Quantum Barro-Gordon Game in Monetary Economics

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Abstract

Classical game theory addresses decision problems in multi-agent environment where one rational agent’s decision affects other agents’ payoffs. Game theory has widespread application in economic, social and biological sciences. In recent years quantum versions of classical games have been proposed and studied. In this paper, we consider a quantum version of the classical Barro-Gordon game which captures the problem of time inconsistency in monetary economics. Such time inconsistency refers to the temptation of weak policy maker to implement high inflation when the public expects low inflation. The inconsistency arises when the public punishes the weak policy maker in the next cycle. We first present a quantum version of the classical Barro-Gordon game which captures the problem of time inconsistency in monetary economics. Such time inconsistency refers to the temptation of weak policy maker to implement high inflation when the public expects low inflation. The inconsistency arises when the public punishes the weak policy maker in the next cycle. We first present a quantum version of the Barro-Gordon game. Next, we show that in a particular case of the quantum game, time-consistent Nash equilibrium could be achieved when public expects low inflation, thus resolving the game.

Key words: Barro-Gordon Game, Quantum Game Theory, Time Inconsistency

PACS: 03.65 Aa, 03.65.Ud, 03.65.w, 01.80.+b, 02.50.Le

HIGHLIGHTS

\begin{itemize}
  \item We reformulate Barro-Gordon Game using quantum game theory.
  \item We use Marinatto-Weber approach for quantization of game theory.
  \item We find that the well-known time inconsistency in the classical game is removed after quantization.
\end{itemize}

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1 Introduction

Game theory is a mathematical formulation of situations where, for two or more agents, the outcome of an action by one of them depends not only on the particular action taken by that agent but on the actions taken by the other (or others) [1]. Game theory has gained much attention in recent years as indicated by the many texts written on it [2,3,4,5]. After Neuman and Morgenstern’s book [6], which was the first important text in game theory, John Nash made important contributions to this theory [7,8]. However, application of game theory to different areas such as economics, politics, biology, started in 1970’s and has been growing ever since [1].

In recent years, game theory has attracted the attention of many physicists as well. Among the many contributions that has come along from the physics community is the inclusion of the rules of quantum mechanics in game theory where quantum effects such as superposition and entanglement can play a role [9,10]. Quantum game was introduced by Eisert, Wilkens, and Lewenstein (EWL) [11], where the role of entanglement was considered first. Subsequently, Marinatto and Weber (MW) [12] offered a more general scheme for quantization of games based on Hilbert space approach. It is now believed that quantization of games can offer interesting situations where various dilemma present in classical games can be removed. As a result of the above-mentioned works, many authors have employed the quantization of various games. For example, Cheon and Tsutsui [13], Fliteny and Hollenberg [14], Makowski [15] and Landsburg [16] have used EWL approach. On the other hand, Arfi [17], Deng et. al [18] and Frackiewicz [19], based their approach on MW scheme. Much of the attention in the above-mentioned works has been paid to well-known games such as prisoner’s dilemma game [11,13,14,17], while others have considered various other games with social implications.

However, despite the growing interest in econophysics [20], little attention has been paid to quantization of classical games in finance and economics. The present work offers a step in this direction. Here, we intend to quantize the classical game of Barro and Gordon (BG) in monetary economics employing the MW scheme. We then study some specific cases of the quantum game in order to find Nash equilibrium and the advantages quantization may offer. BG game, to be explained in details in the following, is a classical game which illustrates the problem of time inconsistency in monetary policy. Time inconsistency was first introduced by Kydland and Prescott [21] and later by Barro and Gordon [22]. The main idea is that when the output is inefficiently low, policy maker can increase it by applying discretionary policy, causing surprising inflation. In this situation, although the output increases which is beneficial, it causes inflation, which is costly. Therefore, we encounter an inflationary bias. Since BG introduced a noncooperative game between public and
central bank, many researchers [23,24,25,26,27] have been using game theory to study time inconsistency. All of these studies are based on the classical game theory, while the principles of quantization has not been applied so far.

