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A Study on Temperature Distribution, Efficiency and Effectiveness of longitudinal porous fins by using Adomian Decomposition Sumudu Transform Method

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Abstract

In this paper, we studied the variation of temperature distribution, efficiency and effectiveness of porous fin for different fractional order $\alpha$, porous parameter $\xi$ and convection parameter $\delta$ by using Adomian Decomposition Sumudu Transform Method (ADSTM). Here the geometry considered is that rectangular porous fin and the passage velocity in heat transfer through porous media is simulated by using Darcy’s model.

Keywords: Adomian Decomposition Sumudu Transform Method; Porous Fins; Temperature Distribution; Fin Efficiency; Fin Effectiveness

1. Introduction

Fins are extensively used to improve the rate of heat dissipation from a hot surface especially in thermal engineering applications [1]. There are many applications of heat transfer in porous media in thermal engineering problems such as heat exchangers, reactor cooling and solar collectors [2]. Kiwan and Al-Nimr [3] introduced the concept of porous fin and then considered the formulation of Darcy’s model [4] in heat transfer in porous fin. Several attempts have been made so far for accurate understanding of heat transfer in extended surfaces made of porous materials. Saedodin and Sadeghi [5] used fourth order Runge–Kutta method to analyzed at transfer in a cylindrical porous fin and concluded that the heat dissipation rate from a porous fin is greater than that of a solid fin. Similarly, Cuce and Cuce [6] studied the efficiency and effectiveness assessment of longitudinal porous fins by using Homotopy perturbation method.

The fractional calculus has gained significant importance over the last few decades mainly due to its useful applications in diverse fields of science and engineering [7]. In this paper, Adomian Decomposition Sumudu Transform Method has been applied to find the analytical solution of fractional order nonlinear energy balance equation.

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2. Problem Description

Here, in this study, a rectangular porous fin profile has been considered having cross section area as constant which is mentioned in Fig. 1. The dimensions of the fin are represented by length $L$, width $W$ and thickness $t$. The consideration of porous nature enables the flow of infiltration through the fin.

Assume the porous medium is isotropic, homogeneous and saturated with single-phase fluid. The fin temperature depends on distance $x$ and temperature variation across the fin thickness has been neglected. Both solid matrix and fluid are kept at local thermal equilibrium and the interactions between the porous medium and the clear fluid is simulated by Darcy’s formulation.

The Energy balance equation of rectangular porous fin [6] is

$$q_x - q_{x+dx} = mc_p(T(x) - T_\infty) + hA(T(x) - T_\infty)$$

(1)

where $q$ is the heat flux, $m$ is the mass flow rate of the fluid passing through the porous material, $c_p$ is the specific heat capacity of the fluid, $h$ is the convective heat transfer coefficient, $A$ is the heat transfer surface area, $T$ is the fin temperature and $T_\infty$ is the ambient temperature.

The velocity of fluid passing through the fin at any point, can be determined from Darcy’s model [8] as:

$$V = \frac{gkb(T - T_\infty)}{v}$$

(2)

where $g$ is the gravitational acceleration, $k$ is the thermal conductivity of the fluid, $b$ is the volumetric thermal expansion coefficient and $v$ is the kinematic viscosity of the fluid.

By introducing the dimensionless parameters:

$$\theta = \frac{T - T_\infty}{T_p - T_\infty}, \quad \zeta = \frac{x}{L}, \quad \xi = \frac{DaR_d}{k_r} \left(\frac{L}{t}\right)^2 \quad \text{and} \quad \delta = \left(\frac{hp}{ksA}\right)^\frac{1}{2}$$

(3)

The energy balance equation at steady state condition can be stated as [6]

$$\frac{d^2 \theta}{d\zeta^2} - \xi(\theta(\zeta))^2 - \delta \theta(\zeta) = 0$$

