Comment on the $\Theta^+$-production at high energy

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We show that the cross sections of the $\Theta^+$-pentaquark production in different processes decrease with energy faster than the cross sections of production of the conventional three-quark hyperons. Therefore, the threshold region with the initial energy of a few GeV or less seems to be more favorable for the production and experimental study of $\Theta^+$-pentaquark.

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The discovery of the $\Theta^+$-pentaquark by the LEPS at SPring-8 has its subsequent confirmation in a series of other experiments performed mainly at low energy. This Rapid Communication we analyze the high-energy limit for the $\Theta^+$-production in three kinematical regions: exclusive production at large momentum transfers, exclusive production at diffractive region and the $\Theta^+$-production in inclusive processes in the fragmentation region. We show that at all cases the $\Theta^+$ production cross section is suppressed compared to the production of the "conventional" three-quark hyperons. In order to remove dimensional parameters we will consider ratios of cross sections taken at two energies: relatively small energy $s_0$ (which is the reference point) and at large energy $s$.

\[ R_Y = \frac{d\sigma_Y(s)}{d\sigma_Y(s_0)}, \quad R_\Theta = \frac{d\sigma_\Theta(s)}{d\sigma_\Theta(s_0)}, \quad R_{\Theta Y} = \frac{R_\Theta}{R_Y}. \]

**I. EXCLUSIVE $\Theta^+$-PRODUCTION AT LARGE MOMENTUM TRANSFERS**

For definiteness sake, let us consider the $\pi N \rightarrow \Theta^+ \bar{K}$ and $\pi N \rightarrow Y K$ processes. The energy dependence of the invariant amplitude of exclusive hadronic process $AB \rightarrow CD$ at large momentum transfers has the "automodel" (scale) behaviour which is defined by the dimension of the connected Born amplitudes. The corresponding diagrams for $\pi N \rightarrow Y K$ and $\pi N \rightarrow \Theta^+ \bar{K}$ reactions are shown in Figs. 1(a) and (b), respectively. The dimension of the invariant amplitude $T_{\pi N \rightarrow Y K}$ is

\[ [\text{length}]^{n_h + 4 N + 2 N_{Y} - 4} = [\text{length}]^6, \]

where $n_h$ is a minimal number of the constituent (quarks) in a hadron $h$ involved in the process and the Dirac spinor are normalized as $\bar{u}(p)\gamma_\alpha u(p) = 2p_\alpha$. This results in power decreasing of the corresponding cross sections

\[ \frac{d\sigma}{dt}(Y) \propto f_Y \left( \frac{t}{s} \right)^8, \quad (4) \]

where $s, t$ are large and the ratio $t/s$ is fixed. Analysis of high-energy processes like $\pi p \rightarrow \pi p, pp \rightarrow pN^*$ etc., at fixed values of $t/s$ shows rather good agreement between the data and power-law exponent $8$. This indicates weak energy dependence of the function $f(t/s)$ in Eq. (4) which can be considered as a constant. These predictions hold when $s$ and $t$ are much larger than the masses of the hadrons involved in the processes.

The dimension of the invariant amplitude $\pi N \rightarrow \Theta^+ \bar{K}$ shown in Fig. 1(b) is $[\text{length}]^8$ which results in

\[ \frac{d\sigma}{dt}(\Theta^+) \propto f_\Theta \left( \frac{t}{s} \right)^{10}. \]

Therefore, for the ratio $R_{\Theta Y}$, we find

\[ R_{\Theta Y} \propto \left( \frac{s_0}{s} \right)^2. \]

This estimation has rather illustrative sense because the cross sections both for $Y$ and $\Theta^+$ production decrease steeply with $s$. Nevertheless, one can see that the production rate of $\Theta^+$ is suppressed at sufficiently high $s$ as compared to that of the conventional three quark hyperon $Y$.

**II. EXCLUSIVE PRODUCTION IN DIFFRACTIVE REGION**

The dual quark diagram for the hyperon production is shown in Fig. 2. At low energy this diagram may...
be interpreted as the \( t \)-channel \( K \) and \( K^* \) meson exchange processes. At large energy they transform to the Regge trajectories where the dominant contribution comes from the lowest \( K^* \)-trajectory with the intercept \( \alpha_{K^*} \simeq 0.3 \) [4]. This results in

\[
\pi N \rightarrow K N (5q) \Theta^+.
\]

FIG. 2: The hyperon \((a)\) and \(\Theta^+ \) \((b)\) production in diffractive region.

\[
R_Y = \left( \frac{50}{s} \right)^{1.4}.
\]  

