Distributed Compression for the Uplink of a Backhaul-Constrained Coordinated Cellular Network

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Abstract

We consider a backhaul-constrained coordinated cellular network. That is, a single-frequency network with \(N+1\) multi-antenna base stations (BSs) that cooperate in order to decode the users’ data, and that are linked by means of a common lossless backhaul, of limited capacity \(R\). To implement receive cooperation, we propose distributed compression: \(N\) BSs, upon receiving their signals, compress them using a multi-source lossy compression code. Then, they send the compressed vectors to a central BS, which performs users’ decoding. Distributed Wyner-Ziv coding is proposed to be used, and is optimally designed in this work. The first part of the paper is devoted to a network with a unique multi-antenna user, that transmits a predefined Gaussian space-time codeword. For such a scenario, the compression codebooks at the BSs are optimized, considering the user’s achievable rate as the performance metric. In particular, for \(N = 1\) the optimum codebook distribution is derived in closed form, while for \(N > 1\) an iterative algorithm is devised. The second part of the contribution focusses on the multi-user scenario. For it, the achievable rate region is obtained by means of the optimum compression codebooks for sum-rate and weighted sum-rate, respectively.

EDICS: WIN-INFO, MSP-CAPC, MSP-MULT, WIN-CONT.

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I. INTRODUCTION

Inter-cell interference is one of the most limiting factors of current cellular networks. It can be partially, but not totally, mitigated resorting to frequency-division multiplexing, sectorized antennas and fractional frequency reuse [1]. However, a more spectrally efficient solution has been recently proposed: coordinated cellular networks [2]. They consist of single-frequency networks with base stations (BSs) cooperating in order to transmit to and receive from the mobile terminals. Beamforming mechanisms are thus deployed in the downlink, as well as coherent detection in the uplink, to drastically augment the system capacity [3], [4]. Hereafter, we only focus on the uplink channel.

Preliminary studies on the uplink performance of coordinated networks consider all BSs connected via a lossless backhaul with unlimited capacity [5] [6]. Accordingly, the capacity region of the network equals that of a MIMO multi-access channel, with a supra-receiver containing all the antennas of all cooperative BSs [7]. Such an assumption seems optimistic in short-mid term, as operators are currently worried about the costs of upgrading their backhaul to support e.g., HSPA traffic load. To deal with a realistic backhaul constraint, two approaches have been proposed: i) distributed decoding [8], [9], consisting on a demodulating scheme distributely carried out among BSs, based on local decisions and belief propagation. Decoding delay appears to be its main problem. ii) Quantization [10], where BSs quantize their observations and forward them to decoding unit. Its main limitation relies on its inability to take profit of signal correlation between antennas/BSs; thus, introduces redundancy into the backhaul.

This paper considers a new approach for the network: distributed compression. The cooperative BSs, upon receiving their signals, distributely compress them using a multi-source lossy compression code [11]. Then, via the lossless backhaul, they transmit the compressed signals to the central unit (also a BS); which decompresses them using its own received signal as side information, and finally uses them to estimate the users’ messages. Distributed compression has been already proposed for coordinated networks in [12]–[14]. However, in those works, authors consider single-antenna BSs with ergodic fading. We extend the analysis here to the multiple-antenna case with time-invariant fading.

The compression of signals with side information at the decoder is introduced by Wyner and Ziv in [15], [16]. They show that side information at the encoder is useless (i.e., the rate-distortion tradeoff remains unchanged) to compress a single, Gaussian, source when it is available at the decoder [16, Section 3]. Unfortunately, when considering multiple (correlated) signals, independently compressed at different BSs, and to be recovered at a central unit with side information, such a statement can not be claimed. Indeed, this is an open problem, for which it is not even clear when source-channel separation
applies [17]. To the best of authors knowledge, the scheme that performs best (in a rate-distortion sense) for this problem is Distributed Wyner-Ziv (D-WZ) compression [18]. Such a compression is the direct extension of Berger-Tung coding to the decoding side information case [19], [20]. In turn, Berger-Tung compression can be thought as the lossy counterpart of the Slepian-Wolf lossless coding [21]. D-WZ coding is thus the compression scheme proposed to be used, and is detailed in the sequel.

**Summary of Contributions.** This paper considers a single-frequency network with $N + 1$ multi-antenna BSs. The first base station, denoted BS$_0$, is the central unit and centralizes the users’ decoding. The rest, BS$_1, \cdots, BS_N$, are cooperative BSs, which distributely compress their received signals using a D-WZ code, and independently transmit them to BS$_0$ via the common backhaul of aggregate capacity $R$. In the network, *time-invariant, frequency-flat* channels are assumed, as well as transmit and receive channel state information (CSI) at the users and BSs, respectively.

The first part of the paper is devoted to a network with a single user, equipped with multiple antennas. It aims at deriving the optimum compression codebooks at the BSs, for which the user’s transmission rate is maximized. Our contributions are the following:

- First, Sec. II revisits Wyner-Ziv coding [16, Section 3] and Distributed Wyner-Ziv coding [19], and adapts them to our compression scenario.
- For the single user transmitting a given Gaussian codeword, Sec. III proves that the optimum compression codebooks at the BSs are Gaussian distributed. Accordingly, the compression step is modelled by means of Gaussian “compression” noise, added by the BSs on their observations before retransmitting them to the central unit.
- Considering a unique cooperative BS in the network (*i.e.*, $N = 1$), Sec. IV derives in closed form the optimum “compression” noise for which the user’s rate is maximized. We also show that conditional Karhunen-Loève transform plus independent Wyner-Ziv coding of scalar streams is optimal.
- The compression design is extended in Sec. V to arbitrary $N$ BSs. The optimum “compression” noises (*i.e.*, the optimum codebook distributions) are obtained by means of an iterative algorithm, constructed using dual decomposition theory and a non-linear block coordinate approach [22], [23]. Due to the non-convexity of the noises optimization, only local convergence is proven.

The second part of the paper extends the analysis to a network where multiple users transmit simultaneously. For it, the achievable rate region is described resorting to the weighted sum-rate optimization:

- First, the sum-rate of the network is derived in Sec. VI, adapting previous results a single-user. Later, the weighted sum-rate, and its associated optimum compression “noises”, are obtained by means of
an iterative algorithm, constructed using dual decomposition and Gradient Projection [23].

**Notation.** \( E \{ \cdot \} \) denotes expectation. \( A^T \), \( A^\dagger \) and \( a^* \) stand for the transpose of \( A \), conjugate transpose of \( A \) and complex conjugate of \( a \), respectively. \( [a]^+ = \max \{ a, 0 \} \). \( I (\cdot;\cdot) \) denotes mutual information, \( H (\cdot) \) entropy. The derivative of a scalar function \( f (\cdot) \) with respect to a complex matrix \( X \) is defined as in [24], i.e., \( \frac{\partial f}{\partial X}_{i,j} = \frac{\partial f}{\partial x_{i,j}} \). In such a way, e.g., \( \frac{\partial \text{tr} \{ AX \}}{\partial X} = A^T \). Moreover, we compactly write \( Y_t^1: N = \{ Y_t^1, \cdots, Y_N^1 \} \), \( Y_G^1 = \{ Y_i^1 | i \in G \} \) and \( Y_n^c = \{ Y_i^1 | i \neq n \} \). A sequence of vectors \( \{ Y_t^1 \}_{t=1}^n \) is compactly denoted by \( Y_t^n \). Furthermore, to define block-diagonal matrices, we state \( \text{diag} (A_1, \cdots, A_n) \), with \( A_i \) square matrices. \( \text{coh} (\cdot) \) stands for convex hull. Finally, the covariance of random vector \( X \) conditioned on random vector \( Y \) is denoted by \( R_{X|Y} \) and computed \( R_{X|Y} = E \left\{ (X - E \{ X \mid Y \}) (X - E \{ X \mid Y \})^\dagger \mid Y \right\} \).

II. COMPRESSION OF VECTOR SOURCES

The aim of compression within coordinated networks is to make the decoder extract the more mutual information from the reconstructed signals. Known rate-distortion results apply to this goal as follows.

**A. Single-Source Compression with Decoder Side Information**

Consider Fig. 1 with \( N = 1 \). Let \( Y_1^n \) be a zero-mean, temporally memoryless, Gaussian vector to be compressed at BS\(_1\). Assume that it is the observation of the signal transmitted by user \( s \), i.e., \( X_s^n \). BS\(_1\) compresses the signal and sends it to BS\(_0\), which makes use of its side information \( Y_0^n \) to decompress it. Finally, once reconstructed the signal into vector \( \hat{Y}_1^n \), the decoder uses it to estimate the message transmitted by the user. Wyner’s results [16] apply to this problem as follows.

**Definition 1 (Single-source Compression Code):** A \((n, 2^{n\rho})\) compression code with side information at the decoder \( Y_0^1 \) is defined by two mappings, \( f_n(\cdot) \) and \( g_n(\cdot) \) and three spaces \( \mathcal{Y}_1^1, \hat{\mathcal{Y}}_1^1 \) and \( \mathcal{Y}_0^1 \), where

\[
\begin{align*}
  f_n : \mathcal{Y}_1^1 &\rightarrow \{1, \cdots, 2^{n\rho}\} \\
  g_n : \{1, \cdots, 2^{n\rho}\} \times \mathcal{Y}_0^1 &\rightarrow \hat{\mathcal{Y}}_1^1.
\end{align*}
\]

**Proposition 1 (Wyner-Ziv Coding [16]):** Let the random vector \( \bar{Y}_1 \) with conditional probability \( p (\bar{Y}_1 \mid Y_1) \) satisfy the Markov chain \( Y_0 \rightarrow Y_1 \rightarrow \bar{Y}_1 \), and let \( Y_0 \) and \( Y_1 \) be jointly Gaussian. Then, considering a sequence of compression codes \((n, 2^{n\rho})\) with side information \( Y_0^1 \) at the decoder:

\[
\frac{1}{n} I (X_s^n; Y_0^n, g_n (Y_0^n, f_n (Y_1^n))) = I (X_s; Y_0, \bar{Y}_1) \quad (1)
\]

as \( n \rightarrow \infty \) if:
• the compression rate $\rho$ satisfies
  \[ I \left( Y_1; \hat{Y}_1 | Y_0 \right) \leq \rho, \]  
  \[ I \left( Y_1; \hat{Y}_1 | Y_0 \right) \leq \rho, \] 
  \[ \text{(2)} \]

• the compression codebook $\mathcal{C}$ consists of $2^{n\rho}$ random sequences $\hat{Y}_1^n$ drawn i.i.d. from $\prod_{i=0}^{n-1} p \left( \hat{Y}_1 \right)$, where $p \left( \hat{Y}_1 \right) = \sum_{Y_1} p \left( Y_1 \right) p \left( \hat{Y}_1 | Y_1 \right)$.

