Triple collisions $e^- p^7\text{Be}$ in solar plasma

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Abstract

Several nuclear reactions involving the $^7\text{Be}$ nucleus, not included into the standard model of the $pp$–chain, are discussed. A qualitative analysis of their possible influence on the fate of the $^7\text{Be}$ in solar plasma and of their role in the interpretation of the solar neutrino experiments is given. As an example, the reaction rate of the nonradiative production of $^8\text{B}$ in the triple collision $p + e^- + ^7\text{Be} \rightarrow ^8\text{B} + e^-$ is estimated in the framework of the adiabatic approximation. For the solar interior conditions the triple collision reaction rate is approximately $10^{-4}$ of that for the binary process $p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma$.

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I. INTRODUCTION

The so–called standard model of the sun has been developed during several decades by a collective effort of many researchers. At present it is an elaborate theory which accounts for many, though not all, observable characteristics of the sun. One of the exceptions is the flux of neutrinos which is several times higher than what is actually seen in experiments (for details on the neutrino problem see, for example, Ref. [1]). There are several possible reasons for this discrepancy which are associated either with the nuclear processes involved or with the neutrino propagation towards the earth.

The solar neutrinos emerge from nuclear reactions which occur in plasma consisting of light nuclei (mainly protons) and electrons. The sequence of these reactions begins with the proton–proton collision and is, therefore, called the pp–chain. In the standard model it is
assumed that this chain consists of the following reactions

\[
p + p \rightarrow ^2\text{H} + e^+ + \nu \\
p + p + e^- \rightarrow ^2\text{H} + \nu \\
p + ^2\text{H} \rightarrow ^3\text{He} + \gamma \\
^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + p + p \\
^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma
\]

and then the $^7\text{Be}$ is destroyed in two ways

\[
e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu \\
p + ^7\text{Li} \rightarrow ^8\text{Be} + \gamma \\
^8\text{Be} \rightarrow ^4\text{He} + ^4\text{He} \\
p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma \\
^8\text{B} \rightarrow ^{8}\text{Be}^* + e^+ + \nu \\
^8\text{Be}^* \rightarrow ^4\text{He} + ^4\text{He}
\]

The fate of $^7\text{Be}$ in this chain is of special interest since a combined analysis of all experiments, in which the neutrino flux from the sun was measured, has come to a paradoxical conclusion: the production of $^7\text{Be}$ nuclei (more precisely, the flux of neutrinos due to $^7\text{Be}$ reactions) must be strongly suppressed or even be negative $^{[2,3]}$. This implies that something is wrong either in the standard model or in the experimental data.

In this work we present a qualitative as well a quantitative analysis of the nuclear reactions involving $^7\text{Be}$, which may contribute to a more adequate description of the behavior of this nucleus in the sun. Our approach differs from the standard model in that we take into account also the triple collisions in solar plasma. In contrast the standard pp–chain consists of a sequence of reactions of two–body initial states, the only exception being the ppe$^-$ collision.

There are two essential differences between the nuclear reactions caused by binary and triple collisions. They can be classified as kinematical and dynamical. The former are related to the selection rules prevailing in the two– and three–body reactions while the latter stem from the inter–dependence of different nuclear processes. Thus, some binary nuclear reactions suppressed by conservation laws (concerning angular momentum, parity, isospin etc.) can be eased due to the presence of a third particle. Therefore, the three–body mechanism of such reactions which is less restricted kinematically, may play a significant role in the nuclear burning in stellar plasma where the probability of triple collisions could be quite high due to the high density of matter. The dynamics of the three particle motion may also lead to a completely different physical picture: Processes considered as independent in binary collisions become dependent when triple collisions are considered. For example, the $e^- + ^7\text{Be}$ and $p + ^7\text{Be}$ processes, become dependent to each other when the $e^- + p + ^7\text{Be}$ collision is taken into account.

