Abstract—In this work, we consider both channel estimation and reflection coefficient design problems in point-to-point reconfigurable intelligent surface (RIS)-aided millimeter-wave (mmWave) MIMO communication systems. First, we show that by exploiting the low-rank nature of mmWave MIMO channels, the received training signals can be written as a low-rank multi-way tensor admitting a canonical polyadic (CP) decomposition. Utilizing such a structure, a tensor-based RIS channel estimation method (termed TenRICE) is proposed, wherein the tensor factor matrices are estimated using an alternating least squares method. Using TenRICE, the transmitter-to-RIS and the RIS-to-receiver channels are efficiently and separately estimated, up to a trivial scaling factor. After that, we formulate the beamforming and RIS reflection coefficient design as a spectral efficiency maximization task. Due to its non-convexity, we propose a heuristic non-iterative two-step method, where the RIS reflection vector is obtained in a closed form using a Frobenius-norm maximization (FroMax) strategy. Our numerical results show that TenRICE has a superior performance, compared to benchmark methods, approaching the Cramér–Rao lower bound with a low training overhead. Moreover, we show that FroMax achieves a comparable performance to benchmark methods with a lower complexity.

Index Terms—Reconfigurable intelligent surface, channel estimation, RIS reflection design, CP tensor decomposition.

I. INTRODUCTION

Reconfigurable intelligent surfaces (RISs) have been proposed recently as a cost-effective technology for reconfiguring the propagation channels in wireless communication systems [1]. An RIS is a 2D surface equipped with a large number of tunable units that can be realized using, e.g., inexpensive antennas or metamaterials and controlled in real-time to influence the communication channels without generating its own signals. Among its many applications, an RIS can be utilized as a solution to the signal-blockage problem in millimeter-wave (mmWave)-based communications by providing alternative and tunable RIS-aided channels.

Recently, RIS-aided communications have attracted great attention, due to their potential of improving the efficiency of wireless mobile communications. RIS reflection design, in particular, have been extensively investigated under various setups and objectives, see [2]–[5] and reference therein. However, due to the non-convexity of the involved problems, relaxations and alternating optimization techniques are commonly used to obtain a locally optimal solution. For example, the authors in [2] considered the capacity maximization and proposed an alternating optimization approach to find a locally optimal solution by iteratively optimizing the transmit covariance matrix or one of the RIS reflection coefficients with the others being fixed. However, such an alternating approach increases the computational complexity and becomes a limiting factor in practice, especially in a massive RIS setup.

The vast majority of the existing works assume perfect channel state information (CSI) at the transceivers, see [2]–[5], which can never be obtained in practice. Recently, RIS-aided channel estimation (CE) methods have been proposed, e.g., in [6]–[9]. These works, however, require that the number of training subframes is, at least, equal to the number of RIS reflection units to obtain an accurate CSI estimate, which increases the training overhead and complexity. To overcome these issues, several approaches have been studied, e.g., by exploiting the low-rank nature of mmWave channels and the multidimensional (i.e., tensor) structure of the received signals. The former allows the CE to be formulated as a sparse-recovery problem and solved using compressed sensing (CS) tools [10]–[12], which are known to require a few measurements to have an accurate estimate, see [13]–[15]. In [13], by exploiting the low-rank nature of the mmWave channels, we have proposed the TRICE framework, which formulates the CE in RIS-aided mmWave MIMO systems as a two-stage multidimensional sparse-recovery problem. On the other hand, tensor-based signal modeling and processing methods offer fundamental advantages over their bilinear (matrix) counterparts, since they have the ability to improve the identifiability of the parameters due to the powerful uniqueness properties of tensor decompositions [16]. In [17], it is shown that the received signals in RIS-aided MIMO communication systems can be written as a 3-way tensor admitting a canonical polyadic (CP) decomposition. However, the proposed method in [17] assumes sub-6 GHz systems and, thus, requires a large number of training subframes, similarly to [6]–[9].

In this paper, we extend our TRICE framework in [13] and propose a CP Tensor decomposition method for RIS-aided CE in mmWave MIMO systems, termed TenRICE, by jointly exploiting the tensor structure of the received signals and the low-rank nature of mmWave channels. Using

Title: Tensor-Based Channel Estimation and Reflection Design for RIS-Aided Millimeter-Wave MIMO Communication Systems

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the TenRICE method, the transmitter-to-RIS and the RIS-to-
receiver channels can be estimated separately, up to a trivial
scaling factor. After that, we formulate the beamforming and
the RIS reflection coefficient design as a spectral efficiency
(SE) maximization problem. Due to its non-convexity, we
propose a heuristic non-iterative two-step solution, where the
RIS reflection vector is obtained, in contrast to [2], in a
closed form using a Frobenius-norm Maximization (FroMax)
strategy. Our numerical results show that TenRICE has a
superior performance, compared to the TRICE framework,
approaching the Cramér–Rao bound (CRB). Moreover, we
show that FroMax achieves a comparable performance to
benchmark methods with a lower complexity.

