A Galois-Connection between Cattell’s and Szondi’s Personality Profiles

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Abstract

We propose a computable Galois-connection between, on the one hand, Cattell’s 16-Personality-Factor (16PF) Profiles, one of the most comprehensive and widely-used personality measures for non-psychiatric populations and their containing PsychEval Personality Profiles (PPPs) for psychiatric populations, and, on the other hand, Szondi’s personality profiles (SPPs), a less well-known but, as we show, finer personality measure for psychiatric as well as non-psychiatric populations (conceived as a unification of the depth psychology of S. Freud, C.G. Jung, and A. Adler). The practical significance of our result is that our Galois-connection provides a pair of computable, interpreting translations between the two personality spaces of PPPs (containing the 16PFs) and SPPs: one concrete from PPP-space to SPP-space (because SPPs are finer than PPPs) and one abstract from SPP-space to PPP-space (because PPPs are coarser than SPPs). Thus Cattell’s and Szondi’s personality-test results are mutually interpretable and inter-translatable, even automatically by computers.

Keywords: applied order theory; comparative, computational, and mathematical psychology; machine translation; personality tests; 16PF.

1 Introduction

According to [2] Page 3, most studies have found Cattell’s comprehensive 16-Personality-Factor (16PF) Profiles [3] “to be among the top five most commonly used normal-range instruments in both research and practice” with culturally adapted translations into over 35 languages world-wide. Further, “[t]he 16PF traits also appear in the PsychEval Personality Questionnaire [4], a comprehensive instrument which includes both normal and abnormal personality dimensions.” Note that according to [2] Page 4, “[i]nstead of being developed to measure preconceived dimensions of interest to a particular author, the instrument was developed from the unique perspective of a scientific quest to try to discover the basic structural elements of personality.” Notwithstanding, and further exemplifying our general methodology introduced in [7], we propose in
the present paper a computable Galois-connection [5] between PsychEval Personality Profiles (PPPs), which contain the 16PFs, and Szondi’s Personality Profiles (SPPs) [9], a less well-known but, as we show, finer personality measure for psychiatric as well as non-psychiatric populations, and conceived as a unification [10] of the depth psychology of S. Freud, C.G. Jung, and A. Adler. This paper being a further illustration of our general methodology introduced in [7], our presentation here thus closely follows the one in [7], even in wording. The generality of our mathematical methodology may be obvious to the (order-theoretic) mathematician, but may well not be so to the general psychologist.

Just like [7], our present result is a contribution to mathematical psychology in the area of personality assessment. It is also meant as a contribution towards practicing psychological research with the methods of the exact sciences, for obvious ethical reasons. The practical significance of our result is that our Galois-connection provides a pair of computable, interpreting translations between the two personality spaces of PPPs and SPPs (and thus hopefully also between their respective academic and non-academic communities): one concrete translation from PPP-space to SPP-space (because SPPs are finer than PPPs) and one abstract translation from SPP-space to PPP-space (because PPPs are coarser than SPPs). Thus Cattell’s and Szondi’s personality-test results are mutually interpretable and inter-translatable, even automatically by computers. The only restriction to this mutuality is the subjective interpretation of the faithfulness of these translations. In our interpretation, we intentionally restrict the translation from SPP-space to PPP-space, and only that one, in order to preserve (our perception of) its faithfulness. More precisely, we choose to map some SPPs to the empty set in PPP-space (but every PPP to a non-empty set in SPP-space). Of course just like in [7], our readers can experiment with their own interpretations, as we explain again in the following paragraph.

We stress that our Galois-connection between the spaces of PPPs and SPPs is independent of their respective test, which evaluate their testees in terms of structured result values—the PPPs and SPPs—in the respective space. Both tests are preference-based, more precisely, test evaluation is based on choices of preferred questions in the case of the PsychEval-test [4] and on choices of preferred portraits in the case of the Szondi-test [9, 8]. Due to the independence of our Galois-connection from these tests, their exact nature need not concern us here. All what we need to be concerned about is the nature of the structured result values that these tests generate. (Other test forms can generate the same form of result values, e.g. [6].) We also stress that our proposed Galois-connection is what we believe to be an interesting candidate brain child for adoption by the community, but that there are other possible candidates, which our readers are empowered to explore themselves. In fact, not only do we propose a candidate Galois-connection between PPP-space and SPP-space, but also do we further illustrate the whole methodology introduced in [7] for generating such candidates. All what readers interested in generating such connections themselves need to do is map their own intuition about the meaning of PPPs to a standard interlingua, called Logical Pivot Language (LPL) here, and check that their mapping has a single simple property, namely the one stated
as Fact 1.1 about our mapping \( f \) in Figure 1. Their desired Galois-connection is then automatically induced jointly by their chosen mapping and a mapping, called \( p \), from SPP-space to LPL that we chose in [7] once and for all possible Galois-connections of interest. What is more, and as already mentioned in [7] and evidenced here, our methodology is applicable even more generally to the generation of Galois-connections between pairs of result spaces of other personality tests. SPPs just happen to have a finer structure than other personality-test values that we are aware of, and so are perhaps best suited to play the distinguished role of explanatory semantics for result values of other personality tests. Of course our readers are still free to choose their own preferred semantic space.