The paper is organized as follows. In Sect. II we discuss the role of the three–body collisions qualitatively. In Sect. III we describe our formalism and in IV and V we outline the procedure employed to evaluate the various ingredients used to obtain the reaction rate. In Sect. VI we present our results and conclusions. Some details concerning the derivation of the reaction rate formula are given in the Appendix.
II. BINARY AND TRIPLE COLLISIONS

According to the standard model of the sun the $^7\text{Be}$ nucleus, produced in the pp–cycle, is destructed via the following two binary reactions

\begin{align*}
e^- + ^7\text{Be} &\to ^7\text{Li} + \nu, \quad (1) \\
p + ^7\text{Be} &\to ^8\text{B} + \gamma. \quad (2)
\end{align*}

Due to the high density of the plasma, proton and electron can always be found in the vicinity of $^7\text{Be}$ and form a three–body initial state

$$\left| p + e^- + ^7\text{Be} \right\rangle,$$

which can give rise not only to the above binary transitions (1) and (2) but also to the processes

$$p + e^- + ^7\text{Be} \longrightarrow \begin{cases} 
    ^7\text{Li} + p + \nu \\
    ^8\text{B} + \gamma + e^- \\
    ^8\text{B} + e^-
\end{cases}.$$  

The first reaction corresponds to the weak transition (1) provided the nearby proton does not participate in the reaction. Similarly, the second of these reactions corresponds to the radiative capture (2) if the electron is a spectator particle. It is obvious that the three processes (3) are not independent, since they are generated from the same initial state. Therefore, the neutrino fluxes related to $^8\text{B}$ and $^7\text{Be}$ nuclei are not independent either. This means that the procedure of comparison of the Homestake and Kamiokande experiments, which gave patently wrong negative amount of $^7\text{Be}$, could be incomplete and should be reconsidered.

The question then arises on how strong is the mutual dependence of the processes (3). This question can be formulated in an alternative way: To what extent the two–body subsystems, involved in (3), can be considered as independent? Formally such an independence would imply that the total wave function of the initial state (3) could be written as a direct product of the wave functions of the $e^- + ^7\text{Be}$, $p + ^7\text{Be}$, and $p + e^-$ subsystems,

$$\left| p + e^- + ^7\text{Be} \right\rangle \approx \left| e^- + ^7\text{Be} \right\rangle \otimes \left| p + ^7\text{Be} \right\rangle \otimes \left| p + e^- \right\rangle.$$  

The possibility of such a factorization for three charged particles was investigated in Refs. [4,5], where it was shown that the total wave function reduces to a product of the type (5) only when all three particles are far away from each other. Such a configuration, however, can not contribute much to the transitions (3), for they are caused by the strong and weak interactions which vanish at large distances. The description of the processes (3) as independent needs therefore an additional substantiation.

There are also other processes which, in principle, can change the balance between the neutrino fluxes from $^7\text{Be}$ and $^8\text{B}$ without changing the abundance of $^7\text{Be}$ nuclei. Two such examples are

\begin{align*}
e^- + e^- + ^7\text{Be} &\to ^7\text{Li} + e^- + \nu, \\
p + p + ^7\text{Be} &\to ^8\text{B} + p.
\end{align*}
In this article, however, we investigate only the processes leading to the $^8\text{B}$ formation as described by (4). This requires the reaction rate of the process

$$p + e^- + ^7\text{Be} \rightarrow ^8\text{B} + e^-$$

which is discussed next.

III. REACTION RATE

To estimate the reaction rate of (6) we assumed that the nucleus $^7\text{Be}$ can be considered as a particle and, therefore, the $^8\text{B}$ nucleus is a bound state of $^7\text{Be}$ with the proton. Of course such an approximation is inadequate if a full description of the spectrum and the structure of $^8\text{B}$ is required. Since, however, its ground state is only 0.138 MeV below the $(p+^7\text{Be})$–threshold [6] (see Fig. 1) this cluster representation should describe it fairly well. In the problem under consideration the other states of $^8\text{B}$ are not needed since the collision occurs at ultra-low energies of the order of $\sim 1 \text{ keV}$ (at temperatures of $\sim 10^7 \text{ K}$) and thus transitions via excited states can be safely neglected and the formation of $^8\text{B}$ proceeds directly from the continuum $(p+^7\text{Be})$ through a kind of Auger transition in which the electron carries away the excess energy.

