Strong quasi-particle tunneling study in the paired quantum Hall states

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The quasi-particle tunneling phenomena in the paired fractional quantum Hall states are studied. A single point-contact system is first considered. Because of relevancy of the quasi-particle tunneling term, the strong tunneling regime should be investigated. Using the instanton method it is shown that the strong quasi-particle tunneling regime is described as the weak electron tunneling regime effectively. Expanding to the network model the paired quantum Hall liquid to insulator transition is discussed.

1. Introduction

The integer and fractional quantum Hall effect was observed in a two-dimensional electron systems subjected to a strong perpendicular magnetic field.\(^8\) In these experiments, the Hall resistivity showed plateau behavior and the longitudinal resistivity vanished in odd denominator filling of the Landau level.\(^8\) Contrary to the odd denominator filling fractions, the quantum Hall effect doesn’t appear in \(\nu = 1/2\).\(^8\) This state is regarded as Fermi liquid state of the composite fermions which have charge \(e\) and the flux \(2\phi_0\) \((\phi_0 = \hbar c/e)\).\(^8\) At mean field level, the fictitious fluxes attached to composite fermions cancel the external magnetic field and the fermions system is described as that in the absence of the magnetic field.\(^8\) On the other hand, the even denominator quantum Hall effect was observed in single layer systems at \(\nu = 5/2\),\(^9\) and in the double layer systems at \(\nu = 1/2\).\(^9\) A large number of theoretical studies have been made on the grand state of the even denominator quantum Hall systems. To date three possible states have been proposed to explain such incompressible regimes at even denominator filling factor. They are Halperin(331),\(^9\) Moore-Read(Pfaffian),\(^10\) Haldane-Rezayi states.\(^11\) These states are regarded as the BCS states in the composite fermion theory. The order parameter symmetry is triplet p-wave with ABM and \(A_1\) type in \(^3\)He for 331 and Pfaffian state respectively, and d-wave for Haldane-Rezayi state.\(^11\)\(^17\) The wave functions of the grand state and the excited states for these paired states are represented as the conformal block of proper conformal field theories. In the case of the Pfaffian, 331, and Haldane-Rezayi states, corresponding theories have a central charge \(c = 1 + 1/2, 1 + 1, 1 - 2\), respectively.\(^10\)\(^11\)\(^17\) These conformal theories also describe the edge excitations on the boundaries of the sample.

Edge quasi-particle tunneling is vital to study the quantum transport phenomena in the quantum Hall systems. Tunneling experiments in the point-contact systems have played a central role for detecting the non-Fermi liquid properties of fractional quantum Hall liquids. Milliken et al. studied the transmission through a point contact in a gated structure.\(^19\) At \(\nu = 1/3\) fractional quantum Hall plateau regime, the temperature dependence of the two-terminal conductance is non-linear \(G \propto T^\alpha\) where \(\alpha = 4\) for \(\nu = 1/3\). Another important experiment is the measurement of the shot noise which can measure the fractional charge. In such experiments, the quasi-particle tunneling between the edges is substantial. This can be described by a Tomonaga-Luttinger liquid model with a scattering potential at \(x = 0\).\(^20\)\(^21\) Many theorists have studied these effects and expanded into more complex situations.\(^22\)\(^24\) In recent work, we also have carried this problems to the hierarchical states and explained Reznikov et al’s experiment.\(^22\)\(^24\) In addition, we have constructed an edge state network model for the hierarchical state and discussed the transition from a quantum Hall liquid to another quantum Hall liquid or an insulator.\(^24\) In this paper we study quasi-particle tunneling phenomena in the paired fractional quantum Hall states. Because of relevancy in the renormalization group analysis, we have to study quasi-particle tunneling non-perturbatively as well as the Laughlin state and the hierarchical state. Using the instanton method we show that in the strong quasi-particle tunneling regime quasi-particle tunneling is described as weak electron tunneling effectively in the Pfaffian, and 331 states. These results are very similar to the Laughlin and hierarchical states. We introduce the edge state network model and discuss the paired quantum Hall liquid to insulator transition.

