Ring wormholes via duality rotations

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Abstract

We apply duality rotations and complex transformations to the Schwarzschild metric to obtain wormhole geometries with two asymptotically flat regions connected by a throat. In the simplest case these are the well-known wormholes supported by phantom scalar field. Further duality rotations remove the scalar field to yield less well known vacuum metrics of the oblate Zipoy-Voorhees-Weyl class, which describe ring wormholes. The ring encircles the wormhole throat and can have any radius, whereas its tension is always negative and should be less than $-c^4/4G$. If the tension reaches the maximal value, the geometry becomes exactly flat, but the topology remains non-trivial and corresponds to two copies of Minkowski space glued together along the disk encircled by the ring. The geodesics are straight lines, and those which traverse the ring get to the other universe. The ring therefore literally produces a hole in space. Such wormholes could perhaps be created by negative energies concentrated in toroidal volumes, for example by vacuum fluctuations.

Wormholes are bridges or tunnels between different universes or different parts of the same universe. They were first introduced by Einstein and Rosen (ER) \cite{1}, who noticed that the Schwarzschild black hole actually has two exterior regions connected by a bridge. The ER bridge is spacelike and cannot be traversed by classical objects, but it has been argued that it may connect quantum particles to produce quantum entanglement and the Einstein-
Pololsky-Rosen (EPR) effect [2], hence ER=EPR [3]. Wormholes were also considered as geometric models of elementary particles – handles of space trapping inside an electric flux, say, which description may indeed be valid at the Planck scale [4]. Wormholes can also describe initial data for the Einstein equations [5] (see [6] for a recent review) whose time evolution corresponds to the black hole collisions of the type observed in the recent GW150914 event [7].

An interesting topic is traversable wormholes – globally static bridges accessible for ordinary classical particles or light [8] (see [9] for a review). In the simplest case such a wormhole is described by a static, spherically symmetric line element

$$ds^2 = -Q^2(r)dt^2 + dr^2 + R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2),$$  \hspace{1cm} (1)

where $Q(r)$ and $R(r)$ are symmetric under $r \to -r$ and $R(r)$ attains a non-zero global minimum at $r = 0$. If both $Q$ and $R/r$ approach unity as $r \to \pm\infty$ then the metric describes two asymptotically flat regions connected by a throat of radius $R(0)$. The Einstein equations $G^\mu_\nu = T^\mu_\nu$ imply that the energy density $\rho = -T^0_0$ and the radial pressure $p = T^r_r$ satisfy at $r = 0$

$$\rho + p = -2 \frac{R''}{R} < 0, \quad p = -\frac{1}{R^2} < 0.$$  \hspace{1cm} (2)

It follows that for a static wormhole to be a solution of the Einstein equations, the Null Energy Condition (NEC), $T^\mu_\nu v^\mu v^\nu = R^\mu_\nu v^\mu v^\nu \geq 0$ for any null $v^\mu$, must be violated. Another demonstration [8] of the violation of the NEC uses the Raychaudhuri equation [10] for a bundle of light rays described by $\theta, \sigma, \omega$: the expansion, shear and vorticity. In the spherically symmetric case one has $\omega = \sigma = 0$ [9], hence

$$\frac{d\theta}{d\lambda} = -R^{\mu\nu} v^\mu v^\nu - \frac{1}{2} \theta^2.$$  \hspace{1cm} (3)

If rays pass through a wormhole throat, there is a moment of minimal cross-section area, $\theta = 0$ but $d\theta/d\lambda > 0$, hence $R^{\mu\nu} v^\mu v^\nu < 0$ and the NEC is violated.

If the spacetime is not spherically symmetric then the above arguments do not apply, but there are more subtle geometric considerations showing that the wormhole throat – a compact two-surface of minimal area – can exist if only the NEC is violated [11, 12]. As a result, traversable wormholes
are possible if only the energy density becomes negative, for example due to vacuum polarization [8], or due to exotic matter types as for example phantom fields with a negative kinetic energy [13, 14]. Otherwise, one can search for wormholes in the alternative theories of gravity, as for example in the Gauss-Bonnet theory [15, 16], in the brainworld models [17], in theories with non-minimally coupled fields [18], or in massive (bi)gravity [19].

