CP violation in Compton scattering

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I consider Compton scattering off the nucleon in the presence of $T$ violation. I construct the Compton tensor which possesses these features and consider low energy expansion (LEX) of the corresponding amplitudes. It allows to separate out the Born contribution which only depends on the static properties of the nucleon, such as the electric charge, the mass, the magnetic moment, and the electric dipole moment (EDM). I introduce new structure constants, the $T$-odd nucleon polarizabilities which parametrize the unknown non-Born part. These constants describe the response of the $T$-violating content of the nucleon to the external quasistatic electromagnetic field. As an estimate, I provide a HBChPT calculation for these new polarizabilities and discuss the implications for the experiment.

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I. INTRODUCTION

The first proposal of experimental search for $CP$ violation effects in atoms was made almost 40 years ago [1]. The modern advanced experimental techniques realized in the experiments on electron's electric dipole moment (EDM) are based on that idea and have the sensitivity of $d_e \sim 10^{-26}$ e cm [2].

Apart from the electron EDM, experimental searches for the EDM of the neutron are on-going [3]. Current sensitivity allows for detection of electric dipole moment (EDM) of the neutron at the level of $d_n \sim 10^{-26}$ e cm. From theoretical point of view, a non-zero EDM could imply non-zero values for the QCD $\theta$-term as the latter can induce an EDM [4].

An attractive idea to enhance the experimental sensitivity to the electron EDM was proposed in [5]. In an atom, the electrons are moving in the electromagnetic field of the nucleus. If the electron possess an EDM, the electric Coulomb field of the nucleus would induce a magnetic dipole moment, proportional to the electron EDM and the electromagnetic field strength, leading to the magnetization of the sample. Since the electromagnetic field strength inside the nucleus can be several orders of magnitude larger than those achievable in the experiment, the effect of a non-zero electron EDM is expected to be magnified. Due to chaotic relative orientation of atoms in a sample, these elementary magnetisations sum up to zero, therefore one would need a polarizing electric field to be applied to the sample, in order to observe the sample magnetization. At the same time, the authors of [5] indicate that the effect of these atomic $T$-odd polarizabilities might interfere with the effect of nuclear EDM, and has to be taken into account. If a non-zero neutron EDM is discovered in the near future, further tests of our understanding of QCD and fundamental interactions will bring us to the study of the microscopic structure of the $CP$-violating content of the nucleon, which can be described in terms of $T$-odd polarizabilities of the nucleon. In this paper I address the following questions. Can the presence of these $CP$-violating structure constants lead to a substantial interference with the measurement of the EDM? How and whether they can be measured?

The concept of the polarizabilities was first introduced in classical electrodynamics and characterizes the ability of the elementary charges within a given system to be displaced from their positions in the presence of an external electric field. The electric dipole moment resulting from such a displacement is proportional to the strength of the applied field, and the coefficient of proportionality is called the (electric dipole) polarizability. This constant quantitatively describes the forces that put the system together. For instance, the atomic or molecular electric dipole polarizability is known to be of the order of the atom’s or molecule’s volume [6].

Instead, the nucleon electric dipole polarizability $\alpha_N \sim 10^{-3}$ fm$^3$ is only about 1/1000 of its volume, $V_N \sim 1$ fm$^3$, which characterizes the strong forces that hold the proton together considerably stronger than the electromagnetic forces holding the electron in the atom.

The $T$-violating polarizabilities of the nucleon result from two pieces: the short range $T$-violating physics ($T$-violation is generated well above the electroweak scale by unknown new physics) responsible, for instance, for the QCD $\theta$-term, and the (mostly) long range pion physics which is quite analogous to the usual Compton scattering case. Thus, the natural size of the nucleonic $T$-violating polarizabilities is expected to be

$$\delta^T \sim 10^{-3} g_0 \text{fm}^3, \quad (1)$$

where $g_0 \lesssim 10^{-11}$ is the strength of the $\theta$-term and $\delta^T$ denotes a $T$-violating nucleon polarizability. One can compare this to the estimates for the atomic $CP$-violating

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polarizability $\beta_{C^P}$ \cite{5},
\[
\frac{\beta_{C^P}}{d_e} \sim 10^{-2} \text{ a.u.}, \quad \frac{\beta_{C^P}}{d_n} \sim 10^{-12} \text{ a.u.},
\]
where atomic units were adopted, and $d_n$ is the EDM of the neutron.

