The Moving Glass Phase of Dirty Type II Superconductors

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We study numerically the motion of vortices in dirty type II superconductors. In two dimensions at strong driving currents, vortices form highly correlated “static channels”. The static structure factor exhibits convincing scaling behaviour, demonstrating quasi long range translational order in the transverse direction. However order in the longitudinal direction is only short range. We clearly establish the existence of a finite transverse critical current, suggesting strong barriers against small transverse driving forces. These results are most consistently interpreted in terms of the recently proposed moving glass picture, modified to account for the strong anisotropy.

(30 April 1996)

PACS numbers: 74.25.Dw,74.60.Ge,74.60.Jg

Many condensed matter systems reach higher levels of organization by forming periodic media. Examples range from crystalline solids to Wigner crystals, charge density waves and vortex lattices in type II superconductors. A central issue is the effect of disorder on the stability of such systems. In the case of flux lattices, for weak disorder it seems sufficient to concentrate on elastic deformations [1] which were argued to lead to a power law decay of lattice correlations, when the periodicity is properly taken into account. This phase was christened a “Bragg-glass” [1]. The irrelevancy of topological excitations has been recently confirmed in three dimension, while the situation is marginal in two dimension [2]. For strong disorder topological excitations such as vortex loops can become relevant, giving rise to a Vortex glass [3]. While numerous additional scenarios have been proposed [4], there is growing experimental evidence supporting the basic picture of two types of glasses as the disorder or the magnetic field is increased [3,4].

Upon increasing the external force beyond a certain depinning strength, these periodic (or glassy) media become mobile. Early work developed perturbation studies at high velocities $v$ in powers of $1/v$ [5,6]. Recently Koshelev and Vinokur argued that the effect of the random potential is seriously weakened at high velocities, as it “averages out”, thus reestablishing the long range solid order [5]. Their numerical simulations suggested that this change occurs abruptly, giving rise to a genuine dynamic phase transition. On the other hand Giamarchi and Le Doussal pointed out that some components of the disorder remain unsuppressed, as they represent static perturbations [6]. These destroy the moving solid, and stabilize a glassy phase instead with quasi-long range order (QLRO) only, giving rise to a moving glass. Balents and Fisher, in their study of the related charge density wave systems also find that the moving phase possesses QLRO only, but may nevertheless support true long range temporal order [7,8]. The physics of the moving glass phase is [10] that the vortices form highly correlated static channels. This picture leads to a logarithmic decay at large distances for the displacement correlations. It also gives rise to diverging potential barriers and consequently a finite critical current against an additional transverse current.

In this Letter, we report a detailed numerical study of the moving glass phase. We establish that the basic picture of “static channels” is indeed characteristic of the system. By studying the static structure factor we demonstrate the existence of quasi long range translational order with an algebraic decay in the transverse direction but find only short range longitudinal order, giving rise to a very anisotropic glass. As a direct consequence we predict the absence of a narrow band noise for moving glasses in the 2D vortex systems. Finally the existence of a critical transverse current will be clearly established.

We employ overdamped Molecular Dynamics (MD) simulations at zero temperature to study two dimensional interacting vortices in the presence of point disorder,

$$\gamma \frac{d\mathbf{r}_i}{dt} = \sum_{j \neq i} \mathbf{F}_v(\mathbf{r}_i - \mathbf{r}_j) + \sum_j \mathbf{F}_{pin}(\mathbf{r}_i - \mathbf{r}_j) + \mathbf{F}_L \quad (1)$$

Here $\gamma$ is the damping parameter, $\mathbf{R}_j$ specifies the pinning center positions, $\mathbf{r}_i$ denotes the location of the $i$-th vortex, and $\mathbf{F}_L$ is the Lorentz force, exerted by the external driving current. The force between vortices is given by

$$\mathbf{F}_v(\mathbf{r}) = F_0(1 - \tilde{r}^2)^2 \frac{\mathbf{r}}{r^2}, \quad (2)$$

where $\tilde{r} = r/R_{cut}$, $F_0 = V_0/R_{cut}$, and we choose $R_{cut} = 3.6a_0$, where $a_0$ is the mean vortex spacing. Here $\gamma, a_0$, and $V_0(\approx \Phi_0^2/8\pi^2\lambda^2)$ define the units of time, length and energy, respectively. The pinning force is taken as

$$\mathbf{F}_{pin}(\tilde{r}) = -4F_p(1 - \tilde{r}^2)^2 \tilde{r} \quad (3)$$

Here $\tilde{r} = r/R_{pin}$ and $R_{pin} = 0.25a_0$. We worked with a fixed density of pinning centers of $5.77/a_0^2$. This set
of parameters was chosen to optimize the convergence of the numerical procedure.

