Abstract—This paper investigates the problem of sparse signal recovery in the presence of additive impulsive noise. The heavy-tailed impulsive noise is well modelled with stable distributions. Since there is no explicit formulation for the probability density function of $\alpha$-Stable distribution, alternative approximations like Generalized Gaussian Distribution (GGD) are used which impose $\ell_p$-norm fidelity on the residual error. In this paper, we exploit a Continuous Mixed Norm (CMN) for robust sparse recovery instead of $\ell_1$-norm. We show that in blind conditions, i.e., in case where the parameters of noise distribution are unknown, incorporating CMN can lead to near optimal recovery. We apply Alternating Direction Method of Multipliers (ADMM) for solving the problem induced by utilizing CMN for robust sparse recovery. In this approach, CMN is replaced with a surrogate function and Majorization-Minimization technique is incorporated to solve the problem. Simulation results confirm the efficiency of the proposed method compared to some recent algorithms in the literature for impulsive noise robust sparse recovery.

Index Terms—Impulsive noise, symmetric $\alpha$-stable distribution, robust sparse recovery, continuous mixed norm

I. INTRODUCTION

In a Compressive Sensing (CS) problem, the objective is to reconstruct a sparse signal from compressed or low-dimensional linear measurements. This has found many applications in various fields in signal processing within the last decade [1]. In simple words, a CS problem is solved by optimizing a cost function, which in case of unconstrained optimization approach, is generally comprised of two terms. The first term penalizes the residual error signal and $\ell_2$-norm is usually used for this purpose. This is an optimal choice in the presence of AWGN noise. The second term is used to promote sparsity constraint on the representation coefficients and $\ell_0$-norm [2] or $\ell_1$-norm [3] is often employed for this regularization. However, sometimes, there exists some heavy-tailed impulsive noise in the measurements which degrades the performance of CS reconstruction. Hence, there are a class of robust sparse recovery algorithms introduced. In [4], the impulsive noise is treated as a sparse vector and a joint sparse recovery method is proposed to reconstruct the original signal. In [5], an iterative hard thresholding algorithm is suggested based on Lorentzian-norm as the fidelity criterion. Another algorithm introduced in [6], applies Alternating Direction Method of Multipliers (ADMM) to solve the $\ell_1$-norm minimization problem. In addition to the popular $\ell_2$-norm fidelity, $\ell_p$-norm is incorporated for robust sparse recovery, specifically in case of impulsive noise corruption. Robust greedy pursuit algorithms based on $\ell_p$-correlation in $\ell_p$-space are also devised in [7]. Recently, an $\ell_p$-$\ell_1$ optimization approach named Lp-ADM is proposed in [8], in which $\ell_p$-norm is used as penalty on the residual error and $\ell_1$-norm is employed for sparsity regularization. There are, in addition, some robust Bayesian sparse recovery algorithms proposed in [9] and [10].

In this paper, we use the Continuous Mixed Norm (CMN) proposed in [11] as the error penalty function. The CMN mixes all $\ell_p$-norms within the range ($p_s \leq p \leq p_f$) by weighted integration over $p$. Since the closed form PDF, modelling the impulsive noise with $\alpha$-Stable distribution does not exist, alternative PDFs are employed for the approximation of this heavy-tailed distribution. In [8], a zero-mean Generalized Gaussian Distribution (GGD) with shape parameter $p$ ($0 < p < 2$) is used to model the PDF of impulsive noise which leads to an $\ell_p$-$\ell_1$ minimization algorithm. The shape parameter is chosen to be equal to $\alpha$ which requires this parameter to be known. In this paper, we consider the case where the distribution parameters are unknown. Treating these parameters as unobserved latent variables, an Expectation Maximization (EM) method is used for progressively optimal estimation of the measurement signal corrupted with impulsive noise. This results in an optimization problem in which CMN is used as the fidelity criterion. To solve the problem corresponding to this formulation, we use ADMM to split the problem into simpler sub-problems. We also apply Majorization-Minimization (MM) technique proposed in [12]. The advantage of using CMN compared to $\ell_p$-norm (as in Lp-ADM) is that it needs not to fine tune the shape parameter $p$ and thus achieves better performance in blind settings.

