Charge and Color Breaking and D-terms in String Theory

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Abstract

In four dimensional superstring models, the gauge group normally contains extra $U(1)$s that are broken at a high energy scale. We show that the presence of the extra $U(1)$s is crucial for the phenomenological viability of string scenarios, since the contribution to the scalar masses from the D-terms can lift the unbounded from below directions that usually appear when the gauge group at high energies is just the Standard Model group. In particular, we show that the dilaton dominated scenario can be allowed in large regions of the parameter space if there exists an anomalous $U(1)$ in the theory, and the charges of the particles are the appropriate ones. We also analyze the parameter space of some explicit string constructions, imposing a correct phenomenology and the absence of dangerous charge and color breaking minima or unbounded from below directions.

1 Introduction

In a supersymmetric (SUSY) theory every fermion has a corresponding scalar partner. As a result, the effective potential is much richer than in a non-supersymmetric theory, and usually there are more minima apart from the electroweak (physical) vacuum in which we live. The requirement that the electroweak vacuum is the deepest minimum of the effective potential imposes very strong constraints on the parameter space. For example, large regions of the Constrained MSSM parameter space are excluded by the requirement of the absence of charge and color breaking (CCB) minima and/or unbounded from below (UFB) directions.

In this paper, we would like to concentrate on four-dimensional string models, which have definite predictions on the soft SUSY breaking terms. In these class of models, there are some massless chiral superfields whose auxiliary components can acquire vacuum expectation values (VEVs), and therefore break supersymmetry. These are the dilaton ($S$) and the moduli ($T_i$) fields, although some other fields could also contribute in some particular models. There are several proposals to trigger SUSY breaking in a

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hidden sector, leading to different soft terms. However, we prefer to follow \[7,8,9\] and we will assume that the underlying theory is a four dimensional string, and that the fields that dominate the SUSY breaking are the dilaton and the moduli. With these minimal assumptions, it is possible to parametrize the soft terms just with the dilaton and moduli auxiliary fields \((F_S, F_T)\), and thus obtain low energy predictions without addressing the problem of how is supersymmetry broken. A particularly interesting limit in this parametrization corresponds to the case in which the dilaton is the field that dominates SUSY breaking. The dilaton field, whose VEV determines the tree-level gauge coupling, is present in any four-dimensional string model and couples at tree-level universally to all particles. As a consequence, the soft terms are universal and independent of the four dimensional string considered. Furthermore, the universality of the soft terms guarantees the absence of large flavour changing neutral currents and large CP effects.

A common feature of the string scenarios studied in the literature is that only in some small windows our electroweak vacuum is the deepest minimum of the effective potential. Generically, the global minimum lies in a direction in which the stau acquires a VEV, thus breaking charge \[4\]. The case of the dilaton dominated SUSY breaking limit is especially disappointing, since the whole parameter space has a charge breaking global minimum \[7\]. This is certainly a blow against these string scenarios, and some possible ways-out have been proposed. It is possible that we live in a metastable minimum, as long as its life time is larger than the age of the Universe \[6\]. However, even if this is the case, it is difficult to explain why the cosmological evolution drove the Universe to a false vacuum instead of the true vacuum. A second possibility assumes that the fundamental theory is strongly coupled \[10,11\]. In that case, the fundamental scale can be lowered and the CCB/UFB bounds on the dilaton dominated scenario are relaxed, thus rescuing some regions of the parameter space \[12\].

Many of these analyses assume that only the dilaton and moduli auxiliary fields contribute to the soft terms. However, most string constructions predict the existence of extra gauge groups broken at a high energy scale that can contribute to the scalar masses via D-term condensation \[13\]. The goal of this paper is to study the impact of this contribution on the CCB and UFB bounds in string scenarios.

Let us briefly review the basic ingredients for our analysis. The general form of the soft SUSY-breaking Lagrangian in the MSSM is given by

\[
\mathcal{L}_{soft} = \frac{1}{2} \sum_{a=1}^{3} M_a \bar{\lambda}_a \lambda_a - \sum_i m_i^2 |\phi_i|^2 - (A_{ijk} W_{ijk} + B \mu H_1 H_2 + h.c.),
\]

where \(W_{ijk}\) are the usual terms of the superpotential of the MSSM with \(i = Q_L, u_R, d_R, L_L, e_R, H_1, H_2, \) and \(\phi_i, \lambda_a\) are the canonically normalized scalar and gaugino fields, respectively. On the other hand, \(m_i, M_a\) and \(A_{ijk}\) are the scalar masses, the gaugino masses and the trilinear terms, respectively. String scenarios give definite predictions for these soft terms. However, this is not the case for the bilinear term \(B\), which depends crucially on the mechanism that generates the \(\mu\) term in the superpotential \((\mu H_1 H_2)\). Although there are several interesting proposals to explain the origin of the \(\mu\) term, the actual mechanism is still unknown. Therefore, we prefer to leave \(B\) as a free parameter in our analysis. As usual, the value of \(\mu\) will be fixed by the requirement of
a correct electroweak symmetry breaking, i.e. by imposing the existence of a realistic minimum with the correct mass for the Z boson.

Now, we turn to the issue of the CCB and UFB bounds, discussed in detail in [3]. There, it was found that the strongest constraint corresponds to the direction labeled as UFB-3, which involves the fields \( \{ H_2, \nu_{L_i}, e_{L_j}, e_{R_j} \} \), with \( i \neq j \), and thus leads to electric charge breaking. For conciseness we will only write explicitly this one, since it is the most relevant one in our analysis. By simple analytical minimization of the relevant terms of the scalar potential, it is possible to determine the value of the \( \nu_{L_i}, e_{L_j}, e_{R_j} \) fields in terms of the \( H_2 \) one [3]. Then, for any value of \( |H_2| < M_{\text{string}} \) satisfying:

\[
|H_2| > \sqrt[2]{\frac{\mu^2}{4\lambda_{e_j}^2} + \frac{4m_{L_i}^2}{g^2 + g_2^2}} - \frac{|\mu|}{2\lambda_{e_j}^2}, \tag{2}
\]

the value of the potential along the UFB-3 direction is simply given by

\[
V_{UFB-3} = (m_2^2 - \mu^2 + m_{L_i}^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}^2}(m_{L_j}^2 + m_{e_j}^2 + m_{L_i}^2)|H_2| - \frac{2m_{L_i}^2}{g^2 + g_2^2}. \tag{3}
\]

Otherwise

\[
V_{UFB-3} = (m_2^2 - \mu^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}^2}(m_{L_i}^2 + m_{e_j}^2)|H_2| + \frac{1}{8}(g^2 + g_2^2) \left| |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}^2}|H_2|^2 \right|^2. \tag{4}
\]

In eqs. (3,4) \( \lambda_{e_j} \) is the leptonic Yukawa coupling of the \( j \)-generation and \( m_2^2 \) is the sum of the \( H_2 \) soft mass squared, \( m_{H_2}^2 \), plus \( \mu^2 \). Then, the UFB-3 condition reads

\[
V_{UFB-3}(Q = \hat{Q}) > V_{\text{real min}}, \tag{5}
\]

where \( V_{\text{real min}} = -\frac{1}{8}(g^2 + g_2^2)(v_2^2 - v_1^2)^2 \), with \( v_{1,2} \) the VEVs of the Higgses \( H_{1,2} \), is the realistic minimum evaluated at \( M_S \) (see below) and the \( \hat{Q} \) scale is given by \( \hat{Q} \sim \text{Max} \{ g_2 |e|, \lambda_{top} |H_2|, g_2 |H_2|, g_2 |L_i|, M_S \} \) with \( |e| = \sqrt{\frac{|\mu|}{\lambda_{e_j}^2}}|H_2| \) and \( |L_i|^2 = \frac{4m_{L_i}^2}{g^2 + g_2^2} \) + (\( |H_2|^2 + |e|^2 \)). Finally, \( M_S \) is the typical scale of SUSY masses (normally a good choice for \( M_S \) is an average of the stop masses). Notice from (3,4) that the negative contribution to \( V_{UFB-3} \) is essentially given by the \( m_2^2 - \mu^2 \) term, which can be very sizeable in many instances. On the other hand, the positive contribution is dominated by the term \( \propto 1/\lambda_{e_j} \), thus the larger \( \lambda_{e_j} \) the more restrictive the constraint becomes. Consequently, the optimum choice of the \( e \)-type slepton is the third generation one, i.e. \( e_j = \text{stau} \).

