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An Adaptive Hybrid Control of Grid Tied Inverter for the Reduction of Total Harmonic Distortion and Improvement of Robustness against Grid Impedance Variation

Tila Muhammad 1,*, Adnan Umar Khan 1, Muhammad Tajammal Chughtai 2, Reyyan Ahmad Khan 3, Yousra Abid 1, Muhammad Islam 4 and Sheroz Khan 4,*

1 Department of Electrical Engineering, International Islamic University Islamabad, Islamabad 44000, Pakistan; adnan.umar@iiu.edu.pk (A.U.K.); yousraabid88@gmail.com (Y.A.)
2 Department of Electrical Engineering, College of Engineering, University of Hail, Hail 55427, Saudi Arabia; mt.chughtai@uoh.edu.sa
3 Department of Electrical Engineering, Soongsil University, Seoul 06978, Korea; reyyanahmad266@gmail.com
4 Department of Electrical Engineering, College of Engineering and Information Technology, Onaizah Colleges, Onaizah 2053, Saudi Arabia; islam.eeng@gmail.com
* Correspondence: eng.tila@gmail.com (T.M.); cnar32.sheroz@gmail.com (S.K.)

Abstract: Background: Grid-tied inverters play an efficient role in the integration of renewable energy resources with utility grids. Motivation: However, the interconnection between power converters and the grid has been seen to be responsible for various stability issues such as weak grid, and under weak grid conditions the injection of power to the grid becomes a challenging task due to continuously varying grid impedance affecting the stability margins as a result. Additionally, the grid impedance-related issues boost the voltage harmonics which further devalue its performance. These voltage harmonics propagate through the Phase-Locked Loop (PLL) circuit to the control unit of the inverter which in turn amplifies the low order harmonics of the inverter. Method: The aim of this research is to introduce a novel strategy that decreases the effect of grid impedance-variations on the performance and stability of an inverter. Hence, an adaptive hybrid mode control technique consisting of two parts is proposed in this research. The current regulator part is implemented in a synchronous reference frame for its gain and time parameters to improve the performance, stability, and response time. The adaptive harmonic compensators are implemented in a stationary reference frame for harmonic compensation purposes. This adaptive nature of harmonic compensator can effectively work in the case of frequency variation where the fixed value based harmonic compensator fails. Results: The adaptive harmonic compensators improve the performance by reducing total harmonic distortion (THD), reduce computation and improve stability when the grid has distorted voltage, variation in grid impedance and frequency. Impact and utility: Our results show that the system becomes less sensitive to grid impedance variations which makes the proposed technique very relevant to the stability performance applications.

Keywords: hybrid control; adaptive control; grid-tied inverter; stability performance; total harmonic distortion

1. Introduction

Grid-tied inverters are commonly used in distributed generation by bringing renewable energy resources on-board with the existing grid networks as additional power on the basis of generating electricity at or near where it is being used. A lot of work has been done on the design of inverters for injecting power to the grid. The nature of the grid is not always same everywhere; in some regions it is strong while in others it becomes weak. Hence, the inverters designed under strong grid assumptions may become unstable if it is tied to the grid at the point where it is weak.
In recent studies, it has been found that the stability of grid-connected inverters depend on the ratio of the grid impedance to the output impedance of an inverter, and that this ratio must satisfy the Nyquist criteria of stability [1,2]. The weak grids have varying impedance which further complicates the system [3–5]. Under these conditions, the voltage harmonics of the grid also fluctuate with the flow of load current. The harmonics amplify with the increase of load current and vice versa. If these harmonics are not compensated within the inverters it can result in further amplification at the point of common coupling (PCC) and can make the system unstable. Therefore, the varying characteristics of the grids require inverters to be designed with larger stability margins, enabling the inverters for stable dynamics under varying grid conditions.

In the case of weak grids, the impedance of the grid depends on some parameters such as line frequency transformers, length of distribution and transmission lines. The number of grid-connected inverters also changes grid impedance. If grid-tied inverters are connected nearer to each other and the impedance between them is ignored, then these inverters become parallel to each other. Such grid-tied inverters are modelled for the analysis of harmonics and the resulting stability of the distributed power generation systems [6]. This configuration varies impedance in two ways: (a) the output impedance of the inverter decreases because it is in parallel with others; (b) the grid impedance is increased by a factor of N when N identical inverters are connected at the PCC, though the admittance of single inverter cannot be directly summed to obtain the admittance of multiple parallel grid-connected inverters [7,8]. Thus, the harmonics in grid currents increases and stability margins decrease. This work focuses on current controller, harmonic compensator, and voltage feed forward loop to improve the performance and stability of a grid tied inverter while responding adaptively to changing grid conditions.

