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A Model for Fermion Mass Hierarchies and Mixings

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We present a model which contains three Abelian symmetries beyond the standard model. One is anomalous à la Green-Schwarz, and family-independent; the other two are family symmetries. All are broken at a large scale by stringy effects. The model is predictive in the neutrino sector: large mixing between $\nu_\mu - \nu_\tau$, and $O(\lambda^3) \nu_e - \nu_\mu$ mixings. Its natural cut-off is the gauge unification scale, orders of magnitude below the perturbative string cutoﬀ.

1 Introduction

Simplicity in the minimal supersymmetric standard model appears only at the scale at which its gauge couplings unify, $M_U$, and where its Yukawa couplings display two different hierarchies: interfamily hierarchy which relates particles of different families in each charge and color sector, as well as intrafamily hierarchy which relates particles of the same family. They have different theoretical origins. The former is due to non-anomalous family symmetries, the latter to family-independent anomalous symmetries.

Our general framework is that of a low-energy effective theory with cut-off, $M_U$. Anomalous gauge symmetries can exist in this framework as long as there is at cut-off an interaction which compensates for the Noether anomalies induced in the Lagrangian. String theories are such an example. The Green-Schwarz mechanism provides a dimension-five interaction term, whose structure demands a specific pattern among the anomaly coefficients, namely that the combinations

$$\alpha_{\text{color}} C_{\text{color}} = \alpha_{\text{weak}} C_{\text{weak}} = \alpha_Y C_Y = \cdots = \alpha_i C_i ,$$

be universal, where $\alpha_i$ is the coupling constant of the gauge group $i$. Here

$$C_i = \text{Tr}(X G_i G_i) ,$$

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are the anomaly coefficients which appear in the divergence of the anomalous $X$-current. At $M_U$, these imply for the standard model

$$\alpha_{\text{color}} = \alpha_{\text{weak}} \rightarrow C_{\text{color}} = C_{\text{weak}} ,$$

as well as

$$\tan^2 \theta_w = \frac{\alpha_{\text{weak}}}{\alpha_Y} = \frac{C_Y}{C_{\text{weak}}} .$$

This relation relates an ultraviolet parameter, the Weinberg angle, to infrared properties, since the anomaly coefficients are determined from the couplings of the massless particles in the theory.

String theories with an anomalous Green-Schwarz $U(1)$, generate a Fayet-Iliopoulos $D$-term, which triggers the breaking of the anomalous symmetry and in general others at a scale that is computably lower than the cut-off. These models therefore generate a small expansion parameter, the ratio of the scale at which the anomalous symmetry is broken to the cut-off.

The phenomenological requirement that neither supersymmetry nor standard model gauge symmetries be broken at a high scale severely restricts any theory that contains an anomalous $U(1)$. In particular, one can relate the absence of dangerous flat directions to the presence of certain interaction terms in the superpotential. Remarkably, these are compatible with the invariants of the so-called minimal supersymmetric standard model. For example, the seesaw mechanism was shown to imply the absence of $R$-parity violating interactions.

Moreover, the anomalous $U(1)$ offers a natural mechanism for supersymmetry breaking, once one assumes dilaton stabilization, and a non-Abelian gauge interaction other than QCD.

2 The Framework

We consider models which have a gauge structure broken in two sectors: a visible sector, and a hidden sector, linked by the anomalous symmetry and possibly other Abelian symmetries (as well as gravity).

$$G_{\text{SM}} \times U(1)_X \times U(1)_{Y^{(1)}} \cdots \times U(1)_{Y^{(M)}} \times G_{\text{hidden}} ,$$

where $G_{\text{hidden}}$ is the hidden gauge group, and $G_{\text{SM}}$ is the standard model gauge group. Only $X$, is anomalous in the sense of Green-Schwarz. $X$, $Y^{(a)}$ are spontaneously broken at a high scale by the Fayet-Iliopoulos term generated by the dilaton vacuum. This DSW vacuum is required by phenomenology to preserve both supersymmetry and the standard model symmetries.
The application of the Green-Schwarz structure to the standard model is consistent with many of its phenomenological patterns. There are also intricate anomaly requirements as not all anomalies are accounted for by the Green-Schwarz mechanism. The anomalies are of the following type:

- The first involve only standard-model gauge groups $G_{SM}$, with coefficients $(G_{SM}G_{SM}G_{SM})$, which cancel for each chiral family and for vector-like matter. Also the hypercharge mixed gravitational anomaly $(YT T)$ vanishes.

