Abstract

The Schalkwijk-Kailath (SK) scheme, which achieves the capacity of the point-to-point white Gaussian channel with feedback, is secure by itself and also achieves the secrecy capacity of the Gaussian wiretap channel with feedback, i.e., the SK scheme is a self-secure capacity-achieving (SSCA) feedback scheme for the Gaussian wiretap channel. For the multi-user wiretap channels, recently, it has been shown that Ozarow’s capacity-achieving feedback scheme for the two-user Gaussian multiple-access channel (GMAC) is the SSCA feedback scheme for the two-user Gaussian multiple-access wiretap channel (GMAC-WT). In this paper, first, we propose a capacity-achieving feedback scheme for the two-user GMAC with degraded message sets (GMAC-DMS), and show that this scheme is the SSCA feedback scheme for the two-user GMAC-WT with degraded message sets (GMAC-WT-DMS). Next, we extend the above scheme to the two-user GMAC-DMS with noncausal channel state information at the transmitters (NCSIT), and show that the extended scheme is capacity-achieving and also a SSCA feedback scheme for the two-user GMAC-WT-DMS with NCSIT. Finally, we derive outer bounds on the secrecy capacity regions of the two-user GMAC-WT-DMS with or without NCSIT, and numerical results show the rate gains by the feedback.
Index Terms

Degraded message sets, feedback, Gaussian multiple-access channel, noncausal channel state information, secrecy capacity region, wiretap channel.

I. INTRODUCTION

The multiple-access channel (MAC), which characterizes the up-link of wireless communication, has received extensive attention in the literature. The capacity regions of MAC and Gaussian MAC (GMAC) were determined by [1] and [2], respectively. Unlike the well known fact that feedback does not increase the capacity of a discrete memoryless channel, [3]-[4] found that feedback increases the capacity region of the MAC by proposing inner bounds on the capacity region of the MAC with feedback. The capacity region of the MAC with feedback remains open, and it is only determined for some special cases:

- For the two-user GMAC with feedback, Ozarow [5] proposed a hybrid scheme which combines the cooperative scheme in [3] and the Schalkwijk-Kailath (SK) scheme [6] for the point-to-point Gaussian channel with feedback, and showed that this scheme is capacity-achieving\(^1\). Subsequently, [7] investigated the two-user GMAC with feedback and noncausal channel state information at the transmitters (NCSIT), and showed that a variation of Ozarow’s scheme [5] is capacity-achieving.

- For the two-user MAC with degraded message sets (DMS), where two independent messages are sent from two sources to a common destination, the uninformed encoder only has access to one message, while the informed encoder has access to both messages. Though it has already been shown that feedback does not increase the capacity region of the MAC with DMS (MAC-DMS) [8], [9] proposed a capacity-achieving scheme for the MAC-DMS with feedback, which is an extension of the posterior matching scheme for the point-to-point discrete memoryless channel with feedback [10].

The physical layer security (PLS), which captures the fundamental limit of secure transmission over communication channels, was first investigated by Wyner in his landmark paper on the wiretap channel (WTC) [11]. The secrecy capacities (channel capacities with perfect secrecy constraint) of the discrete memoryless WTC (DM-WTC) and the Gaussian WTC (G-WTC) was determined in [11]-[12] and [13], respectively. In recent years, the PLS in multiple-access

\(^1\)Here note that for the \(N\)-user (\(N \geq 3\)) GMAC with feedback, the capacity region remains open.
channels receives much attention. Specifically, [14] studied the two-user Gaussian multiple-access wiretap channel (GMAC-WT), and proposed an inner bound on the secrecy capacity region. [15] investigated arbitrarily varying MAC with strong secrecy constraint, and provided bounds on its secrecy capacity region. [16]-[18] studied variations of the MAC with secrecy constraint, and proposed bounds on the corresponding secrecy capacity regions. [19] proposed cooperative jamming schemes for the multiple-access wiretap channel (MAC-WT), which enhance the secrecy capacity region. [20] studied the MAC-WT with NCSIT, and provided bounds on its secrecy capacity region. [21] investigated the effect of feedback delay on the secrecy capacity of the finite state MAC-WT. [22] studied the secure relay schemes for the MAC-WT.

Channel feedback has been proved to be a useful tool to enhance the PLS in communication systems. Recently, [23] showed that the secrecy capacity of the G-WTC with feedback equals the capacity of the same model without secrecy constraint, and it is achieved by the classical SK scheme [6] which is not designed with the consideration of secrecy, i.e., the SK scheme is a self-secure capacity-achieving (SSCA) feedback scheme for the G-WTC. Based on the surprising finding of [23], [24] and [25] respectively showed that variations of the classical SK scheme are also SSCA feedback schemes for the colored G-WTC and the G-WTC with NCSIT. Very recently, [26] showed that Ozarow’s scheme [5] and its variation [7] are also SSCA feedback schemes for the two-user GMAC-WT with or without NCSIT.

Although the SSCA feedback schemes have been well studied in the Gaussian wiretap channels and the Gaussian multiple-access wiretap channels, such a topic remains open for the multiple-access wiretap channels with DMS. In this paper, we focus on the two-user GMAC-WT with DMS, and with or without NCSIT, and study how to design SSCA feedback schemes for these models. We summarize our contribution as follows.

1) Since Ozarow’s scheme is a SSCA feedback scheme for the two-user GMAC-WT [26], it is natural to ask: is this kind of scheme also a SSCA feedback scheme for the two-user GMAC-WT with DMS (GMAC-WT-DMS)? Unfortunately, we find that though Ozarow’s scheme is secure by itself, it cannot achieve the capacity region of the two-user GMAC with DMS (GMAC-DMS) and feedback, hence it is not a SSCA feedback scheme for the two-user GMAC-WT-DMS. In this paper, we propose a novel two-step SK-type feedback scheme for

2In [9], a capacity-achieving feedback scheme is proposed for the MAC-DMS with feedback, but whether this scheme is self-secure or not remains unknown.
the two-user GMAC-DMS, and show that this new feedback scheme is SSCA for the two-user GMAC-WT-DMS. The novelty of this new scheme is explained below.

In the two-user GMAC-DMS with feedback, since the informed encoder has access to both messages, we split this encoder into two parts, where one part encodes the message with rate \( R_2 \) as the codeword \( V^N \), and the other part together with the uninformed encoder encode the message with rate \( R_1 \) as the codewords \( U^N \) and \( X_1^N \). For the receiver, \( U^N \) and \( X_1^N \) are decoded first, and after successfully decoding \( U^N \) and \( X_1^N \), the receiver subtracts them from his/her received signal and further decodes \( V^N \).

Since \( U^N \) and \( X_1^N \) are known by the informed encoder, they can be perfectly canceled when the informed encoder encodes \( V^N \), i.e., the noise of the equivalent channel for \( V^N \) is the original white Gaussian channel noise \( \eta_1^N \) of the GMAC. Hence we directly apply the classical SK scheme \[6\] for the point-to-point white Gaussian channel with feedback to \( V^N \), and from \[23\], we know that the coding scheme of \( V^N \) is SSCA. However, different from the encoding scheme of \( V^N \), since \( V^N \) is not known by the uninformed encoder, for the encoding scheme of \( U^N \) and \( X_1^N \), the noise of their equivalent channel is \( V^N + \eta_1^N \), which is non-white Gaussian noise due to the reason that \( V^N \) is generated by classical SK scheme \[6\] and it is not independent identically distributed (i.i.d.) generated. In general, it is difficult to design a SSCA SK-type scheme for the non-white Gaussian channel. However, by letting the encoder of \( V^N \) work first (starting from time 1), and the encoder of \( U^N \) and \( X_1^N \) work later (starting from time 2), we find that the SK-type scheme of \( U^N \) and \( X_1^N \) is also SSCA, and the key step to the corresponding proof is Lemma 1 in Section III, i.e., for time instant \( 3 \leq k \leq N \), \( E[\epsilon'_{k-1}\eta_{1,k}] = 0 \), where \( \eta_{1,k} = \eta_{1,k} + V_k \), \( \eta_{1,k} \) and \( V_k \) are the \( k \)-th components of \( \eta_1^N \) and \( V^N \), respectively, and \( \epsilon'_{k-1} \) is a deterministic function of \( U_k \) and \( X_{1,k} \), which are the \( k \)-th components of \( U^N \) and \( X_1^N \), respectively. Here note that Lemma 1 is surprising and novel since both \( V_k \) and \( \epsilon'_{k-1} \) depend on the previous noises \( \eta_{1,1}, ..., \eta_{1,k-1} \). By using this surprising property in Lemma 1, we show that the two-step SK-type feedback scheme is SSCA for the two-user GMAC-WT-DMS.

2) We extend the above new feedback scheme to the two-user GMAC with NCSIT and DMS (GMAC-NCSIT-DMS), and show that this extended feedback scheme is also SSCA for the two-user GMAC-WT with NCSIT and DMS (GMAC-WT-NCSIT-DMS). The novelty of this new scheme is explained below.

In the previous two-step SK-type scheme for the two-user GMAC-DMS with feedback, after decoding one message \( W_1 \), the receiver knows \( U^N \) and \( X_1^N \). Hence in the decoding of the
other message $W_2$, the receiver directly subtracts $U^N$ and $X_1^N$ from his/her received signal and does a similar SK-type decoding to obtain $W_2$. However, in the two-user GMAC-NCSIT-DMS with feedback, since the receiver does not know the state interference, after decoding $W_1$, the receiver cannot obtain $U^N$ and $X_1^N$, which leads to the failure of subtracting $U^N$ and $X_1^N$ from the receiver’s received signal. Fortunately, we find that after decoding $W_1$, though the receiver only obtains partial information about $U^N$ and $X_1^N$, by introducing proper offsets into the construction of $V^N$, $U^N$ and $X_1^N$, the receiver’s final estimations of the transmitted messages are the same as those in the previous two-step SK-type scheme for the GMAC-DMS with feedback, which indicates that this modified scheme is also a SSCA feedback scheme for the two-user GMAC-WT-NCSIT-DMS.

3) Outer bounds on the secrecy capacity regions of the GMAC-WT-DMS and the GMAC-WT-NCSIT-DMS are given, and numerical results show the rate gains by the feedback.

Throughout this paper, a random variable (RV) is denoted by an upper case letter (e.g., $X$), its value is denoted by an lower case letter (e.g., $x$), the finite alphabet of the RV is denoted by calligraphic letter (e.g., $\mathcal{X}$), and the probability distribution of an event $\{X = x\}$ is denoted by $P_X(x)$. Random vectors and their values are denoted by a similar convention. For example, $X^N$ represents a $N$-dimensional random vector $(X_1, \ldots, X_N)$, and $x^N = (x_1, \ldots, x_N)$ represents a vector value in $\mathcal{X}^N$ (the $N$-th Cartesian power of the finite alphabet $\mathcal{X}$). In addition, define $A_j^N = (A_{j,1}, A_{j,2}, \ldots, A_{j,N})$ and $a_j^N = (a_{j,1}, a_{j,2}, \ldots, a_{j,N})$. Finally, throughout this paper, the base of the log function is 2.

The remainder of this paper is organized as follows. Formal definitions of the models studied in this paper and preliminary are given in Section II. The SSCA feedback scheme for the GMAC-WT-DMS is given in Section III. The SSCA feedback scheme for the GMAC-WT-NCSIT-DMS is given in Section IV. Section V includes the summary of all results in this paper and discusses future work.

II. Model Formulation and Preliminary

A. Model Formulation-type I: the GMAC-DMS with or without feedback and secrecy constraint

1) Model I: The GMAC-DMS with or without feedback: For the GMAC-DMS with or without feedback, the $i$-th ($i \in \{1, 2, \ldots, N\}$) channel input-output relationship is given by

$$Y_i = X_{1,i} + X_{2,i} + \eta_{1,i}, \quad (2.1)$$
where $X_{1,i}$ and $X_{2,i}$ are the channel inputs subject to average power constraints $P_1$ and $P_2$ (i.e., $\frac{1}{N} \sum_{i=1}^{N} E[X_{1,i}^2] \leq P_1$, $\frac{1}{N} \sum_{i=1}^{N} E[X_{2,i}^2] \leq P_2$), respectively, $Y_i$ is the channel output of the receiver, and $\eta_{1,i} \sim \mathcal{N}(0, \sigma_1^2)$ are the channel noises and are i.i.d. across the time index $i$. The message $W_j$ ($j = 1, 2$) is uniformly distributed in $W_j = \{1, 2, ..., |W_j|\}$. For the GMAC-DMS without feedback, the channel input $X_{1,i}$ is a function of the message $W_1$, and the channel input $X_{2,i}$ is a function of the messages $W_1$ and $W_2$. For the GMAC-DMS with feedback, $X_{1,i}$ is a function of the message $W_1$ and the feedback $Y_{i-1}$, and $X_{2,i}$ is a function of the messages $W_1$, $W_2$ and the feedback $Y_{i-1}$. The receiver generates an estimation $(\hat{W}_1, \hat{W}_2) = \psi(Y^N)$, where $\psi$ is the legitimate receiver’s decoding function, and the average decoding error probability equals

$$P_e = \frac{1}{|W_1| \cdot |W_2|} \sum_{w_1 \in W_1, w_2 \in W_2} \Pr\{\psi(y^N) \neq (w_1, w_2) | (w_1, w_2) \text{ sent}\}. \quad (2.2)$$

A rate pair $(R_1, R_2)$ is said to be achievable if for any $\epsilon$ and sufficiently large $N$, there exists channel encoders and decoder such that

$$\frac{\log |W_1|}{N} = R_1, \quad \frac{\log |W_2|}{N} = R_2, \quad P_e \leq \epsilon. \quad (2.3)$$

