Ranging-Measurements-Based Maneuver Detection Method for Space Target Tracking

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Abstract. Maneuver detection is crucial to improve the performance of space target tracking. A real-time maneuver detection method based on ranging measurements is proposed in this paper. The prediction residual of ranging measurement is selected as the test statistics. Then the maneuver and the outliers can be distinguished by setting multiple hypothesis tests. The superiority of the proposed method is verified by a simulation case. Simulation results demonstrate that the new method can provide higher tracking accuracy than the traditional methods when the maneuver and the outliers both exist, and the maneuver and the outliers can be detected exactly.

1. Introduction
With the increasing number of in-orbit spacecraft, the crowded space environment poses a great threat to the implementation of space missions. Space situation awareness systems can track the space targets and broadcast collision warnings, which is significant to space security [1]. For the non-cooperative targets tracking, the unknown orbital maneuver will reduce the tracking accuracy of the target and even lead to tracking failure. In order to achieve continuous tracking of the target, it is necessary to detect the target orbit maneuver event timely.

The detection methods of space target maneuvers can be divided into two categories: detection method based on historical data analysis and detection method based on real-time observation data [2-4]. The detection based on the analysis of historical data is carried out on the premise that the orbit of the target has been determined. The historical orbit data of the target is an important data source to detect the target maneuver. By detecting the maneuver in historical data, the maneuvering pattern and maneuvering ability of the target can be analyzed, which can provide important references for orbit data processing [5-7]. However, this kind of method has a disadvantage in real-time. The strong tracking filters (STF) are often applied in maneuvering target tracking [8, 9]. The fading factor used in STF can provide a reference for maneuver detection. This detection method is nearly real-time, but the outliers will lead to the false alarm.

In this paper, a real-time maneuver detection method based on ranging measurements is proposed. The prediction residual of ranging measurement is selected as the test statistics. Then the maneuver and the outliers can be distinguished by setting multiple hypothesis tests. Otherwise, the dynamics and measurement models of space target tracking system are described in Section 2; The implementation process of the proposed maneuvering detection method is illuminated in Section 3; In Section 4, the
tackling performances of the proposed method and the traditional method are compared by the simulation cases; Finally, the conclusions are given.

2. Space Target Tracking Model

2.1. Dynamics Model

The dynamics model is expressed by

\[
\begin{bmatrix}
\dot{\mathbf{r}} \\
\dot{\mathbf{v}}
\end{bmatrix} = \begin{bmatrix}
\mathbf{v} \\
\mathbf{a}
\end{bmatrix}
\]  

(1)

and

\[
a = \mathbf{a}_T + \mathbf{a}_J + \mathbf{w}
\]  

(2)

\[
a_T = -\mu_e \frac{\mathbf{r}}{||\mathbf{r}||}
\]  

(3)

\[
a_J = \frac{3}{2} J_2 \left( \frac{R}{||\mathbf{r}||} \right)^2 \begin{bmatrix}
\mu_x \\ 5 \frac{z}{||\mathbf{r}||} - 1 \\
\mu_y \\ 5 \frac{y}{||\mathbf{r}||} - 1 \\
\mu_z \\ 5 \frac{z}{||\mathbf{r}||^2} - 3
\end{bmatrix}
\]  

(4)

where \( \mathbf{r} \) and \( \mathbf{v} \) are the position and velocity of the target in the Earth-centered inertial coordinate, \( \mathbf{r} = [x, y, z]^T \) and \( ||\mathbf{r}|| = \sqrt{x^2 + y^2 + z^2} \); \( \mathbf{a} \) is the acceleration and \( J_2 \approx 0.00108263 \).

2.2. Measurement Model

As shown in Figure 1, the measurement of ground-based radar is expressed by \( \rho, \theta \) and \( \phi \). In which, \( \rho \) is the range from the observation station to the target, \( \theta \) and \( \phi \) are the azimuth and elevation, respectively. The measurement model is given by

\[
\begin{align*}
\rho &= \sqrt{(x - x_{\text{radar}})^2 + (y - y_{\text{radar}})^2 + (z - z_{\text{radar}})^2} + \nu_\rho \\
\theta &= \tan^{-1}\left( \frac{y - y_{\text{radar}}}{x - x_{\text{radar}}} \right) + \nu_\theta \\
\phi &= \tan^{-1}\left( \frac{z - z_{\text{radar}}}{\sqrt{(x - x_{\text{radar}})^2 + (y - y_{\text{radar}})^2}} \right) + \nu_\phi
\end{align*}
\]  

