Left-right asymmetries and exotic vector–boson discovery in lepton-lepton colliders

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Abstract

By considering left-right (L-R) asymmetries we study the capabilities of lepton colliders in searching for new exotic vector bosons. Specifically we study the effect of a doubly charged bilepton boson and an extra neutral vector boson appearing in a 3-3-1 model on the L-R asymmetries for the processes $e^-e^- \rightarrow e^-e^-$, $\mu^-\mu^- \rightarrow \mu^-\mu^-$ and $e^-\mu^- \rightarrow e^-\mu^-$ and show that these asymmetries are very sensitive to these new contributions and that they are in fact powerful tools for discovery this sort of vector bosons.

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I. INTRODUCTION

Any extension of the electroweak standard model (ESM) \[1\] implies necessarily the existence of new particles. We can have a rich scalar-boson sector if there are several Higgs-boson multiplets \[2\] or have more vector and scalar fields in models with a larger gauge symmetry as in the left–right symmetric \[3\] and in 3-3-1 models \[4\], or we also can have at the same time more scalar, fermion, and vector particles as in the supersymmetric extensions of the ESM \[5\].

If in a given model all the new particles contribute to all observables, it will be very difficult to identify their contribution in the usual and exotic processes. In some models \[4,6\] the contributions of the scalar-bosons cannot be suppressed by the fermion mass and they can have same strength of the fermion–vector-boson coupling. Hence, we can ask ourselves if there exist observables and/or processes which allow us to distinguish between the contributions of charged and neutral scalar-bosons from those of the vector-bosons. In Ref. \[7\] it was noted that the left-right (L-R) asymmetries in the lepton–lepton diagonal scattering are insensitive to the contribution of doubly-charged scalar fields but are quite sensible to doubly-charged vector field contributions. On the other hand, in non-diagonal scattering (as \(\mu^−e^-\)) those asymmetries are sensible to the existence of an extra neutral vector-boson \(Z'\) \[8,9\].

Here we will extend our previous analysis by considering a detailed study of the L-R asymmetries in order to analyse their capabilities in detecting new physics. The outline of the paper is the following: In Sec. II we define the asymmetries; in Sec. III we show the lagrangian interaction of the models we are considering here. The results and experimental considerations are given in Sec. IV and our conclusions appear in the last section.

II. THE L-R ASYMMETRIES

The left-right asymmetry for the process \(l^-l'^-\rightarrow l^-l'^-\) with one of the particles being unpolarized is defined as

\[
A_{RL}(ll'\rightarrow ll') \equiv A_{RL}(ll') = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L},
\]

(1)

where \(d\sigma_{R(L)}\) is the differential cross section for one right (left)-handed lepton \(l\) scattering on an unpolarized lepton \(l'\) and where \(l,l' = e,\mu\). That is

\[
A_{RL}(ll') = \frac{(d\sigma_{RR} + d\sigma_{RL}) - (d\sigma_{LL} + d\sigma_{LR})}{(d\sigma_{RR} + d\sigma_{RL}) + (d\sigma_{LL} + d\sigma_{LR})},
\]

(2)

where \(d\sigma_{ij}\) denotes the cross section for incoming leptons with helicity \(i\) and \(j\), respectively, and they are given by

\[
d\sigma_{ij} \propto \sum_{kl} |M_{ij;kl}|^2, \quad i, j; k, l = L, R.
\]

(3)

Notice that when the scattering is diagonal, \(l = l' = e,\mu\), \(d\sigma_{RL} = d\sigma_{LR}\), so the asymmetry in Eq. (2) is equal to the asymmetry defined as \((d\sigma_{RR} - d\sigma_{LL})/(d\sigma_{RR} + d\sigma_{LL})\). For practical
purposes, for the non-diagonal \((e\mu \rightarrow e\mu)\) case of the \(ARL\) asymmetry, we will focus on the scattering of polarized muons by unpolarized electrons.

Another interesting possibility is the case when both leptons are polarized. We can define an asymmetry \(AR_{RL}\) in which one beam is always in the same polarization state, say right-handed, and the other is either right- or left-handed polarized (similarly we can define \(AL_{LR}\)):

\[
AR_{RL} = \frac{d\sigma_{RR} - d\sigma_{RL}}{d\sigma_{RR} + d\sigma_{RL}}; \quad AL_{RL} = \frac{d\sigma_{LR} - d\sigma_{LL}}{d\sigma_{LL} + d\sigma_{LR}}. \tag{4}
\]

In this case, when the non-diagonal scattering is considered, we will assume that the muon beam has always the same polarization and the electron one can have both, the left and the right polarizations.