The capacity regions of the GMAC-DMS with or without feedback are composed of all such achievable rate pairs, and they are denoted by $C_{\text{gmac-dms}}^f$ and $C_{\text{gmac-dms}}$, respectively.
2) Model II: The GMAC-WT-DMS with or without feedback: For the GMAC-WT-DMS with or without feedback, the $i$-th ($i \in \{1, 2, ..., N\}$) channel input-output relationships are given by

$$Y_i = X_{1,i} + X_{2,i} + \eta_{1,i}, \quad Z_i = Y_i + \eta_{2,i}, \quad (2.4)$$

where $X_{1,i}$ and $X_{2,i}$ are the channel inputs subject to average power constraints $P_1$ and $P_2$, respectively, $Y_i$ and $Z_i$ are the channel outputs of the legitimate receiver and the wiretapper, respectively, and $\eta_{1,i} \sim \mathcal{N}(0, \sigma_1^2)$, $\eta_{2,i} \sim \mathcal{N}(0, \sigma_2^2)$ are the channel noises and are independent identically distributed (i.i.d.) across the time index $i$. The message $W_j$ ($j = 1, 2$) is uniformly distributed in $W_j = \{1, 2, ..., |W_j|\}$. For the GMAC-WT-DMS without feedback, the channel input $X_{1,i}$ is a (stochastic) function of the message $W_1$, and the channel input $X_{2,i}$ is a (stochastic) function of the messages $W_1$ and $W_2$. For the GMAC-WT-DMS with feedback, $X_{1,i}$ is a (stochastic) function of the message $W_1$ and the feedback $Y_{i-1}$, and $X_{2,i}$ is a (stochastic) function of the messages $W_1$, $W_2$ and the feedback $Y_{i-1}$. The legitimate receiver generates an estimation $(\hat{W}_1, \hat{W}_2) = \psi(Y^N)$, where $\psi$ is the legitimate receiver’s decoding function, and the average decoding error probability $P_e$ is defined the same as that in (2.2). The wiretapper’s equivocation rate of the messages $W_1$ and $W_2$ is defined as

$$\Delta = \frac{1}{N} H(W_1, W_2|Z^N). \quad (2.5)$$

A rate pair $(R_1, R_2)$ is said to be achievable with perfect weak secrecy if for any $\epsilon$ and sufficiently large $N$, there exists channel encoders and decoder such that

$$\frac{\log |W_1|}{N} = R_1, \quad \frac{\log |W_2|}{N} = R_2, \quad \Delta \geq R_1 + R_2 - \epsilon, \quad P_e \leq \epsilon. \quad (2.6)$$

The secrecy capacity regions of the GMAC-WT-DMS with or without feedback are composed of all such achievable rate pairs, and they are denoted by $C_{s,gmac-dms}^{\ell}$ and $C_{s,gmac-dms}$, respectively.

B. Model Formulation-type II: The GMAC-NCSIT-DMS with or without feedback and secrecy constraint

1) Model III: The GMAC-NCSIT-DMS with or without feedback: For the GMAC-NCSIT-DMS with or without feedback, at each time $i$ ($i \in \{1, 2, ..., N\}$), the channel input-output relationships are given by

$$Y_i = X_{1,i} + X_{2,i} + S_i + \eta_{1,i}, \quad (2.7)$$
where $X_{1,i}$, $X_{2,i}$, $\eta_{1,i}$ and $Y_i$ are defined in the same fashion as those in Subsubsection II-A1, and $S_i \sim \mathcal{N}(0, Q)$ is the independent Gaussian state interference and is i.i.d. across the time index $i$. The message $W_j$ ($j = 1, 2$) is uniformly distributed in $\mathcal{W}_j = \{1, 2, \ldots, |\mathcal{W}_j|\}$. For the GMAC-NCSIT-DMS without feedback, the channel input $X_{1,i}$ is a function of the message $W_1$ and the state interference $S_i^N$, and the channel input $X_{2,i}$ is a function of the messages $W_1$, $W_2$ and the state interference $S_i^N$. For the GMAC-NCSIT-DMS with feedback, $X_{1,i}$ is a function of the message $W_1$, the state interference $S_i^N$ and the feedback $Y_{i-1}$, and $X_{2,i}$ is a function of the messages $W_1$, $W_2$, the state interference $S_i^N$ and the feedback $Y_{i-1}$. The receiver’s decoding function, the average decoding error probability and the achievable rate pair are defined in the same fashion as those in Subsubsection II-A1. The capacity regions of the GMAC-NCSIT-DMS with or without feedback are denoted by $C_{gmac-ncsit-dms}^f$ and $C_{gmac-ncsit-dms}$, respectively.

2) **Model IV: The GMAC-WT-NCSIT-DMS with or without feedback:** For the GMAC-WT-NCSIT-DMS with or without feedback, the $i$-th ($i \in \{1, 2, \ldots, N\}$) channel input-output relationships are given by

$$Y_i = X_{1,i} + X_{2,i} + S_i + \eta_{1,i}, \quad Z_i = Y_i + \eta_{2,i},$$

(2.8)

where $X_{1,i}$, $X_{2,i}$, $S_i$, $\eta_{1,i}$ and $Y_i$ are defined in the same fashion as those in Subsubsection II-B1, and $Z_i$ and $\eta_{2,i}$ are defined in the same fashion as those in Subsubsection II-A2. The message $W_j$ ($j = 1, 2$) is uniformly distributed in $\mathcal{W}_j = \{1, 2, \ldots, |\mathcal{W}_j|\}$. For the GMAC-WT-NCSIT-DMS
without feedback, $X_{1,i}$ is a (stochastic) function of the message $W_1$ and the state interference $S^N$, and $X_{2,i}$ is a (stochastic) function of the messages $W_1, W_2$ and the state interference $S^N$. For the GMAC-WT-NCSIT-DMS with feedback, $X_{1,i}$ is a (stochastic) function of the message $W_1$, the state interference $S^N$ and the feedback $Y_{i-1}$, and $X_{2,i}$ is a (stochastic) function of the messages $W_1, W_2$, the state interference $S^N$ and the feedback $Y_{i-1}$. The legitimate receiver’s decoding function, the wiretapper’s equivocation rate and the achievable rate pair with perfect weak secrecy are defined in the same fashion as those in Subsubsection II-A2. The secrecy capacity regions of the GMAC-WT-NCSIT-DMS with or without feedback are denoted by $C_{s,gmac-ncsit-dms}^f$ and $C_{s,gmac-ncsit-dms}$, respectively.

C. Preliminary: the SK scheme for the point-to-point white Gaussian channel with feedback

For the white Gaussian channel with feedback, at each time $i$ ($i \in \{1, 2, ..., N\}$), the channel input-output relationship is given by

$$Y_i = X_i + \eta_{1,i}, \quad (2.9)$$

where $X_i$ is the channel input subject to an average power constraint $P$, $Y_i$ is the channel output of the receiver, and $\eta_{1,i} \sim \mathcal{N}(0, \sigma_1^2)$ is the white Gaussian noise and it is i.i.d. across the time index $i$. The message $W$ is uniformly distributed in $\mathcal{W} = \{1, 2, ..., |\mathcal{W}|\}$. The channel input $X_i$ is a function of the message $W$ and the feedback $Y_{i-1}$. The receiver generates an estimation $\hat{W} = \psi(Y_N)$, where $\psi$ is the receiver’s decoding function, and the average decoding error probability equals

$$P_e = \frac{1}{|\mathcal{W}|} \sum_{w \in \mathcal{W}} Pr\{\psi(y_N) \neq w | w \text{ sent}\}. \quad (2.10)$$

The capacity of the white Gaussian channel with feedback is denoted by $C_{g}^f$, and it equals the capacity $C_{g}$ of the white Gaussian channel, which is given by

$$C_{g}^f = C_{g} = \frac{1}{2} \log(1 + \frac{P}{\sigma_1^2}). \quad (2.11)$$

In [6], it has been shown that SK scheme achieves $C_{g}^f$, and this classical scheme is briefly described below.

Since $W$ takes values in $\mathcal{W} = \{1, 2, ..., 2^{NR}\}$, we divide the interval $[-0.5, 0.5]$ into $2^{NR}$ equally spaced sub-intervals, and the center of each sub-interval is mapped to a message value.
in $W$. Let $\theta$ be the center of the sub-interval w.r.t. the message $W$ (the variance of $\theta$ approximately equals $\frac{1}{12}$). At time 1, the transmitter sends

$$X_1 = \sqrt{12P}\theta, \quad (2.12)$$

The receiver obtains $Y_1 = X_1 + \eta_{1,1}$, and gets an estimation of $\theta$ by computing

$$\hat{\theta}_1 = \frac{Y_1}{\sqrt{12P}} = \theta + \frac{\eta_{1,1}}{\sqrt{12P}} = \theta + \epsilon_1, \quad (2.13)$$

where $\epsilon_1 = \hat{\theta}_1 - \theta = \frac{\eta_{1,1}}{\sqrt{12P}}$. Let $\alpha_{1} \triangleq Var(\epsilon_1) = \frac{\sigma^2_1}{12P}$.

At time $2 \leq k \leq N$, the receiver obtains $Y_k = X_k + \eta_{1,k}$, and gets an estimation of $\theta_k$ by computing

$$\hat{\theta}_k = \hat{\theta}_{k-1} - \frac{E[Y_k\epsilon_{k-1}]}{E[Y_k^2]} Y_k, \quad (2.14)$$

where $\epsilon_k = \hat{\theta}_k - \theta$. (2.14) yields that

$$\epsilon_k = \epsilon_{k-1} - \frac{E[Y_k\epsilon_{k-1}]}{E[Y_k^2]} Y_k. \quad (2.15)$$

Meanwhile, for time $2 \leq k \leq N$, the transmitter sends

$$X_k = \sqrt{\frac{P}{\alpha_{k-1}}\epsilon_{k-1}}, \quad (2.16)$$

where $\alpha_{k-1} \triangleq Var(\epsilon_{k-1})$.

In [6], it has been shown that if $R < \frac{1}{2} \log(1 + \frac{P}{\sigma^2_1})$, $P_e \to 0$ as $N \to \infty$.

III. THE SSCA FEEDBACK SCHEME FOR THE GMAC-WT-DMS

In this section, first, we propose a two-step SK-type feedback scheme that achieves the capacity of GMAC-DMS with feedback. Second, we show that the proposed feedback scheme is secure by itself and also achieves the secrecy capacity region $C_{s,gmac-dms}^f$ of the GMAC-WT-DMS with feedback. Finally, in order to show the rate gains by the feedback, an outer bound on the secrecy capacity region $C_{s,gmac-dms}$ of GMAC-WT-DMS is provided, and the capacity results given in this section are further explained via a numerical example.

A. A capacity-achieving two-step SK-type scheme for the GMAC-DMS with feedback

The model of the GMAC-DMS with feedback is formulated in Section [II-A1]. In this subsection, first, we introduce capacity results on GMAC-DMS with or without feedback. Then, we propose a two-step SK-type scheme and show that this scheme achieves the capacity of GMAC-DMS with feedback.
1) Capacity results on GMAC-DMS with or without feedback: The following Corollary characterizes the capacity region \( C_{\text{gmac-dms}} \) of the GMAC-DMS.

**Corollary 1:** The capacity region \( C_{\text{gmac-dms}} \) of the GMAC-DMS is given by

\[
C_{\text{gmac-dms}} = \bigcup_{0 \leq \rho \leq 1} \left\{ (R_1 \geq 0, R_2 \geq 0) : \begin{array}{c}
R_2 \leq \frac{1}{2} \log \left( 1 + \frac{P_2(1-\rho^2)}{\sigma_1^2} \right), \\
R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2\rho}}{\sigma_1^2} \right) \end{array} \right\}.
\] (3.1)

**Proof:**

**Achievability of** \( C_{\text{gmac-dms}} \): From [27], the capacity region \( C_{\text{mac-dms}} \) of the discrete memoryless MAC-DMS (DM-MAC-DMS) is given by

\[
C_{\text{mac-dms}} = \{(R_1, R_2) : R_2 \leq I(X_2;Y|X_1), \ R_1 + R_2 \leq I(X_1,X_2;Y)\} \tag{3.2}
\]

for some joint distribution \( P_{X_1X_2}(x_1, x_2) \). Then, substituting \( X_1 \sim N(0, P_1) \) and \( X_2 \sim N(0, P_2) \) and (2.1) into (3.2), defining \( \rho = \text{E}[X_1X_2]/\sqrt{P_1P_2} \), and following the idea of the encoding-decoding scheme of [27], the achievability of \( C_{\text{gmac-dms}} \) is proved.

**Converse of** \( C_{\text{gmac-dms}} \): the converse proof of \( C_{\text{gmac-dms}} \) follows the idea of the converse part in GMAC with feedback [5, pp. 627-628] (see the converse proof of the bounds on \( R_2 \) and \( R_1 + R_2 \)), and hence we omit the details here. The proof of Corollary 1 is completed.

In [8], it has been shown that feedback does not increase the capacity region \( C_{\text{gmac-dms}} \) of the GMAC-DMS, i.e.,

\[
C_{\text{f-gmac-dms}} = C_{\text{gmac-dms}}, \tag{3.3}
\]

where \( C_{\text{gmac-dms}} \) is given in (3.1). Here note that though the capacity region \( C_{\text{gmac-dms}} \) of the GMAC-DMS with feedback is determined, the SK-type feedback scheme that achieves \( C_{\text{gmac-dms}} \) remains unknown. In the remainder of this section, first, a two-step SK-type feedback scheme is proposed for the GMAC-DMS with feedback, and it is shown to be capacity-achieving. Then, we will show that this two-step SK-type scheme also achieves the secrecy capacity region \( C_{\text{s,gmac-dms}} \) of the GMAC-WT-DMS with feedback.

