Strategyproof Mechanisms for Additively Separable Hedonic Games and Fractional Hedonic Games

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\begin{abstract}
Additively separable hedonic games and fractional hedonic games have received considerable attention. They are coalition forming games of selfish agents based on their mutual preferences. Most of the work in the literature characterizes the existence and structure of stable outcomes (i.e., partitions in coalitions), assuming that preferences are given. However, there is little discussion on this assumption. In fact, agents receive different utilities if they belong to different partitions, and thus it is natural for them to declare their preferences strategically in order to maximize their benefit. In this paper we consider strategyproof mechanisms for additively separable hedonic games and fractional hedonic games, that is, partitioning methods without payments such that utility maximizing agents have no incentive to lie about their true preferences. We focus on social welfare maximization and provide several lower and upper bounds on the performance achievable by strategyproof mechanisms for general and specific additive functions. In most of the cases we provide tight or asymptotically tight results. All our mechanisms are simple and can be computed in polynomial time. Moreover, all the lower bounds are unconditional, that is, they do not rely on any computational or complexity assumptions.
\end{abstract}

\section{Introduction}

Teamwork, clustering and group formations, have been important and widely investigated issues in computer science research. In many economic, social and political situations, individuals carry out activities in groups rather than by themselves. In these scenarios, it is of crucial importance to consider the satisfaction of the members of the groups. For example, the utility of an individual in a group sharing a resource, depends both, on the consumption level of the resource, and on the identity of the members in
the group; similarly, the utility for a party belonging to a political coalition depends both, on the party trait, and on the identity of its members.

Hedonic games, introduced in [17], model the formation of coalitions (groups) of players (or agents). They are games in which agents have preferences over the set of all possible agent coalitions, and the utility of an agent depends on the composition of the cluster she belongs to.

In this paper we consider *additively separable hedonic games* (ASHGs), which constitute a natural and succinctly representable class of hedonic games. Each player in an ASHG has a value for any other player, and the utility of a coalition to a particular player is simply the sum of the values she assigns to the members of her coalition. Additive separability satisfies a number of desirable axiomatic properties [3] and ASHGs are the non-transferable utility generalization of graph games studied by Deng and Papadimitriou [16]. We further consider *fractional hedonic games* (FHGs), introduced in [2], which are similar to ASHGs, with the difference that the utility of each agent is divided by the size of her cluster. This allows to model behavioral dynamics in social environments that are not captured by ASHGs: one usually prefers having a couple of good friends in a cluster composed by few other people rather than being part of a crowded cluster populated by uninteresting agents.

Coalition formation in ASHGs and FHGs, has received growing attention, but mainly from the perspective of coalition stability, i.e., core, Nash equilibria, etc, or from a classical offline optimization point of view, i.e., where solutions are not necessarily stable (see Related Work), with little emphasis on mechanism design. We consider such games where agents have private preferences. A major challenge is to design algorithms that work well even when the input is reported by selfish agents aiming only at maximizing their personal utility. An interesting approach is to use strategyproof mechanisms [18, 26], that is designing algorithms (not using payments) where selfish utility maximizing agents have no incentive to lie about their true preferences.

**Our Contribution.**

We present strategyproof mechanisms for ASHGs and FHGs, both for general and specific additive valuation functions. In particular, we consider: i) *general valuations* where additive valuations among agents can get any values; ii) *non-negative valuations* where additive valuations among agents can only get positive values; iii) *duplex valuations* where additive valuations among agents can only get values in \{-1, 0, 1\} (we can think about setting where each agent can express for any other agent if she is an enemy, neutral or a friend); iv) *simple valuations* where additive valuations among agents can only get values in \{0, 1\} (we can think about setting where each agent can express for any other agent if she is neutral or a friend). The latter setting has been also considered in other papers since it models a basic economic scenario referred to in the literature as Bakers and Millers [2, 10]. See Section 2 for more details about the considered valuations.

We focus on the classical utilitarian social welfare, that is the sum of individual utilities of the players in a coalition, and provide several lower and upper bounds on the performance achievable by strategyproof mechanisms.

We are mainly interested in deterministic mechanisms, however we also provide some randomized lower bounds (notice that randomized lower bounds are stronger than deterministic ones). Our results are summarized in Table 1. In most of the cases
(except the case of duplex valuations) we provide tight or asymptotically tight results.

We point out that, on the one hand, all our mechanisms are simple and can be computed in polynomial time. On the other hand, all the lower bounds (some of them randomized) are unconditional, that is, they do not rely on any computational or complexity assumptions.

**Related Work.**

In the literature, a significant stream of research considered hedonic games (see [5]), and in particular ASHGs, from a strategic cooperative point of view [7,12,20], with the purpose of characterizing the existence and the properties of coalitions structures such as the core, and from a non-cooperative point of view [11,21] with special focus on pure Nash equilibria. Computational complexity issues related to the problem of computing stable outcomes have been considered in [3,22,24,25,29]. Finally, hedonic games have also been considered in [6,8,14,15,16] from a classical optimization point of view, i.e., where solutions are not necessarily stable. Concerning FHGs, Aziz et al. [2], give some properties guaranteeing the (non-)existence of the core. Moreover, Brandl et al. [13], study the computational complexity of understanding the existence of core and individual stable outcomes. From a non cooperative point of view, the papers [9,10], study the existence, efficiency and computational complexity of Nash equilibria. Other stability notions have been also investigated, like in [1,19], where the authors focused on Pareto stability. Finally, Aziz et al. [4], consider the computational complexity of computing welfare maximizing partitions (not necessarily stable).

The design of truthful mechanisms, that is of algorithms that use payments to convince the selfish agents to reveal the truth and that then compute the outcome on the basis of their reported values, has been studied in innumerable scenarios. However, there are settings where monetary transfers are not feasible, because of either ethical or legal issues [23], or practical matters in enforcing and collecting payments [26]. A growing stream of research focuses on the design of the more applicable strategyproof mechanisms, that lead agents to report their true preferences, without using payments.

