Pitfalls of the Gram Loss for Neural Texture Synthesis in Light of Deep Feature Histograms

Eric Heitz  Kenneth Vanhoey  Thomas Chambon  Laurent Belcour

{eric,kennethv,thomas.chambon,laurent}@unity3d.com

Unity Technologies

Abstract

Neural texture synthesis and style transfer are both powered by the Gram matrix as a means to measure deep feature statistics. Despite its ubiquity, this second-order feature descriptor has several shortcomings resulting in visual artifacts, ill-defined interpolation, or inability to capture spatial constraints. Many previous works acknowledge these shortcomings but do not really explain why they occur. Fixing them is thus usually approached by adding new losses, which require parameter tuning and make the problem even more ill-defined, or architecting complex and/or adversarial networks. In this paper, we propose a comprehensive study of these problems in the light of the multi-dimensional histograms of deep features. With the insights gained from our analysis, we show how to compute a well-defined and efficient textural loss based on histogram transformations. Our textural loss outperforms the Gram matrix in terms of quality, robustness, spatial control, and interpolation. It does not require additional learning or parameter tuning, and can be implemented in a few lines of code.

1 Introduction

Gatys et al. introduced neural texture synthesis \[\text{Gatys} \] and its extension to style transfer \[\text{Gatys}\]. The goal of neural texture synthesis is to synthesize a new (possibly larger) texture without verbatim copying. One of the key components to achieve this is a loss function that captures the essence of what is a texture. Gatys et al. discovered that the statistics of the feature activations in pretrained Convolutional Neural Networks (CNNs), represented by their Gram matrices, yield impressive results compared to prior work \[\text{WKL}\]. Furthermore, its simplicity and immediate usability without tedious training made it attractive teaching material for countless classes or online tutorials. It is nowadays ubiquitous in neural texture synthesis and style transfer \[\text{Gatys} \], \[\text{GBHS}\], \[\text{ULV}\], \[\text{LGX}\], \[\text{LFY}\], \[\text{Sne}\], \[\text{SCO}\], \[\text{ZB}\], \[\text{YBS}\].

Nonetheless, the Gram-matrix loss is subject to several limitations in quality or textural control \[\text{RWB}\], \[\text{SCO}\], \[\text{KSS}\], \[\text{LZY}\] that brought recent works to switch to more sophisticated paradigms such as training Generative Adverserial Networks (GANs) \[\text{BJV}\], \[\text{ZB}\], \[\text{FAW}\], or adding multiple other loss functions. While these limitations are well-known experimentally, we are not aware of a true in-depth study of why they occur. In this paper, we make the point that several of these shortcomings can be easily understood and fixed by considering the histogram of the deep feature activations. The insights gathered by looking at the histogram allow us to revisit and significantly improve this approach without compromising its original simplicity:

- In Section 3, we argue that there is no reason to stick with the Gram matrix, which is only a second-order statistic, i.e. an incomplete descriptor of the deep feature histogram. We reach the same conclusion as color-transfer methods: transferring the full histogram significantly improves the visual quality and robustness of neural texture synthesis.
We place our study in the context of Gatys et al. single well-defined loss function. It results in contrast. The Gram loss is an incomplete statistical descriptor for texture. Thanks to, the common point of these sections is that the insights into the problems and the resulting solutions all boil down to histogram transformations. For this purpose, we transpose the slicing algorithm typically used for color histogram transfer [PKD05] to the goal of deep feature histogram transfer. We end up with a single well-defined loss function that fits all needs.

Finally, we believe that what makes our analysis and solution appealing is its quality/complexity ratio. Typical improvements to Gatys et al.’s original method are often elaborated, math-heavy and not always truly insightful. In contrast, histogram transformations are simple to understand and to implement even for non-expert readers, and outperform more sophisticated previous works.

