Local Model of Entangled Photon Experiments Compatible with Quantum Predictions Based on the Reality of the Vacuum Fields

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Abstract

Arguments are provided for the reality of the quantum vacuum fields. A polarization correlation experiment with two maximally entangled photons created by spontaneous parametric down-conversion is studied in the Weyl–Wigner formalism, that reproduces the quantum predictions. An interpretation is proposed in terms of stochastic processes assuming that the quantum vacuum fields are real. This proves that local realism is compatible with a violation of Bell inequalities, thus rebutting the claim that it has been refuted by experiments. Entanglement appears as a correlation between fluctuations of a signal field and vacuum fields.

Keywords Local realism · Bell inequalities · Entangled photons · Weyl–Wigner · Loopholes · Vacuum fields

1 Introduction

The purpose of this article is to rebut the standard wisdom that “local realism” has been refuted by experiments [1,2] measuring polarization correlation of entangled photon pairs produced via spontaneous parametric down-conversion in non-linear crystals. For the proof I will present a local realistic model of the experiments resting on the assumption that the quantum vacuum fields are real stochastic fields. In Sect. 2 I will provide arguments for the reality of the vacuum fields. After that I will briefly recall the quantum interpretation of the experiments, firstly using the common Hilbert-space formalism of quantum theory applied to quantum optics in Sect. 3. Then using the Weyl–Wigner formalism, which is physically equivalent to the former, in Sect. 4. Finally in Sect. 5 I will show that the latter formalism may be interpreted in terms of random variables and stochastic processes thus providing a local realistic model.
of the experiments. In this introductory section I include a short discussion about the relevance of local realism and the Bell inequalities.

1.1 The Conflict Between Quantum Mechanics and Local Realism

The interpretation of quantum mechanics has been the subject of continuous debate from the very beginning of the theory. Initially the question was whether quantum mechanics is complete or not, but later the celebrated EPR article [3] showed an incompatibility between completeness and locality (in the sense of relativistic causality). The authors supported locality, something that Einstein strongly backed until his death [4]. In 1964 Bell showed that locality may be empirically tested via his celebrated inequalities [5,6]. Indeed the standard wisdom is that any violation of a Bell inequality refutes locality, meaning that not all natural phenomena are compatible with relativistic causality. Thus the results of the alleged loophole-free empirical tests [1,2] has been interpreted as the “death by experiment for local realism”, this being the hypothesis that “the world is made up of real stuff, existing in space and changing only through local interactions ... about the most intuitive scientific postulate imaginable” [7]. In the present article I prove that local realism has not been refuted if we accept that the quantum vacuum fields are real.

1.2 The Bell Inequalities

Bell defined as “local hidden variables” model, later named “local realistic”, any model of an experiment where the results of all correlation measurements may be interpreted according to the formulas

$$\langle A \rangle = \int \rho (\lambda) d\lambda M_A (\lambda, A), \quad \langle B \rangle = \int \rho (\lambda) d\lambda M_B (\lambda, B),$$

$$\langle AB \rangle = \int \rho (\lambda) d\lambda M_A (\lambda, A) M_B (\lambda, B),$$

(1)

where $\lambda \in \Lambda$ is one or several random (“hidden”) variables, $\langle A \rangle, \langle B \rangle$ and $\langle AB \rangle$ being the expectation values of the results of measuring the observables $A, B$ and their product $AB$, respectively. Here we will consider that the observables correspond to detection, or not, of some signals (e.g. photons) by two parties, say Alice and Bob, attaching the values 1 or 0 to these two possibilities. In this case $\langle A \rangle, \langle B \rangle$ agree with the single and $\langle AB \rangle$ with the coincidence detection rates respectively in typical experiments consisting in repeated trials. The following mathematical conditions are assumed

$$\rho (\lambda) \geq 0, \int \rho (\lambda) d\lambda = 1, \ M_A (\lambda, A) \in \{0, 1\}, \ M_B (\lambda, B) \in \{0, 1\}.$$ 

(2)

A constraint of locality is included, namely $M_A (\lambda, A)$ should be independent of the choice of the observable $B, M_B (\lambda, B)$ independent of $A$, and $\rho (\lambda)$ independent of both $A$ and $B$. From these conditions it is possible to derive empirically testable (Bell)
inequalities [8,9]. The tests are most relevant if the measurements performed by Alice and Bob are spacially separated in the sense of relativity theory.

For experiments measuring polarization correlation of photon pairs the Clauser-Horne inequality [8] may be written

$$\langle \theta_1 \rangle + \langle \phi_1 \rangle \geq \langle \theta_1 \phi_1 \rangle + \langle \theta_1 \phi_2 \rangle + \langle \theta_2 \phi_1 \rangle - \langle \theta_2 \phi_2 \rangle,$$

(3)

where $\theta_j$ stands for the observable “detection of a photon with the Alice detector in front of a polarizer at angle $\theta_j$”. Similarly $\phi_k$ for Bob detector. For simplicity in this paper I will study the case of maximally entangled photons, although in the mentioned experiments [1,2] the photon pairs had partial entanglement that made the experiments easier. I will present a local model that predicts the following single and coincidence rates by Alice and Bob

$$\langle \theta_j \rangle = \langle \phi_k \rangle = K,$$

$$\langle \theta_j \phi_k \rangle = K \cos^2(\theta_j - \phi_k) = \frac{1}{2} K \left[ 1 + \cos(2\theta_j - 2\phi_k) \right],$$

(4)

where $K$ is a constant that depends on the particular experimental setup. It is easy to check that the prediction Eq. (4) violates the inequality Eq. (3) for some choices of angles. For instance the choice

$$\theta_1 = \frac{\pi}{4}, \phi_1 = \frac{\pi}{8}, \theta_2 = 0, \phi_2 = \frac{3\pi}{8},$$

violates the inequality Eq. (3) leading to

$$K \not\geq \frac{1}{2} \left( 1 + \sqrt{2} \right) K \simeq 1.207 K.$$

So far we have assumed ideal detectors, for real detectors the above predicted single rates $\langle \theta_j \rangle$ and $\langle \phi_k \rangle$ should be multiplied times the detection efficiencies $\eta_A$ and $\eta_B$, respectively, and the coincidence rate $\langle \theta_j \phi_k \rangle$ times $\eta_A \eta_B$, whence the empirical violation of the inequality Eq. (3) would require high detection efficiencies, that is

$$\eta_A + \eta_B < (1 + \sqrt{2}) \eta_A \eta_B \Rightarrow \eta > 0.828 \text{ if } \eta_A = \eta_B = \eta.$$

Experiments with some non-maximal entanglement need only $\eta > 2/3$ [9], that was the reason for using such entanglement in the actual experiments [1,2].

