Average power density spectrum of long GRBs detected with *Beppo*SAX/GRBM and with *Fermi*/GBM

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ABSTRACT

From past experiments the average power density spectrum (PDS) of GRBs with unknown redshift was found to be modelled from 0.01 to 1 Hz with a power–law, $f^{-\alpha}$, with $\alpha$ broadly consistent with $5/3$. Recent analyses of the *Swift*/BAT catalogue showed analogous results in the 15–150 keV band. We carried out the same analysis on the bright GRBs detected by *Beppo*SAX/GRBM and *Fermi*/GBM. The *Beppo*SAX/GRBM data, in the energy range 40–700 keV and with 7.8 and 0.5 ms time resolutions, allowed us to explore for the first time the average PDS at very high frequencies (up to 1 kHz) and reveal a break around 1–2 Hz, previously found in *CGRO*/BATSE data. The *Fermi*/GBM data, in the energy band 8–1000 keV, allowed us to explore for the first time the average PDS within a broad energy range. Our results confirm and extend the energy dependence of the PDS slope, according to which harder photons have shallower PDS.

Key words: gamma-rays: bursts — timing analysis: power density spectra

1 INTRODUCTION

Together with the energy spectrum, the temporal behaviour of gamma–ray burst (GRB) light curves holds the key to both the physical mechanism responsible for the production of the prompt gamma rays and the distance from the stellar progenitor at which the energy dissipation into gamma–rays takes place. More than a decade after the first GRB afterglow discoveries, these key questions concerning the GRB prompt emission are yet to be answered. The typical observed durations of pulses span from hundreds milliseconds up to several seconds (e.g., Norris et al. 1996). A proper characterisation of the temporal properties at different energy bands is crucial to provide clues to the energy dissipation process at the origin of the gamma–rays. In this context, the average power density spectrum (PDS) provides a way to characterise the phenomenon in terms of a stochastic process starting from the null hypothesis that each long GRB is a different realisation of a general unique process. In other words, we assume that the same mechanism can explain the variability observed in different light curves, while the observed variety is due to different conditions, which may vary from different GRBs.

The question whether GRB light curves might entirely be explained in terms of different realisations of a unique stochastic process characterised by a pure red noise, is still open. Interestingly, recent analyses have found evidence for the presence of deterministic components (as opposed to pure stochastic noise) ruling the evolution of a GRB light curve and giving rise to a chaotic behaviour (Greco et al. 2011).

In the context of a pure stochastic process entirely characterised by red noise, Beloborodov, Stern & Svensson, in 1998 and 2000 (hereafter, BSS98 and BSS00), studied the average PDS of 527 GRBs detected by the Burst and Transient Source Experiment (BATSE; Paciesas et al. 1999) aboard the Compton Gamma Ray Observatory (CGRO) in 25–2000 keV energy band, revealing a typical power–law behaviour spanning almost two orders of magnitude in frequency, from a few $10^{-2}$ to $\sim 1$ Hz. The power–law index they found is compatible with $5/3$, which is what one expects for the Kolmogorov spectrum of velocity fluctuations within a medium characterised by fully developed turbulence. They also found a sharp break around 1–2 Hz. These results were also supported by the INTEGRAL data analysis of a sample of 10 bright GRB (Ryde et al. 2003).

A recent analysis of the average PDS of the *Swift* Burst Alert Telescope (BAT; Barthelmy et al. 2005) data set in the 15–150 keV energy band was carried out for the first time in the GRB rest-frame average, thanks to the large number of

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GRBs detected by Swift with measured redshift. No significant differences were found between the observer and the rest-frame behaviour (Guidorzi et al. 2012, hereafter, G12). Notably, no evidence for the break around 1–2 Hz was found in the 15–150 keV band. In the present work we aim to study the average PDS in two different unexplored regimes with two different data sets. The goal of this analysis is twofold: i) we address the same average PDS analysis through two additional data sets from independent satellites and detectors; ii) these data sets allow us to study the average PDS at very high frequency (up to 1 kHz) with the BeppoSAX/Gamma-Ray Burst Monitor (GRBM) and across a broad energy band such that of Fermi/Gamma-ray Burst Monitor (GBM) from 8 keV to 1 MeV.

In Section 2 we report the sample selection criteria and the data analysis procedure. Results are presented in Section 3, followed by discussion and conclusions respectively in Sections 4 and 5. Uncertainties on best-fitting parameters are given at 90% confidence for one interesting parameter unless stated otherwise.

2 DATA ANALYSIS

2.1 Fermi/GBM data selection

We initially started with 829 GRBs detected and covered by GBM from July 2008 to December 2011. For each GRB we took the two most illuminated NaI detectors, for which we extracted the corresponding light curves with 64 ms resolution, which we then added to have a single light curve. In this early stage we considered the Time Tagged Event (TTE) files, which hold information about trigger time and energy channel of each detected photon. We excluded all GRBs with no TTE file. In some cases the TTE data do not cover the whole event and thus were not considered for the present analysis. The GRBs durations were expressed in terms of $T_{90}$, which we estimated from the background-subtracted light curves (Figure 1). Background subtraction was performed through interpolation using a polynomial of either first or second order.

We excluded short duration bursts by requiring $T_{90} > 3$ s. At this stage we were left with a sample of 650 GRBs. We then rejected all the GRBs with a poor signal–to–noise ratio (S/N) excluding those with peak rate less than 50 count s$^{-1}$. Spikes caused by radiative decay of some particles dragged in the Earth magnetic field that interact with the spacecraft payload were observed in 22 light curves, whose GRBs were therefore rejected from our sample (Meegan et al. 2009).

The extraction of the light curves for each GRB in different energy ranges was made retrieving the data and processing them with the HEASoft package (v6.12) following the Fermi team threads. We considered the total energy range of the NaI detectors (8–1000 keV) and three main sub-bands (8–40, 40–200, 200–1000 keV). Light curves were extracted using the gtbin tool. Finally we calculated the PDS for each GRB of the resulting sample in the time interval from the earliest to the latest bin whose counts exceed the 5σ signal threshold above background (hereafter, $T_{5\sigma}$).

