A noisy quantum channel model for the cosmological horizon
in de Sitter spacetime

Sang-Eon Bak\textsuperscript{1}, Paul M. Alsing\textsuperscript{2}, Warner A. Miller\textsuperscript{3} and Doyeol Ahn\textsuperscript{1,3*}

\textsuperscript{1}Department of Electrical and Computer Engineering, University of Seoul, 163 Seoulsiripdae-ro, Tongdaimoon-Gu, Seoul 02504, Korea
\textsuperscript{2}Air Force Research Laboratory, Information Directorate, Rome NY 13441, USA
\textsuperscript{3}Physics Department, Charles E Schmidt College of Science, Florida Atlantic University, Boca Raton, FL 33431-0991, USA

* Corresponding author: dahn@uos.ac.kr

Abstract

Quantum correlation between two observers across the cosmological horizon and retrieval of the information behind the cosmological horizon are investigated in the frame of de Sitter spacetime. It is also shown that the effect of the Universe’s expansion can be described by a noisy quantum channel model having a complete positive map with an operator sum representation. This investigation shows that entanglement is degrading with increasing expansion rate of the Universe. We also show that there is a relation between bipartite mutual information and tripartite mutual information with respect to information theoretic context of the scrambling. This implies that there is a scrambling effect in the quantum task across the cosmological horizon in de Sitter spacetime similar to the case of black hole event horizon.
Introduction

Hawking’s argument [1,2] about the information loss problem in black hole evaporation process generated several discussions [3-6] about the unitarity of black hole evolution. In this context, Susskind suggested the black hole complementarity [7]. The black hole complementarity implies that the black hole event horizon has a scrambling process for infalling matters and then emits its information outside the horizon. This is because scrambling effect prohibits outside observer from decoding the full information of freely falling matters until he cannot afford to meet the inside observer when he starts to fall into the black hole. Another suggestion in the reference [7] is that freely falling observer into a black hole can see nothing strange. If these suggestions are true, an observer outside the black hole can retrieve the information of an observer who is freely falling to the black hole without the violation of no-cloning theorem. Also, it is argued that the observer outside the black hole cannot see the signal from an observer inside the black hole when he knows about enough black hole information [8]. The black hole complementarity for black hole information loss problem has been developed in anti-de Sitter spacetime [9, 10]. It is remarkable to find that the causal structure of the cosmological horizon in de Sitter spacetime is similar to the causal structure of the anti-de Sitter black hole horizon. Thus, we suggest that discussions on complementarity can be applied to the quantum phenomena near the cosmological horizon as well. The application of complementarity for de Sitter horizon has been discussed for many years [11-14]. This approach may be useful when we would like to find the property of the cosmological horizon and beyond horizon. It has not yet been fully understood, however, that cosmological horizon has a scrambling effect and it would be important to inquire about the meaning of the spacetime beyond the cosmological horizon in our universe [11]. The latter question is caused from the problem of measurement beyond the horizon. In dealing with this problem, a quantum entanglement is a useful tool because by using this notion we can define the change of quantum correlation with a curved spacetime. The quantum entanglement and their correlation in de Sitter spacetime have been studied in [15-17], and the phonon detector of quantum fields in de Sitter space was studied in [18]. Recently, the quantum fluctuation has been observed by using this kind of setting [19]. Also, there is a result that quantumness of the correlation is decreased by the effect of the curved spacetime [20]. In this work, we will consider scrambling effect across the cosmological horizon by studying the correlation of entangled state across the horizon. Especially, we study a noisy quantum channel model for probing the information exchange with the observer beyond the cosmological horizon. Our model explains the quantum phenomena across de Sitter horizon, especially scrambling behavior.

Simultaneously, in the context of quantum information, quantum entanglement and quantum
Teleportation has been studied for many years [21, 22]. This is because quantum information theoretic quantities such as entanglement entropy and fidelity can be introduced. Also, we introduce the schematic point of view with quantum channel or circuit when we need to study the quantum behavior across the horizon. In this manner, the entanglement in non-inertial frames was studied in [23, 24, 25] with respect to the von Neuman entropy and mutual information. This proved that there is degradation behavior of entanglement depending on the acceleration effect. In the specific limit, it can be viewed as the phenomenon near the event horizon of the Schwarzschild black hole. Furthermore, it was proved that this quantum information theoretic language can give a correspondence between Unruh effect and noisy quantum channel [26]. Hawking effect in Schwarzschild spacetime can be explored by a noisy quantum channel as well [27].

