A non local unitary vector model in 3-D

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Abstract
We present a unified analysis of single excitation vector models in 3D. We show that there is a family of first order master actions related by duality transformations which interpolate between the different models. We use a Hamiltonian (2+1) analysis to show the equivalence of the self-dual and topologically massive models with a covariant non local model which propagates also a single massive excitation. It is shown how the non local terms appears naturally in the path integral framework.

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1 Introduction

Three dimensional vector and gravity models present the most simple set up where duality transformations allow an explicit equivalence of different systems. They also have intrinsic interest for the particular mechanism which generates the mass of the excitations by the incorporation of topological terms [1, 2]. Both these facts have been present in the recent discussions of alternative higher order actions for three dimensional massive gravity [3, 4, 5, 6] which generalize the Fierz-Pauli theory and the parity sensible models. These models provide exceptions [7] to the standard association of higher order actions with ghost propagation generally expected in both the bosonic [8] and the fermionic [9] cases. This should prompt interest to investigate the limitations of that otherwise very useful guiding principle as already proposed in Ref. [10] in a different context. The form in which the intermediate master actions appear in the analysis of vector and gravity models in 3D suggests a mechanism for generating models with higher derivatives which may be unitary. In this paper we consider this possibility for the case of vector fields and construct with this goal, a hierarchy of master actions. We show that the mechanism which generates the higher order equations saturates in this case at the second order. Nevertheless instead of a third order unitary model, we find that a non local unitary model which describes a single massive excitation appears. This is interesting since non-local models, which were introduced early in quantum field theory [11, 12] and play an important role through the effective action in the functional approach [13, 14], have been found in recent years to be of importance in defining the dynamics of higher spin fields [15].

2 Vector models in 3-D

Local, covariant unitary massive vector models in 3D may describe either one or two excitations with definite parity. The usual Proca model (PM) with a Fierz-Pauli mass term whose action is

\[ I_{FP}[A] = \langle -\frac{1}{4} F^{\mu\nu}(A) F_{\mu\nu}(A) - \frac{m}{2} A^\mu A^\mu \rangle. \]  

(1)

describes two parity sensible excitations. The self dual model (SDM) [16]

\[ I_0[a] = \langle -\frac{m}{2} a^\mu a^\mu + \frac{1}{2} a^\mu \epsilon^{\mu\nu\rho} \partial_\nu a_\rho \rangle. \]  

(2)

and the topologically massive model (TMM) [1, 2]

\[ I_{TM}[A] = \langle \frac{1}{2m} \epsilon^{\mu\nu\rho} \partial_\nu A_\rho \epsilon_{\mu\sigma\tau} \partial^\sigma A^\tau - \frac{1}{2} A^\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho \rangle. \]  

(3)

describes each a single excitation of definite parity.

Locally, these last two models are canonically equivalent [17], the SDM being a gauge fixed representation of the gauge invariant TMM [18, 19, 20, 21].
The TMM and the SDM are also related by a duality transformation. This equivalence is most compactly encoded in the master action [17] which allows to connect them in a covariant way. It is given by

\[ I_1[a, A] = \left< -\frac{m}{2} a_\mu a^\mu + A_\mu \epsilon^{\mu \nu \rho} \partial_\nu a_\rho - \frac{1}{2} A_\mu \epsilon^{\mu \nu \rho} \partial_\nu A_\rho \right> . \]  

(4)

Taking variations with respect to \( A_\mu \) one obtains the identity

\[ \epsilon^{\mu \nu \rho} \partial_\nu a_\rho = \epsilon^{\mu \nu \rho} \partial_\nu A_\rho , \]  

(5)

which assures that the transverse parts of \( A_\mu \) and \( a_\mu \) are equal. Substitution in the master action leads to the self dual action \( I_1[a, A(a)] = I_0[a_\mu] \). This procedure may be justified using the gauge invariance of the action. On the other hand, the equation which results from taking variations with respect to \( a_\mu \) establishes that \( a_\mu \) is transverse. We should write it in the form

\[ a_\mu(A) = \frac{1}{m} \epsilon^{\mu \nu \rho} \partial_\nu A_\rho , \]  

