A model for the $Q^2$ dependence of polarized structure functions.

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Abstract

We present an update of a phenomenological model for the spin dependent structure functions $g_1(x, Q^2)$ of the proton and neutron. This model is based on a broken SU(6) wavefunction parametrized by the unpolarized structure functions. The two free parameters of the model are chosen to fulfill the Bjorken and Ellis–Jaffe sum rules. The model respects isospin symmetry and has zero strange sea polarization. Using new values for $F/D$ from hyperon beta decay the resulting $Q^2$ dependent asymmetries $A_1$ are in perfect agreement with the existing data. Therefore we do not see any evidence for a “spin crisis”. With two choices for $g_2$ the $Q^2$ dependence of $A_1(x, Q^2)$ and $A_2(x, Q^2)\sqrt{Q^2/M}$ is predicted and shown to be small for both cases.

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1 Introduction

The first measurements of $g_1^p$ suggested very surprisingly that the quarks carry much less of the protons spin than what was called the “naive expectation”. Subsequent experiments and theoretical papers stimulated a lot of discussion concerning this so called “spin crisis”, much of which centered around the correctness of the Ellis–Jaffe and the Bjorken sum rule. It became clear over the years that knowledge of the precise $Q^2$-dependence is crucial for attaining unambiguous results. Luckily the finite $Q^2$ corrections to the theoretical predictions for the sum rules get slowly under control.

To derive moments of the structure functions from experiment it is necessary to find the right extrapolation of the structure functions to small $x$ for fixed $Q^2$. As low $x$ experimentally corresponds to low $Q^2$ it is very difficult to treat this systematic effect correct other than by much improved measurements over a much wider kinematic range.

At present in all experiments the asymmetry $A_1$ is taken to be $Q^2$ independent. We only know of one previous data analysis that takes the $Q^2$ dependence into account, however the authors of this analysis use an approximation for the numerical evolution of the $Q^2$ dependence of $g_1$ which differs for low $Q^2$ from our results.

In studying the $Q^2$ dependence of the measured asymmetry $A_1$ (c.f. eq. (38)) we proceed in two steps: In section 2 we present a modification of the Carlitz Kaur model to get $g_1^p(x, Q_0^2)$, $g_1^n(x, Q_0^2)$ and a quantity $\Delta\tilde{G}(x, Q_0^2)$, which is a lower bound for the real $\Delta G(x, Q_0^2)$ for a fixed virtuality, e.g. $Q_0^2 = 10$ GeV$^2$. All parameters of the model are fixed by the unpolarized structure functions, the Bjorken and the Ellis–Jaffe sum rule.

In section 3 we present a modification of our model that does not show the leading particle picture. We show that it is able to fit the data as well.

In section 4 the structure functions are evolved in $Q^2$ using first order GLAP equations.
Resulting Asymmetries $A_1^{p/n}$ are compared with data and predictions are made for $A_2^{p/n}(x,Q^2)$.

Section 5 gives a summary and conclusions.

2 A model for $g_1^p(x,Q_0^2)$, $g_1^n(x,Q_0^2)$ and $\Delta \tilde{G}(x,Q_0^2)$ at $Q_0^2$

The model which we present tries to preserve $SU(6)$ symmetry as far as possible. However it is known that $SU(6)$ symmetry is broken in (at least) two aspects. First, the $u(x)$ and $d(x)$ distribution functions for the nucleon are different, and second, there is a mass splitting between the nucleon ground state and the delta resonances.

An $x$ dependent $SU(6)$ symmetric wavefunction for the proton [13], which describes the structure of the valence quarks is:

$$|p(x)\uparrow\rangle = \hat{u}^\dagger(x)\uparrow |I = 0, I_3 = 0, S = 0, S_3 = 0\rangle \sqrt{A_0(x)/x}$$
$$+ \left( \sqrt{2/3} \hat{d}^\dagger(x)|I = 1, I_3 = 1\rangle - \sqrt{1/3} \hat{u}^\dagger(x)|I = 1, I_3 = 0\rangle \right)$$
$$\times \left( \sqrt{2/3} \downarrow |S = 1, S_3 = 1\rangle - \sqrt{1/3} \uparrow |S = 1, S_3 = 0\rangle \right) \sqrt{A_1(x)/x}. \tag{1}$$