II. PROBLEM FORMULATION

The problem explored in this paper is to recover a sparse signal given its linear random measurements corrupted with impulsive noise. Suppose $x \in \mathbb{R}^n$ is the original sparse signal and $A \in \mathbb{R}^{m \times n}$ is the measurement matrix. The signal of linear measurements $Ax$, is then, added with impulsive noise signal modelled with symmetric $\alpha$-stable i.i.d. components, i.e., each component is a random variable denoted by $N$ having $\alpha$-Stable distribution ($\beta = 0$) with the scale parameter $\gamma$ and the location parameter $\delta = 0$. We assume the PDF for this distribution is approximated with GGD as follows:

$$N \sim \mathcal{S}(\alpha, 0, \gamma, 0), \quad f_N(n) \approx \frac{\alpha}{2\sigma_n \Gamma(\frac{\alpha}{2})} \exp\left(-\frac{|n|^\alpha}{\sigma_n^\alpha}\right)$$

where $\sigma_n$ is a constant (function of $\gamma$) and $\Gamma$ denotes the gamma function.

The objective is to recover $x$, given the noisy measurements $y = Ax + n \in \mathbb{R}^m$. If the problem is blind, the parameters of the $\alpha$-Stable distribution modelling the impulsive noise are
unknown. If we treat these unknown parameters, denoted by \( \Theta \), as unobserved latent variables, we can apply EM algorithm to find the original sparse signal. The EM algorithm [13] is an iterative method comprising of Expectation (E) and Maximization (M) steps in each iteration, i.e. we have:

\[
\text{E-step: } \quad Q(x|x^{(t)}) = E_{\Theta|y,x} \log \{ f(y, \Theta|x)f(x) \} \\
\text{M-step: } \quad x^{(t+1)} = \arg \max_x Q(x|x^{(t)})
\] (2)

For \( x^{(t+1)} \) obtained from the M-step (2), it can be proven that \( f(y|x^{(t+1)})f(x^{(t+1)}) > f(y|x^{(t)})f(x^{(t)}) \). So the algorithm gradually maximizes the posterior probability \( f(y|x) \) in each iteration. Now, we expand \( E_{\Theta|y,x} \log f(y, \Theta|x) \) as follows:

\[
E_{\Theta|y,x} \log f(y, \Theta|x) = \int f(\Theta|y,x^{(t)}) \log f(y, \Theta|x) d\Theta \\
= \int f(\Theta|y,x^{(t)}) \log f(y|\Theta,x) d\Theta \\
+ \int f(\Theta|y,x^{(t)}) \log f(\Theta|x) d\Theta
\] (3)

where the latter equality is implied using the Bayes’ rule. Now, assume the unknown noise parameters (\( \Theta \)) are independent from \( x \). Therefore, \( f(\Theta|x) = f(\Theta) \) and the term \( E_2 \) may be discarded while maximizing \( Q(x|x^{(t)}) \) over \( x \). Now, assume \( \sigma_n \) and \( \gamma \) are given and \( \alpha \) is the only unknown parameter. Thus \( \Theta = \alpha \) and the probability density \( f(y|\Theta,x) \) is equal to \( f(y|\alpha,x) \). Now, using (1) for i.i.d. components of noise signal, we may write:

\[
f(y|\alpha,x) = \prod_{i=1}^{m} f_N(n_i|\alpha) = \left( \frac{\alpha}{2\sigma_n \Gamma(\frac{1}{n})} \right)^m \exp\left( -\frac{\|n_i\|^2}{\sigma_n^2} \right)
\]

where \( n = y - Ax \). Therefore, the optimization problem in M-step (2) is finally reduced to:

\[
x^{(t+1)} = \arg \max_x Q(x|x^{(t)}) = E_1 + E_2 + E_{\Theta|y,x} \log f(x)
\]

E-step: \( Q(x|x^{(t)}) = E_{\Theta|y,x} \log \{ f(y, \Theta|x)f(x) \} \)

