Flux qubit as a sensor for a magnetometer with quantum limited sensitivity

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We propose to use the quantum properties of a superconducting flux qubit in the construction of a magnetometer with quantum limited sensitivity. The main advantage of a flux qubit is that its noise is rather low, and its transfer functions relative to the measured flux can be made to be about 10mV/Φ₀, which is an order of magnitude more than the best value for a conventional SQUID magnetometer. We analyze here the voltage-to-flux, the phase-to-flux transfer functions and the main noise sources. We show that the experimental characteristics of a flux qubit, obtained in recent experiments, allow the use of a flux qubit as magnetometer with energy resolution close to the Planck constant.

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Josephson-junction qubits are known to be candidates for solid-state quantum computing circuits. However, owing to their unique quantum properties these devices undoubtedly can be used as sensitive detectors of different physical quantities, such as quantum environmental noise or low frequency fluctuations of the junction critical current. Here we propose to use a Josephson-junction flux qubit as a sensitive detector of magnetic flux. We show that the present state-of-art allows one to obtain the energy sensitivity of such a detector in the order of the Planck constant.

A flux qubit consists of three Josephson junctions in a loop with very small inductance L, typically in the pH range. This ensures an effective decoupling from the environment. Two junctions have an equal critical current I_c and (effective) capacitance C, while those of the third junction are slightly smaller: αI_c and αC, with 0.5 < α < 1. At sufficiently low temperatures (typically T ≈ (10 ~ 30) mK) when Φ_x, the external flux applied to the qubit loop, is in the close vicinity of Φ₀/2 (Φ₀ = h/2e is the flux quantum, h is the Planck constant) the system has two low-lying quantum states E_- and E_. The energy gap of the flux qubits, Δ/ħ = (E_+ − E_-)/ħ is of the order of several GHz. Below we assume k_BT << Δ (k_B is the Boltzmann constant), so that the qubit is definitely in its ground state E_-.

For experimental characterization the flux qubit is inductively coupled through a mutual inductance M to an LC tank circuit with known inductance L_T, capacitance C_T, and quality Q (Fig. 1).

The resonant characteristics of the tank circuit (frequency, phase shift, etc.) are sensitive to the qubit inductance and therefore to the external flux Φ_x.

It was shown in that the amplitude v and the phase χ of the output signal V(t) = v cos(ωt + χ) are coupled by the equations:

\[ v^2 \left( 1 + 4Q^2\xi^2(f_x) \right) = I_0^2\omega^2L_T^2Q^2 \]

\[ \tan \chi = 2Q\xi(f_x), \]

where I_0 is the amplitude of the driving current (I_0(t) = I_0 cosωt), and ω and ω_T are a driving frequency and the resonant frequency of the tank circuit, respectively.

It is worth noting that the the scheme in Fig. 1 and Eqs. (1), (2) are similar to those for a conventional RF SQUID. The only difference is in the expression for a flux-dependent frequency detuning \( \xi(f_x) \). This depends on the qubit parameters as:

\[ \xi(f_x) = \xi_0 - k_B^2L_T^2\frac{L}{\Delta} \left( \frac{\lambda}{2\pi} \right)^2 F(f_x), \]

\[ \xi_0 = (\omega_T - \omega)/\omega_T, \]

and

\[ F(f_x) = \frac{1}{\pi} \int_0^{2\pi} d\phi \frac{\cos^2 \phi}{\left[ 1 + \eta^2 (f_x + \gamma \sin \phi)^2 \right]^{3/2}}, \]

with η = 2E_Jλ/Δ and γ = MI_0Q/Φ₀. The expression for λ, which depends on α, I_c and C, and is given in the text. Therefore, the main effect of the qubit-tank interaction is a shift of the tank resonance. This results in a dip in the voltage-to-flux and phase-to-flux characteristics which have been confirmed by experiment.

