STOCHASTIC ACCELERATION OF $^3$He AND $^4$He IN SOLAR FLARES BY PARALLEL-PROPAGATING PLASMA WAVES: GENERAL RESULTS

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ABSTRACT

We study the acceleration in solar flares of $^3$He and $^4$He from a thermal background by parallel-propagating plasma waves with a general broken power-law spectrum. The exact dispersion relation for a cold plasma is used to describe the relevant wave modes, and the Coulomb collision loss and escape processes are included. Under the quasi-linear approximation, the pitch-angle–averaged acceleration time of $\alpha$-particles is at least 1 order of magnitude longer than that of $^3$He ions at low energies and starts to approach that of $^4$He beyond a few tens of keV nucleon$^{-1}$. Because their loss and escape times are comparable, the acceleration of $^4$He is suppressed significantly at low energies, and the spectrum of the accelerated $\alpha$-particles is always softer than that of $^3$He. Quantitative results depend primarily on the wave generation and damping length scales, the electron plasma to gyrofrequency ratio, and the intensity of turbulence. The model gives a reasonable account of the observed low-energy $^3$He and $^4$He fluxes and spectra in the impulsive solar energetic particle events observed with the Advanced Composition Explorer. Other acceleration processes and/or stochastic acceleration by other wave modes seem to be required to explain the occasionally observed decrease of $^3$He to $^4$He ratio at energies beyond a few MeV nucleon$^{-1}$.

Subject headings: acceleration of particles — plasmas — Sun: abundances — Sun: flares — turbulence

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1. INTRODUCTION

Stochastic acceleration (SA) of particles by plasma waves or turbulence (PWT), a second-order Fermi acceleration process, plays an important role in understanding the energy release processes and the consequent plasma heating and particle acceleration in solar flares (e.g., Ramaty 1979; Möbius et al. 1980, 1982; Miller et al. 1997; Petrosian & Liu 2004, hereafter PL04). This theory was applied to the acceleration of nonthermal electrons (Miller & Ramaty 1987; Hamilton & Petrosian 1992; Park et al. 1997; Petrosian & Donaghy 1999), which are responsible for the microwave and hard X-ray emissions and for type III radio bursts during the impulsive phase of solar flares. In this respect, it has achieved some degree of success (see PL04). It has also been advocated for the production of nonthermal electrons and ions observed near the Earth in association with solar flares (Reames et al. 1985; Mason et al. 1989; Van Hollebeke et al. 1990; Bieber et al. 1994). The accelerated ions show charge-to-mass-ratio–dependent enhancements relative to the photospheric values, which are readily explained in the context of SA (Miller 2003; Hurford et al. 1975; Reames et al. 1994; Mason et al. 1986, 2002a, 2002b, 2004; Reames & Ng 2004). In a few gamma-ray flares, where gamma-ray line emissions due to ion nuclear interactions are clearly observed, there is also evidence of an anomalous abundance pattern of the accelerated ions (Share & Murphy 1998; Hua et al. 1989). The competing models of diffusive shock acceleration and/or direct acceleration by parallel (to magnetic fields) electric fields have not been subjected to quantitative tests. In addition, maintaining large electric fields appears to be problematical (see, however, Holman 1985), and there is no convincing evidence for the presence of low coronal shocks for the impulsive flares.

The most critical challenge to these models including the SA model arises from the extreme enhancement of $^3$He observed in some scatter-free impulsive events (Hsieh & Simpson 1970; Serlemitsos & Balasubrahmanyan 1975; Mason et al. 2000). All models proposed for this are based on either resonant plasma heating or resonant particle acceleration (Fisk 1978; Temerin & Roth 1992; Miller & Viñas 1993; Zhang 1999; Paesold et al. 2003). We recently carried out a quantitative study and showed that the interplay of the acceleration, energy loss, and escape processes in the SA of $^3$He and $^4$He by parallel-propagating plasma waves can account for the $^3$He enhancement, its varied range, and the spectral shape (Liu et al. 2004, hereafter LPM04). The primary feature of our new finding, which comes from our use of the exact dispersion relation, is that the acceleration of $^4$He from a low-energy thermal background is suppressed due to its lack of resonance with multiple waves, a result very similar to the suppression of proton acceleration relative to the electron acceleration (Schlickeiser 1989; PL04).

Earlier studies of SA of low-energy thermal background electrons and protons revealed an acceleration rate larger than the scattering rate (Pryadko & Petrosian 1997; PL04). We show here that, for coronal conditions, this is also true for the SA of $^3$He and $^4$He. Consequently, in impulsive events SA by PWT at low energies (determined by the acceleration rate) is more efficient than the acceleration by diffusive shocks, whose rate of acceleration is proportional to the scattering rate. In addition, recent observations of gradual events, which may involve acceleration by interplanetary shocks, suggest that the source population is preaccelerated by a distinct acceleration process operating in impulsive or gradual flares (Mason et al. 1999; Desai et al. 2001, 2003, 2004). SA may very well be the agent for this acceleration as well.

In this paper we explore more realistic models and quantify the dependence of $^3$He enhancement on basic model parameters. In the next section we discuss the basic features of resonant wave-particle interaction. After showing that the scattering rate...
is usually lower than the acceleration rate at low energies, we calculate the pitch-angle–averaged acceleration timescales of \(^3\)He and \(^4\)He under different plasma conditions and for different turbulence spectra, taking into account the uncertainties associated with the generation of turbulence, its cascade, and damping. Surprisingly, at low energies the acceleration time of \(^4\)He is always longer than that of \(^3\)He. In § 3 the model of SA is briefly summarized and applied to observations of six impulsive solar energetic particle events (SEPs) observed with the Advanced Composition Explorer (ACE), where the low-energy ion spectra have convex shapes, which are referred to as “rounded spectra” (Mason et al. 2002a, 2002b). For parameters typical of impulsive solar flares, the acceleration time of \(^3\)He is always shorter than its energy loss time. Almost all of the background \(^3\)He ions are therefore accelerated to the \(\sim 1\) MeV nucleon\(^{-1}\) energy range, where the escape process starts to dominate. The acceleration of \(^4\)He to high energies, on the other hand, are suppressed significantly since its acceleration time can exceed its loss or escape time at low energies. Reasonable fits to the observed \(^3\)He and \(^4\)He spectra are obtained by adjusting the temperature of the background plasma, the turbulence-generation length scale, the intensity of the turbulence, and a normalization factor. Because the required level of turbulence is low and only a small fraction of the background \(\alpha\)-particles are accelerated to high energies, we believe that this mechanism still works even when the nonlinear effects of the wave–particle interaction are taken into account. In § 3.2 we explore the model parameter space to see how observations can be used to constrain properties of the turbulence and the flaring plasma. We show that the acceleration of low-energy \(^4\)He can still be suppressed, even without wave-damping effects, because \(^4\)He interacts mostly with small-scale waves that have relatively low phase velocities. The damping effects introduced phenomenologically just amplify this suppression for a given temperature of the background plasma. The latter statement, however, likely results from the lack of a self-consistent treatment of the wave–particle coupling due to our use of the dispersion relation for a cold plasma. In real plasmas particles gain energy from the waves. Faster damping of the waves by the \(\alpha\)-particles then implies more efficient heating and/or acceleration of them. The generation length scale of waves in the proton cyclotron (PC) branch determines the acceleration rates at high energies and thus mostly affects the high-energy cutoffs. On the other hand, we show that in more strongly magnetized plasmas the accelerations of \(^4\)He and \(^3\)He become comparable, and \(^3\)He enhancement decreases. This is also true with the increase of the background plasma temperature and/or the intensity of the turbulence. In § 4 we summarize the main results and emphasize the importance of studying the properties of the PC branch in understanding the relative acceleration of \(^3\)He and \(^4\)He. The model limitation and future developments of the SA theory are also discussed.