The corresponding dual diagram for the \(\Theta^+\) production is shown in Fig. 2b. Together with \(K\) and \(K^*-\)exchange processes, there is some additional suppression factor proportional to the amplitude of picking up the fast moving quark-antiquark pair in the projectile (pion) by the slowly moving quark(s) in the target (nucleon). In the non-relativistic limit, this amplitude is related to the wave function of the bound quark-antiquark pair at the large relative momentum \( q = q_1 - q_2 \). For the estimation of this effect in the relativistic case, one can use the light cone representation for the wave functions of the colliding hadrons. Neglecting the transverse momentum distributions, one can find the following expression

\[
\sqrt{\delta R_{\Theta^+}} \propto \int_0^1 dx \int_0^1 dy \varphi_\pi(x) \varphi_N(y) \delta((x + y)^2 - \Delta^2),
\]

\[
\propto \Delta^{M+N},
\]

where \( \varphi_\pi \) and \( \varphi_N \) are the light cone wave function

\[
\varphi_\pi(x) \propto x^M(1-x)^M, \quad \varphi_N(y) \propto y^N(1-y)^N,
\]

and

\[
x \simeq \frac{2p_i}{s}, \quad y \simeq \frac{2p_j}{s}, \quad \Delta \simeq \frac{\sqrt{m_q^2}}{\sqrt{s}}.
\]  

For giving an upper bound we can choose \( M = N = 1 \) which leads to

\[
R_{\Theta^+}^{\text{diff}} \leq \left( \frac{50}{s} \right)^2.
\]  

Another estimation can be performed as it is suggested that \(\Theta^+\) may be produced through the five-quark admixture in the nucleon wave function [10]. The relevant dual diagram for this process is shown in Fig. 3b. Here we have no dynamical suppression factor as discussed above, but this process is suppressed due to the small probability of the 5-quark component in a nucleon. For a numerical estimation, let us assume that the isospin and spin-parity of \(\Theta^+\) are \(I(J^P) = 0(2^+)\), and introduce the parameter \(\xi\) which is the amplitude of the 5-quark admixture in a nucleon. Then, for the \(\Theta NK\) and \(\Lambda NK\) couplings we can write the ratio

\[
\left| \frac{g_{\Theta NK}}{g_{\Lambda NK}} \right| = |\xi| \left( \frac{\Theta \mid N(5q)K}{\Lambda \mid N(3q)K} \right),
\]

and therefore,

\[
|\xi| \simeq \left| \frac{g_{\Theta NK}}{g_{\Lambda NK}} \right|.
\]

Here we have taken into account that \(I(J^P) = 0(2^+)\) and assumed \(\langle \Theta^+ \mid N(5q)K \rangle \simeq \langle \Lambda \mid N(3q)K \rangle\). For \(\Lambda NK\) coupling, one can use the SU(3) relation \(g_{\Lambda NK} = -(3F + D)/\sqrt{3(F + D)}g_{\pi NN}\) with \(F/D \approx 0.575\) [11], which gives \(g_{\Theta NK} \approx -g_{\pi NN}\). For \(\Theta NK\) coupling, we can use the relation between \(g_{\Theta NK}\) and \(\Theta \) decay width

\[
\Gamma_{\Theta^+} = \frac{|g_{\Theta NK}|^2 p_F}{2\pi m_{\Theta}} (\sqrt{M_N^2 + p_F^2} - M_N),
\]

where \(p_F\) is the \(\Theta\) decay momentum, which results in \(g_{\Theta NK} \approx 1\) at \(\Gamma_{\Theta} \approx 1\) MeV [12]. This gives the following estimation

\[
R_{\Theta^+}^{\text{diff}}(5q) \approx 5.6 \times 10^{-3}.
\]

III. \(\Theta^+\)-PRODUCTION IN FRAGMENTATION REGION

Consider now the \(\Theta^+\)-production in inclusive reaction \(AB \to \Theta^+X\) together with the hyperon production: \(AB \to YX\). The cross section of these reactions may be estimated on the base of fragmentation-recombination model, which assumes the elementary sub-processes as depicted in Fig. 4. Thus, it is assumed that at the first stage the colliding hadrons fragmentate into partons (quark, gluon, di-quarks, etc). The probability to find \(i\)-th constituent (parton) is described by the ”fragmentation” function \(F_{i/A}(x)\), where \(x = p_i/p_A\). At the second stage there is some soft (quasi-elastic) interaction between parton \(i\) and some other constituent from the hadron \(B\). Finally, the parton \(i\) is recombined into the observable hadron \(h\) (\(\Theta^+\) or \(Y\)). The probability of
this process is defined by the "recombination" function $R_{h/i}(y)$, where $y = p_h/p_i$. Thus, the cross section of $AB \to \Theta^+ X$-reaction is controlled by the folding

