Theory and simulations of radiation friction induced enhancement of laser-driven longitudinal fields

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Abstract
We consider the generation of a quasistatic longitudinal electric field by intense laser pulses propagating in a transparent plasma with radiation friction (RF) taken into account. For both circular and linear polarization of the driving pulse we develop a 1D analytical model of the process, which is valid in a wide range of laser and plasma parameters. We define the parameter region where RF results in an essential enhancement of the longitudinal field. The amplitude and the period of the generated longitudinal wave are estimated and optimized. Our theoretical predictions are confirmed by 1D and 2D PIC simulations. We also demonstrate numerically that RF should substantially enhance the longitudinal field generated in a plasma by a 10 PW laser such as ELI Beamlines.

Keywords: radiation friction, radiation pressure, laser-plasma interaction, charge separation, ion acceleration

(Some figures may appear in colour only in the online journal)

1. Introduction

The modern laser systems have recently reached the multi-Petawatt power level\textsuperscript{[1–4]}. These facilities already provide extremely strong fields \(\approx 10^{13} \text{ V cm}^{-1}\)\textsuperscript{[5]}, but even more powerful 10 PW lasers are now under construction in Czech Republic (ELI Beamlines [6]), Romania (ELI NP [7]) and France (Apollon [8]). However, since laser fields oscillate with very high frequency, their application to charged particle acceleration is not straightforward. By separating charges, a laser pulse propagating in plasma creates a wake (longitudinal) wave. As suggested many years ago\textsuperscript{[9]} and demonstrated in a recent experiment\textsuperscript{[10]}, this wake wave can efficiently accelerate electrons up to several GeVs.

Several models were suggested also for ion acceleration in various laser-plasma setups\textsuperscript{[11, 12]}, see the reviews\textsuperscript{[13]} and the overview of the recent experimental achievements in [14]. In all such schemes the laser pulse pushes plasma electrons forward via the ponderomotive force, thus leading to charge separation and creation of a quasistatic longitudinal electric field\textsuperscript{[15]} capable for particle acceleration. However, as recently demonstrated\textsuperscript{[16]}, besides the usual ponderomotive mechanism (PM) of charge separation in plasma, there also exists a competing radiation friction mechanism (RFM), which for strong and long laser pulses propagating in low density plasmas should dominate over PM.

To clarify its origin, consider an electron in a circularly polarized plane wave. If radiation friction (RF) is neglected, the electron circles with the field frequency, so that its velocity remains perpendicular to the electric field and parallel to the magnetic field\textsuperscript{[17]}, with no force acting in the direction of pulse propagation. RF in transverse direction changes the angle between the electron velocity and the magnetic field\textsuperscript{[16, 18]}, resulting in the longitudinal Lorenz force \(v \times B\)\textsuperscript{3}, which accelerates the electron in the direction of pulse propagation, thus separating charges in plasma.

\textsuperscript{3} We use CGS-like units but with speed of light \(c = 1\).
The role of RF in laser-plasma interaction was previously a subject of many investigations [19–25], but mostly for overcritical plasmas. However, if plasma is so dense that it is opaque, then electrons never get inside the pulse, where the field is so strong that RF affects the electron transverse motion. As for the overcritical but relativistically transparent case, in a high density plasma RF results in a fast depletion of the laser pulse. Therefore we consider plasma density not exceeding the critical value $n_c = m\omega^2/4\pi e^2$, where $\omega$ is the laser frequency, $m$ and $e$ are the electron mass and charge.

In such a regime charge separation and generation of the quasistatic longitudinal electric field by an intense laser pulse in a plasma with immobile ions are perfectly described by the following 1D model [16]. A pulse entering a plasma accelerates the electrons forward via RFM and/or PM, piling them up into a moving density spike. Thereupon the longitudinal electrostatic field of the naked immobile ions is gradually growing, and eventually starts decelerating the electrons. Finally a breakdown occurs, when a part of electrons from the spike penetrates through the pulse backward partially screening the ion field, see figure 1. The amplitude and the period of the thus generated longitudinal field are defined by the position of the spike at the moment of breakdown, and to find it we consider the motion of the leftmost particle in the electron spike (see figure 1) reads:

$$\frac{dv}{ds} = \omega^2 \left( \frac{dA_i}{d\varphi} \frac{dA_i}{d\varphi \cos \gamma} \right) u_i^3 + k_i F_i^\parallel,$$

(1)

where $u_\parallel = k_i u_i/\omega = \gamma - u_i$, From $u_i u_\parallel = 1$ and $dr = \gamma ds = (\gamma/\omega u_\parallel) d\varphi$ one can easily derive the formulas

$$\frac{du}{ds} = u \left( \frac{\omega^2}{2\gamma} \frac{dA_i}{d\varphi} - \frac{du}{d\varphi} \right),$$

$$u^2 = (1 + u^2)(1 - v_i)\left(1 + v_i\right),$$

using which follows

$$\frac{du}{d\varphi} = \frac{\omega}{2\gamma} \frac{dA_i}{d\varphi} - \frac{2e^2}{3m^3} \left( \frac{dA_i}{d\varphi} \right)^2 \left(1 + u_i^2\right) \left(1 - v_i\right)\left(1 + v_i\right)$$

- \frac{4\pi e^2 n_c}{m} \chi.$$

(3)

An advantage of the above derivation is its generality with respect to polarization of the driving laser field, on which no assumptions have been made yet. We now consider the most important cases of circular (CP) and linear (LP) polarization.

