Studies of the cathode sheath of a low pressure hollow cathode discharge

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Abstract. In this work, a theoretical model for the cathode sheath using experimental data of a low pressure hollow cathode discharge (HCD) is derived considering it to be non-collisional. Secondary electrons emitted from the cathode surface are taken into account in the discharge model and their influence on the theoretical sheath potential profile is investigated. The plasma parameters and the floating potential profile along the discharge axis were inferred from the current-voltage characteristics of a single Langmuir probe positioned at the inter-cathode space of the HCD. For a low pressure HCD, typical values of the electron density and electron temperature are \( n_e \approx 10^{16} \text{ m}^{-3} \) and \( T_e = 4 \text{ eV} \), respectively. By using the probe data the floating potential profile was determined to verify the position of the plasma-cathode sheath interface and to promote a qualitative discussion between theoretical and experimental results.

1. Introduction
The more important effects in a glow discharge occur next to the cathode, i.e. in the cathode sheath and negative-glow regions. The cathode sheath supports almost all the discharge voltage and is the main responsible for the processes of production of neutral and charged species. The electrons crossing the cathode fall and streaming into the negative glow acquire enough energy to promote more intense ionization and excitation rates as compared with the positive-column plasma. This makes the negative glow based discharges a useful tool in laser, lighting, and surface-processing applications [1-3].

HCD are the most common examples of discharges in which the negative glow plasma is generated. The hollow cathode device consists of a cathode with hollow structure (hole, aperture, etc.) and an arbitrarily shaped anode [4]. Compared to a conventional glow discharge between parallel plates, the hollow cathode geometry is characterized by higher current densities and larger concentrations of high-energy electrons. These properties are consequences of the phenomenon known as hollow cathode effect (HCE) [3,4].

A simplified theoretical model of non-collisional cathode sheath of a plane hollow cathode discharge is usually made by only considering the contribution of electrons and ions from the plasma volume. From this model we conclude that the sheath region should have a net positive electrical charge [5]. However, in real situation, we must take into account the contribution of secondary electrons produced as a result of the impact of ions, photons or neutral species or metastable against the cathode surface.
2. Description of the model

We consider a plane of abscissa $x = 0$ coincident with the flat surface of the cathode. The sheath region is confined between the plane $x = 0$ and another one of abscissa $x_b > 0$. The electric potential in the sheath, as well as in the plasma, is described by the function $V = V(x)$, and at any point in the plasma is attributed $V(x) = 0$ to be consistent with the quasi-neutral conditions. The potential of the cathode in $x = 0$ is $V(0) = V_c < 0$ because, in this case, the plasma is the region of highest potential [5].

According to the condition of quasi-neutrality, the plasma density $n_o$ is equal to the ions density $n_{oi}$ and to the electron density $n_{oe}$. The secondary electrons are supposed to travel as a beam, leaving the cathode at $x = 0$. The electron beam is accelerated through the cathode sheath towards the plasma. The ions from plasma are accelerated by the cathode field, and the electrons from the plasma are decelerated by this field.

We consider that the secondary electrons are generated by the impact of ions, photons and neutral species - both stable and metastable - remembering that in the cathode region the emission by electron impact is not very significant [5]. Thus, we can make use of a dimensionless coefficient denoted by the effective (or apparent) electron secondary emission coefficient [6, 7], since it includes the effect of all relevant mechanisms in electron emission. This effective secondary electron emission coefficient is given by:

$$\gamma = \frac{J^-}{J^+ \text{cathode}}$$  \hspace{1cm} (1)

Based on (1), and with the knowledge of the effective coefficient, we can determine the current density of secondary electrons $J^-$. Therefore, it is necessary to determine $J^+$. In particular, by the continuity equation, we have $J^+ = n_{oe} v_{oe}$, so that (1), by applying the continuity equation, results in:

$$J^- = \gamma n_o v_o = en_{ose} v_{se}$$ \hspace{1cm} (2)

where $v_o$ is the ion velocity at the sheath edge, the $n_{ose}$ and $v_{se}$ are the secondary electron density and secondary electron velocity, respectively. Thus, we have:

$$n_{ose} = \gamma n_o v_o \left(\frac{-2eV_c}{m}\right)^{-\frac{1}{2}}$$ \hspace{1cm} (3)

