Deconstructing the Planck TT Power Spectrum to Constrain Deviations from $\Lambda$CDM

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Abstract

Consistency checks of Lambda cold dark matter ($\Lambda$CDM) predictions with current cosmological data sets may illuminate the types of changes needed to resolve cosmological tensions. To this end, we modify the CLASS Boltzmann code to create phenomenological amplitudes, similar to the lensing amplitude parameter $A_L$, for the Sachs–Wolfe, Doppler, early Integrated Sachs–Wolfe (eISW), and polarization contributions to the cosmic microwave background temperature anisotropy, and then we include these additional amplitudes in fits to the Planck TT power spectrum. We find that allowing one of these amplitudes to vary at a time results in little improvement over $\Lambda$CDM alone suggesting that each of these physical effects are being correctly accounted for given the current level of precision. Further, we find that the only pair of phenomenological amplitudes that results in a significant improvement to the fit to Planck temperature data results from varying the amplitudes of the Sachs–Wolfe and Doppler effects simultaneously. However, we show that this model is really just redefining the $\Lambda$CDM + $A_L$ solution. We test adding our phenomenological amplitudes as well as $N_{eff}$, $Y_H$, and $h_{run}$ to $\Lambda$CDM + $A_L$ and find that none of these model extensions provide significant improvement over $\Lambda$CDM + $A_L$ when fitting Planck temperature data. Finally, we quantify the contributions of both the eISW effect and lensing on the constraint of the physical matter density from Planck temperature data by allowing the phenomenological amplitude from each effect to vary. We find that these effects play a relatively small role (the uncertainty increases by 3.5% and 16% respectively) suggesting that the overall photon envelope has the greatest constraining power.

Unified Astronomy Thesaurus concepts: Observational cosmology (1146); Cosmic microwave background radiation (322); Cosmological parameters (339); Hubble constant (758); Cosmological models (337)

1. Introduction

Lambda cold dark matter ($\Lambda$CDM) is the standard model of cosmology because with only six parameters, it successfully explains a wide range of cosmological and astrophysical phenomena. However, in recent years, tensions have emerged between the preferred values of cosmological parameters resulting from fits to cosmological data sets assuming the $\Lambda$CDM model and direct measurements of those cosmological parameters. In particular, there is a 4.4σ tension in the preferred value of the Hubble constant, $H_0$, between the cosmological distance ladder measurement by SH0ES, $H_0 = 74.02 \pm 1.42$ km s$^{-1}$ Mpc$^{-1}$ (Riess et al. 2019), and the inferred value from the most precise measurements to date of the cosmic microwave background (CMB) provided by Planck, $H_0 = 67.37 \pm 0.54$ km s$^{-1}$ Mpc$^{-1}$ (Planck Collaboration et al. 2020a).

The $H_0$ tension can be divided into a discordance between the preferred values by early universe observations assuming $\Lambda$CDM and direct measurements in the late universe. While this tension is usually expressed as a disagreement between Planck and the cosmological distance ladder, Addison et al. (2018) show that a similar discordance is found when combining baryon acoustic oscillation (BAO) data with Planck CMB measurements, CMB measurements from experiments other than Planck, or with primordial deuterium abundances using no CMB anisotropy data (see also, e.g., Aubourg et al. 2015; Cuceu et al. 2019; eBOSS Collaboration et al. 2020).

On the late universe side, this tension persists even if different calibrators are used for the cosmological distance ladder (Huang et al. 2020). Using the tip of the red giant branch as a calibrator results in $H_0 = 69.6 \pm 1.9$ km s$^{-1}$ Mpc$^{-1}$ (Freedman et al. 2019), but Yuan et al. (2019) argue that this analysis overestimates the Large Magellanic Cloud extinction and instead determine $H_0 = 72.4 \pm 2.0$ km s$^{-1}$ Mpc$^{-1}$. Completely independent of the cosmological distance ladder, strong gravitational lensing time delays by $H_0$ lenses in COSMOSGRAL’s Wellspring (HOLiCOW) determine $H_0 = 73.3 \pm 1.7$ km s$^{-1}$ Mpc$^{-1}$, which is in 3.9σ tension with Planck (Wong et al. 2020).

Because the $H_0$ tension exists between multiple data sets and breaks down by cosmological epoch instead of observational technique, it is unlikely to be resolved by an underestimated or unmodeled systematic, suggesting the need for physics beyond the standard model of cosmology. Finding extensions to $\Lambda$CDM that resolve the Hubble tension yet stay consistent with the multitude of cosmological data sets is challenging (see, e.g., Knox & Millea 2020). For example, it has been proposed that incorporating a form of dark energy that comprises about 10% of the energy density of the universe around matter-radiation equality and then decays away before recombination can alleviate the $H_0$ tension (Lin et al. 2019; Poulin et al. 2019; Berghaus & Karwal 2020). However, fitting these current early dark energy models to Planck data results in an increase in the cold dark matter density that is disfavored by large-scale structure measurements (D’Amico et al. 2020; Hill et al. 2020; Ivanov et al. 2020). In the absence of a clear theoretical direction, it can be useful to perform consistency checks of $\Lambda$CDM predictions with current data sets to determine what kinds of changes to the standard model are necessary or even allowed (see, e.g., Kable et al. 2020; Motloch 2020). It has been shown that in addition to the $H_0$ tension with direct measurements, Planck data prefers a 2–3σ larger value of $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$, which measures matter clustering, than weak lensing experiments (Abbott et al. 2018; Hildebrandt et al. 2020; Joudaki et al. 2018; Hikage et al. 2019) and clustering abundance surveys (e.g., Lin & Ishak 2017; McCarthy et al. 2018).
Additionally, there is a ~2.5σ tension between the preferred values of parameters like the physical cold dark matter density, \( \omega_c \), for Planck TT \( \ell \leq 1000 \) and Planck TT \( \ell > 1000 \) (or similarly for Planck TT \( \ell \leq 800 \) and Planck TT \( \ell > 800 \)), which can be resolved by allowing the amplitude of the lensing contribution to the CMB power spectra to vary (e.g., Addison et al. 2016; Planck Collaboration et al. 2020a). This is done by extending \( \Lambda \)CDM to include a phenomenological amplitude, \( A_L \), which rescales the amplitude of the lensing power spectrum as

