Diffusion of chemically reactive species in MHD oscillatory flow with thermal radiation in the presence of constant suction and injection

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Abstract. In this project, it is proposed to investigate the effect of suction/injection on the unsteady oscillatory flow of an incompressible viscous electrically conducting fluid through a channel filled with porous medium and non-uniform wall temperature. The fluid is subjected to a uniform magnetic field normal to the channel and the velocity slip at the cold plate is taken into consideration. With the assumption of magnetic Reynolds number to be very small, the induced magnetic field is neglected. Assuming pressure gradient to be oscillatory across the ends of the channel, resulting flow as unsteady oscillatory flow. Under the usual Boussinesq approximation, a mathematical model representing this fluid flow consisting of governing equations with boundary conditions will be developed. Closed form solutions of the dimensionless governing equations of the fluid flow, namely momentum equation, energy equation and species concentration can be obtained. The effects of heat radiation and chemical reaction with suction and injection on temperature, velocity and species concentration profiles will be analysed with tables and graphs.

Keywords: Oscillatory flow, Magneto-Hydrodynamic (MHD), Slip flow, Thermal radiation, Suction/Injection, Chemical reaction.

1. Introduction

Heat and mass transfer on magneto hydrodynamic (MHD) oscillatory flow along vertical channel exist in many physiological flows, astrophysics and in many fields of engineering. MHD is the study of the interaction of the magnetic field and the plasma treated as a fluid. Several authors have investigated MHD oscillatory channel flow of an electrically conducting fluids. Muthuraj [1] investigated heat transfer effect on MHD oscillatory flow through asymmetric wavy channel and Satya Narayana [2] presented heat and mass transfer effects on MHD oscillatory flow through irregular channel. The study of fluid flow through porous medium process exist in many applications such as underground water resources, water filtration and purification, seepage of water in river beds and filtering gasses.

In agricultural engineering porous media plays an important role in germination of seeds. In addition, few authors have studied the oscillatory flow problem with porous medium [3, 4]...
Makinde and Falade have presented heat transfer to MHD oscillatory flow in channel filled with porous medium and MHD oscillatory flow through a porous channel saturated with porous medium respectively. This phenomenon is common role in human body especially breathing and discharge of excretes through porous skin. The study of bio-mathematics has provided many applications in biological engineering and medicine.

Several authors analysed blood flow as electrically conducting fluid experimentally and theoretically. Misra [5] have presented slip velocity in blood flow through stenosed arteries. In addition Ramachandra Rao and Ogulu [6, 7] reported MHD oscillatory flow of blood through channels of variable cross section and heat transfer on oscillatory blood flow in indented porous artery. The study of the combined heat and mass transfer process by convection in a porous medium used in field of chemical engineering. Free convection define in the fluid, the change in temperature cause density variation leading to buoyancy forces acting on the fluid elements Sasikumar [8] have analyzed free convective MHD oscillatory flow past parallel plates in a porous medium with heat source and chemical reaction. Hossain [9] and K.D. Singh [10] have presented some problems of mixed convection and heat transfer in the vertical channel.

The study of slip effect process exist with several applications in the fields of engineering, geophysics and agriculture. The slip effect on MHD oscillatory flow of fluid in a porous channel with heat and mass transfer and chemical reaction has applications in the fields of engineering such as oil recovery, cooling of nuclear reactor and geothermal reservoirs. Several authors calculated slip effect on MHD oscillatory flow with heat and mass transfer with chemical reaction. To mention just few Sasikumar [11] have investigated effects of heat and mass transfer on MHD oscillatory flow of fluid in porous medium with chemical reaction and slip conditions in asymmetric wavy channel. Adesanya [12] studied MHD oscillatory slip flow and heat transfer filled with porous media. In addition, Daniel and Sinha [13, 14] have discussed slip effect on MHD oscillatory flow of fluid in porous medium with chemical reaction and slip velocity on the oscillatory flow of blood through porous vessel in the presence of heat source and chemical reaction. Mehmood [15] have considered effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Suction or injection at the surface of a porous disk is generally used in chemical engineering to increase the electrochemical reaction time during electrolytic process.

