Holographic Space-time, Newton’s Law, and the Dynamics of Black Holes

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Abstract

We revisit the construction of models of quantum gravity in $d$ dimensional Minkowski space in terms of random tensor models, and correct some mistakes in our previous treatment of the subject. We find a large class of models in which the large impact parameter scattering scales with energy and impact parameter like Newton’s law. These same models also have emergent energy, momentum and angular conservation laws, despite being based on time dependent Hamiltonians. Many of the scattering amplitudes have a Feynman diagram like structure: local interaction vertices connected by propagation of free particles (really Sterman-Weinberg jets of particles). However, there are also amplitudes where jets collide to form large meta-stable objects, with all the scaling properties of black holes: energy, entropy and temperature, as well as the characteristic time scale for the decay of perturbations. We generalize the conjecture of Sekino and Susskind, to claim that all of these models are fast scramblers. The rationale for this claim is that the interactions are invariant under fuzzy subgroups of the group of volume preserving diffeomorphisms, so that they are highly non-local on the holographic screen. We review how this formalism resolves the Firewall Paradox.
1 Introduction

Some time ago we showed how models based on the principles of Holographic Space Time (HST) could reproduce a number of features expected from a quantum theory of gravity[1]. None of those models was completely satisfactory, and in addition we’ve discovered a number of errors in our original treatment. In this paper we revisit those questions and find simple derivations of Newton’s law of gravitational attraction, the temperature and fast scrambling properties of black holes and the correct time scale for the decay of quasi-normal modes. We do all of this in a Minkowski space-time with an arbitrary number of dimensions. We still have not found an HST model, which gives rise to a boost invariant S matrix.

In Section I, we briefly review the formalism of HST[2] and construct the matrix/tensor models which implement scattering theory in Minkowski space. In Section II we derive the above properties of matrix model scattering. We also recall the HST description of black holes, and the way in which it resolves the firewall problem. Section III presents our conclusions.

2 HST and the Variables of Quantum Gravity

The Wheeler DeWitt equation shows that any classical theory of gravity is holographic: time evolution in a causal diamond is a gauge transformation so that the evolving variables live on the boundary. As a consequence, the variables on the boundary of a small causal diamond must be a sub-algebra of those on the boundary of a larger diamond, which subsumes it. The full algebra of variables lives on the conformal boundary.

In a theory of quantum gravity in Minkowski space, it’s reasonable to identify the variables with gauge transformations that act non-trivially on the conformal boundary of Minkowski space. These include the commuting BMS[3] generators, whose spectrum, the light cone $P^2 = 0$, may be viewed[4] as the Fourier dual space to the conformal boundary. Other operators can be viewed as generalized functions on this space. They describe the flow of other quantum numbers, in particular helicity and charge, through and on the conformal boundary. We have conjectured that among the correct helicity bearing operators are the AGS[5] supersymmetric BMS generators, which satisfy

\[
\begin{align*}
[Q_{\alpha}(P), \tilde{Q}_{\beta}(P')]_+ &= \gamma^{\mu}_{\alpha\beta} M_{\mu}(P, P') \delta(P \cdot P').

P_{\mu} \gamma^{\mu}_{\alpha\beta} Q_{\beta}(P) &= 0.
\end{align*}
\]

The last equation says that the generators are spinors on the sphere at infinity. The delta function in the first equation says that the momenta are collinear, and $M_{\mu}$ is the smaller of the two.

String theory teaches us that there may be consistent models of quantum gravity with many other kinds of quantum numbers, which can flow through and on the conformal boundary. Many of these are carried only by massive particles, and the spectrum of these particles varies with the parameters (moduli) of the model. We can describe the flow of such massive currents by introducing a copy of the space reflected null cone $\tilde{P}^2 = 0$, $\tilde{P} \cdot P = 1$. Generators depending on both null variables represent currents that can flow along the null direction of the conformal boundary, as well as through it. This preserves a property of finite causal diamonds, whose null boundary can be penetrated by both massive and massless particles.
Scattering theory takes place in a representation of this algebra such that at finite momentum, the generators vanish outside of a finite number of spherical caps of finite opening angle. These are Sterman-Weinberg jets of particles. The zero momentum generators vanish only in small annuli surrounding each jet, as in Figure 1.

Figure 1: Jets are separated from zero momentum cloud by jet isolation annuli, where variables describing cloud vanish. Jets are constrained states of horizon DOF.

The Scattering operator relates the representations of the AGS algebra on the positive and negative energy cones

\[
\int_+ Q_\alpha(P) f^+_{\alpha}(P) = S^\dagger \int_+ Q_\alpha(-P) f^-_{\alpha}(P) S, \tag{3}
\]

where the integrals are over the positive energy part of the null cone. These somewhat fuzzy definitions are made precise by retreating to the fuzzy geometry of finite causal diamonds.

The Covariant Entropy Principle (CEP) of Fischler-Susskind and Bousso states that the Hilbert space associated with a finite causal diamond in Minkowski space is finite dimensional. We proposed that this finite dimensionality be imposed by an angular momentum cutoff of the AGS algebra. Spinors on the \(d - 2\) sphere, with a cutoff on angular momentum may be displayed as

\[
\psi_{[a_1...a_{d-2}]}, \tag{4}
\]

and the algebra becomes

\[
[\psi_{[a_1...a_{d-2}]}, \psi_{[b_1...b_{d-2}]}^\dagger] = \delta^{ab}\delta_{[a_1...a_{d-2}]}, \tag{5}
\]

The anti-symmetrized indices run from 1 to \(N\), with \(N-1/2\) the angular momentum cutoff. The representation of this algebra has entropy proportional to \(N^{d-2}\) for large \(N\). Identifying this entropy with the area of the holographic screen of the diamond, we see a UV/IR correspondence reminiscent of AdS/CFT. The magnitude of the null momenta in the AGS algebra, which seems

\[\text{1The best argument for finite dimensionality appeared in the prescient paper of Jacobson three years before [7].}\]
to have disappeared from these formulae, will appear as an asymptotic emergent quantum number below.

3 The Models

In $d > 4$ the variables we’ve described above resemble those in large $N$ tensor models, which have been used in attempts to model fluctuating higher dimensional geometries\(^2\). In \([11]\) and some subsequent work, we tried to use some results of these models, without properly understanding the differences between them and conventional matrix models. This led to some incorrect statements, as well as confusions about how to reconcile the calculation of Newton’s law with the scrambling properties of black holes, and with the definition of energy in terms of constraints (see below).

Here we resolve all these difficulties. The main part of the Hamiltonian for these models is constructed as a single trace of a polynomial in the bilinear matrices

$$M^j_i = \psi_{i_1 \ldots i_{d-3}} \psi_{i_1 \ldots i_{d-3} j}^\dagger,$$

where spinor indices are summed over and left implicit. The large $N$ scaling of the coupling

$$g_{2p} \text{Tr } M^p,$$

is

$$g_{2p} \sim N^{-1} \times N^{-(d-3)(p-1)}.$$

We’ve separated out a factor of $N^{-1}$, which does not appear in the conventional treatment of large $N$ tensor models\([10]\). This factor is necessary for recovering a number of different features of gravitational physics. With this scaling, all vacuum bubble Feynman diagrams scale like $N^{d-3}$.

We will not attempt to give a systematic treatment of higher order terms in the $1/N$ expansion. For us, $N$ is a proper time or associated distance and we are only trying to reproduce the long time, long distance properties of gravity. To leading order, it is very easy to understand the scaling laws above. We can treat the totally antisymmetric $\psi$ tensors as rectangular $N \times K$ matrices, where $K \sim N^{d-3}$. This trick, which only works to leading order, turns the problem into a conventional matrix model. The leading terms correspond to a subset of planar diagrams, with the following property: In rectangular matrix models, the double lines in diagrams carry different colors and only lines of the same color can be contracted together to make loops. Every planar vacuum diagram has a single bounding closed loop. The subset of diagrams which dominates is the subset for which the bounding loop is a trace of order $N$, while all the internal loops have traces of order $K \sim N^{d-3}$ (see Figure 2). In $d = 4$, all planar diagrams contribute equally, while in higher dimensions only this melonic subset dominates.

There is a useful geometric interpretation of the rules of this class of large $N$ tensor models. Think of each variable $\psi$ as a $d-2$ - cube or sphere. The matrices $M$ glue two such hypercubes together along a common boundary, as in Figures 3 and 4.

We can think of a typical interaction in the Hamiltonian by opening up the trace, and thinking of this as picking a north and south pole on the $d-2$ sphere. The first $\psi$ on the left is

\(^2\)We emphasize that HST does not admit fluctuating space-time geometries. The spinor variables describe fluctuations in the properties of the holographic screen of a fixed causal diamond.
a patch near the north pole. Think of this patch as a fibration of $S^{d-3}$ over an interval in polar angle. This is glued to another ribbon of $S^{d-3}$s and another, for the length of the polynomial. The trace then eliminates the special choice of polar axis and the interaction is in fact invariant under the fuzzy version of the group of area preserving diffeomorphisms\footnote{Of course the word diffeomorphism is misleading. Continuity and smoothness of functions have to do with the behavior of the large angular momentum Fourier coefficients and cannot be assessed in finite dimensional approximations.} on $S^{d-2}$. A geometric
Figure 4: The matrix $M$ obtained by gluing hypercubes together along a common boundary in $d = 4$, the front face has been made transparent to exhibit the gluing procedure.

picture of a monomial interaction is shown in Figure 5.