2 The Gram-Loss Zoo in the Literature

We place our study in the context of Gatys et al.’s texture synthesis and style transfer work [GEB15, GEB16]. That is, we use a loss function computing statistics over deep features extracted from a texture using the convolutional part of a pre-trained object classifier network. Gatys et al. [GEB15] define and use the Gram-Matrix loss (or Gram loss in short), which we note \( L_{\text{Gram}} \). This formulation generates interesting results provided that \( \alpha \) and \( \beta \) has been achieved. Moreover, the 1D histogram loss \( \alpha L_{\text{hist1D}} \) uses a histogram discretization relying on binning. The number of bins is yet another sensitive parameter to tune: insufficient bins result in poor accuracy while the opposite results in vanishing gradient problems. We also propose a histogram loss in Section 3, but will define and optimize for it in multiple dimensions directly, requiring no other losses to combine with.

| Quality | Periodicity | Interpolation | Miscellaneous |
|---------|-------------|---------------|---------------|
| [RWB17] | \( \alpha L_{\text{Gram}} + \beta \sum L_{\text{hist1D}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{PSD}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{DCorr}} + \gamma L_{\text{Div}} + \delta L_{\text{Smooth}} \) |
| [LX16]  | \( \alpha L_{\text{Gram}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{DCorr}} + \gamma L_{\text{L1}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{Discr.}} \) |
| [SCO17] | \( \alpha L_{\text{Gram}} \) | \( \alpha L_{\text{Gram}} + \gamma L_{\text{L1}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{Discr.}} + \gamma L_{\text{L2}} \) |
| [ZBB+18] | \( \alpha L_{\text{Gram}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{Discr.}} \) | \( \alpha L_{\text{Gram}} + \gamma L_{\text{L2}} \) |
| [YBS+19] | \( \alpha L_{\text{Gram}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{Discr.}} + \gamma L_{\text{L2}} \) | \( \alpha L_{\text{Gram}} + \beta L_{\text{Discr.}} + \gamma L_{\text{L2}} \) |

The Gram loss is an incomplete statistical descriptor for texture [RWB17]. It results in contrast oscillations, especially when producing outputs larger than inputs. As a solution, Risser et al. propose to sum \( n \) 1D histogram losses, one for each feature independently [RWB17]. As this misses out on cross-feature correlations, which are well captured (up to the second order) by Gram matrices, they need to combine it with \( L_{\text{Gram}} \). This formulation generates interesting results provided that careful tuning of the relative weights (here: \( \alpha \) and \( \beta \) has been achieved. Moreover, the 1D histogram loss \( L_{\text{hist1D}} \) uses a histogram discretization relying on binning. The number of bins is yet another sensitive parameter to tune: insufficient bins result in poor accuracy while the opposite results in vanishing gradient problems. We also propose a histogram loss in Section 3, but will define and optimize for it in multiple dimensions directly, requiring no other losses to combine with.
The Gram loss does not capture spatial structure such as periodicity \cite{Liu2016,Lu2017,Zhang2018}. Liu et al. add a loss on the power spectrum of the texture so as to preserve the frequency information \cite{Liu2016}. Sendik et al. propose to add a loss function capturing deep feature correlations with shifted versions of themselves, which makes the optimization at least an order of magnitude slower \cite{Lu2017}. For stability reasons, they added two additional regularizing losses to the equation. All losses require painstaking tuning of both inter-loss weights and inter-layer weights within each loss. In Section 4 we propose to add periodicity constraints in our nD histogram loss in order to solve this problem in a single loss term.

The Gram loss does not interpolate well \cite{Li2017,Yin2019,Yin2020}. While dedicated solutions were proposed, we are unaware of any clear explanation of why Gram matrix interpolation fails. Li et al. build a dedicated auto-encoder architecture in which they modify the deep features by a whitening and coloring transform, then decode it with a new decoder per layer specifically trained at transforming features back to textures \cite{Li2017}. Yu et al. build a GAN combining an adversarial with $L_{\text{Gram}}$ and then still interpolate in the naive (and erroneous) way \cite{Yin2019}. Xue et al. propose an alternative parametric interpolation of the deep statistics based on optimal transport of Gaussians \cite{Xue2020}. The experiment in appendix C shows that the success of their method is largely due to not following Gatys et al.’s advice of using a normalized VGG-19. Why interpolating deep feature statistics fails is thus still not yet understood nor solved. In Section 5 we propose an explanation and solution to the interpolation problem that works in the classical setting and with a single loss function.

In our work, we build an understanding of what is captured by the Gram loss, statistically, and come to the conclusion that Gram matrices do not capture a sufficient set of statistics to avoid the aforementioned issues. As opposed to above-mentioned approaches, we build on a single well-defined loss fits all needs philosophy, so as to avoid tedious tuning of meta-parameters and complex architectures. We are able to achieve this by analyzing the problem through the spectrum of the multi-dimensional (nD) histogram of deep features.

