Heavy Pentaquarks

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We construct the spin-flavor wave functions of the possible heavy pentaquarks containing an anti-charm or anti-bottom quark using various clustered quark models. Then we estimate the masses and magnetic moments of the \(J^P = \frac{1}{2}^+\) or \(\frac{3}{2}^+\) heavy pentaquarks. We emphasize the difference in the predictions of these models. Future experimental searches at BESIII, CLEOc, BELLE, and LEP may find these interesting states.

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I. INTRODUCTION

After LEPS Collaboration [1] announced the discovery of the narrow \(\Theta^+\) pentaquark state around 1540 MeV, many other experimental groups [2, 3, 4, 5, 6, 7, 8] claimed that they confirmed the existence of this exotic baryon with the minimal quark content \(u u d \bar{s}\). The third component of its isospin is \(I_z = 0\). The \(pK^+\) spectrum is featureless [6, 7, 8]. So the \(\Theta^+\) pentaquark is an iso-scalar if it is really a member of the anti-decuplet. At present, the possibility of this state being a member of another multiplet is not completely excluded. Hence its total isospin is probably zero. All the other quantum numbers remain undetermined. Later, another narrow pentaquark candidate \(\Xi_{5}^{-}\) with strangeness \(S = −2\), baryon number \(B = 1\), and isospin \(I = \frac{3}{2}\) around 1862 MeV was observed by NA49 Collaboration [9]. However this state has not been confirmed by other groups, and its existence is not established up to now [10].

These exotic baryons are definitely beyond the conventional quark model, in which ordinary baryons are composed of three quarks and mesons are composed of a pair of quark and anti-quark. Although the simple quark model has been extremely successful in the classification of hadron states, its foundation has not been derived from Quantum Chromo-dynamics (QCD) so far. QCD allows a much richer spectrum than that in quark model. For example, non-conventional hadrons such as glueballs, hybrid mesons, other multi-quark hadrons are expected in QCD. Convinced that the quark model can not be the whole story, people have been looking for these exotic hadrons for decades. None of them was established without controversy [11] until the discovery of the \(\Theta^+\) pentaquark.

Theoretical study of pentaquark states dated back to the early days of QCD using MIT bag model [12]. A few years ago, Diakonov et al. predicted the masses and widths of the anti-decuplet baryons using the chiral soliton model (CSM) and suggested several reaction channels to look for them [13], which partly motivated the experimental search. But the resulting masses and widths of the anti-decuplet baryons are very sensitive to the inputs in this model [13, 14, 15]. For example, either adopting the commonly used value 45 MeV for the \(\sigma\)-term or identifying \(N(1710)\) as a member of the anti-decuplet will lead to a \(\Xi_{5}^{-}\) pentaquark with a mass of 2070 MeV, which is 210 MeV higher than that observed by NA49 Collaboration [13, 16]. Very recently, Ellis et al. used the new value \((79 \pm 7)\) MeV and \((64 \pm 7)\) MeV from two recent analysis [17] for the \(\sigma\)-term and obtained a fairly good description of both \(\Theta^+\) and \(\Xi_{5}^{-}\) masses [13].

But the theoretical foundation of the treatment of the pentaquarks in the chiral soliton model is challenged with the large \(N_c\) formalism by Refs. [13, 10].

Since early last year, there appeared many theoretical papers trying to interpret these exotic states. Among them, Jaffe and Wilczek’s (JW) diquark model is a typical one [20]. In their model, the \(\Theta^+\) pentaquark is composed of a pair of diquarks and a strange anti-quark. The flavor anti-decuplet is always accompanied by an octet which is nearly degenerate and will mix with the decuplet. Shuryak and Zahed’s suggested that the pentaquark mass be lower by replacing one scalar diquark with one tensor diquark in JW’s model [21]. Karliner and Lipkin’s (KL) proposed a diquark-triquark model which also led to a flavor anti-decuplet and octet [22]. In all the above clustered quark model, the resulting angular momentum and parity \(J^P\) of the pentaquark can be either \(\frac{3}{2}^+\) or \(\frac{5}{2}^+\). Dudek and Close first estimated the \(J = \frac{5}{2}\Theta^+\) pentaquark mass in JW’s and KL’s model by considering the spin-orbital force [23].

