Review on vortex beams with low spatial coherence

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ABSTRACT

Vortex beams with helical phase, carrying phase singularity and orbital angular momentum, have attract great attention in the past decades due to their wide applications in optical communications, optical manipulation, super-resolution imaging and so on. Vortex beams with low spatial coherence, i.e. partially coherent vortex beams, carrying correlation singularity, display some unique properties during propagation, e.g. self-shaping, self-splitting and self-reconstruction. Partially coherent vortex beams exhibit some advantages over coherent vortex beams in some applications, such as remote sensing, laser radar and free-space optical communications. This review summarizes research progress on partially coherent vortex beams, including theoretical models, propagation properties, generation and topological charge determination.

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1 Introduction

Since laser beam was invented in the 1960s, it became one of the most powerful instruments and has been used widely in various fields [1]. Beams with prescribed amplitude, coherence, phase and polarization have been...
studied in depth. Among them, coherence and phase are two extremely important properties that have been investigated extensively in the past decades [2–5]. Coherence is regarded as a consequence of the correlation between two or more points in the fluctuating electric field. Laser beam with low spatial coherence named partially coherent beam, which is characterized by the cross-spectral density (CSD) function, has advantages over coherent beam in some applications. For example, decreasing the spatial coherence of a light beam can increase the signal–noise ratio and reduce the bit-error rate in free-space optical communications [6–11]. Partially coherent beam can be used to overcome speckle effectively in laser nuclear fusion [12], reduce noise in photograph [13], and realize classical ghost imaging [14]. In addition, partially coherent beam also displays advantages in particle trapping [15], atom cooling [16], second-harmonic generation [17,18], optical scattering [19,20] and laser scanning [21]. Before a class of novel partially coherent beam with nonconventional correlation function was introduced by Gori and co-workers [22–24], most of the literatures were focused on conventional partially coherent beam (i.e. Gaussian Schell-model beam), whose intensity and degree of coherence satisfy Gaussian distributions [14–16] Some extraordinary properties of the nonconventional partially coherent beams have been found, e.g. self-shaping [25–28], self-splitting [29], self-focusing [30,31] and self-reconstruction [32], which are expected to be useful in many applications. A review on generation and propagation of partially coherent beams with nonconventional correlation functions can be found in [33].

The phase of light can be characterized by its wave front on propagation. In general, there are three types of wave fronts, i.e. planar, spherical and helical wave fronts [34]. Vortex beam with helical phase carries phase singularity and orbital angular momentum (OAM) [34,35]. Phase singularity demonstrates wave front dislocation, which means the point in the field has a zero amplitude with indefinite phase. Phase singularity was first introduced by Nye and Berry in 1974 [35], since then, much work on phase singularity has been introduced, and a new branch of optics named singular optics was formed [36,37]. On the other hand, in 1992, Allen and coworkers discovered that light beam with helical phase [i.e. \( \exp(i\lambda \phi) \)] carries an OAM of \( \lambda h \) per photon with \( \lambda \) being the topological charge, \( \phi \) being the azimuthal angle and \( h \) being Planck’s constant divided by \( 2\pi \) [38]. Due to their extraordinary properties, vortex beams have been applied for particle trapping and manipulations [39–44], optical measurement [45,46], optical communications [47–50], super-resolution imaging [51–53] and so on. Various vortex beams have been introduced, e.g. Bessel beam [54–56], Gaussian-like beam [57], anomalous vortex beam [58] and perfect vortex beam [59,60]. Besides scalar vortex
beams, vector vortex beams, e.g. radial-polarized vortex beam [61] and circularly polarized vortex beam [62], were also introduced. Different methods, e.g. spiral phase plate (SPP) [63–65], computational hologram [66–68], mode converter [69–71], long-period fiber gratings [72] and plasmonic metasurface [73] have been introduced to generate vortex beam. Many methods for detecting the topological charge of a vortex beam have been developed [74–99], e.g. Shack Hartmann wave front sensor [74–77], interference [78–80], diffraction [81–90], scattering [91], Fourier patterns of the intensity [92], mode transformation [93], OAM density [94] and rotational Doppler effect [95].

