Direct evidence for the Maldacena conjecture for $\mathcal{N} = (8, 8)$ super Yang–Mills theory in 1+1 dimensions

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Abstract

We solve $\mathcal{N} = (8, 8)$ super Yang–Mills theory in 1+1 dimensions at strong coupling to directly confirm the predictions of supergravity at weak coupling. We do our calculations in the large-$N_c$ approximation using Supersymmetric Discrete Light-Cone Quantization with up to $3 \times 10^{12}$ basis states. We calculate the stress-energy correlator $\langle T^{++}(r)T^{++}(0) \rangle$ as a function of the separation $r$ and find that at intermediate values of $r$ the correlator behaves as $r^{-5}$ to within errors as predicted by weak-coupling supergravity. We also present an extension to significantly higher resolution of our earlier results for the same correlator in the $\mathcal{N} = (2, 2)$ theory and see that in this theory the correlator has very different behavior at intermediate values of $r$. 
1 Introduction

We give a direct evidence of the duality between supersymmetric field theory at strong coupling and supergravity at weak coupling. This conjectured duality, known as the Maldacena conjecture [1], has been studied extensively, and there is considerable qualitative support for it. Most, if not all, of these tests are in some way indirect, because of the difficulty of solving supersymmetric field theories at strong coupling. The result we present here is unique, in that it is based on a numerical solution of an exactly supersymmetric field theory at strong coupling.

Our method is Supersymmetric Discrete Light-Cone Quantization (SDLCQ) [2, 3]. It is a well established tool for calculations of physical quantities in supersymmetric gauge theory and has been exploited for many supersymmetric Yang–Mills (SYM) theories. Briefly, the SDLCQ method rests on the ability to produce an exact representation of the superalgebra but is otherwise very similar to Discrete Light-Cone Quantization (DLCQ) [4, 5]. In DLCQ, we compactify the $x^- \equiv (t - z)/\sqrt{2}$ direction by putting the system on a circle with a period of $2L$, which discretizes the longitudinal momentum as $p^+ = n\pi/L$, with $n$ a positive integer. The total longitudinal momentum $P^+$ becomes $K\pi/L$, where $K$ is an integer known as the harmonic resolution [4]. The positivity of the light-cone longitudinal momenta then limits the number of possible Fock states for a given $K$, and, thus, the dimension of Fock space becomes finite, enabling us to do some numerical computations. It is assumed that as $K$ approaches infinity, the solutions to this large, finite problem approach the solutions of the field theory. The difference between DLCQ and SDLCQ lies in the choice of discretizing either $P^-$ or $Q^-$ to construct the matrix approximation to the eigenvalue problem $M^2|\Psi\rangle = 2P^+P^-|\Psi\rangle = 2P^+(Q^-)^2/\sqrt{2}|\Psi\rangle$, with $P^+ = K\pi/L$. For more details and additional discussion of SDLCQ, we refer the reader to Ref. [3].

The $\mathcal{N} = (8, 8)$ supersymmetric theory in 1+1 dimensions in the large-$N_c$ limit is discussed in Refs. [6, 7, 8]; however, the published results are primitive compared to what can be obtained today, because of our greatly improved hardware and software. In the present paper we are able to present results for a resolution of $K = 11$. At this resolution the problem has $3 \times 10^{12}$ basis states. By fully exploiting the symmetries of this theory, we only need to deal with a subset of $3 \times 10^7$ such states.

We will look at the two-point correlation function of the stress-energy tensor, namely $\langle T^{++}(r)T^{++}(0)\rangle$. The expected behavior in the ultraviolet (UV) and infrared (IR) regions is $1/r^4$, and the predicted behavior in weak-coupling supergravity theory in the intermediate region is $1/r^5$. We find the power behavior in $\mathcal{N} = (8, 8)$ theory in the intermediate-$r$ region to be consistent with $1/r^5$.

An interesting effect in the calculation is that the finite-dimensional representations in SDLCQ with odd and even values of $K$ result in very distinct solutions of the SYM theory. We saw this result in our work on $\mathcal{N} = (2, 2)$ SYM theory in two dimensions [10]. Only at infinite resolution are the solutions identical. One might initially think that having two numerical representations of the supersymmetry is a shortcoming of the SDLCQ approach, but it turns out to be an advantage because it provides an internal measure of convergence. We present new results for the $\mathcal{N} = (2, 2)$ theory, up to resolution $K = 14$, which clearly demonstrate this.

