Parameter estimation of a breaking wave slamming load model using Monte Carlo simulation
Paper
Wang, Shaofeng; Juul Larsen, Torben; Tobias Gudmestad, Ove

Published in:
Journal of Physics: Conference Series

Link to article, DOI:
10.1088/1742-6596/1104/1/012012

Publication date:
2018

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Wang, S., Juul Larsen, T., & Tobias Gudmestad, O. (2018). Parameter estimation of a breaking wave slamming load model using Monte Carlo simulation: Paper. Journal of Physics: Conference Series, 1104(1), Article 012012. https://doi.org/10.1088/1742-6596/1104/1/012012

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
Parameter estimation of a breaking wave slamming load model using Monte Carlo simulation

To cite this article: Shaofeng Wang et al 2018 J. Phys.: Conf. Ser. 1104 012012

View the article online for updates and enhancements.
Parameter estimation of a breaking wave slamming load model using Monte Carlo simulation

Shaofeng Wang\textsuperscript{1}, Torben Juul Larsen\textsuperscript{1}, Ove Tobias Gudmestad\textsuperscript{2}

\textsuperscript{1} DTU Wind Energy, Frederiksborgvej 399, 4000 Roskilde, Denmark
\textsuperscript{2} University of Stavanger, Kjell Arholmsgate 41, 4036 Stavanger, Norway

E-mail: shfe@dtu.dk

Abstract. For offshore wind turbines (OWTs) located in relatively shallow water, the design is influenced by the occurrence of breaking waves. The strongly nonlinear properties associated with the wave breaking process result in challenges in modelling their impact loads on the structures. The total impact loads are normally calculated as the sum of a slowly varying quasi-static load and an impulsive slamming load. The quasi-static load is normally calculated using Morison’s equation and the slamming load is approximated by the Goda model or the Wienke-Oumeraci model. Given the dynamic properties of OWTs, structural resonances might be excited by the impulsive slamming load. Therefore, there is a clear need to evaluate the response effect excited by the slamming load. In this paper, the response of a vertical pile subjected to a severe breaking wave case is investigated by a combination of data from a large-scale experiment and numerical simulations. The slowly varying quasi-static load obtained in a non-breaking wave packet is modelled using Morison’s equation with the wave kinematics obtained from a fully nonlinear potential flow solver OceanWave3D. The governing parameters used in a slamming load model are estimated using the Monte Carlo method and verified by comparing the experimental data with the numerical simulation results. It is found that the slamming coefficient and the curling factor are close to the values found by the Wienke-Oumeraci model, however the impact duration is significantly larger than the values found by the Goda model and the Wienke-Oumeraci model, which is important for the assessment of the dynamic responses of OWTs.

1. Introduction

For the design of offshore wind turbines (OWTs), aero-hydro-elastic simulations should be performed to evaluate the structural integrity during its lifetime. Morison’s equation has been widely used to calculate the wave forces acting on the substructure if it is composed of slender cylindrical members. However, some offshore wind turbines are installed in relatively shallow water regions and in risk of being subjected to breaking waves. Plunging breakers are highly important as they are associated with high impact loads influencing the design loads significantly.

It is challenging to model breaking waves and their associated impact loads because the breaking process is a nonlinear phenomenon with significant variability. Basically, a wave starts breaking when it becomes steep enough such the water particle velocity near the wave surface exceeds the wave celerity. During the breaking process, a large amount of energy in a wave starts to dissipate resulting in an impulsive load also referred to as the slamming load. Wave breaking in a realistic ocean environment has large variability in breaking wave conditions, which are determined by the site-specific conditions such as bathymetry, current and wind. Several
wave-breaking limits established by McCowan (1894), Miche (1944), Goda (1974) and Battjes (2000) are reported in [1]. The standard IEC 61400-3 [2] suggests using the Battjes limit for the design.

The impact loads induced by breaking waves are also difficult to estimate as the physical process is complicated with air-wave, wave-structure and wave-seabed interactions. To date, several laboratory experimental studies of slamming load on vertical and inclined cylindrical structures have been carried out showing significant variations with respect to the force intensity, the force time history and the impact duration [3][4][5][6]. In addition, a large number of Computational Fluid Dynamics (CFD) studies have been performed on this topic. Bredmose et al. used a 3D CFD model to compute the extreme wave loads on a monopile foundation from breaking waves using focused wave groups and compared them to the loads estimated from Morison’s equation [7]. Kamath et al. used the program REEF3D, developed at NTNU, to simulate plunging wave forces on a vertical cylinder and compared them against the experimental data and investigate the effect of breaker location on the impact loads [8]. Choi et al. used CFD to study the effects of impact loads on vertical and inclined piles by taking structural vibrations into consideration [9]. Nevertheless, application of CFD to engineering design within the industry is limited due to its complexity and computational requirements.

