Holonomous Plasma: $A_0 \neq 0$. Polyakov loop $l < 1$

Necessary to describe the “semi” QGP, near $T_c$.

Holonomous Potential: *old* story at one and two loop order

@ 2 loop order, *need* a gauge invariant source

For weak holonomy ($g A_0 \sim m_{\text{Debye}}$), *need* a source with an infinite # loops

Free energy $F_3 \sim g^3$ are *not* continuous as $l \rightarrow 1$ for fixed source

Need *dynamical* fields: e.g., two dimensional massless ghosts

Strong constraints on effective theory by computing to $\sim g^3$!
Lattice, matrix models for a Holonomous Plasma

$T^2$ term in the free energy for pure glue: *deconfined strings*

Polyakov loop’s from the lattice: *broad transition region*  
from confined to perturbative regime

Matrix models of a Holonomous Plasma: *narrow transition region*
Pure glue: *deconfined* strings above $T_d$.

$T_d \rightarrow 4 T_d$: for pressure, leading correction to ideal gas $T^4$ is *not* a bag constant, but $\sim T^2$

$$\frac{e - 3p}{T^2 T_d} \uparrow$$

For $T$: $1.2 T_d \rightarrow 4 T_d$,

$$p(T) \approx \# (T^4 - T^2 T_d^2)$$

$T^2$ term: *deconfined* strings?

For $T$: $T_d \rightarrow 1.2 T_d$, involved transition. *Narrow region*

Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184
Pure glue: deconfined strings in 2+1 dim.’s

In 2+1 dimensions, leading correction to ideal gas $T^3$ is again $T^2$, $N = 2, 3, 4, 5$

$$p(T) \sim \#(T^3 - T^2 T_d)$$

Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580

$\frac{1}{N^2 - 1} \frac{e - 2p}{T^3}$

$T_d/T \rightarrow$

$\uparrow 10 T_d$ $\uparrow 2 T_d$ $\uparrow 1.1 T_d$
Lattice: Polyakov Loop without and with quarks

Without quarks: *exact* order parameter for global $Z(3) = \text{Polyakov loop}$

Dynamical quarks *always* break $Z(3)$. But in QCD, loop *small* at $T_{\chi}$, $\sim 0.1$?

*Broad* transition from confined to deconfined phase

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**Diagram:**

- $L_{\text{ren}}(T)$ vs. $T$ (in MeV)
- $\langle \text{loop} \rangle \uparrow$
- $QCD \Rightarrow$
- $T_d^{SU(3)} \downarrow$ \quad $T_d^{SU(2)} \downarrow$
- $T_{\text{chiral}}^{QCD} \uparrow$
- Lattice: Bazavov & Petreczky, 1110.2160
Holonomous Potential @ 1 loop order

Holonomy = constant $A_0$. No potential classically, nonzero at 1 loop order.

Gross, RDP, Yaffe, ’81; Weiss ‘82 For two colors:

$$A_0^{cl} = \frac{\pi T}{g} q \sigma_3$$

$$\ell = \frac{1}{2} \text{tr} \mathcal{P} e^{ig \int_0^{1/T} A_0} = \cos(\pi q)$$

$Z(2)$ degenerate vacua: $q = 0, 1$.

Confining vacuum, $q = 1/2$, is maximum of perturbative potential.
Non-perturbative Holonomous Potential

To model deconfinement, add - by hand - a non-perturbative potential for q:

Dumitru, Guo, Hidaka, Korthals-Altes & RDP 1011.3820; 1205.0137.

\[ V_{\text{non}}(q) = \frac{4\pi^2}{3} T^2 T_d^2 \left( -\frac{c_1}{5} q(1 - q) - c_2 q^2 (1 - q)^2 + \frac{c_3}{15} \right) \]

\[ V_{\text{tot}}(q) = V_{\text{pert}}(q) + V_{\text{non}}(q) \], determine \(<q>\) from minima, fit to \(p(T) = - V_{\text{tot}}(<q>)\).

Find: \(<q>\neq 0\) in narrow region, \(T: T_d \rightarrow 1.2 \ T_d\).

