COMPARISON OF FORCE-FREE CORONAL MAGNETIC FIELD MODELING USING VECTOR FIELDS FROM HINODE AND SOLAR DYNAMICS OBSERVATORY

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ABSTRACT

Photospheric magnetic vector maps from two different instruments are used to model the nonlinear force-free coronal magnetic field above an active region. We use vector maps inferred from polarization measurements of the Solar Dynamics Observatory/Helioseismic and Magnetic Imager (HMI) and the Solar Optical Telescope’s Spectropolarimeter (SP) on board Hinode. Besides basing our model calculations on HMI data, we use both SP data of original resolution and scaled down to the resolution of HMI. This allows us to compare the model results based on data from different instruments and to investigate how a binning of high-resolution data affects the model outcome. The resulting three-dimensional magnetic fields are compared in terms of magnetic energy content and magnetic topology. We find stronger magnetic fields in the SP data, translating into a higher total magnetic energy of the SP models. The net Lorentz forces of the HMI and SP lower boundaries verify their force-free compatibility. We find substantial differences in the absolute estimates of the magnetic field energy but similar relative estimates, e.g., the fraction of excess energy and of the flux shared by distinct areas. The location and extension of neighboring connectivity domains differ and the SP model fields tend to be higher and more vertical. Hence, conclusions about the magnetic connectivity based on force-free field models are to be drawn with caution. We find that the deviations of the model solution when based on the lower-resolution SP data are small compared to the differences of the solutions based on data from different instruments.

Key words: Sun: corona – Sun: magnetic topology – Sun: photosphere – Sun: surface magnetism

Online-only material: color figures

1. INTRODUCTION

Most recent studies dealing with the magnetic structure of the solar corona above active regions use different force-free model approaches (see recent review of Wiegelmann & Sakurai 2012) and base the modeling on photospheric vector magnetic field data from either the Helioseismic and Magnetic Imager (HMI) on board the Solar Dynamics Observatory (SDO) or the Spectropolarimeter (SP) of the Solar Optical Telescope (SOT) on board the Hinode spacecraft. Such model approaches are used to compensate for the lack of routine direct measurements of the coronal magnetic field vector.

Hao et al. (2012) used SP data as a lower boundary condition to an optimization approach to analyze the coronal magnetic field associated with a white light flare. The modeling suggested that the flare originated from sheared and twisted field lines with low altitudes bridged by a set of higher magnetic field lines. Inoue et al. (2012) applied an MHD relaxation method based on SP data to investigate the buildup and release of magnetic twist and suspected the importance of the relative handedness of twisted field lines and the ambient field.

Sun et al. (2012b) employed an optimization method based on HMI data to model the temporal evolution of the coronal field of an active region over five days. The modeling displayed distinct stages of the buildup and release of magnetic energy and analyzed the association with changes in the magnetic field. In a subsequent study, Sun et al. (2012a) used the same method for a detailed analysis of the field topology during a series of eruptions observed from HMI. Above the apexes of cusp-like loops observed in coronal images, the modeling result suggested the presence of a coronal null point.

Comparisons of the outcome of different force-free model algorithms based on the same lower boundary conditions have been performed in the past too (e.g., DeRosa et al. 2009; Gilchrist et al. 2012). These studies revealed that order-of-magnitude estimates of these models based on large enough fields of view (FOVs) and high enough spatial resolution of the vector magnetic field data can be expected to be reliable. The model outcome of the same reconstruction algorithm based on data from different instruments has been studied recently by Thalmann et al. (2012), using data from HMI and the Vector-SpectroMagnetograph (VSM) of the SOLIS project (Keller et al. 2003). They found agreements of the resulting force-free models in the form of, e.g., the relative amount of energy to be set free during an eruption but also found a considerable difference in the absolute model energy estimates. In particular, they found the estimated energy content of the VSM model to be about twice that of the HMI model.

In the present study, we use vector magnetic field data of HMI and SP as an input to the same force-free model and compare the results. This is motivated by the data of these two instruments being widely used recently and in the future expected to be frequently used as an input for the modeling of the coronal magnetic field. We regard this as important in order to test the consistency of the model solutions and at the same time to give a feeling about the accuracy of the estimated physical quantities and magnetic field topology based on those models. We also investigate the effect that a binning of the SP instrument data to a lower resolution has on the model outcome. We, however, do not search for explanations of the differences of the HMI and SP inversion products itself, considering this as to be out of the scope of this study.