The Hamiltonian, which governs the evolution of the three–body initial state (3), can be written as

$$H = H_0 + h_0 + V_n + V_e,$$

where $H_0$ and $h_0$ are the kinetic energy operators associated with the Jacobi variables $\vec{r}$ and $\vec{\rho}$ respectively (see Fig. 2), $V_n$ is the $p^7\text{Be}$ potential, which includes strong and Coulomb parts, and $V_e$ is the sum of the $e^-p$ and $e^-^7\text{Be}$ Coulomb potentials. The amplitude, describing various transitions in this three–body system, is proportional to the $T$–operator obeying the Lippmann–Schwinger equation

$$T(z) = V_n + V_e + (V_n + V_e) \frac{1}{z - H_0 - h_0} T(z).$$

The initial asymptotic state (3) is defined by the $p^7\text{Be}$ relative momentum $\vec{p}$, the total nuclear spin $s$ and its third component $m_s$, and by the electron momentum with respect to center of mass of the nuclei $\vec{k}$. The final asymptotic state consist of the bound state of a proton and $^7\text{Be}$, with spin $\sigma = 2$ and third component $m_\sigma$, and a freely moving electron with momentum $\vec{k}'$ which differs from $\vec{k}$ not only in its direction but also in its absolute value, since the electron picked up the energy released in the nuclear process. We note that we neglect the spin of the electron since we assume that its interaction can be satisfactorily described by purely coulombic potentials without spin–orbit or other forces involving magnetic moments. Therefore, the reaction (6) can be described as a transition between these asymptotic states,

$$\left| \vec{p}, s, m_s; \vec{k} \right\rangle \xrightarrow{V_n + V_e} \left| \Psi_{\text{bound}}^{\sigma m}, \vec{k}' \right\rangle .$$

The corresponding three–body reaction rate $R_3$ is given by (see Ref. [7])
\[ \mathcal{R}_3(\vec{p}, s, m_s, \vec{k} \to \sigma m_\sigma, \vec{k}') = \frac{1}{4\pi^2} \delta(E_f - E_i) \left| \left\langle \Psi_{\sigma m_\sigma}^{\text{bound}}, \vec{k}' \left| T \left| \vec{p}, s, m_s; \vec{k} \right. \right. \right\rangle \right|^2 n_e n_p n_{\text{Be}}^7, \]  

where the \( \delta \)-function secures the energy conservation while \( n_e n_p n_{\text{Be}}^7 \) is the product of the densities of electrons, protons, and \(^7\text{Be} \) nuclei in the plasma. The coefficient \( 1/4\pi^2 \) in Eq. (10) stems from the following normalizations of the wave functions

\[ \langle \vec{p}, s, m_s | \vec{p}', s', m'_{s} \rangle = (2\pi)^3 \delta(\vec{p} - \vec{p}') \delta_{ss'} \delta_{m_sm'_{s}}, \]

\[ \langle \Psi_{\sigma m_\sigma}^{\text{bound}} | \Psi_{\sigma' m'_{\sigma}}^{\text{bound}} \rangle = \delta_{\sigma\sigma'} \delta_{m_\sigma m'_{\sigma}}, \]

\[ \langle \vec{k} | \vec{k}' \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}'). \]

Both possible values of the \( p^7\text{Be} \) spin, \( s = 1 \) and \( s = 2 \), as well as all its orientations are represented by different \( p^7\text{Be} \)-pairs in the plasma with equal probabilities. The momenta \( \vec{p} \) and \( \vec{k} \), however, are distributed at the temperature \( \theta \) according to Maxwell’s law

\[ N_\vec{p}(\theta) = (2\pi m \kappa_B \theta)^{-\frac{3}{2}} \exp \left( -\frac{\vec{p}^2}{2m \kappa_B \theta} \right), \]

\[ N_\vec{k}(\theta) = (2\pi \mu \kappa_B \theta)^{-\frac{3}{2}} \exp \left( -\frac{\vec{k}^2}{2\mu \kappa_B \theta} \right), \]

where \( \kappa_B \) is the Boltzmann constant and the reduced masses \( m \) and \( \mu \) are defined by \( m^{-1} = m_p^{-1} + m_{\text{Be}}^{-1} \) and \( \mu^{-1} = m_e^{-1} + (m_p + m_{\text{Be}})^{-1} \).

