Simulatable Security for Quantum Protocols

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Abstract. The notion of simulatable security (reactive simulatability, universal composability) is a powerful tool for allowing the modular design of cryptographic protocols (composition of protocols) and showing the security of a given protocol embedded in a larger one. Recently, these methods have received much attention in the quantum cryptographic community (e.g. [RK04,ROBL+04]).

We give a short introduction to simulatable security in general and proceed by sketching the many different definitional choices together with their advantages and disadvantages. Based on the reactive simulatability modelling of Backes, Pfitzmann and Waidner [BPW04], we then develop a quantum security model. By following the BPW modelling as closely as possible, we show that composable quantum security definitions for quantum protocols can strongly profit from their classical counterparts, since most of the definitional choices in the modelling are independent of the underlying machine model.

In particular, we give a proof for the simple composition theorem in our framework.

Keywords: quantum cryptography, security definitions, simulatable security, universal composability.

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| Version 1, 18 Sep 2004 | Initial version |
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1 Introduction

1.1 Overview

In Section 1.2 we state what the contribution of this work is.
In Section 1.3 we give a show statement about the use of terminology in the field of simulatable security.
In Section 1.4 we try to give an overview of the historical development of simulatable security.
Sections 1.5 and 1.6 give a short introduction to the notion of simulatable security.
Sections 1.7–1.13 gives an overview on design decisions appearing in the modelling of simulatable security.
Section 1.14 gives a short comparison between our model and that of [BOM04].
Section 1.15 very tersely recapitulates the quantum mechanical formalism used in this work.
Section 2 is concerned with the actual definition of quantum machines and quantum networks.
Based on these definitions, Section 4 gives a security definition for quantum protocols.
Section 4 introduces the definition of composition and shows the simple composition theorem.
Finally, in Section 5 some concluding remarks can be found.

1.2 Our contribution

Our contribution in this work is threefold:

– In the introduction we give survey on design decision that have to be made when designing a simulatable security model. We do not only expose the decisions involved in our definition, but try to look at other models, too. The problems in the definitions of simulatable security models are often underestimated, we hope that our survey will give an impression what problems lies ahead on the (probably still quite long) route of finding a simple and convincing model of security.

– Our second contribution is to show, that when defining quantum security models, many of the decisions to be made are not related to the quantum nature of the communication, but would already appear in a classical modelling. To emphasise that point, we develop our model in strong similarity to the classical modelling of [BPW04b], our quantum model can therefore be seen as a quantum extension of that modelling.
By this we hope to show that a quantum and classical security models should be developed hand in hand. First solve the problems appearing in the classical modelling, and then try and lift the classical model to the quantum case.

– Third we give a concrete model of security. We hope that this model will show problems and possibilities in simulatable security, and be a step on the way towards a simple and yet general security definition.
The current version seems to represent a consistent modeling of security, however
the author has to admit that the generality in the modelling of scheduling had a
prize: complexity. We fear that in the present modelling a complete formal proof of
security might be quite unwieldy. This can be seen in the present work in Section 4.1
where the author was unable to find a readable proof for the statements there and
therefore took recourse to a rather vague sketch. We hope that future security models
will solve this problem without loss of generality. (See also Section 5).

1.3 A word on terminology

Some confusion exists when it comes to actually finding a name for the concept of
simulatable security. In order to prevent misunderstandings and allow the reader to
compare the present modelling with others, we will shortly comment on the terminology
used in the present work.

The most widely known term is universal composability. This notion was introduced
by Canetti [Can01]. Since then, the notion universal composition and especially UC
framework became strongly associated with the model of Canetti. However, the word
universal composability is used in two other ways: first, it is used to denote the property
to be secure according to definitions similar to that of [Can01] or [BPW04b], without
meaning the model of Canetti in particular. Second, universal composability often de-
notes the applicability of the Composition Theorem. However, since there are different
flavours of the Composition Theorem, sometimes universal composability means concur-
rent and simple composability1 (e.g. in [Can01]), while on other occasions it is used for
simple composability.

Besides the model of Canetti, another model found much interest in the last few
years: the model of Backes, Pfitzmann, and Waidner [PW01,BPW04b]. While the term
of universal composability in its most general meaning can be applied to that model, too,
Backes, Pfitzmann, and Waidner [BPW04b] prefer the use of reactive simulatability.

In order to avoid such confusion, we will adhere to the following convention in the
present exposition: for the modelling by Canetti we will use the term UC framework.
The modelling by BPW we shall name reactive simulatability (or shorter RS frame-
work), while the overall concept of security notions using simulation and guaranteeing
composition (encompassing these two modelings) we will call simulatable security.

The different flavors of the composition theorem we will differentiate by using the
attributes simple, concurrent, and the combination of both (see Section 1.9). In this
nomenclature the Universal Composition Theorem of [Can01] would be called Simple
and Concurrent Composition Theorem, while the Composition Theorem of [BPW04b]
would be named Simple Composition Theorem. We restrain from using the shorter term
universal composition for simple and concurrent composition to prevent confusion with
the other meanings described above.

A further term which is noteworthy in this context is that of the honest user and the
environment, resp. Both denote the same idea, the first being used in the RS framework,

1 See Section 1.9 for an analysis of the difference between concurrent and simple composability.
the second in the UC framework. We will use these two notions in an interchangeable way, preferring *environment* when trying to motivate or explain on an intuitive level, while sticking to *honest user* when giving formal definitions or proofs. The same holds for the terms *trusted host* (RS framework) and *ideal functionality* (UC framework).

More terms will appear in the course of this exposition which have different translations in different frameworks. We will mention these when introducing the notions in our exposition. The reader is strongly encouraged to use the index (p. 48) to find these translations.

### 1.4 A brief historical account

To the best of our knowledge, the notion of a simulator to define security of a protocol was first introduced in the definition of zero-knowledge proofs [GMR85]. Here the simulation paradigm was used to ensure that the verifier could not learn anything except the truth of the statement to be proven. This was done by requiring that any transcript of the interaction between prover and verifier could also be generated by the simulator (without knowing a proof witness for the statement). If this is possible, then the interaction of course does not allow to learn anything about the proof.

However, it turned out that this definition, while capturing very well the idea that the verifier learns nothing about the proof, does not guarantee the possibility to compose two zero-knowledge protocols in parallel (without losing the zero-knowledge property) [GK96].

Note further, that here the simulation paradigm was only used for one of the required properties (being zero-knowledge), the soundness condition (i.e., that the protocol indeed is a proof) was defined in another manner. In this it differs from today’s simulatable security which aspires to capture all security properties in one single definition.

In [Bea92] the notion of *relative resilience* was introduced. This notion allowed to say that one protocol $\pi$ was at least as secure as another protocol/trusted host $\rho$ by requesting that for all protocol inputs and all adversaries, there is a simulator, so that the outputs of $\pi$ with the adversary and $\rho$ running with the simulator are indistinguishable. This was a major step towards today’s notion of simulatability, since a protocol’s security was now defined in comparison to an ideal specification. [Bea92] showed, that the security definition was closed under sequential composition (executing one protocol after another). However, when executing protocols in parallel, no guarantee was given.

At the same time [MR92] announced another model based a notion of simulatability, even achieving some kind of composition (*reducibility*). It is unknown to the author, whether this idea was further pursued.

Later [PSW00] and [Can00] introduced independent models with synchronous scheduling (see Section 1.9.3). The model of [PSW00] (a predecessor of the BPW-model [BPW04b] underlying our model) already had a simple composition theorem, i.e. one could use one protocol as a sub-protocol of another, and the sub-protocol could run simultaneously with the calling protocol (see Section 1.9). [Can00] achieved a similar result, however the security of a composed protocol could only be ensured in [Can00] if the calling protocol was suspended until the sub-protocol terminated. When the calling and the called
protocol where executed simultaneously, no guarantee would be made. So this composition theorem may be classified as being a sequential composition theorem, similar (but somewhat more powerful) to that of [Bea92].

Shortly afterwards, both Pfitzmann, Waidner [PW01], and Canetti [Can01] presented asynchronous versions of their models. [PW01] had—in our nomenclature—a simple composition theorem, and [Can01] a simple and concurrent composition theorem (see Section 1.9). In [BPW04a] it was shown, that not only simple but also concurrent composition is also possible in their model.

To the best of our knowledge, the first simulation based quantum security models were [vdG98] and [Smi01]. Like [Bea92], both did not have the notion of an honest user/environment, therefore they too could only guarantee sequential composition. The first models having an honest user/environment were the independent works [BOM02] and [Unr02]. Both based on Canetti’s model, they both provided simple and concurrent composition (see Section 1.9).

1.5 On the necessity of simulatable security

A question that may arise is, why do we need another security definition. Are there not sufficiently many definitions like privacy, correctness, robustness, non-maleability, etc. (the list of security properties found in the literature is very long)? Why add another one?

There are several good reasons for this:

– There are (admittedly constructed) protocols, where the privacy requirement is fulfilled, where the correctness requirement is fulfilled, but where in information nevertheless leaks. The example goes back to Mical and Rogaway and we cite here the version found in the introduction of [Graaf:1998:Towards]: Let $x$ and $y$ be the inputs of Alice and Bob, resp. They want to evaluate the function $g$ given by

$$g(x, y) := \begin{cases} 0x, & \text{if } \text{lsb}(y) = 0, \\ 1y, & \text{if } \text{lsb}(y) = 1, \end{cases}$$

Here $\text{lsb}(y)$ stands for the last bit of $y$, and $0x$ and $1y$ for concatenation.

Consider the following protocol for this task:

1. Alice and Bob commit to the bits of their inputs using some secure bit commitment scheme.
2. Bob unveils $\text{lsb}(y)$.
3. If $\text{lsb}(y) = 0$, Bob unveils $x$. If $\text{lsb}(y) = 1$, Alice unveils $y$. Now the both parties can calculate $g(x, y)$.
4. Additionally (just for the sake of the counterexample), Alice sends $x$ to Bob. Clearly, the protocol is correct (i.e. the function $g(x, y)$ is always evaluated correctly). Further, the privacy condition is fulfilled, since in the specification of the protocol there always is a way for Bob to learn Alice’s input, so the fact that $x$ is sent to Alice does formally not violate the privacy condition (see [Graaf:1998:Towards] for details).
But intuitively, the function is not evaluated in a secure fashion, since in the specifica-

tion Bob would learn Alice’s input only for $\text{lsb}(y) = 0$, while in the protocol he

always learns it.

So a new definition is needed to capture both privacy and correctness in one go.

Simulatable security has this advantage.

- The second problem is that the list of desirable security properties is ever growing, it

is not restricted to just privacy and correctness. The simulatable security encompasses

many definitions at once using a very general approach (see Section 1.6), so one

can have more confidence that the intuitively security is guaranteed by simulatable

security. (Note however, that some special security properties like incoercibility are

not guaranteed by simulatable security [MQ03].)

- The probably most striking problem of many security properties is incomposability.

If a protocol $\pi$ is given for some cryptographic primitive $X$ (say a bit commitment),

and another protocol $\rho$ using $X$ securely accomplishes something great, then it would

seem natural to combine $\pi$ and $\rho$ to get a protocol accomplishing the great thing

without recurse to any primitives. However, it turns out that there is no guarantee

that the composed protocol is still secure.

To remove this lack of certainty, and to allow the modular construction of protocols,

we need a security definition that is composable, i.e. that allows design protocols for

small task, and using these as building blocks for bigger protocols, without having

to prove the security of the bigger protocols from scratch.

Fortunately, simulatable security provides such composability (see Section 1.9).

1.6 What is simulatable security?

The basic idea is a follows: Assume, that we have some given cryptographic application,

and we can specify some reference protocol (the ideal protocol) or trusted party $\text{TH}$,

which does implement the wanted behaviour in a secure way. Of course, we do now have

to design that trusted host, but this is usually easier than designing the protocol, since

we do not have to bother with details like insecure communication, or who should carry

out computations etc., since $\text{TH}$ can by definition be regarded as trusted.

If we then accept, that the trusted party is secure (though not necessarily feasible), we

can define that some protocol $\pi$ (the real protocol) is as secure as $\text{TH}$, if replacing $\text{TH}$ in

any situation does not result in any disadvantage for any person (except the adversary,

of course). But how do we formalise situations and how do we formalise disadvantages?
We simply introduce the concept of the environment and the (real) adversary. The

environment is some entity, which does interact with the protocol and the adversary

as a black box and at the end may decide, whether something harmful has happened.

When we quantify over all possible environments (possibly restricting the computational

power), and for no environment (i.e. in no situation) some harm happened using $\pi$ which

could not have happened using $\text{TH}$, too, then replacing $\text{TH}$ by $\pi$ is clearly a sensible

course of action, at least it will surely do no harm (none that could be detected by the

environment, at least), thus $\pi$ can be considered to be secure.
The preceding explanation still has some great drawbacks: An adversary (for which the protocol's internals are no black box, of course) could simply vary its output depending on whether we use $\pi$ or $\text{TH}$. Then the environment might simply consider the output “$\pi$ is in use” as harmful, and suddenly $\pi$ would be insecure. But, when considering our requirement, that *everything harmful, which can happen using $\pi$, can also happen using $\text{TH}$*, we can reasonably interpret it as *everything harmful, which can happen using $\pi$ with some adversary $A_{\text{real}}$, can also happen using $\text{TH}$ with some (other) adversary $A_{\text{sim}}$, the simulator*. If we accept this formulation that implies, that in any situation the adversary may freely choose its strategy to be as “harmful” as possible, we get the security definition which we will develop and examine in this work.

One simplification we may still introduce: So far we have required, that using $\pi$, we get at most as much harm as when using $\text{TH}$. In fact, we can change this definition so that we require that we get the *same amount* of harm. This simplifies the definition and is in fact equivalent, since if one environment defines something as harm, then some other environment could define the opposite as harm, such bounding the amount of harm from both sides and resulting in a claim of equality. After introducing this simplification, the word “harm” is of course inadequate, so we will use the more neutral notion of the *output of the environment*. Further it is a matter of taste, whether we should say that the environment outputs exactly one bit, one may also allow the environment to generate a whole stream of output, which should then be indistinguishable in runs with the real protocol and the trusted host (ideal protocol), see Section 1.8.

So if we put these considerations together, we get a first definition sketch, that runs in the following lines: For any adversary $A_{\text{real}}$, there is some adversary $A_{\text{sim}}$, such that for any environment $H$, the output of $H$ when running with $A_{\text{real}}$ and $\pi$ is indistinguishable from that of $H$ running with $A_{\text{sim}}$ and $\text{TH}$.

In the RS framework of [PW01, BPW04b], the environment is instead called the *honest user*, reflecting the intuition that its view represents anything (e.g. any harm) that an honest user of the protocol may experience. Further instead of the notion *trusted host*, it is also very common to speak of an *ideal functionality*, e.g. in the model of [Can01].

Of course, the above definition sketch still leaves many open questions, like what a protocol formally is, what indistinguishable means, how messages and machines are scheduled, etc. We will try and discuss these questions in the following paragraphs.

1.7 On the order of quantifiers

In Section 1.6 we “defined” simulatable security approximately as follows: For all adversaries there is a simulator s.t. for all environments the real and ideal protocol are indistinguishable, in symbols: $\forall A_{\text{real}} \exists A_{\text{sim}} \forall H \ldots$. However, rereading our motivation one might ask whether the following order of quantifiers does not have as much justification: $\forall A_{\text{real}} \forall H \exists A_{\text{sim}} \ldots$, i.e. should the simulator be allowed to depend on the environment/honest user or not?

In fact, both order are common. [BPW04b] calls security with respect to the $\forall A_{\text{real}} \exists A_{\text{sim}} \forall H$-ordering *universal security*, and security with respect to the $\forall A_{\text{real}} \forall H \exists A_{\text{sim}}$-ordering *stan-
The model of \cite{Can01} uses the stricter notion of universal security, while the \cite{BPW04b} model defines both notions.

One might wonder, whether standard and universal security are equivalent. It was however shown in \cite{?}, that for statistical and polynomial security (see Section \ref{sec:views-output}), there are examples separating these two notions.

There is also a very practical point separating these two notions: while for showing a simple composition theorem (see Section \ref{sec:composition}), it is sufficient to have standard security. However all proof for concurrent composition theorems known to the author need universal security. Note that to the best of our knowledge it is unknown whether concurrent composability could be proven using standard simulatability.

