Two-Zero Symmetric Neutrino Mass Matrices in Minimal Supersymmetric SO(10)

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Abstract

The phenomenological neutrino mass matrix for two-zero symmetric texture has been obtained and used to rule out all possible two-zero symmetric neutrino mass matrices obtained from Yukawa couplings to $\mathbf{10}$ and $\mathbf{126}$ Higgs representations within the framework of minimal supersymmetric SO(10).

1 Introduction

The origin of fermion masses and mixings alongwith the related problem of CP violation constitute a formidable challenge for elementary particle physics. Leaving apart extremely small neutrino masses, even the charged fermion mass hierarchy ranges over at least five orders of magnitudes. Since the fermion masses and the mixing angles are derived from the Yukawa couplings, which are free parameters within the Standard Model (SM), these Yukawa couplings must span several orders of magnitude to accommodate the strongly hierarchical pattern of fermion masses and mixings. However, the currently available data on fermion masses and mixing are insufficient for an unambiguous reconstruction of fermion mass matrices. To make matters worse, radiative corrections can obscure the underlying structure. Thus, the existing data cannot, without some additional assumptions, determine all the elements of the Yukawa coupling matrices for quarks and leptons. Some of these assumptions, invoked to restrict the form of fermion mass matrices include the presence of texture zeros [1], requirement of zero determinant [2] and zero trace condition [3] to name just a few. The main motivation for invoking different mass matrix ansatze is to relate fermion masses and mixing angles in a testable manner which reduces the number of free parameters in the Yukawa sector of SM. The recent evidence for non-zero neutrino masses and mixings leads to a further proliferation of free parameters in the Yukawa sector. In the absence of a significant breakthrough in the theoretical understanding of fermion flavors, the phenomenological approaches are bound to play a crucial role in interpreting new experimental data on quark and lepton mixing. These approaches are expected to provide useful

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hints toward unraveling the dynamics of fermion mass generation, CP violation and identification of possible underlying symmetries of the fermion flavors from which viable models of fermion mass generation and flavor mixing could, hopefully, be constructed.

The strong fermion mass hierarchy should be apparent in the fermion mass matrices themselves with the contribution of smaller elements to physical masses and mixing angles expected to be negligibly small. Thus, these elements can, effectively, be neglected and replaced by zeros: the so-called texture zeros. However, the current neutrino oscillation data are consistent only with a limited number of texture schemes [1]. Specifically, the available neutrino oscillation data disallow all neutrino mass matrices with three or more texture zeros [1] in the flavor basis. The texture zeros in different positions in the neutrino mass matrix, in particular, and fermion mass matrices, in general, could be the consequence of some underlying symmetry. Such universal textures of fermion mass matrices can be realized within the framework of Grand Unified Theories (GUTs). Though Grand Unification on its own does not shed any light on the flavor problem, the GUTs provide the optimal framework in which possible solutions to the flavor problem could be embedded. As mentioned earlier, texture zeros in different positions of the fermion mass matrices could result because of an underlying symmetry. Grand Unified models attempt to explain the masses and mixings in both the quark and lepton sectors simultaneously. The textures for the mass matrices obtained in these models can either be assumed at the very outset or can be derived from the observed mixing matrix in the flavor basis. Alternatively, the textures of the mass matrix can be obtained by embedding some family symmetry within the chosen Grand Unification group. One particularly interesting class of models is that based upon the SO(10) grand unification group. There are two kinds of minimal models in this class: those based upon Higgs representations with dimension $10, 126, 16, 45$ and possibly also $120$ and/or $210$ and those based upon $10, 16, 16, 45$ representations. The former choice, generally, has symmetric and/or antisymmetric texture mass matrices while the latter type generally imply lopsided mass matrices for the down type quarks and charged leptons. In the present work, we restrict ourselves to the former choice i.e. we adopt the so called symmetric two-zero texture for the up-type quark mass matrices within the SUSY SO(10) GUT framework. Within this framework, we not only have a relation between down type quark mass matrices ($M_d$) and charged lepton mass matrices ($M_l$) but also a relation between the up-type quark mass matrices ($M_u$) and Dirac neutrino mass matrices ($M_{\nu_D}$). Thus, once we fix the representation of the Higgs field corresponding to each element, $M_l$ and $M_{\nu_D}$ are uniquely determined from $M_d$ and $M_u$ respectively.