2. RESONANT WAVE-PARTICLE INTERACTION

SA of particles by parallel-propagating waves has been explored in PL04. For the sake of completeness, we summarize here the main results relevant to the current study.

2.1. Dispersion Relation and Resonance Condition

To simplify the investigation, we assume that the turbulent plasma waves propagate in a “cold” fully ionized plasma.\(^4\)

\[^4\) As argued in PL04, the inclusion of thermal effects does not change the main conclusions of SA models. These effects in most cases will be overshadowed by the uncertainties in the model parameters describing the spectrum of the turbulence.

\[ \alpha = \omega_{pe}/\Omega_e = 3.2(n_e/10^{10} \text{ cm}^{-3})^{1/2}(B_0/100 \text{ G})^{-1}, \]

where \(n_e\) is the electron number density and \(B_0\) is the large-scale magnetic field. In the case of solar flare plasmas, modifications to the dispersion relation due to \(^4\)He and elements heavier than \(^4\)He are negligible (Steinacker et al. 1997). For waves propagating parallel to the large-scale magnetic fields, one therefore has

\[ \frac{k^2}{\omega^2} = 1 - \frac{\alpha^2}{\omega^2} \left( \frac{1}{\omega^2 - 1} + \frac{1-2\gamma}{\omega + 1} + \frac{\gamma}{\omega + 1/2} \right), \]

where the fraction of \(^4\)He number abundance \(Y_{He} = 0.08\) and \(\delta = m_e/m_p\) is the electron-to-proton mass ratio. The wave frequency and wavenumber are given by \(\omega\) and \(k\) in units of the proton gyrofrequency \(\Omega_p\) and \(\Omega_p/c\), respectively, where \(c\) is the speed of light. The left polarized waves are designated with a negative sign, reflecting the left-handed polarization.

Then the dispersion relation is determined by the ion abundance and the plasma parameter \(\alpha\), the ratio of the electron plasma frequency \(\omega_{pe}\) to its gyrofrequency \(\Omega_e\):

\[ \omega = k_{\mu}B_{0}/c \]
gyrofrequency. The straight solid and dashed lines in Figure 1 indicate this relation for $^3$He (thick line) and $^4$He (thin line) with $\mu = 1.0$ and two energies. The intersection points of these lines with the dotted lines for the dispersion relation with $\alpha = 0.5$ designate waves that satisfy the resonance condition. The Fokker-Planck coefficients for the particles are calculated by adding contributions from these waves (Dung & Petrosian 1994). The dashed line near the bottom gives the resonance condition for 0.5 keV protons with $\mu = 1$, which will be useful in understanding the damping effects discussed below.

2.2. Turbulence Spectrum

There has been limited work on a complete kinetic theory description of plasma turbulence detailing the wave generation, cascade, and eventually damping by background particles at small scales. Most of the investigations of plasma turbulence are limited to the MHD regime (Bieber et al. 1994; Dröge 2003; Hirose et al. 2004), where the wave frequency is much lower than the particle gyrofrequency, and the turbulence cascade is not isotropic for most of the wave modes (Cho & Lazarian 2003, 2004). A formulation of these processes for application to solar flares is still under development. Here we study the SA by assuming a power spectrum for the PWT, taking into account the knowledge obtained from recent observations of plasma turbulence in the interplanetary medium and from theoretical investigations. The main features of PWT can be formatted by two length scales, the turbulence-generation and the wave-damping length scales, and by the related spectral indexes.

The turbulence-generation length scale is usually related to the large-scale dynamical evolution of the system, such as the size of the current sheet between reconnecting magnetic fields. For high-frequency wave branches, such as the PC branch mentioned above, waves can also be generated via coupling with low-frequency MHD waves, like those in the HeC branch (Xie et al. 2004) or fast-mode waves (André 1985; Cho et al. 2003). Although the magnetic reconnection and the subsequent energy release and turbulence-generation processes seem to involve complex micro- and macroscopic physics, the turbulence-generation length size must be smaller than the typical dimension of the system. For flare conditions, such a scale generally implies low wavenumber or frequency waves, which resonate with relativistic particles that is not quite relevant to observations of SEPs with ACE. This is the case for the HeC branch so that the low-wavenumber cutoff of this branch simply determines the energy content in the turbulence. The PC branch, on the other hand, mostly interacts with nonrelativistic particles. As discussed in PL04 and LPM04, large-scale waves in this branch are very efficient accelerators of hundreds of MeV protons, $^3$He and $^4$He ions. Observations of SEPs in this energy range can be used to constrain the wave generation at these length scales. We therefore take the low-wavenumber cutoff $k_{\text{min}}$ of the PC branch as one of the primary model parameters.

The high-wavenumber cutoff of the turbulence spectrum is determined by the damping rate of these waves by the background plasma. Thermal damping involves complicated nonlinear effects (Swanson 1989, pp. 122, 291, and 305), which can change the wave-damping rate and modulate the evolution of the wave spectrum at high $k$ and the particle distribution in the tail of a thermal background plasma. These determine the injection process of SA. Thermal damping of parallel-propagating waves by the background ions has been studied in detail by Steinacker et al. (1997). One of the major findings of this work is that waves that resonate with the high-energy tail of a thermal distribution are subjected to strong Landau damping. Because the abundances of $^3$He and elements heavier than $^4$He are very low, their contributions to the thermal damping effects can be ignored. But for the short-scale waves in the HeC and PC branches, which are damped by the background $^3$He and protons, respectively, the decay time could be comparable to the proton gyroperiod. Because we use the dispersion relation in a cold plasma to describe the wave modes, these damping effects can only be included phenomenologically by introducing a steep turbulence spectrum in the relevant wavenumber range.