There are, however, a variety of reasons to apply the roles of quantum mechanics to various disciplines outside of traditional physics. For example, in psychology and decision making theory, quantum cognition has gained much attention [28,29]. In most such approaches, quantum probabilities are considered in order to represent certain uncertainty in decision making process. Furthermore, a quantum Hamiltonian approach to various fields of social sciences has also been considered [30,31,32,33,34,35] where operator-valued dynamical variables are governed by a general Hamiltonian which includes all possible interaction. However, we must note that while our approach in this paper is similar in spirit, it is different in its basic assumption and methodology. The problem of strategy selection which is the essence of game theory could be formulated using the laws of quantum mechanics instead of classical probability theory. In fact, it is the purely quantum mechanical concept of superposition (of strategies) which provides the key ingredient in our approach to game theory here.

Furthermore, micro-evolution teaches us that our selfish (microscopic) genes may make our decision on a fundamental level, where quantum mechanics may be relevant. Perhaps from a more practical point of view, recent advances in quantum computational and quantum communication technology [36] may help us in creating quantum devices which must take on quantum strategies in order to solve problems [37]. It is with such motivations in mind that one may consider quantum game theory in various fields of social sciences, and consequently monetary economics in our present case.

The rest of this paper is organized as follows: In Section 2, we briefly describe the problem of time inconsistency in monetary policy. The classical BG game is then described in details in Section 3. In Section 4, we present our main results of quantization of BG game and consider some specific cases of the quantum game. The last section offers concluding remarks.

2 Time Inconsistency

A policy is time inconsistent or dynamic inconsistent when it is considered as the best policy for particular time in the future, but it does not remain so, when that particular time actually arrives. There are two possible mechanisms that have been considered for such a time inconsistency. (i) Strotz [37] explains that time inconsistency is because of changing preferences, (ii) Kydland and Prescott [21] consider another explanation that is based on agent’s rational
expectation. The main idea is, when people expect low inflation, central bank finds the incentive for high inflation. If the public understand this incentive and predict high inflation, the central bank finds it optimal to deliver the public’s expectation and therefore implements high inflationary policy. Therefore, while low inflation is the optimal policy for both banker and the public to begin with, high inflation is eventually implemented. We use this second mechanism in this paper as it is the concept of time inconsistency associated with the Barro-Gordon game.

Barro and Gordon \[22\] have explained time inconsistency of monetary policy as follow: in a discretionary regime, central banker can print more money and make more inflation than people’s expectations. Benefits of this surprising inflation might provide more economic activities or reduce government’s debt. However, if people, due to their rational expectation, understand it and adjust their expectations with it, then policy maker will not reach his goal at all. This is simply due to the fact that inflationary advantages is best when it is unanticipated. Besides that, due to increased money supply, the level of prices will grow, which will have negative consequences for the policy maker. The classical Barro-Gordon game captures the essence of this type of time-inconsistency in the context of decisions (strategy) that a policy maker must make and the expectations that the public can have. The actual game is represented in two different formats where the strong policy maker implements low inflation which is time consistent, while in the case of weak policy maker a time inconsistent strategy is the alternative to a Nash equilibrium.

3 BG Classical Game

In BG game, similar to prisoners’s dilemma, there are two players; public and central bank. In this game, one assumes that the public has rational expectations. The public then predicts inflation by solving out the policy maker’s optimization problem. On the other hand, the policy maker selects inflation policy by considering the public’s inflationary expectations. In this paper, by following Backus and Driffill \[38\] and Storger \[39\], we use a special version of BG game. This version is easier to convert to quantum game due to having a definitive payoff matrix. In this version, there are two types of policy maker: weak policy maker, which uses discretionary policies and gains benefit by making unanticipated inflation. In the other words, a weak policy maker can cheat the public when they formed their low inflationary expectation at the start of the period. Strong policy maker, on the other hand, commits to zero inflation and is not interested in unanticipated inflation.
The utility function of these policy makers is as follows:

\[ U_{pol}^t = \theta (\pi_t - \pi_e^t) - \frac{a \pi_t^2}{2} \]  

