Mathematical Modeling of Intrasoil Condensation in Frozen Soils

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Abstract. Mass exchange in soils occurs with the absorption of heat during evaporation and, conversely, with the release of heat during condensation. This article is devoted to mathematical modeling of heat and humidity transfer taking into account the condensation and evaporation of subsoil humidity during seasonal thawing of permafrost. Humidity motion can be described using the transfer equation in the humid or potential (Richards equation) forms in saturated and unsaturated soils. Based on the experimental and field data, we carried out verification of the parameters of condensation and evaporation of pore humidity. The numerical realization of the nonlinear heat and humidity transfer problem is carried out by the method of finite differences using the Newton’s method. The adequacy of the proposed model is shown in the case of seasonal thawing of permafrost. As a result of a numerical experiment in the conditions of Central Yakutia, it has been established that the total humidity content of the soil increases with evaporation and condensation of intra-soil. In the spring-summer period there is an intense process of evaporation of intra-soil and a decrease in temperature, and the summer-autumn period shows an increase in the amount of the fall-out of the condensate of water vapor with an increase in the heat content of the soil. At the end of the summer season, as well as the end of September and the beginning of October, there is a “blocking” of the wet thawed layer. The suggested mathematical model more adequately reflects the process of freezing of the “blocking” zone.
Keywords: heat and humidity regimes of soils, subsoil condensation, permafrost.

Introduction

The process of condensation of water vapor of air is extremely widespread in nature. It occurs in the atmosphere, on open surfaces and in the upper layers of the lithosphere. The process of condensation is an exothermic process (with heat release), which proves the need for it to be taken into account in balance calculations practically in any natural and technogenic conditions. Taking into account this fact, it becomes especially important to consider condensation of water vapor in the areas of development of the southern mountainous cryolithozone, where insignificant violations of the thermal balance cause a drastic change in the permafrost conditions. This in turn leads to the development of dangerous cryogenic geological processes. Problems of condensation and evaporation of ground water were studied by many scientists, conducted various experimental and field studies. As a result, various hypotheses appeared. Some researchers have established that condensation and evaporation proceed very intensively, while others suggest that these processes can be neglected in practical calculations.

This work is devoted to mathematical modeling of these processes during seasonal thawing-freezing of permafrost soils.
Condensation of water vapor in the ground, its role in the water and heat balance was studied by many researchers [1-16], etc. Some of the researchers claim that in the permafrost zone this process proceeds quite intensively, while others suggest neglecting this process.

1. Verifying parameters

By intra-soil condensation, scientific literature refers to the process of transfer of steam within the soil, due to the presence of a temperature drop, in particular, the vapor pressure gradient inside the substance. The main method of measuring condensation is the method of measuring changes in humidity content within a given volume of soil. However, the humidity reserve varies in the latter simultaneously for a variety of reasons and in various forms (both in droplet liquid and in vapor forms) [14]. Experimentally measuring the total change in humidity, it is very difficult to separate the part that is associated with the condensation of vapors. A variety of methods are available for measuring the amount of condensate precipitation. The theoretical description of this process is even more difficult.

In the scientific literature there is practically no mathematical description of the theory of condensate formation. In the theory of heat transfer, a description of the heat transfer is given only for film-type condensation on the surface, and the theory of heat transfer in the case of drop condensation is absent [17-20]. Proceeding from the imperfection of the theoretical development of heat transfer during drop condensation and the absence of a mathematical description of the formation of condensate in porous media, we propose a different approach.

Knowing that the process of condensation is the reverse process of evaporation and that the heat expended on these processes is the same, we assume that in the equilibrium state, in the first approximation to maintain the balance, the evaporation intensity is equivalent to the intensity of condensate precipitation.

Of recent work on the study of the nature and mechanism of evaporation of humidity from the ground, we can mention the work of Korolyov V.A. and Bludushkina L.B. [21], in which we experimentally discovered the linear dependence of the evaporation rate on the average humidity content in various types of soils.

In actual conditions, the condensation process, in addition to the humidity content, also depends on the temperature and humidity potential. The dependence of the condensation of water vapor on temperature and pressure in soils has been fairly well studied [22, 23].

Taking into account the above, the functional dependence of the amount of condensate \( W_k \) on humidity and soil temperature can be described as follows:

\[
W_k = W_k(T, W_a) = \begin{cases} (W_a - W_c)W_{RH}(T/T_{max}), & T > 0, \\ 0, & T \leq 0, \end{cases}
\]

where \( W_a, W_c \) are free and bound humidity of soil; \( W_{RH} \) – empirical parameter; \( T_{max} \) – maximum soil temperature.