Many models (e.g. [24]) including all the above clustered quark models were constructed to ensure the pentaquarks possess positive parity as in the original chiral soliton model [13, 14]. But the \(\Theta^+\) pentaquark parity is still a pending issue. For example, QCD sum rule approach [24, 24] and lattice QCD simulation favor negative parity [27]. Some other models favor negative parity as well [28, 29]. Recently, many theoretical papers proposed interesting ways to determine \(\Theta^+\) parity.
In a recent paper \[38\], Jaffe and Wilczek pointed out that the decay mode $\Xi^- \rightarrow \Xi^* + \pi$ observed by NA49 signals the existence of an octet around 1862 MeV together the anti-decuplet since the latter can not decay into a decuplet and an octet in the $SU(3)_f$ symmetry limit. If further confirmed, this experiment disfavors a $J^P = \frac{3}{2}^-$ assignment for $\Xi_{5^-}$ and poses a serious challenge for the chiral soliton model since there is no baryon pentaquark octet in the rotational band in this model. But it may be hard to exclude it since there always exist excited vibrational octet modes. These modes can not be calculated rigorously within the chiral soliton model [17].

Besides its mass, the pentaquark magnetic moment is another very important quantity encoding the underlying quark structure and dynamics. Practically speaking, it is essential in the calculation of the cross-sections of the pentaquark electro- and photo-production processes. Different models may yield the same mass, especially when experimental data are available. But they may yield very different results for the magnetic moment, which provides a very good way to distinguish various models.

We have employed the light cone QCD sum rule technique to calculate the $\Theta^+$ pentaquark magnetic moment [39]. In Ref. [40, 41] we have calculated magnetic moments of both $\frac{1}{2}^+$ and $\frac{3}{2}^+$ octet and anti-decuplet pentaquark states in the framework of various clustered quark models. Several other groups have also discussed the light pentaquark magnetic moments, especially $\mu_{\Theta^+}$ [42, 43, 44, 45].

In this work we extend the same formalism to discuss the heavy pentaquarks containing an anti-charm or anti-bottom quark systematically. In the framework of the above clustered quark models, we get one $SU(3)_f$ anti-sextet in JW’s model, both a sextet and triplet in KL’s models with either $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$. Then we estimate their masses and magnetic moments. Experimentally, several groups have also studied the light pentaquarks at LEP and B factories [46], which call for theoretical efforts on this topic. There have been some discussions of heavy pentaquarks in Ref. [20, 22, 23, 47].

As in the case of conventional heavy hadrons, the presence of the heavy anti-quark makes the treatment of the system simpler. If the light quarks are really strongly correlated as proposed in the clustered quark models, the heavy pentaquark system is the ideal place to study this kind of correlation without the additional complication due to the extra light anti-quark. Exploration of these heavy exotic states will deepen our knowledge of strong interactions.

This paper is organized as follows. In Section II we review this rapidly developing field briefly. In the following sections, we calculate the masses and magnetic moments using various clustered quark models. Finally, we compare the results from different models and discuss the experimental searches of these interesting heavy pentaquarks.

II. HEAVY PENTAQUARK STATES IN JAFFE AND WILCZEK’S MODEL

Jaffe and Wilczek proposed that there exists strong correlation between the light quark pair when they are in the anti-symmetric color $(3_s)$, flavor $(\bar{3}_f)$, isospin $(I = 0)$ and spin $(J = 0)$ configuration [20, 38]. The lighter the quarks, the stronger the correlation, which helps the light quark pair form a diquark. For example, the $ud$ diquark behaves like a scalar with positive parity. Such correlation may arise from the color spin force from the one gluon exchange or the flavor spin force induced by the instanton interaction. In order to accommodate the $\Theta^+$ pentaquark, Jaffe and Wilczek required the flavor wave function of the diquark pair to be symmetric $6_f$ and their color wave function to be antisymmetric $3_s$. Bose statistics of the scalar diquarks demands an odd orbital excitation between the two diquarks while there is no orbital excitation inside each diquark, which ensures that the resulting pentaquark parity is positive. The heavy anti-quark is a $SU(3)_f$ flavor singlet. Hence the heavy pentaquarks containing an anti-charm or anti-bottom form a $SU(3)$ flavor anti-sextet as shown in Figure 1. The flavor wave functions of all members of $6_f$ are listed in Table I.

