A New Radiation Hydrodynamics Code and Application to the Calculation of Type Ia Supernovae Light Curves and Continuum Spectra

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Abstract

A new, fully dynamic and self-consistent radiation hydrodynamics code, suitable for the calculation of supernovae light curves and continuum spectra, is described. It is a multigroup (frequency-dependent) code and includes all important $O(v/c)$ effects. It is applied to the model W7 of Nomoto, Thielemann, & Yokoi (1984) for supernovae of type Ia. Radioactive energy deposition is incorporated through use of tables based upon Monte Carlo results. Effects of line opacity (both static or line blanketing and expansion or line blocking) are neglected, although these may prove to be important. At maximum light, models based upon different treatments of the opacity lead to values for $M_{B,max}$ in the range of -19.0 to -19.4. This range falls between the values for observed supernova claimed by Leibundgut & Tammann (1990) and by Pierce, Ressler, & Shure (1992).

Subject headings: supernovae — radiation hydrodynamics
1 Introduction

Until very recently, no unified approach to the modeling of supernovae light curves and spectra has been pursued. For stars (even those with steady winds) sophisticated treatments of the atmosphere and line formation are by now routine. In the case of supernovae there is dynamical coupling of the radiation and the matter (at maximum light $t_{\text{diffusion}} \sim t_{\text{dynamic}} \sim R/v \sim 10^6$ s), much of the ejecta are semi-transparent (and the radiation and matter are not in thermal equilibrium), there are non-trivial $O(v/c)$ effects, and there are uncertainties about the opacity. Add to this uncertainties about the specificity of the model of the exploded star and restrictive limitations on computer resources, and it is unsurprising that few attempts have been made to develop and use a fully-integrated and self-consistent radiation hydrodynamics code (but see Falk & Arnett 1977). Instead, depending on the goal of the study, the radiation dynamics has been reduced to either gas dynamics with radiation diffusion or radiative transfer in the approximation of a quasi-steady wind. The former approach (see, for example, Sutherland & Wheeler 1984) has been used to calculate light curves for assumed models of the explosion, for which the key issues are rise time, maximum brightness and color, and rate of decline from the peak. In such calculations the opacity is often taken to be gray, uniform, and constant (typically, $\kappa \sim 0.1$ cm$^2$ g$^{-1}$ independent of density, temperature, composition, and frequency). The latter approach (Branch 1990, Harkness 1986, Wheeler & Harkness 1990) has been used to generate realistic synthetic line spectra that can serve as diagnostics for the velocity and density profiles of the supernova. For both the work of Branch, in which the Sobolev or escape-probability approximation is adopted in calculating P Cygni profiles, and the work of Harkness, which solves the relativistic, comoving frame radiative transfer equations as a function of frequency, an inner boundary condition is required. This must either be guided by observations or be given by some other, dynamical, calculation.
Recently Ensman (1991) (see also Ensman & Burrows 1992), Höflich, Khokhlov, & Müller (1991) (see also Khokhlov, Müller, & Höflich 1992, 1993; Höflich, Müller, & Khokhlov 1993) have presented calculations based upon new radiation hydrodynamics codes. In both cases the dynamical equations are solved in the comoving frame and include all $O(v/c)$ effects. Ensman works with the frequency-integrated moment equations and Rosseland mean opacities. The Eddington factor (which relates the second radiation moment to the zeroth moment and is essential for closure) is obtained by solving a steady model transfer equation, which also provides the surface boundary condition for the flux. In effect this code is a "two-temperature" code, one each for the matter and radiation, and is suited to light curve calculations but not to the calculation of colors and continuum spectra. The code of Höflich et al. (1991) deals with the full frequency-dependent equations, and thus yields both light curves and continuum spectra. It also incorporates the expansion opacity (Karp et al. 1977). The expansion of the matter is assumed to be simply homologous; consequently no dynamical interaction of the radiation and gas is allowed. This is probably an adequate approximation for Type Ia supernovae (SNIa’s) but may be vitiated for Type II supernovae (SNII’s) models based upon extended red supergiant progenitors.

In this paper, we present a one-dimensional, comoving frame, Lagrangian radiation hydrodynamics code that will (1) solve a monochromatic model radiative transfer equation (correct to $O(v/c)$) to obtain the frequency-dependent Eddington and sphericity factors and the flux surface boundary condition; (2) solve the frequency-integrated radiation moments equation – correct to $O(v/c)$ – for radiation energy density and flux; (3) solve the system of monochromatic radiation moments equations to form the radiation energy and flux spectra; (4) use the computed radiation and flux spectra and precalculated, composition sensitive opacity tables, averaged in a series of frequency bins, to form time-dependent frequency means of opacities; and (5) coupled with an equation of state for ideal electron/ion gas and a self-consistent solution of the Saha equation for ion balance, solve the radiation hy-
drodynamics equation for gas energy and radial velocity. This represents an improvement over both the work of Ensman and Höflich et al. in that the treatment is both fully dynamic and frequency dependent. In addition, the frequency means of the opacity are more self-consistent because the computed radiation energy and flux spectra were used in these means. In §II, we summarize the formalism employed in our light curve calculations and the important physics inputs to the radiation hydrodynamics code.

