Models of inflation liberated by the curvaton hypothesis

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Abstract

It is usually supposed that inflation is of the slow-roll variety, and that the inflaton generates the primordial curvature perturbation. According to the curvaton hypothesis, inflation need not be slow-roll, and if it is the inflaton generates a negligible curvature perturbation. We find that the construction of slow-roll inflation models becomes much easier under this hypothesis. Also, thermal inflation followed by fast-roll becomes viable, with no slow-roll inflation at all.

1 Introduction

The primordial density perturbation, responsible for the origin of structure in the Universe, is dominated by its adiabatic component though significant iso-curvature components are not ruled out. The adiabatic component is determined by the curvature perturbation \( \zeta \) of uniform-density slices of spacetime, which has an almost flat spectrum. The normalization of the spectrum at the scales explored by the CMB anisotropy is given by [1]

\[
P_\zeta^{1/2} \simeq 5 \times 10^{-5}.
\] (1)

The spectral index \( n \equiv 1 + d \ln P_\zeta / d \ln k \) is in the \((1-\sigma)\) range [1, 2]

\[
n = 0.97 \pm 0.03.
\] (2)

Since it is present on super-horizon scales, the primordial curvature perturbation originates presumably during an era of inflation, at the beginning of which the whole observable Universe is inside the horizon. The usual hypothesis (which we shall call the inflaton hypothesis) is that the curvature perturbation comes from the vacuum fluctuation of the inflaton field, defined in this context as the one whose value determines the end of inflation. This makes it quite difficult to construct sensible models of slow-roll inflation [3, 4], and of course it rules out completely the possibility that the curvature perturbation might originate during thermal inflation [5, 6, 7, 8, 9] which does not have an inflaton.

According to the inflaton hypothesis, the curvature perturbation has already reached its observed value at the end of inflation and does not change thereafter. The simplest alternative is to suppose that the curvature perturbation is negligible at the end of inflation, being generated later from the perturbation of some ‘curvaton’ field different from the inflaton [10]. (This possibility was actually noticed much earlier in two papers
This curvaton paradigm has attracted a lot of attention [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48] because it opens up new possibilities both for observation and for model-building.¹ There is another aspect of the curvaton hypothesis though, that has received hardly any attention so far. This is the fact that the task of building a viable model of inflation becomes much easier, if the model is liberated from the requirement that the inflaton be responsible for the curvature perturbation.

The layout of the paper is as follows. In Section 2 we recall the basics of slow-roll inflation. In Section 3 we recall the expected form of the potential in field theory. In Section 4 we examine specific slow-roll models to see what is the effect of liberating them. In Section 5 we ask whether cosmological scales can instead leave the horizon during fast-roll inflation, and in Section 6 we ask the same question for thermal inflation. We conclude in Section 7. Throughout the paper we use units such that \( \bar{h} = \bar{c} = 1 \), in which Newton’s gravitational constant is \( 8\pi G = M_P^{-2} \), where \( M_P = 2.4 \times 10^{18} \text{ GeV} \) is the reduced Planck mass.

## 2 Slow-roll inflation

### 2.1 The basic equations

Slow-roll inflation, with a single-component inflaton field, is described by the following basic equations [3, 4]. In these equations, \( \phi \) is the inflaton field whose value determines the end of inflation, and \( V = V(\phi) \) is the potential during inflation. The other quantities are the scale factor of the Universe \( a \), the Hubble parameter \( H = \dot{a}/a \), and the wavenumber \( k/a \) of the cosmological perturbations.

The potential satisfies the flatness conditions

\[
\epsilon \ll 1 \tag{3}
\]
\[
|\eta| \ll 1 \tag{4}
\]

with \( \epsilon \equiv \frac{1}{2}M_P^2\left(V'/V\right)^2 \) and \( \eta \equiv M_P^2V''/V \), where the prime denotes derivative with respect to the inflaton field \( \phi \). The inflaton’s trajectory will then be an attractor, satisfying the slow–roll approximation

\[
3H\dot{\phi} \simeq -V'. \tag{5}
\]

The energy density is \( \rho \simeq V \) leading to

\[
V \simeq 3M_P^2H^2. \tag{6}
\]

¹According to the scenario developed in the above papers, the curvature perturbation is generated by the oscillation of the curvaton field. A different idea [49] is that the field causing the curvature perturbation does so because its value determines the epoch of reheating, and another is that it does so through a preheating mechanism [50]. For the purpose of the present paper, the term ‘curvaton’ covers all three cases.
Given the potential, Eqs. (5) and (6) can be integrated to give the inflaton trajectory $\phi(t)$ up to the choice of $t = 0$.

The inflaton generates a Gaussian curvature perturbation with spectrum

$$
\frac{4}{25} \mathcal{P}_R(k) = \frac{1}{75 \pi^2 M_p^6 V^2} \bigg|_{*}
$$

where the star denotes the epoch of horizon exit $k = aH$. Eq. (5) determines the number $N(k)$ of e-folds of slow-roll inflation occurring after horizon exit to be

$$
N(k) = M_p^{-2} \int_{\phi_{\text{end}}}^{\phi_*} \left( \frac{V}{V'} \right) d\phi.
$$

On cosmological scales, $N(k)$ is given by

$$
N(k) = 67 - \ln \left( \frac{k}{H_0} \right) - \ln \left( \frac{M_p}{V_0^{1/4}} \right) - \Delta
$$

$$
\Delta \equiv \frac{1}{3} \ln \left( \frac{V_0^{1/4}}{T_{\text{reh}}} \right) + N_0.
$$

Here $H_0$ is the Hubble parameter at present, $T_{\text{reh}}$ is the reheating temperature after inflation, and $N_0 = 0$ if the Universe remains radiation-dominated after reheating, until the onset of the present matter-dominated era. The number $N_0$ is positive if there is more inflation after the almost-exponential era, or if radiation-domination is interrupted by one or more matter-dominated eras. There is no reasonable cosmology for which $N_0$ is negative and $a$ fortiori none for which $\Delta$ is negative. Knowing $N(k)$ and $\phi_{\text{end}}$, Eq. (8) determines $\phi_*$ and then Eq. (7) determines the curvature perturbation generated by the inflaton.

### 2.2 The inflaton hypothesis

According to the inflaton hypothesis, the curvature perturbation on cosmological scales remains constant as long as these scales are far outside the horizon. Comparing Eqs. (1) and (7), this requires at the epoch when the CMB scale leaves the horizon the CMB normalization,

$$
V^{1/4} = 0.027 e^{1/4} M_p.
$$

Differentiating Eq. (7) and using the slow-roll expression $3H\dot{\phi} = -V'$ gives the spectral index

$$
n = 1 + 2\eta - 6\epsilon.
$$

(From now on, $\eta$ and $\epsilon$ will always be evaluated at horizon exit.) In nearly all inflation models, $\phi/M_p$ is very small while cosmological scales leave the horizon, making $\epsilon$ completely negligible [3, 4] so that for practical purposes

$$
n = 1 + 2\eta.
$$
In a large class of models, $\eta$ has the form

$$\eta(\phi) = \text{const } \phi^{p-2}$$

with $p < 1$ or $p > 2$. This leads to

$$n(k) - 1 = -\frac{p - 1}{p - 2N(k)} \frac{2}{N(k)} < 0,$$

making $n$ on cosmological scales almost scale-independent and significantly below 1. Within a few years $n$ will be determined with an accuracy of order $\pm 0.01$, allowing this relation to be confronted with observation [51].

2.3 The curvaton hypothesis

In this paper we adopt the curvaton hypothesis, that the curvature perturbation comes primarily from the vacuum fluctuation of some curvaton field different from the inflaton. Instead of the CMB normalization we therefore have the CMB bound,

$$V^{1/4} \ll 0.027e^{1/4}M_P.$$

Requiring that the curvature perturbation be, say, less than 1% of the observed value, the left-hand-side of this expression must be less than 10% of the total giving

$$V^{1/4} < 2 \times 10^{15} \text{ GeV},$$

which means that a gravitational wave signal will never be detected in the CMB anisotropy [52].

In the curvaton model, the condition $|\eta_{\sigma\sigma}| \lesssim 1$ is needed so that the vacuum fluctuation of $\sigma$ is converted into a classical one, where

$$\eta_{\sigma\sigma} \equiv \frac{M_P^2}{V} \frac{\partial^2 V}{\partial \sigma^2}$$

Taking $|\eta_{\sigma\sigma}| \ll 1$, the spectral index in the curvaton model is given by [10]

$$n = 1 + 2\eta_{\sigma\sigma} - 2\epsilon,$$

where the right hand side is evaluated at the epoch of horizon exit. Discounting an accidental cancellation, Eq. (2) requires

$$\epsilon \lesssim 1/20$$

In the large class of inflation models where $\epsilon$ is completely negligible, $n - 1$ is determined by $\eta_{\sigma\sigma}$ which specifies the second derivative of the potential in the curvaton direction. Since there is no reason why this quantity should increase during inflation, it is reasonable to suppose that it is negligible on cosmological scales, leading to a spectral index indistinguishable from 1.
3 The expected form of the inflaton potential

According to the usual assumption (see Section 4.6 for a possible exception) a single effective field theory holds from the epoch of inflation until the present. To keep the Higgs stable this field theory presumably respects supersymmetry (SUSY), which presumably is local corresponding to supergravity (SUGRA). In the vacuum, SUSY is obviously broken, and it is also broken in the early Universe because of the nonzero energy density. The breaking at the level of SUGRA must be spontaneous, which strongly suggests $N = 1$ SUGRA since it seems very difficult to spontaneously break $N > 1$ to the Standard Model. Although the detailed form of the SUGRA theory is not known, certain features are expected [53, 54, 3, 4] on the basis of generic ideas about field theory, and about the presumed underlying string theory involving extra dimensions that have been integrated out. In this section we recall the main feature that are relevant for inflation model-building.