An SPP can be conceived as a tuple of eight, so-called signed factors whose signatures can in turn take 12 partially ordered values. So SPPs live in an eight-dimensional space. On the other hand, a PPP can be conceived as a \((16 + 12 = 28)\)-tuple of so-called personality traits, which can take 10 totally ordered values. So PPPs live in an apparently finer, 28-dimensional space. Nevertheless, we are going to show that actually the opposite is true, that is, SPPs are finer than PPPs. In particular, SPPs can account for ambiguous personality traits thanks to the partiality of their ordering, whereas PPPs cannot due to the totality of theirs. Moreover, a lot of Cattell’s personality traits turn out to be definable in terms of a combination of Szondi’s signed factors, which means that a lot of Cattell’s personality traits can be understood as (non-atomic/-primitive) psychological syndromes. SPPs being finer than PPPs, the translation from SPPs to PPPs must be a projection (and thus surjection) of SPP-space onto PPP-space. Another insight gained in the finer referential system of SPPs is that PPPs are confirmed to be non-orthogonal or not independent as also mentioned in [2]. Of course our readers are still free to disagree on the value of these insights by giving a convincing argument for why SPP-space would be an inappropriate semantics for PPP-space. After all, Szondi conceived his theory of human personality as a unifying theory. We now put forward our own argument for why we believe SPP-space is indeed an appropriate—though surely not the only—semantics for PPP-space. In Section 2.1 we present the defining mathematical structures for each space, and in Section 2.2 the defining mathematical mappings for their translation. No prior knowledge of either PPPs or SPPs is required to appreciate the results of this paper, but the reader might appreciate them even more when comparing them also with those in [7].

2 The connection

In this section, we present the defining mathematical structures for PPP-space, the interlingua LPL, and SPP-space, as well as the defining mathematical mappings for the concrete translation of PPP-space to SPP-space and the abstract translation of SPP-space back to PPP-space, both via LPL, see Figure 1.
2.1 Structures

In this section, we present the defining mathematical structures for PPP-space, the interlingua LPL, and SPP-space. We start with defining PPP-space.

**Definition 1** (The PsychEval Personality Profile Space). Let

- \(16\text{PF} = \{ \text{A, B, C, E, F, G, H, I, L, M, N, O, Q1, Q2, Q3, Q4} \} \) be the set of the 16 Personality Factors (the normal traits), with A meaning “warmth,” B “reasoning,” C “emotional stability,” E “dominance,” F “liveliness,” G “rule-consciousness,” H “social boldness,” I “sensitivity,” L “vigilance,” M “abstractness,” N “privateness,” O “apprehension,” Q1 “openness to change,” Q2 “self-reliance,” Q3 “perfectionism,” and Q4 “tension;”

- \(\text{PEPF} = \{ \text{PS, HC, ST, AD, LE, SR, AW, PI, OT, AP, TS, TI} \} \) be the set of the 12 PsychEval abnormal traits, with PS meaning “psychological inadequacy,” HC “health concerns,” ST “suicidal thinking,” AD “anxious depression,” LE “low energy state,” SR “self-reproach,” AW “apathetic withdrawal,” PI “paranoid ideation,” OT “obsessional thinking,” AP “alienation/perceptual distortion,” TS “thrill seeking,” and TI “threat immunity;”

- \(\text{PF} = 16\text{PF} \cup \text{PEPF} \).