Vortex beam with low spatial coherence named partially coherent vortex beam was first proposed by Gori and coworkers in 1998 [100], and such beam displays some unique properties [101–115], e.g. correlation singularity. Correlation singularity is defined as the point in the field whose CSDor degree of coherence equals to zero while the corresponding phase is indefinite. In general, the number of the correlation singularities depends on the topological charge of the partially coherent vortex beam. Partially coherent vortex beam possesses the extraordinary properties of both partially coherent beam and vortex beam, e.g. lower beam scintillation and beam wander during propagation in random media [116–122], better self-reconstruction ability [32], and is expected to be useful in free-space optical communications, optical imaging and information transfer. Most of the literatures have been focused on the conventional partially coherent vortex beams whose correlation functions are of Gaussian distributions [101–122]. In recent years, partially coherent vortex beams with nonconventional correlation functions have been introduced [123,124]. In this review, we introduce research progress on partially coherent vortex beams, including theoretical models, propagation properties, generation and topological charge determination.

2 Theoretical models for partially coherent vortex beams

The statistical properties of a scalar partially coherent beam can be characterized by the mutual coherence function in the spatial-time domain or the CSD in the space-frequency domain [3,4]. The CSD of a partially coherent vortex beam in the source plane can be expressed in the following general form

\[ W(r_1, r_2) = \langle E(r_1)E^*(r_2) \rangle = A(r_1)A(r_2) \exp[il(\varphi_1 - \varphi_2)]g(r_1 - r_2). \]  

(1)

where \( E(r) \) and \( A(r) \) are the electric field and the amplitude, respectively, the angular bracket and the asterisk denote ensemble average and complex
conjugate, respectively. \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) are two arbitrary vector coordinates in the source plane, \( \phi = \arctan(2y/x) \) denotes the angular coordinate, \( g(\mathbf{r}_1 - \mathbf{r}_2) \) is the correlation function between two arbitrary points. By varying the amplitude distribution \( A(\mathbf{r}) \), various partially coherent vortex beams have been proposed. For a Collet-Wolf source [3,4], the correlation function between two arbitrary points satisfies Gaussian distribution, i.e.

\[
g(\mathbf{r}_1 - \mathbf{r}_2) = \exp \left[ -\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{2\delta_0^2} \right],
\]

where \( \delta_0 \) denotes the initial coherence width. When \( \delta_0 \to \infty \), Equation (1) reduces to a coherent vortex beam, and when \( \delta_0 \to 0 \), Equation (1) reduces to an incoherent vortex beam. Gaussian Schell-model (GSM) vortex beam is a classical partially coherent vortex beam, whose CSD function in the source plane is written as [112]

\[
W_{\text{GSMV}}(r_1, r_2, \phi_1, \phi_2) = \exp \left[ -\frac{r_1^2 + r_2^2}{4\sigma_0^2} - \frac{r_1^2 + r_2^2}{2\delta_0^2} + \frac{2r_1 r_2 \cos(\phi_1 - \phi_2)}{2\delta_0^2} + il(\phi_1 - \phi_2) \right],
\]

where \( r \) and \( \phi \) are the radial and angle coordinates, respectively. \( \sigma_0 \) denotes the transverse beam width. For \( \delta_0 \to \infty \), a GSM vortex beam reduces to a coherent Gaussian vortex beam. Furthermore, partially coherent vortex beams with more complicated amplitude were proposed in [118–122], e.g. partially coherent Laguerre-Gaussian (LG) beam, partially coherent Bessel-Gaussian beam, partially coherent Airy vortex beam, and some extraordinary propagation properties were found, e.g. non-diffraction, self-acceleration and self-reconstruction. Partially coherent LG beam is a classical partially coherent vortex beam with complicated amplitude whose CSD function in the source plane reads as [113]:

\[
W_{\text{LG}}(r_1, r_2, \phi_1, \phi_2) = \left( \frac{\sqrt{2}r_1}{\omega_0} \right)^l \left( \frac{\sqrt{2}r_2}{\omega_0} \right)^l L_p^l \left( \frac{2r_1^2}{\omega_0^2} \right) L_p^l \left( \frac{2r_2^2}{\omega_0^2} \right) \exp \left( -\frac{r_1^2 + r_2^2}{\omega_0^2} \right) \times \exp \left[ -\frac{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\phi_1 - \phi_2)}{2\delta_0^2} \right] \exp \left[ il(\phi_1 - \phi_2) \right],
\]

