Singlets in Supersymmetry

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Abstract

It is argued that singlet extensions of the MSSM at the weak scale are indicative of either gauged-$R$ symmetry or target space duality in a string effective action at the Planck scale. The criteria used are satisfactory primordial nucleosynthesis, absence of fine-tuning, and absence of cosmological problems such as domain walls. Models which have a global discrete symmetry such as the NMSSM may only be accommodated within rather complicated cosmological scenarios which are also discussed.
1 Introduction

There has lately been some interest in the problem of how to accommodate an extra gauge singlet field into the minimal supersymmetry standard model (MSSM). This is the simplest extension which is consistent with a lightest higgs boson whose mass exceeds the upper bound found in the MSSM [1]. Previously it was thought that, by acquiring a vacuum expectation value of $O(M_W)$, such a singlet could also provide a simple solution to a fine-tuning problem in the MSSM, the so-called ‘$\mu$–problem’ [2, 3]. Because of difficulties with cosmology (specifically the appearance of domain walls) this now no longer appears to be the case [4, 5]. In fact, it was shown in ref. [5] that models with singlets are likely to require symmetries in addition to those in the MSSM if they are to avoid problems with either domain walls or fine-tuning. In this respect models with gauge singlets are singularly less efficient at solving fine-tuning problems. However since they allow for more complicated higgs phenomenology, it is still worth pursuing them. This paper concentrates on the task of building an MSSM extended by a singlet, which avoids reintroducing the hierarchy problem, fine-tuning, and domain walls. These three constraints turn out to place severe restrictions on the type of theory which can be accommodated at the Planck scale. Conversely the form of the low energy higgs sector can give important indications of the physics occurring at the Planck scale.

Let us take as our starting point a low-energy effective theory which includes all the fields of the MSSM, plus one additional singlet $N$. The superpotential is assumed to be the standard MSSM Yukawa couplings plus the higgs interaction

$$W_{\text{higgs}} = \mu H_1 H_2 + \mu' N^2 + \lambda N H_1 H_2 - \frac{k}{3} N^3,$$

and the soft supersymmetry breaking terms are taken to be of the form

$$V_{\text{soft higgs}} = B\mu h_1 h_2 + B'\mu' n^2 + \lambda A \lambda n h_1 h_2 - \frac{k}{3} A_k n^3 + \text{h.c.} + m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + m_N^2 |n|^2,$$

where throughout scalar components will be denoted by lower case letters. For the moment let us put aside the question of how the $\mu$ and $\mu'$ terms get to be so small (i.e. $O(M_W)$ instead of $O(M_{Pl})$), and return to it later. From a low-energy point of view the only requirement is that the additional singlet should significantly alter the higgs mass spectrum. This means that $\lambda \neq 0$. There are four possibilities which can arise:

If all the other operators are absent, then in the low energy phenomenology there is an apparent (anomalous) global $\tilde{U}(1)$ symmetry (orthogonal to the hypercharge), which leads to a massless goldstone boson. This situation is briefly reviewed in section 2, with the conclusion that a significant complication is required in order that axion bounds are satisfied. In fact such models can work if the $\tilde{U}(1)$ symmetry is only approximate, being broken to a high order, discrete symmetry by non-renormalisable terms. In this case an additional singlet can provide a solution to the strong CP problem [6].

There are two cases which lead to a discrete symmetry. These are $\mu = 0$, $k = 0$ which leads to a $Z_2$ symmetry, and $\mu = 0$, $\mu' = 0$ which leads to a $Z_3$ symmetry. The latter is usually referred to as the next-to-minimal supersymmetric standard model (NMSSM) [7, 8], and will be the main focus of this paper.
Thus the second possibility is that there is an exact discrete symmetry, and thus a domain wall problem associated with the existence of degenerate vacua after the electroweak phase transition. Weak scale walls cause severe cosmological problems (for example their density falls as $T^2$ whereas that of radiation falls as $T^4$ so they eventually dominate and cause power law inflation) [3]. This is not true however, if the discrete symmetry is embedded in a broken gauge symmetry. In this case the degenerate vacua are connected by a gauge transformation in the full theory [4]. After the electroweak phase transition, one expects a network of domain walls bounded by cosmic strings to form and then collapse [9]. (There are a number of more baroque solutions outlined in ref. [5] which will not be pursued here.) In addition there is no gauge singlet to destabilise the gauge hierarchy [10]. This situation is examined in section 3, where it is found that bounds from primordial nucleosynthesis (essentially on the reheat temperature after inflation) require that the potential be very flat. Such potentials occur naturally as the low energy approximation to string theory and two examples will be examined. The first, involving an extra $U(1)_X$ gauge symmetry has difficulty satisfying anomaly cancellation conditions. The second example, in which the $Z_3$ is embedded in $SU(3)_C^3$ looks more promising, although the presence of the $N^3$ term necessitates large multiplets which do not exist in the simpler string models. In addition this mechanism depends rather strongly on the cosmology which is not yet fully known for string effective actions, and so models with discrete symmetry (such as the NMSSM) remain questionable.

The third possibility is that the discrete symmetry is broken by gravitationally suppressed interactions [8, 12]. This was the case considered and rejected in ref. [3]. Here the very slight non-degeneracy in the vacua, causes the true vacuum to dominate once the typical curvature scale of the domain wall structure becomes large enough. However one must ensure that the domain walls disappear before the onset of nucleosynthesis and this means that the gravitationally suppressed terms must be of order five. It was shown in ref. [3] that, no matter how complicated the full theory (i.e. including gravity), there is no symmetry which can allow one of these terms, whilst forbidding the operator $\nu N$, where $\nu$ is an effective coupling. Furthermore, any such operator large enough to make the domain walls disappear before nucleosynthesis generates these terms at one loop anyway (with magnitude $\sim M_W^2 M_{Pl} N$), even if they are set to zero initially. This constitutes a reintroduction of the hierarchy problem as emphasised in ref. [10].

The final case which is discussed in section 4, is when there is no discrete symmetry at the weak scale (exact or apparent). This is true when either $\mu \neq 0$ or both $\mu' \neq 0$ and $k \neq 0$. Here the arguments of ref. [3] will again imply fine-tuning, except in the case that there is either a target space duality symmetry, again coming from the low energy approximation to string theory or an $R$-symmetry at the Planck scale. For the reasons discussed in ref. [13], gauged $R$-symmetry [13, 14] might be favoured over global, although the arguments presented will apply to either case. In these cases, fine tuning can be avoided since the transformation of the superpotential is different from that of the Kähler potential. Thanks to the experimental signatures discussed in ref. [13], the case of gauged $R$-symmetry should be easily distinguishable. It is also established that the higher loop contributions to the effective potential (in the framework of $N = 1$ supergravity), do not destabilise the gauge hierarchy either.

Thus it can be argued quite generally that, provided one considers lack of fine-tuning to be a legitimate constraint, gauge singlets at the weak scale are likely to be embedded
in either a string effective action, or a gauged $R$-symmetry. It should be stressed that this is not simply a restatement of the conclusion drawn in ref. [10]. There the absence of destabilising divergences implied that there are no singlets in the full theory including gravity. In fact any symmetry (such as the discrete $Z_3$ symmetry in the NMSSM) is sufficient to ensure this, as was demonstrated earlier.

2 The Global $\tilde{U}(1)$ Case

Models in which $\lambda NH_1H_2$ is the only operator appearing in the higgs superpotential have a continuous $\tilde{U}(1)$ symmetry which is both global and anomalous. This section merely reviews this situation, which is unworkable without the addition of extra suppressed terms, since the model fails to satisfy axion bounds. One may rectify this by adding gravitationally suppressed terms breaking the $\tilde{U}(1)$ symmetry to a high-order discrete symmetry. In this case, for the (pseudo) goldstone boson to provide a solution to the strong CP problem, an additional singlet is required.

