Study of $CP$ violation in $D \rightarrow VV$ decay at BES-III

Xian-Wei Kang$^{1}$ and Hai-Bo Li$^{2}$

$^{1}$Institute of High Energy Physics, P.O.Box 918, Beijing 100049, China
$^{2}$Department of Physics, Henan Normal University, Xinxiang 453007, China

In this paper, we intend to study the problem of $CP$ violation in $D$ meson by $D \rightarrow VV$ decay mode in which the $T$ violating triple-product correlation is examined. That would undoubtedly be another excellent probe of New Physics beyond Standard Model. For the neutral $D$, we focus on direct $CP$ violation without considering $D^{0} - \bar{D}^{0}$ oscillation. Experimentally, by a full angular analysis one may obtain such $CP$ violating signals, and particularly it is worth mentioning that the upcoming large $D$ data samples at BES-III in Beijing will provide a great opportunity to perform it.

PACS numbers: 13.25.Ft, 13.40.Hq, 13.75.Lb, 14.40.Lb

The topic of $CP$ violation in the $D$-meson sector has been the subject of extensive studies involving both charged and neutral $D$ meson decays these years [1]–[12]. In a recent reference [13], they exploited the angular and quantum correlation in the $D^{0} - \bar{D}^{0}$ pairs produced through the decay of the $\psi(3770)$ resonance at BES-III to investigate $CP$ violation. They build $CP$ violating observables in $e^+e^- \rightarrow \psi(3770) \rightarrow D^{0}\bar{D}^{0} \rightarrow (V_1V_2)(V_3V_4)$ ($V$ denotes vector meson) to isolate specific New Physics effects in the charm sector [13].

In practice, one can also probe the $CP$ violation in $D$ meson by one $D$ decay without considering the quantum correlation, which is the motivation of this paper. Among the various kinds of $D$ decay modes, $D \rightarrow V_1V_2$ and subsequently decaying to two pseudoscalars for each vector meson is a particularly interesting one in the perspective of the copious kinematics of final state interaction (FSI). As the same case in $B$ meson [14]–[17], the new type of $T$ violating signal involving so-called triple-product (TP) will emerge by comparing a pair of $CP$ conjugate processes, where TP is composed of the momentum of one vector meson and two polarizations. Assuming $CPT$ invariance, $T$ violating TP asymmetry is equivalent to $CP$ violating. We shall see in this letter such TP asymmetries are related to the helicity components in the angular distribution of the $D \rightarrow V_1V_2$ process. Moreover, performing a full angular analysis is feasible and realistic in experiment.

On the other hand, TP asymmetry (in the latter section we sometimes also refer to TP asymmetry as $CP$ asymmetry assuming $CPT$ invariance) is a sensitive signal of New Physics. As is well known that Standard Model (SM) predictions for $CP$ violation in the charm sector are very small, thus any significant such signals would be exciting. Currently, the BES-III experiment is collecting data at $\psi(3770)$ peak. In the short future, lots of events of $D$ decay will be accumulated, which will provide a great opportunity to perform a full angular analysis to further achieve a valuable information for the question discussed here.

Let us first consider the process $D(p) \rightarrow V_1(k, \epsilon_1)V_2(q, \epsilon_2)$, where the two vectors $V_1, V_2$ are characterized as their four-momenta and polarizations $(k, \epsilon_1)$ and $(q, \epsilon_2)$, respectively. We can write the most general invariant amplitude as a sum of three terms that we will call $s, d, p$ [14]–[17],

$$\mathcal{M} \equiv as + bd + icp$$

$= a\epsilon_1^* \cdot \epsilon_2^* + b \frac{\mu_1 m_2}{m_1 m_2} (p \cdot \epsilon_1^*)(p \cdot \epsilon_2^*)$

