Self-Referencing 3D Characterization of Ultrafast Optical-Vortex Beams Using Tilted Interference TERMITES Technique

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Femtosecond light pulses carrying optical angular momentums (OAMs), possessing intriguing properties of helical phase fronts and ultrafast temporal profiles, enable many applications in nonlinear optics, strong-field physics, and laser micromachining. While generation of OAM-carrying ultrafast pulses and their interactions with matters are intensively studied in experiments, 3D characterization of ultrafast OAM-carrying light beams in spatiotemporal domain has, however, proved difficult to achieve. Conventional measurement schemes rely on the use of a reference pulsed light beam, which needs to be well characterized in its phase front and to have sufficient overlap and coherence with the beam under test, largely limiting practical applications of these schemes. Here a self-referencing set-up is demonstrated based on a tilted interferometer that can be used to measure complete spatiotemporal information of OAM-carrying femtosecond pulses with different topological charges. Through scanning one interferometer arm, the spectral phase over the pulse spatial profile can be obtained using the tilted interference signal, and the temporal envelope of the light field at one particular position around its phase singularity can be retrieved simultaneously, enabling 3D beam reconstruction. This self-referencing technique, capable of measuring spatiotemporal ultrafast optical-vortex beams, may find many applications in fields of nonlinear optics and light–matter interactions.

1. Introduction

Optical-vortex beams carrying orbital angular momentums (OAMs), with phase singularities in their centers and hollow-structured intensity distributions,[1–3] have proved important in many areas of optics and physics.[4–7] In the past several decades, intensive studies on the helical phase fronts of optical-vortex beams have deepened our fundamental understanding on light fields, and meanwhile the unique phase structures of optical-vortex beams, carrying OAMs, have enabled wide-ranging applications in microparticle manipulation,[8–10] optical microscopy,[11,12] quantum optics,[13–15] and optical telecommunications.[16–18] Due to their great application potential, ultrafast light fields carrying OAMs, being powerful tools in studies of nonlinear optics and light–matter interactions, have also attracted extensive interest, adding a major new dimension in nonlinear laser–frequency conversion,[19–21] high-harmonic generation,[22,23] laser micromachining,[24–26] and strong-field physics.[27,28]

In the field of nonlinear optics, self-referencing measurements of ultrafast light fields are of particular importance and have been widely used in many experiments.[29–33] Fast and accurate characterization of ultrafast pulses in three dimensions, in one side, is crucial for studies on ultrafast lasers, providing useful information on the key performance of ultrafast light sources.[34–36] In the other side, accurate measurements of ultrafast light fields, generated from optical experiments, are essential for understanding the mechanisms of the ongoing nonlinear-optics processes.[37,38] While several self-referencing techniques, such as frequency-resolved optical gating (FROG) and spectral phase interferometry for direct electric-field reconstruction (SPIDER) set-ups,[39,40] have been successfully used to measure spectra-temporal information of ultrafast pulses,[39–42,43,44] an advanced self-referencing scheme that is capable of reconstructing ultrafast optical-vortex beams in both spatial and temporal dimensions has, however, proved difficult to achieve. In previous set-ups, for measuring ultrafast optical-vortex pulses,
one reference beam was generally needed to characterize the phase front of the light beam under test using interferometry. In such measurements, however, the reference beam needs to be fully coherent with the one under test, and the two beams should have sufficient overlaps in all the spatial, temporal, and spectral dimensions.\[41–44\] Additionally, the spatial-phase information of the reference beam should be well known before measurements in order to retrieve precisely the phase front of the beam under test.\[41–44\] All these requirements restrict, to some extent, the practical application of these schemes. Recently, an elegant means of in-focus 2D-FFT technique based on the combination of spatially resolved Fourier-transform spectroscopy with an alternate-projection phase-retrieval algorithm, called “INSIGHT,” has been demonstrated for achieving successfully spatio-temporal characterization of optical-vortex beams.\[45\] This self-referenced scheme relies on the recording of a fast beam evolution along its propagation near its focus, and uses an adapted version of the well-known Gerchberg–Saxton phase-retrieval algorithm to extract the spatial phase at each frequency, which makes, however, the phase-retrieval procedure a bit complicated.\[45\]

