Magnetic transitions and radiative decays of singly heavy baryons

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(Dated: December 28, 2021)

A pion mean-field approach allows one to investigate light and singly heavy baryons on an equal footing. In the large $N_c$ limit, the light and singly heavy baryons are viewed respectively as $N_c$ and $N_c - 1$ valence quarks bound by the pion mean fields created self-consistently, since a heavy quark can be regarded as a static color source in the limit of the infinitely heavy quark mass. The transition magnetic moments of the baryon sextet are determined entirely by using the parameters fixed in the light-baryon sector without any additional parameters introduced. Assuming that the transition $E2$ moments are small, we are able to compute the radiative decay rates of the baryon sextet. The numerical results are discussed, being compared with those from other approaches.

Keywords: Heavy baryons, radiative decays, pion mean fields, the chiral quark-soliton model

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I. INTRODUCTION

In the large $N_c$ (the number of colors) limit, an ordinary baryon can be viewed as a state of $N_c$ valence quarks bound by the meson mean fields, which are self-consistently created by the presence of the $N_c$ valence quarks $|12\rangle$. The chiral quark-soliton model ($\chi$QSM) $|3\rangle$ realizes effectively this picture (for a review, we refer to Refs. $|4,5\rangle$). On an equal footing, a singly heavy baryon, which consists of one heavy quark and two light quarks, can be regarded as a state of $N_c-1$ valence quarks bound by the pion mean fields, if the heavy-quark mass is taken to be infinity $(m_Q \to \infty)$ $|6,7\rangle$ (see also a recent review $|5\rangle$). The masses of the lowest-lying singly heavy baryons in both the charmed and bottom flavor sectors were successfully described within the framework of this mean-field approach, and in particular the mass of the $\Omega_c$ was predicted $|7,9\rangle$ with all parameters fixed in the light baryon sector. More recently, the five narrow $\Omega_c$ resonances newly observed by the LHCb Collaboration $|10\rangle$ were studied within the same framework and two of them were advocated as exotic pentaquark baryons belonging to the anti-decapentaplet $|15\rangle$. This classification of the narrow $\Omega_c$ resonances as the members of the baryon anti-decapentaplet has been further supported by computing strong decays of the newly found two narrow $\Omega_c$ $|12\rangle$. Very recently, the electromagnetic form factors of the singly heavy baryons have been also investigated without any new additional parameters $|13\rangle$. While there is no experimental information on radiative decays of heavy baryons, the CLEO Collaboration identified the two resonances of the singly heavy baryons, $\Xi_c^{'+}$ and $\Xi_c^0$, using their radiative decays $\Xi_c^{'+} \to \Xi_c^+ \gamma$ and $\Xi_c^0 \to \Xi_c^0 \gamma$, since the masses of the $\Xi_c^0$ isodoublet lie below threshold for their strong decays $|15\rangle$. Theoretically, radiative decays of singly heavy baryons have been studied within various different approaches: The nonrelativistic quark model (NRQM) $|16,17\rangle$, the bag models $|20,21\rangle$, potential models $|22,23\rangle$, QCD sum rules $|24,27\rangle$, heavy-quark effective theories $|28,29\rangle$, chiral perturbation theories $|30,32\rangle$, relativistic quark models $|33\rangle$, the heavy quark symmetry with a diquark picture $|34\rangle$, the chiral constituent quark model $|35,36\rangle$, and lattice QCD $|37,38\rangle$. In the present work, we want to investigate the transition magnetic moments and radiative decays of singly heavy baryons belonging to the baryon sextet with both spin 1/2 and 3/2. The main virtue of the present approach lies in the fact that all the dynamical parameters, which are required to compute the transition magnetic moments of the singly heavy baryons, have already been fixed in the light-baryon sector $|13,39\rangle$. Thus, we can predict the transition magnetic moments and the radiative decay rates of the lowest-lying singly-heavy baryons in a robust manner.

The present work is organized as follows: In Section II, we explain a general formalism of the $\chi$QSM to compute the transition magnetic moments of the heavy baryons. In Section III, we explicitly calculate the transition magnetic moments within the present framework. In Section IV, we present the numerical results of the transition magnetic moments and radiative decays of the heavy baryons. The last Section is devoted to the summary and conclusions of the present work.

II. GENERAL FORMALISM

We first introduce the electromagnetic (EM) current that consists of both the light and heavy quark currents

$$J_\mu(x) = \bar{\psi}(x) \gamma_\mu \hat{Q} \psi(x) + e_Q \bar{Q} \gamma_\mu Q,$$  \hspace{1cm} (1)

where $\hat{Q}$ stands for the charge operator of the light quarks, defined as

$$\hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \end{pmatrix}. \hspace{1cm} (2)$$

Here, $\lambda_3$ and $\lambda_8$ represent the well-known flavor SU(3) Gell-Mann matrices. The second part of Eq. (1) is the heavy-quark EM current with heavy-quark charge $e_Q$ ($e_c = 2/3$ for the charm quark or as $e_b = -1/3$ for the bottom quark). However, the heavy-quark EM current is not involved in the infinitely heavy-quark mass limit $m_Q \to \infty$, since the magnetic moment of a heavy quark is proportional to the inverse of the corresponding mass, i.e. $\mu \sim (e_Q/m_Q) \sigma$. We expect that the corrections from the next-to-leading order in the $1/m_Q$ expansion should be rather small in comparison with the light-quark contributions to the transition magnetic moments. So, we consider only the first term of Eq. (1) when we calculate the transition magnetic moments of heavy baryons. Since we employ the limit of $m_Q \to \infty$, we obtain the same results for both the charmed and bottom baryons, which is the consequence of heavy flavor symmetry.