Constant term, \(c_3\), gives \(p(T) \sim T^2\) for \(T: 1.2 \ T_d \rightarrow 4 \ T_d\).

Linear term at small \(q\), \(\sim c_1 q\), is crucial to ensure holonomy turns on smoothly.

\(c_2\) is like the \(V_{\text{pert}}(q)\).

Use to compute Polyakov and ‘t Hooft loops.
Matrix model for three colors

Start with three parameters. Require transition occurs at $T_d$, and $p(T_d) \sim 0$. Leave one free parameter, adjust to agree with $(e-3p)/T^4$.

$$T_d = 270 \text{ MeV} , \ c_1 = 0.315 , \ c_2 = 0.83 , \ c_3 = 1.13$$

Lattice:
Beinlich, Peikert, Karsch
lat/9608141
Datta, Gupta 1006.0938
Polyakov loop: model vs lattice?

Polyakov loop *much* smaller than the matrix model

Transition region: matrix model *narrow*, to $\sim 1.2 \, T_d$. Lattice *wide*, to $\sim 4.0 \, T_d$.

But: if one fits to lattice loop, ’t Hooft loop is *much* too small.

![Graph showing Polyakov loop behavior with different models and parameters.](image)
For pure gauge, 't Hooft loop $\sigma = Z(N_c)$ interface tension.

Compute $\sigma$ as tunneling problem in $V_{\text{tot}}(q)$, from $q = 0$ to 1.

Using $V_{\text{pert}}(q)$: Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

With loop as in matrix model, *excellent* agreement with lattice data.

Lattice: de Forcrand, D’Elia, Pepe, lat/0007034
Holonomous Model with quarks

RDP & Skokov, 1604.00022. Add linear sigma model + quarks. Quarks generate contributions to Holonomous Potential, break Z(3) symmetry. Keep $T_d = 270$ MeV, tune Yukawa interaction to get $T_{\text{chiral}} = 154$ MeV.

Non-trivial: pressure remains positive for $T < T_d$.

Polyakov loop *much* narrower than lattice, but: baryon suscept.’s ~ agree.

How broad is the transition regime with quarks? What’s up c Polyakov loop?

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**Graphs:**

- **Left Graph:**
  - $\frac{e - 3p}{T^4}$ vs $T$
  - Lines: $\chi$-M, HTL, LQCD
  - Regions:
    - Light grey: Model
    - Dark grey: Lattice

- **Right Graph:**
  - $\chi^2$ vs $T$
  - Lines: $\chi$-M, HTL, LQCD
  - Points: LQCD
  - Regions:
    - Light grey: Model
    - Dark grey: Lattice

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**Legend:**

- $\chi$-M
- HTL
- LQCD
- $\chi^2$ (model and lattice)
Perturbative computations in a Holonomous Plasma

One loop order, \( \sim 1 \) in free energy: easy peasy

Two loop order, \( \sim g^2 \) in free energy:
- gauge dependent source \( \Rightarrow \) gauge variant free energy
- gauge invariant source \( \Rightarrow \) gauge invariant potential
  \( \Rightarrow \) transverse gluon self energy

\( \sim g^3 \) in free energy for soft Q
- need gauge invariant source with infinite sum over loops
- free energy for off-diagonal gluons discontinuous as \( Q \rightarrow 0 \)?!

generating Holonomous Plasma with dynamical fields
- massless, 2D ghosts (\( \sim \) deconfined strings)
- free energy for off-diagonal gluons continuous as \( Q \rightarrow 0 \)
Holonomous potential to one loop order

For SU(N), take:

\[ A_\mu = A^{cl}_\mu + A^{qu}_\mu, \quad A^{cl}_0 = \frac{2\pi T}{g} q \]

\[ (q^{ab} = q^a \delta^{ab} \sum_{a=1}^{N} q^a = 0) \]

Work in background field gauge. In momentum space, \( n = 0, \pm 1 \ldots \)

\[ D^{cl}_\mu = \partial_\mu - ig[A^{cl}_\mu , *], \quad i D^{cl}_0 \rightarrow p^{ab}_0 = -i2\pi T(n + q_a - q_b) \]

