Baryon masses and $\sigma$ terms in SU(3) BChPT $\times 1/N_c$

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Outline

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4) Gell-Mann-Okubo (GMO) Relation violation

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8) Summary
1) The value of the pion-Nucleon sigma term ranges from 45 MeV to 64 MeV

\[ \sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N} \langle N \mid \bar{u}u + \bar{d}d \mid N \rangle \]

2) There is a long lasting “puzzle” associated with a combination of baryon masses (in SU(3)) in the iso-spin symmetric limit, to obtain the pion-Nucleon sigma term, assuming the contribution by strange quark mass to the nucleon mass is negligible (OZI).

3) The connection between the pion-Nucleon sigma term and size of the correction to the Gell-Mann-Okubo relation

Can one explain these from the ChPT point of view?
Non relativistic version of the BChPT or HBChPT is based on the expansion in terms of the “baryon mass”

\[ p_\mu = m_B v_\mu + k_\mu \quad v^\mu v_\mu = 1 \]
\[ \frac{1}{p^2 - m_B^2} \to \frac{1}{2m_B (v.k)} + \mathcal{O} \left( \frac{1}{m_B^2} \right) \]

The issue of experiencing a slower rate of convergence compare to the Goldstone Boson Sector

Solution: Inclusion of the decuplet baryons in one-loop corrections to physical observables, has been showing a great improvement!

On the other hand, studying the baryons in the large \( N_c \) limit of QCD emerges a dynamical symmetry called “spin-flavor symmetry” which requires the possibility of having degenerate baryon multiplets of higher spin in the intermediate state/s.
Spin-flavor symmetry of Baryons in large $N_c$

Since, $\pi N$ amplitude is $O(N_c^0)$

$$A = -i \frac{k^i k^j}{k_0} N_c^2 g^2 f^2 \pi [X^i, X^j] \Rightarrow [X^i, X^j] \leq O(1/N_c)$$

This result is a tree level, so definitely we can apply it to the case of one loop

Large $N_c$ consistency condition

$[X_0^{ia}, X_0^{ib}] = 0$

This spin-flavor symmetry requires the existence of degenerate baryon multiplets with different spins (a dynamical symmetry) : leads to the consideration of both octet and decuplet contributions in the intermediate state

This symmetry is broken at sub-leading orders in $1/N_c$
$L(\text{Lagrangian}) = x^0 L_{LO} + x^1 L_{NLO} + x^2 L_{NNLO} + x^3 L_{NNNLO} + \ldots$

**Combined approach**

Combining the HBChPT with $1/N_c$ provides a well-behaved expansion in the low energy phenomenology.

$\xi - \text{expansion: } 1/N_c = \mathcal{O}(p)$
### Leading order Lagrangian

\[
\mathcal{L}_B^{(1)} = B^\dagger \left( iD_0 + \hat{g}_A u^i a G^{i a} - \frac{C_{HF}}{N_c} \hat{S}^2 + \frac{c_1}{2\Lambda} \hat{\chi}^+ + \frac{1}{N_c} \right) B
\]

Only the hyperfine mass splitting term breaks symmetry at \( O(\frac{1}{N_c}) \):

\[
- \frac{C_{HF}}{N_c} B^\dagger \hat{S}^2 B
\]

\[
\mathcal{L}_B^{(2)} = B^\dagger \left( -\frac{1}{2m} D^2 + \frac{c_2}{\Lambda} \chi^0 \frac{C_A}{N_c} u^i a S^i A^a + \frac{C_A}{N_c} \epsilon^{ijk} u^i a \{S^j, G^k a\} + \frac{1}{m} (\vec{B}_+^0 + \vec{B}_+^a I^a) \cdot \vec{S} \right.
\]

\[+ \frac{1}{2m} \left( 2(\kappa_0 \vec{B}_+^0 + \kappa_1 \vec{B}_+^a I^a) \cdot \vec{S} + 6 \frac{\kappa_2}{5} B^i a G^{i a} \right) + \rho_0 \vec{E}_-^0 \cdot \vec{S} + \rho_1 E_-^a G^{i a} \]

