The semileptonic decays of $B/B_s$ meson in the perturbative QCD approach:
A short review

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Abstract
In this short review, we present the current status about the theoretical and experimental studies for some important semileptonic decays of $B/B_s$ mesons. We firstly gave a brief introduction for the experimental measurements for $B/B_s \to P$ ($l^+l^-, l^-\bar{\nu}_l, \nu\bar{\nu}$) decays, the BaBar’s $R(D)$ and $R(D^*)$ anomaly, the $P'_5$ deviation for $B^0 \to K^{*0}\mu^+\mu^-$ decay. We then made a careful discussion about the evaluations for the relevant form factors in the light-cone QCD sum rule (LCSRs), the heavy quark effective theory, and the perturbative QCD factorization approach. By using the form factors calculated in the perturbative (pQCD) approach, we then calculate and show the pQCD predictions for the decay rates of many semileptonic decays of $B/B_s$ mesons. We also made careful phenomenological analysis for these pQCD predictions and found, in general, the following points: (a) For all the considered $B/B_s$ semileptonic decays, the next-to-leading order (NLO) pQCD predictions for their decay rates agree well with the data and those from other different theoretical methods; (b) For $R(D)$ and $R(D^*)$, the pQCD predictions agree very well with the data, the BaBar’s anomaly of $R(D^{(*)})$ are therefore explained successfully in the standard model by employing the pQCD approach; and (c) We defined several new ratios $R_{l\tau}^{D}$ and $R_{l\tau}^{D_s}$, they may be more sensitive to the QCD dynamics which controls the $B/B_s \to (D^{(*)}, D_s^{(*)})$ transitions than the old ratios, we therefore strongly suggest LHCb and the forthcoming Super-B experiments to measure these new ratios.

Key Words $B/B_s$ meson semileptonic decays; The pQCD factorization approach; Form factors; Branching ratios; LHCb experiments

1. INTRODUCTION

As is well-known, the semileptonic (SL) decays of $B$ and $B_s$ meson are very important processes in testing the standard model (SM) and in searching for the signal and/or evidence of the new physics (NP) beyond the standard model: such as the extractions of the Cabbibo-Kobayashi-Maskawa matrix elements $|V_{ub}|$ and $|V_{cb}|$, and the determinations of the form factors $F_{0,+}(q^2)$ for the $B/B_s$ transitions to the pion, kaon or other light mesons [1–4]. Since the Spring of 2012, the BaBar’s anomaly about the ratio $R(D^{(*)})$ [5, 6] invoked intensive studies for $B \to D^{(*)}l^-\bar{\nu}_l$ decays in the framework of the SM and various new physics (NP) models, for example, in Refs. [7–15]. The $B \to K^*\mu^+\mu^-$ anomaly observed by LHCb experiments [16–18] also stimulate many interesting studies [19–23]. We here will present a short review about the experimental measurements.

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and the theoretical studies for the following $B/B_s$ semileptonic decay modes:

$$B/B_s \rightarrow P(l^+l^-, l\nu, \nu\bar{\nu}), \quad (D^{(*)}, D_s^{(*)})l^-\bar{\nu}_l;$$

$$B \rightarrow K^*\mu^+\mu^-;$$

where $l = (e, \mu, \tau)$ are leptons and $P = (K, \pi, \eta, \eta')$ are light pseudoscalar mesons. For those considered $B/B_s \rightarrow P l\nu$ decays, the "Tree" Feynman diagrams provide the dominant leading order (LO) contribution. For those $B/B_s \rightarrow P l^+l^-$ and $P \nu\bar{\nu}$ decays, however, the dominant LO standard model contributions come from those electroweak penguin diagrams and $W^+W^-$ box diagrams. For $B/B_s \rightarrow D^{(*)}l^-\bar{\nu}_l$ decays, the $b \rightarrow c l^-\bar{\nu}_l$ transition at the quark level provide the dominant contribution.

As for the relevant experimental measurements, some considered decays of $B \rightarrow P(l^+l^-, l\nu, \nu\bar{\nu})$ have been measured by the Belle, BaBar, CLEO and/or LHCb experiments [24–30]. The LHCb and the forthcoming Super-B experiments [31, 32] will measure the $B_s \rightarrow P(l^+l^-, l\nu, \nu\bar{\nu})$ decays in the near future.

As for the $B \rightarrow D^{(*)}l\bar{\nu}_l$ decays, they have been measured by both BaBar and Belle collaboration [33–35]. Very recently, the BaBar collaboration reported their measurements for the ratios $R(D^{(*)})$ of the corresponding branching ratios [5, 6]:

$$\mathcal{R}(D) \equiv \frac{\mathcal{B}(B \rightarrow D\tau^-\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow Dl^-\bar{\nu}_l)} = 0.440 \pm 0.072,$$

$$\mathcal{R}(D^*) \equiv \frac{\mathcal{B}(B \rightarrow D^*\tau^-\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D^*l^-\bar{\nu}_l)} = 0.332 \pm 0.030.$$  

These BaBar results are surprisingly larger than the SM predictions as given, for example, in Ref. [36]:

$$\mathcal{R}(D)^{SM} = 0.296 \pm 0.016, \quad \mathcal{R}(D^*)^{SM} = 0.252 \pm 0.003,$$

The combined BaBar results disagree with the SM predictions by $3.4\sigma$ [5, 37]. The type-II two-Higgs-doublet model (2HDM) with a charged-Higgs boson is excluded at 99.8% confidence level for any value of $\tan \beta/m_H$ [6].

For the $B \rightarrow K^*\mu^+\mu^-$ decays, recent LHCb measurements show a good agreement with the SM predictions for most physical observables [16–18]. Some deviations of the angular observables from the SM have been observed yet [19–23]. With $3.7\sigma$ the most significant discrepancy arises in the variable $P_5$ [38]. Further LHCb studies based on more luminosity will be necessary to clarify whether the observed deviations are a real sign of NP or simply statistical artifacts [20, 21].

On the theory side, we know that the central issues for the considered Ssemileptonic $B/B_s$ decays are the estimations of the values and shapes of the relevant form factors for $B/B_s \rightarrow (P, D^{(*)})$ transitions. The traditional methods or approaches to calculate the relevant transition form factors are the light cone QCD sum rules (LCSR) [39–51], the heavy quark effective theory (HQET) [1, 15, 52–55] and the lattice QCD (LQCD) techniques [56–58]. In the pQCD approach, however, one can make direct perturbative calculation for the form factors for $B \rightarrow (\pi, K, etc)$ transitions [59–64]. Since the hadronic form factors always involve the non-perturbative QCD dynamics [65], the QCD factorization approach [66] based on the collinear factorization cannot be applied to compute the heavy-to-light form factors directly, but take the soft form factors as input.

In the pQCD factorization approach [67–69], in fact, one can write the form factors conceptually as a convolution of a hard kernel with the distribution amplitudes of those mesons involved...
in the decays. Since the longitudinal momentum $k_L$ approaches zero in the end point region, the parton transverse momenta $k_T$ here become non-negligible. The resummation of the large double logarithmic term $\alpha_s \ln^2(k_T)$, or the large logarithms $\alpha_s \ln^2(x)$ can lead to the famous Sudakov form factors [70–75]. Such Sudakov factors can strongly suppress the endpoint singularity, which in turn can help one to make the perturbative calculation reliably.