Engineering models for calculating the impact loads from breaking waves are important for industrial use. The total impact loads are normally divided into a slowly varying quasi-static load and an impulsive slamming load. The quasi-static load is normally calculated using Morison’s equation. The easiest way for estimating the slamming load is to modify the drag coefficient in Morison’s equation, however, it is not widely used as the impact duration of the slamming load is significantly different from the slowly varying part calculated by Morison’s equation. Alternative engineering models have been proposed by different researchers. Among these, the Goda model [3] and the Wienke-Oumeraci model [4] have been widely used in the industry due to their simplicities. Goda et al. came up with a model based on momentum analysis and experiments using four test piles including circular sections, square sections and triangular sections [3]. Wienke et al. kept the basic assumptions of the Goda model and proposed a new model which takes the pile inclination angle into consideration. The slamming load model established by Wienke et al. is more impulsive than the Goda model with higher force intensity and shorter impact duration [4]. An engineering model proposed recently by Hansen et al. [10] uses wave surface elevation to calculate the slamming loads from breaking waves rather than the wave celerity used in [3][4]. The latest model from Burmester et al. is similar to the Wienke-Oumeraci model [5].

Previous studies [3][4][5][10] revealed significant uncertainties on modelling slamming loads from breaking waves, especially on the load intensity. However, there is a lack of understanding its effect on the structural dynamic response. Given the dynamic properties of an OWT, the temporal development of the slamming force (force shape) must be accounted. Therefore, there is a clear need to describe the slamming loads from the aspect of equivalent structural response. In this paper, the response of a vertical pile subjected to a severe breaking wave case is investigated by a combination of large scale experimental data and numerical simulations. The slowly varying quasi-static load obtained from a non-breaking wave packet is verified by using Morison’s equation with the wave kinematics obtained from the fully nonlinear potential flow solver, OceanWave3D [12]. The governing parameters used in a slamming load model are estimated using the Monte Carlo method and verified by comparing the measured data with the numerical simulation results. It is found that the slamming coefficient and the curling factor are close to the values established by Wienke-Oumeraci, however, the impact duration is significantly larger than the values found by the Goda model and the Wienke-Oumeraci model, which is important for the assessment of the dynamic responses of OWTs.
2. Methodology

As shown in Figure 1, most of the existing engineering slamming load models are based on the assumptions that the breaker front is vertical over the height $\lambda \cdot \eta_b$ and hits the structure simultaneously with a constant wave celerity $C$. The height of the impact area is described as $\lambda \cdot \eta_b$ where $\lambda$ is called curling factor and $\eta_b$ is the wave elevation.

Thus, the slamming load is calculated as the integration of the in-line force acting on the impact area. It demonstrates that the slamming loads are mainly governed by three factors: the area of impact, the in-line force intensity and the impact duration. It should be noted that the spatial and temporal shapes of the force are assumed as a uniform distribution and a triangular time history, respectively. Thus, the breaking wave slamming load model is written in this format:

$$F(t) = f_i(t) \cdot \lambda \cdot \eta_b = \lambda \cdot \eta_b \cdot C_s \cdot \rho \cdot R \cdot C^2 \cdot (1 - \frac{t}{T})$$  \hspace{1cm} (1)

Equation 1 is governed by three parameters: the slamming coefficient $C_s$, the curling factor $\lambda$ and the time duration $T$, which reflect the in-line force intensity, the impact area and the impact duration respectively. Besides, $\eta_b$, $\rho$, $R$ and $t$ are wave elevation, water density, pile radius and time point respectively. Previous studies mainly focus on the force magnitude showing significant variabilities on the slamming coefficient $C_s$. The objective of this study is to obtain the temporal development of the slamming force which is able to excite the equivalent structural response. The governing parameters are tuned by a combination of data from a large-scale experiment and numerical simulations performed using the Monte Carlo method as shown in Figure 2.

