Decay of Nuclear Giant Resonances: Quantum Self-similar Fragmentation

A. Z. Górski, R. Botet, S. Drożdż and M. Płoszajczak

Abstract

Scaling analysis of nuclear giant resonance transition probabilities with increasing level of complexity in the background states is performed. It is found that the background characteristics, typical for chaotic systems lead to nontrivial multifractal scaling properties.

The statistical study of spectra of quantum systems is almost as old as quantum mechanics. In this context, the random-matrix theories (RMT) have been proposed as a natural theoretical framework for describing level fluctuations in stationary systems. The Gaussian orthogonal ensemble of random matrices, which leads to the Wigner distribution, can describe statistical properties (the nearest neighbor spacing (NNS) distribution) of nearly all time-reversal-invariant quantum systems whose classical counterparts are chaotic. On the other hand, the NNS distribution of levels in quantum systems whose classical dynamics is regular is given by the Poisson distribution. Those 'empirical' observations gave rise to 'quantum chaology' and lead to the definition of 'chaotic', 'regular' or 'mixed' quantum systems. The fundamental limit of concepts based on the RMT is their stationary connotation.

The nuclear giant resonance (or plasmon), which at time \( t = 0 \) is associated with simple \((1p-1h)\) configurations and fragments (decays) at later times into the complicated \((np-nh)\) configurations, is only one of many examples of short time quantum phenomena which necessitate the time-dependent formulation of their local statistical properties. The effect of coupling between \((1p-1h)\) and more complicated \((np-nh)\) configurations can be represented as a noise or fluctuating force acting on the observable ('macroscopic' degrees of freedom) in the \((1p-1h)\) space. These fluctuations arise from the elimination of the irrelevant degrees of freedom associated with the environment of the \((1p-1h)\) space, in favor of a small number of macroscopic variables. In this way one arise at the Langevin formulation of the initial multidimensional problem in which macroscopic variables (e.g. \(1p-1h\) collective excitations of various quantum numbers etc.) are driven by the fluctuating force. In general, nature of the stochastic process depends on the choice of the macroscopic variable. It is however worth noticing that e.g. fluctuation properties of quantal spectra (NNS distribution : \(p(x)\)) are correctly given as the limit \( t \to \infty \) of a nonlinear stochastic process with multiplicative fluctuations and is associated with the Fokker-Planck equation:

\[
\frac{\partial}{\partial t} p(x,t) = -\frac{\partial}{\partial x} [(Dx - Bx^{1+\gamma} + \frac{1}{2}Qx)p(x,t)] + \frac{Q}{2} \frac{\partial^2}{\partial x^2} (x^2 p(x,t)) \quad ,
\]

where parameters \(B, D, \gamma\) can be identified with parameters of the fundamental Hamiltonian of the system, and \(\gamma \equiv 2D/Q\) equals 1 or 2 for Poisson or Wigner distributions respectively.
In between those two limiting cases, i.e. for \(1 < \gamma < 2\), one obtains from (1) the Brody distribution\(^6\): 

\[ p(x) = N x^{\gamma - 1} \exp(-bx^{\gamma}) \text{ with } b \equiv B/D, \]

which describes well the NNS in mixed quantum systems\(^7\). This stochastic process\(^\square\) resembles the binary, self-similar and conservative random fragmentation process\(^\square\) which is known to yield 'universal' behaviors, independently of the precise fragmentation mechanism\(^\square\). Generality of this stochastic process makes plausible the hypothesis of universality also in the strength fragmentation associated with the giant resonance decay. This work reports about the investigation of this intriguing quantum phenomenon.

The generic statistical framework for the description of fragmentation aspects of the giant resonance decay is provided by the Nakajima - Zwanzig rate equations:

\[ i \frac{\partial}{\partial t} f_1'(t) = \sum_{1'} H_{11'} f_1'(t) = \sum_{1'} \int_0^t d\tau f_1'(t-\tau) v_{11'}(\tau), \]

where the non-local collision term:

\[ v_{11'}(\tau) = -i \sum_2 H_{12} H_{21'} \exp(-iH_{22}\tau) \]

is expressed in terms of the matrix elements of the Hamiltonian in the \(1p-1h\) (\(|1\rangle\)) and \(2p-2h\) (\(|2\rangle\)) background spaces\(^\square\). \(f_1'\) in (2) are the time-dependent probability amplitudes of the macroscopic, collective state in the space \(|1\rangle\).

