Comments on gauge unparticles

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Abstract
A field model for a quark and an antiquark binding is described. Quarks interact via a gauge unparticle ("ungluons"). The model is formulated in terms of Lagrangian which features the source field $S(x)$ which becomes a local pseudo-Goldstone field of conformal symmetry - the pseudodilaton mode and from which the gauge non-primary unparticle field is derived by $B_\mu(x) \sim \partial_\mu S(x)$. Because the conformal sector is strongly coupled, the mode $S(x)$ may be one of new states accessible at high energies. We have carried out an analysis of the important quantity that enters in the "ungluon" exchange pattern - the "ungluon" propagator.

PACS 11.10.Cd, 11.15.Ex

1 Introduction

It is evidently that unparticle phenomenon [1] and its phenomenology have been widely overlooked and discussed in the literature (see, e.g., the recent papers in [2] and the references therein).

To develop a model for quark and an antiquark binding we follow the Georgi’s idea [1] that a nontrivial scale invariant sector of scale dimension $d$ might manifest itself at low energy as a nonintegral number $d$ of massless unparticles. One of the physical realization of unparticle imagination is to look at unparticle as a limiting case in which the unparticle fields and their mass spectra are given in the tower of infinite number of particles [3].

We assume that the fields of the hidden sector undergo dimensional transmutation at scale $\Lambda$ generating scale invariant unparticle field. It means that $\Lambda$ defines a border energy scale where unparticle field(s) can interact with the Standard Model (SM) ones.

It is worth recalling at this stage that the interaction of the hidden sector given by the scale invariant unparticle operator $O_U$ with dimension $d$ and the SM operator $O_{SM}$ of dimension $n$ is

$$\left( \frac{\Lambda}{M_U} \right)^{d+n-4} \frac{O_U O_{SM}}{\Lambda^{d+n-4}},$$

(1)
where $M_U$ is the mass of messenger in the ultra-violet (UV) hidden sector of dimension $d_{UV}$ possessing the infra-red (IR) fixed point. If $O_U$ is a vector field operator $O_\mu$ it could couple to the matter field(s) and its exchange between particles in the SM could lead to additional effect of interactions.

We investigate the effects on the conformal sector from the gauge sector, and will show that this leads to surprising new bounds on unparticle physics. The main attention is to the effective operator of the type $(\hat{A} = \gamma_\mu A^\mu)$

$$g^* \bar{\psi}(x) \hat{O}(x) \psi(x) \Lambda^{d-1}, \quad g^* = g \left( \frac{\Lambda}{M_U} \right)^{d_{UV}},$$  \hspace{1cm} (2)

where $g$ is dimensionless and $\psi(x)$ being the prototype spin-1/2 spinor (quark) field. We assume that $O_\mu(x)$ transforms like a vector operator under the gauge transformations, and thus, the term with (2) gives an action which is invariant under this transformations. The interaction given in (2) implies that the unparticle can be exchanged between massive spinor particles, and this exchange creates a new force, which we call "un gluon" force added to the standard gluon force.

We shall not consider the quantization of gauge unparticle field in the standard conventional manner for following reasons:
- to be consistent with experiment where no such unparticle has been identified;
- to avoid the IR problems in perturbation theory.

We attribute no dynamical degrees of freedom to the gauge un particle. Instead we regard the unparticle field as the object of a direct interaction between quark and antiquark. We shall investigate the effects of the scale invariant sector from the gauge field sector, and we will show that this leads to new bounds on un particle physics.

The paper is organized as follows. In section 2 we introduce the effective Lagrangian of the model. The "un gluon" propagator function in four-dimensional space-time (4D) is given in section 3. In the last section we conclude with some remarks.

2 Scale invariance and (pseudo)dilaton mode

We start by discussing a model of un particle physics, which is different from the previously suggested models. The gauge-invariant operator $Q_{\alpha\beta}(1,2)$ for quark 1 and antiquark 2 as $4 \times 4$ matrix in Dirac spinor indexes $\alpha$ and $\beta$ is

$$Q_{\alpha\beta}(1,2) = -\frac{1}{N} \bar{\psi}_\beta(2) U(2,1) \psi_\alpha(1),$$  \hspace{1cm} (3)
where the unparticle operator $U(2,1)$ is given by unparticle field $B_\mu(x)$:

$$U(2,1) = \exp \left\{ -ig \int_0^1 ds B_\mu(x_s) \frac{dx_\mu}{ds} \right\}.$$  (4)

Here, the straight path integration is taken through $x_s = x_1 + sr$, where $r = x_2 - x_1$ is the relative distance between fermi-particles.

