Classical solutions in lattice gauge theories

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Abstract

The solutions of the classical equations of motion on a periodic lattice are found which correspond to abelian single and double Dirac sheets. These solutions exist also in non-abelian theories. Possible applications of these solutions to the calculation of gauge dependent and gauge invariant observables are discussed.

1 Introduction

The lattice approach gives the possibility to use powerful numerical methods to calculate gauge invariant as well as gauge dependent objects [1]. However very often the interpretation of the results of numerical calculations needs analytical methods on a lattice, e.g. perturbation theory. The importance of certain classical configurations for understanding of the nonperturbative physics was stressed in [2]. The very existence of non-zero solutions of the classical equations of motion can explain some effects observed in simulations and provide insight into the relation between continuum and lattice theories. The perturbative expansion about the non-zero solutions of the classical equations of motion permits to take into account nonperturbative contributions, at least, partially.

This work is devoted to the study of the classical solutions on a periodic lattice and some applications of these solutions. The special accent in this work is made on the discussion of gauge dependent objects, e.g. photon correlator, especially in the connection with the Gribov ambiguity problem [3].

In many practical situations in lattice gauge theory it is rather useful to calculate gauge variant quantities, e.g., fermionic and gauge field correlators. For example, calculating the gauge variant gluon correlators one can attempt to obtain information about gauge invariant observables, like energies and masses. The study of the Gribov problem on a lattice has demonstrated the existence of the gauge-fixing ambiguities both in abelian and nonabelian theories. The non-uniqueness of the solutions of the commonly used gauge conditions, e.g. Lorentz gauge, can strongly affect the numerical computation of gauge dependent quantities.

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Let $S(U)$ be the lattice action with gauge symmetry group $G$. Under the gauge transformations $V_x$ the link variables $U_{x\mu}$ transform $U_{x\mu} \rightarrow U_{x\mu}^V \equiv V_x U_{x\mu} V_{x+\mu}^\dagger$, $U_{x\mu}, V_x \in G$. The partition function $Z$ is

$$Z \equiv \int \prod_{x\mu} dU_{x\mu} e^{-S(U)} ,$$

where $dU_{x\mu}$ is the Haar measure. The usual way to fix the gauge is to insert in the functional integral in eq.(1) the identity $1 = J(U) \int \prod_x dV_x \prod_x \delta(F_x(U^V))$, where $F_x(U)$ is a gauge fixing functional. The Faddeev–Popov determinant $J(U)$ is

$$J^{-1}(U) = \sum_{V'} \det^{-1}\left( \frac{\partial F^V_x}{\partial V_y} \bigg|_{V=V'} \right) ,$$

where $V'$ satisfies $F_x(U^{V'}) = 0$. The average of any (gauge invariant or not) functional $\Phi$ is defined as

$$\langle \Phi(U) \rangle \equiv Z^{-1} \int \prod_{x\mu} dU_{x\mu} \ J(U) \cdot \prod_x \delta(F_x(U)) \cdot \Phi(U) \cdot e^{-S(U)} .$$

Assuming that there are no gauge copies one can represent eq.(3) in the following form

$$\langle \Phi(U) \rangle = Z^{-1} \int \prod_{x\mu} dU_{x\mu} \ \Phi(U^{V^0}) \cdot e^{-S(U)} ,$$

where $V^0 \equiv \{V_4^0\}$ is the gauge transformation for which $F(U^{V^0}) = 0$. This means that the expected value of any functional $\Phi(U)$ may be calculated from the ensemble of configurations weighted without gauge constraint, but that each configuration should be gauge transformed into $F(U)=0$ gauge before evaluating $\Phi$. Only if the field $U$ has no gauge copies, i.e., $F_x(U^{V'})$ equals to zero for only one value of the gauge parameter $V'$, then $J(U)$ is determined by fluctuations about a single field $U^{V'}$, and eq.(3) provides an unambiguous definition of the average functional $\langle \Phi \rangle$.

On the perturbative language the existence of the gauge copies can be interpreted as a problem of the gauge copies of the 'vacuum' configurations, i.e. the solutions of the classical equations of motion. Assuming that every configuration is some small fluctuation about the corresponding 'vacuum' one can find all gauge copies of this configuration provided all gauge copies of the 'vacuum' are known.