More recently, a novel total electrical-field reconstruction using a Michelson interferometer temporal scan (TERMITES) technique was developed, enabling the spatio-temporal characterization of high-energy femtosecond laser pulses in a self-referencing manner.\[35\] However, in previous studies, the application of this TERMITES technique for reconstructing ultrashort optical-vortex beam with relatively complex phase and amplitude distributions is still challenging.\[23,37\] First, the radial-shearing interferometer in conventional TERMITES set-up is not applicable of measuring hollow-structured optical-vortex beams, since the dark centroid of the vortex beam, i.e., phase singularity, can no longer be used as the reference point of the shearing interference. Second, the relatively large beam size and complicated focusing condition of optical-vortex pulses cause some difficulties in performing high-signal-to-noise-ratio FROG measurements. In order to solve these limitations, we develop a novel tilted-interferometer set-up, which can be used to retrieve the spatial phase information of hollow-structured vortex beams; meanwhile, the FROG measurement using a type-II crystal is integrated in the set-up, measuring simultaneously the temporal profile with a high signal-to-noise ratio. Using this advanced TERMITES technique based on tilted interference, we achieved successfully self-referenced spatio-temporal reconstruction of OAM-carrying ultrashort pulses with topological charges of |\(l| = 1, 2, \text{ and } 3\). The retrieved results show striking agreement with the numerical simulations, highlighting the great application potential of this scheme in studies of nonlinear optics and strong-field physics concerning ultrashort vortex light.\[27,28,46,47\]

2. Measurement Set-Up

The modified TERMITES set-up we build up in the experiment is illustrated in Figure 1a. As shown in Figure 1a, when an optical-vortex pulse with a linear polarization state was launched into the system, it was divided into two parts by a 50:50 beam splitter (BS1). One part of the pulse was reflected by a convex mirror (CM1), leading to a slightly divergent light beam (plotted as the pink beam in Figure 1a). The other part of the pulse (plotted as the green beam in Figure 1a) passed through a birefringent quartz crystal (BQC), and then was reflected by the scanning mirror (SM) mounted on a piezoelectrical moving stage. The principal axis of the BQC can be adjusted to slightly deviate from the polarization direction of the incident pulse, resulting in the generation of two adjacent pulses with orthogonal polarization states (see Figure 1b). The intensity ratio of the two pulses depends on the deviation angle, while the pulse separation can be controlled through varying the thickness of the BQC. The two reflected light beams were then focused onto the entrance of a spectrometer, and the scattered light was finally recorded by a CCD camera.
beams were recombined at BS1, and the output beam was divided into two optical paths at the second 50:50 beam splitter (BS2); see Figure 1a. For the first path that passed through BS2, the interference signal between the two pulses (reflected by CM1 and SM) with the same polarization states was measured using the CCD camera. In the second path, the light beam was focused by a concave mirror (CM2), and a type-II beta-barium borate (BBO) crystal was placed near the focusing point of the beam. The sum-frequency process between the two pulses at different polarization states can be obtained due to the type-II phase matching, and the resulting sum-frequency signal at shorter wavelengths was filtered out by an optical filter and then recorded using a fiber-coupled spectrometer for the FROG measurement; see Figure 1a. Please note that in the current experiment, the birefringence of the type-II BBO crystal leads to ≈15 fs walk-off between the two beams, which can be neglected when it is much less than the duration of the pulse under test. When measuring pulses with shorter durations, this walk-off effect could be further suppressed through using thinner nonlinear crystal or adopting proper correction algorithm in the FROG-trace retrieving.

