The continuity of prime numbers can lead to even continuity

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The continuity of prime numbers can lead to even continuity

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**Abstract:** n continuous prime numbers can combine a group of continuous even numbers. If an adjacent prime number is followed, the even number will continue. For example, if we take prime number 3, we can get even number 6. If we follow an adjacent prime number 5, we can get even numbers by using 3 and 5: 6, 8 and 10.

If a group of continuous prime numbers 3, 5, 7, 11,... P, we can get a group of continuous even numbers 6, 8, 10, 12,..., 2n. Then if an adjacent prime number q is followed, the original group of even numbers 6, 8, 10, 12,..., 2n will be finitely extended to 2 (n + 1) or more adjacent even numbers. My purpose is to prove that the continuity of prime numbers will lead to even continuity as long as 2 (n + 1) can be extended.

If the continuity of even numbers is discontinuous, it violates the Bertrand Chebyshev theorem of prime numbers.

Because there are infinitely many prime numbers: 3, 5, 7, 11,...
We can get infinitely many continuous even numbers: 6, 8, 10, 12,...

**Key words:** prime even continuity; Bertrand Chebyshev theorem; Ascending and descending; Extreme law; Mathematical complete induction

### 1 Preface

1.1 The whole proof begins with the ascent

Rule ①: the minimum odd prime number is 3

Rule ②: add two odd prime numbers.(any combination of two odd primes).

Rule ③: odd prime number can be quoted repeatedly.

Rule ④: meet the previous provisions, and all prime combination to the maximum(for
example, $10 = 5 + 5$ must be: $10 = 5 + 5 = 3 + 7$.
For another example, the combination of 90 must be: $90 = 43 + 47 = 37 + 53 = 31 + 59 = 29 + 61 = 23 + 67 = 19 + 71 = 17 + 73 = 11 + 79 = 7 + 83$.

Rule ⑤: $P_a + P_b = 2S$, and $P_b + P_a = 2S$, Delete one and leave only one.

1.2 I can quote the minimum odd prime number 3.
The results are as follows
$3 + 3 = 6$.
$5 + 1 = 6$ (it is stipulated that 1 is not a prime number, which is deleted because it violates regulation 2).
∴ The unique formula: $3 + 3 = 6$. (comply with Rule 4: all prime numbers are quoted to the maximum).

According to the rules, the prime number 3 uses the maximum combination, and can only get:
$3 + 3 = 6$.
How to make 6 continuous to 8?
If we take the adjacent prime number $P$ greater than 3, we can get $P - 3 \geq 2$ Special record is (a).
From $3 + 3 = 6$, $\Rightarrow 3 + (3 + 2) = 6 + 2$
If $(3 + 2) = P$, the proposition holds when the even number is 8.
Extreme law $\Rightarrow$ Assumption: $(3+2) \neq p$
∴ $p-3 \neq 2$.
∴ (a): $(p-3 \geq 2, p-3 \neq 2) \rightarrow (p-3 > 2) \rightarrow (p-3 \geq 4)$
∴ $p > 2 \times 3$
∴ $p > 2 \times 3 > 3$

The results of Bertrand Chebyshev theorem are as follows:
$(2 \times 3) > (\text{prime number } p_1) > 3$.
∴ $(p > 2 \times 3 > 3) \Rightarrow (p > p_1 > 3) \Rightarrow (P$ and 3 are adjacent prime numbers) contradiction. Negation hypothesis.
∴ $(3+2)=p$, ∴ 3+p=8.
∴ 6→8

The proof process uses the known prime number, and the sieve method is not used in the derivation process,
Prime number theorem and Bertrand Chebyshev theorem.
By quoting the prime number theorem, we get a finite number of continuous prime numbers, and then generate even numbers according to the requirements of this manuscript.
Use the extreme rule again, Force even numbers to continue.
nouns and definitions, citing theorems

2.1 \( \phi_1 \): Definition of prime number: prime number refers to the natural number with no other factors except 1 and itself in the natural number greater than 1.

2.2 \( \phi_2 \): Extreme law: A may or may not be true. What conclusion can we get if we only prove that A is not true.

\{A \mid A=x, A=y\}, (A=x) \implies (QED). Take: A \neq x, only prove the A=y conclusion.

2.3 \( \phi_3 \): [References cited \([1]\)] Bertrand Chebyshev theorem: if the integer \( n > 3 \), then there is at least one prime \( P \), which conforms to \( n < p < 2n-2 \). Another slightly weaker argument is: for all integers \( n \) greater than 1, there is at least one prime \( P \), which conforms to \( n < p < 2n \).