Then,

\[
\text{PPP} = \{ (\text{A, } v_1), (\text{B, } v_2), (\text{C, } v_3), (\text{E, } v_4), (\text{F, } v_5), (\text{G, } v_6), (\text{H, } v_7), (\text{I, } v_8), (\text{L, } v_9),
\text{(M, } v_{10}), (\text{N, } v_{11}), (\text{O, } v_{12}), (\text{Q1, } v_{13}), (\text{Q2, } v_{14}), (\text{Q3, } v_{15}), (\text{Q4, } v_{16}),
\text{(PS, } v_{17}), (\text{HC, } v_{18}), (\text{ST, } v_{19}), (\text{AD, } v_{20}), (\text{LE, } v_{21}), (\text{SR, } v_{22}),
\text{(AW, } v_{23}), (\text{PI, } v_{24}), (\text{OT, } v_{25}), (\text{AP, } v_{26}), (\text{TS, } v_{27}), (\text{TI, } v_{28}) | v_1, \ldots, v_{28} \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \} \]

is the set of PsychEval Personality Profiles (PPPs) \([3][4]\), and

\[
\text{PPP} = (2^{\text{PPP}}, \emptyset, \cap, \cup, \text{PPP}, \subseteq)
\]

defines our *PsychEval Personality Profile Space*, that is, the (inclusion-ordered, Boolean) powerset algebra \([5]\) on PPP (the set of all subsets of PPP).
Note that we do need to define $\mathcal{PPP}$ as the set of all subsets of PPP and not simply as the set of all elements of PPP. The reason is the aforementioned fact that in the finer referential system of SPP-space (see Definition 2), PPPs turn out to be non-orthogonal or not independent, and thus a PPP may have to be mapped to a proper set of SPPs (see Table 2). So the proper setting for SPP-space is a set of subsets of SPPs, which in turn, via the backward translation from SPP-space to $\mathcal{PPP}$, means that the proper setting for $\mathcal{PPP}$, as the target of a mapping of subsets, is also a set of subsets.

We continue to define SPP-space.

**Definition 2** (The Szondi Personality Profile Space). Let us consider the Hasse-diagram [5] in Figure 2 of the partially ordered set of Szondi’s twelve signatures [9] of human reactions, which are:

- approval: from strong $+!!!$, $+!!$, and $+!$ to weak $+$;
- indifference/neutrality: $0$;
- rejection: from weak $-$, $-$!, and $-!!$ to strong $-!!!$; and
- ambivalence: $\pm!$ (approval bias), $\pm$ (no bias), and $\pm!$ (rejection bias).

(Szondi calls the exclamation marks in his signatures *quanta.*
Further let us call this set of signatures $S$, that is,

$$S = \{-!!!, -!!, -!, -!, 0, +, +!, +!!!, +!!m, \pm!, \pm!\}.$$  

Now let us consider *Szondi’s eight factors and four vectors* of human personality [9] as summarised in Table 1 (Their names are of clinical origin and need not concern us here.) And let us call the set of factors $F$, that is,

$$F = \{ h, s, e, hy, k, p, d, m \}.$$  

Then,

$$\text{SPP} = \{ ((h, s_1), (s, s_2), (e, s_3), (hy, s_4), (k, s_5), (p, s_6), (d, s_7), (m, s_8)) \mid s_1, \ldots, s_8 \in S \}$$

is the set of Szondi’s personality profiles, and

$$\text{SPP} = (2^{\text{SPP}}, \emptyset, \cup, \text{SPP}, \setminus, \subseteq)$$

defines our *Szondi Personality Profile Space*, that is, the (inclusion-ordered, Boolean) powerset algebra [5] on SPP (the set of all subsets of SPP).

As an example of an SPP, consider the *norm profile* for the Szondi-test [9]:

$$( (h, +), (s, +), (e, -), (hy, -), (k, -), (p, -), (d, +), (m, +) )$$

Spelled out, this norm profile describes the personality of a human being who approves of physical love, has a proactive attitude, has unethical but moral behaviour, wants to have and be less, and is unfaithful and dependent.

We conclude this subsection with the definition of our interlingua LPL.

**Definition 3** (The Logical Pivot Language). Let

$$A = \{ h s_1, s s_2, e s_3, h y s_4, k s_5, p s_6, d s_7, m s_8 \mid s_1, \ldots, s_8 \in S \}$$

be our set of atomic logical formulas, and LPL($A$) the classical propositional language over $A$, that is, the set of sentences constructed from the elements in
A and the classical propositional connectives ¬ (negation, pronounced “not”), ∧ (conjunction, pronounced “and”), ∨ (disjunction, pronounced “or”), etc.