With respect to the scale invariance, the Noether theorem tells about the existence of the corresponding conserved dilatation current $J^\mu_{dil}$, where $\partial_\mu J^\mu_{dil} = \theta^\mu_{\mu}$ being the energy-momentum tensor. In conformally invariant theories the last expression is equal to zero, however due to quantum effects conformal invariance is broken [4]

$$\theta^\mu_{\mu} = \sum_{q, quarks} m_q \bar{\psi} \psi + \frac{\beta(g)}{2g} F^{a\mu\nu} F^a_{\mu\nu}.$$  (5)

Here, the function $\beta(g) = \mu \partial g(\mu)/(\partial \mu)$ governs the behaviour of the running coupling $g$ with the scale $\mu$. In the case of $SU(3)$ gauge theory coupled to $n_f$ massless fermions in the fundamental representation [5]

$$\beta(g) = \left( \beta_0 - \frac{g^3}{16 \pi^2} + \beta_1 - \frac{g^5}{(16 \pi^2)^2} \right),$$  (6)

where $\beta_0 = -[11 - (2/3)n_f]$, $\beta_1 = -[102 - (10 + 8/3)n_f]$. If $g$ is small enough at high energy, it will increase as the renormalization scale $\mu$ decreases until the fixed point $\beta(g) = 0$ is encountered at some $g = g^*$. This is an IR fixed-point of the renormalization group flow. Hence, at energy $E < \Lambda_U < M_U$ the effective theory becomes scale-invariant.

It has been demonstrated [6] that the coupling of gluodynamics to the conformal background gravity, described by a single scalar field (dilaton), leads to the fact that theory is conformally invariant in any dimension.

In the Lagrangian framework we introduce two vector fields: $B_\mu(x)$ and $C_\mu(x)$, of which only one - an unparticle gauge ("ungluon") $B_\mu(x)$ - field will interact directly with the quark field $\psi(x)$ with mass $m$ ($C_\mu(x)$ being the auxiliary field). The Lagrangian density is

$$L = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi} (i \hat{\partial} - m - \frac{g^*}{\Lambda^{d-1}} \hat{B}) \psi - \frac{\alpha^2}{2 \Lambda^{d-1}} B_{\mu\nu} C^{\mu\nu} - \xi \frac{1}{\Lambda^{d-1}} (\partial_\mu B^\mu) (\partial_\nu C^\nu) - \frac{1}{2} \mu^2 C_\mu C^\mu + \frac{m^2}{2} \left( \frac{1}{\Lambda^{d-1}} B_\mu - \partial_\mu S \right)^2,$$  (7)

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$; $\alpha$ is dimensionless; $\xi$ is the gauge parameter; $\mu$ is a dimensional coupling constant - a mass parameter.
The scalar field $S(x)$ serves as the conformal compensator with mass $m_S$. In real world the scale invariance is lost, particles possess finite mass and sizes. It is therefore tempting to formulate an effective theory of broken scale invariance also in terms of the corresponding pseudo-Goldstone boson of spontaneously broken (approximate) scale invariance. For example, a light Higgs boson $h$ itself can be identified with the pseudodilaton through the relation $S = \sqrt{h} h^\top$. Since the scale transformation does not affect any quantum numbers, the corresponding particle can be, e.g., a scalar glueball or, perhaps, a $\sigma$- or $f_0$-mesons.

In this letter, we suppose that electroweak symmetry breaking is triggered by a spontaneously breaking of scale symmetry (near conformal sector) at the energy scale $f \geq v$ [7,8], where $v$ being the vacuum expectation value of the Higgs boson in the SM. The spectrum of states at the electroweak (EW) scale $\Lambda_{EW} \sim 4 \pi v$ contains a scalar particle (resonance), the pseudo-Goldstone boson, pseudodilaton, of conformal symmetry breaking. This particle is associated with the EW singlet scalar field $S(x)$, the dilaton mode. The typical pattern is provided by new strongly coupled, nearly conformal dynamics at a scale of Conformal field theory (CFT) $\Lambda_{CFT} \sim 4 \pi f$ which then flows into EW sector at $\Lambda_{EW}$. The mass $m_S$ is naturally light, $m_S \sim \gamma f$, where $\gamma$ is the parameter that controls deviations from exact scale invariance. The dilaton becomes massless when conformal symmetry is recovered. Hence, the light scalar resonance is a distinguishing feature of nearly conformal dynamics.

