Error Analysis of High Precision Two-way Time Difference Measurement in Satellite-ground Links

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Abstract. High precision time difference measurement between satellite and earth is an important part of space-based time-frequency system application in the future. With the on-orbit operation of high-precision atomic clocks, the accuracy of satellite-ground time difference measurement link has become the bottleneck of space-based time-frequency distribution. In this paper, the basic principle of two-way time difference measurement is derived. The system error and equipment error involved are analyzed, and the error control strategy is given. The above work will provide engineering guidance for high-precision two-way time difference measurement in the design of satellite-ground links.

1. Introduction

The method of time difference measurement between satellite and ground station includes single frequency one-way measurement, dual frequency one-way measurement, dual frequency two-way time difference measurement etc.[1-4]. Taking the traditional GNSS time service as an example, the typical method is that the satellite continuous broadcasts navigation signal, and the ground user receives the single frequency signal to realize the time synchronization at 10 nanoseconds. In order to further improve the accuracy of time difference measurement, the dual frequency signal sent by GNSS is received to achieve time synchronization at nanoseconds, where the dual frequency signal can correct the ionospheric error introduced in the spatial transmission of navigation signal. However, for the needs of higher accuracy time difference measurement, such as the European Atomic Clock Ensemble in Space (ACES) plan, the measurement accuracy of picoseconds level is the upper limit, which is limited by the tropospheric dispersion effect in the high frequency satellite-ground link.

In this paper, the basic time difference measurement principle of microwave satellite-ground link is derived. The errors are analyzed and the corresponding control strategies are given, which will provide engineering guidance for high-precision two-way time difference measurement between satellite and ground, and technical means for application of space-based time-frequency system.

2. Two-way time difference measurement

The corresponding two-way ranging process[5] is shown in Figure 1, continuous receiving and sending ranging signal between satellite and ground station. When the two-way measurement is taken
out, the clock difference solution can be completed. Assuming that the extracted downlink ranging signal is sent by satellite at $t_{S_{	ext{snd}}}^d$ and received by the ground station at $t_{G_{	ext{rev}}}^r$, and the uplink ranging signal is sent by the ground station at $t_{G_{	ext{snd}}}^d$ and received by the satellite at $t_{S_{	ext{rev}}}^r$.

Figure 1. Process of two-way ranging between satellite and ground

The one-way distance measurement between satellite and ground station is calculated by the local time difference at the time of transmitting and receiving, including the clock difference information of satellite and ground station. The measurement between satellite and ground station is established as follows.

\[
\begin{align*}
\rho_{SG} &= \left| R_G - R_S \left( t_{S_{\text{ snd}}}^d \right) \right| + \delta_{\text{Sagnac}} + c x_S \left( t_{S_{\text{ snd}}}^d \right) + c \delta_{G_{\text{ rev}}} + c \delta_{S_{\text{ snd}}} + \delta_{\text{ion}} \left( R_S \left( t_{S_{\text{ snd}}}^d \right), R_G \left( t_{G_{\text{ rev}}}^r \right) \right) \\
&\quad + \delta_{\text{rel-period}} + \delta_{\text{MP}} + \delta_{\text{PCO}} + \delta_{\text{Gravitational}} + \delta_{\text{Sagnac}} + \delta_{\text{MP}} + \delta_{\text{PCO}} + \delta_{\text{Gravitational}} \\
\rho_{GS} &= \left| R_S - R_G \left( t_{G_{\text{ snd}}}^d \right) \right| + \delta_{\text{Sagnac}} + c x_G \left( t_{G_{\text{ snd}}}^d \right) + c \delta_{G_{\text{ rev}}} + c \delta_{G_{\text{ snd}}} + \delta_{\text{ion}} \left( R_G \left( t_{G_{\text{ snd}}}^d \right), R_S \left( t_{S_{\text{ rev}}}^r \right) \right) \\
&\quad + \delta_{\text{rel-period}} + \delta_{\text{Gravitational}} + \delta_{\text{MP}} + \delta_{\text{PCO}} + \delta_{\text{Gravitational}} + \delta_{\text{Sagnac}} + \delta_{\text{MP}} + \delta_{\text{PCO}} + \delta_{\text{Gravitational}}
\end{align*}
\]