We need the average reaction rate due to all possible initial states with different \( \vec{p}, s, m_s, \vec{k} \) and the total rate of transition to the final states with any allowed \( m_\sigma \) and \( \vec{k}' \). This can be obtained by averaging the rate (10) over all initial and summing it over all final quantum numbers. The resulting reaction rate depends only on the temperature of the plasma

\[ < \mathcal{R}_3(\theta) > = \sum_{s_m, m_s} \frac{1}{2s + 1} \int d^3 \vec{k}' \int d^3 \vec{p} \int d^3 \vec{k} N_\vec{p}(\theta) N_\vec{k}(\theta) \mathcal{R}_3(\vec{p}, s, m_s, \vec{k} \to \sigma m_\sigma, \vec{k}'). \]  

In what follows we consider the various approximations and ingredients needed to evaluate the reaction rate by means of this general formula.

### IV. ADIABATIC APPROXIMATION

The average kinetic energy of the particles in the plasma, \( < E^{\text{kin}} > \sim \kappa_B \theta \approx 1 \text{ keV} \), is the same for nuclei and electrons, but the velocity of an electron is three orders of magnitude higher than that of a proton or \(^7\text{Be} \). Therefore, while the proton approaches \(^7\text{Be} \) very slowly, the electron dashes nearby picking up the energy and leaving the heavy particles in a bound state. Due to this we can significantly simplify the problem by treating the relative \( p^7\text{Be} \) motion adiabatically. Such an approximation means that the electron bypasses the heavy particles very fast and thus it has no time to ‘observe’ any changes in their space position. Moreover, to participate in such three-body reaction, the heavy particles must be close to each other when the electron arrives, because they are too slow as compared to the electron. Hence, while the electron starts from its asymptotic state \( |\vec{k}\rangle \), the heavy particles have already interacted to each other via the potential \( V_n \) and formed the two-body scattering state.
\[ |\vec{p}, s, m_s \rangle \xrightarrow{V_n} |\Psi_{scat}^{\vec{p},s,m_s} \rangle. \] (12)

Therefore, instead of the transition (9) caused by both \( V_n \) and \( V_e \), in the adiabatic approximation we may consider the transition

\[ |\Psi_{scat}^{\vec{p},s,m_s}, \vec{k} \rangle \xrightarrow{V_e} |\Psi_{bound}^{\vec{p},s,m_s}, \vec{k}' \rangle, \] (13)

where the interaction \( V_n \) is taken into account by (12).

Formally, this leads to some simplifications in the Eq. (8). Indeed, since \( \Psi_{scat} \) and \( \Psi_{bound} \) can be considered as ‘excited’ and ground states of the nuclear target respectively, we can describe the transition (13) using another \( T \)-matrix defined as

\[ \tilde{T}(z) = V_e + V_e \frac{1}{z - h_0 - H_A} \tilde{T}(z), \] (14)

where \( H_A = H_0 + V_n \) is the total Hamiltonian of the nuclear subsystem. Within the above approximation we have

\[ \langle \Psi_{\sigma m_s}^{\vec{p}}, \vec{s}, s, \vec{k}' \rangle | T | \vec{p}, s, m_s, \vec{k} \rangle \approx \langle \Psi_{\sigma m_s}^{\vec{p}}, \vec{s}, s, \vec{k}' \rangle | \tilde{T} | \Psi_{scat}^{\vec{p},s,m_s}, \vec{k} \rangle. \] (15)

It is known (see Ref. [8]) that the adiabatic approximation is equivalent to the closure approximation that is when all excited states of the target are assumed to be degenerated and the Hamiltonian \( H_A \) is replaced by a constant, viz.

\[ H_A = \sum_n \mathcal{E}_n |\psi_n \rangle \langle \psi_n | + \int dE \mathcal{E}_0 |\psi_E \rangle \langle \psi_E | \]

\[ \approx \mathcal{E}_0 \left\{ \sum_n |\psi_n \rangle \langle \psi_n | + \int dE |\psi_E \rangle \langle \psi_E | \right\} = \mathcal{E}_0. \]

Therefore, in the adiabatic approximation \( \tilde{T} \approx \tilde{T}^0 \), where \( \tilde{T}^0 \) is defined by the following equation

\[ \tilde{T}^0(z) = V_e + V_e \frac{1}{z - h_0 - \mathcal{E}_0} \tilde{T}^0(z). \] (16)

