Abstract

The role of monopoles in quenched compact QED has been studied by measuring the cluster susceptibility and the order parameter \( n_{\text{max}}/n_{\text{tot}} \) previously introduced by Hands and Wensley in the study of the percolation transition observed in non-compact QED. A correlation between these parameters and the energy (action) at the phase transition has been observed. We conclude that the order parameter \( n_{\text{max}}/n_{\text{tot}} \) is a sensitive probe for studying the phase transition of pure gauge compact QED.
Some time ago Polyakov[1] and Banks, Myerson and Kogut[2] showed that lattice (compact) QED can be written as a theory of photons and monopoles. This theory exhibits a unique confining phase for $D = 3$ and a two phase structure for $D = 4$. The monopoles play a central role in the explanation of the phase-structure. In fact, they produce ”disorder” and give rise to confinement via the dual superconductor mechanism [3], [4]. That is, the gauge vacuum behaves as a ‘magnetic’ superconductor (a monopole condensate) which confines the electric field into flux tubes (a dual Meissner effect).

The first numerical simulations studying the effect of monopoles in pure gauge lattice QED were performed by DeGrand and Toussaint[5]. They searched for monopoles by using Gauss’s law, i.e. measuring the total magnetic flux emanating from a closed surface on the lattice. In $D = 3$ monopoles are pointlike excitations while in $D = 4$ they are closed loops. The density of monopole loops in four dimensions was measured as a function of the coupling $\beta$ and a fall-off of the density across the phase transition was observed. This model was also studied by Barber, Shrock and Scharer [6]. They simulated the pure gauge compact $U(1)$ theory but used techniques to suppress monopoles. They concluded that without monopoles there is no phase transition. This result has been confirmed by more recent numerical simulations [7]. However, it has proved difficult to establish the order of the phase transition: initial measurements on small lattices indicated a second order transition [8] but simulations on larger lattices which collected greater statistics [9], [10] favor a first order transition.

Gupta, Novotny and Cordery[11] confirmed that loops of monopole current are the mechanism driving the phase transition, but conjectured that the apparent discontinuities in observables might be due to finite-size effects. In the same spirit, Grady[12] pointed out the possibility that the ”latent heat” in the phase transition could be a spurious topological effect due to the imposition of periodic boundary conditions. This idea is supported by the recent simulations of the compact action on ”closed topology” lattices by Lang and Neuhaus [13]. In contrast to studies of compact QED on hypercubic lattices with periodic boundary conditions, they found no metastability signals at the phase transition on the lattices with the topology of a sphere. Numerical simulations with fixed boundary conditions also confirm the continuity of the transition [14]. Additionally, in a recent paper, Rebbi et al.[15] have studied the phase structure of pure compact $U(1)$ lattice action supplemented by a monopole term. They have found that the strength of the first order tran-
sition decreases with the weight of the added term in such a way that the transition ultimately gets of second order.

Lattice monopoles can also occur in simpler models. For example, Kogut, Kocić and Hands\cite{16} showed that in pure gauge non-compact lattice QED monopoles percolate and satisfy hyperscaling relations characteristic of an authentic second order phase transition even though the underlying lattice field theory is trivial. The non-local character of the monopole observables is an essential ingredient in having a phase transition in an otherwise trivial theory. In the non-compact case dynamical considerations are subsumed by ”geometrical” considerations. As this case illustrates, percolation is not necessarily connected with condensation or with other bulk, dynamical properties of the theory. Similar phenomena are common in statistical mechanics. For example, many spin glass models are based on free (Gaussian) dynamics, but they experience glassy transitions in appropriate non-local observables which are well studied in laboratory experiments. In addition, there are models where percolation is not related to bulk transitions of primary interest. For example, the 3d Ising model has no phase transitions at non-zero magnetic field despite the presence of percolation thresholds. In addition, a recent study of scalar lattice QED with noncompact gauge fields\cite{17} showed that monopole percolation and the bulk Higgs-Coulomb phase transition occur at separate couplings and are not directly related.