It is apparent from eq. (3) that the UFB-3 bound is relaxed when the slepton masses are large compared to the rest of the parameters in the scalar potential [4]. There is also a second order effect that relaxes the UFB-3 bound, namely when the squark masses are small. The reason is that \( m_{H_2}^2 \) is driven negative by the stop contribution to its RGE, \( Qd m_{H_2}^2/dQ = 6(\lambda_{top}/4\pi)^2(m_{L_i}^2 + m_{R_i}^2) + \ldots \), hence the smaller \( m_{L_i}^2, m_{R_i}^2 \), the weaker the UFB-3 bound becomes [4]. Some of the superstring scenarios analyzed in [4]

\[1\] Also, a departure of gaugino universality in the direction \( M_2^2, M_1^2 > M_3^2 \) helps to rescue regions in the parameter space. Incidentally, this is the pattern expected in weakly coupled string scenarios, when the one loop corrections to the gauge kinetic functions are included. Nevertheless, throughout the paper we will consider universal gaugino masses, neglecting the above-mentioned effect.
did not have this kind of spectrum, and therefore the UFB-3 bound was devastating. However, the D-term contribution to the scalar masses, which appears in most explicit superstring constructions when some extra gauge groups are broken at a high energy scale, can modify the spectrum of scalar masses to produce one with this characteristics. If this is the case, large regions of the parameter space can be rescued.

The paper is organized as follows. In Sect. 2 we analyze the D-term contribution to the scalar masses in a superstring inspired scenario with an anomalous $U(1)$, and examine the implications for CCB minima and UFB directions. In Sect. 3 we perform a similar analysis for the case of a non-anomalous $U(1)$. Sect. 4 is devoted to the analysis of two realistic superstring scenarios, with several extra $U(1)$s. Finally, the conclusions are presented in Sect. 5.

## 2 Scalar masses in the presence of an anomalous $U(1)$

In many explicit four-dimensional heterotic string constructions, the massless spectrum includes matter chiral superfields that transform under an anomalous $U(1)$ symmetry according to \( \phi_k \to e^{iq_k \Lambda} \phi_k \), and a vector superfield that transforms as \( V_A \to V_A + i(\Lambda - \Lambda^*) \), where \( \Lambda \) is the transformation parameter. The anomalies are compensated by a non-trivial transformation of the dilaton under the $U(1)_A$, \( S \to S + i\Lambda \delta_{GS} \), according to the Green-Schwarz mechanism \(^{14}\). In a string theory, the coefficient \( \delta_{GS} \) can be computed explicitly and turns out to be proportional to the apparent chiral anomaly \(^{15}\)

\[
\delta_{GS} = \frac{1}{192\pi^2} \sum_k q_k. \tag{6}
\]

The charges of the fields under the anomalous $U(1)$ are severely constrained by the Green-Schwarz mechanism. The anomalies are cancelled only if the coefficients \( A_i \) of the mixed anomalies of the $U(1)_A$ satisfy the condition \( A_i/k_i = \delta_{GS} \), where \( k_i \) is the Kac-Moody level of the gauge factor. In particular, the mixed anomalies with the Standard Model groups, $SU(3)$, $SU(2)$ and $U(1)_Y$, must be in the ratio \( A_3 : A_2 : A_1 = k_3 : k_2 : k_1 \). Normally (one level case) one takes \( k_3 = k_2 = \frac{3}{5} k_1 = 1 \) to account for the unification of the gauge couplings, which translates into \( A_3 = A_2 = \frac{3}{5} A_1 = 1 \) \(^{16}\). If the only fields charged under the SM gauge group are the usual fields of the MSSM, it can be proved that there are only two family independent $U(1)$ symmetries \(^2\) for which \( A_3 = A_2 = \frac{3}{5} A_1 = 1 \). Following ref. \(^{17}\), we will denote them by $U(1)_X$ and $U(1)_{XX}$. The most general “anomalous” $U(1)$ symmetry is a combination of those two, plus all the possible traceless $U(1)$, since they do not modify the conditions for anomaly cancellation. With the minimal particle content of the MSSM, there is only one such symmetry (apart from the weak hypercharge) which we denote by $U(1)_H$. The charges

\(^2\)We restrict ourselves to family independent symmetries for reasons that will be explained later.
of the MSSM fields under those groups are

|       | $Q_L$ | $u_R$ | $d_R$ | $L_L$ | $e_R$ | $H_2$ | $H_1$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $U(1)_H$ | 0     | 0     | 0     | 0     | 0     | 1     | -1    |
| $U(1)_X$ | 1     | 1     | 0     | 0     | 1     | 0     | 0     |
| $U(1)_{XX}$ | 0   | 0     | 1     | 1     | 0     | 0     | 0     |

Then, the most general “anomalous” $U(1)$, with anomalies cancelled by the Green-Schwarz mechanism, is a linear combination of those symmetries:

$$U(1)_A = z U(1)_H + x U(1)_X + y U(1)_{XX}.$$  (8)

Now, we turn to describe the corresponding supergravity theory. It is defined by three functions, namely the Kähler potential $K(\phi_k, \phi_k^*)$, the gauge kinetic function $f_a(\phi_k)$ and the superpotential $W(\phi_k)$. Gauge invariance requires the Kähler potential to be a function of $S + S^* - \delta_{GS} V_A$. In particular, for orbifold compactifications, the tree-level Kähler potential reads

$$K = -\log(S + S^*) - 3\log(T + T^*) + (T + T^*)^{n_{\phi_k}} e^{2q_{\phi_k} V_A} \phi_k^* \phi_k,$$  (9)

where $n_{\phi_k}$ is the modular weight of the field $\phi_k$. We assume that in the massless spectrum there is one chiral superfield, denoted by $\Phi$, with charge $Q$ under $U(1)_A$ and singlet with respect to the Standard Model group, that will eventually acquire a VEV, thus breaking the anomalous symmetry. On the other hand, the gauge kinetic function is, at tree level, $f_a = k_a S$, where $k_a$ were defined above. Finally, let us discuss the superpotential in this class of models. Commonly, the terms that appear in the MSSM superpotential, that we denote generically by $\phi_i \phi_j \phi_k$, are not gauge invariant under $U(1)_A$. Instead, the superpotential reads

$$W \sim \Theta[-(q_i + q_j + q_k)/Q] \left( \frac{\Phi}{M_{Pl}} \right)^{-\frac{-(q_i + q_j + q_k)}{Q}} \phi_i \phi_j \phi_k,$$  (10)

where $\Theta(x)$ is a step function, defined as 1 when $x \geq 0$ and 0 otherwise, to account for the requirement of holomorphicity. Then, if we want to end up at low energies with the MSSM superpotential, the charges of the particles under $U(1)_A$ must satisfy

$$\left( q_{QL} + q_{u_R} + q_{H_2} \right)/Q \leq 0,$$
$$\left( q_{QL} + q_{d_R} + q_{H_1} \right)/Q \leq 0,$$
$$\left( q_{LL} + q_{e_R} + q_{H_1} \right)/Q \leq 0.$$  (11)

These impose further constraints on the charges of the particles under the anomalous $U(1)$, apart from the condition of anomaly cancellation. A similar holomorphicity constraint applies to the bilinear term, $(q_{H_1} + q_{H_2})/Q \leq 0$.

In the low energy theory, the scalar particles receive two different contributions to their masses. The first one depends on which are the fields responsible for the SUSY breaking, $S$ and/or $T$. These have been computed in [4] in terms of the goldstino angle, defined as $\tan \theta = F_S/F_T$, and the gravitino mass $m_{3/2}$. Assuming cancellation of the
cosmological constant, one obtains from eq.(9) a contribution to the scalar masses given by

\[ [m_k^2]_F = m_{3/2}^2(1 + n_k \cos^2 \theta). \]  

(12)

The second contribution to the scalar masses comes from the breaking of the \(U(1)_A\) symmetry, and contributes through the expectation value of the D-term associated to \(U(1)_A\). The anomalous \(U(1)\) generates a Fayet-Iliopoulos (F-I) D-term in the scalar potential that can break supersymmetry [19]:

\[ D_A = -g_A^2(\xi^2 + q_k |\phi_k|^2), \]  

(13)

where \(\xi^2 = -\delta_{GS}/2(S + S^*)\) is the F-I term. Large-scale supersymmetry breaking is avoided through the chiral field \(\Phi\) acquiring a vacuum expectation value that nearly cancels the F-I term. Due to the low-scale supersymmetry breaking, the cancellation is not exact, and this amounts to a contribution to the scalar masses, given by

\[ [m_k^2]_D = -q_k \langle D_A \rangle. \]  

(14)

Assuming that \(U(1)_A\) is broken at a scale much smaller than the gravitational scale, but much larger than the electroweak scale \(3\), one obtains [20,21]

\[ \langle D_A \rangle = -\frac{1}{Q} m_{3/2}^2(1 - 6 \sin^2 \theta - n_\Phi \cos^2 \theta). \]  

(15)

At this point, some comments are in order. First, notice that the positivity of the scalar masses is not guaranteed after the breaking of \(U(1)_A\). This could lead to the breaking of some of the Standard Model symmetries at a scale \(\sim \xi\), including charge and/or color. This in turn imposes constraints on the goldstino angle and the charges of the particles under \(U(1)_A\). Secondly, if the anomalous \(U(1)\) symmetry is horizontal, the family-dependent contributions to the scalar masses from the D-terms produce rates for the flavour changing processes that could be sizeable. Therefore, to avoid potential problems with flavour violation, we will assume that the anomalous \(U(1)\) symmetry is family independent.

