The impact of wake models on wind farm layout optimization

Jonas Schmidt and Bernhard Stoevesandt
Fraunhofer IWES, Ammerländer Heerstraße 136, 26129 Oldenburg, Germany
E-mail: jonas.schmidt@iwes.fraunhofer.de

Abstract. Results for nine gradient-based layout optimization runs of a wind farm with 25 turbines in flat terrain are presented, varying three different choices of the underlying wake model and three inflow scenarios. In all cases the AEP is maximised and the constraints are purely geometrical. A single inflow vector, a uniform wind rose and a realistic synthetic wind rose are studied, and the final layouts for the Jensen, the Ainslie and a CFD-based numerical wake model are compared. From this an estimate of the average variation of the turbine position due to the different wake models is obtained. All calculations were carried out with the in-house software flapFOAM.

1. Introduction
Wind farm layout optimization is crucial for advancing wind energy in Europe, since often either the available area at a site is limited or the meteorological conditions do not clearly prefer a single instance of inflow conditions. Wake situations may therefore be significant and should not be ignored during the process of layout optimization. For a recent review on the topic of wind farm optimization and more than 20 years of related research see [1], also [2, 3]. For a recent summary on the related topic of optimised wind farm control see [4].

The optimal wind farm layout depends on many site-specific parameters and constraints, especially for on-shore wind farms. Examples are limitations of the available area due to urban neighbourhood, land ownership, forest, inaccessible terrain or unsuitable soil, the requirement of an achievable and realisable cable network and grid connection, restrictions due to access roads, environmental constraints, etc. Such issues imprint on the objective function as well as the optimization constraints. The implementation of a multi-fidelity layout optimization software with complex objective functions beyond annual energy production (AEP) by DTU is described in [5].

In this work, however, we focus on the impact of the wake effect on the layout optimization of wind farms. It induces an interaction between the individual wind turbines and can therefore play a major role, especially at sites where the available area is limited and the wind rose is not dominated by a single wind direction. We therefore leave the above mentioned issues aside and idealise the problem by only considering the maximisation of the AEP of the wind farm.

Wind farm layout optimization relies on iterative methods, which are based on a large number of evaluations of the objective function. This involves the repeated calculation of the wake effect for each turbine, which may be expensive, depending on the method and the invoked models. While 0-equation wake models like the Jensen model [6] or the Frandsen model [7] are fast and...
therefore natural candidates for this purpose, more complex models like the Ainslie model [8] or the CFD-based numerical wake model [9] of the software flapFOAM require more numerical or algorithmic attention. However, these issues are purely technical and therefore either solved or solvable, in contrast to the principle complexity issue of full CFD simulations during run-time of the optimization routine.

It is the main purpose of this work to address the impact of the choice of wake model on the outcome of a layout optimization that purely focuses on the wake effect. Besides studying this question it also serves as a proof-of-principle for the coupling of the wind farm modelling code flapFOAM, that is being developed at the Fraunhofer Institute for Wind Energy and System Technology IWES, and the open source optimization tool box Dakota [10].

2. Methods

2.1. Wind farm modelling with flapFOAM

All wake modelling and wind farm calculations in this work are obtained with the software flapFOAM, which has been developed at Fraunhofer IWES since 2011. It was inspired by the software FLapF, which had been developed earlier at the University of Oldenburg (cf. [11]), without including code of the latter. flapFOAM is based on the idea of single-wake superposition, fully written in C++, and can read OpenFOAM simulation results. Its strictly modular structure allows the developer to extend and improve models independently of the core functionality of the code, and the user to select between a broad range of models and settings. The proof-of-principle of the numerical wake model based on CFD-RANS results was presented in [9], and progress on the inclusion of complex terrain effects was reported in [12]. A detailed description of the software will be given elsewhere, here we only provide a brief summary of the parts that are directly relevant for the current work.

The basic calculation algorithm of flapFOAM for the calculation of the wind vector \( \mathbf{v} \) at a point \( p \) and the effective wind velocity at the \( i \)-th rotor \( v_{\text{rotor}}^{(i)} \) in the presence of wakes is sketched in Fig. 1 (left). The CombinedWindWakeField class contains the background wind field, which represents the wind field at the site without the wake effect, the deficit wind field, which provides all wake deficit data, the wake overlap rule, which is called WakeAdditionModel, and the DownwindOrder, which contains the information of the interdependence of the individual wind turbines.

