Dust-Acoustic Rogue Waves in Opposite Polarity Dusty Plasma Featuring Nonextensive Statistics

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Abstract—Modulational instability of dust-acoustic waves, which propagate in an opposite polarity dusty plasma system containing inertial warm negatively and positively charged massive dust grains as well as nonextensive \(q\)-distributed electrons and ions has been theoretically investigated. The nonlinear Schrödinger equation is derived by employing the reductive perturbation method. The nonlinear Schrödinger equation predicts the conditions of the modulational instability of dust-acoustic waves and the formation of dust-acoustic rogue waves in a nonlinear and dispersive plasma medium. It is observed that the basic features of the dust-acoustic rogue waves (viz., amplitude and width) are significantly modified by the various plasma parameters such as nonextensivity of electrons and ions, electron number density, electron temperature, ion number density, ion temperature, the mass and number density of the dust grains, etc. The application of the results in space and laboratory opposite polarity dusty plasma is briefly discussed.

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INTRODUCTION

A low-temperature plasma including dust grains with sizes ranging from 1 to \(10^3\) \(\mu\)m is usually referred to as dusty or complex plasma [1]. In recent years, dusty plasma (DP) has received enormous attention due to their crucial role in supporting electrostatic density perturbations and potential structures that are observed in planetary rings [1–4], cometary tails [1], early universe, supernova, interstellar cosmic matter, dense molecular clouds [2], Martian atmosphere [1], Earth’s mesosphere [2], and laboratory experiments [5, 6], etc. In addition, DP has recently enabled to achieve the importance in industries due to the contaminants formed during the plasma processing (deposition and etching) of thin films [1]. The opposite polarity (OP) dust grains co-exist in both space [1–3] and laboratory plasma [5, 6], which introduce a new DP model known as OP DP (OPDP). The main ingredients of OPDP are OP dust grains as well as electrons and ions [1–3]. Depending on the plasma environments, the significant or insignificant number of free electrons and ions, which are left over after their absorption by negative and positive dust grains, are always found in OPDP medium [1–3]. Dust grains are normally regarded as negatively charged because they collect electrons from the background plasma [1–3]. However, the existence of positively charged massive dust grains has also been noticed in different regions of space (viz., cometary tails [2], Jupiter’s magnetosphere [7], Earth’s polar mesosphere [8], Martian atmosphere [9], and some plasma experiments [10, 11], etc.). Three principal mechanisms, i.e., photo-emission under ultraviolet (UV) photons, thermionic emission induced by radiative heating, and secondary emission of electrons from the surface of the dust grains are observed by which a dust grains acquire positive charge [1, 12].

Particles from space and laboratory plasma don’t always follow the equilibrium Maxwellian distribution because the Maxwellian distribution is inappreciable to describe systems in a nonequilibrium state with long range interactions. In space and astrophysical environments when the plasma particles move very fast compared to their thermal velocities than these particles are supposed to follow the non-Maxwellian distribution such as nonextensive \(q\)-distribution [13–16]. Nonextensive generalization, which has achieved enormous attention during last few decades, of the Boltzmann–Gibbs–Shannon entropy was first presented by Renyi [13] and subsequently proposed in [14]. The nonextensive \(q\)-distribution is suitable for the statistical description of long-range interacting plasma system [15, 16]. Nonextensive plasma has been a fascinating research topic due to its relevance in cosmological and astrophysical scenarios (viz., stellar
polytropes, Earth’s bow-shock, magnetospheres of Jupiter and Saturn [17–20] etc. as well as laboratory applications like nano-materials, micro-devices, and micro-structures [21], etc.

Moreover, an unexpected, rare, and mysterious collective behavior which is known as rogue waves (RWs) has been observed in many plasma systems [15]. RWs are short lived but high energy event which appear suddenly and increase up to a very high amplitude, i.e., two, three, or even more times the height of the surrounding waves and finally disappear without any trace. It was first observed in ocean [22] and also found later in optical systems [15], fiber optics [24], Bose–Einstein condensates [25], superfluid helium [26], optical cavities [27], atmospheric physics, plasma physics [15], stock market crashes [28], and even in biology. However, a possible mechanism to understand the RWs is the rational solution of nonlinear Schrödinger equation (NLSE). In [15], ion-acoustic RWs (IARWs) in a four components plasma medium having nonextensive plasma species were investigated, and the amplitude and width of the IARWs was found to be independent on the sign of the q but dependent to the magnitude of q. Authors of [23] studied dust-acoustic (DA) RWs (DARWs) in presence of q-distributed electrons and ions in a three components DP medium.

