Collapse/Flattening of Nucleonic Bags in Ultra-Strong Magnetic Field

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It is shown explicitly using MIT bag model that in presence of ultra-strong magnetic fields, a nucleon either flattens or collapses in the direction transverse to the external magnetic field in the classical or quantum mechanical picture respectively. Which gives rise to some kind of mechanical instability. Alternatively, it is argued that the bag model of confinement may not be applicable in this strange situation.

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1. INTRODUCTION

One of the oldest subject of physics- "the effect of strong magnetic field on dense matter" has gotten a new life after the observational discoveries of a few strongly magnetized exotic stellar objects- known as magnetars [1–5]. These uncommon objects are believed to be strongly magnetized young neutron stars and their strong magnetic fields are supposed to be the possible sources of X-rays from anomalous X-ray pulsars (AXP) and low energy γ-radiation form the soft gamma-ray repeaters (SGR). It is believed that such objects may also act as the central engine for gamma ray bursts (GRB). The measured value of magnetic field strength at the surface of these objects are \(10^{14} - 10^{15}\) G. Then it can very easily be shown by scalar virial theorem that the magnetic field strength at the core region may go up to \(10^{18}\) G. These objects are also assumed to be too young compared to the decay/expulsion time scale of magnetic fields from the core region. Now in presence of such intense magnetic fields, most of the physical properties of dense stellar matter, e.g., equation of states, quark-hadron phase transitions etc., must change significantly [6–8]. Not only that, some of the physical processes [9,10], in particular, weak and electromagnetic decays and reactions, neutrino opacities etc., at the core region of compact neutron stars will also be affected in presence of ultra-strong magnetic fields. The transport properties (e.g, shear and bulk viscosities, thermal and electrical conductivities) of dense neutron star matter also change both qualitatively and quantitatively in presence of strong magnetic field [11,12]. Furthermore, these intense magnetic fields could cause structural deformation of the exotic objects. In the classical general relativistic theory, it is shown by using Maxwell stress tensor that such exotic objects get flattened [13–15] for the macroscopic field \(B_m\), whereas in the quantum mechanical scenario they collapse in the direction transverse to the magnetic field [16,17]. In the case of ultra-strong magnetic field, the structure of these objects could become either disk like (in classical picture) or cigar like (in quantum mechanical scenario) from their usual spherical shapes. In the extreme case, they may be converted to black disks or black strings. Therefore, in some sense these strange stellar objects become mechanically unstable in presence of ultra-strong magnetic field. Long ago Chandrasekhar and Fermi in their studies on the stability of magnetized white dwarfs explained the possibility of such strange behavior [18]. Those conclusions are also valid for strongly magnetized neutron stars, where the white dwarf parameters have to be replaced by typical neutron star parameters; the upper limit of magnetic field strength for a stable neutron star of typical character is found to be \(10^{15}\) G. In a recent work we have shown that if the magnetic field is extremely high to populate only the zeroth Landau level (with fully polarized spin states) of electrons, then stable neutron star/proto-neutron star matter can not exist in the \(\beta\)-equilibrium condition [19,20]. It was also shown by Bander and Rubinstein in the context of stability of neutron and protons in a strong magnetic field that in presence of extremely strong magnetic field, protons becomes unstable by gaining effective mass, whereas neutrons, loosing effective mass and becomes stable [21]. In their calculations a delicate interplay between the anomalous magnetic moments of neutron and proton makes the neutron stable and proton becomes unstable; decays into neutron via \(e^+\) and neutrino emission.

In this article following the recent work of Martínez et al [16,17] and Kohri et al [22], we shall show that even the nucleonic (proton or neutron) bags can not be stable in presence of ultra-strong magnetic field- they either collapse...
or elongated in the transverse direction of ultra-strong external magnetic field. We have shown that either the nucleons are mechanically unstable or the bag model calculations can not be well suited for the conditions referred to above. In this work we have therefore studied the mechanical stability of a neutron/proton placed in an ultra-strong magnetic field. On the other hand in ref. [21], Bander and Rubinstein have studied the stability of these objects from the effective mass point of view and showed that neutrons are much more stable energetically than protons in this situation. The paper is organized in the following manner: in section 2, we have reviewed very briefly the MIT bag Lagrangian approach of color confinement. In section 3, we have studied the collapse of nucleons in the transverse direction following the ideas of Martínez et al [16,17]. In section 4, following the model proposed by Kohri et al [22] in the context of anisotropic $e^+e^-$ pressure, we have shown that nucleons get flattened in the transverse direction. The conclusions and discussions are presented in the last section.