Neutrino mass matrices with two texture zeros in the charged lepton basis have only one degree of freedom [4, 5]. The presence of two texture zeros in the neutrino mass matrix imply four conditions on the nine free parameters of the model. The four parameters out of the remaining five parameters are determined by the neutrino data for the values of two squared-mass differences $\Delta m^2_{12}$ and $\Delta m^2_{23}$ and two mixing angles $\theta_{12}$ and $\theta_{23}$. So, we are left with only one free parameter in the neutrino mass matrix [4, 5]. However, we have one more experimental measurement by the CHOOZ experiment which establishes an upper bound on the third mixing angle $\theta_{13}$. A lower bound on $\theta_{13}$ is inherent in the nature of two texture zero neutrino mass matrices [4, 5]. So, even the remaining one degree of freedom is constrained. In fact, the neutrino mass matrix with two texture zeros can be completely determined [7] with the help of the presently available neutrino data for the seven allowed texture zero schemes of Frampton, Glashow and Marfatia [1]. Therefore, the present neutrino data, which
is sufficient to determine the neutrino mass matrix with two texture zeros, will be able to overrule various neutrino mass models with two texture zeros if the model parameters are known and to constrain the model parameters if they are unknown.

In the class of GUT models under consideration, the mass matrices $M_{\nu D}$ for the Dirac neutrinos which gives rise to neutrino masses via see-saw mechanism can be taken identical to $M_u$ except for the accompanying CG coefficients which depend upon the representations of the coupling Higgs field [8, 9]. Therefore, these mass models predict the neutrino mass matrix at GUT scale. This theoretical mass matrix should be consistent with the phenomenological mass matrix at weak scale calculated for the texture scheme consistent with the model. However, the neutrino mass matrix calculated at GUT scale has to be run down to the weak scale before comparing it with the neutrino mass matrix calculated phenomenologically from its texture scheme from the neutrino data at the weak scale. The effect of renormalization group (RG) running will be small for the neutrino mass matrices with normal hierarchy. However, it can be significantly large for the mass matrices with inverted or quasi-degenerate hierarchy.

In the present work, we first derive the phenomenological neutrino mass matrix for a particular texture scheme proposed by Frampton, Glashow and Marfatia [1] and confront it with a class of two-zero symmetric texture GUT models based upon minimal supersymmetric SO(10) [3, 9].

2 Phenomenological neutrino mass matrix with a two zero symmetric texture

We take the neutrino mass matrix to be the real symmetric matrix with two texture zeros:

$$M_\nu = \begin{pmatrix} 0 & A & 0 \\ A & D & B \\ 0 & B & C \end{pmatrix}$$

(1)

where

$$C = m_1 - m_2 + m_3 - D, \quad A^2 = m_1m_2m_3/C,$$

$$B^2 = (m_3 + m_1 - D)(m_3 - m_2 - D)(m_2 - m_1 + D)/C$$

(2)

since we are interested in obtaining only the relative magnitudes of the neutrino mass matrix elements without phases or signs. This is the texture $\mathcal{A}_2$ of Frampton, Glashow and Marfatia [1]. The eigenvalues of $M_\nu$ are $m_1$, $-m_2$ and $m_3$. The real orthogonal matrix $O$ which diagonalizes the neutrino mass matrix $M$ according to the relation

$$O^T M O = diag(m_1, -m_2, m_3)$$

(3)
Another important relation is

\[
O_{11} = \sqrt{\frac{m_2 m_3 (m_3 - m_2 - D)}{(m_1 - m_2 + m_3 - D)(m_1 + m_2)(m_3 - m_1)}}
\]

\[
O_{12} = \sqrt{\frac{m_3 m_1 (m_3 + m_1 - D)}{(m_1 - m_2 + m_3 - D)(m_1 + m_2)(m_3 + m_2)}}
\]

