Superfluidity in asymmetric electron-hole systems

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The pairing in a system of electrons and holes in two spatially separated parallel planes is studied in the case of electron-hole asymmetry caused by the difference in the carriers masses and their chemical potentials. It is predicted that the system may exhibit two critical temperatures for some range of the asymmetry parameters. The lower critical temperature corresponds to the superfluid transition induced by thermal fluctuations. It is found that the superfluid state is possible in a wide range of the asymmetry parameters, because the asymmetries can effectively compensate each other. In the asymmetric system a coexistence of the normal and superfluid states is possible even at zero temperature.

Superfluid transition in systems of two parallel planes with spatially separated electrons and holes has been a subject of interest for many years. Drag currents and zero charge dipolar current were predicted in such systems. In spatially separated system of electrons and holes in two parallel planes the chemical potentials and the mass of the positive and negative carriers can be controlled independently, making them a good testbed for studies of the electron-hole asymmetry on the formation of the superfluid state. It is common that in chemically doped semiconductor systems the positive and negative charge carriers may have very different effective masses. The ratio between the electron and hole masses \( m_e/m_h \) may vary, dependent on the material and the doping levels. Recently there were suggestions to engineer the effective mass value. It was assumed that for a given electron-hole mass ratio the superfluidity exists and the change of this parameter would change the results only quantitatively. Besides, it was usually assumed that the most favorable condition for pairing occurs at the equal concentrations of electrons and holes that for equal carrier masses corresponds to the equal chemical potentials. Even a small carriers concentration difference would lead to a destruction of the superfluid state, because for nonequal carrier concentrations not all particles can be paired. It is expected that the mismatch between the chemical potentials \( \mu_e \neq \mu_h \) may significantly reduce the critical temperature of the superconducting transition. For sufficiently large mismatch between the chemical potentials, comparable to the order parameter, the superfluid transition may be impossible. This situation was extensively studied in the context of asymmetric nuclear matter, where the different number of protons and neutrons results in different Fermi energies, and in cold Fermi gases with two or more atomic species. In nuclear matter the asymmetry comes from the different concentrations of protons and neutron, while their masses are usually assumed identical. The asymmetric nuclear matter may exhibit coexistence of paired and unpaired components at zero temperature and two critical temperatures, that results in superfluidity in some temperature range. These studies predicted that even a tiny proton-neutron asymmetry destroys the superfluid state, when the chemical potential difference becomes comparable to the magnitude of the order parameter in the symmetric system.

In this work we study an asymmetric electron-hole system, where the electrons reside in a two-dimensional layer and the holes in another two-dimensional layer, which can exhibit superfluidity in a very wide range of the electron-hole mass and chemical potential asymmetry. We found a range of the parameters, where the system has two critical temperatures, and superfluidity exists in a finite temperature interval. We show that the superfluid state survives because the electron-hole mass asymmetry can be effectively compensated by the chemical potential difference. We also predict that at zero temperature with the change of the asymmetry parameters the system may undergo the quantum phase transition of the first order.

Let us consider a system of two parallel planes separated by a dielectric with spatially separated electrons and holes confined on each plane. On one hand even in the case of perfectly equal carrier concentrations and zero temperature, the Fermi energy levels may be significantly different due to the difference in the carrier masses. And on the other hand, for unequal masses and equal Fermi energies the difference in concentrations of the carriers in the planes will result in a considerable amount of the carriers, which can not find their pairs. We are presenting the study how these two types of asymmetry interfere with each other and affect the formation of the superfluid state. Let us assume the carrier concentrations in the system are \( \rho_e \) and \( \rho_h \) for electrons and holes, respectively. The effective Hamiltonian can be written as:

\[ H_{\text{eff}} = \sum_p (\xi_p - \mu_e) b_p^\dagger b_p + \sum_{p'} (\xi_{p'} - \mu_h) a_{p'}^\dagger a_{p'} + \sum_p \left[ \Delta_p b_p^\dagger b_{p'}^\dagger + H.c. \right], \]

where \( a_{p'} \) is the operator of annihilation of a hole on one plane, and \( b_p \) is the operator of annihilation of an electron on another plane. The single-particle energies of electrons are given by \( \xi_p = \frac{p^2}{2m_e} = \frac{E^2}{2m_e}. \) Similarly
the single-particle energies of holes are presented as \( \xi_p = \frac{|p|^2}{2m_h} = \frac{\hbar^2|k|^2}{2m_h} \). The masses of the electrons and holes \( m_e \) and \( m_h \) and the chemical potentials \( \mu_e \) and \( \mu_h \) we treat as independent parameters. In Eq. (1) the nonzero order parameter \( \Delta_p \) shows that the system is in the superfluid phase. One can diagonalize Eq. (1) using Bogoliubov unitary transformations \( a_p = u_p \xi_p + v_p \xi_p^\dagger \) and \( b_p = v_p \beta_p - u_p \alpha_p \), with the amplitudes \( u_p, v_p \). The self-consistency condition for the order parameter has the form

