Electronic dephasing in wires due to metallic gates.

Francisco Guinea
Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, E-28043 Madrid, Spain.

The dephasing effect of metallic gates on electrons moving in one quasi–one–dimensional diffusive wires is analyzed. The incomplete screening in this geometry implies that the effect of the gate can be described, at high energies or temperatures, as an electric field fluctuating in time. The resulting system can be considered a realization of the Caldeira-Leggett model of an environment coupled to many particles. Within the range of temperatures where this approximation is valid, a simple estimation of the inverse dephasing time gives $\tau^{-1}_G \propto T^{1/2}$.

PACS numbers: 73.21.-b, 73.22-f, 73.23.-b

I. INTRODUCTION.

The low temperature dephasing time of electrons in diffusive metals has attracted a great deal of attention\textsuperscript{1,2,3,4}. Different mechanisms has been proposed to explain the anomalous dephasing properties. Some of them are extrinsic, like dynamical defects\textsuperscript{5}, two level systems\textsuperscript{6}, or magnetic impurities\textsuperscript{7}. Alternatively, intrinsic effects have also been proposed\textsuperscript{8}. Screening effects in a quasi–one–dimensional geometry are significantly reduced, leading to a breakdown of Fermi liquid theory at low temperatures\textsuperscript{9,10} (see also\textsuperscript{11}). In the following we study the dephasing induced by metallic gates on quasi–one–dimensional diffusive wires. The study follows the analysis in\textsuperscript{12}, where dephasing effects in ballistic quantum dots was considered.

As discussed in more detail below, the presence of the gate implies the existence of two regimes: i) For distances $L \ll z$, or time scales larger than $D_w^{-1} z^2$, where $D_w$ is the diffusion coefficient of the wire, and $z$ is the distance to the gate, the dephasing time is the sum of a contribution from charge fluctuations within the wire and another due to the fluctuations at the gate. The fluctuating potential induced by the gate varies at scales comparable or larger than $z$. Hence, the gate potential at these scales can be calculated within the dipolar approximation. Because of the one dimensional geometry of the wire, this potential is not screened by the charge fluctuations of the wire. As discussed in\textsuperscript{12}, this coupling can be considered a generalization to a many particle system of the Caldeira-Leggett model of ohmic dissipation\textsuperscript{13}. This model shows anomalous dephasing in many situations\textsuperscript{14,15,16,17}. ii) At distances $L \gg z$ or time scales lower than $D_w^{-1} z^2$ the distance between the wire and the gate can be neglected. The screening by the gate leads to an effective short range potential along the wire. This effect, however, only includes logarithmic corrections in the standard expression for the inverse dephasing time in wires\textsuperscript{9,10,11} (see below).

These different regimes of the model are discussed in section III. The next section generalizes the results to gates with different geometries. Finally, section V contains a discussion of the most relevant results. The units are such that $\hbar = 1$.

II. THE MODEL.

We study the setup sketched in Fig. I. A wire of width $w$ is located at height $z$ over a two dimensional metallic gate of width $w'$. The effects of the finite width of both systems is included through the densities of states, $\nu_w$ and $\nu_G$, defined as number of states per unit length and per unit area respectively. We study the contribution to dephasing from the gate using the scheme proposed in\textsuperscript{12}. The probability of transition of a particle at the Fermi level to other states, after time $t$, using second order perturbation theory, at temperature $T = \beta^{-1}$, is:

$$P^{(2)}(t) \simeq \int_0^t d\tau \int_0^t d\tau' \int dq_\parallel \int_{|\omega|>1/t} d\omega e^{i \langle q \mid (r(\tau) - r(\tau')) + i\omega (\tau - \tau') \rangle + i\omega (\tau - \tau') + \frac{|\omega|}{1 - e^{-\beta \omega}} \Im \left[ \nu_{\text{sec}}(q, \omega) \right]}$$