(1)

where, \( \pi_t \) and \( \pi_e^t \) are actual inflation and expected inflation rates, respectively. Inflationary cost is assumed to be proportional to the square of inflation and therefore \( \frac{a \pi_t^2}{2} \) is the cost of inflation where \( a \) is an arbitrary cost parameter. \( \theta \) is a dummy variable that is equal to 1 for weak policy maker and 0 for strong policy maker and \( b \) is a coefficient for benefit of inflation term with \( b > 0 \). If \( \pi_t > \pi_e^t \) then policy maker can decrease unemployment (according to Philips curve \(^1\)) and gain benefit using the first term in Eq.(1). Public’s utility function is as follows:

\[ U_{pub}^t = -(\pi_t - \pi_e^t)^2. \]  

(2)

This function shows that every deviation from expected inflation causes disutility for the public.

We next briefly review the payoff matrix for weak and strong policy maker as obtained by [38,39]: First we use weak policy maker optimization. The weak policy maker optimizes Eq.(1) without any constraint. By taking derivative of Eq.(1) with respect to \( \pi_t \), optimal inflation will be \( \hat{\pi}_t = \frac{b}{a} \). If \( \pi_e^t = \frac{b}{a} \), replacing it in Eq.(1) and (2) will result in \( U_{pol}^t = -\frac{b^2}{2a} < U_{pub}^t = 0 \). Therefore, in this case, weak policy maker cannot gain any benefit and this strategy will not be chosen. On the other hand, optimizing unconstraint Eq.(2), with the assumption of public rational expectations, results in \( \pi_t = \pi_e^t \). If weak policy maker commits to zero inflation, both players will receive zero payoff. However, if the public expects zero inflation, and the weak policy maker implements \( \pi_t = \frac{b}{a} \), then he can gain some benefit (equal to \( \frac{b^2}{2a} \)) and the public will incur losses of \( -(\frac{b}{a})^2 \). Therefore, we have \( U_{pol}^t = \frac{b^2}{2a} > U_{pub}^t = -(\frac{b}{a})^2 \) and the weak policy maker therefore prefers this strategy. Even if the public expects \( \pi_e^t = \frac{b}{a} \) and the policy maker chooses \( \pi_t = \frac{b}{a} \), he can get more payoff than choosing zero inflation. Thus, \( \pi_t = 0 \) is a dominated strategy and will never be selected. Following [38], normalization condition \( (a = b = 2) \), leads to a simple payoff matrix for the weak policy maker as shown in Table 1. Note that the case of \( \pi_t = 1 \), \( \pi_e^t = 1 \) is a Nash equilibrium in this case. However, the actual equilibrium is the case of \( \pi_t = 1 \), \( \pi_e^t = 0 \) if the policy maker is successful in cheating the public. The key point here is that this equilibrium \( (\pi_t = 1 \) , \( \pi_e^t = 0 ) \) is time inconsistent because \( \pi_e^t = 0 \) is announced but \( \pi_e^t = 1 \) is implemented.

In the case of strong policy maker \( (\theta = 0 ) \), \( \pi_t = \frac{b}{a} \) is never chosen as it is a dominated strategy. Strong policy maker will incur a loss equal to \( -\frac{b^2}{2a} \) in both

\(^1\) Philips curve shows an inverse relation between the unemployment and inflation.
θ = 1

| Public | \( \pi_t^e = 0 \) | \( \pi_t^e = 1 \) |
|---|---|---|
| Weak policy maker | \( \pi_t = 0 \) | (0,0) (-2,-1) |
| | \( \pi_t = 1 \) | (1,-1) (-1,0) |

Table 1
Weak policy maker payoff

Barro and Gordon [22] showed that the weak policy maker loses his reputation for cheating the public. In fact, in the next period, public plays “tit for tat” game and punish the weak policy maker by adjusting their expectations. In other words, if \( \pi_{t-1} = \pi_t^e \) then \( \pi_t = \pi_t^e = 0 \); otherwise, \( \pi_t = \pi_t^e = \frac{b}{a} \). Therefore, weak policy maker compares marginal cost and benefit of cheating the public, and then decides to make unanticipated inflation. Consequently, classical BG game needs two time periods to solve the game between the public and weak policy maker. In this paper we generalize this classical game to a quantum framework and ask if it can be made more efficient.

