Numerical Approach toward Ternary Hybrid Nanofluid Flow Using Variable Diffusion and Non-Fourier’s Concept
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ABSTRACT: In the current study, the pseudoplastic model is used to analyze the mass and energy transmission through trihybrid nanofluid flow across a stretched permeable surface. The Darcy–Forchheimer relation is employed in the momentum equation to examine the influence of porosity. Energy and mass diffusion expressions are obtained by employing the double diffusion theories, which were proposed by Cattaneo and Christov and is broadly used by several researchers. The thermal efficiency of the trihybrid nanocrystals is evaluated by integrating them with a pseudoplastic substrate. The study of titanium dioxide (TiO$_2$), cobalt ferrite (CoFe$_2$O$_4$), and magnesium oxide (MgO) nanocomposite base hybrid nanofluids across a stretchable sheet is receiving considerable interest in innovation and research due to their extensive spectrum of applicability. For this reason, the phenomena are modeled in the form of a system of PDEs with the effects of a heat source, magnetic field, natural convection, and chemical reaction. Through resemblance substitutions, these are reduced to an ODE system. The resultant first-order differential equations are further processed using the computational approach PCM. For authenticity and reliability, the values are reviewed against the existing literature. The findings are displayed through figures. When compared to the simple nanofluid, the hybrid and trihybrid nanofluid have a greater tendency for fluid energy and velocity propagation rate. The velocity and heat transition rate enhance 11.73% by varying nanoparticles’ values from 0.01 to 0.04, while the thermal conductivity of base fluid boosts with the addition of hybrid and trihybrid nanocomposites, up to 32% and 61%, respectively.

1. LITERATURE REVIEW
The study of simple and hybrid nanofluid flow across a stretching sheet has numerous applications in various fields, i.e., polymer engineering, extrusion of polymer, plastic sheets compression, glass production, fiber, and in metallic furnace. Numerical computations were carried out by Gul et al. to observe the efficiency of ferrofluid flow over an extending/shrinking slip. Bilal et al. utilized a stretchy substance with sucking and intravenous effects to replicate Maxwell NF flow using the PCM approach. Using directly the meshless local Petrov–Galerkin approach and the Dirac function, Wijayanta et al. reported the laminar natural convection heat transport in triangular cavities. The quantitative results with the suggested method and those derived with the traditional methods described in the literature were compared. Makarim et al. employed the numerical analysis to study the Marangoni convection that occurs when steam is absorbed into an aqueous lithium bromide solution. It was discovered that by including a fixed object at the surface, the convection flow rate can be increased. Shuaib et al. revealed the viscous fluid flow with mass and energy communication generated due to the inconsistent wave of an elastic sheet. Ullah et al. assessed the heat transference and flow of a liquid containing pseudoplastic nanoparticles through a vertical, narrow cylinder. As the curvature factor, mixed convection, and Weissenberg number are elevated, the flow velocity decreases. Moraveji and Toghraie examined the energy and flow patterns in the vortex tube, which are affected by the numbers of inlets, diameter, and tube length. It was determined that as the radius of a cold outflow increases, so does the flow velocity, and that as the length of the vortex tube extended. Alazwari and Safaei used a mixture model to simulate a unique design of a spinning tube under the nonisothermal hydrological conditions. It was discovered that raising the Reynolds number would improve the system’s thermal efficiency. Ahmad et al. evaluated a 3D MHD Maxwell nanofluid flow across a slendering exponentially stretching sheet with energy dissipation using the bvp4c method. Abu-Hamdeh et al. computed the quantity of entropy generation when Powell–Eyring nanofluid flows over permeable media in a horizontal surface under thermal jump conditions. Some further

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explanations and applications of fluid flow along an extending substrate have been discoursed in refs 12–18.

