Meta-Stable Supersymmetry Breaking Vacua on Intersecting Branes

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We identify configurations of intersecting branes that correspond to the meta-stable supersymmetry breaking vacua in the four-dimensional \( \mathcal{N} = 1 \) supersymmetric Yang-Mills theory coupled to massive flavors. We show how their energies, the stability properties, and the decay processes are described geometrically in terms of the brane configurations.
1. Introduction

Although there is a growing body of evidences for existence of a large landscape of string vacua, with current technology it is not straightforward to construct an example of a meta-stable supersymmetry breaking vacuum where one can demonstrate that all the moduli are stabilized in a controlled approximation. It is therefore remarkable that such vacua can be found in simple field theory models such as the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory coupled to massive flavors $[1]$, suggesting that this is a generic phenomenon in field theories. One may hope that this lesson can be applied to gravity theories with finite Planck mass. A possible approach toward this goal would be to embed these field theory models in string theory and to try to gain geometric insights into the existence of supersymmetry breaking vacua.

The result of $[1]$ has recently been extended to several field theory models that can be constructed geometrically in string theory $[2,3,4,5,6]$. In particular, the model studied in $[3]$ has a landscape of meta-stable vacua where there are no massless scalar fields and the R symmetry is broken to $\mathbb{Z}_2$. However, construction and analysis of the meta-stable vacua in these papers have been done using field theory techniques.

In this paper, we will revisit the original model studied in $[1]$, embed it in string theory, and describe the meta-stable vacua of the model as geometric configurations of intersecting branes. The energies of the meta-stable vacua are reproduced by computing the volumes of the branes multiplied by the brane tension, unstable modes for certain field configurations are identified with open string tachyons, the pseudo-moduli are stabilized by closed string exchanges, and the decay process of the meta-stable vacua are described as geometric deformations of the brane configurations.

The construction of the brane configuration in this paper follows the approach initiated in $[7]$ and applied to the model relevant to this paper in $[8]$. For an extensive review of this approach, see $[9]$. The intersecting brane construction is related by a chain of duality to the geometric engineering initiated in $[10]$. It is likely that most of what we will discuss in this paper can be stated in the language of local Calabi-Yau geometry by reverse engineering the latter approach. The geometric engineering description of the meta-stable vacua may involve non-Kähler geometries, and study in this direction may lead to new insights into string theory on such non-supersymmetric geometries. It may also be possible to uplift our result to M theory, where D4 and NS5 branes become M5 branes and D6 branes are replaced by the Taub-NUT geometry $[11,12,13,14]$, along the line of the approach in $[15]$. 


2. Brane configurations for the meta-stable vacua

We propose a configuration of intersecting branes in type IIA string theory which correspond to the meta-stable vacuum with broken supersymmetry in the $\mathcal{N} = 1 U(N_c)$ super Yang-Mills theory coupled to $N_f$ chiral multiplets, which we will refer to as quarks, in the fundamental representation of $U(N_c)$. Throughout of this paper, we assume $N_f > N_c$.

\begin{center}
\text{electric description} \quad \text{magnetic description}
\end{center}

**Fig.1** The supersymmetric brane configurations for the $\mathcal{N} = 1$ supersymmetric gauge theory with massless flavors. The vertical axis represents the holomorphic coordinate $v = x^4 + ix^5$, and the horizontal axis is for $x^6$. The $w = x^7 + ix^8$ and $x^9$ coordinates are suppressed in these diagrams. All the branes share the (0123) plane, where the four-dimensional gauge theory is defined.

In the electric description, the theory can be realized on the network of branes consisting of the following:

- One NS5 brane stretched in the (0123) and (78) directions and located at $v,x^6,x^9 = 0$, where $v = x^4 + ix^5$. We call this as the NS brane.
- One NS5 brane stretched in the (0123) and (45) directions and located at $w,x^9 = 0$ and $x^6 = L$, where $w = x^7 + ix^8$ and $L > 0$. We call this as the NS’ brane.
- $N_f$ D6 branes stretched in the (0123) and (789) directions and located at $v = m_i$ and $x^6 = L' > L$. Here $m_i$ with $i = 1,...,N_f$ are complex numbers that are identified with the masses of the quarks.
- $N_c$ D4 branes stretched in the (0123) directions and going between the NS and NS’ branes along the $x^6$ axis. They are located at $v,w,x^9 = 0$.
- $N_f$ D4 branes extended in the (0123) directions and going between the NS’ brane at $x^6 = L$ to the D4 branes at $x^6 = L'$ along the $x^6$ axis. They are located at $v = m_i$ and $w,x^9 = 0$. 