\[+ i \frac{\tau_{1+}}{N_c} (u_0^a G^{i a} D_i + D_i u^a_0 G^{i a}) + \frac{\tau_{1-}}{N_c} [D_i, u^a_0] G^{i a} \right) B
\]

\[
\mathcal{L}_B^{(3)} = B^\dagger \left( \frac{c_3}{N_c} \frac{\Lambda^3}{\Lambda^3} \hat{\chi}^+ + \frac{h_1}{N_c} \hat{S}^4 + \frac{h_2}{N_c} \hat{\chi}^+ \hat{S}^2 + \frac{h_3}{N_c} \chi^0 \hat{S}^2 + \frac{h_4}{N_c} \chi^a \{S^i, G^{i a}\} \right.
\]

\[+ \frac{C_A}{N_c} u^i a \{\hat{S}^2, G^{i a}\} + \frac{C_A}{N_c^2} u^i a S^i S^j G^{j a} \]

\[+ \frac{D_A}{\Lambda^2} \chi^0 u^i a G^{i a} + \frac{D_A}{\Lambda^2} \chi^a u^i a S^i + \frac{D_A (d)}{\Lambda^2} d^{abc} \chi^a u^i b G^{i c} + \frac{D_A (f)}{\Lambda^2} f^{abc} \chi^a u^i b G^{i c} \]

\[+ g_E [D_i, F_{+0}] + \alpha_1 \frac{i}{N_c} \epsilon^{ijk} F_{+0}^i G^{i a} D_k + \beta_1 \frac{i}{N_c} F_{-ij} G^{i a} D_j + \cdots \right) B
\]
Intermediate Octet and Decuplet baryon contributions are included.

\[ I_{1-loop}(Q, M_\pi) = \int \frac{d^d k}{(2\pi)^d} \frac{i \vec{k}^2}{k^2 - M_\pi^2 + i\epsilon} \frac{1}{k^0 - Q + i\epsilon} \]

\[ = \frac{i}{16\pi^2} \left\{ Q \left( (3M_\pi^2 - 2Q^2)(\lambda_\epsilon - \log \frac{M_\pi^2}{\mu^2}) + (5M_\pi^2 - 4Q^2) \right) \right. \]

\[ + 2\pi(M_\pi^2 - Q^2)^{3/2} + 4(Q^2 - M_\pi^2)^{3/2} \tanh^{-1} \frac{Q}{\sqrt{Q^2 - M_\pi^2}} \left. \right\}, \]

\[ Q = \delta m_n - p^0, \quad \lambda_\epsilon = \frac{1}{\epsilon} - \gamma + \log 4\pi \]
Baryon Masses to $\mathcal{O}(\xi^3)$ in SU(3)

$$\mathcal{L}_B = \mathbf{B}^\dagger \left( iD_0 + \hat{g}_A u^ia G^ia - \frac{C_{HF}}{N_c} \hat{S}^2 - \frac{1}{2\Lambda} c_2 \hat{\chi}_+ + \frac{c_3}{N_c \Lambda^3} \hat{\chi}^2_+ 
+ \frac{h_1}{N_c^3} \hat{S}^4 + \frac{h_2}{N_c^2 \Lambda} \hat{\chi}_+ \hat{S}^2 + \frac{h_3}{N_c \Lambda} \chi^0_+ \hat{S}^2 + \frac{h_4}{N_c \Lambda} \chi^a_+ \{ S^i, G^ia \} + \alpha \hat{Q} + \beta \hat{Q}^2 \right) \mathbf{B}$$

$$m_B = M_0 + \frac{C_{HF}}{N_c} \hat{S}^2 - \frac{c_1}{\Lambda} 2B_0 (\sqrt{3}m_8 Y + N_c m_0) - \frac{c_2}{\Lambda} 4B_0 m_0$$

