Synchronization mode transitions induced by chaos in modified Morris–Lecar neural systems with weak coupling

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Received: 26 October 2021 / Accepted: 19 February 2022 / Published online: 2 March 2022
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Abstract Based on a modified Morris–Lecar neural model, the synchronization modes transitions between two coupled neurons or star-coupled neural network connected by weak electrical and chemical coupling are, respectively, investigated. For the two coupled neurons, by increasing the calcium conductivity, it is found that the period-2 synchronization of the action potential is transformed to desynchronization first, and then to period-3 synchronization. By increasing the potassium conductivity, however, the synchronization mode transition is a reversal direction process as mentioned above. The corresponding inter-spike interval shows the synchronization modes transition is induced by the chaos. The stronger the coupling strength is, the smaller the period-2 synchronization region in the parameters plane is, while the larger the period-3 synchronization region will be. For the star-coupled neural network, in the presence of weak electrical coupling, it can exhibit the completely synchronized mode, desynchronized mode, and drum head mode under different parameter values, respectively. In the presence of chemical synapse, however, the completely synchronized mode cannot be observed. Our results might provide novel insights into synchronization modes transition and related biological experiments.

Keywords Synchronization · Chaos · Morris–Lecar neuron · Weak coupling

1 Introduction

In recent decades, studying the nervous system through neuronal mathematical models has become a very common and effective method. Morris–Lecar (M-L) model [1] is one of the simplified versions of the classic Hodgkin–Huxley (H–H) model [2, 3]. The M-L model can behave like a real neuron by adjusting different system parameters. However, due to the lack of a slow variable, the original two-dimensional M-L model cannot generate bursting behavior. In order to observe and investigate several kinds of bursting in the M-L model, a feedback current which can be treated as a slow variable is introduced by many researchers [4–8]. Also, there are other reasonable ways to introduce slow variables. For example, Hu et al. proposed a modified M-L model in which the magnetic flux is introduced as a slow variable [9]. In addition, a complete electronic circuit of the 2D M-L neuron model has been established and discussed [10], which provides a novel way to study M-L neuron dynamics.

Neuronal activity is related to the transport of sodium and potassium ions across the membrane. Factors such as temperature changes have essential
influences on neuronal activity, which is caused by changes in ionic conductance [11]. With the temperature increasing, it is found that the firing frequency is higher, while the firing amplitude becomes smaller [12], which is consistent with the conclusions drawn in numerical simulation of the H–H model [13]. It is reported that a higher temperature makes it difficult for a neuron to transmit information along the entire axon [14]. The function of potassium and sodium ion channel can also be affected by some toxins. Experimental investigations have shown that the number of operating potassium and sodium ion channel is reduced by tetraethylammonium (TEA) and tetrodotoxin (TTX), respectively [15, 16]. The effects of ion channel blocks are that the number of opened channels or maximum ionic conductivity is reduced [17–21]. In terms of neural discharging, the firing activity is enhanced under the potassium ion channel block, while the firing activity is suppressed when the sodium ion channel is blocked [22].

In the real nervous system, there are two types of couplings between two neurons, the electrical synapse [23–25] and the chemical synapse coupling [26, 27]. For coupling topology of nonlinear systems, nearest coupling, global coupling and non-local coupling were widely investigated [28–32]. In addition, biological experiments and numerical simulations have shown that synchronization is the most common behavior in the nervous system [33–36]. It is found that chaos break can be regarded as a precursor of synchronization in Hindmarsh–Rose neurons under electromagnetic induction [37]. Pal et al. investigated energy consumption of two synchronous coupling H–H neurons and reported that blockage of ion channels also affects neuronal synchronization [38]. Kazantsev et al. investigated two inhibitory coupled H–H neurons with the phase map and found transient in-phase and anti-phase synchronization can be triggered by proper Poisson spike trains [39]. Simonov et al. found phase difference between two excitatory unidirectional connected neurons which is caused by depolarization parameters, i.e., applied currents [40].