\subsection*{2.1. Circular polarization}

If the laser pulse is circularly polarized $A_i(\varphi) = (m/e)a(\varphi)$ [0, cos $\varphi$, sin $\varphi]$ then $(dA_i/d\varphi)^2 = \frac{m^2 a^2}{e^2}$, where $a(\varphi)$ is a slowly varying dimensionless pulse envelope. Denoting the ions immobile, derive the estimations for various regimes of longitudinal field generation, and identify an optimal regime with generation of the highest attainable longitudinal field. Furthermore, by 2D PIC simulations we demonstrate that our 1D model is valid for wide pulses (of comparable length and width), and that RFM can dominate over PM also for tightly focused pulses with the parameters of the upcoming facilities such as ELI Beamlines [6].

The model developed in [16] provides estimations for the amplitude and the period of the longitudinal field generated by a circularly polarized laser pulse via both PM and RFM under the additional assumption of ultrarelativistic electron longitudinal motion, that are in a rather good agreement with PIC simulations. Here we discuss this model in more detail and generalize it to the cases of arbitrary polarization of the laser pulse and arbitrary electron longitudinal velocities, thus extending its validity to a wider range of laser parameters. We formulate and study the limits of applicability of the model (in particular establish the conditions required to consider the
dimensionless variables \( \tau = \omega t, \xi = \omega x \) and parameters \( \bar{n} = n/n_e, \mu = 2\pi \varepsilon_0/\lambda^2 \approx 3.18 \times 10^{-8} \), where \( \varepsilon_0 = e^2/m \) is the classical electron radius, we can cast (3) into the form

\[
\frac{d\mu}{d\tau} = \frac{1}{2\gamma} \left( \frac{d(\xi^2)}{d\varphi} + \mu a^2 (1 + u^2) \right) \frac{1 - v_x}{1 + v_x} - \bar{n} \xi. \tag{4}
\]

Equations (3) and (4) are the exact implications of (1) and hence up to this moment no approximations except for the Landau–Lifshitz approximation and the assumption of a dilute plasma have been made. But if we further assume that damping due to RF is small (see section 2.3 for discussion) then \( n_e \approx \bar{n} a \), and, neglecting also \( 1 \) against \( a^2 \), we can further reduce (4) to the form

\[
\frac{d\mu}{d\tau} = \frac{1}{2\gamma} \frac{d(a^2)}{d\varphi} + \mu a^4 \frac{1 - v_x}{1 + v_x} - \bar{n} \xi. \tag{5}
\]

This is precisely the 1D model suggested in [16]. As mentioned in the Introduction, there are two mechanisms of electron longitudinal acceleration and charge separation in a plasma: PM (corresponding to the first term in the RHS of (5)), and RFM (corresponding to the second term proportional to \( \mu \)).

Since the laser field under consideration is strong, \( a \gg 1 \), electron velocity is close to the speed of light. Still, its longitudinal motion can be either nonrelativistic (\( u_x \ll a \) and, accordingly, \( v_x \ll v_{\parallel} \approx 1 \)) or ultrarelativistic (\( u_x \gg a \) and \( v_x \ll v_{\parallel} \approx 1 \)), where \( v_{\parallel} \) is the velocity component in the direction transverse to laser pulse propagation. In order to further simplify (5) we consider these limiting cases.

2.1. Nonrelativistic longitudinal electron motion. In the nonrelativistic case \( v_x \ll 1 \) we have \( \gamma \approx a, u_x \approx v_x a, \varphi \approx \tau \), and (5) is the same as for a damped harmonic oscillator with slowly varying parameters and shifted equilibrium position:

\[
\ddot{\xi} + 2\dot{\xi} + \Omega^2 \xi = f, \quad \varepsilon(\tau) = \mu a^3 + \frac{\dot{\alpha}}{2a},
\]

\[
\Omega(\tau) = \sqrt{\bar{n} a}, \quad f(\tau) = \mu a^3 + \frac{\dot{\alpha}}{a}, \tag{6}
\]

where the dots abbreviate the derivatives with respect to \( \tau \). In dilute plasmas under consideration here this effective oscillator is always over-damped (\( \varepsilon \gtrsim \Omega \)) and very rapidly (within \( \tau \lesssim \varepsilon/\Omega^2 \)) approaches its equilibrium

\[
\xi_{0}(\tau) \approx \frac{f(\tau)}{\Omega^2(\tau)} = \frac{\mu a^4(\tau) + \dot{\alpha}(\tau)}{\bar{n}}. \tag{7}
\]

Particle motion is nonrelativistic, if \( \xi_{0}(\tau) \ll 1 \), or

\[
T \gg \max \left\{ \frac{\mu a_0^4}{\bar{n}}, \frac{a_0}{\bar{n} T} \right\}, \tag{8}
\]

where \( T = \omega_{\text{pulse}} \) is the dimensionless pulse duration, \( a_0 = a(0) \) is the maximum value of laser field, and we estimated \( \dot{\alpha} \sim a_0/T \). As a matter of fact, this also means, that \( T \gg x/\Omega^2 \), which implies that the oscillator almost immediately reaches an equilibrium.

