Binary parameters from astrometric and spectroscopic errors, and candidate massive dark companions in \textit{Gaia eDR3}

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ABSTRACT

We show how astrometric and spectroscopic errors introduced by an unresolved binary system can be combined to give estimates of the binary period and mass ratio. This can be performed analytically if we assume we see one or more full orbits over our observational baseline, or numerically for any periods. We show how these errors behave for a wide range of systems and what we can infer from them. We apply this method to Gaia eDR3 data, combining the radial velocities from the second data release with the most recent astrometric data. We compare predicted periods and mass ratios to known binaries in APOGEE, finding good agreement between inferred values calculated with our method and measured values. Finally, we use this method to search the whole Gaia RVS dataset for compact object candidates. We select sources with significant astrometric and spectroscopic errors (RUWE$_{ast}$ > 1.25 and RUWE$_{spec}$ > 3), with large inferred mass ratios, and large inferred companion masses ($q > 1$ and $M_c > 3 M_\odot$) giving a catalogue of 2,601 candidate Main Sequence+Compact Object pairs. We apply more stringent cuts, and impose low levels of photometric variability to remove triples (RUWE$_{phot}$ < 2) producing a gold sample of 182 candidates.

Key words: astrometry, binaries: general, binaries: spectroscopic, binaries: close, stars: black holes

1 INTRODUCTION

The Gaia survey (Gaia Collaboration et al. 2016, 2021) has and will continue to provide astrometric (Lindegren et al. 2021), spectroscopic (Seabroke et al. 2021) and photometric (Riello et al. 2021) measurements for a population of stars orders of magnitude larger than any that existed before. The starting assumption for each source is that it is a single star, or at least behaves as such. However we expect around half of all stars to be in binaries, especially for more massive brighter systems (Offner et al. 2022). Long period (∼10 years) binaries will barely change over the time baseline of the current Gaia data releases (22 months for DR2 and 34 for eDR3), though some sufficiently wide binaries, especially in the vicinity of the sun, may be resolvable as two separate but bound point sources (see for example the catalog of El-Badry et al. 2021).

At shorter periods, however, the binary can cause extra motion, both in the plane of the sky (detectable via astrometry) and along the line of sight (encoded in spectroscopic measurements). Very short period systems may be close enough to tidally distort the sources of light (see Morris 1985) and cause a measurable excess of photometric error, although other forms of variability are expected to be common in the dataset as well (e.g. Gaia Collaboration et al. 2019).

In Penoyre et al. (2020), Belokurov et al. (2020), Penoyre et al. (2022a) and Penoyre et al. (2022b) we have explored in detail the astrometric contribution of binaries. In this paper we extend a similar line of reasoning, analytically, numerically and observationally, to the spectroscopic and photometric contributions. In particular, we show that for systems which have both significant astrometric and spectroscopic excess—if we assume that these are both caused by the same binary—we can infer the period, masses and semi-major axis of the system.

One of the most exciting uses for this is the selection of candidate Main Sequence (MS) Compact Object (CO) binaries, where the companion is dark (typically, a neutron star or a black hole) but very massive. As shown by others Breivik et al. (2017); Mashian & Loeb (2017); Andrews et al. (2019); Chawla et al. (2021), Gaia is the best available instrument for detecting these systems within our Galaxy.

Binary-induced astrometric perturbations depend linearly on the semi-major axis of the orbit, and hence become vanishingly small for low period systems (months or less).
Thus, the detectable MS+CO systems are not the very close binaries which may become interesting gravitational wave sources on human timescales (see Peters & Mathews 1963; Hurley et al. 2002), but do inform us about potential progenitors to these systems, as well as encoding information about the stellar evolution of the binary (see e.g. Belczynski et al. 2002).

Any nearby MS+CO systems (with periods ranging from months to years) will likely cause a large and easily detectable excess astrometric and spectroscopic error. However MS+MS binaries can also cause similar excess noise. Given that these are expected to be many thousand times more common (Breivik et al. 2017; Mashian & Loeb 2017) the difficulty becomes choosing selection criteria which have a high level of specificity.

In this work, we will show that combining astrometric and spectroscopic measurements can give this high level of specificity. Furthermore, this method can tell us about some of the most physically relevant properties of the system. In Section 2 we outline the analytical framework for translating between binary properties and their associated error, including the dependence on (generally unknown) viewing angles and eccentricity. In Section 3, we apply this to simulated systems, inferring mass ratios and periods based on synthetic Gaia-like observations. We do this in part by constructing statistics for spectroscopy and photometry analogous to the astrometric (renormalised) unit weight error\(^1\), both for simulated and observational data in Appendix B. In Section 4 we apply our method to Gaia data and compare our inferences to known binaries from the APOGEE catalog (Price-Whelan et al. 2020), for which the period and mass ratio have already been measured. Finally in Section 5 we apply our analysis to the full available Gaia RVS sample and specifically select sources with high inferred mass ratio and companion mass, producing an initial catalog of 2,601 candidate MS+CO systems. We then apply more stringent cuts on this sample, motivated by estimates of the error on our inferred parameters and removing samples with a large degree of photometric error to give a gold sample of 182 candidate MS+CO systems.

2 METHOD

To give an initial context to this discussion we start by showing Figure 1, which gives the excess noise in astrometry, radial velocity and photometry, as parameterised by RUWE\(_{\text{ast}}\) (as given in Gaia eDR3) as well as our constructed RUWE\(_{\text{spec}}\) and RUWE\(_{\text{phot}}\) (as detailed in Appendix B) across the Hertzsprung Russell diagram. Each of these is a measure of an excess of noise compared to a fit performed assuming they are a single non-variable star.

We will leave a detailed discussion of each of these statistics for later in the paper, but we can already see that in the HRD many of the same regions contain high astrometric, spectroscopic, and photometric excess noise. Most clearly, sitting above the spine of the Main Sequence (MS), we see the MS multiples - binaries and higher multiples which are

\(^1\) We will use the name RUWE\(_{\text{ast}}\) for the astrometric (renormalised) unit weight error and introduce RUWE\(_{\text{spec}}\) and RUWE\(_{\text{phot}}\) as well.
Let us imagine we observe a system over a time baseline $B$. Time averaged deviations out. Consider binaries in this paper, though we will note when higher markedly changing their colour). We will only directly contemplate multiples. These systems, containing more stars, are generally binary orbits needed to produce significant tidal distortions. Figure 1 compared to astrometric measurements – the tight significant photometric variability further above the MS in larger/more ubiquitous signals than binaries. Systems of higher multiples may have even larger/more ubiquitous signals than binaries. Systems of three or more bodies are quasi-stable, and any long-lived multiple is likely hierarchically arranged (Tokovinin 2021). This means that for systems of increasing multiplicity it is increasingly likely that one component of the multiple has an orbit within any period window. This may be why we see significant photometric variability further above the MS in Figure 1 compared to astrometric measurements – the tight binary orbits needed to produce significant tidal distortions may only become ubiquitous in triples or even higher multiples. These systems, containing more stars, are generally therefore brighter and sit further above the MS (without markedly changing their colour). We will only directly consider binaries in this paper, though we will note when higher multiples may be particularly relevant and they should be considered a major (and interesting) contaminant throughout.

2.1 Time averaged deviations

Let us imagine we observe a system over a time baseline $B$.2 If the system is a binary, with period $P$, we observe $N_{\text{orb}} = \frac{P}{B}$ orbits. If $N_{\text{orb}} \geq 1$, we can well approximate the behaviour of that system by integrating over one complete orbit, under the assumption of many similarly spaced observations. We will also address the $N_{\text{orb}} < 1$ case more approximately below, as well as discussing the effects of sparser measurements. This will give a relatively simple analytic form for the expected deviations introduced by a binary – including both the dimensional scaling with the physical parameters of the system, and a geometric factor (often of order unity) relating to the specific angle and phase at which the system is viewed.

The relevant parameters of the binary, which may affect any observed quantity, are the period $P$, semi-major axis $a$, and eccentricity, $e$. If we designate the brighter star as the primary (as it is the one we observe) with luminosity ratio $l = a^3$ and mass ratio $q = M_2 = \frac{M_2}{M_1}$ where $M_1$ is the mass of the companion and $M$ the mass of the primary. Through Kepler's third law any two of the three parameters $P$, $a$, and $M(1 + q)$ (the total mass) are sufficient to set the value of the third via

$$\left( \frac{P}{2\pi} \right)^2 = \frac{a^3}{GM(1 + q)} \quad (2)$$

Similarly, $M_2$ is specified entirely by $M$ and $q$ and need not enter in to our calculations (though may be interesting to examine as an end-product).