The feed-forwarding of the grid voltage \( V_g \) at PCC plays an important role in reducing total harmonic distortion and making the system response faster in addition to decreasing the burden on the current controller. A stability enhancing voltage feed-forward inverter control method is proposed in [9] to reduce the effect of PLL and grid-impedance. Similar feed-forward techniques for a grid-connected inverter with an LCL filter is discussed in [10], while relatively extensive analysis of feed-forward strategies on the robustness in terms of harmonic suppression is discussed in [11]. Therefore, in the literature, there exists different strategies that demonstrate the importance of voltage feedforwarding in grid-tied inverters. Proportional, Second-Order Generalised Integrator (SOGI)-based and adaptive feedforward techniques named here as Strategy 1, Strategy 2 and Strategy 3, respectively, are considered in this paper for analysis and comparison. Proportional voltage feed-forward reduces total harmonics distortion, but also decreases the phase margin. Therefore, in most cases, a feedforward of fundamental voltage is used [12]. Although feedforward of fundamental voltage increases the phase margin, it is less helpful in the reduction of total harmonic distortion. In [13], the grid voltage is passed through various filters and then the voltage harmonics, which do not make the system unstable, are feed-forwarded and the rest are prevented from entering the control unit.

The PLL is used to synchronize the phase of inverter output current \( i_g \) with the phase of \( V_g \) by continuously estimating it but in the case of weak grids the estimated phase is distorted due to the presence of voltage harmonics. The harmonics propagate through PLL circuits and reach the control unit of the inverter [14]. These harmonics propagate further through the control unit and add to the low order harmonics present in the current feeding to the grid. An optimally designed PLL can be used with filters to prevent harmonics from entering the control unit, but still some low-order harmonics lie in the bandwidth of the PLL and make it to the control unit. SOGI filters play a characteristic role in such cases [6]. Although SOGI filters for PLL remove harmonics or tend to reduce voltage harmonics from grid signals and extract the phase of grid voltage with very little content of low order harmonics, it is still not possible to estimate the harmonics-free phase. Furthermore, inverter switching also generates low order harmonics. Therefore, it becomes necessary to add controllers for harmonic compensation. These compensators reduce total harmonic
distortion in the current feeding the grid and ultimately improve the performance and stability of the system.

There are different techniques used for harmonic compensators. Some of these are working in a synchronous reference frame while others are in a stationary reference frame [15–19]. Some compensators even operate in a hybrid frame where the current regulators are synchronous, and the compensators are in a stationary reference frame. In [20], the author introduced a new concept in power electronics which is known as a Lock-in Amplifier. This technique is quite useful in the extraction of low order harmonics and its compensation, but it does not work if there is any variation in the fundamental frequency. Although, the large-scale grids have higher inertia but still the line frequency can vary by 2 percent of its rated value. Thus, a compensator designed based on the fixed values fails to reduce harmonics under these conditions. An LCL filter is usually used to generate a pure sine wave, but the capacitor in an LCL filter generates resonance. Two methods are used to dampen this resonance [21], one is the active technique and the other is the passive based using resistors [22]. The resistor-based damping results in losses, therefore the active damping technique is preferable [23]. In this research, the resonance was reduced by using the active damping technique.

The contributions of this research are:

1. Reduced total harmonic distortion using adaptive resonance compensators while keeping stability margins for the current system unchanged;
2. The adaptive filters (Notch and Resonant) are implemented to improve the performance of the harmonic compensators and reduce the burden on the current regulator;
3. The synchronous and stationary reference framed controllers are utilized simultaneously to reduce computational complexity and enhance performance.

The architecture of the grid tied inverter with proposed controllers is given in Figure 1. Moreover, this paper is organized as: Section 2 presents system modeling along with analysis by using an open loop system. In Section 3, impedance ratio-based stability is discussed with the help of using Bode and Nyquist plots. In Section 4, the current regulator and adaptive harmonic compensators are designed. In Section 5, Results and the evaluation of the entire system are summarised. In Section 6, the conclusions of the entire research is presented.

2. System Modeling

The architecture of the grid-tied inverter is given in Figure 1 which consists of five main parts: 1. DC Source, 2. Power Switches, 3. Filter, 4. AC grid and 5. Control unit. Here, for ease of analysis the DC source is assumed as ripple free ideal voltage source.
with an output voltage $V_{DC}$. The full H-Bridge configuration is used for power switches taking DC voltage $V_{DC}$ and four PWM signals as input to gates and converting the input DC power into PWM-based alternating $V_{inv}$ and $I_{inv}$ output. The LCL filter converts $V_{inv}$ and $I_{inv}$ into a smooth $V_{g}$ and $I_{g}$ wave form. The LCL filter components are: inverter side inductor $L_1$, grid side inductor $L_2$, and Capacitor $C_f$ along with parasitic resistors $R_1$, $R_2$, and $R_c$, respectively. The parasitic effects on the stability of the system is negligible and can be ignored. The AC grid has an impedance $Z_g(s)$ and voltage source $V_s$ and is connected to an LCL filter at PCC where it has a voltage $V_g$. The grid voltage has a fundamental frequency of 50 Hz along with some variations and low order odd voltage harmonics to model the characteristic of weak grid. The hybrid control part consists of a current regulator and adaptive harmonic compensators. The current regulator is implemented in the synchronous reference frame while the harmonic compensators are implemented in the stationary reference frame which is discussed later in detail, in their respective sections. The values of system parameters used here are listed in Table 1.