The gaugino masses and the trilinear soft terms do not receive any contribution from the D-terms. At tree level in the gauge kinetic function and the Kähler potential, and neglecting phases, they are given by [3]

\[ M = \sqrt{3} m_{3/2} \sin \theta, \]  

(16)

\[ A_{ijk} = -\sqrt{3} m_{3/2} [\sin \theta + \cos \theta \omega_{ijk}(T, T^*)], \]  

(17)

where

\[ \omega_{ijk}(T, T^*) = \frac{1}{\sqrt{3}} \left( 3 + n_i + n_j + n_k - (T + T^*) \frac{\partial \lambda_{ijk}}{\partial T} \right), \]  

(18)

and \(\lambda_{ijk}\) are the Yukawa couplings in the superpotential.

Next, we study the impact of the anomalous \(U(1)\) symmetry on different well-motivated four-dimensional string scenarios.

\footnote{For an anomalous \(U(1)\), the scale of symmetry breaking can be read from the condition of D-flatness. It yields \(\langle \Phi \rangle \sim 0.02 \sqrt{\sum_k q_k/Q M_{Pl}}\), that typically satisfies these conditions.}
2.1 The dilaton limit

The dilaton limit is a very attractive scenario, since it alleviates the flavour and the CP problems in supersymmetric theories. In this scenario, the different terms in the soft SUSY-breaking Lagrangian have a particularly simple form:

\[
m_k^2 = m_{3/2}^2 (1 + 5q_k),
M = \pm \sqrt{3} m_{3/2},
A = - M.
\]

where we have assumed, without loss of generality, that \( Q = -1 \).

In the Introduction, we remarked that the UFB-3 bound is relaxed when the slepton masses are large and the squark masses are small, the former being the dominant effect. It is interesting to note that the sleptons can be charged under the most general anomaly-free \( U(1) \), eq.(8), and their masses can receive a contribution from the D-terms that could lift the UFB-3 direction. To decide if that is possible or not, we study first some particular choices for \( U(1)_A \) and later on, we will discuss the most general case.

The assignment of charges for the \( U(1)_{XX} \) symmetry yields \( m_{3R}^2 = m_{LL}^2 = 6m_{3/2}^2 \), while the rest of the masses remain unchanged. Also, with this assignment of charges, the requirement of holomorphicity, eqs. (11), is satisfied, and we will get at low energies all the Yukawa couplings of the Standard Model Lagrangian. We plot the regions of the parameter space forbidden from the different UFB and CCB bounds, as well as from the condition of a correct top mass and the experimental lower bounds on the supersymmetric masses. Here we choose the positive sign for the gaugino mass, although the results for negative gaugino masses can also be read from the figures, due to the invariance of the analysis under the transformation \( B, A, M \to -B, -A, -M \).

Then, the only free parameters are the gravitino mass (\( m_{3/2} \)) and the bilinear soft term (\( B \)), so, we will use these variables to span the parameter space. In Figure 1, we show the results assuming that the only contribution to the scalar masses comes from the dilaton and moduli auxiliary fields (or equivalently, when the gauge group at high energies is just the Standard Model group, without an extra anomalous \( U(1) \) in the theory). This was the case studied in [3], where it was found that the whole parameter space is excluded. On the other hand, in Figure 2, left plot, we show the results for the assignment of charges corresponding to \( U(1)_{XX} \). All the points that were forbidden by the UFB-3 bound now become allowed. The reason is that \( m_{LL}^2 \) is large enough to compensate the negative contribution from \( m_2^2 - \mu^2 \) in eq.(3). Also, \( m_{LL}^2 \) appears in the coefficient linear with \( |H_2| \), which is positive, and this further alleviates the UFB-3 constraint. However, this effect is not as strong as the above-mentioned one. Notice also that there are no points forbidden by CCB bounds. The bound that was not satisfied in Figure 1 was

\[
|A_\tau|^2 \leq 3(m_{LL}^2 + m_{3R}^2 + m_{H_1}^2),
\]

which is obviously relaxed when the slepton masses are large. This example illustrates the tremendous impact that the D-term contribution to the scalar masses can have on the low energy phenomenology.
Figure 1: Excluded regions of the parameter space assuming SUSY breaking dominated by the dilaton, in a scenario where the gauge group at high energies is just the Standard Model gauge group. The black regions are excluded because it is not possible to reproduce the experimental mass of the top quark. The small squares indicate regions excluded by unbounded from below bounds, while the circles, regions excluded by charge and color breaking constraints. Finally, the filled diamonds correspond to regions excluded by the experimental lower bounds on supersymmetric masses.

On the other hand, the charges for $U(1)_X$ give rise to a different spectrum of scalar masses, namely, $m_{Q_L}^2 = m_{u_R}^2 = m_{e_R}^2 = 6m_{3/2}^2$ and the rest of the scalar masses equal to $m_{3/2}^2$. This spectrum also has a heavy slepton, which helps to lift the UFB-3 direction. Unfortunately, $m_{e_R}^2$ only enters in the term linear with $|H_2|$ in eqs. (3,4), and is not enough to rescue points in the parameter space. A large $\tilde{e}_R$ mass is useful, though, to rescue points forbidden by the CCB constraint eq. (20), but still the whole parameter space is excluded by UFB constraints. The numerical results are shown in Figure 2, right plot.

The remaining symmetry is $U(1)_H$. We do not analyze this symmetry separately because by itself it is not able to implement the Green-Schwarz mechanism (all the mixed anomalies with the SM gauge group are vanishing). Also, the fact that at least one Higgs has negative charge under $U(1)_H$ implies that the corresponding scalar mass squared can be negative, unless $|z| \leq 1/5$. In terms of charges, this translates into $|q_{H_2}/Q| \leq 1/5$, which does not seem very natural. We should stress that the presence of a negative Higgs mass squared does not represent a problem for charge breaking, since in the MSSM the minimum of the Higgs potential always lies at $\langle H_2^+ \rangle = \langle H_1^- \rangle = 0$. It could be a problem, though, for the electroweak symmetry breaking. A correct electroweak symmetry breaking imposes the following constraint on the parameters entering in the Higgs potential:

$$m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 - 2|\mu B| > 0. \quad (21)$$
Then, if the scalar masses are negative already at the high energy scale, this bound is more difficult to fulfill. However, it should be noted that it could be possible to find some regions in the parameter space where the bound eq. (21) is satisfied, and the electroweak symmetry is broken in the correct way. For simplicity, we will not consider this possibility here and we will impose that at high energies all the scalar masses are positive. So, a sensible choice would be to assume $z = 0$, although our conclusions for this scenario are not very different if $|z| \leq 1/5$.

Then, the most general anomaly-free $U(1)$ is just a combination of $U(1)_X$ and $U(1)_{XX}$. We argued that the UFB-3 direction was lifted when the $\tilde{L}_L$ mass was large compared to the rest of the scalar masses, especially $m_{H_2}$. This was achieved by the charge assignment of the $U(1)_{XX}$ symmetry, whereas the $U(1)_X$ symmetry did not have any significant effect on the UFB bounds. Then, we expect that if $U(1)_{XX}$ is present in $U(1)_A$ to some extent, some regions of the parameter space could be rescued, as long as the requirements of positivity and holomorphicity are fulfilled. To show this, we study

$$U(1)_A = xU(1)_X + (1-x)U(1)_{XX},$$

and we plot the results for a fixed gravitino mass ($m_{3/2} = 100$ GeV). Taking $x = 0$ corresponds to the limiting case in which the anomalous symmetry is just $U(1)_{XX}$ (no points forbidden by UFB or CCB bounds), while $x = 1$ corresponds to $U(1)_X$ (the whole parameter space is excluded). If $x > 1$ or $x < 0$ positivity is not satisfied. We find that all our phenomenological requirements are satisfied for $x \lesssim 0.4$, as is shown in Figure 3. If the gravitino mass is larger, or the charges of all the MSSM fields are larger [for example, if $U(1)_A = 2xU(1)_X + 2(1-x)U(1)_{XX}$], the allowed region becomes larger as well.

