Observation of Gaussian quantum correlations existence of photons under linear beam splitter

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Abstract

In this paper, we investigate the creation of quantum correlations such as entanglement, the Gaussian quantum discord and quantum steering in a quantum system under consideration via the linear beam splitter with squeezed thermal and a single-mode Gaussian states at the inputs. This quantum system generates bright entangled output two-mode light. Specifically, the quantum entanglement can be enhanced through increasing the purity of the Gaussian state and the squeezing parameter at the input states. The quantum correlations reveal similar characteristics for each case. That means when we increase the non-classicality, purity and squeezing parameter, the quantum correlations show enhanced profile. On the other hand, upon increasing the mean thermal photon number at the input, the quantum correlations indicate inverse relation.

1. Introduction

In quantum theory of light, the nonclassical behavior of light generated from different optical systems have been duly explored [1–9]. During the last decades, the quantification of quantum correlations has been investigated using the nonclassical behaviors of light in both theoretically and experimentally [10–17]. Therefore, the characterization of quantum correlations, such as Bell non-locality, quantum entanglement, quantum steering, quantum coherence and quantum discord attract researcher’s attention in the area of quantum optics [18–26]. On the other hand, quantum correlations, specifically, quantum entanglement is a crucial ingredient of quantum information processing that characterize the nonclassical properties between subsystems in a global system. A quantum entanglement also plays a central role to characterize the nonclassical properties between two inseparable states in a two-party system, however, it can not be applicable to explain the nonclassicality of a single-party system. Furthermore, for a two-party system, quantum steering is another quantum correlation measurement parameter to detect the nonclassicality of a bipartite system. Quantum steering has many roles ranging from sub-channel discrimination to one side device independent protocols [27–29]. Furthermore, quantum steering has foundational significance in secure teleportation [27, 28, 30, 31]. For two parties marked as party A and party B, quantum steering explains the quantum communication of party A and party B, after local measurement on one party. Unlike, quantum entanglement, it is described that quantum steering is asymmetric behavior which is based on the relative measurement taken to parties A and B. In addition, quantum steering can be implemented to verify fidelity of teleportation [27] that exceeds 2/3 randomness certification [28, 30, 31].

On the other hand, the quantum coherence and quadrature squeezing are applicable to the nonclassicality for both single-party and multi-party quantum systems. To be specific, quantum coherence plays a key role in high efficiency achievement for a certain quantum system [10]. The quantum coherence can be explained by the off-diagonal elements of the density matrix of a system. It can be also measured in relation with the entropy of a system as the entropy difference between the density matrix of a system and the off-diagonal elements of the orthogonal matrix. It is well known fact that quantum system can be classified as discrete and continuous-variable (CV) Gaussian systems. Due to the essence of its accessibility and controllability nature at experiment, continuous variable quantum states are best candidate to study quantum correlations intensively [32], however, quantification of quantum correlations of continuous variable are more complicated [21, 33].
In this paper, the Gaussian quantum correlations generated by a linear optical beam splitter with Gaussian state and squeezed thermal input states have been analyzed in detail. In particular, we consider a single-mode Gaussian input state incident to one port of the beam splitter on one hand and a single-mode squeezed thermal state incident to the beam splitter through the second port of the beam splitter on the other hand. In fact, an optical beam splitter is a modest essential component in classical optics \cite{34}, quantum optics \cite{35}, and quantum information processing \cite{36}. In this regard, the symmetrical beam splitter is assumed to be used in most existing investigations to reduce the complexity of problems \cite{37-39}, however, asymmetrical optical beam splitter is more applicable since it is difficult to have symmetrical beam splitter for practical purpose.

With such pillar concepts, we preferred asymmetrical beam splitter to reduce the loss of generality. Studying of such optical systems have been pave the way to explore the quantum correlations between two spatially distant systems. To be specific, in this work, we study the nonclassical correlations between different modes produced in spatial separated systems subjected to be propagated through a beam splitter. The complete structure of the system under investigation is provided in figure 1. As shown in the figure, the two independent light beams at an asymmetrical beam splitter interfere with each other. Therefore, $S_a$ represents non-classical light source with single-mode Gaussian state and $S_b$ represents a light source with single-mode squeezed thermal state.