Then,
\[ \mathcal{LPL} = \{ \text{LPL}(A), \Rightarrow \} \]
defines our logical pivot language, with \( \Rightarrow \) being logical consequence.

Logical equivalence \( \equiv \) is defined in terms of \( \Rightarrow \) such that for every \( \phi, \varphi \in \text{LPL}(A) \), \( \phi \equiv \varphi \) by definition if and only if \( \phi \Rightarrow \varphi \) and \( \varphi \Rightarrow \phi \).

2.2 Mappings between structures

In this section, we present the defining mathematical mappings for the concrete translation \( ^o \) of \( \text{PPP} \) to \( \text{SPP} \) via \( \mathcal{LPL} \) and the abstract translation \( ^a \) of \( \text{SPP} \) back to \( \text{PPP} \) again via \( \mathcal{LPL} \) by means of the auxiliary mappings \( f \) and \( p \). We also prove that the ordered pair \( (^o, ^a) \) is a Galois-connection, as promised.

**Definition 4 (Mappings).** Let the mapping (total function)
- \( f \) be defined in
  - the function space \( ([\mathbb{P}\mathbb{F} \times \{1, \ldots, 10\}] \rightarrow \text{LPL}(A)) \) as in Table 2
  - the function space \( (\text{PPP} \rightarrow \text{LPL}(A)) \) such that
    \[
    \begin{align*}
    f((A, v_1), (B, v_2), (C, v_3), (E, v_4), (F, v_5), (G, v_6), (H, v_7), (I, v_8), (L, v_9),
    & (M, v_{10}), (N, v_{11}), (O, v_{12}), (Q1, v_{13}), (Q2, v_{14}), (Q3, v_{15}), (Q4, v_{16}),
    & (PS, v_{17}), (HC, v_{18}), (ST, v_{19}), (AD, v_{20}), (LE, v_{21}), (SR, v_{22}),
    & (AW, v_{23}), (OT, v_{24}), (AP, v_{25}), (TS, v_{26}), (TI, v_{27})) = \\
    & f((A, v_1)) \land f((B, v_2)) \land f((C, v_3)) \land f((E, v_4)) \land \\
    & f((F, v_5)) \land f((G, v_6)) \land f((H, v_7)) \land f((I, v_8)) \land \\
    & f((L, v_9)) \land f((M, v_{10})) \land f((N, v_{11})) \land f((O, v_{12})) \land \\
    & f((Q1, v_{13})) \land f((Q2, v_{14})) \land f((Q3, v_{15})) \land f((Q4, v_{16})) \land \\
    & f((PS, v_{17})) \land f((HC, v_{18})) \land f((ST, v_{19})) \land f((AD, v_{20})) \land \\
    & f((LE, v_{21})) \land f((SR, v_{22})) \land f((AW, v_{23})) \land f((AP, v_{24})) \land \\
    & f((OT, v_{25})) \land f((AP, v_{26})) \land f((TS, v_{27})) \land f((TI, v_{28})),
    \end{align*}
    \]
  - the function space \( (2^{\text{PPP}} \rightarrow \text{LPL}(A)) \) such that for every \( F \in 2^{\text{PPP}} \),
    \[
    f(F) = \bigwedge \{ f(f) \mid f \in F \};
    \]
- \( p \) be defined in the function space \( (\text{SPP} \rightarrow \text{LPL}(A)) \) such that
  \[
  p(((h, s_1), (s, s_2), (e, s_3), (by, s_4), (k, s_5), (p, s_6), (d, s_7), (m, s_8))) = \\
  hs_1 \land ss_2 \land es_3 \land bys_4 \land ks_5 \land ps_6 \land ds_7 \land ms_8
  \]
and in the function space \( (2^{\text{SPP}} \rightarrow \text{LPL}(A)) \) such that for every \( P \in 2^{\text{SPP}} \),
\[
  p(P) = \bigvee \{ p(p) \mid p \in P \}.
\]
Table 2: The translation f of $\mathbb{F} \times \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ to $\text{LPL}(A)$

| Low Range | PF | High Range |
|-----------|----|------------|
| $h^{-1}$  | h+ | $h^{-1}$   |
| $k \wedge p \equiv$ | k+ p | k+ p | k+ p | k+ p | k+ p | k+ p | k+ p | k+ p | k+ p | k+ p |
| $d^{-1}$  | d+ | d+ | d+ | d+ | d+ | d+ | d+ | d+ | d+ | d+ |
| $s^{-1}$  | s+ | s+ | s+ | s+ | s+ | s+ | s+ | s+ | s+ | s+ |
| $k^{-1}$  | k+ | k+ | k+ | k+ | k+ | k+ | k+ | k+ | k+ | k+ |
| $e^{-1} \wedge h^{-1} \wedge k^{-1}$ | e- h- k- | e- h- k- | e- h- k- | e- h- k- | e- h- k- | e- h- k- | e- h- k- | e- h- k- | e- h- k- | e- h- k- |

