Generalization of data on heat exchange within a gas mixture with a small value of Prandtl numbers in channels of different forms

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Abstract. The article presents an analysis of data on heat transfer in the channels of a round and triangular shape with a flow of a gas mixture with a small Prandtl number. It is shown that for the Reynolds number in the interval from 9500 to 49000 the heat-transfer coefficient depends from the length a heat exchange section in tube. Using the dependence of the Nusselt number Nu(Ψ) from a dimensionless parameter Ψ=1/Pe×x/Dh, we can describe the experimental data for the input region of the round and triangular channels, taking into account the channel length and the distances from the input. The adequacy of the model was checked by comparing the proposed dependence with the experimental data of various authors.

1. Introduction

To improve the recuperative heat exchange equipment in various applications, the development of compact heat exchanges with a complex spatial structure of the heat exchange surface is urgent. In such channels, the heat exchange capacity is much higher, due to the fact that the flow of coolant inside the mini-channels is organized in such a way that heat exchange takes place on the hydrodynamic and thermal entrance length where the heat exchange capacity is much higher than in the fully developed laminar flow. The prismatic shape of the channels, typical of most core fuel assemblies in the nuclear power industry, gas reactors in the chemical industry and other similar devices, does not allow the use of well-known theoretical and experimental dependences obtained for circular channels for evaluation. Numerical studies of such flows are not always adequate, since the number of reliable input parameters is not enough.

Use of new a heat conductor with regulable properties allows improving efficiency of the working behaviours of the heat exchangers. Such heat conductor can be create in form two-component a mixtures of gases or fluids. The special attention has of the mixtures of gases with essentially various molecular weights, in particular such as a mixture of hydrogen or helium with heavy gases. For some concentrations of the heavy gas the Prandtl number Pr for a mixture of the hydrogen or the helium can have small value between 0.1 and 0.3. At such Prandtl numbers, thermal boundary layers are thicker than the velocity boundary layers, and consequently Thermal boundary layers merge earlier than hydrodynamic. Therefore the thermal stabilization occurs on a shorter channel section than the of flow stabilization length. However, for fluids with small value of the Prandtl number, at calculation of the heat-transfer coefficient there is problem in precision and validity of the computational result [1, 2]. In particular, identification of the influence degree of the duct entrance region for a turbulent flow in the channels of the intricate cross-section form is required [1, 3].
The Nusselt number \( \text{Nu} \) for a gas laminar flow in the short-length tubes depend on the dimensionless parameter \( \Psi = 1/\text{Pe} \times x/D_h \), where \( \text{Pe} \) – Peclet number, \( x \) – length of the duct, \( D_h \) – hydraulic diameter of the duct [4]. For \( \Psi < 0.01 \) dependence \( \text{Nu} (\Psi) \) asymptotically approaches the solution \( \text{Nu} \sim \text{const} \times \Psi^{1/3} \). For a laminar fluid flow in circular and triangular ducts there are exact solutions for dependence of the Nusselt number \( \text{Nu} (\Psi) \) from parameter \( \Psi \) [4]. In Paper [5] presents the general equations for heat transfer calculations for constant wall temperature in laminar developed flow in ducts of arbitrary cross sections. The results obtained from these equations compare well with the theoretically-calculated values available in the literature for circular, rectangular, triangular, elliptical and parallel plate ducts. But the such dependence \( \text{Nu} (\Psi) \) for a turbulent fluid flow usually is not used. It is associated with a fact, that as a rule for a turbulent flow a ordinary gases, \( \text{Pr} > 0.6 \), length the hydrodynamic and thermal entrance regions of the developing flow in pipes \( x \) does not exceed \( 10 \div 20 \) hydraulic diameters \( D_h \), and for developed flow state the heat-transfer coefficient \( \alpha \) and Nusselt number depend from a Reynolds number \( \text{Re} \) and not depend from a \( x/D_h \).

In the [6] shows that the experimental heat transfer data for helium–xenon mixture flow in triangular cross-section channel can be described by empirical correlation [7] for the turbulent flow regime:

\[
\text{Nu}_{D_h} = 0.022 \cdot \text{Re}^{0.19} \cdot \text{Pr}^\kappa, \quad \kappa = 0.595 \cdot \text{Pr}^{-0.126}
\]

(1)

where \( \text{Re}_{D_h} \) – Reynolds number. The formula of (1) generalizes adequately the obtained experimental data despite the fact that the recommended range of Prandtl numbers for this formula \( 0.3 \div 1 \), therefore, the range applicability of this formula can be extended to Prandtl numbers equal to 0.2. In this paper, we presented the results of the analysis of experimental heat transfer data for gas mixture with a small Prandtl number at flow in the short channels with circular and triangular form.