It is well known that static de Sitter spacetime has the line element similar to that of Schwarzschild spacetime. By using this property, quantization of the massless scalar field in de Sitter spacetime have been done by Gibbons and Hawking [28], Lohiya and Panchapakesan [29]. We argue that this approach will and give us tools to explore the entanglement behavior near de Sitter horizon because the squeezed state appears in the field quantization conducted on de Sitter spacetime. Consequently, the quantum theoretical approach for matter-gravity coupling in de Sitter spacetime becomes meaningful.

The purpose of this paper is to determine the behavior of quantum entanglement across the cosmological horizon in de Sitter space time. For this topic, we analyze the maximally entangled tripartite quantum states across the horizon and study the measure of the entanglement of the states they share. These measures contain fidelity, bipartite mutual information, tripartite mutual information and negativity which appears in quantum information theory. We emphasize the correspondence between the quantum phenomenon across the horizon and the noisy quantum process. Such interpretation gives us a mean to understand the information exchange across the horizon more strictly. Finally, we conclude that there is decoherence and scrambling like quantum computing processes.

**Theoretical Model**

We will consider the massless scalar field in (3+1) dimensional static de Sitter spacetime.

\[
 ds^2 = - \left(1 - \frac{r^2}{a^2}\right) dt^2 + \left(1 - \frac{r^2}{a^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \tag{1}
\]

where \( a \) is the distance between origin of the coordinate and cosmological horizon of the de Sitter spacetime. In this coordinate, the light signal which starts at the position \( r > a \) cannot reach to an
observer at \( r = 0 \). For this reason, the region \( r = a \) is known as cosmological event horizon.

We consider the massless scalar field in de Sitter spacetime. The field quantization in this background is constructed by Gibbons and Hawking [28], and Lohiya and Panchapakesan [29]. They used the Kruskal coordinate in de Sitter spacetime because it is convenient to develop similar quantization with massless scalar field in Schwarzschild spacetime. Following by Lohiya and Panchapakesan [29], we use the tortoise coordinate.

\[
r^* = \frac{a}{2} \ln \left( \frac{a + r}{a - r} \right)
\]

Then we can construct the light cone coordinate and Krusal coordinate by the following coordinization

\[
\bar{u} = t - r^*, \bar{v} = t + r^*
\]

\[
u = -2a e^{\bar{u}}, v = 2ae^{-\bar{v}}
\]

where \( \bar{u}, \bar{v} \) are light cone coordinates, and \( u, v \) are Kruskal coordinates. Then, we can construct a Kruskal diagram in Figure 1 by using the following coordinates.

\[
T = \frac{u + v}{2} = -2ae^{\bar{u}} \sinh \frac{t}{a}, R = \frac{u - v}{2} = 2ae^{-\bar{v}} \cosh \frac{t}{a}
\]

Interestingly, these Kruskal coordinates are conformally Minkowski. Thus, we follow the mode expansion in the Birrell and Davies [30] and Unruh [31] for solving the massless Klein Gordon equation in static de Sitter spacetime. In their approach, the light cone modes for region I and II are not analytic a \( u = v = 0 \). As a result, it makes the light cone modes to have different vacuum states with Kruskal mode. However, we can express the Kruskal mode which is analytic at \( u = v = 0 \) as a particular combination of the light cone modes for region I and II. This approach is suggested by Unruh [31] for deriving the relation between two different vacuums for each mode in Rindler spacetime consideration. With this analysis, the Bogoliubov transformation for the relation between vacuum state for Kruskal mode and vacuum state for light cone mode can be derived as the following formula.