(4)

where, $\xi$ is a porous parameter that specifies the buoyancy effect as well as the effect of permeability of the porous medium and $\delta$ is a convection parameter that specifies the convection effect of the porous fin. As the fin tip is insulated, no heat transfer occurs at the insulated tip, and hence the boundary conditions of the governing differential equation are as follows:

$$\theta(\zeta = 1) = 1, \quad \frac{d\theta}{d\zeta}(\zeta = 0) = 0$$

(5)
3. Formulation for Adomian Decomposition Sumudu Transform Method (ADSTM)

To illustrate the basic idea of this method, we consider a general fractional nonlinear nonhomogeneous differential equation as,

\[ D^\alpha \theta(\zeta) + R \theta(\zeta) + N \theta(\zeta) = g(\zeta) \]  

(6)

with initial condition \( \theta(0) = \theta_0 \).

where \( D^\alpha \) is the Caputo fractional derivative Operator, \( R \) represents the reminder operator, \( N \) represents the nonlinear operator, and \( g(\zeta) \) is the source term.

Taking Sumudu Transform on both sides of Eq. (14), it gives

\[ S[D^\alpha \theta(\zeta)] + S[R \theta(\zeta)] + S[N \theta(\zeta)] = S[g(\zeta)] \]

(7)

Using the property of Sumudu Transform of Caputo fractional derivatives [9], we get

\[ S[\theta(\zeta)] = \theta(0) + u^\alpha S[g(\zeta)] - u^\alpha S[R \theta(\zeta) + N \theta(\zeta)] \]

(8)

Taking inverse sumudu transform [10] of Eq. (8), gives

\[ \theta(\zeta) = G(\zeta) - S^{-1}[u^\alpha S[R \theta(\zeta) + N \theta(\zeta)]] \]

(9)

where \( G(\zeta) = S^{-1}[\theta(0) + u^\alpha S[g(\zeta)] \] represents the term that arises from the given initial conditions.

The approximate solution of Eq. (9) can be written of the form,

\[ \theta(\zeta) = \sum_{n=0}^{\infty} \lambda^n \theta_n(\zeta) \]

(10)

and the nonlinear terms present in Eq. (9) can be expressed as a sum of Adomian Polynomials [11]

\[ N \theta(\zeta) = \sum_{n=0}^{\infty} \lambda^n A_n(\theta) \]

(11)

Where the Adomian polynomials \( A_n(\theta) \) be defined as

\[ A_n(\theta_0, \theta_1, \theta_2, ..., \theta_n) = \frac{1}{n!} \frac{\partial^n}{\partial \lambda^n} \left[ N \left( \sum_{i=0}^{\infty} \lambda^n \theta_i \right) \right] \text{, for } n = 0, 1, 2, ... \]

(12)

On substituting Eqs. (10) and (11) in Eq. (9), it obtain

\[ \sum_{n=0}^{\infty} \lambda^n \theta_n(\zeta) = G(\zeta) - \lambda \left[ S^{-1}[u^\alpha S[R \sum_{n=0}^{\infty} \lambda^n \theta_n(\zeta) + \sum_{n=0}^{\infty} \lambda^n A_n(\theta)]] \right] \]

(13)

which is a combination of Sumudu Transform and Adomian Decomposition Method.

On comparing the coefficients of various power of \( \lambda \), a recurrence relation for \( \theta_0, \theta_1, \theta_2, ..., \) can be obtained which gives the resulted approximate solution \( \sum_{k=0}^{m} \theta_k(\zeta) \) that converges to the exact solution \( \theta(\zeta) \) as \( m \to \infty \).

4. Solution for porous fin by using ADSTM

To understand the anomalous behavior of this system we fractionalize the energy balance Eq. (4) into fractional order \( (\alpha > 0) \) in order to find the fin temperature in rectangular porous fins as,

\[ \frac{d^\alpha \theta}{d\zeta^\alpha} - \xi(\theta(\zeta))^2 - \delta \theta(\zeta) = 0 \text{, where } 1 < \alpha \leq 2 \text{ and } 0 \leq \zeta \leq 1 \]

(14)

having boundary conditions \( \frac{d\theta}{d\zeta}\big|_{\zeta=0} = 0 \text{ and } \theta(1) = 1 \).