During energy transfers, a hybrid nanofluid surpasses conventional fluids such as acetone, water, nanofluids, and acetylene. The capacity to freeze at high temperatures is one of the many thermal features of hybrid nanofluids.19–22 Power production, heat exchange, heating systems, air conditioners, the automobile sector, electronic equipment, generators, reactors, and energy transmission in spacecraft are all applications of hybrid nanocomposites.23–25. The working fluid in this study contained titanium dioxide (TiO2), magnesium oxide (MgO), and cobalt ferrite (CoFe2O4) NPs. TiO2 is an inorganic substance that has been utilized in several products for a long period. It relies on it due to its non-noxious, fluorescent, and non-sensitive qualities, which improve the radiance and illumination of materials without causing harm. It is the whitest pigmentation recognized, with insightful properties and scattering abilities, and the capacity to defend against cancer cells. CoFe2O4 is frequently utilized in sensors, catalysts, and microbiology because of these properties.26,27 Chu et al.28 addressed the flow mechanics and heat transference in the context of Al2O3 and TiO2 NPs that were utilized to boost the thermal properties of the base solution. Kristiawan et al.29 conducted an experimental investigation into a hybrid technique using a microfine tunnel and a TiO2/water nanoliquid with different nanoparticulate concentrations. Purnama et al.30 analyzed the molecular binding reaction and excitation energy of a hybrid nanoliquid flow with the inclusion of CoFe2O4 and Fe3O4 NPs. Ullah et al.31 performed a mathematical analysis of the Darcy–Forchheimer flow of magnetized nanoparticles with zero mass flux and observed that endothermic/exothermic reactions raise the thermostat of nanostructures. Purnama et al.32 prepared strontium-substituted cobalt ferrite nanomaterials utilizing the coprecipitation technique and thermogravimetric evaluation. Ullah et al.33 researched the physicochemical parameters of trihybrid CuO/TiO2/SiO2 nanofluids. Depending on the temperature input, it was discovered that the trihybrid liquid had the highest thermal characteristics at roughly 55 °C. Sahu et al.34 analyzed the natural circulatory loop’s transitory and constant features utilizing a variety of water-based trihybrid nanofluids. Using trihybrid nanomaterials improves efficiency while lowering the rate of entropy production. The shape of nanocrystals has a considerable impact and exhibits the best performance. Many researchers have recently scrutinized the physics and use of TiO2, MgO, and CoFe2O4 hybrid NF over various geometries.

The current model is based on the rheological characteristics of a pseudoplastic composite with trihybrid nanocrystals. The steady and incompressible trihybrid (TiO2 + MgO + CoFe2O4/EG) nanofluid flow along a horizontal stretching sheet is addressed. The theories of non- and Fourier’s Darcy’s Forchheimer are examined in the context of heat emission, natural convection, and chemical reaction. PCM is a computational method used for the numerical simulations of the system of PDEs. Our main objective is to examine the characteristics of trihybrid NF flow for industrial and biomedical applications. In the long run, this research could be used to develop an optimal thermal process, such as refrigerants and heat pumps, using appropriate physical sources.

2. MATHEMATICAL FORMULATION

We assumed the fluid flow of trihybrid nanoparticulates in pseudoplastic fluid over a heated stretching surface. The heat energy and particles solute are examined using the CC model theory. Processes of heat generation and chemical reaction are executed in the existence of CC theory. The consequences of Darcy–Forchheimer, natural convection, heat generation are also considered along with velocity and energy equations. The boundary layers associated with thermal and momentum are generated due to the surface stretching. Tables 1 and 2 reveal the experimental outcomes and physical model for the trihybrid nanofluid. A physical illustration of the proposed model is highlighted in Figure 1. The basic equations that control the trihybrid nanofluid flow are characterized as follows

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= \nu_\text{Thnf} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) - \frac{F_D}{(k')^\frac{1}{2}} u^2 + g(\beta_\text{Thnf}(T - T_w)) + \frac{K_\text{Thnf}}{(C - C_w)} \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) + v \frac{\partial}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \frac{\partial T}{\partial y} - \frac{Q}{(\rho C_p)^\text{Thnf}} \\
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= \frac{\partial}{\partial x} \left( \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} \right) + v \frac{\partial}{\partial y} \frac{\partial C}{\partial x} + 2u \frac{\partial^2 C}{\partial x \partial y} + \frac{\partial}{\partial y} \frac{\partial C}{\partial y} - K_c (\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y})
\end{align*}
\]
Table 2. Thermal Properties of the Hybrid Nanofluid ($\varphi_1 = \varphi_{\text{TiO}_2}, \varphi_2 = \varphi_{\text{CoFe}_2\text{O}_4}, \varphi_3 = \varphi_{\text{MgO}}$)\(^{42}\)