A further ordering of quantifiers appears in \cite{BPW04b}: black-box security. This notion, which is even stricter than universal security, is roughly defined as follows: There is a simulator s.t. for all real adversaries and all environments, the real protocol running with environment and honest user is indistinguishable from the ideal protocol running with environment and simulator, where the simulator has access to the adversary in a black-box manner. In other words, the simulator is not allowed to depend on the real adversary in any manner, but only by using the adversary as a black-box (without rewinding). Since no properties are known to the author which give an advantage of black-box over universal security, we will ignore this notion in the present work, concentrating on universal and standard security.

### 1.8 Views versus output bits - the verdict of the honest user

In Section \ref{sec:views-output} we required that the views or outputs of the environment is the indistinguishable in the run of the real and the ideal protocol. We will now discuss this indistinguishability in more detail.

The first definitional choice to be made is whether the environment outputs only one bit (as done in \cite{Can01}) or has a continuous stream of output, the view of the environment (as done in \cite{BPW04b}). We follow the choice of \cite{BPW04b} and define the view of the honest user to be the transcript of all its classical in- and outputs.

We can now define probability distributions $\text{Real}_k$, $\text{Ideal}_k$ for the view/output bit in the run of the real or ideal protocol, indexed by the security parameter.

We sketch the following major notions of indistinguishability:

- **Perfect.** The distributions $\text{Real}_k$ and $\text{Ideal}_k$ are identical. If further environment, adversary and simulator are computationally unbounded, we talk of perfect security.

- **Statistical.** There is a negligible function\(^2\) $\nu$, s.t. the statistical distance\(^3\) of $\text{Real}_k$ and $\text{Ideal}_k$ is bounded by $\nu(k)$ for all security parameters $k$.

If further environment, adversary and simulator are computationally unbounded, we talk of **strict statistical security**.

\(^2\) There are different possibilities what functions are accepted as negligible ones. Usually a function is called negligible if it asymptotically gets smaller than $1/p$ for any polynomial $p$.

\(^3\) Intuitively the statistical distance describes how good an optimal test can distinguish between two distributions.
We will only use the notion of strict statistical security in the present work. Another variant of security using the statistical indistinguishability is that of statistical security as defined in [BPW04b]. Here we require that for any polynomial \( l \) the prefixes of length \( l(k) \) of Real\(_k\) and Ideal\(_k\) are indistinguishable. However the simple composition theorem does not hold using this security notion [?].

- **Computational.** For any algorithm \( D \) (the distinguisher) that is polynomial-time in \( k \) and has one-bit output, the outputs of \( D \) given Real\(_k\) resp. Ideal\(_k\) as input is statistically indistinguishable. When restricting honest user, adversary and simulator to be computationally bounded (see Section 1.12) we talk of computational indistinguishability. A more detailed definition can be found e.g. in [BPW04b].

In the case of one-bit output of the environment, computational and statistical indistinguishability obviously coincide.

For quantum protocols, computational indistinguishability have to be redefined to incorporate the power of quantum distinguishers.

### 1.9 Flavours of composition

As already stated, one of the major advantages of simulatable security is the possibility of composition. A composition theorem states roughly the following: suppose we are given a protocol \( \pi \) implementing a trusted host TH, and we have another protocol \( \rho \) using TH as a primitive that implements some other trusted host CPLX. Then protocol \( \rho \) using \( \pi \) also implements CPLX. As a formula (where \( \geq \) means implements):

\[
\pi \geq \text{TH} \quad \text{and} \quad (\rho \text{ using } \text{TH}) \geq \text{CPLX} \quad \Longrightarrow \quad (\rho \text{ using } \pi) \geq \text{CPLX}
\]  

(1)

However, it turns out that this formula can be interpreted in two different ways. To illustrate this, we give two examples of composition.

- Assume that \( \pi \) is a protocol implementing a key exchange (e.g. [BBS84]), i.e. \( \pi \geq \text{KE} \).
  
  Assume further that \( \rho \) that generates one key using KE and then uses this key to implement a secure message transmission SMT (applying some key-reuse-strategies).

- Assume that \( \pi \) again implements key exchange. Assume further that \( \tilde{\rho} \) is a protocol that implements a secure channel approximately as follows: For each message to be sent, a new key exchange is invoked. The resulting key is then used for transmitting an encrypted and authenticated message (e.g. using a one-time-pad and authentication [RSMQ04]).

We are now tempted to say, that in both examples we can use (1) to show

\[
(\rho \text{ using } \pi) \geq \text{SMT} \quad \text{and} \quad (\tilde{\rho} \text{ using } \pi) \geq \text{SMT}.
\]

But considering the examples in more detail, we note that there is a very important difference: While \( \rho \) uses only one copy of KE, the protocol \( \tilde{\rho} \) employs many copies of KE (when we assume the environment to be limited to a polynomial amount of messages (in the security parameter \( k \)), we can say that at most polynomially many invocations of KE...
are performed). So whether we can apply \( \Pi \) in the second example depends on whether “\( \rho \) using \( \text{TH} \)” means “using at most one copy of \( \text{TH} \)” or “using at most polynomially many copies of \( \text{TH} \).”

To be able to differentiate more clearly between these two notions of “using,” we introduce the following conventions: Let \( \rho^X \) denote \( \rho \) using one copy of \( X \). Let further denote \( \text{TH}^* \) the machine simulating arbitrarily many copies of \( \text{TH} \), and let \( \rho^* \) denote the machine simulating arbitrarily many copies of \( \rho \). Then we can state the following two variants of the composition theorem:

\[
\text{simple:} \quad \pi \geq \text{TH} \quad \text{and} \quad \rho^{\text{TH}} \geq \text{CPLX} \quad \implies \quad \rho^\pi \geq \text{CPLX} \quad (2)
\]

and

\[
\text{concurrent:} \quad \pi \geq \text{TH} \quad \implies \quad \pi^* \geq \text{TH}^* \quad (3)
\]

Clearly, for the first example it is sufficient to use the simple composition theorem \((2)\). To prove security of the protocol constructed in the second example, we would proceed as follows: first, by the concurrent composition theorem \((3)\), we get \( \pi^* \geq \text{KE}^* \). Then, our assumption that \( \tilde{\rho} \) using \( \text{KE} \) implements \( \text{SMT} \) must be written more concretely as \( \tilde{\rho}^{\text{KE}^*} \geq \text{SMT} \). Now we can apply the simple composition theorem \((2)\) (substituting \( \pi \) by \( \pi^* \) and \( \text{TH} \) by \( \text{KE}^* \)) and get \( \tilde{\rho}^{\pi^*} \geq \text{SMT} \).

So we have seen that for composition as in the first example, simple composition is sufficient, while for the second example simple and concurrent composition is needed.

The “Universal Composition Theorem” from [Can01] provides simple and concurrent composition, the “Secure Two-system Composition Theorem” from [BPW04b] only provides simple composition, but is supplemented by the “General Composition Theorem” from [BPW04a].

It turns out that a simple composition theorem can be shown for standard and for universal security (cf. Section 1.7 for an introduction of these notions). To the best of our knowledge, no proof of the concurrent composition theorem that does not need universal security has been published so far. Here we will only present the proof sketch of the simple composition theorem, for a sketch of the concurrent composition theorem the reader may refer to [Can01].

**Proof sketch for the simple composition theorem.** The basic idea of the proof of the simple composition theorem goes as follows (we follow the proof idea from [PW01], but stay very general so that this proof idea should be applicable for most simulatable security models):

Assume that \( \pi \geq \text{TH} \) and that \( \rho^{\text{TH}} \geq \text{CPLX} \). Consider any real adversary \( A_{\text{real}} \) and honest user \( H \). The network resulting from \( \rho^\pi, H \) and \( A_{\text{real}} \) running together is depicted in Figure 11a. Here we consider the protocol \( \rho^\pi \) to consist of machines executing \( \rho \) and machines executing \( \pi \), the former connected to the latter by secure connections.

Now consider a machine \( H_\rho \) simulating \( H \) and all the machines in \( \rho \). We can now replace \( H \) and \( \rho \) by this simulation and get the network depicted in Figure 11b. Since \( H_\rho \) is a faithful simulation, the view of the original and of the simulated \( H \) are identical.
Note now that the network in Figure 1b is represents the protocol $\pi$ running with honest user $H_\rho$ and real adversary $A_{\text{real}}$. Since we assumed $\pi \geq TH$ we know, that there is a simulator $A_{\text{sim}}$ s.t. the view of $H_\rho$ is indistinguishable in the networks of Figures 1b and 1c. From this we then conclude that the view of the simulates $H_\rho$ are also indistinguishable in these two networks.

Now we replace $H_\rho$ again by the machines it simulated and get the network in Figure 1d. Again, as $H_\rho$ is a faithful simulation, we now that the views of the simulated and the original $H$ in the networks in Figures 1c and 1d are identical.

We can then conclude that the view of $H$ in the networks in Figures 1c and 1d are indistinguishable, so we found a simulator $A_{\text{sim}}$ for the composed protocol $\rho^{\pi}$. Since we showed this for arbitrary $H$ and $A_{\text{real}}$, it follows $\rho^{\pi} \geq \rho^{TH}$.

Using the transitivity of $\geq$, from this and the assumption $\rho^{TH} \geq \text{CPLX}$ we conclude $\rho^{\pi} \geq \text{CPLX}$. So the simple composition theorem is shown for the case of standard security.

In the case of universal security, we additionally have to show that the constructed simulator $A_{\text{sim}}$ does not depend on the honest user $H$. However, since in Figure 1b $A_{\text{sim}}$ does not depend on $H_\pi$, only on $A_{\text{real}}$, this follows trivially.

---

The transitivity is obvious if the security notion is defined symmetrically, i.e. if for the real and the ideal protocol the same adversaries are allowed and the scheduling etc. are defined in the same for real and ideal execution. This is the case in the model of [BPW 04b], unfortunately not in the model of [Can01], so that a formal proof in that model has to take care of more details.
1.10 Models of corruption

Another important point is the modelling of corruption. Since we cannot assume all parties partaking in the protocol to be honest, we have to assume that the adversary can corrupt some parties, which afterwards are under his control.

We distinguish two main flavours of corruption: static and adaptive corruption.

In the case of static corruption, the adversary may choose a set of parties to corrupt (within some limits, e.g. at most \( t \) parties) before the protocol begins. This models the idea parties are either dishonest or honest, but do not change between these two states. (Static corruption is the default modelling in the model of [BPW04].)

In the case of adaptive corruption, the adversary can at any time during the execution of the protocol corrupt further machines (as long as the set of machines does not exceed the given limits). In particular, the choice which party to corrupt may depend on what is intercepted from a run of the protocol. This model captures the view that an adversary may at any time try to “persuade” some parties to do his bidding, e.g. by using force or hacking into their computer. (The model of [Can01] captures this notion of corruption.)

It is common (both in [BPW04] and [Can01]) that the honest user/environment is informed, which parties are corrupted. While it may seem strange at a first glance, informing the environment has the following advantage: Since the simulator tries to mimic the adversary exactly, the same parties are corrupted in the ideal model. Assume now that we have shown some protocol to be secure in the presence of at most \( t + 1 \) corrupted parties. Then by using the fact that the simulator is restricted to corrupting no more parties than the real adversary, we can conclude that the protocol is also secure in the presence of at most \( t \) corrupted parties. This simple and quite intuitive result would not hold if the real adversary and the simulator would not be forced to corrupt the same number of parties.

When considering static corruption and requiring adversary and simulator to corrupt the same parties, the modelling of a protocol with corruptible parties can be reduced to that of a protocol with only incorruptible ones using the following approach from [BPW04]: For each set \( C \) of parties that may be corrupted, let \( \pi_C \) denote the protocol where these parties are removed and all connections these parties had are instead connected to the adversary. Let further \( \text{TH}_C \) denote the trusted host, where all connections associated with the parties from \( C \) are connected to the adversary. Then \( \pi \) implementing \( \text{TH} \) (with possible corruption) is simply defined as \( \pi_C \geq \text{TH}_C \) (without corruption) for all allowed sets \( C \) of corrupted parties.

In the case of adaptive security, another important design choice appears: whether parties are able to delete information. If they are non-deleting, an adversary corrupting a party does not only learn all information the party still wants to use in later protocol phases, but also all information that ever came to the attention of that party. This of course gives the adversary some additional advantage (but also the simulator is given some advantage, so these two security with and without non-deleting parties are probably incomparable). In [Can01] non-deleting parties are used, while in [BPW04] deleting ones are used.
It is difficult to transport the notion of non-deleting parties to the quantum case. Non-deleting parties cannot be required, since this would imply cloning all information passing through the party. The nearest analogue would seem to be non-measuring parties [MQ02], which instead of measuring would entangle the state to be measured with some ancillae. However, this does not completely capture the notion of non-deleting parties, since a party may now circumvent this restriction by basing some scheduling related decisions on some bit. If the scheduling is classical (see Section [1.13] this would destroy information.

Due to these problems in the modelling of non-deleting parties in the quantum case, we will only model deleting parties in the sense that parties are allowed to perform any measurements or erasures. Further we will omit a discussion of adaptive corruption in the quantum case.

1.11 Modelling machines

Another point which can make subtle differences between different simulatable security models, is the question how a machine is modelled.

In classical models, the following two approaches are most common:

– In [BPW04b] a machine is modelled in a most general way. It is defined by a transition function that for each state of the machine, and each set of inputs gives a probability distribution over the output and state after that activation. Here the machine model does not a priori have the notion of a single computational step (like e.g. a Turing machine would have), but only that of an activation. One can then however describe certain subsets of machines like the machines realisable by a Turing machine, or the machines realisable by a polynomial Turing machine etc. The advantage of this model is that one is not forced to formulate all arguments in terms of Turing machines, but in terms of the mathematically much easier transition functions.

– In [Can01] a machine is generally assumed to be a polynomial interactive Turing machine. The non-deletion property is achieved by keeping a copy of the current state in each Turing step.

If adaptive corruption (see Section [1.10]) is used, another point must be taken care of: if a machine is corrupted, the adversary learns its state, which means that there must be a canonical interpretation how the state of a machine is encoded into a message sent to the adversary.

In a quantum setting, one more choice has to be made:

– Should machines be forced to be unitary (and simulate measurements by entangling with auxiliary qubits), or should they be allowed to perform measurements. At first it may seem, that unitary machines yield the simpler mathematical model. However, as soon as some kind of classical scheduling is involved (see Section [1.13], measurements will take place anyway, so the overall behaviour of the network lacks a natural unitary description. When choosing machines to be allowed to perform measurements, then machines and network fall in a natural way into one mathematical framework. Further
no options are lost by such a choice, since unitary transformations are a special case of quantum superoperators (see Section 1.15).

In this work we have chosen to follow the approach of [BPW04b] and try to achieve a modelling of machines which captures any process not disallowed by physical laws. This yielded the following notion of machines:

- Following the [BPW04b]-approach, each machine has a set of in- and out-ports, modelling the connections to other machines. A port can be classical or quantum.
- The transition of a machine is modelled by a quantum superoperator. It takes the state and inputs of the machine to the new state and outputs.
- Before and after each activation, the classical ports are measured, the results of these measurements over the whole life of the machine constitute the view of the machine.
- Before each activation a special (machine dependent) repeatable measurement is performed on the machine’s state to decide, whether the machine is in a final state or not. This measurement may be trivial (for non-terminating machines).

1.12 Computational security

The design choice which turns out to be the most difficult and error-prone in definitions of simulatable security is how to model computational security.

The simplest and most common approach is to require all machines in a network (adversary, simulator, honest user, protocol machines) to be polynomially limited in the sense that there is a polynomial limiting the overall number of Turing steps (or gates, etc. depending on the underlying model of computation). Even this seemingly simple approach can lead to unexpected problems. Assume some trusted host is modelled as a polynomial machine. Since reading a message needs time, the machine will terminate after a given number of inputs on port a. Then however it will not be able to react to queries on port b, even if in the intuitive specification these ports are completely unrelated. This is an artefact of the modelling and should be avoided. Both [PW01] and [Can01] have these problems, the security proofs in these models sometimes being strictly spoken incorrect, e.g. most of the functionalities in [Can01] were not polynomial time, and therefore could not be composed (see below).

