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Structural integrity of tied arch bridges affected by instability phenomena

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Abstract

A numerical study is developed to investigate out-of-plane buckling instability of tied arch bridges due to vertical loads. In particular, a numerical model is implemented to evaluate initial configuration under dead loads and nonlinearities due to nonlinear geometric effects in bridge components arising from instability phenomena. The main aim of the paper is to identify the most relevant bridge components, which are affected out-of-plane instability behavior of the structure. The nonlinear behavior of tied arch bridges is evaluated by means of a three-dimensional finite element model, in which several wind bracing system layouts and cable system configurations are considered. A comparative analysis between Elastic Buckling Analysis and Nonlinear Elastic Analysis methodologies is developed to achieve a more accurate evaluation of the maximum capacity of the structure against instability phenomena. Comparisons in terms of buckling assessment between numerical evaluations and simplified methodologies reported in current codes on bridge structures are proposed. Results show that the simplified methodologies overestimate the instability capacity in most of the bridge configurations, which have been dimensioned according to the preliminary design rules commonly adopted in current applications.

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## Nomenclature

| Symbol | Description |
|--------|-------------|
| $L$    | Bridge span length |
| $L^a$  | Arch rib length |
| $L^b$  | further nomenclature continues down the page inside the text box |
| $W_d$  | Bridge width |
| $f$    | Arch rise |
| $h$    | Height of the end portal |
| $\alpha$ | Cable slope |
| $DL$   | Dead Load |
| $LL$   | Live Load |
| $H^a$  | Height of the arch rib cross-section |
| $B^a$  | Width of the arch rib cross-section |
| $t^w_a$ | Web thickness of the arch rib cross-section |
| $t^f_a$ | Flange thickness of the arch rib cross-section |
| $H^f$  | Height of the tie girder cross-section |
| $B^f$  | Width of the tie girder cross-section |
| $t^w_f$ | Web thickness of the tie girder cross-section |
| $t^f_f$ | Flange thickness of the tie girder cross-section |
| $D^r$  | External diameter of the arch cross beam |
| $t^r$  | Pipe thickness of the arch cross beam |
| $m$    | Number of cables |
| $m^c$  | Number of Arch cross-beam |
| $p$    | Spacing step of the cables |
| $p^{cr}$ | Step of the arch cross-beam along the arch rib |
| $A^c$  | Cable cross-section |
| $S^c$  | Cable initial stress |

### 1. Introduction

Tied arch bridges represent an effective solution to overcome short, medium and long spans since combine structural, economic and aesthetic advantages (Latif and Saka (2019), Tan and Yao (2019)). Tied arch bridges are widely used in railway applications since they contribute to reducing vibrations of both structure and transient vehicles (Abd Elrehim et al. (2019), Greco et al. (2018)).

Tied arch bridges consist of two arch ribs, which sustain a bottom deck by means of a cable system. The deck is composed of two longitudinal tie girders and several transversal beams, which support a concrete slab. Moreover, the tie girders are rigidly connected to the arch rib extremities thus sustaining the arch thrust (Lonetti et al. (2016)). The cable system configuration identifies the typology of tied arch bridge, so that it is possible to distinguish essentially (i) moment tied and (ii) network configurations. The moment tied configuration consists of several vertical hangers equally spaced along the tie girders, whereas the network system is composed by the union of two specular planes of inclined hangers forming a net configuration. The arch ribs are subjected to relevant compression forces, which may lead the whole structure to be affected by in-plane or out-of-plane buckling mechanisms. The performances against out-of-plane buckling mechanisms are usually improved by using a wind bracing system. The wind bracing system can be arranged in several geometric configurations, whose the most commons are the Vierendeel, X-shaped, and K-shaped layouts.

