Realistic coasting cosmology from the Milne model

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Abstract

In the context of the recent synchronicity problem in ΛCDM cosmology, coasting models such as the classic Milne model and the $R_h = ct$ model have attracted much attention. Also, a very recent analysis of supernovae Ia data is reported to favour models with constant expansion rates. We point out that the nonempty $R_h = ct$ model has some known antecedents in the literature. Some of these are published even before the discovery of the accelerated expansion and were shown to have none of the cosmological problems and also that $H_0 t_0 = 1$ and $\Omega_m/\Omega_{\text{dark energy}}$ is some constant of the order of unity. In this paper, we also derive such a model by a complex extension of scale factor in the Milne model.

Key words: cosmology:observations – cosmology:theory

1 INTRODUCTION

The synchronicity problem (Avelino & Kirshner 2016) is the most recent among a series of problems that cropped up in the ΛCDM cosmology during the past few decades. Several problems were identified in cosmology during the 1980s, but the theory of inflation (Guth 1981) solved most of them at one stroke. This theory, which speculates quantum field theoretical effects in the early epochs, suggests that the present universe is flat. But when the condition of flatness is combined with the measured high value of the Hubble parameter, there arose an ‘age problem’ in the middle of the 1990s. The discovery of the accelerated...
expansion in 1998 predicted a cosmological constant $\Lambda$ or some unknown dark energy in the present universe. This discovery resolved the age problem, but in its turn, seeks an explanation to the near equality of the energy density corresponding to such $\Lambda$ or dark energy and the matter density in the present universe. This is the ‘coincidence problem’ in cosmology, for which the $\Lambda$CDM model offers no satisfactory solution yet. Recently, three important cosmological observations, namely, the apparent magnitude and redshift of Type Ia supernovae (SN Ia), CMB power spectrum, and baryon acoustic oscillations, together predict that the dimensionless age $H_0 t(a)$ of the present universe is very close to unity (Avelino & Kirshner 2016).

This is puzzling since according to the $\Lambda$CDM model, the product $H_0 t$ could have values very different from unity; it can be anywhere in the range $0 < H_0 t(a) < \infty$. Except during the period of inflation, in the past or future of the universe, this value shall not be unity either. Just like the coincidence problem, this synchronicity problem in the $\Lambda$CDM model is also regarding the special status ascribed to the epoch in which we live today.

On the other hand, the classic Milne model (Milne 1935) has no synchronicity problem. When viewed as a Friedmann model, its scale factor obeys $a \propto t$. In this case, the present dimensionless age of the universe $H_0^{-1}$ would exactly correspond to the actual age $t_0$, and this result will always remain valid. Hence in this cosmological model, $H_0 t_0 = 1$ is not a problem; instead, this is its prediction. In (Avelino & Kirshner 2016), it is mentioned that the Milne model is the most suitable one to explain the above observational result, but it is soon rejected on grounds that the model is empty and has negatively curved space sections (i.e., $\rho = 0$ and $k = -1$). This is quite reasonable, for an empty model is not a realistic one.

Very recently, an analysis of Type Ia supernova data (Nielsen et al. 2016) has appeared with the result that there is only marginal evidence for the widely accepted claim of the accelerated expansion of the universe. By a rigorous statistical analysis using the Joint Lightcurve Analysis (JLA) catalogue of 740 SN Ia, it is found that the SN Ia Hubble diagram appears consistent with a uniform rate of expansion. This brings the Milne model again to the centre-stage of cosmology, albeit in some new avatar.

Recently, an $R_h = ct$ cosmological model (Melia & Shevchuk 2012), which has an expansion history coinciding with that of the Milne model, is suggested as the true model of the universe (Melia 2012; Melia & Maier 2013). It is claimed that one-on-one comparative tests carried out between this model and the $\Lambda$CDM model using over 14 cosmological measurements and observations give conclusive evidence in support of the $R_h = ct$ model (Melia 2015). In the context of the latest synchronicity problem in cosmology and the analysis of JLA catalogue mentioned above, this model assumes great significance. Possibly due to this, in the literature, there are growing concerns regarding the fundamental basis of the theory itself (Bilicki & Seikel 2012; Mitra 2014). For instance, the latest paper on this subject (Mitra 2014) contends that $R_h = ct$ is a vacuum solution.
Addressing this criticism, it is shown in (Melia 2015) that the $R_h = ct$ model is not empty and hence not the same as the Milne model.

