Effect of collisions on plasma sheath in the presence of a gradient magnetic field

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Abstract. A low-pressure magnetized plasma is studied to find out the dependency of sheath properties on ion-neutral collisions in the presence of a gradient magnetic field. A single fluid hydrodynamic model is considered and the system of equations is solved numerically. The electrons are assumed to follow Boltzmann relation. The study reveals that the width of the plasma sheath expands and presheath length decreases with collisions. The ion-neutral collisions and gradient magnetic field restrict the ions to move towards the wall. The movement of the ions towards the wall can be controlled by choosing a suitable configuration of the magnetic field and ion-neutral collision frequency. The outcome of the study is supposed to help to understand the complex dynamics of ions in plasma confinement and plasma processing of materials.

1. Introduction

The plasma sheath is a very old area of research in plasma physics and it still remains to be one of the important fields due to its crucial role in many of plasma applications such as plasma processing, fabrication of semiconductor devices, etching, etc. Moreover, the study of this space charge layer has acquired more attention due to the growing importance of fusion reactors [1, 2]. In magnetically confined devices like tokamak, particle transport in the edge region often takes place in the presence of gradient magnetic field. Therefore, such applications strive many researchers to study the behavior of the sheath region and its influence on particle transport.

The properties of the plasma sheath have been studied for both electrostatic and magnetized environment [3]. However, for the magnetized case, most of the studies are done with an oblique magnetic field. The dynamics of the plasma sheath is observed to be influenced in the presence of an external magnetic field [1]. On the other hand, the ion-neutral collision plays an important role in the formation and stability of sheath structure. The collisional plasma sheath in the presence of a magnetic field has been investigated by many researchers [3, 4]. It has been observed that the collisional force affects the distribution of ion density and drift velocity [4, 6]. The study of collisional
plasma sheath has shown that collision alters the condition for the formation of the sheath. In the presence of collision, the sheath may be formed with ions having velocity much less than the ion-acoustic speed, violating the Bohm’s criterion \([3, 7, 8]\). Bohm’s criterion in a collisional magnetized plasma is explored in the Ref. \([9, 10]\). Such studies examined the possibility of a modified Bohm’s criterion, which limits both maximum and minimum allowable sheath entrance velocity of the positive ion at the sheath edge. In some other works, the thickness of magnetized presheath was found angle-dependent, in low pressure collisional environment \([3]\). The magnetic pre-sheath ceases to exist with the increase of angle of incidence. A similar study had been carried out by Hatami et al. \([11]\) on the collisional effects in magnetized plasma sheath with two species of positive ions. The findings of this study became particularly important as the kinetic energy of both ion species was found decreasing with the increase of ion-neutral collision frequency.

Collisional plasma sheath is usually modeled in two different ways \([3, 12]\):

- Constant collision frequency scheme. It assumes that the collision frequency does not depend on the ion velocity.
- The constant collision cross-section scheme, where the collision frequency varies with the ion velocity.

The present study follows the latter \([3]\), since a vast amount of literature support the same.

An extensive amount of study has been accomplished on plasma sheath in the presence of uniform and oblique magnetic field. The oblique magnetic field configuration has acquired intense interest with the inception of fusion research \([1–3, 13, 14]\). In particular, such studies under the influence of collision have become a trend in recent times. In a recent study \([15]\), it has been shown that the collisional force dominates over the effect of magnetic field on ion velocity. Again, Pandey et al. \([16]\) reported that the width of the plasma sheath depends not only on the collision and plasma magnetization but also on the orientation of the magnetic field. While these literature provided enough insight on plasma sheath in the presence of uniform and oblique magnetic field, they did not report anything on spatially varying magnetic field so far. In this article, we concentrate on the sheath formation near a wall in the presence of a magnetic field gradient. The geometry of the problem is shown in figure 1. Magnetic field is directed along x-axis, while its magnitude varies along z. We consider a linear profile of this variation. The problem has a perspective, where, a magnetic field is put parallel to the wall in order to restrict the plasma contact with the wall. The motivation lying behind the present work is to understand the nature of the plasma sheath in a collisional environment with the presence of a gradient magnetic field, which might be of use in material processing as well as in magnetically confined plasma devices.