We can integrate over the scattering angle and define the asymmetry \(\int A_{RL}\) as

\[
\int A_{RL} = \frac{(\int d\sigma_{RR} + \int d\sigma_{RL}) - (\int d\sigma_{LL} + \int d\sigma_{LR})}{(\int d\sigma_{RR} + \int d\sigma_{RL}) + (\int d\sigma_{LL} + \int d\sigma_{LR})}, \tag{5}
\]

where \(\int d\sigma_{ij} \equiv \int_{0}^{175^\circ} d\sigma_{ij}\). A similar expression can be written for \(AR_{RL}\).

III. THE MODELS

We are studying here the asymmetries defined above in the context of two models: the electroweak standard model (ESM) and in a model having a doubly charged bilepton vector field \((U_{\mu}^{--})\) and an extra neutral vector boson \(Z'\) [4]. The latter model has also two doubly charged scalar bileptons but since their contributions cancel out in the numerator of the asymmetries we are not consider them on this study. We identify the case under study by using the \{ESM\}, \{ESM + U\}, and \{ESM + Z'\} labels in cross sections and asymmetries.

In the context of the electroweak standard model, at the tree level, the relevant part of the ESM lagrangian is

\[
\mathcal{L}_F = -\sum_i \frac{g m_i}{2M_W} \bar{\psi}_i \gamma^\mu \psi_i H^\mu - e \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu - \frac{g}{2\cos\theta_W} \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma^5) \psi_i Z_\mu, \tag{6}
\]

\(\theta_W \equiv \tan^{-1}(g'/g)\) is the weak mixing angle, \(e = g \sin \theta_W\) is the positron electric charge with \(g\) such that

\[
g^2 = \frac{8G_F M_W^2}{\sqrt{2}}, \quad \text{or} \quad g^2/\alpha = 4\pi \sin^2 \theta_W, \tag{7}
\]

with \(\alpha \approx 1/128\); and the vector and axial neutral couplings are

\[
g_V^i \equiv t_{3L}(i) - 2q_i \sin^2 \theta_W, \quad g_A^i \equiv t_{3L}(i), \tag{8}
\]

where \(t_{3L}(i)\) is the weak isospin of the fermion \(i\) and \(q_i\) is the charge of \(\psi_i\) in units of \(e\).

The charged current interactions in a model having a doubly charged vector boson [4], in terms of the physical basis, are given by
\[-\frac{g}{\sqrt{2}} \left[ \bar{\nu}_L E_L^\dagger l_L W^+ - \bar{l}_L \gamma^\mu E R^\dagger \nu_L V^+ \right] + H.c., \quad (9)\]

with \( l'_L = E_L^\dagger l_L, \quad l'_R = E_R^\dagger l_R, \quad \nu'_L = E_L^\dagger \nu_L \); the primed (unprimed) fields denoting symmetry (mass) eigenstates. We see from Eq. (10) that for massless neutrinos we have no mixing in the charged current coupled to \( W^+ \mu \) but we still have mixing in the charged currents coupled to \( V^+ + U^{++} \). That is, if neutrinos are massless we can always choose \( E_L^\dagger E_L^\dagger = 1 \). However, the charged currents coupled to \( V^+ + U^{++} \) are not diagonal in flavor space and the mixing matrix \( K = E_R^\dagger E_L^\dagger \) has three angles and three phases. (An arbitrary \( 3 \times 3 \) unitary matrix has three angles and six phases. In the present case, however, the matrix \( K \) is determined entirely by the charged lepton sector, so we can rotate only three phases (10)).

The total width of the \( U \)-boson (\( \Gamma_U^{\text{total}} \)) is a calculable quantity in the model once we know all the \( U \)-boson couplings which are derived from the 3-3-1 gauge-invariant lagrangian. However, we find that a complete computation of \( \Gamma_U \) is out of the scope of this paper because in this case some realistic hypotheses concerning the masses of the exotic scalars and quarks should be made. Thus, we will only consider the partial width due to the \( U^{--} \to l^-l^- \) decay. In the limit where all the lepton masses are negligible we have:

\[
\Gamma_U^{\text{total}} \sim \Gamma(U^{--} \to \text{leptons}) = \sum_{i,j} \frac{G_F}{6\sqrt{2}\pi} M_W^2 M_U |K_{ij}|^2
\quad (10)
\]