2. Quasi-hole tunneling model

We consider the model which describes a two-terminal Hall Bar geometry where a two-dimensional electron systems between the left and right terminals has upper and lower edges with a scattering at \(x = 0\). We start to analyze the \(\nu = 1/q\) generalized Pfaffian state. The \(c = 1/2\) part in the edge modes belong to the same universality class as the critical point of the two-dimensional Ising model which is described by the real fermion field \(\psi\) and the spin operator \(\sigma\).\(^11\) These modes are interpreted as the pair breaking and the half quantum vortex excitations. On the other hand \(c = 1\) part describes the Laughlin type excitations which corresponds to the free boson theory.\(^27\)\(^28\) So, the edge state theory is written as
The electron, and the quasi-hole operators are given by \( \psi_e(z) = \psi e^{-i\phi/\nu} \) and \( \psi_{qp}(z) = \sigma e^{-i\phi/2} \), respectively. The renormalization group equation for quasi-particle tunneling is written as

\[
H_{\text{QPT}} = t_{\text{QPT}}(\psi^\dagger_{\text{QPT}} \psi_{\text{QP}} + \text{h.c.}) = \Gamma_{\text{QPT}} \sigma(x = 0) \cos(\phi(x = 0)/2),
\]

and

\[
H_{\text{ET}} = t_{\text{ET}}(\psi^\dagger_{\text{ET}} \psi_{\text{EL}} + \text{h.c.}) = \Gamma_{\text{ET}} i \psi^\dagger \psi \cos(\phi(x = 0)/\nu) \]

respectively, where \( t_{\text{QPT}}(t_{\text{ET}}) \) is the tunneling amplitude of the quasi-particle (electron). We set the value of \( t_{\text{QPT}} \) positive.

Counting the conformal dimension we can derive the renormalization group equation for quasi-particle tunneling as

\[
\frac{d\Gamma_{\text{QPT}}}{dl} = \left[ 1 - \left( \frac{\nu}{4} + \frac{1}{8} \right) \right] \Gamma_{\text{QPT}},
\]

where \( e^l = \frac{A_0}{\Lambda_0} \), \( \Lambda_0 \) and \( \Lambda' \) are bare, renormalized cut-off, respectively. The \( \beta \)-function (right hand side of (4)) is positive for any \( \nu \leq 1 \), so we have to study the strong coupling regime non-perturbatively.

### 3. Instanton method

To study this problem, we introduce an effective theory for the non-linear degree of freedom \( \phi(x = 0) \equiv \theta \) in (2). as following:

\[
Z = \int D\phi D\psi^\dagger D\bar{\psi} \exp \left( -S_B - S_F - \Gamma_{\text{QPT}} \int d\tau \sigma \cos \phi/2 \right)
\]

\[
= \int D\phi D\psi^\dagger e^{-S_{\text{QPT}}} \int D\phi D\theta \delta(\theta - \phi(x = 0)) \exp \left( -S_B - \Gamma_{\text{QPT}} \int d\tau \sigma \cos \theta/2 \right)
\]

\[
= \int D\phi D\psi^\dagger e^{-S_{\text{F}}} \int D\phi D\theta \int D\lambda e^{-\int d\tau \lambda(\theta - \phi(x = 0))} \exp \left( -S_B - \Gamma_{\text{QPT}} \int d\tau \sigma \cos \theta/2 \right)
\]

where \( S_F \) and \( S_B \) are the action for the real fermion and boson respectively. Integrating out \( \phi \) and \( \lambda \), we have;

\[
Z = \int D\phi D\psi^\dagger e^{-S_{\text{F}}} \int D\theta \exp \left( -\sum_{\omega} \frac{|\omega|}{4\pi \nu} \theta(-\omega)\theta(\omega) - \Gamma_{\text{QPT}} \int d\tau \sigma \cos \theta/2 \right).
\]

In the view point of the dynamics for \( \theta \), the first term in (11) is the friction term in the Caldeira-Legget theory, and the second term is regarded as the periodic potential. On the other hand, in terms of the Ising model, the second term in (11) is interpreted as the "magnetic field" which is proportioned to \( \cos \theta/2 \). In the strong coupling limit, due to the "magnetic field" the saddle point solution of the spin field \( \sigma(x = 0) = \) zero no longer. If \( \sigma < 0 \) the \( \theta \) has a value \( 4\pi n \), If \( \sigma > 0 \) \( \theta \) is \( 2\pi (2n + 1) \), where \( n \) is an integer. The excitations for the strong coupling limit are described by the instanton solutions same as the Laughlin state or the problems of single impurity in non-chiral Luttinger liquid. In this situation, however, there is an intrinsic difference with them. It is the degree of freedom for the spin operator \( \sigma \). Specifically usual instanton solutions (we say it "standard instanton") are defined as a step from a minimum to another minimum of the potential. The other process carry spin flip which causes the reversal of the sign of the potential, so the minima and maxima are interchanged. We say the latter solution "half instanton". In the strong coupling regime, the latter contributions are dominant. We have to note that in the half instanton process the dynamics of Ising systems should be considered.