Measured quantities also depend on observational parameters that do not alter the fundamental physics of the system: these are the viewing angles $(\theta, \phi, \alpha_0)$, parallax, $\pi$, time of periapse $t_p$, and the relevant observational errors, $\sigma_{\text{spec}}$ and $\sigma_{\text{ast}}$ for spectroscopic and astrometric measurements respectively. One of the viewing angles, $\alpha_0$, specifies the orientation of the binary relative to the measurement co-ordinate system, and thus will not change any observable results unless the measurements are anisotropic (another assumption that can be violated in practice by Gaia’s irregular scanning law, but which we will assume is satisfied for now).

Thus, there are 5 independent physical parameters of the system $(P, M, a, q, t_p, \pi)$. 4 viewer specific parameters $(\theta, \phi, t_p, \alpha_0)$ and 2 errors $(\sigma_{\text{ast}}, \sigma_{\text{spec}})$ which define the excess error introduced by a binary system and its significance over our measurement sensitivity.

2.2 Error introduced by a binary

We will focus on two error terms, $\sigma_{\theta, \phi}$ and $\sigma_{t_p, \alpha_0}$ corresponding to the error introduced by a binary in spectroscopic and astrometric measurements.

If we imagine a known binary, and assume it is frequently and isotropically scanned, we can derive analytic forms for these assuming $P = B$. As we will show, this result agrees well for all $P < B$ but deviates at higher periods.

2.2.1 Spectroscopic error

The standard deviation of measurements of the radial velocity of a binary system, which we derive in Appendix A, follows the form

$$\sigma_{\theta, \phi} = \frac{q}{1 + q}\frac{2a}{P} \cdot \zeta(P, e, t_p, \theta, \phi) \quad (3)$$

where $P$ denotes the binary period, $a$ denotes the semi-major axis, and $\zeta$ is a function that depends on the viewing angle, eccentricity, orbital period, and observational baselines. In the case that $P = B$, the analytic expression for $\zeta$, which we will call $\zeta_0$, is

$$\zeta_0(e, \theta, \phi) = \frac{1}{e} \sqrt{\kappa_{c}^2 e(1 - e)} - \kappa_{c}^2(\epsilon^2 + e^2(3e - 5) + 4(1 - e))} \quad (4)$$

where $\kappa_{c} = \sin \theta \sin \phi$, $\kappa_{c} = \cos \phi$, and $\epsilon = \sqrt{1 - e^2}$.

Following the naming convention given in Penoyre et al. (2020), $\theta_0$ is defined to be the inclination angle between the orbital plane and the line-of-sight vector, and $\phi_0$ is defined to be the angle between the axis of periapse and the line-of-sight vector projected onto the orbital plane. The top row of Figure 2 shows $\zeta_0$ as a function of viewing angle and eccentricity. It is always small for face-on systems ($\sin \theta_0$ close to zero) and also for eccentric systems where the observed

\[ \text{For example, for the Gaia survey’s second and third data release } B_{\text{DR2}} = \frac{1}{2} \text{ years and } B_{\text{DR3}} = \frac{1}{4} \text{ years respectively.} \]
The behaviour of the simple forms of the astrometric ($\beta_0$) and spectroscopic ($\zeta_0$) projection terms on viewing for different eccentricities. We also show the factor $\zeta_0/\beta_0$ which appears in the formula for period $P$ (equation 8), and $\zeta_0^2/\beta_0$ which directly affects the mass ratio $q$ (equations 10 and 11). $\zeta_0$ tends towards zero when the binary is viewed face-on ($\sin \theta_v = 0$) and is maximised at values of $\phi_v$ when most of the binary motion is radial. We see the inverse behaviour with $\beta_0$, which is maximised when most of the binary motion is planar and minimised when the binary motion is primarily along the line of sight. $\zeta_0/\beta_0$ is in general of order unity, while $\zeta_0^2/\beta_0$ can take on very small ($\sim 10^{-2}$) values.

2.2.2 Astrometric error

As shown in Penoyre et al. (2020) and Penoyre et al. (2022a), the expected astrometric error introduced by a binary is

$$\sigma_{b,\theta} = |\Delta_q| \cdot \sigma \cdot \frac{a}{A} \cdot \beta(P, e, t_p, \theta_v, \phi_v)$$

(5)

where $A$ is one astronomical unit, $\beta$ is a factor of order unity for $P \lesssim B$. $\beta$ encodes all dependence on the viewing angle, and

$$\Delta_q = \frac{q - l}{(1 + q)(1 + l)}$$

(6)

is the relative offset between the photocenter and the center of mass.

In the case that $P = R$, the analytic expression for $\beta$, which we will call $\beta_0$, was shown to be

$$\beta_0(e, \theta_v, \phi_v) = \sqrt{1 - \frac{\sin^2 \theta_v}{2} - e^2 \frac{3 + \sin^2 \theta_v \cos^2 \phi_v}{4} - 2}$$

(7)

The second row of Figure 2 shows $\beta_0$ as a function of motion is along the line of sight at apoapsis and periapse ($\cos \phi_v$ is small).
The mass ratio can also be expressed, in the form of a cubic, as
\[ q^3 - \alpha q^2 - 2\alpha q - \alpha = 0 \]  

where
\[ \alpha = \frac{A_0^2}{GM} \frac{\sigma_{b,0}}{\sigma} \frac{1}{\beta} \frac{\zeta}{\bar{\zeta}}. \]  

This has one real root (given that \( \alpha > 0 \) always) which can be found through translating to a depressed cubic and using Cardano’s formula to give
\[ q = \frac{\alpha}{3} + \left( \frac{\lambda}{2} \right)^{1/3} \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4\mu^3}{27\lambda^2}} \right) + \left( 1 - \sqrt{1 + \frac{4\mu^3}{27\lambda^2}} \right) \right] \]  

where
\[ \mu(\alpha) = -6 + \frac{\alpha}{3} \]  

and
\[ \lambda(\alpha) = \frac{27 + 18\alpha + 2\alpha^2}{27} \alpha. \]  

For extreme values of alpha, \( q \) is asymptotically equivalent to:
\[ q \sim \alpha^{1/3} \quad (\text{as } \alpha \to 0) \]  

\[ q \sim \alpha \quad (\text{as } \alpha \to \infty). \]  

In general calculating \( \alpha \), and thus \( q \), requires a known value of the mass of the primary \( M \). However, it’s interesting to note that as \( \alpha \propto M^{-1} \) for large values of the mass of the secondary \( M_2 = qM \) is independent of primary mass.

The third and fourth rows of Figure 2 shows \( \zeta_0/\bar{b}_0 \) and \( \zeta_0^2/\bar{b}_0^2 \) respectively, i.e. the expected dependence on the viewing angle factors that appear in the period and mass ratio estimations in the short period limit. The former can be significantly larger than the latter for eccentric orbits when \( \bar{b}_0 \) can take very small values. Both go to zero for face-on orbits (when there is effectively no radial motion and \( \sigma_{b,\alpha} \to 0 \)) and \( \zeta_0^2/\bar{b}_0^2 \) has a complex ‘donut’-like structure at high eccentricities.

In the more general case that \( l \neq 0 \), instead we have
\[ p = \frac{2\pi A_0}{\sigma_{b,0}} \frac{\sigma_{b,0} \zeta (1 + l)}{\bar{\beta} |q - l|} \]  

and for the mass ratio
\[ q^2|q - l| - \alpha(1 + q^2)(l + 1) = 0 \]  

In Figure 3, we use these more general equations to find the regions in \( l - q \) space in which Equations 8 and 9 underestimate and overestimate the true periods and mass ratios. For MS-MS binaries (\( l \approx q^{1.5} \)), we expect periods on average to be underestimated and mass ratios to be overestimated. In contrast, for binaries with large light ratios relative to their mass ratios (i.e. \( l >> q \)), we expect periods to be underestimated and mass ratios to be overestimated.

When we apply our method to real data in section 5 we will work under the assumption of \( l = 0 \). We flag candidate MS-CO binaries based on the criteria that inferred mass ratios and secondary masses are large: for MS-MS binaries (which we expect to be the most ubiquitous type of binary in

**Figure 3.** Top: Inferred period assuming \( l = 0 \) (Equation 8) over the true period (Equation 16) in \( l - q \) space. A black dotted-line is drawn at \( l = q \) and a gray dotted-line is drawn at \( l = q^{1.5} \) for MS-MS binaries to guide the eye. Bottom: same as above, except color-coded by inferred mass ratio assuming \( l = 0 \) (Equation 9) over true mass ratio.

viewing angle and eccentricity. As discussed in Penoyre et al. (2022a), the value is significantly greater than 0 for all but very eccentric systems aligned such that the motion is primarily along-the line of sight (\( \sin\theta_1 \) close to 1, and \( \cos\phi_0 \) close to zero).