Figure 5: A monomial interaction, to simplify the picture, we did not attempt to illustrate the invariance under area preserving mappings, which could turn these regular slices into amoeba.

In particular, if we break the indices up into a group $(i)$ whose number is $\ll N$, and the rest $A$, whose number is $o(N)$ then the variables $\psi_{1\ldots d-3A}$ represent the glue that connects a small $d - 2$ cube to the rest of the volume. Setting them equal to zero will be the finite version of the jet state constraint at infinity, which sets the variables in an annulus to zero.
The models we will study therefore have time dependent Hamiltonians of the form

$$H(t) = P_0 + \frac{1}{t} \text{Tr} \ P(\frac{M_n}{t^{(d-3)}}) + H_{\text{out}}(t).$$

We are modeling asymptotically flat space, so the relation between $t$ and the radius of a causal diamond is linear. We impose the CEP by insisting that $t = n$, the range of indices on the $\psi$ variables from which $M_n$ is constructed. The middle term in the Hamiltonian is called $H_{\text{in}}(t)$ and represents dynamics between the time slices $t-1$ and $t$, and $-t$ and $-t+1$. If we restrict the indices on $\psi_{i_1...i_{d-2}}$ to values $\leq n$ we are talking about the variables describing a causal diamond for the proper time interval $[-n,n]$ for some particular time-like geodesic. Note that the Hamiltonian for fixed time is invariant under $U(t)$ transformations on the indices of $\psi$. This is a fuzzy approximation to the group of area preserving transformations on the sphere at null infinity. That group has many $SO(d-1)$ subgroups, but the nesting of smaller $t$ Hilbert spaces inside larger ones picks out a particular subgroup and defines spherical geometry on the screen at infinity. Thus, although we preserve a large group of symmetries, including many $SO(d-1)$ subgroups, in any finite causal diamond, the true $SO(d-1)$ symmetry of the S-matrix, is an emergent asymptotic symmetry. We will see below that the space and time translation symmetries of Minkowski space have a similar emergent character in HST models.

So far, we have not discussed the terms $H_{\text{out}}(t)$ and $P_0$ in the Hamiltonian. $P_0$ describes the evolution of asymptotic jets of particles, and we will discuss it below, after we’ve established that the model indeed has such jets. As for $H_{\text{out}}(t)$, we claim that it is determined by the HST consistency conditions relating the descriptions along different time-like geodesics in Minkowski space. At the moment, we’ve only solved the consistency conditions for trajectories that are all at rest in some fixed Lorentz frame. They are thus related by spatial and temporal translations. We’ll see that the consistency conditions impose asymptotic momentum conservation, and guarantee a Feynman-diagram-like structure of amplitudes, which implements locality of interactions.

The consistency conditions are stated in terms of the principle that causal diamonds of finite area correspond to tensor factors in Hilbert space. Given two time-like trajectories and a choice of proper time intervals around their respective hyperplanes of time symmetry, the causal diamonds $D_{1,2}$ corresponding to those intervals have an overlap (which might be empty) and that overlap region contains a maximal area causal diamond. The principles of HST assign Hilbert spaces $\mathcal{H}_{1,2}$ to each diamond, and another Hilbert space $\mathcal{O}_{12}$ to the maximal diamond in the overlap. $\mathcal{O}_{12}$ should be embedded as a tensor factor in both $\mathcal{H}_1$ and $\mathcal{H}_2$. Given the Hamiltonians and initial conditions in the two quantum systems, and the choice of embedding, each system predicts a density matrix $\rho_{1,2}$ for the tensor factor $\mathcal{O}_{12}$ and the consistency condition states that these two density matrices should have the same eigenvalues, for every choice of pairs of trajectories and intervals.

For homogeneous universes, this infinite set of consistency conditions is simplified, because we take each geodesic in a family that is at relative rest, to have, up to a unitary transformation, the same sequence of time dependent Hamiltonians and the same initial conditions. We will see that this ansatz, together with a Feynman diagram like construction of amplitudes containing multiple localized events, leads to a solution of the constraints, at least when the events are separated by distances much larger than Planck scale.
3.1 Jets

The indices on $\psi_{I_1...I_{d-3}}$ can represent states of fixed angular momentum and Cartan generators of $SO(d-1)$, or any other basis related to this one by unitary transformation. In particular, there is a basis consisting of functions (actually spinor sections) localized around a discrete set of points. The fuzzy delta function on the $d-2$ sphere is

$$\delta_N(\Omega, \Phi) = \sum_L f_L^*(\Omega) f_L(\Phi).$$

(8)

The $f_L$ represent all components of the spherical harmonics, with angular momentum cutoff $N + 1/2$. Think of this as a function of $\Phi$ for fixed $\Omega$. Two of these functions are orthogonal if

$$\delta_N(\Omega_i, \Omega_j) = 0,$$

(9)

if $i \neq j$. Thus, there is a basis in the space of indices, which we can think of as localized at points on the sphere. Note however that the interactions in $H_{in}(t)$ are invariant under changes of basis. This means that a small spherical cap localized around some point, can be changed into an amoeba of equal area, without affecting the Hamiltonian. The interactions are highly non-local on the holographic screens of finite causal diamonds.

The jet constraints we discussed in the previous section can be made precise by the following rules. Divide the index space $I$ into a small group of size $n \ll N$ and a disjoint group of size $N - n$. Denote the small group indices by $i$ and the larger group by $A$. Consider the variables $\psi_{i_1...i_{d-2}}$. They appear in the interactions with the larger group of indices only via two types of terms. The first involves terms in the matrix $M_i^A$ that have the form

$$M_i^{(0)} A = \psi_{i_1...i_{d-3}} \psi_{i_1...i_{d-3}} A.$$

(10)
The superscript (0) appears because the matrix $M^A_i$ has terms where $p$ of the contracted $\psi$ indices come from the large $A$ group. The terms with $p > 0$ do not contain $\psi_{i_1 \ldots i_{d-2}}$. Thus, if we impose the constraints
\[ \psi^{\dagger i_1 \ldots i_{d-3} A} |j e t\rangle = 0, \]
on the initial state at large negative time $|t| \sim N$ then the small block variables do not interact with the rest, in states obeying the constraint. We’ll also insist that the state of the non-vanishing variables is a tensor product of states of the small and big block variables. Given the constraints, and the linearity of quantum mechanics, this is not really a restriction. Any initial state can be expanded in the basis of unentangled tensor product states.

The other type of interaction comes in terms proportional to
\[ M^j_{j_1} M^{j_2}_{j_1} \ldots M^j_{j_k}, \]
because the matrix $M^j_i$ with only small block indices has contributions of the form
\[ \psi^{i A_1 \ldots A_p j_1 \ldots j_{d-3-p}} \psi^{\dagger j A_1 \ldots A_p j_1 \ldots j_{d-3-p}} \]
where $0 \leq p \leq d - 3$. Thus there are products of the $p = 0$ term, which involves $\psi_{i_1 \ldots i_{d-3}}$, with terms containing all other values of $p$. Let’s apply an operator of this type to a state obeying the constraint. The state of the variables in the big block is generic. For $p > 0$ all of the matrices $\psi^{A_p}$ are very long rectangular matrices of quantum operators. The expectation value of $M^{(p)}_{i j}$, which is a product of them, in a generic state will, essentially by the same theorem in linear algebra which gives rise to Page’s theorem[12], be close to the unit matrix, with corrections that are inverse powers of $N$. The order of magnitude of the coefficient of
\[ \delta_i^j \]
in the expectation value of $M^{(p)}_{i j}$ will be $N^{p-1}$. In the Hamiltonian at time $N$, the trace of $M^i$ is multiplied by $N^{-(1+l(d-3))}$, at large $N$. Writing the decomposition
\[ M^j_i = \sum_{p=0} M^{(p)}_{i j}, \]
we see that the effect of evaluating the expectation value of these terms in the Hamiltonian in a generic state of the large block variables is to produce an effective Hamiltonian for the small block, which contains only traces of $M^{(0)}_{i j}$. The leading order terms in this effective Hamiltonian come from terms where we omit all but the $p = 0$ term in the expansion of $M$. Corrections with one additional factor of $M^{(p)}$ are down by a factor of $N^{p-(d+2)}$, and multiple powers of the higher $p$ components of $M$ are suppressed by the corresponding power of this factor. As a consequence, these “interactions” give small corrections to the self interactions of the small block. This argument was carried out in first order perturbation theory, but since the effects are small for large $N$, this approximation is justified and higher order terms will be even more suppressed. There is an “adiabatic switching off of the interactions”.

Now let’s turn to the interactions between small and large blocks, which are mediated by the matrices $M^{(0)}_{i A}$, and their conjugates. We’ll assume the individual $\psi$ variables satisfy canonical Fermion anti-commutation relations, though we believe similar scaling behavior will be found for more general super-algebras. Acting with $M^{(0)}_{i A}$ on a state satisfying the constraint gives
zero, but acting with its conjugate removes the constraint, and if we then act with $M_i^{(0)} A$ we generate interactions that mix up the small and large blocks.