II. SYSTEM MODEL

In this paper[1] we consider an RIS-aided mmWave MIMO
communication system as depicted in Fig. 1 where a transmitt-
er (TX) with $M_T$ antennas is communicating with a receiver
(RX) with $M_R$ antennas via an RIS-aided MIMO channel.
The direct channel between the TX and the RX is assumed
unavailable or too weak, e.g., due to blockage. The RIS
has $M_S$ inexpensive reflecting elements arranged uniformly
with half-wavelength inter-element spacing on a rectangular
surface with $M_S^v$ vertical and $M_S^h$ horizontal elements such
that $M_S = M_S^v \cdot M_S^h$.

Let $H_T \in \mathbb{C}^{M_T \times M_S}$ be the TX to RIS channel and $H_R \in \mathbb{C}^{M_S \times M_R}$ be the RIS to RX channel with $\mathbb{E}\{\|H_T\|^2\} = M_S M_T$
and $\mathbb{E}\{\|H_R\|^2\} = M_S M_R$. We assume a block-fading channel
scenario, where $H_T$ and $H_R$ remain constant during every
channel coherence block and change from block to block. We
assume that every block is divided into two sub-blocks: one
for CE and another for data transmission (DT), see Fig. 2.

In the CE phase, we conduct a channel training procedure
that occupies $K = K_T \cdot K_S$ subframes. The received signal at
the RX at the $(s, t)^{th}$ subframe is given as

$$y_{s, t} = W^H H_R \text{diag}(\phi_s) H_T f_{s, t} s_t + W^H z_{s, t} \in \mathbb{C}^{K_S},$$

where $W \in \mathbb{C}^{M_S \times K_S}$ is a fixed training decoding matrix
with $K_S$ beams, $f_{s, t} \in \mathbb{C}^{M_T}$ is the $t^{th}$ training vector of the
TX with $\|f_{s, t}\|_2^2 = 1$, $t \in \{1, \ldots, K_T\}$, $\phi_s \in \mathbb{C}^{M_S}$ is the
$s^{th}$ training vector of the RIS with $\|\phi_s\|_m = \frac{1}{\sqrt{M_S}}$, $s \in \{1, \ldots, K_S\}$, $s_t \in \mathbb{C}$ is the unit-norm pilot symbol,
and $z_{s, t} \in \mathbb{C}^{M_S}$ is the additive white Gaussian noise vector

1Notation: The transpose, the conjugate transpose (Hermitian), the Moore-
Penrose pseudoinverse, the Kronecker product, and the Khati-Rao product
are denoted as $A^T$, $A^H$, $A^\dagger$, $A \otimes$, and $A \circ$, respectively. Moreover, $1_N$ is
the all ones vector of length $N$, $I_N$ is the $N \times N$ identity matrix, diag$(\alpha)$ forms a
diagonal matrix $A$ by putting the entries of the input vector $a$ in its main
diagonal, diag$(A)$ is the reverse of the diagonal operator, vec$(A)$ forms a
vector by stacking the columns of $A$ over each other, and the $n$-mode product
of a tensor $A \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$ with a matrix $B \in \mathbb{C}^{J \times I_N}$ is denoted as
$A \ast_{n} B$. Throughout this paper, we assume that the singular values of a
given diagonal singular matrix are arranged in a decreasing order. Moreover, the
following properties are used: Property 1: vec$(ABC) = (C^H \otimes A)$vec$(B)$,
Property 2: $AB \odot CD = (A \otimes C)(B \otimes D)$, Property 3: $(A \otimes C)(B \otimes D) = AB \odot CD$. Property 4: Let $A_1 \in \mathbb{C}^{I_1 \times L_1}$ and $A_2 \in \mathbb{C}^{I_2 \times L_2}$. Then
$A_1 \otimes A_2 = A_1 \Omega_1 \otimes A_2 \Omega_2$, where $\Omega_1 = I_{L_1} \otimes 1_{I_2}$ and $\Omega_2 = 1_{L_2} \otimes I_{I_2}$
so that $\Omega_1 \otimes \Omega_2 = I_{L_1 L_2}$. Property 5: vec$(A \text{diag}([b] C)) = (C^T \circ A)[b]$.

having zero-mean circularly symmetric complex-valued entries
with variance $\sigma^2$. We stack $\{y_{s, t}\}_{s=1}^{K_S}$ on top of each other as
$y_s = [y_{s, 1}^T, \ldots, y_{s, K_T}^T]^T$ and after that we stack $\{y_s\}_{s=1}^{K_S}$ next
to each other as $Y = [y_1, \ldots, y_{K_S}]$. Then, using Properties 2 and
5, the above measurement matrix $Y$ can be written as

$$Y = (F^T \otimes W^H) H_C \Phi + Z \in \mathbb{C}^{K_S \times K_T \times K_S},$$

where $H_c = H_T^H \circ H_R$ represents the cascaded channel matrix,
$Z = [z_1, \ldots, z_{K_S}]$, $z_s = [(W^H z_{s, 1})^T, \ldots, (W^H z_{s, K_T})^T]^T$, $F = [f_1, \ldots, f_{K_T} s_{K_S}]$, and $\Phi = [\phi_1, \ldots, \phi_{K_S}]$. Given the
measurement matrix $Y$, the main goal of Section III is to
obtain an accurate estimate of $H_T$ and $H_R$, while keeping the
number of training subframes $K$ as small as possible.