| Symbol | Description | Values |
|--------|-------------|--------|
| $P$    | Rated power | 5 kW   |
| $V_{DC}$ | DC-link voltage | 400 V |
| $V_g$  | Grid voltage (RMS) at PCC | 220 V |
| $f_o$  | Grid frequency(fundamental) | 50 Hz |
| $L_1$  | Inverter side inductance | 0.75 mH |
| $L_2$  | Grid side Inductance | 0.45 mH |
| $C_f$  | Filter capacitor | 6.01 uF |
| $k_c$  | Active damping constant | 10.6/400 |
| $Z_g(s)$ | Grid impedance | 0–15 mH |
| $k_o$  | SOGI filter damping factor | 0.5 |
| $k_{or}$ | Compensator damping factor | 0.001 |
| $k_{on}$ | Notch filter damping factor | 7 |
| $K_r$  | Compensator gain | 250 |
| $K_p$  | Proportional gain | 50 |
| $T_i$  | Integral time constant | 741 |
| $f_n$  | Low pass filter natural frequency | 150 Hz |
| $T_L$  | Low pass filter time constant | $3.18 \times 10^5$ |
| $K_{pPLL}$ | Proportional gain PLL | 10 |
| $T_{PLL}$ | Integral time constant PLL | $20 \times 10^{-6}$ |

The average switch control model block diagram of the grid-tied inverter is given in Figure 2. The capacitor of the LCL filter generates a resonance which makes the system unstable. The feedback loop having gain $k_c$ is used for active damping. The (1) represents its mathematical form.

![Figure 2. The average switch control model of grid tied inverter.](image-url)
\[
\left[ (i_{\text{ref}} - i_g) G_c(s) - i_{c1} kC \right] k_{\text{pwm}} = i_{L1} L_1 s + i_g L_2 s + V_g \tag{1}
\]

where,
\[
V_g = V_s + Z_g(s) i_g \tag{2}
\]
\[
G_c(s) = k_p \left( 1 + \frac{1}{T_s i_s} \right). \tag{3}
\]

The voltage feed-forward is an additional loop from grid voltage to the node between the current regulator and modulator. In addition to the transient response, the voltage feed-forward improves steady-state performance in the form of a better power factor and produces less distortion in the current feeding to grid. Usually, two voltage feed forwarding techniques are used. One is proportional voltage feed-forward; we refer to it as Strategy 1, and the other is an SOGI based feed-forward technique, which we refer to as Strategy 2 as shown in Figures 3a and 4a, respectively. The feed-forward in Figure 3a is the proportional feed-forward method which reduces the phase margin of the system and is effective in a much less distorted strong grid or where grid impedance is low. The mathematical representation of this system is expressed in (4). Its open loop system gain is given in (6) and its simulated response by using values of Table 1 is given in Figure 3b.

\[
\left[ (i_{\text{ref}} - i_g) G_c(s) - i_{c1} kC + V_g G_f(s) \right] k_{\text{pwm}} = i_{L1} L_1 s + i_g L_2 s + V_g \tag{4}
\]
where, $V_g$ and $G_c(s)$ are from (2) and (3) respectively and $G_f$ is

$$G_f(s) = \frac{1}{k_{pwm}}$$

(5)

$$G_{\text{SOGI}}^{S1}(s) = \frac{k_{pwm}G_c(s)}{[L_1C_1L_2s^3 + (L_1C_1Z_g(s) + L_2C_1k_{pwm})s^2 + (C_1Z_g(s)k_{pwm} + L_1 + L_2)s + Z_g(s) - Z_g(s)k_{pwm}G_f(s)]}$$

(6)

The proportional feed-forward of Figure 3a with an additional SOGI filter is given in Figure 4a. It improves the phase margin and makes the system more robust specifically in the case of weak grid applications but due to an SOGI-based voltage feed-forward, the system slows down the response and increases the harmonic distortion. The mathematical expression of this system is given in (7). The open loop gain of the system is presented in (9) and Figure 4b shows its Bode plot response. The Bode plot reveals that the phase margin of the system decreases with the increase of grid impedance, but the system remains stable for the larger values of grid impedance as compared to contemporary works. By comparing the phase margin of both Figures 3 and 4, it can be concluded that SOGI-based feed-forward is a better option for more robust grid-tied inverters.

$$\left[ (i_{\text{ref}} - i_g) G_c(s) - i_{\text{ref}} k_c + V_g G_f(s) G_h(s) \right] k_{pwm} = i_{L1} L_1 s + i_g L_2 s + V_g$$

(7)

$$G_h(s) = \frac{k_o \omega_0 s}{s^2 + k_o \omega_0 s + \omega_0^2}$$

(8)

$$G_{\text{SOGI}}^{S2}(s) = \frac{k_{pwm}G_c(s)}{[L_1C_1L_2s^3 + (L_1C_1Z_g(s) + L_2C_1k_{pwm})s^2 + (C_1Z_g(s)k_{pwm} + L_1 + L_2)s + Z_g(s) - Z_g(s)k_{pwm}G_f(s)G_h(s)]}$$

(9)
In addition to Strategies 1 and 2 given above, another topology is introduced in [13]; we refer to it as Strategy 3, shown in Figure 5a. In this technique, multiple band-pass filters are used in the path of voltage feed-forward. These filters allow few low order harmonics to reach the node between current regulator and modulation while blocking the ones that reduce the phase margin by using an adaptive algorithm. The mathematical form of this system is shown in (10). The open loop gain of this system is shown in (12) and the Bode plot of the open loop gain is given in Figure 5b. The main feature of this technique is to reduce the harmonic distortion and keep the stability margins in between the proportional and SOGI-based voltage feed-forward.

