Efficiency Algorithm for Solving Some Models of Nonlinear Problems

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Abstract: In this paper, a numerical algorithm proposed by composing the Aboodh transform (AT) and Adomian decomposition method (ADM) and it has been named (ATADM). This algorithm is tested for solving some models of nonlinear problems. The modification gives robust tool for large size of enumerations. The obtained results of comparing approximate solution with exact solution for this scheme appeared high accuracy and efficiency for solving nonlinear problems.

1. Introduction

Many scientific problems can be formulated by a mathematical model of partial differential equations (PDEs). Solutions of PDEs have a large and effective role in many fields of science such as in a physics [1, 2, 3], in dynamics [ 4], quantum [5], engineering [6].

Recently, during these modern years, many researchers have developed methods and techniques to find analytical solutions to PDEs. We will review some methods such as Homotopy transform analysis method [7], Homotopy perturbation method [8], Expansion method [9], method of auxiliary equations [10], and others.

Many researchers have focused on using ADM to solve many linear and non-linear problems, as this method is characterized by producing a serial solution using Adomian polynomials [11]. The beginnings of the ADM method were in the eighties of the last century, it had developments due to the interest of researchers and their attempts to improve the accuracy of this method and expand the possibility of applying it to various differential equations and nonlinear systems [12-15].

During these last years, a new transformation appeared by the researcher Khalid Aboodh in the year 2013[16], who wrote the Aboodh transform from a classical Fourier integral and presented important properties of this transformation and found the values of the basic functions with respect to this transformation [16]. Many linear standard and partial time domain differential equations were solved, as well as integral and integro-differential equations and delay differential equations [17-19].

However, this transformation is not used to solve nonlinear problems, so we suggested a new algorithm for solving nonlinear problems by combining this transformation with the Adomian decomposition method.

In this paper, we submitted an algorithm to obtain numerical solution of some classes for one (two)-dimensional nonlinear partial differential equations and nonlinear systems. In fact, we solved these equations in our proposed method without need to convert them into linear systems or simpler equations.

We study the nonlinear partial differential equation of the form:

\[ \mathcal{H}(w, w_x, w_y, w_{xx}, p, q, t) = 0 \] (1)

With initial and boundary conditions

\[ w(p, q, 0) = \alpha(p, q), \forall p, q \in \partial \theta, \theta \in R^2 \] (2)

\[ w(p, q, t) = \beta(p, q, t), \forall p, q \in \partial \theta \] (3)

Where \( \theta \) is the solution region and \( \partial \theta \) is the boundary of \( \theta \).

In the present paper, the Aboodh transform Adomian decomposition method (ATADM) was proposed.

We used the Aboodh transformation to solve the linear terms of the variable \( t \), while the nonlinear terms are solved by using Adomian polynomials and were calculated solution series components by
using iterative algorithm. The paper is made up of four sections: in the section two, the Aboodh transformation and its properties were explained briefly. The section three proposed a new algorithm which includes the Aboodh transform with Adomian decomposition method. Finally, we applied the proposed method for some models of one (or two)-dimensional nonlinear PDFs and two dimensional nonlinear system of PDFs.

2. Basic concepts to Aboodh transform (AT)

The present section includes the definition of the Aboodh transform and some basic features of this transform.

**Definition 2.1 [16, 17]:** Let $D$ be a set, we defined a function of exponential order in a set $D$ as follows:

$$D = \{ h(t) : \exists M, k_1, k_2 > 0, |h(t)| \leq Me^{-\nu t}, t \geq 0, k_1 \leq \nu \leq k_2 \}$$

$\exists M$ is a finite number, $k_1$, $k_2$ are finite or infinite numbers.