\[
d_k = \cosh \gamma b^\dagger_k - \sinh \gamma b^\dagger_k
\]

\[
d_k^\dagger = \cosh \gamma b^\dagger_k - \sinh \gamma b^\dagger_k
\]

where \( \tanh \gamma = e^{-\pi a \omega}, a = \sqrt{3}/\Lambda \), \( \Lambda \) = cosmological constant. \( d_k, d_k^\dagger \) are the annihilation and creation operator for the Kruskal mode, \( b^\dagger_k, b^\dagger_k \) is the annihilation and creation operator for the light cone mode in region I. The creation and annihilation operator for the light cone mode in the region II are
defined by changing upper its indices. These annihilation operators and creation operators generate their corresponding vacuum states respectively. The vacuum state of Kruskal mode is defined by

\[ d_k |0 \rangle_K = 0 \]  

where \( |0 \rangle_K \) is the vacuum state for Kruskal mode. The vacuum state of light cone mode is defined by

\[ b_k^I |0 \rangle_I = b_k^\II |0 \rangle_{\II} = 0 \]  

where \( |0 \rangle_I, |0 \rangle_{\II} \) are the vacuum states for light cone mode in region I and II respectively. These yields the corresponding Fock spaces. By using the Bogoliubov transformation, vacuum state of the acceleration frame can be described as a two-mode squeezed state for the observer staying at the origin. We follow the mathematical manipulation in [23, 24]

\[ |0 \rangle_K = \frac{1}{\cosh \gamma} \sum_{n=0}^{\infty} \tanh^n \gamma |n \rangle_I |n \rangle_{\II} \]  

This result is interpreted as the phenomenon that the observer in region I with light cone mode and its corresponding vacuum state can detect particle creation near the cosmological horizon. Similarly, we derive the first excited state in the Fock space for the Kruskal mode as the following result [23, 24].

\[ |1 \rangle_K = \frac{1}{\cosh^2 \gamma} \sum_{n=0}^{\infty} \tanh^n \gamma \sqrt{n + 1} |n + 1 \rangle_I |n \rangle_{\II} \]  

By using these results, we analyze the quantum system across the cosmological horizon.

In our model, there are three observers who initially share their quantum states. Firstly, Bob is spatially stationary at origin and feels Hawking radiation in the cosmological horizon. His physical state corresponds to the quantum state generated by the creation operator for light cone mode. Secondly, Alice moves away from Bob with the speed same as the expanding rate of de Sitter universe and passes through the horizon with respect to the Bob’s observation. Thirdly, Charlie also move away from Bob, but he does not pass through the horizon with respect to the Bob’s point of view. Alice and Charlie share a state that corresponds to the quantum state generated by the creation operator for the Kruskal mode. Therefore, Alice and Charlie cannot detect the effect of radiation and stay in our expanding universe in Bob’s point of view. We consider this kind of trajectory for Alice because we want to consider not only the information processing across the horizon but also the simulation of the measurable physics beyond the cosmological horizon. Furthermore, we consider that kind of trajectory for Charlie to consider about the scrambling effect in this channel. Initially, we consider two kinds of tripartite entangled states which are written as the following forms.
\[ |\psi^{(W)}\rangle = \frac{1}{\sqrt{3}} (|100\rangle + |010\rangle + |001\rangle) \] (10)

\[ |\psi^{(GHZ)}\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle) \] (11)

where we denote the notation as \(|000\rangle = |0_A\rangle_k|0_B\rangle_k|0_C\rangle_K\) and the state \(|0_A\rangle_k, |0_B\rangle_k, |0_C\rangle_K\) mean that the Kruskal vacuum states which can only be detected by Alice, Bob, Charlie respectively. One is called GHZ state, and the other is called W state. Due to the effect of expanding universe, Bob’s vacuum state and first excited states in (10), (11) evolve to two mode squeezed state in Fock space for light cone modes (8), (9). Then Bob feels the particle creation when the expansion of de Sitter universe starts, and three observers start their moving respectively. We treat this evolved state as a final state. Using the final quantum state, the result can be described by the density matrix of mixed state.