Wright et al. [30] focus on strategyproof mechanisms for ASHGs. They only consider positive preferences. Under this assumption, a trivial optimal strategyproof mechanism just puts all the agents in the same grand coalition. Therefore, they consider coalition size constraints and (approximate) envy-freeness. Their main contribution is a mechanism that, despite not having theoretical guarantees, achieves good experimental performance.

Vallée et al. [28] consider classical hedonic games with general preference relationships, and characterize the conditions of the game structure that allow rational false-name manipulations. However, they do not provide mechanisms. Aziz et al. [1] show that the serial dictatorship mechanism is Pareto optimal, and strategyproof for general hedonic games when appropriate restrictions are imposed on agents. Finally, Rodríguez-Álvarez [27], studies strategyproof core stable solutions properties for hedonic games.

**Paper organization.** The paper is organized as follows. In Section 2 we formally describe the problems and introduce some useful definitions. The studies on the performance of strategyproof mechanisms are then presented in Section 3, 4, 5, and 6, which address, respectively, general, non-negative, duplex and simple valuations. Fi-
nally, in Section 7 we resume our results and list some interesting open problems.

|        | [-1,1] | [0,1] | {-1,0,1} | {0,1} |
|--------|--------|-------|----------|-------|
| ASHGs  | L. B.  | Unbounded* | OPT | \(\Omega(n), 2 - \varepsilon\) | OPT |
| U. B.  | \(-\frac{\varepsilon}{2}\) | \(2 - \varepsilon\) | \(\frac{\varepsilon}{2}\) | \(\Omega(n)\) | 2 |
| FHGs   | L. B.  | Unbounded* | OPT | \(\Omega(n), 2 - \varepsilon\) | OPT |
| U. B.  | \(-\frac{\varepsilon}{2}\) | \(2 - \varepsilon\) | \(\frac{\varepsilon}{2}\) | \(\Omega(n)\) | 2 |

Table 1: Our results for the different cases. * stands for randomized mechanisms. L. B. stands for lower bounds. U. B. stands for upper bounds.

2 Preliminaries

In additive separable hedonic games (ASHGs) and fractional hedonic games (FHGs), we are given a set \(N = \{1, \ldots, n\}\) of selfish agents. The objective or outcome of the game is a partition of the agents into disjoint coalitions \(C = \{C_1, C_2, \ldots\}\), where each coalition \(C_j\) is a subset of agents and each agent is in exactly one coalition. Let \(\mathcal{C}\) be the collection of all the possible outcomes. Given a partition \(C \in \mathcal{C}\), we denote by \(|C|\) the number of its coalitions and by \(C^i\) the coalition of \(C\) containing agent \(i\). Similarly, given a coalition \(C\), we let \(|C|\) be the size or number of agents in \(C\). The grand coalition is the outcome in which all the agents are in the same coalition, i.e., \(|C| = 1\). We assume that each agent has a privately known valuation \(v_i : N \rightarrow \mathbb{R}\), mapping every agent to a real (possibly negative) value. In ASHGs, for any \(C \in \mathcal{C}\), the preference or utility of agent \(i\) is \(u_i(C) = \sum_{j \in C} v_i(j)\), that is, it is additively induced by her valuation function. Similarly, in FHGs, for any \(C \in \mathcal{C}\), the utility of agent \(i\) is \(u_i(C) = \frac{\sum_{j \in C} v_i(j)}{|C|}\).

We are interested in four basic classes of valuation functions. Namely, for any pair of agents \(i, j \in N\), we consider: General valuations: \(v_i(j) \in [-1, 1]\); Non-negative valuations: \(v_i(j) \in [0, 1]\); Duplex valuations: \(v_i(j) \in \{-1, 0, 1\}\); Simple valuations: \(v_i(j) \in \{0, 1\}\). In every case, we assume that \(v_i(i) = 0\), for every \(i \in N\). Notice that any valuation function can be represented by using values in the range \([-1, 1]\).

Agents are self-interested entities. Thus, they may strategically misreport their valuation functions in order to maximize their utilities. Let \(\mathcal{d}\) denote the preferences (valuation functions) declared by all the agents.

A deterministic mechanism \(M\) maps every set (or list) of preferences \(\mathcal{d}\) to a set of disjoint coalitions \(M(\mathcal{d}) \in \mathcal{C}\). We denote by \(M^i(\mathcal{d})\) the coalition assigned to agent \(i\) by \(M\). The utility of agent \(i\) is given by \(u_i(M(\mathcal{d}))\). Let \(\mathcal{d}_{-i}\) be the valuation functions declared by all agents except agent \(i\) and \(d_i\) be a possible declaration of valuation function by \(i\). A deterministic mechanism \(M\) is strategyproof if for any \(i \in N\), any list of preferences \(\mathcal{d}_{-i}\), any \(v_i\) and any \(d_i\), it holds that \(u_i(M(\mathcal{d}_{-i}, v_i)) \geq u_i(M(\mathcal{d}_{-i}, d_i))\). In other words, a strategyproof mechanism prevents any agent \(i\) from benefiting by declaring a valuation different from \(v_i\), whatever the other declared valuations are.

A randomized mechanism \(M\) maps every set of agents’ preferences \(\mathcal{d}\) to a distribution \(\Delta\) over the set of all the possible outcomes \(\mathcal{C}\). The expected utility of agent
is given by \( E[u_i(M(d))] = E_{C \sim \Delta}[u_i(C)] \). A randomized mechanism \( M \) is strategyproof (in expectation) if for any \( i \in N \), any preferences \( d_{-i} \), any \( v_i \) and any \( d_i \), \( E[u_i(M(d_{-i}, v_i))] \geq E[u_i(M(d_{-i}, d_i))] \).