\[
O_{13} = \sqrt{\frac{m_1 m_2 (m_2 - m_3 + D)}{(m_1 - m_3 + m_3 - D)(m_2 + m_3)(m_3 - m_1)}}
\]

\[
O_{21} = \sqrt{\frac{m_1 (m_3 - m_2 - D)}{(m_1 + m_2)(m_3 - m_1)}}
\]

\[
O_{22} = \sqrt{\frac{m_2 (m_3 + m_1 - D)}{(m_2 + m_3)(m_1 + m_2)}}
\]

\[
O_{23} = \sqrt{\frac{m_3 (m_3 - m_1 + D)}{(m_3 - m_1)(m_2 + m_3)}}
\]

\[
O_{31} = \sqrt{\frac{m_1 (m_2 - m_1 + D)(m_3 + m_1 - D)}{(m_1 - m_2 + m_3 - D)(m_2 + m_1)(m_3 - m_1)}}
\]

\[
O_{32} = \sqrt{\frac{m_2 (m_2 - m_1 + D)(m_3 - m_2 - D)}{(m_1 - m_2 + m_3 - D)(m_3 + m_2)(m_1 + m_2)}}
\]

\[
O_{33} = \sqrt{\frac{m_3 (m_3 - m_2 - D)(m_3 + m_1 - D)}{(m_1 - m_2 + m_3 - D)(m_2 + m_3)(m_3 - m_1)}}
\]

where we have given only the magnitudes of the elements of \(O\). This diagonalization scheme is taken from [10] where the references to earlier works can, also, be found.

Many interesting relations can be extracted from the above expressions for the elements of the neutrino mass matrix and its eigenvalues. The two mass ratios \(\frac{m_1}{m_2}\) and \(\frac{m_1}{m_3}\) can be written as

\[
\frac{m_1}{m_2} = \frac{O_{12} O_{21}}{O_{11} O_{22}} \tag{5}
\]

and

\[
\frac{m_1}{m_3} = \frac{O_{13} O_{21}}{O_{11} O_{23}} \tag{6}
\]

The elements \(A\), \(B\) and \(C\) are given by

\[
A = \sqrt{m_1 m_2 \frac{O_{11} O_{12}}{O_{21} O_{22}}},
\]

\[
B = \frac{(m_1 + m_2)(m_2 + m_3)(m_3 - m_1)}{m_1 m_3} O_{12} O_{21} O_{23} \tag{8}
\]

and

\[
C = m_3 \frac{O_{21} O_{22}}{O_{11} O_{12}} \tag{9}
\]

Therefore, the ratios \(\frac{A}{C}\) and \(\frac{B}{C}\) become

\[
\frac{A}{C} = \frac{O_{11} O_{12} O_{13}}{O_{21} O_{22} O_{23}} \tag{10}
\]

and

\[
\frac{B}{C} = O_{11} O_{12} O_{13} \frac{(m_1 + m_2)(m_2 + m_3)(m_3 - m_1)}{m_1 m_2 m_3} \tag{11}
\]

Another important relation is

\[
O_{13} = \sqrt{\frac{m_1 m_2}{m_3}} O_{23} \sqrt{\frac{O_{11} O_{12}}{O_{21} O_{22}}} \tag{12}
\]
Eqs. (5)-(12) are exact and have been derived from Eqs. (4). These relations imply very interesting consequences when written approximately as Taylor series in the powers of $s_{13}$ coming from the orthogonal mixing matrix $O$ which can be written as

\[
O = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix}
\]  

(13)

where the symbols have their usual meaning. We substitute these elements in equations (5)-(12) and expand the results in the powers of $s_{13}$. The mass ratios in Eqs. (5) and (6) are given by

\[
m_1/m_2 = s_{12}^2 c_{12} + O(s_{13})
\]  

(14)

and

\[
m_1/m_3 = s_{12}^2 c_{23}s_{13} c_{12} + O(s_{13}^2)
\]  