The aim of this Letter is to present a method of constructing traversable wormholes starting from the Schwarzschild metric and applying duality rotations and complex transformations. As a first step this gives solutions with a non-trivial scalar field, which, after a complexification, reduce to the well-known wormhole solution of Bronnikov and Ellis (BE) [13, 14]. A further duality rotation puts the scalar field to zero again, yielding less well known vacuum metrics describing ring wormholes. They circumvent the above no-go arguments because they are not spherically symmetric and secondly because they are not globally regular and exhibit a conical singularity along the ring. The ring encircles the wormhole throat and has a negative tension which should be less than $-c^4/4G$. Strikingly, if the tension reaches the maximal value, the geometry becomes exactly flat, but the topology remains non-trivial and corresponds to two copies of Minkowski space glued together along the disk inside the ring. The geodesics are straight lines, and those which intersect the ring get to the other universe. The ring therefore literally creates a hole in space.

It is well-known that the Kerr black hole contains inside the horizon a ring singularity which can also be viewed as a wormhole [20]. However, its surrounding region contains closed timelike curves (CTCs). One should emphasize that our ring is different – it is static and does not create any CTCs and moreover is not shielded by any horizons.

1. Model

We consider the theory with a scalar field,

$$\mathcal{L} = [R - 2\epsilon(\partial\Phi)^2]\sqrt{-g}. \quad (4)$$

Here the parameter takes two values, either $\epsilon = +1$ corresponding to the conventional scalar field, or $\epsilon = -1$ corresponding to the phantom field. We shall denote $\Phi = \phi$ for $\epsilon = 1$ and $\Phi = \psi$ for $\epsilon = -1$. The phantom field $\psi$ can formally be viewed as $\phi$ continued to imaginary values. We use units in which $G = c = 1$, unless otherwise stated.
Assume the spacetime metric to be static,

\[ ds^2 = -e^{2U} \, dt^2 + e^{-2U} \, dl^2, \]  

(5)

where \( dl^2 = \gamma_{ik} dx^i dx^k \) and \( U, \gamma_{ik}, \Phi \) depend on the spatial coordinates \( x^k \). Denoting by \( U_k \equiv \partial_k U \), the Lagrangian becomes

\[ L = \left[ (3) R - 2\gamma^{ik}(U_i U_k + \epsilon \Phi_i \Phi_k) \right] \sqrt{\gamma}, \]  

(6)

where \( (3) R \) is the Ricci scalar for \( \gamma_{ik} \). The field equations are

\[ (3) R_{ik} = 2(U_i U_k + \epsilon \Phi_i \Phi_k), \quad \Delta U = 0, \quad \Delta \Phi = 0. \]  

(7)

We notice that for \( \epsilon = 1 \) these equations are invariant under

\[ U \leftrightarrow \Phi = \phi, \quad \gamma_{ik} \rightarrow \gamma_{ik}. \]  

(8)

If the 3-metric \( \gamma_{ik} \) is chosen to be of the Weyl form,

\[ dl^2 = e^{2k}(d\rho^2 + dz^2) + \rho^2 d\phi^2, \]  

(9)

where \( k \) and also \( U, \Phi \) depend only on \( \rho, z \), then the equations reduce to\(^1\)

\[ U_{\rho \rho} + \frac{1}{\rho} U_{\rho} + U_{zz} = 0, \quad \Phi_{\rho \rho} + \frac{1}{\rho} \Phi_{\rho} + \Phi_{zz} = 0, \]

\[ k_z = 2\rho [U_\rho U_z + \epsilon \Phi_\rho \Phi_z], \quad k_\rho = \rho [U_\rho^2 - U_z^2 + \epsilon (\Phi_\rho^2 - \Phi_z^2)]. \]  

(10)

These equations admit a scaling symmetry mapping solutions to solutions,

\[ U \rightarrow \lambda U, \quad \Phi \rightarrow \lambda \Phi, \quad k \rightarrow \lambda^2 k, \]  