Hence the amplitude of longitudinal charge separation field \( a_{\parallel}^{(\text{RFM})} \approx \bar{n} \xi_{\text{max}} \) can be estimated as

\[
a_{\parallel}^{(\text{RFM})} = \bar{n} \xi_{\text{max}} \approx \max [\mu a^4(\varphi) + \dot{\alpha}(\varphi)]
\]

\[
= \begin{cases} 
\mu a_0^4, & \mu a_0^4 T \gg 1, \\
0.858 a_0/T, & \mu a_0^4 T \ll 1. 
\end{cases} \tag{9}
\]

For a Gaussian envelope \( a = a_0 e^{-\varphi^2/T^2} \) used further for simulations the final maximization cannot be done analytically, but only numerically, because of occurrence of a transcendental equation. The limiting cases above are also given for this particular case. Note that in the nonrelativistic case RF becomes dominant if

\[
\mu a_0 T \gtrsim 1, \tag{10}
\]

i.e. for intensities \( I \gtrsim 10^{22} \text{ W cm}^{-2} \) if we assume the FWHM pulse duration 150 fs and \( \lambda = 1 \mu\text{m} \) as announced for ELI Beamlines [6].

2.1.2. Ultrarelativistic electrons: RF-induced charge separation. From now on assume that longitudinal electron motion is ultrarelativistic, \( \gamma \approx u_x \approx a_0 \). Then we have:

\[
1 - v_x \approx a^2/2u_x^2, \quad 1 + v_x \approx 2, \quad \xi \approx \tau, \quad \text{so that (5) reduces to}
\]

\[
\frac{d\mu}{d\tau} = \frac{1}{2\gamma} \frac{d(a^2)}{d\varphi} + \mu a^6 \frac{1 - v_x}{4u_x^2} - \bar{n} \tau. \tag{11}
\]

Let us consider first the RFM of charge separation and neglect the first (ponderomotive) term in the RHS of (11). The process obviously splits into two stages. During the acceleration stage \( 0 < \tau < \tau_{\text{acc}} \), the last electrostatic term in the RHS remains small and can be also neglected, hence

\[
\int_{0}^{\tau_{\text{bd}}} (1 - v_x) d\tau = T.
\]

The breakdown time can be estimated from the condition that the total phase acquired by the electron in the pulse equals the dimensionless pulse duration:

\[
\tau_{\text{RFM}} \sim (\mu a_0^6/\bar{n}^3)^{1/5}. \tag{12}
\]

When the electrostatic term grows to the same order as the RF term. Next, during the deceleration stage \( \tau_{\text{acc}} < \tau < \tau_{bd} \), electron steadily decelerates until the breakdown at \( \tau = \tau_{bd} \), when it finally leaves the laser pulse and rapidly accelerates backward by the electrostatic field. For the deceleration stage by neglecting the LHS in (11) we obtain \( u_x \approx \sqrt{\mu a^2 a_0^4}/2 \). The breakdown time can be estimated from the condition that the total phase acquired by the electron in the pulse equals the dimensionless pulse duration:

\[
\int_{0}^{\tau_{\text{bd}}} (1 - v_x) d\tau = T. \tag{13}
\]

Since for the laser and plasma parameters providing an efficient RF-induced charge separation we have \( \tau_{\text{acc}} \ll \tau_{\text{bd}} \) (see below), in our estimation we can neglect the duration of
Unlike the case of CP, we can neglect in this stage the derivative of the approximate solution to (14), together with
\[ a_{||}^{(\text{RFM})} = \tilde{n} \tau_{\text{bd}}^{(\text{RFM})} \approx \sqrt{\mu_0 n T a_0^2}, \]
define the period and the amplitude of a longitudinal wave generated in a plasma.

### 2.1.3. Ultrarelativistic electrons: ponderomotive charge separation.

Now let us assume instead that charges are separated by the PM and neglect the second term in the RHS of (3). Estimating the derivative of the envelope by \( da^2/d\varphi \sim a_0^3/T \), we can estimate \( u_\tau \sim a_0 \sqrt{\tau/T} \) for the acceleration stage and \( u_\tau \sim a_0^2/\langle \tilde{n}\tau T \rangle \) for the deceleration stage. The acceleration and breakdown times can be estimated in the same manner as above:
\[ \tau_{\text{bd}}^{(\text{PM})} \sim \tau_{\text{acc}}^{(\text{PM})} \sim \left( \frac{a_0^2}{\tilde{n}^2 T} \right)^{1/3}. \]

Since in this case the acceleration and breakdown times are of the same order, we can use \( \tau_{\text{bd}}^{(\text{PM})} \) as a rough order-of-magnitude estimation for the period of a longitudinal wave. The amplitude of the longitudinal field is then given by (see also [16])
\[ a_{||}^{(\text{PM})} \approx \tilde{n} \tau_{\text{bd}}^{(\text{PM})} \approx \left( \frac{a_0^2}{\tilde{n} T} \right)^{1/3}. \]

Comparing (15) and (17) we conclude that in a transparent plasma longitudinal wave generation is enhanced by RF (RFM dominates over PM) if
\[ \mu_0^3 \tilde{n} T a_0^8 \gtrsim 1, \]
i.e. the stronger and longer the laser pulses are, the more pronounced the effect is.