In this paper, we are interested in strategyproof mechanisms that perform well with respect to the goal of maximizing the classical utilitarian social welfare, that is, the sum of the utilities achieved by all the agents. Namely, the social welfare of a given outcome \( C \) is \( SW(C) = \sum_{i \in N} u_i(C) \). We denote by \( SW(C) = \sum_{i \in C} u_i(C) \) the overall social welfare achieved by the agents belonging to a given coalition \( C \). We measure the performance of a mechanism by comparing the social welfare it achieves with the optimal one. More precisely, the approximation ratio of a deterministic mechanism \( M \) is defined as \( r^M = \sup_d \frac{SW(M(d))}{OPT(d)} \), where \( OPT(d) \) is the social welfare achieved by an optimal set of coalitions in the instance induced by \( d \). For randomized mechanisms, the approximation ratio is computed with respect to the expected social welfare, that is \( r^M = \sup_d \frac{E[SW(M(d))]}{E[OPT(d)]} \).

We say that a deterministic mechanism \( M \) is acceptable if it always guarantees a non negative social welfare, i.e., \( SW(M(d)) \geq 0 \) for any possible list of preferences \( d \). Similarly, a randomized mechanism \( M \) is acceptable if \( E[SW(M(d))] \geq 0 \) holds for every \( d \). In the following, we will always implicitly restrict to acceptable mechanisms. In fact, a simple acceptable strategyproof mechanism for all the considered classes of valuations can be trivially obtained by putting every agent in a separate singleton coalition, regardless of all the declared valuations.

**Graph representation.** ASHGs and FHGs have a very intuitive graph representation. In fact, any instance of the games can be expressed by a weighted directed graph \( G = (V, E) \), where nodes in \( V \) represent the agents, and arcs or directed edges are associated to non null valuations. Namely, if \( v_i(j) \neq 0 \), an arc \((i, j)\) is contained in \( E \) of weight \( w(i, j) = v_i(j) \). As an example, in case of simple valuations, if \((i, j) \notin E\) then \( v_i(j) = 0 \), while if \((i, j) \in E\) then \( w(i, j) = v_i(j) = 1 \).

Throughout the paper we will sometimes describe an instance of the considered game by its graph representation. In the following sections, we provide our results for all of the four considered classes of valuation functions.

### 3 General valuations

In this section, we consider the setting where agents have general valuations. We are able to prove that there is no randomized strategyproof mechanism with bounded approximation ratio both for ASHGs and FHGs. Clearly, the theorem applies also to deterministic mechanisms, since they are special cases of randomized ones.

**Theorem 1.** For general valuation functions, there is no randomized strategyproof acceptable mechanism with bounded approximation ratio both for ASHGs and FHGs.

**Proof.** We prove the claim only for ASHGs. However, the same arguments directly apply also to FHGs.

Let \( M \) be a given randomized strategyproof mechanism. Provided that \( M \) is strategyproof, we implicitly assume that the agents’ declared preferences \( d \) correspond to the true valuation functions. Let us then consider the instance \( I_1 \) depicted
in Figure 1a, and let $p$ be the probability that $\mathcal{M}$ returns an outcome for $I_1$ where agents 2 and 3 are together in the same coalition. Then, the expected social welfare is $E[SW(\mathcal{M}(d))] \leq p(\epsilon - 0.1) + (1 - p)\epsilon = \epsilon - 0.1p$, while the optimal solution has social welfare $\epsilon$. Therefore, the randomized mechanism has bounded approximation ratio only if $\epsilon - 0.1p > 0$, that implies $p < 10\epsilon$. Let us now consider the instance $I_2$ depicted in Figure 1b, and let $q$ be the probability that mechanism $\mathcal{M}$ returns an outcome where agents 2 and 3 are together in the same coalition. Then the expected social welfare is $E[SW(\mathcal{M}(d))] \leq 0.9q + (1 - q)\epsilon$. We notice that $\mathcal{M}$ can be strategyproof only if $p \geq q$, otherwise agent 2 could improve her utility by declaring value $-1$ for agent 3, since in such a case she would get utility $-pe > -qe$. The optimal solution of instance $I_2$ has value $0.9$. Thus, the approximation ratio of $\mathcal{M}$ is $\frac{OPT(d)}{E[SW(\mathcal{M}(d))]} \geq \frac{0.9}{0.9q + (1 - q)\epsilon} \geq \frac{0.9}{0.9q + \epsilon} \geq \frac{0.9}{10\epsilon}$. As $\epsilon$ can be arbitrarily small, we can then conclude that $\mathcal{M}$ has an unbounded approximation ratio. The claim then follows by the arbitrariness of $\mathcal{M}$. 

\section{Non-negative valuations}

In this section, we consider the setting where agents have non-negative valuations. Let us first present a simple optimal mechanism for non-negative valuations in ASHGs.

**Mechanism $\mathcal{M}_1$.** Given as input a list of agents’ valuations $d = \langle d_1, \ldots, d_n \rangle$, the mechanism outputs the grand coalition, i.e. $\mathcal{M}(d) = \{\{1, \ldots, n\}\}$.

It is trivial to see that, in ASHGs with non-negative valuations, the above mechanism $\mathcal{M}_1$ is acceptable, strategyproof, and achieves the optimal social welfare. Therefore, we now focus on FHGs. We are able to show that any deterministic strategyproof mechanism cannot have an approximation better than $\frac{n}{2}$.

**Theorem 2.** For FHGs with non-negative valuations, no deterministic strategyproof acceptable mechanism can achieve approximation ratio $r$, with $r < \frac{n}{2}$.

