Summary of Session D1(i), Quantum General Relativity

Donald Marolf
Physics Department
Syracuse University, Syracuse, NY 13244
(February, 1998)

I. INTRODUCTION

Looking back over the list of speakers from the session on Quantum General Relativity, the most striking feature is the huge breadth covered by the the two afternoons (and 17 speakers!) of the session. Speakers from ten different countries spoke on topics ranging from measures to mapping class groups, from anomalies to asymptotic structure, and from strings to solitons. Clearly, there is not sufficient space here to discuss all of these works in detail, but I have tried to include at least a few comments on each talk, together with appropriate references. Many thanks to the speakers for sending me brief summaries of their presentations from which to work.

I will attempt to group the talks by subject, though close relations between the talks will not exist in general. The one clear exception is in the set of talks on loop gravity (by Thiemann, Lewandowski, Loll, and Pullin), which I will discuss at the end of this summary.

II. SUMMARIES

The session opened with “The measure in Simplicial Gravity” by Ruth Williams, giving a solid review of what is known about that issue. A good introduction to this subject can be found in Williams’ previous review [1] with H.W. Hamber. Whether a measure can be chosen so that simplicial gravity has the proper continuum limit is a question under active investigation, but a necessary criterion for this to happen is for the lattice measure to agree with its continuum counterpart in the limit of weak fields and low momentum. Williams uses this to argue that one should carefully study discretizations of the continuum measure. This is in contrast to another strategy in which a discrete form of the supermetric is sought first, and then the determinant of this discretized metric is evaluated. Other important concerns, such as those based on locality and on factorization and composition laws, were also discussed. Williams’ was the only presentation to discuss discretized approaches to quantum gravity.

The work “Models of Coherent State Evolution in Quantum Gravity” presented by Michael Ryan was a continuation of his interest in the question of to what extent quantum minisuperspace models could actually approximate a full theory of quantum gravity [2]. The work presented in Pune was part of an on-going program of research, but some earlier results and background may be found in [3]. The idea was to use coherent states to probe related issues in \( \lambda \phi^4 \) field theories. In \( \lambda \phi^4 \) theory on a compact space, the homogeneous sector can play the part of a minisuperspace model. The issue of course is, if one begins with a coherent state of the full theory which well approximates a state in the minisuperspace model, whether the evolution of this state will then track the evolution of the minisuperspace state or whether, due to nonlinearities and dispersion, the evolutions will rapidly diverge. He is also currently pursuing such issues in other field theories, such as the Sine-Gordon model, which can be directly related to 1+1 gravity.

The connection between sine-Gordon theory and 1+1 gravity was in fact the subject of another talk, “Black Holes and Solitons” by Gabor Kunstatter. He discussed work [4] showing the connection between black holes in (Lorentzian signature) Jackiw-Teitelboim gravity and Euclidean sine-Gordon solitons. It was previously known that the metrics that solve this theory also solve the sine-Gordon equation in an appropriate sense. Kunstatter and his collaborator Gegenberg have now shown that the dilaton, which generates Killing vectors in this gravity theory, also generates symmetries of the sine-Gordon soliton. While the gauge structure of the two theories is vastly different on the surface, Kunstatter expressed the hope that this connection could lead to a better understanding of black hole entropy, perhaps following the ideas of Carlip [5].

Kunstatter’s was not the only talk to touch on black hole entropy. Hans Kastrup also discussed his ideas [6] for studying the entropy of black holes by first canonically quantizing the (3+1) Schwarzschild metric in a certain way and then relating the resulting states to the Ising droplet nucleation model in 2 dimensions. His talk “Quantum Statistics of Schwarzschild Black Holes and Ising Droplet Nucleation” also showed that this connection can be extended to higher dimensional black holes and higher dimensional droplet models.

