Average Kinetic Energy of Heavy Quark and Virial Theorem

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Abstract

We derive the virial theorem of the relativistic two-body system for the study of the $B$-meson physics. It is also shown that the solution of the variational equation always satisfies the virial theorem. From the virial theorem we also obtained $\mu_\pi^2 \equiv -\lambda_1 \equiv \langle \vec{p}^2 \rangle = 0.40 \sim 0.58 \text{ GeV}^2$, which is consistent with the result of the QCD sum rule calculations of Ball et al.

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I. Introduction

The $B$-physics provides many important and attractive physics informations, and is under active experimental and theoretical investigations. The $B$-factories at KEK and SLAC will give valuable clues for the better understanding of the Standard Model (SM) and the beyond. We expect the $B$-meson system will show the CP-violation phenomena $[1]$, for which we have only the $K_L \rightarrow \pi\pi$ decay for more than 30 years. The mechanism of CP-violation through the complex phase of the Kobayashi-Maskawa three family mixing matrix $[2]$ in the Weinberg-Salam model is presently the CP-violation within the SM. In order to understand and precisely test the SM, it is essential to know the values of the Kobayashi-Maskawa matrix elements, especially through the determinations of $V_{ub}$ $[3]$ and $V_{td}$, and confirm the unitarity triangle $[4]$, both of which will be best probed at the forthcoming $B$-factories.

Recently, it has been an important subject to obtain an accurate value of the kinetic energy, $\mu_\pi^2 (\equiv -\lambda_1 \equiv \langle p^2 \rangle)$, of the heavy quark inside $B$-meson in connection with the heavy quark effective theory (HQET) $[5, 6]$. Ball et al. $[7]$ calculated $\mu_\pi^2$ using the QCD sum rule approach and obtained $\mu_\pi^2 = 0.50 \pm 0.10$ GeV$^2$ for $B$-meson, while Neubert $[8]$ obtained $-\lambda_1 = 0.10 \pm 0.05$ GeV$^2$. Neubert also derived $[9]$ the field-theory version of the virial theorem within the HQET framework. Based on the theorem, he implied that the result $\mu_\pi^2 \sim 0.5$ GeV$^2$ of the QCD sum rule calculations of Ball et al. is too large. However, it should be noted that Refs. $[7]$ and $[8]$ differ in the choice of the 3-point correlation functions used to estimate the matrix elements of interest. The difference in the numerical values obtained in these two calculations is understood in terms of the contributions of excited states, which in principle must be subtracted in any QCD sum rule analysis. In practice, this subtraction can only be done approximately. Therefore, the numerical differences between Refs. $[7]$ and $[8]$ indicate the limited accuracy...
of the QCD sum rule approach. Bigi et al. [10] derived an inequality between the expectation value of the kinetic energy of the heavy quark inside the hadron and that of the chromomagnetic operator, $\langle p^2 \rangle \geq \frac{3}{4} (M_V^2 - M_P^2)$, which gives $\mu_x^2 \geq 0.36 \text{ GeV}^2$ for $B$-meson system. However, Kapustin et al. [11] showed later that this lower bound could be significantly weakened by higher order perturbative corrections. Previously we also calculated [12] the value of $\langle p^2 \rangle$ by applying the variational method to the relativistic Hamiltonian, and obtained $\langle p^2 \rangle = 0.44 \text{ GeV}^2$. Similarly de Fazio [13] computed the matrix elements of the kinetic energy operator by means of a QCD relativistic potential model, and found $\mu_x^2 = 0.46 \text{ GeV}^2$.