Recently, in [2], nonlinear DA waves (DAWs) in a four components inhomogeneous OPDP medium were investigated. Authors of [16] studied modulational instability (MI) of low-frequency electrostatic DAWs propagating in a three components DP medium containing inertial negatively charged dust grains as well as q-distributed electrons and ions. To extend work [16], we consider four component OPDP medium containing inertial warm negatively and positively charged massive dust grains as well as q-distributed electrons and ions, and will examine the MI of the DAWs and the formation of associated DARWs propagating in a dispersive and nonlinear OPDP medium.

**MODEL EQUATIONS**

A collisionless and fully ionized four-component unmagnetized plasma system which consists of inertial warm negatively and positively charged massive dust grains as well as inertialless q-distributed electrons and ions is considered. The quasi-neutrality condition (at equilibrium) can be expressed as $Z_n n_0 + n_0 = Z_n n_0 + n_0$; where $Z_n, n_0, n_0, Z_n, n_0$, and $n_0$ are the number densities of the electrons living on a warm negatively charged dust grains, negatively charged dust grains, $q$-distributed electrons, protons living on a warm positively charged dust grains, positively charged dust grains, and $q$-distributed ions, respectively. The normalized governing equations of the DAWs can be represented by

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (n u_x) = 0, \quad (1)
\]

\[
\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + \lambda n \frac{\partial n}{\partial x} = -\frac{\partial \varphi}{\partial x}, \quad (2)
\]

\[
\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x} (n_+ u_x) = 0, \quad (3)
\]

\[
\frac{\partial u_+}{\partial t} + u_+ \frac{\partial u_+}{\partial x} + \theta n_+ \frac{\partial n_+}{\partial x} = -\alpha \frac{\partial \varphi}{\partial x}, \quad (4)
\]

Here, $n_+$ ($n_+$) is normalized by its equilibrium value $n_0$ ($n_+$); $u_+$($u_+$) is the negative (positive) charged dust fluid speed normalized by $C_- = (Z_ B k_B T_i/m_+)^{1/2}$; where $k_B$ is the Boltzmann constant, $T_i$ is the temperature of $q$-distributed ions, and $m_+$ ($m_+$) is the mass of the negatively (positively) charged warm dust grains; $\varphi$ is the electrostatic wave potential normalized by $k_B T_i/e$ (where $e$ is the magnitude of an electron charge); the time and space variables are normalized by $\omega_{pe}^{-1} = (m_+/4\pi e^2 Z e n_0)^{1/2}$ and $\lambda_{pe}^{-1} = (k_B T_i/4\pi e^2 Z e n_0)^{1/2}$; $\lambda = 3 T_i/Z m_+$ and $\theta = 3 T_i m_+ / Z T m_+$, where $T_i$ ($T_i$) is the temperature of negatively (positively) charged warm dust grains; $\alpha = Z_+ m_+ / Z m_+$, $\beta = n_0 / Z n_0$, and $\gamma = Z n_+ / Z n_0$. It may be noted here that we have considered for our numerical analysis $m_+ > m_+$, $n_+ > n_0$ and $Z_+ > Z_+$. The number density of inertialless $q$-distributed electrons and ions can be expressed as [15]

\[
n_+ = [1 + (q - 1)\delta \varphi |_{\varphi = 0}^{\varphi_{\text{thr}}}], \quad (6)
\]

\[
n_+ = [1 - (q - 1)\delta \varphi |_{\varphi = 0}^{\varphi_{\text{thr}}}], \quad (7)
\]

where $\delta = T_i / T$, $(T_i$ is the temperature of $q$-distributed electrons), and it should be noted here that $T_e, T_i \gg T_e, T_i$.

