Effect of distortional hardening behaviour on material responses of pure titanium sheets during hydraulic bulge test

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Abstract. Plastic deformation characteristics of pure titanium sheets are identified by conducting tensile tests and hydraulic bulge test. Distortional hardening behaviour of the tested material is examined experimentally by comparing the uniaxial tensile results of three specimens taken from different orientations. This study verifies the effect of the distortional hardening behaviour on the material responses for pure titanium sheet by conducting a finite element analysis for the bulge test. For this purpose, a material model is calibrated based on the experimentally measured data. In this model, both yield surface and potential surface were well described by the Yld2000-2d function under the theory of associated flow rule. To match with experimental observation, all parameters of the yield function are implied as functions of the equivalent plastic work during plastic deformation. Additionally, a modified Voce hardening function was applied to reproduce the hardening behaviour for the examined material over large strains. Based on simulation results, it is concluded that the distortional hardening behaviour strongly affects the evaluation of the apex height during the bulge test.

1. Introduction
Hexagonal close packed material such as commercial pure titanium (CP Ti) sheet presents strong anisotropic hardening behavior and distortional hardening (DH) behavior as such yield surface of the tested material was distorted during plastic deformation. Conventionally, the DH behavior was ignored in numerical analysis for such material to reduce computational time [1, 2]. Besides the anisotropic yielding behavior, it has been recently reported that the DH behavior was significantly influenced the surface strain evolution of deformed parts [3]. This study verifies the effect of the DH behavior on numerical analysis for bulging a CP Ti sheet experimentally. A material model for describing CP Ti sheet in finite element analysis (FEA) was developed. Experimental data obtained from uniaxial tensile tests and hydraulic bulge test were used to calibrate flow curve and yield function. Due to the observation of distortional hardening behavior of CP Ti sheet, an evolutionary constitutive model was then operated under the condition of equivalent plastic work. The validity of developed material model is verified by comparing simulation results of bulge test with experimental measurement data.

2. Experimental procedure
2.1. Uniaxial tensile tests
Standard tensile tests were carried out according to the ASTM-E8 standard for the CP Ti sheet material. Tensile specimens were cut out from a sheet of 0.5 mm thickness by a water jet cutting machine along three angles, 0°, 45°, and 90°, from the rolling direction. All the tests were conducted at a constant speed of 3 mm/min, which corresponds to an average strain rate of $10^{-3}$ s$^{-1}$ during uniform deformation. Strains were measured using an extensometer with 50 mm gauge length. Additionally, the Lankford coefficients (or R-values) were measured in different orientations to represent the anisotropy of sheet metal. Therefore, the R-values of CP Ti specimens taken from different orientations from the rolling direction were measured at several levels of equivalent plastic strain.

![Figure 1. Uniaxial tensile test data of CP Ti sheet (a) Engineering stress-strain curves (b) True stress-strain curves](image)

Figure 1 shows uniaxial tensile test results for the CP Ti sheet and Table 1 lists the mechanical properties obtained from these tests. It is seen that the CP Ti is a strong anisotropic material in which the differences among the flow curves in different directions are significant, especially in term of initial yield stress, slope of hardening curve, tensile strength, and elongation. In detail, the initial yield stress of 90° specimen was 20% higher than that of 0° specimen, while R_{90} was mostly three times larger than R_{0}.

| Table 1 Material properties of CP Ti sheets obtained from uniaxial tensile tests |
|---------------------------------|-----------|-----------|-----------|
| Orientations                   | 0 deg     | 45 deg    | 90 deg    |
| Young’s modulus [GPa]          | 101       | 101       | 102       |
| Initial yield stress [MPa]     | 193       | 206       | 232       |
| Ultimate tensile strength [MPa]| 311       | 270       | 288       |
| Lankford coefficient           | 1.7       | 3.98      | 5.5       |

According to Figure 1a, material orientations are strongly sensitive to the strain hardening behaviour of CP Ti sheets. In detail, ultimate tensile strength (UTS) of the specimen taken from the rolling direction is 311 MPa at an engineering strain of 34% (equivalent to 62% of its elongation). However, the UST of 45° specimen and 90° specimen are 270 MPa and 288 MPa, respectively. Their uniform elongations are 14.7% and 9.1%, respectively, which correspond to the elongation of 22.6% and 20.1%. Furthermore, the failure on the specimen was delayed fairly after the maximum force was reached. The observation requires advanced modelling of CP Ti characteristics at large deformation due to the material’s instability.