We can understand the scaling of the interaction of two small blocks, mediated by the large block variables $M_A^B$ by thinking about melonic diagrams. A typical melonic diagram describing the first order action of the Hamiltonian gives a scaling $N^{d-2}$, which we have multiplied by $N^{-1}$ in our definition of the asymptotic time dependence of the Hamiltonian. This comes from a diagram like that of Figure ??, where the bounding loop gives a factor of $N$, and each internal loop should be thought of as a $d-3$ sphere, or face of a $d-2$ cube, and gives a contribution proportional to the “area” of this sphere.

![Figure 7: Melonic diagram leading to Newton’s law.](image)

The ’t Hooft/Gurau limiting behavior of the couplings is designed to divide through by all but one of these areas, leaving a total contribution proportional to $N \times N^{d-3}$. Our asymptotic normalization reduces this to $N^{d-3}$. This is the size of the Hamiltonian when acting on a generic state. The effect of our constraints is to replace two of the $d-3$ spheres of radius $\sim N$, with spheres of radii $n_1$ and $n_2$. Thus, the total strength of the interaction is

\[
\frac{n_1^{d-3} n_2^{d-3}}{N^{d-3}}. \tag{14}
\]

This looks like the $d$ dimensional Newtonian interaction, if we define the energy of a block of size $n$, satisfying the constraints decoupling it from the bulk of the variables, to be proportional to $n^{d-3}$. To justify the interpretation of this calculation as a Newtonian interaction, we have to interpret $N$ as the impact parameter in a collision. We will do that below.

Note however that there are many terms in the Hamiltonian which give rise to a long distance interaction of this form. The condition that Newton’s law be attractive is an inequality involving many coefficients in the Hamiltonian. We will see that there are a large number of consistency checks, which support the definition of energy in terms of constraints, which we used above.

First and foremost, if we consider an asymptotic state characterized by $E N$ constraints, then $E$ is a conserved quantum number as $N \to \infty$. There are two features of our Hamiltonian which
guarantee this. First, at any proper time $t$, with causal diamond size $n \sim t$, the Hamiltonian decomposes into $H_{in}(t) + H_{out}(t)$ where $H_{in}(t)$ acts only only $\sim t^{d-2}$ of the fermion variables. The full evolution operator for the system (concentrating always on the tensor factor inside the causal diamond) is

$$U(N, -N) = \prod_{t=-N}^{N} e^{-iH_{in}(t)}, \quad (15)$$

where we impose the time reflection symmetry $H_{in}(t) = H_{in}(-t)$. For most of the time interval, the Hamiltonian $H_{in}(t)$ does not act on most of the $EN$ variables involved in the constraint, whereas for large $t$, $H_{in}(t)$ goes to zero.

Of course, we cannot neglect the action of $H_{out}(t)$ on the system. This is where the HST consistency conditions come into play. We describe Minkowski space by an infinite number of quantum systems, each associated with a timelike trajectory, and a choice of time symmetry point on that trajectory. The systems have identical Hamiltonians, both $H_{in/out}(t)$, reflecting the space and time translation isometries of the space-time. The different systems are related by locating the largest causal diamond in the intersection of the causal diamonds at intervals along the trajectories. This determines the size of a common tensor factor in the Hilbert spaces of the two systems. The initial states, and $H_{out}(t)$, must be chosen such that the density matrices assigned to the common tensor factor by the two systems, have the same eigenvalues. This infinite set of constraints knits the systems together into a quantum coordinate system on space-time and implements the principle of relativity in the HST formalism. We’ll see below that these constraints impose space and time translation symmetry on the S-matrix. In principle, we should also impose these constraints for trajectories with non-zero relative velocity, which would impose Lorentz invariance. We have not yet understood how to do this.

At any given time, any subset of the fermions, whose number is of order $t^{d-2}$ is acted on by the $H_{in}(t)$ of some trajectory. The meaning of this sentence is that for causal diamonds that are not in causal contact, the easiest way to solve the consistency relations is to make a copy of $H_{in}^{(1)}(t)$ and the state of that subset of the fermions, in the Hilbert space acted on by $H_{out}^{(2)}(t)$.

We’ve identified the times along the two trajectories, using the fact that they’re at relative rest, and have the same time symmetry point. We can then use the arguments outlined above to show that the $H_{out}$ evolution cannot remove a number of constraints of order $EN$, so that $E$ is an asymptotically conserved quantum number. Note that for diamonds that are in causal contact, the consistency conditions are solved in a different manner, as we will see below. The HST formalism can accomodate both the picture of independent DOF for causally separated diamonds, AND the area counting of entropy.

In general, given of order $EN + L$ constraints, we can organize the initial state into a set of small blocks of size $n_i \ll N$ with

$$E = \sum n_i^{d-3}. \quad (16)$$

$L$ counts the number of antisymmetric tensors with mixed indices spanning two blocks. The number of blocks and their individual sizes change between initial and final states, but $E$ is conserved. We see the emergence of a picture of scattering of some set of initial jets into some set of final jets. The vanishing components of $\psi$ in initial and final states are a precise definition

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4The actual eigenvalues of the Hamiltonian grow like $N^{d-3}$ in the large $N$ limit, but the 't Hooft-Gurau scaling arguments show that energy differences scale like $o(1/N)$. This fact will be crucial to the understanding of horizon dynamics below. It is one of the things we missed in [11].
of the annuli of vanishing $Q_\alpha(P)$, which we described at null infinity. We’ve made that fuzzy notion of annuli precise, by making the geometry of the holographic screen fuzzy\footnote{Here we’re using a definition\textsuperscript{9} of fuzzy geometry via a cutoff on the spectrum of the Dirac operator on the screen. This has the advantage, when compared to definitions involving cutoffs on the algebra of functions, that it does not require a symplectic structure on the manifold. It can be exactly invariant under symmetries of the screen, and incorporate features of the topology, which are encoded in the zero mode spectrum of the Dirac operator.}. A second indication that our definition of energy is on the right track is that it fits with the emergence of the magnitude $p$ of the lightlike spectrum $P = p(1, \Omega)$, of asymptotic BMS generators in the large $N$ limit. The candidates for $Q_\alpha(P)$ within a fixed spherical cap are the operators $\psi$ in the small blocks. As $N$ goes to infinity, we can take the individual $n_i$ to infinity at fixed ratio, always satisfying the inequality $\sum n_i^{d-3} \ll N$ (so that the number of constraints scales like $N$). We should expect that as $N \to \infty$, we can put an infinite number of angular momentum modes into each cap, but since the caps are dynamically independent in the limit, the rate at which $n_i$ grows is independent. It is natural to expect an emergent symmetry rescaling the $n_i$ in the limit, and this is one of the arguments that we should get a Lorentz invariant S-matrix.

We’ve reserved the most important argument in favor of our definition of energy for last. The definition of energy in terms of a number of constraints automatically gives us a prediction that, in the maximally uncertain ensemble of our system, the probability\footnote{This is the ensemble average of the quantum probability to be in the constrained state.} of being in a state of energy $E$ is $e^{-cEN}$. Together with our identification of $N$ as a size in Planck units this is the parametric equation expected from quantum gravity, if we believe generalized connection between entropy and area advocated by\textsuperscript{22} and\textsuperscript{7}. That is, we’ve defined energy in terms of entropy in such a way that a state chosen from the maximally uncertain ensemble of quantum states, has a quantum probability of satisfying the constraints corresponding to energy $E$, which is (parametrically) the Boltzmann weight corresponding to a temperature $N^{-1}$.

The fact that the maximally uncertain ensemble also has a fixed finite temperature is connected to the fact that the physics inside a causal diamond in Minkowski space can be viewed from the point of view of many different timelike trajectories in the diamond. Jacobson’s argument makes it clear that the maximally accelerated trajectory sees the entire Hilbert space as an almost degenerate thermal ensemble. Trajectories with smaller acceleration see smaller temperatures. The energy we have defined in terms of entropy in such a way that a state chosen from the maximally uncertain ensemble of quantum states, has a quantum probability of satisfying the constraints corresponding to energy $E$, which is (parametrically) the Boltzmann weight corresponding to a temperature $N^{-1}$. The notion of energy and temperature along this trajectory is thus an emergent one.

We first encountered this connection between energy and constraints when pondering the formulae for Schwarzschild black holes in dS space. These have the remarkable property that the total area of cosmological and black hole horizons shrinks as the black hole mass grows. This leads to a maximal size black hole in dS space, the Nariai solution. When the black hole size is small compared to the maximum, the entropy deficit is precisely the Boltzmann factor $-M/T$, where $T$ is the Gibbons-Hawking temperature of dS space. Note that this derivation of the GH temperature of dS space is entirely classical, and independent of the original Gibbons-Hawking\textsuperscript{13} argument from quantum field theory in curved space-time. Like the Euclidean path integral derivation of black hole entropy it is an illustration of the Jacobsonian idea that GR is a hydrodynamic approximation to the quantum theory of gravity. In\textsuperscript{14} we provided a
crude model of dS space that encompassed the full range of allowed black hole masses. We’ll see below that the relation between constraints and energy also lies at the core of black hole physics and the resolution of the firewall paradox.

