A simple model for the quenching of pairing correlations effects in rigidly deformed rotational bands

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Using Chandrasekhar’s S-type coupling between rotational and intrinsic vortical modes one may simply reproduce the HFB dynamical properties of rotating nuclei within Routhian HF calculations free of pairing correlations yet constrained on the relevant so-called Kelvin circulation operator. From the analogy between magnetic and rotating systems, one derives a model for the quenching of pairing correlations with rotation, introducing a critical angular velocity—analogous to the critical field in superconductors—above which pairing vanishes. Taking stock of this usual model, it is then shown that the characteristic behavior of the vortical mode angular velocity as a function of the global rotation angular velocity can be modelled by a simple two parameter formula, both parameters being completely determined from properties of the band-head (zero-spin) HFB solution. From calculation in five nuclei, the validity of this modelled Routhian approach is assessed. It is clearly shown to be very good in cases where the evolution of rotational properties is only governed by the coupling between the global rotation and the pairing-induced intrinsic vortical currents. It therefore provides a sound ground base for evaluating the importance of coupling of rotation with other modes (shape distortions, quasiparticle degrees of freedom).

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I. INTRODUCTION

In a previous paper (hereafter referred as I) we have provided some evidence that the dynamical effects of pairing correlations at finite spins could be very well represented indeed by an intrinsic flow being both non-deforming and counter-rotating (with respect to the global rotation) \( \mathbf{J} \). This has been borne out from the study of rotational bands in three heavy nuclei chosen to represent rather different cases (as far as nucleon numbers, deformation, spin values or pairing correlation content are concerned). To achieve this, we have performed two sets of microscopic calculations under a Routhian-type constraint (namely including in the variational content are concerned). To achieve this, we have performed two sets of microscopic calculations under a Routhian-type constraint (namely including in the variational quantity for the rotating case a \(-\mathbf{Ω} \cdot \mathbf{J}\) term with usual notation). The first calculations have included pairing correlations within the Hartree-Fock-Bogoliubov (HFB) formalism. The second ones were of the Hartree-Fock type (i.e., without any pairing correlations) with a double Routhian-type constraint, as presented in Refs. [2, 3, 4] and recalled here in Appendix C, which we will refer to below as HF+V calculations. The “measuring stick” for the latter was the so-called Kelvin circulation operator \( \hat{K}_1 \) (see, e.g., Ref. [2]) which is well suited for the description of collective modes dubbed after Chandrasekhar as S-type ellipsoids [6]. Indeed, upon imposing via the Routhian double-constraint that the second type of solutions should have the same \( \langle \hat{K}_1 \rangle \) expectation value at a given value of the spin \( I \) as those corresponding to the HFB calculations, it has been demonstrated in I that both calculations yielded the same rotational properties.

As shown also in I, the intensity of such counter-rotating currents (measured, e.g., by \( \langle \hat{K}_1 \rangle \) ) behaves as a function of the global rotation angular velocity \( \mathbf{Ω} \) in a very characteristic way. One may figure out that such a reactive mode of the fluid should be proportional to the excitation velocity field intensity (i.e., \( \mathbf{Ω} \)). Moreover it is also safe to assume that \( \langle \hat{K}_1 \rangle \) should be an increasing function of the pairing correlations measured by some power of \( -E_{\text{corr}} \) where \( E_{\text{corr}} \) is the (negative) pair correlation energy. Now, it is well known (see Ref. [7]) that pairing correlations tend to decrease upon increasing \( \mathbf{Ω} \). When \( \mathbf{Ω} \) gets larger therefore, one should expect a balance between two competing phenomena resulting in a maximum of \( \langle \hat{K}_1 \rangle \) as a function of \( \mathbf{Ω} \) between two limiting cases. One is \( \mathbf{Ω} = 0 \) and the other is obtained for a critical value \( \mathbf{Ω}_c \) corresponding to the alleged transition between a “superfluid” and a “normal” phase of nuclear matter.

A word of caution is worth adding at this point. It

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is absolutely not clear that a transition between normal and superfluid nuclear phases should occur within a physically acceptable range of \( \Omega \) values for such finite Coulombian systems as atomic nuclei. In this respect it is well known that the usual Bogoliubov treatment (or its Bogoliubov-Valatin or BCS reduction) tends to overestimate the instability of the superfluid solution in low pairing regimes in general, and at high \( \Omega \) values in particular. Only approaches which would conserve explicitly the particle number, could cast a priori some light on the existence of such a transition. Such calculations are currently achieved \( \hat{\mathbf{K}} \) within an extension of the approach developed in Ref. \( \hat{\mathbf{K}} \) to describe correlated states in even-even nuclei (there, of course, the time-reversal symmetry was not broken). In the present paper however, we will overlook this problem and stick to the usual HFB approach which entails naturally that this particular part of our description in the limit of high \( \Omega \) regime (i.e., close to the critical \( \Omega_c \) value) should be considered as rather schematic indeed.

