Physico-Mathematical Model for Theoretical One-Stage Heat-Driven Compressor

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Abstract. The concept design and construction of a one-stage heat-driven compressor is presented. A physico-mathematical model used for analyzing the heat-driven compressor stage is presented. The model is a theoretical one, that takes into account a perfect gas agent and no thermodynamic losses. Performances and output of the theoretical one-stage heat-driven compressor are calculated and analyzed. An exemplification by numerical simulation was performed in this regard. The device could use solar energy for heating. Multi-stage units could obtain larger compression ratios.

1. Introduction

A compressor is a mechanical device that increases the pressure of a gaseous working agent. Based on the working principle, there are several methods used for compressing gases. These methods could be highlighted using the equation of state written for a perfect gas at stagnation point:

\[ p^* = \frac{m}{V^*} R T^* = R \left( \frac{T}{v^*} + \frac{c^2}{2 c_p} \right), \]

where \( p^* \) is the stagnation pressure, \( m \) - mass, \( R \) - specific gas constant, \( T^* \) - stagnation temperature, \( V^* \) - stagnation volume, \( v^* \) - stagnation specific volume, \( T \) - temperature, \( c \) - gas speed and \( c_p \) is the specific heat capacity at constant pressure.

From eq. (1) three possibilities of increasing stagnation pressure appear:

- decreasing the stagnation specific volume \( v^* \);
- increasing \( c \), the speed of the working agent
- increasing the temperature \( T \).

Stagnation specific volume \( v^* \) can decrease by reducing the volume \( V \) at constant mass, by increasing the mass of the working agent at constant volume or even in both ways. Devices based on this working principle are termed “positive displacement compressors”, and could be reciprocating or rotary type.

The functioning of so call “dynamic compressors” is based on increasing of the agent speed. First, the agent total energy and its kinetic energy are increased through its acceleration, by various means, e.g. by transfer of work from a bladed rotor - as in centrifugal, axial of diagonal compressors. Second, the kinetic energy is reduced inside channels having diffuser shape, consequently the agent pressure rises. The category of the dynamic compressors also includes the ejectors (also known as “injectors”, “thermal compressors” or “thermocompressors”), devices requiring two jets of agent to operate. Inside
such apparatus a jet of agent is first accelerated through momentum transfer and than is decelerated inside a diffuser. Another jet of agent carries the needed energy for the process.

The third possibility to increase the pressure of an agent is to increase its temperature. Such compressors are using heat directly for increasing pressure, and could be called “thermal compressor”. Due to a possible confusion with ejectors, authors preferred the term “heat-driven compressor” (HDC) for such a device.

It must be mentioned that there are several other devices that use heat to increase the pressure of a working agent. This is the working principle of explosive weapons as bombs and grenades, which are military applications. A charge of explosive material is releasing a large amount of heat in a very small amount of time, tremendously increasing the inside pressure until the fragmenting of the case. In reciprocating internal combustion engines also, the heat released by the fuel combustion leads not only to the temperature increase, but also to the increase of the working agent pressure.

Recently (2016), a patent application for a device named “thermal compressor” was filled [1]. The device has at least two working spaces periodically isolated and heated. The working agent could be in gaseous or liquid state or both. The thermal energy is used directly for compression.

In 2017 Pan et al. [2] published a paper dealing with a different type of “thermal compressor” that uses heat to generate a pressure wave. This thermal compressor is used in conjunction with a pulse tube cryocooler, and the device aims to reach 4 K more efficiently than other solutions.

Investigations about the possibility to use heat for compressing a gas by the means of using a modified Stirling engine was analyzed in [3] for an alpha-type engine powering a two-stage positive displacement compressor and in [4] for a beta-type engine powering a single-stage compressor. The working agent of the compressor must be the same as the working agent of the Stirling engine.

