Flatness without CMB: The Entanglement of Spatial Curvature and Dark Energy Equation of State

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Abstract

The cosmic spatial curvature parameter \( \Omega_k \) is constrained, primarily by cosmic microwave background data, to be very small. Observations of the cosmic distance ladder and the large-scale structure can provide independent checks of the cosmic flatness. Such late-universe constraints on \( \Omega_k \), however, are sensitive to the assumptions of the nature of dark energy. For minimally coupled scalar-field models of dark energy, the equation of state \( w \) has nontrivial dependence on the cosmic spatial curvature \( \Omega_k \). Such dependence has not been taken into account in previous studies of future observational projects. In this paper we use the \( w \) parameterization proposed by Miao & Huang, where the dependence of \( w \) on \( \Omega_k \) is encoded, and perform a Fisher forecast on mock data of three benchmark projects: a Wide Field InfraRed Survey Telescope-like SNe Ia survey, a Euclid-like spectroscopic redshift survey, and a Large Synoptic Survey Telescope-like photometric redshift survey. We find that the correlation between \( \Omega_k \) and \( w \) is primarily determined by the data rather than by the theoretical prior. We thus validate the standard approaches of treating \( \Omega_k \) and \( w \) as independent quantities.

Key words: cosmic background radiation – cosmological parameters – cosmology: observations – dark energy – inflation – large-scale structure of universe

1. Introduction

The plethora of observational data in the past three decades has led to a concordance model of cosmology—a general-relativity-governed universe composed of about 69% dark energy, 26% dark matter, and 5% standard model particles, with small inhomogeneities that originated from vacuum fluctuation during the early-universe inflation. Despite the unknown microscopic nature of dark energy and dark matter, the concordance model has been confronted with, and passed, a host of observational tests—the temperature and polarization anisotropy in cosmic microwave background (CMB) radiation (Fixsen et al. 1996; Hinshaw et al. 2013; Planck Collaboration et al. 2016, 2018), the SNe Ia light curves (Riess et al. 1998; Perlmutter et al. 1999; Betoule et al. 2014; Scollnic et al. 2018), the large-scale structure (LSS) of galaxies (Beutler et al. 2011; Ross et al. 2015; Gil-Marín et al. 2016; DES Collaboration et al. 2018). The most concise version of the concordance model is the ΛCDM model, where Λ represents the cosmological constant as an interpretation of dark energy, and CDM is the acronym for cold dark matter.

The early-universe inflation, first proposed to solve the horizon and flatness problems (Kazanas 1980; Guth 1981; Sato 1981a, 1981b), has now become a part of the concordance model. The background spatial curvature of the universe, often characterized by a parameter \( \Omega_k \), is closely related to inflation models. While models that predict a detectable \( \Omega_k \) do exist (Gott 1982; Hawking 1984; Ratra 1985; Bucher et al. 1995; Ratra & Peebles 1995; Cornish et al. 1996; Aslanyan & Easther 2015; Ratra 2017), several major classes of inflationary theories favor very small \( \Omega_k \). For instance, the eternal inflation paradigm in general predicts \( |\Omega_k| \lesssim 10^{-4} \) (Guth & Nomura 2012; Kleban & Schillo 2012). The recently measured temperature and polarization power spectra of CMB, however, give a 99% confidence level detection of a negative \( \Omega_k = -0.044^{+0.018}_{-0.015} \) which corresponds to a positive spatial curvature (Planck Collaboration et al. 2018). This constraint is derived with a general power-law primordial power spectrum. In a specific inflation model, however, the spatial curvature can be related to the primordial power spectrum, and a full consistent treatment is required. Works along this direction can be found in Ooba et al. (2018a, 2018b, 2018c) and Park & Ratra (2017, 2018b, 2019), where spatially closed hypersurfaces are also favored. The preference of a closed universe can be weaken by an addition of CMB lensing reconstruction, which pulls \( \Omega_k \) back into consistency with zero to well within 2σ. Further inclusion of baron acoustic oscillations (BAO) data gives a constraint \( \Omega_k = 0.0007 \pm 0.0019 \), strongly supporting a spatially flat universe favored by the eternal inflationary paradigm (Planck Collaboration et al. 2018).

