Modeling comminution processes in ball mills as a canonical ensemble

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Abstract

A new approach to describe comminution processes in general ball mills as a macroscopic canonical ensemble is proposed. Using hamiltonian method, the model is able to take simultaneously into account the internal dynamics from mechanical motions inside the vial and external effects like electromagnetic and gravitational forces. Relevant physical observables are extracted using statistical mechanics approach through partition function at finite temperature. The method enables numerical calculation using Monte Carlo technique to obtain, for instance particle number evolution in term of system temperature. It is argued that the method is experimentally more verifiable than the conventional approaches based on geometrical displacements. An example of simulation for typical spex mill is also given.

Key words: comminution, modeling, ball mill, hamiltonian, canonical ensemble

1. Introduction

The comminution processes in recent years attract the attention among scientists and engineers due to the increasing demand of ultrafine powders for nanotechnology applications in many areas. The demand then requires the improvement of comminution equipments like ball mills, roller mills and so
on. Unfortunately, the development of such comminution equipments always contains a lot of uncertainties due to a wide range of unknown parameters. These, in fact, lead to significant statistical errors. In order to overcome such problems, several models have been developed to quantitatively describe comminution process in various types of mills [1, 2, 3, 4, 5].

On the other hand, mathematical modeling and simulation may provide prior information and constraint to the unknown parameter ranges which should be useful to develop more optimized experimental strategy in comminution processes. However, in most cases of mathematical models, the physical observables like grain-size etc are extracted from a set of equation of motions (EOMs). Such EOM’s are considered to govern as complete as possible the dynamics of the system, from the mechanical motions to the evolution of grain-size distribution. This approach is obviously suffered from the nonlinearities of the equations under consideration, and then the requirement of high computational power to solve them numerically. This fact often discourages a quantitative and deterministic approach for the simulation of such system. These nonlinear effects like chaotic behavior of the sphere motions within the mill encourages some works modeling the system using semi-empirical approaches [6]. However most of semi-empirical models require a large number of experimental data based on prior observations [7], or measured variables obtained from simulation results by other authors [8, 9].

More empirical approach is based on the physically realistic modelization of the ball mill system [10, 11]. This approach in general deals with three aspects : (1) evaluation of milling bodies dynamics and energetic inputs transferred to powders; (2) description of the effects of such inputs on powders breakage; (3) description of powders evolution in terms of particle size distribution [12]. In a recent work [12], a comprehensive study on this line for the case of spex mixer / mill was performed by deploying the 3D simulation for milling bodies motion, and the population balance method to describe the granulometric evolution. Then, both are related through the energetic inputs in the population balance formalism which is obtained from the simulation of milling bodies motion.

In this paper we propose a novel model and approach combining the deterministic approach for milling bodies motion, and the statistical approach to relate them with considerable macroscopic physical observables. This work is devoted to overcome the following problems in some conventional approaches:
• Experimentally it is almost impossible to trace the geometrical displacements of all matters in a vial with proper time resolution to verify the models which are based on the classical EOM. This problem is getting worse as one simulates a system with matters at few nanometers scale with comparable size of time-space resolution.

• Solving a set of EOMs numerically, and then performing a simulation with high accuracy (enough time resolution) require huge efforts on both computing capacity and running time.

• Taking into account the external circumstances around the vial like electromagnetic field and so forth. This might be interesting when one considers a comminution process which can reach the level of few nanometers.

Therefore this work is intended to provide a tool for a nanometer system in a vial by developing direct relations between the vial internal dynamics with some external physical observables which should be more measurable. We should remark here that the vial internal dynamics is yet described empirically using physical modelization approach.