Easy to compute, just \( p_0 \rightarrow p^{ab}_0 \). Result independent of gauge fixing:

\[ S_{pert,1} = -2 \text{tr} \log((p^{ab}_0)^2 + p^2) = -\frac{T^4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^4} |\text{tr} L^n|^2 \]

\[ = \frac{2\pi^2 T^4}{3} \sum_{a,b=1} B_4(q_a - q_b), \quad B_4(x) = -\frac{1}{30} + x^2(1 - |x|)^2 \]

The \( q_a \) respect \( Z(N) \) symmetry: Weyl chamber. Term \( \sim q_a^2 = \) Debye mass sq’d
Holonomous Potential, two loop order

Add gauge variant source $\sim \text{tr } J_0 A_0$. Enqvist & Kajantie ‘90.

Use background field gauge with gauge fixing parameter $\xi$

$$\nu_{\text{pert,2}}(q) = \frac{g^2 T^3}{4} \sum_{a,b,c=1}^N B_2(q_a - q_c)B_2(q_b - q_c) + (1 - \xi)B_1(q_a - q_c)B_3(q_b - q_c)$$

Bernoulli polynomials $B_1$, ...

$$B_1(x) = -s(x)/2 + x ; \ B_2(x) = 1/6 - |x| + x^2$$

$$s(x) = \text{sign}(x)$$

$$B_3(x) = x/2 - 3s(x)x^2/2 + x^3$$

Holonomous Potential is $\xi$-dependent!

Can also show that apparently $<q_a> \sim \xi$: spontaneously breaks CP?!

But the source is gauge variant, so....
Holonomous potential, two loop order, redux

Belyaev '91 Under a gauge transformation $\Omega$,

$$\mathbf{L}(x) = \mathcal{P} \exp(ig \int_0^{1/T} A_0(\tau, x)d\tau) \rightarrow \Omega(1/T, x)^\dagger \mathbf{L}(x)\Omega(0, x)$$

Thermal Wilson line $\mathbf{L}$ is gauge dependent; eigenvalues, $q_a$, are gauge invariant. For SU(2), eigenvalue $q$ renormalizes at one loop order:

$$q_{\text{ren}} = -(3 - \xi) g^2 / (8\pi^2)(q - 1/2)$$

Including this, for SU(N)

$$\mathcal{V}_{\text{pert,2}}(q_a) = -5g^2 T^3 / 24 \sum_{a,b=1}^{N} B_4(q_a - q_b)$$

Manifestly gauge invariant, perturbative vacuum $q_a = 0$ stable

Bhattacharya, Gocksch, Korthals-Altes, RDP '90, '92: $\mathbb{Z}(N)$ interface tension at NLO

Dumitru, Guo, Korthals-Altes, 1305.6846; Guo 1409.6539. General result in SU(N)
Consistent analysis of Holonomous Potential

Korthals-Altes ‘93 Compute gluon self energy perturbatively to 1 loop:

\[ P_{\mu}^{ab} \Pi_{\text{pert}}^{ab;\mu\nu} = +\delta^{\nu0} 4\pi g^2 T^3 / 3 \sum_{a,b,c=1}^{N} \left( B_3(q_a - q_c) + B_3(q_c - q_b) \right) \]

With gauge invariant sources; consistent with BRST identities

Severe problem in computing to higher loop order, esp. \( \sim g^3 \) in free energy.

Resolution: \( B_3 = 4 \frac{d}{dx} B_4(x) \sim \) derivative of the Holonomous Potential.

We show: expanding about a consistent stationary point, the gluon self energy is transverse.
Gauge invariant sources

$SU(N)$ sources for first $N$ Polyakov loops:

$$S_J = \frac{1}{V} \int d^3x \sum_{r=1}^{N} J_r \text{tr} \mathbf{L}^r(x)$$

Terms linear in $A_{\mu}^{qu} = 0$

$$\Rightarrow \text{equations of motion} \quad \frac{1}{V} \sum_{r=1}^{N} 2\pi i J_r \, r \, e^{2\pi i r q_a} + 16\pi^2 T^3/3 \sum_{b=1}^{N} B_3(q_a - q_b) = 0$$