2. DATA AND ANALYSIS METHODS

2.1. Data Sources

The HMI on board SDO (Schou et al. 2012) obtains filtergrams at the photospheric Fe I 617.3 nm spectral line. The full
Stokes vector is retrieved from filtergrams averaged over about 12 minutes and inverted using the Milne–Eddington (ME) inversion algorithm of Borrelo et al. (2010). The 180° azimuth ambiguity of the transverse field is resolved using a minimum energy algorithm (Metcalfe 1994; Metcalfe et al. 2006; Leka et al. 2009) and the resulting vector magnetograms have a plate scale of 0.5 arcsec.

In its fast scan mode, SP as part of the SOT (Tsuneta et al. 2008; Sueutsu et al. 2008; Ichimoto et al. 2008; Shimizu et al. 2008) on board the Hinode spacecraft (Kosugi et al. 2007) observes the FeI spectral line couplet at 630.15 nm and 630.25 nm. Full Stokes profiles are obtained with a spectral sampling of 2.25 × 10^{-3} nm and a slit scan time of 1.6 s. The physical parameters from the full Stokes profiles were obtained using the MERLIN ME inversion algorithm (Skumanich & Lites 1987; Lites et al. 2007). The 180° azimuth ambiguity is resolved in the same way as for HMI data.

2.2. Event Selection and Data Set

To perform a study as described above, simultaneous observations from HMI and SP are required. Both HMI and SP vector magnetic field data are available to analyze the magnetic structure of the active region 11382 on 2011 December 22. SP scanned this active region from 04:46 UT to 05:29 UT (i.e., scanned ∼3 Mm minute^{-1} from solar east to west). The HMI vector map used for this study was retrieved at t_{rec} = 05:00 UT, approximately at half of the scanning time of SP. The HMI data have a spatial resolution of ∼0.32 arcsec. The FeI spectral line couplet obeys a spatial resolution of ∼0.32 arcsec. The Alfvén travel time is v_{A} ∼ 10 km s^{-1}. The Alfvén travel time is and thus the evolution of the photospheric magnetic field over a characteristic distance in the present study (∼100 Mm) is ∼1 hr. Therefore, we assume that the temporal changes of the photospheric field over ∼20 minutes, during which SP scanned before and after the time when HMI recorded, are negligible. Regarding the longitudinal magnetic field we assume this as justified since, e.g., Wang et al. (2009) also found that the average ratio of the longitudinal magnetic flux densities measured by SP and SOHO/MDI (Scherrer et al. 1995) was not strongly influenced by the evolution of the photospheric magnetic field during the scanning time of SP.

The magnetic field vectors are transformed to heliographic coordinates (Gary & Haygard 1990). We correct the SP data set for the effect of differential rotation where we use t_{rec} as reference time. Given present computational capabilities, high-resolution data are sometimes binned to a lower resolution in order to allow for near real-time magnetic field modeling and/or computational domains of feasible dimensions (see, e.g., DeRosa et al. 2009). Thus, in the course of co-alignment of the HMI and SP data, we also bin the original-resolution SP (SP_{orig}) data to the resolution of HMI which involves a two-dimensional linear interpolation. The binned SP data are hereafter denoted as SP_{bin} data.

The FOV used to study active region 11382 is mainly determined by the area covered by the SP scan (∼140 × 85 Mm² centered around S19W07; see Figure 1). Within the HMI data which cover roughly ∼400 × 250 Mm², we define a window equally sized as the FOV of SP and calculate the cross-correlation between its vertical field component and that of the SP data. By shifting the position of the HMI window we search for the highest cross-correlation to find the corresponding HMI sub-field.

2.3. Uncertainty Estimation

The noise level of the HMI data is on the order of 1 mT/10 mT for the longitudinal/transverse magnetic field (X. Sun 2013, private communication). The average uncertainty for both the longitudinal and transverse fields, estimated from the SP inversion error maps, is ±1 mT. Besides seeming rather low especially for the transverse field, these standard error estimates may not be reliable (B. Lites 2013, private communication). Thus, we employ a consistent measure of the uncertainty level for the data from the two instruments, as described in the following.

