Crossing statistics of scattered Laser Light through Nanofluid

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In this paper, we investigate the crossing statistics of speckle patterns formed in Fresnel diffraction region by a laser beam scattering through a nanofluid. We extend zero—crossing statistics to assess dynamical properties of nanofluid. According to joint probability density function of laser beam fluctuations and its time derivative, theoretical framework for Gaussian and non-Gaussian processes are revisited for determining the magnitude of the moving particle’s velocity. We count number of crossings not only at zero level but also for all available thresholds. Using probabilistic framework in determining crossing statistics, *a priori* Gaussianity is not essentially considered, therefore even in presence of deviation from Gaussian fluctuation, this modified approach is capable to compute relevant quantities such as mean value of velocity more precisely. Generalized total crossing which represents the weighted summation of crossings for all thresholds to quantify small deviation from Gaussian statistics is introduced. This criterion can manipulate the contribution of noises and trends to infer reliable physical quantities. The characteristic time scale for having successive crossings at given threshold is defined. In our experimental setup, we find that increasing sample temperature leads to more consistency between Gaussian and perturbative non-Gaussian prediction. The maximum number of crossing does not necessarily occur at mean level indicating that we should take into account other levels in addition to zero level to achieve more accurate assessments.

Complexities are ubiquitous in various phenomena due to initial conditions and evolutions. Quantifying relevant observable quantities essentially depends on exploiting robust statistical approaches. Probabilistic framework corresponding to a typical statistical measure includes joint probability density function (JPDF) of relevant dependent parameters and we should determine mentioned JPDF for further statistical inference.

Among different statistical quantities to characterize the morphology of a stochastic fluctuation in various dimensions, a promising class is devoted to crossing statistics. This method can be used for 1, 2 and 3 dimensional fluctuations corresponding to crossing statistics, length or contour statistics and area statistics, respectively. Mentioned measures have been extensively utilized and improved after pioneering study done by S. O. Rice in the context of mathematical analysis of random noise [1]. This notion has also opened new trends in examining stochastic fields [2–6] ranging from condensed matter and surface physics [7–9], optics [10–22] to astronomy and cosmological random fields [23–25].

A motivation behind crossing statistics is devoted to possibility derivation of an explicit theoretical prediction based on JPDF of random variables and associated derivative with respect to a dynamical variable [5–8, 20–22, 27, 28]. A main goal to utilize crossing statistics in various field of researches is examining the capability of number of crossing at a typical threshold to quantify underlying fluctuations. As explained in details by Yura et al., in optical communication and sensors systems, the statistical properties of fade and surge play key role in characterizing associated physical behavior [21].

In recent years, extensive researches have been conducted on applications of nano-materials in nanofluids technologies. Nanofluid is a new kind of heat transfer fluid prepared by dispersing nanoparticles in traditional based fluids [29, 30]. It is shown that they can enhance effective heat transfer properties of the original base fluid [31, 32]. The most common nanoparticles for nanofluids are generally metallic and nonmetallic materials such as Al$_2$O$_3$, Ag, CuO, Cu, SiO$_2$ and TiO$_2$ [33–35]. The movement of nanoparticles plays a significant role in the anomalously increased heat transfer properties of the nanofluids [36, 37]. So, it is great importance to find out a robust method to measure the movements of nanoparticles in nanofluids, in order to understanding the heat transfer enhancement mechanism.

Due to the complexity of the movements of nanoparticles in nanofluids, calculating the velocity of each particles in the fluid is impossible, but using statistical methods, the average speed of particles can be computed with proper experimental configuration [15]. When a coherent laser light passes through a fluid, a non-uniformly illuminated image (speckle pattern) can be formed. The speckle pattern appearances due to the interference of the scattered light with different phases and amplitudes by the nanoparticles. This pattern changes in time as a consequence of the nanoparticle motion due to the aggregation, sedimentation and Brownian motion [38, 37]. Therefore, the speckle pattern analysis can be used to characterize the dynamical behavior of nanofluids.