2 The Assumption That the Vacuum Fields are Real

2.1 The Quantum Vacuum in Field Theory

The existence of some equilibrium radiation energy in space, even at zero Kelvin, appears for the first time in Planck’s second radiation theory of 1912. This zeropoint
energy of the electromagnetic field (ZPF) was rejected because it is divergent, although
the consequences of its possible reality were soon explored by several authors [10].
The ZPF reappeared in 1927 when Dirac quantized the electromagnetic field via an
expansion in normal modes, that is plane waves in the case of free space. In fact the
Hamiltonian of the field may be written
\[ H = \frac{1}{2}h \sum_j \omega_j \left( a_j + a_j^\dagger \right)^2 = \frac{1}{2}h \sum_j \omega_j \left( a_j^2 + a_j^{\dagger 2} + 2a_j^{\dagger}a_j + 1 \right), \tag{5} \]
where \( \omega_j \) is the frequency of a normal mode, \( a_j \) and \( a_j^{\dagger} \) being the annihilation and
creation operators of photons in that mode, and we have taken the commutation rules
into account in the later equality. The energy is given by the vacuum expectation of
the Hamiltonian, that is
\[ \langle 0 | H | 0 \rangle = \frac{1}{2}h \sum_j \omega_j \left( \langle 0 | a_j^2 + a_j^{\dagger 2} + 2a_j^{\dagger}a_j | 0 \rangle \right) + \frac{1}{2}h \sum_j \omega_j \langle 0 | 1 | 0 \rangle = \sum_j \left( \frac{1}{2}h \omega_j \right), \tag{6} \]
the former expectation being nil. The result corresponds to a mean energy \( \frac{1}{2}h \omega_j \)
per mode, whence the total energy density in space diverges when we sum over all
(infinitely many) normal modes.

The standard solution to the divergence problem is to remove the term that con-
tributes in Eq. (6), a procedure which is known as “normal ordering”. It consists of
writing the annihilation operators to the right, that is to assume that the correct Hamil-
tonian is not the former expression in Eq. (5), but the latter with unity removed. It may
be realized that the normal ordering is equivalent to choosing the zero of energies at the
level of the vacuum. It provides a practical procedure useful in quantum-mechanical
calculations, but it is not a good solution in the opinion of many authors. They see it
as an “ad hoc” assumption, just aimed at removing unpleasant divergences. For these
authors the ZPF is a logical consequence of quantization and the solution of the diver-
gence problem should come from a more natural mechanism. Furthermore it has been
shown that the assumption of reality of the ZPF combined with the classical laws of
electrodynamics allows explaining some phenomena usually taken as purely quantal,
an approach known as stochastic electrodynamics [11].

Around 1947 two new discoveries reinforced the hypothesis that the quantum vac-
uum fields are real, namely the Lamb shift and the Casimir effect. Willis Lamb observed
an unexpected absorption of microwave radiation by atomic hydrogen, that was soon
explained in terms of the interaction of the atom with the quantized electromagnetic
field, that involves the vacuum radiation (ZPF). Indeed Lamb claimed to be the dis-
coverer of the ZPF by experiment. Furthermore he wrote that “photons are the quanta
of the electromagnetic field, but they are not particles” [12]. Lamb discovery led in a
few years to the development of quantum electrodynamics (QED), a theory that allows
predictions in spectacular agreement with experiments, and it was the starting point

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for the whole theory of relativistic quantum fields. The success of QED rests on renormalization techniques, namely assuming that physical particles, e.g. electrons, are dressed with “virtual fields” making their physical mass and charge different from the bare quantities. In my opinion these assumptions behind renormalization are actually a reinforcement of the reality of the quantum vacuum fields, although people avoid commitment with that conclusion using the word “virtual” as an alternative to “really existing”.

The Casimir effect [13] consists of the attraction between two parallel perfectly conducting plates in vacuum. The force \( F \) per unit area depends on the distance \( d \) between the plates,

\[
F = -\frac{\pi^2 \hbar c}{240d^4},
\]

a force confirmed empirically [10]. The reason for the attraction may be understood qualitatively as follows. In equilibrium the electric field of the zeropoint radiation, ZPF, should be zero or normal to any plate surface, otherwise a current would be produced. This fact constrains the possible low frequency normal modes of the radiation, that is those having wavelengths of the order the distance between plates or larger, although the distribution of high frequency modes is barely modified by the presence of the plates. Therefore attaching an energy \( \frac{1}{2} \hbar \omega_j \) to every mode, the total energy of the ZPF in space becomes a function of the distance between plates and the derivative of that function with respect to the distance leads to Eq. (7). Actually the calculated energy of the ZPF diverges if we sum over all radiation modes, including those with arbitrarily high frequency, but there are regularization procedures that give the correct result [10]. They essentially subtract the field energy with the plates present minus the energy with the plates removed. The physical picture of the phenomenon is that the radiation pressures in both faces of each plate are different and this is the reason for a net force on the plate. The Casimir effect is currently considered the most strong argument for the reality of the quantum vacuum fields. For us it is specially relevant because it provides an example of the fact that the difference between the radiation arriving at the two faces of a plate is what matters, rather than the total radiation acting on the plate. We will make a similar assumption in the model of detector to be proposed in Sect. 5 below.

2.2 The Quantum Vacuum in Astrophysics and Cosmology

In laboratory physics, where gravity usually plays no role, the reality of the quantum vacuum fields is not too relevant a question. In fact, their possibly huge, or divergent, energy may be ignored choosing the zero of energy at the level of the vacuum, that is using the normal ordering rule. However this choice is no longer innocuous in the presence of gravity because, according to relativity theory, energy gravitates whence a huge energy would produce a huge gravitational field. Therefore the possible existence of a vacuum energy is a relevant question in astrophysics and cosmology. In the following my personal view is presented about the possible rebuttal of objections to the reality of the vacuum fields.
From long ago the quantum vacuum has been related to the cosmological constant, a term that Einstein introduced in general relativity in order to make possible his 1917 model of universe. The reason for the relation is that the vacuum, even if it possesses some energy density, \( \rho \), should be Lorentz invariant. Then it should exists also pressure, \( P \), with the equation of state \( P = -\rho \), that is equivalent to a cosmological constant term. Actually there was no empirical evidence for a cosmological constant until 1999 [14], but even before that date many authors speculated about the possibility that the quantum vacuum fields give rise to a cosmological term. However there was a big problem, namely the vacuum energy density appears to be infinite if no cut-off exists, and the only natural cut-off seems to be at the Planck scale. This cut-off would give a cosmological term about \( 10^{124} \) times the value derived from observations, a huge discrepancy known as “cosmological constant problem” [15], whose most simple solution is to assume a cancelation between positive and negative terms of the vacuum energy. However, as Weinberg argued [15], it seems difficult to believe that the cancelation is not exact but it is fine tuned to reduce the disagreement by 124 orders of magnitude. The problem might be an argument against the reality of the vacuum fields. Nevertheless the difficulty with the fine tuning is solved in a natural way if we assume that the mean vacuum energy and pressure are exactly balanced but the cosmological constant derives from the vacuum fluctuations. In fact, at a difference with Newtonian gravity, a linear theory where a fluctuating mass with zero mean would produce a fluctuating field also with zero mean, general relativity is non-linear and therefore a fluctuating energy density with zero mean may give rise to a gravitational field (space-time curvature) with mean different from zero [16].

Actually a possible cancelation of the mean vacuum energy might exist between the positive contribution of bose fields and the negative one of the fermi fields. The positive contribution of the best known bose field, the electromagnetic radiation, was exhibited in Eq. (6), now I illustrate the negative contribution of fermi fields with the example of the positron-electron field. In this case the free Hamiltonian may be written [17]

\[
H = \sum_{p, s} E_p \left( b_p^{(s)\dagger} b_p^{(s)} - d_p^{(s)\dagger} d_p^{(s)} \right) = \sum_{p, s} E_p \left( b_p^{(s)\dagger} b_p^{(s)} + d_p^{(s)\dagger} d_p^{(s)} - 1 \right), \tag{8}
\]

where \( b \) (\( d \)) are electron (positron) creation or annihilation operators, \( p \) is the momentum, \( s \) the spin projection and \( E = \sqrt{m^2 + p^2} \) is a (positive) energy. The latter Eq. (8) consists of terms in normal ordering, that do not contribute to the vacuum expectation value, plus a term that gives a negative energy contribution, so that the vacuum expectation of the Hamiltonian is \( -\sum_{p, s} E_p \), a negative divergent quantity. The results Eqs. (6) and (8) are illustrative of the possible cancelation of positive (bose fields) and negative (fermi fields) contributions to the quantum vacuum energy. Of course the assumed cancelation should involve all possible vacuum fields and their interactions. Similarly the positive and negative pressures of bose and fermi vacuum fields also should cancel each other.

