Measurement of higher cumulants of net-charge multiplicity distributions in Au plus Au collisions at root $s_{\text{inN}}$ = 7.7-200 GeV

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Measurement of higher cumulants of net-charge multiplicity distributions in Au plus Au collisions at root s(NN)=7.7-200 GeV

Abstract
We report the measurement of cumulants (C-n,n = 1, ..., 4) of the net-charge distributions measured within pseudorapidity (vertical bar eta vertical bar < 0.35) in Au + Au collisions at root s(NN) = 7.7-200 GeV with the PHENIX experiment at the Relativistic Heavy Ion Collider. The ratios of cumulants (e.g., C-1/C-2, C-3/C-1) of the net-charge distributions, which can be related to volume independent susceptibility ratios, are studied as a function of centrality and energy. These quantities are important to understand the quantum-chromodynamics phase diagram and possible existence of a critical end point. The measured values are very well described by expectation from negative binomial distributions. We do not observe any nonmonotonic behavior in the ratios of the cumulants as a function of collision energy. The measured values of C-1/C-2 and C-3/C-1 can be directly compared to lattice quantum-chromodynamics calculations and thus allow extraction of both the chemical freeze-out temperature and the baryon chemical potential at each center-of-mass energy. The extracted baryon chemical potentials are in excellent agreement with a thermal-statistical analysis model.

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Measurement of higher cumulants of net-charge multiplicity distributions in $Au + Au$ collisions at $\sqrt{s_{NN}} = 7.7$–200 GeV
We report the measurement of cumulants ($C_{n,n}$, $n = 1, \ldots, 4$) of the net-charge distributions measured within pseudorapidity ($|\eta| < 0.35$) in Au + Au collisions at $\sqrt{s_{NN}} = 7.7–200$ GeV with the PHENIX experiment at the Relativistic Heavy Ion Collider. The ratios of cumulants (e.g., $C_1/C_2$, $C_3/C_1$) of the net-charge distributions, which can be related to volume independent susceptibility ratios, are studied as a function of centrality and energy. These quantities are important to understand the quantum-chromodynamics phase diagram and possible existence of a critical end point. The measured values are very well described by expectation from negative binomial distributions. We do not observe any nonmonotonic behavior in the ratios of the cumulants as a function of collision energy. The measured values of $C_1/C_2$ and $C_3/C_1$ can be directly compared to lattice quantum-chromodynamics calculations and thus allow extraction of both the chemical freeze-out temperature and the baryon chemical potential at each center-of-mass energy. The extracted baryon chemical potentials are in excellent agreement with a thermal-statistical analysis model.

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One of the main goals in the study of relativistic heavy ion collisions is to map the quantum chromodynamics (QCD) phase diagram at finite temperature $T$ and baryon chemical potential $\mu_B$ [1]. Although the exact nature of the phase transition at finite baryon density is still not well established, several models suggest that, at large $\mu_B$ and low $T$, the phase transition between the hadronic phase and the quark-gluon-plasma (QGP) phase is of first order [2,3] and that at high $T$ and low $\mu_B$ there is a simple cross over from the QGP to hadronic phase [4–8]. The point at which the first-order phase transition ends in the $T$–$\mu_B$ plane is called the QCD critical end point (CEP), which is one of the central targets of the Relativistic Heavy Ion Collider (RHIC) beam-energy-scan program. Several calculations also reported the possible existence of the CEP in the $T$–$\mu_B$ phase diagram [6,7,9].

RHIC at Brookhaven National Laboratory has provided a large amount of data from Au + Au collisions at different colliding energies, which gives us a unique opportunity to scan the $T$–$\mu_B$ plane and investigate the possible existence and location of the CEP. In the thermodynamic limit, the correlation length $\xi$ diverges at the CEP [1]. Event-by-event fluctuations of various conserved quantities, such as net-baryon number, net charge, and net strangeness are proposed as possible signatures of the existence of the CEP [10–12]. It has been shown in lattice QCD that with a next-to-leading-order
Taylor series expansion around vanishing chemical potentials, the cumulants of charge fluctuations are sensitive indicators for the occurrence of a transition from the hadronic to QGP phase [13,14]. Typically, the variances of net-baryon, net-charge, and net-strangeness distributions are proportional to \( \xi \) as \( \sigma^2 = \langle (\delta N)^2 \rangle \sim \xi^2 \) [19], where \( N \) is the multiplicity, \( \delta N = N - \mu \), and \( \mu (c=1) \) is the mean of the distribution.