\[
C_\ell^L \rightarrow A_L C_\ell^L, \tag{1}
\]

where \( A_L \) has a physical value of 1 (Calabrese et al. 2008). The combined Planck TT, TE, and EE power spectra prefer \( A_L > 1 \) at 2.8σ, which is driven largely by an improvement to the fit for multipoles 1100 \( \leq \ell \leq 2000 \) in the Planck TT power spectrum, though there is also improvement to the fit for Planck TT \( \ell < 30 \) (Planck Collaboration et al. 2020a). However, the lensing power spectrum reconstructed from higher-order statistics of the Planck maps is in good agreement with standard \( \Lambda \)CDM predictions (e.g., Planck Collaboration et al. 2020b; Simard et al. 2018; Motloch & Hu 2020). Moreover, the South Pole Telescope Polarimeter (SPTpol) TE and EE power spectra prefer \( A_L < 1 \) at 1.4σ, and the Atacama Cosmology Telescope (ACT) DR4 is consistent with \( A_L = 1 \) within 1σ (Henning et al. 2018; Aiola et al. 2020). While the Planck TT power spectrum prefers greater peak smoothing consistent with \( A_L > 1 \), other cosmological data sets disfavor changing the physical amount of lensing.

In this paper, we create phenomenological amplitudes analogous to \( A_L \) for the early integrated Sachs–Wolfe (eISW), Sachs–Wolfe, Doppler, and polarization effects,\(^1\) which all source the CMB temperature anisotropy. We fit these new phenomenological amplitudes to Planck data to determine if there are any deviations from standard \( \Lambda \)CDM favored by Planck. In this way, we deconstruct the TT power spectrum into its constituent sources, which provides a test for where potential model extensions are allowed or are necessary. While scaling these physical effects can affect the CMB TE power spectrum, we choose to fit only the Planck TT power spectrum as we are primarily interested in quantifying deviations from \( \Lambda \)CDM predictions in the temperature anisotropy, which is already known to have internal differences in the preferred \( \Lambda \)CDM parameter values between Planck TT \( \ell \leq 800 \) and Planck TT \( \ell > 800 \). The Planck Collaboration performed a similar exercise and found that these phenomenological amplitudes are consistent with expectations (see Footnote 30 of Planck Collaboration et al. 2020a). We quantify this consistency and extend the analysis to include combinations of the phenomenological amplitudes.

There is a well-known degeneracy in the CMB temperature data between the scalar amplitude, \( A_s \), and the optical depth, \( \tau \). This degeneracy is broken by the reionization bump measured by \( \ell \lesssim 20 \) EE data (see, e.g., Figure 8 in Planck Collaboration et al. 2020c). For all cases in this paper, we include a Gaussian prior of \( \tau = 0.0506 \pm 0.0086 \) to account for the constraint from Planck Low \( \ell \) EE data as described by Planck Collaboration et al. (2020a). We tested the impact of changing both the mean value and width of the Gaussian prior on \( \tau \) and found that our conclusions were insensitive to these changes.

This paper is organized as follows. In Section 2 we define the phenomenological amplitudes for the eISW, SW, Doppler, and polarization effects and discuss how each phenomenological amplitude affects the TT power spectrum. In Section 3 we show the constraints provided by the Planck 2018 TT power spectrum when we allow one or more of the phenomenological amplitudes to vary. In Section 4 we test possible extensions to \( \Lambda \)CDM + \( A_L \) to determine where if any further improvement in the fit can be found. Finally in Section 5, we provide conclusions.

2. The Phenomenological Amplitudes

2.1. Definitions of Phenomenological Amplitudes

In this section, we define the phenomenological amplitudes that we will use for the rest of the paper. The perturbation away from a blackbody spectrum of the CMB photon distribution, \( \Theta \equiv \delta T / T_c \), can be quantified by integrating the various cosmological perturbations along the path of the photons. This distribution can be expanded in terms of spherical Bessel functions, \( j_k \), and wavenumbers, \( k \), for a given perturbation as

\[
\Theta_l(k, \eta_0) = A_{SW} \times \int_0^{\eta_0} d\eta_2 [\Theta_0(k, \eta) + \Phi(k, \eta)j_l[k(\eta - \eta_0)]] \\
- A_{Dop} \int_0^{\eta_0} d\eta_2 [\Phi(k, \eta)j_l[k(\eta - \eta_0)]] \\
+ A_{Pol} \int_0^{\eta_0} d\eta_2 \left[ \frac{g(\eta)\Pi}{4} + \frac{3}{4k^2\eta^2} \frac{d^2}{d\eta^2}[g(\eta)\Pi]j_l[k(\eta - \eta_0)] \right],
\]

following Dodelson (2003). In this equation, \( \eta \) is conformal time, \( \tau \) is the optical depth at a given conformal time, \( g(\eta) \equiv -e^{-\tau} \) is the visibility function, \( \Psi \) is the spatial perturbation to the metric, \( v_\theta \) is the velocity of the baryons, and \( \Pi \) is the polarization tensor.

The visibility function is a probability density of the photon path along the line of sight. The third term is the CMB polarization contribution to the CMB temperature anisotropy. This results from the directional dependence of Compton

\(^1\) Note that this polarization effect refers to the contribution to the total intensity that is sourced by CMB polarization. We define this in more detail in Section 2.
scattering and the coupling of the CMB polarization to the quadrupole moment of $\Theta$, which is discussed by Hu & Sugiyama (1996).

