Calculation of the power value of additional heat sources for a cylindrical biogas reactor

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Abstract. When processing organic waste into biogas, it is necessary to take into account the design features of the equipment, the composition of the raw materials and the technological modes of its fermentation. If there is a lack of heat produced during the chemical fermentation reaction, additional (third-party) heat sources are used to ensure the technological regime. It can be various heat exchangers or electric heaters. The article presents the results of theoretical studies evaluating the power value of additional (third-party) heat sources for heating raw materials in a biogas reactor. The analytical dependences for estimating the power of external heat sources for the considered physical model were obtained on the basis of solving the Fourier heat equation in layered media. The analysis of the influence of design parameters and thermophysical characteristics on the amount of required power of additional (third-party) heat sources, as well as on the distribution of the temperature field in a biogas reactor, is carried out.

1. Introduction

When processing organic waste into biogas, it is necessary to take into account the design features of the equipment, the composition of the raw materials and the technological modes of its fermentation [1-9]. In this case, psychrophilic (20–25 °C), mesophilic (25–40 °C), and thermophilic (over 40 °C) fermentation modes are distinguished.

The technological regulations also need to withstand certain requirements for temperature fluctuations over a period of time, which, depending on the recommended modes, can range from ± 0.5 °C/h (with thermophilic mode) to ± 2 °C/h (with psychrophilic mode). The temperature range is determined by the specific temperature during fermentation.

If there is a lack of heat produced during the chemical fermentation reaction, additional (third-party) heat sources are used to ensure the technological regime [1, 3-5]. It can be various heat exchangers or electric heaters. The problem is the correct choice of power of additional heat sources.

2. The main results

Fermentation of organic raw materials is carried out in biogas reactors, the simplest design of which is a cylindrical tank equipped with mixing devices, as well as devices for supplying fresh raw materials, biogas selection and unloading of spent raw materials.

The calculation of the power of additional heat sources can be carried out on the basis of solving the Fourier heat equation in layered media [10].
For the mathematical formulation of the problem, the physical model of the bioreactor can be represented as a solid cylinder of radius \( R_1 \) (working volume of the reactor) and a height \( H \) surrounded by a cylindrical shell (wall) with a thickness \( \Delta \). In this case, the outer radius of the structure will be equal to \( R_2 = R_1 + \Delta \).

The power of additional (third-party) heat sources that can be located inside the working volume of the reactor will depend on the distribution of the temperature field inside the structure and the conditions of heat exchange outside.

To a first approximation, the technological range of temperature changes during fermentation can be determined in two ways: as the difference in the temperature field between the center of the bioreactor \( T_1(0) \) and the inner wall of the bioreactor \( T_1(R) \):

\[
\Delta T_1 = T_1(0) - T_1(R),
\]

or as the difference in the temperature field between the temperature on the axis of the bioreactor \( T_{ce} = T_1(0) \) and the temperature outside (environment) \( T_o \):

\[
\Delta T = T_i(0) - T_o.
\]

Thus, the calculation model of a biogas reactor is represented by a two-layer cylinder with an internal radius \( R_1 \) (working volume of the reactor), an external radius \( R_2 \) (taking into account the reactor wall thickness \( \Delta \)) and a height \( H \), and for calculating the temperature we take the axisymmetric distribution of the temperature field when the temperature inside the reactor depends only from the coordinate \( R \), i.e. consider a one-dimensional problem. On the outer surface of the wall we accept the boundary conditions of the third kind, and between the inner surface of the wall and the internal volume of biomass inside the reactor we provide the conditions for the conjugation of temperature fields and heat flows of the fourth kind.

Based on the foregoing, the temperature field distribution in each layer is determined by the general Fourier heat equation [10]:

\[
\frac{\partial T_i(r, r)}{\partial \tau} = a_i \nabla^2 T_i(r, r) + \frac{q_i(r, r)}{\rho c}, \quad i = 1, 2,
\]

where \( a_i, \rho_i, c_i \) - are the thermal diffusivity, density, and heat capacity of the material in each cylindrical layer, respectively; \( q_i(r) \) - is the power of internal (additional (third-party)) heat sources in each cylindrical layer; \( \nabla^2 T_i(r, r) \) - is the Laplace operator, in the case of an axisymmetric distribution of the temperature field in each layer:

\[
\nabla^2 T_i(r, r) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_i(r, r)}{\partial r} \right), \quad i = 1, 2.
\]

For the steady state (stationary case), equation (3) takes the form:

\[
\nabla^2 T_i(r) = - \frac{q_i(r)}{\lambda_i}, \quad i = 1, 2.
\]

where \( \lambda_i \) - is the thermal conductivity of the material of each layer.

We will also assume that the object is isotropic, i.e. Thermophysical parameters are constant and uniform throughout the volume they occupy.