The canonical quantization of black holes in particular and of spherically symmetric gravity more generally was discussed in some depth by John Friedman. Friedman’s talk “Topological Geons in the context of spherically symmetric minisuperspace” was submitted with two coauthors, Jorma Louko and Steven Winters-Hilt, and the work of all three
was discussed. They were interested both in vacuum wormholes of various topologies and in such spacetimes with an additional degree of freedom corresponding to a collapsing shell of massive dust [3]. Using the quantizations they found to be natural (associated, for example, with the proper time along the shell), they found a discrete energy spectrum for the black hole.

Another talk concerning black holes, “Non-commutative Black Hole Algebra and String Theory from Gravity,” was given by Sebastian de Haro Olle. Using the framework of ’t Hooft, he generalized an action which describes gravitational scattering between point particles and found that it has just the form of a string theory action, complete with an antisymmetric tensor term and a dilaton. At the quantum level, he found that this leads to four noncommuting coordinates.

The final talk related to black holes black holes was given by Sukanta Bose. While the black holes themselves were not his central topic, his talk, “On different approaches to quantizing two-dimensional dilaton-gravity models” concentrated on models that are famous for containing 1+1 dimensional black holes. The point of this work was to compare the perturbative path integral quantization of such models with an exact canonical quantization. He found that the anomaly in the canonical commutator algebra was related to the Polyakov-Liouville term of the path integral method, and that the choice of canonical anomaly potential is related to the choice of local covariant counterterms in the one-loop action and to the quantum state of the matter fields. The talk is based on unpublished work, but see [4] for related work by Bose.

A second talk on 1+1 dimensional theories was “Quantum Fields at any time by Madhavan Varadarajan, for which Charles Torre was a coauthor. The purpose of this delightful talk was to address issues of unitarity when studying quantum fields propagating on curved spacetime backgrounds. In fact, they studied only flat backgrounds, but considered arbitrary slicings. They asked the question of whether the fields (and thus the Schrödinger picture Hilbert space) associated with some given hypersurface are unitarily related to those of an arbitrary hypersurface. They find that such evolution is unitary on the manifold $\mathbb{R} \times S^2$ [10], but that it is generically not so on $\mathbb{R} \times \mathbb{R}$ or on higher dimensional manifolds. The phenomenon is analogous to the lack of unitarity that arises in curved spacetime quantum field theory due to an infinite creation of particles.

A third lower dimensional talk was Jeanette Nelson’s “Constants of motion and the conformal anti-De Sitter algebra in (2+1)-Dimensional Gravity.” Nelson reported on work [11] with her collaborator, Vincent Moncrief, in which constants of motion were calculated for the case where the spacetime topology is $\mathbb{R} \times T^2$. The algebra of these observables was related to the conformal group and the action of large diffeomorphisms on this algebra was discussed.

Large Diffeomorphisms were also addressed by Dominico Giulini in his talk “Mapping Class Groups and their reduction in Quantum Gravity.” He was interested in the structure of the mapping class groups of three dimensional manifolds and of how to reduce a classical or quantum observable algebra with respect to such diffeomorphisms. He reported a number of results concerning manifolds that can be represented as connected sums of prime manifolds. When the connected sum contains no handles ($S^1 \times S^2$), the mapping class group has the structure of an iterated semi-direct product. He also showed that, for a large class of (and perhaps all) three-manifolds, this group is residually finite. Some of Giulini’s past work on this subject can be found in [12].

On a completely different note, Carlos Kozameh presented work on “The phase space of Radiative Spacetimes” which was done together with Malcolm Ludvigsen. Some of this work has already appeared [13]. By studying the covariant symplectic structure of general relativity, they were able to show that the phase space of GR is in fact foliated into an infinite number of leaves, each labeled by the value of the mass aspect ($\psi_2$) at spatial infinity. In a quantum theory, these leaves would correspond to superselection sectors. The ensuing discussion suggested that, as usual, these superselection rules may be related to gauge transformations that are not generated by constraints in the corresponding canonical theory.