In view of the mounting supportive evidences obtained from the above analyses of most recent cosmological data (Avelino & Kirshner 2016; Nielsen et al. 2016; Melia 2015), one notes that models such as the above deserve continued scrutiny, both from the conceptual and observational fronts. In this paper, we first point out that there exist some antecedents (John & Joseph 1996, 1997, 2000) to the $R_h = ct$ cosmological model, though they were not referenced in (Nielsen et al. 2016; Melia & Shevchuk 2012; Melia 2012; Melia & Maier 2013; Melia 2015). As in the $R_h = ct$ model, the expansion in these nonempty, real models is ‘always coasting’ (i.e., in accordance with scale factor $a \propto t$, except possibly during or immediately after the Planck epoch) and this is due to the vanishing of gravitational charge $\rho + 3p$, rather than the vanishing of $\rho$. The most closely related antecedent is the one proposed in (John & Joseph 2000) (where $a = ct$ for $k = 0$). However, its difference in evolution history with that in (John & Joseph 1996, 1997) is only around the Planck epoch and is insignificant at the observational front. The vanishing of cosmological problems (John & Joseph 1996, 1997, 2000) and comparison of this model with the $\Lambda$CDM model using the then available SN Ia data (John & Narlikar 2002; John 2005, 2010) were discussed at length. It is unfortunate that such known published works are ignored in the recent literature on the possibility of constant expansion rate of the universe.

Following this discussion, in this paper we also show that the Milne model can lead to a nonempty, coasting cosmological model. This is done by a complex extension of scale factor in the former model. If we take the energy density in the nonempty model as the sum of matter density and a time-variable dark energy density and do not assume any arbitrary conservation law for these individual components, the coincidence problem can be seen to vanish in it. The new model is realistic and nonsingular and is always coasting after the Planck epoch. As a next step, a quantum cosmological treatment of this model is made, which provides the result that the minimum radius $a_0$ in it is of the order of the Planck length. The complexified Milne model has Euclidean (++++) signature at $t = 0$, but it changes to the usual Lorentzian one (+ - - -) immediately after the Planck epoch. We find that this leads to the widely speculated ‘signature change’ (Ellis et al. 1992; Hayward 1992) in the early universe.

The organisation of this paper is as follows. In Sec. 2, we discuss the antecedents to the $R_h = ct$ model and in Sec. 3, a new derivation of the flat, nonempty coasting model is presented. The quantum cosmological treatment of this model is made in Sec. 4. The last section comprises a discussion of the results.
2 ANTECEDENTS TO THE $R_H = CT$ MODEL

In a well-known work, Kolb (1989) has discussed at length the implications of a coasting cosmological model. In this, the universe is presently dominated by some exotic K-matter and is coasting, with equation of state $p_K = -\rho_K/3$. However, the model is different from the $R_h = ct$ or other ‘always coasting’ models, for during the major part of its history in early epochs, the universe is dominated by radiation and matter and hence its expansion rate is not uniform.

A nonempty, closed cosmological model that is coasting throughout the history of the universe after the Planck epoch was first proposed in 1996 (John & Joseph 1996, 1997). This model coincided with the widely discussed Ozer-Taha model (Ozer & Taha 1986, 1987) at the earliest epochs. The attempt by Ozer and Taha was to put forward an alternative to the theory of inflation, which solves the flatness problem, horizon problem, etc. They obtained a bouncing, nonsingular solution with $a = \sqrt{(a_0^2 + t^2)}$, and speculated that $a_0$, the minimum radius of the universe, has some small value. After the bounce, the model reaches the $a \propto t$ phase for some time, but soon deviates from it to enter the decelerating standard big bang evolution and continued in it. On the contrary, the antecedent to the $R_h = ct$ model mentioned above in (John & Joseph 1996, 1997) has the evolution $a = \sqrt{(a_0^2 + t^2)}$ throughout the cosmological history. Moreover, a quantum cosmological treatment gave the result that the minimum radius $a_0$ is of the order of Planck length. These models were shown to have none of the cosmological problems, such as singularity, horizon, flatness, monopole, cosmological constant, size, age, etc.