2. Mathematical model

Numerical modeling of heat and humidity transport processes in frozen soils has widely used models in the temperature spectrum. The energy equation in the temperature spectrum has the form [24]:

\[
c \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial \tau} \left( \lambda \frac{\partial T}{\partial x} \right) - c_w V \frac{\partial T}{\partial x} + L \frac{\partial W_k}{\partial \tau} + DI_k,
\]
the movement of ground humidity, using the migration model, can be written as:

\[
\frac{\partial W_w}{\partial \tau} = \frac{\partial}{\partial x} \left( k \frac{\partial W_w}{\partial x} \right) - \frac{\partial W}{\partial \tau} + I_k
\]  

(3)

or applying the Richards equation [25] in saturated-unsaturated soils:

\[
\frac{\partial \theta_w}{\partial \tau} = \frac{\partial}{\partial x} \left( k_f \frac{\partial h}{\partial x} \right) - \frac{\partial \theta}{\partial \tau} + I_k
\]  

(4)

The system of equations (2) - (4) is closed by the equilibrium function of the amount of unfrozen water:

\[
W_w = W_w(T) = W_{ww}(T, W)
\]  

(5)

where \(c, c_w\) – volume heat capacity of soil and water, J/(m\(^3\)-K); \(T\) – temperature, K; \(\tau\) – time, s; \(x\) – spatial coordinate (depth), m; \(\lambda\) – thermal conductivity of soil, W/(m-K); \(L\) – bulk heat of the water phase transition, J/m\(^3\); \(W = W_i + W_w\) – total weight humidity, content of weight ice \((W_i)\) and water \((W_w)\); \(D\) – bulk heat of vaporization, J/m\(^3\); \(I_k = \partial W_k / \partial \tau\); \(V\) – filtration rate, m/s; \(k\) – diffusion coefficient, m\(^2\)/s; \(\theta = \theta_i + \theta_w\) – total volumetric humidity content of bulk ice \(\theta_i\) and water \(\theta_w\); \(k_f\) – filtration rate, m/s; \(h = P - z\) – pressure, m; \(P\) – suction pressure of humidity, m.

Equation (3) describes the diffusion process of porous humidity transfer in unsaturated soils, and the Richards equation (4) is the filtration process of humidity in saturated – unsaturated soils. The system of equations (2)-(4) nonlinear and numerical realization is realized by an implicit difference scheme using iteration [24].

3. Solution methods

For the numerical solution of the system of differential equations (2)-(5) in the region \(\Omega = [0, H] \times [0, \tau_m]\) we introduce an uneven grid. On the set of internal nodes \(w_h = \{x_i, i = 1, N-1\}\) we introduce the system of equations is approximated by an implicit difference scheme [24, 26]:

\[
c_{of} T_\tau + D_1 T + c_w C_i T = \varphi_1
\]  

(6)

\[
W_\tau + D_2 W = \varphi_2
\]  

(7)

\[
\frac{\mu^i P_i^{i+1} - P_i^*}{\tau} + \frac{\theta_i^{i+1} - \theta_i^*}{\tau} = D_3 P
\]  

(8)

where

\[
D_1 T = -\left(\lambda^i T_{x i}^{i+1}\right)_x, \quad D_2 W = -\left(k (\mu_i^{i+1} W_x) x\right)_x,
\]
\[
D_i P = \left( k_f (1 - i(T^{e+1})) P^{e+1}_i \right) \\
C_i T = \left( V^+ T_i + V^+ T_r \right) \quad \varphi_i = L i W_r \\\nW = W_i + W_w \quad \theta = \theta_i + \theta_w \\\nc_{ef} = c + L \frac{\partial W_w(T)}{\partial T} + D \frac{\partial W_i}{\partial T} \quad W_w = W_{ww}(T,W) \\
V = V^+ + V^- \quad V^+ = 0.5(V + |V|) \geq 0 \quad V^- = 0.5(V - |V|) \leq 0 \\
i = i(T) = W_i / W, \quad \eta_i = \eta_i(T) = 1 - i(T)
\]