In the DT phase, given the estimated channels $\hat{H}_R$ and
$\hat{H}_T$, the TX first designs the precoding matrix $P \in \mathbb{C}^{M_T \times N_i}$,
the decoding matrix $Q \in \mathbb{C}^{M_R \times N_i}$, and the RIS reflection
coefficient vector $\omega \in \mathbb{C}^{M_S}$ with $|\omega|_m = \frac{1}{\sqrt{M_S}}$, $\forall m$, to
transmit the vector $s \in \mathbb{C}^{N_i}$, of $N_i$ data streams with $\mathbb{E}[ss^H] = I_{N_i}$
to the RX. Therefore, the received signal vector at the RX is
given as

$$y = Q^H H_c P s + Q^H z \in \mathbb{C}^{N_i},$$

where $H_c = H_R \text{diag}(\omega) H_T$ is the effective channel matrix.
The system SE is given as

$$SE = \log_2 \det(I_{N_i} + R^{-1} Q^H H_c P P^H H_c^H Q),$$

where $R = \sigma^2 Q^H Q$ is the noise covariance matrix. In
Section IV we propose a non-iterative beamforming and
RIS reflection coefficient design method to maximize the SE,
where the RIS reflection vector is obtained in a closed form
using a FroMax strategy.

Channel model: In mmWave-based communications [18],
it was observed that the number of paths $L_T$ and $L_R$ for $H_T$
and $H_R$ respectively, are very small compared to the number
of antenna elements. This implies that rank$(H_T) \leq L_T$
and rank$(H_R) \leq L_R$. Therefore, similarly to [13], by assuming
that the TX and the RX employ uniform linear arrays (ULAs)

2The extension of the proposed methods to scenarios where the TX
and/or the RX are equipped with uniform rectangular arrays (URAs) is
straightforward.
where $g_{\ell,\ell} \sim \mathcal{CN}(0,1)$ is the $\ell$th path gain, $\psi_{\ell,\ell} \in [0, 2\pi]$ is the $\ell$th direction-of-arrival (DoA) spatial frequency from the TX, $\psi_{\ell,\ell} \in [0, 2\pi]$ is the $\ell$th direction-of-arrival (DoA) spatial frequency at the RX, $\mu^h_{\ell,\ell} \in [0, 2\pi]$ and $\mu^v_{\ell,\ell} \in [0, 2\pi]$ are the $\ell$th horizontal and vertical DoA spatial frequencies from the RIS. In (5), the 1D and 2D array steering vectors are given as $v_{1D}(\nu) = [1, e^{j\nu}, \ldots, e^{j(M-1)\nu}]^T \in \mathbb{C}^M$ and $v_{2D}(\nu, \psi) = v_{1D}(\nu) \odot v_{1D}(\psi)$, respectively, where $v_{1D}(\nu) \in \mathbb{C}^M$ and $v_{1D}(\psi) \in \mathbb{C}^M$. Moreover, $H_T$ and $H_R$ are in a form of diagonal matrices by letting $A_X = [v_{1D}(\psi_x,1), \ldots, v_{1D}(\psi_x,L)], B_X = [v_{1D}(\psi_y,1), \ldots, v_{1D}(\psi_y,L)], C_X = [v_{1D}(\nu_x,1), \ldots, v_{1D}(\nu_x,L)],$ and $G_X = \frac{1}{\sqrt{v_{\ell,\ell}}} \text{diag}\{g_{\ell,1}, \ldots, g_{\ell,L}\}$ for $X \in \{T, R\}, Y \in \{v, h\}$.

### III. Phase 1: The Proposed CE Method (TenRICE)

In this section, we propose our Tensor-based RIS-aided CE (TenRICE) algorithm by jointly exploiting the low-rank nature of mmWave channels and the tensor structure of received signals. By utilizing the channel model in (5), the cascaded channel matrix $H_c = H_T \odot H_R$ in (2) can be rewritten as

$$H_c = (A_T G_T B_T^T \odot A_R G_R B_R^T) \equiv (A_T \odot A_R) G_B,$$

where $G = G_T \otimes G_R \in \mathbb{C}^{L \times L}$, $B = B_T \otimes B_R \in \mathbb{C}^{L \times M_h}$, $L = L_T \cdot L_R$, and $\equiv$ is obtained from Property 2. In [13], we have shown that $B$ can be expressed as $B = [B_{\psi} \odot B_{\nu}]^T$, where $B_{\psi} = [v_{1D}(\mu_x), \ldots, v_{1D}(\mu_y)] \in \mathbb{C}^{M \times L}, B_{\nu} = [v_{1D}(\mu^h_x), \ldots, v_{1D}(\mu^h_y)] \in \mathbb{C}^{M \times L}, \mu_x = \mu^v_{\ell,\ell} + \mu^h_{\ell,\ell},$ and $n = (L-1) \cdot L_R + k \in \{1, \ldots, L\}$. Then, using Property 2, (6) can be rewritten as

