Flavour physics within and beyond the Standard Model

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Summary. — I review recent progress in theoretical calculations related to the CKM unitarity triangle. After briefly discussing hints for new physics in $B_d$–$\bar{B}_d$ and $B_s$–$\bar{B}_s$ mixing I present three topics of MSSM flavour physics: First I discuss new $\tan\beta$-enhanced radiative corrections to flavour-changing neutral current (FCNC) amplitudes which go beyond the familiar Higgs-mediated FCNC diagrams and may enhance the mixing-induced CP asymmetry in $B_d \to \phi K_S$. The second topic is a reappraisal of the idea that flavour violation originates from the soft supersymmetry-breaking terms. Finally I discuss how $\mu \to e\gamma$ can be used to constrain the flavour structure of the dimension-5 Yukawa interactions which appear in realistic grand unified theories.

12.60.Jv,13.20.He,12.10.-g

1. – Introduction

Flavour physics addresses the transitions between fermions of different generations. Within the Standard Model these transitions originate from the Yukawa couplings of the Higgs field to the fermion fields. In the case of quarks the responsible term of the Lagrangian reads

$$\sum_{j,k=1}^{3} Y_{jk}^u \bar{u}_j^L u_k^R (v + H) - \sum_{j,k=1}^{3} Y_{jk}^d \bar{d}_j^L d_k^R (v + H) + \text{h.c.}$$

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(2) Invited talk at Les Rencontres de Physique de la Vallée d’Aoste, La Thuile, Italy, Feb 28 – Mar 6, 2010.
Here $H$ denotes the field of the yet-to-be-discovered physical Higgs boson and $v = 174\text{ GeV}$ is the corresponding vacuum expectation value. The indices $j$ and $k$ label the generations and $L$ and $R$ refer to the chirality of the quark fields. The Yukawa couplings for up-type and down-type quarks are $3 \times 3$ matrices in flavour space, denoted by $Y^u$ and $Y^d$, respectively. Eq. (1) entails the mass matrices

$$m^u = Y^u v \quad \text{and} \quad m^d = Y^d v.$$ 

The diagonalisations of $m^u$ and $m^d$ involve four unitary rotations in flavour space, one for each $u^L$, $u^R$, $d^L$, and $d^R$. Since the left-handed fields $u^L_k$ and $d^L_k$, which were originally members of a common SU(2) doublet, undergo different rotations, the electroweak SU(2) symmetry is no more manifest in the physical basis in which mass matrices are diagonal. The mismatch between the rotations of the left-handed fields defines the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$

$$(3) \quad V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. $$

The CKM elements occur in the couplings of the $W$ boson, because the $W$ e.g. couples the $\pi^L$ field to the linear combination $V_{cd}d^L + V_{cs}s^L + V_{cb}b^L$ as a consequence of the unitary rotations in flavour space. $V$ can be parametrised in terms of three angles and one phase, the CP-violating Kobayashi-Maskawa phase $\gamma$ [2]. With a history of more than 50 years, research in quark flavour has been essential for the construction of the Standard Model, having guided us to phenomena which were “new physics” at their time: Highlights were the breakdown of the discrete symmetries P [3] and CP [4, 2], the prediction of the charm quark [5] and its mass [6], and a heavy top quark predicted from the size of $B_d - \bar{B}_d$ mixing [7]. In the decade behind us the asymmetric B factories BELLE and BaBar have consolidated the CKM picture of quark flavour physics. With the advent of the LHC era, the focus of flavour physics has shifted from CKM metrology to physics beyond the Standard Model. In the Standard Model flavour-changing neutral current (FCNC) processes (such as meson-antimeson mixing, $B \rightarrow X_s \gamma$ or $K \rightarrow \pi \nu \nu$) are forbidden at tree-level and only occur through highly suppressed one-loop diagrams. FCNC processes are therefore excellent probes of new physics. This is a strong rationale to complement the high-$p_T$ physics programs at ATLAS and CMS with precision flavour physics at LHCb, NA62, BELLE-II, Super-B, BES-III, J-PARC and the future intense proton source Project X at Fermilab.

