Massive Graviton on a Spatial Condensation Web

Chunshan Lin
Kavli Institute for the Physics and Mathematics of the Universe (WPI),
Tokai Institutes for Advanced Study, University of Tokyo,
5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan

INTRODUCTION

In gauge field theory, the Higgs mechanism spontaneously breaks the gauge symmetry, giving the gauge field a mass. Whether such mechanism can be applied to gravity and get a self-consistent massive spin-2 field theory is a basic question in the classical field theory. After the pioneering attempt at 1939 [1], this direction has been attracting a great deal of interest, but its consistency has been a challenging problem for several decades.

One of the most profound problems of the massive gravity is the ghosty sixth mode on the gravity sector, which was found by Bolware and Deser in 1972 [2]. The BD ghost generally appears at the nonlinear massive gravity theory, where those nonlinear terms in the action were introduced to heal the discontinuity problem of the Fierz-Pauli theory [3][4]. Because of the BD ghost, the Hamiltonian of the system is unbounded from below, which spoils the stability of our theory.

An important breakthrough on the way of conquering the BD ghost was in 2002 [5]. As pointed out by the authors of [6], by adopting the effective field theory at the decoupling limit, in principle we can eliminate the BD ghost by the construction of our massive gravity theory. Indeed, such type of theory was achieved in 2010, which now is dubbed as dRGT gravity [7].

However, the following up cosmological perturbations analysis revealed a new ghost instability among the rest five degrees of freedom [8][9][10][11][12]. On the other hand, this theory may also suffer from the acausality problem [13][14].

In this short notes, we propose a simple and elegant scenario of the spatial condensation, which can be considered as a massive gravity theory. The idea is to introduce 3 canonical free scalar fields to gravity sector, and these 3 scalars have nontrivial background vacuum expectation value. The graviton gets a mass by “eating” the Goldstone excitations of these 3 scalars.

As an example of the application, we apply our massive gravity to early universe. It is known that Inflationary paradigm [15] has become a very convincing scenario of the early universe. The quantum fluctuation during inflation seeds the large scale structure and CMB anisotropies nowadays. However, the power spectra of the primordial perturbation suffers from the infrared (IR) divergence and ultraviolet (UV) divergence, if we take into account the contributions from the loop correction. These divergences were firstly noticed in the early work [16][17][18], and has been bothering the theorist for couple of decades (see the recent reviews [19][20] and the references therein).

Although we can always remove such divergence by introducing the IR and UV cutoff, loop diagram still remain growing at super horizon scale. In addition to this “brute force” cutoff regularization [21][22][23], people have invented some other ways to get rid of those divergences, E.g., dimensional regularization [24], Pauli-Villars regularization [25]. All those regularizations should lead to the same results [26].

In this short notes, we focus on the IR divergence. It is known that the scale invariant spectrum in the de sitter space time leads to the logarithmically divergence in the IR. Under the super horizon approximation, all those IR divergence terms cancel out at the leading order of $O(\epsilon)$ [19], where $\epsilon$ is the slow roll parameter of inflation. However, this result may not apply if we include the higher order term with $O(\epsilon^2)$. On the other hand, away from the super-Hubble approximation, such magic cancellation doesn’t happen either. Thus the IR divergence problem is still remained as an open question.

However, in our spatial condensation scenario, thanks to the graviton mass, the inflationary loop diagram converges at IR side.

SPATIAL CONDENSATION

Firstly, let’s write down such a simple action with Einstein-Hilbert term and 3 canonical massless scalar fields,

$$S = M_p^2 \int \sqrt{-g} \left( \frac{R}{2} - \frac{1}{2} m^2 g^{\mu \nu} \partial_\mu \phi^a \partial_\nu \phi^b \delta_{ab} - \Lambda \right),$$