Notation. In layer $l$, we note $M_l$ the number of pixels (spatial dimensions) and $N_l$ the number of features (depth). We note $F^l_m \in \mathbb{R}^{N_l}$ the feature vector located at pixel $m$ and $F^l_m[n] \in \mathbb{R}$ its $n$-th component ($n < N_l$). The entry $(i,j)$ of the Gram matrix $G^l \in \mathbb{R}^{N_l \times N_l}$ of layer $l$ is the second-order cross-moment of features $i$ and $j$ over the pixels:

$$G^l_{ij} = \mathbb{E} \left[ F^l_m[i] F^l_m[j] \right] = \frac{1}{M_l} \sum_m F^l_m[i] F^l_m[j].$$  

The Gram loss between two images $I$ and $\tilde{I}$ is defined by:

$$L_{\text{Gram}}(I, \tilde{I}) = \sum_l \frac{1}{N_l^2} \left\| G^l - \tilde{G}^l \right\|^2,$$  

where $G^l$ (resp. $\tilde{G}^l$) is the Gram matrix of the deep features extracted from $I$ (resp. $\tilde{I}$) at layer $l$. In practice, we use the VGG-19 network \cite{Simonyan2015} and compute our statistics on the output of all but the last few convolution layers (i.e. ranging from block1_conv1 to block5_conv2) unless stated otherwise. $N_l$ varies from 64 to 512 within VGG-19.

Figure 1: Visualizing feature vectors. We visualize two features of the same layer with the colors red and blue. Each pixel can be seen as a point in a 2D feature space and the feature histogram of this layer is the point cloud given by all the pixels. The loss $L_{\text{Gram}}$ optimizes the features to be distributed along the same major directions but this is insufficient to represent the precise shape of the histogram. A complete histogram loss $L_{\text{hist}}$ should guarantee that the feature vectors are exactly the same.
3 How can a Loss Capture the Complete Set of Textural Statistics?

Many works observe that texture synthesis using the Gram-matrix loss show various artefacts (cf. Sec. 2). In Figure 4, we illustrate one of them: contrast oscillation arising especially when synthesizing images larger than the input. Risser et al. document why this is directly related to the incompleteness of this loss as a statistical descriptor for texture \cite{RWB17}. Indeed, the Gram matrix encodes second-order statistics (cross-moments) of intra-layer deep features. This is rather arbitrary and we question the choice of second-order statistics. Why not include higher-order (cross-)moments? Figure 1 intuitively illustrates that second-order statistics can only capture the main directions and extent of nD features, and not its full distribution: having an identical Gram matrix does not prevent from having different feature sets, thus different textures (Figure 1-middle).

We wish to build a single well-defined loss capturing the full essence of a texture. We argue that it is necessary for a well-defined textural loss to capture the full nD distribution in a position-agnostic manner, including (cross-)correlations of any order. The nD histogram is a sampling of this full distribution. We wish our loss to capture the difference between such sets in an efficient manner, as in Figure 1-right. A practical solution to this problem was proposed by Pitie et al. in the context of color transfer \cite{PKD05}: the full histogram loss is defined as the expectation of the histogram losses of marginal 1D distributions over the set of nD directions. More recently, this idea has been reinvented in the context of optimal transport: histogram distances can be measured with the Wasserstein Distance. Much like Pitie et al. \cite{PKD05}, the Sliced Wasserstein Distance is the expectation over random 1D marginals \cite{KMR18, DZS18, WHA19}.

```
# slicing
Vns = random_directions()
def Slicing(F):
    # project each pixel feature onto directions
    proj = dot(F, Vns)
    # flatten pixel indices to [H \times W]
    H, W, N = proj.shape
    proj_flatten = reshape(proj,(H*W,N))
    # sort projections for each direction
    return sort(proj_flatten, axis=0)

def HistogramLoss(F, F_):
    diff = Slicing(F) - Slicing(F_)
    return mean(square(diff))
```

Figure 2: Implementation of a full nD histogram loss with slicing. To compute the sliced histogram loss, we project nD features onto random directions, sort the 1D projections, and compute the $L^2$ difference between the sorted lists. The variable directions is a matrix whose columns are normalized random directions in feature space. The depiction on the right is for a single vector $V$.

