Modifications on parameters of $Z(4430)$ in a dense medium

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Abstract

The charmonium-like resonance $Z_c(3900)$ and its excited state $Z(4430)$ are among the particles that are serious candidates for double heavy tetraquarks. Calculations of different parameters associated with these states both in the vacuum and the medium with finite density are of great importance. Such investigations help us clarify their nature, internal quark-gluon organization and quantum numbers. In this accordance, we extend our previous analyses on the ground state $Z_c(3900)$ to investigate the medium modifications on different parameters of the excited $Z(4430)$ state. In particular, we calculate the mass, vector self-energy and current coupling of $Z(4430)$ in terms of density, up to a density comparable to the density of the core of neutron stars. The obtained results may help experimental groups aiming to study the behavior of exotic states at higher densities.

1 Introduction

Over the past two decades, the results of many experimental observations on some resonances have shown that these states cannot be put in the class of the standard mesons and baryons since their spectrum and decay patterns differ from the standard hadrons, considerably. These results have led us to collect these states under the new title: the exotic states made of more quarks, antiquarks and valence gluons. Among these states, the tetraquarks have been relatively more in the focus of the experimental and theoretical studies. The charmonium-like resonances made of a $c\bar{c}$ and a light $q\bar{q}$ pair receive much attentions as they demonstrate different properties. Investigation of these states can help us not only clarify the nature and internal structures of these states but also get useful knowledge on the nature of strong interaction inside these states.

The observation of the ground states $Z_c^{\pm}(3900)$ were reported simultaneously by the BESIII [1] and Belle [2] Collaborations in 2013. Many aspects of these states have been previously investigated. For details see for instance Refs. [3–7] and references therein. Hence, we won’t spend time on this state in the present work and will focus on the state $Z(4430)$. For a first time in 2008, the Belle Collaboration reported a distinct peak in the $\pi^+\psi'$ invariant mass distribution in $B \to K\pi^+\psi'$ decay with the statistical significance of 6.5σ. The measured mass and width were $M = 4433 \pm 4\text{(stat)} \pm 2\text{(syst)}$ MeV and $\Gamma = 45_{-13}^{+18}\text{(stat)}_{-13}^{+30}\text{(syst)}$ MeV [8]. In an amplitude analysis of $B^0 \to \psi'K^{+}\pi^-$ decays [9], the quantum numbers of the $Z(4430)$ were set to $J^P = 1^+$ and the results for the mass and width were obtained as $4485_{-22}^{+22} \pm 28\text{ MeV}/c^2$ and $200_{-46}^{+41} \pm 26\text{ MeV}$, respectively. The Belle Collaboration then [10], during the observation of a new charmonium like state $Z_c^{+}(4200)$, in addition, they found evidence for $Z_c^{+}(4430) \to J/\psi\pi^+$. In the first independent confirmation by LHCb Collaboration [11], its spin-parity was assigned as $1^+$. The mass of the resonance, $4475 \pm 7_{-12}^{+15}\text{ MeV}$, and its width, $172 \pm 137_{-54}^{+57}\text{ MeV}$, were measured. In the same mass region, an alternative and model-independent confirmation of the existence of a $\psi(2S)\pi$ resonance is later presented by the LHCb, as well [12].

In theoretical side, various assignments have been made on the nature and quark-gluon structure of $Z(4430)$ resonance. Using different phenomenological and computational models, the mass and some other quantities of this state have been studied. Considering a molecular structure, the $Z(4430)$ state was studied in [13–21]. The compact tetraquark or diquark-antidiquark structure was assigned for this state in Refs. [5, 22–29] to calculate various observables associated with the $Z(4430)$ resonance. Other theoretical interpretations can be collected as cusp effects [30–32] and hadrocharmonium [33, 34]. For more information, one can see Refs. [35–43].