where $\Lambda$ is the bare cosmological constant and $a, b = 1, 2, 3$. The background solution spontaneously breaks...
the Lorentz invariance in terms of two different patterns. One is by spontaneously generating a preferred time direction, for example, the effective field theory of inflation\[27\], where
\[
\langle \phi^a \rangle = f(t),
\]
and \( f(t) \) is some function of time. In this case the graviton is still massless and thus it isn’t the main interest of this paper. The second pattern spontaneously generates a preferred spatial frame,
\[
\langle \phi^a \rangle = x^a,
\]
which gives us a spatial condensation scenario (see \[28\] for a similar idea and its application in inflation). Please notice that at the l.h.s of above eq. \( (\text{5}) \), the up index ‘a’ is the internal index of scalar fields and \( \phi^a \) remain invariant under the general coordinate transformation. However, at the r.h.s of equation, ‘a’ is the space time index, under the general coordinate transformation it changes as follows,
\[
x^a \rightarrow x^a + \xi^a.
\]
In order to maintain the eq.\( (\text{5}) \) under the coordinate transformation, we introduce a Goldstone excitation \( \pi^a \), which transforms in the opposite way,
\[
\phi^a = x^a + \pi^a, \quad \pi^a \rightarrow \pi^a - \xi^a.
\]
The Goldstone excitations \( \pi^a \) non linearly realize the diffeomorphisms and they describe the perturbations of 3 scalars.

Our Goldstone excitations of such spatial condensation are actually a vector field, which can be decomposed into 3 independent components: one longitudinal mode and two transverse modes,
\[
\pi^a = \delta^{ab}(\partial_b \varphi + A_b).
\]
In the unitary gauge, we can see those Goldstones are “eaten” by the massless spin-2 field. After that, massless spin-2 particle gets weight and become massive, with 5 degrees of freedom on spectrum. In order to see how does this happen explicitly, let’s do our honest perturbation calculations on the FRW background. Under the FRW ansatz, the metric reads
\[
ds^2 = -N^2 dt^2 + a(t)^2 dx^2.
\]
By taking the variation of the action with respect to the lapse and scale factor, we get the following two background Einstein equations,
\[
3H^2 = \frac{3m^2}{2a^2} + \Lambda, \quad \frac{\dot{H}}{N} = -\frac{m^2}{2a^2}.
\]
Then we perturb the space-time metric and define the metric perturbations by
\[
g_{00} = -N^2(t)[1 + 2\phi_0], \quad g_{0i} = N(t)a(t)(S_i + \partial_i \beta), \quad g_{ij} = a^2(t)[\delta_{ij} + 2\psi \delta_{ij} + (\partial_i \partial_j - \frac{1}{3} \partial^2)E + \frac{1}{2} (\partial_i F_j + \partial_j F_i + \gamma_{ij})].
\]
where
\[
\partial_i S^i = \partial_i F^i = \gamma_i^i = 0.
\]
Noting that the vector field defined by
\[
Z^i \equiv \frac{1}{2} \delta^{ij} (\partial_j E + F_j)
\]
transforms as
\[
Z^i \rightarrow Z^i + \xi^i.
\]
Thus the combination \((Z^i + \pi^i)\) is a gauge invariant quantity. In the unitary gauge, \(Z^i\) eats \(\pi^i\), and survives in the linear perturbation theory. It is constrast to the general relativity, where \(E\) and \(F_i\) both are non-dynamical and we can just simply integrate them out.

**Scalar Perturbation** Now let’s expand the action upto quadratic order in the unitary gauge, where \(\phi^a = x^a\). For the scalar sector, we found that \(\phi^a, \beta \) and \(\psi\) are non-dynamical. After integrating out those non-dynamical modes, the quadratic action for the scalar perturbation reads
\[
\mathcal{L}_s \supset M_p^2 \int dt d^3k \left( \frac{k^4 m^2 a^3 N}{8k^2 + 12m^2 N^2} \frac{E^2}{8k^2 + 12m^2 N^2} - \frac{k^2 m^2 (k^2 + 2m^2)a N}{8k^2 + 12m^2 N^2} E^2 \right)
\]
Noted that background equations eq.\( (\text{8}) \) are used to get the above results. As we expected, after eating the longitudinal mode of our Goldstone, the scalar metric perturbation \(E\) survives and becomes a dynamical degree, propagates on the FRW space time background. By looking at the coefficient of the kinetic term, we can see it is always positive, as long as \(m^2\) is positive. Thus our scalar mode is free from the ghost instability.