**Proof.** Assume $\frac{1}{n} \gg \alpha \gg \beta$. Let us consider the instance $I_1$ with an even number $n$ of agents, where the valuation functions are as follows:

- for any $i = 1, 3, \ldots, n - 1$, $v_i(j) = \alpha$ if $j = i + 1$ and $v_i(j) = 0$ otherwise;
- for any $i = 2, 4, \ldots, n - 2$, $v_i(j) = \beta$ if $j = i + 1$ and $v_i(j) = 0$ otherwise;
- $v_n(1) = \beta$ and $v_n(j) = 0$ for any $j \neq 1$. 

![Figure 1: The lower bound instance for general valuations.](image)
The optimal outcome is given by the set of coalitions $C = \{C_1, C_2, \ldots, C_n\}$, where $C_j = \{2j-1, 2j\}$ for any $j = 1, \ldots, \frac{n}{2}$, and achieves social welfare $\frac{n}{4} \alpha$. We now show that any deterministic strategyproof mechanism with an approximation ratio lower than $\frac{n}{2}$ has to output the grand coalition. In fact, the grand coalition has social welfare $\frac{\alpha + \beta}{2}$, which has approximation ratio tending to $\frac{n}{2}$ when $\beta/\alpha$ tends to 0, thus proving the claim. Assume then that a deterministic strategyproof mechanism $M$ with approximation ratio strictly less than $\frac{n}{2}$ outputs an outcome different from the grand coalition. In this case, there must be at least one agent $k$ having null utility. But then $k$ might improve her utility by declaring $v_k(k+1) = 1$, as in this case $M$, since $\alpha \ll \frac{1}{n}$, in order to achieve approximation less than $\frac{n}{2}$ must give an outcome in which agents $k$ and $k+1$ are in the same coalition. Hence, agent $k$ improves her utility by declaring $v_k(k+1) = 1$. Therefore $M$ for the instance $I_1$ has to output the grand coalition, thus proving the theorem.

Given the above result, it is easy to show that, returning the grand coalition is the best we can do.

**Proposition 1.** For FHGs with non-negative valuations, Mechanism $M_1$ is a deterministic strategyproof acceptable mechanism with approximation ratio $\frac{n}{2}$.

**Proof.** As valuations are non-negative and Mechanism $M_1$ always outputs the grand coalition, the mechanism is clearly acceptable and strategyproof. Let us now focus on its approximation ratio for the social welfare. Notice that, given any $d$, then $\text{OPT}(d) \leq \sum_{i \in N} \sum_{j \in N} v_i(j)$. This is because any coalition in the optimal coalitions with positive social welfare consists of at least two agents. Otherwise, the coalition has zero social welfare since $v_i(i) = 0$ for any $i \in N$. On the other hand the grand coalition has social welfare equal to $\sum_{i \in N} \sum_{j \in N} v_i(j)$. The approximation ratio follows.

## 5 Duplex valuations

In this section, we consider the setting where agents have duplex valuations. We first present deterministic lower bounds for ASHGs and FHGs.

**Theorem 3.** For ASHGs with duplex valuations, no deterministic strategyproof acceptable mechanism has approximation ratio less than $n - 2$.
Proof. Let us consider the instance $I_1$ depicted in Figure 3(a) where the valuations of the $n$ agents are as follows:
- for $i = 1, \ldots, n-2$, $v_i(j) = 1$ if $j = n-1$ and $v_i(j) = 0$ otherwise;
- $v_{n-1}(j) = 1$ if $j = n$ and $v_{n-1}(j) = -1$ otherwise;
- $v_{n}(j) = -1$ for $j = 1, \ldots, n-2$ and $v_n(n-1) = 0$.

In the optimal outcome agents $n-1$ and $n$ are in the same coalition and all other agents are in different coalitions. The resulting social welfare is 1, and in particular it is due to agent $n-1$ having utility 1. It is easy to see that any mechanism having bounded approximation has to return the optimal outcome, as any other solution would have social welfare at most zero. Let us now consider the other instance $I_2$ depicted in Figure 3(b) where agent $n-1$ is the only one with a different valuation function with respect to $I_1$, that is $v_{n-1}(n) = 1$ and $v_{n-1}(j) = 0$ for $j \neq n$. Any strategyproof mechanism with bounded approximation ratio for $I_2$ has to put agents $n-1$ and $n$ in the same coalition, otherwise $n-1$ would have null utility and could increase her utility by declaring her valuation function as it is in instance $I_1$. Moreover, any outcome in which $n-1$ and $n$ are together, independently from the other coalitions, has social welfare 1. However, the optimal outcome, by putting 1, 2, $\ldots$, $n-1$ all together in a same coalition and agent $n$ alone, achieves social welfare $n-2$. This proves the $n-2$ lower bound for any deterministic strategyproof mechanism.

Theorem 4. For FHGs with duplex valuations, no deterministic strategyproof acceptable mechanism can achieve approximation $2 - \epsilon$, for any $\epsilon > 0$.

Proof. The proof is very similar to Theorem 3 but here the optimal solution has value $\frac{n-2}{n-1}$ and the best strategyproof acceptable mechanism returns an outcome of social welfare $\frac{1}{2}$. It follows that for big value of $n$, the ratio tends to 2, and thus proving the theorem.

We are also able to prove the following randomized lower bound.