4 Quantum BG Game

4.1 Quantization of the game

There is essentially two different ways to quantized classical games in the literature. EWL [13,14,15,16] took the original steps in this regard. However, the approach of MW [17,18,19] has been more widely used recently and we intend to use their approach in quantizing BG game.
Frackiewicz [19] argued that, in EWL method, the result of the game depends on many parameters because each player’s strategy is a unitary operator. Therefore it has cumbersome calculation. But in MW, player’s local operators were performed on some fixed entangled state $|\psi\rangle$. It seems that MW is simpler than EWL [40]. We therefore use the MW method to quantize the game between weak policy maker and the public. In this game, there are two players: weak policy maker (M) and public (U). Each player has two strategies: high inflation (H) and low inflation (L). Consider a four-dimensional Hilbert space, $\mathcal{H}$, as the strategy space in ket notion:

$$\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_U = \{ |LL\rangle, |LH\rangle, |HL\rangle, |HH\rangle \}$$

where the first qubit is related to the state of the policy maker and the second one to that of public. Kets show a given strategy in strategy space which in quantum version is a Hilbert space. Therefore, we use an arbitrary quantum strategy as a normalized state vector, $|\psi_i\rangle$.

$$|\psi_i\rangle = \alpha |LL\rangle + \gamma |LH\rangle + \delta |HL\rangle + \beta |HH\rangle$$

Where $|\alpha|^2, |\beta|^2, |\gamma|^2, |\delta|^2$ are probability of observing the strategies of (L, L), (H, H), (L, H) and (H, L), respectively, with $|\alpha|^2 + |\gamma|^2 + |\delta|^2 + |\beta|^2 = 1$. Density matrix is written as $\rho_i = |\psi_i\rangle \langle \psi_i|$. Let $C$ be a unitary Hermitian operator (i.e., $C^\dagger = C = C^{-1}$), such that $C |H\rangle = |L\rangle$ and $C |L\rangle = |H\rangle$ and $I$ is the identity operator such that $I |H\rangle = |H\rangle$ and $I |L\rangle = |L\rangle$. In the game, policy maker and [public] use operators $I$ and $C$ with probabilities of $p, (1-p)$, $q, (1-q)$.

Final density matrix for this system is as follows:

$$\rho_f = pq \left[ (I_M \otimes I_U) \rho_i (I_M \otimes I_U^\dagger) \right]$$

$$+ p(1-q) \left[ (I_M \otimes C_U) \rho_i (I_M \otimes C_U^\dagger) \right]$$

$$+ (1-p)q \left[ (C_M \otimes I_U) \rho_i (C_M \otimes I_U^\dagger) \right]$$

$$+ (1-p)(1-q) \left[ (C_M \otimes C_U) \rho_i (C_M \otimes C_U^\dagger) \right]$$

and the two payoff operators are given as follows:

$$P_M = 0 |LL\rangle \langle LL| - 2 |LH\rangle \langle LH| + |HL\rangle \langle HL| - |HH\rangle \langle HH|$$

$$P_U = 0 |LL\rangle \langle LL| - |LH\rangle \langle LH| - |HL\rangle \langle HL| + 0 |HH\rangle \langle HH|.$$