The obvious solution to allow real or ideal protocol to be computationally unlimited and only restricting honest user, adversary and simulator to be polynomial time, does not work either. An inspection on the proof sketch of the simple composition theorem (Section 1.9) shows, that if the calling protocol ($\rho$ in the proof sketch) is not polynomially limited, then the combination of $\rho$ and the honest user $H$ is not polynomial time any more. So the assumption $\pi \geq TH$ cannot be used to establish equivalence of the networks in Figures 1b and 1c and the proof fails. So such an approach to computational security would lose the composition theorem and is thus not viable.

In [Bac02] this problem was solved for the RS framework by introducing so-called length functions. These allowed a machine to selectively switch off ports, thus being able to ignore inputs on these ports without losing computation time. This solved the problem.
described above. To the best of our knowledge, in the UC framework the problem has not yet been successfully approached.

However, the modelling using length functions still leaves some problems. For example, it is not possible to model trusted hosts that are able to process an arbitrary amount of data. An example for such a trusted host would be a secure channel that is not a priori restricted to some maximal number of messages. In the modelling using length functions every trusted host (and every corresponding real protocol) has to be parametrised in a “life time polynomial”, which may be arbitrarily chosen, but has to be fixed in advance.

A further possible solution is to allow parties to be polynomially bounded in the number of activations or in the length of their inputs. But this approach may lead to a modelling where two parties may gain unbounded computation time by sending messages to each other in a kind of “ping-pong” game.

A solution to this problem is presented in [HMQU04]. Using the modelling presented there one can model parties that are able to process a not-a-priori-bounded amount of data, without losing the property of being in an intuitive way computationally bounded, and without losing the possibility of composition. This additional generality is however bought by additional complexity.

Other points that have to be detailed are e.g. how much time is consumed by reading a message, is a message read from the beginning (disallowing to read the end of very long messages) or is there random-access to the message content (i.e. the machine can start reading at the end, in the middle of a message, etc.5).

A noteworthy point is that (strict) statistical security does not necessarily imply computational security. To see this, imagine an ideal protocol that gives a computationally hard problem to the simulator, and only if the simulator solves this problem, then the ideal protocol will behave exactly as does the real protocol. Then a computationally unbounded simulator will be able to mimic the behaviour of the real protocol, so we have strict statistical (even perfect) security. But a computationally bounded simulator will be unable to “unlock” the ideal protocol, so we do not have computational security.

A possible remedy to this problem has been proposed by [Can00]: the simulator’s runtime must be polynomially bounded in the runtime bound of the real adversary. Then statistical security implies computational security.

In the present work we concentrate on unconditional security. However, we model the length functions of [Bac02], so computational security should be easily definable following the example of [BPW04b]. The approach of [HMQU04] should be easily adaptable to the model presented here, too.

### 1.13 Scheduling and message delivery

Besides specifying how machines operate and send or receive messages, it is necessary to model the behaviour of the overall network. It turns out that here the details may

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5 In the quantum case even the access model would have to be distinguished, where a polynomial machine can in superposition access all bits of a message (the message is given to the machine as an oracle). Then the machine could e.g. determine whether a message of exponential length is balanced or not [DJ92], which clearly would not be possible if it would have to read the message symbol-by-symbol.
get very complicated. We sketch several general approaches to scheduling and message delivery:

- **No scheduling.** Here the order of messages and activations is determined in advance. For example, in a two party protocol it makes sense to say that the parties are alternatingly activated and upon each activation a message is sent to the other party. This is the simplest form of scheduling, it may however be unable to model the behaviour of more complex protocols where the decision whom to sent a message may depend on the protocol input or on some prior messages.

- **Message driven scheduling with immediate message delivery.** Any machine can only send one message. This message is immediately delivered to the recipient (which may be the adversary in the case of an insecure channel). This scheduling is very easy to model, but of course not every protocol can be described in such a model, and some realistic attacks may not be modelled. Further it seems difficult to model an authenticated channel (who should be activated next? the recipient? the adversary?).

- **Message driven scheduling with adversarially controlled delivery.** Here a machine can send one or several messages in any activation. The adversary may then decide whether and when to deliver that message. Several flavours exist:
  - *Fair vs. asynchronous delivery.* It has to be chosen, whether the adversary is required to eventually send a message (*fair delivery*) or whether it may drop messages at will. In case of fair delivery, great care has to be taken with respect to the exact modelling, since otherwise the definition could e.g. allow the adversary to “deliver” after the protocol’s end. Another question is whether parties may or may not know an upper bound for the time it takes a message to be delivered. This problem is elaborated in more detail in [BHMQU04]. Both [BPW04b] and [Can01] model asynchronous delivery. In [Can01] so-called *non-blocking adversaries* are mentioned which are required to eventually deliver, but no definition is given and the above mentioned questions are not answered.

  Following [BPW04b], we adopt asynchronous delivery, however the discussion from [BHMQU04] can easily be adapted to our modelling.

  - *Blind or transparent delivery.* Is the adversary notified on whether a message is to be delivered on some connection? The adversary may have to schedule the connection without knowing whether some message is waiting (*blind delivery*) or he is informed of the fact prior (*transparent delivery*). The blind delivery captures the idea of a channel in which some reordering occurs, but which is not accessible to the adversary (e.g. an ethernet connection). The transparent delivery captures the idea of a connection which is routed through adversarially controlled routers. In [BPW04b] blind delivery is used, while [Can01] adopts transparent delivery. We follow [BPW04b] and use blind delivery.

  - *Symmetric or asymmetric approach.* In the [BPW04b] model each connection has a designated scheduler. These are usually the sending machine (modelling an immediately scheduled connection), the adversary (an adversarially controlled connection), or the recipient (a fetching connection). We call this the *symmetric*
approach.\footnote{However, the symmetry is slightly broken by the fact that there is a designated machine called the \textit{master-scheduler}, which is activated if the activation token is lost.} On the other hand \cite{Can01} pursues an \textit{asymmetric approach}. Here many different rules of scheduling are defined depending on whether the machine is sent from functionality to parties or vice versa and environment and adversary have special uninterchangable roles.

We adapt the symmetric approach from \cite{BPW04b} since we believe that this makes the details of the modelling easier and is additionally of great advantage in detailed proofs, since much less different kinds of delivery are to be distinguished.

– \textit{Non-message driven scheduling}. By this we mean the idea that all machines may run in parallel, and messages do not influence the scheduling. In particular, a machine may execute tasks without the need of being activated by an incoming message. A scheduling falling into that class has been described in \cite{Unr02}. However, no satisfying and easy model has yet been defined using that approach.

It is the author’s personal opinion that such a model would capture reality much better, especially when allowing the machine to measure time in some way. However it seems that coming to an easy and intuitive modelling is a still unsolved problem.

– \textit{Synchronous scheduling}. This means that the protocol proceeds in rounds, and in each round all machines are activated. This scheduling underlies e.g. the early models \cite{PSW00} and \cite{Can00}. However assuming synchronous scheduling means to assume a very strong synchronisation of the protocol participants and is only justified in special cases. This assumption was dropped in \cite{PW01} and \cite{Can01}.

A further interesting issue is whether the scheduling and message delivery is \textit{quantum} or \textit{classical}. By quantum scheduling we mean that events of scheduling (e.g. the recipient of a message, or the fact whether a message is sent at all, or which machine is activated) can be in a quantum superposition. In contrast, with classical scheduling the state of the system would always collapse to one of the possible decisions.

An advantage of quantum scheduling would be the possibility to model protocols that explicitly make use of the superposition between sending and not sending a message. For example, there is a protocol that is able to detect if an eavesdropper tries to find out whether communication takes place at all \cite{SJB01,MQS03} (traffic analysis).

However, modelling quantum scheduling turns out to be quite difficult. This is due to the fact that if the scheduling is to be non-measuring, this means it has to be defined in an unitary way. But then it would have to be reversible, thus disallowing many sensible machine definitions. A suitable combination of measuring and non-measuring scheduling would have to be found, capturing the possibilities of both worlds.

Since how to model quantum scheduling is as far as we know an open problem, we will here present a modelling of security using a classical scheduling.

\section{The Ben-Or-Mayers model}

In this section we compare our model to the modelling of \cite{BOM04}. We organise this comparison into several short topics, according to different sections of the introduction.
– **Machine model (cf. Section 1.11).** In our modelling, machines are modelled by giving a transition operator, which can model any quantum-mechanically possible operation of the machine, including all types of measurements. In contrast, in the BOM04 model machines are modelled as partially ordered sets of gates, i.e., as circuits (the partial order gives the order of execution of the gates).

– **The order of quantifiers (cf. Section 1.7).** In the BOM04 model, no explicit real life adversary exists. Instead the environment communicates directly with the protocol, while in the ideal model a simulator is placed between the protocol and the environment (i.e., in the real model, a dummy adversary is used). The idea behind such a dummy-adversary-approach (which is also given as an alternative formulation in Can01) is that if there are distinguishing adversary and environment, we could put the adversary into the environment and leave an empty hull in place of the adversary, the dummy adversary. Then the honest user would still distinguish. Therefore we can restrict our attention to such dummy-adversaries. Now, in the model of BOM04, the order of quantifiers is as follows: For all environments, there is a simulator, such that real and ideal models are indistinguishable. So the simulator is chosen in dependence of the environment, so we have standard security (while the notion of universal security is not specified in that modelling).

– **Composition (cf. Section 1.9).** The BOM04 shows a composition theorem which is comparable to what we call a simple composition theorem. I.e., the concurrent composition of a non-constant number of copies of the protocols is not allowed by the composition theorem. Since the order of quantifiers is that of standard security, this seems natural.

In the discussions in BOM04, some natural conditions for universal composition are given, which seem to allow for concurrent composition of a polynomial number of protocol copies. However, these conditions require that the statistical distance between real and ideal protocol execution is bounded by a negligible function which must be independent of the environment in use. This condition is a very strict one, since most computationally secure protocols violate that condition (consider some protocol where the adversary must find the pre-image of some function to break the protocol. Then the probability of breaking the protocol may go up if the adversary does a longer search for pre-images). In fact, the author knows of no protocol that is computationally but not statistically secure that would still be computationally secure with respect to these additional conditions.

– **Models of corruption (cf. Section 1.10).** Corruption is modelled in the Ben-Or-Mayers model as follows: There are some so-called classical control registers which may control the order and application of the gates of a circuit. A corruptible machine will then execute its own gates if a special control register, the corruption register, is set to 0. If it is set to 1, gates specified by the adversary are executed instead. Therefore in BOM04, adaptive corruption is modelled, since the adversary might change the value of a control register at a later time.

– **Computational security (cf. Section 1.12).** In BOM04 computational security is modelled by requiring that for any polynomial f, any family of environments with less than f(k) gates (where k is the security parameter) achieves only negligible statistical
distance between ideal and real protocol execution. This is roughly equivalent to requiring security against non-uniform environments, adversaries and simulators.

- **Scheduling and message delivery (cf. Section 1.13).** We have tried to realise a very general scheduling using the concept of buffers, which are scheduled at the discretion of the adversary. In contrast, in the [BOM04]-model a more simple scheduling was chosen: The delivery of messages and activation of machines is represented by the ordering of the gates. Since the ordering of gates within one machine is fixed up to commuting gates, and the access of gates of different machines to a shared register (a channel register) is defined to alternate between sender and recipient, all scheduling and message delivery between machines is fixed in advance. Only when the adversary gates are executed, the scheduling may have some variability. Further there might be situations (like a channel shared by more than two parties) where the scheduling is not completely fixed.

However, the partial order on the gates is fixed by the environment in advance, i.e., may not depend of information gathered by the adversary in the course of a protocol execution.

This model of scheduling seems adequate for protocols with a very simple communication structure (like two-party protocols proceeding in rounds), more complex protocols however tend to contain the possibility of race conditions\(^7\) and message reordering. Therefore a security guarantee given in the [BOM04]-model for such a protocol should be taken with care, since attacks based on such scheduling issues would not be taken into account by that model (at least, if the attack is based on information gathered by the adversary).

Further it does not seem possible even to model protocols which contain instructions like “toss a coin; on outcome 0 send a message to Alice, on outcome 1 send a message to Bob”, since the sending of messages is fixed in advance.

- **Service ports (cf. Section 3.1).** Often it is useful to dedicate some in- and outgoing connection of the protocol to be used only by the adversary (in particular, the simulator can lie to the environment about the information on these connections). Other connections again should be used by the environment (since they constitute the protocol in- and outputs). We have realised this concept by specifying a set of protocol ports that can be used by the honest user, the service ports. All other ports are restricted to be used only by the adversary.

Such a distinction is particularly useful when specifying a trusted host, where some side-channels (like the size of a transmitted message or similar) are intended only for the adversary.

It is unclear to us, whether the [BOM04] model provides a possibility for specifying such a distinction (i.e., for telling the simulator which ports it may access and which it may not).

It would be very interesting to study, how security in the [BOM04]-model relates to security in our model. E.g., a theorem like “if the protocol \(\pi\) is secure in [BOM04] and

\(^7\) I.e., the protocol is sensitive to the order of events. A typical race condition would be if some event happens exactly between two delicate steps of the protocol, causing confusion and insecurity.
satisfies such-and-such conditions, then the protocol $\pi'$, resulting from translating $\pi$ into our model in such-and-such a way, is secure in our model” would seem very nice. (The other direction might be more difficult, since many protocols in our model that have more than three parties would not have an intuitive counterpart in the \[\text{[BOM04]}\]-model.)

### 1.15 Quantum mechanical formalism used in this work

In this work, we adopt the formalism that states in quantum mechanics can be described using density operators. Given a Hilbert space $\mathcal{H}$, the set $\mathcal{P}(\mathcal{H})$ of positive linear operators on $\mathcal{H}$ with trace 1.

If $\mathcal{H} = \mathcal{C}^X$ for some countable set $X$, we say that $\{|x\rangle : x \in X\}$ is the computational basis for $\mathcal{H}$.

Any operation on a system can then described by a so-called superoperator (or quantum operation). A mapping $\mathcal{E} : \mathcal{P}(\mathcal{H}) \rightarrow \mathcal{P}(\mathcal{H})$ is a superoperator on $\mathcal{P}(\mathcal{H})$ (or short on $\mathcal{H}$), iff it is convex-linear, completely positive and it is $0 \leq \text{tr} \mathcal{E} (\rho) \leq \text{tr} \rho$ for all $\rho \in \mathcal{P}(\mathcal{H})$.

Superoperators $\mathcal{E}$ on a system $\mathcal{H}_1$ can be extended to a larger system $\mathcal{H}_1 \otimes \mathcal{H}_2$ by using the tensor product $\mathcal{E} \otimes 1$ (where 1 is the identity).

We will only use a special kind of measurement, so-called von-Neumann or projective measurements. A von-Neumann measurement on $\mathcal{H}$ is given by a set of projections $P_i$ on $\mathcal{H}$, s.t. $\sum_i P_i = 1$ and all $P_i$ are pairwise orthogonal, i.e. $P_i P_j = 0$ for $i \neq j$. Given a density operator $\rho \in \mathcal{P}(\mathcal{H})$, the probability of measurement outcome $i$ is $\text{tr} P_i \rho$, and the post-measurement state for that outcome is $P_i \rho P_i$.

We say a von-Neumann measurement is complete, if every projector has rank 1. A complete von-Neumann measurement in the computational basis denotes a von-Neumann measurement where each projector project onto one vector from the computational basis of $\mathcal{H}$.

Von-Neumann measurements too can be extended to a larger system by using the tensor-product.

For further reading we recommend the textbook [NC00].

Additionally we fix that $\mathbb{N}$ denotes the natural number excluding 0, $\mathbb{N}_0$ the natural number including zero, $\mathbb{R}$ the real numbers, and $\mathbb{R}_{\geq 0}$ the non-negative real numbers.
In this following we will sketch how networks of quantum machines can be defined, while trying to closely mimic the behaviour of the classical networks of the RS framework of [BPW04b].