| Property          | Expression                                                                 |
|-------------------|-----------------------------------------------------------------------------|
| Viscosity         | $\mu_{\text{Thnf}} = \frac{1}{(1 - \varphi_{\text{MgO}})\mu_{\text{MgO}}} + \varphi_{\text{CoFe}_2\text{O}_4}\frac{\mu_{\text{CoFe}_2\text{O}_4}}{\mu_{\text{MgO}}} + \varphi_{\text{TiO}_2}\frac{\mu_{\text{TiO}_2}}{\mu_{\text{MgO}}}$ |
| Density           | $\rho_{\text{Thnf}} = (1 - \varphi_{\text{CoFe}_2\text{O}_4})\rho_{\text{CoFe}_2\text{O}_4} + \varphi_{\text{MgO}}\rho_{\text{MgO}}$ |
| Specific heat     | $(\rho c_p)_{\text{Thnf}} = (1 - \varphi_{\text{CoFe}_2\text{O}_4})\rho c_p_{\text{CoFe}_2\text{O}_4} + \varphi_{\text{MgO}}\rho c_p_{\text{MgO}}$ |
| Thermal conduct.  | $k_{\text{Thnf}} = \frac{k_{\text{MgO}} + 2k_{\text{MgO}} - 2\varphi_{\text{MgO}}(k_{\text{Thnf}} - k_{\text{MgO}})}{k_{\text{MgO}} + 2k_{\text{MgO}} + \varphi_{\text{MgO}}(k_{\text{Thnf}} - k_{\text{MgO}})}$ |
| Electrical cond.  | $\sigma_{\text{Thnf}} = \frac{\sigma_{\text{Ni}} + 3(\sigma_{\text{Ni}} - 1)\varphi_{\text{Ni}}}{\sigma_{\text{Ni}} + 2(\sigma_{\text{Ni}} - 1)\varphi_{\text{Ni}}}$ |

Here, $Q_0$ is the heat source term, $K$ is the rate of a chemical reaction, $D$ is the mass diffusion, $C$ is the concentration, $F_D$ is the inertia coefficient, $k^*$ is the porous medium, Thnf is the trihybrid nanofluid, $C_p^*$ is the specific heat capacity, $\lambda$ is the time relaxation term, and $\rho$ is the density.

The initial and boundary conditions are as follows

\[ u = u_w, \quad v = -u_w, \quad T = T_w, \quad C = C_w \text{ at } y = 0 \]
\[ u \to u_w, \quad T \to T_w, \quad C \to C_w \text{ when } y \to \infty \]  
\[ \text{ (5) } \]

Figure 1. Trihybrid pseudoplastic nanofluid flow over a stretching surface.
\[ \xi = y \left( \frac{u^{2-m}}{x \nu_t} \right)^{1/m+1}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \]  

(6)

Variable mass and diffusion and thermal conductivity are defined as follows:

\[ D_{\text{Thaf}} = D_{\text{Thaf}} \left( 1 + \varepsilon f \left( \frac{C - C_\infty}{C_w - C_\infty} \right) \right), k_{\text{Thaf}} \]

\[ = k_{\text{Thaf}} \left( 1 + \varepsilon f \left( \frac{T - T_\infty}{T_w - T_\infty} \right) \right) \]  

(7)

By incorporating eq 6, we get

\[ (y^{m-1}f^{1/m})' + \frac{1}{m+1}f'f - \varepsilon f - \frac{\nu_t f_k}{\nu_{\text{Thaf}}} f^{(2)} - (Gr \phi + Gc \phi) = 0 \]  

(8)

\[ (1 + \varepsilon f)\phi' + \varepsilon (\theta')^2 + \frac{Pr}{m+1}\theta' - \frac{k_C (\rho C_p)_h}{k_{\text{Thaf}}} \phi' = 0 \]

\[ \text{Pr} \Omega_k \left( f' \phi' + f^2 \phi'' + K_C \phi \right) = 0 \]  