Noise is a manifestation of randomness and fluctuations for a nonlinear system. Many kinds of resonances can be triggered by noise and other simulations [24, 41]. It is interesting to notice that synchronization can also be triggered by different kinds of noise. Wang et al. found that a relatively stronger Gaussian white noise can trigger complete synchronization between two M-L neurons [42]. Similarly, an appropriate intensity of bounded noise can induce complete synchronization between two weak coupled FitzHugh–Nagumo (FHN) neurons [43]. It also reported that multiplicative noise enhances synchronization between two thermosensitive FHN neurons, but coupling can damage it [44]. The synchronization is widely studied not only in two neuron coupling systems but also in some complex neural networks. For example, Budzinski et al. studied the synchronization of identical thermally sensitive Hodgkin–Huxley neurons in a small-world network, and it found that continue to increase temperature, the synchronization degree becomes worse at first, then it becomes better [45]. A complex H–H neural network with excitatory coupling and inhibitory coupling is established and studied, and numerical simulation results show that a balance between excitative and inhibitory currents should be built to maintain the in-phase synchronization [46]. Wang et al. investigated synchronization in a ring network with time delay, it is found that synchronization can be induced by diffusion phenomena [47]. In addition, an interesting topology named the star-coupled system was proposed by Pecora and Carroll [48–50]. Pecora reported an interesting desynchronization state named drum head mode (DHM), which indicates all nodes are synchronized, but the dynamics of the hub are different with nodes. Usha et al. established a star-coupled system with Hindmarsh–Rose (HR) neuron model, and it reported that the DHM state could be found in the unidirectional and bidirectional coupling [51, 52].

Although there are many excellent works which investigate synchronization modes transition between neurons, few of them focus on the effects of weak coupling and ion conductivity on M-L neurons. There are still many interesting phenomena to be discovered in the star-coupled network. For example, the DHM has never been observed in weak electrical bidirectional coupling so far. Also, the effects of adjusting ion conductivity on synchronization modes transition have seldom been studied up to now.

In this paper, a modified M-L neuron model is introduced to study the synchronization modes transition. Section 2 gives a brief introduction of the modified M-L neuron model and electrical and chemical coupling. We discussed the synchronization modes transition in two coupled neurons first in Sect. 3. Even in the case of weak coupling, two neuron
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2 Model

In order to make the traditional M-L neuron model exhibit bursting behavior, Hu et al. proposed a modified M-L model with a slow variable [9]. This modified M-L model is described as follows:

\[
\begin{align*}
C_m \frac{dV_i}{dt} &= -g_{Ca}M_\infty(V)(V - V_{Ca}) - g_K W(V - V_K) - g_l(V - V_l) + k_1(\alpha + \beta \phi^2) V, \\
\frac{dW_i}{dt} &= \lambda(V)(W_\infty(V) - W), \\
\frac{d\phi}{dt} &= k_2 V + k_3 \phi_{ext},
\end{align*}
\]

with

\[
\begin{align*}
M_\infty(V) &= \frac{1}{2} \left( 1 + \tanh \frac{V - V_a}{V_b} \right), \\
W_\infty(V) &= \frac{1}{2} \left( 1 + \tanh \frac{V - V_c}{V_d} \right), \\
\lambda(V) &= \frac{1}{3} \cosh \frac{V - V_c}{2V_d},
\end{align*}
\]

where \( V \) is membrane potential and \( W \) represents recovery variable, respectively. \( \phi \) represents magnetic flux which is a slow variable in this model. The parameters \( V_{Ca}, V_K, \) and \( V_l \) are reversal potential of calcium, potassium, and leak ion. \( g_{Ca}, g_K, \) and \( g_l \) are maximal conductivity of calcium, potassium, and leak ion, respectively. The term \( k_1(\alpha + \beta \phi^2) V \) presents the feedback current caused by magnetic flux changes, and \( k_1 \) represents the feedback gain. \( k_2 \) and \( k_3 \) define the interplay between magnetic flux and membrane potential. The term \( \phi_{ext} \) describes a constant external magnetic flux. \( M_\infty \) and \( W_\infty \) describe the steady values of gating variables of calcium and potassium ion, \( \lambda_\infty \) is the relaxation time. \( V_a, V_b, V_c, \) and \( V_d \) are system parameters.