$$- \frac{c_3}{N_c \Lambda^3} \left( 4B_0 (\sqrt{3}m_8 Y + N_c m_0) \right)^2$$

$$- \frac{h_1}{N_c^2 \Lambda} \hat{S}^4 - \frac{h_2}{N_c \Lambda} 4B_0 (\sqrt{3}m_8 Y + N_c m_0) \hat{S}^2 - \frac{h_3}{N_c \Lambda} 4B_0 m_0 \hat{S}^2$$

$$- \frac{h_4}{N_c \Lambda} \frac{4B_0 m_8}{\sqrt{3}} \left( 3\hat{\chi}^2 - \hat{S}^2 - \frac{1}{12} N_c (N_c + 6) \right)$$

$$+ \frac{1}{2} (N_c + 3) Y - \frac{3}{4} Y^2 \right) + \delta m^\text{loop}_B$$

$$\delta m^{1-\text{loop}}_B = \frac{g_A^2}{F_\pi^2} \frac{1}{d-1} \sum_n G^{ia} \mathcal{P}_n G^{ia} I_{1-\text{loop}}(\delta m_n - p^0, M_\pi)$$

$$\hat{\chi}_+ = N_c \chi^0_+ + \hat{\chi}_+$$

$$\chi^0_+ \to 4B_0 m_0$$

$$\hat{\chi}_+ \to 8B_0 \delta^{a8} m_8$$

$$\hat{\chi}_+ \to 4B_0 (m^8 T^8 + N_c m^0)$$

$m_0 = (2\hat{m} + m_s)/3$

$m^8 = 2/\sqrt{3}(\hat{m} - m_s)$
At tree level the GMO relation is exact for any $N_c$ up to $\mathcal{O} (\xi^3)$

$$
\Delta_{\text{GMO}} = - \left(\frac{g_A}{4\pi F_\pi}\right)^2 \left(\frac{2\pi}{3}\right) \left( M_K^3 - \frac{1}{4} M_\pi^3 - \frac{2}{\sqrt{3}} \left( M_K^2 - \frac{1}{4} M_\pi^2 \right)^{3/2} \right) 
+ \frac{C_{\text{HF}}}{2N_c} \left( 4M_K^2 \log \left( \frac{4M_K^2 - M_\pi^2}{3M_K^2} \right) - M_\pi^2 \log \left( \frac{4M_K^2 - \frac{1}{3} M_\pi^2}{3M_\pi^2} \right) \right) 
+ \mathcal{O} \left( \frac{1}{N_c^3} \right).
$$

The breaking / violation to the GMO relation is only coming through the loop corrections and it behaves like $1/N_c$ with $N_c$.
Fit results to Experimental & Lattice QCD masses

TABLE II. Results for LECs: the ratio $g_A/F_\pi = 0.0122$ MeV$^{-1}$ is fixed by using $\Delta_{G_{SO}}$. The first row is the fit to LQCD octet and decuplet baryon masses [48] including results for $M_\pi \leq 303$ MeV (dof = 50), and second row is the fit including also the physical masses (dof = 58). Throughout the $\mu = \Lambda = m_\pi$.

| $\chi^2_{\text{dof}}$ | $m_0$ [MeV] | $C_{\text{HP}}$ [MeV] | $c_1$  | $c_2$  | $h_2$  | $h_3$  | $h_4$  |
|----------------------|-------------|----------------------|--------|--------|--------|--------|--------|
| 0.47                 | 221(26)     | 215(46)              | −1.49(1) | −0.83(5) | 0.03(3) | 0.61(8) | 0.59(1) |
| 0.64                 | 191(5)      | 242(20)              | −1.47(1) | −0.99(3) | 0.01(1) | 0.73(3) | 0.56(1) |

LQCD baryon masses: C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou. Phys. Rev. D90:074501, (2014)
Baryon matrix elements of scalar quark densities give us the information on the amount of baryon mass originates from the quark masses