The quantum numbers of the higgs fields may be taken as follows:

|   | $U(1)_Y$ | $U(1)$ |
|---|----------|--------|
| $N$ | 0        | -2     |
| $H_1$ | 1        | 1      |
| $H_2$ | -1       | 1      |

with the right and left-handed quarks transforming appropriately to keep the Yukawa terms invariant. The anomaly may be expressed as the non-conservation of the current generated under such a rotation:

$$\partial_\mu \tilde{J}^\mu = 3\theta_N \frac{F_{\mu\nu} \tilde{F}^{\mu\nu}}{32\pi^2}$$

where $N \rightarrow \exp(i\theta_N)N$. There are a number of well rehearsed problems which now arise, and it is worth examining them in some more detail for this case.

The first problem is due to the appearance of domain walls. Eq.(3) tells us that the partition function is invariant when $\theta_N = 0, \pm 2\pi/3$, and that there are therefore three degenerate vacua after the $\tilde{U}(1)$ symmetry has been broken by QCD instantons. Domain walls appear across which $\theta_N$ changes by $\pm 2\pi/3$. In fact the residual $Z_3$ symmetry is precisely the anomaly free discrete symmetry of the NMSSM. This difficulty could potentially be resolved by the Lazarides-Shafi mechanism [9] (i.e. embedding the discrete symmetry in a gauged continuous symmetry) which will be discussed later in the context of the NMSSM.

However a second and more serious problem is the existence of an axion associated with the $\tilde{U}(1)$ symmetry. In fact this model is just a particular case of the DFSZ axion model [14] except here one is hampered considerably by the fact that there is only one parameter, namely $\lambda$, which can be adjusted. Anticipating that the anomalous $\tilde{U}(1)$ can
provide a realisation of the Peccei-Quinn mechanism, in DFSZ models the mass of the axion is given by
\[ m_a = \sqrt{\frac{m_u m_d}{m_u + m_d}} m_{\pi f_a}, \] (4)
where \( f_a \) is the axion decay constant \[16\]. Astrophysics and cosmological considerations place lower and upper bounds respectively on the value of \( f_a \) \[16\]:
\[ 10^9 \lesssim f_a \lesssim 3 \times 10^{12} \text{GeV}. \] (5)
Since \( \langle h_1^0 \rangle \) and \( \langle h_2^0 \rangle \) are constrained to be of order \( M_W \), this means that \( f_a \approx \langle n \rangle \) (where lower case letters denote scalar components). Thus one requires the parameter \( \lambda < \sim 10^{-6} \) for suitable electroweak symmetry breaking to occur (taking \( \lambda |n| < 1 \text{TeV} \)). On the other hand the scalar potential is of the form
\[ V_{\text{neutral-scalars}} = B \lambda^2 \left( |nh_1^0|^2 + |nh_2^0|^2 + |h_1^0 h_2^0|^2 \right) + \lambda A \lambda n h_1^0 h_2^0 + \text{h.c.} + m_1^2 |h_1^0|^2 + m_2^2 |h_2^0|^2 + m_N^2 |n|^2. \] (6)
Because the soft supersymmetry breaking scalar masses and trilinear terms are expected to be of order \( M_W \), one cannot expect \( \langle n \rangle \) to be much more than \( \lambda M_W \). Thus \( f_a \geq 10^9 \text{GeV} \) is incompatible with electroweak symmetry breaking.

Plainly some extension to the superpotential is required, and, at the risk of complicating things considerably, one might consider adding undetectable, non-renormalisable terms such as \( N^m M_{Pl}^{-m-3} \) to the superpotential. Now a minimum can appear for \( \langle n \rangle \sim M_{Pl}(M_W/M_{Pl})^{1/(m-2)} \), lying roughly in the desired range if \( m = 4 \) or 5 (where \( M_{Pl} \) implies the reduced Planck mass). Also there is now no domain wall problem, since the remaining discrete symmetry (i.e. a \( Z_m \) symmetry) is anomalous. This model with one singlet does not resolve the strong CP problem however. This is because the effective potential in \( \bar{\theta} = \theta_{QCD} + 3 \theta_N + \arg \det(\lambda_u \lambda_d) \) which is of the form (see Cheng in ref.\[16\] and references therein),
\[ V_{\text{PQ}} = m_a^2 f_a^2 (1 - \cos(\bar{\theta})), \] (7)
is outweighed by the dependence of the potential on \( \theta_N \),
\[ V_{\text{PQ-breaking}} = -\frac{M_W M_{Pl}^{-m-3}}{m} |n|^m \cos(\theta_N) \] (8)
which drives \( \theta_N \) to 0, \( 2\pi/m \), \( 4\pi/m \ldots \) or \( 2\pi \), rather than the value which gives \( \bar{\theta} = 0 \). There is then no hope of using the Peccei-Quinn mechanism to solve the strong CP problem.

This situation is similar to one which has been studied in some detail by Casas and Ross \[13\]. With the aid of discrete \( Z_2 \times Z_3 \) symmetry and an extra singlet \( (M) \), they constructed models in which the terms responsible for breaking the global \( \tilde{U}(1) \) symmetry were of dimension-12. This gives a successful implementation of the (pseudo) Peccei-Quinn mechanism with a suitably low value for \( \lambda \).

So the case of continuous, global \( \tilde{U}(1) \) seems potentially promising, if the symmetry is only the approximate, low energy manifestation of a high order, anomalous, discrete symmetry. However, the question of successful (dynamical) realisation of electroweak symmetry breaking has not yet been addressed for these models.
3 The Discrete Symmetry Case

This section demonstrates how models with discrete symmetry may be embedded in a gauge symmetry which is spontaneously broken at some high scale. In this case the apparently distinct degenerate vacua of the low energy theory are connected by a gauge transformation. At all times the main aim is to keep the model as ‘minimal’ as possible, however it will become apparent that constraints coming from primordial nucleosynthesis require that the potential to be very flat, leading us in the following section to consider string effective supergravity models.

This scenario was suggested by Lazarides and Shafi [9], and for definiteness (and simplicity) first consider adding an extra $U(1)_X$ gauge symmetry to the the NMSSM, (so that $\mu = 0$ and $\mu' = 0$), which is spontaneously broken at some high scale. The discrete symmetry is an apparent $Z_3$ symmetry when every chiral superfield is rotated by $e^{2\pi i/3}$ (i.e. only trilinear terms appear in the superpotential). This rotation may be embedded in $U(1)_X$ by introducing an additional singlet field $\Phi$ which generates the superpotential as effective interactions via non-renormalisable terms,

$$W_{\text{higgs}} = \lambda \frac{\Phi}{M_{\text{Pl}}} N H_1 H_2 - \frac{k}{3} \frac{\Phi}{M_{\text{Pl}}} N^3. \quad (9)$$

The field $\Phi$ will acquire a VEV of order $M_{\text{Pl}}$, forming strings as it does so. The latter involve physics at distances much smaller than the weak scale. Because of this it is useful to think in terms of the previous $\tilde{U}(1)$ rotation as an additional approximate global symmetry operating orthogonally to the two gauge symmetries since it does not involve $\Phi$ (becoming exact when $k = 0$). The quantum numbers of the higgs fields are as follows:

|        | $U(1)_X$ | $U(1)_Y$ | $U(1)$ |
|--------|----------|----------|--------|
| $N$    | 1        | 0        | -2     |
| $H_1$  | 1        | 1        | 1      |
| $H_2$  | 1        | -1       | 1      |
| $\Phi$ | -3       | 0        | 0      |

The global $\tilde{U}(1)$ symmetry and local $U(1)_X$ symmetry have a non-trivial intersection which is precisely the $Z_3$ operation defined above. One is forced to involve the two higgs fields as well as $N$, which complicates the job of anomaly cancellation. For the moment however this question can be neglected, since the physics we are presently concerned with is purely classical. Eventually anomalies will be cancelled by assigning non-zero $U(1)_X$ numbers to the remaining fields in the visible sector.