The remaining non-loop talk, “Global Anomalies in Canonical Gravity,” was given by Sumati Surya. She and her collaborator, Sachindeo Vaidya, were interested in using topological arguments to study the possibility of certain gravitational anomalies that might arise in the presence of chiral fermions. The point here is that known anomalies in theories with chiral fermions can be related to the existence of non-contractible curves in what, in the GR context, would be called the space of semi-classical theories: that is, in the space of theories defined by quantizing matter fields on a given classical background. The anomalies arise when the adiabatic fermion vacuum changes by a nontrivial phase under transport around such a loop. For technical reasons, they introduced an SU(2) gauge field and found that, when coupled to gravity on a manifold $\Sigma \times \mathbb{R}$, all such phases are trivial when $\Sigma$ is a Lens space [14]. As a result, such a theory should be free of this kind of anomaly.

The remaining five talks concerned the loop approach to quantum gravity. Since these were in fact closely related, I will discuss them together. First, however, it is appropriate to simply list the speakers and their titles: Thomas Thiemann – “Towards Solving the Quantum Einstein Equations: A Status report,” Roberto de Pietri – “The Matrix Elements of the Constraint Operators in Loop Quantum Gravity,” Jerzy Lewandowski – “Loop constraints: A habitat, their algebra, and solutions,” Renate Loll – “Lattice Methods as a Tool for Understanding Quantum Effects,” and Jorge Pullin – “The conflict between diffeomorphism invariance and the field theoretic nature of quantum gravity.”
A common theme of many of these talks was the constraints and dynamics of quantum (loop) gravity. Thiemann gave an overview of his past work [13] on the subject and of his current thoughts for constructing improved constraints. De Pietri discussed some details of the matrix elements of constraints that have been proposed by Thiemann [16] and a related, but perhaps alternative formulation of loop gravity through path integrals which has been described by Reisenberger and Rovelli [11]. For De Pietri’s past work on the subject, see [18].

Lewandowski discussed some difficulties that have recently been found [19] with past proposals for the constraints. In particular, the algebra of these constraints appears to be a rather truncated version of the hypersurface deformation algebra. He also discussed a general incompatibility between having well-defined Hamiltonian constraints and having as much Hilbert space structure as one might like. The point here is that if the Hamiltonian constraints were well-defined operators in some Hilbert space and if their domains contained states \( \langle \psi | H(N)H(M) | \psi \rangle \) which were invariant under spatial diffeomorphisms, then the inner product \( \langle \psi | H(N)H(M) | \psi \rangle \) would provide a diffeomorphism invariant bilinear form on the fields \( N \) and \( M \). However, the first paper of [13] shows that, unless \( N \) and \( M \) are densities of weight 1/2, the only such bilinear form is the trivial one (zero).

The talks of Thiemann and Pullin also referred to the constraint algebra difficulties found in [19], and both speakers expressed the belief that the problem could be avoided by some modification of the loop approach. Thiemann favored changing the definition of spin network states to allow an infinite number of vertices, while Pullin suggested that more attention should be paid to results associated with the Chern-Simons state and certain knot invariants (the Vassiliev invariants) that are associated with Chern-Simons theory. He and Rodolfo Gambini have introduced [20] a well-defined loop derivative on the space of such invariants and have generalized the notion of Vassiliev invariants to spin-networks. The idea is that this may allow the development of a new class of proposals for the Hamiltonian constraint.

The talk by Loll had a somewhat different flavor. While inspired by loop techniques, she is interested in the idea that the discreteness present in the loop approach may not be fundamental in an of itself, but may form a useful approximation to a continuum limit. In this context, for example, the discreteness of areas and volumes in loop quantum gravity may also not be fundamental. She reported the results of a recent calculation [21] suggesting that the quantum algebra of discrete diffeomorphisms in such an approach closes in the limit of vanishing lattice spacing.

References:

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