Soft supersymmetry breaking terms from $D_4 \times Z_2$ lepton flavor symmetry

Hajime Ishimori$^1$, Tatsuo Kobayashi$^2$, Hiroshi Ohki$^3$, Yuji Omura$^3$, Ryo Takahashi$^1$ and Morimitsu Tanimoto$^4$

$^1$Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan
$^2$Department of Physics, Kyoto University, Kyoto 606-8502, Japan
$^3$Department of Physics, Kyoto University, Kyoto 606-8501, Japan
$^4$Department of Physics, Niigata University, Niigata 950-2181, Japan

Abstract

We study the supersymmetric model with $D_4 \times Z_2$ lepton flavor symmetry. We evaluate soft supersymmetry breaking terms, i.e. soft slepton masses and A-terms, which are predicted in the $D_4$ flavor model. We consider constraints due to experiments of flavor changing neutral current processes.
1 Introduction

Understanding the origin of fermion flavor structure, i.e. quark/lepton masses and mixing angles, is one of important issues in particle physics. Indeed, various types of mechanisms have been proposed to realize realistic mass matrices of quarks and leptons. Hereafter, we refer to such mechanisms as flavor mechanisms. Non-Abelian discrete flavor symmetries are interesting proposals and several types of models with non-Abelian discrete flavor symmetries have been constructed \([1]\).

Origins of such non-Abelian discrete flavor symmetries are not clear in 4D field theory, but those may be originated from geometrical structures of extra dimensional field theories and superstring theories on 6D compact spaces. In Ref. \([2, 3, 4]\) it has been shown that certain flavor structures can be derived from heterotic string models on orbifolds. One of typical non-Abelian discrete flavor symmetries, which can appear from heterotic orbifold models, is \(D_4\) symmetry and it can appear from factorizable orbifolds including the \(Z_2\) orbifold. Indeed, several semi-realistic string models with the \(D_4\) flavor symmetry have been constructed in Ref. \([2, 5]\), where three families consist of \(D_4\) singlets and doublets. Thus, it is important to study several phenomenological aspects of \(D_4\) flavor models.

Grimus and Lavoura proposed first the \(D_4\) flavor model for the lepton sector \([6]\) and subsequently other several models were studied \([7,8]\). Recently, the authors proposed a new \(D_4\) lepton flavor model \([9]\), which has only a single electroweak Higgs field, while the Grimus-Lavoura model as well as other models has three electroweak Higgs fields.

Supersymmetric extension of the standard model (SM) is one of interesting candidates for new physics beyond a TeV scale. Supersymmetry (SUSY) can stabilize the Higgs mass against radiative corrections due to heavy modes around high energy scales, e.g. the right-handed Majorana neutrino mass scale, the GUT scale, the Planck scale, etc. Supersymmetric standard models also have a good candidate for dark matter. Furthermore, in the minimal supersymmetric standard model (MSSM) with a pair of up and down Higgs supermultiplets, three gauge couplings are unified at the GUT scale \(M_X \simeq 2 \times 10^{16} \text{ GeV}\) in a good accuracy.

In supersymmetric models, quarks and leptons have their superpartners, i.e. squarks and sleptons. If we have a flavor mechanism to lead to realistic masses and mixing angles of quarks and leptons, such a flavor mechanism would affect mass matrices of squarks and sleptons. Furthermore, if a spe-
pecific pattern of sfermion masses is derived by a certain flavor mechanism, that would become the prediction of a certain flavor mechanism, which could be tested by measuring squark/slepton masses in future experiments. Thus, it is important to study patterns of sfermion masses, which are obtained by flavor mechanisms leading to realistic fermion masses and mixing angles. Although squarks and sleptons have not been detected yet, their mass matrices are constrained severely by experiments of flavor changing neutral current (FCNC) processes [10]. Off-diagonal elements of sfermion mass matrices must be suppressed in the super-CKM basis, where fermion masses are diagonalized, when sfermion masses are of order of the weak scale. At any rate, it is important to study which patterns of sfermion mass matrices, i.e. soft scalar masses and A-terms, are derived from each flavor mechanism.

In this paper, we study supersymmetric extension of the $D_4$ model of Ref. [9] as well as the Grimus-Lavoura model. The $D_4$ model of Ref. [9] has a single electroweak Higgs field, while other models have more than one electroweak Higgs fields. This difference is important in supersymmetric extensions. The former becomes the MSSM with a pair of up and down Higgs fields at low energy, but the latter would have more pairs of Higgs fields. The latter may violate the gauge coupling unification unless we introduce extra colored supermultiplets. That is the reason why we study mainly supersymmetric extension of the $D_4$ model of Ref. [9]. We evaluate soft SUSY breaking terms of sleptons, which are derived in supersymmetric extension of the $D_4$ model [9] as well as the Grimus-Lavoura model. We compute soft scalar masses and A-terms for the lepton sector in these supersymmetric models within the framework of the gravity mediation of SUSY breaking. We examine constraints due to FCNC experiments.

The paper is organized as follows. In Section 2, we supersymmetrize the $D_4$ model of [9] and show values of parameters consistent with neutrino oscillation experiments. In Section 3, we evaluate soft SUSY breaking terms of sleptons, i.e. soft scalar mass matrices and A-terms. We examine FCNC constraints on those SUSY breaking terms as mass insertion parameters. Section 4 is devoted to conclusion and discussion. In Appendix, we also discuss supersymmetric extension of the Grimus-Lavoura model. We evaluate soft SUSY breaking terms in the supersymmetric Grimus-Lavoura model and show that it is almost the same as the results obtained in Section 3.

---

1 See also e.g. Ref. [11] and references therein.
2 See Ref. [4] for a similar analysis on the quark sector.
2 Supersymmetric model with $D_4$ flavor symmetry

In this section, we study our supersymmetric $D_4$ model, based on [9]. The $D_4$ symmetry has five irreducible representations, i.e. one doublet $2$ and four singlets $1_{++}$, $1_{+-}$, $1_{-+}$ and $1_{--}$, where $1_{++}$ is the trivial singlet and the others are non-trivial singlets. Their tensor products are obtained as

$$2 \otimes 2 = 1_{++} \oplus 1_{+-} \oplus 1_{-+} \oplus 1_{--}, \quad 1_{ab} \otimes 1_{cd} = 1_{ef}, \quad (1)$$

where $a, b, c, d = \pm$ and $e = ac$ and $f = bd$.