(1)
where \( q_\parallel \) is the momentum in the direction parallel to the wire, and \( v_{\text{scr}}(q_\parallel, \omega) \) is the screened potential between points within the wire. In general, we can write the screened potential as \( v_{\text{scr}}(\vec{z}, q_\parallel, q_\perp, \omega) \), where \( \vec{z} \) is the vertical coordinate. We assume that the wire is at \( \vec{z} = z \), and the gate is at \( \vec{z} = 0 \). The screened potential at these two vertical coordinates can be written as:

\[
v_{\text{scr}}(z, q_\parallel, q_\perp, \omega) = v_0(z, q_\parallel, q_\perp, \omega) + \frac{2\pi e^2}{|q|} \chi_w(q_\parallel, \omega) \int dq_\perp v_{\text{scr}}(z, q_\parallel, q_\perp, \omega) + \frac{2\pi e^2 e^{-|q|z}}{|q|} \chi_G(q_\parallel, q_\perp, \omega) v_{\text{scr}}(0, q_\parallel, q_\perp, \omega)
\]

\[
v_{\text{scr}}(0, q_\parallel, q_\perp, \omega) = v_0(0, q_\parallel, q_\perp, \omega) + \frac{2\pi e^2}{|q|} \chi_w(q_\parallel, \omega) \int dq_\perp v_{\text{scr}}(z, q_\parallel, q_\perp, \omega) + \frac{2\pi e^2 e^{-|q|z}}{|q|} \chi_G(q_\parallel, q_\perp, \omega) v_{\text{scr}}(0, q_\parallel, q_\perp, \omega)
\]

where \( q_\parallel \) and \( q_\perp \) are the momenta parallel and perpendicular to the wire, and \( \chi_w \) and \( \chi_G \) are the polarizabilities of the wire and the gate. They are given by:

\[
\chi_w(q_\parallel, \omega) = -\frac{\nu_w D_w q_\parallel^2}{i\omega + D_w q_\parallel}
\]

\[
\chi_G(q, \omega) = -\frac{\nu_G D_G q^2}{i\omega + D_G q^2}
\]

where \( \nu_w, \nu_G, D_w \) and \( D_G \) are the densities of states and diffusion coefficients of the wire and the gate, respectively.

From the second of the equations in (2), we can write:

\[
v_{\text{scr}}(0, q_\parallel, q_\perp, \omega) = \frac{v_0(0, q_\parallel, q_\perp, \omega) + 2\pi e^2}{|q|} \chi_w(q_\parallel, \omega) \int dq_\perp v_{\text{scr}}(z, q_\parallel, q_\perp, \omega) + \frac{2\pi e^2 e^{-|q|z}}{|q|} \chi_G(q_\parallel, q_\perp, \omega) v_{\text{scr}}(0, q_\parallel, q_\perp, \omega)
\]

(4)

For a point charge at a point in the wire, we have:

\[
v_0(z, q_\parallel, q_\perp, \omega) = \frac{2\pi e^2}{|q|}
\]

\[
v_0(0, q_\parallel, q_\perp, \omega) = \frac{2\pi e^2 e^{-|q|z}}{|q|}
\]

(5)

Using this expression, and inserting eq. (4) into the first equation in (2), we find:

\[
v_{\text{scr}}(q_\parallel, \omega) = \frac{e^2 \left[ \log \left( \frac{\omega}{q_\parallel} \right) + F(q_\parallel, \omega) \right]}{1 + e^2 \chi_w(q_\parallel, \omega) \log \left( \frac{\omega}{q_\parallel} \right) + F(q_\parallel, \omega)}
\]

\[
F(q_\parallel, \omega) = \int dq_\perp \frac{4\pi^2 e^2 e^{-2|q|z}}{|q_\parallel|^2} \chi_G(q_\parallel, \omega) \quad \frac{1}{1 - \frac{2\pi e^2}{|q_\parallel^2} \chi_G(q_\parallel, \omega)}
\]

(6)

where \( q_* \) is a high momentum cutoff proportional to the inverse of the width of the wire.