In this picture the proton state $|p(x)\uparrow\rangle$ consists of one active quark (parton) carrying the momentum fraction $x$ and two spectator quarks coupled to the indicated quantum numbers. As usual $\hat{q}^\dagger(x)$ denotes a creation operator of quark type $q$ with momentum fraction $x$, $q_v(x)$ denotes a valence quark distribution function of the proton:

$$q_v(x) := q_0^p(x) = \langle p(x)\uparrow |\hat{q}^\dagger(x)\uparrow \hat{q}(x)\uparrow + \hat{q}^\dagger(x)\downarrow \hat{q}(x)\downarrow |p(x)\uparrow \rangle \tag{2}$$

an similar for $\Delta q_v(x)$:

$$\Delta q(x) = \Delta q_v(x) := \Delta q_0^p(x) = \langle p(x)\uparrow |\hat{q}^\dagger(x)\uparrow \hat{q}(x)\uparrow - \hat{q}^\dagger(x)\downarrow \hat{q}(x)\downarrow |p(x)\uparrow \rangle \tag{3}$$

In the following we assume the validity of isospin symmetry i.e.:

$$u^p(x) = d^n(x), \quad d^p(x) = u^n(x), \quad \Delta u^p(x) = \Delta d^n(x), \quad \Delta d^p(x) = \Delta u^n(x). \tag{4}$$
The strange quarks and antiquarks are assumed to be unpolarized ($\Delta s = \Delta \bar{s} = 0$). This assumption is contrary to the usual interpretation of the experimental data which suggests $\int dx (\Delta s(x) + \Delta \bar{s}(x)) \approx -0.1$. This interpretation is, however, far from rigorous. It depends crucially on the question whether the F/D ratio in hyperon beta decays is affected by flavour–symmetry breaking or not, see the discussion in [16]. While the amount of symmetry breaking to be expected is strongly model dependent, it seems to be generally accepted that an appreciably effect can not be excluded by any rigorous undisputable argument. The real data can be fitted excellently with $\Delta s = \Delta \bar{s} = 0$ as we shall show and as was also observed by the Durham group [17]. We are aware of theoretical considerations [18, 19, 20] which predict an isospin asymmetry for the unpolarized case of the order of 10 % for large x and the nonleading parton distribution (i.e. $d^p(x) \neq u^n(x)$, see also [20]). For the polarized case it is common to spin–dilution models to break isospin symmetry [14, 21]. But as we are able to fit the data we do not see experimental evidence for the breaking of isospin symmetry in the polarized case.

Using the parton model expressions for the structure functions:

$$F_2(x) = 2xF_1(x) \quad (5)$$

$$F_1(x) = \frac{1}{2} \sum_f e_f^2 q_f(x) \quad (6)$$

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x) \quad (7)$$

one identifies $A_0$ and $A_1$ with the following combinations of $u$ and $d$ valence quark densities in order to fulfill equation (6):  

$$\frac{A_0(x)}{x} = u_v(x) - \frac{1}{2} d_v(x), \quad \frac{A_1(x)}{x} = \frac{3}{2} d_v(x). \quad (8)$$

Note that $d_v(x)/u_v(x)$ and therefore also $A_1(x)/A_0(x)$ goes to zero for $x \to 1$. For that reason for large x the proton state is dominated by

$$|p(x) \uparrow \rangle \approx \hat{u} \uparrow (x) \uparrow |I = 0, I_3 = 0, S = 0, S_3 = 0 \rangle \sqrt{A_0(x)/x}.$$

$$\quad (9)$$
This formula (9) reflects the intuition of the leading particle picture, namely that for large $x$ the hidden parton will carry the spin and isospin quantum numbers of the proton.

As Carlitz and Kaur [13] discussed it is necessary to introduce a “spin dilution function” in equation (1) to fulfill the Bjorken sum rule. For the spin dilution functions we follow the approach of reference [14] and define:

$$ F_q(I, I_3, S, S_3 = 0; x) = 1 - \frac{P(q \downarrow (x) G \uparrow (xG), I, I_3, S, S_3 = 0)}{P(q \uparrow (x), I, I_3, S, S_3 = 0)} $$

$$ F_q(I, I_3, S, S_3 = 1; x) = 1 - \frac{P(q \uparrow (x) G \downarrow (xG), I, I_3, S, S_3 = 1)}{P(q \downarrow (x), I, I_3, S, S_3 = 1)} $$

with $P(\ldots)$ denoting the probability of finding a state with the quantum numbers and particles in brackets. Following the arguments in reference [14] the spin dilution functions do not depend on $I, I_3$. But there is a dependence on $S_3$, reflecting the $SU(6)$ symmetry breaking between the delta resonance and the nucleon. If one assumes no dependence on $S_3$ one gets the original Carlitz Kaur model [13], which corresponds to the $SU(6)$ value for $F/D$: $F/D = 2/3$. The $F/D$ value we extract from experiment [16] is however smaller $F/D = 0.49 \pm 0.08$.