$$H_c = (A_T \odot A_R) G(B_{\psi} \odot B_{\nu})^T,$$

which is characterized by the following spatial frequency vectors: $\psi_{\ell} = [\psi_{\ell,1}, \ldots, \psi_{\ell,L}]^T$, $\psi_{\ell} = [\psi_{\ell,1}, \ldots, \psi_{\ell,L}]^T$, $\mu^h_{\ell,\ell} = [\mu^h_{\ell,1}, \ldots, \mu^h_{\ell,L}]^T$, and $\mu^v_{\ell,\ell} = [\mu^v_{\ell,1}, \ldots, \mu^v_{\ell,L}]^T$ that define $A_T$, $A_R$, $B_{\psi}$, and $B_{\nu}$ respectively. Therefore, to obtain an estimate of $H_c$, it is sufficient to obtain an estimate of the above vectors from the measurement matrix $Y$ in (2), including the path gain vector $g = \text{undag}(G)$. In [13], we have proposed a two-stage framework, termed TRICE, which estimates $\psi_{\ell}$ and $\psi_{\ell}$ in the first stage as well as $\mu^h_{\ell,\ell}$, $\mu^v_{\ell,\ell}$, and $g$ in the second stage using any efficient multidimensional sparse-recovery technique, like CS [12] and ESPRIT [19]. To further improve the performance of the TRICE framework, we propose in the following the TenRICE method by exploiting the tensor structure of the measurement matrix $Y$.

We assume that the RIS reflection coefficient matrix during the training phase has a Kronecker structure given as $\Phi = \Phi_{\psi} \otimes \Phi_{\nu}$, where $\Phi_{\psi} \in \mathbb{C}^{M^\psi \times K^\psi}$, $\Phi_{\nu} \in \mathbb{C}^{M^\nu \times K^\nu}$, and $K^\psi \cdot K^\nu = K$. By substituting (6) into (2), the vectorized form of $Y$, i.e., $y = \text{vec}(Y)$ can be written as

$$y = \text{vec}\{F^T A_T \otimes W^H A_R [G(B_{\psi} \odot B_{\nu})^T \Phi] + z$$

$$\equiv \text{vec}\{(F^T A_T \Omega_T \otimes W^H A_R \Omega_R [G(B_{\psi} \odot B_{\nu})^T \Phi] + z$$

$$\equiv \text{vec}\{(F^T A_T \Omega_T \otimes W^H A_R \Omega_R [G(B_{\psi} \odot B_{\nu})^T \Phi] + z,$$

where $z = \text{vec}(Z)$ and $g = \text{undag}(G)$. Moreover, $\odot$, $\equiv$, and $\equiv$ are obtained by applying Properties 1, 2, and 4, respectively. From (8), we observe that $y$ is the vectorized form of the transposed 4-mode unfoldings of a 4-way tensor $Y \in \mathbb{C}^{K^\psi \times K^\nu \times K^\psi \times K^\nu}$, i.e., $y = [Y]^T$ that admits a constrained CP decomposition as

$$Y = Y_{4,L} \times_{1} A_R \Omega_{R} \times_{2} A_T \Omega_{T} \times_{3} B_{\psi} \times_{4} B_{\nu} + Z,$$

where $Z$ is the noise tensor, $T_{4,L} \in \mathbb{C}^{L \times L \times L \times L}$ is a super-diagonal tensor with ones on the super diagonal, and

$$A_R = W^H A_R = W^H[v_{1D}(\psi_{\ell,1}), \ldots, v_{1D}(\psi_{\ell,L})],$$

$$A_T = F^T A_T = F^T[v_{1D}(\psi_{\ell,1}), \ldots, v_{1D}(\psi_{\ell,L})],$$

$$B_{\psi} = \Phi_{\psi}^T B_{\psi} \equiv [v_{1D}(\mu^h_{\ell,1}), \ldots, v_{1D}(\mu^h_{\ell,L})]$$

The n-mode unfoldings of tensor $Y$, for $n \in \{1, 2, 3, 4\}$ can be expressed as

$$[Y]_{(1)} = A_R \Omega_{R} (B_{\psi} \odot B_{\psi} \odot A_T \Omega_{T})^T + [Z]_{(1)},$$

$$[Y]_{(2)} = A_T \Omega_{T} (B_{\psi} \odot B_{\psi} \odot A_R \Omega_{R})^T + [Z]_{(2)},$$

$$[Y]_{(3)} = B_{\psi} (B_{\psi} \odot A_T \Omega_{T} \odot A_R \Omega_{R})^T + [Z]_{(3)},$$

$$[Y]_{(4)} = B_{\psi} (B_{\psi} \odot A_T \Omega_{T} \odot A_R \Omega_{R})^T + [Z]_{(4)}.$$
Algorithm 1 Tensor-based RIS-aided CE (TenRICE)

