Gravitational wave constraints on multi-brane inflation

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Abstract. A class of non-canonical inflationary models is identified, where the leading-order contribution to the non-Gaussianity of the curvature perturbation is determined by the sound speed of the fluctuations in the inflaton field. Included in this class of models is the effective action for multiple coincident branes in the finite $n$ limit. The action for this configuration is determined using a powerful iterative technique, based upon the fundamental representation of $SU(2)$. In principle the upper bounds on the tensor–scalar ratio that arise in the standard, single-brane DBI inflationary scenario can be relaxed in such multi-brane configurations if a large and detectable non-Gaussianity is generated. Moreover models with a small number of coincident branes could generate a gravitational wave background that will be observable in future experiments.

Keywords: string theory and cosmology, inflation, physics of the early universe

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1. Introduction

The quest to realize inflation within string/M-theory continues to attract considerable attention. The Dirac–Born–Infeld (DBI) scenario of the compactified type IIB theory is a well-motivated model, in which inflation is driven by one or more D-branes propagating in a warped ‘throat’ background [1]–[10]. Such a background is generated by the non-trivial form-field fluxes over the internal dimensions. (For recent reviews, see [11]–[17].) In the simplest version of the scenario, the inflaton parametrizes the radial position in the throat of a single D3-brane. The brane dynamics are determined by the DBI action in such a way that the inflaton’s kinetic energy is bounded from above by the warped brane tension. The regime where this bound is nearly saturated is known as the ‘relativistic’ limit.

Recently relativistic DBI inflation has come under considerable pressure when confronted with cosmological observations. Baumann and McAllister (BM) and Lidsey and Huston (LH) have shown that the ratio of the amplitudes of the tensor and scalar perturbations generated during inflation is bounded from above by \( r \lesssim 10^{-7} \) [18,19]. However, in ultraviolet (UV) versions of the scenario, where the D-brane is moving towards the tip of the throat, the tensor–scalar ratio is also bounded from below, \( r \gtrsim 0.1(1 - n_s) \), where \( n_s \) denotes the spectral index of the scalar perturbation spectrum [19]. The two bounds on \( r \) are incompatible if \( n_s \sim 0.95 \), as currently favoured by cosmic microwave background (CMB) observations [20,21].

The purpose of the present paper is to investigate whether the upper bounds on \( r \) can be relaxed in more general DBI inflationary scenarios. A natural extension to the single D3-brane model is to consider a Dp-brane wrapped around a \((p - 3)\)-cycle of the internal space. For example, Becker et al [22] have proposed a model where inflation is driven by a D5-brane. In this case, the range of allowed values for the inflaton becomes independent of the throat charge, \( N \), which weakens the upper bound on the tensor–scalar ratio to \( r \lesssim 0.04 \). Strictly speaking, this is only true for the D7-brane case, since the wrapped

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D5-brane imposes $\Delta \phi \sim N^{-1/4}$. However, in arriving at this bound, it was assumed that the backreaction effects of any fluxes in the throat were negligible. Kobayashi et al. [23] considered both D5- and D7-brane models, but concluded that the former case required an excessively large background charge in order to relax the bounds on $r$. Whilst this is highly constraining, it is still much better than the case for single D3-branes which require a much higher background charge and therefore are effectively ruled out as a predictive model. Thus wrapped brane configurations are preferable to single-brane models. However the difficulty in these models is that the backreaction is no longer under control.

Alternative ways to relax these bounds have been proposed, including theories based upon multi-field models [24], the addition of angular momentum as another degree of freedom [25] and using different throat geometries [26]. However, it must be noted that the extra degrees of freedom introduced in these models do not solve the problem. The bounds are relaxed only by a small fraction, and therefore these models should still be regarded as being unsatisfactory since they require an extreme amount of fine tuning in order to work.

Another alternative possibility is to consider multiple brane configurations\(^4\). In the case where $n$-branes are localized initially at equal distances $l > l_0$ and subsequently follow the same trajectory, the effective theory is equivalent to that of $n$ copies of the action for a single brane. A more general initial condition, particularly for branes created in the infrared (IR) region of the throat [4,28,29], is that the branes should be separated over a range of scales, with a subset being coincident and the remainder being widely separated.

Our approach in this paper is twofold. We begin in sections 2 and 3 by noting that the upper bounds on the tensor–scalar ratio arise due to the special algebraic properties of the DBI action. We then adopt a phenomenological approach in section 4 and identify a general class of non-canonical inflationary models, where the leading-order contribution to the non-Gaussianity of the curvature perturbation is determined entirely by the speed of sound of the inflaton fluctuations. In these models, the bounds on $r$ can be relaxed if significant non-Gaussianities are generated. This class of models includes the relativistic limit of the action for $n$ coincident D3-branes, which originates from a UV complete theory. This motivates us to develop the theory of multiple coincident branes further in section 5. We find that, for a finite number of branes, the effective action for $n$ coincident branes can be derived and the backreaction kept firmly under control. In section 6 we find that such models can in principle lead to a detectable gravitational wave background if the number of coincident branes is sufficiently small.

Units are chosen such that $\hbar = c = 1$ and $M_P \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{-18}$ GeV denotes the reduced Planck mass.

**2. Non-canonical inflation**

The low energy, world-volume dynamics of a D3-brane in a warped background is determined by an effective action of the form

$$S = \int d^4x \sqrt{|g|} \left[ \frac{M_P^2}{2} R + P(\phi, X) \right],$$

\(^4\) In certain limits this approach is actually dual to considering wrapped branes [27].
where $R$ is the Ricci curvature scalar, $X \equiv -\frac{1}{2} g^\mu\nu \nabla_\mu \phi \nabla_\nu \phi$ denotes the kinetic energy of the inflaton field $\phi$ and the function $P(\phi, X)$ is referred to as the ‘kinetic function’.

We assume that the four-dimensional universe is spatially flat and isotropic and sourced by an homogeneous inflaton field, $\phi = \phi(t)$, with energy density $E = 2X P_X - P$, where a subscripted comma denotes partial differentiation. We further assume that the inflaton dynamics generates a quasi-exponential expansion of the universe, where $\epsilon \equiv -\dot{H}/H^2 \ll 1$.