The Aboodh transform is denoted as $A(.)$ and defined as:

$$A[h(t)] = \frac{1}{\nu} \int_{0}^{\infty} h(t)e^{-\nu t} \, dt \quad t \geq 0, \ k_1 \leq \nu \leq k_2$$

**Aboodh transform for some special functions:**

| $h(t)$ | $A[h(t)]$ |
|--------|------------|
| $1$    | $\frac{1}{\nu^2}$ |
| $t$    | $\frac{1}{\nu^3}$ |
| $t^n, \ n \geq 1$ | $\frac{n!}{\nu^{n+2}}$ |
| $e^{at}$ | $\frac{1}{\nu^2 - a\nu}$ |
| $\sin at$ | $\frac{1}{\nu(\nu^2 + a^2)}$ |
| $\cos at$ | $\frac{1}{\nu^2 + a^2}$ |

**Table (1): Aboodh transform of some functions**

**Aboodh transform for some partial derivatives [17]:**

1) $A \left[ \frac{\partial^{n} w(p, \epsilon)}{\partial p^{n}} \right] = \nu^{n} A[w(p, \epsilon)] - \frac{w(p, 0)}{\nu}$

2) $A \left[ \frac{\partial^{n+1} w(p, \epsilon)}{\partial p^{n+1}} \right] = \nu^{n+1} A[w(p, \epsilon)] - \frac{w(p, 0)}{\nu^{2-n}}$

3) $A \left[ \frac{\partial w(p, \epsilon)}{\partial p} \right] = \frac{dA[w(p, \epsilon)]}{dp}$

4) $A \left[ \frac{\partial^{2} w(p, \epsilon)}{\partial p^{2}} \right] = \frac{d^{2}A[w(p, \epsilon)]}{dp^{2}}$

3. Analysis of ATADM:

We reformulate the Eq. (1), in the operator form:

$$L(w) + R(w) + N(w) = h,$$  \hspace{1cm} (6)

With auxiliary conditions, the function $w$ is unknown, $L = \frac{\partial^{n}}{\partial \epsilon^{n}}$ is the highest order, $R$ is a linear differential operator of order less than $L$, $N$ is nonlinear term, and $h$ represents the source term.
By applying the Aboodh transform for Eq. (6), and using the linear property to the transformation:

\[ A[w] = \frac{1}{v^n} \sum_{m=0}^{n-1} \frac{1}{v^n} \frac{\partial^m w}{\partial t^m}|_{t=0} + \frac{1}{v^n} A[h] - \frac{1}{v^n} A[R(w)] - \frac{1}{v^n} A[N(w)], \]  

(7)

Let

\[ w = \sum_{k=0}^{\infty} W_k \]  

(8)

And the nonlinear term series

\[ N(w) = \sum_{m=0}^{\infty} B_m \]  

(9)

Where \( B_m \) are Adomian’s polynomials which are defined as \([ 11 ]\),

\[ B_m = \frac{1}{m!} \frac{d^m}{d\rho^m} \left[ N\left( \sum_{k=0}^{\infty} \rho^k W_k \right) \right]_{\rho=0} ; \ m = 0,1,2, ... \]  

(10)

Substituting Eqs. (8) & (9) into Eq. (7), we get:

\[ \sum_{k=0}^{\infty} A[w_k] = \frac{1}{v^n} \sum_{m=0}^{n-1} \frac{1}{v^n} \frac{\partial^m w}{\partial t^m}|_{t=0} + \frac{1}{v^n} A[h] - \frac{1}{v^n} A[R \sum_{k=0}^{\infty} W_k] - \frac{1}{v^n} A[\sum_{k=0}^{\infty} B_k], \]  

(11)

Now, we apply inverse Aboodh transform to the Eq. (11)

\[ \sum_{k=0}^{\infty} W_k = A^{-1} \left[ \frac{1}{v^n} \sum_{m=0}^{n-1} \frac{1}{v^n} \frac{\partial^m w}{\partial t^m}|_{t=0} + A^{-1} \left[ \frac{1}{v^n} A[h] \right] \right] - A^{-1} \left[ \frac{1}{v^n} A[R(\sum_{k=0}^{\infty} W_k)] \right] \]  

(12)

By considering the few first terms, we get:

\[ w_0 = A^{-1} \left[ \frac{1}{v^n} \sum_{m=0}^{n-1} \frac{1}{v^n} \frac{\partial^m w}{\partial t^m}|_{t=0} + A^{-1} \left[ \frac{1}{v^n} A[h] \right] \right] \]  

(13)

Finally, we obtain the series solution \( w \) which consist of the components \( w_0, w_1, w_2, ... \)

4. Numerical Experiments

In this section, we will apply the ATADM for some types of PDEs:

Case One: One-dimensional nonlinear PDEs

Problem 1: Consider the following nonlinear PDE

\[ \frac{\partial w(p,t)}{\partial t} = \left( \frac{\partial w(p,t)}{\partial p} \right)^2 + w(p,t) \frac{\partial^2 w(p,t)}{\partial p^2}, \]  

(14)

\[ w(p,0) = p^2, \]  

(15)

The exact solution is:

\[ w(p,t) = \frac{p^2}{1-\epsilon t} \]  

(16)

Solution:

Applying AT and using initial condition:

\[ \Rightarrow A[w(p,t)] = \frac{p^2}{v^2} + \frac{1}{v^2} A \left( \frac{\partial^2 w(p,t)}{\partial p^2} \right)^2 + w(p,t) \frac{\partial^2 w(p,t)}{\partial p^2}, \]  

(17)

By using Eq. (11)

\[ \Rightarrow \sum_{k=0}^{\infty} A[w_k] = \frac{p^2}{v^2} + \frac{1}{v^2} A[\sum_{k=0}^{\infty} B_k], \]  

(18)

Taking inverse AT

\[ \Rightarrow \sum_{k=0}^{\infty} W_k = A^{-1} \left[ \frac{p^2}{v^2} + \frac{1}{v^2} A[\sum_{k=0}^{\infty} B_k] \right]. \]  

(19)

By using Eq. (13),

\[ w_0(p,t) = p^2 \]  

\[ w_{m+1}(p,t) = A^{-1} \left[ \frac{1}{v} A[B_m] \right], \quad m \geq 0 \]  

The first few components
\[ w_1(p, t) = A^{-1} \left[ \frac{1}{v} A \left( \frac{\partial w_0(p, t)}{\partial p} \right)^2 + w_0(p, t) \frac{\partial^2 w_0(p, t)}{\partial p^2} \right] = 6p^2 t \]

\[ w_2(p, t) = A^{-1} \left[ \frac{1}{v} A \left( 2 \frac{\partial w_0(p, t)}{\partial p} \frac{\partial w_1(p, t)}{\partial p} + w_0(p, t) \frac{\partial^2 w_1(p, t)}{\partial p^2} + w_1(p, t) \frac{\partial^2 w_0(p, t)}{\partial p^2} \right) \right] = 6p^2 t^2 \]

\[ w(p, t) = \sum_{m=0}^{\infty} w_m(p, t) = p^2 + 6p^2 t + 6p^2 t^2 = p^2(1 + 6t + 6t^2 + \ldots) \]

As \( m \to \infty \), then the last equation gives the exact solution to Eq. (14).

**Problem 2:** Consider the following nonlinear PDE

\[ \frac{\partial w(p, t)}{\partial t} + w(p, t) \frac{\partial w(p, t)}{\partial p} = p + pt^2; \quad (17) \]

\[ w(p, 0) = 0, \quad (18) \]

The exact solution is:

\[ w(p, t) = pt \quad (19) \]

**Solution:**

Taking the AT and using initial condition:

\[ \Rightarrow A[w(p, t)] = \left( \frac{p}{v^3} + \frac{2pt}{v^3} \right) - \frac{1}{v} A \left[ w(p, t) \frac{\partial w(p, t)}{\partial p} \right]. \]

By using Eq. (11)

\[ \Rightarrow \sum_{k=0}^{\infty} A[w_k(p, t)] = \frac{p}{v^3} + \frac{2pt}{v^3} - \frac{1}{v} A[\sum_{k=0}^{\infty} B_k]. \]

Applying inverse AT

\[ \Rightarrow \sum_{k=0}^{\infty} w_k(p, t) = A^{-1} \left[ \frac{p}{v^3} + \frac{2pt}{v^3} \right] - A^{-1} \left[ \frac{1}{v} A[\sum_{k=0}^{\infty} B_k] \right]. \]

By using Eq. (13),

\[ \Rightarrow w_0(p, t) = pt + \frac{pt^3}{3} \]

\[ w_{m+1}(p, t) = -A^{-1} \left[ \frac{1}{v} A[B_m] \right], \quad m \geq 0 \]