3. Experiment set up

The experiment campaign was carried out by Irschik [6] using the Large Wave Flume (GWK) of the Coastal Research Center in Hannover, Germany. The wave channel is 309 m long, 5 m wide and 7 m high with a 23 m long 1:10 slope reaching a height of 2.3 m, which is placed 180 m from the wavemaker. One of the test piles is a vertical cylinder with diameter of 0.7 m and length of 5 m, which was installed at the edge of the slope. The experimental set up is shown in Figure 3. The cylinder was mounted on a transverse structure crossing the flume and the total horizontal response forces were measured by two force transducers located at the top and at the bottom of the pile. The free surface elevations at different locations in the wave tank were measured using wave gauges, but only the measured elevation at the pile location is used in this study. It should
**Figure 2.** Flowchart of the methodology combining numerical simulation results and large scale experimental data. OceanWave3D is used to reproduce the wave kinematics and the resulting quasi-static force calculated by Morison’s equation is validated with measured quasi-static force from a non-breaking wave case. Then, the decomposed measured slamming response is used to tune the parameters from Monte Carlo samplings.

It be noted that all the recorded data is sampled with a time step 0.005 s, which corresponds to 200 Hz sampling frequency.

**Figure 3.** Configuration of the experimental set up in GWK [9]

The experiment was originally carried out for investigation of vertical and inclined piles subjected to different wave breaking scenarios. Therefore, a number of wave tests were performed for regular wave conditions. In this study, only a severe breaking wave scenario is discussed as listed in Table 1, because it is the main design driver for the engineering design. The same wave case has been used by [8][9][11] to validate their CFD models. As shown in Figure 4, this wave
test consists of multiple similar wave packets. Both non-breaking cases and breaking cases were present in the wave test. Whether structural vibrations are shown in a force measurement is used to detect whether a wave is breaking or not as shown in Figure 4.

| Case | Water Depth [m] | Wave Height [m] | Wave Period [s] |
|------|----------------|----------------|-----------------|
| 1    | 1.5            | 1.3            | 4.0             |

Figure 4. Measured surface elevation at pile location and total response force. In total, 10 breaking wave cases are detected and labelled with Case No. except 1 abnormal one. The non-breaking wave case is the wave just before the breaking wave Case No.1. The detail of Case No.3 is shown at the top right and significant dynamic response is shown.

4. Numerical simulation tools
4.1. OceanWave3D
The breaking wave impact loads are divided into two loads: the quasi-static load and the slamming load. Therefore, underestimation or overestimation of the quasi-static load results in opposite estimation of the slamming load. In order to validate the slowly varying quasi-static load, wave kinematics are obtained in order to use Morison’s equation. The commonly used weakly nonlinear stream function wave theory is not capable to model a highly nonlinear wave [5]. A validated fully nonlinear potential flow solver OceanWave3D [12] is used in this paper, which is able to simulate fully nonlinear waves in a relatively rough domain with fast speed. The numerical domain is smaller than the experimental tank size as shown in Figure 5 and the stream function wave is applied in the wave generation zone to speed up the simulation.
4.2. **HAWC2**

The structural responses are simulated using DTU Wind Energy’s developed aero-hydro-elastic code HAWC2 [13][14], which is based on a multibody formulation with a floating frame. HAWC2 uses Morison’s equation to calculate the quasi-static load and the slamming load model has been implemented recently. Morison’s equation is calculated as the sum of three force terms: the Froude-Krylov force, the water added mass and the drag force. The formulation for flooded members is written as Equation 2:

\[
F_M(t) = \frac{1}{2} C_d \rho D |u_{rel}| + \rho (A - A_i) \ddot{u} + \rho (C_a A + A_i) \dot{u}_{rel}
\]  

(2)

\(C_d\) and \(C_a\) are drag and added mass coefficients, both values are chosen as 1.0 because the Keulegan Carpenter number is less than 10. \(u\) is the water particle velocity, \(u_{rel}\) and \(\dot{u}_{rel}\) represent the relative velocity and acceleration respectively. \(\rho\) is the water density and \(D\) is the diameter of slender piles. \(A\) and \(A_i\) are the total and the inner reference cross sectional areas.

The vertical pile is modelled as beam elements in HAWC2 with material properties listed in Figure 6. The force transducers at the top and at the bottom of the pile are modelled as a spring damper system. The structural motion is constrained in the vertical direction and free in other directions. The model in HAWC2 should be dynamically equivalent with the experimental system. The first natural frequency and the damping ratio have been measured from the experiments at around 19 Hz and 0.05 respectively, which are set in the model by tuning the spring-damper system parameters.