Figure 1: Isovector strength distribution for the regular (A) and chaotic (B) case.

In the limit \(t \to \infty\) this has been studied recently\(^8\) for the giant quadrupole resonances of \(^{40}\)Ca in the truncated subspace of \(1p-1h\) (26 states) and \(2p-2h\) (3014 states) excitations. Fig. 1 shows the resulting isovector quadrupole strength distribution \(\rho_i\) \((\rho_i = |\langle \hat{f} | i \rangle|^2; \hat{f}\) is the one–body operator and \(|i\rangle\) diagonalizes \(\hat{H}\) in \(|1\rangle \oplus |2\rangle\)) in the two model cases: (A) no residual interaction in \(2p-2h\) subspace which leads to the Poisson level fluctuations and can be associated with regular dynamics and (B) full residual interaction included which results in Wigner fluctuations and thus can be interpreted in terms of chaotic dynamics\(^9\). Significant redistribution of the strength is observed when going from (A) to (B). The most interesting effect is more uniform distribution, even resembling a certain kind of self–similar structure regarding the clustering and the relative size of the transitions. This points to the need of more systematic, multifractal analysis.

To this end we have investigated the sets of \(\textit{pairs}\) of points: the energies and the transition probabilities \(\{E_i, \rho_i\}\). The energy spectrum of a typical quantum systems has the fractal dimension equal to one. However, taking into account probabilities \(\rho_i\) we have more
complicated structure. Because to each energy corresponds only one probability the whole structure should have dimension in the range: $0 \leq d_q \leq 1$.

Therefore, we shall treat the value $\rho_i$ as a number of "points" in the corresponding ($i$–th) box. Hence, the fractal structure of the set $\{E_i\}$ will be modified by the transition probabilities. To have measure with the proper normalization we define our measure $P_i(l)$ as:

$$p_i(l) = P_i(l) \equiv \left[ \sum_{\text{all}} E_i \, \rho_i \right]^{-1} \times \sum_{E_i \in i\text{-th box}} \rho_i,$$

where in the numerator summation goes over probabilities whose energies are included in the $i$-th box. Here again the measure $P_i(l)$ is properly normalized: $\sum_i P_i(l) = 1$. The scaling exponent ("fractal dimension") with the measure (4) we will denote by $D_q$, while $d_q$ we reserve for the standard fractal dimension. From (4) it is clear that for $q = 0$ (capacity dimension) probabilities $\rho_i$ do not contribute to $\chi_0(l)$, as $M(l)$ is the number of boxes with any non-zero number of data points (energies in our case). Hence, we have: $D_0 = d_0$, the last being the standard capacity dimension of the energy spectrum.

The input data consist of the order $\sim 2^{11}$ data points, the number sufficient to display exponential scaling but one should have in mind that some statistical errors will be present, as the fractal dimension formula contains the $l \to 0$ (or, equivalently, the $n \to \infty$) limit.
In fact, for the chaotic case we have got a fairly good scaling in the range of about 8 points in the log–log plot as can be seen from Figs. 2(B). This nice scaling has been considerably worsened in the regular case (Fig. 2(A)). In the special case of the capacity dimension we get \( D_0 \approx 1 \), as in this case the scaling exponents are determined solely by the energy distribution and this has not been plotted in Fig. 2(B). The regular case is also plotted for comparison, even though scaling is very poor and choice of the scaling exponents is difficult in this case. Hence, for \( q > 2 \) the linear fits have not been plotted on Figs. 2(A).

Further studies are still needed to disclose salient features of short time quantal phenomena in the framework of the conservative and self-similar random fragmentation process \([3]\), which in the asymptotic limit \( t \to \infty \) provides a good description of the local statistical properties of quantum systems both in their chaotic and regular limits as well as in the mixed limit. The ubiquity of this fragmentation process \([3]\), both at small (microscopic) and large (macroscopic) scales, makes possible a new insight into the relation between chaoticity or regularity of classical systems and the corresponding statistical properties of the associated quantum systems.

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