Some previous studies have almost focused on fluctuations of light intensity around mean level which is so-called zero-crossing [15, 19]. In mentioned method, the spatially integrated speckle intensity fluctuations produced by the moving diffuse particles in a plane is investigated by counting the crossing at mean level. In addition, direction of moving particles can be determined by using

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I. THEORETICAL MODEL: UP AND DOWN CROSSING STATISTICS

S. O. Rice introduced level crossing or generally crossing statistics [1]. Soon after, in various disciplines ranging from complex systems [2,9], material sciences [7,9] optics [10,22] to cosmology and early universe [23,27], mentioned method has been used and improved. Crossing statistics represents geometrical properties of a typical stochastic process, therefore, it has proper capability in order to quantify fluctuations in a robust manner.

A. Mathematical framework

For a typical phenomenon, suppose that following discrete set is recorded: \( \{ x(t_i) \}, \ i = 1, ..., N, \) and \( \Delta t \equiv t_{i+1} - t_i. \) For convenient we set the mean of recorded data to zero (\( \langle x(t) \rangle = 0. \) In continuous limit, referring to Fig. 1, up and down crossing are defined according to crossing points with positive and negative slopes at an arbitrary threshold, \( \vartheta \equiv \alpha/\sigma_0, \) respectively. Where \( \alpha \) is a typical value in domain of \( x_i \) and \( \sigma_0^2 = \langle x(t)^2 \rangle. \)

![FIG. 1: A sketch for positive and negative slope crossings of a typical time series at a given threshold \( \vartheta \equiv \alpha/\sigma_0. \)](image)

The crossing number density of time series at a given threshold, \( \vartheta, \) can be written as \( n_{\pm}^x(t; \alpha) \) (see Fig. 1 for more details). We consider “+” for up-crossing and “−” for down-crossing. Mathematical description of ensemble average of crossing point is:

\[
\langle n^x_{\pm}(t; \alpha) \rangle = \sum_{t_{\pm}} \delta_D(t - t_{\pm}) = \int dx \int d\eta \delta_D(t - t_{\pm}) P(x, \eta)
\]

(1)

here \( \delta_D \) is Dirac delta function, \( \eta \equiv \partial_x x \) and \( P(x, \eta) \) is the joint probability density function (JPDF) of recorded data sets and associated time derivative. For crossing at \( \alpha, \) we should set \( x(t_{\pm}) = \alpha. \) An important task to do is making a connection between Dirac delta function and corresponding variables in Eq. (1). To this end, we can

directional detecting aperture [17]. Central limit theorem is a key assumption for deriving theoretical zero-crossing function leading to an expression including the moving particle’s velocity in simple form and now becomes almost useful real time method. It is worth noting that, in general case, deviation from Gaussianity is expected, consequently, we should take into account non-Gaussian contributions.

In this paper, we construct an experimental setup to create speckle patterns. Such patterns are formed in Fresnel diffraction region by illumination of a laser beam on the nanoparticles suspended in nanofluid. As a movement of the nanoparticles, the speckle pattern is not static and varies with time. In this experimental setup, we use a He-Ne laser light at 633 nm wavelength as a light source. By measuring the intensity of the speckles and its variations with time, we are able to examine the movement characteristics of the nanofluids [15,19].

Relying on previous statistical analysis of speckle pattern due to scattering laser light from moving diffuse particles used to determine the magnitude and direction of moving objects [15,18]. Present research has following advantages and novelties: we try to use generalized version of zero-crossing which is so-called crossing statistics enumerating crossing for all available thresholds to achieve more precise measurement of relevant quantities. Non-Gaussianity due to the physics of moving diffuse particles as well as experimental setup can now be taken into account, theoretically, to find feasible results. Perturbation approach will be utilized to compute the mean velocity of diffuse particles in weak non-Gaussian case. It is worth noting that for optical applications, previous efforts have been concentrated on deriving joint probability density function (JPDF) for relevant fluctuations in closed form resulting in analytical expression for level-crossing statistics. While in the present paper, we exploit the perturbative expansion of JPDF known as Edgeworth expansion [27,35,40]. Therefore, this approach enables us to investigate various processes irrespective to existence of necessary knowledge concerning functional form of JPDF. In addition we introduce generalized total crossing which is a suitable benchmark for assessment of large and small fluctuations, separately. Finally we will also introduce characteristic time scales in such experimental setup referring to crossing statistics framework.

