Elastic Modulus Prediction of Corrugating Medium under Different Temperature and Relative Humidity

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Abstract: The influence of the temperature and relative humidity on the moisture content of corrugating medium was studied by experiments. The ideal three-dimensional diagram between the moisture content and the temperature and relative humidity was constructed by the GAB model. The prediction equation of the moisture content and the relative elastic modulus was fitted after analyzing the test results by MATLAB software. The results show GAB model can predict the moisture content of corrugating medium in different temperature and humidity conditions, and the fitting equation can describe the relation between the elastic modulus of corrugating medium and the moisture content. The predicted elastic modulus can be used to evaluate the cushioning properties of corrugated sandwich paperboard.

1. Introduction

More and more corrugated sandwich, hexagonal honeycomb core products, which are made from corrugating medium, were applied to prevent the products from damage in real-life logistics environment[1]. People start to pay attention to the mechanical properties of corrugating medium, such as grammage, ring crush, bursting strength and so on. However, the elastic property of corrugating medium was neglected. Elastic modulus is commonly used to characterize the elastic property of materials.

The corrugating medium, mainly composed of plant fibers, is porous and can absorb moisture from the external environment at any time[2]. Environmental conditions greatly affect the moisture content of corrugating medium[3], and then affect the mechanical performance[4]. The mechanical properties of paper structural materials can be predicted and the packaging can be optimized by evaluating the elastic modulus of corrugating medium under different temperature and relative humidity. Therefore, it is significant to study the influence of temperature and relative humidity on the elastic modulus of corrugating medium.

At present, there are some literatures on the moisture content and the elastic modulus of corrugating medium. Nissan[5] took the paper as porous structural material made of hydrogen bond, and proposed an exponential relationship between elastic modulus and moisture content, described by Em=E0/em, where Em is the elastic modulus at any moisture content, E0 is the elastic modulus at m=0, m is moisture content. However he did not study the relationship between moisture content and temperature and relative humidity, nor set up the equation of elastic modulus corresponding to temperature and relative humidity. Petch[6] considered the effect of temperature on the elastic modulus and put the parameter of temperature into the exponential equation mentioned above, but he did not consider the influence of relative humidity on the elastic modulus. Allaoui[7] studied the relationship between the elastic modulus of corrugating medium and relative humidity, and found that the elastic modulus decreases as the relative humidity increases, especially in high humidity...
conditions (RH > 70%), the elastic modulus decreases dramatically. But he did not induce the equation of the relationship between the elastic modulus of corrugating medium and relative humidity. On the basis of J/BT 6544-93, Wang[8] designed the method of measuring the elastic modulus of corrugating medium. The method can be a reference for studying the influence of environmental conditions on the elastic modulus of corrugating medium. As an application, Wang[9] studied a mathematical model in which the structure factors and temperature and relative humidity are concerned is developed through the cooperation of theories and experiments. But the moisture content is not considered.

The primary aim of this study is to evaluate the relationship between the moisture content of corrugating medium and environmental temperature and relative humidity. The second aim of this study is to explore the relationship between the elastic modulus and the moisture content of corrugating medium, and then to seek an equation of the relationship between the elastic modulus of corrugating medium and environmental temperature and relative humidity to predict the mechanical properties of paper structural materials in different environmental conditions.

2. Experiments

2.1 Materials

The corrugating medium is supplied by the HONICEL Paper Honeycomb Products Company (Guangzhou, China). It was made from recycled fiber. The parameters of materials are shown in table 1.

Table 1. Parameters of corrugating medium.

| Size(mm×mm) | Grammage(g/m²) | Poisson ratio | Direction       | Thickness(mm) |
|-------------|----------------|--------------|-----------------|--------------|
| 150×12.5    | 120            | 0.28         | cross-direction | 0.25         |

2.2 Experimental methods

2.2.1 Pre-condition of temperature and relative humidity. All specimens were pre-conditioned in accordance with GB/T 4857.2-2005[10], the experimental conditions are shown in table 2. The experiment equipment is a temperature and humidity programmable controller (CS/CP-KMH-1000R, Komeg Technology Co.Ltd., Dongguan, China). The specimen is placed within the working space of the experiment, and is exposed in the specified conditions for 48h. During the test, the conditioning atmosphere has free access to the top, sides and at least 75% of the base.

Table 2. Experimental conditions.

| Temperature $T(°C)$ | 5   | 20  | 30  | 40  | 50  |
|----------------------|-----|-----|-----|-----|-----|
| Relative Humidity $RH(%)$ | 30  | 50  | 70  | 80  | 90  |

P.S. As the temperature is below zero, the relative humidity cannot be determined. Therefore the temperature which is below zero is not considered.