Recent calculations reveal that higher cumulants of the fluctuations are much more sensitive to the proximity of the CEP than earlier measurements using second cumulants [12,15]. The skewness \( S \) and kurtosis \( \kappa \) are related to the third and fourth moments \( S (=C_1/C_2^{3/2}) = \langle (\delta N)^3 \rangle / \sigma^3 \sim \xi^{1.5} \) and \( \kappa (=C_4/C_2^2) = \langle (\delta N)^4 \rangle / \sigma^4 - 3 \sim \xi \). The ratio of the various order \( n \) of cumulants \( C_n \) and conventional values \( (\mu, \sigma, S, \kappa) \) can be related as follows: \( \mu/\sigma^2 = C_1/C_2^2 \), \( S/\sigma = C_3/C_2 \), \( \kappa/\sigma = C_4/C_2^2 \), and \( S/\mu = C_3/C_1 \). Because \( \xi \) diverges at the CEP, the ratios of cumulants \( S/\sigma \) and \( \kappa/\sigma^2 \) should rise rapidly when approaching the CEP [16,17].

The cumulants of conserved quantities of net baryon, net charge, and net strangeness obtained from lattice QCD calculations [13,14,17] and a hadron resonance gas (HRG) model [18] are desirable for a full understanding of the theory. Fluctuations to the theoretical parameters, both measurements cover a broad range of \( n \) and net-strangeness distributions are proportional to [13,14,17] and a hadron resonance gas (HRG) model [18].

Typically, the variances of net-baryon, net-charge, and net strangeness obtained from lattice QCD calculations are related to the conserved quantum numbers as \( \mu/\sigma = C_1/C_2 \), \( S/\sigma = C_3/C_2 \), and \( \kappa/\sigma = C_4/C_2^2 \). One advantage of measuring these results is that the volume dependence of \( \mu/\sigma \) cancels out in the ratios; hence theoretical calculations can be directly compared with the experimental measurements. These cumulant ratios can also be used to extract the freeze-out parameters and the location of the CEP [14]. Net-electric charge fluctuations are more straightforward to measure experimentally than net-baryon number fluctuations, which are partially accessible via net-proton measurement [19]. While net-charge fluctuations are not as sensitive as net-baryon fluctuations to the theoretical parameters, both measurements are desirable for a full understanding of the theory.

We report here precise measurements of the energy and centrality dependence of higher cumulants of net-charge multiplicity \( \langle \Delta N_{\text{ch}} = N^+ - N^- \rangle \) distributions measured by the PHENIX experiment at RHIC in Au + Au collisions at \( \sqrt{s_{NN}} = 7.7, 19.6, 27, 39, 62.4, \) and 200 GeV. These measurements cover a broad range of \( \mu_B \) in the QCD phase diagram.

The PHENIX detector is composed of two central spectrometer arms, two forward muon arms, and global detectors [20]. In this analysis, we use the central arm spectrometers, which cover a pseudorapidity range of \( |\eta| \leq 0.35 \). Each of the two arms subtends \( \pi/2 \) radians in azimuth and is designed to detect charged hadrons, electrons, and photons. For data taken at \( \sqrt{s_{NN}} = 62.4 \) and 200 GeV in 2010 and 2007, respectively, the event centrality is determined using the total charge deposited in the beam-beam counters (BBC), which are also used for triggering and vertex determination. For lower energies (\( \sqrt{s_{NN}} = 39 \) GeV and below) the acceptance of the BBCs (3.0 < \( |\eta| < 3.9 \)) are within the fragmentation region, so alternate detectors must be employed. For data taken at \( \sqrt{s_{NN}} = 39 \) and 7.7 GeV in 2010, centrality is determined using the total charge deposited in the outer ring of the reaction plane detector (RXNP), which covers 1.0 < \( |\eta| < 1.5 \) [21]. For data taken at \( \sqrt{s_{NN}} = 19.6 \) and 27 GeV in 2011, the RXNP was absent, so centrality is determined using the total energy of electromagnetic calorimeter (EMCal) clusters to minimize the correlation with the charge of the tracks measured in the same acceptance. More details on the procedure are given in [22].

The analyzed events for the above mentioned energies are within a collision vertex of \( |Z_{\text{vertex}}| < 30 \) cm. The number of analyzed events are 2M, 6M, 21M, 154M, 474M, and 1681M for \( \sqrt{s_{NN}} = 7.7, 19.6, 27, 39, 62.4, \) and 200 GeV Au + Au collisions, respectively.