where \( i, j \) run over the \( e, \mu, \) and \( \tau \) leptons and \( K_{ij} \) is a mixing matrix in the flavor space. For the expression above we can write \( \Gamma_U^{\text{total}} = \sum_i \Gamma_{ii} + \frac{1}{2} \sum_{i \neq j} \Gamma_{ij} \) and assuming that the matrix \( K \) is almost diagonal we can neglect \( \Gamma_{ij} \) for \( i \neq j \) and consider for practical purposes that \( \Gamma_U^{\text{total}} = 3 \times \Gamma_{ii} \). In our numerical applications \( \Gamma_U^{\text{total}} \) is a varying function of the \( U \)-boson mass. For instance, for \( M_U = 300 \text{ GeV} \) we have \( \Gamma_U^{\text{total}} \sim 2.5 \text{ GeV} \).

In the model there is also a \( Z' \) neutral vector boson which couples with the leptons as follows

\[
\mathcal{L}_{NC}^{\text{Z'}} = -\frac{g}{2c_W} \left[ \bar{l}_{aL} \gamma^\mu l_{aL} + \bar{l}_{aR} \gamma^\mu r_{aR} + \bar{\nu}_{aL} \gamma^\mu \nu_{aL} \right] Z'_\mu, \quad (11)
\]

with \( L_L = L_{\nu} = -(1 - 4s_W^2)^{1/2} / \sqrt{3} \) and \( R_i = 2L_i \). Notice the leptophobic character of \( Z' \) (11). In this case we have no concerns about the \( Z' \)-width because this neutral boson is only exchanged in the \( t \)-channel.

We will consider the process

\[
l^-(p_1, \lambda) + l'^-(q_1, \Lambda) \to l^-(p_2, \lambda') + l'^-(q_2, \Lambda'), \quad (12)
\]

where \( q = p_2 - p_1 = q_2 - q_1 \) is the transferred momentum. As we said before, we will neglect the electron mass but not the muon mass \( i.e., \ E = |\vec{p}_e| \) for the electron and \( K^2 - |\vec{q}_\mu|^2 = m_\mu^2 \) for the muon. In the non-diagonal elastic scattering in the standard model we have only the \( t \)-channel contribution. The relevant amplitudes for the ESM, \{ESM + U\} and \{ESM + Z'\} models are in the appendices of Ref. (3) (Ref. (3)) for the diagonal (non-diagonal) case.

**IV. RESULTS**
A. The $U$ boson

We start this section by considering the contributions of the doubly charged vector boson $U$ to the asymmetries which contributes uniquely via the $s$-channel for a doubly charged initial state. In Fig. 1 we see that the angular dependence of the $A_{RL}$ asymmetry, taken into account the $U$ contribution, presents a relatively different behavior with respect to the ESM for a wide range of $U$-masses. Notice that the lines are considerably separated even for those values of the $U$-mass that are not close to $\sqrt{s}$. Notice also that for $U$-mass lower than $\sqrt{s}$ we basically reproduce the ESM result for $\theta \approx 0$ and $\pi/2$; the largest difference with the ESM occurs for $\theta$ in the interval $0.5–1$. The behavior of $A_{RL}$ for ESM+$U$ as a function of $M_U$ is showed in Fig. 2 for a fixed scattering angle and for several values of $\sqrt{s}$. We see that this asymmetry is essentially negative and that its maximum value is zero and occurs at the resonance point $M_U = \sqrt{s}$. This is due to the fact that at the resonance the numerator of $A_{RL}$, as defined in Eq. (1), cancels out no mater the value of $M_U$. On the other hand the value of $M_U$ governs the width of the curves around the resonance point. This particular feature is better seen in Fig. 3. In this figure we show the $A_{RL}$ asymmetry as a function of the center-of-mass energy $E_{CM} = \sqrt{s}$ for some values of the $U$-mass and it is clearly seen that not only at the peak but also for a considerably large range of masses around the peak, the curves representing the respective $U$ contribution are significantly separated from the ESM one. It means that this asymmetry is very sensitive to the $U$-boson even in the case where the $U$-mass is larger than $\sqrt{s}$; when the $U$-mass is lower than $\sqrt{s}$ we reproduce the ESM results.

In Fig. 4 and Fig. 5 we show the effects of the $U$-boson on the $A_{R;RL}$ asymmetry, defined in Eq. (4), and as it behaves qualitatively like $A_{RL}$ we come to the same conclusions we did for $A_{RL}$. We must note that near the $U$-resonance the $A_{R;RL}$ asymmetry is negative. However, in this case, polarization for both beams must be available.

