta$ density could be larger than $N$ density.
FIG. 9. Matter constituents for (a) \( g_{\Delta\Delta} = 0.8g_{\omega NN} \) and (b) \( g_{\omega\Delta\Delta} = 0.5g_{\omega NN} \) with fixed value of \( m_{N0} = 500 \) MeV in symmetric matter. The horizontal axis shows the baryon number density scaled by the normal nuclear matter density, and the vertical axis shows the value of \( m_{\Delta0} \). The red area indicates the “\( N \) liquid-gas coexistence”, the green area the “stable \( N \) matter”, the blue area the “coexistence of \( \Delta-N \) matter and nuclear matter”, the cyan area the “stable \( N-\Delta \) matter”. In the black, pink and yellow areas, \( N^*(1535) \), \( \Delta(1700) \), and both of them enter into matter, respectively.

obtained in Ref. 12. We would like to note that neither \( N^*(1535) \) nor \( \Delta(1700) \) appears in density region below \( 6\rho_0 \).

Let us now consider the \( \Delta \) baryon in asymmetric nuclear matter. In the following analysis, we restrict ourselves to study the property for \( g_{\rho\Delta\Delta}/g_0 > 0 \) and \( \mu_I < 0 \). From Eq. 19, the effective mass of \( \Delta^- \) becomes smallest, and Eq. 18 shows that the chemical potential for \( \Delta^- \) is the largest. Then, one can easily see that \( \Delta^- \) enters the matter first among the four \( \Delta \) baryons as in Refs. 13 [15] [18]. In Fig. 11 we show some examples of matter constituents in asymmetric dense matter for \( \mu_I = -60 \) MeV, by taking \( (g_{\rho\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (1, 1), \ (g_{\rho\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (0.8, 1) \) and \( (g_{\rho\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (1, 0.8) \) with fixed \( m_{N0} = 500 \) MeV and \( g_{\rho\Delta\Delta} = g_{\rho NN} \). This shows that, for large values of \( m_{\Delta0} \), the “stable \( \Delta^-N \) matter” indicated by the pink area appears around \( \rho_B/\rho_0 \sim 3-3.5 \), in which \( \Delta^- \) baryon enters into the matter in addition to the nucleon. When the density is increased, \( \Delta^0 \) and \( \Delta^+ \) enter as shown by the cyan and yellow areas, respectively. The values of onset density in this smooth appearance of \( \Delta \) matter are smaller than the ones in symmetric matter. For small values of \( m_{\Delta0} \), on the other hand, there exists the “coexistence of \( \Delta-N \) matter and nuclear matter” indicated by blue area. The values of onset density are similar to the ones in symmetric matter. For any value of \( m_{\Delta0} \), the “stable \( N-\Delta \) matter” appears in the high density region around \( \rho_B/\rho_0 \sim 4 \), where all four \( \Delta \) baryons exist in the matter. The onset density of the “stable \( N-\Delta \)

matter" is smaller than in symmetric matter. From the above, one can say that $\Delta$ matter appears in the smaller density region for asymmetric matter than for symmetric matter. This qualitative feature seems similar to the one shown in Ref. [13].

We should stress that similarly to matter constituents for symmetric matter, we observe the appearance of the chiral partners $N^*(1535)$ and $\Delta(1700)$ for $\rho_B/\rho_0 \sim 4 - 5$, reflecting the partial chiral symmetry restoration. The values of the onset density for asymmetric matter are also smaller than those for symmetric matter.

In Fig. 12, we plot matter constituents for $m_{N0} = 700\text{ MeV}$.

Comparing this with Fig. 11, we observe that the values of the onset density for the appearance of $\Delta$ matter are smaller than for $m_{N0} = 500\text{ MeV}$, similarly to symmetric matter. From Figs. 8-10 and Figs. 11-12 we observe that the phase structures changes significantly from symmetric to asymmetric matter.

