The universe time-asymmetry is essentially produced by its low-entropy unstable initial state. Using qualitative arguments, Paul Davies has demonstrated that the universe expansion may diminish the entropy gap, therefore explaining its low-entropy state, with respect to the maximal possible entropy at any time. This idea is implemented in a qualitative way in a simple homogeneous model.

Some rough coincidence with observational data are found

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I. INTRODUCTION.

The time-asymmetry problem can be stated in the following way:

How can we explain the obvious time-asymmetry of the universe and most of its subsystems if the fundamental laws of physics are time-symmetric?

Physicists usually answer this question first observing that, if the initial state of the universe (or any of its subsystems) would be an equilibrium state, the universe (or the subsystem) will remain forever in such state, making impossible to find any time-asymmetry. Thus we must solve two problems:

i.- To explain why the universe (or the subsystem) began in a non-equilibrium (unstable, low-entropy) state, at a time that we shall call \( t = 0 \).

ii.- To define, for the period \( t > 0 \), a Lyapunov variable, namely a variable that never decreases (e.g., entropy), an arrow of time, and also irreversible evolution equations, despite the fact that the main laws of physics are time-symmetric.

Let us comments these two problems:

i.- The set of irreversible processes that began in an unstable non-equilibrium state constitute a branch system [1], [2]. That is to say, every one of these processes began in a non-equilibrium state, which state was produced by a previous process of the set. E.g.: Gibbs ink drop (initial unstable state) spreading in a glass of water (irreversible process) it is only possible if there was first an ink factory which extracted the necessary energy from an engine, where coal (initial unstable state) was burned (branched irreversible process); in turn, coal was created with energy coming from the sun, where \( H \) (initial unstable state) is burned (branched irreversible process); finally, \( H \) was created using energy obtained from the unstable initial state of the universe (the absolute initial state of the branch system).

Therefore using this hierarchical chain, all the irreversible processes are related to the cosmological initial condition, the single one that must be explained.

ii.- Once we have understood the origin of the initial unstable state of each irreversible process within the universe (even if we have not yet explained the origin of the initial state of the whole universe) it is not difficult to obtain a growing entropy (and irreversible evolution equations), in any subsystem within the universe. With this purpose we can consider, e.g., that forces of stochastic nature penetrate from the exterior of each subsystem adding stochastic terms [3]. Alternatively, taking into account the enormous amount of information contained in the subsystem we can neglect some part of it [4], [5]. Thirdly, or we can use more refined mathematical tools [6], [7]. With any one of this tools we can solve this problem.

It remains only one problem: Why the universe began in an unstable low-entropy state?

If we exclude a miraculous act of creation we have only three scientific answers:

i.- The unstable initial state of the universe is a law of nature.

ii.- This state was produced by a fluctuation.

iii.- The expansion of the universe (coupled to the nuclear reactions in it) produces a decreasing of the (matter-radiation) entropy gap.
The first solution is only a way to bypass the problem, while the fluctuation solution is extremely improbable. In fact, the probability of a fluctuation diminish with the number of particles of the considered system and the universe is the system with the largest number of particles.

The third solution was sketched by Paul Davies in reference [2], only as a qualitative explanation. The expansion of the universe is like an external agency (namely: external to the matter-radiation system of the universe) that produces a decreasing of its entropy gap, with respect to the maximal possible entropy, $S_{\text{max}}$ (and therefore an unstable state), not only at $t = 0$ but in a long period of the universe evolution. We shall call this difference the entropy gap $\Delta S$, so the actual entropy will be $S_{\text{act}} = S_{\text{max}} + \Delta S$ (fig. 1.). In this essay we will try to give a quantitative structure to Davies solution using an oversimplified cosmological model, which, anyhow, yields a first rough numerical coincidence with observational data.

II. THE ENTROPY GAP.

It is well known that the universe isotropic and homogeneous expansion is a reversible process with constant entropy [3]. In this case the matter and the radiation of the universe are in a thermic equilibrium state $\rho_*(t)$ at any time $t$. As the radiation is the only important component, from the thermodynamical point of view, we can chose $\rho_*(t)$ as a black-body radiation state [8], i.e. $\rho_*(t)$ will be a diagonal matrix with main diagonal:

$$\rho_*(\omega) = ZT^{-3} \frac{1}{e^{\omega T} - 1}$$

where $T$ is the temperature, $\omega$ the energy, and $Z$ a normalization constant ( [3], eqs. (60.4) and (60.10)). The total entropy is:

$$S = \frac{16}{3} \sigma VT^3$$

( [3], eq. (60.13)) where $\sigma$ is the Stefan-Boltzmann constant and $V$ a comoving volume.