2) A capacity-achieving two-step SK-type feedback scheme for the GMAC-DMS with feedback: The main idea of the two-step SK-type feedback scheme is briefly illustrated by the following Figure 3. In Figure 3, the common message \( W_1 \) is encoded by both transmitters, and the private message \( W_2 \) is only available at Transmitter 2. Specifically, Transmitter 1 uses power \( P_1 \) to
encode $W_1$ and the feedback $Y^N$ as $X_1^N$. Transmitter 2 uses power $(1 - \rho^2)P_2$ to encode $W_2$ and $Y^N$ as $V^N$, and power $\rho^2P_2$ to encode $W_1$ and $Y^N$ as $U^N$, where $0 \leq \rho \leq 1$,

$$X_2^N = U^N + V^N,$$  

(3.4)

and the average transmission power of $X_2^N$ tends to $P_2$ for large $N$ will be explained later. Here note that since $W_1$ is known by Transmitter 2, the codewords $X_1^N$ and $U^N$ can be subtracted when applying SK scheme to $W_2$, i.e., for the SK scheme of $W_2$, the equivalent channel model has input $V^N$, output $Y'^N = Y^N - X_1^N - U^N$, and channel noise $\eta_1^N$.

In addition, since $W_1$ is known by both transmitters and $W_2$ is only available at Transmitter 2, for the SK scheme of $W_1$, the equivalent channel model has inputs $X_1^N$ and $U^N$, output $Y^N$,
and channel noise $\eta_1^N + V^N$, which is non-white Gaussian noise since $V^N$ is not i.i.d. generated.

Furthermore, observing that

$$Y_i = X_{1,i} + U_i + V_i + \eta_{1,i} = X_i^* + V_i + \eta_{1,i},$$

where $X_i^* = X_{1,i} + U_i$, $X_i^*$ is Gaussian distributed with zero mean and variance $P_i^*$,

$$P_i^* = P_1 + \rho^2 P_2 + 2\sqrt{P_1 P_2 \rho \rho_i'} \leq P_1 + \rho^2 P_2 + 2\sqrt{P_1 P_2 \rho} = P^*,$$

(3.6)

$$\rho_i' = \frac{E[X_i, U_i]}{\rho \sqrt{P_1 P_2}}$$

and $0 \leq \rho_i' \leq 1$. Hence for the SK scheme of $W_1$, the input of the equivalent channel model can be viewed as $X_i^*$. Since $X_{1,i}$ is known by Transmitter 2, let

$$U_i = \rho \sqrt{\frac{P_2}{P_1}} X_{1,i}.$$  

(3.7)

Then we have $\rho_i' = 1$, which leads to

$$P_i^* = P^* = P_1 + \rho^2 P_2 + 2\sqrt{P_1 P_2 \rho},$$

(3.8)

and $X_i^* \sim \mathcal{N}(0, P^*)$. The encoding and decoding procedure of Figure [3] is described below.

Since $W_j$ ($j = 1, 2$) takes values in $\mathcal{W}_j = \{1, 2, \ldots, 2^{NR_j}\}$, divide the interval $[-0.5, 0.5]$ into $2^{NR_j}$ equally spaced sub-intervals, and the center of each sub-interval is mapped to a message value in $\mathcal{W}_j$. Let $\theta_j$ be the center of the sub-interval w.r.t. the message $W_j$ (the variance of $\theta_j$ approximately equals $\frac{1}{12}$).

**Encoding:** At time 1, Transmitter 1 sends

$$X_{1,1} = 0.$$  

(3.9)

Transmitter 2 sends

$$V_1 = \sqrt{\frac{12(1-\rho^2)}{P_2}} P_2 \theta_2,$$

(3.10)

and

$$U_1 = \rho \sqrt{\frac{P_2}{P_1}} X_{1,1} = 0.$$  

(3.11)

The receiver obtains $Y_1 = X_{1,1} + X_{2,1} + \eta_{1,1} = X_{1,1} + V_1 + U_1 + \eta_{1,1} = V_1 + \eta_{1,1}$, and sends $Y_1$ back to Transmitter 2. Let $Y_1' = Y_1 = V_1 + \eta_{1,1}$. Transmitter 2 computes

$$\frac{Y_1'}{\sqrt{\frac{12(1-\rho^2)}{P_2}}} = \theta_2 + \frac{\eta_{1,1}}{\sqrt{\frac{12(1-\rho^2)}{P_2}}} = \theta_2 + \epsilon_1.$$  

(3.12)

Let $\alpha_1 = \text{Var}(\epsilon_1) = \frac{\sigma_1^2}{12(1-\rho^2)P_2}$. 
At time 2, Transmitter 2 sends

\[ V_2 = \sqrt{(1 - \rho^2) P_2} \epsilon_1. \]  

(3.13)

On the other hand, at time 2, Transmitters 1 and 2 respectively send \( X_{1,2} \) and \( U_2 = \rho \sqrt{P_2 X_{1,2}} \) such that

\[ X_{2}^* = U_2 + X_{1,2} = \sqrt{12P^*} \theta_1. \]  

(3.14)

Once receiving the feedback \( Y_2 = X_{2}^* + V_2 + \eta_{1,2} \), both transmitters compute

\[ \frac{Y_2}{\sqrt{12P^*}} = \theta_1 + \frac{V_2 + \eta_{1,2}}{\sqrt{12P^*}} = \theta_1 + \epsilon_2', \]  

(3.15)

and send \( X_{1,3} \) and \( U_3 = \rho \sqrt{P_2 X_{1,3}} \) such that

\[ X_{3}^* = U_3 + X_{1,3} = \sqrt{\frac{P^*}{\alpha_2}} \epsilon_2'. \]  

(3.16)

where \( \alpha_2' \triangleq Var(\epsilon_2') \). In addition, subtracting \( X_{1,2} \) and \( U_2 \) from \( Y_2 \) and let \( Y_2' = Y_2 - X_{1,2} - U_2 = V_2 + \eta_{1,2} \), Transmitter 2 computes

\[ \epsilon_2 = \epsilon_1 - \frac{E[Y_2' \epsilon_1]}{E[(Y_2')^2]} Y_2', \]  

(3.17)

and sends

\[ V_3 = \sqrt{(1 - \rho^2) P_2} \epsilon_2'. \]  

(3.18)

where \( \alpha_2 \triangleq Var(\epsilon_2) \).

At time \( 4 \leq k \leq N \), once receiving \( Y_{k-1} = X_{1,k-1} + U_{k-1} + V_{k-1} + \eta_{1,k-1} \), Transmitter 2 computes

\[ \epsilon_{k-1} = \epsilon_{k-2} - \frac{E[Y_{k-1}' \epsilon_{k-2}]}{E[(Y_{k-1}')^2]} Y_{k-1}', \]  

(3.19)

where

\[ Y_{k-1}' = Y_{k-1} - X_{1,k-1} - U_{k-1}, \]  

(3.20)

and sends

\[ V_k = \sqrt{(1 - \rho^2) P_2} \epsilon_{k-1}. \]  

(3.21)
where $\alpha_{k-1} \triangleq Var(\epsilon_{k-1})$. In the meanwhile, Transmitters 1 and 2 respectively send $X_{1,k}$ and $U_k = \rho \sqrt{\frac{P_2}{P_1}} X_{1,k}$ such that

$$X_k^* = U_k + X_{1,k} = \sqrt{\frac{P^*}{\alpha'_{k-1}}} \epsilon'_{k-1}, \quad (3.22)$$

where

$$\epsilon'_{k-1} = \epsilon'_{k-2} - \frac{E[Y_{k-1}\epsilon'_{k-2}]}{E[(Y_{k-1})^2]} Y_{k-1}, \quad (3.23)$$

and $\alpha'_{k-1} \triangleq Var(\epsilon'_{k-1})$.

The following Lemma 1 is crucial for the analysis of the average transmission power of $X_2^N$ and the decoding error probability.

**Lemma 1:** For $3 \leq k \leq N$,

$$E[\epsilon'_{k-1}\eta'_{1,k}] = 0, \quad (3.24)$$

where

$$\eta'_{1,k} = \eta_{1,k} + V_k. \quad (3.25)$$

**Proof:** See Appendix A.

**Analysis of the average transmission power of $X_2^N$:**

The above Lemma 1 indicates that for $3 \leq k \leq N$,

$$E[\epsilon'_{k-1}\eta'_{1,k}] = E[\epsilon'_{k-1}(\eta_{1,k} + V_k)] = E[\epsilon'_{k-1}\eta_{1,k}] + E[\epsilon'_{k-1}V_k] = 0, \quad (3.26)$$

where (1) follows from the fact that $\eta_{1,k}$ is independent of $\epsilon'_{k-1}$ ($\epsilon'_{k-1}$ is a function of $(\eta_{1,1}, ..., \eta_{1,k-1})$).

Since

$$U_k \overset{(2)}{=} \frac{\rho \sqrt{\frac{P_2}{P_1}}}{\rho \sqrt{\frac{P_2}{P_1}} + 1} \sqrt{\frac{P^*}{\alpha'_{k-1}}} \epsilon'_{k-1}, \quad (3.27)$$

where (2) follows from (3.22) and $U_k = \rho \sqrt{\frac{P_2}{P_1}} X_{1,k}$, substituting (3.27) into (3.26), we conclude that

$$E[U_kV_k] = 0, \quad (3.28)$$

for $3 \leq k \leq N$. In addition, from (3.10), (3.11), (3.13), (3.14) and the fact that $\theta_1$ is independent of $\eta_{1,1}$, we conclude that

$$E[U_1V_1] = E[U_2V_2] = 0, \quad (3.29)$$
and hence $E[U_kV_k] = 0$ for $1 \leq k \leq N$.

Here note that for $1 \leq k \leq N$,

$$E[X_{2,k}^2] = E[(U_k + V_k)^2] \overset{(3)}{=} E[U_k^2] + E[V_k^2],$$  \hspace{1cm} (3.30)

where (3) follows from $E[U_kV_k] = 0$ for $1 \leq k \leq N$. From the above encoding procedure, we conclude that $E[X_{2,1}^2] = E[U_1^2] + E[V_1^2] = (1 - \rho^2)P_2$, and $E[X_{2,k}^2] = E[U_k^2] + E[V_k^2] = P_2$ for $2 \leq k \leq N$, which means that the average transmission power of $X_2^N$ tends to $P_2$ for large $N$.

**Decoding:**

The receiver uses a two-step decoding scheme. First, from (2.14), we observe that at time $k$ ($3 \leq k \leq N$), the receiver’s estimation $\hat{\theta}_{1,k}$ of $\theta_1$ is given by

$$\hat{\theta}_{1,k} = \hat{\theta}_{1,k-1} - \frac{E[Y_k\epsilon'_{k-1}]}{E[(Y_k)^2]}Y_k,$$  \hspace{1cm} (3.31)

where $\epsilon'_{k-1} = \hat{\theta}_{1,k-1} - \theta_1$ and it is computed by (3.23), and

$$\hat{\theta}_{1,2} = \frac{Y_2}{\sqrt{12P_s}} = \theta_1 + \frac{V_2 + \eta_{1,2}}{\sqrt{12P_s}} = \theta_1 + \epsilon'_2.$$  \hspace{1cm} (3.32)

Second, after decoding $W_1$ and the corresponding codewords $X_{1,k}$ and $U_k$ for all $1 \leq k \leq N$, the receiver subtracts $X_{1,k}$ and $U_k$ from $Y_k$, and obtains $Y'_k = V_k + \eta_{1,k}$. At time $k$ ($1 \leq k \leq N$), the receiver’s estimation $\hat{\theta}_{2,k}$ of $\theta_2$ is given by

$$\hat{\theta}_{2,k} = \hat{\theta}_{2,k-1} - \frac{E[Y_k'\epsilon_{k-1}]}{E[(Y'_k)^2]}Y'_k,$$  \hspace{1cm} (3.33)

where $\epsilon_{k-1} = \hat{\theta}_{2,k-1} - \theta_2$ and it is computed by (3.19), and

$$\hat{\theta}_{2,1} = \frac{Y'_1}{\sqrt{12(1 - \rho^2)P_2}} = \theta_2 + \frac{\eta_{1,1}}{\sqrt{12(1 - \rho^2)P_2}} = \theta_2 + \epsilon_1.$$  \hspace{1cm} (3.34)

**Decoding error probability analysis:**

The decoding error probability $P_e$ of the receiver is upper bounded by

$$P_e \leq P_{e1} + P_{e2},$$  \hspace{1cm} (3.35)

where $P_{ej}$ ($j = 1, 2$) is the receiver’s decoding error probability of $W_j$. Observing that the transmission of $W_2$ is through an equivalent white Gaussian channel with power $(1 - \rho^2)P_2$ and Gaussian noise variance $\sigma_1^2$, hence from the classical SK scheme [6], we conclude that the decoding error probability $P_{e2}$ of $W_2$ tends to 0 as $N \to \infty$ if

$$R_2 < \frac{1}{2} \log(1 + \frac{(1 - \rho^2)P_2}{\sigma_1^2}),$$  \hspace{1cm} (3.36)
and hence we omit the derivation here. Now it remains to bound $P_{e1}$, see the followings.