3.1 The potential near the vacuum

In the vacuum, phenomenology demands that the sector of the theory in which spontaneous breaking occurs (the SB sector) be distinct from the sector containing the Standard Model and its minimal extension (the MSSM). To a good approximation, the theory in the MSSM sector should respect global SUSY with explicit (soft) SUSY breaking terms. The SB sector may communicate with the MSSM sector by interactions which are of gravitational strength (gravity mediated SUSY breaking), or stronger and typically involving a gauge symmetry (gauge, gaugino mediated supersymmetry breaking). There is also the case of anomaly-mediated SUSY breaking, where the effect of spontaneous SUSY breaking is felt only through the gravitational anomaly.

In the next section we are going to consider some supersymmetric models of inflation. In them, as in all known models, the inflaton belongs to a sector of the theory which (at least during inflation) is to some extent decoupled from the MSSM sector. Depending on the model, the SB sector during inflation may or may not be the same as the SB sector in the vacuum. Where it is different, the possibilities for communication between the SB sector and the inflaton sector are the same as those already mentioned for the case of the vacuum (gravity-mediated etc.). There is also the additional possibility that the SUGRA during inflation is broken in the inflaton sector itself, with no separate SB breaking sector. We shall encounter examples of all these cases.

The form of the scalar field potential is

$$V_\phi = V_+ (\phi_i) - 3M_0^2m_{3/2}^2(\phi_i)$$ (21)

where $\phi_i$ are the scalar fields. Both terms are positive, and in the vacuum $m_{3/2}$ is the gravitino mass. The first term $V_+$ is a measure of the strength of spontaneous supersymmetry breaking generated by the potential.
Consider first the form of the potential near the vacuum, where $V$ (practically) vanishes. The scale of supersymmetry breaking in the vacuum is denoted by $M_S$;

$$\langle V_+ \rangle \equiv M_S^4$$

(22)

Since $V$ (practically) vanishes in the vacuum, the gravitino mass is given by

$$\sqrt{3}m_\frac{3}{2} = M_S^2 / M_P.$$  

(23)

In the direction of a canonically-normalized real field $\phi$, let us for the moment take $\phi = 0$ as the vacuum value (VEV). Assuming for simplicity a symmetry $\phi \rightarrow -\phi$, the tree-level potential will have the form

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \sum_{\text{even} i} M_P^{4-i} \lambda_i \phi^i.$$  

(24)

For squark and slepton fields, the mass vanishes in the limit of unbroken SUSY,\(^2\) and SUSY breaking gives a value

$$m \sim \sqrt{3}CM_S^2 / M_P = Cm_\frac{3}{2},$$

(25)

(26)

where the number $C$ depends on the mediation from the hidden to the MSSM sector;

$$C \sim 1$$ (gravity – mediated)

$$C \gg 1$$ (gauge/gaugino – mediated)

$$C \sim 10^{-3}$$ (anomaly – mediated).  

(27)

To avoid detection and yet keep the Higgs mass under control one needs $m \sim 100$ GeV to 1 TeV, giving

$$\sqrt{C}M_S \sim 10^{10} \text{ GeV}. $$

(28)

In a generic direction among the Higgs, squark and slepton fields, $\lambda$ is of order 1 but there are ‘flat’ directions where $\lambda$ is practically zero because it vanishes in the limit of unbroken SUSY. It is generally assumed that $\lambda_i \sim 1$ even in the flat directions.

There may be other light fields, in particular moduli whose entire potential vanishes in the limit of unbroken SUSY. We shall make the usual assumption that the potential of a modulus has the form $V = M_S^4 f(\phi/M_P)$ with $f(x)$ and its derivatives of order 1 at a generic field value in the range $\phi \lesssim M_P$. This corresponds to $m \sim m_\frac{3}{2}$ and $|\lambda| \sim |\lambda_i| \sim (m/M_P)^2$.

\(^2\)The following is true for the Higgs fields too, except that there is a small mass ($\mu$-term) in the limit of unbroken SUSY.
3.2 The potential during inflation

During inflation, the potential $V$ is large so at least one field must be far from the vacuum. If the only field with significant displacement is the inflaton we have non-hybrid inflation and Eq. (24) applies with $\phi$ the inflaton. Hybrid inflation is the case where a second field is significantly displaced, being responsible for $V_0$. We focus on slow-roll inflation, which requires the flatness conditions Eqs. (3) and (4). One possibility (‘chaotic inflation’) is a quadratic or quartic potential with $\phi \gg M_p$, corresponding to extreme suppression of the relevant non-renormalizable terms in Eq. (24). Barring this case, all proposed models of inflation suppose that the tree-level inflaton potential during inflation is of the form

$$V(\phi) = V_0 \pm \frac{1}{2} m^2 \phi^2 + \sum_{i=3}^{\infty} \lambda_i M_p^{4-i} \phi^i,$$  

with $\phi \lesssim M_p$. The origin is here taken to be a maximum or a minimum, the former option being mandatory for non-hybrid inflation. (Typically, the origin is the fixed point of the relevant global and/or gauge symmetries.) During slow-roll inflation, the constant term $V_0$ dominates, and only one or two terms in the expansion are supposed to be significant. Usually these are renormalizable terms (quadratic, cubic or quartic). To achieve inflation over a range of $\phi$, each term separately should satisfy Eq. (4), and then $\phi \ll M_p$ ensures that Eq. (3) is satisfied.

During inflation, SUSY is broken by the energy density $\sim V_0$, and to keep the potential as flat as possible one assumes that the relevant $\phi$-dependent terms in Eq. (29) come entirely from the SUSY breaking. (If global SUSY is a good approximation this can be achieved by choosing $\phi$ to be a flat direction. Alternatively it can be achieved by choosing $\phi$ to be a modulus.) This, however, is not enough to guarantee Eq. (4). On the contrary, for a generic field during inflation, the mass generated by SUSY breaking is at least$^3$

$$|m^2| \sim V_0/M_p^2.$$  

If there were a significant degree of cancellation between the two terms of Eq. (21), this estimate would strongly violate Eq. (4). Such a cancellation would be expected if $V_0 \ll M_S^4$, and accordingly such low values of $V_0$ are disfavoured (see Sections 4.2 and 4.5). For bigger values of $V_0$, the estimate Eq. (30) violates Eq. (4) only marginally. A sufficiently small mass can in this case occur through and accident, through a running mass, through a special form for the potential during inflation, or because of a global symmetry [4].

$^3$The mass is bigger for gauge/gaugino-mediated case, making that case definitely unsuitable for the inflaton. In contrast with the vacuum case, the mass during inflation is not generically smaller in the anomaly-mediated case (corresponding to no-scale SUGRA) [55, 56]. We are assuming that SUSY is broken by $F$ terms, but breaking it by $D$ terms may not help. This is because [57] the loop correction then requires that inflation takes place at $\phi \sim M_p$, making it difficult to control the non-renormalizable terms.
4 Liberated models of slow-roll inflation

In this section we examine some models of slow-roll inflation. Under the inflaton hypothesis, reviews of most of them have been given already [3, 4], to which we refer for further detail and references. We omit some models which have too many parameters for a definite statement to be made about the effect of liberating them, and some proposals which were not carried through to the point of producing a specific model, but otherwise our coverage of extant models is fairly comprehensive.

4.1 Slow-roll modular inflation

Starting with [5], many authors (eg. [58, 59, 60]) have considered the possibility of slow-roll non-hybrid inflation where the inflaton is a modulus with the kind of potential we described earlier. Inflation would take place around a maximum, where

\[ V = V_0 - \frac{1}{2} m^2 \phi^2 + \cdots, \tag{31} \]

with the remaining terms negligible in the regime \( V \sim V_0 \) and

\[ m^2 \sim V_0/M_P^2, \tag{32} \]

which will also be roughly the mass-squared of \( \phi \) in the vacuum at \( \langle \phi \rangle \sim M_P \).

In the regime \( \phi \ll M_P \) where Eq. (31) is valid, the flatness parameter \( \epsilon \) is practically zero but the other flatness parameter \( \eta \) is given by

\[ \eta \simeq -\frac{m^2 M_P^2}{V_0} \simeq -\frac{m^2}{3H^2}. \tag{33} \]

The generic estimate Eq. (32) would make \( |\eta| \sim 1 \), but one can imagine that the actual value of \( \eta \) calculated for some modulus in a specific string theory is significantly smaller. This is the hypothesis of slow-roll modular inflation.

