On the Collective Mode Spectrum in FQHE

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Abstract

We consider the Bethe Salpeter Equation (BSE) for a fractionally filled Landau level. A phenomenological discussion of the 1/3 Laughlin's state is performed by assuming an ansatz for the one-particle propagator. The BSE is solved in this approach and it predicts an instability under the formation of charge density oscillations for a wide range of the one-particle gap parameter values in contrast with previous single mode approximation results. However, the conclusion is compatible with the one obtained within a composite fermion description done by us before and with the saturation of the zero momentum oscillator strength sum rule by the cyclotronic resonance. Further studies should be done in order to understand the discrepancy.

72.20.My, 71.45Gm, 73.40Lq
I. INTRODUCTION

The collective mode properties of the ground state within the FQHE research activity has been the subject of interest in the literature [1–4]. A pioneering paper on this theme [1] is based on some ideas already introduced by Feynman [5] in the theory of Superfluidity. The general conclusion of this work indicated the stability of the considered ground state due to the appearance of a gap in the spectrum of the lowest energy collective excitation.

More recently, in [2], the subject has been considered through a Chern-Simons formulation of the composite fermion model. However, these results only cover the small wavevector range and cannot furnish appreciable information on the ground state stability properties.

In a previous paper [4] we studied the collective mode excitation for composite fermions at 1/3 filling factor. In that work the Bethe-Salpeter equation was considered and through it the dispersion relation was calculated. The result obtained was unexpected since instead of being consistent with to the one obtained by Girvin et. al. [1], we observed an unstable behavior.

Seemingly this result would be a consequence of the known perturbation theory difficulties of the composite fermions model. On the other hand such conclusion could be compatible with recent argument about the possible role of cristalline properties in the ground state [6–9].

In order to get a better understanding of the problem, in this note we want to explore what would be the result in the case of considering phenomenologically the two dimensional electron gas in a Laughlin state at 1/3 filling factor. The technique to be used was similar to the one employed in the previous work Ref. [4].

The conclusion coincided with the one in Ref [4]. It was detected an instability which appears for a wide range of values of the gap parameter Δ describing phenomenologically the one-particle gap in the Laughlin state propagator (4).

It is better emphasized that [4] leads to conclusions matching satisfactorily with the spectrum calculated numerically starting from the study of systems of small number of particles [10], and with the experimental results [11]. However, the determination or not for the presence of crystalline properties in the ground state can be expected to require higher number of particles than the ones considered up to now. Moreover, the assumptions employed in [4] in order to conclude the ground state stability are physically argumented and motivating, but not by far unquestionable, that is why the convenience remains yet for further investigations on the collective mode properties.

We divide the present letter in four sections. As a matter of checking the procedures, the second section is devoted to reproduce the dispersion relation in the already known situation when the first Landau is fully occupied. The result obtained correctly resembles the spectrum obtained in Ref. [12].

In the second part the fractional Landau level occupied at 1/3 filling factor is discussed after using a suitable dependence on the frequency and coordinates for the one particle propagator. This construction was based on the general results and arguments given in Refs. [13] [14]. Employing the magnetoroton basis it follows the diagonalization of the BSE in the approximation of retaining only the first Landau level states.

In the fourth section the inclusion of the first excited Landau level in the considerations is done. The outcome indicates that the collective mode frequency approaches the cyclotronic resonance at zero momentum as in Ref. [4]. Therefore, the obtained results reproduce
the general properties of the collective mode following from the composite fermion BSE investigation in our previous paper [4]. Finally, in the last section possible extensions of the work and connections with related research are commented.

II. COLLECTIVE MODE FOR ONE FILLED LANDAU LEVEL

The BSE is considered here in order to reproduce the known collective excitation results in the situation when only one Landau level is completely filled. The aim is to check the results of the technical procedures to be used in the following sections in this solved problem. The discussion closely follows the one in Ref [4] the difference being related with the absence of anyon like interactions. In such case the equation has the form

\[ \mathcal{F}(1, 1', 2, 2') = \mathcal{F}_o(1, 1', 2, 2') \]

\[ + \int \int \int d^4 d^5 d^6 \mathcal{F}_o(1, 1', 3, 4) \mathcal{W}(3, 4, 5, 6) \mathcal{F}(5, 6, 2', 2), \]

where \( \mathcal{F}_o \) is the interaction free four-point function and the interaction kernel in the first approximation is taken as the following functional derivative

\[ \mathcal{W} = i \frac{\delta \mathcal{W}_{HF}(3, 4)}{\delta G(5, 6)} \]

where \( \mathcal{W}_{HF} \) is the mass operator in the Hartree-Fock approximation.