The present work is arranged in six sections. In section 2, the basic equations and the theoretical model is discussed. In section 3, the normalization of the variables is done. Section 4 takes into account the required numerical scheme for the study. Section
2. Theoretical model and basic equations

A steady-state low-pressure \((T_i << T_e)\) plasma consists of positive ions and electrons is considered here. A schematic diagram of the theoretical model is shown in Fig. 1. In the theoretical model, it is assumed that the z-axis is perpendicular to the wall and the sheath parameters, such as plasma density, potential, electric field, etc. are varying along z-axis \([9,14,17]\). The magnetic field is taken along the x-axis, and there is a spatial variation of the magnetic field along the system length, which increases linearly towards the wall. The profile of the magnetic field is given by,

\[
B = B_0 (1 + z)
\]

Here, \(B_0\) is the value of the magnetic field at \(z=0\).

The positive ions are described by the fluid equations. The momentum equation for the ions is expressed as,

\[
m_i n_i v_z \frac{d\vec{v}}{dz} = -n_i e \frac{\partial \phi}{\partial z} \hat{k} + n_i e (\vec{v} \times \vec{B}) - m_i \vec{v} S_i - m_i n_i \vec{v} v_i
\]

Here, \(\phi, n_i,\) and \(v_i\) represent electric potential, ion-density, and ion-neutral collision frequency respectively. The rest of the symbols have their usual meaning. The velocity of the ions has three components \(v_x, v_y,\) and \(v_z\) along \(x, y,\) and \(z\) axes respectively. Accordingly, the momentum equation can be resolved into three components.
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\[ v_z \frac{dv_x}{dz} = -\frac{n_e}{n_i} v_y Z - v_z v_i \]  
(3)

\[ v_z \frac{dv_y}{dz} = \frac{eB}{m_i} v_z - \frac{n_e}{n_i} v_y Z - v_y v_i \]  
(4)

\[ v_z \frac{dv_z}{dz} = -\frac{e}{m_i} \frac{\partial \phi}{\partial z} - \frac{eB}{m_i} v_y - \frac{n_e}{n_i} v_z Z - v_z v_i \]  
(5)

Here, \( Z \) stands for the ionization frequency of the plasma. The source term \( S_i \) in the momentum equation accounts for the difference between the number of ions that are created and annihilated per unit volume per unit time in the plasma. Considering the source term, the continuity equation for the ions takes the form,

\[ \frac{\partial}{\partial z} (n_i v_z) = S_i \]  
(6)

There are usually three different forms of source term found in the literature,

- Zero source term \( i.e., \) the electrons and ions are neither produced nor lost throughout the plasma.

\[ S_i = 0 \]  
(7)

- Constant source term,

\[ S_i = \frac{n_0}{\tau} \]  
(8)

Where \( \tau \) is called ionization time.

- Exponential source term,

\[ S_i = \frac{n_0}{\tau} \exp \left( \frac{e\phi}{kT_e} \right) \]  
(9)

In this work, the exponential source term is taken assuming the main ionization mechanism is governed by electron-neutral collisions.

The electrons in the plasma are considered to be isothermal, and expressed by the Boltzmann relation,

\[ n_e = n_0 \exp \left( \frac{e\phi}{kT_e} \right) \]  
(10)

Here, \( n_0 \) and \( k \) is the bulk plasma density and Boltzmann constant respectively.

Finally, the set of equations is closed by the Poisson’s equation,

\[ \frac{\partial^2 \phi}{\partial z^2} = -\frac{e}{\epsilon_0} (n_i - n_e) \]  
(11)

Here, \( \epsilon_0 \) is the permittivity of free space.
3. Normalized parameters and the scale of simulation

To solve the basic set of equations numerically, the physical variables have been normalized with the help of the parameters given below,

\[ u = \frac{v_x}{c_s}, \quad v = \frac{v_y}{c_s}, \quad w = \frac{v_z}{c_s} \]

(12)

\[ \eta = \frac{e\phi}{kT_e}, \quad \xi = \frac{z}{L}, \quad L = \lambda_{ni} = \frac{c_s}{Z} \]

(13)

\[ N_i = \frac{n_i}{n_0}, \quad c_s = \sqrt{\frac{kT_e}{m_i}}, \quad N_e = \frac{n_e}{n_0} \]

(14)