It proves convenient to define two parameters in terms of the kinetic function $P$ and its derivatives [30,31]:

$$c_s^2 \equiv \frac{P_X}{P_X + 2XP_{XX}}, \quad (2)$$

$$\Lambda \equiv \frac{X^2P_{XX} + \frac{2}{3}X^3P_{XXX}}{XP_X + 2X^2P_{XX}}. \quad (3)$$

The first parameter, $c_s$, determines the sound speed of fluctuations in the inflaton field. This can be significantly less than unity, in contrast to slow-roll inflation driven by a canonical field such that $P_X = 1$.

The amplitudes of the scalar and tensor perturbations generated during inflation are given by [32]

$$P_S^2 = \frac{H^4}{8\pi^2 X c_s P_X}, \quad (4)$$

$$P_T^2 = \frac{2H^2}{\pi^2 M_P^2}, \quad (5)$$

respectively, and the ratio of these amplitudes is defined as [32]

$$r \equiv \frac{P_T^2}{P_S^2} = 16c_s\epsilon. \quad (6)$$

The WMAP3 normalization of the CMB power spectrum implies that $P_S^2 = 2.5 \times 10^{-9}$ and the experimental upper bound on the tensor–scalar ratio is $r < 0.55$ [20].

Deviations from Gaussian statistics in the curvature perturbation, $R$, are parametrized in terms of the nonlinearity parameter, $f_{\text{NL}}$, which is defined by $R = R_G + \frac{3}{5} f_{\text{NL}}(R_G - \langle R_G^2 \rangle)$, where the quadratic component represents a convolution and $R_G$ denotes the Gaussian contribution [33]. In the limit where the three momenta have equal magnitude (corresponding to the equilateral triangle limit), the leading-order contribution to the nonlinearity parameter is given by [34,31]

$$f_{\text{NL}} = -\frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right) + \frac{5}{81} \left( \frac{1}{c_s^2} - 1 - 2\Lambda \right). \quad (7)$$

One should note that the sign convention is that employed by the WMAP dataset. Data from WMAP3 imposes the bound $|f_{\text{NL}}| < 300$ on this parameter [20]. The corresponding bounds for other triangle configurations may be much tighter than this and this may be particularly relevant if non-Gaussian signatures have indeed been detected in the CMB [35,36]. The more recent WMAP5 dataset [21] improves on this bound somewhat,
and also indicates that it is distinctly asymmetric. At the 95% confidence level, the bound on the equilateral triangle becomes $-151 < f_{\text{NL}} < 253$.

Equations (4) and (5) imply that the variation of the inflaton field during inflation is related to the tensor–scalar amplitude by [37,18]

$$\frac{1}{M_p^2} \left( \frac{d\phi}{dN} \right)^2 = \frac{r}{8c_s P_{X}}, \tag{8}$$

where $N \equiv \int dt H$ denotes the number of e-foldings. We will refer to the epoch of inflation that can be directly constrained by cosmological observations as ‘observable inflation’ and will assume that this phase occurred when the brane was located within a throat region$^5$. Observable inflation corresponds to no more than about 4 e-foldings of inflationary expansion, $\Delta N_\ast \simeq 4$. The total variation in the inflaton field between the epoch of observable inflation and the end of inflation is then given by

$$\Delta \phi_{\text{inf}} \equiv \left( \frac{r}{8c_s P_{X}} \right)_{\ast}^{1/2} N_{\text{eff}}, \tag{9}$$

where

$$N_{\text{eff}} \equiv \left( \frac{c_s P_{X}}{r} \right)_{\ast}^{1/2} \int_0^{N_{\text{end}}} \left( \frac{r}{c_s P_{X}} \right)^{1/2} dN. \tag{10}$$

If $r/(c_s P_{X})$ varies sufficiently slowly during observable inflation, the corresponding change in the value of the inflaton field is given approximately by [37,18]

$$\left( \frac{\Delta \phi}{M_p} \right)^2 \simeq \frac{(\Delta N_{\ast})^2}{8} \left( \frac{r}{c_s P_{X}} \right)_\ast. \tag{11}$$

3. Theoretical upper bounds on the tensor–scalar ratio

The ten-dimensional metric of the warped deformed conifold inside a throat region has the form

$$d_{10}^2 = h^2(\rho) ds_4^2 + h^{-2}(\rho) \left( d\rho^2 + \rho^2 ds_5 \right), \tag{12}$$

where the ‘warp factor’ $h(\rho)$ is a function of the radial coordinate $\rho$ along the throat and $X_5$ denotes a five-dimensional, Sasaki–Einstein manifold. In many scenarios, the ten-dimensional manifold (12) can be approximated by the product $\text{AdS}_5 \times X_5$, where $\text{AdS}_5$ represents five-dimensional, anti-de Sitter space. In this case, the warp factor is given by

$$h = \frac{\rho}{L}, \quad L^4 = \frac{4\pi^4 g_s N}{\text{Vol}(X_5) m_s^4}, \tag{13}$$

where $L$ denotes the $\text{AdS}_5$ radius of curvature, $\text{Vol}(X_5)$ is the volume of the 5-manifold $X_5$ with unit radius, $N$ is the D3-brane charge in the throat, $g_s$ is the string coupling and $m_s$ is the string mass scale. Generally the value of the inflaton field is determined by the radial position of the D3-brane in the throat, $\phi \equiv \rho \sqrt{T_3}$, where $T_3 \equiv m_s^4/[(2\pi)^3 g_s]$ is the brane tension.