There are several reasons to calculate the light curves for models of SNIa’s using the best radiative dynamics code and the best input physics. Observationally, the light curves of SNIa’s appear to be remarkably homogeneous (see the atlas of Leibundgut et al. 1991b) and are strong candidates as “standard candles.” There continues to be some dispute over the normalization of these objects. On the one hand, Leibundgut & Tammann (1990) find for SNIa’s in the Virgo cluster that at B maximum light $M_B = -19.8 \pm 0.12$. (Jacoby et al. 1992 survey the calibration of SNIa’s and reports similar results for other methods and samples.) On the other hand, a new determination of the distance to IC 4182 based upon I-band and K-band photometry of the brightest red supergiants implies $M_{B,\text{max}} = -18.8 \pm 0.34$ for SN 1937C. These two contrasting results correspond to approximate values for $H_0$ of 50 and 85 km s$^{-1}$ Mpc$^{-1}$. Thus light curve calculations (and by this is meant, at the least, light curves for each of the principle bands, U, B, V, ...) have the potential to constrain, perhaps reject, either the assumed model for the SN or a range of values for $H_0$ if SNIa’s have uniform properties. The other major reason for calculating SNIa light curves with modern codes is that the paradigm for SNIa’s is well entrenched and well defined. The prevailing paradigm is the explosion of a carbon-oxygen white dwarf near the Chandrasekhar limit. For this model it is essential that $\sim 1.0 M_\odot$ of C/O be partially or fully incinerated in order to explain the velocity profile implied by the observed spectra (see, for example, Branch et al. 1982 for the case of SN 1981B). Furthermore, $\gtrsim 0.1 M_\odot$ must be only partially incinerated to intermediate mass elements. What remains unresolved in the model is the exact amount of C/O fully
incinerated to $^{56}\text{Ni}$. Is the material consumed by a detonation (Arnett 1969), a deflagration (Nomoto, Sugimoto, & Neo 1976) or a deflagration that turns into a detonation (Woosley 1990, Khoklov 1991)? The total mass of $^{56}\text{Ni}$, through energy input by the trapping of $\gamma$-rays released in its decay to $^{56}\text{Co}$ and thence to $^{56}\text{Fe}$, is responsible for the “normalization” of the light curve; the rise time and width of the peak reflect the kinetic energy of the ejecta which is already constrained by the observed velocities. Light curve calculations based upon overly simple approximations to the opacity (see, for example, Sutherland & Wheeler 1984) lead to the conclusion that $M_{Ni} \sim 0.8 \ M_\odot$ if $M_{B,max} \sim -19.8$. This is to some extent confirmed by the semiempirical claim advanced by Arnett, Branch, & Wheeler (1985) that, at maximum light, the bolometric luminosity equals the instantaneous rate of energy deposition by $^{56}\text{Ni}$ decay. These conclusions need to be tested by more rigorous calculations, which may in turn lead to tighter constraints on the nature of the deflagration and/or detonation and the amount of $^{56}\text{Ni}$ produced.

In §III, we describe briefly the initial hydrodynamic model used in our calculation and present the results of a few test runs using this model for both “gray” and full frequency-dependent computations.

2 Description of the code

The code described here is a one-dimensional (spherically symmetric), Lagrangian one. All equations are expressed and solved in the comoving frame (actually a series of frames, each instantaneously at rest with respect to the local matter). The radiation hydrodynamics equations for the matter and radiation, and the radiative transfer equations, are solved simultaneously following the prescription of Mihalas & Mihalas (1984). All terms important to $O(v/c)$, where $v$ is the radial velocity and $c$ is the speed of light, are retained. The emergent flux and its spectrum are transformed to the frame of the observer only at the very
2.1 Radiation Hydrodynamics

In solving radiation hydrodynamic equations for SN light curves, a large range of optical depth must be covered: from a mostly optically thick envelope shortly after the explosion to almost transparent ejecta at late times. Near maximum light, the photosphere has already receded significantly into the ejecta. It is not correct to assume at this point that the radiation is still in thermal equilibrium with the gas, a condition when the diffusion approximation applies. In order to faithfully follow the dynamic evolution of both gas and the radiation field, it is imperative to treat them individually, without any assumption about their thermal equilibrium. The equations for the evolution of the momentum and energy of the matter are

\[
\frac{Dv}{Dt} = -\left(\frac{1}{\rho}\right) \frac{\partial (p_g + Q)}{\partial r} + \left(\frac{\chi_F}{c}\right) F_r - \frac{Gm(r)}{r^2} \tag{1}
\]

\[
\frac{De}{Dt} + (p_g + Q) \frac{D}{Dt} \left(\frac{1}{\rho}\right) = c\kappa_E E_r - 4\pi\kappa_P B(T) + \epsilon \tag{2}
\]

where \(D/Dt\) is the Lagrangian time derivative, \(\rho\), \(e\), \(p_g\) and \(T\) are the density, energy per gram, thermal pressure and temperature in the ejecta, \(m(r) \equiv \int 4\pi r^2 \rho(r)dr\) is the total mass within the sphere of radius \(r\), and \(\epsilon\) is the heat source due to radioactive decay. The gas absorbs momentum and energy from the radiation at the rates \((\chi_F F_r/c)\) and \(c\kappa_E E_r\), respectively, and radiates energy at the rate \(4\pi\kappa_P B(T)\). \(B(T) \equiv (ac/4\pi)T^4\) is the Planck function at the matter temperature, \(F_r\) and \(E_r\) are the radiation flux and energy density, \(\kappa_E\) is the energy mean of the absorptive opacity \(\kappa_\nu\), defined in equation (18) later in this section, \(\kappa_P\) is the Planck mean of the absorptive opacity, and \(\chi_F\) is the flux mean of the total opacity \(\chi_\nu\), defined in equation (19) also later in this section. Our one departure from the conventions of Mihalas & Mihalas (1984) is that all opacities have the units \(\text{cm}^2 \text{g}^{-1}\)
rather than simply \( \text{cm}^{-1} \); that is, our opacities need to be multiplied by \( \rho \) to equal theirs. To handle shocks due to the initial model, an artificial viscosity term for zone \( k + 1/2 \) and at time \( n \),