![Figure 1: Kelvin circulation mean value in \( \hbar \) units for neutrons (upper panel) and protons (lower panel) in the three rotational bands studied in I. The convention in use is the following: circles for \( ^{150}\text{Gd} \), squares for \( ^{192}\text{Hg} \) and triangles for \( ^{254}\text{No} \). Calculations are performed within the HFB formalism for \( ^{254}\text{No} \) and within the HFB+LN formalism for \( ^{150}\text{Gd} \) and \( ^{192}\text{Hg} \).](image)

The above described competition pattern has been illustrated in I by curves (see Fig. 1 there) exhibiting \( \langle \hat{K}_1 \rangle \) as a function of \( \Omega \). The main features of these results are sketched here in Fig. 1 for the isoscalar values of \( \langle \hat{K}_1 \rangle \) in the three considered rotational bands (yrast superdeformed bands of \( ^{150}\text{Gd} \) and \( ^{192}\text{Hg} \), and ground-state band of \( ^{254}\text{No} \)). As seen on this Figure, it is found only in the case of the \( ^{192}\text{Hg} \) nucleus that the range of \( \Omega \) values spanned by our calculations (or more precisely by corresponding available experimental data) allows to exhibit a clean cut extremum in the corresponding curve. For the \( ^{150}\text{Gd} \) nucleus one merely displays the up-going part of the curve (i.e., after the minimum of \( \langle \hat{K}_1 \rangle \) and before \( \Omega_c \)) while, quite on the contrary, for the \( ^{254}\text{No} \) nucleus, one only sees in Fig. 1 the down-going part of the curve (i.e., just above \( \Omega = 0 \)) reaching its minimum, only for protons.

In addition, in the \( ^{150}\text{Gd} \) case, a very interesting phenomenon is present and is worth discussing. As presented in I, one sees that substantial \( \langle \hat{K}_1 \rangle \) values are present for both neutrons and protons (namely about \(+1 \hbar \) and \( -2 \hbar \) respectively) even when pairing correlations have disappeared around \( \hbar \Omega = 0.6 \text{ MeV} \). This phenomenon, also present for the proton Kelvin circulation in \( ^{192}\text{Hg} \) (whose pairing correlations vanish around \( \hbar \Omega = 0.4 \text{ MeV} \)), is clearly a consequence of shell effects as it has been shown in I to be also present within purely rotating Routhian HF calculations. The behavior of \( \langle \hat{K}_1 \rangle \) in \( ^{254}\text{No} \) is also worth a closer look. Indeed, as the neutron pairing energy is almost vanishing (see Fig. 2 in I) even for \( \hbar \Omega = 0 \), a large negative Kelvin circulation mean value is observed in Fig. 1 for neutrons. This phenomenon also could be attributed partly to shell effects, but it is probably mostly related to a dragging effect of the protons.

Another way of representing the same trends is to plot, as done here in Fig. 2 for the three considered nuclei, the values of the Lagrange multiplier \( \omega(\Omega) \) of the Routhian determined in I to obtain the relevant HFB \( \langle \hat{K}_1 \rangle \) values as a function of \( \Omega \). As seen here they exhibit a parabolic-like behavior. Being related explicitly to the extra-contribution to \( \langle \hat{K}_1 \rangle \) due to pairing correlations, it is not surprising that such \( \omega(\Omega) \) curves are all located in our case in the \( \omega < 0 \) part of the plane.

![Figure 2: Kelvin circulation velocity \( \omega \) (in keV) as a function of the rotation angular velocity \( \Omega \) in MeV for the three nuclei considered in I and calculated within the constrained HF+V formalism. The symbol conventions are the same as those in Fig. 1.](image)

It is the aim of the present paper to understand quantitatively the pattern of these \( \omega(\Omega) \) curves. To put it in more operational terms we want to find whether we are able to deduce from the geometrical and pairing correlation properties of HFB solutions at zero-spin, the properties of rotational states. This of course would be only possible in cases where one deals with a “good” rotational band behavior, namely wherever the rotation does not couple significantly with other degrees of freedom like vibrations (e.g., no rotational stretching or anti-stretching).
or single-particle excitations (e.g., no back-bending).

The paper will be organized as follows. In Section II we will briefly develop a simple magnetic-like model inspired from Ref. [4] using the well-known superconductor critical field concept (see, e.g., [10]) to describe the reduction of pairing correlations upon increasing $\Omega$. Section III will be devoted to the proposition and discussion of an ansatz for the relation between $\omega$ and $\Omega$. We will present in Section IV, the prescriptions to determine the parameters of the above relation from the zero-spin HFB solution of a given nucleus. Finally, the validity of the model so established will be tested against actual HFB Routhian calculations for five nuclei (including the three already considered in I) and some conclusions will be drawn in Section V.

II. A SIMPLE MAGNETIC-LIKE ANALOG

In the simple model presented here (which is directly inspired from the rotational quenching mechanism of pairing correlations proposed by Mottelson and Valatin [5], the so-called Coriolis anti-pairing effect), we assume that the action of a global rotation on a pair of particles moving in opposite directions on a given orbital may be described roughly in terms of two interacting magnetic dipoles ($\vec{D}_1$, $\vec{D}_2$) plunged in an external magnetic field $\vec{B}$. The total energy of such a system is made of a term $E_{\text{corr}}$ describing the dipole-dipole coupling and a term $E_{\text{m}}$ taking into account the coupling with the external field. However, while the latter is understood as a magnetic field-dipole interaction term (see, e.g., Ref. [11], Chapter 5)

$$E_{\text{m}} = -\beta \vec{B} \cdot (\vec{D}_1 + \vec{D}_2) \quad ; \quad \beta > 0 ,$$

the former does not originate, in our case, from a magnetic dipole-dipole interaction (which actually would tend to align $\vec{D}_1$ and $\vec{D}_2$), but rather from the strong interaction favoring quite on the contrary an anti-alignment of $\vec{D}_1$ and $\vec{D}_2$. Therefore $E_{\text{corr}}$ (which will be later called the correlation energy) is not given by the standard magnetic dipole-dipole expression (see, e.g., Ref. [11], Chapter 4) but rather, assuming the simplest possible form compatible with the physical requirements, as:

$$E_{\text{corr}} = \alpha (\vec{D}_1 \cdot \vec{D}_2 - |\vec{D}_1||\vec{D}_2|) \quad ; \quad \alpha > 0 .$$