For describing the evolution with respect to the entanglement, we consider the density matrix \( \rho = |\psi\rangle\langle\psi| \). We trace out the Charlie’s mode because we are interested in the entanglement property between Bob and Alice. Then, by using the equation (10), (11) density matrices of the initial states is given as the following form.

\[ \rho_{AB}^{(GHZ)} = Tr_C(|\psi\rangle\langle\psi|) = \frac{1}{2}(|00\rangle\langle00| + |11\rangle\langle11|). \] (12)

\[ \rho_{AB}^{(W)} = Tr_C(|\psi\rangle\langle\psi|) = \frac{1}{3}(|10\rangle\langle10| + |10\rangle\langle01| + |01\rangle\langle10| + |01\rangle\langle01| + |00\rangle\langle00|) \] (13)

Since there are two causally disconnected regions in de Sitter spacetime and Bob can exist only one region, we should trace out one of the Bob’s mode (II) in the density matrices of final state. Then the final state we define above is

\[ \rho'_{AB}^{(GHZ)} = \frac{1}{2\cosh^2\gamma} \sum_{n=0}^{\infty} \tanh^{2n}\gamma \left[ |0\ n\rangle\langle0\ n| + \frac{n + 1}{\cosh^2\gamma} |1\ n + 1\rangle\langle1\ n + 1| \right]. \] (14)

\[ \rho'_{AB}^{(W)} = \frac{1}{3\cosh^2\gamma} \sum_{n=0}^{\infty} \tanh^{2n}\gamma \left[ |1\ n\rangle\langle1\ n| + |0\ n\rangle\langle0\ n| + \frac{n + 1}{\cosh^2\gamma} |0\ n + 1\rangle\langle0\ n + 1| \right. 

\left. + \frac{\sqrt{n + 1}}{\cosh\gamma} |1\ n\rangle\langle0\ n + 1| + \frac{\sqrt{n + 1}}{\cosh\gamma} |0\ n + 1\rangle\langle1\ n| \right]. \] (15)

Then we construct the relation between initial state and final state. It means that the expanding effect can be expressed as an operator sum representation.

\[ \epsilon(\rho_{AB}) = \sum A_n \rho_{AB} A_n^\dagger = \rho'_{AB} \] (16)

The density matrices \( \rho_{AB} \) and \( \rho'_{AB} \) describe the initial and final state in our model. Here, we can
derive the Kraus operator as the following form for both W state case and GHZ state case.

\[ A_n = \frac{1}{\sqrt{n!}} \frac{\tanh^2 \gamma}{\cosh \gamma} (b_i^\dagger)^n \otimes \frac{1}{(\cosh \gamma)^b_i b_i} \]  

(17)

We confirm that these maps preserve trace and positive semi definite. For this reason, we can treat these maps as a noisy quantum channel [22]. It means that the noisy quantum channel between Bob and Alice are affected by the cosmological horizon in de Sitter spacetime.

In this setting, we calculate the fidelity of the channel. The Kraus operator is used as an easier approach to calculate the fidelity of the channel which can be represented by the formula \( F_e = \sum_n \left( \text{tr}(\rho_{AB} A_n) \text{tr}(\rho_{AB} A_n^\dagger) \right) \).

\[ F_e^{(GHZ)} = \frac{1}{4 \cosh^2 \gamma} \left( 1 + \frac{1}{\cosh^2 \gamma} \right)^2 \]  

(18)

\[ F_e^{(W)} = \frac{1}{9 \cosh^2 \gamma} \left( 2 + \frac{1}{\cosh^2 \gamma} \right)^2 \]  

(19)

To determines whether the channel is reliable or not, we study the fidelity \( F_e = \text{tr}_A(\rho_{AB} | \rho_{AB} A_n^\dagger \rangle \langle A_n |) \) which means the preservation of entanglement when they transformed from initial state to final state. In the Figure 2, we show that the fidelity of the channel for both the GHZ state model and the W model case decreases when the expansion rate grows. This can be explained by the behavior of entanglement that observers in de Sitter space prepared. The reason is that the entanglement cannot be preserved by the thermal radiation in Bob’s observation in de Sitter space.

