We investigate nonlinear dispersive mode coupling between the flexural in- and out-of-plane modes of two doubly clamped, nanomechanical silicon nitride string resonators. As the amplitude of one mode transitions from the linear response regime into the nonlinear regime, we find a frequency shift of two other modes. The resonators are strongly elastically coupled via a shared clamping point and can be tuned in and out of resonance dielectrically, giving rise to multimode avoided crossings. When the modes start hybridizing, their polarization changes. This affects the nonlinear dispersive coupling in a non-trivial way. We propose a theoretical model to describe the dependence of the dispersive coupling on the mode hybridization.

I. INTRODUCTION

Nonlinear mechanical mode coupling affects the dynamics of driven systems of coupled nanomechanical resonators, even in situations where the modes are tuned far from resonance or from internal resonances [1, 2]. It is apparent as a dispersive eigenfrequency shift of the coupled mode which depends on the oscillation amplitude of the driven mode. Nonlinear mode coupling has been demonstrated between the harmonics of doubly clamped beams or strings, between the orthogonal flexural modes of a singly clamped nano- or microcantilever, and between the vibrational modes of a torsional resonator and one of its suspension springs [2-9].

Here we present nonlinear dispersive mode coupling between the fundamental flexural modes of two adjacent, intrinsically coupled nanomechanical resonators. The resonators under investigation are doubly clamped, strongly pre-stressed silicon nitride string resonators with fundamental eigenfrequencies in the range of 6 MHz. A simple theoretical model is developed to describe the behavior of the coupled nonlinear system under eigenfrequency tuning. The model reveals that both nonlinear coefficients, the Duffing nonlinearity and the dispersive coupling, depend on the hybridization of the underlying modes. The prospect of controlling the nonlinear coefficients of a system of two or more resonators may be of interest for future applications of nonlinear nanomechanical networks.

II. ACTUATION AND DETECTION

The two freely-suspended silicon nitride strings forming the resonator pair are depicted in Fig. 1. They share one clamping point, which has been engineered to enable strong linear mechanical coupling between the two string resonators [10]. The strings have a nominal length of 55 μm, a width of 180 nm and a thickness of 100 nm.

The mechanical (flexural) modes have Q-factors in the range of 200 000 as a result of the high tensile stress (1.46 GPa) in the silicon nitride nanostrings. One of the two resonators (resonator 1 in Fig. 1) is dielectrically controlled [11, 12]. Actuation is achieved by applying a RF drive tone as well as a DC voltage \( U_{dc} \) to one of the electrodes. At the same time, the DC voltage allows to tune the resonator’s eigenfrequencies. The second gold electrode is connected to a \( \lambda/4 \) microwave cavity for cavity-assisted, dielectric displacement detection [13]. To minimize dielectric crosstalk, the RF and DC voltages are applied to the outer electrode, whereas the microwave-cavity-assisted displacement detection is performed on the inner electrode situated between the two string resonators. All measurements are performed at a pressure below \( 10^{-4} \) mbar and at room temperature.

III. CHARACTERIZATION IN THE LINEAR REGIME

The two lowest flexural modes of a nanostring correspond to its fundamental out-of-plane (OP) and in-plane mode (IP). For the case of two nanostrings, one has to
a breaking of symmetry in the suspended nanowire and generally negligible. Furthermore, we ignore the external drive, the damping and the noise. We will include these terms in the subsequent sections.

In this section we focus on the linear regime by disregarding, for the moment, the nonlinearities,

$$\ddot{v}_i + \omega_i^2 v_i + \sum_{j \neq i} \kappa_{ij} (v_i - v_j) = 0. \quad (2)$$

