Influence of the optical axis orientation of the Glan-Taylor prism and the incident angle on the extraordinary ray

Peng Gao\textsuperscript{1,2}, Xiaochen Hu\textsuperscript{1}, Guanghai Guo\textsuperscript{1}, Yichao Wang\textsuperscript{1} and Jingjing Ai\textsuperscript{1,3}

\textsuperscript{1}College of Mathematical and Physical sciences, Shandong Advanced Optoelectronic Materials and Technologies Engineering Laboratory, Qingdao University of Science and Technology, Qingdao 266061, China
\textsuperscript{2}Institute of Space Optics, School of Electronic and Information Engineering, Xi’an Jiaotong University, Xi’an 710049, China
\textsuperscript{3}Email: jjaiqust@163.com

Abstract. According to the propagation characteristic of the plane wave in the Glan-Taylor prism, using the method of the ray tracing, the deflection angle of the \( e \)-ray wave vector and its position on the exiting plane as a function of different parameters are calculated for the first time. In order to analyze the influences of the incident angle \( i \) together with the angle \( \alpha \) of the optical axis of the Glan-Taylor prism and the crystal interface on the position of the extraordinary ray on the exiting plane \( L \) and \( L’ \), the variations of \( L \) and \( L’ \) with \( i \) and \( \alpha \) are simulated, and the effects of \( \alpha \) on \( L \) and \( L’ \) are very small except for \( i = 90^\circ \). To ensure the incident angle having no effects on the position of the \( e \) beam on the exiting plane, the incident angle should be controlled in a range of \( [0^\circ, 45^\circ] \).

1. Introduction
Since 1808, the polarization light technology has become a specialized and systematic method for the optical information processing [1-2], and the performance and imaging mechanism of the polarizer have also become the research hotspot [3-4].

In 2000, the tempo-spatially modulated polarization interference imaging spectroscopy is proposed by Zhang Chunmin et al. [5], and the static polarization interference imaging spectrometer [6] and static large field of view polarization interference imaging spectrometer [7], based on a Savart polariscope, are developed. These spectrometers use the field diaphragm instead of the slit in the traditional interference imaging spectrometer, thus it has the advantages of high steady state, high flux together with large field of view and high resolution [8-9].

The Glan-Taylor prism is one of the important polarization devices in the polarization interference imaging spectrometer, which is made up of the iceland crystal, and it has the perfect optical performance, wide spectral transmission range, and large birefringence index [10]. Besides, the Glan-Taylor prism adopts the air gap as gluing, which makes its extinction ratio better than \( 10^2 \), and the spectral range varies from 300 nm to 500 nm [11]. These significant advantages make the Glan-Taylor prism widely used in the national defense, scientific research and teaching equipment.

In this paper, using the method of the ray tracing, the deflection angle of the \( e \)-ray wave vector and its position on the exiting plane as a function of different parameters are calculated for the first time.
By computer simulations, the influences of the incident angle and the angle of the optical axis and the crystal interface on the position of the beam on the exiting plane are analyzed and discussed.

2. Theoretical analysis

2.1. Beam-split principle of the Glan-Taylor prism
The Glan-Taylor prism is composed of two identical iceland crystals with the wedge angle of $\beta$, and there is an air gap with a certain thickness on the interface of left-right crystals, as shown in figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The Glan-Taylor prism, where (a) and (b) are corresponding to the physical image and optical structure, respectively.}
\end{figure}

Ideally, the optical axes of the left-right shear plates for the Glan-Taylor prism are parallel to the incident surface. When the incident ray from the objective source reaches the left shear plate, due to the double refraction effect of the uniaxial crystal, it is divided into two linearly polarized light beams whose intensities are equal and the vibration direction is perpendicular to each other, namely, the ordinary ray $o$ and the extraordinary ray $e$. After passing through the air gap, the $o$-ray exits from the bottom of the left shear plate. So, the propagation of the $o$-ray in the Glan-Taylor prism is neglected. The $e$-ray enters into the right shear plate, and then split up into two beams, namely, $ee$-ray and $eo$-ray. Because the transmittance of the $ee$-ray is very small, only the propagation of the $eo$-ray is considered.

2.2. Ray tracing of the extraordinary light
Because the optical axis is not ensured as the ideal state in the processing of the Glan-Taylor prism, there is a certain machining tolerance existing. In this part, the influences of the tilting error of the optical axis of the Glan-Taylor prism on the deflection angle of the $e$-ray wave vector and its position on the exiting plane are analyzed and discussed.
Figure 2. The optical path of the o-ray and e-ray in the Glan-Taylor prism, when the incident ray is (a) above the normal to the incident plane and (b) below the normal to the incident plane.