Throughout this work the 4$d$ lattice with periodic boundary conditions is considered. $N_\mu$ is the lattice size in the direction $\mu$, and $V_4 = N_1 N_2 N_3 N_4$. The lattice derivatives are $\partial_\mu f(x) = f(x + \hat{\mu}) - f(x)$ and $\bar{\partial}_\mu f(x) = f(x) - f(x - \hat{\mu})$, and the lattice spacing is chosen to be unity.
2 Classical solutions in the compact $U(1)$ theory

In the case of the $U(1)$ gauge group $U_{x\mu} = e^{i\theta_{x\mu}} \in U(1)$, and the Wilson action $S_W(U)$ is

$$S_W(U) = \frac{1}{g^2} \sum_x \sum_{\mu\nu=1}^4 (1 - \cos \theta_{x,\mu\nu}) \quad ; \quad \beta = \frac{2}{g^2} ,$$

where $\theta_{x,\mu\nu} = \partial_\mu \theta_{x\nu} - \partial_\nu \theta_{x\mu}$ are the plaquette angles. It is important to note that all link angles $\theta_{x\mu}$ are compact variables ($-\pi < \theta_{x\mu} \leq \pi$), and the gauge transformations $\theta_{x\mu} \to \theta_{x\mu} - \partial_\mu \Omega_x$ are understood modulo $2\pi$.

The plaquette angle $\theta_P \equiv \theta_{x;\mu\nu}$ can be split up:

$$\theta_P = [\theta_P] + 2\pi n_P ,$$

where $[\theta_P] \in (-\pi;\pi]$ and $n_P = 0, \pm 1, \pm 2$. The plaquettes with $n_P \neq 0$ are called Dirac plaquettes. The dual integer valued plaquettes $m_{x,\mu\nu} = \frac{1}{2} \varepsilon_{\mu\rho\sigma} n_{x,\rho\sigma}$ form Dirac sheets [4].

The classical equations of motion are

$$\left. \frac{\partial S_W(U)}{\partial \theta_{x\mu}} \right|_{\theta=\theta^{cl.}} = \frac{2}{g^2} \sum_\nu \bar{\partial}_\nu \sin \theta_{x,\mu\nu} = 0 .$$

The $x$–independent solutions of these equations $\theta^{cl.}_{x\mu} = \phi_\mu$ (zero–momentum modes) and their influence on the gauge–dependent correlators were discussed in [5].

Evidently, any function $\bar{\theta}_{x\mu}$ is the solution of the eq.(6) if the corresponding plaquette angle $\bar{\theta}_P$ satisfies the following condition

$$\bar{\theta}_P = \begin{cases} 
2\pi + \Delta & \text{if } P = P^* ; \\
\Delta & \text{if } P \neq P^* ,
\end{cases}$$

where $P^*$ is some plaquette in the plane $(x_\rho;x_\sigma)$, and $\theta_{x\mu} = 0$ for $\mu \neq \rho;\sigma$. Periodicity presumes

$$\sum_{P \in (x_\rho;x_\sigma)} \bar{\theta}_P = 0 , \quad \Delta = -\frac{2\pi}{N_\rho N_\sigma} .$$

Therefore, $P^*$ is a Dirac plaquette.

To derive the explicit solution of the eq.(6) satisfying the condition in eq.(7) it is convenient to start with a 4$d$ volume in the continuum. Let us choose $(\rho;\sigma) = (1;2)$, and let $B_3^{cont}(\vec{x}_\perp)$ be the third component of the magnetic field :

$$B_3^{cont}(\vec{x}_\perp) = H \left[ \delta(\vec{x}_\perp - \vec{x}_\perp^{(1)}) - \frac{1}{N_1 N_2} \right] , \quad \vec{x}_\perp = (x_1;x_2) ,$$

where $\vec{x}_\perp^{(1)}$ denotes the position of the string. The total flux through the plane $(x_1;x_2)$ equals to zero. It is an easy exercise to find the corresponding vector–potential $A_\mu^{cont}(x)$. Embedding the continuum solution into the lattice one can
define the link variables \( \theta_{x\mu} \) via an integral of the \( gA^\text{cont}_\mu(x) \) along the corresponding links. The constant \( H \) is defined to fulfill eq’s.(7,8). Choosing the Lorentz (or Landau) gauge