The three generations of left-handed, right-handed charged lepton and right-handed neutrino chiral superfields are denoted by $L_I, R_I, N_I$, ($I = e, \mu, \tau$), respectively, and are assigned to $D_4$ trivial singlets $1_{++}$ and doublets $2$. We also introduce a $D_4$ doublet, $(\chi_1, \chi_2)$, a $D_4$ non-trivial singlet, $\chi_{--}$, and a $D_4$ trivial singlet, $\chi$. They are all SM-gauge singlet chiral superfields.

Moreover, we introduce additional $Z_2$ symmetry and we assume that the superfields $R_e, (R_\mu, R_\tau), \chi, \chi_{--}$ have $Z_2$-odd charges. The others are $Z_2$-even. Higgs chiral superfields for both up and down sectors, $H^{u,d}$, are assumed to be $D_4$ trivial singlets and $Z_2$-even. These assignments are shown in Table 1. Hereafter, we follow the conventional notation that superfields and their scalar components are denoted by the same letters.

|     | $L_e$ | $L_1$ | $R_e$ | $R_1$ | $N_e$ | $N_1$ | $H^{u,d}$ | $\chi$ | $\chi_{--}$ | $(\chi_1, \chi_2)$ |
|-----|-------|-------|-------|-------|-------|-------|-----------|-------|-------------|-------------------|
| $D_4$ | $1_{++}$ | 2 | $1_{++}$ | 2 | $1_{++}$ | 2 | $1_{++}$ | 1 | $1_{++}$ | 1 | $(1, 1)$ |
| $Z_2$ | + | + | $-$ | $-$ | $-$ | + | + | $-$ | $-$ | + |

Table 1: $D_4$ and $Z_2$ charges. $I$ corresponds to $I = \mu$ and $\tau$.

The $D_4 \times Z_2$ invariant superpotential of the charged leptons is given as

$$W^{(4)}_L = \frac{y_e}{M_p} L_e R_e H^d \chi + \frac{y_\mu}{M_p} (L_\mu R_\mu + L_\tau R_\tau) H^d \chi + \frac{y'_\mu}{M_p} (L_\mu R_\mu - L_\tau R_\tau) H^d \chi_{--},$$

up to 5-point couplings, where $M_p$ is the Planck scale, $M_p = 2.4 \times 10^{18}$ GeV. We assume that scalar components of $\chi_1, \chi_2, \chi$ and $\chi_{--}$ develop their vacuum expectation values (VEVs) and the $D_4 \times Z_2$ symmetry is broken at a high energy scale. Then, the above superpotential (2) leads to a diagonal charged
lepton mass matrix after the electroweak symmetry breaking by Higgs VEVs, 
\[ v^d = \langle H^d \rangle \] and \[ v^u = \langle H^u \rangle. \] On the other hand, higher-order operators can be also included in the superpotential,
\[ W^{(5)}_L = \frac{y_{\mu e}}{M_p^2} L_\mu (R_\mu \chi_1 + R_\tau \chi_2) \chi H^d + \cdots, \]
and these generate off-diagonal elements of the charged lepton mass matrix. As a result, after \[ D_4 \times Z_2 \] symmetry breaking, the effective Yukawa matrix \( \tilde{y}_\ell \) among the charged leptons and the Higgs field \( H^d \) is found to be
\[ (\tilde{y}_\ell)_{IJ} = \begin{pmatrix} y_{e \alpha a} & y_{e \mu \alpha a} - y'_{e \mu \alpha b} & y_{e \tau \alpha a} + y'_{e \tau \alpha b} \\ y_{\mu e \alpha a} - y'_{\mu e \alpha b} & y_{\mu \alpha a} - y'_{\mu \alpha b} & \gamma_{\mu \tau \alpha a} \alpha^2 + \gamma_{\mu \tau \beta \alpha} \beta^2 \\ y_{\mu e \alpha a} + y'_{\mu e \alpha b} & y_{\mu \alpha a} - y'_{\mu \alpha b} & \gamma_{\mu \tau \alpha a} \alpha^2 + \gamma_{\mu \tau \beta \alpha} \beta^2 \end{pmatrix}, \]
where \( \alpha = \langle \chi_i \rangle / M_p, \alpha_a = \langle \chi \rangle / M_p, \alpha_b = \langle \chi_{-+} \rangle / M_p \). Here, it has been assumed that \[ \langle \chi_1 \rangle = \langle \chi_2 \rangle \] \[ 9 \]. The mass matrix of the charged leptons is obtained as \[ M_\ell = v^d \tilde{y}_\ell. \] If \( \alpha \) is allowed sufficiently small compared with \( m_\mu / m_\tau \), charged lepton masses are estimated by the diagonal elements of \( \tilde{y}_\ell \) as follows,
\[ m_e \sim y_{e \alpha a} v^d, \quad m_\mu \sim (y_{\mu \alpha a} - y'_{\mu \alpha b}) v^d, \quad m_\tau \sim (y_{\mu \alpha a} + y'_{\mu \alpha b}) v^d. \]
We need the fine-tuning of parameters, \( y_{\mu}, y'_{\mu}, \alpha_a \) and \( \alpha_b \), such that we realize the mass ratio \( m_\mu / m_\tau \) as \( (y_{\mu \alpha a} - y'_{\mu \alpha b}) / (y_{\mu \alpha a} + y'_{\mu \alpha b}) = \mathcal{O}(m_\mu / m_\tau) \). That is similar to Ref. \[ 6 \]. Also, the coupling \( y_e \) must be suppressed to lead to \( y_{e \alpha a} / (y_{\mu \alpha a} + y'_{\mu \alpha b}) = \mathcal{O}(m_e / m_\tau) \). Off-diagonal elements of the diagonalizing matrix of \( \tilde{y}_\ell \) are determined by the parameter \( \alpha \). For example, the (1,2) element of the diagonalizing matrix is estimated as
\[ \theta_{12}^\ell \sim \frac{y_{e \mu \alpha a} - y'_{e \mu \alpha b}}{m_\mu} \alpha v^d, \]
and it becomes
\[ \theta_{12}^\ell \sim \alpha m_\tau / m_\mu, \]
when \( y_{e \mu}, y'_{\mu} \sim 1 \). At any rate, when \( \alpha \leq \mathcal{O}(10^{-2}) \), the mixing angle \( \theta_{12}^\ell \) is small\[ 3 \]. Such a small value is not important for the neutrino oscillation in

\[ ^3 \text{Small deviations from the diagonal form are important in a certain case (see e.g. [12]).} \]
our model at the current level of experiments, but important for soft SUSY breaking terms as we will discuss.