### 2.3 Physical parameters from measured errors

We can invert Equations 2, 3, and 5 to recover the period of a binary system. This is particularly easy to do in cases where the companion is dark (\( l = 0 \)) at which point \( |\Delta q/l| = q/\sqrt{q} \) and the period is
\[ P = \frac{2\pi A_0 \sigma_{b,0} \zeta q(1 + l)}{\sigma_{b,\alpha} \bar{\beta} |q - l|}. \]
3 INFERRING MASS RATIOS AND PERIODS WITH SIMULATED SYSTEMS

The inferred mass ratios and periods using Equations 11 and 8 versus the true values are shown in Figures 6 and 7. We see that the inferred values are well-correlated with true values when the binary contribution to RUWE is significant ($RUWE_{\text{spec}} > 3, RUWE_{\text{Eph}} > 1.25$). At short periods (when $RUWE_{\text{Eph}}$ is insignificant), mass ratios are overestimated while at long periods (when $RUWE_{\text{spec}}$ is small) mass ratios are underestimated, confirming the predictions based on Figure 4.

To test this method, we have simulated two million main-sequence + compact-object (MS+CO) systems within 2 kpc using astromet.py 3, a software developed by Penoyre et al. (2022a) which emulates Gaia’s single-body astrometric fitting pipeline. We also use the PYTHON package SCANNINGLAW 4 (Everall et al. 2021, Boubert et al. 2021, Green 2018)) that predicts Gaia observation times and scanning angles as a function of locations on the sky.

The parameters for our simulated sources are drawn from the distributions given in Table 1. All stars are uniformly distributed across the sky and out to a distance of 2 kpc. A normal distribution is used to approximate the observed distribution of proper motions for the bulk of stars in Gaia data. All possible viewing angles and orbital eccentricities are assumed to be equally probable. We hold the mass of the primary fixed at $1M_\odot$ and vary the mass of the secondary by sampling from a log-uniform mass ratio distribution. The log-uniform period distribution was chosen to capture the behaviour of very short ($P \sim 1$ day) and long ($P > 10$ yr) period binaries. While many of our distributions are simple and approximate, the purpose of this section is just to illustrate the effectiveness of inferring mass ratios and periods from measured astrometric parameters alone. A flat along-scan astrometric error $\sigma_{\text{spec}}$ of 0.3 mas was adopted based on the expected uncertainties for sources with apparent magnitudes $m_0 < 16$ shown in Figure A.1 of Lindgren et al. (2021), and radial velocity noise floor of 1.0 km/s was chosen based off the distribution of $\sigma_{\text{rv}}$ for Gaia data shown in Figure B1.

The top panel of Figure 4 shows $\zeta$ and $\beta$ calculated inverting Equations 3 and 5 as a function of period for our simulated systems. All measurements were taken over the baseline of DR3, with the exception of radial velocities which were measured over the baseline of DR2. We see that $\beta$ and $\zeta$ are of order unity between ~5 days and 22 months, and drop significantly for sources with periods longer than the observational baseline. When we eventually apply this to observational data, where the viewing angles are in general unknown, we will adopt a constant value (with some error) for both $\beta$ and $\zeta$, which is a major yet mostly unavoidable source of error.

The second row of Figure 4 also shows that $\zeta$ and $\beta$ normalised by $\zeta_0$ and $\beta_0$ (calculated using Equations 4 and 7). The spread is significantly reduced, and $\zeta/\zeta_0$ is approximately 1 for sources with periods less than 22 months. $\beta/\beta_0$ is noticeably less than 1, which follows from the fact that the simple form of astrometric deviations calculated in Penoyre et al. (2020) and Penoyre et al. (2022a) do not account for the capacity of a binary to bias the single body solution (most notably altering the proper motion) which will in turn reduce the residuals between the observed motion and the fit. Thus, our predicted $\beta$ should be seen as an upper limit.

We also show the distribution of the factors $\zeta^2 \beta$, which go into the inferred mass ratios, and $\frac{\beta}{P}$, which is directly proportional to the observed period. The values of these parameters, even when normalising by $\zeta_0$ and $\beta_0$, change precipitously for periods beyond the baseline of the survey. One way to understand this is to approximate the behaviour at long periods as a polynomial in $\frac{1}{P}$, for which the low order terms will dominate for $N_{\text{orb}} \ll 1$. For the spectroscopic measurements, the constant term is subsumed by the fitted radial velocity and hence the first order term $\propto P^{-1}$ is the lowest order remaining term. For astrometry, this first order is similarly subsumed into proper motion, so the remaining excess is $\propto P^{-2}$. From these scalings, we can also estimate $\zeta^2 \beta \propto P^{-4}$ and $\frac{\beta}{P} \propto P$. This latter result is significant because now both sides of equation 8 are proportional to $P$ and we can no longer find a reliable estimate for the period. We can clearly see this behaviour in the third row of Figure 4 where the inferred periods seem to prefer values just above the baseline of the survey.

At long periods, the noise floor dominates the binary contribution to the radial velocity error, and an uptick in $\zeta$ occurs (since $\zeta \propto P^{1/3}$ when $\sigma_{\text{rv}}$ and $q$ are constant) for $P > 10$ years. Similarly at low period the astrometric signal

### Table 1. Simulated Parameter Distribution

| Parameter | Description | Distribution |
|-----------|-------------|--------------|
| $\alpha$ (deg) | right ascension | $U(0,360)$ |
| $\delta$ (deg) | declination | $U(0,180) \sin^{-1}(U(-1,1))$ |
| $\pi$ (mas) | parallax | $0.5U(0,1)U(-1,1)$ |
| $\mu_k$ (mas/yr) | proper motion along dec | $\xi \cdot U(0,6.67)$ |
| $\mu_r$ (mas/yr) | proper motion along ra | $\xi \cdot U(0,6.67)$ |
| $\phi$ (rad) | azimuthal viewing angle | $U(0,2\pi)$ |
| $\theta$ (rad) | polar viewing angle | $\cos^{-1}U(-1,1)$ |
| $\omega$ (rad) | planar projection angle | $U(0,2\pi)$ |
| $e$ | eccentricity | $U(0,1)$ |
| $T_0$ (yr) | time of periapse | $P \cdot U(0,1)$ |
| $P$ (yrs) | period | $10^{\beta-3.3\beta} U(0,1)$ |
| $M_1$ ($M_\odot$) | mass of primary | $1M_\odot$ |
| $q$ | mass ratio | $10^{\beta-3.3\beta} U(0,1)$ |

Note. — Distribution of parameters for simulated MS+CO binary systems. The light ratio $l$ is set to zero for all binaries, and the mass of the secondary $M_2$ calculated from the mass of the primary $M_1$ and the mass ratio $q = M_2/M_1$. The semi-major axis is calculated using Kepler’s 3rd law along with the period and total mass of each binary.

3 https://github.com/zpenoyre/astromet.py
4 https://github.com/gaiaverse/scanninglaw
Figure 4. Top panels: True values of $\beta$, $\zeta$ as a function of period for simulated systems, calculated by inverting Equations 3 and 5. Vertical solid lines are plotted at 5 days, 1 year, and 22 months (observational baseline for radial velocity measurements). Horizontal lines are plotted at 1. Second and third row of panels: True values of $\beta$, $\zeta$ normalised by $\beta_0$ and $\zeta_0$ (given by Equations 7 and 4) as a function of period (second row of panels) and inferred period (third row of panels) for simulated systems. Dashed blue lines are plotted in 2nd row of panels to extrapolate the behaviour of $\beta$ and $\zeta$ at periods beyond the observational baseline; $\beta$ decays approximately as $P^{-2}$, and $\zeta$ decays approximately as $P^{-1}$ until the noise floor is reached. The bottom three rows of panels are repeats of the top three rows of panels with cuts of $RUWE_{ast} > 1.25$ and $RUWE_{spec} > 3$. Horizontal red lines are drawn at the approximate median values of $\beta = 0.62, \zeta = 0.55, \zeta/\beta = 2, \zeta/\beta = 1$ on the 4th row of panels.
Figure 5. Distribution of $\beta$, $\xi/\beta$, $\xi^2\beta$ for simulated sources with $RUWE_{ast} > 1.25$ and $RUWE_{spec} > 3$. We see an excess in low values of $\xi$ and $\beta$ for long period ($> 22$ months) systems that survive the $RUWE_{ast} > 1.25$ and $RUWE_{spec} > 3$ cuts. For sources with $P < 22$ months, the values of $\xi$ and $\beta$ (when all eccentricities and viewing angles are equally probable) are well-represented by the same distributions.

Figure 6. Top panels: Inferred period versus true period for simulated data color coded by $RUWE_{spec}$ (left) and $RUWE_{ast}$ (right). Vertical lines are drawn at 5 days, 1 year, 22 months, and 34 months. Bottom panels: Inferred $q$ versus $q$ for simulated data.

becomes noise dominated, a behaviour we start to see below periods of a few days.