The more closely related antecedent in (John & Joseph 2000) is an always coasting cosmology with $a \propto t$, obtained by extending the dimensional argument of Chen & Wu (1990). This modified Chen-Wu model, which can have $k = 0, \pm 1$, has both matter and a time-varying cosmological constant. Comparison of this model with the $\Lambda$CDM model was performed using the SNe Ia data, with the help of the Bayesian theory (John & Narlikar 2002). The results showed that the evidence against the coasting model, when compared to the $\Lambda$CDM model, is only marginal. In 2005, an analysis was again performed (John 2005) using the then available SNe data to see whether the data really favours a decelerating past for the universe. Again, the conclusion was that the evidence is not strong enough to discriminate this case from a coasting cosmology. Recently, a quantum cosmological treatment (John 2015) showed that a coasting solution is unique, since it has identical classical and quantum evolution, as in the case of free particles described by plane waves in ordinary quantum mechanics. The $R_h = ct$ model coincides with the coasting model in (John & Joseph 2000), for $k = 0$.

Some other realistic coasting models have also appeared in the literature since then. Nucleosynthesis in a different coasting model was discussed by Lohiya and co-workers in (Sethi et al. 1999; Lohiya & Sethi 1999; Batra et al. 2000). This issue is
pursued in several works, including some recent ones (Singh & Lohiya 2015). They have also investigated the status of realistic coasting models with regard to other cosmological observations, such as the SNe Ia data (Dev et al. 2001).

The synchronicity problem does not arise in these coasting models due to their unique expansion history $a \propto t$. Here it is noted that such a model has vanishing gravitational charge $\rho + 3p$, or equivalently an equation of state $p = -\rho/3$. As described explicitly in the antecedents (John & Joseph 1996, 1997, 2000), the model is devoid of any coincidence problem too, when a proper accounting of its energy densities is made. One notes that the Einstein equation in general relativity implies a conservation law for energy-momentum, but this is valid only for the total energy density. If we assume that the cosmic fluid comprises of matter and a vacuum energy, and if we do not assume any arbitrary conservation law for the individual components, then there can be some interaction between them. In the present case, this may lead to creation of matter from vacuum energy, as can be seen from the fact that here both energy densities vary as $a^{-2}$. (However, the creation rate shall be so small that it would be impossible to detect such events at the present level of precision in observations.) Thus the coasting model, when assumed to contain a decaying vacuum energy, naturally leads to a constant ratio between the energy densities of matter and vacuum, and this ratio shall be of the order of unity. That this resolves the coincidence problem is noted also in (Melia & Fatuzzo 2016).

3 COMPLEX EXTENSION OF THE MILNE MODEL

Let us now extend the scale factor of the Milne model to the complex plane and denote it as $\hat{a}$. We can now show that this results in a coasting cosmology with real scale factor $a \equiv |\hat{a}|$. One can write the Friedmann equations corresponding to the empty Milne model as

\begin{equation}
\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2} = 0
\end{equation}

and

\begin{equation}
2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2} = 0.
\end{equation}

Using the polar form $\dot{a} = ae^{i\phi}$ in these equations and then equating their real parts, we get

\begin{equation}
\frac{\dot{a}^2}{a^2} = \dot{\phi}^2 + \frac{1}{a^2} \cos 2\phi,
\end{equation}
\[ \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = 3\dot{\phi}^2 + \frac{1}{a^2} \cos 2\phi. \] (4)

These appear as the Friedmann equations for a flat \((k = 0)\) model, filled with a homogeneous scalar field \(\phi\). This model is quite different from that of Milne, since the geometry of its space sections and the energy density are different. The right hand sides of the equations imply the presence of a scalar field with kinetic energy \(\dot{\phi}^2\) and potential energy \(\cos(2\phi)/a^2\). That is, we have a new cosmological model with real scale factor \(a = | \dot{a} |\), whose space sections are flat and which is nonempty.

