Yang–Mills–Chern–Simons system in the presence of a Gribov horizon with fundamental Higgs matter

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Abstract
In this work we study the behaviour of Yang–Mills–Chern–Simons theory coupled to a Higgs field in the fundamental representation by taking into account the effects of the presence of the Gribov horizon. By analyzing the infrared structure of the gauge field propagator, both confined and de-confined regions can be detected. The confined region corresponds to the appearance of complex poles in the propagators, while the de-confined one to the presence of real poles. One can move from one region to another by changing the parameters of the theory.

Keywords: topologically massive Yang–Mills theory, Higgs mechanism, Gribov quantization, confinement and deconfinement regimes

(Some figures may appear in colour only in the online journal)

1. Introduction

It is widely known that the issue of the Gribov copies [1] in non-Abelian gauge theories is deeply related to the problem of color confinement. In recent years the Gribov–Zwanziger (GZ) approach [2, 3] has become a promising framework in order to describe several features of the infrared regime of Yang–Mills theory. For instance, the inclusion of dimension two condensates [4, 5] provides a refinement of the GZ framework allowing the description of the infrared behaviour of the gluon propagator to be in very good agreement with the most recent lattice data [6, 7]. Within this approach, it is possible to investigate the spectrum of Yang–Mills theories by constructing local gauge invariant composite operators.
suitable to estimate the masses of the lightest glueball states [8, 9]. Also, the so-called refined GZ scheme has been used in the computation of the Casimir energy in the context of the MIT–bag model [10], producing the correct sign for the Casimir force. A very important problem related to the non-perturbative behavior of Yang–Mills theory is the transition between confining and non-confining phases in the presence of Higgs fields, see [11–13]. For instance, in three dimensions, Polyakov’s seminal work [11] shows that in the Georgi–Glashow model monopole configurations in the Euclidean space give a dominant contribution in the functional integral, providing a successful mechanism for confinement, in agreement with the dual superconductivity picture.

Gauge–Higgs systems for the gauge group $SU(2)$ in the context of the Gribov problem have been addressed in [14–16]. A very interesting aspect to be mentioned is that regarding the physical consequences in the phase structure of the theory due to the choice of the adjoint or the fundamental representation for the Higgs field. In particular, in the case of the Georgi–Glashow model, i.e. of the three-dimensional Yang–Mills theory with Higgs fields in the adjoint representation, the third component of the gauge field, $A_3^\alpha$, is always confined for all values of the gauge coupling $\alpha$ and of the vacuum expectation value (vev) $v$ of the Higgs field. Within the Gribov approach, this feature turns out to be encoded in the corresponding gluon propagator which is of the Gribov type, i.e. it displays two unphysical complex conjugate poles. Moreover, the off-diagonal gauge propagator $A_\alpha^a(q)A_\beta^b(q)$, with $\alpha, \beta = 1, 2$, decomposes into the sum of two Yukawa propagators with real and positive masses, though only the heaviest mass component of this decomposition has a positive residue and can be regarded as a physical mode which is, however, decoupled from the infrared dynamics due to its large mass. In the case of Higgs fields in the fundamental representation, the gauge group $SU(2)$ is completely broken. At weak coupling, all propagators decompose into a sum of two Yukawa propagators with positive masses. One of the components is unphysical due to a negative residue. However, the component with the largest mass is a physical mode. Therefore, at weak coupling all gauge modes display a massive physical component. This is what can be called a Higgs phase. In the strong coupling, the propagator of all gauge modes is of the Gribov type, exhibiting complex conjugate poles. This is the confining phase. Therefore, when the Higgs field is in the fundamental representation, we have a weak coupling Higgs phase and a strong coupling confining phase. These results are in nice agreement with the behavior reported by lattice simulations [17, 19].

A similar behaviour in the infrared region is observed when considering a Yang–Mills field coupled to a Chern–Simons topological term in the Landau gauge [21]. The addition of this term has been largely motivated, see for instance, [18]. Unlike pure three-dimensional Yang–Mills theory, where the effect of the Gribov horizon confines all degrees of freedom of the theory, the addition of the Chern–Simons topological term allows for the possibility of having physical poles in the resulting propagator gauge for certain values of the coupling constant $\alpha$ and the topological mass $M$, making it possible to identify regions of confinement and de-confinement in the parameter space of the model. The present analysis might be relevant for the study of QCD at high temperatures. Indeed, as is well known, in this case the theory can be described with an effective three-dimensional gauge theory in which the Chern–Simons term appears upon integrating out the fermions.