---

Note: The table continues with similar entries for different ranges and translations.
Then, the mapping

- \( \star : \mathcal{PPP} \rightarrow \mathcal{SPP} \) defined such that for every \( F \in 2^{\mathcal{PPP}} \),

\[
F^\star = \{ p \in \mathcal{SPP} | p(p) \Rightarrow f(F) \}
\]

is the so-called right polarity and

- \( \check{\star} : \mathcal{SPP} \rightarrow \mathcal{PPP} \) defined such that for every \( P \in 2^{\mathcal{SPP}} \),

\[
P^\check{\star} = \{ f \in \mathcal{PPP} | p(P) \Rightarrow f(f) \}
\]

is the so-called left polarity of the ordered pair \((\star, \check{\star})\).

Spelled out, (1) the result of applying the mapping \( f \) to a set \( F \) of PPPs \( f \) as defined in Definition 4 is the conjunction of the results of applying \( f \) to each one of these \( f \), which in turn is the conjunction of the results of applying \( f \) to each one of the factor-value pairs in \( f \) as defined in Table 2; (2) the result of applying the mapping \( p \) to a set \( P \) of SPPs \( p \) as defined in Definition 4 is the disjunction of the results of applying \( p \) to each one of these \( p \), which simply is the conjunction of all signed factors in \( p \) taken each one as an atomic proposition; (3) the result of applying the mapping \( \star \) to a set \( F \) of PPPs is the set of all those SPPs \( p \) whose mapping under \( p \) implies the mapping of \( F \) under \( f \); (4) the result of applying the mapping \( \check{\star} \) to a set \( P \) of SPPs \( p \) whose mapping under \( p \) is implied by the mapping of \( P \) under \( p \). Thus from a computer science perspective [5, Section 7.35], PPPs are specifications of SPPs and SPPs are implementations or refinements of PPPs. The Galois-connection then connects correct implementations to their respective specification by stipulating that a correct implementation imply its specification. By convention, \( \land \emptyset = \top \) and \( \lor \emptyset = \bot \), that is, the conjunction over the empty set \( \emptyset \) is tautological truth \( \top \), and the disjunction over \( \emptyset \) is tautological falsehood \( \bot \), respectively.

Note that an example of an SPP that maps to the empty set under \( \check{\star} \) happens to be the Szondi norm profile mentioned before, because its mapping under \( p \)

\[
p(((h, +), (s, +), (e, -), (hy, -), (k, -), (p, -), (d, +), (m, +))) = h+ \land s+ \land e- \land hy- \land k- \land p- \land d+ \land m+,
\]

does not meet our translation of Cattell’s personality trait B, G, H, M, Q3, PS, ST, LE, SR, PI, OT, AP, TS, nor T1, as can seen by inspecting Table 2.

As can also be seen in Table 2 our interpretation of Cattell’s scale is mostly the following: Cattell’s value 1 becomes Szondi’s signature \(-!!\), 2 becomes \(-!\), 3 and 4 become \(-\), 5 and 6 become 0, 7 and 8 become \(+\), 9 becomes \(+!\), and 10 becomes \(+!!\). This corresponds to how Szondi accounts for the corresponding number of portrait choices of the same kind in his test [9]: the low range 1–5 corresponds to the numbers 1–5 of antipathy choices (portrait dislikes), respectively, and the high range 6–10 to the numbers 1–5 of sympathy choices (portrait likes), respectively. Of course, our readers may experiment with their own interpretation and accounting. For example, they might want to take into
account also Szondi’s signatures —!!! and +!!! for pathologically strong, unambiguous negative and positive choices, respectively, and adapt the scale accordingly. Szondi’s signatures — and + account for normally strong, unambiguous negative and positive choices, respectively. Szondi’s test also allows for ambiguous sets of (portrait) choices (noted—“signed” in Szondi’s terminology—as ±, ±!, and ±`). This ambiguity turns out to be also useful in our translation in Table 2. Observe that (1) in the translation of the low-high range opposition, we have made use of signature opposition (polarity, e.g., h− versus h+); (2) abnormal personality traits translate all into psychological syndromes, that is, conjunctions of signed factors; and (3) any conjunctive low-range translation is the conjunction of the opposed factors of the corresponding high range translation. This last observation makes PPPs appear quite rigid, but is justified by the (natural-language) definition—“descriptors” in Cattell’s terminology—of Cattell’s personality traits [2, Table 7.1], which we recall by annotating them with Szondi’s signed factors (Cattell’s commas correspond to conjunctions here):