Now we consider the neutrino sector. The $D_4 \times Z_2$-invariant superpotential is given as

$$W^{(3)}_N = y_1 N_e L e H^u + y_2 (N_\mu L_\mu + N_\tau L_\tau) H^u + y_a N_e (N_\mu \chi_1 + N_\tau \chi_2) + M_1 N_e N_e + M_2 (N_\mu N_\mu + N_\tau N_\tau),$$

up to 4-point couplings. The higher-order operators should be also considered

$$W^{(4)}_N = \frac{y_{21}}{M_p} N_e (L_\mu \chi_1 + L_\tau \chi_2) H^u + \frac{y_{12}}{M_p} L_e (N_\mu \chi_1 + N_\tau \chi_2) H^u + \cdots.$$  \hspace{1cm} (8)

With the above superpotential, the Majorana mass matrix, $M_R$, and the Dirac mass matrix, $M_D$, of the right-handed neutrinos are written down as

$$M_R = \begin{pmatrix} M_1 & y_a M_p \alpha & y_a M_p \alpha \\ y_a M_p \alpha & M_2 & y_b M_p \alpha^2 \\ y_a M_p \alpha & y_b M_p \alpha^2 & M_2 \end{pmatrix},$$

$$M_D = \begin{pmatrix} y_1 & y_{12} \alpha & y_{12} \alpha \\ y_{21} \alpha & y_2 & y_{23} \alpha^2 \\ y_{21} \alpha & y_{32} \alpha^2 & y_2 \end{pmatrix},$$

after the $D_4 \times Z_2$ symmetry and the electroweak symmetry are broken. The light neutrino mass matrix is obtained as,

$$M_\nu = M_D^T M_R^{-1} M_D = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix},$$

where $x$, $y$, $z$ and $w$ are given in terms of parameters of Eq.(10). This form of $M_\nu$ can realize the realistic neutrino mixing without fine-tuning [6, 7, 8, 9]. The light neutrino masses are written as

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T,$$  \hspace{1cm} (11)

These values are obtained at the energy scale $M_R$ and radiative corrections are, in general, important to evaluate values at low energy. However, radiative corrections are negligible in this $D_4$ flavor model [8].
\[
V = \begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12}/\sqrt{2} & \cos \theta_{12}/\sqrt{2} & 1/\sqrt{2} \\
-\sin \theta_{12}/\sqrt{2} & \cos \theta_{12}/\sqrt{2} & -1/\sqrt{2}
\end{pmatrix},
\] (13)

\[
m_1 = \frac{1}{2} \cdot \frac{y_1^2 r^2 + y_2^2 r + \sqrt{2} y_1 y_2 y_α k r / (\cos \theta_{12} \sin \theta_{12})}{r^2 - 2 y_α^2 k^2 r} \cdot \frac{(v_u)^2}{M_1},
\]

\[
m_2 = \frac{1}{2} \cdot \frac{y_α^2 r^2 + y_2^2 r - \sqrt{2} y_1 y_2 y_α k r / (\cos \theta_{12} \sin \theta_{12})}{r^2 - 2 y_α^2 k^2 r} \cdot \frac{(v_u)^2}{M_1},
\]

\[
m_3 = \frac{y_2^2 (r - 2 y_α^2 k^2)}{r^2 - 2 y_α^2 k^2 r} \cdot \frac{(v_u)^2}{M_1},
\] (14)

where we define as \( r \equiv \frac{M_1}{M_2} \) and \( k \equiv \frac{\alpha M_p}{M_1} \), and neglect higher order terms of \( \alpha \) such as \( \alpha^2 M_p \) and \( \alpha^2 M_1 \) appeared in \( M_R^{-1} \). The justification of this approximation and derivation of Eq. (14) are given in Ref. [9]. The mixing angle \( \theta_{12} \) is written as

\[
\cot 2\theta_{12} = \frac{y_1^2 r - y_2^2}{2 \sqrt{2} y_1 y_2 y_α k}.
\] (15)

When \( y_1, y_2 \) and \( y_α \) are of \( \mathcal{O}(1) \), the above mixing angle \( \theta_{12} \) is of \( \mathcal{O}(1) \) and the effect due to \( \theta_{12}^\prime \) is negligible in the neutrino oscillation. Thus, the atmospheric neutrino mixing angle is maximal and the Chooz mixing angle is vanishing, while the solar neutrino mixing is of \( \mathcal{O}(1) \). This also holds if the mass matrices were complex, i.e. if the CP violating case would be studied.

Now, let us consider the realization of experimental values. We use the best fit values of mass squared differences and solar mixing angle as [14]

\[
\Delta m^2_{\text{atm}} = 2.4 \times 10^{-3} \text{ eV}^2, \quad \Delta m^2_{\text{sol}} = 7.6 \times 10^{-5} \text{ eV}^2, \quad \sin^2 \theta_{12} = 0.32.
\] (16)

First we consider the simple case with \( y_1 = y_2 = y_α = 1 \). The above experimental values [16] are obtained by taking [5]

\[
M_1 = 4.9 \times 10^{15} \text{GeV}, \quad M_2 = 6.2 \times 10^{14} \text{GeV}, \quad |\alpha| = 1.6 \times 10^{-3},
\] (17)

If \( M_1, M_2, M_p \alpha \) satisfy the relation \( M_1 + M_p \alpha = M_2 \), \( \cot 2\theta_{12} = 1/(2\sqrt{2}) \) is realized at this approximation level and the mixing matrix \( V \) in Eq. (13) is the so-called tri-bimaximal matrix [15].
for $\nu^\mu \simeq 174$ GeV\(^6\). In this parametrization, the parameter $\alpha$ is sufficiently small, so that the charged lepton mass matrix is diagonal as we assumed in the beginning of this section. When we vary $y_1$, $y_2$ and $y_a$ around $y_1$, $y_2$, $y_a = \mathcal{O}(1)$, the above experimental (16) values are realized for $\mathcal{O}(1)$.

\[ M_2 \sim 0.2 \times \left(\frac{y_2}{y_1}\right)^2 M_1, \quad M_1 \sim 3 \times 10^{15} \times y_1^2 \text{ GeV}, \]
\[ \alpha \sim 0.001 \times \frac{y_1 y_2}{y_a}. \] (18)

Thus, it is found that the value of $\alpha$ is predicted around $\mathcal{O}(10^{-4}) - \mathcal{O}(10^{-2})$ as long as couplings, $y_1, y_2, y_a$ are of $\mathcal{O}(1)$.