II. COMPTON SCATTERING AT LOW PHOTON ENERGIES

Real Compton scattering can be described, under the assumption of invariance under parity, charge conjugation and time reversal, by means of 6 structure dependent amplitudes $A_i(\omega, \theta)$, $i = 1 \ldots 6$, with $\omega = \omega'$ the c.m. energy of the initial and outgoing photons and $\theta$ being the c.m. scattering angle:
\[
T = \chi^\dagger \left\{ A_1(\omega, \theta)(\varepsilon^\dagger \cdot \varepsilon) + A_2(\omega, \theta)(\varepsilon^\dagger \cdot \hat{k}')(\varepsilon \cdot \hat{k'}) + A_3(\omega, \theta)i\sigma \cdot (\varepsilon^\dagger \times \varepsilon) + A_4(\omega, \theta)i\sigma \cdot (\hat{k}' \times \hat{k})(\varepsilon^\dagger \cdot \varepsilon) + A_5(\omega, \theta)i\sigma \cdot [((\varepsilon^\dagger \times \hat{k}')(\varepsilon \cdot \hat{k'}) - (\varepsilon \times \hat{k}')(\varepsilon^\dagger \cdot \varepsilon)] \right\} \chi,
\]
where $\varepsilon, \hat{k}(\varepsilon', \hat{k}')$ stand for the polarization vector, direction of the initial (final) photon, and $\sigma$ is the spin polarization of the nucleon.

Following Refs. \cite{7}, the functions $A_i(\omega, \theta)$ can be expanded into a series in powers of (small) photon energy $\omega$:

\[
A_{1:m}^-(\omega, \theta) = -\frac{e^2 e_N^2}{M_N^2} - \frac{e^2 e_N^2}{4M_N^2}(1 - \cos \theta)\omega^2 + 4\pi (\alpha_E^N + \cos \theta \beta_M^N)\omega^2 + \frac{4\pi}{M_N^2}(\alpha_E^N + \beta_M^N)(1 + \cos \theta)\omega^3 + O(\omega^4)
\]
\[
A_{2:m}^-(\omega, \theta) = \frac{e^2 e_N^2}{M_N^2}\omega - 4\pi \beta_M^N\omega^2 - \frac{4\pi}{M_N^2}(\alpha_E^N + \beta_M^N)\omega^3 + O(\omega^4)
\]
\[
A_{3:m}^-(\omega, \theta) = \left[ \mu_N^2(1 - \cos \theta) - \kappa_N^2 \right] \frac{e^2}{2M_N^2}\omega - \frac{e^2(e_N^2 + 2\kappa_N)}{8M_N^2}\cos \theta\omega^3 + 4\pi \left[ \gamma_1^N - (\gamma_2^N + 2\gamma_4^N) \right] \cos \theta\omega^3 + O(\omega^4)
\]
\[
A_{4:m}^-(\omega, \theta) = -\frac{e^2 \mu_N^2}{2M_N^2}\omega + 4\pi \gamma_2^N\omega^3 + O(\omega^4)
\]
\[
A_{5:m}^-(\omega, \theta) = \frac{e^2 \mu_N^2}{2M_N^2}\omega + 4\pi \gamma_4^N\omega^3 + O(\omega^4)
\]
\[
A_{6:m}^-(\omega, \theta) = -\frac{e^2 \mu_N^2}{2M_N^2}\omega + 4\pi \gamma_5^N\omega^3 + O(\omega^4)
\]

For each Compton amplitude, the leading terms in the $\omega$ expansion are given by model-independent Born contributions which are completely defined by the static properties of the nucleon as a spin-1/2 particle with the electric charge $e_N$, anomalous magnetic moment $\kappa_N$ (magnetic moment $\mu_N = e_N + \kappa_N$) and mass $M_N$ \cite{7}. The higher order terms describe internal structure-dependent effects and are parametrized in terms of 6 polarizabilities.

Two of them, $\alpha_E$ and $\beta_M$, are spin-independent electric and magnetic polarizabilities which enter the amplitude at $O(\omega^2)$ and measure the deformation of the charge and magnetisation distributions in the presence of quasi-static electric $\vec{E}$ and magnetizing $\vec{H}$ external fields,
\[
\vec{d} = 4\pi \alpha_E \vec{E}, \quad \vec{m} = 4\pi \beta_E \vec{H},
\]
with $\vec{d}(\vec{m})$ denoting the induced electric (magnetic) dipole moment.