The canonical normalized scalar perturbation is defined by
\[
\mathcal{E} \equiv \frac{k^2 M_p m \cdot E}{\sqrt{4k^2 + 6m^2}},
\]
where \(m\) is demanded to be positive. In terms of this canonical variable, the quadratic action for scalar perturbation can be rewritten as
\[
\mathcal{L}_s \supset \frac{1}{2} \int dt d^3k N a^3 \left( \frac{\dot{\mathcal{E}}^2}{N^2} - \omega_s^2 \mathcal{E}^2 \right),
\]
where

$$\omega_s^2 = \frac{k^2}{a^2} + \frac{2m^2}{a^2}.$$  \hspace{1cm} (18)

From this dispersion relation, we can see the sound speed of scalar mode is unity, and there is a mass gap on the scalar spectrum.

**Vector Perturbation** Now let’s turn to the vector perturbation. We find the vector perturbation $S_i$ is non-dynamical and we can simply integrate it out. After that, the quadratic action of vector perturbation reads,

$$\mathcal{L}_v \supset M_p^2 \int dt d^3k \left( \frac{k^2 m^2 a^3 N}{8k^2 + 16m^2 N^2} \tilde{F}_i \tilde{F}_i - \frac{k^2 m^2 a N}{8} F_i F_i \right).$$

(19)

Similar to the scalar perturbation, in the unitary gauge, vector perturbation $F_i$ eats the transverse mode of our Goldstone, becomes a dynamical degree and propagates on the FRW space time background. By looking at the coefficient of kinetic term, our vector perturbation is also ghost free when $m^2$ is greater than zero.

Then we canonical normalized the action by defining such canonical variable,

$$F_i \equiv \frac{k M_p m \cdot F_i}{2 \sqrt{k^2 + 2m^2}},$$

and the quadratic action can be rewritten in terms of canonical variable as follows,

$$\mathcal{L}_v \supset \frac{1}{2} \int dt d^3k N a^3 \left( \tilde{F}_i \tilde{F}_i - \omega_s^2 F_i F_i \right),$$

(21)

where

$$\omega_s^2 = \frac{k^2}{a^2} + \frac{2m^2}{a^2}.$$  \hspace{1cm} (22)

Due to the SO(3) symmetry of our scalar fields’ configuration, the dispersion relation of vector mode is exactly the same as the one of scalar mode.

**Tensor Perturbation** Now let’s look at the final sector of our linear metric perturbation. After using the background equations, the quadratic action of our tensor modes reads,

$$\mathcal{L}_T \supset M_p^2 \int dt d^3k \left[ \frac{a^3}{4N} \tilde{\gamma}_{ij} \tilde{\gamma}^{ij} - \frac{(2^2 + 2m^2) a N}{4} \tilde{\gamma}_{ij} \tilde{\gamma}^{ij} \right].$$

(23)

Again, we do the canonical normalization,

$$\tilde{\gamma}_{ij} \equiv \frac{M_p}{2} \gamma_{ij},$$

(24)

and the action can be rewritten as

$$\mathcal{L}_T \supset \frac{1}{2} \int dt d^3k N a^3 \left( \frac{\tilde{\gamma}_{ij} \tilde{\gamma}^{ij}}{N^2} - \omega_T^2 \tilde{\gamma}_{ij} \tilde{\gamma}^{ij} \right),$$

(25)

where

$$\omega_T^2 = \frac{k^2}{a^2} + \frac{2m^2}{a^2}.$$  \hspace{1cm} (26)

Surprisingly! The dispersion relation of our tensor mode is exactly the same as the one of scalar mode and vector mode. On the other hand, our tensor mode receives a mass correction on the dispersion relation, which is contrast to the general relativity.