1: Input: Measurement tensor $\mathbf{Y} \in \mathbb{C}^{R \times K \times K \times L \times L}$ and $I_{\text{max}}$
2: Output: Estimated channels $\hat{H}_R$ and $\hat{H}_T$
3: Initialization: $\hat{B}^{(0)}_R$, $\hat{B}^{(0)}_h$, and $\hat{A}_T$, e.g., randomly
4: while not converged or $i < I_{\text{max}}$ do
5: $\hat{A}^{(i)}_R = [\gamma]_1 \left[ \Omega_R (\hat{B}^{(i-1)}_h \odot \hat{B}^{(i-1)}_h \odot \hat{A}^{(i-1)}_T \Omega_T) \right]^T$
6: $\hat{A}^{(i)}_T = [\gamma]_2 \left[ \Omega_T (\hat{B}^{(i-1)}_h \odot \hat{B}^{(i-1)}_h \odot \hat{A}^{(i-1)}_R \Omega_R) \right]^T$
7: $\hat{B}^{(i)}_h = [\gamma]_3 \left[ \hat{B}^{(i-1)}_h \odot \hat{A}^{(i)}_T \Omega_T \odot \hat{A}^{(i)}_R \Omega_R \right]^T$
8: $\hat{B}^{(i)}_h = [\gamma]_4 \left[ \hat{B}^{(i-1)}_h \odot \hat{A}^{(i)}_T \Omega_T \odot \hat{A}^{(i)}_R \Omega_R \right]^T$
9: end while
10: Recover $\hat{\psi}_R, \hat{\psi}_T, \hat{\mu}^h, \hat{\mu}^v$ using, e.g., [19] or NOMP [25]
11: Compute $\hat{g} = \hat{\Phi} \hat{B}_h \odot \hat{B}_h \odot \hat{F}^T \hat{A}_T \Omega_T \odot \hat{W}^H \hat{A}_R \Omega_R \hat{y}$
12: Reconstruct $\hat{H}_R = (\hat{A}_R \odot \hat{A}_T) \text{diag} (\hat{g}) (\hat{B}_h \odot \hat{B}_h)^T$
13: Estimate $\hat{H}_R$ and $\hat{H}_T$ from $\hat{H}$ using [17, Algorithm 1]

example, the $k$th entry of $\psi_R$, i.e., $\psi_{R,k}$ associated with the $k$th column vector of $\hat{A}_R$, i.e., $\hat{a}_{R,k}$ can be recovered as

$$\hat{\psi}_{R,k} = \arg \max_{\psi \in [0,2\pi]} \left| \mathbf{a}_{R,k}^H \mathbf{W}^H \mathbf{P}_{ID}(\psi) \mathbf{a}_{R,k} \right|,$$  \hspace{1cm} (19)

which can be efficiently implemented by first employing a coarse grid and then gradually refining it around the maximizing grid points. Alternatively, [19] can be interpreted as an off-grid sparse recovery problem, where efficient methods like, Newtonized OMP (NOMP) [25] can be readily applied to recover $\hat{\psi}_{R,k}$ with high accuracy and low complexity. A similar approach can be used to recover the vectors $\hat{\psi}_T$, $\hat{\mu}^h$, and $\hat{\mu}^v$ from $\hat{A}_T$, $\hat{B}_h$, and $\hat{B}_h$, respectively.

Next, using the estimated vectors $\hat{\psi}_R, \hat{\psi}_T, \hat{\mu}^h$, and $\hat{\mu}^v$ in step 10 we reconstruct $\hat{A}_R, \hat{A}_T, \hat{B}_h$, and $\hat{B}_h$, Then, the path gain vector $\hat{g}$ can be estimated from (8) (or $[\gamma]_4$) using a LS method as shown by step 11. Finally, the cascaded channel matrix $\hat{H}_c$ can be reconstructed as in step 12, which can be used to estimate $\hat{H}_T$ and $\hat{H}_R$, up to trivial scaling factors, using the LS Khatari-Rao factorization (LSKRF) method [17].

Uniqueness and identifiability conditions: It is well known that the CP decomposition is unique up to scaling and permutation ambiguities under mild conditions [24], [26]–[29]. In general, the uniqueness of a CP decomposition is guaranteed by Kruskal’s condition [27], which is also known as the k-rank. However, due to the definitions of $\Omega_R$ and $\Omega_T$, the first two factor matrices, i.e., $\hat{A}_R \Omega_R = \hat{A}_R$ and $\hat{A}_T \Omega_T = \hat{A}_T$ contain repeated columns, where every column of $\hat{A}_R$ is repeated $L_R$ times and every column of $\hat{A}_T$ is repeated $L_T$ times. This implies that the $k$-rank of $\hat{A}_R$ and $\hat{A}_T$ is equal to one. Therefore, the sufficient condition of [27] fails [29]. As for Algorithm 1 which is an ALS-based algorithm, the identifiability in the LS sense requires that each of the following matrices: $\hat{C}_R = \Omega_R (\hat{B}_h \odot \hat{B}_h \odot \hat{A}_T \Omega_T)^T \in \mathbb{C}^{L_R \times L_R}$, $\hat{C}_T = \Omega_T (\hat{B}_h \odot \hat{B}_h \odot \hat{A}_R \Omega_R)^T \in \mathbb{C}^{L_T \times L_T}$, $\hat{C}_h = (\hat{B}_h \odot \hat{A}_T \Omega_T \odot \hat{A}_R \Omega_R)^T \in \mathbb{C}^{L_R \times L_T}$, and $\hat{C}_v = (\hat{B}_h \odot \hat{A}_T \Omega_T \odot \hat{A}_R \Omega_R)^T \in \mathbb{C}^{L_R \times L_T}$ to have a unique right Moore-Penrose pseudo-inverse, i.e., full row-rank, where $J_R = K_T K_R$, $J_T = K_R K_T$, $J_h = K_R K_T K_h$, and $J_v = K_R K_T K_h$. This requires that $J_R \geq L_R$, $J_T \geq L_T$, $J_h \geq L$, and $J_v \geq L$, where $L = L_R \cdot L_T$. Since $L_R$ and $L_T$ are practically very small (i.e., $\max \{L_R, L_T\} \approx 3$ [18]), the above conditions are easily satisfied. For example, assuming that the TX is in-line-of-sight with the RIS, we have that $L_T = 1$, as it has been assumed in [8].