$^5$ We denote the values of all parameters evaluated during observable inflation by a subscript ‘$\ast$’.
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The four-dimensional Planck mass is related to the volume of the compactified Calabi–Yau three-fold, $V_6$, such that $M_P^2 = V_6 \kappa_{10}^{-2}$, where $\kappa_{10}^2 \equiv \frac{1}{2} (2\pi)^7 g_s^2 / m_s^8 = \pi / T_3^2$ for a D3-brane. Hence the Planck mass is bounded from below by the volume of a throat region, $M_P^2 > V_{6,\text{th}} \kappa_{10}^{-2}$, where $V_{6,\text{th}} \lesssim V_6$ denotes the throat volume. For an $\text{AdS}_5 \times X_5$ throat, Baumann and McAllister (BM) exploited this inequality to derive an upper limit on the maximum variation of the inflaton field in the throat, $|\Delta \phi_{\text{max}}| < 2M_P / \sqrt{N}$, which leads to the corresponding limit $|\Delta \phi|_* < 2M_P / \sqrt{N}$ [18]. Combining this with the constraint (11) therefore yields an upper limit on the tensor–scalar ratio:

$$r_* < \frac{32}{N N_{\text{eff}}^2} (c_s P_X)_*.$$ (14)

Two of the authors (Lidsey and Huston, LH) derived a complementary bound on the tensor–scalar ratio by noting that during observable inflation the brane spans a fraction of the throat volume [19]

$$|\Delta V_{6,*}| \simeq \text{Vol}(X_5) \frac{|\Delta \rho_*| \rho_*^5}{h_*^4}$$ (15)

and, since $|\Delta V_{6,*}| < V_{6,\text{th}}$, it follows that

$$\left( \frac{\Delta \phi}{M_P} \right)_*^2 < \frac{T_3 \kappa_{10}^2 (\Delta \rho_*)^2}{|\Delta V_{6,*}|}.$$ (16)

It was then assumed that the fractional change in the value of the inflaton field during observable inflation was less than unity:

$$|\Delta \phi_*| < \phi_*.$$ (17)

This condition is necessarily satisfied in UV versions of the scenario, where the brane is moving towards the tip of the throat, but must be assumed as a further constraint in IR versions where the brane moves out of the throat, since in these latter cases $\phi_*$ could be very small. Combining the limit (17) with equation (15) then implies that

$$|\Delta V_{6,*}| > \text{Vol}(X_5) \frac{(\Delta \rho_*)^6}{h_*^4}$$ (18)

and substituting this constraint into the bound (16) yields the condition

$$\left( \frac{\Delta \phi}{M_P} \right)_*^6 < \frac{\pi T_3}{\text{Vol}(X_5)} \left( \frac{h_*}{M_P} \right)^4.$$ (19)

Finally substituting the constraint (11) yields the upper limit [19]

$$r_* < \frac{10}{(\Delta N)^2} \left( \frac{T_3}{\text{Vol}(X_5)} \right)^{1/3} \left( \frac{h_*}{M_P} \right)^{4/3} (c_s P_X)_*.$$ (20)

6 We parametrize the Planck scale in terms of the D3-brane tension out of convenience and note that there is no physical relationship between the two.
Comparison of the limits (14) and (20) implies that the LH bound is the stronger of the two when
\[ h_s^{4/3} N < 20 \left( \text{Vol}(X_5) g_s \right)^{1/3} \left( \frac{m_s}{M_P} \right)^{-4/3} \left( \frac{(\Delta N)^2}{N_{\text{eff}}^2} \right). \] (21)

For typical field-theoretic values Vol\( (X_5) \approx 0(\pi^3) \), \( m_s \sim 0.1 M_P \) and \( g_s \sim 10^{-2} \), this implies
\[ h_s^{4/3} N < 300 \left( \frac{(\Delta N)^2}{N_{\text{eff}}^2} \right). \] (22)

In the following section, we identify a class of models in which these bounds could be relaxed.

4. Relaxing the upper bounds on the tensor–scalar ratio

In the standard DBI scenario, the kinetic function defined in equation (1) takes the form
\[ P(\phi, X) = -T(\phi) \sqrt{1 - 2T^{-1}(\phi)X} + T(\phi) - V(\phi), \] (23)

where \( T(\phi) = T_3 h^4(\phi) \) is the warped brane tension and \( V(\phi) \) is the inflaton potential. Typically in warped compactifications of IIB supergravity, this potential is determined by the relevant fluxes and brane interaction terms. We will ignore the precise origin and form of this potential, but simply note that it is highly sensitive to the string-theoretic construction. For the purpose of this paper we will simply treat it as an arbitrary function of the inflaton field. (See, for example, [9] for a discussion on the precise form that the inflaton potential may take.)

The standard DBI scenario (23) is algebraically special, in the sense that the kinetic function satisfies the constraints
\[ c_s P_{,X} = 1, \quad \Lambda = \frac{1}{2} \left( \frac{1}{c_s^2} - 1 \right). \] (24)

It follows that the bounds (14) and (20) on the tensor–scalar ratio could in principle be significantly relaxed in models where \( (c_s P_{,X})_+ \gg 1 \). In view of the second relation in equation (24), it is of interest to take a phenomenological approach and consider the more general class of models where
\[ \frac{1}{c_s^2} - 1 = \alpha \Lambda, \] (25)

for some positive constant \( \alpha \). Moreover since a large non-Gaussian signature in the curvature perturbation is typically generated in models where the sound speed of fluctuations is small, we will begin by considering scenarios where the kinetic function satisfies the inequalities
\[ X^2 P_{,XXX} \gg XP_{,XX} \gg P_{,X}. \] (26)

In these limits the constraint (25) reduces to the third-order, nonlinear, partial differential equation
\[ P_{,XX}^2 = \frac{\alpha}{6} P_{,X} P_{,XXX}. \] (27)
Changing the dependent variable to \( Q \equiv P_{,XX}/P_{,X} \) reduces equation (27) to
\[
\alpha Q_{,X} = (6 - \alpha)Q^2,
\]
and it is straightforward to integrate equation (28) exactly. The remaining integrations can also be performed analytically and the general solution to equation (27) for \( \alpha \neq 6 \) is given by\(^7\)
\[
P(\phi, X) = f_1(\phi) [1 - f_2(\phi)X]^m - f_3(\phi),
\]
where \( f_i(\phi) \) are arbitrary functions of the scalar field and
\[
m = \frac{2(\alpha - 3)}{\alpha - 6}.
\]
It can be verified that the inequalities (26) are satisfied in the ‘relativistic’ limit, where \( X \simeq 1/f_2 \). We consider the inflationary dynamics in this limit in what follows. For completeness, we note that equation (25) can be solved analytically in full generality and the solution is presented in the appendix.