\[
\left( i_{\text{ref}} - i_g \right) G_c(s) - i_{c1} k_c + V_g G_f(s) G_{\text{adapt}}(s) \right) k_{\text{pwm}} = i_{L1} L_1 s + i_8 L_2 s + V_g \tag{10}
\]

\[
G_{\text{adapt}}(s) = [x_1 G_{h-1}(s) + \ldots + x_7 G_{h-7}(s)] \tag{11}
\]

\[
G_{S3}^{i_g-i_{\text{ref}}}(s) = \frac{k_{\text{pwm}} G_c(s)}{[L_1 C_1 L_2 s^3 + (L_1 C_1 Z_g(s) + L_2 C_1 k_c k_{\text{pwm}}) s^2 + (C_1 Z_g(s) k_c k_{\text{pwm}} + L_1 + L_2) s + Z_g(s)] + Z_g(s) k_{\text{pwm}} G_f(s) G_{\text{adapt}}(s)]} \tag{12}
\]

![Figure 5. Strategy 3: Adaptive grid voltage feedforward (a) Average switch control model. (b) Open loop system response.](image-url)

In (1), (4), (6), (7), (9), (10) and (12), the transfer function $G_c$ as given in (3) is the Proportional Integral (PI) controller implemented in the synchronous reference frame and works as current regulator, and its parameters are listed in Table 1. The PI is used here, due to wider bandwidth and simple implementation as compared to the Proportional Resonant (PR) controller.

The analysis of their respective Bode-plots shows that the grid impedance has a strong impact on its stability margins. It can also be observed that Strategy 2 has larger phase margins as compared to other strategies but it has less capability to reduce the total
harmonic distortion. The strategies other than Strategy 2 provide good immunity against total harmonic distortion but are less robust against the increase of grid impedance and other variations. Therefore, Strategy 2 is the best option for the system to be robust, hence SOGI-based voltage feed-forward is chosen for further improvements in this research. In Section 3, the problem of total harmonic distortion is solved with the help of adaptive feed-forwarding technique. Hence, the system becomes robust and proves to reduce total harmonic distortion considerably.

3. Impedance Based Stability

Impedance based stability is an easy and authentic way to find the stability of grid connected inverters, which is derived from the reduced average switch model of grid tied inverter presented in the Norton and Thevenin form of circuit in Figure 6. By using the network solving technique it can be transferred to the form of conventional control techniques used for finding the stability of a specific system. Although this idea is older in other power electronic converters, for a grid tied inverter the benchmark is [1]. The circuit in Figure 6 consists of two parts; one is the Norton circuit containing the feature of the grid tied inverter in the form of a dependent current source $I_{inv}(s)$ and inverter output impedance $Z_o(s)$ while the other part is the Thevenin circuit consisting of grid impedance $Z_g(s)$ along with grid voltage $V_s$. By using superposition theorem on Figure 6, we get:

$$i_g = \frac{I_{inv}(s)Z_o(s)}{Z_o(s) + Z_g(s)} - \frac{V_s(s)}{Z_o(s) + Z_g(s)}$$

(13)

$$i_g = \left[ \frac{I_{inv}(s) - V_s(s)}{Z_o(s)} \right] \frac{1}{1 + Z_g(s)Z_o(s)}.$$  

(14)

From (14) the impedance ratio $Z_g(s)/Z_o$ can be used for impedance based stability to meet the Nyquist criteria of stability. Now, the stability of the three strategies can be analyzed by using the impedance-based stability test. The inverter output impedance can be derived from Figures 3–5 for Strategies 1, 2 and 3 respectively by ignoring $Z_g(s)$, shown by a dotted path in each figure.

Figure 6. Norton and Thevenin models of inverter and grid in grid tied application.

From the average model, the output impedances of the above three strategies are given in (15), (17) and (19) respectively. The impedance-based stability is checked by using the ratio of grid impedance to inverter output impedance. The combined Bode plots of Strategies 1, 2 and 3 are given in Figure 7a and the Nyquist plots are given in Figure 7b–d accordingly. In the regions of Bode plots where the gain of $Z_g(s)$ is less than $Z_{in}(s)$ (where $n = 1, 2$ and 3), the system is stable while in other regions where it is greater or at the crossing point, the stability conditions are mentioned in [1,2,24]. Thus the results of the three strategies are given in Figure 7.

Admittance of Strategy 1:

$$G_{U_{inv}-i_g}(s) = \frac{-L_1C_1s^2 - C_1k_pk_{pwm}s - 1 + k_{pwm}(s)G_f(s)}{L_1C_1L_2s^3 + L_2C_1k_pk_{pwm}s^2 + (L_1 + L_2)s + k_{pwm}G_c(s)}.$$  

(15)
Z_s^1(s) = \frac{1}{G_{U_{L} - i_{g}}(s)} \quad (16)

Admittance of Strategy 2

\begin{align*}
G_{U_{L} - i_{g}}^{S2}(s) &= \frac{-L_1C_1s^2 - C_1k_{pwm}C_1s - 1 + k_{pwm}(s)G_f(s)G_h(s)}{L_1L_2C_1s^3 + L_2C_1k_{pwm}s^2 + (L_1 + L_2)s + k_{pwm}G_c(s)} \\
Z_s^{S2}(s) &= \frac{1}{G_{U_{L} - i_{g}}^{S2}(s)} \quad (17)
\end{align*}

