A modified partially averaged Navier-Stokes model for the turbulent flows over a backward facing step

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Abstract. In current study, a partially averaged Navier-Stokes model based on a modified SST $k-\omega$ model is proposed (MSST PANS) to predict the turbulent flows with flow separations, recirculation and reattachment. A benchmark case, turbulence flows over the backward facing step ($Re=50000$), is treated to evaluate the capacity of the MSST PANS model. Several other turbulence models, i.e., the standard $k-\varepsilon$ model, SST $k-\omega$ model, the standard $k-\varepsilon$ PANS (SKE PANS) model and SST $k-\omega$ PANS (SST PANS) model are also implemented for comparisons with some available experimental data. Results show that the MSST PANS model performs the closest results in predicting the reattachment length as well as the corner vortex. Furthermore, the MSST PANS model yields improved statistics on the skin frictions, pressures, velocity profiles together with Reynolds stresses. In contrast to the SST PANS models, the modifications on the eddy viscosity and $\omega$ equation with consideration of streamline curvature show promising in capturing the small-scale flow separations, recirculation and reattachments in some industrial applications.

1. Introduction

Flow separation, recirculation and reattachment occur in many engineering configurations, such as the draft tube of Francis turbines, combustors, airfoils and cascades. Owing to the complexity of the geometries, it becomes difficult to visualize the internal flows in detail by means of the traditional experimental measurements. The numerical research has gradually been a valuable tool with the development of the Computation Fluid Dynamics (CFD). The backward facing step is regarded as the simplest benchmark case to investigate the flow separation near the step corner. In 1985, Driver and Seegmiller [1] carried out the incompressible turbulence flow analysis at a high $Re=50000$ over a...
rearward-facing step in a diverging channel flow by a laser Doppler velocimetry (LDV). Several parameters as well as the triple products are compared with different turbulence models. It is indicated in this study that the modifications of the algebraic-stress model (ASM) substantially improved numerical calculations. Kim et al. [2] provided a validation study for different near-wall treatment methods and turbulence models for the backward facing step at Re=38000. In his study, the non-equilibrium wall functions with RNG $k$-$\varepsilon$ and Realizable $k$-$\varepsilon$ models showed the best results. To explore the anisotropy effect of eddy viscosity, Yang et al. [3] presented a numerical study of turbulent flow past a backward-facing step with six linear and non-linear turbulence models. In the case of fully developed turbulent flow, the non-linear models tested in this work yield considerably better predictions than those obtained by the linear models owing to the turbulent non-equilibrium effect.

As mentioned above, although the Reynolds-averaged Navier–Stokes (RANS) models have been applied successfully in some industrial configurations, it still shows insufficient in achieving an accurate predictions in massively separated flows [4]. Except for the Direct Numerical Simulation (DNS) and the Large eddy simulation (LES), whose computation cost is too large to afford for the turbulent flows at a high Reynolds number [5], the non-linear RANS model is more suitable to capture the detailed separated flows. The Detached eddy simulation (DES), which was conceived for aerodynamic flows, is regarded as a hybrid turbulence model [6]. It solves the entire boundary layer with RANS, and the freestream away from solid surface with LES. Inspired by the turbulence bridging methods, the Partially-averaged Navier–Stokes model (PANS) is proposed recently by Girimaji [7]. It is regarded as a bridge from RANS to DNS by two parameters: the unresolved-to-total ratios of kinetic energy ($f_k$) and dissipation ($f_\varepsilon$). Relevant comparisons on these two hybrid models can be referred in [1].

The Partially-averaged Navier–Stokes model (PANS) is a relative advanced turbulence model, which has been utilized in many test cases including the flow around a cylinder [8], cavitating flows around a hydrofoil [9], turbulence flow over a backward facing step [10], and internal flow in a centrifugal pump [11]. It is demonstrated in these researches that the PANS model is suitable to simulate turbulence flows with a reasonable computational expense. Recently, Luo [4] developed a SST PANS models based on the Menter SST $k$-$\omega$ turbulence model [12], and it shows advantage in predicting the flow separation in the region the near the solid wall. Whereas, the accuracy is still inadequate, especially near the flow separation region with consideration of streamline curvature. Thus, further modifications of the turbulence model are needed to improve the predictions.

