Search for the decay $B^0 \to a_1^\pm \rho^\mp$
We present a search for the rare $B$-meson decay $B^0 \rightarrow a_1^+ \rho^-$ with $a_1^+ \rightarrow \pi^+ \pi^- \pi^\pm$. We use $(110 \pm 1.2) \times 10^6 \Upsilon(4S) \rightarrow B\bar{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC. We obtain an upper limit of $30 \times 10^{-6}$ (90% C.L.) for the branching fraction product $B(B^0 \rightarrow a_1^+ \rho^+)B(a_1^- \rightarrow \pi^+ \pi^- \pi^\pm)$, where we assume that the $a_1^-$ decays exclusively to $\rho^0 \pi^\pm$.

PACS numbers: 13.25.Hw, 12.15.Hh, 11.30.Er

In the Standard Model, $CP$-violating effects in the $B$-meson system arise from a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. The decay $B^0 \rightarrow a_1^+ \rho^+$ proceeds via a $b \rightarrow u \bar{m} d$ transition [2], and interference between direct decay and decay after $B^0 \bar{B}^0$ mixing results in a time-dependent decay-rate asymmetry that is sensitive to the angle $\alpha \equiv \arg[-V_{tb}V_{tb}^*/V_{ub}V_{ub}^*]$ [3] in the unitarity triangle of the CKM matrix. An additional motivation for studying $B^0 \rightarrow a_1^+ \rho^+$ is that this is a significant background to $B \rightarrow \rho \rho$ decays, e.g. [4–8], which currently provide the most accurate measurement of $\alpha$. The ARGUS experiment previously searched for the decay $B^0 \rightarrow a_1^+ \rho^+$, which resulted in an upper limit of $B(B^0 \rightarrow a_1^+ \rho^+) < 3.4 \times 10^{-3}$ (90% C.L.) [9]. This paper presents the result of a search for $B^0 \rightarrow a_1^+ \rho^+$ with $a_1^+ \rightarrow \pi^+ \pi^- \pi^\pm$, where we assume that the $a_1^+$ decays exclusively to $\rho^0 \pi^\pm$. A theoretical prediction of the branching fraction $B(B^0 \rightarrow a_1^+ \rho^+)B(a_1^- \rightarrow (\pi \pi)\pi)$ has been made by Bauer, Stecher and Wirbel [10] within the framework of the factorization model. They predict a value of $43 \times 10^{-6}$, assuming $|V_{ub}/V_{cb}| = 0.08$.

The data used in this analysis were collected with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC during the years 2003-2004. This represents a total integrated luminosity of 100 fb$^{-1}$ taken at the $\Upsilon(4S)$ resonance (on-peak), corresponding to a sample of $110 \pm 1.2$ million $B\bar{B}$ pairs. An additional 21.6 fb$^{-1}$ of data, collected at approximately 40 MeV below the $\Upsilon(4S)$ resonance (off-peak), were used to study background from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events.

The BABAR detector is described in detail elsewhere [11]. Surrounding the interaction point is a silicon vertex tracker (SVT) with 5 double-sided layers which measures the impact parameters of charged particle tracks in both the plane transverse to, and along the beam direction. A 40 layer drift chamber (DCH) surrounds the SVT and provides measurements of the transverse momenta for charged particles. Both the SVT and the DCH operate in the magnetic field of a 1.5 T solenoid. Charged hadron identification is achieved through measurements of particle energy-loss in the tracking system and the Cherenkov angle obtained from a detector of internally reflected Cherenkov light. A CsI(Tl) electromagnetic calorimeter (EMC) provides photon detection, electron identification, and $\pi^0$ reconstruction. Finally, the instrumented flux return of the magnet allows discrimination of muons from pions.