This paper is organized as follows: In Section II we will explain theoretical background of crossing statistics and its generalizations. Gaussian and non-Gaussian fluctuations will be considered in the context of crossing statistics. Data description and experimental setup to collect data sets will be demonstrated in Section III. Section IV is devoted to applications of crossing statistics on intensity fluctuation of scattered laser light through a nano-fluid. Summary and concluding remarks will be given in section IV.
expand $x(t)$ around a typical time associated with up- or down-crossings as follows:

$$
x(t) = x(t_\pm) + (t - t_\pm) \eta(t_\pm) + \mathcal{O}(\delta t^2) \tag{2}
$$

therefore, one can find a useful expression for \( \delta_D(t-t_\pm) = \delta_D(x(t) - \alpha)|\eta(t_\pm)| \). Inserting mentioned expression in Eq. \((1)\) results in:

$$
\langle n^+_x(t; \alpha) \rangle = \int dx \int d\eta \, \delta_D(x(t) - \alpha) \, |\eta(t_\pm)| \, \mathcal{P}(x, \eta)
= \int d\eta \, |\eta| \, \mathcal{P}(\alpha, \eta) \tag{3}
$$

To have up-crossing (crossing with positive slope) additional constraint should be taken into account on first derivative of record data as \( \Theta(\eta) \) (\( \Theta(\cdot) \) is step function), while for down-crossing (crossing with negative slope) we have \( \Theta(-\eta) \). Using the functional form of JPDF, theoretical prediction for up- and down-crossings are achieved. According to general definition of JPDF with respect to underlying variables:

$$
\mathcal{P}(x, \eta) = \mathcal{P}(\eta|\Theta(\pm\eta)) \mathcal{P}(x)
$$

we must point out that, \( \langle . \rangle_{\alpha} \) means averaging with conditional PDF as a kernel of integration and in principle depends on \( \alpha \). The total expected number of crossings of underlying time series per unit time at the threshold \( \vartheta = \alpha/\sigma_0 \) for all possible values of \( \eta \) reads as:

$$
N_\alpha = \int_{-\infty}^{\infty} d\eta |\eta| \mathcal{P}(\alpha, \eta)
= \langle n^+_x(\alpha) \rangle + \langle n^-_x(\alpha) \rangle = \mathcal{P}(\alpha) \langle |\eta| \Theta(\pm\eta) + |\eta| \Theta(-\eta) \rangle_{\alpha} \tag{4}
$$

Above equation represents the expected statistical crossings at threshold \( \vartheta = \alpha/\sigma_0 \) with both positive and negative slopes as shown in Fig. \([1]\). Number of crossing at zero level which is so-called zero-crossing is given by:

$$
N_0 = \langle n^+_x(0) \rangle + \langle n^-_x(0) \rangle = \mathcal{P}(0) \langle |\eta| \Theta(\pm\eta) + |\eta| \Theta(-\eta) \rangle_{\alpha=0} \tag{5}
$$

If \( \mathcal{P}(x, \eta) = \mathcal{P}(\eta)\mathcal{P}(x) \), then by using Eqs. \((5)\) and \((6)\) we have:

$$
N_\alpha = N_0 \mathcal{P}(\alpha) \mathcal{P}(0) \tag{6}
$$

For the generalized total level crossing, \( N_{\text{total}} \), we have:

$$
N_{\text{total}}(q) = \int_{-\infty}^{\infty} N_\alpha |\alpha - \tilde{\alpha}|^q d\alpha \tag{7}
$$

where \( q \) represents the order of generalized moment. It turns out, for \( q = 0 \), the quantity \( N_{\text{total}}(0) \) equates to total number of crossing for all thresholds. Therefore \( N_{\text{total}}(0) \) reveals a measure for roughness of underlying signal. Above equation represents the general definition of crossing statistics. In what follows, we are going to examine Gaussian and non-Gaussian processes.