Taking Sumudu Transform on both sides of Eq. (14), it gives

\[ S \left[ \theta(\zeta) \right] = K + S^{-1}\left[ u^\alpha S\left[ \xi(\theta(\zeta))^2 + \delta \theta(\zeta) \right] \right] \]

(15)
By applying Adomian Decomposition Method, it obtains the following equation

$$\sum_{n=0}^{\infty} \lambda^n \theta_n(\zeta) = K + \lambda \left[ S^{-1} \left[ \mu^\alpha \left[ \sum_{n=0}^{\infty} \lambda^n \left[ \delta \theta_n(\zeta) \right] \right] \right] + \sum_{n=0}^{\infty} \lambda^n \left[ \delta \theta_n(\zeta) \right] \right]$$

where \( \theta_n(\zeta) \) represents Adomian’s polynomial for the nonlinear terms and it can be obtain by using Eq. (12).

The approximate solution of Eq. (14) up to five term can be written as

$$\theta(\zeta) = \theta_0(\zeta) + \theta_1(\zeta) + \theta_2(\zeta) + \theta_3(\zeta) + \theta_4(\zeta) + \ldots$$

$$= K + \left( K^2 \zeta + \delta^2 K \right) \frac{\zeta^{2\alpha}}{\Gamma(\alpha + 1)} + \left( K^4 \zeta^3 + 2 K^3 \delta^2 \zeta^2 + K^2 \delta^4 \right) \frac{\Gamma(2\alpha + 1) \zeta^{3\alpha}}{(\Gamma(\alpha + 1))^2 \Gamma(3\alpha + 1)}$$

$$+ \left( 2 K^3 \zeta^2 + 3 K^2 \delta^2 \zeta + K \delta^4 \right) \frac{\zeta^{2\alpha}}{\Gamma(2\alpha + 1)} + \left( 4 K^4 \zeta^3 + 8 K^3 \delta^2 \zeta^2 + 5 K^2 \delta^4 \zeta + K \delta^6 \right) \frac{\Gamma(3\alpha + 1) \zeta^{4\alpha}}{(\Gamma(\alpha + 1))^2 (4\alpha + 1)}$$

$$+ \left( 4 K^4 \zeta^3 + 2 K^3 \delta^2 \zeta + K \delta^4 \right) \frac{\Gamma(2\alpha + 1) \zeta^{3\alpha}}{(\Gamma(\alpha + 1))^2 (4\alpha + 1) \Gamma(3\alpha + 1)}$$

$$+ \left( 2 K^3 \zeta^2 + 4 K^2 \delta^2 \zeta + 2 K \delta^4 \right) \frac{\Gamma(3\alpha + 1) \zeta^{4\alpha}}{(\Gamma(\alpha + 1))^2 (4\alpha + 1)}$$

$$+ \left( 8 K^3 \zeta^3 + 20 K^2 \delta^2 \zeta^2 + 18 K^3 \delta^4 \zeta + 7 K^2 \delta^4 \zeta + 7 K^2 \delta^6 \right) \frac{\Gamma(2\alpha + 1) \zeta^{3\alpha}}{(\Gamma(\alpha + 1))^2 (4\alpha + 1)} + \cdots$$

(17)

Which represents an expression for temperature distribution of porous fin, where the fin tip temperature \( K \) can be determined at \( \theta = 1 \) for \( \zeta = 1 \), and it must be lies within the interval \([0, 1]\).

5. Results and Discussion

5.1. Temperature Distribution

In this section, results obtained from Adomian Decomposition Sumudu Transform Method (ADSTM) are presented for the classical and fractional order values \( \alpha \) in detail. Accuracy of the proposed method has been tested for the case of constant porous parameter \( \xi = 1 \) and classical order solution as shown in Fig. 2, which shows good agreement with Homotopy Perturbation Method (HPM) solution as obtained by [6].

Fig. 2. Comparison of dimensionless temperature distribution along the length of porous fin for different values of convection parameter and classical order \( \alpha \).