When investigating the properties of the HMI and SP data in the course of the force-free modeling, we only consider (1) pixels with values of the vertical field, B_{z}, and horizontal field, B_{h}, above the respective 2σ uncertainty levels (2σ_{B_{z}} and 2σ_{B_{h}}, respectively) or (2) pixels with values of B_{z} > 2σ_{B_{z}} and B_{h} < 2σ_{B_{h}}, the latter in order not to disregard the strong vertical fields in the center of the active region.

The respective 2σ uncertainty levels are calculated for the quiet-Sun area, outlined by the dashed rectangles in Figure 1, where we find 2σ_{B_{z}} = 3.3/6.7 mT and 2σ_{B_{h}} = 5.2/12.2 mT for the HMI/SP_{bin} data. The uncertainty levels for the SP_{orig} data are calculated from a quiet-Sun region, equivalent to that outlined in Figure 1. Here, we find 2σ_{B_{z}} = 8.8 mT and 2σ_{B_{h}} = 12.3 mT, i.e., slightly higher than what was found for the SP_{bin} data. The latter estimates, in fact, conform with the findings of Orozco Suárez et al. (2007), who had estimated the measurement uncertainties of SP for the quiet-Sun inter network field strengths and fluxes as <15 mT.

2.4. Magnetic Field Modeling

Photospheric polarization signals originate from atmospheric layers which are known not to be force-free. For instance,
Magnetic pressure, Gary (2001) was able to estimate the height level. Modeling the interchanging dominance of plasma and considered to be force-free above 0.4 Mm above a photospheric measurements of an active region magnetic field, that it can be 

\[ B_{\text{HMI}} = \text{HMI model.} \]

Figure 2. Relative occurrence of the absolute (a) vertical field, \( B_z \), and (b) horizontal field, \( B_h \). The power of \( B_z \) and \( B_h \) as a function of length scale, \( l \), and wavenumber, \( k \), is shown in panels (c) and (d), respectively. HMI/SPbin data are represented by black/gray lines. Only pixels with values \( B_z > 2\sigma_{B_z} \) and \( B_h > 2\sigma_{B_h} \) or \( B_z > 2\sigma_{B_z} \) and \( B_h < 2\sigma_{B_h} \) are taken into account.

Therefore, the inferred HMI and SP magnetic vector maps are not force-free consistent and need to be preprocessed to achieve suitable consistent force-free boundary conditions (Wiegelmann & Inhester 2006). From the vertical component of the preprocessed field vector, a potential field is calculated and used as start equilibrium and to prescribe the boundaries of the cube computational domain. The bottom boundary is replaced by the preprocessed vector field and the set of force-free equations for the nonlinear case solved in Cartesian coordinates (Wiegelmann & Inhester 2010). A boundary layer of \( \sim 10 \text{ Mm} \) is introduced toward the lateral and top boundaries where the nonlinear force-free (NLFF) solution drops to the prescribed boundary field. For our analysis, we discard this layer and only consider the inner (physical) \( \sim 120 \times 60 \times 70 \text{ Mm}^3 \) domain and the corresponding bottom boundary field. Since this method involves the relaxation of the magnetic field not only inside the computational domain but also on its bottom boundary, we compute a potential field from the relaxed lower boundary based on the fast Fourier method described by Alissandrakis (1981).

Hereafter, we refer to the 3D NLFF field model based on the HMI vector map as an input to the “HMI model.” Similarly, the “SPbin model” and “SPorig model” result from using the binned and original-resolution SP vector magnetic field, respectively, as input data to our preprocessing and force-free reconstruction algorithms. We compare the HMI and SPbin models in Sections 3.1–3.3 and summarize the effects of binning on the model outcome in Section 3.4.