\[
Q_{k+1/2}^n = \begin{cases} 
2a^2(v_{k+1}^n - v_k^n)^2 \rho_{k+1/2}^n & \text{for } \frac{D}{Dt} \left( \frac{1}{\rho} \right) < 0, \frac{dv}{dr} < 0 \\
0 & \text{otherwise.}
\end{cases}
\]  

(3)

is also included. Here \( a \) is an adjustable parameter, generally set at a value slightly larger than 1.

The evolution, in the comoving frame, of the frequency integrated radiation moments is given by

\[
\frac{D}{Dt} \left( \frac{E_r}{\rho} \right) + \left[ f \frac{D}{Dt} \left( \frac{1}{\rho} \right) - (3f - 1) \frac{v}{\rho r} \right] E_r = 4\pi \kappa_p B - c\kappa E_r - \frac{\partial (4\pi r^2 F_r)}{\partial m}
\]  

(4)

\[
\frac{1}{c^2} \frac{DF_r}{Dt} + \frac{1}{q} \frac{\partial (f q E_r)}{\partial r} = -\frac{\chi r \rho}{c} F_r - \frac{2}{c^2} \left( \frac{v}{r} + \frac{\partial v}{\partial r} \right) F_r
\]  

(5)

These equations, and those that follow, are essentially identical to those presented in §95 and §98 of Mihalas & Mihalas (1984); see their equations 95.18, 95.19, 98.1 and 98.2. There is one reduction that we do not do: the term in each flux moment equation with coefficient \( 2 \left( \frac{v}{r} + \frac{\partial v}{\partial r} \right) /c^2 \) is \textit{retained} for the following reasons. This term is normally dropped because it is \( O(v/c) \) relative to the other terms (except for the \( D/Dt \) term which is also of this order). However, this coefficient is essentially just \( 4/(tc^2) \) because the expansion is almost perfectly homologous, and near and definitely after the light curve peak the \( DF/Dt \) term is also \( O(1/t) \) because the radiation field is changing on a dynamical timescale.

In addition to the various opacities which require evaluation from knowledge of (or assumptions about) the energy and flux spectral profiles, there are two other elements that
must be specified: the Eddington factor \( f \) (and the related sphericity factor \( q \)) and a surface boundary condition on the relationship of the flux to the radiation energy density. [The boundary condition at \( r = 0 \) is just \( F_r = 0 \).] These factors (at frequency \( \nu \)) are defined by:

\[
f_\nu = \frac{\int I_\nu \mu^2 d\mu}{\int I_\nu d\mu}
\]

(6)

\[
\ln(q_\nu) = \int_{r_c}^{r} [(3f_\nu - 1)/(r'f_\nu)] dr'
\]

(7)

where \( I_\nu \) is the monochromatic radiation intensity at frequency \( \nu \), and \( \mu \equiv \cos \theta \) the direction cosine of light rays with respect to the outward radial direction. The “core” radius \( r_c \) that enters the equation for \( q_\nu \) is simply that radius interior to which the matter is sufficiently optically thick so that \( f_\nu = 1/3 \). These factors are to be obtained as functions of frequency, in view of the strong frequency-dependence of the opacity. To obtain these essentially geometrical factors, the approach adopted is to solve the frequency-dependent version of the model radiation transfer equation proposed by Mihalas & Mihalas (1984). This model transfer equation, which neglects ray curvature and Doppler shifts, can be solved along parallel tangent rays through zone centers. It decomposes into two equations for the symmetric and anti-symmetric combinations:

\[
j_\nu(\mu) \equiv \frac{[I_\nu(r, \mu) + I_\nu(r, -\mu)]}{2} \quad 0 \leq \mu \leq 1
\]

(8)

\[
h_\nu(\mu) \equiv \frac{[I_\nu(r, \mu) - I_\nu(r, -\mu)]}{2} \quad 0 \leq \mu \leq 1
\]

(9)

yielding:

\[
\frac{1}{c} \frac{Dj_\nu}{Dt} + \frac{\partial h_\nu}{\partial s} = \kappa_\nu \rho B_\nu(T) - \left[ \kappa_\nu \rho + (1 - 3\mu^2) \frac{v}{rc} \frac{1 + \mu^2 D\ln\rho}{c \frac{D\ln\rho}{Dt}} \right] j_\nu
\]

(10)

\[
\frac{1}{c} \frac{Dh_\nu}{Dt} + \frac{\partial j_\nu}{\partial s} = -\chi_\nu \rho h_\nu - \frac{2}{c} \left( \frac{v}{r} + \frac{\partial v}{\partial r} \right) h_\nu
\]

(11)
Here $s$ is the distance along the ray from the front-back symmetry plane ($s^2 = r^2 - p^2$ where $p$ is the impact parameter for a given ray). The boundary conditions used for these equations are that $h_\nu$ is zero at the origin and at $\theta = \pi/2$ (the symmetry plane), and that at the outer boundary ($r = R$) $h_\nu = j_\nu$ (no incident radiation at the surface). Then the frequency-dependent Eddington factor $f_\nu$ is:

$$f_\nu = \int_0^1 j_\nu \mu^2 d\mu / \int_0^1 j_\nu d\mu$$ (12)

and the ratio at the surface of flux to energy density, at frequency $\nu$ is:

$$F_\nu/(cE_\nu) = \int_0^1 h_\nu(\mu) \mu d\mu / \int_0^1 j_\nu(\mu) d\mu = \int_0^1 j_\nu(\mu) \mu d\mu / \int_0^1 j_\nu(\mu) d\mu$$ (13)

because of the surface boundary condition.