Z_s^2(o)(s) = \frac{1}{G_{U_{L} - i_{g}}^{S}(s)} \quad (18)

\begin{align*}
Z_s^{S3}(s) &= \frac{-L_1C_1s^2 - k_{pwm}C_1s - 1 + k_{pwm}(s)G_f(s)G_h(s)G_{adapt}(s)}{L_1L_2C_1s^3 + L_2C_1k_{pwm}s^2 + (L_1 + L_2)s + k_{pwm}G_c(s)} \\
Z_s^{S3}(s) &= \frac{1}{G_{U_{L} - i_{g}}^{S3}(s)} \quad (19)
\end{align*}

Figure 7. Impedance base stability analysis. (a) Bode plot of grid impedance vs. the inverter output impedance. (b) Nyquist plot for Strategies 1, 2 and 3 when \(Z_g(s)\) is 5 mH. (c) Nyquist plot for Strategies 1, 2 and 3 when \(Z_g(s)\) is 10 mH. (d) Nyquist plot for Strategies 1, 2 and 3 when \(Z_g(s)\) is 15 mH.

Admittance of Strategy 3:

\begin{align*}
G_{U_{L} - i_{g}}^{S3}(s) &= \frac{-L_1C_1s^2 - k_{pwm}C_1s - 1 + k_{pwm}(s)G_f(s)G_h(s)G_{adapt}(s)}{L_1L_2C_1s^3 + L_2C_1k_{pwm}s^2 + (L_1 + L_2)s + k_{pwm}G_c(s)} \\
Z_s^{S3}(s) &= \frac{1}{G_{U_{L} - i_{g}}^{S3}(s)} \quad (20)
\end{align*}

where \(G_{U_{L} - i_{g}}^{Sn}(s)\) and \(Z_s^{Sn}(s)\) denotes admittance and impedance of \(n\)th strategy.
4. Proposed Model

The proposed system uses the hybrid control of a grid tied inverter. The PI controller is used as the current regulator designed in the synchronous reference frame and the adaptive resonant compensators are used for harmonic compensation designed in a stationary reference frame. Hence, the technique is referred to as a hybrid controller which uses both synchronous and stationary reference frames simultaneously.

The proposed system reduces total harmonic distortion and provides robustness against the effect of variations in grid impedance. SOGI-based voltage feed-forward and PI controller values play an important role in the enhancement of stability margin. Robustness is improved by introducing the SOGI filter in the path of the voltage feed-forward, which permits the fundamental grid voltage only as given in (7). The orthogonal signals of DQ transformation used in the PLL and PI current regulator are generated with the help of SOGI filters instead of low-pass delayed filters which further enhances the capability of the system to reduce the total harmonic distortion to improve stability.

The averaged model of the proposed system under discussion is as shown in Figure 8. The system can be divided into two parts as A and B. Part A is a current regulator and other circuit parameters, while part B, surrounded by dotted lines, is related to adaptive harmonic compensators.

![Figure 8](image)

Figure 8. The proposed design of GTI with adaptive hybrid control technique.

4.1. Current Regulator

The current regulator is implemented in the synchronous reference frame and is designed by using the MATLAB SISO Tool. The extracted values for gain and time constant are listed in Table 1. The current regulator allows the desired current to enter the grid through tracking the reference signal. The phase and gain margin of the open-loop system depend on the constants selected for PI controller \( G_c \). Furthermore, the proportional gain of the current regulator suppresses the even harmonics accordingly.

As a single-phase system is considered here, orthogonal signals are generated with SOGI filters, which are used in the PLL to estimate the phase of grid voltage, which is required to synchronize the inverter output current. The phase is also used for the DQZ transformation of the grid current. The grid current is converted to orthogonal signals \( I_a \) and \( I_b \) with the help of cascaded two low-pass filters that each have a phase delay of 45°. Then, the orthogonal signals are converted to \( I_d \) and \( I_q \) with the help of DQZ transformation. Later, the \( I_d \) and \( I_q \) generated signals are compared with their respective reference signals \( I_{ref} \) and zero respectively to get the error signals \( E_D \) and \( E_Q \) to pass it through \( G_c \) controller. The \( I_d \omega_c (L_1 + L_2) \) and \( I_q \omega_c (L_1 + L_2) \) are added and subtracted respectively with controlled output signals of \( G_c \) D-axis and Q-axis respectively. The generated signals are passed
through inverse DQZ to generate controlled orthogonal signals of current $V_{\text{inv}\alpha}$ and $V_{\text{inv}\beta}$, which are used for PWM generation as shown in Figure 9.

$$G_c = K_p + T_i \frac{s}{s} (21)$$

$$V_{\text{inv}d} = V_{c_d} - \omega_o (L_1 + L_2) i_q + V_{g_d} (22)$$

$$V_{\text{inv}q} = V_{c_q} + \omega_o (L_1 + L_2) i_d + V_{g_q} (23)$$

$$V_{c_q} = (G_c) (I_q^* - I_q) \quad (24)$$

$$V_{c_d} = (G_c) (I_d^* - I_d). \quad (25)$$

The values of $K_p$ and $T_i$ can be selected on the basis of stability margin. The open loop gain of the proposed system is plotted and the phase margin is analyzed. Then the system is opened in SISO TOOL to find the suitable value of $K_p$ and $T_i$ by selecting the desired stability margins.