2.4 \( \phi_4 \): In this paper, the even number generation rules are as follows

1. Only odd primes are allowed as elements.
2. Only two prime numbers can be added (any combination of two odd primes).
3. Two prime numbers can be used repeatedly: \((3 + 3)\), or \((3 + 5)\), or \((P + P)\).
4. Meet the previous provisions, and all prime combination to the maximum (for example, \(5 + 5\) must be: \(10 = 5 + 5 = 3 + 7\),
   For another example, the combination of 90 must be: \(90 = 43 + 47 = 37 + 53 = 31 + 59 = 29 + 61 = 23 + 67 = 19 + 71 = 17 + 73 = 11 + 79 = 7 + 83\).
5. Take only one of \((p_a + p_b)\) and \((p_b + p_a)\).

2.5 \( \phi_5 \): [References cited \([2]\)] The theorem of infinite number of primes:

the n bit after each prime can always find another prime.

For example, 3 is followed by 5 and 13 is followed by 17; There must be an adjacent prime \( p_i \) after the prime \( P \).
2.6 \( \phi_s \): Here we only discuss the following cases: prime number sequence and even number sequence

\[(\phi_s) \implies \text{Prime number sequence: } 3, 5, 7, 11, 13, 17, 19, 23, \ldots \]

Even number sequence: \( 6, 8, 10, 12, 14, 16, 18, 20, \ldots \)

Explain \( \phi_s \) in today's words: the prime number in the prime number sequence discussed in this paper is adjacent and continuous, and the first number is 3. An even number in an even number sequence is contiguous and the first number is 6.

2.7 \( \phi_t \): Remember: all the primes I'll talk about below refer to odd primes (excluding 2).

Prime symbol \( P \), different primes use \( P_1, P_2, P_3, \ldots, P_x \)

3 Logical argument

3.1 \( \Theta_1 \): Goldbach conjecture: \( 3 \leq \forall n \in \mathbb{N}, 2n=p_x+p_y \) set up \( \Delta_1: \ p_x \geq p_y \)

3.2 \( \Theta_2 \): Theorem: the continuity of prime numbers leads to the continuity of even numbers

In mathematical language:
Known: \{3, 5, 7, 11, 13, \ldots, p_2, p_1, p_0\} \( \in \) (prime). The next neighbor of \( p_1 \) is \( p_0 \), \( 3<5<7<11<13<\ldots< p_2< p_1< p_0 \).
\{6, 8, 10, 12, 14, 16, \ldots, 2n\} \( \in \) (continuous even number).
If: \{3, 5, 7, 11, 13, \ldots, p_2, p_1\} \( \implies \) \{6, 8, 10, 12, 14, 16, \ldots, 2n\}. Inevitable: \{3, 5, 7, 11, 13, \ldots, p_2, p_1, p_0\} \( \implies \) \{6, 8, 10, 12, 14, 16, \ldots, 2n, 2(n+1)\}.

3.2.1 prove:
Humans use computers to calculate a finite number of even numbers: \{6, 8, 10, 12, 14, \ldots, 2n\} every even number satisfies \( \Theta_1 \).
The computer process is finite \{6, 8, 10, 12, 14, ..., 2n\}
It is not logically proved that any even number greater than 4 satisfies \( \Theta \).

\((\phi_5 + \phi_6)\) Take odd prime sequence: 3, 5, 7, 11, 13, 17, 19, 23, ...

Take the minimum prime number 3 from the front of the prime sequence,

\( A_1: \{ \phi_1 + \phi_4 + \{3\} \} \Rightarrow \quad 3+3=6 \)
\( \quad \rightarrow 6 \)
It is recorded as \( A_1 \)

According to the rule, 3 can only get \{3+3=6\}

Nonexistence: \( 5+p=6 \)

【 Because 1 in \( 5+1=6 \) is not defined as a prime number. If 1 is defined as a prime number, this paper will come to the same conclusion 】

Note: \( 5 \notin \{3\} \).

Prime number 3, limit is used according to \( \phi_4 \), cannot be: \( 6 \rightarrow 8 \).

\{ \phi_1 + \phi_4 + \{3\} \} \Rightarrow \quad Quoting prime number 3 can only get even number 6.

If you want to: \( 6 \rightarrow 8 \), you must add an adjacent prime number 5.