Theorem 5. For ASHGws with duplex valuations, no randomized strategyproof acceptable mechanism can achieve approximation $2 - \epsilon$, for any $\epsilon > 0$. 

\[\]
Proof. Let us consider the instance $I_1$ depicted in Figure 3a. Let $p$ be the probability that a randomized mechanism returns the outcome where agents $n-1$ and $n$ are together in the same coalition and all the other agents are alone. Notice that in such a case agent $n-1$ has expected utility equal to $p$. Let us call $rm$ the outcome of the randomized mechanism. Then the expected social welfare in this case is such that $\mathbb{E}[rm] \leq p$. Let us now consider the instance $I_2$ depicted in Figure 3b. Let $q$ be the probability that a randomized mechanism returns an outcome where agents $n-1$ and $n$ are together in the same coalition (possibly with other agents). Notice that the social welfare of any outcome where agents $n-1$ and $n$ are together is always 1, independently from the coalitions of the other agents are member of. Moreover, notice that in such a case agent $n-1$ has expected utility equal to $q$. On the other hand, the mechanism with probability $1-q$ put agents $n-1$ and $n$ not together in the same coalition. In such a case, i.e., with probability $1-q$, the social welfare is at most equal to $n-2$. Let us call $rm'$ the outcome of the randomized mechanism. It turns out that the expected social welfare in this case is such that $\mathbb{E}[rm'] \leq q + (1-q)(n-2)$. We notice that such mechanism is strategyproof only if $q \geq p$. In fact, if $p > q$, then agent $n-1$ can improve her utility by declaring value $v_{n-1}(j) = -1$, for any $j = 1, \ldots, n-2$, and $v_{n-1}(n) = 1$ (thus reconstructing the instance $I_1$), since in such a case she would get expected utility $p > q$. Therefore, the expected social welfare of the mechanism of $I_1$ is maximized when $p = q$. We now equalize the expected approximation ratio of the mechanisms of both instances (where we set $p = q$), where 1 is the optimal value for the instance depicted in Figure 3a and $n-2$ is the optimal value for the instance depicted in Figure 3b.

$$\frac{1}{\mathbb{E}[rm]} = \frac{q-2}{\mathbb{E}[rm']} \implies \frac{1}{q} = \frac{n-2}{q+(1-q)(n-2)} \implies q = \frac{q+(1-q)(n-2)}{n-2}$$

It follows that for big value of $n$, $q$ tends to $\frac{1}{2}$, and thus proving the theorem.

We now present a deterministic strategyproof acceptable mechanism $M_2$ with approximation $O(n^2)$ for ASHGs and $O(n)$ for FHGs. We doubt the existence of deterministic strategyproof acceptable mechanisms with approximation ratio $O(n)$ for ASHGs and $O(1)$ for FHGs. We provide some discussion supporting it, at the end of the section. Closing the gap for duplex valuations, is one of the main open problem.

The following definition is crucial for Mechanism $M_2$.

Definition 1. Given $d = \langle d_1, \ldots, d_n \rangle$ declared by the set of agents $N$, we say that an agent $i \in N$ is a sink if there is no agent $j \in N$ such that $d_i(j) = 1$ and $d_j(i) \neq 1$.

The idea of the mechanism is as follows. It considers the agents in an arbitrary ordering. If the considered agent $i$ has value 1 for some other agent $j$, such that $j$ also has value 1 for $i$, or $j$ is a sink, or $j$ is before $i$ in the ordering, then it returns agents $i$ and $j$ together in a coalition, and each other agent in a coalition alone. If, after considering all the agents, the mechanism does not create the coalition with two agents, then returns each agent in a coalition alone. It follows the formal description of the mechanism $M_2$.

Mechanism $M_2$. Given any declared valuation $d = \langle d_1, \ldots, d_n \rangle$, the mechanism performs as follows:
1 Consider any ordering of the agents and, for the sake of simplicity, let $i$ be the $i$-th agent in such ordering.

2 For $i = 1$ to $n$:
   a) If there exists $j \in N$ such that $d_i(j) = 1 \land d_j(i) = 1$: put agents $i$ and $j$ together into a coalition and any other agent alone, and terminate.
   b) If there exists $j \in N$ such that $d_i(j) = 1 \land d_j(i) = 0 \land j$ is a sink: put agents $i$ and $j$ together into a coalition and any other agent alone, and terminate.
   c) If there exists $j \in N$ such that $d_i(j) = 1 \land d_j(i) = 0 \land j < i$: put agents $i$ and $j$ together into a coalition and any other agent alone, and terminate.

3 If no coalition of two agents has been created during the step 2: return each agent in a coalition on its own.

Theorem 6. For ASHG and FHGs with duplex valuations, Mechanism $\mathcal{M}_2$ is a deterministic strategyproof acceptable mechanism. The approximation ratio is $O(n^2)$ for ASHG and $O(n)$ for FHGs with duplex valuations.

Proof. The mechanism $\mathcal{M}_2$ returns at most one coalition composed by two agents and all the other coalitions are composed by one agent alone. Moreover, no agent $i$ is put together with another agent $j$ in the same coalition if there is a value of $-1$ between them, that is if $d_i(j) = -1$ or $d_j(i) = -1$. This implies that no agent gets negative utility in the outcome returned by $\mathcal{M}_2$, i.e., $\mathcal{M}_2$ is acceptable. More specifically, if a coalition of two agents is created, then such a coalition has positive (i.e., strictly greater than zero) social welfare. In particular, in ASHG every agent gets utility 1 or zero, while in FHG $\frac{1}{2}$ or zero. Furthermore notice that, given the valuations declared by agents, if all the agents are sinks, then the optimal solution has social welfare zero and also $\mathcal{M}_2$ returns the outcome where each agent is in a coalition alone. On the other hand, if there is at least one agent that is not a sink, then it is not difficult to see that the optimal solution has positive social welfare. We now prove that, in such a case $\mathcal{M}_2$ would return a coalition with two agents together with positive social welfare.

Lemma 7. Given the valuations $d = (d_1, \ldots, d_n)$ declared by agents, if there exists an agent $i$ that is not a sink, then Mechanism $\mathcal{M}_2$ returns an outcome where two agents are put together in the same coalition, thus yielding positive social welfare.

