Stationary determinism in Observed Time Series: the earth’s surface temperature

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Abstract

In this work we address the feasibility of estimating and isolating the stationary and deterministic content of observational time series, $\text{Ots}$, which in general have very limited characteristics. In particular, we study the valuable earth’s surface mean temperature time series, $\text{Tts}$, by applying several treatments intended to isolate the stationary and deterministic content. We give particular attention to the sensitivity of results on the different parameters involved. The effects of such treatments were assessed by means of several methods designed to estimate the stationarity of time series. In order to strengthen the significance of the results obtained we have created a comparative framework with seven test time series of well-known origin and characteristics with a similar small number of data points. We have obtained a greater understanding of the potential and limitations of the different methods when applied to real world time series. The study of the stationarity and deterministic content of the $\text{Tts}$ gives useful information about the particular complexity of global climatic evolution and the general important problem of the isolation of a real system from its surroundings by measuring and treating the obtained observations without any other additional information about the system.

1 Introduction

Correlations in time series have different origins. The origin of short term correlations of real systems with low-frequency divergences, called 1/f noise or colored noise, is unknown [1, 2]. Artificial colored noise time series can be generated by introducing in some manner short term correlations into an otherwise uncorrelated stochastic time series [3]. Strong correlations at all time scales are typical of Brownian motion and can be artificially constructed by integrating a stochastic time series that has a Gaussian distribution. These different correlations are very common in nature and compete with the expected correlations generated by deterministic rules from the continuous processes underlying an Observational time series ($\text{Ots}$). $\text{Ots}$ have in general several important limitations such as a small and sparse number of data points, poor resolution and accuracy, dynamical and measurement

\footnote{Part of this work was performed as a Ph.D. dissertation at the Physics Department at New York University in the Applied Science program.}
noise contamination. Dynamical noise affects each time step which may change the nature of the dynamics itself. Contamination with colored noise reinforces low frequencies in the power spectrum producing similar difficulties as those encountered when the time span of observation is too short to cover all the characteristic times of the system. When contamination comes from Brownian motion the time correlations extend to most of the data points of the time series and there is no time scale at which the static properties can be estimated. Therefore, the lack of stationarity in Ots is a very common problem although, it is in general an implicit condition for the application of most of the methods used to identify and characterize determinism in time series. This implicit stationarity condition corresponds to the physical aspect of a defined time scale of observation within which all the relevant processes are followed and sufficiently sampled.

The mentioned limitations of most Ots combined with the actual complexity of the observed process and the strength of the interactions with other systems, frequently produce effects that can be seen as contaminations and lack of stationarity. These artifacts of the observation may be cleaned in some extent and hence, important dynamical information of the process may be isolated, characterized and eventually modeled.

From a dynamical point of view nonstationarity may correspond to nonautonomous systems, changes of the evolution equations, changes of parameters with time, or even changes of the shape of the attractor effectively leaving unchanged the dynamical invariants. Therefore, we may be interested in the problem of extracting a meaningful stationary deterministic time series from the original Ots. In order to achieve this goal we first must be able to estimate the stationarity of an Ots. Since Ots are irreproducible, the importance of stationarity in Ots corresponds to the integrity and permanent identity of the system within the time it is observed with respect to the time resolution in which it is observed. Since we do not know a priori the deterministic rules and parameters characterizing the observed system, the Ots is considered stationary if probabilities of transition between different states of the system are constant during the observed period of time [4]. From a practical point of view stationarity ensures the feasibility of a statistical analysis, since the probabilities are well defined as occurrence limits [4]. However, linear statistics of chaotic time series such as the variance and mean do not approach finite values since they do not have renormalizable probability density distributions. Usually, Ots have small a number of data points and the statistical analysis has in general additional limitations.

An appropriate period of time and frequency of sampling may adequately manage the complex compromise between the system and the observational characteristics and we may consider the Ots stationary in practical terms. Unfortunately such particular time “lens” is not easy if ever possible to determine, as several coexistent and nonseparable time scales may be involved.

The earth’s mean surface temperature time series, Tts, is a particularly complicated case of Ots in two ways. As a time series it is a highly contaminated and relatively small set of low resolution data points. From the point of view of the system that Tts represents, the complexity of the
underlying process may be excessive to be able to be characterized with the available data and the capability of the existing methods. However, the fact that each data point is a monthly earth’s surface mean, considerably reduces the space and time scales concerned. Under such conditions many temporal and local complex processes of the earth’s surface are conveniently rounded out. In this context it may be possible to identify certain global low dimensional dynamics where several complex processes make contributions without being direct deterministic aspects (active degrees of freedom) of such global dynamics. This perspective permits to exploit some potential information about the earth’s climatic evolution contained in this valuable time series obtained in more than a hundred years of measurement and data processing effort. Nevertheless, if there is not a particular set of scales and certain treatments permitting to obtain a time series where the “signal” dominates the different “contaminations” the system is not observable.

Stationarity is a property which can never be positively established in a finite time series. In practice there are some methods which give us partial measures of how strong is the violation of stationarity of a given time series. Contamination and nonstationarity are not disconnected problems in particular for nonlinear dynamical systems. In general, for nonlinear time series it is not possible to filter out contaminations without affecting the actual dynamics because in some sense most of them are part of the dynamics itself. Consequently, we must try to measure how robust the dynamics is to external effects, how discernible these effects are from the actual signal and thus, estimate to which extent such dynamics can be extracted from the Ots. For nonlinear systems when nothing substantial is known about the signal nor about the contamination, we need to characterize the signal and/or the contamination in some fashion which allows us to differentiate them by making certain assumptions. However, this goal has not been as successful as in the case of linear systems. There are only general criteria which can be used to distinguish which assumptions are better than others. For example, in Ots of natural phenomena where in general colored noise is present, it is more efficient to make a local low pass filter than to assume white noise [5].

Most of the modern methods of nonlinear dynamics used to study time series involve a reconstruction of the scalar time series into $m$-dimensional data points in a Euclidean space. A norm is then used to measure distances and geometrical properties of such reconstruction. Embedding is a method to reconstruct the attractor of a system by transforming the scalar time series of observations $x_i, i = 1, ..., N$, where $N$ is the total number of data points, into $m$-dimensional reconstructed vectors or points $\tilde{x}_i = (x_i, x_{i+\tau}, ..., x_{i+\tau(m-1)})$, where $i + \tau(m - 1) \leq N \quad [5]$. Such a reconstruction involves two important parameters, the time delay between each component of a reconstructed vector, $\tau$, and the embedding dimension $m$. The demands on $\tau$ are not precisely defined by the embedding theorem. In practice, for Ots, the choice of a correct $\tau$ may be very important because of the compromise between the relevant time scales of the system and the characteristics of the time series. When the attractor is correctly reconstructed any static geometrical characteristic of the system can be estimated. The embedding theorem is based on correlations from deterministic rules
of the system that in principle exist in any macroscopic physical process. In the case of a chaotic system the trajectories remain confined to the attractor after temporal behaviors have died out. This attractor is a static structure, but when the temporal deterministic correlations are affected by parameter variations or changes in the deterministic rules of the dynamic or any other disturbing correlation, the stationarity of the system may be destroyed and the attractor is not static; it cannot be defined. The maximum norm takes the absolute value of the maximum component as the norm of the vector or \( m \)-dimensional point. The maximum norm is very economic in computational time but may involve certain difficulties when \( m \) is small and the characteristics of the time series are limited as used to be the case for \( Ots \). This problem will be discussed within the application of some nonlinear methods to identify nonstationarity in a time series.

The characteristics of the system itself and those of the corresponding \( Ots \) determine how successful and how complicated could be the process of obtaining a meaningful stationary and deterministic time series from an \( Ots \). For nonlinear time series the success in such process has been very limited although, it is in constant development. In order to attempt such isolation for the \( Tts \) we have chosen those methods with more general applicability or especially designed for nonlinear time series. Methods based on strong assumptions may generate artificial results when applied to \( Ots \). A particular limitation of \( Ots \) is that we cannot manipulate certain parameters such as the sampling time and time span in order to improve stationarity without compromising the number of data points for statistical sufficiency and/or reducing the frequency of observations introducing contaminating effects. The methods used to obtain a meaningful stationary-deterministic time series from an \( Ots \) can be classified in two groups. First, the methods applied in the time domain:

- Polynomial detrending which is natural to the time series and do not assume nonstationarity produced by low frequencies as is the case for detrending with dominant frequencies.
- Low-frequency filter by introducing a decaying memory function in an integral-differential embedding [11, 12].
- Smoothing with a different number of neighbors, or other moving average or FIR filters, which do not have a direct effect on stationarity but increase resolution and reduce fast variations [3, 13–16].
- First differences of data points removes slowly varying trends in the time series.
- The method of the correlation matrix filters out noise in the time domain in particular high frequency contaminations [11, 17–20].
- Noise reduction by nonlinear methods or local maps using neighborhood to neighborhood information [4, 5, 14, 21–27].