At this point, we should remind the reader that we have analyzed a string inspired
scenario, with just the MSSM content plus an additional superfield, Φ, singlet under the SM gauge group, to break the anomalous $U(1)_A$. In realistic string models, there are in general more fields, with masses of $\mathcal{O}(\xi)$, charged under the Standard Model symmetries, that modify the conditions for mixed anomaly cancellation. Therefore, it could happen that the $U(1)_A$ symmetry is not of the form eq. (8), and in consequence, the analysis should be done case by case. Nevertheless, it is encouraging that the dilaton limit can be rescued by the presence of an anomalous $U(1)$. We will study some explicit string constructions in Section 4.

## 2.2 Modular weights equal to $-1$

In this subsection we consider a scenario where all the relevant particles have modular weights equal to $-1$. This is the case when the observable sector belongs to the untwisted sector of an orbifold compactification, although the results are also valid for a Calabi-Yau compactification in the large-$T$ limit. For the purposes of this paper, this case is interesting because it permits the study of the effect of the goldstino angle on the region forbidden by the UFB-3 bound, after including the D-term contribution to the scalar masses. Note that different modular weights for different particles would produce by itself a non-universality in the scalar masses that could obscure the effect we want to study. For this scenario, the Yukawa couplings $\lambda_{ijk}$ are T-independent since a cubic operator of matter fields has exactly the appropriate modular weight $(-3)$. Therefore, the expression for the trilinear term given in eq. (17) becomes independent of $\lambda_{ijk}$ and are universal. Under the usual assumptions, the different terms appearing

Figure 3: The same as Figure 1, but extending the Standard Model gauge group with the anomalous symmetry $U(1)_A = xU(1)_X + (1-x)U(1)_{XX}$, for a gravitino mass of 100 GeV. The meaning of the symbols is the same as in Figure 1.
in the soft SUSY-breaking Lagrangian are:

\[ m_k^2 = m_{3/2}^2 [\sin^2 \theta + \frac{q_k}{Q} (1 - 6 \sin^2 \theta + \cos^2 \theta)] , \quad (23) \]

\[ M = \sqrt{3} m_{3/2} \sin \theta , \quad (24) \]

\[ A_{ijk} = -\sqrt{3} m_{3/2} \sin \theta . \quad (25) \]

If the \( U(1)_A \) charges of the MSSM fields are all set to zero, we find that these expressions satisfy the relation \( M = -A = \pm \sqrt{3} m \), independently of \( \theta \). Since the CCB and UFB bounds depend, for a given \( B \), only on \( M/m \) and \( A/m \), in every point of the parameter space the results are identical to the results at \( \theta = 0 \), i.e. the dilaton limit, which is excluded.

The presence of the anomalous \( U(1) \) in this scenario has several consequences. The first one is a non-trivial dependence of the results with the goldstino angle, as is evident from eq.(23) and the previous discussion. Also, in this scenario it is possible to have negative scalar masses squared at the high energy scale, in contrast to the case with \( q_k = 0 \), where the positivity was guaranteed in every point. As was mentioned in [20], the condition for positivity is

\[(1 - 7q_k/Q) \sin^2 \theta + 2q_k/Q \geq 0 . \quad (26)\]

It is interesting to note that in the moduli limit, \( \sin \theta = 0 \), positivity requires \( q_k/Q \geq 0 \), which is in conflict with the holomorphicity constraints, eqs.([11]), except in the trivial case \( q_k/Q = 0 \).

Most of the conclusions drawn for the dilaton dominated scenario are also valid here: the existence of an anomalous group can rescue large regions of the parameter space, when the charges are such that the \( \tilde{L}_L \) mass is larger than the rest. To illustrate this, we study in detail the assignment of charges corresponding to the \( U(1)_{XX} \) symmetry in eq.(7). Positivity of the masses squared at the high energy scale requires \( \sin \theta > 1/2 \) (as before, we are setting \( Q = -1 \)), and accordingly we present the results only for that range of the goldstino angle. We have just plotted for \( \theta \) in the \([0, \pi]\) range because the results in the \([\pi, 2\pi]\) range can also be read from the figures, due to the invariance of the analysis under the transformation \( B, A, M \rightarrow -B, -A, -M \). The regions allowed by all our phenomenological requirements are shown in Figure 4. There are large regions that have become allowed, while there are still some windows forbidden by the UFB-3 bound. These correspond to points where the non-degeneracy of the masses is not large enough to lift the UFB-3 direction. On the other hand, the parameter space for the \( U(1)_X \) symmetry is entirely excluded, as expected from the results for the dilaton limit. In a more general case, when the “anomalous” \( U(1) \) symmetry is a combination of \( U(1)_X \) and \( U(1)_{XX} \), as in eq. (8), there appear allowed regions for a wide range of goldstino angles when \( x \lesssim 0.4 \) (or may be more, depending on \( m_{3/2} \) and the charges).

### 2.3 ILR model

This model was proposed by Ibañez, Lüst and Ross [22] and has the nice feature that it provides string threshold corrections large enough to fit the joining of the gauge
Figure 4: Allowed regions for a scenario where all the matter fields have modular weight $-1$ and the Standard Model gauge group has been extended with the $U(1)_{XX}$ symmetry. We have fixed the gravitino mass to 100 GeV and the meaning of the symbols is the same as in Figure 1.

coupling constants at a scale $\simeq 10^{16}$ GeV. This is achieved by assigning the following modular weights to the MSSM fields:

$$n_{Q_L} = n_{d_R} = -1, \quad n_{u_R} = -2, \quad n_{L_L} = n_{e_R} = -3, \quad n_{H_1} + n_{H_2} = -5, -4. \quad (27)$$

The above values together with a Re$T \simeq 16$ lead to a good agreement for $\sin^2 \theta_W$ and $\alpha_3$. This case is also interesting because there are two sources of non-universality in the scalar masses: in the contribution from the moduli auxiliary field, due to the different modular weights of the MSSM fields, and in the contribution from the D-term, due to the different charges under $U(1)_A$. The assignment of modular weights makes the slepton soft masses smaller than the rest, hence, if the gauge group at high energies was just the SM gauge group, the UFB-3 direction would be rather deep (in fact, in that case, the whole parameter space is excluded [4]). We want to study here if the D-term contribution to the scalar masses can be large enough to lift the UFB-3 direction, even in this particularly unfavorable scenario.

As in the previous subsection, we extend the SM gauge group with the $U(1)_{XX}$ symmetry, defined in eq.(7), and the matter content of the MSSM with a field $\Phi$, only charged under $U(1)_{XX}$. The scalar masses can be readily computed from eqs.(12, 14, 15). On the other hand, the computation of the trilinear soft terms, eqs.(17,18), deserves some more explanation. In this case, the Yukawa couplings, $\lambda_{ijk}$, are non-trivial functions of $T$ [23], which apparently causes a difficulty when determining the trilinear soft terms. However, the top Yukawa coupling, the only relevant one for the evaluation of the UFB-3 bound, represents a remarkable (and fortunate) exception. The reason is the following. The twisted Yukawa couplings are in general given by a series of terms, all of them suppressed by $e^{-c_i T}$ factors. Only when the fields involved
Figure 5: Allowed regions for the ILR scenario where the Standard Model gauge group has been extended with the $U(1)_{XX}$ symmetry. The gravitino mass is fixed to 100 GeV and the meaning of the symbols is the same as in Figure 1.

belong to the same fixed point (or fixed torus) the first term in the series is $O(1)$ and independent of $T$, otherwise the coupling is suppressed. Consequently, the top Yukawa coupling, being $O(1)$, must be of this type. So, for this particular case we can ignore the $\partial \lambda / \partial T$ factors in (18), thus getting, for the assignment of modular weights corresponding to the ILR model, \( \omega = n_{H_2}/\sqrt{3}, |A| = -m_{3/2}(\sqrt{3}\sin\theta + n_{H_2}\cos\theta). \)

The assignment given in eq. (27) allows different choices of the modular weights of the Higgs doublets. We show the results for the case $n_{H_1} = -2$ and $n_{H_2} = -3$, and $n_{\Phi_1} = n_{\Phi_2} = -1$, although the conclusions for other choices are completely analogous. In Figure 5 we show the numerical results for $\cos^2\theta \leq 1/3$, to avoid negative scalar masses squared at the high energy scale (in particular, $m_{\tilde{e}_R}^2$). The contribution to the scalar masses coming from the D-terms succeeds to modify the spectrum of scalar masses from one with light sleptons, unfavorable from the point of view of the UFB-3 bound, to one with heavy sleptons. In consequence, large regions of the parameter space become allowed after the inclusion of the D-term contribution to the scalar masses. In a more general case, in which the anomalous gauge group is a combination of $U(1)_X$ and $U(1)_{XX}$, as in eq.(22), this scenario becomes allowed for $x \lesssim 0.2$. As expected, it is more difficult to rescue this scenario than the one with $n = -1$, since at the gravitational scale, before the breaking of the anomalous $U(1)$, the scalar masses are already non-universal and have a spectrum which is rather unfavorable from the point of view of the UFB bounds.
3 Scalar masses in the presence of a non-anomalous $U(1)$

The presence of non-anomalous $U(1)$s is also quite common in four-dimensional string constructions. Hence, it is worth studying the effect of the breaking of these groups on the charge and color breaking bounds.