The first few components

\[ w_1(p, t) = -A^{-1} \left[ \frac{1}{v} A \left[ w_0(p, t) \frac{\partial w_0(p, t)}{\partial p} \right] \right] \]

\[ = -A^{-1} \left[ \frac{1}{v} A \left[ \left( pt + \frac{pt^3}{3} \right) (t + pt^2) \right] \right] = -\frac{pt^3}{3} - \frac{2pt^5}{15} - \frac{pt^7}{63} \]

\[ \Rightarrow w(p, t) = \sum_{m=0}^{\infty} w_m(p, t) = pt + \frac{pt^3}{3} - \frac{pt^2}{3} - \frac{2pt^5}{15} - \frac{pt^7}{63} + \ldots \]

**Case two:** Two-dimensional nonlinear PDEs

**Problem 3:** Consider the following two –dimension nonlinear PDE
The exact solution is:
\[ w(p, q, t) = \frac{p-q}{1-t} \]  
(22)

**Solution:**

Applying the AT and using initial condition:

⇒ \[ A[w(p, q, t)] = \frac{p-q}{v^2} + \frac{1}{v} A \left[ \frac{1}{2} \left( w(p, q, t) \frac{\partial w(p, q, t)}{\partial p} + w(p, q, t) \frac{\partial w(p, q, t)}{\partial q} \right) \right]. \]

By using Eq. (11)

⇒ \[ \sum_{k=0}^{\infty} A[w_k(p, q, t)] = \frac{p-q}{v^2} + \frac{1}{v} A[\sum_{k=0}^{\infty} B_k]. \]

Taking inverse AT

⇒ \[ \sum_{k=0}^{\infty} w_k(p, q, t) = A^{-1} \left[ \frac{p-q}{v^2} + \frac{1}{v} A[\sum_{k=0}^{\infty} B_k] \right] \]

By using Eq. (13),

⇒ \[ w_0(p, q, t) = p - q \]

⇒ \[ w_{m+1}(p, q, t) = -A^{-1} \left[ \frac{1}{v} A[B_m] \right], \quad m \geq 0 \]

The first few components

\[ w_1(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left[ \frac{1}{2} \left( w_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial p} - w_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial q} \right) \right] \right] \]

\[ = A^{-1} \left[ \frac{1}{v} A \left[ \frac{1}{2} \left( (p-q)(1) - (p-q)(-1) \right) \right] \right] = (p-q)t \]

\[ w_2(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left[ \frac{1}{2} \left( w_1(p, q, t) \frac{\partial w_1(p, q, t)}{\partial p} - w_1(p, q, t) \frac{\partial w_1(p, q, t)}{\partial q} \right) \right] \right] \]

\[ + w_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial p} - w_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial q} \]

\[ = (p-q)t^2 \]

\[ \vdots \]

\[ w(p, q, t) = \sum_{m=0}^{\infty} w_m(p, q, t) = (p-q) + (p-q)t + (p-q)t^2 + \cdots \]

As \( m \to \infty \), then the last equation gives the exact solution to Eq. (20).

**Problem 4:** Consider the following two–dimension nonlinear system of PDEs

\[ \frac{\partial w(p, q, t)}{\partial t} = w(p, q, t) \frac{\partial w(p, q, t)}{\partial p} + z(p, q, t) \frac{\partial w(p, q, t)}{\partial q} \]

\[ \frac{\partial z(p, q, t)}{\partial t} = w(p, q, t) \frac{\partial z(p, q, t)}{\partial p} + z(p, q, t) \frac{\partial z(p, q, t)}{\partial q} \]

(23)

\[ w(p, q, 0) = z(p, q, 0) = p + q. \]

(24)

The exact solution is:

\[ w(p, q, t) = z(p, q, t) = \frac{p+q}{1-2t} \]

(25)

**Solution:**

Taking the AT and using initial condition:
\[ A[w(p, q, t)] = \frac{p + q}{v^2} + \frac{1}{v} A \left\{ \frac{1}{2} \left( w(p, q, t) \frac{\partial w(p, q, t)}{\partial p} + z(p, q, t) \frac{\partial w(p, q, t)}{\partial q} \right) \right\} \]

\[ A[z(p, q, t)] = \frac{p + q}{v^2} + \frac{1}{v} A \left\{ \frac{1}{2} \left( w(p, q, t) \frac{\partial z(p, q, t)}{\partial p} + z(p, q, t) \frac{\partial z(p, q, t)}{\partial q} \right) \right\} \]

