Hypergraph-based Object-oriented Model and Hypergraph Theory for GIS

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1 Introduction

The GIS is used in many areas, such as the dynamic management of huge GIS data, the automatic generalization of GIS spatial entities and features, temporal GIS model and vector-raster-integrating model, etc. Generally speaking, the traditional GIS spatial model has the following primary problems:

- Object-oriented model with approaches is one of the general methods, which are lacking for spatial relation guideline in modeling spatial entities and features.
- Attributive data or thematic data and spatial data are mostly only an ID relation.

To solve the above problems, we use hypergraph data model and object-oriented model and approaches to define spatial entities and features, called hypergraph and object-oriented model (HOOOM). Hypergraph model and hypergraph data structure inherit the characteristics of the object model. Depending on hypergraph model, complex and composite spatial relations can be described more easily; spatial, attributive and temporal data can also be integrated more efficiently.

2 Theories of GIS spatial data model

The GIS spatial entities and features can be viewed in three abstractive levels: spatial concepts, spatial data models, and spatial data structures.

Cognizing and organizing spatial entities and features we form GIS spatial concepts, such as feature-based and geo-relation-based spatial concepts. The formalization of the spatial concepts leads to spatial data models. The most frequent spatial
formalization for geographic phenomena is based on the Euclidean geometry and geo-relation or thematic data. The formalization includes the basic elements of the spatial concepts such as the point, line, area, polygon, volume, and pixel and attributes of these objects such as location in some spatial reference system, and their interrelations such as topology.

3 HBDS

HBDS (Hypergraph-based data structure)\[4,7\] is a general model, which has been used for building an integrated object-oriented platform, as a kernel of GIS. It is a knowledge model based on six basic abstract data types: class, attribute of class, link between classes, object, attribute of object, and link between objects, with some extensions. Hyperclass, hyperattribute, hyperlink, embryo, prototype, and all structure types are implicitly persistent. These types have a graphical representation allowing an easy and immediate understanding of the real world and provide a convenient communication between geographers.

These basic ADTs have allowed developing some extensions. The first three ones generate group elements of the skeleton. Hyperclasses, hyperattributes, hyperlinks form an easy, clear, and synthetic representation of the phenomenon, and provide a very interesting economy of programming statements.

A second set of extensions are respectively named embryo, prototype and structure. Any ADT may support a condition, which will be automatically tested each time when the ADT will be invoked; likewise, it may support an action, which will be automatically performed each time when the ADT will be invoked; if both exist, the action will be launched only when the condition is verified. A condition is a Boolean expression, as complex as required, but may be a very fuzzy expression, and the action is a list of statements, without exclusion, any number possibly contains fuzzy statements.

HBDS makes it easy and immediate to understand the real world and provide a convenient communication for geographers, but it lacks support in algorithm.

The concept of attribution in hypergraph spatial data model has many meanings. It may be the following items:

- a number
- an \( r \) dimensional array
- the program or law
- a structure that can be described by hypergraph data model

Therefore, the attribution described the static and dynamic character of object. Hypergraph class and object have many relations, such as hierarchical structure and non-hierarchical structure. The hierarchical structure among hypergraph class shows class inheritance, association, and aggregation very clearly. The non-hierarchical structure is the base of distributed calculating and object generalization.

One of the major advantages of HBDS is the generalization of graph theory\[8\]. The vertices may be complicated objects themselves (they may even be graph), and the edges of a hypergraph represent the relationship among subsets of vertices. Therefore, in an HBDS the connections between objects in the same set and between classes of objects are allowed with both hierarchical and non-hierarchical relationships. When used for modeling, a hypergraph usually has objects with common attributes in the same substructures. Objects in a substructure might have relationship among themselves and relationships with objects outside. At one level, for example, internal relationships are not visible. Relationships with outside objects appear as hyperlinks. Basically, relationships between any objects at any levels can be expressed. Whether an object should be placed at a higher level or a lower level depends on significant factors to be considered, such as efficiency of data retrieval. All of these are only spatial data organizations.

4 HOOM

HOOM stands for Hypergraph-based GIS spatial data model (Fig. 1). HOOM has similar semantics with object-oriented model. To define hypergraph spatial data model, first, we must define the class and object of hypergraph spatial data model. Ac -
According to object-oriented model and the characteristics of hypergraph, complex and composite spatial world can be defined as follows.