FIG. 11. Matter constituents for
(a) $(g_{\omega\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (1, 1)$,
(b) $(g_{\omega\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (0.8, 1)$ and
(c) $(g_{\omega\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (1, 0.8)$ with the fixed value of $m_{N0} = 500\text{ MeV}$ in asymmetric matter with $\mu_I = -60\text{ MeV}$. The horizontal axis shows the baryon number density scaled by the normal nuclear matter density, and the vertical axis shows the value of $m_{\Delta0}$. The red area indicates the "neutron matter", the green area the "stable $N$ matter", the blue area the "coexistence of $\Delta$-$N$ matter and nuclear matter", the black area the "stable $N$-$\Delta$ matter". In the pink area the matter is created from $\Delta$ and the nucleon, in the cyan area there exists the $\Delta^0$ in addition, and the $\Delta^+$ enters the matter in the yellow area. In the gray area, $N^*(1535)$ and/or $\Delta(1700)$ enter into matter.
FIG. 12. Matter constituents for
(a) \((g_{\omega\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (1, 1)\),
(b) \((g_{\omega\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (0.8, 1)\) and
(c) \((g_{\omega\Delta\Delta}/g_{\omega NN}, g_{\rho\Delta\Delta}/g_{\rho NN}) = (1, 0.8)\) with the fixed
value of \(m_{N0} = 700\) MeV in asymmetric matter with
\(\mu_I = -60\) MeV. The horizontal axis shows the baryon
number density scaled by the normal nuclear matter density,
and the vertical axis shows the value of \(m_{\Delta 0}\). The red area
indicates the “neutron matter”, the green area the “stable N
matter”, the blue area the “coexistence of \(\Delta-N\) matter and
nuclear matter”, the black area the “stable \(N-\Delta\) matter”. In
the pink area the matter is created from \(\Delta^-\) and the nucleon,
in the cyan area there exists the \(\Delta^0\) in addition, and the \(\Delta^+\)
enters the matter in the yellow area.
So far we have considered the constituents of matter of our model with the focus on $\Delta$ matter. In this section, we study how the chiral structure is affected by the existence of $\Delta$ matter.

We plot the baryon chemical potential dependence of the chiral condensate $\sigma$ in Fig. 13. Here, the red curve is obtained without $\Delta$ baryons and the blue one is with $\Delta$. The blue curve shows a jump at $\mu_B \sim 970$ MeV, where $\Delta$ enters into the matter. This shows that the partial restoration of chiral symmetry is accelerated by the existence of $\Delta$. Then, at the chemical potential slightly above $970$ MeV, $N^*(1535)$ appears in matter. For the red curve (without $\Delta$), $N^*(1535)$ enters at $\mu_B \sim 1060$ MeV, which was identified with the first order chiral phase transition in Ref. 11. For the blue curve (with $\Delta$), on the other hand, the jump at $\mu_B \sim 970$ MeV is not the chiral phase transition, but implies the appearance of $\Delta$ matter. Actually, the chiral phase transition is cross-over transition which occurs at $\mu_B \sim 1140$ MeV. This implies that the critical chemical potential is increased when the existence of $\Delta$ is considered. In other word, the chiral symmetry restoration itself is delayed by $\Delta$ matter.

In the following, to check that the small bump around $\mu_B \sim 1140$ MeV implies the cross-over chiral phase transition, we investigate the phase structure in the chiral limit where the pion is massless. In Fig. 14, we plot the chemical potential dependence of the chiral condensate $\sigma$ at the chiral limit, where we set $\bar{m} = 0$ in Eq. (21) with all the other parameters unchanged. Here, the red curve is without $\Delta$, and the blue curve is with $\Delta$. The red curve jumps at $\mu_B \sim 970$ MeV, which corresponds to the first order chiral phase transition. The blue curve jumps at $\mu_B \sim 930$ MeV and $\mu_B \sim 1070$. Since the $\sigma$ vanishes at $\mu_B \sim 1070$ MeV, the first order chiral phase transition occurs at this point. The first order transition at $\mu_B \sim 930$ MeV just indicates the appearance of $\Delta$ matter.

To show that the first order chiral phase transition at $\mu_B \sim 1070$ MeV in Fig. 14 is connected to the crossover at $\mu_B \sim 1140$ MeV in Fig. 13, we study the chiral condensate $\sigma$ with changing the $\bar{m}$ in Eq. (21). In Fig 15, we draw the chiral phase structure, where the horizontal axis shows the value of $\mu_B$ and the vertical axis shows the value of $m_\pi$ corresponding to $\bar{m}$. This shows that the first order chiral phase transition (pink solid curve) at $m_\pi = 0$ and $\mu_B \sim 1070$ MeV is actually connected to the cross-over transition at $m_\pi = 140$ MeV and $\mu_B \sim 1140$ MeV. Furthermore, two critical chemical potentials for the ordinary liquid-gas phase transition and the appearance of
both the effective masses of $N(939)$ and $\Delta(1232)$ suddenly change their values associated with the appearance of the $\Delta$ baryons in matter.

