On the particle production threshold for ultra-relativistic accelerated protons

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Abstract. Non-electromagnetic emission from cosmic ray particles accelerated in extreme environments has been studied using different variations of semi-classical formalisms. As the energy loss mechanisms of such particles is of great interest, one must improve in some sense the previously intuitive description of classical sources in the presence of fields. In this brief note we evaluate the role played by the classical and dimensional proper acceleration of a particle in radiation processes. One intends to show that the introduction of the acceleration as a measure parameter gives an adequate scale for considering the processes of massive particle emission. If the acceleration threshold is not attained one is considering a regime where the processes of massive particle production are suppressed. The analysis is performed in a semiclassical formalism once applied in different contexts and already checked for consistency in a serie of papers. As a further application of the results we evaluate the possibility of a non negligible meson production by protons accelerated in the framework of polar cap models operating in electromagnetic fields of pulsars. It is shown that inside a systems endowed with magnetic fields \( B \geq 10^{12} \) G the meson emission by protons must not be disregarded

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1. Introduction

The rates and emitted powers of processes in the presence of (classical) $F_{ab}$ fields are usually obtained in the context of classical or semiclassical approaches. It is argued (based on the calculations of the electromagnetic spectra of radiation) that to disregard quantum effects it is mandatory that the source of mass $m$ obeys (1:2:3),

$$\chi = \frac{B^*}{B_{cr}} < 1,$$

where $B^* \equiv \gamma B$ is the magnitude of the magnetic component of the electromagnetic field as measured in the instantaneous reference frame of the source; $\gamma$ is the usual Lorentz factor and $B_{cr} = m^2/e$ is the critical field, which gives the value of the magnetic field where the quantum effects must be taken into account. If $\chi \geq 1$, the backreaction effects become important and a full quantum formalism must be employed. Otherwise if condition (1) is satisfied the classical calculations are accurate and the associated emitted power depends only on the proper acceleration of the source (4). For instance in the presence of a dipolar magnetic field the proper acceleration of the (ultra-relativistic) emitting source is $a \approx \gamma eB/m$ and consequently the limit given by the parameter (1) can be rewritten as

$$\chi = \frac{a}{m} < 1 \quad \longleftrightarrow \quad a < m.$$

(2)

The $\chi$ invariant is usually defined in a frame independent way as $\chi \equiv \sqrt{(eF^{ab}p_a)^2/m^3}$ and depends on the electromagnetic field ($F_{ab}$) and on the momentum ($p_a$) of the source.

In this brief research note we explore the role played by the classical acceleration on processes of emission of massive particles. Essentialy, we specify the ranges of the proper acceleration of the source where a given emission channel of massive fields becomes relevant. At the same time we introduce a semi-quantitative argument for the validity of the classical approximation when one is interested in such a class of processes. Furthermore we expect that the present framework can give some insight about the previous arguments on the applicability of effective approaches. As a further application we analyze the possibility of extra mechanisms of particle production inside a particular engine of ultra-high energy cosmic rays (UHECR) acceleration that may be useful for some astrophysical observations.

The range of proper acceleration where the process becomes favored is a natural consequence of the basic assumptions of the formalism. It is shown that if the threshold for the massive particle production is not taken as a measure parameter it can lead to different conclusions about the relevance of such class of process.

We adopt natural units where $c = \hbar = 1$ unless stated otherwise and Minkowski signature of the metric $\eta_{ab} = (+, -, -, -)$

2. The Formalism and Meson Emitted Powers

After a brief review of the semi-classical formalism of emission, we obtain the formulas for the radiated power of massive scalar and vector mesons. The details of the calculations can be found in (5:6).
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We refer here to the following class of processes:

\[ p_1 \to p_2 + \sum_i p_i. \]  \hspace{1cm} (3)