![FIG. 1: The six members of the SU(3) flavor anti-sextet.](image)

| \(Y, I, I_3\) | Flavor wave functions \((Q = \bar{c} \text{ or } b)\) |
|-----------------|---------------------------------------------------|
| $\Theta^+_0, \Theta^+_{\bar{c}}$ | $[ud]^2 Q$ |
| $\Sigma^+_{5c}, \Sigma^+_{5b}$ | $[ud[us]]^+ Q$ |
| $\Sigma^-_{5c}, \Sigma^-_{5b}$ | $[ud[ds]]^- Q$ |
| $\Xi^0_{5c}, \Xi^0_{5b}$ | $[us]^2 Q$ |
| $\Xi^-_{5c}, \Xi^-_{5b}$ | $[us[ds]]^- Q$ |
| $\Xi^0_{5c}, \Xi^0_{5b}$ | $[ds]^2 Q$ |

TABLE I: Flavor wave functions in Jaffe and Wilczek’s model [21]. $[q_1 q_2][q_3 q_4]_\pm = \sqrt{\frac{1}{2}}([q_1 q_2][q_3 q_4] + [q_3 q_4][q_1 q_2])$ or $[q_1 q_2]^2 = [q_1 q_2][q_1 q_2]$ is the diquark-diquark part.
The magnetic moment of a compound system is the sum of orbital and spin contributions from all of its constituents. Only the orbital motion of the scalar diquarks contributes to the pentaquark magnetic moment. Each diquark’s orbital angular momentum reads

\[
l_1 = r_1 \times p_1 = (R + \frac{m_2}{m_1 + m_2} r) \times (\frac{m_1}{m_1 + m_2} P + p) \tag{1}
\]

where \( R \equiv (m_1 r_1 + m_2 r_2)/(m_1 + m_2) \) is the position of the center of mass of the diquark-diquark system and \( r = r_1 - r_2 \) is the relative position between the two diquarks. Since the pentaquark is a bound state of multi-quarks, \( \langle p_1 \rangle = 0 \). So we have \( \langle P \rangle = \langle p_1 \rangle + \langle p_2 \rangle = 0 \) and \( \langle p \rangle = \langle (m_2 p_1 - m_1 p_2)/(m_1 + m_2) \rangle = 0 \), from which we get

\[
\langle l_1 \rangle = \frac{m_2}{m_1 + m_2} \langle r \times p \rangle + \frac{m_1}{m_1 + m_2} \langle R \times P \rangle
\]

\[
= \frac{m_2}{m_1 + m_2} \langle l \rangle + \frac{m_1}{m_1 + m_2} \langle L \rangle \tag{2}
\]

where \( l = r \times p \) is the diquark-diquark’s relative orbital angular momentum. \( L = R \times P \) is the diquark-diquark’s orbital angular momentum for the motion of center of mass. In the center of mass frame \( \langle L \rangle = 0 \). Therefore we get

\[
\langle l_1 \rangle = \frac{m_2}{m_1 + m_2} \langle l \rangle, \tag{3}
\]

\[
\langle l_2 \rangle = \frac{m_1}{m_1 + m_2} \langle l \rangle. \tag{4}
\]

For the magnetic moment of the diquark-diquark system we have

\[
\mu_i \vec{l} = \frac{m_2 \mu_1}{m_1 + m_2} \vec{l} + \frac{m_1 \mu_2}{m_1 + m_2} \vec{l},
\]

\[
\mu_i = \frac{m_2 \mu_1}{m_1 + m_2} + \frac{m_1 \mu_2}{m_1 + m_2}, \tag{5}
\]

where \( \mu_i = e_i/2m_i \) is the magneton of the \( i \)-th diquark. \( \vec{l} = \vec{J} \) is the angular moment between diquarks.