The opacities that enter equations (1) and (2) require various spectral moments of the frequency-dependent opacity $\kappa_\nu$. The model transfer equations above are suited to the calculation of geometrical factors but are oversimplified for the calculation of the spectra because of the neglect of a critical frequency derivative (see equation [16] below). To estimate the radiation energy and flux spectral profiles

$$e_\nu \equiv E_\nu / E_r$$ (14)

$$\phi_\nu \equiv F_\nu / F_r$$ (15)

it is necessary to solve the monochromatic radiation moments equation,

$$\frac{D}{Dt} \left( \frac{E_\nu}{\rho} \right) + \left[ f_\nu \frac{D}{Dt} \left( \frac{1}{\rho} \right) - (3f_\nu - 1) \frac{v}{\rho r} \right] E_\nu$$

$$- \frac{\partial}{\partial \nu} \left\{ \left[ f_\nu \frac{D}{Dt} \left( \frac{1}{\rho} \right) - (3f_\nu - 1) \frac{v}{\rho r} \right] \nu E_\nu \right\} = \kappa_\nu (4\pi B_\nu - cE_\nu) - \frac{\partial (4\pi r^2 F_\nu)}{\partial m}$$ (16)
\[
\frac{1}{c^2} \frac{DF_\nu}{Dt} + \frac{1}{q_\nu} \frac{\partial (f_\nu q_\nu E_\nu)}{\partial r} = -\frac{\chi_\nu \rho}{c} F_\nu - \frac{2}{c^2} \left( \frac{v}{r} + \frac{\partial v}{\partial r} \right) F_\nu \tag{17}
\]

The boundary conditions for these equations are that \( F_\nu = 0 \) at the origin and \( F_\nu/(cE_\nu) \) at the surface is given by equation (13). For the frequency derivative term, the “upwind” scheme in frequency space is used for stability. Once the spectrum is found, we form the energy means and the flux means of the opacity as in the following,

\[
\kappa_E \equiv \int \kappa_\nu e_\nu d\nu, \quad \tag{18}
\]

\[
\chi_F \equiv \int \chi_\nu \phi_\nu d\nu. \quad \tag{19}
\]

In principle, one could use equations (1), (2), (16), and (17) as the main system of dynamic equations and solve them together for the hydrodynamic, thermal and radiation evolution. Because all the important physics are included in equations (19) and (17), the frequency-integrated, or the bolometric, radiation energy density and flux can be readily found from \( E_\nu \) and \( F_\nu \) by integrating them over frequency. However, this will increase the computational cost tremendously. For economic reasons, we follow the suggestions in Mihalas & Mihalas (1984) and solve the “gray” (frequency-integrated) radiation moments equations instead. The main system of dynamic equations will be (1), (2), (4), and (5). At each time step, we solve them implicitly and iterate to convergence to determine the thermal, hydrodynamic, and radiation structures inside the ejecta as accurately as possible. This information is then used to solve equations (10), (11), (8), and (9) implicitly for \( j_\nu, h_\nu, E_\nu \), and \( F_\nu \). From \( E_\nu \) and \( F_\nu \), estimates for \( e_\nu \) and \( \phi_\nu \) are obtained. Although equations (4) and (5) are the frequency-integrated form of equations (16) and (17) and the solution of equations (4) and (5) should be equivalent to the solutions of equations (16) and (17) integrated over frequency, it is inevitable that numerical errors in these two forms of the radiation moments will accumulate at different speed and eventually, the difference between \( \int E_\nu d\nu \) and \( E_r \), for example, will be too big for equations (16) and (17) to be consistent with equations (4) and (5). Because it is more economical to implement more strict error
control in solving equations (4) and (5) than in equations (16) and (17), we treat solutions of equations (4) and (5) to be the “true” solution and treat any departure of $\int E_\nu d\nu$ from $E_r$ as an accumulated error on the part of $E_\nu$. When that happens, we rescale $E_\nu$ or $F_\nu$ according to $E_r$ or $F_r$, keeping the spectral profiles unchanged.

2.2 Microphysics

The microphysics components incorporated in the code are (1) the equation of state (EOS), including ionization equilibrium, for the matter, (2) the frequency-dependent opacities, and (3) the rate of energy input from radioactive decay.

For the matter, the ideal gas EOS is valid shortly after the explosion and certainly at all times when radiation transport is of interest. (At early times, during and immediately after the explosion, the EOS must allow for relativistic and partial degeneracy, etc.; for this stage a modified version of the code of Wheeler & Hansen (1971) was used. Radiation transport then is insignificant or can be handled in the diffusion approximation.) Ionization equilibrium is obtained from the Saha equation, although the partition function was simplified to include only ground state contributions. For a given composition, the free electron fraction and ionization energy are precomputed and tabulated for the density and temperature ranges appropriate to the light curve calculation.

