Spontaneous Analogy by Piggybacking on a Perceptual System

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Abstract

Most computational models of analogy assume they are given a delineated source domain and often a specified target domain. These systems do not address how analogs can be isolated from large domains and spontaneously retrieved from long-term memory, a process we call spontaneous analogy. We present a system that represents relational structures as feature bags. Using this representation, our system leverages perceptual algorithms to automatically create an ontology of relational structures and to efficiently retrieve analogs for new relational structures from long-term memory. We provide a demonstration of our approach that takes a set of unsegmented stories, constructs an ontology of analogical schemas (corresponding to plot devices), and uses this ontology to efficiently find analogs within new stories, yielding significant time-savings over linear analogy retrieval at a small accuracy cost.

1 Spontaneous Analogy

In our day-to-day experience, we often generate analogies spontaneously (Wharton, Holyoak, & Lange, 1996; Clement, 1987). That is, with no explicit prodding, we conjure up analogs to aspects of our current situation. For example, while reading a story, we may recognize a plot device that is analogous to one used in another story that we read long ago. The shared plot device may be a small part of each story, it is usually not explicitly delineated for us or presented in isolation from the rest of the story, and we may recognize the analogy of the plot device even if the general plots of the two stories are not analogous. Somehow, we segment out the plot device and retrieve the analog from another story in long-term memory. Spontaneous analogy is the process of efficiently retrieving an analog from long-term memory given an unsegmented source domain such that part of the source shares structural similarity with the analog, though they might not share surface similarity. This process differs from standard models of analogy, which are given a delineated source concept, and often a target concept. Given a pair of analogs, analogical mapping is relatively straightforward. The more difficult problem is finding the analogs to begin with. As Chalmers, French, and Hofstadter (1992) argue “when the program’s discovery of the correspondences between the two situations is a direct result of its being explicitly given the appropriate structures to work with, its victory in finding the analogy becomes somewhat hollow”.

1 In our terminology, an analog is a substructure of a domain that is structurally similar to a substructure of another domain, and an analogical schema is a generalization of an analog. For example, an input domain might be the entire story of Romeo & Juliet, an analog would be the part of the story where Romeo kills Tybalt, who killed Romeo’s friend, Mercutio (like in Hamlet where Hamlet kills Claudius, who killed Hamlet’s father), and an analogical schema would be the generalized plot device of a “revenge killing”.

2 Spontol is short for “spontaneous analogy using the Ontol ontology learning and inference algorithm”.

Figure 1: An analog of Analogical Mapping vs. Spontaneous Analogy. In Analogical Mapping (a), we are given an explicit source and target, free from interfering context. In spontaneous analogy (b), the analogs are spontaneously retrieved from long-term memory.

The process of spontaneous analogy shares some properties with low-level perception, as exemplified in Figure 1. Within seconds of being presented with a visual image of a pterodactyl flying over a canyon, one can typically describe the image using the word “pterodactyl”, even if one has had no special explicit recent priming for this concept, indeed even if one has not consciously thought about pterodactyls for several years. For us to produce the word “pterodactyl”, we must segment the pterodactyl from the canyon and retrieve the “pterodactyl” concept from the thousands of concepts stored in memory. We must have learned the “pterodactyl” concept to begin with from unsegmented images. This perceptual process is robust to noise: The pterodactyl in the image could be partially occluded, ill-lit, oddly colored, or even drawn as a cartoon, and we are still able to correctly identify this shape (to a certain point). Likewise, many details of the plot devices from the above story example could be altered or obfuscated, but this analogy would degrade gracefully.

Our primary technical contribution in this paper is an algorithm called Spontol that solves the problem of spontaneous analogy: efficient parsing, storage, and retrieval of analogs from long-term memory. That is, given a corpus of many large unsegmented relational structures, Spontol discovers analogical schemas that are useful for characterizing the corpus and efficiently retrieves analogs given a new structure. E.g., given a set of narratives in predicate form, Spontol discovers plot...
devices and analogs between the stories. We know of no prior work that scales to this task when the number of narratives and statements per narrative are both in the hundreds.

In the remainder of this paper, we describe related work (Section 2), give background on perceptual systems (Section 3), describe the Spontol algorithm, which transforms the problem of spontaneous analogy into a “perceptual” problem (Section 4), demonstrate Spontol’s performance on a story database (Section 5), discuss implications and shortcomings of Spontol, and conclude (Section 6).