The integrated asymmetry $\int A_{RL}$ defined in Eq. (8) is shown in Fig. 8. There we can see that while the ESM curve keeps an almost constant value (0.025-0.031) for $0.5 < \sqrt{s} < 2$ TeV, the ESM+$U$ curves go from zero, for $M_U \neq \sqrt{s}$, to a very pronounced peak ($\sim -0.25$) at the resonance points. In Fig. 9 for the sake of detectability, we show the quantity $\delta\%$ defined by:

$$\delta \% = \frac{\int A_{RL}^{ESM+U} - \int A_{RL}^{ESM}}{\int A_{RL}^{ESM}} \times 100,$$

which in this case stands for the percent deviation of $\int A_{RL}^{ESM+U}$ from $\int A_{RL}^{ESM}$. There we can see that there is a wide range of $U$-masses that can be probed at $e^-e^-$ colliders.

Next we study the effect of non-negligible initial and final fermion masses by considering the $\mu^-\mu^- \rightarrow \mu^-\mu^-$ process in a muon collider. The results are given in Fig. 8. There we can see that, for $M_U = 500$ GeV, below 300 GeV the muon mass effect is in evidence differing sensibly from the electron-electron case, independently of the $U$-contribution for higher energies the lepton mass has no effect at all for both models. Between 300 and 400 GeV all the curves are coincident and we cannot distinguish among both models or leptons. Above 400 GeV it is the $U$-effect which dominates and it is the same for electrons and muons. The effect of the $U$-resonance is well evident and even above the resonance there is an almost constant difference between the ESM+$U$ and ESM asymmetries showing
that the $A_{RL}$ asymmetry is still a sensitive parameter for the $U$ discovery for $\sqrt{s} > M_U$ in lepton-lepton colliders.

B. The $Z'$ boson

In order to search for new physics in the neutral-vector boson sector it is worth to consider de non-diagonal process $\mu^- e^- \rightarrow \mu^- e^-$. In this case, assuming that the couplings of the $U$-boson with leptons are almost diagonal ($K_{ii} \sim 1$ as in the $\Gamma_U$), the $s$-channel $U$-boson exchange will be negligible and provided that the $Z'$ couples with leptons diagonally, the only contributions to this process for ESM+$Z'$ will be the $t$-channel ones, i.e., the contributions of $\gamma$, $Z$, and $Z'$. The angular dependence of $A_{RL}$ is showed in Fig. 9 where we can see that a $Z'$ contribution is clearly distinguished from the ESM one for a wide range of $Z'$-masses around a given $E_\mu = \sqrt{s}/2$. (In the figures concerning the $Z'$-boson, once there is no $s$-channel, we specify the energy by the muon-beam energy $E_\mu$.) As expected the $A_{RL}$ asymmetry is more sensitive to relatively light $Z'$ boson. We come to the same conclusion from Fig. 10 in which we show the $A_{RL}$ asymmetry as a function of $E_\mu$. The sensitivity of the this asymmetry with the $Z'$-mass is showed in Fig. 11.

Contrarily to the case of the search for the $U$-boson, the asymmetry $A_{RL}$ is not sensitive to the extra neutral vector boson. In this case, the potential capabilities of the asymmetries in discovering new neutral vector bosons are better explored by considering the integrated asymmetry $\int A_{RL}$. The angular integration over the scattering angle of the $t$-channel $Z'$ exchange contribution produces curves that are clearly separated, depending on the $Z'$-mass, which are also clearly distinguishable from the ESM curve for a wide range of masses. See Fig. 12.

We have also computed the asymmetries taking into account a $Z'$ which couples to leptons with the same couplings of the standard $Z$ boson but with a different mass. Although these ESM couplings are stronger than those of the 3-3-1 model they have no substantial effect on the asymmetries: The results are very similar to the ones showed in Fig. 11.

For the $e\mu$ scattering there are also contributions coming from the neutral scalar sector of the model. However as in all the scalar contributions the pure scalar terms cancel out in the numerator of the asymmetry and the interference terms are numerically negligible.