(15)

which are consistent with our earlier results [4, 5]. The mass ratio $m_1/m_2$ is much smaller than one. Therefore, the neutrino mass matrix considered here has a normal hierarchy. This fact is important since the neutrino oscillation parameters change very little in the RG evolution from the weak scale to GUT scale for the normal hierarchy. The matrix elements $A$, $B$ and $C$ are given by

\[
A = \frac{\sqrt{m_1 m_2}}{c_{23}} + O(s_{13}),
\]  

(16)

\[
B = m_3 c_{23}s_{23} + O(s_{13})
\]  

(17)

and

\[
C = m_3 c_{23}^2 + O(s_{13}).
\]  

(18)

Moreover, the element $D$ can be expressed as

\[
D = m_3 s_{23}^2 + O(s_{13}).
\]  

(19)

The ratios $A/C$ and $B/C$ can be written as

\[
A/C = \frac{s_{13}}{c_{23}s_{23}} + O(s_{13}^2)
\]  

(20)

and

\[
B/C = 1 + O(s_{13}).
\]  

(21)

Eq. (12) for $O_{13}$ gives

\[
s_{13} = \frac{\sqrt{m_1 m_2}}{m_3} + O(s_{13}^2).
\]  

(22)

This equation is consistent with the analytical results obtained by Desai et al. [7]. Eq. (14) can be used to obtain the relation

\[
m_1 = s_{12}^2 \sqrt{\frac{\Delta m_{12}^2}{\cos 2\theta_{12}}}
\]  

(23)
as shown in our earlier work \[11\]. Using Eq. (23) and neglecting the higher order terms in \( s_{13} \), Eq. (22) can be written as

\[
  s_{13} \approx \left( \frac{\Delta m_{12}^2}{\Delta m_{13}^2} \right)^{\frac{1}{2}} \frac{s_{12}c_{12}}{\sqrt{c_{12}^2 - s_{12}^2}}. \tag{24}
\]

For the tri-bi-maximal value \[12\] \( s_{12}^2 = \frac{1}{3} \), the trigonometric factor in Eq. (24) is given by

\[
  \frac{s_{12}c_{12}}{\sqrt{c_{12}^2 - s_{12}^2}} = \sqrt{\frac{2}{3}}. \tag{25}
\]

Substituting the present best fit values of the oscillation parameters in Eq. (24) \( (\Delta m_{12}^2 = 7.9 \times 10^{-5}, \Delta m_{13}^2 = 2.4 \times 10^{-3}, \text{ and } s_{12} = 0.3 \ [13]) \), we find that \( \theta_{13} = 7.3^\circ \) which is consistent with the earlier estimates \[4, 5\] for this mass matrix texture.

Using the above leading order estimates of the elements \( A, B, C \) and \( D \), the neutrino mass matrix given in Eq. (1) can be written as

\[
  M = \frac{m_3}{2} \begin{pmatrix}
    0 & \epsilon & 0 \\
    \epsilon & 1 + \mathcal{O}(\epsilon) & 1 + \mathcal{O}(\epsilon) \\
    0 & 1 + \mathcal{O}(\epsilon) & 1 + \mathcal{O}(\epsilon)
  \end{pmatrix} \tag{26}
\]

where the parameter \( \epsilon \) is given by

\[
  \epsilon = 2\sqrt{2}s_{13} \tag{27}
\]

for maximal atmospheric mixing. For the present best fit values of the neutrino oscillation parameters, \( \epsilon \sim 0.36 \). So, the hierarchical structure of neutrino mass matrix should be as follows:

\[
  M \sim \frac{1}{2}\sqrt{\Delta m_{13}^2} \begin{pmatrix}
    0 & 0.36 & 0 \\
    0.36 & 1 & 1 \\
    0 & 1 & 1
  \end{pmatrix} \tag{28}
\]

