Geometric And Computer Modeling of Forming the Gear Wheels with Elliptical Centroid

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Abstract. A fairly simple and accurate methodology of automated forming of the teeth of a wheel with an elliptical centroid is proposed. This methodology is based on computer simulation of the real process of forming and does not require the application of complex mathematical body used in most traditional methods. The key point of the methodology is the proposed solution for determining the instantaneous centers of the cutting tool and the angles of its rotation around these centers. The relative position of the cutting tooth profiles is determined by two arrays of point coordinates: one on the elliptical centroid of the non-circular wheel, the other - on its equidistant curve. Compared to the traditional methods, this methodology is a universal and much more efficient and accurate one, especially for wheels with an elliptical centroid. The proposed automated solid-state computer simulation of the forming allows to obtain not only the envelope part of the profile, but also to observe possible formation of transition curves on it and undercutting, which leads to a qualitative solution of the profiling problem. According to the developed algorithms and programs, a computer model of a non-circular wheel is obtained, on the basis of which a corresponding item used in hydraulic unit pumps is made on a CNC machine.

Keywords: non-circular wheels, elliptic centroid, forming, solid modeling

1. Introduction

Products with non-circular wheels are increasingly used in various industries. Thus, non-circular wheels are widely used in textile machines, flow meters and some other mechanisms and machines [1-4]. Non-circular wheels are increasingly used in hydraulic machine pumps used for pumping various liquids, as well as in dosing pumps [5, 6].

The wide use of gear items with non-circular wheels is constrained by the two not completely solved problems: 1) the complexity of manufacturing of teeth of non-circular wheels by the method of running using a variety of additional devices; 2) significant difficulties in creating models of the wheel teeth, because each tooth has its own profile, and the side profiles of one tooth are different.

The main way solving the first problem is the use of processing equipment with numerical control and implementation of electro-physical and electrochemical processing methods.

The studies to solve the second problem of the formation of non-circular wheels are carried out in several directions. One of them is based on the classical theory of envelopes [1]. If the centroid of the wheel is an ellipse, it is necessary to calculate the profile of each tooth, given that its sides are not equal. In this case, a complex mathematical body is used, involving numerical integration of the differential equation, which describes the movement of the contact point of the profiles along the line of action [7–11]. Traditional methods of profiling are advisable to use when high requirements are not required from the accuracy of the tooth profiles and the centroid consists of circumferences arcs [1, 5, 11]. The difficulties here are related to the need of applying different computational dependencies for each section of the centroid.
At present, the direction of forming the item with a tool is successfully developing, based on automated computer simulation of the real process of forming using CAD [11–14], followed by their integration with CAM systems.

The well-known publications do not focus on the fact that for the items with an elliptical centroid not only teeth, but also their sides are different due to the variable rolling speed of the circular centroid on the non-circular one. The modeling of the rolling round centroids of the generating unit over the ellipse is being carried out using complex mathematical algorithms or relevant mathematical software tools. Therefore, the creation of a methodology for the formation of a non-circular wheel, which would automate the simulation of real profiles of the teeth and reduce the production cycle of manufacturing these wheels, is relevant.

2. Problem Statement

The aim of this work is to automate the process of geometric and computer simulation of the formation of wheels teeth with an elliptical centroid in the CAD environment without the involvement of complex mathematical apparatus. The objectives of the study are: development of an algorithm for geometric and computer simulation of a circumference rolling over the elliptical centroid using the capabilities of modern CAD; development of a kinematic model of forming a gear with an elliptical centroid; implementation of the resulting kinematic model in an automated mode based on the methods of computer solid modeling to obtain a digital model of the item adapted to CAM-systems; providing the possibility of obtaining a real profile of the item, including special elements – transition curves and undercutting; creation of a full-scale sample of a non-circular wheel based on the results of the computer simulation.

3. Theory

1. Formation of initial modeling data

Let the centroid of the non-circular wheel of the detail is an ellipse - I, and the centroid of the generating tool is a circumference II of radius R (figure 1). The circumference rolls without sliding over the ellipse, and its centers are located on the curve III, which is the equidistant of the ellipse. Figure 1 shows the initial and an intermediate position of the circumference. It follows from the figure that the length of the arc A1B of the ellipse is equal to the length of the arc AB of the circumference. Then the kinematic scheme of simulating a circumference rolling over the ellipse, given in figure 1, allows us to propose the following algorithm for its implementation: 1) the circumference is moved to a new position so that its center is at the point O2 on equidistant III; 2) point A1 is determined by the contact of the circumference and the ellipse; 3) the length A1B of the ellipse arc is calculated; 4) the angle α is determined: 
\[
\alpha = \frac{AB}{R};
\]
5) the angle ε is calculated; 6) the circumference is rotated around the point O2 by angle α+ε.