The frequency-dependent opacities were provided by Los Alamos National Laboratory (LANL) (see Huebner et al. 1977; Magee, Merts, & Huebner 1984). For a given composition, the multigroup opacities are tabulated on a grid of 39 frequencies (0.5 eV – 200 eV) by 11 temperatures (1 – 10 eV) by 25 densities ($1.15 \times 10^{-13} – 1 \times 10^8$ g cm$^{-3}$) although the temperature-density plane is not fully sampled. Separate tables were provided for scattering and absorption. The LANL opacities are not ideal for supernovae calculations because they do not extend to sufficiently low densities or temperatures. Opacity calculations that
incorporate better physics and have a wider domain of application have been implemented (Rogers & Iglesias 1992) but tables for appropriate compositions have not yet been made available. The major shortcoming in the treatment of opacities in the present calculations is the neglect of lines. The large number of lines in the UV will be responsible for both line blanketing (an essentially “static” effect dependent upon the high density of lines and the overlap of successive lines by their natural and/or thermal widths) and the expansion opacity (Karp et al. 1977), also called line blocking. Harkness (1991) has successfully modeled the spectra for SN 1981B near maximum light with the model W7, using an approximate treatment of only line blanketing. The expansion opacity is an effective continuum opacity due to scattering by a large number of lines in a medium with a large velocity gradient and may play a significant role in the UV. The expansion opacity, even as calculated by Karp et al. (1977) by treating the expansion of the medium (relative to a chosen point) as infinite, homogeneous, and isotropic (“Hubble flow”), places extraordinary demands on computing resources. For the composition of SNIa’s, \( \gtrsim 10^6 \) lines need to be retained, the ionization and excitation of many species and levels must be obtained (for each zone in the model, and at different times as conditions change significantly), and the effects of all lines must be summed for each value of the expansion parameter \( s \propto \rho/(dv/dr) \). The necessary computer resources were not available to pursue these calculations. However, the basic assumption of Karp et al. in treating SNs ejecta in the “Hubble flow” approximation is limited. A local continuum opacity results from the sum of probabilities for scattering in lines. A nominal scattering in a strong line that contributes to the local expansion opacity at a certain frequency, however, may take place at a remote point because of the redshift required for the scattering atom. Then the assumptions of isotropy and homogeneity in the flow may be unwarranted. In short, when expansion opacity effects are important, they may not be simply calculable in terms of a local opacity.

The final microphysics ingredient in the code is the rate of energy deposition due to
radioactive decay. In the decay of $^{56}\text{Ni}$ and $^{56}\text{Co}$, the $\gamma$-rays released by the daughter nuclei Compton scatter (perhaps repeatedly) off free and bound electrons, and less commonly produce electron-positron pairs which are presumed to annihilate locally. The result of the decays is thus a number of high-energy ($\lesssim 1 \text{ MeV}$) electrons. In all light curve calculations, except possibly at late times when the ejecta are nearly transparent, it is a reasonable assumption that these energetic electrons thermalize locally since the electron mean free path is considerably shorter than that of the originating $\gamma$-ray. What remains to be determined, then, is the distribution throughout the ejecta of the energy deposited by the Compton scattering of the $\gamma$-rays. Two approaches to the calculation of the so-called deposition function have been employed in the past: (1) a pure absorption model for $\gamma$-ray transport, as advocated by Sutherland & Wheeler (1984), and (2) a detailed, and in principle exact, calculation through use of Monte Carlo methods (see Ambwani & Sutherland 1988). The latter requires many CPU cycles to achieve a reasonable level of accuracy. The former can be dealt with very efficiently by using a simple model gray atmosphere code. (The source function is just the local mass fraction of radioactive material and its distribution can be arbitrary.) We have compared the results of the two approaches (for the model gray atmosphere calculation we used the code of Swartz, Harkness, & Wheeler 1991) and found that at no epoch of interest and nowhere within the ejecta did the deposition functions differ by more than 5%, and the global means were even closer. In view of the other uncertainties in modeling SNe, it is clear that the deposition function can be calculated with sufficient accuracy with the model gray atmosphere approach; the Monte Carlo approach is overkill.

3 Calculation of Model Light Curves and Continuum Spectra

The SNIa model chosen for the calculation of light curves and spectra is a somewhat mixed version of model W7 of Nomoto, Thielemann, & Yokoi (1984). This is a carbon deflagration model that yields an acceptable light curve (as calculated in the diffusion ap-
proximation with a uniform, gray, opacity) and for which synthetic spectra based on P Cygni lines calculated for the velocity and composition profiles of W7 (the latter requires some mixing) compares favorably with those observed for SN 1972E and SN 1981B near maximum light (Branch et al. 1985). W7 is an exploded white dwarf of total mass \(1.38 \, \text{M}_\odot\) of which \(0.63 \, \text{M}_\odot\) is incinerated to \(^{56}\text{Ni}\); the total kinetic energy is \(1.3 \times 10^{51}\) ergs. The initial model comprised 172 zones and represented conditions \(\sim 3\) s after the explosion. To reduce subsequent computational effort, the model was evolved to \(t \sim 1\) hr (using gray radiative transfer with everywhere \(\kappa = 0.02\ \text{cm}^2\ \text{g}^{-1}\)) and it was then rezoned down to 40 zones. This rezoning, which conserved total thermal and kinetic energy, was nonuniform: in particular, the outermost three zones were left unaltered in order to maintain resolution out to a maximum velocity of \(2.3 \times 10^9\ \text{cm s}^{-1}\). The density and velocity profiles of the model at \(t \sim 1\) hour are shown in Figure 1. Also indicated is the distribution of zones. Even with only 40 zones, it was felt that further compromise with respect to composition was necessary. Accordingly, eight representative compositions were used, and each of the 40 zones was assigned to one of these compositions. In the actual computations, some interpolation between compositions was performed, using \(< \frac{Z}{A} >\). Though arbitrary, this interpolant proved adequate. The compositions used for our 40 zone model are given in Table 1.