Table 1. Time and Peak count rate. Fermi/GBM full sample including 205 GRBs. The PDS is calculated in the time interval reported. This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance.

| Trigger | $t_{\text{start}}$ (s) | $t_{\text{stop}}$ (s) | Peak rate (count s$^{-1}$) | $T_{90}$ (s) |
|---------|------------------------|------------------------|-----------------------------|--------------|
| 090714745 | 1.76 | 31.77 | 69.3 ± 4.8 | 39 |
| 090723557 | 0.13 | 119.42 | 460 ± 17 | 77 |
| 090723895 | 0.29 | 52.89 | 127.6 ± 8.1 | 43 |
| 090724401 | 0.11 | 48.34 | 268 ± 20 | 42 |
| 090736758 | 0.91 | 18.54 | 233 ± 14 | 18 |
| 090806896 | 6.98 | 40.63 | 113.1 ± 7.9 | 44 |
| 090807993 | 0.01 | 49.86 | 267 ± 20 | 20 |
| 090810549 | 10.59 | 102.31 | 50.0 ± 4.5 | 58 |
| 090816503 | 0.47 | 69.35 | 122 ± 11 | 65 |
| 090816989 | 0.04 | 29.10 | 98 ± 11 | 6 |

(a) Referred to the Fermi/GBM trigger time.

We then subtracted the white noise and checked its significance criterion for each grouped bin. Table 4 reports the time interval and peak count rate for each selected GRB in the 8–1000 keV band. Moreover, we also selected a subsample of events with S/N $\geq$ 60 to better explore the high-frequency behaviour. For this sample we extracted the light curves with a time resolution of 0.5 ms (hereafter, very high resolution or VHR curves) both in the same energy band explored by the GRBM (40–700 keV) and in the total NaI energy band (8–1000 keV).

We then subtracted the white noise and checked its Poissonian nature related to the statistical fluctuations observed in light curves. To check the Poissonian character of noise we estimated the mean power at $f > 6$ Hz (Table 2) and compared it against the value of 2, namely the expected value of a $\chi^2$-distribution for pure Poissonian variance in the Leahy normalisation (Leahy et al. 1983). Furthermore, we grouped the background-subtracted PDS along frequency so as to fulfil a 3σ significance criterion for each grouped bin.

Figure 1. $T_{90}$ distributions of a sample of 786 GRBs detected by Fermi/GBM in the 8–1000 keV energy band and of the subsample of 205 long GRBs selected for the analysis of the present work. 126 GRBs have $T_{90} < 3$ s, corresponding to $\sim$ 16% of the whole sample.
data, the selection excluded in each sample (total, low, middle and high energy range) the GRBs whose grouped PDS collected less than 4 grouped frequency bins.

We ended up with 205 GRBs that will be referred to as the Fermi sample with a 64 ms time resolution in the total energy range and, respectively, we ended up with 155, 201 and 74 in the three energy sub-bands: 8–40, 40–200, and 200–1000 keV (low, middle and high energies). The VHR sample includes 96 GRBs whose light curves were extracted in the 8–1000 keV and 40–700 keV energy bands. For each of these samples we calculated and modelled the average PDS.

2.2 BeppoSAX/GRBM data selection

For the BeppoSAX/GRBM GRB sample we started from the GRB catalogue (Frontera et al. 2009) by selecting the GRBs fully covered by the high time resolution mode, available only for those which triggered the GRBM on-board logic. We then excluded the GRBs whose light curves were hampered by gaps in the time profiles. Finally we selected the GRBs with the highest S/N (≫ 40) and ended up with a sample of 89 GRBs. This requirement was motivated by the need of having very good statistical quality even at high frequencies.

Two different kinds of time resolution are available in the GRBM data: i) light curves with 7.8125 ms resolution from −8 to 98 s from the on-board trigger time (hereafter, these curves are referred to as high-resolution or HR curves); ii) light curves with ∼0.5 ms for the first 10 s from the trigger time (VHR curves). Therefore the corresponding Nyquist frequencies are respectively 64 Hz and 1 kHz. The VHR light curve can be obtained only for a sub-sample of 74 GRBs. For each GRB we extracted the PDS in two different time intervals, depending on the type of light curve: the PDS of the HR curves was extracted on the $T_{\text{3sr}}$, like in the case of Fermi/GBM data (Section 2.1), whereas that of the VHR curves was forcibly bound to the first 10 s from the trigger time. Table 3 reports the time interval and peak count rate for each selected GRB of the HR set. Also for BeppoSAX data the final PDS obtained for each GRB of each sub-sample was grouped according to a 3-σ significance criterion excluding the events of the HR sample with fewer than 4 grouped bins and those with of the VHR sample with fewer than 10 bins. Consequently, the final samples include 42 GRBs with HR data and 25 GRBs with VHR data. Hereafter, the two samples are referred to as the BeppoSAX HR and the VHR sample, respectively.

2.3 PDS calculation

Each PDS was calculated through the mixed-radix FFT algorithm implemented within the GNU Scientific Library (Galassi et al. 2003) which does not require the total number of bins to be a power of 2 (Temperton 1983) similarly to what was done for the Swift/BAT sample (G12). We calculated the PDS for each GRBs adopting the Leahy normalisation. For each individual PDS the background level, corresponding to the white noise due to counting statistics, was initially estimated by fitting with a constant the high-frequency range, where the signal is negligible with respect to the statistical noise.

Within the Leahy normalisation, a pure Poissonian noise corresponds to a power value of 2. Therefore we checked the high-frequency constant value for the power averaged out among all the PDSs. For Fermi sample the mean value of white noise level is estimated at 1.99 ± 0.02 for $f > 6$ Hz, fully consistent with a Poissonian variance. For the BeppoSAX samples the PDS shows evidence for the presence of a small, significant extra-Poissonian variance of $(3.7 ± 1.2)%$ and $(0.94 ± 0.35)%$ for the HR and the VHR samples, respectively, in addition to the statistical white noise. These values were estimated in the frequency range above 50 Hz.