The standard DBI scenario is recovered for \( m = 1/2 \). More generally, however, equation (29) implies that
\[
c_s P_{,X} \simeq -\frac{m f_1 f_2}{\sqrt{2(1 - m)}} (1 - f_2 X)^{(2m - 1)/2},
\]
\[
c_s^2 \simeq \frac{1 - f_2 X}{2(1 - m)},
\]
when \( X \simeq 1/f_2 \). Self-consistency therefore requires \( m < 1 \). Moreover, we find from equation (7) that
\[
f_{NL} \simeq -\frac{\beta}{1 - f_2 X}, \quad \beta \equiv \frac{5(59 - 55m)}{486},
\]
\[
f_{NL} \simeq -\frac{\sigma}{c_s^2}, \quad \sigma \equiv \frac{5}{972} \left( \frac{59 - 55m}{1 - m} \right).
\]
Hence substituting equations (31) and (33) into the BM bound (14) and the LH bound (20) implies that
\[
r_* < \frac{32}{N_{\text{eff}}^2} \frac{(-m)f_1 f_2}{\sqrt{2(1 - m)}} \left( -\frac{f_{NL}}{\beta} \right)^{(1 - 2m)/2}
\]
and
\[
r_* < \frac{10}{(\Delta N)^2} \left( \frac{T_3}{\text{Vol}(X_5)} \right)^{1/3} \left( \frac{h_*}{M_5} \right)^{4/3} \sqrt{2(1 - m)} \left( -\frac{f_{NL}}{\beta} \right)^{(1 - 2m)/2},
\]
respectively.

\(^7\) The special case \( \alpha = 6 \) results in an exponential dependence of the kinetic function on \( X \), and is therefore an example of a higher derivative theory. However, we do not consider this model further, since it does not lead to a weakening of the gravitational wave constraints.

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We conclude, therefore, that the upper limit on the tensor–scalar ratio could be
significantly relaxed if $m < 1/2$, since the nonlinearity parameter is at present only weakly
constrained at $f_{NL} > -151$. Although it is possible to phenomenologically construct a
model which has a value of $m$ in this range, it is clearly preferable to identify UV complete
models that satisfy this requirement within a string theory context. Unfortunately this is
quite difficult to achieve since the inflaton will either be associated with an open or closed
string mode. The open strings are governed by relativistic actions of the DBI form, whilst
closed strings arise from compactification of Einstein gravity and are typically put into
canonical form. However, there do exist classes of open string models which satisfy the
above requirement, namely those associated with multiple coincident branes.

More specifically, if the branes are spatially separated, the effective action is
algebraically equivalent to that of a single brane. It will therefore not satisfy the bound
on $m$. Similarly it was shown in [38] that $n$ coincident branes, in the large $n$ limit, will
also fall into this class of models. On the other hand, if it is assumed that $n$ is finite, the
special properties associated with the matrix degrees of freedom become important and
this results in a kinetic function satisfying $m \leq 1/2$. We will discuss this in more detail
in the following section.

5. Action for multiple coincident branes

We have seen how the form of the kinetic function $P$ can significantly change the strength
of the LH bound on the tensor–scalar ratio, depending on its explicit form. One model in
which a suitable form for $P$ is realized is the multiple coincident brane model as outlined
by Thomas and Ward [38].

The world-volume theory for coincident branes is not fully known, although a number
of proposals have been made. We will restrict our analysis to Myers’ prescription, since
this has been extensively discussed in the literature $^9$ [40, 41]. In general the open string
degrees of freedom for $n$ coincident branes combine to fill out representations of $U(n)$
(as opposed to $U(1)^n$ in the case of separated branes). This introduces a non-Abelian
structure into the theory. In the single-brane case, the fluctuations of the brane are
characterized by induced scalar fields on the world volume. However, for multiple branes
these scalars must be promoted to matrix representations of some gauge group.

Typically the transverse space of any given compactification will always admit an
$SO(3)$ isometry. We can therefore choose our scalars to transform under representations
of the algebra of $SO(3) \sim SU(2)$ by making the identifications

$$\phi^i = R\alpha^i \quad i = 1, \ldots, 3,$$

where $R$ is some scale with canonical mass dimension, and the $\alpha^i$ are specified to be the
irreducible generators satisfying the commutator

$$[\alpha^i, \alpha^j] = 2i\epsilon^{ijk}\alpha^k,$$  \hspace{1cm} (38)

$^8$ In this discussion, we are ignoring the non-trivial backreaction of these branes on the background, and therefore
one should be careful about the range of validity of the effective action.

$^9$ There is also a proposal by Tseytlin [39] for the non-Abelian theory of coincident D-branes.
and the conditions
\[
\frac{1}{n} \text{Tr}(\alpha^i \alpha^j) = C \delta^{ij} = (n^2 - 1) \delta^{ij},
\]
where \( C \) is the quadratic Casimir of the gauge group. The irreducibility condition corresponds to the configuration being in the lowest energy state. It is therefore an additional fine tuning of the initial conditions.

The Myers prescription requires a symmetrized trace (denoted \( \text{STr} \)) to be made over the gauge group. This implies that the symmetric averaging must be taken over all the group dependence before taking the trace. For \( n \gg 1 \), the symmetric trace can be approximated with a trace, which results in the usual DBI action multiplied by a potential term (as described in [38, 29]). However, for finite \( n \), the symmetrization clearly becomes more important and it is essential that we have some means of performing this operation. Recently a prescription for the symmetric trace at finite \( n \) was proposed [42, 43], using highest weight methods and chord diagrams.