### Table 1

| Data       | \( |\phi| \) (x10\(^{22}\) Mx) | \( B_h \) (mT) | \( |\tilde{F}_x|/|\tilde{F}_z| \) | \( |\tilde{F}_y|/|\tilde{F}_z| \) | \( |\tilde{F}_z| \) |
|------------|------------------|-------------|----------------|----------------|------------|
| HMI        | 1.323            | 18.4        | 0.13           | 0.08           | 0.29       |
| SPbin      | 2.440            | 22.7        | 0.03           | 0.06           | 0.81       |
| SPorig     | 2.510            | 21.5        | 0.03           | 0.06           | 0.84       |

Notes. Listed are the total unsigned vertical flux \( |\phi| \), the average horizontal field \( B_h \), as well as the net Lorentz force components \( |\tilde{F}_x|, |\tilde{F}_y| \), and \( |\tilde{F}_z| \) normalized to the magnetic pressure force \( |\tilde{F}_z| \). Only pixels with values \( B_z > 2\sigma_{B_z} \) and \( B_h > 2\sigma_{B_h} \) or \( B_z > 2\sigma_{B_z} \) and \( B_h < 2\sigma_{B_h} \) are taken into account.

### 3. RESULTS

#### 3.1. Magnetic Flux and Force-freeness

After performing the data preparation as described in Section 2.2, we are able to investigate how the vertical and horizontal field components of the HMI and SPbin vector maps compare to each other and to check the force-free consistency. We only consider data points where the criteria outlined in Section 2.3 are fulfilled.

We find on overall stronger vertical fields in the SPbin than the HMI data (especially for \( B_z \geq 100 \text{ mT} \); see Figure 2(a)) and stronger horizontal field (except for \( B_h \leq 10 \text{ mT} \); see Figure 2(b)). This can be seen also when comparing the area-integrated unsigned vertical flux, \( |\phi| \), and the average horizontal field, \( B_h \), the HMI data host \( \sim 54\% \) of \( |\phi|_{SPbin} \) and \( \sim 81\% \) of \( B_h \) (see Table 1). HMI and SPbin have a comparable
sensitivity on long scales, \( l \), i.e., at small wavenumbers, \( k \), especially on scales \( l \gtrsim 5 \) Mm (Figures 2(c) and (d)). With decreasing scale the amount of detected field increasingly differs: SP data show considerably stronger fields on smaller scales.

A necessary condition for a magnetic field to be force-free is that the components of the net Lorentz force are considerably smaller than a characteristic magnitude of the total Lorentz force in case of a non-force-free magnetic field. The latter can be approximated by the magnetic pressure, \( \widetilde{p}_F \), on the lower boundary (Low 1985). The ratio \( |\tilde{F}_1|/|\tilde{F}_i| \) with \( i = \{x, y, z\} \) in Table 1 shows that these conditions are met only to a certain degree. The ratios found here agree with the values found by, e.g., Moon et al. (2002) and Tiwari (2012) and we also find \( |\tilde{F}_i| > |\tilde{F}_z| > |\tilde{F}_r| \). Preprocessing, however, certainly improves the force-free as it smooths the vertical field and, additionally, alters the horizontal field in order to minimize the net force and torque and to gain boundary conditions compatible with the force-free assumption (showing ratios that are clearly smaller than unity). The effect of smoothing can be seen when comparing the spectral power of the raw and preprocessed HMI and SPbin data (Figure 3): the signal on shorter scales (i.e., for \( k \) smaller than a characteristic magnitude of the total Lorentz force) differs: SP data show considerably stronger fields on smaller scales.

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**Figure 3.** Power of the absolute vertical field, \( B_z \), and horizontal field, \( B_h \), before and after applying a preprocessing to it (black and gray curves, respectively). Panels (a) and (c) show the power of \( B_z \) of the HMI/SPbin data, while panels (b) and (d) show that of \( B_h \) as a function of length scale \( l \), and wavenumber \( k \).

| \( \Delta E/E_{nlff} \) | SPbin | HMI |
|----------------------|-------|-----|
| 0.19                 | 0.22  | 0.20|

Notes. Given are the total, potential, and excess magnetic energies of the 3D magnetic model fields, listed as \( E_{nlff} \) and \( E_{pot} \), respectively. The ratio \( \Delta E/E_{nlff} \) gives the relative amount of excess energy. The statistical error of these estimates is \( \sim 10\% \) for \( E_{pot} \) and \( E_{nlff} \) and \( \sim 10\% \) for \( \Delta E \).

while iteratively seeking for the force- and divergence-free field solutions in the volume. Its modification to the preprocessed lower boundary data, as listed in Table 1, is that \( |\phi_z| \) is reduced by \( \sim 5\% \) and \( |\phi_{i,SP}| \) is increased by \( \sim 10\% \). Thus, the HMI NLFF lower boundary hosts \( \sim 55\% \) of \( |\phi_{i,SP}| \) and \( \sim 62\% \) of \( B_{h,SPbin} \) of the SPbin NLFF lower boundary data, comparable to the ratio we found for the preprocessed data.