Moreover, it is shown in Figure 1b that the stress ratios $\sigma_{90}/\sigma_0$ and $\sigma_{45}/\sigma_0$ were decreased extremely according to the increment of the equivalent plastic strain. The decrease is more significant after the Maximum Tensile Force Point (MTFP) that is the point in the true stress-strain curve corresponding to...
the UTS in the nominal stress-strain curve. Change of the stress ratios verifies the distortion of yield surface of CP Ti sheet during plastic deformation.

2.2. Hydraulic bulge test

Hydraulic bulge tests have been widely performed in order to experimentally identify the stress-strain relation at equi-biaxial tension mode. In this study, hydraulic bulge tests according to the ISO 16808:2014 standard were conducted for the CP Ti sheets. Based on the membrane theory, the evolution of the flow stress at an equi-biaxial tension, \( \sigma_b \), and the through-thickness strain, \( \varepsilon_t \), at the apex region were derived from three measurable factors in the bulge test: hydraulic pressure, \( p \); bulge radius, \( R_d \); and pole region thickness, \( t_d \) [4].

![Figure 2. Calibrated flow curve](image)

**Figure 2.** Calibrated flow curve (a) Comparison between bulge test data and uniaxial tensile test data (b) Equivalent stress-strain curve

Figure 2a shows a comparison between calculated stress-strain curve obtained from the bulge test and that obtained from the uniaxial tensile test in the rolling direction. It is clear that flow stresses derived from the bulge test are always higher than that of the uniaxial tensile test. Additionally, the bulge testing process provides the stress-strain relationship to a larger range of strains.

Moreover, the derived stress-strain data from the bulge test are converted to an equivalent stress-strain curve by using a convert factor which is a function of equivalent plastic work and be calculated based on the principle of equivalent plastic work. Details on converting process can be found in the work of Kuwabara et al. [5]. Figure 2b shows the calculated equivalent stress-strain data and their approximation. Unlike steel or aluminium sheets, stress-strain curve of CP Ti sheet presents a phenomenon called as three-stage of deformation as shown in this figure. The observation confirms the results of the previous work of Becker and Pantleon [6]. A modified Voce hardening model is therefore applied to capture the phenomenon in whole strain range.

Strain hardening function:  
\[
\sigma^Y = 287.83 - 80.84 \exp(-24.20 \varepsilon) + 323.32 \varepsilon(1-\varepsilon) \text{ (MPa)} \tag{1}
\]

3. Constitutive modelling

Applying the finite element analysis requires three constitutive hypotheses consisting of flow curve, yielding criterion and flow rule. Since the flow curve of the CP Ti sheet was calibrated in the previous section, this section is set out to identify the flow rule and yield function for the tested material. Associated flow rule was widely accepted for sheet metal forming analysis. Under the theory of associated flow rule, the plastic strain increment can be defined as:

\[
d\varepsilon^P = d\lambda \frac{\partial f}{\partial \sigma}
\]

\( \lambda \) is the second Piola-Kirchhoff stress tensor.
where $d\lambda$ is a scalar measuring amount of the plastic flow rate; $f=f(\sigma)$ is the yield function as well as the potential function. In this study, yld2000 yield function [7] was applied to describe yield surface of the CP Ti sheet. Definition of the equivalent stress in this function can be expressed as:

$$\bar{\sigma} = \left\{ \frac{1}{2}\left( |X'_1 - X'_2|^m + |2X''_1 + X''_2|^m + |2X'_2 + X'_1|^m \right) \right\}^{1/m}$$

(3)

where $m$ is a yield surface exponent. The components $X'_{1,2}$ and $X''_{1,2}$ are the principal values of two linear transformations $X'$ and $X''$ of stress tensor. There are eight parameters (i.e. $\alpha_1$~$\alpha_8$) needed to calibrate in the yld2000 yield function. Details on the definition of these parameters can be found in the work of Barlat et al. [7].

Figure 3. Calibration of yield function (a) Evolution of stress ratios (b) Evolution of parameters of yld2000 yield function

Figure 4. Evolution of normalized yield surface

In order to calibrate parameters of the yield function, eight experimental data including $\sigma_0$, $\sigma_{45}$, $\sigma_{90}$, $\sigma_b$, $R_0$, $R_{45}$, $R_{90}$, and $R_b$ are required. All the strain terms (i.e. $R_0$, $R_{45}$, $R_{90}$, and $R_b$) could be assumed to be constants during plastic deformation, as suggested in the previous work of Hama et al. [8]. However, the stress terms (i.e. $\sigma_0$, $\sigma_{45}$, $\sigma_{90}$, and $\sigma_b$) changed during plastic deformation. Figure 3a show the evolution of normalized stresses according to the increment of the equivalent plastic work. These evolutions were approximated by the following equation:

$$y = a_1 + b_1 \exp(-c_1 x) + b_2 \exp(-c_2 x)$$

(4)
where $y$ denotes for the approximations; $a_1, b_1, c_1, b_2, c_2$ are material constants; $x$ denotes for the equivalent plastic work.

