Implementing invariant mass cuts and finite lifetime effects in top-antitop production at threshold

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The effects of the finite top quark width in the top pair production cross section close to the threshold are discussed in this talk. We introduce a $t\bar{t}$ cross section with a cut on the invariant masses of the top and antitop that can be calculated theoretically with effective field theory (EFT) methods. The matching procedure to implement the physical phase-space boundaries in the NRQCD framework ("phase-space matching") is briefly outlined.

The measurement of the $\sigma(e^+e^- \rightarrow t\bar{t})$ line shape at a linear collider (LC) operating at energies around the top-antitop threshold ($\sqrt{s} \sim 350$ GeV) would allow for very accurate determinations of the mass, the width and the couplings of the top quark \cite{1}. In view of the accuracy obtainable at the LC the theoretical uncertainties for the cross section predictions should be lowered to a level of a few percent \cite{2}.

Close to threshold the top quark pairs are produced with small velocities $v \ll 1$. In the standard QCD perturbative expansion singular terms $\sim \alpha_s/v, \alpha_s \log v$ arise from the ratios of the physical scales involved: the top mass $m_t$, the relative three-momentum $p \sim m_t v$ and the nonrelativistic energy $E \sim m_t v^2$. The summation of these terms can be achieved systematically by means of nonrelativistic QCD (NRQCD), the low-energy EFT that describes the quantum fluctuations of full QCD for the kinematic situation of top quarks close to threshold. Renormalization group improved calculations of the $t\bar{t}$ total cross section with NNLL order accuracy for the QCD effects have become available in this framework \cite{3}.

A study of electroweak corrections in $t\bar{t}$ production at threshold beyond leading order has not been systematically undertaken until recently \cite{4,5}. The large top width, being of the same order as the nonrelativistic energy, is essential in the description of the $t\bar{t}$ threshold dynamics. The dominant top decay mode in the Standard Model is $t \rightarrow bW^+$, which gives $\Gamma_t \sim 1.5$ GeV. It was shown \cite{6} that in the nonrelativistic limit the top-quark width can be consistently implemented by the replacement $E \rightarrow E + i\Gamma_t$ in the results for the total cross section for stable top quarks. This replacement rule accounts for the LL electroweak corrections to the total $t\bar{t}$ cross section, but a complete treatment at NLL and NNLL requires the use of an extended NRQCD formalism accounting for the top quark instability.

In the NRQCD framework the quark width effects are incorporated at leading order through the bilinear quark operators

$$\delta\mathcal{L} = \sum_p \bar{\psi}_p \frac{i}{2} \Gamma_t \psi_p + \sum_p \bar{\chi}_p \frac{i}{2} \Gamma_t \chi_p. \tag{1}$$

They represent a LL effect as compared to the typical energy of a Coulombic top quark, of the order of a few GeV. The terms in Eq. (1) describe an absorptive on-shell process that has been integrated out from the theory. In this way, the EFT does not contain any details and differential information of the decay, but can predict the total cross section, an inclusive observable. We can use the optical theorem to compute the total cross section from the imaginary part of the $e^+e^-$ forward scattering amplitude (see Fig. 1). The

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coefficients in Eq. (1) are determined from the matching procedure of the EFT to the full theory (QCD and the electroweak theory). The contributions from real $bW$ final states are thus included through matching conditions of the EFT. The LO bilinear quark terms in the NRQCD Lagrangian lead to the propagator

$$\frac{i}{p_0 - p^2/(2m_t) + i\Gamma_t/2}$$

(2)

for a top/antitop with momentum $(p_0, p) \sim (m_t v^2, m_t v)$. The inclusion of a finite width in the top propagator modifies the high-energy behaviour of the phase-space integrations in the EFT. Let us illustrate this with an example. Consider the double-resonant diagram of Fig. 1 in the full theory with center of mass momentum $q = (2m_t + E, 0)$. When the quark lines are close to their mass shell the 4-particle phase space integration reduces to

$$\sigma_{tt} \sim \int \frac{dp_0}{2\pi} \int \frac{d|p|^2}{2\pi} \frac{\Gamma_t^2}{|t_1 + ie|^2 |t_2 + ie|^2}$$

(3)

where the variables $t_{1,2} \equiv t_{1,2}(p_0, p) = 2m_t(E \pm p_0 - \frac{p^2}{2m_t})$ retain the leading order terms in the nonrelativistic expansion of the top and antitop off-shellness. The $\Gamma_t^2$ factor in the numerator arises from the phase-space integration of the $bW^+$ and $bW^-$ subsystems because in the nonrelativistic limit the invariant masses of the top/antitop quark can be set to $m_t^2$.