Figure 9. Block diagram of synchronous framed current regulator.

There exist very minimal tolerances of frequency variation in the grid frequency. In most of the cases the acceptable tolerance is not more than ±1 Hz. The synchronous framed PI controller has a wide bandwidth relative to the resonant controller thus the small variation of frequency does not affect the performance of the PI controller.

4.2. Adaptive Harmonic Compensators

The main contribution of this research is the design and implementation of an adaptive resonant controller for harmonic compensation. The controllers in a synchronous reference frame are not suitable options due to computational cost and a tedious procedure to implement multiple reference frames for each harmonic. Therefore, the synchronous reference frame controller is not an optimal solution for harmonic compensation. Hence, a hybrid mode control and the design of the fixed reference frame controller for harmonic compensation is specified. However, due to the very narrow bandwidth of the resonant controller, the implementation of the resonant controller is a difficult task. The ideal resonant controller is given in (26). Due to the no damping factor, its bandwidth is narrow and its implementation on a digital processor is almost impossible. To expand its bandwidth, a damping factor is used but it should be kept to a minimum level. The damping factor and digital processor make its hardware implementation possible. The value of the damping factor depends on the processor used in the controller. The larger the processing speed of digital controller the narrower the possible bandwidth of the resonant controller will be and the selection of notch filter features is done accordingly.

$$G_{rn} = \frac{k_d \omega_0 s}{s^2 + (n \omega_0)^2}. \quad (26)$$

If there is a small variation in the fundamental frequency at the grid, then the frequencies of the 3rd, 5th, 7th and 9th harmonics have variations 3, 5, 7 and 9 times of the
variation in the fundamental frequency respectively. Now if the harmonics compensators are designed for fixed 150 Hz, 250 Hz, 350 Hz and 450 Hz respectively, then even a small change in grid frequency severely affects the performance of harmonic compensators. To overcome this problem an adaptive resonance compensator is proposed for harmonic compensation which adapts its characteristics in accordance with the grid frequency by extracting grid frequency from the PLL circuit used for phase extraction.

The compensators are designed to reduce low order odd harmonics because high order harmonics can be easily removed using the LCL filter. Therefore, the first four odd harmonics from 3rd to 9th are considered here for reduction which are produced by the dead time and non-linearities of switches.

The proposed solution for harmonic compensation is shown in Figure 8, highlighted in part B where the compensator consists of two components: one is being an adaptive notch filter while the other is an adaptive harmonic resonant controller.

The damping effect of the resonant controller increases the bandwidth of harmonic compensators which also increases the fundamental current content in the output. The reason for this is the large value of the fundamental component as compared to another individual harmonic. To counteract this effect, a notch filter \( G_{\text{notch}} \) is added in a cascade with a harmonic compensator. Due to the notch filter, the harmonic compensator’s performance is better even with larger damping values. However, as discussed earlier, the frequency of the grid can variate to a certain limit, and notch filters have a very narrow bandwidth, therefore correct estimation of fundamental frequency and its tracking is also necessary to adjust the notch filter accordingly. Hence, an adaptive notch filter is used here for tracking the fundamental frequency and is capable of adapting its values accordingly. Figure 10d,e shows the comparison between the harmonics with and without using the notch filter. The expression of the adaptive notch filter is given in (27) and its circuit is given in Figure 10a.

The frequency response of the notch filter is given in Figure 10c.

\[
G_{\text{notch}}(s) = \frac{s^2 + 0s + \omega_o^2}{s^2 + k_{\text{on}}s + \omega_o^2}, \tag{27}
\]

where \( k_{\text{on}} \) is the damping constant mentioned in (27) and its value is given in Table 1. The damping constant makes the narrow bandwidth notch filter into a wider bandwidth and makes it from ideal into executable form.

Similarly, the adaptive SOGI filters have a damping factor \( k_{\text{or}} \) as given in (28). The value of \( k_{\text{or}} \) is listed in Table 1 and the same value of \( k_{\text{or}} \) is used here for all resonant controllers to keep the design simple. The frequency extracted through PLL(\( \omega_o \)) is used here and the resonant frequencies are selected as odd multiple of extracted fundamental frequency. Here, only the first four odd harmonics are compensated for. Therefore, the resonant controllers \( G_{r1}, G_{r2}, G_{r3} \) and \( G_{r4} \) are used for 3rd, 5th, 7th and 9th harmonics reduction respectively as given in (28). All the outputs of resonant controllers are amplified through \( K_R \) multiplication. The value of \( K_R \) can be selected on the basis of harmonics present in the current \( i_g \). Hence, it is the number which should be multiplied with a harmonic which is detected through the resonant filter to make it nearer to that harmonic present in the current \( i_g \). Thus, collectively the final harmonic compensator \( G_R \) is given in (30).

\[
G_{rn}(s) = \frac{k_{\text{or}}\omega_0s}{s^2 + k_{\text{or}}\omega_0s + (n\omega_0)^2}, \tag{28}
\]

where \( n \) shows the 3rd, 5th, 7th and 9th harmonics.

\[
G_R(s) = K_RG_{\text{notch}}(s)(G_{r1}(s) + G_{r2}(s) + G_{r3}(s) + G_{r4}(s)) \tag{29}
\]

\[
G_R(s) = K_RG_{\text{notch}}(s) \sum_{i=3}^{n} G_{ri}(s), \tag{30}
\]

where \( n = 9 \) and \( i \) is an odd integer starting from 3.
Figure 10. Adaptive resonant controller for harmonic compensation. (a) adaptive notch filter for blocking of fundamental current (b) adaptive resonant harmonic compensator (c) Frequency response of notch filter and resonant harmonic compensator (d) without using notch filter detection of 3rd Harmonic in $i_g$ (e) After using notch filter detection of 3rd Harmonic in $i_g$.