2. COLOR CONFINEMENT- A BRIEF OVERVIEW

To study the mechanical stability of neutron/proton bags in presence of ultra-strong magnetic fields, in the flat space time coordinate, we have considered the MIT bag model of quark confinement [23–25]. We have taken into account both the gluonic interaction of quarks and the bag pressure $B$ to confine quarks within the bag. Before we go into the detailed discussion on the mechanical instability problem of nucleonic bags in presence of intense magnetic fields, we give a brief overview of bag model Lagrangian approach to obtain the pressure balance at the surface of the nucleons. The usual form of bag Lagrangian density is given by

$$L_{\text{MIT}} = i\{\bar{\psi}\gamma^\mu \partial_\mu \psi - (\partial_\mu \bar{\psi})\gamma^\mu \psi\} + g\bar{\psi} \lambda_\alpha V^\alpha_\mu \psi - \bar{\psi} m \psi - \frac{1}{4} F^a_\mu \nu F^{\mu \nu a} - B\theta_v(x)$$

$$- \frac{1}{2} \bar{\psi} \Delta_s$$  \hspace{1cm} (1)

where $g$ is the strong coupling constant, $\lambda_\alpha$’s are the $SU(3)$ generators, with $a = 1, 2, \ldots, 8$, the gluonic color index, $V^a_\mu$ is the gluonic field four vector, $F^{a}_\mu \nu$ is the corresponding field tensor, $m$ is the current mass of quarks, $B$ is the bag constant, $\theta_v = 1$ inside the bag and $= 0$ outside the bag, $\partial \theta_v/\partial x^\mu = n_\mu \Delta_s$, $\Delta_s$ is the surface delta-function and $n_\mu$ is the space-like unit vector normal to the surface. The sum over flavors and color quantum numbers carried by quarks have not been shown explicitly. To obtain the pressure balance at the bag surface, we consider the energy momentum tensor of the bag, given by

$$T^{\mu \nu} = -g^{\mu \nu} L + \left( \frac{\partial L}{\partial \partial_\mu \psi} \partial_\nu \bar{\psi} + \partial_\nu \bar{\psi} \frac{\partial L}{\partial \partial_\mu \psi} \right)$$

$$= -g^{\mu \nu} L + \frac{i}{2} \left( \bar{\psi} \gamma^\mu \partial_\nu \psi - (\partial_\nu \bar{\psi})\gamma^\mu \psi \right) \theta_v$$ \hspace{1cm} (2)

and using the energy momentum conservation, given by $\partial_\mu T^{\mu \nu} = 0$, we have

$$B\Delta_s n^\nu + \frac{i}{2} \left( \bar{\psi} \gamma^\mu \partial_\nu \psi - (\partial_\nu \bar{\psi})\gamma^\mu \psi \right) n_\mu \Delta_s = 0$$ \hspace{1cm} (3)

and

$$\partial_\mu \left( \bar{\psi} \psi \Delta_s \right) = 0$$ \hspace{1cm} (4)

Now considering the surface boundary condition, given by (obtained from standard Euler-Lagrange equation)

$$in_\mu \gamma^\mu \psi = \psi$$ \hspace{1cm} (5)

we obtain on the bag surface

$$Bn^\mu = \frac{1}{2} \bar{\psi} \partial_\mu (\bar{\psi})$$ \hspace{1cm} (6)

This equation is nothing but the pressure balance equation. Since $n^\mu n_\mu = -1$, we have on the bag boundary
In the case of spherical bag, \( n^\mu \equiv (0, \hat{r}) \) and this pressure balance equation reduces to

\[
B = -\frac{1}{2} n^\mu \partial_\mu (\bar{\psi} \psi) \tag{7}
\]

Which means that outward pressure of the quarks is exactly balanced by the inward vacuum pressure \( B \) on the surface of the bag.

