Passenger Flow Control Method for Network-Wide Operation of Urban Rail Transit System: a case study in Guangzhou

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Abstract. A passenger flow control method for network-wide operation of urban rail transit system is proposed in this paper. In order to find a better trade-off between the delayed time and the increasing number of controlled passengers, the objective function of passenger flow control is formulated based on the problem description. The constraints about the passenger behaviour and capacity limitation are described and the model is formulated successfully. Finally, the proposed method is applied to control the passenger flow of GuangZhou Metro Line 1. The comparison results show that the proposed control model can be used to produce the optimal control plan.

1. Introduction
The passenger flow control problem has become popular in large cities of China. For example, more than 90 metro stations in Beijing Metro network have implemented passenger flow control measures. Inbound passenger volume will be restricted by station managers once the total passenger volume reaches 70% of the station capacity in Beijing Subway. However, the control practice will increase the journey time of passengers. Therefore, we should find a trade-off between the level of service and safety[1].

Many researchers have studied the problem of passenger flow control. The optimal passenger flow control strategy is to transport the most passengers while the security and level of service are within an accepted level. The passenger flow control decision-making method of urban rail transit station has been investigated, however, the microscopic behaviour has been ignored[4]. Therefore, a feedback control model has been formulated to suggest the optimal passenger flow control plan in real time by considering the route choice behaviour of passengers[2][3]. From the perspective the metro network, the coordinated passenger inflow control model has been proposed and the reinforcement learning algorithm is used to find the solution[5]. The bus-bridging service was also proposed to convey the waiting passengers to reduce the level of congestion[6]. Therefore, the coordinated and comprehensive control has become one important method to obtain the control plan[7].

In this paper, a passenger flow control method that seeks the optimal inflow of station facilities to support efficient crowd management in metro network during rush hours is proposed. The model is validated by a case study in GuangZhou of China.
2. **Mathematical background and method development**

In this section, the objection and the constraints are analysed. The passenger flow control model is formulated.

### 2.1. Model assumptions and notations

In order to simplify the formulation process of model, some assumptions are described as follows:

1. During the control period, the passengers arrives the entrances and the platform of the metro stations uniformly.
2. The passengers will not leave the subway station and transfer to the other transportation systems during the control period.
3. The walking process of passengers is simplified. The effect of station capacity on the walking time has been ignored.
4. The gathering and scattering process of passengers at stations can be composed of two individual procedures according to different OD (origin-destination): inbound, outbound process, which is conducted on two sub-networks: inbound network and outbound network. Therefore, the interaction between the gathering and scattering process is ignored because managers can separate the bidirectional passenger flow in corridor physically.[8]
5. The inbounding passengers leave the metro stations only by the train. The don’t return the original station.
6. The interval time of train departure is fixed and the trains run based on the timetable because the punctuality rate of metro system is acceptable if no severe failure happens.
7. The passengers will not stay at the platform if there is free capacity in the train.

The following notations are defined for formulating the proposed model (Table 1).

| Notations | Description |
|-----------|-------------|
| $s$       | No of metro stations, $s = 1, 2, 3, \ldots S$. |
| $r$       | No of trains, $r = 1, 2, 3, \ldots R$. |
| $t$       | No of control period, $t = 1, 2, 3, \ldots T$. |
| $\Delta t$ | The duration of each control period. |
| $R_{i,t}^s(t)$ | The arrival rate of passengers at time $t$. |
| $Q_{i,t}^s$ | The number of passengers entering the metro station $s$ at time $t$. |
| $P_{i,s}$ | The proportion of passengers from station $i$ to station $s$ in the alighting passengers of station $i$. |
| $\gamma$ | The enlarging coefficient of inbounding passengers. |
| $f$       | The interval time of train departure. |
| $\Delta T_{s+1}^s$ | The running time from station to station $s + 1$. |
| $Q_s^p$ | The capacity of station $s$. |
| $T_{delay}$ | The total delayed time of passengers. |
| $T_{qi}$ | delayed time resulting from control measures at the entrance. |
| $T_{qr}$ | delayed time resulting from the waiting on the platform. |
| $\theta_1$ | The penalty coefficient of $T_{qi}$. |
| $\theta_2$ | The penalty coefficient of $T_{qr}$. |
| $q_{i,t}^s(t)$ | The number of arrival passengers of station entrance during time $t$. |
| $q_{i,t}^s(t)$ | The number of accumulated passengers at entrance during time $t$. |
| $q_{i,t}^s(t)$ | The number of passengers that can’t enter the station during time $t$. |
| $q_{i,t}^s(t)$ | The number of arrival passengers of platform entrance during time $t$. |
| $q_{i,t}^s(t)$ | The number of waiting passengers on platform during time $t$. |
| $q_{i,t}^s(t)$ | The number of alighting passengers on platform of station $s$ during time $t$. |
| $q_{i,t}^s(t)$ | The number of boarding passengers on platform of station $s$ during time $t$. |
2.2. Objective function formulation
The model is to investigate the relationship between the number of controlled passengers at each station and the delayed time. Therefore, the control variable is the number of controlled passengers \( q_s(t) \). \( q_s(t) \) denotes the number of passengers that cannot enter the station due to the control measures. Because the holding capacity of platform and train is limited, the station managers must control the number of passengers entering the station in order to ensure the safety of passenger and normal operation of metro network.

