Sum rules on quantum hadrodynamics

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Abstract

The development on relativistic nuclear many-body theories is reviewed. The second order self-energies of hadrons are calculated from $\hat{S}_2$ matrix, and then an effective method to solve nuclear many-body problems, sum rules on quantum hadrodynamics, is summarized. The differences between this method and quantum hadrodynamics are discussed. The effective nucleon mass in the nuclear matter is redefined in the relativistic Hartree approximation, and a self-consistent relativistic mean-field model based on quantum hadrodynamics \textsuperscript{2} is proposed. It is pointed out that the original definition on effective nucleon mass in quantum hadrodynamics \textsuperscript{2} is not self-consistent, and all the parameters in the relativistic mean-field approximation should be fixed again according to the new definition.

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I. INTRODUCTION

Although quantum field theory has succeeded greatly in the scope of particle physics, people have met some difficulties when the methods of the quantum field theory are applied to nuclear systems. The ground state of the nuclear system is filled with interacting nucleons, which is different from the vacuum in perturbation theory. In addition, the coupling constants of strong interaction between nucleons are far larger than the fine structure constant in the quantum electrodynamics, so the perturbation method could not be performed perfectly on nuclear systems. At last, the nucleons and mesons consist of quarks and gluons, and are not fundamental components in the level of modern knowledge. All these factors determine the perturbation method in quantum field theory can only be generalized to solve nuclear many-body problems approximately and effectively.

Walecka and his group attempted to solve the nuclear many-body problems in the framework of quantum field theory, and developed the method of relativistic mean-field approximation [1, 2, 3]. In this method, the field operators of the scalar meson and vector meson are replaced with their expectation values in the nuclear matter, respectively. Therefore, the calculation is simplified largely. Until now, there have been some excellent review articles giving a more detailed description on this topic [3, 4, 5, 6]. In the earliest relativistic mean-field theory, the resultant compression modulus is almost 550 MeV [1], which is far from the experimental data range of 200 – 300 MeV. To solve this problem, the nonlinear self-coupling terms of scalar mesons are introduced to produce the proper equation of state of nuclear matter [7]. No doubt, additional parameters would give more freedoms to fit the saturation curve of nuclear matter. Zimanyi and Moszkowki developed the derivative scalar coupling model yielding a compression modulus of 225 MeV without any additional parameter [8].

With the bare nucleon-nucleon interaction, the properties of symmetric nuclear matter at various densities can be determined in the relativistic Brueckner-Hartree-Fock calculation. For each value of the density, the relativistic mean-field equations are solved and the corresponding coupling constants are fixed to the results of Brueckner calculation. Therefore, a relativistic mean-field model with density-dependent coupling constants is obtained [9, 10]. Meanwhile, the Debye screening masses of mesons in the nuclear matter are calculated in the relativistic mean-field approximation [11], and it shows all the screening meson masses
increase with the nucleon number density. With the meson masses replaced by their corresponding screening masses in Walecka-1 model, the saturation properties of the nuclear matter are fitted reasonably, and then a density-dependent relativistic mean-field model is proposed. The nonlinear self-coupling terms of the mesons are not included in both of these density-dependent relativistic mean-field models. In the nuclear matter, the screening meson masses increase with the density of the nuclear matter, and it is equivalent to the statement that the coupling constants decrease with the density increasing while the masses of mesons retain constant. At this point, these two models are consistent with each other.

In the past decade, the relativistic mean-field theory has been extended in various directions to provide a more realistic description of nuclei.

Pair corrections in the nucleus are included in the framework of relativistic Hartree-Fock-Bogoliubov theory\cite{5, 12}. Until now, this theory has been used in the calculations of the properties of both the stable nucleus and the nucleus far from $\beta$-stable line\cite{5, 13}.

Some attempts have been made to develop a model to solve nuclear many-body problems in the quark level. The most representative models are quark meson coupling model based on current quarks\cite{14, 15} and quark meson field model based on constituent quarks\cite{16, 17}.

Quantum chromodynamics (QCD) has two very important properties at low energies: chiral symmetry spontaneous breaking and confinement. It is believed that the two properties closely relate to the vacuum characteristics of QCD. From the Goldstone’s theorem, Goldstone bosons appear as the chiral symmetry is spontaneously broken. To carry out the constraint by appearance of Goldstone bosons, several relativistic many-body models considering the chiral symmetry spontaneous breaking have been developed\cite{6, 18, 19, 20}.

