Double quantum dot as a probe of nonequilibrium charge fluctuations at the quantum point contact

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Absorption of energy quanta generated by quantum point contact results in the inelastic current through the double quantum dot placed nearby. In contrast to a single quantum dot, the inelastic current through the double quantum dot is sensitive to the energy dependence of the quantum point contact transmission, which can explain the experimentally observed features. We calculate the inelastic current as a function of microscopic parameters of the circuit.

Double quantum dot (DQD) has been recently proposed as a detector of nonequilibrium noise generated by nearby mesoscopic devices [1]. This idea was experimentally realized in measurements of the nonequilibrium noise spectrum of a quantum point contact (QPC) detected by DQD [2, 3]. The experiments provided a lot of interesting and in some respect puzzling results.

The noise detection is based on the generation of inelastic current through DQD assisted by absorption of energy quanta emitted by QPC. To implement the noise measurement, DQD is brought into the Coulomb blockade regime with the highest energy electron localized in one of its dots hereafter referred to as dot 1. The exchange of the electron between the two quantum dots brings DQD into the excited state, the excitation energy $\Delta$ being fixed by the gate voltages. Absorbing an energy quanta, an electron tunnels from the low energy state in quantum dot 1 to the excited state localized in quantum dot 2. The tunnel barrier between the quantum dots is tuned to be much higher than the barriers between the dots and the adjacent leads, so that after each interdot tunneling event the electron almost immediately escapes into the adjacent electron reservoir. Another electron occupies quantum dot 1, and the system returns to the ground state, the unit of charge having been transferred through DQD [2]. The generated current is therefore proportional to the noise power on the excitation frequency of DQD. The nonequilibrium noise is generated by QPC, which is brought in a strongly nonequilibrium transport regime by application of transport voltage. At the same time, the plunger voltage applied to QPC controls its transmission.

Theoretical calculations of the generated inelastic current have been performed in Ref. [1], where it has been related to the nonequilibrium noise power $S_I(\Delta/h)$ generated by QPC at the frequency $\Delta/h$. This noise power is given by the local current fluctuations in an arbitrary spatial point of QPC,

$$S_I^{\text{local}}(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{i\omega \tau} \langle \delta I(x, \tau) \delta I(x, 0) \rangle.$$  (1)

Based on the energy conservation law one concludes that increasing the QPC transport voltage $V_{QPC}$, the current through DQD will start at the point $V_{QPC}^{\ast} = \Delta/e$, when the quanta with energy $\Delta$ appear in the nonequilibrium noise spectrum [4]. A puzzling feature of the experimental measurements is the independence of the threshold voltage $V_{QPC}^{\ast}$ of the DQD excitation energy $\Delta$ for a finite range of energies, contrary to the expectations based on the energy conservation law [2].

In this Letter we provide a theoretical description of QPC–DQD system in the nonequilibrium regime that allows to relate experimental measurements of the inelastic current to microscopic parameters. We show that since DQD is an object extended over both sides of QPC, the local noise power $S_I(\omega)$ is not the relevant quantity for the inelastic DQD current. Rather, the noise power absorbed by DQD includes spatially nonlocal correlations of current fluctuations at positions of two quantum dots. The relevant voltage power is given by

$$S_V(\omega) = \left\langle \left| Z(x_1, \omega) \delta I(x_1, \omega) + Z(x_2, \omega) \delta I(x_2, \omega) \right|^2 \right\rangle,$$  (2)

where $Z(x_i, \omega)$ is the spatially dependent transimpedance of the circuit, $x_i$ denotes the position of the corresponding quantum dot, and $\omega = \Delta/h$ is the absorption frequency. In the case of the symmetric coupling of both quantum dots to QPC, which is called a symmetric circuit in what follows, the trans-impedance becomes independent of spatial coordinate. Then the inelastic current is given by the expression similar to that obtained in Ref. [1]

$$I_{\text{DQD}} = \frac{e^2}{\hbar^2} \frac{|Z(\Delta/h)|^2}{\Delta^2} S_I \left( \frac{\Delta}{h} \right),$$  (3)

but with the nonlocal current noise power $S_I(\omega)$ following from

$$S_I(\omega) = \sum_{i, j=1}^2 \int_{-\infty}^{\infty} d\tau \, e^{i\omega \tau} \left\langle \delta I(x_i, \tau) \delta I(x_j, 0) \right\rangle.$$  (4)