Assuming $\vec{B}$ (whose norm will be denoted as $B$) to lie on the $z$-axis and $\vec{D}_1$ to belong to the $(1,3)$ plane, one gets for the total energy the following expression (where $\theta_1$, $\phi_1$ are the usual spherical coordinates determining the $\vec{D}_1$ direction):

$$E_{\text{tot}} = \alpha d^2 \left[ \sin(\theta_1) \sin(\theta_2) \cos(\phi_1) + \cos(\theta_1) \cos(\theta_2) - 1 \right]$$

$$- \beta B d \left[ \cos(\theta_1) + \cos(\theta_2) \right]$$

where $d$ is the common norm of $\vec{D}_1$ and $\vec{D}_2$.

The variation of $E_{\text{tot}}$ with respect to $\phi_2$ leads to an extremum either at $\sin \phi_2 = 0$ or else at $\sin \theta_1$ and/or $\sin \theta_2 = 0$. In the latter case, it is easy to check that to get an extremal energy, the vanishing of $\sin \theta_1$ entails the vanishing of $\sin \theta_2$ and conversely. One is thus left with $\sin \theta_1 = \sin \theta_2 = 0$.

Discarding for a while this solution let us concentrate now on the former case. Clearly to get a minimal energy $\phi_2 = \pi$ is to be preferred to $\phi_2 = 0$. The variation of $E_{\text{tot}}$ with respect to $\theta_1$ and $\theta_2$ leads now to

$$\beta B \sin \theta_1 = \beta B \sin \theta_2 = \alpha d \sin(\theta_1 + \theta_2)$$

which admits two classes of solutions:

$$\theta_1 = \theta_2 = 0 ,$$

$$\theta_1 + \theta_2 = \pi .$$

In the first case, the remaining variational equation

$$\beta B \sin \theta = \alpha d \sin(2\theta)$$

has two solutions. The first one is defined by

$$\cos \theta = \frac{\beta B}{2\alpha d}$$

whose corresponding equilibrium energy is

$$E_{\text{tot}} = -\alpha d^2 - \frac{\beta^2 B^2}{2\alpha} .$$

The second solution is defined by $\sin \theta = 0$. Therefore, all variational solutions other than the one of eq. (7) correspond to $\sin \theta_1 = \sin \theta_2 = 0$, namely :

$$\theta_1 = \theta_2 = 0 \quad ; \quad E_{\text{tot}} = -2\beta Bd ,$$

$$\theta_1 = \theta_2 = \pi \quad ; \quad E_{\text{tot}} = 2\beta Bd ,$$

$$\begin{cases} \theta_1 = 0 , \theta_2 = \pi \quad ; \quad E_{\text{tot}} = -2\alpha d^2 . \end{cases}$$

While searching for the most stable equilibrium solution, it is found upon comparing the total energies given in eqs. (8) and (9a) that the former is to be retained (since in particular $\alpha d^2 + \beta^2 B^2/(2\alpha) - 2\beta Bd$ is clearly positive). For the solution of eq. (8), as expected, there is an upper limit for the norm $B_c$ of $\vec{B}$ given by

$$B_c = \frac{2\alpha d}{\beta}$$

corresponding to a full alignment ($\theta = 0$). The corresponding correlation energy $E_{\text{corr}}$ at equilibrium assumes the following form:

$$E_{\text{corr}} = 2\alpha d^2 \left[ \left( \frac{B}{B_c} \right)^2 - 1 \right] .$$
One may note, parenthetically, that the critical field value is obtained upon equating twice the correlation energy without field with the dipole magnetic interaction in the aligned case

\[ -4\alpha d^2 = -\beta Bd \] (12)

which is very similar in spirit with the way in which one determines the critical magnetic field in the classical theory of superconductivity (see, e.g., Ref. [10]).

III. MODEL ASSUMPTIONS IN THE CASE OF NUCLEAR ROTATIONAL STATES

As well known, the equations of motion of a particle coupled to a magnetic field by its charge, are similar to those of a massive particle moving in a rotating (non-inertial) frame upon identifying, with a proportionality constant, the magnetic field \( \vec{B} \) and the angular velocity \( \vec{\Omega} \). It allows therefore to schematically model the coupling of a single-particle orbital motion in the rotating frame with the global rotation (in the laboratory frame) as the magnetic dipole interaction considered in the previous section [see eq. (1)]. Similarly, the corresponding interaction energy between two particles moving in opposite directions on a same orbit could be approximated as:

\[ E_{\text{corr}} = \lambda \left[ \left( \frac{\Omega}{\Omega_c} \right)^2 - 1 \right], \] (13)

introducing thus a critical angular velocity \( \Omega_c \) which will be defined below.

The preceding could pertain, a priori, to the description of the internal and external energies of a single pair. It is our contention, however, that such a quantity behaves, up to a multiplicative constant, as the energies of all the pairs to be considered in the nucleus. Therefore, after suitably redefining the phenomenological parameter \( \lambda \), one may consider the expression above given in eq. (13) as the total nuclear pairing correlation energy.

