Remarks on CLEO New Measurements for $\Upsilon$(1S) Decays to Charmonium Final States and Investigations on Associate Strange Particle Enhancement in $\Upsilon \to J/\Psi + X$

Wei Han
Department of Physics, Shandong University, Jinan, 250100, P.R. China

Shi-Yuan Li
Institute of Particle Physics, Huazhong Normal University, Wuhan, 430079, P.R. China

(Dated: March 26, 2022)

The recent measurements by CLEO for the inclusive $J/\Psi$ and $\Psi(2S)$ production in $\Upsilon$(1S) decay and our previous calculation are analyzed. The $J/\Psi$ momentum spectrum and the production ratio of $\Psi(2S)$ versus $J/\Psi$ favour $\Upsilon \to J/\Psi(\Psi(2S)) + c\bar{c}$g as the dominant contribution. We point out that the differences between the experimental data and our previous results are mainly originated from the setting of the parameter charm quark mass. Encouraged by this result, we investigate the associate strange particle enhancement as a probe for the open charm particles in $\Upsilon \to J/\Psi(\Psi(2S)) + c\bar{c}$.

PACS numbers: 13.25.Gv, 13.87.Fh, 12.38.-t

Besides the important rôle in the electroweak and CP violation research, heavy quark (b or c) physics is also the traditional arena for Quantum Chromodynamics (QCD), both in perturbative (P) and non-perturbative (NP) aspects. The production and annihilation of heavy quarks via strong interactions can be calculated by PQCD for their large masses naturally set the hard scales, while the structures of heavy hadrons are ruled by NPQCD. Since heavy quarks can hardly be produced in NPQCD phase, one can compare the quantities of them calculated by PQCD with those of the corresponding heavy hadrons to study the NPQCD effects. Hence, heavy quarks act as bridges linking the P and NP phases of QCD. Nevertheless, such a distinct character requires a careful investigation on both PQCD and NPQCD phases for a certain process, as one has to do in the study of quarkonium production.

Non-Relativistic QCD (NRQCD) [1] provides a factorization framework [2] to calculate the heavy quarkonium production and decay. For example, the inclusive production of charmonium $H_c$ in the $p\bar{p}$ collision can be written as

$$d\sigma(p\bar{p} \to H(P)_{c} + X) = \sum \sum d\sigma(p\bar{p} \to c\bar{c}(P)_{t} + X) < O_{t}^{H_c} >,$$ (1)

which is a sum of various contributions. Each contribution is the product of two factors: One is the cross section of the quark pair production in a definite (colour, angular momentum, etc.) state, the other is the NRQCD matrix elements to describe the transition probability of the quark pair to the relevant quarkonium. This formula clearly implies that one has to take into account all the competing contributions in calculating the cross section measured by experiments. For the inclusive charmonium production in $p\bar{p}$ interaction at Tevatron [3], the colour-octet process [4] seems necessary to explain the total cross sections as well as the transverse momentum spectra (But it is not clear yet why the polarization of $J/\Psi$ not properly described [5], because only the cross section of all the colour-singlet processes summed up is of one order lower than the experimental data. This is one example that teaches us to include all kinds of contributions, but does not mean colour-octet process(es) always the dominant. Until now, the values of colour-octet matrix elements can only be extracted via fitting the data but are generally believed to be smaller than the colour-singlet ones, based upon the NRQCD velocity counting rule. Furthermore, when the same matrix elements determined at CDF are applied to photoproduction of $J/\Psi$ at HERA, the colour-octet contribution is about a factor of ten too large [6]. This indicates that these matrix elements may be even smaller. So, for a concrete process, one can not judge ‘a priori which kind of process is the main contribution and can not truncate the hard part ($d\sigma$ in Equation (1)) to a certain order of $\alpha_s$, without distinction for colour-singlet or colour-octet partonic processes. The reason is that the suppression from $\alpha_s$ for higher order colour-singlet partonic process(es) can be compensated by the larger colour-singlet matrix element(s) (The colour-singlet matrix elements of vector mesons can be determined unambiguously from their leptonic decays). So higher order (in $\alpha_s$) colour-singlet process could also contribute significantly. This is the key point [7] to understand the inclusive $J/\Psi$ production data in $\Upsilon$ decay.