These results for $\mathcal{N} = (2, 2)$ also correct a sign error in our earlier numerical results for $\mathcal{N} = (2, 2)$. The quantitative change is very small and has no qualitative effect.
The structure of this paper is the following. In Sec. 2 we review $\mathcal{N}=(8,8)$ SYM theory. In Sec. 3 we discuss the symmetries of this theory, which play a critical role in allowing us to greatly reduce the basis set we need to retain. We discuss the two-point correlation function of the stress-energy tensor in Sec. 4. Finally, in Sec. 5, we present our numerical results. A summary and some additional discussion are given in Sec. 6.

## 2 Review of $\mathcal{N}=(8,8)$ SYM Theory

Before giving the numerical results, let us quickly review some analytical work on $\mathcal{N}=(8,8)$ SYM theory, for the sake of completeness. For more details, see Ref. [6]. This theory is obtained by dimensionally reducing $\mathcal{N}=1$ SYM theory from ten dimensions to two dimensions. In light-cone gauge, where $A_-=0$, we find for the action

$$ S_{1+1}^{\text{LC}} = \int dx^+ dx^- \text{tr} \left[ \partial_+ X_I \partial_- X_I + i \theta_R^T \partial^+ \theta_R + i \theta_L^T \partial^- \theta_L \right] + \frac{1}{2} (\partial^- A_+)^2 + g A_+ J^+ + \sqrt{2} g \theta_L^T \beta_I [X_I, \theta_R] + \frac{g^2}{4} [X_I, X_J]^2, $$

where $x^\pm$ are the light-cone coordinates in two dimensions, the trace is taken over the color indices, the $X_I$ with $I=1, \ldots, 8$ are the scalar remnants of the transverse components of the ten-dimensional gauge field $A_\mu$, the two-component spinor fields $\theta_R$ and $\theta_L$ are remnants of the right-moving and left-moving projections of the sixteen-component spinor in the ten-dimensional theory, and $g$ is the coupling constant. We also define the current $J^+ = i[X_I, \partial_- X_I] + 2 \theta_R^T \theta_R$ and use the matrices $\beta_i$ given in Ref. [11].

After using the equations of motion to eliminate all the non-dynamical fields, we find for $P^- = \int dx^- T^{--}$ the expression

$$ P^- = g^2 \int dx^- \text{tr} \left( -\frac{1}{2} J^+ \frac{1}{\partial_-^2} J^+ - \frac{1}{4} [X_I, X_J]^2 + \frac{i}{2} \beta_I [X_I, \theta_R] \frac{1}{\partial_-} \beta_J [X_J, \theta_R] \right). $$

The supercharges are found by dimensionally reducing the supercurrent in the ten-dimensional theory. We find

$$ Q^- \alpha = g \int dx^- \text{tr} \left( -2^{3/4} J^+ \frac{1}{\partial_-} u_\alpha + 2^{-1/4} i [X_I, X_J] (\beta_I \beta_J)_{\alpha \eta} u_\eta \right), $$

where $\alpha, \eta = 1, \ldots, 8$ and the $u_\alpha$ are the components of $\theta_R$. We expand the dynamical fields $X_I$ and $u_\alpha$ in Fourier modes as

$$ X_{lpq}(x^-) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk^+}{\sqrt{2k^+}} [A_{lpq}(k^+) e^{-ik^+ x^-} + A_{lpq}^\dagger(k^+) e^{ik^+ x^-}], $$

$$ u_{apq}(x^-) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk^+}{\sqrt{2k^+}} [B_{apq}(k^+) e^{-ik^+ x^-} + B_{apq}^\dagger(k^+) e^{ik^+ x^-}], $$

where $p, q = 1, 2, \ldots, N_c$ stand for the color indices, and $A$ and $B$ satisfy the usual commutation relations

$$ [A_{lpq}(k^+), A_{l's}^\dagger(k'^+) ] = \delta_{l,l'} \delta_{ps} \delta(k^+ - k'^+), $$

$$ \{ B_{apq}(k^+), B_{b'rs}^\dagger(k'^+) \} = \delta_{a,b'} \delta_{ps} \delta(k^+ - k'^+). $$
We work in a compactified $x^-$ direction of length $2L$ and ignore zero modes. With periodic boundary conditions, momenta are restricted to a discrete set of values $[2]$, \( k^+ = \pi k/L \), with $k$ a positive integer. The integrals over $k^+$ are replaced by sums: $\int dk^+ \to \frac{\pi}{L} \sum_{k=1}^{\infty}$, and Dirac delta functions become Kronecker deltas: $\delta(k^+ - k'^+) \to \frac{\pi}{L} \delta_{kk'}$. We then rescale the annihilation operators

