Model of stressed-strained state of multi-layered reinforced concrete structure with the use of composite reinforcement

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Abstract. The development in calculating reinforced concrete structure method covers a wide range of issues, including the expansion of fields such as application of new innovative materials which consist of concrete and reinforcement. In order to apply modern construction of multi-layer structure using concrete with different mechanical and physical characteristics, it is compulsory to conduct researches on the stressed-strained state of these structures under various types of load, which is determined not only by the stressed-strained state of the multi-layer reinforced concrete structure, but also by the properties of the reinforcement.

The object of the study is multi-layer reinforced concrete structures of concrete with various physical and mechanical characteristics of materials - concrete and reinforcement under the influence of various types of loading. The subject of this study is the fluctuation in the stress-strain state under loadings of multi-layer reinforced concrete structures with the use of composite reinforcement. Studies related to the use of composite reinforcement in multi-layer structures have not been carried out, which significantly limits the scope of its application.

The methods of investigation are based on numerical modeling of stresses and deformations of multi-layer reinforced concrete elements under the action of different combinations of loads. The use of modern software systems allows a number of studies, combining different combinations of loads and the variability of the strength and deformation characteristics of materials in structural concrete or in low strength concrete for the middle layer and various types of metal or composite reinforcement. In order to precisely calculate and measure these materials, the physical and mechanical properties of materials are used, which are determined on the basis of normative data or experimental studies. Numerical simulation makes it possible to compare the results obtained and construct theoretical dependencies in a wide range of specified parameters for constructing cross sections of multi-layer reinforced concrete elements. The range of this research is limited in the short-term load effect of reinforced concrete.

The obtained scientific results allow determine rational parameters for modeling various design solutions of multi-layer reinforced concrete structures. The practical importance of the research will expand the areas of rational use of multi-layer reinforced concrete structures and ensure safe operation throughout the life cycle of buildings and structures using multi-layer structures.

Keywords: multi-layer structure, modern construction, stressed-strained, strength, multi-layer reinforced, concrete elements, composite reinforcement.
1. Introduction
The development of the theory and calculation of reinforced concrete construction, combining with the development of new technologies of materials from concrete, has allowed inventions of new types of construction corresponding to new materials from concrete.

Concrete is an inhomogeneous material, so when loading reinforced concrete structures, the distribution of stresses and deformations inside is a rather complex process, especially in multi-layer slabs. Over-lappings (or coatings), at present, use low thermal conductivity concrete as the middle layer, so the study and calculation of such reinforced concrete slabs is an urgent task.

The results of previous studies [1, 4] show that, in multi-layer reinforced concrete structures, cracks do not appear until the structure works elastically and the stresses in the stretched zone do not reach the ultimate tensile strength and the ratio of stresses and deformations obeys Hooke's law. Also in the above-mentioned publications of the author, an analysis was made of the interconnection between the modul of elasticity of the middle and outer layers, which affected the stresses and deformations of a multilayer reinforced concrete slab.

To determine the VAT of a multi-layer reinforced concrete structure, it is advised to consider not only the physical and mechanical characteristics but also the operation of the reinforcement itself in the structure. The load-bearing capacity of a multi-layer reinforced concrete structure is greatly improved when reinforced with composite materials.

Composite materials have appeared for a long time, and the science of composite materials has developed strongly since the 1900s until now. Composite materials are being introduced in various spheres: construction, textile industry, machine building, national defense. In the construction sector, the combination of composite materials with concrete reinforces the load-bearing capacity and rigidity of the reinforced concrete structure. But the VAT of a multi-layer reinforced-concrete structure when using composite materials is a complex state.

There are a number of studies on the repair and reinforcement of reinforced concrete structures with composite materials made of fiberglass, carbon plastic. These studies focus on the load-bearing capacity of the reinforced concrete structure [17, 18, 19, 20, 21, 22], rather than specific studies that examine the use of composite materials to reinforce the multi-layer reinforced concrete structure.

This article describes the use of composite reinforcement from fiberglass to reinforce a multi-layer reinforced concrete structure.

2. Research materials
In three-layer slabs, for external layers, concrete of class B12.5-B30 is usually used. For the middle layer, lightweight concrete with low thermal conductivity is used, with a compression resistance of the order of 1.5 MPa [1]. The thickness of the outer layers is from 40 to 60 mm, to ensure minimum deformation and protection of the reinforcement [1].