Many efforts have been devoted to investigate the behavior of entanglement of mixed states. In connection to this, several measures of of entanglement have been proposed \cite{40-42}. These measures are including, for instance, entanglement formation or its normalized version \cite{40}, the entanglement cost \cite{41} and distillable entanglement \cite{42} are used to quantify the entanglement properties of the pure state to create a quantum state and extracted from the a certain state, respectively. So far, the relative entropy of the entanglement has been also another measure of entanglement that interpolate between entanglement formation and distillable entanglement \cite{43}. However in practice, it is not well known that how effectively calculate the entanglement measures analytically for general mixed states because they involve vibrational expressions. On the other hand, the observation of a multi-partite pure state correlations is a complex problem in the study of entanglement and is of interest because one hopes gain a better understanding of the correlations between different registers of quantum computer. But to fill this problem, one could adopt logarithmic negativity which is based on the trace form of the partial transpose of the bipartite mixed state \cite{24, 44, 45}. The determination of the logarithmic negativity is completely straightforward using standard algebraic expressions. Henceforth, the logarithmic negativity can be applied to study the nature of degree of entanglement in pure quantum states, mixed quantum states and the Gaussian states of light. Remarkably, the degree of entanglement in a mixed state can be characterized by the logarithmic negativity. With such natures of the logarithmic negativity, we deliberately utilize the logarithmic negativity as entanglement measure of output superposed light for the system proposed in this work.

It has been verified that the quantum correlations can arise in photons interacting via a beam splitter, example, employing single-mode Gaussian and thermal states at the inputs of the beam splitter \cite{46}. Furthermore, Brunelli et al \cite{14}, considered the single-mode and two-mode Gaussian and thermal state (classical state) at the input of the beam splitter to investigate the quantum correlations establishment after the action of the beam splitter and they obtained that maximum number of thermal photons can be mixed without eliminating the quantum entanglement. Apart from these previous similar works, in this paper, it is the first time to consider the squeezed thermal state (mixed quantum state) with controlled squeezing and the Gaussian state
at the inputs of asymmetrical beam splitter to observe natures of the three non-classical correlations namely quantum entanglement, quantum discord and steering in a continuous variable regime.

In line with the interest of creating quantum correlations in a set of systems, we are motivated to show the possibility of creating quantum correlations from different quantum sources through the linear beam splitter enables us to grasp the properties of the superposed output light after the action of the beam splitter. This leads to the proposed system to be applicable for different purpose, for instance, the output entangled light may be potentially good for quantum information processing. In addition to this, apart from the revealing the mathematical formulations for the output light beams, the observed quantum correlations effects can be applicable for quantum state engineering, for example, secured non-localized state preparation and quantum state teleportation.

2. Description of input states

2.1. Single-mode Gaussian state

The superposition principle via optical beam splitter has been applicable for many quantum systems [47–50]. As an example, we consider a single-mode Gaussian state to the first port of the beam splitter. This state can be characterized by the non-classical depth denoted by $\tau$ [51] and purity represented by $\beta$. A Gaussian state can be classical, for instance, coherent state and thermal state or it can be also non-classical (squeezed) state. The Gaussian state can also be characterized by the covariance matrix say $V_G$ which satisfies the following condition

$$V_G + (\tau - 1/2)I > 0,$$

where the covariance matrix $V_G$ is given in Williamson’s form as [52]

$$V_G = \begin{pmatrix} a & b \\ b & a \end{pmatrix},$$

in which $a$ is real and $b = |b|e^{i\phi}$ is complex. For Gaussian state with covariance matrix $V_G$, the purity of the state becomes [53]

$$\beta = \frac{1}{\sqrt{\det V_G}}.$$

In terms of the degree of non-classicality $\tau$ and the purity $\beta$, one can express the elements of the covariance matrix $V_G$ of the Gaussian state as

$$a = \frac{1}{4\beta^2(1 - 2\tau)} + \frac{1}{2}(1 - 2\tau),$$

$$b = \frac{1}{4\beta^2(1 - 2\tau)} - \frac{1}{2}(1 - 2\tau).$$

2.2. Squeezed thermal state

We let to consider the second input mode be single-mode squeezed thermal state, the covariance matrix for which can be expressed as

$$V_T = \begin{pmatrix} a' & 0 \\ 0 & a' \end{pmatrix},$$

with the matrix element $a' = n \cosh^2 r + \cosh 2r/2$, where $r$ represents the squeezing parameter and $n$ is the average thermal photon number and can be expressed by

$$n = \frac{1}{\exp\left(\frac{\hbar c}{\kappa_B T}\right) - 1},$$

where $\kappa_B$ is the Boltzmann constant and $T$ is the absolute temperature.