The statistical noise was removed in two different way for different cases. For the Fermi sample, noise was assumed to be perfectly Poissonian, compatibly with what we found above. Instead, for the BeppoSAX samples it was obtained from fitting the PDS with a constant value estimated at sufficient high frequencies ($f > 50$ Hz) for each event of the HR sample. The estimated background levels are reported in Table 4. As can be seen in Table 4 for VHR data, the white noise becomes dominant already at $f > 30$ Hz (at higher frequency compared to the Fermi case). Indeed, we did not find significantly different values for the mean power between the two following frequency ranges: $f > 30$ Hz and $f > 50$ Hz.

After calculating the white noise level for each GRB, we subtracted it and renormalised the PDS by the corresponding net variance (G12). This choice ensures that all GRBs have equal weights in the average PDS.

The binning scheme used to average the PDS is different for each considered sample. In the Fermi case with 64-ms binning time the Nyquist frequency is 7.8125 Hz, so we defined a uniform frequency binning scheme with a step of 0.01 Hz. At $f < 0.01$ Hz we considered two bins, 0.001 Hz $\leq f < 0.005$ Hz and 0.005 Hz $\leq f < 0.01$ Hz. The same step is used in the frequency grid defined for the average PDS of the HR BeppoSAX data. In the BeppoSAX case the PDS have correspondingly more frequency bins, due to the higher Nyquist frequency. We took only one single bin

Table 2. White noise level (Leahy normalisation) for the full Fermi sample. The mean value of this sample is 1.99 ± 0.02. Uncertainties at 1σ. This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance.

| Trigger | $(P)$ | $(f > 6 \text{ Hz})$ |
|---------|-------|---------------------|
| 080714745 | 1.63 ± 0.42 |
| 080723557 | 2.00 ± 0.24 |
| 080723985 | 2.50 ± 0.38 |
| 080724401 | 2.26 ± 0.39 |
| 080730786 | 2.44 ± 0.63 |
| 080806896 | 1.67 ± 0.35 |
| 080807993 | 2.05 ± 0.24 |
| 080810549 | 1.98 ± 0.24 |
| 080816503 | 2.09 ± 0.31 |
| 080816909 | 2.31 ± 0.51 |

(a) Too low statistic at $f > 6$ Hz. In this case white noise start at lower frequency, so we have estimated the $(P)$ level above 4 Hz.

http://www.gnu.org/s/gsl/
from 0.001 Hz and 0.01 Hz. The frequency grid changes for the VHR data: we chose a broader frequency step of 1 Hz because the total PDS extraction time is limited to 10 s for each BeppoSAX light curve and this implies a coarser frequency resolution. For the VHR PDS we considered 4 bins with step of 0.2 Hz at \( f < 1 \) Hz. For each individual GRB we calculated the average power in each frequency bin of the corresponding grid described above. Finally, for each frequency bin of the grid we determined the average power over all GRBs of a given sample after they had been renormalised. Finally the frequency bins of the average noise–subtracted PDS were grouped by requiring at least 3σ significance to reduce the uncertainties at high frequencies.

2.4 PDS fitting

The average PDS was modelled using a smoothly broken power-law in the same parametrisation as that adopted by G12,

\[
PDS(f) = 2^{1/n} F_0 \left[ \left( \frac{f}{f_b} \right)^{\nu_1} + \left( \frac{f}{f_b} \right)^{\nu_2} \right]^{-1/n}, \tag{1}
\]

where the parameters left free to vary are the break frequency \( f_b \), the two power-law indices \( \nu_1 \) and \( \nu_2 \) (\( \nu_2 > \nu_1 \)) and the normalisation parameter, \( F_0 \). The smoothness parameter \( n \) could not be effectively constrained in all cases, thus it was fixed to \( n = 10 \), corresponding to a relatively sharp break around \( f_b \), for all cases to ensure a more homogeneous comparison between the best-fit values obtained over different sets as well as with previous results obtained from the Swift data. Thanks to the central limit theorem, we can assume these variables to be normally distributed. This allows us to determine the best-fitting model by minimising the following un-normalised negative log-likelihood function,

\[
L = \frac{1}{2} \sum_{i=1}^{N_f} \left( \frac{P_i - PDS(f_i)}{\sigma_i} \right)^2, \tag{2}
\]

where \( P_i \) and \( f_i \) are the observed power and frequency of the \( i \)-th bin. \( N_f \) is the number of frequency bins, excluding the Nyquist frequency.

3 RESULTS

3.1 Average PDS at different energy bands

Table \( a \) reports the best-fit parameters estimated for the average PDSs for the different GRB samples considered.

For the average Fermi PDS extracted in the total energy range 8–1000 keV (Figure 2) with 64-ms binning time the best-fitting parameters are \( \nu_1 = 1.06^{+0.05}_{-0.07} \), a break at 5.5x10^{-2} Hz above which the PDS steepens to \( \nu_2 = 1.75 \pm 0.03 \). This slope of the spectra is very similar to the previous values found in the literature related to the GRBs detected with BATSE in similar energy bands (BSS98, BSS00), and in agreement with the value of 5/3 of a Kolmogorov spectrum. Indeed BSS00 have found an index ranging from 1.50 to 1.72 in the frequency range 0.025 < \( f < 1 \) Hz fitting the average PDS resulted from the BATSE sample (20–2000 keV) with a simple power–law. Moreover, also for the average PDS of Swift/BAT data (15–150 keV) we see a typical slope described with a low–frequency index \( \nu_1 = 1.03 \pm 0.05 \) up to a break frequency around 3 \( \times \) 10^{-2} Hz, followed by and an index \( \nu_2 = 1.73^{+0.04}_{-0.03} \) (G12). Since the break frequency \( f_b \) is sensitive to the average characteristic time \( \tau \) of typical individual shots roughly as \( f_b \sim 1/(2\pi\tau) \) (Frontera & Fuligini 1974; Bell 1992; Lazzati 2002), the value we found in the Fermi data corresponds to a mean characteristic time of about 3 s.