The relativistic mean-field results may be derived by summing the tadpole diagrams self-consistently in nuclear matter, retaining only the contributions from nucleons in the filled Fermi sea in the evaluation of the self-energy and energy density, i.e., the relativistic mean-field method is consistent to the relativistic Hartree approximation\cite{3}. It is correct only in the framework of original Walecka model (QHD-1). In the most popular QHD-2 model, in which the nonlinear self-coupling terms of the scalar meson are included, whether the relativistic mean-field method is still consistent to the relativistic Hartree approximation has not be studied. In Sect. II, we obtained the self-energies of hadrons by calculating the second order $\hat{S}$ matrix in the nuclear matter, then an effective method to solve nuclear many-body problems is evaluated. In Sect. III, the effective nucleon mass in the nuclear
matter is discussed with the nonlinear self-coupling terms of the scalar meson included in the Lagrangian density, and then a self-consistent relativistic mean-field scheme on quantum hadrodynamics 2 is proposed. The summary is given in Sect. IV.

II. THE SELF-ENERGIES OF HADRONS IN THE NUCLEAR MATTER

According to Walecka-1 model, the nucleons $\psi$ interact with scalar mesons $\sigma$ through a Yukawa coupling $\bar{\psi}\psi\sigma$ and with neutral vector mesons $\omega$ that couple to the conserved baryon current $\bar{\psi}\gamma_\mu\psi$. the Lagrangian density can be written as

$$\mathcal{L} = \bar{\psi} (i\gamma_\mu\partial^\mu - M_N) \psi + \frac{1}{2} \partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu\omega^\mu
- g_\sigma \bar{\psi}\sigma\psi - g_\omega \bar{\psi}\gamma_\mu\omega^\mu\psi,$$

(1)

with $M_N, m_\sigma$ and $m_\omega$ the nucleon, scalar meson and vector meson masses, respectively, and

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$

the vector meson field tensor.

The momentum-space propagators for the scalar meson, vector meson and the nucleon take the forms of

$$i\Delta(p) = \frac{-1}{p^2 - m_\sigma^2 + i\varepsilon},$$

(2)

$$iD_{\mu\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_\omega^2 + i\varepsilon},$$

(3)

$$iG_{F\alpha\beta}(p) = (\gamma_\mu p^\mu + M_N)_{\alpha\beta} \left( -\frac{1}{p^2 - M_N^2 + i\varepsilon} \right).$$

(4)

As the effect of Fermi sea is considered, an on-shell part

$$iG_{D\alpha\beta}(p) = (\gamma_\mu p^\mu + M_N)_{\alpha\beta} \left( -\frac{i\pi}{E(p)} \delta(p^0 - E(p))\theta(p_F - |\vec{p}|) \right),$$

(5)

is included in the nucleon propagator besides the Feynman propagator in Eq. (4), where $E(p) = \sqrt{\vec{p}^2 + M_N}$, and $p_F$ is the Fermi momentum of nucleons.

Since the vector meson couples to the conserved baryon current, the longitudinal part in the propagator of the vector meson will not contribute to physical quantities. Therefore, only the transverse part in the propagator of the vector meson is written in Eq. (3).

According to the Feynmann diagrams shown in Fig.1 and 2, the self-energies of the nucleon, the scalar and vector meson in the nuclear matter can be calculated with the Feynman rules.
in Ref. [3]. It should be noticed that there is a more factor of \((i)\) in each of the propagators of the hadrons in our manuscript than those propagators in Ref. [3].

In this section, we will calculate the self-energies of hadrons in the nuclear matter from Wick’s theorem of quantum field theory.

The momentum-space propagator of the nucleon takes the form of Feynman propagator [21]. Therefore, the Pauli blocking effect of Fermi sea is excluded in the propagator of the nucleon, and the on-shell propagator is no use in the following calculations.

The interaction Hamiltonian can be expressed as

\[
\mathcal{H}_I = g_\sigma \bar{\psi} \sigma \psi + g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi.
\] (6)

In the second order approximation, only

\[
\hat{S}_2 = \frac{(-i)^2}{2!} \int d^4x_1 \int d^4x_2 T [\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)]
\] (7)

in the S-matrix should be calculated in order to obtain the self-energy corrections of the nucleon and mesons.

A. The nucleon self-energy in nuclear matter

The second order self-energy of the nucleon coupling to the scalar meson is discussed firstly.

In order to obtain the second order self-energy correction of the nucleon in Fermi sea, only the normal ordering product

\[
N \left[ \bar{\psi}(x_1)\sigma(x_1)\psi(x_1)\bar{\psi}(x_2)\sigma(x_2)\psi(x_2) \right]
\] (8)

in the Wick’s expansion of the time-ordered product in Eq. (7) should be considered, where the overbrace "\(\overbrace{\cdots}\)" denote the contraction of a pair of field operators.

