Space-Time Uncertainty and Noncommutativity in String Theory

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We analyze the nature of space-time nonlocality in string theory. After giving a brief overview on the conjecture of the space-time uncertainty principle, a (semi-classical) reformulation of string quantum mechanics, in which the dynamics is represented by the noncommutativity between temporal and spatial coordinates, is outlined. The formalism is then compared to the space-time noncommutative field theories associated with nonzero electric $B$-fields.

1. Motivations

What is string theory? This is a question we have been continually asking ourselves in exploring string theory as a hint towards the ultimate unified theory of all interactions including quantum gravity.

One of the most characteristic features of string theory is the existence of a fundamental constant, string length $\ell_s \sim \sqrt{\frac{2\pi\alpha'}{\alpha}}$, which sets a natural cutoff scale for the ultraviolet part of quantum fluctuations for particle fields associated with the spectrum of string states. This implies that string theory must necessarily exhibit some nonlocality and/or certain fuzziness with respect to the short distance structure of space-time.

From this point of view, it is quite remarkable that string theory gives a completely well-defined analytic S-matrix which essentially satisfies all the axioms for physically acceptable theory satisfying, at least perturbatively, Lorentz invariance, (macro) causality and unitarity. In contrast to this, various past attempts toward nonlocal field theories failed to give sensible results, because of lack of self-consistency or of suitable guiding principles for constructing nontrivially interacting theories. It thus seems an important task in uncovering its underlying principles to characterize the nonlocality of string theory. Our attitude is that, given string theory, we should learn how to formulate the idea of fundamental length from the structure of string theory, rather than postulating some arbitrary principles from scratch.

Recent development on the connection of string theory with the external $B$-field to non-commutative geometric field theories might provide a good hint for pursuits in this direction. However, we should keep in mind that the nature of nonlocality originated from the $B$-field is nothing to do with the extendedness of strings. In the present article, I would like to, first, review briefly the old proposal of a ‘space-time uncertainty principle’ as a possible general characterization of the space-time structure of string theory at short distances, and then to discuss some ideas toward a reformulation of string theory in such a way that the noncommutativity between
space and time is manifest. I hope that the comparison of the nature of the latter noncommutativity to the one of the typical noncommutative field theories which have the algebra of space-time coordinates of the Moyal-type product associated with the electric B-field is useful for deepening our understanding of string theory.

2. Space-Time Uncertainty

The main idea for proposing the space-time uncertainty relation comes from a simple analogy concerning the nature of string quantum mechanics. The crucial requirement of the ordinary string perturbation theory is the world-sheet conformal invariance. Indeed, most of the important merits of string theory as a possible unified theory are due to the conformal invariance. In particular, the elimination of the ultraviolet of divergence in the presence of gravity is essentially due to the modular invariance, which is the part of the conformal symmetry. From the viewpoint of generic two-dimensional field theory, the conformal invariance forces us to choose a very particular class of all possible two-dimensional field theories, corresponding to the fixed points of Wilsonian renormalization group. This is quite analogous to the imposition of Bohr-Sommerfeld quantization conditions to classical mechanics, in which the adiabatic invariance of action variables to be quantized can be regarded as a characterization of the quantization condition. In the final formulation of quantum mechanics, the quantization condition was replaced by the more universal framework such as Hilbert space and operator algebra acting in it. This analogy suggests us the importance of reinterpreting the conformal invariance requirement by elevating it to a more universal form, which may ultimately be formulated in a way that does not depend on perturbative methods.