Multilayer reinforced concrete test beams with a size of 160x3000x250 mm with the parameters shown in table 1 [1,7,10, 12, 13].

| Structure of the layers | External (1) | Interior | External (2) |
|-------------------------|-------------|----------|--------------|
| Thickness of layer, m   | 0.04        | 0.170    | 0.04         |
| Critical crack stress of concrete $R_{cb}$, MPa | 21.5        | 1.54     | 21.5         |
| Crush stress of concrete $R_{ct}$, MPa | 1.8         | 0.36     | 1.8          |
| Initial modulus of elasticity, MPa | 12100      | 1310    | 12100        |
| The Poisson's ratio $v = 0.00189 | [R_{ct}]+ 0.12$ | 0.16 | 0.122 | 0.16 |
| The sheer modulus $G=E/(2(1 + v))$, MPa | 5216        | 397      | 5216         |
The armature is obtained with a diameter of 8 mm (A-400), $\sigma_y = 475.2$ MPa, $\sigma_u = 660.5$ MPa, Modulus of elasticity of the armature $E_s = 206,000$ MPa, percentage is 0.29%.

**Figure 1.** Stress – strain curve for concrete in compression

**Figure 2.** Stress – strain curve for concrete in tension

**Figure 3.** Stress-strain figure for steel in tension

**Figure 4.** Stress – strain figure for composite reinforcement in tension

**Figure 5.** Periodic fiber glass ribbed bars (composite reinforcement)

Composite reinforcement has a diameter of 6 mm, a periodic section. The characteristics of the composite reinforcement are shown in table 2 and in figure 5 [17].

**Table 2.** Parameters illustrating characteristics of the reinforcement using periodic fiber glass ribbed bars

| Ultimate elongation ($\varepsilon_u$) | Ultimate tensile strength ($\sigma_u$) | Modulus of elasticity ($E$) | Weight of meter per linear (gram/m) | Concentration coefficient of thermal conductivity |
|-----------------------------------------|--------------------------------------|-----------------------------|-------------------------------------|-----------------------------------------------|
| 2.2%                                     | 900 MPa                               | 30 GPa                      | 38                                  | 0.5 W/mK                                       |

In the test Epoxy 380 epoxy resin is used to fix the composite reinforcement to the multi-layer reinforced concrete beams in the stretched zone of the beam itself. Some characteristics of epoxy Epoxy-380: Compressive strength - 83MPa; The tensile strength is 29.5 MPa; The modulus of elasticity is 4.9 GPa.
3. Methods of research

The use of modern software complexes allows carrying out numerous variational studies combining different combinations of loads and variability of strength and deformation characteristics of materials - structural concrete, low strength concrete for the middle layer and various types of reinforcement - metal and composition. For calculation, the physical and mechanical properties of materials are used, which are determined on the basis of normative data or experimental studies.

Numerical simulation makes it possible to compare the results obtained and construct theoretical dependencies in a wide range of specified parameters for constructing cross sections of multilayer reinforced concrete elements.

In this type of construction, made of heterogeneous materials, under stress, the distribution of stresses and deformations is a rather complex process, so the relationship between the moduli of elasticity of the middle and outer layers exert a strong influence on the distribution of stresses and deformations.

In SP 63.13330.2012 and in the publication of the authors [1, 4, 11, 20], the design scheme for the distribution of forces and deformations in a three-layer reinforced concrete section with a monolithic bond of layers is shown in figure 6.

