Parametric synthesis of stabilizing neurocular control of a technological module

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Abstract. The article proposes a new approach to combined parametric synthesis (synthesis in real or accelerated time) of neuroregulators of multidimensional and multi-connected physical systems and technological processes based on the application of the velocity gradient method in differential form and the theory of sliding modes.

1. Introduction
The adaptive approach assumes the presence of a linear mathematical model based on physical phenomena and an estimate of the unknown parameters included in this model. Neural networks (NN) perform a similar function. However, the main drawback of neural networks organized according to a sequential scheme - the slow convergence of adaptation algorithms (learning and control) in real time - significantly limits the practical application of neural control. Existing neural control schemes turn out to be quasi-adaptive with tuning out of the pace of control processes, or, being in fact adaptive, insufficiently substantiated due to the complexity of analyzing the performance, stability, quality and achievement of control goals [1]. In addition, the classical formulations of neuro-control problems are weakly connected with the concepts of multi-connectivity and multidimensionality of real physical systems, control algorithms are reduced either to the implementation of program control with an empirical choice of the number of layers of neural networks and the number of neurons in each layer [1, 2], or to setting parameters classical PID - controller [3]. The most effective is the adaptive-critical scheme close to dynamic programming, which has found application in modern SMART GRID technologies [4]. Therefore, further prospects of neural control [4, 5] are associated with the indication of parallels between the classical theory of adaptive control and approaches to the construction of neuroregulators (NR) based on NN and the study of neuromorphic structures. NS are nonlinear systems suitable for solving practical control problems, in principle associated with the presence of nonlinear characteristics. Neural networks make it possible to eliminate the quantitative uncertainty of information, since after training they can, due to interpolation (emulation) and extrapolation (adaptation and forecasting) of the input - output characteristics of a physical object, give the correct solution for obtaining new information that is not included in the training set.

Overcoming these difficulties is possible due to the step-by-step solution of the problem of synthesis of an adaptive parametric neuroregulator: initially the selected class of nonlinear systems is investigated for stability; then a stabilizing control $u$ is synthesized to ensure the goal of adaptation: $\lim_{t \to \infty} x(t) = 0$; according to the stability conditions, the NN parameters are initialized.

The report considers the second and third stages of the synthesis problem.
2. Parametric synthesis of the stabilizing neurocontrol

As an effective algorithm that implements the method of reverse propagation of signal errors, the report proposes to use the scheme of the velocity gradient method (VGM) [5].

The formulation of the problem of synthesizing the stabilizing control for the VGM is reduced to the following.

The state of the \( j \)-th TM is \( x \in R^n \), the control \( u \) is scalar, a scalar value \( z = c^T x \in R^l \) is observed. The evolution of the \( j \)-th TM is described by a differential system:

\[
\dot{x} = Ax + B(u + \sigma(t,z,u)) + \xi, \tag{1}
\]

\[
z = c^T x, \tag{2}
\]

\[
u = L z, \tag{3}
\]

where the measured piecewise differentiable function \( \sigma(t,z,u) \) is an additive addition to the scalar control \( u \) and is subject to a constraint of the form

\[
0 \leq zu \sigma \leq \tilde{q}_1 \alpha z^2 + \tilde{q}_2 \beta u^2, \tag{4}
\]

\( B \) - constant \( n \)-vector of adjustable input parameters (coefficients of synoptic connections of a single-layer NN). Control linearly depends on the observed value: \( u = L z \).

The matrix \( A \) and vector \( B \) are not known in advance. A vector \( c \), that takes into account the "contributions" of state variables \( x \) to observation \( z \), is specified.

It is required to synthesize a parametric controller \( L = L(t) \).

The purpose of the control - the fulfillment of the condition \( \lim_{t \to \infty} x(t) = \bar{0} \) - corresponds to minimization in the limit of the local functional \( J(t) = 0,5 x^T H x \), where \( H \) is a positive definite, symmetric matrix of dimension \( n \times n \).

For the synthesis of a nonlinear parametric controller, we use the VGM scheme in differential form [5].

To do this, we define the total derivative of the local functional

\[
\dot{J}(t) = 0,5 (x^T H \dot{x} + x^T \dot{H} x) = x^T H [(A + L B c^T) x + B(\tilde{q}_1 \alpha + \tilde{q}_2 \beta L)]
\]

and calculate the gradient by parameter \( L \)

\[
\nabla_L \varphi(x,L) = x^T HB(z - \frac{\tilde{q}_1 \alpha}{L} + \tilde{q}_2 \beta).
\]

According to the VGM scheme, the nonlinear scalar controller synthesized in differential form has the form

\[
\dot{L} = -\gamma \nabla_L \varphi(x,L)
\]

For system (1) - (3) with constraints on the activation function (FA) of type (4), the adaptation (learning) algorithm \( j \)-th TM is written in the form

\[
\dot{L} = -\gamma x^T HB(z - \frac{\tilde{q}_1 \alpha}{L} + \tilde{q}_2 \beta), \tag{5}
\]

where \( \gamma > 0 \) is a positive number that determines the rate of decrease of the gradient along the parameter \( L \).

According to the statement of the problem, the control procedure should depend only on the observed value \( z \). Therefore, in (5), we require that the equality

\[
HB = c. \tag{6}
\]

Then, taking into account the fact that \( c^T = B^T H \), in the final form, we obtain the adaptation algorithm of the \( j \)-th TM.
\[ \dot{L} = -\gamma (z - \frac{\bar{q}_1 \alpha + \bar{q}_2 \beta}{L^z})z, \]  

(7)

where \( \bar{q}_z \in [0, 1] \), \( \bar{q}_1 = 1 - \bar{q}_z \) are parameters that determine the operating mode of the NN.