To obtain a single nD histogram loss, we thus compute the expectation over many directions of the 1D histogram distance after projecting the nD feature points onto these directions:

$$L_{\text{hist}}(H^l, \tilde{H}^l) = \mathbb{E}_V[L_{\text{hist1D}}(H^l_V, \tilde{H}^l_V)], \tag{3}$$

where $H^l_V = \{<F^l_m, V>\}, \forall m$ is the unordered scalar set of dot products between the $m$ feature vectors $F^l_m$ and the direction $V \in \mathbb{R}^N$. We choose directions randomly on the unit nD hypersphere, thanks to which feature cross-correlations are captured. Also, in order to avoid discretization (i.e. binning), we define the 1D histogram loss continuously on an unordered set of scalars $S$ as the element-wise $L^2$ distance over this list of elements after ordering:

$$L_{\text{hist1D}}(S, \tilde{S}) = \frac{1}{|S|} \|\text{sort}(S) - \text{sort}(\tilde{S})\|^2. \tag{4}$$

Our implementation in Figure 2(left) shows that is boils down to projecting the features on random directions (i.e. unit vectors of dimension $N$), sort the projections and measure the $L^2$ distance on the sorted lists, as depicted in Figure 2(right). We obtain a single and homogeneous textural loss that we sum up over the layers:

$$L(I, \tilde{I}) = \sum_l L_{\text{hist}}(H^l, \tilde{H}^l). \tag{5}$$
**Statistical completeness.** This texture loss brings many benefits. It is defined to minimize histogram loss \([KMR18, DZS18, WHA + 19]\), which is the ultimate position-agnostic statistic: it encodes the full distribution, encompassing correlations of any order. Also, it encompasses both the Gram matrix loss and the per-feature 1D histogram loss of Risser et al. \([RWB17]\): if our loss is zero, theirs is too. Figure 3 shows this for \(L_{\text{Gram}}\). Therefore it does not need additional losses to be well-behaved and it overcomes the qualitative issues of the Gram loss (see Figures 4 and 5).

This loss comes at the cost of a contained increase in computation time of about \(2.5 \times\) when compared to \(L_{\text{Gram}}\): Figure 4 (left) took 4min25s to optimize with \(L_{\text{Gram}}\) and 10min49s with \(L_{\text{hist}}\) using 20 steps of SciPy’s L-BFGS-B implementation (this is overkill and for timing purposes only, good results appear earlier) in Python and Tensorflow 2.0 on an Intel Core i7 CPU and NVidia Titan RTX 2080 GPU. This factor that this is a small price to pay given the increased quality and ease of implementation and setup (i.e. no parameter tuning).

Finally, note that we compute expectations over \(N_l\) random directions, \(i.e.\) as many as there are features in layer \(l\), which we found largely sufficient (note that this can be reduced if one wishes to decrease computation times). To create images larger than the reference texture, we simply duplicate each entry in the latter’s sorted list when evaluating the loss of Equation 4.

**Figure 3:** Loss curves for the images of Figure 4-Left: optimizing for \(L_{\text{Gram}}\) (blue) or \(L_{\text{hist}}\) (red). We monitor the evolution of the values of \(L_{\text{Gram}}\) (left) and \(L_{\text{hist}}\) (right) in either case. \(L_{\text{hist}}\) encompasses \(L_{\text{Gram}}\): minimizing \(L_{\text{hist}}\) impacts the value of \(L_{\text{Gram}}\). The opposite is not true.

**Figure 4:** Texture synthesis. Optimizing textures by minimizing \(L_{\text{Gram}}\) results in contrast variation and other artifacts. Conversely, our textural loss \(L_{\text{hist}}\) avoids these problems.

**Figure 5:** Style transfer. Just like for texture synthesis, we note a superior quality achieved by \(L_{\text{hist}}\) over \(L_{\text{Gram}}\). To make a fair comparison we did not add a content loss with a tuneable parameter like \(\text{GEB16}\). We simply optimize the content image with the texture losses.
How to Homogeneously Unify Spatial Control and Texture Statistics?

