Frictional self-oscillations of the one - two degree of freedom systems

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Abstract. The problem of frictional self-oscillations (FSO) appears for devices and mechanisms, friction units of which operate in conditions of the friction process instability at rather small speeds of slippage. Such oscillations arise at operation of arresting arrangement, friction engagement mechanisms, drilling machines, etc. In the paper, FSO have been analyzed in terms of existence and attraction areas of different conditions on parameter plane both for one- and two-mass models.

1. Introduction

Vibrations in machines and mechanisms are caused by different reason, but frictional self-oscillations (FSO) are the cause of a special importance. Being one of the most widespread types of mechanical self-oscillations, they result in instability of process of a sliding friction at rather small sliding speeds. Considering of FSO is very important in machine-tool construction because they define two important criteria of operational quality of operation of machines: uniform of sluggish movements and accuracy of adjusting movements. Such oscillations arise at operation of arresting arrangement, friction engagement mechanisms, drilling machines, etc.

The study of friction self-oscillations is a rather complicated problem because of friction force nonlinear nature. Such processes are usually described by non-linear equations and analyzed by methods of nonlinear mechanics: the averaging method, the multi-scale method, the fitting method and a number of others. The authors research results of friction auto-oscillations both of one- and two-mass models with relative slip of bodies are presented in the report [1-3, 6].

In the paper, FSO have been analyzed in terms of existence and attraction areas of different conditions on parameter plane both for one- and two-mass models. Various types founded in practice oscillations are considered, and ways of their decreasing by variation of parameters of a system are investigated.

2. Theoretical calculations

Let's give the research FSO in one-mass system on the example of traditional model here. Let's consider a spring-fastened solid body on the plane (see figure 1(a)) moving with constant speed. Dry non Coulomb friction force exerts between a body and a plane (a dry friction in a combination with viscid one) [5]. The friction characteristic is accepted in the form of a cubic parabola [1] (see figure 1(b)).

The equation of motion of the considered system at \( \dot{x} \neq 0 \) can be written in view of
\[ m \ddot{x} + c(x - Vt) = -P f(\dot{x}), \]  

where \( x \)- the absolute coordinate of a body, \( P \)-pressure force on contact, and dependence of a sliding friction on speed is accepted in the form of (figure 3)

\[ f(\dot{x}) = f^* \text{sign} \ \dot{x} - f_1 \dot{x} + f_3 \dot{x}^3, \ \dot{x} \neq 0; \quad -f^* < f(\dot{x}) < f^*, \ \dot{x} = 0, \]  

(\( f^* = \min f(\dot{x}) \) when \( \dot{x} > 0 \)).

It is supposed that the size of a friction coefficient changes slightly when driving, small parameter is entered: \( \varepsilon = (f^* - f_*)/2f^* \). Other constitutive parameters when studying FSO is \( \sigma = V/V^* \).

Research of these equations eq. (1)-(2) allowed to receive the following main results about nature of oscillations of system.

1. On the rising branch of the friction behavior at \( \sigma > 1 \) equilibrium position is stable and self-oscillations in system are impossible.
2. In the narrow range of change of a translational velocity of body \( 0.89 < \sigma < 1 \) in system there are quasiharmonic non-stop self-oscillations.
3. At \( 0 < \sigma < 0.89 \) in system the following modes can be realized:
   a) oscillations with stagnation zones - relaxational oscillations;
   b) oscillations with the instantaneous change of a sign of speed;
   c) oscillations of the mixed type.

For the research FSO in two-mass system of bodies (figure 2) the model in which bodies are directly tied by forces of a dry non-Coulomb friction is offered [4, 5]. Thus one of bodies (top) is stretched through a resilient element on lower body, is elastic fixed on the horizontal basis. Such scheme can serve as model for actual friction units - coupling devices, friction clutch couplings, slider bearings and braking mechanisms. Equations of motion are

\[ m_1 \ddot{x} + c_1(x - Vt) = -f(u)P, \quad m_2 \ddot{y} + c_2 y = f(u)P. \]  

Figure 1. One-mass model of frictional self-oscillatory system (a); cubic friction behavior (b).

