Towards the solution of the \( C_P/C_A \) anomaly in shell-model calculations of muon capture

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Recently many authors have performed shell-model calculations of nuclear matrix elements determining the rates of the ordinary muon capture in light nuclei. These calculations have employed well-tested effective interactions in large scale shell-model studies. For one of the nuclei of interest, namely \( ^{28}\text{Si} \), there exists recent experimental data which can be used to deduce the value of the ratio \( C_P/C_A \) by using the calculated matrix elements. Surprisingly enough, all the abovementioned shell-model results suggest a very small value (\( \approx 0 \)) for \( C_P/C_A \), quite far from the PCAC prediction and recent data on muon capture in hydrogen. We show that this rather disturbing anomaly is solved by employing effective transition operators. This finding is also very important in studies of the scalar coupling of the weak charged current of leptons and hadrons.

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The calculation of the nuclear matrix elements involved in the ordinary (non-radiative) capture of stopped negative muons by atomic nuclei has been of considerable interest because they enable to access the structure of the effective weak baryonic current. Due to the large mass of the captured muon the process involves a large energy release (roughly 100 MeV) and thus surveys the baryonic current deeper than ordinary beta decay or electron capture. In particular, the role of the induced pseudoscalar coupling \( C_P \) becomes prominent. Furthermore, the same nuclear matrix elements can be used in the context of studies of the fundamental structure of the weak charged current of leptons and hadrons, in particular concerning contributions coming from the yet undetected scalar coupling of the current.

In the past there have been many calculations of nuclear matrix elements involved in the muon-capture processes. These calculations have been either very schematic ones \([1,3]\) or more realistic ones using various nuclear models \([2,4]\). Ultimately, all these calculations have aimed at predicting the ratio \( C_P/C_A \) of the induced pseudoscalar and axial-vector coupling strengths of the weak baryonic current by exploiting the scarce experimental information on muon-capture rates in light nuclei. These calculations seem to suggest wide ranges of values \( 4 \leq C_P/C_A \leq 39 \) for \( ^{12}\text{C} \), \( 3 \leq C_P/C_A \leq 20 \) for \( ^{16}\text{O} \) and \( ^{28}\text{Si} \), \( C_P/C_A \geq 13 \) for \( ^{16}\text{N} \), for this ratio. For reference the nuclear-model independent Goldberger-Treiman value is \( C_P/C_A = 6.8 \), which is obtained using the partially conserved axial current hypothesis (PCAC). It should be reasonable to assume that this relation between the induced pseudoscalar and axial-vector coupling constants will survive in finite nuclei, although corrections may occur e.g., due to mesonic corrections in the weak vertices. In particular, this result should be roughly recovered using nuclear-structure calculations assuming the impulse approximation. Renormalizations within the impulse approximation have already been extensively discussed for the \( C_A \) coefficient in the context of beta decay and electron capture. On the experimental side the interval \( 6.8 \leq C_P/C_A \leq 10.6 \) was obtained in the ordinary muon capture in hydrogen \([5]\) and the interval \( 8.8 \leq C_P/C_A \leq 10.8 \) was measured at TRIUMF for the radiative muon capture in hydrogen \([6]\).

Recently, also the nuclear shell-model has been used to calculate the needed nuclear matrix elements for muon capture \([7]\). The calculation of \([7]\) was performed for \( ^{12}\text{C} \) in the 0s and 0p shells but no definitive conclusions for the \( C_P/C_A \) ratio could be reached on the basis of their computed matrix elements. In \([7]\) the muon capture transitions in \( ^{11}\text{B} \) and \( ^{12}\text{B} \) were studied but no conclusions were made with respect to the possible range of the \( C_P/C_A \) value. In \([7]\) a range of \( 4.1 \leq C_P/C_A \leq 8.9 \) was obtained for the ordinary muon capture in \( ^{23}\text{Na} \) by fitting several partial capture rates to the various final states in \( ^{23}\text{Ne} \). In \([7]\) several light nuclei in the 0p, 1s0d and 0p-1s0d shell-model spaces were studied by applying well-established effective two-body interactions in these spaces. In Ref. \([7]\) a comparison of the calculated partial muon-capture rates with the available data indicated a notable suppression (when compared with the above mentioned Goldberger-Treiman value \( C_P/C_A = 6.8 \)) for \( C_P/C_A (\sim 3.0 \leq C_P/C_A \leq 2.5) \) in the case of \( ^{12}\text{C} \) but nothing definitive (the interval \( -7 \leq C_P/C_A \leq 15 \) was examined) could be said in case of the other studied nuclei \( ^{16}\text{O}, ^{20}\text{Ne}, ^{23}\text{Na}, ^{28}\text{Si} \) and \( ^{32}\text{S} \) due to controversial information coming from different partial capture rates in the same nucleus (no best fit procedure was attempted as in \([7]\)).