First, from (A10) and the fact that $\alpha'_k = Var(\epsilon'_k) = E[(\epsilon'_k)^2]$, we have

$$\alpha'_k \overset{(a)}{=} \alpha'_{k-1} \left( \frac{\sqrt{\frac{P^*}{\alpha'_{k-1}}} E[\epsilon'_{k-1}\eta'_{1,k}] + (1 - \rho^2)P_2 + \sigma_1^2}{P^* + 2\sqrt{\frac{P^*}{\alpha'_{k-1}}} E[\epsilon'_{k-1}\eta'_{1,k}] + (1 - \rho^2)P_2 + \sigma_1^2} \right)^2$$

$$- 2\left( \frac{P^*}{\alpha'_{k-1}} E[\epsilon'_{k-1}\eta'_{1,k}] + (1 - \rho^2)P_2 + \sigma_1^2 \right)(\sqrt{P^* \cdot \alpha'_{k-1}} + E[\epsilon'_{k-1}\eta'_{1,k}])$$

$$+ (\sigma_1^2 + (1 - \rho^2)P_2) \left( \frac{\sqrt{P^* \cdot \alpha'_{k-1}} + E[\epsilon'_{k-1}\eta'_{1,k}]}{P^* + 2\sqrt{\frac{P^*}{\alpha'_{k-1}}} E[\epsilon'_{k-1}\eta'_{1,k}] + (1 - \rho^2)P_2 + \sigma_1^2} \right)^2$$

$$\overset{(b)}{=} \frac{\alpha'_{k-1}r^2(r^2 + P^*)}{(P^* + r^2)^2} = \frac{\alpha'_{k-1}r^2}{P^* + r^2},$$

(3.37)

where (a) follows from (A2), and (b) follows from Lemma 1 and the definition in (A17).

Then, from (3.37), we can conclude that

$$\sqrt{\alpha'_N} \overset{(c)}{=} \left( \frac{r}{\sqrt{r^2 + P^*}} \right)^{N-2} \sqrt{\alpha_2} \overset{(d)}{=} \left( \frac{r}{\sqrt{r^2 + P^*}} \right)^{N-2} \frac{r}{\sqrt{12P^*}},$$

(3.38)

where (c) follows from (3.37), and (d) follows from $\alpha'_2 = Var(\epsilon'_2)$, (A14) and (A17).

Finally, we bound $P_{e1}$ as follows. From $\epsilon'_N = \hat{\theta}_{1,N} - \theta_1$ and the definition of $\theta_1$, we have

$$P_{e1} \leq Pr \left\{ |\epsilon'_N| > \frac{1}{2(|W_1| - 1)} \right\}$$

$$\overset{(e)}{\leq} 2Q \left( \frac{1}{2 \cdot 2NR_1} \cdot \frac{1}{\sqrt{\alpha'_N}} \right)$$

$$\overset{(f)}{=} 2Q \left( \frac{1}{2} \cdot 2^{-NR_1} \left( \frac{r}{\sqrt{r^2 + P^*}} \right)^{N-2} \sqrt{\frac{12P^*}{r^2}} \right)^2$$

$$= 2Q \left( \frac{1}{2} \sqrt{\frac{12P^*}{r^2}} 2^{-NR_1} \left( \frac{r}{\sqrt{r^2 + P^*}} \right)^{N-2} \right)^2$$

$$= 2Q \left( \frac{1}{2} \sqrt{\frac{12P^*}{r^2}} 2^{-2NR_1} 2^{(N-2)\log \frac{\sqrt{r^2 + P^*}}{r}} \right)^2$$

$$= 2Q \left( \frac{1}{2} \sqrt{\frac{12P^*}{r^2}} 2^{-2N(2NR_1 - \log \frac{\sqrt{r^2 + P^*}}{r})} \right)^2,$$

(3.39)
where (e) follows from $Q(x)$ is the tail of the unit Gaussian distribution evaluated at $x$, and (f) follows from (3.38) and the fact that $Q(x)$ is decreasing while $x$ is increasing. From (3.39), we can conclude that if
\[
R_1 < \log \frac{\sqrt{r^2 + P^*}}{r} = \frac{1}{2} \log (1 + \frac{P^*}{r^2}) \equiv \frac{1}{2} \log (1 + \frac{P_1 + \rho^2 P_2 + 2\sqrt{P_1 P_2 \rho}}{(1 - \rho^2) P_2 + \sigma_1^2}), \tag{3.40}
\]
where (g) follows from (3.8) and (A17), $P_{e1} \to 0$ as $N \to \infty$.

Now we have shown if (3.36) and (3.40) are satisfied, the decoding error probability $P_{e}$ of the receiver tends to $0$ as $N \to \infty$. In other words, the rate pair $(R_1, R_2)$ in $C_{gmac-dms}^f$ are achievable. Hence the proposed two-step SK-type feedback scheme achieves the capacity region $C_{gmac-dms}^f$ of GMAC-DMS with feedback.

B. Capacity result on the GMAC-WT-DMS with feedback

The model of the GMAC-WT-DMS with feedback is formulated in Section II-A2. The following Theorem [1] establishes that the secrecy constraint does not reduce the capacity of GMAC-DMS with feedback.

**Theorem 1:** $C_{s,gmac-dms}^f = C_{gmac-dms}^f$, where $C_{s,gmac-dms}^f$ is the secrecy capacity region of the GMAC-WT-DMS with feedback, and $C_{gmac-dms}^f$ is given in Corollary [1].

**Proof:** Since $C_{s,gmac-dms}^f \subseteq C_{gmac-dms}^f = C_{gmac-dms}^f$, we only need to show that any achievable rate pair $(R_1, R_2)$ in $C_{gmac-dms}^f$ satisfies the secrecy constraint in (2.6).

In the preceding subsection, we introduce a two-step SK scheme for the GMAC-DMS with feedback, and show that this scheme achieves $C_{gmac-dms}^f$. In this new scheme, the transmitted codewords $X_{1,i}, U_i$ and $V_i$ at time $i$ ($1 \leq i \leq N$) can be expressed as

\[
X_{1,1} = 0, \quad U_1 = 0, \quad V_1 = \sqrt{12(1 - \rho^2) P_2 \theta_2},
\]
\[
X_{1,2} = \frac{\sqrt{12P^* \theta_1}}{\rho \sqrt{P_1 P_2} + 1}, \quad U_2 = \rho \sqrt{P_2 P_1} X_{1,2}, \quad V_2 = \sqrt{(1 - \rho^2) P_2} \eta_{1,1},
\]
\[
X_{1,3} = \frac{\sqrt{P^* P_2 (1 - \rho^2)} \eta_{1,1} + \sqrt{P^* \sigma_1}}{r \rho \sqrt{P_2 P_1} + 1} \eta_{1,2}, \quad U_3 = \rho \sqrt{\frac{P_2}{P_1}} X_{1,3},
\]
\[
V_3 = \sqrt{(1 - \rho^2) P_2} \eta_{1,1} - \frac{(1 - \rho^2) P_2}{r \sigma_1} \eta_{1,2},
\]
...
\[
X_{1,N} = \frac{1}{\rho \sqrt{P_2/P_1}} + 1 \sqrt{\frac{P^*}{\alpha_{N-1}}} \left( \epsilon_{N-2} \frac{r^2}{P^* + r^2} - (\eta_{1,N-1} + \sqrt{\frac{\alpha_{N-3} \sigma^2_1}{\alpha_{N-2} r^2}} V_{N-2} \right)
- \sqrt{\frac{\alpha_{N-3} (1 - \rho^2) P_2}{\alpha_{N-2} r^2}} \eta_{1,n-2} \right) \cdot \sqrt{\frac{P^* \cdot \alpha_{N-2}'}{P^* + r^2}}), \quad U_N = \rho \sqrt{\frac{P_2}{P_1}} X_{1,N}
\]
\[
V_{N} = \sqrt{\frac{\alpha_{N-2} \sigma^2_1}{\alpha_{N-1} r^2}} V_{N-1} - \sqrt{\frac{\alpha_{N-2} (1 - \rho^2) P_2}{\alpha_{N-1} r^2}} \eta_{1,n-1}
\]
(3.41)
where \( r \) is defined in (A17) and \( P^* \) is defined in (3.8).

From (3.41), we can conclude that for \( 3 \leq k \leq N \), \( X_{1,k} \), \( U_k \) and \( V_k \) are functions of \( \eta_{1,1}, \eta_{1,k-1} \), and they are independent of the transmitted messages. For convenience, for \( 3 \leq k \leq N \), define
\[
X_{1,k} = f_{1,k}(\eta_{1,1}, \ldots, \eta_{1,k-1}), \quad U_k = f_{u,k}(\eta_{1,1}, \ldots, \eta_{1,k-1}), \quad V_k = f_{v,k}(\eta_{1,1}, \ldots, \eta_{1,k-1})
\]
(3.42)
By using (3.41) and (3.42), the equivocation rate \( \frac{1}{N} H(W_1, W_2 \mid Z^N) \) of any achievable rate pair in \( C_{gmac-dms}^f \) can be bounded by
\[
\frac{1}{N} H(W_1, W_2 \mid Z^N) = \frac{1}{N} H(\theta_1, \theta_2 \mid Z^N)
\geq \frac{1}{N} H(\theta_1, \theta_2 \mid Z^N, \eta_{1,1}, \ldots, \eta_{1,N}, \eta_{2,3}, \ldots, \eta_{2,N})
\]
(a) follows from (2.1), (3.41) and (3.42), (b) follows from the fact that \( \theta_1, \theta_2, \eta_{2,1} \) and \( \eta_{2,2} \) are independent of \( \eta_{1,1}, \ldots, \eta_{1,N}, \eta_{2,3}, \ldots, \eta_{2,N} \), (c) follows from the fact that \( \theta_1, \theta_2, \eta_{2,1} \) and \( \eta_{2,2} \) are independent of each other, and (d) follows because \( H(\theta_j) = NR_j \) \((j = 1, 2)\), the variance of \( \theta_j \) equals \( \frac{1}{12} \) as \( N \) tends to infinity, and \( \theta_j \) is independent of \( \eta_{2,j} \). Choosing sufficiently large \( N \), the secrecy constraint in (2.6) is guaranteed, which indicates that any achievable rate pair \( (R_1, R_2) \)
in $C_{gmac-dms}^f$ is achievable with perfect weak secrecy, and hence $C_{s,gmac-dms}^f = C_{gmac-dms}^f$. The proof of Theorem 1 is completed.

For comparison, the following Corollary 2 establishes an outer bound on the secrecy capacity region $C_{s,gmac-dms}$ of GMAC-WT-DMS without feedback.

**Corollary 2:** $C_{s,gmac-dms} \subseteq C_{s,gmac-dms}^{out}$, where $C_{s,gmac-dms}^{out}$ is given by

$$C_{s,gmac-dms}^{out} = \bigcup_{-1 \leq \rho \leq 1} \{(R_1 \geq 0, R_2 \geq 0) : R_2 \leq \frac{1}{2} \log \left(1 + \frac{(1 - \rho^2)P_2}{\sigma_1^2}\right),$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2\rho}}{\sigma_1^2}\right) - \frac{1}{2} \log \left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2\rho}}{\sigma_1^2 + \sigma_2^2}\right)\}.$$ (3.44)

**Proof:** See Appendix B.

The following Figure 4 shows the rate gains by using channel feedback for $P_1 = 1$, $P_2 = 1.5$, $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 1.2$.

![Fig. 4: Capacity results on GMAC-WT-DMS with or without feedback](image)

**IV. THE SSCA FEEDBACK SCHEME FOR THE GMAC-WT-NCSIT-DMS**

In this section, first, we extend the two-step SK-type feedback scheme of the preceding section to the GMAC-NCSIT-DMS with feedback, and show that this extended scheme is also
capacity-achieving. Second, we show that this extended feedback scheme is secure by itself and also achieves the secrecy capacity region $C_{s,gmac-ncsit-dms}^f$ of the GMAC-WT-NCSIT-DMS with feedback. Finally, in order to show the rate gains by the feedback, an outer bound on the secrecy capacity region $C_{s,gmac-ncsit-dms}$ of the GMAC-WT-NCSIT-DMS is provided, and the capacity results given in this section are further explained via a numerical example.

A. A capacity-achieving SK-type scheme for the GMAC-NCSIT-DMS with feedback

The model of the GMAC-NCSIT-DMS with feedback is formulated in Section II-B1. In this subsection, first, we introduce capacity results on the GMAC-NCSIT-DMS with or without feedback. Then, we propose a corresponding capacity-achieving feedback scheme.

1) Capacity results on the GMAC-NCSIT-DMS with or without feedback: The following Corollary 3 characterizes the capacity region $C_{gmac-ncsit-dms}$ of the GMAC-NCSIT-DMS.

**Corollary 3:** The capacity region $C_{gmac-ncsit-dms}$ of the GMAC-NCSIT-DMS is given by

$$C_{gmac-ncsit-dms} = \bigcup_{0 \leq \rho \leq 1} \left \{ (R_1, R_2) : R_2 \leq \frac{1}{2} \log \left( 1 + \frac{P_2(1 - \rho^2)}{\sigma_1^2} \right), \right.$$ 

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2\rho}}{\sigma_1^2} \right) \right \}. \quad (4.1)$$

**Proof:** In [28], it has been pointed out that $C_{gmac-ncsit-dms}$ equals $C_{gmac-dms}$, which indicates that for the GMAC-NCSIT-DMS, the state interference can be pre-cancelled by both the transmitters and the receiver.

The following Corollary 4 determines the capacity region $C_{gmac-ncsit-dms}^f$ of the GMAC-NCSIT-DMS with feedback, which indicates that feedback does not increase the capacity of the GMAC-NCSIT-DMS, see the followings.