Contrary to our earlier work [14] we assume now that the difference in current quark mass between $u$ and $d$ quarks does not generate an appreciable isospin asymmetry of the spin distributions.

The remaining four independent spin dilution functions are parametrized as follows:

$$ F_q(S_3; x) = \frac{f_q(S_3; x) + 1}{2} , $$

$$ f_q(S_3; x) = \frac{1}{1 + a(S_3)x^{-\alpha_q}(1 - x)^2} . $$

(11)

Here the idea is the following: for small $x$ the relative polarization, e. g. of the $u\uparrow(x) \uparrow$...
\[ |0, 0, 0, 0\rangle \text{ state is proportional to} \]
\[
\frac{F_u(0; x) - (1 - F_u(0; x))}{F_u(0; x) + (1 - F_u(0; x))} = 2F_u(0; x) - 1 = f_u(0; x). \tag{12}
\]

The rational for the choice of the dilution functions \( f_q(S_3; x) \) can be found in [13, 14]. Basically it is just the easiest interpolation between the small-\( x \) and large-\( x \) limits. This leads to the following proton state consisting of three valence quarks and one gluon:
\[
|p(x, x_G) \rangle = \hat{u}^\dagger(x) \uparrow |0, 0, 0, 0\rangle \sqrt{F_u(0; x)} A_0(x)/x
\]
\[
+ 2/3 \hat{d}^\dagger(x) \downarrow |1, 1, 1, 1\rangle \sqrt{F_d(1; x)} A_1(x)/x
\]
\[
- \sqrt{2}/3 \hat{d}^\dagger(x) \uparrow |1, 1, 0\rangle \sqrt{F_d(0; x)} A_1(x)/x
\]
\[
- \sqrt{2}/3 \hat{u}^\dagger(x) \downarrow |1, 0, 1, 1\rangle \sqrt{F_u(1; x)} A_1(x)/x
\]
\[
+ 1/3 \hat{u}^\dagger(x) \uparrow |1, 0, 1, 0\rangle \sqrt{F_u(0; x)} A_1(x)/x
\]
\[
+ \hat{u}^\dagger(x) \downarrow \tilde{G}(x_G) \uparrow |0, 0, 0, 0\rangle \sqrt{(1 - F_u(0; x)) A_0(x)/x}
\]
\[
+ 2/3 \hat{d}^\dagger(x) \uparrow \tilde{G}(x_G) \downarrow |1, 1, 1, 1\rangle \sqrt{(1 - F_d(1; x)) A_1(x)/x}
\]
\[
- \sqrt{2}/3 \hat{d}^\dagger(x) \downarrow \tilde{G}(x_G) \uparrow |1, 1, 1, 0\rangle \sqrt{(1 - F_d(0; x)) A_1(x)/x}
\]
\[
- \sqrt{2}/3 \hat{u}^\dagger(x) \uparrow \tilde{G}(x_G) \downarrow |1, 0, 1, 1\rangle \sqrt{(1 - F_u(1; x)) A_1(x)/x}
\]
\[
+ 1/3 \hat{u}^\dagger(x) \downarrow \tilde{G}(x_G) \uparrow |1, 0, 1, 0\rangle \sqrt{(1 - F_u(0; x)) A_1(x)/x}
\]

The quantum numbers of the spectator quarks in kets are \( I, I_3, S, S_3 \). The part of the proton state which consists of three quarks and a gluon depends on the fractional momentum of the active quark \( x \) and on the fractional momentum of the gluon \( x_G \). \( x_G \) is an additional unknown quantity. Before the polarized gluon component was neglected in Carlitz–Kaur type models. As we are interested in the \( Q^2 \) evolution we have to take it into account. It seems naturally to assume \( x_G \ll x \). For explicit calculations we shall use the two choices \( x_G = x/10 \) and \( x_G = x \), which both give very similar asymmerties \( A_{1/2} \).
Note that these nucleon states give zero polarization for the strange sea and nonzero gluon polarization. This reflects our intuition that some of the valence spin is transformed to gluons. As gluons also carry spin balanced by angular momentum our results for \(| \Delta \tilde{G} |\) are rather lower bounds than absolute estimates for \(| \Delta G |\).