The existence of positive and negative energy density and pressure contributions of the quantum vacuum fields suggests that the vacuum could be gravitationally polarized, in a way similar to the electric polarization that gives a contribution to the Lamb shift
in heavy atoms. The electric force, being attractive between opposite charges, produces a screening that diminish the effective charge of either a ion in water or a nucleus in vacuum. However gravity is always attractive thus enhancing the field near a mass, whence the said polarization would produce an additional gravitational field in the neighbourhood of big masses like galaxies or clusters. A simplified model has been proposed suggesting that this mechanism might be an alternative to the assumed dark matter [18].

In summary there are strong arguments for the reality of the quantum vacuum fields possessing energy and pressure. Firstly the vacuum fields appear naturally in quantization and should not be removed artificially. Secondly the action of the vacuum fields in laboratory experiments may be weak, usually not observable, due to an almost complete cancelation between radiation traveling in opposite directions, but it may be measured in some delicate breakings of balance like the Casimir effect. In astrophysics and cosmology the alleged huge energy and pressure of the vacuum fields may not hold, because a cancelation could exists between positive and negative contributions.

3 Entangled Photon Experiments in the Hilbert-Space Formalism of Quantum Optics

Spontaneous parametric down-conversion (SPDC), where a crystal having nonlinear electric susceptibility is pumped by a laser, is the common method to produce entangled photon pairs. Amongst the radiation emitted from the opposite side of the crystal two beams are selected, named “signal” and “idler”, with appropriate apertures and lens system. These beams are currently interpreted as consisting of a flow of entangled photon pairs derived from the photons of the laser, with energy $\hbar \omega_P$ each, that are split by the coupling with the crystal, giving rise to two photons with energies $\hbar \omega_s$ and $\hbar \omega_i$; $\omega_s + \omega_i = \omega_P$, this equality interpreted as energy conservation. In contrast the photon momenta are not conserved because the crystal takes a part of the momentum. Indeed the emerging signal and idler photons travel in different directions so that their joint momentum cannot be zero.

In order to study the phenomenon within quantum optics it is standard to consider that the laser and two “vacuum beams” enter the crystal, interact with the laser and emerge as “signal and idler” beams [19,20]. In a simplified two-modes treatment the incoming and emerging beams are represented by two radiation modes with associated field operators

$$in\text{going} : \hat{a}_s, \hat{a}_i; out\text{going} : \hat{a}_s + D\hat{a}_i^\dagger, \hat{a}_i + D\hat{a}_s^\dagger, \quad (9)$$

where $D$ is a small coupling parameter, i. e. $|D| \ll 1$. The incoming modes, corresponding to vacuum fields, are “inactive” and so represented by annihilation operators whilst the outgoing modes have inactive parts plus actual photons, i. e. these with associated creation operators.

The use of two modes as in Eq. (9) provides a bad representation of the physics. In fact a physical beam corresponds to a superposition of the amplitudes, $\hat{a}_k^\dagger$, of many modes with frequencies and wavevectors close to $\omega_s$ and $k_s$, respectively. For instance
we may represent the positive frequency part of an idler beam created in the crystal, to first order in $D$, as follows

$$
\hat{E}_i^+(\mathbf{r}, t) = D \int f_i(k) \, d^3k \hat{a}_k^+ \exp \{i (k - k_s) \cdot \mathbf{r} - i (\omega - \omega_s) t\} + \hat{E}_{ZPF}^+, \quad (10)
$$

where $\omega = \omega(k)$ and $f_i(k)$ is an appropriate function, with domain in a region of $k$ near $k_s$. The field $\hat{E}_{ZPF}^+$ is the sum of amplitudes of all vacuum modes, including the one represented by $\hat{a}_s$ in Eq. (9). Nevertheless the two-mode approximation is generally good enough for calculations. An exception will appear in Sect. 5, when we evaluate Eq. (43). In the rest of the calculations we will ignore the space-time phase factor present in Eq. (10).

With appropriate devices the signal and idler beams may be combined giving rise to two beams with photons entangled in polarization. These beams travel (usually a long path) until Alice and Bob respectively. Alice possesses a polarization analyzer and a detector in front of it, and similarly Bob. Thus the beam fields arriving at these two detectors may be represented, in our two modes approximation, by the operators

$$
Alice : \hat{E}_A^+ = \hat{a}_s \cos \theta + i \hat{a}_i \sin \theta + D[\hat{a}_i^\dagger \cos \theta + i \hat{a}_s^\dagger \sin \theta],
$$

$$
Bob : \hat{E}_B^+ = -i \hat{a}_i \cos \phi + \hat{a}_s \sin \phi + D[-i \hat{a}_i^\dagger \cos \phi + \hat{a}_s^\dagger \sin \phi], \quad (11)
$$

where $\theta$ and $\phi$ are the polarizer’s angles of Alice and Bob respectively. The Hermitian conjugate of these field operators will be labelled $\hat{E}_A^- \equiv (\hat{E}_A^+)^\dagger$, $\hat{E}_B^- \equiv (\hat{E}_B^+)^\dagger$.

From Eq. (11) it is straightforward to get the quantum predictions for the experiment using the standard Hilbert-space formalism (HS in the following). Alice single detection rate is proportional to the following vacuum expectation

$$
R_A = \langle 0 | \hat{E}_A^- \hat{E}_A^+ | 0 \rangle = |D|^2 \langle 0 | \hat{a}_i \hat{a}_i^\dagger \cos^2 \theta + \hat{a}_s \hat{a}_s^\dagger \sin^2 \theta | 0 \rangle = |D|^2, \quad (12)
$$

where I have neglected terms with creation (annihilation) operators appearing on the left (right). A similar result may be obtained for the single detection rate of Bob, that is

$$
R_B = \langle 0 | \hat{E}_B^- \hat{E}_B^+ | 0 \rangle = |D|^2. \quad (13)
$$

The coincidence rate is obtained via the vacuum expectation value of the product of four field operators in normal order. In our case we have two terms, that is

$$
R_{AB} = \frac{1}{2} \langle 0 | \hat{E}_A^- \hat{E}_B^- \hat{E}_B^+ \hat{E}_A^+ | 0 \rangle + \frac{1}{2} \langle 0 | \hat{E}_B^- \hat{E}_A^- \hat{E}_A^+ \hat{E}_B^+ | 0 \rangle. \quad (14)
$$
which would be equal to each other if $\hat{E}_A^+$ and $\hat{E}_B^+$ commuted. The former expectation may be evaluated to order $|D|^2$ as follows

$$\langle 0 \mid \hat{E}_A^- \hat{E}_B^- \hat{E}_B^+ \hat{E}_A^+ \mid 0 \rangle = \langle 0 \mid \hat{E}_A^- \hat{E}_B^- \hat{E}_B^+ \hat{E}_A^+ \mid 0 \rangle = \left| \langle 0 \mid \hat{E}_B^+ \hat{E}_A^+ \mid 0 \rangle \right|^2,$$