- The second type is where the new symmetries appear linearly, of the type $(Y^{(i)}G_{SM}G_{SM})$. If we assume that the $Y^{(i)}$ are traceless over the three chiral families, these vanish over the three families of fermions with standard-model charges. Hence they must vanish on the Higgs fields: with $G_{SM} = SU(2)$, it implies the Higgs pair is vector-like with respect to the $Y^{(i)}$. It also follows that the mixed gravitational anomalies $(Y^{(i)}TT)$ are zero over the fields with standard model quantum numbers.

- The third type involve the new symmetries quadratically, of the form $(G_{SM}Y^{(i)}Y^{(j)})$. These vanish by group theory except for those of the form $(YY^{(i)}Y^{(j)})$. In general two types of fermions contribute: the three chiral families and standard-model vector-like pairs.

- The remaining vanishing anomalies involve the anomalous charge $X$.

- With $X$ family-independent, and $Y^{(i)}$ family-traceless, the vanishing of the $(XY Y^{(i)})$ anomaly coefficients over the three families is assured: so they must also vanish over the Higgs pair. This means that $X$ is also vector-like on the Higgs pair. Hence the standard-model invariant $H_uH_d$ (the $\mu$ term) has zero $X$ and $Y^{(i)}$ charges. In string theory, mass terms do not appear in the superpotential, but only in the Kähler potential. After supersymmetry-breaking, this generates an effective $\mu$-term, of weak strength, as suggested by Giudice and Masiero.

- The coefficients $(XY Y^{(i)}Y^{(j)})$, $i \neq j$. Since standard-model singlets can contribute to these anomalies, we expect cancellation to come about through a combination of hidden sector and singlet fields.

- The coefficient $(XX Y^{(i)})$. This imposes an important constraint on the $X$ charges on the chiral families.

- The coefficients $(XX Y^{(i)})$; with family-traceless symmetries, they vanish over the three families of fermions with standard-model charges, but contributions are expected from other sectors of the theory.
In the standard model, we have the three anomalies associated with its three gauge groups,

\[ C_{\text{color}} = (XSU(3)SU(3)) \ ; \ C_{\text{weak}} = (XSU(2)SU(2)) \ ; \ C_Y = (XYY) \ , \]  

where () stands for the trace. They can be expressed in terms of the X-charges of the invariants of the MSSM

\[ C_{\text{color}} = \frac{1}{2} \sum_i \left[ X_{ii}^{[u]} + X_{ii}^{[d]} \right] - 3X^{[\mu]} \ , \]  

\[ C_Y + C_{\text{weak}} - \frac{8}{3} C_{\text{color}} = 2 \sum_i \left[ X_{ii}^{[e]} - X_{ii}^{[d]} \right] + 2X^{[\mu]} \ , \]

where \( X_{ij}^{[u,d,e]} \) are the X-charges of \( Q_i \bar{u}_j H_u \), \( Q_i \bar{d}_j H_d \), \( L_i \bar{\nu}_j H_d \) respectively, and finally \( X^{[\mu]} \) that of the \( \mu \)-term \( H_u H_d \), where \( i,j \) are the family indices.

A top quark Yukawa mass coupling at tree-level, we have \( X_{33}^{[u]} = X_{33}^{[d]} = 0 \).