Here, \( \lambda_{ni} \) represents ionization length and \( c_s \) is the ion-acoustic speed. After normalization, the equations (3-6) and (10-11) take the following form,

\[ \frac{du}{d\xi} = - \left( \frac{LZ}{c_s} \right) \left( \frac{N_e}{N_i} \right) \left( \frac{u}{w} \right) - \left( \frac{L\nu_i}{c_s} \right) \left( \frac{u}{w} \right) \]

(15)

\[ \frac{dv}{d\xi} = \gamma - \left( \frac{LZ}{c_s} \right) \left( \frac{N_e}{N_i} \right) \left( \frac{v}{w} \right) - \left( \frac{L\nu_i}{c_s} \right) \left( \frac{v}{w} \right) \]

(16)

\[ \frac{dw}{d\xi} = - \left( \frac{1}{w} \right) \left[ \frac{\partial \eta}{\partial \xi} + \gamma v + \left( \frac{L\nu_i}{c_s} \right) w \right] - \left( \frac{LZ}{c_s} \right) \left( \frac{N_e}{N_i} \right) \left( \frac{N_e}{N_i} \right) w \]

(17)

\[ \frac{dN_i}{d\xi} = \left( \frac{N_i}{w^2} \right) \left[ \frac{\partial \eta}{\partial \xi} + \gamma v + \left( \frac{L\nu_i}{c_s} \right) w + 2 \left( \frac{LZ}{c_s} \right) \left( \frac{N_e}{N_i} \right) w \right] \]

(18)

\[ \frac{d^2\eta}{d\xi^2} = q \left( N_e - N_i \right), \quad \text{where} \quad a = \left( \frac{L}{\lambda_D} \right)^2 \]

(19)

\[ N_e = \exp (\eta) \]

(20)

In this article, the constant collision cross-section is assumed for modelling the ion-neutral collision. The collision parameter for the present model can be defined as, \[3\]

\[ K = \frac{L}{\lambda_i}; \quad \lambda_i = \frac{|v_i|}{\nu_i} \]

(21)

Here, the parameter \( \lambda_i \) is the ion mean free path, which is considered constant on a case-by-case basis. \( |v_i| \) is the magnitude of the ion velocity. The equation (21) leads to,

\[ \frac{L\nu_i}{c_s} = K \frac{|v_i|}{c_s} = K \sqrt{u^2 + v^2 + w^2} \]

(22)

If \( d \) is the position of the wall for each numerical execution then the normalized coordinate of the wall is \( \xi_w = \left( d/L \right) \). This makes the graphical visualization easier and provides a new normalized scale between 0 and 1. The domain is \( 0 \leq \xi \leq \xi_w \) and due to new normalization, \[3\] we get \( 0 \leq (\xi/\xi_w) \leq 1 \).
4. Numerical analysis

The equations (15) - (19) is a set of ordinary differential equations that can be treated as an initial value problem, as we truncate our solution at the wall. For bulk plasma ($\xi = 0$), the three components of ion velocity are considered to be zero and the densities of ions and electrons are equal to the bulk plasma density. The value of the electric field and potential is also zero at $\xi = 0$. Therefore, the numerical solution of the set of equations cannot be started at this point. The problem is removed by shifting the initial point by a small amount to a new location towards the wall using Taylor’s series expansion. In this method, the plasma parameters, such as density, velocity, potential, etc. are expanded in a Taylor series to obtain the coefficients of the series. The method \[3,19,20\] provides precise and accurate initial values to start the numerical calculation. The plasma parameters are expressed in Taylor series as follows,

\begin{align*}
N_p &= N_{p_0} + N_{p_1}\xi^2 + N_{p_2}\xi^4 + \cdots \quad (23) \\
V_p &= V_{p_1}\xi + V_{p_2}\xi^3 + \cdots \quad (24) \\
\eta &= \eta_1\xi^2 + \eta_2\xi^4 + \cdots \quad (25)
\end{align*}

$N_p$, and $V_p$ depict the density, and velocity for each species in the plasma, and $\eta$ represents plasma potential. The equations (23) - (25) are substituted back in equations (15) - (19) to obtain the coefficients of each term in the series. Using these coefficients initial values are calculated. In the present work, MATLAB routine, ode45 is used for the numerical solution of the system of equations. The following plasma parameters are used for the numerical execution,

\begin{align*}
T_e &= 2eV, \quad m_i = 40 \text{ amu, } Z = 5 \times 10^5 s^{-1}, \quad n_0 = 10^{16} m^{-3}
\end{align*}