Starting from the transition matrix element of the EM current sandwiched between the heavy-baryon states, we are able to derive the collective operator for the magnetic moments within the framework of the $\chi$QSM. In fact, they have
been already derived in previous works \[39]–\[43]. Considering the rotational $1/N_c$ and linear strange current-quark mass $m_s$ corrections, we obtain the collective operator for the magnetic moments as

$$\hat{\mu} = \hat{\mu}^{(0)} + \hat{\mu}^{(1)},$$  

(3)

where $\hat{\mu}^{(0)}$ and $\hat{\mu}^{(1)}$ denote the leading and rotational $1/N_c$ contributions, and the linear $m_s$ corrections respectively, which are expressed as

$$\hat{\mu}^{(0)} = w_1 D^{(8)}_{Q3} + w_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3,$$

$$\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{Qp} D^{(8)}_{Qq} + w_5 \left( D^{(8)}_{Q3} D^{(8)}_{88} + D^{(8)}_{Q8} D^{(8)}_{88} \right) + w_6 \left( D^{(8)}_{Q3} D^{(8)}_{88} - D^{(8)}_{Q8} D^{(8)}_{88} \right).$$

(4)

The indices of symmetric tensor $d_{pq3}$ run over $p = 4, \cdots, 7$. $\hat{J}_3$ and $\hat{J}_p$ correspond respectively to the third and the $p$th components of the spin operator acting on the soliton. $D^{(\nu)}_{ab}(R)$ are the SU(3) Wigner matrices in the representation $\nu$, which arise from the quantization of the soliton. $D^{(8)}_{Q3}$ is defined by the combination of the SU(3) Wigner $D$ functions

$$D^{(8)}_{Q3} = \frac{1}{2} \left( D^{(8)}_{33} + \frac{1}{\sqrt{3}} D^{(8)}_{38} \right),$$

(6)

which comes from the SU(3) rotation of the EM octet current. The coefficients $w_i$ in Eq. (5) contain a specific dynamics of the chiral soliton and are independent of baryons involved. $w_{1,2,3}$ are the SU(3) symmetric parts that are of order $O(m_s^0)$. Though $w_1$ includes the $m_s$ correction, it is not explicitly involved in the breaking of flavor SU(3) symmetry. So, we will consider $w_1$ as the SU(3) symmetric part, when the transition magnetic moments are derived. On the other hand, $w_{4,5,6}$ are of the leading order in the $m_s$ expansion, i.e. of order $O(m_s)$. The dynamical coefficients $w_i$ can be explicitly computed within a specific chiral solitonic model such as the $\chi$QM \[12, 40, 42\].

The structure of the collective operator $\hat{\mu}$ in Eq. (5) is in fact very general and is considered as a model-independent one. It is deeply related to hedgehog symmetry and the embedding of the SU(2) soliton into SU(3) \[2\], which provides a minimal way of combining the ordinary space with the internal flavor space. Because of this embedding, the symmetry we have is SU(2)$_T \times$ U(1)$_Y$. Therefore, the collective operator for the magnetic moment is expressed in terms of the SU(2)$_T \times$ U(1)$_Y$ invariant tensors

$$d_{abc} = \frac{1}{4} \text{tr}(\lambda_a \{ \lambda_b, \lambda_c \}), \quad S_{abc} = \sqrt{\frac{3}{2}} (\delta_{a3} \delta_{b8} + \delta_{b3} \delta_{a8}), \quad F_{abc} = \sqrt{\frac{3}{2}} (\delta_{a3} \delta_{b8} - \delta_{b3} \delta_{a8}),$$

(7)

which yield the general expression for the collective operator given in Eq. (5). Hence, instead of computing $w_i$ in a specific model, we can determine $w_i$ by the experimental data on the magnetic moments of the baryon octet as done in Refs. \[33, 41, 42\].

To derive the transition magnetic moments of the heavy baryons, we need to compute the matrix elements of the collective operator $\hat{\mu}$ in Eq. (5) between heavy baryon states. Moreover, the presence of $w_{4,5,6}$ comes from the perturbative treatment of $m_s$, so that the collective wavefunctions for the soliton consisting of the light-quark pair are not pure states but mixed ones with higher representations. In order to derive the collective wavefunctions, we need to diagonalize the collective Hamiltonian for flavor SU(3) symmetry breaking, expressed as

$$H_{ab} = \alpha D^{(8)}_{88} + \beta \hat{Y} + \gamma \sum_{i=1}^{3} D^{(8)}_{8i} \hat{J}_i,$$

(8)

where $\alpha$, $\beta$, and $\gamma$ are the dynamical coefficients for the lowest-lying singly heavy baryons, of which the explicit expressions can be found in Refs. \[7, 9\]. Diagonalizing Eq. (8), we obtain the wavefunctions for the baryon anti-triplet ($J = 0$) and the sextet ($J = 1$) respectively \[9\] as

$$|B_{30}\rangle = |\overline{3}_0, B\rangle + \frac{\sqrt{15}}{\sqrt{5}} |1\overline{5}_0, B\rangle,$$

$$|B_{61}\rangle = |6_1, B\rangle + \frac{\sqrt{5}}{\sqrt{15}} |1\overline{5}_1, B\rangle + \frac{\sqrt{2}}{\sqrt{10}} |2\overline{4}_1, B\rangle,$$