We are finally ready to define the extra term $P_0$ in $H_{in}(t)$. This is the asymptotic Hamiltonian defining the propagation of jets. A jet state is defined by insisting that asymptotically, at time $-T \to -\infty$ some set of small blocks with $\sum n_i^{d-3} = E$, are decoupled from the rest of the variables. Our argument shows that the final state will also be a collection of such jets, with the same total energy. The $U(t)$ invariance of the time dependent Hamiltonian, allows us to define the basis and the $SO(d-1)$ subgroup so that the jets are localized states in the fuzzy delta function basis.

$P_0$ is defined separately on asymptotic subspaces satisfying different constraints. If there are a number of small blocks of size $n_i$ then

$$P_0 = \sum n_i^{d-3} + \sum \frac{1}{n_i} \Tr P\left(\frac{M(I)}{n_i^{d-3}}\right),$$

(17)

Here $P$ is the same polynomial, with coefficients of order 1 in the large $n_I$ limit, which appears in the main part of the Hamiltonian. However, the matrix $M(I)$ is built only from the fermionic variables in the $I$th small block.

The second term in the Hamiltonian plays two quite distinct roles. When describing an asymptotic jet state it describes the jet substructure, which changes dynamically. The flow of energy and other quantum numbers concentrated in a spherical cap has probability to be concentrated in a non-uniform way inside the cap, and that probability distribution changes with time, according to the dictates of the second term in $P_0$. On the other hand, we will see below that the Hamiltonian $P_0$ of a small block can also describe the horizon dynamics of a black hole, whose radius is $n_i$. Heuristically, we can describe this dual role of $P_0$ by the phrase, “the near horizon degrees of freedom of a black hole are soft modes of gravitons and other massless particles propagating along the horizon”. The HST formalism gives a definite mathematical meaning to this phrase, but it should be used with caution. The real lesson of HST is that the notion of particle is emergent, and the real degrees of freedom on any apparent horizon are fuzzified versions of the currents at null infinity. A fully equilibrated black hole rarely has particle excitations at all, because its degrees of freedom rarely satisfy any constraints. When the black hole gets into a constrained state, a Hawking jet is emitted.

When using the Hamiltonian $P_0$ to describe asymptotic jets, we must take $n_i \to \infty$, always satisfying $\sum n_i^{d-3} \ll N^{d-3} \sim$ in order to have a hope of defining a Lorentz invariant scattering operator. It is not clear to us how rapidly the $n_i$ are allowed to go to infinity. However, it’s clear that this gives us an adiabatic switching off of the dynamics of jet substructure.

### 3.2 The Emergence of Particle Trajectories and the Limits of Effective Field Theory

In order to understand the emergence of particles and particle collisions, we first study a process in which all of the drama takes place inside a relatively small causal diamond. That is, we want to study an S-matrix element for which a particle collision takes place inside a diamond $D_1$, converting some set of incoming jets into some, generally different, set of outgoing jets. We’ll call the size, $n_{D_1}$, of $D_1$ the impact parameter of the collision. Outside of this diamond, only the
inevitable processes of bremstrahlung and other soft particle emission occur. It’s convenient to choose a trajectory at rest in the center of mass system of the collision, whose point of time symmetry coincides with the time at which all participating jets are contained within the smallest possible diamond, and such that $D_1$ is one of the sequence of diamonds that define the trajectory.

Let us begin in the asymptotic past $t = -N$, when the state of the system satisfies $\sum n_i^{d-3}N + L = EN + L$ constraints. We may think of the variables $\psi_{1...n}$ as forming a hypercube. The jet variables form $M_j$ small hypercubes arrayed inside a large hypercube of side $N$. However, this geometrical picture should be taken with a grain of salt. The interaction is invariant under unitary transformations on the variables $\psi_{I_1...I_{d-2}}$ which act in the antisymmetrized tensor product of the fundamental representation of $U(N)$. Its complex conjugate acts on $\psi^\dagger$. This is the fuzzy version of the volume preserving diffeomorphisms of the $d-2$ sphere, considered as a measure space, but without an a priori metric. In the large $N$ limit, the variables become half measures on the sphere and their commutation relations reflect the measurable structure of the space of square integrable functions. There are many copies of $SO(d-1)$ inside the group of volume preserving diffeomorphisms.

The choice of a particular $SO(d-1)$ is determined by our insistence that increasing $N$ adds a particular spinor spherical harmonic to the operator algebra. That is, it is determined by the relation between a sequence of causal diamonds, rather than the dynamics in any given diamond. By applying a volume preserving diffeomorphism, we can turn a small hypercube/hypersphere into a hyperamoeba, and the dynamics within a particular diamond is indifferent to the choice of shape. This is, we believe, the fundamental structure that guarantees the fast scrambling property, which Hayden and Preskill and Sekino and Susskind\cite{16} have argued must be a property of black hole horizons in Minkowski space\cite{14}.

It is thus the constraints we have imposed on initial states, and the adiabatic switching off of the scrambling Hamiltonian, which enable us to associate a particular small block with a spherical cap of fixed angular opening on the sphere at infinity. The variables $\psi_{i_1...i_{d-3}A}$ which are constrained to vanish, are associated with the annulus of vanishing soft energy flow, which we described heuristically above. This association can be preserved, to a certain extent, as we follow the system to past times smaller than $-N$, though the angular localization becomes fuzzier as a consequence of the paucity of spherical harmonics on smaller diamonds.

Recall that we are trying to describe a process where all of the c.m. energy $E$ goes into the diamond $D_1$, without any substantial interaction taking place. The translation of that phrase is that for all $-n$ earlier than $-n_{D1}$ the constrained variables all belong to the Hilbert space of $H_{in}(-n)$ and we are still in a state where the $En_{D1} + L$ constraints are satisfied. We now compute the amplitudes for various final states in the Hilbert space. A conventional particle scattering amplitude corresponds to a final state satisfying of order $En_{D1}$ constraints. There is no reason for $L$ to be conserved. This corresponds to a redistribution of energy among the particles, and perhaps to a change in the number of jets.

To understand the angular localization of different jets participating in this process, we have to refer to the evolution of the large number $E(N - n_{D1})$ of constraints in the remote past. The statement that nothing of interest happens outside of the diamond $D1$ corresponds to the statement that none of the $E(N - n_{D1})$ fermions that are frozen by those constraints

\footnote{It is not a property of black holes in AdS space with radius large compared to the AdS radius. The dynamics of these black holes is controlled by quantum field theory on the sphere, and scrambling is diffusive.}
appears in any of the Hamiltonians $H_m(-n,x)$ for $-N \ll -n$. Here $x$ labels any other timelike geodesic at rest with respect to the one on which $D1$ is centered, and which has no overlap with $D1$. The consistency conditions then show that $H_{\text{out}}(n)$ for the $D1$-piercing geodesic evolves these constrained variables only over time scales $\sim 1/N$ (by ’t Hooft Gurau scaling of energy differences). During the course of a scattering process beginning at $-N$ and ending at $N$, this can change at most $o(1)$ constraints.

The freezing of the constraints on the infinite sphere, and the mapping of the sphere at $-N$ into that at any $-n > -N$ via our choice of $SO(d-1)$ subgroup, gives us an angular localization of the energy flowing into any of those smaller spheres (albeit a fuzzier localization because there are fewer angular momentum modes on smaller diamonds). Following this in through the time development defines jet trajectories in space-time. One might worry that these trajectories are a little too well defined to be identified with the particle trajectories of Feynman diagrams, but we believe the replacement of particles by jets accounts for this difference.

Figure 8: Jets entering and exiting single diamond.

The Hilbert space of a jet with fixed direction and opening angle is infinite dimensional. In Fock space language, which is appropriate for the description of a jet at finite orders in perturbation theory in quantum field theory, it contains superpositions of states with an arbitrarily large number of particles whose momentum sums up to something in the cone of the jet. The direction and opening angle of the jet are thus collective coordinates of a many particle system, and their quantum mechanical probability distribution is expected to be equivalent to that of a decoherent statistical ensemble. HST is constructed in terms of variables that describe jets directly, without the intervention of Fock space or perturbation theory, but this difference raises the question of how one will “reproduce the results of QFT in regimes where they are supposed to be good approximations”. Our answer to this question is likely to irritate you, and interferes with the flow of the discussion, so we will relegate it to an appendix.

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8The lines in Feynman diagrams represent particle propagators, which are not exactly localized along a particular trajectory.
The result of this analysis is that one can assign a picture (Fig. 8) to the kind of amplitude we have just discussed. The picture consists of lines, representing the incoming and outgoing jets, entering into the diamond $D_1$, and undergoing a transformation. When the diamond is large in Planck units, the amplitude of the transition grows like a power as the diamond shrinks. These pictures do not capture the dynamics of the internal substructure of the jets.

Now consider a more complicated process in which the incoming energy at infinity can be divided into two groups of jets, which enter into two diamonds $D_{1,2}$ separated by a large time-like interval or null interval. The description of each diamond, from the point of view of geodesics centered on the individual diamonds, is identical to the single diamond amplitude discussed above. However, we now have a new possibility, namely that one of incoming jets in the later diamond is also an outgoing jet from the early diamond (Fig. 9).

