A SYMMETRY APPROACH TO CP VIOLATION

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ABSTRACT

One of the greatest challenges for particle physics in the 1990’s is understanding the broken symmetry of CP violation. It is now almost 30 years since the discovery in 1964 of the $K_L \rightarrow 2\pi$ decay. What has happened since? Why has there been no significant new experimental input in this long period? The original $K_L \rightarrow 2\pi$ decay experiment is described by two parameters $\epsilon$ and $\epsilon'$. Today $\epsilon \approx$ its 1964 value while $\epsilon'$ still consistent with zero, and there is no new evidence for CP violation outside the kaon system. Why is it so hard to find CP violation? How can B Physics Help? We present a symmetry approach to these questions.

1. Introduction

1.1. Two approaches to detection of CP Violation

There are two kinds of experimental phenomena which can exhibit CP violation.

(1) Charge asymmetries between the decays of charge conjugate states $M^\pm$ into charge conjugate exclusive final states $f^\mp$; i.e. $M^+ \rightarrow f^+ \neq M^- \rightarrow f^-$. So far no such charge asymmetries have been found.

(2) CP violation in Neutral Meson Mixing. The two mass eigenstates resulting from mixing both decay into same the CP eigenstate. This is found in the neutral kaon system - both mass eigenstates, $K_S$ and $K_L$ decay into two pions. So far this is the only experimental evidence for CP violation.\(^1\)

By using a symmetry approach we can understand why it is so difficult to observe charge asymmetries, and see the essential features of neutral meson mixing.

As a guide to experimenters, symmetries show what works, what doesn’t work and why.
1.2. What does $K_L \to 2\pi$ imply? CP before and after

Before 1964 the two kaon flavour eigenstates $K^o$ and $\bar{K}^o$ carrying strangeness $\pm 1$ were known to be produced in strangeness conserving strong interactions; e.g.

$$K^+ + n \to K^o + p, \quad K^- + p \to \bar{K}^o + n$$

and believed to go into one another under a conserved $CP$ operation.

$$CP \ |K^o\rangle = - |\bar{K}^o\rangle$$

The transition matrix elements for the $CP$-conserving $\pi^+\pi^-$ decay satisfy the relation,

$$\langle \pi^+\pi^- | T | K^o \rangle = - \langle \pi^+\pi^- | T | \bar{K}^o \rangle$$

The mass eigenstates are the $CP$ eigenstates

$$|K_1\rangle = (1/\sqrt{2})(|K^o\rangle - |\bar{K}^o\rangle); \quad \tau_1 = 9 \times 10^{-11} \text{sec}$$

$$|K_2\rangle = (1/\sqrt{2})(|K^o\rangle + |\bar{K}^o\rangle); \quad \tau_2 = 5 \times 10^{-8} \text{sec}$$

The two states have very different lifetimes because the dominant decay mode with the largest phase space is allowed by $CP$ for $K_1$ and forbidden for $K_2$.

$$\langle \pi^+\pi^- | T | K_2 \rangle = \langle \pi^+\pi^- | T | K^o \rangle + \langle \pi^+\pi^- | T | \bar{K}^o \rangle = 0$$

The discovery that the long-lived kaon also decayed into $|\pi^+\pi^-\rangle$ immediately showed $CP$ violation, which was described by defining the following parameters:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | T | K_L \rangle}{\langle \pi^+\pi^- | T | K_S \rangle} \equiv \epsilon + \epsilon' \approx 2.27 \times 10^{-3}$$

$$\eta_{oo} \equiv \frac{\langle \pi^o\pi^o | T | K_L \rangle}{\langle \pi^o\pi^o | T | K_S \rangle} \equiv \epsilon - 2\epsilon' \approx 2.25 \times 10^{-3}$$

$$\frac{\epsilon'}{\epsilon} \approx (2.2 \pm 1.1) \times 10^{-3}$$

where the numerical values are qualitatively correct but may not be exactly up to date. The value of $\epsilon'$ is still consistent both with zero and the value predicted by the standard model.
1.3. Use of the EPR effect in neutral meson mixing.