### 2.2. Linear polarization

Assuming that radiation damping is small, for a linearly polarized laser pulse \( A(\varphi) = (m/\epsilon)a(\varphi)\{0, \cos \varphi, 0\} \) we have \( u \approx \{u_x, a(\varphi)\sin \varphi, 0\} \). Hence the equation for longitudinal motion in ultrarelativistic case takes the form
\[ \frac{du_x}{d\tau} \approx \frac{1}{2u_x} \frac{da_x^2}{d\varphi} \sin^2 \varphi + \frac{a_x^2}{2u_x} \sin 2\varphi + \frac{\mu a_0^6 \sin^4 \varphi \cos^2 \varphi}{4a_0^2} - \tilde{n} \tau. \]

Unlike the case of CP, the motion is accompanied by fast oscillations, but their envelope behaves qualitatively very similar to that case. During the deceleration stage an approximate solution to (19) can be represented in the form
\[ u_x \approx C(\tau) \sin^2 \varphi. \]
As we will shortly see, for \( |\varphi| \sim 1 \) we have \( \tau \varphi \approx \varphi \gg 1 \), indicating that \( \tau \) (and hence \( C \)) is a slowly varying function of \( \varphi \). Hence to find \( C(\tau) \) we can average (19) over the laser oscillations. The results are \( C(\tau) = a_0^3 \sqrt{\mu/(8\tilde{n} \tau)} \) for RFM and \( C(\tau) = a_0^3/(2\tilde{n} \tau) \) for PM. Now we can calculate \( \tau(\varphi) \) for both cases from
\[ \frac{d\tau}{d\varphi} \approx \left\{ \frac{2u_x}{a_0 \sin \varphi} \right\}^2 - \frac{C^2(\tau)}{a_0^2}, \]
thus obtaining \( \tau(\varphi) \propto \varphi^{1/2} \) for RFM and \( \tau(\varphi) \propto \varphi^{1/3} \) for PM. Power dependence of \( \tau \) on \( \varphi \) validates the assumption of slow variation of \( \tau \) for \( |\varphi| \sim 1 \). From \( \tau_{\text{bd}} = \tau(T) \) we obtain that after the substitution \( a_0 \rightarrow a_0/\sqrt{2} \) equations (14)–(17) remain valid for LP as well. Hence in a dilute plasma the charge separation field strength, expressed in terms of laser intensity,
\[ a_{||}^{(\text{RFM})} \approx \sqrt{\mu_0 n \tilde{T}} \sqrt{I_{\text{LP}}}, \quad a_{||}^{(\text{PM})} = \left( \frac{\tilde{n}}{T} \right)^{1/3}, \]
where, \( I_{\text{LP}} = \pi m^2 / (e^2 \lambda^3) \approx 2.74 \times 10^{18} \text{ W cm}^{-2} \) (for \( \lambda = 1 \mu\text{m} \), is independent of the laser polarization.

### 2.3. Analysis of used approximations

Let us discuss in more detail and summarize the conditions of validity of the approximations we made while deriving the estimations (15) and (17):

- The laser field is strong, \( a_0 \gg 1 \).
- Longitudinal motion is ultrarelativistic, \( u_x(\tau_{\text{bd}}) \gg a_0 \), or
\[ \left( \frac{\mu}{\tilde{n} T} \right)^{1/4} a_0 \gg 1, \quad \text{or} \quad \left( \frac{a_0}{\tilde{n} T^2} \right)^{1/3} \gg 1 \]
for RFM and PM cases, respectively (as they should, these conditions are opposite to the ones in (8)). Note that while the period of the created quasistatic longitudinal wave \( \approx T \) in the regime of nonrelativistic longitudinal electron motion, under the conditions (22) according to (14) and (16) it is \( \gg T \).
- Transverse radiation damping is small, i.e. the dimensionless transverse RF force \( F_{\text{RF}} \approx \mu a_0^2 \gamma^2 (1 - v_x) v_x \) is much smaller than the dimensionless transverse Lorentz force \( F_L = a_0 \). In the ultrarelativistic case it is equivalent to \( \mu a_0^2/a_x \ll 1 \) or, taking into account that \( u_x \gg u_x(\tau_{\text{bd}}^{(\text{RFM})}) \), to
\[ \left( \frac{\mu}{\tilde{n} T} \right)^{1/4} a_0 \ll 1. \]
- For RFM, the acceleration time is much smaller than the breakdown time. In fact, this assumption turns out to be literally equivalent to the condition (18) of dominance of RFM over PM, see (12) and (14).
- Depletion of the transverse laser field is neglected, i.e. we assume that the (dimensionless) energies \( \varepsilon_L \) gained by plasma electrons and \( \varepsilon_L \) accumulated in the quasistatic longitudinal field are much smaller than the (dimensionless) energy of the laser pulse \( \varepsilon_L \approx a_0^2 T \). The number of accelerated electrons can be estimated as \( \tilde{n} \tau_{\text{bd}} \) and the
section 2.3 for plasma densities: below ultrarelativistic

Figure 2. A map of various regimes of the longitudinal field generation with respect to laser pulse intensity and duration according to section 2.3 for plasma densities: (a) $n = 0.01n_e$ and (b) $n = 0.1n_e$. Markers on the lines: black triangles and blue diamonds bound from below ultrarelativistic (UR) electron motion for PM and RFM, respectively (see (22)); purple squares bound from above the region of low damping of the laser pulse (see (23)) and red circle bound from below the region of RFM dominance over PM (see (10) and (18)). The yellow unmarked lines bound from above the region of nonrelativistic (NR) $\gamma_{\text{bd}}\mu_e$ ions according to (24) with respect to (the right) intensity axis for the cases of linear (upper solid line) and circular (lower dashed line) polarization.