For sufficiently low momenta, \( |q| \ll \nu_G^{-1} \), we can write:

\[
F(q_\parallel, \omega) \approx F_1(q_\parallel) - i\omega F_2(q_\parallel)
\]

\[
F_1(q_\parallel) = \int dq_\perp \frac{2\pi e^2 e^{-2|q|z}}{|q|} \sim \log(q_\parallel z)
\]

\[
F_2(q_\parallel) \approx -\frac{1}{e^2 \nu_G D_G} \int dq_\perp \frac{e^{-2|q|z}}{|q|^2} \sim \frac{1}{e^2 \nu_G D_G} \frac{1}{q_\parallel}
\]

(7)
For small frequencies, $\omega \leq D_w q_{||}^2$, we find:

$$\text{Im} \left[ v_{\text{scr}}(q_{||}, \omega) \right] \approx \frac{\omega c^4 \nu_{w}^2}{D_w q_{||}^2} \left\{ \log \left( \frac{q_{||}}{q} \right) + \mathcal{F}_1(q_{||}) \right\}^2 + D_w q_{||}^2 \mathcal{F}_2(q_{||})$$  \hspace{1cm} (8)

### III. RESULTS.

#### A. Dipolar approximation ($\tau^{-1}(T) \gg D_w/z^2$).

The integral over the time difference $\tau - \tau'$ in eq. (11) is bounded by the inverse of the temperature, $\beta$. Both $z$ and the value of $\langle |r(\tau) - r(\tau')|^2 \rangle$ for $\tau - \tau' \approx \beta$ act as a lower cutoff in the integrals over $q_{||}$, so that $|q_{||}|^{-2} \ll \min \left\{ \langle |r(\tau) - r(\tau')|^2 \rangle, z^2 \right\}$. When $\langle |r(\tau) - r(\tau')|^2 \rangle \ll z^2$, we can limit the integral over $q_{||}$ to $|q_{||}| \leq z^{-1}$ and substitute:

$$e^{i q_{||} [r(\tau) - r(\tau')]} \approx - q_{||} [r(\tau) - r(\tau')]^2$$  \hspace{1cm} (9)

Within these approximations, and setting $\eta_{c}^{-1} \approx \omega$, eq. (8) becomes:

$$\text{Im} \left[ v_{\text{scr}}(q_{||}, \omega) \right] e^{i q_{||} [r(\tau) - r(\tau')]} \approx v_{1D}(q_{||}, \omega) e^{i q_{||} [r(\tau) - r(\tau')]} + v_{2D}(q_{||}, \omega)$$

$$v_{1D}(q_{||}, \omega) = \frac{\omega}{c^2 \nu_{w} D_w q_{||}^2}$$

$$v_{2D}(q_{||}, \omega) = \frac{\omega |q_{||}| [r(\tau) - r(\tau')]^2}{c^4 \nu_{w}^2 \nu_{G} D_G \log^2(z/w)}$$  \hspace{1cm} (10)

Inserting eq. (12) in eq. (11) one obtains that $P^{(2)}_G(t)$ can be written as the sum of two contributions, $P^{(2)}_w(t)$ and $P^{(2)}_G(t)$ arising from $v_{1D}$ and $v_{2D}$ (note that screening effects from the gate are included in the denominator of $v_{1D}$ in eq. (11)). The first term when inserted in eq. (11), give the contribution to the function $P^{(2)}_G(t)$ calculated in $\mathbb{\Pi}$, leading to the standard expression for the dephasing in one dimensional wires.