Here, $c$ is the speed of light, $R_S$ and $R_G$ are the position vectors of satellite and ground in the earth fixed system, $x_S$ and $x_G$ are the clock difference of satellite and ground station respectively, $\delta_{\text{Sagnac}}$ and $\delta_{\text{MP}}$ are the hardware transmission delay of signal transmission channel and receiving channel; $\delta_{\text{rel-period}}$ is the equivalent delay of relativistic periodic effect, $\delta_{\text{Gravitational}}$ is the signal transmission delay caused by the gravitational field of the earth, $\delta_{\text{Sagnac}}$ is the equivalent delay of Sagnac effect, $\delta_{\text{MP}}$ is the equivalent delay of multipath effect, $\delta_{\text{PCO}}$ is the equivalent delay of phase center offset, and $\varepsilon$ is ranging noise.

In the time comparison between the satellite and the ground station, the time of the ground atomic clock is taken as the reference time, so the ground clock difference can be considered as $x_S = 0$. In the earth fixed system, the position coordinate of the ground station is constant, so the measurement equation can be simplified as:

\[
\begin{align*}
\rho_{SG} &= \left| R_G - R_S \left( t_{S_{\text{ snd}}}^d \right) \right| + \delta_{\text{Sagnac}} - c x_S \left( t_{S_{\text{ snd}}}^d \right) + c \delta_{G_{\text{ rev}}} + c \delta_{S_{\text{ snd}}} + \delta_{\text{ion}} \left( R_S \left( t_{S_{\text{ snd}}}^d \right), R_G \left( t_{G_{\text{ rev}}}^r \right) \right) \\
&\quad + \delta_{\text{rel-period}} + \delta_{\text{Gravitational}} + \delta_{\text{MP}} + \delta_{\text{PCO}} + \delta_{\text{Gravitational}} \\
\rho_{GS} &= \left| R_S \left( t_{S_{\text{ rev}}}^r \right) - R_G \left( t_{G_{\text{ rev}}}^d \right) \right| + \delta_{\text{Sagnac}} + c x_S \left( t_{G_{\text{ rev}}}^d \right) + c \delta_{G_{\text{ rev}}} + c \delta_{G_{\text{ snd}}} + \delta_{\text{ion}} \left( R_G \left( t_{G_{\text{ rev}}}^d \right), R_S \left( t_{G_{\text{ rev}}}^d \right) \right) \\
&\quad + \delta_{\text{rel-period}} + \delta_{\text{Gravitational}} + \delta_{\text{MP}} + \delta_{\text{PCO}} + \delta_{\text{Gravitational}}
\end{align*}
\]
When the time of satellite receiving and transmitting ranging signal is inconsistent, the two unidirectional ranging signals contain the satellite clock difference at different times. However, it needs to be calculated to the same time.

\[ x_S(t_{\text{rev}}) = x_S(t_{\text{end}}) + \Delta t_S \] (3)

Here, \( \Delta t_S \) is the satellite clock difference correction quantity. The clock difference between the satellite and ground station can be calculated by making the difference between the two-way ranging equation.

\[
\begin{align*}
  x_S(t_{\text{end}}) &= \left( \rho_{GS} - \rho_{SG} \right) + \left| R_G - R_S(t_{\text{end}}) \right| - R_S(t_{\text{rev}}) - R_G - c\Delta t_S + c \left( \delta_{GS}^{\text{rev}} - \delta_{GS}^{\text{end}} \right) - c \left( \delta_{S}^{\text{rev}} - \delta_{S}^{\text{end}} \right) \\
  &+ \left[ \delta_{GS}^{\text{end}} (R_S(t_{\text{end}}), R_G) - \delta_{GS}^{\text{end}} (R_G, R_S(t_{\text{end}})) \right] + \left[ \delta_{GS}^{\text{rev}} (R_S(t_{\text{end}}), R_G) - \delta_{GS}^{\text{rev}} (R_G, R_S(t_{\text{rev}})) \right] \\
  &+ \delta_{\text{sagnac}}^{\text{end}} - \delta_{\text{sagnac}}^{\text{rev}} + \delta_{\text{S}^{\text{period}}}^{\text{end}} - \delta_{\text{S}^{\text{period}}}^{\text{rev}} + \delta_{\text{S}^{\text{grav}}}^{\text{end}} - \delta_{\text{S}^{\text{grav}}}^{\text{rev}} + \delta_{\text{MP}}^{\text{end}} - \delta_{\text{MP}}^{\text{rev}} + \delta_{\text{PCO}}^{\text{end}} - \delta_{\text{PCO}}^{\text{rev}} + \epsilon_{\text{SG}} - \epsilon_{\text{GS}} \right] / 2c
\end{align*}
\] (4)