Being position-agnostic is a feature for a texture loss as it allows to generate visual variety. Our histogram loss presented in Sec. 3 measures the full set of position-agnostic statistics. Similarly to $L_{\text{Gram}}$, this means one has no spatial control over the synthesized texture. Textures with a strong structure, like a periodic pattern, can thus not be reproduced natively. Figure 6 illustrates the case of a checkerboard texture. Preserving the nD histogram does not preserve the checkerboard pattern (left).

Building on our single well-defined loss philosophy, we wish to introduce spatial control without adding further losses like [SCO17]. Our idea is to introduce spatial information in the histogram by adding a new dimension to the feature space that stores a spatial tag. In Figure 6, right, the 2D feature space becomes 3D and the third dimension stores a binary tag that encodes the checker pattern. As a result, the only way for the optimized features to match the histogram is to be a checkerboard. In Figure 7 we apply this concept to real textures. We let the user decide on the spatial information he wishes to encode, depending on the texture at hand.

In practice, we concatenate spatial tags to the feature vectors $(F_{m1}^1, \ldots, F_{mN}^1, \text{tag})$, concatenate 1 to the normalized projection direction in feature space $(V_1, \ldots, V_{N^L}, 1)$, and optimize for our histogram loss of Sec. 3 without further modifications. This directly results in a preservation of the period, while letting fine-scale details (like tile color, rust patterns or local noise) vary spatially. We found it sufficient to add periodicity dimensions to the first two VGG-19 layers only.

Homogeneity. We use spatial tags that are strictly larger than the other dimensions in feature space such that the sorting groups the pixels in clusters that have the same tag. The tags only change the sorting order and vanish after subtraction in Equation (4). This is equivalent to solving separate $nD$ histogram losses for each cluster, but it is more practical since it requires no more than a concatenation. As a result, the introduction of the spatial dimension does not break the homogeneity of our loss that remains a feature-space $L^2$ between sorted features and no meta-parameter tuning is required.

Figure 6: Texture synthesis with spatial constraints. Histogram matching is not sufficient to reproduce a checker pattern. We enforce it by adding a spatial dimension to the feature histogram.

Figure 7: Texture synthesis with spatial constraints. $L_{\text{hist}}$ straightforwardly accounts for spatial tags concatenated to the deep features. Top: 1D periodic tags (color-coded). Bottom: binary tags. We show comparisons with [SCO17] in appendix A.
5 How to Ensure Texture Statistics Consistency across Layers?

Parametric interpolation is the process of interpolating two textures in its parametric space, i.e. its statistical description. The Gram matrices of deep features are known to form a bad interpolation space: Figure 8 shows that optimizing for interpolated Gram matrices results in a heterogeneous texture composed of patches of either original textures, as opposed to an intermediate and homogeneous visual pattern, i.e. truly interpolated. Many works report this behavior and propose to remedy it with added complexity: additional regularizing losses or more complex architectures like GANs or Variational Auto-Encoders [LFY+17b, YBS+19]. Despite these reports, we are unaware of any explanation about the source of this problem. Indeed, as any second-order statistic, we expect the interpolation of the Gram matrix to be well-behaved. In this section, we conjecture that this is due to inconsistencies when interpolating multiple deep feature layers separately and not to the Gram loss. In Figure 8, we use $\mathcal{L}_{\text{Gram}}$ to unit test this hypothesis but the conclusion is the same with $\mathcal{L}_{\text{hist}}$ (see appendix D). Using our solution with $\mathcal{L}_{\text{hist}}$ achieves our single well-defined loss goal.

![Figure 8: Texture interpolation. Optimizing for $\mathcal{L}_{\text{Gram}}$ results in a patch compositing of the inputs. Forward Consistency (FC) achieves a textural interpolation with a lower residual optimization loss.](image)

The problem of forward-inconsistent interpolation. We argue that the main problem with the interpolation of deep feature statistics (e.g. the Gram matrices) is that each layer is interpolated separately and this produces inter-layer inconsistencies. Indeed, if the layers are interpolated separately there is no guarantee that the statistics of the interpolated activations at layer $l + 1$ can still match the statistics of the interpolated activations at layer $l$ after the convolution. Figure 9 illustrates this phenomenon. In this toy example, the input layer represents an input in RGB space, the convolution acts like a color classifier, and the output layer is a one-hot-encoded color. Interpolating RGB colors and one-hot-encoded colors do not produce the same results and the interpolated input and output are inconsistent. On a larger scale, i.e. when taking into consideration the numerous convolutions and the non-linearities within VGG-19, the inconsistencies are probably widespread. As a result, the optimizer converges towards a composition of the input images. In Figure 8 the residual Gram losses of these composed interpolations is systematically larger than the residual Gram losses obtained when optimizing the input images. This numerically objectifies that no real image can truly satisfy the Gram loss of the inconsistent interpolated statistics.