2 Related Work

There has been earlier work on the problem of analogy in the absence of explicitly segmented domains. The COWARD system (Baldwin & Goldstone, 2007) addresses this problem by searching for mappings within a large graph, essentially searching for isomorphic subgraphs. SUBDUE (Holder, Cook, & Djoko, 1994) compresses large graphs by breaking them into repeated subgraphs, but is limited in that its output must be a strict hierarchy, and would be unable to discover the lattice structure of the concepts in Figure 2. Nauty (McKay, 1981) uses a number of heuristics to efficiently determine whether one graph is a subgraph of another, but this must be given source and target graphs to begin with. We can also apply The Chunker (described in Section 3) to feature bag graphlet kernels (Shervashidze, Vishwanathan, Petri, Mehlhorn, & Borgwardt, 2009), which are related to Spontol’s transform T in that both represent partial graphs, but this earlier work applies only for cases where there is one kind of entity, one kind of relation, and only binary relations, while Spontol works for multiple kinds of entities and relations, including relations of large arity.

The MAC phase of MAC/FAC (Forbus, Gentner, & Law, 1995) bears some relation to our spontaneous analog retrieval. MAC uses vectors of content, such as the number of nodes and edges in a graph, as a heuristic for analog retrieval. However, in cases where the subgraph in question is a part of a much larger graph, the heuristics that MAC uses are drowned out by the larger graph. Likewise, ARCS (Thagard, Holyoak, Nelson, & Gochfeld, 1990) also assumes that analogs have been delineated (i.e., it matches an entire source domain, rather than a substructure). SEQL (Kuehne, Forbus, Gentner, & Quinn, 2000) generalizes relational concepts, but doesn’t build a hierarchical ontology of analogical schemas.

There has been some work on representing structures as feature vectors. For example, Holographic Reduced Representations have been used to implement Vector Symbolic Architectures in which there is a correlation between vector overlap and structural similarity (Gayler & Levy, 2009). This work is limited in that it requires vectors of length 10,000 to represent very small graphs (≤10 nodes), and only represents binary relations of a single type, so this approach is not directly extendable to relational structures such as the stories in our demonstration. This is also a limitation for the system proposed by Rachkovskij, Kussul, and Baidyk (2012). Both these systems are also limited in that they are unable to exploit partial analogical schemas. That is, a partial overlap in these systems’ vectors does not correspond to a common subgraph in the corresponding structures. These systems stand in contrast to Spontol, which is able to represent larger structures and efficiently find common substructures.

3 Background: Perceptual Systems

Spontol transforms relational structures into feature bags so that their surface similarity corresponds to the structural similarity of the relational structures. After Spontol has made this transformation, the problem of spontaneous analogy is reduced to the problem of feature overlap, and any of several existing “perceptual” systems can be used to find and exploit patterns in feature vectors. Our implementation of Spontol uses a model inspired by the human sensory cortices (auditory, visual, tactile) called Ontol (Pickett, 2011). Ontol is a pair of algorithms, both of which are given “sensor” inputs (fixed-length, real-valued non-negative vectors). The first algorithm constructs an ontology that concisely encodes the inputs. For example, given a set of vectors representing visual windows from natural images, Ontol produces a feature hierarchy loosely modeled on that seen in the visual cortex. The second algorithm takes as input an ontology (produced by the first algorithm) and a new vector, and parses the vector. That is, it produces as output the new vector encoded in the higher-level features of the ontology. In addition to “bottom-up” parsing, the second algorithm also makes “top-down” predictions about any unspecified values in the vector.

Ontol is ignorant of the modality of its input. That is, Ontol is given no information about what sensory organ is producing its inputs. Because of this ignorance, we are able to leverage Ontol to find patterns in abstract “sensory” inputs that are actually encodings of relational structures.