Inflation with the potential Eq. (31) (with the remaining terms negligible in the regime \( V \simeq V_0 \)) has also been considered without any specific hypothesis about the identity of the inflaton (‘natural’ [60, 61] and ‘topological’ [62] inflation). The CMB bound for such a potential is [4] \( V_0^{1/4} \ll 10^{16} \text{GeV} \), corresponding to mass

\[ m \ll 10^{14} \text{GeV}. \tag{34} \]

With \( \phi \) identified as a modulus, it is usually supposed that \( m \) is of order the gravitino mass which is expected to be at most of order 100 TeV (corresponding to anomaly-mediated [63] supersymmetry breaking). This is in strong conflict with the inflaton hypothesis, according to which the CMB bound is saturated. In contrast, if we adopt the curvaton hypothesis there is no problem.
It has been proposed instead [59] that the modulus mass during inflation is of order $10^{14}$ GeV so that the inflaton hypothesis is satisfied. However, in all models invoking the potential Eqs. (31) and (32) the inflaton hypothesis presents another possible problem. This is the fact that even if $|\eta|$ is suppressed sufficiently to allow slow-roll inflation, the spectral index $n \simeq 1 + 2\eta$ may turn out to be too far below 1 to be compatible with observation. This problem cannot occur under the curvaton hypothesis, since the spectral index Eq. (19) is then independent of $\eta$.

We mention in passing the existence of a quite different model, which also gives $n$ too far below 1 on the inflaton hypothesis but is liberated by the curvaton hypothesis. It comes up as an example of a model [64] in which the flatness of the inflaton potential in hybrid inflation is protected by a non-Abelian global symmetry, broken only by non-renormalizable terms. The example, the only one worked out in detail so far [65], gives $\epsilon$ very small but a spectral index too far below 1 on the inflaton hypothesis. (The spectral index on the inflaton hypothesis is actually given by a more complicated formula than Eq. (13) in this model, because the inflaton in this model has two components.)

4.2 The gauge–mediated model of Dine and Riotto

In the previous model the mass-squared term is supposed to dominate until almost the end of inflation. An alternative is to have the mass-squared term dominating while cosmological scales leave the horizon, but to have a $-\lambda\phi^4$ term coming in well before the end of inflation. If $m$ and $\lambda$ are regarded as free parameters this represent fine-tuning, but we shall now consider a model in which the parameters are related so as to produce the inflaton evolution that we just described.

The model [66] works in the approximation of global supersymmetry, and it assumes that SUSY breaking in the MSSM sector is gauge-mediated according to the following standard scheme. In a secluded sector, a field $X$ and its auxiliary field $F_X \equiv \partial W/\partial X$ (with $W$ the superpotential) are both supposed to have nonzero VEVs, the latter determining the scale of supersymmetry breaking,

$$\langle F_X \rangle \equiv M_S^2$$

One requires $\Lambda \equiv F_X / \langle X \rangle$ of order $10^5$ GeV, so that the radiatively-generated soft masses in the MSSM sector have the desired magnitude $\sim 10^2$ GeV.

The inflaton field $\phi$ in this model is the real part of a gauge singlet field $S$, and its VEV generates the $\mu$ term of the MSSM [67]. During inflation, $X$ and $F_X$ as well as the gravitino mass are supposed to be close to their vacuum values, implying some degree of cancellation between the two terms of Eq. (21).

The superpotential is

$$W = -\frac{\beta XS^4}{M_P^2} + \frac{S^5}{M_P} + M_P^{-n}S^{n+1}H_U H_D + \cdots.$$  

This structure can be enforced by discrete symmetries. We have exhibited a coefficient $\beta \lesssim 1$ though eventually we shall favour a value of order 1. All other coefficients are
assumed to be of order 1 from the outset, and throughout the calculation we shall ignore numerical factors of order 1. The third term of the superpotential generates the $\mu$ term of the MSSM, but plays no role during inflation.

The dots represent the contributions to $W$ that do not involve $S$. They generate, among other things, a contribution to $F_X$, which is assumed to be the dominant contribution. Since we are assuming that $F_X$ is close to its VEV during inflation, this requires for consistency

$$\beta\langle \phi^4 \rangle \ll M_P^2 M_S^2$$

(37)

The corresponding potential is

$$V \simeq V_0 - m^2 \phi^2 - \frac{1}{4} \lambda \phi^4 + \left( M_P^4 \phi^8 + \beta^2 M_P^{-4} X^2 \phi^6 \right) + \cdots$$

(38)

with

$$\lambda = \beta M_P^{-2} M_S^2$$

(39)

and

$$m^2 \equiv \alpha V_0 / M_P^2.$$ (40)

(Note that the slow–roll condition Eq. (4) requires $\alpha \ll 1$.) We have dropped a term which is significant only if the two terms in the bracket have a similar magnitude.

The VEV $\langle \phi \rangle$ is determined by minimizing this potential. In the case $\beta M_S^2 \lesssim (\Lambda^2 M_P)^{2/3} \sim (10^9 \text{ GeV})^2$, one finds $\langle \phi^4 \rangle \sim \beta M_S^2 M_P^2$ (marginally consistent with Eq. (37) ) and

$$V_0 \sim \beta^2 M_S^4$$

(41)

In the opposite case $M_S^2 \gtrsim (10^9 \text{ GeV})^2$ one finds instead $\langle \phi \rangle^2 \sim M_P^2 \Lambda^2 / \beta M_S^2$ (comfortably consistent with Eq. (37) ) and

$$V_0 \sim \left( \frac{\Lambda^4 M_P^4}{\beta^3 M_S^6} \right) \beta^2 M_S^4$$

(42)

During inflation the bracketed term in Eq. (38) is negligible, while both of the first two terms are in general significant. A careful calculation reveals [4] that the CMB bound is in all cases $\lambda \ll 10^{-15}$, corresponding to

$$\sqrt{\beta M_S^2} \ll 10^{10.5} \text{ GeV}$$

(43)

On the inflaton hypothesis the bound is saturated, which practically kills the model for two reasons. First, gauge–mediated supersymmetry breaking requires $M_S$ to be significantly less than $10^{10}$ GeV, to validate the assumption that gravity–mediated breaking is negligible. This is in mild conflict with the expectation $\beta \lesssim 1$. Second, even with $\beta \sim 1$, Eq. (42) requires $V_0 \sim 10^{-7} M_S^4$ which means that the two terms of Eq. (21) must cancel.

\footnote{The following results are taken from [4], correcting the erroneous treatment of [66] which misses the second of the two cases below.}
with an accuracy $10^{-7}$, and means also that $\alpha$ has to be suppressed by a more than a factor $10^{-7}$ below its natural value. Both of these problems disappear if we adopt the curvaton hypothesis while retaining $\beta \sim 1$. In particular, the only requirement on $\alpha$ is that it be small enough for slow-roll, implying only mild tuning of $\alpha$ below the natural value of order 1.

### 4.3 Constraints on hybrid inflation

Now we turn to hybrid inflation models. Before considering a couple of specific models, we consider the constraints on the parameter space [68] arising from the fact that hybrid inflation necessarily involves an interaction between the inflaton field $\phi$ and some other field $\chi$, which in turn generates a loop correction that might violate the flatness conditions.

The potential for hybrid inflation is basically of the form

$$V(\phi, \chi) = V_0 + \Delta V(\phi) - \frac{1}{2} m_\chi^2 |\chi|^2 + \frac{1}{2} \lambda' \chi^2 \phi^2 + \frac{1}{4} \lambda \chi^4.$$

(Modifications of the last two terms are sometimes considered, which typically do not affect the following considerations.) Inflation takes place in the regime $\phi^2 > \phi_c^2$, where

$$\phi_c \equiv |m_\chi|/\sqrt{\lambda'}.$$

In this regime, $\chi$ vanishes and the inflaton potential is

$$V = V_0 + \Delta V(\phi).$$

The constant term $V_0$ is assumed to dominate during inflation.

The last term of Eq. (44) serves only to determine the VEV of $\chi$, achieved when $\phi$ falls below $\phi_c$. Using that fact that $V_0$ vanishes in the vacuum, one learns that the VEV is

$$\langle \chi \rangle = 2V_0^{1/2}/|m_\chi|,$$

and that

$$\lambda = \frac{4V_0}{\langle \chi \rangle^4} = \frac{m_\chi^4}{4V_0}.$$

The main difference between hybrid inflation models is in the form of $\Delta V$. In the original model [69],

$$\Delta V(\phi) = \frac{1}{2} m^2 \phi^2.$$

Later, models were proposed where $\Delta V$ is instead dominated by the loop correction coming from the interaction of the inflaton with $\chi$ and its superpartner. These models were formulated in the approximation of spontaneously broken global SUSY (involving either an $F$ term [55, 70] or a $D$ term [71, 72, 73]) giving a loop correction of the form

$$\Delta V = V_0 \frac{g^2}{8\pi^2} \ln \frac{\phi}{Q},$$

(45)
with the renormalization scale $Q$ chosen to make the second term small. The coupling $g$ is of order 1 for $D$-term inflation (gauge coupling) but may be smaller for $F$-term inflation (Yukawa coupling).

Alternatively, it may be that softly broken global SUSY is a good approximation during inflation. In that case the loop correction, coming from $\chi$ and its superpartner or from any other supermultiplet, giving

$$\Delta V = f^2 \phi^2 \ln \frac{\phi}{Q},\quad (51)$$

where $f$ is a measure of the coupling strength. This is the starting point for the running-mass model of Section 4.4.

