K$^+ \to \pi^+\pi^0\gamma$ in the Standard Model and Beyond

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In this note we show how improved theoretical analysis combined with recent experimental data coming from NA48/2 concerning $K^+ \to \pi^+\pi^0\gamma$ decay shed light on the dynamics of the $s \to d\gamma$ transition. Consequences on NP analysis are also presented.

1 Introduction

In the search for New Physics (NP) the $s \to d\gamma$ process is complementary to $b \to s\gamma$ and $\mu \to e\gamma$, as the relative strength of these transitions is a powerful tool to investigate the NP dynamics. However, since $s \to d\gamma$ takes place deep within the non-perturbative regime of QCD we have to control hadronic effects and find observables sensitive to the short-distance dynamics, and thereby to possible NP contributions. The purpose of this note is to show how this can be achieved using the $K^+ \to \pi^+\pi^0\gamma$ observable [1].

In section 2, the anatomy of the $s \to d\gamma$ process in the Standard Model (SM) is shortly detailed. In section 3, we analyse the $K^+ \to \pi^+\pi^0\gamma$ decay in the SM whereas section 4 is devoted to show how, in the MSSM, rare and $K^+ \to \pi^+\pi^0\gamma$ decays, as well as $\text{Re}(\varepsilon'_K/\varepsilon_K)$ can be exploited to constrain NP.

2 The $s \to d\gamma$ anatomy

In the SM, the flavour changing electromagnetic process $s \to d\gamma$ is a loop effects which at low energy scale is described by the effective $\Delta S = 1$ Hamiltonian [2]

$$H_{\text{eff}}(\mu \approx 1 \text{ GeV}) = \sum_{i=1}^{10} C_i(\mu) Q_i(\mu) + C_{\gamma\gamma'} Q_{\gamma\gamma'} + C_\gamma Q_\gamma + h.c.,$$

(1)

where the $Q_i$ are effective four-quarks operators whereas the quark-bilinear electric $Q_{\gamma\gamma'}^\pm$ and magnetic $Q_{\gamma}^\pm$ operators are respectively given by $\sum Q_{\gamma}^\pm = (\bar{s}_L \gamma^\nu d_L \pm \bar{s}_R \gamma^\nu d_R) \partial^\mu F_{\mu\nu}$ and $Q_{\gamma}^\pm = (\bar{s}_L \sigma^{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} d_L) F_{\mu\nu}$. In the non perturbative regime of QCD this Hamiltonian is hadronized into an effective weak Lagrangian that shares the chiral properties of the operators contained in $H_{\text{eff}}$. The chiral structures of $Q_i$ and $Q_{\gamma}^\pm$ allow the usual $O(p^2)$ weak Lagrangian $L_W = G_S O_8 + G_{27} O_{27} + G_{cw} O_{cw}$ (detailed in [10]) whereas the chirality flipping $Q_{\gamma}^\pm$ operators induce more involved $O(p^4)$ local interactions (detailed in [1,10]). The non-trivial dynamics corresponding to

\*By definition : $2\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]$. 
the low-energy tails of the photon penguins arise at $\mathcal{O}(p^4)$ (the $\mathcal{O}(p^2)$ dynamics being completely predicted by Low’s theorem [3]) where they are represented in terms of non-local meson loops, as well as additional $\mathcal{O}(p^4)$ local effective interactions, in particular the $\Delta I = 1/2$ enhanced $N_{14}, ..., N_{18}$ octet counterterms [4,5].

3 $K^+ \to \pi^+\pi^0\gamma$ in the SM 

For the $K^+ \to \pi^+\pi^0\gamma$ decay, the standard phase-space variables are chosen as the $\pi^+$ kinetic energy $T^*_c$ and $W^2 \equiv (q_\gamma \cdot P_K)(q_\gamma \cdot P_{\pi^+})/m^2_{\pi^+}m^2_K$ [6]. Indeed, pulling out the dominant bremsstrahlung contribution, the differential rate can be written

$$\frac{\partial^2 \Gamma}{\partial T^*_c \partial W^2} = \frac{\partial^2 \Gamma_{IB}}{\partial T^*_c \partial W^2} \left( 1 - 2 \frac{m^2_{\pi^+}}{m_K} \text{Re} \left( \frac{E_{DE}}{eA_{IB}} \right) W^2 + \frac{m^4_{\pi^+}}{m^2_K} \left( \frac{|E_{DE}|^2}{e^2 A_{IB}} + \frac{|M_{DE}|^2}{e^2 A_{IB}} \right) W^4 \right). \quad (2)$$

In this expression both electric $E_{DE}$ and magnetic $M_{DE}$ direct emission amplitudes are functions of $W^2$ and $T^*_c$ and appear at $\mathcal{O}(p^4)$. To a very good approximation we can identify these direct emission amplitudes with their first multipole for which the $\pi^+\pi^0$ state is in a $P$ wave. The main interest of $K^+ \to \pi^+\pi^0\gamma$ is that its bremsstrahlung component $A_{5B} = A(K^+ \to \pi^+\pi^0)\gamma$ is pure $\Delta I = 3/2$ hence suppressed, making the direct emission amplitudes easier to access. The magnetic amplitude $M_{DE}$ is dominated by the QED anomaly and will not concern us here.