(dimensionless) average energy of a single electron as $u_0(\tau_{\text{bd}})$, hence for RFM we have $\varepsilon_{\parallel} \approx (\mu^2\gamma T)^{1/4} a_0^6$ and $\varepsilon_{\perp} \approx a_0^2 \tau_{\text{bd}} \approx \sqrt{\mu^2 \gamma T} a_0^6$. Note that:

- In the condition (18) of RFM dominance is fulfilled, then $\varepsilon_{\parallel} \gtrsim \varepsilon_{\perp}$, meaning that RFM is an efficient mechanism for longitudinal field generation;
- The condition (23) of weak damping is equivalent to $\varepsilon_{\perp} \gg \varepsilon_{\parallel}$ of negligibility of the laser field depletion.

- Immobility of the ions. In reality, the generated longitudinal electric field not only decelerates the electrons, but also accelerates the ions. We can neglect the ion motion in the estimations (14)–(17) only if ions remain nonrelativistic until the electron breakdown. In such a case the equation of ion motion is of the form

$$\xi_i(\tau) = \xi_i(\tau_0) + \frac{1}{2} \frac{e}{m_p} \left( A_m \right)_{\text{int}} t^2,$$

and its solution reads $\xi_i(\tau) \approx \xi_0 e^{\varepsilon_i / \tau_i}$, where

$$\tau_i = \frac{Am_p}{Ze\hbar n_0},$$

$\xi_0$ is the initial position of an ion, $m_p$ is the nucleon mass, $A$ and $Ze$ are the ionic weight and charge. An estimation of the ionic charge which we used in this paper is discussed in the appendix. Since ion acceleration is initially exponential, the ions become relativistic within the time of order $\tau_i$. Hence the condition that ions remain nonrelativistic (and can be considered immobile) is written as $\tau_i \gg \tau_{\text{bd}}$ or, explicitly,

$$a_0 \ll \left( \frac{m_p A}{Ze \mu T} \right)^{1/4}. \tag{24}$$

When the ions become relativistic the charge separation and the longitudinal field should get saturated.

Note that the dependence on ion charge and mass appears in (24) in the form $(A/Z)_{\text{int}}^{1/4}$. Since in strong fields under consideration even heavy atoms are almost fully ionized (see appendix), for different sorts of ions it deviates in the range $1$–$1.5$. In order to demonstrate maximal longitudinal field amplification with modest required laser intensity, in this paper we present all the numerical simulations for uranium, for which the restriction (24) is mildest.

The resulting domains of validity of all the independent assumptions discussed above for two plasma densities $n = 0.01n_e$ and $n = 0.1n_e$ are illustrated in figure 2 together with the line separating the dominance of RFM over PM. For clarity, the target region between this line and the topmost line of strong pulse depletion is shaded. The values of intensities from the right axis are converted to the equivalent values of $a_0$ for CP and $\lambda = 1 \mu m$ on the left axis. One can see that for relatively long pulses ($t_{\text{pul}} \gtrsim 100$ fs) RF is dominant already at intensities exceeding $10^{22}$ W cm$^{-2}$. Since ionization degree depends on the field strength but not on the intensity, the bottom orange line without markers, separating the relativistic and nonrelativistic regimes of ion motion for CP, is drawn with respect to both left and right axes, but for LP with respect to the right axis should be replaced with the top one.

2.4. Optimal density and maximally attainable charge separation field

According to (15) and (17), for a dilute plasma and given laser parameters $a_0$ and $T$, the charge separation field is growing monotonically with the plasma density. However, in virtue of (8) and/or (22), for

$$n \gtrsim n_{\text{opt}} \approx \mu \frac{a_0^4}{T} \tag{25}$$

the electron longitudinal motion becomes nonrelativistic and hence for higher densities the charge separation field is
Figure 3. Comparison of our theoretical predictions for an amplitude of the generated longitudinal field with the 1D PIC simulation data (with red filled markers) and without (blue empty markers) RF. The markers of square and circle shapes correspond to the results of simulation with circularly and linearly polarized driving pulses of the given peak intensity, respectively. The red solid line—estimation (15); red dashed line—approximation (9); blue dash dotted line corresponds to the first term from the RHS in (9). FWHM pulse duration 150 fs, \( \lambda = 1 \mu m \), plasma density \( n = 0.01 n_c \), immobile ions.

Plasma density independent\(^7\), see (9). Even though further increase of plasma density can no more affect the maximal charge separation field, it may enhance the laser pulse damping. Hence the longitudinal charge separation field generated in a plasma is maximal for the optimal density (25) and is bounded by

\[
a_{\parallel} \lesssim a_{\parallel}^{\text{max}} \approx \mu a_0^3 \ll a_0,
\]

where we used that in the case of optimal density (25) the condition of negligible damping (23) reduces to \( \mu a_0^3 \ll 1 \). The upper limit \( a_{\parallel} \lesssim a_0 \) could be probably approached when damping is moderate \( \varepsilon_{\parallel} \lesssim \varepsilon_L \), i.e. beyond the applicability of our current model.