\[
\Delta_p = \sum_p V(p-p')u_p^* v_p^*(1 - f(E_+ - f(E_-)),
\]

where \( V(p) \) is the screened Coulomb attractive interaction between electrons and holes. Here we use the notation \( E_\pm = E \pm \eta_p, E = \sqrt{\epsilon_p^2 + \Delta^2} \), \( \epsilon_p = (\xi_p + \xi_p' - \mu_e - \mu_h)/2 \), \( \eta_p = (\xi_p - \mu_e - \xi_p' + \mu_h)/2 \). In Eq. (2) the Fermi-Dirac distribution function is given by \( f(x) = \exp(x/(k_B T)) + 1 \)^{-1}, where \( k_B \) is the Boltzmann constant, and \( T \) is temperature. The amplitudes \( u_p, v_p \) are given by:

\[
u_p^2 = \frac{1}{2}(1 + \frac{\Delta}{\epsilon_p}), \quad \nu_p^2 = \frac{1}{2}(1 - \frac{\Delta}{\epsilon_p}).
\]

It is convenient to introduce new variables. First, we introduce the mass asymmetry parameter \( \alpha = (m_e - m_h)/m_h \) and the chemical potential difference \( \delta \mu = \mu - \mu_h \). We also introduce the average kinetic energy \( \xi = (\xi_p + \xi_p')/2 \), the average chemical potential \( \mu^* = \frac{\mu - \mu_h}{2} \), and the effective asymmetry variable \( r = \alpha \mu^* + \delta \mu \). In the new variables the quasiparticle energies can be represented as:

\[
E_\pm = \sqrt{(\xi - \mu^*)^2 + \Delta^2} \pm (\alpha \xi + \delta \mu).
\]

One can use the result for the interaction potential obtained for the electron-hole pairing in two parallel plane layer, given by Eq. (7) in Ref. [1]. Assuming the constant attraction of the paired particles one can replace the interaction term \( V(p-p') \) by a constant effective interaction \( U \propto \frac{1}{2\pi\sigma_{\Delta} \kappa^2} \exp(-DKr) \), where \( kr \) is the Fermi wavevector, \( \sigma_{\Delta} \) is the characteristic dielectric constant of the dielectric between the planes, and \( D \) is the separation distance between the planes. The cut-off for the effective interaction is set by the characteristic plasma frequency. To simplify the calculations we assume that the order parameter is independent on the momentum: \( \Delta_p = \Delta \). The chemical potentials for electrons \( \mu_e \) and holes \( \mu_h \) are coupled self-consistently to the given surface densities \( \rho_e \) and \( \rho_h \) of the carriers:

\[
\rho_e = \frac{2}{S} \sum_p \left[ f(E_+) \nu_p^2 + (1 - f(E_-)) \nu_p^2 \right],
\]

\[
\rho_h = \frac{2}{S} \sum_p \left[ f(E_-) \nu_p^2 + (1 - f(E_+)) \nu_p^2 \right],
\]

where \( S \) is the unit of the surface area. The chemical potentials \( \mu_e \) and \( \mu_h \), and the gap \( \Delta \) can be determined by the simultaneous solution of Eqs. (2) and (3). If the chemical potentials are much larger than the order parameter, one can neglect their changes due to the superfluid transition. This significantly simplifies the calculations. Without losing generality let us assume the density of electrons is higher than the density of holes: \( \rho_e > \rho_h \). The excesses electrons can not find their pairs, and they may form the normal phase. This phase occupies certain energy levels. Analyzing the density difference

\[
\delta \rho = \rho_e - \rho_h = \frac{2}{S} \sum_p [f(E_+) - f(E_-)],
\]

one notes that in the limit of low temperatures \( T \to 0 \) the Fermi distribution function becomes the step function \( f(x) \to \theta(-x) \). Because of the algebraic structure only one out of the quasiparticle energies \( E_e \) and \( E_h \) can take both positive and negative values, the other quasiparticle energy keeps its sign. Let us assume \( E_+ > 0 \) for all values of the momentum \( p \), and \( \theta(-E_+) \equiv 0 \). In this case the unpaired electron fraction occupies the energy levels within \( E_- < 0 \). Considering the inequality \( E_- = \sqrt{\nu_p^2 + \Delta^2} - \eta_p < 0 \), one can observe that the unpaired particles occupy the energy interval \([\xi_1, \xi_2]\)