Now consider what happens when these symmetries are spontaneously broken down to $SU(3)_c \times U(1)_{em}$. Obviously for the effective low-energy theory to resemble the NMSSM, this must happen with $\langle|\phi|\rangle = \rho_\phi = O(M_{\text{Pl}})$. For the moment therefore, let us simply assume that at some point a Lagrangian develops for the scalar component ($\phi$) of the $\Phi$ superfield of the form,

$$L_\phi = (D\phi)^\dagger D\phi + M^2|\phi|^2 - L|\phi|^4 \quad (10)$$
where one would a priori expect $|M| = \mathcal{O}(M_{Pl})$. When $M^2 > 0$, the potential turns over at the origin, developing a minimum at $\langle |\phi| \rangle = \rho_\phi = \sqrt{M^2/2L}$. The goldstone mode can be identified as the field coupling linearly to the $X^\mu$ current:

$$G_\mu = (3\sqrt{2}g_X/M_X) \left( \rho_\phi^2 \delta_\mu \delta_\phi - \frac{1}{3} \left( \rho_n^2 \delta_\mu \delta_n + \rho_1^2 \delta_\mu \delta_1 + \rho_2^2 \delta_\mu \delta_2 \right) \right)$$  \hspace{1cm} (11)

where,

$$M_X^2 = 18g_X^2 \left( \rho_\phi^2 + \frac{1}{9} \left( \rho_n^2 + \rho_1^2 + \rho_2^2 \right) \right),$$  \hspace{1cm} (12)

and where the scalar components are defined such that, $\phi = |\phi| e^{i\theta_\phi}$, $n = |n| e^{i\theta_n}$, $h_1 = |h_1| e^{i\theta_1}$, $h_2 = |h_2| e^{i\theta_2}$, and $\langle |n| \rangle = \rho_n$, $\langle |h_1| \rangle = \rho_1$, $\langle |h_2| \rangle = \rho_2$. Once the weak scale higgs fields have acquired a vacuum expectation value, there is therefore a tiny amount of mixing between the heavy and light bosons. In fact defining the usual mass of the $Z$ by

$$M_Z^2 = \left( g_X^2 y + g_2^2 \right) \left( \rho_1^2 - \rho_2^2 \right) / 2,$$

and also $m^2 = g_X \sqrt{g_2^2 + g_2^2 (\rho_1 - \rho_2^2)}$, one finds that the mass eigenstates are given by

$$Z'^\mu = \cos \theta_X Z^\mu - \sin \theta_X X^\mu \approx Z^\mu - \frac{m^2}{2M_X^2} X^\mu,$$

$$X'^\mu = \sin \theta_X Z^\mu + \cos \theta_X X^\mu \approx X^\mu + \frac{m^2}{2M_X^2} Z^\mu,$$  \hspace{1cm} (13)

where

$$\theta_X = \frac{1}{2} \tan^{-1} \left( \frac{m^2}{M_X^2 - M_Z^2} \right).$$  \hspace{1cm} (14)

The mass of the $Z$ is virtually unchanged;

$$M_{Z'}^2 = M_Z^2 - 3 \frac{m^2}{M_X^2} m^2.$$  \hspace{1cm} (15)

To describe the topological defects that result from this pattern of breaking, consider first what happens after the first stage, once the $U(1)_X$ symmetry is spontaneously broken. Vortex solutions appear of the form

\begin{align*}
\text{Lim}_{r \gg M_X^{-1}} \langle \phi \rangle &= \rho_\phi e^{-i\alpha(\theta)} \\
\text{Lim}_{r \gg M_X^{-1}} \langle X_\mu \rangle &= \frac{1}{3g_X} \partial_\mu \alpha(\theta)
\end{align*}  \hspace{1cm} (16)

with winding number $s = 3t + u$ where $u = \{0, 1, 2\}$, $t$ is an integer, and $\alpha(\theta)$ is a continuous function of the azimuthal angle $\theta$ such that

$$\alpha(\theta + 2\pi) = \alpha(\theta) + 2s\pi$$  \hspace{1cm} (17)

(assuming translational symmetry in the $z$ direction). This solution corresponds to a gauged $U(1)_X$ rotation through $2s\pi/3$ as one goes around the string and therefore has zero energy density as $r \to \infty$. The integer $u$ labels the elements of $\pi_1(G/H) = Z_3$. As one parallel transports a test particle, $i$, around a string it picks up a phase of $\exp(2\pi i u X_i/3)$. 


Now consider what happens around a string at the second stage of symmetry breaking in which the $n$, $h_1$ and $h_2$ fields acquire a VEV. The VEVs are described by functions which are single valued, such that the $\theta$ dependence of the condensates obeys $\langle n(\theta + 2\pi) \rangle$, $\langle h_1(\theta + 2\pi) \rangle$, $\langle h_2(\theta + 2\pi) \rangle = \langle n(\theta) \rangle$, $\langle h_1(\theta) \rangle$, $\langle h_2(\theta) \rangle$. They may be written as the sum of the above $U(1)_X$ rotation and an additional global $U(1)$ rotation through $-2\pi/3$, so that for the remaining higgs fields,

\begin{align}
    \text{Lim}_{r \to \infty} \langle n \rangle &= \rho_n(r)e^{i(\alpha(\theta)+2\tilde{\alpha}(\theta))/3} \\
    \text{Lim}_{r \to \infty} \langle h_1 \rangle &= \rho_1(r)e^{i(\alpha(\theta)-\tilde{\alpha}(\theta))/3} \\
    \text{Lim}_{r \to \infty} \langle h_2 \rangle &= \rho_2(r)e^{i(\alpha(\theta)-\tilde{\alpha}(\theta))/3},
\end{align}

where

$$\tilde{\alpha}(\theta + 2\pi) = \tilde{\alpha}(\theta) + 2\pi,$$

and $\tilde{s} = 3\tilde{t} + u$ (same $u$). This combination of gauge and global rotations is similar to the hybrid string configuration described in ref. [9] (except here of course the global symmetry is already explicitly broken at tree level). By simple scaling arguments, one expects the string to have a width of $O(L^{-1/2} \rho_\phi^{-1})$ and a mass per unit length of $O(\rho_\phi^2)$.

This description is useful, since (as $r \to \infty$), the $\alpha(\theta)$ dependence may be taken out of the low energy (that is near $M_W$) effective Lagrangian, by expressing the theory around the string in terms of the constants $\lambda = \lambda \rho_\phi/M_{P1}$ and $k = k \rho_\phi/M_{P1}$, and the superfields

$$\begin{align}
    \hat{N} &= N e^{-i\alpha(\theta)/3} \\
    \hat{H}_1 &= H_1 e^{-i\alpha(\theta)/3} \\
    \hat{H}_2 &= H_2 e^{-i\alpha(\theta)/3}.
\end{align}$$

The effective Lagrangian in terms of the roofed parameters is simply the NMSSM. Close to the string for $M_W^{-1} \gg r \gg L^{-1/2} \rho_\phi^{-1}$, the gradient energy density ($\sim \rho^2/r^2$) dominates over the potential energy ($\sim \rho^4$). Here the scalar fields approximately obey the Laplace equation, with solutions,

$$\begin{align}
    \langle \hat{n} \rangle &= \rho_n \sum_l \left( A_n^l z^{l+2u/3} + B_n^l z^{-2u/3} \right) \\
    \langle \hat{h}_1 \rangle &= \rho_1 \sum_l \left( A_1^l z^{l-u/3} + B_1^l z^{l+u/3} \right) \\
    \langle \hat{h}_1 \rangle &= \rho_2 \sum_l \left( A_2^l z^{l-u/3} + B_2^l z^{l+u/3} \right),
\end{align}$$

where $l$ is an integer and $z = (r/r_0)e^{i\theta}$.

On larger scales, $r \gtrsim O(M_W^{-1})$, the potential energy density becomes important. If $u = 0$ (i.e. when the winding number is divisible by three) the minimum energy solution for $r \gg M_W^{-1}$ is simply constant $\langle \hat{n} \rangle$, $\langle \hat{h}_1 \rangle$ and $\langle \hat{h}_1 \rangle$, which can be trivially matched onto the $l = 0$ solution above. This case is just the usual cosmic string of mass/unit length $\eta = O(\rho_\phi^2)$. But if $u = \pm 1$ the minimum energy solution is a single wall of exactly the same form as in ref. [9], which has thickness $\sim M_W^{-1}$, and across which the fields acquire a phase

$$\langle \hat{n} \rangle \to e^{4ui\pi/3} \langle \hat{n} \rangle ; \langle \hat{h}_1 \rangle \to e^{-2ui\pi/3} \langle \hat{h}_1 \rangle ; \langle \hat{h}_2 \rangle \to e^{-2ui\pi/3} \langle \hat{h}_2 \rangle.$$
The configuration in this case is a domain wall of mass/unit area \( \sigma = \mathcal{O}(M_W^3) \), bounded by a cosmic string of mass/unit length \( \eta = \mathcal{O}(\rho_2^3) \).