We assume that the turbulence is unpolarized and has a broken power-law spectrum for each of five wave branches:

$$E(k) = (q - 1)E_0 \left(\frac{k}{k_{\text{min}}}\right)^{-q}, \quad \text{for } k < k_{\text{min}},$$

$$E(k) = (q - 1)E_0 \left(\frac{k}{k_{\text{max}}}\right)^{-q}, \quad \text{for } k_{\text{min}} < k < k_{\text{max}},$$

$$E(k) = (q - 1)E_0 \left(\frac{k_{\text{min}}}{k_{\text{max}}}\right)^{(q - 1)\left(\frac{k_{\text{min}}}{k_{\text{max}}}\right)}, \quad \text{for } k > k_{\text{max}},$$

(4)

where $E_0$ indicates the intensity of the turbulent plasma waves and $k_{\text{min}}$ must be larger (perhaps much larger) than $k_L = 2\pi c/L\Omega_p$ for a system with size $L$. Note that $E_0$, $k_{\text{min}}$, and $k_{\text{max}}$ could be different for different wave branches. However, we assume that the spectral indexes and $E_0$ are the same for all branches so that the wave intensity depends only on the wavenumber $k$ in the inertial ranges, i.e., between $k_{\text{min}}$ and $k_{\text{max}}$. From the resonance conditions for $^3$He and protons with $E = 0.5$ keV nucleon$^{-1}$ and $\mu = 1.0$ in Figure 1, we see that these thermal background particles couple strongly with waves with high wavenumbers. We set the high-wavenumber cutoff $k_{\text{max}} = 2\pi b^{-1/2}$ for the PC branch and one-half of this value for the HeC branch. This characterizes the damping of waves under plasma conditions typical for solar flares (see Xie et al. 2004). Because the PC branch is the dominant branch here, in what follows the corresponding quantities refer to this branch unless specified otherwise. We choose $q = 2.0$ and $q_0 = 4.0$ as the fiducial parameters, which are consistent with solar neighborhood observations (Bieber et al. 1994; Dröge 2003).

For $q = 2.0$ the acceleration and scattering rates of relativistic particles are independent of energy (corresponding to the so-called hard sphere approximation), which gives rise to a power-law particle distribution that is cut off at an energy where the particle resonates with waves with $k \leq k_{\text{min}}$, or where a loss process becomes dominant (PL04). Clearly, the total turbulence energy density $E_{\text{tot}} \approx 2 \sum_j E_0 \left[\frac{q}{q - 1}\right] \sum_k \left(k_{\text{min}}^2\right)^{q - 1}$, where $\sigma$ indicates the wave branches and the factor of 2 arises from the two propagation directions of the waves.

2.3. Fokker-Planck Coefficients and Scattering and Acceleration Times

With the turbulence spectrum specified, one can proceed to calculate the Fokker-Planck coefficients (Dung & Petrosian 1994):

$$D_{ji} = \frac{1}{2} \frac{\mu_j}{\mu_i} \left(\mu_j^2 - 1\right) \frac{2\pi}{2\pi} \sum_{x_j} \chi(k) \mu_j \mu_i (1 - x_j)^2,$$

$$D_{ji} = \frac{1}{2} \frac{\mu_j}{\mu_i} \left(\mu_j^2 - 1\right) \frac{2\pi}{2\pi} \sum_{x_j} \chi(k) \mu_j \mu_i x_j (1 - x_j),$$

$$D_{ji} = \frac{1}{2} \frac{\mu_j}{\mu_i} \left(\mu_j^2 - 1\right) \frac{2\pi}{2\pi} \sum_{x_j} \chi(k) \mu_j \mu_i x_j^2,$$

(5)

where $\chi(k)$ is the resonance condition, $q_0$ is unimportant as long as it is much larger than 1. We set $q_0 = 2$ in our calculation.

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where

\[
\chi(k_j) = \frac{E(k_j)/E_0}{(\beta k_j - \beta_j k_j)}, \quad x_j = \mu \omega / \beta k_j,
\]

\( \frac{\beta}{\beta_j} \) are the particle momentum and gyrofrequency, respectively, and \( \beta_j = d\omega / dk \) is the group velocity of the wave. The sum over \( j \) is for the resonant points discussed above. Following previous studies (Dung & Petrosian 1994; Pryadko & Petrosian 1997; PL04), we describe the turbulence intensity with a characteristic timescale \( \tau_p \)

\[
\tau_p^{-1} = \pi \Omega_p \left( \frac{E_0}{B_0^2/8\pi} \right), \quad \text{with} \quad \Xi_0 = \frac{(q-1)C_{\text{tot}}}{2 \sum_\sigma (k_{\text{min}}^2)\delta^{(1-q)}}.
\]

Figure 2 gives the dimensionless acceleration rate \( \left( D_{pp}/p^2 \right) \tau_p \) (solid lines) and scattering rate \( D_{pp}/\tau_p \) (dashed lines), as functions of \( \mu \), for \( ^3\text{He} \) (thick lines) and \( ^4\text{He} \) (thin lines) at \( E = 1 \text{ MeV nucleon}^{-1} \) (left) and 10 keV nucleon\(^{-1} \) (right) in a plasma with \( \alpha = 0.5 \). The low-wavenumber cutoff of the PC branch \( k_{\text{min}} = 0.1 k_{\text{max}} = 0.02 \alpha^{-1/2} \). These values of the other model parameters introduced above are indicated in the figure. It is obvious that the scattering rate is usually lower than the corresponding acceleration rate. SA is therefore more efficient than acceleration by shocks at this energy. Because of this, to the extent that the acceleration rate depends on \( \mu \), the particle distribution will be anisotropic, unless there are other scattering agents. More importantly, the acceleration rates of \(^3\text{He} \) and \(^4\text{He} \) are comparable at 1 MeV nucleon\(^{-1} \), but the \(^4\text{He} \) acceleration rate is more than 10 times lower than that of \(^3\text{He} \) at lower energies, for example, at 10 keV nucleon\(^{-1} \) (see Fig. 2, right). This is because \( D_{pp} \) is proportional to the phase velocity square of the resonant waves, and at low energies waves resonating with \(^4\text{He} \) have relatively lower phase velocities. Thus, compared with \(^3\text{He} \), the acceleration of \(^4\text{He} \) from a low energy can be suppressed significantly.

To illustrate the effects of introducing the high- and low-wavenumber cutoffs, the left panel also shows the rates for a pure power-law turbulence spectrum \( k_{\text{max}} \rightarrow \infty \) and \( k_{\text{min}} \rightarrow k_j \), i.e., \( q = q_0 = 2 \) for the model. The small differences at small \( \mu \) from the model with a general broken power-law spectrum are due to the damping effects introduced in eq. (4). The large differences at large \( \mu \) are due to the low-wavenumber cutoff \( k_{\text{max}} \). One sees that the introduction of \( k_{\text{min}} \ll k_j \) can reduce the interaction rates of particles at this energy significantly and the acceleration rate is usually higher than the corresponding scattering rate. Right: Same as the left panel, but for \( E = 10 \text{ keV nucleon}^{-1} \) particles. The results for a pure power-law turbulence spectrum are not shown. Here the acceleration rate of \(^3\text{He} \) is more than 1 order of magnitude higher than that for \(^4\text{He} \). The open circles indicate breaks caused by \( k_{\text{max}} \). [See the electronic edition of the Journal for a color version of this figure.]