The assessment of the tied arch bridges against buckling mechanisms represents a fundamental issue that designers have to address (Lonetti and Maletta (2018)). However, exhaustive guidelines to check buckling capacity are currently limited. As a matter of fact, current design codes on arch bridges provide simplified procedures to evaluate buckling capacity of the structure. These approaches may involve erroneous predictions of the buckling capacity of the structure thus leading to unsafe structural configurations (Lonetti et al. (2019)). Advanced investigations on the buckling...
behavior of tied arch bridges are reported in limited research works reported in literature. Palkowski (Palkowski (2012)) analyzed the in-plane buckling behavior of moment tied arch bridges and proposed comparative results in terms of the buckling length factor between numerical analysis and values reported in Figure D.4 of EC3 (European Committee for Standardisation (2006)). In this framework, buckling length factor provided by EC3 recommendations in many cases may lead to unsafe conditions. Romeijn and Bouras (Romeijn and Bouras (2008)) analyzed how the presence of damage mechanisms of the cable elements can affect the in-plane nonlinear behavior of tied arch bridges. Ju (Ju (2003)) developed a statistical study with the aim to define analytical formulas to evaluate the buckling length factors of arch bridges, considering upper and lower deck configurations. More recently, De Backer et al. (De Backer et al. (2014)) investigated out-of-plane nonlinear behavior of steel tied-arch bridges by means of advanced numerical analyses with the aim to assess the Eurocode prescriptions. They denoted that evaluations obtained by means of advanced finite element analysis are less conservative than conventional results determined by using EC3 procedures. They also developed a practical formulas for the evaluation of the buckling length factor.

Note that, previous studies mainly focused on moment tied arch configurations, based on a series of vertical hangers. Contrarily, network arch bridges have received less attention and, to the Author’s knowledge, no detailed works on buckling behavior are reported in the literature. Moreover, simplified methods provided by current codes of arch bridges do not consider network configurations. Consequently, investigations on the out-of-plane buckling behavior of network arch bridges are much required.

The main aim of the present paper is to investigate the nonlinear behavior of tied arch bridges considering both moment tied and network configurations and to identify structural parameters affecting the out-of-plane buckling behavior. Moreover, comparisons between numerical analyses and simplified methodologies prescribed by codes are developed. Finally, the applicability of the simplified methodologies for out-of-plane buckling in the case of network arch bridges is discussed.

The outline of the paper is as follows: In section 2, a review of the simplified method proposed by EC3 to evaluate the critical out-of-plane buckling force of tied arch bridges is presented. The numerical implementation is reported in Section 3 and Section 4 numerical results are discussed.

### 2. A review of EC3 prescriptions for the evaluation of the out-of-plane critical buckling force

EC3 provides prescriptions on the buckling design of tied arch bridges in Annex D.3 (European Committee for Standardisation (2006)), by means of simplified approaches to assess the buckling performances of arch ribs. However, EC3 prescriptions consider exclusively moment tied arch bridges and no guidelines are provided for network configurations. Moreover, simplified approaches cover limited cases of moment tied configurations. This occurs especially for the out-of-plane buckling assessment of tied arch bridges with wind bracing system, in which exclusively the K-shaped configuration is considered. In this framework, EC3 prescribes to perform buckling assessment by checking the stability of the bridge end portals, which correspond to the portion of arch ribs without bracing elements (Fig. 1).

The bridge end portals present a depth coinciding with that of the deck ($W_d$) and an equivalent height ($h_e$), which is defined by the following expression:

$$h_e = \frac{1}{\sin \alpha_k} \sum_{i}^m h_{i,i}$$

(1)

where $m$ is the total number of hangers, $h_{i,i}$ is the height of the $i$-th hanger and $\alpha_k$ is the angle formed by the arch and the horizontal line.

The critical buckling force of the bridge end portals is defined as follows:

$$N_{cr} = \frac{\pi^2 EI}{(\beta h)^2}$$

(2)
where $I$ is the out-of-plane moment of inertia of the arch rib cross-section and $h$ is the height of the end portals. The buckling length factor ($\beta$) is evaluated by using the graphs reported in Table D1 of EC3 (European Committee for Standardisation (2006)). These graphs express the buckling length factor as a function of two additional variables, namely $\eta$ and $\chi$, which are expressed by means of the following expressions:

$$\eta = \frac{EIW}{E_0 I_0 h}$$  \hspace{1cm} (3)

$$\chi = \frac{h}{h_c}$$  \hspace{1cm} (4)

where $E_0 I_0$ and $EI$ are the bending stiffness of the transversal beam and columns of the end portal, respectively.