We introduce a voltage dependence for the intrinsic eigenfrequencies $\omega_i$ of the bare modes in the system. As only resonator 1 is surrounded by the gold electrodes, we expect to control only the modes of resonator 1 by our dielectric actuation and tuning technique. However, we find experimentally that mode OP2 of resonator 2 is also slightly affected by the DC voltage. Therefore, we assume

$$\omega_i(U_{dc})/(2\pi) = \omega_{i0}/(2\pi) + c_i(U_{dc} - U_{i0})^2 + d_i(U_{dc} - U_{i0})^3$$

with $\omega_{i0}$ ($i = 1, 2, 3$) being the (theoretical) intrinsic eigenfrequencies of the bare modes in absence of coupling and vanishing voltage. We have considered a quadratic tuning factor $c_i$, along with a cubic correction $d_i$. To be more general, we have also assumed an offset voltage $U_{i0}$ of the vertices of the tuning parabolas. A voltage offset can arise as a consequence of a buildup of static polarization. Additionally we assume the linear coupling $\kappa_{ij}$ to be independent of the DC voltage, neglecting dielectric effects.

The equations of motion can be re-written as

$$\ddot{v}_i = -\Theta \dot{v}_i,$$ 

with the mode matrix $\Theta$ defined as

$$\Theta = \begin{bmatrix}
\omega_1^2 + \kappa_{12} + \kappa_{13} & -\kappa_{12} & -\kappa_{13} \\
-\kappa_{12} & \omega_2^2 + \kappa_{23} & -\kappa_{23} \\
-\kappa_{13} & -\kappa_{23} & \omega_3^2 + \kappa_{13} + \kappa_{23}
\end{bmatrix}.$$ 

(4)

The eigenvalues $\Omega_i$ of the matrix $\Theta$ in Eq. (4) denote the eigenfrequencies of the three hybridized modes. They depend on the DC voltage, the DC tuning strength and the eigenfrequencies of the bare modes. We used a genetic fit \[10\] to obtain the parameters entering in the frequencies $\Omega_i$, which we extracted from the experimental data in Fig. 2. The result of the fitting yields the following parameters: $\kappa_{12}/(2\pi)^2 = 4.138 \text{ MHz}^2$, $\kappa_{13}/(2\pi)^2 = 1.9098 \text{ MHz}^2$, $\kappa_{23}/(2\pi)^2 = 2.546 \text{ MHz}^2$, $\omega_{10}/(2\pi) = 5.8606 \text{ MHz}$, $\omega_{20}/(2\pi) = 5.9897 \text{ MHz}$, $\omega_{30}/(2\pi) = 6.1117 \text{ MHz}$, $U_{i0} = 0.656 \text{ V}$, $U_{20} = -1.1 \text{ V}$, $U_{30} = 0.485 \text{ V}$, $c_1/(2\pi) = 165.59 \text{ Hz}/\text{V}^2$, $c_2/(2\pi) = 1 \text{ Hz}/\text{V}^2$, $c_3/(2\pi) = -282.5 \text{ Hz}/\text{V}^2$, $d_1/(2\pi) = 1.368 \text{ Hz}/\text{V}^3$, and $d_3/(2\pi) = -2.2 \text{ Hz}/\text{V}^3$.

In Fig. 3a we show the three calculated hybridized eigenfrequencies $\Omega_i$ (blue solid line, red short dashed line, green long dashed line) based on the above parameters for voltages between 0 V and 32 V. As a comparison we show the extracted data from Fig. 2 (blue, red and green dotted lines). As the DC voltage dependence of the eigenfrequencies is (almost) symmetric for positive and negative
Finally, the bare modes strongly hybridize in the area of the
avoided crossings between 15 V and 28 V. Note that
the microwave cavity-assisted readout technique is sen-
tive only to the modes of resonator 1 which is situated
between the electrodes. Therefore a given hybridized
eigenmode can only be detected if it has a sufficiently
strong contribution from a bare mode of resonator 1.