In the experiment, while the SM was adjusted to be perpendicular to the incident light beam, CM1 was rotated to slightly deviate from the perpendicular position, leading to a tilted shearing interference measured at the CCD camera (see Figure 1c). This rotation is necessary for measuring optical-vortex beams due to the following two reasons. First, it can lead to the centroid offset of the two interfering beams at the CCD camera, mitigating the signal fading effect due to dark centers of the beams. Second, the rotation can also result in a sampling point offset in the BBO crystal over the sum-frequency process (see Figure 1c). Due to the convex surface of CM1, the divergent light beam (marked as pink in Figure 1a) has a longer focusing length than the beam reflected by the flat SM (marked as green in Figure 1a) after CM2. When the BBO crystal is placed near the focusing point of the green beam, this focusing point works as a sampling spot on the spatial profile of the pink beam (see Figure 1c). In the experiment, the rotation angle of CM1 was carefully adjusted so as to locate the focusing point of the green beam at one bright ring of the pink beam, ensuring a high signal-to-noise ratio of the FROG measurement.

In order to improve the accuracy of the measurement, the axis-angle and thickness of the BQC need to be carefully selected (see Figure 1c). In the experiment, the angle deviation of the BQC principal axis from the polarization state of the incident pulse was ≈3%, leading to a portion of merely 0.3% light energy coupled to the orthogonal polarization state for FROG measurement, and this portion of light was focused on the BBO crystal for sampling (see Figure 1b). The almost-equal light intensities of the two pulses at the focusing point ensure that the sum-frequency signal output from the crystal has a high signal-to-noise ratio, suppressing the self-frequency multiplication signal generated by each of the beams. The rest light beam with 99.7% energy was used for the tilted shearing-interference measurement, resulting in a high interferometric contrast as shown in Figure 1c. An 8 mm thick BQC was used in the set-up, giving a time delay of 200 fs which is long enough for measuring pulses with a width of <100 fs. The scanning length of the SM is 210 μm (1400 fs scanning delay), sufficient for both spatial phase and FROG measurements.

3. Generation of Ultrafast Light Beams Carrying OAMs

In order to prepare ultrafast optical-vortex beams for testing the capability of this diagnosis set-up, we used a commercial Ti:sapphire laser system (Legend Elite, Coherent Inc.) as the seed laser, which can deliver linearly polarized ultrafast light pulses with 1 kHz pulse repetition rate, ≈45 fs pulse duration, and 790 nm central wavelength. The laser output was collimated to have a Gaussian-shaped profile with a full-width half-maximum bandwidth of ≈9 mm. As shown in Figure 2, this Gaussian light beam passed through, in turn, a quarter-wave plate (QWP), a vortex retarder (VR), and a wire grid polarizer (WGP), resulting in the generation of linearly polarized, ultrafast light beams carrying OAMs. The topological charges of optical-vortex pulses can be altered through using different VRs (see Figure S1 in the Supporting Information).

Using the Collins formula, we simulated theoretically the light evolution over free-space propagation, unveiling the formation process of phase singularity in the centroid of the light beam. The simulated results of the optical-vortex beam with a topological charge of |l| = 1 are illustrated in Figure 2. It is shown that when a Gaussian-shaped light beam is launched into the system, a phase singularity in the centroid of the beam starts to appear after the beam passing through the VR. The size of the dark area increases gradually over light propagation, giving rise to a hollow beam structure (see Figure 2). At the same time, the phase front of the optical-vortex beam starts to rotate, leading to a spiral phase profile with gradually increasing bending radius. Simulations on light evolutions of optical-vortex beams with topological charges of |l| = 2 and 3 were also performed (see Section S1 in the Supporting Information).