4.1 Geometrical object model

Geometrical object model only describes the geometrical characteristics of spatial entity and feature. It describes the shape of feature, spatial location relation, spatial topologic relation, and geometrical measure information. Geometrical object can be divided to node, arc, polygon or pixel, and voxel.

4.2 Feature object model

Feature object model is used to explain the geometrical object model. In other words, Feature object model is defined by adding attributive values to geometrical object. So the same geometrical object may have different geographical attribution because different attributive value is added to the geometrical object. Feature object class is divided into point feature, line feature, polygon feature, and complex feature.

The spatial relation of feature object is derived from its relevant geometrical object.

4.3 Spatial representative object model

GIS spatial representative object model is divided to physical and logical representation. Physical representative model represents GIS distributed characteristics and operating, and physical storage. GIS distributed operation may produce many problems, such as concurrence control and consistency handle. To solve GIS distributed operating, we must add distributed ID label to geometrical and geographical object.

GIS logical representation means that we usually need to calculate and draw thematic map from GIS spatial database and attributive database accord-
ing to the certain conditions and formats. Also we need to manage and query GIS spatial data and attribute data according to the application demands. The association functions are used to associate the graphical object and geographical object.

The main goal that we abstract the graphical class is facility in multi-scale map generalization. The main methods are described as follows.

4.3 Define dynamic association

The simplest rule that defines dynamic association is that from small to great for a given group discrete scales, scale\(_1\), scale\(_2\), ..., scale\(_n\), defining obey follow scale rules, when scale pertains to the scale extend (scale\(_1\), scale\(_2\)), geographical objects are associated with the graphical objects according to the association attributes and association functions. It is not displayed when the association graphics are null. The more complex rules that define dynamic association are establishing map generalization rules and multi-scale and multi-resolution rules on feature objects.

4.3.2 Define the display rules

1) Define the display order, first display polygon feature, then line feature, finally point feature, etc.
2) When the extend of graphics is smaller than the given extend, the graphics are not displayed.
3) Symbolization rules.
4) Map generalization rules.

4.4 Temporal object model

Both GIS spatial data and attribute data have temporal characteristics. Without time stamping GIS data cannot be used efficiently.

The typical data abstract object-oriented technology is classification, generalization, association, and aggregation. The object-oriented characteristics are that they have abstraction, inheritance, and polymorphism. We can use these characteristics to realize our goal\(^9\).

The spatial data in the hypergraph-based object-oriented model can link an attribute class of hypergraph, for example, geometrical object links feature object model. This is a very important character for modeling GIS spatial phenomena. We can label hypergraph class according to the feature of class, such as road, road segment, the grade of road, etc.

5 Hypergraph theory

Hypergraph are the theoretical basis of hypergraph spatial data model. Hypergraph expands the graph theory\(^10\). In general graph, edge set is defined as duality elements of vertex set. But in hypergraph, edge set is defined as any of elements of vertex set and is closed by Jordan curve.

Let \( X = \{ x_1, x_2, \ldots, x_n \} \) be a finite set, and let \( \varepsilon = (E_i/i \in I) \) be a family of subsets of \( X \). The family \( \varepsilon \) is said as a hypergraph on \( X \) if \( E_i \neq \emptyset \) (\( \emptyset \) represents null set) and \( \bigcup E_i = X, i \in I \). The couple \( H = (X, \varepsilon) \) is called a hypergraph, \( |X| = n \) is called the order of this hypergraph. The elements \( x_1, x_2, \ldots, x_n \) are called the vertices and the sets \( E_1, E_2, \ldots, E_m \) are called the edges.

In a hypergraph, an edge \( E_i \) with \( |E_i| > 2 \) is drawn as a curve encircling all the vertices of \( E_i \). An edge \( E_i \), with \( |E_i| = 2 \) is drawn as a curve connecting its two vertices. An edge \( E_i \) with \( |E_i| = 1 \) is drawn as a loop in a graph. Two vertices are said to be adjacent if there is an edge \( E_i \) that contains both of these vertices. Two edges are said to be adjacent if their intersection is not empty.

The incidence matrix of hypergraph \( H = (X, \varepsilon) \) is a matrix \((a_{ij})\) with \( m \) rows which represent the edges of \( H \) and \( n \) columns which represent the vertices of \( H \). Then,

\[
a_{ij} = 1 \text{ if } x_j \in E_i \\
a_{ij} = 0 \text{ if } x_j \notin E_i
\]

each \((0,1)\)-matrix is the incidence matrix of hypergraph if no row or column contains only zeros. Fig. 2 is an example of a hypergraph.