The \( p_i \) are the emitted fields, \( p_1 \) and \( p_2 \) are the states of a two level system following a classical trajectory. The respective masses \( m_i \) of \( p_i \) satisfies \( m_i < m_{1,2} \) being \( m_{1,2} \) the masses of \( p_{1,2} \). The arguments for obtaining the classical limit (1) can be found in (3). Here we begin with the intuitively statement that if the source is supposed to follow a prescribed trajectory, it is assumed that the tri-momentum \( k_{\text{rf}} \) of the radiated “\( i \)” field must be constrained to

\[ |k_{\text{rf}}| \ll m_{1,2} \]  \hspace{1cm} (5; 6). The tri-momentum \( |k_{\text{rf}}| \) is measured in the rest frame of the source. Furthermore in (5) it is shown that the mean energy \( \omega_{\text{rf}} \) of the emitted particles are of the order of the acceleration of the current, i.e.,

\[ \omega_{\text{rf}} \sim a, \]  \hspace{1cm} (4)

(see Fig. (3) in (5)). Then, from the condition \( m_i \ll m_{1,2} \) and \( \omega_{\text{rf}}^2 = |k_{\text{rf}}|^2 + m_j^2 \) one has \( a \ll m_{1,2} \) in agreement with (2).

Further, from (4) one can argue that if the acceleration of the source is

\[ a \geq \sum_i m_i \]  \hspace{1cm} (5)

the general (inertially forbidden) process (3) becomes energetically favoured. Consequently from condition (4) the class of processes (3) becomes more relevant when \( a > \Delta m + \sum_i m_i \) where \( \Delta m = m_2 - m_1 \). Then one has the range

\[ \Delta m + \sum_i m_i \leq a < m, \]  \hspace{1cm} (6)

from the point of view of energetics and the validity of the classical approach in a frame independent form as stated before. Consequently the emission of a \( \pi^0 \) meson by an accelerated proton,

\[ p^+ \to p^+ \pi^0, \]  \hspace{1cm} (7)

will be favoured when \( a \geq m_\pi \approx 140 \text{ MeV} \). This statement about a lower limit for the acceleration is the main contribution of the formalism devised in the previous papers. As we are going to check in the last section, if this quantitative threshold is not taken into account the respective reaction rates becomes irrelevant when compared to the non-massive case.

In the physical relevant range the emitted power of the process (5) for a proton in circular motion can be easily adapted from the formulas of the scalar emission previously calculated in (6).

\[ W_{p^+ \to p^+ \pi^0} \approx \frac{G_{\text{eff}} a^2}{12\pi}. \]  \hspace{1cm} (8)

In order to obtain that the power (8) from Eq.(4.7) of (6) one must perform the following modifications on the due particle states: \( |\pi^+\rangle \leftrightarrow |\pi^-\rangle \) and \( |n\rangle \leftrightarrow |p\rangle \). Those substitutions ensures the charge conservation and that the initial and final charged nucleon states are coupled to the magnetic field, respectively.
Now we apply the same formalism to consider the emission process of massive vector mesons

\[ p^+ \rightarrow p^+ \rho, \]  

by a non-inertial proton in circular motion. We omit the details of the derivations of the formulas once it can be performed in a similar fashion as it is done in previous papers. The semi-classical current in this case is

\[ \hat{j}^a(x) = \hat{q}(\tau) \frac{\delta^3[x - x(\tau) - \frac{u^a(\tau)}{\sqrt{-g}u^0(\tau)}u^a(\tau)]}{\sqrt{-gu^0(\tau)}}, \]  

\[ u^a = dx^a/d\tau \] where \( \tau \) is the proper time and \( \hat{q}(\tau) = e^{i\hat{H}_0\tau}\hat{q}_0e^{-i\hat{H}_0\tau}. \) \( \hat{H}_0 \) is the proper Hamiltonian \( \langle \hat{H}_0|p_{1,2} \rangle = m_{1,2}|p_{1,2} \rangle \) and \( \hat{q}_0 \) is a self-adjoint operator. The coupling between the current and the second quantized meson field is given by the action

\[ \hat{S}_I = \int d^4x \hat{j}^a(x) \left[ \hat{A}_a^\lambda(x) + \hat{A}_a^{\lambda\dagger}(x) \right]. \]  

