Measurement of $\theta_{13}$ by reactor experiments

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Abstract. I describe how reactor measurements of $\sin^2 2\theta_{13}$ can be improved by a near-far detector complex. I show that in the Kashiwazaki plan it is potentially possible to measure $\sin^2 2\theta_{13}$ down to 0.02.

Introduction. After the successful experiments on atmospheric and solar neutrinos and KamLAND, the next step in neutrino oscillation physics is to determine $\theta_{13}$. It has been known that the oscillation parameters $\theta_{jk}$, $\Delta m^2_{jk}$, $\delta$ cannot be determined uniquely even if the appearance probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ are measured precisely from a long baseline accelerator experiment due to so-called parameter degeneracies, and several ideas have been proposed to solve the problem. Among others, combination of a reactor measurement and a long baseline experiment offers a promising possibility [2, 3, 1, 4]. In this talk I briefly explain how measurements of $\sin^2 2\theta_{13}$ in reactor experiments can be improved by a near-far detector complex. I also show that in the Kashiwazaki plan [5] the sensitivity to $\sin^2 2\theta_{13}$ is approximately 0.02.

Reactor measurements of $\theta_{13}$. In the three flavor framework the disappearance probability of the reactor neutrinos is given by

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m^2_{13} L}{4E} \right),$$

if the contribution from $\Delta m^2_{21}$ is negligible. So the analysis is reduced to that of the conventional two flavor framework in vacuum.

Let me start with the derivation of $\chi^2$ used in [1] (See also [6]). For simplicity I assume one reactor and two detectors at near and far distances. $\chi^2$ is defined as

$$\chi^2 \equiv \min_{\alpha, \alpha^n, \alpha^f} \left\{ \frac{M^n - T^n(1 + \alpha + \alpha^n)}{T^n \sigma_{stat}^n} \right\}^2 + \left\{ \frac{M^f - T^f(1 + \alpha + \alpha^f)}{T^f \sigma_{stat}^f} \right\}^2 + \left( \frac{\alpha}{\sigma_e} \right)^2 + \left( \frac{\alpha^n}{\sigma_{ul}} \right)^2 + \left( \frac{\alpha^f}{\sigma_{ul}} \right)^2 \right\}, \quad (1)$$

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1 Based on the work [1] in collaboration with H. Minakata, H. Sugiyama, K. Inoue and F. Suekane. Talk presented at the 5th International Workshop on Neutrino Factories & Superbeams (NuFact’03), Columbia University, New York, USA, June 5-11, 2003.
where the superscripts $n$ and $f$ stand for the quantities at the near and far detectors, $M$ and $T$ stand for the measured and theoretical total numbers of events, $\sigma_{stat}^n = (T^n)^{-1/2}$ and $\sigma_{stat}^f = (T^f)^{-1/2}$ stand for the statistical errors, and $\alpha$, $\alpha^n$, $\alpha^f$ are the variables to introduce the correlated systematic error $\sigma_c$ and the uncorrelated systematic error $\sigma_u$ (I am assuming that the uncorrelated errors for the two detectors are the same). It is understood that the right hand side of Eq. (1) is minimized with respect to the three variables. After eliminating them I get

$$\chi^2 = \left( y^n, y^f \right) \left( \frac{\sigma_c^2 + \sigma_u^2 + (\sigma_{stat}^n)^2}{\sigma_c^2 + \sigma_u^2 + (\sigma_{stat}^f)^2} \right)^{-1} \left( y^n, y^f \right),$$  

(2)

where I have defined $y^n \equiv (M^n - T^n)/T^n$, $y^f \equiv (M^f - T^f)/T^f$. When the statistical errors are negligible, the error matrix in Eq. (2) is easily diagonalized and is expressed as

$$\chi^2 = \frac{1}{2\sigma_u^2}(y^f - y^n)^2 + \frac{1}{2\sigma_u^2 + 4\sigma_c^2}(y^f + y^n)^2.$$  

(3)