This implies that the X-charge of the down quark Yukawa is proportional to the color anomaly, and thus cannot vanish: the down Yukawa is necessarily smaller than the top Yukawa, leading to the suppression of \( m_b \) over \( m_t \), after electroweak breaking! The presence of the color anomaly implies suppression of the bottom mass relative to the top mass. With \( C_{\text{color}} = C_{\text{weak}} \), the second anomaly equation becomes

\[ C_Y - \frac{5}{3} C_{\text{weak}} = 6 \left[ X^{[e]} - X^{[d]} \right] \ , \]

stating that the relative suppression of the down to the charged lepton sector is proportional to the difference of two anomaly coefficients. Since \( m_b = m_\tau \) near \( M_U \), this implies that

\[ \frac{3}{5} = \frac{C_{\text{weak}}}{C_Y} = \tan^2 \theta_w \ . \]

This happens exactly at the phenomenologically preferred value of the Weinberg angle: the \( b - \tau \) unification is related to the value of the Weinberg angle.

3 A Three-Family Model

We can see how some of the features we have just discussed lead to phenomenological consequences in the context of a three-family model, with three Abelian symmetries broken in the DSW vacuum. The matter content of
the theory is inspired by $E_6$, which contains two Abelian symmetries outside of the standard model: the two $U(1)$, $V'$, $V$, appear in the embeddings

$$E_6 \subset SO(10) \times U(1)_{V'}, \quad SO(10) \subset SU(5) \times U(1)_{V}.$$  \hspace{1cm} (11)

Over the three chiral families, the two non-anomalous symmetries are

$$Y^{(1)} = \frac{1}{5} (2Y + V)(2, -1, -1)$$  \hspace{1cm} (12)

$$Y^{(2)} = \frac{1}{4} (V + 3V')(1, 0, -1),$$  \hspace{1cm} (13)

and $Y^{(1,2)}$ are family-traceless. Since $\text{Tr}(YY'') = 0$, there is no appreciable kinetic mixing between the non-anomalous $U(1)$s. The $X$ charges on the three chiral families in the $27$ are of the form

$$X = (\alpha + \beta V + \gamma V')(1, 1, 1),$$  \hspace{1cm} (14)

where $\alpha$, $\beta$, $\gamma$ are expressed in terms of the $X$-charges of $\bar{\mathbf{5}}$, $(-3/2)$, that of $Q_d \bar{H}_d$ $(-3)$, and that of the vector-like pair mass term $EE$ $(-3)$.

The matter content of this model is the smallest that reproduces the observed quark and lepton hierarchy while cancelling the anomalies associated with the extra gauge symmetries:

- Three chiral families each with the quantum numbers of a $27$ of $E_6$. This means three chiral families of the standard model, $Q_i$, $u_i$, $d_i$, $L_i$, and $\bar{\tau}_i$, together with three right-handed neutrinos $\bar{\nu}_i$, three vector-like pairs denoted by $E_i + \bar{D}_i$ and $E_i + D_i$, with the quantum numbers of the $\mathbf{5} + \mathbf{\bar{5}}$ of $SU(5)$, and finally three real singlets $S_i$.

- One standard-model vector-like pair of Higgs weak doublets.

- Chiral fields that are needed to break the three extra $U(1)$ symmetries in the DSW vacuum. We denote these fields by $\theta_a$. In our minimal model with three symmetries that break through the FI term, we just take $a = 1, 2, 3$. The $\theta$ sector is necessarily anomalous.

- Hidden sector gauge interactions and their matter, and other standard model singlet fields.

Finally, the charges of the three $\theta$ fields are given in terms of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix},$$  \hspace{1cm} (15)
showing that all three fields acquire the same vacuum value. Below we display some noteworthy features of this model.