The objective of the proposed model is to minimize the total delayed time of passengers \( T_{delay} \). The total delayed time of passengers is the sum of the delayed time of all passenger in the metro network. Therefore, \( T_{delay} \) is composed of two parts: one part refers to the delayed time resulting from the control measures \( T_{ql} \) and the other part is the delayed time of passengers on the platform \( T_{qr} \). \( T_{ql} \) and \( T_{qr} \) can be described as follows:

\[
T_{ql} = \sum_{t=1}^{T_{ql}} \sum_{s=1}^{S} q_s(t) \Delta t
\]

\[
T_{qr} = \sum_{t=1}^{T_{qr}} \sum_{s=1}^{S} q_s(t) f_t
\]

If the delayed time exceeds the tolerance of passengers, the passenger will not be satisfied with the metro service. Therefore, the penalty coefficients \( \theta_1 \) and \( \theta_2 \) are used to formulate the total delayed time \( T_{delay} \), that is:

\[
T_{delay} = [\theta_1 T_{ql} + \theta_2 T_{qr}]
\]

The passenger flow control strategy is to adjust the arrival time of passengers on the platform by controlling the number of passengers entering the stations in the metro network. From the perspective of station operation, the delayed time resulting from the control measure will increases if the number of controlled passengers increases, however, the delayed time of passengers on the platform will be reduced because the number of passengers on the platform is limited by the control measures. From the perspective of metro network operation, the remaining capacity of trains is larger if we limit more passengers in the upstream stations. As a result, more passenger in the downstream stations can get on the trains and the number of controlled passengers and delayed passengers in these stations will be reduced. Therefore, we can find a better trade-off between the delayed time and the increasing number of controlled passengers.

2.3. Constraints formulation

2.3.1. Arrival passenger constraints
The arrival passengers are the input of the metro network. Therefore, the management measures are applied to control the number of arrival passengers going into the stations in order to achieve the control objective. Therefore, the number of passengers is determined by the control measures and the arrival distribution of passengers. The constraints about the passengers entering the metro station \( s \) can be described as follows:

\[
q_s^0(t) = R_s^a(t) \Delta t
\]

\[
q_s^a(t) = q_s^a(t) + q_s^i(t-1)
\]

\[
q_s^i(t) = q_s^i(t) - q_s^i(t)
\]
Eq.(4) describes the arrival distribution of passengers which is uniform. Eq.(5) computes the number of accumulated passengers which is the sum of the new arrival passengers at time $t$ and the controlled passengers at time $t-1$. Eq.(6) determines the number of passengers that can be allowed to enter station $s$. It is denoted by the difference between the number of accumulated passengers and the number of controlled passengers at time $t$.

2.3.2. Decision variable about train position
The details about the train operation can be ignored because we only concern when the passengers at station $s$ can get on the train $r$ at each time interval. Let $L_{s,r}(t)$ denote whether train $r$ stop at station $s$ at time $t$, $L_{s,r}(t) = 1$; otherwise, $L_{s,r}(t) = 0$, that is:

$$L_{s,r}(t) = \begin{cases} 
1 & \text{if train } r \text{ stop at station } s \text{ at time } t \\
0, & \text{otherwise} 
\end{cases}$$

2.3.3. Boarding and alighting behavior formulation
The behaviour of passengers on the platform includes alighting and boarding processes. When $L_{s,r}(t) = 1$, the alighting and boarding behaviour is intriguied. The number of alighting passengers can be obtained based on the origin-destination matrix. If the number of waiting passengers on the platform is less than the remaining capacity of the train, all the passengers can get on the train; otherwise, some passengers should wait for the next train. The behaviour of passengers on the platform can be described as follows:

$$q_s^p(t) = \begin{cases} 
q_s^p(t-1) + q_s^e(t) & \\
q_s^l(t), \sum_{r=1}^{R} L_{s,r}(t) = 1 
\end{cases}$$

$$q_s^a(t) = \sum_{r=1}^{R} \sum_{i=1}^{S} q_{s,i}^b(t)P_{i,s} L_{s,r}(t)$$

$$q_s^b(t) = \begin{cases} 
\sum_{r=1}^{R} L_{s,r}(t)[q_s^p(t-1) + q_s^a(t)], q_s^p(t-1) + q_s^a(t) \leq q_s^{pu}(t-1) + q_s^a(t) & \\
q_s^l(t)[q_s^p(t-1) + q_s^a(t)], q_s^p(t-1) + q_s^a(t) > q_s^{pu}(t-1) + q_s^a(t) & 
\end{cases}$$