In Refs. [59, 62], for instance, Li et al. calculated the form factors for $B \to (\rho, \pi)$ and $B \to S$ transitions at the full leading order by using the pQCD approach and found that their pQCD predictions for the corresponding form factors are consistent well with those obtained by employing the light-cone sum rules or other different theoretical methods [39–41, 44, 45, 50, 76–78]. In Ref. [79], very recently, Li, Shen and Wang calculated the NLO twist-2 corrections to the $B \to \pi$ transition form factors at leading twist (i.e. LO twist-2 contribution and LO twist-3 contribution) in the $k_T$ factorization theorem. They found that the NLO twist-2 contributions can amount up to 30% of the value of the form factors at the large recoil region of the pion. The calculation for the NLO twist-3 contributions to the form factors of $B \to \pi$ transition will be completed very soon [80].

2. THE FORM FACTORS IN THE PQCD FACTORIZATION APPROACH

As mentioned in last section, the central issue of the $B/B_s$ SL decays considered in this paper are the evaluation of the relevant form factors of the $B \to (P, V)$ transitions, in which $P = (\pi, K, \eta, \eta', D, D_s)$ are pseudo-scalars and $V = (K^*, D^*, D_s^*)$ are the vector mesons. In this section, we take $B \to \pi$ transitions as an example, to show how to calculate the form factors in the pQCD factorization approach. For more details for other cases, one can see the original papers, for example, in Refs. [67–69, 79, 81–91].

2.1 The form factors $F_{0,+}(0)$ for $B \to \pi$ transition: One example

In this paper, for the sake of simplicity, we generally use $B$ to denote both $B$ and $B_s$ meson and $P$ for the pseudo-scalar mesons, such as the pion, kaon and $\eta^{(0)}$, where it is appropriate. In the rest frame of $B$ meson, we define the $B$ meson momentum $p_1$ and the final meson $P$ momentum $p_2$ in the light-cone coordinates: $p_1 = m_B(1, 1, 0_T)/\sqrt{2}, p_2 = m_B(0, 1, 0_T)/\sqrt{2}$, where the parameter $\eta = 1 - q^2/m_B^2$ is the energy fraction of the final state meson, and $q = p_1 - p_2$ in the momentum carried by the final state leptons. The momenta $k_1$ and $k_2$ are parameterized as those in Ref. [81].

For the final state $\pi$ meson, we adopt the distribution amplitudes $\phi^A_\pi(x)$ (the twist-2 part) and
FIG. 2. The Feynman diagrams responsible for the extraction of the LO twist-2 and LO twist-3 contributions to the form factor of $B \rightarrow \pi$ transition. The symbol $\otimes$ refers to the weak vertex.

The form factors $\phi_{n}^{P,T}(x)$ (the twist-3 part) as defined in Refs. [44, 92, 93]:

$$
\phi_{n}^{P}(x) = \frac{f_{n}^{P}}{2\sqrt{6}} x(1-x) \left[ 1 + \left( 30\eta_{3} - \frac{5}{2}\rho_{n}^{2} \right) C_{2}^{1/2}(t) - \frac{9}{20}\rho_{n}^{2}(1 + 6a_{2}^{\pi}) \right] C_{4}^{1/2}(t),
$$

(7)

$$
\phi_{n}^{T}(x) = \frac{f_{n}^{T}}{2\sqrt{6}} x(1-x) \left[ 1 + \left( 5\eta_{3} - \frac{1}{2}\eta_{3}\omega_{3} - \frac{7}{20}\rho_{n}^{2} - \frac{3}{5}\rho_{n}^{2}a_{2}^{\pi} \right) C_{2}^{3/2}(t) \right],
$$

(8)

where $t = 2x - 1$, $\rho_{n} = m_{\pi}/m_{n}^{0}$ is the mass ratios with $m_{n}^{0} = 1.4 \pm 0.1$ GeV is the chiral mass of pion, $a_{i}^{\pi}$ are the Gegenbauer moments, while $C_{n}^{2}(t)$ are the Gegenbauer polynomials [81]. The values of $C_{i}^{\pi,K}$ can be found in Eq. (13) of Ref. [81]. It is worth of mentioning that a new progress about pion form factor in the $\pi\gamma^{*} \rightarrow \gamma$ scattering has been made in Ref. [94] very recently, where the authors made a joint resummation for the pion wave function and the pion transition form factor and proved that the $k_{T}$ factorization is scheme independent.

For the wave functions of the B and $B_{s}$ meson, there are a lot of studies for their structure and shapes, form example, in the framework of the heavy quark limit [95–97]. In Ref. [98], the authors studied the rapidity resummation improved B meson wave function and found that the resummation effect keeps the normalization of the B meson wave functions and strengths their convergent behavior at small spectator momentum. For more details about the wave functions of $B/B_{s}$ meson, one can see a new review paper [99] and references therein. We here still use the $B/B_{s}$ wave functions as defined in Refs. [59–62]. For the distribution amplitudes (DA’s) of $B/B_{s}$ meson, we adopt the same form as being used in Refs. [81, 85–89]:

$$
\phi_{B}(x,b) = N_{B} x^{2}(1-x)^{2} \exp \left[ -\frac{1}{2} \left( \frac{xm_{B}}{\omega_{B}} \right)^{2} - \frac{\omega_{B}^{2}b^{2}}{2} \right],
$$

(9)

where the normalization factors $N_{B}$ ($N_{B_{s}}$) are related to the decay constants $f_{B}$ ($f_{B_{s}}$) through the normalization relation $\int_{0}^{1} dx \phi_{B_{s}}(x,b = 0) = f_{B_{s}}/(2\sqrt{6})$. The shape parameter $\omega_{B} = 0.40 \pm 0.04$ GeV and $\omega_{B_{s}} = 0.50 \pm 0.05$ GeV were estimated by using the rich experimental measurements and setting $f_{B} = 0.21$ GeV and $f_{B_{s}} = 0.23$ GeV.

The form factors $F_{0,+}(q^{2})$ and $F_{T}(q^{2})$ for $B \rightarrow P$ transitions with $P = \pi$ or $K$ are defined in the usual way as in Ref. [44, 92, 93]. In order to cancel the poles at $q^{2} = 0$, $F_{+}(0) = F_{0}(0)$ must be satisfied. For the sake of convenience, one usually define $F_{0,+}(q^{2})$ as a summation of the
auxiliary form factors $f_1(q^2)$ and $f_2(q^2)$:

$$F_+(q^2) = \frac{1}{2} [f_1(q^2) + f_2(q^2)], \quad (10)$$

$$F_0(q^2) = \frac{1}{2} f_1(q^2) \left[ 1 + \frac{q^2}{m_B^2 - m_P^2} \right] + \frac{1}{2} f_2(q^2) \left[ 1 - \frac{q^2}{m_B^2 - m_P^2} \right]. \quad (11)$$

In the pQCD approach, one can calculate perturbatively the LO twist-2 and LO twist-3 contributions to the form factors, through the analytical calculations for the two factorizable emission Feynman diagrams as shown in Fig. 2. By taking the Sudakov form factors and the threshold resummation effects into account, we calculated and found the form factors $f_{1,2}(q^2)$ and $F_+$ for $B \to P$ transitions, as given for example in Eqs. (19-21) of Ref. [81].