3. Numerical implementation

3.1. Structural scheme of the bridge

Fig. 2 depicts a generalized tied arch bridge, in which arch ribs and tied beams are fixed at their extremities and mutually connected along their lengths by means of the cable system. The cable system characterizes the structure and two geometric configurations are typically adopted: (i) the moment tied configuration, which consists of a series of vertical hangers equally spaced along the tie girder and (ii) the network configuration, which is formed by the combination of two specular plains of inclined hangers of constant slope $\alpha$ on the horizontal line. Usually, hollow rectangular cross-sections are adopted for the arch, the girder, and the deck transversal beams, whereas pipes are used for arch transverse beams. Arch ribs are braced against out-of-plane displacements by means of a wind bracing system, which can be arranged in Vierendeel, X-shaped, and K-shaped layouts. Finally, the bridge presents external boundary conditions based on in-plane hinged or simply restraints at right and left ends respectively, whereas along out-of-plane direction fixed conditions are considered.

The structure is usually dimensioned by means of preliminary design rules commonly adopted in the context of bridge design. The design rules size all the structural components of the bridge reasonably from an engineering point of view. The set of preliminary design rules, together with allowable ranges of values, is reported in Table 1. Note that, the range of values have been defined according to the analysis of several existing tied arch bridges built in the last 70 years (Hedgren (1994)).
where $I$ is the out-of-plane moment of inertia of the arch rib cross-section and $h$ is the height of the end portals. The buckling length factor ($\beta$) is evaluated by using the graphs reported in Table D1 of EC3 (European Committee for Standardisation (2006)). These graphs express the buckling length factor as a function of two additional variables, namely $\eta$ and $\chi$, which are expressed by means of the following expressions:

$$d_{OO} = \frac{EI W}{EI h}$$

$$r = \frac{h}{h}$$

where $E_{OO}$ and $E_I$ are the bending stiffness of the transversal beam and columns of the end portal, respectively.

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Fig. 1. Buckling of portals for arches: Schematic of the Figure D.5 reported in EC3 (European Committee for Standardisation (2006)) and end portals boundary conditions

Table 1. Main design variables and feasibility sets (Hedgren (1994))

| Design Variables                                      | Minimum | Maximum | Mean  |
|-------------------------------------------------------|---------|---------|-------|
| Bridge length $L$ [m]                                  | 75      | 250     | 162   |
| Bridge width $W$ [m]                                   | 10      | 20      | 15    |
| Rise to span ratio $f / L$                             | 0.16    | 0.20    | 0.18  |
| Height of the arch rib cross-section to span length ratio $H^e / L$ | 1/190   | 1/140   | 5/806 |
| Width of the arch rib cross-section to span length ratio $B^e / L$ | 1/190   | 1/140   | 5/806 |
| Height of the tie girder cross-section to span length ratio $H^l / L$ | 1/70    | 1/50    | 3/175 |
| Cables slope $\alpha$ [$^\circ$]                       | 40      | 90      | 65    |
| Step of the cables along the tie girder $p^c$ [m]      | 5       | 10      | 7.5   |
| Step of the arch cross beam to bridge width ratio $p^\omega / W_o$ | 1/4     | 3/4     | 1/2   |
| Height of the end portal to arch rib length ratio $h/L^e$ | 0.024   | 0.271   | 0.147 |
| Dead Load $DL$ [N/m$^2$]                              | 6600    | 20000   | 13350 |
| Cable cross-section $A^c$ [cm$^2$]                     | 23.079  | 53.851  | 38.465|