When a nucleon with momentum \(k\) and spin \(\delta\) is considered in the nuclear matter, its field operator \(\psi(k, \delta, x)\) and conjugate field operator \(\bar{\psi}(k, \delta, x)\) can be expressed as

\[
\psi(k, \delta, x) = A_{k\delta} U(k, \delta) \exp(-ik \cdot x) + B^\dagger_{k\delta} V(k, \delta) \exp(ik \cdot x)
\] (9)

and

\[
\bar{\psi}(k, \delta, x) = A^\dagger_{k\delta} \bar{U}(k, \delta) \exp(ik \cdot x) + B_{k\delta} \bar{V}(k, \delta) \exp(-ik \cdot x),
\] (10)
respectively. In the calculation of the self-energy of the nucleon with the momentum $k$ and the spin $\delta$, a pair of the nucleon field operator and the conjugate operator in the normal ordering product of Eq. (8) should be replaced with Eqs. (9) and (10), while the other pair of the nucleon field operator and the conjugate operator connected with the underbrace "\( \bar{\psi}(x_2)\psi(x_2) \)" in the following normal ordering products denote the nucleon in the Fermi sea, and would be replaced with their expansion forms of a complete set of solutions to the Dirac equation, respectively.

\[
N \left[ \bar{\psi}(x_1)\sigma(x_1)\psi(x_1) \bar{\psi}(x_2)\sigma(x_2)\psi(x_2) \right] \tag{11}
\]

\[
\to 2 \sigma(x_1)\sigma(x_2)\left\{ N \left[ \bar{\psi}(k,\delta,x_1)\psi(k,\delta,x_1) \bar{\psi}(x_2)\psi(x_2) \right] + N \left[ \bar{\psi}(k,\delta,x_1)\psi(x_1) \bar{\psi}(x_2)\psi(k,\delta,x_2) \right] \right\}.
\]

Suppose there are no antinucleons in the ground state of nuclear matter and the Fermi sea is filled with interacting nucleons, only positive-energy components are considered in the expansion forms of the nucleon field operator and its conjugation. The expectation value of $\hat{S}_2$ in the nuclear matter on the first term in Eq. (11) can be written as

\[
\langle G | \hat{S}_2 | G \rangle = ig^2(2\pi)^4\delta^4(p_1 + k_1 - p_2 - k_2)
\]

\[
\sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3 E(p)} M_N \theta(p_F - |\vec{p}|) \bar{U}(p,\lambda)U(p,\lambda)
\]

where $k_1 = k_2 = k$, and $p_1 = p_2 = p = (E(p),\vec{p})$, and $\theta(x)$ is the step function.

According to Dyson equation, the nucleon propagator in the nuclear matter can be derived as

\[
\frac{i}{\vec{k} - M_N - \Sigma_1^\sigma + i\varepsilon} = \frac{i}{\vec{k} - M_N + i\varepsilon} + \frac{i}{\vec{k} - M_N + i\varepsilon}
\]

\[
i g_\sigma^2 \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3 E(p)} M_N \theta(p_F - |\vec{p}|) \bar{U}(p,\lambda)U(p,\lambda) \frac{i}{\vec{k} - M_N + i\varepsilon}, \tag{12}
\]

then the second order self-energy of the nucleon in the nuclear matter from the first term in Eq. (11) can be written as

\[
\Sigma_1^\sigma = -g_\sigma^2 \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3 E(p)} M_N \theta(p_F - |\vec{p}|) \bar{U}(p,\lambda)U(p,\lambda)
\]

\[
= -g_\sigma^2 \rho_S \tag{13}
\]

with

\[
\rho_S = \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3 E(p)} M_N \theta(p_F - |\vec{p}|) \tag{14}
\]
the scalar density of protons or neutrons.

The second order self-energy of the nucleon relevant to the second term in Eq. (11) can be obtained similarly

$$\Sigma_{\sigma}^{2} = g_{\sigma}^{2} \sum_{\lambda=1,2} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M_{N}}{E(p)} \theta(p_{F} - |\vec{p}|) \left[ U(p, \lambda) i \Delta(k - p) \bar{U}(p, \lambda) \right]$$

$$= -g_{\sigma}^{2} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M_{N}}{E(p)} \theta(p_{F} - |\vec{p}|) \left[ \frac{\bar{p} + M_{N}}{2M_{N}} \frac{1}{(k - p)^{2} - m_{\omega}^{2}} \right].$$