Substituting Eqs. (6) and (7) into Eq. (5), and expanding it up to third order in $\varphi$, we get

\[
\frac{\partial^2 \varphi}{\partial x^2} = \beta - 1 + n_+ - \beta n_+ + A_1 \varphi + A_2 \varphi^2 + A_3 \varphi^3 + \cdots, \quad (8)
\]

where

\[
A_1 = [(q + 1)(\delta \varphi + \beta - 1 + \mu_i)]/2,
\]

\[
A_2 = [(q + 1)(3 - q)(\delta \varphi - \beta - 1 + \mu_i)]/8,
\]

\[
A_3 = [(q + 1)(q - 3)(3q - 5)(\delta ^3 (\varphi + \beta - 1 + \mu_i))/48.
\]

We may employ the reducible perturbation method to derive the NLSE and hence, to study the MI of the DAWs plasma system. So, at first the stretched coordinates can be written as [29]
\[ \xi = \epsilon(x - v_g t), \quad (9) \]
\[ \tau = \epsilon^2 t, \quad (10) \]

where \( v_g \) is the group velocity of the DAWs, and \( \epsilon \) is a small parameter. Then a general expression for the dependent variables namely, the number density of inertial components and their fluid speed as well as electrostatic potentials of the plasma system can be represented as

\[ \gamma(x, t) = Y_0 + \sum_{m=1}^{\infty} \epsilon^m \sum_{n=1}^{\infty} Y^{(m)}_n(\xi, \tau) \exp(i \Theta), \quad (11) \]

where \( Y^{(m)}_n = [n_{x1}^{(m)}, u_{x1}^{(m)}, n_{x2}^{(m)}, u_{x2}^{(m)}, \phi_{x1}^{(m)}, \phi_{x2}^{(m)}]^T, \quad \gamma_0 = [1, 0, 1, 0, 0]^T, \quad \Theta = (kx - \omega t), \quad \text{and} \quad k (\omega) \) is the fundamental carrier wave number (frequency). All elements of \( Y^{(m)}_n \) satisfy the reality condition \( Y^{(m)*}_n = Y^{(m)}_n \), where the asterisk indicates the complex conjugate. Now, Eqs. (9) and (10) can be represented by following derivative operators:

\[ \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \epsilon v_x \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau}, \quad (12) \]
\[ \frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \xi}. \quad (13) \]

Substituting Eqs. (9)–(13) into Eqs. (1)–(4) and (8), and equating the coefficients of different powers of \( \epsilon \) for \( m = 1, l = 1 \), one obtains

\[ n_{x1}^{(1)} = \frac{k^2}{S} \phi_{x1}^{(1)}, \quad n_{x2}^{(1)} = \frac{k\omega}{A} \phi_{x2}^{(1)}, \]
\[ u_{x1}^{(1)} = \frac{\alpha k}{A} \phi_{x1}^{(1)}, \quad u_{x2}^{(1)} = \frac{\alpha k\omega}{A} \phi_{x2}^{(1)}, \quad (14) \]

where \( S = \lambda k^2 - \omega^2 \) and \( A = \omega^2 - \theta k^2 \). Thus the dispersion relation for the DAWs is obtained as follows

\[ \omega^2 = \frac{k^2 A_A \pm k^2 \sqrt{A_A^2 - 4 A_A A_B}}{2 A_A}, \quad (15) \]

where \( A_A = 1 + \alpha \beta + (\theta + \lambda)(k^2 + A_x), \quad A_B = A_x + k^2 \), and \( A_x = \theta(\lambda k^2 + 1) + \lambda(\theta A_x + \alpha \beta) \). In Eq. (15), the positive (negative) sign corresponds to the fast (slow) DA mode when \( A_x^2 > 4 A_A A_B \). The second-order \( (m = 2) \) reduced equations with \( (l = 1) \) give the group velocity \( v_g \) of DAWs

\[ v_g = \frac{\partial \omega}{\partial k} = \frac{A_x^2 (\lambda k^2 + \omega^2 - 2 \omega^2 - S) + A_A}{2 k \omega A_x^2 + 2 \alpha \beta \omega k S^2}, \quad (16) \]

where \( A_x = \alpha \beta S^2 (\theta k^2 + \omega^2 + A) \). The amplitude of the second-order harmonics is found to be proportional to \( |\phi_{x1}^{(1)}|^2 \),

\[ n_{x1}^{(2)} = F_1 |\phi_{x1}^{(1)}|^2, \quad n_{x2}^{(2)} = F_6 |\phi_{x2}^{(1)}|^2, \]
\[ u_{x1}^{(2)} = F_2 |\phi_{x1}^{(1)}|^2, \quad u_{x2}^{(2)} = F_6 |\phi_{x2}^{(1)}|^2, \]
\[ n_{x2}^{(2)} = F_8 |\phi_{x2}^{(1)}|^2, \quad n_{x2}^{(2)} = F_8 |\phi_{x2}^{(1)}|^2, \]
\[ u_{x2}^{(2)} = F_4 |\phi_{x2}^{(1)}|^2, \quad u_{x2}^{(2)} = F_4 |\phi_{x2}^{(1)}|^2, \]
\[ \phi_{x2}^{(2)} = F_3 |\phi_{x2}^{(1)}|^2, \quad \phi_{x2}^{(2)} = F_3 |\phi_{x2}^{(1)}|^2. \quad (17) \]