We reconstruct $B^0 \rightarrow a_1^+ \rho^-$ candidates from combinations of $a_1^+ \rightarrow \pi^+ \pi^- \pi^\pm$ and $\rho^- \rightarrow \pi^0 \pi^\pm$ candidates. The $a_1(1260) \rightarrow 3\pi$ decay proceeds mainly through the intermediate states $((\pi \pi)\rho \pi)$ and $(\pi \pi)\pi [12]$. We do not distinguish between the dominant P-wave ($\pi \pi)\rho$ and S-wave ($\pi \pi)\pi$ in the channel $\pi^+ \pi^- \pi^-$. The Monte Carlo (MC) signal events are simulated as $B^0$ decays to $a_1^+ (1260)\rho^-$ with $a_1^+ \rightarrow \rho^0 \pi^\pm$ using the GEANT4-based [13] BABAR MC simulation. Possible contributions from $B^0$ decays to $a_2^\pm (1320)\rho^-$ and $\pi^+ (1300)\rho^-$ are investigated.

We only consider events that have a minimum of one $\pi^0$ and four charged tracks, where the charged tracks are required to be inconsistent with lepton, proton and kaon hypotheses.

We form $\pi^0 \rightarrow \gamma \gamma$ candidates from pairs of photon candidates that have been identified as localized energy deposits in the EMC that have the lateral energy distribution expected for a photon. Each photon is required to have an invariant mass of $0.10 < m_{\gamma\gamma} < 0.16$ GeV/c$^2$. The $\rho^-$ mesons are formed from one track that is consistent with a $\pi^-$ and the aforementioned $\pi^0$ candidate. The candidate $\rho^-$ is required to have an invariant mass of $0.5 < m_{\rho^-} < 1.1$ GeV/c$^2$. We also constrain the cosine of the angle between the $\pi^0$ momentum and the direction opposite to the $B^0$ in the $\rho^-$ rest frame ($\cos \theta_{\rho^-}$) to be between $-0.9$ and 0.98. This removes backgrounds which peak at the extremes of the distribution where the signal reconstruction efficiency also falls off.

We form the $a_1^+$ candidate from combinations of three charged pions. We first form a $\rho^+ \rightarrow \pi^+ \pi^- \pi^+$ candidate from two oppositely charged tracks. This combination is required to have an invariant mass of $0.4 < m_{\rho^+} < 1.1$ GeV/c$^2$. The $a_1^+$ candidate is then formed by adding another charged track to the $\rho^+$, and requiring that the mass of the $a_1^+$ satisfies $0.6 < m_{a_1^+} < 1.5$ GeV/c$^2$. The vertex of the $B$-candidate is constrained to originate from the beam spot. In order to reduce background from continuum events we require that $|\cos(\theta_T)| < 0.7$, where $\theta_T$ is the angle between the $B$ thrust axis and that of the rest of the event (ROE).

We use two kinematic variables, $m_{ES}$ and $\Delta E$, in order to isolate any signal. We define the beam-energy substituted mass $m_{ES} = \sqrt{(\sqrt{s}/2)^2 - (p_T^B)^2}$, where $\sqrt{s}$ is the $e^+e^-$ center-of-mass (CM) energy. The second kinematic variable, $\Delta E$, is the difference between the $B$-candidate energy and the beam energy in the CM frame. We require $m_{ES} > 5.25$ GeV/c$^2$ and $-0.15 < \Delta E < 0.1$ GeV.
Additional separation between signal and continuum is obtained by combining several kinematic and topological variables into a Fisher discriminant \((F)\) [14]. The variables \(L_0, L_2, \) and \(| \cos \theta_{TR} |\), and the output of a multivariate tagging algorithm [15] are used as inputs to \(F\). \(L_0\) and \(L_2\) are defined as

\[
L_0 = \sum_{\text{ROE}} | p_i^* |, \quad L_2 = \sum_{\text{ROE}} | p_i^* | \cos(\theta_i)^2, \tag{1}
\]

where the sum is over the ROE, \(p_i^*\) is the particle momentum in the CM frame, \(\theta_i\) is the angle of the particle direction relative to the thrust axis of the \(B\)-candidate, and \(\cos \theta_{TR}\) is the cosine of the angle between the \(B\) thrust axis and the beam axis. The multivariate tagging algorithm identifies the flavor of the other \(B\) in the event to be either a \(B^0\) or \(\bar{B}^0\). The output of this algorithm is ranked into categories of different signal purity.