Now we construct the phase diagram in the driving force - pinning strength plane, at zero temperature. With increasing driving currents three phases emerge: a pinned glass, a plastic flow regime, and some kind of an ordered phase. At low forces the vortices remain pinned, forming a glassy phase. As the Lorentz force is increased beyond a critical value $F_d$, the vortices depin. This transition, and in particular the value of $F_d$ can be well captured by studying the current-voltage (IV) characteristics. The resulting values of $F_d$ were used to construct the lower phase boundary in Fig.1. For strong disorder, just above $F_d$ vortices form a pattern of pinned and unpinned regions, often described as “plastic flow”. In this regime $F_d$ scales linearly with the pinning strength, whereas for weak disorder the relation is quadratic. The near-linearity of the phase boundary in Fig.1 indicates that we concentrated on the regime of strong disorder.

Upon further increase of the driving force, Koshelev and Vinokur argued that the effects of the disorder on the lattice displacements “average out”, leading to solid ordering in 3D, and quasi long range translational order in 2D. Alternatively, Giamarchi and Le Doussal suggested that the vortex system forms a moving glass. To distinguish between these propositions we first determine the location of the phase boundary, then explore the physics of the high-velocity phase.

The phase boundary $F_g$ between the plastic flow regime and the putative moving solid can be established by measuring the static structure factor $S(k)$. In the plastic flow regime the absence of ordering manifests itself in a central peak and a structureless ring (see lower panel in Fig.2). In the high velocity phase $F_L > F_g$, one expects to see six-fold coordinated Bragg peaks, if a moving solid is formed. We do indeed observe a sharp transition into a phase with well developed peaks; however the peak pattern is strongly anisotropic (upper panel in Fig.2).

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**FIG. 1.** The dynamic phase diagram. $F_p$ is the pinning strength, $F_L$ is the Lorentz force. Open circles represent $F_g$, solid circles $F_d$.

**FIG. 2.** The static structure factor $S(k)$. $F_L = 0.6$ in the upper and $0.2$ in the lower panel. The number of vortices is 960 and the disorder strength $F_p = 0.16$. The critical force $F_g$ indicating the transition from the plastic flow regime to the moving glass is about 0.35.

The upper phase boundary in Fig.1 was determined by mapping out $F_p$ for several disorder strengths. One can see that for strong disorder indeed all three expected phases are observed, whereas for weak disorder there is no robust evidence for an intervening plastic flow regime. Either that phase occupies a very slim region in parameter space, or there is a direct pinned Bragg glass-to-moving glass transition. This transition is much harder to identify because both phases exhibit quasi long range order, and thus the structure factors are very similar in the two phases.
The central issue of our paper is to elucidate whether at high velocities the system supports a moving solid or a moving glass. To address this issue we first analyse $S(k)$, 

$$S(k) = \frac{1}{L^d} \sum_{i,j} e^{i k \cdot (r_i(t) - r_j(t))}. \tag{4}$$

The pinning strength $F_p$ is fixed to be 0.16 and the applied force $F_L = 0.6$ is well above the corresponding critical force $F_G \approx 0.35$. We simulate five different system sizes with fixed vortex density and number of vortices ranging from 240 to 1500. The initial configurations are chosen randomly. We let the MD simulations evolve with time, make sure that the system reaches its steady state, then freeze the vortex configuration and measure $S(k)$. In the steady state the vortices form an orderly array. Its principal lattice vector in most cases is aligned with the direction of motion. It was argued that the system chooses such an orientation to minimize the power-dissipation [8]. However the details of this alignment have yet to be understood. The peaks at the reciprocal lattice vectors perpendicular to the motion exhibit quite convincing power law behavior. The height of the other peaks however decay very rapidly with increasing system sizes, suggesting that the corresponding correlations are short ranged.

$$S(\delta k, L) = L^{2-\nu_s} G(\delta k L), \tag{5}$$

with $\delta k = |k - G_0|$. In Fig. (3) the scaling function $G(x)$ is plotted with respect to the dimensionless scaling variable $x = \delta k L$ for the five system sizes. The peak amplitude scales with the system size as $L^{2-\nu_s}$ with $\nu_s = 0.53 \pm 0.1$, as shown in the inset of Fig. (3). Using this value of $\nu_s$ the normalized structure factor exhibits a convincing data collapse onto a single curve. This confirms the scaling behaviour $S(k) \approx |k - G_0|^{-1.47}$ around the peaks.