\[
\sum_\mu \bar{\partial}_\mu \theta_{x\mu} = 0 ,
\]

one arrives after some algebra at the expression for the single Dirac sheet

\[
\bar{\theta}_{x1}(\vec{R}) = \frac{2\pi i}{N_1N_2} \sum_{\vec{q}_\perp \neq 0} \frac{K_2}{K_2^\perp} \cdot e^{i\vec{q}_\perp(\vec{x}_\perp - \vec{R}) - \frac{1}{2}q^2} ; \\
\bar{\theta}_{x2}(\vec{R}) = -\frac{2\pi i}{N_1N_2} \sum_{\vec{q}_\perp \neq 0} \frac{K_1}{K_2^\perp} \cdot e^{i\vec{q}_\perp(\vec{x}_\perp - \vec{R}) - \frac{1}{2}q^2} ; \\
q_j = \frac{2\pi}{N_j} n_j; \quad n_j = -\frac{1}{2}N_j + 1; \ldots; \frac{1}{2}N_j ,
\]

where \( K_\mu = 2\sin \frac{q_\mu}{2} \) and \( K_2^\perp = K_1^2 + K_2^2 \). The twodimensional vector \( \vec{R} = (R_1; R_2) \) corresponds to the position of the Dirac plaquette in the \((x_1; x_2)\) plane:

\[
\bar{\theta}_{x:12} = 2\pi \cdot \delta_{x:1:R} - \frac{2\pi}{N_1N_2} .
\]

Of course, \( \bar{\theta}'_{x\mu} = \phi_\mu + \bar{\theta}_{x\mu} \) is also the solution of eq’s.(3,9).

It is easy to check that the single Dirac sheet solution \( \theta_{x\mu}(\vec{R}) \) corresponds to the local minimum of the action, i.e. it is stable with respect to small fluctuations. The existence of the long–living metastable states corresponding to single Dirac sheets was observed in simulations in the pure gauge \( U(1) \) theory \[4\].

In Figure 1(b) the dependence of the first component \( \bar{\theta}_{x1}(\vec{R}) \) on \( x_1(x_2) \) is shown for different values \( x_1(x_2) \). It demonstrates a characteristic kink–like behavior along the axis \( x_2 \). The second component \( \bar{\theta}_{x:2} \) shows the similar behavior along the axis \( x_1 \).

The classical gauge action \( S_{cl.} = S(\bar{\theta}) \) is

\[
S(\bar{\theta}) = \frac{2V_4}{g^2} \left( 1 - \cos \Delta \right) .
\]

The value \( S(\bar{\theta}) \) depends on the geometry of the lattice. Let us choose \( N_s = N_1 = N_2 = N_3 \to \infty \). On a symmetric lattice \( N_4 = N_s \) the action \( S(\bar{\theta}) \) is non–zero and finite :

\[
S(\bar{\theta}) = \frac{4\pi^2}{g^2} < \infty .
\]
In the zero–temperature limit, i.e. \( N_4 \to \infty, N_s/N_4 \to 0 \), the classical action is

\[
S(\bar{\theta}) = \frac{4\pi^2 N_s}{g^2 N_4} \to 0 ,
\]

if the Dirac plaquette is time–like, i.e. \((\rho; \sigma) = (4; i), \ i = 1, 2, 3\). In the finite–temperature limit, i.e. \( N_4/N_s \to 0 \), the action is

\[
S(\bar{\theta}) = \frac{4\pi^2 N_4}{g^2 N_s} \to 0 ,
\]

if the Dirac plaquette is space–like.

