Double (implicit and explicit) dependence of the electromagnetic field of an accelerated charge on time: Mathematical and physical analysis of the problem

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(January 25, 2022)

Abstract

We considered the electromagnetic field of a charge moving with a constant acceleration along an axis. We found that this field obtained from the Liénard-Wiechert potentials does not satisfy Maxwell equations if one considers exclusively a retarded interaction (i.e. pure implicit dependence this field on time). We show that if and only if one takes into account both retarded interaction and direct interaction (so called “action-at-a-distance”) the field produced by an accelerated charge satisfies Maxwell equations.

PACS numbers: 03.50.-z, 03.50.De
1. Introduction

The problem of a calculation of the potentials and the fields created by a point charge moving with an acceleration was raised for the first time about 100 years ago by A.Liénard and E.Wiechert [1] and has not lost relevance nowadays. The question concerning the choice of a correct way to obtain these fields - seems to have been solved finally (see, e.g. well-known book by L.D.Landau [2]). But lately many authors (see e.g. [3-6] and others in References, and this list one could continue) have time and again resorted to this problem - a problem which was given up by contemporary physics long ago. We think that there must be something behind it that makes the problem still actual from both the scientific and pedagogical points of view.

It is well-known that the electromagnetic field created by an arbitrarily moving charge

\[ E(r, t) = q \left\{ \frac{(R - R_cV)(1 - \frac{V^2}{c^2})}{(R - R_cV)^3} \right\}_{t_0} + q \left\{ \frac{[R \times [(R - R_cV) \times \frac{V}{c^2}]]}{(R - R_cV)^3} \right\}_{t_0} \]

was obtained directly from Liénard-Wiechert potentials [2]:

\[ B(r, t) = \left\{ \frac{R}{R_c} \times E \right\}_{t_0} \]

\[ \varphi(r, t) = \left\{ \frac{q}{(R - R_cV)} \right\}_{t_0}, \quad A(r, t) = \left\{ \frac{qV}{c(R - R_cV)} \right\}_{t_0}. \]

The notation \( \{ ... \}_{t_0} \) means that all functions of \( x, y, z, t \) into the parenthesis \( \{ \} \) are taken at the moment of time \( t_0(x, y, z, t) \) [2] (the instant \( t_0 \) is determined from the condition (8), see below).

Usually, the first terms of the right-hand sides (rhs) of (1) and (2) are called “velocity fields” and the second ones are called “acceleration fields”.

It was recently claimed by E.Comay [7] that “... Acceleration fields by themselves do not satisfy Maxwell’s equations [8]. Only the sum of velocity fields and acceleration fields satisfies Maxwell’s equations.” We wish to argue that this sum does not satisfy Maxwell’s equations.
\[ \nabla \cdot \mathbf{E} = 4\pi \varrho, \quad (4) \]
\[ \nabla \cdot \mathbf{B} = 0, \quad (5) \]
\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (6) \]
\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}. \quad (7) \]

in the case when one takes into consideration \textit{exclusively} a retarded interaction.

The remainder of our paper is organized as follows: In Section 2 we derive the fields \( \mathbf{E} \) and \( \mathbf{B} \) taking into account exclusively the \textit{implicit} dependence the potentials \( \varphi \) and \( \mathbf{A} \) on time \( t \). In Section 3 we prove that the field obtained from the Liénard-Wiechert potentials does not satisfy Maxwell equations if one considers exclusively a retarded interaction (in the other words, the \textit{implicit} dependence the potentials on time of observation \( t \) only). In Section 4 we consider another way to obtain the fields \( \mathbf{E} \) and \( \mathbf{B} \). This way is based on a different type of calculation of the derivatives \( \partial \{ \} / \partial t \) and \( \partial \{ \} / \partial x_i \) in which the functions \( \varphi \) and \( \mathbf{A} \) are considered as functions with a \textit{double} dependence on \((t, x, y, z)\): implicit and explicit \textit{simultaneously}. By this way one obtains formally the \textit{same} expressions (1) and (2) for the fields. If one uses \textit{this} manner to verify the validity of Maxwell’s equations, one finds that fields (1) and (2) satisfy these equations. In this Section we shall show that this way does not correspond to a \textit{pure} retarded interaction between the charge and the point of observation. Section 5 closes the paper.