(6)

stressing its structure as a kind of change of variables. Note that for \( a = A \) this is the equation of motion of the SDM. Upon substitution in (4) the second order gauge invariant TMM action (3) is generated. Since both (5) and (6) involve time derivatives one may in principle wonder if this procedure guarantees canonical equivalence. In this case it is indeed true that the two models (2) and (3) are canonically equivalent but, as we discuss below, a similar strategy may in other cases connect non-equivalent models.

In order to search for higher order models or alternative formulations of the TMM and the SDM, one may use again the trick relating \( I_1[a, A] \) with \( I_0[a_\mu] \) and introduce a second intermediate equivalent action given by,

\[ I_2[a, A, B] = \left< -\frac{m}{2} a_\mu a^\mu + a_\mu \epsilon^{\mu \nu \rho} \partial_\nu A_\rho - A_\mu \epsilon^{\mu \nu \rho} \partial_\nu B_\rho + \frac{1}{2} B_\mu \epsilon^{\mu \nu \rho} \partial_\nu B_\rho \right> . \]  

(7)

The equation of motion obtained by taking variations with respect to \( B \) relates \( A \) and \( B \) in the same form as (5) relates \( a \) and \( A \) in \( I_1 \). Thus taking into account the gauge invariance, \( I_2[a, A, B] \) is also equivalent to \( I_1[a, A] \). If instead we use (6) which is again the equation obtained by taking variations with respect to \( a_\mu \), we end up with

\[ I_2^{TM}[A, B] = \left< \frac{1}{2m} \epsilon^{\mu \nu \rho} \partial_\nu A_\rho \epsilon_{\mu \sigma \tau} \partial^\sigma A^\tau - A_\mu \epsilon^{\mu \nu \rho} \partial_\nu B_\rho + \frac{1}{2} B_\mu \epsilon^{\mu \nu \rho} \partial_\nu B_\rho \right> . \]  

(8)

By the discussion just presented, which will be complemented by the Hamiltonian analysis of section (4) this action is also equivalent to \( I^{TM} \). As discussed in Refs. [22, 23] the introduction of these new fields corresponds to a duality transformation done in the quantum mechanical generating functional. Although the different models are seen to be locally equivalent they are not equivalent on topologically non trivial manifolds [24]. In the path integral formulation this
is reflected in that the generating functionals of the models differ by a factor which is a power of the pure Chern Simons generating functional. We also note that an explicit Chern Simons term $-\frac{1}{2} A_\mu \epsilon^{\mu \nu \rho} \partial_\nu A_\rho$ for the field $A$ may be included in (6) or (8) without affecting the symmetries of the action. In fact it can be seen that this amounts to a shift of the fields and a redefinition of the mass.

We can generalize this procedure and introduce a family of equivalent first order actions

$$ I_N[A'] = < -\frac{m}{2} A^{(0)}_\mu A^{(0)\mu} + A^{(0)}_\mu \epsilon^{\mu \nu \rho} \partial_\nu A^{(1)}_\rho - A^{(1)}_\mu \epsilon^{\mu \nu \rho} \partial_\nu A^{(2)}_\rho + \cdots + (-1)^N \frac{1}{2} A^{(N)}_\mu \epsilon^{\mu \nu \rho} \partial_\nu A^{(N)}_\rho >, $$

with alternating signs for the coupling terms and $N \geq 1$. Each of these families is generated by applying iterate duality transformations either to the SDM or the TMM.