Let us consider an isotropic and homogeneous model of universe with radius (or scale) $a$. As $V \sim a^3$, and, from the conservation of the energy-momentum tensor and radiation state equation, we know that $T \sim a^{-1}$, we can verify that $S = \text{const.}$ Thus the irreversible nature of the universe evolution is not produced by the universe expansion, even if $\rho_*(t)$ has a slow time variation.

Therefore, the main process that has an irreversible nature after decoupling time is the burning of unstable $H$ in the stars (that produces $He$ and, after a chain of nuclear reactions, $Fe$). This nuclear reaction process has certain mean life-time $t_{NR} = \gamma^{-1}$ and phenomenologically we can say the estate of the universe, at time $t$, is:

$$\rho(t) = \rho_*(t) + \rho_1 e^{-\gamma t} + 0[(\gamma t)^{-1}]$$

where $\rho_1$ is certain phenomenological coefficient constant in time, since all the time variation of nuclear reactions is embodied in the exponential law $e^{-\gamma t}$. We can foresee, also on phenomenological grounds, that $\rho_1$ must peak strongly around $\omega_1$, the characteristic energy of the nuclear process.

All these reasonable phenomenological facts can also be explained theoretically: Eq. [3] can be computed with the theory of paper [10] or with rigged Hilbert space theory [5]. In reference [11] it is explicitly proved that $\rho_1$ peaks strongly at the energy $\omega_1$.

The normalization conditions at any time $t$ yields:

$$tr \rho(t) = tr \rho_*(t) = 1, ..., tr \rho_1 = 0$$

The last equations show that $\rho_1$ is not a state but only the coefficients of a correction around the equilibrium state $\rho_*(t)$. It is explicitly proved in paper [11], that $\rho_1$ has a vanishing trace.

We are now able to compute the entropy gap $\Delta S$ with respect to the equilibrium state $\rho_*(t)$ at any time $t$. It will be the conditional entropy of the state $\rho(t)$ with respect to the equilibrium state $\rho_*(t)$ [3]:

$$\Delta S = -tr[\rho \log(\rho_1^{-1} \rho)]$$

Using now eq. [3] and considering only times $t \gg t_{NR} = \gamma^{-1}$ we can expand the logarithm to obtain:

$$\Delta S \approx -e^{-\gamma t} tr \left( \rho_1^{-1} \rho_1^2 \right)$$
where we have used eq. [4]. We can now introduce the equilibrium state \([\omega \gg T]\). Then:

\[
\Delta S \approx -Z^{-1} T^3 e^{-\gamma t_{tr}(e^{\omega T} \rho_1^2)}
\]

(7)

where \(e^{\omega T}\) is a diagonal matrix with this function as diagonal. But as \(\rho_1\) is peaked around \(\omega_1\) we arrive to a final formula for the entropy gap:

\[
\Delta S \approx -CT^3 e^{-\gamma t} e^{\omega_1 T}
\]

(8)

where \(C\) is a positive constant.

### III. EVOLUTION OF THE ENTROPY GAP \(\Delta S\).

We have computed \(\Delta S\) for times larger than decoupling time and therefore, as \(a \sim t^{3/2}\) and \(T \sim a^{-1}\), we have:

\[
T = T_0 \left( \frac{t_0}{t} \right)^{\frac{3}{2}}
\]

(9)

where \(t_0\) is the age of the universe and \(T_0\) the present temperature. Then:

\[
\Delta S \approx -C_1 e^{-\gamma t} t^{-2} e^{\omega_1 T_0 \left( \frac{t_0}{t} \right)^{\frac{3}{2}}}
\]

(10)

where \(C_1\) is a positive constant. Fig. 2 is the graphic representation of curve \(\Delta S(t)\). It has a maximum at \(t = t_{cr1}\) and a minimum at \(t = t_{cr2}\). Let us compute these critical times. The time derivative of the entropy reads:

\[
\Delta S \approx \left[ -\gamma - 2t^{-1} + \frac{2}{3} \frac{\omega_1}{T_0} \left( \frac{t_0}{t} \right)^{\frac{3}{2}} \right] \Delta S
\]

(11)

This equation shows two antagonistic effects. The universe expansion effect is embodied in the second and third terms in the square brackets an external agency to the matter-radiation system such that, if we neglect the second term, it tries to increase the entropy gap and, therefore, to take the system away from equilibrium (as we will see the second term is practically negligible). On the other hand, the nuclear reactions embodied in the \(\gamma\)-term try to convey the matter-radiation system towards equilibrium. These effects becomes equal at the critical times \(t_{cr}\) such that:

\[
\gamma t_0 + 2 \frac{t_0}{t_{cr}} = \frac{2}{3} \frac{\omega_1}{T_0} \left( \frac{t_0}{t} \right)^{\frac{3}{2}}
\]