Two different algorithms are implemented for the downwind order. The updating routine for the iterative model is sketched in the right panel of Fig. 1. In this case the respective wake deficits at any two turbines caused by one another are compared. From this the turbine dependencies and the global order of evaluation are determined. After the re-calculation of the effective wind speeds at the wind turbines, this process is repeated until convergence is reached.

A simpler model determines a dominant wind direction and orders the turbines according to their corresponding coordinate with respect to this direction.

The crucial quantity for all wind farm calculations is the list of effective wind velocities at the rotors \( v_{\text{rotor}} \). This quantity determines the thrust coefficient and therefore the wake strength for each wind turbine rotor. Depending on the RotorModel it is calculated as an average over the rotor disk, where the wake addition model specifies which power of the local wind speed is used as the integral kernel during the calculation of the mean.

The contribution of rotor \( i \) to the total local wind deficit, measured in a volume element of the disk that represents rotor \( j \), depends on the current results of the dependencies of rotor \( j \) and \( v_{\text{rotor}}^{(i)} \), and on the selected SingleWakeModel for wind turbine \( i \). The wake models are formulated with respect to the rotor reference frame, and their defining property is to return the velocity deficit at any point \( p \) given the effective inflow velocity vector, the rotor model and the required constant model parameters. Several wake models are implemented in flapFOAM, and the three relevant for this work are briefly introduced in Section 2.3.
Figure 1. Sketch of local velocity calculation with *flapFOAM* (left) and the updating routine for the iterative downwind order model (right). The calculated velocities at the rotors, here denoted as $v_{\text{rotor}}$, is crucial for all wind farm calculations.

### 2.2. Wind farm model

The numerical wind turbine model that is considered in this work has $D = 120$ m rotor diameter, $H = 120$ m hub height and 2.53 MW nominal power. In what follows the effective wind speed at the rotor is obtained directly from the centre point of the disk, i.e., the simplest rotor model without disk integration is applied.

The model wind farm consists of 25 wind turbine of the above rotor type, and initially arranged in a regular $5 \times 5$ pattern of size $3 \times 3$ km$^2$. The available area for the layout optimization is a square of $8 \times 8$ km$^2$. In order to break the symmetry, the axes which define the initial wind farm layout are chosen as $(1, -0.01)$ and $(-0.01, 1)$ in the xy-plane. This is a simple way to obtain non-vanishing derivatives of the power yield in y-direction for wind vectors that are parallel to the x-axis, using a centred finite-difference gradient scheme.

### 2.3. Wake models

Three wake models are compared in this work, the Jensen, the Ainslie and a CFD-based wake model. The addition of wake deficits is preformed quadratically under the square root, and no partial wake or meandering models are applied.

#### 2.3.1. Jensen model

A classic wake model is the model by Jensen [6]. Its sole parameter is the constant wake expansion parameter, which determines the linear expansion of the wake radius with downstream distance, and is chosen as $k = 0.07$ throughout this work. The model furthermore ignores radial variations of the deficit inside the wake, and the application of mass conservation in a control volume then yields an inverse-square law with respect to distance for the axial deficit. The Jensen model is a zero-equation model, which makes it fast and therefore efficient for layout optimization applications.

#### 2.3.2. Ainslie model

The Ainslie model [8] is a 2-equation eddy-viscosity model, which is based on the reduction of the time-averaged Navier-Stokes equations by imposing axial symmetry. The eddy viscosity is the combination of an ambient turbulence contribution and a wake-induced term that depends on the centre line deficit. The Ainslie model is widely applied since it captures more of the flow dynamics than the Jensen model but is less complex than full RANS equations.

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1 Notice however that this is a matter of sensitivities and the chosen step size, rather than principle, and there are other options to achieve that goal. One example is the application of several optimisation runs with initial gradient step size of $5 \ D$ and subsequently smaller values.
In *flapFOAM* the Ainslie model is solved on-the-fly during run time on a specified 2D grid in the radial coordinate $r$ and the disk orthogonal coordinate $x$, for discrete inflow wind speeds between which interpolation to user-specified numerical accuracy is applied. For this, a Crank-Nicolson discretization scheme is applied to yield a tri-diagonal matrix system, whose solutions are written to a data base in order to avoid redundant calculations.

Here the grid size of the regular finite difference grid in $(x, r)$ was chosen as $9000 \times 4500$ m$^2$ with $900 \times 450$ grid points. The ambient turbulence intensity was chosen uniformly as 10%. All derivatives and interpolations are performed at second order numerical accuracy.