Secondly, methods applied in the frequency domain:

- Windowing or low and high frequency filters are extensively used [12].
- Hilbert transform by using the FFT which is like the generalized zero derivative also used to interpolate small time series [11].
- Maximum entropy or all poles method, also used to detrend time series, estimates the power spectrum by representing the data in terms of a finite number of complex poles of discrete frequency instead of the essential polynomial fit used by the Fourier transform, especially useful when the data looks very noisy [28].
- Fourier interpolation permits to extend and clean out the data sets [11, 12].

The most appropriate method or combination of methods which best isolates the stationary determinism potentially contained in a given Ots, minimizing the damages of the dynamical information, is not easy to find and strongly depends on each particular case. For example, if the power spectrum is not small near the Nyquist frequency, frequency domain processing is a bad idea and should be abandoned [11]. We may economize a lot of effort if we make the assumption that the outcome of the temperature measurements of the Tts, or any other Ots, is a random variable implying a probabilistic approach which turns out to be very fruitful. However, it will not give as much information of the underlying mechanisms as a deterministic model may give.

Given the strong difficulties to define a definitive method to measure and isolate stationary and determinism in Ots, we have performed a comparative analysis of the Tts and several treated Tts altogether with some representative test time series of the same length and well known characteristics. In section 2 we describe the Tts and the test time series. In section 3 we make a short presentation of several methods used to estimate stationarity in finite time series placing particular attention on the problems generated by the limiting characteristics of the Ots. In section 4 we present the results of applying several methods of filtering and detrending to the Tts. We then, use a weak stationarity criterion in order to choose the most promising treated Tts (TTts). In section 5 we apply the different methods presented in section 3 to the Tts, the test time series and the chosen TTts obtained in section 4. In section 6 the results are summarized and discussed. In section 7 we present some conclusions.

2 The Temperature time series (Tts) and test time series

OAK Ridge National Laboratory provides synopsis of frequently used global change data [29]. Of this valuable information, we will use the global temperature anomalies obtained from instrumental surface air temperature records compiled by K. Ya. Vinnikov, P. Ya. Groisman and K. M. Lugina. This data has been mainly taken from the world Weather Records, Monthly Climatic Data for the World, and Meteorological Data for Individual Years over the North Hemisphere Excluding the USSR. This information was completed and improved using other data and methods that are explained and referred to in reference [29]. The original time series of 1356 data points corresponds
to the 12 monthly mean temperatures from 1881 to 1993, in chronological order. This data is given with two decimal digits of accuracy. For a more detailed discussion of the data and its limitations, noise and possible effects on the estimated averages the reader is referred to the original article of Jones, Wigley and Kelly [30].

We are going to use seven test time series that will be identified by the names: Noise, Random, Pnoise, Chaos, Brownian, Eeg and Ekg. These test time series, all with the same 1356 data points, have been obtained from CDA pro, a software for nonlinear analysis of time series [31] where more details of the test time series can be consulted. Noise is a time series of random numbers with Gaussian or normal distribution centred on zero and a standard deviation of 1 called Gaussian or white noise. Random is a time series of random numbers uniformly distributed. Pnoise is a time series of random numbers with Gaussian distribution centred on zero with 1/f power law. Chaos is a time series of the $x$ variable of the Lorenz attractor. Brownian is a Brownian time series generated by the integral of a Gaussian noise time series. Eeg and Ekg are two real world time series with expected nonstationarity. The first consists of measurements of the voltage in the electrodes of an electroencephalogram. The second consists of the intervals between successive interbeats of an electrocardiogram.

The Ots are real world time series corresponding to measurements of a particular observable of a natural system which in many cases are not repeatable and are irreproducible. They differ from experimental time series because they are not produced by an experimental set up which can be controlled to some extent, and from numerical time series because we do not know the equations which generate them. Many Ots, in particular Tts, correspond to systems that never will be in the same circumstances and therefore we have to limit ourselves to observe them as they evolve. We know that no real world system can be completely isolated from its surroundings; to some extent the environment and the observation process are part of the system itself. Similarly, no particular time scale can be completely isolated from the others. Different parts of the system evolve at different frequencies with complex interrelations but conforming the same dynamics. In the case of the Tts we have an external periodic forcing force of astronomical origin, that of the seasonal oscillation determined by the earth’s orbit around the sun and the tilt of the rotation axis (precession). It is known that strong periodicities tend to hide underlying fractal structures [1, 32] and important nonlinear and potentially chaotic dynamics may not be easily detected. Formally speaking, a periodic external force acting on the system may be included as a variable recovering a stationary point of view of the system. The importance of the periodicity observed in local seasonal variations of the temperature is evident. What is not so evident is how strong it is in the sense of global earth’s surface mean temperatures. From the point of view of global means these seasonal cycles are indirectly generated by the asymmetry of those geological masses of the earth which permit the transformation of the incoming solar energy into thermal energy on the surface of the earth. The most important of such geological masses are earth, sea, ice, clouds and some factors
related with the existence of life and particularly with human activities. These different masses and different activities strongly interact in complex ways.

3 Linear statistics and weak stationarity

One reason for the actual incapacity to precisely measure the stationarity of a time series is that there is no uniform definition of stationarity. Loosely speaking, physical stationarity means the constancy of the main features of the system, dynamical stationarity means constancy of equations and parameters involved in an autonomous dynamical system, and statistical stationarity is defined for stochastic processes when joint probability functions are independent of time shifted realizations [4, 5]. Nevertheless, these different perspectives of stationarity have close links. The deterministic aspects of a physical system are not conflictive with the contemplation of its signal as a realization of a stochastic process on which the statistical stationarity is based. In the language of dynamical systems statistical stationarity is equivalent to the existence of an invariant ergodic measure [33, 34]. If linear statistics like the mean, standard deviation and variance, estimated from different parts of a time series do not vary beyond statistical fluctuations, the time series may be considered weakly stationary. Weak stationarity does not imply dynamical stationarity but may be a useful first indication of it [35, 36]. Vice-versa, weak nonstationarity not necessarily implies dynamical nonstationarity, it may be produced for example, by a variety of artifacts in the processes of acquisition and manipulation of data.

We cannot be certain of the effects of most of the detrending and cleaning treatments on the potential nonlinear dynamical information contained in a $O_{ts}$. However, we may consider weak stationarity as an initial criterion to assess the effects of different treatments applied to the $T_{ts}$. We use the variance $V$, as the linear statistics which is more sensitive than the mean, mean deviation and standard deviation but not too sensitive to statistical changes in the time series. The sensitivity of the linear statistics is important because of the limited number of data points but can become a problem if the time series is highly contaminated. In order to test weak stationarity we divide each time series in three consecutive and nonoverlapping segments of equal number of data points. Comparing the first with the second segment of the $T_{ts}$ we observe that $V$ changes in 53%, and comparing the second with the third segment and the third with the first segment, the absolute changes of $V$ are 70% and 26% respectively. These results represent an important weak nonstationarity of the $T_{ts}$. The time series $\text{Chaos}$ present good weak stationarity with unimportant variations of $V$ for the three segments of the data. The time series $\text{Noise}$ and $\text{Random}$ present similar results. This is expected because these three time series are stationary, deterministic and stochastic respectively. The time series $\text{Pnoise}$ presents mild absolute variations of $V$ as expected because of the short term correlations of this stochastic time series. The time series $\text{Eeg}$ and $\text{Ekg}$ give the expected weak nonstationarity more remarkable in the second case but, in both cases, it is
considerably smaller than that of \textit{Tts}. Table 1 summarizes these results.

Within this context we have applied several methods to the \textit{Tts} in order to obtain Treated Temperature time series, \textit{TTts}, with improved weak stationarity. All of the \textit{TTts} were divided in three segments and analyzed as mentioned above. Those \textit{TTts} which present certain reduced absolute variations of V compared with the corresponding values for \textit{Tts}, are considered \textit{TTts} with improved weak stationarity. We have considered six methods which do not need to make strong assumptions about the system and contamination of the time series. The first three methods mainly treat the noise contamination. The following two methods mainly treat the trends of the time series and, the last method removes slow trends and reduces some noise contamination. The six methods used (and their parameters) are:

i) filtering with a correlation matrix (the size of the matrix),
ii) smoothing (number of neighbors and variable intensity for different parts of the time series),
iii) nonlinear filters (\(\tau\), \(m\) and number of neighbors),
iv) detrending with the maximum-entropy or all poles method (number of poles),
v) polynomial detrending (degree of the polynomial),
vi) and first differences.

From an extensive list of more than seventy \textit{TTts} obtained by different combinations of the six methods with different values of the parameters involved, we have chosen the 21 \textit{TTts} with most important improvement of weak stationarity. These 21 \textit{TTts} are described below from 2 to 22 leaving 1 for the original \textit{Tts}. The results of weak stationarity are presented in table 1.

\textit{TTts} 2 is \textit{Tts} smoothed with one neighbor considering a decreasing factor from 1 to 0.001 to compensate the assumption that the quality in obtaining and processing the \textit{Tts} has been improved with time.

\textit{TTts} 3 is \textit{Tts} detrended with the maximum-entropy method using 128 poles.