In our case \( \mathcal{E}_0 \) is the ground state energy of the \(^8\text{B}\) nucleus, \( \mathcal{E}_0 = -0.138 \text{ MeV} \). Substituting into Eq. (16) the total energy \( z = E_e^{\text{kin}} + \mathcal{E}_0 \), where \( E_e^{\text{kin}} \) is the kinetic energy of the electron with respect to the center of mass of the heavy particles, we obtain the Lippmann–Schwinger equation describing the scattering of an electron from fixed centres. For the solar plasma electrons \( e^2/hv < 1 \), which is a sufficient condition to treat Coulomb interactions in Eq. (16) perturbatively, i.e.,

\[ \tilde{T}^0(z) = V_e + V_e \frac{1}{E_e^{\text{kin}} - h_0} V_e + V_e \frac{1}{E_e^{\text{kin}} - h_0} V_e \frac{1}{E_e^{\text{kin}} - h_0} V_e + \cdots, \]

Furthermore, the average potential energy of the electron is of atomic order of magnitude, \( < V_e > \sim 10 \text{ eV} \), while its kinetic energy in solar plasma is two orders of magnitude higher, \( < E_e^{\text{kin}} > \sim 1 \text{ keV} \), which implies that the above iterations should converge very fast. Therefore, we may retain only the first (Born) term [3].
Finally, the exact formula (10) for the reaction rate can be replaced by the following approximate one
\[
\mathcal{R}_3(\vec{p}, s, m_s, \vec{k} \to \sigma m_s, \vec{k}') = \frac{1}{4\pi^2} \delta(E_f - E_i) \left| \langle \Psi_{\sigma m_s}^{\text{bound}}, \vec{k}' | V_c | \Psi_{\vec{p}sm_s}^{\text{scat}}, \vec{k} \rangle \right|^2 n_e n_p n^{7}\text{Be} ,
\] (17)
which can be considered as a three-body generalization of the distorted wave Born approximation (DWBA) which is widely used in the theory of the two–body nuclear reactions. The explicit formula for the reaction rate, which involves \(\Psi_{\vec{p}sm_s}^{\text{scat}}(\vec{r})\) and \(\Psi_{\sigma m_s}^{\text{bound}}(\vec{r})\) described in the next section, is derived in the Appendix.

V. NUCLEAR WAVE FUNCTIONS

A. Scattering state

As mentioned earlier at the ultra-low energies considered (\(\sim 1\) keV) the collisions between protons and \(^7\text{Be}\) nuclei are dominated by the \(S\)–wave state. Therefore, in the partial wave expansion of the corresponding scattering wave function we may retain only the term with zero angular momentum
\[
\Psi_{\vec{p}sm_s}^{\text{scat}}(\vec{r}) = \frac{1}{pr} \sum_{\mu_1 \mu_2} < \frac{1}{2} \mu_1 \frac{3}{2} \mu_2 | sm_s > \chi_{\mu_1}^p \chi_{\mu_2}^{^7\text{Be}} u_p^{\text{scat}}(r) ,
\] (18)
where \(\chi_{\mu_1}^p\) and \(\chi_{\mu_2}^{^7\text{Be}}\) are the spin functions of the proton and \(^7\text{Be}\) nucleus. The \(S\)–wave radial function \(u_p^{\text{scat}}(r)\) is a regular solution of the radial equation
\[
\left\{ \frac{d^2}{dr^2} + p^2 - 2m [V_C(r) + V_S(r)] \right\} u_p^{\text{scat}}(r) = 0
\] (19)
with the asymptotic boundary condition
\[
u_p^{\text{scat}}(r) \xrightarrow[r \to \infty]{} \sin [pr - \eta \ln (2pr) + \delta_0] ,
\] where \(\eta\) is the Sommerfeld parameter. The potentials \(V_C\) and \(V_S\) in Eq. (13) describe the Coulomb and strong \(p^{^7\text{Be}}\) interactions respectively. In order to take into account the nonzero size of \(^7\text{Be}\) nucleus we assume that it is a uniformly charged sphere of radius \(R = 2.95\) fm. Thus the potential \(V_C\) is
\[
V_C(r) = \begin{cases} 
\frac{1}{r} , & r \geq R \\
\frac{3}{2R} - \frac{r^2}{2R^3} , & r < R 
\end{cases}
\] (20)
The strong potential \(V_S\) is assumed to have a Woods–Saxon form
\[
V_S(r) = - \frac{V_0}{1 + \exp [(r - R)/a_N]}
\] (21)
with \(V_0 = 34\) MeV, \(a_N = 0.52\) fm, and the same nuclear radius \(R\).
B. Bound state