2.2.2 The experiment of moisture content. The experimental equipment is a moisture test apparatus (SFY20A, Houwang Technology Co.Ltd., Shenzhen, China). The specimen which was pre-conditioned is placed into the equipment. The time is set for five minutes; the temperature is 100°C. After the adjustment, the equipment is started. Five minutes later, the moisture content of specimens is recorded.

2.2.3 The experiment of elastic modulus. Since there is no clearly defined experimental method for elastic modulus of paper, the experimental method is referenced to GB/T 1447-2005[11]. The experiment equipment is computer-controlled electronic universal testing machine (M-3050, Shenzhen Reger Instrument Co, Ltd, Shenzhen, China). During the test, the gauged distance is set into 100mm; the stretching speed is 1mm/min. At the end of the experiment, the load value corresponding to the stain was 0.07% and 0.08% is recorded, and the elastic modulus of corrugating medium is calculated.
in accordance with GB/T 1447-2005[12], Fiber-reinforced plastics composites-Determination of tensile properties.

3. Results and discussion

3.1 GAB model
On the basis of unimolecular layer adsorption theory and the BET multi-molecular adsorption theory, Guggenheim, Anderson and De Boe propose the GAB model. After constantly modified and the parameter of temperature increases, GAB model becomes a three-parameter model. Below is the mainly composed of plant fibers,

\[
GAB(T, RH) = m = \frac{a \cdot b \cdot \left(\frac{c}{T}\right) \cdot RH}{(1 - b \cdot RH) \cdot \left[1 - b \cdot RH + b \cdot \left(\frac{c}{T}\right) \cdot RH\right]} \tag{1}
\]

Where \(m\) is the equilibrium moisture content; \(T\) is temperature; \(RH\) is relative humidity; \(a\), \(b\) and \(c\) are the model parameters.

The GAB model is one of the best models which include theoretical significance of thermodynamics. It is suitable for a variety of items. This paper fits the moisture content of corrugating medium with the GAB model.

3.2 The relationship between the moisture content of corrugating medium and temperature and humidity
The experimental data of the moisture content tests is shown in table 3. In order to perfectly characterize the relationship between the moisture content of corrugating medium and environmental temperature and humidity, the three-dimensional diagram of the relationship between the moisture content of corrugating medium and environmental temperature and humidity is drawn in light of the experimental data. The experimental diagram is shown in figure 1a.

| Relative Humidity RH (%) | 5    | 20   | 30   | 40   | 50   |
|-------------------------|------|------|------|------|------|
| 30                      | 10.41| 9.32 | 7.31 | 6.67 | 6.33 |
| 50                      | 12.03| 10.15| 8.88 | 9.09 | 8.24 |
| 70                      | 12.59| 10.99| 10.88| 10.87| 9.24 |
| 80                      | 13.16| 12.03| 11.06| 11.38| 10.22|
| 90                      | 17.00| 13.89| 13.19| 12.39| 11.42|

Table 3. Experimental results of the moisture content of corrugating medium in different temperature and relative humidity.
Making use of MATLAB to fit a curve, estimate parameter and describe the diagram. Error estimates were calculated by Levenberg-Marquardt's algorithm, the value of convergence of variance and model parameters is $10^{-20}$, and there is no limit in iteration times. The fitting results are listed as follows:

$$a=10.1157, \quad b=0.00366, \quad c=421.1393, \quad R^2=0.9916.$$ Combining the results with equation (1), the GAB model of the moisture content of corrugating medium is obtained,

$$m = \frac{15.6RH}{(1-0.00366RH)(1-0.00366RH +1.54RH/T)}T$$

(2)

The ideal three-dimensional diagram of the moisture content of corrugating medium is described in accordance with equation (2). The ideal diagram is shown in figure 1b.

According to equation (2) or figure 1b, the ideal moisture content of corrugating medium in a specific environment temperature and humidity can be identified. The three-dimensional diagram of the moisture content is simple and intuitive. It is easier to obtain information from the diagram, which is important for studying the relationship between mechanical property of corrugating medium and environmental temperature and relative humidity.

3.3 The relationship between the elastic modulus and moisture content of corrugating medium

The corrugating medium's tensile stress-strain curves are similar under different environmental conditions. They all show three stages, that is, linear elastic stage, tensile yield stage and fracture stage. A typical tensile stress-strain curve of corrugating medium is shown in figure 2.
The variety of environmental temperature and relative humidity affects the moisture content of corrugating medium, and then affects the mechanical properties of corrugating medium. For this purpose, the relationship between elastic modulus and moisture content of corrugating medium was studied.

The elastic modulus of corrugating medium is to a great extent influenced by the material parameters, such as thickness, grammage and so on. Therefore, the results of this study would not apply to any other corrugating medium. In order to characterize the influence of moisture content on elastic modulus perfectly and compare the mechanical properties of paper made from different materials conveniently, a relative elastic modulus measure can be utilized, in which the experimental elastic modulus is divided by a standard elastic modulus corresponding to a given equilibrium moisture content found at standard conditions (23°C and 50% RH). Therefore, all experimental elastic modulus are related to the elastic modulus determined for an identical corrugating medium conditioned at 23°C and 50% RH.