**3.2. Magnetic Energy**

From the 3D model fields, we can estimate the magnetic field energy content of the potential field \( (E_{pot}) \) and of the NLFF field (total energy; \( E_{nlff} \)). An upper limit for the energy that can be released (excess energy) is given by \( \Delta E = E_{nlff} - E_{pot} \). We can estimate the statistical accuracy of our volume-integrated energy estimates by adding different artificial noise models to the HMI and SP magnetograms, consecutive application of the preprocessing and extrapolation algorithms, and comparison of the resulting energy values. This yields a statistical error of \( \sim 1\% \) for both \( E_{pot} \) and \( E_{nlff} \) and \( \sim 10\% \) for \( \Delta E \).

The estimated absolute potential energy of the HMI model is \( \sim 48\% \) of that of the SPbin model (see Table 2). This is a direct consequence of the HMI NLFF bottom boundary hosting only \( \sim 55\% \) of the unsigned vertical flux of the SPbin bottom boundary (see Table 1). A similar trend is found for the absolute total and excess energy of the HMI model.

However, the relative excess energy is about \( 20\% \) of the total energy in both models (given by the ratio \( \Delta E/E_{nlff} \) in Table 2). An excess energy on the order of \( 10^{23} - 10^{24} \) J is assumed to be sufficient for powering C-class flaring, which was actually
Figure 4. Squashing degree, $Q$, at a height of $\sim 5$ Mm above the NLFF lower boundary as determined from the (a) HMI and (b) SPbin models, shown in the range (55 Mm < $x, y$). Field lines originating from locations where $Q > 100$ within the ROI (rectangular outlined area) and which have both footpoints located on the lower boundary of the HMI/SPbin model are shown in (c) and (e)/(d and f). The magnetic configuration when viewed along the surface normal is shown in (c) and (d). The gray-scale background reflects the vertical magnetic field, $B_z$, of the NLFF lower boundary. Black and white contours are drawn at [+50 mT, +100 mT] and [−50 mT, −100 mT], respectively. The view from solar south (along $y$) is shown in (e) and (f). Color-coded is the absolute height of the calculated field lines in Mm. (A color version of this figure is available in the online journal.)

observed for the active region analyzed here on the days before. Similar values related to C-class flaring activity were found by, e.g., Régnier & Priest (2007) and Thalmann et al. (2008) and lately by Gilchrist et al. (2012).

3.3. Magnetic Field Topology

Besides comparing a volume-integrated quantity like the magnetic energy, we are also interested how the modeled magnetic field configurations compare to each other. To do so and to ensure that we are looking at the same topological structure, we look for regions of strong gradients in the magnetic connectivity. They are thought of as being linked to the creation of strong current concentrations in the solar corona and believed to represent the footprint of quasi-separatrix layers (Priest & Démoulin 1995). We quantify the magnetic connectivity following Titov & Hornig (2002) and calculate the squashing degree $Q$ at a height of $\sim 5$ Mm above the NLFF lower boundary (Figures 4(a) and (b)). The squashing degree quantifies the eccentricity of an elliptical cross-section of a flux tube into which a flux tube of initially circular cross-section is transformed. Wherever $Q$ is large, the magnetic field connectivity changes drastically over short distances. According to this pattern, we choose a region of interest (ROI) around a clearly distinguishable pattern of high values of $Q$ (rectangular outline in Figures 4(a) and (b)). Though clearly visible in both models, the $Q$-ridge appears more diffuse in the SPbin model and its location differs up to $\Delta x \simeq 5$ Mm for a given $y$.