We plot the density dependence of the pressure in symmetric matter in Fig. 18 by the blue curve. For comparison, we show the pressure obtained by assuming that only nucleons exist in matter by the red curve. The region between two vertical lines at $p_B/p_0 \sim 2.7$ and 4 is in the "coexistence of $\Delta$-N matter and nuclear matter." Figure 18 shows that the pressure indicated by the blue curve suddenly changes to decrease around $p_B/p_0 = 3.5$, where the $\Delta$ baryons enter the matter. Furthermore, $N^*(1535)$ enters into matter slightly above $p_B/p_0 \sim 4$, reflecting the partial chiral symmetry restoration. For red curve, the density region of $p_B/p_0 \simeq 4.3 - 4.7$ is unstable since $N^*(1535)$ enters into matter in this region.

We plot the density dependence of the pressure in asymmetric matter in Fig. 19. The region between two vertical lines at $p_B/p_0 \sim 2.7$ and 4 is in the "coexistence of $\Delta$-N matter and nuclear matter." In this region there is a jump around $p_B/p_0 \sim 3.3$ where $\Delta$ enter the matter. The density dependence of pressure in Figs. 18 and 19 show that the pressure of $N-\Delta$ matter is smaller than that of the ordinary $N$ matter, and that the $\Delta$ softens the equation of state as expected.

In Fig. 20 we plot the density dependence of the symmetry energy. With fixing $m_{N_0} = 500$ MeV, $g_{\omega N N} = g_{\rho N N}$ and $g_{\rho \Delta \Delta} = g_{\omega \Delta N}$, we use three inputs for the chiral invariant mass of the $\Delta$: $m_{\Delta_0} = 530$ MeV (red curve), 550 MeV (green curve) and 570 MeV (blue curve). The region between two red vertical lines at $p_B/p_0 \sim 2$ and 4 is in the "coexistence of $\Delta$-N matter and nuclear

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure16}
\caption{Chemical potential dependence of the chiral condensate $\sigma$ in asymmetric matter. The red curve shows the one with assuming no $\Delta$ in matter. Horizontal axis shows the baryon number chemical potential in unit of MeV, while vertical axis shows the value of the chiral condensate $\sigma$ in unit of MeV. The parameters are chosen as $m_{N_0} = 500$ MeV, $m_{\Delta_0} = 550$ MeV and $g_{\omega \Delta \Delta} = g_{\omega NN}$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure17}
\caption{Density dependence of the effective masses of $N(939)$ (red curve) and $\Delta(1232)$ (blue curve) in symmetric matter for $(m_{N_0}, m_{\Delta_0}) = (500, 550)$ MeV. Horizontal axis shows the baryon number density $\rho_B$, and the vertical axis shows the value of masses in unit of MeV. Two vertical lines at $p_B/p_0 \sim 2.7$ and 4 indicate that the region between them is in the "coexistence of $\Delta$-N matter and nuclear matter".}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure18}
\caption{Density dependence of pressure in symmetric matter (green curve) for $(m_{N_0}, m_{\Delta_0}) = (500, 550)$ MeV. The red curve shows the pressure when we assume no $\Delta$ baryons in matter. The horizontal axis shows the baryon number density $\rho_B$ normalized by the normal nuclear matter density $\rho_0$, and the vertical axis shows the pressure in unit of MeV-fm$^{-3}$. The parameters are chosen as $m_{N_0} = 500$ MeV, $m_{\Delta_0} = 550$ MeV and $g_{\omega \Delta \Delta} = g_{\omega NN}$. Two green vertical lines at $p_B/p_0 \sim 2.7$ and 4 indicate that the region between them is in the "coexistence of $\Delta$-N matter and nuclear matter". For red curve, the density region of $p_B/p_0 \simeq 4.3 - 4.7$ is unstable since $N^*(1535)$ enters into matter in this region.}
\end{figure}
FIG. 19. Density dependence of pressure in asymmetric matter of $\mu_I = -60\, \text{MeV}$ (blue curve). The red curve shows the pressure when we assume no $\Delta$ baryons in matter. The horizontal axis shows the baryon number density $\rho_B$ normalized by the normal nuclear matter density $\rho_0$, and the vertical axis shows the pressure in unit of $\text{MeV} \cdot \text{fm}^{-3}$. The parameters are chosen as $m_{N0} = 500\, \text{MeV}$, $m_{\Delta 0} = 550\, \text{MeV}$ and $g_{\omega \Delta \Delta} = g_{\omega NN}$. Two vertical lines at $\rho_B/\rho_0 \sim 2.7$ and 4 indicate that the region between them is in the “coexistence of $\Delta$-N matter and nuclear matter”. For red curve, the density region of $\rho_B/\rho_0 \sim 4.3 - 4.7$ is unstable since $N^*(1535)$ enters into matter in this region.