4. Experimental Results

In the experiments, we measured the ultrafast optical-vortex beams with topological charges of |l| = −1, 2, and 3 at the measurement position of ≈0.7 m behind the VRs. The spatial-phase profiles of these vortex beams, measured using the tilted shearing interferometer, are illustrated in the Figure 3. As shown in Figure 3a–c, at the same measurement position the diameter of the first ring of the hollow structure increases as the topological charge increases, agreeing well with the simulation results (see Figure S1 in the Supporting Information).

The tilted shearing-interference signal was recorded using the CCD camera, exhibiting clear interference patterns with high contrasts (see Figure 3d–f). With a single scan of the time delay between the two interfering light beams, a series of interferograms measured at the CCD camera can be written as

$$S(x, y, r) = \int |E(x, y, r) + E_d(x, y, t - r)|^2 dt$$

(1)

where $E(x, y, r)$ and $E_d(x, y, t - r)$ represent the electrical fields of green and pink (divergent) light beams shown in Figure 1a. The Fourier transform of $S(x, y, r)$ with respect to the delay time $r$ can be used to calculate directly the spectral phase difference between the two beams, which can be expressed as $\Delta \varphi(x, y, t)$ =
**Figure 2.** The evolution of the light beam over free-space propagation, simulated using the Collins formula. A linearly polarized, Gaussian-shaped light beam passes through in turn a quarter-wave plate (QWP), a vortex retarder (VR), and a wire grid polarizer (WGP). The intensity and phase profiles of the simulated OAM-carrying light beam ($|l| = 1$) at different positions behind the vortex retarder are presented in this plot.

**Figure 3.** (a–c) Measured intensity profiles of different optical-vortex beams ($l = −1, 2, \text{ and } 3$). (d–f) Typical results of tilted shearing-interference signal, measured using the CCD camera. (g–l) Retrieved phase profiles of the corresponding optical-vortex beams at two wavelengths of 790 and 800 nm.
\( \varphi(x, y, \omega) - \varphi_0(x, y, \omega) \), where \( \varphi(x, y, \omega) \) and \( \varphi_0(x, y, \omega) \) are, respectively, the spectral phase distributions of the two beams. In this self-referencing scheme, these two phase distributions are closely correlated and can be expressed as

\[
\varphi_2(x, y, \omega) = \varphi(x_0 + \frac{x - x_0}{\beta}, y_0 + \frac{y - y_0}{\beta}, \omega) - \varphi_0(x, y, \omega)
\]

where \((x_0, y_0)\) and \((x_0', y_0')\) represent the centroid positions of the pink and green beams, and \(\beta\) is the beam-expansion rate due to the beam divergence. \(\varphi\), is a spatially varied phase due to CM1, which is induced by the curvature of the mirror. While \(\Delta \varphi(x, y, \omega)\) can be obtained using the measured interferogram data, the precise values of \((x_0, y_0)\), \((x_0', y_0')\), and \(\beta\) can be obtained in the experiment as \((2140.2, 350.6)\), \((1586.7, 313.6)\), and 1.4 for the case of \(l = -1\). Then, we can retrieve the spectral phase distribution \(\varphi(x, y, \omega)\) of the optical-vortex beam using an iterative algorithm. Some detailed information of the retrieval process is presented in Section S2 (Supporting Information).

The retrieved spectral phase distributions of optical-vortex beams with different topological charges, at two wavelengths of 790 and 800 nm, are illustrated in Figure 3g–l, which is mainly due to the fact that the limited spatial resolution (386 by 290 pixels in current measurements) of the interferogram signal caused some errors in phase retrieving, especially at some positions with high phase gradients. Since the size of the error lines is at the single-pixel level, they have negligible influence on the reconstruction of the ultrafast optical-vortex beams (see Figure 5). In practice, these error lines can be effectively eliminated through using a higher spatial resolution in the interferogram measurement; however, this will increase consequently the data size and time consumption in the phase-retrieving process.