The $s$-rule of [7] states that it is not possible to suspend more than one D4 branes between one NS5 brane and one D6 brane while maintaining supersymmetry. Thus, we need a D6 brane for each one of $N_f$ D4 branes. The D4 branes are located at $v = m_i$, and they are parallel to the $N_c$ D4 branes between the NS and NS’ branes as required by supersymmetry. The resulting brane configuration, when all the quarks are massless, $m_i = 0$, is shown on the left-side of Figure 1.

The massless sector of open strings going between the $N_c$ D4 branes gives rise to the vector multiplet for the gauge group $U(N_c)$. In addition, $N_f$ flavors of quarks $(Q_i, \tilde{Q}_i)$ with masses $m_i$ arise from open strings between the $N_c$ D4 branes and the $N_f$ D4 branes. The open strings on the $D6$ branes decouple since the $D6$ branes have infinite volumes in the $(789)$ directions. Thus, the low energy effective theory on the branes is the supersymmetric Yang-Mills theory couples to $N_f$ flavors on the 4-dimensional plane in the (0123) directions shared by all the branes. It was argued in [11] that an infrared singularity on the NS5 branes freezes the diagonal $U(1)$ factor in the $U(N_c)$ group. In the following, we will discuss as if the gauge group is $U(N_c)$ since the $U(1)$ factor is infrared free and decouples from the $SU(N_c)$ dynamics in any case.

The brane configuration has $U(1)_{78}$ global symmetry corresponding to the phase rotation of the coordinate $w = x^7 + ix^8$. This is identified with the R symmetry in the gauge theory. In addition, if all quark masses are zero, we have $U(1)_{45}$ symmetry corresponding to the phase rotation in the coordinate $v = x^4 + ix^5$. In this case, there is also the flavor $U(N_f) \times U(N_f)$ symmetry, where the diagonal $U(N_f)$ is generated by exchanges of the D6 branes.

The brane configuration for the magnetic dual for this theory was identified in [8]. Let us assume for the moment that all the quark masses are zero. To go from the electric description to the magnetic dual, one exchanges the locations of the NS and NS’ branes in the $(69)$ plane. When $N_f > N_c$, the resulting configuration consists of:

- NS’ brane at $(w, x^6, x^9) = 0$.
- NS brane at $(v, x^9 = 0$ and $x^6 = L''$ with $0 < L'' < L'$.
- $N_f$ D6 branes at $v = 0$ and $x^6 = L'$. (We are setting $m_i = 0$.)
- $(N_f - N_c)$ D4 branes between the NS’ and NS branes.
- $N_f$ D4 branes between the NS brane and D6 branes.

\footnote{In [7], the $s$-rule was postulated to reproduce moduli spaces of gauge theories correctly. Subsequently this rule has been derived from various points of view in [10,12,17,18].}
The low energy physics is described by the $U(N_f - N_c)$ vector multiplet coupled to $N_f$ quarks $(q_i, \bar{q}_i)$ in the fundamental representation, and a gauge neutral meson $M_{ij}$, which transforms in the adjoint representation in the $U(N_f)$ flavor symmetry. The extra meson degrees of freedom correspond to the motion of the $N_f$ D4 branes in the $w = x^7 + ix^8$ directions along the NS and D6 branes. The superpotential $W$ in the magnetic description is proportional to $\text{tr} \, \bar{q}Mq$, where $\text{tr}$ is over the fundamental representation of $U(N_f - N_c)$ and the sum over the flavor indices is implicit. This reproduces the magnetic dual of the theory identified by Seiberg [19,20].

