Vehicle path tracking Based on Strong Tracking Cubature Kalman Filter While Encountering An Emergency Collision Avoidance

Bi Wei1,2, Wang Wei3, Bei Shaoyi3, Jin Ben3, Li Bo3

1 School of Mechanical Engineering, Jiangsu University of Technology, Changzhou, 213001
2 School of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Hangzhou, 310018
3 School of Automotive and Traffic Engineering, Jiangsu University of Technology, Changzhou, 213001

Abstract: Considering the high speed emergency avoidance situations, the vehicle path tracking was studied in this paper. The three degrees of freedom nonlinear vehicle model and the cubature Kalman filter are applied to tracking vehicle emergency avoidance path. For the degradation of adaptive tracking performance in the vehicle emergency avoidance, an improved strong tracking cubature Kalman filter algorithm is proposed by introducing the fading factor into filtering process which is learnt from strong tracking filter. The algorithm has a simple implementation, high estimation accuracy and good robustness. Different target paths are selected and the cubature Kalman filter is applied to simulate path tracking in the vehicle emergency avoidance. The simulation results show that the proposed path tracking method can control the vehicle tracking the ideal collision avoidance path rapidly without collisions.

1. Introduction
In recent years, the highway traffic accident remains high, including rear-end collision caused by accidents accounted for more than 75%. High-speed emergency avoidance technology can reduce the incidence of accident, increasing the active safety of vehicle. Vehicle high-speed emergency avoidance system uses modern information technology and sensor technology to transmit information to the control unit, and then the vehicle actively avoids obstacles ahead to ensure the safety of vehicle.

Path tracking is to make the vehicles driving along the expected path, and ensure driving safety and comfort of the vehicle at the same time [1-3]. The operation control layer in the path tracking controller includes path following control, the speed tracking control, logical component control, emergency management and control of the actuator modules and components, among which the path tracking control is the core, and the most studied content at present. When vehicles in the process of emergency avoidance, high speed, strong nonlinear and uncertainty, high speed vehicle path tracking is intelligent vehicle development the key technical problems to be solved in the process of [4-5].

This article uses the idea of the strong tracking filtering algorithm into CKF, using the fading factor for real-time adjustment of gain matrix, thus enhances the system ability of adaptive tracking [6-9]. In this paper, the nonlinear vehicle dynamics model is established, and then the strong tracking kalman filter algorithm and kalman filter volume algorithm are used for the expected avoiding path of
emergency for adaptive tracking respectively. The results show that the strong tracking measurement kalman filter algorithm has higher tracking precision than volume of kalman filter algorithm.

2. Nonlinear 3-DOF vehicle dynamic model
Nonlinear 3-DOF model takes into account three degrees of freedom which are longitudinal movement along the x-axis of , lateral movement along the y-axis and yaw motion along z-axis to establish three degrees of freedom nonlinear model. Three state variables which are longitudinal, lateral yaw variables are mutually coupling in this model.

Limited vehicle lateral acceleration at 0.4g or less, in this situation cornering characteristics of tire is in linear range, and assuming the value of driving force is not great ,without regrading to the influence tangential force from the ground to tire’s cornering characteristics, ignoring the influence to tire’s concerning characteristics and aligning torque caused by left and right tire. There is no aerodynamic effect [12], the resulting model is shown in figure 1.

Vehicle’s movement differential equations show as following:

\[ I \dot{\omega}_x = (a k_1 - b k_2) \beta + \frac{(a^2 k_1 + b^2 k_2)}{v_x} \omega_x - a k_1 \delta \]
\[ m (\dot{v}_y + v_x \omega_x) = (k_1 + k_2) \beta + \frac{(a k_1 - b k_2)}{v_x} \omega_x - k_1 \delta \]
\[ a_x = \dot{v}_x - v_x \omega_x \]
\[ a_y = \dot{v}_y + v_x \omega_x \]

The state equation can be described by equation (5), and the observation equation is shown in formula (6).

\[
\begin{align*}
\dot{\omega}_x &= \frac{(a^2 k_1 + b^2 k_2)}{I_z} \omega_x v_x + \frac{(a k_1 - b k_2)}{I_z} \beta - \frac{a k_1}{I_z} \delta \\
\dot{\beta} &= \frac{(a k_1 - b k_2)}{m} - u^2 \frac{\omega_x}{v_x^2} + \frac{k_1 + k_2}{m} \beta - \frac{k_1}{m} \delta \\
\dot{v}_x &= a_x + \beta v_x \omega_x \\
a_y &= \frac{a k_1 - b k_2}{m} \omega_x + \frac{k_1 + k_2}{m} \beta - \frac{k_1}{m} \delta
\end{align*}
\]