### 3 Soft SUSY breaking terms in supersymmetric $D_4$ model

We consider soft SUSY breaking terms of slepton mass matrices within the framework of supergravity theory, i.e. the gravity mediation.\(^7\) In our model, the $D_4$ symmetry restricts not only the fermion mass matrices but also the scalar matrices. Now we assume chiral superfields $\Phi_k$ to cause SUSY breaking by their non-vanishing F-components. The F-components are given by

\[ F^{\Phi_k} = -e^{K/2M_p} K^{\Phi_k\bar{T}} \left( \partial_{\bar{T}} \bar{W} + \frac{K_{\bar{T}T}}{M_p^2} \right), \] (19)

where $K$ denotes the Kähler potential, $K_{\bar{T}T}$ denotes second derivatives by fields, i.e. $K_{\bar{T}T} = \partial_{\bar{T}} \partial_T K$ and $K^{\bar{T}T}$ is its inverse. In general, the fields $\Phi_k$ in our notation include $D_4 \times Z_2$-singlet moduli fields $Z$ and $\chi$, $\chi_+, \chi_i$. Furthermore VEVs of $F_{\Phi_k}/\Phi_k$ are estimated as $\langle F_{\Phi_k}/\Phi_k \rangle = \mathcal{O}(m_{3/2})$, where $m_{3/2}$ denotes the gravitino mass, which is obtained as $m_{3/2} = \langle e^{K/2M_p^2} W/M_p^2 \rangle$.

First let us discuss the scalar mass matrices given by using the second-order Kähler potential of left-handed and right-handed leptons,

\[ K^{(2)} = a_e(\Phi_k)L_e^\dagger L_e + a_\mu(\Phi_k)(L_\mu^\dagger L_\mu + L_\tau^\dagger L_\tau) + b_e(\Phi_k)R_e^\dagger R_e + b_\mu(\Phi_k)(R_\mu^\dagger R_\mu + R_\tau^\dagger R_\tau), \] (20)

\(^6\)The numerical result has been given in the non-SUSY case.\(^7\) SUSY breaking may be realized as the gauge mediation or anomaly mediation. These mediation mechanisms would lead to the flavor-blind soft SUSY breaking terms.
where $a_{e,\mu}(\Phi_k)$, $b_{e,\mu}(\Phi_k)$ are $D_4 \times Z_2$-invariant generic functions. Then the soft SUSY breaking scalar masses are given by \[13\]

$$m^2_{\text{IJ}} = m^2_{3/2}K^{}_{\text{IJ}} - |F^{\Phi_k}|^2 \partial_{\Phi_k} \partial_{\Phi_k} K^{}_{\text{IJ}} + |F^{\Phi_k}|^2 \partial_{\Phi_k} K^{}_{\text{IJ}} \partial_{\Phi_k} K^{}_{\text{MM}}K^{}_{\overline{M}M},$$ \[(21)\]

for the scalar fields with the Kähler metric $K^{}_{\text{IJ}}$, where we have assumed the vanishing vacuum energy. Evaluating $\langle F^{\Phi_k}/\Phi_k \rangle$ for all fields as $\langle F^{\Phi_k}/\Phi_k \rangle = \mathcal{O}(m_3/2)$, the slepton mass matrices can be found to be

$$m^2_L = \begin{pmatrix} m^2_{L1} & 0 & 0 \\ 0 & m^2_{L2} & 0 \\ 0 & 0 & m^2_{L2} \end{pmatrix}, \quad m^2_R = \begin{pmatrix} m^2_{R1} & 0 & 0 \\ 0 & m^2_{R2} & 0 \\ 0 & 0 & m^2_{R2} \end{pmatrix},$$ \[(22)\]

where $m_{Li}$, $m_{Ri} = \mathcal{O}(m_3/2)$. These contributions give the leading-order of the slepton mass matrices. Since the second and third families are $D_4$ doublets, these forms \[(22)\] can be easily expected from the $D_4$ flavor symmetry and it is the prediction of our model that the second and third families of sleptons have degenerate masses at this level.

Eq. \[(22)\] is written in the $D_4$ flavor basis. The super-CKM basis is convenient to examine constraints due to FCNC. For example, the $(1,2)$ elements are obtained in the super-CKM basis as $(\delta^L_{12})(\text{SCCM})_{12} \sim \theta^L_{12}(m^2_{L(R)} - m^2_{L(R)2})$. The mass insertion parameters are defined as

$$\left(\delta^L_{12}\right)_{ij} \equiv \frac{\left(m^2_{L(R)}\right)_{ij}^{(\text{SCCM})}}{m^2_{\text{SUSY}}},$$ \[(23)\]

where $m_{\text{SUSY}}$ denotes the average mass of sleptons. The $(1,2)$ elements are constrained severely by the $\mu \rightarrow e\gamma$ experiments as $(\delta^L_{12}\text{RR})_{12} \leq \mathcal{O}(10^{-3})$ when $m_{\text{SUSY}} = \mathcal{O}(100)$ GeV, while the others have no strong constraints. That requires

$$\theta^L_{12} \leq \mathcal{O}(10^{-3}).$$ \[(24)\]

For $y_1 = y_2 = y_a = 1$, we obtain $\alpha = \mathcal{O}(10^{-3})$ and $\theta^L_{12} = \mathcal{O}(10^{-2})$, and such a parameter region will be ruled out. The value of $\alpha$ is obtained as $\alpha = 0.001 \times y_1 y_2 / y_a$ for $y_1, y_2, y_a \sim 1$ and we estimate $\theta^L_{12} = \alpha y_{e\mu}$ for $y_{e\mu} \sim y'_{e\mu}$.

---

8 When three families consist of a singlet and a doublet under a non-Abelian discrete symmetry, a similar structure would appear, that is, two of three families of scalar masses corresponding to the doublet would be degenerate at a certain level. See e.g. Ref. \[16\]. However, the exactly same structure as our model has not been predicted in other models.
as discussed in Section 2. Possibly, we can get $\theta_{12} = O(10^{-3})$ in a certain parameter region, e.g. $y_1 = y_2 = y_{e\mu} = y'_{e\mu} = 1/y_a = 0.5$. Hence, our model is marginal for FCNC constraints, that is, a certain parameter region is ruled out already but the remaining parameter region is still wide. Of course, if the couplings $y_{e\mu}, y'_{e\mu}$, which are irrelevant to realization for the neutrino oscillation experiments, are suppressed as $y_{e\mu}, y'_{e\mu} \ll O(1)$, a quite wider region for $y_1, y_2, y_a$ is allowed.