• the encoding $f_n(\cdot)$ outputs the bin-index of codewords $\hat{Y}_1^n$ that are jointly typical with the source sequence $Y_1^n$. In turn, $g_n(\cdot)$ outputs the codeword $\hat{Y}_1^n$ that, belonging to the bin selected by the encoder, is jointly typical with $Y_0^n$.

**Proof:** The proposition is proven in [16, Lemma 5] using joint typicality arguments.

**B. Multiple-Source Compression with Decoder Side Information**

Consider Fig. 1. Let $Y_i^n$, $i = 1, \cdots, N$ be $N$ zero-mean, temporally memoryless, Gaussian vectors to be compressed independently at BS$_1, \cdots, $ BS$_N$, respectively. Assume that they are the observations at the BSs of the signal transmitted by user $s$, i.e., $X_s^n$. The compressed vectors are sent to BS$_0$, which decompresses them using its side information $Y_0^n$ and uses them to estimate the user’s message. Notice that the architecture in Fig. 1 imposes source-channel separation at the compression step, which is not shown to be optimal. However, it includes the coding scheme with best known performance: Distributed Wyner-Ziv coding [18]. It applies to the setup as follows.

**Definition 2 (Multiple-source Compression Code):** A $(n, 2^{n\rho_1}, \cdots, 2^{n\rho_N})$ compression code with side information at the decoder $Y_0$ is defined by $N + 1$ mappings, $f_n^i(\cdot)$, $i = 1, \cdots, N$, and $g_n(\cdot)$, and $2N + 1$ spaces $Y_i, \hat{Y}_i$, $i = 1, \cdots, N$ and $Y_0$, where

\[ f_n^i : Y_i^n \rightarrow \{1, \cdots, 2^{n\rho_i}\}, \quad i = 1, \cdots, N \]

\[ g_n : \{1, \cdots, 2^{n\rho_1}\} \times \cdots \times \{1, \cdots, 2^{n\rho_N}\} \times Y_0^n \rightarrow \hat{Y}_i^n \times \cdots \times \hat{Y}_N^n. \]

**Proposition 2 (Distributed Wyner-Ziv Coding [18]):** Let the random vectors $\hat{Y}_i$, $i = 1, \cdots, N$, have conditional probability $p \left( \hat{Y}_i | Y_i \right)$ and satisfy the Markov chain $(Y_0, Y_i^c, \hat{Y}_i^c) \rightarrow Y_i \rightarrow \hat{Y}_i$. Let $Y_0$ and $Y_i$, $i = 1, \cdots, N$ be jointly Gaussian. Then, considering a sequence of compression codes $(n, 2^{n\rho_1}, \cdots, 2^{n\rho_N})$ with side information $Y_0$ at the decoder:

\[ \frac{1}{n} I \left( X_s^n, Y_0^n, g_n \left( Y_0^n, f_n^1 \left( Y_1^n \right), \cdots, f_n^N \left( Y_N^n \right) \right) \right) = \frac{1}{n} I \left( X_s; Y_0, \hat{Y}_1:N \right) \]

as $n \rightarrow \infty$ if:

- the compression rates $\rho_1, \cdots, \rho_N$ satisfy

\[ I \left( Y_G; \hat{Y}_G | Y_0, \hat{Y}_G \right) \geq \sum_{i \in G} \rho_i \quad \forall G \subseteq \{1, \cdots, N\}, \]

\[ \text{(4)} \]
• each compression codebook $\mathcal{C}_i$, $i = 1, \cdots, N$ consists of $2^{n\rho_i}$ random sequences $\hat{Y}_i^n$ drawn i.i.d. from $\prod_{i=1}^n p \left( \hat{Y}_i \right)$, where $p \left( \hat{Y}_i \right) = \sum_{Y_i} p(Y_i) p \left( \hat{Y}_i | Y_i \right)$.

• for every $i = 1, \cdots, N$, the encoding $f_n^i \left( \cdot \right)$ outputs the bin-index of codewords $\hat{Y}_i^n$ that are jointly typical with the source sequence $Y_i^n$. In turn, $g_n \left( \cdot \right)$ outputs the codewords $\hat{Y}_i^n$, $i = 1, \cdots, N$ that, belonging to the bins selected by the encoders, are all jointly typical with $Y_0^n$.

**Proof:** The proposition is proven for discrete sources and discrete side information in [18, Theorem 2]. Also, the extension to the Gaussian case is conjectured therein. The conjecture can be proven by noting that D-WZ coding is equivalent to Berger-Tung coding with side information at the decoder [19]. In turn, Berger-Tung coding can be implemented through time-sharing of successive Wyner-Ziv compressions [20], for which introducing side information $Y_0$ at the decoder reduces the compression rate as in (4). Due to space limitations, we limit the proof to this sketch.

Now, we can present the coordinated cellular network with D-WZ coding.

### III. System Model

Let a single source $s$, equipped with $N_t$ antennas, transmit data to base stations $\text{BS}_0, \cdots, \text{BS}_N$, each one equipped with $N_i$, $i = 1, \cdots, N$ antennas. The BSs, as in typical 3G networks, are connected (through radio network controllers) to a common lossless backhaul of aggregate capacity $R$, and $\text{BS}_0$ is selected to be the decoding unit. This user-to-BSs assignment is assumed to be given by upper layers and out of the scope of the paper\(^1\).

The source transmits a message $\omega \in \{1, \cdots, 2^{nR_s}\}$ mapped onto a zero-mean, Gaussian codeword $X_s^n$, drawn i.i.d. from random vector $X_s \sim \mathcal{CN} \left( 0, Q \right)$ and not subject to optimization. The transmitted signal, affected by *time-invariant, memory-less* fading, is received at the BSs under additive noise:

$$Y_i^n = H_{s,i} \cdot X_s^n + Z_i^n, \quad i = 0, \cdots, N$$

(5)

where $H_{s,i}$ is the MIMO channel matrix between user $s$ and BS$_i$, and $Z_i \sim \mathcal{CN} \left( 0, \sigma^2 R_i I \right)$ is AWGN. Channel coefficients are known at both the BSs and at the user, while BS$_0$ has centralized knowledge of all the channels within the network.

\(^1\)The derivation of the optimum set of BSs to decode the user is out of the scope of our study. We refer the reader to e.g., [6] for assignment algorithms and selection criteria.
A. Problem Statement

Base stations $BS_1, \cdots, BS_N$, upon receiving their signals, distributely compress them using a D-WZ compression code. Later, they transmit the compressed vectors to $BS_0$, which recovers them and uses them to decode. Considering so, the user’s message can be reliably decoded if [12, Theorem 1]:

$$R_s \leq \lim_{n \to \infty} \frac{1}{n} I \left( X^n_s; Y^n_0, g_n \left( Y^n_0, f^n_1(Y^n_1), \cdots, f^n_N(Y^n_N) \right) \right)$$  \hspace{1cm} (6)

Second equality follows from (3) in Prop. 2. However, equality only holds for compression rates satisfying the set of constraints (4). As mentioned, in the backhaul there is only an aggregate rate constraint $R$, i.e., $\sum_{i \in G} \rho_i \leq R, \forall G \subseteq \{1, \cdots, N\}$. Therefore, the set of constraints (4) can be all re-stated as:

$$I \left( Y_G; \hat{Y}_G \mid Y_0, \hat{Y}_c^G \right) \leq R, \forall G \subseteq \{1, \cdots, N\}.$$  \hspace{1cm} (7)

Furthermore, from the Markov chain in Prop. 2, the following inequality holds

$$I \left( Y_G; \hat{Y}_G \mid Y_0, \hat{Y}^c_G \right) \leq I \left( Y_{1:N}; \hat{Y}_{1:N} \mid Y_0 \right) \forall G \subseteq \{1, \cdots, N\}.$$  \hspace{1cm} (8)

Therefore, forcing the constraint $I \left( Y_{1:N}; \hat{Y}_{1:N} \mid Y_0 \right) \leq R$ to hold makes all constraints in (7) to hold too. Accordingly, the maximum transmission rate $C$ of user $s$ is obtained from optimization:

$$C = \max_{\Phi_1, \cdots, \Phi_N \geq 0} \log \det \left( I + \frac{Q}{\sigma^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^N H_{s,n}^\dagger (\sigma^2 I + \Phi_n)^{-1} H_{s,n} \right)$$

s.t. $I \left( Y_{1:N}; \hat{Y}_{1:N} \mid Y_0 \right) \leq R.$ \hspace{1cm} (9)

**Theorem 1:** Let $X_s \sim CN(0, Q)$. Optimization (9) is solved for Gaussian conditional distributions $p \left( \hat{Y}_i \mid Y_i \right), \ i = 1, \cdots, N$. Thus, the compressed vectors can be modelled as $\hat{Y}_i = Y_i + Z^c_i$, where $Z^c_i \sim CN(0, \Phi_i)$ is independent, Gaussian, ”compression” noise at $BS_i$. That is,

$$C = \max_{\Phi_1, \cdots, \Phi_N \geq 0} \log \det \left( I + \frac{Q}{\sigma^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^N H_{s,n}^\dagger (\sigma^2 I + \Phi_n)^{-1} H_{s,n} \right)$$

s.t. $\log \det \left( I + \text{diag} \left( \Phi_1^{-1}, \cdots, \Phi_N^{-1} \right) R_{Y_{1:N} \mid Y_0} \right) \leq R.$ \hspace{1cm} (10)

where the conditional covariance $R_{Y_{1:N} \mid Y_0}$ follows (54).

**Proof:** See Appendix II for the proof. \hspace{1cm} \blacksquare

**Remark 1:** The maximization above is not concave in standard form: although the feasible set is convex, the objective function is not concave on $\Phi_1, \cdots, \Phi_N$.  

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B. Useful Upper Bounds

Prior to solving (10), we present two upper bounds on it.

**Upper Bound 1:** The achievable rate \( C \) in (10) is upper bounded by

\[
C \leq I(X_s; Y_0, Y_{1:N}) = \log \det (I + \frac{Q}{\sigma_r^2} \sum_{n=0}^{N} H_{s,n}^H H_{s,n}) .
\] (11)

**Upper Bound 2:** The achievable rate \( C \) in (10) satisfies

\[
C \leq I(X_s; Y_0) + R = \log \det (I + \frac{1}{\sigma_r^2} H_{s,0} Q H_{s,0}^\dagger + I) + R .
\] (12)

**Proof:** See Appendix III for the proof.

**Remark 2:** Notice that, independently of the number of BSs, the achievable rate is bounded above by the capacity with BS_0 plus the backhaul rate.

