Generalized model of seismic pulse

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Abstract. The paper presents data on a pulse model, suitable for generalizing models of known seismic pulses. It is shown that for each of the known models it is possible to obtain a very accurate quadratic approximation using the proposed model. For example, the fragment of a real seismic trace is approximated by a pulses set formed on the basis of the proposed model, with a high accuracy.

1. Introduction

All modern seismic studies about researching the deep structure of the earth's interior and excited or spontaneous seismic processes are based on the recording of elastic acoustic waves propagating in the medium.

Registration is carried out by special devices – geophones. These devices create a seismic record and respond to vibrations of the medium, which are a mixture of useful signal and noise. Registration involves a lot of geophones to form seismograms - time fields of seismic signals of various spatial configurations and sizes.

A real seismic signal model is considered to be the distorted recording of seismic pulses on the time sequence. It characterizes the wave profiles excited by the oscillation source and reaching the seismic receiver.

The major role is assigned to the construction of appropriate physical and mathematical models to increase the effectiveness of seismic research. These models use the dynamic characteristics of waves, amplitudes, pulse spectra, etc., which is especially important in solving inverse seismic problems [1].

Accurate knowledge of the shape and excited pulse’s spectral width determines the quality, the accuracy of the geological interpretation and subsequent inverse transformations.

Different spectra types of seismic waves are determined by the properties of the vibration source and the environment’s adjacent area. In the environment, the propagation of a seismic wave source spectrum is changed, the pulse shape is distorted and speed heterogeneity leads to variation in the peak frequency of the pulse. This happens due to the influence of absorption or dispersion of rock properties and nonlinear distortion. Often there are situations when the local dynamics of the seismic record and, accordingly, the impulse parameters are affected by local inhomogeneities of the medium.

When developing methods and tools for disturbance suppression in the seismograms processing, it is necessary to provide the maximum signal-to-noise ratio and high resolution of recorded pulses.
When identifying waves carrying useful information during seismograms processing, it is necessary to control shape’s deformation of the detected pulses. This is done to determine more accurately the arrival time of the seismic signal.

The mathematical description of actual seismic pulse shapes used a number of simplified ideas about the seismic signal’s form and is based mainly on the formal resemblance to the real pulses. These are the formulas proposed by Berlage, Gelfand, Ricker, Puzyrev, as well as the formula of an exponentially decaying sinusoid.

In this paper, let us propose a seismic pulse model based on experimental modeling and using solutions of known nonlinear evolution equations. This model can serve as a generalization of known models of seismic pulses and can be used to approximate real seismograms.

2. Theory

The seismic pulse (wave packet) is the solution of the corresponding nonlinear evolution equation describing the stress-strain states of the propagation medium. Usually the solutions of such equations are sought in the nonlinear wave’s form of stationary profile, in which the dependence on the coordinate and time is determined by unified expression $(X-Vt)$, where $V$ – the group velocity of the wave moving in the direction of $X$.

The proposed model of a seismic pulse is in the form of [2]:

$$s(X,t) = cA \cos(kX - \omega t + \theta) \frac{\cosh(X - Vt)}{\cosh(T/2)},$$

where $c$ – coefficient of proportionality, depending on the type of the evolution equation; $A$ and $T$ – amplitude and duration of the pulse; $X$ – distance between the geophone and the source of the pulse; $\omega = kv$ angular frequency, which is the product of wave number $k$ and the phase velocity of wave $v$; $\theta$ – phase shift of the wave.

The pulse spectrum is calculated by the formula:

$$S(X,\Omega) = \frac{\pi c A (\cosh(\Omega/2V))}{V} \frac{\cosh(T/2V)}{\cosh(\Omega/2V)} \left( \exp(i(k - \Omega/k)Vt) + \exp(i(k + \Omega/k)Vt) \right) \exp(-i\Omega X/V).$$

The energy is calculated by the formula:

$$E = \int_{-\infty}^{\infty} s^2(X,t)dt = \frac{c^2 A^2}{V} \left( \frac{\cos(2(k \Omega/V)t)}{\sinh(\pi T/\Omega V)} + 1 \right).$$

Depending on the dispersive properties of the propagation medium, the solutions of three evolution equations are considered.

The wave dispersion is absent. The solution is the wave packet of the stationary wave equation profile with parameters: $c = A = 1$, $T = 0.049\omega$, $V = v = \pm 1$.

Dispersion of waves is low. The solution is solving the modified Korteweg-de Vries equation with the parameters: $c = 2\sqrt{6}$, $A = \frac{1}{T}$, $V = 3k^2$.

The dispersion of waves is high. The solution is solving the nonlinear Schrödinger equation with the parameters: $c = \sqrt{2}$, $A = \frac{1}{T}$, $V = 2k$.