C. Observability

Based on the figures we have shown throughout the text we have claimed that the values of the asymmetries, when there is an extra contribution of a new vector boson, are different enough from those of ESM ones to allow for the discovery of the referred bosons. However, we must be sure that there is enough statistics to measure these asymmetries. In order to provide some statistical analysis we assume a conservative value for the luminosity: $\mathcal{L} = 1 \text{fb}^{-1} \text{yr}^{-1} = 10^{32} \text{cm}^{-2} \text{s}^{-1}$ for the $e^- e^-$, $\mu^- \mu^-$ and the $\mu^- e^-$ colliders, and compute the number of the expected number of events ($N$) based on the unpolarized integrated cross section for each process.

For the $e^- e^- \rightarrow e^- e^-$ or $\mu^- \mu^- \rightarrow \mu^- \mu^-$ processes, the ESM cross section is relatively small, it goes from 0.05 nb at $\sqrt{s} = 0.5 \text{ TeV}$ to $1.5 \times 10^{-3}$ nb at $\sqrt{s} = 2 \text{ TeV}$. These values
correspond to $5 \times 10^4$ and $1.5 \times 10^3$ events/yr respectively. Then, computing $\sqrt{N}/N$ we get $\sim 4 \times 10^{-3}$, for the first case, and $\sim 2 \times 10^{-2}$, for the second one. This is an indication that the ESM asymmetries can be measured. Note that the asymmetries we have computed here are relatively large: $A_{RL}^{ESM} \sim A_{RL}^{ESM} \sim \mathcal{O}(10^{-1})$ for a fixed scattering angle and of the order $\mathcal{O}(10^{-2})$ for the integrated ones as showed in Figs. 1-5 and Fig. 6, respectively. On the other hand, this reaction is so sensitive to the $U$-boson contribution that there is an enormous enlargement in the cross section and consequently in the statistics. In Tab. 1 we show the relevant parameters depending on the $U$-boson mass and for only two values of $\sqrt{s}$ for shortness. There we can see that there is enough precision to measure the asymmetries, for the range of masses and energies we have considered. For the $U$-boson discovery we can study the cross section directly provided that it is considerably different from that of the ESM \cite{12}. However, the study of the asymmetry gives us more qualitative information once, contrarily to the cross section, it filters the vector-nature contribution of the $U$-boson: it was previously shown \cite{7} that the scalar-boson contributions, which are present in the 3-3-1 model, cancel out in the asymmetry numerator.

For the $\mu^-e^- \rightarrow \mu^-e^-$ process, the integrated cross sections for the ESM are larger than for the electron diagonal process. We find 3 nb for $E_\mu = 0.5$ TeV and $\sim 0.2$ nb for $E_\mu = 2.0$ TeV. The corresponding number of events are $3 \times 10^6$ and $\sim 2 \times 10^5$, respectively, for the same luminosity used before. In this case the ratio $\sqrt{N}/N$ gives $5.7 \times 10^{-4}$ and $2.2 \times 10^{-3}$, respectively, which provide enough precision to measure the $A_{RL}^{ESM}$ once it is of the order $\mathcal{O}(10^{-2})$, as can be seen from Fig. 1-12. For the ESM+Z' case we have that although the Z'-contribution can affect significantly the values of the asymmetry $A_{RL}$, as it has been shown, it only slightly modifies the cross section values. It means that the number of events in for ESM+Z' are similar to those of the ESM and hence we have enough precision to measure $A_{RL}^{ESM+Z'}$ too. Once again we note that the asymmetries are more sensitive than the cross section itself in looking for the Z'-discovery.

V. CONCLUSIONS

Here we have generalized the analysis of Ref. \cite{1,8} and have shown that the L-R asymmetries in the diagonal ($e^-e^-, \mu^-\mu^-$) lepton scattering can be the appropriate observable to discover doubly charged vector bosons $U$ even for values of $M_U$ and $\sqrt{s}$ far away from the resonance condition. Although the cross sections may have important contributions from the scalar fields (doubly charged Higgs bosons) these contributions cancel out in the numerator of the L-R asymmetries. On the other hand, the contribution of the extra neutral Z', leptophobic or with the same couplings of the standard model, gives small contributions to the diagonal $e^-e^-, \mu^-\mu^-$ scattering but it gives an important contribution to non-diagonal $\mu^-e^-$ case.