One of the crucial properties related to modular invariance is expressed as the ‘reciprocity relation’ of the ‘extremal length’. The extremal length is a conformally invariant notion of length associated with families of curves on general Riemann surfaces. If we consider some finite region $\Omega$ and a set $\Gamma$ of arcs on $\Omega$, the extremal length of $\Gamma$ is defined by $\lambda_\Omega(\Gamma) = \sup_\rho \frac{L(\Gamma, \rho)^2}{A(\Omega, \rho)}$ with $L(\Gamma, \rho) = \inf_{\gamma \in \Gamma} L(\gamma, \rho)$, $A(\Omega, \rho) = \int_{\Omega} \rho^2 dz d\bar{z}$ where $\rho$ is the possible metric function giving the length $L(\gamma, \rho) \equiv \int_{\gamma} |\rho dz|$ on $\Omega$ of a curve in $\Gamma$ in the conformal gauge. Since any Riemann surface can be composed of a set of quadrilaterals pasted along the boundaries (with some twisting operations, in general), it is sufficient to consider the extremal length for an arbitrary quadrilateral segment $\Omega$. Let the two pairs of opposite sides of $\Omega$ be $\alpha, \alpha'$ and $\beta, \beta'$. Take $\Gamma$ be the set of all connected set of arcs joining $\alpha$ and $\alpha'$. The set of arcs joining $\beta$ and $\beta'$ is called the conjugate set of arcs, denoted by $\Gamma^*$. We then have two extremal lengths, $\lambda_\Omega(\Gamma)$ and $\lambda_\Omega(\Gamma^*)$. Then the reciprocity relation is that

$$\lambda_\Omega(\Gamma)\lambda_\Omega(\Gamma^*) = 1. \quad (1)$$

The simplest example is just the rectangle with the Euclidean sizes $a$ and $b$ in the Gauss plane. In this case, we can easily prove that $\lambda(\Gamma) = a/b$, $\lambda(\Gamma^*) = b/a$. For details, we refer the reader to a more extensive review and the mathematical
references cited there.

To see how the reciprocity of the extremal length reflects to target space-time, let us consider the Polyakov amplitude for the mapping from the rectangle on a Riemann surface to a rectangular region in space-time with the side lengths $A, B$ with the boundary condition $(0 \leq \xi_1 \leq a, 0 \leq \xi_2 \leq 1)$ $x^\mu(0, \xi_2) = x^\mu(a, \xi_2) = \delta^\mu_2 B \xi_2 / b, x^\mu(\xi_1, 0) = x^\mu(\xi_1, b) = \delta^\mu_1 A \xi_1 / a$. Then the amplitude contains the factor

$$\exp \left[ -\frac{1}{\ell_s^2} \left( \frac{A^2}{\lambda(\Gamma)} + \frac{B^2}{\lambda(\Gamma^*)} \right) \right],$$

multiplied by a power-behaved prefactor. Thus the fluctuations of two space-time lengths $A$ and $B$ satisfy an ‘uncertainty relation’ $\Delta A \Delta B \sim \ell_s^2$. For general and more complicated boundary conditions, it is not easy to establish a simple relation such as above between the extremal lengths and the space-time lengths, since there are various ambiguities in defining space-time lengths in terms of string variables. After all only legitimate observables allowed in string theory is the on-shell $S$-matrix. However, it seems natural to conjecture that the above relation sets a limitation, in some averaged sense, on the measurability of the lengths in space-time in string theory, since conformal invariance must be valid to all orders of string perturbation theory and the random nature of boundaries generally contributes to further fuzziness on the space-time lengths. Note that this reciprocity relation exhibits one of the most important duality relations in string amplitudes between ultraviolet and infrared structures. Since in the Minkowski metric one of the lengths is always dominantly time-like, we propose the following uncertainty relation on the space-time lengths

$$\Delta T \Delta X \gtrsim \ell_s^2$$

as a universal characterization of the short-distance space-time structure of string theory. This relation was originally proposed by the present author in 1987 independently of other proposals of similar nature, for example, the notion of ‘minimal distance’.