3. Covariance matrix of the output mode

As we can see in figure 1, having a single-mode generalized Gaussian state of non-classicality $\tau$ and purity $\beta$ incident to the first port of the beam splitter and a single-mode squeezed thermal state incident to the second port of the beam splitter, it is easy to formulate the covariance matrix of the two-mode output state by applying the action of the beam splitter as
\[ V_{\text{out}} = M_{\text{BS}}(\theta, \varphi) V_{\text{in}} M_{\text{BS}}(\theta, \varphi), \]  
where \( M_{\text{BS}}(\theta, \varphi) \) is the transformation matrix of the beam splitter and then it can be given by
\[
M_{\text{BS}}(\theta, \varphi) = \begin{pmatrix}
\cos \theta & 0 & -\sin \theta e^{i\varphi} & 0 \\
0 & \cos \theta & 0 & -\sin \theta e^{i\varphi} \\
\sin \theta e^{i\varphi} & 0 & \cos \theta & 0 \\
0 & \sin \theta e^{i\varphi} & 0 & \cos \theta
\end{pmatrix},
\]
in which \( \cos^2 \theta \) denotes the transmittance of fields and \( \varphi \) represents the phase difference between the transmitted and reflected light modes. Moreover, the covariance matrix of the input fields can be given by the vector product \( V_{\text{in}} = V_{\text{G}} \otimes V_{\text{T}} = \text{diag}(U_1, U_2) \), where \( U_1 \) and \( U_2 \) are \( 2 \times 2 \) matrices. Therefore, the covariance matrix of the output mode, composed of Gaussian and squeezed thermal states can be expanded as \( 2 \times 2 \) block matrix
\[
V_{\text{out}} = \begin{pmatrix}
\alpha & \gamma^* \\
\gamma & \eta
\end{pmatrix},
\]
The matrices \( \alpha \) and \( \eta \) explains the properties of the individual systems corresponding to generalized Gaussian and squeezed thermal states, respectively, whereas the matrix \( \gamma \) contains information about the correlations between the output modes induced by the optical beam splitter and their expressions are given below
\[
\alpha = \begin{pmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{12} & \alpha_{11}
\end{pmatrix},
\eta = \begin{pmatrix}
\eta_{11} & \eta_{12} \\
\eta_{12} & \eta_{11}
\end{pmatrix},
\gamma = \sin \theta \cos \theta \begin{pmatrix}
\gamma_{11} & \gamma_{12} \\
\gamma_{12} & \gamma_{11}
\end{pmatrix},
\]
where \( \alpha_{11} = a \cos^2 \theta + (n \cosh^2 r + \cosh 2r) \sin^2 \theta, \alpha_{12} = b \cos^2 \theta, \eta_{11} = a \sin^2 \theta + (n \cosh^2 r + \cosh 2r) \cos^2 \theta, \eta_{12} = b \sin^2 \theta e^{-i\varphi}, \gamma_{11} = a - n \cosh^2 r - \cosh 2r \) and \( \gamma_{12} = be^{-i\varphi} \) are the matrices elements.

4. Quantum correlations

In this section, we quantify the quantum correlations for two-mode output state with help of the covariance matrix obtained in equation (10). In this work, we exploit the Gaussian quantum entanglement, quantum discord, and quantum steering of photons salvaged via linear beam splitter using different single-mode Gaussian state choice initially at the input. Entangled state is a two party system property which can be explained in terms of the density operator of a system. Employing a convex sum of product states notation
\[
\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B,
\]
the state of the system is separable with \( 0 \leq p_i \leq 1 \) and \( \sum p_i = 1 \), otherwise, the state of the system is entangled. Obviously, the presence of quantum entanglement between two fields can be verified by different mechanisms [54–59]. In particular, inseparability criteria between two fields in the continuous variable regime has been proposed by several authors [57, 59]. Moreover, Duan et al [59] criteria has been also sufficient to detect the quantum entanglement between parities in continuous variable state. Instead of using these criteria, we consider the logarithmic negativity to notice the appearance of the Gaussian quantum entanglement between the squeezed thermal and Gaussian output states superposed through a beam splitter [60]. The logarithmic negativity of the mode out of the beam splitter can then be defined as [24, 45]
\[
E_N = \max \{0, -\log_2(2V_i)\},
\]
where \( V_i \) is the smaller symplectic eigenvalue under partial transposed state and defined to be [61]
\[
V_i = \left[ \frac{\mu - \sqrt{\mu^2 - 4D}}{2} \right]^{1/2},
\]
in which \( \mu = \text{det} \alpha + \text{det} \eta - 2 \text{det} \gamma \) and \( D = \text{det} V_{\text{out}} \).