Two benchmark calculations were done for our representation of W7 with a hydrodynamics code that implemented radiative transfer in the flux-limited diffusion approximation. This code is essentially the same as that used by Sutherland & Wheeler (1984). The first calculation employed a constant (throughout the ejecta, and in time) opacity of \(\kappa = 0.1\ \text{cm}^2\ \text{g}^{-1}\). This value is typical for “successful” carbon deflagration models of SNIa’s as calculated and judged in the early 1980’s. The second model used the tables of Rosseland means for the LANL continuum opacities. The essential results of these benchmarks are given in the first two lines of Table 2 and in Figure 2. The B flux and color given in the table (and plotted as B’ in Figure 2) are obtained as in Sutherland & Wheeler (1984) by the
following prescription: (1) the photosphere is found by integrating inward until $\tau = 2/3$ is reached; (2) the flux there, $L/4\pi R_{ph}^2$, is equated with that of a blackbody spectrum truncated shortward of $\lambda = 400$ nm (the temperature of this spectrum must be determined iteratively because of the truncation), and (3) the B and V fluxes are computed for this emitting area and temperature. This prescription increases the B flux by $\sim 1$ mag. However, without this or a similar rule, there would be no way to mimic the observed strong uv deficiency and the calculated colors would always be significantly too red. Part of the motivation for the new code described in this paper is that light curves calculated with gray opacities were either inadequate or uncertainly dependent upon somewhat arbitrary prescriptions for computing B and V. The second benchmark calculation, with the Rosseland means of the LANL opacities, shows that $\kappa \sim 0.03$ cm$^2$ g$^{-1}$ at the photosphere. The rise to bolometric maximum is particularly rapid (8.2 days) and reflects this lower opacity.

In the following model calculations done with the new code, the wavelength (frequency) range used for the radiation was 7–2000 nm ($1.5 \times 10^{14} – 4.3 \times 10^{16}$ Hz). The colors associated with the emergent fluxes (in the frame of the observer) were computed with the filters given by Bessel (1990) with zero-points determined by the computed spectrum of Vega given by Dreiling & Bell (1980). The colors computed in this manner never differed by more than 0.1 magnitude from those computed with single-point filters (Allen 1973, p. 202).

Four variants were calculated for W7, reflecting different assumptions about or treatments of the frequency dependence of the opacity. The results are given in Figures 3-8 and summarized in Table 2. The first variant, the “full opacity” model, uses all the information available for the absorptive and scattering opacities. For the second and third models, a simple form for the frequency dependence of $\kappa_\nu$ was adopted:

$$\kappa_\nu = \kappa_0[\theta(\lambda - \lambda_c) + 30\theta(\lambda_c - \lambda)]$$

and the critical wavelength was taken to be either $\lambda_c = 100$ nm or $\lambda_c = 400$ nm. The
enhancement factor of 30 for $\lambda < \lambda_c$ is arbitrary and could have been much higher (see, for example, Fig. 3 and the curve of the “real” $\kappa_\nu$ at the photosphere at maximum light). The coefficient $\kappa_0$ is set by requiring that the flux-weighted mean of the model $\kappa_\nu$ is the same as the tabulated Rosseland mean for the real opacity. (Whenever flux- or energy-weighted means are required in the code, they are obtained from the frequency-dependent radiation moments as evaluated at the previous time step.) The $\lambda_c = 100$ nm choice is meant to simulate the very significant increase in $\kappa_\nu$ due to bound-free transitions at short wavelengths. The $\lambda_c = 400$ nm choice is an attempt to model the effect upon the UV of the line opacity (line blocking and expansion). However, in the absence of a detailed calculation of the enhancement due to this opacity, $\kappa_0$ was still set by requiring the flux-weighted opacity to match the known, static, Rosseland mean. As a consequence of the step up in $\kappa_\nu$ for $\lambda < \lambda_c = 400$ nm (where much of the radiation flux is to be found), the opacity at longer wavelengths is suppressed relative to that for the $\lambda_c = 100$ nm or full opacity models. The fourth and final model used a gray opacity. That is, the code was run as before (multigroup radiative dynamics) but with a constant $\kappa_\nu = \kappa_R$, the Rosseland mean. This calculation is then very similar in spirit to those discussed by Ensman & Burrows (1992), although here the radiation moments are obtained at all frequencies/colors.

Table 2 gives the essential results for the properties of the light curves. All four models show essentially the same rise time to maximum light ($\sim 11.5$ days for the bolometric curves with additional delays $\ll 1$ day in B and V). This is consistent with what is known for rise times of observed SNIa’s, with the exception of the slow-rising SN 1990N which differs in other, spectroscopic, ways (Leibundgut et al. 1991a) from standard SNIa’s such as SN 1972E and SN 1981B, and may not be interpretable with a model for the explosion like W7. However, there is a growing consensus in the SN community that the rise times for canonical SNIa’s may be $\gtrsim 15$ days. If this is true, then W7 may not be an acceptable model for SNIa’s except insofar as its velocity and composition profiles are appropriate for spectra
near maximum light.