The difference scheme (6) of the temperature problem was obtained as follows. Using the function of unfrozen water \( W_w = W_{ww}(T,W) \) and taking into account the independence of the potentials \( T, W \), multiplier of phase transition heat \( \partial W_i / \partial \tau \) equation (2) can be written in the form

\[
\frac{\partial W_i}{\partial \tau} = \frac{\partial W_i}{\partial T} \frac{\partial T}{\partial \tau} + \frac{\partial W_i}{\partial W} \frac{\partial W}{\partial \tau}
\]  

(9)

Hence, using the partial derivatives of the function \( W_i = i(T)W = W - W_w \)

\[
\frac{\partial W_i}{\partial T} = \frac{\partial(W - W_w)}{\partial T} = - \frac{\partial W_w(T)}{\partial T} = - \frac{\partial W_{ww}(T)}{\partial T}, \quad \frac{\partial W_i}{\partial W} = \frac{\partial i(T)W}{\partial W} = i(T),
\]

we obtain

\[
\frac{\partial W}{\partial \tau} = - \frac{\partial W_w}{\partial T} \frac{\partial T}{\partial \tau} + i(T) \frac{\partial W}{\partial \tau}.
\]  

(10)

The first term of expression (10) is taken into account in the effective heat capacity \( c_{ef} \), the second is on the right side \( \varphi_i \) of the equation (6).

The equation of humidity transfer (4) is approximated by the difference scheme (8), on the left side of which you see the Newton’s method (or a method for localizing the water retention curve \( \mu_i = \partial P / \partial W \)) [27]. Numerical realization of implicit difference schemes is performed by the sweep method taking into account the equilibrium function of the amount of unfrozen water and iteration.

4. Numerical experiment

The numerical experiment was implemented in relation to the natural and climatic conditions of Central Yakutia in the area with mixed meadow vegetation. Taking into account the heterogeneity of soils, from the surface up to 15 cm there is a soil-vegetative layer, from 15 to 50 cm there are sandy loamy rocks with a low coefficient of filtration and below 10 m there are fine and medium-grained sands. The experiment took into account the processes of heat and humidity transfer by atmospheric precipitation, evaporation from the surface, and intra-soil condensation.
Average monthly rainfall, evaporation, air temperature and effective heat transfer coefficient \( \alpha_{ef} \), were taken from meteosites using official data of Rosgidromet (Federal Service of Russia on Hydrometeorology and Monitoring of the Environment) [28, 29] and climate data books [30], as well as from field data [6, 31]. On the surface of the ground we take into account the influence of the external temperature \( T_e(\tau) \) and precipitation \( q_w(\tau) \):

\[
\lambda \frac{\partial T}{\partial x} = \alpha_{ef} (T - T_e(\tau)),
\]

\[
k \frac{\partial W}{\partial x} = q_w(\tau)
\]

At the lower limit (with \( x = H \)) conditions of thermal insulation for temperature and humidity permeability are fulfilled:

\[
\lambda \frac{\partial T}{\partial x} = 0,
\]

\[
k \frac{\partial W}{\partial x} = 0. \quad (k_f \frac{\partial h}{\partial x} = 0)
\]

Initial distributions of temperature and total humidity:

\[
T(x,0) = T_0(x), \quad W(x,0) = W_0(x) , \quad x \in [0, H], \quad \tau > 0.
\]

Thermophysical and mass-exchange characteristics are given according to the data of “Tuymaada” Heat Balance Station, taking into account the functional dependence on temperature, total humidity and ice content for various types of soils.

An analysis of the dynamics of the depth of thawing of the heat and humidity model with and without intra-soil condensation, in comparison with full-scale measurements, is shown in Figure 1.
Figure 1. Depth of thawing by: 1 – full-scale data; 2 – heat and humidity model without taking into account the process of intra-soil condensation; 3 – heat and humidity model taking into account the process of intra-soil condensation.

The results of models describing the course of thawing depth differ (see Figure 1). Heat and humidity model, without taking into account the process of intra-soil condensation, quite clearly shows the course of thawing to a maximum (end of September). However, it describes the reverse freezing process rather roughly, closing the active layer a month earlier. Conversely, the model, taking into account the process of intra-soil condensation, more adequately reflects the process of freezing of the “blocking” zone. It practically coincides with the actual data, very realistically describing the process of thawing and freezing of the active layer. The improved model calculates the averaged data, so the depth of thawing is smoothed, but the thawing and freezing trajectories in depth and in time are the same.