The meson field $M_{ij}$ is identified with the bilinear combination $Q_i \bar{Q}_j$ of the quarks in the electric description. Thus, turning on the quark masses in the electric description corresponds to deforming the superpotential by adding a term linear in $M$ as

$$W = \text{tr} \, \bar{q}Mq + \text{tr}' \, mM,$$  \hspace{1cm} (2.1)

where $\text{tr}$ and $\text{tr}'$ are the traces over the gauge and the flavor indices respectively and $m$ is the $N_f \times N_f$ quark mass matrix. The $F$-term conditions are

$$q\bar{q} + m = 0,$$
$$Mq = 0, \quad \bar{q}M = 0.$$  \hspace{1cm} (2.2)

Since the ranks of $q$ and $\bar{q}$ are at most $(N_f - N_c)$, the first equation cannot be satisfied if the rank of the $N_f \times N_f$ mass matrix $m$ exceeds $(N_f - N_c)$. In particular, turning on masses for all the quarks breaks the supersymmetry in the magnetic description. This is the rank condition mechanism of [1].

2.1. Brane configurations for the meta-stable vacua

For simplicity, let us use the flavor symmetry to arrange the mass matrix $m$ so that

$$m = \text{diag}(m_1, m_2, \cdots, m_{N_f}),$$  \hspace{1cm} (2.3)

with

$$|m_1| \geq |m_2| \geq \cdots \geq |m_{N_f}|.$$  \hspace{1cm} (2.4)

\footnote{We assume that $m$ obeys $[m, m^\dagger] = 0$ so that it is diagonalizable with the eigenvalues given by the quark masses $m_i$.}
The local minima of the tree-level $F$ and $D$-term potential can be parametrized as

$$q = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \tilde{q} = (\varphi_0, \ 0)$$

$$M = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \end{pmatrix},$$

where $\varphi_0$ and $\tilde{\varphi}_0$ are $(N_f - N_c) \times (N_f - N_c)$ matrices satisfying

$$\varphi_0 \tilde{\varphi}_0 = -\text{diag}(m_1, m_2, \ldots, m_{N_f-N_c}),$$

and $M_0$ is an arbitrary $N_c \times N_c$ matrix. It is important that we choose the $(N_f - N_c)$ largest masses for the eigenvalues of $q\tilde{q}$. Otherwise, the configuration is unstable under small perturbations [1]. We will show how this instability effect can be seen in the brane configuration.

In the brane configuration, turning on the quark masses corresponds to moving the D6 branes from $v = 0$ to $v = m_1, \ldots, m_{N_f}$. Suppose we turn on masses for $n \leq (N_f - N_c)$ quarks. To move the $n$ D6 branes while maintaining supersymmetry, we need to connect $n$ pairs of D4 branes across the NS brane at $x^6 = L''$ and move them together with the D6 branes since the D4 branes must be parallel to each other. The resulting configuration contains:

- $n$ D4 branes stretched between the NS’ and D6 branes at $v = m_i$ with $i = 1, \ldots, n$.
- $(N_f - N_c - n)$ D4 branes between the NS’ and NS branes.
- $(N_f - n)$ D4 branes between the NS and D6 branes at $v = 0$.

Since the D6 branes impose the Dirichlet boundary condition on the (0123) components of the gauge fields on the D4 branes, the gauge symmetry is broken to $U(N_f - N_c - n)$. This is consistent with the field theory fact that the quark bilinear $q\tilde{q}$ gets a vacuum expectation value of rank $n$ and spontaneously break the gauge symmetry.

This works until $n$ hits $(N_f - N_c)$ when one finds that there is no more D4 brane left between the NS’ and NS branes. If we add mass terms for more quarks and move the corresponding D6 branes away from the origin of the $v$ plane, the D4 branes connecting them with the NS brane will have to be tilted in the $v$-$x^6$ plane. In particular, these D4 branes will not be parallel with the $(N_f - N_c)$ D4 branes going along the $x^6$ axis. See Figure 2.
We claim that the resulting brane configuration corresponds to the field configuration given by (2.5) and (2.6). Note that the quark bilinear $-q \tilde{q}$ specifies the $v$ coordinates of the D4 branes at $x^6 = L''$. This follows from the fact that the quarks in the magnetic description are defined as open strings going between the two types of D4 branes across the NS brane, which is located at $x^6 = L''$. From the brane configuration, we can read off that