**Corollary 4:** $C_{gmac-ncsit-dms}^f = C_{gmac-ncsit-dms}$, where $C_{gmac-ncsit-dms}$ is given in (4.1).

**Proof:** Note that $C_{gmac-ncsit-dms}^f \subseteq C_{gmac-dms}^f = C_{gmac-dms} = C_{gmac-ncsit-dms}$ directly follows from the converse proof of the bounds on $R_2$ and $R_1 + R_2$ in $C_{gmac}^f$ [5, pp. 627-628], and hence we omit the converse proof here. On the other hand, note that $C_{gmac-ncsit-dms} \subseteq C_{gmac-ncsit-dms}^f$ since non-feedback model is a special case of the feedback model, and $C_{gmac-ncsit-dms} = C_{gmac-dms}$ (see (4.1)), and hence the proof of Theorem 4 is completed.
2) A capacity-achieving two-step SK-type feedback scheme for the GMAC-NCSIT-DMS with feedback: Similar to the two-step SK-type feedback scheme in Section III, Transmitter 1 uses power $P_1$ to encode $W_1$, $S^N$ and the feedback $Y^N$ as $X_1^N$. Transmitter 2 uses power $(1 - \rho^2)P_2$ to encode $W_2$, $S^N$ and $Y^N$ as $V^N$, and power $\rho^2P_2$ to encode $W_1$, $S^N$ and $Y^N$ as $U^N$, where $0 \leq \rho \leq 1$. Moreover, let $\theta_j$ be the center of the sub-interval w.r.t. the message $W_j$ (the variance of $\theta_j$ approximately equals $\frac{1}{12}$).

Encoding: At time 1, Transmitter 1 sends

$$X_{1,1} = 0.$$  \hfill (4.2)

Transmitter 2 sends

$$V_1 = \sqrt{12(1 - \rho^2)P_2} (\theta_2 - \frac{S_1}{\sqrt{12(1 - \rho^2)P_2}} + A_2),$$  \hfill (4.3)

and

$$U_1 = \rho \sqrt{\frac{P_2}{P_1}X_{1,1}} = 0,$$  \hfill (4.4)

where $A_2$ is a linear combination of $S_1,...,S_N$, and it will be determined later.

The receiver obtains

$$Y_1 = V_1 + X_{1,1} + U_1 + S_1 + \eta_{1,1} = V_1 + S_1 + \eta_{1,1} = \sqrt{12(1 - \rho^2)P_2}\theta_2 + \sqrt{12(1 - \rho^2)P_2}A_2 + \eta_{1,1},$$  \hfill (4.5)

and gets an estimation $\hat{\theta}_{2,1}$ of $\theta_2$ by computing

$$\hat{\theta}_{2,1} = \frac{Y_1}{\sqrt{12(1 - \rho^2)P_2}} = \theta_2 + A_2 + \frac{\eta_{1,1}}{\sqrt{12(1 - \rho^2)P_2}} = \theta_2 + A_2 + \epsilon_1,$$  \hfill (4.6)

where $\epsilon_1$ is in the same fashion as that in Section III and define $\alpha_1 \triangleq Var(\epsilon_1) = \frac{\sigma_1^2}{12(1 - \rho^2)P_2}$.

Then the receiver sends $Y_1$ back to Transmitter 2. Let $Y'_1 = Y_1 = V_1 + S_1 + \eta_{1,1}$, Transmitter 2 computes

$$\frac{Y'_1}{\sqrt{12(1 - \rho^2)P_2}} = \theta_2 + A_2 + \frac{\eta_{1,1}}{\sqrt{12(1 - \rho^2)P_2}} = \theta_2 + A_2 + \epsilon_1.$$  \hfill (4.7)

Since $A_2$ is known by the transmitters, Transmitter 2 obtains $\epsilon_1$ from (4.7).

At time 2, Transmitter 2 sends $V_2$ exactly in the same fashion as that in (3.13), i.e., $V_2 = \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_1}}\epsilon_1$. On the other hand, at time 2, Transmitters 1 and 2 respectively send $X_{1,2}$ and $U_2 = \rho \sqrt{\frac{P_2}{P_1}X_{1,2}}$ such that

$$X^*_2 = U_2 + X_{1,2} = \sqrt{12P^*}(\theta_1 - \frac{S_2}{\sqrt{12P^*}} + A_1),$$  \hfill (4.8)

where $P^*$ is defined in the same fashion as that in (3.8) and

$$A_1 = \sum_{i=3}^{N} \beta_{1,i} S_i, \quad (4.9)$$

and $\beta_{1,i}$ will be defined later. The receiver obtains

$$Y_2 = X_2^* + V_2 + S_2 + \eta_{1,2} = \sqrt{12P^*}\theta_1 + \sqrt{12P^*}A_1 + V_2 + \eta_{1,2}, \quad (4.10)$$

and gets an estimation $\hat{\theta}_{1,2}$ of $\theta_1$ by computing

$$\hat{\theta}_{1,2} = \frac{Y_2}{\sqrt{12P^*}} = \theta_1 + A_1 + \frac{V_2 + \eta_{1,2}}{\sqrt{12P^*}} = \theta_1 + A_1 + \epsilon'_2, \quad (4.11)$$

where $\epsilon'_2$ is in the same fashion as that in Section III and define $\alpha'_2 \triangleq Var(\epsilon'_2)$. Then the receiver sends $Y_2$ back to both transmitters.

At time 3, once receiving the feedback $Y_2 = X_2^* + V_2 + S_2 + \eta_{1,2}$, both transmitters compute

$$\frac{Y_2}{\sqrt{12P^*}} = \theta_1 + A_1 + \frac{V_2 + \eta_{1,2}}{\sqrt{12P^*}} = \theta_1 + A_1 + \epsilon'_2, \quad (4.12)$$

and send $X_{1,3}$ and $U_3 = \rho \sqrt{\frac{P^*}{P_1}} X_{1,3}$ such that

$$X_3^* = U_3 + X_{1,3} = \frac{P^*}{\alpha_2} \epsilon'_2, \quad (4.13)$$

In addition, subtracting $X_{1,2}$, $U_2$ and $S_2$ from $Y_2$ and let $Y'_2 = Y_2 - X_{1,2} - U_2 - S_2 = V_2 + \eta_{1,2}$, Transmitter 2 computes

$$\epsilon_2 = \epsilon_1 - \frac{E[Y'_2 \epsilon_1]}{E[(Y'_2)^2]} Y'_2, \quad (4.14)$$

and sends

$$V_3 = \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_2}} \epsilon_2, \quad (4.15)$$

where $\alpha_2 \triangleq Var(\epsilon_2)$.

At time $4 \leq k \leq N$, once receiving $Y_{k-1} = X_{1,k-1} + U_{k-1} + V_{k-1} + S_k + \eta_{1,k-1}$, Transmitter 2 computes

$$\epsilon_{k-1} = \epsilon_{k-2} - \beta_{2,k-1} Y'_{k-1}, \quad (4.16)$$

where

$$Y'_{k-1} = Y_{k-1} - X_{1,k-1} - U_{k-1} - S_{k-1}, \quad (4.17)$$
\[ \beta_{2,k-1} = \frac{E[Y'_{k-1}\epsilon_{k-2}]}{E[(Y'_{k-1})^2]}, \]  

(4.18)

and sends

\[ V_k = \sqrt{(1 - \rho^2)P_2 \alpha_{k-1}} \epsilon_{k-1}, \]  

(4.19)

where \( \alpha_{k-1} \triangleq Var(\epsilon_{k-1}) \). In the meanwhile, Transmitters 1 and 2 respectively send \( X_{1,k} \) and \( U_k = \rho \sqrt{\frac{P_2}{P_1}} X_{1,k} \) such that

\[ X_k^* = U_k + X_{1,k} = \sqrt{\frac{P^*}{\alpha_{k-1}^*}} \epsilon_{k-1}^*, \]  

(4.20)

where

\[ \epsilon_{k-1}^* = \epsilon_{k-2}^* - \beta_{1,k-1}(Y_{k-1} - S_{k-1}), \]  

(4.21)

\[ \beta_{1,k-1} = \frac{E[(Y_{k-1} - S_{k-1})\epsilon_{k-2}^*]}{E[(Y_{k-1} - S_{k-1})^2]}, \]  

(4.22)

and \( \alpha_{k-1}^* \triangleq Var(\epsilon_{k-1}^*) \).

Here note that though the use of \( S^N \) at time instants 1 and 2 causes the transmission power of the first two time instants to be larger than the average power constraint. However, for \( k \geq 3 \), the transmission power equals the average power constraint (see the analysis of the average transmission power in Section III-A2), and hence for sufficiently larger \( N \), the power constraint is preserved.

**Decoding:**

The receiver uses a two-step decoding scheme which is similar to that in Section III. Specifically, first, from (2.14), we observe that at time \( k \) \((3 \leq k \leq N)\), the receiver’s estimation \( \hat{\theta}_{1,k} \) of \( \theta_1 \) is given by

\[ \hat{\theta}_{1,k} = \hat{\theta}_{1,k-1} - \beta_{1,k} Y_k, \]  

(4.23)

where \( \beta_{1,k} = \frac{E[(Y_k - S_k)\epsilon_{k-1}^*]}{E[(Y_k - S_k)^2]} \). Combining (4.21) with (4.23), we have

\[
\begin{align*}
\hat{\theta}_{1,k} & = \hat{\theta}_{1,k-1} + \epsilon_k^* - \epsilon_{k-1}^* - \beta_{1,k} S_k \\
& = \hat{\theta}_{1,2} + \epsilon_2^* - \epsilon_{k-2} - \sum_{j=3}^{k} \beta_{1,j} S_j \\
& \overset{(a)}{=} \theta_1 + \epsilon_1^* + \epsilon_2^* + \epsilon_k^* - \epsilon_2^* - \sum_{j=3}^{k} \beta_{1,j} S_j \\
& = \theta_1 + \epsilon_k^* + A_1 - \sum_{j=3}^{k} \beta_{1,j} S_j,
\end{align*}
\]

(4.24)
where (a) follows from (4.11). From (4.24), we can conclude that for \( k = N \),

\[
\hat{\theta}_{1,N} = \theta_1 + \epsilon'_N + A_1 - \sum_{j=3}^{N} \beta_{1,j} S_j
\]

\[\overset{(b)}{=} \theta_1 + \epsilon'_N + A_1 - A_1 = \theta_1 + \epsilon', \quad (4.25)\]

where (b) follows from (4.9). Note that (4.25) indicates that the receiver’s final estimation of \( \theta_1 \)
is in the same fashion as that in Section III and observing that \( \epsilon'_k (2 \leq k \leq N) \) is exactly in the
same fashion as those in Section III we can directly apply Lemma 1 to show that the decoding
error probability \( P_{e1} \) of \( \theta_1 \) tends to 0 as \( N \to \infty \) if \( R_1 < \frac{1}{2} \log(1 + \frac{P_{1} + P_{2} + 2 \sqrt{P_{1} P_{2}}}{(1 - \rho^2) P_{2} + \sigma_{1}^2}) \)
is satisfied.

Second, after decoding \( W_1 (\theta_1) \), the receiver obtains \( \epsilon'_k + A_1 - \sum_{j=3}^{k} \beta_{1,j} S_j \) \((3 \leq k \leq N)\)
from (4.24), and obtains \( \epsilon'_2 + A_1 \) from (4.11). Furthermore, from (4.20) and the fact that \( \sqrt{P_{12}} \)
is a constant value, we can conclude that for \( 3 \leq k \leq N \), the receiver knows

\[
\sqrt{\frac{P_s}{\alpha_k}} (\epsilon'_k + A_1 - \sum_{j=3}^{k} \beta_{1,j} S_j) = X'_{k+1} + \sqrt{\frac{P_s}{\alpha_k}} (A_1 - \sum_{j=3}^{k} \beta_{1,j} S_j). \quad (4.26)
\]

In addition, for \( k = 2 \), the receiver knows

\[
X'_{3} + \sqrt{\frac{P_s}{\alpha_2}} A_1 \quad (4.27)
\]
since \( X'_{3} = \sqrt{\frac{P_s}{\alpha_2}} \epsilon'_2 \) and \( \sqrt{\frac{P_s}{\alpha_2}} \) is a constant value. Here for \( k = 2 \), define \( \sum_{j=3}^{k} \beta_{1,j} S_j = 0 \). Then we can conclude that the receiver knows the terms in (4.26) for \( 2 \leq k \leq N \). Here note that in the decoding procedure of the two-step SK-type scheme for the GMAC-DMS, after decoding \( \theta_1 \), the receiver knows \( X_{1,k} \) and \( U_k \) for all \( 3 \leq k \leq N \), and hence he/she subtracts \( X_{1,k} \) and \( U_k \) from \( Y_k \), and further using SK-type decoding scheme to obtain \( \theta_2 \). While, in this extended scheme for the GMAC-NCSIT-DMS, after decoding \( \theta_1 \), the receiver does not know \( X_{1,k} \) and \( U_k \), instead, he/she only knows \( X'_{k+1} + \sqrt{\frac{P_s}{\alpha_{k-1}}} (A_1 - \sum_{j=3}^{k-1} \beta_{1,j} S_j) \), then the key to further decode \( \theta_2 \) is how to choose \( A_2 \) (a linear combination of \((S_1, \ldots, S_N)) \) to precancel the offset of the receiver’s final estimation of \( \theta_2 \).

