Analysis of Energy Conversion Law in Vehicle Collision Accident

Hongpeng CHI*, Bing GONG
BGRIMM Technology Group, Beijing, 100160, China
*Corresponding author’s e-mail: hongpengchi@163.com

Abstract. The characteristics of automobile collision are introduced. The dynamic analysis of automobile collision accident process is carried out. The whole collision process is reproduced. The energy conversion law of collision accident process is analyzed qualitatively and quantitatively. The calculation of collision deformation of experimental vehicle under three conditions is completed.

1. Introduction
Both the safety design of automobile structure and the scientific analysis of traffic accidents require mastering the basic rules of automobile accident characteristics and collision. The difficulty of the problem lies in that in the process of collision, the body structure of a car produces rapid non-linear large deformation under the action of transient force. It is impossible to deduce the vehicle speed before collision solely from the kinematics and dynamics of rigid body. It is necessary to study the elastic-plastic performance of the car structure and the variation law of the related deformation, energy, speed, acceleration and impact force in the course of collision [1]. The non-linear relationship between these characteristic parameters and collision velocity is determined.

2. Characteristic of automobile collision
Collision refers to the process in which two vehicles or vehicles are in violent contact with obstacles, exchanging momentum, deforming (mainly plastic deformation) and consuming the kinetic energy of the vehicle. Usually, the process of automobile collision involves: displacement, rotation, deformation of automobile shell structure; plastic flow and hardening of materials; contact friction of boundary. Collision time is very short, it carries a lot of information about the collider [2]. Strictly speaking, car crashes have the following characteristics:
- The phenomenon of exchanging moving energy between vehicles;
- Mutual extrusion, through the body damage and fixture damage to consume part of the movement energy;
- It is a phenomenon that some parts are mutually damaged while others are mutually exclusive.
- There is not only the exchange of motion energy, but also the phenomenon of converting part of motion energy into angular motion.
- There is intense relative movement between vehicles and passengers and cargo.
- Collision usually takes place in 0.1-0.2 seconds.
3. Dynamics analysis of vehicle collision accidents

The collision phenomenon in automobile accident is a complicated mechanical process. As far as the mechanical properties of automobile itself are concerned, there are both the rigid side of steel structure and the consequence of plastic deformation under certain impact force [3]. At the same time, due to the differences in the quality, speed, structure and shape of the car in the collision, the damage degree and the motion condition of the car after the collision will be very different [4].

For the mechanics process involved in automobile collision accident, the basic mechanics theorem is taken as the premise to simplify the model and make corresponding analysis and calculation. In order to establish the dynamic equation, the following assumptions are made: the collision process satisfies the momentum conservation and ignores the external forces; the mass distribution and geometric parameters of the vehicle remain unchanged during the collision process; the frontal collision, and the collision impact distributes uniformly on the contact surface[5].

Based on the frontal collision hypothesis, the vehicle collision process can be simplified to a one-dimensional collision model. Therefore, according to the law of conservation of momentum, it can be obtained that:

\[ m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad \Delta E = \frac{1}{2} (1 - \epsilon^2) \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2 \] (1)

Among them, \( m_1 \) and \( m_2 \) are the mass of two vehicles in collision; \( v_1 \) and \( v_2 \) are the velocities before collision; \( v'_1, v'_2 \) are the velocities after collision; \( \epsilon \) is the recovery coefficient between two vehicles, and \( \Delta E \) is the loss of mechanical energy during collision.

3.1 Reappearance of collision process

According to the vehicle collision process, it is divided into two stages: braking process before collision; collision process [6].

3.1.1 Braking process: The braking process can be divided into three stages.

- When the driver is aware of the collision accident, there is a certain reaction time \( t_1 \) before the braking measures are taken, and the car still moves at a uniform speed at this stage.
  - Drivers take braking measures. Before braking is complete (when the wheel is locked, the braking is considered to be complete), the resistance of the vehicle includes rolling friction and sliding friction. The duration of this stage is \( t_2 \).
  - The resistance of the vehicle is pure sliding friction. The vehicle moves in a straight line with uniform deceleration until collision occurs. The time is \( t_3 \).