Proof. First suppose that Mechanism $\mathcal{M}_2$ does not consider agent $i$ (line 2). It means that $\mathcal{M}_2$ has created a coalition with two agents before considering $i$ (with at least one agent of the coalition appearing before $i$ in the ordering). Suppose now that agent $i$ is considered by $\mathcal{M}_2$. Then two scenarios are possible: i) agent $i$ is put together with another agent, still getting a positive social welfare, or ii) $i$ is put alone. This means that, for any agent $j$ such that $d_i(j) = 1$ and $d_j(i) \neq -1$, agent $j$ is not a sink and she appears after $i$ in the ordering. Thus we can now consider the agent $j$ as the new one that is not a sink and apply the same argument as above. Summarizing we have that at any step $s$ of the mechanism, if a coalition of two agents is not created, then there exists an agent that is not a sink and that is not considered at step $s$ yet. Therefore a coalition of two agents will be for sure created by $\mathcal{M}_2$ at some step after $s$. \(\square\)
We are now ready to show that Mechanism $M_2$ is strategyproof. The following argument is valid for both ASHGs and FHGs. The proof relies on the analysis of different cases.

Assume an agent $i$ gets positive (i.e., greater than zero) utility when she declares her valuations truthfully. Then, agent $i$ cannot improve her utility by declaring valuations $d_i \neq v_i$. In fact getting positive utility, that is utility $1$ or $\frac{1}{2}$ depending on whether we consider ASHGs or FHGs respectively, is the best she can obtain.

Assume now that an agent $i$ gets utility zero when she declares her valuations truthfully. We show that agent $i$ cannot improve her utility by declaring valuations $d_i \neq v_i$. If the agent $i$ is a sink then she has no incentive to lie. In fact, in this case $i$ would get positive utility only if she is put together an agent $j$ such that $d_i(j) = 1$ and $d_j(i) = -1$. However the outcome returned by Mechanism $M_2$ is such that no agent gets negative utility. Moreover $i$ has no incentive to declare a value of 1 for some agent $j$ (in order to become not a sink anymore) if the real value is indeed different than 1. It remains to consider the case where the agent $i$ is not a sink. By Lemma 7 we know that in this case our mechanism always returns a coalition of two agents. Let us first suppose that such coalition, that we call $C_{j,z}$, is formed by agents $j$ and $z$ together. If $i$ has not been considered by the mechanism, that is, for instance the coalition $C_{j,z}$ has been created while considering agent $j$ that appears before $i$ in the ordering, then there is nothing that agent $i$ can do in order to get positive utility. Indeed the only thing that $i$ could do is (mis)-declaring $d_i(j) = 1$ (if we suppose that $v_i(j) \neq 1$). In such a case, if also $d_j(i) = 1$, the mechanism could return the coalition with $i$ and $j$ together. However agent $i$ would still not get positive utility. If $i$ has been considered by the mechanism but has not been put in a coalition together with another agent, then it means that while $M_2$ was considering agent $i$, for any $j$ such that $d_i(j) = 1$, $j$ is not a sink and $j$ was not considered by the mechanism yet. We notice that $j$ has no incentive to declare a value of 1 for some agents $z$ if the real value is not 1 (i.e., $v_j(z) \neq 1$). Still there is nothing that $i$ can do.

Let us finally suppose that the coalition of two agents returned by $M_2$ contains agent $i$ (but still $i$ gets utility zero). This is only possible if, while mechanism $M_2$ was considering agent $i$, it was not able to put $i$ together with another agent and (for the same reasons as in the previous case), there is nothing that agent $i$ can do to change it. In fact, agent $i$ could be put together another agent $j$, that appears after $i$ in the ordering, when the mechanism considers $j$. In this case the mechanism could put $i$ together with $j$ only if $d_j(i) = 1$. However it must be that $d_i(j) \neq 1$, otherwise the mechanism would have put $i$ and $j$ together while considering $i$, and therefore agent $i$ still does not get positive utility.

We now show the approximation ratio of the mechanism. If the optimal solution has social welfare zero, then also our mechanism returns an outcome (i.e., all the agents alone) with social welfare zero. If the optimal solution has positive social welfare (and thus there exists an agent that is not a sink), then by Lemma 7 we know that our mechanism returns an outcome with social welfare at least $\frac{1}{2}$ for ASHGs, and at least $\frac{1}{2}$ for FHGs. The theorem follows by noticing that, any agent can get utility at most $n-1$ for ASHGs and at most 1 for FHGs.

We point out that, if we consider ASHGs, there exists an instance and an ordering
of the agents for that instance, such that the optimal solution has value order of $n^2$, while $[M_2]$ puts two agents in a coalition in the last iteration of the loop $For$. Thus, even if $[M_2]$ does not terminate after putting two agents in a coalition, the analysis cannot be improved. Clearly, $[M_2]$ could perform more loops $For$ in order to match more than one pair of agents. However, in such a case we can show that the mechanism is not strategyproof anymore. In fact, consider a cycle of 4 nodes with arcs $\{(1, 2), (2, 3), (3, 4), (4, 1)\}$, and all the weights 1. The ordering is $1, 2, 3, 4$. If the mechanism iterates the loop $For$, it would return in the first iteration agents $\{4, 1\}$ in a coalition, and then, in a second iteration of the $For$, agents $\{2, 3\}$ together. Notice that agent 1 has utility zero. However, agent 1 can improve her utility by declaring a further arc of weight $-1$ to agent 4. In fact, in this case, in the first iteration the mechanism would put agents $\{3, 4\}$ together, and then, in the second one, agents $\{1, 2\}$.

6 Simple valuations

Exactly as in the case of non-negative valuations, for ASHG with simple valuations, Mechanism $M_1$ is acceptable and strategyproof and it also achieves the optimal social welfare. Therefore, we focus on FHGs. We first prove that any deterministic strategyproof mechanism cannot approximate better than $\frac{6}{5}$ the social welfare.