The strong decay of $\Upsilon$ is dominantly via 3 gluons, because of charge parity conservation. So the straightforward partonic process for the charmonium production is $\Upsilon \to gggg \to ggc\bar{c}$ [10], which is of lowest order PQCD to create charm quark pair. Here the charm quark pair is produced via the virtual gluon splitting and is inherently colour-octet. When the colour-octet matrix element extracted from Tevatron is applied, one can get the...
branching ratio comparable to the experimental data \[^8\]. However, because one of the real gluon is preferred to be soft, the \(J/\Psi\) momentum spectrum peaks at the maximum value, like a 2-body final state configuration. This was not consistent with the old CLEO measurements \[^8\], but the error bar was too large to draw a final conclusion, then. Another colour-octet process which is also of the 2-body final state configuration was suggested \[^11\], ignorant of the \(J/\Psi\) momentum spectrum, too. Recently, the CLEO collaboration make a new measurement for the \(\Upsilon(1S)\) decays to charmonium final state \[^9\]. The data sample is 35 times larger. The more precise measurement confirm a rather soft spectrum of the \(J/\Psi\) in the \(\Upsilon \to J/\Psi + X\) process, which is consistent with the colour-singlet process prediction \[^7\] both in the branching ratio as well as the \(J/\Psi\) spectrum but clearly conflict with the colour-octet processes in \(J/\Psi\) momentum spectrum.

As is discussed in \[^7\] and in above, because the much larger colour-singlet matrix element(s) can compensate the suppression from powers of \(\alpha_s\), one should not ignore the colour-singlet process(es) if the suppression from powers of \(\alpha_s\) is not too large (e.g. here, only one extra \(\alpha_s\)). Hence we calculate the contribution of \(\Upsilon \to J/\Psi + c\bar{c}\) to the inclusive \(J/\Psi\) production of \(\Upsilon\) decay \[^7\]. This is dominant of colour-singlet process and can be treated within the traditional non-relativistic wave function approach \[^12, 13, 14\]. For this process, it is very clear that the more final state particles, especially the open charm hadrons from the unbound charm quark pair make the \(J/\Psi\) spectrum much more soft. That the dominant contribution of this channel is via colour-singlet process requires that the bound \(c\) and \(\bar{c}\) should come from different virtual gluons and in nearly the same momentum. This configuration in phase space eliminates the 2-body like configuration that the real gluon is soft and the 2 virtual gluon splitting into 2 back to back \(c\bar{c}\) pair with minimum invariant mass. So the momentum spectrum of \(J/\Psi\) can not peak at the largest value. Though the qualitative analysis is perfectly consistent with data, the new data \[^8\] and our calculation \[^7\] does not coincide completely. One may be interested whether there are other dynamical reasons ignored. Hence in the following, we first clarify that the “inconsistency” between the data and our calculation comes from the setting of the parameter charm mass in numerical calculation. We emphasize on the \(J/\Psi\) momentum spectrum and the production ratio of \(\Psi(2S)\) versus \(J/\Psi\). Furthermore, we investigate the associate strange particle enhancement as a way to probe the charm quark accompanying with \(J/\Psi\).

The reason for the “harder” spectrum of our previous calculation than that of the experiment

The CLEO collaboration claims that the momentum spectrum is “closer to, although softer than” our colour-singlet prediction \[^8\]. However, this is just because we did not take into account the open charm threshold effect in the partonic level calculation. In our calculation, we let the charm quark mass equals to half of \(J/\Psi\) mass. This leads to the unphysical region when the \(c\bar{c}\) cluster mass smaller than \(2m_c\). If taking into account the threshold effect, the peak in our calculation should move to the left about 0.1 unit of \(x\) (scaled momentum). This means the position of the peak of our calculated \(J/\Psi\) momentum spectrum (see, FIG. 3 of \[^7\] or FIG. 10 of \[^9\]) should move to around \(x = 0.4\) rather than \(x = 0.5\). Considering that the width of the bins of the experimental data is 0.2 unit, the prediction of the colour-singlet model is consistent rather than softer than the data.

The “wrong” result of our calculation of \(\Psi(2S)\) production rate

If we only consider the production rate of \(\Psi(2S)\) itself, it may be natural to choose \(m_c = M_{\Psi(2S)}/2\) and use the \(\Psi(2S) \to e^+e^-\) decay width to extract the wave function (In fact sometimes one needs to tune the quark mass to get the proper absolute value of the cross section). This is what we did in our previous calculation. In \[^7\], when calculating the partonic cross section of charm pair creation for \(J/\Psi\) production, we use \(m_c = M_{\Psi(2S)}/2\); while calculating that for the \(\Psi(2S)\) production, we use \(m_c = M_{\Psi(2S)}/2\). As can be seen from the Table demonstrating the charm mass dependence, this leads to a difference of about 2.5 times. The ratio of the wave functions square and the branching ratio of \(\Psi(2S)\) to \(J/\Psi\) lead to another factor about 4. This is why in our paper we only predicted about 10 per cent of \(J/\Psi\) are from the decay of \(\Psi(2S)\). However, if we calculate the production ratio(es) between \(J/\Psi, \Psi(2S)\), etc., and adopt that they are different eigenstates of \(c\bar{c}\) bound state, their differences should be only inherent in the wave functions. The \(M_{\Psi(2S)} - M_{J/\Psi}\) is explained as originated from difference of binding energies. Hence the charm quark mass should be taken as the same. This makes the calculated results consistent and simple for the colour-singlet processes. In the framework of non-relativistic wave function approach \[^12, 13, 14\], one can easily derive, for the S-wave particles,

