Resonantly enhanced tunneling of Bose-Einstein condensates in periodic potentials

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We report on measurements of resonantly enhanced tunneling of Bose-Einstein condensates loaded into an optical lattice. By controlling the initial conditions of our system we were able to observe resonant tunneling in the ground and the first two excited states of the lattice wells. We also investigated the effect of the intrinsic nonlinearity of the condensate on the tunneling resonances.

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Resonantly enhanced tunneling (RET) is a quantum effect in which the probability for tunneling of a particle between two potential wells is increased when the quantized energies of the initial and final states of the process coincide. In spite of the fundamental nature of this effect [1] and the practical interest [2], it has been difficult to observe experimentally in solid state structures. Since the 1970s, much progress has been made in constructing solid state systems such as superlattices [3, 4, 5] and quantum wells [6] which enable the controlled observation of RET [7].

In recent years, ultra-cold atoms in optical lattices [8] have been increasingly used to simulate solid state systems. Optical lattices are easy to realize in the laboratory, and their parameters can be perfectly controlled both statically and dynamically. Also, more complicated potentials can be realized by adding further lattice beams [9]. This makes them attractive as model systems for crystal lattices, and in the last few years cold atoms and Bose-Einstein condensates (BECs) in optical lattices have been used to simulate phenomena such as Bloch oscillations [10] and the Mott insulator transition [11]. In this Letter we show that BECs in accelerated optical lattice potentials are ideally suited to studying RET. While in solid state measurements of RET only a few potential wells were used and the periodic structures had to be grown for each realization, in our experiment the condensate is distributed over several tens of wells and the parameters of the lattice can be freely chosen. Moreover, we are able to control the initial conditions of the system and thus observe RET in any chosen energy level and can also add nonlinearity to the system.

A schematic representation of RET is shown in Fig. 1. In a tilted periodic potential, atoms can escape by tunneling to the continuum via higher-lying levels. The tilt of the potential is proportional to the force \( F \) acting on the atoms, and in general the tunneling rate \( \Gamma_{LZ} \) can be calculated using the Landau-Zener formula [12]. However, when the tilt-induced energy difference \( F d_L \Delta i \) between wells \( i \) and \( i + \Delta i \) matches the separation between two quantized energy levels, the tunneling probability is resonantly enhanced and the Landau-Zener formula no longer gives the correct result, as previously investigated in [13] for cold atoms in optical lattices. While for the parameters of our experiment the enhancement over the Landau-Zener prediction was around a factor of 2 (see theoretical and experimental results of Fig. 2(a)), in general it can be several orders of magnitude.

The starting point of our experiments is a BEC of \(^{87}\)Rb atoms, held in an optical dipole trap whose frequencies can be adjusted to realize a cigar-shaped condensate. The BECs are created using a hybrid approach in which evaporative cooling is initially effected in a magnetic time-orbiting potential (TOP) trap and subsequently in a crossed dipole trap. The dipole trap is realized using two intersecting gaussian laser beams at 1030 nm wavelength and a power of around 1 W per beam focused to waists of 50 \( \mu \)m. After obtaining pure condensates of around \( 5 \times 10^4 \) atoms the powers of the trap beams are adjusted in order to obtain an elongated condensate with the desired trap frequencies (\( \approx 20 \) Hz in the longitudinal direction and \( 80 - 250 \) Hz radially).

Subsequently, the BECs held in the dipole trap are loaded into an optical lattice created by two gaussian laser beams (\( \lambda = 852 \) nm) with 120 \( \mu \)m waist intersecting at an angle \( \theta \). The resulting periodic potential \( V(x) = V_0 \sin^2(\pi x/d_L) \) has a lattice spacing \( d_L = \lambda/(2\sin(\theta/2)) \) and its depth \( V_0 \) is measured in units of the recoil energy \( E_{rec} = \hbar^2 \pi^2/(2md_L^2) \), where \( m \) is the mass of the Rb atoms. In the present experiment, we
used $d_L = 0.426\, \mu m$ (for $V_0/E_{rec} = 6, 4, 9$ and 16) and $d_L = 0.620\, \mu m$ (for $V_0/E_{rec} = 2.5, 10, 12$ and 14). By introducing a frequency difference $\Delta \nu$ between the two lattice beams (using acousto-optic modulators which also control the power of the beams), the optical lattice can be moved at a velocity $v = d_L \Delta \nu$ or accelerated with an acceleration $a = d_L \frac{\Delta \nu}{dt}$.