\( A_2: \{ \phi_1 + \phi_4 + \{3, 5\} \} \Rightarrow \quad 3+3=6 \)
3+5=8

\( 5+5=3+7=10 \) \quad \therefore \quad \phi_4 \quad \{(5,5),(3,7)\} \in 10 \quad \therefore \quad \{ \phi_1 + \phi_4 + \{3,5,7\} \} \Rightarrow \quad 7+5=12 \)

\( 7+7=11+3=14 \) \quad \therefore \quad \phi_4 \quad \{(7,7),(3,11)\} \in 14 \quad \therefore \quad \{ \phi_1 + \phi_4 + \{3,5,7,11\} \} \Rightarrow \quad 11+5=13+3=16 \) \quad \therefore \quad \phi_4 \quad \{(11,5),(13,3)\} \in 16 \quad \therefore \quad \{ \phi_1 + \phi_4 + \{3,5,7,11,13\} \} \Rightarrow \quad 11+7=13+5=18 \)

\( 13+7=17+3=20 \) There is a new prime number 17 in continuity.
Note: \{ \{(13,7),(17,3)\} \in 20 \) even numbers \( \Rightarrow :6,8,10,12,14,16,18,20 \)
Prime numbers are continuous, and there is a new prime number 17.
even numbers \( \Rightarrow :6,8,10,12,14,16,18,20 \).

Note the key point: \( A_1 \) broken, increase the adjacent prime number 5 to have \( A_2 \)

From \( A_1 \quad \rightarrow \quad A_2 \quad \) is it always infinite? Or will it stop?
Here's the wonderful thing:

(analysis I):
Always Unlimited: \( A_1 \to A_2 \to \ldots \)

There are: \((\phi_8 + \phi_6) \Rightarrow \{ 3, 5, 7, 11, 13, 17, 19, 23, \ldots \) \)

Get: \( 6, 8, 10, 12, 14, 16, 20, 22, \ldots \)

Conclusion: \( \emptyset_2 \) (QED).

\((\phi_2) \Rightarrow \) Stop at \( A_n \), cannot continue.

(analysis II): Stop at \( A_n \), not to be continued.

Stop at \( A_n : \ A_1 \to A_2 \to \ldots \to A_n \)
\( \kappa \)-line with continuous Prime: \( 3, 5, 7, 11, 13, 17, 19, 23, \ldots, p_2, p_1 \)

Get: \( 6, 8, 10, 12, 14, 16, 20, 22, \ldots, 2(n-1), 2n \)

\( \emptyset_3 : \ \{ \{ \phi_1 + \phi_4 + \{3,5,7,11,\ldots,p_2,p_1\} \Rightarrow \{6, 8, 10, 12, \ldots, 2(n-1), 2n\} \) \)

\( \Rightarrow \{6, 8, 10, 12, \ldots, 2(n-1), 2n\} \in A_n \} \)

\( \emptyset_4 : \ \{ \{ \phi_1 + \phi_4 + \{3,5,7,11,\ldots,p_2,p_1\} \not\Rightarrow 2(n+1) \) \)

\( \Rightarrow \{2(n+1) \notin A_n \} \)

\{ Let: prime \( p \) satisfy: \( p \notin \{3,5,7,11,\ldots,p_2,p_1\} \), \( \emptyset_3, \emptyset_4 \).

\wave \{ p_1 < p, (p + \forall p) \notin \{6, 8, 10, 12, \ldots, 2(n-1), 2n\}, (p + \forall p) \notin A_n \} \}

(1)

Take the prime number that is greater than \( p_1 \) and adjacent to \( p_1 \) as \( p_0 \),
\wave (1) \Rightarrow \{ p_0 + 3 \not\approx 2n \ , \ p_0 + 3 \leq 2n \} \)
∴ \( p_0 + 3 > 2n \)
∴ \( p_0 + 2 \geq 2n \)  • (odd) ≠ (even)
∴ \( p_0 + 2 > 2n \)  ⇒  \( p_0 + 1 \geq 2n \)  Special record is (W)

Starting from (Analysis II)
The principle of mathematical complete induction:
it is correct in the front, until \( A_n \).