Penoyre et al. (2020) showed that the astrometric error introduced by a binary is related to the unit weight error as

$$UWE = \sqrt{\frac{N}{N-5}} \sqrt{1 + \frac{1}{2} \left(\frac{\sigma_{ast}}{\sigma_{b,fr}}\right)^2}$$

where $N$ is the number of observations and $\sigma_{ast}$ is the astrometric error. Penoyre et al. (2020) also showed that a cut of $RUWE_{ast,DR3} > 1.25$ can be used to identify sources with “significant” astrometric deviations that are inconsistent with single source astrometry. We define an analogous $RUWE$ for $\sigma_{b,fr}$ which we denote $RUWE_{spec}$, and derive a similar cut of $RUWE_{spec} > 3$ in C1. The bottom three rows of Figure 4 show that, when removing sources with $RUWE_{ast} < 1.25$ and $RUWE_{spec} < 3$, a significant amount of the scatter is removed, indicating that this method is only suitable for sources with significant astrometric and spectroscopic errors.

We can also estimate some fiducial values for $\beta$ and $\xi$ from this cleaner subsample. We find $\beta_{median} = 0.62$ and $\xi_{median} = 0.55$. If we used information about the viewing angle for each system, we could find more precise and motivated values of $\beta$ and $\xi$, but in general these are not known, and we will assume no foreknowledge of the viewing angle throughout the rest of this work.

Since $\xi$ and $\beta$ are generally unknown for real data, we can randomly draw from the distributions of $\xi$ and $\beta$ for
sources with $\text{RUWE}_{\text{ast}} > 1.25$, $\text{RUWE}_{\text{spec}} > 3$, and $P < B_{\text{DR}} < 100,000$ times for each source, and use the median inferred mass ratio and period as our final values with uncertainties given by 68% confidence intervals. These distributions of $\zeta$ and $\beta$ are shown in Figure 5.

We note that because our aim is to identify systems with high mass ratios, we do not attempt to remove sources with underestimated $q_s$. However, Figure 4 shows that a cut on inferred period can be used to remove sources with low values of $\zeta$ and $\beta$ (and therefore sources with underestimated mass ratios) if desired.

Finally, we also note that when the true mass ratio $q$ is very small ($< 10^{-2}$), Equation A4 predicts that the radial velocity error contribution from the binary will tend towards

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**Figure 7.** Inferred periods and mass ratios versus true values for entire simulated data (left column) and after $\text{RUWE}_{\text{ast,DR}} > 1.25$ and $\text{RUWE}_{\text{spec,DR}} > 3$ cuts (right column). Vertical lines are shown at 5 days, 1 year, 22 months, and 34 months for inferred versus true period plots.

**Figure 8.** Distribution of combinations of $\text{RUWE}_{\text{spec}}$, $\text{RUWE}_{\text{ast}}$ and $\text{RUWE}_{\text{phot}}$ for simulated MS+CO systems colour coded by orbital period. Vertical and horizontal lines show the cuts for significant values for each type of $\text{RUWE}$. 
0, in which case the spectroscopic noise should dominate the measurements. As Figure 7 shows, this limits our ability to probe binaries with very small mass ratios (e.g. brown dwarfs and exoplanets), since the inferred mass ratios will be overestimated as a result of the noise floor.

3.1 Ellipsoidal variation and photometric RUWE

Because luminous stars with compact object companion can be tidally distorted such that the light curve shows ellipsoidal modulation, we find it useful to define a photometric RUWE to describe the photometric variability of a source:

\[
\text{RUWE}_{\text{phot}} = \sqrt{\frac{F N}{F N - 1}} \sigma_{\text{phot}}
\]

(19)

where \( F N \) is the number of observations contributing to the measured flux, \( \sigma_{\text{phot}} \) is photometric noise, \( \sigma_{\text{FG}} \) is the standard deviation of the measured flux. Using Equation 78 of Penoyre & Stone (2019), we calculate the relative flux variability caused by tides as

\[
\sigma_{\text{tidal,FG}}^2 = F^2 \cdot \text{Var} \left[ \left( \frac{M_c}{M} \left( \frac{R}{a} \right) \left( \frac{3 \sin^2 \theta \cos^2 \phi - 1}{(1 - e \cos \eta)^3} \right) \right) \right]
\]

(20)

where \( R \) is the radius of the primary, \( F \) is the measured flux of the primary, \( \tau \) is a pre-factor of order unity, and \( \text{Var} \) denotes the variance.

To measure photometric RUWE with our simulated systems, we use a constant photometric error of \( \sigma_{\text{phot}} = 10^4 \text{es}^{-1} \) based on the typical values seen in Gaia data (this is discussed in more detail in Appendix C2). Figure 8 shows the distribution of \( \text{RUWE}_{\text{phot}} \) versus \( \text{RUWE}_{\text{phot}} \) for our simulated systems, and we see virtually no overlap between systems with high astrometric \( \text{RUWE} \) and photometric \( \text{RUWE} \), since the orbital separations at which significant ellipsoidal variations occur are too small to produce detectable \( \text{RUWE}_{\text{phot}} \) signals (recall \( \sigma_{\text{phot}} \propto a \) from Equation 5). Because we do not expect candidates selected with our method to be at orbital separations small enough to exhibit ellipsoidal variation, we can instead apply a \( \text{RUWE}_{\text{phot}} \) cut to remove variable sources which we expect to be potential contaminants.

4 AGREEMENT WITH KNOWN SYSTEMS

To apply our method, we cross-match Gaia eDR3 astrometry with DR2 radial velocity measurements through a procedure detailed in Appendix A. Our sample – which we refer to as our radial velocity scatter (RVS) sample – contains 6,188,052 total sources. To infer the value of \( \sigma_{\text{phot}} \) from Gaia data, we follow the procedure detailed in Appendix A. In order to obtain \( \sigma_{\text{phot}} \), we invert Equation 18, substituting in the mean value of \( \text{AL} \) for \( \sigma_{\text{phot}} \) – which is not known for each source – as a function of \( G \) magnitude, which is provided in Lindegren et al. (2021) (solid blue curve of Figure A.1).

We test our method by cross-matching our RVS sample to the gold binary catalogue presented in Price-Whelan et al. (2020), which provides measured periods and mass ratios using spectroscopic data from APOGEE Data Release 16.

5. We find that 303 of the 1,032 sources in the gold binary catalogue exist in our RVS sample.

Figure 9 shows the inferred periods and mass ratios for binaries in the gold binary catalogue versus their measured values. The catalogue only contains estimates for the minimum masses of the secondaries, so we can only compare our inferred \( q \) to the quoted 99th percentile value of \( q_{\text{min}} \).

We see a good agreement with inferred periods and true periods, though the scatter is large (an order of magnitude). Sources that have overestimated mass ratios also tend to have overestimated periods. The periods are on average higher than the measured values, which is consistent with what we would expect for most binaries with \( f \neq 0 \) as discussed in Section 2.3 and Figure 3.

There is also reasonable agreement between the APOGEE mass ratios and the values we infer. Again the agreement is best when the period is also well-estimated.

It is interesting to note that even for systems without significant \( \text{RUWE} \) the period and mass ratios are reasonably well-estimated. Their (marginal) astrometric and spectroscopic errors are still measures of the binary properties, but without them being externally selected via APOGEE we would not be able to pick them out as significantly different from single stars.

5 THE GAIA SURVEY

5.1 Candidate systems

Here we describe the selection procedure we have adopted for our dark-remanent binary candidates. We use the reddening maps of Schlegel et al. (1998) for sources with \( |b| > 15^\circ \) and the 3D reddening maps of Green et al. (2019) for sources with galactic latitude \( |b| < 15^\circ \) along with the extinction coefficients presented in Gaia Collaboration et al. (2018) to correct magnitudes for dust extinction. In addition to the cuts described in Appendix B1, the following cuts are applied to the Gaia RVS dataset to obtain our initial bronze sample of candidates:

\[
\begin{align*}
\text{DR3 RUWE}_{\text{spec}} &> 3 \\
\text{DR3 RUWE}_{\text{ast}} &> 1.25 \\
\text{ipd}_{\text{frac multi peak}} &< 1 \\
E(\text{B-V}) &< 0.5 \\
\text{cnt}_{\text{4C2}} &< 1 \\
q_{\text{inferred}} &> 1 \\
M_{c,\text{inferred}} &> 3M_\odot
\end{align*}
\]

The cut on \( \text{RUWE}_{\text{spec}} \), derived in Appendix B1, is used to separate binaries from single sources, and the analogous cut of 1.25 on \( \text{RUWE}_{\text{ast}} \) is taken from Penoyre et al. (2022a). The cut on \( \text{ipd}_{\text{frac multi peak}} \) is meant to remove binaries with two partially resolved luminous components, and the maximum reddening limit is to remove stars with potentially over-corrected extinction values. Additionally, we require no neighbouring stars within 4º (\( \text{cnt}_{\text{4C2}} < 2 \)) to avoid contamination due to blending. This leaves 128,373 sources.