Through a single scan of the interferometer arm, the pulse FROG trace at the sampling point of the vortex beam can also be measured using the spectrometer placed after the optical filter (see Figure 1a, b). A typical FROG trace, measured at \(l = -1\), is illustrated in Figure 4a. A super-Gaussian filter was first used to extract useful information from the original data, eliminating high-frequency noise as shown in Figure 4b. Then, the retrieved FROG trace was obtained using the extended ptychographic iterative engine (ePIE) algorithm; see Figure 4c. The retrieved temporal intensity and phase profile (Figure 4d) yield a pulse duration of 53 fs together with an almost flat temporal phase curve. The optical spectrum retrieved from the FROG measurement exhibits good agreement with the pulse optical spectrum measured directly using the spectrometer (see Figure 4e). With the relative spectral amplitude and phase distribution measured respectively by the CCD camera and tilted shearing interferometer, the spatio-temporal information over the whole spatial light beam can be obtained using the FROG trace at the sampling point as a reference. Therefore, the 3D reconstruction of the ultrafast optical-vortex beams can be performed through a simple inverse Fourier-transform procedure. In the experiment, the complete 3D electrical fields of ultrafast pulses with \(l = -1, 2, \) and 3 are reconstructed, and the results are illustrated in Figure 5. In order to make the vortex structure more obvious, in these plots we set the isosurface to be 40% of the peak amplitude of the electrical field and the carrier frequency was reduced to be 50% of the original one. The reconstructed structures of optical-vortex beams exhibit good consistency with the simulated results (see Section S4 in the Supporting Information).

5. Discussion and Conclusion

The current set-up can merely be used to measure OAM-carrying ultrafast pulses with linear polarization states, which are mainly limited by the use of birefringent element and type-II phase-matching crystal in the system (see Figure 1a). However, this self-referencing technique could be extended further to characterize OAM-carrying pulses with more complicated polarization features. For simplest case, using a polarization beam splitter and two identical set-ups, simultaneous measurements of spatio-
temporal structures of light beams at two orthogonal polarization states can be achieved. The remaining step is to build up the phase correlation between the two beams at orthogonal polarization states, which can be realized, in principle, through an additional interference measurement. Moreover, the application of this advanced measurement technique could be extended for measuring more complicated spatial-structured light beams, and the tilted shearing interferometer used in this set-up could also be modified according to different beam structures. Some further improvements on this TERMITE scheme are also worth investigations. For example, through scanning the centroid off-set in the interferogram, the signal fading effect due to phase singularities could be largely suppressed, increasing further the signal-to-noise ratio of the measurement.

The use of type-II phase matching in the FROG measurement is critical for characterizing ultrafast optical-vortex beams. Comparing with conventional Gaussian-shaped light beam, optical-vortex beams with phase singularities generally have larger beam sizes. In order to sample one particular point of the optical-vortex beam in the FROG measurement, the pulse-energy difference between two beams has to be carefully selected (997:3 in this experiment) so as to ensure almost-equal light intensities of the two beams at the sampling spot. If a conventional type-I crystal was used in current set-up, the self-frequency-multiplication signal from the high-energy pulse would be much stronger than the sum-frequency one, leading to low signal-to-noise ratio of the FROG measurement. Some detailed results are presented in Section S3 (Supporting Information). When type-II crystal is used, the self-frequency-multiplication noise is strongly suppressed as a result of phase mismatch, ensuring a high signal-to-noise ratio of the measurement.

In conclusion, we demonstrated an advanced TERMITE scheme based on tilted shear-interference and type-II FROG measurement, being capable of characterizing ultrafast optical-vortex beam carrying OAMs in a self-referencing manner. The reconstructed 3D electrical fields of ultrafast optical-vortex pulses with topological charges of \(|l| = 1, 2, \) and \(3\) exhibit striking agreement with numerical simulations, verifying the high performance of the scheme. The set-up reported here provides a novel means of characterizing spatially structured ultrafast pulses, and may find many applications in OAM-related experiments and studies on nonlinear optics, ultrafast lasers, and light–matter interactions.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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