2. Deriving the fields \( \mathbf{E} \) and \( \mathbf{B} \) taking into account the retarded interaction only

Let us try to derive the formulas (1), (2) for the electric (\( \mathbf{E} \)) and magnetic (\( \mathbf{B} \)) fields taking into account that the \textit{state} of the fields \( \mathbf{E} \) and \( \mathbf{B} \) at the instant \( t \) must be \textit{completely} determined by the \textit{state} of the charge at the instant \( t_0 \). The instant \( t_0 \) is determined from the condition (see Eq.(63.1) of Ref.[2]):

\[ t_0 = t - \tau = t - \frac{R(t_0)}{c}. \quad (8) \]

Here \( \tau = R(t_0)/c \) is the so called “retarded time”, \( R = |\mathbf{R}| \), \( \mathbf{R} \) is the vector connecting the site \( \mathbf{r}_0(x_0, y_0, z_0) \) of the charge \( q \) at the instant \( t_0 \) with the point of observation \( \mathbf{r}(x, y, z) \).
All the quantities on the rhs of (3) must be evaluated at the time $t_0$ (see [2], the text after Eq.(63.5)), which, in turn, depends on $x, y, z, t$:

$$t_0 = f(x, y, z, t).$$

(9)

Let us, to be more specific, turn to Landau and Lifshitz who write ([2], p.161)\footnote{We use here our numeration of formulas: our (3) is (63.5) of [2], (8) is (63.1) of [2].}

"To calculate the intensities of the electric and magnetic fields from the formulas

$$E = -\nabla \varphi - \frac{1}{c} \frac{\partial A}{\partial t}, \quad B = [\nabla \times A],$$

(10)

we must differentiate $\varphi$ and $A$ with respect to the coordinates $x, y, z$ of the point, and the time $t$ of observation. But the formulas (3) express the potentials as a functions of $t_0$, and only through the relation (8) as implicit functions of $x, y, z, t$. Therefore to calculate the required derivatives we must first calculate the derivatives of $t_0$".

Now, following this note of Landau, we can construct a scheme of calculating the required derivatives, taking into account that $\varphi$ and $A$ must not depend on $x, y, z, t$ explicitly:

$$\begin{align*}
\frac{\partial \varphi}{\partial x_i} &= \frac{\partial \varphi}{\partial t_0} \frac{\partial t_0}{\partial x_i} \\
\frac{\partial A}{\partial t} &= \frac{\partial A}{\partial t_0} \frac{\partial t_0}{\partial t} \\
\frac{\partial A_k}{\partial x_i} &= \frac{\partial A_k}{\partial t_0} \frac{\partial t_0}{\partial x_i}
\end{align*}$$

(11)

To obtain Eqs. (1) and (2), let us rewrite Eqs.(10) taking into account Eqs.(11)\footnote{In Eqs. (12),(13) we have used the well-known formulas of the vectorial analysis:

$$\nabla u = \frac{\partial u}{\partial \xi} \nabla \xi \quad \text{and} \quad [\nabla \times f] = \left[ \nabla \xi \times \frac{\partial f}{\partial \xi} \right]$$

where $u = u(\xi)$, $f = f(\xi)$ and $\xi = \xi(x, y, z)$.}

$$E = -\nabla \varphi - \frac{1}{c} \frac{\partial A}{\partial t} = -\frac{\partial \varphi}{\partial t_0} \nabla t_0 - \frac{1}{c} \frac{\partial A}{\partial t_0} \frac{\partial t_0}{\partial t},$$

(12)
\[ \mathbf{B} = [\nabla \times \mathbf{A}] = \left[ \nabla t_0 \times \frac{\partial \mathbf{A}}{\partial t_0} \right]. \] (13)

