A student experiment on error analysis and uncertainties based on mobile–device sensors

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Abstract

Science students must deal with the errors inherent to all physical measurements and be conscious of the necessity to express their as a best estimate and a range of uncertainty. Errors are routinely classified as statistical or systematic. Although statistical errors are usually dealt with in the first years of science studies, the typical approaches are based on performing manually repetitive observations. Here, based on data recorded with the sensors present in many mobile devices a set of laboratory experiments to teach error and uncertainties is proposed. The main aspects addressed are the physical meaning of the mean value and standard deviation, and the interpretation of histograms and distributions. Other activities focus on the intensity of the fluctuations in different situations, such as placing the device on a table or held in the hand in different ways and the number of measurements in an interval centered on the mean value as a function of the width expressed in terms of the standard deviation. As applications to every day situations we discuss the smoothness of a road or the different positions to take photographs both of them quantified in terms of the fluctuations registered by the accelerometer. This kind of experiments contributes to gaining a deep insight into modern technologies and statistical errors and, finally, to motivate and encourage engineering and science students.
I. INTRODUCTION

In many experimental situations when a measurement is repeated, for example when we measure a time interval with a stopwatch, or the distance at which a ball launched with a spring-loaded projectile launcher falls or a voltage with a digital multimeter, successive readings, under identical conditions, give slightly different results. This occurs beyond the care we take to always launch the balls in exactly the same way or to connect the components of the circuit so that they are firmly attached. In effect, this phenomenon is due to the fact that most measurements in the real world present statistical uncertainties. When facing repeated observations with different results it is natural to ask ourselves what is the most representative value and what is the confidence that we can have in that value. The International Standard Organization (ISO) defines the errors evaluated by means of the statistical analysis of a series of observations as type A in contrast with other, systematic, sources of errors, type B, whose evaluation is estimated using all available non-statistical information such as instrument characteristics or observer’s individual judgment. In this work, we focus on the teaching of statistical errors in the first years of engineering and science studies using modern sensors.

The study of error analysis and uncertainties plays a prominent role in the first years of all science courses. Perhaps the most important message is to persuade students that any measurement is useless unless a confidence interval is specified. It is expected that after finishing their studies, students are able to discuss whether a result agrees with a given theory, or if it is reproducible, or to distinguish a new phenomenon from other already known. With this objective, various experiments are usually proposed in introductory laboratory courses. These experiments usually involve a great amount of repetitive measurements such as dropping small balls or measuring the length of hundred or thousands of nails using a vernier caliper. The measurements obtained are usually examined from the statistical viewpoint plotting histograms, calculating mean values and standard deviations and, eventually, compared with those expected from a known distribution, typically a normal distribution. Though these experiments are illustrative, most of them are tedious and do not adequately reflect the present state of the art.

The importance for their careers of a physicist being able to design a measurement procedure, select the equipment or instruments, perform the process and finally express the results
as the best estimation and its uncertainties has been remarked. However, recent studies\cite{9,11}, suggest that students lack these abilities. Several difficulties have been described\cite{9}: the lack of understanding of the need to make several measurements, or insight into the notion of confidence interval or the ability to distinguish between random and systematic errors.

Mobile devices such as smartphones or tablets which usually include several sensors (accelerometer, magnetometer, ambient light sensor, among others) appear as modern and versatile alternatives to deal with statistical errors. In fact, the use of smartphone sensors has been proposed in many science experiments\cite{12,13}, ranging from experiments with quadcopters\cite{14} to shadowgraph imaging\cite{15}. The inevitable noise of the sensors, so annoying in any measurement, can be used, however, favorably, to illustrate basic concepts of statistical treatment of measurements. It is possible, using these sensors, to acquire hundreds or thousands of repeated values of physical magnitude in a few seconds that can be analyzed in the mobile device or transferred to a PC. Thanks to their sensitivity these values clearly display statistical fluctuations. In this paper we propose a set of laboratory activities to teach error analysis and uncertainties using modern technologies in a stimulating approach. In the next Section we review some basic concepts about error analysis, while in Section III we describe the proposed activities. Finally, in Section IV we present the summary and conclusion.