Roughly, a quantum network (collection in the terminology of [BPW04b]) consists of a set of machines. Each machine has a set of ports, which can be connected to other machines to transmit and receive messages. A port can be of the following kinds:

- A simple out-port \( p! \). This is a port on which a machine can send a message.
- A simple in-port \( p? \). This is a port on which a machine can receive a message.
- A clock out-port \( p\triangleright! \). Using this port a machine can schedule a message, i.e. messages sent from port \( p! \) are not delivered to \( p? \) until they are scheduled via \( p\triangleright! \).
- Clock in-ports \( p\leftrightarrow? \), buffer in-ports \( p\leftrightarrow! \) and buffer out-ports \( p\leftrightarrow! \). These ports are special to the so-called buffers and are explained below.
- The master-clock-port \( p\triangleright\text{clk} \). This special case of a clock in-port is special in that it is not connected to another machine. Instead, a special machine called the master scheduler is activated via this port if the activation token is lost, i.e. if no machine would be activated via an incoming message.

Each port is additionally classified as being either a quantum or a classical port. (This of course is an addition to the modelling of [BPW04b].)

There are further special machines called buffers. These have the task of storing messages while they are waiting to be delivered. A buffer \( \tilde{p} \) necessarily has ports \( p\leftrightarrow? \), \( p\leftrightarrow! \) (buffer in-/out-port) and \( p\triangleright? \) (clock in-port). It is further associated to the ports \( p! \), \( p? \) and \( p\leftrightarrow! \) by its name \( p \). Therefore we get the situation depicted in Figure 2.

A message delivery now takes place as follows: The sending machine sends a message by writing a string (preparing a quantum state) on the out-port \( p! \). This message is then immediately transferred into the buffer. In the buffer it appended to a queue. In this queue it stays until the scheduler for buffer \( \tilde{p} \) sends a natural number \( n \) on \( p\triangleright! \). If \( n \geq 1 \) and there are at least \( n \) message in the buffer’s queue, the \( n \)-th message is removed from the queue and transmitted to the simple in-port \( p? \) of the receiving machine. The receiving machine is then activated next.

Fig. 2. A connection
The three machines in Figure 2 are not necessarily different ones. In particular, it may e.g. be that the sending machine schedules its messages itself, realising immediate delivery, or that a machine sends messages to itself.

Note that if a machine has several clock out-ports and sends a number on several of these, all but the first one (in some canonical ordering of the ports) are ignored, since otherwise it would be unclear which machine to activate next.

If no machine would be activated next (e.g., because the machine last activated did not write on a clock out-port, or because the number $n$ sent through the clock out-port $p!$ was higher than the number of messages queued in the buffer $\bar{p}$) a special designated machine is activated, the master scheduler. This machine is characterised by having the master-clock-port $\text{clk}!$ on which it then gets the constant input 1.

We have now seen that in a network there are three types of machines

- A simple machine. This machine is characterised by having only simple in- and out-ports and clock out-ports.
- A master scheduler. This machine may have the same ports as a simple machine, but additionally has the master-clock-port $\text{clk}!$. Also the master scheduler is the machine to be activated at the beginning of the execution of the network.
- A buffer. This machine which has a completely fixed behaviour exists merely to store and forward messages. (The buffer will be more exactly defined below.)

Upon each activation of some simple machine or master scheduler, a record of this activation is added to the so-called trace or run of the network (or collection). This record consists of the name of the machine, of all its classical inputs (contents of the classical in-ports before activation), all its classical outputs (contents of the classical out-ports after activation), and the classical state (see below) before and after activation.$^8$

From the run we can easily extract the so-called view of some machine $M$. It consists of all records in the run containing the name of $M$.

The scheduling as described above will be transformed into a formal definition (while strongly drawing from the definitions from [BPW04b] where the introduction of quantum mechanics does not necessitate an alteration).

2.1 Quantum machines

First, for self-containment, we restate the unchanged formal definition of a port from [BPW04b]:

**Definition 1 (Ports [BPW04b]).** Let $P := \Sigma^+ \times \{\varepsilon, \leftrightarrow, \sqcup\} \times \{!, ?\}$ (here $\varepsilon$ denotes the empty word). Then $p \in P$ is called a port. For $p = (n, l, d) \in P$, we call $\text{name}(p) := n$ its name, $\text{label}(p) := l$ its label, and $\text{dir}(p) := d$ its direction.

$^8$ Here we slightly deviate from the modelling of [BPW04b] where the complete in-/output and the complete state is logged. This of course is not possible in the quantum case since it would imply measuring the quantum state and all the in-/outputs in every activation.

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Usually we do not write \((n, l, d)\), but \(nld\), i.e., a port named \(p\) with label \(\triangleleft\) and direction \(!\) would simple be written \(p^{\triangleleft!}\), if the label was \(\varepsilon\), we would write \(p!\).

Note that label \(\varepsilon\) denotes a simple port, label \(\leftrightarrow\) a buffer port and label \(\triangleleft\) a clock port. Further direction \(!\) denotes an out-, and direction \(?\) an in-port.

Further, if \(P\) is a set or sequence of ports, let \(\text{in}(P)\) and \(\text{out}(P)\) denote the restriction of \(P\) to its in- or out-ports, resp.

We can now proceed to the definition of a machine. Since our model shall encompass quantum machines, we will here deviate from the modelling of machines in \[BPW04\]. However, to simplify comparisons, we shortly recapitulate the definition of a machine in \[BPW04\] (which we will call a BPW-machine):

- A machine \(M\) is a tupel \(M = (\text{name}, \text{Ports}, \text{States}, \delta, \text{Ini}, \text{Fin})\). Here \(\text{name}\) is the unique name of the machine, \(\text{Ports}\) the sequence of the ports of this machine, \(\text{States} \subseteq \Sigma^*\) the set of its possible states.
- \(\text{l}\) is the length function of this machine, see below.
- \(\text{Ini}\) is the set of initial states. Since in the security definitions in \[BPW04\] only the states of the form \(1^k\) are used (where \(k\) is the security parameter), we can w.l.o.g. assume \(\text{Ini} = \{1^k : k \in \mathbb{N}\}\).
- \(\text{Fin}\) is the set of final states. A machine reaching a state in \(\text{Fin}\) will never be activated again.
- \(\delta\) is the state-transition function. For a given state \(s\) of the machine, and inputs \(I\), \(\delta(s, I)\) gives the probability distribution of \((s', O)\), where \(s'\) is the state of the machine after activation, and \(O\) its output.

In comparison, we define a machine in our quantum setting as follows (cf. also the discussion after this definition):

**Definition 2 (Machine).** A machine (or quantum machine) is a tuple

\[ M = (\text{name}, \text{Ports}, \text{CPorts}, \text{QStates}, \text{CStates}, \Delta, \text{l}, \text{Fin}) \]

where

- \(\text{name} \in \Sigma^+\) is the name of the machine.
- \(\text{Ports}\) is the sequence of the ports of the machine.
- \(\text{CPorts} \subseteq \text{Ports}\) is the set of the classical ports of the machine. There must be no clock-ports in \(\text{Ports} \setminus \text{CPorts}\) (i.e. all clock-ports are classical).
- \(\text{QStates} \subseteq \Sigma^*\) is the basis of the space of the quantum states, i.e. the states of \(M\) live in \(\mathbb{C}^{\text{QStates}}\). It must be \(0 \in \text{QStates}\).
- \(\text{CStates} \subseteq \Sigma^*\) is the set of the classical states of the machine. It must be \(1^k \in \text{CStates}\) for all \(k \in \mathbb{N}\) (i.e. the classical states must be able to encode the initial states).
- The state-transition operator \(\Delta_M\) is a trace-preserving superoperator operating over the Hilbert space \(\mathbb{C}^{\text{QStates}} \otimes \mathbb{C}^{\text{CStates}} \otimes \mathbb{C}^{I} \otimes \mathbb{C}^{O}\). Here \(I := (\Sigma^*)^{\text{in}(\text{Ports})}\) is the set of all possible inputs of \(M\) (strictly spoken the basis of the state of all inputs), and \(O := (\Sigma^*)^{\text{out}(\text{Ports})}\) analogously for the outputs of \(M\).
– The function \( l : \text{CStates} \times \text{in}(\text{Ports}) \rightarrow \{0, \infty\} \) is called the length function of \( M \). For each classical state \( c \) and each in-port \( p \), this tells whether input on this port should be ignored (\( l(c, p) = 0 \)) or not (\( l(c, p) = \infty \)).

– \( \text{Fin} \subseteq \text{CStates} \) is the set of final states. If the classical state of the machine is a final state, then the machine will not be activated any more.

Given a machine \( M \), we denote the different entries of the tuple defining \( M \) by the name of the entry and a subscript \( M \). E.g., \( \text{CStates}_M \) are the classical states of \( M \).

We will now discuss the elements of this definition. The field name simply defines a unique name of the machine which is used to know with which machine the entries in the trace are associated.

The sequence \( \text{Ports} \) denotes, which ports a given machine has. A subset of these are the classical ports \( \text{CPorts} \). All messages written to or read from the classical ports are measured in the computational basis (before or after the activation of \( M \), depending on whether it is an in- or an out-port). Note that the machine definition above does not handle classical ports differently from quantum ports, the measuring will take place in the run-algorithm (see Section 2.2). We do not allow clock-ports to be quantum ports, since the clock ports contain numbers of the are to be scheduled. Since our scheduling is classical (cf. Section 1.13), these numbers have to be classical.

One notices, that the above machine definition has two sets of states, the quantum states \( \text{QStates} \) and the classical states \( \text{CStates} \). We may imagine the machine to be a bipartite system, consisting of a classical part and a quantum part. However, since the quantum part not restricted to unitary operations, this does not imply a strict separation of a controlling classical machine and a controlled pure quantum machine. This distinction is necessary, since some events in the scheduling etc. depend on the state of the machine. Since these events are of classical nature (see discussion in Section 1.13), they may not depend on the quantum state. Note again, that the state-transition operator does not treat the classical and the quantum state differently, the run-algorithm (Section 2.2) takes care of measuring the classical state.

The most interesting part of the machine is probably the state-transition operator, since it specifies the behaviour of the machine. We try to make the machine definition as general as possible, therefore we want to allow any quantum mechanically possible operation of the space accessible to the machine. This state consists of the inputs \( \mathcal{C}^\mathcal{I} \), the outputs \( \mathcal{C}^\mathcal{O} \), and of course of the bipartite state \( \mathcal{C}^{\text{QStates}} \otimes \mathcal{C}^{\text{CStates}} \) of the machine. The most general operation on such a space is described by a trace-preserving superoperator (cf. Section 1.15). \(^9\) Therefore we do not impose any more restrictions on the state-transition operator \( \Delta \) than to be such a superoperator. Note that \( \Delta \) is formally even allowed to read its output space or write to its input space. However, since the input space is erased after activation, and the output space is initialised to a known state before activation (cf. Section 2.2), this does not pose a problem.

The [BPW04b] model introduced so-called length functions to cope with some problems occurring when modelling computational security (see 2.2). These length functions

\(^9\) We do, of course, neglect advanced physics like special and general relativity. However, formally modelling these in a security model is probably still far off future.
allow machines to set the maximal length of a message which can be received on a given in-port (longer input is truncated). So in particular, a machine can switch off some in-port completely, which has the effect that this machine is not activated any more on input on that port. Since whether the machine is activated or not is here defined as a classical decision, the length function should be classically defined, too. Therefore the length function must only depend on the classical state of the machine. In our setting the length function only plays an inferior role, since we are only concerned with unconditional security. However, since this model is designed to be easily extendable to the computational case (by defining a computational model for the machines and then using a straightforward adaption of the definition of computational security from [BPW04b]), we included the length functions into our model. The only important values of length functions are 0 (ignore the message), and $\infty$ (do not ignore it), all other values (integers greater 0) just truncate the message, but even a computationally limited machine could simply ignore anything longer than indicated by the length function. Since further truncating would imply at least a partial measurement of the length of the message, we have restricted the length functions to take only the values 0 and $\infty$.

Finally the decision whether a machine has terminated or not should be classical, this should only depend on the classical state of the machine. Therefore the set $\text{Fin}$ of final states is a subset of $\text{CStates}$.

A special kind of machine is the so called buffer (see the informal description in Section 2). The state of the buffer contains a (possibly empty) queue of messages. Note that the buffer does not measure these messages, they are simply moved. A buffer $\tilde{p}$ can be called due to two different reasons:

- A message arrived on its buffer in-port $p^{\leftrightarrow}?$. Then this message is appended to the queue.
- A number $n$ was written to its clock in-port $p^{\triangleright}?$. Then the $n$-th message is taken from the queue (if existent) and moved to the buffer out-port $p^{\leftrightarrow}!$.

For completeness we give a formal definition of buffers:

**Definition 3 (Buffers).** Let $\text{Queue}$ denote the set of all possible queue contents:

$$\text{Queue} := \{(n; m_1, \ldots, m_n) : n \in \mathbb{N}_0, m_i \in \Sigma^*\}$$

We assume the elements of $\text{Queue}$ to be encoded as words in $\Sigma^*$, so that (0) (the empty queue) is encoded as the empty word $\varepsilon \in \Sigma^*$.

The buffer $\tilde{p}$ is defined by

$$\tilde{p} := (n\sim, (p^{\leftrightarrow}?, p^{\leftrightarrow}!, p^{\triangleright}?), \{p^{\triangleright}!\}, \text{Queue}, \{1^k\}, \Delta_{\text{buff}}, \infty, \emptyset)$$

That is, the buffer is named $n\sim$, has ports $p^{\leftrightarrow}?$, $p^{\leftrightarrow}!$, $p^{\triangleright}?$, of which $p^{\triangleright}?$ is classical, the classical states are the required initial states $1^k$, the quantum states are the possible queue contents (where the queue initially is empty). The length function is set to constant $\infty$, i.e. no truncating takes place. The set of final states is $\emptyset$, so the buffer will never terminate.

The state-transition operator $\Delta_{\text{buff}}$ is defined by the following measurement process on the buffer’s state (in $\mathcal{H}_{\tilde{p}} = \mathbb{C}\text{Queue} \otimes \mathbb{C}\{1^k\} \otimes \mathcal{H}_{p^{\leftrightarrow}?} \otimes \mathcal{H}_{p^{\triangleright}?} \otimes \mathcal{H}_{p^{\leftrightarrow}!}$):
Measure whether $\mathcal{H}_{p^\leftrightarrow} \ni |\varepsilon\rangle$ (\varepsilon being the empty word). If no $(p^\leftrightarrow \ni$ is nonempty), perform the linear operation given by
\[
|(n;m_1,\ldots,m_n)\rangle \otimes |\text{in}\rangle \longrightarrow |(n+1;m_1,\ldots,m_n,\text{in})\rangle \otimes |\varepsilon\rangle
\]
on $\mathcal{C}^\text{Queue} \otimes \mathcal{H}_{p^\leftrightarrow}$. (I.e., if there is input on port $p^\leftrightarrow$, append it to the queue.)

Perform a complete von-Neumann measurement in the computational basis (from now on called complete measurement) on $\mathcal{H}_{p^\leftrightarrow}$. Let $i$ be the outcome.

Prepare state $|\varepsilon\rangle$ in subsystem $\mathcal{H}_{p^\leftrightarrow}$.

Measure the first component in $\mathcal{C}^\text{Queue}$ (i.e. project $\mathcal{C}^\text{Queue}$ onto one of the spaces $S_n$ where $S_n$ is spanned by the vectors $|(n,m_1,\ldots,m_n)\rangle, m_i \in \Sigma^*$. Let $n$ be the outcome of this measurement (i.e. $n$ is the current queue length).

If $i \in \mathbb{N} \subseteq \Sigma^*$ (we assume natural numbers to be encoded as nonempty words in $\Sigma^*$) and $i \leq n$, then perform the following linear operation on $\mathcal{C}^\text{Queue} \otimes \mathcal{H}_{p^\leftrightarrow}$:
\[
|(n;m_1,\ldots,m_n)\rangle \otimes |\varepsilon\rangle \longrightarrow |(n;m_1,\ldots,m_{i-1},m_{i+1},\ldots,m_n)\rangle \otimes |m_i\rangle.
\]
That is, the $i$-th message is moved to the buffer out-port $p^{\leftrightarrow!}$.