The contribution from $v_{2D}$ in eq. (12) to eq. (11) can be written as:

$$P^{(2)}_G(t) \simeq \frac{T D_w t^2}{c^4 \nu_{w}^2 \nu_{G} D_G \log^2(z/w) z^2} \int_0^t d\tau' \int_{1/t}^T d\omega \left( |r(\tau) - r(\tau')|^2 \right) e^{i \omega (\tau - \tau')}$$  \hspace{1cm} (11)

Using $\langle |r(\tau) - r(\tau')|^2 \rangle = D_w (\tau - \tau')$, we finally obtain:

$$P^{(2)}_G(t) \simeq \frac{T D_w t^2}{c^4 \nu_{w}^2 \nu_{G} D_G \log^2(z/w) z^2}$$  \hspace{1cm} (12)

From this equation we can define a dephasing time due to the presence of the gate, $P^{(2)}_G(\tau_G) \approx 1$, as:

$$h \tau^{-1}_G \approx \frac{T D_w}{c^4 \nu_{w}^2 \nu_{G} D_G \log^2(z/w) z^2}$$  \hspace{1cm} (13)

On the other hand, the dephasing time due to intrinsic processes can be written as (see also $\mathbb{\Pi}$):

$$h \tau^{-1}_w \approx \frac{T^{2/3}}{D_w^{1/3} \nu_{w}^{2/3}}$$  \hspace{1cm} (14)

Because of the different temperature dependence, $\tau^{-1}_G$ is greater than $\tau^{-1}_w$ at temperatures below a value $T'$ given by:

$$T' \approx \frac{D_w^5}{D_w^3 \nu_{G}^2 \nu_{w}^2 c^4 \nu_{w}^2 \log^6(z/w) z^6}$$  \hspace{1cm} (15)
The approximations leading to this result are valid provided that \( \tau_G^{-1} \geq D_w/z^2 \). This condition breaks down below a temperature \( T'' \) given by:

\[
T'' \simeq \frac{D_w}{z^2} e^{4} \nu_w^2 \nu_D G D_G \log^2(z/w)
\]  

(16)

The dephasing due to the gate dominates if a temperature range \( T'' \leq T \leq T' \). The inequality \( T'' \leq T' \) implies:

\[
1 \leq f = \frac{D_w}{e^{4} \nu_w \nu_D G D_G z \log^2(z/w)}
\]  

(17)

We also have:

\[
e^{2} \nu_w \simeq r_{sw}(k_F^w w_w)^2 \approx \frac{1}{r_{sw}(k_F^w l_G)} \left( \frac{l_w}{z} \right)
\]  

(18)

where \( k_F^w \) and \( k_F^G \) are the Fermi wavevectors at the wire and gate, \( l_w \) and \( l_G \) are the elastic mean free paths in the wire and gate, \( w_w \) is the width of the wire, and \( r_{sw} \sim (e^{2} k_F^w) / (\hbar^2 k_F^w) / m \) is inversely proportional to the electronic density in the wire. In the presence of a dielectric between the wire and gate with dielectric constant \( \epsilon_0 \), one has to replace the electric charge \( e \) by \( e^2 / \epsilon_0 \) in all expressions. Hence, eq. (17) can only be satisfied for very clean wires, such that \( z \ll l_w \), or in the presence of a large dielectric constant, \( \epsilon_0 \).

B. Low temperature regime (\( \tau^{-1}(T) \ll D_w/z^2 \)).

At low temperatures the electrons diffuse coherently over distances much larger than \( z \). The dipolar approximation, eq.(19) cannot be made, and the cutoff in the integrals over \( q || \) is \( [D_w(\tau - \tau')]^{-1} \). Then, using eq.(5) we obtain,

\[
\mathcal{P}_G^{(2)}(t) \approx \frac{T t}{e^{4} \nu_w^2 \nu_D G D_G \log^2(q_e z)} \log \left( \frac{D_w}{z^2 t} \right)
\]  

(19)

Neglecting logarithmic corrections, this result leads to:

\[
h \tau_G^{-1} \approx \frac{T}{e^{4} \nu_w^2 \nu_D G D_G \log^2(q_e z)}
\]  

(20)

In this regime there is also a contribution from the intrinsic processes, given by eq. (14). These processes will dominate at sufficiently low temperatures, \( T \leq T''' \) where:

\[
T''' = \frac{e^{12} \nu_w^4 \nu_D^3 G^3 D_G \log^6(q_e z)}{D_w}
\]  

(21)

Note that when \( T'' \leq T''' \), where \( T'' \) is given in eq.(16) the contributions from the gate are always smaller than those coming from fluctuations in the wire. The condition \( T''' \leq T'' \) reduces to eq.(17).