Besides the above theoretical calculations of $\mu_x^2$, Gremm et al. [14] extracted the average kinetic energy by comparing the prediction of the HQET [15] with the shape of the inclusive $B \rightarrow Xl\nu$ lepton energy spectrum [16] for $E_l \geq 1.5$ GeV, in order to avoid the contamination from the secondary leptons of cascade decays of $b \rightarrow c \rightarrow sl\nu$. They obtained $-\lambda_1 = 0.19 \pm 0.10 \text{ GeV}^2$. Combining the experimental data on the inclusive decays of $D \rightarrow Xe\nu$, $B \rightarrow Xe\nu$ and $B \rightarrow X\tau\nu$, Ligeti et al. [17] derived the bound of $\mu_x^2 \leq 0.63 \text{ GeV}^2$ if $\bar{\Lambda} \geq 0.240 \text{ GeV}$, or $\mu_x^2 \leq 0.10 \text{ GeV}^2$ if $\bar{\Lambda} \geq 0.500 \text{ MeV}$. Li et al. [18] obtained the value of $-\lambda_1$ centered at $0.71 \text{ GeV}^2$ from the analysis of the inclusive radiative decay $B \rightarrow X_{s}\gamma$ [19] within the perturbative QCD framework. Related with the comparison of various theoretical calculations of $\mu_x^2$, we note that Ref. [20] emphasizes that one has to be careful when comparing the values of $-\lambda_1$ obtained using different theoretical methods. For instance, QCD sum rule determinations of $-\lambda_1$ for the ground state heavy mesons and baryons are affected by a renormalon ambiguity problem [20].

Concerned with the phenomenological importance of the numerical value for the kinetic energy of the heavy quark, we would like to derive the virial theorem of the two-body system within the relativistic potential model approach for the study of the $B$-meson system. We show at the end that $\mu_x^2 \sim 0.50 \text{ GeV}^2$ of Ball et al. [7] is consistent with our virial theorem result. In Section II we present the variational
analysis of the relativistic quark model for $B$-meson system, and compare it with the HQET. In Section III we derive the virial theorem of the relativistic two-body system which is appropriate for $B$-meson system, and we show that the solutions of the variational equation always satisfy the virial theorem automatically. Section IV contains discussions and conclusions.

II. Relativistic Quark Model

For the study of the bound state properties of hadrons which contain both heavy and light quarks like $B$-meson, it is appropriate to use the relativistic quark model \cite{12, 21}, which is a potential model approach with relativistic kinematics. In the relativistic quark model, the Hamiltonian for $B$-meson is given by

$$H = \sqrt{p^2 + M^2} + \sqrt{p^2 + m^2 + V(r)} \approx M + \frac{p^2}{2M} + \sqrt{p^2 + m^2 + V(r)}$$

in the $B$-meson rest frame, where $M$ and $m$ are heavy and light quark mass respectively. For the potential in (1), we use the Cornell potential \cite{21, 22, 23},

$$V(r) = -\frac{\alpha_c}{r} + Kr + V_0, \quad \alpha_c \equiv \frac{4}{3} \alpha_s, \quad V_0 = \text{constant}.$$  

It is difficult to solve the eigenvalue equation of the Hamiltonian operator (1), so we use the variational method. The variational method is particularly useful in the present work, since we will show in the following section that the solutions of the variational equation always satisfy the virial theorem.

In the variational method, the expectation value of the Hamiltonian is calculated with some trial wave function which has a variational parameter. The value of the variational parameter is determined by the stationary condition (variational equation or so-called gap equation). We take the variational parameter which has the dimension of mass. Then from the dimensional analysis, the expectation value of each term in the Hamiltonian can be expressed as

$$\langle p^2 \rangle = C \mu^2,$$
\[ \langle \sqrt{p^2 + m^2} \rangle = a_1 \mu + a_2 m^2 / \mu + O\left((m^2 / \mu)^2\right), \]
\[ \langle -\frac{\alpha_c}{r} + Kr + V_0 \rangle = \alpha_c (-b_1 \mu) + K(b_2 / \mu) + V_0, \tag{3} \]

where \( \mu \) is the variational parameter of mass dimension, and \( C, a_1, a_2, b_1 \) and \( b_2 \) are dimensionless numerical constants. Collecting the terms in (3), we have
\[ \langle H \rangle_{\mu} = E(\mu) = M + [V_0 + (a_1 - b_1 \alpha_c) \mu + (a_2 m^2 + b_2 K) / \mu] + \frac{C}{2M} \mu^2 \]
\[ = M + [V_0 + \beta \mu + \gamma / \mu] + \frac{C}{2M} \mu^2, \tag{4} \]

where
\[ \beta \equiv a_1 - b_1 \alpha_c \quad \text{and} \quad \gamma \equiv a_2 m^2 + b_2 K. \tag{5} \]