In the present work, we shall adopt the compact tetraquark/diquark-antidiquark interpretation of the state $Z(4430)$ and consider it as the redial excitation of the ground state $Z_c(3900)$ with the same spin-parity. In Ref. [25], the authors have made the hypothesis that the spacing (mass difference) in radial excitations in these channels
could closely resemble those observed in the standard P-wave charmonia. Therefore, the authors have considered the state $Z(4430)$ as the first radial excitation of the $Z_c(3900)$ state. The same scenario has been applied in different studies [5,26,44,45], as well. We calculate the mass and current coupling of $Z(4430)$ state in the medium with high density, comparable with the density of the neutron stars' core. We discuss the modifications on the considered physical quantities due to the nuclear dense medium. The medium effects are represented by various operators that enter the nonperturbative parts of the calculations (for details see, for instance, Ref. [46] and references therein). We calculate the values of the mass and current coupling at saturation density. These quantities are also obtained in vacuum setting the density of the medium to zero. We compare the obtained results with the existing theoretical predictions and the experimental data.

2 Parameters of $Z(4430)$ in cold nuclear medium

In this section, we aim to calculate the mass and current coupling as well as the scalar and vector self energies of the exotic $Z(4430)$ (in what follows we denote it by $Z$ ) state as an exited state of the $Z_c(3900)$ ($Z_c$) in cold nuclear medium. To this end, the positively charged $Z^+(4430)$ state with quark content $car{c}u$ is considered. But, the parameters of the negatively charged state with $car{c}d$ content do not change as a result of Chiral limit used in the calculations. In this context, our starting point is to consider the following density dependent two-point correlation function:

$$
\Pi_{\mu\nu}(p) = i \int d^4 x e^{ipx} \langle \psi_0 | T [J^{Z(c)}_{\mu}(x) J^{Z(c)*}_\nu(0)] | \psi_0 \rangle,
$$

(1)

where $| \psi_0 \rangle$ is the ground state of nuclear matter and $J^{Z(c)}_{\mu}$ is the interpolating current of $Z/Z_c$ states that couples to these states, simultaneously. For $J^P = 1^+$, the interpolating current is given by

$$
J^{Z(c)}_{\mu}(x) = \frac{i\epsilon^{\mu\nu\alpha\beta} C_{\alpha\beta}}{\sqrt{2}} \left[ [u^T_{\tau}(x) C \gamma_5 c_b(x)] [\bar{d}_d(x) \gamma_\nu C \bar{c}_c(x)] - [u^T_{\tau}(x) C \gamma_5 c_b(x)] [\bar{d}_d(x) \gamma_\nu C \bar{c}_c(x)] \right],
$$

(2)

where, $a, b, c, d, e$ are color indices and $C$ is the charge conjugation operator.

We need to derive the in-medium sum rules for the mass $m_Z$ and current coupling $f_Z$ of the excited state $Z$. For this purpose, the previously obtained expressions for the mass and current coupling of $Z_c$ (see Ref. [6]) are considered as input parameters. The phenomenological side of the sum rules are obtained by inserting the complete sets of hadronic states with the same quantum numbers as the interpolating current. Isolating the ground state $Z_c$ and $Z$ resonances contributions and integrating over $x$, we obtain the phenomenological side of the correlation function in momentum space as

$$
\Pi^{Phe}_{\mu\nu}(p) = - \frac{\langle \psi_0 | J_{\mu} | Z_c(p) \rangle \langle Z_c(p) | J^{\dagger}_{\nu} | \psi_0 \rangle}{p^*^2 - m^2_{Z_c}} - \frac{\langle \psi_0 | J_{\mu} | Z(p) \rangle \langle Z(p) | J^{\dagger}_{\nu} | \psi_0 \rangle}{p^*^2 - m^2_Z} + ..., \quad (3)
$$

where $p^*$ is the in-medium momentum; and $m_{Z_c}$ and $m_Z$ are the masses of $Z_c$ and $Z$ states, respectively. In Eq. (3), contributions arising from higher resonances and continuum are represented by the dots. The current meson couplings of the states under consideration are expressed in terms of the polarization vectors $\varepsilon_\mu$ and $\bar{\varepsilon}_\mu$, respectively, as

$$
\langle \psi_0 | J_{\mu} | Z_c(p) \rangle = f_{Z_c} m^2_{Z_c} \varepsilon_\mu, \quad \langle \psi_0 | J_{\mu} | Z(p) \rangle = f_{Z_c} m^2_Z \bar{\varepsilon}_\mu. \quad (4)
$$