### 2.2 Quantum networks

So far we have only modelled single machines. From here, the definition of a network is not very far away. In the [BPW04b] modelling a network simply is a set of machines (called a collection). The connections between the machines are given by the names of the ports, as described in Section 2 and depicted in Figure 2. Some restriction have to apply to a collection to form a sensible network, e.g. there must not be any dangling connection (free ports), and no ports must be duplicated. The formal definition of collections is literally identical to that in [BPW04b], we cite it for selfcontainedness:

**Definition 4 (Collections [BPW04b]).**

- A collection $\hat{C}$ is a finite set of machines with pairwise different machine names, pairwise disjoint port sets, and where each machine is a simple machine, a master scheduler, or a buffer.
- ports($\hat{C}$) denotes the set of all ports of all machines (including buffers) in $\hat{C}$.
- If $\hat{n}$ is a buffer, $\hat{n},M \in \hat{C}$ and $n^! \in \text{Ports}_M$ then we call $M$ the scheduler for buffer $\hat{n}$ in $\hat{C}$, and we omit “in $\hat{C}$” if it is clear from the context.

Note that a collection is not necessarily a complete network, since it is not required that each port has its counterpart. This is important since we will need these “non-closed” collections for defining the notion of protocols (Section 3).

Prior to introducing the notion of a closed collection, which will represent quantum networks, we have to introduce the notion of the free ports. The low-level complement $p^c$ of some port $p$ is the port which in Figure 2 is directly connected to $p$. That is, $(p!)^c = p^{\leftrightarrow?}, (p?!)^c = p^{\leftrightarrow!}, (p^{\leftrightarrow})^c = p^?, (p^{\leftrightarrow!})^c = p^!, (p^{\leftrightarrow?})^c = p^!, (p^{\leftrightarrow!})^c = p^{\leftrightarrow},$ and $(p^?)^c = p^{\leftrightarrow}$ Then we can define the set of free ports of a collection $\hat{C}$ to be the set of ports in $\hat{C}$ that do
not have a low-level complement in \( \hat{C} \), formally \( \text{free}(\hat{C}) = \{ p \in \text{ports}(\hat{C}) : p^c \notin \text{ports}(\hat{C}) \} \). Intuitively, the free ports are those that have a “dangling” connection.

We can now define a closed collection:

**Definition 5 (Closed collection, completion [BPW04b]).**

- The completion \( [\hat{C}] \) of \( \hat{C} \) is defined as
  \[
  [\hat{C}] := \{ \tilde{n} \mid \exists l, d : (n, l, d) \in \text{ports}(\hat{C}) \setminus \{ \text{clk}^c? \} \}.
  \]

- \( \hat{C} \) is closed iff \( \text{free}([\hat{C}]) = \{ \text{clk}^c\} \).

Intuitively, the completion of \( \hat{C} \) results from adding to \( \hat{C} \) all missing buffers. That is, if some machine has a port \( p! \), \( p? \), or \( p^c! \), then the buffer \( \tilde{p} \) is added if not yet present. Note that no buffer is added for the master-clock-ports \( \text{clk}^c? \), since this port should be left free (it is only used to activate the master scheduler in case of a lost activation token).

A collection is then called closed, if only buffers are missing, i.e. if after completing it, there is no “dangling connection” (note that of course the master-clock-port must stay unconnected). Such a closed collection is now a quantum network ready to be executed.

We will now proceed to defining the run of a network. In contrast to the definitions of collections etc. at the beginning of this section the run algorithm is inherently quantum, so the definition given here differs from that in [BPW04b]. However, we try to capture the structure and the behaviour of the scheduling.

Explanations on the individual steps of the algorithm below can be found after the definition.

**Definition 6 (Run).** Let a closed collection \( \hat{C} \) and some security parameter \( k \in \mathbb{N} \) be given. Let \( X \) denote the master scheduler of \( \hat{C} \).

For any machine \( M \in [\hat{C}] \) let

\[
\mathcal{H}_M := \mathbb{C}^{Q\text{States}_M} \otimes \mathbb{C}^{C\text{States}_M} \otimes \mathbb{C}^{I_M} \otimes \mathbb{C}^{O_M}.
\]

\( \mathcal{H}_M \) is the space accessible to \( M \), including the space of its inputs and outputs.

Note that \( I_M \) has the structure \( I_M = \prod_{p \in \text{in}(\text{Ports}_M)} \Sigma^* \), therefore \( \mathbb{C}^{I_M} \) decomposes into

\[
\mathbb{C}^{I_M} = \bigotimes_{p \in \text{in}(\text{Ports}_M)} \mathcal{H}_p \quad \text{with} \quad \mathcal{H}_p := \mathbb{C}^{\Sigma^*}
\]

Below we will sometimes refer to these subsystems directly via their name \( \mathcal{H}_p \).

Let further

\[
\mathcal{H}_{\hat{C}} := \bigotimes_{M \in \hat{C}} \mathcal{H}_M
\]

In the following, when we say that some operation \( X \) (a superoperator or a measurement) is applied to some subsystem \( \mathcal{H} \) of \( \mathcal{H}_{\hat{C}} =: \mathcal{H}_a \otimes \mathcal{H} \otimes \mathcal{H}_b \), we formally mean that
1 ⊗ X ⊗ 1 is applied to \( P(\mathcal{H}_a \otimes \mathcal{H} \otimes \mathcal{H}_b) \) (the set of density operators on \( \mathcal{H}_C \)). Here 1 denotes the identity.

Consider the following measurement process on \( \mathcal{H}_C \) (formally, on the set \( P(\mathcal{H}_C) \) of density operators over \( \mathcal{H}_C \)).

1. Prepare the state

\[
\bigotimes_{M \in C} \rho_{M}^{\text{ini},k} \in P(\mathcal{H}_C)
\]

where

\[
\rho_{M}^{\text{ini},k} := |\varepsilon\rangle\langle\varepsilon| \otimes |1^k\rangle\langle1^k| \otimes |\varepsilon, \ldots, \varepsilon\rangle\langle\varepsilon, \ldots, \varepsilon| \otimes |\varepsilon, \ldots, \varepsilon\rangle\langle\varepsilon, \ldots, \varepsilon| \in P(\mathcal{H}_M)
\]

(This means we initialised all machines to initial quantum state \(|\varepsilon\rangle\langle\varepsilon|\), classical initial state \(1^k\), and empty in- and output-spaces.)

2. Initialise the variable \( M_{\text{CS}} \) (the current scheduler) to have the value \( X \). Prepare \(|1\rangle\langle1|\) in \( \mathcal{H}_d \).\(^{10}\)

3. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on \( C^{|\text{States}_{M_{\text{CS}}}|} \). Let \( s \) denote the outcome.

4. If \( s \in \text{Fin}_{M_{\text{CS}}} \) and \( M_{\text{CS}} = X \), exit (the run is complete). If \( s \in \text{Fin}_{M_{\text{CS}}} \) but \( M_{\text{CS}} \neq X \), proceed to Step 5.

5. For each port \( p \in \text{in}(\text{Ports}_{M_{\text{CS}}}) \) s.t. \( l_{M_{\text{CS}}}(s, p) = 0 \), prepare \(|\varepsilon\rangle\langle\varepsilon|\) in \( \mathcal{H}_p \).

6. For each \( p \in \text{in}(\text{CPorts}_{M_{\text{CS}}}) \), perform a complete measurement on \( \mathcal{H}_p \). Let the outcome be \( I_p \).

7. For each \( p \in \text{in}(\text{Ports}_{M_{\text{CS}}}) \), measure whether \( \mathcal{H}_p \) is in state \(|\varepsilon\rangle\) (whether it is empty). If all ports were empty, proceed to Step 8. Otherwise let \( P \) be the set of the ports that were nonempty.

8. Switch the current scheduler, i.e. apply the state-transition operator \( \Delta_{M_{\text{CS}}} \) to \( \mathcal{H}_{M_{\text{CS}}} \).

9. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on \( C^{|\text{States}_{M_{\text{CS}}}|} \). Let \( s' \) denote the outcome. For each \( p \in \text{out}(\text{CPorts}_{M_{\text{CS}}}) \), perform a complete measurement on \( \mathcal{H}_p \). Let the outcome be \( O_p \).

10. Let \( I := (I_p)_{p \in \text{in}(\text{CPorts}_{M_{\text{CS}}})} \) and \( O := (O_p)_{p \in \text{out}(\text{CPorts}_{M_{\text{CS}}})} \). Add \((\text{name}_{M_{\text{CS}}}, s, I, s', O, P)\) to the trace (which initially is empty).

11. Let \( \text{MOVE} \) denote the superoperator over \( C^{|\Sigma^*|} \otimes C^{|\Sigma^*|} =: \mathcal{H}_1 \otimes \mathcal{H}_2 \) defined by \( \rho \mapsto |\varepsilon\rangle\langle\varepsilon| \otimes \text{tr}_2 \rho \).\(^{11}\) Then for each simple out-port \( p! \in \text{Ports}_{M_{\text{CS}}} \) perform the following: Measure, whether \( p! \) is empty, i.e. measure whether \( \mathcal{H}_{p!} \) is in state \(|\varepsilon\rangle\). If nonempty, apply \( \text{MOVE} \) to \( \mathcal{H}_{p!} \otimes \mathcal{H}_{p!^*} \). Then (if \( p! \) was nonempty) switch buffer \( \bar{p} \), i.e. apply \( \Delta_{\text{buff}} \) to \( \mathcal{H}_{\bar{p}} \).

12. For each \( p \in \text{Ports}_{M_{\text{CS}}} \), prepare \(|\varepsilon\rangle\langle\varepsilon|\) in \( \mathcal{H}_p \). (With \( \varepsilon \) being the empty word.)

---

\(^{10}\) Formally, we apply the superoperator \( \rho \mapsto |1\rangle\langle1| \) to \( \mathcal{H}_{d\text{ink}} \).

\(^{11}\) I.e., the second subsystem is prepared to be \(|\varepsilon\rangle\langle\varepsilon|\), and then then the subsystems are swapped.
13. Let $s^{>\downarrow}$ be the first clock out-port from \( CPor{t}_{MCS} \) (in the ordering given by the port sequence \( \text{Ports}_{MCS} \)) with $I_{s^{>\downarrow}} \neq \varepsilon$. If there is no such port, proceed to Step 2.

14. Prepare $|I_{s^{>\downarrow}}\rangle|I_{s^{>\downarrow}}\rangle$ in \( H_{s^{>\downarrow}} \). Then switch buffer $\tilde{s}$ (apply $\Delta_{\text{buff}}$ to $H_{\tilde{s}}$). Measure whether $s^{>\downarrow}$ is empty (measure whether $H_{s^{>\downarrow}}$ is in state $|\varepsilon\rangle$). If it is empty, proceed to Step 3.

15. Apply \textsc{Move} to $H_{s^{>\downarrow}} \otimes H_{s^{<\uparrow}}$. Let $M_{CS}$ be the unique machine with $s^{<\uparrow} \in \text{Ports}_{MCS}$. Proceed to Step 3.

This measurement process induces a probability distribution on the trace (see Step 10). We call this probability distribution \( \text{run}_{\hat{C},k} \) (the run/trace of $\hat{C}$ on security parameter $k$). We will also use \( \text{run}_{\hat{C},k} \) for a random variable with that distribution.

We will now comment on the individual steps of the measurement process above.

In Step 1 the initial states of the machines are prepared. It consists of the security parameter $k$ (encoded as $1^k$) as the classical state, and the empty word $|\varepsilon\rangle$ as the quantum state.

Then, in Step 2 the master scheduler is chosen to be the next machine to be activated, and its clk$^{>\downarrow}$ port gets the content 1. The loop will jump to this step whenever the activation token is lost, see below.

In Step 3 the classical state $s$ of the current scheduler $M_{CS}$ (the machine to be activated in this iteration of the loop) is measured for inclusion in the trace (Step 10). So $s$ is the state before activation.

In Step 4 it is checked, whether the current scheduler is in a final state. If so, the master scheduler must be activated, so we go back to Step 2. If the current scheduler is the master scheduler and has terminated, the whole process terminated and the run is complete.

In Step 5 for each in-port the length function is evaluated, and if it is 0, the content of that port erased (so that messages on this port are ignored).

Then, in Step 6 the contents of all classical ports are measured for inclusion in the trace (Step 10).

In Step 7 it is checked, whether there is at least one port containing data. If not, the current scheduler is not activated, and the master scheduler is activated by going to Step 2.

Step 8 is probably the most important step in the run. Here the current scheduler’s state-transition operator is finally applied.

Then in Step 9 the classical state $s'$ of the current scheduler and its classical outputs are measured for inclusion into the trace.

In Step 10 the classical state of the master scheduler before and after execution, and its classical in- and outputs are appended to a variable called the trace. This variable describes the observable behaviour of the network and its final value (or the possibly infinite sequence if the loop does not terminate) gives rise to a probability distribution of observable behaviour, the \( \text{run} \) or \( \text{trace} \) on security parameter $k$: \( \text{run}_{\hat{C},k} \).

Then, in Step 11 for each simple out-port of $M_{CS}$, that is nonempty, the content of this port is moved to the corresponding buffer in-port. Then the buffer is activated, with the effect that it stores the incoming message into its queue (cf. Definition 3).
Now, in Step 12, the contents of the ports of $M_{CS}$ are erased (the contents of the clock out-ports are still needed, but they have been measured above are therefore are still accessible via the variables $O_{p_{\sigma}}$.

In Step 13 we choose the first clock-out port of $M_{CS}$ that contained output. If there is not such clock-out port, it means that $M_{CS}$ did not want to schedule any connection, so the master scheduler is activated again via Step 2.

Then in Step 14 if some connection is to be scheduled, the corresponding buffer is activated with the number of the message as input on its clock in-port $p_{\sigma}$. This has the effect of moving the message from the queue to the buffer out-port.

Finally, in Step 15, the message is moved from the buffer’s out-port to the recipients corresponding simple in-port, and the recipient is noted to be the next machine to be activated (current scheduler). Then the loop proceeds with Step 3 (not Step 2 since this would activate the master scheduler).

We encourage the reader to compare this formal definition with intuitive description of the scheduling given in Section 2.

So finally we got a random variable $\hat{run}_{C,k}$ describing the observable behaviour of the network and can proceed to the next section, where the actual security definitions will be stated.
3 Quantum security definitions

In the preceding section we have defined what a quantum network is (formally, as closed collection \( \hat{M} \)). Further we have described the evolution of such a network over time, and the defined a random variable \( \text{run}_{\hat{M},k} \) representing the observable behaviour of such a network when the security parameter is \( k \). Using these prerequisites, it is now easy to define quantum simulatable security. In particular, nothing specially related to the quantum nature of our protocols has to be taken into account any more, so most of this section is very similar to \([BPW04b]\).

3.1 Protocols

Remember, that we “defined” simulatable security in Section 1.6 approximately as follows (we will talk of an ideal protocol instead of the special case of a trusted host here, for greater generality):

A real protocol \( \pi \) is as secure as an ideal protocol \( \rho \) if there for each real adversary \( A_{\text{real}} \) there is a simulator \( A_{\text{sim}} \) s.t. for all honest users \( H \) the view of \( H \) in runs with the real protocol and real adversary is indistinguishable from runs with the ideal protocol and the simulator.

The first point of this “definition” we will elaborate on, is the notion of a protocol. From Section 2.2 we already have the notion of a collection. We remember that a non-closed collection is one where some ports (the free ports) are still unconnected. So such an open collection can be regarded as a protocol, where the in- and outputs of the protocols go over the free ports to some still to be specified outer world.

![Fig. 3. A simple protocol](image)

Let us consider an example: Two parties \( A \) and \( B \) form a protocol. They have a connection \( \text{net} \) between them (so \( A \) and \( B \) have ports \( \text{net}! \) and \( \text{net}? \), resp.), and for getting their in- and outputs they have ports \( \text{in}_A \) and \( \text{out}_B \). Now the collection describing that protocol would simply be \( \hat{M} := \{A, B\} \) (cf. Figure 3). But when we look at the free ports of \( \hat{M} \) (strictly spoken of the completion of \( \hat{M} \)), we note that

\[
\text{free}(\hat{M}) = \{\text{in}^{++}, \text{out}^{++}, \text{in}^{\circ}, \text{out}^{\circ}, \text{net}^{\circ}\}.
\]

This means, that some protocol user (honest user) would in principle be able to connect e.g. to the protocols \( \text{net}^{\circ} \)-port. So a protocol user would “see” and even “control” the
internal scheduling mechanisms of the protocol. But this ability we would reserve for the adversary.