3.1 Differential rate

Given its smallness, we can assume the absence of CP-violation when discussing this observable. Experimentally, the electric and magnetic amplitudes (taken as constant) have been fitted in the range $T^*_c \leq 80$ MeV and $0.2 < W < 0.9$ by NA48/2 [7]. For the electric amplitude, using their parametrization, we obtain at $\mathcal{O}(p^4)$ :

$$X_E = -\text{Re} \left( \frac{E_{DE}/eA_{IB}}{m^2_K \cos(\delta_1 - \delta^2_0)} \right) = \frac{3G_8/G_{27}}{40\pi^2 F^2_{\pi^+}m^2_K} \left[ E^l(W^2, T^*_c) - \frac{m^2_K \text{Re} \bar{N}}{m^2_K - m^2_\pi} \right] \equiv X^l_E - X^{CT}_E , \quad (3)$$

where $\delta^1_1 (\delta^2_0)$ is the strong phase of $E_{DE} (A_{IB})$. The $E^l$ represents $O_8$ and $O_{27}$ induced loop contributions (loop contributions from $O_{ew}$ are sub-leading) and $\bar{N}$ corresponds to local counterterms and $Q^-_c$ contributions. Naively we would expect the $O_{27}$ contributions to be sub-dominant, however, they are dynamically enhanced by $\pi\pi$ loops. Since experimentally, no slope were included in $X_E$, we average $E^l$ over the experimental range and find $X^l_E = -17.6$ GeV$^{-4}$. Knowing $X^l_E$ and using the experimental measurement of $X_E = (24 \pm 4 \pm 4)$ GeV$^{-4}$ we can extract the local contributions

$$X^{CT}_E/X^l_E = 0.37 \pm 0.32 \rightarrow \text{Re} \bar{N} = 0.095 \pm 0.083 . \quad (4)$$

To our knowledge it is the first time that $K^+ \to \pi^+\pi^0\gamma$ counterterms contributions are extracted from experiment. The value we found is much smaller than the $\mathcal{O}(1)$ expected for the $N_i$ on dimensional grounds or from factorization [8]. Note that the required amount of counterterm contribution would have been bigger if $O_{27}$ loops were neglected since then $X^l_E = -10.2$ GeV$^{-4}$. This result is important since it implies that the counterterms combination $\bar{N}$, which appears in other radiative $K$ decays, is now under control and further reliable theoretical investigations can be carried on, in particular concerning the CP violating observables.
3.2 Direct CP-violating asymmetry

Since the bremsstrahlung and direct emission amplitudes interfere and carry different strong and weak phases, a non vanishing CP violating asymmetry can be generated. The asymmetry measures direct CP violation since $K^\pm$ do not mix. Besides and because the long-distance bremsstrahlung amplitude dominates the branching, this CP asymmetry is the simplest window on short-distance physics and a fortiori on possible NP effects. CP-violation in $K^+ \rightarrow \pi^+\pi^0\gamma$ is quantified by the parameter $\varepsilon'_{+0\gamma}$, defined from

$$\text{Re} \left( \frac{E_{DE}}{\epsilon A_{IB}} \right) \left( K^\pm \rightarrow \pi^\mp \pi^0\gamma \right) \approx \frac{\text{Re} E_{DE}}{\text{Re} A_{IB}} \left[ \cos(\delta_1 - \delta_0) - \sin(\delta_1 - \delta_0)\varepsilon'_{+0\gamma} \right],$$

as $\varepsilon'_{+0\gamma} \equiv \text{Arg}E_{DE} - \text{Arg}A_{IB}$ (see [11]). Both $Q^-_\gamma$ and $Q_i$ (through loops and counterterms) contribute to this parameter and we find

$$\varepsilon'_{+0\gamma}(Q_i) = -0.55(25)\frac{\sqrt{3}\varepsilon'_K}{\omega} \quad \text{and} \quad \varepsilon'_{+0\gamma}(Q^-_\gamma) = +2.8(7)\frac{\text{Im}C^{-}_{\gamma}}{G_{F}m_{K}},$$

respectively[6]. Sadly, these contributions interfere destructively implying that $\varepsilon'_{+0\gamma}|_{SM} = 0.5(5) \times 10^{-4}$. This large uncertainty is driven by a large uncertainty on counterterms and on estimated $O(p^6)$ effects. However, contrary to what happens in $\varepsilon'_K$, $\varepsilon'_{+0\gamma}$ is rather insensitive to isospin breaking effects, conservatively taken into account in [11]. Expressing $\varepsilon'_{+0\gamma}(Q_i)$ in term of the experimental $\varepsilon'_K$ allows us to keep possible NP effects in $Q_i$ under control. As a consequence, the only way for NP to affect $\varepsilon'_{+0\gamma}$ is via its $\text{Im}C^{-}_{\gamma}$ component. The current bound obtained by NA48/2 [12] is rather weak and allows very large NP effects in $\varepsilon'_{+0\gamma}$:

$$\text{Im}C^{-}_{\gamma}|_{NP}/G_{F}m_{K} = -0.08 \pm 0.13.$$ (7)

