Calculation of Internal Force and Deformation of Overlying Soil Corrugated Arch Bridge Based on Plate and Shell Theory

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Abstract. The shell is the general name for the flat plate and the shell. The mechanics of the plate and shell are similar to those of the elastic mechanics. The differential plate element is taken at any point in the plate or shell. From the perspectives of statics, geometry and physics, the equilibrium differential equations and geometric equations corresponding to the plate or shell are derived. And physical equations, these equations reflect the relationship and constraints between internal and external forces, strain and displacement, strain and stress. Under certain boundary conditions, the unknowns are determined by the above related basic equations.

1. Introduction
A shell is an object with two curved boundaries, and the distance between the two surfaces is much smaller than the other dimensions of the object. These two surfaces are called shell faces, the distance between the two surfaces is the thickness of the shell, and the surface that divides the thickness is called the middle surface. According to the ratio of the thickness of the casing to the minimum radius of curvature of the mid-surface of the casing, the casing can be divided into a thin shell and a thick shell. According to different shapes, the shell can be divided into a spherical shell, a cylindrical shell, a rotating shell, a translating shell and the like.

2. Equivalent simplified model of orthotropic shell of corrugated steel plate
For corrugated steel cylinder shells, where the presence of corrugations is similar to transverse ribs, the bending and tensile stiffness of the cylindrical shell in the direction of the arching is enhanced. Therefore, a simplified algorithm can be used to evenly distribute this reinforcement over the entire span width to form a structural anisotropy. This paper mainly performs the orthotropic equivalent of orthogonal steel plates.

For corrugated steel sheets, the corrugation direction is sinusoidal and the corrugation height function is:

\[ z = f \sin \left( \frac{\pi x}{l} \right) \]  

(1)

In the formula: \( f \) is the wave height, \( l \) and \( s \) are the chord length and arc length of the half wave, respectively, and \( s = l \left[ 1 + \left( \frac{\pi^2}{4} \right) \cdot \left( \frac{f^2}{l^2} \right) \right] \), as shown in Fig. 1.
3. The theory of cylindrical shell bending of corrugated steel arch bridge (culvert)

The example chosen in this paper is a test model of a covered corrugated steel arch bridge. It is established according to the actual project with a 1:2 ratio, as shown in Figure 2. The footprint is 12m × 5.2m, and the covering soil is silty sand. The minimum height of soil and covering soil is 0.9m, the weight of covering soil is 19kN/m³, the span is 3.7m, and the width of pavement is 5.2m. According to the design of first-class highway, the arch is connected with the foundation with 100mm×100mm×10mm angle steel.

The upper structure is a corrugated steel arch with a span of 3.7 m. The arch circle is a circular arc with a diameter of 4 m, a vector height of \( f_0 = 1.25m \), a sagittal ratio of \( f_0 / L_0 = 0.337 \), a bridge deck width of 5.2 m, and an abutment platform height of 1.25 m. The arch fill height is 0.9m, and the sand is backfilled. The model has a concrete retaining wall with a thickness of 25cm at both ends of the axis.

The steel plate used in the model is galvanized Q235A steel, the yield strength is 235 MPa, the allowable stress is 140 MPa, the wave form is 200 mm×55 mm, and the thickness is 5 mm.

4. Three-dimensional cylindrical shell bending theory verification

In order to verify the theory described above, firstly, the experimental model is used as a prototype, and the corrugated steel plate is replaced by a flat steel plate of equal thickness, which is calculated by MATLAB software, and the deformation and stress state of the structure under the soil load are obtained. The finite element analysis software is used to establish the model calculation and analysis. The analytical solution and the numerical solution are compared under the condition that the structural size, mechanical properties, boundary conditions and load forms are the same to verify the correctness of the theory.

In the process of MATLAB programming, the basic numerical calculation and matrix operation function of MATLAB software are mainly applied. After inputting various parameters and performing basic transformation operations, the linear non-homogeneous equations of the relevant formula are solved, as follows: Shown as follows:
B=[A1,B1,C1;A2,B2,C2;A3,B3,C3];
c=[-Oxnm,-Oynm,-Oznm]';
C=[B c]
n=3;
R_B=rank(B);
R_C=rank(C);
If R_B==R_C& R_B==n
   X=B\c
Elseif R_B==R_C& R_B<n
   X=B\c
   D=null(B,'r')
Else X=' No solution to the equations '
End

Run the above program to obtain the solution X, the three elements in X are the coefficients in the displacement function, so that the deformation and law of the structure can be obtained through the expressions of the three displacement functions, and then through the geometric equation and The operation of the physical equation gives the stress state of the structure.

The finite element analysis of the model is carried out by the large-scale finite element software ANSYS. The model of the steel plate in the model is shell 93 shell unit, and the four sides of the shell are simply supported according to the boundary conditions. According to the current highway bridge code, the soil is transformed into vertical and lateral loads, and then the loads in the two directions are transformed into the forces loaded on the casing according to the ANSYS mid-surface load, as shown in Fig. 3. Show.