\textit{TTts} 4 is \textit{Tts} filtered with the correlation matrix where the three dominant modes have been used.

\textit{TTts} 5 is \textit{TTts} 3 smoothed with the decreasing factor.

\textit{TTts} 6 is \textit{TTts} 4 smoothed with the decreasing factor.

\textit{TTts} 7 is \textit{TTts} 4 detrended with the method of the maximum-entropy using 128 poles.

\textit{TTts} 8 is \textit{TTts} 4 detrended with the method of the maximum-entropy using 128 poles and finally smoothed with the decreasing factor.

\textit{TTts} 9 is \textit{Tts} filtered with the nonlinear method in one dimensional embedding and using two neighbors.

\textit{TTts} 10 is \textit{Tts} filtered with the nonlinear method in six dimensional embedding and using four neighbors.
TTts 11 is Tts detrended with the method of the maximum-entropy using 128 poles and then filtered with the nonlinear method in one dimensional embedding and using two neighbors.

TTts 12 is Tts detrended with the method of the maximum-entropy using 128 poles and then filtered with the nonlinear method in one dimensional embedding and using four neighbors.

TTts 13 is Tts detrended with the method of the maximum-entropy using 128 poles and then filtered with the nonlinear method in six dimensional embedding and using four neighbors.

TTts 14 is Tts detrended with a polynomial of order 2.

TTts 15 is Tts detrended with a polynomial of order 3.

TTts 16 is Tts smoothed once and then detrended with a polynomial of order 2.

TTts 17 is TTts 4 detrended with a polynomial of order 2.

TTts 18 is the first differences of Tts.

TTts 19 is Tts with a nonlinear filter with embedding 1 and 1 neighbor.

TTts 20 is Tts with a nonlinear filter with embedding 3 and 2 neighbors.

TTts 21 is TTts 19 detrended with a polynomial of order 2.

TTts 22 is TTts 20 detrended with a polynomial of order 2.

Some comments about the results of all the TTts studied are in order to justify the 21 selected TTts. The best results of weak stationarity using polynomial detrending are obtained for polynomials of order two and three, for higher order polynomials the weak stationarity degrades very fast. Smoothing degrades weak stationarity when we use more than one neighbor. This is an expected result because smoothing mainly cleans up homogeneously distributed contaminations which is not necessarily the case for nonstationary effects. In principle the first difference of the time series gives greater density in phase space and clarifies nonlinearities reducing the effects of any short-time linear correlation giving some times the same results than the original time series [23]. For Tts it clearly improves weak stationarity but other undesirable effects are produced by the low accurate and other limitations of such Otts. Nonlinear filters have been conceived for potentially chaotic time series and they do not assume any particular knowledge about contamination. However, the scalar time series have to be reconstructed by an embedding procedure which demands information about the dynamics involved, in particular the embedding dimension which is not easy to estimate for Otts. We have explored nonlinear filters of Tts with embedding dimensions from 1 to 6. The TTts obtained with a nonlinear filter present improved weak stationarity for small embeddings, in particular for dimension 1 and 3, and for small number of neighbors, 1 and 2. For larger values of these two parameters the results degrade quickly. TTts obtained by combinations of different methods in general do not improve the results because of the low resolution and highly contaminated data points constituting the original Tts. The difficulty to discriminate between signal and noise is exacerbated when the Tts is manipulated repeatedly and round-off errors propagate indiscriminately. Therefore, the pretention to isolate the stationary deterministic content of a Otts by
applying successive treatments on the data may produce the opposite effect hiding most of such relevant and valuable information.

4 Methods to estimate stationarity in finite time series

In accordance with the difficulties presented above we cannot expect a simple method to diagnose the stationarity of a given Ots. Practical and conceptual problems are all related with the compromises between the relevant scales of the observed system and the process of observation. Together with linear statistics, used to estimate weak stationarity, we apply some of the most important methods to identify and estimate the stationarity of a finite time series. We succinctly describe these methods giving particular attention to the problems encountered when we applied them to Ots:

1- Polynomial Fit: The most natural method to identify nonstationarity is the direct inspection of the time series, we may identify the presence of trends and bursts. Bursts are very short time and large amplitude changes. Such bursts may correspond to faster dynamics than the time resolution of the present time series and are only captured occasionally as eventual coincidences. Alternatively, the time series under study may have some dynamical information that can only be captured with a longer period of observation where a sufficiently large number of such bursts are included. An evident trend on the plot of the time series may correspond to slow variations that cannot be resolved by the time spanned by the Ots. Trends of a time series can be quantitatively estimated by a polynomial fit and then the time series can be detrended by subtracting such fit. For an Ots we do not know the appropriate order of the polynomial to be used and we are not certain of the effects on the nonlinear information of the time series.

2- The correlation function or autocorrelation function $C(t)$, is a measure of time correlations between the different data points of a time series. $C(t)$ is a linear measure of temporal correlation in the sense of least squares prediction of data points at time $t$ from the knowledge of data points at a previous time $[1, 5]$. The correlation time $t'$, is commonly considered the time at which $C(t)$ goes to $1/3$ or $1/e$ of its value at $t = 0$. When $t'$ is larger or on the order of the duration of the time series, it indicates certain nonstationarity. The linear correlation measured by $t'$ is not necessarily the most relevant correlation in a nonlinear dynamical system $[37]$. Uncorrelated stochastic time series present the expected $t' \sim 0$. The short term correlations introduced into an otherwise stochastic uncorrelated time series to produce artificial colored noise time series, is detected by $C(t)$ giving a non negligible $t'$. The continuity and derivativity of dynamical systems of differential equations generate time series with strong correlations which correspond to large values of $t'$. These characteristic results strongly depend on the sampling time or frequency of measurements. Chaotic time series obtained from nonlinear systems of differential equations show in general relatively large $t'$ of the order of the inverse of the Lyapunov exponent. However, the corresponding $C(t)$ decreases very fast consistently with the divergent nature of chaotic behavior. The value of $t'$ for Ots depends
| time series | V 1-2 | V 2-3 | V 3-1 | trend | period. | $t'$ | freq. | sat. time |
|------------|-------|-------|-------|-------|--------|------|------|----------|
| Chaos      | 2     | 2     | 2     | 3     | C      | 6    | .03  | 10*      |
| Noise      | 1     | 1     | 1     | 2     | /      | 1    | -    | 1        |
| Random     | 3     | 5     | 1     | 1     | /      | 1    | -    | 1        |
| Brownian   | 13    | 38    | 42    | 26    | C+     | 169  | 0    | non      |
| Pnoise     | 15    | 23    | 25    | 15    | C-     | 11   | 0    | 10       |
| Eeg        | 16    | 15    | 5     | 1     | C      | 6    | .032 | 10       |
| Ekg        | 36    | 27    | 23    | 6     | C      | 7    | .002 | 30       |
| Tts 1      | 53    | 70    | 26    | 13    | C      | 77   | 0    | 12       |
| Tts 2      | 1     | 15    | 13    | 1     | /      | 1    | -    | 1        |
| Tts 3      | 21    | 19    | 55    | 1     | /      | 1    | -    | 1        |
| Tts 4      | 54    | 125   | 4     | 19    | C      | 90   | 0    | 20       |
| Tts 5      | 10    | 2     | 8     | 7     | /      | 1    | -    | 1        |
| Tts 6      | 12    | 5     | 14    | 5     | /      | 1    | -    | 1        |
| Tts 7      | 23    | 21    | 64    | 1     | /      | 1    | -    | 1        |
| Tts 8      | 3     | 9     | 5     | 1     | /      | 1    | -    | 1        |
| Tts 9      | 31    | 28    | 14    | 12    | C      | 14   | 0    | 20       |
| Tts 10     | 44    | 97    | 10    | 16    | C      | 132  | 0    | 20       |
| Tts 11     | 4     | 5     | 2     | 1     | /      | 1    | -    | 1        |
| Tts 12     | 4     | 8     | 11    | 1     | /      | 1    | -    | 1        |
| Tts 13     | 2     | 8     | 9     | 1     | /      | 1    | -    | 1        |
| Tts 14     | 56    | 53    | 53    | 1     | C      | 6    | .01  | 20       |
| Tts 15     | 57    | 31    | 77    | 1     | C      | 5    | .01  | 20       |
| Tts 16     | 58    | 102   | 17    | 1     | C      | 10   | .01  | 20       |
| Tts 17     | 57    | 80    | 29    | 1     | C      | 8    | .01  | 20       |
| Tts 18     | 56    | 37    | 228   | 1     | C      | 1    | .25  | 1        |
| Tts 19     | 20    | 20    | 4     | 10    | C      | 14   | 0    | 20       |
| Tts 20     | 40    | 62    | 2     | 32    | C      | 76   | 0    | 20       |
| Tts 21     | 22    | 14    | 12    | 1     | C-     | 2    | .01  | 20       |
| Tts 22     | 45    | 49    | 21    | 1     | C-     | 4    | .007 | 20       |

Table 1: Column 1: name of the time series. Columns 2 to 4: absolute percentage change of the variance $V$, for the three pairs of consecutive equal segments of the corresponding time series respectively. Column 5: the maximum difference of the second order polynomial trend with the average value of the corresponding time series; the value is given as a percentage of the range of values of the time series. Column 6: characteristic shape of the periodogram: “/” indicates a flat power spectrum with periodogram along the 45-degree diagonal characteristic of an uncorrelated stochastic time series, and “C” indicates curved or far from the 45-degree diagonal periodogram representative of a fast decreasing power spectrum. “C+” and “C-” are used to indicate that the periodogram is further or closer to the 45-degree diagonal which are characteristics of colored noise and Brownian time series respectively. Column 7: the correlation time $t'$ defined by $C(t') \sim C(0)/e$. Column 8: the dominant frequency of the power spectrum estimated by the method of minimum entropy using 128 poles; the Nyquist frequency, 0.5, is the maximum possible value and “-” indicates that no dominant frequency is observed. Column 9: saturation time of the STSP; * for the time series Chaos the first maximum is considered the saturation time.
on several aspects concerning the system and the corresponding time series. In some cases $C(t)$ also permits to identify certain nonstationarities associated with intermittency if such intermittency is captured by time correlations of temporal neighbors in the form of striking recurrent revivals while decaying on the average.