$$-q \tilde{q} = \text{diag}(m_1, \ldots, m_{N_f-N_c}, 0, \ldots, 0)$$

since the $(N_f - N_c)$ D4 branes have been moved to $v = m_1, \ldots, m_{N_f-N_c}$ along with the D6 branes while the $N_c$ D4 branes are still ending on the NS brane at $v = 0$. Since these $(N_f - N_c)$ D4 branes are frozen at $w = 0$ while the $N_c$ D4 branes can be moved to any locations along the $w$ direction at the string tree level, the expectation value of the meson $M$ can be expressed as

$$M = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \end{pmatrix},$$

with an arbitrary $N_c \times N_c$ matrix $M_0$. This reproduces the field configuration given by (2.5) and (2.6). The R symmetry of the gauge theory is realized as the phase rotation of the $w$ coordinate. It is clear that the brane configuration corresponding to the meta-stable vacuum preserves this R symmetry when $M_0 = 0$ since all the branes are invariant under the rotation. In the following, we will show that this brane configuration captures various other features of the meta-stable vacuum.
2.2. Tachyons

Since the brane configuration proposed in the above does not preserve supersymmetry, we need to make sure that the configuration is locally stable. Let us verify that there are no tachyons in the open string spectrum.

Extrema of the tree-level potential is parametrized as (2.3) with \( \varphi_0 \) given by (2.6). Suppose we did not order the masses as in (2.4), and \( |m_{N_f}| > |m_1| \) for example. One can show that, in this case, fluctuations of the dual quarks \((q, \tilde{q})\) at \( M = 0 \) contain tachyonic modes with \( \text{(mass)}^2 \) given by

\[
\text{(mass)}^2 = -\frac{|m_{N_f}| - |m_1|}{Z \Lambda},
\]

where \( Z \) is the normalization factor in front of the kinetic term of the meson field \( M \) in the magnetic theory when the superpotential is normalized as \( tr m M + \cdots \), and \( \tilde{\Lambda} \) is some mass parameter of the magnetic dual of the gauge theory that cannot be determined by the information in the electric theory alone (see the comment below (5.6) in \([1]\)). Thus, the tree level stability of the field configuration at \( M = 0 \) requires that the quark masses should be lined up as in (2.4).

This instability can be seen in the brane configuration as follows. For simplicity, let us assume that all the masses are real. The first \((N_f - N_c)\) masses \( m_1, \ldots, m_{N_f - N_c} \) are the location of the D6 branes which are connected to the NS’ brane through D4 branes. The remaining \( N_c \) masses \( m_{N_f - N_c + 1}, \ldots, m_{N_f} \) are the locations of the D6 branes connected to the NS brane through D4 branes. The latter D4 branes are in angles in the \( v-x^6 \) plane and the angles are determined by the quark masses. Suppose \( m_1 < m_{N_f} \) for example. As

Fig.3 When \( m_{N_f} > m_1 \), the two D4 branes intersect. The tachyon condensation recombines the two branes.
we can see in the left-side of Figure 3, the D4 branes connected to the D6 branes at \( m_1 \) and \( m_{N_f} \) intersect when both branes are located at \( w = 0 \). It is well-known that, when a pair of branes are intersecting at an angle as described in the above, the spectrum of the open string stretched between the branes contains a tachyon and that the end point of the tachyon condensation is a recombination of the branes \([21,22,23,24]\) as in the right-side of Figure 3. After the recombination, the D6 brane at \( m_{N_f} \) is connected to the NS’ brane through a D4 brane and the D6 brane at \( m_1 \) is connected to the NS brane brane through a D4 brane at an angle in the \( v-x^6 \) plane. After a series of recombinations, the brane configuration will settle down to the configuration that corresponds to \( (2.5) \) in the field theory with the quark masses ordered as \( m_1 \geq \cdots \geq m_{N_f} \). This matches well with what we expect in the field theory.