Firstly, assuming that the incident angle is above the normal ray direction, as depicted in figure 2(a), the exact expressions of the deflection angle of the e-ray wave vector and its position on the exiting plane are derived. Here, the incident angle is denoted as $i$, and the refractive angles of the o-ray and e-ray are denoted by $\theta_o$ and $\theta_e$, respectively; the angle between the optical axis of the Glan-Taylor prism and the o-ray is $\theta$. Where the refractive directions of the o-ray and e-ray together with the optical axis of the prism are denoted by the pink, blue and red solid lines, respectively.

According to the characteristics of the double refraction crystal, the angle between the o-ray and e-ray is expressed as:

$$\tan \gamma = \frac{(n_e^2 - n_o^2) \tan \theta}{n_e^2 + n_o^2 \tan^2 \theta}, \quad (1)$$

where $n_o$ and $n_e$ represent the refractive indices of the o-ray and e-ray, respectively.

Taking equation (1) into the relationship $\theta_e = \theta_o + \gamma$, one has

$$\tan \theta_e = \frac{\tan \theta_o + \tan \gamma}{1 - \tan \theta_o \tan \gamma} = \frac{\tan \theta_o (n_e^2 + n_o^2 \tan^2 \theta) + \tan \theta (n_e^2 - n_o^2)}{(n_e^2 + n_o^2 \tan^2 \theta) - \tan \theta_o \tan \theta (n_e^2 - n_o^2)}. \quad (2)$$

On the basis of the Fresnel law and the geometrical relationship in figure 2(a), the following conditions are obtained:

$$\theta = \theta_o + \alpha - \frac{\pi}{2}, \quad (3)$$

and

$$\theta_o = \arcsin \left( \frac{\sin i}{n_o} \right), \quad (4)$$

where $\alpha$ is the angle of the optical axis for the Glan-Taylor prism and its incident interface.

Substituting equations (3) and (4) into equation (2), the deflection angle of the e-ray wave vector is written as:

$$\tan \theta_e = \frac{\arcsin \left( \frac{\sin i}{n_o} \right) \left[ n_e^2 + n_o^2 \tan^2 \arcsin \left( \frac{\sin i}{n_o} \right) + \alpha \right] - (n_e^2 - n_o^2) \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) + \alpha \right]}{n_e^2 + n_o^2 \tan^2 \arcsin \left( \frac{\sin i}{n_o} \right) + \alpha} + (n_e^2 - n_o^2) \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) \right] \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) + \alpha \right]. \quad (5)$$

Letting the thickness of the Glan-Taylor prism as $d$, the exiting position of the e-ray is expressed as follows:
$$L = d \tan \theta_e$$

$$\tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) \right] \left[ n_e^2 + n_o^2 \tan^2 \left( \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right) \right] - \left( n_e^2 - n_o^2 \right) \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right]$$

$$= d \left\{ n_e^2 + n_o^2 \tan^2 \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right] \right\} + \left( n_e^2 - n_o^2 \right) \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right]$$

When the incident angle is $i = 0^\circ$, the expression of the exiting position $L$ can be simplified as:

$$L = \frac{-d \left( n_e^2 - n_o^2 \right) \sin \alpha \cos \alpha}{n_e^2 \sin^2 \alpha + n_o^2 \cos^2 \alpha}.$$  \hfill (7)

Secondly, assuming that the incident angle is below the normal ray direction, as depicted in figure 2(b), the deflection angle of the $e$-ray wave vector together with its position on the exiting plane as a function of different parameters are obtained.

Similarly, the following result can be obtained from figure 2(b).

$$\theta = \theta_e + \frac{\pi}{2} - \alpha.$$  \hfill (8)

Based on equations (1), (2), (4) and (8), we have

$$\tan \theta'_e = \frac{n_o^2 + n_e^2 \tan^2 \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right] - \left( n_e^2 - n_o^2 \right) \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right]}{n_e^2 + n_o^2 \tan^2 \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right]}$$

and

$$L' = d \left\{ n_e^2 + n_o^2 \tan^2 \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right] \right\} + \left( n_e^2 - n_o^2 \right) \tan \left[ \arcsin \left( \frac{\sin i}{n_o} \right) - \alpha \right]$$

For $i = 0^\circ$, equation (10) is changed as:

$$L' = \frac{d \left( n_e^2 - n_o^2 \right) \sin \alpha \cos \alpha}{n_e^2 \sin^2 \alpha + n_o^2 \cos^2 \alpha}.$$  \hfill (11)

3. Computer simulation

In order to describe the influences of various parameters on the deflection angle of the $e$-ray wave vector together with its position on the exiting plane, the simulation of a selected design example is carried out in detail. Here, the main refractive indices of the calcite crystal are provided with $n_e = 1.65836$ and $n_o = 1.48641$, and the thickness of the crystal is $d = 25\text{mm}$.