We neglected \( O\left((m^2 / \mu)^2\right) \) terms in (4). Then the variational equation reads
\[ \frac{\partial}{\partial \mu} E(\mu) = \beta - \gamma / \mu^2 + \frac{C}{M} \mu = 0. \tag{6} \]

Rather than solving this equation numerically, we obtain the solution as a power series in \( 1/M \), since \( M \) is large. Noting that the solution is given by \( \bar{\mu} \sim \sqrt{\gamma / \beta} \) for very large value of \( M \), we expand
\[ \bar{\mu} = h_0 + h_1 \frac{1}{M} + h_2 \frac{1}{M^2} + \cdots. \tag{7} \]
Substituting (7) into (3) and matching order by order, we get
\[ h_0 = \sqrt{\frac{\gamma}{\beta}}, \quad h_1 = -\frac{C}{2} \left(\frac{\gamma}{\beta^2}\right), \quad h_2 = \frac{5C^2}{8} \sqrt{\frac{\gamma}{\beta}} \left(\frac{\gamma}{\beta^3}\right), \quad \cdots. \tag{8} \]
Then we get \( E(\bar{\mu}) \), the \( B \)-meson mass \( M_B \), as a power series in \( 1/M \),
\[ M_B = E(\bar{\mu}) = M + \left(V_0 + 2\sqrt{\gamma \beta}\right) + \frac{C}{2} \left(\frac{\gamma}{\beta}\right) \frac{1}{M} + O\left(\frac{1}{M^2}\right). \tag{9} \]
Therefore we have performed the \( 1/M \) expansion for \( M_B \) in the relativistic quark model in the framework of the variational method. Then, let us compare the series expansion in (9) with that of the HQET [4, 5] which is written as
\[ M_B = M + \tilde{\Lambda} + \frac{1}{2M} (T + \nu_B \Omega) + O\left(\frac{1}{M^2}\right), \tag{10} \]
where $\nu_B = 1/4$ and $-3/4$ for vector and pseudoscalar $B$-meson respectively. The zeroth order term $\Lambda$ in (10) which is the contribution from the light degrees of freedom corresponds to the sum of the expectation value of the light quark kinetic energy and that of the potential energy,

$$\Lambda \longleftrightarrow \langle \sqrt{\mathbf{p}^2 + m^2 + V(r)} \rangle = V_0 + 2\sqrt{\gamma/\beta}. \tag{11}$$

The heavy quark kinetic energy term $T$ in (10) has the correspondence

$$T \longleftrightarrow \langle \mathbf{p}^2 \rangle = C \frac{\gamma}{\beta}. \tag{12}$$

If we had included a spin dependent potential which is inversely proportional to the heavy quark mass, the expectation value of which would have corresponded to the chromomagnetic interaction term in (10) as

$$\frac{\nu_B \Omega}{2M} \longleftrightarrow \langle V_s \rangle = \frac{1}{M} \langle v_s \rangle \quad \text{with} \quad v_s = \frac{2}{3m} \mathbf{s}_1 \cdot \mathbf{s}_2 \nabla^2 \left(-\frac{\alpha_c}{r}\right), \tag{13}$$

where $v_s$ is the nonrelativistic spin-spin interaction potential.