Towards forward-consistent interpolation. Avoiding inter-layer statistical inconsistencies requires a forward-consistent (FC) interpolation that is such that the features of layer $l + 1$ are statistically likely to be the result of the convolution applied on the features of layer $l$. However, we acknowledge that this claim is rather an intuition than a well-defined proposition. What does “statistically likely” mean in this context? Formalizing the concept of FC interpolation and providing algorithms with mathematical guarantees can be approached in multiple ways and deserves an in-depth study per se.

Empirical approach. In order to support the idea of forward-consistent interpolation, we propose a simple practical approach that already significantly improves the interpolation. Our idea is to interpolate according to the chosen statistic (e.g. nD histograms) in each feature layer before computing the next layer’s features, i.e. we compute a feed-forward interpolation. In practice, we add a histogram interpolation block after each convolutional layer (HI in Figure 10 detailed on the right-hand side). This block takes the nD features of the two inputs to interpolate and operates a nD histogram transfer to one another with the algorithm of Pitie et al. [PKD05], then interpolates each nD features with its
histogram-transferred variant, using an interpolation parameter $p \in [0, 1]$. As a result, we now have two new nD features that have the same interpolated nD histogram but different spatial arrangements. We obtain a space of deep features that is more likely to be consistent and a real input texture that matches such a whole network’s deep features and statistics is more likely to exist. While we cannot provide a formal guarantee that the layers’ statistics are perfectly consistent, our experiments show significant numerical and visual improvements. In Figure 8, the FC interpolated results obtained with our approach are homogeneous intermediate textures, as expected. Furthermore, the residual Gram loss of the interpolated images is now closer to the residual Gram losses obtained with the input images. It means that the interpolated statistics do not exhibit significant unsolvable inconsistencies anymore, which makes this interpolation arguably more well-defined.

### Figure 9: Forward-inconsistent interpolation.
We consider a toy example in which an RGB input image is convolved by color detector filters. Outputs are then one-hot encoded for readability. Convolving the interpolated input generates features that are statistically different than the interpolated output. This results in inter-layer inconsistencies.

### Figure 10: Empirical forward-consistent interpolation.
Instead of interpolating the layers separately, we match their statistics with a histogram interpolation in a feed-forward way. That is, the histogram interpolation module (right) ensures that both input images are first transferred to the same (interpolated) histogram while keeping their specific spatial pixel arrangement. Thanks to this, the interpolated outputs are more likely to be statistically consistent with the interpolated inputs.

## 6 Conclusion

We have shown that common pitfalls in neural texture synthesis can be avoided by ensuring completeness, homogeneity, and consistency of the textural statistics. With multi-dimensional histogram matching, we easily capture complete statistics (Sec. 3), introduce homogeneous spatial control (Sec. 4), and ensure inter-layer consistency (Sec. 5). The major outcome is a single well-defined loss function that captures the essence of visual texture, does not require any other loss to be stable, is straightforward to implement and practical to use in an optimisation framework. With this loss function, we synthesized more qualitative textures, handled periodic textures, and interpolated cleanly between textures. Unlike many related work, this can be achieved all without designing complex network architectures, optimisation methods or sums of loss functions. Many loss functions that were combined with the Gram loss can thus be simplified and improved using our loss, which opens a field for future work. Given its simplicity, we hope our loss will be as widely adopted as the Gram loss.
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A Training TextureNet with our Histogram Loss

In our paper, we have shown the use of our nD Histogram loss $L_{\text{hist}}$ in the scope of the optimization of a single image using the backpropagation algorithm, as introduced by Gatys et al. [GEB15]. Our loss function can be used as is with no modification to train a generative convolutional neural network that outputs textures, as introduced by Ulyanov et al. [ULVL16].