Take the continuous prime number \( (3, 5, 7, 11, \ldots, p_2, p_1) \) from small to large.

\[ A_n : \{ \phi_1 + \phi_4 + \{ 3, 5, 7, 11, \ldots, p_2, p_1 \} \} \Rightarrow : \]

\[ \{3+3=6 \]
\[ 5+3=8 \]
\[ 7+3=5+5=10. \text{ set up: } (7+3=5+5) \text{ sequence: } 7 > 5 \]
\[ 7+5=12 \]
\[ 11+3=7+7=14. \text{ set up } \Delta_2 : (11+3=7+7) \text{ sequence: } 11 > 7 \]
\[ 13+3=11+5=16. \text{ set up } \Delta_2 : (13+3=11+5) \text{ sequence: } 13 > 11 \]
\[ 13+5=11+7=18. \text{ set up } \Delta_2 : (13+5=11+7) \text{ sequence: } 13 > 11 \]
\[ 17+3=13+7=20. \text{ set up } \Delta_2 : (17+3=13+7) \text{ sequence: } 17 > 13 \]
\[ 19+3=17+5=11+11=22. \text{ set up } \Delta_2 : (19+3=17+5=11+11) \text{ sequence: } 19 > 17 > 11 \]
\[ 19+5=17+7=13+11=24. \text{ set up } \Delta_2 : (19+5=17+7=13+11) \text{ sequence: } 19 > 17 > 13 \]

\[ \ldots \ldots \]
\[ p_{c1} + p_{c2} = p_{c3} + p_{c4} = \ldots = 2(n-2) \]
\[ p_{h1} + p_{h2} = p_{h3} + p_{h4} = \ldots = 2(n-1) \]
\[ p_{a1} + p_{a2} = p_{a3} + p_{a4} = \ldots = 2n \} \text{ It is recorded as } A_n \]
⇒ :6,8,10,12,14,16,18, ... , 2(n-1),2n.

In \( A_n \), it is specified that : \( p_{a1} \geq p_{a2} \)
In $A_n$ it is specified that $p_{a_1} > p_{a_3}$ [Reason: $p_{a_1} + p_{a_2} = 2n$ must exist.]

$p_{a_3} + p_{a_4} = 2n$, not necessarily. If $p_{a_3} + p_{a_4} = 2n$ exists $p_{a_1}$ and $p_{a_3}$ if one of them is big, put the big one in the first place according to the regulations.

If: $p_{a_1} = p_{a_3} \Rightarrow \{p_{a_1} + p_{a_2} = 2n\} \subseteq \{p_{a_3} + p_{a_4} = 2n\}$

$\phi_4 \Rightarrow \{p_{a_3} + p_{a_4} = 2n\}$ is deleted $\therefore p_{a_1} > p_{a_3}$

$A_n$ is simplified as $B_n$.

\[
\begin{align*}
3 + 3 &= 6 \\
5 + 3 &= 8 \\
7 + 3 &= 5 + 5 = 10. \\
7 + 5 &= 12. \\
11 + 3 &= 7 + 7 = 14. \\
13 + 3 &= 11 + 5 = 16. \\
13 + 5 &= 11 + 7 = 18. \\
17 + 3 &= 13 + 7 = 20. \\
19 + 3 &= 17 + 5 = 11 + 11 = 22. \\
19 + 5 &= 17 + 7 = 13 + 11 = 24. \\
\end{align*}
\]

$\ldots \ldots$

$p_{c_1} + p_{c_2} = p_{c_3} + p_{c_4} = \ldots = 2(n-2)$

$p_{b_1} + p_{b_2} = p_{b_3} + p_{b_4} = \ldots = 2(n-1)$

$p_{a_1} + p_{a_2} = p_{a_3} + p_{a_4} = \ldots = 2n\}$ It is recorded as $B_n$

Change $B_n$ to $C_n$.

\[
\begin{align*}
\text{Level } 2(n-2): & 3 + 2(n-2) + 3 = 2(n+1) \\
\text{Level } 2(n-3): & 5 + 2(n-3) + 3 = 2(n+1) \\
\text{Level } 2(n-4): & 7 + 2(n-4) + 3 = 5 + 2(n-4) + 5 = 2(n+1) \\
\text{Level } 2(n-5): & 7 + 2(n-5) + 5 = 2(n+1) \\
\text{Level } 2(n-6): & 11 + 2(n-6) + 3 = 7 + 2(n-6) + 7 = 2(n+1) \\
\text{Level } 2(n-7): & 13 + 2(n-7) + 3 = 11 + 2(n-7) + 5 = 2(n+1) \\
\text{Level } 2(n-8): & 13 + 2(n-8) + 5 = 11 + 2(n-8) + 7 = 2(n+1) \\
\text{Level } 2(n-9): & 17 + 2(n-9) + 3 = 13 + 2(n-9) + 7 = 2(n+1) \\
\text{Level } 2(n-10): & 19 + 2(n-10) + 3 = 17 + 2(n-10) + 5 = 11 + 2(n-10) + 11 = 2(n+1) \\
\text{Level } 2(n-11): & 19 + 2(n-11) + 5 = 17 + 2(n-11) + 7 = 13 + 2(n-11) + 11 = 2(n+1)
\end{align*}
\]
Level 6: \( p_{c1} + 6 + p_{c2} = p_{c3} + 6 + p_{c4} = \ldots = 2(n+1) \)