In order to quantitatively analyze the accuracy level of two-way time distance measurement between satellite and ground station, the orbit related error, transmission path error and equipment characteristic error involved in the satellite orbit height of 400km and Ka band link are analyzed.

3. Error analysis related to orbit

Orbit related error items are related to the physical position of the satellite and the earth, including geometric delay correction error, clock error correction error, relativistic periodic term error, and gravitational delay.

3.1. Geometric delay correction error

In the geocentric earth fixed coordinate system, the space non-symmetry distance introduced by the relative motion of the satellite and the ground station can be equivalent to:

\[ C_{\text{o}} = \left( |R_G - R_S(t^*_G)| - |R_G - R_S(t^*_S)| \right) / 2c \] (5)

According to the above formula, the space distance can be corrected by using the satellite precise orbit. If \( d(t) = |R_G - R_S(t)| \) is the geometric distance between satellite and ground at time t, the above formula can be rewritten as follows:

\[ C_{\text{orb}} = \left[ d(t_{\text{end}}) - d(t_{\text{rev}}) - + \delta d(t_{\text{end}}) - \delta d(t_{\text{rev}}) \right] / 2c \] (6)

Here, the superscript ‘\(^*\)’ represents the geometric distance calculated by the precise orbit, \( \delta d(t) \) is the spatial distance error of the precise orbit at time \( t \). Therefore, the space distance correction error at time \( t \) is:

\[ C_{\text{orb}} = \left[ \delta d(t_{\text{end}}) - \delta d(t_{\text{rev}}) \right] / 2c \] (7)

The above formula shows that the space distance correction error is affected by the change rate of space distance error. \( \sigma(C_{\text{orb}}) \) is better than 0.23ns when the precision orbit is centimeter scale and the error is random. In fact, the orbit error of the smoothed satellite is not completely shown as random error, that is, the space distance error \( \delta d(t_{\text{end}}) \) and \( \delta d(t_{\text{rev}}) \) at the two time points have correlation, which can be processed by two-way cancellation.

The equivalent time delay of earth rotation effect is the main geometric time delay in two-way time comparison.

\[ C_{\text{sagnac}} = \frac{\omega}{c^2} \left[ Y_G X_S(t_0) - X_G Y_S(t_0) \right] \] (8)

The correction accuracy of Sagnac effect is affected by satellite orbit accuracy. Calculate the variance of the above formula.

\[ D(C_{\text{sagnac}}) = \frac{\omega^2}{c^4} \left[ Y_G^2 D(X_S(t_0)) + X_G^2 D(Y_S(t_0)) \right] \] (9)
Taking Xi’an station as an example, if the orbit accuracy of the satellite is 10 cm (1σ), it can be concluded that the two-way time synchronization error introduced by Sagnac effect is 4.3e-4ps.

3.2. Clock error correction
The uplink and downlink pseudo ranges contain the satellite clock difference at different receiving and transmitting times. In order to solve the clock difference, the uplink and downlink clock difference should be calculated as the clock difference at the same time, and the correction of clock difference is:

$$\Delta t_s = x_s (t_s^{\text{up}}) - x_s (t_s^{\text{down}})$$  \hspace{1cm} (10)

When $t_s^{\text{up}} = t_s^{\text{down}}$, the uplink and downlink pseudo distance includes the satellite clock difference at the same time of receiving and transmitting, where $\Delta t_s$ is 0.

