Infrared behavior of gluon and ghost propagators from asymmetric lattices

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We present a numerical study of the lattice Landau gluon and ghost propagators in three-dimensional pure SU(2) gauge theory. Data have been obtained using asymmetric lattices \((V = 20^2 \times 60, 40^2 \times 60, 8^2 \times 64, 8^2 \times 140, 12^2 \times 140 \text{ and } 16^2 \times 140)\) for the lattice coupling \(\beta = 3.4,\) in the scaling region. We find that the gluon (respectively ghost) propagator is suppressed (respec. enhanced) at small momenta in the limit of large lattice volume \(V\). By comparing these results with data obtained using symmetric lattices \((V = 60^3 \text{ and } 140^3),\) we find that both propagators suffer from systematic effects in the infrared region \((p \lesssim 650\text{MeV}).\) In particular, the gluon (respec. ghost) propagator is less IR-suppressed (respec. enhanced) than in the symmetric case. We discuss possible implications of the use of asymmetric lattices.

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I. INTRODUCTION

Gluon and ghost propagators are powerful tools in the (non-perturbative) investigation of the infra-red (IR) limit of QCD and are thus of central importance for understanding confinement (see e.g. \([1, 2]).\) In fact, according to the Gribov-Zwanziger \([3, 4, 5, 6, 7]\) and to the Kugo-Ojima \([8, 9]\) confinement scenarios in Landau gauge, the ghost propagator must show a divergent behavior — stronger than \(p^{-2}\) — for vanishing momentum \(p\). This strong infrared divergence corresponds to a long-range interaction in real space, which may be related to quark confinement. At the same time, according to the former scenario, the gluon propagator must be suppressed and may go to zero in the IR limit \(\approx p^{-2}\). This would imply that the real-space gluon propagator is positive and negative in equal measure, i.e. reflection positivity is maximally violated \([13, 14]\), indicating absence of gluons from the physical spectrum (gluon confinement). These predictions are also valid for the case of pure SU(2) gauge theory and for three space-time dimensions.

The nonperturbative evaluation of gluon and ghost propagators can be achieved by first principles methods such as Dyson-Schwinger equations (DSE’s) \([1, 2]\) or the numerical simulation of lattice QCD. Landau-gauge studies of DSE’s (see e.g. \([15, 16, 17]\)) have found an IR behavior of the form \(D(p^2) \sim p ^{4\kappa - 2} = p^{2a_D - 2}\) [implying \(D(0) = 0\) if \(\kappa > 0.5\)] for the gluon propagator and of the form \(G(p^2) \sim p ^{2\kappa - 2} = p ^{2a_G - 2}\) for the ghost propagator, with the same exponent \(\kappa\) (with \(a_D = 2a_G\)). The available predictions for the IR exponent point towards \(\kappa \gtrsim 0.5\) for pure SU(\(N_c\)) gauge theory in the four dimensional case. For the 3d case the exponents are \(a_G \approx 0.4\) and \(a_D \approx 1.3\). Note that in the \(d\) dimensional case \([13, 16]\) the relation between \(a_D\) and \(a_G\) is given by \(a_D = 2a_G + (4 - d)/2,\) implying for the quantity \(\alpha(p^2) = (g^2/4\pi)D(p^2)G^2(p^2)\) the infrared behavior \(p^{2(a_G - 2a_G)} = p^{4-d}\). Thus, in the 4-dimensional case the running coupling \(\alpha(p^2)\) displays and infrared fixed point.

Numerical studies of lattice gauge theories confirm the IR divergence of the Landau ghost propagator \([15, 16, 21]\) and have now also established that the Landau gluon propagator shows a turnover in the IR region, attaining a finite value for \(p \approx 0\). (A reliable extrapolation of \(D(0)\) to the infinite-volume limit is still lacking \([22]\).) More precisely, indications of a decreasing gluon propagator for small \(p\) have been obtained in the 4d SU(2) and SU(3) cases for the strong-coupling regime \([13, 21, 24]\), in the 3d SU(2) case (also in the scaling region) \([24, 26, 27]\), in the 3d SU(2) adjoint Higgs model \([20]\) and in the 4d SU(2) case at finite temperature \([25]\). The actual turning of the gluon propagator has been seen clearly for the equal-time three-dimensional transverse gluon propagator in 4d SU(2) Coulomb gauge \([24, 30]\), for the 3d SU(2) Landau case using very large lattices \([22]\) (of 140 lattice sites per direction) and — recently — in the 4d SU(3) Landau case with the use of asymmetric lattices \([31, 32, 33, 34]\).