Comparing the average PDS of the whole Fermi sample with that of the high–quality (S/N \( \geq 60 \)) subsample extracted with a 0.5–ms resolution, the latter data set shows evidence for a further break around 1–2 Hz with respect to the best–fitting model obtained for the former data set (bottom data in Fig. 2). The behaviour of the average PDS at high frequency is thoroughly discussed in Section 3.3.1 together with BeppoSAX data.

The analysis of the average PDS at different energy channels reveals a clear trend of the spectral shape when we move from soft to hard energy ranges. Figure 3 displays the average PDS corresponding to three different energy channels: 8–40, 40–200, and 200–1000 keV. The index \( \nu_2 \) decreases from 1.95 to 1.47 moving from 8–40 to 200–1000 keV. This reflects the known narrowing of pulses with energy, according to which the same GRB pulse appears to be narrower and spikier at higher energies (Fenimore et al. 1993; Norris et al. 1996; Piro et al. 1998). The same trend was observed in the BATSE average PDS (BSS00), for which the power–law index decreases from 1.72 in the 25–55 keV to 1.50 above 320 keV. Furthermore, a similar behaviour is observed in the Swift data, with \( \nu_2 \) varying from 1.75^{+0.05}_{-0.04} to 1.49^{+0.05}_{-0.07} passing from 15–50 to 50–150 keV.

We also extracted the light curves in the common energy bands with other instruments so that we can compare results limiting the systematic differences connected with different energy passbands. The average Fermi/GBM PDS obtained in the typical Swift/BAT energy range (15–150 keV) are per-
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3.2 FRED sub-sample

We investigated whether the GRBs whose light curves can be described as a single fast rise exponential decay (FRED) show distinctive features in the average PDS. To this aim, we selected 10 GRB of this kind out of the Fermi sample by visual inspection and calculated the corresponding average PDS. The best-fit parameters in this case are \( \alpha_1 = 1.32 \pm 0.10 \) and \( \alpha_2 = 2.53\pm0.39 \) with a break at about 6 \( \times 10^{-2} \) Hz (see Table 3). That the high-frequency tail of the PDS for the FRED sample is steeper than that of the whole sample of GRBs, agrees with the PDS expected for a single FRED (e.g., see Lazzati 2002). This in turn suggests that the average PDS of multiple–pulse GRBs is shallower because of the presence of various characteristic times. The sum of several PDS with different break frequencies would therefore result in a simple power–law with no dominant break in the explored frequency range.

3.3 Average PDS up to high frequency

The average PDS for the HR light curves provided by the BeppoSAX/GRBM shows a second break at high frequency \( (f_2 = 1.91\pm0.43 \text{ Hz}) \). The observed slope can be parametrised with two indices, \( \alpha_2 = 1.49 \pm 0.04 \) and \( \alpha_3 = 2.46\pm0.17 \) (we did not use \( \alpha_1 \), which has previously been used to denote the slope below a few \( 10^{-3} \) Hz). The break is likely to be real because the improvement is significant compared to the same model without it. The \( F \)-test yields a probability of 1.26% that the break is not required. The value itself of this break frequency as well as the values of the corre-

\[ \frac{\alpha_1}{\alpha_2} = 0.59 \]

Figure 5. The 10 FRED used in our PDS analysis. Each curve has a 64–ms bin time and is expressed in units of counts s \(^{-1}\) as a function of time.

Figure 4. Average PDS of the 15–150 keV energy range (circles) compared with the Swift/BAT result (squares) provided by G12. Both are calculated from 64–ms binned light curves. The two independent measures are compatible. The dashed line shows the best fit model for Fermi data.

Figure 3. Different slopes observed in the average PDS extracted on the three main energy ranges using a time resolution of 64 ms. Upside–down triangles, circles, and triangles show the 8–40, 40–200, and 200–1000 keV energy bands, respectively. The spectrum becomes shallower moving from low to high energies.
sponding power–law indices indicate that this feature has a different origin from the other one observed at lower frequency. This motivated us to adopt a different notation for the power-law index above this break, $\alpha_3$. Overall, the different slopes refer to the corresponding frequency ranges: $\alpha_1$ below a few $10^{-2}$ Hz, $\alpha_2$ holds in the range $10^{-2} < f < 1$ Hz, and $\alpha_3$ for $f \gtrsim 1$ Hz.

When we limit our PDS analysis to the first 10 s of the GRBM trigger time of each GRB light curve and use the VHR data, a very similar result is found for the average PDS, which now extends up to 1 kHz. The best–fitting parameters for these data are $\alpha_2 = 1.52 \pm 0.17$, $\alpha_3 = 2.91^{+0.51}_{-0.41}$ with a clear break at $f_{b2} = 2.59^{+0.04}_{-0.04}$ Hz (Fig. 4). Also in this case a break in the model is required to fit the data, with a probability of 0.47% that the improvement obtained with the break is due to chance according to the F-test. Furthermore in Fig. 2 the average PDS obtained from the Fermi VHR sample looks like it also requires a break at $f \gtrsim 1$ Hz. To check the mutual compatibility of these data with a broken power–law model, we extracted the Fermi VHR average PDS over the same energy range covered also by GRBM, 40–700 keV. To fit these data above 0.02 Hz we used a simple power–law as well as a broken power–law and used the F-test to evaluate the improvement one obtains moving from the former to the latter. We estimate a probability of 3.4% that such improvement is just by chance. We found two different slopes, $\alpha_2 = 1.65 \pm 0.03$ and $\alpha_3 = 2.41^{+0.34}_{-0.30}$, with a break at $f_{b2} = 1.1^{+0.3}_{-0.2}$ Hz ($\chi^2$/dof = 1.07). We excluded from the fit the lowest frequency point in the BeppoSAX HR PDS and in the Fermi VHR PDS (40–700 keV), because it clearly lies below the extrapolation of a double broken power–law, since it is clearly affected by the presence of the low–frequency break.