\[
\xi_{1,2} = \mu^* \pm (\alpha r \pm \sqrt{r^2 - g^2 \Delta^2})g^{-2},
\]

where we use the notation \( g = \sqrt{1 - \alpha^2} \). In general case the interval \([\xi_1, \xi_2]\) is asymmetric with respect to \( \mu^* \). If the condition \( \Delta > \frac{\nu_p}{2} \) is satisfied, then the superfluid and unpaired normal phase may coexist even at zero temperature. The presence of unpaired component partially blocks the pairing near the Fermi surface. The energy levels occupied by the unpaired electrons are automatically excluded in Eq. (2). At low temperatures \( T \to 0 \) it becomes \( 1 = V \sum_{p} \frac{1 - \theta(-E_-)}{2E} \), i.e. only states with \( E_- > 0 \) participate in the pairing.

In this interesting case the standard Bardeen-Cooper-Schrieffer variational ground state wavefunction is not useful. To describe the asymmetric system one may use the following variational wavefunction:

\[
\Psi = \prod_{k} a_{k}^\dagger \prod_{k} (u_{k_j} + u_{k_j}^* a_{k_j}^\dagger a_{-k_j}^\dagger)|0>.
\]

The wavefunction describes the paired electron-hole component, as well as the unpaired component that in our case are the excess electrons. The summation over \( k_i \) and \( k_j \) correspond to \( E_- < 0 \) and \( E_+ > 0 \), respectively. It is necessary to note that the condition \( E_+ < 0 \) may have no solutions in the case of relatively small asymmetries. It means that while the densities of holes and electrons are not exactly equal, they all still form the superfluid condensate.

In the case of infinite planes one replaces the summation over \( p \) by two dimensional integration. One can make a standard change of variables in the self-consistency equation transforming from the momentum to the energy integration:

\[
1 = N(0)V \int_{\mu^* - \xi_c}^{\mu^* + \xi_c} d\xi \frac{\theta(\xi_1 - \xi) + \theta(\xi_2 - \xi)}{2E},
\]
where $\xi_c$ is the high-energy cut-off, $N(0)$ is the density of states at the Fermi level, which in the case of two dimensional systems is a constant $N(0) = m^*/\pi \hbar^2$, where $m^*$ is the reduced mass of the electron-hole pair. The expression on the left in Eq. (7) can be represented as $N(0) V \ln(\Delta/\Delta_0)$. The self-consistency Eq. (7) has two possible solutions. The first solution is a constant gap $\Delta = \Delta_0$, where $\Delta_0$ is the gap in the symmetric case $\alpha = \delta \mu = 0$ at zero temperature. The second non-trivial solution can be found analytically in some limiting cases. If one assumes $\mu^* \gg \xi_c \gg \Delta$, then $-\xi_c + \sqrt{(\xi_c)^2 + \Delta^2} \approx \frac{1}{2} \xi_c$ and $(\xi_c + \sqrt{(\xi_c)^2 + \Delta^2}) \approx 2 \xi_c$. With these simplifications the self-consistency condition becomes

$$1 = N(0) V \ln \left[ \frac{4\xi_c^2 (\xi_2 - \mu^*) - \sqrt{(\xi_2 - \mu^*)^2 + \Delta^2}}{\Delta^2 (\xi_1 - \mu^*) - \sqrt{(\xi_1 - \mu^*)^2 + \Delta^2}} \right]. \tag{8}$$

Using that $\sqrt{(\xi_i - \mu^*)^2 + \Delta^2} = \alpha \xi_i + \delta \mu$, $i = 1, 2$ one can further simplify Eq. (8):

$$(\xi_2 - \mu^* + \alpha \xi_2 + \delta \mu)(\xi_1 - \mu^* - \alpha \xi_1 - \delta \mu) = -1/\Delta_0^2. \tag{9}$$