In the pQCD approach at LO level, the form factors $F_{0,+}(q^2)$ and $F_T(q^2)$ as defined in Eqs. (10,11) include the LO twist-2 and LO twist-3 contributions only. They, of course, are the dominant part of the form factors in consideration [79]. In Ref. [79], the authors compared the relative strength of the LO twist-2 part and LO twist-3 part, and then calculated the NLO twist-2 contribution to the form factors of $B \to \pi$ transition, found the corresponding NLO form factor $F_{\text{twist-2}} = F(x_1, x_2, \eta, \mu_f, \mu, \zeta_1)$. The explicit expression of the NLO factor $F(x_1, x_2, \eta, \mu_f, \mu, \zeta_1)$ can be found easily in Refs. [79, 81]. At the NLO level, consequently, the NLO hard kernel $H$ can be written as the form of [79]

$$H = H^{(0)}(\alpha_s) + H^{(1)}(\alpha_s^2) = [1 + F_{\text{twist-2}}(x_1, x_2, \mu_f, \eta, \zeta_1)] H^{(0)}(\alpha_s), \quad (12)$$

where the hard kernel $H^{(0)}(\alpha_s)$ contains the LO twist-2 and LO twist-3 contributions. The NLO twist-3 contribution $F_{\text{twist-3}}$ is another part of the NLO contribution to the form factors in the framework of the pQCD factorization approach, which is still absent now but in the process of analytical calculation [80]. Based on the $SU(3)_F$ flavor symmetry, we can find the similar expressions for the form factors of other final state pseudoscalar mesons, such as $K, \eta$ and $\eta'$ meson [81, 82].

### 2.2 The form factors for $B \to D_{(s)}, D^*_{(s)}$ transitions

For the pseudoscalar $D$ meson and the vector $D^*$ meson, their wave functions can be chosen as [83, 100]

$$\Phi_D(p, x) = \frac{i}{\sqrt{6}} \gamma_5 (p_D + m_D) \phi_D(x), \quad (13)$$

$$\Phi_{D^*}(p, x) = \frac{-i}{\sqrt{6}} \left[ f_L(p_{D^*} + m_{D^*}) \phi_{D^*}^L(x) + f_T(p_{D^*} + m_{D^*}) \phi_{D^*}^T(x) \right]. \quad (14)$$

For the distribution amplitudes of $D^{(*)}$ meson, we adopt the one as defined in Ref. [100]

$$\phi_{D^{(*)}}(x) = \frac{f_{D^{(*)}}}{2\sqrt{6}} 6x(1-x) [1 + C_{D^{(*)}} (1-2x)] \cdot \exp \left[ -\frac{\omega b^2}{2} \right]. \quad (15)$$

From the heavy quark limit, we here assume that $f_{D^*} = f_{D^*}^T = f_{D^*}, \phi_{D^*}^L = \phi_{D^*}^T = \phi_{D^*}$, and set $C_D = C_{D^*} = 0.5, \omega = 0.1$ GeV as Ref. [83, 100].

For $B \to D$ transition, the form factors $F_{0,+}(q^2)$ can be written in terms of $f_{1,2}(q^2)$ as in Eq. (10,11). The explicit expressions of $f_{1,2}(q^2)$ for $B \to D$ transition can be found easily in Eqs. (14-18) of Ref. [83]. For $B \to D^*$ transitions, the relevant form factors are $V(q^2)$ and $A_{0,1,2}(q^2)$ [101], and have been given explicitly in Eqs. (20-23) of Ref. [83]. For $B_s \to (D_s, D^*_s)$ transitions, the explicit expressions of the form factors $F_{0,+,}(q^2), V(q^2)$ and $A_{0,1,2}(q^2)$ can be found directly in Eqs.(17-19,24-27) of Ref. [84].
2.3 The extrapolation of the form factors

As mentioned in previous section, the central issue for the theoretical calculations of the semileptonic $B/B_s$ decays are the evaluation of the values and the shape of the relevant form factors $F_{0,+}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$. For the $B/B_s \to P$ transition with $P = (\pi, K, etc)$ the light pseudoscalar mesons, the two traditional methods of evaluating the form factors are the LCSRs in the low $q^2$ region and the Lattice QCD for the high $q^2$ region of $q^2 \approx q_{max}^2$. For the form factors of $B \to \pi, \tau$ transitions, the relevant experiments also provide some help to determine their value and the shape [102, 103]. The pQCD predictions for values of those form factors in low $q^2$ region are consistent well with those from LCSRs [81, 82, 100, 101].

For $B \to (D, D^*)$ transitions, the traditional methods to evaluate the form factors are the HQET [1, 15, 52–55, 104] in the low $q^2$ region and the LQCD techniques [56–58] in the high $q^2$ region. In Refs. [100, 105, 106], the authors examined the applicability of the pQCD approach to $B \to (D, D^*)$ transitions, and have shown that the pQCD approach with the inclusion of the Sudakov effects is applicable to the $B \to D(D^*)_\ell \nu$ decays in the lower $q^2$ region (i.e. the $D$ or $D^*$ meson recoils fast). Since the pQCD predictions for the relevant form factors are reliable in the low $q^2$ region, we will calculate explicitly the values of the form factors $F_{0,+}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ in the lower range of $m_l^2 \leq q^2 \leq m_s^2$ with $l = (e, \mu)$ by using the expressions as given in previous subsection.

In the low $q^2$ region of $m_l^2 \leq q^2 \leq m_s^2$, we firstly calculate the form factors $F_i(q^2)$ for $B \to P, D(D^*)_\ell$ transitions at some points by employing the pQCD approach respectively. Secondly we make an extrapolation for the form factors $F_i(q^2)$ from the low $q^2$ region to the high $q^2$ region. In Refs. [81, 82], we use different parametrization for $F_0(q^2)$ and $F_{+,T}(q^2)$ respectively. For the form factor $F_0(q^2)$ of $B/B_s \to (\pi, K)$ transitions, we use the classical pole model parametrization to make the extrapolation

$$F_0(q^2) = \frac{F_0(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2},$$

where the parameter $a$ and $b$ will be determined by the fitting procedure as described in Refs. [81, 82].

For $F_{+,T}(q^2)$, we use the Ball/Zwicky(BZ) parametrization to do the extrapolation [39–41, 44, 107]

$$F_i(q^2) = \frac{F_i(0)}{1 - q^2/m_{B(s)}^2} + \frac{F_i(0) r q^2/m_{B(s)}^2}{1 - a q^2/m_{B(s)}^2},$$

where the shape parameters $\alpha$ and $r$ could be determined by the fitting procedure the same as in Ref. [81, 82].

In Table I, we collect the LO and NLO pQCD predictions for the transition form factors $F_{0,+}(0)$ and $F_T(0)$ for the considered decay modes. The total errors are obtained by adding the individual errors in quadrature. In this table, we also show the LO pQCD predictions for the form factors $F_{0,+}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ for $B \to D(D^*)$ transitions with $q^2 = 0, m_s^2$, respectively. For more details see Refs. [81–84]. One can see from the theoretical predictions as listed in Table I that the pQCD predictions for the form factors of $B \to (\pi, K, D)$ and $B_s \to K$ transitions at $q^2 = 0$ generally agree well with those from LCSRs [44, 108] within one standard deviation.