By using Eq. (11)

\[ \sum_{k=0}^{\infty} A[w_k(p, q, t)] = \frac{p + q}{v^2} + \frac{1}{v} A \left\{ \sum_{k=0}^{\infty} B_k \right\} \]

\[ \sum_{k=0}^{\infty} A[z_k(p, q, t)] = \frac{p + q}{v^2} + \frac{1}{v} A \left\{ \sum_{k=0}^{\infty} C_k \right\} \]

Applying inverse AT

\[ \sum_{k=0}^{\infty} w_k(p, q, t) = p + q + A^{-1} \left[ \frac{1}{v} A \left\{ \sum_{k=0}^{\infty} B_k \right\} \right] \]

\[ \sum_{k=0}^{\infty} z_k(p, q, t) = p + q + A^{-1} \left[ \frac{1}{v} A \left\{ \sum_{k=0}^{\infty} C_k \right\} \right] \]

By using Eq. (13),

\[ w_0(p, q, t) = p + q, \quad w_{m+1}(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left\{ B_m \right\} \right], \quad m \geq 0 \]

\[ z_0(p, q, t) = p + q, \quad z_{m+1}(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left\{ C_m \right\} \right], \quad m \geq 0 \]

The first few components

\[ w_1(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left\{ w_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial p} + z_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial q} \right\} \right] = (p + q)(2t) \]

\[ z_1(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left\{ w_0(p, q, t) \frac{\partial z_0(p, q, t)}{\partial p} + z_0(p, q, t) \frac{\partial z_0(p, q, t)}{\partial q} \right\} \right] = (p + q)(2t) \]

\[ w_2(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left\{ \frac{1}{2} \left( w_0(p, q, t) \frac{\partial w_0(p, q, t)}{\partial p} + (p, q, t) \frac{\partial w_1(p, q, t)}{\partial q} \right) \right\} \right] = (p + q)(2t)^2 \]

\[ z_2(p, q, t) = A^{-1} \left[ \frac{1}{v} A \left\{ \frac{1}{2} \left( w_0(p, q, t) \frac{\partial z_1(p, q, t)}{\partial p} + z_0(p, q, t) \frac{\partial z_1(p, q, t)}{\partial q} \right) \right\} \right] = (p + q)(2t)^2 \]

\[ \vdots \]

\[ w(p, q, t) = \sum_{m=0}^{\infty} w_m(p, q, t) = (p + q) + (p + q)(2t) + (p + q)(2t)^2 + \ldots \]

\[ = (p + q)(1 + 2t + (2t)^2 + \ldots) \]
\[ z(p, q, t) = \sum_{m=0}^{\infty} z_m(p, q, t) = (p + q) + (p + q)(2t) + (p + q)(2t)^2 + \ldots = (p + q)(1 + 2t + (2t)^2 + \ldots) \]

As \( m \to \infty \), then the last equation gives the exact solution to Eq. (23).

### Numerical Results and Discussions

This section describes the numerical results by comparing between the exact solution and approximate solution which obtained by the ATADM. The second table explained the absolute error for \( w_1, w_2, w_5 \) of the first problem. Third table deals with the absolute error for the first and second terms of the second problem, where the difference between exact and Numerical solutions is very small despite of we took two terms only. In the fourth table, we got the absolute error for \( w_1, w_3, w_5 \) of the third problem. Finally, we considered the absolute error in fifth table for \( w_1, w_2, w_3 \) to the fourth problem. We noted that the rate of convergence was excellent with few terms, where \( p = 0.1, 0.5 \), \( q = 0.01 \) and \( t = 0, 0.02, 0.04, 0.06, 0.08, 0.1 \).