Level 4: \( p_{b1} + 4 + p_{b2} = p_{b3} + 4 + p_{b4} = \ldots = 2(n+1) \)

Level 2: \( p_{a1} + 2 + p_{a2} = p_{a3} + 2 + p_{a4} = \ldots = 2(n+1) \)  It is recorded as: \( C_n \)

"Line \( \beta \)" in \( C_n: 2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow \ldots \rightarrow 2(n-5) \rightarrow 2(n-4) \rightarrow 2(n-3) \rightarrow 2(n-2) \)

In \( C_n \) (Level 2): \( \{ p_{a1}, p_{a2}, p_{a3}, p_{a4}, \ldots, p_{an} \} \), any element is modeled as \( p_a \), and each prime is abbreviated as \( p_a \).

In \( C_n \) (Level 4): \( \{ p_{b1}, p_{b2}, p_{b3}, p_{b4}, \ldots, p_{bn} \} \), any element is modeled as \( p_b \), and each prime is abbreviated as \( p_b \).

The same analogy follows (at each level).

Important note: in \( C_n \), I didn't specify \( p_{a1} > p_{b1} \), I didn't specify their size relationship.

Theorem (\( \omega_1 \)) : \( \{ p_1 \) and \( p_0 \) are adjacent prime numbers. \( p_0 > p_1 \} \Rightarrow p_0 > 2p_1. \)

Proof:
Assumptions: \( p_0 > 2p_1. \)
\[ \Rightarrow : p_0 > 2p_1 > p_1. \]
\[ \phi, +\{ p_0 > 2p_1 > p_1. \} \Rightarrow p_0 > 2p_1 > p_1 > p_1. \]
\[ \Rightarrow \{ p_0 > p_1 > p_1. \} \text{ contradiction. } *(p_1 \text{ and } p_0 \text{ are adjacent prime numbers.)} \]
\[ \therefore p_0 > 2p_1. \]

(QED).

Theorem (\( \omega_2 \)) : \( A_n, B_n, C_n \), if \( \{ 2n+2 = p_x + p_y \text{ set up } \Delta_1: p_x > p_y \} \)

There must be: \( p_x = p_0 \)

Proof:
If: \[ p_x + p_y = 2(n+1) \] (2)

(w)+(2): \[ p_0 + 3 \geq p_x + p_y \] (3)

\[ \text{【(w):} p_0 + 1 \geq 2n \Rightarrow p_0 + 1 + 2 \geq 2n + 2 = p_x + p_y \Rightarrow p_0 + 3 \geq p_x + p_y \text{】} \]

∵ (the smallest prime in \( \kappa \) is 3)

∴ \[ p_y \geq 3 \]

∴ (3) \[ p_0 \geq p_x \] (4)

∵ \[ \{ (4) \} + (6) \]

Because \( p_0 > p_1 \), and the prime number: \( p_0 \) and \( p_1 \) adjacent.

\[ \{ p_0 \text{ and } p_1 \text{ adjacent.} + (7) \} \Rightarrow \therefore p_x = p_0 \]

(\( \omega_2 \)) (QED).

**Theorem(\( \omega_3 \)) :**

Known: \{ \( A_n \), \( B_n \), \( C_n \), \( \phi_2 \), \( \phi_4 \), \( p_1 \) and \( p_0 \) are adjacent prime numbers , \( p_0 > p_1 \), \}

\( \beta \) line: \[ 2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow \ldots \rightarrow 2(n-6) \rightarrow 2(n-5) \rightarrow 2(n-4) \rightarrow 2(n-3) \rightarrow 2(n-2) \]

\( \kappa \) line: \[ 3 \rightarrow 5 \rightarrow 7 \rightarrow 11 \rightarrow 13 \rightarrow 17 \rightarrow 19 \rightarrow \ldots \rightarrow p_2 \rightarrow p_1 \]

\( \phi_2 \) Extreme law : Not allowed: \[ 2n + 2 = p_x + p_y \} \therefore \text{ There must be: } p_0 - p_1 > 2(n-2) \]