$+ i c \frac{\alpha \beta \gamma \delta}{m_1 m_2} \epsilon_1^\alpha \epsilon_2^\beta \epsilon_3^\gamma \epsilon_4^\delta \cdot k_{\alpha \beta \gamma \delta}$

where $m_1$ ($m_2$) is the mass of $V_1$ ($V_2$), and the scalar coefficients $a, b$ and $c$ are generally complex and can receive contributions from several amplitudes with different phases. Thus, one can parameterize the coefficients as [14]

$$a = \sum_j a_je^{i\delta_{s,j}}e^{i\phi_{s,j}}$$

$$b = \sum_j b_je^{i\delta_{d,j}}$$

$$c = \sum_j c_je^{i\delta_{p,j}}e^{i\phi_{p,j}}$$

where $a_j, b_j$ and $c_j$ are the modulus of their corresponding complex quantities, and $\delta_j$ denotes strong phase (also called unitary phase in Ref. [14]), and $\phi_j$ is the weak phase which is the necessary condition for occurring of $CP$ violation on the basis of Cabibbo-Kobayashi-Maskawa (CKM) mechanism [18]–[19] in the SM. Squaring the matrix element of Eq. (1), one obtains...
After a simple calculation by inserting Eq. (2) and Eq. (5), we see that arises from interference terms involving the CP triple correlation for $A$ with the antiparticle decay $\bar{T}$, where the subscript $D$ CPT invariance, the matrix element for the antiparticle decay $\bar{D}(p) \rightarrow V_1(k, \epsilon_1)V_2(q, \epsilon_2)$ can be written as

$$\bar{M} = \bar{a} \epsilon_1^* \cdot \epsilon_2^* + \frac{\bar{b}}{m_{1m_2}^2}(p \cdot \epsilon_1^*)(p \cdot \epsilon_2^*)$$

$$-i \frac{\bar{c}}{m_{1m_2}^2} \epsilon_{\alpha} \epsilon_{\beta} \epsilon_{\gamma} \epsilon_{\delta} (1^*_\alpha \epsilon_{2^*\beta}, 1^*_\gamma \epsilon_{2^*\delta})$$

with

$$\bar{a} = \sum_j a_j e^{i \delta_j} e^{-i \phi_j},$$

$$\bar{b} = \sum_j b_j e^{i \delta_j} e^{-i \phi_j},$$

and

$$\bar{c} = \sum_j c_j e^{i \delta_j} e^{-i \phi_j}.$$ 

Note that $CP$ operator leaves strong phases invariant and only changes the sign of weak phase. From Eq. (1) and Eq. (4), we will find that the $p$ wave amplitude in $\bar{M}$ changes the sign comparing with $\bar{M}$, which will introduce an interesting property between $|\bar{M}|^2$ and $|\bar{M}|^2$. To be clear, we would square the matrix element for the antiparticle decay in Eq. (4):

$$|\bar{M}|^2 = |\bar{a}|^2 |\epsilon_1^* \cdot \epsilon_2^*|^2 + \frac{|\bar{b}|^2}{m_{1m_2}^2}(k \cdot \epsilon_1^*)(q \cdot \epsilon_2^*) + \frac{|\bar{c}|^2}{m_{1m_2}^2} \epsilon_{\alpha} \epsilon_{\beta} \epsilon_{\gamma} \epsilon_{\delta} (1^*_\alpha \epsilon_{2^*\beta}, 1^*_\gamma \epsilon_{2^*\delta})^2 + 2 \frac{Re(\bar{a} \bar{b}^*)}{m_{1m_2}^2} (\epsilon_1^* \cdot \epsilon_2^*) (k \cdot \epsilon_1^*)(q \cdot \epsilon_2^*)$$

$$-2 \frac{Im(\bar{a}c^*)}{m_{1m_2}^2} (\epsilon_1^* \cdot \epsilon_2^*) \epsilon_{\alpha} \epsilon_{\beta} \epsilon_{\gamma} \epsilon_{\delta} (1^*_\alpha \epsilon_{2^*\beta}, 1^*_\gamma \epsilon_{2^*\delta}) - 2 \frac{Im(bc^*)}{m_{1m_2}^2} (k \cdot \epsilon_2^*)(q \cdot \epsilon_1^*) \epsilon_{\alpha} \epsilon_{\beta} \epsilon_{\gamma} \epsilon_{\delta} (1^*_\alpha \epsilon_{2^*\beta}, 1^*_\gamma \epsilon_{2^*\delta}).$$

