Analysis of the static load test results referred to limit the bearing capacity of a pile

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Abstract. In the paper the authors used results of static load tests of piles that have been conducted in a full range of loads in order to verify the analytically calculated parameters obtained for a continuous load-settlement curve. For a description of continuous static load test curve of a pile the authors have used Meyer-Kowalow method (M-K method). The result of a static load test is a load-settlement set \( \{N_i, s_i\} \) which is the background for further calculations of M-K curve parameters. Calculations that are presented in the paper were carried out using an original computer program and the authors compared M-K curve parameters, specifically limit the bearing capacity of a pile calculated and measured values obtained from the static load test performed in a full range of load. The research was done in order to determine the optimal method of calculating bearing capacity of pile based on static load test which later can be practically used in the M-K method to obtain full range continuous Q-s curve. Results obtained during the analysis performed by the authors show that the values of the limit bearing capacity of a pile can be calculated with sufficient for practical use accuracy from different parts of the data set, as an example using just the chosen part of the set. The research conducted by the authors points the fact that the limit bearing capacity which is the most important parameter determining safety, can be effectively obtained for different M-K curves using least squares method, even in cases when static load tests were not performed in a full load range. After determining M-K curve parameters it can later be used for further analysis of the pile-soil interactions.

1. Introduction
In most practical cases static load tests are performed only in limited load range which makes it necessary to calculate the limit bearing capacity of a pile. The result of the static load test is a load-settlement set \( \{N_i, s_i\} \). This data set can be used to extrapolate Q-s relation in order to determine the value of load corresponding to uncontrolled growth of settlement which can be referred to as a limit bearing capacity of a pile. The aim of this study is to conduct an analysis of results of such extrapolations, which can provide verification of the accuracy of assumed models. Stresses resulting from pile settlement can be schematically presented, as shown in figure 1.
In literature it is possible to find different approaches to solving a problem of accurately predicting the values of limit bearing capacity calculated based upon a data set that is the result of the static load test conducted in a limited range of loads. It has been described among others in [1-4]. In the further part of this paper authors decided to use one of the method that allows for full description of continuous load-settlement curve that was proposed by Meyer and Kowalow in 2010 [5]. M-K curve can be described both graphically as shown in figure 2 and numerically as shown in equation (1) and (2). In the further part of the work it will be referred to as the M-K method or M-K curve.

From figure 2 the analytical values come:
\[
\lim_{N \to 0} s(N) = C_2 \cdot N_2 \quad (1)
\]
\[
\lim_{N \to N_{gr2}} s(N) = \infty \quad (2)
\]
Following above mention assumptions it is possible to write full range equation describing continuous M-K curve:
\[
s(N_2) = C_2 \cdot N_{gr2} \cdot \left(1 - \frac{N}{N_{gr2}}\right)^{\frac{-x_2}{\kappa_2}} - 1 \quad (3)
\]
3. Estimation of limit bearing capacity of a pile.

In order to describe the M-K curve it is necessary to determine the parameters of M-K equation $\kappa_2, C_2, N_{gr2}$. Most commonly used are statistical approaches based on the least square method described among others in works of Szmechel [4], who also shows that it is possible to extrapolate the values of M-K parameters even in cases when static tests were performed only in limited range of loads. Other research concerning plasticized zone under the pile toe points the fact that CPTU results can also be used with regard to calculating the values of $\kappa_2$ [7]. For small loads it is also possible to use the method of Q-s curve conversion described in [8] in order to determine parameters presented in the M-K method.

Analysis conducted in this paper is based upon using statistical mathematics, specifically least square method and it concerns just the limit bearing capacity $N_{gr2}$ values of other M-K parameters $\kappa_2, C_2$ will not be analyzed. In this paper authors used static load test that has been performed in a full range of loads on reinforced concrete piles executed in medium sands for about 10 meters and loam beneath that, which allowed for verification of estimated values obtained using statistics. As a result of the static load test we receive a set of load values and corresponding settlements \{Ni, si\} which can be used for estimation. If in the equation (3) we substitute:

\[
A = C_2 \cdot N_{gr2}
\]

and:

\[
Y_i = \left(1 - \frac{N_i}{N_{gr2}}\right)^{-\kappa_2} - 1
\]

we get the following:

\[
N_2(s) = N_{gr2} \cdot \left[1 - \left(1 + \frac{\kappa_2 \cdot s}{C_2 \cdot N_{gr2}}\right)\right]^{1/\kappa_2}
\]

(4)

where:

$s$ – pile settlement [mm], $C_2$ – inverse aggregated Winkler modulus [mm/kN], $N_{gr2}$ – limit bearing capacity fo a pile [kN], $\kappa_2$ – parameter that indicates proportion between skin and toe resistance of a pile, $N_2$ – load at the head of a pile [kN]