We also performed a combined analysis of the two and three samples, BeppoSAX (HR + VHR) (i.e., BeppoSAX data alone), and BeppoSAX (HR + VHR) plus Fermi VHR, fitting all the spectra simultaneously with the same model, apart from allowing each set a different normalisation term. For the BeppoSAX data alone, the resulting break frequency is found to be $f_{b2} = 2.11^{+0.42}_{-0.33}$ Hz, while the two slopes have indices respectively $\alpha_2 = 1.50^{+0.03}_{-0.04}$ and $\alpha_3 = 2.69^{+0.27}_{-0.20}$. This treatment implicitly assumed the two data sets to be statistically independent. Although this is not completely true, since the 10 s data of the VHR curves are part of the full profile of about 100 s of HR data, on average the common data amount to 10–20% or so. Consequently, the expected correlation between the two data sets affects the results within a comparable fraction. By adding the VHR sample extracted with Fermi, we found $\alpha_2 = 1.60^{+0.02}_{-0.03}$, $\alpha_3 = 2.33^{+0.15}_{-0.12}$, with a break at $f_{b2} = 1.4 \pm 0.3$ Hz ($\chi^2$/dof = 1.37). We tried to see whether the quality of the fit could be improved by allowing the smoothness parameter to vary (eq. 1), thus allowing a smooth transition from one power–law regime to the following one, with no appreciable result though.

Although the white noise subtraction was done through a careful estimation of the high frequency power (Section 2.3), we examined whether the break could be an artifact of a small bias in the white noise subtraction. More specifically, overestimating the white noise could mimic the appearance of an artificial break. To test this possibility, we extracted the average PDS without noise subtraction, keeping the same relative normalisation for each GRB as that of the noise– subtracted case. We fixed the best–fitting model of the noise– subtracted PDS obtained above and fitted the white noise with a constant. Figure 5 clearly shows that the break in the average PDS occurs when the average signal still dominates the white noise level (by more than one order of magnitude in the VHR data). This rules out the possibility of the break around 1–2 Hz being the result of biased white noise subtraction and suggests it to be a genuine feature of the average PDS at energies above 40 keV.
4 DISCUSSION

In general, two distinct sources of time variability have been found to characterise the GRB variability: a fast component dominated by the presence of relatively short (<1 s) pulses and a slow component linked to pulses lasting several seconds (Scargle et al. 1998; Vetere et al. 2006; Margutti 2009; Gao et al. 2012). These two kinds of dominant time scales should be produced by different mechanisms involved in the physical process, and different explanations in different scenarios are available in the literature (Morsony et al. 2011; Zhang & Yan 2011; Titarchuk et al. 2012). The simple power–law modelling the average PDS and encompassing nearly two orders of magnitude in frequency is suggestive of some kind of scale invariance within the same frequency range, thus confirming the coexistence of multiple characteristic timescales.

The study of the average PDS in different energy ranges made possible by Fermi/GBM provides clues to better characterise the different aspects of GRB time variability. The observed energy dependence of the power–law index of the average PDS, $\alpha_2$, in the frequency range $10^{-2} < f < 1$ Hz confirms and extends the results found with previous work and data sets. Indeed, in the 8–1000 keV band the average PDS of long GRBs detected with GBM show a broken power–law behaviour ($\alpha_1 = 1.06^{+0.05}_{-0.07}$, $\alpha_2 = 1.73^{+0.04}_{-0.03}$ and $f_0 = 5.5 \times 10^{-3}$ Hz) with $\alpha_2$ very close to the slope of average PDS observed in the BATSE analysis ($\alpha \approx 1.67$).

More specifically, the average PDS slope undergoes a steep–to–shallow evolution passing from soft to hard energy channels, as shown in Fig. 8. This behaviour is consistent with the narrowing of pulses with energy: Fenimore et al. (1993) found a dependence of the average pulse width $w$ on energy $E$ as $w \propto E^{-0.4}$, estimated by measuring the average auto-correlation function (ACF) width for a sample of BATSE bursts as a function of the energy channel. In addition to the energy dependence of the average pulse width, also the shape itself and, in particular, the peakedness of the average ACF depends on energy (BSS00). Indeed, the energy dependence of the shape of the pulse profile explains the energy dependence of the power–law index: if the shapes of a given pulse at different energies were the same, only the break frequency in the average PDS should change correspondingly, while the slope should remain unaffected. Since this is not what is observed, the evolution with energy of the average power–law index in the PDS confirms the change in the shape itself of the energy pulse as a function of energy.

Another important result that emerged from the present analysis is the break revealed around 1–2 Hz in the BeppoSAX average PDS. Although the evidence for it in the Fermi data alone is less compelling because of the lower S/N in that frequency range, the Fermi average PDS is fully compatible with it. The joint BeppoSAX–Fermi analysis of such high–frequency break shows that this may significantly vary between 1 and 2 Hz, depending on the GRB sample and on its average S/N. Together with results obtained on Swift data by G12, this break becomes evident at harder energies.

This feature in the average PDS and its possible dependence on energy provides an important clue to constraining theoretical models proposed to explain the physical mechanism involved in GRBs and confirms and strengthens the analogous result obtained by BSS00 on BATSE data. The break could be related to an average intrinsic variability time scale, $\Delta t \lesssim 0.1$ s, below which the temporal power changes regime. This may link directly to the central engine. Alternatively, it could be related to the variation of the outflow Lorentz factor, or it could depend on the radius at which the expanding shell becomes optically thin $R_\ast$. In this latter scenario we could observe variability only on time scales longer than a characteristic time $t_\ast = R_\ast/cR^2$ (BSS00).