II. STATISTICAL ERRORS

In this work we focus on the teaching of statistical errors which due to a multitude of causes are inherent to all physical measurements\cite{1}. We assume that in a given experiment an observation is repeated $N$ times under identical conditions obtaining different results $x_i$, with $i = 1, ..., N$. It can be shown that the best representative or estimate of the set of values is given by the mean value $\bar{x}$ defined as

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i. \quad (1)$$

The deviation with respect to the mean value is identified with $\epsilon_i = x_i - \bar{x}$. It can be shown that the mean value defined as above minimizes the sum of the squared deviations. Intuitively, it can be regarded as the center-of-mass of the set of the observations or equivalent to the value closest to all the other values. In statistical errors it is of interest to quantify
the dispersion of the values around the mean value or, informally, the width of the cloud. The standard deviation defined as

\[ \sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \]  

(2)

can be seen as a measure of this dispersion. If the number of observations, \( N \), is large enough, \( \sigma \) it is characteristic of the set of all the possible observations and does not depend on the specific set of observations. In practice, the uncertainty in the determination of a physical magnitude depends on the number of repeated measurements we have done.

The standard error, or standard deviation of the mean, is defined as \( \sigma_x = \sigma / \sqrt{N} \) and it is demonstrated that it represents the margin of uncertainty of the mean value obtained in a particular set of measurements. The result of a specific measurement is usually expressed in terms of the mean value and the standard error as

\[ \bar{x} \pm \sigma_x \]  

(3)

representing the best estimate and the confidence in that value. It is worth highlighting that the standard deviation is related to the degree to which an observation deviated from the mean value whereas the standard error is an estimate of the uncertainty of the mean value. In a practical situation the standard error depends on the number of measurements taken with \( N^{-1/2} \). Then, given a set of \( N \) measures the standard deviation gives an idea of the dispersion of an ideal set of infinite measures while the standard error represents the uncertainty of our set. This margin can be reduced by increasing the number of measurements, however, the square-root implies that this reduction is relatively slow.

It is an empirical fact that when the uncertainties of a continuous magnitude do not have a preferred direction they follow a normal or Gaussian distribution. The probability distribution function resembles the well-known bell-shaped curve centered around the mean value observed in many phenomena in natural and social sciences. The width of the bell is given by the standard deviation, the inflection points are located at \( \bar{x} \pm \sigma \).

III. A LABORATORY BASED ON MOBILE DEVICES

The vast majority of smartphones and tablets have several built-in sensors, in particular, triaxial accelerometers capable of measuring the acceleration of the device in the three inde-
pendent spatial directions. Though it is possible to use all the components simultaneously, here, for the sake of clarity, the following experiments are based on the z direction which is defined as perpendicular to the screen. To access the values registered by the sensors a specific piece of software or app is necessary.

From the many apps available in the digital stores we selected Physics Toolbox Suite\textsuperscript{10}, Androsensor and PhyPhox\textsuperscript{17} whose screenshots are shown in Fig. 1. Using these apps it is possible to select the relevant sensors, and to setup the parameters such as the duration of the time series and the sampling frequency. The registered data can be analyzed directly on the smartphone screen or transferred to the cloud and studied on a Personal Computer using a standard graphics package. Others useful characteristics present in these apps are the delayed execution and the remote access via wi-fi or browser. These capabilities allow the avoidance of touching or pushing the mobile device when the experiments has started.

![Screenshot of Physics Toolbox Suite and PhyPhox](image)

**FIG. 1.** Screenshots of the most used apps: Physics Toolbox suite (left) and Phyphox (center and right). The right panel shows a Phyphox screenshot of the experiment *Statistical Basics* including a temporal series of the vertical component of the acceleration (top) and the corresponding histogram (bottom) overlapped with a Gaussian curve with the same mean and standard deviation indicated in the image.
A. A first approach to fluctuations

The first experiment consists of recording the fluctuations of the vertical component of the accelerometer sensor with the mobile device standing on a table, during a time lapse. In this experiment, and all the described above, it is possible to use an app and download the data or use the PhyPhox app or to choose in the menu the experiment Statistical physics which automatically displays temporal series and histograms. In our case, we choose, unless stated otherwise, a delay of 3 s and register $a_z$ for 10s. The 3 s delay is important to avoid touching the device at the moment the register starts and introducing spurious values. The screenshot is displayed in Fig. 1 (right). In this case the number of measurements and the sampling period are $N = 2501$ and $\Delta t = 0.004$ respectively.