III. Virial theorem

In classical mechanics, the virial theorem for a system with kinetic energy $K(p)$ and potential energy $V(r)$ is based on the relation

$$\frac{d}{dt}(\mathbf{r} \cdot \mathbf{p}) = \mathbf{v} \cdot \mathbf{p} + \mathbf{r} \cdot \frac{d}{dt}\mathbf{p} = (\frac{\partial}{\partial \mathbf{p}} K(\mathbf{p})) \cdot \mathbf{p} + \mathbf{r} \cdot (-\nabla V(\mathbf{r})), \tag{14}$$

where $K(\mathbf{p})$ is the kinetic energy given by $K(\mathbf{p}) = \int \mathbf{F} \cdot d\mathbf{r} = \int \mathbf{v} \cdot d\mathbf{p}$. The time average of the left hand side of (14) vanishes for a periodic motion or a bounded motion in an infinite time interval, hence we get the virial theorem,

$$\langle \mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} K(\mathbf{p}) - \mathbf{r} \cdot \frac{\partial}{\partial \mathbf{r}} V(\mathbf{r}) \rangle_{\text{time average}} = 0. \tag{15}$$

This theorem holds for both nonrelativistic and relativistic kinematics which have the following relations respectively,

$$\mathbf{p} = m\mathbf{v}, \quad K = \frac{\mathbf{p}^2}{2m}, \quad \mathbf{p} \cdot \frac{\partial K}{\partial \mathbf{p}} = \frac{\mathbf{p}^2}{m} = 2K; \tag{16}$$
\[ p = \frac{mv}{\sqrt{1 - v^2}}, \quad K = \sqrt{p^2 + m^2}, \quad \mathbf{p} \cdot \frac{\partial K}{\partial \mathbf{p}} = \frac{p^2}{\sqrt{p^2 + m^2}}. \]

In quantum mechanics, the virial theorem for a system with Hamiltonian \( H = K(p) + V(r) \) is based on the relation
\[ \frac{d}{dt} \langle \mathbf{r} \cdot \mathbf{p} \rangle = \frac{\hbar}{i} \langle [\mathbf{r} \cdot \mathbf{p}, H] \rangle = \frac{\hbar}{i} \langle \mathbf{r} \cdot (-\frac{\partial V}{\partial \mathbf{r}}) + (\frac{\partial K}{\partial \mathbf{p}}) \cdot \mathbf{p} \rangle. \quad (17) \]

For stationary states represented by eigenfunctions of \( H \), the left hand side of (17) is zero. Hence we get the virial theorem written as
\[ \langle \mathbf{r} \cdot (-\frac{\partial V}{\partial \mathbf{r}}) + (\frac{\partial K}{\partial \mathbf{p}}) \cdot \mathbf{p} \rangle = 0. \quad (18) \]

However, the functions which satisfy the virial theorem (18) are not restricted to the eigenfunctions of \( H \). The solutions of the variational equation also satisfy the theorem (18). With the variational parameter \( \mu \) of the mass dimension, the expectation value of the Hamiltonian is expressed as
\[ E(\mu) = \langle \psi(\mu)|H|\psi(\mu)\rangle = \langle K(p) + V(r) \rangle_\mu, \quad (19) \]
for central potential \( V = V(r) \). In general, the kinetic and potential energy functions in (19) can be expanded in the Laurent series as
\[ K(p) = \sum_n k_n p^n, \quad V(r) = \sum_n v_n r^n. \quad (20) \]
The expectation values of each terms are written in terms of \( \mu \),
\[ \langle p^n \rangle_\mu = a_n \mu^n, \quad \langle r^n \rangle_\mu = b_n \mu^{-n}, \quad (21) \]
where \( a_n, b_n \) are numerical constants which have no dimensions. The expressions in (21) satisfy the relations,
\[ \mu \frac{\partial}{\partial \mu} \langle p^n \rangle_\mu = \langle p \frac{d}{dp} p^n \rangle_\mu, \quad \mu \frac{\partial}{\partial \mu} \langle r^n \rangle_\mu = -\langle r \frac{d}{dr} r^n \rangle_\mu. \quad (22) \]
From (19)–(22), we get
\[ \mu \frac{\partial}{\partial \mu} E(\mu) = \langle p \frac{d}{dp} K(p) - r \frac{d}{dr} V(r) \rangle. \quad (23) \]
The equation (23) means that the variational equation, \( \partial E(\mu)/\partial \mu = 0 \), is equivalent to the virial theorem (18). If we have used the parameter \( \mu \) having the dimension of length, we would have gotten the same result except for the overall sign, which is irrelevant.