Formula (7) is valid in areas for which \( L > L^+ \), \( L < L^- \), and the function \( \sigma(L) \) is close to linear dependence. The parametric controller \( L \) in these areas of variation of the argument is quasilinear.

The parameter \( \gamma \) in the control law (7) is recommended to be chosen from the condition: \( \gamma \geq \frac{\bar{q}_1 \alpha}{\bar{q}_2 \beta} \).

In the region bounded by the vertical \( L^\pm \) and horizontal \( \sigma^\pm \) asymptotes of the activation function \( \sigma(t, z, u) = \sigma(L) = \frac{\alpha}{L} + \bar{q}_2 \beta L \), sliding modes arise, to ensure the asymptotic stability of which, when measuring the first \( r \) phase coordinates, instead of formulas (3), (7) of the parametric controller, a control law with discontinuous coefficients is used \([6, p. 249]\). In our case, we take the solution for the stabilization problem \( r = 1 \) and the control law of the form

\[
\dot{u} = -Lz - \delta_0, \quad L = \begin{cases} L^+, & zs > 0, \\ L^-, & zs < 0, \end{cases}
\]

(8)

\[
\delta_0 = \begin{cases} \delta_0, & s > 0, \\ -\delta_0, & s < 0, \end{cases} \quad \delta_0 = \left| \delta_0 \right| \text{sign}(c^T B).
\]

(9)

Here \( \delta_0 \) is an arbitrarily small nonzero number, the sign of which coincides with the sign of the scalar \( c^T B \), and the switching surface is determined by the second factor of formula (7)

\[
s(z) = z - \frac{\bar{q}_1 \alpha}{L^z} + \bar{q}_2 \beta. \]

(10)

The time derivative of the function \( s \) by virtue of equations (10) has the form

\[
\dot{s}(z) = \dot{z} + \frac{3\bar{q}_1 \alpha}{L^z} \dot{L}.
\]

(11)

The conditions for sliding under the axiomatic approach are determined by the inequalities \([6]\):

\[
\dot{s}^+ = \lim_{s(x) \to 0^+} \dot{s} < 0, \quad \dot{s}^- = \lim_{s(x) \to 0^-} \dot{s} > 0.
\]

(12)

The indicated conditions (12) are separated from each other by the manifold \( \dot{s}^+ = 0 \) or \( \dot{s}^- = 0 \), or in the general case \( \dot{s} = 0 \).

The condition for the representative point to hit the sliding surface has the form \([6]\)

\[
s(z)\dot{s}(z) < 0.
\]

(13)

and meets the conditions: \( z \dot{z} < 0, \ L \dot{L} < 0. \)

The required manifold \( \dot{s}(z) = 0 \) is determined from the formula (11)

\[
\dot{s}(z) = \dot{z} + \frac{3\bar{q}_1 \alpha}{L^z} \dot{L}.
\]

(14)

Taking into account that in the region of sliding modes at \( L = L^+ \) or \( L = L^- \), the control law (7) takes the form

\[
\dot{L} = -\gamma z^2,
\]

(15)

after substituting formula (12) into equality (11), we obtain the equation of sliding motion.
where the parameter $L = L^\pm$ according to formula (8) has one of the possible values $L^+$ or $L^-$. The solution of the sliding motion equation (16) can be obtained in an analytical form through the separation of variables

$$\frac{dz}{z^2} = \frac{3\tilde{q}_1\alpha}{L^3} \, dt,$$

after integrating its right and left sides

$$z = \frac{(L^\pm)^3}{3\tilde{q}_1\alpha \gamma t} + C = \pm \lambda_1 + C,$$

where $\lambda_1 = \frac{1}{3\tilde{q}_1\alpha \gamma}$ is variable coefficient of proportionality, $C$ - constants determined by various initial conditions $z(0)$.

The family of curves (17) are functions of inversely proportional time dependences $z = f(t)$ (Fig. 1), in I and IV quadrants relative to the time axis. An analysis of these curves shows that at $t \to \infty$, the function $z$ tends to zero. Thus, when organizing sliding motion, the goal of adaptation is achieved: $\lim_{t \to \infty} z(t) = z_\ast = 0$ by the output of system (1) - (3) in the stabilization problem. Since at $t \to \infty$ is the observed value $z \to 0$, then by virtue of Eqs. (17), (3) in the region of sliding modes, the parameter $L$ and control $u$ also tend to zero.

The phase portrait of the motion of the representative point according to the equation of sliding motion (16) is determined by a family of parabolas $\dot{z} = f(z) = \pm \lambda_2 z^2$ symmetrical up and down relative to the axis $z$ (Fig. 2). Here $\lambda_2 = \frac{3\tilde{q}_1\alpha}{(L^\pm)^3}$ is a variable proportionality coefficient.

### 3. Conclusion

Achievement of the natural goal of adaptation $z$ by the system (1) - (3): $\lim_{t \to \infty} z(t) = z_\ast = 0$, i.e. reaching a bounded limit set, determines the main property of absolutely stable systems - dissipativity [7, p. 403]. In our case, the dissipativity property ensures the system's roughness to external disturbances due to the introduction of a dead zone $\sigma^\pm$ in the functions $\sigma(L)$, the system roughness to parametric disturbances due to the introduction of a controller parameter $L$ deadband $L^\pm$ (Fig. 2) and the system roughness to dynamic disturbances due to the introduction of additional output feedback $z$. 
Figure 1. Organization of sliding motion of the system (1) - (3)

Figure 2. Phase portrait of the sliding motion of the system

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