The boundary conditions on the inner surface \( r = R_1 \) are defined as the boundary conditions of the fourth kind:

\[
T_1(R_1) = T_2(R_1),
\]

\[
\lambda_1 \frac{dT_1(R_1)}{dR} = \lambda_2 \frac{dT_2(R_1)}{dR}.
\]

The heat transfer conditions on the outer surface \( r = R_2 \) are defined as the boundary conditions of the third kind:
\[ T_2(R_2) + (\lambda_2/\alpha) \frac{dT_2(R_2)}{dR} = T_0 \]  
\[
\text{where} \alpha \text{ - is the heat transfer coefficient on the outer surface} \ r = R_2; T_0 \text{ - is the ambient temperature.} 
\]

The solution to equation (5) is a function of the form:

\[ T_i(r) = g_i(r) + A_iP_i(r) + B_i, i = 1,2, \]

where \( A, B \) - are constant coefficients determined by the boundary conditions,

\[ P_i(r) = \ln r, \]

\[ g_i(r) = - \int \frac{dr}{r} \int r \frac{q_i(r)}{\lambda} dr. \]

For heat sources independent of the spatial coordinate \( r \) (uniformly distributed over the volume) \( q_i(r) = q_i \) and the function defined by integral (11) has the form:

\[ g_i(r) = - \frac{q_i}{4\lambda_i} r^2. \]

Given the boundedness of the solution at \( r = 0 \), we should set \( A_1 = 0 \).

In this case, the solutions for the temperature fields in each layer are the functions:

\[ T_1(r) = T_0 - \frac{q_1}{4\lambda_1} r^2 + B_1, 0 \leq r \leq R_1, \]

\[ T_2(r) = T_0 - \frac{q_2}{4\lambda_2} r^2 + A_2 \ln r + B_2, R_1 \leq r \leq R_2. \]

If internal heat sources are present only inside the reactor volume \( (q_2 = 0) \), then we obtain:

\[ T_1(r) = T_0 - \frac{q_1}{4\lambda_1} r^2 + B_1, 0 \leq r \leq R_1, \]

\[ T_2(r) = T_0 + A_2 \ln r + B_2, R_1 \leq r \leq R_2. \]

Expressions (14), (15) can be represented in the form:

\[ T_1(r) = T_0 + \frac{q_1}{4\lambda_1} (C_3 - r^2), 0 \leq r \leq R_1, \]

\[ T_2(r) = T_0 + \frac{q_1}{4\lambda_1} (C_2 \ln r + C_2), R_1 \leq r \leq R_2. \]

The values of the coefficients \( C_1, C_2, C_3 \) are determined from conditions (6), (7), (8):

\[ C_1 = -2(\lambda_1/\lambda_2) R_2^2, \]

\[ C_2 = -C_1 \ln R_2 - C_1 (\lambda_2/\alpha) (1/R_2), \]

\[ C_3 = C_1 \ln R_1 + C_2 + R_1^2. \]

Thus, expressions (17) - (21) determine the distribution of the temperature field in the object.

The recommendations on the temperature regimes of biomass fermentation usually indicate the recommended temperature or temperature range, for example, for the mesophilic temperature regime - 34 - 37 °C. In this case, the temperature range can be taken as a first approximation as the values of the temperature field at the walls of the reactor \( T_1(R) \) and in the center of the reactor \( T_3(0) \).

Then, to maintain the temperature range \( \Delta T_1 = T_1(0) - T_3(R) \) from expression (17) we obtain:

\[ \Delta T_1 = T_1(0) - T_3(R) = \frac{q_1}{4\lambda_1} R_1^2. \]

The power of additional (third-party) heat sources is determined by the expression:

\[ q_1 = 4\lambda_1 \Delta T_1 / R_1^2. \]
Taking into account the assumptions that the power of the sources $P$ is distributed throughout the entire volume of the reactor $V$, then for $q$ we get:

$$ q_1 = P/V = 4\lambda \Delta T_1 / R_1^2. \quad (24) $$

The volume of a cylindrical reactor is equal to:

$$ V = \pi R_1^2 H, \quad (25) $$

where $H$ - is the height of the reactor.

Finally, taking into account (24) and (25), we obtain the expression for calculating the thermal power of sources $P$:

$$ P = 4\pi \lambda_1 H \Delta T_1. \quad (26) $$

From expression (26) it follows that the power of the uniformly distributed additional (third-party) heat sources necessary to maintain the temperature difference $\Delta T_1$ between the wall and the center of the reactor depends on the height of the reactor $H$ and the thermal conductivity of the biomass $\lambda_1$ and does not depend on its radius $R_1$.

3. Process analysis

Figure 1 shows the calculated values of the heat source power depending on the height of the reactor $H$ for various values of $\Delta T_1$ (°C) for biomass with thermal conductivity $\lambda_1=0.6$ Bt;/(m K).

![Figure 1. Power of additional (third-party) heat sources depending on the height of the reactor $H$ at various values of $\Delta T_1$ (°C)](image)

The next important point is the maintenance of optimal temperatures inside the reactor depending on the temperature outside (environment) $T_0$, on the thermophysical properties of the wall $\lambda_2$, wall thickness $\Delta = R_2 - R_1$ and heat transfer intensity $\alpha$.