(9)

with the mixing coefficients

$$p_{\overline{3}5}^B = p_{\overline{4}5} \begin{bmatrix} -\sqrt{15}/10 \\ -3\sqrt{5}/20 \end{bmatrix}, \quad q_{\overline{3}5}^B = q_{\overline{4}5} \begin{bmatrix} \sqrt{5}/5 \\ \sqrt{30}/20 \\ 0 \end{bmatrix}, \quad q_{\overline{6}5}^B = q_{\overline{2}4} \begin{bmatrix} -\sqrt{10}/10 \\ -\sqrt{15}/10 \\ -\sqrt{15}/10 \end{bmatrix}.$$

(10)
respectively, in the basis $[\Lambda_Q, \Xi_Q]$ for the anti-triplet and $[\Sigma_Q, \bar{Q}, \Xi_Q, \Omega_Q]$ for the sextets. The expressions for the parameters $p_{\Sigma\Xi}$, $q_{\Xi\Xi}$, and $q_{\Xi\Omega}$ are also found in Refs. 6, 13 and the corresponding numerical values are given as

$$p_{\Sigma\Xi} = -0.104 \pm 0.003, \quad q_{\Xi\Xi} = 0.238 \pm 0.005, \quad q_{\Xi\Omega} = -0.049 \pm 0.002.$$  

Note that the mixing coefficients are proportional to $m_s$ linearly. So, the effects of flavor SU(3) symmetry breaking arise also from the collective wavefunctions for the heavy baryons in addition to the collective operator for the magnetic moments, which contains $w_{4,5,6}$.

The complete wavefunction for a heavy baryon can be constructed by coupling the soliton wavefunction to the heavy quark such that the heavy baryon becomes a color singlet, which are expressed as

$$|B_\mu; (J', J_3)(T, T_3)\rangle = \sum_{J_3, JQ} C_{J', J_3 JQ} \chi_{JQ3} |B_\mu; (J, J_3)(T, T_3)\rangle$$

where $\chi_{JQ3}$ stand for the Pauli spinors and $C_{J', J_3 JQ}$ denote the Clebsch-Gordan coefficients. $J'$ and $J_3$ represent the spin and its third component of the heavy baryons whereas $T$ and $T_3$ are the corresponding isospin and its third component. The wavefunctions $|B_\mu; (J, J_3)(T, T_3)\rangle$ with representation $\mu$ are given in Eq. (9). Calculating the matrix elements of $\hat{\mu}$ becomes just the simple $D$-function algebra. Thus, we can easily obtain the transition magnetic moments of the heavy baryons.

In Ref. [13], we discussed in detail how $w_1, w_4, w_5,$ and $w_6$ are modified for the singly heavy baryons that consist of $N_c - 1$ light valence quarks. Accordingly, the first coefficient $w_1$ was revised as

$$\bar{w}_1 = \left[ \frac{N_c - 1}{N_c} (w_1 + w_3) - w_3 \right] \sigma,$$

where $\sigma$ is introduced to compensate the deviation arising from the nonrelativistic limit used in the course of deriving $\bar{w}_1$. The numerical value of $\sigma$ was already determined in Ref. [12]: $\sigma \sim 0.85$. The other two coefficients $w_2$ and $w_3$ are kept to be intact in the course of reducing the number of valence quarks from $N_c$ to $N_c - 1$ whereas $w_{4,5,6}$ need to be modified as

$$\bar{w}_i = \frac{(N_c - 1)}{N_c} w_i, \quad i = 4, 5, 6.$$  

Using the numerical values of $w_i$ determined in Ref. [13] and Eqs. (13) and (14), we obtain the following values [13]

$$\bar{w}_1 = 10.08 \pm 0.24,$$
$$w_2 = 4.15 \pm 0.93,$$
$$w_3 = 8.54 \pm 0.86,$$
$$\bar{w}_4 = -2.53 \pm 0.14,$$
$$\bar{w}_5 = -3.29 \pm 0.57,$$
$$\bar{w}_6 = -1.34 \pm 0.56.$$  

Thus, once we know the numerical values of these dynamical parameters, we can straightforwardly compute the transition magnetic moments of singly heavy baryons, which will be shown in the next Section.

III. TRANSITION MAGNETIC MOMENTS AND PARTIAL DECAY WIDTHS OF RADIATIVE DECAYS

The matrix elements of the EM current between the baryon sextet with spin 1/2 to the baryon antitriplet can be parametrized by the transition EM form factors $F_i(q^2)$ as follows:

$$\langle B_{1/2}'(p')|J_\mu(0)|B_{1/2}(p)\rangle = \bar{w}_{B_{1/2}'(p')} \left[ F_1(q^2) + \frac{q''}{M_{B_{1/2}}^2 + M_{B_{1/2}}^2} F_2 \right] u_{B_{1/2}}(p, \lambda),$$

where $q^2$ is the square of the four-momentum transfer, and $M_{B_{1/2}}$ and $M_{B_{1/2}}$ denote respectively the masses of the baryon antitriplet and sextet with spin 1/2. The $u_{B(B')_{1/2}}$ is the Dirac spinor corresponding to the baryon $B(B')$. 