A ramp from 0 to $V_0$ in around 1 ms loads the BEC adiabatically into the optical lattice [14]. For loading the ground state levels, the lattice velocity is $v = 0$ during the ramp. For the first and second excited levels, during the ramp the lattice is moved at a finite velocity calculated from the conservation of energy and quasimomentum [16]. Finally, the optical lattice is accelerated with acceleration $a$ for an integer number of Bloch oscillation cycles. In the rest frame of the lattice, this results in a force $F = ma$ on the condensate. Atoms that are dragged along by the accelerated lattice acquire a larger final velocity than those that have undergone tunneling, and are spatially separated from the latter by releasing the BEC from the dipole trap and lattice at the end of the acceleration period and allowing it to fall under gravity for 5–20 ms. After the time-of-flight, the atoms are detected by absorptive imaging on a CCD camera using a resonant flash.

From the dragged fraction $N_{drag}/N_{tot}$, we then determine the tunneling rate $\Gamma_n$ in the asymptotic decay law

$$N_{drag}(t) = N_{tot} \exp(-\Gamma_n t)$$

where the subscript $n$ indicates the dependence of the tunnelling rate on the local energy level $n$ in which the atoms are initially prepared (ground state: $n = 1$, first excited state: $n = 2$, etc.). In the experiments reported in this work, the number of bound states in the wells was small (2-4, depending on the lattice depth), so after the first tunneling event, the probability for tunneling to the next bound state or the continuum was close to unity.

The resolution of our tunneling measurement is given by the minimum number of atoms that we can distinguish from the background noise in our CCD images, which varies between 500 and 1000 atoms, depending on the width of the observed region. With our condensate number, and taking into account the minimum acceleration time limited by the need to spatially separate the two fractions after time-of-flight and the maximum acceleration time limited by the field of view of the CCD camera, this results in a maximum $\Gamma_n/\nu_{rec}$ of $\approx 1$ and a minimum of $\approx 1 \times 10^{-2}$, with the recoil frequency $\nu_{rec} = E_{rec}/\hbar$.

A typical plot of the tunneling rate $\Gamma_1$ out of the ground state as a function of $F_0^{-1}$ (where $F_0 = F d_L/E_{rec}$ is the dimensionless force) in the linear regime is shown in Fig. 2(a). This regime is reached either by choosing small radial dipole trap frequencies or by releasing the BEC from the trap before the acceleration phase and thus letting it expand. In both cases, the density and hence the interaction energy of the BEC is reduced. Superimposed on the overall exponential decay of $\Gamma_1/F_0$ with $F_0^{-1}$, one clearly sees the resonant tunneling peaks corresponding to $\Delta i = 2, 3$ and 4 (for this choice of parameters, the $\Delta i = 1$ peak lay outside our experimental resolution). In order to highlight the deviation from the Landau-Zener prediction, in the inset of Fig. 2(a) we plot $\Gamma_1/\Gamma_{LZ}$, where the Landau-Zener tunnelling rate $\Gamma_{LZ}$ is given by [12,16]

$$\Gamma_{LZ} = \nu_{rec} F_0 e^{-\frac{\nu_{rec} F_0}{2\Delta i E_{rec}}}. \quad (2)$$

The experimental results are in good agreement with numerical solutions obtained by diagonalizing the Hamiltonian of the open decaying system [17,18]. Figure 2(b) summarizes our results for the positions of the ground-state resonances $\Delta i = 1, 2$ and 3 as a function of the lattice depth together with a theoretical fit assuming the separation of the lowest energy levels to be

$$\Delta E = \alpha E_{rec} \sqrt{V_0/E_{rec}}. \quad (3)$$
Independently of $\Delta i$, the best fit is achieved for $\alpha = 1.5$, to be compared with $\alpha = 2$ for the harmonic oscillator approximation. A value $\alpha < 2$ is to be expected since our lattice wells only contain a few bound states and are, therefore, highly anharmonic.