![Figure 5. Configuration of numerical set up in the computer model OceanWave3D](image)

![Figure 6. Sketch and system properties of the pile modelled in HAWC2](image)
5. Results

5.1. Verification of the HAWC2 model

Verification of the HAWC2 model is carried out using a free decay test. A small initial displacement at the pile top results in a free decay response matching the pre-determined natural frequency and damping ratio as shown in Figure 7.

![Figure 7. Verification of HAWC2 model using free decay test](image)

5.2. Non-breaking wave

Several non-breaking waves are identified in the wave test as the force experiments do not show any structural vibration. The non-breaking wave before the first breaking wave case is found to have similar wave shape to breaking waves as shown in Figure 4. The non-breaking wave results in only the slowly varying load, which is used to validate the numerical simulation results from HAWC2 based on Morison’s equation.

Wave generation is done using OceanWave3D and the wave surface elevation and the wave kinematics data are obtained at the pile location. Then, the extracted wave kinematics data are used in HAWC2 to simulate the response forces at the top and bottom force transducers respectively. The measured wave surface elevation is compared with the OceanWave3D simulation results and the measured quasi-static forces from the non-breaking wave are compared with HAWC2 simulation results. Figure 8 shows that the wave elevation simulated by OceanWave3D agrees well with the experimental data. Details around the trough are not fully captured because the high frequency wave components are damped out when using OceanWave3D. Figure 9 shows comparison for forces at the pile top and bottom, respectively. The simulated responses are in good agreement with the experimental data. However, the secondary load cycle is not shown in the simulation results. It is in agreement with previous findings, which show that the secondary load cycle is caused by the fluid motion at the downstream side of the cylinder [15]. This complicated process is not accounted for within the simple Morison’s equation.

5.3. Breaking wave

5.3.1. Decomposition of slamming load response

The time histories of measured forces for breaking waves show significant structural vibrations, which are excited by the slamming loads. The difference between the measured force caused by a breaking wave and the time history of measured force caused by a non-breaking wave results in the dynamic response for the slamming loads as shown in Figure 10. The measured dynamic
Figure 8. Validation of wave elevation from OceanWave3D using experimental data

Figure 9. Validation of quasi-static force for a non-breaking wave using HAWC2

responses due to the slamming loads are then compared with the numerical simulation results to estimate the governing parameters in the slamming load model formulated by Equation 1.

5.3.2. Monte Carlo samplings
Parameters are sampled using the Monte Carlo method, where a uniform probability distribution of the parameters is assumed as shown in Table 2. 5000 random combinations of the three parameters are picked as the input for HAWC2 simulations. The fitted decay curves after the peak response hit by a breaking wave for the simulated results and experimental data are
Figure 10. Decomposition of the slamming response from measured total dynamic response by subtracting the quasi-static force part compared against each other in order to highlight the response peaks in the time history as shown in the Figure 11.

Table 2. Distribution of governing parameters using the Monte Carlo method

| Parameters       | Slamming Coefficient $C_s [-]$ | Impact Duration $T [s]$ | Curling Factor $\lambda [-]$ |
|------------------|---------------------------------|--------------------------|-----------------------------|
| Range            | 0.5$\pi$-2.5$\pi$               | 0.02-0.26                | 0.3-0.5                     |
| Distribution     | Uniform                         | Uniform                  | Uniform                     |

In this paper, the parameters are tuned to reproduce the equivalent slamming response with the measurements. The difference between simulated and measured slamming response are represented by comparing the decay curves using the root mean square error (RMSE). Among the 5000 simulations, 100 combinations of the three parameters which give smallest RMSE are selected as the estimated parameters. The mean values of these estimated parameters are substituted into the slamming load model in HAWC2 to simulate their responses. The comparison between measurements and simulation results shows a good agreement for the peak values as shown in Figure 12.
Figure 11. Decaying curve fitting for comparison between simulation and experimental data

Figure 12. Comparison of the simulated and measured total dynamic response for the breaking wave Case No.3. The measured quasi-static force is added on the slamming response to get the total dynamic response for both of simulation and measurements.