$$\sqrt{\frac{L}{\pi}} a(k) = A(k^+ = \frac{\pi k}{L}), \quad \sqrt{\frac{L}{\pi}} b(k) = B(k^+ = \frac{\pi k}{L}),$$

so that

$$[a_{Ipq}(k), a_{Jrs}^\dagger(k')] = \delta_{IJ} \delta_{pq} \delta_{kk'}, \quad \{b_{opq}(k), b_{brs}^\dagger(k')\} = \delta_{\alpha\beta} \delta_{pq} \delta_{kk'}.$$  \hspace{1cm} (9)

The expansions become

$$X_{Ipq}(x^-) = \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{1}{\sqrt{2k}} [a_{Ipq}(k)e^{-i\frac{\pi}{L}kx^-} + a_{Iqp}^\dagger(k^+)e^{i\frac{\pi}{L}kx^-}],$$

$$u_{opq}(x^-) = \frac{1}{\sqrt{2L}} \sum_{k=1}^{\infty} \frac{1}{\sqrt{2}} [b_{opq}(k)e^{-i\frac{\pi}{L}kx^-} + b_{qop}^\dagger(k)e^{i\frac{\pi}{L}kx^-}].$$  \hspace{1cm} (10)

In terms of $a$ and $b$, the supercharge is given by

$$Q^-_{\alpha} = \frac{i2^{-1/4}g}{\pi} \sqrt{\frac{L}{\pi}} \sum_{k_1, k_2, k_3=1}^{\infty} \delta_{(k_1+k_2),k_3} \left\{ \right.$$  

\begin{align*}
&\times \left[ \frac{1}{2\sqrt{k_1k_2}} \left( \frac{k_2 - k_1}{k_3} \right) \left[ b_{\alpha ij}(k_3)a_{Iim}(k_1)a_{I mj}(k_2) - a_{Iim}(k_1)a_{I mj}(k_2)b_{\alpha ij}(k_3) \right] \\
&+ \frac{1}{2\sqrt{k_1k_3}} \left( \frac{k_1 + k_3}{k_2} \right) \left[ a_{Iim}(k_1)b_{\alpha mj}(k_2)a_{I ij}(k_3) - a_{Iij}(k_3)a_{Iim}(k_1)b_{\alpha mj}(k_2) \right] \\
&+ \frac{1}{2\sqrt{k_2k_3}} \left( \frac{k_2 + k_3}{k_1} \right) \left[ a_{I ij}(k_3)b_{\alpha im}(k_1)a_{I mj}(k_2) - b_{\alpha im}(k_1)a_{I mj}(k_2)a_{I ij}(k_3) \right] \\
&\left. - \frac{1}{k_1} [b_{\beta ij}(k_3)b_{\alpha im}(k_1)b_{\alpha mj}(k_2) + b_{\alpha im}(k_1)b_{\beta mj}(k_2)b_{\beta ij}(k_3)] \\
&- \frac{1}{k_2} [b_{\beta ij}(k_3)b_{\alpha im}(k_1)b_{\alpha mj}(k_2) + b_{\alpha im}(k_1)b_{\beta mj}(k_2)b_{\beta ij}(k_3)] \\
&+ \frac{1}{k_3} [b_{\alpha ij}(k_3)b_{\beta im}(k_1)b_{\beta mj}(k_2) + b_{\beta im}(k_1)b_{\alpha mj}(k_2)b_{\alpha ij}(k_3)] \\
&\right. \\
&\left. + \frac{2}{4\sqrt{k_1k_2}} \left[ b_{\alpha ij}(k_3)a_{Iim}(k_1)a_{I mj}(k_2) + a_{Iim}(k_1)a_{I mj}(k_2)b_{\alpha ij}(k_3) \right] \\
&\left. + \frac{1}{4\sqrt{k_2k_3}} \left[ a_{Iij}(k_3)b_{\alpha im}(k_1)a_{I mj}(k_2) + b_{\alpha im}(k_1)a_{I mj}(k_2)a_{I ij}(k_3) \right] \\
&\left. + \frac{1}{4\sqrt{k_3k_1}} \left[ a_{Iij}(k_3)a_{Iim}(k_1)b_{\alpha mj}(k_2) + a_{Iim}(k_1)b_{\alpha mj}(k_2)a_{I ij}(k_3) \right] \right\},
\end{align*}

where we have used the relation $([\beta_I, \beta_J])_{\alpha \beta} = \delta_{\alpha \beta} \delta_{IJ}$.  \hspace{1cm} (12)
3 Symmetries