Using the instanton method we construct an effective action for the strong coupling regime. We introduce a function defined as

\[
\frac{d\theta_{\text{ins}}}{d\tau} = h(\tau),
\]

where \( \theta_{\text{ins}} \) is the solution for single half instanton. So the differential of the \( n \) instanton solution and its Fourier coefficient are written as

\[
\frac{d\theta_{n}}{d\tau} = \sum_{i=1}^{n} e_i h(\tau - \tau_i)
\]

\[
-\omega \theta_{n}(\omega) = \sum_{i=1}^{n} e_i \tilde{h}(\omega) e^{i\omega \tau_i},
\]

where \( e_i \) is the charge of the instantons, and \( \tilde{h}(\omega) \) is the Fourier coefficient of \( h(\tau) \). Especially the value;

\[
\tilde{h}(\omega = 0) = \frac{1}{\sqrt{\beta}} \int d\tau h(\tau) = \frac{2\pi}{\sqrt{\beta}}
\]

will be important. To derive the effective action, we consider a grand canonical ensemble of the half instantons. The contributions of the standard instantons are not important in the strong coupling regime. We neglect the \( \omega \)-dependence of \( h(\omega) \) in the first term in (11) as

\[
S_D \equiv \sum_{\omega} \frac{|\omega|}{4\pi \nu} \theta(-\omega)\theta(\omega)
\]

\[
\cong \sum_{i,j} \left( \frac{\pi}{\nu \beta} \sum_{\omega} \frac{1}{|\omega|} e^{-i\omega(\tau_i - \tau_j)} \right) e_i e_j.
\]
The grand canonical partition function is given as
\[
Z = \int D\psi D\bar{\psi} e^{-S_F} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \int_0^\beta d\tau_n \int_0^{\tau_n} d\tau_{n-1} \cdots \int_0^{\tau_2} d\tau_1 \prod_{i=0}^{n-1} z(\tau_i) \right. \\
\exp \left[ -\sum_{ij} \left( \frac{\pi}{\nu} \beta \sum_\omega e^{-i\omega(\tau_i-\tau_j)} \right) \right]
\]
where \(z(\tau) = \omega_0 e^{-S_{ms}}\) is the fugacity of the instanton, \(\omega_0\) is a characteristic energy of the tunneling process. The value must be directly proportional to the energy operator \(\epsilon\), i.e., \(\omega_0 = z_0(\tau)\). Where \(z_0\) is a constant. Introducing Stratonovich-Hubbard field \(\theta\) we get
\[
Z = \int D\psi D\bar{\psi} e^{-S_F} \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\beta d\tau \int_0^{\beta} d\tau_{n-1} \cdots \int_0^{\tau_2} d\tau_1 \prod_{i=0}^{n-1} z(\tau_i) \\
\exp \left[ -\sum_{ij} \left( \frac{\pi}{\nu} \beta \sum_\omega e^{-i\omega(\tau_i-\tau_j)} \right) \right]
\]
(12)

In the charge-spin basis, \(\phi_c, \phi_s = \phi_\uparrow \pm \phi_\downarrow\), the two component can be separated as
\[
\mathcal{L}_0 = \left( \frac{i}{4\pi K_c} \frac{\partial \phi_\uparrow^+}{\partial \tau} - \frac{\partial \phi_\downarrow^-}{\partial x} + \frac{\nu_c}{8\pi K_s} \left( \frac{\partial \phi_\uparrow^+}{\partial x} + \frac{\partial \phi_\downarrow^-}{\partial x} \right)^2 \right)
\]
(18)

The electron, quasi-hole operators are given by
\[
\psi_{xc} = e^{-3i\phi_x - i\phi_\downarrow}, \psi_{x\downarrow} = e^{-i\phi_x - 3i\phi_\downarrow}
\]
(19)
\[
\psi_{qh} = e^{-i\phi_\uparrow}, \psi_{qh} = e^{-i\phi_\downarrow}
\]
(20)