### 3 Models with higher derivatives

The actions considered till now and others which appear in the discussion below are arranged in Fig.1. Straight arrows connect physically equivalent actions and dashed arrows denote relations between actions when at least one of them is not unitary. To continue, we observe that in the same way that $I_1$ leads to the second order topologically massive model $I^{TM}_2$, the action $I_2$ should generate a third order model $I^{3th}$. Substituting the equation of motion

$$ \epsilon^{\mu \nu \rho} \partial_\nu B_\rho = \frac{1}{m} \epsilon^{\mu \nu \rho} \partial_\nu \epsilon_{\rho \sigma \tau} \partial^\sigma A^\tau $$

in $I^{TM}_2$ one obtains

$$ I^{3th}[A] = < -\frac{1}{2m^2} \epsilon^{\mu \nu \rho} \partial_\nu \epsilon_{\rho \sigma \tau} \partial^\sigma A^\tau - \frac{1}{2m^2} \epsilon^{\mu \nu \rho} \partial_\nu A_{\rho} \epsilon_{\rho \sigma \tau} \partial^\sigma A^\tau >. $$

The curved arrow in Fig.1 between $I_0$ and $I^{3th}$ points out that they are also related by the covariant, but not canonical change of variables, $a(A)$ defined by (6). Due to the higher order derivatives present in the actions none of these procedures allow to establish the canonical equivalence between the actions considered and in fact $I^{3th}$ and $I^{TM}_2$ are not equivalent (hence, the corresponding arrow in the graph above is dotted). The model defined by $I^{3th}$ is not unitary.
as was shown in [25] where the propagation of a massive transverse mode and a spurious massless ghost was demonstrated. Since the self dual action depends on the longitudinal part of \( a_\mu \) perhaps it is not surprising that the change of variables (9), which only define the transverse part of the field \( a_\mu \) does not generate a unitary model. In section (5) we show, using the Hamiltonian analysis, that instead there exists a non local unitary model which is equivalent to \( I_{2}^{TM} \) and hence to \( I^{TM} \) and \( I_{0} \).

The models related to \( I_{2} \) do not exhaust the set of master actions of potential interest. As already said, all the actions \( I_{N} \) defined by (9) are equivalent to \( I_{0} \).

We now check if they generate other unitary equivalent actions. Consider then \( I_{3} \) which renaming the fields takes the form,

\[
I_{3}[a, A, B, C] = \left< \frac{m^2}{2} a_\mu a^\mu + a_\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho - A_\mu \epsilon^{\mu\nu\rho} \partial_\nu B_\rho
+ B_\mu \epsilon^{\mu\nu\rho} \partial_\nu C_\rho - \frac{1}{2} C_\mu \epsilon^{\mu\nu\rho} \partial_\nu C_\rho \right>.
\] (13)

The action \( I_{3}^{th,F_{P}} \) is obtained by unfolding the Maxwell term of \( I_{3}^{th} \) with the aid of an auxiliary field, and reads,

\[
I_{3}^{th,F_{P}}[a, A] = \left< \frac{1}{2} a^\mu a_\nu - a^\mu \epsilon_{\mu\alpha\beta} \partial_\alpha A_\beta + \frac{1}{2m^2} \epsilon^{\mu\nu\rho} \partial_\nu \epsilon_{\rho\alpha\beta} \partial_\alpha A^\beta \epsilon_{\mu\sigma\tau} \partial_\sigma A^\tau \right>.
\] (14)

The action \( I_{2}^{th} \) is obtained eliminating \( B \) in \( I_{3}^{TM}[A, B, C] \) using the equa-
tions of motion. It is given by

\[ I_{2}^{3th}[A, C] = \left< \frac{1}{2m} \epsilon^{\mu\nu\rho} \partial_\nu A_\rho \epsilon_{\mu\sigma\tau} \partial^\sigma A^\tau \right> \]

(15)

Note that the coupling here is second order. By resolving \( C \) in terms of \( A \) using the equations of motion in (15) one obtains \( I_{2}^{3th} \), but no fourth order model may be generated from \( I_{2}^{3th} \) in any evident way.

Instead, insisting in eliminate \( A \) covariantly one recovers \( I_{T M} \) without of course establishing canonical equivalence. The Hamiltonian analysis of \( I_{2}^{3th} \) which we discuss in the following section shows that it is equivalent to the ghost propagating \( I_{3}^{3th} \). Generalizations of \( I_{3}^{3th} \) in the spirit of (9) and (10) are straightforward to define and would appear related to \( I_{N} \) with \( N > 3 \), but no promising unitary models appear from such analysis.