When $t_s^{\text{up}} \neq t_s^{\text{down}}$, $\Delta t_s$ depends on the frequency performance of satellite atomic clock and the time difference $\Delta t = t_s^{\text{up}} - t_s^{\text{down}}$ between receiving and transmitting. For an atomic clock with a frequency accuracy of e-12, the correction value of the clock difference corresponding to the time difference of 10 ms is 0.01ps.

3.3. Relativistic periodic term correction error
The measurement of pseudo range and carrier phase between satellite and ground will be affected by general and special relativity effects. With the periodic motion of the satellite in the earth's gravitational field, the velocity and the gravitational potential change periodically, the equivalent time delay of the relativistic period term could be represented as:

$$\tau_{\text{rel, period}}^{\text{GS}} = -2 \frac{\sqrt{GM_e \cdot e \sin E}}{c^2}$$  \hspace{1cm} (11)

It needs to be corrected in the process of calculating the clock difference between satellite and ground. Its equivalent form is:

$$\tau_{\text{rel, period}}^{\text{GS}} = -2 R_s \cdot V_s / c^2$$  \hspace{1cm} (12)

$R_s$ and $V_s$ are the position and velocity vector of the satellite in the inertial system. The equivalent path delay of the relativistic effect can be calculated according to the orbit parameters of the satellite in the inertial reference system. For the special relativistic effect of the receiver and transmitter of the uplink and downlink pseudo range, the correction amount is as follows:

$$C_{\text{rel, period}} = \left( \tau_{\text{rel, period}}^{\text{SG}} - \tau_{\text{rel, period}}^{\text{GS}} \right) / 2 = 2 R_s \cdot V_s (t_0) / c^2$$  \hspace{1cm} (13)

When the three-axis position accuracy of satellite orbit is 10cm (1σ) and the three-axis speed accuracy is 1mm/s (1σ), the simulation calculation of the two-way time synchronization error introduced by the relativistic periodic effect is shown in Figure 2(a). The standard deviation of error is 0.15ps.

![Figure 2. Relativistic periodic effect correction error](image-url)
When the three-axis position accuracy of satellite orbit is 10cm (1σ) and the three-axis speed accuracy is 0.1mm/s (1σ), the simulation calculation of the two-way time synchronization error introduced by the relativistic periodic effect is shown in Figure 2(b). The standard deviation of error is 0.02ps. The correction accuracy of relativistic effect is affected by the accuracy of satellite orbit.

3.4. Gravitational delay
The gravitational field will change the propagation speed of light or electromagnetic wave in space. The difference between signal propagation time in gravitational field and signal propagation time in vacuum (non-gravitational field) is gravitational delay. The time delay of general relativity caused by the gravitational field of the earth is:

\[
C_\nu = \frac{2GM_E}{c^2c} \ln \frac{r_s + r_G + d}{(r_s + r_G - d)}
\]  

(14)

Taking Xi’an station as an example, the gravitational delay between the satellite and the ground is shown in Figure 3. The maximum value can reach 300 ps, so the gravitational delay needs to be corrected. The gravitational delay correction is calculated by satellite and ground coordinates, so the gravitational delay correction error will be introduced along with the satellite orbit error. If the three-axis position accuracy of satellite orbit is 10cm (1σ), the correction error of gravitational delay is shown in Figure 4. The correction error of gravitational delay introduced by satellite orbit is at the level of e-5ps.

4. Transmission path error analysis

4.1. Tropospheric delay error analysis
Based on mesoscale meteorological model, the existing data is analyzed. From October 1 to 30, 2019, the nondispersive delay of the troposphere in Xi’an and Shanghai is in the range of 5-8ns as shown in Figure 5, which can be corrected by the model.
Due to the continuous rotation of water vapor molecules, settling particles and oxygen molecules in the troposphere, the tropospheric dispersion delay is 11ps as shown in Figure 6, which could be corrected by atmospheric model or real-time temperature and humidity pressure data. The tropospheric dispersion should be comprehensively considered in high accuracy measurement.