Many researchers Sahin Ahmed [16] have investigated suction effects on MHD oscillatory flow through porous channel. Acharya [17] have presented magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and heat flux. Devika and Misra [18, 19] have analyzed MHD oscillatory flow of a visco-elastic fluid in a porous channel with chemical reaction and MHD oscillatory channel flow heat and mass transfer in a physiological fluid in presence of chemical reaction respectively.

After thorough survey of the literature it is observed that the effect on suction/injection and thermal radiation on the slip flow of oscillatory hydromagnetic fluid through a channel filled with saturated porous medium in presence of chemical reaction has not been discussed.

2. Mathematical Formulation

- Consider suction/injection on the unsteady oscillatory flow of an incompressible viscous electrically conducting fluid through a channel filled with porous medium.
- The flow is subjected to suction at the cold wall and injection at the heated wall.
- An external magnetic field is applied normal to the channel. Magnetic Reynolds number is assumed to be much less than unity, so that the induced magnetic field is negligible in comparison to the applied magnetic field.
- Assuming pressure gradient to be oscillatory across the ends of the channel, resulting flow
is unsteady oscillatory.

- All the fluid properties are assumed to be constant except the influence of density variation with temperature.
- The basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium.

Choose the rectangular coordinate system \((x',y')\) with \(x'\)-axis lying along middle of the channel and \(y'\)-axis along width of the channel having boundary walls at \(y' = 0\) and \(y' = h\) as shown in the figure. Fluid injected through the heated wall and sucked through cold wall with constant velocity \(\nu_0\).

Under the usual Boussinesq approximation the equations governing the flow are as follows:

Assuming Boussinesq approximation, the governing equations of the flow i.e momentum equation, energy equation and species concentration equation are formulated as follows:

\[
\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{dP'}{dx'} + v \frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{k} u' - \frac{\nu B_0^2}{\rho} u' + \frac{\nu B_0^2}{\rho} u' + g\beta T (T' - T_0) + G\beta_C (C' - C_0) \tag{1}
\]

\[
\frac{\partial T'}{\partial y'} - v_0 \frac{\partial T'}{\partial y'} = k_f \rho \frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho C_p} (T' - T_0) \tag{2}
\]

\[
\frac{\partial C'}{\partial y'} - v_0 \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - k_C (C' - C_0) \tag{3}
\]

with the boundary conditions

\[
u' = \sqrt{\frac{k}{\alpha}} \frac{du'}{dy'}; \quad T = T_0; \quad C = C_0 \text{ on } y' = 0 \tag{4}
\]

\[
u' = 0; \quad T' = T_1; \quad C' = C_1 \text{ on } y' = h \tag{5}
\]

We now introduce the following non-dimensional variables:

\[
(x,y) = \left(\frac{x'}{h}, \frac{y'}{h}\right), \quad u = \frac{hu'}{\nu}, \quad t = \frac{u' t'}{\nu}, \quad Pr = \frac{\rho C_p \nu}{k}, \quad p = \frac{h^2 \rho u'}{\nu^2} \tag{6}
\]

\[
G_r = \frac{g\beta (T_1 - T_0) h^3}{\nu^2}, \quad G_c = \frac{g\beta C (C_1 - C_0) h^3}{\nu^2}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \phi = \frac{C - C_0}{C_1 - C_0}, \quad N = \frac{4\alpha^2 h^2}{\rho C_p}, \quad Da = \frac{k}{\nu^2}, \quad Ha^2 = \frac{\sigma_B h^2}{\rho \nu^2}, \quad Sc = \frac{\nu}{D}, \quad kC = \frac{Dk_C (C_1 - C_0)}{\nu^2}
\]
Equations (1),(2), and (3) transformed into The dimensionless equations are:

\[ \frac{\partial u}{\partial t} - s \frac{\partial u}{\partial y} = \frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2} - \left( Ha^2 + \frac{1}{Da} \right) u + Gr\theta + Gc\varphi \]  