Feynman-Hellman theorem

$$\sigma_i(B) = m_i \frac{\partial}{\partial m_i} m_B$$

Baryon mass dependencies on quark masses

$$\sigma_{\pi N} = \hat{\sigma} + 2 \frac{\hat{m}}{m_s} \sigma_s$$

$$\sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

$$\hat{\sigma} = \frac{\hat{m}}{2m_N} \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle$$

$$\sigma_s = \frac{m_s}{2m_N} \langle N | \bar{s}s | N \rangle$$

$$| \sigma_s | \lesssim 50 \text{ MeV}$$

$$\sigma_{\pi N} \sim \hat{\sigma}$$
\[ \sigma_{\pi N} = \hat{\sigma} + 2\frac{\hat{m}}{m_s}\sigma_s \]

\[ \hat{\sigma} = \sqrt{3}\frac{\hat{m}}{m_8}\sigma_8 \]

\[ \sigma_8 = \frac{1}{3}(2m_N - m_{\Sigma} - m_{\Xi}) \]

\[ m_3 = m_u - m_d \]
\[ m_8 = \frac{1}{\sqrt{3}}(\hat{m} - m_s) \]

\[ \sigma_8 = \frac{1}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle \]

\[ \Delta \sigma_8 = \sigma_8 - \frac{1}{9}\left(\frac{5N_c - 3}{2}m_N - (2N_c - 3)m_{\Sigma} - \frac{N_c + 3}{2}m_{\Xi}\right) \]

\[ \Delta_{GMO} = 3m_\Lambda + m_{\Sigma} - 2(m_N + m_{\Xi}) \sim 25 \text{ MeV} \]

There is a (hidden) large correction \( \sim 44 \text{ MeV} \) from non-analytic contributions from baryon self-energies.

A long lasting puzzle!

\[ \sim 26 \text{ MeV} \]

\[ \hat{\sigma} = \frac{\hat{m}}{m_s - \hat{m}}(m_\Xi + m_{\Sigma} - 2m_N) \]

\[ \Delta \hat{\sigma} \]
\[ \sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N} \langle N | \bar{u}u + \bar{d}d | N \rangle \]

### Table: Sigma Terms (Results)

| Fit | \( \frac{A}{F_\pi} \) MeV\(^{-1} \) | \( M_0 \) MeV | \( C_{HF} \) MeV | \( c_1 \) | \( c_2 \) | \( h_2 \) | \( h_3 \) | \( h_4 \) | \( \alpha \) MeV | \( \beta \) MeV |
|-----|-----------------|-----------|-------------|------|------|------|------|------|-------|-------|
| 1   | 0.0126(2)       | 364(1)    | 166(23)     | -1.48(4) | 0    | 0    | 0.67(9) | 0.56(2) | -1.63(24) | 2.16(22) |
| 2   | 0.0126(3)       | 213(1)    | 179(20)     | -1.49(4) | -1.02(5) | -0.018(20) | 0.69(7) | 0.56(2) | -1.62(24) | 2.14(22) |
| 3   | 0.0126*         | 262(30)   | 147(52)     | -1.55(3) | -0.67(8) | 0    | 0.64(3) | 0.63(3) | -1.63* | 2.14* |

### Diagram

**LQCD data from:** ALEXANDROU et al. (PRD 90, 074501 (2014))

![Graph showing LQCD data comparison](image)

Summary of the determinations of \( \sigma_{\pi N} \) from \( \pi N \) scattering

\[ \sigma_{\pi N} = 69(8)(6) \text{ MeV} \]
• The value of $\hat{g}_A/F_\pi$ can be fixed by $\Delta_{GMO}$, and it is consistent with the other calculations.

• Octet baryons in the intermediate states contribute 43% to $\Delta_{GMO}$ and 33% to $\Delta\sigma_8$.

• One can realize that this is a well behaved expansion by considering the contribution to the baryon mass from each LEC.