In this study, a modified SST $k$-$\omega$ partially averaged Navier-stokes (MSST PANS) model is proposed to evaluate its capacities in simulating the turbulent flows over a backward-facing step. The standard $k$-$\varepsilon$ model, SST $k$-$\omega$ model, standard $k$-$\varepsilon$ PANS (SKE PANS) model and SST $k$-$\omega$ PANS (SST PANS) model are also treated for comparisons. Detailed analyses are carried out for the calculated results and the available experimental data.

2. Modeling formulations

The PANS model was described in detail in [13]. In this section, formulations of the modified SST $k$-$\omega$ Partially-averaged Navier–Stokes (MSST PANS) model is introduced.

Shu [14] proposed a modified SST $k$-$\omega$ (PRSST) model, which combines the Wilcox $k$-$\omega$ model with the realizable $k$-$\varepsilon$ model [15]. The two equations are shown below:
where $k$ is the turbulent kinetic energy, $\omega$ is the turbulent eddy frequency. $P_k$ is the production term, $S$ represents the invariant measure of the strain rate. $\nu$ is the kinematic viscosity, $\mu$ is the dynamic viscosity and $\mu^*$ is the turbulent eddy viscosity, defined in equation (3). $F_1$ is the first blending function and $F_2$ is a second blending function. $C_1$ is a fundamental parameter in the realizable $k$-$\varepsilon$ model defined in equation (4).

\[
\mu^* = \min(\mu_{rea}, \frac{P_{u_k}}{S_{k^2}}); \mu_{rea} = \rho C_\mu \frac{k^2}{\varepsilon}; C_\mu = \left( A_0 + A_1 k^2/\varepsilon \right)^{-1};
\]

\[
A_0 = 4.0; A_1 = \sqrt{6} \cos \phi; \phi = \frac{1}{3} \cos^{-1}(\sqrt{6} W); W = \frac{E_y E_y}{(E_y E_y)^{3/2}}; E_y = \frac{1}{2} \left( \frac{\partial U_j}{\partial x_j} + \frac{\partial U_j}{\partial x_j} \right);
\]

\[
U^* = \left( E_y E_y + \Omega_y \Omega_y \right)^{1/2}; \Omega_y = \frac{1}{2} \frac{\partial U_j}{\partial x_j} - \frac{1}{2} \frac{\partial U_j}{\partial x_j}.
\]

where $\mu_{rea}$ is the eddy viscosity defined in the realizable $k$-$\varepsilon$ model [15], which is based on the non-linear and anisotropy hypothesis. $a_1=0.31$.

\[
C_1 = \max(0.43, \frac{\eta}{\eta + 5}); \eta = (2 E_y E_y)^{3/2} \frac{k}{\varepsilon}; E_y = \frac{1}{2} \frac{\partial U_j}{\partial x_j} + \frac{\partial U_j}{\partial x_j}
\]

All the coefficients are calculated by blending via $\alpha=F_1\alpha_1+(1-F_1)\alpha_2$. The model constants are: $\gamma_1=5/9$, $\gamma_2=0.44$, $\beta_1=0.075$, $\beta_2=0.0828$, $\sigma_1=1.1765$, $\sigma_2=1.0$, $\sigma_{\omega_1}=2$, $\sigma_{\omega_2}=1.168$, $\beta=0.09$ and $C_2=1.9$.

In the PANS model, the unresolved turbulent kinetic energy $k_u$ and its eddy frequency $\omega_u$ are determined by two parameters, $f_k$ and $f_\omega$.

\[
f_k = \frac{k_u}{k}; f_\omega = \frac{\omega_u}{\omega} = \frac{f_u}{f_k}
\]

where $f_k$ and $f_\omega$ represent the unresolved-to-total ratios of turbulence kinetic energy and turbulence eddy frequency, respectively.