Complexity analysis: Assuming that the complexity of calculating the Moore-Penrose pseudo-inverse of a $n \times m$ matrix is on the order of $O(\min(n,m))^3$. Then, the complexity of the ALS steps in Alg. 1 is on the order of $O(I_{\text{max}} (L_R^3 + L_T^3 + 2L^3))$. Moreover, assuming that the NOMP method from [25] is used in step 10 then the complexity of recovering the channel parameters is on the order of $L (L_R + L_T + 2L)$, where $L$ denotes the number of grid points used by NOMP in the sparse-coding stage. In comparison, the complexity of TRICE-SC [13] is on the order of $O(L L_R K_T (L_T^2 + L^2) + 2L^3 + L K_R K_T L_T^2)$ and the Joint-CS method [14] is on the order of $O(L (N_R K_T K_S (L^3 + L + L^2) + L^3)$). Clearly, TenRICE has a much lower complexity compared to both methods. The main reason is that TRICE and Joint-CS require multidimensional (xD) dictionaries (2D for TRICE and 4D for Joint-CS) compared to the 1D dictionary required by TenRICE. Moreover, in contrast to the TenRICE, TRICE and Joint-CS methods require a dictionary orthogonalization operation during the parameter recovery [30], which is very complex especially with large dictionaries.

IV. PHASE 2: THE PROPOSED RIS REFLECTION DESIGN METHOD (FroMAX)

In this section, given the estimated channels $\hat{H}_R$ and $\hat{H}_T$, we design the TX and the RX beamforming matrices and the RIS reflection coefficient vector as a solution to the following SE maximization problem:

$$\max_{Q, \mathbf{P}, \omega} \log_2 \det (\mathbf{I}_N + R^{-1} Q^H \hat{H}_c \mathbf{P} \mathbf{P}^H \hat{H}_c^H Q)$$

s.t. $\| \mathbf{P} \|^2_F \leq P_{\text{max}}$ and $| \omega_m | = 1/\sqrt{M_S}, \forall m,$

where $\hat{H}_c = \tilde{H}_R \text{diag} (\omega) \tilde{H}_T$ and $P_{\text{max}}$ is the transmit power at the TX. Note that (20) is non-convex, since the objective function is non-concave with $\omega$ and the constant modulus constraints are non-convex functions. Moreover, $\mathbf{P}, \mathbf{Q},$ and $\omega$ depend on each other, which makes (20) a difficult problem to solve. In the following, we propose a non-iterative solution to (20), which has a comparable performance to that of [2], but with much lower complexity.

Initially, it is not hard to see that for any given $\omega$, (20) reduces to a single-user multi-stream MIMO communication system. Let $\hat{H}_c = U \Sigma_{\mathbf{H}} \hat{V}_{\mathbf{H}}$, be the singular value decomposition (SVD) of $\hat{H}_c$. Then, the optimal fully-digital
solutions to $Q$ and $P$, for fixed $\omega$, are given as
\begin{equation}
Q = U_s \text{ and } P = V_s \text{diag}\{\sqrt{p_1}, \ldots, \sqrt{p_{N_s}}\},
\end{equation}
where $U_s = [U_{\hat{H}_T}]_{[1:1:N_s]}$, $V_s = [V_{\hat{H}_R}]_{[1:1:N_s]}$, and $(p_i)_{i=1}^{N_s}$ are the power allocations found using the waterfilling method. Thus, $\sum_{i=1}^{N_s} p_i = P_{\text{max}}$. Consequently, $Q^H Q = I_{N_s}$.

\[ \Sigma_s = U_s^H \hat{H}_R \text{diag}(\omega) \hat{H}_T V_s = \text{diag}(\alpha_1, \ldots, \alpha_{N_s}) \text{, and the SE expression in (4) simplifies to} \]
\begin{equation}
SE = \sum_{i=1}^{N_s} \log_2(1 + \frac{1}{\sigma^2} \alpha_i^2 p_i),
\end{equation}
where $\alpha_i$ is the $i$th dominant singular value in $\Sigma_s$. In the following, we turn our attention to the RIS reflection coefficient design and propose an efficient non-iterative solution to find $\omega$ based on a FroMax design strategy.