If the tachyonic mode in the field theory is to be identified with the open string tachyon at the intersection of the two D4 branes, we should be able to understand the mass formula \( (2.7) \) from the string theory point of view also. The mass of the open string tachyon on the branes at an angle \( \theta \) is given by \([21]\)

\[
(mass)^2 = - \frac{\theta}{l_s^2}.
\]  

(2.8)

To compute the angle \( \theta \), we note that one of the D4 branes is at \( v = m_1 \) and the other is going from \( v = 0 \) to \( v = m_{N_f} \). The physical distance for the separation of the two end points of the second D4 brane when projected on the \( v \) plane is \( m_{N_f} l_s^2 \). On the other hand the length of the D4 brane is proportional to the normalization factor \( Z \) of the meson kinetic term. Since the meson field \( M \) is identified with the quark bilinear \( Q \tilde{Q} \) in the electric variable, the factor \( Z \) should have the dimensions of \((\text{length})^2\). Thus, the angle \( \theta \) of the D4 brane can be expressed as

\[
\tan \theta = \frac{m_{N_f} l_s^2}{Z N'},
\]  

(2.9)

with some dimensionful parameter \( N' \). With this, the tachyon mass in the field theory limit \( l_s \to 0 \) is estimated as

\[
(mass)^2 = - \frac{\theta}{l_s^2} \to \frac{m_{N_f}}{Z N'}.
\]  

(2.10)

This reproduces the first half of the tachyon mass formula \( (2.7) \) if we identify \( N' \) with the unknown parameter \( \Lambda' \) in the magnetic theory.
The above formula (2.10) is obtained by ignoring the fact that the D4 branes end on the D6 branes. We claim that the second half of the tachyon mass formula (2.7) is due to the D6 brane boundary condition. The formula shows in particular that the tachyon becomes massless in the limit of $m_{N_f} = m_1$. Let us try to understand this phenomenon by taking into account the boundary conditions at the D6 branes.

According to [23], if we take the field theory limit $l_s \to 0$ so that $\theta/l_s^2$ remains finite as in our case, the string theory tachyon can be described as a classical configuration of the $U(2)$ gauge theory along the $x^6$ direction as follows. Consider the gauge field $A_6$ in the $x^6$ direction and the scalar field $\phi$ in the adjoint of $U(2)$ corresponds to the oscillation of the branes in the $v$ direction. The fact that the branes intersect at the angle $\theta$ can be expressed in the gauge theory language as the expectation value of the scalar field $\Phi$ as follows,

$$\Phi(x^6) = \sigma^3 \frac{\theta}{l_s^2} x^6,$$

where we assume that the intersection is at $x^6 = 0$ and $\sigma^{1,2,3}$ are the Pauli matrices. We can evaluate the fluctuation spectrum of the D4 branes around this background by expanding the gauge theory action in quadratic order in $A_6$ and $\Phi$. If we ignore the boundary conditions at the NS and D6 branes, the gauge theory equations of motion combined with the Gauss law constraint for the gauge $A_0 = 0$ imply that the lowest energy excitation is of the form,

$$A_6 = \sigma^2 \exp \left[ -\frac{|\theta|}{l_s^2} (x^6)^2 \right],$$

$$\delta \Phi = \sigma^1 \exp \left[ -\frac{|\theta|}{l_s^2} (x^6)^2 \right].$$

(2.12)

There is another excitation with the same energy obtained by exchanging $(\sigma^1, \sigma^2) \to (-\sigma^2, \sigma^1)$. The (mass)$^2$ for this configuration correctly reproduces the formula (2.8).