The two equations of the MSST PANS are then displayed herein.

\[
\frac{\partial (\rho k_u)}{\partial t} + \frac{\partial (\rho k_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu^*_u}{\sigma_{\omega_u}} \frac{\partial \rho k_u}{\partial x_j} \right) + P_{\omega_u} - \beta' \rho \omega_u k_u
\]
\[
\frac{\partial (\rho \omega_u)}{\partial t} + \frac{\partial (\rho \omega_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu \frac{\partial \omega_u}{\partial x_j} \right) + \frac{\gamma}{k^2} \frac{\partial}{\partial x_j} \rho \omega_u P_u - \beta \frac{\partial \omega_u^2}{\partial x_j} + (1 - F_2) \rho \left( C_1 \frac{\beta' S \omega_u}{k} - C_2 \frac{k}{\sqrt{\beta' v_k \omega_u}} \right) + 2(1 - F_2) \sigma_{\omega^2} \frac{\rho \frac{\partial k}{\partial x_j} \frac{\partial \omega_u}{\partial x_j}}{\omega}
\]

(7)

The modified model coefficients are:

\[
\beta' = (1 - f_\kappa) \gamma \beta' + \beta f_\kappa; P_u = \min \left( P_c, 10 \rho \beta' \kappa \omega \right); \sigma_{\omega_u} = \sigma_{\kappa} \frac{f_\kappa}{f_k}; \sigma_{\omega^2} = \sigma_{\kappa} \frac{f_\kappa}{f_k}
\]

(8)

At high-Reynolds number cases, Girimaji proposed a spatially varying function to determine the optimum value of \( f_\kappa \) upon the local grid size and turbulence length scale similar to Kolmogorov scale in advance.

\[
f_\kappa (x) = \min \left( 1, \frac{1}{\sqrt{C_\mu}} \left( \frac{\Lambda^2}{\Lambda} \right) \right)
\]

(9)

where \( C_\mu \) is model coefficient. \( \Lambda \) is the Taylor turbulence length scale, which defined as \( \Lambda = k^{1.5}/\varepsilon \). \( \Delta \) stands for the local grid size, defined as \( \Delta = (\Delta x^* \Delta y^* \Delta z^*)^{1/3} \).

3. Computational details

3.1 Computational domain

The computation domain of the backward facing step [1] is a two-dimensional step with a large aspect ratio (tunnel-width-to-step-height ratio of 12), which could minimize three-dimensional effects in the separated region. In order to save the computational expenses, it is available to focus on the flow separations at the middle section. Hence, the step span is set to \( H \) wide [10]. As depicted in figure 1(a), the step height is \( H = 0.0127m \), the inlet duct is \( 8H \) high×4\( H \) long, and outlet tunnel is \( 9H \) high×50\( H \) long. The small expansion ratio \( [(8H + H)/8H = 1.125] \) is set to minimize the freestream pressure gradient owing to a sudden expansion. According to the experimental data, the Reynolds number is 50000 at inlet section, which make the flow in a fully turbulent state over the step.

![Figure 1](image-url)
The refined hexahedral meshes near the step is carefully treated to satisfy the requirement of the \( k-\omega \) model in the region of boundary layer as shown in figure 1(b). In order to achieve satisfying flow structures and accurate results, six sets of meshes schemes are employed for the entire computational domain (shown in the figure 1(a)) to test the mesh independence with MSST PANS model by monitoring the reattachment length \( (X_r) \). Results are shown in the Table 1. Take the solution accuracy and computation resources into consideration, Mesh 5 is employed as the final mesh scheme in the following simulations, which is relative finer than that utilized by Huang et al. [10].

### Table 1. Mesh independence test.

| Mesh case | Grid       | \( X_r \) (m) |
|-----------|------------|---------------|
| Mesh1     | 350×70×20  | 0.072         |
| Mesh2     | 450×100×20 | 0.074         |
| Mesh3     | 600×140×20 | 0.077         |
| Mesh4     | 800×170×20 | 0.078         |
| Mesh5     | 970×230×20 | 0.080         |
| Mesh6     | 1050×300×20| 0.080         |

#### 3.2 Boundary conditions

Current study performs the steady incompressible three-dimensional simulations with different turbulence models by finite volume method in the commercial CFD code ANSYS CFX. The high resolution was employed in the advection scheme and turbulence numerics. Pressure based coupled solver was performed in the ANSYS CFX. As for the boundary conditions, a velocity profile measured by the experiment [1] was prescribed at the inlet section, which matches the experimental condition. Based on the wall boundary-layer thickness \((1.5H)\), the “Intensity and Length Scale” was chosen for the turbulence option at inlet plane, which contributes to achieving a better convergence (lower than \(1\times10^{-6}\)). A static pressure was imposed at the outlet section. For the lateral side walls, a translational periodicity boundary was set to remove the sidewall effects. In addition, non-slip boundary conditions are assigned on the solid walls. Steady calculation was adopted with a fixed time step of \(2.87\times10^{-4}\)s.