Equations (7), (8), (13) give for the proton
\[
2xg^p_1(x) = \frac{4}{9} xu_v(x) f_u(0; x) - \frac{1}{27} xd_v(x) \left(4f_u(0; x) + 4f_u(1; x) - f_d(0; x) + 2f_d(1; x)\right)
\]  
(14)

for the neutron
\[
2xg^n_1(x) = \frac{1}{9} xu_v(x) f_u(0, x) - \frac{1}{27} xd_v(x) \left(f_u(0; x) + f_u(1; x) - 4f_d(0; x) + 8f_d(1; x)\right)
\]  
(15)

and for the gluon polarization:
\[
x_G \Delta \tilde{G}(x_G) = \frac{1}{2} xu_v(x) \left(1 - f_u(0; x)\right) - \frac{1}{6} xd_v(x) \left(3 + f_d(0; x) - f_u(0; x) - 2f_d(1; x) - f_u(1; x)\right)
\]  
(16)

It is not obvious from equation (16) that \(| \Delta \tilde{G}(x) | \leq | G(x) |\), but we checked it explicitly.

The parameters \(\alpha_q\) are fixed by the assumption, that for \(x \to 0\) all degrees of freedom get equally populated. More precise,
\[
\lim_{x \to 0} f_q(S_3, x) \sim \frac{q_v(x)}{G(x)} \sim x^{\alpha_q}.
\]  
(17)

For the unpolarized structure functions we use the first order \((\alpha_s)\) parametrization of Glück, Reya, Vogt \([22]\). For \(Q_0^2 = 10 \text{ GeV}^2\) they give: \(\alpha_u = 0.326\) and \(\alpha_d = 0.505\). These (new) values are substantial smaller than the ones used in reference \([14]\). The remaining two parameters \(a(0) = a_0\) and \(a(1) = a_0 \cdot a_{10}\) are chosen in a way that \(g^{p/n}_1\) fulfills the Bjorken \([8]\) and Ellis Jaffe \([7]\) sum rules for \(Q_0^2 = 10 \text{ GeV}^2\). i.e.:
\[
\int_0^1 dx \left( g^p_1(x, Q_0^2 = 10 \text{ GeV}^2) - g^n_1(x, Q_0^2 = 10 \text{ GeV}^2) \right) = 0.187
\]  
(18)

\(^1\) Unfortunately there are misprints in the corresponding equation (17) of reference \([14]\).
and

\[ \int_0^1 dx g_1^n(x, Q_0^2 = 10 \text{ GeV}^2) = -0.042. \]  

(19)

This value includes the O(\(\alpha_s^3\)) corrections and the O(\(\alpha_s^4\)) estimate for the nonsinglet part and the O(\(\alpha_s^2\)) correction and the O(\(\alpha_s^3\)) estimate for the singlet part [9]. It also includes the small higher twist corrections from [23]. We use \(\alpha_s(10\text{ GeV}^2) = 0.24\) from [24]. Our F and D values of \(F = 0.415, D = 0.843\), with the constraint \(F + D = 1.257\) are taken from our analysis [16]. The resulting values (for \(Q_0^2 = 10 \text{ GeV}^2\)) for quark and gluon polarization are: \(\Delta u(Q^2 = 10 \text{ GeV}^2) = 0.746\), \(\Delta d(Q^2 = 10 \text{ GeV}^2) = -0.377\), \(\Delta s := 0\) (by construction). This gives \(a_0 = 0.225\) and \(a_{10} = 0.15\) and \(\Delta \tilde{G}(Q^2 = 10 \text{ GeV}^2) = 0.315\). Note that in our model \(\Delta u(Q_0^2 = 10 \text{ GeV}^2) + \Delta d(Q_0^2 = 10 \text{ GeV}^2) + 2\Delta \tilde{G}(Q_0^2 = 10 \text{ GeV}^2) = 1\), as some of the quarks spin is transferred to the gluons, but no angular momentum is generated. Figure 1 shows the polarized parton densities \(\Delta u\), \(\Delta d\), \(\Delta \tilde{G}\), for \(Q_0^2 = 10 \text{ GeV}^2\). The behavior for \(x \to 0\) and \(x \to 1\) of the distributions is as follows:

\[
\lim_{x \to 1} x \Delta u(x) = x u_v(x) \]  

(20)

\[
\lim_{x \to 1} x \Delta d(x) = -\frac{1}{3} x d_v(x) \]  

(21)

\[
\lim_{x_G \to 1} x_G \Delta \tilde{G}(x_G) = \frac{a_0}{2} x u_v(x)(1 - x)^2 \]  

(22)

\[
\lim_{x \to 0} x \Delta u(x) = x^{2\alpha_u} \approx x^{0.652} \]  

(23)

\[
\lim_{x \to 0} x \Delta d(x) = x^{2\alpha_d} \approx x^{1.010} \]  

(24)

\[
\lim_{x_G \to 0} x_G \Delta \tilde{G}(x_G) = \frac{1}{2} x u_v(x) \approx x^{0.326} \]  

(25)

\(\Delta u\) shows the behavior predicted by the quark counting rules [25], however \(\Delta d\) does
not. In the framework of the spin dilution model it is impossible to get a counting rule like behaviour for both $\Delta u$ and $\Delta d$ if one assumes $\lim_{x \to 1} F_q(S_3, x) = 1$. A counting rule like behaviour of $\Delta u$ is essential in order to obtain the leading particle picture. The precise behavior of $\Delta d(x)$ is unimportant for the leading particle picture as $|\Delta d(x)| \leq d(x)$. Change the behaviour of $\Delta d$ ad hoc in order to fulfill the quark counting rules leads to results which contradict existing data.