In Equation (2), we defined phenomenological amplitudes for each of these effects. We adopt the same convention as Hou et al. (2013) where the phenomenological amplitudes scale the sources of the photon distribution. Additionally, we define $f(z(t), A_{\text{ISW}}) = A_{\text{ISW}}$, when $z > 30$ and unity for $z < 30$ as was done in Hou et al. (2013). We could additionally define a phenomenological amplitude to account for the late time ISW effect, but we find that this is too poorly constrained by the CMB data to provide a meaningful test.

2.2. Effects of Varying Phenomenological Amplitudes on Theory TT Power Spectrum

Before we discuss results of extending $\Lambda$CDM to include these phenomenological amplitudes when fitting to Planck TT data, we illustrate the general effects on the TT power spectrum of varying each of these phenomenological amplitudes. To do so, we modify the source function in the CLASS Boltzmann code (Blas et al. 2011; Lesgourgues 2011) to include these new parameters. In Figure 1, we show the effect on the CMB power spectrum of changing each of the four phenomenological amplitudes as well as $A_L$. In each case, we employ a fiducial cosmology resulting from a Markov Chain Monte Carlo (MCMC) using Monte Python (Audren et al. 2013; Brinckmann & Lesgourgues 2018) of Planck 2018 TT data with a prior of $\tau = 0.0506 \pm 0.0086$.

Changing $A_{\text{SW}}$ has the largest effect on the overall amplitude of the power spectrum of the parameters varied in Figure 1. While increasing $A_{\text{SW}}$ increases both acoustic peaks and troughs, it increases the heights of the peaks by a larger fraction. The effect is stronger on the compression modes (odd peaks), where the baryon-photon fluid is at the bottom of the gravitational potential, than the rarefaction modes (even peaks). Increasing the Sachs–Wolfe effect leads to deeper potentials allowing for greater compression. Increasing $A_{\text{SW}}$ also leads to a small phase shift toward larger scales.

Increasing $A_{\text{Dop}}$ also results in an overall increase to the power spectrum and a small phase shift to larger scales, but unlike the Sachs–Wolfe effect, it disproportionately impacts the troughs and even peaks of the power spectrum. In particular,
the ratio of the heights of the peaks to troughs decreases as $A_{\text{Dop}}$ increases. The Doppler effect is proportional to the baryon velocity as shown in Equation (2). In the absence of baryon loading, the baryon velocity would peak when $\Theta_2 + \Psi = 0$, which corresponds to the troughs of the CMB power spectrum (see, e.g., Section 5.2 of Hu 1995). With the baryon loading, the baryon velocity still peaks near the troughs and therefore increasing $A_{\text{Dop}}$ fills in the troughs. The rarefaction modes get more power than the compression modes because increasing the baryon velocity increases the pressure, which makes it easier for photons to escape the gravity wells.

Changing $A_{\text{ISW}}$ primarily affects the first peak, but also makes small contributions to the higher peaks with a preference for the odd acoustic peaks. $A_{\text{ISW}}$ has the largest effect on the first acoustic peak because it has the largest effect on modes that enter the horizon when the universe is dominated by matter but still has a sizable radiation density (see, e.g., Section 8.6 of Dodelson 2003). Increasing $A_{\text{ISW}}$ causes an increase in power because it increases the radiation density, which hastens the decay of the gravity wells. There is also a slight filling in of the second trough.

Finally, Figure 1 shows that changing $A_{\text{Pol}}$ makes the smallest change to the amplitude of the power spectrum. Increasing $A_{\text{Pol}}$ results in a phase shift to smaller scales. This phenomenological amplitude is coupled to the CMB quadrupole moment, $\Theta_2$, which sources photon diffusion damping (see, e.g., Section 8.4 of Dodelson 2003). Hence, increasing $A_{\text{Pol}}$ results in increased damping.

### 3. Results from Varying Phenomenological Amplitudes

In the previous section, we defined phenomenological amplitudes for the Sachs–Wolfe, eISW, Doppler, and polarization effects that source the CMB temperature anisotropy. In this section, we explore how these phenomenological amplitudes are constrained by the CMB by running MCMC fits on Planck 2018 TT data. To sample the posterior distributions for the model parameters, we use our modified CLASS Boltzmann code, which includes the amplitudes defined in Equation (2) as additional model parameters, and run MCMCs using Monte Python.

We use the likelihoods for Planck 2018 TT High $\ell$ Lite corresponding to $30 \leq \ell \leq 2508$ and Planck 2018 TT Low $\ell$ corresponding to $\ell < 30$ provided by the Planck Collaboration. We choose to use the Lite likelihoods, where foreground parameters have already been marginalized over, because we are not investigating the impact of altering the foreground model in this work. Hereafter, we will refer to this likelihood as Planck TT.

For certain models, we also explore splitting the Planck data to highlight the discrepancy between the parameter posteriors resulting from sampling Planck TT $\ell \leq 800$ and Planck TT $\ell > 800$. We choose to split the Planck data at $\ell = 800$ because this corresponds to the point where each split of the Planck data has roughly equivalent constraining power (e.g., Planck Collaboration Li 2017). We refer to these data split likelihoods as Planck TT $\ell \leq 800$ and Planck TT $\ell > 800$, respectively.

Finally, unless otherwise specified, we use a Gelman–Rubin convergence statistic of $R - 1 = 0.05$ for the least constrained parameter to define the point where our MCMC chains have converged (Gelman & Rubin 1992).