**B. Gaussian fluctuation**

In previous subsection we derived integration form for crossing statistics. In this subsection, we assume a Gaussian process with variables \( \mathbf{A} = \{x, \eta\} \), subsequently, the functional form of JPDF can be considered as bivariate Gaussian function:

$$
\mathcal{P}(\mathbf{A}) = \sqrt{\frac{\det\mathcal{M}}{(2\pi)^2}} e^{-\frac{1}{2}(\mathbf{A}^T \mathcal{M} \mathbf{A})} \tag{8}
$$

where \( \mathcal{M} \) is the inverse of the covariance matrix of underlying variables:

$$
\mathcal{M}^{-1} \equiv \text{Cov} = \begin{bmatrix}
\langle x^2 \rangle & \langle x \eta \rangle \\
\langle x \eta \rangle & \langle \eta^2 \rangle
\end{bmatrix} \tag{9}
$$

Each element of covariance matrix can be computed using the power spectrum, \( S(\omega) \), of given process, accord-
In our experimental setup.

\[ C_{xx}(\tau) \equiv \langle x(t)x(t+\tau) \rangle = \frac{1}{2\pi} \int d\omega \ e^{i\omega \tau} S(\omega) \]  
\[ C_{xx}(0) \equiv \sigma_0^2 = \langle x^2 \rangle = \frac{1}{2\pi} \int d\omega S(\omega) \]  
\[ C_{\eta\eta}(0) \equiv \sigma_1^2 = \langle \eta^2 \rangle = \frac{1}{2\pi} \int d\omega \omega^2 S(\omega) \]  
where \( \sigma_0 \) and \( \sigma_1 \) are called spectral parameters. In addition \( C_{\eta\eta} = -C_{xx} = -\langle x(t)\xi(t+\tau) \rangle \), here \( \xi(t) = \partial^2 x(t)/\partial t^2 \). For stationary time series \( \langle x\eta \rangle = 0 \), consequently, Eq. (4) becomes:

\[ \langle n^+_x(\alpha) \rangle = \frac{1}{2\pi \sigma_0} e^{-\alpha^2/2\sigma_0^2} \]  

Therefore Eqs. (6) and (7) become:

\[ N_0 = \frac{1}{\pi} \sigma_1 \]  
\[ N_\alpha = N_0 \ e^{-\alpha^2/2\sigma_0^2} \]  

Plug in \( \sigma_0 \) and \( \sigma_1 \) into Eq. (15), we have:

\[ N_0 = \frac{1}{\pi} \left( -\frac{C_{xx}(0)}{C_{\eta\eta}(0)} \right)^{\frac{1}{2}} \]  

This relation shows that the number of zero-crossing per unit time is proportional to square root of second derivative autocorrelation function divided by autocorrelation function. For number of crossing at a given threshold one can write:

\[ N_\alpha = \frac{1}{\pi} \left( -\frac{C_{xx}(0)}{C_{\eta\eta}(0)} \right)^{\frac{1}{2}} \ e^{-\alpha^2/2\sigma_0^2} \]  

In addition the generalized total crossing introduced by Eq. (8) for Gaussian process is:

\[ N_{total}(q) = \left( \frac{2C_{xx}(0)}{\pi} \right)^{q+1/2} \ e^{-\alpha^2/2C_{xx}(0)} \]  

C. Non-Gaussian fluctuations: Perturbation approach

In pervious subsection we derived the analytical function for \( \langle n^+_x(\alpha) \rangle \) for a perfect Gaussian process. There are many reasons causing to non-Gaussianity, therefore in context of perturbation approach T. Matsubara tried to calculate an expansion for crossing statistics in terms of various order of \( \sigma_0 \). The Edgeworth expansion for probability density function for weak non-Gaussianity reads as [27]:

\[ P(\alpha) = \frac{e^{-\alpha^2/2\sigma_0^2}}{\sqrt{2\pi \sigma_0}} \left[ 1 + \sigma_0 S^{(0)}(\alpha/\sigma_0) + \mathcal{O}(\sigma_0^2) \right] \]  

where \( S^{(1)} \equiv -\frac{1}{2} \langle \alpha^2 \xi(0) \rangle/(\sigma_0^2 \sigma_1^2) \) and \( \Gamma(\cdot) \) is gamma function. In this section we derived general definition of crossing statistics for a given threshold, subsequently, an extension for computing relevant physical quantities is achieved. In order to use perturbative expansion, second cumulant should be remained finite. In section III we apply this approach on experimental data sets and compute mean velocity of diffuse objects as well as we will examine the Gaussianity nature of recorded fluctuations in our experimental setup.