It is worth noting that currently there is no fully independent check with comparable accuracy (\( \sigma(\Omega_k) \lesssim 0.01 \)) on the cosmic flatness from other cosmological probes. The constraints on \( \Omega_k \) by low-redshift observations, such as SNe, BAO, and Hubble constant, often rely on some injection of CMB priors, and are sensitive to the assumptions about the nature of dark energy (Faroq et al. 2017; Wang et al. 2017; Park & Ratra 2018a; Ryan et al. 2018; Yu et al. 2018). Strong lensing time delay is a novel tool that in principle can give more model-independent measurements of the cosmic spatial curvature (Denissenya et al. 2018). Currently available strong lensing data, however, may contain systematic biases that are yet to be understood better (Li et al. 2018).

In the far future, the spatial curvature is expected to be precisely constrained by a combination of a variety of cosmological probes (Knox 2006). For instance, Witzemann et al. (2018) investigated 21 cm intensity mapping experiments, whereas Wei (2018) studied gravitational-wave standard sirens and cosmic chronometers. Assuming a CMB prior and good BAO reconstruction on quasinonlinear scales, Takada & Doré (2015) showed an all-sky, cosmic-variance-limited galaxy survey covering the universe up to \( z \geq 4 \) can determine \( \Omega_k \) to a remarkable accuracy of \( \sigma(\Omega_k) \lesssim \text{a few} \times 10^{-4} \). This
forecast can be considered as an ideal limit for future BAO constraints.

In this work we are interested in an explicitly CMB-independent check of the cosmic flatness. More specifically, we consider three experiments that had been proposed—the Wide Field InfraRed Survey Telescope (WFIRST) supernovae survey, the Euclid spectroscopy redshift survey, and the Large Synoptic Survey Telescope (LSST) photometric redshift survey—as our benchmarks. The major configurations of our forecast are taken from the publicly available documents (LSST Science Collaboration et al. 2009; Laureijs et al. 2011; Spergel et al. 2015). We do not follow all the details and the recent advances of these projects (see, e.g., Amendola et al. 2018). Thus, we dub the data sets WFIRST-like, Euclid-like, and LSST-like, to distinguish between our work and official studies of these projects.

For late-universe observables, \( \Omega_m \) has significant degeneracy with dark energy parameters. The standard approach in the literature is to treat \( \Omega_k \) and dark energy parameters as independent quantities, and to marginalize over the dark energy parameters. However, this approach is in principle problematic as the evolution of dark energy, and hence its equation of state, may depend on the spatial curvature. For instance, if dark energy is quintessence (a minimally coupled canonical scalar field) with a smooth potential (Ratra & Peebles 1988; Wetterich 1988; Caldwell et al. 1998; Zlatev et al. 1999), its equation of state will depend on the spatial curvature. Such dependence is explicitly calculated in Miao & Huang (2018).

Thus, we use the dark energy parameterization proposed by Huang et al. (2011) and later improved by Miao & Huang (2018), where the equation of state of dark energy is given by

\[
w = -1 + \frac{2}{3} \left( \sqrt{\epsilon_{\phi \infty}} + \sqrt{\epsilon_s - \frac{2\epsilon_{\phi \infty}}{1 - \Omega_k}} \right) \times \left[ F \left( \Omega_k, \frac{a}{a_{\text{eq}}}, \frac{a}{a_{\text{eq}}}, \frac{a}{a_{\text{eq}}} \right) + \zeta_F F_2 \left( \frac{a}{a_{\text{eq}}} \right) \right]^2, \tag{1}
\]

where \( a \) is the scale factor of the universe, normalized to unity today. The pivot \( a_{\text{eq}} \) is defined as the scale factor at the equality of dark energy and dark matter densities. The ratio of dark matter density to the critical density, \( \Omega_m \), enters the formula through its impact on the Hubble drag on the scalar field. The three additional parameters \( \epsilon_s, \zeta_F \) and \( \epsilon_{\phi \infty} \) characterize the slope (first derivative) and curvature (second derivative) of the scalar-field logarithm potential at the pivot, and the initial velocity of the scalar field, respectively. The functions \( F \) and \( F_2 \), given by

\[
F(\lambda, x) \equiv \frac{3}{x^3} \int_0^x \sqrt{\frac{t^2}{1 + \lambda t + t^2}} dt, \tag{2}
\]

and

\[
F_2(x) \equiv \sqrt{2} \left[ 1 - \ln \left( \frac{1 + x^3}{x^3} \right) \right] - \sqrt{\frac{1 + x^3}{x^3}} + \ln \left[ x^3 + \sqrt{1 + x^3} \right], \tag{3}
\]

can be derived from dynamic equations of the scalar field. The calculation is tedious but straightforward and can be found in Huang et al. (2011) and Miao & Huang (2018).