The number of positively charged \( N^+ \) and negatively charged \( N^- \) particles measured on an event-by-event basis are used to calculate the net-charge \( \Delta N_{\text{ch}} \) distributions for each collision centrality and energy. The charged-particle trajectories are reconstructed using information from the drift chamber and pad chambers (PC1 and PC3). A combination of reconstructed drift-chamber tracks and matching hits in PC1 are used to determine the momentum and charge of the particle. Tracks having a transverse momentum between 0.3 and 2.0 GeV/c are selected for this analysis. The ring imaging Čerenkov detector is used to reduce the electron background resulting from conversion photons. To further reduce the background, selected tracks are required to lie within a 2.5\( \sigma \) matching window between track projections and PC3 hits, and a 3\( \sigma \) matching window for the EMCal.

Figures 1(a) and 1(b) show \( \Delta N_{\text{ch}} \) distributions in Au + Au collisions for central (0%–5%) and peripheral (55%–60%)
collisions at different collision energies. These \( \Delta N_{\text{ch}} \) distributions are not corrected for reconstruction efficiency. The centrality classes associated with the average number of participants, \( \langle N_{\text{part}} \rangle \), are defined for each 5% centrality bin. These classes are determined using a Monte-Carlo simulation based on Glauber model calculations with the BBC, RXNP, and EMCal detector responses taken into account \[22,23\].

The \( \Delta N_{\text{ch}} \) distributions are characterized by cumulants and related quantities, such as \( \mu, \sigma, S, \) and \( \kappa \), which are calculated from the distributions. The statistical uncertainties for the cumulants are calculated using the bootstrap method \[24\]. Corrections are then made for the reconstruction efficiency, which is estimated for each centrality and energy using the HIJING1.37 event generator \[25\] and then processed through a GEANT simulation with the PHENIX detector setup. For all collision energies, the average efficiency for detecting the particles within the acceptance varies between 65%–72% and 76%–85% for central (0%–5%) and peripheral (55%–60%) events, respectively, with a 4%–5% variation as a function of energy. The efficiency correction applied to the cumulants is based on a binomial probability distribution for the reconstruction efficiency \[26\]. The efficiency corrected \( \mu, \sigma, S, \) and \( \kappa \) as a function of \( \langle N_{\text{part}} \rangle \) are shown in Figs. 1(c)–1(f).

The \( \mu \) and \( \sigma \) for net-charge distributions increase with increasing \( \langle N_{\text{part}} \rangle \), while \( S \) and \( \kappa \) decrease with increasing \( \langle N_{\text{part}} \rangle \) for all collision energies. At a given \( \langle N_{\text{part}} \rangle \) value, \( \mu, \sigma, S, \) and \( \kappa \) of net-charge distributions decrease with increasing collision energy. However, the width \( \sigma \) of net-charge distributions increases with increasing collision energy indicating the increase of fluctuations in the system at higher \( \sqrt{s_{\text{NN}}} \).

The systematic uncertainties are estimated by: (1) varying the \( Z_{\text{vertex}} \) cut to less than \( \pm 10 \) cm; (2) varying the matching parameters of PC3 hits and EMCal clusters with the projected tracks to study the effect of background tracks originating from secondary interactions or from ghost tracks; (3) varying the centrality bin width to study nondynamical contributions to the net-charge fluctuations due to the finite width of the centrality bins \[27–29\]; and (4) varying the lower cut. The total systematic uncertainties estimated for various cumulants for all energies are: 10%–24% for \( \mu \), 5%–10% for \( \sigma \), 25%–30% for \( S \), and 12%–19% for \( \kappa \). The systematic uncertainties are similar for all centralities at a given energy and are treated as uncorrelated as a function of \( \sqrt{s_{\text{NN}}} \). For clarity of presentation, the systematic uncertainties are only shown for central (0%–5%) collisions.

Figure 2 shows the \( \langle N_{\text{part}} \rangle \) dependence of \( \mu/\sigma^2 \), \( S\sigma \), \( \kappa\sigma^2 \), and \( S^3/\mu \) \( \equiv \langle S\sigma \rangle /\langle \mu/\sigma^2 \rangle \) extracted from the net-charge distributions in Au + Au collisions at different \( \sqrt{s_{\text{NN}}} \). The results are corrected for the reconstruction efficiencies. Statistical uncertainties are shown along with the data points. The systematic uncertainties are constant fractional errors for all centralities at a particular energy; hence they are presented for the central (0%–5%) collision data point only. The systematic uncertainties on these ratios across different energies varies as follows: 20%–30% for \( \mu/\sigma^2 \), 15%–34% for \( S\sigma \), 12%–22% for \( \kappa\sigma^2 \), and 17%–32% for \( S^3/\mu \). It is observed in Fig. 2 that the ratios of the cumulants are weakly dependent on \( \langle N_{\text{part}} \rangle \) for each collision energy; the values of \( \mu/\sigma^2 \) and \( S\sigma \) decrease from lower to higher collision energies, while the \( \kappa\sigma^2 \) and \( S^3/\mu \) values are constant as a function of \( \sqrt{s_{\text{NN}}} \) within systematic uncertainties.