### 3.1 Quark and Charged Lepton Masses

The Yukawa interactions in the charge $2/3$ quark sector are generated by operators of the form

$$Q_i \bar{u}_j H_u \left( \frac{\theta_1}{M} \right)^{n_{1ij}} \left( \frac{\theta_2}{M} \right)^{n_{2ij}} \left( \frac{\theta_3}{M} \right)^{n_{3ij}},$$

in which the exponents must be positive integers or zero. Assuming that only the top quark Yukawa coupling appears at tree-level, a straightforward computation of their charges yields in the DSW vacuum the charge $2/3$ and $-1/3$ Yukawa matrices

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim \lambda^3 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}. \quad (17)$$

where $\lambda = |\theta_a|/M$ is the common expansion parameter, and we have used $X^{(d)} = -3$. Diagonalization of the two Yukawa matrices yields the CKM matrix

$$U_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \quad (18)$$

This shows the expansion parameter to be of the same order of magnitude as the Cabibbo angle $\lambda_c$. The eigenvalues of these matrices reproduce the geometric interfamily hierarchy for quarks of both charges

$$\frac{m_u}{m_t} \sim \lambda_c^8, \quad \frac{m_c}{m_t} \sim \lambda_c^4. \quad (19)$$

$$\frac{m_d}{m_b} \sim \lambda_c^4, \quad \frac{m_s}{m_b} \sim \lambda_c^2. \quad (20)$$

while the quark intrafamily hierarchy is given by

$$\frac{m_b}{m_t} = \cot \beta \lambda_c^3. \quad (21)$$

implying the relative suppression of the bottom to top quark masses, without large $\tan \beta$.  

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The analysis in the charged lepton sector is similar. Since \( X[e] = -3 \), there are supersymmetric zeros in the (21) and (31) position, yielding

\[
Y[e] \sim \lambda^3 \left( \begin{array}{ccc}
\lambda_4 & \lambda_5 & \lambda_3 \\
0 & \lambda_2 & 1 \\
0 & \lambda_2 & 1
\end{array} \right) .
\]

(22)

Its diagonalization yields the lepton interfamily hierarchy

\[
\frac{m_e}{m_\tau} \sim \lambda^4 ; \quad \frac{m_\mu}{m_\tau} \sim \lambda^2 .
\]

(23)

Our choice of \( X \) insures \( X[d] = X[e] \), which yields at cut-off

\[
\frac{m_b}{m_\tau} \sim 1 ; \quad \sin^2 \theta_w = \frac{3}{8} \leftrightarrow X[d] = X[e] .
\]

(24)

A remarkable feature of this type of model that both inter- and intra-family hierarchies are linked not only with one another but with the value of the Weinberg angle. In addition, the model predicts a natural suppression of \( m_b/m_\tau \), which suggests that \( \tan \beta \) is of order one.

3.2 Neutrino Masses

Neutrino masses are naturally generated by the seesaw mechanism if the three right-handed neutrinos \( \nu_i \) acquire a Majorana mass in the DSW vacuum. The flat direction analysis indicates that their \( X \)-charges must be negative half-odd integers, with \( X_{\nu} = -3/2 \). One finds three massive right-handed neutrinos with masses

\[
m_{\nu_e} \sim M\lambda^3_e ; \quad m_{\nu_\mu} \sim m_{\nu_\tau} \sim M\lambda^7_e .
\]

(25)

In our model, \( X(L_iH_u\bar{N}_j) \equiv X[v] = 0 \). The seesaw mechanism yields the light neutrino Yukawa matrix \((\nu_u \equiv \langle H_0^u \rangle)\)

\[
\frac{v_u^2}{M\lambda^3_e} \left( \begin{array}{ccc}
\lambda_6 & \lambda_5 & \lambda_3 \\
\lambda_3 & 1 & 1 \\
\lambda_3 & 1 & 1
\end{array} \right) .
\]

(26)

A characteristic of the seesaw mechanism is that the charges of the \( \bar{N}_i \) do not enter in the determination of these orders of magnitude as long as there are no massless right-handed neutrinos. Hence the structure of the neutrino mass matrix depends only on the charges of the invariants \( L_iH_u \), already fixed by phenomenology and anomaly cancellation. In particular, the family structure
is determined by the lepton doublets $L_i$. In our model, since $L_2$ and $L_3$ have the same charges, we have no flavor distinction between the neutrinos of the second and third family. Its diagonalization yields the neutrino mixing matrix:

$$U_{\text{MNS}} = \begin{pmatrix}
1 & \lambda^3_c & \lambda^3_c \\
\lambda^2_c & 1 & 1 \\
\lambda^3_c & 1 & 1
\end{pmatrix},$$

so that the mixing of the electron neutrino is small, of the order of $\lambda^3_c$, while the mixing between the $\mu$ and $\tau$ neutrinos is of order one. Remarkably enough, this mixing pattern is precisely the one suggested by the non-adiabatic MSW explanation of the solar neutrino deficit and by the oscillation interpretation of the reported anomaly in atmospheric neutrino fluxes (which has been recently confirmed by the Super-Kamiokande and Soudan collaborations).