We can also show the relations more explicitly. For the expectation value of the potential,

\[
\langle V(r) \rangle_{\mu} = \int d^3r \ | \psi(r; \mu) |^2 V(r) = \mu^3 \int dr r^2 A(\mu r) V(r),
\]

(24)

where we defined \( A(\mu r) \) such that it satisfies

\[
\int d^3r \ | \psi(r; \mu) |^2 = \int dr r^2 | R(r; \mu) |^2 = \mu^3 \int dr r^2 A(\mu r) = 1.
\]

(25)

Changing the integration variable in (24) by \( x = \mu r \),

\[
\langle V(r) \rangle_{\mu} = \int dx x^2 A(x) V(x/\mu),
\]

(26)

then we get the relation

\[
\mu \frac{\partial}{\partial \mu} \langle V(r) \rangle_{\mu} = \int dx x^2 A(x) \left\{ -\frac{x}{\mu} V'(x/\mu) \right\}
\]

\[
= \mu^3 \int dr r^2 A(\mu r) \left\{ -r V'(r) \right\}
\]

\[
= -\langle r \frac{d}{dr} V(r) \rangle,
\]

(27)

For the expectation value of the kinetic energy, we can follow a similar procedure in the momentum space to get the desired result \( \mu \frac{\partial}{\partial \mu} \langle K(p) \rangle_{\mu} = \langle p \frac{d}{dp} K(p) \rangle \). Therefore we obtain the relation (23) again.

In the above, we have shown that \( \psi(\bar{\mu}) \) with \( \bar{\mu} \) being the solution of the variational equation, \( \partial E(\mu)/\partial \mu = 0 \), automatically satisfies the virial theorem given by

\[
\langle \psi(\bar{\mu}) | p \frac{\partial}{\partial p} K(p) | \psi(\bar{\mu}) \rangle = \langle \psi(\bar{\mu}) | r \frac{\partial}{\partial r} V(r) | \psi(\bar{\mu}) \rangle,
\]

(28)

even though it is not an eigenfunction of \( H \). We emphasize that the solution set of the virial theorem contains the functions determined by the variational method as
well as the eigenfunctions of the Hamiltonian. Repko et al. [24] previously proved that variational principle implies the virial theorem, and Lucha et al. [25] derived the relativistic form of the virial theorem. Let us show explicitly that the virial theorem (28) is satisfied for the system given by the Hamiltonian of (1) and (2). For $B$-meson which has the Hamiltonian (1) with $K(p) = p^2/(2M) + \sqrt{p^2 + m^2}$, the virial theorem (28) is written as

$$\langle \frac{p^2}{M} + \frac{p^2}{\sqrt{p^2 + m^2}} \rangle = \langle r \frac{\partial}{\partial r} V(r) \rangle. \quad (29)$$

The left hand side of (29), which is related with the kinetic energy, can be calculated from (3),

$$\langle \frac{p^2}{M} + \frac{p^2}{\sqrt{p^2 + m^2}} \rangle = C\mu^2 \langle \sqrt{p^2 + m^2} \rangle + \langle \sqrt{p^2 + m^2} - \frac{m^2}{\sqrt{p^2 + m^2}} \rangle$$
$$= C\mu^2 \langle \sqrt{p^2 + m^2} \rangle + (a_1\mu + a_2 m^2/\mu) - 2m^2 \frac{\partial}{\partial (m^2)} (a_1\mu + a_2 m^2/\mu)$$
$$= C\mu^2 \langle \sqrt{p^2 + m^2} \rangle + (a_1\mu - a_2 m^2/\mu). \quad (30)$$

The right hand side of (29), which is related to the potential energy, is also given from (3),

$$\langle r \frac{\partial}{\partial r} V(r) \rangle = \langle \frac{\alpha_c}{r} + Kr \rangle = \alpha_c(b_1\mu) + K(b_2/\mu). \quad (31)$$

Putting (31) and (31) in (29) and dividing by $\mu$, we get the variational equation (6). Therefore we have shown that the virial theorem is satisfied for the solution of the variational equation of the Hamiltonian given by (1) and (2).