Now we come to the source of the constraint on the parameter space of hybrid models. The loop correction coming from $\chi$ and its superpartner will presumably not be accurately canceled by other terms over the relevant range of $\phi$. The contribution of this loop provides an approximate lower bound for $\Delta V$ and $\Delta V'$, and the constraint on the parameter space comes from the requirement that this bound should respect the flatness conditions Eqs. (3) and (4). This constraint has been evaluated in [68]. We consider only the weakest constraint, corresponding to Eq. (50). Under the inflaton hypothesis, it may be written in the form

$$\langle \chi \rangle^4 \phi_{\text{CMB}} \gtrapprox \left(10^9 \text{GeV}\right)^5,\quad (52)$$

where the subscript CMB refers to the epoch at which the scales explored by the CMB anisotropy leave the horizon. This becomes an explicit constraint on the parameter space, once a form for $\Delta V$ is specified which allows $\phi_{\text{CMB}}$ to be calculated. For example, with the original form Eq. (49),

$$\langle \chi \rangle^3 \lesssim 5 \times 10^{-5} \sqrt{\lambda} M^3_P\quad (53)$$

$$\lambda' \lesssim (\eta/22)^{3/2}\quad (54)$$

$$\eta \lesssim (90 \langle \chi \rangle M_P)^4,\quad (55)$$

where $\eta = m^2 M^2_P/V_0$.

These constraints are very powerful. In particular, Eq. (52) means that un–liberated hybrid inflation cannot be expected to work if the ultra–violet cutoff is below $10^9$ GeV (since both $\langle \chi \rangle$ and $\phi_{\text{CMB}}$ are expected to be below this value). In particular, one cannot expect un–liberated hybrid inflation to work if there is an extra dimension with size $\gtrapprox (10^9 \text{GeV})^{-1}$.

In contrast, for liberated hybrid inflation Eq. (52) becomes

$$\langle \chi \rangle^2 \phi_{\text{CMB}} \gtrapprox \left(10^4 \text{GeV}\right)^3 \sqrt{N_{\text{CMB}}}\quad (56)$$

which in the case of Eq. (49) becomes

$$\lambda' V_0^{1/2} \lesssim 16\pi^2 m \langle \chi \rangle\quad (57)$$

These constraints are far weaker, and in particular Eq. (56) makes liberated hybrid inflation feasible for an ultra–violet cutoff as low as $10^4$ GeV.
4.4 Running-mass hybrid inflation

The running-mass model [74, 75] assumes that the inflaton sector can be described by global SUSY with explicit (soft) breaking, as opposed to the spontaneous breaking that gives Eq. (50). The inflaton mass is supposed to run significantly, corresponding to a gauge or Yukawa interaction between the inflaton and some supermultiplet (in the case of a Yukawa, the interaction might be the one involving $\chi$). At some high scale the inflaton mass-squared is supposed to have the generic magnitude of order $V/M_P^2$. At some lower scale $Q$ the running mass is supposed to pass through zero, generating a maximum or minimum of the potential at some field value $\phi_* \sim Q$. Near this value the potential is well-approximated [51] by the 1-loop correction Eq. (51). The resulting potential may be written

$$V = V_0 \left\{ 1 - \frac{c}{2M_P^2} \left[ \ln(\phi/\phi_*) - \frac{1}{2} \right] \right\}. \quad (58)$$

The CMB bound for this model is

$$\frac{V_0^{1/2}}{M_P} \ll 2 \times 10^{-5} c \phi_{\text{CMB}} \left| \ln \left( \frac{\phi_*}{\phi_{\text{CMB}}} \right) \right| \quad (59)$$

Under the inflaton hypothesis, the spectral index predicted by the running mass model is

$$n(k) - 1 = 2c \left\{ \ln \left( \frac{\phi_*}{\phi_{\text{CMB}}} \right) \exp[c\Delta N(k)] - 1 \right\} \quad (60)$$

$$\Delta N(k) \equiv N(k_{\text{CMB}}) - N(k). \quad (61)$$

The requirement that $n(k)$ be within observational bounds even now constrains the parameter space [51, 76, 77, 78], and in the future it might rule out the model altogether. Saturating the CMB bound may also make the value of $\phi_*$ rather low compared with theoretical expectations [77, 79]. Both of these possible problems disappear if the model is liberated.

4.5 The hybrid inflation model of Bastero-Gil and King

Like the model of Section 4.2, this one assumes that supersymmetry is broken during inflation by the same mechanism as in the vacuum, at the same scale $M_S$ [80]. In contrast with that model though, supersymmetry breaking is supposed to be transmitted with only gravitational strength, both to the inflaton and to the MSSM sector; in other words, we are dealing exclusively with gravity-mediated supersymmetry breaking, and $M_S \sim 10^{10}$ GeV.

In this model, the inflaton generates the $\mu$ term only indirectly. The superpotential is

$$W = \lambda N H_U H_D - \kappa SN^2 + \cdots \quad (62)$$
where $N$ and $S$ are gauge singlets. They respect the Peccei-Quinn global symmetry, whose pseudo-Goldstone boson is the axion which ensures the CP invariance of the strong interaction.

The axion is practically massless, and can be chosen such that $S$ is real. The inflaton is the canonically–normalized quantity $\phi = \sqrt{2} \text{Re} S$. During inflation $H_U H_D$ is negligible. Writing $\sqrt{2} N = N_1 + i N_2$, and including a soft supersymmetry breaking trilinear term $2A \kappa \phi N^2 + \text{c.c}$ (with $A$ taken to be real) as well as soft supersymmetry breaking mass terms, the potential is

$$V = V_0 + \kappa^2 |N|^4 + \frac{1}{2} \sum_i m_i^2(\phi) N_i^2 + \frac{1}{2} m_\phi^2 \phi^2,$$  

(63)

where

$$m_1^2(\phi) = m_1^2 - 2 \kappa A \phi + 4 \kappa^2 \phi^2,$$  

(64)

$$m_2^2(\phi) = m_2^2 + 2 \kappa A \phi + 4 \kappa^2 \phi^2.$$  

(65)

$$m_3^2(\phi) = m_3^2 - 2 \kappa A \phi + 4 \kappa^2 \phi^2.$$  

(66)

The soft supersymmetry breaking parameters $m_i$ and $A$ are supposed to have the typical values for gravity–mediated supersymmetry breaking,

$$m_i \sim A \sim M_S^2/M_P (\sim 100 \text{ GeV})$$  

(67)

In contrast, in order to achieve slow–roll inflation, the mass $m_\phi$ is supposed to be

$$m_\phi^2 \sim \alpha V_0/M_P^2$$  

(68)

with $\alpha \ll 1$.

The VEVs are given by

$$\langle \phi \rangle = \frac{A}{4 \kappa},$$  

(69)

$$\langle N_1 \rangle = \frac{A}{2 \sqrt{2} \kappa} \sqrt{1 - 4 m_1^2/A^2}$$  

(70)

$$\langle N_2 \rangle = 0.$$  

(71)

where we ignored the tiny effect of $m_\phi$. It is assumed that $4m_1^2$ is somewhat below $A^2$, so that

$$A \sim \kappa \langle N_1 \rangle \sim \kappa \langle \phi \rangle \sim 1 \text{ TeV}.$$  

(72)

To have the VEVs at the axion scale, say $10^{13} \text{ GeV}$, we require $\kappa \sim 10^{-10}$. Also, $\lambda$ should have a similar value, since $\lambda \langle N_1 \rangle$ will be the $\mu$ parameter of the MSSM. The tiny couplings $\kappa$ and $\lambda$ are supposed to be products of several terms like $(\psi/M_P)$ where $\psi$ is the VEV of a field that is integrated out.
During inflation, the fields $\mathcal{N}_i$ are trapped at the origin, and

$$V = V_0 + \frac{1}{2} m_i^2 \phi^2.$$  \hfill (73)

The field $\mathcal{N}_1$ is destabilized if $\phi$ lies between the values

$$\phi^\pm_c = \frac{A}{4\kappa} \left(1 \pm \sqrt{1 - \frac{4m_i^2}{A^2}}\right) \sim A/\kappa.$$  \hfill (74)

If $m_i^2$ is positive the model gives ordinary hybrid inflation ending at $\phi^+_c$, but if it is negative it gives inverted hybrid inflation ending at $\phi^-_c$. The height of the potential is

$$V_0^{1/4} \sim A/\sqrt{\kappa} \sim 10^8 \text{ GeV}$$  \hfill (75)

As $V_0$ is a factor $10^{-8}$ below $M_S^4$, the flatness condition $\alpha \lesssim 1$ requires that the $|m_i|^2$ is more than a factor $10^{-8}$ below the generic value given by Eq. (30).

The CMB bound for this model is

$$A \ll 5 \times 10^{-4} \alpha e^{\pm N_{\text{CMB}}} M_P$$  \hfill (76)

On the inflaton hypothesis, the bound is saturated, corresponding to $\alpha \sim 10^{-12}$ which requires that $|m_i|^2$ is a factor $10^{-12-8} = 10^{-20}$ below its generic value. Adopting instead the curvaton hypothesis, we require only the milder suppression by a factor $10^{-8}$ required by the flatness condition. This suppression comes from the mismatch between the height of the potential $V_0^{1/4} \sim 10^8 \text{ GeV}$ and the supersymmetry breaking scale $M_S \sim 10^{10} \text{ GeV}$. It would be interesting to see if the model could be modified to reduce this mismatch, for instance by lowering $M_S$ and generating $m_i$ and $A$ through interactions with the supersymmetry breaking sector which are of more than gravitational strength.\footnote{After the above words were written, a modification of the model has been formulated \cite{50} doing just that. In this model the Higgs field is responsible for the curvature perturbation, through a modified version of the curvaton mechanism.}

### 4.6 Inflation from a moving brane

In all of the models considered so far, the Universe is supposed to be described by a single effective field theory from inflation until the present day. In particular, this field theory is supposed to describe the reheating process that replaces the inflaton field with thermalized radiation.