It is also possible to describe M-K curves concerning toe $N_1(s)$ and skin $T(s)$ resistance with equations (5) and (6) by analogy to load-settlement curve $N_2(s)$, which allows for detailed analysis of pile-soil interaction. We have:

\[
N_1(s) = N_{gr1} \cdot \left[1 - \left(1 + \frac{\kappa_1 \cdot s}{C_1 \cdot N_{gr1}}\right)\right]^{1/\kappa_1}
\]

(5)

\[
T(s) = N_2(s) - N_1(s)
\]

(6)
we obtain the following linear relation describing i-th settlement:
\[ s_i = A \cdot Y_i \]  
(9)
in which for a given set of load-settlement values parameter \( A \) is a constant and can be calculated from the set of \( \{N_i, s_i\} \) using the following relation:
\[ A = \frac{\sum (s_i \cdot Y_i)}{\sum (Y_i)^2} \]  
(10)
The deviation square can then be determined as follows:
\[ \delta_i^2 = (s_i - A \cdot Y_i)^2 \]  
(11)
Using equations (7)-(11) listed above it is possible to describe the procedure for M-K curve parameter approximation for the set of \( \{N_i, s_i\} \):
step 1: assume starting the value of \( \kappa_2 = \kappa_{2,j} \), for \( j = 1 \);
step 2: assume starting the value of \( N_{gr2} = N_{gr2,k} \), for \( k = 1 \);
\[ Y_{i,j,k} = \frac{(1 - \frac{N_{i,j}}{N_{gr2,k}})^{-\kappa_{2,j}} - 1}{\kappa_{2,j}} \], for \( i = 1 \ldots n; j = 1; k = 1 \);
step 3: calculate the value of \( A_{j,k} = \frac{\sum (s_i \cdot Y_{i,j,k})}{\sum (Y_{i,j,k})} \), for \( i = 1 \ldots n; j = 1; k = 1 \);
step 4: calculate deviation square values of \( \delta_{i,j,k}^2 = (s_i - A_{j,k} \cdot Y_{i,j,k})^2 \), for \( i = 1 \ldots n; j = 1; k = 1 \);
step 5: calculate deviation square values of \( \delta_{i,j,k}^2 = (s_i - A_{j,k} \cdot Y_{i,j,k})^2 \), for \( i = 1 \ldots n; j = 1; k = 1 \);
step 6: this means that for every pair of \( j \) and \( k \) there is one value of the sum of deviation squares for the data set \( \{N_i, s_i\} \); step 7: repeat steps 2-6, for \( k = 1 \ldots n \);
step 8: find for which \( k \) the value of \( \sum (\delta_{i,j,k}^2)_{j,k} \) for \( i = 1 \ldots n; j = 1; k = 1 \ldots n \) is minimal, this means that the value of \( N_{gr2,opt,j} \) corresponding to \( k \) for which the value of \( \sum (\delta_{i,j,k}^2)_{j,k} \) is minimal is optimal parameter for assumed value of \( \kappa_2 = \kappa_{2,j} \), value of \( C_{2,j} \) can be calculated from equation (7); step 9: repeat steps 1-8 for \( j = 1 \ldots n \); this means that for every assumed value \( \kappa_{2,j} \) we obtain the optimal value of limit bearing capacity \( N_{gr2,opt,j} \) and \( C_{2,j} \);
step 10: find for which \( j \) the value of \( \min \sum (\delta_{i,j,k}^2) \) is minimal, corresponding parameters \( \kappa_{2,opt,j} \), \( N_{gr2,opt,j} \) and \( C_{2,j} \) are the optimal M-K parameters for the set of \( \{N_i, s_i\} \).
As have been previously stated authors used for the analysis piles tested in a full range of loads. Pile parameters are shown in table 1. The values of the limit bearing capacity $N_{gr,2,meas}$ stated in table 1 are measured values.

### Table 1. Pile parameters.

| Pile   | H[m] | D[m] | $N_{gr,2,meas}$ [kN] |
|--------|------|------|----------------------|
| 36N-10L | 27,5 | 1    | 6490                 |
| 31-10L  | 27,5 | 2    | 7700                 |
| 38-10P  | 31,5 | 1,5  | 11900                |
| 21N-10L | 27,5 | 2    | 8260                 |
| 25N-10L | 27,5 | 1    | 6350                 |
| 38-12P  | 33,5 | 1,5  | 11785                |
| 38-12L  | 33,5 | 1,5  | 12150                |

The set of load-settlement values $\{N_i, s_i\}$ for the purpose of this analysis had been divided into three parts, as shown in figure 3. Since the entire data set consists of 23 values ($n = 23$) authors decided to divide it into three following parts:

- Part 1 consists of $n = 1...9$
- Part 2 consists of $n = 8...16$
- Part 1+2 consists of $n = 1...16$
- Part 3 consists of $n = 1...23$

![Figure 3. Division of the data set $\{N_i, s_i\}$.](image)

For each of the piles and for each part of the load-settlement data set authors carried out the statistical calculation using the procedure mentioned above in this paper to estimate optimal values of limit bearing capacity $N_{gr,2,opt}$. Results are shown below in table 2 and table 3.
Table 2. Calculated and measured the values of limit bearing capacity.