**IV. DISPERSE NONLINEAR COUPLING**

We now fix the DC voltage to 15 V and apply a si-
musoidal drive tone $\omega_d$ close to the eigenfrequency $\Omega_2$
of the intermediate hybridized eigenmode. In addition
white noise (power of -5dBm, bandwidth of 7 MHz) is
applied to enhance the thermal motion of the resonators.
We sweep the drive frequency $\omega_d$ and measure the power
spectrum which is reported in Fig. 4(a). Far away from
resonant drive, i.e. $\omega_d \neq \Omega_2$, the three horizontal lines
correspond to the (enhanced) thermal motion of the three
hybridized eigenmodes at the DC voltage 15 V. As the
drive tone approaches the eigenfrequency, $\omega_d \approx \Omega_2$ (viz.
$\Omega_2 - \omega_d$ becomes comparable to the linewidth), one starts
to drive eigenmode 2. This is clearly visible in Fig. 4(b)
which displays the measured frequency response of the
driven eigenmode 2, namely its vibration amplitude as a
function of the drive frequency. The vibrational state of
eigenmode 2 is clearly in the nonlinear regime and the vi-
bibration amplitude shows the characteristic shape of the
Duffing oscillator (with negative, i.e. softening Duffing
nonlinearity). In our high quality resonators operating
at room temperature, the nonlinear regime of the driven
eigenmode manifests in the power spectrum as the for-
mation of two satellite peaks [14] around the drive tone
(almost a delta peak), clearly resolved in the frequency
range $\omega_d \approx \Omega_2$.

Here we focus our attention on the other two undriven
hybridized eigenmodes which are apparent in the power
spectrum as a result of the presence of the noise-enhanced
thermal fluctuation. Figure 4(a) reveals a clear frequency
shift for both undriven eigenmodes 1 and 3, as the drive
frequency approaches the eigenfrequency of eigenmode 2.
This occurs when the vibration amplitude of the driven
eigenmode strongly increases as we cross the resonant
condition $\omega_d \approx \Omega_2$. Far away from any internal reso-
nance, the minimal theoretical model that describes the
experimental data is given by assuming a dispersive cou-
pling between the driven eigenmode, see Eq. (4), and
the undriven, fluctuating eigenmodes. We consider the
following equations of motion: for the driven eigenmode
($\omega_d \approx \Omega_2$) we set

$$\ddot{q}_2(t) = -\Omega_2^2 q_2(t) - \Gamma_2 \dot{q}_2(t) - \tilde{\gamma}_{22} q_2^3(t) + F_d \cos(\omega_d t) + \delta F_2(t),$$

(see Fig. 3), which is only indirectly affected by the tun-
ing of resonator 1.

Using the transformation

$$v_i = \sum_{j=1}^{3} e_{ji} (V_{dc}) q_j$$

(5)

we set the variables $q_j$ as the vibration amplitude of
the hybridized eigenmodes. Figures 3a, c and d show
the (voltage dependent) components $e_{ji}$ of the calculated
eigenvectors of the three hybridized eigenmodes, respec-
tively. One can observe that the three bare modes are
always partially hybridized even in the limit of vanishing
DC voltage, since $e_{ji}$ remains non-zero even for $j \neq i$. On
the other hand, in the limit $U_{dc} \rightarrow 0$, each hybridized
mode is dominated by the contribution of one particu-
lar bare mode, $e_{jj}$, of the system. For example, Fig. 3b
shows that the hybridized eigenmode 1 is dominated by
the bare mode OP1 whereas Fig. 3c and d show that
eigenmodes 2 and 3 are dominated by the contribution
of OP2 and IP1, respectively. This is also deducible from
Fig. 3a, which shows that eigenmode 1 and 3 both depend
strongly on the DC voltage which is applied on resonator
1. On the other hand, mode 2 is almost flat. This mode
has the strongest contribution from the bare mode OP2
As mentioned above, the dispersive coupling manifests itself in a frequency shift of one mode, if the vibration amplitude of another mode is changed.

Figures 4 and 5 show the shift of the frequencies $\tilde{\Omega}_1^2$ of eigenmode 1 (blue dots) and $\tilde{\Omega}_3^2$ of eigenmode 3 (green dots) as a function of the amplitude of the driven eigenmode. We carry out a fit of Eq. (9) and Eq. (10) to the data (black lines) to obtain the dispersive coupling (including a calibration factor $c_2$ which accounts for the displacement sensitivity of eigenmode 2) and find $\tilde{c}_2\tilde{\gamma}_{12} = -6.694 \cdot 10^{19} \text{Hz}^2/V^2$ and $\tilde{c}_2\tilde{\gamma}_{23} = -2.368 \cdot 10^{19} \text{Hz}^2/V^2$.