![Finite element model of ANSYS](image)

**Fig. 3** Finite element model of ANSYS

The analytical solution calculated according to the three-dimensional plate-shell bending theory shows that the structure has the largest deformation in the span, the deformation value is -0.32mm; the maximum stress value also appears at the center of the structure, which is 22.5MPa; the result of the finite element analysis is in the deformation. The regularity and stress state agree well with the analytical solution, but the numerical value is deviated. The maximum displacement value is -0.61mm and the maximum stress value is 14.3MPa. The main factors causing this error are: First, the analytical derivation process uses a lot of simplification, and ignores the strain and compressive stress perpendicular to the mid-surface direction, which is different from the fineness of the finite element analysis; secondly, the two the treatment method of the soil-covering load is different. The finite element is processed according to the unit surface force, and the theoretical derivation is processed by the triangle series, which also causes the calculation result to deviate.
5. Actual engineering calculation results

Through the previous calculations and the finite element calculation results, the analytical solution of the three-dimensional cylindrical shell bending problem based on the shell-and-shell theory proposed in this paper can be proved. It has certain computational reliability and precision. Based on this theory, the real bridge experimental model mentioned is calculated and analyzed.

In order to use this calculation method, the corrugated steel plate is first equivalent to the orthotropic plate according to the method proposed in the present paper. Taking the elastic modulus in two directions separately. In order to use this calculation method, the corrugated steel plate is first equivalent to the orthotropic plate according to the method proposed in the present paper. According to the corrugated plate type of the test arch bridge, the elastic modulus in two directions is calculated as: 257 MPa, 40.6×106 MPa.

The construction process of the overlying corrugated steel plate arch culvert is to excavate the foundation pit, the foundation and the abutment construction, assemble the corrugated steel arch and cover the soil backfill. After the corrugated steel plate is assembled, the soil will be backfilled layer by layer. During the backfilling process, the corrugated steel shell structure will also undergo a series of stressing processes. When backfilling, in the cross section, the soil is backfilled from both sides of the arch. The standard stipulates that the height difference between the sides of the arch should not exceed 40cm. If the height difference between the two sides is too large, the structure will be greatly deformed by the lateral load. In the axial direction of the arch culvert, the soil is backfilled from the two holes in the middle direction.

Each time the backfill is backfilled, it has a total of 9 stages and 9 layers of backfill. From the first stage to the fifth stage, before the top of the soil, each backfilling height is about 25cm, and the sides of the arch are backfilled at the same time; after the capping, the backfill is filled according to each layer of 20cm, the last layer is 30cm, and the total height of the backfill soil is 0.25×5+0.2×3+0.3=3.15m. The hierarchical diagram is shown in Figure 4:

![Fig. 4 Layered backfill](image)

In this paper, the MATLAB software is used to simulate the backfilling process. The above theory is used to calculate the bending problem of the corrugated steel bridge. The relationship between the displacement of the mid-surface of the corrugated steel shell L/2 and the height of the overlay is shown in Fig. 5.
By comparing the above theoretical calculations with the experimental data, it can be found that the stress values of the two are relatively close, and the development and change laws of the structural displacement response are also consistent. In the process of backfilling, when the backfill reaches a certain height and the shell In the case where there is no soil covering above, the two sides of the shell structure are subjected to large extrusion to produce a large reverse arching upturn, and the warping height reaches a maximum when the covering cover is near the top of the casing, with the shell When the overburden is increased, the pressure above is gradually increased, and the upturned deformation is gradually reduced. The use of corrugated steel plate greatly increases the structural rigidity, and the deformation value is smaller than that of the flat steel plate arch. This is reasonable, but there is a large error compared with the actual measurement. The main factors causing this situation are: (a). The theoretical model is based on the principle of elastic mechanics and fails to consider the complexity of the actual structural material properties; (b). In the process of processing the boundary conditions of the structure, the ideal four-sided simple support boundary is adopted, which is also different from the actual structure; (c). When the corrugated steel plate is equivalent to an orthotropic flat plate, the mechanical properties of the actual corrugated plate are different; (d). The calculation model does not consider the influence of the soil-junction interaction, but simply loads it equivalent to the soil load on the structure, which also produces some error; (e). The connection between the steel plates of the experiment itself, the structure of the arches, the deformation of the foundation, and the measurement accuracy also bring about certain errors.

In summary, the three-dimensional cylindrical shell bending theory proposed in this paper has certain guiding significance in analyzing the displacement response law and stress state of corrugated steel bridges and culverts, but for different structural linear shapes and different boundary conditions, it is more in the mathematical processing process. For the complexity, the difficulty of analytical derivation is increased. The analytical solution obtained in this paper can be applied to the basic calculation and reference in the engineering selection stage. The design and calculation of the actual complex structure are mostly based on the finite element method.

6. Summary
According to the boundary conditions of the corrugated steel arch bridge, the double triangular series solution of the simply supported cylindrical shell under the soil load is proposed. Taking a corrugated steel plate bridge built in the experiment as an engineering example, the above theory is used to verify the calculation and compare with the experimental data. The results show that the developmental variation of the structural displacement response and the stress state are more consistent. The method can be applied to the basic calculation and reference of the engineering selection stage.
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