With the aim of obtaining more knowledge about the role of monopole percolation and condensation phenomena in lattice quantum electrodynamics, we have considered the original pure gauge compact U(1) case, studying the behaviour of the cluster susceptibility already introduced by Hands and Wensley\cite{18}. Our main result is that in compact pure gauge QED, the percolation threshold occurs at the same point as the deconfining phase transition, i.e. the point of monopole condensation. Moreover, the $n_{\text{max}}/n_{\text{tot}}$ order parameter used to characterize monopole percolation proves to be an excellent order parameter to study the location of the apparent first order phase transition. The quantity $n_{\text{max}}/n_{\text{tot}}$ has been advocated recently in a study of the Villain model of QED \cite{19}, and, in a different context, in three-dimensional quantum gravity coupled to gauge fields\cite{20}.

The lattice action used in our simulation is the original Wilson (compact)
action

\[ S_{\text{gauge}} = \sum_{n\mu\nu} \cos(\theta_\mu(n) + \theta_\nu(n + \mu) - \theta_\mu(n + \nu) - \theta_\nu(n)) = \sum_{n\mu\nu} \cos \Theta_{\mu\nu}(n). \]  

(1)

Following \cite{[5]} we can separate the plaquette angle \( \Theta_{\mu\nu} \) into two pieces: physical fluctuations which lie in the range \(-\pi\) to \(\pi\) and Dirac strings which carry \(2\pi\) units of flux. Introducing an electric charge \(e\) we define an integer-valued Dirac String by,

\[ e\Theta_{\mu\nu} = e\tilde{\Theta}_{\mu\nu}(\tilde{n}) + 2\pi S_{\mu\nu}, \]  

(2)

where the integer \(S_{\mu\nu}\) determines the strength of the string threading the plaquette and \(e\Theta_{\mu\nu}\) represents physical fluctuations. The integer-valued monopole current \(m_{\mu}(n)\) defined on links of the dual lattice, is then

\[ m_{\mu}(\tilde{n}) = \frac{1}{2} \epsilon_{\mu\nu\kappa\lambda} \Delta^+_{\nu} S_{\kappa\lambda}(n + \hat{\mu}), \]  

(3)

where \(\Delta^+\) is the forward lattice difference operator, and \(m_{\mu}\) is the oriented sum of the \(S_{\mu\nu}\) around the faces of an elementary cube. This definition, which is gauge-invariant, implies the conservation law \(\Delta^- m_{\mu}(\tilde{n}) = 0\) which means that monopole world lines form closed loops.

Analogous to ref.\cite{[16]} which considered the non-compact U(1) model, we have used the constructions and concepts of percolation in order to clarify the confinement/deconfinement phase transition in the compact U(1) model. We also have borrowed the idea of a connected cluster of monopoles on the dual lattice: one counts the number of dual sites joined into clusters by monopole line elements. We also have used the order parameter \(M = \frac{n_{\text{max}}}{n_{\text{tot}}}\) where \(n_{\text{max}}\) is the number of sites in the largest cluster and \(n_{\text{tot}}\) is the total number of connected sites. Its associated susceptibility reads,

\[ \chi = \frac{\langle \sum_{n_{\text{min}}}^{n_{\text{max}}} g_n n^2 - n_{\text{max}}^2 \rangle}{n_{\text{tot}}}, \]  

(4)

where \(n\) labels the size of a cluster occurring \(g_n\) times on the dual lattice. In general \(n_{\text{min}} = 2\), but for monopoles \(n_{\text{min}} = 4\) because of the conservation law.

We have applied the standard Metropolis algorithm to simulate the compact U(1) lattice gauge theory with standard periodic boundary conditions.
As it was mentioned, this system exhibits a first order phase transition. The lattice sizes used have been from $8^4$ to $14^4$. We have made measurements of the monopole observables near the phase transition which occurs very close to $\beta = 1.0$. Two different kinds of measurements have been done, first some simple thermal cycles on the smaller lattice sizes in order to detect hysteresis phenomena associated with an apparent first order phase transition. Secondly, a more detailed analysis on larger lattices in order to confirm the persistence of the discontinuity in the values of the observables.