Let us assume a simplified scenario in which there is only one non-anomalous $U(1)$, whose gauge supermultiplet we denote by $V$. Then, for orbifold compactifications, the tree level Kähler potential is

$$K = -\log(S + S^*) - 3\log(T + T^*) + (T + T^*)^n_{\phi_k} e^{2q_k V} \phi_k^* \phi_k,$$

where $q_k$ and $n_{\phi_k}$ are, respectively, the $U(1)$ charge and the modular weight of the chiral superfield $\phi_k$. Let us also assume that in the massless spectrum there are two chiral superfields, $\Phi_1$ and $\Phi_2$, with charges $Q$ and $-Q$ and modular weights $n_{\Phi_1}$ and $n_{\Phi_2}$, respectively, whose scalar components acquire a vacuum expectation value and break the $U(1)$ symmetry at a scale much smaller than the gravitational scale but much larger than the electroweak scale. Under these assumptions, the D-term contribution to the scalar masses is:

$$[m^2_k]_D = \frac{q_k m^2 / 2 \cos^2 \theta (n_{\Phi_2} - n_{\Phi_1})}{2}.$$

From this formula, it is apparent that the D-term contribution to the scalar masses vanishes when the modular weights of $\Phi_1$ and $\Phi_2$ (or equivalently, their soft masses) are identical. If that is the case, the vacuum is invariant under the interchange $\Phi_1 \leftrightarrow \Phi_2$ and hence $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle$, yielding $\langle D \rangle = 0$. The D-term also vanishes when $\cos \theta = 0$, i.e. the dilaton limit. This contrasts with the anomalous $U(1)$ case, where the non-trivial transformation of the dilaton with the anomalous symmetry translated into a contribution to the scalar masses that depended on the dilaton auxiliary field, generating a contribution even in the dilaton dominated SUSY breaking limit. We conclude, then, that the dilaton limit can only be rescued when the light fields are charged under an anomalous gauge group, broken at a high energy scale, and the charges are the appropriate ones. Nevertheless, when $\langle D \rangle$ is not vanishing, the D-term contribution to the scalar masses is comparable to the soft masses, and should be considered in the analyses.

As was mentioned in Section 2, if the only fields charged under the SM group are the MSSM fields, there is only one anomaly-free $U(1)$, apart from the weak hypercharge. Under that symmetry, denoted by $U(1)_H$, the Higgs doublets transform with opposite charges, while the rest of the particles are neutral. In particular, the sleptons are.

---

4This is a tree-level argument, and certainly, if $\Phi_1$ and $\Phi_2$ had different interactions, radiative corrections could produce some differences between $m^2_{\Phi_1}$ and $m^2_{\Phi_2}$ and the vacuum would not be invariant under $\Phi_1 \leftrightarrow \Phi_2$ anymore. However, the effects of the running between the gravitational scale and the $U(1)$ breaking scale are in general too small to have any significant effect. They might be relevant, though, for a horizontal $U(1)$ symmetry, since the (flavour dependent) D-term contribution to the scalar masses represents a source of flavour violation.
neutral, so it is not possible to lift the UFB-3 direction by the D-term contribution to their scalar masses, as we did in the previous section. Furthermore, the fact that the Higgs doublets have opposite charges implies that either $m_{H_1}^2$ or $m_{H_2}^2$ could be negative at the high energy scale, with the subsequent potential problems for the electroweak symmetry breaking. Nevertheless, this scenario deserves a careful analysis, since a departure of universality in the Higgs masses could help to rescue points from the UFB-3 bound. To be precise, if $H_2$ is heavier than the rest of particles, at the appropriate scale to compute the effective potential, $m_{H_2}^2$ is less negative than in the universal case. Therefore, the UFB-3 bound is alleviated.

We study in detail the case with $q_{H_2} = -q_{H_1} = Q$, and $n_{\Phi_2} - n_{\Phi_1} = 1$. We also fix all the modular weights of the MSSM fields to $-1$, so that the only effects of non-universality come from the D-terms. That assignment of charges produces a sparticle spectrum where, out of the dilaton limit, $H_2$ has the largest soft mass and $H_1$ the smallest. The departure of universality opens some small allowed windows, as is apparent from Figure 6. It should be noted that in the plots we have varied the goldstino angle between 0 and $\pi$, and thus positivity is not guaranteed in the whole parameter space (positivity requires $\cos^2 \theta \geq 2/3$). As a matter of fact, in the allowed regions, the $H_1$ soft mass squared is negative. The reason is that the Higgs masses satisfy at high energies the relation $m_{H_1}^2 + m_{H_2}^2 = 2m_{\text{rest}}^2$, where $m_{\text{rest}}^2$ is the common soft mass of the rest of the particles. Then, had we required positivity of all the scalar masses squared, the condition $m_{H_1}^2 \geq 0$ would have translated into $m_{H_2}^2 \leq 2m_{\text{rest}}^2$, and this non-degeneracy is not strong enough to lift the UFB-3 direction. As a result, the regions where all the scalar masses squared are positive are excluded. Allowing $m_{H_1}^2$ to be negative, $m_{H_2}^2$ can be very large, thus relaxing the UFB-3 bound. On the other hand, and as was mentioned before, if the Higgs masses squared are already negative at high energies, it is more difficult to fulfill the bound eq.(21). However, there are some small windows where $m_{H_1}^2 < 0$, but the electroweak symmetry is broken in the correct way, and the non-degeneracy is strong enough to lift the UFB-3 direction.

4 Application to realistic string models

In the previous sections we have studied “string inspired” scenarios, with the SM gauge group extended to include one extra Abelian symmetry, anomalous or non-anomalous, and the matter content of the MSSM plus some additional fields, to break the extra group. However, in a realistic model, there appear several extra $U(1)$s, one of them usually anomalous, and many chiral fields. Those extra symmetries are broken when certain fields acquire a vacuum expectation value, yielding at low energies the SM group, the fields of the MSSM and some other fields that are not observable, either because they are very heavy, or because they are not charged under the SM gauge group, i.e. they belong to the hidden sector. In this section, we will analyze some of these realistic models to illustrate the impact of the D-term contribution to the scalar masses in a model with several $U(1)$s and several fields acquiring vacuum expectation values, as happens generically in explicit string constructions.

The gauge group in realistic models is of the form $SU(3)_C \times SU(2)_L \times U(1)^{N+1} \times....$
Figure 6: Excluded regions for a scenario where the Standard Model gauge group has been extended with the non-anomalous $U(1)_H$ symmetry. The arrows indicate the limits of the regions where positivity is not satisfied. The gravitino mass has been fixed to 100 GeV and the meaning of the symbols is the same as in Figure 1.

Usually, there is a combination of those $U(1)$s which is anomalous, $U(1)_A$, with vector superfield $V_A$, and whose anomalies are cancelled by the Green-Schwarz mechanism. On the other hand, the remaining $U(1)$s are non-anomalous, and the corresponding vector superfields are $V_a$, $a = 1, \ldots, N$. Concerning the matter content, there appear many chiral superfields, $\phi_k$, that have a charge $q^A_k$ under $U(1)_A$ and $q^a_k$ under $U(1)_a$. Then, for orbifold compactifications, the tree level Kähler potential reads:

$$K = -\log(S + S^* - \delta_{GS} V_A) - 3 \log(T + T^*) + (T + T^*)^{n_{\phi_k}} e^{2(q^A_k V_A + \sum_a q^a_k V_a)} \phi_k^* \phi_k,$$

where we have omitted the non-Abelian part.

To obtain the correct phenomenology, only one of the $N$ non-anomalous $U(1)$s should remain unbroken at low energies, and should be identified with the weak hypercharge. Also, both the anomalous and the other $N - 1$ non-anomalous combinations must be broken along F- and D-flat directions, to preserve supersymmetry, when certain scalars acquire a vacuum expectation value. We will denote these fields by $\Phi_k$.

After the breaking of the extra $U(1)$ symmetries, the scalar masses receive a contribution from the D-terms given by,

$$[m^2_k]_D = -q^A_k \langle D_A \rangle - \sum_a q^a_k \langle D_a \rangle,$$

where

$$\langle D_A \rangle = -m^2_{3/2} \frac{\sum_k Q^A_k \langle \Phi_k \rangle^2 (1 - 6 \sin^2 \theta - n_{\Phi_k} \cos^2 \theta)}{\sum_k (Q^A_k)^2 \langle \Phi_k \rangle^2},$$

and

$$\langle D_a \rangle = m^2_{3/2} \frac{\sum_k Q^a_k \langle \Phi_k \rangle^2 n_{\Phi_k} \cos^2 \theta}{\sum_k (Q^a_k)^2 \langle \Phi_k \rangle^2}.$$
The scalar masses depend crucially on the charges of the fields involved, and on the vacuum expectation value of the fields that break the $U(1)$s. In the next subsections, we analyze in some detail two explicit string constructions that give definite predictions for these quantities.