The other quantifier to measure the quantum correlation between the Gaussian and squeezed thermal output states is quantum discord [62, 63]. It can be expressed as
\[
D_h = f(\sqrt{\text{det} \eta}) - f(\lambda_+) - f(\lambda_-) + f(h),
\]
where

\[
\lambda_{\pm} = \left( \frac{\zeta - \sqrt{\zeta^2 - 4D}}{2} \right)^{1/2},
\]

in which \( \zeta = \det \alpha + \det \eta + 2 \det \gamma \) and the parameter \( h \) is given by

\[
h = \frac{\sqrt{\det \eta} + 2 \sqrt{(\det \alpha)(\det \eta)} + 2 \det \gamma}{1 + 2 \sqrt{\det \eta}},
\]

and we use the notation

\[
f(k) = \left( k + \frac{1}{2} \right) \log_2 \left( k + \frac{1}{2} \right) - \left( k - \frac{1}{2} \right) \log_2 \left( k - \frac{1}{2} \right).
\]

The quantum state is either separable or inseparable when the measurement obeys the condition \( 0 \leq D_{\alpha} \leq 1 \) \cite{62, 63}.

Next, we concentrate on the quantum steering which is the central point to fill the space between entanglement and quantum discord \cite{21}. Quantum steering is asymmetric in general i.e \( S^{A:B} \neq S^{B:A} \), where the quantum steering from party \( A \) (say Gaussian state in this case) to party \( B \) (squeezed thermal state) can be expressed as

\[
S^{A:B} = \max \left\{ 0, \frac{1}{2} \log_2 \left( \frac{\det \alpha}{4D} \right) \right\},
\]

5. Results and discussions

As shown in figure 2, the quantum entanglement is plotted against the beam splitter angle \( \theta \) using non-classicality of the Gaussian state \( \tau = 0.02, \varphi = \pi \), the mean thermal photon number \( n = 2 \), and the squeezing parameter \( r = 0.2 \) for various values for the purity of the Gaussian state \( \beta \). Moreover, for a purity \( \beta = 1.0 \), it is observed that the quantum entanglement between the output states is attained the maximum value for the choice of \( \theta = \pi/2 \), however, it decreases eventually when the angle of the beam splitter \( \theta \) far from \( \theta = \pi/2 \).

In figure 3, the dynamics of quantum discord as a function of the beam splitter angle \( \theta \) with identical conditions used in figure 2 has been shown. From the figure, we notice that the optimum quantum discord is occurred at \( \theta = \pi/2 \) for the case of pure Gaussian state, \( \beta = 1.0 \). Moreover, The quantum discord increases as the purity increases. This result is in agreement with previous reports \cite{45, 60}.

In figure 4, it is observed that quantum steering has been plotted as a function of the beam splitter angle \( \theta \) with the same conditions used in figures 2 and 3. The quantum steering has similar characteristics with quantum entanglement and discord.

In figures 5–7, it is shown that the dynamics of quantum entanglement, quantum discord and steering against the squeezing parameter has been plotted, respectively. The three quantum correlations reveal similar characteristics in their own perspectives that is the entangled output states are steerable while the quantum discord indicates enhanced quantum profile for quantum correlations produced by the linear beam splitter. As can be seen from the figures, the quantum correlations grown monotonically as the squeezing parameter
increases. The system exhibits the quantum discord due to the presence of entangled states. Moreover, the quantum steering is the witness of quantum entanglement. We note that the characteristics of the three quantum correlations reveal inverse relation with thermal photons with mean $n$. In this case, the non-classical correlations degraded as the thermal mean photon number increased at the input state of the beam splitter.