2. Data analysis and results discussion

For the analysis heat transfer at gas flow in a short channel, experimental data from the works [6, 8–11] were used. Experiments were carried out with a gas mixture \( \text{Pr} \sim 0.2 \) and air \( \text{Pr} \sim 0.7 \) in direct a tubes circular and triangular shape. The cylindrical pipe made from stainless-steel A2 with a wall thickness of 1.3 mm, the length 0.875 m, internal diameter \( D = 4.6 \) mm and \( D = 5.5 \) mm. The plain triangle channel was made from a stainless-steel thin-wall plain pipe with the outer diameter 5 mm and the wall thickness 0.15 mm. The working length of the triangle heated channel was 670 mm. The hydraulic diameter of the equilateral triangle channel with the side equal to 4.8 mm was \( D_h = 2.814 \) mm. The pipe was heated by passing through it an alternating electric current.

The channel inlet temperature was 300 K. The inlet pressure ranged from 392 to 433 kPa, and at the outlet from 382 to 413 kPa. The inlet Reynolds numbers varied over the range from 9500 to 49000, and the \( x/D_h \) from 10 to 120. Reynolds number \( \text{Re}_{D_h} \) and Nusselt number \( \text{Nu}_{D_h} \) for distance \( x \) from the entrance was determined by the hydraulic diameter \( D_h \):

\[
\text{Re}_{D_h} = \frac{U_m \cdot D_h}{\nu}
\]

\[
\text{Nu}_{D_h}(x) = \frac{1}{x} \int_0^x \alpha(\ell) \cdot D_h \cdot \ell \cdot d\ell
\]

where \( S \) – cross section area, \( \Pi \) – length wetted perimeter channel, \( U_m \) – average rate flow velocity at the entrance to the channel, \( \alpha \) – local heat transfer coefficient. The local circumferential-average heat transfer coefficient for the heated walls is defined here as

\[
\alpha = \frac{(Q/\Pi) / (T_h - T_w)}
\]

where \( Q \) is the rate of convective heat transfer per unit axial length from the heated walls to the fluid. The temperatures \( T_h \) and \( T_w \) respectively represent the values for the heated wall and the bulk. The values viscosity \( \nu \), gas density \( \rho \), thermal conductivity \( \lambda \) of the gas were calculated by data from the works [1, 12, 13].

On Figure 1 Nusselt number \( \text{Nu}_{D_h} \) varying with dimensionless distance from the entrance \( x/D_h \), the Reynolds number \( \text{Re}_{D_h} \) and Prandtl number \( \text{Pr} \) for the gases mix flow in circular tube and triangular
duct for various flow regime ($9000 < \text{Re}_{Dh} < 50000$) are shown. As can be seen from Figure 1 the relationship between the value $\text{Nu}_{Dh}$ and the parameters $\text{Re}_{Dh}$, Pr and $x/D_h$ has an uncertain correlation. The two vertical lines on Figure 1 are show the range values $\text{Nu}$ calculated by formula Tetelbaum (1) [7] for Pr = 0.23, Pr = 0.7 and Reynolds numbers between 9500 and 49000. It is possible see on a graph that the range of a calculated values $\text{Nu}$ does not comport with the experimental data.

![Figure 1](image-url)

**Figure 1.** Nusselt number $\text{Nu}_{Dh}$ varying with dimensionless distance from the entrance $x/D_h$ for various flow regime (Reynolds number $\text{Re}_{Dh}$) at gasmix flow with different Prandtl number Pr in circular tube [8, 9] and triangular duct [6, 9, 10, 11].