### 3.1. Fits to $\Lambda$CDM Plus One Phenomenological Amplitude

In this subsection, we compare the MCMC fits to Planck TT assuming $\Lambda$CDM + one phenomenological amplitude to the MCMC fits to Planck TT assuming $\Lambda$CDM. The results of these MCMC fits are summarized in Table 1 and Figures 2 and 3. In Table 1, we show that no variations of the phenomenological amplitudes that we introduced in Section 2 are able to fit Planck TT significantly better than $\Lambda$CDM. Moreover, no variations of these phenomenological amplitudes are able to alleviate the $H_0$ tension.

Varying $A_{\text{ISW}}$ results in the largest improvement over standard $\Lambda$CDM of these new phenomenological amplitudes. Nevertheless, this variation results in a $<2\sigma$ shift in the posterior distribution for $A_{\text{ISW}}$ away from the fiducial value of 1. Moreover, the difference in $\chi^2$ found by adding $A_{\text{ISW}}$ corresponds to a probability to exceed (PTE) of 0.13 further indicating that including $A_{\text{ISW}}$ is not a significant model improvement over $\Lambda$CDM. Considering that we tested four model extensions to standard $\Lambda$CDM, it is not surprising that one of them resulted in a $>1\sigma$ improvement to the fit.

To understand where this minor improvement is coming from, we fit $\Lambda$CDM + $A_{\text{ISW}}$ to Planck TT but excluded multipoles $\ell < 30$ and found that the preference for $A_{\text{ISW}} > 1$...
was reduced to $< 0.5\sigma$. This suggests that the primary improvement over $\Lambda$CDM when fitting $\Lambda$CDM + $A_{\text{SW}}$ to Planck TT comes from multipoles $\ell < 30$. In particular, we find that the TT power spectrum resulting from the best-fit cosmology for $\Lambda$CDM + $A_{\text{SW}}$ has less power than standard $\Lambda$CDM for $\ell < 30$ when fit to Planck TT, which allows this model to fit the well-known deficit of power at $\ell < 30$ in WMAP and Planck TT data (Bennett et al. 2013; Planck Collaboration et al. 2020a). When $A_{\text{SW}}$ is allowed to vary, Planck TT prefers a decrease in the preferred value of $A$, and an increase in the preferred value of $n_s$, which results in a reduction in power for $\ell < 30$ for the the best-fit TT power spectrum.

From Table 1, we see that the improvement found by $\Lambda$CDM + $A_{\text{SW}}$ over standard $\Lambda$CDM is primarily compensated by a 0.043 shift downward in the value of $100 \times \omega_b$ (100 times the physical baryon density), which corresponds to almost twice the original uncertainty. On a related note, the uncertainty of the baryon density...
when varying the amplitude of the eISW effect increases by roughly 60%, which illuminates how powerful the relative peak heights, and in particular the height of the first acoustic peak, are in constraining the physical baryon density.

After $A_L$ and $A_{\text{ISW}}$, allowing $A_{\text{SW}}$ to vary results in the next most significant improvement over just $\Lambda$CDM, which can be seen by the approximately 1σ shift in the value of $A_{\text{SW}}$ from the fiducial value of unity. While the uncertainties on the $\Lambda$CDM parameters increase, such as the near doubling of the uncertainty of $A_s e^{-2\tau}$, most parameter shifts are <0.5σ. Adding $A_{\text{Dop}}$ to $\Lambda$CDM when fitting Planck TT results in a <0.5σ shift of the posterior distribution of $A_{\text{Dop}}$ from the fiducial value of unity. From a phenomenological perspective, these tests provide no significant evidence for an improvement over $\Lambda$CDM by solely modifying the monopole or dipole contributions to the CMB photon distribution.

The $\Lambda$CDM + $A_{\text{Pol}}$ fit to Planck TT generally results in no substantial shifts in the central value of the posteriors. The
largest such shift is a 0.41 km s\(^{-1}\) Mpc\(^{-1}\) shift upward in the mean value of \(H_0\). Nevertheless there are substantial increases in the uncertainties of the parameters over the \(\Lambda\)CDM case. In particular, note that the uncertainties of \(H_0\) and \(\omega_b\) increase by roughly 35% and 50%, respectively, over the \(\Lambda\)CDM case. This highlights the importance of the polarization effect even when determining parameters from the TT spectrum.

In Figure 2 we compare the two-dimensional posterior distributions for \(\Lambda\)CDM + either \(A_{\text{SW}}\) or \(A_{\text{Dop}}\) to the \(\Lambda\)CDM case. The correlations between either \(A_{\text{SW}}\) or \(A_{\text{Dop}}\) and the \(\Lambda\)CDM parameters have an opposite sign for these two models because these phenomenological amplitudes disproportionately add power to either odd or even acoustic peaks of the power spectrum as discussed in Section 2. For example, increasing \(A_{\text{SW}}\) disproportionately adds power to the odd peaks which must then be compensated by decreasing the baryon density. In contrast, increasing \(A_{\text{Dop}}\) disproportionately adds power to even peaks, which must then be compensated for by increasing the baryon density. In Figure 3, we show the constraints for \(\Lambda\)CDM and \(\Lambda\)CDM + one of \(A_L\), \(A_{\text{SW}}\), or \(A_{\text{Dop}}\). In all of these cases, the size of the contours increase dramatically over \(\Lambda\)CDM, which should be contrasted with the relatively minor changes when varying either \(A_{\text{SW}}\) or \(A_{\text{Dop}}\).

In summary, these tests show that \(\Lambda\)CDM is able to correctly account for the Sachs–Wolfe, eISW, Doppler, and polarization effects measured by Planck with the known caveat that there is an internal tension in the Planck data between low \(\ell\) and high \(\ell\), which can be relieved by allowing a parameter like \(A_L\) to vary. Because the cosmological parameters do not shift much when the amplitudes for the Sachs–Wolfe, Doppler, eISW, or polarization effects are varied, the parameter constraints from these physical processes are internally consistent. Finally we note that even when allowing the amplitudes for any one of the physical effects that source the CMB temperature anisotropy to vary, Planck TT is still able to place strong constraints on the \(\Lambda\)CDM parameters.