Further, rather solving the EOM’s governing the whole dynamics, we use the hamiltonian approach to model all relevant interactions, and extract the physical observables through partition function by treating the system as a canonical ensemble at finite temperature. As a consequence, instead of observing the geometrical evolution of matters in term of time in a ball mill, we can investigate the particle number distribution in term of temperature. This introduces a novel method relating the internal dynamics with the macroscopic physical parameter like temperature, rather than time and geometrical displacements which are in most cases difficult to realize. The preliminary work but with incomplete hamiltonian on this matter has been reported in our previous work [13], however the present paper comprises more complete theoretical formalism including the external forces and the real simulation using Monte Carlo integration method.

The paper is organized as follows. First, after this introduction we present the model and explain the underlying knowledge in detail. Before summarizing the results, numerical analysis and simulation for the case of ball mill with a structure similar to the well-known spex mixer / mill are discussed.
2. The model

The whole system is modeled empirically using hamiltonian method. First we construct the total hamiltonian describing the dynamics in the ball mills. It is further followed by formulating the partition function and extracting the relevant thermodynamics observables.

2.1. The dynamics

In our model, the dynamics of each 'matter' in the system, i.e. balls and powders inside the vial, are described by a hamiltonian \( H_m(\vec{r}, t) \). The index \( m \) denotes the powder (p) or ball (b) and \( \vec{r} = (x, y, z) \). The hamiltonian contains some terms representing all relevant interactions working on the matters inside the system as follow,

\[
H_m = H_0 + V_{m-m} + V_{m-v} + V_{m-m'} + V_{\text{ext}} ,
\]

with \( v \) denotes the vial, while \( H_0 \) is the free matter hamiltonian containing the kinetic term,

\[
H_0 = \frac{1}{2m_m} \sum_{i=1}^{n_m} |(\vec{p}_m)_i|^2 ,
\]

where \( n_m \) is the matter number, \( m_m \) and \( \vec{p}_m \) are the matter mass and momentum respectively. Throughout the paper we assume that the mass or size evolution of matters is uniform for the same matters.

The matter self-interaction \( V_{m-m} \), the matter–vial interaction \( V_{m-v} \) and the interactions between different matters may be induced by, for instance, impact (\( V^{\text{imp}} \)) and Coulomb (\( V^{\text{Coul}} \)) potentials,

\[
V^{\text{imp}}_{m-m'}(\vec{r}, t) = - \sum_{i=1}^{n_m} \sum_{j=1}^{n_m'} \int_0^t (\xi_{mni})_{ij} d (\xi_{mn'j})_{ij} \vec{n} \cdot (\vec{F}_{mn'})_{ij} ,
\]

\[
V^{\text{Coul}}_{m-m'}(\vec{r}) = Q_m Q_{m'} \sum_{i=1}^{n_m} \sum_{j=1}^{n_{m'}} \frac{1}{(|\vec{r}_m)_i - (\vec{r}_{m'})_j|} ,
\]

with \( Q_m \) is the matter charge, while \( m, m' : v, p, b \) and \( \vec{n} \) is the unit normal vector. These potentials are considered describing the mechanical and static electrical properties of the matters. The first potentials should in fact represent the whole classical dynamics among the matters, i.e. the impact forces among balls and powders. This form will be clarified soon below. The
Coulomb potential disappears if the interacting matters have neutral charges. Also it works only in a short range of distance. Therefore, it should be negligible for some physics at nanometers scale as considered in the present case. The impact forces between the vial surface and balls or powders are treated in the same way using Eq. (3) by considering that the surface is formed by a set of fixed spheres [12]. This is inline with the simulation in the last section where the space displacement in the vial is at a distance comparable with the desired powder size, i.e. few tens nanometers at the largest.