Another possible solution of the classical equation of motion – double Dirac sheet – consists of the two single Dirac sheets with an opposite orientation of the flux :

\[
\bar{\theta}_{x_i}(\vec{R}^a; \vec{R}^b) = \bar{\theta}_{x_i}(\vec{R}^a) - \bar{\theta}_{x_i}(\vec{R}^b), \quad i = 1; 2 , \tag{12}
\]

where vectors \( \vec{R}^a \) and \( \vec{R}^b \) correspond to the two Dirac plaquettes in the plane \((x_1; x_2)\). It is easy to see that

\[
\bar{\theta}_{x;12}(\vec{R}^a; \vec{R}^b) = 2\pi \cdot \left[ \delta_{\vec{x}_\perp;\vec{R}^a} - \delta_{\vec{x}_\perp;\vec{R}^b} \right] . \tag{13}
\]

The double Dirac sheet \( \bar{\theta}_{x;i}(\vec{R}^a; \vec{R}^b) \) has a zero action: \( S(\bar{\theta}) = 0 \).

Gauge transformations can shift the Dirac sheets and change their form. For example, the ‘big’ gauge transformation function \( \Omega_x \)

\[
\Omega_x = -\frac{2\pi}{N_1 N_2} \sum_{\vec{q}_\perp \neq 0} \frac{e^{i\vec{q}_\perp \cdot (\vec{x}_\perp - \vec{R})}}{K_{\vec{q}_\perp}^2} \cdot \left( 1 - e^{-iq_2} \right) ; \tag{14}
\]

\[
\Box \Omega_x = 2\pi \cdot \left[ \delta_{\vec{x}_\perp;\vec{R}} - \delta_{\vec{x}_\perp;\vec{R}^{-1}} \right] , \quad \Box = \sum_\mu \partial_\mu \partial_\mu ,
\]

shifts the single Dirac sheet in the \( x_1 \)–direction :

\[
\bar{\theta}_{x_1}(\vec{R}) = \bar{\theta}_{x_1}(\vec{R} - \hat{1}) ;
\]

\[
\bar{\theta}_{x_2}(\vec{R}) = \bar{\theta}_{x_2}(\vec{R} - \hat{1}) - \left[ 2\pi \cdot \delta_{\vec{x}_\perp;\vec{R}} - \frac{2\pi}{N_1 N_2} \right]. \tag{15}
\]

The gauge transformation \( \Omega_x \) in eq.(14) applied to the zero–field creates a double Dirac sheet as in eq.(12) with \( \vec{R}^a = \vec{R} \) and \( \vec{R}^b = \vec{R} - \hat{1} \) :
\[ \partial_1 \Omega_x = -\tilde{\theta}_{x1}(\vec{R}; \vec{R} - \hat{1}) ; \]
\[ \partial_2 \Omega_x = -\tilde{\theta}_{x2}(\vec{R}; \vec{R} - \hat{1}) - 2\pi \cdot \delta_{x_{\perp};\vec{R}} + \frac{2\pi}{N_1N_2} . \]

Therefore, \( \tilde{\theta}_{xi}(\vec{R}^a; \vec{R}^b) \) is a Gribov copy of the zero solution \( \theta_{x\mu}^d = 0 \).

It is not difficult now to obtain the general Dirac sheet solutions, i.e. the Dirac sheets curved in the four-dimensional space. As an example, let us define the ‘big’ gauge transformation \( \tilde{\Omega}_x \) depending on \( (\vec{x}_{\perp}; x_3) \):

\[ \tilde{\Omega}_x = \delta_{x_{\perp};\xi_3} \cdot \Omega_x , \quad (16) \]

where \( \Omega_x \) is defined in eq. (14), and \( \xi_3 \) is some number. After gauge transformation

\[ \tilde{\theta}_{x\mu}(\vec{R}) \xrightarrow{\tilde{\Omega}} \tilde{\theta}_{x\mu} = \tilde{\theta}_{x\mu}(\vec{R}) - \partial_{\mu} \tilde{\Omega}_x , \quad (17) \]

the flat Dirac sheet \( \tilde{\theta}_{x\mu}(\vec{R}) \) (Figure 2a) changes its shape as shown in Figure 2b. The new field \( \theta^\Omega_{x\mu} \) does not fulfill the Lorentz gauge:

\[ \sum_{\mu} \partial_{\mu} \tilde{\theta}^\Omega_{x\mu} = -\Omega_x \cdot \vec{\partial}_3 \delta_{x_{\perp};\vec{R}} \neq 0 \]