(11)

where \( \lambda \) is a constant parameter. In addition, they are invariant under

\[ U \leftrightarrow \Phi, \quad k \rightarrow \epsilon k. \]  

(12)

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\(^1\)One can check that this reduction is indeed consistent, which would not be the case if the scalar field had a potential. The Weyl formulation is also consistent for an electrostatic vector field, so that it applies, for example, within the electrostatic sector of dilaton gravity.
2. BE wormhole

We start from the Schwarzschild metric (here $d\Omega^2 = d\vartheta^2 + \sin^2 \vartheta d\varphi^2$)

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2d\Omega^2. \tag{13}$$

Introducing $x = r - m$ it can be put to the form (5) with

$$U = \frac{1}{2} \ln \left(\frac{x - m}{x + m}\right), \quad dl^2 = dx^2 + (x^2 - m^2) d\Omega^2, \quad \Phi = 0. \tag{14}$$

Applying to this the symmetry (8) gives a new solution which is ultrastatic but has a non-trivial scalar field

$$U = 0, \quad dl^2 = dx^2 + (x^2 - m^2) d\Omega^2, \quad \phi = \frac{1}{2} \ln \left(\frac{x - m}{x + m}\right). \tag{15}$$

If we now continue the parameter $m$ to the imaginary region, $m \to i\mu$, then the metric remains real, while

$$\phi \to \frac{1}{2} \ln \left(\frac{x - i\mu}{x + i\mu}\right) = i\psi, \tag{16}$$

where $\psi$ is real. This gives the solution for the phantom field,

$$ds^2 = -dt^2 + dx^2 + (x^2 + \mu^2)(d\vartheta^2 + \sin^2 \vartheta d\varphi^2),$$

$$\psi = \arctan \left(\frac{x}{\mu}\right), \tag{17}$$

which is precisely the BE wormhole [13, 14]. Here $x \in (-\infty, +\infty)$ and the limits $x \to \pm \infty$ correspond to two asymptotically flat regions, while $x = 0$ is the wormhole throat – the 2-sphere of minimal radius $\mu$. The geodesics can get through the throat from one asymptotic region to the other.

Passing to the coordinates

$$z = x \cos \vartheta, \quad \rho = \sqrt{x^2 + \mu^2} \sin \vartheta \leftrightarrow x \pm i\mu \cos \vartheta = \sqrt{\rho^2 + (z \pm i\mu)^2}, \tag{18}$$

This is a member of the Fisher-Janis-Newman-Winicour solution family [21, 22]. The rest of this family can be similarly obtained from the Schwarzschild metric by applying, instead of the discrete symmetry (8), continuous $SO(2)$ rotations in the $U, \phi$ space.
the wormhole metric assumes the Weyl form,

\[ ds^2 = -dt^2 + e^{2k}(d\rho^2 + dz^2) + \rho^2 d\varphi^2 \]  

(19)

with

\[ e^{2k} = \frac{x^2 + \mu^2}{x^2 + \mu^2 \cos^2 \vartheta}. \]  

(20)

Since

\[ d\rho^2 + dz^2 = \frac{x^2 + \mu^2 \cos^2 \vartheta}{x^2 + \mu^2} \left[ dx^2 + (x^2 + \mu^2)d\vartheta^2 \right], \]  

(21)

the metric (19) is indeed equivalent to the one in Eq.(17).

### 3. Ring wormhole

Let us apply to the solution (19) the symmetry (12), \( U \leftrightarrow \psi, k \to -k \), and then act on the result with the scaling symmetry (11) with \( \lambda = \sigma \). This gives the axially symmetric metrics with \( \Phi = 0 \),

\[ ds^2 = -e^{2U} dt^2 + e^{-2U} dl^2, \quad U = \sigma \arctan \left( \frac{x}{\mu} \right), \]  

(22)

\[ dl^2 = \left( \frac{x^2 + \mu^2 \cos^2 \vartheta}{x^2 + \mu^2} \right)^{\sigma^2+1} \left[ dx^2 + (x^2 + \mu^2)d\vartheta^2 \right] + (x^2 + \mu^2)\sin^2 \vartheta d\varphi^2. \]

We note that the 3-metric \( dl^2 \) is invariant under \( x \to -x \) while the Newtonian potential \( U \) changes sign. As we shall see below, these solutions describe ring wormholes. We thought initially these solutions were new, but actually they were described before although remain very little known.