(7)

\[ \frac{\partial \theta}{\partial t} - s \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + N\theta \]  

(8)

\[ \frac{\partial \varphi}{\partial t} - s \frac{\partial \varphi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \varphi}{\partial y^2} - kc\varphi \]  

(9)

with the appropriate boundary conditions

\[ u = \gamma \frac{du}{dy}, \theta = 0, \varphi = 0, \text{ on } y = 0 \]  

(10)

\[ u = 0, \theta = 1, \varphi = 1, \text{ on } y = 1 \]  

(11)

3. Method of Solution

Taking pressure gradient for oscillatory flow

\[ -\frac{dp}{dx} = \lambda e^{i\omega t} \]

For oscillatory flow, assuming the solution as

\[ u(t, y) = u_0(y)e^{i\omega t} \]

\[ \theta(t, y) = \theta_0(y)e^{i\omega t} \]

\[ \varphi(t, y) = \varphi_0(y)e^{i\omega t} \]

Equation (7), (8), and (9) becomes,

\[ u''_0 - su'_0 - \left( H a^2 + \frac{1}{D a} + i\omega \right) u_0 = -\lambda - Gr\theta - Gc\varphi \]  

(12)

\[ \theta''_0 + sPr\theta'_0 + (N - i\omega)Pr\theta_0 = 0 \]  

(13)

\[ \varphi''_0 + sSc\varphi'_0 + (-kc - i\omega)Sc\varphi_0 = 0 \]  

(14)

with the boundary conditions

\[ u_0 = \gamma \frac{du}{dy}, \theta_0 = 0, \varphi_0 = 0, \text{ at } y = 0 \]

\[ u_0 = 0, \theta_0 = 1, \varphi_0 = 1, \text{ at } y = 1 \]

Solving equation (13) with boundary conditions

\[ \theta_0 = \frac{1}{e^{m_1} - e^{m_2}} \{ e^{m_1y} - e^{m_2y} \} \]

\[ \theta = \frac{1}{e^{m_1} - e^{m_2}} \{ e^{m_1y} - e^{m_2y} \} e^{i\omega t} \]
Solving equation (14) with boundary conditions
\[
\phi_0 = \frac{1}{e^{m_3} - e^{m_4}} \{ e^{m_3 y} - e^{m_4 y} \}
\]
\[
\phi = \frac{1}{e^{m_3} - e^{m_4}} \{ e^{m_3 y} - e^{m_4 y} \} e^{i\omega t}
\]

Solving equation (12) with boundary conditions
\[
u_0 = \left\{ A_1 e^{m_5 y} + B_1 e^{m_6 y} + Q_0 + Q_1 e^{m_1 y} + Q_2 e^{m_2 y} + Q_3 e^{m_3 y} + Q_4 e^{m_4 y} \right\}
\]
\[
u = \left\{ A_1 e^{m_5 y} + B_1 e^{m_6 y} + Q_0 + Q_1 e^{m_1 y} + Q_2 e^{m_2 y} + Q_3 e^{m_3 y} + Q_4 e^{m_4 y} \right\} e^{i\omega t}
\]

The rate of heat transfer is given by
\[
Nu = \frac{\partial \theta}{\partial y} = (A_0 m_1 e^{m_1 y} + B_0 m_2 e^{m_2 y}) e^{i\omega t}
\]
and the shear stress is given by
\[
S_f = \frac{\partial \nu}{\partial y} = (A_1 m_5 e^{m_5 y} + B_1 m_5 e^{m_6 y} + Q_1 m_1 e^{m_1 y} + Q_2 m_2 e^{m_2 y} + Q_3 m_3 e^{m_3 y} + Q_4 m_4 e^{m_4 y}) e^{i\omega t}
\]

All constants are defined in Appendix.