This parameterization contains a cosmological constant model (\( w = -1 \)) as a special case (when \( \epsilon_s = \zeta_F = \epsilon_{\phi \infty} = 0 \)) and covers a wide class of models that can be described by a minimally coupled canonical field. There are models beyond the scope of this parameterization, such as k-essence (Armendariz-Picon et al. 2000, 2001), \( f(R) \) gravity (Capozziello et al. 2003; Carroll et al. 2004; Nojiri & Odintsov 2006; Hu & Sawicki 2007), etc. The increasing complexity of dark energy model, which we will not cover in this work, may lead to more degeneracy between dark energy parameters and \( \Omega_k \).

In addition to the physical parameterization for quintessence model, we also use for comparison purpose a phenomenological dark energy parameterization \( w = w_0 + w_1 (1 - a) \) (Chevallier & Polarski 2001; Linder 2003), where the dark energy equation of state does not depend on \( \Omega_k \).

This article is organized as follows. In Section 2 we describe the Fisher forecast method and the mock data. In Section 3 we give the results and discuss their implications. Section 4 concludes. Unless otherwise specified, we work with the natural units \( c = h = 1 \).

### 2. Fisher Forecast

In this section we give a detailed description of the mock data sets and the Fisher forecast technique.

#### 2.1. WFIRST-like SNe Mock Data

The WFIRST-like SNe mock data are generated in 17 uniform redshift bins spanning a redshift range from \( z = 0 \) to...
\textbf{Table 3}  
Redshift Bins and Wavenumber Bounds for the LSST-like Mock Data

| \(z_{\text{mean}}\) | \(z\)-range | \(\delta [h^3 \text{Mpc}^{-3}]\) | \(k_{\text{max}}[h/\text{Mpc}]\) | \(k_{\text{max}}[h/\text{Mpc}]\) |
|----------------|-------------|----------------|----------------|----------------|
| 0.31           | 0.2–0.46    | 0.15            | 0.0071         | 0.08           |
| 0.55           | 0.46–0.64   | 0.10            | 0.0050         | 0.09           |
| 0.84           | 0.64–1.04   | 0.064           | 0.0040         | 0.11           |
| 1.18           | 1.04–1.32   | 0.036           | 0.0035         | 0.14           |
| 1.59           | 1.32–1.86   | 0.017           | 0.0030         | 0.17           |
| 2.08           | 1.86–2.3    | 0.0069          | 0.0028         | 0.23           |
| 2.67           | 2.3–3       | 0.0022          | 0.0026         | 0.31           |

Note. Other fiducial parameters are efficiency \(\epsilon = 0.5\), bias \(b = 1 + 0.84z\); sky coverage \(f_{\text{sky}} = 0.58\); photometric redshift error \(\sigma_{\text{phi}} = 0.04\), and r.m.s. radial motion parameter \(\sigma = 0.0019\).

\(z = 1.7\), with the number of mock samples in each bin listed in Table 1.

The distance modulus, namely, the difference between the apparent magnitude \(m\) and the absolute magnitude \(M\), of a supernova at luminosity distance \(d_L\) is given by

\[
\mu = 5 \log_{10}\left( \frac{d_L}{\text{Mpc}} \right) + 25.
\]

The luminosity distance \(d_L\) as a function of redshift \(z\) for a given cosmology is calculated with publicly available code CAMB (Lewis et al. 2000), with minor modification for the dark energy parameterization as done in Huang et al. (2011) and Miao & Huang (2018). The uncertainty of the distance modulus at redshift \(z\) is modeled as

\[
\sigma = \sqrt{\sigma_{\text{mean}}^2 + \sigma_{\text{int}}^2 + \sigma_{\text{lens}}^2 + \sigma_{\gamma}^2},
\]

where \(\sigma_{\text{mean}} = 0.08\) is the photometric measurement error, \(\sigma_{\text{int}} = 0.09\) is the intrinsic dispersion error, and \(\sigma_{\text{lens}} = 0.07z\) represents gravitational lensing error. Finally, \(\sigma_{\gamma} \approx \frac{5}{\sqrt{z}}\) is due to the redshift uncertainty from the random line-of-sight motion of the sample, assuming an r.m.s. projected peculiar velocity \(v_{\text{pec}} = 400\) km s\(^{-1}\). We ignore possible systematic errors such as correlation between samples, assuming these effects can be properly calibrated. Finally, we marginalize over the absolute magnitude \(M\) with a flat prior.