3 A minimal supersymmetric SO(10) realization of the two-zero symmetric texture

The phenomenological neutrino mass matrix obtained here has to be confronted with the neutrino mass matrices given by various neutrino mass models. As an example, we confront a class of minimal supersymmetric SO(10) GUT models with two-zero symmetric texture \[8, 9\] with the phenomenological mass matrix obtained above. In this class of models, each element of \( M_u \) is dominated by contribution either from 10 or 126 Higgs representation and the Yukawa couplings of Dirac neutrinos \( \nu_D \) are that of corresponding up quarks multiplied by a factor of 1 or -3 respectively. Therefore, the Dirac neutrino mass matrix for this class of models can be written as

\[
  M_{\nu_D} = \begin{pmatrix}
    0 & a & 0 \\
    a & b & c \\
    0 & c & d
  \end{pmatrix} m_t \tag{29}
\]
which is related to the up-quark mass matrix $M_u$

$$M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u & c_u \\ 0 & c_u & 1 \end{pmatrix} m_t$$

(30)

through the relations

$$a = x_{12} a_u, \ b = x_{22} b_u, \ c = x_{23} c_u, \ d = x_{33}$$

(31)

where

$$a_u = \sqrt{m_u m_c} / m_t, \ b_u = m_c / m_t, \ c_u = \sqrt{m_u / m_t}$$

(32)

and $m_u, m_c$ and $m_t$ are the masses of the up quarks $u, c$ and $t$ at the GUT scale. This mass matrix is consistent with the quark masses and mixing data. The coefficients $x_{12}, x_{22}, x_{23}$ and $x_{33}$ are the CG factors which connect the respective elements of $M_u$ and $M_{\nu_D}$ and are equal to 1 or -3 depending upon whether the corresponding Higgs representation is 10 or 126. The Higgs representation for $M_u$ which has the best agreement with the data is given by

$$M_u : \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}.$$  (33)

For this $M_u$, the CG coefficients are given by

$$x_{12} = -3, \ x_{22} = 1, \ x_{23} = 1, \ x_{33} = -3.$$  (34)

The link with the neutrino mass matrix is established through the see-saw mechanism using the right handed neutrino mass matrix $M_R$:

$$M_R = \begin{pmatrix} 0 & rm_R & 0 \\ rm_R & 0 & 0 \\ 0 & 0 & m_R \end{pmatrix}.$$  (35)

This particular form of $M_R$ is consistent with the 126 dimensional representation for Higgs field which couples with up quarks. It gives two degenerate right handed neutrino states of mass $rm_R$ and a third neutrino state of mass $m_R$. Thus, $r$ is the ratio of two right handed neutrino mass scales. The neutrino mass matrix $M_\nu$ can be obtained from the right-handed Majorana mass matrix ($M_R$) and the Dirac mass matrix ($M_{\nu_D}$) via the see-saw relation

$$M_\nu = M_{\nu_D}^T M_R^{-1} M_{\nu_D}$$  (36)

and is given by

$$M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & \frac{2ab + c}{r} & \frac{c(a + 1)}{d^2} \\ \frac{a^2}{r} & \frac{2ab + c^2}{r} & \frac{c(a + 1)}{d^2} & \frac{m^2}{m_R} \\ \frac{2ab + c}{r} & \frac{c(a + 1)}{d^2} & \frac{m^2}{m_R} \end{pmatrix}.$$  (37)

The above neutrino mass matrix, to a very good approximation, can be written as

$$M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & \frac{2ab}{r} & \frac{ac}{d^2} \\ \frac{a^2}{r} & \frac{2ab}{r} & \frac{ac}{d^2} & \frac{m^2}{m_R} \end{pmatrix}.$$  (38)
It will be convenient to parameterize $M_\nu$ in the following way \[8, 9\]
\[
M_\nu = \begin{pmatrix}
0 & \beta & 0 \\
\beta & \alpha & h \\
0 & h & 1
\end{pmatrix} \frac{d^2 m_i^2}{m_R} 
\] (39)
where
\[
\alpha = \frac{2ab}{rd^2} = \frac{x_{12}x_{22}}{x_{33}^2} \left( \frac{2a_u b_u}{r} \right),
\] (40)
\[
\beta = \frac{a^2}{rd^2} = \frac{x_{12}^2}{x_{33}^2} \left( \frac{a_u^2}{r} \right),
\] (41)
and
\[
h = \frac{ac}{rd^2} = \frac{x_{12}x_{23}}{x_{33}^2} \left( \frac{a_u c_u}{r} \right). 
\] (42)