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Because the acceleration rate is usually higher than the scattering rate at low energies, one can define the pitch-angle-averaged acceleration and scattering times (Pryadko & Petrosian 1997):

\[
\tau_{ac} = \frac{2\mu^2}{\int_{-1}^{1} d\mu \, D_{pp}(\mu)}.
\]

\[
\tau_{sc} = \int_{-1}^{1} d\mu \frac{(1 - \mu^2)^2}{D_{pp}(\mu)}.
\]

Note that here we define \(\tau_{ac}\) differently from that in LPM04, where we assumed isotropy and that \(D_\mu \gg D_{pp}\) \(\rho^2\), which is always the case at high energies. But as shown above this is not true at low energies, so the above relation is more accurate. This makes only a quantitative (not a qualitative) difference. Figure 3 shows the energy dependence of these times (acceleration time: solid lines, scattering time: dashed lines) for \(^4\)He (thick lines) and \(^3\)He (thin lines) for the model depicted in Figure 2. The scattered time is longer than the corresponding acceleration time below a few MeV nucleon\(^{-1}\). First, this implies that at low energies SA is more efficient than diffusive shock acceleration, whose acceleration time is comparable to the scattering time. We also expect some anisotropy in accelerated particle distributions with a pitch-angle distribution similar to the pitch-angle dependence of the acceleration rates. However, as shown in Figure 2 the expected anisotropy will not be large, and other scattering processes, such as Coulomb collisions, may further isotropize the accelerated particle distribution. Beyond a few tens of MeV nucleon\(^{-1}\), scattering becomes more efficient, and isotropy of the particle distribution is established by wave scatterings.

Below a few tens of keV nucleon\(^{-1}\) the acceleration time of \(^4\)He is more than 1 order of magnitude longer than that of \(^3\)He. The sharp decrease of the \(^4\)He acceleration time near 40 keV nucleon\(^{-1}\) is due to the interaction of \(^4\)He with large-scale waves in the PC branch, which also dominate the acceleration of \(^3\)He (see Fig. 1). The acceleration times of the two particles become comparable beyond 100 keV nucleon\(^{-1}\) and are identical at relativistic energies.

2.4. Dependence of Acceleration Times on Model Parameters

The physical conditions for different solar flares are expected to be different, and in § 2.2 we showed that at least six parameters, namely, \(k_{min}, k_{max}, q_h, q_b, q_f, \) and \(E_0\), are required to describe the spectrum of the turbulence. Because for \(q_f \geq 1\) waves below \(k_{min}\) do not contribute to the particle acceleration significantly (so that \(q_f\) does not affect the spectrum of the accelerated particles) and \(E_0\) determines the characteristic timescale \(\tau_p\), to have a general view of the model parameter space one only needs to consider how the acceleration times in units of \(\tau_p\) depend on \(k_{min}, k_{max}, q_h, q_b,\) and \(\alpha\).

As discussed above, we parameterize the wave spectrum by several parameters, with two of them characterizing the phenomenological damping; namely, the high-wavenumber cutoff \(k_{max}\) and the turbulence spectral index \(q_b\). Figure 4 shows how the acceleration times of \(^3\)He and \(^4\)He change with these parameters. The left panel is for different \(k_{max}\), and the right panel has different \(q_b\). Here the solid lines represent our fiducial model, which is analogous to that in Figure 3 except that the low-wavenumber cutoff \(k_{min}\) is now 2 times lower: \(k_{min} = 0.1\alpha^{-1/2}\).

One obvious feature is that the damping does not affect the acceleration time of \(^3\)He. As mentioned above, this is because \(^3\)He mostly interacts with large-scale waves in the PC branch, which cannot be damped by the low-energy protons and \(\alpha\)-particles in the background. The lower \(k_{max}\) and the larger \(q_b\), the stronger the damping, and, as expected, the longer the acceleration time of \(^4\)He. The acceleration time of higher energy particles is not affected by the damping because these particles interact with low-wavenumber waves. We also note that the acceleration time of \(^4\)He at low energies is more sensitive to \(k_{max}\) than to \(q_b\). Even in the case \(q_b = 2\), which corresponds to the absence of damping effects, the \(^4\)He acceleration time is still more than 10 times longer than the \(^3\)He acceleration time at low energies. So the \(^4\)He acceleration from a low energy can be suppressed even without the phenomenological damping effects. This suppression is purely because of the modification to the dispersion by the presence in the plasma of a significant population of \(\alpha\)-particles and depends only on the plasma parameter \(\alpha\) and the \(^4\)He abundance \(Y_{He}\).

As mentioned earlier, the generation length of waves in the PC branch, corresponding to \(k_{min}\), is another important parameter. The left panel of Figure 5 shows how \(k_{min}\) affects the acceleration times. (The low-wavenumber cutoffs of other branches are assumed to be far lower so that they do not affect the acceleration of nonrelativistic particles.) Obviously, these large-scale waves mostly interact with particles in the MeV nucleon\(^{-1}\) energy range so that the acceleration times at low energies remain unchanged. However, the acceleration of MeV nucleon\(^{-1}\) particles is very sensitive to this parameter for both \(^3\)He and \(^4\)He. With the decrease of \(k_{min}\), particle acceleration at high energies becomes more efficient, but the relative rate of the two ions changes very little. In the following sections we show that the high-energy cutoffs of the accelerated particle spectra are very sensitive to this parameter, and observations with ACE can be used to constrain it directly.

The right panel of Figure 5 shows the dependence of the acceleration time on the plasma parameter \(\alpha\). The most prominent feature of these curves is the decrease of \(\tau_{ac}\) for both ions at low energies with the decrease of \(\alpha\). This does not affect the \(^3\)He acceleration but has important effects on the \(^4\)He acceleration due to its long \(\tau_{ac}\) (see § 3.2). Because \(k_{min}\) is fixed, the acceleration
times in the MeV nucleon$^{-1}$ energy range do not change significantly. Therefore, the plasma parameter mostly affects the acceleration rates at low energies and the relative acceleration of the ions.

Figure 6 shows how the acceleration times change with the turbulence spectral indexes $q$. Here we keep $q_{h} - q = 2$ and other model parameters unchanged in these calculations. As expected, the acceleration of high-energy particles becomes more efficient when the turbulence spectrum becomes steeper, since high-energy particles mostly resonate with large-scale waves (eq. [3]). (Note that the normalization time $\tau_p$ changes with $q$.) Surprisingly, however, for all the model parameters considered here, the acceleration time of $^4$He is always longer than the $^3$He acceleration time. This is one of the key features of SA by...
parallel-propagating waves and has profound implications on the enhancement of $^3$He and the relative acceleration of the two species.

3. MODEL DESCRIPTION AND APPLICATION TO SEPs

In the SA theory ions gain energy via their resonant interactions with the PWT and lose energy via Coulomb collisions with the background thermal particles. The ions also diffuse spatially and can escape from the acceleration site. These processes can be described by the Fokker-Planck equation, whose coefficients have been calculated in §2 for interactions with parallel-propagating waves, under the quasi-linear approximation and where the perpendicular diffusion across magnetic field lines is ignored. When the wave-ion scattering rate is comparable to the ion acceleration rate, in principle one needs to solve the four-dimensional equation for the particle phase space distribution with the time $t$, momentum $p$, pitch-angle cosine $\mu$, and the location along the large-scale magnetic field line $s$ as variables. But the problem can be simplified considerably when one of the rates is much higher than the other (Pryadko & Petrosian 1997). This is the case in the low- and high-energy limits considered here, where the acceleration rate and scattering rate respectively dominate. Moreover, if the pitch-angle dependence of the diffusion coefficients is weak (e.g., peaked in a narrow range of $\mu$, see Fig. 2), then the pitch-angle distribution of the particles will be relatively smooth and the anisotropy will be small so that one can use pitch-angle–averaged quantities. Furthermore, because we are mostly interested in the spatially integrated spectra, we can also integrate the particle distribution spatially (along the field lines). Then the acceleration of particles from a low-energy thermal background to relativistic energies can be treated fairly accurately by the well-known equation

$$\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial \epsilon^2} (D_{\epsilon} N) + \frac{\partial}{\partial \epsilon} \left[ (\hat{\epsilon} - A(\epsilon)) N \right] - \frac{N}{T_{\text{esc}}} + \dot{Q},$$