In total, 92/83 magnetic field lines in the HMI/SPbin model (1) start from locations where $Q > 100$ within the ROI and (2) connect back to the lower boundary (see Figures 4(c)–(f)). They qualitatively outline two neighboring connectivity domains which connect the negative polarity region to its neighboring positive polarity surrounding. The locations where the field lines connect back to the lower boundary are shown color-coded based on the local vertical field in Figure 5. We assume that the field lines we calculated represent thin flux tubes. For each flux tube, we choose its cross-section at the location from where we started the field line calculation, $dA$, as the size of one pixel (i.e., $dA \sim 360^2$ km$^2$). Furthermore, we assume the vertical

Figure 5. NLFF lower boundary of the (a) HMI and (b) SPbin models in the range (55 Mm < $x, y$). The vertical magnetic field is shown as a gray-scale background. Black and white contours are drawn at [+50 mT, +100 mT] and [−50 mT, −100 mT], respectively. The locations where field lines originate from regions of $Q > 100$ within the ROI (rectangular outline) re-enter the lower boundary are color-coded according to the vertical magnetic field there. (A color version of this figure is available in the online journal.)
field there determines the flux of the flux tube. Summation over all considered thin flux tubes gives an estimate of the total absolute shared flux for the HMI/SP\textsubscript{bin} model, where we find $|\phi| \approx 1.0/1.3 \times 10^{20} \text{Mx}$. We find a lower value of connected flux in the HMI model though more closed magnetic field lines are considered due to our selection criterion. However, the relative amount of connected flux linked by these field lines is comparable: $|\phi'|$ comprises $\sim 1\%$ of the unsigned vertical NLFF lower boundary flux as listed in Table 1.

The connectivity pattern of the field lines, as shown in Figure 5, explains the difference found for the connected flux. Numerous SP\textsubscript{bin} model field lines connect to the weak-field regions ($B_z \lesssim 50 \text{mT}$) between the two major positive polarity patches centered around $(x \approx 65 \text{ Mm}, y \approx 35 \text{ Mm})$ or to the weak-field surrounding (90 Mm $\lesssim x$, 10 Mm $\lesssim y$). However, none of the selected field lines of the HMI model does so; instead they re-enter the NLFF lower boundary at locations of strong vertical fields ($B_z \gtrsim 50 \text{mT}$). From Figures 4(c)–(f), one recognizes that a larger number of field lines connects to the positive polarity at $(x, y) \approx (120 \text{ Mm}, 5 \text{ Mm})$ and the highest field lines of the SP\textsubscript{bin} model reach up to greater heights than those in the HMI model.

To investigate if the latter represents a general trend or is biased due to our restrictive selection of start locations for field line calculation, we consider all field lines starting from a pixel location on the NLFF bottom boundary in the range (60 Mm $< x$, y) where $B_z > 2 \sigma_{B_h}$ and $B_y > 2 \sigma_{B_R}$ or $B_z > 2 \sigma_{B_h}$ and $B_y < 2 \sigma_{B_R}$. We then find very similar relationships of the length of the calculated field lines, $l_{FL}$, to the height of their apex, $h_{\text{max}}$, for both models (see Figure 6); the relation is well defined and to a first approximation linear (see also, e.g., Schrijver & Aschwanden 2002). The SP\textsubscript{bin} model field lines follow a steeper distribution, i.e., seem to be higher overall. It appears that the field lines carrying the strongest vertical fluxes are neither the shortest ones nor the longest ones. Instead, it seems that in both models, field lines with a length of 20 Mm $\lesssim l_{FL} \lesssim 100$ Mm and an apex height of 5 Mm $\lesssim h_{\text{max}} \lesssim 35$ Mm carry the most magnetic flux.

The longest and highest closed field lines in the HMI model are found to be $\sim 150$ Mm and 50 Mm, respectively. This is lower than the values found for the longest/highest closed field lines of the SP\textsubscript{bin} model ($\sim 170$ Mm/60 Mm). It is not surprising that the SP\textsubscript{bin} model field lines tend to be longer and reaching higher in the model atmosphere when looking at the magnetic field distribution on the lower boundary. There, we find an average of $(B_z/B_y) = 0.8/0.9$ for the HMI/SP\textsubscript{bin} model, i.e., the SP\textsubscript{bin} model boundary field is on average more vertical.

### 3.4. Effects of SP Data Binning on the Modeling

The binning of the original-resolution SP data (SP\textsubscript{orig}) to the resolution of HMI involves a two-dimensional interpolation and the resulting changes are discussed in the following. As before, we only consider data values fulfilling the criteria outlined in Section 2.3.