Similarly, the transition EM form factors from the baryon sextet with spin 3/2 to the sextet or antitriplet with spin 1/2, i.e. \(6_{3/2} \rightarrow 3_{1/2} \) or \(6_{3/2} \rightarrow 6_{1/2} \), are defined as \(48\):

\[
\langle B_{1/2}(p') | J_\mu | B_{3/2}(p) \rangle = i \sqrt{\frac{2}{3}} u_{B_{3/2}}(p') \left(\gamma_\mu p - q \cdot \gamma \right) \left[ G_M^s(q^2) K_M^{M \rightarrow B_{1/2}} + G_E^s(q^2) K_E^{M \rightarrow B_{1/2}} + G_C^s(q^2) K_C^{M \rightarrow B_{1/2}} \right] u_{B_{1/2}}(p, \Lambda),
\]

where \(G_M^s, \ G_E^s, \ \text{and} \ \ G_C^s \) stand respectively for the transition magnetic dipole, electric quadrupole, and Coulomb form factors. The \(K_{M,E,C}^{M \rightarrow B_{1/2}} \) are the covariant tensors defined in Ref. \(48\):

\[
K_M^{\beta \mu} = -i \frac{3(M_{3/2} + M_{1/2})}{2M_{1/2}(M_{3/2} + M_{1/2})^2 - q^2} \epsilon_{\beta \mu \lambda \sigma} P^\lambda q^\sigma,
\]

\[
K_E^{\beta \mu} = -K_M^{\beta \mu} - \frac{6(M_{3/2} + M_{1/2})}{M_{1/2} \Delta(q^2)} \epsilon_{\beta \sigma \lambda \mu} P^\lambda q^\sigma \epsilon_{\mu \nu \delta} P^\nu \delta q^\gamma,
\]

\[
K_C^{\beta \mu} = -i \frac{3(M_{3/2} + M_{1/2})}{M_{1/2} \Delta(q^2)} q^\beta (q^2 P^\mu - q \cdot P q^\mu) \gamma^5.
\]

with

\[
\Delta(q^2) = [(M_{3/2} + M_{1/2})^2 - q^2][(M_{3/2} - M_{1/2})^2 - q^2].
\]

\(u_{B_{3/2}}\) represents the Rarita-Schwinger spinor for the baryon sextet with spin 3/2. At \(q^2 = 0\), note that the transition magnetic dipole form factors \(F_2(q^2)\) and \(G_M^s(q^2)\) are the same as the transition magnetic moments. Since the magnetic dipole transitions \((M1)\) are experimentally dominant over the electric quadrupole transitions \((E2)\) in hyperon radiative decays, one can neglect the contribution from the \(E2\) transitions to the radiative decays of heavy baryons. This is a plausible approximation, since the the size of the \(E2/M1\) ratio for the \(\Delta\) isobar is yielded in the range of \((1 - 3)\%\) experimentally \(49\). Though there is no experimental data on the \(E2/M1\) ratio for the singly heavy baryons, we can safely assume that the size of the \(E2/M1\) ratio is very small. It indicates that the effects of the \(E2\) are expected to be negligible on the radiative decay widths of the singly heavy baryons, because even the squared values of the \(E2\) contribute to them. In fact, a recent lattice study has predicted the magnitude of the \(E2\) form factor of \(\Omega_c^+\) to be much smaller than those of the \(M1\) form factors \(37\): \(M1(\Omega_c^+) = -0.657\) and \(E2(\Omega_c^+) = -0.012\). A very recent calculation of the electromagnetic transition form factors of the singly heavy baryons within the self-consistent SU(3) \(\chi\) QSM has come to a similar conclusion \(53\).

So, we can express the radiative decay rates in terms of the transition magnetic moments. Using Eqs. \(16\) and neglecting the \(E2\) transitions, we obtain the radiative decay rates for \(B_{1/2} \rightarrow B'_{1/2}\) and for \(B_{3/2} \rightarrow B'_{1/2}\), respectively:

\[
\Gamma(B_{1/2} \rightarrow B'_{1/2}, \gamma) = 4 \alpha_{EM} \frac{E_\gamma^3}{(M_{B'_{1/2}} + M_{B_{1/2}})^2} \left( \frac{\mu_{B'_{1/2}B_{1/2}}}{\mu_N} \right)^2,
\]

\[
\Gamma(B_{3/2} \rightarrow B'_{1/2}, \gamma) = \frac{\alpha_{EM}}{2} \frac{E_\gamma^3}{M_{B'_{1/2}}^2} \left( \frac{\mu_{B'_{1/2}B_{1/2}}}{\mu_N} \right)^2,
\]

respectively, where \(M_{B_j}\) is the mass of baryon \(B\) with spin \(J\) and \(\mu_{B_j' B_j}\) is the transition magnetic moments for the radiative decay \(B_j \rightarrow B'_{j'}, \gamma\). In Eq. \(20\) \(\alpha_{EM}\) denotes the fine structure constant and \(E_\gamma\) the energy of the produced photon:

\[
E_\gamma = \frac{M_{B_j}^2 - M_{B'_j}^2}{2M_{B_j}}.
\]
for the baryon from the baryon sextet with spin 1/2 to the antitriplet, $6_{1/2} \rightarrow \bar{3}_{1/2}$, as

