A novel baseline-free approach for acousto-ultrasonic crack monitoring of rotating axles

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Abstract
Axles are widely used mechanical components in rotating machines. In many applications, axles are exposed to varying environmental and operational conditions, including for example, temperature, load and rotational speed. To apply acousto-ultrasonics-based structural health monitoring for axles, it is in some cases not economically feasible to collect data and to construct a baseline model for each combination of all possible environmental and operational conditions. In consideration of such a practical limit, a novel baseline-free method, called Dynamic Reference Method, is proposed in this article for the detection of transverse cracks in axles. Piezoelectric wafer active sensors are applied for exciting and sensing ultrasonic waves. In case a transverse crack is present in an axle, it is noticed that the crack will sequentially open and close within one revolution of the axle when it is mechanically loaded in the radial direction. Such an opening and closing phenomenon of the transverse crack will influence the propagation of the ultrasonic waves. During the short time of data acquisition, the environmental and operational conditions remain nearly constant and the changes in the propagation of the ultrasonic waves may indicate the appearance of a transverse crack on the axle, and a baseline model for the environmental and operational conditions is not needed. To validate the proposed baseline-free method, an axle is put on a rotating bending test rig to first initiate a fatigue crack on the surface of the axle and second to observe its growth. During the test, the ultrasonic waves are measured by piezoelectric wafer active sensor in situ and are analysed online using the proposed baseline-free method. According to the test results, not only can the crack be effectively detected, but also the progressive changes of the crack size can be differentiated. Such results demonstrate the potential of the proposed baseline-free method in condition-based maintenance and predictive maintenance for critical axle components.

Keywords
Axle crack detection, acousto-ultrasonics, structural health monitoring, baseline-free, piezoelectric wafer active sensors, condition-based maintenance, predictive maintenance, environmental and operational conditions, acousto-elastic effect

Introduction
In order to prevent catastrophic failures, safety critical components in civil infrastructure, aircrafts and mechanical systems need to be monitored. For this reason, structural health monitoring (SHM) and non-destructive testing (NDT) techniques are moving into the focus of research communities and potential industrial users. This article presents an acousto-ultrasonics (AU)-based method which has the potential to monitor permanently the health condition of mechanically loaded and rotating axles during operation. Thus, the presented method actually provides the possibility to realize condition-based maintenance (CBM) and predictive maintenance. This in turn will significantly reduce the maintenance cost and increase availability.

AU-based methods are a group of well-established active methods for SHM with a wide range of applications. These methods use the principle that ultrasonic waves travel through the material and interact with discontinuities in the structure. Such kinds of SHM systems need at least one actuator which is capable of exciting the ultrasonic waves and at least one sensor which is used to measure the structural response. In
this study, piezoelectric wafer active sensors (PWASs) are applied considering its small size and reasonable price. PWASs operate on the piezoelectric principle and can function either as actuators or as sensors. They are permanently bonded to the structure. Each structural response has a characteristic wave pattern which is caused by reflections at discontinuities. Possible cracks in the structure will change this pattern. To extract these changes, most SHM systems first need to collect data from the structure in healthy condition to build the so-called baseline. With the help of signal processing and feature extraction, the structural health condition is assessed through the comparison of the baseline and the newly acquired data.\(^2,3\) The method which is presented in this study is based on the difference in the propagation of ultrasonic waves that is generated between the opening and the closing of a fatigue crack, and it does not need to collect data from a healthy structure in advance. In this sense, the presented method is baseline-free.

Mechanically loaded and rotating axles often work under varying environmental and operational conditions (EOCs), for example, temperature, rotational speed and load. In some earlier studies, it has been reported that temperature has dominant effects on the ultrasonic wave propagation. In order to compensate such temperature effects, methods like optimal baseline selection and baseline signal stretching have been developed by for example, Lu and Michaels,\(^7\) Konstantinidis et al.\(^8\) and Croxford et al.\(^9\) These methods work well for the cases which exhibit a homogeneous temperature change within the monitored structure. However, this is not always the case. In many applications, the distribution and the change of the temperature on axles are not always homogeneous. Furthermore, it might be not economically feasible to collect data and to construct baselines for different types of axles under all different combinations of EOCs. In consideration of such practical factors, this article presents a baseline-free method which is based on the difference that is generated in the ultrasonic wave propagation between the opening and the closing of a fatigue crack. The recorded structural response data are analysed with a principal component analysis (PCA)-based damage detection method described by Mujica et al.\(^10\)

**Methodology**

The ultrasonic waves can travel through a structure and interact with discontinuities within the structure. Even in healthy condition, the structure has discontinuities, for example, edges, holes or thickness changes, which influence the propagation of the ultrasonic waves.\(^11\) In this article, the focus is on the difference in the propagation of ultrasonic waves which are generated between the opening and the closing of a fatigue crack in mechanically loaded and rotating axles. The opening and the closing of a fatigue crack during one revolution of an axle is described as the crack breathing mechanism.\(^12\)