### 3.2. \(\Lambda\)CDM + \(A_{\text{SW}}\) + \(A_{\text{Dop}}\)

In the previous subsection, we showed results for extending \(\Lambda\)CDM to include one of the phenomenological amplitudes that we introduced in Section 2. In this subsection, we discuss adding pairs of the phenomenological amplitudes. In general, we find that much like adding one phenomenological amplitude, adding pairs of phenomenological amplitudes does not result in either an improved fit to Planck TT or a reduction in the \(H_0\) tension with late universe measurements. We find that only \(\Lambda\)CDM + \(A_{\text{SW}}\) + \(A_{\text{Dop}}\) exhibits a significant improvement to the fit to Planck TT.

We summarize the results of the MCMC sampling for \(\Lambda\)CDM + \(A_{\text{SW}}\) + \(A_{\text{Dop}}\) to Planck TT in Table 2. With two parameters, it becomes more complicated to define when there is a significant shift in the posterior, but \(A_{\text{SW}}\) and \(A_{\text{Dop}}\) are both more than 2\(\sigma\) below the fiducial value of unity when simultaneously allowed to vary. Additionally, the PTE of the \(\Delta \chi^2\) assuming two degrees of freedom is 0.03 indicating a significant improvement over the \(\Lambda\)CDM case. Note that adding both \(A_{\text{SW}}\) and \(A_{\text{Dop}}\) together results in a significant improvement over standard \(\Lambda\)CDM when fitting to Planck TT because when only one at a time was added there was much less improvement. Allowing both \(A_{\text{SW}}\) and \(A_{\text{Dop}}\) to vary simultaneously does not also increase the Planck TT preferred value of \(H_0\) like when adding \(A_L\).

Note that since \(A_{\text{SW}}\) and \(A_{\text{Dop}}\) appear to be acting in unison, we should recover approximately the same model if we use a single phenomenological amplitude to scale both the Sachs–Wolfe and Doppler effects. Taking a step back, if we had used a single amplitude to rescale all of the effects that source the CMB TT anisotropy in Equation (2), then this new phenomenological amplitude would have been almost completely degenerate with \(A_L\), up to corrections from lensing, when fitting to Planck TT. In this case, \(A_L\) becomes a proxy for \(A_L\) because of how \(A_L\) explicitly enters the equations for lensing (see, e.g., Sections 3.1–3.2 of Lewis & Challinor 2006). Varying both \(A_{\text{SW}}\) and \(A_{\text{Dop}}\) simultaneously increases the uncertainty of \(A_L e^{-2\pi i}\) by a factor of 4 relative to the \(\Lambda\)CDM case, which allows sufficient freedom for \(A_L\) to become a proxy for \(A_L\).

Additionally in Table 2, we include the constraints when \(A_{\text{SW}}\) is added to \(\Lambda\)CDM + \(A_{\text{SW}}\) + \(A_{\text{Dop}}\). For this MCMC run, we only used a convergence criteria of \(R = 1 = 0.1\) because convergence was difficult to achieve. While this \(\Lambda\)CDM + \(A_{\text{SW}}\) + \(A_{\text{Dop}}\) + \(A_{\text{SW}}\) result gives a significant improvement over \(\Lambda\)CDM with a PTE of 0.03 assuming a \(\Delta \chi^2\) with three degrees of freedom, it is
not a significant improvement over $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ with a PTE of 0.13 assuming one degree of freedom. This improved $\Delta \chi^2$ is roughly equivalent to the improved $\Delta \chi^2$ when adding only $A_{eISW}$ to $\Lambda$CDM as shown in Table 1, but note that Planck TT prefers $A_{eISW} < 1$ for this model to be more in line with the preferred values for $A_{SW}$ and $A_{Dop}$. While adding $A_{eISW}$ to $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ does not result in a significant improvement, there is an increase in the preferred value of $H_0$ similar to the $\Lambda$CDM + $A_L$ preferred $H_0$ value.

In Figure 4, we compare the 2D posteriors for the one parameter extensions, $\Lambda$CDM + $A_{SW}$, $A_{Dop}$, and $A_{eISW}$, and the combinations $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ and $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ + $A_{eISW}$. Note the strong degeneracies between the phenomenological amplitudes and the scalar amplitude when more than one phenomenological amplitude is varied. Adding $A_{eISW}$ results in a substantial increase in the degeneracy between the phenomenological amplitudes and the scalar amplitude. This, in turn, allows for parameters like $H_0$ to access a broader parameter space.

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In Figure 5, we demonstrate how the Sachs–Wolfe and Doppler effects work together to rescale the power spectrum by plotting the quotient of the $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ case to the $\Lambda$CDM case. In particular note that for $\ell > 400$, the quotient is flat, up to some wiggles that result from not additionally rescaling $A_{Pol}$. In the middle panel of Figure 5, we show that the slope in the quotient for $\ell < 400$ results from not also rescaling the ISW effect. For $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$, it is this ability to rescale the TT power spectrum on scales $\ell > 400$ that degrades the precision of $A_L$ allowing it to become a proxy for $A_L$.

Because the degeneracy between $A_{SW}$, $A_{Dop}$, and $A_L$ when fitting $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ breaks down for multipoles $\ell < 400$, we use fits to Planck TT and Planck TT $\ell > 800$ to illustrate that $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ is approximately finding the $\Lambda$CDM + $A_L$ solution. In Figure 6, we show the residuals of the theory TT power spectrum calculated using the best-fit parameters for the $\Lambda$CDM + $A_L$ and $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ fits to both Planck TT and Planck TT $\ell > 800$ with the theory TT power spectrum calculated using the best-fit parameters for the $\Lambda$CDM fit to Planck TT. Additionally, we include the residual of the measured Planck TT data with the $\Lambda$CDM fit to Planck TT. To increase the clarity of the plot, we rescale the Planck TT data using new super bins of $\Delta \ell \approx 65$. Note that there are high levels of correlation, often at the 80% level, between the bins for Plik Lite, which result from marginalizing over the foregrounds.