V. VOLTAGE DEPENDENCE OF DUFFING NONLINEARITY AND DISPERSIVE COUPLING

We repeat the evaluation discussed so far for the other modes, namely by driving either eigenmode 1, 2 or 3 and detecting the frequency shifts of the respective other eigenmodes. This allows to extract all dispersive coupling coefficients $\tilde{c}_i\tilde{\gamma}_{ij}$ and $\tilde{c}_j\tilde{\gamma}_{ij}$ as well as Duffing nonlinearities $\tilde{c}_i\tilde{\gamma}_{ii}$ of the eigenmodes (with $i, j = 1, 2, 3$ and $i \neq j$).

Note that we detect different modes, each having a different (and voltage-dependent) calibration factor $c_i$ as a result of the mode-polarization dependent displacement sensitivity of our detection scheme.

We repeat the whole procedure for different DC voltages in a range of 0 V to 22 V to find the DC voltage dependence of the dispersive coupling $\tilde{c}_i\tilde{\gamma}_{ij}$ and $\tilde{c}_i\tilde{\gamma}_{ii}$. The results are summarized in Fig. 5a and Fig. 5b, which depict the experimentally determined Duffing nonlinearities and dispersive coupling coefficients, both including the appropriate calibration factor, respectively.

![Figure 4](image-url)  
**FIG. 4.** (color online) a) Power spectra for varying drive frequency at a DC voltage of 15 V. The (enhanced) thermal fluctuations of the three hybridized modes $\Omega_1$, $\Omega_2$ and $\Omega_3$ are clearly apparent. When the drive frequency $\omega_d$ (indicated by white line) approaches the eigenfrequency $\Omega_2$, this mode is excited and vibrates at the frequency $\omega_d$. b) Frequency response of the driven eigenmode 2 reflecting the typical Duffing curve. c) and d) Dependence of the squared renormalized eigenfrequencies of the two undriven eigenmodes 1 (blue dots) and 3 (green dots) as a function of the vibration amplitude of the driven eigenmode 2. The black lines depict the fit of Eq. (9) and Eq. (10).

whereas for the other two eigenmodes

$$\ddot{q}_1(t) = -\left[\Omega_1^2 + \tilde{\gamma}_{12}q_2(t)\right]q_1(t) - \Gamma_1 \dot{q}_1(t) + \delta F_1(t),$$

$$\ddot{q}_3(t) = -\left[\Omega_3^2 + \tilde{\gamma}_{23}q_2(t)\right]q_3(t) - \Gamma_3 \dot{q}_3(t) + \delta F_3(t).$$

In Eq. (6) and Eq. (7) $\Gamma_i$ represents the damping of the $i$-th eigenmode, and $\delta F_i(t)$ the (enhanced) thermal noise. The parameter $\tilde{\gamma}_{22}$ is the Duffing nonlinearity of the driven eigenmode at the DC voltage 15 V. Since the two undriven eigenmodes are weakly fluctuating only due to the (enhanced) thermal noise, we neglect their effect of nonlinear coupling on the driven eigenmode (this term should correspond to a noise effect in the frequency of the driven eigenmode). As the driven mode is oscillating at the drive frequency, far away from the eigenfrequency $\Omega_1$ and $\Omega_3$ of the other two eigenmodes, we have to take only the time average $\overline{q_2^2(t)}$ of the driven mode to account for its effect on the other two modes. We set the latter time average as $\overline{q_2^2(t)} = c_2x_2^2$, with $x_2$ the measured signal and $c_2$ a calibration factor. Then we can write the renormalized eigenfrequencies of eigenmodes 1 and 3 as a function of the amplitude of the driven eigenmode 2 as

$$\tilde{\Omega}_1^2 = \Omega_1^2 + \frac{1}{2}\tilde{\gamma}_{12}c_2x_2^2,$$

$$\tilde{\Omega}_3^2 = \Omega_3^2 + \frac{1}{2}\tilde{\gamma}_{23}c_2x_2^2.$$
VI. HYBRIDIZATION-DEPENDENT DUFFING NONLINEARITY AND DISPERSIVE COUPLING

In this section we present a theoretical model that describes the voltage dependence of the Duffing nonlinearities and of the dispersive coupling coefficients. It is based on the DC-voltage dependent hybridization of the modes, see Fig. 3.

