Lorentz violations in canonical quantum gravity

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This is a summary of a talk given at the CP01 meeting on possible Lorentz anomalies in canonical quantum gravity. It briefly reviews some initial explorations on the subject that have taken place recently, and should be only be seen as a pointer to the literature on the subject, mostly for outsiders.

I. INTRODUCTION

It has been recognized for some time that applying the rules of quantum mechanics to general relativity is problematic \[3\]. The fact that general relativity has a dimensionful coupling constant immediately implies that a straightforward perturbative approach will lead to a non-renormalizable theory. The current majority-held point of view towards the problem of quantum gravity is that general relativity is pathological and it has to be replaced by a more general theory to be quantized. Current candidates include “M-Theory”, a non-perturbative superset of string theories \[2\].

A group of researchers has over the last decade devoted themselves to reexamine the original question: is general relativity impossible to quantize? The main rationale for this is that there is a feeling that the question has not been examined in a detailed enough way. Yes, apparent non-renormalizability suggests a problem, but it is not clear if this is a problem of the theory itself, or of the way in which perturbation theory is being implemented. A striking example of this distinction is given by gravity in 2 + 1 dimensions \[3\]. In 2 + 1 dimensions the Einstein equations imply space-time is locally flat. The only possible degrees of freedom of gravity have to do with global properties of the space-time. The resulting theory is not even a field theory proper: its reduced phase space is finite dimensional. Nevertheless, if one does a straight application of perturbation theory to the quantization of general relativity in 2 + 1 dimensions, the theory appears non-renormalizable for the same reasons as gravity in 3 + 1 dimensions. Witten was the first to point out that a non-perturbative quantization should be possible, and after that was accomplished, other researchers showed that one could indeed study the theory perturbatively. The overriding question is: could we be in the same situation with gravity in 3 + 1 dimensions? Other examples of theories that do not appear to exist perturbatively but do non-perturbatively are known \[4\].

It is not surprising that quantizing general relativity should be tricky. The theory is invariant under space-time diffeomorphisms, reflecting the coordinate invariance of general relativity. Most traditional quantum field theory techniques are not well suited for diffeomorphism invariant theories, since they break such invariance. We have only comparatively recently started to understand how to quantize diffeomorphism invariant theories, but most of the results have been for “topological field theories” (again, theories with a finite number of degrees of freedom) \[5\]. With almost no exception, in general relativity is the first time we confront the quantization of a theory that is both diffeomorphism invariant and also has an infinite number of degrees of freedom.

Non-perturbative quantizations of general relativity (both using canonical and path integral techniques) were first attempted in the 60’s and 70’s. Most of these attempts ended unsuccessfully. The expressions encountered either for the quantum Hamiltonian constraint or the path integral were ill-defined and there was no idea of how to regularize them. A revival of interest took place when Ashtekar \[6\] discovered a new set of variables in terms of which general relativity resembled a Yang–Mills theory. This opened the possibility of applying techniques of non-perturbative quantization that were successfully applied in the Yang–Mills context. The new variables introduced by Ashtekar consist of replacing the spatial metric (in canonical quantization, the theory is based on studying a slice of space as it evolves in time) by a set of three vector fields (triads), \(E_i^a\) (the reader can pretend one is dealing with the three electric fields of an \(SU(2)\) Yang–Mills theory), and the conjugate momentum behaves like an \(SU(2)\) connection \(A_i^a\). In terms of these variables the constraints become polynomial and include among them a Gauss law. That is, one can view the phase space of general relativity as a submanifold of that of Yang–Mills theory.

Having the theory cast in terms of a Yang–Mills-like connection, allows one to expand the wavefunctions in terms of traces of holonomies (Wilson loops). The coefficients of the expansion constitute the “loop representation”, which had been studied for Yang–Mills theories by Gambini and Trias \[7\] in the 80’s and was introduced into the gravitational context by Rovelli and Smolin \[8\]. Expressing things in terms of loops is quite attractive in the gravitational context, since one can embody diffeomorphism invariance in a remarkably clear way: one simply considers functions of loops.
that are invariant under deformations of the loops. Such functions have been studied by mathematicians since the
time of Gauss, and are called knot invariants [9].
Ashtekar and Lewandowski [10] introduced a set of mathematical tools to deal with gauge-invariant diffeomorphism
invariant functions of a connection. These tools consist of a set of simple functions (cylindrical functions) that
can be rigorously endowed with a measure. This is highly non-trivial, since it is a measure of integration in an
infinite-dimensional space. There are few examples of such measures available. One now therefore has a definite,
mathematically controlled setting in which to construct a theory of quantum gravity.
The basis of Wilson loops is overcomplete, and for many years this caused problems when working in the loop
representation. Rovelli and Smolin [11] showed that one could find an elegant way of labeling the independent elements
of the basis of loops in terms of “spin networks”. The latter are a mathematical construction first introduced by Penrose
consisting of embedded graphs of lines intersecting at multivalent vertices, each line labeled by a representation of
$SU(2)$. The Ashtekar–Lewandowski measure is particularly simple to understand in terms of spin networks.
Several attractive results have been obtained using these techniques. For instance, the spectrum of the quantum
operators associated with the area of a surface and of the volume [12] of a region were studied in detail, and found to
be discrete. Expressions for the Hamiltonian constraint of gravity that are finite and well defined were also proposed
[13]. This constitutes the first proposal ever of a non-trivial, well defined theory of quantum gravity. The present
efforts are concentrating on demonstrating that the theory constructed has the correct classical and semi-classical
limit.
There has also been progress in applying these mathematical tools to define a path-integral quantization of general
relativity [14]. Impressive recent results have found a certain sense in which the Lorentzian path integral can be made
finite.