Thus, the two nonperturbative approaches above seem to support the Gribov-Zwanziger and the Kugo-Ojima confinement scenarios in Landau gauge. However, the agreement between the two methods is still at the qualitative level, at least when considering the gluon and the ghost propagators. Moreover, recent lattice studies \([32, 30]\) seem to indicate a null IR limit for \(\alpha(p^2)\), instead of a finite nonzero value. Of course, in this case, special care must be taken to eliminate finite-size effects, especially when the IR region is considered \([21, 22, 24, 26, 37, 38]\). This issue has also been studied analytically using DSE’s \([33]\), showing that torus and continuum solutions are qualitatively different. This suggests a nontrivial relation between studies on compact and on noncompact manifolds. Finally, the nonrenormalizability of the ghost-gluon vertex — proven at the perturbative level \([40]\) — confirmed on the lattice \([36, 41]\) (for \(p \gtrsim 200\text{MeV}\)) and used in DSE studies to simplify the coupled set of equations — has been recently criticized in Ref. \([35]\). Thus, clear quantitative understanding of the two confinement scenarios is still an open problem.

In this work we extend the study presented in \([22]\) for the 3d SU(2) case, by including results for the ghost...
propagators from very large lattices and by considering also asymmetric lattices, as done in [31, 32, 33, 34, 42]. Our aim is to verify possible systematic effects related to the use of asymmetric lattices (as suggested in [30]), by comparing the results to the ones obtained for symmetric lattices. In particular, we focus on the determination of the IR critical exponents $a_D$ and $a_G$ and on their extrapolation to the infinite-volume limit. The study of gluon and ghost propagators in three space-time dimensions is computationally much simpler than in the four-dimensional case and it may help to get a better understanding of the 4d case [35, 36]. Note that the 3d case is also of interest in finite-temperature QCD (see for example [40, 47]).

In the following section we describe our simulations and present our results. We end with our conclusions.

II. SIMULATIONS AND RESULTS

Simulations have been done using the standard Wilson action for $SU(2)$ lattice gauge theory in three dimensions with periodic boundary conditions. We consider $\beta = 3.4$ and several lattice volumes, up to $V = 140^3$. In Table I we report, for each lattice volume $V$, the parameters used for the simulations. All our runs start with a random gauge configuration. We used the hybrid overrelaxed algorithm (HOR) [48, 49] for thermalization and the stochastic overrelaxation algorithm or the so-called Cornell method for gauge fixing thermalized configurations to the lattice Landau gauge [50, 51, 52]. The numerical code is parallelized using MPI [53]. For the random number generator we use a double-precision implementation of RANLUX (version 2.1) with luxury level set to 2. Computations were performed on the PC clusters at the IFSC-USP. We ran part of the $60^3$ lattices on four or eight nodes.

In order to set the physical scale we consider $3d$ $SU(2)$ lattice results for the string tension and the input value $\sqrt{\sigma} = 0.44 GeV$, which is a typical value for this quantity in the $4d$ $SU(3)$ case. With $\hbar = c = 1$, this implies $1 fm^{-1} = 0.4485\sqrt{\sigma}$. For $\beta = 3.4$ we obtain the average plaquette value $\langle W_{1,1} \rangle = 0.672730(3)$. Then, considering the tadpole-improved coupling $\beta_I \equiv \beta(W_{1,1})$, we can evaluate the string tension $\sqrt{\sigma}$ in lattice units using the fit reported in Eq. 2 and Table IV of Ref. [54]. (The fit is valid for $\beta \gtrsim 3.0$, i.e. the coupling $\beta$ considered here is well above the strong-coupling region.) This gives $\sqrt{\sigma} = 0.506(5)$, implying a lattice spacing $a = 0.227(2)$ fm, i.e. $a^{-1} = 0.869(8)$ GeV. Also, we are able to consider here momenta as small as $p_{\text{min}} = 39.0(4)$ MeV and physical lattice sides of almost 32 fm. Finally, let us notice that, if we compare the data for the string tension (in lattice units) with the data reported in Ref. [55] for the $SU(3)$ group in four dimensions (see also Ref. [21]), then our value of $\beta$ corresponds to $\beta \approx 2.21$ in the 4d case.