It was argued [29] that important new information can be obtained from the analysis of semi–inclusive processes. For the $\pi$–asymmetry $A_\pi$ defined as

$$A_\pi(x) = \frac{N_{\pi^+} - N_{\pi^-}}{N_{\pi^+} + N_{\pi^-}}$$

the pion asymmetry for proton, deuterium and $^3$He is given by:

$$A_{p\pi}(x) = \frac{4\Delta u^v(x) - \Delta d^v(x)}{4u^v(x) - d^v(x)},$$

$$A_{d\pi}(x) = \frac{\Delta u^v(x) + \Delta d^v(x)}{u^v(x) + d^v(x)},$$

$$A_{^3He\pi}(x) = \frac{4\Delta d^v(x) - \Delta u^v(x)}{7u^v(x) + 2d^v(x)}.$$  

Figure 2 shows our model prediction for this asymmetries.

3 A modified spin dilution model

If one does not insist to get a counting rule like behaviour for $\Delta u$ it is also possible to construct a proton state e. g. according to:

$$|p(x) \uparrow \rangle = \left( \hat{u} \uparrow(x) \uparrow |0, 0, 0, 0\rangle + \frac{1}{3} \hat{u} \uparrow(x) \uparrow |1, 0, 1, 0\rangle - \frac{\sqrt{2}}{3} \hat{u} \downarrow(x) \downarrow |1, 0, 1, 1\rangle \right) \sqrt{\hat{A}_0(x)/x}$$

$$+ \left( \frac{2}{3} \hat{d} \uparrow(x) \uparrow |1, 1, 1, 1\rangle - \frac{\sqrt{2}}{3} \hat{d} \downarrow(x) \downarrow |1, 0, 1, 0\rangle \right) \sqrt{\hat{A}_1(x)/x}$$

This behaviour is natural if one has a spin dilution in mind, thinking of the $F_q$’s as coefficients of a general Melosh (spin) rotation [26, 27] this constraint does not necessarily exist.

3Note the papers of Artru [28], where he derives an alternative to the dimensional counting rules with different predictions.
instead of equation (1), thus giving up approximate SU(6) symmetry. In equation (30) \( \tilde{A}_0(x) \) is proportional to \( u_v(x) \) and \( \tilde{A}_1 \) is proportional to \( d_v(x) \). Therefore the probability to find a quark of flavor \( q \) with definite spin will only depend on the unpolarized distribution function of the same flavor \( q(x) \), which seems to be much more natural compared to equation (1), were e.g. the probability to find a \( u \) quark with spin down depends only on \( d_v(x) \), but not on \( u_v(x) \). (This mixing of up and down distribution functions obviously reflects the SU(6) symmetry.)

Introducing the spin dilution functions of equation (11) gives:

\[
|p(x, x_G) \uparrow \rangle = \left( \left( \hat{u}^\dagger(x) \uparrow |0, 0, 0, 0\right) \sqrt{F_u(0; x)} \\
+ \frac{1}{3} \hat{u}^\dagger(x) \uparrow |1, 0, 1, 0\right) \sqrt{u(x)/4} \\
- \sqrt{2}/3 \hat{u}^\dagger(x) \downarrow |1, 0, 1, 1\right) \sqrt{u(x)/4} \\
+ \left( \left( \hat{d}^\dagger(x) \downarrow |1, 1, 1, 1\right) \sqrt{d(x)/2} \\
- \sqrt{2}/3 \hat{d}^\dagger(x) \uparrow |1, 0, 1, 0\right) \sqrt{d(x)/2} \\
+ \left( \left( \hat{u}^\dagger(x) \downarrow \tilde{G}^\dagger(x_G) \uparrow |0, 0, 0, 0\right) \sqrt{1 - F_u(0; x)} \\
+ \frac{1}{3} \hat{u}^\dagger(x) \downarrow \tilde{G}^\dagger(x_G) \uparrow |1, 0, 1, 0\right) \sqrt{1 - F_u(1; x)} \right) \sqrt{u(x)/4} \\
- \sqrt{2}/3 \hat{u}^\dagger(x) \uparrow \tilde{G}^\dagger(x_G) \downarrow |1, 0, 1, 1\right) \sqrt{u(x)/4} \\
+ \left( \left( \hat{d}^\dagger(x) \uparrow \tilde{G}^\dagger(x_G) \downarrow |1, 1, 1, 1\right) \sqrt{1 - F_d(0; x)} \\
- \sqrt{2}/3 \hat{d}^\dagger(x) \downarrow \tilde{G}^\dagger(x_G) \uparrow |1, 0, 1, 0\right) \sqrt{1 - F_d(1; x)} \right) \sqrt{d(x)/2} \right) (31)
\]