Combining the results in this subsection and in the preceding one, we can write:

\[
\mathcal{P}_G^{(2)}(t) \approx \left\{ \begin{array}{ll}
\frac{T t}{e^{4} \nu_w^2 \nu_D G D_G \log^2(q_e z)} & \text{if } T'' \leq T \leq T'''
\\
\frac{T t}{e^{4} \nu_w^2 \nu_D G D_G \log^2(q_e z)} \log \left( \frac{D_w}{z^2 t} \right) & \text{if } T''' \leq T
\end{array} \right.
\]  

(22)

where \( f \) is defined in eq. (17). These expressions lead to:

\[
\tau_G^{-1} \approx \left\{ \begin{array}{ll}
\frac{D_w}{z^2} T^{-1/2} & \text{if } T \geq T''
\\
\frac{D_w}{z^2} T^{-2/3} & \text{if } T \leq T''
\end{array} \right.
\]  

(23)

IV. EXTENSIONS TO OTHER GEOMETRIES.

A. One dimensional gate.

The analysis in the two previous sections can be extended, in a straightforward way, to the case where the gate is another quasi–one–dimensional wire. The gate polarizability, defined in eq. (8) depends only on the momentum parallel to the wire. The function \( \mathcal{F} \) in eq. (6) becomes:

\[
\mathcal{F}(q ||, \omega) \approx e^{4} K_0^2(q || z) \frac{\chi_G(q ||, \omega)}{1 - e^{2} K_0(q || z) \chi_G(q ||, \omega)}
\]  

(24)

where \( K_0(q || z) \) is a modified Bessel function:

\[
K_0(q || z) = \int_0^\infty dq_{\perp} \frac{\cos(q_{\perp} z)}{q_{\perp} + q_{\perp}^2}
\]  

(25)

We can, as in the preceding section, study separately the dipolar regime, \( \tau^{-1} \ll D_w/z^2 \), and the long time regime, \( \tau^{-1} \gg D_w/z^2 \). In the dipolar regime, using eq. (15), we obtain:

\[
\tau_G^{-1} \approx \frac{T \nu_{\nu_D} D_w}{e^{4} \nu_D G D_G (\nu_w + \nu_G)^2}
\]  

(26)

(note that now \( \nu_w \) is a quasi–one–dimensional density of states). The restriction \( \tau^{-1} \leq D_w/z^2 \) implies that this result is only valid for temperatures \( T \geq T'' \) where:

\[
T'' = \frac{D_w e^{2} \nu_D G D_G (\nu_w + \nu_G)^2}{z^3 \nu_w}
\]  

(27)

At high temperatures, \( T \geq T' \), we find \( \tau_G^{-1} \leq \tau_{\nu_D}^{-1} \), where:

\[
T' = \frac{D_w^2 \nu_D^2}{e^{6} \nu_D D_G^3 (\nu_w + \nu_G)^2 z^3}
\]  

(28)
The condition required for the relevance of the dephasing due to the gate, $T'' \leq T'$ implies:

$$1 \leq \frac{D_w \nu_w^2}{e^2 \nu_G D_G (\nu_w + \nu_G)^2}$$  \hspace{1cm} (29)$$

It is interesting to note that this condition does not depend on the distance between the wire and the gate.

At temperatures below $T''$, such that $\tau^{-1} \leq D_w/z^2$, the combined system acts like an effective one dimensional wire, leading to a $\tau^{-1} \propto T^{2/3}$ dependence.