We start from the nonlinear potential of fourth order which couples the three bare modes of the system (OP1, OP2, IP1)

\[ \mathcal{V}[\{v_i\}] = \frac{1}{4} \sum_{i=1}^{3} \gamma_{ii} v_i^4 + \frac{1}{2} \left( \gamma_{12} v_1^2 v_2^2 + \gamma_{13} v_1^2 v_3^2 + \gamma_{23} v_2^2 v_3^2 \right) \]

(11)

with amplitudes \( v_i \). Using the transformation Eq. (5), we change the basis and we express the quartic potential \( \mathcal{V} \) of Eq. (11) in terms of the hybridized eigenmodes of the system which are linearly independent. The result reads as follows:

\[ \tilde{\mathcal{V}}[\{q_i\}] = \frac{1}{4} \sum_{i=1}^{3} \gamma_{ii} q_i^4 + \frac{1}{2} \left( \gamma_{12} q_1^2 q_2^2 + \gamma_{13} q_1^2 q_3^2 + \gamma_{23} q_2^2 q_3^2 \right) \]

\[ + \sum_{i \neq j} \psi_{ij} q_i q_j + \sum_{i,j,k} \kappa_{ijk} \phi_i q_j q_k \, , \]

(12)

with the Levi-Civita antisymmetric tensor \( \epsilon_{ijk} \). The nonlinear potential \( \tilde{\mathcal{V}} \) of Eq. (12) for the amplitudes of the hybridized eigenmodes contains cubic nonlinearities and even three-body interactions. However, far from any internal resonances between the frequencies \( \Omega_i \) of the hybridized eigenmodes, the cubic and the three-body interaction terms can be neglected if we apply a single drive frequency. In other words, we only consider the Duffing nonlinearity of the driven mode, and the dispersive coupling to describe the frequency-shift of the non-driven, thermally activated modes. Table I summarizes the notation for the different modes and their interactions.

| Bare modes | Hybridized modes |
|------------|------------------|
| Amplitude  | \( v_i \) \( q_i \) \( (e_i (V_{dc})) \) |
| Linear interaction | \( \kappa_{ij} \) - |
| Nonlinear interaction | \( \gamma_{ij} \) \( \tilde{\gamma}_{ij} (V_{dc}) \) |

TABLE I. Summary of the notations used for the bare intrinsic modes and the hybridized eigenmodes.

Similarly we obtain the relation for the dispersive coupling coefficients of the hybridized eigenmodes

\[ \tilde{\gamma}_{ij} = 6 \sum_k e_{ik}^2 e_{jk}^2 \gamma_{kk} + \sum_{k \neq m} (e_{ik}^2 e_{jm}^2 + e_{jk}^2 e_{im}^2) \gamma_{km} \]

\[ + 4 \sum_{k \neq m} e_{ik} e_{jm} e_{jk} e_{im} \gamma_{mk} \quad \text{for} \quad i \neq j \, , \]

(14)

with \( i,j,k,m = 1, 2, 3 \). By varying the DC voltage, we modify the hybridization of the eigenmodes of the systems, namely the DC voltage-dependent components of the eigenvectors \( e_{ij}(V_{dc}) \). In consequence, the Duffing nonlinearity of each mode as well as their dispersive coupling to the other modes are altered. In other words, in the model as given by Eqs. (13) and (14), the Duffing nonlinearities and the dispersive coupling coefficients of the hybridized eigenmodes are voltage-dependent, \( \tilde{\gamma}_{ij} (V_{dc}) \), since they depend on the mode polarization.

Using the theoretical mode polarization discussed in Fig. 3 we compute the DC-voltage dependence of the Duffing nonlinearities \( \tilde{\gamma}_{ij} \) and of the dispersive coupling coefficients \( \tilde{\gamma}_{ij} \).