Equating also the imaginary parts, we get two supplementary equations

\[ \ddot{\phi} + 2\dot{\phi} \frac{\dot{a}}{a} = 0, \] (5)
and

\[ 2\dot{\phi} \frac{\dot{a}}{a} = -\frac{1}{a^2} \sin 2\phi. \] (6)

These shall be of help in solving the system of equations (3) and (4). However, one can directly solve equations (1) and (2) to obtain \(\dot{a} = (c_1 \pm it) + ic_2\). Choosing the origin of time such that the constant of integration \(c_1 = 0\) and relabelling \(c_2 \equiv a_0\), we get the solution for \(\dot{a}\) as

\[ \dot{a} = \pm t + ia_0. \] (7)

The solution for \(a = | \dot{a} |\) can be seen to be

\[ a = \sqrt{a_0^2 + t^2}, \] (8)
and the argument \(\phi\) can be obtained as

\[ \phi = \tan^{-1} \left( \frac{a_0}{t} \right). \] (9)

This is a bouncing, nonsingular evolution as in the early phase of Ozer-Taha model (Özer & Taha 1986, 1987), with \(a_0\) as the minimum value for the scale factor. The time at which this minimum occurs for \(a\) is taken as \(t = 0\).

The energy density \(\rho\) and pressure \(p\) in the new model, as can be deduced from equations (3) and (4), have the following
evolution. At the epoch near $t = 0$, the kinetic energy of the field $\phi$ is dominant, leading to the nonsingular behaviour. This phase of evolution is capable of solving cosmological problems such as that related to the horizon. For large $t$, we have $\rho \propto a^{-2}$ and $\rho + 3p \approx 0$. The contribution for $\rho$ in the late universe comes almost entirely from the potential energy term. It may also be noted that its magnitude is very nearly equal to the critical density for the universe. One can explicitly write the variation of total density and pressure with scale factor as

$$\rho = \frac{3}{8\pi G} \left( \frac{1}{a^2} - \frac{a_0^3}{a^4} \right) \quad (10)$$

and

$$p = -\frac{1}{8\pi G} \left( \frac{1}{a^2} + \frac{a_0^2}{a^4} \right), \quad (11)$$

so that

$$\rho + 3p = -\frac{3}{4\pi G} \frac{a_0^2}{a^4}, \quad (12)$$

The term that causes the bounce to happen corresponds to a negative energy density, which can now be separated as

$$\rho_{-} = -\frac{3}{8\pi G} \frac{a_0^2}{a^4}, \quad (13)$$

The pressure due to this is given by $p_{-} = (1/3)\rho_{-}$, which is an equation of state characteristic of relativistic energy densities. This energy density becomes negligible for $a \gg a_0$, when compared to the rest of the energy densities.

Thus the real model is flat, with the total energy density and pressure obeying $\rho + 3p \approx 0$ for $a \gg a_0$. After this initial epoch, if the potential energy of the field comprises energy corresponding to radiation/matter and a time-variable dark energy, we can write $\rho = \rho_m + \rho_{d.e.}$. Taking $p_m = w \rho_m$ and $p_{d.e.} = -\rho_{d.e.}$, one obtains

$$\Omega_m \equiv \frac{\rho_m}{\rho_c} = \frac{2}{3(1 + w)}, \quad \Omega_{d.e.} \equiv \frac{\rho_{d.e.}}{\rho_c} = \frac{1 + 3w}{3(1 + w)}. \quad (14)$$

Since this universe is flat, the total density parameter $\Omega = \rho/\rho_c = 1$. When matter in the present universe is considered to be nonrelativistic with $w = 0$, the above equations predict $\Omega_m = 2/3$ and $\Omega_{d.e.} = 1/3$. The ratio between these densities is a constant of the order of unity throughout the expansion history (for $a \gg a_0$), and this avoids the coincidence problem. Being
a coasting evolution, it naturally has no synchronicity problem. It solves all other cosmological problems, as demonstrated in
the previous works.