Another “thermal compressor”, coined as “boostHEAT”, was recently studied by Ibsane et al. [5] and [6]. A stage is comprises a single cylinder divided in two working spaces by a displacer piston. The two spaces are connected through a line of heat exchangers – heater, regenerator and cooler – like in a Stirling engine. Intake and discharge valves control the gas exchange. Heat provided by the heater is used to raise the agent pressure up to the discharge pressure. The discharge process takes place at constant pressure, as in a regular compressor.

Heat is used for compression also in hydride compressors, devices used for compressing hydrogen. At low pressures hydrogen is absorbed by metal hydrides. The desorption of hydrogen will take place at high pressure, after the heating process. Such compressors were also studied recently [7], [8].

A theoretical one-stage HDC is the subject of this paper.

2. Heat-driven compressor physical model

The heat-driven compressor (HDC) schematic diagram is presented in figure 1. A HDC unit comprises the following main components: a cylinder 5 containing the working space 4, equipped with suction valve 3 and discharge valve 6. Through suction line 2 and suction valve the working space could communicate with suction tank 1. Through discharge line 7 and discharge valve the working space could communicate with the receiver tank 8. The HDC is exchanging heat through the walls of the cylinder, which materialize both heater and cooler. The heater could use any type of heat (e.g. from solar energy) and the cooler could use any cold agent (e.g. atmospheric air).

The working agent is placed inside a unique working space. When HDC is filled with agent and heated at constant volume, with both valves closed, temperature and pressure inside the working space are rising. This is the thermal compression process, or heating. After reaching the discharge pressure the discharge valve opens, allowing the agent to flow inside the receiver tank. The discharge process continues at constant pressure until the working space is heated up to the maximum temperature of the cycle. During the discharge process a certain mass of agent leaves the working space. The thermal expansion process (or cooling) follows next. After the discharge valve closing, the agent is cooled at constant volume. The process continues until the pressure inside the working space goes down to the suction pressure (this happens at a temperature greater than the suction temperature) and the suction valve is opening. It follows the suction process, when a certain mass of agent enters the working space.
at constant pressure. The cooling of the working space continues during suction, until the suction temperature is reached and the cycle is closed.

The HDC cycle described above comprises two isochoric processes with constant mass, connected by two isobaric processes with variable mass (figure 1).

![Diagram of the HDC cycle](image)

**Figure 1.** Schematic diagram and cycle of the heat-driven compressor: 1 – suction tank; 2 – suction line; 3 – suction valve; 4 – working space; 5 - cylinder; 6 – discharge valve; 7 – discharge line; 8 – receiver tank.

Since the heat exchange is the primal cause behind all processes, the characterization “heat-driven” used for this type of thermal compressor is justified.

3. Hypotheses of the theoretical HDC

The model chosen for analyzing the functioning of the HDC belongs to the family of the theoretical models. The theoretical model does not take into account any possible loss accompanying the real processes. The following hypotheses were assumed:

- the working agent is a perfect gas;
- all processes are quasistatic (at any moment, the thermodynamic parameters are the same in any point of the working space);
- suction and discharge valves are opening and closing instantaneously, at zero pressure difference;
- curtain areas of the valves are large enough that the speed of the agent and the corresponding kinetic energy are negligible in these areas;
- there are no gas leaks during functioning;
- suction and receiver tanks have very large volumes, so suction and discharge pressures are constant;
- the walls of the cylinder enclosing the working space are very thin, so the volumetric heat capacity and the thermal inertia of the cylinder are negligible.

It can be noticed a certain similarity of the hypotheses of the HDC with the hypotheses usually used for the theoretical positive displacement reciprocating compressor [9].

4. Mathematical model

The mathematical model aims to express the thermodynamic parameters in any point of the cycle and to obtain relations for the energies exchanged in each process. Also aims to estimate the performances of the theoretical HDC.