On the other hand, instead of Eq. (3) we can consider a simpler 'effective potential' like the harmonic oscillator \( V_{m-m'}^{\text{osc}}(\vec{r}) = \frac{1}{2} k_{mm'} \Delta r^2 \) to represent the whole mechanical properties in terms of 'effective coupling' \( k_{mm'} \). In this case, if \( m_m \gg m_{m'} \), the potential can be rewritten as \( V_{m-m'} = \frac{1}{2} m_m \omega_m^2 \Delta r^2 \) since \( \omega_m \equiv \sqrt{k_{mm'}/m_m} \). Actually this is the case of ball–powder interactions since \( m_b \gg m_p \) by the order of namely \( O(10^2) \). Nevertheless, in contrast with its simplicity, \( V_{osc} \) absorbs the time dependencies and another interesting physical parameters characterizing the material properties like viscoelasticity, Young modulus etc. The time dependency is important to directly relate the system temperature with physical observables through finite temperature partition function as shown in the next subsection. Therefore, in this paper we take the impact potential to represent the mechanical properties in the system.

The impact potential in Eq. (3) is induced by the impact force consists of normal and tangential components [12], \( \vec{F}_{\text{imp}}^{nm} = \vec{F}_{\text{imp}}^N + \vec{F}_{\text{imp}}^T \). The normal component may be written [14],

\[
\vec{F}_{nm}^N(\vec{r}, t) = \left[ \frac{2 \Upsilon_{mm'}}{3(1 - \nu_{mm'})} \sqrt{R_{eff}_{mm'}^2 \left( \xi_{mm'}^{3/2} + \frac{3}{2} A_{mm'} \sqrt{\xi_{mm'} \frac{d\xi_{mm'}}{dt}} \right)} \right] \vec{n} .
\]

Here the first term is the elastic part based on the Hertz contact law, and the second term is the dissipative one that takes into account material viscosity. \( \Upsilon_{mm'} \) is the Young modulus and \( \nu_{mm'} \) represents the Poisson ratio of the sphere material. The term \( R_{eff}_{mm'} = (R_m R_{m'})/(R_m + R_{m'}) \) represents the effective radius, while \( \xi_{mm'} = R_m + R_{m'} - |\vec{r}_m - \vec{r}_{m'}| \) is the displacement with \( R_m \) is the radius of interacting matter. \( A \) is a dissipative parameter [14, 15, 16],

\[
A_{mm'} = \frac{3 \eta_{mm'} - \eta_m^2}{3(3 \eta_{mm'} + 2 \eta_m)} \left[ \frac{(1 - \nu_{mm'}^2)(1 - 2 \nu_{mm'})}{\Upsilon_{mm'} \nu_{mm'}^2} \right] .
\]
The viscous constants $\eta_m$ and $\eta_{m'}$ relate the dissipative stress tensor to the deformation tensor \([14, 15]\).

There are several proposed formulations for the tangential component $F^T_{mm'}(\vec{r}, t)$. However it always assumes that the material slide upon each other in the case where the Coulomb condition $\mu |F^N_{mm'}| \leq |F^T_{mm'}|$ holds, otherwise some viscous frictions occur \([17]\). In particular it follows that $F^T_{mm'}(\vec{r}, t) \propto m_{\text{eff}}$, where the effective mass is $m_{\text{eff}} \equiv m_mm_{m'}/(m_m + m_{m'})$ \([12]\). Obviously, in our case with large mass discrepancy between powder and ball, i.e. $m_p/m_b \sim 0$ which leads to $m_{\text{pl}} \sim m_p$, the tangential impact force may be neglected for a good approximation. So, let us from now consider the normal component dominated impact force, that is $F^N_{mm'}(\vec{r}, t) \sim \vec{F}^N_{mm'}(\vec{r}, t)$. This result simply yields the impact potential as written in Eq. \((3)\) due to the Euler-Lagrange equation,

$$\vec{F} = -\frac{dV}{dt} + \frac{d}{dt} \left( \frac{dV}{d\vec{v}} \right), \quad (7)$$

since the dependency on matter velocity appears only in the tangential component $F^T_{mm'}(\vec{r}, t)$ \([12]\).