In the above estimation we restricted the Kähler potential to $K^{(2)}$, but we can write more general terms in the Kähler potential after $D_4 \times Z_2$-symmetry breaking. The VEVs of $\chi_i$ are large compared with VEVs of the other fields $\chi$ and $\chi_{--}$ as the result of Section 2. Thus, corrections including $\chi_i$ are more important in correction terms of the Kähler potential. Therefore, we include such terms in the Kähler potential. For example, the following terms

$$\Delta K_L^{(e)} = \frac{\beta_{L1}(Z)}{M_p} L_e^\dagger (L_\mu \chi_1^\dagger + L_\tau \chi_2^\dagger) + \frac{\beta_{L2}(Z)}{M_p} L_e^\dagger (L_\mu \chi_1 + L_\tau \chi_2) + h.c.,$$

(25)

where $\alpha_{L\pm}(Z)$ and $\beta_{L\pm}(Z)$ are dimensionless generic functions of moduli fields $Z$, appear for the mixing between $L_e$ and $L_{\mu,\tau}$. In addition, we have the following corrections

$$\Delta K_L^{(\tau \mu)} = \frac{\alpha_{L-}(Z)}{M_p^2} (L_\mu^\dagger L_\tau - L_\tau^\dagger L_\mu) \sigma_{--} + \frac{\alpha_{L+}(Z)}{M_p^2} (L_\mu^\dagger L_\tau + L_\tau^\dagger L_\mu) \sigma_{+-} + \cdots,$$

(26)

for the mixing between $L_\mu$ and $L_\tau$, where $\alpha_{L\pm}(Z)$ and $\beta_{L\pm}(Z)$ are generic dimensionless functions of moduli fields $Z$. Here, $\sigma_{--}$ and $\sigma_{+-}$ denote

$$\sigma_{--} = \chi_1^\dagger \chi_2 - \chi_2^\dagger \chi_1, \quad \sigma_{+-} = \chi_1^\dagger \chi_2 + \chi_2^\dagger \chi_1,$$

(27)

and the ellipsis in (26) denotes terms including other bi-linear combinations of $\chi_1, \chi_2$ and their conjugates. The contribution of the $D_4$ singlets $\chi, \chi_{--}$ are suppressed by $Z_2$ symmetry and do not contribute to the leading order of off diagonal matrix elements. Also, the Kähler potential for the right handed charged leptons $R_{e,\mu,\tau}$ has similar corrections. Including these corrections, scalar masses squared are estimated in the $D_4$ flavor basis as

$$m_L^2 = \begin{pmatrix}
    m_{L1}^2 & O(\alpha m_{3/2}^2) & O(\alpha^2 m_{3/2}^2) \\
    O(\alpha m_{3/2}^2) & m_{L2}^2 & O(\alpha^2 m_{3/2}^2) \\
    O(\alpha^2 m_{3/2}^2) & O(\alpha m_{3/2}^2) & m_{L2}^2
\end{pmatrix},$$

(28)
\[
m_R^2 = \begin{pmatrix}
m_{R_1}^2 & \mathcal{O}(\alpha m_{3/2}^2) & \mathcal{O}(\alpha m_{3/2}^2) \\
\mathcal{O}(\alpha m_{3/2}^2) & m_{R_2}^2 & \mathcal{O}(\alpha^2 m_{3/2}^2) \\
\mathcal{O}(\alpha m_{3/2}^2) & \mathcal{O}(\alpha^2 m_{3/2}^2) & m_{R_2}^2
\end{pmatrix}
\]

(29)

where corrections of \(\mathcal{O}(\alpha^2 m_{3/2}^2, \alpha^2 m_{3/2}^2, \alpha \alpha_1 m_{3/2}^2, \alpha \alpha_2 m_{3/2}^2)\) are omitted in the diagonal elements. Even though we include these corrections, we have almost the same constraint due to FCNC as the previous estimation (24).

Also we examine the mass matrix between the left-handed and the right-handed sleptons, which is generated by the so-called A-terms. The A-terms are trilinear couplings of two sleptons and one Higgs field, and are obtained as [13]

\[
h_{IJ} D_I R_J H^d = \tilde{h}_{IJ} D_I R_J H^d - (\tilde{y}_L)_{IJ} M_J \partial_{\Phi_k} K_{\bar{T}I} F_{\Phi_k} K_{\bar{T}I}
\]

\[
-(\tilde{y}_L)_{1M} D_I R_J H^d F_{\Phi_k} K_{\bar{M}I} \partial_{\Phi_k} K_{\bar{M}I},
\]

(30)

where \(\tilde{h}_{IJ} = F_{\Phi_k} \partial_{\Phi_k} (\tilde{y}_L)_{IJ}\). Note that effective Yukawa couplings \((\tilde{y}_L)_{IJ}\) include \(\chi, \chi_{-+}, \chi_i\) as Eq. (4). After electroweak symmetry breaking, these provide us with the left-right mixing mass squared \((m_{LR}^2)_{IJ} = h_{IJ} v^d\).

Now, let us discuss the first term in Eq. (30), \(\tilde{h}_{33}\). For simplicity, we assume that Yukawa couplings are independent of moduli fields \(Z\). For example, the \((3,3)\) element, \(\tilde{h}_{33}\) is obtained as

\[
\tilde{h}_{33} = y_\mu \frac{F^x}{M_p} + y'_\mu \frac{F^{x-+}}{M_p}.
\]

(31)

Then, we can estimate \(\tilde{h}_{33} v^d = \mathcal{O}(m_\tau m_{3/2})\). Similarly, we obtain the \((2,2)\) element as

\[
\tilde{h}_{22} = y_\mu \frac{F^x}{M_p} - y'_\mu \frac{F^{x-+}}{M_p}.
\]

(32)