3. COLLAPSE OF NUCLEONIC BAGS

Now we shall consider the nucleonic bag (either neutron or proton) as an interacting thermodynamic system in equilibrium. The constituents are valance quarks, sea quarks and gluons. Then the total kinetic pressure of the system is given by

\[
P_{\text{in}} = P_{\text{in}}^{(v)} + P_{\text{in}}^{(s)} + P_{\text{in}}^{(g)} \tag{9}
\]

where \( v, s \) and \( g \) represent the valance quarks, sea quarks and gluonic contributions respectively. As discussed before, this internal kinetic pressure has to be balanced by the external bag pressure to maintain the stability of the system. Then we can write down the effective thermodynamic potential per unit volume of the system as

\[
-\Omega = P_{\text{in}} - B \tag{10}
\]

and it should be zero. Then following Martínez et al [16,17], we have in presence of ultra-strong magnetic field of strength \( B_m \), the thermodynamic potential per unit volume (we have chosen the gauge \( A^\mu \equiv (0, -yB_m/2, xB_m/2, 0) \), so that \( B_m \) is a constant magnetic field along \( Z \)-axis)

\[
T^\nu_\mu = \left( \frac{T\partial\Omega}{\partial T} + \sum_r \mu_r \frac{\partial\Omega}{\partial \mu_r} \right) g_4^\nu g_4^\mu + 4F_{\mu\lambda}F^{\nu\lambda} \frac{\partial\Omega}{\partial F^2} - g_\mu^\nu \Omega \tag{11}
\]

Hence the longitudinal component of pressure (along the direction of field) is given by

\[
T_{zz} = P_{||} = -\Omega = 0 \tag{12}
\]

and the transverse part of total pressure

\[
T_{xx} = T_{yy} = P_{\perp} = -\Omega - \mathcal{M}B_m = P_{||} - \mathcal{M}B_m \tag{13}
\]

where \( \mathcal{M} \) is the effective magnetic dipole moment density of the bag. Since \( \Omega = 0 \), nucleons will therefore be inflated or collapsed in the transverse direction in presence of ultra-strong magnetic field depending on the overall sign of \( \mathcal{M} \). The system will collapse if \( \mathcal{M} \) is positive, else it will be inflated in the transverse direction. In order to have an order of magnitude estimate of extra in/out-ward pressure, we choose the contribution to \( \mathcal{M} \) from valance quarks only (in fact the valance quarks only contribute in the evaluation of magnetic dipole moment of the nucleons). The magnetic dipole moment density of the \( i \)th. component \( (i = u \) or \( d \)-quarks) is given by

\[
\mathcal{M}_i = -\frac{\partial\Omega_i}{\partial B_m} \tag{14}
\]

and the total value is given by

\[
\mathcal{M} = \sum_{i=u,d} \mathcal{M}_i \tag{15}
\]
where

\[
\Omega_i = \frac{g_i q_i B_m}{4 \pi^2} \sum_{\nu=0}^{n_{\text{max}}} \sum_{s=\pm 1} (\mu_i (\mu_i^2 - M^2_{i,\nu,s})^{1/2} - M^2_{i,\nu,s}) \ln \left( \frac{\mu_i + (\mu_i^2 - M^2_{i,\nu,s})^{1/2}}{M_{i,\nu,s}} \right)
\]

(16)

is the thermodynamic potential density of the component \( i \), \( g_i \) and \( q_i \) are respectively the degeneracy and charge of the \( i \)th. species, \( M^2_{i,\nu,s} = \left((\mu_i^2 + m_i^2)^{1/2} + sQ_i B_m\right)^2 \), \( m_i \) is the current quark mass (=5MeV), \( p_\perp = (2\nu q_i B_m)^{1/2} \) is the transverse component of momentum and \( Q_i \) is the anomalous magnetic dipole moment of the \( i \)th. quark species \((Q_u = 1.852\mu_N \text{and} Q_d = -0.972\mu_N, \mu_N \text{is the nuclear magneton})\). The maximum value of Landau quantum number is given by

\[
\mu^{(i)}_{\text{max}} = \left[ \frac{\mu_i^2 - sQ_i B_m^2 - m_i^2}{2q_i B_m} \right]
\]