Finally, payoff functions are calculated according to:

$$\tilde{S}_M(p, q) = Tr(P_M \rho_f)$$

$$\tilde{S}_U(p, q) = Tr(P_U \rho_f).$$
This can be written as:

\[ \tilde{\mathbf{s}}_M(p, q) = \Phi \Omega \gamma_M^T \]
\[ \tilde{\mathbf{s}}_U(p, q) = \Phi \Omega \gamma_U^T \]

where

\[ \Phi = [pq, p(1 - q), (1 - p)q, (1 - p)(1 - q)] \]

\[ \Omega = \begin{bmatrix}
\alpha^2 & \gamma^2 & \delta^2 & \beta^2 \\
\delta^2 & \beta^2 & \alpha^2 & \gamma^2 \\
\gamma^2 & \alpha^2 & \beta^2 & \delta^2 \\
\beta^2 & \delta^2 & \gamma^2 & \alpha^2 \\
\gamma_M = [0, -2, 1, -1] \\
\gamma_U = [0, -1, -1, 0].
\end{bmatrix} \]

The payoff functions for policy maker and public are therefore calculated as:

\[ \tilde{\mathbf{s}}_M(p, q) = 2p(\alpha^2 - \beta^2 + \delta^2 - \gamma^2) + q(\delta^2 - \alpha^2 - \gamma^2 + \beta^2) - \alpha^2 + \gamma^2 - 2\delta^2 \]

\[ \tilde{\mathbf{s}}_U(p, q) = (1 - 2(\delta^2 + \gamma^2))(q(2p - 1) - p) - (\delta^2 + \gamma^2) \]

In order for Nash equilibrium to exist one needs to implement the following conditions [12]:

\[ \tilde{\mathbf{s}}_M(p^*, q^*) - \tilde{\mathbf{s}}_M(p, q^*) \geq 0, \quad \forall p \in [0, 1] \]

\[ \tilde{\mathbf{s}}_U(p^*, q^*) - \tilde{\mathbf{s}}_U(p^*, q) \geq 0, \quad \forall q \in [0, 1] \]

which in our case lead to:

\[ 2(p^* - p)(\alpha^2 - \beta^2 + \delta^2 - \gamma^2) \geq 0 \]

\[ (1 - 2(\delta^2 + \gamma^2))(q^* - q)(2p^* - 1) \geq 0. \]

4.2 Analysis of the game

Equation (13-14) and (17-18) are our main results. Following MW’s approach, we consider the validity of three possible situations below:

(a) \( p^* = q^* = 1 \)

In this case the payoffs are as follows:

\[ \tilde{\mathbf{s}}_M(1, 1) = -\beta^2 - 2\gamma^2 + \delta^2 \]

\[ \tilde{\mathbf{s}}_U(1, 1) = -\gamma^2 - \delta^2 \]
And Nash equilibrium conditions are:

\[ 2(1 - p)(\alpha^2 - \beta^2 + \delta^2 - \gamma^2) \geq 0 \quad (21) \]

\[ (1 - 2(\delta^2 + \gamma^2))(1 - q) \geq 0 \Rightarrow \gamma^2 + \delta^2 \leq 1/2. \quad (22) \]

If \( \alpha^2 + \delta^2 > \beta^2 + \gamma^2 \), then the first Nash equilibrium condition will be satisfied. In fact, this condition will most likely be satisfied for the weak policy maker which we are considering here, since according to Table (1), he prefers (L,L) or (H,L) strategies rather than (H,H) or (L,H) strategies. We thus assume that for weak policy maker the condition of \( \alpha^2 + \delta^2 > \beta^2 + \gamma^2 \) is always true. The second Nash equilibrium condition may or may not be satisfied depending on the choice of the quantum strategy. We will return to this point later on in this paper.