The ground state of $^7$Be has the quantum numbers $3/2^−$. Therefore to construct the ground state of the Boron isotope $^8$B with angular momentum 2 and positive parity, in the two–body $p^7$Be–model, the angular momentum $l$ of $p^7$Be relative motion must be $l = 1$. The wave function of $^8$B can then be written as

$$\Psi_{\text{bound}}^{σm} (\vec{r}) = \frac{1}{r} \sum_{\mu_1 \mu_2 s_m} \frac{1}{2} \langle \frac{3}{2} - \sigma_m 1m | 2m_\sigma > \chi_{\mu_1}^p \chi_{\mu_2}^{^7}\text{Be} | Y_{1m}(\hat{r}) \phi_{\text{bound}}^m (r) \rangle .$$

(22)

The radial wave function $\phi_{\text{bound}}^m (r)$ is a square–integrable solution of the equation

$$\left\{ \frac{d^2}{dr^2} + \frac{2}{r} \frac{dE}{dE} - \frac{2}{r^2} - 2m [V_C(r) + V'_S(r)] \right\} \phi_{\text{bound}}^m (r) = 0 ,$$

(23)

where the potential $V'_S$ has the same form as (21) but the strength parameter is slightly modified, $V'_0 = 32.62$ MeV to reproduce the bound state of $^8$B, $E_0 = -0.138$ MeV.

VI. RESULTS AND DISCUSSIONS

The average production of $^8$B nuclei per unit volume per second via the triple collisions (6) at a temperature $θ$ can be written as (in units cm$^{-3}$ sec$^{-1}$)

$$\langle R_3(θ) \rangle = \langle Σ(θ) \rangle n_en_p n_7\text{Be}$$

where $\langle Σ(θ) \rangle$ is analogous to the so–called transport cross-section $\langle σ_2v \rangle$ used in the theory of two–body reactions [11]. Both $\langle Σ(θ) \rangle$ and $\langle σ_2v \rangle$ are the reaction rates corresponding to unit densities of particles. The explicit form of $\langle Σ(θ) \rangle$ is given in the Appendix.

The average production of $^8$B via the radiative capture (2) is (in units cm$^{-3}$ sec$^{-1}$)

$$\langle R_2(θ) \rangle = \langle σ_2v \rangle n_p n_7\text{Be}$$

Therefore, the ratio of the triple to binary rate depends only on the density of electrons and the temperature

$$\frac{\langle R_3(θ) \rangle}{\langle R_2(θ) \rangle} = \frac{\langle Σ(θ) \rangle}{\langle σ_2v \rangle} n_e .$$

(24)

We calculated this ratio using $n_e = 100N_A$ cm$^{-3}$ where $N_A = 6.022 \times 10^{23}$ is the Avogadro constant. This density corresponds to solar plasma conditions [3]. The results of our calculations for several values of $θ$ are given in Table 1. For comparison, this table contains also the absolute values of the reaction rates of the radiative and nonradiative processes (2) and (6) calculated for $n_p = n_7\text{Be} = N_A$. The data for the radiative proton capture by $^7$Be are taken from Ref. [11]. It is seen that the triple collision (6) plays, apparently, a minor role similarly to the nonradiative capture reaction $e^- + p + ^2\text{H} \rightarrow ^3\text{He} + e^- [12]$. However, at the early stages of the universe when $n_e/N_A \gg 100$, this reaction, might have been significant.
The temperature dependence of these reaction rates, normalized to the same value at $\theta = 14 \times 10^6 \, \degree K$, are shown in Fig. 3. It is seen that the curves for the radiative and nonradiative processes practically coincide. This is not surprising, for in both cases the temperature dependence is determined mainly by the same exponential function stemming from the Maxwellian distribution of the $p^7$Be momentum $p$ (see formula (A9) in the Appendix). Indeed, a change of the temperature shifts the maximum of the distribution which changes the most probable energy of $p^7$Be collision. This in turn affects the values of $\Psi_{\text{scat}}(\vec{r})$ near the point $r = 0$. Since the Coulomb interaction between the proton and $^7$Be is repulsive and the collision energy is very low, the scattering wave function at small distances depends on the temperature very strongly. This dependence manifests itself via the corresponding behaviour of the reaction rates because both the radiative and nonradiative transitions occur at short distances and their rates depend on the overlapping of $\Psi_{\text{scat}}(\vec{r})$ with the bound state wave function $\Psi_{\text{bound}}(\vec{r})$.