5. Results and Discussion

In the present work, an attempt has been made to establish the dependence of plasma sheath properties on ion-neutral collision in the presence of a gradient magnetic field. To have an exact idea about the sheath width, it is essential to study the space charge profile and ion-electron density profiles together. The bifurcation point in the ion-electron density can approximately be assumed as the starting point of the sheath \[21\]. The onset of space charges may also be considered as the sheath entrance point. From Figs. 2 and 3, it can be seen that the sheath width increases with the increase of ion-neutral collision. Collisions try to restrict the ions from moving towards the wall, which reduces the shielding of the negative potential at the wall causing plasma sheath to expand. Similar result can also be found in the following reference for unmagnetized plasmas \[21\]. For a particular value of collision parameter, the gradient magnetic field is found to prevent the ion movement towards the wall in a similar fashion. Hence, it is expected that the field is likely to have an indirect effect on the expansion of the sheath.
Figure 2. Variation of space charge with collision parameter \((K)\) for a gradient magnetic field with \(B_0 = 0.3T\).

Figure 3. Profile of ion-electron density for various values of collision parameter \((K)\) in the presence a gradient magnetic field with \(B_0 = 0.3T\).

Figure 4. Variation of ion density with collision parameter \((K)\) for a gradient magnetic field with \(B_0 = 0.3T\).
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Fig. 4 shows the variation of ion densities. It is observed that for the collision parameter $K = 100$ to 1200, the ion densities are almost falling together, as we go from the bulk plasma towards the wall up to a point $(\xi/\xi_w) \approx 0.6$. After that, the ion densities fall slowly with the increase of the collision. This can be explained with the help of Fig. 5, i.e., from the velocity component $w$. The density of ions along the sheath depends on the $z$-component of velocity. For the collision parameter $K = 100$ to 1200, as we go from the bulk plasma towards the wall, the $z$-component of velocity has no significant change up to a point $(\xi/\xi_w) \approx 0.6$. Thereafter, the $z$-component of velocity falls rapidly with the increase of collision parameter. Due to this, the ion density falls slowly to conserve the ion flux inside the sheath [15]. For the collisionless case ($K = 1$), the $z$-component of velocity is greater than the collisional case, and therefore the ion density falls faster than the collisional case. For a particular value of collision parameter, the velocity along the $z$-direction grows under the effect of,

- Collisional drag force which always tends to reduce the velocity of ions.
- Magnetic force along the negative direction of the $z$-axis.
- Accelerating force towards the wall due to the existing electric field.

The velocity component $w$ falls with the increase of the collision parameter. The increasing ion-neutral collisions inhibit the ions to move towards the wall. Therefore, the ions are slowed down and eventually, the ions strike the wall with a lower speed.

To have an idea about the variation of presheath length the parameters electric field, potential and electron density are plotted in the normalised scale. It can be seen from Figs. 2, 3, 6 and 8 that although the plasma sheath expands, the presheath length decreases with the increase of ion-neutral collision. Due to the presence of gradient magnetic field and increasing ion-neutral collision frequency, the Bohm’s criterion gets modified and the sheath forms at a lower value of ion velocity. Kim et al. [22] have experimentally shown that the length of presheath decreases with the increase of ion-
neutral collisions and strength of the magnetic field. So, the result obtained is in agreement with the experiment.

Figs. 6 and 7, respectively give the evolution of the electric field and potential inside the sheath. As the presheath length decreases, the separation between the wall and the bulk plasma appears to shrink with the increase of collision parameter. As a result, the electric field appears to fall more rapidly with collision. The collisions, on the other hand, slow down the ions and causes the ion density inside the sheath to fall slowly towards the wall. This in turn increases the difference between the amount of positive charge of ions and the negative charge nearby the wall causing the increase of the electric field inside the sheath.

Fig. 7 depicts that the potential is falling rapidly with the increase of collision parameter. This is again caused by the decrease in the distance of the wall from bulk plasma with collision. The potential at the wall is almost independent of the collision parameter since there is no apparent change of electron density near the wall with an increase of ion-neutral collision. Fig. 8 shows the electron density profiles for different collision parameters. For $K = 100$ to 1200, the electron density seems to fall rapidly as compared to the collisionless case ($K = 1$). Since the electrons are considered Boltzmann, they are expected to follow the potential variation across the length.