2.3.3. CFD wake model

Basically the 3D-RANS equations applied to an isolated actuator disk define a $(4+\mathrm{x})$-equation wake model, where $\mathrm{x}$ represents the turbulence model equations. Hence it can be treated similar to the Ainslie model described above. However, due to the complexity of CFD simulations they are obtained before run time of *flapFOAM* and should span the range of inflow wind speeds of interest. Details of the implementation of a numerical wake model based on pre-calculated CFD-RANS results are given in [9].

For the purpose of this work we performed eight CFD-RANS simulations of a single uniform actuator disk in neutral stratification with OpenFOAM’s *simpleFoam* solver (version 2.3.1), at inflow wind speeds 3, 5, 8, 10, 12, 15, 18 and 20 m/s at hub height 120 m.

The mesh has dimensions $8.8 \times 1.5 \times 1.0$ km$^3$ and was created using the IWES in-house tool *terrainMesher*, which is a follow-up of the open-source *terrainBlockMesher* [13]. It consists of 2.05 million cells, including the actuator disk with 1892 cells, cf. Fig. 2. The first cell height at the ground is 1 m and standard wall functions with roughness length $z_0 = 5$ cm were used. Both grading and refinement were applied to improve the resolution of the wake and the near-disk region.

We applied the $k-\epsilon-f_p$ turbulence model [14] with parameters as recommended there. Compared to the standard $k-\epsilon$ model this version includes a correction of turbulent viscosity that depends on the change of velocity gradients due to the presence of the actuator disk, enhancing the wake deficit. All boundary conditions at the inlet were obtained by a one-dimensional cyclic precursor run, given the mass flow according to a standard log-profile. The desired profile and the inflow wind speed at hub height were well matched by the results. All variables converged to residuals below $10^{-5}$ in all simulations. The converged results were mapped to a fully structured coarser grid with 277200 cells and minimal cell size of 10 m in order to speed up the optimization.

2.4. Inflow models

Inflow model specify for which set of inflow conditions the wind farm calculations are performed. In cases different from the single wind vector case, the obtained results are mean results over the corresponding background wind fields. Each individual contribution is therefore added with the associated statistical weight. Three different inflow cases are considered in this work.
2.4.1. Single wind vector The single vector inflow case (SWV) is defined by a uniform background wind with velocity vector $v = (8, 0, 0)$ m/s, chosen in the linear regime of the considered turbine power curve. The SWV case has a large number of local and global minima, since there are many ways to arrange 25 wind turbines inside the $8 \times 8$ km$^2$ area such that they have almost free-stream conditions.

2.4.2. Uniform wind rose A uniform wind rose with 360 sectors (WR360U) and 8 m/s wind speed at hub height is used as a test scenario that has no trivial free-stream solutions. It can be expected that layout optimization results show the corresponding symmetry, possibly broken by the rectangular wind farm boundary.

2.4.3. Realistic wind rose This case is a synthetic non-uniform wind rose with 120 sectors (WR120R), each equipped with a wind speed histogram with maximal 5 bins in the range from 3 to 20 m/s, cf. Fig. 3. As for the WR360U scenario there are no free-stream solutions for the wind turbines. However, there is a strong bias from south-westerly wind directions. In total this inflow case comprises 510 non-vanishing wind inflow vectors, with mean wind speed 7.4 m/s and mean direction 230°.

2.5. Optimization

For efficient optimization, flapFOAM has been coupled to the open source optimization tool box Dakota (version 6.0.0), developed by Sandia National Laboratories [10]. The coupling has been achieved internally, i.e., in terms of linked libraries and the implementation of wrapper classes that communicate between the two software packages. This is efficient, because flapFOAM can call the optimization routine, and the slow process of reloading data into memory during optimization iterations is avoided.

Since we focus on the wake effect in this work, the objective function is the park efficiency without any further contributions. Here park efficiency is defined as the total power yield of the wind farm, normalized by the number of turbines and the power that an isolated turbine would produce in the specific inflow case. Notice that the latter varies depending on the scenario. For the chosen machine model it is 1.674 MW for the SWV and WR360U cases and 1.247 MW for the WR120R inflow case.
Table 1. The parameters used for CONMIN optimization method, as given to Dakota. The scaling parameter activates the normalisation and requires no value in the method specification.

| Parameter                  | Value       |
|----------------------------|-------------|
| method                     | conmin_mfd  |
| max_iterations             | 5000        |
| convergence_tolerance      | $10^{-5}$   |
| constraint_tolerance       | 0.003       |
| max_function_evaluations    | 10000       |
| scaling                    |             |

Figure 4. The SWV inflow case for the Jensen wake model. The left panel shows the initial condition, the right panel the final layout. The park efficiency improved from 68.5% to 97.5%.