In order to investigate the synchronization modes transition, two coupled M-L neurons are described as follows:

\[
\begin{align*}
C_m \frac{dV_i}{dt} &= -g_{Ca}M_\infty(V_i)(V_i - V_{Ca}) - g_K W_i(V_i - V_K) - g_l(V_i - V_l) + k_1(\alpha + \beta \phi_i^2) V_i + g_{(e/c)}(\delta_{(e/c)}(V_i, V_j), \nonumber \\
\frac{dW_i}{dt} &= \lambda(V_i)(W_\infty(V_i) - W_i), \\
\frac{d\phi_i}{dt} &= k_2 V_i + k_3 \phi_{ext},
\end{align*}
\]

where \( i = 1, 2 \) and \( j = 2, 1 \). The term \( g_{(e/c)} \) is the coupling intensity of electrical or chemical coupling. \( \delta(V_i, V_j) \) defines different types of coupling terms. In this work, the electrical coupling is described as:

\[
\delta_e(V_i, V_j) = V_i - V_j. 
\]

The chemical coupling is described as follows:

\[
\delta_c(V_i, V_j) = \frac{V_i - V_s}{1 + \exp[-\lambda(V_j - \theta)]}, 
\]

where \( V_s \) is reversal potential. In this paper, only the excitatory synapse is adopted, which requires the value of \( V_s \) should be larger than the maximal value of the membrane potential. \( \lambda \) and \( \theta \) represent the ratio parameter and threshold of chemical coupling, respectively.

To quantitatively determine the degree of synchronization between neurons, a statistic index named similar function is introduced [42]. It is given by

\[
S = \sqrt{\frac{\langle (V_i(t) - V_j(t))^2 \rangle}{\langle V_i^2(t) \rangle \cdot \langle V_j^2(t) \rangle}}, 
\]

where \( \langle \cdot \rangle \) represents the average value over time. Based on previous work, a relatively smaller \( S \) means better synchronization between two neurons. The value \( S = 0 \) means that the coupled system is completely synchronized.

3 Results and discussions

In this work, the Euler algorithm is adopted and the time step \( h \) is set as 0.001 ms. Here, \( g_{Ca}, g_K, \) and \( g_{(e/c)} \)
are regarded as control parameters. The remaining system parameters are selected as: $C_m = 1 \text{ uF/cm}^2$, $g_I = 0.5 \text{ mS/cm}^2$, $V_{Ca} = 1 \text{ mV}$, $V_I = -0.5 \text{ mV}$, $V_{K} = -0.7 \text{ mV}$, $V_a = -0.01 \text{ mV}$, $V_b = 0.15 \text{ mV}$, $V_c = 0.1 \text{ mV}$, $V_d = 0.05 \text{ mV}$, $\alpha = 0.1 \text{ mS/cm}^2$, $\beta = -0.09 \text{ mS/mWb}^2 \cdot \text{cm}^2$, $\phi_{ext} = 0.2 \text{ mWb}$, $k_1 = 1.0$, $k_2 = 0.15$, $k_3 = 0.15 \text{ s}^{-1}$, $V_s = 0.4 \text{ mV}$, $\lambda = 30$, and $\theta = 0.1 \text{ mV}$.

3.1 The effects of changing ion conductivity on dynamics of a single M-L neuron.

It is generally believed that ion channel block and temperature can profoundly change the ionic conductivity and thus affect the firing activity of neurons [13–15]. Before studying synchronization modes transition between neurons, we first discuss the effects of changing different ion conductivity on a single neuron. In the absence of coupling, the firing activity of a single M-L neuron exhibits variable dynamical behaviors by adjusting $g_{Ca}$ and $g_K$. The inter-spike interval (ISI) and maximum/minimum values of membrane potential ($V_{max}/V_{min}$) are shown in Fig. 1.

In Fig. 1a1, the phenomenon of period-adding can be observed. As the $g_{Ca}$ increases, the firing state is transformed from the quiescent state to spiking state then to bursting state, and the maximum value of membrane potential $V_{max}$ is increased. With the further increase in $g_{Ca}$, the bursting state is transformed to the quiescent state. It can also be found that the firing behavior requires $g_{K_{Ca}}$ to reach a certain value, while the firing is inhibited by an excessive value of $g_{Ca}$. Conversely, the phenomenon of period-reducing is obtained as $g_K$ increases. It shows that the firing state is transformed from quiescent to bursting then to spiking state, and $V_{max}$ is decreased with $g_K$ increasing. Similarly, a relatively smaller $g_K$ cannot induce firing, either. One can easily observe that increasing $g_{Ca}$ has opposite effects on neural dynamic to increasing $g_K$.

