2. Damage Addition Rule for Fatigue Testing

In the fatigue test the strain oscillation is carried out, at defined amplitude, frequency and temperature, and the number of strain cycles \( n_{\text{crit}} \) to rupture is determined. The procedure is that used in mechanical engineering in the prediction of service life.\(^3\) The cycle number \( n_{\text{crit}} \) enters into the damage sum. The damage addition rule comprises two kinds of sum, one reflecting the damage caused by the accumulated average strain, denoted in the following with \( L_{\varepsilon} \), and the other considering the damage made by the strain oscillations, denoted with \( L_{n} \).

Hence, the resulting damage addition rule is

\[
\sum_{i=1}^{n_{\text{crit}}} \frac{\Delta \varepsilon(i)}{\varepsilon_{\text{crit}}(i)} + \sum_{i=1}^{n_{\text{crit}}} \frac{\Delta n(i)}{n_{\text{crit}}(i)} = L_{\varepsilon} + L_{n} = L \quad \ldots \ldots \ldots (1)
\]

Here, \( \Delta \varepsilon(i) \) is the number of cycles in the time interval \( \Delta t \), and \( n_{\text{crit}}(i) \) the number of cycles till rupture existing under the conditions (temperature etc) of \( t(i) \). The critical cycle number \( n_{\text{crit}} \) depends on temperature, strain amplitude and oscillation frequency. The strain increment \( \Delta \varepsilon(i) \) is related to the strain rate \( \dot{\varepsilon}(i) \) by \( \dot{\varepsilon}(i) \Delta t \) where \( \dot{\varepsilon}(i) \) is the strain rate during the time interval \( \Delta t \).

Equation (1) is valid, of course, also for isothermal conditions with constant average strain rate and constant strain amplitude and cycle frequency, respectively, when \( \varepsilon_{\text{crit}} \) and \( n_{\text{crit}} \) are constant. Hence, for such conditions

\[
\frac{1}{\varepsilon_{\text{crit}}} \sum_{i=1}^{n_{\text{crit}}} \Delta \varepsilon(i) + \frac{1}{n_{\text{crit}}} \sum_{i=1}^{n_{\text{crit}}} \Delta n(i) = L_{\varepsilon} + L_{n} = L \quad \ldots \ldots \ldots (2)
\]

The sum in the first term \( L_{\varepsilon} \) is the accumulated strain and the sum in the second term \( L_{n} \) is the accumulated cycle number. Equation (2) must fulfill the two limiting cases, namely that for the average strain rate being zero, and that for no cycling. In the first case, rupture occurs if the accumulated cycle number is \( n_{\text{crit}} \) and the first term is zero, that is \( L=L_{n} \) is 1. In the second case rupture occurs if the accumulated strain is \( \varepsilon_{\text{crit}} \) and the second term is zero, that is \( L=L_{\varepsilon} \) is 1. It is assumed that also in the general case of cycling with non-zero strain accumulation, rupture occurs at \( L=1 \), that is that Eq. (2) is valid also for conditions between the limiting cases.

Hence, the cracking criterion based on Eq. (1) is

\[
\sum_{i=1}^{n_{\text{crit}}} \frac{\Delta \varepsilon(i)}{\varepsilon_{\text{crit}}(i)} + \sum_{i=1}^{n_{\text{crit}}} \frac{\Delta n(i)}{n_{\text{crit}}(i)} = L_{\varepsilon} + L_{n} \geq 1. \quad \ldots \ldots \ldots (3)
\]

For the use of Eq. (3) for prediction of cracking in continuous casting the dependence of the critical cycle number on the relevant parameters must be known. Unfortunately, investigations of fatigue strength (alternating strain) of carbon steels at continuous casting temperatures have not been carried out systematically. But a few data\(^3\)–\(^5\) are available and can be used for explaining the procedure. Figure 1 shows the number of strain cycles till rupture as a function of temperature for an aluminum deoxidized carbon steel. The data were determined in a tensile machine. Strain amplitude \( \varepsilon_{\text{crit}} \) and cycle time \( t_{\text{cycle}} \) (cycle frequency \( n = 1 / t_{\text{cycle}} \)) were kept constant.

**Fig. 1.** Results of fatigue test (alternating strain in tensile test). The critical cycle number \( n_{\text{crit}} \) is the cycle number at which cracking occurs. Hertel, Litterscheidt, Lotter and Pircher.\(^3\)
at constant values. There is a minimum of cycle number at about 870°C which is caused by the brittleness due to nitride precipitates and is the equivalent to the minimum on the $R_{A}(T)$ curve arising in the usual (non-alternating) tensile experiment.

The main quantity of influence on the cycle number $n_{\text{crit}}$, in addition to temperature, frequency and steel composition, is the strain amplitude $\varepsilon_{a}$. It is well known from the investigations of fatigue strength that $\varepsilon_{a}$ and $n_{\text{crit}}$ can be correlated over wide ranges with each other by a linear logarithmic “law” according to

$$\log \varepsilon_{a} = A + B \log n_{\text{crit}}$$

and this equation can be utilized for describing the relationship between $n_{\text{crit}}$ on $\varepsilon_{a}$. The coefficients $A$ and $B$ depend on temperature and oscillation frequency. In order to determine them two pairs of $\varepsilon_{a}$ and $n_{\text{crit}}$ are required at each chosen temperature and oscillation frequency.