**Self Consistency** With such simple and elegant theory of massive gravity in hand, we still need to check its self-consistency though. The first basic question concerning its self-consistency would be the number of degrees of freedom. Do we have correct number of degrees for a massive spin-2 particle? Or in other word, can we really interpret those 5 degrees as the 5 polarizations of the massive spin-2 particle?

One may get puzzled at this point, because conventionally when we say a massive spin-2 particle should contain 5 degrees of freedom, we imply the Poincare symmetry in the 3+1 dimension Minkowski spacetime. However, in the massive phase of our spatial condensation, i.e. eq. [3], the Poincare symmetry is broken, and we don’t have the Minkowski solution either.

The key to understand this point is to notice that in general relativity, we always have a larger symmetry than the Poincare symmetry. Our theory is invariant under the general coordinate transformation. In general, a massive spin-2 particle is the symmetric and traceless rank-2 tensor representation for a SO(3) group, which is the little group of our general coordinate transformation group. Such representation only contains 5 independent components, thus it is consistent with the number of degrees of our spatial condensation scenario.

The second question concerning its self-consistency would be the vDVZ discontinuity. In the early and famous work of Fierz and Pauli [1], the simplest linear extension to GR suffer from the vDVZ discontinuity, which the theory can not reduce to GR at the massless limit $m \rightarrow 0$ [3][4][5]. Such discontinuity arises from the strong coupling between the scalar and tensor mode at the massless limit.

In our spatial condensation scenario, such strong coupling is absent, thus our theory can smoothly reduce to GR at the massless limit. At the massless limit, the effective action can be written in terms of a massless graviton and scalar mode of massive graviton,

$$\mathcal{L}_{DL} \supset M_p^2 \int \frac{1}{4} \varepsilon_{\mu\nu} \varepsilon^{\alpha\beta} h_{\alpha\beta} +$$

$$m^2 \left( - \frac{1}{2} k^2 \varphi^2 + h k^2 \varphi + h k^4 \varphi^2 + h^2 k^2 \varphi + \ldots \right).$$

(27)

Where $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ and the indices are omitted for the simplicity of handwriting. The canonical normalized
scalar and tensor modes are
\[ h^c \equiv M_p h^c, \quad \varphi^c \equiv M_p m k^c \varphi. \tag{28} \]

In terms of canonical variable, the linear coupling term between scalar and tensor is
\[ m h^c \varphi^c \to 0, \tag{29} \]

it disappears at massless limit. The non-linear coupling terms between scalar and tensor are
\[ \frac{k}{M_p} k h^c \varphi^2 \to 0, \quad \frac{m k^c}{M_p} k^c \varphi^c \to 0, \tag{30} \]

which strongly suppressed by the factor of \( k/M_p \) thus it can be neglected. One can easily check that the higher order coupling terms are also strongly suppressed by such factor. Thus, we conclude that at the massless limit \( m \to 0 \), our spatial condensate scalars decouple from gravity and we recover GR.

It is worth to notice that since our spatial condensation is just free scalar theory, the absence of higher order Goldstone interactions implies that our effective field theory approach is valid up to the energy scale where the quantum gravity effect becomes important, say, Planck scale.

**IR SAFE INFLATION**

Before applying our spatial condensation scenario to the early universe, let’s briefly review the IR divergence problem of inflationary correlation function in the framework of GR. We take the one graviton loop diagram depicted in Fig.1 as an example. This diagram is particularly important because if the inflaton is a free scalar field, such diagram makes the leading contribution to the non-linear correction of the primordial spectrum\(^1\). The graviton interaction vertex corresponding to Fig.1 is
\[ \mathcal{H}_1 \supset \gamma_{ij}^2 (\partial_k \delta \sigma)^2. \tag{31} \]

where \( \delta \sigma \) is the inflaton scalar’s perturbation, and \( \gamma_{ij} \) is the tensor perturbations which is defined by
\[ \delta g_{ij} = a(t)^2 \gamma_{ij}, \tag{32} \]

and it satisfies the transverse condition and traceless condition,
\[ \partial_i \gamma_{ij} = \gamma_i^i = 0. \tag{33} \]