II. EXPERIMENTAL SETUP AND DATA DESCRIPTION

Our versatile experimental setup to examine the intensity of scattered light through nanofluid is conducted by using a combined system represented in upper panel of Fig. 2. It consists of the optical arrangement for the formation of speckles and the electrical processing of the spatially integrated speckle intensity. The Gaussian beam generated by a single mode He-Ne laser with the wavelength \( \lambda = 633\text{nm} \) with beam waist \( w_0 = 1.1\text{mm} \) is focused on the sample. Our sample is located with a distance \( z = 20\text{cm} \) away from the position of the beam waist. The speckle patterns are formed by illuminating the CdS-CdS nanoparticles suspended in ethanol. These
FIG. 3: Upper panel indicates speckles patterns recorded by CCD camera from left to right corresponds to \( T = 50^\circ C \), \( T = 70^\circ C \) and \( T = 90^\circ C \), respectively. Lower panel corresponds to power of integrated speckle versus time for three different experiments.

patterns are detected in the Fresnel diffraction field at the plane, a distance \( \ell = 80 \text{ cm} \) away from the center of sample by a circular aperture with a radius \( d = 5 \text{ mm} \) in front of power meter. To achieve a drift velocity due to convection, a heater is located under the sample. (Fig. 2).

If a coherent light is incident upon a nanofluids containing randomly distributed nanoparticles, a nonuniform illuminated image is obtained. This pattern is generated due to the interference of the scattered lights by the nanoparticles. Subsequently, fluctuations of the image intensity in each location of the interference field, which is so-called "speckle" is appeared. As a movement of the nanoparticles, the speckle pattern is not static and changes with time. So one can consider the time-varying of speckle intensity fluctuation is a stochastic field. Power meter records integrated intensity of all speckles during a given interval time which is called spatially integrated speckle intensity. We define the fluctuations of spatially integrated speckle intensity by \( I(t) - \langle I(t) \rangle \) where \( \langle \rangle \) denotes the ensemble average of the speckle intensity variation. In lower panel of Fig. 2, we indicate the optical system for the formation of speckles under the illumination of a Gaussian laser beam over a transmitting diffuse object moving with constant velocity in a certain direction of a plane perpendicular to the optical axis. Upper panel of Fig. 3 illustrates the formation of speckles intensity recorded by CCD camera. Lower panel of Fig. 3 represents power fluctuations as a function of time for three different experiments with different physical conditions. These data sets will be used as input for crossing statistics explained in previous section in order to determine magnitude and direction measurement of diffuse object’s velocity and relevant statistical properties of intensity fluctuation containing some information about the dynamics of speckle.

III. IMPLEMENTATION ON SCATTERED LASER BEAM

One of the main purpose of this analysis is devoted to determining the average of diffuse velocity of nanoparticle suspended in a solution. The zero-crossing analysis of the dynamic speckles can be used to measure the speed of nanoparticles in nanofluids. According to this method, the crossing statistics of intensity fluctuations of speckles at zero level corresponding to mean value, \( \langle I(t) \rangle \) is enumerated \([15]\). To get reliable result, two following conditions should be satisfied. Firstly, the convection velocity of nanofluid must be constant and perpendicular to the direct of Gaussian laser beam. To achieve such situation, a heater is placed at a position in beneath of sample. Therefore, one can control the temperature of
sample. Secondly, the transmittance amplitude of speckles should be modeled as a stationary random process. Since the corresponding surface area elements are small enough compared to the illumination area on the diffusion screen, consequently, this condition is almost assured for nanoparticles. It turns out that any deviation from above conditions leads to get inaccurate value for velocity of nanoparticles. Therefor, we look for a robust algorithm without changing experimental setup considerably for precise measurement. Suppose that recorded data is represented by \( x(t) \equiv I(t) - \langle I(t) \rangle \). The correlation function of the speckle intensity fluctuations produced at the plane of the Fresnel diffraction field under the illumination of the Gaussian beam for arbitrary shape of detector aperture is written by [17]:

\[
C_{xx}(\tau) = K \frac{1}{\pi} \frac{\lambda}{\ell} e^{-\gamma^2 \tau^2 / \omega^2} \times \int dR D(R) \exp \left( -\frac{(\pi \omega / \lambda r)^2}{R - (\ell / \rho + 1) v r} \right)
\]

where \( K \) is a constant, \( \omega \) and \( \rho \) indicate the width and wavefront curve of the Gaussian beam at the sample position and \( D(R) \) is the spatial correlation function of the detecting aperture [17]. Finally the number of zero-crossing becomes:

\[
N_0 = \frac{v}{\pi} \sqrt{\frac{\gamma^2}{\Delta^2 + d^2} + \frac{1}{\omega^2}}
\]

here \( \gamma \equiv \frac{\ell}{\rho} + 1 \) and \( \Delta \equiv \frac{\lambda}{2 \pi} \) representing the average grain size of speckles at the detecting plane. \( \ell \) is distance between sample and detector while \( d \) is radius of aperture (Fig. 2). Above equation indicates that the number of zero-crossings is directly proportional to the magnitude of the in-plane velocity of diffuse nanoparticle, \( v \). [16]