![Jets entering and exiting multiple diamonds.](image)

The HST consistency conditions are solved, for widely separated events, by copying the dynamics of $H_{in}$ along one of the geodesics into $H_{out}$ of the other geodesic. Note that the consistency conditions for the full set of geodesics interpolating between the two interaction events determine the velocity of the jet that travels between the two.

What we have shown is that certain processes in HST are describable by time ordered Feynman-like diagrams, consisting of localized vertices at which the number and energies of the jets change, tied together by propagation between the jets. These results finally allow us to justify our claim that the interaction

$$\sim \frac{(n_1n_2)^{d-3}}{n_{D1}^{d-3}}$$

between two decoupled blocks of sizes $n_{1,2}$ is indeed the Newtonian interaction. Imagine two incoming jets of energies $E_i \sim n_i^{d-3}$, viewed from a geodesic which passes through the position of
the jet with energy $E_1$, at the time of their closest approach. Take that time to be the moment of 
time symmetry of that geodesic. The time of closest approach is a phrase whose mathematical 
meaning is that the constraints corresponding to the jet of energy $E_2$ are not imposed on 
the Hilbert space of this trajectory for any causal diamond smaller than $D_1$. We’re explicitly 
assuming that the two jets scatter from each other, without binding. The strongest interaction 
between the two sets of degrees of freedom is thus given by our “Newtonian” calculation, with 
$N \rightarrow n_{D1}$. There are of course corresponding contributions to scattering for all diamonds larger 
than $D_1$, which are smaller by the appropriate Newtonian scaling.

There are, of course, other interactions between these two groups of DOF, which fall off 
more rapidly with the distance, as well as retarded interactions coming from the time ordered 
Feynman diagrams described above, which have the same long distance fall-off. In a relativistic 
theory, the time ordered diagrams would combine with the static Newton potential to give a 
vanishing scattering amplitude when the two jets have parallel momenta. It’s quite clear that 
for a general choice of the polynomial $P(M)$, this cancelation will not occur.

3.3 Momentum Conservation

Consider two overlapping causal diamonds, $D_{1,2}$ both much larger than the Planck size, $n_{1,2} \gg 1$. Each diamond is associated with some time-like geodesic in Minkowski space. The in-
tersection between them contains a maximal size causal diamond. In the HST formalism, 
each diamond is associated with a Hilbert space, $\mathcal{H}_{1,2}$ on which the Hamiltonians $H_{in}^{(1)}(t_1)$ and 
$H_{in}^{(2)}(t_2)$ act. The diamonds are defined by the intervals $[-t_i, t_i]$, the time symmetry points 
that identify the zeros of the respective times, and the relative velocity between the geodesics.
The sequence of time dependent Hamiltonians along each geodesic as well as the initial state in 
each full Hilbert space (associated with the time interval $[-N, N]$ where $N$ is eventually taken 
to infinity) are identical, reflecting the homogeneity in space and time of Minkowski space.
Only the split between $in$ and $out$ at each time, and the relation between the different time 
parameters, depends on the trajectory.

The maximal diamond in the intersection between $D_1$ and $D_2$ is associated with tensor 
factors $\mathcal{T}_{1,2}$ of equal dimension in $\mathcal{H}_{1,2}$. The basic HST consistency condition is that, for each pair of diamonds, the reduced density matrices on these two tensor factors have the same 
eigenvalues, or equivalently, that they’re related by a unitary transformation. When $n_{1,2} \gg 1$, 
Page’s theorem[12] gives us a lot of information about how to satisfy this condition. Roughly 
speaking, the theorem says that when dim $\mathcal{T}_1 < (\text{dim } \mathcal{H}_1)^{1/2}$, then, for a pure state picked 
randomly with respect to the $SU(e^{n_{dim}^{d-3}})$ invariant measure on $\mathcal{H}_1$, the density matrix on $\mathcal{T}_1$ is

$$\rho = \rho_{max} + o(e^{-n_{dim}^{d-3}}),$$  \hspace{1cm} (19)$$

where $\rho_{max}$ is the maximally uncertain density matrix in $\mathcal{T}_1$. On the other hand, when dim $\mathcal{T}_1 > (\text{dim } \mathcal{H}_1)^{1/2}$, then with probability $1 - o(e^{-n_{dim}^{d-3}})$ the state in $\mathcal{T}_1$ completely determines the full 
pure state in $\mathcal{H}_1$. These two statements are aspects of the same mathematical fact: given a 
pure state, and a random tensor factorization of a large Hilbert space, it is almost always the 
case that the two tensor factors are maximally entangled. The probability that this is not the 
case is an inverse power of the dimension of the Hilbert space, which in HST means that it is 
exponentially small in the area of the diamond.
Physically, this implies that if two diamonds have only a small overlap relative to their areas then the HST constraints tell us nothing. At the point where the entropy of the overlap Hilbert space is half the entropy of one of the spaces, then the larger diamond has enough quantum information to completely read off the state of the full Hilbert space of the smaller diamond. The transition happens exponentially rapidly, as a function of the area of the overlap.

Let’s apply this intuition to a pair of geodesics, at relative rest, and the sequence of diamonds centered around two different, but fixed, space-time events, with space-like separation. As $n_{1,2}$ are taken to infinity, the area of the overlap between the two diamonds, in Planck units, grows. If $r$ is the spacelike distance between the events and we take $n_{1,2} \to \infty$, at fixed ratio, then the difference between the overlap area and the area of either of the individual diamonds, grows as $n_{1}^{d-3} r$, so the fractional difference in entropy goes to zero. Thus, as the proper time is taken to infinity, we can only satisfy the consistency conditions for all initial states if there is a unitary transformation in the Hilbert space of each system, which commutes with the evolution operator for infinite proper time. Similar arguments hold for geodesics at relative rest and sequences of diamonds whose centers are separated by time-like or null intervals. Thus the HST consistency conditions imply the existence of asymptotic space and time translation symmetries of the scattering amplitudes.

We’ve already identified the quantum number associated with asymptotic time translation and argued that it is conserved for the entire class of Hamiltonians discussed in this paper. The conservation of momentum is not tied to characteristics of the Hamiltonian along a single trajectory, but rather to the consistency conditions, which relate the “out” part of the state along one geodesic, to the “in” part along another. Imposing these conditions for a sequence of geodesics separated in space along the directions of motion of incoming and outgoing jets, we see the flow of energy through space, which is of course just the momentum density. Of course, in the HST formalism, all of these concepts are only approximate. Energy itself is only defined as the number of asymptotic constraints, so the concept of a local flow of energy can have no exact meaning. We believe that it is applicable to the subclass of amplitudes described by Feynman-like diagrams, which we have analyzed above.

Many of our readers will be appalled, as we must admit we are ourselves, at the amount of complicated indirect argument we must go through, in order to implement a condition that is an algebraic identity in quantum field theory. We hope that in the future, a more automatic and transparent implementation of space translation invariance will be found in HST. On the other hand, there are many reasons to believe that the situation must be more complicated than simple field theoretic arguments indicate. Primary among these is the fact that in HST the degrees of freedom describing an approximation to the particles QFT, are a tiny subset of the degrees of freedom of HST, in any finite causal diamond. Furthermore, we have claimed that particles themselves have to be replaced by jets, which means that the final “particle” state in any finite causal diamond is complicated and not described by a single momentum. The vast set of non-particle degrees of freedom, which we believe are the proper description of zero energy or soft modes of massless fields, decouple from the particles in the $N \to \infty$ limit, but are crucial to the dynamics in finite diamonds. For example, we’ve seen that they mediate the “static” part of the long distance interaction. The internal dynamics of the jets freezes out only when we take the $n_i$ to infinity limit. It’s our belief that an “elegant” treatment of momentum conservation in quantum gravity can only be found in a formulation like Matrix Theory or AdS/CFT, which are both non-perturbative and non-local. For Minkowski space, this would consist of a set of rules determining the unitary map between the representations of
current algebra on the past and future momentum light cones.

Arguments similar to those given above show that the HST consistency conditions for geodesics with a non-zero relative velocity, imply that the asymptotic Hilbert space carries a unitary representation of the full Lorentz group, which is preserved by the Scattering operator. This puts strong constraints on the form of the time dependent Hamiltonian, which we have not been able to solve. To see that the constraints are non-trivial, we need only consider the large impact parameter scattering, which we described above. The term we have estimated is the one describing the instantaneous Newtonian potential in a fixed physical gauge. It is tied to a particular Lorentz frame and depends only on the energies of the objects being scattered, which could be black holes, or massless particles. The Feynman diagram contributions can also give rise to an interaction of the same order in the impact parameter, from massless particle exchange. The particle exchange contribution depends on spatial momenta, and in field theory it combines with the Newtonian potential to give a Lorentz invariant amplitude, which is quite different for black holes and massless particles. In particular, graviton graviton scattering vanishes when the graviton momenta are parallel. It is quite clear that such cancellations will not occur for generic choices of the coefficients in the Hamiltonian.

Experience with string theory suggests that the constraint of Lorentz invariance is even more severe. We believe that consistent super-Poincare invariant quantum theories of gravity with 16 or more supercharges are very severely constrained, and that we know them all. Less is known about models with 8 or 4 supercharges, but it’s likely that the latter case is even more severely constrained, once we consider things non-perturbatively in the string coupling. Indeed, it is likely that the string coupling is not actually a continuously variable parameter. In HST, the spectrum of exactly stable states in the theory is determined by the current algebra on the null cone. It is likely that one cannot solve the constraints of Lorentz invariance with an arbitrary choice of algebra. Unfortunately, we have no wisdom to impart about how to solve this problem.