Thus, we evaluate \(\tilde{h}_{22} v^d = \mathcal{O}(m_\tau m_{3/2})\), because we estimate \(F^x/\chi, F^{x-+}/\chi_{-+} = \mathcal{O}(m_3/2)\), but \(F^x/\chi\) and \(F^{x-+}/\chi_{-+}\) are, in general, different from each other. That may cause a problem. If \(|h_{IJ}/\tilde{y}_{IJ}|\) is large compared with slepton masses, there would be a minimum, where charge is broken [17]. In order to avoid this, we assume that the Kähler metric of \(\chi\) and \(\chi_{-+}\) are the

\[
\text{If a decay rate from the realistic minimum to such charge breaking minimum is sufficiently small compared with the age of the universe, that might not be a problem.}
\]
same e.g. canonical, and the non-perturbative superpotential leading to SUSY breaking does not include $\chi$ or $\chi^-$, i.e. $\langle \partial_\chi W \rangle = \langle \partial_{\chi^-} W \rangle = 0$. In this case, we obtain

$$F^\chi/\chi = F^{\chi^-}/\chi^- = -m_{3/2},$$

and we estimate $\hat{h}_{22} v^d = \mathcal{O}(m_{\mu} m_{3/2})$. Similarly, other elements of $\hat{h}_{IJ}$ are estimated as

$$\hat{h}_{IJ} v^d = m_{3/2} \begin{pmatrix} \mathcal{O}(m_{\tau}) & \mathcal{O}(m_{\mu} \theta_{12}) & \mathcal{O}(m_{\tau} \alpha) \\ \mathcal{O}(m_{\mu} \theta_{12}^t) & \mathcal{O}(m_{\mu}) & \mathcal{O}(m_{\tau} \alpha^2) \\ \mathcal{O}(m_{\tau} \alpha) & \mathcal{O}(m_{\tau} \alpha^2) & \mathcal{O}(m_{\tau}) \end{pmatrix}.$$  \hfill (34)

The other terms in Eq. (30) lead to the same order of A-terms as Eq. (34). Thus, we obtain the left-right mixing slepton mass matrix in the $D_4$ flavor basis as

$$m_{LR}^2 \equiv h_{I,J} v^d = m_{3/2} \begin{pmatrix} \mathcal{O}(m_{\tau}) & \mathcal{O}(m_{\mu} \theta_{12}) & \mathcal{O}(m_{\tau} \alpha) \\ \mathcal{O}(m_{\mu} \theta_{12}^t) & \mathcal{O}(m_{\mu}) & \mathcal{O}(m_{\tau} \alpha^2) \\ \mathcal{O}(m_{\tau} \alpha) & \mathcal{O}(m_{\tau} \alpha^2) & \mathcal{O}(m_{\tau}) \end{pmatrix}.$$  \hfill (35)

The pattern of this mass matrix in the super-CKM basis is the same as the above. We define the mass insertion parameters for the left-right mixing as

$$(\delta_{LR}^I)_{ij} \equiv \frac{(m_{LR}^{2(sCKM)_{ij}})}{m_{SUSY}^2},$$  \hfill (36)

where $(m_{LR}^{2(sCKM)})_{ij}$ is the left-right mixing slepton mass squared matrix in the super-CKM basis. Only for the (1,2) element of $(\delta_{LR}^I)_{ij}$, there is a strong constraint due to FCNC as $(\delta_{LR}^I)_{12} \leq \mathcal{O}(10^{-6})$ [10], when $m_{SUSY} = \mathcal{O}(100)$ GeV. This constraint also requires $\theta_{12} \leq \mathcal{O}(10^{-3})$, which is the same as Eq. (24).

The soft SUSY breaking terms, which we have studied, are generated at a high energy scale such as the Planck scale or the GUT scale. In the above

---

10 The $D_4$ flavor structure can be realized in heterotic string models on factorizable orbifolds including $Z_4$ [3]. In those heterotic orbifold models, $D_4$ non-trivial singlets and trivial singlets appear in the same sector and have the same Kähler metric. In such models, our assumption would be justified.
discussion, we have neglected radiative corrections. The gaugino contributions are dominant in radiative corrections to slepton masses, that is, slepton masses at the weak scale are obtained by ones at the GUT scale $M_X$ as

$$
\begin{align*}
m^2_{\tilde{\nu}_L}(M_Z) &= m^2_{\tilde{\nu}_L}(M_X) + 0.5M^2_W + 0.04M^2_B, \\
m^2_{\tilde{\nu}_R}(M_Z) &= m^2_{\tilde{\nu}_R}(M_X) + 0.2M^2_B,
\end{align*}
$$

where $M_B$ and $M_W$ are bino and wino masses, respectively. The above estimation on FCNC constraints does not change drastically when these gaugino masses are comparable with slepton masses. If these gaugino masses are quite large compared with slepton masses, FCNC constraints would be improved.

4 Conclusion

We have studied supersymmetric extension of the $D_4 \times Z_2$ flavor model of [9]. We have evaluated soft SUSY breaking terms about the slepton mass terms. It is remarked that the second and third families of slepton masses are almost degenerate. The difference is tiny as $O(\alpha^2 m^2_{3/2}, \alpha_a^2 m^2_{3/2}, \alpha_b^2 m^2_{3/2}/2, \alpha_a \alpha_b m^2_{3/2})$.

The (1,2) element $\theta^\ell_{12}$ of the diagonalizing matrix for the charged lepton mass matrix is important for the FCNC constraints, in particular $\mu \to e\gamma$ experiments, although it is not important to realize the neutrino oscillation experiments. It is constrained as $\theta^\ell_{12} \leq O(10^{-3})$ from the current bound of $BR(\mu \to e\gamma)$ and our model is marginal. Thus, future improvement on the bound of $BR(\mu \to e\gamma)$, e.g. by the MEG experiment [18] is quite important in our model.

In Appendix, we have also discussed supersymmetric extension of the Grimus-Lavoura $D_4 \times Z_2$ flavor model, which leads to almost the same results for soft SUSY breaking terms.

Finally, we give a comment on realization of our SUSY model by string model building. The $D_4$ flavor symmetry can appear from heterotic string models on factorizable orbifold models including the $Z_2$ orbifold like $Z_2 \times Z_N$ orbifolds [2, 3, 4], unless one does not introduce Wilson lines, which break degeneracy of massless spectra. Indeed, several semi-realistic models have been constructed [2, 5], where three families correspond to $D_4$ trivial singlets and doublets. Stringy realization of our flavor structure would be plausible from this viewpoint. However, such heterotic orbifold models include only $D_4$ trivial singlets and doublets, but not $D_4$ non-trivial singlets as fundamental
modes. On the other hand, the $D_4$ non-trivial singlet $\chi_{-+}$ plays an important role in our model. Such a mode could appear as a composite mode. Alternatively, $D_4$ non-trivial singlets as well as trivial singlets and doublets can appear as fundamental modes in heterotic string models on factorizable orbifolds including the $Z_4$ orbifolds like $Z_4 \times Z_N$ orbifolds. Thus, stringy realization on such orbifolds might be alternative possibility.