**Influence of the crack breathing mechanism on the ultrasonic wave propagation**

During one revolution of an axle, the crack will be fully open/closed or partially open/closed. Assuming an incipient crack on the surface of an axle, the crack will be open at the upper half of the cross-section of the axle when it is under local tensile stress \((\sigma_{xx} \leq 0)\). Figure 1(a) shows the location of the crack for this case. Such an open crack has separated crack faces, which forms a discontinuity and influences the ultrasonic wave propagation. As described in Figure 1(b), the incoming ultrasonic waves, indicated by solid lines, travel with the group velocity \(v\) and are influenced by the open crack, generating some reflected ultrasonic waves, which are indicated by dashed lines, and some transmitted ultrasonic waves, indicated by dotted lines. The above description can be supported with the theory proposed by Giurgiuțiu,\(^2\) Bachschmid et al.\(^12\) and Rose.\(^11\) As the axle rotates, the crack will move from the upper half to the lower half of the cross-section of the axle, as Figure 1(c) indicates. At this location, the crack is under local compressive stress \((\sigma_{xx} \geq 0)\) and set to be closed. But the closed crack, as a discontinuity, has less influence on the ultrasonic wave propagation compared to an open crack. Such a difference can be noticed by comparing Figure 1(b) with Figure 1(d).

In addition, the ultrasonic wave propagation is influenced by the change of the stress distribution surrounding the crack and the crack tip plastic zone.\(^13,14\)

Figure 2 clearly shows the difference of the stress distribution surrounding the crack for an open and a closed crack. The pictures were taken from a numerical simulation model of a semi-elliptical crack with the crack length of \(2a = 15\) mm and the crack depth of \(c = 4.7\) mm at the surface of a hollow steel axle under a rotating bending load condition. Due to the rotating bending, crack breathing occurs. The ultrasonic wave propagation is influenced by the stress distribution because of the acousto-elastic effect, which is described in the next section. It is obvious that the crack breathing mechanism changes the stress distribution and in turn the ultrasonic wave propagation of the ultrasonic waves changes during one revolution of the axle in the presence of a crack.

Clark et al.\(^15\) show that the propagation of ultrasonic waves is influenced by the stress distribution surrounding a crack tip. Furthermore, they determine the entire stress field in the region of interest around the crack tip with an acousto-elastic measurement method.
Influence of stress on the ultrasonic wave propagation

The basic theory of the bulk elastic wave propagation for isotropic material subjected to stress is proposed by Murnaghan.\textsuperscript{16} Based on this theory, Hughes and Kelly\textsuperscript{17} developed the formulation of the influence of stress distribution on the propagation of ultrasonic waves in the theory of acousto-elasticity. They consider the change in wave-speed in isotropic elastic materials in the presence of uni-axially applied stress and hydrostatic pressure. The velocity of the longitudinal wave $c_{xx(x)}$ in the direction of the applied stress $x$ is given in equation (1)\textsuperscript{11}

$$\rho_0 c_{xx(x)}^2 = \lambda + 2\mu + \frac{\sigma}{3K_0} \left[ \frac{\lambda + \mu}{\mu} \left( 4\lambda + 10\mu + 4m \right) + \lambda + 2\mu \right].$$

(1)

The velocity of the shear wave propagation in the direction of the applied stress $x$ and polarized in the $y$- and $z$- direction $c_{xy(x)}$ and $c_{xz(x)}$ are given in equation (2)\textsuperscript{11}

$$\rho_0 c_{xy(x)}^2 = \rho_0 c_{xz(x)}^2 = \mu + \frac{\sigma}{3K_0} \left[ \frac{\lambda n}{4\mu} + 4\lambda + 4\mu + m \right].$$

(2)

Equations (1) and (2) are dependent on the two Lamé constants $\lambda$ and $\mu$, where $K_0 = \lambda + (2/3)\mu$. Furthermore, the third-order elastic constants $l$, $m$ and $n$, which have been proposed first by Murnaghan,\textsuperscript{16} are required. $\rho_0$ denotes the density of the undeformed material. The value of $\sigma$ is positive for an applied tensile stress and negative for an applied compressive stress.

In this article, mechanically loaded and rotating axles are under investigation. Due to the loading condition, a bending stress appears in the structure. The influence of bending stress on the propagation of ultrasonic waves in an elastically deformed material is described in, for example, Si-Chaib et al.\textsuperscript{18} in which numerical and experimental evaluations are made to study the change in the wave velocity of longitudinal and shear waves in a cylindrical sample under three-point bending. For the half of the cross-section of the sample under local tensile stress, the longitudinal wave velocity decreases while the shear wave velocity...
polarized in the $y$- and $z$-direction increases. For the half of the cross-section of the sample under local compressive stress, the longitudinal wave velocity increases while the shear wave velocity polarized in the $y$- and $z$- direction decreases.

For the described sample in this article, the stress distribution will also change during one revolution of the axle, which will lead to a dependence of the ultrasonic wave propagation on the rotation angle of the axle. The influence of stress on the ultrasonic wave propagation are provided in the section on the experimental results.

**Damage detection**

The main steps of the presented baseline-free method are illustrated by the flow chart in Figure 3. The core of the process is the Dynamic Reference Method, which will be described later.