The quadratic action for the tensor mode reads
\[ L_T = \frac{M_p^2}{8} \int dtd^3a \left[ \dot{\gamma}_{ij}^2 - a^{-2} \partial_i \gamma_{ij} \partial_j \gamma_{ij} \right]. \tag{34} \]

Take the variation of the action with respect to the \( \gamma_{ij} \), we get the equation of motion
\[ \gamma_{ij}'' + 2a H \gamma_{ij} - \frac{\nabla^2}{a^2} \gamma_{ij} = 0, \tag{35} \]

where \( \dot{} \equiv \frac{d}{dt} \frac{k}{\pi} \) and \( H \) is the Hubble parameter. For simplicity, we neglect the inflaton slow roll effect and consider the Hubble parameter as a constant. We quantizes the tensor mode as:
\[ \gamma_{ij}(x) = \sum_{s=\pm} \int d^3k \left[ a(k) e_i^s(k, s) \gamma_{ks} e^{ikx} + h.c. \right], \tag{36} \]

where \( a(k) \) is the annihilation operator, the subscript \( s \) of the graviton is the helicity, and \( e_{ij}(k, s) \) is the transverse and traceless polarisation tensor which can be normalized as
\[ e_{ij}(k, s) e^{ij}(k, s') = \delta_{ss'} \tag{37} \]

The mode function in the de-sitter space time is easy to obtain. We assume that the fluctuation is generated at the deep sub-horizon scale, and the vacuum is the standard Bounce-David vacuum. The solution to eq. (35) is
\[ \gamma_{\pm, k} = \frac{H}{(2\pi)^{3/2} \sqrt{k^3}} (1 + ik\eta) e^{-i k \eta}. \tag{38} \]

In the IR limit, the power spectrum of tensor mode has an almost scale-invariant form,
\[ P_{GW}(k) = 2 |\gamma_k(\eta)|^2 = \frac{2H^2}{(2\pi)^3 k^3} \left[ 1 + O(k^2 \eta^2) \right], \tag{39} \]

where the factor 2 comes from the 2 helicity of the tensor mode, and we take the IR limit to drop the \( O(k^2 \eta^2) \) term.

We then calculate the one graviton loop in Fig.1. Using in-in formalism, we find that one graviton loop diagram depicted in Fig.1 obtained from the contraction between the two \( \gamma_s \) with such divergent factor
\[ \langle \zeta(x)\zeta(x) \rangle_{\text{loop}} \propto \int d^3k P_{GW}(k) \propto \int \frac{dk}{k}. \tag{40} \]

\(^1\) The authors of the paper\(^30\) pointed out that such diagram is exactly canceled by another two-vertex loop diagram if \( \epsilon = 0 \), where \( \epsilon \) is the slow roll parameter. However, \( \epsilon \) is not always a constant for the most of inflationary models.
Obviously, such momentum integral diverges logarithmically in the both UV and IR.

Now let’s turn to our spatial condensation scenario, see how does graviton mass heal the IR divergence. In this case, the action of our system can be written as

\[ S = \int \sqrt{-g} \left( \frac{M_p^2}{2} \mathcal{R} - M_p^2 m^2 \frac{1}{2} g^\mu\nu \partial_\mu \phi^a \partial_\nu \phi^b \delta_{ab} - \frac{1}{2} g^\mu\nu \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right), \]  

(41)

where \( \sigma \) is the inflaton scalar. The energy density of the spatial condensation scales as \( a^{-2} \) and gets diluted away rapidly during inflation. Thus one can expect that its Goldstone fluctuations make minor contribution to the primordial curvature perturbation, and the primordial curvature perturbation is scale invariant as long as the Hubble constant changes slowly enough.