The cosmology of this type of string/wall system was discussed in refs.\([9, 17]\). At temperatures high above the electroweak phase transition but below \( M_X \), the strings evolve freely, straightening under their own tension until there is roughly one string per horizon. Once the temperature drops below the electroweak phase transition some of them become connected by domain walls. These also evolve until there is roughly one domain wall bounded by a string per horizon, at which point the system collapses under its own tension \([9, 17]\).

Nucleosynthesis imposes some simple bounds on the parameters \( M \) and \( L \) appearing in eq.\((10)\) as follows. Firstly it should be stressed that since \( M_X \) is much larger than \( M_W \), the domain walls are stable within the lifetime of the universe. This is because the probability for quantum tunnelling a hole bounded by a string in any wall is suppressed by the Boltzmann factor \( \exp(-M_X^4/M_W^3 T)\)\([9]\). (Interestingly, in addition to the usual configuration where two strings appear with opposite winding number giving a hole, there is also the possibility for a three wall vertex to collapse by inserting into the vertex two strings of winding number \(+1\) and one of winding number \(−2\).) This means that any period of inflation cannot have inflated away the strings. The creation and subsequent decay of gravitinos places an upper limit on the reheat temperature after inflation \( T_R \lesssim 10^9 \text{ GeV} \)\([18]\). Thus the \( U(1)_X \) symmetry must be restored at temperatures below \( T_R \), giving
\[
T_c = \frac{\sqrt{48M^2/(16L + 27g_X^2 T^2)}}{32M^2_{\text{Pl}}} \lesssim 10^9 \text{ GeV},
\] (23)
in a high-temperature mean-field approximation. (Note that the temperature in the \( \Phi \) sector need not be the same as in the visible sector since they may not be in equilibrium, however the bound on \( T_R \) applies to all sectors separately.) This bound is avoided if the gravitino is very heavy (heavier than 50 TeV) or extremely light (lighter than 1 KeV), but neither of these possibilities may be easily realised (see Sarkar in ref.\([18]\) and references therein). The condition in eq.\((23)\) is in conflict with the phenomenological requirement that \( \rho_\phi \sim M_{\text{Pl}} \), since together they imply that
\[
L \approx \frac{9g_X^2 T^2 c^2}{32M^2_{\text{Pl}}} \lesssim 10^{-20}
\]
\[
M \approx \frac{3}{4}g_x T^2 c \lesssim 10^{-10} M_{\text{Pl}},
\] (24)
assuming that \( g_X = \mathcal{O}(1) \). This flatness in the potential looks very unnatural unless it is enforced by some symmetry, at least in the context of field theory. However string effective supergravity models naturally have flat directions, and so the next section considers how the mechanism might, in principle, be made to work in these.

4 The Lazarides-Shafi Mechanism in String Theory

Generally, string effective supergravity has many flat directions, some of which correspond to moduli determining the size and shape of the compactified space. Furthermore these moduli have discrete duality symmetries, which at certain points of enhanced symmetry
become continuous gauge symmetries [19]. Thus one would expect the Lazarides-Shafi mechanism to be directly applicable here. (This possibility has also been alluded to in refs.[20, 21].) In this section is shown that this is indeed the case, although anomalies and again nucleosynthesis bounds, force the resulting models to be rather clumsy. In fact anomaly cancellation virtually eliminates $U(1)_X$ as a reasonable symmetry in which to embed $Z_3$. More generally one expects the cosmology to be rather troublesome.

In Calabi-Yau models, abelian orbifolds and fermionic strings the moduli include three Kähler class moduli ($T$-type) which are always present, plus the possible deformations of the complex structure ($U$-type), all of which are gauge singlets. Additionally there will generally be complex Wilson line fields [23, 24]. When the latter acquire a vacuum expectation value they result in the breaking of gauge symmetries. There has been continued interest in string effective actions since they may induce the higgs $\mu$-term [3, 24, 25, 26], be able to explain the Yukawa structure [27, 28], and be able to explain the smallness of the cosmological constant in a no-scale fashion [27, 29]. Since the main objective here is simply to find a route to a low energy model with visible higgs singlets and apparent discrete symmetry, these questions will only be partially addressed.

Typically the moduli and matter fields describe a space whose local structure is given by a direct product of $SU(n, m)/SU(n) \times SU(m)$ and $SO(n, m)/SO(n) \times SO(m)$ factors [23, 24]. As an example consider the Kähler potential derived in refs.[24], which at the tree level is of the form

$$K = - \log(S + \bar{S}) - \log[(T + \bar{T})(U + \bar{U}) - \frac{1}{2}(\Phi_1 + \bar{\Phi}_2)(\Phi_2 + \bar{\Phi}_1)] + \ldots$$

(25)

The $S$ superfield is the dilaton/axion chiral multiplet, and the ellipsis stands for terms involving the matter fields. The fields $\Phi_1$ and $\Phi_2$ are two Wilson line moduli. In ref.[23], these fields were identified with the neutral components of the higgs doublets in order to break electroweak symmetry, and also to provide a $\mu$-term. Here however, their role is to break $U(1)_X$, and so they are instead chosen to have X-charges of $-1$ and $+1$ respectively. Problems such as how the dilaton acquires a VEV, or the eventual mechanism which seeds supersymmetry breaking will not be addressed here.

The moduli space is given locally by

$$\mathcal{K}_0 = \frac{SU(1, 1)}{U(1)} \times \frac{SO(2, 4)}{SO(2) \times SO(4)},$$

(26)

which ensures the vanishing of the scalar potential at least at the tree level, provided that the $S$, $T$ and $U$ fields all participate in supersymmetry breaking (i.e. $G_S$, $G_T$, $G_U \neq 0$). In fact writing the Kähler function as

$$G = K(z, \bar{z}) + \ln |W(z)|^2,$$

(27)

where $z_i$ are generic chiral superfields, the scalar potential becomes

$$V_s = - e^G \left( 3 - G_i G^\mathcal{J}_i G^\mathcal{J}_i \right) + \frac{g^2}{2} \text{Re}(G^A_{iA} T_i z_j)(G^k_{kA} T^A_{jA} z_l),$$

(28)

where $G_i = \partial G/\partial z_i$, and $G^\mathcal{J}_{\mathcal{J}} = (G_{\mathcal{J}i})^{-1}$. The dilaton contribution separates, and gives $G_S G^s \bar{G}_s G_{\bar{s}} = 1$. To show that the remaining contribution is 2, it is simplest to define the vector

$$A^\alpha = a(t, u, h, \bar{h})$$

(29)
where the components are defined as \( \alpha = (1 \ldots 4) \equiv (T, U, \Phi_1, \Phi_2) \), and \( u = U + \overline{U}, \ t = T + \overline{T}, \ h = \Phi_1 + \Phi_2 \). It is easy to show that
\[
G_\alpha A^\alpha = -2a.
\]

The vector \( A^\alpha \) is designed so that \( G_\beta A^\alpha \) is proportional to \( G_\beta \); viz,
\[
G_\beta A^\alpha = -aG_\beta.
\]

Multiplying both sides by \( G_\alpha G_\beta \) gives the desired result, i.e. that \( G_\alpha G_\beta = 2 \). Thus, if the VEVs of the matter fields are zero, the potential vanishes and is flat for all values of the moduli \( T \) and \( U \), along the direction \( \langle |\Phi_1| \rangle = \langle |\Phi_2| \rangle = \rho_\phi \) (since this is the direction in which the \( D \)-terms vanish). The gravitino mass is therefore undetermined at tree level, being given by
\[
m^2/2 = \langle e^G \rangle = \frac{|W|^2}{s(ut - 2\rho^2)}.
\]