This proton state gives:

\[
\Delta u(x) = \frac{1}{6} u_v(x) \left( 5f_u(0; x) - f_u(1; x) \right) \\
\Delta d(x) = \frac{1}{3} d_v(x) \left( f_d(0; x) - 2f_d(1; x) \right) \\
\Delta \tilde{G}(x_G) = \frac{1}{12} \left( 4 - 5f_u(0; x) + f_u(1; x) \right) + \frac{1}{6} d_v(x) \left( 2f_d(1; x) - f_d(0; x) - 1 \right) (32)
\]

With \( a_0 = 0.233 \) and \( a_{10} = 0.145 \) these distributions fulfill equations (18) and (19).
Their behaviour for $x \to 0, 1$ is:

\[
\lim_{x \to 1} x \Delta u(x) = \frac{2}{3} x u_v(x) \quad (33)
\]

\[
\lim_{x \to 1} x \Delta d(x) = -\frac{1}{3} x d_v(x) \quad (34)
\]

\[
\lim_{x \to 0} x \Delta u(x) = x^{2\alpha_u} \approx x^{0.652} \quad (35)
\]

\[
\lim_{x \to 0} x \Delta d(x) = x^{2\alpha_d} \approx x^{1.010} \quad (36)
\]

\[
\lim_{x_G \to 0} x_G \Delta G(x_G) = \frac{1}{3} x u_v(x) \approx x^{0.326} \quad (37)
\]

Figure 3 shows the resulting distributions. The main difference between the two models is the behaviour of $\Delta u$ for large $x$. The model in this section gives $\lim_{x \to 1} = 2/3$ for the asymmetry $A_1$ for proton and neutron. This contradicts the leading particle picture. However we are able to fit the experimental results with this model, as will be seen in the next section. As a consequence it seems to be extremely important to get more accurate data at high $x$ in order to check the validity of the leading particle picture.

### 4 $Q^2$ evolution and comparison with data

Using the first order GLAP equations \cite{15} with $\Delta s(x, Q^2) = 0$, $n_f = 2$, but no additional assumptions we numerically\cite{14} generate from equations (14) - (16), (32) $g_1$ for the experimental values of $Q^2$ by evolution from $Q_0^2 = 10$ GeV$^2$. Figures 4, 5, 6, 7 show a comparison of the virtual photon asymmetry $A_1$\cite{31, 32}:

\[
A_1^{p/n} = \frac{g_1^{p/n}(x, Q^2) - 4g_2^{p/n}(x, Q^2)M^2x^2/Q^2}{F_1(x, Q^2)} \quad (38)
\]

\footnote{We do not have any problems with the stability or accuracy of the numerical solutions for any values of $Q^2$ and $x$.}
with experimental data [1, 2, 3, 4, 5, 6]. $M$ denotes the mass of the proton. As we neglect the effect of twist 4 and higher operators as well as $O(\alpha_s^2)$ corrections we insert the parton model expression (1) for $F_1$ (with sea quarks included) into equation (38), which we take from [22]. Instead we could also substitute $F_1$ by the parton model expression for $F_2$ (7) and $R$: $F_1 = 2xF_2(1 + R)$. Using the standard parametrization for $R$ [30], which includes higher twist and higher order radiative corrections, this would lead to different results for $A_1$. Most notably for large $x$, $R(x,Q^2) > 0$ and therefore for this choice of asymmetry $A_1(x,Q^2) < 1$ for $x \rightarrow 1$ even if the parton densities show leading particle behaviour. As this is hard to conciliate with physical intuition we use $F_1$ (parton) instead.

As there exists no experimental data for $g_2(x,Q^2)$ we considered two rather different assumptions for it.

$$g_2(x,Q^2) = 0 ,$$

which is the simple parton model prediction and implies that the twist three part of $g_2$ is as large as the twist two part and exactly cancels it. And

$$g_2(x,Q^2) = g_{2}^{\text{tw} 2}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2) ,$$

which means that the twist three part of $g_2$ is zero. We expect the physical reality to lie somewhere inbetween these two extreme scenarios.