(10)

where $N(\epsilon)$ is the pitch-angle–averaged and spatially integrated distribution of accelerated particles with kinetic energy $\epsilon$, $D_{\epsilon} = \epsilon^2 \tau_{\text{diff}}^{-1}$ is the pitch angle and spatially averaged diffusion rate, and $A(\epsilon) = dD_{\epsilon}/d\epsilon + D_{\epsilon}(2 - \gamma^{-2})/\epsilon(1 + \gamma^{-1})$ is the direct acceleration rate. The loss rate $\dot{\epsilon}_L$ due to Coulomb collisions is summarized in PL04. The escape time $T_{\text{esc}} = L^2/2\nu + L^2/\epsilon^2 \tau_{\text{esc}}$ describes the spatial diffusion of the particles along the large-scale magnetic field lines. $Q$ is a spatially integrated source term, which we identify as the thermal background plasma. For quantitative modeling, one needs to specify the length of the acceleration region $L$ and the background plasma density $n_0$, magnetic field $B_0$, and temperature $T$, which we assume is the same for all background charged particles.

Each term in equation (10) can be characterized by an energy-dependent timescale. The timescale of source particle injection $Q$ is determined by the large-scale dynamical evolution of the system, which is comparable to the modulation timescale of the solar flare. Because the other interaction times are usually much shorter than the dynamical time of the system, one only needs to consider the steady state solution of the equation with $Q = \text{const.}$ Then the escaping flux $f = N/T_{\text{esc}}$ can be compared with the observed fluxes of SEPs with a normalization factor, taking into account the duration of the flare and the cross-sectional area of the flaring magnetic fields. (Note that for the impulsive SEPs studied here, the transport effects of the ions from the flaring site in the solar corona to $ACE$ in the Earth’s neighborhood are negligible because their scattering mean free path is long [Mason et al. 1989, 2002b].) To appreciate spectral features of the corresponding solutions, besides the escape time mentioned above, we also define a loss time $\tau_{\text{loss}} = \epsilon/\dot{\epsilon}_L$ and a direct acceleration time $\tau_a = \epsilon/\dot{\epsilon}_a \sim \tau_{\text{esc}}$ for the second term on the right-hand side of equation (10). The ion loss time as a function of the ion kinetic energy per nucleon is proportional to the ion atomic number and is inversely proportional to the square of the ion charge (PL04).

3.1. Inverse Spectrum

The left panel of Figure 7 shows these timescales for a model with $L = 2 \times 10^9$ cm, $B_0 = 200$ G, $n_0 = 9 \times 10^9$ cm$^{-3}$, the temperature of the background plasma $k_B T = 0.26$ keV, and the characteristic timescale $\tau_p = 5.5 \times 10^3$ s$^{-1}$. All other model parameters are the same as in Figure 3. The corresponding plasma parameter $x = 0.48$, and $8 \pi e^2 n_0 B_0^2 = 49 \times 10^{-13}$. The total turbulence energy density can be very low if the generation length scale of waves in the HeC branch is much shorter than $L$. All these parameters are typical for solar flares, and a weak level of turbulence also justifies the quasi-linear approximation adopted here and is consistent with the general view that the flares are powered by energies stored in the preflare magnetic fields presumably via magnetic reconnection. The right panel shows the model fit to the observed $^3$He and $^4$He spectra of an impulsive SEP on 1999 September 30 observed with $ACE$. The injected (background) plasma is assumed to have a solar abundance so that the number of $^3$He ions integrated over the whole energy range is 2000 times smaller than that of $^4$He. The model clearly produces the observed extreme enhancement of $^3$He and gives a reasonable fit to the spectra of both ions. However, given the uncertainties in the wave spectrum and the dimension of the model parameter space, we emphasize that the model fit is not unique. For example, an equally good fit to this event can be obtained with a slightly higher temperature and lower $k_{\text{max}}$. A self-consistent treatment of the wave-particle interaction will remove most of these model parameters and give better insights on the particle acceleration processes (see discussion in §3.2).
The steady state spectra of the ions are determined by the interplay of their acceleration, loss, and escape processes. Because the acceleration time of \(^3\)He is always much shorter than its Coulomb loss time, the acceleration and escape terms dominate its acceleration. Injected at a very low energy, almost all thermal \((k_B T = 0.26 \text{ keV})\) \(^3\)He particles are accelerated to the hundreds of keV nucleon\(^{-1}\) energy range, where the escape process starts to dominate over the acceleration process. The sharp spectral cutoff near 1 MeV nucleon\(^{-1}\) results from the quick divergence of the acceleration and the escape times above this energy. The SA acceleration is effectively quenched at higher energies. The acceleration of \(^4\)He, on the other hand, is quite different. The acceleration time of \(^4\)He is comparable with its loss time below a few tens of keV nucleon\(^{-1}\). Near the energy of the injected thermal particles, \(A(\epsilon) \leq \epsilon_L\) and the direct acceleration is difficult; the energy diffusion term on the right-hand side of equation (10) dominates the acceleration processes so that more than half of the injected particles are accelerated to energies \(>1\) keV nucleon\(^{-1}\). However, the escape time becomes much shorter than the other times between 10 and 100 keV nucleon\(^{-1}\), resulting in a very steep spectrum beyond \(\sim 10\) keV nucleon\(^{-1}\). Consequently, most of the accelerated \(\alpha\)-particles stay below \(\sim 10\) keV nucleon\(^{-1}\). This suppresses the acceleration of \(^4\)He ions to higher energies and gives rise to the observed enhancement of \(^3\)He ions in the MeV nucleon\(^{-1}\) energy range. At a few hundreds of keV nucleon\(^{-1}\) the \(^4\)He acceleration time becomes comparable with its escape time due to very efficient acceleration by large-scale waves in the PC branch. This gives rise to a relatively hard accelerated \(^4\)He spectrum. However, the \(^4\)He acceleration time is always longer than that of \(^3\)He, its spectrum is always softer than the spectrum of accelerated \(^3\)He ions. The fact that only a very small fraction (<0.1%) of the background \(^4\)He ions are accelerated to high energies (>50 keV nucleon\(^{-1}\)), where they can resonate with waves required for the acceleration of \(^3\)He ions, shows that the damping of waves in the PC branch by the accelerated \(^4\)He is unimportant, and the acceleration of the abundant background \(^4\)He ions does not have significant effects on the \(^3\)He acceleration.

The achievements of the model in explaining the 1999 September 30 event tempt us to apply it to other impulsive SEPs. We first consider the events with the so-called rounded spectra, as suggested by the model calculations above (Fig. 7). Another five such impulsive events are found during the ACE observation period from 1998 to 2000. These events show quite different \(^3\)He enhancement, and the location of the high-energy cutoff of the spectra varies considerably (Fig. 8). The predicted spectra of the SA theory may depend on the model parameters.