With the discovery of neutrino flavour oscillations, the much younger field of lepton flavour physics has emerged. The Standard Model in its original formulation [8] lacks a right-handed neutrino field and can neither accommodate neutrino masses nor neutrino oscillations. The simplest remedy for this is the introduction of a dimension-5 Yukawa term composed of two lepton doublets $L = (\nu^L, \ell^L)$ and two Higgs doublets leading to Majorana masses for the neutrinos and generating the desired lepton flavour mixing. Alternatively one can mimic the quark sector by introducing right-handed neutrino fields (and imposing $B - L$, the difference between baryon and lepton numbers, as an exact symmetry). With both variants FCNC transitions among charged leptons (such as $\mu \rightarrow e \gamma$) are unobservably small, so that any observation of such a process will imply the existence of further new particles. Charged-lepton FCNC decays are currently searched
for in the dedicated MEG experiment (studying $\mu \to e\gamma$), in B factory data (on e.g. $\tau \to \mu\gamma$) and at three of the four major LHC experiments (searching e.g. for $\tau \to \mu\mu\mu$).

In the following section I briefly review recent theoretical progress on the Standard-Model predictions for FCNC processes. Subsequently I discuss new developments in flavour physics beyond the Standard Model. I limit myself to supersymmetric theories, which reflects my personal research interests. For a recent broader overview, which also covers extra dimensions and Little-Higgs models, see ref. [10]. Exhaustive studies of the flavour sector in a four-generation Standard Model can be found in refs. [11].

2. – Standard Model

The standard unitarity triangle (UT) is a triangle with unit baseline and apex $(\bar{\rho}, \eta)$, which is defined through

$$\bar{\rho} + i\eta \equiv \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}}.$$  \hspace{1cm} (4)

The two non-trivial sides of the triangle are $R_u \equiv \sqrt{\bar{\rho}^2 + \eta^2}$ and $R_t \equiv \sqrt{(1-\bar{\rho})^2 + \eta^2}$. The triangle’s three angles

$$\alpha = \arg \left[ \frac{V_{td} V_{ub}^*}{V_{ud} V_{tb}^*} \right], \quad \beta = \arg \left[ \frac{V_{cd} V_{cb}^*}{V_{cd} V_{cb}^*} \right], \quad \gamma = \arg \left[ \frac{V_{ub} V_{tb}^*}{V_{cd} V_{cb}^*} \right].$$  \hspace{1cm} (5)

are associated with CP-violating quantities. Measurements of flavour-changing quantities imply constraints on $(\bar{\rho}, \eta)$. Last year’s global analysis of the UT performed by the CKMfitter collaboration is shown in fig. 1. For the results of the UTFit collaboration, which uses a different statistical approach see ref. [13]. The figure shows the consistency of the various measurements, which single out the small yellow area as the allowed region for the apex of the triangle. Clearly, the CKM mechanism is the dominant source of flavour violation in the quark sector.
From the quantities entering the global UT analysis in fig. 1 the meson-antimeson mixing amplitudes are the ones most sensitive to generic new physics. While the extraction of the UT angle $\beta$ from the CP phase in $B_d - \overline{B}_d$ mixing is theoretically very clean, all other quantities related to meson-antimeson mixing are plagued by theoretical uncertainties. Namely, the uncertainties in the mass differences $\Delta m_q$ and $\Delta m_s$ of the two $B - \overline{B}$ mixing complexes and in $\epsilon_K$, which quantifies CP violation in $K - \overline{K}$ mixing, completely dominate over the irrelevantly small experimental errors. Note that $\Delta m_s$ is practically independent of $\overline{\eta}$ and $\overline{\eta}$ and is only useful for the UT fit because the ratio $\Delta m_d / \Delta m_s$ has a smaller uncertainty than $\Delta m_d$. The $K - \overline{K}$ mixing mixing amplitude $M_{12}$ involves the matrix element $\langle K^0 | H^{S=2} | \overline{K}^0 \rangle$ of the $\Delta S = 2$ hamiltonian $H^{S=2}$ [18]. $H^{S=2}$ is proportional to the four-quark operator $\overline{d}L_{\gamma_\mu}s^L \overline{d}L_{\gamma_\nu}s^L$ with the relevant matrix element

$$\langle K^0 | \overline{d}^L_{\gamma\mu}s^L \overline{d}^L_{\gamma\nu}s^L (\mu) | \overline{K}^0 \rangle = \frac{2}{3} M_K f_K^2 \frac{\hat{B}_K}{b(\mu)}.$$  

