Abstract

We investigate the polarization properties of vector quarkonia produced in lepton anti-lepton annihilation with special attention to effects due to intermediate Z bosons.

1 Introduction

With the possible availability of $J/\Psi$ factories in the foreseeable future, it is interesting to consider the discovery potential of such machines for new physics effects or, equivalently, for precision tests of the standard model \[1\]. In this article we calculate the polarisation of $J/\Psi$ and other vector quarkonia in resonant production through lepton-antilepton annihilation. If the production proceeds through a single photon, the vector quarkonium is an equal mixture of left and right handed polarization. We also consider the production contribution through a virtual $Z$ leading to calculable deviations from this polarization. As a preliminary application we derive the forward backward asymmetry of the quarkonium decay into a lepton pair. In section II we derive the probabilities that the quarkonium state has each of the three possible polarizations. In section III we consider the forward backward asymmetries.

In its rest frame a massive vector meson has three possible polarization states along any quantization axis.

$$\epsilon(\uparrow)_{\mu} = -(0, 1, +i, 0)/\sqrt{2}$$  \hspace{1cm} (1.1)
The massive vector field has the canonical form

\[ \epsilon(\psi)_{\mu} = (0, 1, -i, 0)/\sqrt{2} \]  \hspace{1cm} (1.2)

\[ \epsilon(0)_{\mu} = (0, 0, 0, 1) \]  \hspace{1cm} (1.3)

In the zero binding approximation, the production of a quark with momentum \( P/2 \) and spin projection \( \lambda/2 \) and an antiquark with the same momentum \( P/2 \) and spin projection \( \lambda/2 \) with \( P^2 = M^2 \), the vector meson mass squared, is proportional to the production of the vector quarkonium with spin projection \( \lambda/2 + \lambda/2 \). Beginning in 1975 \[2\], various production and decay amplitudes have been discussed in this color singlet, zero-binding model. In this approximation, the quark mass and momentum are taken to be one half of the corresponding quarkonium mass and momentum. Quantitatively, we have the identities \[3\]

\[ v_\alpha(P/2, \uparrow)\bar{u}_\beta(P/2, \uparrow) = \frac{1}{2\sqrt{2}} (\varphi^0(\bar{\psi})(P + M))_{\alpha\beta} \]  \hspace{1cm} (1.5)

\[ v_\alpha(P/2, \downarrow)\bar{u}_\beta(P/2, \downarrow) = \frac{1}{2\sqrt{2}} (\varphi^0(\bar{\psi})(P + M))_{\alpha\beta} \]  \hspace{1cm} (1.6)

\[ \frac{1}{\sqrt{2}} (v_\alpha(P/2, \uparrow)\bar{u}_\beta(P/2, \downarrow) + v_\alpha(P/2, \downarrow)\bar{u}_\beta(P/2, \uparrow)) = \frac{1}{2\sqrt{2}} (\varphi^0(0)(P + M))_{\alpha\beta}. \]  \hspace{1cm} (1.7)

These are readily derivable in the quark and antiquark rest frame which, in the zero binding approximation, is the same as the quarkonium rest frame. Then covariance leads us to the above equations in any frame. We use the Bjorken-Drell conventions for spinors, gamma matrices and Lorentz metric except that spinors are normalized to \( \bar{u}u = 2m \). In particular, \( \epsilon_{0123} = 1 \).

The color singlet model is then defined by inserting on the quark line a wave function factor

\[ F_W = \frac{T^0\Psi(0)^*}{\sqrt{12M}} \varphi^0(P + M) \]  \hspace{1cm} (1.8)

when a vector quarkonium is created and the conjugate wave function factor
\[ T_W = \frac{\Psi(0) T^0}{\sqrt{12M}} (P + M) \]  \tag{1.9}

when the vector quarkonium state decays. Here \( T^0 \) is the 3 \( \times \) 3 unit matrix in color space and \( \Psi(0) \) is the wave function averaged over a small volume around the origin. This can be estimated from the leptonic decay of the \( J/\Psi \).

\[ |\Psi(0)|^2 = \Gamma(J/\Psi \rightarrow e^+e^-)M^2/(16\pi\alpha^2 Q_f^2) \approx 0.0044 \text{GeV} M^2 \]  \tag{1.10}

The same wave function at the origin squared, scaling as \( M^2 \), also adequately describes the \( \Phi \) and \( \Upsilon \) leptonic decay.