To calculate Eqs.(12),(13) we use relations \( \frac{\partial t_0}{\partial t} \) and \( \frac{\partial t_0}{\partial x_i} \) obtained in [2]:

\[ \frac{\partial t_0}{\partial t} = \frac{R}{R - R\mathbf{V}/c} \quad \text{and} \quad \frac{\partial t_0}{\partial x_i} = -\frac{x_i - x_{0i}}{c[R - R\mathbf{V}/c]} . \] (14)

From Eqs.(3) we find:

\[ \frac{\partial \varphi}{\partial t_0} = -\frac{q}{(R - R\beta)^2} \left( \frac{\partial R}{\partial t_0} - \frac{\partial R}{\partial t_0} \beta - R \frac{\partial \beta}{\partial t_0} \right), \] (15)

where \( \beta = \mathbf{V}/c \). Hence, taking into account that

\[ \frac{\partial R}{\partial t_0} = -c, \quad \frac{\partial R}{\partial t_0} = -\frac{\partial r_0}{\partial t_0} = -\mathbf{V}(t_0) \quad \text{and} \quad \frac{\partial \mathbf{V}}{\partial t_0} = \dot{\mathbf{V}}, \]

we have (after an algebraic simplification):

\[ \frac{\partial \varphi}{\partial t_0} = \frac{qc(1 - \beta^2 + R\dot{\beta}/c)}{(R - R\beta)^2} . \] (16)

In turn

\[ \frac{\partial \mathbf{A}}{\partial t_0} = \frac{\partial \varphi}{\partial t_0} \beta + \varphi \dot{\beta} . \] (17)

Putting \( \varphi \) from Eqs.(3), Eq.(16) and Eq.(17) together, we have (after simplification):

\[ \frac{\partial \mathbf{A}}{\partial t_0} = q c \frac{\beta(1 - \beta^2 + R\dot{\beta}/c) + (\dot{\beta}/c)(R - R\beta)}{(R - R\beta)^2} . \] (18)

Finally, substituting Eqs. (14), (16) and (18) in Eq.(12) we obtain:

\[ \mathbf{E} = \frac{qc(1 - \beta^2 + R\dot{\beta}/c)}{(R - R\beta)^2} \left( -\frac{R}{c(R - R\beta)} \right) - \]

\[ - q \frac{\beta(1 - \beta^2 + R\dot{\beta}/c) + (\dot{\beta}/c)(R - R\beta)}{(R - R\beta)^2} \left( \frac{R}{R - R\beta} \right) = \]

\[ = q \frac{R(1 - \beta^2 + R\dot{\beta}/c) - R\beta(1 - \beta^2 + R\dot{\beta}/c) - (R\dot{\beta}/c)(R - R\beta)}{(R - R\beta)^3} . \] (19)

\(^3\)This follows from the expressions \( R = c(t - t_0) \) and \( \mathbf{R} = \mathbf{r} - t_0(t_0) \). See [2].
Grouping together all terms with acceleration, one can reduce this expression to

\[
E = q \frac{(R - R \beta)(1 - \beta^2)}{(R - R \beta)^3} + q \frac{(R \dot{\beta}/c)(R - R \beta) - (R \ddot{\beta}/c)(R - R \beta)}{(R - R \beta)^3}. \tag{20}
\]

Now, using the formula of double vectorial product, it is not worth reducing the numerator of the second term of Eq.(20) to \([R \times [(R - R \beta) \times \dot{\beta}/c]]\). As a result we have Eq.(1).

Analogically, substituting Eqs. (14) and (18) in Eq.(13) we obtain

\[
B = \left[ \frac{R}{R} \times q \frac{-R \beta (1 - \beta^2 + R \ddot{\beta}/c) - (R \ddot{\beta}/c)(R - R \beta)}{(R - R \beta)^3} \right]. \tag{21}
\]

If we add \(R(1 - \beta^2 + \dot{\beta}/c)\) to the numerator of the second term of the vectorial product (21)\(^4\) we obtain Eq.(2) (see Eq.(19))