Now we can write down the magnetic moment of a \( J^P = \frac{1}{2}^+ \) pentaquark as follows,

\[
\mu = \langle 2 \mu_Q \vec{J} + \mu_1 \vec{l} \rangle (J_z = \frac{1}{2})
\]

\[
= \left( \langle 0 | \frac{1}{2} | 1 \rangle \frac{1}{2} \frac{1}{2} \rangle^2 - \langle 1 \frac{1}{2} | 2 \frac{1}{2} \rangle^2 \right) \mu_Q
\]

\[
+ \langle 1 \frac{1}{2} | 1 \rangle \langle 1 \frac{1}{2} | 2 \rangle \mu_1
\]

\[
= -\frac{1}{3} \mu_Q + \frac{2}{3} \mu_1. \tag{6}
\]

Similarly, for \( J^P = \frac{3}{2}^+ \) pentaquarks, we have

\[
\mu = \langle 2 \mu_Q \vec{J} + \mu_1 \vec{l} \rangle (J_z = \frac{3}{2})
\]

\[
= \left( \langle 1 \frac{1}{2} | 1 \rangle \frac{3}{2} \frac{3}{2} \rangle^2 \mu_Q + \langle 1 \frac{1}{2} | 2 \rangle \frac{3}{2} \frac{3}{2} \rangle \mu_1
\]

\[
= \mu_Q + \mu_1. \tag{7}
\]

We use heavy quark masses \( m_c = 1710 \text{ MeV}, \ m_b = 5050 \text{ MeV} \) from Ref. \( [22] \) and diquark masses \( m_{ud} = 420 \text{ MeV}, \ m_{us} = m_{ds} = 600 \text{ MeV} \) from Ref. \( [21] \) to compute the magnetic moments of these \( J^P = \frac{1}{2}^- \) and \( J^P = \frac{3}{2}^+ \) pentaquarks. The numerical results are summarized in Table II.

| \( Y, l, I_3 \) | Q=c | Q=b |
|-----------------|-----|-----|
| \( (\frac{1}{2}, \frac{1}{2}) \) | 0.62 | 0.38 |
| \( (\frac{1}{2}, \frac{1}{2}) \) | 0.56 | 0.29 |
| \( (\frac{1}{2}, \frac{1}{2}) \) | 0.13 | -0.36 |
| \( (\frac{1}{2}, 1) \) | -0.19 | -0.20 |
| \( (\frac{1}{2}, 1) \) | -0.57 | -1.41 |
| \( (\frac{1}{2}, -1) \) | -0.72 | -0.98 |

Table II: Numerical results of the \( J^P = \frac{1}{2}^- \) and \( J^P = \frac{3}{2}^+ \) heavy pentaquark magnetic moments in Jaffe and Wilczek’s model with \( m_{ud} = 420 \text{ MeV}, \ m_{us} = m_{ds} = 600 \text{ MeV} \) from Ref. \( [21] \).

Jaffe and Wilczek estimated \( \Theta_0^0 \) and \( \Theta_0^+ \) masses by replacing the \( s \) in the \( \Theta^+ \) with \( c \) and \( b \) respectively. The cost of this replacement is roughly the mass difference between \( \Lambda_c \) and \( \Lambda \) baryons. The \( [ud] \) diquark in the \( \Lambda_c \) and \( \Lambda \) experiences nearly the same environment as in \( \Theta_0^0 \) and \( \Theta^+ \) pentaquarks, especially when the \( [ud] \) diquark is viewed as a tightly bound entity. Thus they got

\[
M(\Theta_0^0) = M(\Theta^+) + [M(\Lambda_c) - M(\Lambda)]
\]

\[
M(\Theta_0^+) = M(\Theta^+) + [M(\Lambda_c^+) - M(\Lambda)].
\]

Here we extend their formalism to estimate the masses of all the other members of \( \bar{6}_c \). The heavy pentaquarks in the same isospin multiplet are degenerate.

\[
M(\Sigma_{0c}^0) = M(\Sigma_{0c}^+ = \Theta_0^0 + [M(\Lambda_c) - M(\Lambda)]
\]

\[
M(\Sigma_{0c}^+) = M(\Sigma_{0c}^+ + [M(\Lambda_c^+) - M(\Lambda)].
\]