To each hypergraph \( H = (X_1, E_1, E_2, \ldots, E_m) \), there is a corresponding hypergraph \( H^* = (E_1, X_1, E_2, \ldots, X_n) \) whose vertices are points \( e_1, e_2, \ldots, e_m \) (that respectively represent \( E_1, E_2, \ldots, E_m \)) and whose edges are sets \( X_1, X_2, \ldots, X_n \) (that respectively represent \( x_1, x_2, \ldots, x_n \)). Hypergraph \( H^* \) is called the dual hypergraph of \( H \). The incidence matrix of hypergraph \( H^* \) is the transpose of that of hypergraph \( H \).
Hypergraph generates relevant concepts and algorithms, such as K-section, transversal of a hypergraph, representative graph of a hypergraph.

5.1 K-section of hypergraph

Given an integer $k > 0$, the K-section of hypergraph $H = (X, e)$ is defined to be the couple $H_k = (X, e_k)$, formed by $X$ and the following:

$$e_k = \{F / F \subseteq X; |F| \leq k; F \cap E \neq \emptyset\}$$

$H_k$ is a hypergraph as well. Its vertex set is $X$, but its edge set changes to not more than $k$ points, and some $s \notin e$. For some $E_i \in e$.

5.2 Transversal of hypergraph

For known $H_k = (X, e_k)$, let $T \subseteq X (X$ is the original vertex set of hypergraph). If $T$ has $T \cap E_i \neq \emptyset$ (at least it includes one point to each edge) to all $i \in m$, here $T$ is called the transversal of hypergraph. In fact, the transversal of hypergraph is part of vertex set. This represents a spatial transverse relation in GIS model.

5.3 Representative graph of hypergraph

Let $H = (E; X_1, X_2, \ldots, X_n)$ be a hypergraph with $n$ edges. The representative graph $H$ is defined to be the simple graph $G = L(H)$ of order $n$ whose vertices $x_1, x_2, \ldots, x_n$ respectively represent the edges $X_1, X_2, \ldots, X_n$ of $H$ and with vertices $x_i$ and $x_j$ joined by an edge if, and only if $x_i \cap x_j \neq \emptyset$. This new graph $G$ is called representative graph of hypergraph $H$. The graph $G$ represents the edge relationships of $H$. In GIS model, the representative graph of hypergraph gives us good ability to analyze the complex spatial relations. $G = L(H)$ means that vertices $x_i$ and $x_j$ are adjacent if, and only if there exists a vertex $e_k \in x_i \cap x_j$ in $H$. This is also equivalent to saying that there exists an edge $E_k$ in $H^*$ that contains both $x_i$ and $x_j$, or $G = (H^*)^2$.

According to basic researches we can draw the following conclusions. To analyze and simplify the basic relationships we can use K-section and representative graph of hypergraph. Representative graph of hypergraph represent major dependent relationship between edges of hypergraph. K-section make us analyze the relationships hierarchically.
The dual hypergraph algorithm is important for analyzing dependent relationships between the vertices and edges. That is, if in hypergraph vertices represent spatial geometrical objects and attributes and edges represent the relationship between objects, then the hypergraph represents the objects and attributes of the relationship dependence and association. But the dual hypergraph reverses these associations. It represents the relationships between objects and attributes of dependence and association.

Applying hypergraph theory in HOOM

No references described how to transform between hypergraph and HBDS or HOOM. Ostensibly, Hypergraph is different from HOOM, but it is only different in vision. If we take $E_3$ in Fig. 2 as the root of HOOM, then in HOOM visualizing mode, Fig. 2 can be visualized as Fig. 4 (Hypergraph edge as class of HOOM, vertices as member of class, and link relationship exists among vertices).

Also we can take the vertices of hypergraph as the class of HOOM and edges as constrained relationship.

Once we have established the transformation relationship between hypergraph and HBDS or HOOM, we can use the algorithm of hypergraph to simplify or analyze the feature relationship.

An example

Here an example of using hypergraph is given to describe the terrain feature, shown in Fig. 5-Fig. 7.

Conclusion and suggestion

Using hypergraph model we can design many GIS application systems such as Urban integrated information system, decision support system of region sustainable development, etc. However, But we did not pay enough attention to hypergraph theory. Hypergraph theory can be used in data organization of multi-scale or multi-resolutions.

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Fig. 5 DEM and its contour map

Fig. 6 Terrain feature

Fig. 7 Hypergraph representation of terrain feature

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