It is possible to simplify the integral equation (1) after introducing magneto-exciton wave functions Ref. [16,17] which have the explicit form

\[ \Psi_{n\alpha} = \frac{(-1)^n}{L} \frac{1}{(2\pi)^n n! n'!} \left( \frac{\partial}{\partial z_1} - \frac{1}{2} z_1 \right)^n \left( \frac{\partial}{\partial z_2} - \frac{1}{2} z_2 \right)^{n'} \exp[-(|z_1|^2 + |z_2|^2 + |z_\alpha|^2)/4] \exp[(z_1^* z_2 + z_1 z_\alpha - z_2 z_\alpha^*)/2], \]

\[ z = x - y i. \]

where \( L \) is the radius of the macroscopic sample. These two-particle states are constructed from the one-body wave functions for electrons and holes in the \( n \)th and \( n' \)th Landau levels. They are characterized by a center of mass momentum which can be written in complex form as \( z_\alpha = i q_x - q_y \). The main simplification introduced by these states is that the \( z_\alpha \) is conserved and all the matrix elements are diagonal in this quantum number Ref. [16].

Then, through the use the basis functions (2), the integral equation (1) can be written as a linear matrix equation in the form

\[ \mathcal{F}_{nm,n'm'}(\alpha, \omega) = \delta_{nn'} \delta_{mm'} \mathcal{F}_{oom}(\omega) \]

\[ + \sum_{kl} \mathcal{F}_{oom}(\omega) < nm\alpha|\mathcal{W}|kl\alpha > \mathcal{F}_{kl,n'm'}(\alpha, \omega), \]

The matrix elements of the free four point function \( \mathcal{F}_o \) and the interaction kernel \( \mathcal{W} \) are given by the expressions
in which the dependence on the $\alpha$ quantum number has been omitted and a vertical representation of the bracket indexes is also used.

The interaction kernel in the here considered first approximation consists in the sum of two contributions coming from the Coulomb interaction and mainly corresponds to the disregarding of the screening corrections

$$W = W_{12}^c + W_{22}^c.$$ 

At experimental FQHE regime this approximation seems to be reasonable because at magnetic length distances the screening of the coulomb interaction should be weak since the mean distance between electrons is of the same order than the magnetic length in experimental samples. The explicit formulae for the $W^c$ terms and for $F_o$ are given in the Appendix.

The collective modes of the system, corresponds with the lowest frequencies leading to zero eigenvalues of the inverse kernel

$$F^{-1} = (F_o)^{-1} - W,$$ 

which in addition can produce singularities of the dielectric response tensor temporal-temporal component [16]. Due to the fact that $F_o$ is qualitatively different for particle-hole and hole-particle channels it becomes useful to divide the matrix equation (4) in four sub-blocks. Basically, the block representation is obtained by restricting, as conceived in Ref. [16], the indexes $m$ and $n$ to empty Landau levels and $m'$ and $n'$ to filled ones and define the operators $E$ and $d$ through their block matrix elements

\begin{align*}
\langle m' \mid W \mid n' \rangle &= \langle m' \mid E \mid n' \rangle , \\
\langle m' \mid W \mid n \rangle &= \langle m' \mid d \mid n \rangle ,
\end{align*}

for which the properties of the magneto-exciton wave function also implies

\begin{align*}
\langle m \mid W \mid n' \rangle &= \langle m \mid E \mid n' \rangle \ast , \\
\langle m \mid W \mid n \rangle &= \langle m \mid d \mid n \rangle \ast .
\end{align*}