1. Reserved [h−], Impersonal [h−], Distant [h−]—Warmth (A)—Warm-hearted [h+], Caring [h+], Attentive To Others [h+];
2. Concrete [k+, having, matter], Lower Mental Capacity [p−, psychological projection, subjectivity]—Reasoning (B)—Abstract [p+, being, ideas], Bright [k−, p+], Fast-Learner [p+, intuition];
3. Reactive [s−, d+], Affected By Feelings [d+, depression]—Emotional Stability (C)—Emotionally Stable [d−], Adaptive Mature [d±];
4. Deferential [s−], Cooperative [s−], Avoids Conflict [s−]—Dominance (E)—Dominant [s+], Forceful [s+], Assertive [s+];
5. Serious [k−], Restrained [k−], Careful [k−]—Liveliness (F)—Enthusiastic [k+], Animated [k+], Spontaneous [k+];
6. Expedient [e−, hy+, k+], Nonconforming [e−, hy+]—Rule-Consciousness (G)—Rule-Conscious [e+, hy−, k−], Dutiful [e+];
7. Shy [hy−], Timid [hy−], Threat-Sensitive [d−]—Social Boldness (H)—Socially Bold [hy+], Venturesome [d+], Thick-Skinned [h0];
8. Tough [h0, hy+, p+], Objective [p+], Unsentimental [h−]—Sensitivity (I)—Sensitive [h+, hy−, p−], Aesthetic [h+], Tender-Minded [h+, p−];
9. Trusting [p+, m+], Unsuspecting [p+], Accepting [k+]—Vigilance (L)—Vigilant [p−], Suspicious [p−], Skeptical [k−], Wary [p−];
10. Practical [p−], Grounded [p−], Down-To-Earth [p−]—Abstractedness (M)—Abstracted [p+], Imaginative [p+], Idea-Oriented [p+];
11. Forthright [hy+], Genuine [hy+], Artless [hy+]—Privateness (N)—Private [hy−], Discreet [hy−], Non-Disclosing [hy−];
12. Self-Assured \([p+],\) Unworried \([p+],\) Complacent \([p+]—\)Apprehension \((O)—\)Apprehensive \([p−],\) Self-Doubting \([p−],\) Worried \([p−];\)

13. Traditional \([d−],\) Attached To Familiar \([d−]—\)Openness to Change \((Q1)—\)Open To Change \([d+],\) Experimenting \([d+];\)

14. Group-Oriented \([d+, m+],\) Affiliative \([d+, m+]—\)Self-Reliance \((Q2)—\)Self-Reliant \([d−, m−],\) Solitary \([d−, m−],\) Individualistic \([d−, d−];\)

15. Tolerates Disorder \([k0],\) Unexacting \([k0],\) Flexible \([k0]—\)Perfectionism \((Q3)—\)Perfectionistic \([k±],\) Organized \([k−],\) Self-Disciplined \([k±];\)

16. Relaxed \([e+],\) Placid \([e+],\) Patient \([e+]—\)Tension \((Q4)—\)Tense \([e−],\) High Energy \([e−],\) Driven \([e−].\)

Cattell’s global personality factors (Cattell’s “Big Five”), defined as groups of 16PF primary traits \([2, Table 7.2]\), can then simply be translated as disjunctions of the translations of the corresponding primary traits. That is, for every value \(v \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}:

\[
\text{Extraversion } v = f((A, v)) \lor f((F, v)) \lor f((H, v)) \lor f((N, 10 - v)) \lor f((Q2, 10 - v))
\]

\[
\text{High Anxiety } v = f((C, v)) \lor f((L, v)) \lor f((O, v)) \lor f((Q4, v))
\]

\[
\text{Tough-Mindedness } v = f((A, 10 - v)) \lor f((I, 10 - v)) \lor f((M, v)) \lor f((Q1, v))
\]

\[
\text{Independence } v = f((E, v)) \lor f((H, v)) \lor f((L, 10 - v)) \lor f((Q1, v))
\]

\[
\text{Self-Control } v = f((F, 10 - v)) \lor f((G, v)) \lor f((M, 10 - v)) \lor f((Q3, v))
\]