The presence of nonlocal current correlations substantially modifies the resulting noise spectrum. So, while the energy dependence of the QPC transmission is not essential for the local current fluctuations [1] [1, 4, 5],
it becomes crucial for the spatially nonlocal fluctuations of the current \[ I \]. As the result, the generated inelastic current becomes sensitive to the energy dependence of QPC transmission. In particular, for the symmetric circuit the inelastic current turns to zero when the QPC transmission is energy independent within the transport voltage window. This can explain the independence of the finite frequency Fano factor \( F \) on the QPC voltage window. This can explain the independence of the inelastic current \( I_I \) on the transport voltage \( V \) seen in experiment [2]. These findings are illustrated in Fig. 1. If the QPC transmission amplitude exhibits a plateau in its energy dependence, the current onset voltage is determined by the width of the plateau as long as that width exceeds the QPC excitation energy \( \Delta \) (solid lines). In contrast, the threshold voltage equals \( \Delta/e \) for the continuously rising QPC transmission (dashed lines).

To emphasize the importance of the energy dependence of QPC transmission for the power absorbed by DQD, it is advantageous to define an analogy of the Fano factor for a finite frequency noise \( F(\omega) \equiv S(\omega)/(2I QPC) \). This value relates the power generated at frequency \( \omega \) and defined by \[ I \] to the direct current through QPC.

Its explicit expression reads

\[
F(\omega) = \left[ 1 + \tanh \left( \frac{h\omega}{2T} \right) \coth \left( \frac{eV - h\omega}{2T} \right) \right] \frac{\int dt |r_x + i\omega| t_x - t_x + i\omega| t_x|^2 (f_{L}^R - f_{R}^L)}{2 \int dt |t_x|^2 (f_{L}^R - f_{R}^L)}.
\]

(5)

where \( t_x \) and \( r_x \) are the transmission and reflection amplitudes of QPC at energy \( \epsilon \). The Fermi distributions in the left (source)/right (drain) reservoirs of QPC are denoted as \( f_{L}^R \). Their chemical potentials differ by the QPC transport voltage and can be written as \( \mu_{L/R} = \pm eV_{QPC}/2 \), the chemical potential in the unbiased QPC being taken as zero. For zero temperature, and for the absorption frequency \( \omega = \Delta/e \), Eq. (5) simplifies to

\[
F(\Delta/e) = \Theta(eV - \Delta) \frac{\int dt |t_x + i\Delta t_x - t_x + i\Delta r_x|^2}{2 \int dt |t_x|^2 (f_{L}^R - f_{R}^L)}.
\]

(6)

It is evident from \[ I \], that the Fano factor is determined by the energy dependence of QPC transmission. The dependence of the finite frequency Fano factor on the QPC transport voltage is shown in the inset to Fig. 1. The energy threshold of the Fano factor corresponds to the width of the plateau in QPC transmission. Furthermore, the Fano factor drops if the applied voltage is larger than the energy interval for the onset of the conducting channel. In that case, the DQD inelastic current reaches saturation while the direct QPC current continues to grow. Using the Fano factor \[ I \], the expression for the generated inelastic current can be cast in the form

\[
I_{QPC} = 2eI_{QPC} F(\Delta/e) \frac{\int dt |t_x + \Delta t_x|^2}{\Delta^2} I_{QPC},
\]

(7)

where \( \Gamma_{QPC} = \frac{se_{QPC}^2}{2e^2} \) is the DQD rectification factor, \( t_0 \) denotes the tunnel amplitude between the two quantum dots, and \( R_Q = \hbar/e^2 \) is the quantum resistance.

In what follows we introduce the theoretical model for the coupled QPC – DQD system and outline the derivation of the presented results. Since the maximal inelastic current is observed when a new conducting channel is opening in QPC, we concentrate on a single conducting channel of QPC. We distinguish two species of electrons in QPC, namely those coming from the right and the left reservoirs, and we describe them by the fermion field operators \( \psi_{R/L}(x) \). The electrons of each sort are in equilibrium with their own reservoir. Taking the position of the QPC potential barrier at \( x = 0 \), we represent the field on each side of it as

\[
\psi(x) = \int \frac{dk}{2\pi} \left\{ \psi_{L}(k) e^{i(p_F+k)x} + r_k e^{-i(p_F+k)x} \right\}, \quad \text{for } x < 0,
\]

(8)

\[
\psi(x) = \int \frac{dk}{2\pi} \left\{ \psi_{R}(k) e^{-i(p_F+k)x} + r_k e^{i(p_F+k)x} \right\}, \quad \text{for } x > 0.
\]

(9)

Here \( p_F \) denotes the Fermi wave vector at zero transport voltage.