In the $A_0 = 0$ gauge, the D6 branes impose the Neumann boundary condition on $A_6$ and the Dirichlet boundary condition on $\delta \Phi$. Since (2.12) does not satisfy these conditions, we expect that the tachyon (mass)$^2$ is raised by the boundary condition. The distance of the intersection point $x^6 = 0$ to the boundary at the D6 branes is proportional to $(m_{N_f} - m_1)$, and the effect of the boundary condition should become greater as $m_{N_f}$ approaches $m_1$. In the limit of $m_{N_f} = m_1$, the tachyon becomes massless. This can be verified directly by solving the gauge theory equations combined with the Gauss law constraint. When $m_{N_f} = m_1$, the boundary condition has the global $U(2)$ symmetry for the exchange of the two D6 branes, but it is spontaneously broken by the field expectation value (2.11). This
gives rise to the Nambu-Goldstone boson, and that is the massless mode that appears in
the limit of the tachyon at \( m_{N_f} = m_1 \). Note that this Nambu-Goldstone mode would be
non-normalizable if the branes were infinitely extended since \( \delta \Phi \) grows linearly in \( x^6 \), but it
is normalizable on our D4 branes of finite lengths and we should count it in the spectrum.
We have also examined other normalizable modes on the half line \( x^6 \leq 0 \). The Dirichlet
boundary condition on \( \delta \Phi \) at \( x^6 = 0 \) can be taken into account by extending the space to
\( x^6 > 0 \) and by requiring that \( \delta \Phi \) be odd under the reflection of \( x^6 \rightarrow -x^6 \). We found that
all the fluctuations other than the Nambu-Goldstone modes have positive (mass)\(^2\). Thus,
in the limit of \( m_{N_f} = m_1 \), the open string tachyon is removed by the D6 brane boundary
condition and the brane configuration becomes stable.

If we move the D6 branes further so that \( m_1 > m_{N_f} \), the Nambu-Goldstone mode
becomes massive as the open string between the D4 branes are separated even at the end
points of the D4 branes at the D6 branes. It is straightforward to check that there are
no other sources of open string tachyons in this configuration when \( m_1 \geq \cdots \geq m_{N_f} \) even
though the brane configuration breaks supersymmetry. To our knowledge, this way of
eliminating tachyons has not been considered in phenomenological model building based
on intersecting branes.\(^3\) It would be interesting to explore possibilities of superstring model
building using configurations like this.

2.9. Vacuum energy

The energy of the meta-stable vacuum is higher than that of the supersymmetric
brane configuration at \( m_i = 0 \) since the D4 branes at angles are longer. The angles are

\[
\theta_i \sim \frac{|m_i|}{Z}, \quad \text{i.e. } N_f - N_c + 1, \ldots, N_f,
\]

and the length of the D4 brane times the brane tension is proportional to the normalization
factor \( Z \) of the meson kinetic term. Thus, if we set the energy of the supersymmetric brane

\(^3\) We thank K. Hashimoto on communication on this point and for sharing his unpublished
notes with us, which were useful for our analysis here.

\(^4\) This is different from the quasi-supersymmetric construction studied in \(^2\), where each
intersection preserves some supersymmetry but the whole configuration breaks supersymmetry.
In the present case, when the quark masses are degenerate, there are D4 branes meeting with angle,
breaking all supersymmetry. The potential tachyon is eliminated by the boundary condition at
the D6 branes.
configuration to be zero, the energy density $V$ for the meta-stable configuration can be estimated as

$$V = \sum_{i=N_f-N_c+1}^{N_f} \left( \frac{1}{\cos \theta_i} - 1 \right) Z \sim \sum_{i=N_f-N_c+1}^{N_f} \theta_i^2 Z \sim \frac{1}{Z} \sum_{i=N_f-N_c+1}^{N_f} |m_i|^2. \quad (2.13)$$

This agrees with the value of the $F$-term potential

$$V = \frac{1}{Z} \left| \frac{\partial W}{\partial M} \right|^2,$$

evaluated for the field configuration (2.5).

2.4. Pseudo-moduli and their one-loop effective potential

At the tree level in the field theory analysis, there is no potential for deformation of $M_0$ in (2.3). Thus, $M_0$ is called pseudo-moduli. Since there are non-compact directions in $M_0$, it is important to find out if these directions are stabilized by quantum effects.

The pseudo-moduli $M_0$ describe locations of the $N_c$ D4 branes in the $w$ direction along the NS and D6 branes. In the field theory, it was shown in [1] that the one-loop Coleman-Weinberg potential lifts these flat directions, except for those protected by the Goldstone theorem. In the string picture, the one-loop computation in the gauge theory is the $l_s \to 0$ limit of the one-loop open string computation. By the worldsheet duality, it is related to exchange of closed strings. If the branes are not in angles, the closed string exchange does not generate a potential because of the cancellation of effects due to exchanges of NS-NS states and RR states. Since the $(N_f - N_c)$ D4 branes and the $N_c$ D4 branes are at angles in our case, the cancellation is not perfect. Thus we expect that a potential is generated for $M_0$. The one-loop field theory analysis predicts that this potential is attractive for all the non-compact directions in $M_0$.