This equation merely defines the parameter $\hat{B}_K$ which is commonly used to parametrise the matrix element of interest. In eq. (6) $M_K = 497.6$ MeV and $f_K = 160$ MeV are mass and decay constant of the neutral Kaon and $b_K(\mu)$ is introduced to render $\hat{B}_K$ independent of the unphysical renormalisation scale $\mu$ and the renormalisation scheme chosen for the definition of the operator $\overline{d}^L_{\gamma\mu}s^L \overline{d}^L_{\gamma\nu}s^L(\mu)$. In the commonly used MS scheme one has $b_K(\mu = 1$ GeV) $= 1.24 \pm 0.02$. The matrix element in eq. (6) must be calculated with lattice gauge theory. A new computation by Aubin, Laiho and Van de Water finds [14]

$$\hat{B}_K = 0.724(8)(29).$$

This result is in good agreement with the 2007 result of the RBC and UKQCD collaborations, $\hat{B}_K = 0.720(13)(37)$ [15]. In view of the superb experimental precision in $|\epsilon_K| = (2.23 \pm 0.01) \times 10^{-3}$ further progress on $\hat{B}_K$ is certainly highly desirable. The Increasing precision in $\hat{B}_K$ has also stimulated more precise analyses of other ingredients of $M_{12}$. Recently a reanalysis of the long-distance contribution to $\text{Im} M_{12}$ has resulted in an upward shift of 2% in $\epsilon_K$ [19]. A similar contribution constituting the element $\Gamma_{12}$ in the decay matrix, affects $\epsilon_K$ at the few-percent level [16, 17].

In the case of $B - \overline{B}$ mixing all long-distance contributions are highly GIM-suppressed and only the local contribution from the box diagram with internal top quarks and $W$ bosons matters. The two mass eigenstates of the neutral $B_q - \overline{B}_q$ system differ in their masses and widths. The mass difference $\Delta m_q, q = d, s$, which equals the $B_q - \overline{B}_q$ oscillation frequency, is given by $\Delta m_q = 2 |M_{12}| = 2 |\langle B_q | H^{S=2} | \overline{B}_q \rangle|$. Lattice calculations are needed to compute $f_{B_q}^2 \overline{B}_{B_q}$, which is defined in analogy to eq. (6). Here I focus on the ratio $\Delta m_d / \Delta m_s$ yielding the orange (medium gray) annulus centred around $(\overline{\eta}, \overline{\eta}) = (1, 0)$ in fig. 1. This ratio involves the hadronic quantity

$$\xi = \frac{f_{B_d} \sqrt{\overline{B}_{B_d}}}{f_{B_s} \sqrt{\overline{B}_{B_s}}} = 1.23 \pm 0.04.$$  

The numerical value in eq. (8) is my bold average of the values summarised by Aubin at the Lattice ’09 conference [20]. With this number and the measured values $\Delta m_{B_d} =
(0.507 ± 0.005) ps⁻¹ [21] and $\Delta m_{B_s} = (17.77 ± 0.10 ± 0.07)$ ps⁻¹ [22] one finds

$$\left| \frac{V_{td}}{V_{ts}} \right| = \frac{\Delta m_{B_d}}{\Delta m_{B_s}} \sqrt{\frac{M_{B_d}}{M_{B_s}}} \xi = 0.210 ± 0.007.$$  \hspace{1cm} (9)

With $|V_{td}/V_{ts}| = 0.228 R_t$ one finds $R_t = 0.92 ± 0.03$ for the side of the UT opposite to $\gamma$. For a pedagogical introduction into meson-antimeson mixing and CKM phenomenology cf. ref. [23].

3. – Beyond the Standard Model

3.1. Phenomenology of new physics in $B - \bar{B}$ mixing. – The plot of the UT in fig. 1 is not the best way to show possible deviations from the Standard Model, because it conceals certain correlations between different quantities. In the LHC era we will more often see plots of quantities which directly quantify the size of new physics contributions. In the case of meson-antimeson mixing new physics can be parametrised model-independently by a single complex parameter [24]. For $B_q - \bar{B}_q$ mixing, $q = d, s$, one defines

$$\Delta_q = \frac{M_{12}^q}{M_{12}^{q, SM}}.$$  \hspace{1cm} (10)