Now, we are proceeding with the major assumption behind our model. Namely, we assume that the vector \( \vec{\omega} \) associated with the intrinsic vortical motion is proportional to the global rotation angular velocity \( \vec{\Omega} \). The corresponding proportionality factor is negative and we propose that the projection \( \omega \) of the vector \( \vec{\omega} \) on the vector \( \vec{\Omega} \) is given by:

\[ \omega = -\alpha (E_{\text{corr}})^\gamma \Omega; \quad \alpha, \gamma > 0. \] (14)

Upon inserting in such an ansatz, the correlation energy expression of eq. (13), one gets for \( \omega \)

\[ \omega = -k \Omega \left[ 1 - \left( \frac{\Omega}{\Omega_c} \right)^2 \right]^{\gamma}; \quad k > 0 \text{ and } \Omega \in [0, \Omega_c]. \] (15)

For \( \gamma \neq 1 \), one gets two extrema for \( \omega \) corresponding to:

\[ \Omega_1 = \frac{\Omega_c}{\sqrt{1 + 2\gamma}}; \quad \Omega_2 = \Omega_c \] (16)

(for \( \gamma = 1 \) one would have only the variational solution corresponding to \( \Omega_1 \)). The correlation energy associated with the \( \Omega_1 \) solution, expressed in units of its value at zero angular momentum (\( \Omega = 0 \)) is:

\[ \frac{E_{\text{corr}}(\Omega_1)}{E_{\text{corr}}(0)} = \frac{2\gamma}{1 + 2\gamma}. \] (17)

IV. DETERMINATION OF THE MODEL PARAMETERS

Let us summarize the modeling which has been performed so far. With eq. (15), we have a three-parameter expression for the intrinsic vorticity parameter \( \omega \) in terms of the global rotation angular velocity \( \Omega \), namely:

i) a scale parameter (\( \Omega_c \)) for the abscissa variable \( \Omega \),

ii) a scale parameter (\( k \)) for the studied quantity \( \omega \), and

iii) a power parameter (\( \gamma \)) expressing the nature of the pairing correlation dependence of \( \omega \).

The former parameter \( \Omega_c \) may be fixed a priori for each considered nucleus as sketched before. In eq. (12), it is given upon equating twice the correlation energy without field (thus, here, without rotation), with the equivalent of the dipole-magnetic coupling energy in the aligned case, namely the rotational energy without pairing. We therefore get

\[ \Omega_c = \sqrt{\frac{4E_{\text{corr}}(\Omega = 0)}{J_{\text{rig}}}}. \] (18)

In the above \( J_{\text{rig}} \) represents the moment of inertia of an unpaired nucleus which could be reasonably well represented by the rigid-body moment of inertia associated with a one-body reduced local density \( \rho(\vec{r}) \) obtained, e.g., in a Hartree-Fock or Hartree-Fock-Bogoliubov calculation as:

\[ J_{\text{rig}} = m \int \rho(\vec{r}) (\vec{r} - (\vec{u} \cdot \vec{r})\vec{u})^2 \, d^3 r, \] (19)

where \( \vec{u} \) is the unit vector \( \vec{\Omega}/\Omega \). However, in doing so one would introduce an unwanted contribution of shell effects and/or pairing correlations to \( J_{\text{rig}} \) through the density function \( \rho(\vec{r}) \). Even though the effect of the latter is generally rather small we propose here, to remain in the spirit of the model (associated with bulk nuclear properties), to rather consider the self-consistent semiclassical moment of inertia \( J_{\text{ETF}} \) (within the Extended
Thomas-Fermi scheme) of Ref. [12] using the same effective interaction at the relevant deformation.

In most current Routhian HFB calculations (including ours [13]) using the Skyrme interaction for the "normal" mean field, there is an inconsistency in the interaction used in the Hartree-Fock ("normal") part and in the part dealing with pairing correlations. As a result, the naturally defined correlation energy, namely the energy difference between the correlated and the non-correlated solutions, is not relevant. Thus we must replace in eq. (18), the correlation energy $E_{\text{corr}}(\Omega = 0)$ by an unequivocal quantity. The "abnormal" part of the total HFB energy—sometimes called (see, e.g., Ref. [14] and Appendix A the "pair condensation energy", $E_{\text{cond}}$—could be considered in practice in this case, as a more reliable index of the amount of pairing correlations. As demonstrated in model calculations performed in the three cases considered in I according to the approach of Ref. [9] which is free from the usual HFB breaking of the particle number symmetry, the pair condensation energy is found to be roughly equal to twice the pair correlation energy (see appendix A for details). The equation (18) becomes thus

$$\Omega_c = \sqrt{\frac{2E_{\text{cond}}(\Omega = 0)}{J_{\text{ETF}}}}. \quad (20)$$

The scale parameter $k$ can be determined in the following way. If we assume that our relation $\omega(\Omega)$ is of any global relevance, it should also be valid in the low angular velocity regime. Namely, one should have for $\Omega \ll \Omega_c$ whatever the choice of the power $\gamma$:

$$k = -\frac{\omega}{\Omega}. \quad (21)$$

In this adiabatic regime, one obtains an expression for the total collective kinetic energy in a HF+V approach which is quadratic in $\Omega$ and $\omega$, as with the notation of Ref. [12]

$$E(\Omega, \omega) = \frac{1}{2} \left( A\omega^2 + 2B\omega\Omega + C\Omega^2 \right). \quad (22)$$

Upon including the above limiting expression for the ratio of $\omega$ and $\Omega$ one gets the expectation value of the total angular momentum as

$$\langle \hat{J}_1 \rangle = \frac{\partial E(\Omega, \omega)}{\partial \Omega} \bigg|_{\omega = -k\Omega} = (C - kB) \Omega, \quad (23)$$

which in turn yields the value of the dynamical moment of inertia within our model:

$$\phi^{(2)}_{\text{mod}} = \frac{d \langle \hat{J}_1 \rangle}{d\Omega} = C - kB \quad (24)$$

as shown in Appendix C.