Where \( m \) is the vehicle mass, \( I_z \) is the moment of inertia about z-axis, \( a_x, a_y \) are the longitudinal acceleration and lateral acceleration measured of center mass, \( \beta \) is the side slip angle of center mass, \( \omega_x \) is the yaw rate, \( a, b \) is the distance from center of mass to front axle and rear axle, \( v_x, v_y \) is the longitudinal velocity and lateral velocity of center mass, \( k_1, k_2 \) are the cornering stiffness of the front axle and rear axle.
3. Strong Tracking CKF
CKF has a high estimated accuracy and it’s easy to implement. However, to the path of adaptive tracking ability, CKF is poor. The reason is that when the system is stable state, the algorithm of gain matrix will be minimum, and when mutations in the state prediction residual, great changes have taken place. However, the system remained for the minimum gain matrix, it doesn’t vary with the change of residual real-time adaptive adjustment, resulting in the unapproachable to the optimal gain matrix. Using the ideas of strong tracking, the fading factor is introduced into the algorithm of one step prediction covariance matrix to reduce the data for the current filter value, thus the adaptive adjustment of gain matrix, achieving the goal of stable tracking.

Calculating the fading factor needs to calculate jacobian matrix \( \Phi \) and \( H_{k+1} \). But both of them cannot directly getting from algorithm, they need to be improved, the literature \[11\] gives the improved fading factor of this, as follows:

\[
N_{k+1} = V_{k+1} - R_{k+1} - H_{k+1}O_k H_{k+1}^T
\]

\[
M_{k+1} = P_{k+1} - V_{k+1} + N_{k+1}
\]

Wherein: \( H_{k+1} = \left[ P_{l}\left| x_{k+1}, k \right. \right] \left[ P_{l}\left| k \right. \right]^{-1}\), (l) said the introduction of fading factor variable.

The literature[12] introducing weakening factor \( \beta \) in the improved strong tracking algorithm is fading factor, increasing determine filtering abnormal threshold value to \( \beta C_{0, k+1} \) ( \( C_{0, k+1} \) is filtering residual), it greatly reduced the probability of misjudgment filtering divergence in the normal working conditions , and also improved the accuracy of the filter at the same time. The calculation process is as follows:

\[
N_{k+1} = V_{0, k+1} - \beta \left( H_{k+1}O_{k+1}H_{k+1}^T + R_{k+1} \right)
\]

\[
M_{k+1} = \beta H_{k+1}F P_k F^T H_{k+1}^T
\]

4. Road model
Double lane and serpentine lines are typical road models which are commonly proposed in vehicle handling stability evaluation. To simulate vehicle state estimation, double lane and serpentine lines are used as the ideal road input. Meanwhile, the road trace takes the actual road centerline trajectory.

4.1. Double lane road input model
Double lane road test model dimensions are shown in figure 2. The actual road segment size parameters in figure 4 are, \( s_0 = s_1 = s_2 = s_4 = 2u\), \( s_3 = u\), \( s_5 = 5u\), \( s_6 = 3u\).

![Figure 2. Double lane road test model diagram.](image)

Where, \( u \) is the car’s driving speed, and the changing lanes distance width \( B = 3.5m \) (assumed to be a standard motor vehicle lane width).

Take the centerline of the test road for the ideal way trajectory to track in the driver mind. Because the way in the corner points has a mutation, the car’s actual travel path don’t have a mutation, so smooth handling is conducted in the mutation department to make the road close to the ideal trajectory. The most simple and effective treatment is carrying out third-order curve to fit to this polyline, so after
fitting the road functions and a derivative at break point are continuous. The ideal path input after third-order curve fit is shown in figure 3.

The abscissa $x$ is the longitudinal displacement of the car, and the ordinate $y$ is the lateral displacement of the car.

$$f(x) = \begin{cases} 
0 & x \in s_1 \\
e_0 + e_1x + e_2x^2 + e_3x^3 & x \in s_2 \\
B & x \in s_3 \\
e_0' + e_1'x + e_2'x^2 + e_3'x^3 & x \in s_4 \\
0 & x \in s_5
\end{cases} \quad (11)$$

Where, parameters are as follows:

$a_0 = s_1, \quad a_1 = s_1 + s_2, \quad a_2 = s_1 + s_2 + s_3, \quad a_3 = s_1 + s_2 + s_3 + s_4, \quad d = a_1 - a_0, d' = a_2 - a_3,$

$e_0 = a_0^2(3a_4 - a_0)B/d^3, \quad e_1 = -6a_0a_1B/d^3, \quad e_2 = 3(a_0 + a_1)B/d^3, \quad e_3 = -2B/d^3,$

$e_0' = a_2(3a_2 - a_3)B/d^3, \quad e_1' = -6a_2a_3B/d^3, \quad e_2' = 3(a_2 + a_3)B/d^3, \quad e_3' = -2B/d^3.$