Parameters at the end of the suction process and at the beginning of the compression, \( T_1 = T_{st}, \ p_1 = p_s \) and \( V_1 = V_{ws} = \text{ct.} \) are known. Here \( T_{st} \) is the temperature of the agent inside the suction tank, \( p_s \) is the suction pressure and \( V_{ws} \) is the constant volume of the working space. Maximum heating temperature \( T_{max} \) determines the maximum possible discharge pressure \( p_{d \ max} \). Actual discharge pressure can be established at any level between \( p_s \) and \( p_{d \ max} \).

Pressure and temperature at the end of the thermal compression process 1-2 are:
\[ p_2 = p_d \] 

and 
\[ T_2 = T_1 \frac{p_2}{p_1}. \] 

At the end of the discharge process 2-3 the thermodynamics parameters are: 
\[ p_3 = p_d \] 

and 
\[ T_3 = T_{\text{max}}. \] 

Parameters at the end of the thermal expansion process 3-4 are: 
\[ p_4 = p_s \] 

and 
\[ T_4 = T_3 \frac{p_4}{p_3}. \] 

Thermal compression and thermal expansion are processes with constant mass, so heats exchanged are calculated with: 
\[ Q_{12} = m_1 c_v (T_2 - T_1) \] 

and 
\[ Q_{34} = m_3 c_v (T_4 - T_3), \] 

where \( c_v \) is the specific heat capacity at constant volume while masses \( m_1 = m_2 \) and \( m_3 = m_4 \) are expressed from the equations of state: 
\[ m_1 = \frac{p_1 V_{w_1}}{R T_1} \] 

and 
\[ m_3 = \frac{p_3 V_{w_3}}{R T_3}. \] 

Discharge and suction processes take place in open systems, with variable mass. For such processes the first law of Thermodynamics written for a control volume and for a period of time has the following form [9]: 
\[ \Delta U = Q - W + m_1 \left( h_i + \frac{c_i^2}{2} + g z_i \right) - m_o \left( h_o + \frac{c_o^2}{2} + g z_o \right), \] 

where: \( U \) – internal energy, \( Q \) – heat, \( W \) – work, \( m_i \) – mass of agent that entered inside the control volume, \( h_i, c_i^2/2 \) and \( g z_i \) – specific enthalpy, specific kinetic energy and specific gravitational potential energy of the entered mass, subscripts “i” and “o”- inlet and outlet, \( g \) - standard acceleration due to gravity, \( z \) – elevation above a reference plane, \( m_o \) – mass of agent that flows outside the control volume, \( h_o, c_o^2/2 \) and \( g z_o \) – specific enthalpy, specific kinetic energy and specific gravitational potential energy of the mass that leaves the control volume.

For the discharge process equation (12) is particularized as 
\[ Q_{23} = \Delta U_{23} + \left( m_o h_o \right)_{23}, \] 

since there is no work exchanged by the control volume itself, no mass enters the control volume and kinetic and gravitational potential energy are negligible.
Because $p_d$, $V_{ws}$ and $R$ are constant, it can be inferred from the equation of state that during discharge the product between mass and temperature is constant, $m_d T_d = m_2 T_2 = \text{ct.}$, where $m_d$ and $T_d$ are the variable mass and the variable temperature during discharge. As a consequence, the variation of the internal energy for the agent inside the control volume is

$$\Delta U_{23} = c_v m_1 T_1 - c_v m_2 T_2 = 0.$$  \hfill (14)

The heat received by the gas inside the control volume during discharge is then equal with the enthalpy that leaves the control volume

$$Q_{23} = \left( m_o h_o \right)_{23} = \int_{T_2}^{T_3} h_o \, dm_o = \int_{T_2}^{T_3} -c_p T_d \, d[m_d(T_d)],$$  \hfill (15)

where $h_o = c_p T_d$ and, from continuity considerations, $dm_o = -dm_d$.