**FroMax-1**: As a baseline method, the RIS reflection vector is found as a solution to
\begin{equation}
\omega = \arg\max_{\omega} \| \hat{H}_R \text{diag}(\omega) \hat{H}_T \|_F^2 = \arg\max_{\omega} \| K \omega \|_2^2,
\end{equation}
subject to $\| \omega \|_m = 1/\sqrt{M_s}, \forall m$,

where $K = \hat{H}_T^H \hat{H}_R$ is obtained by applying Property 1. Note that (24) is non-convex due to the constant modulus constraints. Therefore, we first seek a solution to the following relaxed and convex version of (24) given as
\begin{equation}
\hat{\omega} = \arg\max_{\omega} \| K \omega \|_2^2, \text{ s.t. } \| \omega \|_2 = 1.
\end{equation}

Let $K = U_K \Sigma_K V_K^H$ be the SVD of $K$. Then, the optimal solution to (24) is given as $\hat{\omega} = [V_K]_{[1:1]}$. To satisfy the constant modulus constraints of (23), we use a simple projection function, where the $m$th entry of $\omega$ is given as
\begin{equation}
[\omega_{\text{FroMax-1}}]_m = \frac{1}{\sqrt{M_s}} \cdot \left(\frac{[\hat{\omega}]_m}{[\omega]_m}\right), \forall m.
\end{equation}

However, using computer simulations, we have observed that FroMax-1 mainly maximizes the dominant singular value of $\hat{H}_c$, which makes it limited to single-stream scenarios.

**FroMax-2**: From (22), we can clearly see that $\omega$ should be designed so that the singular values $\alpha_i$ are maximized. Thus, we propose to modify (23) as
\begin{equation}
\omega = \arg\max_{\omega} \| \Sigma_s \|_F^2 = \arg\max_{\omega} \| D \omega \|_2^2
\end{equation}
subject to $\| \omega \|_m = 1/\sqrt{M_s}, \forall m$,

where $D$, due to the diagonal structure of $\Sigma_s$, is given as
\begin{equation}
D \triangleq \begin{bmatrix}
[V_s]^H_{[1:1]} \hat{H}_T \hat{H}_R \\
\vdots \\
[V_s]^H_{[N_s:1]} \hat{H}_T \hat{H}_R
\end{bmatrix} \in \mathbb{C}^{N_s \times M_s}.
\end{equation}

Similarly to (24), (26) can be relaxed to a convex form as
\begin{equation}
\hat{\omega} = \arg\max_{\omega} \| D \omega \|_2^2, \text{ s.t. } \| \omega \|_2 = 1.
\end{equation}

However, differently from (24), we propose a solution that achieves a higher SE, where $\omega$ is obtained by taking the contributions of the dominant $N_r$ right singular vectors of $D$. Specifically, let $D = U_D \Sigma_D V_D^H$ be the SVD of $D$. Then, the proposed solution is given as $\omega = [V_D]_{[1:1]}$. Using $\omega$, the RIS reflection vector $\omega$ is obtained as
\begin{equation}
[\omega_{\text{FroMax-2}}]_m = \frac{1}{\sqrt{M_s}} \cdot \left(\frac{[\hat{\omega}]_m}{[\omega]_m}\right), \forall m.
\end{equation}

**Remark 1**: From (27), it is clear that the unitary matrices $U_s$ and $V_s$ are required to construct $D$. However, since $U_s$ and $V_s$ depend on $\omega$, an iterative two-step algorithm is required, where we update $U_s$ and $V_s$ in one step and $\omega$ in the other step. However, we found that if $U_s$ and $V_s$ are appropriately initialized, then one iteration of such an algorithm is sufficient to have a comparable SE performance to that obtained by the iterative method of (2). Here, we propose to initialize $U_s$ and $V_s$ as follows. Let $\hat{H}_R = U_{\hat{H}_R} \Sigma_{\hat{H}_R} V_{\hat{H}_R}^H$ and $\hat{H}_T = U_{\hat{H}_T} \Sigma_{\hat{H}_T} V_{\hat{H}_T}^H$ be the SVD of $\hat{H}_R$ and $\hat{H}_T$, respectively. Then, we assume that $U_s$ and $V_s$ in (26) are given as $U_s = [U_{\hat{H}_R}]_{[1:1:N_s]}$ and $V_s = [V_{\hat{H}_T}]_{[1:1:N_s]}$.

In summary, the proposed beamforming and RIS reflection coefficient design method is summarized in Algorithm 2.

**Algorithm 2** FroMax-based methods for RIS reflection design.

1. Input: $\hat{H}_T$, $\hat{H}_R$, and $P_{\text{max}}$
2. if FroMax-1 based method then
3. Construct $K$ as in (23) and get $\omega$ from $V_K$
4. Obtain $\omega^* \leftarrow \omega_{\text{FroMax-1}}$ using (25)
5. else if FroMax-2 based method then
6. Compute $U_s = [U_{\hat{H}_R}]_{[1:1:N_s]}$ and $V_s = [V_{\hat{H}_T}]_{[1:1:N_s]$
7. Construct $D$ as in (27) and get $\omega$ from $V_D$
8. Obtain $\omega^* \leftarrow \omega_{\text{FroMax-2}}$ using (29)
9. end if
10. For given $\omega^*$, obtain $Q$ and $P$ as in (21)

**Complexity analysis**: Let the complexity of calculating the SVD of a $n \times m$ matrix on the order of $O(nm^2)$. Then, the complexity of Algorithm 2 steps 3, 6, 7, and 10 is on the order of $O(M_R M_T M_s^2)$, $O(M_R M_s^2 + M_s M_T^2)$, $O(N_s M_s^2)$, and $O(M_R M_T^2)$ respectively. Accordingly, the complexity of FroMax-1 is on the order of $O(M_R M_T M_s^2 + M_R M_T^2)$ and of FroMax-2 is on the order of $O(M_R M_s^2 + M_s M_T^2 + N_s M_s^2 + M_R M_T^2)$. In comparison, the complexity of the alternation maximization (AltMax) method of (2) is on the order of $O(J_{\text{max}}(3M_s^3 + 2M_R^2 M_T + M_s M_T^2 + M_R M_T^2))$, where $J_{\text{max}}$ is the maximum number of iterations.