IV. The Two-Base Stations Case

We first solve (10) for \( N = 1 \). As mentioned, the objective function, which has to be maximized, is convex on \( \Phi_1 \succeq 0 \). In order to make it concave, we change the variables \( \Phi_1 = A_1^{-1} \), so that

\[
C = \max_{A_1 \succeq 0} \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^H H_{s,0} + A_1 \sigma_r^2 + I \right)^{-1} A_1 H_{s,1}
\] (13)

subject to \( \log \det (I + A_1 R_{Y_1|Y_0}) \leq R \).

The objective has turned into concave. However, the constraint now does not define a convex feasible set. Therefore, Karush-Kuhn-Tucker (KKT) conditions become necessary but not sufficient for optimality. To solve the problem, we need to resort to the general sufficiency condition [23, Proposition 3.3.4]: first, we derive a matrix \( A_1^* \) for which the KKT conditions hold. Later, we demonstrate that the selected matrix also satisfies the general sufficiency condition, thus becoming the optimal solution. The optimum compression noise is finally recovered as \( \Phi_1^* = (A_1^*)^{-1} \). This result is presented in Theorem 2:

**Theorem 2:** Let \( X_s \sim CN(0, Q) \) and the conditional covariance (see Appendix I-A):

\[
R_{Y_1|Y_0} = H_{s,1} \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^H H_{s,0} \right)^{-1} Q H_{s,1}^\dagger + \sigma_r^2 I,
\] (14)

with eigen-decomposition \( R_{Y_1|Y_0} = U \text{diag} (s_1, \cdots, s_{N_1}) U^\dagger \). The optimum “compression” noise at BS_1 is \( \Phi_1^* = U (\text{diag} (\eta_1, \cdots, \eta_{N_1}))^{-1} U^\dagger \), with

\[
\eta_j = \left[ \frac{1}{\lambda} \left( \frac{1}{\sigma_r^2} - \frac{1}{s_j} \right) - \frac{1}{\sigma_r^2} \right]^+, \] (15)

and \( \lambda \) is such that \( \sum_{j=1}^{N_1} \log (1 + \eta_j s_j) = R \).

**Proof:** See Appendix IV for the proof

\[2\text{Notice that all feasible points are regular.}\]
A. Practical Implementation

The optimum compression in Theorem 2 can be carried out using a practical Transform Coding (TC) approach. With TC, BS1 first transforms its received vector using an invertible linear function and then separately compresses the resulting scalar streams [25]. We show that the conditional Karhunen-Loève transform (CKLT) is an optimal linear transformation [26]. First, let recall that multiplying a vector by a matrix does not change the mutual information [27], i.e.,

\[ I(X_s; Y_0, \hat{Y}_1) = I(X_s; Y_0, U^\dagger \hat{Y}_1) \]

and

\[ I(Y_1; \hat{Y}_1|Y_0) = I(Y_1; U^\dagger \hat{Y}_1|Y_0). \]

From Theorem 2, the optimum compressed vector satisfies

\[ \hat{Y}_1^* = Y_1 + Z_c^* \]

with

\[ Z_c^* \sim CN(0, U\eta^{-1}U^\dagger) \]

and

\[ R_{\hat{Y}_1|Y_0} = USU^\dagger. \]

Therefore, the following compressed vectors are also optimal

\[ \hat{Y}_1 = U^\dagger Y_1 + U^\dagger Z_c^*, \]

where vector \( U^\dagger Y_1 \) is referred to as the CKLT of vector \( Y_1 \). Notice now that \( R_{\hat{Y}_1|Y_0} = R_{U^\dagger Y_1|Y_0} + R_{U^\dagger Z_c^*} = S + \eta^{-1} \) is diagonal. Therefore, the elements of the compressed vector \( \hat{Y}_1 \) are conditionally uncorrelated given \( Y_0 \). Likewise, so are the elements of vector \( U^\dagger Y_1 \). Due to this uncorrelation, each element \( j = 1, \cdots, N_1 \) of vector \( U^\dagger Y_1 \) can be compressed, without loss of optimality, independently of the compression of the others elements, at a compression rate

\[ r_j = \log (1 + \eta_j s_j), \quad j = 1, \cdots, N_1 \]

[16].

From Theorem 2 we validate that \( \sum_{j=1}^{N_1} r_j = R \). This demonstrates that CKLT plus independent coding of streams is optimal, not only for minimizing distortion as shown in [26], but also for maximizing the achievable rate of coordinated networks.

V. THE MULTIPLE-BASE STATIONS CASE

Consider now BS0 assisted by \( N > 1 \) cooperative BSs. The achievable rate follows (10) where, as previously, the objective function is not concave over \( \Phi_n, n = 1, \cdots, N \). To make it concave, we change the variables: \( \Phi_n = A_n^{-1}, n = 1, \cdots, N \), so that:

\[
C = \max_{A_1, \cdots, A_N \geq 0} \log \det \left( I + \frac{Q}{\sigma^2} \sum_{n=1}^{N} H_{s,n}^\dagger H_{s,n} + \sum_{n=1}^{N} A_n \sigma^2 + I \right)^{-1} A_n H_{s,n}
\]

s.t. \( \log \det \left( I + \diag(A_1, \cdots, A_N) R_{\hat{Y}_{1:N}|Y_0} \right) \leq R \).

Again, the feasible set does not define a convex set. Our strategy to solve the optimization is the following: first, we show that the duality gap for the problem is zero. Later, we propose an iterative algorithm that solves the dual problem, thus solving the primal too. An interesting property of the dual problem is that the coupling constraint in (17) is decoupled [23, Chapter 5].
A. The dual problem

Let the Lagrangian of (17) be defined on $A_n \succeq 0, n = 1, \cdots, N$ and $\lambda \geq 0$ as:

$$
\mathcal{L}(A_1, \cdots, A_N, \lambda) = \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^N H_{s,n}^\dagger \left( A_n \sigma_r^2 + I \right)^{-1} A_n H_{s,n} \right) - \lambda \cdot \left( \log \det \left( I + \text{diag}(A_1, \cdots, A_N) R_{Y_{1:N}|Y_0} \right) - R \right).
$$

The dual function $g(\lambda)$ for $\lambda \geq 0$ follows [22, Section 5.1]:

$$
g(\lambda) = \max_{A_1, \cdots, A_N \succeq 0} \mathcal{L}(A_1, \cdots, A_N, \lambda).
$$

The solution of the dual problem is then obtained from

$$
C' = \min_{\lambda \geq 0} g(\lambda).
$$

**Lemma 1:** The duality gap for optimization (17) is zero, i.e., the primal problem (17) and the dual problem (20) have the same solution.

**Proof:** The duality gap for problems of the form of (17), and satisfying the time-sharing property, is zero [28, Theorem 1]. Time-sharing property is defined as follows: let $C_x, C_y, C_z$ be the solution of (17) for backhaul rates $R_x, R_y, R_z$, respectively. Consider $R_z = \nu R_x + (1 - \nu) R_y$ for some $0 \leq \nu \leq 1$. Then, the property is satisfied if and only if $C_z \geq \nu C_x + (1 - \nu) C_y, \forall \nu \in [0, 1]$. That is, if the solution of (17) is concave with respect to the backhaul rate $R$. It is well known that time-sharing of compressions cannot decrease the resulting distortion [27, Lemma 13.4.1], neither improve the mutual information obtained from the reconstructed vectors. Hence, the property holds for (17), and the duality gap is zero.

We then solve the dual problem in order to obtain the solution of the primal. First, consider maximization (19). As expected, the maximization cannot be solved in closed form. However, as the feasible set (i.e., $A_1, \cdots, A_N \succeq 0$) is the cartesian product of convex sets, then a block coordinate ascent algorithm\(^3\) can be used to search for the maximum [23, Section 2.7]. The algorithm iteratively optimizes the function with respect to one $A_n$ while keeping the others fixed. It has been previously used to e.g., solve the sum-rate problem of MIMO multiple access channels with individual and sum-power constraint [30] [31]. We define it for our problem as:

$$
A_n^{t+1} = \arg \max_{A_n \succeq 0} \mathcal{L}(A_1^{t+1}, \cdots, A_{n-1}^{t+1}, A_n, A_{n+1}^t, \cdots, A_N^t, \lambda),
$$

where $t$ is the iteration index. As shown in Theorem 3, the maximization (21) is uniquely attained.

\(^3\)Also known as Non-Linear Gauss-Seidel Algorithm [29, Section II-C].
Theorem 3: Let the optimization \( A_n^* = \arg \max_{A_n \geq 0} \mathcal{L}(A_1, \cdots, A_N, \lambda) \) and the conditional covariance matrix (See Appendix I-A)

\[
R_{Y_n|Y_0, \hat{Y}_c} = H_{s,n} \left( I + Q \left( \frac{1}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + \sum_{p \neq n} H_{s,p}^\dagger (A_p \sigma_r^2 I + I)^{-1} A_p H_{s,p} \right) \right)^{-1} QH_{s,n}^\dagger + \sigma_r^2 I
\]

with eigen-decomposition \( R_{Y_n|Y_0, \hat{Y}_c} = U_n S U_n^\dagger \). The optimization is uniquely attained at \( A_n^* = U_n \eta U_n^\dagger \), where

\[
\eta_j = \left[ \frac{1}{\lambda} \left( \frac{1}{\sigma_r^2} - \frac{1}{s_j} \right) - \frac{1}{\sigma_r^2} \right]^+, \quad j = 1, \cdots, N_n.
\]

Proof: See Appendix V-A for the proof.

Function \( \mathcal{L}(A_1, \cdots, A_N, \lambda) \) is continuously differentiable, and the maximization (21) is uniquely attained. Hence, the limit point of the sequence \( \{A_1^t, \cdots, A_N^t\} \) is proven to converge to a local maximum [23, Proposition 2.7.1]. To demonstrate convergence to the global maximum, it is necessary to show that the mapping \( T(A_1, \cdots, A_N) = [A_1 + \gamma \nabla_{A_1} \mathcal{L}, \cdots, A_N + \gamma \nabla_{A_N} \mathcal{L}] \) is a block contraction\(^4\) for some \( \gamma \) [32, Proposition 3.10]. Unfortunately, we were not able to demonstrate the contraction property on the Lagrangian, although simulation results suggest global convergence of our algorithm always.