**Data acquisition and denoising**

In the first step, the structural response data are acquired at different rotation angles $\phi_i$ within one revolution of the axle, ranging from $0^\circ$ to $360^\circ$ with a fixed step-size $\Delta \phi$. In order to minimize the effect from measurement noise and to generate the input data for the PCA-Model, data from several revolutions are recorded.

In the denoising step, the recorded data sets are processed with digital filters and discrete wavelet transform (DWT) in order to enhance the useful information in the data. In the present article, the wavelet multi-resolution analysis (WMRA) is used as a kind of low-pass filter to reduce noise in the sensor signals. This method is widely used to reduce noise in sensor signals very efficiently. For filtering, the DWT is terminated after a certain level and the approximation of the signal, which contains the low-frequency components of the signal, is used as the filtered sensor signal. The level of the DWT is chosen so that, based on the sampling frequency, the maximum frequency of the approximation of the signal is close to the highest frequency of the frequency range of the excitation signal. In this article, Daubechies-wavelets $db8$ are used. Remaining undesired low-frequency components in the structural responses are filtered with a Butterworth-filter (high pass). The cut-off frequency of the filter is close to the lowest frequency of the frequency range in the excitation signal. The filter coefficients are calculated with the Signal Processing Toolbox of MATLAB.

**PCA**

To detect the difference in the ultrasonic wave propagation, the following damage detection method based on a statistical approach is used.

The denoised data are compressed with the help of the PCA technique which is an eigenvector-based multivariate statistical approach to reduce a complex data set to a lower dimension and to extract hidden features. Details on the PCA technique can be found in, for example, Jolliffe's study. The damage indicator is calculated in the form of a squared prediction error (SPE) which is sensitive to changes that are not represented by the PCA model. The main equations in PCA and the equation for calculating the damage indicator are provided below.

For the purpose of building the PCA model, $M$ data sets with each having $Q$ sample points of one reference experiment, for example, at one discrete rotation angle of the axle, is arranged in the $(M \times Q)$ data matrix $X_{\text{raw}}$. Before applying the PCA, each data set $X_{\text{raw}}$ is normalized to have zero mean which leads to the $(M \times Q)$ data matrix $X$.

The covariance matrix $C_X$ from the normalized data matrix $X$ is a square symmetric matrix $(Q \times Q)$ and measures the degree of linear relationships between all possible pairs of data samples. The covariance matrix $C_X$ is computed by

$$C_X = \frac{1}{M-1}X^T X.$$

The eigenvectors $p_i$ and eigenvalues $\lambda_i$ of the covariance matrix $C_X$ define the subspace of the PCA with $P = [p_1 \, p_2 \, \ldots \, p_i]$ and $A = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_i)$ for $i \leq Q$ as follows

$$C_X P = PA.$$
The eigenvectors $p_i$ are sorted in $P$ by descending eigenvalues and are called the principal components (PCs). The score matrix $T$ is the projection of the original data set $X$ onto the space which is spanned by the PCs

$$T = XP.$$  \hfill (5)

For the full set of eigenvectors, which means $i = (1, 2, \ldots, Q)$, this projection is invertible and the original data set $X$ can be recovered using the score matrix $T$ and the PCs $P$ by $X = TP^T$. For the PCA model of the reference data set only a reduced number ($K < Q$) of the PCs $P$ is chosen. The eigenvectors $p_i$ with the highest eigenvalues $\lambda_i$ contain the most information of the normalized data matrix $X$ of the acquired structural responses (AU signals). These eigenvectors $p_i$ with $i = 1, 2, \ldots, K$ are used as the extracted features of the acquired data. For the application in this article, it has been shown that the reduced number $K$ of the eigenvectors $p_i$ should be chosen so that the sum of the first $K$ eigenvalues $\lambda_i$ with $i = 1, 2, \ldots, K$ is less than or equal to 0.95. Concerning this reduction, it is not possible to fully recover the original data set $X$ with $T$ and $P$

$$X = TP^T.$$  \hfill (6)

### PCA-based SPE

Based on the difference between $X$ and the projection $X$ there are various statistical approaches that quantity the precision of the PCA model or the difference between the model of the reference data set and other experimental data sets such as the data sets taken at different rotation angles during one revolution of the axle. The Hotelling’s $T^2$-statistic or $Q$-statistic (SPE-statistic) are frequently used. A comparison between $T^2$-statistic and $Q$-statistic is given by Mujica et al. In this article, the SPE-statistic is used, because the SPE is sensitive to changes which are not represented in the PCA model. Therefore, the SPE-statistic is able to simultaneously quantify changes in the structural response in the form of a frequency and amplitude variation, which results from the difference in ultrasonic wave propagation between an open and a closed crack. The residual matrix $X$ is defined as below

$$\hat{X} = X - \bar{X} = X (1 - PP^T).$$  \hfill (7)

The variability of the data projected within the residual subspace is represented by the SPE and is defined for the $j$th experiment vector $x_j$ of $X$ by

$$SPE_j = x_j (1 - PP^T) x_j^T.$$  \hfill (8)

The SPE quantifies the difference between a sample and its projection into the PCA model, which indicates abnormal situations.