### 2.2. LSS Mock Data

We consider a spectroscopic \textit{Euclid}-like redshift survey and an LSST-like photometric redshift survey of galaxies. The galaxy power spectrum is modeled as (Kaiser 1987; Tegmark 1997; Huang 2012; Huang et al. 2012; Chen et al. 2016; Amendola et al. 2018)

\[
P_g(k, \mu; z) = (b + f_1 \mu)^2 P_{\phi}(k) e^{-k^2 \mu^2 R_c^2} - k^2(1 - \mu^2)R_c^2 + \frac{1}{\epsilon \bar{n}_{\text{obs}}},
\]

where \(\mu\), not to be confused with the distance modulus discussed in the previous subsection, is the cosine of the angle between the wave vector \(\mathbf{k}\) and the line of sight. In the last Poisson noise term on the right-hand side, \(\bar{n}_{\text{obs}}\) is the number density of observed galaxies, of which a fraction \(\epsilon\) of galaxies with successfully measured redshift is used to compute the power spectrum. In Fisher analysis, the wavenumber vector \(\mathbf{k}\) and matter power spectrum \(P_m(k)\) are recalibrated to the reference fiducial cosmology that we use to generate the mock data.

The linear galaxy bias \(b\) is parameterized as

\[
b(z, k) = (b_0 + b_1 z) e^{-\alpha k^2},
\]

where \(b_0, b_1, \alpha,\) and \(\beta\) are nuisance parameters. We assume a conservative Gaussian prior \(b_0 = 1 \pm 0.05\) to account for the knowledge of galaxy bias and to avoid singularity of Fisher matrix due to perfect degeneracy between bias and the primordial amplitude of density fluctuations. The weak \(k\) dependence \((e^{-\alpha k^2}\) factor, with \(\alpha \ll \Omega_{\text{m}}^{-2}\)) allows the baryonic matter to decorrelate with the dark matter on very small scales.

The linear matter power spectrum \(P_m(k)\) and the growth \(f,\) for a given cosmology, again can be computed with CAMB; see, e.g., Amendola et al. (2018) for more details.

The radial smearing scale \(R_\parallel\) is given by

\[
R_\parallel = \frac{c \sigma_\parallel}{H},
\]

where \(c\) is the speed of light, \(H\) is the Hubble expansion rate at redshift \(z\), and \(\sigma_\parallel\) is the combined redshift uncertainty due to the photometric redshift error and the random motion of galaxy along the line of sight. The recipe for \(\sigma_\parallel\) is (Huang 2012; Huang et al. 2012; Chen et al. 2016)

\[
\sigma_\parallel = (1 + z)^2 \sigma_{\phi0}^2 + \sigma_{\gamma0}^2.
\]

A Gaussian prior is assumed for the nuisance parameter \(\sigma_{\gamma0} = 0.0019 \pm 0.0009\). For the uncertainty of redshift measurement, we use \(\sigma_{\phi0} = 0.04\) for photometric redshift (LSST-like mock data) and \(\sigma_{\phi0} = 0.001\) for spectroscopic redshift (\textit{Euclid}-like mock data).

The smearing in directions perpendicular to the line of sight can be treated either with spherical harmonics or by converting spherical coordinates in a redshift shell to Cartesian coordinates, the latter approach yields, approximately, the transverse smearing scale

\[
R_\perp = \frac{c^2 \sigma_\parallel \Delta z}{H^2 d_c},
\]

where \(\Delta z\) is the redshift bin size and \(d_c\) is the comoving distance. In the thin-shell limit \(R_\perp \ll R_\parallel\) and the transverse smearing is often ignored in the literature.

