Development of a mathematical model of functioning system communications spacecraft

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Abstract: The paper discusses a mathematical model of the functioning of communication spacecraft, using systems of differential equations for translational and rotational motion, as well as the process of distributing problems in a constellation of three satellites. The model is implemented by means of the python 3.6 language and the computational method library numpy1.19. A series of computational experiments was carried out in order to estimate the energy costs for the operation of grouping with various orbital parameters and external impact models. The presented results of the experiments suggest the possibility of increasing the life of spacecraft by improving the operating system.

Keywords: communication spacecraft, mathematical model, control system

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the spacecraft. Depending on the completeness of the factors taken into account, mathematical models of various levels of complexity are used in the research. In practice, under specified constraints, deterministic or stochastic linear models are usually used in the state area.

The state area is formed by a set of vectors with a minimum number of coordinates containing all the necessary information about the motion of the spacecraft. In the state area given in this way, linear mathematical models have a formal representation of the form [1-3]:

\[
\dot{x} = A(t)x + B(t)u(t),
\]

where \(x^T = (x_1, x_2, ..., x_i)\) - state vector, \(T\) - transpose symbol; \(A(t)\) - matrix of dynamic model features; \(B(t)\) - matrix of intense of control action; \(u^T = (u_1, u_2, ..., u)\) - control action.

The behavior of the system (1) over a period of time \([0, T]\) is determined by setting the initial conditions \(x(t = 0)\) and described by the expression [1,2]:

\[
0 = N(t, 0) x(0) + \int_0^T N(t, \tau) B(\tau) u(\tau) d\tau,
\]

where \(N(t, \tau) = \Phi(\cdot)\Phi^{-1}((\cdot))\) - matrix of initial system influence; \(\Phi(\cdot)\) - fundamental matrix of homogeneous system \(\dot{x} = A(t)x(t)\).

It is not possible to find an analytical solution in a general form, so there is a need to present mathematical models of spacecraft motion in finite differences. In discrete form, provided that the control action \(u(t)\) retains its value unchanged on the segment \([t_n, t_{n+1}]\), the expression (2) will take the form:

\[
x(t_{n+1}) = N(t_{n+1}, t_n)x(t_n) + \int_{t_n}^{t_{n+1}} N(t_{n+1}, \tau) B(\tau) u(\tau) d\tau.
\]

We introduce the following notation:

\[
x(t_{n+1}) = x_{n+1},
\]

\[
u(t_{n+1}) = u_{n+1},
\]

\[
\int_{t_n}^{t_{n+1}} N(t_{n+1}, \tau) B(\tau) d\tau = \Gamma(t_n),
\]

\[
N(t_{n+1}, t_n) = N_{n+1,n}.
\]

Taking into account the entered notation, the expression (3) will take the form [3]:

\[
x_{n+1} = N_{n+1,n}x_n + \Gamma_n u_n.
\]

Researching the motion of a spacecraft in real operation, the assumption of the determinism of external influences and functional parameters is acceptable only for relatively short time sampling intervals.

The works of L.S. Pontryagin and his students are devoted to the description of the properties of dynamical systems subject to random perturbations. The further development of ideas about the management of complex technical systems was reflected in the use of artificial intelligence based on information system redundancy [4]. The problem of optimal control in the information space is solved in [6,7].

The main problem of optimal control of the spacecraft is formulated as follows: among all the permissible controls that transfer the image point in the phase space of the system from the initial position to the final one, it is necessary to find such a control at which the minimum of the following functional is achieved [2].

\[
I'(x,u) = \min \sum_{n=0}^{N-1} G_n(x_n, u_n) + \varphi(x_N),
\]

where \(n\) - number of discrete time step; \(G\) - the functional of the costs of managing the spacecraft; in the special case, when the optimality problem is considered for energy consumption for control, the integrand function \(G(x,u) \equiv |u|; \varphi(x_N)\) - a term that takes into account the ability of the spacecraft to perform the task according to its intended purpose in the final state.

The minimized functional \(I'()\) is an inverse control efficiency. This mapping cannot always be defined as unambiguous, so a more general type of functionality is used.

The solution of the problem by the method of dynamic programming allows us to reduce the problem to a sequential minimization of the
function $r$ of variables (the dimension of the control vector $u$).