The other four polarizabilities $\gamma_i$, $i = 1 \ldots 4$ describe the response of the spin-dependent distributions inside the nucleon to the quasi-static external field. For example, the polarizabilities $\gamma_{1,3}$ quantify the induced spin-dependent electric dipole moment in the external magnetic field,
\[
\vec{d}_s^E = 4\pi \gamma_1 \left\{ \vec{S} \times (\vec{\nabla} \times \vec{B}) \right\}, \quad \vec{d}_s^B = 4\pi \gamma_3 \vec{\nabla} \left( \vec{S} \cdot \vec{B} \right).
\]
Similarly, the polarizabilities $\gamma_{2,4}$ quantify the magnetic dipole moment induced in the external electric field:
\[
\vec{m}_s^E = 4\pi \gamma_2 \vec{\nabla} \left( \vec{S} \cdot \vec{E} \right), \quad \vec{m}_s^E = 4\pi \gamma_4 \left\{ \vec{S} \times (\vec{\nabla} \times \vec{E}) \right\}.
\]
III. COMPTON SCATTERING WITH P AND CP-VIOLATION

Quite in the spirit of the previous section, we will construct the Compton amplitude which violates both parity and time-reversal (and thus CP).

\[
T^C = \chi^+ \left\{ A^T_C(\omega, \theta)(\vec{\varepsilon}^* \cdot \vec{\varepsilon})\sigma \cdot (\vec{k} - \vec{k}') \right. \\
+ A^T_{2}(\omega, \theta)(\vec{\varepsilon}^* \cdot \vec{k})(\vec{\varepsilon} \cdot \sigma)(\vec{k} - \vec{k}') \\
+ A^T_{3}(\omega, \theta)i\sigma \cdot (\vec{k} + \vec{k}') \cdot [\vec{\varepsilon}^* \times \vec{\varepsilon}] \\
+ A^T_{4}(\omega, \theta)i\sigma \cdot [(\vec{k} + \vec{k}') \times (\vec{\varepsilon}^* \times \vec{\varepsilon})] \\
+ A^T_{5}(\omega, \theta)i\sigma \cdot (\vec{\varepsilon}^* \times \vec{k})((\vec{\varepsilon} \cdot \vec{k})(\vec{k} \cdot \vec{\varepsilon}^*)) \right\} \chi. 
\]

In the above formula, I use the notation as close to the CP-even Compton scattering as possible. All the structures we are interested in should contain at most one spin vector since one can always reduce the number of the \( \sigma \)'s in a product through \( \sigma_a \sigma_b = \delta_{ab} + i\epsilon_{abc}\sigma_c \) and the minimal power of photon momenta. I do not change the dependence of the structures in Eq.(4) on the photon polarization vectors, if possible, but modify the dependence on the photons’ momenta and nucleon spin such that the result is CP-odd. The first two structures in Eq.(8) are obtained by multiplying the corresponding C, P and T conserving structures of Eq.(4) by the explicitly P and T violating scalar \( \sigma \cdot (\vec{k} - \vec{k}') \). The third structure results from substituting the P-even, T-odd spin vector \( \sigma \) by P-odd, T-even vector \( (\vec{k} - \vec{k}') \). The fourth structure in Eq.(4) does not allow for a modification to obtain a CP-odd structure distinct from that coming with \( A_T^C \), therefore I use another structure instead. The last two structures are obtained from the corresponding Compton structures by changing the relative sign between two terms. In this way, these combinations become T-odd but are P-even. For completeness, I also give here the amplitudes which are P-odd and T-even,

\[
T^P = \chi^+ \left\{ A^P_4(\omega, \theta)(\vec{\varepsilon}^* \cdot \vec{\varepsilon})\sigma \cdot (\vec{k} - \vec{k}') \right. \\
+ A^P_5(\omega, \theta)(\vec{\varepsilon}^* \cdot \vec{k})(\vec{\varepsilon} \cdot \sigma)(\vec{k} - \vec{k}') \\
+ A^P_6(\omega, \theta)i\sigma \cdot (\vec{k} + \vec{k}') \cdot [\vec{\varepsilon}^* \times \vec{\varepsilon}] \\
+ A^P_7(\omega, \theta)i\sigma \cdot [(\vec{k} + \vec{k}') \times (\vec{\varepsilon}^* \times \vec{\varepsilon})] \right\} \chi. 
\]

Together with the C, P, and T even Compton amplitudes of Eq.(4) and T-odd amplitudes of Eq.(5), these amplitudes form the complete basis for real Compton scattering with \( 2^4 = 16 \) possible polarization states.