The consistency of this ‘space-time uncertainty’ relation with the high-energy behaviors of the perturbative string amplitudes was analyzed in ref. to which I refer the reader for details and relevant references. We find that generically there are many instances where the above uncertainty relation is far from being saturated. However, so far all the known results seem to be consistent with the validity of the space-time uncertainty relation as an inequality. In particular, in the high-energy and high-momentum-transfer limit, both the temporal and spatial uncertainties increase linearly with respect to energy for fixed-genus amplitudes. However, the proportional constant decreases for higher genera and the well-known behavior $|A_{\text{resum}}(s, \phi)| \sim \exp \left( -\sqrt{6\pi\hbar^2 s f(\phi) / \log s} \right)$ of the Borel-summed amplitude is consistent with the saturation of the equality in (3) up to some possible logarithmic corrections that perhaps depend on how to precisely define the space-time uncertainties. This may be an indication that the relation (3) is indeed valid independently of string coupling $g_s$. 
A further support for the validity of the relation is its effectiveness for D-branes. For example, the effective Yang-Mills theories for the low-velocity D-p-branes predict that the characteristic spatial (transverse to D-p-branes) and temporal scales of D-p-brane scattering oppositely scale with respect to the string coupling, namely as $\Delta X \sim g_s^{-1/(3-p)}\ell_s$ and $\Delta T \sim g_s^{1/(3-p)}$ for $p \geq 0$ and for $p \neq 3$. Although the case $p = 3$ is special in that the effective Yang-Mills theory is conformally invariant, the conformal transformation property is actually consistent with the space-time uncertainty relation as discussed in ref. 6. We can also derive these characteristic scales directly without recourse to the effective Yang-Mills theory. For example, the characteristic scales $\Delta X \sim g_s^{1/3}\ell_s$, $\Delta T \sim g_s^{-1/3}\ell_s$ of D-particle-D-particle scattering are a direct consequence of the space-time uncertainty relation and the ordinary quantum mechanical Heisenberg relation, given the fact that the mass of the D-particle is proportional to $1/g_s$. All these properties are natural from the viewpoint of open string theories where the relation (3) must be valid.

Finally, let us discuss one important question. Since the relation (3) is independent of string coupling $g_s$, it seems at first sight that it does not take into account gravity. So what is its relation to the Planck scale which is the characteristic scale of quantum gravity? In string theory, the existence of gravity can also be regarded as an important consequence of the world sheet conformal invariance. This is due to the possibility of deforming the background space-time by a linearized gravitational wave. However, in perturbation theory, the coupling strength of the gravitational wave is an independent parameter determined by the vacuum expectation value of dilaton. In this sense, the string coupling can not be a fundamental constant which appears in the universal nonperturbative property of string theory. Thus in order to take into account the Planck length for the space-time uncertainty relation, we have to put that information by hand. Now we shall show that by combining the Planck scale with the space-time uncertainty relation, we can derive the M-theory scale without invoking D-branes or membranes.

For that purpose, it is useful first to reinterpret the meaning of the Planck length using a similar language of the stringy space-time uncertainties, by considering the limitation of the notion of classical space-time as the background against the possible formation of virtual black holes in the short distance regime. If we want to probe the space-time structure in time direction to order $\delta T$, the quantum mechanical uncertainty relation tells us that the uncertainty with respect to the energy of order $\delta E \sim 1/\delta T$ is necessarily induced. If we further require that the structure of the background space-time is not influenced appreciably by this amount of fluctuation, the spatial scale $\delta X$ to be probed can not be smaller than the Schwarzschild radius associated with the energy fluctuation. Hence, $\delta X \gtrsim (G_D \delta E)^{1/(D-3)}$ in $D$-space-time dimensions. This sets the relation for the characteristic gravitational scales in the form

$$\delta T(\delta X)^7 \gtrsim g_s^2\ell_s^8.$$  

(4)

in $D = 10$ dimensional string theory. This may be called the ‘black-hole uncertainty’ relation. Note, however, that the nature of the scales $\delta T, \delta X$ is different from those
Figure 1: This diagram schematically shows the structure of the space-time uncertainty relation and the uncertainty relation associated with the Planck scale. The critical point is where the two relations meet.