![Figure 6. Calculation schemes for the distribution of forces and deformations in three-layer sections](image)

Stresses in concrete at the level of the extreme compressed fiber at $\lambda_b = 1$ (inelastic deformations of concrete are not taken into account):

$$\sigma_{b,ext}^{1-1} = \varepsilon_{b,ext}^{1-1} E_{b,ext} = \varepsilon_b \frac{x}{h-x} E_{b,ext} = \frac{2R_{bt,ext}}{E_{b,ext}} \frac{x}{h-x} E_{b,ext} = 2R_{bt,ext} \frac{x}{h-x}$$

(1)

Stresses in concrete at the contact level of the upper layer:

$$\sigma_{b,ext}^{2-2} = \varepsilon_{b,ext}^{2-2} E_{b,ext} = \varepsilon_b \frac{x-h_j}{h-x} E_{b,ext} = \frac{2R_{bt,ext}}{E_{b,ext}} \frac{x-h_j}{h-x} E_{b,ext} = 2R_{bt,ext} \frac{x-h_j}{h-x}$$

(2)

Stresses in concrete at the contact level of the lower layer:

$$\sigma_{b,int}^{2-2} = \varepsilon_{b,int}^{2-2} E_{b,int} = \varepsilon_b \frac{x-h_j}{h-x} E_{b,int} = \frac{2R_{bt,int}}{E_{b,int}} \frac{x-h_j}{h-x} E_{b,int}$$

(3)

Stresses in the stretched and compressed armature (taking into account the stresses from shrinkage of concrete), respectively:

$$\sigma_s = \varepsilon_s E_s = \frac{2R_{bt,ext}}{E_{b,ext}} \frac{h-x-a}{h-x} E_s = 2\alpha R_{bt,ext} \frac{h-x-a}{h-x}$$

(4)
\[ \sigma_x = \varepsilon_x E = \frac{2R_{bt,ext}}{E_{b,ext}} \frac{x-a}{h-x} E_x = 2\alpha R_{bt,ext} \frac{x-a}{h-x} \]

(5)

Hence, the forces perceived by concrete and reinforcement immediately prior to the formation of cracks are equal:

\[ N_{b,ext} + N_{b,int} = R_{bt,ext} \left( \frac{2x-h}{h-x} h_x b + \frac{E_{bt,2}}{E_{bt,1}} \frac{(x-h)^2}{h-x} b \right) \]

(6)

\[ N_{bt,ext} + N_{bt,int} = R_{bt,ext} h_x b + \frac{1}{2} \alpha \frac{R_{bt,ext}}{h-x} (h-x-h_x) b \]

(7)

\[ N_x = \sigma_x A_x = (2R_{bt,ext} \frac{x-a}{h-x}) A_x \]

(8)

\[ N_x = \sigma_x A_x = (2R_{bt,ext} \frac{h-x-a}{h-x}) A_x \]

(9)

The moment of crack formation according to scheme 1.a is determined by the equation:

\[ M_{ext} = R_{bt,1} [(h_x b + 2\alpha \frac{h-x-a}{h-x} A_x)(h-x-h_x^2 + y_x') + R_{bt,1} [(h-x-h_x) b \frac{h-x-h_x}{2} + y_x' \]] \]

(10)

At this height of the compressed zone, x is determined from equation

\[ [R_{bt,1} b \left( \frac{R_{bt,2}}{R_{bt,1}} - \frac{E_{bt,2}}{E_{bt,1}} \right)x^2 - [R_{bt,1} b h_x (3+2\alpha (A_x + A_x')) + R_{bt,2} b (2h-h_x)]x + \]

\[ + R_{bt,1} h_x^2 b \left( \frac{h}{h_x} + 1 \right) - E_{bt,2} \frac{E_{bt,1}}{E_{bt,1}} + 2\alpha (A_x + A_x')(h-a)] + R_{bt,2} b h_x (h-h_x) = 0 \]

(11)

Figure 7 [15,18] illustrates scheme and stress diagram in section which is normal to the longitudinal axis of a bent ferro-concrete element with external reinforcement of composite materials, when calculated by strength when strengthening the stretched beam zone with composite reinforcement following SP164.1325800.2014.

![Figure 7](image_url)

Figure 7. Schematic of stresses and stress diagrams in a section normal to the longitudinal axis of a bent ferro-concrete element with external reinforcement of composite materials, when calculating its strength

Calculation of the strength of cross-sections of bent elements reinforced by external reinforcement of composite materials should be carried out from the condition [15, 17, 18, 19].
\[ M \leq M_{\text{ult}} \] (12)

With \( M \) is the bending moment from the external load;
\( M_{\text{ult}} \)-limit bending moment, which can be perceived by the reinforced section of the element.