5 We cross-match on the Gaia DR2 source_id column provided Price-Whelan et al. (2020).
Binary parameters from astrometric and spectroscopic errors

Figure 9. Left: true periods for the gold binary catalogue presented in Price-Whelan et al. (2020) versus those inferred by our method. Vertical lines are drawn at 5 days, 1 year, 22 months, and 34 months. Points are color-coded by inferred versus measured minimum mass ratios. Sources which pass RUWEast $> 1.25$ and RUWEspec $> 3$ cuts are outlined. Right: measured minimum mass ratios versus inferred mass ratios color-coded by the ratio of inferred period to true period. Characteristic (median) error bars are shown in inset panels of both plots.

Figure 10. Distribution of inferred mass ratios (top) and periods (bottom) normalised by median values for all stars in our bronze sample (2,601 sources).

To infer the mass of the primary based on its absolute Gaia G magnitude, we use the main-sequence mass-magnitude relation presented in Pittordis & Sutherland (2019). We subsequently place a cut of $q_{\text{inferred}} > 1$ and $M_{C,\text{inferred}} > 3M_\odot$, leaving 2,601 sources which constitute our bronze sample.

As mentioned in Section 3, we use the Monte Carlo method of error propagation to determine uncertainties on inferred mass ratios and periods by drawing from the distributions in Figure 5 100,000 times; Figure 10 shows the distribution of mass ratios and periods (100,000 for each source) normalised by median values for all sources in the bronze catalogue.

For our gold and silver samples, we also require that the 68% confidence intervals of $q_{\text{inferred}}$ and $M_{C,\text{inferred}}$ lie above 1 and $3M_\odot$:

\[ q_{\text{inferred}} - \sigma_{q_{\text{inferred}}} > 1 \]
\[ M_{C,\text{inferred}} - \sigma_{M_{C,\text{inferred}}} > 3M_\odot \]

which leaves 544 sources for our silver sample. For our gold sample, we apply the following additional cuts:

\[ RUWE_{\text{phot}} < 2 \]
\[ \text{cnt}_8 < 2 \]
\[ \text{MSmask}=\text{True} \]

Because we do not expect our candidates to be at orbital separations small enough to exhibit ellipsoidal variation, we remove variable sources with $RUWE_{\text{phot}} > 2$ (see Appendix C2), leaving 441 sources. This cut is important as unresolved triples may be a significant contamination of our sample, and our analysis (which assumes astrometric and photometric noise come from the same orbit) will break down if the inner pair causes significant radial velocities whilst the outer dominates the astrometric motion. As shown in Figure 8 there is almost no overlap between binary systems which cause significant RUWEphot and those with significant RUWEast. This may also remove some Variability Induced Movers (Wielen 1996) where the astrometric motion comes from a wide binary in which one (or both) of the components is varying in brightness, causing the photocenter to move over time.

We also apply a stricter cut on crowding by removing sources with neighbours within 8", leaving 237 sources.

Figure 11 compares the distribution of remaining candidates on the HR diagram to MIST isochrones (Paxton et al. (2015), Dotter (2016), Choi et al. (2016)) with sub-solar, solar, and super-solar metallicities. Because the discrepancy between theoretical masses calculated by MIST and the primary masses we calculated using the mass-magnitude relation given by Pittordis & Sutherland (2019) is large for sources that lie significantly outside the main sequence, we remove all sources that do not lie within the black mask shown in Figure 11, leaving 182 sources for our gold sample.

As shown in Figure 12 our sources lie mostly within the

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Figure 11. Number density plot of our RVS sample on the HRD (gray). Isochrones taken from Choi et al. (2016) Dotter (2016) and Paxton et al. (2015) are over-plotted assuming sub-solar (top panels) solar (middle) and super-solar (bottom) metallicities. In the right column, our bronze sample is over-plotted, colour-coded by inferred primary masses.

(thin) disk of the Milky Way (spatially and kinematically) and thus likely contain a range of metallicities. However, we can see in Figure 11 that the broad distribution of our sources is consistent with $[\text{Fe/H}]$ somewhere between 0.0 and +0.5, suggesting a young stellar population.

Finally, in Figures 13 and 14 we show some simple properties of our candidates. The former shows the companion mass, mass ratio and periods as a function of position on the HR diagram. Only the mass ratio shows a notable behaviour, with smaller mass ratios for brighter (more massive) systems. This is mostly due to our cut on companion mass being, in general, more stringent than our cut on mass ratio. Lower mass ratio systems are likely more common, but
Figure 12. Properties of the bronze (gold) candidate sample shown as black (golden) points. Left: Azimuthal and radial velocity components in galacto-centric spherical polars are shown for systems with heliocentric distances < 1.5 kpc. Grey-scale density contours correspond to the stars in the Gaia DR2 RVS sample with heliocentric distances < 1.5 kpc. CO candidates in our sample sit firmly within the thin disc portion of the distribution. Middle: Distribution of the candidates in heliocentric Galactic coordinates l and b. Regions with high extinction and high source density typically towards the Galactic centre are avoided. Right: Inferred secondary mass as a function of inferred primary mass. Reassuringly, no obvious correlation is visible.

given the above cuts are only included in our candidate list for massive primaries.

Whilst there are a few Sun-like systems below the Red Giant turnoff (at $M_G \approx 4$), the majority are brighter stars - particularly on the Young Main Sequence (especially after we remove Giants as we believe our estimates for their masses are unreliable). This makes sense from an observational perspective - we expect MS+CO systems to be relatively rare and thus we need to look to large distances before we see a significant number, hence preferring brighter systems for which accurate astrometry and spectroscopy can still be performed.

There is similarly a plausible stellar evolution argument – we expect both components of a binary to have similar masses (Moe & Di Stefano 2017). The compact object in each system was likely once the brighter more massive primary, such that it exhausted its fusible material first and collapsed. In order to form a CO, its mass must have been high, likely more than 10 $M_\odot$ (Heeger et al. 2003) and it’s lifetime relatively short. Thus, we would expect the original secondary (now the visible luminous component) to be similarly young and bright.

Turning to Figure 14 we show the distribution of the inferred parameters of each catalog of our candidates and their distances - as well as the properties of the entire RVS sample and the subset with significant RUWE.

The candidate systems tend to reside at distances of around 1 kpc, with the gold sample in particularly tending to be slightly closer. More distant systems are dimmer and hence the astrometric and spectroscopic error likely becomes too high for significant RUWE, whilst the expected rarity of MS+CO systems likely accounts for the dearth of high parallax candidates.

Our candidates have lower inferred periods, matching our assertion that periods can only be reliably inferred below the baseline of the survey. The mass ratios and companions masses are, by design, high and are larger for each more selective subsample. The mass ratios and companion masses ($q > 10$ and $M_c > 10M_\odot$) though it is hard to gauge if these systems are really this extreme or are just the high-tail end of the distribution of possible $\beta$, $\zeta$ and other (unmodelled) random noise.

6 CONCLUSIONS

In this paper, we explore how spectroscopic and astrometric errors can be used to infer the mass ratios and periods of unresolved binary systems. Specifically, we have developed a method for inferring the mass ratios and periods for binaries with very small light ratios ($l \ll 1$), and have also identified the light-ratio regimes in which our method results in over-estimated and underestimated values.

We also show that the projection terms $\zeta$ and $\beta$ – which parameterise the dependence of spectroscopic and astrometric errors on viewing angle, period, and eccentricity – are period-independent for binaries with $P < B$ and therefore these systems are much easier to constrain. Even for sources in which the viewing angle, eccentricity, and period are unknown, $\zeta$ and $\beta$ – which are needed to calculate the period and mass ratio – can still be approximated by assuming $P < B$ and drawing repeatedly from simulated distributions of $\zeta_0$ and $\beta_0$. We show approximately how $\zeta$ and $\beta$ decay with period for $P > B$, and consequently, that binaries with periods larger than the observational baseline have underestimated mass ratios. However, these sources can mostly be removed with RUWE cuts.

We construct an analogous statistic to the astrometric renormalised unit weight error (RUWE_{ast}) for spectroscopic and photometric errors (RUWE_{spec} and RUWE_{phot}, respectively). We find, after quality cuts on RUWE_{spec} > 3 and RUWE_{ast} > 1.25, there is good agreement between our predicted periods and mass ratios to true values with both simulated systems and known binaries in APOGEE.

We apply our method to Gaia eDR3 data and DR2 radial velocity data in order to obtain a candidate list of luminous stars with massive dark companions. We find 2,601 sources with inferred mass ratios larger than unity and inferred com-
Figure 13. Distribution of bronze (left column) silver (middle) and gold (right) plated candidates on the HRD color-coded by inferred companion mass (top row), mass ratio (middle) and period (bottom). A number-density plot of our full RVS sample is shown in the background.