In the next section we shall consider a charge moving with a constant acceleration along the \(X\) axis and we shall show that the Eq.(7) is not satisfied if one substitutes \(E\) and \(B\) from Eqs.(1) and (2) in Eq.(7) and takes into consideration exclusively a retarded interaction. To verify this we have to find the derivatives of \(x-, y-, z-\)components of the fields \(E\) and \(B\) with respect to the time \(t\) and the coordinates \(x, y, z\). The functions \(E\) and \(B\) depend on \(x, y, z, t\) through \(t_0\) from the conditions (8)-(9). In other words, we shall show that these fields \(E\) and \(B\) do not satisfy the Maxwell equations if the differentiation rules (11) that were applied to \(\varphi\) and \(A\) (to obtain \(E\) and \(B\)) are applied identically to \(E\) and \(B\).

3. Does the retarded electromagnetic field of a charge moving with a constant acceleration satisfy Maxwell equations?

Let us consider a charge \(q\) moving with a constant acceleration along the \(X\) axis. In this case its velocity and acceleration have only \(x\)-components, respectively \(V(V, 0, 0)\) and \(a(a, 0, 0)\). Now we rewrite the Eqs. (1) and (2) by components:

\[
E_x(x, y, z, t) = q \left\{ \frac{(V^2 - c^2)[RV - c(x - x_0)]}{[cR - V(x - x_0)]^3} \right\}_{t_0} + q \left\{ \frac{ac[(x - x_0)^2 - R^2]}{[(cR - V(x - x_0))]^3} \right\}_{t_0}, \tag{22}
\]

\(^4\)The meaning of Eq.(21) does not change because of \([R \times R] = 0\).
\[ E_y(x, y, z, t) = -q \left\{ \frac{c(V^2 - c^2)(y - y_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0} + q \left\{ \frac{a(x - x_0)(y - y_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0}, \quad (23) \]

\[ E_z(x, y, z, t) = -q \left\{ \frac{c(V^2 - c^2)(z - z_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0} + q \left\{ \frac{a(x - x_0)(z - z_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0}, \quad (24) \]

\[ B_x(x, y, z, t) = 0, \quad (25) \]

\[ B_y(x, y, z, t) = q \left\{ \frac{V(V^2 - c^2)(z - z_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0} - q \left\{ \frac{acR(z - z_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0}, \quad (26) \]

\[ B_z(x, y, z, t) = -q \left\{ \frac{V(V^2 - c^2)(y - y_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0} + q \left\{ \frac{acR(y - y_0)}{[cR - V(x - x_0)]^3} \right\}_{t_0}, \quad (27) \]

Obviously, these components are functions of \(x, y, z, t\) through \(t_0\) from the conditions (8)-(9). This means that when we substitute the field components given by Eqs.(22)-(27) in the Maxwell equations (4)-(7), we once again have to use the differentiation rules as in (11):

\[ \frac{\partial E \{or B\}_k}{\partial t} = \frac{\partial E \{or B\}_k}{\partial t_0} \frac{\partial t_0}{\partial t}, \quad (28) \]

where \(k\) and \(x_i\) are \(x, y, z\).

Remember that we are considering the case with \(V = (V, 0, 0)\), so, one obtains:

\[ \frac{\partial t_0}{\partial t} = \frac{R}{R - (x - x_0)V/c} \quad \text{and} \quad \frac{\partial t_0}{\partial x_i} = -\frac{c}{c[R - (x - x_0)V/c]} \cdot \quad (29) \]

Let us rewrite Eq.(7) by components taking into account the rules (28) and Eq.(25):

\[ \frac{\partial E_x \partial t_0}{\partial y} - \frac{\partial E_y \partial t_0}{\partial z} = 0, \quad (30) \]

\[ \frac{\partial E_x \partial t_0}{\partial z} - \frac{\partial E_z \partial t_0}{\partial x} + \frac{1}{c} \frac{\partial B_y \partial t_0}{\partial t} = 0, \quad (31) \]
\[
\frac{\partial E_y}{\partial t_0} \frac{\partial t_0}{\partial x} - \frac{\partial E_x}{\partial t_0} \frac{\partial t_0}{\partial y} + \frac{1}{c} \frac{\partial B_z}{\partial t_0} \frac{\partial t_0}{\partial t} = 0.
\] (32)