An analysis experimental data showed that dependences splitting for Nusselt number Nu (Re) relative to Prandtl number Pr disappears if use the dependence Nu (Pe) on the Peclet number $\text{Pe} = \text{Re} \times \text{Pr}$. Graph in Figure 2 illustrates examples of the comparisons between a dependence of the Nu from the Re (Figure 2a) and dependence of the Nu from the Pe (Figure 2b). Examples for the experimental data in a circular tubes for a different distances from an entry $x/D$ are shown. Can see that for every one of the arguments $x/D_h$ there exist the unique dependences of a Nu from a Pe for all a Pr.
Figure 2. Dependence of the Nusselt number $\text{Nu}_{Dh}$ on Reynolds number $\text{Re}_{Dh}$ (a) and from Peclet number $\text{Pe}$ (b) at gasmix flow with various Prandtl number $\text{Pr}$ in circular tube [8, 9] on different dimensionless distances from an entry $x/D_h$.

Using the hydraulic diameter $D_h$ the effect of channel shape on heat transfer usually can be taken into account [1, 4, 14–16]. Figure 3 shows the data for Nusselt number $\text{Nu}_{Dh}$ at flow of the gas with $\text{Pr} = 0.23$ in circular tube and triangular duct, which have equal wetted perimeter values $\Pi$ of cross-section. In Figure 3a, data for $\text{Nu}_{Dh}$ depending on $\text{Re}_{Dh}$, and in Figure 3b depending on dimensionless parameter $\Theta = \text{Re}_{Dh} \times \pi \times D_h/\Pi$ are presented. It can are seen that for circular tube and triangular duct there is a general dependence for the $\text{Nu}_{Dh}$ from the parameter $\Theta$.

Thus, it is quite possible that dependence of the $\text{Nu}$ from the dimensionless parameter $\Psi = 1/\text{Pr} \times x/D_h$ as well as for the laminar flow of ordinary gases in the short-length tubes, it is valid, and for a turbulent flow of the gas mixes with small a Prandtl number. In [14], an interpolation equation was proposed for heat exchange in a circular pipe with laminar flow and constant temperature of the channel wall

$$\text{Nu} = 3.66 + \frac{0.0668/\Psi}{1 + 0.04 \cdot \Psi^{2/3}}$$  \hspace{1cm} (2)

By analogy with equation (2), the following equation was used for the generalization dependence:

$$\text{Nu} \sim \text{Nu}_0 + \frac{1}{(a \cdot \Psi + b)^v}$$  \hspace{1cm} (3)
The values of the Nusselt number \( \text{Nu} \) as a function of the dimensionless parameter \( \Theta \) for gas mixture flow with the value of the Prandtl number \( \text{Pr} = 0.23 \) in circular tube [9] and triangular duct [10].

The main difference of equation (3) from equation (2) is that for \( \Psi = 0 \) \( \text{Nu} \neq \infty \). To this end, the term \( b \) has been added to equation (3). The value of \( a \) and the exponent \( n \) must satisfy the condition \( \text{Nu} \geq \text{const} \times \Psi^{1/3} \) for \( \Psi < 0.01 \). The value of \( \text{Nu}_0 \) are chosen such that \( \Psi \gg 1 \) equation (3) would correspond to exact solutions for a laminar flow in circular tube and triangular duct [4]. The coefficients \( a \) and \( b \) were found by reducing the dependence to a linear form, according to the well-known standard mathematical rules. At uniform wall heat flux for the circular tube \( \text{Nu}_{\text{lam}} = 4.364 \) and for triangular duct \( \text{Nu}_{\text{lam}} = 2.970 \) [1, 4].

Thus, data on heat transfer in a turbulent flow of gas mixtures with a low Prandtl number are described by the dependence (3). In Figure 4 the dependence of the data for Nusselt number on dimensionless parameter \( \Psi \) from works [6, 8–11] are shown.

**Figure 3.** The values of the Nusselt number \( \text{Nu}_Dh \) as a function of the dimensionless parameter \( \Theta \) for gas mixture flow with the value of the Prandtl number \( \text{Pr} = 0.23 \) in circular tube [9] and triangular duct [10].
Figure 4. Dependence of the Nusselt number $\text{Nu}_{Dh}$ on dimensionless parameter $\Psi$ at gas mixture flow in circular tube [8, 9] and triangular duct [6, 9–11].

The graph also shows the curves of $\text{Nu}(\Psi)$ for the developing laminar regime flow [14, 15]. Can see that all experimental points is grouping along the common curve. On a graph two fit curves for this data are shown. The resulting expression for the approximation function is given by equation for triangular duct:

$$\text{Nu} = 2.970 + \frac{1}{(20 \cdot \Psi + 0.005)^{1.125}}$$

(4),

and for the circular tube:

$$\text{Nu} = 4.364 + \frac{1}{(20 \cdot \Psi + 0.005)^{1.125}}$$

(5).