The number density of secondary electrons in the plasma remains constant, but the same does not occur in the sheath. Such density can be denoted by $n^-$ and also calculated based on the equations of energy conservation and continuity:

$$n^- = \gamma n_o v_o \left[\frac{2e(V - V_c)}{m}\right]^{-\frac{1}{2}}, \hspace{0.5cm} V \neq V_c$$ \hspace{1cm} (4)

Also, the electron density in the plasma volume is given by the difference between $n_o$ and $n_{ose}$, so:

$$n_{oep} = n_o \left[1 - \gamma v_o \left(\frac{-2eV_c}{m}\right)^{-\frac{1}{2}}\right]$$ \hspace{1cm} (5)

The electrons from the plasma are assumed to be in thermal equilibrium, called thermal electrons. The electrons density ($n_e$) in the sheath region satisfies the Boltzmann relation given by:
\[ n_e(x) = n_{\text{eop}} \exp \left( \frac{eV(x)}{kT_e} \right), \]

where \( n_{\text{eop}} \) is the electron density of plasma without the contribution from the electron beam, \( V \) is the electric potential in the sheath, \( e \) is electron charge, and \( kT_e \) is the electron temperature in eV. The ions are cold and monoenergetic. Using the continuity equation and the energy conservation, the ion density \( n_i \) in the sheath can be obtained by:

\[ n_i(x) = n_o \left( 1 - \frac{2eV(x)}{Mv_o^2} \right)^{\frac{1}{2}}, \]

where \( M \) is the ion mass. The potential in the sheath is given by Poisson’s equation

\[ \frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_o} = -\frac{1}{\varepsilon_o} \left( -n_e e - n_i e + n_i e \right), \]

where \( \varepsilon_o \) is the vacuum permittivity and \( \rho \) is the total charge density. By using (4), (5), (6), and (7), we obtain

\[ \frac{\varepsilon_o}{en_o} \frac{d^2V}{dx^2} = \left[ 1 - \gamma \nu_0 \left( \frac{-2eV_C}{m} \right)^{\frac{1}{2}} \right] \exp \left( \frac{eV}{kT_e} \right) + \gamma \nu_0 \left[ \frac{2e(V - V_C)}{m} \right]^{\frac{1}{2}} \left( 1 - \frac{2eV}{Mv_o^2} \right)^{\frac{1}{2}}. \]

Introducing the normalized quantities

\[ \eta = \frac{eV}{kT_e}, \quad \eta_c = \frac{-eV_C}{kT_e}, \quad \xi = \frac{x}{\lambda_D}, \quad \mu = \frac{M}{m}, \quad \nu = \frac{v_o}{c_s}, \]

where \( \lambda_D \) is the electron Debye length at the sheath edge and \( c_s = (kT_e/M)^{1/2} \) is the ion sound speed [8, 9], we obtain

\[ \frac{d^2\eta}{d\xi^2} = -\left[ 1 - \gamma \nu \left( 2\mu \eta_c \right)^{\frac{1}{2}} \right] \exp (-\eta) - \gamma \nu \left( 2\mu \eta_c \right)^{\frac{1}{2}} \left( 1 - \frac{\eta}{\eta_c} \right)^{\frac{1}{2}} + (1 + 2\eta \nu^2)^{\frac{1}{2}}. \]

Equation (12) has as inconvenience the fact that \( \nu \) is an unknown. To circumvent this, it is necessary to establish a relationship involving \( \nu \) and other parameters of this equation, namely, \( \gamma, \mu \) and \( \eta_c \). By imposing equality of charge density at \( \eta \to 0 \) we have ensured one of the matching conditions as \( \xi \) becomes large. Also if \( \eta \) is to evolve smoothly to zero as \( \xi \) increases, then it may be supposed to vary asymptotically as [10]:

\[ \eta = k \xi^{-\alpha} \]

The substitution of (12) in (11), and after performing the Taylor series expansions around \( \eta = 0 \), which is particularly valid in the limiting region between the sheath and the plasma, results in an equation which can be satisfied only if the terms that multiply \( \xi^{2n} \) vanish. This condition leads to the following third-degree polynomial equation in \( \nu \):

\[ k\nu^3 - \nu^2 + 1 = 0 \]

