Non-asymptotic elastoinertial turbulence for asymptotic drag reduction

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Maximum drag reduction (MDR) by polymers is recently depicted as asymptotic convergence to a state of elastoinertial turbulence (EIT) where inertia-driven turbulence (IDT) vanishes. This theory has difficulty explaining the universality of MDR – asymptotic convergence of drag reduction (DR) with increasing elasticity. We confirm that EIT is indeed non-asymptotic in DR. Asymptotic DR is only achieved when IDT is triggered and quenched intermittently from EIT, in a manner similar to the earlier framework based on active-hibernating-bursting (AHB) cycles.

Turbulent friction drag can be substantially reduced with a small amount of polymer additives [1–3]. Intuitively, drag reduction (DR) should increase with fluid elasticity – higher polymer concentration or molecular weight. The effect is, however, bounded by an asymptotic upper limit – the maximum DR (MDR), whose mean flow measurements are insensitive to changing polymer solution properties. MDR remains the most coveted problem in this area – both its existence and universality have puzzled researchers for decades. The former indicates a new self-sustaining process (SSP) where turbulence cannot be further suppressed but maintains at a level with significantly reduced drag. The latter requires a mechanism by which the mean flow of this SSP, achieved under strong influence of polymers, does not vary with their properties. Two promising theories emerged in the past decade but both have unfilled gaps [3–7].

The first considers MDR to be dominated by a form of weak “hibernating” turbulence [4], which is inherently part of Newtonian turbulence but becomes unmasked by polymer elasticity. Hibernation is unstable and stronger active turbulence would always grow [4, 8–11], typically following a bursting event [11–14]. Long-lasting active periods dominate Newtonian turbulence. At MDR, it is postulated that strong elasticity would quickly quench active turbulence upon emergence, keeping the flow statistically close to hibernation. Intermittent active-hibernating-bursting (AHB) cycles reflect state-space transitions between a “kernel” of strong turbulence and the laminar-turbulent boundary region [5, 11, 13, 15, 16]. The latter is barely affected by polymers, which would block laminarization attempts and keep turbulence sustained. Convergence to this invariant boundary would perfectly explain the universality of MDR. However, the conjectured SSP at MDR is not captured in direct numerical simulation (DNS) where the flow laminarizes as elasticity increases [3].

The second considers MDR to be “elastoinertial” turbulence (EIT) where elasticity drives, rather than suppresses, flow instabilities [6, 7, 17]. EIT is a different type of turbulence with distinct spanwise vortex rolls and tilted polymer sheet structures [6, 18]. It can be sustained without spanwise variation [19] and is likely underlay by 2D instabilities [20–22]. Recent experiments showed that at lower Re, increasing polymer concentration first causes laminarization, i.e., complete suppression of classical turbulence, before turbulence resurges in another form – presumably EIT [7, 23]. It is broadly believed that although active turbulence occurs intermittently before MDR, it gradually vanishes as the flow converges to pure EIT [7, 19, 24]. Since EIT is self-sustaining, the existence of MDR is easily addressed. However, its universality is a lingering question: an instability driven partially by elasticity is expected to vary with polymer properties. Meanwhile, asymptotic DR was indeed experimentally observed in the resurgent turbulence regime [7]. This apparent conflict between theoretical expectation and experimental observation motivates the current study.

We perform DNS for channel flow under fixed pressure drop. The streamwise ($x$-) and spanwise ($z$-) directions are periodic and $y$-direction is bounded by no-slip walls. The Navier-Stokes equation is coupled with the FENE-P constitutive equation [25]

$$\frac{Dv}{Dt} = -\nabla p + \frac{\beta}{Re} \nabla^2 v + \frac{2(1-\beta)}{ReWi} (\nabla \cdot \tau_p),$$

$$\nabla \cdot v = 0,$$

$$\frac{Wi}{2} \left( \frac{D\alpha}{Dt} - \alpha \nabla v - (\alpha \cdot \nabla v)^T \right) = -\frac{b}{b+5} \tau_p,$$

$$\tau_p = \frac{b+5}{b} \left( \frac{\alpha}{1 - \text{tr}(\alpha)} \left( \frac{b}{b+2} \right) \delta \right),$$