The structure of the interaction between DQD and QPC-channel plays a crucial role for the generation
of inelastic current, determining the trans-impedance $Z(x_i, \omega)$. Due to the presence of external electrodes, the effective interaction becomes screened and time-retarded. Moreover, the presence of QPC violates the spatial homogeneity of the interaction. Therefore, we can write the interaction term in the action in the form

$$A_{\text{int}} = -\epsilon^2 \sum_{i=1,2} \int dx \int dt dt' U_i(|x|, |x - x_i|, t - t') \times \hat{n}(x, t) \hat{n}(x', t') \tag{10}$$

Here $n_1$ and $n_2$ are the particle number operators in each quantum dot, and $\hat{n}(x)$ is the operator of density fluctuations in the conducting channel at point $x$. We assume that the quantum dots are situated far away from the potential barrier of QPC, one on each side of it (see experimental setup of Ref. [2]). In the detection regime, the total occupation of DQD is fixed to $n_1(t) + n_2(t) = 1$. This allows us to use a pseudo-spin 1/2 description of DQD. We associate the states localized in the quantum dots 1 and 2 with the spin-up and spin-down states respectively. The charge transfer between the two quantum dots corresponds to the spin-flip between the ground state spin-up and the excited state spin-down. The interaction term (10) can be split into the interaction with the total charge of DQD, and the interaction with the $z$-component of DQD pseudo-spin, $\hat{S}^z = \hat{n}_1 - \hat{n}_2$. Note that the product $\hat{\pi} = \epsilon \hat{S}^z$, is proportional to the operator of the DQD dipole moment. Omitting the interaction with the total charge of DQD, the relevant interaction can be represented as the one between the dipole moments of DQD and QPC, which in the Fourier-transformed form reads

$$A_{\text{int}} = -\int \frac{d\omega}{2\pi} \hat{P}(\omega) \hat{\pi}(-\omega), \tag{11}$$

where the QPC dipole moment $\hat{P}(\omega)$ is defined as

$$\hat{P}(\omega) = e \int dx \hat{n}(x, \omega) \left[ U_1(|x|, |x - x_1|, \omega) - U_2(|x|, |x - x_2|, \omega) \right]. \tag{12}$$

The forward and backward inelastic scattering amplitudes are the only relevant ones for the one-dimensional motion of electrons in QPC. They are given in terms of Fourier transforms $\int dx e^{-i q(x-x_1)} U_i(|x|, |x - x_i|)$ at wave vectors $|q| \ll p_F$ and $q = \pm 2p_F$. We assume that the interaction is strongly screened, and it takes place only in a small region of the size of the screening length around each quantum dot. Then the behavior of scattering amplitudes at small wave vectors is smooth, and we can approximate $U_i(|q| \ll p_F) \approx U_i(q = 0) \equiv U_i^f$ for the forward scattering. For the backward scattering we obtain $U_i^b(\pm 2p_F) = U_i(\pm 2p_F) e^{\mp 2ip_F x_i}$. Taking into account the finite size of a quantum dot, one has to integrate over $x_i$ within that size, which greatly diminishes the backscattering amplitude because of the rapidly oscillating factors $e^{\pm 2ip_F x_i}$. On that account we neglect the backscattering amplitudes in what follows.

Using Eqs. (8), (9), (12), we represent the total dipole moment operator $\hat{P}$ in terms of the fields of right- and left-reservoirs as follows

$$\hat{P}(\omega) = e \sum_{\chi, \chi' = R, L} \int \frac{dk dk'}{(2\pi)^2} w_{\chi, \chi'}^x \psi^+_{\chi'}(k') \psi_{\chi}(k) \delta(\epsilon' - \epsilon - \omega). \tag{13}$$

The combined effect of the scattering by QPC potential barrier and interactions with quantum dots is captured by the effective inelastic scattering amplitudes $w_{\chi, \chi'}^x$ between the two species of fermion fields. Further calculation shows that only the inelastic scattering between different species contributes to the nonequilibrium noise power. The amplitude $w_{\chi, \chi'}^x$ is given by

$$w_{\chi, \chi'}^{LR} = U_1^f r_{\chi} e - U_2^f r_{\chi'}, \tag{14}$$

and the amplitude $w_{\chi, \chi'}^{RL}$ is obtained from $w_{\chi, \chi'}^{LR}$ by exchange $U_1^f \leftrightarrow -U_2^f$. Using the representation of the electric current operator in the basis $\hat{s}$, (11) we can express the dipole moment $\hat{P}$, in terms of the Fourier transform of the current operator at frequency $\omega$ [8, 9],

$$\hat{P}(\omega) = \sum_{i=1,2} Z(x_i, \omega) I(x_i, \omega) \int \frac{d\omega_1}{2\pi} \int \frac{d\omega_2}{2\pi} \psi_{\chi^1} \psi_{\chi^2} \delta(\omega_1 - \omega_2 - \omega). \tag{15}$$