All four models have nearly the same $M_{bol}$ at maximum light, but this is consistent with them having identical energy deposition rates, very nearly the same integrated opacities, and consequently similar rise times. Figures 3-6 show, however, that the curves for $S_{rad}$ (the total instantaneous rate of energy deposition from radioactivity) do not pass precisely through the bolometric peaks. The main differences among the models are to be found in their colors at maximum light and the rates of decline of these colors after maximum light. Two of the best-studied, and by presumption most representative, SNIa’s are SN 1972E and SN 1981B for which the following obtain (see Leibundgut et al. 1991b): at maximum light (B-V) = -0.08 and -0.03 (respectively), (U-B) = -0.34 and -0.14, and in both cases the rate of decline of the B curve is 2 mag in about 23 days. The full opacity model is the faintest in B, by 0.5 mag, has too much U flux at maximum light, and is too slow to decline in B. This suggests that if the underlying model of the explosion is reasonable, then an enhancement of the opacity in the UV is essential. This presumably is the effect of line blanketing and line blocking. The properties of the $\lambda_c = 100$ nm model are similar to those of the full opacity model, except for the more rapid (and thus desirable) decline in B. The functional form of $\kappa_\nu$ in this model is realistic, but the neglect of scattering for $\lambda > \lambda_c$ leads to a thermalization of the radiation at lesser depths in the SN atmosphere which in turn leads to a lower color temperature and the more rapid decline of B. The $\lambda_c = 400$ nm model is too extreme in that U is overly suppressed and B-V is too large at maximum light. The B maximum is the largest of the 4 models, but again this is because the assumed form of $\kappa_\nu$ has pushed the flux into the B and V bands. The results for the gray opacity model are virtually identical to those of the $\lambda_c = 100$ nm model. This is because, at and after maximum light, the radiation near the photosphere (always defined by $\tau_F = 2/3$ where $\tau_F$ is the optical depth measured inward for the flux-weighted opacity) has predominantly $\lambda > \lambda_c$ and thus the two opacities are operationally nearly the same.
The conclusion to be reached from these model calculations is that if something like W7 is a good representation of the explosion, an enhancement of the scattering opacity in the uv is probably essential to explain the observations. This may well be the expansion opacity, as has been argued by Höflich, Müller, & Khoklov (1993), coupled with a treatment of line blanketing. A further conclusion is that light curve calculations for model W7 using fully self-consistent, frequency-dependent radiation dynamics yields values of $M_{B,max}$ that are fainter by $\sim 0.5$ magnitudes than earlier work (Sutherland & Wheeler 1984) suggested, although in such earlier calculations the conversion from the bolometric light curve to the B light curve was ad hoc and somewhat suspect. The new results are more in keeping with larger, rather than lower, values of $H_0$ as suggested by the recalibration of the distance to IC 4182 and the brightness of SN 1937C (Pierce, Ressler, & Shure 1992).

We wish to thank Bob Clark of Los Alamos National Laboratory who prepared the extensive opacity tables used in our calculations. Bruno Leibundgut provided us with light curve templates for SNIa’s and commented usefully on the uncertainties associated with UBV colors for SNs. Robert Harkness, Doug Swartz, and Craig Wheeler gave us a set of Rosseland mean opacities that we used in preliminary calculations, and they also had valuable insights regarding the expansion opacity (although this matter remains to some degree confused). The final iteration of this paper was completed at the Aspen Center for Physics, and any improvements can be attributed in part to the workshop on SN spectra held there in 1993 June. This research was supported by a grant from the Natural Sciences and Engineering Research Council of Canada.
## Compositions used for W7 Model