With the aid of the solution (7), one can see that the complex-extended Milne model has Euclidean (++++) signature at \( t = 0 \), but it changes to the usual Lorentzian (+ - - -) one for \( a \gg a_0 \). Hence the complex extension of Milne model leads naturally to a signature change in the early universe, a possibility discussed extensively in the literature (Ellis et al. 1992; Hayward 1992). The famous Hartle-Hawking ‘no boundary’ boundary condition (Hawking 1984) in quantum cosmology envisages a change of signature in the early universe. In this regime, there is no time and the spacetime is purely spatial. Even in the classical Einstein equations, it is argued that the metric is Lorentzian not because it is demanded by the field equations; instead, it is a condition imposed on the metric before one looks for solutions (Ellis et al. 1992). In our case, the complex Milne model undergoes a signature change in a very natural way.

4 WHEELER-DEWITT EQUATION

In this section, we write down the Wheeler-DeWitt equation for the complex-extended Milne model. For this, we first note that the classical Milne model follows from the Lagrangian

\[
L = -\frac{3\pi}{4G}\left(\frac{\dot{\hat{a}}^2\hat{a}}{N} + N\dot{\hat{a}}\right) \tag{15}
\]

Here \( N \) is called the lapse function, for which one can fix some convenient gauge. Writing down the Euler-Lagrange equations with respect to the variables \( N \) and \( \hat{a} \) and fixing the gauge \( N = 1 \) leads to equations (1) and (2), respectively. The canonically conjugate momentum is

\[
\hat{\pi}_a = \frac{\partial L}{\partial \dot{\hat{a}}} = -\frac{3\pi}{2G N} \dot{\hat{a}} \tag{16}
\]

The canonical Hamiltonian can now be constructed as

\[
\mathcal{H}_c = \dot{\hat{a}}\hat{a} - L = N\left(-\frac{G}{3\pi a^2} + \frac{3\pi}{4G}\right) = N\mathcal{H} \tag{17}
\]
Quantisation of a classical system like the one above means introduction of a wave function \( \Psi(\hat{a}) \) and requiring that it satisfies (Kolb & Turner 1990)

\[
\frac{i\hbar}{\partial t} \frac{\partial \Psi}{\partial t} = \mathcal{H}_c \Psi = N \mathcal{H} \Psi. \tag{18}
\]

To ensure that time reparametrisation invariance is not lost at the quantum level, the conventional practice is to ask that the wave function is annihilated by the operator version of \( \mathcal{H} \); i.e.,

\[
\mathcal{H} \Psi = 0. \tag{19}
\]

Eq. (19) is called the Wheeler-DeWitt equation. It is analogous to a zero energy Schrödinger equation, in which the dynamical variable \( \hat{a} \) and its conjugate momentum \( \hat{\pi}_a \) are replaced by the corresponding operators. The wave function \( \Psi \) is defined on the minisuperspace with just one coordinate \( \hat{a} \) and we expect it to provide information regarding the evolution of the universe.

We may note here that the wave function is independent of time; it is a stationary solution in the minisuperspace.

The Wheeler-DeWitt equation for our case can be written by making the operator replacements for \( \hat{\pi}_a \) and \( \hat{a} \) in \( \mathcal{H} \). However, finding the operator corresponding to \( \hat{\pi}_a^2/\hat{a} \) is problematic due to an operator ordering ambiguity. We shall adopt the most commonly used form

\[
\hat{\pi}_a^2 \rightarrow -\hat{a}^{-r-1} \frac{\partial}{\partial \hat{a}} \left( \hat{a}^r \frac{\partial}{\partial \hat{a}} \right), \tag{20}
\]

where the choice of \( r \) is arbitrary and is usually made according to convenience. Using this expression with \( r = -1 \), we obtain the Wheeler-DeWitt equation for the complexified Milne model as

\[
\frac{d^2 \Psi}{d\hat{a}^2} - \frac{1}{\hat{a}} \frac{d \Psi}{d\hat{a}} + \frac{9\pi^2}{4G^2\hat{a}^2} \Psi = 0. \tag{21}
\]