By solving (13) we obtain:
The solution of (14) that we should adopt is one which, considering the fact that $k$ is typically much smaller than unity, gives the value of $\nu$ such that the Bohm criterion is satisfied and that tends to unity when $k$ approaches zero. It can be shown that $\nu$ is a positive real number if $0 < k < \frac{2}{3\sqrt{3}}$. This condition is satisfied by a wide range of typical values of $\gamma$, $\mu$, and $\eta_c$. The electric field and potential in the sheath can be obtained by integrating numerically (12) using $\eta = 0$ and $d\eta/d\xi = 0$, so:

$$\frac{1}{2} \left( \frac{d\eta}{d\xi} \right)^2 = \left[ 1 - \gamma \nu \left( 2 \mu \eta_c \right)^{3/2} \right] \exp \left( -\eta \right) - 1 + 2 \gamma \nu \left( 2 \mu \eta_c \right)^{3/2} \eta_c \left( 1 - \frac{\eta}{\eta_c} \right)^{3/2} - 2 \gamma \nu \left( 2 \mu \eta_c \right)^{3/2} \eta_c + \nu^2 \left( 1 + 2 \eta \nu^{-2} \right)^{3/2} - \nu^2$$

(15)

The boundary condition for determining where the sheath edge is located, at $\xi = s$, is

$$\eta \left( s/L_p \right) = -\frac{eV_b}{kT_e} = \frac{\nu^2}{2}$$

(16)

because the sheath potential ($V_b$) in this position is given by:

$$V_b = -\frac{1}{2e} Mv_0^2 = -\frac{\nu^2 kT_e}{2e}$$

(17)

The mathematical problem constituted by (11) and condition $\eta(0) = \eta_c$ must be solved numerically to determine the theoretical profile of the electrical potential in the cathode sheath. The solution of the problem without the use of dimensionless variables depends on the knowledge of plasma parameters according to (10).

3. Experimental setup

A schematic diagram of the plane hollow-cathode discharge apparatus is illustrated in figure 1. The discharge resembles a glow discharge regime extending from the hollow anode (earth potential) up to flat cathodes made of aluminum covered by Teflon. The hollow anode is made of titanium of 7 mm external diameter which is covered by a ceramic tube terminated by an orifice of 2.0 mm diameter. The internal diameter of the hollow anode cavity is 4 mm. The orifice constrains the anode discharge to form electrostatic layers which contribute to increase the ionization rate in the anodic region. The vacuum glass chamber of 130 mm internal diameter and 300 mm length was preliminary evacuated to a pressure below $10^{-2}$ Pa. The plane hollow-cathode discharge was operated with argon gas for pressures ($P$) in the range of (10 – 50) Pa. The gases were fed through the anode hole at a flow rate in the range of (1 – 40) sccm. Thus, the gas flows through the hollow anode and becomes partially ionized in the process before it reaches the vacuum chamber. The self-sustained cathode-anode voltage ($V_d$) during operation was in the range of (350 – 900) V with a discharge current ($I_d$) in the range of (10 – 800) mA. A cylindrical tungsten probe with 4.0 mm in length and 0.1 mm in diameter was positioned at the axis of the discharge between the cathodes. The axis of the probe is perpendicular to the normal to the cathode surface. Standard technique [5] was used to extract the argon plasma parameters from the
current-voltage characteristic of the probe. The plasma generated between the cathodes is characterized by the presence of impurity metal-ions and gas ions due to intensive ion sputtering of the cathode. A relatively high content of atoms of the cathode material in the plasma is explained by a high kinetic energy (1 – 10 eV) [5] of the sputtered atoms. This effect promotes the contamination of the chamber walls and of the electrostatic probe. The combined effect of the ion bombardment by polarizing the probe with the cathode potential promotes control over the incandescence intensity of the probe to clean it.