We obtained them from the BE wormhole by rotating away the scalar field. However, they can also be obtained from the Schwarzschild metric without introducing any scalar fields at all. Passing to the coordinates

\[ z = x \cos \vartheta, \quad \rho = \sqrt{x^2 - m^2 \sin \vartheta} \leftrightarrow x \pm m \cos \vartheta = \sqrt{\rho^2 + (z \pm m)^2}, \]  

(23)

the Schwarzschild metric (14) assumes the Weyl form with

\[ dl^2 = dx^2 + (x^2 - m^2) d\Omega^2 = e^{2k}(d\rho^2 + dz^2) + \rho^2 d\varphi^2, \]  

(24)
where

\[ e^{2k} = \frac{x^2 - m^2}{x^2 - m^2 \cos^2 \vartheta}, \]  

(25)

since

\[ d\rho^2 + dz^2 = \frac{x^2 - m^2 \cos^2 \vartheta}{x^2 - m^2} \left[ dx^2 + (x^2 - m^2) d\vartheta^2 \right]. \]  

(26)

Acting on this with the scaling symmetry (11) with \( \lambda = -\delta \) gives vacuum metrics of the prolate Zipoy-Voorhees (ZV) class [23, 24]

\[ ds^2 = -\left( \frac{x - m}{x + m} \right)^{-\delta} + \left( \frac{x - m}{x + m} \right)^{\delta} dl^2, \]  

(27)

\[ dl^2 = \left( \frac{x^2 - m^2 \cos^2 \vartheta}{x^2 - m^2} \right)^{1-\delta^2} \left[ dx^2 + (x^2 - m^2) d\vartheta^2 \right] + (x^2 - m^2) \sin^2 \vartheta d\phi^2. \]

These metrics have been relatively well studied (see for example [25, 26]) since they can be used to describe deformations of the Schwarzschild metric.

If we now continue the parameters to the imaginary region,

\[ \delta \to i\sigma, \quad m \to i\mu, \]  

(28)

then metrics (27) remain real and reduce to the oblate ZV solutions [23, 24], which are precisely the metrics (22). Much less is known about these solutions. Their wormhole nature has been discussed [27, 28] but no systematic description is currently available and moreover the solutions remain largely unknown. We shall therefore describe their essential properties and find new surprising features, notably the existence of a non-trivial flat space limit.

4. Properties of the ring wormhole

Similarly to the BE wormhole (17), solution (22) has a throat and two asymptotically flat regions. This can be most easily seen by noting that at the symmetry axis, where \( \cos^2 \vartheta = 1 \), the line element \( dl^2 \) reduces precisely to that in (17), while \( U \) is a bounded function. Therefore, the solution indeed describes a wormhole interpolating between two asymptotically flat regions. One has for \( x \to \pm\infty \) (assuming that \( \mu > 0 \))

\[ e^{2U} \to e^{\pm \sigma} \left( 1 - \frac{2\sigma \mu}{x} + \ldots \right), \]  

(29)
hence the time coordinates in both limits differ by a factor of $e^{2\sigma\pi}$ while the ADM mass $M = \pm \sigma \mu$ is positive when seen from one wormhole side and negative from the other.

The wormhole is traversable, which can be easily seen by considering geodesics along the symmetry axis. A particle of mass $m$ and energy $E$ follows a trajectory $x(s)$ defined by

$$\left(\frac{dx}{ds}\right)^2 + m^2 e^{2U(x)} = E^2,$$

hence $x(s) \in (-\infty, +\infty)$ if $E^2 > m^2 e^{\sigma\pi}$. Since $U(x)$ grows with $x$, it follows that the wormhole attracts particles in the $x > 0$ region but repels them in the $x < 0$ region. Therefore, it acts as a “drainhole” sucking in matter from the $x > 0$ region and spitting it out to the $x < 0$ region.