Assume that all the values of the optimal control $u_n$, except the last one, are found and the system is in the state $x_{N-1}$. According to the optimality principle, the control $u_{N-1}$ must also be optimal.

This control should provide a minimum of functionality, which, taking into account the restrictions imposed by the requirements for ensuring a communication session on a given section of the spacecraft trajectory, for a section of the trajectory ($N - 1$) has the form [2]:

$$I_{N-1}(x_{N-1}, u_{N-1}) = G_{N-1}[x_{N-1}, u_{N-1}] + \varphi[x_N].$$

Define

$$S_{N-1}(x_{N-1}) = \min_{u_n \in U} I_{N-1}(x_{N-1}, u_{N-1}).$$

Then the recurrent formula for determining the minimum value of the functional $I_{K-k}(x, u)$ and the corresponding optimal control $\tilde{u}(K-k)$ at the $k$ step will have the form:

$$S_{k-1}(x_{k-1}) = \min_{u_{k-1} \in U} \{G_{K-k}(x_{k-1}, u_{K-k}) + S_{K-k-1}(x_{K-k})\}.$$  

The optimal control $\tilde{u}(K-k)$ is defined as a function of the coordinates of the state of the system $\tilde{u}(K-k) = \tilde{u}(x_{k-1})$. Calculating the values of the function $S_{k-1}$ sequentially, we obtain the minimum value of the functional for the entire trajectory. At the same time, the optimal control is determined in the function of the coordinates of the system, i.e. the problem of synthesizing the optimal controller is solved. The specified procedure is performed using a computer. Thus, the application of the optimality principle makes it possible to significantly simplify calculations in comparison with the analytical method of solving the problem for a conditional extremum.

2. THE MODELING METHOD
Consider an open discrete spacecraft control system (Fig. 1). The input of the control device receives a setting action $x_n^*$, which determines what the desired state of the object should be. The control device generates an $r$-dimensional control action $u_n$ on the control object (spacecraft). The motion of the spacecraft is influenced by external disturbing factors $w_n$. The value of the state vector of the spacecraft $x_n$ is determined using measuring devices with an error $d_n$ (measurement noise). The system is considered fully observable if the values of all elements of the state vector $x$ can be found from the measurement vector $z$.

The state vector of the spacecraft is characterized by the coordinates [1, 3]:

$$x = x_c, y_c, z_c, v_x, v_y, v_z, m, \psi, \theta, \phi, \omega_x, \omega_y, \omega_z,$$

where $x_c, y_c, z_c$ – coordinates of the center of mass of the spacecraft; $v_x, v_y, v_z$ – projections of the velocity vector on the coordinate axis; $m$ – spacecraft mass; $\psi, \theta, \phi$ – projections of the angular velocity of the spacecraft; $I_{\psi}, I_{\theta}, I_{\phi}$ – projections of the inertia tensor of the spacecraft.

The vector of external disturbances consists of the effects of the unstable gravitational field of the Earth and other cosmic bodies [1, 3]:

$$w = \{F_{Gx}, F_{Gy}, F_{Gz}, \delta F_x, \delta F_y, \delta F_z, \delta M_{\psi}, \delta M_{\theta}, \delta M_{\phi}\},$$

where $F_{Gx}, F_{Gy}, F_{Gz}$ – the deterministic component of the action of the gravitational attraction of the Earth; $\delta F_x, \delta F_y, \delta F_z$ – random impact of linear forces of cosmic bodies; $\delta M_{\psi}, \delta M_{\theta}, \delta M_{\phi}$ – random moments of rotation.