Because the rolling friction coefficient has a certain relationship with the speed of the vehicle, and the braking process of the vehicle is a variable acceleration process, there is a variable acceleration. Under the action of non-inertial force, the pressure produced by the front and rear wheels is different, so the rolling friction coefficient of the front and rear wheels is different. For the second stage of braking process, the friction coefficient is approximately considered to be constant. The average values of rolling resistance coefficient and sliding resistance coefficient are taken as follows:

\[ \mu_3 = (\mu_1 + \mu_2)/2 \] (2)

\( \mu_1 \)- Rolling resistance coefficient, \( \mu_2 \)- Sliding resistance coefficient.

The equation of motion of braking process can be expressed as follows:

- Uniform linear motion: \( S_1 = v_1 t_1, v_1 \) is the normal driving speed before braking, \( t_1 \) is the driver’s reaction time.
- Variable deceleration linear motion process: \( S_2 = v_2 t_2 + 1/2 a_2 t_2^2, v_3 = v_2 + a_2 t_2, a_2 = -\mu_3 g \). In the model, \( v_2 = v_1 \), which is the normal driving speed before braking, \( t \) is the time from start braking to complete braking, \( a = \mu_3 g \).
- Uniform deceleration linear motion process: \( S_3 = (v_4^2 - v_3^2)/2a_3, a_3 = -\mu_2 g \).
In the model, \( s \) is the displacement of the car from just complete braking to collision, \( v_1 \) is the instantaneous speed of the car just complete braking, and \( v_i \) is the instantaneous speed of the car when collision occurs.

And satisfy the following formula.

\[
S = S_1 + S_2 + S_3
\]

\( S \) is the distance between the front of the car and the obstacle when the driver is aware of the collision. Ultimately available

\[
v_4 = \sqrt{2\mu g\left[v_1(t_1 + t_2) - 1/2 \mu g t_2^2 + (v_1 - \mu g t_2)^2/2\mu g - S\right]}
\]

### 3.1.2 Collision process

In the process of automobile collision, the kinetic energy of automobile is consumed by plastic deformation. Plastic deformation refers to the deformation of an object under the action of an external force. When the applied external force is removed, the object can not be restored to its original state. On the one hand, the process of plastic deformation of automobile is related to the size of collision force, on the other hand, it is related to the structural stiffness of automobile. Structural stiffness is different, the impact force and deceleration of vehicle deformation response are different.

### Table 1. Vehicle stiffness coefficient

| Level 1 | Front | Back | Side |
|---------|-------|------|------|
|         | \( f_1/(N \cdot cm^2) \) | \( f_2/(N \cdot cm^2) \) | \( f_3/(N \cdot cm^2) \) |
| 1       | 528.9 | 32.4 | 641.0 | 26.20 | 134.8 | 25.5 |
| 2       | 453.6 | 29.6 | 684.7 | 28.3 | 245.2 | 46.2 |
| 3       | 555.2 | 38.6 | 718.0 | 30.3 | 303.0 | 39.3 |
| 4       | 623.5 | 23.4 | 625.2 | 8.96 | 250.4 | 34.5 |
| 5       | 569.2 | 25.5 | 520.1 | 48.3 | 310.0 | 34.5 |
| 6       | 569.2 | 25.5 | 520.1 | 48.3 | 310.0 | 34.5 |
| 7       | 670.7 | 86.9 | 525.4 | 37.9 | 310.0 | 34.5 |
| 8       | 840.6 | 34.5 | 605.9 | 17.2 | 310.0 | 34.5 |
| 9       | 853.2 | 26.2 |       |      |       |      |

The stiffness coefficients and lateral rotation inertia radius of nine kinds of vehicles were found out by the National Safety Administration from 180 vehicle crash experiments, as shown in Table 1.

On the determination of coordinate system: two vehicles will move at the same speed after collision, and then they will be selected as active coordinates. The vehicle impact width is represented by \( B \), the displacement of vehicle 1 and 2 relative to the contact surface is represented by \( x_1 \) and \( x_2 \), the displacement of contact surface is represented by \( x_3 \), and the impact stiffness coefficients of vehicle 1 and 2 are represented by \( C_{A1}, C_{B1} \) and \( C_{A2}, C_{B2} \) respectively. So, the deformation forces of car 1 and 2 are respectively:

\[
F_1 = [C_{A1} + C_{B1}(x_1 + x_3)]B, \quad F_2 = [C_{A2} + C_{B2}(x_2 - x_3)]B
\]

Motion Equation of Vehicle 1: \( m_1\ddot{x}_1 + F_1 = 0 \)

On the basis of the principle that the displacement of the relative moving coordinate system of two vehicles is inversely proportional to their mass.