### B. Three dimensional gate.

We assume that quasiparticles at the gate are specularly reflected at the boundary. Then, the screening properties of the system can be calculated from the fluctuations of surface charges at the top of the gate. The function $F$ in eq. (6) can be written as:

$$F(q_{\parallel}, \omega) = \int dq_{\perp} \frac{2\pi e^2 e^{-2q_{\perp}} B(q_{\perp}, \omega) - 1}{|q_{\perp}|} B(q_{\perp}, \omega) + 1$$

$$B(q, \omega) = \frac{|q|}{\pi} \int dq_{\perp} \left( \frac{1}{|q|^2 + q_{\perp}^2} \right) \epsilon_G(q, q_{\perp}, \omega)$$

$$\epsilon_G(q, q_{\perp}, \omega) = 1 + \frac{4\pi e^2}{|q|^2 + q_{\perp}^2} \chi_G(q, q_{\perp}, \omega)$$  \hspace{1cm} (30)$$

Using this expression, the parameters $\tau_G, T', T''$ and $T'''$ which characterize the dephasing induced by the gate can be calculated. One finds that the screening dependence is the same as for a quasi-two-dimensional gate, with the only replacement $\nu_G^{3D} \rightarrow \nu_G^{3D} \times z$, similarly to the results discussed in [12]. Qualitatively, a three dimensional gate behaves as a two dimensional gate of width $z$. The constraint which needs to be satisfied for the gate induced dephasing to be dominant is:

$$1 \leq \frac{D_w}{e^4 \nu_w \nu_G D_G z^2 \log^2 (z/w)}$$  \hspace{1cm} (31)$$

### C. Granular gate.

We now analyze the dephasing induced by a gate made up of disconnected metallic grains. We assume that each grain has volume $V$ and it is characterized by a diffusion coefficient $D_G$ and density of states $\nu_G$, leading to an intrinsic d. c. conductivity $\sigma = e^2 \nu_G D_G$. Their response to an applied field is characterized by their polarizability, $P$ and absorption coefficient, $\gamma(\omega) \approx V \omega^2/\nu_G$. The dielectric constant of the granular system can be written as:

$$\epsilon(\omega) = V^{-1} \left( P + i \frac{\omega V}{e^2 \nu_G D_G} \right)$$  \hspace{1cm} (32)$$

We can insert this expression into eqs. (30), and carry out the following steps in order to obtain the dephasing effects of the gate. The inequality which needs to be satisfied for the gate induced dephasing to prevail is:

$$1 \leq \frac{D_w}{e^4 \nu_w \nu_G D_G z^2 (1 + V^{-1} P)^2}$$  \hspace{1cm} (33)$$

This expression, valid for grains larger than the mean free path, is very similar to the corresponding one for a three dimensional gate, eq. (31). The effects of a granular gate, however, can be greatly enhanced by the surface roughness of the grains (see also [22]), or for grains much smaller than the mean free path.

### V. CONCLUSIONS.

We have analyzed, within the standard approach to dephasing in metals [20,11], the effects of a two dimensional diffusive gate on the coherence properties of electrons in quasi one dimensional wires. The method used can be easily generalized to other types of gates, like granular metals or ballistic systems.

The main results are schematically depicted in Fig. 2. At short distances, or low temperatures, the gate induces fluctuations of surface charges at the top of the gate, which fluctuates in time but is approximately constant in space. The resulting model can be considered an extension of the Caldeira-Leggett model to a many particle system. Using the self consistent perturbation theory to calculate the dephasing time, we find a $\tau_G^{-1} \propto T^{1/2}$ dependence.
This regime requires only the validity of the dipolar approximation, see section IIIA. Hence, it is not restricted to the diffusive 1D wire. At lower temperatures, the separation between the gate and the wire becomes irrelevant, and the contribution from the gate changes to a $\tau^{-1}_{G} \propto T$ dependence.