Using Eq. (4) in Eq. (3) and summing over the polarization vectors, we can construct the new form of the function $\Pi^{Phe}_{\mu\nu}(p)$ as

$$
\Pi^{Phe}_{\mu\nu}(p) = - \frac{m^2_{Z_c} f^2_{Z_c} m^2_{Z_c}}{p^*^2 - m^2_{Z_c}} \left[ - g_{\mu\nu} + \frac{p^*_\mu p^*_\nu}{m^2_{Z_c}} \right] - \frac{m^2_{Z} f^2_{Z} m^2_{Z}}{p^*^2 - m^2_{Z}} \left[ - g_{\mu\nu} + \frac{p^*_\mu p^*_\nu}{m^2_{Z}} \right] + ... \quad (5)
$$

At this point, we should note that a particle in nuclear medium gain two kinds of self energies: The scalar self energy, which is defined as the difference between the in-medium and vacuum masses, $\Sigma_s = m^2_{Z_c} - m_Z$, and the vector self energy that enters the expression of the in-medium momentum, $p^*_\mu = p_\mu - \Sigma_v u_\mu$, where $\Sigma_v$ is the vector self-energy and $u_\mu$ is the four-velocity of the nuclear medium. Our calculations take place in the rest frame of the medium,
corresponding to each structure is expressed in terms of perturbative and non-perturbative contributions as follows:

\[
\Pi_{\mu \nu}^{Phe}(p) = - \frac{f_0^2}{p^2 - m_{Z_c}^2} \left[ - g_{\mu \nu} m_{Z_c}^2 + p_\mu p_\nu - \Sigma_{Z_c}^{u} p_\mu u_\nu - \Sigma_{Z_c}^{u} p_\nu u_\mu + \Sigma_{Z_c}^{u,2} u_\mu u_\nu \right] \\
- \frac{f_0^2}{p^2 - m_{Z_c}^2} \left[ - g_{\mu \nu} m_{Z_c}^2 + p_\mu p_\nu - \Sigma_{Z_c}^{u} p_\mu u_\nu - \Sigma_{Z_c}^{u} p_\nu u_\mu + \Sigma_{Z_c}^{u,2} u_\mu u_\nu \right] + \ldots,
\]

where \( m_{Z_c}^2 = m_{Z_c}^2 - \Sigma_{Z_c}^{u,2} + 2p_0 \Sigma_{Z_c}^{u} \). After applying the Borel transformation with respect to the parameter \( p^2 \)
to both sides of Eq. (6), the phenomenological side of the correlation function yields,

\[
\Pi_{\mu \nu}^{Phe}(p) = f_0^2 e^{-\frac{m_{Z_c}^2}{M^2}} \left[ - g_{\mu \nu} m_{Z_c}^2 + p_\mu p_\nu - \Sigma_{Z_c}^{u} p_\mu u_\nu - \Sigma_{Z_c}^{u} p_\nu u_\mu + \Sigma_{Z_c}^{u,2} u_\mu u_\nu \right] \\
+ f_0^2 e^{-\frac{m_{Z_c}^2}{M^2}} \left[ - g_{\mu \nu} m_{Z_c}^2 + p_\mu p_\nu - \Sigma_{Z_c}^{u} p_\mu u_\nu - \Sigma_{Z_c}^{u} p_\nu u_\mu + \Sigma_{Z_c}^{u,2} u_\mu u_\nu \right] + \ldots,
\]

where \( M^2 \) is the Borel mass parameter.