The dependence $\text{Nu}(\Psi)$ (4), (5) differ from the exact analytical solution $\text{Nu}_{\text{lam}}(\Psi)$ [1, 4, 14, 15] for a laminar fluid flow in smooth tube.

To determine the application range equation (3) in Figure 5 presents the experimental data of different authors [16–19] for laminar flow in ducts of arbitrary cross-sections.
Figure 5. The dependence of the Nusselt number \( \text{Nu}_{Dh} \) [16–19] on dimensionless parameter \( \Psi \) for gas and liquid flow in ducts of arbitrary cross-sections.

Geometric and operational parameters for which experimental data on heat transfer were obtained, used in Figure 5 are presented in Table 1.

Table 1. Geometrical and operational parameters at which the obtained experimental data on heat transfer used in Figure 5.

| Points | Channel cross-sections | Pr  | \( D_h \) (mm) | \( \text{Re}_{Dh} \) | Ref. |
|--------|------------------------|-----|----------------|-----------------|-----|
| 1      | circular               | 0.7 | 37.24          | \( 9.2 \times 10^3 \div 1.6 \times 10^5 \) | [16] |
| 2      | parallel plates        | 0.7 | 63.75          | \( 1.2 \times 10^4 \div 6.1 \times 10^4 \) | [16] |
| 3      | triangular             | 0.7 | 22.9           | \( 9.7 \times 10^3 \div 5.9 \times 10^4 \) | [17] |
| 4      | circular               | 0.7 | 37.24          | \( 9.2 \times 10^3 \div 1.6 \times 10^5 \) | [16] |
| 5      | triangular             | 0.7 | 22.9           | \( 4.1 \times 10^3 \div 5.9 \times 10^4 \) | [17] |
| 6      | rectangular 0.39×1     | 7.1 | 7.97           | \( 1.9 \times 10^3 \div 2.9 \times 10^3 \) | [18] |
| 7      | rectangular 0.39×1     | 2.4 | 2.8            | \( 2.0 \times 10^2 \div 2.8 \times 10^3 \) | [18] |
| 8      | parallel plates        | 0.7 | 63.75          | \( 1.2 \times 10^4 \div 6.1 \times 10^4 \) | [16] |
| 9      | quasi-triangular       | 0.7 | 1.31           | \( 7.2 \times 10^2 \div 5.2 \times 10^3 \) | [19] |

Experimental data 19 were obtained in the study heat transfer for air flow in a quasi-triangular channel at a constant out wall temperature of 273 K. The walls of the quasi-triangular channel are made from 50-micron copper foil. The channel shape was set so that the inner curvature of the channel walls was 6.4 mm. During the analysis, all data were divided into two groups according to the dimensionless length of zone heat exchange. First group 1 – 3 for \( x/D_h \) < 5, and second group 4–9 for \( x/D_h > 15 \). Also, the graph shows curves \( \text{Nu}(\Psi) \) for the developing laminar flow [1, 4, 14, 15] of channels of round and triangular form, and flat ducts. On the graph you can see that the data for group \( x/D_h > 15 \) located near the curve (3). The data for group \( x/D_h < 5 \) is located below curve (3) near the curves for the developing laminar flow. Its can be explained by the fact that for small distances from
the entrance transverse and longitudinal pulsations gas (liquid) velocity, typical for developed turbulent flow, were insignificant and did not influence on process heat transfer.

3. Conclusions
Analysing the experimental data on heat transfer for turbulent flow of a gas mixture with small Prandtl numbers may to the conclusion that for the Reynolds number in the interval from 9500 to 49000 the heat-transfer coefficient depends from the length a heating area in tube. For an entrance region of a triangular and circular duct a data on heat transfer may define by the dependence \( \text{Nu}(\Psi) \) from a dimensionless parameter \( \Psi = 1/\text{Pe} \times x/D_h \). The this dependence \( \text{Nu}(\Psi) \) for a turbulent gas flow differ from the exact analytical solution \( \text{Nu}_{\text{lam}}(\Psi) \) for a laminar fluid flow in smooth tube.

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