The prospects for distinguishing the dilaton mode from a minimal Higgs boson at the LHC and ILC is presented in [8].

3 "Ungluon" propagator

In the physical point of view, observable quantities may be defined as those that are invariant under the gauge transformations of the second kind,

$$\psi \rightarrow \psi \exp(i g \lambda), \ B_\mu \rightarrow B_\mu + \Lambda^{d-1} \partial_\mu \lambda, \ C_\mu \rightarrow C_\mu + \partial_\mu \lambda, \ S \rightarrow S + \lambda$$  \hspace{1cm} (8)

and $\lambda$ satisfies $\partial^2 \lambda = 0$.

The equations of motion read

$$\nabla^2 B_\mu - \left(1 - \frac{\xi}{\alpha}\right) \partial_\mu (\partial_\cdot B) = \frac{\mu^2}{\alpha} \frac{1}{\Lambda^{1-d}} C_\mu,$$ \hspace{1cm} (9)

$$\nabla^2 C_\mu - \left(1 - \frac{\xi}{\alpha}\right) \partial_\mu (\partial_\cdot C) + \frac{m_S^2}{\alpha} \left(\frac{1}{\Lambda^{d-1}} B_\mu - \partial_\mu S\right) = J^s_\mu,$$ \hspace{1cm} (10)
\[ B_\mu = \Lambda^{d-1} \partial_\mu S, \tag{11} \]

from which one can easily find the principal equations for fields \( B_\mu \) and \( S \)

\[ (\nabla^2)^2 B_\mu - \left( 1 - \frac{\xi^2}{\alpha^2} \right) \nabla^2 \partial_\mu (\partial \cdot B) = \frac{\mu^2}{\alpha} \frac{1}{\Lambda^{1-d}} J_\mu^*, \tag{12} \]

\[ (\nabla^2)^2 S - \frac{\mu^2}{\xi}(\partial \cdot C) = 0, \tag{13} \]

where the current \( J_\mu^* = \alpha^{-1} g^* \bar{\psi} \gamma_\mu \psi - \partial_\nu B_{\nu \mu} \) is conserved. Note that the field \( C_\mu \) does not commute with \( B_\mu \). If \( \mu \to 0 \), \( S(x) \) becomes the dipole pseudodilaton field obeying the equation

\[ (\nabla^2)^2 S(x) = 0. \tag{14} \]

Few words concerning the higher derivative scalar field theory. We have already mentioned that on the classical level conformal symmetry is manifested by the fact that the stress-energy tensor is traceless. On the other hand, at quantum level the conformal symmetry is broken which gives rise to conformal (trace) anomaly. It was found that for 4-dimensional higher derivative scalar field theory the trace anomaly can be obtained from the action \( E = \int d^4x \sqrt{-g} S \Delta^4 S \), where \( \Delta^4 \equiv (\nabla^2)^2 - 2 R^{\mu \nu} \nabla_\mu \nabla_\nu + (2/3) R \nabla^2 - (1/3)(\nabla_\mu R) \nabla^\mu \). Operator \( \Delta^4 \) contains the conformally covariant structure for a fourth-order differential operator [9]. The field \( S(x) \) in (14) provides an instructive and useful control over UV and IR divergences for its free propagator. More detailed consideration of dipole-field scalars are given in [10-14].