5.3.3. Identified parameters
Ten valid breaking wave packets are identified from the experimental data. The estimated parameters show significant variability from wave to wave as shown in Figure 13, which is consistent with previous findings [4][5][10]. Even though variability exists, the slamming coefficient $C_s$ and the curling factor $\lambda$ are found similar to the values proposed by the Wienke-Oumeraci model. Nevertheless, the slamming impact duration $T$ is significantly larger than the value found by the Goda model and the Wienke-Oumeraci model. This finding is also reported recently by the project WIFI in the Netherlands [5]. The statistics for the first eight breaking waves are listed in Table 3. Note that the last two breaking cases give results far away from the mean values.
Figure 13. Estimated parameters for all breaking wave packets in the experiment (Error bars indicate the standard deviation of the selected 100 parameters).

Table 3. Statistics of the estimated parameters (Case 1-8)

| Parameters             | Slamming Coefficient $C_s$ | Impact Duration $T$ | Curling Factor $\lambda$ |
|------------------------|-----------------------------|---------------------|---------------------------|
| Mean                   | $1.89\pi$                   | $1.95\frac{R}{C}$  | $0.39$                    |
| Standard Deviation     | $0.21\pi$                   | $0.35\frac{R}{C}$  | $0.02$                    |
| Goda Model             | $\pi$                       | $\frac{R}{T}$      | $0.4-0.5$                 |
| Wienke-Oumeraci Model  | $2\pi$                      | $\frac{13}{32}\frac{R}{C}$ | $0.46$ |

6. Discussion

6.1. Importance of impact duration

Even though the estimated impact duration shows quite significant variability, it’s still valid to state that both the Goda model and the Wienke-Oumeraci model significantly underestimate the impact duration of the slamming loads induced by breaking waves. From classic structural dynamic theory [16], it is known that the dynamic amplification factor (DAF) of a single degree of freedom system subjected to triangle loads is decided by the time ratio between impact duration and system natural period. A monopile supported OWT can be modelled as a single degree of freedom system, and its first natural period is relatively larger than the slamming load impact duration. Therefore, a fair estimation of the impact duration is critical to understand the dynamic effects of the slamming loads on design loads as shown in Figure 14. Basically, for OWTs located in areas where breaking waves are present, a flexible structure is recommended to eliminate the dynamic amplification from slamming loads.

6.2. Uncertainty discussion

Determination of the wave celerity is crucial for a proper estimation of the wave slamming load as it decides the load intensity as well as the impact duration. However, the uncertainty determining the wave celerity is still high as it is not straightforward to calibrate from measurements. Three approaches are normally used to calculate the wave celerity. $rac{L}{T}$ is used if the recorded or calculated wave length $L$ and wave period $T$ exist. In case of absence of the information, the water particle velocity at the crest of a breaking point can be used as it is found identical to the wave celerity [6]. The linear or nonlinear dispersion relation can also be used to calculate the wave celerity.

Uncertainties always exist in measurement campaigns due to installation uncertainty, measurement device errors, human factors and so on. The case here is also faced with these uncertainties. In order to minimize the uncertainties, more test cases should be performed to
Figure 14. Dynamic amplification factor in the relation with time ratio. The x axis is the time ratio between impact load duration and natural period of the dynamic system, the y axis is the ratio of maximum dynamic response over maximum static response [16].

guarantee its reliability from the aspect of statistics. The measurement sampling frequency 200Hz is relatively low compared with the high structural frequency around 19Hz.

Even though uncertainties exist in several aspects, the influence is considered as marginal because the repeated wave tests show similar trends. The findings of the governing parameters in this paper will not differ too much even if these uncertainties were dealt with.

7. Conclusions
This paper investigates the governing parameters of a slamming load model by combining large scale experimental data with numerical simulation results performed using the Monte Carlo method. The fully nonlinear potential flow solver OceanWave3D is used, which is able to reproduce the experimental data. The wave surface elevation simulated from OceanWave3D is in good agreement with the measured surface elevation. The wave kinematics extracted from OceanWave3D is used in HAWC2 for simulating the quasi-static load of a non-breaking wave based on Morison’s equation. The simulation results for a non-breaking wave are validated against the experimental data. For breaking waves, 5000 random samplings of the three governing parameters using Monte Carlo method are simulated in HAWC2 to estimate the suitable parameters. It is concluded that the slamming coefficient $C_s$ and the curling factor $\lambda$ are close to the values proposed by the Wienke-Oumeraci model. However, either the Goda model or the Wienke-Oumeraci model underestimate the impact duration. The effect of impact duration on dynamic amplification for OWTs is discussed and it can be concluded that it is critical for the assessment of the dynamic responses of OWTs. Further experiment using a flexible pile with realistic dynamic properties (natural frequency, mode shape etc.) similar to a real OWT would be useful to investigate whether the dynamic response is significant or not. Besides, only well controlled regular wave is studied here which cannot represent the highly nonlinear irregular wave in the open sea. Therefore, the results from this study might be limited for predicting the slamming force in the open sea.
Acknowledgments