Longitudinal displacement of the car can be obtained by $x = ut$, and the function $f(x)$ in formula (5) can be converted into a function of time $f(t)$. The specific expressions are:

$$f(t) = \begin{cases} 
0 & t \in t_1 \\
g_0 + g_1t + g_2t^2 + g_3t^3 & t \in t_2 \\
B & t \in t_3 \\
h_0 + h_1t + h_2t^2 + h_3t^3 & t \in t_4 \\
0 & t \in t_5
\end{cases} \quad (12)$$

Where, $g_0 = e_0, \quad g_j = e_ju^j (j = 1,2,3), \quad h_0 = e_0', \quad h_j = e_j'u^j (j = 1,2,3).$

4.2. Serpentine road input model

Serpentine test road model dimensions are shown in figure 4.

The actual road segment size parameters in figure 4 are: $s_0 = L = 2u, \quad 5L = 10u, \quad s = 3u$.

$u$ is vehicle speed, pole width $B = 2.46m$ (the width of the text is designed according to the car models in the test).

The ideal tracking trajectory by an actual driver shall be shown in figure 5. After a cubic fitting the road, the third-order curve with continuous derivative is:
The serpentine road function of time after fitting is as follows: (a similar derivation with double lane, so will not repeat them, but only give results).

\[
f(t) = \begin{cases} 
0 & t \in t_1 \\
H \frac{(1.0 + \sin(\pi(t + 1)/2))}{2.0} & t \in t_2 \\
H \cos(\pi t/2) & t \in t_3 \\
H \frac{(-1.0 + \sin(\pi(t + 1)/2))}{2.0} & t \in t_4 \\
0 & t \in t_5
\end{cases}
\] (13)

Where, \( H = B/2 \).

5. The results of simulation and analysis

This paper is to simulate vehicle emergency avoidance path tracking simulation experiment, computer simulation model is established based on Matlab, and selects the double line and serpentine road model, the strong tracking kalman filter for a given volume path for precise tracking, kalman filtering and strong tracking filtering respectively with volume contrast research its tracking precision. The model parameters are as follows: the mass \( M = 1818.2 \text{ kg} \), suspension quality \( M_s = 1418 \text{ kg} \), center of mass to the front axle distance \( a = 1.463 \text{ m} \), the center of mass to the rear axle distance \( b = 1.585 \text{ m} \), the rotational inertia of the vehicle \( I_z = 3885 \text{ kg} \cdot \text{m}^2 \), suspension mass moment of inertia of axis of \( I_X = 980 \text{ kg} \cdot \text{m}^2 \), suspension winding quality and product of inertia of \( I_{XZ} = 0 \), the front wheel cornering stiffness \( (2 k_f) \) \( k_f = 62618 \text{ N/ rad} \), and rear wheel cornering stiffness \( (2 k_r) \) \( k_r = 110185 \text{ N/ rad} \), front suspension damping \( D_f = 3430 \text{ N} \cdot \text{m/ s/ rad} \), roll Angle after suspension damping \( D_r = 3430 \text{ N} \cdot \text{m/ s/ rad} \) roll Angle, roll lever \( h = 0.488 \text{ m} \), \( C_f = 100548 \text{ N} \cdot \text{m/ rad} \), dip Angle of the side back rigid \( C_r = 32732 \text{ N} \cdot \text{m/ rad} \) Angle, steering system transmission ratio \( i = 24.9 \), the gravitational acceleration \( g = 9.8 \text{ m/ s}^2 \).

5.1. Double line simulation results

Two algorithms are adopted to simulate the vehicle tracking double line avoiding path, taking speed \( u = 108 \text{ km/h} \) and \( 90 \text{ km/h} \) respectively.

5.1.1. The simulation results of \( u = 108 \text{ km/h} \). Figure 6 (a) is a model’s simulation results of lateral displacement of using ISTCKF and CKF algorithm tracking collision avoidance path for double line, compared with using the algorithm of ISTCKF in the figure 6 (a), CKF algorithm with the desired avoidance path is more consistent. In addition, from the figure 6 (b) the actual lateral displacement and tracking collision avoidance path absolute error, using ISTCKF algorithm has higher tracking precision than CKF. So the ISTCKF algorithm that adopted in this paper can make the car avoid better at a high speed.
5.1.2. The simulation results of $u=90\text{km/h}$. Figure 7 is the simulation results of two kinds of algorithm in $u=90\text{ km/h}$, it can be seen that the entire movement process, the response of the vehicle and the trend of $u=108\text{ km/h}$, verifying again that compared with using the algorithm of ISTCKF, CKF algorithm has higher tracking precision.

5.2. Serpentine line simulation results

Two algorithms are adopted to simulate the vehicle tracking serpentine avoidance path, take speed $u=90\text{ km/h}$ and $u=72\text{ km/h}$ respectively.