Once obtained \( g(\lambda) \) through the Gauss-Seidel Algorithm\(^5\), it remains to minimize it on \( \lambda \geq 0 \). First, recall that \( g(\lambda) \) is a convex function, defined as the pointwise maximum of a family of affine functions [22]. Hence, to minimize it, we may use a subgradient approach as e.g., that proposed by Yu in [31]. The subgradient search consists on following search direction \( -h \) such that

\[
\frac{g(\lambda') - g(\lambda)}{\lambda' - \lambda} \geq h \quad \forall \lambda'.
\]

Such a search is proven to converge to the global minimum for diminishing step-size rules [29, Section II-B]. Considering the definition of \( g(\lambda) \), the following \( h \) satisfies (24):

\[
h = R - \log \det \left( I + \text{diag}(A_1, \cdots, A_N) R_{Y_1|Y_0} \right).
\]

Therefore, it is used to search for the optimum \( \lambda \) as:

\[
\text{increase } \lambda \text{ if } h \leq 0 \quad \text{or decrease } \lambda \text{ if } h \geq 0.
\]

Consider now \( \lambda^0 = 1 \) as the initial value of the Lagrange multiplier. For such a multiplier, the optimum solution of (19) is \( \{A_1^*, \cdots, A_N^*\} = 0 \) and the subgradient (25) is \( h = R \) (See Appendix V-B). Hence,

\(^4\)See [32, Section 3.1.2] for the definition of block-contraction.

\(^5\)Assume hereafter that the algorithm has converged to the global maximum of \( \mathcal{L}(A_1, \cdots, A_N, \lambda) \).
following (26), the optimum value of $\lambda$ is strictly lower than one. Algorithm 1 takes all this into account in order to solve the dual problem, hence solving the primal too. As mentioned, we can only claim convergence of the algorithm to a local maximum.

Algorithm 1 Multiple-BSs dual problem

1: Initialize $\lambda_{\min} = 0$ and $\lambda_{\max} = 1$
2: repeat
3: $\lambda = \frac{\lambda_{\max} - \lambda_{\min}}{2}$
4: Obtain $\{A_1^*, \cdots, A_N^*\} = \arg\max L(A_1, \cdots, A_N, \lambda)$ from Algorithm 2
5: Evaluate $h$ as in (25).
6: if $h \leq 0$, then $\lambda_{\min} = \lambda$, else $\lambda_{\max} = \lambda$
7: until $\lambda_{\max} - \lambda_{\min} \leq \epsilon$
8: $\{\Phi_1^*, \cdots, \Phi_N^*\} = \{(A_1^*)^{-1}, \cdots, (A_N^*)^{-1}\}$

Algorithm 2 Non-linear Gauss-Seidel to obtain $g(\lambda)$

1: Initialize $A_n^0 = 0$, $n = 1, \cdots, N$ and $t = 0$
2: repeat
3: for $n = 1$ to $N$ do
4: Compute $R_{Y_n|Y_0,Y_n} (A_1^{t+1}, \cdots, A_{n-1}^{t+1}, A_{n+1}^{t}, \cdots, A_N^{t})$ from (22).
5: Take its eigen-decomposition $U_n S U_n^\dagger$ and compute $\eta$ as in (23).
6: Update $A_n^{t+1} = U_n \eta U_n^\dagger$.
7: end for
8: $t = t + 1$
9: until The sequence converges $\{A_1^t, \cdots, A_N^t\} \rightarrow \{A_1^*, \cdots, A_N^*\}$
10: Return $\{A_1^*, \cdots, A_N^*\}$

B. Practical Implementation

In the network, Distributed Wyner-Ziv compression can be practically implemented using a simple Successive Wyner-Ziv (S-WZ) approach [20] [33, Theorem 3]. To describe it, let us recall that the optimum compression noises $\Phi_1^*, \cdots, \Phi_N^*$ are obtained from Algorithm 1, and let $\pi(\cdot)$ be a given permutation on $\{1, \cdots, N\}$. For such a permutation, the S-WZ coding is defined as follows:
• **Parallel Compression**: BS$_{\pi(1)}$ compresses its received vector using a single-source Wyner-Ziv code with decoder side information $Y_0$ (following Proposition 1), at a compression rate

$$\rho_{\pi(1)} = I \left( Y_{\pi(1)}; \hat{Y}_{\pi(1)} | Y_0 \right) = \log \det \left( I + \left( \Phi_{\pi(1)}^* \right)^{-1} R_{Y_{\pi(1)} | Y_0} \right).$$  (27)

The conditional covariance is calculated in (53). In parallel, BS$_{\pi(n)}$, $n > 1$, compresses its signal using a single-source Wyner-Ziv code with decoder side information $(Y_0, \hat{Y}_{\pi(1:n-1)})$, at a rate

$$\rho_{\pi(n)} = I \left( Y_{\pi(n)}; \hat{Y}_{\pi(n)} | Y_0, \hat{Y}_{\pi(1:n-1)} \right) = \log \det \left( I + \left( \Phi_{\pi(n)}^* \right)^{-1} R_{Y_{\pi(n)} | Y_0, \hat{Y}_{\pi(1:n-1)}} \right).$$  (28)

In this case, the conditional covariance can be calculated from (56).

• **Successive Decompression**: BS$_0$ first recovers the codeword $\hat{Y}_{\pi(1)}$ using side information $Y_0$; later, it successively recovers codewords $\hat{Y}_{\pi(n)}$, $n > 1$, using $Y_0, \hat{Y}_{\pi(1:n-1)}$ as side information.

It is easy to check the optimality of the S-WZ coding:

$$\sum_{n=1}^{N} \rho_{\pi(n)} = \sum_{n=1}^{N} I \left( Y_{\pi(n)}; \hat{Y}_{\pi(n)} | Y_0, \hat{Y}_{\pi(1:n-1)} \right) = I \left( Y_{1:N}; \hat{Y}_{1:N} | Y_0 \right) = R.$$

Second equality comes from the Markov chain in Proposition 2, and third from the chain rule for mutual information; The fourth follows from the fact that $\Phi_{\pi,1}, \cdots, \Phi_{\pi,N}$ satisfy the constraint (10) with equality. Unfortunately, transform coding is not (generally) optimum for S-WZ with $N > 1$, since the eigenvectors of $\Phi_{\pi(n)}^* = U_n \eta^{-1} U_n^\dagger$, and those of $R_{Y_{\pi(1)} | Y_0, Y_{\pi(1:n-1)}} = V_n S V_n^\dagger$ does necessarily match.

**VI. THE MULTIPLE USER SCENARIO**

In previous sections, we considered a single user within the network. To complement the analysis, we study hereafter multiple (i.e., two) senders transmitting simultaneously. The users, $s_1$ and $s_2$, transmit two independent messages $\omega_u \in \{1, \cdots, 2^{nR_u}\}$, $u = 1, 2$, mapped onto codewords $X_u^n$, $u = 1, 2$, respectively. Codewords are drawn i.i.d. from random vectors $X_u \sim \mathcal{CN}(0, Q_u)$, $u = 1, 2$ and are not
subject to optimization. Hence, now, the BSs receive:

$$Y_i^n = \sum_{u=1}^{2} H_{u,i} X_u^n + Z_i^n, \quad i = 0, \cdots, N.$$  \hfill (30)

Here, $H_{u,i}$ is the MIMO channel between user $s_u$ and BS$_i$, and $Z_i \sim \mathcal{CN} \left(0, \sigma_i^2 I\right)$. As previously, signals at BS$_1, \cdots, BS_N$ are distributely compressed using a D-WZ code, and later sent to BS$_0$, which centralizes decoding. Using standard arguments, the set $\mathcal{C}$ of transmission rates $R_{u}$, $u = 1, 2$ at which messages $\omega_u$, $u = 1, 2$ can be reliably decoded is [27] [14]:

$$\mathcal{C} = \text{coh} \left( \bigcup_{\Pi_{i=1}^{N} p(Y_i|Y_i): \bigcup_{(Y_1, N) \cap (Y_1, N) \cap (Y_1, N)} \leq R} \left\{ \begin{array}{l}
R_1 \leq I(X_1; Y_0, Y_{1,N} | X_2) \\
R_2 \leq I(X_2; Y_0, Y_{1,N} | X_1) \\
R_1 + R_2 \leq I(X_1, X_2; Y_0, Y_{1,N})
\end{array} \right\} \right)$$  \hfill (31)

The union in (31) is explained by the fact that compression codebooks might be arbitrary chosen at the BSs. Notice that the boundary points of the region can be achieved using superposition coding (SC) at the users, successive interference cancellation (SIC) at the BS$_i$, and (optionally) time-sharing (TS). Furthermore, as for the single-user case, the optimum conditional distributions $p \left( \hat{Y}_i | Y_i \right)$, $i = 1, \cdots, N$ at the boundary of the region can be proven to be Gaussian\(^6\). Therefore, the union in (31) can be restricted to compressed vectors of the form $\hat{Y}_i = Y_i^c + Z_i^c$, where $Z_i^c \sim \mathcal{CN} \left(0, \Phi_i\right)$. That is:

$$\mathcal{C} = \text{coh} \left( \bigcup_{\Phi_1, \cdots, \Phi_N \in c \left( R \right)} \left\{ \begin{array}{l}
R_1 \leq \log \det \left( I + \frac{Q_2}{\sigma_2^2} H_{1,0}^\dagger H_{1,0} + Q_1 \sum_{n=1}^{N} H_{1,n}^\dagger \left( \sigma_i^2 I + \Phi_n \right)^{-1} H_{1,n} \right)
R_2 \leq \log \det \left( I + \frac{Q_2}{\sigma_2^2} H_{2,0}^\dagger H_{2,0} + Q_2 \sum_{n=1}^{N} H_{2,n}^\dagger \left( \sigma_i^2 I + \Phi_n \right)^{-1} H_{2,n} \right)
R_1 + R_2 \leq \log \det \left( I + \frac{Q_2}{\sigma_2^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^{N} H_{s,n}^\dagger \left( \sigma_i^2 I + \Phi_n \right)^{-1} H_{s,n} \right)
\end{array} \right\} \right)$$  \hfill (32)

Where $c \left( R \right) = \left\{ \Phi_1, \cdots, \Phi_N : \log \det \left( I + \text{diag} \left( \Phi_1^{-1}, \cdots, \Phi_N^{-1} \right) R_{Y_{1,N}} | Y_0 \right) \leq R \right\}$, $Q = \text{diag} \left( Q_1, Q_2 \right)$ and $H_{s,n} = [H_{1,n}, H_{2,n}]$, for $n = 0, \cdots, N$. Covariance $R_{Y_{1,N}} | Y_0$ is calculated in Appendix I-B. To evaluate such a region, we resort to the weighted sum-rate (WSR) optimization [34, Sec. III-C]. That is, we express

$$\mathcal{C} = \{ (R_1, R_2) : \alpha R_1 + (1 - \alpha) R_2 \leq \mathcal{R} \left( \alpha \right), \forall \alpha \in [0, 1] \},$$  \hfill (33)

with $\mathcal{R} \left( \alpha \right)$ the maximum WSR, given weights $\alpha$ and $(1 - \alpha)$ for user $s_1$ and $s_2$, respectively. Such a WSR is achieved with equality at the boundary of the region. Thus, it can be attained considering SIC at BS$_0$, which consists of first decoding the user with lowest weight, considering second user as interference. Later, once decoded the first user, the decoder subtracts its contribution to the received signal, and then decodes the second user without interference.