To investigate further, we estimate the amount of the correlation between two quantum states by calculating the mutual information \( I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \) where \( S(\rho) = -\text{tr}(\rho \ln \rho) \) is the von Neumann entropy of the density matrix \( \rho \). The bipartite mutual information can be studied to indicates the amount of Alice’s information that Bob can obtain with his measurement on his state. When Alice, Bob, and Charlie share the entangled state, the bipartite mutual information is given as

\[ I^{(GHZ)}(A: B) = 1 + \frac{1}{2 \cosh^2 r} \sum_{n=0}^\infty \tanh^2 \gamma \left[ (W_n - 2) \log_2(W_n - 2) - (W_n - 1) \log_2(W_n - 1) \right], \]  

(20)
\[ I^{(W)}(A:B) = \log_2 3 - \frac{5}{3} - \frac{1}{3} \cosh^2 \gamma \left( \log_2 (3 \cosh^2 \gamma) - \frac{1}{3} \log_2 (\tanh^2 \gamma) \right) \]
\[ - \frac{1}{6 \cosh^2 \gamma} \sum_{n=0}^{\infty} \tanh^{2n} \gamma \left[ M_n^+ \log_2 M_n^+ + M_n^- \log_2 M_n^- - 2W_n \log_2 W_n \right] \]
\[ - \frac{2}{\cosh^2 \gamma} \log_2 \left( \frac{\tanh^{2n} \gamma}{6 \cosh^2 \gamma} \right) \]

where \( M_n^\pm (\gamma) = 1 + \tanh^2 \gamma \pm \frac{n+1}{\cosh^2 \gamma} \left( 1 - \tanh^2 \gamma + \frac{1}{\cosh^2 \gamma} \left( 1 + 3 \tanh^2 \gamma + \frac{n+1}{\cosh^2 \gamma} \right)^2 \right) \)

\[ W_n (\gamma) = \left( 2 + \frac{n}{\sinh^2 \gamma} \right), W'_n (\gamma) = \left( 2 + \frac{n+1}{\cosh^2 \gamma} \right) \]

which we plot in Figure 3. When the expansion rate grows, the bipartite mutual information for the prepared state decreases. The decrease of the bipartite mutual information starts from 2 for the GHZ state case. On the other hand, the decrease of the bipartite mutual information starts from 1 for W state case.

To study further, we consider another measure of correlation of quantum states. It is the tripartite mutual information which is defined by \( I(A:B:C) = I(A:B) + I(A:C) - I(A:BC) \). By some manipulation, the tripartite information for each state is given by

\[ I^{(GHZ)}(A:B:C) = 1 + \frac{1}{2 \cosh^2 \gamma} \sum_{n=0}^{\infty} \tanh^{2n} \gamma \left[ (W_n - 2) \log_2 (W_n - 2) + (W'_n - 2) \log_2 (W'_n - 2) \right] \]
\[ - (W_n - 1) \log_2 (W_n - 1) - (W'_n - 1) \log_2 (W'_n - 1) \]  

(22)

\[ I^{(W)}(A:B:C) = \log_2 3 - \frac{9}{3} - \frac{2}{3 \cosh^2 \gamma} \log_2 (3 \cosh^2 \gamma) - \frac{1}{3} \log_2 (\tanh^2 \gamma) \]
\[ - \frac{1}{3 \cosh^2 \gamma} \sum_{n=0}^{\infty} \tanh^{2n} \gamma \left[ M_n^+ \log_2 M_n^+ + M_n^- \log_2 M_n^- - W_n \log_2 W_n - W'_n \log_2 W'_n \right] \]
\[ - \frac{2}{\cosh^2 \gamma} \log_2 \left( \frac{\tanh^{2n} \gamma}{6 \cosh^2 \gamma} \right) \]

By using this measure, we investigate the amount of information obtained about system A, through each system B and C rather than the system BC thoroughly. This can be interpreted as the scrambling on the system [32]. In this context, the tripartite mutual information of the entangled state has large negative value when there is large magnitude of the scrambling in the quantum system [32]. With respect to the thermalized GHZ state, the tripartite mutual information is decreased monotonically when the expansion rate grows as shown in Figure 4. The result is similar to the behavior of bipartite mutual information since it has decreasing curve with increasing expansion rate \( \gamma \). In other word, the
magnitude of the bipartite mutual information is proportional to the magnitude of the tripartite mutual information in our GHZ state model.