### 4.1 Model I

First, we analyze the model presented in [24], obtained from the compactification of the heterotic string on the $Z_3$ orbifold with Wilson lines [25][26]. This model has several drawbacks that make it phenomenologically unacceptable, but is a good example of what one expects in a realistic string model: several $U(1)$s and several fields that acquire a vacuum expectation value to break the extra groups.

At the gravitational scale, the model corresponds to the one presented in [27], with the gauge group $[SU(3) \times SU(2) \times U(1)^5] \times [SO(10) \times U(1)^3]_{\text{hidden}}$. The spectrum of massless particles consists of three generations of 76 fields, labeled $f_k$, charged under the gauge group of the theory. Remarkably enough, it is possible to find D- and F-flat directions where the gauge group is just $[SU(3) \times SU(2) \times U(1)]_Y \times [SO(10)]_{\text{hidden}}$, i.e., the SM group times a hidden sector group. In particular, this happens for the following direction:

\[
|\langle f_{12} \rangle|^2 = 2|\langle f_{58} \rangle|^2 = 2|\langle \alpha \rangle|^2 ,
\]

\[
|\langle f_{15} \rangle|^2 = |\langle f_{18} \rangle|^2 = 3|\langle \alpha \rangle|^2 ,
\]

\[
|\langle f_{21} \rangle|^2 = 2|\langle f_{32} \rangle|^2 = 6|\langle \alpha \rangle|^2 ,
\]

\[
|\langle f_{43} \rangle|^2 = 4|\langle \alpha \rangle|^2 ,
\]

\[
|\langle f_{54} \rangle|^2 = |\langle f_{65} \rangle|^2 = |\langle \alpha \rangle|^2 ,
\]

where $|\langle \alpha \rangle|^2 = \sqrt{3}/24 \xi^2$. Furthermore, after the breaking of the extra symmetries, many of the 76 fields become heavy, while others remain massless. The spectrum of massless particles is very similar to the MSSM: $3 \times (3,2) + 2(\bar{3},1) + (1,2) + (1) + 3 \times (1,1,2) + 4(1) + (16)' + 11(1)'$, where the prime denotes a representation that is invisible at low energies. The only differences are that there are three generations of Higgses, $3 \times 2$ extra doublets and $3 \times 4$ extra singlets. These fields have not been observed, and so this model is not completely realistic. However, it has some non-trivial predictions, namely $\mathcal{N} = 1$ SUSY, the SM gauge group, three generations, and a matter content that resembles the MSSM. The similarity of this model with the MSSM is remarkable and suggests that the true vacuum might not be very different from the one predicted. In any case, we will use this “semi-realistic” string model to illustrate the impact of the D-term contribution to the scalar masses on the CCB and UFB bounds.

From the charges of the different fields and eq.(34) it is straightforward to compute the scalar masses of the MSSM fields at high energies. In this model, all the fields that acquire a vacuum expectation value belong to the twisted sector, and thus have the same modular weight, $-2$. Hence, the contribution to the scalar masses from the breaking of the non-anomalous $U(1)$s vanishes and the only contribution comes from $U(1)_A$. On the other hand, regarding the MSSM fields, $Q_L$, $u_R$ and $H_2$ belong to the
untwisted sector (modular weight $-1$), and the rest, to the twisted sector (modular weight $-2$). The charges of the relevant fields are:

$$ Q^A = \begin{pmatrix} f_{12} & f_{15} & f_{18} & f_{21} & f_{32} & f_{43} & f_{54} & f_{58} & f_{65} \\ \sqrt{3} & \sqrt{3} & \sqrt{3} & \sqrt{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} $$

(35)

$$ q^4 = \begin{pmatrix} Q & d_R & u_R & L_L & e_R & H_1 & H_2 \\ 0 & \frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{pmatrix} $$

(36)

Therefore, the scalar masses are:

$$ m^2_{Q_L} = m^2_{u_R} = m^2_{H_2} = m^2_{3/2}(1 - \cos^2 \theta) , $$

$$ m^2_{d_R} = m^2_{L_L} = m^2_{H_1} = m^2_{3/2} \left( \frac{3}{7} - \frac{19}{7} \cos 2\theta \right) , $$

$$ m^2_{e_R} = m^2_{3/2} \left( -\frac{9}{7} + \frac{17}{7} \cos 2\theta \right) . $$

(37)

From these formulas it can be checked that at high energies there is always a charged field whose mass squared is negative, thus breaking charge. The problematic field is $e_R$: it has a charge under the anomalous $U(1)$ of opposite sign to the charges of the rest of the MSSM particles. Incidentally, this field is also problematic in the model presented in [24], because there are three singlets under $SU(3) \times SU(2)$ with hypercharge 1, and any of them could play the role of the $e_R$ (of course, one should find the way to make the other two heavy to make the model realistic). It is worth noticing that one of the possible $e_R$s, the combination of $\{ f_8, f_{25}, f_{39}, f_{42}, f_{49}, f_{59}, f_{66}, f_{75}, f_{76} \}$, has charge $4/\sqrt{3}$ under $U(1)_A$, identical to the charges of $d_R, L_L, H_1$. If we identify $e_R$ with this combination of fields, there will be some ranges of $\theta$ where all the scalar masses will be positive. If we do that, the scalar masses of the MSSM fields read:

$$ m^2_{Q_L} = m^2_{u_R} = m^2_{H_2} = m^2_{3/2}(1 - \cos^2 \theta) , $$

$$ m^2_{d_R} = m^2_{L_L} = m^2_{H_1} = m^2_{e_R} = m^2_{3/2} \left( \frac{3}{7} - \frac{19}{7} \cos 2\theta \right) , $$

(38)

which are simultaneously positive for $\cos 2\theta < 3/19$. The positivity of these scalar masses at high energies is a necessary condition for the phenomenological viability of the model. However, one must also impose, among other things, a correct top mass, SUSY masses above the experimental bounds, and the absence of CCB minima and UFB directions. To perform the analysis, we also need to compute the trilinear soft terms. This has the same limitation as in the ILR scenario (see Sect. 2.3), i.e. some of the fields belong to the twisted sector and the trilinear couplings could become $T$-dependent. Nevertheless, for the UFB-3 bound, the most important one for our analysis, only the top trilinear term is relevant, and the corresponding fields, $Q_L, u_R$ and $H_2$, are all untwisted in this model. Consequently, the limitation mentioned above does not apply and the trilinear term is simply $A_t = \sqrt{3} m_{3/2} \sin \theta$.

It is interesting to note that in the dilaton limit the slepton masses are approximately three times larger than the $H_2$ mass, which helps to prevent the appearance of UFB directions. Therefore, one expects to find some regions in the parameter space where all our phenomenological requirements are satisfied (incidentally, if we had ignored the D-term contribution to the scalar masses, the whole parameter space would
be excluded). This is confirmed by the numerical analysis, shown in Figure 7. If we depart from the dilaton limit, it is still possible to find allowed regions in the parameter space. For example, for a gravitino mass of 500 GeV, there are allowed regions for $1.3 \lesssim \theta \lesssim 1.8$.

4.2 Model II

We analyze now the model presented in [28,29,30], constructed in the four-dimensional free fermionic string formulation [31]. This model has several remarkable properties. First, all the exotic fractionally charged states and all the color triplets receive a mass of the order of the string scale. Furthermore, even though initially there are four fields with the same quantum numbers as the up and down Higgs, it is possible to find a flat direction where only one Higgs doublet remains light. At the end of the day, we obtain at low energies a model with just the MSSM spectrum and the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times G_{\text{hidden}}$. It is equally remarkable that the weak hypercharge has the standard $SO(10)$ normalization.