Although the device is at rest on a horizontal surface, the $a_z$ values displayed in Fig. 2 fluctuate steadily around a mean value given by the gravitational acceleration $\pi = 9.776 \text{m/s}^2$ and a standard deviation $\sigma = 0.008 \text{m/s}^2$. The non-zero mean value is due to the fact that accelerometers are in fact force sensors that cannot distinguish between the acceleration and the gravitational field\textsuperscript{18,19}. If, instead of the acceleration sensor, the so-called linear acceleration pseudo-sensor were used, the measurements would fluctuate around 0 m/s$^2$.

The corresponding histogram is displayed in Fig. 3 with, for the sake of comparison, a normal (Gaussian) curve with the same mean value and standard deviation. The vertical scale has been adjusted so that the area under the normal curve and the sum of the bins of the histogram are both equal to 1. From this figure, it can be concluded that the histogram and the normal curve agree very well. By increasing the number of samples $N$ and simultaneously decreasing the width of the bins, it can be seen (not shown here) that the agreement improves even more.

B. Resolution in digital sensors

It can be noticed in Fig. 2 that the sensor values display a clear regularity, the ordinates do not take arbitrary continuous values but only a discrete set. This is more evident in Fig. 4 where, in the left panel, the horizontal axis of Fig. 2 has been zoomed out and, in the right panel, a laid down histogram with the same values is shown. The difference between the discrete values in the vertical axis is the resolution of the instrument, that is, the
FIG. 2. Temporal values of the $z$ component of the accelerometer while the smartphone is standing at rest on a table. The values registered by the sensor are indicated with small circles while the lines are guides for the eye.

The minimum difference that the sensor can register. This is typical of digital instruments, where a continuous magnitude (such as acceleration, in this case) is transformed by a sensor into an analog electrical signal, which is transformed by an analog-to-digital converter (ADC) into a digital signal which can only take certain discrete values. The acceleration sensor of the Samsung S7 is a K6DS3TR, as shown in Table I. The resolution given by the manufacturer (sometimes it appears incorrectly as accuracy), is $\delta = 0.0023942017$ m/s$^2$, which, as can be seen in Fig. 4, corresponds exactly to the difference between the groups of acceleration values.

The resolution of the sensor can be related to other important characteristic of the digital sensors. One is the range of the sensor, $R$, corresponding to the difference between the maximum and minimum value that it is capable of measuring. The maximum number of different values that the sensor can register is $2^n$ where the $n$ is known as the number of bits of the DAC. Resolution is simply the quotient between the range and the total number of
In the sensor used in this experiment Table I shows that the accelerometer used in this case can measure a maximum acceleration of 78.4532 m/s². Since it registers not only positive measures, but also negative accelerations, the range turns out to be twice the maximum value, that is, $R = 156.9064$ m/s². Therefore it can be determined that this sensor is capable of measuring $R/\delta = 65536$ different values and since $65536 = 2^{16}$, this means that it is a 16-bit sensor. These characteristics can be easily verified on the datasheets of the sensors.

C. Different noise intensities

In order to gain insight into the role of noise in different situations in this experiment two sets of data are considered. In the first the smartphone is steady on a table and in the
FIG. 4. Discrete nature of the sensor data. The left panel is similar to Fig. 2 but zoomed out in the horizontal axis to emphasize the discrete nature of the accelerometer values. The right panel shows the same values in a laid down histogram with the same vertical scale.

other the device is held in the hand of the experimenter. In Fig. 5 both temporal series are shown while in Fig. 6 the corresponding histograms are displayed. Moreover, histograms are overlapped with normal curves with their respective mean values and standard deviations.

It is clearly appreciated that the dispersion of data, quantified by the standard deviation, is larger when the smartphone is held in the hands than when the device is on the table. It is also noticeable in both cases that normal curves agree very well with the histograms. This activity can be translated to other settings. In particular, this is one the basic mechanisms of seismographs.

D. Number of observations in a given interval

In general, the fundamental property of distributions is that the area under one sector of the curve represents the probability that a new measurement falls within this interval. In
| Phone            | Sensor | Range (m/s²) | Resolution (m/s²) |
|------------------|--------|--------------|-------------------|
| Samsung Galaxy S7 | K6DS3TR | 78.4532      | 0.0023942017      |
| LG G3            | LGE    | 39.226593    | 0.0011901855      |
| Nexus 5          | MPU-6515 | 19.613297    | 0.0005950928      |
| iPhone 6         | MPU-6700 |              |                   |
| Samsung J6+      | LSM6DSL | 39.2266      | 0.0011971008      |
| Xiaomi Redmi Note7 | ICM20607 | 78.4532      | 0.0011901855      |
| Samsung Galaxy S9 | LSM6DSL | 78.4532      | 0.0023942017      |

TABLE I. Sensor characteristics of the devices used in the different activities obtained with the Androsensor app. In the case of the iPhone the manufacturer does not provide this information.