The given algorithm is the basis for its geometrical and computer simulation of a circumference rolling over the ellipse.

2. Geometric modeling of the process of rolling a circumference in an ellipse.

As it is known, an ellipse can be specified by parametric equations of the form

\[
\begin{align*}
x &= a \cdot \sin t, \\
y &= b \cdot \cos t,
\end{align*}
\]

at \( t \in [0,2\pi] \), where a is the major semi-axis of the ellipse, b is the minor semi-axis of the ellipse, and the center of the ellipse is at the origin of the XY coordinate system. In figure 1 \( a=OC, b=OB \).

Then the equation of the normal to the ellipse at some point \( A_1(x_0, y_0) \) will have the following form:

\[
y - y_0 = \tan \theta (x - x_0),
\]

and the equidistant of the ellipse is determined by the following system of equations:
The arc length of a plane curve defined by parametric equations is calculated by the formula [15]

\[
L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt
\]

The arc length of a plane curve defined by parametric equations is calculated by the formula [15]

\[
x = a \pm \frac{d \cdot b}{\sqrt{a^2 \cdot \cos^2(t) + b^2 \cdot \sin^2(t)}} \sin(t), \\
y = b \mp \frac{d \cdot a}{\sqrt{a^2 \cdot \cos^2(t) + b^2 \cdot \sin^2(t)}} \cos(t),
\]

(3)

Then for the ellipse given by the equations (1) we obtain:

\[
L_x = \int_{t_1}^{t_2} \sqrt{(a^2 \sin^2 t + b^2 \cos^2 t)} \, dt
\]

or in the form

\[
L_x = a \int_{t_1}^{t_2} \sqrt{1 - e^2 \cos^2 t} \, dt,
\]

(4)

where \(e<1\).
This integral is an elliptic integral of the second kind \( E(t,e) \) and is not expressed in elementary functions, which causes significant difficulties both in the implementation of the algorithm and in the solution of the problem by the methods proposed in the known publications of other researchers. These dependences are calculated to obtain the coordinates of the center of the circumference and the angles of its rotation. According to figure 2, if the coordinates of some point \( A(x_0, y_0) \) on the ellipse are known, then the normal (2) to it at this point crosses the coordinate axis \( x \) at point \( M \), for which

\[
ctg \alpha = \frac{x_0 - M}{y_0}\,
\]

\( y = 0 \). Then the angle of inclination of the normal to the \( x \) axis is

\[
\beta = \frac{\pi}{2} - \frac{\alpha}{2}
\]

(figure 1). As a result, the coordinates of the center of the circumference are determined from the dependencies

\[
\begin{align*}
x_{o_2} &= x_0 + h \cos \beta, \\
y_{o_2} &= y_0 + h \sin \beta,
\end{align*}
\]

(5)

where \( h = O_1B \).

Thus, the position parameters of the circumference center and one of the angles of its rotation are determined, but the problem of determining the angle \( \alpha \) remains unsolved, i.e. determining the length of the ellipse arc \( \alpha \) (4), which is one of the main problems of modeling the formation of non-circular wheels. Below a solution to this problem using computer simulation in CAD environment is suggested.

3. Computer modeling of the process of circumference rolling over the ellipse.

Some CAD systems with powerful modeling tools allow, in particular cases, to divide a plane curve into \( n \) equal parts. We use this opportunity to solve the problem that allows us to solve this significant problem of numerous calculations of the ellipse arc lengths. Computer simulation algorithm includes several stages. At the first stage, let us divide the fourth part of the ellipse into \( n \) parts (figure 3) assuming that the resulting model will have two axes of symmetry. The number of parts is selected from the condition of the required modeling accuracy. In the automated mode, an array of their coordinates is formed by these points in the form of a list in the AutoLISP programming language. This is the first array of point coordinates. It is used to create a second array of coordinates of points located on the equidistant of the ellipse. To do this, the normal (2) is calculated at each point on the ellipse and its intersection with equidistant (3) is determined. Figure 3 shows the normal \( N_i \) going through the \( i \)-th point of the ellipse. The second array of points is used both to set the centers of the circular centroid and to calculate the angles \( \beta \) and \( \varepsilon \) of its rotation according to the above dependencies (figure 2).