### 4 Results and discussion

#### 4.1 Reattachment Length

Table 2 shows the predicted reattachment lengths \( (X_r) \) using different turbulence models. As stated by [1], the reattachment length is a sensitive parameter that is utilized to evaluate the predictive capability of different turbulence models. In the reference experiment, the \( X_r \) is measured by the oil-flow laser interferometer. The averaged value of \( X_r/H \) is 6.26, ranging from 6.16 to 6.36. As demonstrated in Table 2, the relative value of reattachment length \( (X_r/H) \) is different. The standard \( k-\varepsilon \) model reaches the poorest result with \( X_r/H=5.35 \). The eddy viscosity is overestimated severely, which causes the shear layer to spread rapidly and the flow to reattach prematurely. Owing to the apparent adverse pressure gradient shown in this experiment, the SST \( k-\omega \) model reaches much better results with the value of \( X_r/H = 5.98 \). However, the SKE PANS model and SST PANS model all over-predicted with the relative error of 5.9% and 8.5%, respectively. In consideration of the strong streamline curvature flows near the
step, the MSST PANS model achieves the most accurate results with $X_r/H=6.29$, which is in the range of the experimental measurement.

Table 2. Results on the reattachment length with different models.

| Turbulence model   | $X_r$ | $X_r/H$ |
|--------------------|-------|---------|
| Standard $k$-$\varepsilon$ | 0.068 | 5.35    |
| SST $k$-$\omega$    | 0.076 | 5.98    |
| SKE PANS            | 0.084 | 6.61    |
| SST PANS            | 0.086 | 6.77    |
| MSST PANS           | 0.080 | 6.29    |
| Exp. [28]           |       | 6.16~6.26 |

The streamlines over the step corner using different turbulence models are also displayed in figure 2. It is noted that the apparent recirculation region is captured in all turbulence models. However, the standard $k$-$\varepsilon$ model as well as the SKE PANS model, fail to predict the small-scale corner vortex. While it is captured successfully by Huang et al. [10] using low $f_k$ value PANS model. This small-scale corner vortex is also captured successfully using the SST $k$-$\omega$ model, SST PANS model and the MSST PANS model. Among these three models, MSST PANS model predicts the most reasonable flow structure considering the predicted $X_r$ value.

Figure 2. Streamlines over the backward facing step using five turbulence models.

4.2 Eddy viscosity distributions

To explain the mechanism of the discrepancies, the comparisons on eddy viscosity are also denoted in figure 3. The eddy viscosity is overestimated severely by the standard $k$-$\varepsilon$ model, causing overprediction of the turbulence shear stress. The streamlines finally reattach prematurely as shown in the figure 2(a). Although the wall-bounded region is well treated by the SST $k$-$\omega$ model, the eddy viscosity is still relatively high. On the contrary, the eddy viscosity is alleviated remarkably when using the PANS models. The eddy viscosity predicted by the SST PANS model is smallest. As a consequence, the recirculation is suppressed too late after the step corner, and the reattachment length reaches the highest. Nonetheless, the eddy viscosity predicted by the MSST PANS model is between the SKE PANS model and the SST $k$-$\omega$ model. The reattachment length value is therefore in between. Thus, the internal flow patterns are strongly related with the eddy viscosity.
4.3 Skin friction and pressure distributions

Figure 4 presents the distributions of skin friction coefficient ($C_f$) and pressure coefficient ($C_p$) along the centerline of the bottom wall. The $C_f$ and $C_p$ are defined in equation (10) and equation (11).

$$C_f = \frac{\tau_w}{0.5 \rho U^2_{\infty}}$$  \hspace{1cm} (10)

$$C_p = \frac{p - p_{\infty}}{0.5 \rho U^2_{\infty}}$$  \hspace{1cm} (11)

where $\tau_w$ is the skin friction, $\rho$ is the density, $U_{\infty}$ and $p_{\infty}$ are the reference velocity and static pressure respectively at the inlet section of the duct ($x/H=-4$).