In order to assess the influence of the intra-soil condensation on the soil heat and humidity regime, let’s compare the results of calculations of the dynamics of the temperature and humidity regimes of the soils in time (late May, June, July, August, September and October) with and without intra-soil condensation (Figure 2a, 2b).

The results show the following, the temperature distributions obtained without and taking into account the condensation are quite different. In the first half of the warm period of the year, accounting for intra-soil condensation gives a negative effect, soil cooling is observed to 2 degrees and in the second half of summer, the heating effect is up to 2.5 degrees compared to calculation without taking into account condensation. This is due to an increase in the intensity of evaporation from the ground due to an increase in their humidity in the spring-summer period as a result of cooling of the soil. As the research shows V.A. Korolyov and L.B. Bludushkina [21], evaporation takes place in soils up to a depth of 0.5 m. Accordingly, in the second half of the warm period, the layer of condensate formation lies to a depth of 0.5 m, and in the second half, after changing the sign of the annual heat flow, this layer passes deeper 0.5 m and the intensity of evaporation decreases.
Figure 2a. Dynamics of soil temperature by humidity (a) and potential form (b) in depth, for the warm period of the year: 1 (red) – without condensation; 2 (green) – taking into account condensation.
Figure 2b. Dynamics of soil humidity by humidity (a) and potential form (b) in depth during the warm period of the year: 1 (red) – without condensation; 2 (green) – taking into account condensation.
Other factors of the heating effect of intra-soil condensation in the second half of summer may be the end of the growing season, the weakening of the energy of the radiant solar radiation. If we compare the temperature distribution by the humidity and potential form of calculation, there is practically no difference. The course and nature of the temperature distribution are identical.

The humid field of the ground, as a result of the comparison of the options with and without intra-soil condensation, differs in a wide range over the entire depth, reaching a difference of 0.4 to 2.2%. Near the front of the phase transitions, in the distribution of humidity there is a zone of sharp desiccation, deviating from the normal from 1 to 2.5% in different periods. This is explained by the inclusion of intra-soil condensation as an internal source of humidity, which accumulates in this layer.

However, if we compare the humidity and potential forms of calculation, then there is a strong divergence. The potential form, in comparison with the humidity content, more clearly describes the distribution of soil humidity, taking into account their heterogeneity and is close to field observations.

As a result of modeling, we have established updated values for the distribution of soil temperature and humidity in the active layer, taking into account intra-soil condensation. They show the heating and cooling effect of this factor on the thermal and humidity regime of soils during the warm season of the year, characterized by certain features of heat exchange in the soils of the upper horizons of the layer of annual heat rotation that were not previously taken into account.

The results of the numerical experiment show the need to take into account the condensation of humidity in balance calculations in virtually any natural and technogenic conditions.

Taking into account the foregoing, the particularly important value of the condensation of water vapor in the regions of development of the southern mountainous cryolithozone becomes understandable, where insignificant violations of the thermal balance cause a drastic change in the permafrost conditions.

Conclusions

A mathematical model of heat and humidity transfer is developed taking into account the condensation and evaporation of intra-soil humidity during seasonal thawing of permafrost. The verification of humidity exchange parameters during condensation is carried out, using the experimental data of thermodynamic processes. The adequacy of the proposed model is shown by comparing the dynamics of the processes of seasonal thawing and freezing of permafrost soils.

As a result of a numerical experiment in the conditions of Central Yakutia, it has been established that when the process of condensation of the subsoil humidity proceeds, the total humidity content of the soil increases. In the spring-summer period there is an intensive process of evaporation of intra-soil and a decrease in temperature, and in the summer-autumn period, an increase in the heat content of the soil with an increase in the formation of condensation of water vapor is observed.

At the end of the summer season (the end of September and the beginning of October) there is a “blocking” of the wet thawed layer. The mathematical model, taking into account the process of intra-soil condensation, more adequately reflects the process of freezing of the “blocking” zone.

For a more accurate description of the humidity field, it is necessary to know the mass exchange characteristics taking into account the heterogeneity of the soils.

The condition for intensive process of condensation of water vapor in soils is a large temperature gradient (heat rotation at the atmosphere-soil interface), the presence of moist air and porosity of the rocks. The most favorable conditions for the progress of this process are possible in regions with a sharply continental climate and a large amount of atmospheric precipitation falling in the short-period regime.

Acknowledgments. The financial support for this work was provided by RFBR according to the research projects N 18-41-140008 and N 18-55-53041.

References