For $D^0 \rightarrow V_1V_2$ decay, one can define an asymmetry $A_T$ with the definite sign for the triple product $(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^*)$ as

$$A_T = \frac{N(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^* > 0) - N(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^* < 0)}{N_{total}},$$

where the subscript $T$ implies triple products and $N$ denotes the corresponding number of events. Eq. (7) above is actually

$$A_T = \frac{\Gamma(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^* > 0) - \Gamma(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^* < 0)}{\Gamma(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^* > 0) + \Gamma(\bar{k} \cdot \epsilon_1^* \times \epsilon_2^* < 0)}.$$ 

Similarly, for $D^0 \rightarrow \bar{V}_1V_2$ decay, $\bar{A}_T$ can also be constructed as the same way. In $|\bar{M}|^2$, a triple-product correlation arises from interference terms involving the $p$ amplitude, and will be present if $Im(ac^*)$ (or $Im(bc^*)$) is non-zero. After a simple calculation by inserting Eq. (2) and Eq. (5), we see that

$$A_T \propto Im(ac^*) = \sum_{i,j} a_i c_j \sin[(\phi_{si} - \phi_{pj}) + (\delta_{si} - \delta_{pj})]$$

Note that a non-zero triple correlation does not necessarily imply $CP$ violation, since final state interactions (FSI) can fake it, namely the strong phase can also produce non-zero $A_T$ (or $\bar{A}_T$) even the weak phases are zero. Yet comparing a triple correlation for $CP$ conjugate transitions allows to distinguish genuine $CP$ violation from FSI effects. Thus we obtain,

$$\frac{1}{2}(A_T + \bar{A}_T) \propto \frac{1}{2} [Im(ac^*) - Im(\bar{a}c^*)] = \sum_{i,j} a_i c_j \sin(\phi_{si} - \phi_{pj}) \cos(\delta_{si} - \delta_{pj}),$$

$$\frac{1}{2}(A_T - \bar{A}_T) \propto \frac{1}{2} [Im(ac^*) + Im(\bar{a}c^*)] = \sum_{i,j} a_i c_j \sin(\phi_{si} - \phi_{pj}) \sin(\delta_{si} - \delta_{pj}).$$
and
\[
\frac{1}{2}(A_T - \bar{A}_T) \propto \frac{1}{2}(Im(ac^*) + Im(\bar{a}c^*)) = \sum_{i,j} a_i c_j \cos(\phi_{si} - \phi_{pj}) \sin(\delta_{si} - \delta_{pj}).
\]  

(11)

So far, a non-zero value $A_T + \bar{A}_T$ will be undoubtedly a clean signal of CP non-conservation, because there must be at least one non-zero weak phase $\phi$. Here, we also note that if there is only one amplitude contributing to each partial wave, one can simultaneously determine the strong phase difference and weak phase difference from Eq. (10) and Eq. (11).

Experimentally, one can perform a full angular analysis to obtain the above TP asymmetry information because the complex coefficients $a, b, c$ are related to the helicity amplitudes $A_0, A_{||}, A_\perp$ as discussed in Refs. [16, 20],
\[
A_0 = -ax - b(x^2 - 1),
A_{||} = \sqrt{2}a, 
A_\perp = \sqrt{2}e\sqrt{x^2 - 1},
\]

(12)

where we have introduced $A_\perp$ with definite odd CP eigenvalue and the CP even partners $A_0, A_{||}$ via

\[
A_0 = A_0, 
A_{||} = \frac{1}{\sqrt{2}}(A_{11} + A_{-1-1}),
A_\perp = \frac{1}{\sqrt{2}}(A_{11} - A_{-1-1}),
\]