If we take the temperature on the axis of the reactor $T_{cz} = T_1(0)$, as a normalized (critical) temperature, then we need to estimate the power of additional heat sources to maintain the temperature difference $\Delta T = T_1(0) - T_0$.

The analysis shows that in this case the power of additional heat sources will be determined according to the expression:

$$ P = 4\pi \lambda_1 H \Delta T F(R_1, \Delta, \lambda_2, \alpha), \quad (27) $$
where $F(R_1, \Delta, \lambda_2, \alpha)$ is a dimensionless function taking into account the properties of the wall and heat transfer conditions:

$$F(R_1, \Delta, \lambda_2, \alpha) = 1/\left[1 + 2\lambda_1/\left[(\alpha(R_1 + \Delta)) - 2(\lambda_1/\lambda_2)ln(R_1/(R_1 + \Delta))\right]\right],$$

where $\Delta$ is the wall thickness; $R_1$ is the inner radius of the reactor; $\alpha$ is the heat transfer coefficient on the outer surface; $\lambda_1$ is the coefficient of thermal conductivity of the biogas medium; $\lambda_2$ is the coefficient of thermal conductivity of the wall of the reactor.

The heat transfer coefficient on the outer surface $\alpha$ can be determined by the formula:

$$\alpha = \lambda_0 N_u/(2R_2),$$

where $\lambda_0$ is the coefficient of thermal conductivity of the environment; $N_u$ is the Nusselt dimensionless criterion for heat transfer conditions.

Then the function $F(R_1, \Delta, \lambda_2, \alpha)$ is determined according to the following expression:

$$F(R_1, \Delta, \lambda_2, \alpha) = 1/\left[1 + 4\lambda_1/\left[(\alpha(R_1 + \Delta)) - 2(\lambda_1/\lambda_2)ln(R_1/(R_1 + \Delta))\right]\right],$$

where $\Delta$ is the wall thickness; $R_1$ is the inner radius of the reactor; $\lambda_0$ is the coefficient of thermal conductivity of the environment; $\lambda_1$ is the coefficient of thermal conductivity of the biogas medium; $\lambda_2$ is the coefficient of thermal conductivity of the wall of the reactor; $N_u$ is the Nusselt dimensionless criterion for heat transfer conditions.

The results of calculating the power of additional (third-party) heat sources evenly distributed inside the bioreactor's working volume when the brick wall thickness $\Delta$ and the outside air temperature $T_o$ change are carried out taking into account expressions (27) - (30) are shown in Figure 2.

![Figure 2. Rated power of additional (third-party) heat sources when changing the thickness of the brick wall $\Delta$ and the outside air temperature $T_o$](image)

The nature of the surface shown in Figure 2 indicates that the required power is practically independent of the reactor wall thickness $\Delta$, but substantially depends on the ambient temperature outside the reactor $T_o$.

Figure 3 below shows the calculated values of the temperature field inside the bioreactor, obtained from expressions (17) - (18), depending on the wall thickness for the following parameters: $\lambda_1 = 0.6$ W/(m K); $\lambda_2 = 0.4$ W/(m K); $\lambda_0 = 0.022$ W/(m K); $H = 5$ m; $R_1 = 5$ m; $T_0 = 45$ °C; $N_u = 2$; $T_o = 0$ °C; $P = 30$ W.
The analysis of the surface shown in Figure 3 shows that the temperature difference between the center and the inner wall of the reactor practically depends on the wall thickness of the bioreactor $\Delta$, the temperature difference is $\Delta T = 0.796 \, ^\circ C$.

Figure 3. Distribution of the temperature field inside the bioreactor when changing the thickness of the brick wall $\Delta$

However, with the increase in the wall thickness of the bioreactor $\Delta$, the absolute temperature inside it, although insignificantly, increases and amounts to: $T_i(0) = 44.27 \, ^\circ C$ at $\Delta = 0.1$; $T_i(5) = 44.45 \, ^\circ C$ at $\Delta = 0.5$.

4. Conclusion
Analytical relations are obtained for calculating the temperature fields inside a cylindrical bioreactor, as well as calculation formulas for estimating the power of additional (third-party) heat sources evenly distributed inside the working volume for heating the mass in the bioreactor.

As a result of theoretical studies, it was found that the power of additional (third-party) heat sources necessary to ensure the temperature difference $\Delta T_1$ does not depend on the radius of the bioreactor working volume $R_1$ and is determined by parameters such as biomass thermal conductivity $\lambda_1$ and bioreactor height $H$.

The required power of additional (third-party) heat sources practically does not depend on the reactor wall thickness $\Delta$, but substantially depends on the ambient temperature outside the reactor $T_o$.

The temperature difference between the center and the inner wall of the reactor practically depends on the wall thickness of the bioreactor $\Delta$, the temperature difference is $\Delta T = 0.796 \, ^\circ C$. However, with an increase in the wall thickness of the bioreactor $\Delta$, the absolute temperature inside it, although insignificantly, increases and amounts to: $T_i(0) = 44.27 \, ^\circ C$ at $\Delta = 0.1 \, m$; $T_i(5) = 44.45 \, ^\circ C$ at $\Delta = 0.5 \, m$.

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