Ontology Learning

Ontol’s ontology formation algorithm, called The Chunker, seeks to find concepts (or chunks) that allow for concise characterization of vectors. Since chunks themselves are vectors, The Chunker is applied recursively to create an ontology. In essence, this algorithm is similar to the recursive block pursuit algorithm described by Si and Zhu (2011) in that both search for large frequently occurring sets of features. The Chunker differs in that it allows for multiple inheritance, while recursive block pursuit creates only strict tree structures. In Section 4, we show the importance of this property for finding multiple analogical schemas within a single relational structure. For simplicity, we describe the discrete binary version of The Chunker algorithm (chunk(B), which takes as input a set B of feature bags and produces an ontology Ω) provided by Pickett (2011), but this can be modified for continuous vectors. In this version, each vector is treated as a set, with a value of 1 for feature f signifying inclusion of f in the set, and a value of 0 signifying exclusion.

The Chunker searches for intersections among existing feature bags and proposes these as candidates for new concepts.
Each candidate is evaluated by how much it would compress the ontology, then the best candidate is selected and added to the set of feature bags, and the process is repeated until no candidates are found that further reduce the description length of the ontology. Figure 2 shows the ontology constructed by this algorithm when applied to an animal dataset, where the “sensory percepts” are features for each animal.

Although the parsing problem is NP-complete, a single bottom-up pass can be performed in logarithmic time (Pickett, 2011). Importantly, Ontol examines only a small subset of the concepts and instances while parsing. This means that, when judging concept similarity, Ontol does not need to compare each of its $n$ nodes. This property is important for spontaneous analog retrieval (described below).

### 4 Analogy as Perception

We now describe a method for transforming relational structures into sparse feature vectors (or feature bags) such that the problem of analog retrieval is reduced to the problem of percept parsing. An example of this process is shown for the Sour Grapes fable in Figure 5. For this process, we rely on a transform $T$ (described below) that takes a small relational structure and converts it into a feature bag (exemplified in Figure 5(c)). The size of relational structure is limited for $T$ because $T$’s runtime is quadratic in the size of the structure. We view this limitation as acceptable because people generally cannot keep all the details of an entire lengthy novel (or all the workings of a car engine) in working memory. Generally, people focus on some aspect of the novel, or some abstracted summary of the novel (or engine). Therefore, we break each large relational structure into multiple overlapping windows. A window is a small set of connected statements, where two statements are connected if they share at least one argument. Spontol exploits a principle akin to one used by the HMax model of the visual cortex (Riesenhuber & Poggio, 1999): as the number of windows for a relational structure increases, the probability decreases that another structure has the same windows without being isomorphic to the first.

The process for building an ontology of analogical schemas from large relational structures, called Spontol-Build, is described in Figure 3. This algorithm extracts $numWindows$ windows from each large relational structure and transforms them into feature bags (exemplified in Figure 5(d)) and chunks these feature bags to create an ontology of windows called windowOntology. Spontol-Build then re-encodes the windows by parsing them using this ontology, and re-encodes the larger structures (from which the windows came) as a feature bag of the parsed windows. Finally, Spontol-Build runs another pass of chunking on the re-encoded structures to generate the schema ontology.

The process of spontaneous analog retrieval, called Sponton-Retrieve, is given in Figure 4. When given a new relational structure $s$, we encode $s$ by extracting windows from it, parsing these using the windowOntology, then parsing the feature bag representation using the schemaOntology. This yields a set of schemas that are contained in $s$.

### Transforming Small Relational Structures

Here, we describe an operation $T$, which transforms a (small) relational structure into a feature bag. In our demonstration, we assume that the relational structure is described in predicate logic, but our approach is not limited to this representation. We consider a relational structure to be a

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A full description and implementation of The Chunker, as well as source code for our demonstration of Spontol can be downloaded at [http://marcpickett.com/src/analogyDemo.tgz](http://marcpickett.com/src/analogyDemo.tgz).
Figure 3: Spontol’s Ontology Learning Algorithm

```
// Finds an ontology of schemas given a set of structures S.
// numWindows is the number of windows to grab per structure.
// windowSize is the number of statements per window.
define Spontol-Build (S, numWindows, windowSize)
// Randomly grab windows from each structure, // and transform them into feature bag form.
foreach s ∈ S; for i = 1, ..., numWindows
    let w_i = grabConnectedStatements (s, windowSize)
    add T (w_i) to allWindows
// Run The Chunker to generate the window ontology windowOntology = chunk (allWindows)
// Re-encode each structure using the reduced-size windows.
foreach s ∈ S; for i = 1, ..., numWindows
    add parse (T (w_i), windowOntology) to bigWindows,
// Run The Chunker to generate the schema ontology.
schemaOntology = chunk (bigWindows)
return schemaOntology, windowOntology
```