More precarious is the situation, if we would like to say that our protocol implements some trusted host. Assume e.g. that the trusted host models some behaviour with an explicit insecurity (as is usual in the modelling of trusted hosts, in most cases the adversary is at least informed about the length of the data). Then the trusted host (its completion) would have another free port called e.g. \texttt{len}^+!. An honest user connecting to that port would of course immediately “notice a difference”, be it only that there are more ports in the ideal than in the real protocol.

Therefore we need a way to specify which ports the honest user may possibly connect to. Following [BPW04b] we will call these ports the service ports, and a non-closed collection together with the set of its service ports will be called a structure. A structure is our notion of a protocol. We cite the formal definition of a structure from [BPW04b]:

**Definition 7 (Structures and service ports [BPW04b]).** A structure is a pair \((\hat{M}, S)\) where \(M\) is a collection of simple machines with \(S \subseteq \text{free}([\hat{M}])\). The set \(S\) is called service ports of \((\hat{M}, S)\).

The notion of service ports allows us to specify the set of ports an honest user must not connect to (we cite again):

**Definition 8 (Forbidden ports [BPW04b]).** For a structure \((\hat{M}, S)\) let \(\bar{S}_\hat{M} := \text{free}([\hat{M}]) \setminus S\). We call \(\text{forb}(\hat{M}, S) := \text{Ports}_{\hat{M}} \cup \bar{S}_{\hat{M}}\) the forbidden ports. \(\text{forb}\) denotes the element-wise low-level complement.

It is sufficient to know, that \(\text{forb}(\hat{M}, S)\) consists of the ports \(\hat{M}\) may not have either because they would connect to non-service ports, or because they are already used by the protocol and would give rise to a name clash.

The next step is to specify, which honest users and adversaries are valid ones. We must e.g. disallow honest users which connect to non-service ports. And we must guarantee that honest user, adversary and structure together give rise to a closed collection, that can then be executed (as in Section 2.2) to give rise to some protocol trace. Here we can again cite [BPW04b]:

**Definition 9 (Configurations [BPW04b]).**

- A configuration of a structure \((\hat{M}, S)\) is a tuple \(\text{conf} = (\hat{M}, S, H, A)\) where
  - \(H\) is a machine called user (or honest user) without forbidden ports, i.e., \(\text{Ports}_H \cup \text{forb}(\hat{M}, S) = \emptyset\).
  - \(A\) is a machine called adversary.
  - the completion \(\hat{C} := [\hat{M} \cup \{H, A\}]\) is a closed collection.
- The set of configurations of \((\hat{M}, S)\) is written \(\text{Conf}(\hat{M}, S)\).
- Let \((\hat{M}_1, S)\) and \((\hat{M}_2, S)\) be structures (with identical service ports). The set of suitable configurations \(\text{Conf}^{\hat{M}_2}(\hat{M}, S) \subseteq \text{Conf}(\hat{M}, S)\) is defined by \((\hat{M}_1, S, H, A) \in \text{Conf}^{\hat{M}_2}(\hat{M}, S)\) iff \(\text{Ports}_H \cap \text{forb}(\hat{M}_2, S) = \emptyset\).
The first part of this definition tells us what honest users and adversaries are admissible for some structure. Honest user $H$ and adversary $A$ are admissible for the structure $(\hat{M}, S)$ exactly if $(\hat{M}, S, H, A) \in \text{Conf}(\hat{M}, S)$.

The last part of the definition is needed further below. Consider an honest user that is admissible for the real protocol $(\hat{M}_1, S)$. Then our security definition will use the same honest user also for the ideal protocol $(\hat{M}_2, S)$. If the honest user has ports that would be forbidden with the ideal protocol, trouble is at hand. Therefore the definition of suitable configurations additionally requires that the honest user has no forbidden ports of either $(\hat{M}_1, S)$ nor $(\hat{M}_2, S)$.

### 3.2 The security relation

Besides the unspecified notion of a “protocol”, there was another under-specified term in the “definition” from Section 1.6: we required the view of $H$ to be indistinguishable in runs with real or ideal protocol. So let us first specify what indistinguishability means in our scenario (we cite again):

**Definition 10 (Small functions [BPW04b])**.

- The class $\text{NEGL}$ of negligible functions contains all functions $s: \mathbb{N} \rightarrow \mathbb{R}_{\geq 0}$ that decrease faster than the inverse of every polynomial, i.e., for all positive polynomials $Q \exists k_0 \forall k > k_0: s(k) < \frac{1}{Q(k)}$.
- The set $\text{SMALL}$ of functions $\mathbb{N} \rightarrow \mathbb{R}_{\geq 0}$ is a class of small functions if it is closed under addition, and with a function $g$ also contains every function $g' : \mathbb{N} \rightarrow \mathbb{R}_{\geq 0}$ with $g' \leq g$.

**Definition 11 (Indistinguishability [BPW04b])**. Two families $(\text{var})_{k \in \mathbb{N}}$ and $(\text{var}')_{k \in \mathbb{N}}$ of probability distributions (or random variables) on common domains $(D_k)_{k \in \mathbb{N}}$ are

- perfectly indistinguishable (“=”) iff $\forall k \in \mathbb{N} : \text{var}_k = \text{var}'_k$.
- statistically indistinguishable (“$\approx_{\text{SMALL}}$”) for a class $\text{SMALL}$ of small functions if the distributions are discrete and their statistical distances, as a function of $k$, are small, i.e.

$$\text{StatDist}(\text{var}_k, \text{var}'_k)_{k \in \mathbb{N}} := \left(\frac{1}{2} \sum_{d \in D_k} |\text{Pr}(\text{var}_k = d) - \text{Pr}(\text{var}'_k = d)| \in \text{SMALL}\right).$$

Mostly we will use $\text{SMALL} := \text{NEGL}$.

The last term in our definition that still has to be defined is that of a view. Intuitively, a view is everything (classical) a machine experiences during the run. Since in the definition of the run, every record in the run is tagged with the name of the corresponding machine, it is now easy to define the view of a machine $M$ by removing all entries not tagged with the name of that machine from the run. Formally:

**Definition 12 (Views)**. Let a closed collection $\hat{C}$ be given. Then Definition 6 gives rise to a family of random variables $\text{run}_{\hat{C}, k}$. Let further $M \in \hat{C}$ be a simple machine
or a master scheduler. Then the view \( \text{view}_{\hat{C},k} \) of \( M \) on security parameter \( k \) is the subsequence of \( \text{run}_{\hat{C},k} \) resulting by taking only the elements \( (n, s, I, s', O, p) \in \text{run}_{\hat{C},k} \) satisfying \( n = \text{name}_M \).

If \( \text{conf} = (M, S, H, A) \) is a configurations, then we define \( \text{view}_{\text{conf},k}(M) := \text{view}_{M \cup \{H, A\},k}(M) \).

We restate our informal “definition”.

A real protocol \( \pi \) is as secure as an ideal protocol \( \rho \) if there for each real adversary \( A_{\text{real}} \) there is a simulator \( A_{\text{sim}} \) s.t. for all honest users \( H \) the view of \( H \) in runs with the real protocol and real adversary is indistinguishable from runs with the ideal protocol and the simulator.

Using the definitional tools developed above, we can capture this definition formally (slightly modified in comparison to [BPW04b] to include the notion of strict statistical security):

**Definition 13 (Security for structures).** Let structures \((\hat{M}_1, S)\) and \((\hat{M}_2, S)\) with identical sets of service ports be given.

\(- (\hat{M}_1, S) \geq^\text{perf}_{\text{sec}} (\hat{M}_2, S)\), spoken \((\hat{M}_1, S)\) is perfectly as secure as \((\hat{M}_1, S)\), iff for every configurations \( \text{conf}_1 = (\hat{M}_1, S, H, A_{\text{real}}) \in \text{Conf}^{\hat{M}_2}(\hat{M}_1, S) \) (the real configuration), there exists a configuration \( \text{conf}_2 = (\hat{M}_2, S, H, A_{\text{sim}}) \in \text{Conf}(\hat{M}_2, S) \) with the same \( H \) (the ideal configuration) s.t.

\[
(\text{view}_{\text{conf}_1,k}(H))_k = (\text{view}_{\text{conf}_2,k}(H))_k.
\]

\(- (\hat{M}_1, S) \geq^\text{SMALL}_{\text{sec}} (\hat{M}_2, S)\), spoken \((\hat{M}_1, S)\) is strictly statistically as secure as \((\hat{M}_1, S)\), for a class of small functions, iff for every configurations \( \text{conf}_1 = (\hat{M}_1, S, H, A_{\text{real}}) \in \text{Conf}^{\hat{M}_2}(\hat{M}_1, S) \) (the real configuration), there exists a configuration \( \text{conf}_2 = (\hat{M}_2, S, H, A_{\text{sim}}) \in \text{Conf}(\hat{M}_2, S) \) with the same \( H \) (the ideal configuration) s.t.

\[
(\text{view}_{\text{conf}_1,k}(H))_k \approx^\text{SMALL} (\text{view}_{\text{conf}_2,k}(H))_k.
\]

In both cases, we speak of universal simulatability (or universal security) if \( A_{\text{sim}} \) in \( \text{conf}_2 \) does not depend on \( H \) (only on \( \hat{M}_1 \), \( S \), and \( A_{\text{real}} \)), and we use the notation \( \geq^\text{perf}_{\text{sec}} \) etc. for this.

### 3.3 Corruption

So far, we have not modelled the possibility of corrupting a party. However, the approach of [BPW04b] applies to our setting with virtually no modifications, so we simply refer to [BPW04b]. (A very short sketch of their approach is also found in Section 1.10)
4 Composition

In the present section we are going to show the simple composition theorem. See Section 1.9 for an overview what this composition theorem is for, and different flavours of composition theorems exist.

4.1 Combinations

Consider a network consisting of some machines $M_1, \ldots, M_n$. Imagine now taking two of these machines (say $M_1, M_2$) and putting them into a cardboard box (without disconnecting any of the network cables). Then (if the machines are not equipped with some special hardware for detecting cardboard boxes, e.g. a light sensor) that no machine will detect a difference, i.e. the views of all machines are unchanged. Even more, we can now consider the cardboard box as a new, more complex machine $\text{Comb}(M_1, M_2)$ (the combination of $M_1, M_2$). The view of $\text{Comb}(M_1, M_2)$ then contains the views of $M_1$ and $M_2$, so we can still claim that the view of no machine (not even $M_1$ or $M_2$) is changed by removing $M_1$ and $M_2$ from the network and replacing them by $\text{Comb}(M_1, M_2)$.

This seemingly trivial observation is—when formally modelled—a powerful tool for reasoning about networks which we will need below.

Before we define the combination, consider the following technical definition:

**Definition 14 (Canonisation).** Let a master scheduler or a simple machine $M$ with state-transition operator $\Delta_M$ be given.

Then

$$
\mathcal{H}_M := \mathbb{C}^{Q\text{States}_M} \otimes \mathbb{C}^{C\text{States}_M} \otimes \mathbb{C}^{I\text{M}} \otimes \mathbb{C}^{O\text{M}}
$$

is the state space of $M$ (together with its inputs and outputs).

Then we define $\tilde{\Delta}_M$ to be the superoperator resulting from the following measurement process on $\mathcal{H}_M$:

- Measure the classical state $s$ of $M$.
- For each in-port of $M$, measure whether it is empty.
- For each in-port $p$ s.t. $l_M(c, p) = 0$, prepare $|\varepsilon\rangle\langle\varepsilon|$ in $\mathcal{H}_p$.
- If $s \notin \text{Fin}_M$, and at least one port was nonempty, apply $\Delta_M$.

Then the canonisation of $M$ is the machine

$$
\tilde{M} := (\text{name}_M, \text{Ports}_M, \text{CPorts}_M, Q\text{States}_M, C\text{States}_M, \tilde{\Delta}_M, l_M, \text{Fin}_M),
$$

i.e. $\tilde{M}$ results from $M$ by replacing its state-transition operator by $\tilde{\Delta}_M$.

Now note that in the run algorithm (Definition 6) the only place where the state-transition operator $\tilde{\Delta}_M$ can be applied is Step 8. But preceding that step, measurements occurred that guarantee that at least one port is nonempty, that $s$ is not a final state, and that all ports with $l_M(c, p) = 0$ are empty. Therefore there is no difference between using $\Delta_M$ and $\tilde{\Delta}_M$ in Step 8. From this insight, the following lemma follows:
Lemma 1. Let $\hat{C}$ be some closed collection and $M \in \hat{C}$ a master scheduler or a simple machine. Let $\tilde{M}$ be the canonisation of $M$, and $\hat{D} := \hat{C} \setminus \{M\} \cup \{\tilde{M}\}$ result from replacing $M$ by $\tilde{M}$. Then $\hat{D}$ is closed, and for all $k \in \mathbb{N}$

$$\text{run}_{\hat{C},k} = \text{run}_{\hat{D},k}.$$ 

Using the canonisation it is now easy to define the combination of two machines:

Definition 15 (Combination). Let two machines $M_1, M_2$ with disjoint sets of ports be given. Assume both them to be either master schedulers or simple machines or a combinations thereof.

Denote by $\tilde{M}_1$ and $\tilde{M}_2$ the canonisations of $M_1$ and $M_2$. Then let

$$\begin{align*}
\text{name}_C &:= \text{name}_{\tilde{M}_1} \text{name}_{\tilde{M}_2} \\
\text{Ports}_C &:= \text{Ports}_{\tilde{M}_1} \cup \text{Ports}_{\tilde{M}_2} \\
\text{CPorts}_C &:= \text{CPorts}_{\tilde{M}_1} \cup \text{CPorts}_{\tilde{M}_2} \\
\text{QStates}_C &:= \text{QStates}_{\tilde{M}_1} \times \text{QStates}_{\tilde{M}_2} \\
\text{CStates}_C &:= \text{CStates}_{\tilde{M}_1} \times \text{CStates}_{\tilde{M}_2} \\
\Delta_C &:= \Delta_{\tilde{M}_1} \otimes \Delta_{\tilde{M}_2} \\
l_C((c_1, c_2), p) &:= \begin{cases} \\
 l_{\tilde{M}_1}(c_1, p), & \text{if } p \in \text{Ports}_{\tilde{M}_1} \\
l_{\tilde{M}_2}(c_2, p), & \text{if } p \in \text{Ports}_{\tilde{M}_2} \end{cases} \\
\text{Fin}_C &:= \text{Fin}_{\tilde{M}_1} \times \text{Fin}_{\tilde{M}_2}
\end{align*}$$

and

$$\text{Comb}(M_1, M_2) := (\text{name}_C, \text{Ports}_C, \text{CPorts}_C, \text{QStates}_C, \text{CStates}_C, \Delta_C, l_C, \text{Fin}_C).$$

We can now state the following combination lemma:

Lemma 2 (Combination). Let a closed collection $\hat{C}$ be given. Assume $M_1, M_2 \in \hat{C}$ to be master schedulers, simple machines or a combinations of both. Let $\hat{D} := \hat{C} \setminus \{M_1, M_2\} \cup \{\text{Comb}(M_1, M_2)\}$.

Then for any machine $M \in \hat{C} \setminus \{M_1, M_2\}$ it is for all $k \in \mathbb{N}$

$$\text{view}_{\hat{C}}(M) = \text{view}_{\hat{D}}(M).$$

Further let $\text{view}_{\hat{D}}(M_1)$ denote the view of $M_1$ extracted from the $v := \text{view}_{\hat{D}}(M_1)$ in the following manner:

For each element $v_i = (\text{name}, (s_1, s_2), (I_1, I_2), (s'_1, s'_2), (O_1, O_2), P)$, do the following

- If $P \cap \text{Ports}_{M_1} \neq \emptyset$, replace $v_i$ by $(\text{name}_{M_1}, s_1, I_1, s'_1, O'_1, P)$.
- Otherwise, remove $v_i$ from the view.
Then
\[ \text{view}_{\hat{C},k}(M_1) = \text{view}_{\hat{D},k}(M_1). \]

Then same holds for \( M_2 \).

The proof of this lemma consists of first applying Lemma 1 to show
\[ \text{run}_{\hat{C},k}(M) = \text{run}_{\hat{C}',k}(M). \]
where \( \hat{C}' \) is \( \hat{C} \) with \( M_1 \) and \( M_2 \) replaced by their canonisations. The rest is a straightforward but long and tedious checking of each step of the run-algorithm in Definition 6 to show
\[ \text{view}_{\hat{C}}(M) = \text{view}_{\hat{D}}(M). \]
A proof is given in Appendix A.1.