## 2 Vector Quarkonium Polarization in Lepton-Antilepton Annihilation

In the zero-binding, color singlet model the polarization state of a produced vector quarkonium is defined by the probabilities to produce at the resonant energy with no relative momentum the corresponding combined spin state of the quark and antiquark. These probabilities are in turn proportional to the squared production amplitudes. In lepton-antilepton annihilation the production amplitudes are

\[
\mathcal{M}(\lambda, \bar{\lambda}) = \pi(p_e) \gamma_{\mu} u(p_e) Q_f Q_q D_\gamma(P) \pi(P/2, s_q) \gamma_{\mu} v(P/2, s_q) + \pi(p_{\bar{e}}) \gamma_{\mu} (V_{\bar{f}} + A_{e\gamma_5}) u(p_{\bar{e}}) D_Z(P) \pi(P/2, s_q) \gamma_{\mu} (V_q + A_q \gamma_5) v(P/2, s_q). \tag{2.1}
\]

We take for the \( Z \) propagator

\[
D_Z(P) = \frac{1}{M_Z^2 - P^2 - i M_Z \Gamma_Z} \tag{2.2}
\]

We use a constant \( Z \) width equal to 2.5 GeV although our results are not sensitive to this. The photon propagator, \( D_\gamma(P) \) is as above with mass and width put to zero. Constant factors in the propagators do not affect our result. The vector and axial vector couplings of the \( Z \) to a fermion of weak isospin \( I_{3f} \) and charge \( Q_f \) are (relative to the proton charge)

\[
V_f = (I_{3f} - 2Q_f \sin^2 \theta_W)/\sin(2\theta_W) \\
A_f = -I_{3f}/\sin(2\theta_W). \tag{2.3}
\]

We define the quantization axis in the quarkonium rest frame to be the direction of the incident lepton. Covariantly, we put
\[ s_{q\mu} = \lambda \frac{\delta_{\mu}}{M} \]

\[ s_{\bar{q}\mu} = \overline{\lambda} \frac{\delta_{\mu}}{M} \]  \hspace{1cm} (2.4)

where

\[ \delta_{\mu} = p_{e\mu} - p_{\bar{e}\mu} \]  \hspace{1cm} (2.5)

and \( \lambda, \overline{\lambda} \) are \( \pm 1 \). The polarization probabilities as a function of \( \lambda, \overline{\lambda} \) are then

\[ P(\lambda, \overline{\lambda}) = \frac{b}{8M^4} \sum |\mathcal{M}(\lambda, \overline{\lambda})|^2 \]  \hspace{1cm} (2.6)

the sum being taken over the initial state lepton spins. The proportionality constant \( b \) is fixed by requiring

\[ P(\uparrow) + P(\downarrow) + P(0) = 1 \]  \hspace{1cm} (2.7)

where

\[ P(\uparrow) = P(1, 1) \]
\[ P(\downarrow) = P(-1, -1) \]  \hspace{1cm} (2.8)
\[ P(0) = \frac{1}{2}(P(1, -1) + P(-1, 1)) \]

Neglecting lepton masses we find that

\[ P(\lambda, \overline{\lambda}) = \frac{b}{8M^4} F_{\mu\nu} \left( M^2 g_{\mu\nu} (1 + \lambda \overline{\lambda}) - i(\lambda + \overline{\lambda}) \varepsilon(P, \delta, \mu, \nu) \right) \]  \hspace{1cm} (2.9)

where

\[ F_{\mu\nu} = -\frac{Q^2_e}{2} L^{\gamma}_{\mu\nu} |D_{\gamma}(P)|^2 - \frac{V^2}{2} L^{Z}_{\mu\nu} |D_{Z}(P)|^2 - 2 \frac{Q q e}{2} L^{\gamma Z}_{\mu\nu} Re(D_{\gamma}(P)D_{Z}(P)^*) \]  \hspace{1cm} (2.10)