In this work we pursue our previous investigation [21] by studying Yang–Mills–Chern–Simons–Higgs systems when the the presence of the Gribov horizon is taken into account.
Following the seminal paper by Gribov [1], we shall work in three-dimensional Euclidean space-time. There is a deep reason for this. Our present knowledge of the issue of the Gribov copies in Minkowski space is very poor, due to the highly nontrivial difficulties of understanding the topology of the geometry of the gauge orbits in Minkowski space-time. So far, all mathematical results achieved in the properties of the so called Gribov region $\Omega$ [1] have been established in Euclidean space-time, see [3]. Moreover, in the following, we shall make use of the original semiclassical approach outlined in the original work [1], which has been formulated in Euclidean space-time. At the end, one expects that the correlation functions of local gauge invariant composite operators should exhibit an analytic structure allowing for a smooth continuation from Euclidean to Minkowski space. A first evidence of this nontrivial feature might be found in [22].

The paper is organized as follows. In section 2 we study the Yang–Mills–Chern–Simons–Higgs system with scalar fields in the fundamental representation. Section 3 collects our conclusion.

2. Yang–Mills–Chern–Simons–Higgs model in the fundamental representation of $SU(2)$

Let us start this section with a brief reminder of Gribov’s procedure [1] in order to take into account the presence of gauge copies in the functional Euclidean integral. It amounts to restricting the domain of integration in the path integral to the so called Gribov region $\Omega$, defined as the set of all field configurations fulfilling the Landau gauge, $\partial_0 A^a_\mu = 0$, and for which the Faddeev–Popov operator $\mathcal{M}^{ab} = - \partial^2 \delta^{ab} - g f^{abc} A^b_\mu$ is strictly positive, namely

$$\Omega = \{ A^a_\mu; \partial_0 A^a_\mu = 0; \mathcal{M} = - \partial^2 \delta^{ab} - g f^{abc} A^b_\mu > 0 \}.$$  \hspace{1cm} (1)

The region $\Omega$ is known to be bounded in all directions in field space and to be convex. The boundary $\partial \Omega$ of the Gribov region is known as the Gribov horizon, where the first vanishing eigenvalue of the operator $\mathcal{M}^{ab}$ shows up. Furthermore, it has been shown that every gauge orbit crosses the region $\Omega$ at least once, thus giving a well defined support to Gribov’s original proposal to cut off the functional integral at the Gribov horizon.

In order to implement the restriction to the region $\Omega$, Gribov required the no-pole condition [1] on the Faddeev–Popov ghost propagator, which is nothing but the inverse of the operator $\mathcal{M}^{ab}$. More precisely, following [1], one parametrizes the ghost propagator $\mathcal{G}(q, A)$ as

$$\mathcal{G}^{ab}(q, A) = \frac{\delta^{ab} 1}{q^2 1 - \sigma(q, A)},$$  \hspace{1cm} (2)

and one imposes the condition

$$\sigma(q, A) < 1,$$ \hspace{1cm} (3)

which ensures that the inverse of the Faddeev–Popov operator $\mathcal{M}^{ab}$ is always positive, so that one always remains inside the Gribov region $\Omega$, i.e. the Gribov horizon $\partial \Omega$ is never crossed. As the form factor $\sigma(q, A)$ turns out to be a decreasing function of the momentum $q$[1], it is sufficient to require

$$\sigma(0, A) < 1,$$ \hspace{1cm} (4)
which is known as the Gribov no-pole condition [1]. According to the no-pole prescription (4), the Faddeev–Popov quantization formula gets modified as [1]:

\[
d\mu_{FP} = \mathcal{D}A \delta(\partial A) \det(M^{ab}) e^{-S_{YM}}
\]

\[
\rightarrow \mathcal{D}A \delta(\partial A) \det(M^{ab}) \theta(1 - \sigma(0, A)) e^{-S_{YM}}
\]

(5)

where \(S_{YM}\) is the Yang–Mills action

\[
S_{YM} = \frac{1}{4} \int d^4x \ F_{\mu\nu}^a F_{\mu\nu}^a.
\]

(6)

and \(\theta(x)\) stands for the step function. Making use of the integral representation

\[
\theta(x) = \int_{-i\infty + \epsilon}^{+i\infty + \epsilon} \frac{d\beta}{2\pi i\beta} e^{-\beta x},
\]