The action \( I_{2}^{3th} \) may also be obtained using the self-dual change of variables (6) in the form \( (a \to a(A), A \to C) \) in \( I_{1}[a, A] \). Substituting the second field in the same action with a similar change of variables one obtains,

\[ I_{4}^{3th}[a, C] = \left< \frac{1}{2m} a^\rho \epsilon_{\rho\sigma\tau\nu} \partial_\nu C_\sigma \epsilon^{\mu\sigma\tau} \partial^\mu \right> \]  

(16)

By eliminating covariantly \( a \) in this action one gets, after some rearrangements,

\[ I_{4}^{3th}[C] = \frac{1}{m^2} \left< \frac{1}{2m} \epsilon^{\mu\nu\rho} \partial_\nu C_\rho \epsilon_{\rho\sigma\tau} \partial^\sigma C^\tau - \frac{1}{2} C_\mu \Box \epsilon^{\mu\nu\rho} \partial_\nu C_\rho > \right). \]

(17)

This fourth order gauge invariant action may also be generated using the self-dual change of variables \( A(C) \) in the topologically massive action (3). The action (17) is simply the \( T M M \) with an interpolated D'Alembertian operator and propagates ghosts. Iterating the process we rise the order of the action by two each time and we can generate the actions

\[ I_{2n}^{3th}[B_\mu] = \frac{1}{m^{2(n-1)}} \left< \frac{1}{2m} \epsilon^{\mu\nu\rho} \partial_\nu B_\rho \Box^{n-1} \epsilon_{\mu\sigma\tau} \partial^\sigma B^\tau - \frac{1}{2} B_\mu \Box^{n-1} \epsilon^{\mu\nu\rho} \partial_\nu B_\rho > \right), \]

with \( n = 2, 3, \ldots \) The TMM may be identified with the case \( n = 1 \)

4 Hamiltonian analysis

Let us turn to the \((2 + 1)\) analysis of the dynamics in the so called Hamiltonian variables (26, 27, 28). For that consider the operators

\[ \rho \equiv \sqrt{-\partial_i \partial_i}, \quad \rho_\iota \equiv \frac{\partial_i}{\rho}, \quad \sigma_i \equiv \epsilon_{ij} \rho_j, \quad (19) \]

\footnote{We use the metric \( \eta = diag(-1, 1, 1) \), \( \epsilon^{012} = -\epsilon_{012} = 1, \epsilon^{0j} = \epsilon_{ij}, \epsilon_{ijk} \sigma_k = \delta_{jk} \)}
which satisfy
\[ \rho_i \rho_j = -1 = \sigma_i \sigma_j , \quad \epsilon_{ij} \sigma_j = -\rho_i , \quad \rho_i \sigma_i = 0 , \quad \Box = -\partial_0 \partial_0 - \rho^2 , \]  
and define transverse and longitudinal variables of a vector field \( A \) by [27, 28],
\[ A_\mu \to A_0 , \quad A_i = -\sigma_i A^T - \rho_i A^L , \quad A^T = \sigma_i A_i , \quad A^L = \rho_i A_i . \]  
The transverse variable \( A_T \) is gauge invariant. Define also the auxiliary gauge invariant variable
\[ F_A = \rho A_0 + \partial_0 A^L . \]  
Using these variables the self dual action reduces to
\[ I_0[a] = \frac{1}{2m} <a_0 a_0 - a^T a^T - a^L a^L> - <a^T F_A> , \]  
whereas the topologically massive model has the form [1]
\[ I^{TM}[A] = \frac{1}{2m} <A^T(\Box) A^T + F_A F_A> + <A^T F_A> . \]  
Here we see clearly that both actions propagate only a massive mode and that it is the coupling of the transverse mode to the gauge invariant variable \( F_A \) what generates the mass of the systems. Although \( F_A \) includes a time derivative in its definition it may be verified, by looking at the equations generated by \( A_0 \) and \( A^L \) separately, that it can be used safely as a fundamental variable in this case. This occurs because the variations with respect to \( A_0 \) generate a constraint and the variations with respect to \( A^L \) generate the time derivative of this constraint. In some of the models with higher derivatives discussed below this does not happen.