Very recently two important measurements of correlation coefficients of \( \gamma \)-radiation anisotropy in the capture of a polarized negative muon have been reported \([10,11]\).
For the allowed muon capture the angular correlation between the emitted $\gamma$-radiation and the neutrino is scaled by the coefficient $\alpha$ which is related to the coefficient

$$x \equiv M_1(2)/M_1(-1)$$

of $|\alpha|$ by

$$\alpha = \frac{\sqrt{2} x - x^2 / 2}{1 + x^2}.$$  

Here the quantities $M_1(2)$ and $M_1(-1)$ are linear combinations of the coupling coefficients of the baryonic weak current and the nuclear matrix elements for muon capture [101], [121], [011p], [111p] in the notation of [122]. Using the Fuji-Primakoff approximation (FPA) [13] leads to the following nuclear-model independent result [12]

$$(C_P/C_A)_{\text{FPA}} = \frac{68.37 x}{x + \sqrt{2}} - 2.71.$$  

In the two measurements the following parameter values were extracted: $x = 0.254 \pm 0.034$ [122] and $x = 0.315 \pm 0.08$ [10], leading to the ratios $(C_P/C_A)_{\text{FPA}} = 7.7 \pm 1.2$ and $(C_P/C_A)_{\text{FPA}} = 9.7 \pm 2.6$, respectively, compatible with the PCAC prediction.

Abandoning the FPA and determining the ratio $C_P/C_A$ from Eqs. (1) and (2) using the complicated expressions of $M_1(2)$ and $M_1(-1)$, leads to rather contradictory results between different realistic nuclear models. In [122] the following values of $C_P/C_A$ were extracted using the different realistic nuclear models: $C_P/C_A = 3.4 \pm 1.0$ and $C_P/C_A = 2.0 \pm 1.6$ for the matrix elements of [14] and [122], respectively. In the measurement of [10] the corresponding extracted values are $C_P/C_A = 5.3 \pm 2.0$ [14] and $C_P/C_A = 4.2 \pm 2.5$ [122]. These results would indicate a small quenching of the $C_P/C_A$ ratio with respect to the PCAC value. However, using the more realistic nuclear matrix elements obtained from the full 1s0d shell-model calculation with the sd-shell effective interaction (USD) of Wildenthal [14], would lead to the very contradictory result of $C_P/C_A = 0.0 \pm 3.2$ [14] by employing the computed matrix elements of Junker et al. [17]. The same situation occurs for the calculation of [11], verifying thereby the calculation of [17]. This result is rather surprising when one takes into account the fact that the USD interaction is found to be very successful for 1s0d nuclei in reproducing various spectroscopic quantities like energy spectra, Gamow-Teller decay properties, electromagnetic moments and transitions (see e.g., [8,9]) as well as strength functions of charge-exchange reactions.

The above anomaly in the $C_P/C_A$ predictions from state-of-the-art shell-model calculations is rather disturbing when contrasted with the experimental data. To give a deeper insight, we investigate in the present Letter the capture reaction $^{28}\text{Si}(0^+_g) + \mu^- \rightarrow ^{28}\text{Al}(1^+_s) + \nu_\mu$ within the full shell-model framework and try to evaluate the ratio $C_P/C_A$ through the quantity $x$ and its measured values reviewed above. The needed muon-capture formalism is developed in [11] and reviewed in the case of shell-model calculations in [11]. The two-body interaction matrix elements used in the shell-model calculation are given by the USD interaction [14] and a microscopic effective interaction based on the recent charge dependent nucleon-nucleon interaction of Machleidt and co-workers, the CD-Bonn interaction [20]. This is a meson-exchange potential model. Based on this nucleon-nucleon interaction, we derive a $G$-matrix appropriate for the 1s0d shell. This $G$-matrix is in turn used in a perturbative summation of higher-order terms using the so-called $Q$-box approach described in e.g., Ref. [21]. All diagrams through third order in perturbation theory were used to define the $Q$-box, while folded diagrams were summed to infinite order, see Ref. [22] for further details. Below we will label results obtained with this effective interaction by CD-Bonn. We have also performed similar calculations with the Nijmegen I [22] potential. The results were very close to those obtained with the CD-Bonn potential and hence skipped in the discussion below.

These effective interactions are defined within the 1s0d shell, using $^{16}\text{O}$ as closed shell core, and we perform a full shell-model calculation using the code OXBASH [23].

It is important to keep in mind that the USD interaction is fitted to reproduce several properties of 1s0d shell nuclei, whereas the effective interaction based on the CD-Bonn meson-exchange potential model starts from the bare nucleon-nucleon interaction. The effects of the nuclear medium are then introduced through various terms in the many-body expansion.

The single-particle matrix elements were evaluated in the harmonic-oscillator basis by numerical integration of the radial part containing overlap of the initial and final harmonic-oscillator wave functions and the muonic $s$-state wave function. A harmonic oscillator basis was also employed in our calculation of the $G$-matrix which enters the computation of the effective interaction, using an oscillator parameter of 1.72 fm.

In addition to using the traditional bare transition operators in the evaluation of the nuclear matrix elements for muon capture the aim here is to employ effective transition operators as well. Such a calculation will be referred to as the renormalized one in the discussion below. To obtain effective one-body transition operators for muon capture, we evaluate all effective operator diagrams through second-order in the $G$-matrix obtained with the CD-Bonn interaction, including folded diagrams. Such diagrams are discussed in the reviews by Towner [24] and Ellis and Osnes [25]. Terms arising from meson-exchange currents have been neglected and we omit wave function renormalizations in the calculation of the effective operators.