We expect the polarization of the \(a_1^-\rho^-\) final state to be predominantly longitudinal, as was found in the similar decay \(B \to \rho\rho\) [4–8]. We have used both longitudinal and transverse polarized signal MC simulated data in this analysis. After applying the selection cuts above, we have 2.8 (2.3) longitudinal (transverse) polarized signal MC simulated data candidates per event.

We define as self-cross-feed (SCF) the set of candidates that were incorrectly reconstructed from particles in events that contain a true signal candidate. We select one \(B\) candidate per event in which the mass of the reconstructed \(\rho^0\) is closest to that of the true \(\rho^0\) mass [16]. Choosing the candidate using the \(\rho^0\) mass reduces the SCF fraction by 18% relative to a random selection. To avoid potentially biasing our final result, we do not use information from the \(\rho^0\) meson in the remainder of the analysis. After all selection cuts have been applied, the longitudinal and transverse signal SCF fractions are 0.58 and 0.42, respectively. The selection efficiency of longitudinal (transverse) signal is 9.44% (10.15%).

Besides the continuum background we also have background from \(B\) decays. We divide the \(B\)-background into the following four categories according to \(B\)-meson charge and the charm content of the final states: (i) \(B^0 \to\) charm, (ii) \(\bar{B}^0 \to\) charmless, (iii) \(B^\pm \to\) charm and (iv) \(B^{\pm} \to\) charmless. From large samples of inclusive MC simulated data we expect 2394, 424, 3281 and 215 events of these background types, respectively. In addition, a number of exclusive \(B\)-background modes that have a similar final state to the signal were studied. This includes those that have an intermediate \(a_1\) meson in the decay. None of these modes were seen to have a significant efficiency after the selection cuts had been applied.

We perform an extended unbinned maximum likelihood fit to the data. The likelihood model has the following types: (i)-(iv) the four aforementioned inclusive \(B\)-background categories, (v) true signal, (vi) SCF signal and the (vii) \(e^+e^- \to q\bar{q} (q = u, d, s, c)\) continuum background. The probability density function (PDF) for each event \(i\) has the form \(P_{i,c} = P_{i,c}(m_{ES}, \Delta E, \mathcal{F}, m_{\rho^-}, m_{\rho^-}, \cos \theta_{\rho^-})\). From these individual PDFs the total likelihood

\[
L = e^{-n} \prod_{i=1}^{n} \sum_{c} N_c P_{i,c}, \tag{2}
\]

is constructed. The parameters \(n\) and \(n'\) are the numbers of selected on-peak events, and the sum of the yields \(N_c\), where \(c\) is one of the seven types in the likelihood model. Correlations between the \(m_{\rho^-}\) and \(\cos \theta_{\rho^-}\) variables are taken into account for the real (T) and fake combinatorial (F) \(\rho^-\) candidates. All other correlations between fit variables are seen to be small. However, the effect of ignoring them results in a bias on the fitted signal yield which is discussed below.

Each of the signal distributions has a signal yield and a polarization fraction that are denoted by \(N_{sig}(N'_{sig})\) and \(f_L(f'_L)\), for the true (SCF) signal, such that the sum of the true and SCF signal is described by

\[
N_{sig}[f_L f_i^{long, true} + (1 - f_L) f_i^{tr, true}] + N'_{sig}[f'_L f_i^{long, SCF} + (1 - f'_L) f_i^{tr, SCF}]. \tag{3}
\]

The continuum yield, \(N_{sig}, N'_{sig}\), and the parameters of the continuum \(m_{ES}\) and \(\Delta E\) PDFs are allowed to vary in the fit. Under the assumption that no significant signal is observed, the value of \(f_L\) is fixed to 1.0 in the fit. The value of \(f'_L\) is also fixed to 1.0 in the fit since it is highly correlated with \(N_{sig}, N'_{sig}\) and \(f_L\). Only the fitted value of \(N_{sig}\) is used to derive the final result. We also fix the \(B\)-background yields to the aforementioned values.