For the case of normal curves, it is usual to take intervals centered around the mean value and the width in terms of the standard deviation. Then, it is shown that 68% of the observations will be in the "σ" interval, this is the interval between \( \bar{x} - \sigma \) and \( \bar{x} + \sigma \),

\[
P(\bar{x} - \sigma < x < \bar{x} + \sigma) = \int_{\bar{x}-\sigma}^{\bar{x}+\sigma} f(x)dx = 0.682...
\]  

Similarly, the intervals "2σ," "3σ," and "4σ" concentrate, respectively, 95.4%, 99.7%, and 99.9% of the observations. This is a characteristic of normal distributions, i.e., almost all the observations are concentrated around a few "sigmas" around the mean value and graphically the curve is relatively stretched.

To illustrate this phenomenon, Fig. 7 shows the temporal series of Fig. 2 with horizontal lines indicating the σ intervals. It is evident from this figure that most of the values concentrate around the mean value and a few σ intervals. To quantify this relation, two experiments with different noise intensities (on the floor and on an aircraft) are described in Table II. In this table the number of observations in a given interval are compared with the expected values according to the normal distribution. It can be seen that the expected percentages are similar to those according to a normal distribution.

An interesting point is to express these ranges in terms of the resolution of the sensor. In this way it is noted that 68% of the measurements are within a radius interval equal to 3 times the resolution. On the other hand, 99% of the measurements are within a radius interval equal to 10 times the resolution of the sensor.
E. Optimal number of measurements

Accuracy and precision are different concepts. On the one hand, the precision of a measurement, related to the random errors, is characterized by the dispersion of the values, i.e. the standard deviation. The smaller \( \sigma \), the less dispersion and therefore, the greater the precision. On the other hand, accuracy is related to systematic error and quantified according to the agreement with an expected value. As mentioned above, in observations under identical and independent conditions, the standard deviation does not change considerably with the number of observations \( N \). In contrast, the standard error, giving account of the range of confidence in the estimation of the mean value in a particular set of measurements decreases with \( N^{-1/2} \). In Table. [11] the standard deviation and the standard error are shown for different set of observations with different \( N \). It is clear from that data, as mentioned above, \( \sigma \) is nearly constant while \( \sigma_x \) clearly decreases.

As the decrease of the standard error with \( N \) is slow, an important question in practical
situations is about the optimal number of observations \( N_{opt} \). Indeed, if we could repeat the measurements infinite times we could achieve a perfect knowledge of the best estimate and, accordingly, the standard error would vanish. In fact, in addition to the statistical errors, type B errors must be taken into account. In absence of other sources of systematic errors, the optimal number of observations is defined when the standard error is equal to the resolution of the digital instrument \( \sigma_x = \sigma/\sqrt{N} \sim \delta \). In the experiment, depicted in Table [III] given the resolution of S7 model 0.0012 \( \text{m/s}^2 \), the optimal number is \( N_{opt} \sim 250 \).
FIG. 7. Temporal series indicating the vertical intervals in term of units of the standard deviation \( \sigma \).

F. The best position to take a photograph with a cell phone

An interesting experiment is to study the intensities of the fluctuations depending on the way in which the experimenter holds his/her device. This activity can be adapted to be proposed as a challenge to a group of students consisting in trying to hold the device as steadily as possible. Another possibility, not recommended by the authors, is, similarly to Ref.\(^{20}\), to study the fluctuations of the gait of a pedestrian as a function of the alcohol beverage intake.

The steadiness of the device is quantified by the standard deviation of a given temporal series. In Table IV we display the intensities of the fluctuations in different positions. It is evident from these values that keeping the device close to trunk represents a more stable position.
Experiment 1, \( N = 2098 \), \( a_z = (9.776 \pm 0.008) \text{ m/s}^2 \)

| Interval | Theo. (%) | Theo. Exp. (%) | Exp. |
|----------|------------|----------------|------|
| \( \bar{x} \pm \sigma \) | 68.2 | 1431 | 1468 |
| \( \bar{x} \pm 2\sigma \) | 95.4 | 2001 | 2003 |
| \( \bar{x} \pm 3\sigma \) | 99.7 | 2092 | 2090 |

Experiment 2, \( N = 1501 \), \( a_z = (9.65 \pm 0.29) \text{ m/s}^2 \)

| Interval | Theo. (%) | Theo. Exp. (%) | Exp. |
|----------|------------|----------------|------|
| \( \bar{x} \pm \sigma \) | 68.2 | 1024 | 1003 |
| \( \bar{x} \pm 2\sigma \) | 95.4 | 1432 | 1422 |
| \( \bar{x} \pm 3\sigma \) | 99.7 | 1497 | 1497 |

TABLE II. Measurements in a given interval. Expected number of values according to a normal distribution and to the experimental results, respectively blue and red, displayed in Figs. 5 and 6.