Beside the interactions among the matters itself, it is also possible to take into account the external potentials working on the whole system. For instance concerning the ball dynamics with relatively large ball size, the gravitational potential,

$$V^{\text{grav}}_{\text{ext}} = m_m G \sum_{i=1}^{n_m} (z_m)_i, \quad (8)$$

might be important. Here, $G$ is the gravitational constant. On the other hand, this should be less important for the powder dynamics due to its tiny size.

On the other hand, we may also incorporate the effect of external electromagnetic field surrounding the system to the charged matters. The potential is induced by the Lorentz force, $F^\text{EM}_m = Q_m (\vec{E} + \vec{v}_m \times \vec{B})$, which leads to,

$$V^{\text{EM}}_{\text{ext}} = Q_m \sum_{i=1}^{n_m} \left[ \phi - (\vec{v}_m)_i \cdot \vec{A} \right], \quad (9)$$

and satisfies Eq. \((7)\). $\phi$ and $\vec{A}$ are the electromagnetic scalar and vector potentials related to the electric and magnetic fields by $\vec{E} = -\nabla \phi - \partial \vec{A} / \partial t$.
and $\vec{B} = \vec{\nabla} \times \vec{A}$. The inclusion of electromagnetic potential shifts the kinetic term in Eq. (2) as follow,

$$H_0 \rightarrow H_{0+EM} = \frac{1}{2m} \sum_{i=1}^{n_m} \left| (\vec{p}_m)_i - Q_m \vec{A}_i \right|^2 + n_m Q_m \phi,$$  \hspace{1cm} (10)

From now, let us focus only on the dynamics of powders which is our main interest in the sense of comminution process. From Eqs. (1), (2), (3), (4) and (9), the total hamiltonian for the powder in our model is,

$$H_p = \frac{1}{2m} \sum_{i=1}^{n_p} \left| (\vec{p}_p)_i - Q_p \vec{A}_i \right|^2 + n_p Q_p \phi$$

$$- \frac{1}{2} \sum_{i(\neq j)=1}^{n_p} \sum_{j=1}^{n_p} \int_0^\beta d(\xi_{pp})_{ij} \vec{n} \cdot \left( F_{pp}^{imp} \right)_{ij}$$

$$- \sum_{m:b,v} \sum_{i=1}^{n_m} \sum_{j=1}^{n_m} \int_0^\beta d(\xi_{pm})_{ij} \vec{n} \cdot \left( F_{pm}^{imp} \right)_{ij}, \hspace{1cm} (11)$$

for $Q_p \neq 0$. The last two potentials represent the total impact potential among powders; powders and vial; powders and balls respectively. Obviously, we do not need to take into account the ball self-interaction $V_{b-b}^{imp}$ nor ball-vial interaction $V_{b-v}^{imp}$. This is actually the advantage of using hamiltonian method.

### 2.2. Physical observables

As mentioned briefly in introduction, the greatest advantage of deploying the hamiltonian method is one can extract some physical observables without solving the EOM’s governing the system. Instead, in a canonical ensemble of matter $m$ one can consider the partition function,

$$Z_m = \int \prod_{i=1}^{n_m} d\vec{p}_i \ d\vec{r}_i \ \exp \left[ - \int_0^\beta dt H_m \right], \hspace{1cm} (12)$$

governed by a particular hamiltonian $H_m$. Here, $\beta \equiv 1/(k_B T)$ with $k_B$ and $T$ are the Boltzman constant and absolute temperature. Having partition function at hand, we can obtain some thermodynamics quantities in the system through relations namely,

$$F_m = -\frac{1}{\beta} \ln Z_m , \hspace{1cm} (13)$$
for free energy and,

\[ P_m = -\frac{\partial F_m}{\partial V} = -\frac{F_m}{V}, \]  

(14)

for pressure in a vial with volume \( V \).

