Critical exponents for the metal-insulator transition of $^{70}$Ge:Ga in magnetic fields

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Abstract
We have measured the electrical conductivity of nominally uncompensated $^{70}$Ge:Ga samples in magnetic fields up to $B = 8$ T at low temperatures ($T = 0.05 - 0.5$ K) in order to investigate the metal-insulator transition in magnetic fields. The values of the critical exponents in magnetic fields are consistent with the scaling theories.

1 Introduction
Doped crystalline semiconductors are ideal solids to probe the effects of both disorder and electron-electron interaction on the metal-insulator transition (MIT) in disordered electronic systems. Important information about the MIT is provided by the critical exponent $\mu$ for the zero-temperature conductivity $\sigma(0)$ defined by

$$\sigma(0) \propto (N - N_c)^\mu,$$

where $N$ is the dopant concentration and $N_c$ is the critical concentration for the MIT, and $\mu \approx 0.5$ has been found in a number of nominally uncompensated semiconductors including our $^{70}$Ge:Ga.

According to the theories of the MIT, the value of the critical exponents does not depend on the details of the system, but depends only on the universality class to which the system belongs. In this sense, the application of a magnetic field is important because the motion of carriers loses its time-reversal symmetry in magnetic fields, and the universality class changes. In our earlier work, we reported that a different exponent $\mu = \mu' = 1.1 \pm 0.1$ is obtained in magnetic fields.

Here, $\mu'$ characterizes the magnetic-field-induced MIT:

$$\sigma(0) \propto (B_c - B)^{\mu'}.$$

Since this result is based solely on the metallic samples, in this work we perform a finite-temperature scaling analysis which uses the data on the both sides of the transition. Moreover, we investigate the temperature dependence of the conductivity on the insulating side in the context of variable-range-hopping (VRH) conduction.

2 Experiment
All the samples were prepared by neutron-transmutation-doping (NTD) of isotopically enriched $^{70}$Ge single crystals. The NTD

method assures a homogeneous Ga acceptor distribution which is a crucial condition for experimental studies of the MIT. The electrical conductivity was measured at low temperatures between 0.05 and 0.5 K.

3 Results and discussion
We show in Fig. 1 that $\mu' = 1.1$ (Ref. 3) yields an excellent finite-temperature scaling plot

$$\sigma(B, T) T^x \propto f\left(\frac{|B_c - B|}{T^y}\right),$$

where $x/y$ is equivalent to $\mu'$. Here we employ $B_c$ obtained by fitting Eq. (3). The temperature variation of the conductivity is proportional to $T^{1/2}$ even around the critical point in magnetic fields, leading to $x = 1/2$. Note that $y = x/\mu' = 0.45$, i.e., none of these parameters are used as a fitting parameter. Hence, Fig. 1 strongly supports $\mu' = 1.1$.

The temperature dependence of the conductivity on the insulating side of the MIT is shown in Fig. 2. We already reported that VRH conductivity at $B = 0$ obeys Efros and

Fig. 1 Finite-temperature scaling plot for the magnetic-field-induced metal-insulator transition in the sample having $N = 2.004 \times 10^{17}$ cm$^{-3}$.
Conductivity multiplied by $T^{-1/2}$ as a function of $T^{-3/5}$ for the sample having $N = 1.912 \times 10^{17}$ cm$^{-3}$ in magnetic fields. From top to bottom in units of tesla, the magnetic induction is 5, 6, 7, and 8, respectively.

Shklovskii’s (ES) law [3]

$$\sigma(N, 0, T) = \sigma_0(N, 0) \exp[-(T_0/T)^{1/2}]$$

(4)
even in the immediate vicinity of $N_c (0.99N_c < N < N_c)$ when an appropriate temperature dependence of the prefactor $\sigma_0 \propto T^r$ is taken into account [5]. Based on this finding, we analyze in this work the data at $B \geq 5$ T in the context of ES VRH conduction

$$\sigma(N, B, T) = \sigma_0(N, B) \exp[-(T_0/T)^{3/5}]$$

(5)
in a strong field ($\sqrt{\hbar/eB} \ll \xi$, where $\xi$ is the localization length) [3]. By assuming $\sigma_0 \propto T^{1/2}$, which is consistent with the above finite-temperature scaling analysis, the conductivity in magnetic fields is described well by Eq. (5) as seen in Fig. 3. The values of $T_0$ in Eq. (5) satisfy the relations $T_0 \propto (N_c - N)^{\alpha}$ and $T_0 \propto (B - B_c)^{\alpha'}$, and $\alpha = \alpha' \approx 2.9$ is obtained from the data satisfying $T_0 > 0.05$ K. (See Fig. 3.) Note that the condition $T < T_0$ is required for the ES theory to be valid, i.e., $T_0$ has to be evaluated only from the data obtained at temperatures low enough to satisfy this condition. Since $T_0 \propto (\xi\varepsilon)^{-1}$ and both $\xi$ and the dielectric constant $\varepsilon$ diverge at $N_c$, $\alpha = v + \zeta$. Here, $v$ and $\zeta$ are given by $\zeta \propto (N_c - N)^{-1}$ and $\varepsilon \propto (N_c - N)^{-3}$, respectively. The relation $2v \approx \zeta$, which was predicted theoretically [7], has been obtained at $B = 0$ for the present system by measuring magnetoresistance in weak

![Fig. 3](image_url)

$T_0$ determined by $\sigma(T) \propto T^{1/2} \exp[-(T_0/T)^{3/5}]$ as a function of $1 - N/N_c(B)$ in constant magnetic fields of $B = 5, 6, 7$, and 8 T (left data set), and as a function of $B/B_c - 1$ for the sample having $N = 1.912 \times 10^{17}$ cm$^{-3}$ (right data set). The dashed lines represent the best fits to the data satisfying $T_0 > 0.05$ K.

Fields ($\sqrt{\hbar/eB} \gg \xi$). Assuming the relation $2v = \zeta$, $v \approx 1$ is obtained for $B \neq 0$, and hence, the Wegner relation $\mu = v$ [3] holds in magnetic fields.

4 Conclusion

The critical exponent of the zero-temperature conductivity for the metal-insulator transition of doped semiconductors in magnetic fields is confirmed to be close to unity. Other exponents in magnetic fields are also explained within the context of the scaling theories.

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