The metric (22) degenerates at $x = 0$, $\vartheta = \pi/2$, which corresponds to a circle of radius $\mu$. In its vicinity one can define $y = \mu \cos \vartheta$ and then $dl^2$ in (22) becomes

$$dl^2 = \left(\frac{x^2 + y^2}{\mu^2}\right)^{\sigma^2+1} (dx^2 + dy^2) + \mu^2 d\varphi^2$$

$$= \mu^2 r^{2\sigma^2+2} (dr^2 + r^2 d\alpha^2) + \mu^2 d\varphi^2 = \mu^2 (dR^2 + R^2 d\omega^2) + \mu^2 d\varphi^2,$$

where $x = \mu r \cos \alpha$, $y = \mu r \sin \alpha$ while $R = r^{\sigma^2+2}/(\sigma^2 + 2)$ and $\omega = (\sigma^2+2) \alpha$. The metric in the last line in (31) looks flat, however, since $\alpha \in [0, 2\pi)$, the angle $\omega$ ranges from zero to $\omega_{\text{max}} = 2\pi (\sigma^2 + 2) > 2\pi$. Therefore, there is a negative angle deficit

$$\Delta \omega = 2\pi - \omega_{\text{max}} = -(\sigma^2 + 1) 2\pi$$

and hence a conical singularity at $R = 0$. Since the singularity stretches in the $\varphi$-direction, it sweeps a ring of radius $\mu$. Such line singularities are known to be generated by singular matter sources distributed along lines – cosmic strings. Their energy per unit length (tension) $T$ is related to the angle deficit via $\Delta \omega = (8\pi G/c^4) T$ (we restore for a moment the correct physical dimensions), hence the string has a negative tension

$$T = -\frac{(1 + \sigma^2) c^4}{4G}.$$
Therefore, the wormhole solution (22) can be viewed as sourced by a ring with negative tension. The solution carries the free parameter $\mu$, which determines the radius of the ring, and also $\sigma$ determining the value of the ring tension.

It is quite surprising that the self-gravitating ring can be in equilibrium and moreover admits an exact solution. If the string tension was positive then the string loop would be shrinking and the system would not be static (we are unaware of any exact solutions in this case, although one can construct exact initial data for a string loop [29]). For the solution (22) the ring has a negative tension and is similar to a strut, hence it should rather tend to expand, which tendency is counterbalanced by the gravitational attraction. This must be the reason for which such an equilibrium system exists (we do not know if the equilibrium is stable).

Even more remarkable is the following. Setting $\sigma = 0$ in (22) yields

$$ds^2 = -dt^2 + \frac{x^2 + \mu^2 \cos^2 \vartheta}{x^2 + \mu^2} \left[ dx^2 + (x^2 + \mu^2) d\varphi^2 \right] + (x^2 + \mu^2) \sin^2 \vartheta d\varphi^2. \quad (34)$$

This metric is vacuum and ultrastatic hence it must be flat, so one may think this limit is trivial. However, close to the symmetry axis, where $\cos^2 \vartheta \approx 1$, the geometry reduces to that for the BE wormhole (17). The wormhole throat is located at $x = 0$, but, unlike in the BE case, this is not a sphere but rather a sphere squashed to a disk with the geometry

$$\mu^2 (\cos^2 \vartheta d\vartheta^2 + \sin^2 \vartheta d\varphi^2) = \mu^2 (d\xi^2 + \xi^2 d\varphi^2), \quad (35)$$

where $\xi = \sin \vartheta \in [0, 1]$. The disk is encircled by the ring.