In §2.4 we have shown that the relative acceleration of \(^3\)He and \(^4\)He is controlled by the phenomenological damping of waves at small scales, since these damping effects only modify the acceleration rate of \(^4\)He at low energies. In the application of the SA model to the event studied above, we find that the acceleration of \(^3\)He is quite insensitive to the temperature of the injected plasma because its acceleration time is always much shorter than its loss time. However, the acceleration of \(^4\)He can be suppressed dramatically as \(T\) decreases because of the difficulty in the direct acceleration at low energies. The three model parameters, \(T, k_{\text{max}}\), and \(q_h\), thus have the equivalent function of controlling the acceleration of low-energy \(^4\)He and consequently the enhancement of \(^3\)He. These three parameters are all related to the thermal damping processes of plasma waves. For a given energy input rate into the turbulence at large scales, the value of \(k_{\text{max}}\) and \(q_h\) should depend only on \(T\). In what follows we assume that \(k_{\text{max}}\) and \(q_h\) weakly depend on the turbulence energy.
density and $T$ and fix them to the values used for the event shown in Figure 7. The corresponding effective temperature of the injected plasma $T_{\text{eff}}$ is used to adjust the enhancement of $^3\text{He}$ (or more accurately, the suppression of accelerated $^4\text{He}$).

The analyses of the wave-particle interactions in § 3 also suggest that the high-energy cutoff of the accelerated ion spectra is likely related to the generation length of waves in the PC branch. We therefore leave $k_{\text{min}}$ as another primary model parameter to account for the observed variations of the ion high-energy spectral cutoffs. For different solar flares, $\tau_p$ and other model parameters will also change. We adjust $\tau_p$ and the normalization factor for the total amount of the injected particles $N_0 \propto Q$ to finalize the spectral fitting, but in order to reduce the amount of calculation we keep all other model parameters the same as those in the above model. Figure 8 shows the model fit to the spectra of these five events. The corresponding model parameters are listed in Table 1. The models in general give reasonable spectral fits to the five events and reproduce their varied degree of $^3\text{He}$ enhancement. There are, however, obvious discrepancies in the low-energy $^3\text{He}$ spectrum of the 1999 August 7 event and the high-energy $^4\text{He}$ spectra of the 2000 events.

For the six events studied here, the effective temperature of the injected plasma varies within a factor of 4. Because the $^3\text{He}$ enhancement decreases with increasing $T_{\text{eff}}$ (see discussions above and more quantitative studies in § 3.2), and the damping effects are enhanced with temperature, we expect more significant variations in the real temperature of the background plasma for these events except that there is a positive correlation between the temperature $T$ and the turbulence energy density (indicated by the value of $k_{\text{rms}}^2/c_0^2$, which is anticorrelated with the $^3\text{He}$ enhancement, because $^3\text{He}$ can be accelerated to high energies even with large $\tau_p$ while $^4\text{He}$ cannot). The “generation” length scale or $k_{\text{min}}$ of the waves in the PC branch changes within a factor of 2.2, which seems to be a reasonable range. The characteristic timescale $\tau_p$ has much less variation (about 30%), which may be due to the similarities of the rounded spectra of these events and the comparable amplitude of the events as indicated by the relative normalization factor in the last column, which varies within a factor of 3.

### 3.2. Exploring of Model Parameter Space

In the previous section we showed that the observed degree of enhancement and the primary features of the rounded spectra

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**TABLE 1**

| Event        | $k_{\text{min}}/k_{\text{max}}$ | $\tau_p^{-1}/10^{-3}$ (s$^{-1}$) | $k_{\text{rms}} T_{\text{eff}}$ (keV) | $N_0^*$ |
|--------------|---------------------------------|----------------------------------|-------------------------------------|---------|
| 1998 Aug 18  | 0.16                            | 6.0                              | 0.12                                | 0.79    |
| 1999 Mar 21  | 0.075                           | 6.0                              | 0.14                                | 1.6     |
| 1999 Aug 7   | 0.075                           | 6.5                              | 0.12                                | 0.63    |
| 1999 Sep 30  | 0.10                            | 5.5                              | 0.26                                | 1       |
| 2000 Jan 6   | 0.14                            | 5.5                              | 0.10                                | 1.6     |
| 2000 Jan 17  | 0.14                            | 7.5                              | 0.07                                | 0.79    |

*The normalization is with respect to the event on 1999 September 30.*

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Note.—The other model parameters are the same as those for the 1999 September 30 event; i.e., $L = 2 \times 10^7$ cm, $B_0 = 200$ G, $n_e = 9 \times 10^9$ cm$^{-3}$ (therefore $\alpha = 0.48$), $k_{\text{rms}} = 2 \pi \delta^2$ for the PC branch, and one-half of it for the HeC branch, $q_1 = 2, q_2 = 2,$ and $q_3 = 4$. |
of $^3$He and $^4$He can be reproduced in our model with reasonable values of the parameters. Here we describe the dependence of these observed characteristics on various parameters.

3.2.1. Turbulence Spectrum and Wave Damping

One of the most uncertain parts of the model is related to the characteristics of turbulence. The two model parameters $k_{max}$ and $q_h$ characterize the wave-damping process. The background temperature, which plays important roles in this damping process, also has important influences. As mentioned above, thermal damping that involves complicated nonlinear processes determines the connections among these parameters (Petrosian et al. 2005). Investigation of these aspects is beyond the scope of this paper. Here we consider the influence of these parameters separately. In Figure 9 we show the effects of the above-mentioned parameters, while keeping other parameters constant and equal to those in Figure 7. The enhancement of $^3$He (or equivalently the suppression of $^4$He acceleration) decreases rapidly with increasing $T$ at low values of $T$, primarily because the number of $^4$He in the energy range where $\tau_{loss} < \tau_{e}$ decreases exponentially with $T$. However, even with a high temperature, the acceleration of $^4$He is still suppressed because its escape process dominates in the tens of keV energy range, and the $^3$He enhancement eventually saturates. The saturation level depends on the other parameters. For higher values of $k_{max}$ and lower $q_h$ (indicating slower damping) the enhancement of $^4$He is smaller. But for the range of values explored here, there is always a significant enhancement. Even in the case without the phenomenological damping effects, i.e., for $q_h = 2$, strong $^3$He enhancement can still be produced, which underscores the importance of using the exact dispersion relation in modeling the wave-particle interactions (LPM04). The phenomenological damping affects primarily the $^3$He to $^4$He ratio but introduces minor changes in the spectrum of accelerated ions (especially $^3$He) (Fig. 9, middle and right panels).

The right panel of Figure 9 shows how the spectrum of $^4$He is affected by the assumed damping with the temperature of the
background plasma fixed at 0.26 keV. Clearly the $^4\text{He}$ acceleration becomes less efficient with stronger damping ($q = 6$). This is because we have not done a self-consistent analysis of the damping effects, which would show that the energy of the damped waves heats up the plasma (Petrosian et al. 2005). This will increase the acceleration of $^4\text{He}$, as shown in the left panel.

The dot-dashed horizontal line in the left panel of Figure 9 indicates the observed range of $^3\text{He}$ enhancement. The spectral indexes vary from $-1$ to $-4$. The lack of observations with even higher $^3\text{He}$ to $^4\text{He}$ ratio against models with small $k_{\text{max}}$ (or very strong damping) and low temperatures of the background plasmas. More detailed theoretical investigations of the wave-damping should be able to resolve this issue and give better constraints on the SA model.