The \( \hat{A}_a^\lambda(x) \) are obtained from the solutions of the Proca equation. In a rest reference frame, the worldline of the source is that of a particle in a circular trajectory of radius \( R \) and angular frequency \( \Omega \). The emitted power for the process \( (9) \) obtained from this procedure when \( \gamma \gg 1 \) and \( a < m \) is

\[ W_{p^+ \rightarrow p^+ \rho} \approx \frac{2G_{\text{eff}}^{(v)2}a^2}{3\pi}. \]

The effective coupling constant,

\[ |\langle p_2|\hat{q}_0|p_1 \rangle| \equiv G_{\text{eff}}^{(v)}, \]  

is to be associated with the strong coupling constant \( G_{\text{eff}}^{(v)2} \mapsto g^2 \equiv g^2/(4\pi) \approx 14 \) as well as in the former scalar case \( G_{\text{eff}}^{(s)2} \mapsto g^2 \). If the conditions given above are not accomplished in a system, a full quantum calculation of the transition rate is necessary, as done in (2). However, as we are going to see, this range is enough for our discussion and applications.

It can be verified that Eq. (12) can lead to the Larmor formula for photon radiation by means of the direct association: \( \alpha \mapsto 2g^2 \). This happens because the coupling of protons with photons and \( \rho \) mesons are both given by the action (11). Both interactions are of a vector type and the states can be that of a massive or a zero rest mass field distinguishable in the sum over the polarization states. Thus the final form of the emitted powers differs only by the magnitude of the coupling constant. Once we establish the role of the proper acceleration it is argued that in a regime where the acceleration exceeds the rest mass of the emitted field, \( (a/m_\rho \gg 1) \) the fact that this mass differs from zero becomes irrelevant and can be disregarded.

It must be emphasized that the formalism presented here cannot be applied when the charges in the initial and final states of the current are different. Neutral and charged particles follow distinct trajectories in the presence of an electromagnetic field. The related channel \( p^+ \rightarrow n\pi^+ \) for example, requires a different semi-classical formalism. The formalism and the respective calculations can be found in (8).
3. Applications

We now discuss some aspects and perform a practical application of the introduction of the parameter of massive particle production derived from the present formalism.

The distinction between the respective emitted power of π and photon radiation comes from the difference on strong and electromagnetic coupling constants. A comparison between (8) and the Larmor formula for synchrotron radiation shows that \( W_\pi / W_\gamma \sim g'^2/\alpha \gg 1 \) where \( \alpha \) is the electromagnetic coupling constant. Then if the proper acceleration of a proton reach the respective values \( a \sim m_\rho,\pi \) the channels (9) or (5) could be of interest.

As an application of the results we consider now the acceleration of particles in polar cap models of pulsars (9; 10). In those models, a proton interacts with the dipolar magnetic field of the pulsar and is accelerated by a parallel electric field. Firstly the component \( p_\perp \) (transverse to the magnetic fields) of the \( p^\pi \) momenta vanish due to the usual synchrotron radiation before they reach the threshold for the meson emission. Furthermore, the protons with momenta \( p_\parallel \) (parallel to the magnetic and electric fields) are accelerated by the electric potential drop of the pulsar along the magnetic field lines, producing photons by curvature radiation. The pion curvature radiation (5) can also occur (11; 12). The low \( \chi \) regime is usually taken as \( \chi \ll m_\pi/m \) (2; 13), which is equivalent to \( a \ll m_\pi \). In such a range, the respective rates of process (5) decreases exponentially by a \( \chi \) factor and consequently the pion emission will not be relevant in the known astophysical sites (12). This conclusion agrees with the present analysis in the range \( a \ll m_\pi \). As we are going to see further, this is not the case in the due regime where \( a > m_\pi \).