where \( \omega_0 = 2\sigma_0 \), \( L_p^l(\cdot) \) is the Laguerre polynomial with mode orders \( p \) and \( l \). For \( p = l = 0 \), Equation (4) reduces to a classical GSM beam. For \( p = 0 \) and \( l \neq 0 \), Equation (4) reduces to a fundamental GSM vortex beam.

One can modulate not only the amplitude \( A(\mathbf{r}) \) of a partially coherent vortex beam but also its correlation function \( g(\mathbf{r}_1 - \mathbf{r}_2) \). The correlation function of a conventional partially coherent vortex beam is of Gaussian distribution. Partially coherent vortex beams with nonconventional correlation functions were proposed and generated [123,124], and some interesting properties were found. Laguerre-Gaussian correlated Schell-model
(LGCSM) vortex beam is a typical nonconventional partially coherent vortex beam, whose correlation function is of LG distribution, and its CSD function in the source is expressed as \[ W_{LGCSMV}(r_1, r_2) = \exp\left[ -\frac{r_1^2 + r_2^2 - (r_1 - r_2)^2}{4\sigma_0^2} \right] L_p^0 \exp\left[ \frac{(r_1 - r_2)^2}{2\delta_0^2} \right] \exp[il(\theta_1 - \theta_2)], \] where \( L_p^0(\cdot) \) denotes the Laguerre polynomial. Equation (6) reduces to a GSM vortex beam when \( p = 0 \). More recently, partially coherent vortex beam with periodical coherence properties was introduced \[124\] and such beam displays interesting propagation properties, i.e. a Gaussian beam spot evolves into multiple beam spots (i.e. intensity lattices) in the focal plane (i.e. in the far field), which are useful for particle trapping and information transfer.

A vector partially coherent vortex beam has both \( x \)- and \( y \)- components and can be characterized by the CSD matrix in the space-frequency domain \[3,4\]. The degree of polarization and state of polarization of vector partially coherent beam may vary on propagation in free space \[4\], which is much different from a vector coherent beam \[4\]. An electromagnetic Gaussian Schell-model (EGSM) vortex beam is a typical vector partially coherent vortex beam with uniform state of polarization, and the CSD matrix of an EGSM vortex beam in the source plane is expressed as \[125\]:

\[
\hat{W}_{EGSMV}(r_1, r_2) = \begin{pmatrix} W_{xx}(r_1, r_2) & W_{xy}(r_1, r_2) \\ W_{yx}(r_1, r_2) & W_{yy}(r_1, r_2) \end{pmatrix},
\]

and its elements are given as

\[
W_{\alpha\beta}(r_1, r_2) = A_{\alpha}A_{\beta}B_{\alpha\beta} \exp\left[ -\frac{r_1^2}{4\sigma_\alpha^2} - \frac{r_2^2}{4\sigma_\beta^2} - \frac{(r_1 - r_2)^2}{2\delta_{\alpha\beta}^2} \right] \exp[i(\phi_1 - \phi_2)],
\]

where \( A_x \) and \( A_y \) are the amplitudes of \( x \) and \( y \) components of the electric field, respectively. \( B_{xx} = B_{yy} = 1 \), \( B_{xy} = |B_{xy}| \exp\left( i\phi_{xy} \right) \) is the complex correlation coefficient between \( x \) and \( y \) components of the electric field with \( \phi_{xy} \) being the phase difference between the two components. \( \sigma_i \) is the r.m.s width of the intensity distribution along \( i \) direction, \( \delta_{xx}, \delta_{yy} \) and \( \delta_{xy} = \delta_{yx} \) are the r.m.s widths of the autocorrelation functions of the \( x \) component of the electric field, of the \( y \) component of the electric field and of the mutual correlation function of \( x \) and \( y \) components, respectively. The nine real parameters \( A_x, A_y, \sigma_x, \sigma_y, \delta_{xx}, \delta_{yy}, \delta_{xy}, |B_{xy}|, \phi_{xy} \) are shown to satisfy several intrinsic constraints and obey some simplifying assumptions \[4\]. Vector partially coherent vector beam
with uniform state of polarization named partially coherent radially polarized beam was proposed and generated in [126].