As we have seen, the assumption of a single field theory makes it quite difficult to keep the inflaton potential flat enough. Now we come to a proposal which at least in spirit is different. This is the proposal \cite{81, 82, 83, 84, 85} that the inflaton corresponds to the distance between two branes moving in $d$ extra dimensions, reheating occurring when the branes collide. At least in the current implementations of this proposal, the $N = 1$
supergravity that is presumed to hold for the field theory containing the Standard Model is not presumed to hold during inflation, and as a result it is relatively easy to keep the potential sufficiently flat. One interpretation of this state of affairs might be that the field theory during inflation is different from the one after reheating, both theories breaking down during reheating.

The form of the potential depends on the setup in the context of string theory. In the original proposal [81],

\[ V \simeq V_0 (1 - e^{-q\phi/M_P}) \] (77)

with \( q \) of order 1. The scale \( V_0^{1/4} \) of the potential is of order the higher-dimensional Planck scale \( M_P \), related to the size \( R \) of the extra dimensions by \( R^d \sim M_P^2/M^{2+d} \), and the minimal value \( M \sim \text{TeV} \) was taken to be the favoured one. The CMB bound for the potential Eq. (77) is [4]

\[ V_0^{1/4} \lesssim 7 \times 10^{15} \text{GeV} \] (78)

As the authors noted, this places \( M \) far above the TeV. Liberating the model allows instead \( M \sim \text{TeV} \).

Later authors [83, 84, 85] have considered in more detail the form of the potential to be expected on the basis of string theory, finding in different regimes potentials such as [84]

\[ V = V_0 \left( 1 - \frac{V_0}{32\pi^2\phi^4} \right) \] (79)

and [83]

\[ V = V_0 - \lambda\phi^4 \] (80)

Both of these forms have been proposed earlier on the basis of purely four-dimensional field theory, and their predictions for the CMB bound are well known [4]. By relating the parameters to the higher-dimensional quantum gravity scale \( M \), it is again found that the latter needs to be far above the TeV scale. Again, liberating the models allows instead \( M \sim \text{TeV} \).

| Type                  | The effect of liberation                                      |
|-----------------------|----------------------------------------------------------------|
| Modular               | Allows the usual modulus mass \( \sim \text{TeV} \)           |
| Dine/Riotto           | Removes extreme fine-tuning                                  |
| Hybrid (generic)      | Allows extra dimension \( \sim (10^4 \text{GeV})^{-1} \)       |
| Running mass          | Removes danger from future CMB measurements                   |
| Bastero-Gil/King      | Removes of alleviates extreme fine-tuning                     |
| Moving brane          | Allows extra dimension \( \sim (10^3 \text{GeV})^{-1} \)       |

Table 1: Models of slow-roll inflation which benefit from liberation
4.7 Models which do not benefit from liberation

**D-term inflation**  We end with four examples where liberation does nothing to improve the model. The first case is that of ‘D–term inflation’ [71, 72, 73] and a related $F$–term model [55, 70] leading to hybrid inflation dominated by the loop correction with spontaneously broken supersymmetry. The potential is given by Eq. (50), and the CMB bound is

$$ V^{1/4} \ll 6.0 \left( \frac{50}{N} \right)^{1/4} g \times 10^{15} \text{GeV} $$

(81)

In the case of $D$–term inflation, even the upper bound is difficult to reconcile with the expectation from weakly coupled heterotic string theory. Liberating the model obviously does nothing to improve that situation. Nor does it remove another problem of $D$–term inflation, that the value $g \sim 1$ corresponding to a gauge coupling leads to a value of $\phi$ during inflation is of order $M_P$ making the needed suppression of non–renormalizable terms difficult to understand. In the case of $F$–term inflation, $g$ is a Yukawa coupling which can be small, possibly removing the second problem, and also the inflation scale is arbitrary as opposed to being tied to string theory. Liberating the $F$–term model has therefore a neutral effect on both counts.

**Monomial potential (‘chaotic inflation’)**  For the $V = \frac{1}{2} m^2 \phi^2$ the CMB bound is

$$ m \leq 2 \times 10^{13} \text{GeV}.$$  

(82)

During inflation $\phi$ is much bigger than $M_P$, making it again difficult to understand the absence of non–renormalizable terms. Liberating the model does not help with that problem, and has the unfortunate effect of removing its prediction that the primordial gravitational waves will be observable in the foreseeable future through the CMB anisotropy.

**Extended inflation**  Extended inflation [3, 86] gives in its simplest form a potential $V = V_0 \exp(-\sqrt{2/\rho} \phi/M_P)$, leading to $\epsilon = 1/p$ and $\eta = 2/p$. In this model the end of inflation occurs through bubble formation, and to keep the bubbles invisible on the microwave sky requires $p < 10$ or $\epsilon > 0.1$. Under the inflaton hypothesis this gives spectral index $n < 0.8$ which is too low compared with the observational bound Eq. (2). However, liberation probably does not help (in contrast with the situation for modular inflation) because the model has $\epsilon \gtrsim 1/10$. Unless there is cancellation between the terms of Eq. (19), this again makes $n \lesssim 0.8$.

**Inflation from the trace anomaly**  Before the term ‘inflation’ was coined, Starobinsky [87] proposed that effective slow–roll inflation could be caused by higher derivative curvature terms, without an explicit scalar field. The proposal has recently been reexamined by Hawking et al. [88], who estimate that in their version of the model the CMB
bound is
\[ N_S^2 \left( 250 + 240 \beta - 40 \alpha \right) \gg 10^{13} \]  
(83)
where \( \alpha \) and \( \beta \) are the coefficients of higher-order curvature terms and \( N_S \) is the number of scalar fields. This has the unpleasant feature that at least one of the three quantities must be exponentially large even in the un-liberated cases, and liberating the model clearly does not help.

## 5 Fast-roll inflation

In this section we return to inflation with the potential given by Eqs. (31) and (32), but now suppose the flatness parameter \( \eta \) has magnitude of order 1 in accordance with the generic expectation of Eq. (32). Since the flatness condition \( |\eta| \ll 1 \) is now violated, the slow-roll condition Eq. (5) is also violated. However, there can still be inflation, which has been called fast-roll inflation [89]. The exact classical equation for the inflaton field is
\[
\ddot{\phi} + 3H \dot{\phi} + V' = 0
\]  
(84)
Taking \( V' = -m^2 \phi \) from Eq. (31) and making the approximation \( H = \text{constant} \), the solution of this equation is
\[
\phi \propto e^{F H t}
\]  
(85)
where
\[
F \equiv \frac{3}{2} \left( \sqrt{1 + \frac{4}{3}|\eta|} - 1 \right)
\]  
(86)
The approximation \( H = \text{constant} \) is justified because this solution gives
\[
\epsilon \simeq -\frac{\dot{H}}{H^2} = \frac{1}{2} \frac{\dot{\phi}^2}{M_P^2 H^2} = \frac{1}{2} F^2 \frac{\phi^2}{M_P^2} \ll 1.
\]  
(87)

Under the inflaton hypothesis, fast-roll inflation is not viable because the predicted spectral index [90] \( n - 1 = 2\eta + \frac{2}{3} \eta^2 \) will almost certainly be too far from unity.6 Under the curvaton hypothesis this problem with the spectral index does not arise. We must ask, though, whether enough e-folds of inflaton can be generated by fast-roll inflation.

The number of e-folds of inflation is \( N = F^{-1} \ln(M_0/\phi_0) \) where \( M_0 \sim M_P \) is the VEV of \( \phi \) and \( \phi_0 \) is its initial value. There is a lower bound on the latter, coming from the requirement that the classical motion of the field in a Hubble time, \( H^{-1} \dot{\phi} \), be bigger than the quantum fluctuation \( H/2\pi \). Since we are dealing with \( \eta \sim 1 \), this requirement amounts to \( \phi \gtrsim H \), giving [91]
\[
N_{\text{roll}} \simeq \frac{1}{F} \ln \left( \frac{M_P}{m} \right) \simeq \frac{2}{F} \ln \left( \frac{M_P}{v_{1/4}} \right).
\]  
(88)

6For \( \eta = -3 \) the two terms cancel, but this represents fine-tuning and in any case the neglected terms in Eq. (31) will almost certainly change the result and prevent the cancellation from occurring over the whole cosmological range of scales.
The minimum number of e-folds that are needed is given by Eq. (10). Setting $N_0 = 0$, we can use the inequalities $T_{\text{reh}} > 10\,\text{MeV}$ (from nucleosynthesis) and $V_0^{1/4} < 10^{15}\,\text{GeV}$ (from Eq. (17)) to find
\[
\Delta < 15
\]
To ensure that there is no excessive quadrupole contribution to the CMB anisotropy (Grishchuk-Zeldovich effect) the spectrum of the curvature perturbation should extend down to comoving wavenumber \([3] k \sim 10^{-2}H_0\). This is the biggest cosmological scale, which leaves the horizon at
\[
N_{\text{big}} \simeq 72 - \ln \left( \frac{M_P}{V_0^{1/4}} \right) - \Delta.
\]
The smallest cosmological scale presumably is the one enclosing a mass of order $10^6 M_\odot$, corresponding to $k \sim 10^{-5.5}H_0$. It leaves the horizon about 17 e-folds after the biggest cosmological scale.