3. Space-Time Noncommutativity

The validity of the fundamental uncertainty relation of the form (3) suggests the existence of certain noncommutative space-time structure that underlies string theory. Indeed, the expression (2) is strongly reminiscent of the Wigner representation \( \rho(p, q) \sim \exp \left[-\frac{(p/\Delta p)^2}{2} - \frac{(q/\Delta q)^2}{2}\right] \) of the density matrix corresponding to the gaussian wave packet in particle quantum mechanics, suggesting that the
space-time of string theory is something analogous to the classical phase space in particle quantum mechanics. However, the usual quantum mechanics of strings does not directly show any such noncommutativity between space and time. This raises a question: Is there any alternative formulation of string quantum mechanics in which the space-time noncommutativity is manifest? In such a formulation, we expect that the world-sheet conformal symmetry would be translated into a quite different language. There might be, hopefully, a chance of providing a hint toward some nonperturbative formulation of string theory.

Let us start from the following version of the Nambu-Goto-Schild action:

\[ S_{ngs} = -\int_{\Sigma} d^2 \xi \left\{ \frac{1}{4 \pi \alpha'} \left[ -\frac{1}{2} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu \right] + e \right\}. \]  

(5)

We only consider the case of bosonic strings, expecting that the extension to superstrings will not cause any fundamental difficulty. By eliminating nonpropagating auxiliary field \( e(\xi) \), the action reduces to the ordinary Nambu-Goto action. In this form, the conformal invariance of string theory is buried in the existence of the standard Virasoro constraint \( P^2 + \frac{1}{16 \pi \alpha'} \bar{X}^2 = 0, \quad P \cdot \bar{X} = 0 \) which does not explicitly involve the world sheet auxiliary field and the world-sheet metric. It is important for later interpretation that the Hamiltonian constraint comes from the equation

\[ \frac{1}{4 \pi \alpha'} \sqrt{-\frac{1}{2} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu} = e \]  

(6)

for the auxiliary field \( e \). Recall also that causality of string theory is embodied in the time-like nature of the area element \( \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu \) in this formalism. Now in order to rewrite the action such that it becomes quadratic in the space-time coordinates, we introduce another auxiliary field \( b_{\mu \nu}(\xi) \) which transforms as a world-sheet scalar and simultaneously as an antisymmetric tensor with respect to the space-time indices:

\[ S_{ngs2} = -\int_{\Sigma} d^2 \xi \left\{ \frac{1}{4 \pi \alpha'} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu b_{\mu \nu} + e \left( \frac{1}{2} b_{\mu \nu}^2 + 1 \right) \right\}. \]  

(7)

Now the constraint (6) is replaced by that for the new auxiliary field \( b_{\mu \nu} \):

\[ \frac{1}{2} b_{\mu \nu}^2 = -1. \]  

(8)

The first auxiliary field \( e \) only plays the role of Lagrange multiplier for this condition. Namely, the requirement of conformal invariance is essentially reinterpreted as the condition that the world-sheet \( b \) field is time-like or ‘electric’.

Let us consider the quantization of this action by regarding the \( b \)-field as an external field. Since the action is then first order with respect to the world-sheet time \((\tau)\) derivative, the system has second class constraints \( P_\mu = b_{\mu \nu} \partial_\nu X^\nu / 2 \pi \alpha' \), relating the components of the generalized coordinates and momenta directly. The Dirac bracket taking into account this constraint is given as

\[ \left\{ X^\mu(\sigma_1), \partial_\nu b_{\mu \alpha}(\sigma_2) X^\alpha(\sigma_2) \right\}_D = 2 \pi \alpha' \delta^\mu_\nu \delta(\sigma_1 - \sigma_2). \]  