Partially coherent beam can carry not only vortex phase and but also twist phase. Twist phase only exists in a partially coherent beam [127]. In the past decades, the vortex phase and vortex beam have been studied separately, and both phase will induce OAM. More recently, a new partially coherent beam named twisted Laguerre-Gaussian Schell-model (TLGSM) beam was introduced, which carries both vortex phase and twist phase, and its CSD function is given by [128]

\[
W_{TLGSM}(r_1,r_2) = r_1^{l_1}r_2^{l_2}\exp\left(-\frac{r_1^2 + r_2^2}{4\sigma_0^2}\right)\exp\left[-\frac{(r_1 - r_2)^2}{2\delta_0^2}\right] \times \exp\left[-il(\phi_1 - \phi_2)\right] \exp\left(-i\mu_0x_1y_2 + ik\mu_0x_2y_1\right).
\]

The last term in Equation (9) represents the twist phase. \(\mu_0\) is a real-valued twist factor with dimension of an inverse length. \(k = 2\pi/\lambda\) with \(\lambda\) being the wavelength of light. According to [127], the twist factor must satisfy the inequality \(\mu_0^2 \leq (k^2\delta_0^2)^{-1}\). The vortex phase’s and twist phase’s contributions to the OAM are interrelated, which can greatly increase the amount of OAM [128].

3 Propagation properties of partially coherent vortex beams

Propagation of partially coherent beams in free space can be treated by the generalized Huygens-Fresnel diffraction integral [3], and the generalized Collins formula was used to treat the propagation of partially coherent beams through a paraxial ABCD optical system [129]. An efficient tensor method was proposed to treat the propagation of complicated partially coherent beam recently [130]. Based on above methods, the propagation properties of partially coherent vortex beams and correlation vortices have been studied in detail [101–122,131–136]. Both numerical results and experimental results have shown that a partially coherent vortex beam has advantage over a coherent vortex beam or a GSM beam for reducing turbulence-induced degradation and scintillation [118–122,137], which is expected to be useful for free-space optical communications. Recently, Zeng et al. introduced a new kind of partially coherent vortex named partially coherent fractional vortex beam, which has fractional topological charge and its intensity pattern has a controllable opening gap and can be manipulated through varying its coherence width [138].

It is known that it is time-consuming to obtain analytical solution of a partially coherent beam propagating in turbulent atmosphere. In recent years, a simulation method using multiple phase screens has been
introduced to simulate the propagation of coherent beam through turbulent atmosphere. However, this method cannot be applied directly to treat the propagation of partially coherent beams, since they were characterized by the CSD function instead of the electric field. A feasible method is the coherent mode decomposition, we can expand a partially coherent beam as the incoherent superposition of multiple coherent modes [139]. By simulating the propagation of every single mode through turbulence, then superposing them incoherently, one can obtain the statistical properties of a partially coherent beam in turbulent atmosphere. Another method resorts to a random phase screen which satisfies certain statistical properties to simulate partially coherent beams [140]. A pure phase screen can be easily generated by the spatial light modulator (SLM) to simulate the propagation of a conventional-correlated partially coherent beam in turbulent atmosphere, e.g. GSM beam and partially coherent flat-topped beam [141,142]. However, for a partially coherent beam with non-conventional correlation function, a complex phase screen is needed in this case [143]. Hence, one can simulate the propagation of a partially coherent beam through turbulent atmosphere by multiple random phase screens (see Figure 2) [144].