With the described statistical approach, the presented SHM method evaluates the differences during the rotation of the axle caused by the crack breathing mechanism. In this study, a PCA model is built with a data set which is recorded at one reference rotation angle $\phi$. An SPE is calculated for each data set recorded at the other rotation angles $\phi_i$ with reference to $\phi$. Here $\phi_i$ are the discrete rotation angles for which the data sets are recorded during the revolution of the axle.

In order to compare all data sets which are recorded at different rotation angles $\phi_i$, all calculated SPE values $SPE_i$ for one specific rotation angle $\phi_i$ are summarized in its mean value $\bar{SPE}$.

### Dynamic Reference Method

In this study, a Dynamic Reference Method is proposed and applied for the detection of transversal cracks in mechanically loaded and rotating axles. This methodology is based on the acousto-elastic effect and the crack breathing mechanism.

As previously discussed in the section “Methodology”, the stress distribution has a significant influence on the propagation of ultrasonic waves. The difference in ultrasonic wave propagation compared to the ultrasonic wave propagation at $\phi = 0^\circ$ increases as the rotation angle increases from $0^\circ$ to $180^\circ$. The difference in ultrasonic wave propagation reaches a maximum, if the difference between the reference rotation angle $\phi$ and the rotation angle $\phi_i$ is $180^\circ$. Comparing these two rotation angles, the stress distribution in the cross-section of the axle is in reverse to each other. As the rotation angle $\phi_i$ increases from $180^\circ$ to $360^\circ$, the difference in ultrasonic wave propagation decreases. The ultrasonic wave propagation at $\phi = 0^\circ$ and $360^\circ$ are expected to be equal.

For an undamaged mechanically loaded axle, the stress distribution for the rotation angles $0^\circ \leq \phi_i \leq 180^\circ$ is symmetrical to the stress distribution between the rotation angles $180^\circ \leq \phi_i \leq 360^\circ$. Comparing $\phi = 0^\circ$ and $\phi = 360^\circ$ after one revolution of the axle, there is no difference in ultrasonic wave propagation expected, if the EOCs do not change during this revolution.

The changes in ultrasonic wave propagation due to the stress dependence can be quantified by the SPE. Quiroga et al. show that the PCA and in particular the SPE is able to differentiate various stress scenarios in cylindrical steel specimens using PWAS and ultrasonic waves.

For an undamaged mechanically loaded axle, the pattern of the SPE values for one revolution theoretically increases for the rotation angles $0^\circ \leq \phi_i \leq 180^\circ$. The maximum of the SPE values is reached at the rotation
angle \( \varphi_i = 180^\circ \). Due to the symmetry of the stress distribution between the rotation angles \( 0^\circ \leq \varphi_i \leq 180^\circ \) and \( 180^\circ \leq \varphi_i \leq 360^\circ \), the pattern of the SPE values is expected to be nearly symmetrical to \( 180^\circ \). After one revolution of the axle, the SPE value corresponding to the structural response at \( \phi = 0 \) and \( \varphi = 360^\circ \) is theoretically zero.

For a damaged axle, the crack will be fully open/closed or partially open/closed during one revolution. As previously described, the crack breathing mechanism influences the ultrasonic wave propagation and generates a difference in ultrasonic wave propagation between an open and a closed crack. The influence on the ultrasonic wave propagation will lead to a change in the pattern of the SPE values. Due to such a change, the pattern of the SPE values is no longer symmetrical to \( 180^\circ \).

Using only one randomly chosen reference rotation angle \( \phi \), it is possible that the crack is fully open or fully closed at this rotation angle. It is expected for the selection of this reference rotation angle that the symmetry of pattern of the SPE values do not change significantly. This would lead to an unreliable detection of cracks with the proposed SHM method. In order to avoid this issue, since the rotational position of the crack is unknown, it is necessary to rotate the reference rotation angle \( \phi \) which is illustrated in Figure 4.

The Dynamic Reference Method can be divided into different steps. In the first step, the data sets are recorded at different rotation angles \( \varphi_i \), ranging from \( 0^\circ \) to \( 360^\circ \) with a certain step-size \( \Delta \phi \). In the second step, the PCA model is built with the data set of \( \phi = 0^\circ \), which then is compared with all other data sets of the rotation angles \( 0^\circ \leq \varphi_i \leq 360^\circ \) of one full revolution by the SPE. In the next step, the reference rotation angle \( \phi \) is shifted to \( \phi = 10^\circ \). The PCA-Model is built for the data set of \( \phi = 10^\circ \) which is compared to the data sets taken between \( \varphi_i = \phi \) and \( \varphi_i = \phi + 360^\circ \). This step is repeated until \( \phi \) reaches \( 360^\circ \).