As mentioned before, when the detecting aperture is sufficiently large enough in comparison to the average grain size of detected speckles, consequently, the probability density function of the spatially integrated speckle-intensity variation has Gaussian form. In such case, above expression for \( N_0 \) is a good estimation. In real experimental setup, non-Gaussianity is expected to occur. Therefore, to increase the estimation accuracy of nanoparticles velocity, we extend our analysis based on crossing statistics instead of using only zero-crossing method.

In our experimental setup, we measure the fluctuation of spatial intensity at three different temperatures, namely 50, 70 and 90 °C. In Tab. 1 we report the computed average velocity of nanoparticle for various temperature. These values are compatible with those results presented in ref. [11]. The statistical error reduces due to more available measurements. Also referring to Eq. (24), we also determine \( N_{total}(0) \). As represented in this table, by increasing temperature of solution, we expect that fluctuations of recorded signal in this experiment to be increased corresponding to higher roughness of series. These fluctuations are directly related to the high speed movement of nanoparticles.

So far, we could compute the value of mean-velocity of nanoparticles suspended in fluid, but the crossing statistics prepares more complete approach to reduce statistical errors arising in the experiment. In addition to zero-crossing method, we compute crossing statistics for all available values of thresholds and by stacking all results with own weighted errors, we are able to estimate more precise values for relevant physical quantities such as mean value of nanoparticles velocity.

The crossing statistics in terms of threshold for three experiments is represented in Fig. 4. Symbols indicates \( N_\alpha \) versus \( \alpha \) computed numerically, while solid line is devoted to theoretical prediction for Gaussian fluctuations. Dashed line is associated with non-Gaussian theory. These plots demonstrate that our experimental setup produces non-Gaussian fluctuations converging to Gaussian when sample temperature to be increased. This result is compatible with our expectation, because, more randomness leads to have more Gaussian fluctuations. According to theoretical prediction for crossing statistics at a given threshold using Gaussian theory, Eq. (19), we compute mean value of nanoparticle velocity. The quantity \( \langle v(\alpha) \rangle_\alpha \) corresponds to computed mean velocity averaged on all available levels. These values are in agreement with that of determined by only zero-crossing method. Statistical dispersion of mean speed, \( \sigma_v^2 = \langle (v(\alpha) - \langle v(\alpha) \rangle)^2 \rangle_\alpha \), is reported in Tab. 1. Taking into account the non-Gaussianity of fluctuations leads to more precise value for \( v \) as reported in Tab. 1.

In Fig. 5 we indicate the computed particle’s velocity as a function of \( \alpha \) for three samples. In this plot \( \alpha \) for three samples. In this plot values are reported in Tab. 1. In Fig. 5, we indicate the computed particle’s velocity for each level. Our results demonstrate that the consistency interval for threshold between Gaussian and non-Gaussian theory for computing relevant quantity is less than 2\( \sigma \) centered by mean value for low temperature. Mentioned interval increases beyond 2\( \sigma \) for higher temperature. Subsequently for computing mean value of velocity, the weak non-Gaussian model should be considered in order to achieve precise values.

Now we focus on generalized total number of crossing statistics introduced in Eqs. (19) and (22). \( N_{total}(q) \) for \( q = 0 \) characterizes the total number of fluctuations for all thresholds. This value is directly devoted to roughness of underlying fluctuations. As we expect, higher sample temperature corresponds to higher value of \( N_{total}(q = 0) \) (see Fig. 6). The higher value of slope for large \( q \) clarifies the higher probability of having large fluctuations in intensity of light fluctuations detected during the measurement. Fluctuations in light passing through the sample in lower temperature are highly encoded with spike-shape due to more non-homogeneity in velocity of particle, while by increasing the sample temperature, the convection velocity becomes dominant in comparison with other effects. Subsequently, the probability of finding the large fluctuations in beam intensity becomes small as seen in Figs. 3 (lower panel) and 4. In Fig. 7 we plot \( N_{total} \) as a function of \( q \) for three samples. In this plot symbols correspond to numerical computation of generalized total crossings. Solid line predicted by Eq. (22)
TABLE I: The value of particle’s velocity computed in experiments for different temperatures at 1σ confidence interval. In last column the total crossing for \( q = 0 \) has been reported.