A number of theoretical interpretations of the power–law PDS with an index compatible with 5/3 have been put forward in the literature. This is what is expected for a Kolmogorov spectrum within a medium with fully developed turbulence. For instance, in the internal shock model, the parameters of the wind of relativistic shells can be constrained so as to reproduce the observed average PDS (Panaitescu et al. 1998; Spada et al. 2000); or in the context of a relativistic jet making its way out through the stellar envelope of the progenitor star (Zhang et al. 2009).
5 CONCLUSIONS

We studied the properties of the average PDS of GRBs in two unexplored regimes: across a broadband energy range from 8 keV to 1 MeV using Fermi/GBM data and up to very high frequencies (up to 1 kHz) using BeppoSAX/GRBM data. In agreement with previous results obtained from an analogous analysis of CGRO/BATSE and of Swift/BAT data, we also found a clear relation between the average PDS slope from $\sim 0.01$ to $\sim 1$ Hz range and energy, with the index spanning the range from 1.5 to 1.9 from 8 keV through 1 MeV in three channels ($8-40, 40-200$, and $200-1000$ keV). The slope of the average PDS carries information about the spininess of light curve as well as the multiple presence of several characteristic time scales (scale invariance within the two decades of the aforementioned frequency range).

For the first time we extended the study of PDS up to 1 kHz in frequency with the very high time resolution provided by BeppoSAX/GRBM. In this case, the average PDS pinned down a clear break at 1–2 Hz. This provides a strong clue to the dominant minimum variability time, potentially connected with either the intrinsic inner engine variability, or with the dispersion of the bulk Lorentz factor distribution for a wind of relativistic shells, or with the average distance at which internal collisions dissipate energy into gammarays. Combining our results with those obtained from the Swift data set, the presence of this break emerges only in the harder energy channels ($\gtrsim 100$ keV).

The average slope is broadly consistent with the theoretically appealing value of 5/3 expected for a Kolmogorov spectrum of velocities within a fully turbulent medium, as suggested in previous works (BSS98, BSS00). Our results in the frequency range $\sim 10^{-2}$ to $\sim 1$ Hz are in broad agreement with a number of theoretical interpretations within different alternative contexts, encompassing the classical internal shock scenario as well as the magnetically–dominated outflows models. Instead, still missing is a detailed theoretical explanation for the other two properties: i) the presence of the 1–2 Hz break and its energy dependence; ii) the energy dependence of the average power–law index.

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Table 3. Time and Peak count rate. *BeppoSAX*/GRBM HR sample including 42 GRBs.

| GRB     | \(t_{\text{start}}\) (s) | \(t_{\text{stop}}\) (s) | Peak rate (count s\(^{-1}\)) | \(T_{\text{bb}}\) (s) |
|---------|-----------------|-----------------|------------------|-----------------|
| 970111  | −0.34           | 40.02           | 46.53 ± 1.26     | 31.00           |
| 970117B | −0.25           | 19.00           | 61.63 ± 1.38     | 13.00           |
| 970315A | −0.41           | 20.97           | 116.87 ± 8.15    | 15.00           |
| 970517B | −0.75           | 3.61            | 139.11 ± 5.09    | 5.00            |
| 970601  | 6.99            | 41.75           | 74.83 ± 3.13     | 30.00           |
| 970612B | −0.89           | 37.71           | 14.53 ± 2.49     | 38.00           |
| 970625B | −1.35           | 48.98           | 178.88 ± 9.79    | 15.00           |
| 970627B | −0.73           | 15.86           | 100.03 ± 7.58    | 15.00           |
| 970706  | −9.01           | 72.25           | 15.69 ± 0.61     | 59.00           |
| 970816  | −0.06           | 6.61            | 51.43 ± 2.74     | 6.00            |
| 971027A | −1.66           | 12.20           | 26.89 ± 1.46     | 11.00           |
| 971223C | −6.22           | 50.18           | 52.58 ± 4.13     | 47.00           |
| 980209B | 0.38            | 48.73           | 217.07 ± 8.81    | 23.00           |
| 980309C | 0.62            | 28.45           | 79.07 ± 1.95     | 21.00           |
| 980392A | −1.06           | 36.93           | 73.26 ± 4.15     | 19.00           |
| 980428  | −5.05           | 88.46           | 21.72 ± 1.34     | 100.00          |
| 980615B | 0.94            | 97.48           | 85.10 ± 5.01     | 64.00           |
| 980827C | 0.33            | 87.24           | 158.30 ± 5.36    | 51.00           |
| 981111  | −6.39           | 48.81           | 35.91 ± 2.67     | 34.00           |
| 990128  | 0.67            | 11.30           | 121.88 ± 3.11    | 8.00            |
| 990620  | 0.23            | 13.97           | 38.68 ± 1.72     | 16.00           |
| 990705  | −0.23           | 41.19           | 63.92 ± 3.93     | 32.00           |
| 990913A | 0.03            | 44.54           | 183.03 ± 8.08    | 40.00           |
| 991124B | −1.65           | 25.31           | 8.01 ± 0.62      | 28.00           |
| 991216B | 0.46            | 25.42           | 416.88 ± 11.96   | 15.00           |
| 991315  | 0.04            | 25.71           | 200.84 ± 8.42    | 15.00           |
| 990214A | 0.37            | 8.75            | 56.66 ± 3.60     | 8.00            |
| 990218B | 0.26            | 23.70           | 258.43 ± 11.67   | 20.00           |
| 990419  | 0.72            | 20.70           | 21.65 ± 0.82     | 20.00           |
| 990630  | 0.94            | 44.55           | 21.76 ± 2.02     | 26.00           |
| 990718B | −0.19           | 97.05           | 67.51 ± 2.98     | 34.00           |
| 991004  | 1.10            | 11.20           | 191.46 ± 8.26    | 9.00            |
| 991011C | 0.94            | 31.62           | 29.67 ± 1.42     | 24.00           |
| 991212B | 0.64            | 72.46           | 45.83 ± 3.10     | 67.00           |
| 991009  | 0.90            | 22.17           | 293.48 ± 6.62    | 7.00            |
| 991317  | 0.87            | 31.03           | 210.87 ± 8.65    | 30.00           |
| 991408B | 0.23            | 6.40            | 199.33 ± 8.39    | 3.81            |
| 991412  | −1.49           | 65.48           | 24.62 ± 2.47     | 60.00           |
| 990504  | −0.12           | 19.84           | 42.79 ± 4.07     | 15.00           |
| 991020B | 1.06            | 27.05           | 53.73 ± 4.50     | 20.00           |
| 991022  | 0.60            | 41.52           | 19.20 ± 1.30     | 40.00           |
| 991103  | −0.94           | 45.41           | 36.72 ± 1.81     | 34.00           |

Note. — The PDS is calculated in the time interval reported.