M-step: \( x^{(t+1)} = \arg \max_x Q(x|x^{(t)}) \)

where \( \ell(y) = \text{CMN}(y) = \int_0^{p_f} \lambda(p)|y|_p^p dp \) is what we call the continuous mixed norm [11] of vector \( y \). In fact, we have shown that problem (4), under assumptions discussed in the beginning of this section, is equivalent to a sparse recovery problem in which, CMN is incorporated as fidelity criterion. Now, inserting the auxiliary variable \( z = \frac{1}{\sigma_n}(Ax - y) \) into problem (6), we can solve the following multivariate optimization problem, equivalently:

\[
\min_{x,z} C(x,z) = \ell(z) + \mu ||x||_1 \quad \text{s.t. } \quad z = \frac{Ax - y}{\sigma_n}
\] (7)

Using the Augmented Lagrangian Method (ALM), problem (7) is transformed into the following unconstrained optimization problem \( (\sigma > 0) \):

\[
\min_{x,z} \frac{1}{2} \|Ax - y - z\|_2^2 + \mu b \|x\|_1 + \eta^T (\frac{Ax - y}{\sigma_n} - z)
\]

Applying ADMM [14] to solve this problem, we obtain an iterative solution with the following alternating steps:

**z update step**

The optimization problem associated with this step is:

\[
z^{(k+1)} = \arg \min_{z} C^z(x^{(k)}, z, \eta^{(k)})
\]

\[
\equiv \arg \min_{z} \ell(z) + \frac{\sigma}{2} \|Ax - y - z + \frac{\eta^{(k)}}{\sigma}\|^2
\]

The solution to (9) is found by applying Majorization Minimization (MM). In other words, we iteratively optimize a set of surrogate functions instead of \( \ell(z) \) at each iteration.

**Lemma 1. For any \( 0 < p \leq q \) and any real \( z' \), the function \( s(z_i, z'_i) = |z_i|^q \left( \frac{p}{q} |z'_i|^{p-q} \right) + \left(1 - \frac{p}{q}\right) |z_i|^p \) is a surrogate function for \( |z_i|^p \) w.r.t \( z_i \).**

**Proof.** The proof is simple investigating \( s(z_i, z'_i) - |z_i|^p \geq 0 \) using first and second order derivatives with respect to \( z_i \). □

Assume \( q \geq p_f \). Using lemma 1, we may write:

\[
\ell(z) = \sum_i \int_{p_r}^{p_f} \lambda(p) |z_i|^p dp \\
\leq \sum_i \int_{p_r}^{p_f} \lambda(p) \left( |z_i|^q \left( \frac{p}{q} |z'_i|^{p-q} \right) + \left(1 - \frac{p}{q}\right) |z_i|^p \right) dp
\]

\[
= \sum_i |z_i|^q \phi_{p_r, p_f}(z'_i) + \sum_i \psi_{p_r, p_f}(z'_i) = \ell_S(z, z')
\]

The term \( \ell_S(z, z') \) is a surrogate function for \( \ell(z) \) (note that \( \ell_S(z, z') = \ell(z) \) and \( \ell_S(z, z')' = \ell'(z) \) for any \( z \neq z' \)). If we assume uniform distribution for \( \lambda(p) \), we obtain:

\[
\phi_{p_r, p_f}(z'_i) = |z'_i|^{p_f} \left( p_f \log |z'_i|^p - p_f \log |z_i|^p + p_f \log |z_i|^p \right) \\
+ \left( p_f - p_r \right) |z'_i|^{p_f} \log |z'_i|^{p_f} - 1
\]

\[
(\psi_{p_r, p_f}(z'_i) = |z'_i|^{p_f} \left( p_f \log |z'_i|^p - p_f \log |z_i|^p + p_f \log |z_i|^p \right) \\
+ \left( p_f - p_r \right) |z'_i|^{p_f} \log |z'_i|^{p_f} - 1
\]

Now let \( z' = z^{(k)} \); using MM technique, it suffices to solve the following optimization problem at each iteration:

\[
z^{(k+1)} = \arg \min_{z} C^z_S(x^{(k)}, z, \eta^{(k)})
\]

\[
= \arg \min_{z} \ell_S(z, z^{(k)}) + \frac{\sigma}{2} \|Ax^{(k)} - y - z + \frac{\eta^{(k)}}{\sigma}\|^2
\]
Hence, depending on the value of \( q \), we will deal with the following problems:

I.) \( q = 1 \) and \( p_s < p_f \leq 1 \): In this case we have:

\[
\ell_S(x, z^{(k)}) = \sum_i |z_i| \phi_{p_s, p_f}(z_i^{(k)}) + \sum_i \psi_{p_s, p_f}(z_i^{(k)}) \tag{13}
\]

Now, substituting \( \phi_{p_s, p_f}(z_i^{(k)}) \) from (11) and discarding the constant term \( \sum_i \psi_{p_s, p_f}(z_i^{(k)}) \), problem (12) is transformed into an \( \ell_1 \) minimization problem with respect to \( z \) where the solution is obtained via soft-thresholding operator \( S_T \) [15]:

\[
z^{(k+1)}(z^{(k)}) = S_T \left( \frac{Ax^{(k)} - y + \eta^{(k)}}{\sigma} \right) \tag{14}
\]

and \( T^{(k)} = \frac{1}{\sigma} \phi_{p_s, p_f}(z^{(k)}) = \frac{1}{\sigma} [\phi_{p_s, p_f}(s_1^{(k)}), \ldots, \phi_{p_s, p_f}(s_m^{(k)})]T \).

II.) \( q = 2 \) and \( p_s < p_f \leq 2 \): The choice of \( q = 2 \) results in quadratic formulation for \( \ell_S(x, z^{(k)}) \). Thus, substituting \( \phi_{p_s, p_f}(z_i^{(k)}) \) from (11), we can restate problem (12) as follows:

\[
z^{(k+1)} = \text{argmin}_z z^T W(z^{(k)}) z + \frac{\sigma}{2} \left( \frac{Ax^{(k)} - y + \eta^{(k)}}{\sigma} - z \right)^2
\]

where \( W(z^{(k)}) = \text{diag}(\phi_{p_s, p_f}(z_i^{(k)})) \). This problem has a closed form solution obtained by:

\[
z^{(k+1)} = \left( I + \frac{2}{\sigma} \sigma W(z^{(k)}) \right)^{-1} \left( \frac{Ax^{(k)} - y + \eta^{(k)}}{\sigma} \right) \tag{15}
\]

Since the matrix \( I + 2/\sigma \sigma W(z^{(k)}) \) is diagonal, this inverse solution is easily obtained with element-wise scalar divisions.

**x update step**

This step has an optimization problem formulated as:

\[
x^{(k+1)} = \text{argmin}_x C_S^\epsilon(x, z^{(k+1)}, \eta^{(k)}) \tag{16}
\]

\[
= \text{argmin}_x \frac{\sigma}{2} \left( \frac{Ax - y}{\sigma} - z^{(k+1)} + \frac{\eta^{(k)}}{\sigma} \right)^2 + \mu \|x\|_1
\]

solving this LASSO problem MM with \( \lambda_0 > ||A||^2/\sigma_n^2 \) yields [15]:

\[
x^{(k+1)} = S_{\lambda_0}(x^{(k)} - \frac{1}{\lambda_0 \sigma_n} A^T \left( \frac{Ax - y}{\sigma} - z^{(k+1)} + \frac{\eta^{(k)}}{\sigma} \right)) \tag{17}
\]

**Multiplier update step**

Finally we have the update formula for \( \eta \) which is:

\[
\eta^{(k+1)} = \eta^{(k)} + \sigma \left( \frac{Ax^{(k+1)} - y}{\sigma_n} - z^{(k+1)} \right) \tag{18}
\]

Remark

The weight function \( \phi_{p_s, p_f}(z_i) \) given in equation (11) is undefined when \( |z_i| = 0 \). To deal with this problem, a common alternative ([16]) is to consider a small regularizing parameter \( \epsilon \) in the definition of CMN, i.e., problem (7) is restated as:

\[
\min_{x, z} \int_{p_s}^p \lambda(p) \sum_i |z_i| + \epsilon p dp + \mu \|x\|_1 \quad s.t. \quad z = \frac{Ax - y}{\sigma_n}
\]

The derivation for the \( \epsilon \)-regularized problem with approximation, is similar to what is obtained in \( z \)-update step in section III, except that we let \( |z_i^{(k)}| \leftarrow |z_i^{(k)}| + \epsilon \) in the computation of \( \phi_{p_s, p_f}(z_i^{(k)}) \). In addition, to fasten the rate of convergence of the algorithm, we apply continuation method on the regularizing parameter \( \mu \) (as proposed in [17]) with a minimum threshold denoted by \( \mu_{\text{min}} \). Hence, modified steps of the proposed algorithm named CMN-ALM, are given in Alg. 1.