For the master action \( I_1 \), one has
\[ I_1[a,A] = \frac{1}{2m} <a_0 a_0 - a^T a^T - a^L a^L> + <A^T F_A - A^T F_a - a^T F_A> , \]  
which reduces directly to [23] or [24] when either one or the other of the fields is eliminated. The reduced form of \( I_2[a,A,B] \) and \( I^{TM}_2[A,B] \) are given by
\[ I_2[a,A] = \frac{1}{2m} <a_0 a_0 - a^T a^T - a^L a^L> + < -A^T F_a - a^T F_A + A^T F_B + B^T F_A - B^T F_B> , \]  
\[ I^{TM}_2[A,B] = \frac{1}{2m} <A^T(\Box) A^T + F_A F_A> + <A^T F_B + B^T F_A - B^T F_B> . \]  
Eliminating \( B^T \) and \( F_B \) from \( I_2[a,A,B] \) one recovers the form [25] of \( I_1[a,A] \). Alternatively substituting first the expressions for \( a_0, a^L \) and \( a^T \) one gets the form [27] of \( I^{TM}_2[A,B] \). This completes the demonstration of the canonical equivalence of the unitary models considered until now.
There is an alternative approach which can be taken to analyze the dynamical content of $I_2^{TM}[A, B]$ and is to eliminate $B$ in (27). Then, one arrives in one step to (24). Restricting the functional space adequately in order to have the inverse operators well defined and eliminating $A$ in the same action one obtains the new equivalent action

$$I_2^{TM}[A(B), B] = -\frac{m}{2} < B^T B^T + F_B(\Box)F_B > - < B^T F_B >,$$

which, in spite of being non local, should propagate a single massive excitation. This is enforced by the equation of motion for $B^T$,

$$m^2 B^T - B^T = 0$$

which is obtained after eliminating $F_B$. Moreover, including a Chern Simons term for $A$ in (7) or (8) in the form mentioned in the previous section simply results in a shift $m \rightarrow (m + \beta)$ of the mass. In the next section we consider this action further.

For the reduced action of $I_{3th}^{th}$ which appeared in the covariant treatment the canonical analysis gives [25],

$$I_{3th}[A] = -\frac{1}{2m} < A^T \Box A^T + F_A F_A > - \frac{1}{m} < A^T \Box F_A >,$$

which as mentioned in the previous section propagates a massive particle and a massless ghost.

The action $I_{2th}^{th}$ in Hamiltonian variables is given by

$$I_{2th}[A, C] = \frac{1}{m} < -\frac{1}{2} A^T \Box A^T + \frac{1}{2} F_A F_A + A^T \Box C^T + F_A F_C - m C^T F_C >,$$

which may be reduced to

$$I_{2th}[A, C] = \frac{1}{m} < -\frac{1}{2} A^T \Box A^T + A^T \Box C^T - \frac{m}{2} C^T C^T >,$$

This action is equivalent to [30] after a field redefinition and hence is not unitary.

Finally for the ghost propagating $I_{4th}^{th}$ we have the reduction

$$I_{4th}[B] = \frac{1}{2m^2} < \frac{1}{m} B^T (\Box^2) B^T + F_B(\Box)F_B > + < B^T (\Box) F_B >.$$

5 The topologically massive non local model

Due to the linearity of the reduction to the Hamiltonian variables, comparison of (28) and (30) allows us to recognize that the non local covariant action

$$I_{NL}[B] = -\frac{1}{2} < m\epsilon^{\mu
u\rho\sigma} \partial_\nu B_\rho \epsilon_{\mu\sigma\tau} \partial^\tau B^\tau >$$

$$+ \frac{1}{2} < \epsilon^{\mu
u\tau} \partial_\nu \epsilon_{\rho\sigma\tau} B_\rho \epsilon_{\mu\sigma\tau} \partial^\tau B^\tau >$$