*Referred to the *BeppoSAX*/GRBM trigger time.
Table 4. White noise level (Leahy normalisation). BeppoSAX HR sample.

| GRB   | (P) (f > 30 Hz) | (P) (f > 50 Hz) |
|-------|-----------------|-----------------|
| 970111| 2.13 ± 0.09     | 2.11 ± 0.15     |
| 970117B| 2.11 ± 0.14     | 2.17 ± 0.22     |
| 970315A| 2.09 ± 0.13     | 2.09 ± 0.20     |
| 970517B| 2.50 ± 0.31     | 2.19 ± 0.46     |
| 970601 | 2.18 ± 0.10     | 2.24 ± 0.16     |
| 970612B| 2.15 ± 0.10     | 2.14 ± 0.15     |
| 970625B| 2.10 ± 0.08     | 2.04 ± 0.13     |
| 970627B| 2.02 ± 0.15     | 1.94 ± 0.26     |
| 970706 | 2.09 ± 0.07     | 2.13 ± 0.10     |
| 970816 | 1.75 ± 0.22     | 1.88 ± 0.35     |
| 971027A| 1.98 ± 0.16     | 1.95 ± 0.25     |
| 971223C| 2.13 ± 0.08     | 2.22 ± 0.13     |
| 980203B| 2.07 ± 0.09     | 2.04 ± 0.13     |
| 980306C| 1.96 ± 0.11     | 1.97 ± 0.17     |
| 980329A| 2.06 ± 0.10     | 1.98 ± 0.15     |
| 980428 | 2.08 ± 0.06     | 2.14 ± 0.10     |
| 980615B| 2.10 ± 0.06     | 2.08 ± 0.09     |
| 980827C| 2.08 ± 0.06     | 2.18 ± 0.10     |
| 981111 | 2.16 ± 0.08     | 2.19 ± 0.13     |
| 990128 | 2.11 ± 0.19     | 2.08 ± 0.29     |
| 990620 | 2.13 ± 0.16     | 2.12 ± 0.26     |
| 990705 | 2.07 ± 0.09     | 1.97 ± 0.14     |
| 990913A| 1.94 ± 0.09     | 1.95 ± 0.14     |
| 991124B| 2.11 ± 0.12     | 2.14 ± 0.18     |
| 991216B| 2.01 ± 0.12     | 2.01 ± 0.19     |
| 000315 | 2.06 ± 0.12     | 2.03 ± 0.18     |
| 000214A| 2.12 ± 0.21     | 1.91 ± 0.32     |
| 000218B| 2.44 ± 0.13     | 1.81 ± 0.18     |
| 000419 | 2.12 ± 0.13     | 2.05 ± 0.21     |
| 000630 | 2.03 ± 0.09     | 2.03 ± 0.14     |
| 000718B| 2.10 ± 0.06     | 2.05 ± 0.09     |
| 010004 | 2.05 ± 0.19     | 1.99 ± 0.29     |
| 010011C| 2.07 ± 0.11     | 2.00 ± 0.18     |
| 010122B| 2.04 ± 0.07     | 2.12 ± 0.11     |
| 010109 | 1.93 ± 0.13     | 1.87 ± 0.20     |
| 010317 | 2.29 ± 0.11     | 2.05 ± 0.17     |
| 010408B| 1.76 ± 0.23     | 1.4 ± 0.34      |
| 010410 | 2.01 ± 0.07     | 2.07 ± 0.11     |
| 010504 | 2.03 ± 0.14     | 1.98 ± 0.25     |
| 010710B| 2.07 ± 0.12     | 2.00 ± 0.18     |
| 010922 | 2.06 ± 0.09     | 2.09 ± 0.15     |
| 011003 | 2.10 ± 0.09     | 2.17 ± 0.14     |

Note. — Table of white noise level at f > 30 Hz and at f > 50 Hz related to the sub-sample of 42 GRBs detected by BeppoSAX/GRBM with 7.8 ms time resolution. Uncertainties at 1σ.
Table 5. White noise level (Leahy normalisation). *BeppoSAX* VHR sample.

| GRB     | \( \langle P \rangle (f > 30 \text{ Hz}) \) | \( \langle P \rangle (f > 50 \text{ Hz}) \) |
|---------|---------------------------------------------|---------------------------------------------|
| 970315A | 2.00 ± 0.03                                 | 2.00 ± 0.03                                 |
| 970517B | 2.03 ± 0.03                                 | 2.03 ± 0.03                                 |
| 970601  | 2.12 ± 0.03                                 | 2.10 ± 0.04                                 |
| 970625B | 2.02 ± 0.03                                 | 2.01 ± 0.03                                 |
| 970816  | 1.97 ± 0.04                                 | 1.98 ± 0.04                                 |
| 980203B | 2.01 ± 0.03                                 | 2.01 ± 0.03                                 |
| 990128  | 2.04 ± 0.03                                 | 2.04 ± 0.03                                 |
| 990620  | 2.06 ± 0.03                                 | 2.06 ± 0.03                                 |
| 990705  | 2.04 ± 0.03                                 | 2.04 ± 0.03                                 |
| 990913A | 1.97 ± 0.03                                 | 1.97 ± 0.03                                 |
| 991216B | 1.91 ± 0.03                                 | 1.91 ± 0.03                                 |
| 000115  | 2.01 ± 0.03                                 | 2.01 ± 0.03                                 |
| 000214A | 2.03 ± 0.03                                 | 2.03 ± 0.03                                 |
| 000630  | 2.05 ± 0.03                                 | 2.05 ± 0.03                                 |
| 001004  | 1.99 ± 0.03                                 | 1.98 ± 0.03                                 |
| 001212B | 2.06 ± 0.03                                 | 2.06 ± 0.03                                 |
| 010319  | 1.94 ± 0.03                                 | 1.94 ± 0.03                                 |
| 010317  | 2.04 ± 0.03                                 | 2.03 ± 0.03                                 |
| 010408B | 1.99 ± 0.03                                 | 1.99 ± 0.03                                 |
| 010504  | 2.01 ± 0.04                                 | 2.01 ± 0.04                                 |