**Algorithm 1 Proposed robust CS algorithm (CMN-ALM)**

Set \( 0 \leq p_s < p_f \leq 2 \), \( \mu, \sigma, \lambda_0, \zeta > 0 \), \( \mu_{\text{min}}, \epsilon \ll 1 \).

Initialize \( \eta^{(0)} = 0 \), \( x^{(0)} = 0 \), \( z^{(0)} = -y \), \( k = 0 \).

repeat

Update \( z^{(k+1)} \) using (14) or (15) (with \( |z_i^{(k)}| \leftarrow |z_i^{(k)}| + \epsilon \))

Update \( x^{(k+1)} \) using (17)

Update \( \eta^{(k+1)} \) using (18)

Update \( \mu \leftarrow \max(\zeta \mu, \mu_{\text{min}}) \) and \( k \leftarrow k + 1 \)

until A stopping criterion is reached

**IV. SIMULATION RESULTS**

In this section, we conduct experiments to compare the reconstruction quality performance of our proposed method with some state of the art algorithms for robust sparse recovery. In particular, we use Lp-ADM, YALL1 [18], BP-SEP [19] and Huber-FISTA [20]. Based on different choices for \( p_s, p_f \) and \( q \), we obtain different versions of our proposed algorithm. In the following experiments we run 3 versions with \( (p_s = 0, p_f = 1, q = 1) \), \( (p_s = 0, p_f = 1, q = 2) \) and \( (p_s = 0, p_f = 2, q = 2) \). We choose \( \sigma_n = 1 \) and the stopping criterion is set to primal or dual error tolerance of 1e-5 with 100 maximum iterations. The experiments and results are categorized into the following sub-sections:

1) **Average preference ratio:** In this scenario, a set of \( N_t \) \( k \)-sparse vectors of size \( n \times 1 \) are randomly generated where \( N_t = 60, n = 128 \) and \( k = 7 \). For each vector \( x_i \), the elements of the support set are chosen uniformly at random and the values are generated according to normal distribution. This model is indeed used as approximation of the Laplacian prior for the original signal. Each sample vector is then compressed via a random Gaussian measurement matrix \( A_i \) of dimensions \( m \times n \). We choose \( m = 50 \) (nearly 0.4 sampling rate) and the measurement vector \( y_i = A_i x_i \) is then added with \( SNR \) noise with distribution parameters \( \alpha \) and \( \gamma \). These values are chosen from \( \alpha \in \{0.5, 1, 1.5\} \) and \( \gamma \in \{1e-4, 1e-3, 1e-2, 1e-1\} \). The noisy observed vector is finally given to robust sparse recovery algorithms to obtain an estimate \( \hat{x}_i \). We eventually calculate SNR performance and average the results over random turns (random \( x_i \) and \( A_i \)). For this part, we only compare our results with Lp-ADM algorithm. We choose \( \sigma = 1 \), \( \mu_{\text{min}} = 5e-1 \), \( \zeta = 0.95 \), \( \epsilon = 1e-2 \) and \( \lambda_0 = 2 \). We have used the source code for Lp-ADM with default parameters. Since the problem is blind, we do not know the best choice of \( p \) (\( \ell_p \) norm) applied in Lp-ADM algorithm. To compare the performance of the algorithms, we plot the average SNR of Lp-ADM versus \( p \). We then calculate the preference ratio, defined as the percentage of the region \( p \in (0, 2) \) where the SNR curves
corresponding to different versions of our proposed algorithm (these curves are constant lines versus $p$) lie above the Lp-ADM curve. These regions for each curve are highlighted in the preference diagram depicted in Fig. 1. In fact, this experiment demonstrates the probability that our proposed algorithms outperform Lp-ADM when the problem is blind and $p$ is chosen uniformly at random. Furthermore, in a blind problem, neither the original sparse signal nor the parameter $\gamma$ is known. Thus, the regularization path for specifying $\mu$ as proposed in [8] is not applicable. In this case, we choose $\mu = \xi ||A^TX||_\infty$ (as in [17]) with $\xi = 0.1$ for all algorithms. Table I shows these preference ratios for different values of $S_\alpha S$ noise parameters. According to this table, the version of the proposed algorithm with $(p_s = 0, p_f = 1, q = 1)$ has best performance in blind settings. Even the worst case corresponding to the version $(p_s = 0, p_f = 2, q = 2)$ which fails at $\alpha = 0.5$, outperforms Lp-ADM most of the times especially for $\alpha > 0.5$ and $\gamma > 1e-4$.