(9)
Remembering that the $b$-field is dominantly time-like by (8), we see that the space $X^i(\xi)$ and the time $X^0(\xi)$ become indeed noncommutative. In particular, the center-of-mass time $T \equiv (1/2\pi) \int_0^{2\pi} d\sigma X^0$ and the spatial extension $X$ defined by

$$X \equiv -\int d\sigma b_{0i}(\sigma) \partial_\sigma X^i$$  \hspace{2cm} (10)

satisfy $(2\pi\alpha' \to \ell_s^2)$

$$\{T, X\}_D = \ell_s^2. \hspace{2cm} (11)$$

Here for simplicity we have assumed a closed string. That the expression (10) can be adopted justifiably as the measure of spatial (longitudinal) extension of strings can be seen by remembering that in the semi-classical approximation the $b$-field is just proportional to the area element of the world sheet of strings, $b_{\mu\nu} = -\frac{1}{4\pi\alpha'} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu / e$, which is derived by taking the variation of the action with respect to the $b$-field. Note that in this approximation the auxiliary $e$-field is determined by the normalization condition (6) through (8). In this way, we have now reformulated the string mechanics, at least in the semi-classical approximation, in such a way that the noncommutativity between spatial extension and time is manifest. This naturally conforms to the general property (3) of the space-time uncertainties, derived on the basis of the world-sheet conformal symmetry in the previous section, as it should be. For example, the constraint (8) indeed replaces the role of conformal invariance.

Remarks

1. The nature of the noncommutativity discussed above is close, at least formally, to that associated with the antisymmetric space-time external $B_{\mu\nu}(X)$ field in the presence of D-branes. Note, however, an obvious difference that the present noncommutativity is intrinsic to the extendedness of strings and is nothing to do with the choice of the background of string theory. If we add the $B$-field background in considering D-branes, there arises an additional contribution to the noncommutativity, since the first term in the action is deformed as $b_{\mu\nu} \to b_{\mu\nu} + 2\pi\alpha' B_{\mu\nu}$. Of course, when $B_{\mu\nu}$ is constant, it affects only at the end points of open strings. Let us briefly treat the open string boundary in the present formalism. If we allow free variations for $\delta X^\mu$ at the boundary without $B$-field along the D-brane world volume, the boundary condition is $\partial_\tau X^\nu b_{\mu\nu} = 0$. In the semi-classical approximation we are using, we can set $b_{\mu\nu} = -\frac{1}{4\pi\alpha'} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu / e$. It is convenient here to choose the orthonormal world-sheet coordinate satisfying $\dot{X} \cdot X' = 0$ and $X^2 + X'^2 = 0$ where the Lorentz contractions are done only over the directions along the D-brane world volume. Then, using (3) the boundary condition becomes the usual Neumann condition $X'^\mu = 0$. Now suppose we add a constant space-time $B$-field which corresponds to the additional boundary term $(1/2) \int_{\partial\Sigma^\tau} d\tau X^\mu(\tau) \partial_\tau X^\nu(\tau) B_{\mu\nu}$. Then the boundary condition is $X'^\mu + 2\pi\alpha' B_{\mu\nu} \dot{X}_\nu = 0$. If the $B$-field is magnetic, it does not affect the time-like nature of the string coordinate at the boundary. However, if that is electric and the magnitude exceeds a critical value $1/2\pi\alpha'$ such that $b_{0i} + 2\pi\alpha' B_{0i}$
can vanish, the time-like nature of the open string boundary would be lost, leading to a violation of causality in the dynamics of open strings and D-branes. This can be seen as follows. Without losing generality we can assume that the only nonzero component of the $B$-field is $B_{01} = B$, providing the direction 1 is along the D-brane world volume. The boundary condition together with the coordinate condition leads to the relation

$$\dot{X}^2 = -X'^2 = (2\pi\alpha')^2(\dot{X}^1)^2 - (\dot{X}^0)^2.$$  

When $(2\pi\alpha')^2 > 1$, this implies that the vector $\dot{X}^\mu$ is space-like at the open-string boundary. Thus it is impossible to decouple the string scale from that of the noncommutativity associated to electric $B$-field. Namely, we can not define sensible field theory limits satisfying causality and unitarity using electric $B$ field without open-string degrees of freedom. This seems to indicate that the feasibility of the space-time noncommutativity is inextricably connected to stringy nonlocality.