This equation has an exact solution

\[
\Psi(\hat{a}) \propto \exp \left( \pm \frac{3\pi}{4G} \hat{a}^2 \right). \tag{22}
\]

Whether this solution corresponds to the classical evolution of the model can be determined by drawing the de Broglie-Bohm
trajectories for this wave function (John 2015). Identifying $\Psi \equiv R \exp(iS)$, one can draw these quantum trajectories by using the de Broglie equation of motion

\[ \dot{a} = \frac{\partial S}{\partial a} \]  

(23)

In the present case of quantum cosmology, while using (16), this equation of motion simply reads

\[ -\frac{3\pi^2}{2G}\dot{a} = \pm \frac{3\pi^2}{2G}\dot{a} \quad \text{or} \quad \dot{a} = \pm 1, \]  

(24)

which is the same classical equation (1). Hence, here we have the same classical and quantum trajectories, implying identical behaviour as in the case of free particles described by plane waves in ordinary quantum mechanics.

A notable feature here is the appearance of the factor $\sqrt{2G/3\pi}$ in the wave function (22). This has value very nearly equal to the Planck length. In all quantum gravity theories, a natural length scale is the Planck length. Hence, one can deduce that the value of $a_0$, the imaginary constant appearing in the scale factor of the complex-extended Milne model is also of this value. In turn, this is the minimum radius of the bouncing, real, coasting model. Thus we see that the nonsingular behaviour of the present coasting model is due to the quantum effects at the earliest moments within the Planck time.

5 DISCUSSION

The first coasting model for the universe, which was conceived by Milne, has been an attractive idea, primarily for its simplicity. Though it is an empty model and is devoid of any gravitational effects, the model is not forgotten even after three quarters of a century. The nonempty ‘always coasting’ cosmology in (John & Joseph 1996, 1997) was proposed at a time when the universe was considered to be decelerating at the present epoch; i.e., well before the discovery of the unnatural dimming of SN Ia at large redshifts that led to the claim that the universe is accelerating. The solution of the cosmological problems, including the age and coincidence problems, was one of the strong motivations for its prediction. Presently, the model is more relevant in the context of the synchronicity problem, for the currently popular $\Lambda$-CDM model has no easy way out of it. The former model shall always remain a potential rival to the latter, unless more sophisticated data shows that $H_0t_0 \neq 1$. (As mentioned by Avelino & Kirshner (2016), if this value is very near to unity and is not exactly equal to it, there would be an even worst synchronicity problem.)

One can see that the complex extension of Milne model, which led to the nonempty, flat model with $a = \sqrt{a_0^2 + t^2}$, naturally brings in a negative energy which causes the bounce in the real model, and this is what relieves the model from the
singularity problem. As can be expected, this negative energy has an equation of state corresponding to relativistic matter. There are speculations on a universe driven by a Casimir energy, which is negative (Jaffe 2005). Casimir first showed, on the basis of relativistic quantum field theory, that between two parallel perfect plane conductors separated by a distance \( l \), there is a renormalised energy \( E = \frac{\pi^2}{720} l^4 \) per unit area (Casimir 1948). For a static universe and for a massless scalar field, it was calculated that Casimir energy has a density (Elizalde 1994)

\[
\rho_{\text{casimir}} = - \frac{0.411505}{4\pi a^4}. \tag{25}
\]

If we accept the value of \( a_0 = \sqrt{2G/3\pi} \), which is nearly equal to the Planck length, our expression for negative energy (13) can be written as

\[
\rho_- = - \frac{1}{4\pi a^4}. \tag{26}
\]

Thus there is some strong ground to believe that the negative energy appearing in the complex extension of Milne model is of the nature of Casimir energy. However, one must admit that a much deeper justification for this, on the basis of a true quantum field or similar theory, is needed here.

This caveat is there also when the energy of the field \( \phi \) is assumed to contribute to the energy of matter/radiation and the time-variable dark energy. Similar is the case for the creation of matter/radiation from dark energy, as envisaged in this model. One can only view these predictions as providing outlines of a future broader theory. But since we have the result from quantum cosmology that there is exact classical to quantum correspondence for this new model, the above pointers from the classical theory can be expected to be in the right direction.

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