Note. — Table of white noise level at \( f > 30 \text{ Hz} \) and at \( f > 50 \text{ Hz} \) related to the sub sample of 25 GRBs detected by *BeppoSAX/GRBM* with 0.5 ms time resolution. Uncertainties at 1σ.
Table 6. Best fit parameters of the average PDS for different samples of GRBs

| Sample | Size | Norm | $\alpha_1$ | $f_b$ (10$^{-2}$ Hz) | $\alpha_2$ | $f_{b2}$ (Hz) | $\alpha_3$ | $\chi^2$/dof |
|--------|------|------|------------|------------------|------------|--------------|------------|-------------|
| Fermi/GBM (8–1000 keV)$^a$ | 205 | $5.0^{+1.2}_{-0.9}$ | $1.06^{+0.05}_{-0.07}$ | $5.5^{+0.8}_{-0.7}$ | $1.75^{+0.03}_{-0.03}$ | – | – | 110/100 |
| Fermi/GBM (8–40 keV)$^a$ | 155 | $3.9^{+1.1}_{-1.0}$ | $1.20^{+0.08}_{-0.07}$ | $6.4^{+1.1}_{-0.7}$ | $1.95^{+0.07}_{-0.06}$ | – | – | 78/54 |
| Fermi/GBM (40–200 keV)$^a$ | 201 | $5.1^{+0.7}_{-1.1}$ | $1.03^{+0.06}_{-0.04}$ | $5.5^{+1.0}_{-0.5}$ | $1.67^{+0.02}_{-0.03}$ | – | – | 130/115 |
| Fermi/GBM (200–1000 keV)$^a$ | 74 | $7.3^{+5.8}_{-2.7}$ | $1.05^{+0.08}_{-0.09}$ | $3.8^{+3.4}_{-1.3}$ | $1.47^{+0.06}_{-0.04}$ | – | – | 79/72 |
| Fermi/GBM FRED (8–100 keV)$^a$ | 10 | $3.8^{+2.0}_{-1.9}$ | $1.32^{+0.10}_{-0.10}$ | $6.3^{+3.1}_{-1.9}$ | $2.53^{+0.39}_{-0.24}$ | – | – | 16/14 |
| BeppoSAX/GRBM HR (40–700 keV)$^b$ | 42 | $0.32^{+0.013}_{-0.04}$ | – | – | $1.45^{+0.04}_{-0.04}$ | $1.9^{+0.4}_{-0.4}$ | $2.46^{+0.31}_{-0.31}$ | 145/143 |
| BeppoSAX/GRBM VHR (40–700 keV)$^b$ | 25 | $0.04^{+0.004}_{-0.002}$ | – | – | $1.52^{+0.17}_{-0.17}$ | $2.6^{+0.3}_{-0.3}$ | $2.9^{+0.4}_{-0.4}$ | 4/7 |
| BeppoSAX/GRBM HR+VHR (40–700 keV)$^{b,c}$ | 42+25 | $0.016^{+0.006}_{-0.005}$ | $0.05^{+0.017}_{-0.014}$ | – | $1.50^{+0.03}_{-0.03}$ | $2.1^{+0.4}_{-0.3}$ | $2.69^{+0.27}_{-0.26}$ | 165/161 |
| Fermi/GBM VHR (40–700 keV)$^{b,d}$ | 96 | $0.02^{+0.015}_{-0.011}$ | – | – | $1.65^{+0.03}_{-0.03}$ | $1.1^{+0.3}_{-0.3}$ | $2.41^{+0.34}_{-0.34}$ | 213/200 |
| BeppoSAX/GRBM HR+VHR + Fermi/GRBM VHR (40–700 keV)$^{b,e}$ | 42+25+96 | $0.027^{+0.014}_{-0.008}$ | $0.088^{+0.042}_{-0.025}$ | $0.019^{+0.010}_{-0.006}$ | – | $1.66^{+0.02}_{-0.03}$ | $1.4^{+0.3}_{-0.3}$ | $2.33^{+0.15}_{-0.13}$ | 502/365 |
| Fermi/GBM (15–150 keV)$^b$ | 200 | $5.1^{+1.2}_{-1.0}$ | $1.06^{+0.07}_{-0.07}$ | $5.5^{+0.9}_{-0.7}$ | $1.78^{+0.04}_{-0.03}$ | – | – | 95/91 |

Note. — Best–fitting parameters of the average PDS of each sample within different energy bands (Fermi) and time resolution (BeppoSAX).

$^a$Low frequency break

$^b$High frequency break

$^c$Joint fitting of two samples with different time resolutions obtained through the minimization of the joint likelihood. The normalisation parameters refer to $7.8$ and $0.5$–ms time resolution, respectively.

$^d$In this case, the best–fitting parameters were found by fitting the average spectra in the same frequency range considered for BeppoSAX from 0.02 to 1000 Hz.

$^e$Joint fitting of three samples with different time resolutions obtained through the minimization of the joint likelihood. The normalisation parameters refer to $7.8$ and $0.5$ ms time resolution for BeppoSAX and $0.5$ ms for the Fermi, respectively.