| Temperature | \( v(0) \pm \sigma_v \) | \( \langle v(\alpha) \rangle_{\text{Gaussian}} \pm \sigma_v \) | \( \langle v(\alpha) \rangle_{\text{non-Gaussian}} \pm \sigma_v \) | \( N_{\text{total}}(0) \) |
|-------------|----------------|----------------|----------------|-----------------|
| 50 °C       | 2.93 ± 0.10 (mm/s) | 2.89 ± 0.02 (mm/s) | 2.96 ± 0.02 (mm/s) | 9692 |
| 70 °C       | 3.35 ± 0.10 (mm/s) | 3.26 ± 0.02 (mm/s) | 3.36 ± 0.02 (mm/s) | 11892 |
| 90 °C       | 3.57 ± 0.10 (mm/s) | 3.53 ± 0.02 (mm/s) | 3.55 ± 0.02 (mm/s) | 12732 |

FIG. 4: Crossing statistics as a function of threshold for our experimental setups for three different temperatures. Upper panel corresponds to \( T = 50^\circ\text{C} \), middle panel is for \( T = 70^\circ\text{C} \) and lower panel represents crossing statistics for \( T = 90^\circ\text{C} \). Symbols in these plots indicate \( N_\alpha \) computed numerically, solid line shows theoretical prediction of crossing statistics for Gaussian fluctuations. Dashed line represents \( N_\alpha \) in the context of perturbation theory up to \( \mathcal{O}(\sigma_0^2) \) (Eq. 21).

FIG. 5: Measured mean value of particle’s velocity for three temperatures based on various levels. In each panel, filled square symbols are devoted to Gaussian theory while circle symbols represent results based on non-Gaussian perturbative theory up to \( \mathcal{O}(\sigma_0^2) \). Horizontal solid line in each plot represents the particle’s velocity computed by zero level.

while Gaussian model (Eq. (19)) is illustrated by solid line. This measure is very sensitive to existence of rare events. For higher velocity of particle, we expect to have more particle collision causing the intensity of speckles varies rapidly, so the average of speckles intensity fluctuations is almost concentrated around the value of mean fluctuations. Therefore, the slope of \( N_{\text{total}}(q) \) for large value of \( q \) becomes smaller than that of given for low temperature. Using the \( N_{\text{total}}(q) \) quantity, it is possible to examine the contribution of various size of fluctuations. For \( q < 1 \), small fluctuations have dominant impact, on the contrary, for \( q \geq 1 \) large fluctuations play main role in computing generalized crossing statistics. In our experimental setup, the well-known Gaussian theory can not reveal proper view for velocity dispersion of suspended particles in solution which is affected by thermal fluctuations and other relevant phenomena. On the contrary, to achieve more precise characterization of underlying sam-
ple, perturbative crossing statistical approach introduced in context of statistical analysis of random fields [27] is an alternative approach. According to Eqs. (22) and (24), the corresponding velocity for different $q$ can be written as:

$$v(q) = \frac{N_{\text{total}}^{\exp}(q)}{\beta(2\sigma_0^2)^{(q+1)/2} \Gamma\left(\frac{q+1}{2}\right)} \left(1 - \sigma_0 \left\{ \frac{(2+q)\sigma_0^2 - 3}{6} S^{(0)} + \frac{S^{(1)}}{3} \right\} \frac{2\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{q+1}{2}\right)} + O(\sigma_0^2) \right) \right\}$$

TABLE II: The value of threshold for which we get maximum value of crossing ($N_{\alpha}^{\text{max}}$) and corresponding characteristic time scale for three experiments.

| Temperature | $\alpha/\sigma_0$ | $\tau_0$ (sec) |
|-------------|------------------|----------------|
| 50 °C       | 0.23             | 0.13 ± 0.01    |
| 70 °C       | -0.23            | 0.11 ± 0.01    |
| 90 °C       | -0.07            | 0.10 ± 0.01    |