IV. LOW ENERGY EXPANSION AND CP-ODD POLARIZABILITIES OF THE NUCLEON

The next step is to calculate the Born part of the T-violating Compton scattering amplitude. This Born contribution corresponds to an elastic absorption (emission) of the initial (final) photon by either the initial or final nucleon with the single nucleon propagating in the intermediate state. For such an amplitude to violate time-reversal one needs T-violating photon coupling to the nucleon. This can be arranged by including a non-zero EDM term into the electromagnetic vertex of the nucleus,

\[
\Gamma^\mu(q) = \epsilon \left[ e_N \gamma^\mu + \kappa_N i \sigma^\mu a \frac{q_\mu}{2M_N} + \bar{d}_N i \gamma_5 \sigma^\mu \frac{q_\mu}{2M_N} \right] 
\]

where the dimensionless \( \bar{d}_N \) is the electric dipole moment (EDM) of the nucleon measured in units of the nuclear magneton \( \frac{e}{2M_N} \), and the index \( N = p, n \) indicates whether the nucleon is the proton or the neutron, respectively. In the following, I will use the EDM in the usual units,

\[
d_N = \bar{d}_N \frac{e}{2M_N}. 
\]

FIG. 1: Born contributions to CP-violating Compton scattering. The square represents the coupling of the photon to the nucleon EDM. Graphs with opposite ordering of the SM and EDM couplings are not shown.

The direct calculation of the diagrams in Fig[1] leads to the following results:

\[
A^C_T = \frac{ed_N}{M_N}(2e_N + \kappa_N)\omega + 4\pi \frac{\epsilon_T}{(1 - \cos \theta) + \cos \theta \delta_T^2}\omega^3 + O(\omega^4) \\
A^P_T = -\frac{ed_N}{2M_N}(2e_N + \mu_N)\omega^2 - \frac{ed_N}{4M_N} e_N - \kappa_N - 2\cos \theta(2e_N + \mu_N) \\
- 4\pi \delta_T^2 \omega^3 + O(\omega^4) \\
A^T_T = \frac{ed_N}{2M_N} \mu_N \omega^2 + 4\pi \delta_T^2 \omega^2 \\
- \frac{ed_N}{4M_N} e_N - \kappa_N + \cos \theta(2\mu_N + \kappa_N) \right] + O(\omega^4) \\
A^P_T = \frac{ed_N}{M_N}(e_N - \kappa_N)\omega + \frac{ed_N}{2M_N} \mu_N \omega^2 + 3\mu_N \cos \theta - 2e_N \cos^2 \theta \omega^3 + O(\omega^4),
\]
where the four constants were introduced, the T-odd, P-odd polarizabilities of the nucleon $\delta_i^T$, $i = 1 \ldots 4$. As in the case of P-even $T$-even Compton scattering, the spin independent polarizability parametrizes the term which is quadratic in photon energy, while the spin dependent ones come at order $\omega^3$. One should note that the fact that the amplitudes do not possess definite crossing symmetry ($\omega \to -\omega$) is due to the use of the c.m. frame which makes the direct and crossed channels asymmetric by imposing that the nucleon in the intermediate state is at rest in the $s$-channel, $\vec{p} + \vec{k} = 0$, while it is not the case for the $u$-channel, $\vec{p} - \vec{k} \neq 0$.

The polarizabilities introduced above can be interpreted as the deformation of the system under the influence of the external electromagnetic field, as it was done for the usual Compton scattering. The polarizability $\delta_1^T$ quantifies the electric dipole moment induced by the gradient of the external electric field in the direction of the spin, and similarly for the polarizability $\delta_2^T$

$$d_s^E = 4\pi \delta_1^T (\vec{S} \cdot \vec{\nabla}) \vec{E},$$

$$d_s^B = 4\pi \delta_2^T (\vec{S} \cdot \vec{\nabla}) \vec{B}. \quad (12)$$

Furthermore, there is one polarizability which characterizes the electric dipole moment induced by the external magnetic field (without spin),

$$d_s^E = 4\pi \delta_3^B \vec{B}, \quad (13)$$

and finally, another polarizability which quantifies the electric dipole moment induced by the gradient of the projection of the external electric field onto the nucleon spin,

$$d_s^E = 4\pi \delta_4^T (\vec{S} \cdot \vec{E}). \quad (14)$$

V. HBCHPT CALCULATION OF THE T-ODD POLARIZABILITIES

The QCD $\theta$-term leads to a $P$-odd $T$-odd coupling of the pion to the nucleon,

$$L_X = -g_0 \vec{N} \vec{\tau} \cdot \vec{\pi} N. \quad (15)$$

The contribution to the polarizability $\delta_1^T$ arises due to neutral pion exchange in the $t$-channel, as shown in Fig. 2 for which one needs the anomalous $\pi^0\gamma\gamma$ vertex