IV. Discussions and Conclusions

We have shown that the relativistic quark model combined with the variational method can give many useful results. The solutions of the variational equation always satisfy the virial theorem automatically, and the $B$-meson mass $M_B$ is expressible as a power series in $1/M$. Combining (29) and (31), we get the relativistic...
virial theorem within the Cornell potential,

\[
\frac{\mathbf{p}^2}{M} + \frac{\mathbf{p}^2}{\sqrt{\mathbf{p}^2 + m^2}} = \mathbf{r} \cdot \nabla V = \frac{\alpha_c}{r} + K r = \alpha_c (b_1 \mu) + K (b_2 / \mu).
\]  

(32)

With the usual input values \[21, 22, 23\] of \( \alpha_s = 0.21 \sim 0.36, K = 0.19 \text{ GeV}^2, m = 0.15 \text{ GeV} \), we obtain \( \mu_\pi^2 = 0.40 \sim 0.58 \text{ GeV}^2 \) for Gaussian trial functions, which is consistent with the result \( \mu_\pi^2 \sim 0.50 \text{ GeV}^2 \) of the QCD sum rule calculations of Ball et al. \[7\]. If we use exponential trial functions, we get higher values of \( \mu_\pi^2 \). If we rather had used the virial theorem of the nonrelativistic two-body system

\[
\frac{\mathbf{p}^2}{M} + \frac{\mathbf{p}^2}{m} = \mathbf{r} \cdot \nabla V = \frac{\alpha_c}{r} + K r,
\]  

(33)

we would have obtained the value \( \mu_\pi^2 = 0.10 \sim 0.15 \text{ GeV}^2 \), which is very small compared with the above relativistic result.

If we apply the virial theorem to the system of one-body in an external potential, it reads

\[
\frac{\mathbf{p}^2}{M} = \mathbf{r} \frac{\partial}{\partial r} V(r),
\]  

(34)

with the nonrelativistic kinematics for the heavy quark. This corresponds to a one-body (heavy quark) motion in a fixed external potential of the background system which produces the potential in the meson rest frame. Compared with the hydrogen atom, where the nonrelativistic one-body virial theorem (through the reduced mass) is also applied, the roles of the heavy (proton) and light (electron) degrees of freedom are reversed. Within the potential model approach, the background system corresponds to a valence light quark and virtual gluons, the former carries the compensating momentum against the heavy quark motion and the latter contributes as a potential energy. Considering these, the correct virial theorem for \( B \)-meson system within the potential model approach should be the form of the two-body closed system as given in \[92\].

In conclusion, we derived the virial theorem for the relativistic two-body system, and numerically obtained \( \mu_\pi^2 \equiv -\lambda_1 \equiv \langle \mathbf{p}^2 \rangle = 0.40 \sim 0.58 \text{ GeV}^2 \), which is
consistent with the result of the QCD sum rule calculations of Ball et al. It is also shown that the solution of the variational equation always satisfies the virial theorem. As our final comment, we note the recent observation [20] on the mixing of the operator for the heavy-quark kinetic energy with the identity operator, which implies that the parameter $\lambda_1$ of the heavy-quark effective theory is not directly a physical quantity, but requires a non-perturbative subtraction. Concerned with the phenomenological importance of the numerical value for the kinetic energy of the heavy quark, it would be the most urgent to try all the possible attempts on the physical understanding of the kinetic energy of the heavy quark inside $B$-meson system.

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