We notice that though the phase-space in the full theory is cut-off by the top mass, the nonrelativistic limit formally makes $m_t \to \infty$ and the phase-space boundaries are thus taken to infin-

Figure 1. Tree-level double-resonant diagrams in the full and effective theories. The dashed line means we extract the imaginary part or, equivalently, that we perform the phase space inte-

Figure 2. $(e^+e^-)(e^+e^-)$ scattering amplitude with a kinetic energy correction and a Coulomb potential. Its imaginary part gives a NNLO divergent contribution to $\sigma_{tt}$.

The corresponding NRQCD amplitude (diagram on the right in Fig. 1) reproduces Eq. (3) when $\Gamma_t$ is treated as an insertion. The NRQCD power counting, however, tells that the $\Gamma_t \sim E$ term needs to be resummed as part of the top propagator (see Eq. (2)), thus effectively replacing replacing $i\epsilon \to -i\Gamma_t$, to finally give $\sigma_{tt} \propto v/m_t^2$, with $v = \sqrt{(E + i\Gamma_t)/m_t}$.

Despite the integration limits, the tree-level integration in Eq. (3) is finite. However the integrand becomes more sensitive to large momentum regions once we include relativistic corrections $\sim p^2/m_t^2$. Using dimensional regularization these subleading contributions can lead to $1/\epsilon$ singularities if the high energy behaviour of the EFT phase-space integration is logarithmic. This occurs for some of the NNLO matrix elements containing a single insertion of the Coulomb potential and a $p^2/m_t^2$ correction. Figure 2 shows an example. Similar UV divergences had also been noted in Ref. [4], where the imaginary parts of the NRQCD matching conditions at NNLL coming from $Wb$ cuts in the full theory were determined and inserted into NRQCD matrix elements. These UV divergences would not exist for a stable top quark. These phase space divergences can be handled with the usual EFT renormalization techniques. In this specific example the UV divergences are absorbed by $(e^+e^-)(e^+e^-)$ operators that will thus have complex Wilson coefficients $\tilde{C}_V/A(\mu)$ [4]. The imaginary parts of these Wilson coefficients contribute to the $t\bar{t}$ cross section through the optical theorem, $\Delta \sigma_{tt} = \text{Im}[\tilde{C}_V + \tilde{C}_A]$. The renor-

\textsuperscript{2}Note that the arguments of the delta and step functions in the phase-space also have to be expanded out in the spirit of the threshold expansion.
malization procedure of course does not determine the matching conditions at the hard scale, $\tilde{C}_{V/A}(\mu = m_t)$, and a matching prescription is required in order to put back into the EFT the information about the physical phase-space boundaries. A possible approach is to define the matching condition as the difference between the full theory computation for the $bW^+\bar{b}W^-$ total cross section (considering $b$ and $W$ as stable) and the NRQCD result, similarly to the procedure used in Ref. [7] for the case of $W^+W^-$ threshold production. Obviously, the full theory computation with the 4-particle final state is a rather complicated task beyond tree-level, even if an asymptotic expansion technique is employed.

At this point, a different path can be followed, that will reduce the amount of full theory input required and allow us to make contact with the experimentally measured cross section. Among the experimental cuts that need to be applied to select $bW^+\bar{b}W^-$ events in $t\bar{t}$ production, cuts on the top and antitop invariant masses will be essential. Close to the threshold, it is more natural to define the cut on the the top invariant mass squared $p_t^2$ with respect to its on-shell value, i.e.

$$|p_t^2 - m_t^2| \leq \Lambda^2,$$

and similarly for the antitop. In the nonrelativistic limit, the top and antitop off-shellness are characterized by the variables $t_{1,2}(p_0, \pmb{p})$. The cutoff $\Lambda$ divides the phase-space into three different regions: the double-resonant region, with both $|t_{1,2}| < \Lambda^2$, the single-resonant regions, corresponding to $(|t_1| < \Lambda^2, |t_2| > \Lambda^2)$ or $(|t_2| < \Lambda^2, |t_1| > \Lambda^2)$, and the non-resonant region, where both $|t_{1,2}| > \Lambda^2$. The regions are shown in the phase-space diagram in Fig. 3 (gray, light-gray and white, respectively). The red lines in Fig. 3 correspond to the top and antitop on-shell conditions $t_{1,2} = 0$. For the case $\Gamma_t = 0$, the cut through the top and antitop EFT propagators yields a delta function instead of a Breit-Wigner distribution, and the phase-space shrinks to the point given by the intersection of the two on-shell lines ($|\pmb{p}| = \sqrt{mE}$). Restricting the phase-space boundaries to the double-resonant region defines a new threshold $t\bar{t}$ cross section, $\sigma_{t\bar{t}}(\Lambda)$, that corresponds to taking into account only those measured $bW^+$ and $bW^-$ events with invariant masses inside the window given by the condition (4).