We use 30 log-uniform \(k\)-bins and 30 uniform \(\mu\) bins. The result is stable while we increase the number of bins, if we use the following approximate covariance matrix for the galaxy power spectrum in the \(i\)th bin and the \(j\)th bin,

\[
\text{Cov}[P_g(k_i), P_g(k_j)] = \frac{2 \delta_{ij}}{N_h} [P_g(k_i)]^2 + \frac{\sigma_{\phi0}^2}{\sqrt{N_h}} P_g(k_i) P_g(k_j),
\]

where \(N_h\) the number of independent modes in a survey volume \(V_{\text{survey}}\) and \(i\)th Fourier-space bin with wavenumber \(k_i\).
and bin sizes \( dk \) and \( d\mu \), can be written as

\[
N_i = \frac{(2\pi)^3}{V_{\text{survey}}(2\pi k_i^2 dk d\mu)}.
\]

The \( \sigma_{\text{min}} \) terms are approximated non-Gaussian corrections to the covariance matrix (Carron et al. 2015). Since a conservative cutoff of quasilinear scale \( k_{\text{max}} \) is used for each redshift bin, the non-Gaussian corrections are expected to be subdominant. Thus, we simply use \( \sigma_{\text{min}} = 1.5 \times 10^{-4} \) as estimated in Carron et al. (2015) and ignore the dependence of \( \sigma_{\text{min}} \) on survey configurations.

Finally, we summarize the specifications for Euclid-like mock data and LSST-like mock data in Tables 2 and 3, respectively.

For the cosmological parameters, unless otherwise specified, we use the Planck best-fit \( \Lambda \)CDM parameters: Hubble constant \( H_0 = 67.32 \text{ km s}^{-1} \text{ Mpc}^{-1} \), fractional matter density \( \Omega_m = 0.264 \), fractional baryon density \( \Omega_b = 0.0494 \), and amplitude and spectral index of primordial power spectrum \( A_s = 2.10 \times 10^{-9} \), \( n_s = 0.966 \). Moreover, we assume a minimal neutrino mass \( \sum m_\nu = 0.06 \text{ eV} \) and a 0.6% Gaussian prior on \( H_0 \), which is expected to be achievable by future local distance ladder measurements (Riess et al. 2018).

3. Results

We present the forecast results for nonflat \( \Lambda \)CDM, quintessence and \( w_0-w_a \) models in Tables 4–6, respectively. The constraint on the spatial curvature depends on the dark energy model. For the WFIRST-like SNe mock data alone, \( \Omega_k \) has very strong degeneracy with dark energy parameters. The 1\( \sigma \) error on \( \Omega_k \) can vary by more than an order of magnitude for cosmologies with and without dark energy degrees of freedom. For LSST-like and Euclid-like redshift surveys, the variation of \( \sigma(\Omega_k) \) for models with and without dark energy dynamics is about a factor of two. However, the dependence of \( \sigma(\Omega_k) \) on dark energy models becomes weak while we combine the three mock data sets (WFIRST-like, Euclid-like, and LSST-like) together, in which case a percent-level constraint \( \sigma(\Omega_k) \approx 0.01 \) can be achieved. Comparing the results for quintessence parameterization and for \( w_0-w_a \) parameterization, we do not find significant difference in the constraints on other parameters, in particular on \( \Omega_k \). This statement approximately holds true when we consider combined constraints on multiple parameters. Two examples are given in Figure 1.

A cosmological constant does not have dependence on the spatial curvature, whereas a slowly rolling quintessence field does. We have chosen a fiducial \( \varepsilon = 0.3 \), roughly 1\( \sigma \) bound allowed by current data (Miao & Huang 2018), to describe a
Table 5
Same as Table 4, Except for the (Nonflat) Quintessence Model with Fiducial $\varepsilon_s = 0.3$ and $\varepsilon_{\phi\infty} = \zeta_s = 0$

| Parameter | W | E | L | E+W | L+W | E+L+W |
|-----------|---|---|---|------|------|--------|
| $\varepsilon_s$ | 13.85 | 1.02 | 0.23 | 0.18 | 0.11 | 0.11 |
| $\varepsilon_{\phi\infty}$ | 7.71 | 0.51 | 0.20 | 0.37 | 0.17 | 0.15 |
| $\zeta_s$ | 469.3 | 13.52 | 4.27 | 6.95 | 3.25 | 2.9136 |
| $\Omega_L$ | 7.72 | 0.038 | 0.037 | 0.024 | 0.018 | 0.013 |
| $\Omega_m$ | 4.69 | 0.0283 | 0.0151 | 0.0058 | 0.0068 | 0.0045 |
| $\Omega_b$ | ... | 0.0062 | 0.0033 | 0.0015 | 0.0020 | 0.0012 |
| $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | ... | 0.38 | 0.38 | 0.36 | 0.36 | 0.25 |
| $10^6A_s$ | ... | 0.19 | 0.22 | 0.11 | 0.16 | 0.08 |
| $n_s$ | ... | 0.028 | 0.027 | 0.015 | 0.019 | 0.012 |

Note. Bolded values highlight the constraints on the spatial curvature.