\[
x_2 = x_1(m_1/m_2), \quad \dot{x}_1 + (C_B B/m)X_1 + C_B B/m_1 = 0, \quad m = m_1m_2/(m_1 + m_2), \quad p = C_B/m_1, \quad q = C_B/m_2
\]

\( C_A, C_B \) and \( m \) are called conversion stiffness and conversion mass respectively. For solving non-homogeneous differential equations, there are
\[ X_1 = A_1 \sin(\omega_1 t + \alpha_1) - C_1 \left( \frac{C_A}{C_B} \right) \cdot \dot{X}_1 = A_1 \omega_1 \cos(\omega_1 t + \alpha_1), \dot{X}_1 = -A_1 \omega_1^2 \sin(\omega_1 t + \alpha_1) \]  

From the initial conditions \( t = 0, X = 0, \dot{X}_1 = 0 \)

\[ \omega_1 = \sqrt{\frac{100C_A B/m_1}{\alpha_1}} = \tan^{-1} C_A m_1 v_c A_1 = C_1 \sqrt{(v_c^2/\omega_1)^2 + (C_A/C_B)^2}, C_1 = m/m_1, v_c = v_{10} + v_{20} \]  

According to a large number of experimental data in vehicle crash studies in literature, the relationship between crash velocity and deformation is also linear. Therefore

\[ C_2 = m_1/m_2, C_3 = (C_2 q - 1)/(1 + q), C_4 = (p - 1)/(1 + q) \]  

So \( X_2 = C_2 X_1, \dot{X}_2 = C_2 \dot{X}_1, X_3 = C_3 X_1 + C_4 (C_A/C_B), \dot{X}_3 = C_3 \dot{X}_1, X_3 = C_3 \dot{X}_1. \)

So, the deformation of car 1 and 2 in time \( t \) is as follows: \( \Delta t = X_1 + X_3, \Delta t = X_2 - X_3. \)

### 3.2 Collision energy conversion

Most of the kinetic energy in the collision process is converted into the internal energy of the car, because the metal plastic deformation after the collision increases the internal energy a lot. Other kinetic energy is converted into heat energy, sound energy, etc. (Compared with the increase of internal energy, it can be neglected in engineering) [6].

![Figure 1. Vehicle collision energy curve](image)

If it hits a wall (or something else) and the wall is not broken, then kinetic energy is converted into internal energy. If you hit something else, such as another car, and the other car is hit, then part of the kinetic energy is converted into internal energy. The above condition is that the car collides on a horizontal road. If a collision occurs on a slope, the energy conversion part also includes potential energy.

In fact, there must be energy loss in the collision of macroscopic objects (mechanical energy loss, which is kinetic energy loss when potential energy is not involved) because the objects are heated when colliding, which conforms to the conservation of energy. Under ideal conditions, the mechanical energy of the system is conserved. In fact, ideal conditions do not exist. All actual collisions are incomplete elastic collisions. During the collision, inelastic deformation will be caused, and some mechanical energy will be transformed into internal energy. In order to stop the car, all this energy must be converted into other forms. If the brakes stop, they dissipate energy by generating heat through friction. In the case of a crash, energy dissipation is achieved by bending and extrusion of the metal outside the car. Figure 1 shows the energy change curve of vehicle frontal impact.

The results show that the collision force \( F \) of object-vehicle collision coincides with that of vehicle-vehicle collision. \( F = (C_A + C_B x) B \).

In the above formula, \( C \) is the stiffness coefficient and \( B \) is the collision width. From the function relation, we can find the work done by the collision force to the automobile in this process. \( W = \int_0^S F dx \).

Thus, it can be concluded that the energy consumed by plastic deformation is

\[ W = \int_0^S (C_A B + C_B x B) dx = (C_A B x + 1/2 C_B B x^2) \bigg|_0^S = C_A B S + 1/2 C_B B S^2 \]  

(11)
Because the work done by the collision force to the car in this process is equal to the loss of the kinetic energy of the car, so the collision force is equal to the loss of the kinetic energy of the car.

\[ \Delta E_K = 1/2 \, m v^2 = W = C_A S + 1/2 \, C_B S^2 \]  

(12)

Therefore, the corresponding deformation can be estimated as

\[ S = (\sqrt{C_A^2 B^2 + m v^2 C_B} - C_A B) / C_B B \]  

(13)

At the same time, the maximum and minimum acceleration can be known.