Correspondingly, the normal ordering products relevant to the self-energy of the nucleon coupling to the vector meson in the calculation of the $\hat{S}_{2}$ matrix can be written as

$$N \left[ \bar{\psi}(x_{1}) \gamma_{\mu} \omega^{\mu}(x_{1}) \psi(x_{1}) \bar{\psi}(x_{2}) \gamma_{\nu} \omega^{\nu}(x_{2}) \psi(x_{2}) \right]$$

$$\rightarrow 2 \omega^{\mu}(x_{1}) \omega^{\nu}(x_{2}) \left\{ N \left[ \bar{\psi}(k, \delta, x_{1}) \gamma_{\mu} \psi(k, \delta, x_{1}) \bar{\psi}(x_{2}) \gamma_{\nu} \psi(x_{2}) \right] + N \left[ \bar{\psi}(k, \delta, x_{1}) \gamma_{\mu} \psi(x_{1}) \bar{\psi}(x_{2}) \gamma_{\nu} \psi(k, \delta, x_{2}) \right] \right\}.$$

Therefore, the second order self-energies of the nucleon coupling to the vector meson in the nuclear matter corresponding to the first and second terms in Eq. (16) can be calculated as

$$\Sigma_{\omega}^{1} = (-i g_{\omega})^{2} \sum_{\lambda=1,2} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M_{N}}{E(p)} \theta(p_{F} - |\vec{p}|) \gamma_{\mu} i D^{\mu\nu}(0) \left[ \bar{U}(p, \lambda) \gamma_{\nu} U(p, \lambda) \right]$$

$$= \gamma_{0} g_{\omega}^{2} \frac{g_{\omega}}{m_{\omega}^{2}} \rho_{\nu}$$

with

$$\rho_{\nu} = \sum_{\lambda=1,2} \int \frac{d^{3}p}{(2\pi)^{3}} \theta(p_{F} - |\vec{p}|)$$

the number density of protons or neutrons, and

$$\Sigma_{\omega}^{2} = g_{\omega}^{2} \sum_{\lambda=1,2} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M_{N}}{E(p)} \theta(p_{F} - |\vec{p}|) \left[ \gamma_{\mu} U(p, \lambda) i D^{\mu\nu}_{6}(k - p) U(p, \lambda) \gamma_{\nu} \right]$$

$$= g_{\omega}^{2} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{(p^{2} + M_{N}^{2})^{1/2}} \left( k - p \right)^{2} - m_{\omega}^{2}.$$ (19)

Obviously, the second order self-energies of the nucleon $\Sigma_{\sigma}^{1}, \Sigma_{\sigma}^{2}, \Sigma_{\omega}^{1}, \Sigma_{\omega}^{2}$ calculated from Wick’s expansion in this section are same as those results from quantum hadrodynamics, respectively.
B. The self-energies of the scalar and vector mesons

In the Wick expansion of the time-ordered product in Eq. (7), only the normal ordering products including one contraction of a pair of nucleon field operator and its conjugate operator should be studied in order to obtain the second order self-energy of the scalar meson in the filled Fermi sea.

When a scalar meson with determined momentum \( k \) is studied in the nuclear matter, its field operator \( \sigma(k, x) \) can be expressed as

\[
\sigma(k, x) = a_k \exp(-ik \cdot x) + a_k^\dagger \exp(ik \cdot x).
\]  (20)

In order to calculate the self-energy of the scalar meson in the nuclear matter, the scalar field operators \( \sigma(x_1) \) and \( \sigma(x_2) \) in the following normal ordering product should be replaced with Eq. (20),

\[
N \left[ \bar{\psi}(x_1) \sigma(x_1) \psi(x_2) \sigma(x_2) \psi(x_2) \right] + N \left[ \bar{\psi}(x_1) \sigma(x_1) \bar{\psi}(x_2) \sigma(x_2) \psi(x_2) \right] 
\rightarrow 2\psi(x_1)\bar{\psi}(x_2)N \left[ \bar{\psi}(x_1) \sigma(k, x_1) \sigma(k, x_2) \psi(x_2) \right].
\]  (21)

The nucleon field operator \( \psi(x_2) \) and the conjugate field operator \( \bar{\psi}(x_1) \) in the normal ordering product of Eq. (21) should be expanded in terms of the set of solutions to the Dirac equation, respectively. Therefore, the second order self-energy of the scalar meson can be obtained as

\[
\Sigma_\sigma = (-ig_\sigma)^2 \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N}{E(p)} \theta(p_F - |\vec{p}|) \left[ \tilde{U}(p, \lambda) \left( iG(p-k) + iG(p+k) \right) U(p, \lambda) \right]
= g_\sigma^2 \int \frac{d^3p}{(2\pi)^3} \frac{M_N}{E(p)} \theta(p_F - |\vec{p}|) \left[ Tr \left( \frac{1}{\hat{p} - \hat{k} - M_N} \frac{\vec{\hat{p}} + M_N}{2M_N} \right) \right]
+ Tr \left( \frac{\vec{\hat{p}} + M_N}{2M_N} \frac{1}{\hat{p} - \hat{k} - M_N} \right).\]  (22)

