Viscous Hydrodynamics

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Abstract. We study the role of viscosity in the early stages of relativistic heavy ion collisions. We investigate the effects of viscosity on the chemical equilibration of a parton gas. In the presence of viscosity the lifetime of the system is increased. The temperature as well as the parton fugacities evolves more slowly compared to ideal fluid dynamics.

1. Introduction

Ultra-relativistic nucleus-nucleus collisions probe the properties of nuclear matter under extreme conditions [1]. Lattice quantum chromodynamics (QCD) calculations [2] predict that ordinary nuclear matter undergoes a phase transition to quark gluon plasma (QGP). An important question is whether the high-energy density matter formed in ultra-relativistic nuclear collisions lives sufficiently long enough to reach thermodynamical equilibrium. That is does the matter reaches thermal, mechanical and chemical equilibrium? In this work we assume that the matter reaches thermal and mechanical equilibrium after proper time $\tau_0$. We do not, however, assume that the matter is in chemical equilibrium. Under these assumptions, and given initial values for temperature and the quark, antiquark and gluon number densities, we can then employ fluid dynamics to study the subsequent evolution of the kinetically equilibrated quark-gluon phase, coupled to the rate equations which determine the chemical composition of the system far away from equilibrium. This problem has been studied previously in [11, 12, 13] using ideal fluid, in [14] using first order theory of dissipative fluid dynamics, and recently in [15] using parton cascade.

Ideal (Euler) fluid dynamics has been useful in describing most of the observables at RHIC [16, 17]. In the early stages of heavy ion collisions, non–equilibrium effects play a dominant role. A complete description of the dynamics of heavy ion reactions needs to include the effects of dissipation through non–equilibrium/dissipative fluid dynamics. As is well–known [8, 10], second order (or extended) theories (which are hyperbolic and causal) of dissipative fluids due to Grad [5], Müller [6], and Israel and Stewart [7] were introduced to remedy some undesirable features such as acausality. It seems appropriate therefore to resort to hyperbolic theories instead of first order
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2. Viscous hydrodynamics and chemical equilibration

In this work we extend the work of [8] to include chemically non-equilibrium effects. For partially equilibrated plasma of massless particles the equation of state can be written as [11]

\[ \varepsilon = 3p = [a_2\lambda_g + b_2(\lambda_q + \bar{\lambda}_q)] T^4, \]

where \( a_2 = 8\pi^2/15 \), \( b_2 = 7\pi^2 N_f/40 \) with \( N_f \) being the number of quark flavours, and the \( \lambda_i \) are the parton fugacities defined through

\[ n_g = \lambda_g n_g^{eq}, \quad n_q = \lambda_q n_q^{eq}, \quad n_q = \lambda_q n_q^{eq}, \]

where the \( n_i^{eq} \) are the equilibrium parton densities

\[ n_g^{eq} = a_1 T^3, \quad n_q^{eq} = b_1 T^3 = n_q^{eq}, \]

where \( a_1 = (16/\pi^2)\zeta(3) \) and \( b_1 = (9/2\pi^2)\zeta(3) \). The shear viscosity coefficient is given by

\[ \eta = \lambda_g \eta_g + \lambda_q \eta_q, \]

where the shear viscosity coefficients for the quarks and gluons are given by [19] [20] [21]

\[ \eta_q = b_q T^3, \quad \eta_g = b_q T^3, \]

where \( b_q = 0.82(\alpha_s^2 \ln(1/\alpha_s))^{-1} \) and \( b_g = 0.20(\alpha_s^2 \ln(1/\alpha_s))^{-1} \) with \( \alpha_s \) being the strong coupling constant. We take \( \alpha_s = 0.4 \) throughout this analysis, unless otherwise stated.