2. It is perhaps instructive to make a further comparison of the present formalism with the naive field theory model in which the noncommutativity between space and time is introduced explicitly. A scalar field theory in a noncommutative space-time with space-time commutation relation $[x, t] = i\theta$ can be constructed by assuming the product of the fields are defined by the Moyal product

$$\phi(x) \star \phi(x) = e^{i\frac{\theta}{2}(\partial_{x_1}\partial_{t_2} - \partial_{x_2}\partial_{t_1})}\phi(x_1)\phi(x_2)\big|_{x_1=x_2=x, t_1=t_2=t}.$$  

For notational clarity, we consider only $(1+1)$-dimensional part of space-time. Then a 3-point interaction vertex takes the following form in the coordinate representation

$$\int dx dt (\phi_1 \star \phi_2 \star \phi_3)(x, t)$$

$$= (\pi\theta)^{-2} \left( \prod_{i=1}^{3} \int dx_i dt_i \right) \exp \left[ \frac{2}{\theta} \sum_{cyclic} \left( x_1 t_j - t_i x_j \right) \right] \prod_{i=1}^{3} \phi_i(x_i, t_i). \quad (12)$$

The exponent $\frac{2}{\theta} \sum_{cyclic} \left( x_1 t_j - t_i x_j \right)$ in this expression is the formal analogue of the first term of the string action (7) with the identification $\theta \sim \alpha'$. Actually, there is a crucial difference in that the exponential factor in field theory case directly leads to non-causal shifts of the time coordinates in proportion to the momenta of external lines as $t_1 - t_2 \sim \theta p_3, \ldots$. Hence the sign of the time shifts depends on the direction of momenta. In the case of strings, the connection between the external momenta and the shifts, if any, of the time is not so direct as in the case of the simple Moyal product. For example, the center-of-mass spatial coordinates do not directly contribute to the noncommutativity in (11). The dynamics of strings is completely local at each point of world sheet and hence the time-like nature of the area element ensures causality in the evolution of the system. As discussed above, causality is preserved as long as the external space-time electric $B$ field does not exceed the critical value.
However, the above formal analogy prompts us to speculate a possibility of formulating the space-time uncertainty and noncommutativity as a certain kind of 'deformation' from classical space-time geometry to quantum and stringy geometry. It is a major challenge to find some unique characterization of such stringy deformation of space-time geometry.

3. Another remark which might be useful in understanding the nature of the present reformulation is that the counterpart in particle theory of what we have done above is simply the momentum representation of the particle propagator. We can start from the familiar particle action

\[ L = -\frac{1}{2} \int d\tau \left( -\frac{1}{e} \left( \frac{dx^\mu}{d\tau} \right)^2 + em^2 \right) . \]

By introducing another auxiliary field \( p_\mu \) which is now a space-time vector corresponding to the line element of the world line, we can rewrite it as

\[ L_p = \int d\tau \left( p_\mu \frac{dx^\mu}{d\tau} - \frac{1}{2} e(p^2 + m^2) \right) . \]

Obviously, the role of the Virasoro condition in strings is now played by the mass-shell condition \( p^2 + m^2 = 0 \) requiring that the momentum is time-like (or light-like when \( m = 0 \)). In the particle case, the action \( \int d\tau p_\mu \frac{dx^\mu}{d\tau} \) defines a usual Poisson structure. In analogy with this, the string action \( \frac{1}{4\pi \alpha'} \int d^2 \xi \epsilon^{ab} \partial_a X^\mu \partial_b X^{\nu} b_{\mu\nu} \) can be regarded as defining a generalized Poisson structure which is appropriate to strings.*

One natural possibility along this line would be to regard the auxiliary b-field as a sort of momentum variable corresponding to the area element of string world sheet.