Recall that the receiver’s estimation \( \hat{\theta}_{2,1} \) of \( \theta_2 \) is given by (4.6). At time 2, since \( \theta_1 \) is obtained by the receiver, the receiver’s estimation \( \hat{\theta}_{2,2} \) of \( \theta_2 \) is given by

\[
\hat{\theta}_{2,2} = \hat{\theta}_{2,1} - \beta_{2,2} (Y_2 - \sqrt{12P_s} \theta_1)
\]

\[\overset{(c)}{=} \hat{\theta}_{2,1} + \epsilon_2 - \epsilon_1 - \beta_{2,2} \sqrt{12P_s} A_1
\]

\[\overset{(d)}{=} \theta_2 + A_2 + \epsilon_1 + \epsilon_2 - \epsilon_1 - \beta_{2,2} \sqrt{12P_s} A_1 = \theta_2 + \epsilon_2 + A_2 - \beta_{2,2} \sqrt{12P_s} A_1, \quad (4.28)\]
where (c) follows from (4.14), and (d) follows from (4.6). At time \( k \) (\( 3 \leq k \leq N \)), the receiver’s estimation \( \hat{\theta}_{2,k} \) of \( \theta_2 \) is given by
\[
\hat{\theta}_{2,k} = \hat{\theta}_{2,k-1} - \beta_{2,k} \left( Y_k - X^*_k - \sqrt{\frac{P^*}{\alpha_{k-1}}} (A_1 - \sum_{j=3}^{k-1} \beta_{1,j} S_j) \right)
\]
\[
= \hat{\theta}_{2,k-1} - \beta_{2,k} \left( Y_k - X^*_k - \sqrt{\frac{P^*}{\alpha_{k-1}}} (A_1 - \sum_{j=3}^{k-1} \beta_{1,j} S_j) \right)
\]
where (e) follows from the fact that the term in (4.26) is known by the receiver and hence it can be subtracted from \( Y_k \), (f) follows from (4.16), and (g) follows from (4.28). From (4.29), we can conclude that for \( k = N \),
\[
\hat{\theta}_{2,N} = \hat{\theta}_2 + \epsilon_N + A_2 - \beta_{2,2} \sqrt{12P^* A_1} + \epsilon_k + \epsilon_2 + \sum_{i=3}^{k} \left( \beta_{2,i} \sqrt{\frac{P^*}{\alpha_{i-1}^*}} (A_1 - \sum_{j=3}^{i-1} \beta_{1,j} S_j) - \beta_{2,i} S_i \right) \cdot \beta_{2,i} S_i \right), \quad (4.29)
\]
Observing that if
\[
A_2 = \beta_{2,2} \sqrt{12P^* A_1} - \sum_{i=3}^{N} \left( \beta_{2,i} \sqrt{\frac{P^*}{\alpha_{i-1}^*}} (A_1 - \sum_{j=3}^{i-1} \beta_{1,j} S_j) - \beta_{2,i} S_i \right), \quad (4.31)
\]
(4.30) can be re-written as
\[
\hat{\theta}_{2,N} = \hat{\theta}_2 + \epsilon_N, \quad (4.32)
\]
which indicates that the receiver’s final estimation of \( \theta_2 \) is in the same fashion as that in Section III and observing that \( \epsilon_k \) (\( 1 \leq k \leq N \)) is exactly in the same fashion as those in Section III, we can directly apply the same argument in Section III to show that the decoding error probability \( P_{e2} \) of \( \theta_2 \) tends to 0 as \( N \to \infty \) if \( R_2 < \frac{1}{2} \log(1 + \frac{(1 - \rho^2)P_2}{\sigma_1^2}) \) is satisfied.

Finally, note that the decoding error probability \( P_e \) of the receiver is upper bounded by \( P_e \leq P_{e1} + P_{e2} \), and from above analysis, we can conclude that the rate pair \( (R_1 = \frac{1}{2} \log(1 + \frac{P_1 + \rho^2 P_2 + 2\sqrt{P_1 P_2} \rho}{(1 - \rho^2) P_2 + \sigma_1^2}), \quad R_2 = \frac{1}{2} \log(1 + \frac{(1 - \rho^2)P_2}{\sigma_1^2}) \) is achievable for all \( 0 \leq \rho \leq 1 \), which indicates
that all rate pairs \((R_1, R_2)\) in \(C_{\text{gmac–ncsit–dms}}^f\) are achievable. Hence this extended two-step SK-type feedback scheme achieves the capacity region \(C_{\text{gmac–ncsit–dms}}^f\) of GMAC-NCSIT-DMS with feedback.

B. Capacity results on the GMAC-WT-NCSIT-DMS with or without feedback

The model of the GMAC-WT-NCSIT-DMS with feedback is formulated in Section II-B2. The following Theorem 2 establishes that the secrecy constraint does not reduce the capacity of GMAC-NCSIT-DMS with feedback.

**Theorem 2:** \(C_{s,\text{gmac–ncsit–dms}}^f = C_{\text{gmac–ncsit–dms}}^f\), where \(C_{s,\text{gmac–ncsit–dms}}^f\) is the secrecy capacity region of the GMAC-WT-NCSIT-DMS with feedback, and \(C_{\text{gmac–ncsit–dms}}^f\) is given in Corollary 4.

**Proof:** Since \(C_{s,\text{gmac–ncsit–dms}}^f \subseteq C_{\text{gmac–ncsit–dms}}^f\), we only need to show that any achievable rate pair \((R_1, R_2)\) in \(C_{\text{gmac–ncsit–dms}}^f\) satisfies the secrecy constraint in (2.6). In the preceding subsection, we introduce an extended feedback scheme for the GMAC-NCSIT-DMS with feedback, and show that this scheme achieves \(C_{\text{gmac–ncsit–dms}}^f\). In this new scheme, the transmitted codewords \(X_{1,i}, U_i\) and \(V_i\) at time \(i (1 \leq i \leq N)\) can be expressed almost in the same fashion as those in (3.41), except that

\[
V_1 = \sqrt{12(1 - \rho^2)}P_2(\theta_2 - \frac{S_1}{\sqrt{12(1 - \rho^2)}P_2} + A_2),
\]

\[
X_{1,2} = \frac{\sqrt{12P_2^*}(1 - \frac{S_1}{\sqrt{12P_2^*}} + A_1)}{\rho\sqrt{\frac{P_2^*}{P_1} + 1}}, \quad U_2 = \frac{P_1}{P_2^*}X_{1,2}.
\]

From (3.41) and (4.33), we can conclude that for \(3 \leq i \leq N\), \(\theta_1\) and \(\theta_2\) are not contained in the transmitted \(X_{1,i}, U_i\) and \(V_i\). Hence following the steps in (3.43) and choosing sufficiently large \(N\), we can prove that \(\frac{1}{N}H(W_1, W_2|Z^N) \geq R_1 + R_2 - \epsilon\), which completes the proof.

For comparison, the following Corollary 5 establishes an outer bound on the secrecy capacity region \(C_{s,\text{gmac–ncsit–dms}}\) of GMAC-WT-NCSIT-DMS.

**Corollary 5:** \(C_{s,\text{gmac–ncsit–dms}} \subseteq C^\text{out}_{s,\text{gmac–ncsit–dms}}\), where \(C^\text{out}_{s,\text{gmac–ncsit–dms}}\) is given by

\[
C^\text{out}_{s,\text{gmac–ncsit–dms}} = \bigcup_{-1 \leq \rho_{12, \rho_1 s, \rho_2 s} \leq 1} \{(R_1 \geq 0, R_2 \geq 0) : \]

\[
R_2 \leq \frac{1}{2} \log \left(1 + \frac{P_2 + \sigma_1^2 + a^2P_1 + b^2Q - 2a\rho_{12}\sqrt{P_1P_2} - 2b\rho_{2s}\sqrt{P_2Q} + 2ab\rho_{1s}\sqrt{P_1Q}}{\sigma_1^2} \right),
\]

\[
R_1 + R_2 \leq \frac{1}{2} \log \left(1 + \frac{P_1 + P_2 + Q + 2\sqrt{P_1P_2\rho_{12}} + 2\rho_{1s}\sqrt{P_1Q} + 2\rho_{2s}\sqrt{P_2Q}}{\sigma_1^2} \right).
\]
\[-\frac{1}{2} \log \left( 1 + \frac{P_1 + P_2 + Q + 2\sqrt{P_1 P_2 \rho_{12}} + 2\rho_{1s} \sqrt{P_1 Q} + 2\rho_{2s} \sqrt{P_2 Q}}{\sigma_1^2 + \sigma_2^2} \right) \right],
\]

where

\[a = \sqrt{\frac{P_2 \rho_{12} - \rho_{1s} \rho_{2s}}{P_1}}, \quad b = \sqrt{\frac{P_2 \rho_{2s} - \rho_{12} \rho_{1s}}{Q}}.
\]

**Proof:** See Appendix C.

The following Figure 5 shows the rate gains by using channel feedback for \(P_1 = 10\), \(P_2 = 3\), \(Q = 5\), \(\sigma_1^2 = \sigma_2^2 = 20\).

V. CONCLUSION

In this paper, we determine the secrecy capacity regions of the GMAC-WT-DMS with feedback and the GMAC-WT-NCSIT-DMS with feedback by proposing SSCA feedback schemes for these models. Possible future work includes:

- The rate-splitting feature used in [29] might be a good element to identify future strategies that potentially be useful for the GMACs with general (not necessarily degraded) message set, and maybe via that one can reach similar conclusions given above when the rate regions are not degraded by introducing secrecy constraint.
• To explore whether one can identify dualities of some kind between the GMAC and the Gaussian broadcast models when feedback and secrecy constraint are considered.
• The finite blocklength regime also deserves attention even in the single user wiretap case where a modified SK scheme motivated by \[30\] might be useful.

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APPENDIX A
PROOF OF LEMMA

For \(2 \leq k \leq N\), define

\[
\eta_{i,k}' = \eta_{i,k} + V_k. \tag{A1}
\]

Note that

\[
E[(\eta_{i,k}')^2] = E[(\eta_{i,k} + V_k)^2] \overset{(a)}{=} E[(\eta_{i,k})^2] + E[(V_k)^2] \overset{(b)}{=} \sigma_1^2 + (1 - \rho^2)P_2, \tag{A2}
\]

where (a) follows from the fact that \(V_k\) is independent of \(\eta_{i,k}\) since \(V_1\) is a function of \(\theta_1\) and \(V_k\ (2 \leq k \leq N)\) is a function of \(\eta_{i,1}, \ldots, \eta_{i,k-1}\), and (b) follows from (3.21). Furthermore, from (3.19) and (3.21), \(V_k\) can be re-written as

\[
V_k = \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-1}}} \epsilon_{k-1}
\]

\[
= \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-1}}} \left( \epsilon_{k-2} - \frac{E[(V_{k-1} + \eta_{1,k-1})\epsilon_{k-2}]}{E[(V_{k-1} + \eta_{1,k-1})^2]}(V_{k-1} + \eta_{1,k-1}) \right)
\]

\[
\overset{(c)}{=} \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-1}}} \left( \epsilon_{k-2} - \sqrt{\frac{(1 - \rho^2)P_2\alpha_{k-2}}{(1 - \rho^2)P_2 + \sigma_1^2}}(V_{k-1} + \eta_{1,k-1}) \right)
\]

\[
= \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-2}}} \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}}} \left( \epsilon_{k-2} - \sqrt{\frac{(1 - \rho^2)P_2\alpha_{k-2}}{(1 - \rho^2)P_2 + \sigma_1^2}}(V_{k-1} + \eta_{1,k-1}) \right)
\]

\[
\overset{(d)}{=} \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}}} V_{k-1} - \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-1}}} \sqrt{(1 - \rho^2)P_2\alpha_{k-2}}(V_{k-1} + \eta_{1,k-1})
\]
\[
\eta'_{1,k} = \eta_{1,k} + V_k = \eta_{1,k} + \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{\sigma_1^2}{(1 - \rho^2)P_2 + \sigma_1^2}} V_{k-1} - \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{(1 - \rho^2)P_2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta_{1,k-1}
\]

where (c) follows from \(\epsilon_{k-2}\) is independent of \(\eta_{1,k-1}\), \(V_{k-1} = \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-2}}} \epsilon_{k-2}\) and \(\alpha_{k-2} \triangleq Var(\epsilon_{k-2})\), and (d) follows from \(V_{k-1} = \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_{k-2}}} \epsilon_{k-2}\). Substituting (A3) into (A1), we have

\[
\eta'_{1,k} = \eta_{1,k} + V_k = \eta_{1,k} + \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{\sigma_1^2}{(1 - \rho^2)P_2 + \sigma_1^2}} V_{k-1} - \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{(1 - \rho^2)P_2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta_{1,k-1}
\]

\[
+ \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{\sigma_1^2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta_{1,k-1} - \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{(1 - \rho^2)P_2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta_{1,k-1}
\]

\[
= \eta_{1,k} + \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{\sigma_1^2}{(1 - \rho^2)P_2 + \sigma_1^2}} (V_{k-1} + \eta_{1,k-1}) - \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{(1 - \rho^2)P_2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta_{1,k-1}
\]

\[
= \eta_{1,k} + \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{\sigma_1^2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta'_{1,k-1} - \sqrt{\frac{\alpha_{k-2}}{\alpha_{k-1}} \frac{(1 - \rho^2)P_2}{(1 - \rho^2)P_2 + \sigma_1^2}} \eta_{1,k-1}.
\]

From classical SK scheme [6], we know that

\[
\frac{\alpha_k}{\alpha_{k-1}} = \frac{\sigma_1^2}{(1 - \rho^2)P_2 + \sigma_1^2}
\]

for all \(2 \leq k \leq N\). Substituting (A5) into (A4), we obtain

\[
\eta'_{1,k} = \frac{\sigma_1}{\sqrt{(1 - \rho^2)P_2 + \sigma_1^2}} \eta'_{1,k-1} + \eta_{1,k} - \sqrt{\frac{(1 - \rho^2)P_2 + \sigma_1^2}{\sigma_1^2}} \eta_{1,k-1}.
\]