Now, we define the HFB Routhian dynamical moment of inertia $J_{\text{HFB}}$ (in the low angular velocity regime) and require it has the same value as $\phi^{(2)}_{\text{mod}}$:

$$J_{\text{HFB}} = \frac{\langle \hat{J}_1 \rangle}{\Omega} = \phi^{(2)}_{\text{mod}}. \quad (25)$$

One is left therefore, now, with the problem of determining the moments $A$, $B$ and $C$. A natural way to do so is of course to calculate the total energy surface as a function of $\Omega$ and $\omega$ in the vicinity of zero for both parameters through doubly constrained HF calculations (called in I Routhian HF + V calculations) and then perform a quadratic fit. This has been done indeed for the five considered cases (see Appendix B for the results). Even though this does not amount to performing very much time-consuming calculations, we have developed another approach which is much simpler and seems to be quite sufficient for the model calculations tested here.

One may evaluate from the single-particle eigenstates of the corresponding Hartree-Fock solution, the mass parameters $A$, $B$ and $C$ through the Inglis cranking formula (16, 17) where the cranking operators are (see Ref. [14]) the first components of the total angular momentum $\hat{J}_1$ and of the Kelvin circulation operator $K_1$. With usual notation one has thus

$$C = 2 \sum_{p,h} \frac{|\langle p| \hat{J}_1| h \rangle|^2}{\epsilon_p - \epsilon_h}, \quad (26)$$

$$B = 2 \sum_{p,h} \frac{\langle p| \hat{J}_1| h \rangle \langle h| K_1 | p \rangle}{\epsilon_p - \epsilon_h},$$

$$A = 2 \sum_{p,h} \frac{|\langle p| K_1| h \rangle|^2}{\epsilon_p - \epsilon_h}.$$

However it is well known (see e.g. Ref. [13]) that one should not compare dynamical moments of inertia obtained in a Routhian approach with comparable moments in an Inglis cranking approach. Indeed the latter lack the so-called Thouless-Valatin [19] terms which are merely coming from the time-odd density response to the self-consistent time-odd Hartree-Fock mean field. For instance, it has been estimated in Ref. [20] for the usual cranking moment $C$ and in the presence of pairing correlations that one gets with the Inglis cranking formula an under-estimation of the Routhian value of about 1/3.

In view of the similarity of the two operators $\hat{J}_1$ and $K_1$ it is reasonable to further assume that the same multiplying factor should be applied also for the mass parameters $A$ and $B$. These approximations and the value of the Thouless-Valatin correction parameter $\eta$ [such that the actual values should be enhanced from their Inglis cranking crude estimates by a factor $(1 + \eta)$] have been assessed from doubly constrained HF calculations in the five considered cases in Appendix B. As we will see in the next section, two different Skyrme effective interactions have been used in our calculations (SIII [21] and SkM* [22]). The corresponding values for $\eta$ are 0.2 and 0.1 respectively.

The difference between these values as well as with the value proposed in Ref. [21] may have two origins. The first is that the interactions used in the various self-consistent calculations are different. As shown in [22], the Thouless-Valatin corrective terms for Skyrme inter-
actions increase when the difference between the real nucleon mass \( m \) and its effective value in nuclear matter \( m^* \) increases. If this evidence for Skyrme interactions is to be extended to the Gogny force in use in \cite{21}, it appears that at least a part of this difference may be due to the fact that the ratio \( m^*/m \) equals 0.70 for the considered Gogny force, 0.75 for the SIII force, and 0.79 for the SkM* force. The second is that in Ref. \cite{20} one has performed Routhian calculation with pairing correlations.

As a result one gets the following value for \( k \):

\[
k = \frac{C - (1 - \eta)\tilde{\gamma}_{\text{HFB}}}{B}.
\]

The last parameter whose value has to be pinpointed is the power \( \gamma \) of the correlation energy dependence of \( \omega \). By definition if \( \gamma = 1 \) the parameter \( \omega \) depends linearly on the pairing energy (correlation or condensation energies) whereas if \( \gamma = 0.5 \) it would depend linearly on the average pairing gap. We have found no binding arguments to decide a priori on this value. Besides, as seen from equation \((17)\) in the \( \gamma = 0.5-1 \) range the location of the maximum value of \( |\omega| \) is not violently contingent upon a specific figure for \( \gamma \). We have therefore decided not to use the freedom of fitting this parameter and therefore we have somewhat arbitrarily decided to take \( \gamma = 1 \).

V. RESULTS AND CONCLUSIONS

We have applied the previously described protocol to define the model parameters \( \Omega_c \) and \( k \) [see eq. \((15)\) with \( \gamma = 1 \)] for rotational bands in five nuclei, including the three cases studied in I and adding the ground-state bands of \( ^{154}\text{Sm} \) and \( ^{178}\text{Hf} \). The Kelvin circulation mean values for protons and neutrons calculated in the Routhian HFB approach are plotted on Fig. \ref{fig:3} as a function of \( \Omega \) for these two nuclei. The different trends of these curves for \( ^{154}\text{Sm} \) as opposed to \( ^{178}\text{Hf} \) do not reflect the behavior (illustrated on Fig. \ref{fig:3}) of the condensation energies, but some specific dynamical properties which will be discussed below.