Substituting the expressions for $\xi_1$ and $\xi_2$ from Eq. (10) into Eq. (9) one can find an analytical expression for the order parameter

$$\Delta = \Delta_0 \sqrt{1 - \frac{2|\alpha|}{g \Delta_0^2}}. \tag{10}$$

Now let us study the thermodynamical stability of the obtained solutions. Following Ref. 14, we are using the theorem that for small variations of an external parameter of the system the grand potential varies in the same way as the Hamiltonian: $\frac{\partial \Omega}{\partial V} = <\frac{\partial H}{\partial V}>$. Using the expression for the Hamiltonian of the system it is possible to write the expression for the grand potential:

$$\Omega = -\int \frac{dV}{\sqrt{2}} \int dr |\Delta(r)|^2. \tag{11}$$

One can make change of variables, using the result for the gap $\Delta_0$ in the symmetric case: $\frac{d\Delta_0}{\Delta_0} = \frac{2dV}{N(0)V}$. For a homogeneous system the difference between the grand potential of the superconducting and the normal states is

$$\Omega - \Omega_o = -\frac{N(0)}{2} \int_{\Delta_1}^{\Delta_0} |\Delta|^2 \frac{d\Delta_0}{\Delta_0}, \tag{12}$$

where $\Delta_1$ is the value of $\Delta_0$ corresponding to $\Delta = 0$. It is easy to compare the value of the grand potential for two solutions. By substituting the solution with the constant gap $\Delta = \Delta_0$ in Eq. (12) and integrating from $\Delta_1 = 0$ to $\Delta_0$ we obtain $-N(0) \Delta^2_0/4$. If we now substitute Eq. (11) into Eq. (12), integrate from $\Delta_1 = 2|\alpha|/g$ to $\Delta_0$ and compare the obtained value with the previous one, we can conclude that the solution with the constant gap $\Delta = \Delta_0$ always gives the lower grand potential.

In Fig. 1 we plot the gap $\Delta$ as a function of the mass asymmetry parameter $\alpha$ and the chemical potential difference $\delta \mu$ at zero temperature $T = 0$. Note that the line determined by the condition $r \equiv \alpha \mu^* + \delta \mu = 0$ goes as the middle line of the "stripe". System parameters are $\xi_c/\mu^* = 0.1, V/\mu^* = 1.5 \times 10^{-3}$.

![FIG. 1: The gap $\Delta$ as a function of the mass asymmetry parameter $\alpha$ and the chemical potential difference $\delta \mu$ at zero temperature $T = 0$. Note that the line determined by the condition $r \equiv \alpha \mu^* + \delta \mu = 0$ goes as the middle line of the "stripe". System parameters are $\xi_c/\mu^* = 0.1, V/\mu^* = 1.5 \times 10^{-3}$.](image)
FIG. 2: Critical temperature as a function of the mass asymmetry parameter \(\alpha\) and the chemical potential difference \(\delta \mu\). For some parameters values Eq. (13) for the critical temperature has two solutions: (a) the lower critical temperature \(T_{c1}\); (b) the upper critical temperature \(T_{c2}\). Note that the condition \(r = \alpha \mu^* + \delta \mu = 0\) corresponds to the maximum of the upper critical temperature. System parameters are \(\xi_c/\mu^* = 0.1, V/\mu^* = 1.5 \times 10^{-3}\).

of Eq. (13). The lower critical temperature \(T_{c1}\) exists for a range of the asymmetry parameters \(\alpha\) and \(\delta \mu\), represented by two narrow bands shown in Fig. 2(a), which are both parallel to the line \(r = \alpha \mu^* + \delta \mu = 0\). The higher critical temperature \(T_{c2}\) exists for the relatively broader range of the parameters, and is represented by one wide band shown in Fig. 2(b). Note that the condition \(r = 0\) corresponds to the maximum of \(T_{c2}\), when one asymmetry effectively compensates the another one. This situation is very distinct from the previous considerations of just one asymmetry parameter, where even tiny asymmetry in the system adversely affects the superfluid state. In more general case of two simultaneous asymmetries, one can still achieve superfluidity, even for very extreme values of the asymmetry parameters. The width of the “band” in the Fig. 1, where the superfluid state is possible, is controlled by the magnitude of the gap \(\Delta_0\) in the symmetric case.

In conclusion, we have studied the effect of electron-hole asymmetry on formation of the superfluid state in a system of spatially separated electrons and holes in two parallel planes. We predict that for some values of the electron-hole mass asymmetry and mismatch of the chemical potentials two critical temperatures are possible. The lower critical temperature corresponds to the superfluid transition induced by thermal fluctuations making possible the pairing between electrons and holes from distinct Fermi surfaces. We found that the mass asymmetry can effectively compensate the mismatch between the chemical potentials making the superfluid state possible in a much wider range of the system’s parameters than it was expected before. This effect can be used by experimentalists to study superfluidity in a variety of asymmetric systems, including coupled quantum quantum wells. It is found that for some asymmetry parameters the normal and paired phases can coexist even at zero temperature.

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