(15)

with $A_{\lambda_1, \lambda_2}$ denoting the helicity mode of two vector mesons. $\theta_i$s (i=1,2) are the angles between the direction of motion of one of the $V_{1,2} \rightarrow PP$ pseudoscalar final states and the inverse direction of motion of the D meson as measured in the $V_{1,2}$ rest frame, $\phi$ is the angle between the two decay plane of vector mesons in the D rest frame. Figure 1 illustrates the decay kinematics of the process $D \rightarrow V_1 V_2 \rightarrow (P_1 P_2)(P_3 P_4)$ in the rest frame of $V_{1,2}$.

\[
\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} \propto \frac{1}{2} \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \phi |A_{||}|^2 + \frac{1}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi |A_\perp|^2 + \cos^2 \theta_1 \cos^2 \theta_2 |A_0|^2
\]

\[
-\frac{1}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi Im(A_\perp A_0^*) - \frac{\sqrt{3}}{4} \sin 2\theta_1 \sin 2\theta_2 \cos \phi Re(A_{||} A_0^*) + \frac{\sqrt{3}}{4} \sin 2\theta_1 \sin 2\theta_2 \sin \phi Im(A_{||} A_0^*),
\]

(14)

\[
d\phi = \frac{k \cdot q}{m_1 m_2} = \frac{m_1^2 - m_1^2 - m_2^2}{2m_1 m_2},
\]

(13)

where $m_D$ is the mass of D meson.

Now we turn to the full angular dependence of process $D \rightarrow V_1 V_2 \rightarrow (P_1 P_2)(P_3 P_4)$ with $P$ pseudoscalar, after some algebra one can get

\[
A_T = \frac{Im(A_{||} A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_{||}|^2},
\]

(17)

thus we will derive the CP violating observables,
\[
A = \frac{1}{2}(A_T^0 + \bar{A}_T^0) = \frac{1}{2} \left( \frac{Im(A_{||} A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_{||}|^2} + \frac{Im(A_{||} A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_{||}|^2} \right),
\]

(18)

and
\[
A' = \frac{1}{2}(A_T^0 + \bar{A}_T^0) = \frac{1}{2} \left( \frac{Im(A_{||} A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_{||}|^2} + \frac{Im(A_{||} A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_{||}|^2} \right).
\]

(19)
Before this study, there had been attempts to study $CP$ violation of $D$ meson via $T$ violating TP correlation in theoretical viewpoint that differs from our method \cite{12, 23, 24}. But the only reported experimental search for $T$-odd asymmetries is from FOCUS in the $D^0 \to K^+ K^- \pi^+ \pi^-$ and $D_{s}^+ \to K_S^0 K^+ \pi^+ \pi^-$ decay modes \cite{25}, as listed in Table I. No evidence for a $T$ asymmetry is observed. The large BES-III data sample is expected to provide enhanced sensitivity to possible $T$ violating asymmetries.

![Table I: $T$ violating asymmetries in $D$ meson decays from the FOCUS experiment \cite{25}.

| Decay mode | $A(\%)$ |
|------------|---------|
| $D^0 \to K^+ K^- \pi^+ \pi^-$ | $1.0 \pm 5.7 \pm 3.7$ |
| $D^+ \to K_S^0 K^+ \pi^+ \pi^-$ | $2.3 \pm 6.2 \pm 2.2$ |
| $D_{s}^+ \to K_S^0 K^+ \pi^+ \pi^-$ | $-3.6 \pm 6.7 \pm 2.3$ |

In Table II the branching fractions with asterisks have not been measured yet, but some estimates combining naive factorization and models for FSI are available from Refs. \cite{26, 27}. Note that in Table II the estimated efficiencies are average value for the various partial waves by assuming that the magnitude of the longitudinal polarization is half of the decay rate. In the future, a careful measurements at BES-III about the efficiency and fraction for each partial wave are suggested. A more realistic analysis requires a likelihood fit to the full angular dependence of the $D \to V_1 V_2 \to (P_1 P_2)(P_3 P_4)$ mode. Systematics will arise from the mis-reconstruction as $V_1 V_2$ of the events that actually come from other resonances or non-resonance $D \to P_1 P_2 P_3 P_4$ background contributions. In view of the sizable width of the vector resonances, we expect that these systematics will dominate the final result. Their precise estimate in the BES-III experiment is beyond the scope of this paper. However, as pointed out in Ref. \cite{24} by Bigi, the four body decay of $D \to P_1 P_2 P_3 P_4$ both with and without intermediate states can all be used to probe $T$ asymmetry in the frame work of TP.