The superalgebra relations that involve $Q^+_\alpha$, as specified by the anticommutators

$$\{Q^+_\alpha, Q^+_\beta\} = \delta_{\alpha\beta}2\sqrt{2}P^+, \quad \{Q^+_\alpha, Q^-_\beta\} = 0,$$

(13)

are satisfied, but the anticommutators for $Q^-_\alpha$

$$\{Q^-_\alpha, Q^-_\beta\} = \delta_{\alpha\beta}2\sqrt{2}P^-$$

(14)

become in SDLCQ

$$\{Q^-_\alpha, Q^-_\beta\} \neq 0 \text{ if } \alpha \neq \beta, \quad \{Q^-_\alpha, Q^-_\alpha\} = 2\sqrt{2}P^-.$$

(15)

Although we have different $P^-_\alpha$ for different $Q^-_\alpha$, we can find unitary, self-adjoint transformations $C_{\alpha'\alpha}$, such that $C_{\alpha'\alpha}P^-_\alpha C_{\alpha'\alpha} = P^-_{\alpha'}$. Thus the eigenvalues of the different $P^-_\alpha$ are the same, and we may choose any one of the $Q^-_\alpha$'s, at least for our purposes. In what follows we will use $Q^-_8$ and will suppress the subscript unless it is needed for clarity.

To reduce the size of the numerical calculation, we seek symmetries that block diagonalize the $P^-_8$ matrix. One such symmetry is a $Z_2$ symmetry of $Q^-_8$, called $S$-symmetry [12]: $a_{ij} \rightarrow -a_{ij}$, $b_{aij} \rightarrow -b_{aij}$. Extending the work of [8], we also look for permutations and sign changes of the bosonic and fermionic operators that leave $Q^-_8$ unchanged. The terms not involving $\beta$ matrices are preserved under all interchanges that fix $b_8$. The additional condition is that $(\beta_1\beta_2^T - \beta_2\beta_1^T)_{\alpha\beta}b^I_\alpha a^I_\beta$ be invariant with respect to the replacement

$$a_I \rightarrow r(I)a_{p(I)}, \quad b_\beta \rightarrow s(\beta)b_{q(\beta)}.$$

(16)

Defining $N_\alpha(\beta, I, J) = (\beta_1\beta_2^T - \beta_2\beta_1^T)_{\alpha\beta}$, this gives us the condition

$$N_\alpha(\beta, I, J)b^I_\alpha a^I_\beta = s(\beta)r(I)r(J)N_\alpha(\beta, I, J)b^I_{q(\beta)p(I)}a^I_{p(J)}.$$

(17)

Each possible flavor combination appears at most once on each side, so that we can set the coefficients equal, term by term, and obtain the condition

$$N_\alpha(q(\beta), p(I), p(J)) = s(\beta)r(I)r(J)N_\alpha(\beta, I, J).$$

(18)

This is then used to determine the allowed transformations as specified by $r, s, p,$ and $q$. There are $(8!)^22^{16}$ such transformations. When we restrict these to transformations that leave $b_8$ unchanged, and thus have $q(8) = 8$ and $s(8) = 1$, we have instead a set of $8!15 \approx 6.7 \times 10^{12}$ transformations.

To determine the allowed transformations, we first find the permutations such that

$$|N_\alpha(q(\beta), p(I), p(J))| = |N_\alpha(\beta, I, J)|.$$  

(19)

Then, among these we check for choices of $r(I)$ and $s(I)$ that allow the full condition (18) to be satisfied. There are 1344 permutations that satisfy the absolute-value condition (19), and, for each of these, there are 16 choices of $r(I)$ and $s(I)$ that satisfy the full condition (18). From these, we find that the group of transformations that leave $Q^-_8$ unchanged can be generated by 7 operators that square to 1. These generators are listed in Table 1.
of transformations that leave $Q_5^c$ unchanged. The first three transformations are permutations with sign changes, and the last four involve only sign changes.