Some interesting propagation properties of partially coherent vortex beams have been found in [112,114,145,146], e.g. self-shaping, self-splitting and self-reconstruction. Unlike the hollow profile of a coherent vortex beam, the intensity distribution of a partially coherent vortex beam varies during propagation, e.g. dark hollow, flat-topped and Gaussian beam profiles can be formed at different propagation distances. When a partially coherent vortex beam with conventional correlation function (e.g. partially coherent LG_{0l} beam) is focused, one can obtain dark hollow or flat-topped or Gaussian beam spot in the focal plane by choosing suitable initial coherence width (see Figure 3) [112]. The

Figure 1. Focused intensity of optical coherence vortex lattices in the focal plane for different values of the initial coherence width [124].
topological charge of partially coherent vortex beam plays a role of anti-degradation, i.e. the dark hollow beam profile evolves into a Gaussian beam profile more slowly as the topological charge increases. For a partially coherent vortex beam with periodical coherence properties (i.e. optical coherence vortex lattices) focused by a thin lens, one can obtain different intensity lattices in the focal plane through varying initial coherence width (see Figure 3)[124], and each beamlet of the intensity lattices in the focal plane can display dark hollow or flat-topped or Gaussian beam spot. The intensity lattices are useful for simultaneously trapping multiple particles. In addition, for a vector partially coherent vortex beam with uniform state of polarization (i.e. EGSM vortex beam), one

![Figure 2. Simulation of the propagation of light beam through turbulent atmosphere by multiple random phase screens [144].](image1)

![Figure 3. Focused intensity distribution of a partially coherent LG\(_{00}\) beam for different values of the topological charge and initial coherence width [145].](image2)
can control its focused intensity distribution by varying its initial degree of polarization, topological charge and coherence widths [125].

In principle, the beam spot of a partially coherent vortex beam rotates on propagation, which is caused by the topological charge, while it is hard to observe the beam spot rotation directly when the intensity distribution has a circular symmetry. Some special partially coherent vortex beam with non-circular symmetry, e.g. partially coherent fractional vortex beam, displays the phenomenon of beam spot rotation on propagation clearly. The intensity distribution of a partially coherent fractional vortex beam has a gap, and the gap rotates clockwise or anti-clockwise depending on the sign of the topological charge [138]. One also can observe the rotation of the beam spot when a circular partially coherent vortex beam passes through an anisotropic optical system [131], e.g. cylindrical lens. For a partially coherent radially polarized vortex beam, the beam spot of x or y component also demonstrates rotation on propagation [126].

Another interesting and useful property of partially coherent vortex beam is its self-reconstruction ability. In general, the self-reconstruction is considered as the property of coherent diffraction-free beams, e.g. Bessel beam, Bessel-Gaussian beam and Airy beam [147–149], and optical self-reconstruction has been applied for microscopic particle manipulation [150] and human tissue microscopy [151]. Usually diffraction-free beams reconstitute their spatial shapes upon interaction with the obstacle. In [32], Wang et al. found that partially coherent beam can self-reconstruct its intensity profile and state of polarization upon scattering from an opaque obstacle provided the beam coherence area is reduced well below the obstacle area.

For an obstructed partially coherent vortex beam, the distribution of the degree of coherence also reconstructs upon propagation as shown in Figure 4, and the self-reconstruction ability of the intensity and degree of coherence increases as the initial coherence width decreases. The reconstructed intensity does not reveal any information about the topological charge, while the reconstructed degree of coherence contain the information of the topological charge, i.e. the ring dislocation number of the degree of coherence is related to the magnitude of the topological charge.

3. Generation of partially coherent vortex beams

In general, generation of a partially coherent vortex beam mainly include two steps: the first step is to produce a partially coherent beam, and the second step is to load the vortex phase into the generated partially coherent beam. Different methods have been introduced for generating partially coherent beams: one is to increase the coherence width of incoherent beam by using the filter plate (e.g. an aperture filter), which has been widely used before the invention of laser beam. The energy loss of
this method is large and the initial coherence width of the generated beam is not controllable. It is worth pointing out that an incoherent beam can also evolve into a partially coherent beam after long-distance propagation. The second method is to decrease the coherence width of a coherent beam with the help of random media (e.g. rotating ground-glass disk). By using this method, one can precisely control the initial coherence width of the generated partially coherent beam by manipulating the beam spot size on the surface of the rotating-ground glass disk [26], however, the energy loss of this method is also inevitable. The third method is to superpose a sequence of coherent modes because partially coherent beam can be decomposed as a superposition of multiple coherent modes [152,153], and one can increase the power of the generated partially coherent beam with this method. On the other hand, various optical elements and methods have been proposed to generate vortex phase, e.g. SPP, diffractive optical elements, computer-generated hologram spiral fiber, uniaxial crystal and nanostructures.