The result is that the \( \text{STr} \) acts on different spin representations of \( SU(2) \) in the following manner:

\[
\text{STr}(\alpha^i \alpha^i)^q = 2 (2q + 1) \sum_{i=1}^{n/2} (2i)^{2q}, \quad n \text{ even},
\]

\[
\text{STr}(\alpha^i \alpha^i)^q = 2 (2q + 1) \sum_{i=1}^{(n-1)/2} (2i)^{2q}, \quad n \text{ odd}.
\]

In order for the solution to converge in this prescription, it is also necessary to modify the definition of the radius of the \( SU(2) \) sphere. In the large \( n \) limit, this is given by

\[
\rho^2 = \lambda^2 R^2 \frac{1}{n} \text{Tr}(\alpha^i \alpha^i) = \lambda^2 R^2 C,
\]

where \( \lambda \equiv 2\pi l_s^2 = 2\pi m_s^{-2} \), whereas for finite \( n \), it becomes

\[
\rho^2 = \lambda^2 R^2 \lim_{q \to \infty} \left( \frac{\text{STr}(\alpha^i \alpha^i)^{q+1}}{\text{STr}(\alpha^i \alpha^i)^q} \right) = \lambda^2 R^2 (n - 1)^2.
\]

This converges to the large \( n \) result in the appropriate limit. This point is important, since the warp factor of the four-dimensional theory is typically of the form \( h = h(\rho) \).

The resulting kinetic function for \( n \) coincident branes in the finite \( n \) limit is therefore given by

\[
P = -T_3 \text{STr} \left( h^4(\rho) \sum_{k,p=0}^{\infty} (-Z R^2)^k Y^p(\alpha^i \alpha^i)^{k+p} \left( \frac{1}{k} \right) \left( \frac{1/2}{p} \right) + V(\rho) - h^4(\rho) \right),
\]

where

\[
Z \equiv \lambda^2 h^{-4}(\rho), \quad Y \equiv 4\lambda^2 R^4 h^{-4}(\rho), \quad \left( \frac{1/2}{q} \right) \equiv \frac{\Gamma(3/2)}{\Gamma(3/2 - q)\Gamma(1 + q)}.
\]

Note that the second and third terms in equation (44) are singlets under the \( \text{STr} \) and therefore contribute terms proportional to \( n \). The physics of these branes away from the large \( n \) limit is particularly interesting as discussed further in [38, 27].
The simplest case to consider is that of two coincident branes. However, the form of the STr prescription implies that all other solutions for \( n > 2 \) can be deduced entirely from the \( n = 2 \) solution by a recursion relation. In order to see this, let us define

\[
P_2(Z, Y) = -2T_3 h^4 \left( \frac{(1 + 2Y - (2 + 3Y)Z \dot{R}^2)}{\sqrt{1 + Y} \sqrt{1 - Z \dot{R}^2}} \right),
\]

\[
E_2(Z, Y) = 2T_3 h^4 \left( \frac{(1 + 2Y - YZ \dot{R}^2)}{\sqrt{1 + Y} (1 - Z \dot{R}^2)^{3/2}} \right).
\]

These quantities correspond to the pressure and energy density functions when \( n = 2 \) which arise solely from the DBI sector of the action. The full pressure and energy densities are then given by \( P = P_2 - 2T_3(V - h^4) \) and \( E = E_2 + 2T_3(V - h^4) \), respectively. Since the symmetrized trace acts differently on the differing spin representations of \( SU(2) \), we should expect this structure to follow through in the recursion relation. Indeed, we find that for odd \( n \)

\[
P_n^{(O)} = \sum_{k=1}^{(n-1)/2} P_2[(2k)^2 Z, (2k)^2 Y] - nT_3(V - h^4),
\]

\[
E_n^{(O)} = \sum_{k=1}^{(n-1)/2} E_2[(2k)^2 Z, (2k)^2 Y] + nT_3(V - h^4),
\]

while for even \( n \) we find that

\[
P_n^{(E)} = \sum_{k=1}^{n/2} P_2[(2k - 1)^2 Z, (2k - 1)^2 Y] - nT_3(V - h^4),
\]

\[
E_n^{(E)} = \sum_{k=1}^{n/2} E_2[(2k - 1)^2 Z, (2k - 1)^2 Y] + nT_3(V - h^4).
\]

For example, we can employ these recursion relations to obtain the solutions for \( n = 3 \):

\[
P = -2T_3 \left( \frac{h^4(1 + 8Y - 8Z \dot{R}^2(1 + 6Y))}{\sqrt{1 - 4Z \dot{R}^2} \sqrt{1 + 4Y}} \right) - 3T_3(V - h^4),
\]

\[
E = 2T_3 \left( \frac{h^4(1 + 8Y - 2Z \dot{R}^2(1 + 6Y))}{(1 - 4Z \dot{R}^2)^{3/2} \sqrt{1 + 4Y}} \right) + 3T_3(V - h^4),
\]

which agrees precisely with the result computed by direct expansion of the STr prescription. Furthermore the \( n = 4 \) case is given by

\[
P = -2T_3 \left( \frac{h^4(1 + 2Y - Z \dot{R}^2(2 + 3Y))}{\sqrt{1 + Y} \sqrt{1 - Z \dot{R}^2}} \right) - 4T_3(V - h^4),
\]

\[
+ \frac{h^4(1 + 18Y - 9Z \dot{R}^2(2 + 27Y))}{\sqrt{1 + 9Y} \sqrt{1 - 9Z \dot{R}^2}}
\]

\[ - 4T_3(V - h^4), \]
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\[
E = 2T_3 \left( \frac{h^4(1 + 2Y - Y Z \dot{R}^2)}{\sqrt{1 + Y(1 - Z \dot{R}^2)^{3/2}}} + \frac{h^4(1 + 18Y - 81Z \dot{R}^2)}{\sqrt{1 + 9Y(1 - 9X \dot{R}^2)^{3/2}}} \right) + 4T_3(V - h^4).
\]

(50)

It is clear that the relevant functions increase in complexity as \(n\) increases, since there are progressively more terms to include in the STr expansion. However, equations (47) and (48) represent the most general solutions.