In the framework of decomposition scheme [3], the unparticle interaction with the SM quarks in (7) becomes

\[ \frac{1}{\Lambda^{d-1}} g^* \bar{\psi} \gamma_\mu \psi \sum_{k=1}^{k=\infty} f_k \partial_\mu S_k, \tag{15} \]

where

\[ f_k^2 = \frac{A_d}{2 \pi} \left( m_{S_k}^2 \right)^{d-2} \Delta_s^2, \quad A_d = \frac{16 \pi^{5/2}}{(2 \pi)^{2d}} \frac{\Gamma(d + 1/2)}{\Gamma(d - 1) \Gamma(2d)}. \tag{16} \]

The pseudodilaton fields \( S_k \) are characterized by the mass squared \( m_{S_k}^2 = k \Delta_s^2 \) as \( \Delta_s \to 0 \). Therefore the coupling of each \( S_k \) to the SM quarks is proportional to \( \Delta_s \) and vanishes in the continuum limit \( \Delta_s \to 0 \).
The most general form of the commutator for free $B_\mu$ field is (we put $\xi = 1$, $\alpha = 1$ so simplicity)

$$[B_\mu(x), B_\nu(y)] = i g_{\mu\nu} \left[ -\mu^2 \Lambda^{d-1} E(x-y) + c D(x-y) + \text{const} \right], \quad (17)$$

where $c$ is an arbitrary real number; the commutator functions, namely, the invariant function $E(x-y)$ and the Pauli-Jordan function $D(x-y)$ are [15,11]

$$E(x) = i \int 2\pi \text{sign}(p^0) \delta'(p^2) e^{-ipx} \frac{d^4 p}{(2\pi)^4} = (8\pi)^{-1} \text{sign}(x^0) \theta(x^2), \quad (18)$$

$$D(x) = \nabla^2 E(x) = (2\pi)^{-1} \text{sign}(x^0) \delta(x^2) \quad (19)$$

with their properties

$$E(0, \vec{x}) = \partial_0 E(x)|_{x^0=0} = \partial_0^2 E(x)|_{x^0=0} = 0, \quad \partial_\mu^3 E(x)|_{x^0=0} = g_{\mu 0} \delta^3(\vec{x}), \quad (20)$$

$$\nabla^2 D(x) = 0, \quad D(0, \vec{x}) = 0, \quad \partial_0 D(0, \vec{x}) = \delta^3(\vec{x}). \quad (21)$$

The form (17) ensures the equal-time canonical commutation relation (CCR)

$$[B_\mu(x), \pi_{B_\nu}(y)]|_{x^0=y^0} = i g_{\mu\nu} \delta^3(\vec{x} - \vec{y}), \quad (22)$$

where

$$\pi_{B_\nu} = \Lambda^{1-d} \left[ \partial_\mu C_0 - \partial_0 C_\mu - g_{0\mu} (\partial \cdot C) + \partial_\mu B_0 - \partial_0 B_\mu \right]. \quad (23)$$

The next step is to decompose $E(x)$ into its negative ($E^-(x)$) - and positive ($E^+(x) = [E^-(x)]^* = -E^-(x)$) -frequency parts, each of which is analytic in the past and future tubes: $E(x) = E^-(x) + E^+(x)$ with [15]

$$E^-(x) = i \int 2\pi \theta(p^0) \delta'(p^2) e^{-ipx} \frac{d^4 p}{(2\pi)^4} = -i \left( \frac{l^2}{4\pi^2} \right)^\frac{1}{2} \ln \left( \frac{l^2}{-x^2 + i\epsilon x^0} \right) \theta(x^0) \theta(x^2). \quad (24)$$

Here, $l$ is an arbitrary length scale with dimension minus one in mass units and introduced in the logarithmic function $\ln[-(x^0 - i\epsilon)^2 + \vec{x}^2]$ for dimensional reason and $\kappa \sim l^{-1}$ being the mass parameter of the IR regularization. The origin of $l$ becomes more transparent in momentum space. Note that the distribution $\theta(p^0) \delta'(p^2)$ in (24) is well-defined only with the basic functions $u(p)$ having the properties: $u(p) = 0$ at $p = 0$.  

6
The time-ordered two-point Wightman function (TPWF) for $B_\mu$ field is

$$W_{\mu\nu}(x) = \langle 0 | T B_\mu(x) B_\nu(0) | 0 \rangle = \theta(x^0) \omega_{\mu\nu}(x) + \theta(-x^0) \omega_{\mu\nu}(-x), \quad (25)$$

where TPWF $\omega_{\mu\nu}(x) = \langle 0 | B_\mu(x) B_\nu(0) | 0 \rangle$ is

$$\omega_{\mu\nu}(x) = ig_{\mu\nu} \left[ -\frac{\mu^2}{\Lambda^2(1-d)} E^-(x-y) + c D^-(x-y) + \text{const} \right], \quad (26)$$

with $D(x) = D^-(x) - D^-(x)$.