Acknowledgement
T. K. is supported in part by the Grand-in-Aid for Scientific Research, No. 17540251 and the Grant-in-Aid for the 21st Century COE “The Center for Diversity and Universality in Physics” from the Ministry of Education, Culture, Sports, Science and Technology of Japan. The work of R.T. has been supported by the Japan Society of Promotion of Science. The work of M.T. has been supported by the Grant-in-Aid for Science Research of the Ministry of Education, Science, and Culture of Japan Nos. 17540243 and 19034002.

A Appendix

Here, we discuss the supersymmetric extension of the Grimus-Lavoura model [6]. The Grimus-Lavoura non-SUSY model includes three electroweak Higgs fields. Thus, we have to introduce three pairs of Higgs superfields for the up and down sectors, $H_{i}^{u,d}$ ($i = 1, 2, 3$). Also we introduce a $D_4$ doublet $(\chi_1, \chi_2)$. Table 2 shows $D_4$ and $Z_2$ charges for all fields.

|       | $L_e$ | $L_I$ | $R_e$ | $R_I$ | $N_e$ | $N_I$ | $H_1^{u,d}$ | $H_2^{u,d}$ | $H_3^{u,d}$ | $(\chi_1, \chi_2)$ |
|-------|------|------|------|------|------|------|------------|------------|------------|----------------|
| $D_4$ | $1_{++}$ | 2    | $1_{++}$ | 2    | $1_{++}$ | 2    | $1_{++}$  | $1_{++}$  | $1_{+-}$  | 2             |
| $Z_2$ | +    | +    | +    | +    | +    | +    | -          | +          | +          | +             |

Table 2: $D_4$ and $Z_2$ charges for the supersymmetric Grimus-Lavoura model. $I$ corresponds to $\mu$ and $\tau$.

Following these assignments, the superpotential which leads the lepton
mass matrices is found to be
\[ W^{(3)} = y_e L_e R_e H^d_1 + y_{\mu} (L_{\mu} R_{\mu} + D_{\tau} R_{\tau}) H^d_2 + y'_\mu (L_{\mu} R_{\mu} - L_{\mu} R_{\mu}) H^d_3 \\
+ (y_1 L_e N_e + y_2 (L_{\mu} N_{\mu} + L_{\tau} N_{\tau})) H^u_1 + y_3 (L_{\tau} N_{\tau} - L_{\mu} N_{\mu}) H^u_3 \\
+ y_a N_e (N_{\mu} \chi_1 + N_{\tau} \chi_2) + M_1 N_e N_e + M_2 (N_{\mu} N_{\mu} + N_{\tau} N_{\tau}), \] (38)
up to 4-point couplings. Furthermore, higher-order superpotential terms like,
\[ \langle \theta \rangle^O \]
should be also considered, because \( \langle \chi_i \rangle \) must be large to lead the realistic neutrino mixing. Now the charged lepton mass matrix can be evaluated as follows
\[ M_L = \begin{pmatrix}
y_e v^d_1 & (y_{e\mu} v^d_2 - y'_{e\mu} v^d_3) \alpha & (y_{e\mu} v^d_2 + y'_{e\mu} v^d_3) \alpha \\
y_{\mu} v^d_2 & y_{\mu} v^d_2 - y'_\mu v^d_3 & O(v^d_2 \alpha^2) \\
y_{\mu} v^d_3 & O(v^d_3 \alpha^2) & y_{\mu} v^d_2 + y'_\mu v^d_3
\end{pmatrix}, \] (40)
where \( \langle H^d_i \rangle = v^d_i \). If \( \alpha \ll 1 \) is allowed, eigenvalues of lepton masses can be found to be equal to the diagonal elements like Eq. (4). We need fine-tuning Yukawa couplings and VEVs such that \( (y_{\mu} v^d_2 - y'_\mu v^d_3) / (y_{\mu} v^d_2 + y'_\mu v^d_3) = O(m_\mu / m_\tau) \). In addition, we require \( y_{e\mu} v^d_2 \) to be suppressed compared with \( y_{\mu} v^d_2 + y'_\mu v^d_3 \) to lead to the mass ratio \( m_e / m_\tau \), i.e. \( y_e v^d_1 / (y_{\mu} v^d_2 + y'_\mu v^d_3) = O(m_e / m_\tau) \). The (1,2) element of diagonalizing matrix \( \theta^d \) is estimated as \( \theta^d_{12} = (y_{e\mu} v^d_2 - y'_e v^d_3) / m_\mu \) and it reduces to \( \theta^d_{12} \sim y_{e\mu} \alpha m_\tau / m_\mu \) for \( y_{e\mu} \sim y'_{e\mu} \) and \( v^d_2 \sim v^d_3 \). Thus, this is also the same as Eqs. (3) and (7).

On the other hand, the Dirac mass matrix, \( M_D \), and the Majorana mass matrix, \( M_R \), in the neutrino sector are written as
\[ M_D = \begin{pmatrix}
y_1 v^u_1 & (y_1 v^u_2 - y'_1 v^u_3) \alpha & (y_1 v^u_2 + y'_1 v^u_3) \alpha \\
y_2 v^u_1 & y_2 v^u_2 - y_3 v^u_3 & O(v^u_2 \alpha^2) \\
y_3 v^u_1 & O(v^u_3 \alpha^2) & y_2 v^u_2 + y_3 v^u_3
\end{pmatrix}, \] (41)
\[ M_R = \begin{pmatrix}
M_1 & y_a M_p & y_a M_p \\
y_a M_p & M_2 & y_b M_p \alpha^2 \\
y_a M_p & y_b M_p \alpha^2 & M_2
\end{pmatrix}, \]
where \( \langle H^u_i \rangle = v^u_i \). The above pattern is quite similar to Eq. (10), and in particular, the form of \( M_R \) is the same as Eq. (11). Thus, similar values of parameters lead to realistic results [13], i.e., \( M_1, M_2 = O(10^{15}) \)GeV and \( \alpha \sim M_2 / M_p \). Thus the favorable region of \( \alpha \) is of \( O(10^{-4}) - O(10^{-2}) \).
In this model, the soft SUSY breaking terms are also restricted by $D_4 \times Z_2$ symmetry and expected not to be different from the estimation in Section 3. In fact, the scalar mass terms are given by