The generated inelastic current is calculated perturbatively in the lowest order of QPC-DQD interaction employing the Keldysh technique [10, 11]. The total action is given by $A = A_0 + A_{\text{int}}$, with the interaction part given by (11) and the free part

$$A_0 = \int dt \left\{ \sum_{\chi = R, L} \bar{\psi}_{\chi} G_{\chi}^{-1} \psi_{\chi} + \Phi D \Phi' \right\}. \tag{15}$$

In the free part of the action, we used the semifermionic representation of the pseudospin degrees of freedom of DQD [12, 13]. Here $\Phi = (\phi_+, \phi_-)^T$ is the spinor Grassmann field. Each spin component $\phi_\sigma$ is in turn a two-component field in the retarded-advanced space. $D$ is the semifermionic Green's function, its retarded components are given by $D_{\sigma\sigma}(\epsilon) = [\epsilon - \sigma \Delta/2 + i\omega]^{-1}$, $D_{\sigma\sigma'}^R = \frac{i}{\omega} (D_{\sigma+b}^R - D_{\sigma-b}^R)$. The semifermionic spin-1/2 representation imposes special rules of calculus in Keldysh formalism. The Keldysh component is parameterized as $D_k^R(\epsilon) = D_k^R(\epsilon) F_k(\epsilon) - F_k(\epsilon) D_k^A(\epsilon)$ with the function $F_\epsilon(\epsilon) = \tanh \left( \frac{i}{2} \right) \pm \frac{i}{\cosh \left( \frac{\epsilon}{2} \right) \cosh \left( \frac{\epsilon}{2} \right)}$. In the diagrammatic expansion, each diagram is calculated taking $F_k$ once with
the plus and once with the minus sign, and the half-sum of the results is taken at the end of the calculation \( [13] \).

The Green’s function of the fermions from the reservoir \( \chi \) in QPC channel is given by \( G_{R/L}^{\sigma} (\epsilon, k) = [\epsilon \mp v_F k - \mu_{R/L} + i\delta]^{-1} \), the Keldysh component is obtained by usual rules \([10, 11]\). The energy \( \epsilon \) is counted from the Fermi level of the unbiased QPC.

The operator of the current through the double quantum dot can be written in the pseudospin representation as \( \hat{I}_{\text{DQD}} = -e\hbar \hat{S}^y / \hbar \). In the Keldysh formalism, its quantum component is proportional to the matrix \( \sigma^y \) acting in the retarded-advanced space \( \hat{I}_{\text{Keldysh}} = \sigma^y \otimes \hat{I}_{\text{DQD}} \). The current is calculated using a diagrammatic representation derived from the Keldysh path integral with the action \( [15] \) (see Fig. 2). The diagram for the current in the second order of perturbation in the interaction is shown in Fig. 2d), the corresponding analytical expression reads \( I_{\text{DQD}} = Tr \{ \hat{\Gamma} \hat{\Pi} \} \). The polarization operator \( \hat{\Pi} = \langle \hat{\rho}(\omega) \hat{\rho}(-\omega) \rangle \) can be represented in terms of current operators and trans-impedances as \( \hat{\Pi} = \left( \sum_{i=1,2} Z_{\iota}(x_i, \omega) \hat{I}(x_i, \omega) \right)^2 \). For the symmetric circuit we obtain \( \hat{\Pi}(\omega) = |Z(\omega)|^2 S_\iota(\omega) \) with the noise power \( S_\iota(\omega) \) given by \([4]\). Explicit calculation of the diagram shown in Fig. 2d) leads to the final expression for DQD current \( [7] \).

The suppression of the DQD current for \( V_{\text{QPC}} \) exceeding the energy conservation threshold \( \Delta / e \) represents a profound feature of the current voltage characteristics of the double dot system under consideration, which has no analogy in the shot noise induced current through a single quantum dot \([14]\). It can be understood in simple terms, if one considers the generation of DQD current as a kind of a Coulomb drag experiment. Indeed, the diagrammatic representation for the inelastic current Fig. 2d) is almost identical to the diagrams for the drag current, the difference being the presence of a nonequilibrium polarization operator \( \Pi \) for QPC instead of the current operator \([11, 13]\). To realize the drag, the particle-hole \( (p-h) \) symmetry has to be violated in both components of the system. Its violation in the gated DQD is explicit. In the case of symmetric circuit, the violation of the \( p-h \) symmetry in the nonequilibrium QPC is possible only if the transmission amplitudes at energies differing by \( \Delta \) are different. Indeed, the initial bosonic excitation that provides the energy to the DQD is an electron-hole pair in QPC with energy \( \Delta \). The Hamiltonian describing the propagation of such a pair through QPC is \( p-h \) asymmetric only for different transmission amplitudes for the electrons and the holes.

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