The explicit expressions for the matrix elements of the operators $E$ and $d$ are given in the Appendix.
Moreover, again introducing an operator $\Delta \epsilon$ through its matrix elements as
\[
\langle m' | m | \Delta \epsilon | n' \rangle \equiv \delta_{mm'}(\epsilon_n - \epsilon_{n'})
\] (8)
in order to compact the blocks coming from the matrix representation of $\mathcal{F}_o$, the Bethe-Salpeter equation (4) can be reduced to the matrix form
\[
\langle m' | m | \mathcal{F} | n' \rangle \equiv \int \int \int \Psi^{m'}_{m\alpha}(r_1, r_4)\mathcal{F}(r_1, r_4|r_2r_3)\Psi^m_{n\alpha}(r_2, r_3)dr_1dr_2dr_3dr_4
\]
\[
= \langle m' | m | \begin{bmatrix} \omega & -d & -d^{*} & -\omega - \Delta \epsilon - \mathcal{E} + i\eta \\ -d & -\omega & \mathcal{E} + i\eta & \omega - \Delta \epsilon \end{bmatrix}^{-1} | n' \rangle .
\] (9)

This representation was then used to determine numerically the collective mode dispersion relation. For this purpose a finite number of basis functions corresponding to the Landau levels index up to a maximum value were retained in constructing the BS matrix (4). The roots of the determinant of the matrix (9) were found. The spectrum of the collective mode is shown in Fig.1. It can be seen that the behavior is qualitatively similar to the one obtained in Ref. [12]. In the next section the framework for the discussion of the collective mode is somewhat modified intending to describe the 1/3 partially filled Landau level.

### III. PARTIALLY FILLED LANDAU LEVEL

Let us analyze now the situation when one Landau level is partially occupied at 1/3 filling factor. After projecting BSE on the first Landau level is possible diagonalize it by using the magnetoexciton basis states with $n = n' = 0$ in the following way
\[
\mathcal{F}_{00,00}(\alpha, w) = \mathcal{F}_{00}(w) + \sum \mathcal{F}_{00}(\omega) < \alpha, 0, 0 | \mathcal{W} | \alpha, 0, 0 > \mathcal{F}_{00,00}(\alpha, w)
\] (10)

Note that all $\mathcal{F}_{00,00}$, $\mathcal{F}_{00}$ and $\mathcal{W}$ have been projected in the same subspace of the $n = n' = 0$ functions. The one-particle propagator will be also projected in the first Landau level and its spatial structure can be explicitly written after considering the results of Refs. [13–15]. The concrete expression to be used has the form
\[
G(\vec{x}, \vec{x}', w) = \Pi(\vec{x}, \vec{x}')G^e(w),
\]
where $G^e(w)$ will be defined in a phenomenological fashion in order to model the exact structure associated to the Laughlin’s state, and $\Pi$ is the projector operator in the first Landau level (See Appendix). Concretely, it will be assumed that the frequency dependence is given by two poles corresponding to energies shifted in a one-particle gap as follows
\[
G^e(w) = \left[ \frac{1}{3} w + \Delta/2 + i\eta \right]^{-1} \left[ \frac{1}{3} w - \Delta/2 - i\eta \right]^{-1}
\]
where the frequency and energies in this section are measured in units of $e^2/r_o$ and the factors $1/3$ and $2/3$ assures the form of the exact one-particle density matrix for the Laughlin state.
determined in Refs. [13,15]. The one-particle energy reference system is taken at midway between the two energies \( +\Delta/2 \) and \( -\Delta/2 \). It turns out that this dependence seems to be a good description of the exact one according to the arguments given by Rezayi in Ref. [15].

After fixed the \( G^c(w) \) dependence, the quantity \( F_{oo} \) in (10) can be calculated explicitly and takes the form

\[
F_o = \frac{2}{9} \left[ \frac{1}{w-\Delta-i\eta} + \frac{1}{-w-\Delta-i\eta} \right].
\]

(11)

The numerical value for gap parameter \( \Delta \) can be estimated from the results in Ref. [15]. The homogeneous BSE associated to Eq (10) now reduces to a simple scalar condition

\[
F_o^{-1} - <\alpha,0,0|W|\alpha,0,0> = 0.
\]

(12)

Relation (12) can be simplified to write the explicit formula for the dispersion relation

\[
w^2 - \Delta^2 = -\frac{4}{9} \Delta \left[ (\pi/2)^{1/2} e^{-\vec{q}^2/4} I_o(\vec{q}^2/4) - \frac{1}{|\vec{q}|} e^{-\vec{q}^2/2} \right],
\]

(13)

where the term in brackets corresponds with the evaluation of \( W_c \) when \( n = n' = 0 \) from the expressions in the Appendix after setting \( u_c = 1 \).