In Fig. 3, as an example, we illustrate the $q^2$-dependence of the pQCD predictions for the form factors $F_{0,+}(q^2)$ at the LO (dots lines) and the NLO (solid line) for the $B \to \pi$ transition. The
TABLE I. The pQCD predictions for the form factors $F_{0,+}(0)$ and $F_T(0)$ for $B/B_s \to (\pi, K)$ transitions, and $F_{0,+}(q^2), V(q^2)$ and $A_{0,1,2}(q^2)$ for $B \to D^{(*)}$ transitions with $q^2 = 0, m_T^2$, respectively.

| Transitions | $F_{0}(0)_{LO}$ | $F_{+}(0)_{LO}$ | $F_T(0)_{LO}$ |
|-------------|-----------------|-----------------|----------------|
| $B \to \pi$ | $0.22^{+0.04}_{-0.03}$ | $0.22^{+0.04}_{-0.03}$ | $0.23^{+0.04}_{-0.03}$ |
| $B \to K$  | $0.27^{+0.05}_{-0.04}$ | $0.27^{+0.05}_{-0.04}$ | $0.30^{+0.05}_{-0.04}$ |
| $B_s \to K$ | $0.22 \pm 0.04$ | $0.22 \pm 0.04$ | $0.25^{+0.05}_{-0.04}$ |

| $F_{0}(0)_{NLO}$ | $F_{+}(0)_{NLO}$ | $F_T(0)_{NLO}$ |
|-----------------|-----------------|----------------|
| $B \to \pi$     | $0.26^{+0.05}_{-0.04}$ | $0.26^{+0.05}_{-0.04}$ | $0.26^{+0.05}_{-0.04}$ |
| $B \to K$       | $0.31 \pm 0.05$ | $0.31 \pm 0.05$ | $0.34^{+0.06}_{-0.05}$ |
| $B_s \to K$     | $0.26^{+0.05}_{-0.04}$ | $0.26^{+0.05}_{-0.04}$ | $0.28^{+0.06}_{-0.06}$ |

| $B \to D^{(*)}$ | $F_{0}(m_T^2)$ | $F_{+}(m_T^2)$ | $V(m_T^2)$ |
|-----------------|----------------|----------------|----------------|
| $B \to D^{(*)}$ | $0.52^{+0.12}_{-0.10}$ | $0.52^{+0.12}_{-0.10}$ | $0.59^{+0.12}_{-0.11}$ |
| $B \to D^{(*)}$ | $0.64^{+0.14}_{-0.12}$ | $0.70^{+0.16}_{-0.14}$ | $0.79^{+0.15}_{-0.14}$ |

| $B \to D^{(*)}$ | $A_{0}(0)$ | $A_{1}(0)$ | $A_{2}(0)$ |
|-----------------|----------------|----------------|----------------|
| $B \to D^{(*)}$ | $0.46^{+0.10}_{-0.08}$ | $0.48^{+0.10}_{-0.09}$ | $0.51^{+0.11}_{-0.09}$ |
| $B \to D^{(*)}$ | $0.62^{+0.12}_{-0.11}$ | $0.58^{+0.11}_{-0.10}$ | $0.66^{+0.13}_{-0.12}$ |

FIG. 3. The pQCD predictions for the form factors $F_{0,+}(q^2)$ for $B \to \pi$ transition. The solid line denotes the total value of the NLO results and the shaded band describes the total theoretical error.

The shaded band in Fig. 3 illustrates the total error of the pQCD predictions obtained by adding the different theoretical errors in quadrature. For more details about the pQCD predictions for the values and the $q^2$-dependence of the relevant form factors for other $B/B_s$ semileptonic decays considered in this paper, one can see Refs. [81–84].

2.4 Form factors of $B \to D^{(*)}$ transitions in the HQET

The HQET is the traditional method for the evaluations of the form factors for $B \to (D, D^*)$ transitions [1, 15, 52–54]. We here present the formulae for evaluating the form factors for $B \to$
$D^{(*)}l\bar{\nu}_l$ decays, quoted directly from Refs. [11, 36].

$$F_+(q^2) = \frac{m_B + m_D}{2\sqrt{m_Bm_D}} G_1(w),$$

$$F_0(q^2) = \frac{\sqrt{m_Bm_D}}{m_B + m_D} G_1(w) \Delta(w) (1 + w)^{\frac{1+r}{1-r}},$$

with the function $G_1(w)$ in the form of

$$G_1(w) = G_1(1) \left[ 1 - 8\rho^2 z(w) + (51\rho^2 - 10) z(w)^2 - (252\rho^2 - 84) z(w)^3 \right],$$

where $r = m_D/m_B$, $z(w) = (\sqrt{w + 1} - \sqrt{2})/(\sqrt{w + 1} + \sqrt{2})$, the new kinematical variable $w$ is defined as $w = v_B \cdot v_{D^{(*)}} = (m_B^2 + m_{D^{(*)}}^2 - q^2)/(2m_Bm_{D^{(*)}})$ with $q^2 = (p_B - p_{D^{(*)}})^2$. The scalar density $\Delta(w)$ is approximated by a constant value $\Delta(w) = 0.46 \pm 0.02$ [11, 36]. From Refs. [11, 36], we also find

$$G_1(1)|V_{cb}| = (42.64 \pm 1.53) \times 10^{-3},$$

$$\rho_1^2 = 1.186 \pm 0.036 \pm 0.041.$$  \tag{21}

For the $\bar{B} \to D^*$ transition, the form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ are related to the universal HQET form factor $h_{A_1}(w)$ via [36, 109]

$$V(q^2) = \frac{R_1(w)}{R_{D^*}} h_{A_1}(w), \quad A_0(q^2) = \frac{R_0(w)}{R_{D^*}} h_{A_1}(w),$$

$$A_1(q^2) = R_{D^*} \frac{w+1}{2} h_{A_1}(w),$$

$$A_2(q^2) = \frac{R_2(w)}{R_{D^*}} h_{A_1}(w),$$

where $R_{D^*} = 2\sqrt{m_Bm_{D^*}}/(m_B + m_{D^*})$, while $h_{A_1}(w)$ and ratios $R_{0,1,2}(w)$ are of the following [36, 109]

$$h_{A_1}(w) = h_{A_1}(1) \left[ 1 - 8\rho^2 z(w) + (53\rho^2 - 15) z(w)^2 - (231\rho^2 - 91) z(w)^3 \right],$$

$$R_0(w) = R_0(1) - 0.11(w-1) + 0.01(w-1)^2,$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2.$$  \tag{23}

The parameters $\rho^2$, $R_1(1)$ and $R_2(1)$ are determined from the well-measured $\bar{B} \to D^*\ell\bar{\nu}$ decay distributions [110] ($\ell = e, \mu$),

$$\rho^2 = 1.207 \pm 0.026, \quad R_1(1) = 1.403 \pm 0.033,$$

$$R_2(1) = 0.854 \pm 0.020, \quad R_3(1) = 0.97 \pm 0.10,$$

$$h_{A_1}(1)|V_{cb}| = (35.90 \pm 0.45) \times 10^{-3}.$$  \tag{24}

While the parameter $R_0(1)$ can be derived from the equation

$$\frac{R_2(1)(1-r) + r[R_0(1)(1+r) - 2]}{(1-r)^2} = R_3(1).$$  \tag{25}
3. \( B_{(s)} \to (\pi, K, \eta^{(s)})(l^+l^-, l\bar{\nu}, \nu\bar{l}) \) DECAYS: BRANCHING RATIOS

3.1 The formulae of differential decay widths

For the Semileptonic decays \( B \to \pi l^- \bar{\nu}_l \) and \( \bar{B}_{s}^0 \to K^+ l^- \bar{\nu}_l \), the quark level transitions are the \( b \to ul^\pm \bar{\nu}_l \) with \( l^- = (e^-, \nu^-, \tau^-) \), and the corresponding effective Hamiltonian is of the form [111]

\[
\mathcal{H}_{\text{eff}}(b \to ul\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub} \cdot \bar{u} \gamma_\mu (1 - \gamma_5) b \cdot \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l, \tag{26}
\]

where \( G_F = 1.16637 \times 10^{-5} \text{GeV}^{-2} \) is the Fermi coupling constant, \( V_{ub} \) is one of the Cabbibo-Kobayashi-Maskawa quark mixing matrix elements. The differential decay rates can be written as [62, 112]

\[
\frac{d\Gamma(b \to ul\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3 m_B^2} \left( 1 - \frac{m_l^2}{q^2} \right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{ 3 m_t^2 \left( m_B^2 - m_P^2 \right)^2 |F_0(q^2)|^2 + \left( m_t^2 + 2q^2 \right) \lambda(q^2) |F_+(q^2)|^2 \right\}, \tag{27}
\]

where \( m_l \) is the mass of lepton, and \( \lambda(q^2) = (m_B^2 + m_P^2 - q^2)^2 - 4m_B^2 m_P^2 \) is the phase-space factor.