So this lemma states that we can in fact replace any two machines by their combination, without changing the behaviour of the network or the views of the individual machines.

We write \( \text{Comb}(M_1, M_2, \ldots, M_n) \) for \( \text{Comb}(M_1, \text{Comb}(M_2, \text{Comb}(\ldots, \text{Comb}(M_{n-1}, M_n)))) \) to get a combination of more than two machines.

### 4.2 Transitivity

**Lemma 3 (Transitivity).** Let \( (\hat{M}_1, S) \geq (\hat{M}_2, S) \geq (\hat{M}_3, S) \). Then \( (\hat{M}_1, S) \geq (\hat{M}_3, S) \).

Here \( \geq \) may denote perfect, strict statistical, universal perfect and universal strict statistical security.

This lemma is obvious from Definition 13.

### 4.3 The simple composition theorem

In the preceding sections we have tried to get a strong notational and structural similarity to [BPW04b]. We can now harvest the fruits of this program: the definition of simple composition, the simple composition theorem and the proof thereof are almost identical to those in the RS framework. We restate the definition of composition for self-containment:

**Definition 16 (Composition [Bac02]).** Structures \( (\hat{M}_1, S), \ldots, (\hat{M}_n, S_n) \) are composable if no port of \( M_i \) is contained in \( \text{forb}(M_j, S_j) \) for \( i \neq j \), and \( S_1 \cap \text{free}(\hat{M}_2) = S_2 \cap \text{free}(\hat{M}_1) \).

Their composition is then \( (\hat{M}_1, S_1) || \ldots || (\hat{M}_n, S_n) := (\hat{M}, s) \) with \( \hat{M} = \hat{M}_1 \cup \cdots \cup \hat{M}_n \)
and \( S = (S_1 \cup \cdots \cup S_n) \cap \text{free}(\hat{M}) \).

For details on the conditions in the definition of composable, see [Bac02].

We can now state the simple composition theorem (called Secure Two-system Composition in [Bac02]).
Theorem 1 (Simple composition). Let $\hat{M}_0, S_0$, $(\hat{M}_0', S_0)$ and $(\hat{M}_1, S_1)$ be structures, s.t. $(\hat{M}_0, S_0), (\hat{M}_1, S_1)$ are composable, and $(\hat{M}_0', S_0)$ and $(\hat{M}_0', S_0)$ are composable. Assume further $\text{ports}(\hat{M}_0') \cap S_0^c = \text{ports}(\hat{M}_0) \cap S_1^c$. ($\text{ports}(\hat{M})$ is the set of all ports of all machines in $\hat{M}$.)

Then

\[(\hat{M}_0, S_0) \geq (\hat{M}_0', S_0) \implies (\hat{M}_0, S_0) \parallel (\hat{M}_1, S_1) \geq (\hat{M}_0', S_0) \parallel (\hat{M}_1, S_1)\]

Here $\geq$ may denote perfect, strict statistical, universal perfect and universal strict statistical security.

The proof of the composition theorem in [PW01] is completely based on a higher-level view on the network model in the sense that every statement about the view of machines is derived through the combination lemma. So the proof of [PW01] applies in the quantum setting. (In fact the proof in [PW01] covers the more general case of the security of systems, however, the composition theorem stated above is just a special case of the composition theorem in [PW01].) We therefore refer the reader to the proof in [PW01].
5 Conclusions

In the present work we have seen how to “lift” a classical model to a quantum one. However, much possible work still lies ahead:

– Simplicity. In the personal opinion of the author, the most urgent matter is the search for a model that is both simple (meaning both being simple to understand, and simple to use in proofs) and general (so taking recourse to restricting the possibilities of scheduling and message delivery would not be a solution).

We believe that the complexity and the amount of details in the present work (the reader probably noticed them) is mostly due to the use of a message driven scheduling. A glance on the run-algorithm (Definition 6) shows, that most of the steps are actually concerned with finding out which machine is to be activated with which inputs. Only one Step 8 actually executes a machine’s program.

Note however that these complications are not particular to our model. Both in [BPW04b] and [Can01] most of the modelling is concerned with the order of activation of the machines. An effect of this is that security proofs tend to either get complicated and unreadable, or tend to ignore the details of scheduling almost completely and assume that message delivery will take place in a well-behaved and intuitive way.

In the quantum case this problem is amplified by the fact that here one has to take care to explicitly specify any measurements done, instead of just referring to facts about the state of the system as in the classical case.

– Formal proofs and machine verification. Even when the security models have reached a point where proofs may concentrate on the essentials, large protocols may still be quite complex to manage. It would therefore be very helpful to have a hand tools for formally proving (using e.g. rewriting rules for networks or similar) security, and for verifying (or even generating) proof with a machine.

Some effort have already been done in that direction in the RS framework, e.g. already in [BCJP02] the security of a protocol was shown in the theorem prover PVS.

– Concrete security proofs. So far only a few protocols have been shown to be secure in a model of simulatable security. E.g., the only family of quantum protocols to far is that of quantum key distribution.

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A Postponed proofs

A.1 Combination lemma (Lemma 2)

Let \( \hat{C} := \hat{C} \setminus \{M_1, M_2\} \cup \{\tilde{M}_1, \tilde{M}_2\} \), i.e. \( \hat{C} \) results from \( \hat{C} \) by replacing \( M_i \) by their canoni- sations \( \tilde{M}_i \). Then by Lemma 1, we have

\[
\text{view}_\hat{C}(M) = \text{view}_{\hat{D}}(M).
\]

So for proving the first part of Lemma 2 it is sufficient to show

\[
\text{view}_{\hat{C}}(M) = \text{view}_{\hat{D}}(M).
\]  (4)

In order to do this, we will start with the run-algorithm for \( \hat{C} \) and perform a series of rewritings, until we have reached the run-algorithm for \( \hat{D} \).

Let \( A_0 \) denote the run-algorithm for \( \hat{C} \). Then we derive the algorithm \( A_1 \) by replacing Steps 2 and 15 by

2'. Initialise the variable \( M_{CS} \) (the current scheduler) to have the value \( X \). Prepare \( |1\rangle\langle 1| \in H_{clkw} \). Set \( M_{CS} := X \). Prepare \( |1\rangle\langle 1| \in H_{clk} \). Proceed to Step 3.

Let \( M_{CS} \) be the unique machine in \( \tilde{C} \) with \( s? \in \text{Ports}_{M_{CS}} \).

If \( M_{CS} \notin \{\tilde{M}_1, \tilde{M}_2\} \), assign \( M_{CS} := \text{Comb}(M_1, M_2) \), otherwise set \( M_{CS} := M_{CS} \).

In both steps we have only set a hitherto unused variable, but changed nothing else, so the behaviour of \( A_0 \) and \( A_1 \) are identical, i.e. if we define \( O_0 \) to be the output (the trace) of \( A_i \),

\[
\text{run}_\hat{C} = O_0 = O_1.
\]  (5)

Further note, that it now holds after every step of the algorithm, that \( M_{CS} = M_{CS} \)
for \( M_{CS} \notin \{M_1, M_2\} \), and \( M_{CS} = \text{Comb}(M_1, M_2) \) otherwise.

We now derive the algorithm \( A_2 \) from \( A_1 \) by replacing Step 3 by

3'. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on \( C^{\text{State}_{M_{CS}}} \). Let \( \tilde{s} \) denote the outcome. If \( M_{CS} \notin \{M_1, M_2\} \), set \( s := \tilde{s} \), otherwise let \( s \) be the \( i \)-th component of \( \tilde{s} \) (for \( M_{CS} = M_i \)).

We use here \( C^{\text{State}_{\text{Comb}(M_1, M_2)}} := C^{\text{State}_{M_1}} \otimes C^{\text{State}_{M_2}} \).

If \( M_{CS} \notin \{M_1, M_2\} \), this step evidently behaves like Step 3 (except that it sets an variable that is not used in other steps). If \( M_{CS} = M_1 \), the classical state of \( \tilde{M}_2 \) is additionally measured, however, this state was already measured in Step 3 so measuring it does not disturb the state of the system. Further the result the original Step 3 would have yielded \( (s) \) is reconstructed from \( (\tilde{s}) \). Therefore in this case Step 3 behaves like Step 3. Analogously for \( \tilde{M}_2 \). So we have

\[
O_1 = O_2.
\]  (6)

Similarly, we get algorithm \( A_3 \) by successively replacing Steps 5 by

5. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on \( C^{\text{State}_{\text{Comb}(M_1, M_2)}} \). Let \( \tilde{s} \) denote the outcome. If \( M_{CS} \notin \{M_1, M_2\} \), set \( s := \tilde{s} \), otherwise let \( s \) be the \( i \)-th component of \( \tilde{s} \) (for \( M_{CS} = M_i \)).

We use here \( C^{\text{State}_{\text{Comb}(M_1, M_2)}} := C^{\text{State}_{M_1}} \otimes C^{\text{State}_{M_2}} \).

If \( M_{CS} \notin \{M_1, M_2\} \), this step evidently behaves like Step 3 (except that it sets an variable that is not used in other steps). If \( M_{CS} = M_1 \), the classical state of \( \tilde{M}_2 \) is additionally measured, however, this state was already measured in Step 3 so measuring it does not disturb the state of the system. Further the result the original Step 3 would have yielded \( (s) \) is reconstructed from \( (\tilde{s}) \). Therefore in this case Step 3 behaves like Step 3. Analogously for \( M_2 \). So we have

\[
O_1 = O_2.
\]  (6)
1. For each port $p \in \text{in}(\text{Ports}_{\tilde{M}_{CS}})$ s.t. $I_{M_{CS}}(\tilde{s}, p) = 0$, prepare $|\varepsilon\rangle\langle\varepsilon|$ in $H_p$.

2. For each $p \in \text{in}(\text{CPorts}_{\tilde{M}_{CS}})$, prepare a complete measurement on $H_p$. Let the outcome be $I_p$.

3. For each port $p \in \text{in}(\text{Ports}_{\tilde{M}_{CS}})$, measure whether $H_p$ is in state $|\varepsilon\rangle$ (whether it is empty). If all ports were empty, proceed to Step 4. Otherwise let $P$ be the set of the ports that were nonempty.

4. Perform a complete von-Neumann measurement in the computational basis (called a $\tilde{X}$ here) on $\mathbb{C}^{C\text{States}_{\tilde{M}_{CS}}}$. Let $\tilde{s}'$ denote the outcome. For each $p \in \text{out}(\text{CPorts}_{\tilde{M}_{CS}})$, perform a complete measurement on $H_p$. Let the outcome be $O_p$.

   If $M_{CS} \notin \{\tilde{M}_1, \tilde{M}_2\}$, set $s' := \tilde{s}'$, otherwise let $s'$ be the $i$-th component of $\tilde{s}'$ (for $M_{CS} = M_i$).

5. For each simple out-port $p! \in \text{Ports}_{\tilde{M}_{CS}}$ perform the following: Measure, whether $p!$ is empty, i.e. measure whether $H_{p!}$ is in state $|\varepsilon\rangle$. If nonempty, apply MOVE to $H_{p!} \otimes H_{p<\!>!}$. Then (if $p!$ was nonempty) switch buffer $\tilde{p}$, i.e. apply $\Delta_{\text{buf}}$ to $H_{\tilde{p}}$.

6. For each $p \in \text{Ports}_{\tilde{M}_{CS}}$, prepare $|\varepsilon\rangle\langle\varepsilon|$ in $H_p$.

7. Let $s^\text{on}$! be the first clock out-port from $\text{CPorts}_{\tilde{M}_{CS}}$ (in the ordering given by the port sequence $\text{Ports}_{\tilde{M}_{CS}}$) with $I_{s^\text{on}} \neq \varepsilon$. If there is no such port, proceed to Step 4.

Note: In all steps we have replaced $M_{CS}$ by $\tilde{M}_{CS}$, in Step 4 also $s$ by $\tilde{s}$, in Step 6 we have additionally generated $s'$ from the measurement result $\tilde{s}'$.

The reader can easily convince himself (similarly to the reasoning on the replacement of Step 4 above), that each of the replacements does not modify the behaviour of the algorithm, so

$$O_2 = O_3. \quad (7)$$

Now we replace Step 4 by the following, resulting in an algorithm $A_4$

8. If $s \in \text{Fin}_{M_{CS}}$ and $M_{CS} = X$, exit (the run is complete). If $s \in \text{Fin}_{M_{CS}}$, but $M_{CS} \neq X$, erase $H_p$ for all $p \in \text{in}(\text{Ports}_{\tilde{M}_{CS}})$, and proceed to Step 4.

This does not change the behaviour of the algorithm, since if a machine has terminated, its in-ports are not read any more, so they can safely be erased. The in-port of all other machines are either empty or will not be read any more, so they can also safely be erased. Therefore

$$O_3 = O_4. \quad (8)$$

Now we replace Step 4 by the following, resulting in an algorithm $A_5$

9. If $\tilde{s} \in \text{Fin}_{\tilde{M}_{CS}}$ and $\tilde{M}_{CS} = \tilde{X}$, exit (the run is complete). If $\tilde{s} \in \text{Fin}_{\tilde{M}_{CS}}$, but $\tilde{M}_{CS} \neq \tilde{X}$, erase $H_p$ for all $p \in \text{in}(\text{Ports}_{\tilde{M}_{CS}})$, and proceed to Step 4.

Here $\tilde{X} := \text{Comb}(\tilde{M}_1, \tilde{M}_2)$ if $\tilde{M}_1$ or $\tilde{M}_2$ is master scheduler, and $\tilde{X} := X$ otherwise.

To comparing Steps 4 and 4 we have to distinguish the following cases:

- If $M_{CS} \notin \{\tilde{M}_1, \tilde{M}_2\}$, it is $M_{CS} = \tilde{M}_{CS}$, so Steps 4 and 4 exhibit the same behaviour.
– If \( M_{CS} = \tilde{M}_1 \) and \( s \notin \text{Fin}_{\tilde{M}_1} \), it also is \( \tilde{s} \notin \text{Fin}_{\text{Comb}(M_1, M_2)} \), so Steps 4 and 4' exhibit the same behaviour.

– If \( M_{CS} = \tilde{M}_1, s \in \text{Fin}_{\tilde{M}_1} \) and \( \tilde{s} \in \text{Fin}_{\text{Comb}(M_1, M_2)} \), Steps 4 and 4' exhibit the same behaviour.

– If \( M_{CS} = \tilde{M}_1 \), \( \tilde{M}_1 \) is not master scheduler, \( s \in \text{Fin}_{\tilde{M}_1} \) and \( \tilde{s} \notin \text{Fin}_{\text{Comb}(M_1, M_2)} \), then Step 4 will jump to Step 2, while from Step 4' the algorithm \( A_5 \) will proceed through Steps 5, 6 and 7. Since \( l(s, p) = 0 \) for all in-ports of \( \tilde{M}_1 \) (since \( \tilde{M}_1 \) was canonised), in Step 5 all inputs will be erased. So in Step 7, algorithm \( A_5 \) will jump to Step 2. So in this case, Steps 4 and 4' exhibit the same behaviour.

– If \( M_{CS} = \tilde{M}_2 \), we distinguish analogous cases.