Let us now briefly describe the model. At high energies, the gauge group is $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \times U(1)_Y \times G_{\text{hidden}}^6$ in the observable sector, and $SO(4) \times SU(3) \times U(1)_Y^4$ in the hidden sector. There are six anomalous $U(1)_s$, although it is possible to combine them to yield five non anomalous and one anomalous $U(1)$. The corresponding Fayet-Iliopoulos term must be compensated by the VEVs of some fields to preserve supersymmetry. If those fields are also charged under other $U(1)_s$, these other groups will also be broken. The VEVs of all the fields must satisfy the condition that all the D-terms and the F-terms are cancelled simultaneously, to preserve supersymmetry, and that all the $U(1)_s$ in the observable sector are broken except one combination, which should be identified with the weak hypercharge. Obviously, it is highly non-trivial to
fulfill all these conditions. However, this model presents several flat directions where this actually happens. For example, the direction defined by

\[
\left( |\langle \phi_4 \rangle|^2 + |\langle \phi'_4 \rangle|^2 \right) = 0 ,
\]

\[
\frac{1}{2} |\langle \phi_{12} \rangle|^2 = |\langle \phi_{23} \rangle|^2 = |\langle \bar{\phi}_{45} \rangle|^2 = |\langle \bar{\phi}'_{12} \rangle|^2 ,
\]

\[
\frac{1}{2} |(H_{15})|^2 = \frac{1}{2} |(H_{30})|^2 = |(H_{31})|^2 = \frac{1}{2} |(H_{38})|^2 = |\langle \alpha \rangle|^2 ,
\]

\[
(|\langle \phi_4 \rangle|^2 + |\langle \phi'_4 \rangle|^2) - (|\langle \bar{\phi}_{45} \rangle|^2 + |\langle \bar{\phi}'_{12} \rangle|^2) = |\langle \alpha \rangle|^2 ,
\]

where \(|\langle \alpha \rangle| \approx 7 \times 10^{16} \text{ GeV}, only leaves the combination \(\frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L \equiv U(1)_Y \) unbroken [29].

Concerning the matter content, at high energies the massless spectrum includes fields both from the Neveu-Schwarz (NS) sector and the \(b_1, b_2, b_3\) sectors. The NS sector gives rise to the spin-2 states, the gauge bosons in the observable and hidden sectors, and scalar representations belonging to the observable sector (denoted by \(h\) and \(\phi\)). These fields are untwisted and hence have modular weight \(-1\). On the other hand, the \(b_1, b_2, b_3\) sectors give rise to the (twisted) chiral fields of the observable sector (denoted by \(V\), \(H\) and the three families of quarks and leptons). In the free fermionic models there is an underlying \(Z_2 \times Z_2\) orbifold, so, the modular weights of the twisted fields are also \(-1\) [4].

The scalar masses for this model can be readily computed from eqs. (31, 32, 33). All the fields that acquire a VEV have modular weight \(-1\), then, only the breaking of the anomalous \(U(1)\) has an impact on the scalar masses — the non-anomalous contribution vanishes. The charges of those fields under the anomalous \(U(1)_A\) symmetry are:

\[
\begin{array}{c|cccccccccccc}
Q^A & \phi_{12} & \phi_{23} & \bar{\phi}_{56} & H_{15} & H_{30} & H_{31} & H_{38} & \phi_4 & \phi'_4 & \bar{\phi}_4 & \bar{\phi}'_4 \\
-9 & -2 & -2 & -2 & -6 & 3 & -4 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Notice that the conditions eqs. (39) do not fix the VEVs of the \(\{\phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4\}\) fields to any particular value. However, this is not a problem for the calculation of the scalar masses, because those fields are neutral under \(U(1)_A\). Then, a straightforward calculation yields

\[
m_k^2 = m_{3/2}^2 \left( 1 + \frac{145}{204} q_k \right) \sin^2 \theta ,
\]

where \(k\) runs over all the MSSM fields. The scalar soft masses depend both on the goldstino angle and the charges under \(U(1)_A\). These are:

\[
\begin{array}{c|cccccccc}
\alpha & Q_{L_\alpha} & d_{R_\alpha} & u_{R_\alpha} & L_{L_\alpha} & e_{R_\alpha} & h_\alpha & \bar{h}_\alpha \\
1 & 2 & 2 & 2 & 2 & 2 & 4 & -4 \\
2 & 3 & 2 & 3 & 2 & 3 & -5 & 5 \\
3 & 2 & 1 & 2 & 1 & 2 & -3 & 3 \\
\end{array}
\]

where \(\alpha\) indicates the sector \(b_\alpha\) which gives rise to the field (the index \(\alpha\) is not related a priori with the generation of the field). There are two more MSSM-like fields, \(H_{34}\) and \(H_{41}\), that have the same charges under the SM gauge group as the up and down Higgs doublets, and charges 2 and 0 under \(U(1)_A\), respectively. In principle, the presence of
four light Higgs generations would be a problem for the phenomenological viability of the model (the gauge couplings would not unify, for example). However the 4 × 4 mass matrix yields only one massless pair, \((-h_1 + \sqrt{3}h_3)/2 \equiv H_1\) and \(\bar{h}_1 \equiv H_2\), and three with a mass of the order of the string scale.

The renormalizable part of the MSSM superpotential reads:

\[
W_{\text{MSSM}} = g_s \sqrt{2} H_2 Q_1 u_{R_1} + g_s \sqrt{2} \cos \sqrt{3} H_1 [Q_3 d_{R_3} + L_3 e_{R_3}],
\]

(43)

where we have omitted a term involving right handed neutrinos, which is irrelevant for our discussion. The quark and lepton fields in eq. (43) should be identified with the third generation particles (for example, \(u_{R_1} \equiv t_R\)), whereas the two first generations only appear in the superpotential through non-renormalizable terms of order four or six.

It is apparent from eq. (41) that if some field has a \(U(1)_A\) charge \(\lesssim -3/2\), the corresponding scalar mass squared will be negative at high energies. It is interesting that squarks and sleptons have positive charge under \(U(1)_A\) and therefore their masses squared are positive. However, this is not true for the Higgs doublet \(H_2\), whose mass squared is negative at high energies. This result is a consequence of the gauge invariance of the superpotential under \(U(1)_A\). Gauge invariance requires that, in every term of the superpotential, at least one of the fields has a negative charge. If there is a term involving only MSSM fields, some of the MSSM fields will have negative charge. As a matter of fact, this is what happens in this model, where a Yukawa coupling involving \(H_2\) and other MSSM fields appears at the renormalizable level. Then, some of the MSSM fields must have negative charge and the corresponding scalar masses squared are negative at high energies. Things are different, though, for the other Higgs doublet, \(H_1\). \(H_1\) does not have a definite \(U(1)_A\) charge, but is a mixed state of fields with different charges: \(h_1\), with charge 4, and \(h_3\), with charge \(-3\). The component that couples to the MSSM fields in the renormalizable superpotential is \(h_3\), whose mass squared is negative. However, we are interested in \(m^2_{H_1}\), that is a combination of the masses squared of \(h_1\) and \(h_3\). The former is positive and the latter is negative, but \(m^2_{H_1}\) turns out to be positive. This shows that it is possible to have a renormalizable superpotential involving only MSSM fields, and at the same time all the scalar masses squared positive at high energies, provided the Higgs doublets \(H_1\) and \(H_2\) are mixed states with a certain component of a field with positive \(U(1)_A\) charge \(5\).

As was mentioned in Section 2, negative Higgs masses squared represent a potential problem for a correct electroweak symmetry breaking, but they do not prevent it: the contribution to the Higgs masses from the \(\mu\) term can lift the UFB direction in the Higgs potential that would otherwise appear. Therefore, we expect that in some regions of the parameter space the electroweak symmetry breaking occurs in the usual way. In Figure 8 we show the forbidden regions in the parameter space of this model, for a gravitino mass of 100 GeV. From the figure it can be seen that it is possible to break the electroweak symmetry generating a top mass of 174 GeV, but the global minimum

\[5\] A second way of getting positive masses squared is in a model where the MSSM Lagrangian comes from a non-renormalizable superpotential, as the models in Sections 2 and 3. Then, gauge invariance allows a term in the superpotential in which all the MSSM fields have a positive charge, and the negative charge is carried by a field or fields that will eventually acquire a VEV.
always breaks charge. The reason is that the UFB-3 direction is particularly dangerous in this model. First, because $m^2_{H_2}$ is large and negative, and second, because $m^2_L$ is small and cannot compensate the negative contribution from $m^2_{H_2}$ in the first term of eqs. (3, 4).

5 Conclusions

We have performed an analysis of the parameter space of different weakly coupled string scenarios, imposing the requirements of an acceptable electroweak symmetry breaking, a top mass within the experimental range, supersymmetric masses above the experimental bounds, and the absence of dangerous charge and color breaking (CCB) minima or unbounded from below (UFB) directions. We have taken into account the fact that in superstring models there are usually extra $U(1)$s broken at a high energy scale, that contribute to the scalar masses through the vacuum expectation value of the $D$ field associated to the $U(1)$ vector superfield. Then, the spectrum of supersymmetric particles can be very different to the one obtained in a superstring scenario with just the Standard Model gauge group, where the only contribution to the scalar soft masses comes from the vacuum expectation value of the dilaton and/or moduli auxiliary fields.