3. Discussion of modeling results

Different seismic pulse models are used in the exploration seismology to approximate real seismograms:
1) Berlage wavelet: \( w(t) = t^n \exp(-bt) \sin(\omega t); \)

2) Damped sine wavelet: \( w(t) = \exp(-bt) \sin(\omega t); \)

3) Gelfand wavelet: \( w(t) = \exp(-br^2) \sin(\omega t); \)

4) Puzyrev wavelet: \( w(t) = \exp\left(-\frac{\omega}{\pi}t^2\right) \sin(\omega t); \)

5) Ricker wavelet: \( w(t) = \left(1 - 2\left(\frac{\omega}{2}\right)^2\right) \exp\left(-\frac{\omega^2}{2}\right). \)

In Figures 1-5, the normalized time forms and frequency spectra of the seismic pulse models listed above and the proposed model are given for comparison.

The proposed model can take the form of zero-phase (pulses 1, 3 and 5) and minimum-phase (pulses 2 and 4) seismic pulses due to the selection of phase shift \( \theta. \)

Figures show the high similarity of the proposed and all other models of seismic impulses.

The quality of the approximation was estimated by three criteria: \( SSE \) (sum of squares due to error), \( RS \) (coefficient of determination), \( RSME \) (root squared mean error).
Criteria are calculated by formulas:

\[ SSE = \sum_{k=1}^{n} (y_k - \hat{y}_k)^2, \quad SST = \sum_{k=1}^{n} (y_k - \bar{y})^2, \quad R = 1 - \frac{SSE}{SST}, \quad RSME = \sqrt{\frac{SSE}{n-m}}, \]

\( y_k \) – pulse values at points \( x_k; \) \( \hat{y}_k \) – values of the parametric model at points \( x_k; \) \( \bar{y} \) – mean; \( n \) - number of data; \( m \) - number of model parameters.

All 8 parameters of the model were calculated, although the main information about the shape of the pulse is carried by only 3 parameters. There are: \( T \) – parameter responsible for the pulse width; \( \omega \) - parameter specifying the carrier frequency; \( \theta \) – parameter characterizing the deviation from the zero phase.

In Table 1, for each pulse model, the values of the proposed model parameters and those of the proximity criteria are given.

| Wavelet   | A    | T   | V   | X   | c   | k   | \( \theta \) | \( \omega \) | SSE    | RS   | RMSE   |
|-----------|------|-----|-----|-----|-----|-----|------------|------------|--------|-------|--------|
| Berlage   | -0.900 | 0.854 | 1.904 | 1.798 | 1.000 | -1.778 | -0.471 | 6.311 | 3.22e-06 | 0.9987 | 1.05e-04 |
| D. sine   | 1.201 | 0.354 | 1.794 | -0.470 | 1.614 | 1.545 | 2.292 | 6.271 | 6.15e-06 | 1.0000 | 2.57e-04 |
| Gelfand   | 1.025 | 0.312 | 1.317 | -7.99e-08 | 1.103 | -0.084 | 1.571 | 6.114 | 1.20e-02 | 0.9996 | 4.50e-03 |
| Puzyrev    | -0.921 | 0.450 | -1.800 | 1.520 | 0.994 | 2.368 | 1.114 | 6.283 | 2.65e-08 | 1.0000 | 1.36e-05 |
| Ricker    | 1.030 | 0.892 | 3.071 | 2.70e-06 | -0.972 | 2.251 | 3.142 | 6.291 | 5.78e-02 | 0.9981 | 9.87e-03 |

Improving the efficiency of seismic survey is the most complete use of the dynamic wave’s characteristics: amplitude, pulses spectra, etc. [3]. Therefore, information about the shape and the spectrum of the pulses which make up the seismic recording traces can solve a number of problems in the processing and interpretation of seismic data. Let us note two very important problems.

Information on the shape and the impulses spectrum allows one to abandon completely or partially the procedure of deconvolution of seismic traces.

Knowledge of the pulses shape allows us to determine the arrival time of the seismic wave. It is extremely important in the event of tracking and correlation of waves for the coherent lineup.

Figure 6 shows a fragment of a real seismic trace and its approximation in a hundred consecutive time intervals of 15 samples. The average for 100 intervals is \( RS = 0.95 \). For each interval, all 8 parameters of the proposed model are calculated.

![Figure 5. Ricker wavelet](image)

![Figure 6. Approximation of real seismic trace](image)
4. Conclusion
A pulse model suitable for generalizing models of known seismic pulses is proposed. It is shown that for each of the known models it is possible to obtain a very accurate quadratic approximation using the proposed model.

With a high accuracy, the fragment of a real seismic trace is approximated by a set of pulses formed on the proposed model basis.

As a further research of the proposed model applications (and the material of the next publication), the problem of identifying a medium’s model in which the seismic wave propagates is considered. The input of the medium model is supplied with a zero-phase pulse with the specified parameters, from the output one or another pulse of the known model, or real seismic trace is removed. Different output pulses correspond to different medium models, which can be identified by known methods in the form of the certain coefficients of filters. This representation of the wave propagation medium is widely used in seismic studies [4].

References
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