For the suction process equation (12) is particularized as

$$Q_{41} = \Delta U_{41} - m_1 h_i.$$  \hfill (16)

From similar considerations as for discharge, $m_s T_s = m_4 T_4 = \text{ct.}$ and $\Delta U_{23} = 0$. Here $m_s$ and $T_s$ are the variable mass and the variable temperature inside the control volume during suction. It must be pointed here that suction temperature $T_s$ is different from the temperature $T_{st}$ inside suction tank. The suction process was considered to be quasi-static, so $T_s$ is obtained after the isobaric mixing of each portion of sucked gas with the agent found inside the control volume.

The heat removed from the gas inside control volume during suction is then equal with the enthalpy that enters the control volume

$$Q_{41} = -m_i h_i = -\int_{T_4}^{T_1} h_i \, dm_i = -\int_{T_4}^{T_1} c_p T_{st} \, d[m_s(T_{st})],$$  \hfill (17)

where $h_i = c_p T_{st}$ and, from continuity considerations, $dm_i = dm_o$.

Several coefficients for evaluating the performances of the HDC can be defined. Mass transfer ratio is defined as ratio between the discharged mass and the mass inside control volume at the beginning of the discharge process:

$$\eta_m = \frac{m_1 - m_2}{m_2}.$$  \hfill (18)

Efficiency of the HDC is defined as ratio of the enthalpy that leaves the control volume with discharged mass during a cycle and the heat received by the agent inside the control volume, also during a cycle:

$$\eta_i = \frac{\int_{T_2}^{T_3} h_o \, dm_o}{Q_{12} + Q_{23}}.$$  \hfill (19)

The efficiency in term of enthalpy increase of the discharged mass is

$$\eta_h = \frac{\int_{T_2}^{T_3} h_o \, dm_o - (m_2 - m_1) c_p T_{st}}{Q_{12} + Q_{23}}.$$  \hfill (20)

Equations (18), (19) and (20) could be useful for comparing different theoretical HDC.
5. Results of the numerical simulation of the HDC and discussion

For a HDC (figure 1) described by the following values: \( V_{ws} = 1 \text{ m}^3 \), \( p_s = 100000 \text{ Pa} \), \( T_{st} = 283 \text{ K} \), \( T_{max} = 353 \text{ K} \), \( p_d = 0.75 \cdot p_{d_{max}} \), performances presented in figures 2 and 3 were calculated.

Dry air, considered to be a perfect gas, was chosen as working agent for this example. Dry air molar composition \([10]\) is 78.084 \% N\(_2\), 20.9476 \% O\(_2\), 0.9365 \% Ar, 0.0319 \% CO\(_2\). Based on molar gas constant \( R_{μ} = 8314.51 \text{ J kmol}^{-1} \text{ K}^{-1} \), molar composition and component properties, the following values were calculated: average molar mass of air \( μ_{air} = 28.965 \text{ kg kmol}^{-1} \), specific gas constant of dry air \( R_{air} = 287.052 \text{ J kg}^{-1} \text{ K}^{-1} \), specific heats at constant pressure and at constant volume for dry air \( c_p = 1002.04 \text{ J kg}^{-1} \text{ K}^{-1} \) and \( c_v = 714.99 \text{ J kg}^{-1} \text{ K}^{-1} \).

For the entropic diagrams the entropy was calculated using \( S = m \cdot s \) and the specific entropy of the perfect gas \( s \) was calculated with the defining formula \([9]\)

\[
s(T, p) = c_p \ln \left( \frac{T}{T_0} \right) - R \ln \left( \frac{p}{p_0} \right),
\]

where \( T_0 = 200 \text{ K} \) and \( p_0 = 100000 \text{ Pa} \) are thermodynamic parameters of the arbitrary origin of the specific entropy.