A linear combination of the two mass eigenstates can always be constructed for which a given decay mode is forbidden; e.g.

\[ |K^\pm_\nu\rangle = |K_L\rangle - \eta_{+-} |K_S\rangle; \quad \langle \pi^+\pi^- | T | K^\pm_\nu\rangle = 0 \quad (1.7a) \]

\[ |K'^{oo}_\nu\rangle = |K_L\rangle - \eta_{oo} |K_S\rangle; \quad \langle \pi^o\pi^o | T | K'^{oo}_\nu\rangle = 0 \quad (1.7b) \]

The difference between these two states is proportional to the parameter \( \epsilon' \)

\[ |K'^{oo}_\nu\rangle - |K^\pm_\nu\rangle = 3\epsilon' |K_S\rangle \quad (1.8) \]

A \( |K^\pm_\nu\rangle \) beam should not decay to \( \pi^+\pi^- \), while the decay \( K^\pm_\nu \to \pi^o\pi^o \) is proportional to \( \epsilon' \) and could be used in a null experiment to determine \( \epsilon' \).

\[ \langle \pi^o\pi^o | T | K^\pm_\nu\rangle = -3\epsilon' \langle \pi^o\pi^o | T | K_S\rangle \quad (1.9) \]

The Einstein-Podolsky-Rosen effect provides a means for creating a \( |K^\pm_\nu\rangle \) beam. Consider the decay of the \( \phi \) vector meson at rest into two neutral kaons with momenta \( \vec{k} \) and \(-\vec{k}\)

\[ \phi \to K^o(\vec{k})\bar{K}^o(\vec{k}) - K^o(-\vec{k})\bar{K}^o(\vec{k}) \quad (1.10a) \]

This same wave function can also be written in a basis \((K^\pm_\mu; K^\pm_\nu)\) where the state \( K^\pm_\mu \) is defined to be orthogonal to the state \( K^\pm_\nu \)

\[ \phi \to K^\pm_\nu(\vec{k})K^\pm_\mu(-\vec{k}) - K^\pm_\mu(-\vec{k})\bar{K}^\pm_\nu(\vec{k}) \quad (1.10b) \]

If a decay \( K^\pm_\mu \to \pi^+\pi^- \) is detected at \(-\vec{k}\), the wave function collapses to make \( K^\pm_\nu \) beam at \( \vec{k} \). This proposal was called “An experiment for the future” when first suggested in 1968.² Now we hear suggestions for carrying out such experiments at \( \phi \) factories, and the EPR effect is in common use in \( B \) decay experiments using the \( B \) analog of the \( \phi \), the first bottomonium state \( T(4S) \) above \( BB \) threshold.
2. Detecting Charge Asymmetries in Decays

2.1. How CPT complicates detection of CP Violation

Can decays of $K^+$ and $K^−$ be different? For decays to a pair of charge conjugate final states $|f^\pm\rangle$ described by the Fermi Golden Rule,

$$W_{K^\pm\to f} \approx (2\pi/\hbar)|\langle f^\pm | H_{wk} | K^\pm\rangle|^2 \rho(E_f)$$

(2.1)

But from CPT and hermiticity, we see that there can be no asymmetry,

$$|\langle f^- | H_{wk} | K^-\rangle|^2 = |\langle K^+ | H_{wk} | f^+\rangle|^2 = |\langle f^+ | H_{wk} | K^+\rangle|^2$$

(2.2a)

$$W_{K^+\to f^+} \approx W_{K^-\to f^-}$$

(2.2b)

CPT also requires equal total widths of $K^+$ and $K^-$. This is easily seen by noting that s-wave elastic $\pi^\pm\pi^o$ scatterings go into one another under CPT. Thus $\sigma_{el.,s}(\pi^+\pi^o) = \sigma_{el.,s}(\pi^-\pi^o)$ in the neighborhood of the kaon mass and is a very narrow Breit-Wigner resonance with the same width for both charge states,

$$\Gamma_{tot}(K^+) = \Gamma_{tot}(K^-)$$

(2.3)

Thus the following conditions are necessary for observation of charge-asymmetric decays:

1. Golden rule breaks down. This is exact first order perturbation theory and can only break down where higher order contributions are important. Second-order weak contributions are negligible; thus higher order strong contributions are needed.

2. Conspiracy of several decay modes. Total widths must be equal. Any asymmetry in the partial widths of a pair of conjugate modes must be compensated by opposite asymmetries in other modes.