The uncorrelated error $2\sigma_u^2$ in Eq. (3) is the sum of the contributions from the two detectors and $\sigma_{rel} \equiv \sqrt{2}\sigma_u$ is referred to as the relative normalization error in [7]. Assuming that one can extrapolate the reference values for the systematic errors of the Bugey experiment [7] to the CHOOZ detectors [8], the systematic errors are estimated to be $\sigma_u = 0.8\%/\sqrt{2}=0.6\%$ and $\sigma_c = 2.6\%$. Eq. (3) indicates that the contribution from the sum $y^f + y^n$ is much smaller than that from the difference $y^f - y^n$. This was the reason why the $(y^f + y^n)^2$ term in $\chi^2$ was ignored in [1]. It should be emphasized that Eq. (3) shows the advantage of a near-far detector complex, since the correlated systematic error $\sigma_c$ is canceled in the denominator of the $(y^f - y^n)^2$ term$^2$.

From the expression (3) of $\chi^2$ for the rate, let me define the following $\chi^2$ for the spectrum analysis:

$$\chi^2 = \sum_j \frac{1}{\sigma_j^2} \left( \frac{M^f_j - T^f_j}{T^f_j} - \frac{M^n_j - T^n_j}{T^n_j} \right)^2,$$

where $M^n,f$ and $T^n,f$ stand for the measured and expected number of events at the near and far detectors for the $j$-th bin, and $\sigma_j$ is the statistical error plus the uncorrelated systematic error for each bin: $\sigma_j^2 = 1/T^n_j + 1/T^f_j + 2(\sigma_{bin}^j)^2$. Here I assume that the uncorrelated systematic error is the same for all bins: $\sigma_{bin}^j = \sigma_{bin}^u$, so $\sigma_{bin}^u$ is estimated from the uncorrelated systematic error $\sigma_u$ for the total number of events by

$$\left( \sigma_{bin}^u \right)^2 = \sigma_u^2 \frac{(T_{tot}^f)^2}{\sum_j (T_j^f)^2}, \quad T_{tot}^f \equiv \sum_j T_j^f.$$  

$^2$ The Krasnoyarsk proposal [9] also takes advantage of a near-far detector complex.
since the uncertainty squared of the total number of events is obtained by adding up the bin-by-bin systematic errors \( \sigma_{\text{bin}}^2(T_f)^2 \). The ratio \( \sigma_{\text{bin}}^u / \sigma_u \) is approximately 3 in our analysis. Although the sensitivity to \( \sin^2 2\theta_{13} \) is optimized at \( L \approx 1.7\text{km} \) for \( |\Delta m^2_{\text{13}}| = 2.5 \times 10^{-3}\text{eV}^2 \) [1], the longest baseline for the far detector inside the campus of the Kashiwazaki-Kariwa nuclear power plant turns out to be 1.3km [5] (See Fig.1).

In Fig. 2 the 90 \% CL exclusion limits, which corresponds to \( \chi^2 = 2.7 \) for one degree of freedom, are presented for two sets of parameters (data size, \( \sigma_{\text{rel}} \))=(10 ton-year, 1 \%), (40 ton-year, 0.5 \%) and for two baselines \( L = 1.3\text{km}, 1.7\text{km} \), where \( \sigma_{\text{rel}} \equiv \sqrt{2} \sigma_u \) was introduced earlier 3. Fig. 2 shows that the sensitivity to \( \sin^2 2\theta_{13} \) does not decrease very much for \( L = 1.3\text{km} \) and that it is possible to measure \( \sin^2 2\theta_{13} \) down to 0.02, provided the quoted values of the uncorrelated systematic error are realized.

**Summary & Conclusions.** In this talk I emphasized the advantage of a near-far detector complex in reactor measurements of \( \sin^2 2\theta_{13} \). I showed that it is potentially possible to measure \( \sin^2 2\theta_{13} \) down to 0.02 in the Kashiwazaki plan.

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3 For simplicity I have assumed in the calculation that there is only one reactor and two detectors, but the sensitivity with this simplification turns out to be almost the same as that with the exact calculation with seven reactors and three detectors [6].
FIGURE 2. The 90% CL exclusion limits on $\sin^2 2\theta_{13}$ in the Kashiwazaki plan. The light shadowed band is the allowed region at 90%CL for $|\Delta m^2_{13}|$ from the atmospheric neutrino data.

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