![Thermodynamic diagrams](image)

**Figure 2.** HDC cycle represented in thermodynamic diagrams:
(a) – \( T-Δs \); (b) – \( T-ΔS \); (c) – \( p-T \); (d) – \( T-m \); (e) – \( p-v \).
The mass of working agent discharged during a cycle is $\Delta m_d = m_2 - m_3$. The actual discharge pressure $p_d$ was calculated using a factor “$f$”, defined as $f = p_3 + f(p_d \text{ max} - p_d)$. For the numerical example it was considered $f = 0.75$. The compression ratio of the HDC, $R_c$, is the ratio between discharge and suction pressures.

For the initial values mentioned before the following performances of the HDC were calculated: $\Delta m_d = 0.061 \text{ kg}$, $\eta_m = 0.0496$, $\eta_t = 0.3129$ and $\eta_h = 0.0556$. Heats exchanged in each process are $Q_{12} = 46207 \text{ J}$, $Q_{23} = 21042 \text{ J}$, $Q_{34} = -46207 \text{ J}$ and $Q_{41} = -17305 \text{ J}$.

In figure 2 the thermodynamic cycle of the HDC is represented in several diagrams – (a), (b), (c), (d) and (e). The $p-V$ diagram was intentionally omitted, since the total volume of the working space $V_{ws}$ is obviously a constant. All diagrams show the correlations between thermodynamic parameters, and also the variations of these parameters for each process.

First diagram, (a) in figure 2, shows a clockwise $T-\Delta s$ cycle, meaning that the sum of heats added per unit mass is larger than the absolute value of the sum of heats removed from the system per unit mass. The HDC cycle in $T-\Delta S$ diagram, figure 2 (b), goes counterclockwise. This happens due to the entropy variations associated with the matter exchanged by the system, which act besides the variation of entropy associated with the heat transfer.

The clockwise cycle in $p-v$ diagram, figure 2 (e), shows that HDC exchanges specific work with the surroundings during the cycle. This specific work is in fact the specific flow work and has a positive value, because the sum of the specific flow work works performed and received is positive; the work performed refers to the system that is pushing out some mass of agent while the work received refers to the same mass entering the system.

![Figure 3](image-url)

**Figure 3.** Evaluation of the performances of the analyzed HDC: (a) – cyclically discharged mass $\Delta m_d$ function of maximum heating temperature $T_{max}$, with factor “$f$” as parameter; (b) – mass transfer ratio $\eta_m$ function of compression ratio $R_c$, for various values of $T_{max}$; (c) – efficiency of the HDC $\eta_t$ function of compression ratio, for various values of $T_{max}$; (d) – efficiency in term of enthalpy increase of the discharged mass $\eta_h$ function of compression ratio, for various values of $T_{max}$.

Figure 3 (a) shows that the mass discharged per cycle increases with $T_{max}$. Obviously, $\Delta m_d$ decreases with the factor $f$ and consequently with discharge pressure, as in a regular reciprocating compressor. Curves in figure 3 (b) show that mass transfer ratio $\eta_m$ defined by equation (18) linearly decrease with discharge ratio $R_c$. Efficiency of the HDC, $\eta_t$, figure 3 (c), decreases when compression ratio increases. This efficiency takes larger values when maximum heating temperature increases.
Efficiency in terms of enthalpy increase of the discharged mass $\eta_h$ - and also the numerator of the fraction in equation (20) - reaches a maximum value for a certain compression ratio, as can be observed in figure 3 (d). The small increase of $\eta_h$ observed for smaller compression ratios is explained by the dependence of the mass discharged cyclically $m_2 - m_3$ by the discharge pressure.

6. Conclusions
The HDC comprises a non-deformable container (“cylinder”) equipped with suction and discharge valves. The theoretical thermodynamic cycle is made of two isochoric processes connected by two isobaric processes, the last two being performed with variable mass. The variable mass processes are also characterized by the equation $m T = cT$. The increase of pressure is obtained by heating the working agent inside the cylinder. After the opening of the discharge valve the heating continues up to the maximum heating temperature; a portion of the mass inside the cylinder being discharged in this process. After the end of the discharge process the agent inside the cylinder in cooled until the suction valve opens. Suction process takes place at constant pressure, due to the cooling of the agent down to the minimum temperature inside the cycle.