The spectrum (13) predicts instability for a range of the phenomenological one-particle gap parameter \( \Delta \). The Fig.2 shows the momentum dependence of the collective mode frequency for the estimated gap parameter value \( \Delta \equiv 0.2 \) measured in units of \( e^2/(\epsilon l_o) \) as taken from Ref. [15].

It should be mentioned that Giuliani and Quinn [3] had also pointed out a similar behavior through an study of the BSE associated to the Tao-Thouless parent states (TT). However, in that system the instability result was attributed to the ground state degeneration of the TT state. The present description, however, seems to be well designed for the consideration of the collective mode in the Laughlin’s or any translationally invariant temptative ground state.

The instability region for the parameter \( \Delta \) is given by

\[
\Delta < 0.2474.
\]

Therefore for an appreciable range of values for the parameter \( \Delta \) describing the one-particle gap, an instability under the formation of periodic charge density waves results.

**IV. INCLUSION OF ONE HIGHER LANDAU LEVEL**

In Ref. [4] the collective mode dispersion relation at low momenta satisfied the Kohn theorem in an external magnetic field. In other words the low momenta frequency approached the cyclotron frequency. In the previous section, where only the lowest Landau level was considered it should not be possible to test this property. However the validity of such a condition is important because it is related to the general result stating that the zero momentum cyclotron resonance saturates the oscillator strength sum rule once the homogeneity
of the system is assumed. It can be noticed that the collective mode spectrum evaluated in Ref. [4] at low momentum seems to be compatible with such property.

Then, the purpose of this section is to consider corrections for the low momentum behavior of the collective mode, introduced by the inclusion of the next empty Landau level. In this way it will be intended to check for the validity of Kohn theorem in the present approach.

Our procedure will be an slight modification of the one used in the first section. The form of matrix element of $F_0$ will be taken in the approximation consisting in retaining only the lowest two Landau levels $n=0, n=1$. This approximation produces for the relevant matrix elements of $F_0$ among the retained states the expression

$$\langle m' | F_0 | n' \rangle = \delta_{\alpha\beta} \delta_{mn} \delta_{m'n'} \begin{cases} (\omega - (\epsilon_n - \epsilon_{n'}) + i\eta)^{-1} & n = 0, n' = 1 \\ (-\omega - (\epsilon_{n'} - \epsilon_n) + i\eta)^{-1} & n' = 0, n = 1 \\ \frac{1}{2}((w - \Delta' - i\eta)^{-1} + (-w - \Delta' - i\eta)^{-1}) & n = 0, n' = 0, \\ 0 & n = 1, n' = 1, \end{cases}$$

corresponding to neglecting the higher $n > 1$ Landau levels. A new parameter $\Delta' = u c \Delta$ has been introduced because the energy in this section is being measured in units of $\hbar w_c$.

The matrix elements of $W$ have now mixing of contributions $E_{00}, E_{01}, E_{10}, d_{01}, d_{10}$. Note that due to

$$\langle 1, 1 | F_0 | 1, 1 \rangle = 0$$

and $F_0$ being diagonal in the considered particle-hole basis, imply that all the matrix elements of $W$ among the state $n=1, m=1$ and any of the other three retained states decouple from equation (4).

Then, the dispersion relation condition for the collective mode following from the homogeneous BSE takes now the form

$$\text{Det} \begin{bmatrix} \frac{2 \Delta^2}{\Delta'} - E_{00} & -d_{01} & -E_{01} \\ -d_{10} & \omega - \Delta \epsilon - E_{11} + i\eta & -d_{11} \\ -E_{01} & -d_{11} & -\omega - \Delta \epsilon - E_{11}^* + i\eta \end{bmatrix} = 0$$

(14)

The numerical solution of (14) for the same value $\Delta = 0.2$ chosen in Section 3 produces the collective mode dispersion relation shown in Fig.3 where the instability region is again present. However, at low momentum it follows that the frequency of the lowest energy collective mode instead of tending to infinity as in Fig.2, reaches the cyclotron resonance. Therefore, the present discussion reproduces in this limit the conclusion of Ref. [4] and the lowest energy collective mode again satisfies the Kohn Theorem. In a similar way as in Ref. [2] there are two modes that (in the present case approximately) approach to the cyclotron resonance in the low momentum limit. In Fig.4, a magnified part of the low momentum region of Fig. 3, shows how the inter-Landau level collective mode at low momentum gets the divergent frequency behavior which the intra-Landau mode (Fig.1) previously had in the preceding approximation. Observe that in Section 3, as well as here, this divergent behavior at low momentum seems to be forced with the saturation of the oscillator strength sum rule by the cyclotronic resonance.
V. COMMENTS