Fig. 9 gives the velocity profile along the $y$-direction. In the $y$-direction, there exist two drift velocities of the guiding center of the gyrating ions. The drift velocities are caused by the presence of the electric field and spatially varying magnetic field. They are, \[23\]

- $v_{E \times B} = \frac{\vec{E} \times \vec{B}}{B^2}$ i.e. $\vec{E} \times \vec{B}$ drift velocity along the positive $y$-direction.
- $v_{\nabla B} = \left( \frac{\nu_{el}}{2} \right) \frac{\vec{B} \times \nabla B}{B^2}$ i.e. $\nabla B$ drift velocity along the negative $y$-direction.

The magnetic force, acting along the $y$-direction increases linearly towards the wall. For a specific value of collision parameter, the magnetic force tends to increase the
**Figure 7.** Potential profile for various collision parameter ($K$) in the presence of a gradient magnetic field with $B_0 = 0.3T$.

**Figure 8.** Variation of electron density with different collision parameter ($K$) for a gradient magnetic field with $B_0 = 0.3T$.

**Figure 9.** Profile of ion velocity along the y-direction with collision parameter ($K$) for a gradient magnetic field with $B_0 = 0.3T$. 
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y-component of velocity, and ion-neutral collisions try to reduce it. The $\vec{E} \times \vec{B}$ drift velocity being in the positive y-direction contributes to the velocity $v$, and the grad-B drift velocity tends to diminish it. Under the combined effect of all these factors, velocity along the y-direction evolves as depicted in Fig. 9. As the ion-neutral collisions slow down the ions, the y-component of the velocity of ions falls with increasing collisions.

The velocity of ions along the direction of the x-axis is very small. Therefore, the variation in the velocity component $u$ is not properly observed when both velocity and distance are plotted on a linear scale. To overcome this problem velocity $u$ is plotted on a semi-log scale with distance. From Fig. 10, it is evident that the component of ion velocity along the x-direction rapidly falls to zero. It can be observed that there is no significant change in the profile of velocity with the increase of ion-neutral collision. Along the x-direction, there is no force except the collision, which always has a tendency to reduce the velocity of the particles.

**Figure 10.** Evolution of ion velocity along the x-direction with collision parameter ($K$) for a gradient magnetic field with $B_0 = 0.3T$

**Figure 11.** Evolution of ion velocity $u$, $v$ and $w$ along the sheath for various collision parameter ($K$) in the presence of a gradient magnetic field with $B_0 = 0.3T$
Fig. 11 describes the evolution of velocity components \( u, v \) and \( w \) with collision parameter. From the figure, it can be seen that sharing of velocity takes place between \( v \) and \( w \) only. The component of velocity \( u \) along the x-direction is almost zero in comparison to \( v \) and \( w \). As the ion-neutral collision increases, \( w \) gets the more share of velocity. The velocity \( w \) dominates the other components due to the presence of the accelerating electric field along the sheath length.

6. Conclusions

Using a single fluid hydrodynamic approach, the properties of a collisional plasma sheath in the presence of a gradient magnetic field has been explored. It has been seen that with the increase of collision parameter or ion-neutral collision frequency,

- Plasma sheath expands but the presheath length decreases.
- The distance of the wall from the bulk plasma decreases.
- Ions are slowed down.
- The density of ion falls slowly and space charge increases.
- The electric field increases but the potential remain unchanged.
- The \( w \) component of velocity always dominates over \( u \) and \( v \) component.
- The velocity along the x-direction rapidly falls to zero, and there is no significant change in the velocity with collision.

The magnetic field has a tendency to restrict the ions from moving towards the wall. The gradient magnetic field with increasing strength towards the wall, will impose more resistance to the movement of the ions in the direction of the wall. The ion-neutral collision plays a role similar to that of the magnetic field. Therefore, the combination of ion-neutral collision and gradient magnetic field might be useful in confining a plasma. Further, the flow of ions towards the wall may be controlled by optimizing the frequency of collision and the configuration of the gradient magnetic field. The future aspect of the study is to develop an experimental model for this, which is supposed to help in plasma processing of materials. The study of ion dynamics may help to develop the model. However, a more detailed study is required for a better understanding of the problem.

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