3. PIC simulations results

In order to confirm our theoretical findings we performed one and two dimensional PIC simulations using the code EPOCH\(^8\) with the classical RF included in the form of Landau and Lifshitz\(^9\). The numerical approach is similar to the one developed in [19, 20].

The results for the dependence of the longitudinal field amplitude on laser intensity from 1D simulations are presented in figure 3. We used 100 cells per wavelength and 20 particles per cell. The laser pulse envelope was chosen Gaussian with FWHM duration 150 fs, as expected at ELI Beamlines [6]. The initial plasma density was \( n = 0.01 n_c \) and the ions were considered immobile. As one can observe, when expressed in terms of intensity, the longitudinal field generation is insensitive to laser polarization, and the simulation data is in excellent agreement with the approximation (9) and the estimation (15). At the same time, neglecting RF results in substantial (up to an order-of magnitude) underestimation of the longitudinal field already for the intensities \( I \gtrsim 2 \times 10^{22} \text{ W cm}^{-2} \). Note that for such long pulses electron acceleration in longitudinal direction when RF is neglected (i.e. solely due to PM) remains nonrelativistic for the whole shown laser intensity range. However, by considering shorter pulses we have previously justified in [16] the estimation (17) for PM for ultrarelativistic case as well.

In order to strengthen the reliability of our results, we performed also a series of more realistic 2D EPOCH simulations\(^8\) with linearly polarized driving pulses and the heavy mobile ions\(^{10}^{238}\text{U}^{20+}\) (the average ionization degree is estimated according to the appendix). As in [16], in order to reduce transverse expulsion of electrons by the ponderomotive force of the pulse, in all simulations we use the pulses with a symmetric bimodal Gaussian transverse profile shown in the inset of the figure 4(a), with the distance \( d \) between the peaks fixed so that the transverse field amplitude on the \( x \)-axis coincides with the amplitude of a single pulse, \( d = 2w\sqrt{\ln 2} \approx 1.7w \), where \( w \) is the waist of each peak.

Our first simulation (see figure 4) corresponds to the case of a wide (weakly focused of comparable width and length, \( w = t_{\text{pulse}} = 22\lambda \), where the wavelength \( \lambda = 1 \mu m \)) driving pulse. Since in this particular case we are mainly interested in stronger validation of our 1D model, we determined the simulation parameters according to figure 2 to maintain the longitudinal electron motion ultrarelativistic, while keeping ions nonrelativistic: plasma density \( n = 0.1 n_c \), laser intensity \( I = 2.5 \times 10^{23} \text{ W cm}^{-2} \) and FWHM 60 fs—see the cross between the unmarked orange solid line and the blue line marked with diamonds in the figure 2(b). From figures 4(a) and (b), where we compare the longitudinal fields computed with and without accounting for RF, respectively, one can observe that for these parameters the effect of RF-induced enhancement is extremely well pronounced in 2D. Moreover, for such wide pulses the longitudinal field distribution on the \( x \)-axis is in a rather good agreement with both the 1D simulations and the estimations of the preceding section, see figure 4(c). The most notable 2D effect is that some electrons bypass the ion bubble [28], getting inside from its rear side (see figure 4(d)), and in this way screening the quasi-static longitudinal field. Its slight decrease (as compared to the 1D simulation) at the rear side of the resulting longitudinal wave in figure 4(c) is explained partially by this effect, and partially by the nonrelativistic ion motion.

Since the laser pulses discussed in figure 4 are both intense and wide, their total power is extremely high. In order to substantiate the effect in a more realistic setup, we also present the results of simulations with the pulses tightly

\(^7\) Note, however, that for densities (25) the plasma effects ignored in our model can be important.

\(^8\) For 2D PIC simulations we use 50 cells per wavelength and 10 particles per cell.

\(^9\) For 2D simulations with circularly polarized pulses and lighter xenon ions see [16]. Note that violation of the condition (24) resulted there in a notable discrepancy between simulations with mobile and immobile ions.
focused (waist radius \( w = \lambda \)) at the left plasma boundary, see figure 5. Here we assume the parameters announced for ELI Beamlines \([6]\): \( I = 1.7 \times 10^{23} \) W cm\(^{-2}\) (total power \( \approx 10 \) PW) and FWHM 150 fs. In order to suppress strong diffraction by selffocusing, keeping longer the field strong enough for a noticeable effect of RF, we choose here plasma density \( n = n_c \). The resulting longitudinal fields averaged over laser wavelength—with and without RF—are shown in figures 5(a)
and (b) (comparison of a root mean square (rms) of the transverse field distribution with the case $n = 0.1 n_e$ is shown in figures 5(c) and (d)). According to the figures, the effect of RF-induced enhancement approximately by factor of 2 (see the insets in figures 5(a) and (b)) of the longitudinal field generation in a plasma could be observed on the forthcoming 10 PW laser facilities of the near future.