Since, $\alpha$, $\beta$ and $h$ are all proportional to $\frac{1}{r}$, we can eliminate $r$ and work in term of the ratios $\frac{\beta}{\alpha}$ and $\frac{h}{\alpha}$ which are given by
\[
\frac{\beta}{\alpha} = \frac{x_{12}}{x_{23}} \left( \frac{1}{2} \frac{m_u}{m_c} \right) 
\] (43)
and
\[
\frac{\alpha}{h} = \frac{x_{22}}{x_{23}} \left( 2 \frac{m_c}{\sqrt{m_u m_t}} \right). 
\] (44)

The ratio $\frac{h}{\alpha}$ is found to be of the order of unity and is in agreement with the value obtained from the phenomenological neutrino mass matrix [Eq. (26) and Eq. (28)]. However, this ratio depends on the top quark mass $m_t$ which has rather large uncertainty. Moreover, the quark masses have to be run to the GUT scale and the RG running effects are significantly large for this ratio. Therefore, this ratio cannot serve as a good criterion to test this model. On the other hand, the ratio $\frac{\beta}{\alpha}$ depends on the relatively better known quark masses $m_u$ and $m_c$ and the RG running effects in calculating this ratio at the GUT scale are quite small. In fact, this ratio is quite stable against the various input SUSY parameters (like $\tan^2 \beta$ and threshold corrections) which affect the RG evolution \[14\]. Substituting the value of the quark mass ratio $\frac{m_u}{m_c}$ at GUT scale \[14\]:
\[
\frac{m_u}{m_c} = 0.0027 \pm 0.0006,
\] (45)
we find that the model prediction for the ratio $\frac{\beta}{\alpha}$ is about $0.078 \pm 0.009$.

The ratio $\frac{\beta}{\alpha}$ can, also, be obtained from the phenomenological neutrino mass matrix since
\[
\frac{\beta}{\alpha} = \frac{A}{D} = \frac{A}{C} + \mathcal{O}(s_{13}) \sim \epsilon 
\] (46)
and hence $\frac{\beta}{\alpha} \sim 0.4$. This value has to be evolved to the GUT scale by the RG running. However, it can be seen that this ratio is quite stable against RG evolution which can lower this ratio by a factor of about $\frac{\epsilon}{10}$ at the most when going from the weak scale to the GUT scale. This can be seen from the relations \[15\]
\[
\alpha \to \frac{\alpha}{(1 - \epsilon)\beta}, \quad \beta \to \frac{\beta}{(1 - \epsilon)(1 - \epsilon)}, \quad h \to \frac{h}{(1 - \epsilon)} 
\] (47)
where the sign ‘→’ denotes the RG evolution from the weak scale to the GUT scale and $\epsilon_e$ and $\epsilon_\mu$ are the RG parameters which can be at the most 0.1. Hence, the smallest possible value of the ratio $\frac{\beta}{\alpha}$ at the GUT scale is approximately 0.3 which is still larger than the model value for this ratio by a factor of about 4. This disagreement between the phenomenological values of the ratio $\frac{\beta}{\alpha}$ calculated from the neutrino data and the values predicted by the GUT model cannot be reconciled even if the errors of the neutrino oscillation data are taken into account. We have done the exact numerical calculation and found that $\frac{\beta}{\alpha} = 0.36 \pm 0.05$. This value differs from the model value by more than 5 $\sigma$ C.L.. Therefore, the two-zero symmetric texture of the neutrino mass matrix based upon the minimal supersymmetric SO(10) GUT is ruled out by more than 5 $\sigma$ C.L..