Figure 4: Spontol’s Spontaneous Analogy Algorithm

```
// Finds analagous schemas for relational structure s.
// schemaOntology is the schema ontology.
// windowOntology is the window ontology.
// numWindows is the number of windows to grab per structure.
// windowSize is the number of statements per window.
define Spontol-Retrieve (s, windowSize)
// Randomly grab windows from s, // and transform them into feature bag form.
for i = 1, ..., numWindows
    w_i = grabConnectedStatements (s, windowSize)
    // Parse bag, the bag representation of s relevantSchemas = parse (bag, schemaOntology)
return relevantSchemas
```

set of relational statements, where each statement is either a relation (of fixed arity) with its arguments, or the special relation sameAs, which uses the syntax sameAs <name> (<rel> <arg1> <arg2> ...). The sameAs relation allows for statements about statements. E.g., the statements in Figure 5(b) encode (among other things) that “a fox decides that the grapes are sour”. Given a small relational structure s (< 10 statements), T transforms s into a feature bag using a variant of conjunctive coding. That is, T breaks each statement into a set of roles and fillers. For example, the statement want Of3Fox Of3Grapes has two roles and fillers, namely the two arguments of the want relation. So T breaks this statement into want1=Of3Fox and want2=Of3Grapes, where want2 means the 2nd argument of want (i.e., the “wanted”). Then T creates one large set of all the roles and their fillers. If there are multiple instances of a relation, it gives them an arbitrary lettering (e.g., wantB1=Of3Fox). T makes a special case for the sameAs relation. In this case, T uses a dot operator to replace the intermediate variable. For example, the statements sameAs f35 (decide Of3Fox f36) and sameAs f36 (sour Of3Grapes) would yield decide1.dot=Of3Fox soursOf3Grapes. The dot operator allows T to encode nested statements (i.e., statements about statements). Given a set of roles and fillers, T then chains the fillers to get filler equalities. For example, if we have that decide1=Of3Fox and want1=Of3Fox, then chaining gives us decide1=want1. Chaining is essential for recognizing structural similarity between relational structures, and allows us to side-step a criticism of conjunctive coding and tensor products: that the code for wantB1=Of3Fox may have no overlap with the code for want1=Of3Fox (Hummel et al., 2004). Chaining introduces the code for wantB1=want1, which makes the similarity apparent when searching for analogs (these “chained” features are a core difference between MAC’s content vectors and our feature bags). After
chaining the roles and fillers, T treats each of these role-filler bindings as an atomic feature. Note that, when we treat roles and fillers as atomic features, Ontol doesn’t recognize overlap among feature bags unless they share exactly the same feature. For example, the atomic feature wantB1=Of3Fox has no more resemblance to want1=Of3Fox for Ontol than it does for any other feature. Also note that the ordering of the roles in each feature is arbitrary but consistent (T uses reverse alphabetical order), so there is a men1=incapable1 feature, but not an incapable1=men1 feature. The left side of Figure 5(c) shows a window taken from the sour grapes story from Figure 5(b). On the right side is the feature bag transform of this set of 6 statements, consisting of 11 atoms.

5 Demonstration

We applied Spontol to a database of 126 stories provided by Thagard et al. (1990). These include 100 fables and 26 plays all encoded in a predicate format, where each story is a set of unsorted statements. An example story in predicate form is shown in Figure 5(b). Note that although the predicates and arguments have English names, our algorithm treats all these as gensyms except for the special sameAs relation. In this encoding, the smallest story had 5 statements, while the largest had 124 statements, with an average of 39.5 statements.