Figure 2. Two-mass model of frictional self-oscillatory system.
Here x, y, m₁, m₂, c₁, c₂ - the coordinates, masses and rigidities corresponding to a top and bottom body, and f friction behavior depending on their relative speed \( u = \dot{x} - \dot{y} \). In a problem only unceasing oscillations of bodies are considered (\( u > 0 \)). Function \( f \) in eq. (3), as above is approximated by cubic dependence. This traditional choice of friction characteristic corresponds to the experimental data. The averaging method is applied to the analysis. Two various cases - the main resonance and non-resonance are considered. In the first of them the small parameter \( \mu \) of the same order, as \( \varepsilon \) characterizing a difference of partial frequencies of system, and the dimensionless coordinates of bodies of \( \xi, \eta \) is entered.

The relative coordinate of \( u = \xi - \eta \) and variable \( w = (1 - \chi)\xi - (1 - \chi)\eta \), where \( \chi = (m₂ - m₁)/(m₂ + m₁) \). The average equations for a resonance case at a first approximation have an appearance

\[
\begin{align*}
& a' = \frac{e}{2} \left[ \frac{6\xi}{1 - \chi^2} \left( 1 - \sigma^2 - \frac{a^2}{4} \right) a - b \sin \theta \right], \\
& b' = \frac{e}{2} \left[ (1 - \chi^2) a \sin \theta \right], \\
& \theta' = \frac{e}{2} \left[ \frac{(1 - \chi^2)a}{b} \cos \theta - \frac{b}{a} \cos \theta - 2\chi \right].
\end{align*}
\]  

(4)

Here \( a \) and \( b \) are the vibration amplitudes for variables \( u, w \) and \( \theta \) - a difference in phase. This system of equations has four steady stationary points. One solution of the problem is correspond oscillations only of the top body (with the body lower at first approximation is fixed). Oscillations amplitudes on the \( \xi, \eta \) variables thus will be \( A_\xi = a_\star, A_\eta = 0 \). Other solution of a problem eq. (4) is corresponded oscillations of the lower body: \( A_\xi = 0, A_\eta = a_\star \). Here variable of \( a_\star = 2\sqrt{1 - \sigma^2} \) is the dimensionless amplitude of quasiharmonic self-oscillations in one-mass system. It is visible that stationary oscillations are possible also only on the dropping section of the friction behavior, when \( \sigma < 1 \).

By means of a numerical integration of the average equations domains of attraction of two called modes of oscillations are constructed. From the analysis of areas it is visible that if the mass of bodies are sufficiently close also a static friction no small; the lower body oscillate, in an opposite case - top body oscillates. In non-resonance case similar results are obtained.

Thus, effect of dynamic self-oscillation suppression is possible in system of two bodies engaged by forces of a dry non-Coulomb friction both in resonance and in non-resonance case. In a first approximation the translating body is supposed to be fixed and another body oscillates. This effect is caused by small increment of the frictional force operating between oscillated bodies.

![Figure 3. FSO on the plane. Phase trajectories at various values of the angle between the direction of speed and one of flexural axis (x) are presented.](image)

Also FSO spring-fastened body (material point) in the plane is investigated (see figure 3)[20]. The point from the plane is affected by force of a dry non-Coulomb friction
\[ \vec{F} = -f(u) \cdot \vec{u} / u, \]  

(5)

where \( \vec{u} = (\dot{x} - V \cos \alpha) \hat{i} + (\dot{y} - V \sin \alpha) \hat{j} \) - vector of the relative speed of a point; \( u = |\vec{u}| \). The characteristic of a sliding friction in eq. (5) is accepted as cubic (figure 1(b)). Equations of motion of a point can be written down here in the following look

\[
\begin{align*}
    m \ddot{x} + c_1 x &= -f(u) \cdot (\dot{x} - V \cos \alpha) / u, \\
    m \ddot{y} + c_2 y &= -f(u) \cdot (\dot{y} - V \sin \alpha) / u.
\end{align*}
\]

(6)

Also FSO in two-bodies system is investigated (see figure 4). The body I and plate interact by a dry non-Coulomb friction force depended on relative speed by cubic

\[
F(\dot{x}_1) = f(\dot{x}_1)P = f^*(1 - 3 \frac{f^* - f_0 \dot{x}_1}{2f^* v_0} + \frac{f^* - f_0 \dot{x}_1^3}{2f^* v_0^3}) P.
\]

(7)

In this case motion equations are performed:

\[
\begin{align*}
    m_1 \ddot{x}_1 &= -c_1 (x_1 - vt) - c_2 (x_1 - x_2) - F(\dot{x}_1), \\
    m_2 \ddot{x}_2 &= c_2 (x_1 - x_2).
\end{align*}
\]