Figure 6a and b display the Duffing nonlinearities (\( \tilde{\gamma}_{11} \): blue solid line, \( \tilde{\gamma}_{22} \): red short dashed line, \( \tilde{\gamma}_{33} \): green long dashed line) as well as the dispersive coupling coefficients (\( \tilde{\gamma}_{12} \): blue solid line, \( \tilde{\gamma}_{23} \): red short dashed line, \( \tilde{\gamma}_{31} \): green large dashed line), respectively. It is evident that all nonlinear coefficients strongly depend on the mode hybridization.

At this stage two observations can be made. First, by varying the DC voltage, we are also varying a priori the intrinsic nonlinear coefficients of the bare modes, namely \( \gamma_{ij} \) \((i,j = 1, 2, 3)\). This effect is not included in the theoretical model given by Eq. (13) and Eq. (14) and represents an intrinsic property of dielectrically controlled nanomechanical resonators. As a second point, we observe that we have measured the quantities \( c_i \tilde{\gamma}_{ij} \) which

![Fig. 6](color online) Theoretical model for the polarization-dependent nonlinear coefficients. a) Duffing nonlinearities \( \tilde{\gamma}_{ij} \) (\( \tilde{\gamma}_{11} \): blue solid line, \( \tilde{\gamma}_{22} \): red short dashed line, \( \tilde{\gamma}_{33} \): green long dashed line). b) Dispersive coupling coefficients \( \tilde{\gamma}_{ij} \) (\( \tilde{\gamma}_{12} \): blue solid line, \( \tilde{\gamma}_{23} \): red short dashed line, \( \tilde{\gamma}_{31} \): green large dashed line). All plotted quantities have been scaled with respect to \( \gamma_0 = (\tilde{\gamma}_{11} + \tilde{\gamma}_{22})/6 \), and we have used the following realistic parameters for the bare modes \( \gamma_{11}/\gamma_0 = 2.2 \), \( \gamma_{22}/\gamma_0 = 3.8 \), \( \gamma_{33}/\gamma_0 = 0.01 \), and \( \gamma_{12}/\gamma_0 = -6.5 \), \( \gamma_{13}/\gamma_0 = 0.8 \), \( \gamma_{23}/\gamma_0 = 1.3 \).
involves calibration factors \( \tilde{c}_i \). In the present experiment, we have not been able to extract these quantities: the DC-voltage dependence of the calibration factors \( \tilde{c}_i \) is not known. Therefore, we have opted for a qualitative comparison between theory and experiment in order to illustrate the similarities and the differences between the experimental results and the expected DC voltage dependence as predicted by the mode polarization model.

However, the order of magnitude for the variations of the experimentally obtained values are clearly comparable with the predictions of the theory. Moreover, a strong dependence on the voltage is experimentally observed when the modes are strongly hybridized in the vicinity of the avoided crossings.

VII. CONCLUSION

We observed nonlinear dispersive mode coupling in a nanomechanical system formed by three linearly coupled bare eigenmodes of a pair of nanostring resonators.

We also observed a strong voltage dependence of the nonlinear coefficients of the system. Both the Duffing nonlinearities \cite{16} and the dispersive coupling coefficients are modified, particularly in the DC-voltage regime in which strong hybridization occurs. To understand the latter effect, we have analyzed a minimal theoretical model which yields a voltage dependence resulting from the hybridization of the modes.

We showed, that even if a direct comparison between experimental data and theory was not possible, the theoretical model based on the mode hybridization qualitatively reproduces the non-trivial voltage dependence. From the experimental side, for further investigations on the voltage dependence of the nonlinear coefficients, the displacement detection scheme needs to be improved to allow for a full calibration of the measured data. In addition, a second microwave cavity should be included for resonator 2 to enable the direct detection of all modes. From the theoretical side, the effects of the voltage dependence of the intrinsic parameters of the systems for the bare modes have to be taken into account with a microscopic model for the dielectric modulation of the nonlinearities. Future work will address these directions.

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