Two types of optimization constraints apply. First, the positions have to respect the maximally available area for the wind farm, cf. Sec. 2.2, and second, a minimal inter-turbine distance of 500 m, i.e. 4.17 D, is required. For the considered 25 wind turbines within the square boundary there are 50 design variables and 400 constraints. During evaluation, the constraints are normalized with respect to characteristic scales.

All results presented in the following were obtained by the CONMIN optimization method, as provided by Dakota. This gradient-based constraint optimization algorithm was developed by Vanderplaats in 1973 [15] and has been applied to a vast number of problems ever since. It is limited to finding the nearest local minimum. Two variants are implemented in Dakota, a conjugate gradient optimization method (frcg) for unconstrained problems, and a method of feasible directions (mfd). The latter was chosen here, with parameters as listed in Table 1. It is based on the evaluation of gradients of the objective function and the constraints, from which a direction in the parameter space is determined. The variables are then altered according to that direction and a step size, until constraints are hit or the objective function does no longer improve. In that case a new feasible direction is determined, and the procedure is continued until convergence is reached. For all results a central finite-difference gradient scheme with step size 1 D was applied.
3. Results and discussion

3.1. Single wind vector

The initial and optimized layouts for the SWV inflow case and the Jensen wake model are shown in Fig. 4, for the Ainslie and CFD wake models the latter are shown in Fig. 5. The park efficiencies were increased by the optimization from 68.5%, 65.3% and 61.0% to 97.5%, 95.6% and 95.8% for the Jensen, Ainslie and CFD wake models, respectively.

For all wake models almost free-stream solutions were found by the CONMIN optimization method and the chosen initial conditions and parameters. Especially for the Jensen wake model, which has no radial dependence within the wake region, but also for the axisymmetric Ainslie wake model, this requires some tuning and is not automatic. For example, small step sizes may lead to changes of the objective function that are below the tolerance threshold, thereby terminating the optimization. Here slightly rotated initial grid axes were chosen to break the symmetry, cf. Sec. 2.2.

One obvious issue of the method is that it does not explore all of the available parameter space, but only the vicinity at each iteration. Hence the optimal park efficiency of 100% is not reached, and the turbines appear clustered and attached to the neighbour turbine’s wake. For the Jensen model the clustering is less pronounced, since the relatively large choice of the opening angle yields a wide and constantly growing wake radius. In general the issue can be addressed by considering non-trivial rotor models or by increasing the sensitivity. However, for realistic applications the inflow will always involve a distribution of wind directions, which counteracts the effect.

3.2. Uniform wind rose

The left panel of Fig. 6 shows the initial mean wind speed at hub height for the Ainslie wake model and the WR360U inflow case. Clearly the layout is not the optimal configuration, since all turbines are located at positions where the mean wind speed is low compared to the outer regions. The right panel shows the optimized layout. Here no gradients of the mean wind speed are apparent beyond the turbine vicinities, as expected for the uniform wind rose case. One central turbine is surrounded by a ring of turbines, the remaining turbines are located near the wind farm boundary. This pattern is invariant under all common symmetries of the boundary...
Figure 6. The WR360U inflow case for the Ainslie wake model. The left panel shows the mean wind speed at hub height for the initial condition, the right panel for the final layout.

| Case     | Wake | Initial FE | Final FE | Case     | Wake | Initial FE | Final FE |
|----------|------|------------|----------|----------|------|------------|----------|
| WR360U   | Jensen | 88.6% | 97.5% | WR360U   | Jensen | 1.0 D | 4.1 D |
|          | Ainslie | 89.7% | 92.2% |          | Ainslie | -    | 3.2 D |
|          | CFD    | 87.3% | 96.0% |          | CFD    | -    | 1.7 D |
| WR120R   | Jensen | 93.1% | 98.0% | WR120R   | Jensen | 4.7 D | 4.0 D |
|          | Ainslie | 83.7% | 98.3% |          | Ainslie | -    | -    |
|          | CFD    | 92.1% | 97.8% |          | CFD    | -    | -    |

Table 2. Left: Wind farm efficiencies (FE) for the WR360U and WR120R cases. Right: Average turbine distance between two final layouts, cf. Fig. 7.