5.2.1. The simulation results of $u=90\text{km/h}$. Figure 8 (a) a model’s simulation results of lateral displacement of using ISTCKF and CKF algorithm tracking collision avoidance path for serpentine, using the algorithm of ISTCKF compared with CKF algorithm and the desired avoidance path is more consistent. In addition, from the figure 6 (b) the actual lateral displacement and tracking collision avoidance path absolute error, using ISTCKF algorithm has higher tracking precision than CKF. So the ISTCKF algorithm that adopted in this paper can make the car avoid better at a high speed.
5.2.2. The simulation results of $u = 72$ km/h. Figure 9 is the simulation results of two kinds of algorithm in $u = 72$ km/h, it can be seen that the entire movement process, the response of the vehicle and the trend of $u = 90$ km/h, verifying again that compared with using the algorithm of ISTCKF, CKF algorithm tracking precision is higher.

6. Conclusion
Since the low volume precision of traditional kalman filtering algorithm of path tracking, drawing lessons from the theory of strong tracking filter, an improved kalman filtering algorithm of strong tracking volume is put forward. Nonlinear vehicle dynamics model is established, and then use the strong tracking kalman filter algorithm and kalman filter volume algorithm to the expected avoiding path of emergency for adaptive tracking respectively. The results show that the Strong tracking measurement kalman filter algorithm has higher tracking precision than Volume of kalman filter algorithm.

Therefore, vehicle emergency avoidance path tracking problem, that uses strong tracking volume kalman filtering algorithm, can provide certain theoretical guidance to vehicle dynamic control system software design.

Acknowledgments
This work is partially supported by the National Science Foundation of China (Grant no. 51305175 and 51705220) , the Jiangsu Province Higher Education Natural Science Research Project (grant no. 17KJD580001), the Jiangsu Provincial Higher Education Natural Science Research Major Project (grant no. 17KJA580003), the National Science Foundation of JiangSu Province (Grant no.
BK2012586), the National Science Foundation of Jiangsu University of Technology (Grant no. KYY14041), the 333 Project of Jiangsu Province (BRA2015365) and Jiangsu province “six personnel peak” fund projects (Grant no. 2012-ZBZZ-023) and (Grant no. 2013-ZBZZ-039) , Policy Guidance Program (Collaboration)-Prospective Joint Research Project (BY2015028-02). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

References
[1] Lee K., Kim D., Chung W. Car Parking Control Using a Trajectory Tracking Controller[J].IEEE. International Joint Conference. Washington DC:IEEE, 2006:2058-2063.
[2] Hatwal H, Mikuleck E C. Some Inverse Solutions to an Automobile Path-tracking Problem with Input Control of Steering and Braks[J], Vehicle System Dynamics, 1986, 15(2): 61-71.
[3] Aracil J., Heredia G., Ollero A. Global Stability Analysis of Fuzzy Path Tracking Using Frequency Response[J]. Engineering Applications of Artificial Intelligence, 2000(13): 109-119.
[4] Hsu T., Liu J F., Yu P N. Development of an Automatic Parking System for Vehicle[C].IEEE, Vehicle Power and Propulsion conference, Washington DC:IEEE, 2008:1-6.
[5] Muller B, Deutscher J, Grodde S. Trajectory Generation and Feedforward Control for Parking a Car[C].IEEE. International Conference on Control Application. Washington DC:IEEE, 2006:163-168.
[6] Julier S.J., Uhlmann J.K. A new method for the nonlinear transformation of means and covariances in filters and estimators[J]. IEEE Trans. on AC, 45(3), 2000:477-482.
[7] Arasaratnam I, Haykin S. Cubature Kalman filters[J]. IEEE Trans. on Automatic Control, 2009, 54(6):1254-1269.
[8] Hiroshi Mouri. Investigation of automatic path tracking using an extended Kalman filter [J]. Society of Automotive Engineers of Japan, 2002, 23:61-67.
[9] Zhang Xinchun, Guo Chengjun. Square-root imbedded cubature Kalman filtering[J]. Control Theory & Applications, 2013, 30(9):1116-1121.
[10] Bai Yan, Jia Xin, Zong Changfu, Guan Xin. Experimental Identification of Transfer Function for Vehicle Handling Stability Evaluation[J]. Automotive Engineering, 2014, 36(9):1074-1079.
[11] Wang Xiaoxu, Zhao Lin, Xia Quanxi, Hao Yong. Strong tracking filter based on unscented transformation[J]. Control and Decision, 2010, 25(7):1063-1068.
[12] Xu Shusheng, Lin Xiaogong, Li Xinfei. Strong tracking adaptive square-root cubature kalman Filter algorithm[J]. Acta Electronica Sinica, 2014, 42(12):2394-2400.