3- The power spectrum (PS), is closely related with $C(t)$. The information provided by these two methods may be very similar in particular for long-time regimes \[38\]. In the case of Ots, the long-time regime is rarely achieved and each method may give different information about certain stationary aspects of the underlying system. The standard power spectrum defined by the squared Fourier transform of the time series can also be defined in terms of $C(t)$ and both definitions coincide under mild restrictions given by the Wiener-Khinchin theorem \[1\]. This coincidence permits to relate fast decreasing behavior in a wide but finite frequency range of PS with slowly decaying $C(t)$ which is characteristic of long-range memory. For this decreasing range, Chaotic time series sometimes present exponential behavior while stochastic time series present power law behavior \[38\]. For finite time series this distinction is in general very difficult, in particular for certain colored noise time series because the power spectra is not sensitive to the phase information of the Fourier transform and cannot provide a full characterization of the dynamics. There is not a satisfactory explanation for this behavior of the power spectrum of colored noise \[1\]. The sampling theorem due to Nyquist \[10\] and Shanon \[11\], gives the conditions under which a continuous signal is completely determined by the discrete sampled values constituting the time series. The power of frequencies higher than the Nyquist frequency defined as the reciprocal of twice the sampling time must be zero, otherwise the problem of ”alising” will be present \[12\]. To avoid alising we may use a low-pass filter removing high frequencies or, alternatively, increase the sampling frequency. The sampling frequency may be augmented by means of an interpolation method in the time domain or fast Fourier transform-based interpolation \[11, 12\]. These methods are not strongly recommended for potentially nonlinear time series. Since PS reflects the contribution of all periods from the sampling interval up to the total time covered by the time series and the Ots are commonly highly contaminated, it is normal to have the Nyquist and higher frequencies small but different from zero. This may correspond to a contamination amplitude from different noise sources and not necessary to under-sampling. Random (infinite) time series such as experimental error and numerical round-off, have an absolutely continuous PS \[1\]. On the other hand, colored noise has most of its power concentrated in low frequencies. Nonstationarity is commonly present when a considerable part of the spectrum of a time series is in low frequencies implying that most of the variations of the system have very few oscillations during the observation time, i.e. many relevant times scales of the dynamics are of the order of the total observation time. The low frequencies of the PS can also be affected by low resolution and slow forcing forces. If this behavior is persistent down to zero frequency, the system will present a singularity of the power spectrum. This nonintegrable PS, whose integral is proportional to the average energy of the system, implies a nonstationary system.
because it is obtaining energy from the outside. However, dissipation may provide the mechanism to generate a stationary low dimensional attractor in state space. The high frequencies of the PS can also be affected by white noise, short time effects, fast forcing forces and sub-harmonics. Thus, to improve the stationary dynamical content of a Ots, we should in principle, filter both extremes of the power spectrum reducing all possible contaminations and nonstationarity. It is important to remember that for chaotic time series most of the interesting frequencies are low frequencies around the characteristic cycle times and filtering out frequencies a factor of ten greater and a factor of ten smaller than the frequency corresponding to such cycle will work at least qualitatively well most of the time. Such cycle times are unknown in the case of a Ots and are difficult to identify before improving the stationary deterministic content of the time series.

4- Nonlinear Statistics and complexity measures: The convergence of nonlinear statistics is not very well known apart from its very slow convergence making it very difficult to obtain meaningful results for small Ots. The correlation dimension $D_2$ \[4, 5, 43, 44\], is particularly sensitive to nonstationarity from parameter drift and in particular to insufficient sampling. This sensitivity could be useful to estimate the stationarity of a given time series. However, estimates of $D_2$ are also very sensitive to contamination, accuracy and size of the time series. Estimates of $D_2$ for sections of small Ots are therefore very inaccurate and no useful information for stationarity may be obtained from them. The capacity dimension $D_1$ \[4, 5, 45, 46\], is less sensitive to the limitations of Ots than $D_2$. On the other hand, it is not as good an estimate of the attractor dimension. This is because $D_1$ only captures geometrical information while, $D_2$ captures geometrical as well dynamical information of the attractor. Initially we have used five different nonlinear statistics designed to identify determinism in apparently stochastic time series. We applied them in each case to the whole time series, and to only the first and second half of the data points to obtain a better statistical meaning of the results. The five nonlinear statistics are:

i) capacity dimension, $D_1$,
ii) largest Lyapunov exponent, $Le$ \[47, 48\],
iii) algorithmic complexity measured by the Lempel-Ziv method, $LZ$ \[49, 50\],
iv) and $BDS$ statistics which measures deviation from pure randomness \[51\],
v) Hurst exponent, $He$, which is a measure of self similarity \[52–56\].

5- Recurrence Density Plots (RDP): Recurrence plots were introduced to identify nonstationarity in a time series when the underlying dynamical system is not autonomous (time appears explicitly in the evolution equations), and when the characteristics times of such systems are of the order of the length of the time series \[57\]. Later works have extended the utility and applicability of such plots \[58–60\]. In particular there is strong evidence, although not yet formally proved, that most of the qualitative as well as quantitative results do not depend on the embedding parameters at least for
embedding dimensions smaller or equal to 3 \([59, 71]\). This is a very interesting result because good estimates for the embedding parameters are not easily determined from Ots. We have verified this hypothesis with the time series Chaos with embedding dimensions from 1 to 5, and with the Tts for embedding dimension from 1 to 4 and time delays 1 and 9 sampling times where 9 corresponds to the first minimum of the average mutual information \([1, 22]\). Only small numerical variations have been observed. The density of distances, trend or recurrence density plot RDP \([57, 58]\), is a kind of recurrence plot which measures the number of pairs of points within a threshold distance (\(\sim 10\%\) of the range of the time series), as a function of their temporal separation. The RDP is particularly useful for Ots because their characteristic small number and low accurate data points make it very difficult to observe patterns and varying densities in a traditional two dimensional recurrence plot. For an autonomous system with short characteristic times compared with the length of the time series, RDP must look globally constant. This global behavior may be superimposed to a smaller time scale structure which would correspond to cycles of the system. These cycles must be much smaller than the length of the time series in order to indicate stationarity. For a nonstationary time series RDP would present an overall decreasing behavior.

6- Cross Prediction Error (CPE): The nonlinear cross prediction method was introduced by Schreiber in 1997 \([8]\). This method has a very important conceptual contribution because it is based on the similarity between sections of the time series itself rather than similarity of certain statistics estimates from different sections of the time series. This method may be qualitatively independent of the particular statistics used. This method is particularly useful when the nonstationarity emerges from changes of the shape of the attractor while dynamical invariants remain effectively unchanged \([4, 63]\). This method has the practical advantage of a small demand of data and more robustness to contamination. This is because it compares the data with itself using all of the data points and may use nonlinear predictions which demand rather short segments of data points. The nonlinear cross prediction method generates a plot of nonlinear Cross Prediction Errors (CPE), with two indices indicating the predicted and predictee segments of the time series used in each estimate. In this context the predicted segment is used to construct \(m\)-dimensional vectors and the predictee segment is used to construct the neighborhood for a locally constant approximation \([63]\). The whole time series is divided into equal, consecutive and nonoverlapping segments of length \(l\). \(l\) must be larger than the characteristic times of the process where all the typical cycles of the system are completed several times. For stationary time series and an appropriate length for \(l\), we expect that CPE is independent of the indices. A purely random time series must present large and homogeneous CPE for all combinations of segments. Time series with correlations from deterministic, or any other origin, may present certain patterns in the CPE which depend on \(l\) and indicate the time length of such correlations. This method has the disadvantage of having four parameters and the results of the method may be very sensitive for some ranges of their values when applied to Ots. These four parameters are the size of the segments \(l\), in which the time series is divided, the distance to
determine the neighbors of a given data point, $\epsilon$, and the two reconstruction parameters, the time lag $\tau$, and the embedding dimension $m$. The sparseness and resolution of the data are very important for the first two parameters. The range of CPE evidently depends on the range and resolution of the data points. The compromise between $\epsilon$ and the density of data points is important because it determines the number of neighbors from which the prediction is going to be made. If $\epsilon$ is small and the time series is dense the situation is ideal but, if the data is sparse we may not find close neighbors. If $\epsilon$ and $l$ are large and the data is dense we may be using neighbors from dynamically different parts of the system and confusing results can be obtained. Ots are in general sparse time series therefore, $\epsilon$ cannot be small and $l$ cannot be large and we cannot easily reduce the risk of making predictions with points from dynamically different periods of the time series. If $\epsilon$ is too large the predicitve interval dominates the results of CPE and no information is obtained from the predicted interval. When the studied time series is from a known system these problems can be avoided by considering the characteristic cycles involved.