\[
\mu^{(0)} [\Sigma^+_c \rightarrow \Lambda^+_c] = -\frac{1}{4\sqrt{6}} \left( \bar{w}_1 - \frac{1}{2} w_2 \right),
\]
\[
\mu^{(op)} [\Sigma^+_c \rightarrow \Lambda^+_c] = -\frac{1}{20\sqrt{6}} (\bar{w}_4 + 2\bar{w}_5 + \bar{w}_6),
\]
\[
\mu^{(wr)} [\Sigma^+_c \rightarrow \Lambda^+_c] = \frac{1}{120\sqrt{2}} \left( \bar{w}_1 + \frac{3}{2} w_2 \right) p_{T5} - \frac{1}{20\sqrt{3}} \left( \bar{w}_1 + \frac{1}{2} w_2 \right) q_{T5},
\]
\[
\mu^{(0)} [\Xi'_c \rightarrow \Xi_c] = \frac{1}{4\sqrt{6}} \bar{Q} \left( \bar{w}_1 - \frac{1}{2} w_2 \right),
\]
\[
\mu^{(op)} [\Xi'_c \rightarrow \Xi_c] = \frac{1}{20\sqrt{6}} \left( (Q - 1) \bar{w}_4 + 2(Q - 1) \bar{w}_5 - (2Q - 1) \bar{w}_6 \right),
\]
\[
\mu^{(wr)} [\Xi'_c \rightarrow \Xi_c] = -\frac{1}{240\sqrt{2}} (5Q - 4) \left( \bar{w}_1 + \frac{3}{2} w_2 \right) p_{T5} + \frac{1}{40\sqrt{6}} (Q - 2) \left( \bar{w}_1 + \frac{1}{2} w_2 \right) q_{T5}.
\]

(23)

where $Q$ designates the electric charges of the corresponding heavy baryons. $\mu^{(0)} [B_c \rightarrow B'_c]$ denotes the contribution from the leading order. $\mu^{(op)}$ stands for the linear $m_s$ terms from the collective operator [3] and $\mu^{(wr)}$ comes from the collective wavefunctions [3].

Similarly, we can derive the expressions of the transition magnetic moments for the $6_{3/2} \rightarrow \bar{3}_{1/2}\gamma$ decays as

\[
\mu^{(0)} [\Sigma^{++}_c \rightarrow \Lambda^+_c] = \frac{1}{4\sqrt{3}} \left( \bar{w}_1 - \frac{1}{2} w_2 \right),
\]
\[
\mu^{(op)} [\Sigma^{++}_c \rightarrow \Lambda^+_c] = \frac{1}{20\sqrt{3}} (\bar{w}_4 + 2\bar{w}_5 + \bar{w}_6),
\]
\[
\mu^{(wr)} [\Sigma^{++}_c \rightarrow \Lambda^+_c] = -\frac{1}{120} \left( \bar{w}_1 + \frac{3}{2} w_2 \right) p_{T5} + \frac{1}{10\sqrt{6}} \left( \bar{w}_1 + \frac{1}{2} w_2 \right) q_{T5},
\]
\[
\mu^{(0)} [\Xi'_c \rightarrow \Xi_c] = -\frac{1}{4\sqrt{3}} \bar{Q} \left( \bar{w}_1 - \frac{1}{2} w_2 \right),
\]
\[
\mu^{(op)} [\Xi'_c \rightarrow \Xi_c] = \frac{1}{20\sqrt{3}} [(Q - 1) \bar{w}_4 + 2(Q - 1) \bar{w}_5 - (2Q - 1) \bar{w}_6],
\]
\[
\mu^{(wr)} [\Xi'_c \rightarrow \Xi_c] = \frac{1}{240} (5Q - 4) \left( \bar{w}_1 + \frac{3}{2} w_2 \right) p_{T5} - \frac{1}{40\sqrt{6}} (Q - 2) \left( \bar{w}_1 + \frac{1}{2} w_2 \right) q_{T5}.
\]

(24)

and for $6_{3/2} \rightarrow 6_{1/2}\gamma$ as

\[
\mu^{(0)} [\Sigma^*_c \rightarrow \Sigma_c] = -\frac{1}{30\sqrt{2}} (3Q - 2) \left( \bar{w}_1 - \frac{1}{2} w_2 - \frac{1}{3} w_3 \right),
\]
\[
\mu^{(op)} [\Sigma^*_c \rightarrow \Sigma_c] = -\frac{1}{270\sqrt{2}} [(5Q - 7) \bar{w}_4 + 3(4Q - 5) \bar{w}_5],
\]
\[
\mu^{(wr)} [\Sigma^*_c \rightarrow \Sigma_c] = -\frac{1}{45} (Q - 2) \left( \bar{w}_1 + \frac{1}{2} w_2 + w_3 \right) q_{T5} + \frac{1}{180\sqrt{5}} (Q + 1) (\bar{w}_1 + 2w_2 - 2w_3) q_{T5},
\]
\[
\mu^{(0)} [\Xi^*_c \rightarrow \Xi'_c] = -\frac{1}{30\sqrt{2}} (3Q - 2) \left( \bar{w}_1 - \frac{1}{2} w_2 - \frac{1}{3} w_3 \right),
\]
\[
\mu^{(op)} [\Xi^*_c \rightarrow \Xi'_c] = -\frac{1}{270\sqrt{2}} [(7Q - 2) \bar{w}_4 + (6Q - 3) \bar{w}_5],
\]
\[
\mu^{(wr)} [\Xi^*_c \rightarrow \Xi'_c] = \frac{1}{180} (5Q - 4) \left( \bar{w}_1 + \frac{1}{2} w_2 + w_3 \right) q_{T5} + \frac{1}{90\sqrt{5}} (Q + 1) (\bar{w}_1 + 2w_2 - 2w_3) q_{T5},
\]
\[
\mu^{(0)} [\Omega^*_c \rightarrow \Omega_c] = \frac{1}{15\sqrt{2}} \left( \bar{w}_1 - \frac{1}{2} w_2 - \frac{1}{3} w_3 \right),
\]
\[
\mu^{(op)} [\Omega^*_c \rightarrow \Omega_c] = -\frac{1}{90\sqrt{2}} (\bar{w}_4 + 3\bar{w}_5),
\]
\[
\mu^{(wr)} [\Omega^*_c \rightarrow \Omega_c] = \frac{1}{60\sqrt{5}} (\bar{w}_1 + 2w_2 - 2w_3) q_{T5}.
\]