Fig. 3(a)–(c), shows the variation of temperature distribution within porous fin with distance \( \zeta \), for different values of convection parameter \( \delta \) and fractional value \( \alpha \) keeping the porous parameter constant \( \xi = 1 \). Further, the nature of the graphs depicts that considered value of \( \alpha \) represents the point of convergence under the given range of interval between 1 and 2.
The variation of temperature distribution for different values of porous parameter $\xi$ and fractional value $\alpha$ keeping the convection parameter constant ($\delta = 1$) has been depicted in Fig. 4 (a)–(d). It can be observed that as porous and convection parameter increases there is a decrease in temperature of porous fin and the fin temperature quickly reaches the surrounding temperature.

5.2. Fin efficiency

Fin efficiency of an extended surface is given by [1]:

$$\eta = \frac{Q_p}{Q_q}$$

(18)

where $Q_p$ is the heat transfer rate and $Q_q$ is the maximum heat transfer rate which can be achieved from the extended surface. The heat dissipation from the fin can be calculated by using Newton’s law of cooling:

$$Q_p = \int_0^\xi hk(T(x) - T_\infty)dx$$

(19)

where $k$ is the fin perimeter. The maximum heat transfer rate can be calculated as follows:

$$Q_q = hkL(T_b - T_\infty)$$

(20)

where $T_b$ be the temperature at the fin base which is remaining same throughout the fin.

Now Substituting Eq. (19) and Eq. (20) in Eq. (18), we get:

$$\eta = \frac{\int_0^\xi hk(T(x) - T_\infty)dx}{hkL(T_b - T_\infty)} = \int_{\zeta=0}^1 \theta(\zeta)d\zeta$$

(21)
The efficiency of porous fin for different values of porous and convection parameter and for different \( \alpha \)'s values has been depicted in Fig. 5 (a)–(d). It is noted that the fin efficiency considerably reduces with increasing porous and convection parameter. This finding can be attributed to the lower fin temperatures due to the higher cooling influence of \( \delta \) and \( \xi \) at greater values.

![Fig. 5. Efficiency of porous fin for distinct porous parameter, convection parameter and fractional values (a) \( \alpha = 2 \) (b) \( \alpha = 1.75 \) (c) \( \alpha = 1.5 \) (d) \( \alpha = 1.25 \)](image)

### 5.3. Fin effectiveness

The Fin effectiveness of an extended surface can be determined by [1]:

\[
\omega = \frac{Q_p}{Q_{pb}}
\]  

(22)

where \( Q_{pb} \) corresponds to the heat loss from the fin base and is given by:

\[
Q_{pb} = \frac{hkL(T_b - T_\infty)}{2}
\]  

(23)

Here it is assumed that the width of the fin \( W \) be greater than the fin thickness \( t \).

Eqs. (19) and (23) together with Eq. (22) gives:

\[
\omega = \frac{\int_0^L 2hk(T(x) - T_\infty)dx}{hk\frac{T_b - T_\infty}{2}} = \int_{\zeta=0}^1 \psi(\zeta)d\zeta
\]  

(24)

where \( \psi = \frac{2\zeta}{L} \).

As it is clear in Fig. 6 (a)–(d) that the fin effectiveness decreases with increasing convection parameter where as it increases with increasing values of fin length/fin thickness ratio.
6. Conclusion

In this study, ADSTM has been used to evaluate temperature distribution, fin efficiency and fin effectiveness along the length of porous fin as a function of porous and convection parameter. ADSTM is a perturbation based iterative technique which is an effective method for the solution of nonlinear fractional differential equations. The thermal analysis has been performed on a finite-length fin with insulated tip which shows the effect of parameters on temperature distribution, fins efficiency and fin effectiveness for different fractional value’s $\alpha$ using Eqs. (17), (21) and (24). Finally, the comparison of ADSTM with HPM as shown in Fig. 2 reveals that it is enough to come to a conclusion about the efficiency of the method.

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