The relationship between the relative elastic modulus and moisture content of corrugating medium obtained from the experiment is shown in figure 3. It indicates that the relative elastic modulus declines with the increase of moisture content. Base on the theory of hydrogen bonding, Nissan studied the elastic modulus of porous structural material and found that there is an exponential relationship between the elastic modulus and the moisture content of porous structural material. Since the paper is a porous solid, the MATLAB was utilized to fit the relationship between relative elastic modulus and moisture content of corrugating medium exponentially and the predictive model for the relative elastic modulus was assessed. The predictive model is given as follows:

$$\%E = 2.3084e^{-0.0769m}$$  \hspace{1cm} (3)

Where $\%E$ is relative elastic modulus; $m$ is moisture content of corrugating medium.
of determination), $MRD = 3.752\%$ (mean residual deviation). Therefore, the predictive model fit suitably. The model was utilized to predict the elastic modulus and the predicted and experimental values are in good agreement. This shows that the model can be used to describe the relationship between the elastic modulus and the moisture content of corrugating medium.

Substituting equation (2) into equation (3), and the equation which describes the relationship between relative elastic modulus and moisture content was obtained, as follows:

$$\%E = 2.3084e^{\frac{-1.1996\cdot RH}{(1-0.00366\cdot RH)^{1.54}} - \frac{RH}{T}}$$

(4)

The three-dimensional diagram was drawn in accordance with equation (4) and shown in figure 4. The comparison of experimental $\%E$ and predictive $\%E$ is listed in table 4. The comparison diagram of experimental value and predictive value is shown in figure 5, the coefficient of determination $R^2 = 0.9613$. This proves that the model fits well. It is easy to find that both relative humidity and temperature affect relative elastic modulus from the figure 4. As the relative humidity increases, the relative elastic modulus decreases. Conversely, increase of the temperature will strengthen the relative elastic modulus, as the higher temperature will cause evaporation of moisture from corrugating medium, thereby lowering the moisture content, the relative elastic modulus will be higher.

Table 4. The comparison of experimental $\%E$ and predictive $\%E$.

| Relative Humidity $RH$ (%) | 5     | 20    | 30    | 40    | 50    |
|---------------------------|-------|-------|-------|-------|-------|
| 30                        | 1.103/1.039 | 1.123/1.227 | 1.326/1.325 | 1.375/1.408 | 1.447/1.478 |
| 50                        | 0.924/0.933 | 1.039/1.051 | 1.157/1.120 | 1.154/1.181 | 1.283/1.238 |
| 70                        | 0.821/0.838 | 0.904/0.919 | 0.958/0.969 | 1.112/1.015 | 1.113/1.060 |
| 80                        | 0.786/0.791 | 0.874/0.859 | 0.931/0.902 | 0.938/0.942 | 1.003/0.981 |
| 90                        | 0.704/0.742 | 0.819/0.801 | 0.849/0.837 | 0.868/0.872 | 0.891/0.905 |

Figure 4. The three-dimensional diagram of relative elastic modulus predicted from different humidity and temperatures by the model given in equation 4.

Figure 5. The linear relationship between experimental $\%E$ and predicted $\%E$.

The three-dimensional diagram is simple and easy to understand for packaging designers, and provides a quick and convenient method for estimating the elastic modulus of corrugating medium in different environmental conditions. In real-life logistics environment, varying environmental conditions, with both short and long-lasting changes in temperature and relative humidity, are more likely, and it is well known that to a greater extent the varying conditions affect mechanical properties
than constant conditions. Hence, it is important to validate the relative elastic modulus in response to cyclic environmental conditions as well, to obtain a full understanding of the mechanical functionality of corrugating medium used for packaging.

4. Conclusions
(1) Based on the test analysis, the moisture content of corrugating medium alters as the environmental conditions change. When the relative humidity is constant, the moisture content decreases with the temperature increasing. When the temperature is constant, the moisture content increases with the relative humidity increasing.

(2) The relationship between the elastic modulus and the moisture content of corrugating medium is exponential, that is to say, as the moisture content rises, the elastic modulus of corrugating medium decreases exponentially. The predictive model which was fitted is given as follows:

\[ %E = 2.3084e^{\frac{-1.1996RH}{(1-0.00366RH)(1-0.00366RH+1.5e^{\frac{2RH}{T}})}} \]

After verification, the model can predict the elastic modulus of corrugating medium under different temperatures and humidity conditions.

(3) The three-dimensional diagram drawn in accordance with the model is more direct. It can predict the mechanical properties of corrugating medium under different environmental conditions by combining model and three-dimensional diagram.

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