When the $N_c$ D4 brane segments are far from the other $(N_f - N_c)$ D4 branes, main contributions to the potential come from graviton and RR fields exchange. Since the branes are at angles, the effect of the RR field exchange, which is repulsive, is weaker than the effect of the graviton exchange, which is attractive. Thus, the potential should be attractive when the two sets of D4 branes are widely separated. This is consistent with the field theory analysis. Unfortunately, this is not in the field theory regime where the distances between the D4 branes are less than $l_s$. It would be useful to develop general criteria in the language of brane configurations to decide when a potential between branes at angles becomes attractive so that we can tell when supersymmetry breaking configurations are locally stable.
2.5. Decay of the meta-stable vacua

In addition to the meta-stable supersymmetry breaking vacuum, the theory has supersymmetric vacua. We will show how the decay of the meta-stable vacuum into the supersymmetric vacua is described in the brane construction.

![Fig.4](image)

**Fig.4** In order for the meta-stable vacuum to decay into supersymmetric vacua, it has to climb up the potential barrier. In the brane construction, it is described as the bending of the \((N_f - N_c)\) D4 branes.

To go from the meta-stable vacuum to the supersymmetric vacua, we first move the \((N_f - N_c)\) D4 branes down toward the NS brane so that we end up having \((N_f - N_c)\) D4 branes connecting the NS’ and NS branes and \(N_f\) D4 branes connecting the NS and D6 branes. See Figure 4. This process costs energy as the \((N_f - N_c)\) D4 branes are bent and their lengths increase. We can estimate the extra energy density \(\Delta V\) for this configuration as

\[
\Delta V \sim \sum_{i=1}^{N_f-N_c} \left( \frac{1}{\cos \theta_i} - 1 \right) Z \sim \frac{1}{Z} \sum_{i=1}^{N_f-N_c} |m_i|^2. \tag{2.14}
\]

In the field theory, this corresponds to \(q = \tilde{q} = 0\) and \(M = 0\). This configuration was considered in [1] as an intermediate state in the decay of the meta-stable vacuum. The \(F\)-term potential evaluated for this field theory configuration agrees with (2.14).

Since there are \(N_f\) D4 branes connecting the NS and D6 branes, we can move them in the \(w\) direction. The locations of the D4 branes in the \(w\) plane are specified by the expectation value of \(M\). The field theory result shows that there are supersymmetric brane configurations at

\[
M_{ij} = m_{ij}^{-1} (\det m)^{\frac{1}{N_f}} (\Lambda^{3N_c-N_f})^{\frac{1}{N_c}}, \tag{2.15}
\]

where \(\Lambda\) is the strong coupling scale of the electric theory. For a generic mass matrix \(m\) with rank \(m = N_f\), the \(N_f\) D4 branes are all separated and away from the origin of the \(w\)
plane. This is why the brane configuration needs to climb up the potential barrier, reach the stage shown in Figure 4, and let all the $N_f$ D4 brane segments be moved away, before relaxing itself to the supersymmetric configurations.

We can also derive the locations of the $N_f$ D4 branes in the supersymmetric vacua by lifting the brane configurations to M theory. The description of the supersymmetric vacua in the M theory has already been given in [11,12,13,14] and we will not repeat the analysis here. In the M theory description, the NS5 brane and D4 branes are interpreted as M5 branes and the D6 branes are replaced by the Taub-NUT geometry. The M5 brane configurations are supersymmetric if they are holomorphic. From the subsequent analysis of the M5 brane configurations in [26,27,28,29], it is clear that they can be interpreted as the M theory lift of the D4/D6/NS5 brane configurations that can be reached by moving the $N_f$ D4 branes along the NS and D6 branes starting from the configuration in Figure 4. This is how the meta-stable brane configuration decays into the supersymmetric configurations.

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