Some degree of contamination, most notably by unresolved triples (and higher multiples) as well as the occasionally purely spurious Gaia measurement, is inevitable. Particularly rife for misinterpretation are multiple star systems for which large spectroscopic and astrometric RUWEs stem from different pairs with different periods. Our candidates appear to be almost entirely in the disk, and are likely young and metal rich, which is consistent with a MS+CO interpretation.

Even in cases where these systems do not host compact objects they are still some of the most extreme and thus...
Figure 14. Normalized distributions of parallaxes, inferred periods, inferred mass ratios, and inferred companion masses for bronze, silver, and gold-plated samples, as well as our full RVS sample (dashed black line) and sources in our RVS sample with RUWE$_{ast} > 1.25$, RUWE$_{spec} > 3$ (solid gray line).

potentially engaging within the current Gaia data. These candidates form an exciting — but preliminary — sample of possible MS+CO binaries. Future Gaia data releases, including the imminent full third data release (which notably will include 34 months of spectroscopic data) will better constrain the properties of these systems, as well as lowering the noise floor and allowing us to select yet more candidates.

DATA AVAILABILITY
The bronze, silver and gold candidate lists, as well as some relevant Gaia eDR3 fields and parameters inferred by our method is freely available at https://zenodo.org/record/6617544.

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APPENDIX A: RADIAL VELOCITY ERROR

The standard deviation of the line-of-sight velocity is given by
\[
\sigma_v = \sqrt{(v_1^2) - \langle v_1 \rangle^2}. \tag{A1}
\]

The radial velocity of a binary is the vector sum of the radial velocity of the system's center of mass (which we will take to be constant and denote \(v_o\)) and the radial velocity due to the orbit (denoted by \(v_o(t)\)). Then
\[
(v_r)^2 = (v_r + v_o(t))^2 = v_r^2 + 2v_r v_o + v_o^2 \tag{A2}
\]
\[
(v_r)^2 = (v_r + v_o(t))^2 = v_r^2 + 2v_r v_o + v_o^2 \tag{A3}
\]

Thus
\[
\sigma_v = \sqrt{(v_o^2) - \langle v_o \rangle^2} \tag{A4}
\]

We can define a Cartesian coordinate system in which the origin is defined to be the center of mass of the binary and the orbit is confined to the X-Y plane. Assuming the binary is subject to no external forces, then the position of the primary \(\vec{r}_1\) and secondary \(\vec{r}_2\) at orbital phase \(\phi\) is given by
\[
\vec{r}_1 = r(t) \cos \phi \cos \theta \sin \phi, \quad \vec{r}_2 = r(t) \cos \phi \cos \theta \sin \phi, \tag{A5}
\]
\[
\vec{r}_2 = -r(t) \cos \phi \cos \theta \sin \phi \tag{A6}
\]
where \(m_1\) is the mass of the primary, \(m_2\) is the mass of the secondary, \(\dot{M}_\text{tot}\) is the total mass of the binary, \(q \equiv m_1/m_2\), and \(r(t)\) denotes the separation between the two stars as a function of time. It is known that the orbital separation of a binary in a Keplerian orbit with eccentricity \(e\) and semi-major axis \(a\) evolves with time as:
\[
r(t) = a(1 - e^2) \frac{1}{1 + e \cos \phi}. \tag{A7}
\]
Equivalently, the orbital separation can be more succinctly expressed as a function of the eccentric anomaly \(\eta\)
\[
r(\eta) = a(1 - e \cos \eta) \tag{A8}
\]
where \(\eta\) is related to \(\phi\) as
\[
\cos \phi = \cos \eta - e \cos \eta \tag{A9}
\]
\[
\sin \phi = \sqrt{1 - e^2} \sin \eta \tag{A10}
\]
and the time evolution of the binary follows
\[
t = \frac{P}{2 \pi} (\eta - \sin \eta). \tag{A12}
\]
We can define two viewing angles \(\theta_1\) and \(\theta_2\) to describe the line of sight vector \(\vec{k}\) such that \(\theta_1\) is the angle between \(\vec{k}\) and \(\vec{Z}\) (i.e. \(\theta_1 = 0\) when viewing the binary face-on), and \(\theta_2\) is the angle between \(\vec{X}\) (which intersects periapse) and the component of \(\vec{k}\) projected onto the X-Y plane. Then \(\vec{k}\) can be expressed in the X-Y-Z coordinate system as
\[
\vec{k} = \begin{bmatrix}
\sin \theta_1 \cos \phi \\
\sin \theta_1 \sin \phi \\
\cos \theta_1
\end{bmatrix}. \tag{A13}
\]

The radial velocity due to the orbit can be calculated by taking the time derivative of the primary, \(\dot{r}_1\), projected onto the line of sight \(\vec{k}\):

\[
v_o = \frac{d}{dt} \langle \dot{r}_1 \cdot \vec{k} \rangle \tag{A14}
\]

which provides
\[
v_o = \frac{2\pi a - m_2}{P (m_1 + m_2)} \frac{\kappa_\eta \sin \eta - \sqrt{1 - e^2} \kappa_\eta \cos \eta}{1 - e \cos \eta}. \tag{A15}
\]
Substituting Equation A15 into Equation A4, we can re-express the RVS in the form given by equation 3.

**A1  Analytical radial velocity scatter in the short period limit**

Substituting Equation A15 into Equation A4, we can re-express the RVS as

\[
\sigma_r = \frac{m_2}{m_1 + m_2} \frac{2\pi a}{P} \sqrt{(w_0^2 - \langle w \rangle^2)}
\]  

(A16)

where \( w_0 \equiv \nu_0(m_1 + m_2)P/(m_22\pi a) \). Assuming that the binary is frequently and isotropically scanned, we can express the time average of \( w_0 \) as

\[
\langle w_0 \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} dt w_0(t)
\]  

(A17)

\[
= \frac{P}{2\pi a(t_2 - t_1)} \int_{t_1}^{t_2} dt \frac{\tilde{f}_1 - \tilde{k}}{(m_1 + m_2)}
\]  

\[
= \frac{\tilde{f}_1(t_2) - \tilde{k}(t_1) - \tilde{k}}{(t_2 - t_1) \nu_0 m_2} P
\]  

(A18)

(A19)

Moving on to the time average of \( w_0^2 \), we have

\[
\langle w_0^2 \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} dt w_0(t)^2
\]  

(A20)

Using Equation A12, we can change integration variables from \( t \) to \( \eta \):

\[
\langle w_0^2 \rangle = \frac{P}{82\pi^2(t_2 - t_1)} \int_{0}^{\eta_0} \frac{d\eta}{\eta} \left( \frac{-\kappa_0 \sin \eta + \kappa_0 \cos \eta \sqrt{1 - \epsilon^2}}{1 - \epsilon \cos \eta} \right)^2
\]  

(A21)

In the short-period limit where multiple orbits occur over the observational baseline, we can approximate the time average as the time average over one complete orbit. Then \( \langle w_0 \rangle = 0 \) and

\[
\langle w_0^2 \rangle = \frac{P}{82\pi^2(t_2 - t_1)} \int_{0}^{\eta_0 + 2\pi} \frac{d\eta}{\eta} \left( \frac{-\kappa_0 \sin \eta + \kappa_0 \cos \eta \sqrt{1 - \epsilon^2}}{1 - \epsilon \cos \eta} \right)^2
\]  

\[
= \kappa_0^2 \epsilon(1 - \epsilon) \epsilon^2 - \kappa_0^2 \epsilon^2(\epsilon^2 + \epsilon(3\epsilon - 5) + 4(1 - \epsilon))
\]  

\[
\epsilon^2(1 - \epsilon^2)\epsilon^2
\]  

(A22)

(A23)

This provides the expression for \( \xi_0 \) given in Equation 4.

**APPENDIX B: ESTIMATING ERROR AND RUWE FOR SPECTROSCOPY AND PHOTOMETRY**

The precision of the measurements (e.g. \( \sigma_{pec}, \sigma_{ast} \) and \( \sigma_{phot} \)) needs to be known in order to calculate the significance of any extra noise (such as that caused by an unresolved binary). However this is often an unknown quantity which we have to find pragmatically with reference to our observations.

In *Gaia* the precision of most measurements is a function of (at least) the magnitude and colour of the source. We will approach this data preparation step in a manner purposefully similar to the construction of the astrometric renormalised unit weight error (*RUWE*DR) as detailed in *Lindegren* (2018). We will use the precomputed values of *RUWE*DR and calculate \( \sigma_{pec} \) and \( \sigma_{phot} \), and construct an equivalent measure of the significance, *RUWEdr* and *RUWEphot*, here.