We have shown that the presence of family-independent extra $U(1)$s can help to lift the unbounded from below directions that appear in the scalar potential when the gauge group is just the Standard Model gauge group. To be precise, this happens when the charges of the fields under the $U(1)$s are such that the slepton masses are larger than the rest. In particular, we have studied a string motivated scenario where the gauge group of the Standard Model is extended with an extra anomalous (non-anomalous) $U(1)$ and the matter content of the MSSM is extended with one (two) chiral superfields to break the $U(1)$. In the anomalous case, we have found that the
non-universality in the scalar masses, generated by the breaking of the anomalous $U(1)$, can be large enough to open some allowed windows in the parameter space. We have also studied some realistic string scenarios, with several extra $U(1)$s, anomalous and non-anomalous, and several fields that acquire a vacuum expectation value. Again, we find that the D-term contribution to the scalar masses is crucial to lift the UFB directions that otherwise appear.

We have analyzed in detail the case where the dilaton is the field that dominates the supersymmetry breaking. Since the dilaton couples universally to all the particles, the contribution to the scalar masses from the dilaton $F$-term is universal, thus alleviating the supersymmetric flavour and CP problems. If the gauge group at high energies is just the Standard Model, this scenario is essentially ruled out because the global minimum of the effective potential breaks charge. However, in a realistic string theory there are usually extra $U(1)$s broken at high energies that modify the spectrum of scalar masses. We have seen that the non-anomalous $U(1)$s do not contribute to the scalar masses in the dilaton limit, but the anomalous $U(1)$ does. Depending on the charges of the particles under the anomalous $U(1)$, it is possible to find regions of the parameter space where our physical vacuum is the global minimum of the effective potential, namely, when the charges are such that the slepton masses are large. At the same time, if the extra gauge group is family blind, this scenario keeps all the desirable properties mentioned above, regarding the flavour and CP problems in supersymmetric theories. It is indeed remarkable that there are models with allowed windows in this interesting limit, which has important implications for flavour and CP violation physics.

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References

[1] J. M. Frere, D. R. Jones and S. Raby, Nucl. Phys. B 222 (1983) 11. L. Alvarez-Gaume, J. Polchinski and M. B. Wise, Nucl. Phys. B 221 (1983) 495. J. P. Derendinger and C. A. Savoy, Nucl. Phys. B 237 (1984) 307. C. Kounnas, A. B. Lahanas, D. V. Nanopoulos and M. Quiros, Nucl. Phys. B 236 (1984) 438. M. Drees, M. Gluck and K. Grassie, Phys. Lett. B 157 (1985) 164. J. F. Gunion, H. E. Haber and M. Sher, Nucl. Phys. B 306 (1988) 1. H. Komatsu, Phys. Lett. B 215 (1988) 323. P. Langacker and N. Polonsky, Phys. Rev. D 49 (1994) 1454; J. A. Casas, A. Lleyda and C. Muñoz, Phys. Lett. B 389 (1996) 305; J. A. Casas and S. Dimopoulos, Phys. Lett. B 387 (1996) 107.

[2] S. A. Abel and C. A. Savoy, Phys. Lett. B 444 (1998) 119; S. A. Abel and B. C. Allanach, Phys. Lett. B 431 (1998) 339; JHEP 0007 (2000) 037; S. Abel
and T. Falk, Phys. Lett. B 444 (1998) 427; C. Le Mouel, Nucl. Phys. B 607 (2001) 38; U. Ellwanger and C. Hugonie, Phys. Lett. B 457 (1999) 299; A. Datta and A. Samanta, hep-ph/0108056; E. Gabrielli, K. Huitu and S. Roy, hep-ph/0108246.

[3] J. A. Casas, A. Lleyda and C. Muñoz, Nucl. Phys. B 471 (1996) 3.

[4] J. A. Casas, A. Ibarra and C. Muñoz, Nucl. Phys. B 554 (1999) 67.

[5] J. A. Casas, A. Lleyda and C. Muñoz, Phys. Lett. B 380 (1996) 59.

[6] M. Claudson, L. J. Hall and I. Hinchliffe, Nucl. Phys. B 228 (1983) 501; T. Falk, K. A. Olive, L. Roszkowski and M. Srednicki, Phys. Lett. B 367 (1996) 183; A. Riotto and E. Roulet, Phys. Lett. B 377 (1996) 60; A. Kusenko, P. Langacker and G. Segre, Phys. Rev. D 54 (1996) 5824; A. Strumia, Nucl. Phys. B 482 (1996) 24; A. Kusenko and P. Langacker, Phys. Lett. B 391 (1997) 29; T. Falk, K. A. Olive, L. Roszkowski, A. Singh and M. Srednicki, Phys. Lett. B 396 (1997) 50; S. A. Abel and C. A. Savoy, Nucl. Phys. B 532 (1998) 3; I. Dasgupta, R. Rademacher and P. Suranyi, Phys. Lett. B 447 (1999) 284;

[7] L. E. Ibáñez and D. Lust, Nucl. Phys. B 382 (1992) 305;

[8] V. S. Kaplunovsky and J. Louis, Phys. Lett. B 306 (1993) 269;

[9] A. Brignole, L. E. Ibáñez and C. Muñoz, Nucl. Phys. B 422 (1994) 125 [Erratum-ibid. B 436 (1994) 747]; A. Brignole, L. E. Ibáñez, C. Muñoz and C. Scheich, Z. Phys. C 74 (1997) 157;

[10] E. Witten, Nucl. Phys. B 471 (1996) 135;

[11] J. D. Lykken, Phys. Rev. D 54 (1996) 3693;

[12] S. A. Abel, B. C. Allanach, F. Quevedo, L. Ibáñez and M. Klein, JHEP 0012 (2000) 026;

[13] M. Drees, Phys. Lett. B 181 (1986) 279. J. S. Hagelin and S. Kelley, Nucl. Phys. B 342 (1990) 95. Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Rev. D 51 (1995) 1337;

[14] M. B. Green and J. H. Schwarz, Phys. Lett. B 149 (1984) 117.

[15] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289 (1987) 589; J. J. Atick, L. J. Dixon and A. Sen, Nucl. Phys. B 292 (1987) 109;

[16] L. E. Ibanez, Phys. Lett. B 303 (1993) 55 hep-ph/9205234.

[17] L. E. Ibanez and G. G. Ross, Phys. Lett. B 332 (1994) 100 hep-ph/9403338.
[18] E. Witten, Phys. Lett. B 155 (1985) 151; S. Ferrara, C. Kounnas and M. Porrati, Phys. Lett. B 181 (1986) 263; M. Cvetic, J. Louis and B. A. Ovrut, Phys. Lett. B 206 (1988) 227; L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B 329 (1990) 27.

[19] P. Fayet and J. Iliopoulos, Phys. Lett. B 51 (1974) 461; P. Fayet, Nucl. Phys. B 90 (1975) 104.

[20] Y. Kawamura and T. Kobayashi, Phys. Lett. B 375 (1996) 141 [Erratum-ibid. B 388 (1996) 867] [hep-ph/9601365]; Y. Kawamura, Phys. Lett. B 446 (1999) 228 [hep-ph/9811312].

[21] Y. Kawamura and T. Kobayashi, Phys. Rev. D 56 (1997) 3844 [hep-ph/9608233].

[22] L. E. Ibanez, D. Lust and G. G. Ross, Phys. Lett. B 272 (1991) 251.

[23] S. Hamidi and C. Vafa, Nucl. Phys. B 279 (1987) 465; L. J. Dixon, D. Friedan, E. J. Martinec and S. H. Shenker, Nucl. Phys. B 282 (1987) 13.

[24] J. A. Casas and C. Munoz, Phys. Lett. B 214 (1988) 63. J. A. Casas, E. K. Katehou and C. Munoz, Nucl. Phys. B 317 (1989) 171;

[25] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B 261 (1985) 678; L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B 274 (1986) 285.

[26] L. E. Ibanez, H. P. Nilles and F. Quevedo, Phys. Lett. B 187 (1987) 25.

[27] L. E. Ibanez, J. E. Kim, H. P. Nilles and F. Quevedo, Phys. Lett. B 191 (1987) 282.

[28] A. E. Faraggi, D. V. Nanopoulos and K. j. Yuan, Nucl. Phys. B 335 (1990) 347.

[29] G. B. Cleaver, A. E. Faraggi and D. V. Nanopoulos, Phys. Lett. B 455 (1999) 135.

[30] G. B. Cleaver, A. E. Faraggi and D. V. Nanopoulos, Int. J. Mod. Phys. A 16 (2001) 425. G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos and J. W. Walker, Nucl. Phys. B 593 (2001) 471.

[31] I. Antoniadis, C. P. Bachas and C. Kounnas, Nucl. Phys. B 289 (1987) 87. H. Kawai, D. C. Lewellen and S. H. Tye, Nucl. Phys. B 288 (1987) 1.