For each reference rotation angle \( \phi_n \), a damage indicator based on a correlation coefficient is calculated to detect and to localize a potential damage. This is in accordance with the first two levels of SHM defined by Rytter.\textsuperscript{27} In order to quantify the size of a damage, a damage indicator based on a mean squared error (MSE) is calculated. The assessment of a damage corresponds to the third level of SHM defined by Rytter.\textsuperscript{27} Details on the damage indicator based on a correlation coefficient and the MSE are provided in the section “Damage indicator for the Dynamic Reference Method”.

Basically, the Dynamic Reference Method can be broken down into individual analyses. The reference rotation angle \( \phi_n \) for the \( n \)th analysis is defined as

\[
\phi_n = (n - 1)\Delta \phi; \quad \text{for} \quad n = [1, 2, \ldots, N + 1]. \tag{9}
\]

It has to be mentioned that there is no need for new measurements for the Dynamic Reference Method. All analysis is performed based on the same data sets.

For an undamaged mechanically loaded axle, it is expected that the pattern of the SPE values is nearly symmetrical to \( 180^\circ \) and nearly identical for all reference rotation angles \( \phi_n \). This is due to the fact that apart from the influence of the stress distribution on the ultrasonic wave propagation, no other angle-dependent influence occurs.

For a damaged mechanically loaded axle, an angle-dependent influence, such as the crack breathing mechanism, leads to different patterns of the SPE values in dependence of the reference rotation angle \( \phi_n \). As described previously, it is expected that these patterns are not symmetrical to \( 180^\circ \), which indicates the presence of a crack in the axle.

**Damage indicator for the Dynamic Reference Method**

In order to detect and to localize a crack, it is necessary to quantify the symmetry of the patterns of the SPE values. One possible indicator is the correlation coefficient \( r \) between the SPE values \( \overline{\text{SPE}} \) which correspond to rotation angles \( \phi_n \leq \varphi_i \leq \phi_n + 180^\circ \) and the SPE values \( \overline{\text{SPE}}_i \) for rotation angles \( \phi_n + 180^\circ \leq \varphi_i \leq \phi_n + 360^\circ \).
For an undamaged mechanically loaded axle, a correlation coefficient $r = 1$ is expected. For a damaged mechanically loaded axle, a correlation coefficient $r < 1$ may indicate an angle-dependent influence, such as the crack breathing mechanism.

Using the Dynamic Reference Method, the correlation coefficient $r$ is calculated for each pattern of the SPE values of each individual analysis $n$ for the corresponding reference rotation angle $\phi_n$. In the undamaged case, the correlation coefficient $r_n$ for each reference rotation angle $\phi_n$ is theoretically $r_n = 1$. In presence of an angle-dependent influence, such as the crack breathing mechanism, the correlation coefficients $r_n$ are expected to be smaller than $r_n < 1$ and vary in dependence of the reference rotation angle $\phi_n$. If the reference rotation angle $\phi_n$ corresponds to the rotation angle, where the crack is fully open or fully closed, the correlation coefficient $r_n$ is expected to be $r_n = 1$. This in fact can lead to a localization of the approximate circumferential position of the crack.

In order to visualize the health condition of the mechanically loaded and rotating axle during operation, the correlation coefficients $r_n$ can be plotted in dependence of each reference rotation angle $\phi_n$ in a polar plot as shown in the section “Experimental results and discussion”.

A single damage indicator to quantify the symmetry of the patterns of the SPE values for all reference rotation angles $\phi_n$ can be determined with the MSE between the SPE values $\text{SPE}_i$ which correspond to rotation angles $\phi_i \leq \phi_n \leq \phi_i + 180^\circ$ and the SPE values $\text{SPE}_i$ for rotation angles $\phi_i + 180^\circ \leq \phi_i \leq \phi_i + 360^\circ$. The MSE of the SPE values $\text{SPE}_i$ for the analysis of one reference rotation angle $\phi_n$ is determined by

$$\text{(MSE)}_n = \frac{1}{N/2 + 1} \sum_{i=1}^{N/2 + 1} \left( \text{SPE}_i - \text{SPE}_{(N+2) - i} \right)^2.$$

(11)

$\text{(SPE)}_n$ is the mean value of the SPE for all data sets taken for each of the rotation angles $\phi_i$ for each reference rotation angle $\phi_n$. The MSE for all reference rotation angles $\phi_n$ of one experiment is given by

$$\text{MSE} = \frac{1}{N + 1} \sum_{n=1}^{N+1} (\text{MSE})_n.$$

(12)

In such a manner, cracks can be detected with the proposed Dynamic Reference Method, and previously collected baselines for different EOCs are not needed. The reference data sets are all taken during a short time of data acquisition during a few revolutions of the axle. During the short time of data acquisition, the EOCs remain nearly constant, and the methodology focuses only on the difference in ultrasonic wave propagation due to the changing stress distribution and the angle-dependent influence of the crack breathing mechanism. In this sense, the Dynamic Reference Method is baseline-free.