![Diagram](image)

FIG. 2: $\pi^0$ pole contribution to $T$-odd Compton scattering.

provided by Wess-Zumino-Witten Lagrangian,

$$L_{\pi^0\gamma\gamma}^{WZW} = -\frac{e^2}{32\pi^2 F_\pi} \epsilon_{\mu\nu\alpha\beta} F_{\mu\alpha} F^{\nu\beta} \pi^0, \quad (16)$$

with $F_\pi$ the pion decay constant. Furthermore, we will need the usual $CP$-conserving pion-nucleon Lagrangian. The lowest order ChPT Lagrangian in the heavy baryon formalism is well known and we refer the reader to Ref. [8] for the details.

![Graphs](image)

FIG. 3: Representative one loop graphs for the HBChPT calculation of the T-odd polarizabilities. The square represents the $CP$-violating pion-nucleon coupling as described in text. Graphs with opposite coupling ordering are not shown.

The representative diagrams at one loop are shown in Fig. 3. Keeping the leading terms in $\omega/M_N$ only, we obtain the following results for the $T$-odd polarizabilities:

$$\delta_1^T = -\frac{\alpha_{em}}{4\pi} \frac{g_A}{8F_\pi} m_\pi^2 g_0,$$

$$\delta_2^T = \frac{\alpha_{em}}{4\pi} \frac{g_A}{24F_\pi} m_\pi^2 g_0,$$

$$\delta_3^T = -\frac{\alpha_{em}}{8\pi} \frac{1}{F_\pi} m_\pi^2 g_0,$$

$$\delta_4^T = 0 + O(\omega/M_N), \quad (17)$$

where $g_A$ stands for the axial coupling of the nucleon. We note that that the polarizability $\delta_4^T$ arises due to nucleon recoil effects and is thus of order $\omega/M_N$.

VI. EXPERIMENTAL ACCESS TO THE T-ODD POLARIZABILITIES

In principle, one might try to measure these new structure constants of the nucleon using the well established experimental techniques used in the EDM-type experiments. In such experiments, one measures the difference in the precession frequency of the spin in the external magnetic and electric fields depending on the field orientation. However, these experiments use static fields. Under these conditions, Compton effect is undetectable, as the corresponding Compton frequency shift is $\sim \omega/M_N$.

As one can see from Eqs. (11)-(17), the polarizabilities contributions arise as corrections in powers of $\omega/m_\pi$ to the leading Born contributions. To see these corrections one has to go to photon energies (frequencies) comparable to the pion mass. Such experimental conditions may be accomplished in a Compton scattering experiment. One of the possibilities would be to scatter circularly polarized photons off unpolarized nucleon target. One can flip the polarization of the photon and without detecting the polarization state of the photon in the final state, measure
the difference in the signal. The general expression of such a single spin asymmetry in terms of the amplitudes defined above is somewhat lengthy. Making use of the LEX of the PCTC Compton amplitude for the proton, one has

$$\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} = -\frac{8}{1 + \cos^2 \theta} \left( \cos \theta \frac{A_1^P}{2A_1} + \sin \theta \frac{A_2^T}{2A_1} \right), \quad (18)$$

where the LEX of the amplitudes $A_1$ and $A_2^T$ are given in Eqs. (4) and (11), while the LEX of the P-odd amplitude $A_2^T$ can be found in Ref. [9].