where $\tau_p$ and $\alpha$ are polymer stress and conformation tensors: polymer extension $\propto \sqrt{\text{tr}(\alpha)}$. Velocity, length, and time are nondimensionalized by the Newtonian laminar centerline velocity $U_{CL}$, half channel height $l$, and $l/U_{CL}$, respectively. Dimensionless parameters include: Reynolds number $\text{Re} \equiv \rho U_{CL} l / \eta$ ($\rho$ and $\eta$ are fluid density and viscosity), Weissenberg number $\text{Wi} \equiv 2\lambda U_{CL} / \lambda$ ($\lambda$ is polymer relaxation time), viscosity ratio $\beta \equiv \eta_p / \eta_s$ ($\eta_s$ is solvent viscosity), and finite extensibility parameter $b \equiv \max(\text{tr}(\alpha))$. Results are also reported in turbulent inner units [26], denoted by “$*$”, where the friction velocity $u_\tau \equiv \sqrt{\tau_w / \rho}$ ($\tau_w$ is wall shear stress) and viscous length scale $l_v \equiv \eta / (\rho u_\tau)$ are used. For instantaneous quantities denoted by “$*$”, instantaneous $\tau_w$ (without time average) is used [4]. The domain size is $L_x^* \times L_y^* \times L_z^* = 720 \times 2Re_\tau \times 230$ for 3D and
From the balance of average turbulent kinetic energy, which is unequivocally determined by a negative $\langle \epsilon_p \rangle_Y$ and $\langle \epsilon_k \rangle_Y$ are negative [11, 30, 31]. Low $\langle \mathcal{P}^k \rangle_Y$ reflects a diminishing role of inertia in the SSP. We will, however, resist the urge to label EIT as a purely elastic instability because (1) instantaneous magnitudes of $\langle \mathcal{P}^k \rangle_Y$ are still substantial and (2) there is no evidence for its existence at the vanishing Re limit. Flow structures in fig. 1(b) agree well with previous studies [19, 22]. Distinct thin bands of highly stretched polymers (high tr$(\alpha)$) tilt in an acute angle to the flow direction. Vortices are identified by positive $Q$ values based on the $Q$-criterion [32, 33]. Spanwise rolls line up near both walls with thinner threads in between. By contrast, in IDT near-wall vortices align largely with the flow and high tr$(\alpha)$ regions wrap around vortices [34–37].

At $b = 5000$, 2D EIT is found for Wi down to 40 (fig. 2(a)), where polymers are stretched, on average, to $\approx 30\%$ of full extension $(\sqrt{\langle \mathcal{P}^k \rangle_Y}/b \approx 0.3$; $\langle \gamma \rangle_Y$ denotes volume and time average) and $-\langle \epsilon_p \rangle_Y$, is just above 0. Its drag, measured by the Fanning friction factor $C_f \equiv 2\tau_w/(\rho v_\infty^2 y^+)$, is barely above the laminar level. Both polymer extension and $C_f$ increase with Wi. There is no apparent tendency for $C_f$ to converge even at the highest Wi = 800 where polymers are nearly 90% stretched. Meanwhile, $-\langle \epsilon_p \rangle_Y$ also increases as instability amplifies but seems to plateau as $(\text{tr}(\alpha))/Y \rightarrow b$. This is rationalized considering that when polymer chains are nearly fully stretched everywhere and relaxation is slow: $\tau'_p$, on which $\epsilon_k$ depends, may decrease. At $b = 2000$, the whole $C_f$ curve is higher, which again rises monotonically with Wi without convergence. For mean velocity profiles (fig. 2(b)), although certain cases can be close to the empirical Virk MDR profile [1], $U_{\infty}^2(y^+)$ keeps declining with both Wi and b to much lower levels.

Ever growing drag enhancement (DE) with Wi and b in elastic or elastoinertial instabilities should come as no surprise, which is common in other flow types [40]. It however reveals that, contrary to common belief, MDR cannot be this single EIT state as its $C_f$ should not vary with polymer parameters. We have indeed found asymptotic DR but only in 3D flow (fig. 2(a)). Contrary to DE in 2D EIT, 3D DNS shows DR whose $C_f$ stays nearly constant for a wide Wi range of 64 to 400. $U_{\infty}^2(y^+)$ profiles of Wi = 64 and 400 are also inseparable (fig. 2(c)). The converged DR level slightly exceeds Virk MDR, which may be attributed to the small simulation domain used. Indeed, mean velocity exceeding Virk MDR is commonly seen in recent DNS of comparable regimes [18, 24].