| Pile     | $N_{gr2,\text{opt},1}$ [kN] | $N_{gr2,\text{opt},2}$ [kN] | $N_{gr2,\text{opt},1+2}$ [kN] | $N_{gr2,\text{opt},3}$ [kN] | $N_{gr2,\text{meas}}$ [kN] |
|----------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 36N-10L  | 6700                          | 6560                          | 6660                          | 6590                          | 6490                          |
| 31-10L   | 6600                          | 7800                          | 9450                          | 8700                          | 7700                          |
| 38-10P   | 15800                         | 12000                         | 14000                         | 12500                         | 11900                         |
| 21N-10L  | 6910                          | 8300                          | 8350                          | 8270                          | 8260                          |
| 25N-10L  | 6350                          | 6600                          | 6650                          | 6780                          | 6350                          |
| 38-12P   | 17600                         | 12550                         | 12400                         | 11820                         | 11785                         |
| 38-12L   | 18800                         | 12100                         | 12300                         | 12200                         | 12150                         |

Table 3. Deviation of the calculated values of limit bearing capacity.

| Pile     | $N_{gr2,\text{opt},1}$ | $N_{gr2,\text{opt},2}$ | $N_{gr2,\text{opt},1+2}$ | $N_{gr2,\text{opt},3}$ |
|----------|-------------------------|-------------------------|--------------------------|-------------------------|
|          | $N_{gr2,\text{meas}}$   | $N_{gr2,\text{meas}}$   | $N_{gr2,\text{meas}}$   | $N_{gr2,\text{meas}}$   |
| 36N-10L  | 1,03                    | 1,01                    | 1,03                     | 1,02                     |
| 31-10L   | 0,86                    | 1,01                    | 1,23                     | 1,13                     |
| 38-10P   | 1,33                    | 1,01                    | 1,18                     | 1,05                     |
| 21N-10L  | 0,84                    | 1,00                    | 1,01                     | 1,00                     |
| 25N-10L  | 1,00                    | 1,04                    | 1,05                     | 1,07                     |
| 38-12P   | 1,49                    | 1,06                    | 1,05                     | 1,00                     |
| 38-12L   | 1,55                    | 1,00                    | 1,01                     | 1,01                     |

3. Results and discussions

Estimated values of limit bearing capacity obtained by the authors as a results of conducted statistical analysis are in most cases very in close to the measured values. The worst accuracy can be observed in $N_{gr2,\text{opt}}$ calculated using just the first part of the data set ($n = 1...9$). Most likely it is a result of measurement inaccuracies in the set of values $\{N_i, s_i\}$ caused by deflection of space around the pile, as well as soil adapting to the pile. This had been the subject of the research conducted by Meyer and Wasiluk [9] and they proposed to use in such cases modified form of M-K equation, which takes into account those effects.

Analysis conducted by the authors points the fact that when we take into account both part 1 and part 2 of the data set ($n = 1...16$) in all cases it reduces deviation in the values of limit bearing capacity, as shown in table 3, which can be practically used in static load tests performed in limited load range. If we take into account safety factor recommended in Eurocode 7, which is equal to 2,0 we can see that all results of the approximation will be on the safe side. However it is not recommended to use just values from part 1 ($n = 1...9$) for small $Q-s$ and classical form M-K equation (3), since in some cases, as shown especially for piles 38-12P and 38-12L we can observe about 50% overestimated values of $N_{gr2,\text{opt}}$. For small load-settlement it is recommended to use approach take into account measurement inaccuracies, as described in [9].

Limit bearing capacity is the most important factor determining the safety of the construction and can be used to calculate other M-K parameters. For example using the relation between limit bearing capacity and toe resistance of a pile which was subject of research conducted among others by Żarkiewicz [10,11] allows calculating toe resistance of a pile from estimate values of limit bearing capacity $N_{gr2,\text{opt}}$. 


\[ \frac{N_{gr2}}{N_{gr1}} = 2^{\epsilon_2} \]  

(12)

4. Conclusions

1) It is possible to obtain the values of the limit bearing capacity of a pile with enough for practical use accuracy using the statistical procedure described in this paper.

2) It is recommended to use the entire data set from the static load test \( \{N_i, s_i\} \) for estimation of the limit bearing capacity \( N_{gr2} \). However accurate enough for practical use results can also be effectively obtained from the middle part of \( Q-s \) curve \( (n = 8...16) \) or from the first two thirds \( (n = 1...16) \) of the data set \( \{N_i, s_i\} \) in case of static tests performed in a limited range of loads.

3) Accurately estimated values of \( N_{gr2,opt} \) can later be used for further calculations of other M-K curve parameters.

4) Values for small load-settlements had the biggest impact of measurement inaccuracies resulting from the deflection of space around the pile, as well as the time of effects of soil adapting to the pile [9].

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