\(^6\)Recall that $X_u \sim \mathcal{CN} \left(0, Q_u\right)$, $u = 1, 2$. We omit the proof due to space limitations.
A. Useful Outer Regions

Prior to solving the WSR optimization, we present two outer regions on (32).

**Outer Region 1:** Rate region (32) is contained within the region

\[
R_1 \leq \log \det \left( I + \frac{Q_1}{\sigma_r^2} \sum_{n=0}^{N} H_{1,n}^\dagger H_{1,n} \right)
\]

\[
R_2 \leq \log \det \left( I + \frac{Q_2}{\sigma_r^2} \sum_{n=0}^{N} H_{2,n}^\dagger H_{2,n} \right)
\]

\[
R_1 + R_2 \leq \log \det \left( I + \frac{Q}{\sigma_r^2} \sum_{n=0}^{N} H_{s,n}^\dagger H_{s,n} \right)
\]

(34)

**Remark 3:** It is the capacity region when \( Y_i, i = 1, \cdots, N \) are available at BS_0.

**Outer Region 2:** The sum-rate satisfies

\[
R_1 + R_2 \leq \log \det \left( I + \frac{1}{\sigma_r^2} H_{s,0} Q H_{s,0}^\dagger + R \right).
\]

(35)

**Proof:** It is equivalent to the proof of upper bound 2.

B. Sum Rate Maximization

The sum-rate of (32) is identical to the maximum transmission rate of a single user \( s \) transmitting a vector \( X_s = [X_{1,s}^T, X_{2,s}^T]^T \), with equivalent channel \( H_{s,n} = [H_{1,n}, H_{2,n}], n = 0, \cdots, N \). Hence, to maximize it we resort to Algorithm 1.

C. Weighted Sum Rate Maximization

Let consider the WSR optimization with \( \alpha > \frac{1}{2} \) (i.e., higher priority to user 1, which is decoded last at the SIC). With such a decoding, the maximum rate of user 1 is

\[
R_1 = I \left( X_1; Y_0, \hat{Y}_{1:N}|X_2 \right)
\]

\[
= \log \det \left( I + \frac{Q_1}{\sigma_r^2} H_{1,0}^\dagger H_{1,0} + Q_1 \sum_{n=1}^{N} H_{1,n}^\dagger (\sigma_r^2 I + \Phi_n)^{-1} H_{1,n} \right).
\]

(36)

On the other hand, the rate of user 2, which is decoded first, follows:

\[
R_2 = I \left( X_2; Y_0, \hat{Y}_{1:N} \right)
\]

\[
= I \left( X_1, X_2; Y_0, \hat{Y}_{1:N} \right) - I \left( X_1; Y_0, \hat{Y}_{1:N}|X_2 \right)
\]

\[
= \log \det \left( I + \frac{Q_2}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^{N} H_{s,n}^\dagger (\sigma_r^2 I + \Phi_n)^{-1} H_{s,n} \right) - R_1,
\]

where \( Q = \text{diag}(Q_1, Q_2) \) and \( H_{s,n} = [H_{1,n}, H_{2,n}] \). The WSR, \( \alpha R_1 + (1 - \alpha) R_2 \), which has to be maximized is convex on \( \Phi_1, \cdots, \Phi_N \). To make it concave, we use the change the variables \( \Phi_n = A_n^{-1}, \)
n = 1, \cdots, N. Then, plugging (36) and (37) into (33), the WSR optimization turns into

\[ R(\alpha) = \max_{A_1, \cdots, A_N} \alpha \cdot R_1 + (1 - \alpha) \cdot R_2 \]

s.t. \( \log \det \left( I + \text{diag} (A_1, \cdots, A_N) R_{Y_1:Y_N|Y_0} \right) \leq R \) \quad (38)

As previously, the constraint does not define a convex feasible set. To solve the optimization, we follow the strategy presented previously: first, we show that the optimization has zero duality gap. Later, we propose an iterative algorithm that solves the dual problem, thus solving the primal too.

**Lemma 2:** The duality gap for the WSR optimization (38) is zero.

**Proof:** Applying the time-sharing property in [28, Theorem 1] the zero-duality gap is demonstrated. \( \blacksquare \)

Let then solve the dual problem. The Lagrangian for optimization (38) is defined as:

\[ L_\alpha (A_1, \cdots, A_n, \lambda) = \alpha \cdot R_1 + (1 - \alpha) \cdot R_2 - \lambda \cdot \left( \log \det \left( I + \text{diag} (A_1, \cdots, A_N) R_{Y_1:Y_N|Y_0} \right) - R \right) \] \quad (39)

The first step is to find the dual function [23, Section 5]

\[ g_\alpha (\lambda) = \max_{A_1, \cdots, A_n \succeq 0} L_\alpha (A_1, \cdots, A_n, \lambda) \] \quad (40)

In previous sections, we showed that such an optimization can be tackled using a block-coordinate algorithm. Unfortunately, now, the maximization with respect to a single \( A_n \) cannot be solved in closed-form, and is not clear to be uniquely attained. Hence, to solve (40), we propose another algorithm: the gradient projection method (GP) [23, Section 2.3]. GP has been used to e.g., compute transmit covariances for MIMO interference channels, and the WSR of MIMO broadcast channels [35, Section IV-C] [36]. It is defined as follows: let (40), and consider the initial point \( \{A_1^0, \cdots, A_N^0\} \succeq 0 \). It iteratively updates [23, Section 2.3.1]:

\[ A_n^{t+1} = A_n^t + \gamma_t \left( \bar{A}_n^t - A_n^t \right), \quad n = 1, \cdots, N \] \quad (41)

where \( t \) is the iteration index and \( 0 < \gamma_t \leq 1 \) is the step size. Also,

\[ \bar{A}_n^t = \left[ A_n^t + s_t \cdot \nabla_{A_n} \mathcal{L}_\alpha (\lambda, A_1^t, \cdots, A_N^t) \right] \succeq 0, \quad n = 1, \cdots, N \] \quad (42)

with \( s_t \geq 0 \) an scalar and \( \nabla_{A_n} \mathcal{L}_\alpha (\lambda, A_1^t, \cdots, A_N^t) \) the gradient of \( \mathcal{L}_\alpha (\cdot) \) with respect to \( A_n \), evaluated at \( A_1^t, \cdots, A_N^t \). Finally, \( [\cdot] \succeq 0 \) denotes the projection (with respect to the Frobenius norm) onto the cone of positive semidefinite matrices. Whenever \( \gamma_t \) and \( s_t \) are chosen appropriately, the sequence \( \{A_1^t, \cdots, A_N^t\} \) is proven to converge to a local maximum of (40) [23, Proposition 2.2.1]. (For global convergence to hold, the contraction property must be satisfied. Unfortunately, we were not able to prove this property for our optimization). In order to make the algorithm work for the problem, we need to: i) compute the
projection of a Hermitian matrix $\mathbf{S}$, with eigen-decomposition $\mathbf{S} = \mathbf{U} \eta \mathbf{U}^\dagger$, onto the cone of positive semidefinite matrices. It is equal to [37, Theorem 2.1]:

$$[\mathbf{S}]_{\geq 0} = \mathbf{U} \text{diag} \left( \max \{ \eta_1, 0 \}, \cdots, \max \{ \eta_m, 0 \} \right) \mathbf{U}^\dagger. \quad (43)$$

ii) Obtain the gradient of $L_\alpha (\cdot)$ with respect to a single $\mathbf{A}_n$, which is twice the conjugate of the partial derivative of the function with respect to such a matrix [24]:

$$\nabla_{\mathbf{A}_n} L_\alpha (\mathbf{A}_{1:N}, \lambda) = 2 \left( \left[ \frac{\partial L_\alpha (\mathbf{A}_{1:N}, \lambda)}{\partial \mathbf{A}_n} \right]^T \right)^\dagger \quad (44)$$

The Lagrangian is defined in (39). To obtain its partial derivative, we make use of (79):

$$\left[ \frac{\partial \log \det (\mathbf{I} + \text{diag} (\mathbf{A}_1, \cdots, \mathbf{A}_N) \mathbf{R}_{\mathbf{Y}_n|\mathbf{Y}_0})}{\partial \mathbf{A}_n} \right]^T = \left[ \frac{\partial \log \det (\mathbf{I} + \mathbf{A}_n \mathbf{R}_{\mathbf{Y}_n|\mathbf{Y}_0})}{\partial \mathbf{A}_n} \right]^T \quad (45)$$

$$= \mathbf{R}_{\mathbf{Y}_n|\mathbf{Y}_0} (\mathbf{I} + \mathbf{A}_n \mathbf{R}_{\mathbf{Y}_n|\mathbf{Y}_0})^{-1}. \quad (46)$$

The conditional covariance is computed in Appendix I-B. Furthermore, we can also derive that

$$\frac{\partial \mathbf{R}_1}{\partial \mathbf{A}_n} = \frac{\partial \mathbf{I} \left( \mathbf{X}_1; \mathbf{Y}_0, \hat{\mathbf{Y}}_{1:N} | \mathbf{X}_2 \right)}{\partial \mathbf{A}_n} \quad (47)$$

where second equality follows from the chain rule for mutual information and noting that $\mathbf{I} \left( \mathbf{X}_1; \mathbf{Y}_0, \hat{\mathbf{Y}}_{n}^c | \mathbf{X}_2 \right)$ does not depend on $\mathbf{A}_n$. The mutual information above is evaluated as:

$$\mathbf{I} \left( \mathbf{X}_1; \hat{\mathbf{Y}}_{n}^c | \mathbf{X}_2, \mathbf{Y}_0, \hat{\mathbf{Y}}_{n}^c \right) = \mathbf{H} \left( \hat{\mathbf{Y}}_{n}^c | \mathbf{X}_2, \mathbf{Y}_0 \right) - \mathbf{H} \left( \hat{\mathbf{Y}}_{n} | \mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}_0, \hat{\mathbf{Y}}_{n}^c \right) \quad (48)$$