(9)

\[ \left( 1 + \varepsilon f \right) \phi' + \varepsilon (\theta')^2 - \frac{(1 - \phi_f)^{-2.5} Sc}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2} \text{Pr} \Omega_k} \times \left( f' \phi' + f^2 \phi'' + K_C \phi \right) = 0 \]

\[ = \frac{(1 - \phi_f)^{-2.5} Sc}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2} \phi'} = 0 \]  

(10)

Here, Pr is the Prandtl number, \( m \) is the power-law number, Fr is the Forchheimer number, \( \varepsilon \) is Darcy’s number, \( \varepsilon \) is the variable thermal conductivity, Sc is the Schmidt number, \( \Omega_k \) is the parameter generated due to non-Fourier’s theory, \( K_C \) is the parameter formulated due to Cattaneo–Christov’s theory, and \( K_C \) is the chemical reaction rate. It is assessed that the chemical reaction is a nonreactive reaction for \( K_C = 0 \).

The mathematical expression for skin friction is

\[ C_f = \frac{2 \tau_w}{U^2 \rho_t} \]  

(11)

\[ \text{Re}^{1/m+1} C_f = -\frac{(1 - \phi_f)^{-2.5} Sc}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2} [F'(0)F'(0)]^{m-1}} \]  

(12)

The temperature gradient is modeled as follows

\[ Nu = \frac{x Q_w}{(T_w - T_\infty)k_f}, \quad \text{Re}^{-1/m+1} Nu = -\frac{k_{\text{Thaf}}}{k_f} \theta'(0) \]  

(13)

The rate of mass diffusion is as follows

\[ Sc = \frac{x M_w}{(C_w - C_\infty)D_t}, \quad (\text{Re})^{1/m+1} Sc = -\frac{(1 - \phi_f)^{-2.5}}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2} \phi'(0)} \]  

(14)

The Reynolds number is

\[ \text{Re} = \left( \frac{x^{2-m}}{n} \right) \]  

3. NUMERICAL SOLUTION

Several researchers have used different types of numerical procedures for the solution of highly nonlinear PDEs. The fundamental steps involved in the PCM solution methodology, while dealing with the system of ODEs (8–10), are as follows.

Step 1: Reducing the BVP to a first-order system of ODEs

\[ h_1 = f(\eta), \quad h_2 = f'(\eta), \quad h_3 = \theta(\eta), \quad h_4 = \theta'(\eta), \quad h_5 = \phi(\eta), \quad h_6 = \phi'(\eta) \]  

(15)

By putting eqn 13 in eqs 7–9 and 10, we get

\[ (h_2 f^{m-1} f_2) + \frac{1}{m+1} h_2 f_2 - \varepsilon h_2 - \frac{\nu_t}{\nu_{\text{Thaf}}} f(\eta) \]  

\[ - (Gr h_3 + Gc h_3) = 0 \]  

(16)

\[ (1 + \varepsilon h_3) h_4 + \varepsilon (h_3)^2 + \frac{Pr}{m+1} h_4 h_4 - \frac{k_C (\rho C_p)_h}{k_{\text{Thaf}}} h_4 h_6 \]  

\[ \text{Pr} \Omega_k [h_1 h_2 h_4 + f^2 h_4 + H_h \text{Pr} h_3] = 0 \]  

(17)

\[ (1 + \varepsilon h_6) h_6 + \varepsilon (h_6)^2 - \frac{(1 - \phi_f)^{-2.5} Sc}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2} \text{Pr} \Omega_k} \times \left( (h_1 h_2 h_6 + h_1 h_2 h_6 + K_C h_6) \right) \]

\[ = -\frac{1}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2}} \]  

\[ K_C h_6 + \frac{(1 - \phi_f)^{-2.5} Sc}{(1 - \phi_f)^{-2}(1 - \phi_f)^{-2}} = 0 \]  

(18)