The PDFs used for each component are given in Table I. The signal and \(B\)-backgrounds are parameterized using MC. We use a non-parametric smoothing algorithm [17] when defining some of the background PDFs (as indicated by the abbreviation NP). We account for the difference between \(F\) and \(T\) \(\rho^-\to \pi^-\pi^0\) distributions in the background using the product of 1D PDFs, denoted in Table I as ‘BG m-hel’, such that

\[
P_i(m_{\rho^-}, \cos \theta_{\rho^-}) = (1 - a_T) P_{T,i}(m_{\rho^-}) P_{F,i}(\cos \theta_{\rho^-}) + a_T P_{T,i}(m_{\rho^-}) P_{T,i}(\cos \theta_{\rho^-}), \tag{4}
\]

where \(a_T\) is the fraction of \(T\) events. The continuum shape for \(\cos \theta_{\rho^-}(m_{\rho^-})\) is derived from off-peak (on-peak) data. The true \(\rho^-\) resonance Breit Wigner shape uses \(m_{\rho^-} = 777 \text{ GeV}/c^2\), and \(\Gamma = 0.150 \text{ GeV}\) [16]. The parameterizations used for this PDF are summarized in Table II.

The results from the fit are \(N_{sig} = 90 \pm 38\) (stat), \(N'_{sig} = 42 \pm 98\) and a continuum yield of 25798±182 events. The bias on the fitted signal yield is evaluated by performing ensembles of mock experiments using signal MC embedded into MC samples of background generated from
TABLE I: The types of PDFs used to model the different variables for each component in the likelihood fit, where the PDFs underlined have their parameters varying in the nominal fit. The abbreviations are: G = Gaussian, G2 = Double Gaussian, G3 = Triple Gaussian, CB = Crystal Ball (a Gaussian with a low side exponential tail) [18], ARGUS = ARGUS function \(x\sqrt{1-x^2}\exp\left[-\xi(1-x^2)\right]\), with \(x \equiv 2m_{ES}/\sqrt{s}\) and parameter \(\xi\) [19], which is allowed to vary in the fit, Pn = Polynomial of order n, BW = Breit-Wigner, helicity = \(\cos^2\theta_{\rho^-}\) or \(\sin^2\theta_{\rho^-}\) depending on partial wave which is modified by a quadratic acceptance function, BG m-hel = Background \(\cos\theta_{\rho^-}\) and \(m_{\rho^-}\) PDF of Eq. 4, off-peak = PDF taken from off-peak data, and 1D = smoothed 1D histogram.

| Component | \(m_{ES}\) | \(\Delta E\) | \(\mathcal{F}\) | \(m_{a_1^+}\) | \(\cos\theta_{\rho^-}\) | \(m_{\rho^-}\) |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|
| signal (long/ trans./true/SCF) | CB | CB+G | G2 | G3 | helicity | BW+P4 |
| \(q\bar{q}\) | ARGUS | P1 | G2 (off-peak) 1D (off-peak) | BG m-hel | BG m-hel |
| \(B^0 (B^\pm)\rightarrow charm (charmless)\) | NP | NP | NP | BG m-hel | BG m-hel |

TABLE II: The types of PDFs used to model the different background \(\cos\theta_{\rho^-}\) and \(m_{\rho^-}\) PDF shapes. The abbreviations Pn and BW are defined in the caption of Table I.

| Component | \(T\ \cos\theta_{\rho^-}\) | \(T\ m_{\rho^-}\) | \(F\ \cos\theta_{\rho^-}\) | \(F\ m_{\rho^-}\) |
|-----------|-----------------|-----------------|-----------------|-----------------|
| \(q\bar{q}\) | P2 | BW + P1 | P5 | P4 |
| \(B^\pm\rightarrow charmless\) | P4 | BW | P5 | P3 |
| \(B^0\rightarrow charmless\) | P4 | BW | P5 | P3 |
| \(B^\pm\rightarrow charm\) | P2 | BW | P5 | P3 |
| \(B^0\rightarrow charm\) | P2 | BW | P5 | P3 |