Using BECs in optical lattices allows us to explore resonant tunneling in regimes that are difficult or even impossible to access in solid state systems. First, we can prepare the condensates in the excited levels of the lattice wells before the acceleration. Again, tunneling resonances are clearly visible, and the experimental results agree with theoretical calculations. The accessibility of higher energy levels allows us to experimentally determine the decay rates at resonance of two strongly coupled levels. Although our experimental resolution does not allow us to measure the decay rates in two different levels for the same set of parameters $F_0$ and $V_0$, we are able to compare the ground and excited state decay rates $\Gamma_1$ and $\Gamma_2$ with the theoretical predictions for two different parameter sets, as shown in Fig. 3. This figure reveals the anti-crossing of the decay rates of strongly coupled levels as a function of our control parameter $F_0$. These results demonstrate a peculiar behaviour of the Wannier-Stark states studied theoretically [6, 19] and more recently rephrased within a general context of crossings and anti-crossings for the real and imaginary parts of the eigenvalues of a non-hermitian Hamiltonians [20]. Our data confirm the predictions of [17] that the anti-crossings modify the decay rates of the two perturbing states in different ways.

Additionally, by exploiting the intrinsic nonlinearity of the condensate due to atom-atom interactions, we can study RET in the nonlinear regime, as simulated in [21]. In order to realize this regime, we carry out the acceleration experiments in radially tighter traps (radial frequency $\gtrsim 100$ Hz) and hence at larger condensate densities. Figure 4(a) shows the results for increasing values of the nonlinear parameter $22$

$$C = \frac{n_0 a_s d^2}{\pi},$$

where $n_0$ is the peak condensate density and $a_s$ the $s$-wave scattering length. Two effects are visible: First, the overall (off-resonant) level of $\Gamma_1$ increases linearly.

![Graph](image-url)  
FIG. 3: Anti-crossing scenario of the RET rates. (a) Theoretical plot of $\Gamma_{1,2}$ for $V_0 = 2.5 E_{\text{rec}}$ with experimental points for $\Gamma_1$. (b) Theoretical plot of $\Gamma_{1,2}$ for $V_0 = 10 E_{\text{rec}}$ with experimental points for $\Gamma_2$. For clarity, the vertical axes have been split and the $\Gamma_n$ plotted on a linear scale, and only one representative error bar is shown.

![Graph](image-url)  
FIG. 4: Resonant tunneling in the nonlinear regime. (a) Resonance $\Delta i = 3$ for $V_0 = 2.5 E_{\text{rec}}$ with $C = 0.024$ (squares), $C = 0.035$ (circles) and $C = 0.057$ (triangles). The solid line is the theoretical prediction for $C = 0$; the dashed lines are guides to the eye. (b) Dependence on $C$ of the tunneling rate at the position of the peak $F_0 = 1.21$ (solid symbols) and of the trough $F_0 = 1.03$ (open symbols). The dashed lines are fits to guide the eye. For clarity, in (a) and (b) only one typical error bar is shown.
In the nonlinear regime, i.e. when the radial trap frequency is large and hence the interaction energy of the BEC is appreciable, the speed of the ramp has to be reduced and loading times can be as long as tens of milliseconds [15].

In summary, we have measured resonantly enhanced tunneling of BECs in accelerated periodic potentials in a regime where the standard Landau-Zener description is not valid. Our results in the linear regime agree with numerical calculations, and the possibility to observe RET for arbitrary initial conditions and parameters of the periodic potential underlines the advantage of our system over solid state realizations. Furthermore, we have explored RET in the nonlinear regime and demonstrated that, as theoretically predicted, the tunneling resonances disappear for large values of the nonlinearity.