To restore the Lorentz gauge one should make an additional gauge transformation \( \tilde{\theta}^\Omega_{x\mu}(\vec{R}) \xrightarrow{\omega} \tilde{\theta}^{\tilde{\Omega}}_{x\mu}(\vec{R}) - \partial_{\mu} \omega_x \), where

\[ \omega_x = \frac{2\pi i}{V_3} \sum_{\vec{q}_{\perp} \neq 0} \sum_{q_3} e^{i\vec{q}_{\perp}(\vec{x}_{\perp} - \vec{R}) + iq_3(x_3 - \xi_3) - \frac{1}{2}q_3} \cdot \frac{K_2 \vec{K}_{\perp}^2}{K^2 \vec{K}_{\perp}^2} \quad (18) \]

where \( \vec{K}^2 = \vec{K}_{\perp}^2 + K_3^2 \). Therefore, the successive application of the ‘big’ gauge transformations creates a Dirac sheet of any possible geometry in the 4d space.

The symbolic representation of the dependence of the action \( S(U) \) on the gauge field configuration \( \{U_{x\mu}\} \) is shown in Figure 3. The absolute minima correspond to double Dirac sheets, and the local minima correspond to single Dirac sheets. Of course, there are also the local minima corresponding to the two single Dirac sheets, etc. (are not shown). In principle, the existence of non–trivial classical solutions of other type can not be excluded.

It is interesting to discuss the role of the lattice (DeGrand–Toissaint) monopoles \[4\] in connection with the stability of the Dirac sheet solutions. The breaking of a periodically closed single Dirac sheet means the appearence of monopole–antimonopole pairs. Therefore, the tunneling from one ‘vacuum’ to another, i.e. the changing of the number of the single Dirac sheets, is accompanied by the creation of monopoles. The modification of the compact action which suppresses the monopoles \[6, 8\] prevents the breaking of the sheet and makes the tunneling impossible. In this case the choice of the initial configuration in numerical calculations will correspond to the choice of the ‘vacuum’.
It is easy to see that the single and double Dirac sheets are also the classical solutions of the non–abelian theories. For example, in the case of the $SU(2)$ gauge group the link variables

$$U_{x\mu}(\vec{R}_1; \vec{R}_2) \equiv \exp\left\{i\bar{\theta}_{x\mu}(\vec{R}_1; \vec{R}_2) \cdot \sigma_3\right\} \in SU(2)$$ (19)

satisfy evidently the classical equations of motion and correspond to the zero action.

### 3 The gauge–dependent photon correlator

As an example of the importance of the classical solutions $\theta_{x\mu}^{cl.} \neq 0$ in lattice calculations let us consider the gauge–dependent photon correlator $\Gamma_\mu(\tau; \vec{p})$

$$\Gamma_\mu(\tau; \vec{p}) = \langle O^*_\mu(\tau; \vec{p}) O_\mu(0; \vec{p}) \rangle = \frac{1}{N_4} \sum_{t=0}^{N_4-1} \langle O^*_\mu(t \oplus \tau; \vec{p}) O_\mu(t; \vec{p}) \rangle ;$$

$$O_\mu(\tau; \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x} - \frac{1}{2} \nu^2} \cdot \sin \theta_{x\mu} , \quad \mu = 1, 2, 3 ,$$ (20)

where $t \oplus \tau = t + \tau \mod N_4$. Evidently, $\langle O_\mu \rangle = 0$.

In [4] it was shown that in the Coulomb phase some of the gauge copies produce a photon correlator with a decay behavior inconsistent with the zero mass behavior. Numerical study [10] has shown that there is a connection between ‘bad’ gauge copies and the appearance of configurations with double Dirac sheets. Now we can explain this effect.