The next step is to calculate the QCD side of the correlation function. This is done via the usage of the explicit form of the interpolating current in the correlation function. The quark fields are contracted in the presence of the dense medium via the Wick’s theorem. This results in the expression of the correlation function in terms of the in-medium heavy and light propagators:

\[
\Pi_{\mu \nu}^{QCD}(p) = - i \frac{\bar{\epsilon}_{abc} \bar{\epsilon}_{a'\nu} \epsilon \bar{\epsilon}_{dc} \epsilon_{d'\mu}}{24 \pi^4} \int d^4x e^{ipx} \left\{ Tr \left[ \gamma_5 \tilde{S}_{u}^{a'a'}(x) \gamma_5 S_{c}^{b'b'}(x) \right] - Tr \left[ \gamma_5 \tilde{S}_{c}^{a'a'}(x) \gamma_5 \tilde{S}_{u}^{b'b'}(x) \right] \right\} |_{\psi_0},
\]

where \( \tilde{S}_{q(c)} = C \bar{s} S_{q(c)}^T C \). The in-medium heavy and light quarks’ propagators are used in coordinate space and the calculations are transferred to the momentum space by performing the Fourier integrals. To suppress the contributions of the higher states and continuum the Borel transformations are applied based on the standard prescriptions of the method. To further suppress the contributions of the unwanted states, continuum subtraction supplied by the quark-hadron duality assumption is applied. All of these procedures are described in Ref. [6]. Thus, the QCD side of the correlation function in terms of the selected Lorentz structures is written as

\[
\Pi_{\mu \nu}^{QCD}(M^2, s_0) = \mathcal{Y}_1^{QCD}(M^2, s_0)(-g_{\mu \nu}) + \mathcal{Y}_2^{QCD}(M^2, s_0)p_\mu p_\nu + \mathcal{Y}_3^{QCD}(M^2, s_0)(-p_\mu u_\nu) \\
+ \mathcal{Y}_4^{QCD}(M^2, s_0)(-p_\nu u_\mu) + \mathcal{Y}_5^{QCD}(M^2, s_0)u_\mu u_\nu.
\]

The Borel transformed invariant functions \( \mathcal{Y}_i^{QCD}(M^2, s_0) \) in Eq. (9) are represented in terms of the two-point spectral densities, \( \rho_i^{QCD}(s) \), related to the imaginary parts of the selected coefficients:

\[
\mathcal{Y}_i^{QCD}(M^2, s_0) = \int_{4m_0^2}^{s_0} ds \rho_i^{QCD}(s) e^{-\frac{s}{M^2}},
\]

where \( s_0 \) is the in-medium continuum threshold parameter separating the contributions of the ground state \( Z_c \) and the first excited state \( Z \) from the higher resonances and continuum. \( i \) runs from 1 to 5. The spectral density corresponding to each structure is expressed in terms of perturbative and non-perturbative contributions as follows:

\[
\rho_i^{QCD}(s) = \rho_i^{pert}(s) + \rho_i^{qq}(s) + \rho_i^{gg}(s) + \rho_i^{qg}(s),
\]

where \( q, g, gq, gg \) denote the two quark, two gluon and mixed quark-gluon condensates as the non-perturbative effects, respectively. As an example, the spectral density corresponding to the structure \( g_{\mu \nu} \) is given in Ref. [6].
After all these lengthy calculations, when the phenomenological and QCD results of each structure are compensated, the sum rules for the mass and current coupling constant of $Z_c$ state are obtained as follows:

\[
\begin{align*}
  m_{Z_c}^2 f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} + m_{Z_c}^2 f_{Z_c} e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} &= \Upsilon_{1}^{QCD}(M^2, s_0), \\
  f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} &= \Upsilon_{2}^{QCD}(M^2, s_0), \\
  \Sigma_{Z_c}^2 f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} + \Sigma_{Z_c} f_{Z_c} e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} &= \Upsilon_{3}^{QCD}(M^2, s_0), \\
  \Sigma_{Z_{u}}^2 f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} + \Sigma_{Z_{u}} f_{Z_c} e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} &= \Upsilon_{4}^{QCD}(M^2, s_0), \\
  \Sigma_{Z_{u}}^2 f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} + \Sigma_{Z_{u}} f_{Z_c} e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}} &= \Upsilon_{5}^{QCD}(M^2, s_0).
\end{align*}
\]