3.2. Numerical model and methods of analysis

The structure is analyzed by means of an advanced three-dimensional FE numerical model, in which arch ribs, tie girders and transversal beams are modelled as Timoshenko beams elements, whereas hangers are schematized as truss elements. In particular, each hanger is subdivided into a series of truss elements according to the Multi Element Cable System (MECS) approach, which permits to reproduce cable sag effect properly. The nonlinear behavior of the tied arch bridges is usually investigated by means of two methods, which are eigenvalue buckling analysis (EBA) and nonlinear elastic analysis (NEA). EBA analysis evaluates the critical mode shapes of the structure and corresponding critical load multipliers, but it does not consider any nonlinear effect arising from cable elements. Contrarily, NEA predicts the nonlinear behavior of the structure more accurately since it takes into account any kind of nonlinear effect, but initial displacements have to be considered to reproduce elastic collapse mechanisms. Moreover, for complex...
structures, NEA may require considerable computational honors. On the base of the previous observations, in the present study a combined approach, which combines EBA and NEA have been adopted to investigate the nonlinear behavior of tied arch bridges. The approach can be summarized in the following steps:

1. Definition of the structure.
2. EBA is performed to identify the critical mode shapes of the structure.
   2.1. Identification of the initial configuration of the structure under the action of Dead Loads (DL);
   2.2. Solution of the eigenvalue buckling problem.
3. NEA is performed by means of the following consecutive steps:
   3.1. Identification of the initial configuration of the structure under the action of DL;
   3.2. Initial out-of-plane imperfections are imposed to the structure, according to the results of point 2.
   3.3. Increasing live loads (LL) are imposed to the structure and the maximum loading capacity is evaluated.

An important task to achieve in both analysis methods consists in the definition of the initial configuration of the bridge structure under the action of dead loads (DL) (points 2.1 and 3.1), which involves the evaluation of stress distribution in hangers and other structural components. This task is considerably important in the present study since the nonlinear behavior of the structure is quite affected by stress and strain distribution. The procedure to define the initial configuration of the structure is based on the “zero displacement method” approach, which is typically adopted in the framework of long-span cable-supported bridges (Greco et al. (2013), Lonetti and Pascuzzo (2014a), Lonetti and Pascuzzo (2014b), Lonetti and Pascuzzo (2014c)). The method identifies the initial stress distribution of hangers, arch and girder to minimize the deformations of the structure under the action of dead and permanent loads. For sake of brevity, this procedure is not discussed here, but details regarding the numerical implementation can be found in (Bruno et al. (2016), Lonetti and Pascuzzo (2016)).

Once that the initial configuration of the structure is defined, for NEA (see point 3.2), initial imperfections are imposed to the structure with the aim to take into account out-of-plane buckling mechanisms. In particular, the structure is deformed consistently with the first critical buckling mode shape with a maximum magnitude of L/8000. Note that, the maximum magnitude of L/8000 is considerably smaller that L/300, which is the maximum value that EC3 prescribes to reproduce the effect of geometric imperfections. This is because L/300 may affect considerably the nonlinear behavior of the structure thus leading to highly conservative predictions of the maximum buckling capacity of the structure. EBA and NEA analyses evaluate the live load multiplier ($\lambda$), which leads to buckling instability of the structure. In particular, NEA performs an incremental step-by-step analysis, in which the equilibrium equations of the structure are solved by imposing at the generic loading step the following equations:

$$\begin{bmatrix} K_L + \lambda K_{NL} \end{bmatrix} \begin{bmatrix} u + \Delta u \end{bmatrix} = g_0 + \lambda q$$