The binning causes a decrease of $|\phi|$ by $\approx 3\%$ (see Table 1). It also causes an increase of $\overline{B_z}_{\text{SP\textsubscript{bin}}}$ by $\approx 6\%$. The force-freeness remains basically the same, as do the relative occurrence of the vertical/horizontal field (Figures 7(a) and (b)) and the respective power distributions (Figures 7(c) and (d)) remain similar. Now it is also evident that the binning is only to a minor degree responsible for the differences between the SP\textsubscript{bin} and HMI data. This agrees with, e.g., Wang et al. (2009), who found that scaling the data to a lower resolution does not significantly alter the results of their particular analysis.

The SP\textsubscript{orig} vertical field is overall stronger than that of the HMI data as is the horizontal field. The HMI data host $\sim 53\%$ of $|\phi|_{\text{SP\textsubscript{orig}}}$ and $\sim 86\%$ of $\overline{B_z}_{\text{SP\textsubscript{orig}}}$ (see Table 1). Naively, one would suspect that the difference in $|\phi|$ arises from the different resolution limits of the two instruments, i.e., that the magnetic field is partially on scales that HMI cannot resolve. In contrast with the relative occurrence of $|\phi|$ discussed in Thalmann et al. (2012), however, this cannot be concluded since it was found that HMI vertical fields are overall weaker than those of VSM data (which with a plate scale of $\sim 1$ arcsec has a lower resolution than HMI). Therefore, the different occurrence rates must have reasons besides the different resolution limit of the instruments.

Preprocessing the SP\textsubscript{orig} data leads to a decrease of $|\phi|_{\text{SP\textsubscript{orig}}}$ by $\sim 3\%$ and increase of $\overline{B_z}_{\text{SP\textsubscript{orig}}}$ by $\sim 40\%$, which is comparable to the changes of the SP\textsubscript{bin} data due to preprocessing. The modification to the SP\textsubscript{orig} lower boundary data during solving for the force- and divergence-free fields, as listed in Table 1, is $|\phi|_{\text{SP\textsubscript{orig}}}$ decreases by $\sim 3\%$ and $\overline{B_z}_{\text{SP\textsubscript{orig}}}$ increases by $\lesssim 1\%$.

Summarizing, we find a similar behavior of the modifications to the SP\textsubscript{orig} data during solving for the force-free and divergence-free fields, as listed in Table 1, is $|\phi|_{\text{SP\textsubscript{orig}}}$ decreases by $\sim 3\%$ and $\overline{B_z}_{\text{SP\textsubscript{orig}}}$ increases by $\lesssim 1\%$.

The potential energy of the SP\textsubscript{bin} model is $\sim 1\%$ lower and the total energy is $\sim 2\%$ higher than that of the SP\textsubscript{orig} model (see Table 2). In other words, basing the magnetic field modeling on the SP\textsubscript{bin} data results in the modifications of the absolute energy estimates on the order of the statistical error of these estimates itself. The excess energy, $\Delta E$, is enhanced by $\sim 10\%$, i.e., also on the order of the statistical error. The relative amount of $\Delta E$, however, remains approximately the same.

Repeating the analysis of the magnetic connectivity within the SP\textsubscript{orig} model, we find that in total 228 magnetic field lines originating from locations of $Q > 100$ within the ROI in Figure 8(a) connect back to the lower boundary (see Figures 8(b) and (c)). Again, we assume that those represent thin flux tubes with a cross-section of $A \sim 220^2 \text{ km}^2$ and assume that the vertical field at the footpoint from which we started the field line calculation determines its flux. Summation over all the considered thin flux tubes gives an estimated total shared flux of $|\phi'| = 1.2 \times 10^{22} \text{ Mx}$ which comprises about $1\%$ of the unsigned vertical flux of the NLFF lower boundary of the SP\textsubscript{orig} model. This is almost identical to what was found for the model based on the SP\textsubscript{bin} data, as is the connectivity pattern (compare
Figure 7. Relative occurrence of the absolute (a) vertical field $B_z$ and (b) horizontal field $B_h$. The power of $B_z$ and $B_h$ as a function of length scale $l$ and wavenumber $k$ is shown in panels (c) and (d), respectively. SP bin data and original-resolution data (SP orig) are represented by gray and black lines, respectively. Only pixels with values $B_z > 2\sigma_{B_z}$ and $B_h > 2\sigma_{B_h}$ or $B_z > 2\sigma_{B_z}$ and $B_h < 2\sigma_{B_h}$ are taken into account.