4. Discussion

A laser pulse propagating in plasma separates the charges thus generating a quasistatic longitudinal electric field, which, as discussed in this paper, under certain conditions can be substantially enhanced due to modification of the electron transverse oscillations by even weak RF. To describe the effect, we develop a 1D model and propose explicit analytical estimates of an amplitude and a period for the enhanced effect, we develop a 1D model and propose explicit analytical estimates of an amplitude and a period for the generated longitudinal field according to a particular regime of the process (nonrelativistic versus ultrarelativistic longitudinal electron motion, and RFM versus PM dominance). In particular we show that, being expressed in terms of the laser intensity, these parameters are in fact independent of polarization of the driving pulse. Our theoretical analysis is limited by the assumptions that the damping of the driving laser pulse is small and that ions are immobile, and we provide a detailed analysis of their validity for a wide range of laser and plasma parameters. Due to energy conservation, the amplitude of the generated longitudinal field can never exceed the amplitude of a driving laser field and, according to our theory, for given pulse intensity and duration, the highest longitudinal field is achieved with such an optimal plasma density that the electron longitudinal motion is mildly relativistic.

We have validated our predictions by comparing them to PIC simulations with both immobile and mobile high-Z ions in 1D, as well as with 2D wide driving pulses. However, consideration of concurrently strong and wide pulses implies unrealistically huge laser power, therefore in order to realize the proposed regime of RF-induced enhancement of the longitudinal field generation within the limitations imposed by the current or foreseeable experimental capabilities, tight focusing is required. For tightly focused pulses our model is no longer valid literally on a quantitative level, because such purely 2D effects as fast diffraction, various plasma instabilities, and the alternating longitudinal field of a tightly focused pulse (which can in fact be as strong as the transverse field itself), are vital. Increase of the plasma density, in addition to boosting the magnitude of the generated quasi-static longitudinal field as explained above, also facilitates to suppress pulse diffraction by self-focusing, but is limited by laser radiative damping and by the condition of plasma transparency, as in an opaque plasma electrons cannot penetrate deep inside the pulse to experience a strong enough field for a noticeable RF. According to simulations, by certain tuning of the parameters (plasma density close to $n_e$ and tight focusing to obtain high intensity), the effect of the RF-induced longitudinal field enhancement can be still revealed even for the parameters attainable at the ELI Beamlines facility in the near future [6].

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Appendix. Estimation of ionization degree

In order to deal with the ion motion, we need first to estimate the ionization degree (average ionic charges) for a field of a given strength. Even though now EPOCH contains a subroutine for realistic simulation of ionization, for highly-charged ions it generates a large number of species, and correct account for them requires significant computer resources and can seriously slow down simulations. A simplified approach is also needed if we want to make estimations analytically. Hence here we used for this purpose the following heuristic approach.

In a high intensity laser field ionization occurs via the tunneling mechanism, for which general theory is rather well developed (see, e.g. [29]). One can use the known formulas for ionization probability $w_i(a_0)$ to estimate the ionization degree from the condition $w_i(a_0) \cdot t_{pulse} \approx 1$. Since $w_i(a_0)$ depends on the field $a_0$ exponentially, and also because the laser pulse duration is large enough on the atomic scale, this criterion can be roughly reformulated as that ionization of an atomic level takes place when the laser field strength exceeds about 10% of the atomic field strength at the corresponding outer shell (the actual fraction weakly (logarithmically) depends on the parameters and can be specified more accurately if needed, see below).

Let us consider an atom with atomic number $Z$ and denote its residual ionic charge by $Z_*$, meaning that $Z - Z_*$ electrons remain, while $Z_*$ electrons have escaped due to ionization. We can roughly estimate the principle quantum number $n_{max}$ of the outer shell by using the Pauli principle
and the known degeneracies of a hydrogen-like ion:

\[ Z - Z_a = \sum_{n=1}^{n_{\text{max}}} 2n^2 \approx \frac{2}{3} n_{\text{max}}^3. \]

Assuming further that the corresponding fraction of the nuclei charge is completely screened by residual electrons, we can estimate the outer shell radius as \( R_{\text{out}} \sim (\hbar^2/2mZ_a e^2) n_{\text{max}}^2 \) and the corresponding atomic field strength as \( Z_a e^2/R_{\text{out}} = m^2 e^5 z_a^3/(\hbar^2 n_{\text{max}}^4) \). With all that our criterion can be formulated as

\[ a_0 \approx 0.1 \frac{m}{\hbar^2} \left( \frac{\alpha Z_a}{3} \right)^{1/3}, \quad \text{(A.1)} \]

where \( \alpha \approx 1/137 \) is the fine structure constant. The solution to (A.1) for uranium \((Z = 92)\) and xenon \((Z = 54)\) ions that were used in the work on this paper are plotted in figure A1.

A more advanced approach could include: more accurate consideration of the mentioned large logarithmic factor (resulting from the known pre-exponential factor of the tunneling formula), an account for relativistic corrections, and a two-stage procedure of first determining an ionization potential from the tunneling formula, and next selecting the corresponding ion potential using the tabular data (e.g. from [30]).

We have checked, however, that all these complications do not change our heuristic results within \( \pm 10\% \), which is enough for the rough estimates presented in the paper.