Similarly, the normal ordering products relevant to the self-energy of the vector meson in the nuclear matter can be written as

\[
N \left[ \bar{\psi}(x_1) \gamma_\mu \gamma_\nu \gamma_\mu \psi(x_1) \psi(x_2) \gamma_\nu \gamma_\mu \psi(x_2) \right] 
+ N \left[ \bar{\psi}(x_1) \gamma_\mu \gamma_\nu \gamma_\mu \psi(x_1) \psi(x_2) \gamma_\nu \gamma_\mu \psi(x_2) \right] 
\rightarrow 2\psi(x_1)\bar{\psi}(x_2)N \left[ \bar{\psi}(x_1) \gamma_\mu \gamma_\nu \gamma_\mu \gamma_\nu \psi(x_2) \right].\]  (23)
with
\[ \omega_\mu(k, \delta, x) = b_k \delta \epsilon_\mu(k, \delta) \exp(-ik \cdot x) + b_k^\dagger \delta \epsilon_\mu(k, \delta) \exp(ik \cdot x). \] (24)

The self-energy of the vector meson in the nuclear matter can be calculated similarly as
\[ \Sigma_\omega = (-i g_\omega)^2 \sum_{\lambda=1,2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \theta(p_F - |\vec{p}|) \]
\[ \left[ \bar{U}(p, \lambda) \left( \gamma_\nu \gamma_\mu(p - k) \gamma_\mu + \gamma_\mu \gamma_\mu(p + k) \gamma_\nu \right) U(p, \lambda) \right] \]
\[ = g_\omega^2 \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \theta(p_F - |\vec{p}|) \left[ \text{Tr} \left( \gamma_\nu \frac{1}{\not{p} - \not{k} - M_N} \gamma_\mu \not{p} + M_N \right) \right] \]
\[ + \text{Tr} \left( \gamma_\nu \frac{1}{2M_N} \gamma_\mu \frac{1}{\not{p} + \not{k} - M_N} \right) . \] (25)

It is no doubt that the second order self-energies of the scalar and vector mesons calculated from \( \hat{S}_2 \) matrix directly are same as those from quantum hadrodynamics, respectively.

C. Feynman rules

The second-order self-energies of the nucleon, the scalar and vector meson in the nuclear matter are calculated from Wick’s expansion. It shows the same results as those in quantum hadrodynamics, and then an effective many-body method based on vacuum propagators has been evaluated. Feynman rules on this effective method can be summarized similarly as those in quantum hadrodynamics. In the new Feynman rules, a factor of
\[ \sum_{\lambda=1,2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \theta(p_F - |\vec{p}|) \]
is included for each pair of crosses, which denote the initial and final states of the nucleon in the Fermi sea. Moreover, the momentums and spins of external lines with a cross or without a cross take the same values with each other, respectively. In addition, include a factor of \((-1)\) in the calculation of exchange diagrams.

The loop diagrams, which relate to the contribution of Dirac sea and cause divergences, are not necessary to be considered in no Dirac sea approximation. Therefore, only the diagrams with crosses should be studied in the calculation of self-energies of particles. Because there do not exist antinucleons in the ground state of nuclear matter, the diagrams with an external line of antinucleons should be excluded, too.

The Feynman diagrams for the second order self-energy of the nucleon in the nuclear matter in Section A are shown in Fig. 3. The first diagram in Fig.3 corresponds to the...
tadpole contribution, and the second corresponds to the exchange term in the relativistic Hartree-Fock approximation. The self-energies of the scalar meson, vector meson or the photon in the nuclear matter can be calculated with the Feynman diagrams in Fig.4. It shows the same results as the one-fermion-loop approximation in quantum hadrodynamics, i.e., the same effective masses of the photon and mesons in Ref. [11, 22] can be obtained in the one-fermion-loop approximation in quantum hadrodynamics.

In our formalism, the effects of the nuclear medium come from the nucleon condensation, i.e., the scalar density of nucleons. In Walecka’s formalism, the propagators of hadrons are defined in the ground state of the nuclear matter and different from the propagators defined in vacuum, and the loop diagrams are considered although the meanings are different from those in quantum field theory. Therefore, Feynman rules in our formalism are different from those in Ref. [3, 23], and the condensation of the nucleon is embodied in the integrals of three-momentum space in the new Feynman rules.