Due to the graviton mass, according to the eq. (26), the mode function of the tensor perturbations must be rewritten as

\[ \gamma_{\pm k} = \frac{H}{(2\pi)^{3/2} \sqrt{k^3}} (1 + ik\eta) e^{-i k \eta}, \]  

(42)

where

\[ k \equiv \sqrt{k^2 + 2m^2}, \]  

(43)

and the power spectrum reads

\[ P_{GW}(k) = \frac{2H^2}{(2\pi)^3 k^3} \left[ 1 + O(k^2 \eta^2) \right]. \]  

(44)

Thanks to the graviton mass, the factor in eq. (40) converges at the IR side,

\[ \langle \zeta(x) \zeta(x) \rangle_{\text{1-loop}} \propto \int_0^{a(t)H(t)} d^3k P_{GW}(k) \propto Ht + \log (H/m), \]  

(45)

where the integration upper bound is chosen to be consistent with the IR limit, which is used to simplify our calculation in this section. Away from IR limit, the result will receive an additional term which depends on the UV cut-off. Our graviton mass has nothing to do with the UV physics, thus it isn’t our main interest and we are not going to discuss the UV divergence issue in this paper.

As we can see from the eq. (45), although the graviton mass removes the IR divergence, but the secular growth remains. Compare to the tree diagram of power spectrum, the loop diagram is suppressed by a factor of \( 10^{-10} \). Thus the loop correction becomes important only if the inflation lasts at least \( 10^{10} \) e-folding number, which is much more than the 60 e-folding number we need for solving the flatness and horizon problem of the standard hot big bang cosmology. Thus in the most of cases, we don’t have to worry about such secular growth.

**CONCLUSION AND DISCUSSION**

In this short notes, we consider a novel pattern of spontaneously Lorentz symmetry breaking. The background solution spontaneously generates a preferred spatial frame, which gives us a spatial condenensation scenario. The equation of state of the spatial condensation is \(-1/3\), and the energy density scales as \( a^{-2} \). In the unitary gauge, massless graviton eats the Goldstone excitations of spatial condensation, gets weight and becomes a massive graviton. Our massive graviton is a multiplet particle, its 5 polarizations have exactly the same dispersion relation, with a mass gap on the spectrum.

We then apply our massive gravity theory to inflation, and find graviton mass removes the IR divergence of inflationary loop diagram, but the secular growth still remains. In addition to the virtue of IR safe, we would expect our model has some other interesting features. The primordial vector perturbations may lead to the CMB large scale anomalies [31]. On the other hand, the graviton mass also changes the primordial tensor mode, we expect to find some interesting feature on the B mode polarization of CMB.

Although we only checked the stability of our theory at FRW background, we expect it has the universal healthy nature since our theory is nothing but Einstein-Hilbert action and 3 canonical free scalars. More generally, taking the SO(3) symmetry of scalars’ configuration as our building principle, we can write down a most general action with non-derivative graviton potential terms as

\[ S = M_p^2 \int \sqrt{-g} \left[ \frac{\mathcal{R}}{2} - m^2 \mathcal{U}(g^{\mu\nu}, f_{\mu\nu}) \right]. \]  

(46)

where \( f_{\mu\nu} \equiv \partial_\mu \phi^a \partial_\nu \phi^b \delta_{ab} \) and \( \mathcal{U}(g^{\mu\nu}, f_{\mu\nu}) \) is a general function of \( g^{\mu\nu} \) and \( f_{\mu\nu} \). Besides the non-derivative potential terms, we are also able to introduce the derivative coupling terms, e.g. the Horndeski term \( G^{\mu\nu} f_{\mu\nu} \), where \( G^{\mu\nu} \) is the Einstein tensor. The stability of such theory is checked in the ref. [29].