Hence, both $U^-$ and Z' vector bosons can be potentially discovered in these sort of processes by measuring the L-R asymmetries. Since the couplings of both particles with matter are known in a given model, once their masses were known other processes like exotic decays could be used to study the respective contributions of the scalar fields present in the model.
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FIG. 1. The $A_{RL}$ asymmetry for an $e^-e^-$ collider with $\sqrt{s} = 2$ TeV for the ESM (solid line) and for the ESM+U for several U–masses as a function of the scattering angle.
FIG. 2. The $A_{RL}$ asymmetry for a fixed scattering angle $\theta = 0.5$ rad and several values of $\sqrt{s}$ of $e^-e^-$ colliders for ESM+U as a function of $M_U$. 
FIG. 3. The $A_{RL}$ asymmetry for a fixed scattering angle $\theta = 0.5$ rad for the ESM (solid line) and for the ESM+U for several U-masses as a function of $E_{CM}$. 
FIG. 4. The same as in Fig. 1 for the $A_{R,RL}$ asymmetry.
FIG. 5. The same as in Fig. 3 for the $A_{R,RL}$ asymmetry.
FIG. 6. The integrated asymmetry \( \int A_{RL} \) for \( e^-e^- \) collider for the ESM (solid line) and for the ESM+U for several U-masses as function of \( E_{CM} \).
FIG. 7. The quantity $\delta\%$ as defined in Eq. (13) for $e^-e^-$ collider for several $\sqrt{s}$-values as a function of the $M_U$. 

16
FIG. 8. The $A_{RL}$ asymmetry for $M_U = 0.5$ TeV and $\theta = 0.5$ rad for $e^-e^-$ and $\mu^-\mu^-$ colliders for the ESM and for the ESM+U as a function of $E_{CM}$. 

FIG. 8. The $A_{RL}$ asymmetry for $M_U = 0.5$ TeV and $\theta = 0.5$ rad for $e^-e^-$ and $\mu^-\mu^-$ colliders for the ESM and for the ESM+U as a function of $E_{CM}$. 

17
FIG. 9. The $A_{RL}$ asymmetry for a $\mu^−e^−$-collider of $E_\mu = 1$ Tev ($\sqrt{s} = 2$ TeV) for the ESM (solid line) and for the ESM+$Z'$ for several values of $M_{Z'}$ as function of the scattering angle.
FIG. 10. The $A_{RL}$ asymmetry for a $\mu^-e^-$-collider for a fixed scattering angle, $\theta = 0.5$ rad, for the ESM (solid line) and for the ESM+$Z'$ for several values of $M_{Z'}$ as function of $E_\mu$. 
FIG. 11. The $A_{RL}$ asymmetry for a fixed scattering angle, $\theta = 0.5$ rad, and several values of $\sqrt{s}$ of $\mu^-e^-$ colliders for ESM+$Z'$ as a function of $M_{Z'}$. 
FIG. 12. The integrated asymmetry $\int A_{RL}$ for the ESM (solid line) and for the ESM+Z' for several $M_{Z'}$ as function of $E_\mu$ for $\mu^-e^-$ colliders.
TABLE I. The quantities \( \sigma, N, \) and \( \sqrt{N}/N \) for the unpolarized \( e^-e^- \rightarrow e^-e^- \) process with \( \mathcal{L} = 1 \text{fb}^{-1}\text{yr}^{-1} \) for the ESM and the ESM+U (\( U \)-mass dependent); masses and energies are in TeV units.

| \( \sqrt{s} \) | \( \sigma \) (nb) | \( N \) (events) | \( \sqrt{N}/N \) | \( \sigma \) (nb) | \( N \) (events) | \( \sqrt{N}/N \) |
|---|---|---|---|---|---|---|
| 0.5 | \( 5 \times 10^{-2} \) | \( 5 \times 10^4 \) | \( 4 \times 10^{-3} \) | \( 1.5 \times 10^{-3} \) | \( 1.5 \times 10^4 \) | \( 2 \times 10^{-2} \) |
| 2.0 | | | | | | |
| ESM+U | | | | | | |
| \( M_U = 0.5 \) | 427 | \( 4 \times 10^8 \) | \( 5 \times 10^{-6} \) | 1.2 | \( 1.2 \times 10^6 \) | 9 \( \times 10^{-4} \) |
| \( M_U = 1.0 \) | 25 | \( 2.5 \times 10^6 \) | \( 2 \times 10^{-4} \) | \( \sim 1.2 \) | \( \sim 1.2 \times 10^6 \) | \( \sim 9 \times 10^{-3} \) |
| \( M_U = 2.0 \) | \( \sim 25 \) | \( \sim 2.5 \times 10^7 \) | \( \sim 2 \times 10^{-4} \) | 26 | \( 2.6 \times 10^7 \) | \( \sim 2 \times 10^{-4} \) |