7- The Space Time Separation Plot (STSP), was introduced by Provenzale et al [3]. It can be used to estimate the correlation time of a scalar time series. This is made by comparing geometrical and temporal correlations giving information about the stationarity of the time series. STSP is robust to most of the limiting characteristics of Ots and has been used to obtain good geometrical estimates such as the correlation dimension. STSP measures the effects of time correlations on the probability of finding two points within a given distance. In practice, STSP estimates the correlation times existent in a time series by measuring the spatial distance to be covered in order to find certain predetermined proportion of points for different time separations [4]. If such spatial distance does not saturate for increasing time separation, all the data points of the time series are temporally correlated and no stationarity can be obtained for any time scale. If saturation is obtained for a certain time separation the time series can be considered stationary for time scales larger than this time separation. Despite the general robustness of STSP with respect to the conditions imposed by Ots, the method is very sensitive to the percentage of points and the size of the spatial partition. These two parameters give the resolution of the plot to resolve the actual behavior from numerical variations. As a scatter plot of the spatial separation versus the time separation between points, STSP must not depend strongly on the embedding dimension $m$ [59, 61]. It was first verified with the results for the time series Chaos where only small numerical variations were observed as $m$ was varied. The cycles of the Lorenz attractor around the two fixed points are seen as certain periodicity of the STSP on the order of $\sim$ 14 sampling times, see figure 5a. For Ots with low sampling rates we can eventually find near returns closer to a given point than its dynamical next neighbor. In this situation the nonstationarity of the time series given by long term correlations cannot be detected by the STSP, i.e. the plots saturate fast and remain constant for any value of time. For Ots with low resolution data points it is still easier to find return points and even in the case of short Ots the STSP may not be able to detect nonstationarity for long correlation times.
We now mention some other important methods that are not going to be used in this work:

i) Comparison of different linear or nonlinear tests for several consecutive and overlapping segments of the time series increases the number of data points but the independence of each segment is reduced [34, 64, 65]. These strategies may not give useful results when applied to OtS with a small number of data points.

ii) The concepts introduced by the nonlinear cross prediction method can be applied in different forms and with different statistics. For example, the identification of neighbors to make the nonlinear prediction can be weighted with different criteria [23, 66]. However, these criteria are not general and may work for particular situations which must be identified in advance depending on the knowledge of the system which is not available for most OtS.

iii) The surrogate data method to evaluate the results of nonlinear analysis of time series has been tested against non-stationarity, in particular for cyclostationarity [67]. Surrogate data is a robust method of general applicability that could be exploited in order to test more directly the stationarity of OtS.

iv) An important method to test stationarity in time series that has not been applied in this work uses the information in the time distribution of points in a state space reconstruction [58]. The need of accurate estimates of the embedding parameters makes this method potentially problematic for OtS. Another related method based on time domain information in reconstructed phase space [69], presents no qualitative dependence on the embedding parameters and seems to work well for small time series. These are two important desirable qualities for a method used to estimate stationarity of OtS. These methods have to be included in further analysis to understand OtS and in particular TtS. However, it is important to measure their sensitivity to contamination, data resolution and other important limitations of OtS.

v) Two novel methods to test stationarity in time series may give relevant results when applied to OtS. They involve wavelets multi-scale ideas which are natural tools to study stationarity [70] and, powerful and well-studied data compression techniques which are robust to noise [71]. Wavelet transforms can also be used to remove nonstationarity from time series that were affected at certain specific time scales or ranges of time scales by some particular circumstance [72]–[74].

5 Results

The results obtained with the test time series give validity to the algorithms used for each method. These results also contribute to define a practical framework for the interpretation of the sensitive results corresponding to TtS and the different TTtS.

Polynomial fit: The value of the linear and quadratic constants in a polynomial fit give a quantitative measure of the trend of the analyzed time series. If these values are large compared to the range of the time series, the linear and quadratic corrections are important compared with a de-
trended behavior. The stationary stochastic time series \textit{Noise} and \textit{Random} present the expected small values and no evident trend is observed. The real world time series \textit{Eeg} and \textit{Ekg} do not present clear trends, only in the second case a small trend is observed. The stationary chaotic time series \textit{Chaos} presents a negligible trend probably caused by the small size of the time series. The nonstationarity of the time series \textit{Pnoise} and \textit{Brownian} is evident with large linear and quadratic constants. No trends are observed for the \textit{TTts}\textsubscript{2,3,7,8,11,12,13,14,15,16,17,18,21 and 22}. Evident trends exist for \textit{Tts} and \textit{TTts}\textsubscript{4,9,10,19 and 20} which were not detrended by any method, only filtered. \textit{TTts}\textsubscript{5 and 6} present less evident trends. In table 1 column 5 we give a quantitative measure of the trends for all of the studied time series consisting on the maximum difference between the average value of the time series with the polynomial fit. This value is given as a percentage of the total range of the data of the corresponding time series. Apart from the time series \textit{Brownian} and \textit{Pnoise} which present very large constants in the quadratic term of the polynomial fit, all other time series studied present almost linear trends when nonnegligeable.

\textbf{Correlation function:} In table 1 column 6, we present the results of the correlation time $t'$ in units of the sampling time, for the seven test time series, the \textit{Tts} and the 21 \textit{TTts}. The continuous deterministic origin of the time series \textit{Chaos} generates a correlation, $t'=5.99$, which is small enough to be consistent with the chaotic nature of the time series. The real world time series \textit{Eeg} and \textit{Ekg} present similar values $t'=5.62$ and 7.12, respectively. The correlation time of the stochastic but temporally correlated time series \textit{Pnoise} is of the same order of magnitude with a slightly larger value, $t'=11.33$. The time series \textit{Brownian} presents a very large correlation time, $t'=169.37$, consistent with the construction of such stochastic time series. The stochastic and uncorrelated time series \textit{Random} and \textit{Noise} present the expected result $t' \sim 1$ and $C(t) \sim 0$ for all $t > 1$. \textit{Tts} presents a large correlation time $t' = 76.58$. The low resolution of the data points of \textit{Tts} may produce apparently long term correlations as measured by $C(t)$. The \textit{TTts}\textsubscript{2,3,5,6,7,8,11,12,13 and 18} have all stochastic uncorrelated-like behavior with $t' \sim 1$ and $C(t) \sim 0$ for all $t > 1$. The treatments to obtain \textit{TTTs} 4 and 10 increase the value of $t'$ obtained for \textit{Tts} and that of \textit{TTTs} 20 leaves it unchanged. The rest of the \textit{TTTs}: 9,14,15,16,17,19,21 and 22 present $t'$ values close to those of the time series \textit{Chaos}, \textit{Pnoise}, \textit{Eeg} and \textit{Ekg}.