Here \( N_{0c}^+ \) has a quark content \( [u d] [u s] \bar{s} \). Its mass was estimated to be around 1700 MeV \( [20] \). For the heavy pentaquark containing two strange quarks, we use

\[
M(\Xi_{0c}^0) = M(\Xi_{0c}^-) + [M(\Xi_c) - M(\Sigma)]
\]

\[
M(\Xi_{0c}^+) = M(\Xi_{0c}^-) + [M(\Xi_b) - M(\Sigma)].
\]

We use 1860 MeV for the \( \Xi_{0c}^- \) mass from NA49 experiment \( [7] \). Another simple way to estimate \( \Xi_{0c}^- \) (\( \Xi_{0b}^- \)) mass is

\[
M(\Xi_{0c}^0) = M(\Theta^+) + 2[m_{us} - m_{ud}] + m_c - m_s
\]

\[
M(\Xi_{0c}^+) = M(\Theta^+) + 2[m_{us} - m_{ud}] + m_b - m_s.
\]

With \( m_{ud} = 420 \text{ MeV}, \ m_{us} = 600 \text{ MeV} \) \( [20] \), \( m_c = 1710 \text{ MeV} \) and \( m_b = 5050 \text{ MeV} \) \( [22] \), the \( \Xi_{0c}^- \) and \( \Xi_{0b}^- \) mass from the second estimate is 3110 MeV and 5460 MeV respectively, which is only 25 MeV below those from the


| $Q = \bar{e}$ | Masses | $Q = \bar{b}$ | Mass |
|---------------|--------|---------------|------|
| $\Theta^0_{c}$ | 2710MeV | $\Theta^+_c$ | 6050MeV |
| $\Sigma^{0,S}_{5c}, \Sigma^+_{5c}$ | 2870MeV | $\Sigma^{0,S}_{5b}, \Sigma^+_{5b}$ | 6210MeV |
| $\Xi^{0,S}_{5c}, \Xi^+_{5c}, \Xi^-_{5c}, \Xi^{±}_{5c}$ | 5135MeV | $\Xi^{0,S}_{5b}, \Xi^+_{5b}, \Xi^-_{5b}, \Xi^{±}_{5b}$ | 6475MeV |

**TABLE III:** Masses of all members of $\bar{6}_{T}$ in Jaffe and Wilczek’s model.

First estimate. Given that the errors of both estimates are around 100 MeV, these two approaches yield quite consistent results, which are collected in Table III

The mass splitting between $J^P = \frac{3}{2}^+$ light pentaquarks and their $J^P = \frac{3}{2}^+$ partners are estimated to around tens of MeVs according to Dudek and Close [23]. The mass splitting between $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ multiplets scales as the product of the inverse constituent mass. Hence for heavy pentaquarks, the mass splitting between $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ multiplets will be even smaller, around 10 MeV.

**III. HEAVY PENTAQUARK STATES IN KARLINER AND LIPKIN’S MODEL**

In Karliner and Lipkin’s (KL) model [22], the pentaquark is divided into two color non-singlet clusters: a scalar diquark and a triquark. There is a $P$-wave excitation between the two clusters. The angular momentum barrier between the two clusters prevents them from rearranging into the usual baryon and meson system.

The two quarks in the triquark are in the symmetric $6_T$ representation. They couple with the anti-quark to form an $SU(3)_c$ triplet $3_T$. The two quarks are in the anti-symmetric flavor $\bar{3}_T$ representation. The anti-charm or anti-bottom is a $SU(3)_f$ flavor singlet. Hence the triquark belongs to a flavor anti-triplet. The spin wave function of the two quarks inside the triquark is symmetric. The spin of the triquark is one half.

The direct product of the $3_T$ of diquark and the $3_T$ of triquark leads to $6_T$ and $\bar{3}_T$ pentaquarks. There is one orbital angular momentum $L = 1$ between the diquark and the triquark. The resulting $J^P$ of the pentaquark can be either $\frac{3}{2}^+$ or $\frac{1}{2}^+$. We list the flavor wave functions of these two multiplets in Table [IV] and [V].