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | $a_1$ | $a_8$ | $-a_5$ | $-a_4$ | $-a_3$ | $a_6$ | $-a_7$ | $a_2$ | $b_1$ | $b_4$ | $-b_3$ | $b_2$ | $b_7$ | $-b_6$ | $b_5$ |
| 2     | $a_2$ | $a_1$ | $-a_5$ | $-a_6$ | $-a_3$ | $-a_4$ | $a_8$ | $-a_7$ | $b_4$ | $b_3$ | $b_2$ | $b_1$ | $b_6$ | $-b_6$ | $-b_7$ |
| 3     | $a_2$ | $a_1$ | $-a_6$ | $a_8$ | $a_7$ | $-a_3$ | $a_5$ | $a_4$ | $b_1$ | $-b_2$ | $b_6$ | $b_5$ | $b_4$ | $b_3$ | $-b_7$ |
| 4     | $-a_1$ | $-a_2$ | $-a_3$ | $-a_4$ | $-a_5$ | $-a_6$ | $-a_7$ | $-a_8$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $-b_7$ |
| 5     | $a_1$ | $a_2$ | $a_3$ | $-a_4$ | $-a_5$ | $a_6$ | $-a_7$ | $-a_8$ | $b_1$ | $b_2$ | $-b_3$ | $-b_4$ | $-b_5$ | $b_6$ | $-b_7$ |
| 6     | $-a_1$ | $a_2$ | $a_3$ | $-a_4$ | $a_5$ | $-a_6$ | $-a_7$ | $a_8$ | $b_1$ | $-b_2$ | $b_3$ | $-b_4$ | $-b_5$ | $b_6$ | $-b_7$ |
| 7     | $a_1$ | $-a_2$ | $a_3$ | $a_4$ | $-a_5$ | $-a_6$ | $-a_7$ | $a_8$ | $-b_1$ | $b_2$ | $b_3$ | $-b_4$ | $b_5$ | $-b_6$ | $-b_7$ |

4 Correlation functions

One of the physical quantities that we can calculate nonperturbatively is the two-point function of the stress-energy tensor. Previous calculations of this correlator in this and other theories can be found in [7, 8, 13]. Ref. [8] gives results for the theory considered here but only for resolutions $K$ up to 6. We can now reach $K = 11$, with its $3 \times 10^{12}$ basis states, by careful use of the symmetries discussed in the previous section. The largest matrices we have to consider are $3 \times 10^7$ by $3 \times 10^7$.

We find that there are distinct behaviors in the correlation function for even and odd $K$. This is a familiar aspect of SYM theories with extended supersymmetry, and we have argued [10] that we have two different classes of representations at finite $K$, which become identical as $K \to \infty$.

Let us first recall that there is a duality that relates the results for the two-point function in $\mathcal{N} = (8,8)$ SYM theory to the results in string theory [8]. The correlation function on the string-theory side, which can be calculated with use of the supergravity approximation, was presented in [7], and we will only quote the result here. The computation is essentially a generalization of that given in [14, 15]. The main conclusion on the supergravity side was reported in [9]. Up to a numerical coefficient of order one, which we have suppressed, it was found that

$$\langle T^{++}(r)T^{++}(0) \rangle = \frac{N_c^{3/2}}{g_{YM} r^5}. \quad (20)$$

This result passes the following important consistency test. The SYM theory in two dimensions with 16 supercharges has conformal fixed points in both the UV and the IR regions, with central charges of order $N_c^2$ and $N_c$, respectively. Therefore, we expect the two-point function of the stress-energy tensor to scale like $N_c^2/r^4$ and $N_c/r^4$ in the deep UV and IR regions, respectively. According to the analysis of [16], we expect to deviate from these conformal behaviors and cross over to a regime where the supergravity calculation can be trusted. The crossover occurs at $r = 1/g_{YM} \sqrt{N_c}$ and $r = \sqrt{N_c}/g_{YM}$. At these points, the $N_c$ scaling of (20) and the conformal result match in the sense of the correspondence principle [17]. We should note here that this property for the correlation functions is expected only for $\mathcal{N} = (8,8)$ SYM theory. However, it would be natural to expect some similarity between $\mathcal{N} = (8,8)$ and $\mathcal{N} = (2,2)$ theories.
We wish to compute the expression $F(x^- , x^+) = \langle T^{++}(x^- , x^+)T^{++}(0, 0) \rangle$, where we fix the total momentum in the $x^-$ direction. It is more natural to compute the Fourier transform and express the transform in a spectrally decomposed form [7, 8]

\[ \hat{F}(P_-, x^+) = \frac{1}{2L} \langle T^{++}(P_-, x^+)T^{++}(-P_-, 0) \rangle \]

\[ = \sum \frac{1}{2L} \langle 0 | T^{++}(P_-, 0) | i \rangle e^{-iP_+ x^+} \langle i | T^{++}(-P_-, 0) | 0 \rangle. \] (21)