Here note that (A6) holds for \(3 \leq k \leq N\), and

\[
\eta'_{1,2} = \eta_{1,2} + V_2 = \eta_{1,2} + \sqrt{\frac{(1 - \rho^2)P_2}{\alpha_1}} \epsilon_1 = \eta_{1,2} + \frac{\eta_{1,1} \sqrt{(1 - \rho^2)P_2}}{\sigma_1}.
\]

On the other hand, from (3.23), we have

\[
E[Y_{k-1}\epsilon'_{k-2}] = E[(X^*_{k-1} + \eta'_{1,k-1})\epsilon'_{k-2}] = E(\sqrt{\frac{P^*_{\epsilon'_{k-2}}}{\alpha'_{k-2}}} + \eta'_{1,k-1}) = \sqrt{\alpha'_{k-2}} + E[\eta'_{1,k-1}\epsilon'_{k-2}],
\]

and

\[
E[Y_{k-1}^2] = E[(X^*_{k-1} + \eta'_{1,k-1})^2] = E[(\sqrt{\frac{P^*_{\epsilon'_{k-2}}}{\alpha'_{k-2}}} + \eta'_{1,k-1})^2] = P^* + 2 \frac{P^*}{\alpha'_{k-2}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2.
\]
where (e) follows from (3.22), and (f) follows from (A2). Substituting (A8) and (A9) into (3.23), \( \epsilon'_{k-1} \) can be re-written as

\[
\epsilon'_{k-1} = \epsilon'_{k-2} - \frac{E[Y_{k-1}\epsilon'_{k-2}]}{E[Y_{k-1}]} Y_{k-1}
\]

\[
= \epsilon'_{k-2} - \frac{P^* \alpha'_{k-2} + E[\epsilon'_{k-2}\eta'_{1,k-1}]}{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2} \left( \sqrt{P^* \epsilon'_{k-2} + \eta'_{1,k-1}} \right)
\]

\[
= \epsilon'_{k-2} - \frac{\epsilon'_{k-2}(P^* + E[\epsilon'_{k-2}\eta'_{1,k-1}] \sqrt{\frac{P^*}{\alpha'_{k-2}}}) + \eta'_{1,k-1} \left( \sqrt{P^* \cdot \alpha'_{k-2} + E[\epsilon'_{k-2}\eta'_{1,k-1}]} \right)}{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}
\]

\[
= \epsilon'_{k-2} \frac{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}{\frac{P^* \cdot \alpha'_{k-2} + E[\epsilon'_{k-2}\eta'_{1,k-1}]}{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}}
\]

\[
- \frac{\eta'_{1,k-1}}{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}
\]

From (A10), we observe that \( \epsilon'_{k-1} \) depends on \( E[\epsilon'_{k-2}\eta'_{1,k-1}] \). Combining (A6) with (A10), we can conclude that

\[
E[\epsilon'_{k-1}\eta'_{1,k}] = E\left[ \left( \frac{\sigma_1}{\sqrt{(1 - \rho^2)P_2 + \sigma_1^2}} \eta'_{1,k-1} + \eta_{1,k} - \sqrt{(1 - \rho^2)P_2 + \sigma_1^2} \eta_{1,k-1} \right) \right]
\]

\[
\epsilon'_{k-2} \frac{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}
\]

\[
- \eta'_{1,k-1} \frac{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}{P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2}
\]

\[
(\epsilon'_{k-2}) E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2 \cdot \frac{\sigma_1}{\sqrt{(1 - \rho^2)P_2 + \sigma_1^2}} E[\epsilon'_{k-2}\eta'_{1,k-1}]
\]

\[
- \sqrt{P^* \cdot \alpha'_{k-2} + E[\epsilon'_{k-2}\eta'_{1,k-1}]}
\]

\[
P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2
\]

\[
+ \sqrt{P^* \cdot \alpha'_{k-2} + E[\epsilon'_{k-2}\eta'_{1,k-1}]}
\]

\[
P^* + 2 \sqrt{\frac{P^*}{\alpha'_{k-2}}} E[\epsilon'_{k-2}\eta'_{1,k-1}] + (1 - \rho^2)P_2 + \sigma_1^2
\]
where (g) follows from \( E[\epsilon'_{k-2} \eta_{1,k-1}] = E[\epsilon'_{k-2} \eta_{1,k-1}] = E[\eta'_{1,k-1} \eta_{1,k}] = 0 \), and (h) follows from (A6), which indicates that

\[
E[\eta'_{1,k-1} \eta_{1,k-1}] = E \left[ \left( \frac{\sigma_1}{\sqrt{(1 - \rho^2) P_2 + \sigma_1^2}} \eta'_{1,k-2} + \eta_{1,k-1} - \sqrt{\frac{(1 - \rho^2) P_2 + \sigma_1^2}{\sigma_1^2}} \eta_{1,k-2} \right) \eta_{1,k-1} \right] = E[(\eta_{1,k-1})^2] = \sigma_1^2, \tag{A12}
\]

where (i) follows from \( E[\eta'_{1,k-2} \eta_{1,k-1}] = E[\eta_{1,k-2} \eta_{1,k-1}] = 0 \).

Observing that the first item of \( E[\epsilon'_{k-1} \eta_{1,k}] \) is \( E[\epsilon'_{2} \eta'_{1}] \), and it is given by

\[
E[\epsilon'_{2} \eta'_{1}] = E[\epsilon'_{2}(V_3 + \eta_{1,3})] = E \left[ \epsilon'_{2}(\eta_{1,3} + \sqrt{\frac{(1 - \rho^2) P_2}{\alpha_2}} \eta_{1,2}) \right] = E \left[ \frac{\sqrt{(1 - \rho^2) P_2}}{\sigma_1} \eta_{1,1} + \eta_{1,2} \right] \left( \eta_{1,3} + \frac{1}{r} \sqrt{(1 - \rho^2) P_2} \eta_{1,1} - \frac{(1 - \rho^2) P_2}{r \sigma_1} \eta_{1,2} \right]
\]

\[
= \frac{(1 - \rho^2) P_2}{r \sigma_1} \frac{\sigma_1^2}{\sqrt{2 P^*}} - \frac{(1 - \rho^2) P_2}{r \sigma_1} \frac{\sigma_1^2}{\sqrt{2 P^*}} = 0, \tag{A13}
\]

where (j) follows from

\[
\epsilon'_{2} = \frac{V_2 + \eta_{1,2}}{\sqrt{2 P^*}} = \frac{\sqrt{(1 - \rho^2) P_2}}{\sigma_1^2} \eta_{1,1} + \eta_{1,2}, \tag{A14}
\]
\[ \epsilon_2 = \epsilon_1 - \frac{E[Y_2' \epsilon_1]}{E[Y_2']} \]
\[ = \frac{\eta_{1,1}}{\sqrt{12(1-\rho^2)P_2}} - \frac{E[(\sqrt{\frac{(1-\rho^2)P_2}{\sigma_1^2}} \eta_{1,1} + \eta_{1,2})]}{E[(\sqrt{\frac{(1-\rho^2)P_2}{\sigma_1^2}} \eta_{1,1} + \eta_{1,2})]^2} (\sqrt{\frac{(1-\rho^2)P_2}{\sigma_1^2}} \eta_{1,1} + \eta_{1,2}) \]
\[ = \frac{\sigma_1^2}{\sqrt{12(1-\rho^2)P_2r^2}} \eta_{1,1} - \frac{\sigma_1}{\sqrt{12r^2}} \eta_{1,2}, \] (A15)

\[ \alpha_2 = \frac{\sigma_1^4}{12(1-\rho^2)P_2r^2}, \] (A16)

\[ r = \sqrt{(1-\rho^2)P_2 + \sigma_1^2}, \] (A17)

and (k) follows from \( E[\eta_{1,3}\eta_{1,1}] = E[\eta_{1,3}\eta_{1,2}] = E[\eta_{1,1}\eta_{1,2}] = 0. \) Now substituting (A13) into (A11), we can conclude that

\[ E[\epsilon_{k-1}' \eta_{1,k}] = 0 \] (A18)

for all \( 3 \leq k \leq N, \) which completes the proof.

**APPENDIX B**

**PROOF OF COROLLARY**[2]

We begin with the sum rate bound \( R_1 + R_2 - \epsilon \leq \frac{1}{N} H(W_1, W_2|Z^N) \leq \epsilon, \) which is bounded by

\[ R_1 + R_2 - \epsilon \leq \frac{1}{N} H(W_1, W_2|Z^N) \]
\[ = \frac{1}{N} (H(W_1, W_2|Z^N) - H(W_1, W_2|Z^N, Y^N) + H(W_1, W_2|Z^N, Y^N)) \]
\[ \stackrel{(b)}{=} \frac{1}{N} (I(W_1, W_2; Y^N|Z^N) + \delta(\epsilon)) \]
\[ \leq \frac{1}{N} (I(X_1^N, X_2^N; Y^N|Z^N) + \delta(\epsilon)) \]
\[ \stackrel{(c)}{=} \frac{1}{N} (I(X_1^N, X_2^N; Y^N) - I(X_1^N, X_2^N; Z^N) + \delta(\epsilon)) \]
\[ = \frac{1}{N} \sum_{i=1}^{N} (H(Y_i|Y^{i-1}) - H(Y_i|X_{1,i}, X_{2,i}) - H(Z_i|Z^{i-1}) + H(Z_i|X_{1,i}, X_{2,i})) + \frac{\delta(\epsilon)}{N} \]
\[ \leq \frac{1}{N} \sum_{i=1}^{N} (H(Y_i|Y^{i-1}, Z^{i-1}) - H(Y_i|X_{1,i}, X_{2,i}) - H(Z_i|Z^{i-1}) + H(Z_i|X_{1,i}, X_{2,i})) + \frac{\delta(\epsilon)}{N} \]

for all \( 3 \leq k \leq N, \) which completes the proof.
\[ (f) \leq \frac{1}{N} \sum_{i=1}^{N} (H(Y_i) - H(Y_i|X_{1,i}, X_{2,i}) - H(Z_i) + H(Z_i|X_{1,i}, X_{2,i})) + \frac{\delta(\epsilon)}{N} \]

\[ = \frac{1}{N} \sum_{i=1}^{N} (I(X_{1,i}, X_{2,i}; Y_i) - I(X_{1,i}, X_{2,i}; Z_i)) + \frac{\delta(\epsilon)}{N}, \quad (A19) \]

where (b) follows from Fano’s inequality and \( P_\epsilon \leq \epsilon \), (c) follows from \( H(W_1, W_2|X_1^N, X_2^N) = 0 \), (d) follows from \( (X_i^N, X_i^N) \rightarrow Y^N \rightarrow Z^N \), (e) follows from \( Y_i \rightarrow Y_{i-1} \rightarrow Z_{i-1} \), and (f) follows from \( Z_{i-1} \rightarrow Y_i \rightarrow Z_i \), which indicates that \( I(Z_i; Z_{i-1}) \leq I(Y_i; Z_{i-1}) \), i.e., \( H(Y_i|Z_{i-1}) - H(Z_i|Z_{i-1}) \leq H(Y_i) - H(Z_i) \).

Then substituting \( Y_i = X_{1,i} + X_{2,i} + \eta_{1,i} \) and \( Z_i = Y_i + \eta_{2,i} \) into (A19), we have

\[ R_1 + R_2 - \epsilon \leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{N} \sum_{i=1}^{N} h(Y_i|X_{1,i}, X_{2,i}) - \frac{1}{N} \sum_{i=1}^{N} h(Z_i) + \frac{1}{N} \sum_{i=1}^{N} h(Z_i|X_{1,i}, X_{2,i}) + \frac{\delta(\epsilon)}{N} \]

\[ = \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{N} \sum_{i=1}^{N} h(\eta_{1,i}) - \frac{1}{N} \sum_{i=1}^{N} h(Z_i) + \frac{1}{N} \sum_{i=1}^{N} h(\eta_{1,i} + \eta_{2,i}) + \frac{\delta(\epsilon)}{N} \]

\[ \leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log 2\pi e\sigma_1^2 - \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} \log(2^{2h(\eta_{1,i})} + 2^{2h(\eta_{2,i})}) + \frac{1}{2} \log 2\pi e(\sigma_1^2 + \sigma_2^2) + \frac{\delta(\epsilon)}{N} \]

\[ \leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log 2\pi e\sigma_1^2 - \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} \log(2^{2h(\eta_{1,i})} + 2\pi e\sigma_2^2) + \frac{1}{2} \log 2\pi e(\sigma_1^2 + \sigma_2^2) + \frac{\delta(\epsilon)}{N} \]

\[ \leq \frac{1}{2} \log 2\pi e(P_1 + P_2 + 2\sqrt{P_1 P_2 \rho} + \sigma_1^2) - \frac{1}{2} \log 2\pi e(\sigma_1^2 + \sigma_2^2) + \frac{\delta(\epsilon)}{N} \]

\[ \leq \frac{1}{2} \log(1 + \frac{P_1}{\sigma_1^2} + \frac{P_2}{\sigma_2^2} + \frac{2\sqrt{P_1 P_2 \rho}}{\sigma_1^2 + \sigma_2^2}) \]

where (a) follows from \( \eta_{1,i} \) and \( \eta_{2,i} \) are independent of \( X_{1,i} \) and \( X_{2,i} \), (b) follows from the entropy power inequality, (c) follows from the fact that \( \log(2^x + c) \) is a convex function and Jensen’s inequality, and (d) follows from \( \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log(2^{2h(\eta_{1,i})} + 2\pi e\sigma_2^2) \) is increasing while \( \frac{1}{N} \sum_{i=1}^{N} h(Y_i) \) is increasing and

\[ \frac{1}{N} \sum_{i=1}^{N} h(Y_i) \]

\[ \leq \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} \log 2\pi e(P_{1,i} + P_{2,i} + 2E[X_{1,i}X_{2,i}] + \sigma_1^2) \]

\[ \leq \frac{1}{2} \log 2\pi e(\frac{1}{N} \sum_{i=1}^{N} (P_{1,i} + P_{2,i} + 2E[X_{1,i}X_{2,i}] + \sigma_1^2)) \equiv \frac{1}{2} \log 2\pi e(P_1 + P_2 + 2\sqrt{P_1 P_2 \rho} + \sigma_1^2), \]
where (e) follows from the definitions

\[ E[X_{1,i}^2] = P_{1,i}, \quad E[X_{2,i}^2] = P_{2,i}, \quad P_1 = \frac{1}{N} \sum_{i=1}^{N} P_{1,i}, \quad P_2 = \frac{1}{N} \sum_{i=1}^{N} P_{2,i}, \quad \rho = \frac{1}{N} \sum_{i=1}^{N} E[X_{1,i}X_{2,i}]. \]

(A22)

Letting \( \epsilon \to 0 \), \( R_1 + R_2 \leq \frac{1}{2} \log(1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2 \rho}}{\epsilon}) \) is proved.