The neutrino mass matrix $M_\nu$ given in Eq. (39) corresponds to the up-quark mass matrix $M_u$ with the Higgs representation given in Eq. (33) which is only one of the possible Higgs representations. $M_u$, a symmetric matrix with two texture zeros, contains four independent elements which can couple with either 10 or 126 dimensional Higgs fields. There are 16 such representations listed in Table 1 where we have adopted the classification scheme of [9] for comparison. Only eight textures out of the total sixteen were found to be allowed in earlier studies [8]. Just like the basic texture based upon the Higgs representation given in Eq. (33) for $M_u$, the other fifteen possible textures with different possible Higgs representation for $M_u$ are also ruled out by the phenomenological values for the ratios $\frac{\beta}{\alpha}$ and $\frac{\alpha}{\alpha}$. The representation chosen for $M_u$ in Eq. (33) is $S_1$ and so we can call the $M_\nu$ obtained in Eq. (39) to be of type $S_1$. Similarly, one can calculate the neutrino mass matrix for all the sixteen Higgs field representations of $M_u$ which will differ only in the values of the CG factors $x_{12}, x_{22}, x_{23}$ and $x_{33}$. These factors for all the sixteen type of neutrino mass matrices have been tabulated in Table 2 where we also calculate the parameters $\alpha, \beta$ and $h$ in the units of $(\frac{2a_u a_c}{r}), (\frac{a_u a_r}{r})$ and $(\frac{a_u}{r})$ respectively and their ratios $\frac{\beta}{\alpha}$ and $\frac{\alpha}{\alpha}$ in the units of $(\frac{\sqrt{m_u}}{m_c})^2$ and $(\frac{2m_c}{m_u m_t})$ respectively. In these units, the ratio $\frac{\alpha}{\alpha}$ should be about 1 and the ratio $\frac{\beta}{\alpha}$ should be about 15 to explain the present neutrino data. It can be seen that the ratio $\frac{\beta}{\alpha}$ is 1 in agreement with its phenomenological value only for the categories $S, A$ and $B$. These categories were favored by the experimental data in some earlier studies [8] [9]. However, the ratio $\frac{\beta}{\alpha}$ is approximately smaller by the factors of 5, 15 and 45 from its phenomenological value for the classes $S, A$ and $B$, respectively, ruling out these categories. The categories $C$ and $F$ are ruled out in a straightforward manner since they do not reproduce either of the two ratios correctly. Hence, the entire class of two-zero symmetric texture of neutrino mass matrices is ruled out.

4 Conclusions

In conclusion, we have obtained the phenomenological neutrino mass matrix for the two-zero symmetric texture and found that it is not compatible with all possible two-zero symmetric texture neutrino mass matrices obtained from the Yukawa couplings with 10 and 126 Higgs representations within the framework of minimal supersymmetric SO(10). Therefore, the theoretical motivation for two-zero symmetric texture of the neutrino mass matrix, no longer, exists.
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Table 1: The sixteen possible Higgs representations for $M_u$ as classified in Ref. [9].

|      | $S_1$                              | $S_2$                              |
|------|------------------------------------|------------------------------------|
|      | $\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$ | $\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$ |

|      | $A_1$                              | $A_2$                              |
|------|------------------------------------|------------------------------------|
|      | $\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix}$ | $\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix}$ |
|      | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$ | $\begin{pmatrix} 10 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$ |

|      | $B_1$                              | $B_2$                              |
|------|------------------------------------|------------------------------------|
|      | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix}$ | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix}$ |

|      | $C_1$                              | $C_2$                              |
|------|------------------------------------|------------------------------------|
|      | $\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix}$ | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix}$ |
|      | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix}$ | $\begin{pmatrix} 126 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix}$ |

|      | $F_1$                              | $F_2$                              |
|------|------------------------------------|------------------------------------|
|      | $\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix}$ | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix}$ |
|      | $\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}$ | $\begin{pmatrix} 126 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}$ |
Table 2: The CG coefficient, the parameters $\alpha$, $\beta$ and $h$ and their ratios for the sixteen categories of neutrino mass matrices. We have given $\alpha$, $\beta$ and $h$ in the units of $\left(\frac{\alpha u^2}{a} \right)$, $\left(\frac{\beta a}{r} \right)$ and $\left(\frac{\alpha u c}{r} \right)$ respectively.