#### Table 2: Comparison of Exact Solution with Approximate Solution for Problem 1

| \( p \) | \( t \) | \( 0.1 \) | \( 0.5 \) | \( 0.0 \) | \( 0.5 \) | \( 0.0 \) | \( 0.5 \) | \( 0.0 \) | \( 0.5 \) | \( 0.0 \) | \( 0.5 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( w - w_1 \) | \( w - w_2 \) | \( w - w_3 \) | \( w - w_4 \) | \( w - w_5 \) | \( w - w_6 \) | \( w - w_7 \) | \( w - w_8 \) | \( w - w_9 \) | \( w - w_10 \) | \( w - w_11 \) | \( w - w_12 \) |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.02 | 2.66667 \times 10^{-7} | 4.26687 \times 10^{-11} | 1.33333 \times 10^{-6} | 2.13333 \times 10^{-10} |
| 0.04 | 2.13333 \times 10^{-6} | 1.36559 \times 10^{-9} | 0.000106667 | 6.82797 \times 10^{-9} |
| 0.06 | 7.2 \times 10^{-6} | 1.03724 \times 10^{-8} | 0.000036 | 5.18622 \times 10^{-8} |
| 0.08 | 0.0000170667 | 4.3724 \times 10^{-8} | 0.0000853333 | 2.1862 \times 10^{-7} |
| 0.1 | 0.0000333333 | 1.33492 \times 10^{-7} | 0.000166667 | 6.6746 \times 10^{-7} |

#### Table 3: Comparison of Exact Solution with Approximate Solution for Problem 2

| \( p \) | \( 0.1 \) | \( 0.5 \) | \( 0.0 \) | \( 0.5 \) | \( 0.0 \) | \( 0.5 \) |
|---|---|---|---|---|---|---|
| \( w - w_0 \) | \( w - w_1 \) | \( w - w_0 \) | \( w - w_1 \) | \( w - w_0 \) | \( w - w_1 \) |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.02 | 2.66667 \times 10^{-7} | 4.26687 \times 10^{-11} | 1.33333 \times 10^{-6} | 2.13333 \times 10^{-10} |
| 0.04 | 2.13333 \times 10^{-6} | 1.36559 \times 10^{-9} | 0.000106667 | 6.82797 \times 10^{-9} |
| 0.06 | 7.2 \times 10^{-6} | 1.03724 \times 10^{-8} | 0.000036 | 5.18622 \times 10^{-8} |
| 0.08 | 0.0000170667 | 4.3724 \times 10^{-8} | 0.0000853333 | 2.1862 \times 10^{-7} |
| 0.1 | 0.0000333333 | 1.33492 \times 10^{-7} | 0.000166667 | 6.6746 \times 10^{-7} |
Table 4: Comparison of Exact Solution with Approximate Solution for Problem 3

| ρ | 0.1 | 0.5 |
|---|---|---|
| | | |
| 0 | 0 | 0 |
| 0.0 | 0.00036734 | 0.00001833 |
| 2 | 2.9333×10^-7 | 1.36×10^-6 |
| 0.0 | 0.0018 | 0.00002592 |
| 4 | 4.89739×10^-6 | 3.73248×10^-7 |
| 6 | 0.0018 | 4.69333×10^-7 |
| 8 | 0.00335238 | 2.19702×10^-6 |
| 0.1 | 0.0055 | 8.8×10^-6 |

Table 5: Comparison of Exact Solution with Approximate Solution for Problem 4

| ρ | 0.1 | 0.5 |
|---|---|---|
| | | |
| 0 | 0 | 0 |
| 0.0 | 0.00018333 | 0.00001833 |
| 2 | 2.9333×10^-7 | 1.36×10^-6 |
| 0.0 | 0.0018 | 0.00002592 |
| 4 | 4.89739×10^-6 | 3.73248×10^-7 |
| 6 | 0.0018 | 4.69333×10^-7 |
| 8 | 0.00335238 | 2.19702×10^-6 |
| 0.1 | 0.0055 | 8.8×10^-6 |

Conclusions

During this research, a new algorithm called ATADN was formulated. This algorithm includes the Aboodh transformation, which deals with linear part of the differential equation with respect to the variable $t$ and Adomian polynomial were applied to the nonlinear part, we calculated the series solution components by using iterative algorithm. One of the main features of this method is that there is no necessity to transform the problem into, linear, ordinary or algebraic problems or using the concepts of perturbation and discretization.