(7)

it turns out that the ghost form factor \(\sigma(0, A)\) can be brought into the exponential of the Yang–Mills measure \(d\mu_{FP}\), i.e.

\[
e^{-S_{YM}} \rightarrow e^{-S_{YM}} e^{\beta\sigma(0, A)}.
\]

(8)

Moreover, making use of the saddle point approximation in order to evaluate the integral over \(\beta\) [1], for the partition function \(\mathcal{Z}\), one writes

\[
\mathcal{Z} = \int \mathcal{D}A \delta(\partial A) \det(M^{ab}) e^{-S_{YM}} e^{\beta\sigma(1 - \sigma(0, A))},
\]

(9)

where, to the first order, \(\beta^*\) is determined by the gap equation [1]

\[
1 = \frac{3Ng^2}{4} \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 + \gamma^4}, \quad \gamma^4 = \frac{g^2N}{2(N^2 - 1)} \beta^*.
\]

(10)

Having given a short account of the main steps of Gribov’s construction, let us focus on the action of the topologically massive Yang–Mills [18] coupled to a Higgs field in the fundamental representation. As we have seen, the Gribov problem is originally formulated in four dimensions, nevertheless, it has also been analyzed in three-dimensional Yang–Mills theory, within the refined GZ approach, describing a similar non-perturbative behaviour [20].

The action of our model reads

\[
S = S_{CS} + S_{FP} + S_0
\]

\[
= -iM \int d^4x \epsilon_{\mu\nu\rho\sigma} \left( \frac{1}{2} A^\mu_a \partial_\nu A^\rho_a + \frac{1}{3!} g^{abc} A^\mu_a A^\rho_b A^\sigma_c \right) + \frac{1}{4} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a
\]

\[
+ \int d^4x \left( b^a \partial_\mu A^\mu_a + c^a \partial_\mu D^\mu_a c^b \right) + \int d^4x \left( D^i_\mu \Phi^j D^k_\nu \Phi^l + \left( \Phi \Phi - \nu^2 \right)^2 \right)
\]

(11)

where \(b^a\) stands for the Lagrange multiplier implementing the Landau gauge, \(\partial_\mu A^\mu_a = 0\), and \((c^a, c^b)\) are the Faddeev–Popov ghosts. In the fundamental representation, the covariant derivative is defined by

\[
D^i_\mu \Phi^j = \partial_\mu \Phi^j - ig \frac{(e^a)^{ij}}{2} A^\mu_a \Phi^j
\]

(12)

where \(i, j = 1, 2\), refer to the fundamental representation of \(SU(2)\) and \(e^a\) are the Pauli matrices. When \(\Phi\) acquires a vev, we can use the freedom of the \(SU(2)\) rotations to write this expectation value in the form

\[4\]
\[ \langle \Phi \rangle = \frac{1}{\sqrt{2}} \left( 0 \right) \]  

(13)

As a consequence, all components of the gauge field acquire the same mass \( m^2 = \frac{g^2 \nu^2}{4} \).

2.1. Infrared behaviour gauge field propagator

After the spontaneous symmetry breaking, the quadratic part of the gauge field action is

\[ S_{\text{quad}} = \int d^4x \left\{ \frac{1}{4} \left( \partial_\mu A_\nu^a - \partial_\nu A_\mu^a \right)^2 - \frac{M}{2} \epsilon_{\mu\nu\rho\sigma} A_\mu^a \partial_\nu A_\rho^a + b^a \partial_\mu A_\mu^a + \frac{8g^2\nu^2}{8} A_\mu^a A_\mu^a \right\} \]  

(14)

The generalization of the implementation of the restriction to the Gribov region \( \Omega \) to the action (11) can be done by following the procedure outlined at the beginning of this section. Taking into account the effects of the Gribov horizon, for the gauge propagator one obtains

\[ \langle A_\mu^a(q)A_\nu^b(-q) \rangle = \delta^{ab} \frac{q^2(\gamma^4 + q^4) + g^2\nu^2q^4}{M^2q^4 + (\gamma^4 + q^4)^2 + 2g^2\nu^2q^2(\gamma^4 + q^4)} \times \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{M\epsilon_{\mu\nu\rho\sigma}q_\rho}{\gamma^4 + q^4 + g^2\nu^2q^2} \right) \]  

(15)