The CKMfitter collaboration has found that the Standard-Model point $\Delta_d = 1$ is ruled out at 95% CL (left plot in fig. 2), if all other quantities entering the global UT analysis are assumed free of new physics contributions. This discrepancy is largely driven by $B(B^+ \to \tau^+ \nu)$ and, if interpreted in terms of new physics, may well indicate non-standard physics in quantities other than $B_d - \bar{B}_d$ mixing. A tension on the global UT fit was also noted by Lunghi and Soni [27] and by Buras and Guadagnoli [17]. The situation is much simpler in the case of $B_s - \bar{B}_s$ mixing, which shows a deviation from the Standard Model expectation of similar size (right plot in fig. 2). The allowed region for $\Delta_s$ is essentially independent from input other than the $B_s - \bar{B}_s$ mixing amplitude $M_{12}^s$. The quantities entering the analysis are primarily $\Delta m_s$, the width difference $\Delta \Gamma_s$ [28, 24], the time-dependent angular distribution in $B_s \to J/\psi \phi$ (with access to the mixing-induced CP asymmetry $A_{CP}^{mix}(B_s \to J/\psi \phi)$ if the $B_s$ flavour is tagged), and the CP asymmetry in flavour-specific decays $a_{fs}^s$ [28, 24]. The first global analysis of these quantities, which used improved Standard-Model predictions, was performed in 2006 [24] showing a $2\sigma$ deviation from the Standard-Model value $\Delta_s = 1$. At the time of this talk the discrepancy from the combined DØ and CDF data on $B_s \to J/\psi \phi$ alone was between 2.0$\sigma$ and 2.3$\sigma$, depending on details of the statistical analysis [29]. After this conference the discrepancy in $a_{fs}^s$ has increased due to a new DØ measurement of the dimuon asymmetry in a mixed $B_d, B_s$ data sample [30]. On the other hand, new CDF data on $A_{CP}^{mix}(B_s \to J/\psi \phi)$ have pulled the result towards the Standard Model [31]. Still all measurements favour $\Delta_s > 0$.

3.2. Supersymmetry with large $\tan \beta$. – Extensions of the Standard Model typically come with new sources of flavour violation, beyond the Yukawa couplings in eq. (1). In the Minimal Supersymmetric Standard Model (MSSM) the soft supersymmetry breaking terms a priori possess a flavour structures which is unrelated to $Y^u$ and $Y^d$. To avoid excessive FCNCs violating experimental bounds the MSSM is often supplemented with
the assumption of Minimal Flavour Violation (MFV), which amounts to a flavour-blind supersymmetry-breaking sector. In the MFV-MSSM supersymmetric FCNC transitions are typically smaller than the error bars of today’s experiments, unless the parameter $\tan \beta$ is large. Probing values around $\tan \beta = 60$ tests the unification of top and bottom Yukawa couplings. Importantly, loop suppression factors can be offset by a factor of $\tan \beta$ and may yield contributions of order one, with most spectacular effects in $B(B_s \to \mu^+\mu^-)$ [32]. The $\tan \beta$-enhanced loop corrections must be summed to all orders in perturbation theory. In the limit that the masses of the SUSY particles in the loop are heavier than the electroweak vev and the masses of the five Higgs bosons, $M_{\text{SUSY}} \gg v, M_{A^0}, M_{H^\pm}, \ldots$, one can achieve this resummation easily: After integrating out the heavy SUSY particles one obtains an effective two-Higgs doublet model with novel loop-induced couplings [33]. In supersymmetric theories, however, it is natural that $M_{\text{SUSY}}$ is not much different from $v$ and further $M_{\text{SUSY}} \gg M_{A^0}$ involves an unnatural fine-tuning in the Higgs sector. Phenomenologically, large-$\tan \beta$ scenarios comply with the experimental bound from $B(B_s \to \mu^+\mu^-)$ more easily if $M_{A^0}$ is large, which may easily conflict with $M_{\text{SUSY}} \gg M_{A^0}$. To derive resummation formulae valid for arbitrary values of $M_{\text{SUSY}}$ one cannot resort to the method of an effective field theory. Instead one should work strictly diagrammatically in the full MSSM to identify $\tan \beta$-enhanced corrections. This procedure requires full control of the renormalisation scheme: The analytical results for the resummed expressions differ for different schemes and not all renormalisation schemes permit an analytic solution to the resummation problem. The diagrammatic resummation has been obtained for the flavour-diagonal case in ref. [34] and recently for flavour-changing interactions in ref. [35]. This opens the possibility to study $\tan \beta$-enhanced corrections also to supersymmetric loop processes which decouple for $M_{\text{SUSY}} \gg v$ and to collider processes involving supersymmetric particles. In ref. [35] a novel large effect, which does not involve Higgs bosons, in the Wilson coefficient $C_8$ has been found, with interesting implications for the mixing-induced CP asymmetry $S_{\phi K_S}$ in $B_d \to \phi K_S$ (see fig. 3).
3.3. Radiative flavour violation. – A symmetry-based definition of MFV starts from the observation that the MSSM sector is invariant under arbitrary unitary rotations of the (s)quark multiplets in flavour space. This $[U(3)]^3$ flavour symmetry ($[U(3)]^5$ if (s)leptons are included) is broken by the Yukawa couplings, and MFV can be defined through the postulate that the Yukawa couplings are the only spurion fields breaking the $[U(3)]^3$ flavour symmetry [36]. Interestingly, there is a viable alternative to MFV to solve the supersymmetric flavour problem: We may start with a Yukawa sector in which all Yukawa couplings of the first and second generation are zero. That is, the MSSM superpotential possesses an exact $[U(2)]^3 \times U(1)$ symmetry. Then we postulate that the trilinear SUSY breaking terms $A^{q}_{ij}$ and $A^{d}_{ij}$ are the spurion fields breaking this symmetry. The observed off-diagonal CKM elements and the light quark masses are generated radiatively through squark-gluino loops, explaining their smallness in a natural way. In ref. [37] it has been found that this setup of Radiative Flavour Violation (RFV) complies with all FCNC bounds, if the squark masses are larger than roughly 500 GeV. By contrast, the bilinear SUSY breaking terms cannot be the spurion fields breaking $[U(2)]^3 \times U(1)$ without violating the constraints from FCNC processes. The idea that SUSY breaking could be the origin of flavour violation is not new [38, 39], remarkably the absence of tree-level light-fermion Yukawa couplings substantially alleviates the supersymmetric CP problem associated with electric dipole moments [39].