We ran Spontol-Build on these stories using numWindows = 100 and windowSize = 20 which produced an ontology of stories, part of which is shown in Figure 6. In this figure we see a “Double Suicide” analogical schema found in both Romeo & Juliet and in Julius Caesar. In the former, Romeo thinks that Juliet is dead, which causes him to kill himself. Juliet, who is actually alive, finds that Romeo has died, which causes her to kill herself. Likewise, in Julius Caesar, Cassius kills himself after hearing of Titinius’s death. Titinius, who is actually alive, sees Cassius’s corpse, and kills himself. The largest schema found (in terms of number of outgoing edges) was that shared by Romeo & Juliet and West Side Story, which are both stories about lovers from rival groups. The latter doesn’t inherit from the Double Suicide schema because Maria (the analog of Juliet), doesn’t die in the story, and Tony (Romeo’s analog) meets his death by murder, not suicide. Some of the schemas found were quite general. For example, the oval on the lower right with 6 incoming edges and 3 outgoing edges corresponds to the schema of “a single event has two significant effects”. And the oval above the Double Suicide oval corresponds to the schema of “killing to avenge another killing”.

Spontol-Retrieve uses this schema ontology to efficiently retrieve schemas for a new story, which can be used to make inferences about the new story in a manner analogous to the “goldfish” example from Section 3. To evaluate the efficiency of Spontol-Retrieve, we randomly split our story dataset into 100 training stories and 26 testing stories. We then used an ontology learned from the training set, and measured the number of comparisons needed to retrieve schemas (during parse) for the testing set. We compare this approach to MAC/FAC, which, during the MAC phase, visits each of the 100 training stories. Whereas MAC/FAC returns entire stories, Spontol-Retrieve returns analogical schemas (just as a visual system would return a generic “pterodactyl” concept rather than specific instances of pterodactyls). For comparison, we modify Spontol-Retrieve to return the set of instances that inherit from relevantSchemas, rather than just the schemas.

|          | Accuracy          | Average # Comparisons |
|----------|-------------------|-----------------------|
| MAC/FAC  | 100.00% ± .00%    | 100.00 ± .00          |
| Spontol  | 95.45% ± .62%     | 15.43 ± .20           |

Results are shown in Table 1, averaged over 100 trials. We show accuracy (and standard error) for both systems mea-
sured as the percentage of stories correctly retrieved, where a story was determined to be correct if it was retrieved by MAC/FAC. Spontol effectively improves on a linear (in the number of structures) case-by-case comparison to an “indexed” logarithmic-time look-up at a slight cost of accuracy. Therefore, Spontol requires orders of magnitude fewer comparisons than MAC/FAC, or any linear look-up algorithm (for a survey, see (Rachkovskij et al., 2012)). For larger datasets, we hypothesize that these differences will be even more pronounced. Although each comparison by both MAC and Spontol-Retrieve is a fast vector operation, for very large datasets (e.g., $10^9$ relational structures), even a linear number of vector operations becomes impractical. In future work, we will test these systems on a broader range of relational datasets to help elucidate the conditions under which Spontol yields high accuracy and very-low retrieval cost.

6 Conclusion

The chief contribution of this paper is a demonstration of a system, Spontol, that is able to solve the problem of spontaneous analogy. That is, we have demonstrated how Spontol can efficiently store and retrieve analogs without the need of human delineation of schemas.

Our representation also offers a new solution for the binding problem for long-term (static) memory that allows for efficient analog retrieval in the absence of explicitly segmented domains. The binding problem asks how we can meaningfully represent bindings between roles and fillers. Most solutions to the binding problem in connectionism do so in terms of temporal synchronicity (e.g., LISA (Hummel & Holyoak, 2005)). Temporal synchronicity only works for knowledge in working memory, and these models typically address storage in long-term memory by relying on some form of conjunctive coding or tensor products. Though these systems fail to address how relational structures can be efficiently retrieved from long-term memory, we hypothesize that a working-memory system, such as LISA, is necessary for the “chaining” process on which our system relies.

Spontol may offer evidence in support of a uniform “substrate” of intelligence (Mountcastle, 1978). In particular, we’ve shown how a system that was designed to process perceptual data (Ontol) can be leveraged to process “symbolic” data (i.e., relational structures). This may provide insight into how species capable of higher-order cognition might have evolved from species capable of only low-level perception.

Although Spontol addresses some outstanding problems in Computational Analogy, there is still ample room for future work. Our implementation for characterizing a relational structure as a set of windows might not scale well to very large structures without some modifications. An open problem is how windows might be managed in a sensible way. Spontol currently uses “bags of windows” for medium-sized structures. We propose extending Spontol by allowing hierarchies of progressively higher-order bags to represent larger structures (e.g., bags of bags of bags of windows).

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