4 $K^+ \rightarrow \pi^+\pi^0\gamma$ beyond the SM

Once combined with other short-distance sensitive observables, any experimental improved measurement of $\varepsilon'_{+0\gamma}$ will be greatly rewarding. The main problem when probing NP is the issue of disentangling correlations between various NP sources in a fully model-independent way. In [10], we analysed broad classes of NP scenarios defined as model-independently as possible and identified corresponding strategies to constrain and disentangle NP sources using experimental informations on $K_L \rightarrow \pi\ell^+\ell^-$, $K \rightarrow \pi\nu\bar{\nu}$ decays and $\text{Re}(\varepsilon'_K/\varepsilon_K)$. Doing so we highlighted the complementary informations that could be obtained from radiative decays.

In the MSSM [11][10], NP can affect all the operators in [11] as well as gluon-penguin (denoted by $Q_g^\pm$) and semi-leptonic operators, in particular $Q_{V,I} = s_i\gamma_\mu d\otimes \bar{\ell}\gamma^\mu\ell$. In this particular model the irreducible correlations are two fold. First $Q_{V,I}^+$ and $Q_{V,I}^\pm$ always interfere in $K_L \rightarrow \pi\ell^+\ell^-$ in and beyond the SM and second, $\text{Re}(\varepsilon'_K/\varepsilon_K)$ receives NP contributions from many different sources. The corresponding bounds are displayed in Figure 1 where we see that a large but not impossible cancellation between NP in gluon-penguin and electroweak operators in $\text{Re}(\varepsilon'_K/\varepsilon_K)$ allows for $\text{Im}C^{-}_{\gamma}$ to reach the percent level if we impose $\text{Im}C^{-}_{\gamma} = \pm 1.5 \text{Im}C^{+}_{\gamma}$. This value will correspond to a saturation of the current $K_L \rightarrow \pi^0e^+e^-$ upper bound and since in the MSSM $Q_g^\pm$ and $Q_{V,I}^\pm$ mix under renormalization this $\text{Im}C^{-}_{\gamma}$ upper bound provides also an lower bound for $\text{Im}C^{-}_{\gamma}$. From (10) this implies that NP can push $\varepsilon'_{+0\gamma}$ up to roughly two orders of magnitude above its SM prediction. The parameter $\varepsilon'_{+0\gamma}$ provides therefore a very good probe for NP $\gamma$-penguin effects and furthermore reveals NP cancellations occurring inside $\text{Re}(\varepsilon'_K/\varepsilon_K)$.

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[6] Numerically, in the SM, the Wilson coefficient of the magnetic operator in $b \rightarrow s\gamma$ can be used for $\text{Im}C^{+}_{\gamma}$, since the CKM elements for the $u$, $c$, and $t$ contributions scale similarly and we find $\text{Im}C^{+}_{\gamma}(2\text{ GeV})_{SM}/G_{F}m_{K} = \mp 0.31(8) \times \text{Im}\lambda_t$. 

Figure 1: Loop-level FCNC scenario, with all the electroweak operators as well as $Q_1^\pm$ simultaneously turned on, but imposing $\text{Im } C_1^\pm = \pm 1.5 \text{ Im } C_5^\gamma$ and $|\text{Re}(\varepsilon_K^\prime/\varepsilon_K)| < 2 \text{ Re}(\varepsilon_K^\prime/\varepsilon_K)^\text{exp}$. (a) The $\text{Im } C_1^\pm$ range as a function of the fine-tuning between $\text{Re}(\varepsilon_K^\prime/\varepsilon_K)_{\text{EW}}$ and $\text{Re}(\varepsilon_K^\prime/\varepsilon_K)_{\gamma}$. (c) The corresponding contours in the $\text{Im } C_{\nu,L} - \text{Im } C_{\nu,\gamma}$ plane. In (b), the lighter (darker) colors denote destructive (constructive) interference between NP $\gamma^*$-penguin and $Q_1^\pm$ in $K_L \to \pi^0\ell^+\ell^-$.

5 Conclusion

We exemplify in $K^+ \to \pi^+\pi^0\gamma$ that the stage is now set theoretically to fully exploit the $s \to d\gamma$ transition. The SM predictions are under good control, the sensitivity to NP is excellent, and signals in rare and radiative $K$ decays not far from the current experimental sensitivity are possible. Thus, with the advent of the next generation of $K$ physics experiments (NA62 at CERN, K0TO at J-Parc, ORKA at Fermilab and KLOE-II at the LNF), the complete set of flavor changing electromagnetic processes, $s \to d\gamma$, $b \to (s,d)\gamma$, and $\ell \to \ell'\gamma$, could become one of our main windows into the flavor sector of the NP which will hopefully show up at the LHC.

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