It is generally considered (see, e.g., \cite{22}) that the SIII interaction provides extremely good spectroscopic properties for normally deformed nuclei, while the SkM* interaction, due to its better surface tension, is well suited to the description of superdeformed or heavy (i.e., with a fissility close to 1) nuclei. Therefore we have used the former for the calculations of the ground-state bands of \( ^{154}\text{Sm} \) and \( ^{178}\text{Hf} \), and the latter for the three others nuclei already considered in I.

First, we have obtained values of the critical angular velocities \( \Omega_c \) according to eq. \((20)\).

For the five nuclei, HFB calculations at zero-spin have been performed to yield the corresponding pair condensation energies \( E_{\text{cond}}(\Omega = 0) \). They have already been shown on Fig. \ref{fig:2} in I for \( ^{192}\text{Hg} \) and \( ^{254}\text{No} \). In the \( ^{150}\text{Gd} \) case, we have “artificially” constructed a zero-spin solution consistent with the deformation of the superdeformed band under consideration by constraining the HFB solution to have the value of the axial mass quadrupole moment of the SD rotational band states (of course, without constraint one would have obtained the normally deformed \( ^{150}\text{Gd} \) equilibrium solution). The retained values of \( E_{\text{cond}}(\Omega = 0) \) are listed in table \ref{tab:1}.

The values of the semiclassical moment of inertia to be used in eq. \((20)\) (see above the discussion in Section IV) are also listed in table \ref{tab:1} together with the resulting \( \Omega_c \) values. It is encouraging to note that these values are actually rather close to the expected values of \( \Omega \) which

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Same as Fig. \ref{fig:1} for \( ^{154}\text{Sm} \) (diamonds) and \( ^{178}\text{Hf} \) (stars).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Condensation energies (in MeV) as functions of the angular velocity \( \Omega \) (in MeV) for \( ^{154}\text{Sm} \) (upper panel) and \( ^{178}\text{Hf} \) (lower panel). Proton (neutron resp.) condensation energies are represented as a dashed line (dotted line resp.) and the total condensation energy is represented as a full line. Calculations are performed within the HFB formalism.}
\end{figure}
duced from the plots of calculated in the close vicinity of $\omega = 0$ and may be de-
in $\hbar^2$ MeV$^{-1}$ units) as well as the Kelvin circulation velocity $\omega$ (in keV). The results obtained within various formalisms are represented as follows: dotted line for the (pure) HF formalism, full line (dashed line resp.) for the HFB (HF+V resp.) formalisms and full line with opened circles for our model. Experimental data for the moments of inertia are represented as filled circles. The Lipkin-Nogami correction has been applied for the Hg isotope.

![Graphs of $\omega(\Omega)$](image)

FIG. 5: Results of our calculations for $^{192}$Hg (upper panels) and $^{254}$No (lower panels). From left to right we show (as functions of the rotation angular velocity $\Omega$ in MeV) for each nuclei the kinetic moment of inertia and the dynamical moment of inertia (in $\hbar^2$ MeV$^{-1}$ units) as well as the Kelvin circulation velocity $\omega$ (in keV). The results obtained within various formalisms are represented as follows: dotted line for the (pure) HF formalism, full line (dashed line resp.) for the HFB (HF+V resp.) formalisms and full line with opened circles for our model. Experimental data for the moments of inertia are represented as filled circles. The Lipkin-Nogami correction has been applied for the Hg isotope.

TABLE I: Input parameter $E_{\text{cond}}$ (in MeV), $\tilde{\hbar}E_{\text{TF}}$ and $\tilde{\hbar}H_{\text{FB}}$ (in $\hbar^2$ MeV$^{-1}$ units) for our model (see text). The output parameter $\hbar\Omega_0$ (in MeV) and $k$ are also given.

|       | $E_{\text{cond}}$ | $\tilde{\hbar}E_{\text{TF}}$ | $\tilde{\hbar}H_{\text{FB}}$ | $\hbar\Omega_0$ | $k$     |
|-------|------------------|------------------------------|-------------------------------|-----------------|--------|
| $^{150}$Gd | 10.47            | 88.47                        | 69.74                         | 0.4864          | 0.3443 |
| $^{154}$Sm | 10.03            | 77.85                        | 32.23                         | 0.5077          | 0.6629 |
| $^{176}$Hf | 16.14            | 97.77                        | 26.14                         | 0.5745          | 0.6285 |
| $^{192}$Hg | 6.99             | 120.08                       | 85.94                         | 0.3411          | 0.3159 |
| $^{254}$No | 9.45             | 164.53                       | 68.45                         | 0.3389          | 0.3196 |

would correspond to a vanishing of the isoscalar pair correlation energy as exhibited (see Fig. 2 of I and Fig. 8 here).

The values of $A$, $B$, $C$ and $\gamma$ have been discussed and given in Appendix B. The values of $\tilde{\hbar}H_{\text{FB}}$ have been calculated in the close vicinity of $\Omega = 0$ and may be deduced from the plots of $\tilde{\hbar}$ of Fig. 1 in I for $^{192}$Hg and $^{254}$No. Again in the case of $^{150}$Gd this value is yielded by a Routhian HFB calculation with an appropriate constraint on the axial mass quadrupole moment (as above discussed). The values of these moments are also listed in Table I together with the resulting values of the $k$ parameter. It is worth noting in Table I that the $k$ values corresponding to a given effective force are not very much varying from one nucleus to another. However they seem to be strongly dependent of the considered force (about 0.3 for SkM* and 0.6 for SIII). We do not yet understand this feature.