The collision energy dependence of \( \mu/\sigma^2 \), \( S\sigma \), \( \kappa\sigma^2 \), and \( S^3/\mu \) of the net-charge distributions for central (0%–5%) Au + Au collisions are shown in Fig. 3. The statistical and systematic uncertainties are shown along with the data points. The experimental data are compared with negative-binomial-distribution (NBD) expectations, which are calculated by computing the efficiency corrected cumulants for the measured \( N^+ \) and \( N^- \) distributions fit with NBD’s respectively, which also describe total charge \( (N^+ + N^-) \) distributions very well.
TABLE I. Freeze-out $T_f$ and $\mu_B$ vs $\sqrt{s_{NN}}$ in the range $27 \leq \sqrt{s_{NN}} \leq 200$ GeV. The “PHENIX + Refs. [14,36]” values are from this Rapid Communication using lattice QCD calculations from Refs. [14,36]; the “PHENIX + Ref. [37]” values use the continuum limit calculations from Ref. [37]. The “STAR + Ref. [35]” values are the $\mu_B$ values from Ref. [35], which used STAR net-charge cumulant measurements from Ref. [32] for $\mu_B$ with 140 MeV $\leq T_f \leq$ 150 MeV, obtained from the STAR net-proton measurement in Ref. [33] by averaging $S\sigma^3/\mu$ over $\sqrt{s_{NN}} = 27, 39, 62.4$ and 200 GeV.

| $\sqrt{s_{NN}}$ (GeV) | PHENIX + Refs. [14,36] | PHENIX + Ref. [37] | STAR + Ref. [35] |
|-----------------------|------------------------|---------------------|-------------------|
| $T_f$ (MeV)           | $\mu_B$ (MeV)          | $T_f$ (MeV)         | $\mu_B$ (MeV)     |
|                       |                        |                     |                   |
| 27                    | 164 ± 6               | 181 ± 21            | 160 ± 6           |
| 39                    | 158 ± 5               | 114 ± 13            | 156 ± 5           |
| 62.4                  | 163 ± 5               | 71 ± 8              | 159 ± 5           |
| 200                   | 163 ± 8              | 27 ± 5              | 159 ± 8           |

The various order $(n = 1, 2, 3, 4)$ of net-charge cumulants from NBD are given as $C_n(\Delta N_{ch}) = C_n(N^+) + (-1)^n C_n(N^-)$, where $C_n(N^+)$ and $C_n(N^-)$ are cumulants of $N^+$ and $N^-$ distributions, respectively [30,31].

The $\mu/\sigma^2$ and $S\sigma$ values in Figs. 3(a) and 3(b), respectively, both decrease with increasing $\sqrt{s_{NN}}$. The NBD expectation agrees well with the data. The $\kappa\sigma^2$ values in Fig. 3(c) remain constant and positive, between 1.0 $< \kappa\sigma^2 < 2.0$ at all the collision energies within the statistical and systematic uncertainties. However, there is $\sim$25% increase of $\kappa\sigma^2$ values at lower energies compared to higher energies above $\sqrt{s_{NN}} = 39$ GeV, which is within the systematic uncertainties. These data are in agreement with a previous measurement [32], but provide a more precise determination of the higher cumulant ratios, verified by the NBD method of correcting for efficiency, which is simple and analytical for all cumulant ratios with the standard binomial correction [26]. The $S\sigma^3/\mu$ values in Fig. 3(d) remain constant at all collision energies within the uncertainties and are well described by the NBD expectation. From the energy dependence of $\mu/\sigma^2$, $S\sigma$, $\kappa\sigma^2$, and $S\sigma^3/\mu$, no obvious nonmonotonic behavior is observed. Although both previous measurements by the STAR Collaboration [32,33] use the pseudorapidity range $|\eta| \leq 0.5$, compared to the present measurement spanning $|\eta| \leq 0.35$, these measurements are all within the central rapidity region and are expected to be valid for comparison to lattice QCD calculations. The efficiency corrected results for the cumulant ratios $\mu/\sigma^2$, $S\sigma$, and $\kappa\sigma^2$ remain the same within statistics whether each single arm of the PHENIX central spectrometer (azimuthal aperture $\delta\phi = \pi/2$) or both arms ($\delta\phi = \pi$) are used. This is a clear verification of the insensitivity of measured cumulant ratios to volume effects.