We study the lattice gluon and ghost propagators [respectively, $D(p^2)$ and $G(p^2)$] as a function of the magnitude of the lattice momentum $p(k)$ (see Ref. [27, 50] for definitions). We consider all vectors $k = (k_x, k_y, k_z)$ with only one component different from zero. For asymmetric lattices, the non-zero component has been taken along the elongated direction; for symmetric lattices, when evaluating the gluon propagator, we average over the three directions in order to increase statistics. Here, we do not check for possible effects of the breaking of rotational invariance [57, 58]. Nevertheless, as we will see below, systematic effects due to the use of asymmetric lattices are evident mostly in the IR limit, where one expects the breaking of the rotational symmetry to play a small effect (see also Ref. [22]).

In Figures I and II we plot the data obtained for the gluon propagator for different lattice volumes. (Errors represent one standard deviation.) One can clearly see

| Table I: Lattice volumes $V$, configurations, HOR sweeps used for thermalization and between two configurations (used for evaluating both propagators). |
|---|---|---|---|
| $V$ | Configurations | Thermalization | Sweeps |
| $20^3$ | 500 | 550 | 50 |
| $20^2 \times 40$ | 800 | 440 | 40 |
| $20^2 \times 60$ | 600 | 550 | 50 |
| $60^3$ | 400 | 1100 | 100 |
| $8^3 \times 64$ | 100 | 550 | 50 |
| $8^2 \times 140$ | 1000 | 550 | 50 |
| $12^2 \times 140$ | 600 | 660 | 60 |
| $16^2 \times 140$ | 600 | 770 | 70 |
| $140^3$ | 101 | 1650 | 150 |
that the propagator is IR-suppressed for sufficiently large lattice volumes. This behavior is evident both for the asymmetric and the symmetric lattices. However, the shape of the propagator is clearly different in the two cases for momenta $p \lesssim 0.75/a$, i.e. $p \lesssim 0.65$ GeV. Indeed, for asymmetric lattices the gluon propagator starts to decrease only at very small momenta (see Figures 1 and 2). A similar behavior can also be observed in Fig. 2 of Ref. 32 and in Fig. 4 of Ref. 34.

The asymmetric lattices one would estimate a value $M \lesssim 0.25/a \approx 0.22$ GeV as a turnover point in the IR, i.e. the momentum $p = M$ for which the propagator reaches its peak. On the other hand, considering the two largest lattice volumes, i.e. $V = 60^3$ (see Fig. 1) and $V = 140^3$ (see Fig. 2), the gluon propagator is clearly a decreasing function of $p$ for $p \lesssim 0.5/a$, corresponding to $M \lesssim 0.435$ GeV. This is in agreement with Refs. 22, 27 where the value $M = 350^{+100}_{-50}$ MeV is reported. We thus see a difference of almost a factor 2 between the momentum-turnover point in the symmetric and asymmetric cases.

Let us also note that in Figures 1 and 2 the gluon propagator at zero momentum $D(0)$ is monotonically decr...
FIG. 5: Plot of the ghost propagator $G(p^2)$ as a function of $p$ (both in lattice units) for lattice volumes $V = 8^3 \times 140 (\times)$ and $20^3 (\circ)$. Note the logarithmic scale on the $y$ axis.

FIG. 6: Plot of the ghost propagator $G(p^2)$ as a function of $p$ (both in lattice units) for lattice volumes $V = 8^2 \times 140 (\times)$, $12^2 \times 140 (\circ)$, $16^2 \times 140 (\ast)$ and $V = 140^3 (\bigcirc)$. Note the logarithmic scale on the $y$ axis.