To show that our loss function can be used as is, we produced the results of Figure 11. For each texture, we train a TextureNetV1 [ULVL16] using for loss function $L_{\text{hist}}$. This network takes a multiscale noise as input and produces a texture as output. We train with a mini-batch size of 8 and update the directions of projection ($V$ in our paper) after each iteration. We train for $3k$ iterations using a learning rate of $10^{-3}$, which took about an hour: this could be optimized for faster convergence.

The advantage of a generative and fully-convolutional network is that it is pre-trained for a target texture, then can be used for very fast synthesis of an arbitrary-sized output texture. Hence, we trained using textures of size $256 \times 256$ and generated output textures of size $256 \times 2048$ in 58ms on average (on an NVidia Titan Xp GPU) in Figure 11. Output variety is provided by varying the input noise. This can be seen in our banners: the texture varies along the width of the banner.

Figure 11: Generative Texture Synthesis. Left: reference textures of size $256 \times 256$. Right: textures of size $256 \times 2048$ synthesized using TextureNetV1 [ULVL16] pre-trained to minimize $L_{\text{hist}}$ loss of the generated image (once network per texture type).
The main previous work related to parametric spatial control in neural texture synthesis is the deep-correlation method of Sendik and Cohen-Or [SCO17]. Their method adds a spatial correlation loss to the classic Gram loss $L_{\text{Gram}}$ (cf. Table 1 in the main paper). This spatial correlation loss allows to represent spatial constraints such as periodic structures.

The deep-correlation method is theoretically more general than our histogram loss $L_{\text{hist}}$ augmented with the spatial tags (as explained in our Section 4). However, it has a meta-parameter to tune and arguably implies a much more involved implementation. Furthermore, despite it theoretical generality, a large part of the results provided in the deep-correlation paper happen to be covered by our method.

In Figure [12] we show that our simpler method provides a solution for these cases and arguably achieves superior visual results. The advantage of our approach is that the user only needs to provide a tag that encodes the period of the structure to represent, which is a more intuitive information to provide compared to fine-tuning a meta-parameter. Note that providing the period could even be automated by extracting it from the image’s Fourier Spectrum.

Finally, our point is that spatial control should be introduced homogeneously. What makes the deep-correlation method harder to use is that it introduces spatial control as an additional loss. This breaks the homogeneity of the textural loss and the result is a compromise between spatial control and texture quality. In contrast, our method incorporates the spatial control without compromising the definition of the texture. This is because the spatial tags only affect the sorting order in the computation of $L_{\text{hist}}$ and hence do not break its homogeneity as a feature-space $L^2$ loss between sorted features. There is thus no compromise to fine-tune and intuitive information such as periodic structure can be handled in a simple way.
Figure 12: Comparison against Sendik and Cohen-Or’s deep-correlation method [SCO17]. We extracted all the inputs and their results from their paper.
C Case Study of Xue and Wang’s Interpolation Method [XW20]

The main previous work related to parametric interpolation of deep features is the method of Xue and Wang [XW20]. Their method uses a Fourier transform to interpolate the activations of each layer. The property of their approach is that if the activations were a Gaussian random field then the Gaussian histogram of the deep features would be optimally transported.

Their methods operates on each layer separately and thus do not consider forward consistency as explained in our paper. Although the results presented by Xue and Wang seem to indicate that forward consistency is not necessary with their interpolation scheme, we argue that, on the contrary, their method lacks robustness precisely because of this problem. We argue that this problem is not apparent in their results because they did not follow Gatys et al.’s advice of normalizing the pretrained VGG-19 before using it for a textural loss. Our experiments indicate that using their method with a normalized VGG-19 reveals problems that can be explained with the concept of forward consistency.

Normalized VGG-19. Gatys et al. explain that it is important to normalize the layers of the pretrained VGG-19 before using it for neural texture synthesis [GEB15]. They should be scaled such that the average value of the activations is 1 on natural images. This can be done without impacting the propagation through the network if the activations are ReLUs, which is the case for VGG-19. Normalizing the layers guarantees that they all contribute equally to the textural loss. This is important to achieve high-quality outputs.