• $\Delta_{GMO}$ and $\Delta\sigma_8$ can be determined only by $\hat{g}_A/F_\pi$, $C_{HF}$ and the meson masses, whereas the ratio $\Delta\sigma_8/\Delta_{GMO}$ doesn’t depend on $\hat{g}_A/F_\pi$.

• Fit 2 is compatible with Fit 1: implies that the chiral extrapolation of the LQCD to the physical case is consistent.

• LQCD baryon masses have an issue of describing the hyperfine mass shifts between the octet and decuplet.

• Both $\hat{\sigma}$ and $\sigma_{\pi N}$ has mild dependence on $M_K$.

• Determination of $\sigma_s$ was not precise because the LQCD results are at approximately fixed $m_s$.

• Our result for $\sigma_{\pi N}$ is consistent with the larger values obtained from $\pi - N$ scattering analyses.

• Iso spin breaking sigma terms $\sigma_3$ and $\sigma_{(u+d)}$ were estimated.

• With the information we have we can determine the contribution of Nucleon mass due to the mass difference of $m_{u-d}$ and therefore $m_{\text{Proton}}$ and $m_{\text{Neutron}}$ difference.
The discussion can be extended to the rest of the $\sigma$ terms for the different baryons and their various relations (Tree level)

$$\sigma_{Nm_s} = \frac{m_s}{8\hat{m}}(-4(N_c - 1)\sigma_{N\hat{m}} + (N_c + 3)\sigma_{\Lambda\hat{m}} + 3(N_c - 1)\sigma_{\Sigma\hat{m}})$$

$$\sigma_{\Lambda m_s} = \frac{m_s}{8\hat{m}}(-4(N_c - 3)\sigma_{N\hat{m}} + (N_c - 5)\sigma_{\Lambda\hat{m}} + 3(N_c - 1)\sigma_{\Sigma\hat{m}})$$

$$\sigma_{\Sigma m_s} = \frac{m_s}{8\hat{m}}(-4(N_c - 3)\sigma_{N\hat{m}} + (N_c + 3)\sigma_{\Lambda\hat{m}} + (3N_c - 11)\sigma_{\Sigma\hat{m}})$$

$$\sigma_{\Delta m_s} = \frac{m_s}{8\hat{m}}(-4(N_c - 1)\sigma_{\Delta\hat{m}} - 5(N_c - 3)(\sigma_{\Lambda\hat{m}} - \sigma_{\Sigma\hat{m}}) + 4N_c\sigma_{\Sigma^*\hat{m}})$$

$$\sigma_{\Sigma^* m_s} = \frac{m_s}{8\hat{m}}(-(N_c - 3)(4\sigma_{\Delta\hat{m}} + 5\sigma_{\Lambda\hat{m}} - 5\sigma_{\Sigma\hat{m}}) + 4(N_c - 2)\sigma_{\Sigma^*\hat{m}}).$$

I.P. Fernando, J.L. Goity, Phys. Rev. D 97 (2018) 054010, arXiv:1712.01672.

The LO axial charge can be obtained by the fits to axial currents from LQCD, which is shown to have a value lower than 20% of the physical value.

More applications.....
The σ terms of nucleons were calculated using SU(3) BChPT × 1/Nc

Our value for sigma Pi-N is in agreement with similar determinations in calculations that included the decuplet baryons as explicit degrees of freedom

The “σ term puzzle” is understood as the result of large non-analytic contributions to the mass combination, while the higher order corrections to the σ terms have natural magnitude.

The intermediate spin 3/2 baryons play an important role in enhancing \( \hat{\sigma} \) and thus \( \sigma_{\pi N} \)

The analysis carried out here shows that there is compatibility in the description of GMO and the nucleon σ terms

The value of \( \sigma_{\pi N} = 69 \pm 10 \text{ MeV} \) obtained here from fitting to Physical & LQCD baryon masses agrees with the more recent results from \( \pi N \) analyses
Thank you