\textbf{Spectral analysis:} The five test time series \textit{Chaos}, \textit{Pnoise}, \textit{Brownian}, \textit{Eeg} and \textit{Ekg}, and the \textit{Tts} present very small or zero dominant frequency depending weakly on the number of poles used in the minimum entropy method. The time series \textit{Noise} and \textit{Random} do not present a dominant frequency. More quantitative information is obtained from the cumulative periodogram which is the integral of the power spectrum over frequency. A cumulative periodogram following the 45-degree line, as is the case for the two uncorrelated stochastic time series \textit{Noise} and \textit{Random}, indicates the flatness of the spectrum. The other five test time series present far from 45-degree line periodograms except for the temporally correlated time series \textit{Pnoise} which is clearly closer but not along this line. The periodogram for \textit{Tts} is far from the 45-degree line being an important
qualitatively difference with the time series $P_{\text{noise}}$. The $TTts\;2,3,5,6,7,8,11,12$ and 13 present a flat power spectrum verified by the cumulative periodogram following the 45-degree line characteristic of uncorrelated stochastic time series. The procedures generating the $TTts\;4,10,19$ and 20 do not produce any identifiable difference of the power spectrum compared with that of the $Tts$. The power spectrum of the remaining $TTts\;9,14,15,16,17,21$ and 22 present small changes compared to that of the $Tts$ with the exception of $Tts\;18$ where the periodogram is inverted. In table 1 column 6 we summarize these results qualitatively by using “C” for curved or far from the 45-degree line periodograms and “/” for periodograms along this line. When the periodogram of a $TTts$ gets closer to the 45-degree line than the periodogram of the $Tts$, this may indicate that the corresponding treatment generates some losses of the potential dynamical information contained in $Tts$. The case of $Tts\;18$ presents a strongly damped power spectrum in low frequencies with most of the power concentrated in high frequencies. This increases the possibilities of stationarity but also indicates an increase of contamination probably introduced by numerical round-off and the low accuracy of $Tts$. The dominant frequency remains zero only for the $TTts\;4,9,10,19$ and 20. The $TTts\;14,15,16,17,21$ and 22 have a dominant frequency $\sim 1/50$ of the Nyquist frequency. Except for $TTts\;18$, the remaining $TTts$ present no dominant frequency and a flat PS. The dominant frequencies are presented in table 1 column 8. The linear decay in a log-linear scale of the power spectrum corresponds to exponential decay in linear-linear spectrum frequently associated with a chaotic time series which is clearly observed for the time series $\text{Chaos}$. This characteristic is hard to precise objectively for $Ots$ but certain tendency can be identified. The $TTts\;4,14,17,21$ and 22 present an improved linear decay compared with that of the $Tts$. For the test time series the two extremes are clear, linear decay for the time series $\text{Chaos}$ and a flat PS for the uncorrelated stochastic time series $\text{Noise}$ and $\text{Random}$. For the correlated stochastic time series $P_{\text{noise}}, Tts$ and the real world $\text{Eeg}$ and $\text{Ekg}$ the behavior of PS is between these two well defined behaviors with a certain power law or $1/f$ spectrum. The problem of alising and therefore, the applicability of the sampling theorem, has been apparently solved for all of the studied time series except for $TTts\;18$. As far as stationarity is concerned we are more interested in the low frequency part of the power spectrum where we can observe indications of the problems of low sampling and slow trends.

Nonlinear statistics and complexity measures: As anticipated, the small size of the time series studied in this work do not permit to obtain significant results for stationary analysis using nonlinear statistics on sections of such small time series. Only for the Lempel-Ziv complexity measure ($LZ$), Hurst exponent ($He$), and the BDS statistics ($BDS$), we have obtained certain results that deserve to be mentioned. The seven test time series present the expected results with acceptable accuracy giving some validity to the results obtained for the $Tts$ and the $TTts$. These results are presented in table 2 for the whole time series and the corresponding first and second halves indicated by I and II respectively. The uncorrelated stochastic time series $\text{Noise}$ and $\text{Random}$ have $LZ \sim 1$, 

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$H_e \sim 0$ and $BDS << 0$. The correlated time series $P_{\text{noise}}$ has $LZ \sim 0.8$, $H_e \sim 0.16$ and $BDS \sim -0.5$ which indicates a less complex time series, more self-similar and less stochastic than the two previous time series. The long term correlations of the time series $\text{Brownian}$ reduce the complexity of the otherwise uncorrelated stochastic time series to $LZ \sim 0.2$, increases self-similarity to $H_e \sim 0.5$ and destroys its stochastic or random nature to $BDS \sim 2.0$. The time series $\text{Chaos}$ is much less complex $LZ \sim 0.2$, strongly self-similar $H_e \sim 0.5$ and clearly nonrandom $BDS \sim 0.6 > 0$. The two real world time series $\text{Eeg}$ and $\text{Ekg}$ give similar results with mild complexity $LZ \sim 0.5$, relatively strong self-similarity $H_e < 0.4$ and clearly nonrandom $BDS \gg 0$. The different results for the whole time series and the corresponding two halves are consistent within (large) statistical variations. However, the small number of data points used in all cases, in particular when half time series are considered, do not permit to resolve the sensitivity of these statistics to stationary changes from the statistical variations. The results for $\text{TTts}$ 2, 3, 5, 6, 7, 8, 11, 12, 13 and 18, and less clearly for $\text{TTts}$ 9, 19, 20, 21 and 22, present very close results to those characteristics of stochastic time series. That is $LZ \sim 1$, $H_e \sim 0$ and $BDS \ll 0$ for the three values corresponding to the whole and two halves of each $\text{TTts}$. The $\text{Tts}$ and $\text{TTts}$ 4, 10, 14, 15, 16 and 17 definitively survived the stochastic classification from the nonlinear statistics point of view.

Recurrence Density Plots (RDP): The test time series $\text{Chaos}$, $P_{\text{noise}}$, $\text{Noise}$, $\text{Brownian}$ and $\text{Random}$ present the expected results for the RDP. The stationary time series $\text{Chaos}$ presents constant global behavior superimposed to a fast oscillation of large amplitude and period of 14 sampling times. This oscillation corresponds to the wandering of the trajectory from one symmetric spiral to the other around the two fixed points of the Lorenz attractor. The time series $\text{Noise}$ and $\text{Random}$ present also a constant RDP but no structure of any scale can be identified aside from random and fast variations of small amplitude. The time series $\text{Brownian}$ presents very large RDP for short times with a very fast decreasing amplitude as time increases, as expected from the correlations of such time series. The time series $P_{\text{noise}}$ present a decreasing RDP revealing the temporal correlations introduced in this otherwise stationary stochastic time series. The two other real world time series $\text{Eeg}$ and $\text{Ekg}$ present an overall constant RDP which corresponds to stationarity from the point of view of this analysis. In both cases, although more evident in the case of $\text{Eeg}$, a smaller structure is superimposed without well defined cycles or periods. Some representative RDP of the test time series are presented in figure 1.

The RDP for the $\text{Tts}$ presents very clear nonstationarity with monotonic decreasing behavior superimposed to a small structure with no clear periods. The twenty one $\text{TTts}$ can be divided in three qualitatively similar groups. The first group conformed by $\text{TTts}$ 2, 3, 5, 6, 7, 8, 11, 12 and 13 present a stationary-like RDP with some superimposed structure more evident in some cases than in others. This small structure is particularly evident in $\text{TTts}$ 6 with a relatively well defined period of $\sim 100$ sampling times or monthly measurements, i.e. between 8 to 9 years. From this group of nine apparently stationary time series the short time structure is particularly noise-like for the
Table 2: Nonlinear statistics and complexity measures: Algorithmic complexity of Lempel and Ziv (LZ), Hurst exponent as a measure of self-similarity (He), and the BDS statistic as a discriminator from pure stochastic time series (BDS). The value corresponding to the whole time series is placed in the first column in front of the name of the corresponding time series and under the corresponding nonlinear statistics. The values corresponding to the first and second half of the same time series are placed in the following two columns indicated by I and II respectively, under the corresponding nonlinear statistics.
TTts 7, 8 and 13. The TTts 18 has slow decreasing RDP with a strong noise-like small variations. The second group is given by TTts 4,9,10,19 and 20 which seems clearly nonstationary with minor differences compared to the corresponding RDP of the Tts. The third and most interesting group is given by TTts 14,15,16,17,21 and 22. These six time series present good stationarity from the RDP point of view, in particular for TTts 21. The other five TTts present slightly decreasing trends. All six TTts of the third group present relevant small structure without well defined periods but clearly discriminable from noise-like behavior. Representative RDP of the three groups of TTts and Tts are presented in figure 2.

Nonlinear Cross Prediction Error (CPE): Nonstationarity in a deterministic time series may be identified by overall trends in the CPE, as the predicted and predictee segments correspond to more and more distant parts of the time series. The time series Brownian shows the largest range of values and variability of CPE. For the time series Noise and Random, we get large CPE with a homogeneous random appearance. These two characteristics of CPE are representative of uncorrelated stochastic time series because it is irrelevant which segments are used as predictee and predicted. In the case of the time series Pnoise and Brownian CPE is in general large with clear patterns of valleys and crests indicating the corresponding correlations, which are more remarkable in the second case. For the time series Chaos CPE is in general small with regular patterns indicating permanent correlations due to stationary determinism. Increasing trends indicating parameter drifts or changes in the form of the attractor are not observed in the time series Chaos, as expected. In all cases the results are not qualitatively affected by the values of the embedding dimension from 1 to 6. The real world time series Eeg and Ekg present the deterministic characteristic of patterns with general small CPE and do not present clear evidence of nonstationarity like increasing trends as predicted and predictee segments get further apart. However, for these two time series the results are much more sensitive (than the other test time series), to the parameter $\epsilon$ and the uncorrelated stochastic appearance is easily obtained. For Tts the results are also very sensitive on the parameter $\epsilon$ although, strong indications of the mentioned deterministic characteristics persist. The CPE plot for Tts presents four characteristic regions determined by the two halves of the Tts when we use them as a prediction or to be predicted alternatively. The first half is not good for prediction and similarly bad predictions of it are obtained. The second half is good to make predictions and good predictions (small CPE) of it are also obtained apart from the last two or three segments which present larger CPE. These are consistent results with the fact that the first half of Tts is more contaminated than the second, and the last part of the second half may present strong trends. These results are also consistent with those of nonlinear statistics which give indications of less determinism for the first half of Tts.