Importantly, Figure 6 shows that the residuals for $\Lambda$CDM + $A_L$ and $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ are highly correlated when fit to Planck TT $\ell > 800$, which emphasizes that these two models are making the same changes at high $\ell$, and it is the low $\ell$ behavior that restricts the latter model when fit to Planck TT. For $\ell > 1250$, the $\Lambda$CDM + $A_L$ fit to Planck TT also becomes highly correlated to these fits indicating that this is the primary feature of the lensing solution. Further note that the $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ fit to Planck TT does fit the oscillatory residual in the Planck data, albeit without the increased power for multipoles $\ell > 1250$. This is how the $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ fit to Planck TT achieves a significant improvement over $\Lambda$CDM.

In Table 3, we show the results from MCMC runs for $\Lambda$CDM, $\Lambda$CDM + $A_L$, and $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ fits to Planck TT $\ell \leq 800$ and Planck TT $\ell > 800$. Neither $\Lambda$CDM + $A_L$ nor

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**Figure 5.** Top panel: ratio of the TT power spectrum when varying both $A_{SW}$ and $A_{Dop}$ together to the TT power spectrum of standard $\Lambda$CDM. The curves correspond to all phenomenological amplitudes being set to $\{0.90, 0.92, 0.94, 0.96, 0.98\}$ from top to bottom. If $A_{SW}$ and $A_{Dop}$ act in unison, then, at high $\ell$, they can rescale the power spectrum and allow $A_L$ to become a proxy for $A_{Pol}$, which results in a significant improvement over $\Lambda$CDM by mimicking the effect of lensing. Middle panel: when $A_{ISW}$ acts in unison with $A_{SW}$ and $A_{Dop}$, the degeneracy extends to lower multipole moments resulting in a stronger degeneracy. Bottom panel: the wiggles at high $\ell$ result from not including $A_{Pol}$ in the rescaling.

**Figure 6.** Residuals of the best-fit TT power spectrum from $\Lambda$CDM + $A_L$ and $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ fits to both Planck TT and Planck TT $\ell > 800$ with the $\Lambda$CDM fit to Planck TT. We additionally include the residual of the measured Planck TT data (black) points, but we have rebinned them with $\Delta \ell \approx 65$ for visual clarity. When fit to only Planck TT $\ell > 800$, varying the amplitudes for the Sachs–Wolfe and Doppler effects results in a similar residual to the $\Lambda$CDM + $A_L$ residual, which suggests that at high $\ell$ these two models achieve approximately the same effect. The $\Lambda$CDM + $A_{SW}$ + $A_{Dop}$ fit to Planck TT is restricted predominantly by the ISW effect breaking the degeneracy between the $A_{SW}$, $A_{Dop}$, and $A_L$, but it still fits the oscillatory residual in the multipole range $1250 \leq \ell \leq 2000$, which explains the improved fit to the $\chi^2$ over the $\Lambda$CDM fit to Planck TT. Note that the bins provided by the Planck collaboration for Plik Lite are highly correlated at high $\ell$. 

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\( \Lambda \text{CDM} + A_{\text{SW}} + A_{\text{Dop}} \) results in a significantly better fit to the Planck temperature data when only half of the data are included. This highlights that the improvement found when allowing either \( A_L \) or \( A_{\text{SW}} \) and \( A_{\text{Dop}} \) to vary is primarily in bringing the two halves of the Planck power spectrum into better agreement.

Allowing \( A_{\text{SW}} \) and \( A_{\text{Dop}} \) to vary when fitting to either Planck TT \( \ell \leq 800 \) or Planck TT \( \ell > 800 \) results in an increase in the preferred value of \( H_0 \), though notably the uncertainty of \( H_0 \) also increases to be >5 km s\(^{-1}\) Mpc\(^{-1}\). For Planck TT \( \ell > 800 \), the uncertainty on \( H_0 \) increases by a factor of 3.7 when \( A_L \) is allowed to vary indicating that lensing is important in constraining cosmological parameters at high \( \ell \).

In summary, we find that adding the phenomenological amplitudes we introduced in Section 2 in pairs does not result in a significant improvement to the fit to Planck TT over standard \( \Lambda \text{CDM} \). The one exception is when the phenomenological amplitudes for the Sachs–Wolfe and Doppler effects are both allowed to vary, but we show that this solution is really approximately refining the \( \Lambda \text{CDM} + A_L \) solution by allowing \( A_L \) to become a proxy for \( A_L \). Adding more phenomenological amplitudes, such as \( A_{\text{ISW}} \), can make this approximation marginally better but does not result in a significant improvement to the fit to Planck TT.

4. Can Additional Model Freedom Improve over \( \Lambda \text{CDM} + A_L \)?

In the previous section we found that none of the phenomenological amplitudes that we introduced in Section 2 showed any significant deviations from standard \( \Lambda \text{CDM} \) predictions. Moreover, while we found that combining the phenomenological amplitudes for the Sachs–Wolfe and Doppler effects show a \( \sim 2.7\sigma \) preference for \( A_{\text{SW}} \) and \( A_{\text{Dop}} \) both below unity, we noted that this solution was just refining the \( \Lambda \text{CDM} + A_L \) solution.