Intermediate state excitations in each diagram up to $6\hbar\omega$ in oscillator energy were included, as was also the case for the effective shell-model interaction discussed above.

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As such, the determination of both the microscopic shell-model effective interaction and the effective one-body operator is done at the same level of many-body approach. Further details will be presented elsewhere [20]. In general, higher order terms introduce a correction of the order of 10% compared with the bare transition operator. Since the USD interaction is an effective one acting within the $1s0d$ shell only, it is not possible to calculate a corresponding effective operator employing perturbative many-body methods. We will therefore employ the effective operators obtained from the CD-Bonn interaction for the USD calculation as well.

Our results are summarized in Figs. 1 and 2 with $CV/C_A = -1.0$ (for discussion of aspects concerning this ratio see [11] for more details).

Inspection of the nuclear matrix elements for muon capture of Fig. 1 shows that in the case of the USD interaction the renormalization corrections are largest for the largest matrix elements. This, in turn, is reflected in the capture rates, where the renormalized USD calculation shows much better agreement with experiment than the bare one. Thus, the effect of the renormalization is to bridge the gap between the shell-model calculation and experiment. For the CD-Bonn interaction the changes in the reduced matrix elements are more subtle. Generally the bare matrix elements have larger absolute values than the renormalized ones. The USD interaction favours the extreme single-particle limit for the ground state of $^{28}$Si, where all the valence nucleons lie on the $d_{5/2}$ orbit. The CD-Bonn occupancies are more evenly distributed over the various orbits, the largest one being $(d_{5/2})^9 (s_{1/2})^4 (d_{3/2})^2$. The final state of $^{28}$Si has very different structure: the USD calculation indicates that the partition $(d_{5/2})^{10}(s_{1/2})^1(d_{3/2})^3$ should be the most important part in the $1^+_2$ wave function whereas the CD-Bonn interaction predicts the state to be mostly of the $(d_{5/2})^8 (s_{1/2})^2(d_{1/2})^2$ configuration. In general, CD-Bonn calculations favour the configurations which have several particles lifted up from the extreme single-particle shell-model limit which is, on the contrary, favoured by the USD interaction.

To extract an estimate for the ratio $C_P/C_A$, we have plotted $x$ of Eq. (1) as a function of $C_P/C_A$. The experimental value $x = 0.315 \pm 0.08$ is taken from [11]. With this choice we obtain from Fig. 2 the range $-0.4 \leq C_P/C_A \leq 1.2$ for the bare USD and CD-Bonn calculations in agreement with the USD result of [11] cited in [11]. Thus, for both of the adopted interactions the bare result is almost the same in spite of the basic difference in the origin of the used interactions and differences in the resulting wave functions, hinting to a strong suppression of the $C_P/C_A$ ratio for the studied muon-capture transition in the framework of the nuclear shell-model. As discussed before, this contradicts the shell-model calculation of the partial capture rates in $^{23}$Na (with a fitted $C_P/C_A$ ratio) as well as experimental data on hydrogen.

Also the USD and CD-Bonn calculations with renormalized transition probabilities agree almost exactly and both yield a very different value for $C_P/C_A$ than the calculation using bare operators. The renormalized result is $4.4 \leq C_P/C_A \leq 5.9$, close to the PCAC value and the results of the calculations reviewed in [12] and [13]. This result encourages us to believe that the anomaly in the $C_P/C_A$ ratio, inherent in the sophisticated shell-model calculations, has been lifted by introducing effective renormalized transitions operators acting in the muon-capture process. In this way the renormalized shell-model calculations yield a value for $C_P/C_A$ in $^{28}$Si compatible with data and expectations coming from other theoretical approaches.

In conclusion, it is found that the renormalization of the transition operators is essential in the ordinary muon capture even in the case of an empirical effective interaction, which otherwise reproduces the spectroscopy of the involved nuclei extremely well. This is confirmed by the anisotropy data through the quantity $x = M_1(2)/M_1(-1)$. The renormalization has helped to solve the $C_P/C_A$ problem in the shell-model calculations. Further analysis of this observation is in progress for other light nuclei in terms of the capture rates since anisotropy data are only available for $^{28}$Si. For other nuclei of interest the theoretical situation is more complicated than is the case in $^{28}$Si since they are either situated at the interface of the $0p$ and $1s0d$ shells or $1s0d$ and $1p0f$ shells, complicating thereby the evaluation of an effective interaction and increasing the dimensionality of the shell-model calculation. Research along such lines is in progress [20].

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FIG. 1. The values of all the relevant reduced nuclear matrix elements for muon capture computed using the USD and CD-Bonn interactions with and without renormalization of the involved transition operators. The recoil matrix elements $[\ldots p]$ are in units of fm$^{-1}$. 
FIG. 2. The ratio $x = M(2)/M(-1)$ as a function of the ratio $C_P/C_A$. The experimental value with the error limits is indicated by the horizontal lines.