(12)

For almost any reasonable numerical values this equation has two positive roots: \(t_{cr1} \ll t_0 \ll t_{cr2}\). Precisely:

i.- For the first root we can neglect the \(\gamma t_0\)-term and we obtain:

\[
t_{cr1} \approx t_0 \left( \frac{T_0}{3 \omega_1} \right)^{\frac{3}{2}}
\]

(13)

(this quantity, with minus sign, gives the third unphysical root).

ii.- For the second root we can neglect the \(2(t_0/t_{cr})\)-term, and we find:

\[
t_{cr2} \approx t_0 \left( \frac{2 \omega_1 T_{NR}}{3 T_0} \right)^{3}
\]

(14)

### IV. NUMERICAL ESTIMATES.

We must chose numerical values to four parameters: \(\omega_1 = T_{NR}, t_{NR} = \gamma^{-1}, t_0,\) and \(T_0\). \(T_{NR}\) and \(t_{NR}\) can be chosen between the following values [12]:

\[
T_{NR} = 10^6..10^{80} K
\]

(15)
\[ t_{NR} = 10^6 \ldots 10^9 \text{years} \]

while for \( t_0 \) and \( T_0 \) we can take:

\[ t_0 = 1.5 \times 10^{10} \text{years} \]

\[ T_0 = 3^0 K \]

In order to obtain a reasonable result we choose the lower bounds for \( T_{NR} \) and \( t_{NR} \) and we obtain for \( t_{cr_1} \):

\[ t_{cr_1} \approx 1.5 \times 10^3 \text{years} \] (16)

\[ T_{cr_1} = 30 K \]

So \( t_{cr_1} \) is smaller than the decoupling time and, it must not be considered since the physical processes before this time are different than those we have used in our model. Also, we must consider only times \( t > t_{NR} = \gamma^{-1} \), in order to use eq. 6. So, only the r.h.s. from the dashed line of fig. 2 can be taken into account.

For \( t_{cr_2} \) we obtain:

\[ t_{cr_2} \lesssim 10^4 t_0 \] (17)

From eqs. (17) and (18) we can see that \( t_{cr_1} \ll t_0 \ll t_{cr_2} \). Thus:

- From \( t_{NR} \) to \( t_{cr_2} \) the expansion of the universe produces a decreasing of entropy gap, according to Paul Davies prediction. It probably produces also a growing order, and therefore the creation of structures like clusters, galaxies and stars [3].

- After \( t_{cr_2} \) we have a growing of entropy, a decreasing order and a spreading of the structures: stars energy is spread in the universe, which ends in a thermic equilibrium [4]. In fact, when \( t \to \infty \) the entropy gap vanishes (see eq. 6) and the universe reaches a thermic equilibrium final state.

\[ t_{cr_2} \lesssim 10^4 t_0 \] is the frontier between the two periods. Is the order of magnitude of \( t_{cr_2} \) a realistic one? In fact it is, since \( 10^4 t_0 \approx 1.5 \times 10^{14} \text{years} \) after the big-bang all the stars will exhaust their fuel [4], so the border between the two periods most likely have this order of magnitude and must also be smaller than this number. This is precisely the result of our calculations.

V. CONCLUSION.

Clearly we have not a physical argument to choose the lower bounds in eq.13, we have just chosen this values for convenience. Therefore, our model is extremely naive and simplified: an homogeneous isotropic universe. In real universe nuclear reactions take place within the stars, that only can be properly considered in a inhomogeneous geometry. Also there are condensation phenomena that increase \( S_{\text{max}} \), in such a way that, even if \( \Delta S \) first decreases and then increases, \( S_{\text{act}} \) always increases, according to the second law of Thermodynamics (fig.1). Notice that we have only take into account the global expansion effect of the universe and not the local effect of the gravitational field. Perhaps this effect can yield better results. Nevertheless, our model is at the edge of a correct physical prediction.

Summarizing, we have proved that Davies qualitative idea can be implemented quantitatively, preparing the scenario for more detailed calculations.

VI. ACKNOWLEDGMENT.

This work was partially supported by grants: ..................................of the European Community, PID-0150 of CONICET (National Research Council of Argentina), EX-198 of the Buenos Aires University, and 12217/1 of Fundación Antorchas.

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**VIII. FIGURE CAPTION.**

Fig. 1. The evolution of the maximum entropy and the actual entropy.

Fig 2. The evolution of the entropy gap. This figure is only qualitative, the scales are not the real ones.