In addition to the properties described above, there is an \( O(2,4,Z) \) duality corresponding to automorphisms of the compactification lattice \([19, 24]\). This constrains the possible form of the superpotential. The \( PSL(2,Z) \) subgroup implies invariance under the transformations \([19, 24]\),
\[
T \rightarrow \frac{aT - ib}{icT + d}, \\
U \rightarrow U - \frac{ic}{2} \frac{\Phi_1 \Phi_2}{icT + d}, \\
\Phi_i \rightarrow \Phi_i(icT + d)^{n_i},
\]

where \( a, b, c, d \in Z, \ ad - bc = 1 \), and where \( \Phi_i \) stands for general matter superfields with weight \( n_i \) under the modular transformation above. The \( \Phi_1 \) and \( \Phi_2 \) fields have modular weight \(-1\). It is easy to verify the invariance of the Kähler function under this transformation provided that
\[
W \rightarrow (icT + d)^{-1}W.
\]

The superpotential should be defined to be consistent with this requirement in addition to \( X \)-charge invariance, and this leads to a constraint on the modular weights of the Yukawa couplings and matter fields. Anomalies occur here also, and must be cancelled in addition to the gauge anomalies.

Now let us try to include the Lazarides-Shafi mechanism by assuming that the superpotential contains a higgs portion,
\[
W_{\text{higgs}} = \lambda \frac{\Phi_1^{a_1} \Phi_2^{a_2}}{M_1^{a_1} + M_2^{a_2}} NH_1 H_2 - \frac{k}{3} \frac{\Phi_1^{b_1} \Phi_2^{b_2}}{M_1^{b_1} + M_2^{b_2}} N^3,
\]

where, for the purposes of anomaly cancellation, the powers are as general as possible. In addition let the other Yukawa couplings have factors of powers of \( \Phi_1 \) and \( \Phi_2 \),
\[
W_{\text{yukawa}} = \lambda_{ijk} \left( \frac{\Phi_1^{l_{ij}}}{M_1^{l_{ij}}} \right)^{m_{ijk}} \left( \frac{\Phi_2^{k_{ijk}}}{M_2^{k_{ijk}}} \right)^{l_{ijk}} z^i z^j z^k = \hat{\lambda}_{ijk} z^i z^j z^k,
\]
where from now on $z^i$ should be understood as a generic visible sector chiral superfield. The Yukawa couplings will eventually absorb factors of $K^{-1/2}$ when the physical scalars are correctly normalised. The low energy Yukawa couplings are then functions of the VEVs of $S, T, U, \Phi_1$ and $\Phi_2$ fields (and possibly other fields which are uncharged under $U(1)_X$), but they are effectively constants in the MSSM, once the hidden sector has been decoupled.

The cosmology of string actions is still being developed, but let us adhere closely to the current thinking (which has been summarized by Lyth and Stewart in ref. [21]). In order for the strings to be formed, the $U(1)_X$ symmetry must at some point be restored after inflation with $|\Phi| \ll M_{Pl}$. This is possible if the $X$ gauge boson is in thermal equilibrium after inflation, the flat direction being lifted away from the origin by thermal mass terms $\sim |\Phi|^2 T^2$ ($T$ being the temperature here). This term holds the $\Phi_i$ fields (here playing the role of ‘flatons’) at the origin, until the temperature drops below $m_{3/2}$. Thus, during the period where $\sqrt{m_{3/2}\rho_\phi} \gtrsim T \gtrsim m_{3/2}$, there is ‘thermal’ inflation of only a few $e$-folds (10 or so) due to the non-zero vacuum energy density. It is assumed of course that the vacuum energy density is zero where $|\Phi|$ eventually obtains its VEV (with $\langle|\Phi|\rangle = \rho_\phi \sim M_{Pl}$).

After inflation the energy is converted into oscillations of the $\Phi_i$ fields, which eventually decay with a reheat temperature

$$ T_D \sim \left( \frac{10^{11} \text{GeV}}{\rho_\phi} \right) \text{GeV}. \quad (37) $$

This gives rise to the cosmological ‘moduli’ problem [22, 20, 21]. Successful nucleosynthesis requires that $T_D \gtrsim 10 \text{MeV}$ implying that $\rho_\phi \lesssim 10^{14} \text{GeV}$. (Note that the possibility that the minimum remain fairly constant is excluded here by the requirement that $|\Phi| \ll M_{Pl}$ initially and by the form of the Yukawa couplings which require $|\Phi| \sim M_{Pl}$ eventually; there is bound to be a large amount of entropy stored up in moduli oscillations.) The only solution appears to be for a second period of thermal inflation to occur which dilutes the abundance of the $\Phi_i$ particles after they have thermalized, involving a second flaton whose VEV should be less than $10^{12} \text{GeV}$ [21].

A second difficulty, effectively rules out the $U(1)_X$ case entirely, when one considers anomaly cancellation and duality invariance in the effective action. The anomaly cancellation conditions (which may be found in ref. [30] for example) must be taken to be

$$ \frac{3}{5} A_1 = A_2 = A_3 = A'_1 = 0. $$

Since the $\Phi_i$ particles are required to be in equilibrium, one should avoid using the Green-Schwarz mechanism to help cancel these [31]. This is because Green-Schwarz anomaly cancellation involves a Fayet-Iliopoulos term in the potential at the Planck scale, which gives the $X$ gauge-boson a mass of $\mathcal{O}(M_{Pl})$ [22]. (For the same reason it is not possible to choose the $U(1)_X$ symmetry to be a gauged $R$-symmetry.) This provides severe restrictions on the allowed Yukawa couplings. Since there is a $U(1)_X$ symmetry, using $\rho_\phi$ to determine the fermion masses and mixings as in ref. [30], would seem a possibility. However one must be careful not to induce flavour changing neutral currents in the low-energy theory. This can be ensured by assuming that both the modular weights (denoted by $n$) and $X$-charges (denoted by $x$) are generation independent. In addition one can allow (and in fact this will be necessary) a generation-degenerate weight for each of the Yukawa couplings. Thus the best one could hope for here is to explain the hierarchy between $\hat{\lambda}_t \hat{\lambda}_b \hat{\lambda}_\tau$. The embedding of the $Z_3$ symmetry in $U(1)_X$ requires that $x_{h_1} = x_{h_2} = x_n$. First one may solve for the gauge anomalies eliminating $x_q, x_u$ and
The quadratic $A'_1$ anomaly is then linear in $x_e$ which is also eliminated. Since the cubic and gravitational anomalies may be cancelled with some extra hidden sector fields, they will not be considered. Next, the condition eq.(34) may be solved, together with the corresponding anomaly cancellation; $B_1 = B_2 = B_3$ using the notation of ref.[25]. Assuming that the top quark mass is not suppressed by very many powers of $\rho_\phi$, one then finds that the Yukawa couplings must themselves have a modular weight. Finally one can solve for $X$-charge invariance of the Yukawa interactions and eliminate $x_{h_1}$, $x_d$ and the $X$-charge of $\hat{\lambda}_\tau$. This leaves the freedom to choose the weights of $\lambda_t$, $\lambda_b$, $\lambda_\tau$ and $k$ (which for example may be done so that $B_1 = B_2 = B_3 = 0$), and the $X$-charges of $\hat{\lambda}_t$ and $\hat{\lambda}_b$, together with $x_{\Phi_1} = -x_{\Phi_2}$, $u_2$, $w_2$, $n_{h_1}$ and $n_q$. The charges and weights may for instance be chosen as follows,

| $x_i$ | $n_i$ |
|------|------|
| $N$   | 3    |
| $H_1$ | 3    |
| $H_2$ | 3    |
| $\Phi_1$ | -1  |
| $\Phi_2$ | 1   |
| $Q_L$  | -1/3 |
| $U_R$  | -5/3 |
| $D_R$  | 7/3  |
| $L$    | -1   |
| $E_R$  | 1    |
| $\lambda$ | -1  |
| $k$    | -1   |
| $\lambda_t$ | -5/3 |
| $\lambda_b$ | -2/3 |
| $\lambda_\tau$ | 4   |

The total superpotential becomes,

$$W = \lambda \frac{\Phi^9_1}{M_{Pl}} N H_1 H_2 - k \frac{\Phi^9_1}{3 M_{Pl}^3} N^3$$

$$+ \lambda_t \frac{\Phi_1}{M_{Pl}} Q_L H_2 U_R + \lambda_b \frac{\Phi^5_1}{M_{Pl}^5} Q_L H_1 D_R + \lambda_\tau \frac{\Phi^4_1 \Phi_2}{M_{Pl}^5} L H_1 E_R$$

which is invariant and anomaly free under $U(1)_X$ and the transformation in eq.(33). Clearly this example is not the most elegant of solutions; string models with such complicated weights and charges have not been derived, and in addition the higgs couplings are suppressed by a factor of at least $\rho_\phi/M_{Pl}$, irrespective of the values one takes for any of the free parameters above. For reasonable electroweak symmetry breaking $\rho_\phi/M_{Pl}$ will not be small.