Step 2: Introducing parameter \( p \):

\[ (h_2 f^{m-1} f_2) + \frac{1}{m+1} h_2 f_2 - \varepsilon (h_2 - 2) - p \]  

\[ - (Gr h_3 + Gc h_3) = 0 \]  

(19)

\[ (1 + \varepsilon h_3) h_4 + \varepsilon (h_3)^2 + \frac{Pr}{m+1} h_4 (h_4 - 1) p \]

\[ - \frac{k_C (\rho C_p)_h}{k_{\text{Thaf}}} \text{Pr} \Omega_k [h_1 h_2 h_4 + f^2 h_4 + H_h \text{Pr} h_3] \]

\[ = 0 \]  

(20)
Figure 2. Velocity $f(\eta)$ outlines versus (a) Darcy's number $\varepsilon$, (b) Forchheimer number $Fr$, (c) power-law number $m$, (d) thermal Grashof number, and (e) mass Grashof number.
Step 3: Apply Cauchy Principle and discretized eqs 19–21.
After discretization, the obtained set of equations are computed through the MATLAB code of PCM.

4. RESULTS AND DISCUSSION

This section explains the physics behind each figure and table.

4.1. Velocity Profile \( f(\eta) \). Figure 2a–e describes the velocity \( f(\eta) \) outlines versus Darcy’s number \( \varepsilon \), the Forchheimer number \( Fr \), the power-law number \( m \), the thermal Grashof number, and the mass Grashof number, respectively. Figure 2a–c reveals that the field declines with the influence of Darcy’s number \( \varepsilon \), the Forchheimer number \( Fr \), and the power-law number, respectively. Physically, greater values of \( \varepsilon \) generate a frictional effect on fluid flow. So, the frictional effect opposes fluid flow. Hence, an inverse relation occurs among \( \varepsilon \) and flow, as shown in Figure 2a. The Forchheimer term mathematically appeared as a velocity squared in momentum equations. This offered a retardation effect to the flow motion, as elaborated in Figure 2b. It is assessed that the explanation of \( m \) is exhibited due to the addition of the pseudoplastic liquid effect. Furthermore, the grouping of shear thickening, shear thinning, and Newtonian liquid behavior is dependent on the \( m \) values. For \( m > 1 \) and \( m < 1 \), the fluid behaves as shear thinning and thickening among the fluid molecules, respectively. It has been detected that the momentum profile lessens with the increment of \( m \). Figure 2d,e highlights the velocity transfer profile boosts with the rising credit of the thermal and mass Grashof number. The sheet stretching velocity declines with the effect of Gr and Gc, which causes such behavior of the velocity profile.

4.2. Temperature Profile \( \theta(\eta) \). Figure 3a–d demonstrates the presentation of energy \( \theta(\eta) \) contours versus the variable thermal conductivity \( \varepsilon_l \), heat absorption/generation \( H_N \), parameter \( \Omega_a \) (generated due to non-Fourier’s theory), and comparative assessment between simple, hybrid, and trihybrid nanofluid.

\[
(1 + \varepsilon_l h_1 h^*_6 + \varepsilon_l h_2 h^*_6 + 1)p - \frac{(1 - \phi_1)^{2.5}Sc}{(1 - \phi_2)^{2.5}(1 - \phi_3)^{2.5}} \\
Pr\Omega_a \times \left( (h_1 h_2 h_6 + h_1^2 h^*_6 + K_a h_1 h_6) \right) \\
- \frac{(1 - \phi_1)^{2.5}(1 - \phi_5)^{2.5}}(1 - \phi_5)^{2.5}(1 - \phi_6)^{2.5} \\
K_a h_5 + \frac{(1 - \phi_5)^{2.5}Sc}{(1 - \phi_1)^{2.5}(1 - \phi_6)^{2.5}} h_1 h_6 = 0,
\]

(21)
Figure 4. Concentration $\phi(\eta)$ outlines versus (a) parameter $\Omega_c$ (formulated due to Cattaneo–Christov theory), (b) parameter $\varepsilon_2$, (c) chemical reaction $K_c$, and (d) Schmidt number $Sc$.