A naive order of magnitude diagonalization gives a $\mu$ and $\tau$ neutrinos of comparable masses, and a much lighter electron neutrino:

$$m_{\nu_e} \sim m_0 \lambda^6_c; \quad m_{\nu_\mu}, m_{\nu_\tau} \sim m_0; \quad m_0 = \frac{\nu^2}{M\lambda^6_c},$$

At first sight, this spectrum is not compatible with a simultaneous explanation of the solar and atmospheric neutrino problems, which requires a hierarchy between $m_{\nu_\mu}$ and $m_{\nu_\tau}$. However, the estimates (28) are too crude: since the $(2,2)$, $(2,3)$ and $(3,3)$ entries of the mass matrix all have the same order of magnitude, the prefactors that multiply the powers of $\lambda_c$ in (26) can spoil the naive determination of the mass eigenvalues. To take this effect into account, we rewrite the neutrino mass matrix, expressed in the basis of charged lepton mass eigenstates, as:

$$m_0 \begin{pmatrix}
a \lambda^6_c & b \lambda^3_c & c \lambda^3_c \\
b \lambda^3_c & d & e \\
c \lambda^3_c & e & f
\end{pmatrix},$$

where the prefactors $a$, $b$, $c$, $d$, $e$ and $f$, unconstrained by any symmetry, are assumed to be of order one, say $0.5 < a, \ldots, f < 2$. Depending on their values, the two heaviest neutrinos may be either approximately degenerate (case 1) or well separated in mass (case 2). It is convenient to express their mass ratio and mixing angle in terms of the two parameters $x = \frac{d-\epsilon^2}{(d+f)x}$ and $y = \frac{d-f}{d+f}$:

$$\frac{m_{\nu_\mu}}{m_{\nu_\tau}} = \frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}}; \quad \sin^2 2\theta_{\mu\tau} = 1 - \frac{y^2}{1 - 4x}.\quad (30)$$

Case 1 corresponds to both regimes $4x \sim 1$ and $-4x) \gg 1$, while case 2 requires $|x| \ll 1$. Small values of $|x|$ are very generic when $d$ and $f$ have the
same sign, provided that $df \sim e^2$. Since this condition is very often satisfied by arbitrary numbers of order one, a mass hierarchy is not less natural, given the structure (26), than an approximate degeneracy.

**Case 1:** $m_{\nu_2} \sim m_{\nu_3}$. The oscillation frequencies $\Delta m^2_{12} = m^2_{\nu_2} - m^2_{\nu_1}$ are roughly of the same order of magnitude, $\Delta m^2_{12} \sim \Delta m^2_{23} \sim \Delta m^2_{13}$. There is no simultaneous explanation of the solar and atmospheric neutrino data. A strong degeneracy between $\nu_2$ and $\nu_3$, which would result in two distinct oscillation frequencies, $\Delta m^2_{23} \ll \Delta m^2_{12} \simeq \Delta m^2_{13}$, would be difficult to achieve unless additional symmetries are invoked. This case yields only the MSW effect, with $\Delta m^2_{12} \sim \Delta m^2_{13} \sim 10^{-6} eV^2$, and a total electron neutrino oscillation probability

$$P(\nu_e \rightarrow \nu_{\mu,\tau}) = 4 u^2 \lambda^6 \sin^2 \left( \frac{\Delta m^2_{12}L}{4E} \right) + 4 v^2 \lambda^6 \sin^2 \left( \frac{\Delta m^2_{13}L}{4E} \right),$$