Table 6
Same as Table 4, Except for the (Nonflat) $w_0$-$w_a$ Model with Fiducial $w_0 = -1$ and $w_a = 0$

| Parameter | W | E | L | E+W | L+W | E+L+W |
|-----------|---|---|---|------|------|--------|
| $w_0$ | 1.51 | 0.21 | 0.042 | 0.040 | 0.023 | 0.023 |
| $w_a$ | 5.03 | 0.78 | 0.16 | 0.20 | 0.12 | 0.11 |
| $\Omega_L$ | 1.07 | 0.036 | 0.036 | 0.020 | 0.016 | 0.013 |
| $\Omega_m$ | 0.43 | 0.0260 | 0.0152 | 0.0058 | 0.0067 | 0.0046 |
| $\Omega_b$ | ... | 0.0053 | 0.0032 | 0.0015 | 0.0021 | 0.0012 |
| $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | ... | 0.37 | 0.38 | 0.36 | 0.34 | 0.24 |
| $10^6A_s$ | ... | 0.20 | 0.23 | 0.11 | 0.16 | 0.08 |
| $n_s$ | ... | 0.031 | 0.028 | 0.014 | 0.018 | 0.011 |

Note. Bolded values highlight the constraints on the spatial curvature.

slowly rolling field that interacts with the spacetime geometry. For a comparison, we switch to a fiducial $\varepsilon_s = 0$ (a very flat quintessence potential) to freeze the field dynamics and to minimize the dependence of $w$ on $\Omega_k$. We find, again, no significant variation of the $1\sigma$ errors on the parameters or of the error contours for multiple parameters. In this paper we only considered a single type of object as the tracer of the large-scale structure. For a survey targeting multiple tracers, for instance, the upcoming Dark Energy Spectroscopic Instrument (DESI) experiment (Aghamousa et al. 2016; Vargas-Magana et al. 2019), complexities such as tracer-dependent biases and cross-correlation between tracers cannot be captured with our methodology. Nevertheless, the recent impressive engineering advances of DESI motivate us to make at least a qualitative estimation of DESI’s potential constraints on the interplay of spatial curvature and dark energy dynamics. To achieve this goal, we ignore the complexity due to multiple tracers and simply add together the number densities of the tracers listed in Table 2.3 of Aghamousa et al. (2016) and use a simple bias $b = \sqrt{1 + z}$ for all the tracers. For a sky coverage 14,000 square degrees and a five-year integration, the result is similar to that of Euclid-like forecast. This agrees with what has been found in Aghamousa et al. (2016).

We have used conservative cutoffs and almost only used information on linear scales. Methods such as BAO reconstruction techniques will provide us with more information on nonlinear scales and improve the constraint on $\Omega_k$ and other parameters (Takada & Doré 2015). Alcock-Paczynski effect on nonlinear scales may also be a powerful tool to extract information about the geometry of the universe (Zhang et al. 2019). We leave exploration in these directions as our future work.

4. Discussion and Conclusions

The combination of upcoming SNe Ia survey and large-volume redshift surveys will confirm (or reject) the cosmic flatness to a remarkable subpercent precision. Such constraint is not sensitive to the theoretical priors on the connection between dark energy equation of state and the spatial curvature. Thus, we have justified the approaches of treating $\Omega_k$ and $w$ as independent quantities.

In this paper we only considered a single type of object as the tracer of the large-scale structure. For a survey targeting multiple tracers, for instance, the upcoming Dark Energy Spectroscopic Instrument (DESI) experiment (Aghamousa et al. 2016; Vargas-Magana et al. 2019), complexities such as tracer-dependent biases and cross-correlation between tracers cannot be captured with our methodology. Nevertheless, the recent impressive engineering advances of DESI motivate us to make at least a qualitative estimation of DESI’s potential constraints on the interplay of spatial curvature and dark energy dynamics. To achieve this goal, we ignore the complexity due to multiple tracers and simply add together the number densities of the tracers listed in Table 2.3 of Aghamousa et al. (2016) and use a simple bias $b = \sqrt{1 + z}$ for all the tracers. For a sky coverage 14,000 square degrees and a five-year integration, the result is similar to that of Euclid-like forecast. This agrees with what has been found in Aghamousa et al. (2016).

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