Figure 8. (a) Squashing degree, $Q$, at a height of $\sim 5$ Mm above the NLFF lower boundary as determined from the closed SP orig model field lines, shown in the range $x \geq 55$ Mm. Field lines originating from locations where $Q > 100$ within the ROI (outlined rectangular area) and which have both footpoints located on the lower boundary are shown in panels (b) and (c). The magnetic configuration when viewed along the surface normal is shown in (c). The gray-scale background reflects the vertical magnetic field, $B_z$, of the NLFF lower boundary. Black and white contours are drawn at $[+50 \text{ mT}, +100 \text{ mT}]$ and $[-50 \text{ mT}, -100 \text{ mT}]$, respectively. The view from solar south (along $y$) is shown in (c).

(A color version of this figure is available in the online journal.)

Figures 9 and 5(b)). Considering all field lines starting from a pixel location on the NLFF bottom boundary in the range ($60 \text{ Mm} < x, y$) where $B_z > 2\sigma_{B_z}$ and $B_h > 2\sigma_{B_h}$ or $B_z > 2\sigma_{B_z}$ and $B_h < 2\sigma_{B_h}$, we find an identical relationship of $l_{FL}$ and $h_{\text{max}}$ (Figure 10) as was found for the SP bin model (compare Figure 6(b)). The SP orig model field is found to be even more vertical ($\langle B_z/B_h \rangle \sim 0.97$) and, again, the field lines carrying the strongest vertical fluxes are those with an intermediate length and apex height.

4. DISCUSSION AND CONCLUSIONS

Non-identical photospheric vector magnetic fields inferred from polarization measurements of different instruments used as input for NLFF coronal magnetic field models directly translate to substantial differences in the model outcome. To quantify these, we performed force-free magnetic field modeling using active region vector magnetic field data based on measurements of polarization signals by the SDO/HMI and Hinode SOT/SP. The possible causes of the differences of the

Figure 9. NLFF lower boundary of the SP orig model in the range ($55 \text{ Mm} < x, y$). The vertical magnetic field is shown as gray-scale background. Black and white contours are drawn at $[+50 \text{ mT}, +100 \text{ mT}]$ and $[-50 \text{ mT}, -100 \text{ mT}]$, respectively. The locations where field lines originate from regions of $Q > 100$ within the ROI (rectangular outline) re-enter the lower boundary are color-coded according to the vertical magnetic field there.

(A color version of this figure is available in the online journal.)
data products itself, including, e.g., the different intrinsic nature and sensitivity of the instruments, the temporal evolution of the photospheric magnetic field during the ongoing scanning times or the inversion techniques used to infer the magnetic field vector from the measured polarization signals, were beyond the scope of this work. Our aim was to compare force-free model results based on data from these two instruments so we applied all data preparation and modeling routines in exactly the same way to both data sets.

Force-free coronal magnetic field modeling is computationally expensive and high-resolution data are sometimes binned to a lower resolution in order to shorten the computational time. We, therefore, binned the original-resolution SP (SPorig) data to a lower resolution in order to shorten the computational time.

We did not intend to mimic the different spatial resolutions of the instruments by binning the vector data since, as pointed out by Leka & Barnes (2012), any kind of binning does not account for the modifications to the data due to application of our force-free modeling routines. Also, the modifications to the vertical flux was found to be more or less linearly related to the apex height. Also common to the model outcomes is that the shortest and lowest as well as the longest and highest field lines carry the least vertical flux.

In conclusion, caution is needed when analyzing the coronal magnetic field and its connectivity with the help of force-free magnetic field models based on the vector magnetic field products of different instruments, made available to the community. Relative estimates and the overall structure of the model magnetic fields might indeed be reliable while absolute estimates might only be so when concerning their order of magnitude. Moreover, binning of the magnetic vector data to a lower resolution prior to the force-free modeling results only in little differences in the model outcome, which are small compared to the remarkable deviations when basing the modeling on data from the two different instruments, HMI and SP.

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