Theorem 8. For FHGs with simple valuations, no deterministic strategyproof acceptable mechanism has approximation ratio less than $\frac{6}{5}$.

Proof. Let us consider the instance $I_1$ depicted in Figure 4a. The reader can easily check (by considering all the possible coalitions) that an optimal solution has social welfare $\frac{5}{3}$. It is composed by the three coalitions where, two of them contain two consecutive agents, and the remaining one contains three consecutive agents. For instance, an optimal solution could be $C_1 = \{1, 2\}, C_2 = \{3, 4\}, C_3 = \{5, 6, 7\}$. Notice that the grand coalition has social welfare 1. Therefore, a mechanism achieving an approximation better than $\frac{5}{3}$ has to return more than one coalition. In such a solution there always exists at least one agent, say agent $k$, having utility zero. Let us now consider the instance $I_2$ depicted in Figure 4b, where without loss of generality we suppose that $k = 2$. Again the reader can easily check (by considering all the possible coalitions) that an optimal solution has social welfare 2. Such optimal solution is $C_1 = \{2, 3, 4\}, C_2 = \{5, 6\}, C_3 = \{1, 7\}$. Once again the reader can check that any solution where agents 2 and 3 are not in the same coalition, (i.e., any solution where agent 2 has utility equal to 0 in the instance $I_1$) can achieve a social welfare of at most $\frac{2}{3}$, and therefore an approximation not better than $\frac{6}{5}$. We conclude that any mechanism achieving an approximation ratio strictly better than $\frac{6}{5}$, in both instances $I_1$ and $I_2$, is not strategyproof.

We now show a deterministic strategyproof acceptable mechanism with nearly optimal social welfare. Given the preferences declared by the agents $d = d_1, \ldots, d_n$, and the associated directed graph representation $G = (V, E)$ (notice that since we are considering simple valuations, $d_i$ represents (indeed is) the set of arcs outgoing from
node \( i \) in \( G \), we construct an undirected weighted graph \( \bar{G} = (\bar{V}, \bar{E}) \), where \( \bar{V} = V \). There is an (undirected) edge \( \{i, j\} \in \bar{E} \), if \( (i, j) \in E \) or \( (j, i) \in E \). Finally, for each \( \{i, j\} \in \bar{E} \), we have that the weight \( w(i, j) = 1 \) if either \( (i, j) \in E \) or \( (j, i) \in E \), and \( w(i, j) = 2 \) if both \( (i, j) \in E \) and \( (j, i) \in E \) (otherwise \( w(i, j) = 0 \), i.e., \( \{i, j\} \notin E \)).

A matching \( m \) of \( \bar{G} \) naturally induces an outcome for fractional hedonic games, that is, any edge \( \{i, j\} \in m \) induces the coalition \( C_{i,j} = \{i, j\} \), and for any node \( i \) not matched in \( m \) we have the coalition \( C_i = \{i\} \). Notice that the coalitions induced by the matching are such that each agent can have utility either \( \frac{1}{2} \) or 0. It is possible to show that, finding the maximum weighted matching of \( G = (\bar{N}, \bar{E}) \), using a consistent tie-breaking rule, gives a strategyproof mechanism.

The proof of the following lemma is similar to the one proposed in [18], which also shows that \( \prec \)-minimal matching can be found in poly-time.

**Lemma 9.** Fix a total order \( \prec \) on matchings in the complete graph induced by all the agents. Let \( \mathcal{M} \) be the mechanism that, given the input \( d = \langle d_1, \ldots, d_n \rangle \), finds the \( \prec \)-minimal matching \( m \) on \( G = (V, E) \), such that \( \sum_{(i, j) \in m} w(i, j) \) is maximized. Then \( \mathcal{M} \) is strategyproof.

**Proof.** Assume for a contradiction that \( \mathcal{M} \) is not truthful. Then there exists \( \bar{E} \) induced by edges \( E_{-i} \cup E_i \), and \( E'_i \) (inducing the edges set \( E' = E_{-i} \cup E'_i \)), violating the truthfulness. Let \( m = \mathcal{M}(\bar{E}) \) and \( m' = \mathcal{M}(E') \). Agent \( i \) has utility zero in the coalitions induced by \( m \), that is, for any \( \{i, j\} \in m \) we have that \( \{i, j\} \notin E_i \). Yet agent \( i \) has utility \( \frac{1}{2} \) in the coalitions induced by \( m' \). It means that there exists \( \{i, j\} \in m' \) such that \( (i, j) \in E_i \) (and then clearly \( \{i, j\} \in \bar{E} \)). Moreover since the mechanism only uses declared edges and agent \( i \) has utility \( \frac{1}{2} \) in the coalitions induced by \( m' \), it follows that there exists \( \{i, j\} \in m' \) such that \( (i, j) \in E'_i \). It implies that both \( m \) and \( m' \) are in \( M(\bar{E}) \cap M(E') \). Since the mechanism returns the maximum matching it follows that \( m \) and \( m' \) are optimal in both \( M(\bar{E}) \) and \( M(E') \). Recalling that \( \mathcal{M} \) breaks ties consistently, this yields a contradiction, as needed. \( \square \)

Now we prove the approximation ratio of the mechanism. Given an undirected graph \( G = (V, E) \), where \( w \) is the edges weight function, we denote by \( w(E) \) the sum of the weights of the edges belonging to \( E \), i.e., \( w(E) = \sum_{(i, j) \in E} w(i, j) \).
Theorem 10. The deterministic mechanism outputting the maximum matching as described in Lemma 9 is strategy-proof and acceptable with approximation ratio of 2.