We see immediately that it is difficult to satisfy these conditions in the kaon system. At the kaon mass s-wave $\pi^\pm\pi^o$ scattering can only be elastic; no inelastic channels are open. Thus the s-wave $\pi^\pm\pi^o$ state is an exact eigenstate of the strong interaction S-matrix, the golden rule holds for $K^\pm \to \pi^\pm\pi^o$ and no charge asymmetry can be observed. Some possibilities exist in other decay modes, like $3\pi$, where the $\pi^\pm\pi^\pm\pi^\mp$ and $\pi^\pm\pi^o\pi^o$ modes are coupled. However, these are linear combinations of two isospin eigenstates with $I=1$ and $I=3$. The $I=3$ amplitude is expected to be suppressed; it is a $\Delta I = 5/2$ transition and doubly suppressed by the $\Delta I = 1/2$ rule. Thus the $I=1$ amplitude is nearly an eigenstate of the strong interaction S matrix and the golden rule should be a good approximation. A similar situation obtains for different partial wave amplitudes which are coupled. Here the overall s-wave is expected to be dominant and again be an approximate strong S-matrix eigenstate. Thus all charge asymmetry effects in the kaon system are expected to be small.
2.2. Beating CPT for Charge Asymmetries in B Physics

Can decays of $B^+$ and $B^-$ be different? Here many more channels are open, different decay modes can conspire to give the same total width and Final state rescattering can beat the Fermi golden rule via higher order transitions in strong interactions; e.g.

\[
B^- \rightarrow K^0\pi^- \rightarrow K^-\pi^0; \quad B^+ \rightarrow K^0\pi^+ \rightarrow K^+\pi^0
\]  \hspace{1cm} (2.4)

This has no simple counterpart in the kaon system where the only open hadronic channels are $2\pi$ and $3\pi$ and the $I=3$ amplitude is strongly suppressed. Here both $(K\pi)$ isospin eigenstates $I = 1/2$ and $I = 3/2$ are produced by $\Delta I = 1$ transitions and are equally allowed.

How a CP-violating asymmetry can be obtained is very simply illustrated in a toy model where only $K\pi$ decay modes contribute to $B$ decay. The isospin eigenstates $(K\pi)_I$ where $I = 1/2$ and $I = 3/2$ are eigenstates of the strong interaction S-matrix and are both expected to be produced without any suppression. Thus the golden rule applies to decays into these states. Since the strong interactions are exactly diagonalized and the higher order weak interactions are negligible there are no higher order corrections to decays into isospin eigenstates in this model. Thus from CPT and hermiticity there can be no charge asymmetry in decays to isospin eigenstates. For $I=1/2$ and $3/2$,

\[
|\langle (K\pi)_I | H_{wk} | B^- \rangle |^2 = |\langle (K\pi)_I | H_{wk} | B^+ \rangle |^2 \hspace{1cm} (2.5a)
\]

\[
\Gamma\{B^+ \rightarrow (K\pi)_I\} = \Gamma\{B^- \rightarrow (K\pi)_I\} \hspace{1cm} (2.5b)
\]

Then

\[
\Gamma_{tot}(B^+) = \sum_I \Gamma\{B^+ \rightarrow (K\pi)_I\} = \Gamma_{tot}(B^-) = \sum_I \Gamma\{B^- \rightarrow (K\pi)_I\} \hspace{1cm} (2.6)
\]

in agreement with the CPT requirement of charge symmetric total widths.

However, asymmetries can occur for decays into final states which are not strong interaction eigenstates; e.g. $K^\pm\pi^0$:

\[
A\{B^+ \rightarrow K^+\pi^0\} = \sum_I C_I^{+o} |A\{B^+ \rightarrow (K\pi)_I\}| \cdot e^{iW_I} e^{iS_I} \hspace{1cm} (2.7a)
\]

\[
A\{B^- \rightarrow K^-\pi^0\} = \sum_I C_I^{-o} |A\{B^+ \rightarrow (K\pi)_I\}| \cdot e^{-iW_I} e^{iS_I} \hspace{1cm} (2.7b)
\]

where $C_I^{+o}$ denotes Clebsch-Gordan coefficients for isospin couplings. We have written every isospin amplitude as the product of its magnitude, a weak phase factor
\(e^{-iW_I}\) and a strong phase factor \(e^{iS_I}\), and noted that the weak CP-violating phase reverses sign under charge conjugation while the strong CP-conserving phase remains unchanged. Then \(I=3/2 - 1/2\) interference can produce charge asymmetry,

\[
|A\{B^+ \to K^+ \pi^o\}|^2 - |A\{B^- \to K^- \pi^o\}|^2 =
\]

\[
= C_1^+ C_3^+ |A_1 A_3| \cdot \left\{ e^{i(W_1-W_3)} e^{i(S_1-S_3)} - e^{i(W_3-W_1)} e^{i(S_1-S_3)} \right\} + c.c. \quad (2.8)
\]