One notices that this asymmetry obtains contributions both from PVTC and PVTV amplitudes. The contribution of the former originates mainly from the P-odd combination of Compton helicity amplitudes $|T_{1, 1, 1, 1}|^2 - |T_{1, 1, -1, 1}|^2$, which does not violate time reversal. The main source of the latter of the two is the combination of the backward-dominant helicity amplitudes $|T_{1, -1, -1, 1}|^2 - |T_{1, 1, 1, -1}|^2$ which is non-zero in the presence of both P- and T-violation. The parity-violating asymmetry was first considered in [9] and it is expected to be of order $10^{-8}$ at forward angles. If CP-violation is due to a non-zero QCD $\theta$-term that is very tightly constrained by the experimental limit on EDM, the expected value of the single spin asymmetry in Eq. (18) is

$$A^{CP} \lesssim 10^{-11} \quad (19)$$

for $\sim 100$ MeV photons and very backward angles. The effect is quadratic in the photon energy $\omega$. Although this effect is tiny as compared to the parity-violating contribution, by going to very backward angles this latter is highly suppressed due to the $\cos^4 \frac{\theta}{2}$ factor in front. It furthermore has a cubic dependence on the photon energy [9]. Another important background is represented by the analyzing power of Compton scattering. Experimentally, it is impossible to achieve a 100% circular polarization.

The remaining linear polarization component can lead to a similar $T$-odd observable that does not require $T$-violation, but arises from the final state interaction. One has for the Compton cross section [12]

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma_0}{d\Omega} \right) (1 + \lambda P \cos 2\phi) \quad (20)$$

with $\sigma_0$ the usual unpolarized differential cross section and $\phi$ the angle between the linear photon polarization direction and the reaction plane. Finally, $\lambda$ represents the degree of linear polarization, and $P$ the analyzing power. The analyzing power arises as an interference between the purely real leading order Compton amplitude (below the pion threshold), and the imaginary part of the next-to-leading order Compton amplitude, as shown in Fig. 4.

Inserting the leading in LEX Thomson term in place of the blobs in the figure, we obtain:

$$\frac{\text{Im} A_1}{A_1^T} = \alpha_{em} \frac{\omega}{M} + O(\omega^2) \quad (21)$$

where the energy dependent factor is due to the phase space. The analyzing power $P$ is then obtained from interference terms like

$$P \sim \frac{A_2^P \text{Im} A_1}{|A_1|^2} \frac{\sin^2 \theta}{1 + \cos^2 \theta} \sim \frac{\alpha_{em} \omega^2}{M^2} \frac{\sin^2 \theta}{1 + \cos^2 \theta} + O(\omega^3) \quad (22)$$

The above formula gives an adequate result for the leading contribution at low energies, and the corrections due to imaginary parts of other amplitudes arise at the order $\omega^3$.

**FIG. 4:** The mechanism for the analyzing power of Compton scattering at low energies.

Assuming the degree of linear polarization in the photon beam to be of order 1%, and going to very backward angles, so that $\cos \frac{\theta}{2} \leq 0.1$, we see that such QED final state interactions can lead to asymmetries of the order

$$A^{FSI} \sim 0.01 \times 0.1 \times \frac{1}{137} \times \frac{\omega^2}{M^2} \times \cos 2\phi$$

$$\sim 10^{-7} \cos 2\phi \quad (23)$$

As one can see, the analyzing power represents a very substantial background for a measurement of a CP-violating Compton scattering, as well as that of the parity-violating Compton scattering as proposed in [9]. This background can be distinguished experimentally by measuring the complete azimuthal angle dependence of the considered single spin asymmetry. The final result is given by a cosine-modulated FSI contribution plus a constant term in $\phi$ that is the $P$- and CP-odd pieces.

These estimates indicate that whenever $CP$-violation in Compton scattering is due to mechanisms that can directly contribute to the EDM, as well, the corresponding asymmetries cannot exceed roughly $10^{-11}$ at energies just below the pion threshold.

**VII. MODEL INDEPENDENT CONSTRAINTS ONTO CP-VIOLATION IN COMPTON SCATTERING**

I will next examine the upper limit constraint on $CP$-violation in spin-independent Compton scattering.

As a possible, although exotic scenario, one can assume that an unknown source of $CP$-violation may exist, that generates the $T$-odd Compton amplitude but is for some reason forbidden to show up in just one photon vertex (for instance, the spin-independent $CP$-odd term is not present in the one photon coupling, but arises in Compton scattering). An example of such a Lagrangian is a
dimension-7 operator
\[ L = c(\Lambda) \frac{e^2}{\Lambda^3} \epsilon_{\mu \nu \alpha \beta} F^{\mu \nu} F^{\alpha \beta} \hat{N}N, \]
where \( \Lambda \) represents the scale of the unknown \( CP \)-violating New Physics that is integrated out and is replaced by the above effective vertex, and \( c(\Lambda) \) the corresponding Wilson coefficient. This operator generates the non-Born part of the amplitude \( A_T^0 \) only, since the EDM is supposed not to obtain contribution from this mechanism,
\[ 4\pi \delta_3^T = \frac{4e^2c(0)}{\Lambda^3} \]
where the Wilson coefficient should be taken at low energy. The EDM then arises at one-loop level as a QED radiative correction, as shown in Fig. 5