Using results obtained in paper [14] we get the propagator function (25) in 4D momentum space in any local covariant gauge

$$\tilde{W}_{\mu\nu}(p) = \left[ g_{\mu\nu} - \left( 1 - \frac{1}{\xi^2} \right) \frac{p_\mu p_\nu}{p^2} \right] \mu^2 \tau(p; \kappa^2), \quad (27)$$

where

$$\tau(p; \kappa^2) = \sum_{k=1}^{k=\infty} \lim_{\lambda_k \rightarrow 0} f_k^2 \left[ \frac{1}{(p^2 - \lambda_k^2 + i\epsilon)^2} + \frac{i}{(4\pi)^2} \ln \frac{\lambda_k^2}{\kappa^2} \delta^4(p) \right]. \quad (28)$$

The distribution $\lim_{\lambda_k \rightarrow 0} [1/(p^2 - \lambda_k^2 + i\epsilon)^2]$ is defined only on a particular subspace of the space of complex Schwartz test functions on $\mathbb{R}^4$, namely on those (test) functions $f(p)$ such that $f(0) = 0$. The set of its extensions onto the whole space is a one-parameter set of functionals parameterized by $\kappa$. It means that the set of Lorentz-invariant, causal extensions of this distribution to those not vanishing at $p = 0$ constitute a new $\kappa$-parameter family.

In the continuum limit the final result for $\tau(p; \kappa^2)$ is

$$\tau(p; \kappa^2) = \frac{A_d}{2\pi} \left\{ \frac{(2-d)\pi}{(-1)^{d-1}} \csc[(d-1)\pi] \frac{1}{(p^2 + i\epsilon)^{3-d}} \right. + \lim_{\epsilon \rightarrow 0} \frac{i}{(4\pi)^2} \frac{\ln \epsilon - \gamma - \psi \left( \frac{3}{2} - d \right)}{(\frac{3}{2} - d) \epsilon^{3/2-d}} \left. \frac{1}{(\kappa^2)^{3/2-d}} \delta^3(p) \right\}, \quad (29)$$

which is valid for $1 < d < 3/2$; $\gamma = -\Gamma'(1) \simeq 0.577$, $\psi(x) = \Gamma'(x)/\Gamma(x)$ is the digamma-function. This formula accompanied by (27) gives the main result for ”ungluon” propagator which has the asymptotic behaviour in the form $\sim g_{\mu\nu}/(p^2)^{2-d}$ at $\xi = 1$.

There are prices that must be paid for maintaining new result: i) the Fourier transformation of TPWF $\omega_{\mu\nu}(x)$ contains $\delta'(p^2)$-function which is the consequence of non-unitarity character of the model ($\delta'$ is not a measure); ii) the spectral function of the first term in expansion (26) gives an indefinite metric and hence the translations become pseudounitarity (see paper [10] for details).
4 Conclusions

The gauge unparticle model has been studied, using canonical quantization in the framework of decomposition scheme. One of the main objects is the pseudodilaton field \( S(x) \) which governs the "ungluon" field \( B_\mu(x) = \Lambda^{d-1} \partial_\mu S(x) \). We have found the propagator function \( \tilde{W}_{\mu\nu}(p) \) of "ungluon" field in 4D momentum space in any local covariant gauge. The dipole-type "ghost" behaviour of \( \tilde{W}_{\mu\nu}(p) \) is evident. It is shown that no mass of "ungluon" field appeared in (29).

The non-unitarity character of the model is the direct consequence of i) the form of the Lagrangian density (7), ii) scale and gauge invariance of the model, iii) spontaneous breaking of scale invariance, iv) the form of the equation of motion and related commutator for the field \( B_\mu(x) \) (17) which ensures the CCR (22).

It follows from our investigation that the degrees of freedom called "unparticles" in [1] do, indeed, in the gauge sector very much behave like the "ghost" fields.

It would be especially interesting to see how the inclusion of the correction to non-perturbative potential arising from exchange of "ungluons" (at least to lowest order) affects QCD physics. This item is left for future paper.

5 Acknowledgments

It is a pleasure to thank Fermilab I visited during the course of this work.

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