3.2.2. Turbulence Strength and Scale

The generation length scale of waves in the PC branch, characterized by $k_{\text{min}}$, and the turbulence intensity, proportional to $\tau_p^{-1}$, are two other important parameters characterizing the PWT. Figure 10 shows the dependence of the $^3\text{He}$ enhancement and the accelerated spectra of $^3\text{He}$ and $^4\text{He}$ on these parameters. In general, again almost all of the injected $^3\text{He}$ ions are accelerated to high energies, but the acceleration of low-energy $^4\text{He}$ ions is very sensitive to $\tau_p^{-1}$, especially at low $\tau_p^{-1}$. Therefore, the $^3\text{He}$ to $^4\text{He}$ ratio at high energies decreases rapidly with the increase of the turbulence intensity. With very strong turbulence ($\tau_p^{-1} \geq 0.1$), almost all of the injected $\alpha$-particles could be accelerated to high energies, and there is essentially no $^4\text{He}$ enhancement (see Fig. 10, left). The high-energy spectra of the accelerated particles are also very sensitive to $\tau_p^{-1}$ and $k_{\text{min}}$. With the increase of $\tau_p^{-1}$, the spectra become harder (Fig. 10, middle). But for low values of $k_{\text{min}}$, the cutoff energies of the accelerated ions will increase with minimal effects on the shapes of the spectra at lower energies (Fig. 10, right).

3.2.3. Background Plasma Parameters

Besides the temperature, other characteristics of the background plasma also affect the particle acceleration, with the main parameter being the plasma parameter $\alpha$. To change $\alpha$ we keep the density, which affects the loss and escape processes at low energies, constant and change the large-scale magnetic field $B_0$. Figure 11 depicts the influence of $\alpha$. Following the right panel of Figure 5, we keep $k_{\text{max}}/\alpha$ and all other model parameters unchanged. As expected, $\alpha$ predominantly affects the ion acceleration in the low-energy range. The $^3\text{He}$ enhancement decreases with the decrease of $\alpha$, corresponding to more strongly magnetized plasmas. The $\tau_p$ dependence of the $^3\text{He}$ to $^4\text{He}$ ratio at 1 MeV nucleon$^{-1}$ is quite different for different values of $\alpha$, especially when $\tau_p^{-1}$ is small (Fig. 11, left). The high-energy spectral indexes, however, are only weakly dependent on $\alpha$ (Fig. 11, middle). The right panel shows explicitly that the acceleration of low-energy ions, especially $^4\text{He}$, becomes more efficient in more strongly magnetized plasmas.

4. SUMMARY AND CONCLUSION

Recent observations of SEPs and radiations produced by solar flares indicate that stochastic acceleration may play a major role in the acceleration of all particles. It has been applied extensively to impulsive SEPs and may be involved in the preacceleration of particles in gradual flares, where the particles may be further accelerated to even higher energies by interplanetary shocks (Desai et al. 2001, 2003, 2004). We have described a SA model for the acceleration of $^3\text{He}$ and $^4\text{He}$ in SEPs by PWT assuming some characteristics for this turbulence. We have shown that for reasonable values of parameters describing the PWT and the background plasma, the model gives acceptable fit to the spectra and explains the extreme enhancement of $^3\text{He}$. Model fits are obtained for a few events.

In general, the outcome of the acceleration of each species and their relative abundances and spectra are determined by the interplay of the acceleration, Coulomb collision loss, and escape processes. Relative to the loss and escape rates, the $^4\text{He}$ acceleration rate is much higher than that of $^4\text{He}$. Almost all of the $^3\text{He}$ particles are accelerated, while only a small fraction of $^4\text{He}$ ions attain high energies. The primary reason for this is that the presence of a significant abundance (relative to protons) of $\alpha$-particles in the background plasma allows excitation of PC branch waves. Low-energy $^4\text{He}$ ions interact mainly with waves in the HeC branch, which have larger wavenumbers and are subjected to damping by the thermal $\alpha$-particles, while $^3\text{He}$ ions interact not only with waves from the HeC but also with waves from the PC branch. The $^4\text{He}$ acceleration is dominated by their interactions with the PC branch waves because they have smaller wavenumbers (and therefore contain more energy) and
cannot be damped by low-energy $^4$He ions and protons. The damping of these waves by accelerated $^4$He ions is unimportant because they interact with higher energy (>100 keV) $^4$He ions, which are relatively few in number.

In general, nonrelativistic $^3$He and $^4$He ions resonate mostly with waves with frequencies close to the $\alpha$-particle gyrofrequency. To study the SA of these ions, the exact dispersion relation for the plasma waves must be used, resulting in more efficient acceleration than scattering that could lead to anisotropic particle distributions. The presence of cosmic abundance of $\alpha$-particles in the background plasma then plays a key role in determining the relative acceleration of the two ions. Because of their modification to the dispersion relation, low-energy $^4$He ions interact mostly with low-phase velocity waves in the HeC branch, giving rise to a longer acceleration time. In addition, the stronger the damping of the waves, (i.e., the lower the value of the cutoff $\kappa_{\text{max}}$ and/or the larger the cutoff index $q_{\text{c}}$) which we introduced phenomenologically to mimic the thermal damping effects on the turbulence spectrum, the stronger the suppression of the acceleration of $^4$He and the higher the enhancement of $^3$He. On the other hand, a higher background temperature (which could cause a stronger damping) increases the efficiency of $^4$He acceleration up to tens of keV.

1. The turbulence intensity determines the hardness of the accelerated particle spectra. Harder spectra and smaller $^3$He enhancements arise from stronger turbulence.

2. In a strongly magnetized plasma (lower value of $\alpha$), the condition becomes more favorable for acceleration of $^4$He, and the $^3$He enhancement decreases. The $\alpha$ parameter also affects the ion spectra at low energies.

3. The effect of the lower wavenumber $k_{\text{min}}$ (large-scale) cutoff in the spectrum of turbulence is felt at high energies by both ions; the cutoff energy increases for lower values of $k_{\text{min}}$.

We note that although only a small fraction of $\alpha$-particles are accelerated to the MeV energy range, the turbulence energy going into $^4$He is at least 1 order of magnitude higher than that energizing $^3$He (see Figs. 7 and 8). The essential idea behind the model is the depletion of $^3$He from the flaring plasma. The observed $^3$He fluence therefore gives a measure of the amount of plasma involved in the flare. The $^3$He fluence, however, depends primarily on the amount of energy deposited into the flare. Regardless of quantitative details of the model, the $^4$He fluence increases when there is more energy deposited, which suggests more efficient acceleration and harder accelerated particle spectra. We would expect an anticorrelation between the $^3$He to $^4$He ratio and the spectral hardness of the accelerated $^4$He. This prediction may be readily tested in the near future.