(34)
when expressed in the Hamiltonian variables has also the decomposition (28),
\( I^{NL} = I^{M} \), and propagates a single massive
excitation. For the second term in (34) we note the following identity
\[
I^{CSNL}[B, \beta] \equiv \frac{\beta}{2} < \epsilon^{\mu\nu\rho} \partial_\nu \epsilon_{\rho\sigma\tau} \Box B^\beta \epsilon_{\mu\sigma\tau} \partial^\sigma B^\tau > + \frac{\beta}{2} \epsilon^{\mu\rho} \partial_\mu \epsilon_{\rho\sigma\tau} \Box \epsilon_{\sigma\tau} B^\beta \equiv -I^{CS}[B, \beta],
\]
(35)
where in the second line we have the pure Chern-Simons action
\( I^{NL} \) may be
deduced covariantly from (8) by first noting that the equation of motion (11)
implies
\[
\epsilon^{\mu\nu\rho} \partial_\nu A_\rho = m \epsilon^{\mu\nu\rho} \partial_\nu \Box \epsilon_{\rho\sigma\tau} \partial^\sigma B^\tau,
\]
(36)
and propagates a single particle with modified mass \( m/\beta \).

6 The Fierz-Pauli non local model

The natural generalization to the PM of the ideas worked in the previous sections
is to apply the self dual change of variables (6) in (1). This leads to the third
order action,
\[
I^{FPSD}[B] = \frac{1}{4} \epsilon^{\mu\nu} F_{\mu\nu}(B) F^{\mu\nu}(B) - \frac{1}{4m^2} \epsilon^{\mu\nu} F_{\mu\nu}(B) \Box F^{\mu\nu}(B) >
\]
(40)
which propagates a massless ghost together with the two massive modes. The
remaining option is to explore the non-local alternatives. The structure observed
so far suggests the following model,
\[
I^{FPNL}[B] = -\frac{1}{4} \epsilon^{\mu\nu} F_{\mu\nu}(B) F^{\mu\nu}(B) + \frac{m^2}{4} \epsilon^{\mu\nu} F_{\mu\nu}(B) \Box F^{\mu\nu}(B) >
\]
(41)
which has been discussed in connexion with Stueckelberg formalism [36]. The non-local Maxwell term may be viewed as a Fierz-Pauli mass term plus a non-local gauge fixing term. The Hamiltonian analysis establishes that this action may be written in the form

$$I_{FPNL}[B] = \frac{1}{2} < B^T (\Box - m^2) B^T > + \frac{1}{2} < F_B F_B - m^2 F_B (\frac{1}{\Box}) F_B > , \tag{42}$$

which suggests the propagation of two massive modes with the same mass. To compare, the Hamiltonian decomposition of the Proca-Fierz-Pauli action after eliminating $A_0$ takes the form

$$I_{FP}[B] = \frac{1}{2} < A^T (\Box - m^2) A^T > + \frac{m^2}{2} < A^L (\Box - m^2) A^L > . \tag{43}$$

The non-local Maxwell term may be induced by a $BF$ coupling with a massless auxiliary field in a way resembling the mechanism in 2D for the Schwinger model. In that case [29] the non-local Maxwell term of (41) which appears after integrating out the fermions, is expressed in terms of a local scalar field which propagates a massive excitation and the original field which does not propagate. Here we introduce a second vector field with a $BF$ coupling to obtain the same behavior. In terms of the gauge fixed massless Maxwell action

$$I^M(C, \kappa) = < -\frac{1}{4} F_{\mu\nu}(C)(F^{\mu\nu}(C) + \kappa \partial_{\mu} C^{\mu} \partial_{\nu} C^{\nu} > \tag{44}$$

which in 3-D propagates a single degree of freedom, the model that we consider reads

$$I_{CB} = I^M(C, \kappa) + \frac{1}{2} C^{\mu}_{\kappa} \epsilon^{\mu\nu\rho} \partial_\nu B_\rho > + I^M(B, \tilde{\kappa}) . \tag{45}$$