Figures 8 - 11 show the resulting predictions for $A_{2}^{p/n}(x,Q^2)$

$$A_{2}^{p/n}(x,Q^2)\sqrt{Q^2/M} = 2x \frac{g_{2}^{p/n}(x,Q^2) + g_{2}^{n/p}(x,Q^2)}{F_{1}^{p/n}(x,Q^2)}$$

with the assumptions (39) and (40) for $g_2$. 

12
5 Conclusion

We presented two variants of spin dilution models that describe the data. The models give predictions for $\Delta u(x)$ and $\Delta d(x)$ and a kind of lower bound for $\Delta G(x)$. They fit the experimental data very well. These fits fulfill the Bjorken and Ellis–Jaffe sum rules with appropriate choosen F and D values. Therefore we do not see experimental evidence for isospin violation (as originally advocated in [14]) or for a strong strange sea polarization. As we could not fit data with a $\Delta d(x)$ that behaves like the counting rule prediction we doubt the correctness of the counting rules for $\Delta d$. Furthermore it seems us by now impossible to decide from data, whether the counting rule prediction (and the leading particle picture!) for $\Delta u$ is right. Our calculation gives a small, but systematic, dependence of $A_1$ and $\sqrt{Q^2} \cdot A_2$ on $Q^2$.

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References

[1] M.J. Alguard et al., Phys. Rev. Lett. 37 , 1261 (1976).
    M.J. Alguard et al., Phys. Rev. Lett. 41 , 70 (1978).
    G. Baum et al., Phys. Rev. Lett. 45 , 2000 (1980).
    G. Baum et al.; Phys. Rev. Lett. 51 , 1135 (1983).

[2] J. Ashman et al., Phys. Lett. B206 , 364 (1988).
    J. Ashman et. al., Nucl. Phys. B328 , 1 (1989)

[3] P.L. Anthony et al., Phys. Rev. Lett. 71 , 959 (1993).

[4] B. Adeva et al., Phys. Lett. B 302 , 533 (1993).
    B. Adeva et al., Phys. Lett. B320 , 400 (1994).
[5] D. Adams et al., Phys. Lett. B329, 399 (1994).

[6] K. Abe et al., Phys. Rev. Lett. 74, 346 (1995).

[7] J. Ellis and R. Jaffe, Phys. Rev. D 9, 1444 (1974).

[8] J.D. Bjorken, Phys. Rev. 148, 1467 (1966).

[9] S.A. Larin and J.A.M. Vermaseren, Phys. Lett. B 259, 345 (1991), Phys. Lett. B 303, 334 (1993). S.A. Larin, The next-to-leading QCD approximation to the Ellis–Jaffe sum rule. CERN–TH.7208/94.

[10] E. Stein, P. Gornicki, L. Mankiewicz, A. Schäfer and W. Greiner, QCD sum rule calculation of twist–3 contributions to polarized nucleon structure functions. hep-ph 9409212.

I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko Phys. Lett. B242, 245 (1990); B308 648(E) (1993).

[11] G. Altarelli, P. Nason, G. Ridolfi Phys. Lett. B320, 152 (1994) Erratum -ibd B325, 538 (1994).

[12] R. D. Ball, S. Forte, G. Ridolfi, Scale dependence and small $x$ behaviour of polarized parton distributions. CERN-TH/95-31

[13] R. Carlitz and J. Kaur, Phys. Rev. Lett. 38, 673 (1977).

J. Kaur, Nucl. Phys. B 128, 219 (1977).

[14] A. Schäfer, Phys. Lett. B208, 175 (1988).

[15] G. Altarelli and G. Parisi, Nucl. Phy. B126, 298 (1977).

V.N. Gribov and L.N. Lipatov, Yad. Fiz. 15, 781 (1972) [Sov.J.Nucl. Phys. 15, 438 (1972)].
[16] B. Ehrnsperger and A. Schäfer, *Reanalysis of hyperon beta decay data on F/D*. Phys. Lett. **B 323**, 439 (1995).

[17] T. Gehrmann and W.J. Stirling, *Spin-dependent parton distributions from polarized structure function data*. DTP/94/38 and hep-ph 9406212.

[18] E. N. Rodinov, A. W. Thomas and J. T. Londergan, Mod. Phys. Lett. **A 9**, 1799 (1994).

[19] E. Sather, Phys. Lett. **B 274**, 433 (1992).

[20] B. Q. Ma, A. Schäfer and W. Greiner, J. Phys. **G 20**, 719 (1994).

[21] D. de Florian, L. N. Epele, H. Fanchiotti, C. A. Garcia Canal and R. Sassot, Phys. Lett. **B 319**, 285 (1993) and hep-ph 9408363.