It is easy to check that in the cases when \( M = 0, \gamma = 0 \) or \( \nu = 0 \), one recovers the propagators studied previously in [14, 21]. In particular, in [21], we have shown that the Chern–Simons term does not contribute to the Gribov gap equation due to its topological nature. As a consequence, in the present case, for the gap equation determining the value of the Gribov parameter \( \gamma \), we have [14]

\[ \frac{4}{3}g^2 \int \frac{d^4q}{(2\pi)^3} \frac{1}{q^4 + \frac{g^2\nu^2}{4}q^2 + \gamma^4} = 1 \]  

(16)

which gives

\[ \gamma^4 = \frac{1}{4} \left( \frac{g^2\nu^2}{4} - \frac{g^4}{9\pi^2} \right)^2. \]  

(17)

The gap condition (16) determines the Gribov parameter \( \gamma \) in terms of the coupling constant \( g \) and of the vev of the Higgs field \( \nu \).

Therefore, taking into account the condition (17) and looking at the propagator (15), one sees that its analytic structure depends on the three parameters \((M, g, \nu)\). Nevertheless, it turns out to be useful to define dimensionless generalized variables by absorbing the Chern–Simons mass in the quantities \((q_\mu, \gamma, g^2, \nu^2)\) of the propagator, i.e. by introducing the rescaled quantities \((k_\mu, \tilde{\gamma}, \tilde{g}^2, \tilde{\nu}^2)\)

\[
\begin{align*}
q_\mu &= Mk_\mu \\
\gamma &= M\tilde{\gamma} \\
g^2 &= M\tilde{g}^2 \\
\nu^2 &= M\tilde{\nu}^2,
\end{align*}
\]  

(18)
so that for the gauge propagator we have
\[
G^{ab}_{\mu\nu}(k) = \left\{ A^a_{\mu}(k)A^b_{\nu}(-k) \right\} = \delta^{ab} \left\{ \frac{k^2(\tilde{\gamma}^4 + k^4) + \tilde{g}^2\tilde{\nu}^2k^4}{M^2 k^6 + (\tilde{\gamma}^4 + k^4)^2 + 2\tilde{g}^2\tilde{\nu}^2k^2(\tilde{\gamma}^4 + k^4)} \right\} \times \left\{ \delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} + \frac{k^4}{(\tilde{\gamma}^4 + k^4)^2 + 2\tilde{g}^2\tilde{\nu}^2k^2(\tilde{\gamma}^4 + k^4)}M\epsilon_{\mu\nu\rho\sigma}k^\rho \right\} ,
\]
(19)
in which the parameter \( M \) appears as a global factor. Of course, expression (19) is valid only for \( M \neq 0 \). As we have already discussed, the gap equation (17) yields the parameter \( \tilde{\gamma} \) as a function of the parameters \((\tilde{g}, \tilde{\nu})\). This feature allows us to describe the analytic structure of the propagator (19) in terms of two dimensionless parameters \((\tilde{g}, \tilde{\nu})\), by analyzing the poles of expression (19).