Only $N-1$ independent sources:

$$\sum_{a=1}^{N} \sum_{r=1}^{N} r \, J_r \, e^{2\pi i r q_a} = 0$$

Sources $\sim$ Polyakov loops nonlinear in $A_{\mu}^{qu}$, so terms quadratic in $A_{\mu}^{qu}$:

$$\Pi_{J}^{ab;00} = -(1/p_{0}^{ab}) \, 4\pi g^2 T^3/3 \, \sum_{a,b,c=1}^{N} (B_3(q_a - q_c) + B_3(q_c - q_b))$$

Using equations of motion. Valid for arbitrary sources. Self energy transverse

$$P_{\mu}^{ab} (\Pi_{\text{pert}}^{ab;\mu\nu} + \Pi_{J}^{ab;\mu\nu}) = 0$$

Gives same, gauge invariant free energy, to $g^2$. 
Weak Holonomous Potential to $g^3$.

Consider small $q_a \sim g$. Then for $A^{qu}$, $q_a^2 T^2 \sim m_{\text{Debye}}^2$. Weak Holonomous plasma

Perturbatively, need to resum “ring” diagrams. In Holonomous Plasma,

$$\mathcal{F}_3 = -T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \text{tr} \log \left( (P_{\mu}^{ab})^2 + (\xi^{-1} - 1)P_{\mu}^{ab}P_{\nu}^{ab} - \delta \Pi_{\mu\nu}^{ab} \right)$$

Only the static mode, $p_0 = 0$, contributes. Typical momenta are $p \sim g T$.

To be independent of $\xi$, gluon self energy must be transverse.

In Holonomous Plasma, diagonal and off-diagonal gluons contributions.

Find: mass$^2$ of diagonal gluons are negative with sources linear in loops!
Holonomous Plasma for two colors

Consider two colors, add to the perturbative HP two non-perturbative terms:
Nishimura & Ogilvie, 1111.6101; DGHKP, 1205.0137; HKNPS, 1905...

\[ \mathcal{V}(q) = \left(4\pi^2 T^3 / 3\right) \left(q^2 - |q|^3 + q^4\right) + 4j_1 \ell^2 + 16j_2 \ell^4, \ \ell = \cos(\pi q) \]

Add two non-pert. terms, \( \sim j_1 \) & \( j_2 \).

Find that there is always a 1st order transition from pert. vac. to Holonomous Plasma, \( \langle q \rangle \neq 0 \)

Trivial reason: loop is always even in \( q \) about \( q = 0 \)

1-loop HP has cubic term - \( q^3 \)!

- cubic => 1st order transition
Non-perturbative potentials for HP

Usual source \( \sim J \phi \): for any \( J \), even infinitesimal, \( \phi \neq 0 \).

Here: need gauge invariant sources: any finite number of loops \( \sim q_a^2 \), \( q_a \ll 1 \)

For HP @ 1 loop, term cubic in \( q_a \) because sum over \( \infty \) number of loops.

Source linear in \( q_a \), \( q_a \ll 1 \), need sum over \( \infty \) number of loops. We choose

\[
S_{2D} = \sum_{n=1}^{\infty} \frac{1}{n^2} |\text{tr} L^n|^2 = \sum_{a,b=1}^{N} B_2(q_a - q_b), \quad B_2(x) = 1/6 - |x| + x^2
\]

Like the free energy of a massless boson in 1+1 dimensions...

Many others possible: \( B_3, B_5 \)...

With a source \( \sim J_2 S_{2D} \), \( q_a \neq 0 \) for any infinitesimal \( J_2 \).

For diagonal gluons, this source gives positive mass

\[
\text{free energy } \sim g^3 \text{ that is smooth as } J_2 \rightarrow 0
\]
Off-diagonal gluons for free energy $\sim g^3$.

Need: self energy for off-diagonal gluons, $p_0 = 0$, $q_a \sim g$.