(b) \( p^* = q^* = 0 \)

In this case the payoffs are as follows:

\[ \bar{M}(0,0) = -\alpha^2 + \gamma^2 - 2\delta^2 \quad (23) \]

\[ \bar{U}(0,0) = -\gamma^2 - \delta^2 \quad (24) \]

and Nash equilibrium conditions are calculated as:

\[ -2p(\alpha^2 - \beta^2 + \delta^2 - \gamma^2) \geq 0 \quad (25) \]

\[ (1 - 2(\delta^2 + \gamma^2))q \geq 0 \Rightarrow \gamma^2 + \delta^2 \leq 1/2 \quad (26) \]

In this case, Nash equilibrium will be satisfied if \( \alpha^2 + \delta^2 < \beta^2 + \gamma^2 \) which is exactly the opposite of the previous case. Again, since we are considering a weak policy maker here (see above), we will consider this condition as unacceptable. Therefore, we do not consider Nash equilibrium to hold for the weak policy maker in the case of \( p^* = q^* = 0 \).

(c) \( p^* = q^* = 1/2 \)

In this case the payoffs are as follows:

\[ \bar{M}(1/2,1/2) = -1/2 \quad (27) \]

\[ \bar{U}(1/2,1/2) = -1/2 \quad (28) \]

and Nash equilibrium conditions are calculated as:

\[ 2(1/2 - p)(\alpha^2 - \beta^2 + \delta^2 - \gamma^2) \geq 0 \quad (29) \]

\[ (1 - 2(\delta^2 + \gamma^2))(1/2 - q)(1 - 1) = 0 \quad (30) \]

The second Nash equilibrium condition is trivially satisfied. However, the first Nash equilibrium condition is clearly violated for \( p > 1/2 \) for a weak policy
maker (i.e. \(\alpha^2 - \beta^2 + \delta^2 - \gamma^2 > 0\)). But since in this scenario both players have negative payoff, this equilibrium is not preferred.

Therefore among the three possibilities we have considered above, the second scenario (b) could not be satisfied because Nash equilibrium did not exist and the third scenario (c) included dominated strategies. We therefore choose to only consider the first scenario (a). However, the first scenario could satisfy Nash equilibrium depending on the choice of quantum strategies, i.e. choice of \(\alpha, \beta, \delta, \gamma\). In the following we consider two different quantum strategies of weak policy maker where Nash equilibrium could potentially exist:

(i) Suppose that the public has false prediction about inflation. It means that, quantum strategy is a superposition of two strategies, (L, H) and (H, L) where \(\alpha = \beta = 0\):

\[
|\psi_i\rangle = \gamma |LH\rangle + \delta |HL\rangle.
\]

Therefore, payoff functions will be as follows:

\[
\bar{M}(1,1) = -2\gamma^2 + \delta^2
\]

\[
\bar{U}(1,1) = -\gamma^2 - \delta^2 = -1
\]

And Nash equilibrium conditions:

\[
2(1 - p)(\delta^2 - \gamma^2) \geq 0
\]

\[
\gamma^2 + \delta^2 \leq 1/2
\]

In this case, the public would always lose due to false expectations, i.e. Eq. (33). However weak policy maker can earn better a payoff if \(\delta^2 > 2\gamma^2\). However, this scenario cannot imply a stable situation due to Eq. (35) which indicate that Nash equilibrium can never be obtained since \(\gamma^2 + \delta^2 = 1\). This result is reminiscent of the classical version of the game where the weak policy maker can earn positive payoff by cheating the public for just one cycle. Afterwards, the public will punish him and correct their expectation. Here, we showed that a quantum strategy that is superposition of false prediction strategies is not a Nash equilibrium. In other word, it is not a sustainable equilibrium.