$$\text{Im} \tilde{C}_{V/A} \equiv \sigma_{t\bar{t}}(\Lambda) - \sigma_{t\bar{t}}^{\text{EFT}},$$

becomes thus clear in this approach: it corrects for the wrong behaviour of the effective propagators outside the nonrelativistic domain ($\sigma_{t\bar{t}}^{\text{EFT}}$ is the standard NRQCD result, i.e. for $\Lambda \rightarrow \infty$). We can go a step further by realizing that inside the double-resonant region the full theory cutoff cross section $\sigma_{t\bar{t}}(\Lambda)$ can be nonrelativistically expanded since $\Lambda \ll m_t$. The computation of $\sigma_{t\bar{t}}(\Lambda)$ can thus be carried out using NRQCD Feynman rules and, in this way, the whole approach is interpreted as a matching procedure within the EFT itself. There are of course contributions with the same $bW^+\bar{b}W^-$ final state that are not described by the EFT computation (the interferences of amplitudes containing a single top/antitop line, for example). The latter are part of the irreducible background, whose numerical contribution to $\sigma_{t\bar{t}}(\Lambda)$ can be estimated using the tree-level predictions since there is no Coulomb-enhancement for these terms at higher orders in $\alpha_s$.

The coefficients $\tilde{C}_{V/A}$ provide the corrections to
the standard NRQCD result due to phase-space effects at a practical level. From a formal point of view, we have overlooked the fact that the corrections $\tilde{C}_{V/A}$ defined through Eq. (5) are energy dependent, and cannot be cast as the Wilson coefficients of local $(e^+e^-)(e^+e^-)$ operators. A local action is needed in order to use the optical theorem to determine the cross section from the imaginary part of the $e^+e^-$ forward scattering amplitude. The natural solution is to write the corrections $\tilde{C}_{V/A}$ as the coefficients of an operator product expansion in $(e^+e^-)(e^+e^-)$ operators with increasing powers of time derivatives $i\partial_0 \sim \bar{E}$. This is achieved practically by expanding the phase-space effects in powers of $E/\Lambda$. For example, the matching of the $O(\alpha_s^0)$ diagram shown schematically in Fig. 4 yields the phase-space correction:

$$\sum_n \left( \frac{E}{m_t} \right)^n \tilde{C}_{V/A}^{(n)} \sim \frac{i}{m_t^2} \frac{\bar{E}}{\Lambda} \left( 1 + \frac{m_tE}{3\Lambda^2} + \ldots \right)$$  \hspace{1cm} (6)

As expected, the result vanishes for $\Gamma_t \to 0$. We also note that the non-local contribution $\propto v$ in the diagrams of Fig. 4 cancels in the difference. The term proportional to $\Gamma_t/\Lambda$ gives the dominant phase-space correction to the $t\bar{t}$ cross section, whose relevance will depend on the chosen value for the cutoff. We mentioned above that our approach is formally valid for $\Lambda^2/m_t \ll m_t$. This condition may seem necessary due to the appearance of powers of $\Lambda^2/m_t^2$ when higher dimensional operators are considered. However, a numerical analysis of the coefficients of such contributions shows that the OPE series has a good convergence even for values of $\Lambda$ close to $m_t$, although smaller values ($\Lambda \lesssim 0.6m_t$) are required to lower their size below the desired accuracy of $1\%$. We shall use the counting $\Lambda \sim m_t$ to identify the order at which the phase-space matching corrections contribute. In this scheme, the correction from Eq. (6) scales as $\Gamma/\Lambda \sim v^2$, and thus represents a NLO correction.

The NRQCD calculation of the $t\bar{t}$ cross section sums up the insertions of Coulomb potentials to all orders in $\alpha_s$. Since the suggested OPE expansion of the phase-space corrections necessarily involves a matching of diagrams with a given order in $v$ and $\alpha_s$, we have to test also the convergence of the matching corrections arising from the addition of Coulomb potentials. The insertion of a Coulomb potential yields a factor $\alpha_s/v$ for $\Lambda \to \infty$: therefore we shall expect a suppression factor $\alpha_s m_t/\Lambda$ for every Coulomb exchange added to the matching relation of Fig. 4. An optimal range of values for the cutoff is to be determined in order to find the required suppression of both the $(\Lambda^2/m_t^2)^k$ power-counting breaking terms and the $(\alpha_s m_t/\Lambda)^k$ Coulomb phase-space corrections. Let us note that the phase-space matching of the $(e^+e^-)(e^+e^-)$ operators at 2-loops requires the removal of $\Lambda$-dependent non-local terms by 1-loop diagrams with phase-space subtractions in the production currents, in analogy to the process of removal of subdivergences in the usual renormalization procedure.

Results for all the phase-space corrections needed for a determination of the threshold $t\bar{t}$ cross section with NLLL accuracy and a complete numerical analysis shall be presented in a forthcoming publication.

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