Now it remains to show that \( R_2 \leq \frac{1}{2} \log(1 + \frac{(1-\rho^2)P_2}{\epsilon}) \), and the proof is exactly in the same fashion as that in [5, pp. 627-628]. Hence we omit the proof here. The proof of Corollary 2 is completed.

APPENDIX C

PROOF OF COROLLARY 5

We begin with the sum rate bound \( R_1 + R_2 - \frac{1}{N} H(W_1, W_2 | Z^N) \leq \epsilon \), which can be bounded by

\[
R_1 + R_2 - \epsilon \leq \frac{1}{N} H(W_1, W_2 | Z^N) \\
\overset{(a)}{\leq} \frac{1}{N} (I(W_1, W_2; Y^N | Z^N) + \delta(\epsilon)) \\
\overset{(b)}{\leq} \frac{1}{N} (I(X_1^N, X_2^N, S^N; Y^N | Z^N) + \delta(\epsilon)) \\
\overset{(c)}{=} \frac{1}{N} (I(X_1^N, X_2^N, S^N; Y^N) - I(X_1^N, X_2^N, S^N; Z^N) + \delta(\epsilon)) \\
= \frac{1}{N} \sum_{i=1}^{N} (H(Y_i | Y^{i-1}) - H(Y_i | X_{1,i}, X_{2,i}, S_i) - H(Z_i | Z^{i-1}) + H(Z_i | X_{1,i}, X_{2,i}, S_i)) + \frac{\delta(\epsilon)}{N} \\
\overset{(d)}{=} \frac{1}{N} \sum_{i=1}^{N} (H(Y_i | Y^{i-1}, Z^{i-1}) - H(Y_i | X_{1,i}, X_{2,i}, S_i) - H(Z_i | Z^{i-1}) + H(Z_i | X_{1,i}, X_{2,i}, S_i)) + \frac{\delta(\epsilon)}{N} \\
\leq \frac{1}{N} \sum_{i=1}^{N} (H(Y_i | Z^{i-1}) - H(Y_i | X_{1,i}, X_{2,i}, S_i) - H(Z_i | Z^{i-1}) + H(Z_i | X_{1,i}, X_{2,i}, S_i)) + \frac{\delta(\epsilon)}{N} \\
\overset{(e)}{=} \frac{1}{N} \sum_{i=1}^{N} (H(Y_i) - H(Y_i | X_{1,i}, X_{2,i}, S_i) - H(Z_i) + H(Z_i | X_{1,i}, X_{2,i}, S_i)) + \frac{\delta(\epsilon)}{N} \\
= \frac{1}{N} \sum_{i=1}^{N} (I(X_{1,i}, X_{2,i}, S_i; Y_i) - I(X_{1,i}, X_{2,i}, S_i; Z_i)) + \frac{\delta(\epsilon)}{N}, \tag{A23}
\]

where (a) follows from Fano’s inequality and \( P_e \leq \epsilon \), (b) follows from \( H(W_1, W_2 | X_1^N, X_2^N) = 0 \), (c) follows from \( (X_1^N, X_2^N, S^N) \to Y^N \to Z^N \), (d) follows from \( Y_i \to Y^{i-1} \to Z^{i-1} \),
and (e) follows from $Z^{i-1} \to Y_i \to Z_i$, which indicates that $I(Z_i; Z^{i-1}) \leq I(Y_i; Z^{i-1})$, i.e., $H(Y_i|Z^{i-1}) - H(Z_i|Z^{i-1}) \leq H(Y_i) - H(Z_i)$.

Then substituting $Y_i = X_{1,i} + X_{2,i} + S_i + \eta_{1,i}$ and $Z_i = Y_i + \eta_{2,i}$ into (A23), we have

\[
R_1 + R_2 - \epsilon \\
\leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{N} \sum_{i=1}^{N} h(Y_i|X_{1,i}, X_{2,i}, S_i) - \frac{1}{N} \sum_{i=1}^{N} h(Z_i) + \frac{1}{N} \sum_{i=1}^{N} h(Z_i|X_{1,i}, X_{2,i}, S_i) + \frac{\delta(\epsilon)}{N} \tag{f}
\]

\[
\leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{N} \sum_{i=1}^{N} h(\eta_{1,i}) - \frac{1}{N} \sum_{i=1}^{N} h(Z_i) + \frac{1}{N} \sum_{i=1}^{N} h(\eta_{1,i} + \eta_{2,i}) + \frac{\delta(\epsilon)}{N} \tag{g}
\]

\[
\leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log 2 \pi e \sigma_1^2 - \frac{1}{N} \sum_{i=1}^{N} \left( 2 \log(2^{2h(Y_i)} + 2^{2h(\eta_{2,i})}) + \frac{1}{2} \log 2 \pi e (\sigma_1^2 + \sigma_2^2) + \frac{\delta(\epsilon)}{N} \right) \tag{h}
\]

\[
\leq \frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log 2 \pi e \sigma_1^2 - \frac{1}{2} \log(2^{2h(Y_i)} + 2^{2h(\eta_{2,i})}) + \frac{1}{2} \log 2 \pi e (\sigma_1^2 + \sigma_2^2) + \frac{\delta(\epsilon)}{N}
\]

\[
\leq \frac{1}{2} \log 2 \pi e(P_1 + P_2 + 2\sqrt{P_1P_2} + \sigma_1^2) - \frac{1}{2} \log 2 \pi e \sigma_1^2 - \frac{1}{2} \log 2 \pi e(P_1 + P_2 + 2\sqrt{P_1P_2} + \sigma_1^2 + \sigma_2^2)
\]

\[
+ \frac{1}{2} \log 2 \pi e(\sigma_1^2 + \sigma_2^2) + \frac{\delta(\epsilon)}{N} \tag{i}
\]

\[
\leq \frac{1}{2} \log 1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2}}{\sigma_1^2} - \frac{1}{2} \log 1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2}}{\sigma_1^2 + \sigma_2^2} + \frac{\delta(\epsilon)}{N}, \tag{A24}
\]

where (f) follows from $\eta_{1,i}$ and $\eta_{2,i}$ are independent of $X_{1,i}$ and $X_{2,i}$, (g) follows from the entropy power inequality, (h) follows from the fact that $\log(2^x + c)$ is a convex function and Jensen’s inequality, and (i) follows from $\frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log(2^{2h(Y_i)} + 2^{2h(\eta_{2,i})})$ is increasing while $\frac{1}{N} \sum_{i=1}^{N} h(Y_i)$ is increasing and

\[
\frac{1}{N} \sum_{i=1}^{N} h(Y_i) - \frac{1}{2} \log 2 \pi e(P_1 + P_2 + Q + 2E[X_{1,i}X_{2,i}] + 2E[X_{1,i}S_i] + 2E[X_{2,i}S_i] + \sigma_1^2)
\]

\[
\leq \frac{1}{2} \log 2 \pi e(P_1 + P_2 + 2\sqrt{P_1P_2} + 2\sqrt{P_1Q} + 2\sqrt{P_2} + \sigma_1^2), \tag{A25}
\]

where (j) follows from the definitions

\[
E[X_{1,i}^2] = P_{1,i}, \ E[X_{2,i}^2] = P_{2,i}, \ P_1 = \frac{1}{N} \sum_{i=1}^{N} P_{1,i}, \ P_2 = \frac{1}{N} \sum_{i=1}^{N} P_{2,i},
\]
\[ \rho_{12} = \frac{1}{N} \sum_{i=1}^{N} E[X_{1,i}X_{2,i} \sqrt{P_1P_2}, \quad \rho_{1s} = \frac{1}{N} \sum_{i=1}^{N} E[X_{1,i}S_i \sqrt{P_1Q}, \quad \rho_{2s} = \frac{1}{N} \sum_{i=1}^{N} E[X_{2,i}S_i \sqrt{P_2Q}. \]

\begin{equation}
\tag{A26}
\end{equation}

Letting \( \epsilon \to 0 \), the sum rate bound of Theorem 5 is proved.

Now it remains to show the upper bound on the individual rate \( R_2 \), see the details below. First, note that

\[ R_2 - \epsilon \leq \frac{1}{N} H(W_2|Z_1^N) \leq \frac{1}{N} H(W_2) = \frac{1}{N} H(W_2|X_1^N, S^N) \]

\[ \leq \frac{1}{N} (I(W_2; Y_1^N|X_1^N, S^N) + \delta(\epsilon)) \]

\[ \leq \frac{1}{N} (I(X_2^N; Y_1^N|X_1^N, S^N) + \delta(\epsilon)) \]

\[ \leq \frac{1}{N} N \sum_{i=1}^{N} (H(Y_i|X_1,i, S_i) - H(Y_i|X_1,i, X_2,i, S_i)) + \delta(\epsilon) \]

\begin{equation}
\tag{A27}
\end{equation}

where (k) follows from \( W_2 \) is independent of \( X_1^N \) and \( S^N \), (l) follows from Fano’s inequality and \( P_e \leq \epsilon \), and (m) follows from \( H(W_2|X_2^N) = 0 \).

Then substituting \( Y_i = X_1,i + X_2,i + S_i + \eta_{1,i} \) into (A27), and using the fact that \( \eta_{1,i} \) is independent of \( X_1,i, X_2,i \) and \( S_i \), we have

\[ R_2 - \epsilon \leq \frac{1}{N} \sum_{i=1}^{N} (h(X_2,i + \eta_{1,i}|X_1,i, S_i) - h(\eta_{1,i})) + \delta(\epsilon) \]

\[ \leq \frac{1}{N} \sum_{i=1}^{N} (\frac{1}{2} \log 2\pi e (Var(X_2,i|X_1,i, S_i) + \sigma_i^2) - \frac{1}{2} \log 2\pi e \sigma_i^2) + \delta(\epsilon) \]

\[ \leq \frac{1}{N} \sum_{i=1}^{N} (\frac{1}{2} \log 2\pi e (Var(X_2,i - a_iX_1,i - b_iS_i + \sigma_i^2) - \frac{1}{2} \log 2\pi e \sigma_i^2) + \delta(\epsilon) \]

\[ = \frac{1}{N} \sum_{i=1}^{N} (\frac{1}{2} \log 2\pi e (P_{2,i} + a_i^2P_{1,i} + b_i^2Q - 2a_iE[X_{1,i}X_{2,i}] - 2b_iE[X_{2,i}S_i] - 2a_i b_i E[X_{1,i}S_i] + \sigma_i^2) \]

\[ - \frac{1}{2} \log 2\pi e \sigma_i^2) + \delta(\epsilon) \]

\begin{equation}
\tag{A28}
\end{equation}

where (n) follows from \( \eta_{1,i} \) is independent of \( X_1,i, X_2,i \) and \( S_i \), and (o) follows from \( Var(X_2,i|X_1,i, S_i) \) is no greater than the variance of the difference between \( X_2,i \) and its linear MMSE estimation \( \hat{X}_{2,i} = a_iX_{1,i} + b_iS_i \), and

\[ a_i = \frac{E[X_{1,i}X_{2,i}]Q - E[X_{1,i}S_i]E[X_{2,i}S_i]}{P_{1,i}Q - (E[X_{1,i}S_i])^2}, \quad b_i = \frac{E[X_{2,i}S_i]P_{1,i} - E[X_{1,i}S_i]E[X_{1,i}X_{2,i}]}{P_{1,i}Q - (E[X_{1,i}S_i])^2}. \]

\begin{equation}
\tag{A29}
\end{equation}
Observing that in (A28), we can readily check that the logarithm function is concave in $P_{1,i}$, $P_{2,i}$, $E[X_{1,i}X_{2,i}]$, $E[X_{2,i}S_i]$ and $E[X_{1,i}S_i]$ by evaluating the corresponding Hessian matrix. Hence applying Jensen’s inequality, using (A26), defining

$$a = \sqrt{\frac{P_2}{P_1} \rho_{12} - \rho_{1s} \rho_{2s}}, \quad b = \sqrt{\frac{P_2}{Q} \rho_{2s} - \rho_{12} \rho_{1s}},$$

(A30)

and letting $\epsilon \to 0$, the bound on the individual rate $R_2$ is proved.

The proof of Corollary 5 is completed.

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