In particle physics, people are mostly interested in scattering processes, for which the $\hat{S}$ matrix providing the probability of transition from the initial states to final states, is the most suitable framework. In statistical physics, however, we are mainly concerned on the expectation value of physical quantities at finite time. Obviously, these two problems are connected with each other in our formalism. Because vacuum propagators are adopted in our formalism, which are not relevant to the state of the system, it is not difficult to extend this formalism to study the properties of non-equilibrium and finite temperature states. Some works have been done along this direction.

Actually, the propagator including the on-shell part of Eq. (5) is not for the nucleon, but for a kind of quasinucleon, whose creation and annihilation operators satisfy the same anticommutation relations as those of the nucleon. There is a Bogoliubov transformation between the creation and annihilation operators of the quasinucleon and the nucleon.

### III. SELF-CONSISTENT RELATIVISTIC HARTREE APPROXIMATION

When the isospin $SU(2)$ symmetry is considered in the nuclear matter, the $\rho$ meson interaction should be included in the Lagrangian density,

$$\mathcal{L}_{\text{int}}^{\rho} = - g_{\rho} \bar{\psi} \gamma_{\mu} \vec{\tau} \cdot \vec{\rho}_{\mu} \psi$$

(26)
with \( \vec{\tau} \) being the Pauli matrix. Because the \( \rho_+ \) and \( \rho_- \) mesons only contribute to the second order self-energy of the nucleon in the exchange terms, only the \( \rho_0 \) meson interaction is considered in the relativistic Hartree approximation.

With the similar method, the second order self-energy corrections of the proton and neutron coupling to the \( \rho_0 \) meson can be written as

\[
\Sigma_1^\rho = \gamma_0 \frac{g_0^2}{\pm 4m_\rho^2} (\rho_p - \rho_n) \tag{27}
\]

with the plus for the proton and the minus for the neutron, where \( \rho_p \) and \( \rho_n \) are the number density of protons and neutrons, respectively. Obviously, the results in Eq. (27) are same as those in the relativistic mean-field approximation\[3\].

In the calculation of the second order self-energies of hadrons in the nuclear matter in Sect. II, the noninteracting propagators of hardrons are used. Although the second order results can be summed to all orders with Dyson’s equation, this procedure is not self-consistent. Self-consistency can be achieved by using the interacting propagators to also determine the self-energy\[3\]. In the relativistic Hartree approximation, the self-energy of the nucleon in the nuclear matter can be calculated self-consistently with the interacting propagator of the nucleon

\[
iG_H(p) = \frac{-1}{\gamma_\mu \vec{p}^\mu - M_N^* + i\varepsilon}, \tag{28}
\]

where

\[
M_N^* = M_N - \frac{g_\sigma^2}{m_\sigma^2} (\rho_p^S + \rho_n^S), \tag{29}
\]

\[
\vec{p}^0 = p^0 - \frac{g_\omega^2}{m_\omega^2} (\rho_p + \rho_n) - \frac{g_\rho^2}{\pm 4m_\rho^2} (\rho_p - \rho_n) \tag{30}
\]

with the plus for the proton and the minus for the neutron, and

\[
\vec{p} = \vec{p}. \tag{31}
\]

In Eq. (29), \( \rho_p^S \) and \( \rho_n^S \) are the scalar densities of protons and neutrons, respectively. It corresponds to the transformation

\[
M_N \rightarrow M_N^*, \quad E(p) \rightarrow E^*(p), \tag{32}
\]

in the self-energy of the nucleon in Eq. (13) and (17), where \( E^*(p) = (\vec{p}^2 + M_N^*^2)^{1/2} \).
The effective nucleon mass can be defined as the pole of the nucleon propagator in the limit of the space-momentum of the nucleon $\vec{p} \rightarrow 0$, which corresponds to the mass spectra of the collective excitations in the nuclear matter $^{26,27}$. According to Eq. (13), the effective nucleon mass in the nuclear matter is defined in Eq. (29) in the relativistic Hartree approximation. In quantum hadrodynamics $^2$, the nonlinear self-coupling terms of the scalar meson are introduced to replace the mass term $\frac{1}{2}m_\sigma^2\sigma^2$, $^{1}$, 

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4.$$  (33)

Because the boson distribution functions of the mesons are zero in the nuclear matter at zero temperature, the self-coupling terms of the scalar meson have no contribution to the self-energy corrections of the nucleon and the meson when the loop diagrams are ignored. Therefore, the effective nucleon mass still takes the form in Eq. (29) in quantum hadrodynamics $^2$, which is important to conserve the self-consistency in the calculation of relativistic Hartree approximation.