FIG. 20. Density dependence of symmetry energy. Two red vertical lines at $\rho_B/\rho_0 \sim 2$ and 4 indicate that the region between them is in the “coexistence of $\Delta$-N matter and nuclear matter” for $m_{\Delta 0} = 530\, \text{MeV}$. Two green vertical lines at $\rho_B/\rho_0 \sim 2.7$ and 4 and two blue lines at $\rho_B/\rho_0 \sim 3.3$ and 4 indicate the coexistence region for $m_{\Delta 0} = 550\, \text{MeV}$ and $m_{\Delta 0} = 570\, \text{MeV}$, respectively.

VI. SUMMARY AND DISCUSSION

In the framework of an extended parity doublet model with $\Delta$ baryons, we studied the transition from nuclear matter to $\Delta$ matter in terrestrial dense matter that can be created in heavy ion collisions (HIC) with a few hundreds $\text{A MeV}$. We also investigated the effects of the chiral invariant mass and the isospin asymmetry on the transition to $\Delta$ matter and on some physical quantities such as the nuclear symmetry energy.

We explored matter constituents of cold dense matter and showed that, in symmetric matter, $\Delta$ enters into matter at $\rho_B/\rho_0 \sim 1 - 4\rho_0$, and that stable $\Delta$ matter exists for $\rho_B \gtrsim 4\rho_0$. We observed in symmetric dense matter that larger $m_{N0}$ tends to lower the transition density to the “stable N-$\Delta$ matter”. We also showed that the matter constituents change significantly with the finite isospin chemical potentials. In asymmetric matter, $\Delta^-$ appears first similarly to the ones obtained in the neutron star analyses [17][18].

We next investigated the chiral structure by studying the chemical potential dependence of the chiral condensate $\sigma$. We showed that the chiral condensate rapidly decreases when $\Delta$ enters into matter, implying that the partial chiral symmetry restoration is accelerated by $\Delta$ matter. When the chiral invariant mass of nucleon is $500\, \text{MeV}$, as a result of this acceleration, $N^*(1535)$, which is the chiral partner to $N(939)$, enters into matter at the density slightly above the onset density of stable $\Delta$ matter, signaling the partial chiral symmetry restoration. We also showed that the chiral symmetry restoration itself is delayed by $\Delta$ matter.

We finally calculated the effective masses, pressure and symmetry energy. The density-dependences of effective masses and pressure change drastically around the onset density of $\Delta$ matter, which are similar to the ones shown in Refs. [14][15][18]. We also observed a sudden change of symmetry energy around the onset density.

In the present study we do not consider any possibility of having hyperon matter because strangeness is conserved with strong interactions and terrestrial dense matter from heavy ion collisions at a few hundreds $\text{A MeV}$ sustains much short compared to the time scale of weak interactions.

It will be interesting to see how the observations made in this study such as the transition to $\Delta$ matter affect the observables in heavy ion collisions such as neutron-proton collective flows and $\pi^+ / \pi^-$ ratio at low and/or intermediate energy in a transport model simulation, which is relegated to our future study. Also, the fate of pion condensation with $\Delta$ matter in heavy ion collisions will be investigated. For the earlier discussion on the role of the $\Delta$ baryons in pion condensation, we refer to [24][25].
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