**IV. SUMMARY AND CONCLUSION**

We can extract viable physical properties concerning dynamical properties of suspended particles by using. Recording the fluctuation of scattered laser beam through a nanofluid. The complexity nature of underlying system enforces that only robust methods to be able to quantify reliable values for relevant quantities. A
statistics, we are able to evaluate small and large fluctuations in different manners. If fluctuations with larger values are affected by artifacts due to experimental setup and etc. more than smaller fluctuations, therefore considering smaller values of \( q \) can reduce spurious results in determining velocity. On the contrary, larger values of \( q \) is suitable to magnify the contribution of fluctuations far from the mean value, statistically (Eq. (25)). Finally according to crossing statistics, some characteristic time scales which probably contain physical properties enabling us to achieve deep insight associated with underlying sample, are introduced.

We determined crossing statistics of laser light fluctuations for three different sample temperatures, \( T = 50^\circ\text{C}, \ T = 70^\circ\text{C} \) and \( T = 90^\circ\text{C} \). Fig. 4 illustrates crossing statistics by enumeration crossing for various thresholds represented by square symbols. Theoretical predictions based on Gaussian and weakly non-Gaussian models are shown by solid and dashed lines, respectively. Our results demonstrated that by increasing sample tempera-
ture, experimental results, Gaussian and non-Gaussian are consistent together around mean level. In Fig. 5 we computed mean velocity by using various thresholds. We found that for low temperature the consistency interval for threshold between Gaussian and non-Gaussian theory for computing relevant quantity is less than \( 2\sigma \) centered by mean value. Mentioned interval increases beyond \( 2\sigma \) for higher temperature. Subsequently for computing mean value of velocity, the weak non-Gaussian model should be considered in order to achieve robust results. In Tab. 1 we reported the mean value of velocity computed by only zero-crossing \( \langle v(0) \rangle \), based on Gaussian theory \( \langle v(\alpha) \rangle_{\text{Gaussian}} \) and according to weakly non-Gaussian approach \( \langle v(\alpha) \rangle_{\text{non-Gaussian}} \). Obviously the statistical uncertainty for mentioned value when we consider available thresholds decreases. For all case the mean value of velocity determined by weakly non-Gaussian theory is higher than that of computed by Gaussian framework.

Referring to Fig. 4 it was elucidated that for low temperature, the probability to achieve crossing at higher threshold is considerable compare to higher temperature. Indeed at low enough temperature, the value of diffusive particles’s velocity is statistically small. Consequently, the probability of higher value for fluctuations of spatial integrated intensity during the time interval of observations increases.

To quantify the value of deviation from Gaussian theory, we computed \( N_{\text{total}} \) as a function of \( q \) indicated in Fig. 4. For larger value of \( q \), higher fluctuations have dominant contribution, on the contrary for \( q < 1 \), small fluctuations play predominant role. We found that by increasing sample temperature higher value of fluctuations become small. Also, the non-Gaussian terms vanish and non-Gaussian model as well as experimental data converge to Gaussian situation.

Characteristic time scale using crossing statistics was defined by inverse value of \( N_0 \) for a given threshold. If the peak of crossing exits at zero level, therefore the minimum value of time interval for having crossing corresponds to \( N_0 \). While for our results, the associated threshold for minimum time scale was deviated from zero level. The corresponding values have been reported in Tab. 1. Higher temperature leads to more coincidence between peaks of crossing statistics curve determined by different approaches. This means that zero threshold for high enough temperature supports reliable physical properties and for lower temperature, we should also take into account results for other thresholds.

Finally, one must point out that to derive more accurate dynamical properties, crossing statistics which is an extension of zero-crossing method is useful approach. However, in our experimental setup, the computed mean velocity of diffuse objects according to Gaussian and non-Gaussian model are statistically consistent, but it turns out that this consistency is not essentially satisfied for other range of temperatures and different concentrations.

It could be interesting to apply this method for different experimental configurations to assess the capability...
of this modification in crossing statistics for determining mean velocity and its dependency to particle size and viscosities of solution. Also based on directionality nature of crossing statistics, this method enables us to examine statistical isotropy of intensity fluctuations which is of interest and it is outside the scope of present study.

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