|     | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          |
|-----|------------|------------|------------|------------|------------|------------|------------|------------|
| $^4\text{He}$ | 0.211E-08  | 0.162E-04  | 0.145E-01  | 0.158E-07  | 0.100E-09  | 0.100E-09  | 0.100E-09  | 0.100E-09  |
| $^{12}\text{C}$ | 0.100E-09  | 0.100E-09  | 0.805E-05  | 0.769E-08  | 0.122E-05  | 0.977E-05  | 0.100E-04  | 0.807E-03  |
| $^{16}\text{O}$ | 0.100E-09  | 0.100E-09  | 0.844E-08  | 0.466E-07  | 0.154E-04  | 0.664E-01  | 0.579      | 0.582      |
| $^{20}\text{Ne}$ | 0.100E-09  | 0.100E-09  | 0.159E-07  | 0.100E-09  | 0.670E-09  | 0.263E-05  | 0.120E-04  | 0.290E-02  |
| $^{24}\text{Mg}$ | 0.100E-09  | 0.100E-09  | 0.383E-07  | 0.562E-07  | 0.124E-04  | 0.142E-03  | 0.119      | 0.122      |
| $^{26}\text{Al}$ | 0.100E-09  | 0.100E-09  | 0.353E-07  | 0.114E-09  | 0.104E-08  | 0.255E-05  | 0.226E-02  | 0.425E-02  |
| $^{28}\text{Si}$ | 0.666E-08  | 0.100E-09  | 0.780E-06  | 0.876E-02  | 0.499      | 0.561      | 0.222      | 0.210      |
| $^{30}\text{P}$ | 0.198E-09  | 0.100E-09  | 0.480E-07  | 0.282E-06  | 0.283E-05  | 0.674E-03  | 0.535E-03  | 0.676E-03  |
| $^{32}\text{S}$ | 0.533E-07  | 0.100E-09  | 0.223E-05  | 0.143E-01  | 0.298      | 0.276      | 0.636E-01  | 0.632E-01  |
| $^{34}\text{Cl}$ | 0.773E-09  | 0.100E-09  | 0.152E-05  | 0.118E-06  | 0.215E-05  | 0.797E-03  | 0.120E-03  | 0.662E-03  |
| $^{36}\text{Ar}$ | 0.165E-06  | 0.202E-09  | 0.565E-05  | 0.717E-02  | 0.556E-01  | 0.526E-01  | 0.102E-01  | 0.106E-01  |
| $^{38}\text{K}$ | 0.287E-08  | 0.100E-09  | 0.250E-05  | 0.499E-06  | 0.920E-05  | 0.102E-02  | 0.156E-04  | 0.131E-03  |
| $^{40}\text{Ca}$ | 0.271E-07  | 0.189E-07  | 0.288E-04  | 0.156E-01  | 0.439E-01  | 0.226E-01  | 0.677E-03  | 0.650E-03  |
| $^{44}\text{Ti}$ | 0.642E-05  | 0.181E-08  | 0.518E-04  | 0.220E-04  | 0.263E-04  | 0.239E-03  | 0.317E-05  | 0.288E-05  |
| $^{48}\text{Cr}$ | 0.515E-01  | 0.144E-05  | 0.383E-04  | 0.971E-03  | 0.935E-03  | 0.945E-03  | 0.579E-05  | 0.548E-05  |
| $^{50}\text{Mn}$ | 0.339E-02  | 0.802E-06  | 0.533E-07  | 0.710E-04  | 0.641E-04  | 0.157E-03  | 0.332E-06  | 0.144E-05  |
| $^{52}\text{Fe}$ | 0.799      | 0.159E-01  | 0.691E-04  | 0.719E-01  | 0.638E-01  | 0.171E-01  | 0.503E-03  | 0.121E-02  |
| $^{54}\text{Co}$ | 0.146      | 0.200      | 0.815E-01  | 0.323E-01  | 0.832E-02  | 0.106E-02  | 0.111E-02  | 0.402E-03  |
| $^{56}\text{Ni}$ | 0.892E-07  | 0.784      | 0.876      | 0.849      | 0.299E-01  | 0.817E-06  | 0.934E-09  | 0.000E+00  |

Table 1: Mass fractions of significant elements in the eight different compositions used in our representation of W7. In the 40 zone model, the actual initial mass fraction of $^{56}\text{Ni}$ was used, rather than interpolation among the eight values above.
Table 2: Summary of Light Curve Calculations. The first column identifies the opacity model. The second and third columns give the time at, and magnitude for, bolometric maximum light. The fourth and fifth columns give the time and maximum for the B light curve, followed by the colors B-V and U-B at this time. The last column is the additional time beyond that of B maximum light for the B light curve to drop by 2 mag. The first two models were calculated using the flux-limited diffusion (FLD) approximation as implemented by Sutherland & Wheeler (1984); for the first one the opacity is everywhere constant at $\kappa = 0.1 \text{ cm}^2 \text{ g}^{-1}$, while for the second the tabulated Rosseland mean opacities were employed. For these two models the colors were computed based upon an assumed truncated (shortward of 400 nm) blackbody spectrum, so that the U flux is not relevant.
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Figure 1: Density and velocity profiles for model W7 of Nomoto, Thielemann, & Yokoi (1984). Also shown (crosses) are the location of the 40 zone boundaries. Note how the inner and outer zones have been set to preserve the velocity resolution of the original 172 zone model.

Figure 2: Light curves for the two models calculated with the flux-limited diffusion approximation. For the first model (upper panels) the opacity is \( \kappa = 0.1 \text{ cm}^2 \text{ g}^{-1} \) everywhere whereas for the second model the Rosseland mean opacities are employed. The B light curve is based upon a blackbody spectrum appropriate to the photospheric radius and total luminosity whereas the B’ light curve is based upon an assumed truncated (shortward of 400 nm) blackbody spectrum with the same net flux.

Figure 3: Light and color curves and photospheric properties for the “full opacity” model. The upper left panel gives the UBV and bolometric light curves and the total instantaneous rate of deposition of radioactive energy \( (S_{\text{rad}}) \). The lower left panel gives the radius (in \( 10^{15} \text{ cm} \)) of the photosphere and its temperature (in \( 10^4 \text{ K} \)) and the zone (\( \Delta \)) containing the photosphere. The lower right panel gives the opacity (absorption and total) as a function of wavelength at maximum light at the photosphere. That \( \kappa_{\text{abs}} \) slightly exceeds \( \kappa_{\text{total}} \) near 600 A reflects the limited resolution of our interpolation table.

Figure 4: Same as Figure 3, except for the \( \lambda_c = 100 \text{ nm} \) model.

Figure 5: Same as Figure 3, except for the \( \lambda_c = 400 \text{ nm} \) model.

Figure 6: Same as Figure 3, except for gray model.

Figure 7: Continuum spectra for the four models at bolometric maximum light. In each panel a dashed curve for a blackbody spectrum at the matter temperature at the photosphere is also given. The units of \( L_\nu \) are ergs cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\).

Figure 8: A direct comparison of the four continuum spectra at bolometric maximum light.