Let us choose the momentum $\vec{p} = (0; p_2; 0)$ with $p_2 \neq 0$, and $\mu = 1$. The perturbative expansion about the zero solution of the classical equation of motion $\theta_{x\mu}^{cl.} = 0$, i.e. the standard perturbation theory, gives in the lowest approximation

$$\Gamma_1^{stan.}(\tau; \vec{p}) \sim e^{-\tau E_p} + e^{-(N_4-\tau) E_p} ,$$ (21)

where the energy $E_p$ satisfies the lattice dispersion relation

$$\sinh^2 \frac{E_p}{2} = \sum_{i=1}^{3} \sin^2 \frac{P_i}{2} .$$ (22)

The non–zero solutions of the classical equations of motion $\theta_{x\mu}^{cl.} \neq 0$ have to be taken into account. Representing the gauge field in the form $\theta_{x\mu} = \theta_{x\mu}^{cl.} + gA_{x\mu}$ and expanding in powers of $g$ one obtains

$$O_1(\tau; \vec{p}) = \Phi(\tau; \vec{p}) + \frac{g}{V_3} \sum_{\vec{q}} A^q_1(\tau; \vec{q}) \cdot e^{\star a_1} \Psi(\tau; \vec{p} - \vec{q}) + O(g^2) ;$$

$$\Phi(\tau; \vec{q}) = \frac{1}{V_3} \sum_{\vec{x}} e^{-i\vec{q} \cdot \vec{x}} \cdot \sin \theta_{x1}^{cl.} ; \quad \Psi(\tau; \vec{q}) = \frac{1}{V_3} \sum_{\vec{x}} e^{-i\vec{q} \cdot \vec{x}} \cdot \cos \theta_{x1}^{cl.} .$$ (23)
In what follows the $O(g^2)$ term in the r.h.s. in eq.(23) is discarded. The transverse correlator $\Gamma_1(\tau; \vec{p})$ is

$$
\Gamma_1(\tau; \vec{p}) = \Gamma_1^{(0)}(\tau; \vec{p}) + g^2 \cdot \Gamma_1^{(1)}(\tau; \vec{p}) + \ldots ;
$$

$$
\Gamma_1^{(0)}(\tau; \vec{p}) = \frac{1}{N_4} \sum_{t=0}^{N_4-1} \Phi^*(t \oplus \vec{p}) \Phi(t; \vec{p}) ;
$$

$$
\Gamma_1^{(1)}(\tau; \vec{p}) = \frac{1}{2V_3} \sum_{\vec{q}} G(\tau; \vec{q}) \frac{1}{N_4} \sum_{t=0}^{N_4-1} \Psi^*(\tau \oplus t; \vec{p} - \vec{q}) \Psi(t; \vec{p} - \vec{q}) ,
$$

(24)

where $G(\tau; \vec{p})$ is

$$
G(\tau; \vec{q}) = \left[ 1 + \frac{K_2(q)}{2 \sinh E_q} \cdot \frac{d}{dE_q} \right] G_0(\tau; \vec{q}) ;
$$

(25)

$$
G_0(\tau; \vec{q}) = \frac{1}{2 \sinh E_q} \frac{1}{1 - e^{-N_4 E_q}} \cdot \left[ e^{-\tau E_q} + e^{-(N_4 - \tau) E_q} \right] + \ldots .
$$

It is easy to see that the expansion about a Dirac sheet solution gives a contribution to the correlator very different from that in eq.(21).

As an example, let us choose the flat double Dirac sheet with the space–like Dirac plaquettes $\theta_{x\mu} = \theta_{x\mu}(\vec{R}_1; \vec{R}_2)$ as defined in eq.(12). In this case $\Phi$ and $\Psi$ do not depend on $\tau$:

$$
\Phi(\vec{q}) = \frac{\delta_{q_3,0}}{N_1 N_2} \sum_{\vec{x}_\perp} e^{-i\vec{q}_\perp \cdot \vec{x}_\perp} \sin \theta_{x_1} ; \quad \Psi(\vec{q}) = \frac{\delta_{q_3,0}}{N_1 N_2} \sum_{\vec{x}_\perp} e^{-i\vec{q}_\perp \cdot \vec{x}_\perp} \cos \theta_{x_1} ,
$$

(26)

and the correlator in the double Dirac sheet background is

$$
\Gamma_1^{dds}(\tau; \vec{p}) = \left| \Phi(\vec{p}) \right|^2 + \frac{g^2}{2V_3} \sum_{\vec{q}} G(\tau; \vec{q} + \vec{p}) \cdot \left| \Psi(\vec{q}) \right|^2 .
$$

(27)