Then the quasi-hole and the electron tunneling Hamiltonian is written as
\[
\mathcal{H}_{QPT} = \mathcal{H}_{QPT} \sum_{\sigma = \uparrow, \downarrow} \cos \phi_\sigma
\]
(21)
\[
\mathcal{H}_{ET} = 2\mathcal{H}_{ET} \cos 2\phi_c \cos \phi_s
\]
(22)

We derive the effective action for \(\theta_c = \phi_c(x = 0)\) and \(\theta_s = \phi_s(x = 0)\) like eq.(6)
\[
S = \sum_\omega \left[ \frac{\sqrt{\omega}}{4\pi K_c} \theta_c(-\omega) \theta_c(\omega) + \frac{\sqrt{\omega}}{4\pi K_s} \theta_s(-\omega) \theta_s(\omega) \right]
\]
\[
+ \int d\tau \frac{2}{\mathcal{H}_{QPT}} \cos \frac{\theta_c}{2} \cos \frac{\theta_s}{2}
\]
(23)

In the present case, the instantons are specified by the change of the values of the phase \(\phi_c, \phi_s\). Most important type in the strong coupling regime is the \((\pm \pi, \pm \pi)\). Considering the grand-canonical ensemble and introducing the Stratonovich-Hubbard field we can construct the effective action;
\[
S' = \sum_\omega \left[ \frac{\sqrt{\omega}}{4\pi K_c} \theta_c(-\omega) \theta_c(\omega) + \frac{\sqrt{\omega}}{4\pi K_s} \theta_s(-\omega) \theta_s(\omega) \right]
\]
\[
+ \int d\tau \frac{2}{\mathcal{H}_{ET}} \cos 2\theta_c \cos \theta_s.
\]
(24)

Note the last term of (24) is the electron tunneling term again. Therefore in the case of the 331 the quasi-hole tunneling also can be regarded as weak electron tunneling. Temperature dependence of the two-terminal conductance can be calculated as (15), and we have \(G \propto T^4\).

4. Paired quantum Hall liquid to Hall insulator transition

We consider the situation to increase the magnetic field. Increasing the strength the longitudinal resistance arises and the system becomes the Hall insulator. Near the critical point, the phase separation should occur. In
such a condition, the edge state network model is applicable. In previous work, to discuss the quantum Hall plateau-plateau transition and quantum Hall liquid to insulator transition, we introduced a hierarchical filling version of the edge state network model. The model has the dual description: weak quasi-particle tunneling regime can be regarded as the (hierarchical) quantum Hall liquid phase, while the strong quasi-hole tunneling regime (weak electron tunneling regime) corresponds to the insulating phase. Because the former case corresponds to the situation that there are some small vortices condensed regions in the sea of filled quantum Hall liquid, quasi-particle tunneling occurs between edges which enclose the vortices condensed regions. On the other hand the latter case corresponds to the opposite situation. Namely there are some small quantum Hall droplets connected by electron tunneling where the vortices condense regions are dominant. If one increases the strength of randomness, the liquid-insulator transition occurs also. We can apply the model to the pairing state. Reconstructing the instanton method the dual symmetry has been shown. As well as the Laughlin and hierarchical version, the strong quasi-hole tunneling regime should be regarded as the Hall insulator. The dual symmetry found by us in present paper suggests I - V reflection symmetry which is observed in Shahar’s experiment near the critical point of the ν = 1/3 fractional quantum Hall liquid-insulator transition. However, even though in the case of ν = 1/3, no one could show rigidly the fact that if at B > Bc I - V character is given by I = F(V), at B < Bc I = F⁻¹(V). This point is left to a future problem.

Finally we note the transition from a 331 state to another state in the ν = 1/2 double layer quantum Hall systems. In eq.(21) we treated the tunneling amplitude symmetrically for the spin index as ΓQPT↑ = ΓQPT↓. However, in the ν = 1/2 double layer quantum Hall systems, the values of them may be different. If only one of the coupling constant (say it ΓQPT↑) become large and the other ΓQPT↓ left small, the ν = 1/2 plateau to ν = 1/3 plateau transition can occur for the systems without inter-layer tunneling. Because the field ϕ↑ is pinned at the tunneling points, and it cannot contribute to the transport, the effective action for ϕ↑ which should respond to the electric perturbation is written as that of at νeff = 1/3.

In this paper we have investigated quasi-particle tunneling. We showed that we can treat the strong quasi-particle tunneling regime as weak electron tunneling. This dual symmetry hold in the edge state network model. We insist the possibility of observation of the I - V reflection symmetry near the liquid-insulator transition.

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