**Proof:**

\( p_1 \) and \( p_0 \) are adjacent prime numbers , \( p_0 > p_1 \),

Take: \( p_0 - p_1 = 2 \)
It can be proved that in \( C_n \): \((p_1 + \forall p) \notin \{ \text{Level 4, Level 6, Level 8, ..., Level 2(n-2) } \}

\begin{enumerate}
  \item \textbf{level 4:} \( p_1 \in p_{a1} \implies p_1 + 4 + p_{a2} = 2n + 2 \implies p_0 + 2 + 4 + p_{a2} = 2n + 2 \)

\( p_0 + p_{a2} = 2n \)  Contradiction with formula (1). \text{(*)}\( (p_0 + p_{a2}) \notin A_n \)

\item \textbf{level 6:} \( p_1 \in p_{a1} \implies p_1 + 6 + p_{a2} = 2n + 2 \implies p_0 + 2 + 6 + p_{a2} = 2n + 2 \)

\( p_0 + p_{a2} = 2n - 2 \)  Contradiction with formula (1). \text{(*)}\( (p_0 + p_{a2}) \notin A_n \)

\end{enumerate}

Same logic \( p_1 \notin \{ \text{Level 4, Level 6, Level 8, ..., Level 2(n-2) } \} \)

Record as (i)

\text{(*)}  \((p_1 + \forall p) \notin A_n \implies p_1 \notin p_{a1}\) \{ \text{(*)} \Delta_1, \Delta_2 \} \implies p_1 = p_{a1}

\implies p_1 + 2 + p_{a2} = 2n + 2 \implies p_0 + p_{a2} = 2n + 2 \implies \Theta_2 \quad (QED).

\( \phi_2 \) Extreme law: not allowed: \( 2n + 2 = p_x + p_y \implies \{ p_x - p_1 \neq 2 \}

Dispute 1: \( \phi_2 \) Extreme law: not allowed: \( p_1 \in p_{a1} \implies p_1 \notin p_{a1} \implies p_0 - p_1 \neq 2 \)

Dispute 2: \( \phi_2 \) Extreme law: not allowed: \( p_{i-} - p_1 = 2 \implies p_{i-} - p_1 \neq 2 \implies p_1 \in p_{a1} \}

It is indisputable that: \( p_0 - p_1 \neq 2 \implies p_0 - p_1 > 2

I will prove that: \( p_1 \notin p_{a} \)

\text{(*)}  \text{If:} \ p_1 \in p_{a} \quad \{ \text{(*)} \Delta_1, \Delta_2 \} \implies p_1 = p_{a1}

B_3: \( p_{a1} + p_{a2} = 2n \implies p_1 + p_{a2} = 2n \)

\text{(*)}  \text{line:} \ p_{a2} \geq 3.

\{ \text{Take:} \ p_{a2} = 3 \implies p_1 + 3 = 2n \implies p_1 + 5 = 2n + 2 \implies \Theta_2 \quad (QED) \}

\( \phi_2 \) Extreme law: not allowed: \( p_1 + 5 = 2n + 2 \). \text{(*)}  \ p_{a2} \geq 3.

\( p_{a2} \geq 3 \implies (p_1 + 3) < p_1 + p_{a2} = 2n \implies (p_1 + 3) < 2n \)

\text{(*)}  \( p_1 + 3 < 2n \} \implies (p_1 + 3) \in \{ \text{Level 4, Level 6, Level 8, ..., Level 2(n-2) } \}

Take: \( (p_1 + 3) = 2f \)

\( \phi_4: \implies (p_1 + 1) = 2f - 2 \implies (p_1 + \forall p) \notin \{ \text{Level (2f-2), Level (2f-2), Level (2f-4), Level (2f-6), ..., Level 2(n-2) } \}

\( \implies (p_1 + 3) = 2f \implies \{ \phi_4 \}: p_1 \text{ appears for the first time.} \quad \text{Record as (x)} \)

\( \phi_4: \implies (p_1 + 1) = 2f - 2 \implies (p_1 + \forall p) \notin \{ \text{Level (2f-2), Level (2f-2), Level (2f-4), Level (2f-6), ..., Level 2(n-2) } \}

\{ \implies (p_1 + \forall p) \notin \{ (2f-2), (2f-4), (2f-6), ..., 8, 6 \} \}

\text{Record as (e)}
\[ p_2 \in \{3, 5, 7, 11, \ldots, p_2\} \quad \text{Record as (h)} \]

\[ \{x\} \cap (e) + \phi_4 \Rightarrow 6, 8, 10, \ldots, (2f-2) \quad \text{It stopped.} \]