**Experimental setup**

In order to validate the method described in the previous section, a dynamic experiment was performed. The test object is a hollow steel axle with several diameter transitions between 130 and 205 mm and a length of 2200 mm. In the experiment, the axle was put on a rotating bending test rig to first initiate a fatigue crack on the surface of the axle and second to observe its growth. With the rotating bending test rig, it is possible to apply a rotating bending moment as it appears in real operation. A drawing of the test rig is shown in Figure 5. To control the location of the starting crack and to accelerate this process, a small notch (drilled...
hole with a diameter of 1 mm and a depth of 1 mm) is introduced in the axle at a transition between a diameter of 170 and 205 mm, before starting the dynamic test. At the position of the notch, the axle is a hollow cylinder with an outer diameter of 170 mm and an inner diameter of 30 mm. During the operation of the rotating bending test rig, sensor data are recorded and analysed on-line to monitor the health condition of the axle. As a reference, crack propagation gauges are applied to monitor the crack growth during fatigue.

Figure 6 shows the setup used for data acquisition. A PWAS is attached to the front surface of the axle. The PWAS is used as an actuator and as a sensor using a transmit and receive switch (T/R-switch). A high-resolution oscilloscope is used for generating the excitation signal for the actuator and acquiring the measured structural response. A sampling rate of $f_s = 1 \text{ MHz}$ and a acquisition time of $t_s = 6 \text{ ms}$ for each measured structural response is chosen. Therefore, the acquisition time is much shorter than the time for one revolution of the axle. In order to increase the voltage of the excitation signal, a high-voltage amplifier is used. The excitation signal is a Hanning-windowed tone burst with a carrier frequency in the range of 100–200 kHz depending on the size of the PWAS.

The data are recorded at different rotation angles $\phi$, ranging from 0° to 360° with a step-size of $\Delta \phi = 5.625°$. For each of the rotation angles $\phi$, where data acquisition was performed, 25 data sets are recorded and analysed. To control the rotational angle an incremental encoder is used. The incremental encoder is coupled with a triggering system, which operates on an embedded system and starts the measurement chain angle controlled. The triggering system is developed by Wiese et al. A laptop using MATLAB controls the measurement chain and analyzes the recorded data.

The results from the dynamic experiment are provided in the next section.

**Experimental results and discussion**

In the following section, the proposed baseline-free AU-based method for detection of cracks in mechanically loaded and rotating axles is validated. First, the influence on ultrasonic wave propagation due to a rotating bending moment in the axle is considered. Then, the Dynamic Reference Method for crack detection is applied during the rotating bending fatigue test. Different crack sizes are considered. This is followed by an investigation of the influence of different stress levels on the Dynamic Reference Method for crack detection during the rotating bending fatigue test.

**Influence of an applied rotating bending moment on ultrasonic wave propagation**

The following experimental investigation shows the influence on the ultrasonic wave propagation due to a rotating bending moment. With the rotating bending test rig, three different rotating bending moments are realized. The applied rotating bending moments result in stress ranges of ±35, ±50 and ±65 MPa. The stresses are measured by strain gauges at a transition between a diameter of 170 and 205 mm close to the fixed support of the axle at the test rig.

Figure 7 shows the patterns of the calculated SPE values depending on the rotation angle $\phi$ of the axle for a stress level of ±35, ±50 and ±65 MPa. In order to investigate the influence on the ultrasonic wave propagation due to a rotating bending moment for the undamaged case, it is enough to consider only one reference rotation angle $\phi$. Each marker represents the mean value SPE$_i$ of the SPE values of the 25 data sets for each of the rotation angles $\phi_i$. For all three stress levels the profile of the undamaged case in Figure 7 clearly shows that the calculated SPE values are dependent on the rotation angles and are nearly symmetrical to 180°. The SPE values at 0° and 360° are close to zero. This is due to the fact that the model, which is used for calculating the SPE values, is based on the data recorded at $\phi = 0°$. Furthermore, it is noticed that the SPE value increases to a maximum as the rotation angle increases to 180° and then decreases to zero as the rotation angle reaches 360°. The dependence of the SPE value on the rotation angle is described in section “Methodology” by the acousto-elastic effect.
As the stress level increases, the maximum of the SPE values at 180° increases. This dependence is caused by the acousto-elastic effect and is described by the equations (1) and (2).

In real operation of an axle, the stress level may not always be the same. To be able to compare different experiments with different load levels in the following, the maximum of the SPE values is normalized to \( \text{SPE}_i = 1 \) at the rotation angle \( \phi_i = \phi_n + 180° \).

**Application of the Dynamic Reference Method**

In this section, the Dynamic Reference Method for crack detection in mechanically loaded and rotating axles is applied. The data are recorded during the rotating bending fatigue test as described in the section “Experimental Setup.” During the fatigue test on the rotating bending test rig four crack sizes are examined in more detail. The crack sizes are listed in Table 1. The crack length is determined by means of crack propagation gauges. The crack depth \( c \) can only be estimated for this test, as the axle would have been broken up to examine the crack surface. As an approximation, a semi-elliptical surface crack with the ratio of crack depth \( c \) and crack length \( 2a \) of \( c/a = 0.8 \) is assumed. Figure 8 provides an image of the propagated crack which finally reached a length of \( 2a = 100 \text{ mm} \) and an estimated depth of about \( c = 40 \text{ mm} \). It can be seen that the crack propagates approximately 50 mm to the left side and 50 mm to the ride side of the notch. About 25 mm to the left of the notch, the direction of the propagating crack changed slightly upwards. Whereas on the right side, the direction of the propagating crack changed slightly downwards.