The possibility of the described algorithm of computer modeling of the formation of a gear wheel with an elliptical centroid is illustrated by the experiments. They are based both on the above results and the ones obtained earlier by one of the authors [11], [16, 17].

4. Experimental results

In the computer experiment, the generating wheel with a rectilinear tooth profile was chosen, and the half-axes of the centroids of the non-circular wheel are equal: \( a = 30 \) mm, \( b = 22 \) mm (figure 3). The diameter of the centroid of the wheel with periodic teeth is calculated so that the length of the fourth part of the elliptical centroid of the non-circular wheel is a multiple of the teeth step of the generating wheel. According to these parameters, solid models of the generating wheel and the workpiece with a non-circular centroid are designed. They are the source for the automated computer modeling of the direct problem of forming.

Modeling of an item forming is performed according to the developed algorithms and programs in the CAD environment without involving complex mathematics. This modeling is based on Boolean operators. The coordinates of the centers of the circular centroid are determined by dividing the ellipse into equal parts, followed by the construction of normals through them. This approach eliminates the need for multiple calculation of the ellipse arc lengths by complex dependencies. At each point in the center of the circumference, the required rotation angles were calculated to model the rolling of the circular centroid. The modeling result is shown in figure 4a.
Figure 2. Models of ellipse I, its equidistants III and some of their parameters

Figure 3. A computer model of the determination of the moving centroid centres
To compare the obtained tooth profiles on different parts of the non-circular wheel, figure 4b shows its fragment in a larger scale. It follows that the lateral profiles of one tooth are different, and transition curves for some teeth are possible.

The resulting digital model of an item is used for its manufacture on CNC machine. A computer model of a non-circular wheel is shown in figure 5a, while the item made according to it – in figure 5b.

During the modeling, the formation of a non-circular wheel, the developed programs allow to perform modeling and removable layers, which can be used to analyze their qualitative and quantitative characteristics in order to assign optimal values of the tool feed parameter, the number of passes, as well as to identify the load of the cutting edges.

**Figure 4.** Computer solid-state models of generating and non-circular wheels after forming (a) and a fragment of the wheel crown on an increased scale – (b).

**Figure 5.** A gear wheel with an elliptical centroid: a) computer model; b) made of metal

5. Results and discussion

In the process of forming a gear with an elliptical centroid, the generating wheel rolls at a variable speed. As a result, not only all the teeth of the non-circular wheel differ, but also the lateral profiles of these teeth, which was shown by the experiments carried out in the work. The solution of this problem by analytical
methods requires the involvement of complex mathematical apparatus and involves significant difficulties. Even more difficult in this case is the solution of the inverse problem of forming. Solid-state computer modeling, implemented at the virtual simulation level by means of CAD, based on the developed new algorithms and programs allows us to automate the process of forming. At the same time, along with obtaining tooth profiles, it is possible to identify transition curves and undercutting. It gives an idea of the design and technological conditions of their appearance, allows to conduct corresponding research and make the necessary adjustments to the kinematic scheme of formation. The proposed methodology allows to carry out the modeling of non-circular wheels forming when changing the shape of their centroid from a single point of view. Further improvement of the developed methodology is aimed at modeling the formation of items with a composite non-circular centroid.

6. Conclusion
1. A methodology for solving the problem of forming a non-circular wheel is proposed, which uses CAD capabilities to simulate the rolling of a circumference over the ellipse. An algorithm for automated calculation of the coordinates of the centers of the circular centroid and the angles of its rotation when rolling over the ellipse is developed. This solution is effective and does not require the involvement of complex mathematical body.

2. Algorithms and programs of the automated solution of the problem of forming a non-circular wheel of an item with periodic teeth at the virtual simulation level are developed. This solution provides an opportunity to obtain not only the desired tooth profiles as envelopes of line families, but also to model the undercut lines and transition curves, which allows to achieve a qualitative solution to the profiling problem, while reducing the production cycle of manufacturing a non-circular gear wheel. The digital model of a non-circular wheel obtained in the experiment is used to manufacture a full-scale item on a CNC machine.

3. The developed programs let us make solid models of the layers to be removed, based on the analysis of which it is possible to solve technological problems – to assign the optimal value of the tool feed parameter and the number of passes during forming.

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