Then, several sets of reference data of stress terms and strain terms were determined at different levels of the equivalent plastic work. For each set of reference data, all parameters of the yld2000 yield function were identified with a note that the coefficient $m$ was pre-determined i.e. $m=9$ in this study. A Newton-Raphson solver was developed for this purpose. Thereafter, fitting equations within a formulation expressed in Eq. 4 were applied to describe the evolution of parameters according to the increment of the equivalent plastic work. Extending these equations to larger values of the equivalent plastic work is in accordance with extending the evolution of yield surface to larger strains. Figure 4 shows derived yield loci of CP Ti sheet during plastic deformation. It is seen that the function was able to match with measured experimental data until wide ranges of the equivalent strain.

4. Simulation of bulge test

An FE model of the hydraulic bulge test for the CP Ti sheets was built up in ABAQUS/EXPLICIT package version 6.14. The blank was modelled by using shell element (S4R) with 9 integration points through thickness while the die and blank holder were assumed to be rigid bodies. One-quarter of the blank was analysed because it is a symmetrical problem. Mesh generation was shown in Figure 5 where the increment of an element was $2.5^\circ$ in the circumferential direction and 1 mm in the radial direction. In addition, the blank holder force was set as same as experimental that was 80 ton for CP Ti sheet. Hydraulic pressure generated the deformation of the blank is approximated by a nonlinear function. A user’s subroutine VUAMP was developed to describe the pressure in ABAQUS/EXPLICIT. A detail on developing the VUAMP subroutine can be found in our previous study [9].

![Figure 5. Mesh generation on the specimen and boundary condition for bulge test](image)

The material model developed in previous sections were applied to FEA of bulge test for CP Ti sheet by using another user’s subroutine VUMAT. An algorithm of cutting plane method is applied to update the stress components during simulation. In this algorithm, a trial stress increment is calculated to predict the new stress state at each time step. If the new stress state is located outside of the yield surface, a plastic correction is used to take it back into the yield surface. Further detail on the integration process can be found in the work of Pham and Kim [9].

Figure 6 shows comparisons between simulation results and experimental observations. According to Figure 6a, FE model using the yield function calibrated from an early stage of plastic deformation underestimated the evolution of the apex height while the FE model obtained at a large strain overestimated the experimentally measured data. In contrast, the FE model coupling the DH behaviour predicted perfectly the apex evolution. Additionally, Figure 6b confirms the accuracy of the thickness distribution on the surface of the FE model. Consequently, the DH behaviour strongly influences the material’s response during bulge test, especially in the case of CP Ti sheet.
Figure 6. Comparison between experimentally measured data and simulation results (a) height of the apex vs hydraulic pressure (b) thickness distribution

5. Conclusions

Anisotropic yielding behavior and the distortional hardening behavior of CP Ti sheet were verified experimentally by comparing experimental data obtained from uniaxial tensile tests and hydraulic bulge test. An approach to reproduce the distortional hardening behavior was presented meanwhile the yld2000 yield function was used to well describe the anisotropy of the tested material. Additionally, a modified Voce hardening model was applied to capture the strain hardening behavior until a large strain. The developed material model was then utilized to simulate the bulge test.

Simulation results showed that the FE model including the DH behavior provided the best prediction for the evolution of apex’s height and thickness distribution on the surface of the deformed specimen. Therefore, the DH behavior is an important factor that strongly influences the material’s response during plastic deformation. Additional tests are needed to experimentally exhibit complete yield surface evolution for the CP Ti sheet, especially in the plane-strain forming mode.

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References

[1] Gao E, Li H, Kou H, Chang H, Li J and Zhou L, 2010 Rare Materials 29(1) 108
[2] Gatea S, Xu D, Ou H and McCartney G, 2017 Int J Adv Manuf Technol
[3] Aretz H, 2008 Int J Plast 29(9) 1457-1480
[4] Lemoine X, Iancu A, Ferron G, In Proceeding of the 14th International ESAFORM conference
[5] Kuwabara T, Mori T, Asano M, Hakoyama T, Barlat F, 2017 Int J Plast 93 164-186.
[6] Becker H, Panteleon W, 2013 Com Mat Sci 76 52-59.
[7] Barlat F, Brem J C, Yoon J W, Chung K, Dick R E, Lege D J, Pourboghrat F, Choi S H, Chu E, 2003 Int J Plast 19 1297-1319
[8] Hama T, Kobuki A and Takuda H, 2017 Int J Plast 91 77-108
[9] Pham Q T, Kim Y S, 2017 Met Mater Int 2(23) 254-263