Last equality follows from $\Phi_n = \mathbf{A}_n^{-1}$, and $\mathbf{R}_{\mathbf{Y}_n|\mathbf{X}_2,\mathbf{Y}_0,\hat{\mathbf{Y}}_{n}}$ is computed in Appendix I-B. Therefore, the derivative of $\mathbf{R}_1$ remains [24]

$$\left[ \frac{\partial \mathbf{R}_1}{\partial \mathbf{A}_n} \right]^T = \mathbf{R}_{\mathbf{Y}_n|\mathbf{X}_2,\mathbf{Y}_0,\hat{\mathbf{Y}}_{n}} (\mathbf{A}_n \mathbf{R}_{\mathbf{Y}_n|\mathbf{X}_2,\mathbf{Y}_0,\hat{\mathbf{Y}}_{n}} + \mathbf{I})^{-1} - \sigma_r^2 (\mathbf{A}_n \sigma_r^2 + \mathbf{I})^{-1}. \quad (49)$$

Equivalently, we can obtain for the derivative of $\mathbf{R}_2$ that

$$\frac{\partial \mathbf{R}_2}{\partial \mathbf{A}_n} = \frac{\partial \mathbf{I} \left( \mathbf{X}_2; \mathbf{Y}_0, \hat{\mathbf{Y}}_{1:N} \right)}{\partial \mathbf{A}_n} \quad (50)$$

$$= \frac{\partial \mathbf{I} \left( \mathbf{X}_2; \hat{\mathbf{Y}}_{n}^c | \mathbf{Y}_0, \hat{\mathbf{Y}}_{n}^c \right)}{\partial \mathbf{A}_n} \quad (51)$$
Where we evaluate:

\[
I \left( X_2; \hat{Y}_n | Y_0, \hat{Y}_c^c \right) = H \left( \hat{Y}_n | Y_0, \hat{Y}_c^c \right) - H \left( \hat{Y}_n | X_2, Y_0, \hat{Y}_c^c \right)
\]

\[
= \log \det \left( A_n R_{Y_n | Y_0, \hat{Y}_c^c} + I \right) - \log \det \left( A_n R_{Y_n | X_2, Y_0, \hat{Y}_c^c} + I \right)
\]  \hspace{1cm} (50)

Conditional covariances are obtained in Appendix I-B. The derivative of \( R_2 \) thus remains:

\[
\left[ \frac{\partial R_2}{\partial A_n} \right]^T = R_{Y_n | Y_0, \hat{Y}_c^c} \left( A_n R_{Y_n | Y_0, \hat{Y}_c^c} + I \right)^{-1} - R_{Y_n | X_2, Y_0, \hat{Y}_c^c} \left( A_n R_{Y_n | X_2, Y_0, \hat{Y}_c^c} + I \right)^{-1}.
\]  \hspace{1cm} (51)

Plugging (45), (48) and (51) into (44) we obtain the gradient of the function, which is used in the GP algorithm to obtain \( g_\alpha (\lambda) \). Notice that for \( \alpha \leq \frac{1}{2} \), the roles of users \( s_1 \) and \( s_2 \) are interchanged, being user 1 decoded first. This roles would also need to be interchanged in the computation of the gradients of \( R_1 \) and \( R_2 \). Once obtained the dual function, we minimize it to obtain:

\[
\mathcal{R} (\alpha) = \min_{\lambda \geq 0} g_\alpha (\lambda).
\]  \hspace{1cm} (52)

To solve this minimization, we use the subgradient approach as in Section V. Taking all this into account we build up Algorithm 3. As for the previous section, we can only claim local convergence.

**Algorithm 3 Two-user WSR dual problem**

1. Initialize \( \lambda_{\text{min}} = 0 \) and \( \lambda_{\text{max}} \)
2. repeat
   3. \( \lambda = \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{2} \)
4. Obtain \( \{ A_1^*, \ldots, A_N^* \} = \arg \max L_\alpha \left( \{ A_1, \ldots, A_n \}, \lambda \right) \) from Algorithm 4
5. Evaluate \( h \) as in (25), where \( R_{Y_n | Y_0} \) follows Appendix I-B.
6. if \( h \leq 0 \), then \( \lambda_{\text{min}} = \lambda \), else \( \lambda_{\text{max}} = \lambda \)
7. until \( \lambda_{\text{max}} - \lambda_{\text{min}} \leq \epsilon \)
8. \( \mathcal{R} (\alpha) = \alpha R_1 (A_1^*, \ldots, A_N^*) + (1 - \alpha) R_2 (A_1^*, \ldots, A_N^*) \).

**VII. Numerical Results**

We evaluate the performance of D-WZ coding within a single-frequency network composed of a central base station \( \text{BS}_0 \) plus its first tier of six cells. The radius of each cell is 700 m, and BSs have all three receive antennas. On the other hand, users have two antennas, are located at the edge of the central cell and transmit isotropically, i.e., \( Q_i = \frac{P}{2} I \). Transmitted power is set to 23 dBm, and wireless channels are simulated taking into account path loss, log-normal shadowing and Rayleigh fading. Specifically, fading is assumed i.i.d. among antennas, and shadowing uncorrelated among BSs. Two propagation scenarios
Algorithm 4 GP to obtain $g_\alpha (\lambda)$

1: Initialize $A_0^n = 0, n = 1, \ldots , N$ and $t = 0$
2: repeat
3: Compute the gradient $G^t_n = \nabla A_n \mathcal{L}_\alpha (\lambda, A^1_1, \cdots , A^t_N), n = 1, \cdots , N$ from (44).
4: Choose appropriate $s_t$
5: Set $\hat{A}^t_n = A^t_n + s_t \cdot G^t_n$. Calculate $\hat{A}^t_n = U_n \eta U^\dagger_n$. Then, $\bar{A}^t_n = U_n \max \{ \eta, 0 \} U^\dagger_n, n = 1, \cdots , N$.
6: Choose appropriate $\gamma_t$
7: Update $A^{t+1}_n = A^t_n + \gamma_t (\bar{A}^t_n - A^t_n), n = 1, \cdots , N$
8: $t = t + 1$
9: until The sequence converges $\{ A^1_1, \cdots , A^t_N \} \rightarrow \{ A^*_1, \cdots , A^*_N \}$
10: Return $\{ A^*_1, \cdots , A^*_N \}$

are studied: i) Line-of-sight (LOS), with path-loss exponent $\alpha = 2.6$ and shadowing standard deviation $\sigma = 4$ dB. ii) Non Line-of-sight (N-LOS), with $\alpha = 4.05$ and $\sigma = 10$ dB.

Fig. 2 plots the cumulative density function (cdf) of the uplink rate$^7$ for a single-user network, considering different values of the backhaul rate $R$. Particularly, Fig. 2(a) depicts results for LOS propagation, and shows gains up to 6 Mbit/s @ 5% outage, with $R = 15$ Mbit/s. It is clearly shown that BSs cooperation becomes more remarkable for lower outage probabilities. On the other hand, Fig. 2(b) shows results for N-LOS propagation, where rate gains are reduced. In this case, cooperation becomes more convenient for higher outages, showing that @ 50% outage, three-fold gains arise with 15 Mbit/s of backhaul.

Fig 3 plots the uplink rate of a single-user network with $R = 7$ Mbit/s, for different number $N$ of cooperative BSs. First, Fig. 3(a) depicts the cdf of the user’s rate under LOS propagation conditions. We notice that @ 5% outage, with only 1 cooperative BS, a rate gain of 2 Mbit/s is obtained with respect to the non-cooperative case. However, when increasing the number of cooperative BSs to 6, only an additional rate gain of 2 Mbit/s is obtained. That is, the impact of introducing new cooperative BSs in the system diminishes as the network grows. Again, cooperation is more useful for low outages. On the other hand, Fig. 3(b) depicts results for N-LOS propagation. It can be shown that, @ 50% outage, the rate is doubled from 1 cooperative BS to 6 cooperative BS. This fact highlights the relevant role of

$^7$The user is assumed to transmit at 1 Mbaud, i.e., 1 Msymb/s.
macro-diversity on N-LOS conditions, which are most common ones on urban cellular networks. Next, Fig. 4 compares the rate performance of our D-WZ approach with respect to that of Quantization [10], assuming LOS propagation. We consider a simple network with two BSs: BS0 and BS1, and plot its outage capacity with D-WZ and with uniform quantization, respectively. Both are normalized with respect to the outage capacity with infinite backhaul and computed at a probability of outage of $10^{-2}$. Results show significant gains, of up to 12%, for low backhaul rates, and highlights the fact that D-WZ requires half of backhaul rate than Quantization to converge to the $\infty$ backhaul capacity.

Fig 5 depicts the expected sum-rate\(^8\) of the multi-user setup versus the total number of users. Results are shown for different values of the backhaul rate. Although the sum-rate analysis (see Sec. VI-B) was carried out for two users only, the extension to $U > 2$ is straightforward. Fig 5(a) depicts the sum-rate for LOS propagation. We first notice that the sum rate with $\infty$ backhaul capacity (i.e., outer region 1) is far away from the sum-rate with D-WZ compression. This is explained by means of outer region 2: the sum-rate of the system is constrained by the available rate at the backhaul network. On the other hand, for N-LOS propagation (Fig. 5(b)), upper bound 2 is not reached. Indeed, for less than 5 users, the expected sum-rate with only $R = 15$ Mbit/s of backhaul is almost identical to that of $R = \infty$.

Therefore, for practical number of transmitters, the full rate gain due to macro-diversity is obtained via D-WZ compression. Finally, Fig. 6(a) and Fig. 6(b) depict the rate region of a 2-user network, with and without LOS respectively, for different values of the Backhaul rate $R$. It is clearly shown that the region is significantly enlarged with only 5 Mbit/s of backhaul rate.

VIII. CONCLUSIONS

We studied distributed compression for the uplink of a coordinated cellular network with $N+1$ multi-antenna BSs. Considering a constrained backhaul of limited capacity $R$, base stations $BS_1, \cdots, BS_N$ distributely compress their received signal using a Distributed Wyner-Ziv code. The compressed vectors are sent to $BS_0$, which centralizes user’s decoding. Considering single and multiple users within the network, respectively, the D-WZ scheme has been optimized using the users’ rate as the performance metric.