As a result, the curves $\omega(\Omega)$ yielded by the above defined parameters are plotted on Figs. 5-7 in comparison with the self-consistently calculated ones (HF+V). The global agreement is rather satisfactory in a qualitative fashion for $^{150}$Gd, $^{192}$Hg and $^{254}$No. This is not so for $^{154}$Sm and $^{178}$Hf where a sudden raise of $\omega$ occurs. In these three figures, experimental data are taken from Refs. [24] for $^{154}$Sm and $^{178}$Hf, [25] for $^{150}$Gd, [20] for $^{192}$Hg and [21, 28, 29] for $^{254}$No.

The final step of the assessment of our model consists of course in performing Routhian HF+V calculations with the model values of $\omega(\Omega)$ and compare their results with those of corresponding Routhian HFB calculations. They should also be compared with the results of Routhian HF+V calculations where the constrained $\langle K_1 \rangle$ expectation values are those obtained in Routhian HFB calculations. However we knew already from I (see Fig. 1 therein) that both are very close indeed.

The results of the comparison fall into three categories. For the $^{192}$Hg and $^{254}$No nuclei (Fig. 8), the agreement between the HFB (and thus HF+V) and the model results are very good indeed. One exception however should be noted, it concerns the high spin part of the superdeformed band of $^{192}$Hg where pairing correlations are strongly damped or disappearing in the HFB approach and therefore the simple ansatz here is obviously lacking any sound basis.

The $^{154}$Sm and $^{178}$Hf cases (Fig. 9) are interesting in that HFB calculations exhibit a up-bending pattern for both $\tilde{\hbar}^{(1)}$ and $\tilde{\hbar}^{(2)}$. The fact that it corresponds very well to the experimental situation in the latter and is completely absent in the data for the former is irrelevant for our discussion here. It is remarkable that our model
just ignores these bendings. Therefore we must conclude that these patterns are not to be attributed to a pairing-rotation coupling but to something else which is (as well known) the intrusion of quasiparticle degree of freedom in the rotational dynamics.

As for the last nucleus of our study (150Gd), Fig. displays clearly in its \( J^{(1)} \) and \( J^{(2)} \) parts that obviously the quasiparticle degrees of freedom are dominating there in explaining the rotational behavior within this superdeformed band. Nevertheless, it is to be noted that our model works fairly well in the low angular velocity regime and only fails above \( \hbar \Omega \approx 0.4 \text{ MeV} \) after the occurrence of a change in the ordering of quasiparticle states.

Let us summarize what has been learned from such a comparison. Actually, one should conclude at two different levels.

First of all, it appears in some of the studied cases that one could skip Routhian HFB calculations at finite values of the angular momentum and replace them by more handy constrained Routhian HF calculations. For that one needs only to perform a HFB calculation in the close vicinity of \( \Omega = 0 \). This result is of some practical value in that it allows to predict quickly what could be the trend of the moments of inertia with respect to the angular velocity.

However, the limitation of such an approach is clear. It is only valid in cases where the rotational (global) collective mode does not couple either with deformation modes (no rotational stretching, anti-stretching or triaxial modes, etc.) nor with single-particle degrees of freedom (no back-bending for instance). It is only able to give an account which turns out to be rather good quantitatively of the coupling of the pairing degrees of freedom with the rotational mode.

Taking at face value the rough assumption made in previous papers that the collective effects of these correlations could be mocked up by Chandrasekhar \( S \)-type ellipsoid velocity fields together with the Mottelson-Valatin picture for the rotational quenching of pairing correlations, our model provides a good reproduction of fully self-consistent HFB results. It offers thus reasonable grounds to assume that both conjectures rather accurately describe the microscopic coupling at work in rigidly deformed purely collective rotational bands.
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APPENDIX A: COMPARISON BETWEEN CORRELATION AND CONDENSATION ENERGIES

Within the HFB formalism, using different interactions in the particle-hole and particle-particle channel, one cannot obtain a relevant definition of the pair correlation energy which should be the energy difference between the correlated (HFB) and uncorrelated (HF) states. On the contrary, the pair condensation energy which is defined as the “abnormal” part of the HFB energy proportional to \( Tr(\kappa \Delta) \), where \( \kappa \) is the “abnormal” density matrix and \( \Delta \) is the pairing tensor, is free of any ambiguity.

An alternate formalism to describe pairing correlations in nuclei, dubbed as “Higher Tamm-Dancoff Approximation” (HTDA), free of particle number symmetry-breaking, has been recently presented in Ref. [9].

For the three considered nuclear states we have computed the collective kinetic energy surface on a 4x4 mesh for angular velocities \( \Omega \) and \( \omega \) lower in absolute value than 30 keV\(h^{-1} \). This energy surface has then been fitted by the quadratic expression of eq. (22). The mass parameters \( A \), \( B \) and \( C \) so obtained do take into account the Thouless-Valatin self-consistent contributions and are listed in table III. The Inglis approximation of these mass parameters as obtained by eqs. (26) are also given.