So we can conclude

\[ O_4 = O_5. \tag{9} \]

Further (using the same argument as above, where we replaced Step 4 by 4'), we can replace Step 4 by Step 4' and get algorithm \( A_6 \):

\[ 4'. \text{ If } \tilde{s} \in \text{Fin}_{\tilde{M}_{CS}} \text{ and } M_{CS} = \tilde{X}, \text{ exit (the run is complete)}. \text{ If } \tilde{s} \in \text{Fin}_{\tilde{M}_{CS}}, \text{ but } \tilde{M}_{CS} \neq \tilde{X}, \text{ proceed to Step 2'}. \]

so

\[ O_5 = O_6. \tag{10} \]

Now, we make the following change to Step 8 resulting in algorithm \( A_7 \):

\[ 8'. \text{ Apply the state-transition operator } \Delta_{M_{CS}} \text{ to } H_{M_{CS}}. \text{ If } M_{CS} = \tilde{M}_1, \text{ then additionally apply } \Delta_{\tilde{M}_1} \text{ to } H_{\tilde{M}_1}. \text{ If } M_{CS} = \tilde{M}_2, \text{ then additionally apply } \Delta_{\tilde{M}_1} \text{ to } H_{\tilde{M}_1}. \]

If \( M_{CS} \neq \tilde{M}_1 \), then either all in-ports of \( \tilde{M}_1 \) are empty or \( \tilde{M}_1 \) is in a final state. Since \( \tilde{M}_1 \) has been canonised, applying \( \Delta_{\tilde{M}_1} \) in this case behaves like the identity. The same holds for \( \tilde{M}_2 \). So the above changes have no effect, and

\[ O_6 = O_7. \tag{11} \]

Now we replace Step 8 by 8', yielding algorithm \( A_8 \):

\[ 8'. \text{ Apply the state-transition operator } \Delta_{\tilde{M}_{CS}} \text{ to } H_{\tilde{M}_{CS}}. \]

By definition of \( \Delta_{M_{CS}} \), and using \( \text{Comb}(M_1, M_2) := H_{M_1} \otimes H_{M_2} \), we have

\[ O_7 = O_8. \tag{12} \]

Now we replace Step 10 by 10', yielding algorithm \( A_9 \):

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Let \( I := (I_p)_{p \in \text{in}(C\text{Ports}_{\tilde{M}CS})} \) and \( O := (O_p)_{p \in \text{out}(C\text{Ports}_{\tilde{M}CS})} \). Add \( (\text{name}_{\tilde{M}CS}, \tilde{s}, I, \tilde{s}', O, P) \) to the trace (which initially is empty).

Let further \( f \) denote the function that element-wise applies the following mapping to the run

\[
(name, s, I, s', O, P) \mapsto \begin{cases} 
(name, s, I, s', O, P), & \text{if } name \neq \text{name}_{\text{Comb}(M_1, M_2)}, \\
(name_{M_1}, s_1, I_1, s'_1, O_1, P), & \text{if } name = \text{name}_{\text{Comb}(M_1, M_2)} \text{ and } P \cap \text{Ports}_{M_1} \neq \emptyset, \\
(name_{M_2}, s_2, I_2, s'_2, O_2, P), & \text{if } name = \text{name}_{\text{Comb}(M_1, M_2)} \text{ and } P \cap \text{Ports}_{M_2} \neq \emptyset,
\end{cases}
\]

Here \( s_i \) and \( s'_i \) denote the \( i \)-th component of \( s \) and \( s' \), resp.

It is now easy to verify that

\[ O_8 = f(O_9). \quad (13) \]

(Note that \( P \cap \text{Ports}_M \neq \emptyset \) only if \( M_{\text{CS}} = M \).

Since \( \mathcal{H}_{\text{Comb}(M_1, M_2)} = \mathcal{H}_{M_1} \otimes \mathcal{H}_{M_2} \), we can replace \( \mathcal{H}_{\tilde{C}} \) by the isomorphic space \( \mathcal{H}_{\tilde{D}} \), and rewrite Step II as follows, giving algorithm \( A_{10} \):

\textbf{1. Prepare the state}

\[
\bigotimes_{M \in \tilde{D}} \rho_{M}^{\text{ini},k} \in \mathcal{P}(\mathcal{H}_{\tilde{D}})
\]

where

\[
\rho_{M}^{\text{ini},k} := |\varepsilon\rangle\langle\varepsilon| \otimes |1^k\rangle\langle1^k| \\
\otimes |\varepsilon, \ldots, \varepsilon\rangle\langle\varepsilon, \ldots, \varepsilon| \\
\otimes |\varepsilon, \ldots, \varepsilon\rangle\langle\varepsilon, \ldots, \varepsilon| \in \mathcal{P}(\mathcal{H}_M).
\]

Then obviously

\[ O_9 = O_{10}. \quad (14) \]

From \( A_{10} \) we can now derive \( A_{11} \) by removing all instructions that set \( \tilde{M}_{\text{CS}}, \tilde{s} \) or \( \tilde{s}' \). \( A_{11} \) has the following description:

\textbf{1. Prepare the state}

\[
\bigotimes_{M \in \tilde{D}} \rho_{M}^{\text{ini},k} \in \mathcal{P}(\mathcal{H}_{\tilde{D}})
\]

where

\[
\rho_{M}^{\text{ini},k} := |\varepsilon\rangle\langle\varepsilon| \otimes |1^k\rangle\langle1^k| \\
\otimes |\varepsilon, \ldots, \varepsilon\rangle\langle\varepsilon, \ldots, \varepsilon| \\
\otimes |\varepsilon, \ldots, \varepsilon\rangle\langle\varepsilon, \ldots, \varepsilon| \in \mathcal{P}(\mathcal{H}_M).
\]

\textbf{2. Prepare }|1\rangle\langle1| \text{ in } \mathcal{H}_{\text{clk}^0}. \text{ Set } \tilde{M}_{\text{CS}} := \tilde{X}.
5. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on $C^{\text{States}}_{\tilde{M}_{CS}}$. Let $\tilde{s}$ denote the outcome.

6'. If $\tilde{s} \in \text{Fin}_{\tilde{M}_{CS}}$ and $M_{CS} = \tilde{X}$, exit (the run is complete). If $\tilde{s} \in \text{Fin}_{M_{CS}}$, but $M_{CS} \neq \tilde{X}$, proceed to Step 4'.

5'. For each port $p \in \text{in}(\text{Ports}_{\tilde{M}_{CS}})$ s.t. $l_{\tilde{M}_{CS}}(\tilde{s}, p) = 0$, prepare $|\varepsilon\rangle\langle \varepsilon|$ in $H_p$.

6'. For each $p \in \text{in}(C\text{Ports}_{\tilde{M}_{CS}})$, perform a complete measurement on $H_p$. Let the outcome be $I_p$.

4. For each port $p \in \text{in}(\text{Ports}_{M_{CS}})$, measure whether $H_p$ is in state $|\varepsilon\rangle$ (whether it is empty). If all ports were empty, proceed to Step 2'. Otherwise let $P$ be the set of the ports that were nonempty.

5'. Apply the state-transition operator $\Delta_{\tilde{M}_{CS}}$ to $H_{\text{in}}$.

6'. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on $C^{\text{States}}_{\tilde{M}_{CS}}$. Let $\tilde{s}'$ denote the outcome. For each $p \in \text{out}(C\text{Ports}_{\tilde{M}_{CS}})$, perform a complete measurement on $H_p$. Let the outcome be $O_p$.

7'. For each port $p \in \text{in}(\text{Ports}_{\tilde{M}_{CS}})$, measure whether $H_p$ is in state $|\varepsilon\rangle$ (whether it is empty). If all ports were empty, proceed to Step 2'. Otherwise let $P$ be the set of the ports that were nonempty.

8'. Apply the state-transition operator $\Delta_{M_{CS}}$ to $H_{\tilde{M}_{CS}}$.

9'. Perform a complete von-Neumann measurement in the computational basis (called a complete measurement from now on) on $C^{\text{States}}_{\tilde{M}_{CS}}$. Let $s''$ denote the outcome. For each $p \in \text{out}(C\text{Ports}_{\tilde{M}_{CS}})$, perform a complete measurement on $H_p$. Let the outcome be $O_p$.

10'. Let $I := (I_p)_{p \in \text{in}(C\text{Ports}_{\tilde{M}_{CS}})}$ and $O := (O_p)_{p \in \text{out}(C\text{Ports}_{\tilde{M}_{CS}})}$. Add (name $M_{CS}$, $\tilde{s}$, $I$, $s''$, $O$, $P$) to the trace (which initially is empty).

11'. For each simple out-port $p! \in \text{Ports}_{\tilde{M}_{CS}}$, perform the following: Measure, whether $p!$ is empty, i.e. measure whether $H_p!$ is in state $|\varepsilon\rangle$. If nonempty, apply MOVE to $H_p! \otimes H_{p!?}$. Then (if $p!$ was nonempty) switch buffer $\tilde{p}$, i.e. apply $\Delta_{\text{buf}}$ to $H_{\tilde{p}}$.

12'. For each $p \in \text{Ports}_{\tilde{M}_{CS}}$, prepare $|\varepsilon\rangle\langle \varepsilon|$ in $H_p$.

13'. Let $s \triangleright !$ be the first clock out-port from $C\text{Ports}_{\tilde{M}_{CS}}$ (in the ordering given by the port sequence $\text{Ports}_{\tilde{M}_{CS}}$) with $I_{s \triangleright !} \neq \varepsilon$. If there is no such port, proceed to Step 2'.

14'. Apply MOVE to $H_{s \triangleright !} \otimes H_s$. Let $M_{CS}$ be the unique machine in $\hat{D}$ with $s \in \text{Ports}_{M_{CS}}$. Proceed to Step 2.

It is

$$O_{10} = O_{11}.$$  \hspace{1cm} (15)

This algorithm $A_{11}$ is in fact the algorithm for the run of $\hat{D}$, so

$$O_{11} = \text{run}_{\hat{D}}.$$  \hspace{1cm} (16)

By equations (5) (16) we have

$$\text{run}_{\tilde{C}} = f(\text{run}_{\hat{D}}).$$

From this it follows for all $M \in \tilde{C}$,

$$\text{view}_{\tilde{C}}(M) = \text{view}_{\tilde{C}}(M),$$

with $\text{view}_{\tilde{C}}(M_i)$ ($i = 1, 2$) defined as in Lemma 2.

Using (14), the statement of the lemma follows. \hspace{1cm} $\square$
References

[Bac02] Michael Backes. Cryptographically Sound Analysis of Security Protocols. PhD thesis, Universität des Saarlandes, 2002. Online available at http://www.zurich.ibm.com/~mbc/papers/PhDthesis.ps.gz.

[BB84] Charles H. Bennett and Gilles Brassard. Quantum cryptography: Public-key distribution and coin tossing. In Proceedings of IEEE International Conference on Computers, Systems and Signal Processing 1984, pages 175–179. IEEE Computer Society, 1984.

[BCJP02] Michael Backes, II Christian Jacobi, and Birgit Pfitzmann. Deriving cryptographically sound implementations using composition and formally verified bisimulation. In Proceedings of the International Symposium of Formal Methods Europe on Formal Methods - Getting IT Right, pages 310–329. Springer-Verlag, 2002.

[Bea92] Donald Beaver. Foundations of secure interactive computing. In Proceedings of the 11th Annual International Cryptology Conference on Advances in Cryptology, pages 377–391. Springer-Verlag, 1992.

[BHMQU04] Michael Backes, Dennis Hofheinz, Jörn Müller-Quade, and Dominique Unruh. Fair and reliable networks in the context of simulatable security. Unpublished, 2004.

[BOHL^+04] M. Ben-Or, Michael Horodecki, D. W. Leung, D. Mayers, and J. Oppenheim. The universal composable security of quantum key distribution, September 2004. Online available at http://xxx.lanl.gov/abs/quant-ph/0409078.

[BOM02] Michael Ben-Or and Dominic Mayers. Quantum universal composability. Mathematical Sciences Research Institute, November 2002. Presentation at “Quantum Information and Cryptography” Workshop, slides online available at http://www.msri.org/publications/ln/msri/2002/quantumcrypto/mayers/1/meta/aux/mayers.pdf.

[BOM04] M. Ben-Or and D. Mayers. General security definition and composable security for quantum & classical protocols, September 2004. Online available at http://xxx.lanl.gov/abs/quant-ph/0409062.

[BPW04a] Michael Backes, Birgit Pfitzmann, and Michael Waidner. A general composition theorem for secure reactive systems. In Moni Naor, editor, TCC, volume 2951 of Lecture Notes in Computer Science, pages 336–354. Springer, 2004. Electronically available at http://www.zurich.ibm.com/security/publications/2004/BaPfWa2004MoreGeneralComposition.pdf.

[BPW04b] Michael Backes, Birgit Pfitzmann, and Michael Waidner. Secure asynchronous reactive systems. IACR ePrint Archive, March 2004. Online available at http://eprint.iacr.org/2004/082.ps.

[Can00] Ran Canetti. Security and composition of multi-party cryptographic protocols. Journal of Cryptology, 3(1):143–202, 2000. Full version online available at http://eprint.iacr.org/1998/018.ps.

[Can01] Ran Canetti. Universally composable security: A new paradigm for cryptographic protocols. In 42th Annual Symposium on Foundations of Computer Science, Proceedings of FOCS 2001, pages 136–145. IEEE Computer Society, 2001. Full version online available at http://eprint.iacr.org/2000/067.ps.

[DJ92] D. Deutsch and R. Jozsa. Rapid solution of problems by quantum computation. Proc. Roy. Soc. Lond. A, 439:553–558, October 1992.

[GK96] Oded Goldreich and Hugo Krawczyk. On the composition of zero-knowledge proof systems. SIAM J. Comput., 25(1):169–192, 1996.

[GMR85] S Goldwasser, S Micali, and C Rackoff. The knowledge complexity of interactive proof-systems. In Proceedings of the seventeenth annual ACM symposium on Theory of computing, pages 291–304. ACM Press, 1985.

[HMQU04] Dennis Hofheinz, Jörn Müller-Quade, and Dominique Unruh. Polynomial runtime in simulatability definitions, 2004. Submitted to TCC 2005.

[MQ02] Jörn Müller-Quade. Personal communication with the author, 2002.

[MQ03] Jörn Müller-Quade. Personal communication with the author, 2003.
Jörn Müller-Quade and Rainer Steinwandt. On the problem of authentication in a quantum protocol to detect traffic analysis. *Quantum Information and Computation*, 3(1):48–54, 2003.

Silvio Micali and Phillip Rogaway. Secure computation (abstract). In Joan Feigenbaum, editor, *Advances in Cryptology, Proceedings of CRYPTO ’91*, volume 576 of *Lecture Notes in Computer Science*, pages 392–404. Springer-Verlag, 1992.

M. Nielsen and I. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, Cambridge, 2000.

Birgit Pfitzmann, Matthias Schunter, and Michael Waidner. Secure reactive systems. Technical Report RZ 3206, IBM Zurich Research Laboratory, 2000. Online available at [http://www.semper.org/sirene/publ/PfSW00ReactSimulIBM.ps.gz](http://www.semper.org/sirene/publ/PfSW00ReactSimulIBM.ps.gz).

Birgit Pfitzmann and Michael Waidner. A model for asynchronous reactive systems and its application to secure message transmission. In *IEEE Symposium on Security and Privacy, Proceedings of SSP 2001*, pages 184–200. IEEE Computer Society, 2001. Full version online available at [http://eprint.iacr.org/2000/066.ps](http://eprint.iacr.org/2000/066.ps).

Renato Renner and Robert König. Universally composable privacy amplification against quantum adversaries, March 2004. Online available at [http://xxx.lanl.gov/abs/quant-ph/0403133](http://xxx.lanl.gov/abs/quant-ph/0403133).

Dominik Raub, Rainer Steinwandt, and Joern Mueller-Quade. On the security and composability of the one time pad. Cryptology ePrint Archive, Report 2004/113, 2004. Online available at [http://eprint.iacr.org/2004/113/](http://eprint.iacr.org/2004/113/).

Rainer Steinwandt, Dominik Janzing, and Thomas Beth. On using quantum protocols to detect traffic analysis. *Quantum Information and Computation*, 1(3):62–69, 2001.

Adam D. Smith. Quantum multi-party computation. S.m. thesis, Department of Electrical Engineering and Computer Science, August 2001. [Http://theory.lcs.mit.edu/~asmith/PS/masters-main.ps](http://theory.lcs.mit.edu/~asmith/PS/masters-main.ps).

Dominique Unruh. Formal security in quantum cryptology. Student research project, Institut für Algorithmen und Kognitive Systeme, Universität Karlsruhe, December 2002. Online available at [http://www.unruh.de/DmiQ/publications/quantum_security.ps.gz](http://www.unruh.de/DmiQ/publications/quantum_security.ps.gz).

Jeroen van de Graaf. *Towards a formal definition of security for quantum protocols*. PhD thesis, Département d’informatique et de r.o., Université de Montréal, 1998. Online available at [http://www.cs.mcgill.ca/~crepeau/PS/theses-jeroen.ps](http://www.cs.mcgill.ca/~crepeau/PS/theses-jeroen.ps).
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