(5)

where \( \mathbf{r}_i = [x_{\text{radar}}, y_{\text{radar}}, z_{\text{radar}}]^T \) is the position of the ground-based observation station. \( \mathbf{v} = [\nu_\rho, \nu_\theta, \nu_\phi]^T \) is the measurement noise. Suppose \( \nu_\rho, \nu_\theta \) and \( \nu_\phi \) are independent zero-mean Gaussian noises, and their variances are \( \sigma_\rho^2, \sigma_\theta^2 \) and \( \sigma_\phi^2 \).
2.3. Strong Tracking Filter

Let us focus on this nonlinear dynamical system

\[ x_k = f(x_{k-1}) + w_{k-1} \]
\[ z_k = h(x_k) + v_k \]

in which \( x_k \) is the state variable at \( t_k \), \( z_k \) is the measurement, \( w_{k-1} \) and \( v_k \) both are zero-mean Gaussian noise. They are independent and their covariance are \( Q_{k-1} \) and \( R_k \).

Among all kinds of strong tracking filters, strong unscented Kalman filter (SUKF) is the most widely used. The performance degradation of the tracking filter is caused by dynamics model error during maneuvers. In order to solve this problem, the \( P_{i|k-1} \) is inflated by a fading factor \( \alpha_k \) to reduce the contribution of \( x_{i|k-1} \) to \( \hat{x}_k \). Suppose the estimation state and the covariance matrix at \( t_{k-1} \) are \( \hat{x}_{k-1} \) and \( P_{k-1} \). The process of the SUKF is as follows [10]:

Step 1: The sigma points and the weighs update.

The sigma points, \( \{ \delta_{i|k-1} | \tau = 0, L, 2m, k \geq 1 \} \), can be obtained by

\[
\begin{align*}
\delta_{0|k-1} &= \hat{x}_{k-1} \\
\delta_{i|k-1} &= \hat{x}_{k-1} + \sqrt{m+\theta} \cdot \left( \sqrt{P_{k-1}} \right)_\tau & \tau = 1, 2, L, m \\
\delta_{i+n|k-1} &= \hat{x}_{k-1} - \sqrt{m+\theta} \cdot \left( \sqrt{P_{k-1}} \right)_\tau & \tau = m+1, m+2, L, 2m
\end{align*}
\]

in which \( \theta = 3\alpha^2 - m \), \( m \) means the dimension of the state variable, where \( 0 \leq \alpha < 1 \), and \( \left( \sqrt{P_{k-1}} \right)_\tau \) is \( \tau \)th row for the Cholesky factor of \( P_{k-1} \).

The weight for the mean value of the sigma points are \( \omega_i^m \), and the weight for the covariance value of the sigma points are \( \omega_i^c \). They are given by

\[
\begin{align*}
\omega_0^m &= \frac{\lambda}{m+\lambda} \\
\omega_i^m &= \frac{\lambda}{m+\lambda} + \left( 1 - \alpha^2 + \beta \right) \\
\omega_i^c &= \frac{1}{2(m+\lambda)}, \quad \tau = 1, L, 2m
\end{align*}
\]
where $\beta$ is generally set as 2.

Step 2: Time update:

The predicted estimation state and the corresponding covariance matrix are given by

$$\delta_{r,k|k-1} = f(\delta_{r,k-1})$$

$$x_{k|k-1} = \sum_{r=0}^{2n} \omega_r^x \delta_{r,k|k-1}$$

$$P_{kk|k-1} = \sum_{r=0}^{2n} \omega_r^x [\delta_{r,k|k-1} - x_{k|k-1}] [\delta_{r,k|k-1} - x_{k|k-1}]^T + Q_{k-1}$$

Step 3: Fading factor update:

$$\alpha_k = \begin{cases} 
\alpha_{k,0} & \alpha_{k,0} > 1 \\
1 & \alpha_{k,0} \leq 1
\end{cases}$$

$$\alpha_{k,0} = tr[C_k - R_k]/tr[P_{xx,k} - R_k]$$

and

$$P_{xx,k} = \sum_{r=0}^{2n} \omega_r^x [z_{r,k|k-1} - \hat{z}_{r|k-1}][z_{r,k|k-1} - \hat{z}_{r|k-1}]^T + R_k$$

$$C_k = \begin{cases} 
\xi_k (\xi_k)^T & k = 1 \\
\sigma C_{k-1} + \xi_k (\xi_k)^T & k > 1,
\end{cases}$$

$$\xi_k = z_k - h(x_{k|k-1})$$

where $\sigma$ is set as 0.95.