4. Results and discussion

The resolution of several computational problems constituted by (15), with the variation of the parameters involved, shows that the model developed for the cathode sheath is not very sensitive to variations of the coefficient of secondary electron emission, $\gamma$, within the typical range of this parameter. Figure 2 shows the result from numerical integration of the (15). For a certain set of parameters we can conclude that the dimensionless potential profile in the cathode sheath is little influenced by variations of the $\gamma$ coefficient. On the one hand, this low sensitivity may indicate that the same effect is valid in practice due to the relative low densities of secondary electrons. Only an analysis including values of the coefficient $\gamma$ of the order of hundreds shows significant changes in the profiles of the electric potential for the same parameters adopted in figure 2. A relevant theoretical analysis refers to the variation of thickness of the cathode sheath with the plasma density, which can be easily varied by changing some discharge parameters as gas pressure, axial magnetic field, inter-cathode distance and discharge voltage. Figure 3 shows that the thickness of the cathode sheath decreases monotonically with increasing of the plasma density, which is a consequence of the direct dependence of such thickness on the Debye length.

If the electron temperature $T_e$ is expressed in electron volts, and the number density ($n_o$) in electrons per cubic meter, the Debye length ($\lambda_D$) is given by:

$$\lambda_D \approx 7434 \left( \frac{T_e}{n_o} \right)^{1/2}$$

(18)

The Debye length is the characteristic thickness of sheaths which form between a plasma and an electrode, or a plasma and a wall. Equation (18) predicts a Debye shielding distance of approximately 0.15 mm for plasmas with an electron temperature of 4 eV and a number density of $10^{16}$ electrons/m$^3$. The increase of the discharge current and the plasma density gives lower Debye lengths, consequently, the thickness of the cathode sheaths decreases. This effect generates a greater plasma volume and, consequently, it is possible to enhance the discharge current by lowering the inter-cathode distance. In our experimental apparatus the inter-cathode distance was kept fixed at 19 mm. The discharge was operated by using argon as working gas. The mean values of the plasma parameters summarized in table 1 were obtained at different positions inside of the negative glow formed between the flat cathode surfaces for two operational conditions of the hollow cathode discharge:

- Condition I with $P = 15$ Pa, $V_d = 431$ V and $I_d = 15$ mA.
- Condition II with $P = 25$ Pa, $V_d = 428$ V and $I_d = 19$ mA.

The position of the interface region between the negative glow and the cathode sheath is visually identified by intense brightness difference between these two regions. Experimentally we can evaluate this interface region by moving the cylindrical Langmuir probe with respect to the cathode surface along the axis of the discharge. The mean values of the interface positions also are presented in table 1.
Figure 1. Schematic diagram of the plane hollow-cathode discharge apparatus.

Figure 2. The normalized electric potential $\eta$ as a function of distance normalized to the plasma Debye length and secondary electron coefficient, with $kT_e = 4 \text{ eV}$, $n_o = 1.0 \times 10^{16} \text{ m}^{-3}$, $\eta_C = 104$.

Figure 3. Thickness of the cathode sheath as function of the plasma density. The parameters used for making this simulation were: $kT_e = 4 \text{ eV}$, $\eta_C = 104$ and $\gamma = 0.07$.

Table 1. Plasma parameters and the position of the sheath-negative glow interface for conditions I and II.

| Condition | $kT_e$ (eV) | $n_o$ ($10^{16} \text{ m}^{-3}$) | $\lambda_D$ ($10^3 \text{ m}$) | Position of interface ($10^3 \text{ m}$) |
|-----------|-------------|-------------------------------|-----------------|-----------------|
| I         | 7.7±0.9     | 1.9±0.3                       | 0.15±0.08       | 3.3±0.5         |
| II        | 5.9±0.6     | 2.3±0.2                       | 0.12±0.06       | 2.8±0.5         |
The sheath-negative glow interface region can be also evaluated by determining the floating potential profile along the axis of the discharge, as shown in figures 4 and 5 for conditions I and II, respectively, with emphasis on the transition range of brightness at the sheath-negative glow interface region. The floating potential corresponds to the probe polarization potential for which current the probe current vanishes. It is supposed here that the floating potential profile has approximately the same qualitative behavior as the space potential profile, meaning that at the point where the electric potential of the medium begins to change, it is expected the same to happen with the floating potential of the electrostatic probe. The sheath-negative glow interface region can be identified from the numerical solution of (15), see figure 6, for comparisons with the experimental results determined from the floating potential profile along the axis of the discharge.