In this section we test some extensions to \( \Lambda \text{CDM} + A_L \) to seek a model extension that better fits Planck TT. In Section 4.1, we test extending the \( \Lambda \text{CDM} + A_L \) model to include the phenomenological amplitudes that we introduced in Section 2. In Section 4.2, we test extending the \( \Lambda \text{CDM} + A_L \) model to include one of \( N_{\text{eff}}, n_{\text{run}}, \) and \( Y_{\text{He}} \) which all have effects on the high multipole moments of the TT power spectrum.

### Table 3

| Parameter | \( \Lambda \text{CDM} \ell \leq 800 \) | \( A_L \ell \leq 800 \) | \( A_{\text{SW}} + A_{\text{Dop}} \ell \leq 800 \) | \( \Lambda \text{CDM} \ell > 800 \) | \( A_L \ell > 800 \) | \( A_{\text{SW}} + A_{\text{Dop}} \ell > 800 \) |
|-----------|---------------------------------|-----------------|---------------------------------|---------------------------------|-----------------|---------------------------------|
| \( H_0 \)  | 69.95 ± 1.84                    | 71.1 ± 2.1      | 73.3 ± 5.0                      | 64.28 ± 1.33                    | 71.0 ± 4.8      | 69.93 ± 5.3                     |
| \( 100 \times \omega_b \) | 2.252 ± 0.042                  | 2.283 ± 0.050   | 2.345 ± 0.111                   | 2.193 ± 0.039                   | 2.387 ± 0.145   | 2.273 ± 0.145                   |
| \( \omega_c \)  | 0.1145 ± 0.0033                 | 0.1128 ± 0.0036 | 0.1106 ± 0.0068                 | 0.1279 ± 0.0034                 | 0.1152 ± 0.0088 | 0.1162 ± 0.0106                 |
| \( 10^4 A_{L e^{-2 \ell}} \) | 1.859 ± 0.0170                  | 1.847 ± 0.0177  | 1.80 ± 0.31                     | 1.922 ± 0.021                   | 1.896 ± 0.025   | 2.59 ± 0.62                     |
| \( n_s \)     | 0.9756 ± 0.0120                 | 0.9829 ± 0.0137 | 0.9895 ± 0.0186                 | 0.9489 ± 0.0119                 | 0.9579 ± 0.0134 | 0.9823 ± 0.037                  |
| \( A_{\text{SW}} \) | ...                             | 1.64 ± 0.53     | ...                             | ...                             | 1.41 ± 0.30     | ...                             |
| \( A_{\text{Dop}} \)   | ...                             | 1.04 ± 0.104    | ...                             | ...                             | ...             | ...                             |

\( \chi^2 \) 95.48 94.04 94.35 123.28 121.10 121.32

**Note.** For definitions of the phenomenological amplitudes see Section 2. We use a prior of \( \tau = 0.0560 \pm 0.0086 \).

4.1. Testing \( A_L \) Plus One Additional Phenomenological Amplitude

In this subsection, we add the additional phenomenological amplitudes introduced in Section 2 to \( \Lambda \text{CDM} + A_L \) and fit to Planck TT. While the results in Section 3.1 showed no preference for any of these phenomenological amplitudes alone, Section 3.2 highlights the possibility that multiple phenomenological amplitudes working together could result in some improvement to the fit to Planck TT.

The results of adding \( A_{\text{ISW}}, A_{\text{Dop}}, A_{\text{ISW}}, \) or \( A_{\text{Pol}} \) to \( \Lambda \text{CDM} + A_L \) when fitting to Planck TT are summarized in Table 4. From Table 4, it is clear that there is almost no improvement to the \( \chi^2 \) when including these phenomenological amplitudes. Moreover, none of the posterior for the phenomenological amplitudes are more than 1\( \sigma \) away from unity. This is consistent with our results from Section 3.1 but again highlights that each of these physical effects are being correctly accounted for.

In Section 3.1, we showed that \( \Lambda \text{CDM} + A_{\text{ISW}} \) results in a minor improvement of 2.26 in the \( \chi^2 \) fit to Planck TT over \( \Lambda \text{CDM} \) alone. Adding \( A_{\text{ISW}} \) to \( \Lambda \text{CDM} + A_L \) results in almost no change in the \( \chi^2 \) from the \( \Lambda \text{CDM} + A_L \) case or a significant shift in the preferred value of \( A_{\text{ISW}} \) from unity. This suggests that the changes made by varying \( A_{\text{ISW}} \) are no longer necessary when \( A_L \) is already allowed to vary. This is consistent with the primary improvement to the fit to Planck TT found in the \( \Lambda \text{CDM} + A_{\text{ISW}} \) model coming from multipoles \( \ell < 30 \) as \( \Lambda \text{CDM} + A_L \) already makes improvements to fitting these multipoles. Note that the improvement in the multipole range \( \ell < 30 \) when \( \Lambda \text{CDM} + A_L \) is fit to Planck TT results from freeing up the constraints on other cosmological parameters such as allowing the preferred value of \( A_L \) to decrease and the preferred value of \( n_s \) to increase.

The model \( \Lambda \text{CDM} + A_L + A_{\text{ISW}} \) also provides an exploration into how the physical matter density is constrained by the CMB TT power spectrum. Knox & Millea (2020) point out that the physical matter density is predominantly determined by the overall photon envelope, followed by lensing, and then finally by the eISW effect. When \( A_L \) is allowed to vary, the uncertainty of the physical matter density increases by roughly 16.5% over standard \( \Lambda \text{CDM} \). Meanwhile, allowing \( A_{\text{ISW}} \) to vary results in a 3.5% increase in the uncertainty over standard \( \Lambda \text{CDM} \). Allowing \( A_L \) and \( A_{\text{ISW}} \) to vary results in a
19% increase in the physical matter density over standard LCDM. This suggests that the overall photon envelope constrains the physical matter density significantly more than either lensing or the eSW effect, consistent with Knox & Millea (2020). However, note that Table 3 shows that allowing \( A_L \) to vary results in a 240% increase in the uncertainty of the physical matter density when only \( \ell > 800 \) are included. This highlights the importance of lensing to constraining the physical matter density at high \( \ell \).