[22] M. Glück, E. Reya and A. Vogt, Z. Phys. **C 53**, 127 (1992). Phys. Lett. **B306**, 391 (1993).

[23] E. Stein, P. Gornicki, L. Mankiewicz, and A. Schäfer, *QCD Sum Rule Calculation of Twist-4 Corrections to Bjorken ans Ellis-Jaffe Sum Rules*. UFTP preprint 380/1995

[24] Review of Particle Properties, L. Montanet et al., Phys. Rev. **D50**, 1173, Part I (1 August 1994).

[25] S.J. Brodsky and M. Burkardt, *Perturbative QCD constraints on the shape of polarized quark and gluon distributions*. hep-ph/9401328 (1994).

[26] F. E. Close, Nucl. Phys. **B80**, 269 (1974).

[27] H. J. Melosh, Phys. Rev. **D 9**, 1095 (1974).

[28] X. Artru, Phys. Rev. **D 24**, 1662 (1980).

X. Artru, Il Nuovo Cimento, **71 A**, 341 (1982).
[29] L. L. Frankfurt, M. Strickman, L. Mankiewicz, A. Schäfer, E. Rondio, A. Sandacz and V. Papavassiliou, Phys. Lett. B 230, 141 (1989).

[30] L. W. Whitlow, S. Rock, A. Bodek, S. Dasu and E. M. Riordan, Phys. Lett. B250, 193 (1990).

[31] K. Rith, The internal spin structure of the nucleon. In *Erice 1989, Proceedings, The nature of hadrons and nuclei by electron scattering* 363-377 (1989).

[32] M. Anselmino, A. Efremov, E. Leader, The theory and phenomenology of polarized deep inelastic scattering. CERN-TH/7216/94 to appear in Physics Reports.
Figure Caption

Figure 1: solid line $x\Delta u(x, Q_0^2)$, dotted line $x\Delta d(x, Q_0^2)$, dashed line $x\Delta \tilde{G}(x = x_G, Q_0^2)$ for $Q_0^2 = 30$ GeV$^2$, for the model of section 2.

Figure 2: Pion–asymmetries for proton (solid line), deuterium (dotted line) and $^3$He (dashed line).

Figure 3: solid line $x\Delta u(x, Q_0^2)$, dotted line $x\Delta d(x, Q_0^2)$, dashed line $x\Delta \tilde{G}(x = x_G, Q_0^2)$ for $Q_0^2 = 30$ GeV$^2$, for the model of section 3.

Figure 4: $A_1^p(x, Q^2)$: solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$, for the model of section 2. Diamonds SMC, circles EMC, Squares E130 measurements.

Figure 5: $A_1^p(x, Q^2)$: solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$, for the model of section 3. Diamonds SMC, circles EMC, Squares E130 measurements.

Figure 6: $A_1^n(x, Q^2)$: solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, for the model of section 2. dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$, Diamonds E142 measurement.

Figure 7: $A_1^n(x, Q^2)$: solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, for the model of section 3. dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$, Diamonds E142 measurement.

Figure 8: $A_2^p(x, Q^2)$ for $g_2 = 0$ and for the model of section 2. Solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$.

Figure 9: $A_2^p(x, Q^2)$ for $g_2 = g_2^{tw2}$ and for the model of section 2. Solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$
GeV$^2$.

Figure 10: $A_2^g(x, Q^2)$ for $g_2 = 0$ and for the model of section 2. Solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$.

Figure 11: $A_2^n(x, Q^2)$ for $g_2 = g_2^{\text{FW}}$ and for the model of section 2. Solid line $Q^2 = 1$ GeV$^2$, large dashed line $Q^2 = 4$ GeV$^2$, dashed line $Q^2 = 10$ GeV$^2$, dotted line $Q^2 = 40$ GeV$^2$. 
g_2 = 0 \ a_10 = 0.145 \ a_0 = 0.233

Figure 7
\[ g_2 = W W \ a_10 = 0.15 \ a_0 = 0.225 \]

Figure 4
\[ g_2 = WW a_{10} = 0.145 \quad a_0 = 0.233 \]

Figure 5
Figure 8
Figure 9
$Q^2 = 10 \text{ GeV}^2 \quad a_0 = 0.225 \quad a_{10} = 0.15$

Figure 1
\[ Q^2 = 10 \text{ GeV}^2 \quad a_0 = 0.233 \quad a_{10} = 0.145 \]
$Q^2 = 10 \text{ GeV}^2 \ a_0 = 0.225 \ a_{10} = 0.15$

**Figure 2**