$$
\sigma \propto \int F_{i/A}(x) R_{h/i}(y) \delta(z - xy) dx dy,
$$

where $z = p_h/p_A$. By making use of the scale behaviour of $F_{i/A}(x)$ and $R_{h/i}(y)$

$$
F_{i/A}(x) \sim R_0(x)(1 - x)^{b_i}; \quad R_{h/i}(y) \sim R_0(y)(1 - y)^{c_i},
$$

where $F_0(x)$ and $R_0(x)$ are smooth functions of $x$, and keeping the dominant terms we get

$$
\sigma \propto \int_{z}^{1} (1 - x)^{b_i} (x - z)^{c_i} \, dx.
$$

The integral can be performed by an elementary method, and we can estimate the cross section as $(b_i^h = b, c_i^h = c)$

$$
\frac{blc!}{(b+c)!} \frac{1}{b+c+1} (1 - z)^{b+c}.
$$

For further estimation, we have to specify the power $b$ and $c$ in the fragmentation and recombination functions. In the quark-parton picture \[^{18}\], these coefficients are related to the number of the constituent partons in $A$ and $h$: $b_i^h = 2n_A - 3$ and $c_i^h = 2n_h - 3$. Consider now two extreme variants. Firstly, we assume the quark-diquark picture of the hadrons $A$ and $h$. When $A$ is a nucleon and $h$ is a hyperon or $\Theta^+$ we have $b = 1, c(\Theta^+) = 3$ and $c(Y) = 1$. The corresponding ratio of $\Theta^+$ to $Y$-production reads

$$
R_{\Theta^+Y\text{-diquarks}} \simeq \frac{3!2!3}{4!5} (1 - z)^{2} = 0.3(1 - z)^2.
$$

In the quark picture $b = 3, c(\Theta^+) = 7$ and $c(Y) = 3$, and

$$
R_{\Theta^+Y\text{-quarks}} \simeq \frac{6!7!}{3!10!11} (1 - z)^{4} \simeq 0.11(1 - z)^4.
$$

Combining these extreme cases we get the following estimation

$$
R_{\Theta^+Y} \simeq \frac{0.11(1 - z)^4}{3 \times 10^{-2} \text{[diquarks]}}.
$$

Choosing for $z$ the typical value for the fragmentation region $z \simeq 0.7$ we get the following bounds

$$
R_{\Theta^+Y} \simeq \left\{ \begin{array}{l}
9 \times 10^{-4} \text{[quarks]} \\
3 \times 10^{-2} \text{[diquarks]}
\end{array} \right.,
$$

which means that the $\Theta^+$-production in the fragmentation region is strongly suppressed. Notice that $(1 - z)$-power behaviour of the hadron production cross sections in the fragmentation region as a rule starts from $z \simeq 0.4 - 0.5 \,[14]$. At $z = 0.5$, the accuracy of Eqs. \((18)\) and \((19)\) is 20 ad 35%, respectively. For $z \simeq 0.7$ it is 10 ad 20%, respectively and becomes better when $z \to 1$. At $z \leq 0.4$, we have to specify the functions $F_0(x)$ and $R_0(x)$ in Eq. \((15)\), which may be important for the central rapidity region. We also have to include the dependence on the transversal momentum (for the finite $p_\perp$) which is, however, beyond the scope of our present qualitative analysis.

In summary, we have analyzed the high energy limit of the $\Theta^+$-pentaquark production. Our consideration is based on the well-known high energy phenomenology: energy dependence of the Regge trajectories and the scaling behaviour of the hadronic amplitudes. We found distinct decreasing of the ratio of the $\Theta^+$ production compared to the background processes in diffractive processes and exclusive reactions with large momentum transfers. In the fragmentation region at high energy, this ratio is rather small. Our estimation is done on the base of the fragmentation-recombination model but it has a general character and is valid for any model (for example, the relativistic string model). Physically, the $\Theta^+$-pentaquark production in the fragmentation region is accompanied by creation of additional 2 quark-antiquark (diquark-antidiquark) pairs with subsequent pick quarks up by the outgoing hadron. This results in additional suppression factor $(1 - z)\alpha$ with $\alpha \geq 2$. It may be worthwhile the point out that there will be no suppression with increasing energy in the central rapidity regions in inclusive reactions. Nevertheless, the $\Theta^+$ production at low energies seems to be most suitable for the study of the properties of $\Theta^+$.

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