(31)

where the parameters $u$ and $v$ are defined to be $u = \frac{b_f - c_e}{\sqrt{-x}^2}$ and $v = \frac{b_e - c_d}{\sqrt{-x}^2}$. If $\Delta m^2_{12}$ is close enough to $\Delta m^2_{13}$, (31) can be viewed as a two-flavour oscillation with a mixing angle $\sin^2 2\theta_{\mu\tau} \sim (u^2 + v^2) \lambda^6$. The solar neutrino data then require $(u^2 + v^2) \sim 10 - 20$, which is still reasonable in our approach. Although the mixing between $\mu$ and $\tau$ neutrinos is of order one, they are too light to account for the atmospheric neutrino anomaly.

**Case 2:** $m_{\nu_2} \ll m_{\nu_3}$. The two distinct oscillation frequencies $\Delta m^2_{12}$ and $\Delta m^2_{13} \simeq \Delta m^2_{23}$ can explain both the solar and atmospheric neutrino data: the non-adiabatic MSW $\nu_e \rightarrow \nu_{\mu,\tau}$ solution suggest

$$4 \times 10^{-6} eV^2 \leq \Delta m^2 \leq 10^{-5} eV^2 \quad \text{(best fit: } 5 \times 10^{-6} eV^2),$$

(32)

while the atmospheric neutrino anomaly requires

$$5 \times 10^{-4} eV^2 \leq \Delta m^2 \leq 5 \times 10^{-3} eV^2 \quad \text{(best fit: } 10^{-3} eV^2).$$

(33)

To accommodate both, we need $0.03 \leq \frac{m_{\nu_2}}{m_{\nu_3}} \simeq x \leq 0.15$ (with $x = 0.06$ for the best fits), which can be achieved without any fine-tuning in our model. Interestingly enough, such small values of $x$ generically push $\sin^2 2\theta_{\mu\tau}$ towards its maximum, as can be seen from (30). Indeed, since $d$ and $f$ have the same sign and are both of order one, $y^2$ is naturally small compared with $(1 - 4x)$. This is certainly a welcome feature, since the best fit to the atmospheric neutrino data is obtained precisely for $\sin^2 2\theta = 1$.

In both cases, the scale of the neutrino masses measures the cut-off $M$. In case 1, the MSW effect requires $m_0 \sim 10^{-3} eV$, which gives $M \sim 10^{18} GeV$. In case 2, the best fit to the atmospheric neutrino data gives $m_0 (d + f) = m_{\nu_2} +$
\[ m_{\nu_3} \simeq 0.03 \text{eV}, \] which corresponds to a slightly lower cut-off, \[ 10^{16} \text{GeV} \leq M \leq 4 \times 10^{17} \text{GeV} \] (assuming \( 0.2 \leq d + f \leq 5 \)). It is remarkable that those values are so close to the unification scale obtained by running the standard model gauge couplings. This result depends of course on our choice for \( X_N = -3/2 \), favored by the flat direction analysis.

To conclude this section, we note that our model predicts order-one mixing between \( \nu_\mu \) and \( \nu_\tau \), as well as the small angle MSW solution to the solar neutrino deficit. In addition, the scales of the measured mass eigenvalues "measure" the cut-off to be of the order of \( M_U \). Lastly, our model predicts neither a neutrino mass in the few eV range, which could account for the hot component of the dark matter needed to understand structure formation, nor that implied by LSND[2].

### 4 Conclusion

The case for the extension to the standard model to an anomalous \( U(1) \) is very compelling, as it can yield the correct quark and lepton hierarchies, including neutrino masses and mixings in agreement by current experiments. However, our model is not complete, as it only predicts orders of magnitude of Yukawa couplings, not their prefactors. Many of its features are found in free fermion theories[2], which arise in the context of perturbative string theory. Since anomalies are involved, it is hoped that these features extend beyond perturbative string theories. Specifically, the calculation of the Fayet-Iliopoulos term in non-perturbative regimes might reconcile the cut-off from the low energy theory with the string scale.

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