Both the skin friction and pressure coefficient display a valley near the recirculation region, which is predicted by all the turbulence models. The $C_f$ and $C_p$ return to the positive value after the reattachment point. As depicted in figure 4 (a), the MSST PANS model shows the best results within only 8% relative error of $C_f$ in the recovery region ($x/H>15$). On the contrary, the standard $k-\varepsilon$ model overestimates the $C_f$ due to its overestimation in eddy viscosity while the SKE PANS and SST PANS model underestimate the $C_f$ compared with the MSST PANS model owing to its lower eddy viscosity. The results in the figure 4 (b) show that in the recovery region, all the turbulence models predict the same tendency in the $C_p$. However, deviations have been noticed in the recirculation region. The $C_p$ drops in a much lower valley and rises prematurely using the standard $k-\varepsilon$ model. In contrast, compared with the MSST PANS model, the SST PANS model predicts the slightly higher pressure in this region, but the relative error in the region of 5<$x/H<$10 is larger. Thus, it can be concluded that the MSST PANS model yields an improved accuracy among these models.

4.4 Streamwise velocity profiles

Since the streamwise velocity profiles are associated with the internal flow patterns significantly, figure 5 compares the streamwise velocity profiles acquired by the five turbulence models and experimental data. At position of $x/H=2$ and $x/H=4$, the velocity gradient is almost zero in the shear layer where the recirculation occurs. Results predicted by the five turbulence models agree quite well with the experimental data. At $x/H=6$, 8 and 10, the velocity profiles are all underestimated by the SKE PANS model, SST PANS model and MSST PANS model, due to the difference of the reattachment length. As shown in the figure 2, the reattachment length predicted by the SST PANS model reaches the highest, which still affects velocity profiles at $x/H=6$, 8 and 10. On the contrary, the RANS models, i.e., standard $k-\varepsilon$ model and the SST $k-\omega$ model are approaching the experimental data owing to the lower
reattachment length. The MSST PANS model gives a compromise value between SST $k-\omega$ model and SST PANS model, which is the most reasonable results.

![Diagram](image.png)

**Figure 4.** (a) skin friction and (b) wall static pressure distributions.

4.5 Reynolds stresses

Having compared the mean flow information as stated below, higher order moments, i.e., normal Reynolds stresses is further compared with the available experimental data. Figure 5 and presents the normal stresses and shear stresses profiles at different positions. The normal Reynolds stresses behave the same tendency at different positions. As the flow propagates downstream, the maximum of the normal Reynolds stresses ($\langle uu+vv\rangle/2U_r^2$) increases from $x/H=2$ to $x/H=4$ and decreases right after the $x/H=4$. The normal Reynolds stresses vary remarkably near the step corner, i.e., $y/H<1$, where the reattachment occurs. As the $y/H>1$, the normal Reynolds stresses begin to decay and remain is a very low value. The numerical results predicted by different turbulence models show good tendency with the experimental data. At the recirculation region, such as $x/H=2$, MSST PANS simulations give fairly good agreement with the measurements, slightly better than the SKE PANS and SST PANS model. At the position of $x/H=4$ and $x/H=6$, the MSST PANS model underestimates the normal Reynold stresses as the $y/H<0.5$. However, in the region $0.5<y/H<1$, the MSST PANS model matches with the experimental data best. After the reattachment region, the MSST PANS results also show good agreement with the measurements. Thus, the MSST PANS model can simulate the convection of both normal stresses into the far freestream as revealed by the experiments.

5. Conclusions

A modified SST $k-\omega$ PANS (MSST PANS) model is proposed in current study, which treats the modified SST $k-\omega$ model as the parent model with consideration of the streamline curvature. A benchmark case, backward facing step, is calculated using the MSST PANS model as well as other four turbulence models, i.e., standard $k-\varepsilon$ model, SST $k-\omega$ model, the standard $k-\varepsilon$ PANS (SKE PANS) model and SST $k-\omega$ PANS (SST PANS) model. To evaluate their capabilities on predicting the reattachment and recirculation with high shear rate and large curvature, the calculation results are compared with the available experimental
Owing to the strong dissipation of the standard $k$-$\varepsilon$ model, the standard $k$-$\varepsilon$ model and the SKE PANS model fail to reproduce the corner vortex around the step corner. The attachment length predicted by the standard $k$-$\varepsilon$ model is far away from the measurement data.