In the relativistic mean-field approximation of quantum hadrodynamics $^2$, 

$$m_\sigma^2\sigma_0 + g_2\sigma_0^2 + g_3\sigma_0^3 = -g_\sigma(\rho_S^p + \rho_S^n),$$  (34)

then the effective nucleon mass in the nuclear matter can be written as

$$M_N^* = M_N + g_\sigma\sigma_0 + \frac{g_\sigma g_2}{m_\sigma^2}\sigma_0^2 + \frac{g_\sigma g_3}{m_\sigma^2}\sigma_0^3.$$  (35)

The effective nucleon mass in the relativistic mean-field approximation has not been studied carefully although the other four saturation properties of the nuclear matter have been given more attentions. From the relativistic mean-field form of Dirac Equation, the effective nucleon mass, which is called Dirac mass in our manuscript, had been drawn out by adding the $g_\sigma\sigma_0$ on the mass of the nucleon, which had been believed to be correct In QHD-1 and QHD-2. From this definition, The effective nucleon mass is only related to the linear term of the scalar meson. In QHD-1, it is same as the effective nucleon mass derived from the relativistic Hartree approximation in Eq. (29). In the relativistic Hartree approximation of QHD-2, The effective nucleon mass still takes the form of Eq. (29). If the self-consistency of the relativistic mean-field approximation of QHD-2 is considered, which is very important in the calculation of strong interaction systems, the effective nucleon mass must be defined as Eq. (29), then the nonlinear self-coupling terms of the scalar meson will contribute to the
effective nucleon mass directly in the relativistic mean-field approximation. Moreover, the form of the effective nucleon mass in Eq. (35) is consistent with the definition in Ref. [28].

In the quantum hadrodynamics 2, the total energy density and the pressure of nuclear matter can be deduced to

\[ \varepsilon = \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{3} g_2 \sigma_0^3 + \frac{1}{4} g_3 \sigma_0^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2 + \sum_{B=p,n} \frac{2}{(2\pi)^3} \int_0^{p_F(B)} d\vec{p} \left( \vec{p}^2 + M_N^2 \right)^{\frac{1}{2}}, \quad (36) \]

and

\[ p = \frac{1}{3} \sum_{B=p,n} \left( \frac{2}{(2\pi)^3} \int_0^{p_F(B)} d\vec{p} \left( \vec{p}^2 + M_N^{4/3} \right)^{\frac{1}{2}} \right) - \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{3} g_2 \sigma_0^3 - \frac{1}{4} g_3 \sigma_0^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2, \quad (37) \]

where \( M_N^4 = M_N + g_\sigma \sigma_0 \) is the Dirac mass of the nucleon in the nuclear matter defined from the Dirac equation of the nucleon.

Because the original definition of the effective nucleon mass is wrong, all sets of parameters on the relativistic mean-field approximation of QHD-2 based on that wrong definition cannot be used in the new self-consistent scheme mentioned above. Therefore, all the parameters must be readjusted. By fitting the saturation properties of nuclear matter, the parameters in the relativistic mean-field approximation with the effective nucleon mass defined in Eq. (35) can be fixed as

\[ g_\sigma = 8.95, \quad g_\omega = 10.94, \quad g_\rho = 7.2, \]

\[ g_2 = -1.38 \text{fm}^{-1}, \quad g_3 = 23.0 \]

with \( m_\sigma = 532.5 \text{MeV} \), and the masses of the vector and \( \rho \) mesons take the experimental values, respectively.

With these parameters we obtain a saturation density of 0.1655fm\(^{-3}\), a binding energy of 15.771 MeV, a compression modulus of 227 MeV, an asymmetric energy coefficient of 32.44 MeV and an effective nucleon mass of 0.638\( M_N \) for the symmetric saturation nuclear matter. At the saturation point, the Dirac mass of the nucleon is 0.661\( M_N \), larger than the effective nucleon mass under this set of parameters. More accurate parameters should be fixed by studying the properties of finite nuclei.

In Walecka’s model, the effective nucleon mass is defined from the Dirac equation of the nucleon in the relativistic mean-field approximation. However, the effective nucleon mass
is defined as the pole of the nucleon propagator in the relativistic Hartree approximation in our model. Although these two definitions are same as each other in the framework of $QHD - 1$, they are different from each other in $QHD - 2$, in which the self-coupling terms of the scalar meson are included. Since the self-consistency is realized in a different manner, the parameters must be fixed again to fit the saturation properties of nuclear matter.