In order to calculate the derivatives \( \partial E(\text{or } B)_k/\partial t \) we need the values of the expressions \( \partial V/\partial t, \partial x_0/\partial t \) and \( \partial R/\partial t \). In our case we have to use [4]

\[
\frac{\partial R}{\partial t_0} = -c, \quad \frac{\partial x_0}{\partial t_0} = V \quad \text{and} \quad \frac{\partial V}{\partial t_0} = a.
\] (33)

Now, using Eqs. (29) and (33), we want to verify the validity of Eqs. (30)-(32). The result of the verification is as follows [5]:

\[
\frac{\partial E_z}{\partial t_0} \frac{\partial t_0}{\partial y} - \frac{\partial E_y}{\partial t_0} \frac{\partial t_0}{\partial z} = 0,
\] (34)

\[
\frac{\partial E_x}{\partial t_0} \frac{\partial t_0}{\partial z} - \frac{\partial E_z}{\partial t_0} \frac{\partial t_0}{\partial x} + \frac{1}{c} \frac{\partial B_y}{\partial t_0} \frac{\partial t_0}{\partial t} = -\frac{ac(z - z_0)}{(cR - V(x - x_0))^3},
\] (35)

\[
\frac{\partial E_y}{\partial t_0} \frac{\partial t_0}{\partial x} - \frac{\partial E_x}{\partial t_0} \frac{\partial t_0}{\partial y} + \frac{1}{c} \frac{\partial B_z}{\partial t_0} \frac{\partial t_0}{\partial t} = \frac{ac(y - y_0)}{(cR - V(x - x_0))^3}.
\] (36)

The verification [6] shows that Eq. (30) is valid. But instead of Eq. (31) and Eq. (32) we have Eq. (35) and Eq. (36) respectively. A reader has to agree that this result is rather unexpected.

However, another way to obtain the fields (1) and (2) exists. If one uses this manner to verify the validity of Maxwell’s equations, one finds that fields (1) and (2) satisfy these equations. In the next section we shall consider this way in detail and we shall show that it does not correspond to a pure retarded interaction between the charge and the point of observation.

\[5\]See the footnote (3).

\[6\]The expressions (34)-(36) were calculated using the program “Mathematica, Version 2.2”, therefore it is easy to check these calculations.

\[7\]There is another manner to verify the validity of Eqs. (30)-(32). If one substitutes \( E \) and \( B \) from (10) in Eq. (7), one only has to satisfy oneself that the operators “\( \nabla \times \)” and “\( \partial/\partial t \)” commute. In our case, because of \( V = (V, 0, 0) \) and \( A = (A_x, 0, 0) \), it means the commutation of the operators \( \partial/\partial y \) (or \( z \)) and \( \partial/\partial t \). The verification shows that these operators do not commute if one uses the rules (11).
4. Double (implicit and explicit) dependence of $\phi$, $A$, $E$ and $B$ on $t$ and $x_i$.

Total derivatives: mathematical and physical aspects

Let us, at the beginning, consider in detail Landau’s method [2] to obtain the derivatives $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$. Landau considered two different expressions of the function $R$:

$$R = c(t - t_0), \quad \text{where} \quad t_0 = f(x, y, z, t) \quad (37)$$

and

$$R = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}, \quad \text{where} \quad x_0i = f_i(t_0). \quad (38)$$