Figure 9 shows the calculated mean values \( \text{SPE}_i \) of the SPE values of 25 data sets for each of the rotation angles \( u_i \) of the axle for the undamaged case with an applied bending stress level of \( \pm 35, \pm 50 \) and \( \pm 65 \text{ MPa} \).

As the stress level increases, the maximum of the SPE values at 180° increases. This dependence is caused by the acousto-elastic effect and is described by the equations (1) and (2).

![Figure 7](image-url)  
**Figure 7.** Calculated mean values \( \text{SPE}_i \) of the SPE values of 25 data sets for each of the rotation angles \( \phi_i \) of the axle for the undamaged case with an applied bending stress level of \( \pm 35, \pm 50 \) and \( \pm 65 \text{ MPa} \).

![Figure 8](image-url)  
**Figure 8.** Propagated crack from starter notch at a transition between two diameters of the axle. Crack propagation gauges were applied to monitor the crack growth during fatigue.
Figure 9. Calculated mean values $\text{SPE}_i$ of the SPE values of 25 data sets for each of the rotation angles $\phi_i$ of the axle for the reference rotation angles $\phi = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ$ and $360^\circ$ for the undamaged case (0 mm) and for different crack sizes listed in Table I during the fatigue test on the rotating bending test rig.
rotation angles $\phi_n$, the pattern of the SPE values are nearly symmetrical to 180°.

In contrast, the results with a crack in the axle are strongly dependent on the selected reference rotation angle $\phi_n$. The pattern of the SPE values are different for each reference rotation angle $\phi_n$. It is revealed, that the profiles of the SPE values for all crack sizes are not symmetrical to 180° for most reference rotation angles $\phi_n$.

For the reference rotation angles $\phi = 0°$, 180° and 360°, there is a slight symmetry visible. This is due to the fact that the crack is fully closed for the reference rotation angles $\phi = 0°$ and 360° and fully open for the reference rotation angle $\phi = 180°$. In this case, the maximum difference in the ultrasonic wave propagation is between $\phi_n$ and $\phi_n + 180°$. Therefore, the shape of the SPE values for the data sets between the rotation angle from 0° to 180° and the rotation angle from 180° to 360° differ only slightly. The previously described normalization of the maximum of the SPE values at the rotation angle $\phi_n = \phi_n + 180°$ lets the increased SPE vanish to a normalized value of $\text{SPE}_n = 1$. The increase is caused by the difference in the ultrasonic wave propagation between the open and closed crack. However, the normalization is necessary to be able to compare different experiments with different load levels and to be able to use the presented Dynamic Reference Method, when the load level of the axle is unknown. This, in particular, is relevant for an application of the method in real operation.

The patterns of the SPE values for all crack sizes show an obvious deviation from that representing the undamaged case. Furthermore, the deviation increases as the crack size increases. This indicates that the pattern of the SPE values is related with the health condition of the axle, and an unsymmetrical pattern can be considered as an indication of the existence of a crack.

Figure 10 shows a polar plot of the correlation coefficients $r_n$ in dependence of each reference rotation angle $\phi_n$ for the undamaged axle (0 mm) and for different crack sizes listed in Table 1 during the fatigue test on the rotating bending test rig.

As previously described, for the reference rotation angles $\phi = 0°$, 180° and 360°, there is a slight symmetry visible in the patterns of the SPE values due to the relation between the circumferential position of the open/closed crack and the reference rotation angle $\phi_n$. For these reference rotation angles, this leads to correlation coefficients $r_n$ close to 1. Furthermore, this can lead to an identification of the approximate circumferential position of the crack with the help of the previously described dependence of the correlation coefficients $r_n$ on the reference rotation angle $\phi_n$.

Nevertheless, correlation coefficients $r_n < 1$ and the dependence of the correlation coefficients $r_n$ on the reference rotation angle $\phi_n$ may indicate the existence of a crack on the surface of the axle. In such a manner, the crack can be detected, the crack propagation can be observed, and the approximate circumferential position of the crack can be localized by the proposed Dynamic Reference Method using only the calculated correlation coefficients $r_n$. Therefore, a previously collected baseline is not needed.

**Influence of rotating bending moments on the Dynamic Reference Method**

In order to analyse the influence of rotating bending moments on the Dynamic Reference Method, the fatigue test is interrupted at different crack sizes. During
the interruption, data are recorded for different stress levels ranging from 20 up to 155 MPa. The bending stresses in the crack plane are measured with strain gauges.

Figure 11 shows the calculated MSE for all reference rotation angles $\phi_n$ for the undamaged axle (0 mm) and for different crack sizes listed in Table 1 during the fatigue test on the rotating bending test rig in dependence of different bending stresses measured in the crack plane. The MSE is calculated by equation (12).