The asymmetry is seen to vanish unless both \(W_1 \neq W_3\) and \(S_1 \neq S_3\). Thus the condition for observing an asymmetry is that at least two amplitudes must contribute which arise from different strong interaction eigenstates, and that these amplitudes must have both different strong phases and different weak phases.

### 2.3. Charge Asymmetry in Standard Model - Trees and Penguins

In the standard model two different diagrams with different weak phases can contribute to \(B \to K\pi\) decays via two different strong interaction eigenstates. There is therefore a possibility of observing a CP asymmetry.

The tree diagram gives only the \(K^\pm \pi^o\) final states.

\[
B^+ (\bar{b}u) \to \bar{u} + W^+ + u \to \bar{u} + u + s + u \to K^+ + \pi^o \quad (2.9a)
\]

\[
B^- (b\bar{u}) \to u + W^- + \bar{u} \to u + \bar{u} + s + \bar{u} \to K^- + \pi^o \quad (2.9b)
\]

The penguin diagram gives only the \(I=1/2 K - \pi\) final state.

\[
B^+ (b\bar{u}) \to \ell + W^+ + u \to \bar{s} + u \to (K\pi)_{I=1/2} \quad (2.10a)
\]

\[
B^- (b\bar{u}) \to \ell + W^- + \bar{u} \to s + \bar{u} \to (\bar{K}\pi)_{I=1/2} \quad (2.10b)
\]

In this case the tree contribution is strongly suppressed. It involves both suppressed weak vertices, \(b \to u\) and \(s \to u\). There is therefore hope that tree-penguin interference may be observed. So far no penguin contributions have been unambiguously identified.
3. Symmetry Analysis of Neutral Meson $M^o - \bar{M}^o$ Mixing

3.1. A Quasispin Description of Neutral Meson Mixing

It is convenient to describe neutral meson mixing by a quasispin SU(2) picture in which the two flavor eigenstates of the meson system, denoted by $M$ and $\bar{M}$ are defined as eigenstates of $\sigma_z$ with “spin up” and “spin down” respectively. Thus

$$\sigma_z | M^o \rangle = | M^o \rangle; \quad \sigma_z | \bar{M}^o \rangle = -| \bar{M}^o \rangle$$ (3.1)

Strong and electromagnetic interactions conserve quasispin. Weak interactions break quasispin.

If CPT is conserved the mass eigenstates $M_S$ and $M_L$ are equal mixtures of $M^o$ and $\bar{M}^o$. We can then choose quasispin x axis to make

$$\sigma_x | M_S \rangle = | M_S \rangle; \quad \sigma_x | M_L \rangle = -| M_L \rangle$$ (3.2)

If CP is conserved, $M_S$ and $M_L$ are also CP eigenstates.

If CP is violated, $M_S$ and $M_L$ can both decay into the same given CP eigenstate $| f \rangle$. But a basis $(M_\nu; M_\mu)$ can be defined to make $\langle f | H_{\text{weak}} | M_\nu \rangle = 0$. If $M_\nu$ and $M_\mu$ also equal mixtures of $M^o$ and $\bar{M}^o$, as occurs in many cases, they define a direction in the $x - y$ plane at some angle $\theta_f$ with the $x$ axis. The values of $\theta_f$ for two different CP eigenstates are directly to the CP-violation parameters $\epsilon$ and $\epsilon'$. If $CP$ is conserved, $\theta_f = 0$.

3.2. Quasispin Symmetry Breaking by Weak Interaction

There are two symmetry-breaking mechanisms.

Breaking by the lifetime difference is dominant in the kaon system where a pure $K_L$ state can be produced simply by waiting. The breaking is determined by phase space and independent of the Standard model. Lifetime breaking is negligible in heavy quark mesons.