**V. Numerical Results**

In this section, we show simulation results to evaluate the effectiveness of the proposed methods. In all simulation results, we assume that $M_T = 64$, $M_R = 16$, and $M_s = 32$, i.e., the RIS has $M_s = 256$ reflecting elements.
Fig. 3. MSE vs. SNR $[L_T = L_R = 2]$.

Fig. 4. NMSE vs. SNR.

Fig. 5. SE vs. SNR $[L_T = L_R = 2]$. Phase 1: $K_R = K_T = K_S^1 = K_S^2 = 8$]

**Phase 1 - CE:** In the CE phase, we assume that the training matrices $W$, $F$, $\Phi^h$, and $\Phi^v$ in (2) are randomly generated such that the $(i,j)$th entry of $W$ is given as $[W]_{i,j} = \frac{1}{\sqrt{M_R}} e^{j\varphi_{i,j}}$, where $\varphi_{i,j} \in [0, 2\pi]$, and $F$, $\Phi^h$, and $\Phi^v$ are similarly generated. We show results in terms of the mean-squared error (MSE) of $\psi_R$ defined as $\text{MSE}(\psi_R) = \mathbb{E}\{\|\psi_R - \psi_R\|^2\}$, where $\text{MSE}(\psi_T)$, $\text{MSE}(\mu^h)$, and $\text{MSE}(\mu^v)$ are similarly defined, and the normalized MSE (NMSE) of the cascaded channel is defined as $\text{NMSE} = \mathbb{E}\{\|H - \bar{H}\|^2\}/\mathbb{E}\{\|H\|^2\}$. We define the signal-to-noise ratio (SNR) as $\text{SNR} = \frac{\mathbb{E}\{\|\mathbf{Y} - \mathbf{Z}\|^2\}}{\mathbb{E}\{\|\mathbf{Z}\|^2\}}$. For comparison, we include simulation results of the two-stage TRICE-CS framework [13], where the estimation is performed using the classical OMP technique assuming a 2D dictionary of $128 \times 128$ grid points in both stages.

Figs. 3 and 4 show the MSE versus the SNR and the NMSE versus the SNR results, respectively, averaged over 1,000 channel realizations. From Fig. 3, we can see that TenRICE provides more accurate parameter estimates, compared to TRICE-CS, approaching the CRB as the SNR increases. The main reason is that TenRICE not only exploits the low-rank nature of mmWave channels, but also the tensor structure of the received signals when estimating the channel parameters. Moreover, TenRICE employs a high-resolution parameter recovery method in NOMP, while TRICE-CS suffers from quantization errors, due to the on-grid assumption. These advantages lead to more accurate channel estimates, as can be seen from Fig. 4 with less training overhead and lower complexity.

**Phase 2 - DT:** Next, we show simulation results to illustrate the efficiency of the proposed RIS reflection design method, FroMax. For comparison, we include results when the RIS reflection coefficient vector $\omega$ is designed according to the alternating maximization method in [2], termed AltMax, and Random, where the entries of $\omega$ are randomly generated such that the $m$th entry is given as $[\omega]_m = \frac{1}{\sqrt{M_A}} e^{j\omega_m}$, $\omega_m \in [0, 2\pi]$. We define the SNR as $\text{SNR} = P_{\text{max}}/\sigma^2$.

Fig. 5 shows SE versus SNR results, averaged over 1,000 channel realizations. Clearly, we can see that FroMax-1 has an equal performance to that of FroMax-2 and AltMax when $N_s = 1$. However, FroMax-1 experiences a performance loss when $N_s = 2$, since it mainly maximizes the dominant singular value, as it can be seen from Fig. 6. Differently, the AltMax and FroMax-2 methods optimize the dominant $N_s$ singular values of the effective channel such that it maximizes the system SE. Note that, in the low SNR regime, i.e., below 5 dB, all the simulated methods experience a very low SE performance, due to the CE errors. Therefore, a preprocessing denoising step will be required to improve the CE accuracy, which we leave for future work.
VI. CONCLUSIONS

In this work, we have considered the channel estimation and the RIS reflection coefficient design problems in point-to-point RIS-aided mmWave MIMO communication systems. We have proposed a CP tensor-based channel estimation method termed TenRICE, which estimates the transmitter to RIS and the RIS to receiver channels separately, up to a trivial scaling factor. We have shown that by jointly exploiting the low-rank nature of mmWave channels and the tensor structure of the received signals, not only the estimation accuracy can be improved, but also the training overhead and the complexity can be reduced. The proposed non-iterative RIS reflection design method based on a Frobenius-norm maximization (FroMax) design strategy has a comparable performance to a benchmark method but with significantly lower complexity.

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