(ii) Suppose that public has correct prediction about the inflation. Therefore, quantum strategy is a superposition of two strategies, (L, L) and (H, H) where \(\gamma = \delta = 0\):

\[
|\psi_i\rangle = \alpha |LL\rangle + \beta |HH\rangle
\]

\[
\bar{M}(1,1) = -\beta^2
\]

\[
\bar{U}(1,1) = 0
\]

Thus Nash equilibrium conditions in this case are:

\[
2(1 - p)(\alpha^2 - \beta^2) \geq 0
\]
The second Nash equilibrium condition (Eq. 40) is always true. However, the first one (Eq. 39) will be satisfied for \( \alpha^2 > \beta^2 \) which is an acceptable condition for the weak policy maker. This shows that the larger the share of \((L, L)\) strategy is chosen (\(\alpha\)) the smaller the negative payoff of policy maker becomes (\(\beta\)). The important point here is that a Nash equilibrium exist for the weak policy maker where the public is guaranteed not to lose and the policy maker’s loss can be minimized by reducing \(\beta\). In fact, in an extreme case \(\beta \to 0\), where the quantum strategy converges to a (non-superposition) single strategy \((L, L)\), the payoff of both players will be zero as in the classical case (see Table (1)). However the important difference is that Nash equilibrium is satisfied here, where in the classical version it is not. Therefore, the quantum version of the game in this scenario offers a time consistent Nash equilibrium. This constitutes our main result.

5 Concluding Remarks

Barro and Gordon proposed a game between the public and policy maker based on the theory of time inconsistency. In this game, a weak policy maker can earn some benefits in short time by cheating the public about inflation. However, the public will punish him in the next period. Therefore, inflation increases and policy maker will lose his benefits. Thus, in this classical version of BG game, the implementation of low inflation by the weak policy maker is not a Nash equilibrium. In this paper we generalized the BG game by using the quantum game scheme according to Marinatto and Weber. We considered the quantum game as a superposition of four classical strategies, and Nash equilibrium conditions were subsequently calculated. The results showed that among the three possibilities we have considered, the first scenario was more acceptable. Then we considered two different quantum strategies of weak policy maker where Nash equilibrium could potentially exist: (i) public has false prediction and (ii) public has a correct prediction. It was shown that Nash equilibrium was not satisfied when the public has false prediction. However, we obtained a Nash equilibrium that is time consistent in the second scenario where the public has a correct prediction about inflation. Our result is important since it shows that in the quantum version of BG game, unlike its classical version, the low inflation policy is a Nash equilibrium when the public expects low inflation thus removing the time inconsistency and therefore solving the game. We emphasize that the purely quantum effect of superposition entangled states was the key ingredient in solving the game and removing the time inconsistency present in the classical version.

We next briefly comment on some issues regarding our results. The relevance of quantum game may at first glance seem a bit peculiar despite the motiva-
tions provided in the Introduction (Section 1) above. Ever since 1935 when Schrödinger introduced what is now known as the Schrödinger cat, the possibility of macroscopic superposition states was debated in the literature. However, quantum technology has provided for macroscopic superposition states [41], and one can imagine that with sufficiently advanced technology, future machines could employ strategies that could benefit from quantum game theory. Furthermore, one may ask if our results would be different if we had employed other methods besides MW for game quantization. Arefi [17] has shown that one would obtain the same results for prisoner’s dilemma game regardless of the method of quantization. We chose the MW method since it suited our game in a more straightforward way. However, one can imagine that employing the method of EWL would lead to essentially the same results. The important point that seems to be the common point of most game quantization is that quantum games offer an advantage over their classical version because they employ superposition principle and are thus able to resolve the conflict existing in the classical version. We have also obtained the same essential results here, and suspect that our result would be independent of the method of quantization.

Our aim here has been to provide an example of quantum game theory in economics and how the rules of quantum mechanics may offer advantages in this regard. However, one might consider further work along the same line presented here. For example, one can consider the possibility of other equilibria that might exist for the case of various other choices of $p$ and $q$ besides the ones considered here (which were purely motivated by previous studies). Another interesting avenue would be to consider a Hamiltonian formulation and thus the time evolution of various operators along the line of [31, 34, 35]. This might be interesting as dynamical evolution would become quantum mechanical and one might consider the different evolution of an initially (quantum) superposition state vs. its classical analog of a mixed state.

Acknowledgements—Grants from Research Council of Shiraz University is kindly acknowledged. This paper has also benefited from constructive criticism of respected (anonymous) referees.

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