4. Results and Discussions

In this paper, the MHD oscillatory flow through a porous medium with chemical reaction in slip regime under constant suction and injection analyzed. The temperature profiles are drawn in figure 1 - figure 4 for various values of suction parameter, Prandtl number, radiation parameter and oscillation respectively. In figure 1 and figure 3 display results of temperature profile increase due to either an increase in the suction parameter or radiation parameter. Figure 4 shows the effects of oscillation on temperature profile. It is clear from the figure 4 that temperature profile decreases with an increasing values of oscillation. In figure 2 effect of Prandtl number on temperature profile increasing for upto $Pr = 3$ and then starts decreasing.

The concentration profiles are drawn in figure 5 - figure 8 for various values of suction parameter, Schmidt number, chemical reaction parameter, oscillation respectively. In figure 6 and figure 7 respectively shows the effect of Schmidt number and chemical reaction parameter on concentration. It is clear that concentration decreases due to either the Schmidt number increases or the chemical reaction parameter increases. In figure 8 displays the concentration profiles decreases due to oscillation increases. Figure 5 displays the concentration profiles which increase due to the increase in suction parameter values.

In figure 9 - figure 20 for various values of pressure gradient, Grashof and modified Grashof number, Hartmann number, Darcy number, suction parameter, oscillation, slip parameter, Prandtl number, thermal radiation parameter, Schmidt number, chemical reaction parameter respectively.

Figure 9 and figure 13 respectively shows that effect of pressure gradient and Darcy number on velocity profile. It is clear that velocity increases due to either pressure gradient or Darcy number increases.

In figure 10 and figure 11 shows that velocity profile increases due to either Grashof number or modified Grashof number increases.
Figure 16 and figure 18 illustrate the effect of wall slip parameter and thermal radiation parameter. It is noted that the velocity profile increases due to the increase in either slip parameter or thermal radiation values.

In figure 14 displays velocity profile increases as suction parameter increases up to $y = 0.6$ and for $y$ greater than point 0.6 pattern of velocity reversed.

In figure 12 and figure 15 respectively show that the effect of Hartmann number and oscillation on velocity profiles. It is clear that velocity profiles decrease due to either Hartmann number or oscillation increases.

Figure 17 and figure 19 shows that velocity profiles decreases either Prandtl number or Schmidt number increases.

In figure 20 displays the effect of due to chemical reaction parameter increases due to velocity profile decreases.

Now we consider two special analysis one is rate of heat transfer and another one is skin friction. From figure 21 displays the rate of heat transfer increases as suction parameter increases up to $y = 0.4$ and for $y$ greater than point 0.4 pattern of heat transfer rate is reversed.

In figure 22 displays the skin-friction increases as suction parameter increases up to $y = 0.2$ and $y$ greater than point 0.8 and skin-friction decreases between $y = 0.2$ to $y = 0.8$.

**Figure 1.** Effect of suction parameter on fluid temperature

**Figure 2.** Effect of Prandtl number on fluid temperature

**Figure 3.** Effect of thermal radiation on fluid temperature

**Figure 4.** Effect of oscillation on fluid temperature
Figure 5. Effect of suction parameter on fluid concentration

Figure 6. Effect of Schmidt on fluid concentration

Figure 7. Effect of chemical reaction parameter on fluid concentration

Figure 8. Effect of oscillation on fluid concentration

Figure 9. Effect of pressure gradient on fluid velocity

Figure 10. Effect of Grashof number on fluid velocity
Figure 11. Effect of modified Grashof number on fluid velocity

Figure 12. Effect of Hartmann’s number on fluid velocity

Figure 13. Effect of Darcy number on fluid velocity

Figure 14. Effect of suction parameter on fluid velocity

Figure 15. Effect of oscillation on fluid velocity

Figure 16. Effect of wall slip parameter on fluid velocity
**Figure 17.** Effect of Prandtl number on fluid velocity

**Figure 18.** Effect of thermal radiation on fluid velocity

**Figure 19.** Effect of Schmidt number on fluid velocity

**Figure 20.** Effect of chemical reaction parameter on fluid velocity

**Figure 21.** Effect of suction parameter on the rate of heat transfer

**Figure 22.** Effect of suction on the skin-friction across the channel
5. Conclusion

- When suction parameter increases fluid temperature increases.
- When suction parameter increases fluid velocity near the cold wall increases and fluid velocity decreases after middle (y greater than 0.6) of the channel.
- Increase in suction parameter decreases the rate of heat transfer at the heated wall and increases at the cold wall.
- Increase in suction parameter results in increase of skinfriction near the walls and decreases when away from the walls.
- As chemical reaction parameter increases concentration and velocity profiles are decreasing.
- When thermal radiation parameter increases fluid temperature and fluid velocity increases.