Figure 5. Percentage (%) comparison between nanofluids. Hybrid and trihybridized nanofluid.
and nanomaterials improves as well. The dualistic function of thermal potential is shown on the thermal energy profile, with negative values of $H_h$ indicating absorption of heat and positive values indicating energy production. When an exterior heat source is installed at the surface, the thermal energy profile is increased. The conduct of boundary layers depends on thermal impact increment. The application of an external entity of thermal potential, which is situated at the surface wall, produces this growing effect of thermal energy, as revealed in Figure 3b.

Figure 3d displays the comparative assessment of simple, hybrid, and trihybrid nanofluid. It has been perceived that the trihybrid nanofluid has significantly boosted the energy propagation rate as compared to simple and hybrid nanofluid.

**4.3. Concentration Profile $\phi(\eta)$.** Figure 4a–d reports the appearance of concentration $\phi(\eta)$ outlines versus the parameter $\Omega_c$ (formulated due to Cattaneo–Christov theory), parameter $\varepsilon_2$, chemical reaction $K_c$, and the Schmidt number $Sc$, respectively. Figure 3a,b describes that the mass transport $\varphi(\eta)$ field enhances with the positive deviation of parameters $\Omega_c$ and $\varepsilon_2$. Elevated quantities $\Omega_c$ are detected as a massive improvement of species rate when three types of nanostructures are injected into species diffusion. In comparison to the role of $\Omega_c$, a significant efficiency of mass species is found. Because of the influence of Cattaneo–Christov theory, the existence of $\Omega_c$ is defined. When $\Omega_c$ is increased to include trihybrid nanoparticles, a considerable improvement in solute particles is observed in Figure 4a. The chemical change parameter has a positive impact on mass transport since it pushes fluid molecules to travel quickly, resulting in a positive variation (see Figure 4c). The effect of $Sc$ also diminishes the mass transmission rate because it improves the kinetic viscosity of the fluid, which results in such senior as shown in Figure 4d.

Figure 5 emphasizes the relative analysis of simple (TiO$_2$ or MgO or CoFe$_2$O$_4$), hybrid nanofluid (MgO+TiO$_2$/water), and ternary hybrid nanoliquid (MgO + TiO$_2$ + CoFe$_2$O$_4$/EG) for the energy and velocity profiles. When relative to the simple nanofluid, the trihybrid and hybrid nanofluids have a higher tendency for fluid velocity and energy transmission efficiency. For the numerical approximation of the current model, the parametric continuation method is used. In Figure 6, the convergence of the parametric continuation method is considered. The convergence zones of velocities, energy, and mass profiles of the trihybrid nanofluids are reviewed.

| Fr | $H_h$ | Sc | $K_c$ | ref 40 | present work | ref 40 | present work | ref 40 | present work |
|----|-------|----|-------|-------|------------|-------|------------|-------|------------|
| 0.0| 0.9831194688 | 0.9831195589 | 1.273048742 | 1.2730488451 | 0.1738269788 | 0.1738269788 |
| 0.3| 0.9948702726 | 0.9948703627 | 1.263799787 | 1.263799878 | 0.1636186839 | 0.1636186839 |
| 0.6| 1.0055424430 | 1.005545331 | 1.244465792 | 1.244465883 | 0.1434299640 | 0.1434299731 |
| 1.5| 1.048480792 | 1.048481693 | 2.988980397 | 2.988980886 | 0.1582282354 | 0.1582282445 |
| 0.3| 1.021340103 | 1.021341004 | 2.184136239 | 2.184136348 | 0.1468662589 | 0.1468662677 |
| 0.7| 1.010130191 | 1.010131092 | 2.110134671 | 2.110134762 | 0.1380771772 | 0.1380771863 |
| 0.0| 0.978480792 | 0.978481693 | 1.275290361 | 1.275290452 | 0.1059774514 | 0.1059774605 |
| 0.3| 1.018480792 | 1.018481693 | 1.275290361 | 1.275290452 | 0.2836633481 | 0.2836633573 |
| 0.6| 1.018480792 | 1.018481693 | 1.275290361 | 1.275290450 | 0.6105156610 | 0.6105156700 |
| 1.4| 1.018480792 | 1.018481693 | 1.275290361 | 1.275290450 | 0.0831204767 | 0.0831204776 |
| 0.0| 1.018480792 | 1.018481693 | 1.310199330 | 1.310199441 | 0.0912766237 | 0.0912766246 |
| 0.5| 1.018480792 | 1.018481693 | 1.501284349 | 1.501284457 | 0.2961188108 | 0.2961188117 |
Table 3 explains the numerical valuation of the Sherwood number, skin friction, and Nusselt number of the existing literature with current outcomes to ensure the authenticity of the current report.