Deflections from bending in the middle of span of beam samples before the appearance of cracks are calculated by the formula (SP 63.13330.2012).

\[ f_m = \frac{M}{\varphi_h E J_{\text{red}} \rho_m l^2} \] (13)

4. Results and discussion

In order to compare the results of the simulation with the real picture of the deformations of the tested three-layer reinforced concrete beams of the beam type, the parameters of the beam layers were taken for the first model based on the results of previous experimental studies. Multilayer concrete slabs have a size of 160x3000x250 mm with the parameters shown in table 1 and in figure 8.

The armature is obtained with a diameter of 8 mm (A-400), \( \sigma_y = 475.2 \) MPa, \( \sigma_u = 660.5 \) MPa, Modulus of elasticity of the armature \( E_s = 206,000 \) MPa, percentage is 0.29%.

In test number 1, the steel reinforcement was adopted with a diameter of 8 mm (A-400), the percentage of reinforcement made 0.29%.

![Figure 8. Parameters of the experimental model](image)

. The stress diagram and deformation in the middle of the span of the plate during the formation of cracks is shown in figure 9 and in table 3.

![Figure 9. The stress distribution of the plate during cracking, MPa](image)
Table 3. Comparison of the results of calculating the moment of formation of cracks (M_{crc}) in the program Ansys and Calculation according to the scheme in Figure 6

| In the program Ansys | The calculation according to the scheme in Fig. 6 |
|----------------------|-----------------------------------------------|
| 2.15/96.2            | 2.29/100                                      |

From the results of the model it is shown that when the value of the moment M = 2.15 kNm, cracks begin to form. Cracks begin initially in the stretched zone of the central part of the beam when the tensile strength of concrete is exceeded. The difference between the moments of crack formation, obtained from the computer program ANSYS and the theory of calculation in figure 6, is not significant (4%). The results show that the moment in the formation of cracks in the ANSYS model is close to the calculated SP 63.133.30.2012 [15] for the reduced cross-sections according to the scheme in figure 11 and test results.

To examine the effect of composite reinforcement on the operation of a multi-layer reinforced concrete beam, tests of number 2 and number 3 are conducted. For test number 2, the initial data is entered as in test number 1, the diameter of the composite reinforcement is taken to be 6mm with the parameters shown in table 2 (Parameters illustrating characteristics of the reinforcement using periodic fiber glass ribbed bars). Ultimate tensile strength (σ_u) is 900 MPa; Modulus of elasticity (E) is 300 Gpa; Weight of meter per linear is 38 gram/m; Ultimate elongation (ε_u) is 2.2%.

For test number 3, 2 rods of composite reinforcement 6 mm in diameter are attached to the surface of the stretched beam.

Deflections of the beam are described in figure 11.
From the results it is shown that when the beam is reinforced with composite reinforcement, the fracture and deflection are limited in comparison with the test without the reinforcement of the beam by composite reinforcement. When tested without composite reinforcement the moment of crack formation $M_{crc} = 2.15\,\text{kNm}$, When reinforced with composite reinforcement, the moment of crack formation $M_{crc} = 8.34\,\text{kNm}$ (more by 388%).

Destructive moment of multi-layer reinforced concrete beam with reinforcement by composite reinforcement ($M_{ult} = 12.67\,\text{kNm}$) will be significantly more destructive moment of multi-reinforced concrete beam without composite reinforcement ($M_{ult} = 10.89\,\text{kNm}$) (more by 115%).

5. Conclusions
1. The results of the analysis show that the use of the ANSYS PC for modeling the stress-strain state of three-layer concrete structures is a modern method. The results obtained with the ANSYS PC are approximate to the experimental and theoretical results.
2. When reinforcing the multi-layer reinforced concrete structure with composite reinforcement, the bearing capacity and rigidity are increased. The moment of crack Marc increase by about 388%. Destructive moment of Multi-layer reinforced concrete beam with reinforcement by composite reinforcement will be much more destructive moment of Multi-reinforced concrete beam without composite reinforcement more by 115%.
3. The use of composite reinforcement for a multi-layer reinforced concrete beam is an advantageous perspective development. Deformation and deflection of the beam before destruction will be small, so that more should be investigated and tested before implementation.

In this article, the long-term load effect of reinforced concrete is not under consideration in analysis. This problem will be resolve in the other researches.

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