Dynamics in 3D flow must be more complex than just EIT. In fig. 3, a 3D DNS trajectory $(Wi = 40, b = 5000)$ from a perturbed laminar state first goes around the 2D EIT neighborhood and turns towards lower $-\langle \epsilon_p \rangle_Y$. Growth of IDT is mediated by the edge state (ES), a solution on the laminar-IDT (lamIDT) boundary that pivots the dynamics towards the IDT kernel (active tur-
Fig. 2: (a) (left) friction factor (arrows mark Newtonian turbulence [26], MDR [1], and laminar values) and (right) elastic conversion of TKE for 2D EIT at \( b = 5000, 20000 \) (\( Wi = 40, 64, 100, 200, 400, 800 \)) and for 3D DNS at \( b = 5000, 20000 \) (\( Wi = 30, 40, 64, 100, 400, 800 \)); (b) and (c) show mean velocity profiles of 2D EIT and 3D DNS, respectively (instants I and II are marked in fig. 3 and fig. 5(a); reference lines are: (dot-dashed) viscous sublayer \( U_m^+ = y^+ \); (solid) Newtonian log law \( U_m^+ = 2.5 \log y^+ + 5.5 \) [38]; (dashed) MDR \( U_m^+ = 11.7 \log y^+ - 17.0 \) [1].) Error bars use block average [39].

Fig. 3: Projections of (solid) a 3D DNS trajectory initiated from perturbed laminar flow (color maps to time), (dashed) 2D EIT, and (dotted/white) edge state [15] (all at \( Wi = 40 \)); (dash-dotted) IDT kernel sampled by 3D DNS at \( Wi = 30 \). \( A^* \equiv y^*(\partial U_m^*/\partial y^*) \) (i.e., instantaneous mean velocity \( U_m^*(y^*) = A^* \log y^+ + \text{const.}; \) at MDR, \( A^* = 11.7 \) [1]); \( \langle \epsilon_k^x \epsilon_k^y \rangle_{\text{max}} \) is the instantaneous Reynolds shear stress profile peak [11, 15]. The \( -\langle \epsilon_k^y \rangle_{\text{max}} \) axis uses a transformed log scale \( L(x) = \log(1 + 10^7|x|) \) [41, 42].

Fig. 4: Flow structures of 3D DNS at instants I and II marked in fig. 3 and fig. 5(a) and III and IV marked in fig. 5(c) (colors: \( \alpha/b \); isosurfaces: \( Q = 0.004 – \) bottom half only).

The latter is approximately represented by a 3D solution at \( Wi = 30 \), where IDT still dominates. The \( Wi = 40 \) trajectory frequently embarks on two types of cycles. Type A is the aforementioned AHB cycle which runs directly through the ES region (hibernation) [3, 4, 8, 43]. Type B approaches the tip of 2D EIT where it pivots and returns via the ES.

Flow structures are distinctly different between dynamical phases (fig. 4). The IDT kernel (instant I) shows classical streamwise vortices, although residual EIT structures appear near the wall. Instant II features trains of spanwise rolls spaced by threads, which clearly resembles fig. 1(b). The structure is not fully 2D and in fig. 3 the 3D solution also never fully reaches 2D EIT, likely because of the intermittent recurrence of IDT. The instantaneous \( U_m^*(y^*) \) profile of I (fig. 2(c)) is similar to that of drag-reduced IDT [44] (also compare with the \( Wi = 30 \) profile in fig. 2(c)), while that of II approaches 2D EIT (fig. 2(b)).

Instantaneous friction factor \( C_{ins}^f \equiv 2\tau_w(t)/(\rho \langle u_x^2 \rangle_{\text{ref}}) \) (fig. 5(a)) shows intermittent dives separated by spikes.
FIG. 5: Time series of instantaneous friction factor $C_{f}^{\text{ins}}$ and TKE change rates by inertial production $\langle P^k \rangle_V$ and elastic conversion $-\langle \epsilon^k_p \rangle_V$ (scaled by $s$): (a) $Wi = 40$ ($s = 20$), (b) $Wi = 64$ ($s = 12$), (c) $Wi = 400$ ($s = 4$), and (d) enlargement of (c). Arrows mark $C_t$ of Newtonian turbulence and MDR.