6. appendix

\[
\begin{align*}
Q_1 &= \frac{m_3^2 + s m_2 - (H a^2 + \frac{1}{D a} + i \omega)}{K c G c} \\
Q_2 &= -\frac{m_3^2 + s m_2 - (H a^2 + \frac{1}{D a} + i \omega)}{G r B g} \\
Q_3 &= -\frac{m_3^2 + s m_3 - (H a^2 + \frac{1}{D a} + i \omega)}{G r C c}
\end{align*}
\]

\[
\begin{align*}
m_1 &= \frac{-s Pr + \sqrt{s Pr^2 - 4(N - i \omega) Pr}}{2} \\
m_2 &= \frac{-s Pr - \sqrt{s Pr^2 - 4(N - i \omega) Pr}}{2} \\
m_3 &= \frac{-s Sc + \sqrt{s Sc^2 - 4(-K e - i \omega) Sc}}{2} \\
m_4 &= \frac{-s Sc - \sqrt{s Sc^2 - 4(-K e - i \omega) Sc}}{2} \\
m_5 &= \frac{\lambda}{2} \left( \frac{H a^2 + \frac{1}{D a} + i \omega}{D a G d} \right) \\
m_6 &= \frac{\lambda}{2} \left( \frac{H a^2 + \frac{1}{D a} + i \omega}{D a G d} \right)
\end{align*}
\]

\[
\begin{align*}
Q_4 &= -\frac{m_2^2 + s m_4 - (H a^2 + \frac{1}{D a} + i \omega)}{m_2 + s m_4 - (H a^2 + \frac{1}{D a} + i \omega)} \\
A_1 &= B_1 \left( m_6 \gamma - 1 \right) + m_1 - n_0 \\
B_1 &= \left( \frac{m_6 \gamma - 1}{1 - m_6} \right) \\
A_0 &= \frac{1}{e^{m_1} - e^{m_4}} \\
B_0 &= \frac{1}{e^{m_1} - e^{m_4}} \\
C_0 &= \frac{1}{e^{m_3} - e^{m_4}} \\
D_0 &= \frac{1}{e^{m_3} - e^{m_4}}
\end{align*}
\]

7. Nomenclature

- \( Da \) - Darcy number
- \( t' \) - Time
- \( u' \) - Axial velocity
- \( p' \) - Fluid pressure
- \( k \) - Porous permeability
- \( B_0 \) - Magnetic field intensity
- \( g \) - Gravitational acceleration
- \( C_p \) - Specific heat at constant
- \( k_f \) - Thermal conductivity
- \( T' \) - Fluid temperature
- \( T_0 \) - Referenced fluid temperature
- \( k_c \) - Chemical reaction parameter
- \( C' \) - Fluid concentration
- \( C_0 \) - Referenced fluid concentration
- \( s \) - Suction injection parameter
- \( H a^2 \) - Hartmanns number
- \( Gr \) - Grashof number
- \( Gc \) - Modified Grashof number
- \( Pr \) - Prandtl number
- \( N \) - Thermal radiation parameter
- \( D \) - Species concentration
Greek Symbols

\[ \theta \] - Fluid temperature
\[ \beta \] - Volumetric expansion
\[ \beta_T \] - Coefficient of thermal expansion
\[ \nu \] - Kinematic viscosity
\[ \sigma_e \] - Electrical conductivity
\[ \omega \] - Frequency of the oscillation.

\[ \alpha \] - Thermal radiation;
\[ \beta_C \] - Coefficient of mass expansion;
\[ \rho \] - Fluid density;
\[ \nu_0 \] - Constant horizontal velocity;
\[ \lambda \] - Positive constant;

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