\[
\frac{B(\Upsilon \to \Psi(2S) + c\bar{c})}{B(\Upsilon \to J/\Psi + c\bar{c})} = \frac{\Gamma(\Upsilon \to \Psi(2S) + c\bar{c})}{\Gamma(\Upsilon \to J/\Psi + c\bar{c})} = \frac{|\psi_{2S}^{cL}(0)|^2}{|\psi_{1S}^{cL}(0)|^2} = \frac{\Gamma(\Psi(2S) \to e^+e^-)}{\Gamma(J/\Psi \to e^+e^-)}.
\]

In the above equations, \(\psi_{2S}^{cL}(0)\) and \(\psi_{1S}^{cL}(0)\) are wave functions at origin for \(\Psi(2S)\) and \(J/\Psi\), respectively. To get the relations in Equation \[^2\], we notice that \(\Gamma(\Upsilon \to \Psi(nS) + c\bar{c})\) should be, to order \(O(v^0)\),

\[
\frac{1}{2M_\Upsilon} \int \frac{|\psi_{2S}^{cL}(0)|^2}{m_c} \int_{2m_c} \sqrt{E^2 - 4m_c^2} \, dE \, dR' 
\times \left| M(\Upsilon \to c(P/2)c(P/2)c\bar{c}\bar{c}) \right|^2,
\]

where \(dR'\) represents other integral variables than the \(\Psi(nS)\) energy \(E\) for the \(\Psi(nS)c\bar{c}\bar{c}\) phase space ele-
ment. $P$ is the four momentum of $\Psi(nS)$. $M(\Upsilon \rightarrow c(P/2)\bar{c}(P/2)cg)$ is the invariant amplitude for $\Upsilon \rightarrow e^+e^-$ process. In this formula, when taking the same charm quark mass, the only difference for different $S$-wave bound states is the wave function square. For the leptonic decay width of $S$-wave vector charmonia, a very similar formula and hence conclusion can be obtained. Only that one should take the energy of the charmonia at rest frame to be $2m_c$, neglecting their mass differences.

Using the values of 2002 data book [13], which is also used by the experimental group [9], one can calculate the ratio of the $e^+e^-$ decay widths (central value) between $\Psi(2S)$ and $J/\Psi$. It is striking to be 0.416 (the to-date values give 0.45), which is very near the central value of the data 0.41 [9]. However, the colour-octet process cannot simply derive such a relation with the $e^+e^-$ decay width [20].

Unfortunately, we have not got the analytical formula for the partonic process $J/\Psi \rightarrow cc\bar{c}g$, so it is difficult to evaluate the $P$ wave particles which is also measured by CLEO. The calculation needs the derivatives of the partonic amplitude. Encouraged from the $S$-wave productions, we leave the calculation of $P$-wave particles as an interesting separate work to do.

From the above discussions, it is clear not that NRQCD leads to the prediction that the colour-octet processes dominate in $\Upsilon \rightarrow J/\Psi + X$. It is just the NRQCD formulae require a global consideration of all possible partonic processes to explain the data. Frankly, there are many uncertain parameters like quark mass, $\alpha_s$ (scale dependence is significant in the lowest order calculation), etc. So only quantities like the relative production ratio and shape of spectrum which are not sensitive to the absolute values of these parameters are relevant to justify the partonic processes. Only to get a cross section or branching ratio comparable with data sometimes seems tricky.