Can use Holonomous Hard Thermal Loop: Hidaka & RDP 0906.1751

$$\delta \Pi^{ab;ij} \sim \int_0^{\infty} d^3k \frac{k^i k^j}{E_k E_{p-k}} \int \frac{d\Omega}{4\pi} \mathcal{I} \ , \ \mathcal{I} = \frac{n(E_k - iQ_a) - n(E_{p-k} + iQ_b)}{ip_0^{ab} - E_k + E_{p-k}}$$

Usual HTL valid for soft $\omega = -ip_0 \sim p \sim gT$; Holonomous HTL for soft $\omega = -ip_0^{ab}$

$p_0^{ab} = p_0 - (Q_a - Q_b) = p_0 - 2\pi T (q_a - q_b)$: soft when $p_0 = 0$, $q_a \sim g$.

Dominated by hard $k \sim T$. Then

$$\mathcal{I} \approx \frac{1}{-\hat{k} \cdot \vec{p} + i(Q_a - Q_b)} \left( n(k) - n(k - \hat{k} \cdot \vec{p} + i(Q_a - Q_b)) \right) \approx -\frac{d}{dk} n(k)$$

So independent of $Q! \Rightarrow$ constant. Could have been function of $p/Q$.

Implies (off-diagonal) self energy vanishes!

Free energy from off-diag $\sim g^3$ discontinuous: $\neq 0$ when $q_a = 0$; $= 0$ when $q_a \sim g$.

Makes no sense.
Two dimensional fields

Introduce 2-dimensional fields:

\[ x^\mu = (\hat{x}, x_\perp), \quad \hat{x} = (x_0, \vec{x} \cdot \hat{n}); \quad \hat{n}^2 = 1, \quad x_\perp \cdot \hat{n} = 0 \]

Embed isotropically by integrating over all directions of unit vector n.
Anisotropic between along n and perp. to n.

\[
S_{2D} = \int_0^{1/T} d\tau \int \frac{d\Omega_{\hat{n}}}{4\pi} \int_0^\infty \hat{x} \int_0^\infty d^2x_\perp \text{tr} \left( (\hat{D}\phi)^2 + (D_\perp \phi)^2 \right)
\]

Adjoint scalar \( \phi \) is two dimensional at short distances, \( < 1/T_d \), but four dimensional over large distances.

At one loop order, gluon self energy gauge invariant; scalar not, will patch up.
Two dimensional ghosts

Assume $T_d << T$: then momentum integral trivially reduces to 2D

$$S_{2D} = \text{tr} \log (-\hat{D}^2 - D^2_{\perp}) \approx T_d^2 \text{tr} \log (-\hat{D}^2) = -T^2 T_d^2 \sum_{a,b=1}^{N} B_2(q_a - q_b)$$

The $\phi$ field must be a ghost field for $B_2$ to have the proper sign.

Natural: one wants to decrease the pressure from physical gluons.

Find Holonomous HTL's in Euclidean space: $Q_{ab} = 2 \pi T (q_a - q_b)$

$$\delta \Pi_{\text{long}}^{ab} = -1 + \frac{Q_{ab}}{p} \arctan \left( \frac{p}{Q_{ab}} \right),$$

$$\delta \Pi_{\text{tr}}^{ab} = \frac{3}{2} \left( \left( \frac{Q^2_{ab}}{p^2} + 1 \right) \frac{Q_{ab}}{p} \arctan \left( \frac{p}{Q_{ab}} \right) + \frac{Q^2_{ab}}{p^2} \right)$$

Can show: if holonomy generated by 2D ghosts, then free energy $\sim g^3$ is smooth as $q_a \to 0$. 
Using two dimensional ghosts

Previously: constructed effective theory with $B_2$ to fit Euclidean pressure, etc.
Now: Generate $B_2$ dynamically from massless, 2D ghosts

To do: compute transport coefficients using gluons + 2D ghosts in Holonomous Plasma in perturbation theory

Severe constraint: ghosts could drive pressure, transport coefficients negative!
Doesn’t happen for the pressure.

Shear viscosity \textit{suppressed} by loop$^2$ in Holonomous Plasma:
including only gluons, Hidaka & RDP, 0803.0453, 0912.0940

\[ \eta \sim \frac{T^3}{g^4 \log(1/g)} |\ell|^2 \]

Need to include 2D ghosts...