For a scalar conventional partially coherent vortex beam, e.g. a typical GSM vortex beam, the corresponding experimental setup was shown in Figure 5 [112]. A linearly polarized He-Ne laser beam is focused onto a rotating ground glass disk (RGGD) by thin lens (L₁), the transmitted incoherent light then is collimated and filtered by the thin lens (L₂) and Gaussian amplitude filter (GAF), respectively, producing a GSM beam. By adding vortex phase \( \exp(\imath \phi) \) into the GSM beam with help of a SPP, one can generate a GSM vortex beam. The neutral density filter (NDF) is used to modulate the amplitude of the beam. One can modulate the coherence width of the generated beam by changing the focused beam spot size on the RGGD, here the focused beam spot size depends
on the distance between thin lens $L_1$ and RGGD. Based on above experimental setup, if we replace the SPP with a SLM, both the amplitude and the phase of the generated partially coherent beam can be modulated, and then more complicated partially coherent vortex beam with conventional-correlated function can be obtained, e.g. partially coherent LG beam and Bessel Gaussian Schell-model (BGSM) beam [154,155].

For a nonconventional correlated partially coherent vortex beam, e.g. optical coherent vortex lattices, the corresponding experimental setup is shown in Figure 6. A linearly polarized laser beam is expanded by a beam expander and reflected to an amplitude mask by a mirror, then the shaped coherent beam illuminates the RGGD. The transmitted beam becomes a partially coherent beam with nonconventional correlation function after passing through thin lens $L_1$ and GAF. Here the amplitude mask can be controlled by the SLM to generate various beam spots, e.g. doughnut spot, flat-topped beam spot, spot array. When the amplitude mask is a Gaussian beam spot array, optical coherence lattices can be formed in experiment. By adding the vortex phase into the optical coherence lattices with the help of the SPP, one can generate optical coherent vortex

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**Figure 5.** Experimental setup for generating a GSM vortex beam. NDF, neutral density filter; RGGD, rotating ground-glass disk; GAF, Gaussian amplitude filter; SPP, spiral phase plate; $L_1$, $L_2$, thin lenses [112].

**Figure 6.** Experimental setup for generating nonconventional correlated partially coherent vortex beams [124].
lattices. Figure 7 shows the experimental results of the normalized-focused intensity distribution of optical coherence vortex lattices for different initial coherence widths, and we indeed obtain intensity lattices in the focal plane, and the beam profile of each beamlet is controlled through varying the initial coherence width.

We can generate vector partially coherent vortex beam using similar method. Two types of vector partially coherent vortex beams with uniform state of polarization (i.e. EGSM vortex beam) and nonuniform state of polarization (i.e. partially coherent radially polarized vortex beam) have been proposed and generated in [125,126]. The state of polarization and degree of polarization of vector partially coherent vortex beams varies on propagation in free space, and the vortex phase plays a role of resisting coherence-induced degradation and depolarization.

In above introduction, a RGGD has been used to generate various partially coherent vortex beams with the help of a SPP or spatial light modulator. Recently, a digital method using a SLM has been proposed to generate partially coherent vortex beams [156], and the corresponding experimental setup is shown in Figure 8. A series of complex random phase screens are loaded dynamically by the SLM to generate partially coherent vortex beams. However, this method depends on the performance of the liquid crystal (e.g. refresh rate) of the SLM, and could not be used for generating a partially

![Figure 7](image1.png)

**Figure 7.** Experimental results of the normalized-focused intensity distribution of optical coherence vortex lattices for different initial coherence width [124].

![Figure 8](image2.png)

**Figure 8.** Experimental setup for digital generation of partially coherent vortex beams. BE, beam expander; SLM, spatial light modulator; L₁, L₂, thin lens; D, aperture [156].
coherent vortex beam with high energy density. Mode-decomposition method may be used to generate partially coherent vortex beam with high power in future.