So at least for the case of $U(1)_X$, this mechanism does not allow very convincing models to be constructed. The alternative is to embed the $Z_3$ in the centre of a non-abelian group which could be $SU(3n)$ or $E_6$. This certainly eases anomaly cancellation.
although the presence of the $N^3$ term is particularly problematic since, being symmetric, it requires large multiplets. Let us finish this section by considering an example, based on the $SU(3)_c \times SU(3)_L \times SU(3)_R$ unification (see ref. [33] for a summary of the phenomenology).

In this model, the particle content is,

\[
Q = (3, 3, 1) = \begin{pmatrix} u \\ d \\ D \end{pmatrix}_L
\]
\[
q = (\bar{3}, 1, \bar{3}) = \begin{pmatrix} u \\ d \\ D \end{pmatrix}_R
\]
\[
L = (1, \bar{3}, 3) = \begin{pmatrix} H^0_\nu \\ H^+_2 \\ H^0_1 \\ \nu_L \\ \nu_R^c \\ N \end{pmatrix}_L,
\]

where there are three generations, and colour indices on the quarks. The pattern of symmetry breaking is

\[
SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}
\]
\[
\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y.
\]

The hypercharge is given by

\[
Y = T^3_R + \frac{1}{\sqrt{3}}(T^8_L + T^8_R)
\]

where $T^i$ are the standard Gell-Mann matrices for $SU(3)$. Clearly with this assignment of fields, the $Z_3$ symmetry is simply given by a rotation in the

\[
T^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
\]
direction. The superpotential is of the form

\[
W_{\text{higgs}} = \lambda \epsilon^{ijn} \epsilon_{ikm} L^i_j L^k_L m^m_n - \frac{k \Phi^{ijn}_{ikm} L^i_j L^k_L m^m_n}{M_{\text{Pl}}} + \lambda_{\text{yuk}} L^i_j q^j Q_i.
\]

The first term is responsible for the $\lambda NH_1 H_2$ terms as well as the lepton Yukawa couplings. The second term is responsible for the $N^3$ coupling, for which the superfield, $\Phi = (1, 10, \bar{10})$ had to be introduced. Since the anomaly contribution of the 10 is 27 times that of the fundamental representation, the simplest way to cancel anomalies is to include an additional $\Phi' = (1, \bar{10}, 10)$ field.

The $\Phi$ field must get VEV along the flat $\Phi^{333}$ direction, giving the first stage of symmetry breaking. A VEV along $\Phi^{333}$ can give the second. The third term provides the Yukawa couplings for the quarks, and also a mass for the $D$-quark which (through two possible triplet quark couplings which have been omitted here) would otherwise mediate proton decay. This requires a high VEV for at least one of the $N$ fields which, since it
contributes to the first stage of symmetry breaking, must also be in a flat direction. With a suitable choice of generation dependence in the couplings, one can then arrange for the low energy particle content to be that of the NMSSM.

Finally, the stability of the strings requires that $\pi_1(G/H) \neq 0$. Initially the $Z_3$ symmetry is given by rotations through $\exp(2\pi i/3)$ for every $L_j$. As long as only $\Phi_{333}$ has a VEV, this remains as a global symmetry so that

$$\pi_1(G/H) = \pi_1\left(\frac{SU(3)_L \times SU(3)_R}{SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times Z_3}\right) = \pi_0(H) = Z_3$$

since $\pi_1(G) = \pi_0(G) = 0$. (The discrete symmetry may be larger if there are zeroes or degeneracies in the Yukawa coupling.) So one expects $Z_3$ strings to form. However, a VEV for any of the three generations of $N$ break the $Z_3$ as well. This means that no $N$ should get a VEV until $T \sim M_W$ otherwise the strings would be removed by domain walls before electroweak symmetry breaking (since the $Z_3$ symmetry that would remain in the low energy theory would not be embedded in $SU(3)$). For this one can appeal to the cosmological scenario described earlier, with all the $N$ VEVs remaining trapped at the origin until $T \sim m_3/2$, when two of them acquire Planck scale VEVs.

These examples show that, at least in principle, it is possible to make discrete symmetries in models such as the NMSSM innocuous by embedding them in continuous gauge symmetries at the Planck scale. However the success or otherwise of this mechanism depends rather heavily upon the cosmology. Because of this an exact discrete symmetry at the weak scale still seems to be unlikely, and models such as the NMSSM remain questionable.

5 The ‘Indiscrete’ Symmetry Case

In this section the case where there is no discrete symmetry at the weak scale is examined. In ref.[5] it was shown that breaking the discrete symmetry by gravitationally suppressed terms cannot remove the walls before nucleosynthesis without destabilising the gauge hierarchy. This means that either $\mu \neq 0$ or both $\mu' \neq 0$ and $k \neq 0$ in addition to $\lambda \neq 0$. It will be shown that presently there exist two ways in which a gauge singlet may be accommodated at the weak scale without fine-tuning. These are $R$-symmetry and duality symmetry in a string effective action. What these symmetries have in common is that under them one requires different charges for the superpotential and Kähler potential.

First let us recapitulate the arguments of ref.[5], by considering adding an ordinary gauge symmetry to the Lagrangian which exists at the Planck scale. For simplicity, again take this to be a $U(1)_X$ symmetry. The effective couplings at the weak scale are in general arbitrary functions of hidden sector fields which carry charge under the new $U(1)_X$ which shall be referred to collectively as $\Phi$ (with $\xi = \Phi/M_{Pl}$). It is simple to see that one cannot use this symmetry to forbid terms linear in $N$ thereby avoiding fine-tuning (again assuming there is an explanation for the smallness of $\mu$ and $\mu'$ in which all couplings are of order unity).

If $\mu(\xi) \neq 0$ then $\mu(\xi)$ must have the same charge as $\lambda(\xi)N$ and therefore $(\mu(\xi))^\dagger \lambda(\xi)N$ is uncharged. If both $\mu' \neq 0$ and $k \neq 0$ then $\mu'(\xi)$ must have the same charge as $k(\xi)N$ and therefore $(\mu'(\xi))^\dagger k(\xi)N$ is uncharged. Either way an operator of the form

$$15
$f(\xi, \overline{\xi})M_{\text{Pl}}N + \text{h.c.}$ is allowed in the Kähler potential. Typically $\langle e^G \rangle = \langle e^K W \overline{W} \rangle = \mathcal{O}(m_{3/2}^2)$, so that the terms

$$m_{3/2} M_{\text{Pl}} f(\xi) H_1 H_2 ; m_{3/2} M_{\text{Pl}} f(\xi) N^2 ; m_{3/2}^2 M_{\text{Pl}} f(\xi) N$$

also appear in the potential, unless one sets $f(\xi) M_{\text{Pl}} N = 0$ initially.