6 As *Gaia* do not currently publish the value of the astrometric error assumed in order to calculate ASTRONOMIC_CH12_AL we cannot perform the necessary analysis to infer \( \sigma_{ast} \) from the data.

**B1  Selecting the data**

We will start from the *Gaia* eDR3 catalog, crossmatching with the gaiad3r3.dr2_neighbourhood table to find the relevant astrometric values in *Gaia* DR2. We use the radial velocity measurements as recorded in the eDR3 catalog (which has had some spurious sources removed since DR2 but is essentially the same data) and we’ll use the shorthand:

\[
RV_E = DR2\_RADIAL\_VELOCITY\_ERROR
\]

(B1)

and

\[
RV_N = DR2\_RV\_NE\_TRANSITS
\]

(B2)

We select sources for which

- \( RV_E > 0 \)
- \( RV_N \geq 3 \)
- \( RUWE_{ast,DR2} > 0 \)
- \( RUWE_{ast,\rhoDR3} > 0 \)
- \( \sigma_{pec}/\sigma_{ast,DR2} > 10 \)

and for which the G, BP and RP band magnitudes are finite in both DR2 and eDR3. The first cut just ensures that the source is in the RVS sample, and the second that a median and variance of spectroscopic measurements can be meaningfully measured. Cutting on *RUWE*ast ensures that the source has a 5-parameter astrometric solution, and correspondingly that the solution is based on a sufficient number of visibility periods *check value*. The cut on parallax over error will limit us to closer sources, where the parallax is relatively reliable, and cut out any negative parallax solutions. This leaves 6,188,052 sources out of the full 7,224,631 in the *Gaia* DR2 radial velocity catalogue.

**B2  Spectroscopic error**

There are a few transformations on the quoted *Gaia* data needed before we have the equivalent of a variance of measured velocities. Firstly we must subtract the noise floor of \( 0.110m\ s^{-1} \) added to give the error on the median radial velocity:

\[
\sigma_r = \sqrt{RV_E^2 - 0.11^2}
\]

which we can translate back to a standard deviation via

\[
\sigma_r = \frac{\sqrt{2RV N}}{\pi} \sigma_{pec}
\]

(B4)

where the factor of \( \sqrt{2/\pi} \) comes from the extra variance introduced by using the median rather than the mean of \( v_r \).

Figure B1 shows the distribution of sources in magnitude and colour, their median standard deviation and the fraction of scans for which a spectroscopic measurement is made. Understandably the brightest sources can be measured with the lowest errors, though there is a broad range of magnitudes (up to 13 or 14) for which the precision is high, after which the standard deviation increases rapidly due to Poisson noise on a low number of photons. We also see strong trends in which sources get more spectroscopic measurements - below \( \sim 8^{th} \) magnitude roughly two thirds of scans result in a recording (likely corresponding to the relative geometric proportions of the array of astrometric CCDs compared to the slightly smaller spectroscopic array). For dimmer stars this number drops precipitously, with less than a third of scans yielding measurements. For simplicity, we assume a constant one-third chance of any scan having an associated spectroscopic measurement in our simulations (e.g. in Appendix C1).

We can define the spectroscopic reduced chi squared

\[
\chi^2_{pec} = \frac{1}{V} \sum_{i} \frac{[v_{ij} - \langle v_i \rangle]^2}{\sigma_{pec}^2}
\]

\[
= \frac{RV N}{RV N - 1} \sigma_{pec}^2
\]

(B5)

where \( v_{ij} \) is the radial velocity measured on the \( i^{th} \) observation and \( \langle v_i \rangle \) is the mean of these values. \( v = RV N - 1 \) is the number
of degrees of freedom (our model being single constant value for the radial velocity of the star).

Finally we can convert this to the full analogue of \( RUWE_{\text{rad}} \):

\[
RUWE_{\text{spec}} = \sqrt{\chi^2_{\text{spec}}} = \sqrt{\frac{\text{RVN} - \sigma_{\text{spec}}}{\text{RVN} - 1}} \sigma_{\text{spec}}.
\]  

(B6)

If our model of a constant radial velocity is correct for the majority of sources the modal value of the reduced chi squared, and of \( RUWE_{\text{spec}} \) should be equal to one. We expect this to be the case, as even though binaries are ubiquitous only a subset of them will cause noticeable extra radial velocity variation.

\[
\sigma_{\text{spec}}(m_G, m_B, m_R) = \text{mode}\left( RUWE_{\text{spec}}(m_G, m_B, m_R) | \sigma_{\text{spec}} = 1 \text{ km}^{-1}\right) \cdot 1 \text{ km}^{-1}.
\]  

(B7)

Calculating the mode is not entirely straightforward, so we replicate the method used in Lindegren (2018). If our sample were entirely composed of single stars with no other sources of noise we might expect the median value to correspond to the mode. However binaries will bias our distribution to higher values, thus we take the value at the 41st percentile to be the mode of the single star behaviour (i.e. assuming around 20% of sources have been biased high by binarity or other sources of excess noise).

Towards the edge of the parameter space (in \( m_G \) and \( m_R - m_B \)) we have a diminishing number of data points and the question of how to estimate this error at, or beyond, the edge of the parameter space becomes difficult. To do this we use a novel method, an alternative to the analysis in Lindegren (2018) which using splines to estimate the behaviour of the mode beyond the space spanned by the data. Our method is as follows:

- Firstly we define the span of our data (3 < \( m_G < 16 \) and \( -1 < m_R - m_B < 6 \)) and remormalise these coordinates to a unit box spanning 0 to 1 in each dimension. Data points outside of this range are ignored.
- We choose some subset of points, and for every point in the unit box we find which of this subset is its nearest neighbour.
- For each of the subset of points we calculate the 41st percentile of the parameter of interest (our initial guess for \( RUWE_{\text{spec}} \)) from all points for which it is the nearest neighbour.
- This provides a Voronoi-like mesh of cells, with estimates for the parameter of interest spanning the entire unit box, with the inferred value at some point in the space being that calculated for the enclosing Voronoi cell.
- We sample this on an equally spaced 100 by 100 grid (for easy interpolation later) spanning the unit box.
- We repeat the above procedure with different random subsets, and the eventual values taken for the grid are the median across all runs.
- Finally we can map our grid in the unit box back on to the original span of our data, to give a uniform grid of estimates of the local value of our parameter of interest evenly spanning the whole parameter space.

Note that though this process is closely related to the construction of a Voronoi tessellated cells, we do not have to actually construct this tessellation to perform the calculation. We perform all nearest neighbour calculations with scipy.spatial.KDTree which allows quick lookup of which of our roughly 6 million data points are closest to each of our subset of 1000 random points.

There are two (related) complications to the above procedure, the choice of the random subset of points and the number of repetitions. More repetitions will provide a smoother grid, while at the lower limit of no repetitions we get back exactly the Voronoi cell structure of any single iteration of the procedure. The only significant downside to more repetitions is computational cost, but here we find 10 to be a reasonable medium (running in minutes on a single core of a laptop).
Figure B2. Our estimate of the spectroscopic error, as calculated from the value needed to shift the 41st percentile of observed RUWEs to 1. Each column shows a different weighting of the selection function the subset of (1000) random points, corresponding to $n = 0, -\frac{1}{2}$ and $-1$ respectively. Each row is a different number of iterations of the method, from which the final value used for each grid point is the median of all iterations. The bottom middle panel corresponds to the inferred errors used through the rest of this work and from which $\text{RUWE}_{\text{spec}}$ is also calculated. As the grid (of 100 by 100) points is relatively coarse some edges, especially for low numbers of iterations, appear jagged.

We might be tempted to choose the subset of points randomly from the whole dataset - but this has a disadvantage: it is (linearly) weighted to select more points in areas of higher density. This is close to opposite to what we want, which is an estimate that is robust in areas of low density. Instead we adopt an extra step where we first estimate the density of sources and then weight our random selection by the density to some power, i.e.

\[ w_i = \frac{\rho_i^n}{\sum \rho_i^n} \]  

(B8)

where $\rho_i$ is the density of sources in some local region around the $i^{th}$ point.

When $n$ is equal to zero all data points are equally likely to be selected and we are in the regime described above where sampling is proportional to density. Another way of putting this is that every cell can be expected to contain roughly the same number of data points.

At the other extreme, when $n = -1$, the chance of a point falling in some region being chosen is independent of the local density, and hence points are uniformly distributed in space (from the subset of the space spanned by the data). In some regions this will lead to cells which contain very few data points and (at least for small numbers of iterations) may be dominated by noise.