3.2 Weak electrical coupling for two M-L neurons.

In order to investigate the synchronization modes transition caused by adjusting $g_{Ca}$ and $g_K$, the coupling intensity $g_e$ is fixed at $g_e = 0.005 \text{ mS/cm}^2$. The time series of membrane potential is shown in Fig. 2. Even if the coupling is very weak, the complete synchronization of two M-L neurons can be observed clearly. In Fig. 2a1–a4, with calcium conductivity $g_{Ca}$ increasing, the successive synchronization modes transition is realized. It is found that synchronization mode is transformed from period-2 synchronization ($g_{Ca} = 1.25 \text{ mS/cm}^2$) to period-3 desynchronization ($g_{Ca} = 1.305 \text{ mS/cm}^2$), then to phase lock desynchronization ($g_{Ca} = 1.33 \text{ mS/cm}^2$), and finally to period-3 synchronization ($g_{Ca} = 1.4 \text{ mS/cm}^2$). On
the contrary, with increases in $g_K$, synchronization mode is changed from period-3 synchronization ($g_K = 1.33 \text{ mS/cm}^2$) to phase lock desynchronization ($g_K = 1.83 \text{ mS/cm}^2$), then to period-3 desynchronization ($g_K = 1.85 \text{ mS/cm}^2$), finally to period-2 synchronization ($g_K = 2.0 \text{ mS/cm}^2$). In terms of synchronization modes transition, it is interesting to notice increasing $g_{Ca}$ also has opposite effects of increasing $g_K$.

Next, we’ll show chaos caused by adjusting $g_{Ca}$ or $g_K$ can greatly influence the synchronization of two M-L neurons. Figure 3 provides the similar function $S$ and the inter-spike interval (ISI) of $V_1$ in a picture. It should be mentioned that the diagrams of ISI of the two neurons are almost the same. In Fig. 3a, b, one can clearly see that synchronization disappeared at about $g_{Ca} = 1.29 \text{ mS/cm}^2$. In addition, from the view of the ISI, the firing state of the first neuron is also transformed from period-2 bursting to chaotic state at $g_{Ca} = 1.29 \text{ mS/cm}^2$. At $g_{Ca} = 1.29 \text{ mS/cm}^2$, the transition of bursting to chaotic state is the reason for the disappearance of synchronization. When $g_{Ca}$ is greater than 1.31 mS/cm$^2$, the chaotic state is transformed to period-2 bursting state. When $g_{Ca} = 1.32 \text{ mS/cm}^2$, even the chaotic state disappears, synchronization does not occur. Further increasing $g_{Ca}$ to approximately 1.344 mS/cm$^2$, a very narrow chaotic region emerges, and the degree of synchronization is
suddenly getting better (Fig. 3b). If $g_{\text{Ca}}$ keeps increasing, the value of $S$ slowly decreases to 0.

The coupling intensity and time delay are usually regarded as parameters to induce synchronization between two coupled neurons [43, 53]. In this work, when two M-L neurons are in period-2 firing state, even the bidirectional coupling is very weak, synaptic current makes them get complete synchronization. However, the chaos destroys complete synchronization, which is mainly manifested as an obvious phase difference after the chaos. This phase difference cannot be eliminated by such a small synaptic current. Another chaotic region decreases the phase difference helps rebuild the synchronization mode.

From Fig. 3b and d, the effects of increasing $g_K$ on firing states and synchronization modes transition are investigated. When $g_K$ is increased to 1.797 mS/cm$^2$, the synchronization has been destroyed at first. After the chaotic region, period-3 bursting can be observed, but synchronization does not show up. As $g_K$ further increases, a relatively wider chaotic region appeared at $g_K = 1.84$ mS/cm$^2$, and the value of $S$ began to reduce. When $g_K$ is over 1.86 mS/cm$^2$, the chaotic state is changed to period-2 bursting state, and $S$ is dropping rapidly to 0. In short, we conclude that chaos is the reason for the establishment and breaking of the synchronization.

The contour plot of similar function $S$ associated with electrical coupling intensity $g_e$ and ionic conductivity ($g_{\text{Ca}}$ or $g_K$) is depicted in Fig. 4. From Fig. 4a, as $g_e$ increases, the period-2 synchronization region becomes smaller, and the critical value of $g_{\text{Ca}}$ of synchronization modes transition decreases as well, which indicates the period-2 synchronization region is shrunk in the parameter plane. A similar conclusion can be drawn in Fig. 4b.