Breaking by the mass difference is dominant in heavy quark mesons and determined by dynamical effects depending upon standard model. In the quasispin picture this breaking can be described as a “magnetic field” in the quasispin space. The time dependence of the heavy meson mixing is then described as a quasispin precession in the magnetic field.

Experiments can be described as the production of a quasispin polarized beam followed by a subsequent polarization measurement.
The time development of a general neutral $B$ meson which is in the state $|B(0)\rangle$ at time $t = 0$ is given by

$$|B(t)\rangle = e^{-\frac{\Gamma}{2}t} e^{-i\omega\sigma_z(t/2)} |B(0)\rangle = e^{-\frac{\Gamma}{2}t} \cdot \{\cos(\frac{\omega t}{2}) - i\sigma_x \sin(\frac{\omega t}{2})\} |B(0)\rangle$$ (3.3)

where $\Gamma$ is the decay width and $\omega$ the mass difference between the two eigenstates. Then for states which are initially $|B^o\rangle$ and $|\bar{B}^o\rangle$ at $t=0$,

$$|B^o(t)\rangle = e^{-\frac{\Gamma}{2}t} \cdot \{\cos(\frac{\omega t}{2}) |B^o\rangle - i\sin(\frac{\omega t}{2}) |\bar{B}^o\rangle\}$$ (3.4a)

$$|\bar{B}^o(t)\rangle = e^{-\frac{\Gamma}{2}t} \cdot \{\cos(\frac{\omega t}{2}) |\bar{B}^o\rangle - i\sin(\frac{\omega t}{2}) |B^o\rangle\}$$ (3.4b)

Then

$$2e^{\Gamma t} |\langle B^o | B^o(t) \rangle|^2 = \cos^2(\frac{\omega t}{2}) = \frac{1}{2}\{1 + \cos(\omega t)\}$$ (3.5a)

$$2e^{\Gamma t} |\langle B^o | \bar{B}^o(t) \rangle|^2 = \sin^2(\frac{\omega t}{2}) = \frac{1}{2}\{1 - \cos(\omega t)\}$$ (3.5b)

This is just the well known $B^o - \bar{B}^o$ mixing independent of all $CP$ Violation.

### 3.3. Experiments as Quasispin Polarization Measurements

Consider an experiment in which a $B$ meson is prepared in some state $|B_\nu\rangle$ and its decay is observed after a time $t$ in a mode allowed only for $B$ or allowed only for $\bar{B}$; e.g. leptonic modes. The difference between the probabilities of decay into $B$ or $\bar{B}$ allowed modes; e.g. a lepton asymmetry, is just given by the quasispin polarization in the $z$ direction of the prepared state. This is easily evaluated using the Pauli spin algebra.

$$|\langle B^o | e^{-i\omega\sigma_z(t/2)} |B_\nu\rangle|^2 - |\langle B^o | e^{-i\omega\sigma_z(t/2)} |B_\nu\rangle|^2 = \langle B_\nu | e^{i\omega\sigma_z(t/2)} \sigma_z e^{-i\omega\sigma_z(t/2)} |B_\nu\rangle$$

$$= \langle B_\nu | \sigma_z e^{-i\omega\sigma_z} |B_\nu\rangle = \langle B_\nu | \sigma_z \cos(\omega t) - i\sigma_z \sigma_x \sin(\omega t) |B_\nu\rangle =$$

$$= \langle B_\nu | \sigma_z |B_\nu\rangle \cos(\omega t) + \langle B_\nu | \sigma_y |B_\nu\rangle \sin(\omega t) = \sin(\omega t) \sin \theta$$ (3.6)

where the last equality holds for the case where the state $|B_\nu\rangle$ is an equal mixture of $B^o$ and $\bar{B}^o$ and thus has its quasispin in the direction of an axis in the $x - y$ plane. The angle $\theta$ between this axis and the $x$ axis is determined by the relative phase of the $B^o$ and $\bar{B}^o$ components. This situation occurs often in experiments where the state $|B_\nu\rangle$ is prepared by observing a decay into a $CP$ eigenstate. The angle $\theta$ then gives a measure of the $CP$ violation.
A common example of such an experiment is one in which a neutral $B\bar{B}$ pair is created by the decay of the $\Upsilon(4S)$, one $B$ decays in the $K_S\psi$ decay mode, and the other decays into a leptonic mode. Let us define a basis $(B_\mu; B_\nu)$ to make $\langle K_S\psi | T | B_\nu \rangle = 0$. The second $B$ is required to be in the state $| B_\nu \rangle$ at the time that the $K_S\psi$ decay of the other $B$ is observed. The lepton asymmetry observed in the second decay at a time $t$ after the first decay is seen to be given by the expression (3.6). Another interesting identity relevant to this experiment is obtained by use of the quasispin algebra:

$$| \langle B_\mu | e^{-i\sigma_z(t/2)} | B^0 \rangle | = | \langle B_\mu | e^{-i\omega\sigma_z(t/2)} \sigma_z | B^0 \rangle | = | \langle B_\mu | \sigma_z e^{i\omega\sigma_z(t/2)} | B^0 \rangle | =$$

$$= | \langle B_\nu | e^{i\omega\sigma_z(t/2)} | B^0 \rangle | = | \langle B^0 | e^{-i\omega\sigma_z(t/2)} | B_\nu \rangle^* |$$ (3.7)

Thus the probability that a meson created as a $B^0$ at time $t_1$ will be observed as a $B_\mu$ at time $t_2$ is exactly equal to the probability that a meson created as a $B_\nu$ at time $t_1$ will be observed as a $B^0$ at time $t_2$.

$$P\{B^0(t_1) \rightarrow B_\mu(t_2)\} = P\{B_\nu(t_1) \rightarrow B^0(t_2)\}$$ (3.8)

We now see the implications of this identity for the $\Upsilon(4S)$ experiment in which one $B \rightarrow K_S\psi$ and the other decays leptonically.

$$P\{\Upsilon(4S) \rightarrow \bar{B}^0(t_1)B_\mu(t_2)\} = P\{\Upsilon(4S) \rightarrow \bar{B}^0(t_1)B^0(t_1)\} \cdot P\{B^0(t_1) \rightarrow B_\mu(t_2)\}$$ (3.9a)

$$P\{\Upsilon(4S) \rightarrow B_\mu(t_1)B^0(t_2)\} = P\{\Upsilon(4S) \rightarrow B_\mu(t_1)B_\nu(t_1)\} \cdot P\{B_\nu(t_1) \rightarrow B^0(t_2)\}$$ (3.9b)

$$P\{\Upsilon(4S) \rightarrow \bar{B}^0(t_1)B_\mu(t_2)\} = P\{\Upsilon(4S) \rightarrow B_\mu(t_1)B^0(t_2)\}$$ (3.10)

The lepton asymmetry observed at time $t_1$ when a $K_S\psi$ decay is observed at time $t_2$ is seen to be exactly equal and opposite to lepton asymmetry observed at time $t_2$ when a $K_S\psi$ decay is observed at time $t_1$. Thus in this kind of experiment the CP-violating lepton asymmetry cancels out if the results are integrated over time. Since time measurements are difficult in the rest frame of the $\Upsilon(4S)$ where the two $B$ mesons move very slowly “asymmetric B factories” have been proposed to produce the $\Upsilon(4S)$ in flight so that the $B$ mesons traverse a measurable distance before decay.
4. Summary - How $B$ and $K$ physics differ - Good and bad news

1. No Dominant $B$ Decay Mode
   (a) No Lifetime Difference
   (b) Mass Eigenstates Not Separated by Waiting

2. Many $B$ Decay Modes
   (a) Rich Data - Small Branching Ratios $\approx 1\%$
   (b) Final State Rescattering - Beats Golden Rule
   (c) Conspiracies Beat CPT Restrictions

3. $B^0 - \bar{B}^0$ Oscillations During Decay
   (a) Time Dependence Confuses Measurements
   (b) CP Violation Observable in Mixing Phases

4. All Dominant Hadronic $B$ decays involve 3 Generations
   (a) CP violation Observable in $B$ Decays in Direct Diagrams $b \to c\bar{u}d$
   (b) CP Violation Observable in charm and strangeness decays only via diagrams with virtual $t$ and $b$ quarks

5. References

1. J.H. Christensen et al., Phys. Rev. Lett. 13 (1964) 138
2. Harry J. Lipkin, Phys. Rev. 176 (1968) 1715