The effective Hamiltonian for \( b \to ql^- l^+ \) and \( b \to dv\bar{l} \) transitions are of the form

\[
\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{tq} \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \tag{28}
\]

\[
\mathcal{H}_{b \to q\bar{\nu}l} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2 \pi \sin^2(\theta_W)} V_{tb} V_{tq}^* \eta_X X(x_t) \cdot \left[ \bar{q} \gamma^\mu (1 - \gamma_5) b \right] \left[ \bar{l} \gamma^\mu (1 - \gamma_5) \nu \right], \tag{29}
\]

where \( q = (d, s) \), \( C_i(\mu) \) are the Wilson coefficients at the scale \( \mu \) and \( O_i(\mu) \) are the local four-fermi operators [81, 111]. The explicit expressions of the differential decay widths for these two kinds of decays can be found easily in Refs. [62, 111, 113].

3.2 pQCD predictions for \( Br(B_{(s)} \to P(l^+l^-, l\nu, \nu\bar{l})) \)

Since the input parameters used in different papers may be a little different, one can see individual papers for the choices of the input parameter about the masses, decay constants, life-times and CKM elements [110, 114]. By using the formula and the input parameters as given in previous sections, we firstly calculate the branching ratios for the considered charged and neutral current semileptonic decays.

After the numerical integration for \( q^2 \) over the range of \( m_l^2 \leq q^2 \leq (M_B - m_i)^2 \), we obtain the pQCD predictions for the branching ratios of all decay modes in consideration. For \( B/B_s \to (\pi, K)(l^+l^-, l\nu, \nu\bar{l}) \) decays, all numerical results are listed in Table II. The theoretical predictions obtained by employing other methods also in the framework of the SM [115–117], as well as the measured values currently available [24–30, 110, 114, 118], are all included in this Table. The total theoretical error of the pQCD predictions for the branching ratios are obtained by the the combination in quadrature of the individual errors from \( \omega_B \) or \( \omega_B, f_B \) or \( f_{B_s} \), relevant Gegenbauer moments \( a_i \) and the chiral mass \( m_0^{\pi, K} \).
In Table III, we list the NLO pQCD predictions for the branching ratios of the \( B_s \to (\eta, \eta')(l^+l^-, \nu\nu) \) decays with \( l = (e, \mu) \). We considered two mixing schemes: (a) the traditional Feldmann-Kroll-Stech (FKS) \( \eta-\eta' \) mixing scheme \([119, 120]\) in the quark-flavor basis; and (b) the \( \eta-\eta'-G \) mixing scheme as defined in Ref. \([121]\): the physical states \( \eta, \eta' \) and \( G \) are related to \( \eta_l, \eta' \) and \( g \) through the rotation matrix \( U(\theta, \phi, \phi_G) \), which has been defined in Eq. \((4)\) of Ref. \([121]\) with \( \phi = \theta + 54.7^\circ \) and \( \phi_G \sim 30^\circ \).

In Table III, furthermore, we show the pQCD predictions in the FKS mixing scheme in column two (the NLO predictions with the total errors). In Table III we also show the NLO pQCD predictions for the branching ratios in the \( \eta-\eta'-G \) mixing scheme with the choice of the mixing angles: \( (\phi, \phi_G) = (43.7^\circ, 33^\circ) \), the same as in Ref. \([122]\). Currently available two measured values \([114]\) are \( Br(B^+ \to \eta l^- \bar{\nu}_l) = (0.39 \pm 0.08) \times 10^{-4} \) and \( Br(B^- \to \eta' l^- \bar{\nu}_l) = (0.23 \pm 0.08) \times 10^{-4} \).

Table III also includes a comparison between our pQCD predictions and other theoretical results \([116, 123-127]\) obtained by using the different theoretical methods or approaches, including for example the LCSR \([39-41]\), light-front quark model (LFQM) \([116]\), the lattice QCD\([57, 58]\). One can see from the numerical results in Table III that the pQCD predictions agree well with the theoretical predictions obtained from other nonperturbative methods.
TABLE III. The pQCD predictions at the NLO level in both the FKS $\eta$-$\eta'$ mixing scheme and the $\eta$-$\eta'$-$G$ mixing scheme. Other theoretical predictions \[116,123–127\] are listed in last column.

| Decay modes | $\eta$-$\eta'$ | $\eta$-$\eta'$-$G$ | Others |
|-------------|---------------|------------------|--------|
| $Br(B^{-} \to \eta l^- \bar{\nu}_l)(10^{-4})$ | $0.41^{+0.15}_{-0.12}$ $0.33^{+0.12}_{-0.10}$ | $0.43^{+0.08}_{-0.06}$ [106] |
| $Br(B^{-} \to \eta^* l^- \bar{\nu}_l)(10^{-4})$ | $0.24^{+0.07}_{-0.06}$ $0.20^{+0.07}_{-0.05}$ | $0.29^{+0.07}_{-0.06}$ [108] |
| $Br(B^{-} \to \eta' l^- \bar{\nu}_l)(10^{-4})$ | $0.20^{+0.08}_{-0.06}$ $0.16^{+0.08}_{-0.05}$ | $0.21^{+0.04}_{-0.03}$ [106] |
| $Br(B^{-} \to \eta' \tau^- \bar{\nu}_\tau)(10^{-4})$ | $0.10^{+0.04}_{-0.03}$ $0.08^{+0.03}_{-0.02}$ | $0.13^{+0.03}_{-0.02}$ [108] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-8})$ | $0.48^{+0.16}_{-0.14}$ $0.39^{+0.14}_{-0.11}$ | $0.60$ [107] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-9})$ | $0.98^{+0.33}_{-0.28}$ $0.80^{+0.28}_{-0.23}$ | $1.1 \pm 0.1$ [109] |
| $Br(\bar{B}^0 \to \eta' l^+ l^-)(10^{-9})$ | $0.38^{+0.11}_{-0.12}$ $0.31^{+0.11}_{-0.09}$ | $0.30$ [107] |
| $Br(\bar{B}^0 \to \eta' l^+ l^-)(10^{-9})$ | $0.25^{+0.09}_{-0.07}$ $0.20^{+0.05}_{-0.05}$ | $0.25$ [107] |
| $Br(\bar{B}^0 \to \eta' l^+ l^-)(10^{-9})$ | $0.18^{+0.07}_{-0.05}$ $0.14^{+0.05}_{-0.04}$ | $0.24$ [100] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-7})$ | $2.07^{+0.87}_{-0.72}$ $2.59^{+1.00}_{-0.99}$ | $2.4$ [100] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-6})$ | $1.62^{+0.71}_{-0.55}$ $2.03^{+0.89}_{-0.69}$ | $1.4$ [110] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-7})$ | $2.18^{+0.96}_{-0.76}$ $1.45^{+0.64}_{-0.50}$ | $1.8$ [100] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-7})$ | $0.27^{+0.12}_{-0.10}$ $0.18^{+0.07}_{-0.06}$ | $0.28$ [110] |
| $Br(\bar{B}^0 \to \eta l^+ l^-)(10^{-6})$ | $1.71^{+0.75}_{-0.66}$ $1.14^{+0.47}_{-0.46}$ | $1.3$ [110] |

Based on the theoretical predictions for the branching ratios of all considered semileptonic decays of $B$ and $B_s$ meson as shown in Table II and III, and the phenomenological analysis presented in Refs. [81, 82], we have the following observations:

1. For the relevant transition form factors $F_{0,+,-}(q^2)$, the NLO pQCD predictions for their values and the $q^2$-dependence are consistent with those obtained from the LCSR or other theoretical methods [39–41, 44, 45, 50]. The pQCD predictions for the NLO twist-2 contribution to the form factors is $\sim 20\%$ of the total value.