One should also be aware that the backreaction of multiple branes will typically introduce corrections of the form \(n/N\); therefore it is important for this ratio to be small in order for us to trust the supergravity analysis. Typically we can argue that wrapped branes are dual to multi-brane configurations when we are in the limit that \(n \gg 1\). However, since we also wish to keep \(N \gg 1\) we must tune the solution so that \(n/N \ll 1\) is satisfied. Therefore the origin of the backreaction effects is much clearer from this perspective. One can compute the \(1/N\) corrections to the multi-brane action in the large \(n\) limit [27] which, in the dual picture, correspond to backreactive corrections to the wrapped brane models. It would certainly be more useful to develop both these models in more detail.

6. Bounds on the tensor–scalar ratio for multi-brane inflation

The last terms appearing in the summations of equations (47) and (48) correspond to the \(k = (n - 1)/2\) term when \(n\) is odd and to the \(k = n/2\) term when \(n\) is even. This implies that, for all \(n\), these terms can be expressed in the form

\[
P = -2T_3 \left\{ \frac{h^4 \left[ 1 + 2(n - 1)^2Y - [2 + 3(n - 1)^2Y](n - 1)^2Z \dot{R}^2 \right]}{\sqrt{1 + (n - 1)^2Y} \sqrt{1 - (n - 1)^2Z \dot{R}^2}} \right\} - nT_3(V - h^4). \tag{51}
\]

Inspection of equations (46)–(48) implies that the relativistic limit is realized for any finite number of branes when \((n - 1)^2Z \dot{R}^2 \to 1\). In this case, the dominant contribution to the summations appearing in equations (47) and (48) will arise from the last term, equation (51). In this limit, therefore, the kinetic function appearing in the effective action simplifies to

\[
P = 2T_3 \left\{ h^4 \sqrt{1 + (n - 1)^2Y} \left( 1 - \frac{2X}{T_3 h^4} \right)^{-1/2} \right\} - nT_3(V - h^4), \tag{52}
\]

where

\[
Y \equiv \frac{4}{(n - 1)^4 \lambda^2 T_3^2} \left( \frac{\phi}{h} \right)^4, \tag{53}
\]

\[
Z \dot{R}^2 \equiv \frac{2}{(n - 1)^2 h^4 T_3} X, \tag{54}
\]

and we have effectively imposed the relativistic condition

\[
X \simeq \frac{1}{2} T_3 h^4, \tag{55}
\]

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in the numerator of equation (51). For the $n = 2$ and $n = 3$ cases, we have verified by direct calculation that when one calculates the speed of sound (2) and the nonlinearity parameter (7) from the general expressions (46) and (47) and then imposes the relativistic limit (55), one arrives at the identical result by starting explicitly with equation (52).

At this point we should consider the validity of the function in equation (52). Using the recursion relations defined in the previous section, we see that in the large $n$ limit the kinetic function converges to the corresponding function defined in the large $n$ limit in [38]. This is not the same function as that for $n$ separated branes, as the matrix degrees of freedom lead to an additional potential term for the scalars. However, it does belong to the same class of models with $m = 1/2$. We have verified this convergence numerically since the algebraic sums are unfortunately not tractable. The key point is that there must exist some value of $n$ beyond which the function appears to look more like the standard DBI action, rather than the approximate form proposed in (52). For a range of background solutions, the numerics suggest that the approximation is valid up to terms of $\mathcal{O}(10)$. Since there are a large number of parameters in the theory, it is possible to find solutions where $n \gg 10$. However, we will then be forced to generate a larger background flux, which will result in a situation where even the conformal Calabi–Yau condition is no longer valid. In view of this, we focus on the sector of the theory where $n \leq 10$, which implies that the backreaction is under control and that the kinetic function is still of the required form.

Equation (52) is precisely of the form given by the general solution (29), where

$$f_1(\phi) = 2T_3h^4\sqrt{1 + (n - 1)^2Y}, \quad f_2(\phi) = \frac{2}{T_3h^4}. \tag{56}$$

We may therefore immediately conclude from equation (34) that $f_{\text{NL}} \simeq -0.3/c_s^2$. Moreover, since $\beta \simeq 0.9$ in this scenario, equations (31) and (33) reduce to

$$c_s P_X \simeq -1.3\sqrt{1 + (n - 1)^2Y} f_{\text{NL}}. \tag{57}$$

We first consider the LH bound (20). This applies at least for all UV scenarios. It follows after substitution of the relativistic limit (55) into the scalar perturbation amplitude, equation (4), that

$$\mathcal{P}_S^2 \simeq \frac{1}{50T_3h^4\sqrt{1 + (n - 1)^2Y}}\frac{H^4}{f_{\text{NL}}}. \tag{58}$$

Substituting the tensor–scalar ratio (6) into equation (58) then results in a constraint on the magnitude of the warp factor during observable inflation:

$$\frac{h^4}{M_{pl}^4} \simeq \frac{-1}{2T_3\sqrt{1 + (n - 1)^2Y}}\frac{r^2\mathcal{P}_S^2}{f_{\text{NL}}}. \tag{59}$$

Equations (57) and (59) may now be substituted into the LH bound (20) to yield

$$r_* < \frac{1100}{(\Delta N)^6} \frac{[1 + (n - 1)^2Y]}{\text{Vol}(X_5)}\mathcal{P}_S^2 f_{\text{NL}}^2. \tag{60}$$
It is clear that the parameter $Y$ must be sufficiently large if the tensor perturbations are to be non-negligible. For the $\text{AdS}_5 \times X_5$ throat, this parameter takes the constant value

$$Y_{\text{AdS}} \equiv \frac{4\pi^2 g_s N}{(n-1)^2 \text{Vol}(X_5)}. \quad (61)$$