According to the experimental data, the MSST PANS model gives the most reasonable eddy viscosity value, which is between the SKE PANS model and the SST $k$-$\omega$ model. The MSST PANS model gives the most accurate reattachment length and improves the results of the skin friction, pressure coefficient and velocity profiles especially in the recirculation region.

Although some deviations occur near the wall-bounded region in predicting the Reynold stress, the MSST PANS model still performs the best capability to capture the recirculation flows and weak flows.

In general, the current MSST PANS model is a promising and encouraging method in industrial turbomachines for predicting the strong swirling flows with high shear rate and large curvature. Further investigations will be carried out in the high rotation speed and cavitating flow cases.

**Figure 5.** Streamwise velocity profiles at position of $x/H=2$, 4, 6, 8 and 10.

**Figure 6.** Profiles of normal stresses at position of $x/H=2$, 4, 6, 8 and 10.

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Reference

[1] Driver D M and Seegmiller H L 1985 Features of a reattaching turbulent shear layer in divergent channel flow AIAA J. 23 163-71

[2] Kim J Y, Ghajar A J, Tang C and Foutch G L 2005 Comparison of near-wall treatment methods for high Reynolds number backward-facing step flow Int. J. Comput. Fluid Dyn. 19 493-500

[3] Yang X D, Ma H Y and Huang Y N 2005 Prediction of homogeneous shear flow and a backward-facing step flow with some linear and non-linear k-ε turbulence models Commun. Nonlinear Sci. Numer. Simulation 10 315-28

[4] Luo D H, Yan C, Liu H K and Zhao R 2014 Comparative assessment of PANS and DES for simulation of flow past a circular cylinder J. Wind Eng. Ind. Aerodyn. 134 65-77

[5] Dittakavi N, Chunekar A and Frankel S 2010 Large eddy simulation of turbulent-cavitation interactions in a venturi nozzle J. Fluids Eng.-Trans. ASME 132 121301

[6] Schiestel R and Dejoan A 2005 Towards a new partially integrated transport model for coarse grid and unsteady turbulent flow simulations Theor. Comput. Fluid Dyn. 18 443-68

[7] Girimaji S S and Abdol-Hamid K S 2005 Partially-averaged navier stokes model for turbulence: Implementation and validation 43rd AIAA Aerospace Sciences Meeting and Exhibit 10-13 January Reno, Nevada 12887-12900

[8] Lakshmipathy S and Girimaji S S 2010 Partially Averaged Navier-Stokes (PANS) method for turbulence simulations: Flow past a circular cylinder J. Fluids Eng.-Trans. ASME 132 121202

[9] Luo X W, Huang R F and Ji B 2016 Transient cavitating vortical flows around a hydrofoil using k-ω partially averaged Navier-Stokes model Mod. Phys. Lett. B 30 1550262

[10] Huang R F, Luo X W, Ji B and Ji Q F 2017 Turbulent Flows Over a Backward Facing Step Simulated Using a Modified Partially Averaged Navier-Stokes Model J. Fluids Eng.-Trans. ASME 139 044501

[11] Ye W X, Luo X W, Huang R F, Jiang Z W, Li, X J and Zhu Z C 2019 Investigation of flow instability characteristics in a low specific speed centrifugal pump using a modified partially averaged Navier–Stokes model Proc. Inst. Mech. Eng., Part A 233 834-48

[12] Menter F R 1994 2-Equation Eddy-Viscosity Turbulence Models for Engineering Applications. AIAA J. 32 1598-1605

[13] Girimaji S S 2006 Partially-averaged navier-stokes model for turbulence: A reynolds-averaged navier-stokes to direct numerical simulation bridging method J. Appl. Mech.-Trans. ASME 73 413-21

[14] Shu M H 2011 Research on Turbulence Models of Numerical Simulation for Inner Flow in Centrifugal Pumps Jiangsu University (Chinese)

[15] Shih T H, Liou W W, Shabbrir A, Yang Z and Zhu J 1995 A new k-ε eddy viscosity model for high reynolds number turbulent flows Comput. Fluids 24 227-38