(25)
TABLE I. Numerical results of the transition magnetic moments for the charmed baryons in units of \( \mu_N \).

| Reaction          | \( \mu^{(0)} / \mu_N \) | \( \mu^{(\text{total})} / \mu_N \) |
|-------------------|--------------------------|----------------------------------|
| \( \Sigma^+_c \to \Lambda^+_c \) | 1.24 ± 0.05, 1.54 ± 0.06 | 1.63, -0.15 ± 0.04, -1.38 ± 0.02 |
| \( \Xi^{*+}_c \to \Xi^+_c \)  | -1.24 ± 0.05, -1.19 ± 0.06 | 1.56, -0.73 (input)               |
| \( \Xi^{0}_c \to \Xi^0_0 \)  | 0, 0.21 ± 0.03           | -0.07, -0.12                      |
| \( \Sigma^{*+}_c \to \Lambda^+_c \) | -1.76 ± 0.08, -2.18 ± 0.08 | 2.2, 1.70, 2.00 ± 0.53            |
| \( \Xi^{*+}_c \to \Xi^+_c \)  | 1.76 ± 0.08, 1.69 ± 0.08  | 2.03, 1.50, 1.93 ± 0.72           |
| \( \Xi^{0}_c \to \Xi^0_0 \)  | 0, -0.29 ± 0.04          | -0.33, -0.22, 0.22 ± 0.07         |
| \( \Sigma^{*+}_c \to \Sigma^{*+}_c \) | 1.42 ± 0.07, 1.52 ± 0.07  | 1.39, 0.91, 1.33 ± 0.38           |
| \( \Sigma^{*+}_c \to \Sigma^{*0}_c \) | 0.35 ± 0.02, 0.33 ± 0.02  | 0.07, -0.06, 0.57 ± 0.09          |
| \( \Sigma^{*0}_c \to \Sigma^{*0}_c \) | -0.71 ± 0.03, -0.87 ± 0.03 | -1.24, -1.03, 0.24 ± 0.05        |
| \( \Xi^{*+}_c \to \Xi^{*0}_c \) | 0.35 ± 0.02, 0.43 ± 0.02  | 0.09, -0.09, 0.23 ± 0.06          |
| \( \Xi^{*0}_c \to \Omega^0_0 \) | -0.71 ± 0.03, -0.74 ± 0.03 | -1.07, -0.92, -0.59 ± 0.12       |
| \( \Omega^{*0}_0 \to \Omega^0_0 \) | -0.71 ± 0.03, -0.60 ± 0.04 | -0.94, -0.54, 0.49 ± 0.14        |

As explained in the previous Section, the effects of flavor SU(3) symmetry breaking come from both the collective operators with \( \mathbf{3}_c \) and the collective wavefunctions. The total results listed in Table I contain both the contributions, though we do not show them separately. In general, the effects from the collective operators are dominant over those from the wavefunction corrections. However, the wavefunction corrections are not negligible specifically in the transitions \( \Sigma_c \to \Lambda^+_c \), \( \Xi^{*}_c \to \Xi^+_c \), \( \Sigma^{*+}_c \to \Lambda^+_c \gamma \), and \( \Xi^{*0}_c \to \Xi^{0}_c \gamma \). As shown in Eqs. (23) and (24), the transition magnetic moments for the decays \( \Sigma_c \to \Xi^+_c \gamma \) and \( \Xi^{*}_c \to \Xi^{0}_c \gamma \) are proportional to the corresponding charge \( Q \). Thus,
the transition magnetic moments for the radiative decays of the neutral \( \Xi' \) and \( \Xi^+ \) baryons vanish. This means that the linear \( m_s \) corrections take over the leading-order role. In this case, the effects from the wavefunctions are still important. They contribute to the results of \( \mu[\Xi_c^0 \to \Xi_c^0] \) and \( \mu[\Xi_c^0 \to \Xi_c^0] \) by about 30\%. It is interesting to see that the effects of SU(3) symmetry breaking contribute to the transitions of the \( \Sigma_c^+ \to \Lambda_c^+ \gamma \), \( \Sigma_c^+ \to \Lambda_c^+ \gamma \), \( \Sigma_c^0 \to \Sigma_c^0 \gamma \), \( \Xi^+_c \to \Xi^+_c \gamma \), and \( \Omega_c^0 \to \Omega_c^0 \gamma \) by about 20\%, whereas their contributions are rather small to other decay channels (about 6\%). Moreover, the effects of SU(3) symmetry breaking add to the magnitudes of the transition magnetic moments in certain channels but reduce those of the \( \Xi_c^0 \) (about 6\%).

We can analyze other decay channels in the same way. Note that a similar tendency was found in the case of the \( \Xi_c^0 \) decay [43]. On the other hand, the SU(3) symmetric part yields the negative result, so that the magnitude of the transition magnetic moment of the \( \Xi_c^+ \to \Xi_c^+ \gamma \) is diminished by the \( m_s \) corrections. One can understand other decay channels in the same way. Note that a similar tendency was found in the case of the \( \Xi^0 \to \Xi^0 \gamma \) decay [42].

We also compare the present results with those from the NRQM [16–18], the modified bag model [21], the chiral constituent quark model [33], the light-cone QCD sum rules [24, 25], and the Skyrme models with bound state approaches [43–46]. The present are comparable with those from all other works. In particular, it is of great interest to compare the present results with those of a recent simulation of lattice QCD [37, 38], which is listed in the last column. Note that in Refs. [37, 38] a value of the pion mass \( m_\pi = 156 \text{ MeV} \) was used, which is quite close to the experimental value. Apart from the signs, the present results are in qualitative agreement with the lattice data. The result of the \( \Omega_c^0 \to \Omega_c \gamma \) decay is in good agreement with that of Ref. [37], compared to those from other models.