In the present set-up the measurement of the tunneling rate is limited in its dynamic range by the detection geometry. A larger dynamic range can be realized by long-distance transport of BECs [26]. Our method for observing RET can also be generalized in order to study other regular or disordered potentials, the effects of noise and the presence of a thermal fraction in the condensate. Furthermore, one might exploit the tunneling resonances to explore the spatial decoherence processes and to perform precision measurements.

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[1] D. Bohm, Quantum Theory. p. 286 (Dover Publications, New York, 1989).
[2] L.L. Chang, E.E. Mendez, and C. Tejedor (eds), Resonant Tunneling in Semiconductors (Plenum, New York, 1991).
[3] L.L. Chang, L. Esaki, and R. Tsu, Appl. Phys. Lett. 24, 593 (1974).
[4] L. Esaki, IEEE Journal Quant. Electr. QE-22(9), 1611 (1986).
[5] S. Glutsch, Phys. Rev. B 69, 235317 (2004).
[6] M. Wagner and H. Mizuta, Phys. Rev. B 48, 14393 (1993).
[7] K. Leo, High-Field Transport in Semiconductor Superlattices (Springer, Berlin, 2003).
[8] G. Grynberg and C. Robilliard, Phys. Rep. 355, 335 (2001).
[9] L. Santos, M. A. Baranov, J. I. Cirac, H.-U. Everts, H. Fehrmann, and M. Lewenstein, Phys. Rev. Lett. 93, 030601 (2004).
[10] O. Morsch and M. Oberthaler, Rev. Mod. Phys. 78, 179 (2006) and references therein.
[11] M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, and I. Bloch, Nature (London) 415, 39 (2002).
[12] L. Landau, Phys. Z. Sowjetunion 1, 88 (1932); 2, 46 (1932); C. Zener, Proc. R. Soc. London Ser. A 137, 696 (1932).
[13] P. Bharucha, K.W. Madison, P.R. Morrow, S.R. Wilkinson, B. Sundaram, and M.G. Raizen, Phys. Rev. A 55, R857 (1997).
[14] In the nonlinear regime, i.e. when the radial trap frequency is large and hence the interaction energy of the BEC is appreciable, the speed of the ramp has to be reduced and loading times can be as long as tens of milliseconds [15].
[15] T. Gericke et al., cond-mat/0603590.
[16] E. Peik, M.B. Dahan, I. Bouchoule, Y. Castin, and C. Salomon, Phys. Rev. A 55, 2989 (1997).
[17] M. Glück, A.R. Kolovsky, and H.J. Korsch, Phys. Rev. Lett. 83, 891 (1999).
[18] M. Glück, A.R. Kolovsky, and H.J. Korsch, Phys. Rep. 366, 103-182 (2002).
[19] J.E. Avron, Ann. Phys. 143, 33 (1982).
[20] F. Keck, H.J. Korsch, and S. Mossmann, J. Phys. A 36, 2125 (2003).
[21] S. Wimberger, R. Mannella, O. Morsch, E. Arimondo, A.R. Kolovsky, and A. Buchleitner, Phys. Rev. A 72, 063610 (2005).
[22] O. Morsch, J.H. Müller, M. Cristiani, D. Ciampini, and E. Arimondo, Phys. Rev. Lett. 87, 140402 (2001).
[23] M. Jona-Lasinio, O. Morsch, M. Cristiani, N. Malossi, J.H. Müller, E. Courtade, M. Anderlini, and E. Arimondo, Phys. Rev. Lett. 91, 230406 (2003).
[24] D.I. Choi and Q. Niu, Phys. Rev. Lett. 82(10), 2022 (1999).
[25] O. Morsch, J.H. Müller, D. Ciampini, M. Cristiani, P.B. Blakie, C.J. Williams, P.S. Julienne, and E. Arimondo, Phys. Rev. A 67, 031603 (2003).
[26] S. Schmid, G. Thalhammer, K. Winkler, L. Lang, and J.H. Denschlag, New J. Phys. 8(8), 159 (2006).