5. CONCLUSIONS

The pseudoplastic model is used in the current study to analyze the energy communication through trihybrid nanofluid flow consisting of MgO, TiO$_2$, and CoFe$_2$O$_4$ NPs across a stretched permeable surface. Energy and mass diffusion expressions are obtained by employing the double diffusion theories. The thermal efficiency of the trihybrid nanocrystals is evaluated by integrating them with a pseudoplastic substrate. The problem is solved using the computational approach PCM. The main discoveries are as follows:

- The velocity field $f'(\eta)$ declines with the influence of Darcy’s number $\varepsilon_d$, Forchheimer’s number $Fr$, and power law.
- The velocity transfer profile boosts with the rising credit of thermal Gr and mass Grashof number.
- The temperature profile enhances with the significances of $\varepsilon_t$, heat absorption/generation $H_t$, and parameter $\Omega_c$.
- The velocity and heat transition rate enhance 11.73% by varying nanoparticle values from 0.01 to 0.04 (see figure 5), while the thermal conductivity of base fluid boosts with the addition of hybrid and trihybrid nanocomposites, up to 32 and 61%, respectively.
- The trihybrid nanofluid has significantly boosted the energy propagation rate as compared to simple and hybrid nanofluid.
- The mass transport $\phi(\eta)$ outlines develop with the variation of parameters $\Omega_1$ and $\varepsilon_2$, while the chemical reaction and Schmidt number positively affect the mass transmission.
- The skin friction enhances, while the Nusselt number decreases with the variation of $Fr$.

5. Nomenclature

- $\rho$ density
- $D$ mass diffusion
- $F_{Fr}$ inertia coefficient
- $Q_m$ heat source term
- $\lambda_t$ time relaxation term
- $Pr$ Prandtl number
- $\varepsilon_t$ variable thermal conductivity
- $Thnf$ Trihybrid nanofluid
- $C_f$ skin friction
- $Nu$ Nusselt number
- $\varepsilon$ Darcy’s number
- $\phi$ Dimensionless concentration
- $Gr$ Thermal Grashof number
- $Gc$ Mass Grashof number
- $\sigma$ Electrical conductivity
- $k$ thermal conduction
- $\mu$ viscosity
- $PCM$ parametric continuation method
- $M$ power-law coefficient
- $C$ concentration
- $k^c$ chemical reaction
- $k^s$ porous medium
- $C_p$ specific heat capacity
- $Fr$ Forchheimer number
- $Sc$ Schmidt number
- $\Omega_1$ generated due to non-Fourier’s theory
- $\Omega_2$ generated due to Cattaneo–Christov theory
- $K_c = 0$ nonreactive chemical reaction
- $Re$ Reynold number
- $\theta$ dimensionless temperature
- $\phi_1 = \phi_{TiO_2}$ titanium dioxide nanoparticles
- $\phi_2 = \phi_{CoFe_2O_4}$ cobalt ferrite nanoparticles
- $\phi_3 = \phi_{MgO}$ magnesium oxide nanoparticles
- ($\rho C_p)_f$ specific heat of base fluid
- $g$ Gravitational acceleration
- $EG$ (C$_2$H$_4$O$_2$)-ethylene glycol

Author Contributions

M.B. and A.S. wrote the original manuscript and performed the numerical simulation. E.A.A. and H.F.A. reviewed the mathematical results and restructured the manuscript and also responded to the reviewer’s queries. Wajaree Weera has done the data curation and conceptualization and reviewed the manuscript. All of the authors are agreed on the final draft of the submission file.

Notes

The authors declare no competing financial interest.

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