Larger (negative) values of $n$ could be chosen but these would give more samples to areas of lower density which seems counter-productive in most use cases.
Figure B3. Top: Distribution of spectroscopic deviations of sources in our sample. \( \sigma_{vr} \) (purple) is effectively the radial velocity error quoted by Gaia (after removing the 0.11 km s\(^{-1}\) noise floor) - note that sources with values above 50 km s\(^{-1}\) are removed from the sample. We then translate to the standard deviation of spectroscopic measurements \( \sigma_v \) (red). Using our inferred spectroscopic error we can find the excess spectroscopic noise which we might associate with an unmodelled companion, \( \sigma_{\text{excess}} = \sqrt{\sigma_v^2 - \sigma_{\text{spec}}^2} \) (orange). We also show the subset of \( \sigma_{\text{excess}} \) values for sources with \( \text{RUWE}_{\text{spec}} > 3 \) (dashed orange). Bottom: The distribution of \( \text{RUWE}_{\text{spec}} \). The dashed vertical line corresponds to a value of 1, the expected mode. The solid vertical line is at \( \text{RUWE}_{\text{spec}} = 3 \) which gives a rudimentary criteria for significant excess noise. We fit by eye a student-t distribution to the left hand side of the distribution (dashed blue) with 5 degrees of freedom and a width of 0.15.

Figure B4. Similar to Figure B1 but now showing photometric variability (as estimated from Gaia’s G band flux). The top panel shows the distribution of the standard deviation of flux, whilst the bottom shows our grid of estimated values.

We have settled upon \( n = 0.5 \) as a reasonable compromise that well samples both dense and sparse regions.

This procedure can be seen in Figure B2, where we have made the conversion specified in equation B5 to convert directly to the inferred spectroscopic error. We can see the cell-like structure for a single iteration, and the degree to which these cells span the parameter space. Reassuringly as the number of iterations increases the agreement between the different methods of selecting random points improves - suggesting that repeated samplings alone may be sufficient even if we had used the naive selection of points at random (without weights). Thus we have a smooth estimate of our parameter of interest (here the spectroscopic error) which can easily be interpolated anywhere on this grid, and could even be extrapolated beyond.

In Figure B3 we show the distribution of spectroscopic errors and our inferred \( \text{RUWE}_{\text{spec}} \) distribution. Note that as sources with \( RV_E > 50 \) km s\(^{-1}\) are removed from the RVS sample there is a soft upper limit on the possible values of \( \text{RUWE}_{\text{spec}} \) as seen in the figure. There seems to be a clear single star peak, and a significant excess of high \( \text{RUWE}_{\text{spec}} \) sources that likely correspond to binary systems and other sources of contamination.

B3 Photometric error

We can repeat an analogous calculation to that which we’ve applied to spectroscopic errors to Gaia photometry. This allows us to estimate if the flux of any given star is varying by more than we would expect for its colour and magnitude. In some cases this is a measure of unreliable data, whilst in others this variation can be astrophysical, associated with any of a number of sources of variability.

We start from the \PHOT_G_MEAN_FLUX_ERROR\ and \PHOT_G_N_OBS\, analogous to \( RV_E \) and \( RV_N \). There is no noise floor to subtract for photometric errors, but otherwise each step is the perfect analog of those presented above.

Figure B4 shows this process. Because we are working directly in the count of electron per second (the raw data read from each CCD) the error is largest for the brightest stars - this follows from the fact that we probably expect the error on a count of \( N \) photons to be roughly \( \sqrt{N} \) (assuming Poissonian errors). If we
were to calculate the relative error (i.e. dividing through by \( N \)) we would expect the bright stars to be most accurately measured. The dependence on colour is slight, but there is some variation and an apparent change in behaviour above and below a \( m_{BP} - m_{RP} \) of \( \sim 1 \).

**APPENDIX C: THRESHOLD VALUES FOR RUWE SPEC AND RUWE PHOT**

**C1 Distinguishing binaries from single sources using RUWE spec**

To estimate the value of \( RUWE_{spec} \) that can be used to distinguish single sources from binaries, we simulate another population of 100,000 MS-MS binaries and 100,000 single sources using `astromet.py`. With the exception of periods, masses, and light ratios, almost all parameters are randomly drawn from the distributions provided in Table 1; changed parameter distributions are provided in Table C1. The mass distribution for main sequence stars is from Chabrier (2005). We choose a log-uniform period distribution for binaries spanning a day to a 100 years—the regime we expect spectroscopic errors to be sensitive to—and the light ratio \( l = q^{3.5} \) is estimated from the main-sequence luminosity relation.

In Figure C1 we show the distribution of the number of radial velocity measurements (\( RV \)) and astrometric measurements (\( N_{\text{obs}} = \text{ASTROMETRY}\_N\_OBS\_AL \)) where the factor of 9 comes from the 9 rows of CCDs which scan over any target for each visit). They both follow the shape of Gaia’s scanning law (Boubert et al. 2020), though the former is also significantly lower close to the galactic disk. This can be seen even more clearly when looking at the ratio of the two. Whilst there is clearly variation both on sky and with the magnitude of the source, for simplicity we assume a probability of \( \frac{1}{3} \) of each set of astrometric measurements also yielding radial velocity measurements.

Figure C2 shows the distribution of \( RUWE_{spec} \) for simulated single systems and binaries over the DR2 and DR3 time baselines. We see that for both DR2 and DR3, the distribution of single sources peaks is significantly narrower than the distribution of binaries, and peaks at a \( RUWE_{spec} \) of 1. The fact that we see a smaller spread when moving from DR2 to DR3 \( RUWE_{spec} \) reflects the longer baseline (and larger number of observations per source) in DR3. Penoyre et al. (2020) showed that beyond \( RUWE_{ast,DR3} > 1.25 \)—the cut recommended to separate single sources from binaries—approximately one in a million single sources remain. We conduct a analogous analysis by fitting a student t distribution to the \( RUWE_{spec} \) distribution of single sources, and find that only one in a million single sources have \( RUWE_{ast,DR2} > 3.3 \) and \( RUWE_{ast,DR3} > 2.1 \). Because these exact values are sensitive to the number of radial velocity measurements—which we have approximated to be 1/3 of total Gaia observation times per source—we adopt a critical value of \( RUWE_{spec} = 3 \) for DR2 and recommend a value of \( RUWE_{spec} = 2 \) for DR3.

**C2 Distinguishing variable from non-variable sources using RUWE phot**

To estimate the value of \( RUWE_{phot} \) that can be used to distinguish non-variable from variable sources, we plot the distribution of \( RUWE_{phot} \) for our RVS sample, and model the distribution of non-variable sources by reflecting sources with \( RUWE_{phot} < 1 \) about the peak of the distribution (\( RUWE_{phot} = 1 \)). Figure C3 shows that there is good agreement between this modelled distribution and the distribution of simulated, (non-variable) systems with the same parameter distribution of single sources described in C1 and a constant photometric error of \( \sigma_{phot} = 10^{-4\text{e}^{-1}} \).

**Figure C1.** Distribution of DR2 radial velocity transits (top), astrometric observations (middle) and fraction of radial velocity transits to astrometric observations (bottom) as a function of galactic longitude and latitude.
Table C1. MS-MS Parameter Distribution

| Parameter | Description | Distribution |
|-----------|-------------|--------------|
| \(P\) (days) | period | \(10^9(-2.56,2)\) |
| \(M_1\) (M⊙) | mass of primary | \(10^4(-0.66,0.57)\) |
| \(M_2\) (M⊙) | mass of secondary | \(10^4(-0.66,0.57)\) |
| \(q\) | mass ratio | \(M_2/M_1\) |
| \(l\) | light ratio | \(q^{1.5}\) |

Note. — Distribution of changed parameters for the MS-MS binaries used in the analysis provided in Appendix C1. All other parameters are given in Table 1.

We fit a student t distribution to the RUWE\(_{phot}\) distribution of modelled non-variable sources in the RVS sample, and find that only one in a million single sources have RUWE\(_{phot}\) > 1.85. We adopt a critical value of RUWE\(_{phot} = 2\) for distinguishing non-variable from variable sources in DR3.

Figure C2. Top: distribution of DR2 RUWE\(_{spec}\) (black) and DR3 RUWE\(_{spec}\) (yellow) for simulated single sources and binaries, with a grey vertical line at a RUWE\(_{spec}\) of 1. Bottom: survival function for single sources measured by DR2 (black) and DR3 (yellow). Student-t distributions are fit to both curves and shown with the thick gray and yellow lines. The yellow and gray vertical lines are plotted at 2.1 and 3.4, respectively. Horizontal lines are plotted at 0.25 and 0.4.

Figure C3. Top: distribution of DR3 RUWE\(_{phot}\) for RVS sample (orange) and simulated non-variable sources (dark red), with a grey vertical line at 1. The solid black curve shows the modeled distribution of non-variable sources, obtained by reflecting sources in the RVS sample with RUWE\(_{phot} < 1\) about the peak (at RUWE\(_{phot} = 1\)). Bottom: survival functions with a student-t distribution fitted to the modeled distribution of non-variable sources shown in gray. A vertical line is plotted at RUWE\(_{phot} = 1.85\).