We also tried a fit to the data using the fitting functions considered in Ref. 34 (see results in Table III). We see that the IR exponent $a_D$ usually increases with the lattice volume, in agreement with the results reported in Table 2 of Ref. 34. However, it is clear from Figure 4 that the dependence of $a_D$ on $1/V$ is not simple. Indeed, $a_D$ looks almost constant when considering $V = 20^2 \times 40$ and $20^2 \times 60$, but it gets a larger value when using the symmetric — and much larger — lattice volumes $V = 60^3$ and $140^3$. A smaller value for $a_D$ is also found if one tries an extrapolation as a function of $1/V$ for the results obtained using the three asymmetric lattices $V = N_s^2 \times 140$. Again, it could be difficult to have systematic errors under control when trying an extrapolation using only asymmetric lattices.

Strong systematic effects can also be observed in the ghost case. The propagator is less IR divergent when one considers asymmetric lattices (see Figures 5 and 6). As a consequence, the IR exponent $a_G$ is also smaller (see Table III) for asymmetric lattices. This is the case either when comparing an asymmetric lattice to a symmetric one with almost equal lattice volume (i.e. $V = 8^3 \times 140$ and $V = 20^3$) or when considering an asymmetric lattice and a symmetric one with the same largest lattice side (e.g. $V = 16^2 \times 140$ and $V = 140^3$). Moreover, the dependence on the lattice volume is very different in the two cases. Indeed, for symmetric lattices (e.g. $V = 20^3$, $60^3$ and $140^3$), the IR exponent decreases as the lattice volume increases (at fixed $\beta$). This is in agreement with the result obtained in the $4d$ SU(2) case (see Table 4 and Figure 3 in Ref. 10). On the other hand, $a_G$ increases as a function of $V$ for the lattices $V = N_s^2 \times 140$ with increasing $N_s$. Thus, there is no simple relation between $a_G$ and $V$ and an extrapolation in $V$ using asymmetric lattices is probably of difficult interpretation.

Using the above results we can evaluate the quantity $a_D - 2a_G - 1/2$, which is zero using DSE’s [13, 14]. We find $a_D - 2a_G - 1/2 = 0.21(2)$ for $V = 60^3$ and $0.28(2)$ for $V = 140^3$. For the asymmetric lattices this value is

| $V$     | $a_G$  | $\chi^2$/d.o.f. | #d.o.f. |
|---------|--------|-----------------|---------|
| $20^3$  | 0.272  | 0.4             | 1       |
| $20^3 \times 40$ | 0.18 (1) | 4.9          | 6       |
| $20^3 \times 60$ | 0.154 (9) | 3.9         | 11      |
| $60^3$  | 0.21 (1) | 4.6            | 11      |
| $8^2 \times 64$ | 0.053 (7) | 1.7          | 12      |
| $8^2 \times 140$ | 0.025 (2) | 1.9         | 31      |
| $12^2 \times 140$ | 0.060 (4) | 2.0         | 31      |
| $16^2 \times 140$ | 0.093 (6) | 2.0         | 31      |
| $140^3$ | 0.174 (7) | 8.5          | 31      |
somewhat larger. However, let us note that these results depend on the choice of the fitting function. Moreover, the null value for \(a_D - 2a_G - 1/2\) should be obtained only in the infinite-volume and in the continuum limit. We will analyze these limits in a future study.

In this work we did not do a careful study of Gribov-copy effects [10, 54, 61, 62, 63, 64, 65, 66, 67], since we are interested in possible systematic effects due to the use of asymmetric lattices [68]. Nevertheless, for the gluon propagator and \(V = 20^3\) and \(8^2 \times 140\), we compare results obtained using a standard gauge-fixing algorithm to data obtained with the so-called smearing method [31, 32]. (In this case we considered 2000 configurations for each lattice volume and each gauge-fixing method.) We found that for \(V = 20^3\) the value of \(D(0)\) is 0.873(9) when using the smearing method, while one gets 0.906(9) with a standard gauge-fixing method. The two results differ by several standard deviations and the propagator is smaller. This seems to be a systematic effect, which we observe for the first nine smaller momenta, when considering the asymmetric lattice. We believe that this issue deserves a more careful analysis.