\[ L^{\gamma}_{\mu\nu} = Q^2_e Tr(p_{\mu} \gamma_{\mu} p_{\nu} \gamma_{\nu}) \]  \hspace{1cm} (2.11)

\[ L^{Z}_{\mu\nu} = (V^2_e + A^2_e) Tr(p_{\mu} \gamma_{\mu} p_{\nu} \gamma_{\nu}) - 4i V_e A_e \varepsilon(P, \delta, \mu, \nu) \]  \hspace{1cm} (2.12)

\[ L^{Z\gamma}_{\mu\nu} = Q_e V_e Tr(p_{\mu} \gamma_{\mu} p_{\nu} \gamma_{\nu}) - 2i Q_e A_e \varepsilon(P, \delta, \mu, \nu). \]  \hspace{1cm} (2.13)
Thus

\[ P(\uparrow) + P(\downarrow) = b \left( Q_e^2 Q_q^2 |D_\gamma(P)|^2 + V_e^2 (V_e^2 + A_e^2) |D_Z(P)|^2 \\
+ 2Q_e Q_q V_e V_q Re(D_\gamma(P) D_Z^*(P)) \right) \]

(2.14)

\[ P(\uparrow) - P(\downarrow) = 2b V_q A_e \left( V_q V_e |D_Z(P)|^2 + Q_q Q_e Re(D_\gamma(P) D_Z^*(P)) \right) \]

(2.15)

\[ P(0) = 0. \]

(2.16)

The normalization condition eq. 2.7 implies that

\[ b = \left( Q_e^2 Q_q^2 |D_\gamma(P)|^2 + V_e^2 (V_e^2 + A_e^2) |D_Z(P)|^2 + 2Q_e Q_q V_e V_q Re(D_\gamma(P) D_Z^*(P)) \right)^{-1}. \]

(2.17)

3 Forward Backward Asymmetry in Leptonic Quarkonium Decay

The quarkonium polarization calculated in the previous section can be tested in various decay final states. For example, using the wave function factor of eq. 1.9, the amplitude for quarkonium decay into a lepton pair is

\[ \mathcal{M} = 2\pi \alpha \Psi(0) \sqrt{3M p_f} [\not{\epsilon}(V + A\gamma_5)\not{v}(p_f)] \]

(3.1)

where

\[ V = Q_q Q_f D_\gamma(P) + V_{qf} D_Z(P) \]

\[ A = V_q A_f D_Z(P) \]

(3.2)

the \( V_{qf} \) and \( A_{qf} \) being given by eq. 2.3. The matrix element squared is

\[ |\mathcal{M}|^2 = \epsilon_\alpha(\lambda) \epsilon_\beta^*(\lambda) 192\pi^2 |\Psi(0)|^2 \alpha^2 M \left( (|V|^2 + |A|^2) Tr(p_f^\gamma_\alpha \not{v}_f \gamma_\beta) \right) + (2ReVA^*) Tr(\gamma_5 \not{v}_f^\gamma_\alpha \not{v}_f \gamma_\beta), \]

(3.3)

The probability weighted sum over quarkonium helicities is

\[ \sum_\lambda \epsilon_\alpha(\lambda) \epsilon_\beta^*(\lambda) = \frac{1}{2} \left( P(\uparrow) + P(\downarrow) \right) \left( -g_{\alpha\beta} + \frac{P_\alpha P_\beta - \delta_{\alpha\beta} \gamma_\delta}{M^2} \right) + \frac{1}{2} \left( P(\uparrow) - P(\downarrow) \right) \frac{-i\epsilon(P, \delta, \alpha, \beta)}{M^2} \]

\[ + P(0) \frac{\delta_{\alpha\beta}}{M^2}. \]

(3.4)
This can be easily seen in the quarkonium rest frame using eqs. (1.1-1.3) and then generalized to an arbitrary frame using Lorentz covariance. We write the final state fermion momenta in eq. (3.3) as

\[ p_f = \frac{(P + \delta_f)}{2}, \]
\[ p_f = \frac{(P - \delta_f)}{2}. \]  

Then, performing the quarkonium spin average in eq. (3.3) yields

\[ |\mathcal{M}|^2 = 192\pi^2|\Psi(0)|^2\alpha^2 M \left( (P(\uparrow) + P(\downarrow))(|V|^2 + |A|^2)(M^2 + \frac{(\delta \cdot \delta_f)^2}{M^2}) \right. \\
\left. - 4(P(\uparrow) - P(\downarrow))Re(VA^*)\delta \cdot \delta_f + 2P(0)(|V|^2 + |A|^2)(M^2 - \frac{(\delta \cdot \delta_f)^2}{M^2}) \right) \].