The intrinsic magnetic moment of the triquark is defined as

$$g_{tri} \mu_{tri} \vec{I} \frac{1}{2} = 2\mu_1 \vec{I} \frac{1}{2} + 2\mu_2 \vec{I} \frac{1}{2} + 2\mu_3 \vec{I} \frac{1}{2}.$$  

For the orbital part we have

$$\mu_I = \frac{m_{tri} \mu_{di} + m_{di} \mu_{tri}}{m_{tri} + m_{di}},$$

where $m_{di}$ is the mass of the diquark, $m_{tri}$ is the mass of the triquark.

For $J^P = \frac{3}{2}^+$, the magnetic moment of the pentaquark is

$$\mu = \left(\frac{11}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} \right) - \left(\frac{11}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} \right) | \frac{1}{2} g_{tri} \mu_{tri}$$

$$= \frac{2}{3} \mu_I - \frac{1}{3} \times \frac{1}{2} g_{tri} \mu_{tri}. \quad (11)$$

For $J^P = \frac{1}{2}^+$, we have

$$\mu = \left(\frac{11}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} \right) \mu_I + \left(\frac{11}{2} | \frac{1}{2} | \frac{1}{2} | \frac{1}{2} \right) \frac{1}{2} g_{tri} \mu_{tri}$$

$$= \mu_I + \frac{1}{2} g_{tri} \mu_{tri}. \quad (12)$$

The results are listed in Table [VI]. We use $m_u = m_d = 360$ MeV, $m_s = 500$ MeV while $m_c = 1710$ MeV, $m_b = 5050$ MeV as in Ref. [22]. The mass of each diquark (triquark) is the sum of its constituent mass.

Karliner and Lipkin have estimated the masses of $\Theta^+$ and its heavy flavor analogs [22]. The mass of a pentaquark comes mainly from the masses of its constituent

**TABLE IV:** Flavor wave functions of the anti-sixplet pentaquarks in Karliner and Lipkin’s model [22]. $Y$, $I$ and $I_3$ are hypercharge, isospin and the third component of isospin respectively. $\{q_1,q_2,\bar{Q}\} \equiv [q_1,q_2]\bar{Q}$ is the triquark’s flavor wave function.

| pentaquarks | $(Y,I,I_3)$ | Flavor wave functions |
|-------------|-------------|---------------------|
| $\Theta^0_{c}, \Theta^+_{c}$ | $(\frac{1}{3},0,0)$ | $[ud][ud\bar{Q}]$ |
| $\Sigma^{0,S}_{5c}, \Sigma^+_{5c}$ | $(\frac{1}{3},\frac{1}{3},0)$ | $\sqrt{\frac{2}{5}}(u[d][us\bar{Q}]+[us][ud\bar{Q}])$ |
| $\Sigma^{0,S}_{5b}, \Sigma^+_{5b}$ | $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ | $\sqrt{\frac{2}{5}}(u[d][ds\bar{Q}]+[ds][ud\bar{Q}])$ |
| $\Xi^{0,S}_{5c}, \Xi^+_{5c}, \Xi^-_{5c}, \Xi^{±}_{5c}$ | $(-\frac{1}{3},1,0)$ | $[us][us\bar{Q}]$ |
| $\Xi^{0,S}_{5b}, \Xi^+_{5b}, \Xi^-_{5b}, \Xi^{±}_{5b}$ | $(-\frac{1}{3},1,-1)$ | $[ds][ds\bar{Q}]$ |

**TABLE V:** Flavor wave functions of the triplet heavy pentaquarks in Karliner and Lipkin’s model [22]. Notations are the same as in Table [IV].
TABLE VI: Numerical results of heavy pentaquark magnetic moments in unit of $\mu_N$ in Karliner and Lipkin’s model.

| pentaquarks | $Q=c$ | $Q=b$ |
|-------------|-------|-------|
| $\Theta^0_b \Theta_+^b$ | $J^P=\frac{1}{2}^+$ | $J^P=\frac{1}{2}^+$ | $J^P=\frac{1}{2}^+$ | $J^P=\frac{1}{2}^+$ |
| $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$ | -0.031 | 1.01 | 0.079 | 0.96 |
| $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$ | -0.079 | 1.06 | 0.030 | 1.00 |
| $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$, $\Sigma_c^0$ | -0.084 | -0.26 | -0.0028 | -0.35 |
| $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$ | -0.13 | 1.11 | -0.020 | 1.05 |
| $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$ | -0.14 | -0.22 | -0.054 | -0.30 |
| $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$, $\Xi_c^0$ | -0.15 | -1.54 | -0.089 | -1.66 |