**Associate strange particle enhancement** The above arguments confirm the significance of the colour-singlet process. As has been pointed out by [2] and the experimental group [9], a very important probe for the partonic process is to measure the content of the cluster produced associated with the $J/\Psi$ in the $\Upsilon$ decay, which is theoretical-framework-independent. It is interesting to notice that in $e^+e^-$ continuum process at nearly the same energy, BELLE Collaboration [17] reported that about 60 per cent events of $e^+e^- \rightarrow J/\Psi + X$ is from $e^+e^- \rightarrow J/\Psi + \bar{c}c$. This is a challenge for the theory, and also suggests the importance to measure the associate particles in $\Upsilon \rightarrow J/\Psi + X$. However, the data sample seems still not enough. According to the experiment [9, 18], the $e^+e^-$ and $\mu^+\mu^-$ pairs are used to identify and reconstruct the charmonium. Only less than 10$^3$ $J/\Psi$ events are collected. Even if these events are all $\Upsilon \rightarrow J/\Psi + c\bar{c}g$, and employing the standard method to reconstruct all the charm pseudo-scalar and vector mesons that the associate charm quarks fragment to one can only expect to get less than 10 per cent events. On the other hand, it is not efficient to reconstruct all the important charm mesons. If only use the $D^0 (\rightarrow K\pi)$ channel, the number of events to be found could only be around 9 [18]. Then the statistical error may not allow a definite conclusion.

However, a charm quark generally decays into strange quark, hence the strange particles can manifest the ever-production of charm. So the inclusive measurement of kaon may be a good probe [18]. Besides charm decay, the strange quark can be created from the vacuum during the hadronization process. Hence it is not the absolute production rate of strange particles but their enhancement relative to other cases is the very signal for charm. For the processes we are going to investigate, the colour-octet models predict a flavour-singlet cluster (one or two colour-octet gluon(s)) associate with $J/\Psi$, while the colour-singlet model predicts the cluster $c\bar{c}g$, which lead to strange flavour via weak decay. To find a measurable quantity, we study the ratio $\frac{<K>}{<\pi>}$ ($<K>$ and $<\pi>$ are average multiplicity of $K^\pm$ and $\pi^\pm$, respectively) of the cluster accompanying with the $J/\Psi$. The cluster $c\bar{c}g$ is colour-singlet and we can straightforwardly employ JETSET [19] to simulate its hadronization [21]. We can expect the ratio is similar as those of $e^+e^- \rightarrow c\bar{c} \rightarrow h's$ at the same energy (see Table 1). On the other hand, it is not very practical to hadronize the colour-octet 2-gluon cluster with JETSET for the colour-octet process $\Upsilon \rightarrow J/\Psi + gg$. Another “technical” problem is that in the partonic level calculation, this cluster dominantly have a vanishing invariant mass. For providing a reference to the charm enhancement, we give the result of a cluster of $gg$ which is in colour-singlet (its invariant mass is set to be the peak value of the $cc\bar{c}g$ cluster mass). What is in common for the 2-gluon cluster in colour-octet and colour-singlet is that they are both flavour-singlet. We find that the $\frac{<K>}{<\pi>}$ value is not sensitive to the invariant mass for the colour-singlet $gg$ cluster. At the same time, using the JETSET subroutine (hluent), one can investigate the “independent” fragmentation of one gluon in the colour-octet process $\Upsilon \rightarrow J/\Psi + g$. The obtained $\frac{<K>}{<\pi>}$ is insensitive to its energy. We find that the result is near that of colour-singlet $gg$ cluster (Table 1). For 2-gluon system, if we do not care it is in colour-singlet or octet, its $\frac{<K>}{<\pi>}$ value can be obtained from independent fragmentation of each gluon, which is very near that of the colour-singlet 2-gluon cluster from the above discussion. This suggests that the colour-octet 2-gluon cluster should give $\frac{<K>}{<\pi>}$ not far beyond that of the colour-singlet cluster. Besides the above, for a comprehensive comparison, the results of a flavour-singlet virtual photon ($M < 2m_b$) are also given as reference, but we should understand that the electromagnetic interaction breaks the flavour SU(4) invariance (the initial quark effects). The result is also shown in Table 1.
Table 1, the $\frac{<K>}{<\pi>}$ values are given in the form $rot \pm \chi$ for the processes we discuss above. We can conclude that the fluctuations are small and will not smear the difference, so that the strange particle enhancement of the $c\bar{c}g$ cluster is very significant, comparing with the flavour-singlet $g$ (gg) as well as the virtual photon.

In summary, we point out that the recent measurements for $\Upsilon$ decay into charmonium final states by CLEO Collaboration agree well with the $\Upsilon \rightarrow J/\Psi + c\bar{c}g$ calculation, provided that the value of charm quark mass is properly set. This indicates that the colour-singlet process could be the dominant for the inclusive $J/\Psi$ or $\Psi(2S)$ production in $\Upsilon$ decay. Furthermore, by investigating the hadronization of the parton(s) produced accompanying with the $J/\Psi$ in $\Upsilon$ decay, we found that the $c\bar{c}g$ cluster in colour-singlet (but not flavour-singlet) will lead to significant strange particle enhancement comparing to the colour-octet (but flavour-singlet) gluon(s). Thus it is a good signal to probe the process $\Upsilon \rightarrow J/\Psi + c\bar{c}g$.