(5)

where $K_L$ is stiffness matrix, $K_{NL}$ is the stress stiffness matrix, $g_0$ and $q$ are the dead and live load vectors, $u$ and $\Delta u$ are the displacement vector and its incremental quantity, respectively. Note that, the stress stiffness matrix $K_{NL}$ derives from the contribution of nonlinear terms relates to truss and beams elements. In particular, the contribution related to the beams has been implemented by incorporating in the linear variation equations of Timoshenko beam formulation the following weak contribution:

$$- \int L N^C_{i} u'_i \delta u_i' + \int L N^C_{i} u'_i \delta u_i'$$

(6)

where $N^C_{i}$ is the axial force and $u_i$ with $i=2,3$ represent the translational displacements along transverse axes of the element. On the other hand, EBA evaluates the live load multiplier of the structure by solving the following eigenvalue problem associated to governing Eq.(5):

$$\det\left[K_L + \lambda K_{NL}\right] = 0$$

(7)

Note that, the maximum magnitude of L/8000 is considerably smaller than L/300, which is the maximum value that EC3 prescribes to reproduce the effect of geometric imperfections. This is because L/300 may affect considerably the nonlinear behavior of the structure thus leading to highly conservative predictions of the maximum buckling capacity of the structure. The investigation is performed arising from cable system elements. Furthermore, comparisons are proposed between numerical analyses and simplified methods reported in EC3 to assess the buckling capacity of the structure.
In particular, the buckling load is the smallest eigenvalue obtained by solving Eq.(7), and the collapse mode shape is the corresponding eigenvector.

4. Results

The proposed study aims to investigate the out-of-plane nonlinear behavior of tied arch bridges identifying the main structural parameters, which influence the critical buckling force of the structure. The investigation is performed by means of an advanced FE numerical model based on three-dimensional scheme, in which tie girders and arch ribs are schematized by means of beam elements based on Timoshenko nonlinear formulation, whereas the cable are modelled by means of multi-truss elements. In particular, the nonlinear behavior of the structure has been investigated by means of a combined approach based on Eigenvalue Buckling analysis (EBA) and Nonlinear Elastic Analysis (NEA). EBA analysis allows identifying the shape of the critical buckling mode of the structure, whereas NEA provides an accurate evaluation of the buckling load of the structure since it accounts for any nonlinear contributions arising from cable system elements. Furthermore, comparisons are proposed between numerical analyses and simplified methods reported in EC3 to assess the buckling capacity of the structure.

At first, results are proposed to investigate the influence of cable system configuration and wind bracing system layout on the out-of-plane buckling behavior of tied arch bridges. Fig. 3 shows the prediction of the maximum live load multiplier ($\lambda$) for a moment tied arch bridge (Fig. 3.a) and a network system (Fig. 3.b) with $\alpha=65^\circ$. The buckling behavior has been investigated with reference to three bracing system layouts, i.e. Vierendeel, X-shaped, and K. The results of NEA are presented in terms of load multiplier ($\lambda$) versus dimensionless out-of-plane displacement ($\delta/L$), whereas the maximum live load multiplier is reported for EBA. Both the bridges present a span length of $L=150$ m and have been dimensioned according to the mean values of preliminary design rules reported in the third column of Table 1. Moreover, Fig. 3 reports the first critical buckling mode shapes related to wind bracing system layouts investigated.

Fig. 3. Comparisons between moment tied (a) and network (b) configurations in terms of analysis method, i.e. Elastic Buckling Analysis (EBA) and Nonlinear Elastic Analysis (NEA), and wind bracing system.
The results show that EBA overestimates the critical buckling load of the structure in all examined cases, thus denoting how nonlinearities of the structure affect the out-of-plane buckling behavior. Consequently, a proper evaluation of the buckling capacity of the structure can be achieved only by using NEA analysis. The results also show that K or X shaped wind bracing schemes provide the best performances against out-of-plane buckling mechanisms, whereas the Vierendeel scheme presents the lower capacity. This behavior can be explained in view of the considerable stiffness provided by K-shaped and X-shaped layouts due to their geometrical configurations. This aspect allows designing the arch cross-beams and diagonals by slender elements thus saving a relevant amount of material. Finally, the results show that the cable system configuration does not affect the maximum capacity of the structure against out-of-plane buckling mechanisms. However, the cable system configuration mainly affects the shape of the critical buckling mechanism when X or K-shaped bracing system layouts are employed. As a matter of fact, the critical buckling mode of moment tied and network configurations is antisymmetric and symmetric, respectively. Previous results have denoted how the wind bracing system affects the buckling behavior of the bridge. However, other components may increase nonlinearities involved in the structure. In order to identify the structural components that mostly affect the out-of-plane nonlinear behavior, a Variable Screening Analysis (VSA) has been performed with reference to the network arch bridge with K-shaped bracing. In particular, VSA proposed by (Rocha et al. (2012)) has been adopted to identify the design variables in Table 1, which affect the critical buckling force \( N_{cr} \) of the arch ribs. Fig. 4 shows that the most important design variables that affect the critical buckling force of the structure are the width of the arch cross-beam \( B^x \) and the height of the end portal \( h \) since they present an importance indicator higher than 10%. The value of 10% has been selected as the threshold limit to consider a design variable significant for the out-of-plane nonlinear behavior. For completeness, further specific results have been developed to better clarify the role played by \( B^x \) and \( h \).