APPENDIX I

CONDITIONAL COVARIANCES

We derive here conditional covariances used throughout the paper. (See supporting material)

\(^8\)The expected sum-rate is obtained by averaging the sum-rate of the system over the user’s channels.
A. The single user case

\[ R_{Y_0|Y_0} = H_{s,n} \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} \right)^{-1} Q H_{s,n}^\dagger + \sigma_r^2 I, \quad n = 1, \ldots, N. \]  

\[ R_{Y_1,N|Y_0} = \begin{bmatrix} H_{s,1} \\ \vdots \\ H_{s,N} \end{bmatrix} \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} \right)^{-1} Q \begin{bmatrix} H_{s,1} \\ \vdots \\ H_{s,N} \end{bmatrix}^\dagger + \sigma_r^2 I. \]  

\[ R_{Y_n|Y_0,Y_n} = H_{s,n} \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + \sum_{j \neq n} Q H_{s,j}^\dagger (\sigma_r^2 I + \Phi_j)^{-1} H_{s,j} \right)^{-1} Q H_{s,n}^\dagger + \sigma_r^2 I. \]  

B. The multiuser case

Define \( H_{s,n} = [H_{1,n}, H_{2,n}] \) and \( Q = \text{diag}(Q_1, Q_2) \). Then, Conditional covariances \( R_{Y_n|Y_0}, R_{Y_1,N|Y_0} \) \( R_{Y_n|Y_0,Y_n} \) and \( R_{Y_n|Y_0,Y_n} \) follow Subsection I-A. Furthermore, let \( i, j \in \{1, 2\} \) with \( j \neq i \), then:

\[ R_{Y_n|X_i,Y_0,Y_n} = H_{j,n} \left( I + \frac{Q_{i,j}}{\sigma_r^2} H_{j,0}^\dagger H_{j,0} + \sum_{p \neq n} Q_{j,p} H_{j,p}^\dagger (\sigma_r^2 I + \Phi_p)^{-1} H_{j,p} \right)^{-1} Q_{j,n} H_{j,n}^\dagger + \sigma_r^2 I \]  

APPENDIX II

PROOF OF PROPOSITION 1

Let the chain rule for mutual information:

\[ I \left( X_s; Y_0, \hat{Y}_{1:N} \right) = I \left( X_s; Y_0 \right) + I \left( X_s; \hat{Y}_{1:N}|Y_0 \right). \]  

Also, let expand the constraint to obtain:

\[ I \left( Y_{1:N}; \hat{Y}_{1:N}|Y_0 \right) = H \left( \hat{Y}_{1:N}|Y_0 \right) - H \left( \hat{Y}_{1:N}|Y_0, Y_{1:N} \right) \]

\[ = I \left( X_s; \hat{Y}_{1:N}|Y_0 \right) + H \left( \hat{Y}_{1:N}|Y_0, X_s \right) - H \left( \hat{Y}_{1:N}|Y_0, Y_{1:N} \right). \]  

Given the Markov chain in Theorem 2: \( H \left( \hat{Y}_{1:N}|Y_0, Y_{1:N} \right) = H \left( \hat{Y}_{1:N}|Y_0, X_s, Y_{1:N} \right) \), which plugged into (59):

\[ I \left( Y_{1:N}; \hat{Y}_{1:N}|Y_0 \right) = I \left( X_s; \hat{Y}_{1:N}|Y_0 \right) + I \left( Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s \right). \]
Let now $\mathcal{P}$ be the feasible set of conditional probabilities $\prod_{i=1}^{N} p\left(Y_i|Y_i^c\right)$, i.e., the set for which $I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0\right) \leq R$. Hence, making use of (60), the feasible set satisfies:

$$I\left(X_s; Y_{1:N}|Y_0\right) \leq R - I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right). \tag{61}$$

Introducing (61) into (58), we derive that for the feasible set:

$$I\left(X_s; Y_0, \hat{Y}_{1:N}\right) \leq I\left(X_s; Y_0\right) + R - I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right). \tag{62}$$

Now, notice that $I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right) = I\left(Z_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right)$ where $Z_i$ is the AWGN at the BS$_i$. This mutual information is minimized in $\mathcal{P}$ for $p\left(\hat{Y}_{1:N}\right)$ Gaussian. Therefore, $I\left(X_s; Y_0, \hat{Y}_{1:N}\right)$ in (62) is maximum in $\mathcal{P}$ for Gaussian distributed vectors $\hat{Y}_{1:N}$, specifically those satisfying $I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0\right) = R$ (i.e., those for which equality holds in (62) and (61)). As mentioned, the received vectors $Y_i$ are also Gaussian. Therefore, at the optimum, $\hat{Y}_i$ and $Y_i$ are jointly Gaussian, so we can write $\hat{Y}_i = MY_i + Z_i^c$ with $M$ a constant matrix and $Z_i^c$ an independent Gaussian vector. However, as the multiplication by a matrix does not affect mutual information, we can state that vectors $\hat{Y}_i = Y_i + Z_i^c$ are also optimal, with $Z_i^c \sim \mathcal{CN}\left(0, \Phi_i\right)$. Using this relationship, we evaluate

$$I\left(X_s; Y_0, \hat{Y}_{1:N}\right) = \log \det \left( I + \frac{Q}{\sigma_n^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^{N} H_{s,n}^\dagger (\sigma_n^2 I + \Phi_n)^{-1} H_{s,n} \right) \tag{63}$$

Furthermore, we can also obtain:

$$I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0\right) = H\left(\hat{Y}_{1:N}|Y_0\right) - H\left(\hat{Y}_{1:N}|Y_{1:N}, Y_0\right) \tag{64}$$

$$= \log \det \left( I + \text{diag}\left(\Phi_1^{-1}, \ldots, \Phi_N^{-1}\right) R_{Y_{1:N}|Y_0}\right).$$

**Appendix III**

**Proof of Upper Bound 2**

To prove the statement, we first rewrite the objective and constraint of (9) as (58) and (60), respectively. At the optimum point of maximization (9), the constraint is satisfied. Therefore, $I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0\right) \leq R$, which plugged into (60) obtains

$$I\left(X_s; \hat{Y}_{1:N}|Y_0\right) \leq R - I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right), \tag{65}$$

which in turn introduced into (58) allows to bound

$$I\left(X_s; Y_0, \hat{Y}_{1:N}\right) \leq I\left(X_s; Y_0\right) + R - I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right) \tag{66}$$

Since $I\left(Y_{1:N}; \hat{Y}_{1:N}|Y_0, X_s\right) \geq 0$ by definition, we can state that $I\left(X_s; Y_0, \hat{Y}_{1:N}\right) \leq I\left(X_s; Y_0\right) + R$. 
APPENDIX IV
PROOF OF PROPOSITION 2

In this Appendix, we solve the non-convex optimization (13). Let us first expand:

\[
\log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^+ H_{s,0} + Q H_{s,1}^+ \left( A_1 \sigma_r^2 + I \right)^{-1} A_1 H_{s,1} \right)
\]

\[
= \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^+ H_{s,0} \right) + \log \det \left( I + \left( A_1 \sigma_r^2 + I \right)^{-1} A_1 \left( R_{Y_i|Y_0} - \sigma_r^2 I \right) \right)
\]

\[
= \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^+ H_{s,0} \right) + \log \det \left( I + A_1 R_{Y_i|Y_0} \right) - \log \det \left( I + A_1 \sigma_r^2 \right) .
\]

(67)

First equality follows from the value of \( R_{Y_i|Y_0} \) in (53). Notice that \( \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^+ H_{s,0} \right) \) does not depend on \( A_1 \). Therefore, the Lagrangian for the problem can be written as

\[
\mathcal{L} \left( A_1, \lambda, \Phi \right) = (1 - \lambda) \log \det \left( I + A_1 R_{Y_i|Y_0} \right) - \log \det \left( I + A_1 \sigma_r^2 \right) + \lambda R - \text{tr} \left\{ \Phi A_1 \right\},
\]

where \( \lambda \) is the Lagrange multiplier for the explicit constraint and \( \Phi \preceq 0 \) for the semidefinite positiveness constraint. The derivative of the Lagrangian with respect to \( A_1 \) thus reads [24]:

\[
\left[ \frac{\partial \mathcal{L}}{\partial A_1} \right]^T = (1 - \lambda) R_{Y_i|Y_0} \left( I + A_1 R_{Y_i|Y_0} \right)^{-1} - \sigma_r^2 \left( I + A_1 \sigma_r^2 \right)^{-1} - \Phi.
\]

(68)

Accordingly, the KKT conditions for the problem, which are necessary but not sufficient, are:

\[
i) \left[ \frac{\partial \mathcal{L}}{\partial A_1} \right]^T = 0
\]

\[
ii) \lambda \left( \log \det \left( I + A_1 R_{Y_i|Y_0} \right) - R \right) = 0
\]

\[
iii) \text{tr} \left\{ \Phi A_1 \right\} = 0.
\]

Let now the eigen-decomposition \( R_{Y_i|Y_0} = USU^\dagger \). Then, it can be readily shown that matrix \( A_1^* = U \text{diag} \left( \eta_1, \cdots, \eta_{N_i} \right) U^\dagger \), with

\[
\eta_j = \left[ \frac{1}{\lambda^*} \left( \frac{1}{\sigma_r^2} - \frac{1}{s_j} \right) - \frac{1}{\sigma_r^2} \right]^+,
\]

(70)
satisfies the KKT conditions, with multiplier \( \lambda^* \) such that \( \sum_{j=1}^{N_i} \log (1 + \eta_j s_j) = R \) (therefore, \( \lambda^* < 1 \)), and multiplier \( \Phi^* \preceq 0 \) computed from (68). Let now show that \( A_1^* \) satisfies also the general sufficiency condition for optimality, which is presented in the next Lemma.

**Lemma 3:** [23, Proposition 3.3.4] Let the differentiable maximization (13). Consider a pair \( (A_1^*, \lambda^*) \) for which \( \lambda^* \left( \log \det \left( I + A_1^* R_{Y_i|Y_0} \right) - R \right) = 0. \) Then, \( A_1^* \) is the global maximum of (13) if:

\[
A_1^* \in \arg \max_{A_1 \geq 0} \mathcal{L} \left( A_1, \lambda^* \right),
\]

(71)
where the Lagrangian\(^9\) has been defined in (68).

**Lemma 4:** Let \( A, B \succeq 0 \), with ordered eigenvalues \( \Gamma_A, \Gamma_B \) respectively. Then,

\[
\log \det (I + AB) \leq \log \det (I + \Gamma_A \Gamma_B),
\]

(72)

with equality whenever \( A \) and \( B \) have conjugate transpose eigenvectors.