The finding that loop contributions involving $A^{q}_{ij}, q = u, d$, can be large has also consequences for the generic MSSM: In FCNC analyses aiming at constraints on flavour-violating SUSY-breaking terms one must include chirally enhanced higher-order corrections involving $A^{q}_{ij}$ and, if $\tan \beta$ is large, also corrections with bilinear SUSY-breaking terms [40]. The trilinear terms further imply important loop corrections to quark and lepton masses [41] and can induce right-handed $W$ couplings [42].

3.4. MSSM with GUT constraints. – In grand unified theories (GUTs) quarks and leptons are combined into symmetry multiplets. As a consequence, it may be possible to see imprints of lepton mixing in the quark sector and vice versa. In particular, the large atmospheric neutrino mixing angle may influence $b \rightarrow s$ transitions through the mixing of right-handed $\tilde{b}$ and $\tilde{s}$ squarks [43]. Yet the usual small dimension-4 Yukawa interactions of the first two generations are sensitive to corrections from dimension-5 terms which are suppressed by $M_{\text{GUT}}/M_{\text{Planck}}$ [44]. These contributions are welcome to fix the unification of the Yukawa couplings, but may come with an arbitrary flavour
structure, spoiling the predictiveness of the quark-lepton flavour connection. SU(5) and SO(10) models with dimension-5 Yukawa couplings have been studied in great detail [45]. Phenomenologically one can constrain the troublesome flavour misalignment using data on FCNC transitions between the first two generations. Here I present a recent SU(5) analysis exploiting the experimental bound on $B(\mu \to e\gamma)$ [46]. At the GUT scale the Yukawa matrices for down-type quarks, $Y_d$ and $Y_l$, read

$$Y_d = Y_{\text{GUT}} + k_d \frac{\sigma}{M_{\text{Planck}}} Y_{\sigma}, \quad Y_l^\dagger = Y_{\text{GUT}} + k_e \frac{\sigma}{M_{\text{Planck}}} Y_{\sigma}. \quad (11)$$

Here $Y_{\text{GUT}}$ is the unified dimension-4 Yukawa matrix, $\sigma = \mathcal{O}(M_{\text{GUT}})$ is a linear combination of Higgs vevs and the prefactors $k_d$ and $k_e$ differ from each other due to GUT breaking. If the universality condition $A_l = A_d = a_0 Y_{\text{GUT}}$ is invoked at the GUT scale, any misalignment between $Y_{\text{GUT}}$ and $Y_{\sigma}$ will lead to a non-MFV low-energy theory, because $A_l \propto Y_l$ and $A_d \propto Y_d$. We may parametrise this effect as

$$A_l \simeq A_0 \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} Y_l. \quad (12)$$

Now the experimental upper bound on $B(\mu \to e\gamma)$ determines the maximally allowed $|\theta|$ as a function of $A_0$. In ref. [46] it is found that $|\theta|$ can hardly exceed 10 degrees once $|A_0|$ exceeds 50 GeV. An analysis in the quark sector (studying SO(10) models [43]) finds similar strong constraints from $\epsilon_K$ [47]. As a consequence, the dimension-5 terms can barely spoil the GUT prediction derived from the dimension-4 relation $Y_d = Y_l^\dagger = Y_{\text{GUT}}$, unless $|A_0|$ is small. This result may indicate that dimension-4 and dimension-5 Yukawa couplings are governed by the same flavour symmetries.

Acknowledgements

I thank the organisers for the invitation to this wonderful conference and for financial support. The presented work is supported by the DFG Research Unit SFB–TR 9, by BMBF grant no. 05H09VKF and by the EU Contract No. MRTN-CT-2006-035482, “FLAVIAnet”.

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