Step 4: Measurement update:

$$P_{zz,k} = \alpha_k \cdot \sum_{r=0}^{2n} \omega_r^x [z_{r,k|k-1} - \hat{z}_{r|k-1}][z_{r,k|k-1} - \hat{z}_{r|k-1}]^T + R_k$$

$$P_{xz,k} = \alpha_k \cdot \sum_{r=0}^{2n} \omega_r^x [\delta_{r,k|k-1} - x_{k|k-1}][z_{r,k|k-1} - \hat{z}_{r|k-1}]^T$$

$$\hat{P}_{kk,k-1} = \alpha_k \cdot P_{xx,k-1}$$

$$K_k = P_{xz,k} P_{zz,k}^{-1}$$

$$\hat{x}_k = x_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1})$$

$$P_{k} = \hat{P}_{kk,k-1} - K_k P_{xz,k} K_k^T$$

3. Space Target Maneuver Detection

3.1. Analysis of Ranging Measurements
In this paper, the maneuver detection is based on the ranging measurements of ground-based radar. Therefore, we need to analyze the time-varying characteristics of the ranging measurements under different conditions. As shown in Figure 2, the ordinate is the difference between the ranging measurements under different conditions and the measurements without errors. As we can see, the value of the difference is zero-mean Gaussian noise when no outliers and maneuvers occur. The value of the difference is growing when a maneuver occurs. Otherwise, the value of the difference exhibits remarkable jumps when the outliers occur. All above, these time-varying characteristics can provide a reference for maneuver detection.

3.2. Maneuver Detection
The process of the proposed maneuver method is as follow:

Step 1: Prediction residual of ranging measurement update:

The predicted ranging measurement ($\hat{\rho}_{k+N+1}$) at time $t_{k+N+1}$ can be obtained by using quadratic fitting when $\rho_{k+1}, \rho_{k+2}, \ldots, \rho_{k+N}$ is given. In which, $N$ is the length of detection window. Then the prediction residual of measurement can be updated by

$$\delta\rho_{k+N+1} = \rho_{k+N+1} - \hat{\rho}_{k+N+1}$$

Step 2: Single outliers detection:

When there are no outliers and maneuver, the prediction residual of measurement obeys the follow distribution

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**Figure 2.** The difference between the ranging measurements under different conditions and the measurements without errors.
\[ \delta p_{k,N+1} \sim N\left(0, \sigma_p\right) \]  

(24)

where \( \sigma_p \) is the ranging measurement variance.

The single outliers detection can be performed as

\[ 
\lambda_{k,N+1}^{\text{out}} = \begin{cases} 
0, & |\delta p_{k,N+1}| < 5\sigma_p, \text{ no outliers} \\
1, & |\delta p_{k,N+1}| \geq 5\sigma_p, \text{ outliers} 
\end{cases} 
\]

(25)

Step 3: Maneuver or continuous outliers detection:

When \( \delta p_{k,N+1} \) continuously exceed the threshold (\( \lambda_{k,N+1}^{\text{out}}=1 \) and \( \lambda_{k,N+2}^{\text{out}}=1 \)), we need to determine whether maneuver or continuous outliers occur by the following equation.

\[ 
\lambda_{k,N+2}^{\text{man}} = \begin{cases} 
0, & |\delta p_{k,N+2}| - |\delta p_{k,N+1}| < 5\sigma_p, \text{ continuous outliers} \\
1, & |\delta p_{k,N+2}| - |\delta p_{k,N+1}| \geq 5\sigma_p, \text{ maneuver} 
\end{cases} 
\]

(26)

Step 4: Target tracking:

The proposed detection method can be used to improve the performance of SUKF. The outliers will be rejected when outliers detection alarms, while the fading factor will be added when maneuver detection alarms.

4. Simulation Results Analysis

The simulation scenario is to track a low Earth orbit (LEO) target via a ground-based radar. The longitude and latitude of radar are 110° E and 30° N, and its altitude is 500 m. The ranging error is 1 m, and the angle error is 30 arcsec. The simulation time is 1000 s. Moreover, the initial orbital elements for the target are as follows.

| a/km | e  | i/° | \( \Omega \)/° | \( \omega \)/° | \( f \)/° |
|------|----|-----|-------------|-------------|--------|
| 7371 | 0  | 63.41| 0           | 0           | 10     |

In this case, the target operates orbital maneuver at time step of 301 s. The characteristic velocity is 10 m/s. Otherwise, the measurement outliers occur at time steps of 501 s and 701–705 s.

The new filter and the SUKF are compared in Figure 3. As we can see, the new filter can reduce the impact of the outliers by adopting a detection method. The detection results are given in Figure 4. The proposed method can detect the maneuver in nearly real-time, and the single or continuous outliers also can be distinguished. The time histories of the fading factor for the proposed method and SUKF are shown in Figure 5. We know that the effectiveness of the strong tracking filter is improved by adding maneuver detection.
5. Conclusions
In this paper, we propose a real-time maneuver detection method based on ranging measurements when the outliers are considered. The traditional SUKF can be improved by applied this detection method because the maneuver and the outliers can be distinguished and handled. Simulation results show that the new tracking filter performs better than the SUKF when the maneuver and the outliers both exist, and the maneuver and the outliers can be detected exactly.

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