TABLE VII: Masses of all members of $\bar{6}_f$ and $3_f$ in Karliner and Lipkin’s model.

| $\bar{Q} = \bar{c}$ | Mass | $\bar{Q} = \bar{b}$ | Mass |
|---------------------|------|---------------------|------|
| $\Theta^0_b$ | 2990 MeV | $\Theta^+_b$ | 6400 MeV |
| $\Sigma_{10}^0$, $\Sigma_{10}^0$, $\Sigma_{10}^0$, $\Sigma_{10}^0$, $\Sigma_{10}^0$ | 3165 MeV | $\Sigma_{10}^0$, $\Sigma_{10}^0$, $\Sigma_{10}^0$, $\Sigma_{10}^0$, $\Sigma_{10}^0$ | 6570 MeV |
| $\Xi_{10}^0$, $\Xi_{10}^0$, $\Xi_{10}^0$, $\Xi_{10}^0$, $\Xi_{10}^0$ | 3340 MeV | $\Xi_{10}^0$, $\Xi_{10}^0$, $\Xi_{10}^0$, $\Xi_{10}^0$, $\Xi_{10}^0$ | 6740 MeV |

IV. DISCUSSIONS

In this paper we have studied the masses and magnetic moments of heavy pentaquarks in the framework of two clustered quark models. There is only a $SU(3)$ flavor anti-sixtet in Jaffe and Wilczek’s model. In contrast, an additional flavor triplet exists in Karliner and Lipkin’s diquark and triquark model. The masses and magnetic moments of these triplet pentaquarks are equal to their anti-sixtet partners in KL’s model. For the light pentaquarks, all the above two clustered quark models predicted the existence of an anti-decuplet and an accompanying octet.

Another dramatic difference lies in the prediction of the magnetic moments of the $J^P=\frac{1}{2}^+$ heavy sextet pentaquarks. The magnetic moments of the $J^P=\frac{1}{2}^+$ heavy sextet is tiny and much smaller in KL’s model than those in JW’s model. There is strong cancellation between the magnetic moment of the triquark and the orbital magnetic moment in a $J^P=\frac{1}{2}^+$ sextet when they are combined by Clebsch-Gordan coefficients [c.f. Eq. (11)]. This cancellation is almost exact for bottom pentaquarks. For the magnetic moments of the $J^P=\frac{1}{2}^+$ sextet, there is also significant difference between KL’s model and JW’s models.

The third difference is that the masses of the pentaquarks in KL’s model are about 300 MeV larger than those in JW’s model. The reason is that Karliner and Lipkin assumed that the diquark (triquark) mass is simply the sum of its constituents. In contrast, the diquark mass is made much lower than the sum of the two quarks through the strong correlation between light quarks when they are in anti-symmetric configuration in JW’s model.

In JW’s models, the masses of heavy pentaquarks are about 100 MeV below the threshold of the strong decay modes if one believes our rough estimate. Only iso-spin violating strong decays, electromagnetic decays and weak decays can occur in this case. These heavy sextet members should be stable. In the case that their masses are under-estimated by around 100 MeV in our calculation, they are still barely above threshold. Severe phase space suppression will make them very narrow even if they have strong decays.

On the other hand, the pentaquarks in KL’s model lie above threshold and have strong decay modes. For example, the strong decay $\Xi_{10}^0 \rightarrow B^+ \Sigma^+$ and $\Xi_{10}^0 \rightarrow B^+ \Xi^0$ may occur in KL’s model. However, they may still be narrow states as the $\Theta^+$ pentaquark even if strong decay modes exist.

The presence of additional triplet states, different magnetic moments, different masses and decay modes will play a role in distinguishing these two clustered quark models. Future experiments at CLEOc, BESIII, BELLE and LEP will be able to find these very interesting states if they really exist. Hopefully the present study will somehow contribute to the experimental search.
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