3. Example for Construction of Damage Sum for Crack Prediction in Continuous Casting

The data given in Fig. 1 are used for demonstrating the computation of $L_{n}$. One data pair of $\varepsilon_{a}$ and $n_{\text{crit}}$ is given (at each temperature), for the steel investigated, by the curve in Fig. 1. Since another curve for another $\varepsilon_{a}$ is not available, the other pair can be made with the strain for $n_{\text{crit}}=1$. How large must $\varepsilon_{a}$ be so that the specimen ruptures at one cycle? In a heuristic approximation this strain is taken to be that in the tensile test at constant strain rate $\dot{\varepsilon}$ when the time to rupture $t_{\text{rupture}}$ (in the tensile test) is equal to the cycle time $t_{\text{cycle}}=1/\dot{\varepsilon}$ (in the fatigue test). Hence, it is assumed that $\varepsilon_{a}(n_{\text{crit}}=1, \text{fatigue test})=\varepsilon_{\text{rupture}}(t_{\text{cycle}}=t_{\text{cycle}}, \text{tensile test})$. For the data in Fig. 1 the cycle time is 15 s, viz. $t_{\text{cycle}}=t_{\text{rupture}}=\varepsilon_{\text{crit}}/\varepsilon=15$. It can be shown that, on the basis of RA functions according Fig. 4 and Eqs. (15) to (19) of the preceding paper, $\varepsilon_{a}$ for $n_{\text{crit}}=1$ at $t_{\text{cycle}}=15$ s is between 0.20 and 0.40, depending on temperature, in the temperature range between $T_{\text{DB}}$ (temperature of transition during cooling from ductile to brittle state) and $T_{f}$ (temperature of minimal $n_{\text{crit}}$ in Fig. 1) the two value pairs for the determination of $A$ and $B$ in Eq. (4) are available. For instance, at 870°C (temperature of minimal $n_{\text{crit}}$ in Fig. 1) the two value pairs are $\varepsilon_{a}=0.01$ for $n_{\text{crit}}=8$ and $\varepsilon_{a}=0.30$ for $n_{\text{crit}}=1$ which yields Eq. (4) as $\log \varepsilon_{a}(870°C) = 0.523 - 1.636 \log n_{\text{crit}}$.

Since log1 is zero, it follows immediately that

$$A = \log 0.30 = -0.523$$

The experimental data in Fig. 1 can be represented by a parabola of the form

$$n_{\text{crit}}(\varepsilon_{a}=0.01, t_{\text{cycle}}=15\text{ s}) = 8 + 0.00344(T-870)^2$$

Inserting this expression in Eq. (4) and using $A=-0.523$ and $\varepsilon_{a}=0.01$ yields the function for $B$ as

$$B(T) = \frac{-1.477}{\log[8 + 0.00344(T-870)^2]}$$

Thus, $n_{\text{crit}}$ can be computed as a function of $\varepsilon_{a}$ and $T$ using Eqs. (4), (5) and (7). The resulting dependence between cycle number $n_{\text{crit}}$, strain amplitude $\varepsilon_{a}$ and temperature $T$ is illustrated graphically in Fig. 2.

These functions are deduced for the cycle time of 15 s used in the laboratory experiments. But they may be applied also for somewhat higher or lower cycle times because the dependence of $n_{\text{crit}}$ on frequency is comparatively small.

In the computational example the strain amplitude (roll contact) at the center of the wide face of the slab is about 0.0010, Fig. 3, and with this strain value 32.7 cycles would lead to cracking if the temperature were constant and at 870°C. This cycle number is not so high. It is to be inferred that the inclusion of a strain cycling term in the damage accumulation sum (or integral) seems to be rather important.

The damage sum due to strain cycling $L_{n}$ was evaluated, on the basis of the computed strain trace and a smoothed surface temperature curve, for the whole length of the slab cast at 1.3 m min$^{-1}$. The result is shown in Fig. 4. It is evident that $L_{n}$ increases to almost 1 at the strand length of 23 m. If factors for the effects of increased grain size, segregation and notches are introduced in the computation of $n_{\text{crit}}$, see ref. 1), the value of $L_{n}$ would still become larger and
exceed 1. So, for this example it is predicted that the occurrence of transverse surface cracking, due to strain oscillations, is probable.

4. Conclusions

In the Eqs. (1) to (3) the strain cycling is taken to have one single amplitude. In reality there are two kinds of strain (temperature) oscillations, one due to the roll contacts and the other due to the impinging water sprays, see Fig. 3. So, the treatment can be refined further by using two damage accumulation sums for the oscillations according to expression

\[
L_n = \sum_{i=1}^{n} \frac{\Delta n(i)}{\tau_{i, n_{crit}(i)}}
\]

for strain oscillations along center of width of a slab. The considered strain oscillations are those belonging to the roll contacts.

Normally, however, the strain peaks due to roll contact are much higher than those due to the spray so that \(n_{crit}(roll)<<n_{crit}(spray)\). Hence, the term for the sprays is small in comparison to that for rolls and might be neglected.

It should be realized that there is a difference in the strain oscillations in the fatigue test and at the slab surface in continuous casting. In the fatigue test the strain oscillation occurs at constant temperature whereas in continuous casting strain and temperature oscillations are coupled. In the use of such fatigue test data for crack prediction in continuous casting the effect of temperature change on accumulated strain is taken into account by \(L_\varepsilon\) in Eq. (3). Alternatively, a fatigue test could be made in which the cycling of strain is performed by alternating the sample temperature, at constant average strain rate and constant average temperature. That is, average temperature and average strain rate are set constant in the tensile apparatus, controlled temperature fluctuations are superposed and the amplitude of strain oscillation (stress oscillation) is measured. Such kind of test would be ideal for crack prediction in continuous casting.

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