By using the block reduction method, the open loop gain of the proposed system is given in (31). The Bode plot of the open loop gain is given in Figure 11. The open loop gain of the system shows that the minimum phase margin is more than 40°. Hence, the closed loop system is stable even for the grid impedance of 15 mH. Figure 11 also shows that the gain at $3\omega_o$, $5\omega_o$, $7\omega_o$ and $9\omega_o$ is low which means that the system has immunity to stop the low order odd harmonics from entering the grid and reduces the total harmonic distortion.

Figure 11. Open loop response of grid tied inverter with proposed adaptive hybrid control technique. The minimum stability margins are also highlighted.
In comparison with [13], the proposed system has larger stability (phase and gain) margins. The improved margins of the proposed system show that it is more robust to the variation of the grid impedance. Figures 7 and 9 of [13] can be compared with Figure 11. Figure 7 of [13] shows that the system is unable to handle the 4th harmonic and for grid impedance more than 9mH the technique is not able to handle 3rd, 2nd and 1st order odd harmonics too. However, in comparison, the proposed system is tested for the grid impedance having inductance up to 15mH and the system is working effectively which makes the proposed solution robust to grid impedance variation as compared to the other similar techniques.

\[
G_{S^4}^i = \frac{G_c(s)k_{PWM}}{L_1L_2C_1s^2 + L_2C_1k_{PWM}s + (L_1 + L_2)Ls + Z_g(s)(s) - Z_g(s)G_c(s)k_{PWM}G_{PR}(s)}
\]

(31)

\[
G_{S^4}^U_i(s) = \frac{G_f(s)G_h(s)K_{PWM} - C_1L_1s^2 - C_1k_cke^{s_0}S - 1}{L_1L_2C_1s^3 + L_2C_1k_{PWM}s + (L_1 + L_2)s + G_c(s)k_{PWM} + K_rk_{PWM}G_{PR}(s)}
\]

(32)

\[
Z_o^p(s) = \frac{1}{G_{S^4}^o_i(s)}
\]

(33)

The admittance model of the proposed system which is given in Equation (32) and the impedance model in (33) are derived from the average switched model of the proposed system given in Figure 8 by using the block reduction method, keeping the reference current to zero. A similar method of impedance modeling is also used in reference [6,12,13]. The impedance-based stability is checked with the help of an impedance ratio between \(Z_g(s)\) and \(Z_o^p\) (output impedance of proposed inverter). The Bode plot and Nyquist plot of the impedance ratio are given in Figure 12 and Figure 13, respectively. The results verify the stability of the proposed system even for weak grids that have a grid impedance of 15mH.

![Figure 12. Impedance-based stability of grid tied inverter with proposed hybrid control.](image-url)
5. Results and Evaluation

The proposed design is verified for an inverter of 5.5 kW. The rated current injected to the grid is assumed to be 25 A and the grid voltage to be 220 V (rms). The LCL filter is considered to stop high frequency harmonics. The filter and grid parameters are selected on the basis of the existing parameters available in the literature [13]. The reason for this selection is to make the comparison with existing techniques and to evaluate the performance of the proposed solution. The other parameters and values used in the simulation are also listed in Table 1. The proposed model is simulated in the MATLAB Simulink environment. Three values of grid impedance (5 mH, 10 mH and 15 mH) are considered for simulation and the results are compared with the results of existing techniques. Figure 14 shows the working of the proposed technique under distorted grid voltage \( V_s \). This distorted grid voltage has a different THD from \( V_g \), which is the grid voltage at the point of common coupling due to grid impedance. This distorted voltage is used in the next figures for extracted results. Figure 15 shows the results when there is no harmonic compensator while Figure 16 shows the results when four adaptive harmonic compensators along with an adaptive notch filter are used. The first four harmonics in each case are given in Figure 15b and Figure 16b respectively. Table 2 shows the comparative analysis and cross-validation of the proposed technique with contemporary reported techniques. The extracted results show that the proposed solution not only increases the robustness of the grid-tied inverter but also minimizes the total harmonic distortion. The comparative results are tabulated in Table 2.

![Figure 13](image)

**Figure 13.** Impedance-based stability of proposed GTI by using Nyquist Plot (a) Nyquist plot of impedance ratio \( Z_g(s)/Z_o(s) \). (b) Impedance ratio \( Z_g^2(s)/Z_o(s) \). (c) Impedance ratio \( Z_g^3(s)/Z_o(s) \).
Figure 15. Results of Grid Tied Inverter without harmonic compensator. (a) PCC voltage $V_g$ and grid current $i_g$ along with their respective FFT analysis. (b) First four odd harmonics in current $i_g$ when grid impedance $Z_g(s)$ is 15 mH.
Figure 16. Results of Grid Tied Inverter with proposed harmonic compensator. (a) PCC voltage $V_g$ and grid current $i_g$ along with their respective FFT analysis. (b) First four odd harmonics in current $i_g$ when grid impedance $Z_g(s)$ is 15 mH.
Table 2 represents first four odd harmonics extracted from Figures 15 and 16 for grid impedance 5mH, 10mH and 15mH. The table clearly shows that the proposed system reduces the total harmonics distortion by minimizing low order odd harmonics and the effect of grid impedance on total harmonic distortion is minimized.