Requiring $N_{\text{big}} < N_{\text{roll}}$ gives
\[
|\eta| \lesssim 2 \left( \frac{X}{72 - \Delta - X} \right) + \frac{4}{3} \left( \frac{X}{72 - \Delta - X} \right)^2,
\]
where $X \equiv \ln(M_P/V_0^{1/4})$. For $V_0^{1/4} \sim 10^{11}\,\text{GeV}$ and $\Delta = 0$ the above gives $|\eta|_{\text{max}} \approx 0.75$. This can be somewhat relaxed if $\Delta \neq 0$, with $|\eta|_{\text{max}}(\Delta = 15) \approx 1.09$.

We conclude that fast-roll inflation can provide enough e-folds, provided that the inflaton starts out very close to the origin. In the next section, we will see how this condition may be achieved by coupling the inflaton to a thermal bath existing before inflation, which leads us to investigate thermal modular inflation below. As we will show, in the case of thermal modular inflation one also has the extra bonus of somewhat relaxing the bound on $|\eta|$.

\section{Thermal inflation}

The inflation models that we looked at in the previous section all involve an inflaton field. We end by looking at thermal inflation \([5, 6, 7, 8, 9]\). Thermal inflation is maintained by a finite-temperature correction to the potential, and it ends when the temperature $T$ falls below some critical value. There is no inflaton field during thermal inflation, and all previous authors have therefore assumed that the curvature perturbation originates during an earlier era of slow-roll inflation, with perhaps a few e-folds of thermal inflation tacked on later to mop up any unwanted relics.

Under the inflaton hypothesis this set-up is mandatory, but adopting instead the curvaton hypothesis things are not so clear. Might it be that the curvaton field acquires its inhomogeneity during an era of thermal inflation, at least on cosmological scales?
After a very few \(e\)-folds, thermal inflation certainly is of the needed almost-exponential type, since the radiation density \(\rho_\gamma\) falls like \(T^{-4} \propto a^{-4}\) leading to

\[
\epsilon \simeq -\frac{\dot{H}}{H^2} = \frac{2\rho_\gamma}{\rho_\gamma + V_0} = \frac{2}{1 + \exp(4\Delta N_{\text{therm}})}.
\] (92)

However one needs to ask whether cosmological scales can leave the horizon during thermal inflation.

### 6.1 Ordinary and modular thermal inflation

Two sorts of thermal inflation have been considered, depending on whether \(\phi\) is an ordinary field (‘matter field’ in the terminology of string theory) or a modulus. For ordinary thermal inflation \([7]\) the temperature-dependent effective potential is

\[
V(\phi, T) = V_0 + (gT^2 - \frac{1}{2} m^2)\phi^2 + \frac{1}{d} \lambda \frac{\phi^d}{M_p^{d-4}}
\] (93)

where \(d > 4\) is an integer, \(g \sim 1\) is the coupling of \(\phi\) with the particles of the thermal bath and \(\lambda \gtrsim 1\) is the coupling of the leading non-renormalizable term.\(^7\) This expression is supposed to be a good approximation when \(\phi\) is less than its VEV \(M_0\).

We consider first the zero-temperature potential. Setting its derivative equal to zero gives the VEV

\[
M_0 = \left(\frac{1}{\lambda}\right)^{\frac{1}{d-2}} \left(\frac{m}{M_p}\right)^{\frac{2}{d-2}} M_p \ll M_p,
\] (94)

and evaluating the second derivative gives the mass in the vacuum as

\[
m^2_\phi = (d - 2)m^2 \sim m^2.
\] (95)

Demanding that \(V(M_0) = 0\) gives

\[
V_0 = \frac{d - 2}{2d} m^2 M_0^2 \sim m^2 M_0^2.
\] (96)

This gives

\[
|\eta| = \frac{2d}{d - 2} \left(\frac{M_p}{M_0}\right)^2 \sim M_p^2/M_0^2 \gg 1.
\] (97)

where \(\eta = -m^2 M_p^2/V_0\).

For modular thermal inflation \([9]\), \(\phi\) is supposed to be a modulus, with the zero-temperature potential considered in Sections 4.1 and 5. In this case the last term of Eq. (93) is replaced by some unknown function, but on the basis of string theory examples

\(^7\)In terms of the ultra-violet cutoff \(\Lambda \lesssim M_p\) of the effective field theory the expected value is \(\lambda \sim (M_p/\Lambda)^{d-4} \gtrsim 1\).
the order of magnitude estimates of Eqs. (95), (96) and (97) are assumed to be valid. It follows that for modular inflation, $|\eta| \sim 1$.

Now consider the evolution of $\phi$. At $T$ bigger than $T_c \equiv (m/\sqrt{2g})$, the effective mass-squared $m^2(T) \equiv 2gT^2 - m^2$ holds $\phi$ close to the origin, and the energy density is

$$\rho = \frac{\pi^2}{30} g_* T^4 + V_0,$$

where $g_*$ is the effective number of relativistic degrees of freedom. Thermal inflation starts when the second term starts to dominate at temperature

$$T_{\text{start}} = \left(\frac{30}{\pi^2 g_*}\right)^{1/4} V_0^{1/4} \sim V_0^{1/4}$$

Thermal inflation ends when $T = T_c$. To estimate the number $N_{\text{therm}}$ of $e$-folds of thermal inflation, we can take $g_*$ to be constant, giving $T \propto 1/a$ and

$$N_{\text{therm}} \sim \ln(V_0^{1/4}/m) = \ln(M_P/V_0^{1/4}) - \frac{1}{2} \ln |\eta|.$$  

During thermal inflation, the typical field value is $\phi \sim T$. The value at the end of thermal inflation is therefore $\phi \sim m$, and before this value changes much the potential has practically its zero temperature form. Then $\phi$ rolls away from the origin, reaching its VEV after a number $N_{\text{roll}}$ of Hubble times given by Eq. (88). For ordinary thermal inflation, $|\eta|$ is exponentially large and $N_{\text{roll}} \ll 1$. In this case there is no more inflation after thermal inflation ends. In contrast, for modular inflation $|\eta|$ is of order unity and one may have $N_{\text{roll}} \gg 1$. In this case there are $N_{\text{roll}}$ $e$-folds of inflation after thermal inflation ends.

### 6.2 Thermal inflation with $m \ll H$

To complete this discussion of the dynamics of thermal inflation, we need to consider the case $|\eta| \ll 1$ or equivalently $m \ll H$. In order to avoid $M_0 \gg M_P$, the form Eq. (93) must in this case be modified so as to steepen the potential, either in the $\phi$ direction or in the direction of some other field as in inverted hybrid inflation [92].

With $m \ll H$ there is a regime of temperature $m \ll T \ll H$. Before $T$ enters this regime, the effective mass-squared $\sim T^2$ is big enough to hold $\phi$ at the typical value $\phi \sim T$ mentioned earlier. Afterwards though, the effective mass-squared falls below $H^2$, and $\phi$ begins a random walk under the influence of the quantum fluctuation, moving a

---

8By analogy with the case of particles in equilibrium, the field $\phi$ will presumably fall out of thermal equilibrium when the temperature falls below $H$. But also by analogy with that case, one can expect that the form of the effective potential will continue to be the same as if there were equilibrium. One can verify this explicitly for the case of thermal equilibrium with a scalar field $\chi$ through a coupling $g\phi^2\chi^2$, where the thermal average $\chi^2 \sim T^2$ corresponds to the contributions of plane waves representing relativistic particles in thermal equilibrium.
distance $\pm H/2\pi$ during each Hubble time. This continues until the potential becomes steep enough that the random walk is slower than the classical roll given by Eq. (5). The random-walk era is therefore an era of ‘eternal’ inflation (so-called because its duration, in a given region, can be indefinitely long). On scales leaving the horizon during eternal inflation, the curvature perturbation is of order 1, and when these scales start to enter the horizon around half of the energy density of the Universe collapses to black holes.

When eternal inflation has been considered previously, it has been supposed to occur before cosmological scales leave the horizon. Then the scales on which the curvature perturbation is of order 1 are outside the horizon at the present epoch and can be ignored (except for the Grishchuk-Zeldovich effect which is excluded by observation [3]). In our case, we want cosmological scales to leave the horizon during thermal inflation and the eternal inflation would generate a curvature perturbation of order 1 on sub-cosmological scales. The black hole formation would therefore be a disaster, generating an irrevocably matter-dominated early Universe. (We discount the possibility that the black holes evaporate, which would require them to form on a scale which is implausibly small in the present context.) We conclude that thermal inflation with $m \ll H$ is unviable if cosmological scales are required to exit the horizon during this inflation.$^{10}$

### 6.3 Generating the curvature perturbation

We now ask if cosmological scales can leave the horizon during thermal inflation, so that the curvaton may acquire its perturbation then. As we have discussed in Sec. 5 the biggest cosmological scale is given by Eq. (90). We now impose the requirement that cosmological scales leave the horizon during thermal inflation.