The coordinates $x, \vartheta, \varphi$ are global, but to study the geodesics it is convenient to pass to the Weyl coordinates (18). The metric then becomes manifestly flat

$$ds^2 = -dt^2 + d\rho^2 + dz^2 + \rho^2 d\varphi^2. \quad (36)$$

However, this does not mean the solution describes empty Minkowski space. Indeed, the ring is still present and has the tension

$$T = -\frac{c^4}{4G}. \quad (37)$$

As a consequence of this the topology is non-trivial. This follows from the fact that the Weyl coordinates $\rho, z$ cover either only the $x < 0$ part or only
Figure 1: Wormhole topology. The $x, \vartheta$ coordinates cover the whole of the manifold, the throat being at $x = 0$, $\vartheta \in [0, \pi]$ and the ring is at $x = 0$, $\theta = \pi/2$. The Weyl charts $D_+$ and $D_-$ cover, respectively, the $x > 0$ and $x < 0$ regions. Lines of constant $x$ are the (half)-ellipses in the Weyl coordinates, the ring corresponds to the branch points at $z = 0, \rho = \mu$, while the throat corresponds to the branch cuts $O\mu$. The upper edge of the cut in the $D_+$ patch is identified with the lower edge of the cut in the $D_-$ patch and vice-versa. A winding around the ring in the $x, \vartheta$ coordinates corresponds to two windings in Weyl coordinates.

the $x > 0$ part of the wormhole. Indeed, one has

$$\frac{z^2}{x^2} + \frac{\rho^2}{x^2 + \mu^2} = 1,$$

(38)

hence lines of constant $x$ in the $\rho, z$ plane are ellipses insensitive to the sign of $x$, therefore one needs two Weyl charts, $D_{\pm} : \{\rho \geq 0, -\infty < z < \infty\}$, to cover the whole of the manifold – one for $x < 0$ and one for $x > 0$ (see Fig.1) [30]. The coordinate transformation $(x, \vartheta) \rightarrow (\rho, z)$ degenerates at the branch point $z = 0$, $\rho = \mu$ corresponding to the position of the ring. Each chart therefore has a branch cut ending at the branch point, which may be chosen to be $z = 0, \rho \in [0, \mu]$ (see Fig.1). The upper edge of the $D_+$ cut
is identified with the lower edge of the $D_-$ cut and vice-versa. These cuts correspond to the wormhole throat, so that we see once again that the throat is a disk encircled by the ring.

The geodesics in Weyl coordinates are straight lines. Those which miss the ring always stay in the same coordinate chart (geodesic $A$ in Fig.1), while those threading the ring (geodesics $B, C$ in Fig.1) continue from $D_+$ to $D_-$ thus traversing the wormhole. Therefore, the negative tension ring genuinely creates a hole in space through which one can observe another universe as well as get there. This reminds one of Alice observing the room behind the looking glass and next jumping there. An object falling through the ring can be seen from behind, while viewed from the side it is not seen coming from the other side (see Fig.2).

![Figure 2: Particles entering the ring are not seen coming out from the other side](image)

It is striking that the line source is enough to create the wormhole. Usually the negative energy supporting the wormhole is distributed over the 3-volume, as for the BE solution, or at least over a 2-surface, as for the thin-shell wormholes [32]. We see however that it is sufficient to distribute the negative energy only along the one-dimensional ring, which is presumably easier to achieve than in other cases since a smaller amount of the NEC violation is needed.

It is also interesting to note that the arguments based on the Raychaudhuri equation do not apply since the wormhole either does not affect the geodesics at all if they miss it, or it absorbs them if they hit it – its edges

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\(^3\) A similar possibility to create a hole in flat space, although without specifying the precise structure of the matter source needed, was discussed in Ref. [31], following the discussion in Ref. [9].
are sharp. It seems plausible that the wormhole edges could be smoothened without changing the global structure if the singular ring is replaced by a regular hoop-shaped energy distribution of finite thickness and with the same tension. Inside the hoop the energy density is finite hence the geometry must be regular, while outside the energy is zero and the geometry should be more or less the same as for the original ring. This suggests that wormholes could be created by negative energies concentrated in toroidal volumes, for example by vacuum fluctuations.