This model propagates two massive modes of opposite helicity as can be shown by means of the Hamiltonian analysis or doing the decoupling change of variables

$$B_\mu = \frac{1}{\sqrt{2}} [A^1_\mu + A^2_\mu] , \quad C_\mu = \frac{1}{\sqrt{2}} [A^1_\mu - A^2_\mu] . \tag{46}$$

For the analysis in the functional integral introduce the operators associated to the gauge fixed Maxwell action

$$D^{M\nu}(\kappa) = \Box \eta_{\mu\nu} - (\kappa + 1) \partial_\mu \partial_\nu , \quad G^{M\nu}(\kappa) = \frac{\eta_{\nu\rho}}{\Box} - \frac{(\kappa + 1)}{\kappa} \partial_\nu \partial_\rho \tag{47}$$

which satisfy

$$D^{M\nu}(\kappa) G^{M\nu}(\kappa) = \delta^{\rho}_{\nu} , \quad G^{M\nu}(\kappa) \epsilon^{\rho\alpha\beta} \partial_\alpha C_\beta = \epsilon^{\nu\alpha\beta} \partial_\alpha C_\beta . \tag{48}$$

Then we have the identity,

$$\int DCDB e^{-[I^M(C, \kappa) + \frac{1}{2} C^{\mu}_{\kappa} \epsilon^{\mu\nu\rho} \partial_\nu B_\rho > + I^M(B, \tilde{\kappa})]} \quad = \quad \int DC'DB e^{-[I^M(C', \kappa) + \frac{m^2}{2} < F_{\mu\nu}(B)(\frac{1}{\Box}) F^{\mu\nu}(B) > + I^M(B, \tilde{\kappa})]} . \tag{49}$$
where $C'_\mu = C_\mu - mG_\mu^{\nu}(\kappa)\epsilon_{\nu\alpha\beta}\partial^\alpha B^\beta$. One of the vector fields is the physical field which couples to the sources and acquires its mass through the appearance of the non-local Maxwell term. The second field is an auxiliary field which does not couple to the sources and contributes a constant factor to the generating functional which in turn is absorbed in the normalization constant.

Alternatively, the Fierz Pauli non local model emerges from the Stueckelberg model in any dimension

$$I^S(A, \Phi) = -\frac{1}{4} <F_{\mu\nu}(A)F^{\mu\nu}(A)> - \frac{m^2}{2} <(A_\mu + \frac{1}{m}\partial_\mu \Phi)(A^\mu + \frac{1}{m}\partial^\mu \Phi)>$$

(50)

by decoupling $\Phi$ in the functional integral in the usual way. Again a factor corresponding to an uncoupled massless scalar field remains in the generating functional and is absorbed in the normalization constant.

7 Discussion

In this paper we presented a unified analysis of massive vector models in 3D in terms of a set of master actions related by duality transformations. Using the Hamiltonian $(2 + 1)$ canonical variables the equivalence between the various models was checked explicitly. This allowed us to identify a new unitary non local vector model with a single massive excitation. The non-local action emerges as an improvement, suggested by the Hamiltonian analysis of a third order non unitary model which appears naturally in the covariant reduction of one of the first order actions generated by duality. Non-local gauge invariant actions in various dimensions have also been discussed in the literature in relation with the Stueckelberg formalism (see [36] for a recent review). It is then of interest that in 3D complete canonical equivalence with a local model may be demonstrated using the Hamiltonian variables.

The non local terms discussed also may be extracted by decoupling the fields in the path integral formalism as was illustrated with the non-local Fierz Pauli model and are related in this way to the Stueckelberg field [36]. It remains to be explored if interactions could be incorporated consistently to these models. It is not ruled out that the techniques used in this work when applied to an action with higher $N$ in (9) or (10) may generate also non local unitary models.