FIG. 5: New Physics contribution to the nucleon EDM at one loop. The square represents the operator specified in the text.

\[ 4\pi \delta_3^T = \frac{4e^2c(0)}{\Lambda^3} \]

\[ \delta(d_N) \sim \frac{\alpha_{em} c(\Lambda)}{4\pi} \frac{\Lambda}{\Lambda} \]

such that we obtain a limit on this New Physics contributions from the EDM,
\[ c(\Lambda) \lesssim 10^{-10} \frac{\Lambda}{1\text{GeV}} \]

The loop calculation contains a quadratic divergence due to the neglection of the vertex structure within an effective field theory treatment. To obtain an order-of-magnitude estimate of this loop contribution, I use the “naive dimensional analysis” method \([11]\) that is based on the dimensional regularization approach that ensures that the only mass scales arising in such a calculation are physical particle masses. The quadratic divergence itself should cancel exactly, once one specifies the underlying theory that is renormalizable. Unless there exists a symmetry that enforces the exact cancellation, at low photon energies this cancellation should not occur at 100% level. Then, the naive dimensional analysis estimate is still adequate.

Combining this limit with Eqs. (25) and (18) we arrive to the following upper limit for the \( CP \)-violating asymmetry generated by the New Physics:
\[ A^{NP} \lesssim 10^{-11} \frac{\omega^2}{\Lambda^2} \]

To arrive to this result I neglected the running of the Wilson coefficient \( c(\Lambda) \). This running may indeed be substantial, but one would rather expect a logarithmical, and not quadratic running, therefore the \( 1/\Lambda^2 \) will be dominant for the dependence of the above limit on the New Physics scale. An accurate treatment can indeed change the estimate somewhat, and I leave this investigation for a future work, as this calculation goes beyond the scope of the present article.

Since the pion is the lightest hadronic state, this result means that the calculation of the \( CP \)-odd polarizabilities of the nucleon in the ChPT with pions of Eq. (17) represents the dominant contribution due to the \( 1/\Lambda^2 \) suppression of heavier particles contributions. In other words, if a substantial \( CP \)-violating mechanism in nucleon spin-independent Compton scattering exists, it should involve some light particles, like pions or eta’s for instance, to be observable at the same level as the \( CP \)-violating \( \pi NN \) coupling contribution considered here in greater detail.

### VIII. CONCLUSIONS

In summary, I considered Compton scattering in the presence of \( CP \)-violation. I defined the Compton amplitudes that possess this feature and applied the Low Energy Theorem to this amplitude. After the separation of the Born contribution that is defined by static properties of the nucleon and its EDM, I parametrized the unknown non-Born part by introducing the model-independent structure constants, the nucleon \( T \)-odd polarizabilities. I calculated these polarizabilities in the assumption that the \( CP \)-violation is due to non-zero QCD \( \theta \)-term that generates a \( CP \)-violating \( \pi NN \) coupling. I furthermore proposed an observable in Compton scattering that is directly sensitive to \( CP \)-violation that is a single-spin asymmetry with circularly polarized photon beam. Within the model for the \( T \)-odd polarizabilities used in this work, I estimated such an asymmetry to be of order \( 10^{-11} \times \frac{\omega^2}{m^2} \), \( \omega \) being the photon energy, and found that it is picked at backward angles. The above limit originates from the experimental limits on EDM translated into the strength of the QCD \( \theta \)-term. Experimentally, the \( CP \)-odd asymmetry is always accompanied by the \( P \)-violation contribution to Compton scattering which was estimated in the literature as \( 10^{-8} \times \frac{m^3}{m^2} \) and is suppressed at backward angles as \( \cos^4 \theta \). I also considered a background process that involves the linear polarization component in the photon beam which can lead to azimuthal angle-dependent asymmetry due to final state interactions that generate an imaginary part of Compton scattering. At very backward angles, such FSI can lead to asymmetries of order \( 10^{-7} \times \cos 2\phi \), with \( \phi \) the angle between the direction of the linear polarization and the scattering plane.
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