Here,

\[ F_1 = \frac{k^2 (2 F_2 S^2 - 3 \omega^2 k^2 - \lambda k^4)}{2 S^3}, \]
\[ F_2 = \frac{\omega F_r S^2 - \omega k^4}{k S^2}, \]
\[ F_3 = \frac{\omega (F_r A_x^2 - \alpha^2 k^4)}{k A_x^2}, \]
\[ F_5 = \frac{A_x^4 (3 \omega^2 + \lambda k^2) - 2 A_x A_x^3 S^3 + A_A}{2 k^2 A_x^4 (S^2 + 4 S^2) + 2 A_x^2 S^4 (A_x A_x - \alpha \beta k^2)^2}, \]
\[ A_A = \beta \alpha S^2 k^4 (\theta k^2 + 3 \omega^2), \]
\[ F_6 = \frac{k^2 (2 \omega \Delta k + \lambda k^2 + \omega^2) - F_{10} S^2}{S^2 v_g^2 - \lambda S^2}, \]
\[ F_7 = \frac{v_g F_r S^2 - 2 \omega k^3}{S^2}, \]
\[ F_8 = \frac{\alpha F_r A_x^2 + \alpha^2 k^2 (\omega^2 + \theta k^2 + 2 \omega k v_g)}{A_x^2 v_g^2 - \theta A_x^2}, \]
\[ F_9 = \frac{v_g F_r A_x^2 - 2 \omega \alpha S^2 k^3}{A_x^2}, \]
\[ F_{10} = \frac{2 A_x A_x^2 S^2 (v_g^2 - \theta) (v_g^2 - \lambda) + A_A}{A_A}, \]
\[ A_A = A_x^2 k^2 (v_g^2 - \theta) (2 v_g^2 \omega k + \lambda k^2 + \omega^2) - \alpha \beta S^2 k^4 (v_g^2 - \lambda) (2 v_g^2 \omega k + \theta k^2 + \omega^2), \]
\[ A_0 = A_x^2 S^2 \left[ \alpha \beta (v_g^2 - \lambda) + (v_g^2 - \theta) \right] \left[ 1 - A_x (v_g^2 - \lambda) \right]. \]

Finally, the third-order harmonics \( (m = 3) \) and \( (l = 1) \), and with the help of Eqs. (14)–(17), provide a set of equations which can be simplified to the following NLSE

\[ i \frac{\partial \psi}{\partial t} + P \frac{\partial^2 \psi}{\partial \xi^2} + Q |\psi|^2 \psi = 0, \quad (18) \]
where \( \psi = \varphi^{(i)} \) for simplicity and the dispersive coefficient \( P \) of Eq. (18) can be represent as

\[
P = \frac{A_{11}}{2AS\omega k^{2}(A^2 + \alpha\beta S^2)}.
\]

Here,

\[
A_{11} = \omega^2 \left(A^2 - \alpha\beta S^2\right) \left(4k^2 - \omega^2\right) + 2k^2\omega \left(A^2\omega - \alpha\beta S^2\right) (2v_k - \omega) - k^4 \left(A^2 - \alpha\beta S^2\theta^2\right) - A^2 S^3 + \beta S^2 k\left(A^2 + \alpha\beta S^2\right) - \beta S^2 k^2 \left(A^2 + \alpha\beta S^2\right).
\]

Similarly, the nonlinear coefficient \( Q \) in Eq. (18) can be represent as

\[
Q = \frac{A_{12}}{2\omega k^2\left(A^2 + \alpha\beta S^2\right)},
\]

where

\[
A_{12} = A^2 \left(2A_1F_2 + A_2F_{10} + 3A_3\right) - k^2 \left[2\omega k \left(F_1 + F_2\right) + \left(\omega^2 + \lambda k^2\right) \left(F_1 + F_2\right)\right] - \alpha\beta S^2 k^2 \left[2\omega k \left(F_1 + F_2\right) - \left(\omega^2 + \theta k^2\right) \left(F_1 + F_2\right)\right].
\]

It may be noted here that both \( P \) and \( Q \) are function of various plasma parameters such as \( \alpha, \beta, \delta, \theta, \lambda, \mu_i, \omega_i, \omega_s, k \), and \( q \). So, all the plasma parameters are used to maintain the nonlinearity and the dispersion properties of the plasma medium.