Since, comparing to the particle case, the string case has one additional dimension, this line of thought leads to the use of the so-called Nambu bracket. Such a possibility was indeed suggested in [12]. Unfortunately, there seems to be no appropriate quantization procedure based on this interpretation. The interpretation we have given in the present article by means of the ordinary Dirac bracket quantization seems to be the only viable possibility toward quantization. Our discussion, however, remains still at a very formal level. It is an open question whether the above formalism can lead to a new exactly calculable scheme in full-fledged quantum theory. It is tempting to speculate a possibility of some tractable integral representation of string amplitudes where the ordinary moduli parameters of Riemann surfaces are integrated over. Instead of the moduli parameters, we should have some different integration variables corresponding to the ‘area momenta’.

I leave such a possibility for the reader as an interesting new direction in exploring string theory.

4. Another relevant question related to the more precise formulation of the present approach is to discuss the curved background. A natural way of including the space-time metric is by introducing the viel-bein field \( e_\mu^A(X) \) where \( A \) is the local Lorentz index. We can assume that the auxiliary field \( b_{AB}(\xi) \) now transforms

* After writing the previous review [1], the present author came to know by reading reference [12] that a suggestion which is closely related to the present remark was first made by Nambu in [4].
as an antisymmetric tensor at each local Lorentz frame on the space-time point $X^\mu(\xi)$ with the constraint $(1/2)b_{AB} = -1$. The area element is then written as $\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu e^A(\xi)e^B(\xi)b_{AB}(\xi)$. The Dirac bracket relation is more complicated than the flat space, with the right hand side depending on the space-time coordinates. We expect that the requirement of consistent quantization would lead to the condition for the space-time background which should be equivalent to the familiar $\beta$-function condition of renormalization group. It is an important problem to work this out. Of course, once we could arrive at a satisfiable characterization of the deformed geometry as suggested above, such a property would be an evident consequence of the general formalism.

Even apart from further clarification and refinements of the ideas discussed here, there are innumerable many other remaining questions, such as the relevance of the space-time uncertainties and noncommutativity for black-hole physics, the interpretation from the viewpoint of 11 dimensional M-theory, consistency with S- and T- dualities, interpretation of possible other scales than the M-theory scale, the role of supersymmetry, and so on. Some of these questions have been treated partially in ref. 3. Real answers to most of the deeper questions, however, must be left to the future.

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1. T. Yoneya, in “Wandering in the Fields”, eds. K. Kawarabayashi and A. Ukawa (World Scientific, 1987), p. 419.
2. T. Yoneya, Mod. Phys. Lett. A4, 1587(1989).
3. T. Yoneya, Prog. Theor. Phys. 103 (2000) 1081; hep-th/0004074.
4. D. Gross, a talk at the Munich conference in 1987, See Proc. XXIV Int. Conf. High Energy Physics, Munich, Eds. R. Lotthaus and J. Kühn, Springer, Verlag (1989); D. Gross and P. Mende, Nucl. Phys. Nucl. Phys. B303 (1988) 407.
5. P. Mende and H. Ooguri, Nucl. Phys. B303(1988), 407.
6. A. Jevicki and T. Yoneya, Nucl. Phys. B535 (1998) 335.
7. M. Li and T. Yoneya, Phys. Rev. Lett. 78 (1997) 1219; M. Li and T. Yoneya, Chaos, Solitons and Fractals 10(1999) 423–443; hep-th/9806240.
8. T. Yoneya, Lett. Nuovo. Cim. 8(1973)951;Prog. Theor. Phys. 51(1974) 1907; 56(1976)1310.
9. J. Scherk and J. Schwarz, Nucl. Phys. B31(1974)118; Phys. Lett. 57B(1975)463.
10. N. Seiberg, L. Susskind and N. Toumbas, hep-th/0005017; R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, hep-th/0005044; J. L. F. Barbón and E. Ravino- bic, hep-th/0005073.
11. J. Gomis and T. Mehren, hep-th/0005129.
12. I. V. Kanatchikov, Rep. Math. Phys. 40 (1997) 225; hep-th/9710006.
13. Y. Nambu, Phys. Lett. 92B (1980) 327.