(15)

where $\hat{E}_{A1}^+$ is the part of order $|D|$ of $\hat{E}_A^+$ and $\hat{E}_{B1}^+$ the part of order $|D|$ of $\hat{E}_B^+$, see Eq. (11), $\hat{E}_{A0}^+$ and $\hat{E}_{B0}^+$ being the parts of order zero respectively. In the former equality of Eq. (15) we have removed creation operators on the left and annihilation operators on the right, in the second we have removed terms of order $|D|^4$. The latter term of Eq. (14) gives a similar result, with $A(B)$ substituted for $B(A)$. Then the coincidence detection rate becomes

$$R_{AB} = \frac{1}{2} \left| \langle 0 \mid \hat{E}_{B0}^+ \hat{E}_{A1}^+ \mid 0 \rangle \right|^2 + \frac{1}{2} \left| \langle 0 \mid \hat{E}_{A0}^+ \hat{E}_{B1}^+ \mid 0 \rangle \right|^2.$$

(16)

Hence, taking Eq. (11) into account we have

$$\left| \langle 0 \mid \hat{E}_{B0}^+ \hat{E}_{A1}^+ \mid 0 \rangle \right|^2 = |D|^2 |\sin \phi \sin \theta + \cos \phi \cos \theta|^2 = |D|^2 \cos^2(\theta - \phi).$$

The latter term of Eq. (16) leads to a similar contribution whence we get

$$R_{AB} = |D|^2 \cos^2(\theta - \phi).$$

(17)

The results Eqs. (12), (13) and (17) have the form of Eq. (4) and therefore the quantum predictions violate a Bell inequality.

### 4 The Experiments in the Weyl–Wigner Formalism

#### 4.1 The Formalism in Quantum Optics

In the following I shall shortly review the treatment within the Weyl–Wigner (WW) formalism of the polarization correlation experiment. The WW formalism was developed for non-relativistic quantum mechanics, where the basic observables involved are positions, $\hat{x}_j$, and momenta, $\hat{p}_j$, of the particles [21–23]. It may be trivially extended to quantum optics provided we interpret $\hat{x}_j$ and $\hat{p}_j$ to be the sum and the difference of the creation, $\hat{a}_j^+$, and annihilation, $\hat{a}_j$, operators of the $j$ normal mode of the radiation. That is

$$\hat{x}_j \equiv \frac{c}{\sqrt{2}\omega_j} \left( \hat{a}_j + \hat{a}_j^\dagger \right), \quad \hat{p}_j \equiv \frac{i\hbar\omega_j}{\sqrt{2}c} \left( \hat{a}_j - \hat{a}_j^\dagger \right)$$

$$\Rightarrow \hat{a}_j = \frac{1}{\sqrt{2}} \left( \frac{\omega_j}{c} \hat{x}_j + \frac{i\hbar}{\hbar\omega_j} \hat{p}_j \right), \quad \hat{a}_j^\dagger = \frac{1}{\sqrt{2}} \left( \frac{\omega_j}{c} \hat{x}_j - \frac{i\hbar}{\hbar\omega_j} \hat{p}_j \right).$$

(18)
Here $\hbar$ is Planck constant, $c$ the velocity of light and $\omega_j$ the frequency of the normal mode. In the following I will use units $\hbar = c = 1$. For the sake of clarity I will represent the field operators on a Hilbert space with a ‘hat’ as in the previous section, e. g. $\hat{a}_j, \hat{a}^\dagger_j$, but remove the ‘hat’ for the amplitudes in the WW formalism, e. g. $a_j, a^*_j$.

The connection with the Hilbert-space (HS) formalism is made via the Weyl transform as follows. For any trace class operator $\hat{M}$ of the former we define its Weyl transform to be a function of the field operators $\{\hat{a}_j, \hat{a}^\dagger_j\}$, that is

$$W_{\hat{M}} = \frac{1}{(2\pi^2)^n} \prod_{j=1}^{n} \int_{-\infty}^{\infty} d\lambda_j \int_{-\infty}^{\infty} d\mu_j \exp[-2i\lambda_j \text{Re}a_j - 2i\mu_j \text{Im}a_j]$$

$$\times \text{Tr} \left\{ \hat{M} \exp \left[ i\lambda_j (\hat{a}_j + \hat{a}^\dagger_j) + i\mu_j (\hat{a}_j - \hat{a}^\dagger_j) \right] \right\}. \quad (19)$$

The transform is invertible, that is if $f$ is the transform of $\hat{f}$ and $g$ the transform of $\hat{g}$, then the transform of $\hat{f} + \hat{g}$ is $f + g$.

It is standard wisdom that the WW formalism is unable to provide any intuitive picture of the quantum phenomena. The reason is that the Wigner function, that would represent the quantum states of the HS formalism, is not positive definite in general and therefore cannot be interpreted as a probability distribution (of positions and momenta in quantum mechanics, or field amplitudes in quantum optics). However we shall see that in quantum optics the formalism in the “Heisenberg picture” allows the interpretation of the experiments using the Wigner function only for the vacuum state, that is positive definite.

The use of the WW formalism in quantum optics has the following features in comparison with the HS formalism:

1. It is just quantum optics, therefore the predictions for experiments are the same.
2. The calculations using the WW formalism are usually no more involved than the corresponding ones in Hilbert space. However the latter allows significant shortcuts in some cases via a clever use of the noncommutative algebra of operators in HS.
3. The formalism suggests a physical picture in terms of random variables and stochastic processes. In particular the counterparts of creation and annihilation operators look like random amplitudes in a complex representation of radiation.

However I shall stress that the physical picture is possible only if we renounce to get an interpretation in terms of random variables for all alleged states and observables of the standard HS formalism, but we attempt just to interpret (get a physical picture...}
of) actual experiments, either performed or possible. An example is the interpretation offered in Sect. 5 for test of Bell inequalities without photons, although I will use sometimes the common language speaking for instance about “entangled photons” in order to show the connection with the HS formalism. In our interpretation of the experiments the particle behaviour of the quantum electromagnetic field does not appear at all, photocounts being just rapid changes in the detectors due to the interaction with the radiation (wave) field.

4.2 Properties

All properties of the WW formalism in particle systems may be translated to quantum optics via Eq. (18). The transform Eq. (19) allows getting a function of (c-number) amplitudes for any trace-class operator (e.g. any function of the creation and annihilation operators of ‘photons’). In particular we may get the (Wigner) function corresponding to any quantum state of the HS formalism. For instance the vacuum state, represented by the density matrix $|0\rangle\langle0|$, is associated to the following Wigner function

$$W_0 = \prod_j \frac{2}{\pi} \exp\left(-2|a_j|^2\right).$$

Hence the suggested picture that the quantum vacuum of the electromagnetic field (the zeropoint field, ZPF) consists of stochastic fields with a probability distribution independent for every mode, having a Gaussian distribution with mean energy $\frac{1}{2}\hbar \omega$ per mode. That interpretation will be studied in the next section.