**III. CONCLUSIONS**

We have compared data for gluon and ghost propagators (in minimal Landau gauge) obtained using symmetric and asymmetric lattices. We find, for both propagators, clear evidences of systematic effects at relatively small momenta (\(p \lesssim 650\) MeV \(\approx 1.5\sqrt{\sigma}\)). In particular, the gluon (respectively, ghost) propagator is less suppressed (resp. enhanced) in the IR limit when considering asymmetric lattices than for the case of symmetric lattices. This implies that the estimates for the IR critical exponents \(a_G\) and \(a_D\) are systematically smaller in the asymmetric case compared to the symmetric one. Also, for the gluon propagator, the turnover point \(M\) is significantly smaller when considering asymmetric lattices than for the symmetric ones. Let us recall that \(M\) corresponds to the Gribov mass scale in a Gribov-like propagator \(D(p^2) = p^2/(p^4 + M^4)\). Finally, we have seen that the extrapolation to infinite volume of results obtained using asymmetric lattices is also most likely affected by systematic errors. We conclude that, even though using an asymmetric lattice does not modify the qualitative behavior of the two propagators, one should be careful in extracting quantitative information from such studies.

Our data have been obtained in the 3d \(SU(2)\) case. However, the behavior observed for the gluon propagator is very similar to what is obtained in the 4d \(SU(3)\) case [31, 52, 53, 54], where of course a study of this type is more complicated.

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[1] R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001).
[2] A. Holl, C. D. Roberts and S. V. Wright, nucl-th/0601071.
[3] V. N. Gribov, Nucl. Phys. B 139, 1 (1978);
[4] D. Zwanziger, Nucl. Phys. B 412, 657 (1994).
[5] R. F. Sobreiro, S. P. Sorella, D. Dudal and H. Verschelde, Phys. Lett. B 590, 265 (2004).
[6] D. Dudal, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D 72, 014016 (2005).
[7] R. F. Sobreiro and S. P. Sorella, hep-th/0504095.
[8] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. 66, 1 (1979); Erratum – Prog. Theor. Phys. 71, 1121 (1984).
[9] T. Kugo, hep-th/9511033.
[10] D. Zwanziger, Phys. Lett. B 257, 168 (1991).
[11] D. Zwanziger, Nucl. Phys. B 364, 127 (1991).
[12] D. Zwanziger, Nucl. Phys. B 378, 525 (1992).
[13] A. Cucchieri, T. Mendes and A. R. Taurines, Phys. Rev. D 71, 051902(R) (2005).
[14] S. Furui and H. Nakajima, Phys. Rev. D 70, 094504 (2004).
[15] D. Zwanziger, Phys. Rev. D 65, 094039 (2002).
[16] C. Lerche and L. von Smekal, Phys. Rev. D 65 (2002) 125006.
[17] R. Alkofer, W. Detmold, C. S. Fischer and P. Maris, Phys. Rev. D 70, 014014 (2004).
[18] H. Suman and K. Schilling, Phys. Lett. B 373, 314 (1996).
[19] A. Cucchieri, Nucl. Phys. B 508, 353 (1997).
[20] S. Furui and H. Nakajima, Phys. Rev. D 69, 074505 (2004).
[21] J. C. R. Bloch, A. Cucchieri, K. Langfeld and T. Mendes, Nucl. Phys. B 687, 76 (2004).
[22] A. Cucchieri, T. Mendes and A. R. Taurines, Phys. Rev. D 67, 091502(R) (2003).
[23] H. Nakajima and S. Furui, Nucl. Phys. Proc. Suppl. 73, 635 (1999).
[24] A. Cucchieri, Phys. Lett. B 422, 233 (1998).
[25] A. Cucchieri, F. Karsch and P. Petreczky, Phys. Lett. B 497, 80 (2001).
[26] A. Cucchieri, F. Karsch and P. Petreczky, Phys. Rev. D 64, 036001 (2001).
[27] A. Cucchieri, Phys. Rev. D 60, 034508 (1999).
[28] I. L. Bogolubsky and V. K. Mitrjushkin, hep-lat/
Note that evidences of Gribov-copy effects in lattice Landau gauge have been found for gluon and ghost propagators, the horizon tensor, the smallest eigenvalue of the Faddeev-Popov matrix, the Kugo-Ojima parameter and $\alpha_s(p)$ (defined using gluon and ghost propagators).