In what follows, we chose natural field-theoretic values for the volume, $\text{Vol}(X_5) \simeq \pi^3$, and the string coupling, $g_s \simeq 10^{-2}$, and further assume that $(n-1)^2 Y \gg 1$. It is possible that observations will probe a range of scales $\Delta N_* \simeq 1$, but it is more realistic to require that the tensor–scalar ratio should not change significantly over the entire range of scales that are accessible to cosmological observation, which corresponds to $\Delta N_* \simeq 4$. After substitution of the above values, therefore, the bound (60) simplifies to

$$r_* < 2.8 \times 10^{-13} \frac{N}{(n-1)^2} f_{\text{NL}}. \quad (62)$$

Global tadpole cancellation constrains the magnitude of the background charge $N$ in terms of the topology of a Calabi–Yau four-fold such that $N < \chi/24$, where $\chi$ is the Euler characteristic of the four-fold [44]–[49]. The maximal known value of the Euler number for such four-folds arises from hypersurfaces in weighted projective spaces and is given by $\chi = 1, 820, 448$ [49]. This implies the upper limit of

$$N < 75,852 \quad (63)$$

for known solutions, although in principle higher values are possible. Imposing the WMAP5 bound $f_{\text{NL}} > -151$ in (62) and noting that $n \geq 2$ for consistency then implies an absolute upper limit on the tensor–scalar ratio:

$$r_* < 5 \times 10^{-4}. \quad (64)$$

This limit is below the sensitivity of the Planck satellite ($r \gtrsim 0.02$) [50]. On the other hand, the projected sensitivity of future CMB polarization experiments indicates that a background of primordial gravitational waves with $r_* \gtrsim 10^{-4}$ should be observable [51,52]. In view of this, it is interesting to consider whether a detectable gravitational wave background could in principle be generated in this class of multi-brane inflationary models. We find from (62) that this would require

$$n < 1 - 5.3 \times 10^{-5} \sqrt{N} f_{\text{NL}} < 1 - 0.014 f_{\text{NL}}, \quad (65)$$

where the theoretical limit (63) for known compactifications has been imposed in the second inequality. We may deduce, therefore, that, since we require $n \geq 2$ for consistency, a detectable tensor signal will require $f_{\text{NL}} < -70$, which implies that an observation of the tensors should also be accompanied by a sufficiently large—and detectable—non-Gaussianity. In other words, this class of models could be ruled out if tensors are observed in the absence of any non-Gaussianity. On the other hand, the current limit of $f_{\text{NL}} > -151$ implies that $n \leq 3$ is required for the tensors to be observable. Consequently, if tensor perturbations are detected, this would rule out all models with $n \geq 4$ or, alternatively, would require presently unknown configurations with $N$ exceeding bound (63).

In the above analysis we assumed that the string coupling took the value $g_s \simeq 10^{-2}$. For the $\text{AdS}_5 \times X_5$ throat, the bound (60) depends proportionally on $g_s$ and can therefore...
be weakened by allowing for larger values of the string coupling. For example, increasing this parameter by a factor of 4 to $g_s \simeq 0.04$ (so that it is still in the perturbative regime) relaxes the limit on the number of branes for the tensors to be detectable to $n \leq 5$. Similarly, considering a smaller value for the volume of the Einstein manifold $X_5$ will also weaken the upper limit.

Let us reiterate that this limit on $n$ is well within the regime of validity for the theory, which we have argued is self-consistent for $n < 10$. Moreover since the constraint (65) arises using the absolute maximal bound on the known Euler characteristics, it suggests that in realistic scenarios $n$ will always be much smaller than this. Indeed, one could argue that only the $n = 2$ and $n = 3$ theories are likely to be valid over a large distribution of the flux landscape.

We must also ensure that our approximation $(n - 1)^2 Y \gg 1$ is valid for consistency. For the parameter values we have chosen this requires that $g_s N \gg (n - 1)^2$ and this is satisfied if the condition (65) holds. Note also that we require $N \gg n$ for the supergravity approximation to be under control and for backreaction effects to be negligible. This is also satisfied when (65) holds.

For completeness we should also consider the BM bound (14) for this class of models. This is given by

$$r_* < -\frac{42}{NN_{\text{eff}}^2} \sqrt{1 + (n - 1)^2 Y f_{\text{NL}}}$$

and, in the case of an AdS$_5 \times X_5$ throat, simplifies to

$$r_* < -\frac{5f_{\text{NL}}}{N_{\text{eff}}^2 (n - 1)\sqrt{N}}$$

(66)

Comparing the limits (62) and (67) implies that the LH bound is stronger than the corresponding BM bound if

$$n > 1 - 5.5 \times 10^{-14} N^{3/2} N_{\text{eff}}^2 f_{\text{NL}}$$

(68)

and this condition is always satisfied if

$$-5.5 \times 10^{-14} N^{3/2} N_{\text{eff}}^2 f_{\text{NL}} < 1.$$  

(69)

Moreover, the bound (69) will itself be satisfied for all values of $f_{\text{NL}}$ and $N$ if it is satisfied when the limits $f_{\text{NL}} = -151$ and $N = 75852$ are imposed. Hence, we conclude that the LH bound is stronger for $N_{\text{eff}} < 75$. In general, it is difficult to quantify the magnitude of $N_{\text{eff}}$ without imposing further restrictions on the parameters of the models and, in particular, on the functional form of the inflaton potential. However, if the ratio $\epsilon/P_{X}$ remains approximately constant during the final stages of inflation, one would anticipate that $N_{\text{eff}} \lesssim 60$. Nevertheless, if $N \ll 75852$, the bound (68) will only be violated for $n \leq 3$ if $N_{\text{eff}} \gg 60$.

Finally, it should be emphasized that the derivation of the LH bound underestimates the Planck mass by assuming that the volume of the throat is much smaller than the volume of the compactified Calabi–Yau three-fold. It is likely, therefore, that the actual constraint on $r$ would be much stronger. Consequently, although the bound (65) does marginally allow for detectable tensors if $n$ is sufficiently small, in practice this constraint would be further tightened by a more complete calculation. Nonetheless, our analysis does not necessarily rule out these models as viable candidates for inflation. Rather, it suggests that it will be difficult to construct a working model that results in a detectable tensor signal.
7. Discussion

The relativistic DBI brane scenario represents an attractive, string-inspired realization of the inflationary scenario. Recent cosmological data has placed very strong constraints on the simplest models based on a single D3-brane. The strength of these constraints follows from field-theoretic upper limits on the tensor–scalar ratio, \( r \), which in turn arise because the effective DBI action satisfies special algebraic properties. This provides motivation for considering generalizations of the scenario, in particular to multi-brane configurations.