A particle following a circular trajectory of radius \( R_c \) has proper acceleration given by \( a \approx \gamma^2/R_c \). The curvature radius \( R_c \) of the magnetic field line of a pulsar is related to the radius \( r_s \) of the star and to the light cone radius, \( R_L = 2\pi P \) through \( R_c = 4/3(r_sR_L)^{1/2} \), where \( P \) is the star period. Then, one has

\[
a \approx \frac{\eta^2 e^2\phi^2}{m^2 R_c}. \tag{14}
\]

The \( \eta \) factor accounts for the efficiency of the polar cap acceleration mechanism (14; 15) \( \gamma = \epsilon/m = \eta e\phi/m \). Also,

\[
\phi = \left( \frac{2\pi^2 B r_s^3}{P^2} \right) \approx 6.6 \times 10^{18} \left( \frac{B}{10^{12} \text{G}} \right) \left( \frac{1 \text{ms}}{P} \right)^2 \left( \frac{r_s}{10^4 \text{m}} \right)^3 \text{V}, \tag{15}
\]

is the maximum available potential drop near the surface of the star (12; 17). For a typical young pulsar the rotation frequency is \( P \sim 1 \text{ ms} \) and \( r_s \sim 10^4 \text{ m} \). Then, one has from (14) and (5), the condition

\[
\left( \frac{B}{10^{12} \text{G}} \right)^2 \left( \frac{\text{ms}}{P} \right)^{9/2} > 2.6 \eta^{-2} \left( \frac{m_\pi}{m_\rho} \right), \tag{16}
\]

for the meson emission occurs with a non negligible intensity. It is easily verified that for \( \eta = 0.1 \sim 1 \) the condition (16) is fullfilled for typical values of the pairs \((B, P)\) of the known
pulsars \cite{18}. We plot in Fig. (1) the values of the pulsar parameters satisfying (16) for the emission of scalar and vector mesons.

The relevance of the scalar meson emission must of course, be analyzed in the usual fashion from the cooling and acceleration timescales \cite{19,12} in the physical relevant regime. The relationship between the respective cooling times ($t = \epsilon/W$) for the scalar and gamma curvature radiation is

$$t_s = \frac{e^2}{4g^2} t_\gamma.$$  \hfill (17)

The maximum attainable energy is given by the balance condition between the rate of energy gain and the rate of energy loss \cite{20}. Then, from the equality

$$t_{\pi,\rho,\gamma} = t_a,$$  \hfill (18)

where $t_a = \epsilon/\phi$ is the acceleration time, one has

$$\gamma = 6 \times 10^6 \left[ \left( \frac{B}{10^{15} G} \right) \left( \frac{1 \text{ms}}{P} \right) \right]^{1/4},$$  \hfill (19)

$$\gamma = 1 \times 10^7 \left[ \left( \frac{B}{10^{15} G} \right) \left( \frac{1 \text{ms}}{P} \right) \right]^{1/4},$$  \hfill (20)

$$\gamma = 1 \times 10^8 \left[ \left( \frac{B}{10^{15} G} \right) \left( \frac{1 \text{ms}}{P} \right) \right]^{1/4},$$  \hfill (21)

for the vector, scalar meson and photon emission respectively, where the parameters $B$ and $P$ must satisfy (16). We stress that if $\eta(B/10^{15} G)^2 (\text{ms}/P)^{9/2} < 2.6 \times 10^{-6} m_x/m_p$, the $\pi$ and $\rho$ meson emission rate is highly suppressed and the energy losses from usual

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{The values of the ($B, P$) pairs satisfying (16) for a (conservative) efficiency factor, $\eta = 0.1$. The full line corresponds to the values of ($B, P$) satisfying the threshold $m_x = m_\rho = 770$ MeV for the $\rho$ emission; the dashed line is the respective curve for a $\pi$ meson $m_x = m_\pi = 140$ MeV.}
\end{figure}
emagnetic curvature radiation dominates over the related emission of massive fields. Because of the non-recoil condition it is not possible to draw conclusions for a pulsar satisfying \( \eta(B/10^{15} \text{G})^2(\text{ms}/P)^{9/2} \geq 2.3 \times 10^{-6} \).