Similarly there are functions associated to the observables. For instance the following Weyl transforms are obtained

$$\hat{a}_j \leftrightarrow a_j, \hat{a}_j^\dagger \leftrightarrow a_j^*, \frac{1}{2} \left(\hat{a}_j^\dagger \hat{a}_j + \hat{a}_j \hat{a}_j^\dagger\right) \leftrightarrow a_j a_j^* = |a_j|^2,$$

$$\hat{a}_j^\dagger \hat{a}_j = \frac{1}{2} \left(\hat{a}_j^\dagger \hat{a}_j + \hat{a}_j \hat{a}_j^\dagger\right) + \frac{1}{2} \left(\hat{a}_j^\dagger \hat{a}_j - \hat{a}_j \hat{a}_j^\dagger\right) \leftrightarrow |a_j|^2 - \frac{1}{2},$$

$$\left(\hat{a}_j^\dagger + \hat{a}_j\right)^n \leftrightarrow \left(a_j + a_j^*\right)^n, \left(\hat{a}_j^\dagger - \hat{a}_j\right)^n \leftrightarrow \left(a_j - a_j^*\right)^n, n \text{ an integer.}$$

I stress that the quantities $a_j$ and $a_j^*$ are c-numbers and therefore they commute with each other. The former Eq. (21) mean that in expressions linear in creation and/or annihilation operator the Weyl transform just implies ‘removing the hats’. However this is not the case in nonlinear expressions in general. In fact from the latter two Eq. (21) plus the linear property it follows that for a product in the WW formalism the HS counterpart is

$$a_j^k a_j^l \leftrightarrow (\hat{a}_j^k \hat{a}_j^l)^\text{sym},$$

where the subindex $\text{sym}$ means writing the product in all possible orderings and dividing by the number of terms. Hence the WW field amplitudes corresponding to
products of field operators may be obtained putting the operators in symmetrical order via the commutation relations.

*Expectation values* may be calculated in the WW formalism as follows. In the HS formalism they read $Tr(\hat{\rho} \hat{M})$, or in particular $\langle \psi | \hat{M} | \psi \rangle$, whence the translation to the WW formalism is obtained taking into account that the trace of the product of two operators becomes

$$
Tr(\hat{\rho} \hat{M}) = \int W_\rho \left\{ \hat{a}_j, \hat{a}_j^\dagger \right\} W_M \left\{ \hat{a}_j, \hat{a}_j^\dagger \right\} \prod_j d\text{Re}a_j d\text{Im}a_j. \tag{23}
$$

That integral is the WW counterpart of the trace operation in the HS formalism. Particular instances are the following expectations that will be of interest later on

$$
\begin{align*}
\langle |a_j|^2 \rangle &\equiv \int d\Gamma W_0 |a_j|^2 = \frac{1}{2}, \langle a_j^n a_j^{*m} \rangle = 0 \text{ if } n \neq m. \\
\langle 0 | \hat{a}_j^\dagger \hat{a}_j | 0 \rangle &\equiv \int d\Gamma (a_j^* a_j - \frac{1}{2}) W_0 = 0, \\
\langle 0 | \hat{a}_j \hat{a}_j^\dagger | 0 \rangle &\equiv \int d\Gamma (|a_j|^2 + \frac{1}{2}) W_0 = 2 \langle |a_j|^2 \rangle = 1, \\
\langle |a_j|^4 \rangle &\equiv 1/2, \langle |a_j|^n |a_k|^m \rangle = \langle |a_j|^n \rangle \langle |a_k|^m \rangle \text{ if } j \neq k. \tag{24}
\end{align*}
$$

where $W_0$ is the Wigner function of the vacuum, Eq. (20). This means that in the WW formalism the field amplitude $a_j$ (coming from the vacuum) behaves like a complex random variable with Gaussian distribution and mean square modulus $\langle |a_j|^2 \rangle = 1/2$. I point out that the integral for any mode not entering in the function $W_\rho \left\{ \left\{ a_j, a_j^* \right\} \right\}$ gives unity in the integral Eq. (23) due to the normalization of the Wigner function Eq. (20). An important consequence of Eq. (24) is that normal (anti-normal) ordering of one creation and one annihilation operators in the Hilbert space formalism becomes subtraction (addition) of 1/2 in the WW formalism. That is the normal ordering rule is equivalent to subtracting the vacuum contribution, as was commented in Sect. 2.1.

### 4.3 Entangled Photon Pairs in the WW Formalism

All quantum optical phenomena that may be analyzed using the HS formulation of quantum optics may be also studied with the WW formalism. In fact, it is enough to translate the equations to the new formalism by means of the Weyl transform. Here I will apply the WW formalism to the description of the polarization correlation of entangled photon pairs produced via spontaneous parametric down-conversion (SPDC). I will start with the fields that are the WW counterparts of the HS field operators Eq. (11), that I will write in terms of two partial field amplitudes for later convenience,
that is
\[
E_A^+ = E_{A0}^+ + E_{A1}^+, \quad E_B^+ = E_{B0}^+ + E_{B1}^+,
\]
\[
E_{A0}^+ = a_s \cos \theta + ia_i \sin \theta, \quad E_{A1}^+ = D \left[ a_i^* \cos \theta + ia_s^* \sin \theta \right],
\]
\[
E_{B0}^+ = -ia_s \sin \phi + a_i \cos \phi, \quad E_{B1}^+ = D \left[ -ia_i^* \sin \phi + a_s^* \cos \phi \right].
\] (25)

Now we are in a position to derive the WW detection rules via a Weyl transform of the stated rules in the HS formalism. The following transforms may be easily derived from Eq. (24)
\[
\langle 0 \mid \hat{a}_i \hat{a}_i^\dagger \mid 0 \rangle \rightarrow \langle |a_i|^2 + \frac{1}{2} \rangle = 2 \langle |a_i|^2 \rangle,
\]
\[
\langle 0 \mid \hat{a}_i^\dagger \hat{a}_i \mid 0 \rangle \rightarrow \langle |a_i|^2 - \frac{1}{2} \rangle = 0.
\] (26)

Hence it may be realized that the HS expectations with the field operators in antinormal order, as in Eqs. (12) or (16), a factor 2 should be included for every pair of field operators. Therefore Eq. (12) leads to
\[
R_A = \langle 0 \mid \hat{E}_{A1}^- \hat{E}_{A1}^+ \mid 0 \rangle \rightarrow R_A = 2 \langle E_{A1}^- E_{A1}^+ \rangle, \quad R_B = 2 \langle E_{B1}^- E_{B1}^+ \rangle.
\] (27)

With analogous arguments the HS rule for the coincidence rate, Eq. (15), should be multiplied times 4, giving the WW rule
\[
R_{AB} = 2 \left| \langle E_{A0}^+ E_{B1}^+ \rangle \right|^2 + 2 \left| \langle E_{B0}^+ E_{A1}^+ \rangle \right|^2
\]
\[
= 2 \left| \langle E_{A0}^+ E_{B1}^+ \rangle \right|^2 + 2 \left| \langle E_{B0}^+ E_{A1}^+ \rangle \right|^2,
\] (28) (29)

where the latter equality takes into account the identity
\[
\langle E_{A0}^+ E_{B0}^- \rangle = \langle E_{A0}^+ (E_{B0}^- + E_{B1}^-) \rangle = \langle E_{A0}^+ E_{B0}^- \rangle + \langle E_{A0}^+ E_{B1}^- \rangle,
\]
and \( \langle E_{A0}^+ E_{B0}^- \rangle = 0 \) that may be derived from the second Eq. (24) with \( n = 2, m = 0 \).

Equation (28) provides the desired coincidence detection rule, which is rather simple written in terms of field amplitudes arriving at Alice and Bob respectively. By construction it is obvious that, using Eqs. (27) and (28), the WW formalism will give the same predictions as the standard quantum HS formalism for all experiments involving entangled photon pairs produced via SPDC.