2. For the considered decay modes $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$, $B^- \to \pi^0 l^- \bar{\nu}_l$, $\bar{B}^0 \to \bar{K}^0 l^+ l^-$, and $B^- \to K^- l^+ l^-$, the NLO pQCD predictions for their decay rates agree very well with currently available experimental measurements.

3. From the direct comparison between the pQCD predictions for the branching ratio $Br(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l)$ and the measured value, the value of $V_{ub}$ can be extracted directly: $|V_{ub}| = (3.80^{+0.66}_{-0.60}(th.) \pm 0.13) \times 10^{-3}$.

4. For the branching ratios $Br(B^- \to \eta l^- \bar{\nu}_l)$, the NLO twist-2 contribution to the transition form factors can provide $\sim 25\%$ enhancement to the LO pQCD results, which leads to a better agreement of the pQCD predictions with the measured values.

5. Analogous to the ratio $R(D)$, we have also defined several ratios of the branching ratios $R_\nu$, $R_C$ and $R_{N1,N2,N3}$, calculated and listed the pQCD predictions for their values and the errors, these theoretical predictions will be tested by the LHCb experiments and by the Super-B experiments in the near future.
(6) For $B/B_s \to (\eta, \eta')(l^+l^-, l^-\bar{\nu}_l, \nu\bar{\nu})$ with $l = (e, \mu, \tau)$ decays, we considered both the traditional FKS $\eta-\eta'$ mixing scheme and the new $\eta-\eta'$-G mixing scheme, and we found that the relevant pQCD predictions in these two mixing schemes are consistent with each other.

### 3.3 $V_{ub}$ and $V_{cb}$, $B \to K^* \mu^+\mu^-$ decays

As is well-known, the best method to determine $|V_{ub}|$ ($|V_{cb}|$) is to measure semileptonic decay rates for $B \to X_u l\nu$ ($B \to X_c l\nu$), which is proportional to $|V_{ub}|^2$ ($|V_{cb}|^2$). By using the data from the inclusive or exclusive semileptonic decay modes, one can extract out those two CKM elements simultaneously. Since the experimental and theoretical techniques for these inclusive and exclusive method are rather different and largely independent, one can make a cross-check for the consistency of our understanding of the theory and the experimental measurements.

For the experimental measurements of $V_{cb}$, it is now in good shape: the values determined by the exclusive and inclusive processes become consistent. In Ref. [51], for instance, Fu et al. calculated $B \to D$ transition form factors by employing the QCD light-cone sum rule and then estimated the value and the uncertainty of $|V_{cb}|$ from the data for the semi-leptonic $B \to D l\bar{\nu}_l$ decays. Their estimation for $|V_{cb}|$ shows a good agreement with the BABAR, CLEO and Belle measurements. For $V_{ub}$, however, it is still a puzzle, the tension between the exclusive and inclusive values persists at present.

The $B^0 \to K^{*0} \mu^+\mu^-$ decay is a self-tagging process with $K^{*0} \to K^+\pi^-$, mediated by electroweak box and penguin type diagrams in the SM. The shape of the angular distribution of the $(K^+\pi^-)\mu^+\mu^-$ system offers particular sensitivity to the values of $C_{7\gamma}$ and $C_9$, and to the contributions from the new physics beyond the standard model. The differential decay rates of the considered decays also provides useful information on the estimation about the contribution from those new particles appeared in the loops but typically suffers from large theoretical errors due to the large uncertainty of the hadronic form factors. For the semileptonic decays $B \to K^{(*)} l^+l^-$, furthermore, there also exist non-factorizable QCD effects which can not be accounted for in form factors or short-distance Wilson coefficients, both at small and large $Q^2$ region, as discussed in Refs. [128–130].

In Ref. [16], very recently, LHCb collaboration reported their measurements for the differential branching fraction, $dB/dq^2$ of the $B^0 \to K^{*0} \mu^+\mu^-$ decay. Measurements of the angular observables, $A_{FB}(A_{FB}^{T})$, $F_L$, $S_3 (A_2^T)$ and $A_0$ have also been performed in the same $q^2$ bins. The LHCb results [16] are the most precise measurements of $dB/dq^2$ and the angular observables to date. The measured CP asymmetries in $B^0 \to K^{*0} \mu^+\mu^-$ [131, 132], for example, is of the form

$$A_{CP}(B^0 \to K^{*0} \mu^+\mu^-) = -0.072 \pm 0.041,$$

which is consistent with the SM at $1.8\sigma$ [132].

All of the observables are consistent with SM expectations and together put stringent constraints on the contributions from new particles to $b \to s\mu^+\mu^-$ FCNC processes. A bin-by-bin comparison of the measured angular distribution with the SM hypothesis indicates an excellent agreement with p-values between 18% and 72%. The first LHCb measurement for the position of the zero-crossing point of the forward-backward (FB) asymmetry for the decay mode $B^0 \to K^{*0} \mu^+\mu^-$, $q_0^2 = (4.9 \pm 0.9) GeV^2/c^4$, agrees well with the SM prediction [133–135]: $q_{0,SM}^2 \in [3.9, 4.4] GeV^2/c^4$.

For $B^0 \to K^{*0} \mu^+\mu^-$ decays, the previous measurements for the considered observables do suffer from large theoretical errors due to the sizable uncertainties of relevant hadronic form factors. The new observables $P_{4,5,6,7}$ as defined in Ref. [136]: $P_i = S_i / \sqrt{F_L(1 - F_L)}$, which have
small form-factor uncertainties, especially at low $q^2$ region. By using the full 2011 data sample, LHCb presented their first measurements for these new observables [17]. For more details about theoretical studies and experimental measurements of $B^0 \rightarrow K^{(*)}l^+l^-$ decays, one can see Refs. [16, 17, 132–135, 137] and references therein.

4 $B/B_s \rightarrow (D^{(*)}, D_s^{(*)})l^-\bar{\nu}_l$ DECAYS

In this section, we will present the pQCD predictions for the branching ratios of $B/B_s \rightarrow (D^{(*)}, D_s^{(*)})l^-\bar{\nu}_l$ decays [83, 84], and make some comparisons with those from the HQET method or other different approaches [7–15].

For $B \rightarrow Dl\bar{\nu}_l$ decays, the formulae of the differential decay rate $d\Gamma(B \rightarrow Dl\bar{\nu}_l)/dq^2$ can be obtained from Eq. (27) by simple replacements: $m_F \rightarrow m_D$ and $V_{ub} \rightarrow V_{cb}$. For $B \rightarrow D^*l\bar{\nu}_l$ decays, the expressions of the differential decay widths can be found in Refs. [62, 83]. For $B_s \rightarrow D_s^{(*)}l\bar{\nu}_l$ decays, the formulae of the differential decay rates can be found in Ref.[84].

By using the relevant form factors as defined in Sec.2, one can calculate directly the branching ratios for the considered decays by the integrations over the whole range of $q^2$. In Table IV, the pQCD predictions for the branching ratios of the eight considered decay modes are listed in the column two. For the case of light leptons $l = (e, \mu)$, we show the averaged results. In column three, we show the HQET predictions obtained by our direct calculations using the formulae as given in Refs. [11, 36], which agree perfectly with those as given in Ref. [36]. The measured values from BaBar [5] are also listed in last column as a comparison.