The mass sum rules for $Z$ state can be derived by different ways from the above equations. We apply derivative with respect to $-\frac{1}{M_{\pi}^2}$ to both sides of Eq. (12) and eliminate $f_{Z_c}^2$ by dividing both sides of the resultant equation to both sides of the original equation. As a result we get

\[
\mu_{Z_c}^2 = -\frac{\mu_{Z_c}^2}{M_{\pi}^2} \left( \frac{\rho \Upsilon_{1}^{QCD}(M^2, s_0) - m_{Z_c}^2 f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}}}{\Upsilon_{1}^{QCD}(M^2, s_0) - m_{Z_c}^2 f_{Z_c} e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}}} \right),
\]

where, the in-medium mass of $Z$ state is found as

\[
m_{Z_c}^2 = \mu_{Z_c}^2 + \Sigma_{Z_{u}}^2 - 2\rho_0 \Sigma_{Z_{u}}^2.
\]

By rearranging the equations (16) and (13) and dividing both sides of these equations to each other we get $\Sigma_{Z_{u}}^2$ as

\[
\Sigma_{Z_{u}}^2 = \frac{\Upsilon_{5}^{QCD}(M^2, s_0) - \Sigma_{Z_{u}}^2 f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}}}{\Upsilon_{2}^{QCD}(M^2, s_0) - f_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}}},
\]

To obtain the in-medium current coupling constant, $f_{Z_c}^2$, there are different possibilities. One way is to find it from Eq. (12), which leads to

\[
f_{Z_c}^2 = \frac{\Upsilon_{1}^{QCD}(M^2, s_0) - m_{Z_c}^2 f_{Z_c} e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}}}{m_{Z_c}^2 e^{-\frac{\mu_{Z_c}^2}{M_{\pi}^2}}}.
\]

As it is seen, the parameters of $Z_c$ calculated in Ref. [6] are entered as the inputs to the sum rules for $Z$ state in the dense medium.

### 3 Numerical results

The expressions obtained in Eqs. (17-20) for the physical observables in cold nuclear matter contain quark and baryon masses, nuclear matter density and different operators both in vacuum and dense medium as input parameters. The operators appearing in the calculations in a dense medium are presented in Ref. [46]. The numerical results for expectation values of different operators together with the values of other input parameters are collected in table (1). The obtained in-medium sum rules for the mass, current coupling and vector self-energy of $Z$ state contain two more auxiliary parameters, as well. These parameters are fixed based on the standard requirements of the method: mild variations of the physical quantities with respect to the changes in the values of these parameters, dominance of first two resonances over the higher states and continuum and convergence of the OPE. These requirements are all satisfied when

\[
\begin{align*}
  M^2 &\in \{3 - 5\} \text{ GeV}^2, \\
  s_0 &\in \{21.9 - 23.8\} \text{ GeV}^2.
\end{align*}
\]

In Fig. (1), we demonstrate the in-medium mass $m_{Z_c}^2$ as a function of Borel mass parameter $M^2$ at the saturation nuclear mass density, $\rho_{sat} = 0.113$ GeV$^3$, and at three fixed values of the continuum threshold. As is seen, the in-medium mass of the $Z(4430)$ resonance shows pretty good stability against variations of both of Borel mass.
 parameter $M^2$ and in-medium continuum threshold $s_0$. In Fig. (2), the vacuum mass ($m_Z$) which is obtained at $p \to 0$ limit, the in-medium mass ($m_Z^*$) and vector self energy ($\Sigma^V_N$) of $Z$ state are displayed as a function of $M^2$ at average value of continuum threshold and at saturation nuclear matter density. From this figure we see a considerable negative shift in the mass of the state under study due to the medium effects. This shift is called the scalar self-energy. Strictly speaking, the in-medium mass of the $Z$ resonance reduces to approximately 74% of its vacuum value at the saturation nuclear matter density and amounts as $3311_{-185}^{+203}$ MeV. Thus, the negative shift in the mass due to the cold nuclear matter is approximately 26%. This state gains a vector self energy with the value $1310_{-186}^{+301}$ MeV at saturation nuclear matter density. At saturation density the current coupling of the state $Z$ is found as $1.34_{-0.45}^{+0.78}$ GeV. The numerical results for the masses and current couplings obtained in vacuum and medium are collected in table (2). For comparison, we demonstrate the existing values from the experiment and other theoretical studies in the same table, as well. From this table, we see that the vacuum mass obtained in the present study is nicely consistent with the world experimental average [48]. This value is also in accord with other theoretical predictions within the errors. The vacuum current coupling is consistent with other theoretical predictions within the presented uncertainties. Our predictions for the in-medium mass and current coupling as well as the vector self-energy can be tested in future in-medium experiments and by other theoretical studies.