As far as the self-consistency of calculations for strong interaction systems is concerned, the redefinition on the effective nucleon mass is essential. The original definition of effective nucleon mass in quantum hadrodynamics is wrong and the relativistic mean-field calculations with parameters fixed according to this wrong definition are not self-consistent.

Because of the strong coupling between hadrons, the nuclear systems can only be studied approximately and effectively in the framework of quantum field theory, and the renormalization is even meaningless on nuclear systems. In nuclear many-body theories, the self-consistency should be considered, and the different interacting propagators should be adopted in the calculation of different order approaches.

IV. SUMMARY

In a conclusion, an effective formalism to solve nuclear many-body problems is evaluated, and we find this formalism with off-shell propagators gives the same results as those in quantum hadrodynamics in the calculation of self-energies of particles in the nuclear matter. Moreover, Feynman rules is induced for this new method, which is named as sum rules on quantum hadrodynamics by us. In addition, the self-consistency of quantum hadrodynamics is discussed in the relativistic mean-field approximation.

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[1] J. D. Walecka, Ann. Phys.(N.Y.) 83 (1974) 491
[2] S. A. Chin, Ann. Phys.(N.Y.) 108 (1977) 301
[3] B. D. Serot, J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1
[4] P. G. Reinhard, Rept. Prog. Phys. 52 (1989) 439
[5] P. Ring, Prog. Part. Nucl. Phys. 37 (1996) 193
[6] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6 (1997) 515
[7] J. Boguta, A. R. Bodmer, Nucl. Phys. A 292 (1977) 413
[8] J. Zimanyi, S. A. Moszkowski, Phys. Rev. C 42 (1990) 1416
[9] R. Brockmann and H. Toki, Phys. Rev. Lett. 68 (1992) 3408
[10] H. Shen, Y. Sugahara and H. Toki, Phys. Rev. C 55 (1997) 1211
[11] B. X. Sun et al., nucl-th/0206029, Int. J. Mod. Phys. E 12 (2003) 543
[12] H. Kucharek and P. Ring, Z. Phys. A 339 (1991) 23
[13] J. Meng, Nucl. Phys. A 635 (1998) 3
[14] P. A. M. Guichon, Phys. Lett. B 200 (1988) 235
[15] K. Saito, K. Tsushima, A. W. Thomas, Phys. Rev. C 55 (1997) 2637
[16] H. Shen, H. Toki, Phys. Rev. C 61 (2000) 045205
[17] Y. H. Tan, H. Shen, P. Z. Ning, Phys. Rev. C 63 (2001) 055203
[18] S. Weinberg, The quantum Theory of Field, (Cambridge University Press, 1996)
[19] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffer-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 59 (1999) 411
[20] X. F. Lu, B. X. Sun, Y. X. Liu, H. Guo, E. G. Zhao, nucl-th/0112073.
[21] C. Itzykson, J. B. Zuber, Quantum Field Theory , (Mc Graw-Hill Inc, 1980)
[22] B. X. Sun et al., nucl-th/0204013, Mod. Phys. Lett. A 18 (2003) 1485
[23] R. J. Furnstahl and B. D. Serot, Phys. Rev. C 44 (1991) 2141
[24] B. X. Sun et al., nucl-th/0209041
[25] H. Umezawa, H. Matsumoto and M. Tachiki, Thermofield Dynamics and Condensed States, (North-Holland, 1982).
[26] C. Song, Phys. Rev. D 48, (1993) 1375
[27] S. Gao, Y. J. Zhang and R. K. Su, Phys. Rev. C 52, (1995) 380
[28] H. A. Bethe, Ann. Rev. Nucl. Sci. 21, (1971) 125
Figure Captions

**Fig. 1** Feynman diagrams for the second order self-energy of the nucleon in nuclear matter calculated in the quantum hadrodynamics. The double solid lines denote the nucleon propagators defined in the ground state of nuclear matter.

**Fig. 2** Feynman diagram for the second order self-energies of the scalar or vector meson in nuclear matter calculated in the quantum hadrodynamics. Same case as in Fig. 1.

**Fig. 3** Feynman diagrams for the second order self-energy of the nucleon in nuclear matter calculated from $\hat{S}_2$ matrix elements. The wave lines denote the scalar meson or vector meson, while 1 and 2 denote particles of the initial state, 3 and 4 denote particles of the final state.

**Fig. 4** Feynman diagrams for the second order self-energies of the scalar or vector meson in nuclear matter calculated from $\hat{S}_2$ matrix elements. Same case as in Fig. 3.