For W state case, however, there is non-trivial behavior that we can investigate in the plot. There is positive value of tripartite mutual information in thermalized W state on some range. We will analyze this non-trivial behavior by analyzing the relation with negativity [33] which is treated as a measure of entanglement. For the density matrix $\rho_{AB}$, we can perceive that there is distillable entanglement when there is at least one negative eigenvalue of the matrix $\rho_{AB}^{\tau_A}$ which indicates the partial transpose of the density matrix. Otherwise, we cannot determine whether there is an entanglement in the quantum state when there exist only positive eigenvalues. This is known as the partial transpose criterion. For GHZ state, only positive eigenvalue exists among eigenvalues of the matrix $\rho_{AB}^{\tau_A}$. Therefore, there is no distillable entanglement in GHZ state thermalized by expansion of our universe.

On the other hand, eigenvalues of $\rho_{AB}^{\tau_A}$ for W state case are given as the following, for $n = 0, 1, 2, …$

$$\lambda_n = \frac{\tanh^2 \gamma}{6 \cosh^2 \gamma} \left[ (1 + \frac{n}{\sinh^2 \gamma} + \tanh^2 \gamma) \pm \sqrt{(1 + \frac{n}{\sinh^2 \gamma} + \tanh^2 \gamma)^2 - 4 \tanh^2 \gamma + \frac{4}{\cosh^2 \gamma}} \right].$$ (24)

There exists a negative eigenvalue when $4 \tanh^2 \gamma + \frac{4}{\cosh^2 \gamma}$ is greater than 0. We calculate numerically that $4 \tanh^2 \gamma + \frac{4}{\cosh^2 \gamma}$ is greater than 0 when the expansion rate $\gamma$ is less than 0.783 approximately. When the state is in our universe with expansion rate $\gamma$ is less than 0.783…, there remains a distillable entanglement in the state which is thermalized state. We plot the sum of absolute value of negative eigenvalues $\sum_{n=0}^{\infty} |\lambda_n|$ which indicate the amount of negative eigenvalue. The negativity is plotted with Tripartite mutual information in Figure 4 and 5 for GHZ state case and W state case respectively. In the plot, we can show that there are different properties between the ranges below and above $r = 0.783$. It is similar to the behavior of the tripartite mutual information which has different sign between two distinct ranges. Thus, it can be suggested that the tripartite mutual information for W state case is positive in the region which is less than 0.783… because there still remains distillable entanglement. In other word, the scrambling effect does not happen in this region due to the distillable entanglement in thermalized W state.

However, in the region which is greater than 0.783…, there is still some region that has positive tripartite mutual information. In this region, we find that there may exist a bound entangled state or just mixed state because there is no distillable entangled state for these expansion rates. Even though the existence of bound entanglement cannot be totally confirmed by some measures discussed in this paper, we suppose that there is a bound entangled state so that the scrambling effect is prohibited. This is reasonable because the scrambling effect implies the system AB as maximally mixed state which
does not contain entanglement [32]. Thus, it is more probable that there is entangled state for the positive tripartite mutual information. Since there remains difficulty of probing the existence of bound entangled state [34], we would not strictly study the measure of bound entanglement.

For these analyses, we conclude that the significant behavior of the negativity in W state case is related with the non-trivial behavior of tripartite mutual information. Also, for large expansion rate \( \gamma \), without distillable entanglement in thermalized W state, the magnitude of the tripartite mutual information is proportional to the magnitude of the bipartite mutual information. This result is similar to the behavior of tripartite mutual information for thermalized GHZ state.

Interestingly, it was also studied that the retrieval of the information of states between two distinct regions is proportionally related to the scrambling on the total system [32]. It is compatible with our result with an exception for the region that include a distillable entanglement. When there is a small magnitude of the bipartite mutual information, the tripartite mutual information of the entangled state has large negative value which implies scrambling effect that compensate the small amount of information retrieval. In our case, we analyze the information exchange of tripartite entangled state across the cosmological horizon. Thus, we expect that even though there is entanglement degradation, by the scrambling effect there is a possibility of the information retrieval process across cosmological horizon.