4 Black Hole Physics and the Firewall Paradox

Some-times, the final state in a causal diamond into which some jets enter at time \(-n\), will not satisfy any constraints in \(H_{\text{in}}\) at time \(n\). This does not mean that energy has disappeared, because the state in \(H_{\text{out}}\) still satisfies constraints decoupling it from most of the DOF in \(H_{\text{out}}\). Let us assume we are working on a geodesic at rest in the c.m. of the collision. We then have a state of energy \(E\) and vanishing spatial momentum, which is a fairly generic state in \(H_{\text{in}} \otimes H_{\text{out}}\).

The localized state now has energy \(E \sim n^{d-3}\) since all \(n^{d-2}\) of its variables are decoupled from the horizon, and this has happened starting from an initial state that had jets localized in this diamond. The energy \(E\) may be more, but cannot be less, than the sum of the incoming jet energies, because the initial jet energies satisfied the inequality \(\sum E_i < n^{d-3}\). Indeed, once this inequality is saturated, in any causal diamond, then (assuming always that the Hamiltonian is a fast scrambler[16]) the initial state is highly non-generic (has less than maximal entropy) and

\[9\]

If we assume that we are studying the only local process that occurred, given the initial state at time \(-N\) then this state is pure. Otherwise the state in \(H_{\text{in}}\) is likely to be entangled with the states of other localized subsystems, at least because of the Newtonian interaction with them at early times. In fact, exact purity is never achievable, because of the emission of soft radiation by the colliding jets before time \(-n\).
will be equilibrated in the time between $-n$ and $n$. Note that, parametrically, this criterion gives the Penrose criterion for creation of black holes in scattering experiments: a black hole is created with probability one when the impact parameter becomes of order the Schwarzschild radius of the c.m. energy $^{17}$.

Thus, all the models under study have processes in which incoming jets can create an equilibrium state with energy and entropy obeying the relation $S \sim E^{\frac{d-2}{d-3}}$, which, by the laws of thermodynamics implies negative specific heat, and temperature $\sim 1/n$. This temperature can be verified dynamically by noting that as we go into the future, evolution of the generic state will produce a state satisfying $\epsilon n$ constraints with probability $e^{-\epsilon n}$ $^{10}$ That is, the meta-stable state is thermal, with a temperature proportional to $n^{-1}$. Of course we know that the system has states of all energies $\epsilon$ consisting of jets of massless particles, which can propagate away from the black hole. The $SO(d-1)$ invariance of the Hamiltonian guarantees that the emission is isotropic and Fermi’s Golden Rule guarantees that this is indeed a decay of the meta-stable state into jets.

A final property of black holes, that the natural scrambling time scale for disturbances on the horizon is the Schwarzschild radius, is guaranteed by the fact that energy differences in all of our Hamiltonians are of order $1/n$. This was something we got wrong in $^{11}$. The more refined estimate of the dimensionless coefficient $\ln n$ in the scrambling time would follow from an extension of the matrix model conjecture of $^{16}$ to the tensor models introduced in this paper. We believe that the intuitive explanation of why these models are fast scramblers is that they are invariant under the fuzzy remnant of volume preserving diffeomorphisms. There is no sense in which a small set of degrees of freedom is localized on the $d-2$ sphere. $U(n)$ transformations turn a small localized patch into a many armed amoeba, which is in touch with a large region of the sphere.

We believe that the discovery of actual quantum mechanical models, which have a scattering interpretation, and include both amplitudes described by Feynman like diagrams with localized vertices, and meta-stable states $^{11}$ with all of the qualitative properties of black holes, deserves some notice, even if the models do not have Lorentz invariance. The initial and final Hilbert spaces of all of these models do carry representations of the full Poincare group, since they are representations of the covariant AGS algebra satisfying Lorentz covariant constraints. General choices of parameters in the Hamiltonian, and (given string theoretic evidence) general choices for the super-algebra satisfied by the $\psi$ variables, will not lead to a Lorentz covariant scattering operator. This is motivation to work hard on implementing the constraint of Lorentz invariance, rather than a reason to ignore these models.

A possible approach to this problem is to try to formulate a complete set of consistency conditions for $S$ operators, mapping the past and future AGS algebras into each other, in the case of 11 dimensions, where we believe there is a unique answer. This strategy is likely to encounter subtle mathematical issues/divergences in treating the singular point $P = 0$ and the infinite dimensional algebra associated with it. These could be regularized via the formalism studied in this paper, relating the Hamiltonian of the finite system directly to Lorentz covariant quantities. Perhaps in this exotic context, the old S-matrix bootstrap problem can finally be

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$^{10}$This is the statistical probability, to produce a state times the quantum probability that the state satisfies the constraints.

$^{11}$The meta-stable states are like Feynman vertices with much longer extent in time than in space. Alternately, we can say that the vertices of ordinary diagrams describe production and decay of very short lived black holes.
4.1 The Firewall Problem: What Field Theory Gets Wrong

Consider one of the meta-stable excitations of the previous subsection. We will take the liberty of calling it a black hole\(^\text{12}\). Notice throughout our description of black hole production and evaporation we used geodesics in Minkowski space. The space-time picture of the process is that there is a long time-like cylinder, stretching from \(-n\) to something of order \(n^{d-1}\), during which the black hole exists.

There are geodesics which go right through that cylinder and come out the other side and these are just as good as geodesics that stay outside the cylinder. Many workers in quantum gravity are uncomfortable with this idea. They want to see an analysis in terms of geodesics in the background black hole metric, despite the difficulties that the black hole singularity poses for such a description. Our response to this is Jacobsonian\(^\text{13}\): classical GR is hydrodynamics. In the present context it is hydrodynamics of a system whose fundamental quantum formulation is based on the causal structure of Minkowski space. We all know that the causal structure of the black hole is partially incorrect, because the black hole decays. Our contention is that, just as in the perturbative approach to quantum gravity, the Schwarzschild metric is just an approximate resummation of some of the scattering data in Minkowski space.

If we follow a geodesic that enters into the time-like cylinder occupied by the meta-stable state, for large \(n\), we see a collection of jets, which begin to interact with each other, and with heretofore invisible degrees of freedom. They were invisible both because the time scale defined by energy differences in that horizon system is \(1/n\) while the time scale of energies of the jets is \(o(1)\) (as far as powers of \(n\) are concerned), and because the initial state had the crucial \(n\sum n_i^{d-3}\) variables, which mediate interactions between the jet and horizon variables, set equal to zero.

Note that the quantum effective field theory picture of what goes on as the singularity is approached is completely wrong, even though there is a long period when the curvature is still small. QFT would tell us that rapidly varying metrics lead to production of more particles. The HST description says that, by contrast, what is happening is the wiping out of particle identity by turning on the coupling between particles and horizon. Classical GR does get the time-scale for infall to the singularity right, but QFT in the classical background sees uncontrolled particle production, rather than equilibration with the horizon as the outcome near the singularity.

There are two problems with the field theoretic description. First, it assumes that up to some Planck scale cutoff there are particle creation and annihilation operators, including the bosonic creation and annihilation operators of gravitons, of arbitrarily high frequency, and for bosons arbitrarily high occupation number. If we, with \([18]\), impose a cutoff on particle states so that the energy is insufficient to push the black hole horizon further out, we ameliorate this problem somewhat.

The second error of all extant field theory treatments of black holes, is to take insufficient account of the proper treatment of generally covariant field theories in finite regions. The graviton is treated as just another field. In actual fact, if we try to quantize gravity as a field

\(^{12}\)Recalling the old joke: looks like a black hole, smells like a black hole, tastes like a black hole. Good thing we didn’t step in it.

\(^{13}\)Although it must be admitted that Ted Jacobson was completely unconvinced by this argument. His objections can be found recorded in the wikipage of the recent KITP conference on Quantum Gravity from UV to IR. One can read the next few paragraphs as our response to Ted’s worries.

21
theory in a finite causal diamond, we have to deal with modes that are pure gauge everywhere except on the boundary. This has never been done, and the relation between those modes and bulk gravitons never elucidated. While such an approach might have led to progress before the invention of the Covariant Entropy Principle, we do not think it is a complete resolution of the dynamics of black holes, because it still over-counts the degrees of freedom. The boundary gauge transformations include bosonic fields living on the boundary. The first section of this paper elucidated our own take on this problem. The bosonic gauge transformations at infinity are all diagonalized, and other operators (apart from the orbital piece of the Lorentz group) commute with them, and are thus generalized functions on their spectrum. For finite diamonds, the spectrum becomes discrete and the whole operator algebra is finite dimensional.

Our description of black hole production in particle collisions within the HST formalism, shows how the QFT approximation breaks down, and that field theory does not even get the early stages of the breakdown right. The only qualitative feature that is correct is that as one approaches the singularity, a state of a few jets behind the horizon evolves into a state of very high entropy. Of course, the entropy of the initial state in a finite causal diamond was already very high, but most of the degrees of freedom were decoupled from the jets. The approach to the singularity is a signal of the disappearance of the constraints that define localized excitations inside the horizon, rather than of the production of more localized excitations. We’ll have more to say about the geometry inside the horizon below.