The translational and rotational motion of the spacecraft is described by a system of differential equations (SDE) [1]:

$$\dot{x} = x_0 + v_x I, \quad \dot{y} = y_0 + v_y I, \quad \dot{z} = z_0 + v_z I,$$
\[ \dot{v}_x = v_{x0} + \frac{F_{Gx} + \delta F_x}{m} \]
\[ \dot{v}_y = v_{y0} + \frac{F_{Gy} + \delta F_y}{m} \]
\[ \dot{v}_z = v_{z0} + \frac{F_{Gz} + \delta F_z}{m} \]
\[ \psi = \psi_0 + \omega_{\theta} t, \]
\[ \dot{\theta} = \theta_0 + \omega_{\theta} t, \]
\[ \dot{\gamma} = \gamma_0 + \omega_{\gamma} t, \]
\[ \dot{\omega}_v = \omega_{v} + \frac{M_v}{I_v} t, \]
\[ \dot{\omega}_\theta = \omega_{\theta} + \frac{M_{\theta}}{I_\theta} t, \]
\[ \dot{\omega}_\gamma = \omega_{\gamma} + \frac{M_{\gamma}}{I_\gamma} t, \]

where \( x_0, y_0, z_0 \) – coordinates of the center of mass of the spacecraft at the initial moment of time, \( \dot{x}, \dot{y}, \dot{z} \) – coordinates of the center of mass of the spacecraft at a given time, \( \dot{v}_x, \dot{v}_y, \dot{v}_z \) – components of the linear velocity of the spacecraft at a given time, \( \psi_0, \theta_0, \gamma_0 \) – angular position of the spacecraft at the initial moment of time, \( \psi, \dot{\theta}, \dot{\gamma} \) – the angular position of the spacecraft at a given time, \( \dot{\omega}_v, \dot{\omega}_\theta, \dot{\omega}_\gamma \) – the angular velocity of the spacecraft at a given time.

The system is solved using the two-stage Runge-Kutta method (Hoyne method) according to the predictor-corrector scheme [5] of the following type:

**Predictor:**
\[ \hat{b}_i = b_{i-1} + (a_i - a_{i-1}) f(a_{i-1}, b_{i-1}) \]

**Corrector:**
\[ b_i = b_{i-1} + (a_i - a_{i-1}) f(a_{i-1}, b_{i-1}) + f(a_i, \hat{b}_i) \]

Pulsed two-parameter three-component correction is carried out in order to maintain the parameters of the spacecraft’s orbit and maintain orientation during a communication session. The model uses the hypothesis of an instantaneous change in linear and angular velocities, while taking into account only the energy costs of correction:
\[ E_x = m_{KI} \frac{\Delta V^2}{2}, \]
\[ E_w = I_\omega \frac{\Delta \omega^2}{2}. \]

To determine the feed moments and the magnitude of the correction pulses, a quasi-optimal control algorithm was used, which minimizes energy consumption for maintaining the specified orbit parameters.

\[ i_e = 0.02.00^{+0.06.00} \]
\[ \Omega_e = 0.00.00 \]
\[ \omega_e = 0.00.00 \]
\[ e_c = 0.01.16^{+0.0035} \]
\[ p_c = 42050 \]
\[ \nu_e = 30.00.00^{+0.06.00}. \]

An object-oriented model with three entities has been developed for the presented mathematical model: SpaceShip – spacecraft; Earth – planet Earth; Environment – external environment.

The simulation model is implemented by means of the **Python 3.6** language using the **numpy v1.19** library, the visualization is implemented by means of the **matplotlib v3.3.1** library.

### 3. THE MODELING RESULTS

An experiment was conducted with a grouping of three spacecraft that ensure the fulfillment of the target task within one day. The obtained spacecraft trajectories without modeling external influences are verified with the specified orbit characteristics and with a given relative error not exceeding \( 10^{-4} \) for each coordinate.

Taking into account the distribution of the task execution time by purpose (providing signal retransmission) between the three spacecraft, the motion simulation was carried out taking into account the disturbing effects of the external environment (Fig. 2). During the experiment, estimates of energy consumption
for maintaining the specified characteristics of the orbits for spacecraft moving along different trajectories were obtained (Fig. 3).

Based on the empirically obtained average energy consumption of 0.1 MJ/h, an experiment was conducted in which the average operating time of the spacecraft was studied depending on the nature and magnitude of the signal-to-noise ratio in the measuring channel. The results are shown in the graph (Fig. 5).

4. CONCLUSION

Thus, the developed model allows us to study the energy efficiency of control algorithms in various modes of operation of both a single spacecraft and a group of spacecrafts, taking into account the restrictions imposed by the implementation of the communication task. The conducted experiments indicate the possibility of increasing the active life of spacecraft by optimizing the operation system.

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