SCF signal events. The effect of exclusive \(B\) meson decays to final states including \(a_1^-\)-mesons were evaluated using ensembles of mock experiments. In particular, the systematic uncertainty on the signal yield from neglecting \(B\rightarrow a_1^-a_1^-\) modes in the fit is 6 events. We assign a systematic uncertainty from using a relativistic Breit-Wigner with a Blatt-Weisskopf form factor with a range parameter of 3.0 GeV\(^{-1}\) for the \(a_1^-\) meson line shape. In the fit we assume that the \(a_1^-\) meson width, \(\Gamma_{a_1^-}\), is 400 MeV. We evaluate a systematic uncertainty due to this assumption by varying \(\Gamma_{a_1^-}\) over the experimentally allowed range: 250 - 600 MeV [16]. The difference in the distribution of \(\mathcal{F}\) between data and MC is evaluated with a large sample of \(B\rightarrow D^*\rho\) decays. The systematic uncertainties that contribute to the branching fraction only through the efficiency come from charged particle identification (6.0%), \(\pi^0\) meson reconstruction (3.0%), tracking efficiency (3.2%), and the number of \(B\) meson pairs (1.1%). The systematic error contribution from MC statistics is negligible.

When the fit bias correction of \(-22\) events is applied to the signal yield, and one accounts for systematic uncertainties, the significance of the result is...
0.95 standard deviations. Figure 2 shows the distribution of $-\ln(L/L_{\text{max}})$ for the fit, with and without these systematic errors. $L_{\text{max}}$ is the value of the likelihood corresponding to the nominal fit result. The branching fraction value for the fit-bias-corrected signal yield of $68 \pm 38 \text{(stat)} +45^{-56} \text{(syst)}$ is $B(B^0 \rightarrow a_1^+ \rho^-)B(a_1^+ \rightarrow \pi^+\pi^-\pi^+) = (15.7 \pm 8.7 \text{(stat)} +10.3^{-12.8} \text{(syst)}) \times 10^{-6}$. This assumes that $f_L = 1.0$ and that the branching fraction of $a_1^+ \rightarrow \pi^+\pi^-\pi^+ = 0.5$. As the signal yield obtained is not significant, we calculate the upper limit $x_{\text{UL}}$, by integrating the likelihood function (including systematic uncertainties) from 0 to $x_{\text{UL}}$, for different physically allowed values of $f_L$, such that the C.L. of the upper limit is 90%. As the signal efficiency is a function of $f_L$, we report the most conservative upper limit obtained, which corresponds to $f_L = 1.0$. On doing this, an upper limit of $30 \times 10^{-6}$ (90% C.L.) is obtained.

We have performed a search for the decay $B^0 \rightarrow a_1^+ \rho \pi$ in a data sample of 100 fb$^{-1}$. After correcting for fit bias and accounting for systematic uncertainties, the signal yield is $68 \pm 38 \text{(stat)} +45^{-56} \text{(syst)}$ events, with a significance of 0.95$\sigma$. As there is no significant evidence for a signal, we place an upper limit of $30 \times 10^{-6}$ (90% C.L.) on $B(B^0 \rightarrow a_1^+ \rho^-)B(a_1^+ \rightarrow \pi^+\pi^-\pi^+)$, where we assume that the $a_1^+$ decays exclusively to $\rho^0\pi^+$. Assuming $B(a_1^+ \rightarrow \pi^+\pi^-\pi^+)$ is equal to $B(a_1^+ \rightarrow \pi^+\pi^0\pi^0)$, we obtain $B(B^0 \rightarrow a_1^+ \rho^-)B(a_1^+ \rightarrow (3\pi)^+) < 61 \times 10^{-6}$ (90% C.L.). This upper limit corresponds to a significant improvement over the previous bound and is compatible with theoretical expectations [10]. This result is a significant improvement in constraining an important $B$ background contribution in $B \rightarrow \rho \pi$ decays.

**ACKNOWLEDGMENTS**

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and
kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from CONACyT (Mexico), Marie Curie EIF (European Union), the A. P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

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