The variation of the position of the $e$ beam on the exiting plane changing with the incident angle and the angle of the optical axis and the crystal interface is shown in figure 3, where the horizontal axis and vertical axis represent the incident angle $i$ and the angle of the optical axis and the crystal interface $\alpha$, respectively, while the color distribution stands for the exiting position value. From figures 3(a) and 3(b), the exiting position $L$ and $L'$ increase with the increasing of $i$ while they show a small change with the variation of $\alpha$. When the incident angle is varied from $80^\circ$ to $90^\circ$, the values of $L$ and $L'$ decrease with the increasing of $\alpha$. 


Figure 3. The position of the beam on the exiting plane $L$ and $L'$ as a function of the incident angle $i$ and the angle of the optical axis and the crystal interface $\alpha$, where (a) the incident ray above the normal to the incident plane and (b) the incident ray below the normal to the incident plane.

In order to further study the effects of the angles $i$ and $\alpha$ on the $L$ and $L'$, the two-dimensional simulations are carried out. Figure 4 shows the position $L$ and $L'$ as a function of $\alpha$ in the case of $i=0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$, while the position $L$ and $L'$ changing with $i$ are depicted in figure 5 for $\alpha=0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$, respectively.

According to figure 4, it is seen that, the values of $L$ and $L'$ varies little with various $\alpha$ if $i=0^\circ$, $30^\circ$ and $60^\circ$. However, for $i=90^\circ$, the value of $L$ first decreases with the increase of $\alpha$ then increases slowly; the value of $L'$ first increases and reaches a maximum, then decreases with increase of $\alpha$; namely, the influences of $\alpha$ on $L$ and $L'$ are very small in addition to $i=90^\circ$. So, the incident angle $i=90^\circ$ can not be realized in the engineering.

Figure 4. The position of the beam on the exiting plane $L$ and $L'$ as a function of the angle of the optical axis and the crystal interface $\alpha$, for $i=0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$, respectively.

Figure 5. The position of the beam on the exiting plane $L$ and $L'$ as a function of the incident angle $i$, for $\alpha=0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$, respectively.
On the basis of figure 5, the position of the beam on the exiting plane $L$ and $L'$ increase slowly with the increase of $i$ when $\alpha$ remains a constant. In the case of the incident ray above the normal to the incident plane, the influence of $i$ on $L$ is larger for $\alpha=0^\circ$, while the influence of $i$ on $L'$ is larger for $\alpha=30^\circ$ when the incident ray is below the normal to the incident plane. Besides, the values of $L$ and $L'$ have little change when $i=0^\circ$, that is to say, the incident angle has almost no effects on the position of the beam on the exiting plane if it is controlled in a range of $[0^\circ, 45^\circ]$.

4. Conclusions
According to the propagation characteristic of the plane wave in the Glan-Taylor prism, the exact expressions of the deflection angle of the $e$-ray wave vector together with its position on the exiting plane as a function of different parameters are obtained when the incident ray is either above or below the normal ray direction. In order to describe the influences of the incident angle $i$ and the angle of the optical axis and the crystal interface $\alpha$ on the positions of the $e$ beam on the exiting plane $L$ and $L'$, the variations of $L$ and $L'$ with $i$ and $\alpha$ are simulated, and some important conclusions are obtained. The influences of $\alpha$ on $L$ and $L'$ are very small in addition to $i=90^\circ$, and the incident angle has almost no effects on the positions of the $e$ beam on the exiting plane if it is controlled in a range of $[0^\circ, 45^\circ]$. Therefore, the incident angle and the optical axis orientation of the Glan-Taylor prism have an important impact on the $e$-ray tracing.

Acknowledgements
This work was supported by the National Natural Science Foundation of China (Grant No. 61605098; 11704213), the Natural Science Foundation of Shandong Province (ZR2017PD004) and the China's Post-doctoral Science Fund (No. 2017M610630).

References
[1] Zhang C, Huang W and Zhao B 2010 Acta Physica Sinica. 59 5479
[2] Peng Z, Zhang C, Zhao B, Li Y and Wu F 2006 Acta Physica Sinica. 55 6374
[3] Zhang C, Zhao B and Xiangli B 2004 Applied Optics. 43 6090
[4] Jian X, Zhang C, Zhao B, Zhang L and Zhu L 2009 Acta Physica Sinica. 58 2286
[5] Zhang C, Xiangli B and Zhao B 2000 International Society for Optics and Photonics. 957
[6] Zhang C and Jian X 2010 Optics letters. 35 366
[7] Yuan Z, Zhang C, Zhao B 2007 Acta Physica Sinic. 56 6413
[8] Jian X, Zhang C and Zhao B 2007 Acta Physica Sinica. 56 824
[9] Mu T, Zhang C, Jia C and Ren W 2012 Optics Express. 20 18194
[10] Zhang C, Liu N and Wu F 2010 Acta Physica Sinica. 59 949
[11] Zhang C, Bai X, Jing C and Wu H 2011 Optik-International Journal for Light and Electron Optics. 122 1770