Consider first ordinary thermal inflation, where no more inflation takes place after thermal inflation. In this case Eqs. (100) and (90) show that, regardless of $V_0$, the requirement is $\ln(M_P/m) \gtrsim 72 - \Delta$. If one assumes prompt reheating and $N_0 = 0$, then $\Delta = 0$ and the above bound corresponds to

$$m \lesssim 10^{-4} \text{eV}$$

$$V_0^{1/4} \lesssim \text{TeV}$$

(101)

(102)

It is easy to verify that practically the same bound is obtained even if inefficient reheating is allowed, due to the the combined effect of the requirement $|\eta| \gtrsim 1$ and the nucleosynthesis bound $T_{\text{reh}} \gtrsim 10 \text{MeV}$. Such a small mass is completely unviable because it would prevent reheating after thermal inflation. The only effective decay channel would be to the photon with rate $\Gamma \sim m^3/M_P^2$. Thus, since $T_{\text{reh}} \sim \sqrt{\Gamma M_P}$, we find$^{11}$

$^{9}$If the modification drives the motion in the direction of another field it is the classical motion in that direction that is relevant. We are discounting the possibility that the modification of the potential becomes significant before $T \sim H$ since this would lead to a completely different type of model.

$^{10}$Note that, when $m \gtrsim H$, there is also a brief period in which $m^2(T) < H^2$ but it lasts less than a Hubble time so that there is no black hole formation.

$^{11}$Even with $\Delta \neq 0$ one obtains $T_{\text{reh}} \sim 10^{-19+9(\Delta/15)} \text{eV} < 10^{-10} \text{eV}$.  

22
Therefore, we conclude that \textit{cosmological scales cannot leave the horizon during ordinary thermal inflation.}

Now consider modular inflation, where some number \(N_{\text{roll}}\) of inflationary e-folds take place after thermal inflation, given by Eq. (88). Since it takes about \(\Delta N_{\text{cosm}} \sim 17\) e-folds for cosmological scales to leave the horizon, and we want this to happen during thermal inflation, we require

\[
N_{\text{big}} - N_{\text{therm}} < N_{\text{roll}} < N_{\text{big}} - \Delta N_{\text{cosm}}
\]

or

\[
\frac{72 - \Delta}{2 + \frac{2}{F}} < \ln \left( \frac{M_P}{V_0^{1/4}} \right) < \frac{55 - \Delta}{1 + \frac{2}{F}}
\]

There is a solution only for

\[
F > \frac{34}{38 - \Delta}
\]

(For smaller values of \(F\), cosmological scales start to leave the horizon only after thermal inflation is over.) With \(\Delta = 0\) this gives \(|\eta| > 1.16\), and if \(\Delta\) saturates Eq. (89) it gives \(|\eta| > 2.21\).

In the limit where the bound Eq. (106) on \(F\) is saturated, we find independently of \(\Delta\),

\[
\ln \left( \frac{M_P}{V_0^{1/4}} \right) \simeq 17
\]

corresponding to

\[
V_0^{1/4} \simeq 1 \times 10^{11}\ \text{GeV}
\]

\[
m \simeq 4 - 6\ \text{TeV}
\]

Increasing \(F\) decreases \(V_0^{1/4}\). The limit \(F \to \infty\) corresponds to \(V_0^{1/4} \sim \text{TeV}\), in agreement with Eq. (102).

The values represented by Eqs. (108) and (109), are about the ones expected for a modulus in the case of gravity-mediated supersymmetry breaking. Our conclusion therefore is that cosmological scales can leave the horizon during modular thermal inflation.

For any kind of modular inflation (slow-roll, fast-roll or thermal) the subsequent oscillation of the modulus around its VEV must decay well before nucleosynthesis. The usual assumption is that the modulus mass is of order \(m_{3/2}\) with the modulus decaying with only gravitational strength, which requires \(m_{3/2} \gtrsim 10\ \text{TeV}\) corresponding to anomaly-mediated SUSY breaking [63]. This is mildly inconsistent with Eq. (109). However, it is conceivable [9] that the modulus involved with modular inflation correspond to a flat direction of the MSSM, in which case the decay would be prompt. Of course, in the case of thermal inflation this would require that the interactions holding the modulus at its maximum be different from the Standard Model interactions.
6.4 The vacuum assumption during thermal inflation

In the previous subsection, we have implicitly assumed that at the beginning of inflation, the curvaton field is in the vacuum on scales within the horizon. This is because we assumed that cosmological scales could start to leave the horizon as soon as thermal inflation starts, the vacuum assumption then being necessary to obtain the flat spectrum for the curvaton field in the usual way.\(^{12}\) We now ask to what extent this assumption is justified.

There is a model-independent upper limit on the number \(N_{\text{bef}}\) of e-folds which elapse before scales leaving the horizon are the vacuum, from the same consideration that has been invoked under the inflaton hypothesis [93]. At the beginning of inflation, all scalar fields with mass \(m \ll H\) must be in the vacuum on scales \(k/a > V_1^{1/4}\) because they would otherwise dominate the energy density and (having positive pressure) spoil inflation. The upper limit on \(N_{\text{bef}}\) is therefore the number of e-folds which elapse before such scales start to leave the horizon,

\[
N_{\text{bef}} < \ln \left( \frac{V_0^{1/4}}{H} \right) \sim \ln \left( \frac{M_P}{V_0^{1/4}} \right)
\]

But this uses up all of the e-folds of thermal inflation, which blocks the desired objective that cosmological scales leave the horizon during thermal inflation. We need fewer e-folds! On the other hand, the following argument shows that \(N_{\text{bef}}\) cannot be zero.

Suppose first that the radiation domination era which precedes thermal inflation is itself preceded by an inflationary era. That, presumably, must be the case if the radiation dominated Universe is flat and homogeneous to high accuracy. The reason that \(N_{\text{bef}}\) cannot then be zero is that during radiation domination scales are entering the horizon which left the horizon during the previous inflation. Such scales cannot be in the vacuum because their vacuum fluctuation has been converted to a classical perturbation. However, the fate of such perturbations after reentry is not clear. It is plausible that when the perturbations become again causally connected their dynamics will result in suppression of their power at small scales, even though they are weakly coupled.

Another possibility is that there is no substantial earlier inflation and the Universe is only roughly flat, homogeneous and isotropic during the early radiation dominated era. Still, even in that case \(N_{\text{bef}}\) has to be non-zero. This is because we must wait a few e-folds before the the observable Universe leaves the horizon so that it will have the observed extreme flatness, and homogeneity.

From this discussion we learn that some number of e-folds must elapse before cosmological scales leave the horizon. On the other hand we have not found any definite lower limit on this number, so that the estimates that we earlier made by setting it equal to zero may still be valid as an approximation.

\(^{12}\)We shall use the term ‘vacuum state’ only for scales well inside the horizon, where it is unambiguously defined. We do not consider the possibility that a flat spectrum could be obtained starting from some non-vacuum state, since no such state has ever been exhibited.
7 Conclusion

The first viable inflation models, generally termed ‘new inflation’ models [94], involved a single field, the inflaton, which was supposed to perform three tasks; support inflation, end inflation and generate the curvature perturbation. Such a paradigm is economical in terms of the number of fields involved, but it makes model-building a quite difficult task and arguably involves unacceptable fine-tuning.

The hybrid inflation paradigm [69] delegated the task of supporting inflation (i.e., of generating the required potential) to ‘waterfall’ field different from the inflaton. Hybrid inflation models are much easier to construct and can be relatively free of fine-tuning, but they are still quite constrained.

In both of these cases, the hypothesis regarding the origin of the curvature perturbation is the same; it comes from the perturbation of the inflaton field. The curvaton hypothesis is that the curvature perturbation comes from the perturbation of a different field, the curvaton. According to this hypothesis, inflation itself may be of the ‘new’ or ‘hybrid’ variety. (In the hybrid case though, the curvaton cannot be identified with the waterfall field since the latter has mass much bigger than $H$.) Alternatively, inflation may be of a type not involving any rolling field, such as thermal inflation. In this paper, we have revisited most of the inflation models that have been proposed, asking to what extent they become more attractive when they are liberated by the curvaton hypothesis.

For slow-roll inflation, the results are summarized in Table 1. An interesting finding is that a modulus becomes a more attractive candidate. Turning to thermal inflation, we found again an attractive model involving a modulus. When thermal inflation ends, some more $e$-folds of inflation occur while the modulus rolls to the vacuum. The model is perhaps more attractive than modular inflation without thermal inflation, because the initial value of the modulus is explained and because there need not be so many $e$-folds of thermal inflation.

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References

[1] D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).

[2] M. Tegmark et al. [SDSS Collaboration], arXiv:astro-ph/0310723.
[3] D. H. Lyth & A. R. Liddle, *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press (2000).

[4] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999).

[5] P. Binetruy and M. K. Gaillard, Phys. Rev. D 34, 3069 (1986).

[6] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Rev. Lett. 56, 557 (1986).

[7] D. H. Lyth and E. D. Stewart, Phys. Rev. Lett. 75, 201 (1995); D. H. Lyth and E. D. Stewart, Phys. Rev. D 53, 1784 (1996); T. Barreiro, E. J. Copeland, D. H. Lyth and T. Prokopec, Phys. Rev. D 54, 1379 (1996).