Fig. 5 plots the variability of the critical buckling force \( N_{cr} \) in terms of the out-of-plane moment of inertia of the arch rib cross-section \( I_z^x \). The results shows that out-of-plane buckling mechanisms dominate the collapse configuration, mostly with symmetric mode shapes. Only for larger values of \( I_z^x \), antisymmetric mechanisms are observed. The results have also denoted that \( N_{cr} \) varies almost linearly with \( I_z^x \). This linear trend has been assessed by means of a linear regression analysis and a Pearson’s coefficient equal to 0.99 has been determined.
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Fig. 4 shows that the most important design variables that affect the critical buckling force of the structure are the width of the arch cross-beam (R_B) and the height of the end portal (h) since they present an importance indicator higher than 10%. The value of 10% has been selected as the threshold limit to consider a design variable significant for the out-of-plane nonlinear behavior. For completeness, further specific results have been developed to better clarify the role played by R_B and h.

Fig. 5 plots the variability of the critical buckling force (N_{cr}) in terms of the out-of-plane moment of inertia of the arch rib cross-section (R_z I). The results shows that out-of-plane buckling mechanisms dominate the collapse configuration, mostly with symmetric mode shapes. Only for larger values of R_z I, antisymmetric mechanisms are observed. The results have also denoted that N_{cr} varies almost linearly with R_z I. This linear trend has been assessed by means of a linear regression analysis and a Pearson's coefficient equal to 0.99 has been determined.

Fig. 6. Variability of the buckling length factor (\(\beta\)) as a function of the dimensionless height of the bridge end portal (h/L^R) for bridge span lengths (L) of 100, 150 and 200 m.
Fig. 6 shows the variability of the buckling length factor ($\beta$) of end portals of the bridge as a function of the dimensionless height of the bridge end portals ($h/L^r$) for several values of the bridge span in the range between small, medium and large lengths. The results denote that $\beta$ varies with $h/L^r$ in a nonlinear manner. In particular, $\beta$ increases as $h/L^r$ decreases thus revealing how reduced height of the bridge end portals contribute to improve the buckling capacity of the structure against out-of-plane mechanisms. Note that, the bridge span length ($L$) relatively affects the evolution of the buckling length factor $\beta$, since all curves in Fig. 6 are quite closed and evolves similarly.

Finally, a parametric study has been performed to assess the goodness of the simplified approach reported in EC3, which is summarized in section 2. In particular, comparisons are performed between numerical simulations and analytical predictions, which have achieved by means of Eq.s (1)-(4). The comparative study is performed considering more than 30 network arch bridges with a K-shaped wind bracing system, which have been dimensioned randomly according to feasible sets reported in Table 1. Note that, two different approaches have been employed to evaluate the parameter $h_r$ defined in Eq. (1): in case (A), the mean value of the vertical projection of the hangers is assumed, whereas in case (B) the total length of the hangers is considered. The results are presented in Fig. 7. in terms of TT-plot, which relates the evaluations of the critical buckling forces ($N_{cr}$) obtained by FEM analysis and EC3 analytic evaluations. The results show that EC3 simplified approach may involve erroneous predictions of the critical buckling force for both descriptions employed to quantify $h_r$. Large underestimations are observed for most of the investigated cases since most of the points are arranged under the bisector line, which denotes the best agreement between numerical and analytical predictions. With the aims to quantify the differences between the numerical and analytical evaluations, the percentage error $e(\%)$ between FEM and analytical predictions is investigated in terms of the parameter $\eta$ defined in Eq. (3), which is represents the ratio between the bending stiffness of columns and transversal beams of the end portals of the bridge (Fig. 1).