It should first be made clear that the ‘explanation for the smallness of $\mu$ and $\mu'$’ is here taken to be something like the Giudice-Masiero mechanism [3], in which the couplings responsible are of order unity. There also exist models in which the smallness of $\mu$ and $\mu'$ is created by non-renormalisable couplings of the singlet to fields whose VEVs are small. In fact in this case things are worse, because although at tree level the terms in eq. (44) are indeed small, the singlet field $N$ couples to the supersymmetry breaking in the hidden sector through a tadpole diagram. This can be seen from the logarithmically divergent terms appearing at one loop from $[10]$

$$V_{1\text{-loop}} = \frac{\log \Lambda^2}{32\pi^2} \int d^4\theta e^{2K/3 M_{\text{Pl}}^2} \varphi \overline{\varphi} W_{ij} \overline{W}^{ij} + \ldots$$

where here the indices denote differentiation which is covariant with respect to the Kähler manifold and $\varphi$ is the chiral compensator. (Supersymmetry breaking is embodied in the $\theta$ dependence in the VEVs of $K$ and $\varphi$.) These are the divergent terms which lead to logarithmic running of the soft-breaking scalar masses. However, if there is a $\mu$-term produced directly in the superpotential from some product of hidden sector fields ($\mu = \Phi^m / M_{\text{Pl}}^{m-1}$ for example), the contribution above includes

$$\frac{\log \Lambda^2}{32\pi^2} \int d^4\theta \mu(\Phi) \lambda^l \langle N angle = \frac{\log \Lambda^2}{32\pi^2} \lambda^l F_N \frac{m_{\Phi}^{m-1} F_{\Phi}}{M_{\text{Pl}}^{m-1}} \sim \left( \frac{M_{\text{Pl}}}{M_W} \right)^{1/m} M_W F_N^l.$$

where since $\Phi$ is a hidden sector field, it is assumed that $F_{\Phi} \sim M_W M_{\text{Pl}}$, and that also $\langle |\phi|^m \rangle \sim M_W M_{\text{Pl}}^{m-1}$ in order to get $\mu \sim M_W$. This leads to a value of $F_N \gg M_W$ unless $m$ is extremely large, destabilising the gauge hierarchy.

One may easily demonstrate that the above arguments also apply when the additional group is non-abelian. For example consider extending the $SU(3)^3$ example of the previous section by adding extra $\mu$ and $\mu'$ terms in the Kähler potential. The $\mu' N^2$ term may be generated from the coupling $\Theta^{ij l} L_i^j L_l^k / M_{\text{Pl}}$ where $\Theta = (1, \overline{5}, 6)$ and $\Theta_{33}^{33}$ acquires a VEV. But now the linear term $(\Theta \Phi_i^l)^j L_j^l$ is also allowed. Similarly, the $\mu H_1 H_2$ term requires $(\epsilon \epsilon \epsilon \overline{\epsilon})^{ij l} L_i^j L_l^k / M_{\text{Pl}}$ where $\Theta = (1, 3, \overline{3})$ and $\Theta_{33}^3$ acquires a VEV. This then allows the coupling $\Theta_j^l L_j^l = \Theta_{33}^3 N$. (In terms of the component fields, in this case the low energy singlet $N$ mixes with singlets which get a VEV of $\mathcal{O}(M_{\text{Pl}})$.)

One should bear in mind that these terms do not necessarily lead to a destabilisation of the gauge hierarchy since in most (but not all) cases, if one sets the linear coupling to zero in the first place, it remains small to higher order in perturbation theory. So this is merely a fine-tuning problem. One might also argue that this fine-tuning problem is of a less serious nature than the $\mu$-problem, since in the latter the coupling has to be very small, whereas here the coupling may just happen to be absent (as for example are superpotential mass terms in string theory). However, in addition to the simplest operators, there are generally many more (possibly an infinite number of) operators which must be set to zero.
This becomes evident when one continues with the $U(1)_X$ case. Take for example the Kähler function in ref. [3];
\[
G = z_i z_i^\dagger + \Phi \Phi^\dagger + \Phi^\prime \Phi^{\prime \dagger} + \left( \frac{\alpha}{M_{Pl}} \Phi^\dagger H_1 H_2 + \frac{\alpha'}{M_{Pl}} \Phi^{\prime \dagger} N^2 + \text{h.c.} \right) + \log |h(z, \Phi) + g(\Phi, \Phi^\prime)|^2, \tag{47}
\]
where again, $z^i$ are the visible sector fields, and $h(z, \Phi)$ is the superpotential of the NMSSM given in eq. (4) with charge assignments as before. Under the $U(1)_X$, $\Phi^\prime$ must have charge 2 and so the operator
\[
\Phi^\prime \Phi N \tag{48}
\]
is also charge invariant and must be set to zero. Clearly there is a very large number of additional, nonrenormalisable operators $f(\Phi, \Phi^\prime)N$ which should not appear (assuming that $\langle \Phi \rangle = \mathcal{O}(M_{Pl})$) as well as $f(\Phi, \Phi^\prime)\Phi^2 N^2$ and $f(\Phi, \Phi^\prime)\Phi^2 H_1 H_2$. (Here $f$ implies general polynomials with the correct charge, for example $\Phi^2 \Phi^a$.) These are the operators which directly destabilise the gauge hierarchy. In addition there are operators which destabilise the hierarchy through divergences; for example at one loop order the potential receives contributions of the form $\Lambda^2 R^k_n (We^K)_k (We^K)^n$, where $R^k_n$ is the Ricci tensor for the Kähler manifold, and $k$ and $n$ subscripts denote differentiation with respect to the fields (raising and lowering of indices being done by $K_{\alpha}^\dagger$). Jain in ref. [10] has shown that destabilising divergences occur for any couplings of the form $f(\Phi, \Phi^\prime) z z z$. This means that the operators $f(\Phi, \Phi^\prime) N H_i H_i^\dagger$ and $f(\Phi, \Phi^\prime) N N N^\dagger$ are also disallowed. Progressing to higher loop order, the operators $\Phi^4 \Phi^4 N^4$, $\Phi^4 \Phi^4 N^2 H_1 H_2$ and $\Phi^4 \Phi^4 (H_1 H_2)^2$ appearing in the superpotential destabilise the gauge hierarchy through two and three loop diagrams, and so on. Obviously the degree of fine-tuning decreases with higher order since each loop gives a factor $\Lambda^2/(16\pi^2)$ where $\Lambda$ is a cut-off, and involves more Yukawa couplings. It therefore seems reasonable to assume that contributions which are higher than six-loop are unable to destabilise the hierarchy. Upto and including six loop, the following operators could be dangerous if they appear in the superpotential (multiplied by any function of $f(\Phi, \Phi^\prime)$), since one can write down a tadpole diagram using them (together with the trilinear operators of the NMSSM);

| Operator | Loop-order of diagram |
|----------|----------------------|
| $N^2, H_1 H_2$ | 1 |
| $N^3, N^2 H_1 H_2$ | 2 |
| $(H_1 H_2)^2, N(H_1 H_2)^2, N^3(H_1 H_2), N^5$ | 3 |
| $N^4(H_1 H_2)^2, N^4(H_1 H_2), N^7$ | 4 |
| $N(H_1 H_2)^3, N^2(H_1 H_2)^3, N^4(H_1 H_2)^2, N^6(H_1 H_2), N^8$ | 5 |
| $N^2(H_1 H_2)^4, N^4(H_1 H_2)^3, N^6(H_1 H_2)^2, N^8(H_1 H_2), N^{10}$ | 6 |

Notice that, since the leading divergences involve chiral or antichiral vertices only, an operator must break the $Z_3$ symmetry in $h(z, \Phi)$ in order for it to be dangerous (so that for example $N^2(H_1 H_2)^2$ does not destabilise the hierarchy). These are all the operators which lead to a superspace tadpole diagram, but it turns out that only those which have even dimension are dangerous. This can be seen as follows.
First, using the supergraph rules described in refs.[10,34] for the leading rigid supergraphs, it may easily be seen that only diagrams with an even number of vertices are dangerous. A chiral vertex, $A$, with $L_A$ internal legs, throws $L_A - 1$ of the $D^2$ operators onto the surrounding propagators. These may be manipulated in the standard manner by partial differentiation to expose the $\delta(\theta - \theta')$ function on a propagator, allowing the integrations over $\theta$s to be carried out ($\theta$ and $\theta'$ belong to the vertex at either end of the propagator). Products of three or more $D^2$ may be reduced using the identities $D^2 D^2 D^2 = 16 \partial^2 D^2$ and $\overline{D}^2 D^2 \overline{D}^2 = 16 \partial^2 \overline{D}^2$. Pairs of $D^2$ operators are also removed on integration over $\theta$s, since

$$\int d^4 \theta' \delta(\theta - \theta') D^2 \overline{D}^2 \delta(\theta - \theta') = 16.$$ (49)

A single $D^2$ may also be removed by acting on the $(e^{K/3}/\phi \overline{\phi})_{\text{classical}}$ factors on each propagator [10], but this renders the diagram innocuous. The end result is an integral over a single $\theta$, but clearly only if the initial total number of $D^2$ and $\overline{D}^2$ operators was even. This number is given by

$$\sum_A (L_A - 1) = 2P - V = \text{even}$$ (50)

where $P$ is the number of propagators, and $V$ is the number of vertices; hence the number of vertices should be even.