Figure 17 represents Table 2 in bar format, where System 1 is a system with Proposed Harmonic Compensator and System 2 is a system with Proposed Harmonic Compensator. The bar-height represents the percentage of harmonic with reference to grid current $i_g$ (keeping grid current 25 A as 100 percent). $H_3$, $H_5$, $H_7$, and $H_9$ represent 3rd, 5th, 7th and 9th harmonic, respectively.

Table 2. First four odd harmonics comparisons.

| Systems                                      | Z_g  | H3  | H5  | H7  | H9  |
|----------------------------------------------|------|-----|-----|-----|-----|
| System 1 without Harmonic Compensator        | 5 mH | 1.1 | 0.5 | 0.5 | 0.55|
| System 2 with Proposed Harmonic Compensator | 5 mH | 0.2 | 0.1 | 0.1 | 0.15|
| System 1 without Harmonic Compensator        | 10 mH| 1.1 | 0.5 | 0.5 | 0.5 |
| System 2 with Proposed Harmonic Compensator | 10 mH| 0.45| 0.19| 0.1 | 0.1 |
| System 1 without Harmonic Compensator        | 15 mH| 1.4 | 0.65| 0.4 | 0.55|
| System 2 with Proposed Harmonic Compensator | 15 mH| 0.4 | 0.2 | 0.1 | 0.15|

Figures 15 and 16 show the results of the method in [12] and the results of the proposed system tested for a grid frequency of 50 Hz respectively, while Figures 18 and 19 show the results to validate the adaptability of the proposed system under the grid frequency variations. The proposed system is effectively responding to frequencies exceeding 50 Hz (50.5 Hz) or falling behind 50 Hz (49.5 Hz) as shown in Figure 18 and Figure 19 respectively. Results of the proposed adaptive harmonic compensator, fixed value harmonic compensator and without harmonic compensator are produced and compared when the grid frequency is 50.5 Hz and 49.5 Hz; it is found that the result of the proposed adaptive harmonic compensator is more promising while the others lack this ability.
Figure 18. Results of proposed adaptive harmonic compensator, fixed value harmonic compensator and without harmonic compensator when grid frequency is 50.5 Hz.

Figure 19. Results of proposed adaptive harmonic compensator, fixed value harmonic compensator and without harmonic compensator when grid frequency is 49.5 Hz.
As discussed earlier, the PI current regulator has a wide bandwidth and therefore grid frequency variation in the acceptable range does not affect the performance of the current regulator. This is shown to be validated in the results presented in Figures 16 and 17. The comparison of the proposed method and that in reference [6] is given in Table 3. The table shows that the proposed method has a better performance as compared to other existing techniques.

Table 3. THD of $i_g$ (proposed system vs. [6]).

| S. No. | Technique                                      | THD When $Z_g = 5$ mH | THD When $Z_g = 10$ mH | THD When $Z_g = 15$ mH |
|-------|-----------------------------------------------|------------------------|------------------------|------------------------|
| 1     | Proposed (Grid Voltage THD = 4.07%)           | 0.90%                  | 0.67%                  | 0.58%                  |
| 2     | Reference [6] (Table III)                     | 3.50%                  | 3.20%                  | 2.90%                  |

6. Conclusions

The adaptive hybrid control technique is proposed for inverters used in the grid tied applications which make the system performance better by tracking the fundamental frequency of the grid and tuning itself according to any variation. This adaptive nature of compensators improves the performance. It is found that the proposed adaptive system has greater immunity to grid-impedance variation and that the system has a better performance as compared to currently reported techniques. The system is tested using open loop gain and impedance-based stability keeping the Nyquist stability criteria under consideration. It is found that the system has a much better performance as compared to existing techniques. The system is further tested for a highly distorted grid and the system performance is found to be more promising.

Author Contributions: Conceptualization, T.M. and R.A.K.; Formal analysis, M.T.C., Y.A. and S.K.; Funding acquisition, M.T.C., M.I. and S.K.; Investigation, Y.A.; Methodology, T.M., R.A.K., M.I. and S.K.; Project administration, A.U.K. and S.K.; Resources, M.T.C. and M.I.; Supervision, A.U.K.; Writing—original draft, T.M. All authors have read and agreed to the published version of the manuscript.

Funding: The authors gratefully acknowledge Onaizah College of Engineering and Information Technology and Department of Electrical Engineering, College of Engineering, University of Hail, 55427, Hail (Saudi Arabia) for funding the publication charges.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors acknowledge receiving the technical support from International Islamic University Islamabad (Pakistan), Onaizah College of Engineering and Information Technology (Saudi Arabia), and Department of Electrical Engineering, College of Engineering, University of Hail (Saudi Arabia).

Conflicts of Interest: There is no conflict of interest.

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