![Fig. 7. Comparisons between numerical results and analytic evaluation in terms of critical buckling force: (a) TT plot, (b) percentage error.](image-url)
In particular, the percentage error has been defined by means of the following expression:

$$e(\%) = \frac{N_{cr}^{FEM} - N_{cr}^{X}}{N_{cr}^{FEM}} \times 100$$

(8)

where X = [EC3(a) or EC3(b)]. The results denote that, e(%) increases with increments of $\eta$, i.e. when the bending stiffness of the arch ribs became considerable larger than that of the arch cross-beams. This probably occurs because the definition of the buckling length factor ($\beta$) from the graphs in Table D.1 of EC3 can not be performed accurately when $\eta$ is larger than 4. As a matter of fact, the graphs of Table D.1, which express the buckling length factor in terms of $h/h_r$ and $\eta$, presents several curves for values of $\eta$ between 0.25 and 4, whereas exclusively two curves are reported for $4 < \eta < \infty$. This range characterizes most of Network Arch Bridges (NAB) with K-shape bracing system since (i) the cross-section of the arch ribs is stiffener than the ones of arch-cross beams and diagonals and (ii) the bridge end portals usually consist of short columns and long transversal beams. Consequently, simplified approach prescribed by EC3 may lead to erroneous predictions when highly stiff wind bracing systems, such as K-shape and X-shape, are employed.

5. Conclusions

An accurate evaluation of the instability behavior of tied arch bridges requires a proper definition of initial stress configuration in the structure as well as a refined depiction of geometric nonlinearities arising from cable system elements. In fact, overestimations of the maximum capacity of the bridge against out-of-plane instability were obtained by using traditional Elastic Buckling Analysis, in which initial stress distribution and cable nonlinearities are not properly taken into account. The out-of-plane instability behavior of tied arch bridges is usually described in terms of critical buckling load, which denotes the maximum capacity of the structure against out-of-plane instability phenomena, and the corresponding shape of the instability mechanism. The amplitude of critical buckling load is largely influenced by geometric and mechanical properties of the wind bracing system, which is commonly arranged by using Vierendeel, X-shaped, and K-shaped configurations. In this framework, X-shaped and K-shaped configurations represent the best solutions to improve the out-of-plane instability performances since they provide a high stiff connection between arch ribs. The shape of out-of-plane instability mechanisms, which can be symmetric or antisymmetric, is affected mainly by the configuration of the cable system, which is commonly arranged by using vertical or inclined hangers according to moment tied or network configurations, respectively. The moment tied and network configurations involve antisymmetric and symmetric shapes of instability mechanisms, respectively. Detailed investigations were performed with reference to Network arch bridges with K-shaped bracing system with the aim to identify the design variables of the bridge structure that affect the out-of-plane nonlinear behavior. It was found that the height of the end portals of the bridge, i.e. the region of the arch ribs without bracing elements, and the out-of-plane moment of inertia of the arch ribs were the most relevant design variables that affect the buckling capacity of the structure. Finally, comparisons between numerical evaluations and simplified approaches reported in current codes of bridge structures for evaluating the critical buckling load were performed. The simplified approaches involves notable underestimations thus resulting too conservative and inappropriate to be employed for the buckling assessment of network arch bridges. Future work will involve the development of a design method to properly estimate the maximum out-of-plane capacity of network arch bridges.

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