In the undamaged case, the MSE is very small and almost independent on the stress level. Only a slight increase can be recognized by increasing the stress level. In comparison to the damaged cases, the MSE can be neglected.

The MSE for the crack sizes of $2a = 50$ mm, $2a = 80$ mm and $2a = 100$ mm are strongly dependent on the applied stress level in the crack plane. For all three crack sizes, the following behaviour is noticed:

1. The MSE first increases to a maximum.
2. The maximum of the MSE value increases with an increasing crack size.
3. The applied stress, where the MSE is at a maximum, decreases with an increasing crack size.
4. If the applied stress further increases after the MSE reached the maximum, the MSE decreases.

For the crack size of $2a = 50$ mm, the maximum value for the MSE is reached when the bending stress in the crack plane reaches 65 MPa. Whereas, for a crack size of $2a = 80$ mm, the maximum is reached for 60 MPa. At 50 MPa, the MSE for the crack size of $2a = 100$ mm is maximum.

The dependence of the MSE on the applied stress can be explained with the help of the crack breathing mechanism and the theory of acousto-elasticity. For lower stress levels, the crack opening displacement is not enough to fully separate the crack faces due to fatigue crack closure, in particular roughness-induced crack closure, oxide-induced crack closure and plasticity-induced crack closure. The stress, at which the crack opens completely, decreases as the crack size increases. Therefore, the MSE starts to increase for increasing crack sizes at lower stress levels. For a rising crack size, the angle-dependent influences due to the crack breathing mechanism increase, which leads to higher maximum MSE values. By further increasing the applied stress, the change in the ultrasonic wave propagation due to the acousto-elastic effect during one revolution of the axle has a dominant influence on the pattern of the SPE values. As previously described, the maximum of the SPE values at $\phi_i = \phi_n + 180^\circ$ increases with an increasing stress level. The angle-dependent influences on the ultrasonic wave propagation due to the crack breathing mechanism gets small in comparison to the influence due to the rising stress level, which leads to a decreasing MSE value because the asymmetry in the pattern of the SPE values is less pronounced.

This investigation thus shows that the crack is detected with the Dynamic Reference Method starting from a certain stress as a function of the crack size, but is less well detected at higher stresses. In real operation, however, a further increase in stresses would lead to a higher crack growth rate, which in turn would increase the level of the damage indicator and thus lead to an early intervention of the proposed SHM system.

**Conclusion and outlook**

In this article, a novel baseline-free AU-based SHM method for the detection of transversal cracks in...
mechanically loaded and rotating axles is proposed. The described Dynamic Reference Method utilizes the difference that is generated in the ultrasonic wave propagation between the opening and the closing of a fatigue crack during one revolution of an axle. For the Dynamic Reference Method, a previously collected baseline is not needed.

The proposed Dynamic Reference Method is validated on a mechanically loaded axle through a dynamic experiment on a rotating bending test rig. During the fatigue test, different crack sizes are considered and the influence due to different rotating bending moments in the axle is investigated.

The results from the experiments demonstrate the effectiveness of the proposed method. Cracks can be detected and the crack propagation can be observed during the experiment. Furthermore, the approximate circumferential position of the crack can be localized by the proposed method. The experimental results show a dependence of the proposed damage indicator on the applied rotating bending moment.

As next steps, the Dynamic Reference Method will be validated under consideration of varying EOCs and the correlation between the crack size and the change of symmetry in the profiles of the SPE values will be under investigation. Therefore, more fatigue tests are necessary to statistically validate this correlation. Furthermore, the system will be further developed towards an industrial application.

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Appendix 1

Notation

\(a\) crack length, mm
\(c_{xx}(x)\) longitudinal wave velocity, m/s
\(c_{xy}(x)\) shear wave velocity, m/s
\(c_{xz}(x)\) shear wave velocity, m/s
\(c\) crack depth, mm
\(f_s\) sampling rate, Hz
\(l\) third-order elastic constant, N/m²
\(m\) third-order elastic constant, N/m²
\(n\) third-order elastic constant, N/m²
\(P\) eigenvectors
\(P_r\) correlation coefficient
\(t_s\) acquisition time, s
\(X\) row vector of the normalized data matrix
\(C_X\) covariance matrix
\(F\) force, N
\(K\) reduced number of principal components
\(M\) number of data sets
\(N\) number of rotation angles
\(P\) matrix of eigenvectors
\(P_r\) reduced matrix of eigenvectors
\(Q\) number of sample points
\(\bar{\text{SPE}}_i\) mean-squared prediction error
\(T\) score matrix
\(X_{\text{raw}}\) matrix of acquired raw data
\(\check{X}\) normalized data matrix
\(\hat{X}\) projected normalized data matrix
\(\tilde{X}\) residual matrix
\(A\) matrix of eigenvalues
\(\Delta \phi\) step-size of the discrete rotation angles, °
\(\lambda\) Lamé constant, N/m²
\(\lambda_1\) Lamé constant, N/m²
\(\mu\) reference rotation angles, °
\(\rho_0\) density of the undeformed material, kg/m³
\(\sigma\) stress, MPa
\(\phi_1\) discrete rotation angles, °