In Figure 4 the correlator $\Gamma_1^{dds}(\tau; \vec{p})$ is shown in comparison with the ‘standard’ photon correlator $\Gamma_1^{stan.}(\tau; \vec{p})$ on a $12^3 \times 24$ lattice for $\vec{p} = (0; \pi/6; 0)$ and $\beta = 1.1$. Both correlators are normalized to unity at $\tau = 0$. The $\tau$–independent term $\left| \Phi(\vec{p}) \right|^2$ in the r.h.s. in eq.(27) gives the dominant contribution which results in a photon correlator inconsistent with the zero mass behavior. This effect shows rather weak volume dependence.
4 Conclusions and discussions

The solutions of the classical equations of motion on a periodic lattice are found which correspond to the $U(1)$ single and double Dirac sheets. These solutions demonstrate the typical kink–like (or kink–antikink) behavior and have finite energy. The perturbative expansion should take into account the contribution of these stationary points.

The double Dirac sheets are the Gribov copies of the trivial solution $\theta_{x\mu}^d = 0$ with zero action. They can be created from the zero solution by the ‘big’ gauge–transformations $\Omega_x$. Therefore, the gauge invariant objects are not affected by the double Dirac sheets. On the contrary, the influence of the double Dirac sheets on gauge dependent values can be of crucial importance, as it was demonstrated on the example of the photon correlator.

The single Dirac sheets, i.e. the classical solutions corresponding to local minima of the action, deserve a special study. Contrary to the case of the double Dirac sheets the single Dirac sheets give a non–zero contribution to gauge invariant observables, e.g. average plaquette. This contribution is of nonperturbative nature, i.e. $\sim \exp\{-C/g^2\}$, and can mimic the ‘condensate’ contribution.

The tunneling from one ‘vacuum’ to another is accompanied by the creation of lattice monopoles. The proper modification of the action, i.e. the suppression of monopoles, excludes the tunneling making the ‘vacua’ stable. In this case the choice of the initial configuration in numerical calculations will correspond to the choice of the ‘vacuum’.

The single and double Dirac sheets are also the classical solutions of the non–abelian theories. Therefore, the gauge dependent gluon correlators are supposed to be strongly influenced by the Dirac sheets as it happens in the case of the photon correlator. This question as well as question about the connection of the Dirac sheets with confinement needs further clarification. This work is in progress.

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Figure Captions

**Figure 1** The link angle $\theta_{x1}$ as defined in eq. (10) on a lattice with $N_1 = N_2 = 12$. The lines are to guide the eyes.

**Figure 2** The flat Dirac string (a) and distorted Dirac string (b).

**Figure 3** The symbolic representation of the dependence of the action $S(U)$ on the gauge field configuration $\{U_{x\mu}\}$. The absolute minima correspond to double Dirac sheets, and the local minima correspond to single Dirac sheets. There are also the local minima corresponding to the two single Dirac sheets, etc. (not shown).

**Figure 4** The normalized correlators $\Gamma^{dds}_{1}(\tau; \vec{p})$ (circles) and $\Gamma^{stan.}_{1}(\tau; \vec{p})$ (diamonds) on a $12^3 \times 24$ lattice for $\vec{p} = (0; \frac{\pi}{6}; 0)$ and $\beta = 1.1$. 
Figure 1: The link angle $\theta_{x_1}$ as defined in eq.(10) on a lattice with $N_1 = N_2 = 12$. The lines are to guide the eyes.
Figure 2: The flat Dirac string (a) and distorted Dirac string (b).

Figure 3: The symbolic representation of the dependence of the action $S(U)$ on the gauge field configuration $\{U_{x\mu}\}$. The absolute minima correspond to double Dirac sheets, and the local minima correspond to single Dirac sheets. There are also the local minima corresponding to two single Dirac sheets, etc. (not shown).
Figure 4: The normalized correlators $\Gamma^{\text{dds}}_1(\tau; \vec{p})$ (circles) and $\Gamma^{\text{stan.}}_1(\tau; \vec{p})$ (diamonds) on a $12^3 \times 24$ lattice for $\vec{p} = (0; \frac{\pi}{6}; 0)$ and $\beta = 1.1$. 