To summarize: HST describes black holes, their production and decay, as scattering processes in Minkowski space, viewed from any Minkowski geodesic. The black hole is a time-like cylinder, of finite time-like extent, and varying spatial cross section in its rest frame. The classical black hole geometry approximates some of the coarse grained properties of scattering amplitudes in the presence of the black hole, during the period between its production and disappearance. The quantum field theoretic description of the black hole interior is a poor guide to the actual behavior there.

Now consider the description of a multi-jet system as it falls into a black hole. We can view this from the point of view of a variety of Minkowski geodesics, including the one which goes through the center of the black hole cylinder (call this the central trajectory), and one which remains outside the black hole for all time. Remember that the time-like geodesics on which the Minkowski HST formalism is based, are not defined as trajectories followed by actual physical systems. They’re just the quantum version of a coordinate system on space-time.

One might put a localized detector on the central trajectory, but it would be destroyed and come into equilibrium with the horizon a time of order $n \ln n$ after the black hole forms. The view of physics as seen along the Minkowski trajectory which remains outside the black hole, is somewhat analogous to the view we conventionally attribute to a supported observer in the black hole space-time. Localized physical objects cannot stay on this geodesic, without some means of propulsion, which allows them to accelerate. In the microphysics of HST, this is attributed to the scattering amplitudes between the localized object and the black hole, which at long distance are just given by Newtonian attraction. The elegant summary of these amplitudes in terms of particle trajectories in the Schwarzschild metric, is a hydrodynamic (Jacobson) approximation to the underlying quantum theory, which ignores the internal quantum states of the black hole. Quantum field theory in the black hole background captures the fact that the black hole has a myriad of internal states. This property is reflected in field theoretic calculations via the periodicity of the Euclidean continuation of the geometry. We view this as a consequence of the fact that the Einstein action can be derived from the covariant entropy principle. Indeed the
only calculation of the black hole entropy from field theory is the Gibbons-Hawking calculation in terms of the classical Euclidean action. It is not surprising that we can recover the black hole temperature from a QFT calculation. We could use QFT just outside a star, to calculate the surface temperature of the star in terms of the state of the outgoing photons. All we would need is a hydrodynamic model of the surface of the star and a prescription for coupling the hydrodynamic variables to the photon field. QFT in curved space-time gives us such description of a black hole.

Returning to our problem of jets striking a black hole, the in-falling jets can reach the horizon either before or after the Page time. The only difference between these two situations is whether or not the state of the black hole is maximally entangled with distant radiation. The local description of the interaction of infalling matter is independent of this. This description follows: The interaction takes place in a causal diamond larger than the black hole horizon. That diamond includes a segment of the time-like black hole cylinder. Figure 10 show that segment for the central geodesic, and for one that never enters the black hole.

![Figure 10: Black hole history including accretion and evaporation with two trajectories: one propagating inside the blackhole the other hovering outside.](image)

Each of the diamonds, $D_c$ and $D_o$ is described by a Hilbert space $\mathcal{H}_{c,o}$ and we choose the diamonds to have equal area $n_D^{d-3}$. Each Hilbert space has a tensor factor corresponding to the degrees of freedom of the black hole, and it has another (anti)-commuting subset of degrees of freedom representing the infalling particles. Most importantly, each Hilbert space is subjected to of order $n_D(n^{d-3} + E_{jets})$ constraints, which identify the black hole and jets as localized objects. The fact that they are independent localized objects is expressed by another $nE_{jets}$ constraints, where we assume that $n^{d-3} \gg E_{jets} \equiv n_{jet}^{d-3}$. It is this latter set of constraints which are removed by the interaction between the black hole and the jets, after they strike the horizon. The jet variables are little $d-2$ spheres, which impinge on the large $d-2$ sphere of the black hole. Impinge upon is an effective bulk phrase, for the removal of the constraints.
\[ \psi_{[i_1 \ldots i_{d-3} \ldots]}^\text{hole + jet} = 0 \]
by the action of the Hamiltonian acting on the boundary of the diamond \( D_c \) or \( D_0 \), and the equilibration of the jet variables with the existing black hole variables. This results in the creating of a larger black hole, with entropy approximately

\[ S_{\text{new}} \sim n^{d-2} + nn_{\text{jet}}^{d-3}. \tag{20} \]

Note that this is much larger than the jet entropy, assuming that the black hole is large\(^{14}\).

Given the large \( n \) scaling of the Hamiltonian, the time that it takes to remove the constraints and thermalize is \( o(n) \). Thermalization is completed in a time of order \( n \ln n \) if the Hamiltonian is a fast scrambler. If several jets fall through the horizon at angles and times that allow them to interact on a time scale \( t_f \ll n \), then that interaction will occur without major change compared to jets in empty space. From the point of view of the central trajectory, this means that we can envision a portion of space-time behind the horizon, a causal diamond of size some order one fraction of \( n \), in which physics proceeds as if the horizon was not there. From the point of view of the trajectory outside the black hole, the same physics is interpreted, using the membrane paradigm\(^{21}\), in terms of dissipation of perturbations on the horizon. It is well known that the picture on the membrane presents a mirage of what’s going on behind the black hole horizon. For example\(^{20}\) a pair of charged particles penetrating the horizon, leave behind a dipole field. If the particles collide and annihilate before they hit the singularity, the dipole disappears by shrinking in size, before it dissipates into the featureless sphere of the horizon.

This picture is called a mirage, because one can certainly create states in QFT inside the horizon, where the in-falling particles are hit by high energy particles that propagate inside the horizon, before they annihilate. This would change the actual quantum state inside the horizon, without changing the fields on the horizon. In HST, one does not have this freedom. The states of localized excitations are completely determined by the asymptotic boundary conditions at past infinity. The mirage is real, in order to have consistency with the description along the central trajectory. We also learn that the “space behind the horizon” is constantly being recreated by new localized excitations that fall on the black hole.

We believe that the classical singular geometry behind the horizon is a hydrodynamic approximation to the picture above. That geometry is time dependent. If we follow a particular time-like geodesic, then we always reach a singular regime in a time of order \( n \) after the geodesic crosses the horizon. On the other hand, geodesic motion with a time delay \( > n \), along the same path, always encounters a non-singular region, of the same extent \( n \), before hitting the singularity. In the hydrodynamic approximation, a geodesic is a probe particle, whose horizon crossing does not increase the black hole mass. Nonetheless the classical geometry does exhibit the new space created by those infinitesimal infalls.

How does this analysis differ before and after the Page time? The answer is, “Very little”. The Hamiltonians \( H_{in}(D_{c,o}) \) do not act on the radiation at infinity. If the jets encounter the black hole before the Page time, a good approximation to the state of the black hole plus early radiation is a black hole state independent of the radiation, tensored with a state of a small entropy subsystem of the black hole, maximally entangled with the early radiation. After the Page time, the full black hole state is maximally entangled with the early radiation. The

\(^{14}\)There has been a lot of recent literature\(^{19}\) devoted to attempts to go beyond the classical entropy formula by adding some sort of renormalized entanglement entropy of quantum fields. Those formulae do not give a quantum mechanical account of the large entropy increase when a system originally described by quantum field theory falls behind a horizon.
Hamiltonian describing interaction of the black hole with jets acts only on the black hole factor in these entangled states, and entangles it with the state of the $\sim nr_{jet}^{d-3}$ new degrees of freedom, whose excitation is the key ingredient of formation of the new black hole from the black hole plus jet state. The new black hole has entropy much larger than the old one, and its state is no longer maximally entangled with the early radiation. This is true independently of whether we are before or after the Page time.

We believe it should be obvious to the informed reader that the discussion above resolves the infamous Firewall paradox. QFT in curved space-time gives correct answers if it is used correctly. It affirms that the only low energy field theoretic state that resembles the vacuum in the small region near the boundary of a causal diamond, is the field theory vacuum. This is true for non-gauge field theories, but for gauge fields one must take into account gauge transformations with non-trivial action on the boundary. The covariant entropy principle implies that the space of quantum states associated with those degrees of freedom is finite, and not what one might have guessed from bosonic field theory (even with a short distance cutoff). The Firewall discussion has completely neglected these boundary gauge DOF, except of course for, where they account for the full entropy of the Hilbert space.

- It should be invariant under $SO(d - 2)$ and pick out two special antipodal points on the $d - 2$ sphere. For localized excitations, the transverse extent of the state should be the same as that of the same state in the Hamiltonian for geodesic motion in the same diamond.

- There should be a redshift of the energy, relative to its value in the Hamiltonian describing physics along a geodesic in the same diamond. Since the two trajectories share the same diamond, the two Hamiltonians act in the same Hilbert space.