### Table II. Numerical results of the transition magnetic moments for the bottom baryons in units of \( \mu_N \). They are compared with those from the nonrelativistic quark model from Refs. [16–18] and Ref. [19] for \( B_c \) corresponding to \( 6_{1/2} \) and \( 6_{3/2} \), respectively. The values from modified bag model [21], light-cone QCD sum rule [24, 25], chiral perturbation theory [32], Skyrme model (SM) [43], and bound state approach [46] are also listed.

| \( B_b \) → \( B'_b \) | \( \mu^{(0)}[\mu_N] \) | \( \mu^{(t)}[\mu_N] \) | [16–19] | [21] | [24, 25] | [32] | [46] | [45] |
|-----------------|-----------------|-----------------|------|------|------|------|------|------|
| \( \Sigma_b^+ \to \Lambda_b^0 \) | \(-1.24 \pm 0.05 \) | \(-1.54 \pm 0.06 \) | \(-0.63 \) | \(-0.38 \) | \(-1.37 \) | \(-2.24 \) | \(-1.54 \) |
| \( \Xi_b^0 \to \Xi_b^0 \) | \(1.24 \pm 0.05 \) | \(1.19 \pm 0.06 \) | \(-0.75 \) | \(-0.38 \) | \(-0.75 \) | \(-0.38 \) | \(-0.38 \) |
| \( \Xi_b^+ \to \Xi_b^0 \) | \(0 \) | \(0.21 \pm 0.03 \) | \(-0.21 \) | \(-0.14 \) | \(-0.21 \) | \(-0.14 \) | \(-0.14 \) |
| \( \Sigma_b^0 \to \Lambda_b^0 \) | \(-1.76 \pm 0.08 \) | \(-2.18 \pm 0.08 \) | \(2.28 \) | \(1.49 \) | \(2.28 \) | \(1.49 \) | \(1.49 \) |
| \( \Xi_b^0 \to \Xi_b^0 \) | \(1.76 \pm 0.08 \) | \(1.69 \pm 0.08 \) | \(0.06 \) | \(0.06 \) | \(0.06 \) | \(0.06 \) | \(0.06 \) |
| \( \Xi_b^+ \to \Xi_b^0 \) | \(0 \) | \(0.29 \pm 0.04 \) | \(-0.26 \) | \(-0.14 \) | \(-0.30 \) | \(-0.14 \) | \(-0.14 \) |
| \( \Sigma_b^0 \to \Lambda_b^0 \) | \(-1.42 \pm 0.07 \) | \(-1.52 \pm 0.07 \) | \(1.81 \) | \(1.19 \) | \(1.81 \) | \(1.19 \) | \(1.19 \) |
| \( \Sigma_b^0 \to \Sigma_b^0 \) | \(-0.35 \pm 0.02 \) | \(-0.33 \pm 0.02 \) | \(0.49 \) | \(0.35 \) | \(0.49 \) | \(0.35 \) | \(0.35 \) |
| \( \Sigma_b^+ \to \Sigma_b^0 \) | \(0.71 \pm 0.03 \) | \(0.87 \pm 0.03 \) | \(-0.82 \) | \(-0.50 \) | \(-0.82 \) | \(-0.50 \) | \(-0.50 \) |
| \( \Xi_b^0 \to \Xi_b^0 \) | \(-0.35 \pm 0.02 \) | \(-0.43 \pm 0.02 \) | \(0.61 \) | \(0.39 \) | \(0.61 \) | \(0.39 \) | \(0.39 \) |
| \( \Xi_b^+ \to \Xi_b^0 \) | \(0.71 \pm 0.03 \) | \(0.74 \pm 0.03 \) | \(-0.66 \) | \(-0.42 \) | \(-0.66 \) | \(-0.42 \) | \(-0.42 \) |
| \( \Omega_b^0 \to \Omega_b^0 \) | \(0.71 \pm 0.03 \) | \(0.60 \pm 0.04 \) | \(-0.52 \) | \(-0.34 \) | \(-0.52 \) | \(-0.34 \) | \(-0.34 \) |

Table II lists the results of the transition magnetic moments of the bottom baryons. In the present pion mean-field approach, we take the limit of the infinitely heavy quark mass (\( m_Q \to \infty \)). Thus, the results are in fact the same as those of the charmed baryons. Only the signs of the results are different, because charge of the bottom quark \( Q_b = -1/3 \) is different from that of the charm quark \( Q_c = 2/3 \). Nevertheless, the results are quite comparable with those of other works. In particular, we find that the present result of the \( \Sigma_b^0 \to \Lambda_b^0 \gamma \) is almost the same as that from the Skyrme model [43].
TABLE III. Numerical results of the radiative decay rates for the charmed baryons in units of keV. The results are compared with those from the modified bag model \cite{21}, light-cone QCD sum rule \cite{25}, chiral perturbation theory \cite{31,32}, and lattice QCD \cite{37,38}.