### 3.3 Weak excitatory chemical coupling for two M-L neurons.

In this part, a weak excitatory chemical coupling is used to connect two identical M-L neurons. Here, the initial values of $(V_1(0), W_1(0), \varphi_1(0), V_2(0), W_2(0), \varphi_2(0))$ are $(0.1, 0, -0.1, 0, 0, -0.1)$.

The time series of membrane potential is drawn in Fig. 5. We choose the coupling intensity $g_e = 0.005$ mS/cm$^2$. The initial transients completely vanished after $t = 1000$ ms, and the system behavior seems to be stable. As shown in Fig. 5a1–a4, with increase in calcium conductivity $g_{\text{Ca}}$, the synchronization modes undergo a successive transitions, i.e., period-2 synchronization ($g_{\text{Ca}} = 1.25$ mS/cm$^2$) → desynchronization ($g_{\text{Ca}} = 1.309$ mS/cm$^2$) → period-3 phase lock ($g_{\text{Ca}} = 1.35$ mS/cm$^2$) → period-3 synchronization ($g_{\text{Ca}} = 1.4$ mS/cm$^2$). The process of synchronization modes transition by increasing $g_K$ is completely opposite to increasing $g_{\text{Ca}}$.

The similar function $S$ and inter-spike interval (ISI) of the first neuron are provided in Fig. 6. If $g_{\text{Ca}}$ is less than 1.31 mS/cm$^2$, the two neurons are completely

![Fig. 4](image-url)

**Fig. 4** Dependence of similar function $S$ of two M-L neurons with weak electrical coupling on electrical coupling intensity $g_e$ and ionic conductivity ($g_{\text{Ca}}$ or $g_K$). The fixed parameters are: a $g_K = 2.0$ mS/cm$^2$; b $g_{\text{Ca}} = 1.2$ mS/cm$^2$
synchronized. However, when $g_{Ca} = 1.31$ mS/cm$^2$, the firing state is transformed to chaotic state with simultaneous disappearance of synchronization. Although the chaos region vanishes at $g_{Ca} = 1.312$ mS/cm$^2$, there is no complete synchronization before the next chaotic region arrives. From Fig. 6b, when $g_{Ca}$ is greater than 1.355 mS/cm$^2$, $S$ starts to decrease gradually. At $g_{Ca} = 1.37$ mS/cm$^2$, $S$ decreases to 0, and perfect synchronization is formed. We also studied the influence of adjusting $g_{K}$ on synchronization modes transition (Fig. 6c, d). It found that increasing $g_{K}$ almost has the opposite process as increasing $g_{Ca}$.

The contour plot of similar function $S$ is associated with chemical coupling intensity $g_{c}$ and ionic conductivity ($g_{Ca}$ or $g_{K}$) in Fig. 7. In Fig. 7a, b, it is found that as $g_{c}$ increases, the period-2 synchronization region shrinks, but the period-3 synchronization region expands in the parameter plane. In short, whether it is weak electrical or chemical coupling, adjusting $g_{Ca}$ (or $g_{K}$) exerts similar effects on synchronization modes transformation.
3.4 Weak electrical coupling for star-coupled neural network.

In this section, synchronization modes transition is discussed in the star-coupled system. The dynamics of nodes are described as follows:

\[
\begin{align*}
C_m \frac{dV_i}{dt} &= -g_{Ca}M_\infty(V_i)(V_i - V_{Ca}) - g_K W_i(V_i - V_K) \\
&\quad - g_l(V_i - V_l) + k_1(z + \beta \phi_i^2) V_i \\
&\quad - g_{(e/c)} \delta_{(e/c)}(V_i, V_N), \\
\frac{dW_i}{dt} &= \lambda(V_i)(W_\infty(V_i) - W_i), \\
\frac{d\phi_i}{dt} &= k_2 V_i + k_3 \phi_{ext}, \quad i = 1, 2, \ldots, N-1.
\end{align*}
\]

(6)

The dynamics of the hub is described as follows:

\[
\begin{align*}
C_m \frac{dV_N}{dt} &= -g_{Ca}M_\infty(V_N)(V_N - V_{Ca}) - g_K W_N(V_N - V_K) \\
&\quad - g_l(V_N - V_l) + k_1(z + \beta \phi_N^2) V_N \\
&\quad - g_{(e/c)} \sum_{i=1}^{N-1} \delta_{(e/c)}(V_N, V_i), \\
\frac{dW_N}{dt} &= \lambda(V_N)(W_\infty(V_N) - W_K), \\
\frac{d\phi_N}{dt} &= k_2 V_N + k_3 \phi_{ext}, \quad \text{where } N \text{ is the total number of neurons, } i = 1, 2, \ldots, N-1.
\end{align*}
\]