Like in \([7]\), we now prove in two intermediate steps that the pair \((\bowtie, \triangleleft)\) is indeed a Galois-connection. The first step is the following announced fact, from which the second step, Lemma \([1]\), follows, from which in turn the desired result, Theorem \([1]\), then follows—easily. As announced, all that our readers need to check on their own analog of our mapping \(f\) is that it has the property stated as Fact \([1.1]\). Their own Galois-connection is then automatically induced.

**Fact 1** (Some facts about \(f\) and \(p\)).

1. if \(F \subseteq F'\) then \(f(F') \Rightarrow f(F)\)
2. if \(P \subseteq P'\) then \(p(P) \Rightarrow p(P')\)
3. The function \(p\) but not the function \(f\) is injective, and neither is surjective.

**Proof.** By inspection of Definition \([1]\) and Table \([2]\).

Like in \([7]\), we need Fact \([1.1]\) and \([1.2]\) but not Fact \([1.3]\) in the following development. Therefor, note the two macro-definitions \(\bowtie \circ \triangleleft := \bowtie \circ \triangleleft \circ \bowtie \circ \triangleleft\) and \(\triangleleft \circ \bowtie := \triangleleft \circ \bowtie \circ \triangleleft \circ \bowtie\) with \(\circ\) being function composition, as usual (from right to left, as usual too).

**Lemma 1** (Some useful properties of \(\bowtie\) and \(\triangleleft\)).
1. if \( F \subseteq F' \) then \( F^\prec \subseteq F^\preceq \) \((^\prec \text{ is antitone})\)
2. if \( P \subseteq P' \) then \( P^\preceq \subseteq P^\geq \) \((^\geq \text{ is antitone})\)
3. \( P \subseteq (P^\preceq)^\prec \) \((^\prec \text{ is inflationary})\)
4. \( F \subseteq (F^\preceq)^\geq \) \((^\geq \text{ is inflationary})\)

**Proof.** Like in [7].

We are ready for making the final step.

**Theorem 1** (The Galois-connection property of \((^\prec, ^\preceq)\)). The ordered pair \((^\prec, ^\preceq)\) is an antitone or order-reversing Galois-connection between \(\mathcal{PPP}\) and \(\mathcal{SPP}\). That is, for every \(F \in 2^{\mathcal{PPP}}\) and \(P \in 2^{\mathcal{SPP}}\),

\[ P \subseteq F^\prec \text{ if and only if } F \subseteq P^\preceq. \]

**Proof.** Like in [7].

Thus from a computer science perspective [5 Section 7.35], smaller (larger) sets of PPPs and thus less (more) restrictive specifications correspond to larger (smaller) sets of SPPs and thus more (fewer) possible implementations.

Note that Galois-connections are connected to residuated mappings [1]. Further, natural notions of equivalence on \(\mathcal{PPP}\) and \(\mathcal{SPP}\) are given by the kernels of \(^\prec\) and \(^\preceq\), respectively, which are, by definition:

\[ F \equiv F' \text{ if and only if } F^\prec = F'^\prec; \]
\[ P \equiv P' \text{ if and only if } P^\preceq = P'^\preceq. \]

**Proposition 1** (The computability of \((^\prec, ^\preceq)\)).

1. Given \(F \in 2^{\mathcal{PPP}}\), \(F^\prec\) is computable.
2. Given \(P \in 2^{\mathcal{SPP}}\), \(P^\preceq\) is computable.

**Proof.** Similar to [7], but with the difference that the Galois-connection there is efficiently computable, whereas the one here is only so for small sets \(F\) and \(P\) (which in practice usually are singleton sets of only one personality profile).

3 Conclusion

We have proposed a computable Galois-connection between PsychEval Personality Profiles (including the 16PF Personality Profiles) and Szondi’s personality profiles, as promised in the abstract and as a further illustration of our simple methodology introduced in [7] for generating such Galois-connections.

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