The main advantage of the HDC is its pure thermal actioning. The apparatus functions driven by heat exchange only; no work from outside is needed for compressing the working agent. There are several other advantages of the HDC: any heat source can be effectively used for heating, there are no moving parts like pistons and rods, the discharged agent is free of oil. If solar energy is used for heating, HDC becomes an environmentally friendly technology. If the working agent is atmospheric air, a solar heated HDC can power a compressed air energy storage installation. In this case the energy stored inside the compressed air could be termed as green energy.

There are some notable disadvantages of HDC. Cooling and heating take place in the same functional space, so for a real HDC the thermal inertia of the cylinder walls must be taken into account. Unavoidable, such HDC will experience unnecessary heat transfers, and the duration of the cycle will be a very large one. Another disadvantage is the fact that the compression ratio of a HDC stage is limited, being smaller than the compression ratio of a regular stage of a reciprocating compressor. A larger compression ratio could be obtained only in a multi-stage HDC.

In spite of its theoretical character, the proposed model allows to get a clear picture of the functioning of the HDC. The thermodynamic analysis shows that the mass of agent discharged cyclically by HDC becomes larger when maximum heating temperature is higher. Besides, $\Delta m_d$ becomes smaller when discharge pressure is higher. The decrease of the discharged mass when discharge pressure increases is a behavior similar with the behavior of the reciprocating compressor, for which the mass flow diminishes when discharge pressure increases [9].

The three efficiencies proposed for assessing the functioning of the HDC - and mathematically defined inside this paper - demonstrate the need of using maximum heating temperatures as higher as possible. They also show that the maximum heating temperature set the maximum possible value of the discharge pressure.

7. References
[1] Arapkoules N K 2016 Thermal compressor US Patent Application US20160201658A1
[2] Pan CZ, Wang J, Zhang T, Wang JJ and Zhou Y Numerical investigation on the thermoacoustics characteristics of thermal compressor for the pulse tube cryocooler 2017 Applied Thermal Engineering 123 pp 234-42
[3] Homutescu V M Kinematic Stirling motor-driven compressors 2005 Proceedings of CNEI 2005 Conference “Modelling and Optimization in the Machine Building Field”, Bacău, Romania MOCM-11 vol. 3 (Bacău: ALMA MATER Publishing House) pp 49-52
[4] Homutescu V M Maximum performances of the Stirling machine working as motor-driven compressor 2013 Bul. LP.I. LIX 3 pp 27-38
[5] Ibsaine R, Joffroy J-M and Stouffs P 2014 A new heat driven compressor for heat pump application Proceedings of the 16th International Stirling Engine Conference: ISEC 2014:
16th International Stirling Engine Conference, 24-26 September 2014, Bilbao, Spain (Ancona: Stirling International) pp 271-79

[6]  Ibsaine R, Joffroy J-M and Stouffs P Modelling of a new thermal compressor for supercritical CO2 heat pump 2016 Energy 117 pp 530-9

[7]  Popeneciu G, Almășan V, Coldea I, Lupu D, Mișan I and Ardelean O Investigation on a three-stage hydrogen thermal compressor based on metal hydrides 2009 J. Phys.: Conf. Ser. 182 5 pp

[8]  Gkanas E I and Khzouz M Numerical analysis of candidate materials for multi-stage metal hydride hydrogen compression processes 2017 Enewable Energy 111 pp 484-93

[9]  Horbaniuc B 2015 Termodinamică tehnică vol. 1 (Bucharest: AGIR Publishing House)

[10] Gordon S 1982 Thermodynamic and Transport Combustion Properties of Hydrocarbons With Air NASA Technical Paper 1906-pt.1

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9