For the realistic interpretation to be given in the next section it is interesting to write the detection rules Eqs. (27) and (28) in terms of field intensities rather than amplitudes. To do that we may define intensities as follows
\[
I_{A0} = E_{A0}^+ E_{A0}^-, \quad I_{A1} = E_{A0}^+ E_{A1}^- + E_{A1}^+ E_{A0}^-, \quad I_{A2} = E_{A1}^+ E_{A1}^-,
\]
\[
I_{B0} = E_{B0}^+ E_{B0}^-, \quad I_{B1} = E_{B0}^+ E_{B1}^- + E_{B1}^+ E_{B0}^-, \quad I_{B2} = E_{B1}^+ E_{B1}^-,
\]
\[
I_A = E_A^+ E_A^- = I_{A0} + I_{A1} + I_{A2}, \quad I_B = E_B^+ E_B^- = I_{B0} + I_{B1} + I_{B2}.
\] (30)
although $I_{A1}$ and $I_{B1}$ are not actual intensities, in particular they are not positive
definite. I point out that $I_{A0}$, $I_{A1}$ and $I_{A2}$ are of order $1$, $|D|$ and $|D|^2$, respectively,
in the small parameter $|D| << 1$. Equation (27) may trivially obtained in terms of
intensities taking Eq. (30) into account. We get

$$
R_A = 2\langle IA \rangle - 2\langle I_{A0} \rangle = 2\langle I_{A2} \rangle, \\
R_B = 2\langle IB \rangle - 2\langle I_{B0} \rangle = 2\langle I_{B2} \rangle,
$$

(31)

the terms $I_{A1}$ and $I_{B1}$ not contributing, as may be realized. Writing the coincidence
detection rate Eq. (28) in terms of intensities is more involved and will be postponed
to the next section.

We conclude that the predictions for the experiments are the same either in the HS
or in the WW formalism provided in the latter we use for the single rates either Eqs.
(27) or (31) and for the coincidence rates Eq. (28).

5 A Realistic Local Model of the Experiments

5.1 Model of Photodetector

The WW formalism suggests a picture of the quantum optical phenomena. The picture
for experiments involving ‘entangled photon pairs’ provides a local realistic interpre-
tation in terms of random variables and stochastic processes. In the following I present
the main ideas of this stochastic interpretation. It rests upon several assumptions as
follows.

The fundamental hypothesis is that the electromagnetic vacuum field is a real
stochastic field (the zeropoint field, ZPF) as commented in Sect. 2. If expanded in nor-
mal modes the ZPF has a (positive) probability distribution of the amplitudes given by
Eq. (20). Therefore we assume that all bodies are immersed in ZPF, charged particles
absorbing and emitting radiation continuously, in some cases reaching a dynamical
equilibrium with the ZPF (that would correspond to the ground state in the HS for-
malism [11]).

According to that assumption any photodetector would be under the action of an
extremely strong radiation, infinite if no cut-off existed. Thus how might we explain
that detectors are not activated by the vacuum radiation? Firstly the strong vacuum
field is effectively reduced to a weak level if we assume that only radiation within
some small frequency interval is able to activate a photodetector, that is the interval of
sensibility $(\omega_1, \omega_2)$. However the problem is not yet solved because signals involved in
experiments have typical intensities of order the vacuum radiation in the said frequency
interval so that the detector would be unable to distinguish a signal from the ZPF noise.
Our proposed solution is to assume that a detector may be activated only when the
net Poynting vector (i.e. the directional energy flux) of the incoming radiation is
different from zero, including both signal and vacuum fields. More specifically I will
model a detector as possessing an active area, the probability of a photocount being
proportional to the net radiant energy flux crossing that area from the front side during
the activation time, the probability being zero if the net flux crosses the area in the
reverse direction during that time interval. The need of some finite detection time is a known fact in experiments.

These assumptions allow to understand qualitatively why the signals, but not the vacuum fields, activate detectors. Indeed the ZPF arriving at any point (in particular the detector) would be usually isotropic on the average, whence its associated mean Poynting vector would be nil, therefore only the (directional) signal radiation might produce photocounts. A problem remains because the vacuum fields are fluctuating so that the net Poynting vector also fluctuates, and it may point in the wrong direction even in the presence of signals. However a net flux in the wrong direction would be unlikely if the activation time of the detectors is large enough because this would effectively average the vacuum fluctuations, washing their effect.

Our aim is to achieve a realistic local interpretation of the experiments measuring polarization correlation of entangled photon pairs, that we studied with the WW formalism in the previous section. Thus I will consider two vacuum beams entering the nonlinear crystal, where they give rise to a “signal” and an “idler” beams. After crossing several appropriate devices they produce fields that arrive at the Alice and Bob detectors. I will not attempt a detailed model that should involve many modes in order to represent the signals as narrow beams (see Eq. (10)).

In agreement with our previous hypotheses a photocount should derive from the net energy flux crossing the active photocounter surface. Thus I will assume that the detection probabilities per time window, $T$, that are proportional to the single $R_A$, $R_B$ and coincidence $R_{AB}$ detection rates, will be

$$R_A = \langle [M_A]_+ \rangle, \quad R_B = \langle [M_A]_+ \rangle, \quad R_{AB} = \langle [M_A]_+ [M_B]_+ \rangle,$$

$$M_A \equiv T^{-1} \int_0^T \vec{n}_A \cdot \vec{I}_{\text{total}}^A (r_A, t) \, dt, \quad M_B \equiv T^{-1} \int_0^T \vec{n}_B \cdot \vec{I}_{\text{total}}^B (r_A, t) \, dt, \quad (32)$$

where $[M]_+ = M$ if $M > 0$, $[M]_+ = 0$ otherwise, and $\vec{n}$ is a unit vector in the direction of the incoming signal beams, that I assume perpendicular to the active area of the detector. I use units such that both the intensities and the detection rates are dimensionless, the latter because they are defined as probabilities within a time window $T$, this being greater than the photocounter activation time. In Eq. (32) the positivity constraint (i.e. putting $[M_A]_+$ rather than $M_A$) is needed because the detection probabilities must be non-negative for any particular run of an experiment whilst the quantities $M_A$ and $M_B$ are fluctuating and might be negative. Nevertheless the ensemble averages involved in Eq. (32) are positive or zero and the fluctuations will not be too relevant due to the time integration that washes them out. Therefore I will make the approximation of ignoring the positivity constraint in the following, substituting $M_A$ for $[M_A]_+$ and $M_B$ for $[M_B]_+$.