However, not all features of the observed spectra can be described easily by the model. In the MeV nucleon$^{-1}$ energy range both $^2$He and $^4$He ions interact mostly with the same waves in the PC branch; the acceleration time of $^2$He is always longer than that of $^4$He. As a result, the model predicts a $^3$He spectrum that is always harder than that of $^4$He. This is in conflict with the observed decrease of $^2$He to $^4$He ratio with energy above a few MeV nucleon$^{-1}$ in some flares (Mason et al. 2002b; Reames et al. 1997; Möbius et al. 1982, 1980). This may suggest inadequacy of the simple model used here; for example, it is possible that at high energies obliquely propagating waves become more important and the acceleration is dominated by the transit time damping process (we note that at $\sim$1 MeV nucleon$^{-1}$ the ion velocity is higher than the Alfvén velocity so that the resonance condition for the transit time damping can be satisfied). Or this may require a special injection process or a second-phase shock acceleration (Van Hollebeke et al. 1990; Serlemitsos & Balasubramanyan 1975; Reames et al. 1997).

In the model described here the PC branch plays a dominant role in the acceleration of $^3$He and $^4$He ions. Compared to the damping of waves by the abundant thermal background ions, the damping by superthermal particles is less important, and our current approach gives a reasonable account of the observed $^3$He and $^4$He spectra. However, a more detailed study of the generation of waves in this branch and a comprehensive investigation of the thermal damping effects will reduce the model parameters considerably, giving better constraints on the SA models. It is clear that the model can also be used to study the acceleration of heavy ions. The heavy-ion acceleration is dominated by resonant interactions with waves in the HeC branch, and ions with lower charge-to-mass ratio are accelerated more efficiently due to their interactions with larger scale waves (Mason et al. 1986, 2004; Reames & Ng 2004). With simultaneous observations of solar flares by the Wind and ACE spacecrafts, a comprehensive study of acceleration of all charged particles including electrons, protons, and ions by the same PWT will set strict constraints on the power spectrum of PWT, guiding the exploration of the dynamical properties of the plasma turbulence. A complete time-dependent treatment of the wave-particle interaction, including the coupled kinetic evolution of the particle distribution and the turbulence spectrum, is also required.

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REFERENCES

André M. 1985, J. Plasma Phys., 33, 1
Bieber, J. W., Matthaeus, W. H., Smith, C. W., Wanner, W., Kallenrode, M. B., & Wil伯ern, G. 1994, ApJ, 420, 294
Cho, J., & Lazarian, A. 2003, MRNAS, 345, 325
———. 2004, ApJ, 615, L41
Cho, J., Lazarian, A., & Vishniac, E. T. 2003, in Turbulence and Magnetic Fields in Astrophysics, ed. E. Falgarone & T. Passot (Berlin: Springer), 56
Desai, M. I., Mason, G. M., Dwyer, J. R., Mazur, J. E., Smith, C. W., & Skoug, R. M. 2001, ApJ, 553, L89
Desai, M. I., et al. 2003, ApJ, 588, 1149
———. 2004, ApJ, 611, 1156
Dröge, W. 2003, ApJ, 589, 1027
Dung, R., & Petrosian, V. 1994, ApJ, 421, 550
Fisk, L. A. 1978, ApJ, 224, 1048
Hamilton, R. J., & Petrosian, V. 1992, ApJ, 398, 350

Hirose, S., Krol, J. H., De Villiers, J. P., & Havel, J. F. 2004, ApJ, 606, 1083
Holman, G. D. 1985, ApJ, 293, 584
Hsieh, K. C., & Simpson, J. A. 1970, ApJ, 162, L191
Hua, X., Ramaty, R., & Lingenfelter, R. E. 1989, ApJ, 341, 516
Hurford, G. J., Mewaldt, R. A., Stone, E. F., & Vogt, R. E. 1975, ApJ, 201, L95
Liu, S., Petrosian, V., & Mason, G. M. 2004, ApJ, 613, L81 (LPM04)
Mason, G. M., Dwyer, J. R., & Mazur, J. E. 2000, ApJ, 545, L157
Mason, G. M., Mazur, J. E., & Dwyer, J. R. 1999, ApJ, 525, L133
———. 2002a, ApJ, 565, L51
Mason, G. M., Mazur, J. E., Dwyer, J. R., Jokipii, J. R., Gold, R. E., & Krimigis, S. M. 2004, ApJ, 606, 555
Mason, G. M., Ng, C. K., Klecker, B., & Green, G. 1989, ApJ, 339, 529
Mason, G. M., Reames, D. V., Klecker, B., Hovestadt, D., & von Rosenvinge, T. T. 1986, ApJ, 303, 849
Mason, G. M., et al. 2002b, ApJ, 574, 1039
Miller, J. A. 2003, in COSPAR Colloq. 13, Multi-Wavelength Observations of Coronal Structure and Dynamics, ed. P. C. H. Martens & D. P. Cauffman (Amsterdam: Pergamon), 387
Miller, J. A., & Ramaty, D. A. 1987, Sol. Phys., 113, 195
Miller, J. A., & Viñas, A. F. 1993, ApJ, 412, 386
Miller, J. A., et al. 1997, J. Geophys. Res., 102, 14631
Möbius, E., Hovestadt, D., Kleeber, B., & Gloeckler, G. 1980, ApJ, 238, 768
Möbius, E., Scholer, M., Hovestadt, D., Kleeber, B., & Gloeckler, G. 1982, ApJ, 259, 397
Paeold, G., Kallenbach, R., & Benz, A. O. 2003, ApJ, 582, 495
Park, B. T., Petrosian, V., & Schwartz, R. A. 1997, ApJ, 489, 358
Petrosian, V., & Donahay, T. Q. 1999, ApJ, 527, 945
Petrosian, V., & Liu, S. 2004, ApJ, 610, 550 (PL04)
Petrosian, V., Yan, H., & Lazarian, A. 2005, ApJ, submitted (astro/ph-0508567)
Pryadaio, J. M., & Petrosian, V. 1997, ApJ, 482, 774
Ramaty, R. 1979, in AIP Conf. Proc. 56, Particle Acceleration Mechanisms in Astrophysics, ed. J. Arons, C. Max, & C. McKee (New York: AIP), 135
Reames, D. V., Barbier, L. M., von Rosenvinge, T. T., Mason, G. M., Mazur, J. E., & Dwyer, J. R. 1997, ApJ, 483, 515
Reames, D. V., Meyer, J. P., & von Rosenvinge, T. T. 1994, ApJS, 90, 649
Reames, D. V., & Ng, C. K. 2004, ApJ, 610, 510
Reames, D. V., von Rosenvinge, T. T., & Lin, R. P. 1985, ApJ, 292, 716
Schlickeiser, R. 1989, ApJ, 336, 243
Serlemitsos, A. T., & Balasubrahmanyan, V. K. 1975, ApJ, 198, 195
Share, G. H., & Murphy, R. J. 1998, ApJ, 508, 876
Steinacker, J., Meyer, J. P., Steinacker, A., & Reames, D. V. 1997, ApJ, 476, 403
Swanson, D. G. 1989, Plasma Waves (New York: Academic Press)
Temerin, M., & Roth, I. 1992, ApJ, 391, L105
Van Hollebeke, M. A. I., McDonald, F. B., & Meyer, J. P. 1990, ApJS, 73, 285
Xie, H., Ofman, L., & Viñas, A. 2004, J. Geophys. Res., 109, A08103
Zhang, T. X. 1999, ApJ, 518, 954