(17)

where \([ \ ]\) indicates an integer less than the decimal number within the brackets. To obtain the chemical potentials for \( u \) and \( d \) quarks, we have made the following assumptions. The \( i \)th. quark species density within the nucleon is given by

\[
n_i = \frac{g_i q_i B_m}{2 \pi^2} \sum_{\nu=0}^{n_{\text{max}}} \sum_{s=\pm 1} (\mu_i^2 - M^2_{i,\nu,s})^{1/2} = \frac{\text{NO}(i)}{V}
\]

(18)

where \( \text{NO}(i) \) is the number of \( i \)th. quarks species in the system. Therefore, \( \text{NO}(i) = \text{NO}(u) = 1 \) for neutrons and 2 for protons. Similarly, \( \text{NO}(i) = \text{NO}(d) = 2 \) for neutrons and 1 for protons and \( V \) is the nucleonic volume. We further assume that \( r = 0.8\text{fm} \) as the radius of the nucleons. Solving numerically, we have obtained the chemical potentials \( \mu_i \)'s for both \( u \) and \( d \) quarks and hence evaluated the magnetic dipole moment per unit volume for the system from eqns.(13)-(15). In fig.(1) we have plotted \( MB_m \) for various values of \( B_m \) for both neutrons and protons. The product \( MB_m \) is always positive and oscillatory in the strong field regime \(( \geq 10^{17}\text{G}) \). The system will therefore collapse in the transverse direction and becomes ellipsoidal with cylindrical symmetry. The minor axes lengths \( b \) will therefore oscillate with the strength of magnetic field in particular, above \( 10^{17}\text{G} \). Now in the study of mechanical stability of strongly magnetized neutron stars in quantum mechanical scenario, it has been shown that the system will either be inflated or collapsed if the magnetic dipole moment is negative or positive respectively. It has further been shown that neutron matter always behaves like a paramagnetic material with \( M > 0 \), as a result, in the quantum mechanical picture a strongly magnetized neutron star always collapses in the transverse direction. Therefore we can infer that the conclusion drawn for such macroscopic objects like neutron stars is also valid in the microscopic level-e.g., neutrons or protons. We can then conclude that in a strong magnetic field, not only neutron stars, even their constituents, neutrons and protons become mechanically unstable. Alternatively, one could conclude that the bag model is perhaps not applicable in such strange situation, in that case the use of bag model for magnetized quark stars is also questionable. Therefore the investigations of this section show that both neutrons and protons become cigar like and in the extreme case they may reduce to what is called black string.

4. FLATTENING OF NUCLEONS

In this section we shall evaluate the longitudinal and transverse parts of the kinetic pressures following ref. [22]. We choose the gauge \( A^\mu \equiv (0,0, xB_m, 0) \) so that \( B_m \equiv (0,0, B_m) \). Then the solution of the Dirac equation is given by

\[
\psi = \exp(-iE_nt) \begin{pmatrix} \phi \\ \chi \end{pmatrix}
\]

(19)

where \( \phi \) and \( \chi \) are the upper and lower components. The upper component is given by

\[
\phi = \exp(ip_y y + ip_z z)f_n \zeta_n,
\]

(20)

where \( n = 0,1,2,.. \) is the Landau quantum number, \( s = \pm 1 \), the spin quantum number, so that
and

$$f_n(x, p_y) = \frac{1}{(2\pi \nu)^{1/2}} \exp \left( -\frac{\xi^2}{2} \right) H_n(\xi)$$