We have obtained indications of the presence of unstable excitations in a translationally invariant states for FQHE systems corresponding to the developing of crystalline charge density waves. Such a result is coherent with the search in progress for ground states showing periodic charge density oscillations \[6-9\]. The wavefunctions discussed in these works can exhibit relatively weak periodic charge densities in a lattice having one flux quantum passing through its unit cell. The associated wave vector of this periodicity has similar magnitude as the momentum values laying inside the instability region discussed here. In fact the previous consideration of these states motivated our interest in discussing the stability properties of the homogeneous ground states through the investigation of their collective modes.

It is clear that further research about the discrepancy between the results of this work with the conclusions following from the single mode approximation in Ref. \[1\] is in need. In this direction some speculative reasoning which we consider useful to expose will be given below.

Under assuming the existence of a sufficiently weak charge density wave in the true ground state of the system, the correlation functions of this wavefunction would be well approximated by the ones associated to the Laughlin state. Therefore, the single mode approximation for the lowest energy excitations of this real ground state (determined by such correlations) would be expected to produce similar results to the ones in Ref. \[1\]. However, in another hand, if an exactly translationally invariant candidate for ground state (like the Laughlin’s one) is unstable, the single mode approximation should not be valid. But, in such conditions the ansatz wavefunction given by the Fourier components of the density would also become a sort of imposition eventually capable of ruling out the instability result from the outcome in Ref. \[1\]. Henceforth, if this considerations have some foundation the similarity of the results for the single mode approximation for collective mode with the essentially exact data for the energy spectrum for few particle systems would become a possible outcome in spite of presence of a charge density wave instability of the considered state.

The investigation on these questions will be the objective of a future continuation of the work.

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REFERENCES

[1] S. M. Girvin, A. H. MacDonald and P. M. Platzman, *Phys. Rev.* B33 (1986) 2481.
[2] A. Lopez and E. Fradkin, *Phys. Rev.* B47 (1993-I) 7080.
[3] G.F. Giuliani and J.J. Quinn, *Phys. Rev.* B31 (1985) 3451.
[4] A. Perez Martinez, A. Cabo and V. Guerra, *ICTP Preprint* IC/95/295 (1995) (Sent to *Int. J. Mod. Phys.* B).
[5] R.P. Feynman, Statistical Mechanics (Benjamin, Reading, Mass. 1972) Cap 11 and references therein.
[6] A. Cabo, *Phys. Lett.* A171 (1992) 90.
[7] R. Ferrari, *Int. J. Mod. Phys.* B8 (1993) 529.
[8] A. Cabo, *Phys. Lett.* A191 (1994) 323.
[9] A. Cabo, *ICTP Preprint* IC/95/208 (1995) (To appear in *Phys. Lett.* A).
[10] F.D. Haldane, *Phys. Rev. Lett.* 52, (1984) 1583.
[11] A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. West *Phys. Rev. Lett.* 70 (1993) 3983.
[12] C. Kallin and B.I. Halperin, *Phys. Rev.* B30 (1984) 5655.
[13] A. Burke and A. Cabo, *Physica* A173 (1990) 281.
[14] S. M. Girvin and A.H. MacDonald, *Phys. Rev. Lett.* 58 (1987) 1252.
[15] E. H. Rezayi, *Phys. Rev.* B35 (1987) 3032.
[16] Q. Dai, J.L. Levy, A.L. Fetter, C.B. Hanna and R.B. Laughlin, *Phys. Rev.* B46 (1992) 5642.
[17] C. B. Hanna, R.B. Laughlin and A. Fetter, *Phys. Rev.* B43 (1991) 8745.
APPENDIX