(8)

**Figure 4.** Sheme of the two-bodies system.

We will be to explore of subharmonic intrinsic resonance \( \omega_1/\omega_2 \approx p/q \) (\( \omega_1, \omega_2 \) - natural frequencies of system, \( p/q \) - simplified fraction. The averaging method (by high period \( T_q = 2\pi q \) \( (T_q > T_p) \)) in non-resonance case (\( p/q \neq 1, 2, 3 \)) allows to receive in the eq. (8).

\[
\vec{a}' = \delta_1 \left[ a_*^2 - a^2 - 2b^2 \right] \vec{a}, \quad \vec{b}' = \delta_2 \left[ a_*^2 - a^2 - 2b^2 \right] \vec{b}, \quad \vec{a} = 0.
\]

(9)

Here \( a, b \) are slow variables (amplitudes of oscillations). Instead two rapid phases (\( \varphi \) and \( \psi \)) we enter slow variable \( \Theta = q\varphi - p\psi \). The line over letter means average variable.

In eq. (9) are used name of variables: \( a_* = 2\sqrt{1 - \sigma^2} \) - non-dimensional amplitude of FSO in the system with one degree of freedom, and \( \delta_1 = v \frac{3}{8} \gamma n_2 / (n_2 - n_1) > 0, \delta_2 = -v \frac{3}{8} \gamma n_2 / (n_2 - n_1) > 0 \) \( (n_1 \text{ and } n_2 \text{ - shape quotient of free oscillations}).
Note that the first two equations of eq. (1) coincide with the averaged equations in the non-resonant case, when the eigen frequencies are significantly different and the averaging is carried out over two fast phases.

Stationary modes (critical points) of system are

1. \( a_1=0, b_1= a^* \);
2. \( a_2= a^*, b_2 = 0 \);
3. \( a_3=b_3 = a^*/\sqrt{3} \);
4. \( a_4=0, b_4= 0 \).

To investigate the stability of steady regimes form the characteristic equation

\[
\Delta(\lambda) = \begin{vmatrix}
\frac{\partial H_1}{\partial a} |_{\bar{a}} |_{\bar{b}} - \lambda & \frac{\partial H_1}{\partial b} |_{\bar{a}} |_{\bar{b}} \\
\frac{\partial H_2}{\partial a} |_{\bar{a}} |_{\bar{b}} & \frac{\partial H_2}{\partial b} |_{\bar{a}} |_{\bar{b}} - \lambda
\end{vmatrix} = 0.
\] (11)

3. Results and discussions

Research of these equations by a method of averaging allows drawing the following conclusions (figure 5).

1) The one-dimensional regimes of self-oscillations are realized not only when the direction of speed is close to the direction of the corresponding axis of a rigidity but also when the vector of speed is significantly rejected from this axis. And this deviation can be that larger, than more force of static friction.

2) The two-dimensional regime is possible at rather small size of a static friction force.

For two-mass model stationary modes 1., 2. (see eq. 10) the roots of the eq. 11 are negative and these modes are stable (stable node) with \( \sigma < 1 \) (when the slip velocity belongs to the falling part of the friction behavior). For modes 3. and 4. (see eq. 10) the roots are positive and the singular points are unstable. Thus, only single-frequency oscillatory modes are realized in the initial mechanical system. As is known, in the general case, oscillations with two frequencies are characteristic of two-mass systems. Which of the modes will be realized in fact, determined by the values of the system parameters and initial conditions (pertaining to zero).

Figure 5. FSO on the plane. Phase trajectories at various values of the angle between the direction of speed and one of flexural axis (x) are presented.
The domains of attraction of the modes are constructed by numerical integration of the eq. 9. In the figure 6 each boundary mode 1 is represented and below - mode 2 in the region above.

4. Conclusion

The auto-oscillations of one- and two-mass models in arresting arrangement are considered, especially in view of the evolution study of oscillating behavior depending on stationary speed of the system. It is shown, that different types of FSO – relaxational, quasiharmonic, with instantaneous stoppings and unceasing oscillations can arise on a falling part of the friction characteristic. Existence areas of these conditions are built on the parameters plan. In two-mass model the effect of oscillation reduction is analyzed, when one of masses commits oscillations with considerable stationary amplitude and the other is quite motionless at first approximation. Attraction areas of such conditions for a non-resonance case are built by the average method.

References

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