We thank Prof. S. Blusk for informing us the experimental results and following discussions that push forward this investigation. We also thank Dr. Yu-Kun Song for the discussions on gluon fragmentation function.

| processes       | $\frac{<K>}{<\pi>}$ |
|-----------------|---------------------|
| $c\bar{c}g \rightarrow K^\pm s$ | 0.243±0.009         |
| $e^+ e^- \rightarrow c\bar{c} \rightarrow K^\pm s$ | 0.240±0.008         |
| $qg \rightarrow K^\pm s$ | 0.120±0.007         |
| $q \rightarrow K^\pm s$ | 0.113±0.007         |
| $\gamma^* \rightarrow K^\pm s$ | 0.176±0.008         |

TABLE I: The production ratio between $K$ and $\pi$ in various processes.

results. Their definition is based upon the following consideration: The number of collected events by the CLEO Collaboration is only of the order $10^3$. So we should estimate the statistical error, or the fluctuation of some 10$^3$ event samples. Here we use $rot$ to represent the “real” value of $\frac{<K>}{<\pi>}$, which can be estimated by a sample generated by JETSET with event number large enough ($\rightarrow \infty$). At the same time, we generate many $10^3$ event samples, then we calculate

$$\chi = \sqrt{\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{j=1}^{N} (r_j - rot)^2}, \text{ with } r_j = \frac{<K>}{<\pi>}.$$  \hspace{1cm} (4)

The subscript $j$ represents each $10^3$ event sample. In

[1] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51, 1125 (1995). [Erratum-ibid. D 55, 5853 (1997)].
[2] Possible problems in this factorization framework, see, e.g., G. C. Nayak, J. W. Qiu and G. Sterman, Phys. Rev. D 72, 114012 (2005).
[3] F. Abe et al. [CDF Collaboration], Phys. Rev. Lett. 69, 3704 (1992); 71, 2537 (1993); 75, 1451 (1995).
[4] E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995).
[5] T. Affolder et al. [CDF Collaboration], Phys. Rev. Lett. 85, 2886 (2000).
[6] M. Cacciari and M. Krämer, Phys. Rev. Lett. 76, 4128 (1996), and references therein.
[7] Shi-Yuan Li, Qu-Bing Xie and Qun Wang, Phys. Lett. B 482, 65 (2000).
[8] R. Fulton et al. [CLEO Collaboration], Phys. Lett. B 224, 445 (1989).
[9] R. A. Briere et al. [CLEO Collaboration], Phys. Rev. D 70, 072001 (2004).
[10] K. Cheung, W. Keung, and T. Yuan, Phys. Rev. D 54, 929 (1996).
[11] M. Napsuciale, Phys. Rev. D 57, 5711 (1998).
[12] J. H. Kühn, J. Kaplan and E. G. O. Safiani, Nucl. Phys. B 157, 125 (1979).
[13] C. H. Chang, Nucl. Phys. B 172, 425 (1980).
[14] R. Baier and R. Rückl, Phys. Lett. B 102, 364 (1981); and Z. Phys. C 19, 251 (1983).
[15] K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002). The to-date values are from: W.-M. Yao et al., J. Phys. G 33, 1 (2006).
[16] E. Braaten, S. Fleming and T.C. Yuan, Ann. Rev. Nucl. Part. Sci. 46, 197 (1996).
[17] K Abe et al. [BELLE Collaboration], Phys. Rev. Lett. 89, 142001 (2002).
[18] S. Blusk, private communications.
[19] T. Sjöstrand, Comp. Phys. Commun. 82, 74 (1994).
[20] The ratio of the corresponding colour-octet matrix elements of $\Psi(2S)$ and $J/\Psi$ is around 0.3, which is lower than but within the error of the data.
[21] We use the subroutine lu3ent. The invariant mass of this cluster is set between $M_{LD}$ to $M_{\Upsilon} - M_{J/\Psi}$, with the weight calculated by the same programme to calculate the $J/\Psi$ spectrum.

APPENDIX A: THE DISTRIBUTION OF $\frac{<K>}{<\pi>}$

In the following figure, we demonstrate the distribution of the $\frac{<K>}{<\pi>}$, to show that the dispersions are really very small.
FIG. 1: Here \( r = \frac{K \gamma}{\pi \rho} \)