To discuss the properties of the poles of (19), we first rewrite the expression as
\[
G^{ab}_{\mu\nu}(k) = G^{ab}_{\mu\nu}(k)\big|_{\text{par}} + G^{ab}_{\mu\nu}(k)\big|_{\text{par-viol}}
\]
with
\[
G^{ab}_{\mu\nu}(k)\big|_{\text{par}} = \delta^{ab} \left\{ \frac{k^2(\gamma^4 + k^4) + g^2\nu^2k^4}{M^2 k^6 + (\gamma^4 + k^4)^2 + 2g^2\nu^2k^2(\gamma^4 + k^4)} \right\} \left( \delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) ,
\]
(20)
\[
G^{ab}_{\mu\nu}(k)\big|_{\text{par-viol}} = \delta^{ab} \left( \frac{k^4}{M^2 k^6 + (\gamma^4 + k^4)^2 + 2g^2\nu^2k^2(\gamma^4 + k^4)} \right)M\epsilon_{\mu\nu\rho\sigma}k^\rho ,
\]
(21)
where \( G^{ab}_{\mu\nu}(k)\big|_{\text{par}} \) and \( G^{ab}_{\mu\nu}(k)\big|_{\text{par-viol}} \) stand, respectively, for the parity conserving and parity violating part of the gauge propagator (19). Further, we decompose the corresponding denominators in partial fractions, obtaining their pole structure
\[
G^{ab}_{\mu\nu}(k)\big|_{\text{par-viol}} = \delta^{ab} \left( \frac{R_1}{k^2 + m_1^2} + \frac{R_2}{k^2 + m_2^2} + \frac{R_3}{k^2 + m_3^2} + \frac{R_4}{k^2 + m_4^2} \right)M\epsilon_{\mu\nu\rho\sigma}k^\rho ,
\]
(23)
\[
G^{ab}_{\mu\nu}(k)\big|_{\text{par}} = \delta^{ab} \left( \frac{F_1}{k^2 + m_1^2} + \frac{F_2}{k^2 + m_2^2} + \frac{F_3}{k^2 + m_3^2} + \frac{F_4}{k^2 + m_4^2} \right) \left( \delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) ,
\]
(24)
where \((m_1, m_2, m_3, m_4)\) are the roots of the denominators of expressions (21) and (22). The explicit expression of \((m_1, m_2, m_3, m_4)\) turns out to be rather complicated, although they can be computed in closed form in terms of \((\tilde{m}, \tilde{g})\). The factors \((R_1, ..., R_4)\) are
\[
R_1 = \frac{m_1^4}{(m_2^2 - m_1^2)(m_3^2 - m_1^2)(m_4^2 - m_1^2)} ,
\]
(25)
\[
R_2 = -\frac{m_2^4}{(m_1^2 - m_2^2)(m_3^2 - m_2^2)(m_4^2 - m_2^2)} ,
\]
(26)
\[
R_3 = \frac{m_3^4}{(m_1^2 - m_3^2)(m_2^2 - m_3^2)(m_4^2 - m_3^2)} ,
\]
(27)
\[ R_4 = -\frac{m_4^4}{(m_2^2 - m_1^2)(m_4^2 - m_3^2)(m_4^2 - m_2^2)}, \]  
\[ (28) \]

while, for \((F_1, F_2, F_3, F_4)\), we get

\[ F_1 = \frac{m_2^2 \left( s^4 + m_1^2 \left( \tilde{g}^2 v^2 + m_4^2 \right) \right)}{(m_2^2 - m_1^2)(m_3^2 - m_1^2)(m_4^2 - m_2^2)}, \]
\[ (29) \]

\[ F_2 = -\frac{m_2^2 \left( s^4 + m_1^2 \left( \tilde{g}^2 v^2 + m_4^2 \right) \right)}{(m_2^2 - m_1^2)(m_3^2 - m_1^2)(m_4^2 - m_2^2)}, \]
\[ (30) \]

\[ F_3 = \frac{m_2^4 \left( \tilde{s}^4 + m_2^2 \left( \tilde{g}^2 v^2 + m_3^2 \right) \right)}{(m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_4^2 - m_2^2)}, \]
\[ (31) \]

\[ F_4 = -\frac{m_2^4 \left( \tilde{s}^4 + m_2^2 \left( \tilde{g}^2 v^2 + m_3^2 \right) \right)}{(m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_4^2 - m_2^2)}. \]
\[ (32) \]

2.2. Analytic structure of the gauge propagator and the regimes of the theory

In order to study the analytic structure of the gauge propagator, we look at the discriminant of the roots in the denominator of expressions (21), (22), i.e.

\[ P(k) = k^4 + 2g v \left( s^4 + k^2 \right) + \left( s^4 + k^2 \right)^2 \]
\[ (33) \]

where we have performed the change of variables \(\tilde{x} = x^2\), for \(x = k, \tilde{v}, \tilde{g}\). With these new variables, the Gribov parameter reads

\[ s^4 = \frac{1}{4} \left( \frac{g v}{4} - \frac{\tilde{g}^2}{9\pi^2} \right)^2. \]

Since the polynomial \(P(k)\) is a quartic function of \(\tilde{k}\), the discriminant can be determined in a closed form, being given by

\[ \Delta = \frac{\tilde{g}^8 \left( 4\tilde{g} + 9\pi^2 v \right)^6}{2430 \pi^2} \Delta^* \]
\[ (34) \]

with

\[
\Delta^* = \left( 1024\tilde{g}^6 \left( 324\pi^4 v^4 + 1 \right) + 9216\tilde{g}^5 \left( 162\pi^6 v^5 + 36\pi^4 v^3 + \pi^2 \tilde{v} \right) - 10368\pi^4 \tilde{g}^4 \tilde{v}^2 \left( 18\pi^3 \left( 567\pi^2 v^2 - 8 \right) \tilde{v}^2 + 25 \right) - 46656\pi^4 \tilde{g}^3 \tilde{v} \left( 3420\pi^4 v^4 + 27\pi^2 \tilde{v}^2 + 4 \right) - 8748\pi^4 \tilde{g}^2 \left( 9381\pi^6 v^4 + 96\pi^2 \tilde{v}^2 + 4 \right) - 52488\pi^4 \tilde{g} \tilde{v} \left( 274\pi^2 \tilde{v}^2 + 3 \right) - 177147\pi^6 \tilde{v}^2 \right)
\]  
\[ (35) \]