In order to see the contributions from the interactions, it is more convenient to consider the normalized partition function,

\[ Z'_m \equiv \frac{Z_m}{Z_{0m}} = \frac{\int \prod_{i=1}^{n_m} d\vec{p}_i \, d\vec{r}_i \, \exp \left[ -\int_0^\beta dt \, H_m \right]}{\int \prod_{i=1}^{n_m} d\vec{p}_i \, \exp \left[ -\int_0^\beta dt \, H_{0m} \right]}, \]  

(15)

and further,

\[ P'_m = F'_m \equiv \frac{F_m}{F_{0m}} = \frac{\ln Z_m}{\ln Z_{0m}}. \]  

(16)

Performing the integral over time \( (t) \), we immediately obtain a temperature dependent partition function, while the integrals over \( \vec{p}_i \) are the decoupled gaussian integral which can be easily calculated. In the case of Eq. (11) it gives,

\[ Z_p = \left( \frac{2 m_p \pi}{\beta} \right)^{n_p/2} \int \prod_{i=1}^{n_p} d\vec{r}_i \, \exp \left[ -\int_0^\beta dt \, H'_p \right], \]  

(17)

and,

\[ Z'_p = \int \prod_{i=1}^{n_p} d\vec{r}_i \, \exp \left[ -\int_0^\beta dt \, H'_p \right], \]  

(18)

where the interaction hamiltonian is,

\[ H'_p = n_p Q_p \phi - \frac{1}{2} \sum_{i(\neq j)=1}^{n_p} \sum_{j=1}^{n_p} \int_0^\beta d (\xi_{pp})_{ij} \vec{n} \cdot (\vec{F}_{imp})_{ij} \]

\[ - \sum_{m:b,v} \sum_{i=1}^{n_p} \sum_{j=1}^{n_m} \int_0^\beta d (\xi_{pm})_{ij} \vec{n} \cdot (\vec{F}_{imp})_{ij}. \]  

(19)

Obviously, only the scalar potential of external electromagnetic field contributes to the total energy of system under consideration. In other words, we can conclude here that in our model the magnetic field \( \vec{B} \) does not influence the ball mill system, but the electric field \( \vec{E} \) does.
Moreover, we can perform the integration over time \((t)\) and \(\xi_{ij}\) to obtain further,

\[
Z_p' = \int \prod_{i=1}^{n_p} d\vec{r}_i \exp \left[ -\beta \left( Q_p \phi - \frac{2}{15} \sum_{i(\neq j)=1}^{n_p} \sum_{j=1}^{n_p} \gamma_{pp} \frac{R_{pp}^{\text{eff}} (\xi_{pp})_{ij}^{5/2}}{1 - v_{pp}^2} \right) \right. \\
\left. - \frac{4}{15} \sum_{m:b,v} \sum_{i=1}^{n_p} \sum_{j=1}^{n_m} \gamma_{pm} \frac{R_{pm}^{\text{eff}} (\xi_{pm})_{ij}^{5/2}}{1 - v_{pm}^2} \right].
\]  

(20)

From this result, the thermodynamics observables are clearly not affected with the dissipative term, i.e. the second term in Eq. (5). After performing same integration we obtain,

\[
P_p' = 1 - \beta \mathcal{F} \ln^{-1} \left( \frac{2 m_p \pi}{\beta} \right),
\]

(21)

respectively with,

\[
\mathcal{F} \equiv 2 \int \prod_{i=1}^{n_p} d\vec{r}_i \left[ Q_p \phi - \frac{2}{15} \sum_{i(\neq j)=1}^{n_p} \sum_{j=1}^{n_p} \gamma_{pp} \frac{R_{pp}^{\text{eff}} (\xi_{pp})_{ij}^{5/2}}{1 - v_{pp}^2} \right. \\
\left. - \frac{4}{15} \sum_{m:b,v} \sum_{i=1}^{n_p} \sum_{j=1}^{n_m} \gamma_{pm} \frac{R_{pm}^{\text{eff}} (\xi_{pm})_{ij}^{5/2}}{1 - v_{pm}^2} \right].
\]