In this paper we have identified a phenomenological class of effective actions for which the constraints on \( r \) are relaxed if significant (and detectable) non-Gaussian curvature perturbations are generated during inflation. Included in this class is the relativistic limit of the action associated with \( n \) coincident branes in the small \( n \) limit. Moreover we have found that such an effective action for arbitrary, finite \( n \) can be expressed directly in terms of the corresponding action for the \( n = 2 \) model, due to the fact that the spin-1/2 representation of \( SU(2) \) is actually the fundamental one. This allows us to construct models for various values of \( n \) using the two-brane action and the iteration equations. Physically these brane configurations typically have a smaller sound speed than the single-brane models due to the different structure of their action. This differing structure is also manifest in the non-relativistic limit—since the non-Abelian nature of the theory introduces new 'potential terms' that couple to the usual kinetic components of the action. In some cases this extra potential could help to further flatten the inflaton potential, whereas in other cases it will make it significantly steeper. An in-depth analysis of slow roll in such models would be welcome. Their backreaction is also significantly smaller than other multi-brane configurations, and therefore this relaxes the amount of tuning required for the background charge.

We then proceeded to consider the question of whether the upper limits on \( r \) could be relaxed to such an extent that a background of primordial gravitational waves might be detectable in future CMB experiments. The vast majority of string-inspired inflationary models that have been proposed to date generate an unobservable tensor background. We found that a detectable signal is possible, in principle, for typical string-theoretic parameter values if the number of coincident branes, \( n \), is either 2 or 3. This is consistent with known F-theory configurations and current WMAP3 limits on the non-Gaussianity. Furthermore, we found that the level of non-Gaussianity must exceed \( f_{\text{NL}} \lesssim -70 \) if such configurations are to generate a detectable tensor signal. This is well within the projected sensitivity of the Planck satellite.

Our analysis invoked an \( \text{AdS}_5 \times X_5 \) warped throat geometry. However, we made no assumptions regarding the form of the inflaton potential, other than imposing the implicit requirement that the universe underwent a phase of quasi-exponential expansion. In this sense, therefore, we have yet to explicitly establish that these inflationary models will be able to generate a measurable tensor signal. Nonetheless, since such a detection would provide a unique observational window into high energy physics, our results provide strong motivation for considering the cosmological consequences of these multi-brane configurations further when specific choices for the inflaton potential are made. In particular, it would be interesting to employ the techniques developed in [16, 53, 13, 15, 54] to identify the ranges of parameter space that are consistent with current cosmological constraints.
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observations. It would also be interesting to investigate whether the effective action \( (29) \) with values of \( m \neq -1/2 \) arises in other string-inspired settings.

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Appendix. Exact solution

Equation \( (25) \) can be analytically solved in full generality without imposing the limits \( (26) \) on the derivatives of the kinetic function. This allows us to determine the most general class of models where the nonlinearity parameter satisfies the condition \( f_{\text{NL}} \propto 1/c_s^2 \) at leading order.

In general equation \( (25) \) takes the form

\[
(2 - \alpha) P_X P_{XX} + 4 X P_{X XX}^2 = \frac{2 \alpha}{3} X P_X P_{XXX} \tag{A.1}
\]

and this reduces to

\[
\alpha Q_{,X} = (6 - \alpha) Q^2 + \frac{3(2 - \alpha) Q}{2 X}, \tag{A.2}
\]

where \( Q \equiv P_{XX}/P_X \). Equation \( (A.2) \) can be transformed into the linear equation

\[
U_{,X} + \frac{3(2 - \alpha) U}{2 \alpha X} = \frac{\alpha - 6}{\alpha} \tag{A.3}
\]

after the change of variables \( U \equiv 1/Q \) and the general solution to equation \( (A.3) \) is given by

\[
\frac{P_{XX}}{P_X} = \frac{1}{X [f_2(\phi) X^{(\alpha-6)/2\alpha} - 2]} \tag{A.4}
\]

Integrating a second time implies that

\[
P_X = f_1(\phi) \left(1 - f_2(\phi) X^{-s}\right)^{1/(2s)}, \tag{A.5}
\]

where \( s \equiv (\alpha - 6)/(2\alpha) \) and we have redefined the arbitrary integration functions \( f_i(\phi) \). Finally equation \( (A.5) \) can be formally integrated in terms of a hypergeometric function

\[
P = f_1 X \, _2F_1 \left(-\frac{1}{s}, -\frac{1}{2s}; 1 - \frac{1}{s}; f_2 X^{-s}\right), \tag{A.6}
\]

which represents the most general solution for this class of models. Note that we have set the remaining constant of integration to zero to ensure that the kinetic function vanishes in the limit of zero velocity. In fact this expression admits many different classes of solution, arising as limits of the expansion of the hypergeometric function.

The special case of \( \alpha = 2(s = -1) \) implies (after a further redefinition of the functions \( f_i(\phi) \)) that

\[
P = f_1 \sqrt{1 - f_2 X} - f_3 \tag{A.7}
\]

and this corresponds to the standard DBI action \( (23) \) \([34,55]\).
The case $\alpha = 18/5 (s = -1/3)$ can also be expressed in terms of elementary functions, again after redefinition of the $f_i(\phi)$:

$$P = \frac{f_1 \left( 8 - 4 f_2 X^{1/3} - \left( f_2 X^{1/3} \right)^2 \right)}{\sqrt{1 - f_2 X^{1/3}}} - f_3.$$  \hspace{1cm} (A.8)

Note that this expression appears in a slightly different form to that in (51). However, in deriving (51) we assumed the relativistic limit, which in turn imposes a non-trivial relation between $X$ and $\phi$. Using this, and with a suitable redefinition of the functions, we can easily transform the above expression into the required form.

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