Experiment. In Figure 13, we test the method of Xu and Wang in three different setups:

- Fig. 13-(a), we apply their interpolation [XW20] on 5 layers (block1_conv1, block1_pool, block2_pool, block3_pool, block4_pool) of VGG-19 without normalization. → They confirmed this is the implementation that Xu and Wang used for all their results.
- Fig. 13-(b), we apply [XW20] on 12 layers (block1_conv1, block1_conv2, block2_conv1, block2_conv2, block3_conv1, block3_conv2, block3_conv3, block3_conv4, block4_conv1, block4_conv2, block4_conv3, block4_conv4) of VGG-19 without normalization.
- Fig. 13-(c), we apply their interpolation [XW20] on 5 layers (block1_conv1, block1_pool, block2_pool, block3_pool, block4_pool) of VGG-19 with normalization.

Analysis. Fig. 13-(a) is the default setup of Xue and Wang. In this configuration, we obtain results similar to theirs. We noticed that in this configuration, the loss is largely driven by a single layer. The error of block3_pool is up to $10^3$ larger than of the other layers: the contribution of this layer is dramatically predominant in the loss value. However, having a single layer obfuscate all others in their contribution to the loss value does not provide the highest textural quality, which is why Gatys et al. recommended normalizing VGG-19 in the first place [GEB15].

In Fig. 13-(b), we use more layers and in Fig. 13-(c) we normalize the layers. The effect of using more layers or normalizing them is to have more layers contribute to the textural loss, as opposed to having a single layer dominating it. By doing this, we should logically expect an increase in quality as explained by Gatys et al. Unfortunately, the interpolated results show the opposite: the more layers contribute, the lower the quality: the patch-composition artifact (which we document in the main paper) appears.

While this might appear counter-intuitive, the concept of forward consistency provides a plausible explanation to this problem. Xue and Wang’s method is not forward-consistent and is likely to create unsolvable statistical inconsistencies between layers. These inconsistencies are not important if a single layer dominates the loss. However, the more layers contribute to the loss, the more the inconsistencies become important and the more the quality decreases. This is why the interpolation method of Xue and Wang works optimally in the setup where a single layer dominates the texture loss, which is also the setup where the loss provides the lowest textural quality.

In comparison, our forward-consistent method shown in Fig. 13-(d) works with a large number of layers that are normalized and thus allows to interpolate texture with the high quality. This supports our point that the key to interpolating deep features is rather to be found in how to make inter-layer statistics coherent than in the per-layer interpolation modality.
Figure 13: Analysis of Xu and Wang’s method [XW20]. (a) is the same setup as in Xu and Wang’s article. (b) and (c) are variants that we propose to test their method. (d) is our empirical forward-consistent (FC) interpolation. Rows 1 and 2 uses inputs taken from Xu and Wang’s article. Rows 3 and 4 uses inputs taken from our article.
D Analysis of our Empirical Forward-Consistent Interpolation

In this section, we further validate the use of Forward-Consistency (FC) as a mean to achieve stable and consistent interpolation of images.

Experiment. For each result figure, we display the outcome of optimizing an input noise with $L_{\text{Gram}}$, $L_{\text{Gram}}$ with Forward-Consistency, $L_{\text{hist}}$, and $L_{\text{hist}}$ with Forward-Consistency. For each result, we display different interpolation percentages. Note that the leftmost (100/0) and rightmost (0/100) images are the input images to the interpolation and not optimized images. Each image is the result of 15 steps of optimization through L-BFGS.

Analysis. As expected, interpolation without Forward-Consistency interpolates the spatial footprint of the input texture. For example, texture A covers $X\%$ of the interpolated image and texture B covers $100 - X\%$. This issue happens regardless of the loss we use, be it $L_{\text{Gram}}$ or $L_{\text{hist}}$. Using Forward-Consistency, we coherently blend the textures together, regardless of the loss we use.

Note however that, even with forward consistency, using $L_{\text{Gram}}$ is subject to contrast oscillations but this is the typical shortcoming of $L_{\text{Gram}}$ that occurs even without interpolation and that $L_{\text{hist}}$ overcomes. Besides the classic shortcomings of $L_{\text{Gram}}$, the interpolation itself appears to be similar when the optimization is done with $L_{\text{Gram}}$ and $L_{\text{hist}}$.

Again, this observation supports our point that the key to interpolating deep features is to be found in how to make inter-layer statistics coherent rather than in the per-layer interpolation modality or the per-layer loss function.