Our geometric picture of the action of the Hamiltonian suggests that we implement the breaking of $SO(d - 1)$ by inserting an operator into the trace, and the natural operator is $L_{ij}$, the total angular momentum generator in the planes tangent to the antipodal points through which the boost trajectory passes. Thus we replace

$$\text{Tr } P \left( \frac{M}{n^{d-3}} \right) \rightarrow \text{Tr } L_{ij} P \left( \frac{M}{n^{d-3}} \right). \quad (21)$$

We have to be a little more careful in implementing the red shift. The bulk of the degrees of freedom propagate along the boundary of the causal diamond, which is left invariant by the boost. Their energies are not red-shifted. The jet energies $P_0$ are redshifted relative to their values measured along the geodesic. We implement this by $P_0 \rightarrow Z(a)P_0$, where the redshift factor $Z$ decreases with the acceleration, reaching a value of order $1/n$ for maximal acceleration. In the frame of the maximally accelerated trajectory, the time scales associated with particle motion approach those responsible for equilibrating the particles with the horizon degrees of freedom.

Particles viewed from the geodesic interact with a heat bath whose temperature goes to zero with proper time like $1/n$. The constraints on initial states are such that the Hamiltonian cannot equilibrate the jets with the horizon as $n \rightarrow \infty$. The field theoretic Unruh effect is a phenomenon described in the $n \rightarrow \infty$ limit, but taking the redshift factor $Z$ to scale like $1/n$, so that the temperature experienced along the accelerated trajectory remains finite.
5 Conclusions

In this paper we have described a series of quantum mechanical models using large $N$ tensor model technology. These models have variables which can be thought of as describing building blocks of a $d-2$ dimensional holographic screen. In fact they are sections of the spinor bundle over the screen, with an eigenvalue cut-off on the Dirac operator. In this paper we studied only screens of the form $S^{d-2}(n) \times K$, where $K$ is a compact manifold with fixed finite volume of order one in Planck units. $n$ is the radius of the sphere in Planck units. In principle, the super-algebra of the fundamental variables depends on $K$. We’ve taken the variables to be canonical fermions, but our arguments would go through for any super-algebra with a finite dimensional unitary representation.

If $M$ are the $n \times n$ matrix variables described in the text and

$$h(n) = \text{tr} P(M/n^{d-3}),$$

(22)

with $P$ a finite order polynomial, whose coefficients have finite large $n$ limits, then the ’t Hooft-Gurau scaling of energies is that typical states have energies $n^{d-2}$ and typical energy differences are of order 1. The dominant large $n$ Feynman diagrams are melonic planar diagrams, in which a single index loop of order $n$ surrounds a collection of “seeds” which have traces of order $n^{d-3}$.

HST uses this tensor model technology in a way which is entirely peculiar. It describes Minkowski space-time in terms of a time dependent Hamiltonian for propagation in the proper time of a fixed time-like geodesic. That Hamiltonian is

$$H(t) = P_0 + \frac{1}{t} h(t) + H_{\text{out}}(t).$$

(23)

The full Hamiltonian acts on a Hilbert space whose entropy is of order $N^{d-2}$ ($N$ is introduced as a regulator, and is taken to infinity to define the $S$ matrix). $h(t)$ acts on a tensor factor of entropy $t^{d-2}$, which is defined by restricting the large $n$ tensor indices on our spinor variables to take on only $t$ values. $H_{\text{out}}(t)$ acts on the complementary tensor factor.

We’ve demonstrated that, starting from states in which of order $EN$ of the spinor variables are set equal to zero, our models describe a scattering theory in which $E$ is a conserved quantity. The asymptotic states have an interpretation in terms of jets of massless particles, and by following the evolution of the constraints we’ve given a Feynman diagram like picture of many of the amplitudes in the theory. The vertices in the diagrams are causal diamonds for which both the initial and final states satisfy jet constraints, and the propagation between vertices is controlled by the free asymptotic Hamiltonian $P_0$.

When the jet energy entering into a diamond of size $n$ saturates the inequality $E < n^{d-3}$ the final state satisfies no constraints, and becomes a long lived meta-stable excitation. It will emit a jet of energy $\epsilon$ with Boltzmann probability $e^{-\epsilon n}$ and is thus a thermal system with temperature $\sim n^{-1}$. If the equilibrium is perturbed, the time scale for scrambling is of order $n \ln n$, assuming that all of our models are fast scramblers. All of our models have both amplitudes which resemble those of effective field theory (unitary, local interactions of particles/jets) as well a amplitudes for forming meta-stable excitations with all of the scaling properties of black holes. All of our models have large impact parameter scattering dominated by a long range static interaction of Newtonian form, as well as exchange of massless jets. In general, these will not be related in the way guaranteed by Lorentz invariance. We know from string theory
that Lorentz invariance is a very strong constraint in quantum gravity. In the HST context it is likely to constrain both the detailed form of the Hamiltonian, and the commutation relations of the fundamental variables.

6 Appendix on the Answer to a Question Raised in the Text

How does HST reproduce the results of QFT in regimes where they are supposed to be good approximations?

We regret to say that we think this question should be turned around. If a version of HST is found, which produces Lorentz invariant probability amplitudes mapping a space of asymptotic currents into each other, and if the currents can describe flow of helicity $\pm 2$ through a spherical cap at null infinity, then we have a manifestly finite, unitary, local, candidate for a theory of quantum gravity. It is then obvious that, to the extent that one can approximate those amplitudes by amplitudes in Fock space they will reduce at low energy to the amplitudes of the quantized Einstein-Hilbert action. It is no longer clear to us that one should expect to be able to find an all orders low energy expansion of amplitudes in terms of low energy effective field theory.

A very short time ago, we would have argued that the effective field theory expansion had to be valid to all orders in gravity, because all the corrections to Einstein’s Lagrangian all came from high energy short distance degrees of freedom. We began to understand the flaw in that argument in [23] and it was really brought home to us by our struggles with the Firewall Paradox. The only rigorous argument for the effective field theory expansion is the ancient argument of Weinberg[24], which however assumes that the exact answer is an S matrix for graviton scattering in Fock space. We now believe that the true scattering operator exists only in the space of asymptotic currents $Q^\alpha_i(P)$, which we have described in this paper. The currents are operator values (half) measures on the null cone of asymptotic momenta and its spatial reflection. Loosely speaking, Fock space amplitudes are amplitudes where the opening angles of jets are taken to zero. In four dimensions, this limit does not exist, even in perturbation theory, and we see no reason for it to exist in any dimension, in the exact theory.

In regimes of moduli space where string theory has dimensionless expansion parameters, there is no obvious contradiction in assuming that perturbation theory produces an asymptotic expansion of some exact amplitude, for the S-matrix of stable particles, in dimensions high enough that those amplitudes have no IR divergences.\footnote{On the other hand, one has no proof that this is the case. The only examples where these expansions have no UV divergences, are in weakly coupled string theory, and in that regime we do not have a prescription for treating amplitudes involving production and decay of most of the weakly coupled string modes, which occurs in finite orders of perturbation theory. In addition, we have many examples of weakly coupled theories in four dimensions, where it’s been argued that perturbation theory is sensible to all orders, but where we expect the dilaton to be frozen at discrete points by non-perturbative effects. It’s rather unclear how to differentiate the behavior of the perturbation series away from those discrete points, where we don’t expect it to be asymptotic to anything, from its behavior at those points. Even estimates of its large order behavior do not pick out discrete points. The auxiliary field formulation of large $N$ vector models gives an example of how subtle things can be. The auxiliary field path integral defines correlation functions of invariant operators as locally analytic functions of $N$. The simple way to see that these only make sense for integer $N$ is to compute the multiplicities associated with singularities corresponding to non-singlets, and see that they are not integers. In the $1/N$ expansion one} Even in such regimes, it’s clear that
the domain of validity of the perturbation series depends on kinematics in a highly non-uniform way. The series fails badly whenever black holes can be produced, and this can happen both in high energy collisions of a small number of particles\[17\], collisions of high multiplicities of low energy particles, and many regimes intermediate between the two. In many cases one cannot tell what will happen without a detailed solution of the theory. We’re thinking of several bodies which collide and form a bound orbit with large radius, losing their positive energy to gravitational radiation, or the escape of one of the bodies. The bound system continues to radiate, and the question of whether it eventually collapses to a black hole is extremely complicated and depends on an enormous number of details.

In a generic regime of moduli space, the only expansion available is the expansion in kinematic invariants divided by the Planck scale. General arguments in field theory show that any perturbative expansion is divergent, and the behavior of weakly coupled string perturbation theory suggests that it is not Borel summable. This implies essential singularities at vanishing kinematic invariants, which have no conventional interpretation\[25\] in analytic S matrix terms. The only other possibility is that the uncalculable counterterms in the UV divergent power series conspire to make the series converge better than field theory or string theory would lead us to expect.

Our own guess about the resolution of this problem is that the perturbative field and string theoretic calculations of quantum gravity amplitudes, are calculating things that do not actually exist non-perturbatively. The zero momentum singularities in ordinary S matrix elements, which are implied by divergence of the perturbation series are, in our opinion, the signal that the non-perturbative Hilbert space is not Fock space. We do not know how to argue that an all orders effective field theory expansion of these non-Fock amplitudes exists. It may be that the domain of validity of effective field theory can be widened by calculating only inclusive cross sections, for which the detailed quantum state of infinite numbers of soft gravitons is ignored.

We should also point out that the AdS/CFT correspondence avoids this problem, by putting the system in a box. The arguments that one can obtain correct answers by taking a limit of CFT correlators assumes that the limiting answer is an S matrix in Fock space. If it is not, it seems unlikely that correct amplitudes for currents can be extracted from CFT correlators.

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