In Table V, we list the pQCD predictions for the values of the six $R(X)$ ratios in the second column. As a direct comparison, we also show the HQET predictions calculated by ourselves or those as given in Refs. [36], other SM predictions as presented in Refs. [7, 8, 14, 138], and the BaBar measured values [5]. Since the most hadronic and SM parameter uncertainties are greatly canceled in the ratios of the corresponding branching ratios, the theoretical errors of the pQCD predictions for R(X)-ratios are reduced significantly to about 5%, similar in size with those in the HQET.

| Channels | pQCD(%) | HQET(%) | BaBar(%) |
|----------|---------|---------|----------|
| $Br(B^0 \rightarrow D^+\tau^-\bar{\nu}_\tau)$ | 0.67$^{+0.34}_{-0.28}$ | 0.63 ± 0.06 | 1.01 ± 0.22 |
| $Br(B^0 \rightarrow D^+l^-\bar{\nu}_l)$ | 2.03$^{+0.92}_{-0.70}$ | 2.13$^{+0.19}_{-0.18}$ | 2.15 ± 0.08 |
| $Br(B^- \rightarrow D^0\tau^-\bar{\nu}_\tau)$ | 0.95$^{+0.37}_{-0.31}$ | 0.69 ± 0.06 | 0.99 ± 0.23 |
| $Br(B^- \rightarrow D^0l^-\bar{\nu}_l)$ | 2.19$^{+0.99}_{-0.76}$ | 2.30 ± 0.20 | 2.34 ± 0.14 |
| $Br(B^0 \rightarrow D^+\tau^-\bar{\nu}_\tau)$ | 1.36$^{+0.38}_{-0.37}$ | 1.25 ± 0.04 | 1.74 ± 0.23 |
| $Br(B^0 \rightarrow D^+\tau^-\bar{\nu}_\tau)$ | 4.52$^{+1.44}_{-1.31}$ | 4.94 ± 0.15 | 4.69 ± 0.34 |
| $Br(B^- \rightarrow D^0\tau^-\bar{\nu}_\tau)$ | 1.47$^{+0.43}_{-0.40}$ | 1.35 ± 0.04 | 1.71 ± 0.21 |
| $Br(B^- \rightarrow D^0\tau^-\bar{\nu}_\tau)$ | 4.87$^{+1.60}_{-1.41}$ | 5.35 ± 0.16 | 5.40 ± 0.22 |

From the theoretical predictions as collected in Table IV and V we have the following observations:
TABLE V. The theoretical predictions for the six \( R \)-ratios obtained by employing the pQCD approach or other theoretical methods, and the measured values [5].

| Ratio    | pQCD       | HQET       | HQET [36] | SM [7, 8] | SM [14] | SM [138] | BaBar [5] |
|----------|------------|------------|-----------|-----------|---------|----------|-----------|
| \( R(D^0) \) | \( 0.433^{+0.017}_{-0.027} \) | \( 0.297^{+0.017}_{-0.016} \) | –         | –         | –       | 0.429 ± 0.097 |
| \( R(D^+) \) | \( 0.428^{+0.023}_{-0.033} \) | 0.297 ± 0.017 | –         | –         | –       | 0.469 ± 0.099 |
| \( R(D^{*0}) \) | \( 0.302^{+0.012}_{-0.014} \) | 0.253 ± 0.004 | –         | –         | –       | 0.322 ± 0.039 |
| \( R(D^{*+}) \) | \( 0.301^{+0.012}_{-0.015} \) | 0.252 ± 0.004 | –         | –         | –       | 0.355 ± 0.044 |
| \( \mathcal{R}(D) \) | \( 0.430^{+0.021}_{-0.026} \) | 0.297 ± 0.017 | 0.296^{+0.016}_{-0.016} | 0.316 | 0.315 | 0.31 | 0.440 ± 0.072 |
| \( \mathcal{R}(D^*) \) | \( 0.301^{+0.013}_{-0.013} \) | 0.252 ± 0.004 | 0.252^{+0.003}_{-0.003} | – | 0.260 | – | 0.332 ± 0.030 |

(1) The pQCD predictions for \( \text{Br}(B \to D^{(*)} l^- \bar{\nu}_l) \) agree well with other theoretical predictions based on different methods and the measured values within one standard deviation.

(2) The previous SM predictions for \( R(D^{(*)}) \) as given in Refs. [7, 8, 14, 138] are consistent with each other within their errors, but there still exist a clear discrepancy between these predictions and the BaBar’s measurements [5].

(3) For \( R(D) \) and \( R(D^*) \), the pQCD predictions agree very well with the data, the BaBar’s anomaly of \( R(D^{(*)}) \) are explained successfully in the framework of the SM by using the pQCD factorization approach.

 Analogous to \( R(D^{(*)}) \) ratios, we also defined new ratios \( R_D^l \) and \( R_D^r \) [83] and found the pQCD predictions

\[
R_D^l \equiv \frac{B(B \to D^+ l^- \bar{\nu}_l) + B(B \to D^0 l^- \bar{\nu}_l)}{B(B \to D^{*+} l^- \bar{\nu}_l) + B(B \to D^{*0} l^- \bar{\nu}_l)} = 0.450^{+0.064}_{-0.051}, \tag{31}
\]

\[
R_D^r \equiv \frac{B(B \to D^+ \tau^- \bar{\nu}_\tau) + B(B \to D^0 \tau^- \bar{\nu}_\tau)}{B(B \to D^{*+} \tau^- \bar{\nu}_\tau) + B(B \to D^{*0} \tau^- \bar{\nu}_\tau)} = 0.642^{+0.081}_{-0.070}. \tag{32}
\]

These new ratios may be more sensitive to the QCD dynamics than the old ones and therefore should be tested in the forthcoming experiments.

Following the same procedure as for the cases of the decays \( \bar{B}^0 \to D^{(*)} l \bar{\nu} \), one can estimate the decay rates of the four \( \bar{B}^0_s \to D^{(*)} l \bar{\nu} \) decays, and the four \( R(X) \) ratios of the corresponding branching ratios. The pQCD predictions are listed in Table VI and VII. As comparisons, Table VI also include the theoretical predictions for the branching ratios from other SM methods: for example, the constituent quark model [141], the QCD sum rules [142], the LCSRs or the covariant light-front quark model (CLFQM) [143, 144] and other methods [145, 146]. The errors of the pQCD predictions as given in Table VI and VII are the combinations of the major theoretical errors come from the uncertainties of \( \omega_{Bs} = 0.50 \pm 0.05 \) GeV \( (\omega_B = 0.40 \pm 0.04 \) GeV) and \( m_c = 1.35 \pm 0.03 \) GeV, while those induced by the variations of \( f_{Bs} (f_B) \) and \( |V_{cb}| \) are canceled completely in the pQCD predictions for the \( R(X) \)-ratios.