Note. We use a prior of \( \tau = 0.0506 \pm 0.0086 \).

Allowing both \( A_L \) and \( A_{\text{Pol}} \) to vary results in a negligible shift in the central values of the posteriors and a negligible improvement to the \( \chi^2 \) when fitting to Planck TT data. Again, the most significant effect when allowing \( A_{\text{Pol}} \) to vary is an increase in the uncertainty of parameters such as the 30% increase in the uncertainty of \( H_0 \).

### 4.2. Testing \( A_L \) Plus One Additional Nonphenomenological Amplitude

In this subsection we test models for LCDM + \( A_L \) + one of \( N_{\text{eff}} \), \( n_{\text{run}} \), and \( Y_{\text{He}} \). \( N_{\text{eff}} \) is designed to account for the effective number of relativistic degrees of freedom well after electron-positron annihilation. The parameter \( n_{\text{run}} \) accounts for possible linear order deviations from a flat primordial power spectrum with a spectral tilt given by \( n_s \). Finally, the helium fraction, \( Y_{\text{He}} \), affects the free electron density before and during recombination. All of these parameters added to LCDM + \( A_L \) could in principle affect the low \( \ell \) and high \( \ell \) consistency.

The results of adding \( N_{\text{eff}} \), \( n_{\text{run}} \), or \( Y_{\text{He}} \) to LCDM + \( A_L \) when fitting to Planck TT data are shown in Table 5. From Table 5, it
is clear that none of these result in a significant improvement to the fit. There is an increase of about 2.4 km s$^{-1}$ Mpc$^{-1}$ in the preferred value of $H_0$ when allowing both $A_L$ and $N_{\text{eff}}$ to vary. This is accompanied by a roughly 300% increase in the uncertainty of $H_0$ placing the posterior for $H_0$ within 1$\sigma$ of the measured value by the cosmological distance ladder. However, when the Planck TE and EE power spectra are added to the fit, the constraint becomes $H_0 = 68.1 \pm 1.7$ km s$^{-1}$ Mpc$^{-1}$, which corresponds to a 2.7$\sigma$ tension with the distance ladder preferred value for $H_0$. Therefore, $\Lambda$CDM + $A_L$ + $N_{\text{eff}}$ is not a plausible resolution of the Hubble tension.

In this section, we have allowed various additional types of model freedom but found no substantial improvement over $\Lambda$CDM + $A_L$; for whatever reason $A_L$ does seem to do a very effective job at relieving internal Planck tension.

5. Conclusions

We test the impact of allowing phenomenological amplitudes for the Sachs–Wolfe, eISW, Doppler, and polarization effects, which source the CMB temperature anisotropy, to vary when fitting to the Planck TT power spectrum. We find that allowing these amplitudes to vary results in only minimal improvement in the fit over standard $\Lambda$CDM. Moreover, there are only minimal shifts in the preferred values of the $\Lambda$CDM parameters when the amplitudes of these physical effects are varied. We conclude that $\Lambda$CDM correctly accounts for each of these physical effects.

Additionally, we test allowing multiple of these phenomenological amplitudes to vary simultaneously and find that allowing $A_{\text{SW}}$ and $A_{\text{Dop}}$ to vary together was the only combination that results in a significant improvement to the $\chi^2$ when fitting to Planck TT data. However, we also show that allowing these two phenomenological amplitudes to vary simultaneously results in a significant degradation of the precision of $A_L$, which comes from the near rescaling of the power spectrum for multipoles $\ell > 400$ when $A_{\text{SW}}$ and $A_{\text{Dop}}$ are scaled in unison. When only multipoles $\ell > 800$ are included in the fit to the Planck TT spectrum, $\Lambda$CDM + $A_{\text{SW}}$ + $A_{\text{Dop}}$ produces almost the same power spectrum as $\Lambda$CDM + $A_L$. We conclude that $\Lambda$CDM + $A_{\text{SW}}$ + $A_{\text{Dop}}$ is finding the $\Lambda$CDM + $A_L$ solution and therefore does not provide any new evidence for deviations from $\Lambda$CDM predictions.

From our tests where we vary $A_L$ and $A_{\text{SW}}$ both simultaneously and separately, we quantitatively determine that the physical matter density is constrained primarily by the overall photon envelope with smaller contributions from both lensing and the eISW effect when fitting to Planck TT data. These findings are in line with Knox & Milea (2020). However, when only Planck TT $\ell > 800$ data is included, lensing provides the majority of the constraining power for the physical matter density.

Finally, we varied both $A_L$ and one of $N_{\text{eff}}$, $n_{\text{run}}$, and $Y_B$ and fit to Planck TT data. All of these parameters impact the TT power spectrum at high $\ell$ meaning each of these parameter extensions provides a test of whether $A_L$ is fully able to resolve the internal tension between Planck TT $\ell \leq 800$ and Planck TT $\ell > 800$. We find no significant improvement in the fit over the $\Lambda$CDM + $A_L$ case, which suggests that there is little room for improvement from each of these effects.

Allowing these phenomenological amplitudes for the physical effects that source the CMB temperature anisotropy to vary provides a new test of consistency of each of these physical effects with $\Lambda$CDM predictions. While none of our new phenomenological tests provide evidence for deviations in the predictions made by $\Lambda$CDM, this lack of deviations from $\Lambda$CDM highlights that $\Lambda$CDM is generally good at describing the very complex nature of the CMB temperature anisotropy with the caveat that there is a known Planck internal tension between $\ell \leq 800$ and $\ell > 800$. These tests suggest that any new model of cosmology will need to make similar predictions to $\Lambda$CDM for the Sachs–Wolfe, Doppler, eISW, and polarization effects.

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