At last, we consider the potential sensitivity on the $CP$ violating observables $A$ and $A'$ at BES-III. From Eq. (7), for a small asymmetry, there is a general result that its error is approximately estimated as $1/\sqrt{N_{\text{total}}}$, where $N_{\text{total}}$ is the total number of events observed. At BES-III, with an integrated luminosity of 20 fb$^{-1}$ at $\psi(3770)$ peak, about $72 \times 10^6 D^0 \bar{D}^0$ pairs will be collected with four year’s running \cite{26, 27}. Table III lists some promising channels to search for $T$ asymmetry for both neutral and charged $D$ decays and the corresponding expected statistical errors are estimated. The projected efficiencies are extracted from Ref. \cite{26} and branching ratios are obtained from Ref. \cite{28}.

![Table II: The promising (VV) modes with large branching fractions, efficiencies and expected errors on the $T$ asymmetry: the corresponding expected errors are estimated by assuming 20 fb$^{-1}$ data at $\psi(3770)$ peak at BES-III; the branching fractions with asterisk are estimated according to Refs. 26, 27. The last row is from $D^+$ decay.

| VV | Br (%) | Eff. (c) | Expected errors |
|----|--------|---------|-----------------|
| $\rho^0 \rho^0 \to (\pi^+ \pi^-)(\pi^+ \pi^-)$ | 0.18 | 0.74 | 0.004 |
| $K^{*+} \rho^0 \to (K^- \pi^+)(\pi^+ \pi^-)$ | 1.08 | 0.68 | 0.002 |
| $\rho^0 \rho^- \to (\pi^+ \pi^-)(\pi^+ \pi^-)$ | 0.14 | 0.26 | 0.006 |
| $\rho^+ \rho^- \to (\pi^+ \pi^-)(\pi^- \pi^0)$ | 0.6 | 0.65 | 0.002 |
| $K^{*+} K^{*-} \to (K^+ \pi^0)(K^- \pi^-)$ | 0.08 | 0.55 | 0.006 |
| $K^{*0} K^0 \to (K^+ K^-)(K^- K^+)$ | 0.048 | 0.62 | 0.002 |
| $K^{*+} \bar{K}^0 \to (K^+ \pi^-)(\pi^+ \pi^-)$ | 1.33 | 0.59 | 0.001 |

In conclusion, we studied the $CP$ violation in $D \to VV$ decay mode in which the $T$ violating triple-product correlation is examined. That would undoubtedly be another excellent probe of new physics beyond the SM. The $CP$ violating observables in connection with angular distribution are constructed. For neutral $D$ decays, we neglect the $CP$ violation induced by $D^0 - \bar{D}^0$ oscillation. Experimentally, by doing a full angular analysis one may obtain such $CP$ violating signals, and particularly it is worth mentioning that the upcoming large $D$ data sample at BES-III will provide a great opportunity to perform it. The sensitivities for $CP$ violating observables are estimated by assuming 20$^{-1}$ fb data-taking at $\psi(3770)$ peak at BES-III.

One of authors, X. W. Kang, wish to thank Rong-
Gang Ping for the help of drawing tools and Professor Z. z. Xing for many valuable suggestions, and specially thank Professor G. Valencia for stimulating discussions through mail. This work is supported in part by the National Natural Science Foundation of China under contracts Nos. 10521003, 10821063, 10835001, 10979008, the National Key Basic Research Program (973 by MOST) under Contract No. 2009CB825200, Knowledge Innovation Key Project of Chinese Academy of Sciences under Contract No. KJCX2-YW-N29, the 100 Talents program of CAS, and the Knowledge Innovation Project of CAS under contract Nos. U-612 and U-530 (IHEP).