Known: 6, 8, 10, \ldots, (2n-2), (2n-1)\(2n\) Only A1 is broken in the middle.

\[ p_1 = 5 \quad \text{Contradiction with (analysis II)} \]

\[ p_1 \notin A \]

\[ \Rightarrow \{ p_0 - p_1 \neq 2, p_1 \notin p_3 \} \]

\[ \Rightarrow \{ p_0 - p_1 > 2, p_1 \notin p_3 \} \quad (8) \]

Take: \( p_0 - p_1 = 4 \)

Similarly, the logic of (i) can prove that \( C_n: \{ (p_1+\bigvee p_4) \notin \{ \text{Level 6}, \text{Level 8}, \ldots, \text{Level 2} (n-2) \} \}

\[ (8) \Rightarrow \{ p_1 \notin p_3, \quad \therefore p_1 \notin A_n, \quad \therefore p_1 \notin p_3 \Rightarrow p_1 = p_{b1} \quad \text{Conflict with (1)} \}

\[ \text{Level 4: } p_{b1} + 4 + p_{b2} = 2n + 2 \Rightarrow p_1 + 4 + p_{b2} = 2n + 2 \Rightarrow p_0 + p_{b2} = 2n + 2 \Rightarrow \Theta_2 \quad (QED). \]

\[ \phi_2 \quad \text{Extreme law: not allowed: } p_0 - p_1 = 4 \Rightarrow p_0 - p_1 \neq 4 \]

I will prove that: \( p_1 \notin p_3 \)

\[ \text{If: } p_1 \notin p_3, \quad \therefore p_1 \notin p_3 \]

\[ B_n: \quad p_{b1} + p_{b2} = 2n - 2 \Rightarrow p_1 + p_{b2} = 2n - 2 \]

\[ \checkmark \quad \text{line: } p_{b2} \geq 3. \]

Take: \( p_{b2} = 3 \Rightarrow p_1 + 3 = 2n - 2 \Rightarrow p_1 + 5 = 2n \Rightarrow \text{Conflict with (8)} \quad \therefore p_1 \notin p_3 \]

\[ \therefore p_0 > 3 \Rightarrow (p_1 + 3) < p_1 + p_{b2} = 2n - 2 \Rightarrow (p_1 + 3) < 2n - 2 \]

\[ \therefore \{ \phi_4, (p_1 + 3) < (2n - 2) \} \Rightarrow (p_1 + 3) \notin \{ \text{Level 6}, \text{Level 8}, \ldots, \text{Level 2} (n-2) \} \]

\[ \text{Same logic (Reference: (x), (e), (h)) } \Rightarrow p_1 = 5 \quad \text{Contradiction with (analysis II)} \]

\[ \therefore p_1 \notin p_3 \]

\[ \Rightarrow \{ p_0 - p_1 \neq 4, p_1 \notin p_3 \} \]

\[ \Rightarrow \{ p_0 - p_1 > 4, p_1 \notin \{ p_0, p_3 \} \} \quad (9) \]

Take: \( p_0 - p_1 = 6 \)
The same logic can be proved C_n: \( \{ (p_1 + \forall p) \in \text{Level 8}, \ldots, \text{Level 2 (n-2)} \} \)【∵ Conflict with (1).】Record as (iii).

(9) \( \Rightarrow \) : \( p_1 \in \{ p_a, p_b \} \), \( \therefore \) \( p_1 \in A_n \), \( \therefore \) \( p_1 \in p_c \) \( \Rightarrow \) \( p_1 = p_{c1} \)【∵ \( \triangle_1, \triangle_2 \).】 

Level 6: \( p_{c1} + 6 + p_{c2} = 2n+2 \Rightarrow p_1 + 6 + p_{c2} = 2n+2 \Rightarrow p_0 + p_{c2} = 2n+2 \Rightarrow \emptyset_2 \) (QED).