As is well known, for a quartic polynomial, if \(\Delta > 0\), it displays four complex roots; on the other hand if \(\Delta < 0\) the polynomial exhibits two real and two conjugate complex roots. In
this way, we shall be able to characterize a confining and a de-confining region in the parameter space. To do that, firstly, let us note that

\[
\lim_{n \to \infty} \Delta^* = -\infty \tag{36}
\]

\[
\lim_{\bar{g} \to \infty} \Delta^* = +\infty \tag{37}
\]

for \( g \) and \( \nu \) finite respectively.

This means that the discriminant changes sign for either large values of the vev \( \nu \) or of the coupling constant \( g \). The transition line of the discriminant’s sign can be plotted exactly, as is shown in figure 1. In particular, in the limit when both \( \bar{\nu} \) and \( \bar{g} \) tend to infinity, the transition line can be approximated by taking the leading order term of the polynomial, which behaves as

\[
\lim_{\bar{g} \to \infty, \bar{\nu} \to \infty} \Delta^* \propto \pi^4 g^4 \bar{\nu}^4 \left( 4 \bar{g} - 63 \pi^2 \bar{\nu} \right) \left( 4 \bar{g} + 81 \pi^2 \bar{\nu} \right). \tag{38}
\]

Therefore, in the infinite limit \( \nu, g \to \infty \), with \( \frac{\nu}{g} = \frac{4}{63 \pi^2} \), there is a change of sign in the discriminant, as we see from figure 1.

Now, if we consider the limit \( \bar{\nu} \to 0 \), we get

\[
\lim_{\bar{\nu} \to 0} \Delta^* = 16 \left( 64 \bar{g}^6 - 27 \bar{g}^2 (3\pi)^4 \right) \tag{39}
\]

Naturally, in this limit, the discriminant coincides with the one founded in [21], after the proper redefinitions of the parameters. In this case, the behaviour of the discriminant shows that the transition line starts at
Thus, as is apparent from figure 1, for large values of the parameter \( \bar{\nu} \) and small enough values of the parameter \( g \), we can identify two real poles, e.g. \((m_1, m_2)\), and two complex conjugate poles, e.g. \((m_3, m_4)\). This region would correspond to what we would call a weak coupling region, i.e. a small coupling constant and large values of the Higgs vev. The real poles \((m_1, m_2)\) would correspond to Yukawa-like propagators, thus being identifiable with physical excitations, provided the corresponding residues are positive. Using simple computer algebra, it is easy to show that \( R_1 \) and \( F_2 \) are always positive, while \( R_2 \) and \( F_1 \) attain negative values.

On the other hand, for large values of the coupling constant \( g \), i.e. the strong coupling regime, the propagator shows only complex poles, giving rise to the confining sector of the theory.

With the definition (18), all parameters have been converted into dimensionless quantities by means of the mass \( M \). As a consequence, a change in the parameter \( M \) would modify the scale of the diagram of figure 1, but it would not change it qualitatively.

3. Conclusion

In this work we have studied the non-perturbative behaviour of the Yang–Mills–Chern–Simons system by taking into account the Gribov horizon and in the presence of a Higgs field in the fundamental representation of the gauge group. As is well known, in this representation the Higgs mechanism affects all the components of the gauge field, giving rise to three massive gauge fields. By analyzing the structure of the infrared gauge field propagator, we are able to describe confining and de-confined regions in the parameter space. More precisely, we find that for large values of the parameter \( \bar{g}^2 = \frac{\bar{\nu}^2}{M} \), the system shows a confined regime, characterized by complex poles in the gauge propagators. On the other hand, for small values of \( \bar{g} \) and large enough values of the vev of the Higgs field, \( \bar{\nu}^2 = \frac{\nu^2}{M} \), it is possible to observe physical poles with positive masses in the propagator, signalling that we are in the de-confined regime of the theory.
Finally, let us also point out that the case in which the Higgs field is in the adjoint representation of the gauge group can be analyzed in a similar way showing, again, the existence of confined and de-confined regimes for the right range of parameters.

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