Proof. Let $m$ be the matching computed by the mechanism and $C^m$ be the coalitions induced by $m$. Let $C^* = \{C^*_1, \ldots, C^*_p\}$ be optimal coalitions (we do not consider optimal coalitions having social welfare equal to zero, indeed we can ignore them). Let $m = m'_1 \cup \cdots \cup m'_p$, where $m'_h$, $1 \leq h \leq p$, is a maximum matching in the graph induced by the vertices of $C^*_h$. Let $C^m_1$ be the coalitions induced by $m'$. Let $A_h$ be the vertices matched in $m'_h$ and $B_h = C^*_h \setminus A_h$. Notice that $B_h$ is a stable set and that $|A_h|$ is an even number.

Proposition 2. When $|B_h| > 0$, then for any $h = 1, \ldots, p$, and any edge $\{i, j\} \in m'_h$, we have that $\sum_{b \in B_h} w(i, b) + w(j, b) \leq w(i, j)(|B_h| + 1)$.

Proof. First notice that, for any $b \in B_h$, it holds that $w(i, b) \leq w(i, j)$ and $w(j, b) \leq w(i, j)$, since otherwise we can get a better matching by removing the edge $\{i, j\}$ from $m'_h$ and adding the new edge having weight strictly greater than $w(i, j)$. We now distinguish two cases depending on the size of $B_h$. If $|B_h| > 1$, then suppose that $\sum_{b \in B_h} w(i, b) + w(j, b) > w(i, j)(|B_h| + 1)$. It implies there are two distinct edges $\{i, b\}$ and $\{j, b'\}$ for some $b, b' \in B_h$ such that $w(i, b) + w(j, b') > w(i, j)$ and then contradicting the fact that $m'_h$ is a maximum matching in $C^*_h$. If $|B_h| = 1$ then the claim easily follows from the observation that $w(i, b) \leq w(i, j)$ and $w(j, b) \leq w(i, j)$.

Let $\hat{E}_h$ be the set of edges of the graph induced by the vertices of $A_h$ minus the edges belonging to the matching $m'_h$. Moreover, let $w(\hat{E}_h) = \sum_{\{i, j\} \in \hat{E}_h} w(i, j)$.

Proposition 3. For any $h = 1, \ldots, p$, then $w(\hat{E}_h) \leq w(m'_h)(|A_h| - 2)$.

Proof. Assume for a contradiction that $w(\hat{E}_h) > w(m'_h)(|A_h| - 2)$. Let us consider the graph $G_{A_h}$ induced by the vertices of $A_h$ and suppose that $G_{A_h}$ is complete (if it is not complete, we can just add edges of weights zero). It is easy to see that all the edges of such complete graph can be partitioned into $|A_h| - 1$ different perfect matchings (recall that $|A_h|$ is an even number). It implies that must exist a perfect matching in $G_{A_h}$ having weight at least equal to $\frac{w(\hat{E}_h) + w(m'_h)}{|A_h| - 1} > \frac{w(m'_h)(|A_h| - 2) + w(m'_h)}{|A_h| - 1} = w(m'_h)$ thus contradicting the fact that $m'_h$ is a maximum matching.

Then, when $|B_h| > 0$, by using propositions 2 and 3 we can bound the social welfare of $C^*_h$, for any $h = 1, \ldots, p$, as follows:

$$SW(C^*_h) = \frac{1}{|C^*_h|} \left[ \sum_{\{i, j\} \in m'_h} (w(i, j) + \sum_{b \in B_h} w(i, b) + w(j, b)) + w(\hat{E}_h) \right]$$

$\leq \frac{1}{|C^*_h|} [w(m'_h) + w(m'_h)(|B_h| + 1) + w(m'_h)(|A_h| - 2)]$

$= w(m'_h)$. 

14
Similarly, when \(|B_h| = 0\) we can get that \(SW(C^*_h) \leq w(m'_h)\). Therefore, overall we have that \(SW(C^*) \leq w(m')\). Since it is easy to see that \(w(m) \geq w(m')\), then we have that the social welfare of \(C^m\) is

\[
SW(C^m) = \frac{w(m)}{2} \geq \frac{w(m')}{2} \geq \frac{SW(C^*)}{2}.
\]

We point out that, when dealing with FHGs, it is natural to resort on matchings. Many papers (for instance [2, 4, 9, 10]) used them. The challenge is how to exploit their properties, and in this sense we make some steps forward. Indeed, we better exploit properties of maximum weighted matchings. This is proved by the fact that, our analysis can be used to improve from the 4-approximation (Theorem 7 of the paper [4]) of maximum weighted matching for symmetric valuations, i.e., undirected graph, to a 2-approximation. Another remark is that, our results are not only working for the approximation of asymmetric FHGs, i.e., directed graphs, but also include the strategyproofness, which was not considered before for FHGs.

We finally notice that, 2 is the best approximation achievable by using matchings, when dealing with the problem of computing the maximum social welfare in symmetric fractional hedonic games. In fact, consider a complete graph of \(n\) nodes. In the grand coalition, each node has utility \(\frac{n-1}{n}\) (consider big \(n\)), while in a matching, each node has utility at most \(\frac{1}{2}\).

7 Conclusion and future work

In this paper, we studied strategyproof mechanisms for ASHGs and FHGs, under general and specific additive valuation functions. Despite the theoretical interest for specific valuations, for which we were able to show better bounds with respect to generic valuations, specific valuations also model realistic scenarios.

Our paper leaves some appealing open problems. First of all, it would be nice to close the gaps of Table[1] and in particular the gap of deterministic strategyproof mechanisms for duplex valuations. Moreover, it is worth to understand whether randomized strategyproof mechanisms can achieve significantly better performance than deterministic ones. It would be also important to understand what happens when valuations are drawn at random from some distribution (in order to avoid the bad instances), or when there are size constraints to the coalitions. Finally, another research direction, is that of considering more general valuation functions than additive ones.

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