(7)

First, it is necessary to introduce the structure and characteristics of the star-coupled system. The topological structure is illustrated in Fig. 8. The 1st to N-1th and Nth units are named nodes and hub, respectively. All the nodes are connected with the hub bidirectionally, and there is no interaction between any two nodes. Next, system parameters for all the units are completely the same except the initial values. In the star-coupled neural network, there are three different synchronization modes. The first one is the completely synchronized mode, which means both nodes and hub are synchronized. The second one is a special desynchronized mode named drum head mode (DHM) [43–45], i.e., all the nodes are synchronized, but the hub exhibits different dynamic behavior. The third is the desynchronized mode: not all nodes are synchronized.

Investigating the above three different synchronization modes of the star-coupled system is still meaningful [49, 50]. In order to distinguish the different synchronization modes of this system, two similar functions were introduced. The first one is the average similar function over all star-coupled neurons, which is represented by:
is provided in Fig. 9. The coupling intensity and the number of neurons in star-coupled neurons are fixed: the 1st to N-1th neurons are named nodes, and the Nth neuron is regarded as the hub. The membrane potential of all units at \( t = 5000 \) ms is provided in Fig. 9. The parameter space of different synchronization modes for star-coupled neurons with weak electrical coupling is drawn in Fig. 12. Here, completely synchronized mode, DHM, and desynchronized modes are represented by red, white and black, respectively.

In this section, we’ll show synchronization modes transition of star-coupled neurons is caused by chaos (or bifurcation). Before discussing, it should be mentioned that the bifurcation diagrams of all nodes are almost the same. We provide the bifurcation diagram (ISI) of the 1st neuron and two similar functions (\( S_{\text{all}} \) and \( S_{\text{nodes}} \)) in Fig. 10. From Fig. 10a, c, when \( g_{\text{Ca}} \) is less than 1.3426 mS/cm\(^2\), both \( S_{\text{all}} \) and \( S_{\text{nodes}} \) are very close to 0, which indicates star-coupled exhibits completely synchronized mode. When \( g_{\text{Ca}} \) is slightly greater than 1.3427 mS/cm\(^2\), the firing state is transformed to chaotic state, and completely synchronized mode disappears (Fig. 11a, c). Continue to increase \( g_{\text{Ca}} \), the completely synchronized mode or DHM does not appear until \( g_{\text{Ca}} = 1.3436 \) mS/cm\(^2\). If \( g_{\text{Ca}} \) is slightly greater than 1.3437 mS/cm\(^2\), the firing state is transformed to period-3 bursting, and DHM is observed.

For increasing \( g_{K} \) (Fig. 10b and d), the star-coupled system exhibits the DHM when \( g_{K} \) is less than 1.7966 mS/cm\(^2\). When \( g_{K} \) is slightly greater than 1.7967 mS/cm\(^2\), period-3 is turned to the period-4 bursting, and the DHM is transformed to desynchronized mode. As \( g_{K} \) increases, there exist multiple DHM regions, which indicates that synchronization modes transition of the star-coupled system is more sensitive to changes in \( g_{K} \) than \( g_{\text{Ca}} \). If \( g_{\text{Ca}} \) is slightly greater than 1.7982 mS/cm\(^2\), period-3 is changed to period-2 bursting, and the desynchronized mode is transformed to completely synchronized mode (Fig. 11).
increasing $N$ exerts little effect on the DHM region. A similar conclusion can be drawn from Fig. 12b1–b3.

3.5 Weak excitable chemical coupling for star-coupled neural network

In this section, the star-coupled M-L system is connected by weak excitable chemical coupling. The initial values of $(V_i(0), W_i(0), \varphi_i(0))$ are still set as $(0.01 \times i, 0, 0)$. The collective dynamics of all units at $t = 2000$ ms is plotted in Fig. 13. We fixed coupling intensity $g_e = 0.0005$ mS/cm$^2$ and total number $N = 20$. From the left column of Fig. 13, the star-coupled system undergoes transition between with $g_{Ca}$ increasing, i.e., the DHM ($g_{Ca} = 1.342$ mS/cm$^2$) is changed to desynchronized mode ($g_{Ca} = 1.343$ mS/cm$^2$) then to the DHM ($g_{Ca} = 1.344$ mS/cm$^2$). The right column shows the same process (Fig. 13b1, b2 and b3).