This implies that phase slips in the transverse direction are strongly suppressed, justifying the elastic approach. In contrast, the peaks at momenta with nonzero longitudinal components decay rapidly with system size. Several physical mechanisms can lead to such a decay. Extensive study of single snapshots of the spatial distribution of vortices suggests that phase slips between longitudinal boundaries of elastic domains are primarily responsible. This observation questions the elastic theory for this direction. Clearly a more complete understanding is needed on this issue, especially in two dimension [12].

A measurable consequence of the absence of translational order in the longitudinal direction should be the corresponding absence of narrow band noise in 2D driven vortex systems. The same conclusion was reached for the analogous CDW models in 2D in Ref. [11].

FIG. 3. Finite size scaling of the static structure factor $S(k)$. The driving force $F_L = 0.6$ is well above the critical force $F_G \sim 0.35$. The inset shows power law dependence of the peak heights with varying system size: $S_p(L) \sim L^{2-\nu_s}$ with $\nu_s \sim 0.53 \pm 0.1$.

To study the peaks of $S(k)$ at the reciprocal lattice vector $G_0 = (0, \pm 4\pi/\sqrt{3})$, we write down the following finite size scaling form:

$$S(\delta k, L) = L^{d-\nu_s} G(\delta k L), \tag{5}$$

FIG. 4. The static channels in the steady state. The small fluctuations transverse to the channels are due to the perturbation from the rapidly varying time-dependent component of the disorder, viewed in the rest frame of vortices.

The scaling behaviour of $S(k)$ does not distinguish between a quasi-solid and a moving glass. The proposition of the glassy phase rests on the argument that certain components of the disorder do not average out, but present a static perturbation. If so, the moving vortices should form static channels, which do not change their shape with time. To study this we now map out the
trajectories of the vortices. Making sure that the flow reached its steady state, we take a large number of consecutive snapshots, which are then displayed on top of each other. The resulting Fig. 4 clearly demonstrates the formation of static channels. The very existence of these time-independent channels lends strong credence to the moving glass picture.

The moving glass picture also implies the existence of diverging barriers against small transverse currents, leading to a finite transverse critical current. This critical current is the largest when one of the principal lattice vectors is parallel to the motion [10].

We select 50 disorder realizations which lead to a steady state with one of its primitive lattice vectors parallel to the direction of the velocity. After the system reaches the steady state, a small additional transverse force is applied and the transverse velocity $v_y$ is measured. Fig. 5 clearly exhibits a finite critical transverse force $F_{y}^d \sim 0.006 \pm 0.001$. $F_{y}^d$ is much smaller than the longitudinal critical force $F_{y} \sim 0.1$, since the effective disorder strength is inversely proportional to the velocity of the moving glass [10]. Simulations for several different system sizes result in identical critical currents, indicating that this threshold behaviour is not a finite size effect [13][13].

In sum, we explored the moving phase of vortex systems at high velocities. We measured the IV characteristics to determine the critical depinning force $F_{d}$, establishing the phase boundary between the pinned glass and the plastic flow regime. Next we studied the structure factor $S(k)$, which exhibited a sharp transition from its ring shape in the plastic flow regime into a phase with an anisotropic peak pattern. The Bragg peaks suggest the existence of power law order in the transverse directions, but only short range longitudinal order. Thus we expect the absence of a narrow band noise. We demonstrated the formation of static channels, and found a finite critical transverse current. These results are consistent with the suggestion that the driven vortex system in two dimension forms an anisotropic moving glass.

We acknowledge useful discussions with T. Giamarchi, V. Vinokur, L. Balents, A. Koshelev and P. LeDoussal. K. Moon wishes to thank S. M. Girvin for generously allowing access to his computing facilities. This work has been supported by NSF-DMR-95-28535 and by the LACOR program of the Los Alamos National Laboratory.

![Fig. 5. IV characteristics, showing transverse velocity $v_y$ as a function of transverse force $F_y$. Here $F_L = 0.6$. The existence of a finite critical force $F_{y}^d \sim 0.006 \pm 0.001$ is clear. The straight line represents a free flux flow response.](image)

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