The latter reflect returns to the IDT kernel (active turbulence) with strong $\langle P^k \rangle_V$ and negative $-\langle \epsilon^k_p \rangle_V$. Type A cycles display shallow dives (e.g., near $t = 4500$) lasting for $O(100)$ time units (TUs). They still belong to IDT with negative $-\langle \epsilon^k_p \rangle_V$ but their $C_{f}^{\text{ins}}$ can approach the MDR level. They are the same as hibernating turbulence known in [4, 8]. Type B cycles give deeper dives lasting for $O(1000)$ TUs with lower $C_{f}^{\text{ins}}$. They spend most time near EIT where $-\langle \epsilon^k_p \rangle_V$ turns positive: note that instant II is almost 2000 TUs after the dive starts.

Findings here show that the ES and lamIDT boundary do not block all escapes across the IDT kernel as the previous AHB theory presumed [3–5]. At high Wi, they may be bypassed. EIT must step up as a second line of defense to keep turbulence sustained. If the lamIDT boundary is compromised before EIT emerges, a laminarization window would appear, as observed in recent experiments [7, 23]. At Wi = 64 (fig. 5(b)), type A cycles disappear: EIT becomes solely responsible for preventing laminarization. The dives are also significantly prolonged.

Strikingly, at Wi = 400, whose $U_m^+$ and average $C_t$ are indistinguishable from the Wi = 64 case (fig. 2), the dynamics is completely different (fig. 5(c)). Intermittent $C_{f}^{\text{ins}}$ spikes are replaced by smaller but more regular wiggles. Its cycles (type C) differ from both previous types with much higher frequency. Each cycle (fig. 5(d)) starts with the rise of $-\langle \epsilon^k_p \rangle_V$, which quickly sparks a stronger inertial instability, marked by a much higher peak in $\langle P^k \rangle_V$ (in fig. 5(c) & (d), $-\langle \epsilon^k_p \rangle_V$ is scaled by $s = 4$). If we turn off polymer stress at instant III, full IDT would develop, which proves that elasticity suppresses the inertial phase of the cycle. After the suppression, the flow enters a quiescent phase (instant IV), during which removing polymer stress would cause laminarization. EIT is thus required to trigger the next cycle and sustain turbulence. Such cycles are reminiscent of what Xi and Graham [4] initially conjectured for MDR, that the dynamics is statistically confined in hibernation with intermittent bursts of stronger turbulence quickly quenched by polymer stress. What we know now is that at high Wi, it is EIT, not the lamIDT boundary, that prevents laminarization. Dynamics at $Wi = 100$ (not shown) is more chaotic and oscillates at lower frequency – all three cases in fig. 2(a) showing the same $C_t$ have different dynamics.

Distinct hairpin-like structures are observed at instant III (fig. 4). They are likely unrelated with hairpin vortices in Newtonian turbulence at higher Re [33, 45, 46] and appear to stem from the coalescence between streamwise vortices of IDT and spanwise rolls of EIT. The latter initially appear near the wall but expand with Wi, allowing them to interact with IDT structures. Whether such interactions are related with the rise of type C cycles is for future research. Coexistence of EIT structures with IDT at this instant explains its non-negative net $-\langle \epsilon^k_p \rangle_V$. The flow field of instant IV is more similar to typical EIT.

In summary, we conclude that MDR cannot be, as commonly believed, a simple converged EIT state. Rather, EIT is non-asymptotic with increasing DE as elasticity increases. Asymptotic DR is observed only in 3D flow where $C_t$ is invariant over a wide Wi range. Its underlying dynamics is, however, intrinsically intermittent (i.e., not confined in any single state) and non-asymptotic (i.e., evolving with Wi). Such dynamics echoes the earlier AHB framework except that at high Wi, EIT gradually replaces the lamIDT boundary as the main barrier shielding the flow from laminarization. Recurrent bursts of IDT are perpetual and also indispensable, which prevents the flow from fully converging to the non-asymptotic EIT. At our highest Wi, such bursts are quickly suppressed by polymer stress each time they occur, leading to a dynamical balance between EIT and IDT. Finally, the proposed scenario does not contradict the experiments by Choueiri et al. [7]: however, their resurgent turbulence stage, which was believed to be EIT, likely contains recurrent IDT. Indeed, growing intermittency with polymer concentration is noticeable in experimental flow patterns despite the converging $C_t$ [24].

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