**TABLE I**

| Preference ratio (in %) of different versions of the proposed algorithm, as defined in Section IV-1, compared to Lp-ADM. The values are reported in 3-tuples corresponding to $(p_s = 0, p_f = 1, q = 1), (p_s = 0, p_f = 1, q = 2)$ and $(p_s = 0, p_f = 2, q = 2)$ versions, respectively. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\gamma = 1e-4$ | $\gamma = 1e-3$ | $\gamma = 1e-2$ | $\gamma = 1e-1$ |
| $\alpha = 0.5$  | $100$ $100$ $0$  | $100$ $100$ $0$  | $100$ $100$ $0$  | $100$ $0$ $0$    |
| $\alpha = 1$    | $87$ $87$ $0$   | $63$ $87$ $100$ | $100$ $100$ $100$ | $100$ $100$ $100$ |
| $\alpha = 1.5$  | $100$ $100$ $100$ | $100$ $100$ $100$ | $100$ $100$ $100$ | $57$ $78$ $100$ |

2) The influence of noise power: In this part, we would like to examine the effect of noise power on the performance of robust sparse recovery algorithms. The settings and the parameters for the algorithms are just similar to the previous scenario except that $\alpha$ is fixed in this experiment and the SNR performance is depicted versus the parameter $\gamma$, which in some sense specifies the additive noise power. For Lp-ADM we choose $p \in \{0.5, 1, 1.5\}$ and the results are compared with those of our proposed methods as well as YALL1, BP-SEP and Huber-FISTA. Fig. 2a shows the SNR performance of robust CS algorithms in $S_\alpha S$ noise versus the scale parameter $\gamma$. In Fig. 2a we have chosen $\alpha = 0.5$ and Fig. 2b depicts the results for $\alpha = 1$. As shown in these figures, the proposed algorithms clearly outperform competing algorithms specifically for $\alpha = 0.5$.

3) The effect of CS factor: This part demonstrates the performance of the proposed algorithms in terms of the CS factor, i.e., the ratio $m/n$ where $m$ equals the number of measurements and $n$ denotes the size of the sparse vector. Similar to previous sections (using same parameters), we generate random 8-sparse vectors of size $n = 128$, but $m$ varies from 0.1$n$ to 0.9$n$. The sparse signals are then corrupted with $S_\alpha S$ impulsive noise. We consider two cases where in the first, we let $\alpha = 1$ and $\gamma = 1e-3$ and in the second, $\alpha = 1.5$ and $\gamma = 1e-3$ are chosen. The SNR performance of the sparse recovery algorithms are finally depicted versus the ratio $m/n$. Fig. 3a and Fig. 3b show the results for the first and the second scenario, respectively. For $\alpha = 1$ the proposed algorithm with $p_s = 0, p_f = 2, q = 2$ has the best performance while for $\alpha = 1.5$ the version with $p_s = 0, p_f = 1, q = 1$ yields more robust recovery.

V. CONCLUSION

In this paper, we explored the problem of blind sparse signal recovery in the presence of impulsive noise. We modelled the noise signal with symmetric $\alpha$-stable i.i.d. components and we incorporated GGD to approximate the PDF of the distribution. In blind conditions, the parameters $\alpha$ and $\gamma$ of the $S_\alpha S$ distribution model are unknown. Treating these parameters as unobserved latent variables, we applied EM algorithm to obtain an iterative approach to near optimal recovery of the original signal where in each iteration, an optimization problem should be solved. Under assumptions, this problem was shown to be equivalent to a CS problem in which, a continuous mixed norm is employed as fidelity criterion. We incorporated ADMM with Majorization Minimization technique to iteratively solve the proposed CS problem. The performance of the proposed algorithm in blind CS recovery in impulsive noise, was finally examined via simulation experiments.
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