The efficacy and accuracy of this method or algorithm were tested on four nonlinear problems, where the first problem was a homogenous nonlinear partial differential equation of one dimension. The second problem is a nonhomogeneous nonlinear PDE with one dimension, the third problem is a nonlinear PDE with two dimensions and finally a system of nonlinear PDEs with two dimensions. Through the obtained results, we noticed that the method is very efficient in obtaining approximate solutions which are very close to the exact solution.
References

1. Irshad A., Mohyud-Din S. T., Ahmed N., and Khan U., (2017), A new modification in simple equation method and its applications on nonlinear equations of physical nature, Results in Physics, 7, 4232-4240.

2. Durur H., and Yokuş A., (2020), Analytical solutions of Kolmogorov–Petrovskii–Piskunov equation, Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 22(2), 628-636.

3. Ahmad H., Seadawy A. R., and Khan T. A., (2020), Study on numerical solution of dispersive water wave phenomena by using a reliable modification of variational iteration algorithm, Mathematics and Computers in Simulation.

4. Ahmad H., Seadawy A. R., Khan T. A., and Thounthong P., (2020), Analytic approximate solutions for some nonlinear parabolic dynamical wave equations, Journal of Taibah University for Science, 14(1), 346-358.

5. Ali K. K., Yılmazer R., Yokus A., and Bulut H., (2020), Analytical solutions of Kolmogorov-Petrovskii-Piskunov equation, Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 22(2), 628-636.

6. Taher A. N., (2016), Simple equation method for nonlinear partial differential equations and its applications, Journal of the Egyptian Mathematical Society, 24, 204-209.

7. Eman M. N., (2020), Homotopy transforms analysis method for solving fractional Navier- Stokes equations with applications, Iraqi Journal of Sciences, 61(8), 2048-2054.

8. Aqeel F. J., (2015), Solving partial differential equations by Homotopy perturbation method "., Journal of the College of Basic Education, 21(89), 157-164.

9. Ismael H. F., Bulut H., and Baskonus H. M., (2020), Optical soliton solutions to the Fokas–Lenells equation via sine -Gordon expansion method and (m+(G'/G))-expansion method. Pramana, 94(1), 35.

10. Durur H., Tasbozan O., and Kurt A., (2020), New analytical solutions of conformable Time fractional Bad and good modified Boussinesq equations. Applied Mathematics and Nonlinear Sciences, 5(1), 447-454.

11. Asmaa A. A. and Aqeel F. J., (2014), The approximate solution of Newell-Whitehead-Segal and Fisher equations using the Adomian decomposition method, Al-Mustansiriyah J. Sci., 25(4), 45-56.

12. AL-Mazmumy M. and AL-Malki H., (2015), Some modifications of Adomian decomposition methods for nonlinear partial differential equations, IJRRAS, 23(2), 164-173.

13. Mariam A. M and Safa O.A., (2017), Restarted Adomian decomposition method for solving Voltera's population model, Journal of Computational Mathematics, 7, 175-182.

14. Mariam A. M. and Safa O. A., (2016), Solution nonlinear integro differential equations by two- step decomposition method (TSADM), International Journal of Modern Nonlinear Theory and Application, 5, 248-255.

15. Shazad S. A., Shokhan A. H. S., and Mariwan R. A., (2019), Laplace Adomian and Laplace modified Adomian decomposition methods for solving nonlinear integro-fractional differential equations of the Volterra-Hammerstein type, Iraqi Journal of Science, 60(10), 2207-2222.

16. Khalid S. A., (2013), The New Integral form Aboodh Transform, Global Journal of Pure and Applied Mathematics, 9(1), 35-43.

17. Mohamed E. B. and Kacem B., (2019), Application of Aboodh transform for solving first order constant coefficients complex equation, General Letters in Mathematics, 6(1), 28-34.

18. Aboodh K. S., Farah R. A., Almardy I. A. and Osman A. K., (2018), Solving delay differential equations by Aboodh transformation method, International Journal of Applied Mathematics & Statistical Sciences (IJAMSS), 7(1), 21-30.

19. Ali A. I. and Bhatti M. I., (2015), Comparison of Aboodh transformation and differential transform method numerically, Sci. In(Lahore), 27(2), 873-879.