**STABILITY AND ROGUE WAVES**

The space and time evolution of the DAWs in OPDP medium are directly governed by the dispersion \( (P) \) and nonlinear \( (Q) \) coefficients, and indirectly governed by different plasma parameters such as \( \alpha, \beta, \mu_i, \lambda, \theta, \delta, \) and \( q \). Thus, these plasma parameters significantly affect the stability conditions of the DAWs. Modulationally stable DAWs are obtained when \( P/Q \) is negative. On the other hand, for the case of positive value of \( P/Q \), DAWs are modulationally unstable against external perturbations [27–29]. The intersecting point, in which the instability window appears for DAWs, of the \( P/Q \) curve with \( k \)-axis in \( P/Q \) vs \( k \) graph is known as critical wave number \( (k_c) \).

We have depicted the variation of the \( P/Q \) against different values of \( \alpha \) in Fig. 1, and it is found that the instability window or \( k_c \) appears to decrease (increase) with the increase of negative (positive) dust mass and with fixed values of positive and negative dust charge state (via \( \alpha = Z/\nu - Z/\nu_i \)). All figures are presented for fast DA and \( \tau = 0, \lambda = 0.0003, \theta = 0.003 \).

The governing equation of the RWs in the modulationally unstable regime \( (P/Q > 0) \) can be written as [30–34]

\[
\psi(x, t) = \frac{2P}{Q} \left[ \frac{4(1 + 4P\tau)}{1 + 16P^2 \tau^2 + 4\xi^2} - 1 \right] \exp(2i\theta t).
\]

The solution of the NLSE directly depends on \( P \) and \( Q \). So, the dispersion and the nonlinear behavior of the plasma medium highly predict the formation of the DARWs in space and time evolution. Figures 2–6 show how the amplitude and width of the DARWs change with the variation of four important OPDP parameters namely, \( \delta, \beta, \mu_i \), and \( q \). The dependence of the DARWs against the ion and electron temperature (via \( \delta \)) can be shown in Fig. 2. The electron (ion) temperature reduces (increases) the nonlinearity of the OPDP medium by depicting shorter (taller) DARWs (via \( \delta = T_i/T_e \)).

The nature of the formation of DARWs in OPDP medium is crucially depended on the number density of the warm positive and negative massive dust grains, and also their charge state (via \( \beta = Z/n_{i0}/Z/n_{n0} \)). It can be deduced from Fig. 3 that (a) the amplitude and width of the DARWs increase with the increase in the value of positive dust charge state but decrease with an increase in the value of negative dust charge state for constant number density of positive and negative dust; (b) similarly, the nonlinearity of the OPDP medium increases (decreases) with the number density of positive (negative) dust grains when their charge state remains invariant (via \( \beta = Z/n_{i0}/Z/n_{n0} \)). The increase in the value of ion number density does not only cause to increase the amplitude of the DARWs but also causes to increase the width of the DARWs for fixed value of \( Z \) and \( n_{i0} \) (via \( \mu_i \)). Physically, the nonlinearity, which enhances the amplitude and width of the DARWs, of the OPDP medium increases with ion number density.

The effects of nonextensivity of the restoring force providing components (electrons and ions) are shown
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in Figs. 5 and 6, and it is obvious that the amplitude and width of the DARWs is independent (dependent) to the sign (magnitude) of the \( q \), and this result is a good agreement with the results from [15].

CONCLUSIONS

We have investigated the MI of electrostatic DAWs in an unmagnetized four-component OPDP system in presence of \( q \)-distributed electrons and ions. The non-linear and dispersion co-efficient of NLSE directly predict the stable and unstable regime for DAWs. The configuration of the DARWs in the unstable regime is also a function of various plasma parameters namely, the charge state of plasma components, number density, and their associated temperatures, etc. The limitation of our present work is that we have not considered any magnetic field or self-gravitational field for OPDP system. These, however, do not affect our pres-
ent investigation of DARWs in OPDP medium. We may hope that the results of this theoretical investigation, finally, could be helpful for understanding the nonlinear electrostatic DARWs in astrophysical (viz., planetary rings [1, 2], cometary tails [1], early universe, supernova, interstellar cosmic matter [2], etc.) and laboratory environments [10, 11].

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