**Proof:** It is known that \( \log \det (I + AB) = \log \det (I + \Gamma_{AB}) \), where \( \Gamma_{AB} \) are the ordered eigenvalues of \( AB \). Those eigenvalues are logarithmically majorized [38, Definition 1.4] by the product of the separate eigenvalues of \( A \) and \( B \), i.e., \( \Gamma_{AB} \prec \times \Gamma_A \Gamma_B \) [39, Theorem 9.H.1.d]. Let now the function \( f(X) = \log \det (I + X) \) be defined on the set of semi-definite positive diagonal matrices, i.e., \( f(X) = \sum \log (1 + x_i) \). We may apply [38, Theorem 1.6] to prove that \( f(X) \) is a Schur-geometrically-convex function. Accordingly, provided that \( \Gamma_{AB} \prec \times \Gamma_A \Gamma_B \), then \( \log \det (I + \Gamma_{AB}) \leq \log \det (I + \Gamma_A \Gamma_B) \), which concludes the proof. \( \blacksquare \)

Let us prove now that our pair \((A_1^*, \lambda^*)\) satisfies (71). The lagrangian is defined for the problem as

\[
\mathcal{L}(A_1, \lambda^*) = (1 - \lambda^*) \log \det (I + A_1 \Gamma_{Y|Y_0}) - \log \det (I + A_1 \sigma_t^2) + \lambda^* R.
\]

(73)

Recall that \( \lambda^* < 1 \) and \( \Gamma_{Y|Y_0} = USU^\dagger \). Then, using Lemma 4 we can bound:

\[
\max_{A_1 \succeq 0} \mathcal{L}(A_1, \lambda^*) \leq \max_{\eta \succeq 0} (1 - \lambda^*) \log \det (I + \eta S) - \log \det (I + \eta \sigma_t^2) + \lambda^* R
\]

\[
= \lambda^* R + \sum_{j=1}^{N_1} \max_{\eta_j \succeq 0} (1 - \lambda^*) \log (1 + \eta_j s_j) - \log (1 + \eta_j \sigma_t^2)
\]

(74)

where \( \eta \) is the diagonal matrix of ordered eigenvalues of \( A_1 \). The individual maximizations on \( \eta_j \) in (74) are not concave. However, the continuously differentiable functions \( f_j(\eta_j) = (1 - \lambda^*) \log (1 + \eta_j s_j) - \log (1 + \eta_j \sigma_t^2) \) have only two stationary points, i.e.,:

\[
\frac{df_j}{d\eta_j} = 0 \rightarrow \left\{ \begin{array}{l}
\eta_j = \infty \\
\eta_j = \frac{1}{\lambda^*} \left( \frac{1}{\sigma_t^2} - \frac{1}{s_j} \right) - \frac{1}{\sigma_t^2}
\end{array} \right.
\]

(75)

Recalling that \( 0 \leq \lambda^* < 1 \), it is easy to show that \( \lim_{\eta_j \to -\infty} f_j(\eta_j) = -\infty \). Therefore \( \eta_j = \infty \) is the global minimum of the problem. Considering the other stationary point, it can be shown that its second derivative is lower than zero. Accordingly, it is a local maximum, unique because there is no other. However, we restricted the optimization to the values \( \eta_j \geq 0 \). Hence, functions \( f_j(\eta_j) \) take maximum at:

\[
\eta_j^* = \left[ \frac{1}{\lambda^*} \left( \frac{1}{\sigma_t^2} - \frac{1}{s_j} \right) - \frac{1}{\sigma_t^2} \right]^+. \]

(76)

\(^9\)Notice that the semi-definite multiplier \( \Phi \) has been removed of the Lagrangian by constraining the maximization (71) to the set \( A_1 \geq 0 \).
Plugging this optimal values into (74), we bound

\[
\max_{A_1 \geq 0} \mathcal{L}(A_1, \lambda^*) \leq \lambda^* R + (1 - \lambda^*) \sum_{j=1}^{N_1} \log (1 + \eta_j^* s_j) - \sum_{i=1}^{N} \log (1 + \eta_j^* \sigma_r^2)
\]  

(77)

Furthermore, noticing that for \( A_1^* = U \eta^* U^\dagger \):

\[
\mathcal{L}(A_1^*, \lambda^*) = \lambda^* R + (1 - \lambda^*) \sum_{j=1}^{N_1} \log (1 + \eta_j^* s_j) - \sum_{i=1}^{N} \log (1 + \eta_j^* \sigma_r^2),
\]  

(78)

then, it is demonstrated that \( A_1^* = \arg \max_{A_1 \geq 0} \mathcal{L}(A_1, \lambda^*) \). Hence, the general sufficient condition holds, and it is optimum. Finally, \( \Phi_n^* = (A_1^*)^{-1} \), which concludes the proof.

APPENDIX V

A. Proof of Proposition 3

In this Appendix, we solve the non-convex optimization \( A_n^* = \arg \max_{A_n \geq 0} \mathcal{L}(A_1, \ldots, A_N, \lambda) \). First, recall that \( \log \det (I + \text{diag}(A_1, \ldots, A_N) R_{Y_{1:N} | Y_0}) \) is equal to \( I(Y_{1:N}; \hat{Y}_{1:N} | Y_0) \) (as shown in (64), changing \( \Phi_n = A_n^{-1} \forall n \)). Then:

\[
\log \det (I + \text{diag}(A_1, \ldots, A_N) R_{Y_{1:N} | Y_0}) = I(Y_{1:N}; \hat{Y}_{1:N} | Y_0) = I(Y_{1:N}; \hat{Y}_{1:N}^c | Y_0) + I(Y_{1:N}; \hat{Y}_n | Y_0, \hat{Y}_n^c) = I(Y_n^c; \hat{Y}_n | Y_0) + I(Y_n; \hat{Y}_n | Y_0, \hat{Y}_n^c) = \log \det (I + \text{diag}(A_1, \ldots, A_{n-1}, A_{n+1}, \ldots, A_N) R_{Y_n^c | Y_0}) + \log \det (I + A_n R_{Y_n | Y_0, Y_n^c})
\]  

(79)

where second equality follows from the chain rule for mutual information, and the third from the Markov chain in Proposition 2. Finally, the fourth equality evaluates the mutual information as in (64), with \( \Phi_n = A_n^{-1} \). The conditional covariances are computed in Appendix I. Later, using (55) and equivalently to (67):

\[
\log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{n=1}^{N} H_{s,n}^\dagger (A_n \sigma_r^{-2} + I)^{-1} A_n H_{s,n} \right)
\]

\[
= \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{j \neq n} H_{s,j}^\dagger (A_j \sigma_r^{-2} + I)^{-1} A_j H_{s,j} \right) + \log \det (I + A_n R_{Y_n | Y_n^c}) - \log \det (I + A_n \sigma_r^{-2})
\]  

(80)
Therefore, plugging (79) and (80) into (18), we can expand the function under study as:

\[
\mathcal{L}(A_1, \cdots, A_N, \lambda) = \log \det \left( I + \frac{Q}{\sigma_r^2} H_{s,0}^\dagger H_{s,0} + Q \sum_{j \neq n} H_{s,j}^\dagger \left( A_j \sigma_r^2 + I \right)^{-1} A_j H_{s,j} \right)
\]

\[
+ \log \det \left( I + A_n R_{Y_n|Y_0} \right) - \log \det \left( I + A_n \sigma_r^2 \right)
\]

\[-\lambda \left( \log \det \left( I + \text{diag} \left( A_{n-1}, A_{n+1}, \cdots, A_N \right) R_{Y_n|Y_0} \right) + \log \det \left( I + A_n R_{Y_n|Y_0} \right) - R \right)
\]

In order to obtain \( A_n^* = \arg \max_{A_n \geq 0} \mathcal{L}(A_1, \cdots, A_N, \lambda) \), we first notice that the following Lagrangian satisfies \( \arg \max_{A_n \geq 0} \mathcal{L}(A_n, \lambda) = \arg \max_{A_n \geq 0} \mathcal{L}(A_1, \cdots, A_N, \lambda) \), and it is identical to the Lagrangian in (73). Therefore, we can directly apply derivation (73)-(78) to solve it:

Consider first \( \lambda \geq 1 \). For it, (1 - \( \lambda \)) \( \log \det \left( I + A_n R_{Y_n|Y_0} \right) - \log \det \left( I + A_n \sigma_r^2 \right) \leq 0, \forall A_n \geq 0 \). Therefore, it is readily shown that:

\[
0 = \arg \max_{A_n \geq 0} \mathcal{L}(A_1, \cdots, A_N, \lambda) \quad \text{for} \lambda \geq 1.
\]

Let now \( \lambda < 1 \). Applying (73)-(78) we show that

\[
U_n \eta U_n^\dagger = \arg \max_{A_n \geq 0} \mathcal{L}(A_1, \cdots, A_N, \lambda) \quad \text{for} \lambda < 1,
\]

with \( R_{Y_n|Y_0, Y_n} = U_n \text{SU}_n^\dagger \), and

\[
\eta_j = \left[ \frac{1}{\lambda} \left( \frac{1}{\sigma_r^2} - \frac{1}{s_j} \right) - \frac{1}{\sigma_r^2} \right]^+, \quad j = 1, \cdots, N_n.
\]

This concludes the proof.

**B. Solution of (19) with \( \lambda \geq 1 \)**

Applying equivalent arguments to those in (67), we can rewrite the Lagrangian in (19) as:

\[
\mathcal{L}(A_1, \cdots, A_N, \lambda) = (1 - \lambda) \log \det \left( I + \text{diag} \left( A_1, \cdots, A_N \right) R_{Y_1|Y_0} \right)
\]

\[
- \log \det \left( I + \text{diag} \left( A_1, \cdots, A_N \right) \sigma_r^2 \right) - \lambda R,
\]

It is clear that, for \( \lambda \geq 1 \), the Lagrangian takes its optimal value at \( \{ A_1^*, \cdots, A_N^* \} = 0 \).
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Fig. 1. Multiple-source compression with side information at the decoder.

(a) CDF versus R, LOS
(b) CDF versus R, N-LOS

Fig. 2. Single user capacity results with respect to the backhaul rate. BS$_1$, · · · , BS$_6$ cooperate with BS$_0$.

(a) CDF versus N, LOS
(b) CDF versus N, N-LOS

Fig. 3. Single user capacity results with respect to the number of Cooperative BS. Backhaul rate $R = 7$ Mbit/s
Fig. 4. Outage Capacity with D-WZ and with Quantization, respectively, for different values of the backhaul rate $R$. LOS.

Fig. 5. Sum-rate versus number of users. BS$_1, \ldots, BS_6$ cooperate with BS$_0$.

Fig. 6. Rate region for different values of $R$. BS$_1, \ldots, BS_6$ cooperate with BS$_0$. 