Now consider constructing a tadpole diagram beginning with a single vertex with an odd number of legs. When more vertices are added, if the extra non-renormalisable operator has an odd number of superfields then there are only vertices with odd numbers of legs to choose from. Adding a single vertex with an odd number of legs changes the total number of external legs by an odd number. So in order to have a tadpole diagram one has to add an even number of vertices implying that $V$ is odd, and that the diagram is therefore harmless. Hence only operators with even numbers of superfields can be harmful to the gauge hierarchy. Counting in addition the $N$ operator itself, this means that in this case 17 operators (multiplied by any appropriately charged function $f(\Phi, \Phi')$) must be set to zero by hand.

The reason that it has not been possible to forbid divergences linear in $N$ in the models that have been discussed here and in ref.[3], is that the Kähler potential and superpotential have the same charges (i.e. zero). There are however two available symmetries in which the Kähler and superpotentials transform differently. These may accommodate singlet extensions to the MSSM simply and without fine-tuning.

The first is gauged $U(1)_R$-symmetry [3,4]. In this case the Kähler potential has zero $R$-charge, but the superpotential has $R$-charge 2. This means that the standard renormalisable NMSSM higgs superpotential,

$$W_{\text{higgs}} = \lambda N H_1 H_2 - \frac{k}{3} N^3,$$ (51)

has the correct $R$-charge if $R(N) = 2/3$ and $R(H_1) + R(H_2) = 4/3$. So consider the
Kähler potential

$$\mathcal{G} = z^i z^i + \Phi \Phi^\dagger + \Phi' \Phi'^\dagger + \left( \frac{\alpha}{M_{P1}} \Phi' H_1 H_2 + \frac{\alpha'}{M_{P1}} \Phi' N^2 + \text{h.c.} \right) + \log |h(z) + g(\Phi, \Phi')|^2,$$

(52)

where $h(z)$ is the superpotential involving just visible sector fields and $\Phi, \Phi'$ again represent hidden sector fields with superpotential $g(\Phi, \Phi')$ (they may represent arbitrary functions of hidden sector fields in what follows). Both $\Phi$ and $\Phi'^\dagger$ appear here in order to prevent unwanted couplings being allowed in the superpotential which must be a holomorphic function of superfields.

Invariance of the Kähler potential requires that $R(\Phi) + R(\Phi'^\dagger) = 4/3$. If all the $R$-charges are chosen to be positive, then the terms in eq.(51) are clearly the only functions which can appear in the superpotential (since all higher dimension ones have $R$-charge greater than 2). (The $R$-charges of $\Phi$ and $\Phi'^\dagger$ must of course be chosen to be sufficiently obtuse; for example $R(\Phi) = 16/3$, $R(\Phi') = 4$, is adequate, because the lowest $R$-charge one can make with them is $\pm 4/3$.) Moreover it is easy to see that with this set of $R$-charges there can never be a coupling which is linear or trilinear appearing in the Kähler potential. In fact the operators

$$\tilde{O}_{mkl} = f N^{2m+2k-2l} (H_1 H_2)^l (H_2^\dagger H_1^\dagger)^k,$$

(53)

where $f$ is an arbitrary $R$-invariant function, satisfy

$$- m \left( R(\Phi) + R(\Phi'^\dagger) \right) = R(N^{2m}) = R(\tilde{O}_{mkl}).$$

(54)

The quadratic coupling in eq.(52) results when $m = 1$, and when $m = 2$, the couplings $N^4$, $N^2(H_1 H_2)$ and $(H_1 H_2)^2$, but this time appearing only in the Kähler potential, not the superpotential. Such couplings do not destabilise the hierarchy. In particular the dangerous trilinear terms in the Kähler potential have been avoided.

The second symmetry one can use to forbid terms linear in $N$ is target space duality. Consider for example the Kähler potential defined in eq.(25), but now, as in refs.\cite{24, 26}, identifying $\Phi_1$ and $\Phi_2$ with the higgs superfields $H_1$ and $H_2$ both of which have weight $-1$. This identification generates a $\mu H_1 H_2$ term in the low-energy lagrangian \cite{24, 25, 26}. The remaining dependence on the matter fields may be written as

$$\delta K_{\text{matter}} = K^k_k z^i z^j + \ldots$$

(55)

and is invariant under the modular transformation in eq.(53). Here the ellipsis represents terms higher order in the expansion. Since the superpotential must transform as in eq.(54), one can consider the usual NMSSM superpotential if

$$3n_N + n_k = -1$$

$$n_N + n_\lambda = +1,$$

(56)

which, taking $n_k = 0$, gives $n_N = -1/3$ and $n_\lambda = 4/3$.

With this assignment of duality charges the first extra term which can appear in the superpotential is the seventh order $\lambda^3 N^3(H_1 H_2)^2/M_P^4$ operator. Since this operator has an odd number of fields it cannot destabilise the hierarchy by itself for reasons discussed
earlier. Furthermore only odd operators can result with this choice of weights, since the required weight $-1$ is an odd multiple of $n_N$ whilst those of $(H_1H_2)$ and $\lambda$ are both even multiples of $n_N$.

As for the Kähler potential, one expects the extra terms in the expansion of eq. (55) to be multiplied by powers of $(T + \bar{T})$. Thus terms in which the holomorphic and anti-holomorphic weights are the same may be allowed. The $\lambda N^2(H_1H_2)$ coupling (which in fact has weight zero) is the lowest dimension operator which satisfies this criterion.

There are clearly many ways in which one could devise similar models. A perhaps more obvious example would be models in which the superpotential transforms with weight $-3$. There all the physical fields could be given weight $-1$, with the couplings having weight 0. It is then clear that only trilinear couplings can exist in the superpotential, and only even-dimension terms can appear in the Kähler potential.

6 Conclusions

The possibilities for extending the MSSM by a singlet field have been examined, and constraints from fine-tuning, primordial nucleosynthesis and cosmological domain walls have been applied. The most appealing models have no discrete or continuous global symmetries at the weak scale.

For the case of continuous global symmetry, the existence of an axion, and the cosmological bounds associated with it, make a reasonable phenomenology difficult to achieve without fine-tuning. At the very least, one can rule out the Peccei-Quinn mechanism here, unless the continuous symmetry is broken to a high order, discrete symmetry by gravitationally suppressed terms. For this to be successful additional singlets are required.

When there is a discrete symmetry at the electroweak scale, breaking it by gravitationally suppressed terms cannot remove the associated domain walls before nucleosynthesis, without reintroducing the hierarchy problem. A suitable solution to the ensuing domain wall problem may be to embed the discrete symmetry in a gauge symmetry. Although the cosmology is then necessarily rather complicated, it seems that, at least for the $SU(3)^3$ model we discussed, the emerging picture of string cosmology may be able to accommodate it.

The most efficient way to accommodate the singlet is however to impose a gauged $R$-symmetry or a target space duality on the full theory including gravity. In this case one expects all couplings (i.e. $\mu H_1H_2$, $\mu N^2$, $\lambda NH_1H_2$ and $kN^3$) to be possible in the weak scale effective theory. The phenomenological implications of these more general cases, have been discussed recently in ref. [35].

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