The main aim in the present study, is to obtain the behavior of the physical quantities under study with respect to the density, specially at higher densities. In the present study, we consider the range from zero to $5\rho^{\text{sat}}$, corresponding

### Table 1: Input parameters and their numerical values used in calculations.

| Parameter | Value | Unit | Ref. # |
|-----------|-------|------|--------|
| $\rho^{\text{sat}}$ | 0.11$^3$ | - | [47] |
| $p_0$ | $4.478_{-0.015}^{+0.015}$ | GeV | [48] |
| $m_u$ | $2.16_{-0.28}^{+0.18}$ | MeV | [48] |
| $m_d$ | $4.67_{-0.17}^{+0.17}$ | MeV | [48] |
| $m_e$ | $1.27 \pm 0.02$ | GeV | [48] |
| $\langle q^2 q \rangle_\rho$ | $2 \rho$ | GeV$^3$ | [49] |
| $\langle q^2 q \rangle_0$ | $-2 \langle 5 \rangle^3$ | MeV$^3$ | [50-55] |
| $\langle q^2 q \rangle_\rho$ | $\langle q^2 q \rangle_0 + \frac{2 \rho}{\tilde{m}_\rho}$ | MeV$^3$ | [56] |
| $m_\rho$ | 0.00345 | GeV | [48] |
| $(\tilde{m}_\rho^2 G^2)_0$ | 0.012 $\pm$ 0.004 | GeV$^4$ | [50, 57, 58] |
| $(\tilde{m}_\rho^2 G^2)_\rho$ | $(\tilde{m}_\rho^2 G^2)_0 + \rho (\tilde{m}_\rho^2 G^2)_N$ | GeV$^4$ | [50, 59, 60] |
| $M_N$ | 939.49 $\pm$ 0.05 | MeV | [48] |
| $\sigma_{\pi N}$ | 45(6) | MeV | [61] |
| $\sigma_{s N}$ | 21(6) | MeV | [61] |
| $\langle q^2 D_{q} \rangle_\rho$ | $\frac{4}{3} m_\rho$ | GeV$^4$ | [50, 56] |
| $\langle q^2 g \sigma G q \rangle_\rho$ | $m_\rho^2 \langle q^2 q \rangle_\rho$ | GeV$^5$ | [50] |
| $\langle q^2 g \sigma G q \rangle_0$ | $m_\rho^2 \langle q^2 q \rangle_0$ | GeV$^5$ | [50] |
| $\lambda_\rho^2$ | 0.8 $\pm$ 0.2 | GeV$^2$ | [50, 62] |
| $\langle q^2 D_{q} \rangle_0$ | $\frac{\lambda_\rho^2 \sigma_{\pi N}}{2 \tilde{m}_\rho}$ | GeV$^5$ | [50, 56] |
| $\langle q^2 D_{q} \rangle_\rho$ | $\frac{\lambda_\rho^2 \sigma_{\pi N}}{2 \tilde{m}_\rho} - \frac{1}{2} \langle q g \sigma G q \rangle_\rho$ | GeV$^5$ | [50, 56] |
| $\lambda_\rho^2$ | 0.4 $\pm$ 0.1 | GeV$^2$ | [50, 62] |

| Table 2: The vacuum and in-medium masses and current couplings of $Z$ state. PS stands for present study. |

| Parameter | Value | Unit | Ref. # |
|-----------|-------|------|--------|
| $m_Z$ | $4486_{-115}^{+112}$ | MeV | [48] |
| $m_Z^*$ | $3311_{-185}^{+203}$ | MeV | [48] |
| $f_Z \times 10^2$ | $1.27_{-0.36}^{+0.33}$ | GeV$^4$ | [48] |
| $f_{Z^*} \times 10^2$ | $1.34_{-0.45}^{+0.78}$ | GeV$^4$ | [48] |