**References**

[1] Danson C, Hillier D, Hoppes N and Neely D 2015 *High Power Laser Sci. Eng.* 3 2109–14
[2] Chu Y et al 2013 *Opt. Express* 21 29231–9
[3] Sung J H, Lee H W, Yoo J Y, Yoon J W, Lee C W, Yang J M, Son Y J, Jang Y H, Lee S K and Nam C H 2017 *Opt. Lett.* 42 2058–61
[4] Zeng X et al 2017 *Opt. Lett.* 42 2014–7
[5] Bahk S W, Rousseau P, Planchon T A, Chvykov V, Kalintchenko G, Maksimchuk A, Mourou G A and Yanovsky V 2004 *Opt. Lett.* 29 2837
[6] Yanovsky V et al 2008 *Opt. Express* 16 2109–14
[7] Mourou G, Korn G, Sandner W and Collier J 2011 *ELI Extreme Light Infrastructure* (Berlin: Thoss Media)
[8] Zamfir N V 2014 *Eur. Phys. J. Spec. Top.* 223 1221–7
[9] Zou J P et al 2015 *High Power Laser Sci. Eng.* 3 E2
[10] Leemans W P et al 2014 *Phys. Rev. Lett.* 113 245002
[11] Esirkepov T, Borghesi M, Bulanov S V, Mourou G and Tajima T 2004 *Phys. Rev. Lett.* 92 175003
[12] Macchi A, Veghini S and Pegraro F 2009 *Phys. Rev. Lett.* 103 085003
[13] Bulanov S S, Esarey E, Schroeder C B, Bulanov S V, Esirkepov T Z, Kando M, Pegoraro F and Leemans W P 2015 *Phys. Rev. Lett.* 114 105003
[14] Wilks S C, Krer W L, Tabak M and Langdon A B 1992 *Phys. Rev.* 69 1383
[15] Macchi A, Cattani F, Liseykina T V and Cornolli F 2006 *Phys. Rev. Lett.* 94 165003
[16] Qiao B, Zepf M, Borghesi M and Geissler M 2009 *Phys. Rev. Lett.* 102 145002
[17] Naumova N, Schlegel T, Tikhonchuk V T, Labanne C, Sokolov I V and Mourou G 2009 *Phys. Rev. Lett.* 102 025002
[18] Schlegel T, Naumova N, Tikhonchuk V T, Labanne C, Sokolov I V and Mourou G 2009 *Phys. Plasmas* 16 083103
[19] Daido H, Nishiuchi M and Pirozhkov A S 2012 *Rep. Prog. Phys.* 75 056401
[20] Macchi A, Borghesi M and Passoni M 2013 *Rev. Mod. Phys.* 85 751
[21] Wagner F et al 2016 *Phys. Rev. Lett.* 116 205002
[22] Scullion C et al 2017 *Phys. Rev. Lett.* 119 054801
[23] Siminos E, Grech M, Skupin S, Schlegel T and Tikhonchuk V T 2012 *Phys. Rev. E* 86 056404
[24] Gelfer E G, Elkina N V and Fedotov A M 2017 Unusual face of radiation friction: enhancing production of longitudinal plasma waves arXiv:1710.09253
[25] Akhiezer A and Polovin R 1956 *Sov. Phys.-JETP* 3 696–705
[26] Voronin B S and Kolomenskii A A 1965 *Sov. Phys.-Usp.* 88 1027
[27] Zeldovich Y B 1975 *Sov. Phys.—Usp.* 18 79
[28] Fradkin D M 1979 *Phys. Rev. Lett.* 42 1209
[29] Di Piazza A 2008 *Lett. Math. Phys.* 83 305
[30] Zhidkov A, Koga J, Sasaki A and Uesaka M 2002 *Phys. Rev. Lett.* 88 185002
[31] Tamburini M, Pegoraro F, Di Piazza A, Keitel C H and Macchi A 2010 *New J. Phys.* 12 123005
[32] Tamburini M, Pegoraro F, Di Piazza A, Keitel C H, Liseykina T V and Macchi A 2011 *Nucl. Instrum. Methods Phys. Res. A* 653 181
[33] Brady C S, Ridgers C P, Arber T D, Bell A R and Kirk H G 2012 *Phys. Rev. Lett.* 109 245006
[34] Nakamura T, Koga J K, Esirkepov T Z, Kando M, Korn G and Bulanov S V 2012 *Phys. Rev. Lett.* 108 195001
[35] Bashinov A V and Kim A V 2013 *Phys. Plasmas* 20 113111
[36] Stark D J, Tonician T and Areifev A V 2016 *Phys. Rev. Lett.* 116 185003
[26] Landau L D and Lifshitz E M 1975 *The Classical Theory of Fields* vol 2 4th edn (Oxford: Butterworth-Heinemann)

[27] Brady C S and Arber T A 2011 *Plasma Phys. Control. Fusion* **53** 015001

[28] Bulanov S V, Pegoraro F, Pukhov A M and Sakharov A S 1997 *Phys. Rev. Lett.* **78** 4205

Pukhov A and Meyer-ter-Vehn J 2002 *Appl. Phys. B* **74** 355

[29] Karnakov B M, Mur V D, Popruzhenko S V and Popov V S 2015 *Phys.—Usp.* **58** 3

[30] Kramida A, Ralchenko Y, Reader J and NIST ASD Team 2017 NIST Atomic Spectra Database (ver. 5.5.1) [https://physics.nist.gov/asd](https://physics.nist.gov/asd) (5 January 2018), National Institute of Standards and Technology, Gaithersburg, MD