(3.5)

Here we have kept the \( P(0) \) term for generality although it vanishes in quarkonium production by lepton-antilepton annihilation neglecting the lepton mass. In terms of the angle \( \theta \) between the final state lepton and the initial state lepton in the quarkonium rest frame

\[ \delta \cdot \delta_f = -4p_e \cdot p_f = -M^2 \cos \theta \]  

(3.7)

and

\[ |\mathcal{M}|^2 = 192\pi^2|\Psi(0)|^2\alpha^2 M^3 \left( (P(\uparrow) + P(\downarrow))(|V|^2 + |A|^2)(1 + \cos^2 \theta) \right. \\
\left. + 4(P(\uparrow) - P(\downarrow))Re(VA^*) \cos \theta + 2P(0)(|V|^2 + |A|^2) \sin^2 \theta \right) \].

(3.8)

Putting \( P(0) \) to zero, the forward backward asymmetry is

\[ \frac{F - B}{F + B} = 3 \frac{ReVA^*}{2 |V|^2 + |A|^2} (P(\uparrow) - P(\downarrow)). \]

(3.9)

In table 1 we collect at each of four quarkonia, the differences \( \Delta P = P(\uparrow) - P(\downarrow) \) and the forward-backward asymmetries from eq. (3.9). For the vector toponium resonance we assume a mass of 340 GeV. The forward-backward asymmetries in the leptonic decay of the charm and bottom quarkonia might be measurable at future \( J/\Psi \) or Upsilon facilities while the large predicted forward backward asymmetry at toponium should be observable at the next linear collider. The experiments, however, are made difficult by the small branching ratios into lepton pairs.

Off-resonance, or neglecting the quarkonia effects, the forward backward asymmetries are

\[ \frac{(F - B)}{(F + B)}_{\text{off-res}} = \frac{3H}{4G} \]

(3.10)

where

\[ G = |Q_c Q_f D_\gamma(P)|^2 + (V_e^2 + A_e^2)(V_f^2 + A_f^2)|D_Z(P)|^2 + 2Q_e Q_f V_e V_f Re(D_\gamma(P)D_Z(P)) \]
\[ H = 2A_e A_f \left( 2V_e V_f |D_Z(P)|^2 + Q_e Q_f Re(D_\gamma(P)D_Z(P)) \right). \]

(3.11)
|      | $BR_\mu$ | $\Delta P$ | $(F - B)_{asym}$ | $(F - B)_{asym_{off-res}}$ |
|------|---------|------------|------------------|---------------------------|
| $\Phi$ | $(3.7 \pm 0.5) \cdot 10^{-4}$ | $1.82 \cdot 10^{-4}$ | $2.48 \cdot 10^{-8}$ | $-6.58 \cdot 10^{-5}$ |
| $J/\Psi$ | .059 $\pm$ .001 | $4.65 \cdot 10^{-4}$ | $1.62 \cdot 10^{-7}$ | $-6.08 \cdot 10^{-4}$ |
| $\Upsilon$ | .0248 $\pm$ .0006 | $1.59 \cdot 10^{-2}$ | $1.88 \cdot 10^{-3}$ | $-5.73 \cdot 10^{-3}$ |
| Toponium | | $-0.409$ | $0.125$ | $0.496$ |

Table 1: Polarization differences and forward-backward asymmetries of lepton pair decays of quarkonia. Also tabulated are the experimental muon pair branching ratios and the theoretical off-resonance forward-backward asymmetries.

It is predicted that the forward backward asymmetry is opposite in sign and suppressed in magnitude relative to the off-resonance values in the $\Phi$, $J/\Psi$, and $\Upsilon$ regions while in the toponium region the asymmetry retains its sign and is only slightly suppressed in magnitude relative to off-resonance values.

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