In the absence of chemical equilibrium we need the master equations for the evolution of parton densities. We consider only the dominant reactions \( gg \leftrightarrow ggg \), and \( gg \leftrightarrow q\bar{q} \). For longitudinal boost invariant longitudinal flow under the assumed equation of state and transport coefficients the energy equation and shear pressure [8] [18] become

\[ \frac{\dot{\lambda}_g + b(\dot{\lambda}_q + \dot{\bar{\lambda}}_q)}{\lambda_g + b(\lambda_q + q)} + \frac{4\dot{T}}{T} + \frac{4}{3\tau} = \frac{1}{[a_2\dot{\lambda}_g + b_2(\dot{\lambda}_q + \dot{\bar{\lambda}}_q)] T^4} \Phi, \]

\[ \frac{d\Phi}{d\tau} + \frac{2}{9} \frac{[a_2\dot{\lambda}_g + b_2(\dot{\lambda}_q + \dot{\bar{\lambda}}_q)] T^4 \Phi}{\lambda_g b_q + \lambda_q b_q} = -\frac{1}{2} \Phi \left[ \frac{1}{\tau} - \left[ \frac{5\dot{T}}{T} + \frac{\dot{\lambda}_g + b(\dot{\lambda}_q + \dot{\bar{\lambda}}_q)}{\lambda_g + b(\lambda_q + q)} \right] \right] \]

\[ + \frac{8}{27} \frac{[a_2\dot{\lambda}_g + b_2(\dot{\lambda}_q + \dot{\bar{\lambda}}_q)] T^4}{\tau}, \]

and are coupled to the master equations for the fugacities [11] [12] [13]

\[ \frac{\dot{\lambda}_g}{\lambda_g} + \frac{3\dot{T}}{T} + \frac{1}{\tau} = R_3(1 - \lambda_g) - 2R_2 \left( 1 - \frac{\lambda_q \lambda_q}{\lambda_g^2} \right), \]

\[ \frac{\dot{\lambda}_q}{\lambda_q} + \frac{3\dot{T}}{T} + \frac{1}{\tau} = R_2 \frac{a_1}{b_1} \left( \frac{\lambda_g}{\lambda_q} - \frac{\lambda_q}{\lambda_g} \right), \]
where \( b = b_2/a_2 = 21N_f/64 \) with \( N_f \) being the number of quark flavours. The reaction rates \( R_2 \) and \( R_3 \) are given by

\[
R_2 \simeq 0.24N_f\alpha_s^2\lambda_g T \ln \left( \frac{1.65}{\alpha_s \lambda_g} \right), \quad R_3 \simeq 2.1\alpha_s^2T\sqrt{2\lambda_g - \lambda_g^2}.
\]  

(10)

We consider initial conditions relevant for RHIC: \( \tau_0 = 0.25 \text{ fm/c}, T_0 = 0.66 \text{ GeV}, \Phi_0 = p_0/5, \lambda_g0 = 0.34, \) and \( \lambda_q0 = 0.064 \); and for LHC: \( \tau_0 = 0.25 \text{ fm/c}, T_0 = 1.0 \text{ GeV}, \Phi_0 = p_0/5, \lambda_g0 = 0.43, \) and \( \lambda_q0 = 0.082 \). Here \( p_0 \) is the initial pressure. These sets of initial conditions are motivated in [12, 8].

3. Results

In Fig. 1(a,b) we show the well known results (see [8]) of the temperature evolution. The ideal fluid dynamics approximation leads to faster cooling. Due to the reduction of longitudinal pressure, less work is done to the expansion and hence the slow cooling in the presence of viscosity. However the first order theory even predicts heating during the expansion stage. This is in contradiction to the energy conservation laws. Also the first order theory will overestimate the freeze-out temperatures. This in turn might lead to wrong conclusions about the observables. On the other hand the second order theory does not have these undesirable features that are exhibited by first order theory.

In Figs. 2(a,b) and 3(a,b) we show time evolution of the parton fugacities. In the presence of viscosity the parton viscosities evolve more slowly towards chemical equilibration. This will have considerable effects on the observables such as strangeness production. In the early stages of the expansion of the system one sees the difference between first order theory and second order theory. This difference is investigated in more detail [18].
4. Conclusions

We have investigated the effects of shear viscosity on the chemical equilibration of the parton system in relativistic nuclear collisions. Due to slow cooling of the system in the presence of viscosity chemical equilibration is slowed. The effects of transverse expansion \cite{9} and of the mass of strange quark in the chemically non-equilibrated viscous system are being studied and will be published somewhere \cite{18}.

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