\( \phi_2 \) Extreme law: not allowed: \( p_0 - p_1 = 6 \Rightarrow p_0 - p_1 \neq 6 \)

I will prove that: \( p_1 \notin p_c \)

【If: \( p_1 \notin p_c \), \( \{ \therefore \triangle_1, \triangle_2 \}. \) \Rightarrow : p_1 = p_{c1} \】 

B_n: \( p_{c1} + p_{c2} = 2n-4 \Rightarrow p_1 + p_{c2} = 2n-4 \)

\( \kappa \) line: \( p_{c2} \geq 3. \)

Take: \( p_{c2} = 3 \Rightarrow p_1 + 3 = 2n-4 \Rightarrow (p_1 + 3) < 2n-4 \Rightarrow (p_1 + 3) < 2n-4 \)

\( \therefore \) \( \{ p_1, (p_1 + 3) < 2n-4 \} \Rightarrow (p_1 + 3) \notin \{ \text{Level 8}, \ldots, \text{Level 2 (n-2)} \} \)

Same logic(Reference: (x),(e),(h)) \( \Rightarrow : p_1 = 5 \) Contradiction with (analysis II).

\( \therefore : p_1 \notin p_c \)

\( \Rightarrow \{ p_0 - p_1 \neq 6, p_1 \notin p_c \} \)

\( \Rightarrow \{ p_0 - p_1 > 6, p_1 \notin \{ p_a, p_b, p_c \} \} \) (10)

... 

\( \beta \) line: \( 2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow \ldots \rightarrow 2(n-6) \rightarrow 2(n-5) \rightarrow 2(n-4) \rightarrow 2(n-3) \rightarrow 2(n-2) \)

C_n: (Level 2 (n-2)) also follows the same principle: \( p_0 - p_1 > 2 (n + 2) \left( \omega_3 \right) \) (QED).

\( (\omega_3) \) \( \Rightarrow \) \( p_0 - p_1 > 2(n-2) \Rightarrow p_0 - p_1 > 2n-4 \geq p_{i1} + p_{i4} \)【∵ \( 2n \geq p_{i1} + p_{i4} \)】

\( \therefore \) \( p_0 > (2p_1 + p_{i4} - 4) \)

\( \therefore \) \( p_0 \geq (2p_1 + p_{i4} - 3) \)

\( \therefore \) (the smallest prime in \( \kappa \) is 3) \( \Rightarrow p_1 \geq 3 \)【∵ \( p_0 \geq 2p_1 \)

(odd) \( \neq \) (even). \( \therefore \) \( p_0 > 2p_1 \)
\( p_0 > 2p_1 \) contradicts \((\omega_1)\)

It is proved that it is wrong to quote "extreme law \( \phi_2 \)" in this process.

As long as one of them does not quote the "extreme law \( \phi_2 \)"

Must get: \( p_x + p_y = 2(n+1) \) \( \Rightarrow \) \( p_0 + p_y = 2(n+1) \) [Theorem \((\omega_2)\): \( p_0 = p_x \) ]

\( \omega_2 \Rightarrow \) New (\( \kappa \) line): \( 3 \rightarrow 5 \rightarrow 7 \rightarrow 11 \rightarrow 13 \rightarrow 17 \rightarrow 19 \rightarrow ... \rightarrow p_1 \rightarrow p_0 \)

At this time, the complete proof \( A_n \) is followed by \( p_0 + p_y = 2(n+1) \)

\( \Theta_2 \) (QED).

\( \Theta_1 \) and \( \Theta_2 \) are equivalent: \( \Theta_2 \) (QED) \( \Rightarrow \) \( \Theta_1 \) (QED).

4 Conclusion

\( \Theta_1 \) and \( \Theta_2 \) are equivalent: \( \Theta_2 \) (QED) \( \Rightarrow \) \( \Theta_1 \) (QED).

Complete the mathematical complete induction:

\{ \( A_1 \) (\( \kappa \) line:3) \( \Theta_1 \) (QED) \},

\( A_n \) (\( \kappa \) line:3,5,7,...\( p_1 \)) \( \Theta_1 \) (QED),

\( A_{n+1} \) (\( \kappa \) line:3,5,7,...\( p_1,p_0 \)) \( \Theta_1 \) (QED)\}.

There is another situation: it is impossible to stop, it is impossible to stop at \( A_n \), and it is always infinite. \( \Rightarrow \) \( \Theta_1 \) (QED).

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6 References cited:

[1] Bertrand Chebyshev theorem: if the integer n > 3, then there is at least one prime P, which conforms to n < p < 2n−2. Another slightly weaker argument is: for all integers n greater than 1, there is at least one prime P, which conforms to n < p < 2n.
https://www.researchgate.net/publication/228592091_A_Generalization_of_Erdos's_Proof_of_Bertrand-Chebyshev_Theorem

[2] The theorem of infinite number of primes: the n bit after each prime can always find another prime.
https://www.researchgate.net/publication/266044680_THE_NUMBER_OF_PRIMES_IS_INFINITY