Of course, the energy of the electron also depends on the temperature. This however has not much influence on the reaction rate because the electron is attracted to the area of the reaction and its wave function is always large at small $\rho$. An additional increase or decrease of it makes, therefore, no much difference in contrast to the repulsive interaction.

This reasoning leads us to yet another conclusion. A reaction with a three–body initial state and with all particles positively charged, must have very strong temperature dependence. Thus one should expect quite different $\theta$–dependence for the nonradiative $^8$B production with two protons in the initial state

$$p + p + ^7\text{Be} \rightarrow ^8\text{B} + p.$$ 

Since in this case the adiabatic approximations is not expected to work well, a more rigorous treatment of this reaction, for example, using the Faddeev formalism, is required.

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APPENDIX A: EVALUATION OF THE TRANSITION RATE

The potential \( V_e \) describing Coulomb interaction of the electron with the proton and \(^7\text{Be}\) in the coordinate space is

\[
V_e(\vec{\rho}, \vec{r}) = -\frac{4}{|\vec{\rho} + \beta \vec{r}|} - \frac{1}{|\vec{\rho} - \alpha \vec{r}|} \tag{A1}
\]

where

\[
\alpha = \frac{m_{7\text{Be}}}{m_{7\text{Be}} + m_p}, \quad \beta = 1 - \alpha .
\]

The Jacobi vectors \( \vec{\rho} \) and \( \vec{r} \) are depicted in Fig. 2. According to (17), the matrix element between the initial and final 'distorted' waves provides the amplitude of the transition (13)

\[
M(\vec{p}, s, m_s, \vec{k} \rightarrow \sigma m_\sigma, \vec{k'}) = \langle \Psi_{\sigma m_\sigma}^{\text{bound}}, \vec{k'} | V_e | \Psi_{\sigma m_\sigma}^{\text{scat}}(\vec{r}) \rangle = \int d\vec{r} \Psi_{\sigma m_\sigma}^{\text{bound}}(\vec{r}) U_e(\vec{q}, \vec{r}) \Psi_{\sigma m_\sigma}^{\text{scat}}(\vec{r}), \tag{A2}
\]

with

\[
U_e(\vec{q}, \vec{r}) = -\frac{4\pi}{q^2} \left[ 4 \exp(-i\beta \vec{q} \cdot \vec{r}) + \exp(i\alpha \vec{q} \cdot \vec{r}) \right], \tag{A3}
\]

where \( \vec{q} = \vec{k} - \vec{k}' \) is the momentum transferred to the electron, and \( U_e(\vec{q}, \vec{r}) \) is the Fourier transform of the two-center potential (A1) over the electron variable \( \vec{\rho} \). Performing the angular integrations in (A2), we obtain

\[
M(\vec{p}, s, m_s, \vec{k} \rightarrow \sigma m_\sigma, \vec{k'}) = -\frac{i(4\pi)^2}{pq^2} \sum_m <s m_s 1 m | \sigma m_\sigma > Y_{1m}(\vec{q}) \left[ R(\alpha, q, p) - 4R(\beta, q, p) \right] , \tag{A4}
\]

where the radial integrals are given by

\[
R(\gamma, q, p) = \int_0^\infty \phi_{\text{bound}}^\gamma(r) u_p^{\text{scat}}(r) j_1(\gamma qr) \, dr \tag{A5}
\]

with \( j_1 \) being the Riccati–Bessel function [13]. As can be seen from Eq. (A4) and the definition of \( \vec{q} \) all partial waves of the electron motion contribute to the transition amplitude \( M(\vec{p}, s, m_s, \vec{k} \rightarrow \sigma m_\sigma, \vec{k'}) \).