This study is a part of project DeRisk (Grant Number 4106-00038B), which is funded by Innovation Fund Denmark. Further funding is provided by Statoil and the participating partners. All funding is gratefully acknowledged. Henrik Bredmose at DTU Wind Energy is greatly appreciated for guiding OceanWave3D and Jithin Jose at University of Stavanger is appreciated for the fruitful discussion.

References

[1] Hallowell S, Myers AT, Arwade SR. Variability of breaking wave characteristics and impact loads on offshore wind turbines supported by monopiles. Wind Energy. 2016 Feb 1;19(2):301-12.
[2] TC88-MT IE. IEC 61400-3: Wind turbines part 1: Design requirements. International Electrotechnical Commission, Geneva. 2005.
[3] Goda Y, Haranaka S, Kitahata M. Study of impulsive breaking wave forces on piles. Report Port and Harbour Technical Research Institute. 1966 Apr;6(5):1-30.
[4] Wienke J, Oumeraci H. Breaking wave impact force on a vertical and inclined slender pile-theoretical and large-scale model investigations. Coastal Engineering. 2005 May 31;52(5):435-62.
[5] Burmester S, de Ridder EJ, Wehmeyer C, Asp E, Gujer P. Comparing Different Approaches for Calculating Wave Impacts on a Monopile Turbine Foundation. In: ASME 2017 36th International Conference on Ocean, Offshore and Arctic Engineering 2017 Jun 25 (pp. V010T09A063-V010T09A063). American Society of Mechanical Engineers.
[6] Irshik K, Sparboom U, Oumeraci H. Breaking wave characteristics for the loading of a slender pile. InProc. 28th Int. Conf. Coastal Eng., ICCE 2003 (Vol. 2002, pp. 1341-1352).
[7] Bredmose H, Jacobsen NG. Breaking wave impacts on offshore wind turbine foundations: focused wave groups and CFD. In: ASME 2010 29th International Conference on Ocean, Offshore and Arctic Engineering 2010 Jan 1 (pp. 397-404). American Society of Mechanical Engineers.
[8] Kamath A, Chella MA, Bils H, Amntsen A. Breaking wave interaction with a vertical cylinder and the effect of breaker location. Ocean Engineering. 2016 Dec 1;128:105-15.
[9] Choi SJ, Lee KH, Gudmestad OT. The effect of dynamic amplification due to a structure’s vibration on breaking wave impact. Ocean Engineering. 2015 Mar 1;96:8-20.
[10] Hansen HF, Kofod-Hansen H. An Engineering-Model for Extreme Wave-Induced Loads on Monopile Foundations. In: ASME 2017 36th International Conference on Ocean, Offshore and Arctic Engineering 2017 Jun 25 (pp. V03BT02A014-V03BT02A014). American Society of Mechanical Engineers.
[11] Jose J, Choi SJ, Giljarhus KE, Gudmestad OT. A comparison of numerical simulations of breaking wave forces on a monopile structure using two different numerical models based on finite difference and finite volume methods. Ocean Engineering. 2017 June 1;137:78-88.
[12] Engsig-Karup AP, Bingham HB, Lindberg O. An efficient flexible-order model for 3D nonlinear water waves. Journal of computational physics. 2009 Apr 1;228(6):2100-18.
[13] Larsen TJ, Hansen AM. How 2 HAWC2, the user’s manual. target. 2015 July:22.
[14] Kim T, Hansen AM, Branner K. Development of an anisotropic beam finite element for composite wind turbine blades in multibody system. Renewable Energy. 2013 Nov 30;59:172-83.
[15] Paulsen BT, Bredmose H, Bingham HB, Jacobsen NG. Forcing of a bottom-mounted circular cylinder by steep regular water waves at finite depth. Journal of fluid mechanics. 2014 Sept.;755:1-34.
[16] Biggs JM, Biggs JM. Introduction to structural dynamics. McGraw-Hill College; 1964.