\(\xi = (q_i B_m)^{1/2}(x - p_y/(q_i B_m))\) and \(H_n(\xi)\) is the Hermite polynomial of order \(n\). The lower component is given by

$$\chi = \vec{\sigma} \cdot (\vec{p} - q_i \vec{A})$$

The energy eigen value is given by

$$E_n = (p^2 + m^2 + q_i B_m(2n + 1 - s))^{1/2}.$$

Then we have from the first part of eqn. (2),

$$T_\mu^\nu = \text{diag}(E_n, -\hat{P}_x, -\hat{P}_y, -\hat{P}_z)$$

whereas all the off-diagonal terms are zero. Then it is very easy to show

$$\hat{P}_x = \hat{P}_y = \left( n + \frac{1}{2} - \frac{s}{2} \right) q_i B_m, \quad \hat{P}_z = \frac{p^2}{E_n}$$

These are called the dynamic pressure [22]. The ensemble average of these pressures are given by (at \(T = 0\))

$$P_x = P_y = \left( \frac{q_i B_m}{2\pi^2} \right)^{\nu_{\text{max}}} \sum_{\nu=0}^{\nu_{\text{max}}} \nu(2 - \delta_{0\nu}) \ln \left( \frac{\mu_i + (\mu_i^2 - m_{\nu}^2)^{1/2}}{m_{\nu}} \right)$$

which are the transverse part, where \(2\nu = 2n + 1 - s\) and \(m_{\nu}^2 = m_i^2 + 2q_i B_m\nu\). Similarly, we have the longitudinal component

$$P_z = \frac{q_i B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{0\nu}) \left[ \mu_i (\mu_i^2 - m_{\nu}^2)^{1/2} - \ln \left( \frac{\mu_i + (\mu_i^2 - m_{\nu}^2)^{1/2}}{m_{\nu}} \right) \right]$$

Following the same numerical techniques as followed in previous section, in figs. (2) and (3) we have plotted the longitudinal and transverse component of kinetic pressures with various magnetic field strengths for protons and neutrons respectively. The curves in both the figures show that the longitudinal part of kinetic pressure is zero and / or very low for high magnetic field strength. Whereas the transverse part is high for high magnetic field. These two components saturate to some constant value for low or moderate magnetic field strength. Which indicates that the system reduces to pressure isotropic configuration at low magnetic field (as we generally see in conventional thermodynamic system). Therefore, according to this model, at very high magnetic field strength the system (neutron or proton) becomes oblate in shape and in the extreme case it reduces to a black disk.

5. CONCLUSIONS

In conclusion, we have studied the mechanical stability of neutrons and protons in a compact neutron star in presence of strong quantizing magnetic field. We have followed two entirely different approaches. In the so called quantum mechanical picture, in which the interaction of magnetic dipole moments of quark constituents with the external magnetic field has been considered, the shapes of both neutron and proton become prolate type from their usual spherical nature. The effect is more prominent at high field limit \((> 10^{16} \text{G})\). On the other hand, in the classical picture, both the systems acquire oblate shape. The effect is again prominent for high magnetic field. In the classical picture, it has been observed that such anisotropy of kinetic pressure is automatically removed at moderate
($\geq 10^{15-16}$G) values of magnetic field strength and both the systems become mechanically stable. However, in the quantum mechanical picture, there is always an extra in-word pressure in the transverse direction even for moderate values of magnetic field strength. This is because of non-zero finite values for $\mathcal{M}B_m$ of the systems, but the effect is not so significant. Therefore, the behavior of bulk objects like neutron stars and their constituents, e.g., neutrons and protons (which are of microscopic in sizes) are almost identical in an external strong magnetic field.

![Graph](image-url)

**FIG. 1.** The variation of $\mathcal{M}B_m$ with $B_m/B_{m}^{(c)(e)}$ for neutron (indicated by the symbol n) and proton (indicated by the symbol p).
FIG. 2. The variation of kinetic pressure ($P^{1/4}$ in MeV) with $B_m/B_{m(e)}^{(c)}$ for proton. Curve $a$ is for longitudinal and $b$ is for transverse components of kinetic pressures respectively.

FIG. 3. The variation of kinetic pressure ($P^{1/4}$ in MeV) with $B_m/B_{m(e)}^{(c)}$ for neutron. Curve $a$ is for longitudinal and $b$ is for transverse components of kinetic pressures respectively.

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