### 5.2 Realistic Interpretation of Entangled Photons

In order to have a realistic model of the experiments I will consider a simplified treatment involving just two vacuum radiation modes, with amplitudes $a_s$ and $a_i$, as in the WW calculation of the previous section. After crossing several appropriate
devices the fields will arrive at the Alice and Bob detectors. Each one of these two fields consists of two parts, one of order 1 and another of order $|D| \ll 1$, see Eq. (25). It may be realized that the former is what would arrive at the detectors if there was no pumping laser and therefore no signal. It is just a part of the ZPF, whilst the rest of the ZPF consists of radiation not appearing in the equations of the previous section because they were not needed in the calculations. The total ZPF should have the property of isotropy, therefore giving nil net flux in the detector (modulo fluctuations that may contribute to a dark rate that we shall ignore in this paper). The terms of order $|D|$ derive from the signals produced in the nonlinear crystal. In summary the Poynting vectors of the radiation at the (center of the) active area of the detectors may be written

$$
\begin{align*}
\text{Alice} : & \vec{I}_A^{\text{total}}(t) = \vec{I}_A^{\text{ZPF}}(t) + \vec{I}_A(t), \\
\text{Bob} : & \vec{I}_B^{\text{total}}(t) = \vec{I}_B^{\text{ZPF}}(t) + \vec{I}_B(t).
\end{align*}
$$

(33)

$\vec{I}_A$, $\vec{I}_B$, are due to the fields $E_A$, $E_B$, Eq. (25), emerging from the non-linear crystal after they are transformed by lens systems, apertures, beam splitters, etc. The Poynting vectors $\vec{I}_A(t)$ and $\vec{I}_B(t)$ have the direction of $\vec{n}_A$ and $\vec{n}_B$ respectively, see Eq. (32), and their moduli would be field intensities. Furthermore these intensities are time independent in our two modes representation of the fields, the time dependence in actual experiments coming from the interference of many modes, see Eq. (10). Therefore we will write

$$
M_A = I_A^{\text{ZPF}} + I_A, \quad I_A^{\text{ZPF}} \equiv T^{-1} \int_0^T \vec{n}_A \cdot \vec{I}_A^{\text{ZPF}}(r_A,t) \, dt
$$

(34)

and similar for $M_B$.

In order to get the Alice single detection rate we need the average of $M_A$, that we will evaluate by comparison with the case when there is no pumping on the nonlinear crystal. In this case $I_A$ becomes $I_{A0}$ and the Poynting vector of all vacuum fields arriving at the detector of Alice, i.e. $\vec{I}_A^{\text{ZPF}}(t) + \vec{I}_{A0}(t)$, should have nil average due to the isotropy of the total $ZPF$. And similar for Bob. As a consequence the intensities $I_{A0}$ and $I_{B0}$, Eq. (30), should fulfil the following equalities

$$
\left\langle I_A^{\text{ZPF}} + I_{A0} \right\rangle = \left\langle I_B^{\text{ZPF}} + I_{B0} \right\rangle = 0.
$$

(35)

It might appear that this relation could not be true for all values of the angles $\theta$, $\phi$ Eq. (25) because the ZPF Poynting vectors $\vec{I}_A^{\text{ZPF}}$ and $\vec{I}_B^{\text{ZPF}}$ should not depend on our choice of angles whilst $I_{A0}$ and $I_{B0}$ do depend. However the positions of the polarizers do influence also the ZPF arriving at the detectors and it is plausible that the total Poynting vector has zero mean in any case. From Eqs. (34) and (35) we may derive the single rates of Alice and Bob, that is

$$
R_A = \langle M_A \rangle = \langle I_A \rangle - \langle I_{A0} \rangle = \frac{1}{2} |D|^2, \quad P_B = \langle M_B \rangle = \frac{1}{2} |D|^2.
$$

(36)
The result agrees with the quantum prediction Eq. (31) except for a scale factor 2, that will be irrelevant for our purposes as shown later on (it derives from an arbitrary proportionality constant that we have taken as unity in Eq. (32)).

The coincidence detection rate may be got taking Eq. (32) into account, that is

\[ R_{AB} = \langle M_A M_B \rangle = \left( \left[ I_{ZPF}^A + I_A \right] \left[ I_{ZPF}^B + I_B \right] \right). \] (37)

If there was no pumping laser on the nonlinear crystal the joint detection rate should be zero whence Eq. (37) leads to

\[ \left( \left[ I_{ZPF}^A + I_{A0} \right] \left[ I_{ZPF}^B + I_{B0} \right] \right) = 0. \] (38)

From Eqs. (35), (37) and (38) it is possible to get \( R_{AB} \). Firstly I point out that \( I_{ZPF}^A \) and \( I_{ZPF}^B \) could not depend on whether the pumping is on or off. More correctly, the probability distribution of the possible values of \( I_{ZPF} \) would be the same whether the pumping is on or off. Then subtracting Eq. (38) from Eq. (37) we get

\[ R_{AB} = \left\langle (I_B - I_{B0}) I_{ZPF}^2 \right\rangle + \left\langle (I_A - I_{A0}) I_{ZPF}^B \right\rangle + \left\langle I_A I_B \right\rangle - \left\langle I_{A0} I_{B0} \right\rangle. \] (39)

On the average the ZPF intensity \( I_{ZPF}^A \) is independent of whether the pumping is on or off, as assumed above, whence \( I_{ZPF}^A \) and \( (I_B - I_{B0}) \) are uncorrelated. Similarly for \( I_{ZPF}^B \) and \( (I_A - I_{A0}) \). Hence we may write

\[
R_{AB} = \left\langle I_A I_B \right\rangle - \left\langle I_{A0} I_{B0} \right\rangle + \left\langle I_B - I_{B0} \right\rangle \left\langle I_{ZPF}^A \right\rangle + \left\langle I_A - I_{A0} \right\rangle \left\langle I_{ZPF}^B \right\rangle
\]

\[
= \left\langle I_A I_B \right\rangle - \left\langle I_{A0} I_{B0} \right\rangle - \left\langle I_A I_{B2} \right\rangle - \left\langle I_{B2} I_{A0} \right\rangle .
\] (39)

the averages \( \langle I_A \rangle \) and \( \langle I_B \rangle \) being nil. The latter inequality follows taking Eq. (35) into account.

Now we may write Eq. (39) in terms of the fields. For the former term we have, taking Eq. (30) into account

\[
\langle I_A I_B \rangle = \langle (I_A + I_{A1} + I_{A2})(I_B + I_{B1} + I_{B2}) \rangle
\]

\[
= \langle I_{A0} I_{B0} \rangle + \langle I_{A0} I_{B2} \rangle + \langle I_{A2} I_{B0} \rangle + \langle I_{A1} I_{B1} \rangle .
\] (40)

the terms \( \langle I_{A0} I_{B1} \rangle, \langle I_{A1} I_{B0} \rangle, \langle I_{A1} I_{B2} \rangle \) and \( \langle I_{A2} I_{B1} \rangle \) not contributing as may be checked, and the term \( \langle I_{A1} I_{B1} \rangle \) contributes to order \( |D|^4 \), therefore being negligible in our calculation to order \( |D|^2 \). The term \( \langle I_{A0} I_{B0} \rangle \) will cancel with a similar term in Eq. (39) and the four remaining terms of Eq. (40) may be written in terms of amplitudes as follows

\[
\langle I_{A0} I_{B2} \rangle = \langle E^+_{A0} E^-_{A0} E^+_{B1} E^-_{B1} \rangle = \langle E^+_{A0} E^-_{A0} \rangle \langle E^+_B E^-_{B1} \rangle
\]

\[
+ \langle E^+_{A0} E^-_{B1} \rangle \langle E^-_{A0} E^+_B \rangle + \langle E^+_{A0} E^-_{B1} \rangle \langle E^-_{A0} E^+_B \rangle
\]

\[
= \langle I_{A0} \rangle \langle I_{B2} \rangle + \left| \langle E^+_{A0} E^+_B \rangle \right|^2 .
\]
the former equality deriving from the property of the average of four Gaussian random variables and the average $\langle E_{A0}^+ E_{B1}^+ \rangle$ not contributing as may be realized. A similar procedure may be used for $\langle I_{A2} I_{B0} \rangle$ whence Eq. (39) becomes

$$R_{AB} = \left| \langle E_{A0}^+ E_{B1}^+ \rangle \right|^2 + \left| \langle E_{A1}^+ E_{B0}^+ \rangle \right|^2 + \langle I_{A1} I_{B1} \rangle. \quad (41)$$