In Fig. 1 we show the results of a simultaneous measurement of the standard monopole density $\rho$ and the monopole percolation parameter $M = n_{\text{max}}/n_{\text{tot}}$ on a $12^4$ lattice. Each point is a measurement over 10,000 configurations after discarding 5,000 for thermalization. Note that although both observables show a fall-off after the transition, the jump in the $M$ parameter looks particularly abrupt.

The analysis of the cluster susceptibility $\chi$ is shown in Fig. 2, for lattice sizes of 6, 8, 10, 12 and 14 lattice units. The measurements are over sets of 30,000 iterations at each point after a thermalization of 5,000 iterations. From this graph one sees a peak in the susceptibility just beyond the phase transition point. This singular behavior does not in itself indicate the order of the transition. One must measure the volume dependence of the peak height to address this point. In the case of the pure gauge non compact U(1) theory, a true phase transition was detected, and critical indices were measured sufficiently accurately that the transition could be classified as four dimensional bond percolation. A finite size analysis of our compact case shows that the peak grows with a different power exponent than the non compact case but a truly precise determination of the finite size dependence was not made. Higher statistics and measurements on a finer grid of $\beta$ values would be necessary for that. Our conclusion is that both monopole parameters $n_{\text{max}}/n_{\text{tot}}$ and $\chi$ indicate an apparent first order phase transition, just as the internal energy does.

To confirm this scenario we show in Fig. 3 the evolution of the internal energy, the cluster susceptibility $\chi$ and the $n_{\text{max}}/n_{\text{tot}}$ order parameter in "Monte Carlo time", i.e. the iteration number, in a set of measurements for a coupling just beyond the phase transition on a $10^4$ lattice. The "tunneling" between the two phases is very clear for all three observables at the same point. This phenomena is characteristic of a first order phase transition.

As was pointed out recently in [15], we have observed that the tunneling
between the phases is strongly suppressed. This fact difficult the study of the phase transition on larger lattices. Nevertheless we have confirmed the behavior of the monopole percolation parameters on a $14^4$ lattice. Since our main objective is to study the behaviour of monopoles over the phase transition, we have not applied any iterative reweighting procedure to locate the phase transition point. In Fig. 4 we show the evolution of the monopole susceptibility and the $n_{\text{max}}/n_{\text{tot}}$ parameter for $\beta = 1.01040$ (value very close to the critical point, but still in the strong coupling phase) starting from an ordered initial configuration. These results show clearly the change of the phase, indicating that the parameter $n_{\text{max}}/n_{\text{tot}}$ exhibits, greatly amplified, the characteristic discontinuity of a first order transition.

In conclusion, the analysis performed here shows that the observable $n_{\text{max}}/n_{\text{tot}}$, defined by Hands and Wensley [18] in the study of the percolation phenomena of non-compact lattice electrodynamics, proves to be a useful order parameter for the study of the compact U(1) model phase transition. This observable has definite advantages when compared with other order parameters [21], such as the monopole density $\rho$, since it has a larger ”discontinuity” separating the two phases. Finally, these results show that in the pure gauge compact case the percolation threshold occurs at the same point as the bulk phase transition where monopole condensation occurs.

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Figure captions

1. Simultaneous measurements of the monopole density $\rho$ (dashes) and the monopole percolation $M = n_{\text{max}}/n_{\text{tot}}$ parameter (solid) on a $12^4$ lattice. The statistics is 10,000 iterations at each point after discarding 5,000 for thermalization.

2. Measurements of the cluster susceptibility $\chi$ on different lattice sizes (squares $6^4$, diamonds $8^4$, crosses $10^4$, circles $12^4$ and stars $14^4$). Lines are only to guide the eye.

3. Simultaneous measurement of the Internal Energy $E$, the cluster susceptibility $\chi$ and the monopole order parameter $M = n_{\text{max}}/n_{\text{tot}}$ on a single run at $\beta = 1.02$ on a $10^4$ lattice. Each point is an average over 20 consecutive iterations. The total statistics is 25,000 iterations after discarding 5000 for thermalization.

4. Results of the measurement of the monopole susceptibility and the $M = n_{\text{max}}/n_{\text{tot}}$ parameter from a simulation on a $14^4$ lattice at $\beta = 1.01040$. 