(22)

Eq. (21) provides a general behavior for temperature-dependent pressure in the model, while the geometrical structure and motion of vial is absorbed in the function \(\mathcal{F}\). From Eq. (21) clearly the physically meaningful regions are for \(0 < T < (2 m_p \pi k_B)^{-1}\) and \(T \geq T_{th}\). The later is equivalent to the condition,

\[
\mathcal{F} \leq k_B T_{th} \ln \left( 2 m_p \pi k_B T_{th} \right),
\]

(23)

and \(T_{th}\) is always greater than \((2 m_p \pi k_B)^{-1}\). The behavior of the temperature-dependent pressure in the model is depicted in Fig. 1.

3. Simulation

The simulation is done for a typical case of ball mill, that is spex mixer / mill. However, the model under consideration can in principle be extended to deal with another types of ball mills by changing the coordinate system accordingly.
3.1. Coordinate system

In the case of spex mill, the system contains a vial moving in a non-inertial system [10, 12]. Therefore, we can adopt the same coordinate system and schematization of vial surface as Figs. 1 and 2 in [12]. Since the system under consideration is moving on a non-inertial system $\vec{r} = (x, y, z)$, we should transform all coordinates in our formula to the coordinate system of inertial system $(X, Y, Z)$. In the simulation, we make use of previous results on the roto-translation transformation [10, 12],

$$X(t) = x \cos \theta(t) \cos \alpha(t) + y \cos \theta(t) \sin \alpha(t) + z \sin \alpha(t) + L \sin \theta(t),$$  \hspace{1cm} (24)

$$Y(t) = -x \sin \alpha(t) + y \cos \alpha(t),$$  \hspace{1cm} (25)

$$Z(t) = -x \sin \theta(t) \cos \alpha(t) - y \sin \theta(t) \sin \alpha(t) + z \cos \alpha(t) + L \cos \theta(t).$$  \hspace{1cm} (26)

Here, $L$ is the length of mechanical shaft-arm, $\theta$ and $\alpha$ are the angles of rotation around the $Y$- and $z$-axis. Both angles can be written as follows,

$$\theta = \theta_0 \sin(\omega t + \varphi),$$  \hspace{1cm} (27)

$$\alpha = \alpha_0 \sin(\omega t + \varphi).$$  \hspace{1cm} (28)
where $\theta_0$ and $\alpha_0$ are the angular momentum around the respected axis, $\omega$ is the frequency and $\varphi$ is the phase factor which depends on the initial conditions.

In our simulation, for the sake of convenience the Cartesian coordinate is transformed into the cylindrical coordinate system,

$$(x, y, z) \rightarrow (x, r \cos \vartheta, r \sin \vartheta),$$

with $\vartheta$ is the rotation angle around $x$ in vial bases and $r \equiv \sqrt{y^2 + z^2}$.

As already mentioned above, we can apply the proposed model to any types of ball mill. This can be accomplished by replacing the coordinate system of ball mill under consideration like Eqs. (24)$\sim$(26) to the appropriate ones which represent its geometrical motion.

3.2. Technique

The simulation within the present model and its underlying numerical calculation are done using Monte Carlo technique. This is the most appropriate technique to deal with higher dimensional integral of many matters involved in the system like Eq. (22) [18].

We should note that the simulation here is performed to provide a complete picture on the model and its applications rather than showing a comprehensive numerical simulation that is out of the scope of present paper. A comprehensive Monte Carlo integration in the present model requires proper resolution on 3-dimensional space which should be comparable with the powder size, and also enough time resolution within full running period to represent the whole dynamics. Unfortunately, this kind of simulation is very time consuming, while the simulation itself is not the main interest in the present paper. As an illustration, for $100\mu m$ powder size the appropriate resolution should be at least $\sim 10^4 \times 10^4 \times 10^4$. The resolution must be increased accordingly as the powder size is decreasing. Also, the present simulation is done for a particular time, that is equivalent to a particular static position of vial motion. Note that in the full simulation, the time evolution and the frequency of vial rotation are related each other.