By averaging over the initial and summing over the final spin orientations we obtain

\[
\frac{1}{2s + 1} \sum_{m_s m_\sigma} \left| M(\vec{p}, s, m_s, \vec{k} \rightarrow \sigma m_\sigma, \vec{k'}) \right|^2 = \frac{5}{2s + 1} \frac{(4\pi)^3}{16p^2q^4} |R(\alpha, q, p) - 4R(\beta, q, p)|^2 . \tag{A6}
\]

Then the reaction rate (11) can be written as

\[
< R_\beta(\theta) > = < \Sigma(\theta) > n_e n_p n_{7\text{Be}} , \tag{A7}
\]

where
\[ <\Sigma(\theta)> = \sum_s \frac{40\pi^4}{2s+1} \int d^3\vec{k}' \int d^3\vec{p} \int d^3\vec{k} N^s_p(\theta) N^s_\vec{k}(\theta) \]
\[ \times \delta\left( \frac{\vec{p}^2}{2m} + \frac{\vec{k}^2}{2\mu} - \frac{\vec{k}'^2}{2\mu} + \varepsilon_0 \right) \frac{1}{p^2q^4} |R(\alpha, q, p) - 4R(\beta, q, p)|^2 . \]

(A8)

The integration over \( \vec{k}' \) in the above formula can be performed using \( \vec{q} = \vec{k}' - \vec{k} \) instead of \( \vec{k}' \) in the integration. Then the \( \delta \)-function facilitates the integration over the solid angle of \( \vec{q} \).

Finally, the unit density reaction rate \( <\Sigma(\theta)> \) is expressed in the form of a two-dimensional integral

\[ <\Sigma(\theta)> = \sum_s \frac{10(2\pi)^4}{2s+1} \sqrt{\frac{\mu}{\theta^2}} \int_0^\infty dp \exp\left( -\frac{p^2}{2\theta} - \frac{2\pi}{p} \right) \]
\[ \times \int_0^\infty dq \frac{q^3}{q^4} \exp\left[ -\frac{\mu}{2\theta q^2} \left( \frac{p^2}{2} - \frac{q^2}{2\mu} - \varepsilon_0 \right) \right]^2 [Q(\alpha, p, q) - 4Q(\beta, p, q)]^2 \]

(A9)

with

\[ Q(\alpha, p, q) = \int_0^\infty \phi^{\text{bound}}(r) j_1(\alpha qr) u^{\text{scat}}_p(r) \exp\left( \frac{\pi}{p} \right) dr . \]
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TABLE I. Temperature dependence of the reaction rates for the radiative and nonradiative formation of $^{8}\text{B}$ in solar plasma.

| $\theta_6$ ($10^6$ °K) | $<\sigma v>_N A$ (cm$^3$ mol$^{-1}$ sec$^{-1}$) | $<\Sigma(\theta)> N_A^2$ (cm$^3$ mol$^{-1}$ sec$^{-1}$) | $<\mathcal{R}_3(\theta)> / <\mathcal{R}_2(\theta)>$ |
|------------------------|---------------------------------|---------------------------------|-------------------------------|
| 14                     | $1.73 \times 10^{-12}$         | $1.83 \times 10^{-16}$         | $1.06 \times 10^{-4}$         |
| 15                     | $4.34 \times 10^{-12}$         | $4.43 \times 10^{-16}$         | $1.02 \times 10^{-4}$         |
| 16                     | $10.1 \times 10^{-12}$         | $9.80 \times 10^{-16}$         | $0.97 \times 10^{-4}$         |
| 18                     | $44.7 \times 10^{-12}$         | $42.0 \times 10^{-16}$         | $0.94 \times 10^{-4}$         |
| 20                     | $161.0 \times 10^{-12}$        | $143.3 \times 10^{-16}$        | $0.89 \times 10^{-4}$         |
FIG. 1. Spectrum of $^8$B nucleus. Energy levels are in MeV.

FIG. 2. The Jacobi vectors.
FIG. 3. Temperature dependence of the binary and triple reaction rates given in Table 1 and normalized to the same value at $\theta = 14 \times 10^6$ °K.