From the theoretical predictions as collected in Table IV-VII one can find the following points:
TABLE VI. The pQCD predictions for the decay rates (in units of $10^{-2}$) of the decay modes in consideration. The theoretical predictions are given in Refs. [141–146] are listed as a comparison.

| Channel | pQCD | CQM[141] | QCDSRs[142] | IAMF[145] | RQM[146] |
|---------|------|----------|-------------|-----------|---------|
| $Br(B_s^0 \to D_s^+ \tau^- \bar{\nu}_\tau)$ | $0.84^{+0.38}_{-0.28}$ | $-$ | $-$ | $0.33^{+0.14}_{-0.11}$ | $0.47 - 0.55$ | $62 \pm 0.05$ |
| $Br(B_s^0 \to D_s^+ l^- \bar{\nu}_l)$ | $2.13^{+1.12}_{-0.75}$ | $2.73 - 3.00$ | $2.8 - 3.8$ | $1.0^{+0.4}_{-0.3}$ | $1.4 - 1.7$ | $2.1 \pm 0.2$ |
| $Br(B_s^0 \to D_s^0 \tau^- \bar{\nu}_\tau)$ | $1.44^{+0.51}_{-0.42}$ | $-$ | $-$ | $1.3^{+0.2}_{-0.1}$ | $1.2 - 1.3$ | $1.3 \pm 0.1$ |
| $Br(B_s^0 \to D_s^0 l^- \bar{\nu}_l)$ | $4.76^{+1.87}_{-1.49}$ | $7.49 - 7.66$ | $1.89 - 6.61$ | $5.2 \pm 0.6$ | $5.1 - 5.8$ | $5.3 \pm 0.5$ |

(1) For the pQCD predictions for all R(X) ratios of the branching ratios, due to the large cancellation of the theoretical errors in the ratios, the total theoretical errors now become less than 13%, much smaller than those for the branching ratios themselves. All these ratios could be measured at the LHCb experiments or the Super-B experiments in the near future.

(2) The ratio $R(D_s)$ and $R(D_s^*)$ are defined [84] in the same way as the ratios $R(D(s))$ in Refs. [5, 36]. These ratios generally measure the mass effects of heavy $m_{\tau}$ against the light $m_{\tau}$ or $m_{\mu}$.

(3) The new ratios $R_{D_s}^{l,\tau}$ and $R_{D_s}^{R,\tau}$ will measure the effects induced by the variations of the form factors for $B_s^0 \to (D, D^*)$ and $\bar{B}_s^0 \to (D_s, D_s^*)$ transitions. In other words, the new ratios $R_{D_s}^{l,\tau}$ and $R_{D_s}^{R,\tau}$ may be more sensitive to the QCD dynamics which controls the $B/B_s \to (D(s), D(s)^*)$ transitions than the old ratios ratios $R(D(s))$ and $R(D(s)^*)$.

(4) On the limit of the $SU(3)_F$ flavor symmetry, the four ratios defined for $\bar{B}_s^0 \to D_s^{(*)} l\bar{\nu}_l$ decays should be very similar with the corresponding ones for $B \to D^{(*)} l\bar{\nu}_l$ decays. The pQCD predictions as listed in Table V – VII do support this expectation. The breaking of $SU(3)_F$ flavor symmetry is less than 10%.

(5) At present, only the ratio $R(D)$ and $R(D^*)$ have been measured by Belle and BaBar [5, 33–35]. In order to check if the BaBar’s anomaly do exist or not for $\bar{B}_s^0 \to D_s^{(*)} l\bar{\nu}_l$ decays, and to test the $SU(3)_F$ flavor symmetry among $\bar{B}_s^0 \to D_s^{(*)} l\bar{\nu}_l$ and $B \to D^{(*)} l\bar{\nu}_l$ decays, we strongly suggest LHCb and the forthcoming Super-B experiments to measure these four new ratios $R(D_s)$, $R(D_s^*)$, $R_{D_s}^l$ and $R_{D_s}^R$.

5 SUMMARY AND EXPECTATIONS

The semileptonic decays of $B/B_s$ mesons represent a very rich physics. The three or four final state particles are rather special, since they allow for a wealth of angular observables, decay rates and asymmetries: sensitive to new physics, experimentally clean signatures and theoretically well
predicted. The B factory experiments, the LHCb, CMS and ATLAS, and the forthcoming Super-B factories. A large number of events have been collected, and much much more are expected!

In this short review, we present the current status about the theoretical and experimental studies for some important semileptonic decays of $B/B_s$ mesons. We firstly gave a brief introduction for the experimental measurements for some phenomenologically interesting channels, such as the improved measurements for $B/B_s \to P(l^+l^-, l^-\bar{\nu}_l, \nu\bar{\nu})$ mainly from LHCb experiments, the BaBar's $R(D)$ and $R(D^*)$ anomaly, the $P_\perp^{2}$ deviation for $B^0 \to K^{*0} \mu^+\mu^-$ decay. We then made a careful discussion about the evaluations for the form factors relevant for the considered semileptonic decays in the popular methods, such as the LCSRs, the heavy quark effective theory and the new pQCD factorization approach. We listed the pQCD predictions for the form factors $F_{0,+,\tau}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ for $\bar{B}/B_s \to (\pi, K, D, D^*)$ transitions in Table I. We made numerical comparisons and found that the pQCD predictions at the low $q^2$ region agree well with those from other methods.

In Sec.III and IV, using the form factors obtained from the $k_T$ factorization formulism, we calculated and then presented the LO and/or NLO pQCD predictions for the decay rates of all considered SL decays of the $B$ and $B_s$ mesons, for example, the charged current $B/B_s \to (\pi, K, \eta(0), D^{(s)}, D^*(s))\nu\bar{\nu}$ decays, the neutral current $B/B_s \to (\pi, K, \eta(0))(l^+l^-, \nu\bar{\nu})$ processes. We also made careful phenomenological analysis for these pQCD predictions, compared them with those from different methods and the measured values from BaBar, Belle, LHCb and other collaborations. We found, in general, the following points:

(1) For all the considered $B/B_s \to (\pi, K, \eta, \eta')(l^+l^-, l^-\bar{\nu}_l, \nu\bar{\nu})$ decays, the pQCD predictions at the NLO level for their decay rates agree well with the measured values or those obtained by using other popular but different theoretical methods.

(2) For the two ratios $R(D)$ and $R(D^*)$, our pQCD predictions do agree very well with the measured values as reported by BaBar collaboration, the so-called BaBar's anomaly about the ratios $R(D^{(s)})$ are therefore explained successfully in the framework of the pQCD factorization approach.

(3) Besides the ratios $R(D^{(s)})$ and $R(D_s^{(s)})$, we defined several new ratios $R_D^{l,\tau}$ and $R_{D_s}^{l,\tau}$, which will measure the effects induced by the variations of the form factors for $\bar{B}^0 \to (D, D^*)$ and $\bar{B}_s^0 \to (D_s, D^{(s)}_s)$ transitions. The new ratios $R_D^{l,\tau}$ and $R_{D_s}^{l,\tau}$ may be more sensitive to the QCD dynamics which controls the $B/B_s \to (D^{(s)}, D^{(s)}_s)$ transitions than the old ratios $R(D^{(s)})$ and $R(D^{(s)}_s)$, we therefore strongly suggest LHCb and the forthcoming Super-B experiments to measure these four new ratios $R(D), R(D^*), R(D_s)$ and $R(D^{(s)}_s)$.

As is well-known, the heavy flavor b physics is a powerful tool to make a precision test for the standard model theory and for the searches for the signal or evidence of the new physics effects beyond the standard model. Precision measurements for these decays an probe at mass scales not attainable with direct measurements at the high energy frontiers. LHC itself is a flavor factory, other environments/experiments also providing crucial flavoured data. At present, most experimental measurements are consistent with the SM expectations, but some hints or tensions have been seen in a few observables.

At present, the LHCb is the most sensitive heavy flavor physics experiment [147]. So far we have used its first two years sensitivity to rule out some new physics models. But we know there must be new physics. We do believe that the heavy flavor b physics has a bright future ahead, many more exciting results and much high precision are expected, not only with results on existing data,
but also from outstanding prospects with future facilities such as the LHCb upgrade and Super-B factories. We do believe that, again, the heavy flavor b physics has brilliant present and ambitious long-term prospect!

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