The contribution to the dephasing rate due to processes intrinsic to the wire does not change qualitatively from the standard result. The screening by the gate cancels the long range part of the electrostatic potential. The resulting interaction, however, when treated within the RPA, leads to the usual $\tau^{-1}_{\text{int}} \propto T^{2/3}$ dependence.

The contribution from intrinsic processes dominates both at low and high temperatures (see Fig. 2). The existence of an intermediate range of temperatures where the gate determines the inverse dephasing time depends on the inequality in eq.(17). This constraint requires a mean free path $l_w \geq z$, which is probably not satisfied in current experimental situations, where the mean free path of the wire is comparable to its transverse dimensions, a few nanometers. A medium between the wire and the gate with large dielectric constant can enhance the dephasing due to the gate. Note that a non-perturbative treatment shows that the coupling to an environment modelled by the Caldeira-Leggett model strongly suppresses quantum coherence in a variety of situations.

Qualitatively similar effects can be expected for other types of gates. If the gate is another quasi–one–dimensional wire, the condition which determines the relative importance of the gate is independent of the distance between the two wires, eq.(29).

VI. ACKNOWLEDGMENTS.

I am thankful to R. Jalabert and A. Zaikin for helpful comments. I am also thankful to Ministerio de Ciencia y Tecnología (Spain) for financial support through grant MAT2002-0495-C02-01.

---

1. E. J. P. Mohanty and R. Webb, Phys. Rev. Lett. 78, 3366 (1997).
2. F. Pierre and N. O. Birge, Phys. Rev. Lett. 89, 206804 (2002).
3. P. Mohanty and R. A. Webb, Phys. Rev. Lett. 91, 066604 (2003).
4. F. Pierre, A. B. Gougam, H. P. A. Anthore, D. Esteve, and N. O. Birge, Phys. Rev. B 68, 085413 (2003).
5. Y. Imry, J. Fukuyama, and C. Schwab, Europhys. Lett. 47, 608 (1999).
6. A. Zawadovski, J. van Delft, and R. C. Ralph, Phys. Rev. Lett. 83, 2632 (1999).
7. A. Kaminski and L. I. Glazman, Phys. Rev. Lett. 86, 2400 (2001).
8. D. Golubev and A. Zaikin, Phys. Rev. Lett. 81, 1074 (1998).
9. B. Altshuler, A. Aronov, and D. Khmelnitskii, J. Phys. C 15, 7367 (1982).
10. A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A 41, 3436 (1990).
11. G. Montambaux and E. Akkermans (2004), cond-mat/0404361.
12. F. Guinea, R. A. Jalabert, and F. Sols (2004), cond-mat/0402277.
13. A. O. Caldeira and A. Leggett, Phys. Rev. Lett. 46, 211 (1981).
14. D. S. Golubev and A. Zaikin, Physica B 255, 164 (1998).
15. G.-L. Ingold, R. A. Jalabert, and K. Richter, Am. J. Phys. 69, 201 (2001).
16. F. Guinea, Phys. Rev. B 65, 205317 (2002).
17. F. Guinea, Phys. Rev. B 67, 045103 (2003).
18. Y. M. Blanter, Phys. Rev. B 54, 12807 (1996).
19. R. H. Ritchie and A. L. Marusak, Surf. Sc. 4, 234 (1966).
20. B. Mehlig and M. Wilkinson, J. Phys. C: Condens. Matter 9, 3277 (1997).
21. E. J. Austin and M. Wilkinson, J. Phys. C: Condens. Matter 5, 8461 (1993).
22. W. P. Halperin, Rev. Mod. Phys. 56, 533 (1986).
23. It is interesting to note that, for a ballistic system, $\langle |r(\tau) - r(\tau')|^2 \rangle \approx s_p^2 (\tau - \tau')^2$. This result implies that $\tau^{-1}_{G} \propto T^{1/3}$ in this case.