The wind farm efficiency is increased from 89.7% to 92.2% in case of the Ainslie model, cf. Table 2 (left). Notice that the park efficiencies are calculated with the same wake model that is used for the layout optimization. They should not be used directly for comparisons of the final layouts, since their relevance is a matter of validation and therefore model dependent. The issue of objective comparison of optimization results is beyond the scope of this paper and left for future work.

The final layouts for all three wake models and the WR360U inflow case are presented in the left panel of Fig. 7. They all show the expected symmetries, however, the turbine positions do not exactly coincide. As listed in Table 2 (right) the average distance between the positions of one particular turbine in two different final layouts is between 1.0 to 4.1 D. The Jensen and Ainslie models disagree little in the optimal layout, while the CFD based wake model yields a considerably different layout. This concerns the radius of the inner ring of turbines, which varies depending on the decrease of the wind deficit in the wake with distance from the rotor. Furthermore the outer turbines seem to follow a rounder shape for the CFD model case, with every second turbine shifted inwards from the boundary.

3.3. Realistic wind rose
The initial and final mean wind speed fields for the WR120R inflow case and the CFD wake model are shown in Fig. 8. The final layout is a distorted version of the result for the symmetric
Figure 7. The final layouts for the WR360U (left) and WR120R (right) inflow cases and the three studied wake models.

Figure 8. The WR120R inflow case for the CFD wake model. The left panel shows the mean wind speed at hub height for the initial condition, the right panel for the final layout.

WR360U case. A similar observation can be made for the Ainslie wake model, cf. Fig. 7 (right). Table 2 states that the final layouts of these two models coincides up to an average turbine position shift of 1.7 D.

The final layout result for the Jensen wake model is qualitatively different. It cannot be characterized as a distortion of the symmetric case, it rather shows features of a staggered layout orthogonal to the main wind direction south-west. The mean turbine position variation compared to the Ainslie and the CFD wake models is 4.7 and 4.0 D, respectively.
4. Conclusion

In this work we studied wind farm layout optimization of a wind farm with 25 wind turbines in flat terrain, using the software \textit{flapFOAM}, which recently has been coupled to the optimization library \textit{Dakota}. Three different wake models were compared, the Jensen wake model, the Ainslie wake model and a CFD-based numerical wake model that was obtained from pre-calculated actuator disk simulations with \textit{OpenFOAM}. Layout optimization results were obtained with the gradient based CONMIN method for three different inflow cases, a single wind vector (SWV), a uniform wind rose with 360 sectors (WR360U) and a realistic, synthetic wind rose with 120 sectors and up to 5 wind speed bins per sector (WR120R).

The SWV inflow case has a large number of optimization minima, and for the three wake models three different solutions with nearly free-stream conditions for all turbines were found. The gradient based optimiser needs different degrees of tuning in order to avoid unfavourable blocking of a turbine in the bulk between neighbour wakes. Also the usage of non-purely local gradient based optimization methods can overcome this issue. Though the CFD wake model seems to be less sensitive to tuning, a quantitative comparison of the wake model impact is difficult for this inflow case due to the large number of solutions.

The uniform wind rose case has no solution with almost free-stream conditions. For all wake models final configurations were found that show the expected symmetries. The Jensen and Ainslie wake models agree on the turbine position up to 1.0 D on average, the CFD wake model yields variations of up to 4.1 D on average. This may be due to a significantly different decay behaviour of the wake deficit in this model.

For the WR120R inflow case the Ainslie and the CFD wake model are in good agreement, with an average variation of the turbine position of 1.7 D. Both models show a distortion of the symmetric solution that was found for the WR360U case. The Jensen model yields a qualitatively different solution that resembles a staggered layout with lines of turbines orthogonal to the main wind direction. The displacement compared to the other solutions is up to 4.7 D on average.

Our results demonstrate that wake models can have an impact on the outcome of wind farm layout optimization. Based on the our examples and a gradient based optimization method the magnitude of the effect can be estimated as an average variation of the turbine position in the range 1–5 D. An objective decision which wake model is the best for layout optimisation is difficult, since it involves the definition of a metric that ideally does not rely on engineering models and is validated for a large number of wind farm configurations.

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