**Conclusion**

In conclusion, we have studied the behavior of the quantum correlation that is initially prepared by three maximally entangled states. In our model, massless scalar particles coupled with de Sitter spacetime are treated. The effect of expansion can be described by operator sum representation with corresponding Kraus operator. It is shown that this map is trace-preserving and complete positive map. By showing this property, we interpret this phenomenon as a noisy quantum channel. Using the quantum information theoretical language, we investigate that the entanglement is degraded when our universe expands more rapidly. The reason is the fact that Bob is detect the thermal radiation from the de Sitter horizon. Also, there remains asymptotically non-vanishing entanglement. It means that Alice, Bob, and Charlie can interchange their quantum information. This phenomenon implies the quantum information exchange across the de Sitter horizon. Thus, we conclude that the consistent possibility of quantum information processing means that the measurable physics beyond the cosmological horizon of our universe. Finally, we argue that our numerical analysis for the relation between bipartite mutual
information and tripartite mutual information can be viewed as a specific model that shows the relation between retrieval of the information and scrambling of the system. For large expansion rate, there exists a small magnitude of the bipartite mutual information when the magnitude of the tripartite mutual information is large. Since the large negative value of tripartite mutual information indicates the scrambling effect of the horizon, Bob can retrieve the information of Alice who passes through the cosmological horizon even though there is a small bipartite mutual information. It means that there is also a scrambling behavior of the cosmological horizon in de Sitter spacetime like black hole event horizon. It implies that the detailed quantum information theoretic tasks can be given for the retrieval of the information across the horizon. However, for low expansion rate, there is a non-trivial behavior for the tripartite mutual information of W state case. Remarkably, it is implied that remaining distillable or bound entanglement in thermalized W state at low expansion rate is related to the non-trivial behavior of tripartite mutual information at low expansion rate $\gamma$. Moreover, in our analysis, we deduce that the measures which are used to study black hole can be applied to investigate the horizon which means membrane between our universe and the area beyond our universe. Specifically, with the quantum channel model we give an explicit calculation to study the scrambling and retrieval of quantum information across the cosmological horizon. Therefore, it can be suggested that there will be a detailed complementarity consideration in de Sitter spacetime with information theoretic concept such as quantum complexity.

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**Figure captions**

**FIG. 1:** (a) is a Kruskal diagram and (b) is a Penrose diagram of the de Sitter spacetime. Red arrow indicates the Alice’s trajectory, and Blue arrow indicates the Bob’s trajectory. Also, there is a green arrow which corresponds to Charlie’s trajectory.

**FIG. 2:** The red line indicates the fidelity of thermalized state with its initial state as W state, and the blue line indicates the fidelity of thermalized state with its initial state GHZ state. These numerical results of fidelity decrease as a function of expansion rate $\gamma$.

**FIG. 3:** The red line indicates the bipartite mutual information of thermalized state which is initially W state. The blue line indicates the bipartite mutual information of thermalized state which is initially GHZ state. Bipartite mutual information for both states decreases as a function of expansion rate $\gamma$.

**FIG. 4:** The blue line indicates the tripartite mutual information of thermalized state with its initial state as GHZ state. The red line indicates the negativity of the state. The tripartite mutual information is always negative value, but its absolute value increases as a function of expansion rate $\gamma$. There is no negative eigenvalue of $\rho_{AB}^{TA}$ for all expansion rate $\gamma$.

**FIG. 5:** The blue line indicates the tripartite mutual information of thermalized state with its initial state as W state. The red line indicates the negativity of the state. The tripartite mutual information has a region that it is positive value, but its absolute value decreases when the expansion rate $\gamma$ is large. There are negative eigenvalues of $\rho_{AB}^{TA}$ for some expansion rate $\gamma$. 
FIGURES

FIG. 1
FIG. 2
FIG. 3
FIG. 5