| $B_c \rightarrow B'_c \gamma$ | $\Gamma^{(0)}_{\gamma}$ [keV] | $\Gamma^{(\text{total})}_{\gamma}$ [keV] | \cite{21} | \cite{25} | \cite{30} | \cite{32} | \cite{37,38} |
|---|---|---|---|---|---|---|---|
| $\Sigma^+_c \rightarrow \Lambda^+_c \gamma$ | 8.32 ± 0.73 | 12.82 ± 0.95 | \(\cdots\) | \(\cdots\) | 65.6 ± 2 | \(\cdots\) | \(\cdots\) |
| $\Xi^+_c \rightarrow \Xi^+_c \gamma$ | 2.18 ± 0.20 | 2.02 ± 0.20 | \(\cdots\) | \(\cdots\) | 5.43 ± 0.33 | \(\cdots\) | 5.468 |
| $\Xi^0_c \rightarrow \Xi^0_c \gamma$ | 0 | 0.06 ± 0.01 | \(\cdots\) | \(\cdots\) | 1.2 ± 0.7 | \(\cdots\) | 0.46 ± 0.002 |
| $\Sigma^{*+}_{c} \rightarrow \Sigma^{*+}_{c} \gamma$ | 23.04 ± 2.12 | 35.49 ± 2.81 | 21.61 | 29.90 | \(\cdots\) | 161.6 ± 5 | \(\cdots\) |
| $\Xi^{*+}_{c} \rightarrow \Xi^{*+}_{c} \gamma$ | 9.34 ± 0.82 | 8.66 ± 0.81 | 6.82 | 11.29 | \(\cdots\) | 21.6 ± 1 | \(\cdots\) |
| $\Xi^{*0}_{c} \rightarrow \Xi^{*0}_{c} \gamma$ | 0 | 0.25 ± 0.06 | 0.14 | 0.14 | 5.1 ± 2.7 | \(\cdots\) | 1.84 | \(\cdots\) |

It is straightforward to compute the radiative decay rates of the heavy baryons by using Eqs. \[(20)\] and \[(21)\], once we have known the corresponding transition magnetic moments. The results for the charmed baryons are presented in Table III. They are quite similar to those from the modified bag model \cite{21} and light-cone QCD sum rule \cite{25}, whereas they are somewhat deviated from those of Ref. \cite{32} even though the results of the transition magnetic moments are comparable to the present ones. Since Ref. \cite{30} presented only the results of the radiative decay widths from chPT, we compare the results from the present work with them. Those of Ref. \cite{30} are larger than those of all other models as well as the present results. In the last column of Table III, the results from the calculation of lattice QCD are listed \cite{37,38}, which are also comparable to the present ones.

TABLE IV. Numerical results of the radiative decay rates for the bottom baryons in units of keV. The results are compared with those from the modified bag model \cite{21}, light-cone QCD sum rule \cite{25}, and chiral perturbation theory \cite{31,32}.

| $B_b \rightarrow B'_b \gamma$ | $\Gamma^{(0)}_{\gamma}$ [keV] | $\Gamma^{(\text{total})}_{\gamma}$ [keV] | \cite{21} | \cite{25} | \cite{30} | \cite{32} |
|---|---|---|---|---|---|---|
| $\Sigma^+_b \rightarrow \Lambda^+_b \gamma$ | 3.3 ± 0.3 | 5.1 ± 0.4 | 3.38 | 2.48 | \(\cdots\) | 42.5 ± 5 | \(\cdots\) |
| $\Xi^0_b \rightarrow \Xi^0_b \gamma$ | 1.3 ± 0.1 | 1.2 ± 0.1 | 0.72 | 1.20 | \(\cdots\) | 17.2 ± 0.1 | \(\cdots\) |
| $\Xi^{-}_{b} \rightarrow \Xi^{-}_{b} \gamma$ | 0 | 0.03 ± 0.01 | 0.01 | 0.01 | 4.2 ± 2.4 | \(\cdots\) | 1.4 |

In Table IV, we list the results of the radiative decay rates for the bottom baryons. Again, they are comparable with those from the other works. By the same reason, the results from chPT \cite{32} are different from the present ones but the recalculated results with Eqs. \[(20)\] and \[(21)\] are quite comparable with the present ones.
V. SUMMARY AND CONCLUSIONS

In the present work, we have investigated the transition magnetic moments of the lowest-lying heavy baryon sextets within a pion mean-field approach or a “model-independent” chiral quark-soliton model. Since all the dynamical parameters for the magnetic moments of the singly-heavy baryons were fixed in the light baryon sector, we were able to compute the transition magnetic moments of the baryon sextet without any additional parameters introduced. We also derived the various relations among the transition magnetic moments as in the case of the magnetic moments of the heavy baryons. In addition, we found the relations that arise from the $U$-spin symmetry. The results of the transition magnetic moments for the charmed and bottom baryons were compared with those from other works. In particular, the present results are in good agreement with those from a simulation of lattice QCD. We obtained the radiative decay rates of the heavy baryons and compared the results with those from other works. While they are quite comparable with the results from the modified bag model, the light-cone QCD sum rule, and the simulation of lattice QCD, those from chiral perturbation theory seem different from the present ones. The reason arises from the fact that different formulae for the radiative decay rates were used. Using the same formulae, we found that the present results are also in good agreement with those from chiral perturbation theory.

Information on the transition magnetic moments may provide the vector meson coupling to the heavy baryons. Though the experimental data on them are still absent, theoretical investigations may shed lights on how the vector mesons can be coupled to the heavy baryons. In fact, this will provide essential information on the hadronic description of heavy hadron productions. The corresponding study is under way.

ACKNOWLEDGMENTS

The authors are grateful to J.-Y. Kim for valuable discussions. The present work was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (NRF-2019R1A2C1010443 (Gh.-S. Y.), 2018R1A2B2001752, and 2018R1A5A1025563 (H.-Ch.K.).
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