3.3. Results

Now we are ready to perform a preliminary simulation for the powders with various strengths of interactions characterized by the defined parameters inside the potentials, and also in a circumstances with non-zero electric field
Figure 2: $\mathcal{F}$ as a function of the ratio of matter number, $n_p/n_b$ for silica, steel and nickel powders for $R_b/R_p = 50/3$.

$\vec{E}$ as well. The simulation is done for vial length $l = 50$ mm, vial radius $r_v = 10$ mm, shaft-arm length $L = 200$ mm and ball radius $R_b = 5$ mm.

The simulation of $\mathcal{F}$ is performed for various values of the ratio of matter number ($n_p/n_b$) and matter size ($R_b/R_p$). Each case is also simulated for various materials of powders characterized with Young modulus ($\Upsilon$) and Poisson ratio ($v$) of its sphere materials. The results are depicted in Figs. 2 and 3.

Figs. 2 and 3 show logarithmic scale of $\mathcal{F}$ for silica, steel and nickel powders. The error bars are coming from statistical errors due to the uncertainties of Young modulus and Poisson ratio of each material. The errors are significant for small ratio of matter number and large ratio of matter size. These facts are natural since as large as the matter number ratio, and also as small as matter size ratio would increase the probability of collisions between balls and powders. On the other hand, the increasing matter number and
size ratios indicate the on-going comminution processes inside the vial.

Again, it should be remarked that the present simulation has lost the temperature evolution since it is done only for a single point of vial position. The temperature evolution will be recovered in a full simulation within the whole running period of ball mill.

4. Summary

We have proposed and discussed a novel model and approach for top-down mechanical comminution processes to produce nanomaterial using ball mill equipments. The study is focused on investigating the relevant potentials in the hamiltonian for a ball mill system and the formalism to extract relevant physical observables without tracking the geometrical displacements inside the vial. The governing hamiltonian is related to relevant physical observables through partition function of statistical mechanics approach.
From theoretical formalisms developed in this paper, we point out some interesting remarks which hold for any type (and geometrical motions) of ball mills,

- Any ball mills should have the same temperature dependencies of its normalized pressure as shown in Eq. (21). Because the geometrical structure is absorbed in the auxiliary function \( F \) in Eq. (22).

- The magnetic field does not affect the matter dynamics inside the vial, while the electric field does.

- The contribution of dissipative term in the impact force is negligible for small ratio of \( m_p/m_b \) that is almost the case in all ball mills. This fact simplifies the whole analysis since the preKnowledge of material viscosity is not needed anymore.

- The model does not involve the powder number distribution, for instance using the population balance algorithm etc [19], in the formalism. The static powder number is represented by the number of interacting matters inside the vial, \( n_m \). Nevertheless, one can consider dynamic powder number distribution using any breakage functions like \( n_m(t) \sim n_{0m}(t_0) \exp(t/t_0) \) with \( n_{0m} \) is the initial powder number.

According to these results, it can be argued that the model should easily be confirmed by the experiments by verifying the above features. More importantly, the experimental setup is much simpler than the conventional approaches due to no requirement on tracking the geometrical displacements of involving matters inside the vial.

We have also shown the simulation for particular case of spex mill using Monte Carlo technique to calculate the complicated integrals in the formalism. More detail and comprehensive simulations are still under progress. Such works involves some improvements. First of all, increasing the accuracy up to the realistic powder size at the order of \( \sim \) few tens nanometers. This means the Monte Carlo integration is done at the resolution of \( 10^8 \times 10^8 \times 10^8 \). Secondly, performing complete temperature (time) evolution with appropriate accuracy within a realistic running period for various powder number distributions. According to the present preliminary result, the proper time resolution is \( \sim O(\pi/\omega) \).
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