4. Determination the topological charge of partially coherent vortex beams

The photon of partially coherent vortex beam carries OAM, which is closely related to the topological charge and can be used for information encoding and decoding [49]. Generally, to encode the information, we need to know (or determine) the magnitude and sign of the topological charge. One can determine the topological charge of a coherent vortex beam through measuring the intensity distribution after diffraction or interference [77–95]. It is known that the intensity distribution of a partially coherent vortex beam gradually evolve from doughnut to Gaussian distribution with the decrease of the initial coherence width, and the traditional intensity measurement methods for determining the topological charge become invalid. Figure 9 shows the influence of the initial coherence width on the interference pattern of a GSM vortex beam. One sees that one cannot infer any information about the topological charge when the initial coherence width is small.

On the other hand, for a partially coherent vortex beam, the phase singularity gradually disappears on propagation, while the correlation singularity (i.e. ring dislocations) in the correlation function appears [103], and the number of ring dislocations of a partially coherent LG0_l beam in the focal plane or in the far field equals to the magnitude of the topological charge (see Figure 10) [145,155]. For a partially coherent LGpl beam, the number of the ring dislocations equals to 2p + |l| [113], and one can determine p and l through measuring the double-correlation function [157,158].

In addition to the magnitude, the sign of the topological charge also plays an important role for information encoding and transfer. Above mentioned literatures are limited to the measurement of the magnitude of the topological charge. Actually, the sign of the topological charge also can be determined by the correlation function [131]. Figure 11 shows the theoretical simulation (a–e) of the intensity distribution for the GSM vortex beam with different initial coherence widths and topological charge l = 2.

![Figure 9](image_url)

**Figure 9.** Numerical results of the interference pattern of a GSM vortex beam for different values of the initial coherence width with topological charge l = 2.
the logarithm of the correlation function of a partially coherent LG$_{0l}$ beam for different topological charges at certain propagation distance after passing through a couple of perpendicular cylindrical lenses. The anisotropic correlation function pattern rotates anti-clockwise or clockwise corresponding to the positive or negative sign of the topological charge, respectively. In addition, one finds that the number of the bright fringes equals to $2|l| + 1$. More recently, Lu et al. proposed a phase-analysis method for measuring the correlation singularities through introducing a movable perturbation at a certain point in an illumination window of a finite size. Using the proposed method, the correlation

Figure 10. Distribution of the modulus of the degree of coherence of a partially coherent LG$_{0l}$ beam with different values of the topological charge $l$ in the focal plane for different state of coherence [145].

Figure 11. (a–e) Theoretical simulations of the logarithm of the correlation function of a partially coherent LG$_{0l}$ beam for different topological charges at certain propagation distance after passing through a couple of perpendicular cylindrical lenses [131].
singularities of a partially coherent vortex beam in the focal plane were measured. From the results, the magnitude and sign of the topological charge can be determined simultaneously from the phase distribution of the correlation singularities [159].

For practical optical communications with partially coherent vortex beams, more complicated problems will be concerned. How can we determine the topological charge of a multiplexed partially coherent vortex beam? Recently, Chen and Li proposed a new method by using the lensless ghost imaging system to discriminate incoherent LG0 modes [160], which provides an effective way to determine the topological charge of a multiplexed partially coherent vortex beam and can be used for optical communications.

4. Summary

As a summary, we have given a brief review on partially coherent vortex beams including theoretical models, propagation properties, generation and topological charge determination. Different from coherent vortex beams, partially coherent vortex beams exhibit some unique and extraordinary propagation properties, e.g. correlation singularity, self-shaping, self-splitting and self-reconstruction, which are useful for particle manipulations, material process and super-resolution imaging. One can determine the topological charge of a partially coherent vortex beam by measuring the correlation singularity. Partially coherent vortex beams have advantage over coherent vortex beams for reducing turbulence-induced degradation and scintillation [118–122,137], and are expected to be useful for free-space optical communications, imaging and information transfer. We believe this field will grow further and expand rapidly, and more and more interesting results, and potential applications will be revealed.

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