However, the energy density needed to create a ring wormhole is extremely high. The absolute value of the negative tension \( T \) coincides with the highest possible value for a positive tension (force), according to the maximum tension principle in General Relativity conjectured in [33, 34]. This conjecture is supported, for example, by the fact that the angle deficit of a cosmic string cannot exceed \( 2\pi \). Numerically, \( T = -3.0257 \times 10^{39} \) New-trons \( \approx -3 \times 10^{39} \) Tonnes. To create a ring of radius \( R = 1 \) metre, say, one needs a negative energy equivalent to the \( 10^{-3} \) Solar masses, \( 2\pi RT/c^2 \approx -0.001 \times M_\odot \).

At the same time, one can imagine that such rings could by quantum fluctuations appear spontaneously from the vacuum and then disappear again. Particles crossing the ring during its existence would no longer be accessible from our universe after the ring disappears. If true, this would be a potential mechanism for the loss of quantum coherence.

In summary, by applying generating techniques we (re)-discovered solutions describing ring wormholes, computed the ring tension, and showed that their geometry can be precisely flat: the ring creates a hole in space.

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References

[1] A. Einstein, N. Rosen, The Particle Problem in the General Theory of Relativity, Phys. Rev. 48 (1935) 73–77. doi:10.1103/PhysRev.48.73.
[2] A. Einstein, B. Podolsky, N. Rosen, Can quantum mechanical description of physical reality be considered complete?, Phys. Rev. 47 (1935) 777–780. doi:10.1103/PhysRev.47.777.

[3] J. Maldacena, L. Susskind, Cool horizons for entangled black holes, Fortsch. Phys. 61 (2013) 781–811. arXiv:1306.0533, doi:10.1002/prop.201300020.

[4] C. W. Misner, J. A. Wheeler, Classical physics as geometry: Gravitation, electromagnetism, unquantized charge, and mass as properties of curved empty space, Annals Phys. 2 (1957) 525–603. doi:10.1016/0003-4916(57)90049-0.

[5] C. W. Misner, Wormhole Initial Conditions, Phys. Rev. 118 (1960) 1110–1111. doi:10.1103/PhysRev.118.1110.

[6] M. Cvetic, G. W. Gibbons, C. N. Pope, Super-Geometrodynamics, JHEP 03 (2015) 029. arXiv:1411.1084, doi:10.1007/JHEP03(2015)029.

[7] B. P. Abbott, et al., Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116 (6) (2016) 061102. arXiv:1602.03837, doi:10.1103/PhysRevLett.116.061102.

[8] M. Morris, K. Thorne, U. Yurtsever, Wormholes, Time Machines, and the Weak Energy Condition, Phys.Rev.Lett. 61 (1988) 1446–1449. doi:10.1103/PhysRevLett.61.1446.

[9] M. Visser, Lorentzian wormholes: From Einstein to Hawking, AIP, 1996.

[10] S. W. Hawking, G. F. R. Ellis, The Large Scale Structure of Space-Time, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2011. doi:10.1017/CBO9780511524646.

[11] J. L. Friedman, K. Schleich, D. M. Witt, Topological censorship, Phys. Rev. Lett. 71 (1993) 1486–1489, [Erratum: Phys. Rev. Lett.75,1872(1995)]. arXiv:gr-qc/9305017, doi:10.1103/PhysRevLett.71.1486.

[12] D. Hochberg, M. Visser, The Null energy condition in dynamic wormholes, Phys. Rev. Lett. 81 (1998) 746–749. arXiv:gr-qc/9802048, doi:10.1103/PhysRevLett.81.746.
[13] K. Bronnikov, Scalar-tensor theory and scalar charge, Acta Phys.Polon. B4 (1973) 251–266.

[14] H. G. Ellis, Ether flow through a drainhole - a particle model in general relativity, J. Math. Phys. 14 (1973) 104–118. doi:10.1063/1.1666161.

[15] H. Maeda, M. Nozawa, Static and symmetric wormholes respecting energy conditions in Einstein-Gauss-Bonnet gravity, Phys.Rev. D78 (2008) 024005. arXiv:0803.1704, doi:10.1103/PhysRevD.78.024005.

[16] P. Kanti, B. Kleihaus, J. Kunz, Wormholes in Dilatonic Einstein-Gauss-Bonnet Theory, Phys.Rev.Lett. 107 (2011) 271101. arXiv:1108.3003, doi:10.1103/PhysRevLett.107.271101.