From Eqs. (19)-(21) it is possible to verify that the energy scale for the production of mesons (and consequently the respective cooling times for the meson curvature radiation) is less than the respective scale for ordinary curvature emission in pulsars with high magnetic fields. We can argue that the \( \pi \) and \( \rho \) curvature radiation can efficiently drain the proton energy when they reach energies \( \epsilon \approx 10^{15} \text{ eV} \) in millisecond pulsars if the magnetic field is \( 10^{12} \text{ G} \). Since the scalar and vector mesons produces different neutrino flavors from its decay channels, the meson synchrotron emission could be inferred from some detected excess of neutrinos. Such an analysis is not performed here and must be published elsewhere.

It is well known that there are energy loss mechanisms other than curvature and synchrotron radiation, e.g., photopion production and inverse Compton scattering. The present application does not take these channels into account and just considers the relationship between the scalar meson and photon curvature radiation from protons inside the region of particle acceleration in pulsars.

4. Conclusions

In this brief note we have evaluated the role played by the proper acceleration of a source when one is considering the classical limit of field methods. One can see from Eq. (16) and Fig. (1) that the respective threshold for meson production is realized for pulsars endowed with magnetic fields of order \( B = 10^{12} - 10^{15} \text{ Gauss} \) and respective periods, \( P \approx (1 - 10) \text{ ms} \); for higher periods the corresponding magnetic fields would be \( B > 10^{15} \text{ G} \). Those statements follows directly from the lower limit for acceleration of the sources, \( a \geq m_x \); it has not been taken into account in the previous works on emission of massive meson fields (12, 13). The absence of this lower limit for favouring the process has lead to the conclusion that those processes could never take place in an astrophysical environment due the dominance of electromagnetic losses. Then if one is dealing with classical or semiclassical treatment of the emission rates the lower (threshold for massive particle emission) \( a_{\text{source}} > m_{\text{emitted}} \) and the upper (non-recoil condition) limit \( a_{\text{source}} < m_{\text{source}} \) must constrain the range of calculations.

It is desirable to account for the consequences of this extra mechanism of energy loss in models of particle acceleration where high magnetic fields of pulsars are needed.

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References

[1] Ginzburg V L and Syrovatskii S I 1965 Ann. Rev. Astron. Astrophys. 3 297
[2] Zharkov G F 1965 Sov. Phys. JETP 20 1525
[3] Erber T 1966 Rev. Mod. Phys. 38 626
[4] Jackson J D 1999 Classical electrodynamics 3rd ed (New York, NY: Wiley)
[5] Vanzella D A and Matsas G E 2001 Phys. Rev. D63 014010
[6] Fregolente D, Matsas G E A and Vanzella D A T 2006 Phys. Rev. D74 045032
[7] Fregolente D and Saa A 2008 Phys. Rev. D77 103010
[8] Herpay T and Patkós A 2008 J. Phys. G 35 025201
[9] Ruderman M A and Sutherland P G 1975 Astrophys. J. 196 51
[10] Harding A K, Usov V V and Muslimov A G 2005 Astrophys. J. 622 531
[11] Berezinsky V, Dolgov A and Kachelrieß M 1995 Phys. Lett. B 351 261
[12] Herpay T, Razzaque S, Patkós A and Mészáros P 2008 J. Cosm. Astropart. Phys. 8 25
[13] Tokuhisa A and Kajino T 1999 Astrophys. J. 525 L117
[14] Arons J 2003 Astrophys. J. 589 871
[15] Zhang B, Dai Z G, Mészáros P, Waxman E and Harding A K 2003 Astrophys. J. 595 346
[16] Ostrowski M 2002 Astropart. Phys. 18 229
[17] Harding A K and Muslimov A G 2001 Astrophys. J. 556 987
[18] Manchester R N 2004 Science 304 542
[19] Aharonian F A 2004 Very high energy cosmic gamma radiation: a crucial window on the extreme Universe (River Edge, NJ: World Scientific Publishing)
[20] Vietri M 2008 Foundations of High-Energy Astrophysics Theoretical Astrophysics (Chicago, IL: Chicago Univ. Press)