$$q_s^{d}(t) = \sum_{r=1}^{R} L_{s,r}(t)[q_s^{p}(t-1) + q_s^{a}(t)] - q_s^{b}(t)$$

$$q_s^{d}(t) = q_s^{in}(t-1) - q_s^{a}(t) + q_s^{b}(t)$$

$$q_s^{out}(t) = \eta^{max}C - q_s^{in}(t)$$

Eq.(8) updates the number of passengers on the platform. Eq.(9) describes the alighting process of passengers and the number of alighting passengers can be obtained by the boarding passengers of upstream stations and the OD matrix. Eq.(10) describes the boarding process and the relationship between the number of waiting passengers and boarding passengers. If all the passengers on the platform can get on the train, $q_s^p(t-1) + q_s^e(t) = q_s^b(t)$ and then $q_s^d(t) = 0$. If some passengers on the platform cannot get on the train, $q_s^p(t-1) + q_s^e(t) > q_s^b(t)$ and then $q_s^d(t) > 0$. Eq.(12) updates the number of passengers in the train. When the train arrive at a station, the number of passengers in the train should be updated based on the boarding and alighting process which can be described by Eq.(9) and Eq.(10). Eq.(12) updates the remaining capacity of train $r$ based on the capacity of train $\eta^{max}C$ and the current number of passengers in the train $r$.

2.3.4. Capacity constraints
The capacity of metro stations and trains should be limited. Therefore, the following equations are formulated to ensure that the number of passengers cannot exceed the capacity:

$$q_s^{in}(t) \leq C\eta^{max}$$

$$q_s^{b}(t) \leq Q_s^{b}$$

$$q_s^{e}(t) \leq q_s^{d}(t) + q_s^{l}(t-1)$$

$$q_s^{out}(t) \leq q_s^{b}(t)$$
Eq. (14) ensures that the number of passengers in the train \( r \) at each time interval don’t exceed the capacity of train \( n_{\text{max}}^r \). Similarly, Eq. (15) ensures that the number of passengers on the platform of station \( s \) at each time interval don’t exceed the capacity of platform. Eq. (16) ensures that the number of controlled passengers at each time interval don’t exceed the number of accumulated passengers before the entrance of station \( s \). Eq. (17) ensures that the number of boarding passengers at each time interval don’t exceed the number of waiting passengers on the platform of station \( s \). Finally, the passenger flow control model can be formulated based on the collection of objective function and constraints. The software Matlab is used to solve the problem.

3. Case study

3.1. Parameters

We applied the proposed method to control the passenger flow of GuangZhou Metro line 1. There are 16 metro stations in which 7 stations are transfer stations. In order to manage the mass passenger flow, the passenger flow control measure should be applied. The control period is 6:45-8:00 and the input data is the OD matrix. The time interval of passenger flow control is 15 min. The headway of train departure is 3 min. The holding capacity of each train is 1428 while the load factor is 120%. The trains run from XiLang station to HuangSha station. The maximum density of each platform is 2 p/m². The penalty coefficient \( \theta_1 = 0.6, \theta_2 = 0.4 \).

3.2. Model results and comparison

The model is formulated based on the model description in section 2. The software Matlab is used to solve the model. The number of controlled passengers at each time interval is listed in Table 2.

| Time interval | KengKou | FangCun | ChenJiaCi | XiMenKou | ChangShouLu | NongJiangSuo |
|---------------|--------|--------|-----------|----------|-------------|-------------|
| 1             | 1020   | 1200   | 813       | 900      | 610         | 1011        |
| 2             | 1050   | 1400   | 953       | 850      | 698         | 1202        |
| 3             | 1085   | 1500   | 1010      | 850      | 800         | 1003        |
| 4             | 1000   | 1300   | 1088      | 950      | 845         | 1053        |
| 5             | 1050   | 1200   | 1050      | 1000     | 745         | 1109        |

In order to validate the proposed model, we compare the control plan from our model and the control plan in practice. The comparison results are depicted in Figure 1. As can be seen from Figure 1, the distance between our control plan and practical control plan is very small. The accuracy is larger than 90% in most stations.

Figure 1: Model and practice comparison

Figure 2 shows the delayed time of passengers under independent and coordinated control. It can be seen that the delayed time of passengers under coordinated control is less than the time under independent control which also validates the efficiency of the proposed model.
4. Discussion and Conclusion
The passenger flow control problem has become popular in large cities of China. Many metro operation company issues some regulations about passenger flow control. However, the control plans are usually determined by the managers’ experience and simple computation. Therefore, we proposed a passenger flow control model in this paper to produce the optimal control plan which can minimizes the delayed time of passengers.

The proposed passenger flow control model has been validated by the case study. The comparison results show that the proposed control plan is nearly consistent with the practice and the coordinated control can reduce the delayed time compared to the independent control.

In conclusion, the proposed model can be used for practical applications. In the future study, the proposed model may be improved by considering the microscopic behaviour of passengers.

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