\[ a_{\text{max}} = F_{\text{max}} / m = F \mid_{x = S} = (C_A S + C_B) / m, \quad a_{\text{min}} = F_{\text{min}} / m = F \mid_{x = 0} = C_B / m \]  

(14)

Thus, the upper- and lower-time limits of the collision process can be calculated.

\[ t_s = v_4 / a_{\text{min}} = v_4 / (C_B / m), \quad t_x = v_4 / a_{\text{max}} = v_4 / [(C_A S + C_B) / m] \]  

(15)

3.3 Test vehicles

| Vehicle type | Flammable liquid tank semi-trailer |
|--------------|-----------------------------------|
| Total quality | 40000(Kg) |
| Rated load quality | 31600(Kg) |

For asphalt pavement with good weather and good pavement, the longitudinal adhesion coefficient \( \psi = 0.72 \) is taken. Assume that the methanol transport tanker A travels at a speed of 80km/h. The stationary truck B in front, the total mass of the truck B is 25 tons. There was a rear-end collision accident between the tanker and the truck. After the collision, the tanker taxied 15 meters. According to the law of conservation of energy.

The kinetic energy \( E \) of tanker car before the accident is as follows:

\[ E = 1/2 \, m v^2 = 9876543.21J. \]

The slip energy loss of the two vehicles is as follows:

\[ E_h = (m_1 + m_2) \, g \, \psi \, s = 6879600J. \]

Most of the other energy is transformed into the deformation energy of tank car. According to the calculation formula of the deformation energy of tank car:

\[ W = \int_0^S (C_A B + C_B x) \, dx = \left( C_A B x + \frac{1}{2} C_B B x^2 \right) \bigg|_0^S = C_A BS + 1/2 \, C_B B S^2, \quad W = E - E_h \]  

(16)

Referring to the collision stiffness coefficient \( C_A = 840.6 \, N/cm, \quad C_B = 34.5 \, N/cm^2 \) and the width \( B = 250 \, cm \) of the truck head, the deformation of the truck can be calculated as follows:

\[ S = \left( \sqrt{C_A^2 B^2 + 2 C_B B (E - E_h)} - C_A B \right) / C_B B = 11.55cm. \]

If the other conditions remain unchanged and the vehicle glides 10 meters after collision, there is a law of conservation of energy. The kinetic energy \( E \) of the tank car before the accident is as follows:

\[ E = 1/2 \, m v^2 = 9876543.21J. \]

The slip energy loss of the two vehicles is as follows:

\[ E_h = (m_1 + m_2) \, g \, \psi \, s = 4586400J. \]

Vehicle deformations:

\[ S = \left( \sqrt{C_A^2 B^2 + 2 C_B B (E - E_h)} - C_A B \right) / C_B B = 9.83cm. \]

If the other conditions remain unchanged and the vehicle glides 5 meters after collision, there is a law of conservation of energy. The kinetic energy \( E \) of the tank car before the accident is as follows:

\[ E = 1/2 \, m v^2 = 9876543.21J. \]

The slip energy loss of the two vehicles is as follows:

\[ E_h = (m_1 + m_2) \, g \, \psi \, s = 2293200J. \]

Vehicle deformations:

\[ S = \left( \sqrt{C_A^2 B^2 + 2 C_B B (E - E_h)} - C_A B \right) / C_B B = 24.1cm. \]

If the other conditions remain unchanged, the vehicle hits a rigid building and stops immediately after the collision, then there is the law of conservation of energy. The kinetic energy \( E \) of the tank car before the accident is as follows:

\[ E = 1/2 \, m v^2 = 9876543.21J. \]

The slip energy loss of the two vehicles is 0, and the deformation of the vehicle is \( S \).

\[ S = \left( \sqrt{C_A^2 B^2 + 2 C_B B E - C_A B} \right) / C_B B = 29.3cm \]  

(17)
4. Conclusion
The whole crash process is reproduced by analyzing the dynamics of vehicle crash accident process. Qualitative and quantitative analysis of energy conversion law in collision accident process is carried out. Most of the kinetic energy in the collision process is converted into the internal energy of the car. Because of the plastic deformation of the metal after collision, the internal energy increases a lot. Other kinetic energy is converted into heat energy, sound energy and so on. Compared with the increase of internal energy, this part can be neglected in engineering. The calculation of the collision deformation of the experimental vehicle under three conditions is completed.

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