[8] E. D. Stewart, M. Kawasaki and T. Yanagida, Phys. Rev. D 54 (1996) 6032; T. Asaka and M. Kawasaki, Phys. Rev. D 60 (1999) 123509; R. Jeannerot, Phys. Rev. D 59 (1999) 083501; T. Asaka, M. Kawasaki and T. Yanagida, Phys. Rev. D 60 (1999) 103518.

[9] L. Hui and E. D. Stewart, Phys. Rev. D 60 (1999) 023518.

[10] D. H. Lyth and D. Wands, Phys. Lett. B 524, 5 (2002).

[11] S. Mollerach, Phys. Rev. D 42, 313 (1990).

[12] A. D. Linde and V. Mukhanov, Phys. Rev. D 56, 535 (1997).

[13] T. Moroi and T. Takahashi, Phys. Lett. B 522, 215 (2001).

[14] N. Bartolo and A. R. Liddle, Phys. Rev. D 65, 121301 (2002).

[15] T. Moroi and T. Takahashi, arXiv:hep-ph/0206026.

[16] M. Fujii and T. Yanagida, Phys. Rev. D 66 123515, (2002).

[17] D. H. Lyth, C. Ungarelli and D. Wands, arXiv:astro-ph/0208055.

[18] M. S. Sloth, Nucl. Phys. B 656, 239 (2003).

[19] A. Hebecker, J. March-Russell and T. Yanagida, arXiv:hep-ph/0208249.

[20] R. Hofmann, arXiv:hep-ph/0208267.

[21] T. Moroi and H. Murayama, arXiv:hep-ph/0211019.

[22] K. Enqvist, S. Kasuya and A. Mazumdar, arXiv:hep-ph/0211147.

[23] K. A. Malik, D. Wands and C. Ungarelli, arXiv:astro-ph/0211602.

[24] M. Postma, arXiv:hep-ph/0212005.

26
[25] B. Feng and M. Li, arXiv:hep-ph/0212213.

[26] C. Gordon and A. Lewis, Phys. Rev. D 67, 123513 (2003).

[27] K. Dimopoulos, arXiv:astro-ph/0212264.

[28] M. Giovannini, Phys. Rev. D 67 (2003) 123512.

[29] A. R. Liddle and L. A. Urena-Lopez, arXiv:astro-ph/0302054.

[30] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, arXiv:hep-ph/0303154.

[31] K. Enqvist, A. Jokinen, S. Kasuya and A. Mazumdar, arXiv:hep-ph/0303165.

[32] D. H. Lyth and D. Wands, arXiv:astro-ph/0306500.

[33] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, arXiv:hep-ph/0308015.

[34] K. Dimopoulos, D. H. Lyth, A. Notari and A. Riotto, JHEP 0307 (2003) 053.

[35] M. Postma and A. Mazumdar, arXiv:hep-ph/0304246.

[36] J. McDonald, arXiv:hep-ph/0308048.

[37] S. Kasuya, M. Kawasaki and F. Takahashi, arXiv:hep-ph/0305134.

[38] M. Endo, M. Kawasaki and T. Moroi, arXiv:hep-ph/0304126.

[39] K. Hamaguchi, M. Kawasaki, T. Moroi and F. Takahashi, arXiv:hep-ph/0308174.

[40] J. McDonald, arXiv:hep-ph/0308295.

[41] N. Bartolo, S. Matarrese and A. Riotto, arXiv:hep-ph/0309033.

[42] N. Bartolo, S. Matarrese and A. Riotto, arXiv:astro-ph/0309692.

[43] M. Giovannini, arXiv:hep-ph/0310024.

[44] J. McDonald, arXiv:hep-ph/0310126.

[45] A. Mazumdar, arXiv:hep-th/0310162.

[46] M. Axenides and K. Dimopoulos, arXiv:hep-ph/0310194.

[47] C. Armendariz-Picon, arXiv:astro-ph/0310512.

[48] E. J. Chun, K. Dimopoulos & D. H. Lyth, in preparation.
[49] G. Dvali, A. Gruzinov and M. Zaldarriaga, arXiv:astro-ph/0303591; L. Kofman, arXiv:astro-ph/0303614; K. Enqvist, A. Mazumdar and M. Postma, inflaton coupling,” Phys. Rev. D 67, 121303 (2003) [arXiv:astro-ph/0304187]; G. Dvali, A. Gruzinov and M. Zaldarriaga, and Mass Domination,” arXiv:astro-ph/0305548; S. Tsujikawa, arXiv:astro-ph/0305569; S. Matarrese and A. Riotto, the inflaton decay rate,” arXiv:astro-ph/0306416; A. Mazumdar and M. Postma, arXiv:astro-ph/0306509.

[50] M. Bastero-Gil, V. Di Clemente and S. F. King, arXiv:hep-ph/0211011; M. Bastero-Gil, V. Di Clemente and S. F. King, arXiv:hep-ph/0211012; M. Bastero-Gil, V. Di Clemente and S. F. King, in preparation.

[51] D. H. Lyth and L. Covi, Phys. Rev. D 62, 103504 (2000).

[52] L. Knox and Y. S. Song, Phys. Rev. Lett. 89, 011303 (2002); M. Kesden, A. Cooray and M. Kamionkowski, Phys. Rev. Lett. 89, 011304 (2002).

[53] S. Weinberg, *The Quantum Theory Of Fields. Vol. 3: Supersymmetry*, Cambridge University Press (2000).

[54] J. Polchinski, *String Theory. Vol. 2: Superstring Theory And Beyond*, Cambridge University Press (1998).

[55] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D 49, 6410 (1994).

[56] M. K. Gaillard, D. H. Lyth and H. Murayama, Phys. Rev. D 58, 123505 (1998).

[57] C. F. Kolda and J. March-Russell, Phys. Rev. D 60, 023504 (1999).

[58] T. Banks, M. Berkooz, S. H. Shenker, G. W. Moore and P. J. Steinhardt, Phys. Rev. D 52, 3548 (1995).

[59] T. Banks, arXiv:hep-th/9906126; R. Brustein, S. P. De Alwis and E. G. Novak, space,” arXiv:hep-th/0205042.

[60] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. D 47, 426 (1993).

[61] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. 65, 3233 (1990).

[62] A. Vilenkin, Phys. Rev. Lett. 72, 3137 (1994); A. D. Linde and D. A. Linde, Phys. Rev. D 50, 2456 (1994).

[63] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999); T. Moroi and L. Randall, Nucl. Phys. B 570, 455 (2000).
[64] J. D. Cohn and E. D. Stewart, Phys. Lett. B 475, 231 (2000).
[65] E. D. Stewart and J. D. Cohn, gauge symmetries,” Phys. Rev. D 63, 083519 (2001).
[66] M. Dine and A. Riotto, Phys. Rev. Lett. 79, 2632 (1997).
[67] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53, 2658 (1996).
[68] D. H. Lyth, Phys. Lett. B 466, 85 (1999).
[69] A. D. Linde, Phys. Lett. B 259, 38 (1991).
[70] G. R. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. 73, 1886 (1994).
[71] E. D. Stewart, Phys. Rev. D 51, 6847 (1995).
[72] P. Binetruy and G. R. Dvali, Phys. Lett. B 388, 241 (1996).
[73] E. Halyo, Phys. Lett. B 387, 43 (1996).
[74] E. D. Stewart, Phys. Lett. B 391, 34 (1997); E. D. Stewart, Phys. Rev. D 56, 2019 (1997).
[75] L. Covi, D. H. Lyth and L. Roszkowski, Phys. Rev. D 60, 023509 (1999); L. Covi and D. H. Lyth, Phys. Rev. D 59, 063515 (1999).
[76] L. Covi and D. H. Lyth, Mon. Not. Roy. Astr. Soc. 326, 877 (2001).
[77] L. Covi, D. H. Lyth and A. Melchiorri, arXiv:hep-ph/0210395.
[78] L. Covi, D. H. Lyth and A. Melchiorri, in preparation.
[79] L. Covi and D. H. Lyth, in preparation.
[80] M. Bastero-Gil and S. F. King, Phys. Lett. B 423, 27 (1998).
[81] G. R. Dvali and S. H. Tye, Phys. Lett. B 450, 72 (1999).
[82] S. H. Alexander, Phys. Rev. D 65, 023507 (2002).
[83] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, JHEP 0107, 047 (2001); S. Alexander, Y. Ling and L. Smolin, cosmologies,” Phys. Rev. D 65, 083503 (2002).
[84] G. R. Dvali, Q. Shafi and S. Solganik, arXiv:hep-th/0105203.
[85] G. Shiu and S. H. Tye, Phys. Lett. B 516, 421 (2001).
[86] D. La and P. J. Steinhardt, Phys. Rev. Lett. 62, 376 (1989) [Erratum-ibid. 62, 1066 (1989)].
[87] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).

[88] S. W. Hawking, T. Hertog and H. S. Reall, Phys. Rev. D 63, 083504 (2001).

[89] A. Linde, JHEP 0111, 052 (2001).

[90] E. D. Stewart and D. H. Lyth, Phys. Lett. B 302, 171 (1993).

[91] L. Randall, M. Soljacic and A. H. Guth, Nucl. Phys. B 472, 377 (1996).

[92] D. H. Lyth and E. D. Stewart, Phys. Rev. D 54, 7186 (1996).

[93] A. R. Liddle and D. H. Lyth, Phys. Rept. 231, 1 (1993).

[94] A. D. Linde, Phys. Lett. B 108, 389 (1982). A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).