TABLE II: Mass parameters of the collective energy in HF+V calculations for the five considered nuclear states in \( h^2 \text{MeV}^{-1} \) units.

|       | Inglis       | Thouless-Valatin |
|-------|--------------|------------------|
|       | \( A \)      | \( B \)          |
| \( ^{150}\text{Gd} \) | 86.89        | 64.8\%           |
| \( ^{154}\text{Sm} \) | 66.69        | 53.8\%           |
| \( ^{178}\text{Hf} \) | 56.45        | 57.3\%           |
| \( ^{192}\text{Hg} \) | 22.81        | 74.9\%           |
| \( ^{254}\text{No} \) | 12.65        | 89.1\%           |

TABLE III: Enhancement of the Thouless-Valatin mass parameters relative to the Inglis ones (in percent) within the HF+V formalism.

|       | \( A \)      | \( B \)          |
|-------|--------------|------------------|
| \( ^{150}\text{Gd} \) | 9.8 %        | 8.2 %            |
| \( ^{154}\text{Sm} \) | 7.3 %        | 6.7 %            |
| \( ^{178}\text{Hf} \) | 17.5 %       | 17.1 %           |
| \( ^{192}\text{Hg} \) | 7.6 %        | 7.5 %            |
| \( ^{254}\text{No} \) | 11.5 %       | 10.4 %           |

As expected, the Thouless-Valatin values are higher than those obtained with the Inglis approximation. The enhancement (due to the taking into account of all time-odd self-consistent contributions) is shown in table III.
In view of the values exhibited in this table, we can assume that for the three considered nuclear states, the Thouless-Valatin correction roughly enhance the Inglis mass parameters by 10% for the SKM∗ interaction and 20% for SIII.

APPENDIX C: MOMENT OF INERTIA WITHIN DOUBLY-CONSTRAINED ROUThIAN CALCULATIONS

The doubly-constrained Routhian in use in HF+V calculations within the present model writes (see Refs. 2,3,4):

\[ \hat{R} = \hat{H} - \Omega \hat{J}_1 - \omega(\Omega) \hat{K}_1. \]  

(C1)

The stationary condition \( \delta \langle \hat{R} \rangle = 0 \), at a fixed value of \( \Omega \), entails

\[ \frac{1}{\Omega} \frac{d}{d\Omega} \langle \hat{H} \rangle = \frac{d}{d\Omega} \langle \hat{J}_1 \rangle + \frac{\omega(\Omega)}{\Omega} \frac{d}{d\Omega} \langle \hat{K}_1 \rangle. \]  

(C2)

Within a purely rotational formalism, only the first term of the right hand side would be present and would be identified with the dynamical moment of inertia. The second term which arises here is therefore directly related to the added intrinsic vortical mode and one is led to the conclusion that the dynamical moment of inertia in our model should be taken as

\[ J^{(2)} = \frac{d}{d\Omega} \langle \hat{J}_1 \rangle. \]  

(C3)

This is actually what had been assumed in I where it was substantiated by the fact that requiring identical values of \( \langle \hat{J}_1 \rangle \) and \( \langle \hat{K}_1 \rangle \) within both HFB and HF+V formalisms yielded the same angular velocity \( \Omega \) in the two calculations and thus the same moments of inertia.

[1] H. Laftchiev, D. Samsoen, P. Quentin, and I. N. Mikhailov, Phys. Rev. C 67, 014307 (2003).
[2] I. N. Mikhailov, P. Quentin, and D. Samsoen, Nucl. Phys. A627, 259 (1997).
[3] D. Samsoen, P. Quentin, and J. Bartel, Nucl. Phys. A652, 34 (1999).
[4] D. Samsoen, P. Quentin, and I. N. Mikhailov, Phys. Rev. C 60, 014301 (1999).
[5] G. Rosensteel, Phys. Rev. C 46, 1818 (1992).
[6] S. Chandrasekhar, Ellipsoidal Figures of Equilibrium (Dover, New York, 1987).
[7] B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5, 511 (1960).
[8] H. Laftchiev, J. Libert, and P. Quentin, in preparation.
[9] N. Pillet, P. Quentin, and J. Libert, Nucl. Phys. A697, 141 (2002).
[10] C. Kittel, Quantum Theory of Solids (Wiley, New York, 1963).
[11] J. D. Jackson, Classical Electrodynamics (Wiley, New York, 1962).
[12] K. Benchekih, P. Quentin, and J. Bartel, Nucl. Phys. A571, 518 (1994).
[13] H. Laftchiev, D. Samsoen, P. Quentin, and J. Piperova, Eur. Phys. J. A 12, 155 (2001).
[14] H. Flocard, P. Quentin, A. K. Kerman, and D. Vautherin, Nucl. Phys. A203, 433 (1973).
[15] I. N. Mikhailov and P. Quentin, Phys. Rev. Lett. 74, 3336 (1995).
[16] D. R. Inglis, Phys. Rev. 96, 1059 (1954).
[17] D. R. Inglis, Phys. Rev. 103, 1786 (1956).
[18] M. J. Giannoni and P. Quentin, Phys. Rev. C 21, 2060 (1980).
[19] D. J. Thouless and J. G. Valatin, Nucl. Phys. 31, 211 (1962).
[20] J. Libert, M. Girod, and J.-P. Delaroche, Phys. Rev. C 60, 054301 (1999).
[21] M. Beiner, H. Flocard, N. Van Giai, and P. Quentin, Nucl. Phys. A238, 29 (1975).
[22] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Häkansson, Nucl. Phys. A386, 79 (1982).
[23] M. J. Giannoni and P. Quentin, Phys. Rev. C 21, 2076 (1980).
[24] R. B. Firestone et al., Table of isotopes, eighth edition (J. Wiley and sons, New York, 1996), and Refs. quoted therein.
[25] P. Fallon et al., Phys. Lett. 257B, 269 (1991).
[26] B. Gall et al., Z. Phys. A 347, 223 (1994).
[27] P. Reiter et al., Phys. Rev. Lett. 82, 509 (1999).
[28] M. Leino et al., Eur. Phys. J. A 6, 63 (1999).
[29] P. Reiter et al., Phys. Rev. Lett. 84, 3542 (2000).
[30] T. L. Ha, P. Quentin, and D. D. Strottman, in preparation.