**Figure 3.** Plots of quantum discord of single-mode Gaussian and squeezed thermal state against beam splitter angle $\theta$ at the output of beam splitter for fixed values of $\tau = 0.02, r = 0.2, \varphi = \pi$ and thermal mean photon number $n = 2$ for different values of $\beta$.

**Figure 4.** Plots of quantum steering of single-mode Gaussian and squeezed thermal state against beam splitter angle $\theta$ at the output of beam splitter for fixed values of $\tau = 0.02, r = 0.2, \varphi = \pi$ and thermal mean photon number $n = 2$ for different values of $\beta$.

**Figure 5.** Plots of quantum entanglement of single-mode Gaussian and squeezed thermal state against squeezing parameter $r$ at the output of beam splitter for fixed values of $\tau = 0.02, \theta = \pi/2, \varphi = \pi, \beta = 1.0$, for different values of thermal mean photon number $n$. 

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In this paper, we have investigated the three kinds of quantum correlations such as entanglement, quantum steering and discord via linear beam splitter using single-mode Gaussian state and squeezed thermal state initially at the inputs. The dynamics of each quantum correlations has been analyzed with respect to each other using the system parameters. The system exhibits a bipartite quantum entanglement using the beam splitter as entangler of the input states. With the help of logarithmic negativity, we show the dynamics of quantum entanglement for the proposed quantum source by employing the covariance matrix of the output two-mode state. Furthermore, the quantum discord and steering show similar trend like quantum entanglement. In summary, for given parameters of the system, the quantum correllations have positive relation with purity, non-classical depth, and squeezing parameter. However, the three kinds of quantum correlations have inverse relation with the mean photon number of thermal state. In this regard, the quantum entanglement can be enhanced by increasing the squeezing parameter at the squeezed thermal input state while increasing the purity and non-classicality at the Gaussian input state. For example, particularly at $\theta = \pi / 2$, the output field is maximally entangled for $r = 0.2$, $n = 2$ and the pure state that is $\beta = 1$.

Moreover, the thermal mean photon number in the squeezed thermal state has decreasing effect on the quantum correlations of the two-mode output state whereas it has an enhancement effect on the total mean number of photon at the output state. Besides, the squeezing parameter has also enhancement effect on the quantum correlations as expected. Furthermore, the quantum system generates an entangled bright light due to the presence of optical beam splitter. Thus, it can be further applicable for different technologies which are

**Figure 6.** Plots of quantum discord of single-mode Gaussian and squeezed thermal state against squeezing parameter $r$ at the output of beam splitter for fixed values of $\tau = 0.02, \theta = \pi / 2, \varphi = \pi, \beta = 1.0$, for different values of thermal mean photon number $n$.

**Figure 7.** Plots of quantum steering of single-mode Gaussian and squeezed thermal state against squeezing parameter $r$ at the output of beam splitter for fixed values of $\tau = 0.02, \theta = \pi / 2, \varphi = \pi, \beta = 1.0$, for different values of thermal mean photon number $n$. 

6. Conclusions

In this paper, we have investigated the three kinds of quantum correlations such as entanglement, quantum steering and discord via linear beam splitter using single-mode Gaussian state and squeezed thermal state initially at the inputs. The dynamics of each quantum correlations has been analyzed with respect to each other using the system parameters. The system exhibits a bipartite quantum entanglement using the beam splitter as entangler of the input states. With the help of logarithmic negativity, we show the dynamics of quantum entanglement for the proposed quantum source by employing the covariance matrix of the output two-mode state. Furthermore, the quantum discord and steering show similar trend like quantum entanglement. In summary, for given parameters of the system, the quantum correlations have positive relation with purity, non-classical depth, and squeezing parameter. However, the three kinds of quantum correlations have inverse relation with the mean photon number of thermal state. In this regard, the quantum entanglement can be enhanced by increasing the squeezing parameter at the squeezed thermal input state while increasing the purity and non-classicality at the Gaussian input state. For example, particularly at $\theta = \pi / 2$, the output field is maximally entangled for $r = 0.2$, $n = 2$ and the pure state that is $\beta = 1$.

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operated based on beam splitter and quantum optical sources to function. We hope these results on quantum correlation could further develop the applications in quantum information processing.

Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

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