PS stands for present study.
to the density of the core of the neutron stars. To this end, Fig. (3) displays $m^*_Z$ and $\Sigma^Z_\nu$ as functions of $\rho/\rho^{sat}$ at average values of the continuum threshold and Borel parameter. As it is clear, the sum rules for both the in-medium mass and vector self-energy give reliable results up to $\rho = 1.6\rho^{sat}$ GeV$^3$. To extend our calculations for higher densities, we use some fit functions. The best fits for the mass is the following exponential function:

$$m^*_Z(\rho) = 4.384e^{-0.233x} \text{GeV}, \quad (22)$$

where $x = \rho/\rho^{sat}$. For vector self-energy we obtain

$$\Sigma^Z_\nu(\rho) = 0.099x^2 + 0.917x + 0.272 \text{ GeV}. \quad (23)$$

In Fig. (4), we plot the density-dependent current coupling. The following function best describes the current coupling:

$$f^*_Z(\rho) = -0.002x^2 + 0.002x + 0.015\text{GeV}^4. \quad (24)$$

The current coupling becomes zero at $x = 3.2$. The vector self energy increases with the increasing in the density, while the mass decreases with increasing in the density and reaches to roughly $1.4\text{GeV}$ at $x = 5$ in average. Figure 5 shows the $m^*_Z - x$ graphic when all the uncertainties in the auxiliary parameters as well as in other inputs are considered.

4 Summary and conclusions

The vacuum properties of different tetraquark states have been discussed in many theoretical studies as well as different experiments. With the developments in the experimental side, we hope we will be able to study the behavior of these states in in-medium experiments like $\bar{P}$ANDA in near future. Thus, theoretical studies on the spectroscopic properties of exotics can play crucial roles in conducting the related experiments. Such studies can also help us fix the quantum numbers and determine the nature and internal structures of the famous candidates of charmonium-like tetraquarks. In this accordance, we took into consideration the $Z(4430)$ state in the present
study and calculated some spectroscopic parameters of this state both in vacuum and a medium with finite density. Based on some experimental information, we treated this state as the first excited state in the $Z_c(3900)$ channel and constructed the related sum rules based on the standard prescriptions of the method. As we saw, the parameters of the ground state $Z_c(3900)$ were entered as input parameters to the sum rules for $Z(4430)$ state. We fixed the entering auxiliary parameters in accordance with the requirements of the method and used the expectation values of the different in-medium operators available in the literature to extract the values of the parameters at saturation density and determine the behavior of these parameters with respect to the density.

The numerical calculations showed that the mass gains a negative shift (scalar self-energy) due to the medium effects: this shifts amounts $-26\%$ at saturation density. This state gains a repulsive vector self energy which amounts $1310^{+101}_{-86}$ MeV at saturation density. We also found the values of the current coupling as $1.34^{+0.78}_{-0.45}$ GeV$^4$ at saturation nuclear matter density. This value can be used in determinations of different parameters related to the electromagnetic, weak and strong interactions/decays of this state in medium.

We obtained the behavior of the physical quantities in the interval $\rho \in [0, 5]\rho^{sat}$, which its upper limit corresponds to the core density of the neutron stars. We observed that the current coupling becomes zero at $\rho = 3.2\rho^{sat}$, while the vector self-energy increases with the density. The mass exponentially reduces with the increasing in the value of the density and reaches to roughly $1.4\text{GeV}$ at the end point. Such behavior of the melting of the mass of $Z(4430)$ state with respect to density may be checked in future in-medium experiments.

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Figure 3: In-medium mass and vector self energy as functions of $\rho/\rho^{\text{sat}}$ at mean value of the continuum threshold and Borel parameter.

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