This result differs from the quantum prediction got via the WW formalism, Eq. (28), aside from the irrelevant scale factor 2 (see comment after Eq. (36)) due to the presence of the term $\langle I_{A1} I_{B1} \rangle$. Now we argue that this term does not contribute. In fact writing it in terms of fields we have

$$\langle I_{A1} I_{B1} \rangle = \langle \left( E_{A0}^+ E_{A1}^- + E_{A1}^+ E_{A0}^- \right) \left( E_{B0}^- E_{B1}^- + E_{B1}^- E_{B0}^- \right) \rangle. \quad (42)$$

Performing the product there are 4 terms that we may calculate using again the property of the product of four Gaussian variables. The former term leads to a null result, namely

$$\langle E_{A0}^+ E_{A1}^- E_{B0}^- E_{B1}^- \rangle = \langle E_{A0}^+ E_{A1}^- \rangle \langle E_{B0}^- E_{B1}^- \rangle + \langle E_{A0}^+ E_{B1}^- \rangle \langle E_{A1}^- E_{B0}^- \rangle + \langle E_{A0}^+ E_{B0}^- \rangle \langle E_{A1}^- E_{B1}^- \rangle = 0,$$

because all the averages are zero taking Eq. (25) into account. Similarly $\langle E_{A1}^+ E_{A0}^- E_{B1}^- E_{B0}^- \rangle = 0$. The two terms remaining from Eq. (42) are complex conjugate of each other so that we may write

$$\langle I_{A1} I_{B1} \rangle = 2 \text{Re} \left\{ \langle E_{A0}^+ E_{A1}^- E_{B1}^- E_{B0}^- \rangle \right\}. \quad (43)$$

If we evaluated this expectation using the Gaussian property as previously, we would get

$$\langle E_{A0}^+ E_{A1}^- E_{B1}^- E_{B0}^- \rangle = \langle E_{A0}^+ E_{A1}^- \rangle \langle E_{B1}^- E_{B0}^- \rangle + \langle E_{A0}^+ E_{B1}^- \rangle \langle E_{A1}^- E_{B0}^- \rangle$$

$$+ \langle E_{A0}^+ E_{B0}^- \rangle \langle E_{A1}^- E_{B1}^- \rangle = \langle E_{A0}^+ E_{B1}^- \rangle \langle E_{A1}^- E_{B0}^- \rangle + \langle E_{A0}^+ E_{B0}^- \rangle \langle E_{A1}^- E_{B1}^- \rangle,$$

the former term being nil. The latter two terms consist of a product of two averages, or expectation values, each. Every average consists of a field amplitude arriving at Alice times another amplitude arriving at Bob. In these conditions we cannot ignore the space-time factors like $\exp \{ i k \cdot r - i\omega t \}$, see Eq. (10), that would be different in the Alice and Bob beams, and uncorrelated. Therefore it is plausible that the average over these phases should result in a null value for the expectation and consequently the average Eq. (43) will be zero, so ending the proof that the term $\langle I_{A1} I_{B1} \rangle$ of Eq. (41) does not contribute to order $|D|^2$. I point out that the condition is quite different in the other terms of Eq. (41) because they involve absolute values, making the space-time phases irrelevant in the average. Thus we get finally the prediction of our local model for the coincidence detection rate

$$R_{AB} = \left| \langle E_{A0}^+ E_{B1}^+ \rangle \right|^2 + \left| \langle E_{A1}^+ E_{B0}^+ \rangle \right|^2, \quad (44)$$
that agrees with the quantum prediction Eq. (28), except for a scale factor 2 (see comment after Eq. (36)). Indeed taking Eq. (25) into account, we get the coincidence detection rate

\[
R_{AB} = \frac{1}{2} |D|^2 \cos^2(\theta - \phi). \tag{45}
\]

As a conclusion the results of our realistic model, Eqs. (36) and (45), agree with the quantum predictions Eqs. (12), (13) and (17), modulo an scaling parameter 1/2. Indeed the predictions of the model, Eqs. (36) and (45), have the form of Eq. (4) and therefore violate a Bell inequality.

It is interesting for the picture of the phenomenon, to be discussed in the next subsection, to write Eq. (44) in terms of intensities, rather than amplitudes. This may be achieved taking Eqs. (39) and (40) into account and neglecting the term \(\langle I_{A1}I_{B1}\rangle\) which is of order \(|D|^4\). Thus we get

\[
R_{AB} = \langle (I_{A0} - \langle I_{A0}\rangle)I_{B2} \rangle + \langle (I_{B0} - \langle I_{B0}\rangle)I_{A2} \rangle \tag{46}
\]

5.3 A Physical Picture of the Experiments

Our model, resting upon the WW formalism of quantum optics, provides a picture quite different from the one suggested by the HS formalism in terms of photons. We do not assume that the photocount probability within a time window factorizes so that there is a small probability, of order \(|D|^2\), that the pumping laser produces in the crystal an “entangled photon pair” within a given time window, and then there is a detection probability of order unity conditional to the photon pair production. (The latter probability is identified with the detection efficiency). Furthermore the concept of photon does not appear at all, but there are continuous fluctuating fields including a real ZPF arriving at the detectors, that are activated when the fluctuations are big enough.

It is interesting to study more closely the “quantum correlation” qualified as strange from a classical point of view because it is a consequence of the phenomenon of entanglement. The origin is the correlation between the signal \(I_{B1}\) produced by the action of the laser on the crystal and the part \(I_{A0}\) of the ZPF that entered in the crystal. This correlation is essential for the large value of the coincidence rate Eq. (45). In fact the detection probability by Bob, that we might expect to be simply \(\langle I_{B1} \rangle\), is enhanced by the correlation deriving from the fact that the same normal mode appears in both radiation fields, \(E_{A0}^+\) and \(E_{B1}^+\) see Eq. (25). And similarly for \(E_{A1}^+\) and \(E_{B0}^+\). More specifically with reference to Eq. (46) I point out that the ensemble average of \(I_{A0} - \langle I_{A0}\rangle\) being zero, only the fluctuations are involved in the enhancement of detection probability by Bob. Indeed in a fluctuation such that \(I_{A0} - \langle I_{A0}\rangle > 0\) \((< 0)\) Eq. (25) shows that also \(I_{B1} > 0\) \((< 0)\) whence the average of their product is always positive, that is \(\langle I_{B1}(I_{A0} - \langle I_{A0}\rangle)\rangle > 0\). And similarly for the fluctuations of (Bob) term \(I_{B0} - \langle I_{B0}\rangle\) that enhance the detection probability of Alice due to \(I_{A1}\).

This leads to an interesting interpretation of entanglement: it is a correlation between fluctuations involving the vacuum fields.
5.4 Conclusions

I conclude that the hypothesis that the quantum vacuum fields are real allows a concept of locality weaker than the fulfillment of the Bell inequalities. Indeed in our model the signal fields (accompanied by vacuum fields) travel causally from the source (that is the laser pumping beam and the nonlinear crystal followed by a beam-splitter and other devices) to the detectors. Thus I claim that the model of this paper is local. On the other hand the results for the single and coincidence detection probabilities within a time window, Eqs. (36) and (45), violate a Clauser-Horne (Bell) inequality Eq. (3). Therefore it is possible to explain the violation of Bell inequalities in entangled photon experiments without any conflict with relativistic causality.

References

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