**Acknowledgments** The author would like to thank J. Chen, P. Chen, C. Feng, F. Finelli, H. Firouzjahi, A. Emir Gumrukcuoglu, G. Gabadadze, X. Gao, K. Hinterbichler, Q. Huang, R. Kimura, L. Labun, M. Li, X. Meng, S. Mukohyama, R. Saito, M. Sasaki, G. Shiu, N. Tanahashi, T. Tanaka, H. Tye, Y. Urakawa, Y. Wang, W. Xue, Y. Zhang for the useful discussion. The author also would like to thank the hospitality of Yukawa institute, since the idea of this paper was spontaneously generated on the author’s way back from Yukawa institute, after two days’ short visiting. This work is supported by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.
[1] M. Fierz, W. Pauli, Proc. Roy. Soc. Lond. A173, 211-232 (1939).
[2] D. G. Boulware, S. Deser, Phys. Rev. D6, 3368-3382 (1972).
[3] H. van Dam, M. J. G. Veltman, Nucl. Phys. B22, 397-411 (1970).
[4] V. I. Zakharov, JETP Lett. 12, 312 (1970).
[5] A. I. Vainshtein, Phys. Lett. B 39, 393 (1972).
[6] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. 305, 96 (2003) [hep-th/0210154].
[7] C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett. 106, 231101 (2011) [arXiv:1011.1232 [hep-th]].
[8] A. E. Gumrukcuoglu, C. Lin and S. Mukohyama, JCAP 1203, 006 (2012) [arXiv:1111.4107 [hep-th]].
[9] A. De Felice, A. E. Gumrukcuoglu and S. Mukohyama, Phys. Rev. Lett. 109, 171101 (2012).
[10] A. De Felice, A. E. Gumrukcuoglu, C. Lin and S. Mukohyama, arXiv:1303.4154 [hep-th].
[11] A. De Felice, A. E. Gumrukcuoglu, C. Lin and S. Mukohyama, arXiv:1304.0484 [hep-th].
[12] N. Khosravi, G. Niz, K. Koyama and G. Tasinato, arXiv:1305.2950 [hep-th].
[13] S. Deser and A. Waldron, Phys. Rev. Lett. 110, 111101 (2013) [arXiv:1212.5835 [hep-th]].
[14] S. Deser, K. Izumi, Y. C. Ong and A. Waldron, arXiv:1306.5457 [hep-th].
[15] A. H. Guth, Phys. Rev. D23, 347 (1981).
[16] A. Vilenkin, L. H. Ford, Phys. Rev. D26, 1231 (1982).
[17] A. D. Linde, Phys. Lett. B116, 335 (1982).
[18] A. A. Starobinsky, Phys. Lett. B 117, 175 (1982).
[19] D. Seery, Class. Quant. Grav. 27, 124005 (2010) [arXiv:1005.1649 [astro-ph.CO]].
[20] T. Tanaka and Y. Urakawa, arXiv:1306.4461 [hep-th].
[21] M. S. Sloth, Nucl. Phys. B 748, 149 (2006) [astro-ph/0604488].
[22] D. Seery, JCAP 0802, 006 (2008) [arXiv:0707.3378 [astro-ph]].
[23] E. Dimastrogiovanni and N. Bartolo, JCAP 0811, 016 (2008) [arXiv:0807.2790 [astro-ph]].
[24] S. Weinberg, Phys. Rev. D 72, 043514 (2005) [hep-th/0506236].
[25] S. Weinberg, Phys. Rev. D 83, 065008 (2011) [arXiv:1011.1630 [hep-th]].
[26] W. Xue, K. Dasgupta and R. Brandenberger, Phys. Rev. D 83, 083520 (2011) [arXiv:1103.0285 [hep-th]].
[27] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, JHEP 0803, 014 (2008) [arXiv:0709.0293 [hep-th]].
[28] S. Endlich, A. Nicolis and J. Wang, arXiv:1210.0569 [hep-th].
[29] C. Lin, arXiv:1305.2969 [hep-th].
[30] W. Xue, X. Gao and R. Brandenberger, JCAP 1206, 035 (2012) [arXiv:1201.0768 [hep-th]].
[31] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5083 [astro-ph.CO].