II. LORENTZ VIOLATIONS

Why should there be Lorentz violations in non-perturbative quantum gravity? The answer can only be tentative,
since we do not have a definite answer concerning the semi-classical limit of the theory. Fortunately, progress in this
area is happening at a significant pace [15] and I expect in near future CP meetings a more definitive answer will be
present.
To see why one might expect violations, let us consider at a very broad and heuristic sense how would one construct
a semi-classical limit of the theory. When one has a quantum theory under control, one should be able to build
cohherent states that are peaked around a solution of the classical equations of motion, and to therefore study quantum
fluctuations. Since gravity is now under control, we can build such states. However, there is an unusual twist when one
carry out this straightforward idea in the case of gravity. The twist is that presumably one does not want to consider
a coherent state that approximates any classical metric. The reason for this is that one can obviously construct many
classical metrics that do not correspond to a “classical” situation at all (for instance a metric with gravitational waves
of wavelength shorter than the Planck length). Nothing prevents one from mathematically constructing such coherent
states. One should therefore introduce into the semi-classical construction a proviso that one will only consider states
that approximate metrics that are “reasonably smooth” at a certain length scale. This length scale is absolute.
Therefore, unless one elaborates further, Lorentz violations will occur.
As I stated before, this subject is rapidly evolving. Amelino-Camelia has several articles on this point and I would
refer the reader to them [16].

III. CONCRETE CALCULATIONS

It is evidently that more work is needed before we can perform a concrete calculation showing Lorentz violations.
Up to present only a handful of preliminary investigations have been carried out. In a piece of work with Gambini
[17] we studied the coupling of gravity to Maxwell theory when gravity is treated quantum mechanically. We took a
naive point of view in which we assumed that the quantum state was given by a spin network state and studied the
coupling Hamiltonian pretending that the Maxwell fields were in a semi-classical coherent state and only kept the
leading terms. This calculation is inadequate at many levels: first of all we did not consider a quantum state that
satisfies all constraints, and considered a semi-classical limit of them. We just considered a generic spin network state
and treated the Maxwell theory as a regular field theory. Under these assumptions, we showed that if the spin network
state had certain properties (namely it was parity-violating), one got Lorentz violating corrections to Maxwell theory,
that implied a birefringence of space-time at the level of $\ell_p/\lambda$ where $\ell_p$ is Planck’s length and $\lambda$ is the wavelength
of the light. Such effect could influence the light arriving from gamma-ray bursts and be within a few orders of
magnitude of observation (very much like the string inspired violations of Amelino-Camelia et al. [18]). It was later
noted that such birefringence is severely constrained by optical observations of polarized sources [19]. Alfaro et al.
[20] have performed a reexamination of these calculations, with a more careful motivation for the quantum states
considered, and found certain discrepancies in the results with those of reference. The same authors [21] have also
extended these results to propagations of Fermions, finding possibly observable effects in the time arrival of neutrinos.
The Fermionic calculations have a further drawback in that a given expectation value for the connection has to be
assumed in the quantum state. The results are therefore highly dependent on this value, which is not currently fixed
by any a priori argument. There is rather general consensus that effects at higher orders in $\ell_P/\lambda$ than the one we
found are likely be present. Such effects appear to weak to be observed.

IV. CONCLUSIONS

At present it appears possible that the canonical quantization of gravity will, in its semi-classical limit, contain
violations of Lorentz symmetry. Some preliminary heuristic calculations have exhibited this effect. The encouraging
part is that rigorous semi-classical studies are currently under way and it is expected that in the next few years these
questions will be given a definitive answer.

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