[17] K. Bronnikov, S.-W. Kim, Possible wormholes in a brane world, Phys.Rev. D67 (2003) 064027. arXiv:gr-qc/0212112, doi:10.1103/PhysRevD.67.064027.

[18] S. V. Sushkov, R. Korolev, Scalar wormholes with nonminimal derivative coupling, Class.Quant.Grav. 29 (2012) 085008. arXiv:1111.3415, doi:10.1088/0264-9381/29/8/085008.

[19] S. V. Sushkov, M. S. Volkov, Giant wormholes in ghost-free bi-gravity theory, JCAP 1506 (06) (2015) 017. arXiv:1502.03712, doi:10.1088/1475-7516/2015/06/017.

[20] B. Carter, Global structure of the Kerr family of gravitational fields, Phys. Rev. 174 (1968) 1559–1571. doi:10.1103/PhysRev.174.1559.

[21] I. Z. Fisher, Scalar mesostatic field with regard for gravitational effects, Zh. Eksp. Teor. Fiz. 18 (1948) 636–640. arXiv:gr-qc/9911008.

[22] A. I. Janis, E. T. Newman, J. Winicour, Reality of the Schwarzschild Singularity, Phys. Rev. Lett. 20 (1968) 878–880. doi:10.1103/PhysRevLett.20.878.

[23] D. Zipoy, Topology of some spheroidal metrics, J. Math. Phys. 7 (1966) 1137–1143.

[24] B. H. Voorhees, Static axially symmetric gravitational fields, Phys. Rev. D2 (1970) 2119–2122. doi:10.1103/PhysRevD.2.2119.
[25] H. Kodama, W. Hikida, Global structure of the Zipoy-Voorhees-Weyl spacetime and the delta=2 Tomimatsu-Sato spacetime, Class. Quant. Grav. 20 (2003) 5121–5140. arXiv:gr-qc/0304064, doi:10.1088/0264-9381/20/23/011.

[26] G. Lukes-Gerakopoulos, The non-integrability of the Zipoy-Voorhees metric, Phys. Rev. D86 (2012) 044013. arXiv:1206.0660, doi:10.1103/PhysRevD.86.044013.

[27] K. A. Bronnikov, J. C. Fabris, Weyl space-times and wormholes in D-dimensional Einstein and dilaton gravity, Class. Quant. Grav. 14 (1997) 831–842. doi:10.1088/0264-9381/14/4/003.

[28] G. Clement, Selfgravitating cosmic rings, Phys. Lett. B449 (1999) 12–16. arXiv:gr-qc/9808082, doi:10.1016/S0370-2693(99)00079-9.

[29] V. P. Frolov, W. Israel, W. G. Unruh, Gravitational Fields of Straight and Circular Cosmic Strings: Relation Between Gravitational Mass, Angular Deficit, and Internal Structure, Phys. Rev. D39 (1989) 1084–1096. doi:10.1103/PhysRevD.39.1084.

[30] A. I. Egorov, P. E. Kashargin, S. V. Sushkov, Scalar multi-wormholes, arXiv:1603.09552.

[31] S. Krasnikov, The Quantum inequalities do not forbid space-time shortcuts, Phys. Rev. D67 (2003) 104013. arXiv:gr-qc/0207057, doi:10.1103/PhysRevD.67.104013.

[32] N. M. Garcia, F. S. N. Lobo, M. Visser, Generic spherically symmetric dynamic thin-shell traversable wormholes in standard general relativity, Phys. Rev. D86 (2012) 044026. arXiv:1112.2057, doi:10.1103/PhysRevD.86.044026.

[33] C. Schiller, Simple derivation of minimum length, minimum dipole moment and lack of space-time continuity, Int. J. Theor. Phys. 45 (2006) 221–235. doi:10.1007/s10773-005-9018-7.

[34] G. W. Gibbons, The Maximum tension principle in general relativity, Found. Phys. 32 (2002) 1891–1901. arXiv:hep-th/0210109, doi:10.1023/A:1022370717626.