The precise measurement of both $\mu/\sigma^2$ and $S\sigma^3/\mu$ in the present study allows both $\mu_B$ and $T_f$ to be determined, unlike a previous calculation in Refs. [35,37], which was only able to use the $\mu/\sigma^2$ measurement from Ref. [32]. The comparison of $S\sigma^3/\mu$ for different $\sqrt{s_{NN}}$ with the lattice calculations {Fig. 3(b) in Refs. [14,36]} enables us to extract the chemical freeze-out temperature $T_f$. Furthermore, $\mu_B$ can be extracted by comparing the measured $\mu/\sigma^2$ ratios with the lattice calculations of $R_{12} = \mu/\sigma^2$ {Fig. 3(a) in Refs. [14,36]}. The extracted $T_f$ and $\mu_B$ values are listed in Table I. The $T_f$ and $\mu_B$ extracted using the lattice calculations in the continuum limit from Ref. [37] are also depicted in Table I. The extracted freeze-out parameters using different lattice results agree very well. However, the extracted $T_f$ are 2–4 MeV lower using Ref. [37] than with Refs. [14,36], which is well within the stated uncertainties. The detailed freeze-out parameter extraction procedure is given in Refs. [14,35,37]. This is a direct combination of experimental data and lattice calculations to extract physical quantities. The $\sqrt{s_{NN}}$ dependence of $\mu_B$ shown in Fig. 4 is in agreement with the thermal-statistical analysis model of identified particle yields [34]. The $\mu_B$ extracted in the present net-charge measurement and the values reported in [35] are in agreement within stated uncertainties, with some tension at $\sqrt{s_{NN}} = 27$ GeV. Available lattice results allow extraction of $\mu_B$ and $T_f$ from $\sqrt{s_{NN}} = 27$ GeV and higher using the present net-charge experimental data. Other recent calculations [38,39] have used both net-proton and net-charge measurements to estimate the freeze-out parameters.

In summary, fluctuations of net-charge distributions have been studied using higher cumulants ($\mu$, $\sigma$, $S$, and $\kappa$) for $|\eta| < 0.35$ with the PHENIX experiment in Au + Au collisions ranging from $\sqrt{s_{NN}} = 7.7$ to 200 GeV. The ratios of cumulants ($\mu/\sigma^2$, $S\sigma$, $\kappa\sigma^2$, and $S\sigma^3/\mu$) have been derived from the individual cumulants of the distributions studied as a function of $\langle N_{part} \rangle$ and $\sqrt{s_{NN}}$. The $\mu/\sigma^2$ and $S\sigma$ values decrease with increasing collision energy and are weakly dependent on $\sqrt{s_{NN}}$ through $\sqrt{s_{NN}} = 39$ GeV. The ratio $S\sigma^3/\mu$ decreases with increasing collision energy. The $\mu/\sigma^2$ and $S\sigma$ values both decrease with increasing collision energy and are weakly dependent on $\sqrt{s_{NN}}$ through $\sqrt{s_{NN}} = 39$ GeV. The ratio $S\sigma^3/\mu$ decreases with increasing collision energy.

FIG. 4. The energy dependence of the chemical freeze-out parameter $\mu_B$. The dashed line is the parametrization given in Ref. [34], and the SchwerionenSynchrotron (SIS), Alternating Gradient Synchrotron (AGS), and Super Proton Synchrotron (SPS) data are from Ref. [34] and references therein.
centrality, whereas $\kappa\sigma^2$ and $S\sigma^3/\mu$ values remain constant over all collision energies within uncertainties. The efficiency corrected values from the NBD expectation reproduce the experimental data. These data are in agreement with a previous measurement [32], but provide more precise determination of the higher cumulant ratios $S\sigma$ and $\kappa\sigma^2$. In the present study we do not observe any significant nonmonotonic behavior of $\mu/\sigma^2$, $S\sigma$, $\kappa\sigma^2$, and $S\sigma^3/\mu$ as a function of collision energies. Comparison of the present measurements together with the lattice calculations enables us to extract the freeze-out temperature $T_f$ and baryon chemical potential $\mu_B$ over a range of collision energies. The extracted $\mu_B$ values are in excellent agreement with the thermal-statistical analysis model [34].

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