The inter-spike interval (ISI) of the 1st neuron and two kinds of similar function ($S_{all}$ and $S_{nodes}$) are drawn in Fig. 14. When $g_{Ca}$ is less than 1.3428 mS/cm$^2$, the system exhibits the DHM. As $g_{Ca}$ increases, the DHM disappears, and the firing state of 1st neuron is transformed to chaotic state. The DHM reappears at $g_{Ca} = 1.3434$ mS/cm$^2$, where chaotic state is changed to period-3 bursting. The analogous phenomenon can be found in Fig. 14b and d.
The parameter space of different synchronization modes with excitatory chemical coupling is drawn in Fig. 15. One can clearly observe that if $g_{Ca}$ (or $g_K$) is fixed, a higher value of $g_c$ makes the DHM region larger. However, with the number of nodes increasing, the DHM region is reduced significantly. The absence of the completely synchronized mode in an excitatory chemical star-coupled network could be a consequence of the nonlinear feedback form of the coupling.

4 Conclusion

It is widely accepted that the conductivity of ion channels is deeply affected by factors, such as
temperature changes [11, 14]. It seems reasonable that the ionic conductivity is set as the control parameters to study the synchronization mode transition. In this work, synchronization modes transition with weak electrical and excitatory chemical coupling is investigated.

The main conclusion of the two coupled neurons is as follows: If the electrical coupling intensity $g_e$ is fixed, increasing $g_{Ca}$ makes the system undergo a successive transition: from period-2 synchronization to desynchronization, and finally to period-3 synchronization. The phenomenon of synchronization mode transition is caused by chaos. The desynchronized region is mainly sandwiched between two chaotic regions. When two neurons are first in period-2 bursting state, even if the bidirectional coupling is very weak, synaptic current makes them get complete synchronization. As $g_{Ca}$ continues to increase, the firing state is transformed to chaotic state. Meanwhile, chaos destroys complete synchronization, which is manifested as a phase difference between the two neurons after the chaotic state. The next chaotic region decreases the phase difference and rebuilds synchronization modes. An opposite phenomenon of increasing $g_{Ca}$ can be observed by increasing $g_K$. In addition, a relatively stronger electrical coupling makes the period-2 synchronization region smaller but the period-3 synchronization region larger in the
For weak excitatory chemical coupling, the phenomenon and conclusions are very similar to those of electrical coupling.

For a star-coupled neural network with electrical coupling, it was found that: Continue to increase $g_{Ca}$, the completely synchronized mode is changed to desynchronized mode, finally to the DHM. It is the first time that the DHM has been observed in a star-coupled neural network with electrical coupling. These phenomena of synchronization modes transition are created by chaos (or bifurcation). When nodes show chaotic state, the synapse current between the star-coupled network is not powerful to maintain the completely synchronized mode, so that it exhibits desynchronized mode. An opposite modes transition of increasing $g_{Ca}$ can be observed by increasing $g_{K}$. A relatively stronger electrical coupling expands the region of completely synchronized mode and DHM, while the completely synchronized mode region shrinks as the number of nodes increases.

For a star-coupled neural network with weak excitatory chemical synapse, it was found that: by increasing $g_{Ca}$, the DHM is changed to desynchronized mode, and finally to DHM. In this case, we failed to observe the completely synchronized mode. In addition, a relatively greater value of $g_{Ca}$ expands the parameter region of the DHM, but the DHM region shrinks as the number of nodes increases.

We have investigated the synchronization mode transition with electrical and excitatory chemical coupling. To study the effects of inhibitory synapses on the above-mentioned topology is still meaningful. Also, some improved neural model should also be introduced to further investigate star-coupled neural network [54–56].

**Acknowledgements** This project is supported by the National Natural Science Foundation of China under Grants No. 12175080.

**Funding** This study was funded by the National Natural Science Foundation of China under Grant No. 12175080.

**Data availability** All data generated or analyzed during this study are included in this published article.

**Declarations**

**Conflict of interest** The authors declare that they have no potential conflict of interest.
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