A. Definitions of various auxiliary quantities

\[ c(m, n) = (-1)^{m+n} \frac{z_{\alpha}^{n} z_{\alpha}^{m}}{(2^{m+n} m! n!)^{1/2}}. \]

\[ d(m, n) = (-1)^{m+n} \frac{z_{\alpha}^{s(m+n)}}{(2^{m+n} m! n!)^{1/2}}. \]

\[ co(m, n) = \frac{m!}{(m - l)! l!}. \]

\[ b = \frac{1}{2} |z_{\alpha}|^2 \]

The projection operator in the first Landau level \( \Pi \) is defined as

\[ \Pi(1, 2) = \frac{1}{2\pi l_{o}^2} \exp \left\{ -\left( |z_{1}|^2 + |z_{2}|^2 \right)/4 + z_{1}^{*} z_{2}/2 \right\}. \]

with \( l_{o} \) is the magnetic length in the external magnetic field \( B \).

B. The selfenergies and coulomb contributions for each Landau level of index \( n \)

\[ \epsilon_{n} = n + \epsilon_{c}(n) \]

\[ \epsilon_{c}(n) = -u_{c}(\pi/2)^{1/2} \left( -1 + \sum_{l=0}^{n} \frac{(-1/2)^{l} n!(2l - 1)!!}{l! (n - l)!} \right) \]

where \( n \) is the index of the Landau level in the magnetic field and the constant \( u_{c} \) is given by

\[ u_{c} = \frac{e^2}{\epsilon l_{o}} \sqrt{\frac{\hbar e B}{mc}} \]

C. The matrix elements of \( F_{o} \) for fully empty or filled Landau levels

\[ \left\langle m' \right| F_{o} \left| n \right\rangle = \delta_{\alpha \beta} \delta_{m n} \delta_{m' n'} \begin{cases} \frac{1}{1} - (\epsilon_{n} - \epsilon_{n'}) + i\eta)^{-1} & n \text{ empty}, \ n' \text{ occupied} \\ -\frac{1}{1} - (\epsilon_{n'} - \epsilon_{n}) + i\eta)^{-1} & n' \text{ empty}, \ n \text{ occupied} \\ 0 & \text{otherwise} \end{cases} \]

when the first Landau level is not filled a nonzero value for the matrix elements \( < 0, 0|F_{o}|0, 0 > \) can appear.
D. Matrix elements of interaction kernel $W$

$$W_{21}^{C} = - \int d1 \int d2 \frac{u_c}{|1-2|} \Psi_A^*(1,2) \Psi_B(1,2),$$

$$W_{22}^{C} = \int d1 \int d2 \int d3 \frac{u_c}{|1-2|} \Psi_A^*(2,2) \Psi_B(1,1).$$

E. The $E$ block matrix components for all $m, n$

$$E_{21}(m,n) = - u_c c(m,n) e^{-b} \sum_{l=0}^{m} \sum_{k=0}^{n} c(m,l)c(n,k) \Gamma(\frac{(1+k+l+|l-k|)}{2})$$

$$(1/2)^{|l-k|/2-(l+k)/2}(-1)^{-(l+k)}|a|^{|l-k|-(l+k)},$$

$$\text{F}_1\left((1+k+l+|l-k|)/2,1+|l-k|,\frac{|a|^2}{2}\right) 2^{1/2} \Gamma(1+|l-k|),$$

$$E_{22}(m,n) = u_c c(m,n) \frac{e^{-b}}{|a|},$$

$$d_{21}(m,n) = - u_c d(m,n) e^{-b} \sum_{l=0}^{m+n} c(m,l)(1/4)^{|l|/2}(-1)^{-|l|} a^{|l|-l} \Gamma(|l|+1/2)$$

$$\text{F}_1\left(|l|+1/2,|l|+1,\frac{|a|^2}{2}\right) \sqrt{2(1/2)^{|l|} \Gamma(|l|+1)},$$

$$d_{22}(m,n) = u_c d(m,n) \frac{e^{-b}}{|a|}. $$
FIGURES

The collective mode energy for the one filled Landau level

The collective mode energy for 1/3 filled Landau level as disregarding the higher ones

The low momentum behavior of the collective mode for 1/3 filled Landau level including the first excited level

The inter and intra Landau level collective mode energy at 1/3 filled Landau level
Fig. 3

$\frac{\omega}{\omega_c}$ versus $ql_o$

Fig. 4

$\frac{\omega}{\omega_c}$ versus $ql_o$