∗ Electronic address: kangxw@ihep.ac.cn
† Electronic address: lihb@ihep.ac.cn

[1] E. M. Aitala et al., [E791 Collaboration], Phys. Lett. B 403, 377 (1997) [arXiv:hep-ex/9612005].
[2] F. Buccella, M. Lusignoli, G. Mangano, G. Miele, A. Pugliese and P. Santorelli, Phys. Lett. B 302, 319 (1993) [arXiv:hep-ph/9212253].
[3] B. Aubert et al., [BABAR Collaboration], Phys. Rev. D 78, 051102 (2008) [arXiv:0802.4035 [hep-ex]].
[4] G. Boca et al., [Focus Collaboration], AIP Conf. Proc. 717, 576 (2004). Also in “Aschaffenburg 2003, Hadron spectroscopy” 576-580.
[5] D. Cronin-Hennessy et al., [CLEO Collaboration], arXiv:hep-ex/0102006.
[6] M. Nandy and V. P. Gautam, Czech. J. Phys. 46, 905 (1996).
[7] Z. z. Xing, Phys. Rev. D 55, 196 (1997) [arXiv:hep-ph/9606422].
[8] A. Le Yaouanc, L. Oliver and J. -C. Raynal, Phys. Lett. B 292, 353 (1992).
[9] S. L. Alder and D. s. Du, Phys. Rev. D 35, 2252 (1987).
[10] G. L. Kane and G. Senjanovic, Phys. Rev. D 25, 173 (1982).
[11] V. A. Monich, B. V. Struminsky and G. G. Volkov, Sov. J. Nucl. Phys. 34, 245 (1981); Yad. Fiz. 34, 435 (1981).
[12] J. G. Körner, K. Schilcher and Y. L. Wu, Z. Phys. C 55, 479 (1992).
[13] J. Charles, S. Descotes-Genon, X. W. Kang, H. B. Li and G. R. Lu, arxiv[hep-ph/0912.0899].
[14] G. Valencia, Phys. Rev. D 39, 3339 (1989).
[15] A. Datta and D. London, Int. J. Mod. A 19, 2505 (2004).
[16] G. Kramer and W. F. Palmer, Phys. Rev. D 45, 193 (1992); Phys. Lett. B 279, 181 (1992); Phys. Rev. D 46 3197 (1992).
[17] B. Tseng and C. W. Chiang, hep-ph/9905338.
[18] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
[19] M. Kobayashi and T. Maskawa, Prog. Theo. Phys. 49, 652 (1973).
[20] A. S. Dighe, I. Dunietz, H. J. Lipkin and J. L. Rosner, Phys. Lett. B 369, 144 (1996).
[21] J. G. Körner and G. R. Goldstein, Phys. Lett. B 89, 105 (1979).
[22] M. Beneke, J. Rohrer and D. s. Yang, Nucl. Phys. B 774, 64 (2007).
[23] S. Bianco, F. L. Fabbri, D. Benson and I. Bigi, La Rivista del Nuov. Cim. 26, 7-8 (2003).
[24] I. I. Bigi, CP violation in the SM, Quantum Subtleties and the Insights of Yogi Berra, hep-ph/0703132.
[25] J. M. Link et al., [FOCUS collaboration], Phys. Lett. B 622, 239 (2005).
[26] BESIII Collaboration, "The Preliminary Design Report of the BESIII Detector", Report No. IHEP-BEPCCII-SB-13.
[27] D. M. Asner et al., "Physics at BES-III", edited by K. T. Chao and Y. F. Wang, Int. J. Mod. Phys. A 24, Supp. 1(2009).
[28] C. Amsler et al., [Particle Data Group], Phys. Lett. B 667, 1 (2008).
[29] T. Uppal and R. C. Verma, Z. Phys. C 56, 273 (1992).
[30] A. N. Kamal, R. C. Verma and N. Sinha, Phys. Rev. D 43, 843 (1991).
[31] P. Bedaque, A. Das and V. S. Mathur, Phys. Rev. D 49, 269 (1994).
[32] I. Hinchliffe and T. A. Kaeding, Phys. Rev. D 54 914 (1996).