For all time series studied with 1356 data points, we have used 20 nonoverlapping sections which corresponds to $l \sim 67$ data points. For the time series Chaos $l \sim 67$ data points is a time length larger than the “coherence time” of the Lorenz attractor, $\sim 14$ sampling times identified by the RDP.
For the rest of the studied time series $l \sim 67$ data points seems to be a good compromise between the sampling time and the length of the time series. Similarly to $\text{Tts}$, the two groups $\text{TTts}$ 4, 9, 10, 16, 17 and 19, and $\text{TTts}$ 14, 15, 18, 20, 21 and 22 present relatively large ranges of CPE with clearly small regions. The second group looks slightly more stationary with less trends and is less sensitive to $\epsilon$. The remaining $\text{TTts}$ 2, 3, 5, 6, 7, 8, 11, 12 and 13 give CPE with homogeneous random variability with no patterns and no predominantly small regions, i.e. uncorrelated stochastic-like behavior. From the point of view of the CPE, several of the treatments applied to $\text{Tts}$ to isolate the deterministic information produces the contrary effect and noise-like time series are obtained as mentioned above. When this was not the case, the apparently nonstationarity of $\text{Tts}$ was in some cases reduced, in particular for $\text{TTts}$ 9, 10, 14 to 22. In figures 3 and 4 we present representative CPE plots for $\text{Tts}$, some test time series and some $\text{TTts}$.

For all time series studied in this work the sensitivity on $\epsilon$ seems to depend strongly on the complexity of the system as well as on the characteristics of the time series. As more deterministic or correlated and contaminated is the time series more sensitive to $\epsilon$ it is and more easily it gives uncorrelated stochastic-like results. However, $\epsilon = 40\%$ of the range of the time series, works in general well and allows as to compare all of the time series on equal footing. The results of CPE were in all cases weakly dependent on small values of $m$, between 1 and 3, when $\tau = 1$ [61].

**Space Time Separation Plot (STSP):** The stationary and deterministic time series $\text{Chaos}$ numerically generated with well known geometrical and dynamical characteristics, using an appropriated sampling time and good resolution of eight digits data points permits to sufficiently sample the attractor avoiding large temporal correlations. For the time series $\text{Chaos}$ all the curves of the STPS corresponding to different percentages, start at zero and then a regular oscillation, without any trend, is observed for increasing time. Since the cycle length of $\sim 14$ sampling times, is much smaller than the observation period, 1356 sampling times, the oscillation is harmless. We are interested in the first time steps where the plots increase consistently revealing the length of the temporal correlations [3, 4]. The time series $\text{Noise}$, $\text{Random}$ and $\text{Pnoise}$ show the expected saturated behavior corresponding to stochastic uncorrelated time series. In these last three cases the STSP saturates very fast and no other relevant behavior apart from numerical oscillations is observed. The only observable difference is for the STSP of the time series $\text{Pnoise}$ with a slightly slower saturation behavior which corresponds to the short time correlations involved in such time series. The STSP for the time series $\text{Brownian}$ present small numerical variations and no saturation at any time is obtained representing the long term correlations of this time series. The results for the time series $\text{Chaos}$ are qualitative invariant under changes of the embedding parameters for $m = 1$ to 7 with $\tau = 1$, for $m = 1$ to 4, for $\tau = 2$ to 4, and a wide range of distances used to define neighbor points.

The STSP analysis of the time series $\text{Eeg}$ and $\text{Ekg}$ does not show clearly the otherwise expected nonstationarity because the sampling time and time span of these time series have been controlled
for this purpose. The corresponding plots saturate relatively fast but slower than the time series **Noise** and **Pnoise** representing certain correlation present in these real world time series. The STSP for the **Tts** shows good stationarity after a correlation on the order of $\sim 10$ to $15$ sampling times, corresponding approximately to one year. The relatively long time between measurements and low resolution of the **Tts** may be important reasons for such short time correlations. The results of the STSP for the **Tts** are also robust with respect to different embedding dimensions and small values of the time delay $\tau$. For $\tau \geq 3$ the results are strongly altered which is expected for these sparse time series because the time scales are importantly changed when $\tau$ becomes large. For all **TTts** the results of STSP are similar to those of **Tts**. In the cases where **TTts** were obtained by treatments which explicitly damp high frequencies or short term correlations as **TTts** 3,7,8,11,12 and 13, STSP presents faster saturation although, those STSP are difficult to differentiate from those corresponding to uncorrelated stochastic time series. In figures 5 and 6 we present some representative STSP and the approximately saturation times of all the studied time series are presented in table 1 column 9. Since we are mostly interested in the qualitative behavior, we chose a set of parameter values around which the results of STSP are robust to parameter variations and represent well the characteristics of the corresponding time series. We can then compare the results of the different time series on similar footing. For figures 5 and 6 we have fixed $\tau = 1$, $m = 1$ and $1/10$ of the maximum number of spatial partitions that each time series supports (given by the range of the data points divided by their resolution), as the spatial partition used to select near neighbors.

Formally speaking, for a strongly stationary time series the results obtained with different methods that use some kind of statistics to estimate stationarity, must not depend on the embedding parameters. This is because in dynamical systems strong stationarity means that in each conceivable embedding space the statistical properties of the phase flows referring to different pieces of the time series are the same [34]. However, in practice and in cases of less stronger but still valuable stationarity, the estimates of such stationarity may depend on the embedding parameters (at least for values beyond certain ranges), among others aspects of the particular system and the time series at hand.

### 6 Summary and Discussion

From inspection of the plot of the data points as a function of time and from all the seven methods applied to estimate stationarity, none of the studied time series presented indications of bursts as evidence of possible intermittency. Of the seven test time series only **Pnoise, Brownian, Eeg** and **Ekg** presented trends measured by a polynomial fit of order two. Indications of nonstationarity given by relatively large $t'$, larger than $10$ sampling times, are evident for the time series **Pnoise** and **Brownian**. The real world test time series **Eeg** and **Ekg** have $t'$ similar to that of the time...
series Chaos which may be representative of certain determinism. Indications of nonstationarity represented by dominant low frequencies persistent to zero frequency in the power spectrum (PS), are observed for the test time series Brownian and less clear for the time series Pnoise, Eeg and Ekg. The dominant low frequency PS of the time series Chaos is not persistent to frequency zero corresponding to a stationary-like nondivergent PS. The time series Noise and Random have the expected flat PS with no dominant frequency and present the stochastic characteristic $t' \sim 1$ and $C(t) \sim 0$ for all $t > 1$. Qualitatively, PS for the time series Eeg and Ekg seems to be between the power law behavior of the time series Pnoise and the exponential behavior of the time series Chaos with a nonzero dominant frequency. Tts present clear evidences of nonstationarity with a marked trend and a large correlation time $t' \sim 76$ sampling times. The power spectrum of the Tts is very similar to that of the time series Pnoise and persistent to zero frequency as another strong indication of nonstationarity. However, the periodogram of the Tts differs importantly from that of the time series Pnoise although, in the direction of the time series Brownian. We recall that these two correlated stochastic time series are very common in nature and can dominate the dynamical information of Ots.

The twenty one TTts selected by their improved weak stationarity compared with Tts, present interesting and varied results. TTts 4, 9, 10, 19, and 20 were not explicitly detrended by any method and present the expected similar nonstationary results to Tts with evident trends and $t' > 10$ sampling times. TTts 14, 15, 16, 17, 21 and 22 have $1 << t' < 10$ indicating better stationarity and deterministic-like correlations. The remaining ten TTts have $t' \sim 1$ and $C(t) \sim 0$ for all $t > 1$ characteristic of uncorrelated stochastic time series. The periodograms of the TTts 2, 3, 5, 6, 7, 8, 11, 12 and 13 are all along the 45-degree diagonal also characteristic of uncorrelated stochastic time series. From the remaining TTts only TTts 4, 10, 19 and 20 present dominant low frequencies persistent to zero frequency which is an indication of nonstationarity. Therefore, the TTts 14, 15, 16, 17, 18, 21 and 22 seem to be deterministic and stationary from a linear point of view given by PS and $C(t)$.

The results of the nonlinear and complexity measures of such small test time series verify but do not demonstrate the corresponding stationary and determinism or stochastic nature of the five test time series Chaos, Noise, Pnoise, Brownian and Random. The real world test time series Eeg and Ekg present indications of determinism and in the first case certain stationarity depending the method used. In the qualitative framework defined by these results of the nonlinear statistics for the test time series, Tts presents indications of determinism, or at least indications of correlated nonstationary time series with important differences between the first and second half. Notably, the first half of Tts is more uncorrelated and the second half is less stationary. The two groups TTts 2, 3, 5, 6, 7, 8, 11, 12, 13 and 18, and TTts 9, 19, 20, 21 and 22, present uncorrelated stochastic-like results which are less evident for the second group. The remaining TTts seem more deterministic but their indications of stationarity vary and depend on the nonlinear statistic used.
No meaningful conclusions of stationarity can be made from the results of nonlinear statistics.