G. The smartphone as a way to assess road quality

Recently, smartphones’ sensors were proposed to assess road quality\(^{21}\). In this activity, which can be performed outdoors, students can assess the quality of a road. A means of transport, in this case a car, under similar conditions (speed) is employed, but other possibilities, such as a bike, are equally feasible. In Table V the intensities of the fluctuations traveling by car on different roads are listed. To get an insight of the fluctuations due to the road in the first row the noise with the car stopped and engine idle is indicated. Just for the sake of comparison a similar measurement but in a flying aircraft is included.

The intensity of the fluctuations depends on the specific sensor but exhibits in all cases the same trends mentioned above. To summarize the results, all the intensities of the fluctuations using the different built-in sensors in several situations are depicted in Fig. 6.

IV. CONCLUSION

The main conclusion is that modern mobile-device sensors are useful tools for teaching error analysis and uncertainties. In this work we proposed several activities that can be performed to teach uncertainties and error analysis using digital instruments and the built-in
| $N$ | $g$  | $\sigma$ | $\sigma_x$ |
|-----|------|----------|-----------|
| 563 | 9.493| 0,020    | 0,00085   |
| 1156| 9.487| 0,019    | 0,00054   |
| 1746| 9.478| 0,018    | 0,00044   |
| 2348| 9.469| 0,019    | 0,00039   |
| 2941| 9.466| 0,020    | 0,00036   |
| 3535| 9.464| 0,019    | 0,00032   |
| 4166| 9.462| 0,019    | 0,00029   |
| 4733| 9.464| 0,019    | 0,00027   |
| 5327| 9.465| 0,019    | 0,00026   |
| 5919| 9.464| 0,020    | 0,00026   |

**TABLE III.** Mean value, standard deviation and standard error as a function of the number of measures.

**FIG. 8.** Comparative table of the standard deviation $\sigma$ for different mobile devices in different activities as a function of the different models (see Table I).
TABLE IV. Standard deviation of $a_z$ of two smartphone models in different positions.

| Situation           | N   | $\sigma_{G3}$ (m/s$^2$) | N   | $\sigma_{XR7}$ (m/s$^2$) |
|---------------------|-----|--------------------------|-----|--------------------------|
| On the table        | 1746| 0.0184                   | 2407| 0.0052                   |
| Close to the body   | 1190| 0.067                    | 2502| 0.1030                   |
| Selfie position     | 1190| 0.1206                   | 2512| 0.1413                   |

TABLE V. Assessment of the quality of different roads. Standard deviation of $a_z$ while the device is on the floor of the car with the screen orientated upwards.

| Situation         | N    | $\sigma_{G3}$ (m/s$^2$) | N   | $\sigma_{XR7}$ (m/s$^2$) |
|-------------------|------|--------------------------|-----|--------------------------|
| Engine idle       | 1181 | 0.3818                   | 4984| 0.0352                   |
| Smooth pavement   | 1200 | 1.3487                   | 4974| 0.5642                   |
| Stone pavement    | -    | -                        | 4952| 1.1491                   |
| Aircraft          | 1999 | 0.4374                   | -   | -                        |

sensors included in modern mobile devices. It is shown that the distribution of fluctuations obeys normal (Gaussian) statistics. Its main characteristics –mean, standard deviation, histograms– are analyzed. The role of noise intensity, spreading or narrowing the normal bell-shaped curve is revealed. The width of the distribution in terms of units of the standard deviation can be related to the number of measurements in a given interval. Holding the mobile in different ways also gives an idea of how firmly it is held. In this approach, the lengthy and laborious manipulations necessary in traditional approaches based on repetitive measurements, are avoided allowing teaching to focus on the fundamental concepts. These experiments could contribute to motivating students and to showing them the necessity of considering uncertainty analysis.
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