$$m^2_L = \begin{pmatrix}
m^2_{L1} + \mathcal{O}(\alpha^2 m^2_{3/2}) & \mathcal{O}(\alpha m^2_{3/2}) & \mathcal{O}(\alpha m^2_{3/2}) \\
\mathcal{O}(\alpha m^2_{3/2}) & m^2_{L2} + \mathcal{O}(\alpha^2 m^2_{3/2}) & \mathcal{O}(\alpha^2 m^2_{3/2}) \\
\mathcal{O}(\alpha^2 m^2_{3/2}) & \mathcal{O}(\alpha^2 m^2_{3/2}) & m^2_{L3} + \mathcal{O}(\alpha^2 m^2_{3/2})
\end{pmatrix}, \quad (42)$$

and

$$m^2_R = \begin{pmatrix}
m^2_{R1} + \mathcal{O}(\alpha^2 m^2_{3/2}) & 0 & 0 \\
0 & m^2_{R2} + \mathcal{O}(\alpha^2 m^2_{3/2}) & \mathcal{O}(\alpha^2 m^2_{3/2}) \\
0 & \mathcal{O}(\alpha^2 m^2_{3/2}) & m^2_{R3} + \mathcal{O}(\alpha^2 m^2_{3/2})
\end{pmatrix}, \quad (43)$$
in the $D_4$ flavor basis. Thus, we obtain the same constraint on the mass insertion parameters due to FCNC as (24), i.e. $\theta^l_{12} \leq \mathcal{O}(10^{-3})$.

Similarly, the A-terms can also be evaluated. This model would have the same problem about the $(2,2)$ element of A-terms as the model in Section 3, that is, the $(2,2)$-element would be of $\mathcal{O}(m_\tau m_{3/2})$ without tuning about the coefficient. However, when the Kähler metric of $H^d_2$ and $H^d_3$ are the same, the $(2,2)$ element becomes of $\mathcal{O}(m_\mu m_{3/2})$. Then, the left-right mixing slepton mass matrix could be estimated as

$$m^2_{LR} = m_{3/2}/2 \begin{pmatrix}
\mathcal{O}(m_e) & \mathcal{O}(\theta^l_{12} m_\tau) & \mathcal{O}(\alpha m_\tau) \\
\mathcal{O}(\alpha m_e) & \mathcal{O}(m_\mu) & \mathcal{O}(\alpha^2 m_\tau) \\
\mathcal{O}(\alpha m_e) & \mathcal{O}(\alpha^2 m_\tau) & \mathcal{O}(m_\tau)
\end{pmatrix}, \quad (44)$$
in the $D_4$ flavor basis, and its form is almost the same in the super-CKM basis. Thus, the mass insertion parameter is estimated as $(\delta^l_{LR})_{12} = \mathcal{O}(\theta^l_{12} m_\mu/m_{3/2})$, and we have the same constraint as one in Section 3, i.e. $\theta^l_{12} \leq \mathcal{O}(10^{-3})$.

As a result, soft SUSY breaking terms, which are predicted in the supersymmetric Grimus-Lavoura model, are almost the same as those obtained in Section 3. The difference is the number of Higgs pairs, that is, the supersymmetric Grimus-Lavoura model has three pairs of Higgs supermultiplets, while the model in Section 2 has only one pair and it becomes the MSSM at low energy. The former may violate the gauge coupling unification unless one introduces extra colored supermultiplets.
References

[1] See for review, e.g. E. Ma, arXiv:hep-ph/0612013; arXiv:0705.0327 [hep-ph] and references therein.

[2] T. Kobayashi, S. Raby and R. J. Zhang, Nucl. Phys. B 704, 3 (2005).

[3] T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B 768, 135 (2007).

[4] P. Ko, T. Kobayashi, J. h. Park and S. Raby, Phys. Rev. D 76, 035005 (2007) [Erratum-ibid. D 76, 059901 (2007)].

[5] T. Kobayashi, S. Raby and R. J. Zhang, Phys. Lett. B 593, 262 (2004); W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, Nucl. Phys. B 785, 149 (2007); O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, Phys. Lett. B 645, 88 (2007).

[6] W. Grimus and L. Lavoura, Phys. Lett. B 572, 189 (2003).

[7] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, JHEP 0407, 078 (2004); A. Blum, R. N. Mohapatra and W. Rodejohann, Phys. Rev. D 76, 053003 (2007); A. Blum, C. Hagedorn and M. Lindner, arXiv:0709.3450 [hep-ph].

[8] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka and M. Tanimoto, Nucl. Phys. B 713, 151 (2005).

[9] H. Ishimori, T. Kobayashi, H. Ohki, Y. Omura, R. Takahashi and M. Tanimoto, arXiv:0802.2310 [hep-ph].

[10] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996).

[11] P. H. Chankowski, O. Lebedev and S. Pokorski, Nucl. Phys. B 717, 190 (2005).

[12] K. A. Hochmuth, S. T. Petcov and W. Rodejohann, Phys. Lett. B 654, 177 (2007) [arXiv:0706.2975 [hep-ph]].
[13] V. S. Kaplunovsky and J. Louis, Phys. Lett. B 306, 269 (1993).

[14] M. Maltoni, T. Schwetz, M. Tortola, and J.W.F. Valle, New J. Phys. 6, 122 (2004); G.L. Fogli, E. Lisi, A. Marrone, and A. Palazzo, Prog. Part. Nucl. Phys. 57, 742 (2006).

[15] P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B 530, 167 (2002); P.F. Harrison and W.G. Scott, Phys. Lett. B 535, 163 (2002).

[16] T. Kobayashi, J. Kubo and H. Terao, Phys. Lett. B 568, 83 (2003) [arXiv:hep-ph/0303084].

[17] J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B 222, 11 (1983); L. Alvarez-Gaume, J. Polchinski and M. B. Wise, Nucl. Phys. B 221, 495 (1983); L. Alvarez-Gaume, J. Polchinski and M. B. Wise, Nucl. Phys. B 221, 495 (1983); C. Kounnas, A. B. Lahanas, D. V. Nanopoulos and M. Quiros, Nucl. Phys. B 236, 438 (1984); M. Claudson, L. J. Hall and I. Hinchlorfe, Nucl. Phys. B 228, 501 (1983); J. A. Casas, A. Lleyda and C. Munoz, Nucl. Phys. B 471, 3 (1996).

[18] T. Mori, Nucl. Phys. Proc. Suppl. 169 (2007) 166.