Then one calculates the derivatives ($\partial/\partial t$ and $\partial/\partial x_i$) of functions (37) and (38), and equating the results obtains $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$. While Landau uses here a symbol $\partial$ (see the expression before Eq. (63.6) in [2]) in order to emphasize that $R$ depends also on others independent variables $x, y, z$, it is easy to show that he calculates here total derivatives of the functions (37), (38) with respect to $t$ and $x_i$. The point is that if a given function is expressed by two different types of functional dependencies, exclusively total derivatives of these expressions with respect to a given variable can be equated (contrary to the partial ones). Here we adduce the scheme\(^8\) which was used in [2] to obtain $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$:

\[^8\]In this scheme we have used a symbol $d$ for a total derivative. In original text [2] we have

$$\frac{\partial R}{\partial t} = \frac{\partial R}{\partial t_0} \frac{\partial t_0}{\partial t} = -\frac{RV}{R} \frac{\partial t_0}{\partial t} = c \left(1 - \frac{\partial t_0}{\partial t}\right), \quad \text{and} \quad \nabla t_0 = -\frac{1}{c} \nabla R(t_0) = \frac{1}{c} \left(\frac{\partial R}{\partial t_0} \nabla t_0 + \frac{R}{R}\right).$$
If one takes into account that \( \partial t / \partial x_i = \partial x_i / \partial t = 0 \), as a result obtains the same values of the derivatives which have been obtained in (14).

Let us now, as it was mentioned above in the fin of Section 3, calculate the expressions (10) taking into consideration that the functions \( \varphi \) and \( A \) depend on \( t \) (or on \( x_i \)) implicitly and explicitly simultaneously. In this case we have:

\[
\frac{\partial \varphi}{\partial x_i} = -\frac{q}{(R - R\beta)^2} \left( \frac{\partial R}{\partial x_i} - R \frac{\partial R}{\partial x_i} \beta - R \frac{\partial \beta}{\partial x_i} \right),
\]

\[
\frac{\partial \varphi}{\partial t} = -\frac{q}{(R - R\beta)^2} \left( \frac{\partial R}{\partial t} - R \frac{\partial R}{\partial t} \beta - R \frac{\partial \beta}{\partial t} \right),
\]

and

\[
\frac{\partial A}{\partial t} = \frac{\partial \varphi}{\partial t} \beta + \varphi \frac{\partial \beta}{\partial t},
\]

where

\[
\frac{\partial \beta}{\partial t} = \frac{\partial \beta}{\partial t_0} \frac{\partial t_0}{\partial t} \quad \text{and} \quad \frac{\partial \beta}{\partial x_i} = \frac{\partial \beta}{\partial t_0} \frac{\partial t_0}{\partial x_i}.
\]

Now, let us consider all derivatives in (10), (40)-(43) as total derivatives with respect to \( t \) and \( x_i \). Then, if substitute the expressions (40)-(43) in (10) (of course, taking into account either lhs or rhs of the scheme (39)), we obtain formally the same expressions for the fields (1) and (2)! Then if one substitutes the fields (1) and (2) in the Maxwell’s

\[\text{(39)}\]

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9This depends on the choice of the expression for \( R \) in (37), (38)
equation (7), considering all derivatives in (7) as total ones and, of course, considering the functions $E$ and $B$ as functions with both implicit and explicit dependence on $t$ (or on $x_i$), one can see that the equation (7) is satisfied!

5. Conclusion

If we consider only the implicit functional dependence of $E$ and $B$ with respect to the time $t$, this means that we describe exclusively the retarded interaction: the electromagnetic perturbation created by the charge at the instant $t_0$ reaches the point of observation $(x, y, z)$ after the time $\tau = R(t_0)/c$. Surprisingly, the Maxwell equations are not satisfied in this case!

If we take into account a possible explicit functional dependence of $E$ and $B$ with respect to the time $t$, together with the implicit dependence, the Maxwell equations are satisfied. The explicit dependence of $E$ and $B$ on $t$ means that, contrary to the implicit dependence, there is not a retarded time for electromagnetic perturbation to reach the point of observation. A possible interpretation may be an action-at-a-distance phenomenon, as a full-value solution of the Maxwell equations within the framework of the so called “dualism concept” [9,10]. In other words, there is a simultaneous and independent coexistence of instantaneous long-range and retarded short-range interactions which cannot be reduced to each other.

Acknowledgments

We are grateful to Professor V. Dvoeglazov and Drs. D.W. Ahluwalia and F. Brau for many stimulating discussions and critical comments. We acknowledge the paper of Professor E. Comay, which put an idea into us to make the present work.
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