The floating potential profile inferred from the probe characteristic may be qualitatively compared with the theoretical curve of electric potential of the medium (without the interference of the probe), obtained from the numerical solution of (16), as shown in figure 6 simulated for the condition II. In this case, the electron secondary emission coefficient, which is already known not to influence significantly the results for the electrical potential was kept fixed at \( \gamma = 0.38 \) for argon and aluminum electrode \([11]\). For the argon ion, supposedly the only ionic species present in the discharge, \( \mu \approx 72819.7 \). The cathode potential fall, \( -V_{C} \), is theoretically estimated as the sum of the discharge voltage \( V_{d} \) with the anode potential fall, generally near the ionization potential of the gas \([12]\). In the case of argon, this potential is 15.76 V. The electric potential along the cathode sheath region increases monotonically with the distance from cathode and tends to zero in the negative glow region where we have supposed the charge neutrality. The theoretical sheath thickness \( s \) was evaluated to be about 3.13 mm and therefore, within the range of sheath edge position shown in figure 5.

In figure 7, we can observe that the electric field is relatively small at distances larger than the sheath thickness, which is in agreement with the physical concept of the pre-sheath region, where a small electric field must exist to ensure the acceleration of ions coming from the plasma at a speed at least equal to the ion sound speed \( c_{s} \). This electric field, yet of small intensity, is capable of producing significant modifications in charge densities of ions and electrons from plasma shortly before the entry in the cathode sheath region, as can be observed in figures 8 and 9.

Figure 9 also indicates that the electron number density in the sheath suffers a very drastic reduction governed by the ratio of Boltzmann equation (6) and practically vanishes at a distance of 2.5
mm from the cathode. The net electrical charge in figure 6 shows that for the parameters simulated, this density does not become negative at any point. Furthermore, we can explain the behavior of this profile: near the limiting plane between the sheath and negative glow, exists a maximum of net charge density. As the ions coming from the plasma have not yet been sufficiently accelerated, the continuity equation establishes that their density must be still high at this point, which is confirmed in figure 8.

On the other hand, the secondary electrons released from the cathode surface are accelerated through the cathode sheath towards the plasma region followed by an expressive reduction of their charge density, as shown in the figure 10. This observation indicates that the electrons from the plasma region, decelerated by the sheath potential, are the ones that play the dominant role for the strong reduction of the net charge density at the sheath-plasma boundary (see figure 11). On the left side of the maximum of the net charge density profile, the ions density is much higher than the total electron density. The secondary electron charge density decreases with the distance from cathode becoming three orders of magnitude lower than that of the ions in the plasma region. Overall, this means that the contribution of secondary electrons for the electrical potential profiles on the cathode sheath region is not significant, in agreement with the theoretical model.

Figure 6. Electric potential profile along the x axis; $kT_e = 5.9$ eV, $n_0 = 2.3 \times 10^{16}$ m$^{-3}$, $V_C = -444$ V and $\gamma = 0.38$.

Figure 7. Electric field profile along the x axis; $kT_e = 5.9$ eV, $n_0 = 2.3 \times 10^{16}$ m$^{-3}$, $V_C = -444$ V and $\gamma = 0.38$.

Figure 8. Variation of the ion charge density with the distance from the cathode along the x axis; $kT_e = 5.9$ eV, $n_0 = 2.3 \times 10^{16}$ m$^{-3}$, $V_C = -444$ V and $\gamma = 0.38$.

Figure 9. Variation of the electrons charge density with the distance from the cathode along the x axis; $kT_e = 5.9$ eV, $n_0 = 2.3 \times 10^{16}$ m$^{-3}$, $V_C = -444$ V and $\gamma = 0.38$. 
5. Conclusion

The non-collisional model for the electrical potential near the cathode of a HCD in conjunction with experimental data obtained from a Langmuir probe indicates a weak relationship of the cathode sheath potential profile with the secondary electron coefficient ($\gamma$). We include atypical values of $\gamma$ to show that significant changes can occur in the profiles of electric potential only in the case of a net negative charge near the cathode sheath - this means that in these hypothetical cases, the emission of secondary electrons from the cathode is so intense that, in the cathode region, the number density of these electrons exceeds that of ions from the plasma. The floating potential profiles inferred from the current-voltage characteristic of a single Langmuir probe are qualitatively similar to the theoretical electrical potential profiles. The position of the plasma-cathode sheath interface determined from the simulated model shows a good agreement with the experimental data. This transition region allows a good estimative of the thickness of the cathode sheath.

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