Theoretical phase-to-flux \( \chi(f_x) \) (PFC) and voltage-to-flux (VFC) \( v(f_x) \) dependencies at resonance \( \omega = \omega_T \), are shown in Fig. 2 for three values of the amplitude of the bias current I_0. The graphs have been calculated from...
The output signal of the measured flux. In principle two modes of detection are possible: voltage mode, where \( \delta V = V_\Phi \delta \Phi_X \), and the phase mode, where \( \delta V = \chi_\phi \delta \Phi_X \). The qubit transfer functions \( \chi_\Phi = v \partial \chi / \partial \Phi_X \) and \( V_\Phi = \partial \chi / \partial \Phi_X \) are shown in Fig. 4 for the same qubit-tank parameters as those used in Fig. 3. It is seen that qubit transfer functions can exceed 10 mV/\( \Phi_0 \). This value should be compared with 1 mV/\( \Phi_0 \), the best value obtained for a DC SQUID with additional positive feedback.12

The flux and energy sensitivity depend on the main noise sources, which come from the low frequency fluctuations of the junction critical current, \( S_{1C} \), and from the voltage noise, \( S_V \) and the current noise, \( S_I \) of the preamplifier, where \( S_{1C} \), \( S_V \) and \( S_I \) are the corresponding spectral densities. The fluctuations of \( I_C \) result in the fluctuating flux in the qubit loop \( S_{\Phi,1C}^{1/2} = L S_{1C}^{1/2} \). For \( I_C = 400 \) nA, junction area \( A = 0.12 \mu \text{m}^2 \), \( T = 0.1 \) K we estimate for three-junction flux qubit (see Eq.18 in12) \( S_{\Phi,1C}^{1/2} \approx 2 \times 10^{-8} \Phi_0/\text{Hz}^{1/2} \) at 1 Hz. We will see that this value is almost one order of magnitude smaller than the noise from a preamplifier. Therefore, the self noise of the qubit can be neglected. The contribution of the voltage noise of the preamplifier to the flux resolution referred to the input is \( S_V^{1/2} = S_V/V_\Phi^2 \) or \( S_\Phi^{1/2} = S_V/\chi_\Phi^2 \), depending on the detection mode. The current noise of preamplifier which is related to its noise temperature \( T_N \), \( S_I = 4k_B T_N/R_T \), \( (R_T = \omega T Q) \), contributes via two mechanisms. The first one comes from magnetic coupling between the tank inductance and the inductance of the qubit loop \( S_{\Phi}^{1/2} = M^2 Q^2 S_I \). This contribution cannot be separated from the measured flux. The second mechanism contributes through a voltage noise induced by the current noise of the preamplifier across the dynamic resistance of the tank \( S_\Phi^D = R_D^2 S_I/V_\Phi^2 \) or \( S_\Phi^D = R_D^2 S_I/\chi_\Phi^2 \), where \( R_D = \partial \chi / \partial I_0 \). By combining these three mechanisms we obtain for the flux sensitivity:

\[
S_\Phi = M^2 Q^2 S_I + S_V/V_\Phi^2 + R_D^2 S_I/V_\Phi^2 \tag{5}
\]

where \( R_D \) is approximately equal to \( R_T \), the resistance of unloaded tank. In the case of the phase mode detection we should substitute in \( \chi_\Phi \) for \( V_\Phi \).
The main contribution to the flux noise comes from the term $k^2 Q^2 S_I = k^2 Q^4 k_B T N L / \omega_T$ (Eq. 5) on the condition $\omega_T \approx \Phi_0 / h$ should hold. We also made calculations for $\omega_T = 200 \text{MHz}$ with $k=0.01$, $Q=1000$, with other parameters being unchanged. We obtain at $I_0 = 200 \text{pA}$ for the phase detection $S_{k^2}^{1/2} = 1.6 \times 10^{-7} \Phi_0 / \text{Hz}^{1/2}$, which for $L = 40 \text{pH}$ corresponds to the energy sensitivity $\varepsilon = S_{k^2} / 2L = 1.3 \times 10^{-33} \text{J}/\text{Hz} = 2h$. These values should be compared with those for conventional SQUIDs: $S_{k^2}^{1/2} \approx 10^{-6} \Phi_0 / \text{Hz}^{1/2}$, $\varepsilon \approx 10^{-32} \text{J}/\text{Hz}$. 

In summary, we have shown that a superconducting flux qubit can be developed as a sensor of magnetic flux with an energy sensitivity close to the Planck constant.

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