Although the non local formulation may perhaps be only of limited value in an operatorial quantum mechanical description it should be useful in the path integral framework in connection with the dual representation of low dimensional systems. In particular the system defined by (37) may be also of interest in relation with bosonization of fermion fields in 3D [30, 31, 32, 33, 34]. In 3D the bosonized fermion current is identified with a Chern Simons topological current [31].

$$\bar{\psi}\gamma^\mu \psi \sim e^{\mu\nu\rho} \partial_\mu A_\nu \ .$$

(51)

Both in the operatorial [30] and in the functional approaches [33, 34] the effective bosonized action is shown in general to be non local and it could be interesting to determine if there is some relation with the system at hand.
Some of the ideas presented here may be extended to the self-dual, second order [35], topologically massive [1] and fourth order [3, 4] three dimensional massive gravity models which share a part of the structure of the vector models and also display duality relations.

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A Chern Simons self duality

We can get more insight about where the non-local terms come from by considering the dual to the pure Chern Simons action. As mentioned in the introduction this is obtained from the gauge fixed Chern Simon action of the gauge field $A$ by means of $BF$ coupling with a new vector field $B$ [22],

$$I_{gf}^{CSD}[A,B] = < -\frac{\beta}{2} A_\mu \epsilon^{\mu\nu\rho} \partial_{\nu} A_\rho + \frac{\kappa}{2} \partial_{\mu} A^\mu \partial_{\nu} A^\nu $$

$$- A_\mu \epsilon^{\mu\nu\rho} \partial_{\nu} B_\rho + \frac{\kappa}{2} \partial_{\mu} B^\mu \partial_{\nu} B^\nu >$$  \hspace{1cm} (A.1)

where the subscript $gf$ is a remainder that a gauge fixed term has been included for each field. Introduce now,

$$D_{\mu\nu} = \beta \epsilon_{\mu\nu\rho} \partial_{\rho} - \kappa \partial_{\mu} \partial_{\nu} , \quad G_{\nu\rho} = \frac{1}{\beta} \epsilon_{\nu\rho\tau} \partial^\tau - \frac{1}{\kappa} \frac{1}{\Box} \partial_{\nu} \partial_{\rho} ,$$  \hspace{1cm} (A.2)

which satisfy in symbolic Heaviside notation

$$D_{\mu\nu} G_{\nu\rho} = \delta^\mu_\nu , \quad G_{\nu\rho} \epsilon^{\rho\tau\sigma} \partial_{\tau} B_\sigma = \frac{1}{\beta} \epsilon_{\nu\rho\lambda} \partial^\lambda \epsilon^{\rho\tau\sigma} \partial_{\tau} B_\sigma$$  \hspace{1cm} (A.3)

and consider the vacuum amplitude

$$Z^{CSD}[0] = \int \mathcal{D}A \mathcal{D}B e^{-I_{gf}^{CSD}[A,B]} .$$  \hspace{1cm} (A.4)

Then shifting $A_\mu \rightarrow A_\mu + G_{\mu\nu} \epsilon^{\nu\tau\sigma} \partial_\tau B_\sigma$ in the generating functional after completing the square in the action one gets

$$Z^{CSD}[0] = \int \mathcal{D}A e^{-I_{gf}^{CS}[A,\beta]} \int \mathcal{D}B e^{-I_{gf}^{CSNL}[B,\tilde{\beta}]}$$  \hspace{1cm} (A.5)

where $\tilde{\beta} = -\frac{1}{\beta}$. This shows that the action $I_{gf}^{CSNL} = -I^{CS}$ in equation (35) is the dual to the pure Chern Simons model. The appearance of the apparently non-local term in $I_{gf}^{CSNL}$ may be traced to the term $-\frac{1}{\beta} \epsilon_{\nu\rho\tau} \frac{1}{\Box} \partial_{\tau}$ in the covariant propagator.
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