The nonstationarity originated by nonautonomy or by characteristic times of the system of the order of the time span of the time series specially identified by RDP, are not observed for the time series Chaos and the expected overall constant RDP is obtained. Apart from the real world time series Eeg and Ekg, the other four test time series also present the expected results. The stationarity of these two real world time series has been improved by choosing a short enough time span of the time series to reduce nonstationary effects of larger time variations. For Tts we do not have the possibility of adjusting the time span without reducing the already small number of data points. Therefore, the clear nonstationarity observed in its monotonically decreasing RDP cannot be avoided nor even reduced in this way, the time series has to be treated. The apparently stationarity observed in the RDP of the TTts 2, 3, 5, 6, 7, 8, 11, 12, 13 and 18 corresponds to the uncorrelated stochastic-like time series to which Tts was reduced to by the corresponding treatments. In the case of TTts 4, 9, 10, 19 and 20 the nonstationarity of Tts has not been improved from the point of view of the RDP. The remaining TTts 14, 15, 16, 17, 21 and 22 present improved stationarity without becoming uncorrelated stochastic-like time series.

The comparison of the results of CPE for the seven test time series with those of the Tts, permits us to interpret the CPE corresponding to Tts as a possibly deterministic but nonstationary time series in its second half. The first half has strong indications of weakly correlated stochastic time series as a possible consequence of dominating contaminations. The TTts with better stationary deterministic indications from the point of view of the CPE are 14, 15, 18, 20, 21 and 22. In all of these cases the nonstationarity of the Tts has been reduced in 50% to 80% by comparing values of CPE.

From the point of view of the STSP and with the exception of the time series Brownian which does not saturate at any time, the remaining six test time series can all be considered stationary when temporal correlations smaller than 20 sampling times are ruled out. Very small saturation times are characteristic of uncorrelated stochastic time series. STSP identify short term correlations but it does not easily identify long term correlations mostly generated by oversampling and certain contaminations. Thus, the long sampling time and low resolution of the Tts may be some of the reasons why the corresponding STSP does not saturate for longer times. The results of STSP for all TTts are therefore very similar to those of the Tts.

7 Conclusions

The limited capacity of obtaining good estimates of stationarity with the different methods when applied to Tts, has been qualitatively improved by the framework of reference given by the test time series and the TTts. The characteristic limitations imposed by Ots and in particular by Tts, do not permit to obtain quantitative and precise meaningful results from a detailed analysis of such
time series with any particular method. Therefore, the construction of a qualitative criteria with increasing accuracy as more information is acquired, may provide a quantitative measure with a defined accuracy as is the spirit of the statistical perspective. The formal and intuitive arguments relating the problems of contamination and stationarity has been observed in the concrete case of Tts. The intended goal of obtaining a meaningful stationary and deterministic time series from an Ots, in particular Tts, becomes a conflict between the two desirable qualities of the treated time series. This problem becomes evident when we lose deterministic content by applying some treatments to filter noise and improve the stationarity of the Tts. The TTts 2, 3, 5, 6, 7, 8, 11, 12 and 18 are clear cases of this situation as indicated by most of the analysis performed. The most successful results have been obtained by combinations of polynomial detrending with smoothing, filtering with a correlation matrix and in particular with nonlinear local filters. The combination of methods which give the best results correspond to TTts 21, 22, 15, 14, 17 and 16 with decreasing quality as indicated by most of the methods utilized. From these results we corroborate that weak stationarity is not always a clear indication for all the possible origins of nonstationarity considering the strong limitations imposed by the characteristics of Ots and particularly for Tts.

On one hand, the conflictive problem of improving stationarity and reducing contamination in a Ots is closely related with the complexity of the underlying process, its interactions with other processes and the variety of time scales involved. It may not be possible to solve this problem appropriately. On the other hand, such a conflictive situation could strongly depend on the process of observation, in such a case certain treatments of the Ots under study may produce a meaningful deterministic and stationary time series with important dynamical information about the underlying dynamics.

The stationary and deterministic content obtained for some TTts is not completely clear and definitive but, it has been improved compared with Tts. The interpretation of the different TTts, and to which extent they are meaningful improvements of the dynamical information of the original Tts, are not problems that can be solved without a deep understanding of the phenomenology of the system involved. The goal is to reduce the possible explanations from a point of view which manages to capture certain exploitable deterministic characteristics of the Ots under study; the Tts is the illustrative case. This perspective may provide new and complementary information looking toward a progress in the understanding of such complex systems and their corresponding Ots. The use of new and improved already existing methods to measure determinism and stationarity in apparently stochastic time series, will eventually uncover characteristic features of a particular Ots. This work is an initial effort intended to contribute in the definition and limitations of such an ambitious goal as a part of many efforts currently in progress with different perspectives. This new perspective pretends to create a meaningful framework of applicability of important and mature methods to Ots which have been extensively tested in situations where important knowledge of the system and good quality of the corresponding time series are available. In such cases the correct application
and interpretation of results of the different methods are fairly unambiguous although, there are a lot of improvements and new ideas to be developed.

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These results may be an artifact of the reconstruction process and the norm used as indicated by the minimum probability $p$ of finding identical consecutive reconstructed vectors. For $\tau = 1$ the probability $p$ is very small for large $m$ however, it is not small for small $m$ and decreases slowly for increasing but small $m$, $p \geq (m - 1)/m^2$. This work is in progress.
Figure 1: Recurrence Density Plot (RDP), for some test time series. The plot indicates the proportion of points encountered at a given spatial distance as a function of the time between such points. a) Time series Chaos, b) time series Pnoise, c) time series Brownian, and d) time series Ekg. The time series Eeg, Noise and Random present similar RDP to that of the time series Ekg in the sense of an overall constant behavior. However, the two stochastic and stationary time series present smaller and more random-like amplitude oscillations with a faster initial decreasing behavior.
Figure 2: RDP for Tts and some characteristic TTts. The plot indicates the proportion of points encountered at a given spatial distance as a function of the time between such points. a) Tts. b) TTts 6, representative of stochastic-like behavior, i.e. initially small and overall constant density: TTts 2, 3, 5, 6, 7, 8, 11, 12 and 13. In the particular case of TTts 6 certain periodic superimposed structure is observed which is not at all the case for the rest of the TTts of this group. c) TTts 19, representative of clearly nonstationary RDP: TTts 4, 9, 10, 19 and 20. d) TTts 21, representative of nonstochastic stationary RDP, i.e. initially large and decreasing density followed by an overall constant density: TTts 14, 15, 16, 17, 20, 21 and 22. The short but not negligible initial decreasing behavior of RDP, characteristic of nonstochastic time series, is not clearly observed on the RDP since we are mainly interested on the long term overall behavior of RDP needed to assess the stationarity of the time series.
Figure 3: Cross-Prediction Errors, (CPE). The two horizontal axes indicate the index of the segments used to predict and to be predicted. The CPE values are given in arbitrary units proportional to one half of the range of values of the corresponding time series. For clarity, we have fixed CPE to the maximum possible value (one half of the range of the time series) for the point \((0,0)\) and to zero for the rest of the pairs \((predicted, 0)\) and \((0, predictee)\).

a) CPE plot for time series **Chaos**, b) CPE plot for time series **Noise**, c) CPE plot for time series **Brownian**, and d) CPE plot for time series **Pnoise**.
Figure 4: CPE plots for Tts and some representative TTts. For interpretation of the plots see figure 3. (a) CPE plot for Tts. (b) CPE plot for TTts 11 as representative of TTts 4, 9, 10, 16, 17 and 19 which remain similar to Tts 11, 12 and 13. (c) CPE plot for TTts 14 as representative of the uncorrelated and stochastic-like TTts 2, 3, 5, 6, 7, 8, 14, 15, 18, 20, 21 and 22 which become more stationary from the point of view of this method.
Figure 5: STSP for some test time series. The curves indicate the spatial distance obtained as a function of the temporal separation in order to obtain a fixed fraction of pairs of all the possible pairs. The three curves in each plot represent respectively, from bottom to top, 10, 50 and 90% of all possible pairs of points. Each curve was calculated up to 300 or more sampling times to make sure of the overall saturation when present. We show only the first 100 or less sampling times, as indicated by the horizontal axis, in order to clarify the behavior before saturation.

a) Time series Chaos. b) Time series Pnoise which present qualitative equal results to the time series Random and Noise but with a slightly slower saturation. c) Time series Brownian, the only time series which in practice does not present saturation. In practice the saturation time appears before the end of the time series because of the finite resolution and finite spatial partition. This practical fact is certainly an important factor on the saturation behavior of Tts and the TTts. d) Time series Ekg. This STSP is similar to that of time series Ekg but with a slower saturation.
Figure 6: Space Time Separation Plot (STSP), for Tts and some representative TTts. The interpretation of the plot and parameter values are explained in figure 5. a) Tts. b) TTts 5, representative of those TTts with uncorrelated and stochastic-like plots: 2,3,5,6,7,8,11,12,13 and 18. c) TTts 21, representative of those TTts with indications of determinism and stationarity: 9,19,20,21 and 22. d) TTts 4, representative of those TTts with more clear deterministic and stationary results: 4,10,14,15,16 and 17.