4.2. Ionospheric delay error analysis
The ionospheric delay error is related to frequency. The first-order ionospheric error can be eliminated by multi frequency. The second-order effect of the ionosphere is determined by the earth's magnetic field and background electron density. In the year of great solar activity. The second-order dispersion effect of the ionosphere would be about 0.2ps. Considering the complexity of the system and the second-order effect, the ionospheric errors above the second-order can be processed by monitoring the solar activity.

5. Equipment characteristic error analysis

5.1. Receiver processing
Taking carrier measurement as an example, the phase measurement error sources of carrier tracking loop mainly include phase jitter and dynamic stress error. The error sources causing phase jitter include thermal noise $\sigma_{tPLL}$, oscillation frequency jitter $\sigma_v$ and Allan mean square deviation $\sigma_A(\tau)$.

The estimation formula of $\sigma_{tPLL}$ is as follows:

$$\sigma_{tPLL} = \frac{180^\circ}{\pi} \sqrt{\frac{B_L}{C/N_0}} \left( 1 + \frac{1}{2T_{coh}C/N_0} \right)$$ (15)

Here, $B_L$ is the loop noise bandwidth, $C/N_0$ is the carrier noise ratio, $T_{coh}$ is the coherent integration time. The above formula shows that reducing noise bandwidth and improving coherent integration time can reduce the tracking error.

The motion of the satellite and the mechanical vibration of the receiving device will cause the jitter of the reference oscillation frequency. In order to ensure the measurement accuracy, $\sigma_v$ should be designed reasonably according to the carrier frequency.

The phase jitter noise will also be introduced into the oscillation frequency drift of Allan crystal along with the time accumulation. Mean square deviation $\sigma_\lambda$ is related to the Allan mean square deviation $\sigma_A(\tau)$ and coherent integration time $T_{coh}$. The formula is shown as follows.

$$\sigma_\lambda = 360^\circ \frac{c}{\lambda} T_{coh} \sigma_A(\tau)$$ (16)

Here, $c$ is the speed of light, $\lambda$ is the carrier wavelength. Setting $T_{coh}$ is 1ms, when $\sigma_A(\tau) < 10^{-12}$, $\sigma_\lambda < 0.01ps$, it could be ignored.

In conclusion, the total mean square deviation of phase jitter caused by error sources is:

$$\sigma_i = \sqrt{\sigma_{tPLL}^2 + \sigma_v^2 + \sigma_\lambda^2}$$ (17)

5.2. Multipath effect
Multipath effect is a technical problem that has not been solved effectively in the radio communication, and it is also an important factor that affects the high-precision measurement of the time difference between satellite and ground. Correlation output is affected by multipath, which will lead to zero crossing offset. For multipath effect, the most effective way is to control the environment around the equipment. It can also be achieved by improving the receiver's own suppression ability.
5.3. Phase center shift of Antennas
The measurement between the satellite and the ground station is corresponding to the distance between the phase centers of the two antennas. The propagation delay between the phase centers needs to be corrected to the center of mass. Generally, the antenna phase center is a function of antenna azimuth and pitch angle. The biggest change usually does not exceed the geometric size of the antenna, which is always close to the geometric center of the antenna.

5.4. Time delay error of channels
The hardware structure and working temperature will bring variation of channel delay. On the one hand, time delay is uncertain. On the other hand, the delay will change with the ambient temperature. Therefore, it is necessary to control the delay difference in the receiving and sending channels. The following measures should be taken for the control of time delay characteristics of the channels.

- In order to reduce the change of channel delay, the filter which is not sensitive to the change of component parameters could be selected.
- Control the change range of component parameters and minimize the change of parameters.
- A closed-loop self-tuning system should be designed, which could detect and compensate when the delay changes.
- To ensure small temperature change, which is an important error factor in the process of engineering design.

6. Conclusion
According to the above analysis, the following aspects need to be considered in the design of high-precision satellite-ground link.

- Obtaining high-precision satellite orbit data is the key content of satellite ground-link design.
- Transmission path error is an important factor to be considered in high-precision time difference measurement, especially the tropospheric dispersion effect of high-frequency signal.
- The key factors to be controlled in design phase include the equipment error, the satellite ground link margin, the surrounding environment, the stability of the equipment itself and so on.

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