The position-space form of the correlation function is recovered by Fourier transforming with respect to $P_- = P^+ = K\pi/L$. We can continue to Euclidean space by taking $r = \sqrt{2x^+ x^-}$ to be real. The result for the correlator of the stress-energy tensor was presented in [7], and we only quote the result here:

\[ F(x^-, x^+) = \sum \left| \frac{L}{\pi} \langle 0 | T^{++}(K) | i \rangle \right|^2 \left( \frac{x^+}{x^-} \right)^2 \frac{M_i^4}{8\pi^2 K^3} K_4(M_i \sqrt{2x^+ x^-}), \] (22)

where $M_i$ is a mass eigenvalue and $K_4(M_i r)$ is the modified Bessel function of order 4. In [7] we found that the stress-energy operator is given by

\[ T^{++}(x^-, x^+) = \text{tr} \left[ (\partial_- X^I)^2 + \frac{1}{2} (i u^a \partial_- u^a - i (\partial_- u^a) u^a) \right]. \] (23)

When written in terms of the discretized creation operators, $a^\dagger$ and $b^\dagger$, as in Eqs. (10) and (11), we find

\[ T^{++}(K) | 0 \rangle = \frac{\pi}{2L} \sum_{k=1}^{K-1} \left[ -\sqrt{k(K-k)} a^\dagger_{ij}(K-k) a^\dagger_{ji}(k) \right. \]

\[ \left. + \left( \frac{K}{2} - k \right) b^\dagger_{ijkl}(K-k) b^\dagger_{ijkl}(k) \right] | 0 \rangle. \] (24)

The matrix element $\langle L/\pi | 0 | T^{++}(K) | i \rangle$ is independent of $L$ and can be substituted directly to give an explicit expression for the two-point function. We see immediately that the correlator behaves like $1/r^4$ at small $r$, for in that limit, it asymptotes to

\[ \left( \frac{x^-}{x^+} \right)^2 F(x^-, x^+) = \frac{N_c^2 (2n_b + n_f)}{4\pi^2 r^4} \left( 1 - \frac{1}{K} \right). \] (25)

On the other hand, the contribution to the correlator from strictly massless states is given by

\[ \left( \frac{x^-}{x^+} \right)^2 F(x^-, x^+) = \sum_i \left| \frac{L}{\pi} \langle 0 | T^{++}(K) | i \rangle \right|^2 \frac{6}{M_i^3 K^3 \pi^2 r^4}. \] (26)

That is to say, we would expect the correlator to behave like $1/r^4$ at both small and large $r$, assuming massless states have non-zero matrix elements.

The operator used for calculating the correlator, $T^{++}(K)$ as given in Eq. (24), is preserved under all of the $Z_2$ symmetries discussed in Sec. 3. Therefore, we only need to consider states in the singlet sector. Including factors of 2 for supersymmetry and the $S$ symmetry, the dimension of the $Q^-$ matrix is thus reduced by a factor of 86016. Because the dimension of the basis increases by a factor of almost 17 for each unit increment in $K$, symmetry allows us to increase the resolution by more than four without increasing the size of the matrix to be diagonalized.
5 Numerical results

To compute the correlator using Eq. (22), we approximate the sum over eigenstates by a Lanczos [18] iteration technique, as described in [8, 13]. The results are shown in Figs. 1 and 2, which show the log-log derivative $d\log_{10}(f)/d\log_{10}(r)$ of the scaled correlation function

$$f \equiv \langle T^{++}(x^-, x^+) T^{++}(0) \rangle \left( \frac{x^-}{x^+} \right)^2 \frac{4\pi^2 r^4}{N_c^2 (2n_b + n_f)}.$$  \hspace{1cm} (27)

Figure 1: The log-log derivative $d\log_{10}(f)/d\log_{10}(r)$ of the scaled correlation function $f$, as defined in Eq. (27) of the text, versus $\log_{10}(r)$, for the $\mathcal{N} = (2, 2)$ theory. The separation $r$ is measured in units of $\sqrt{\pi/g^2 N_c}$. In (a), the lines correspond to different values of resolution $K$ from 3 to 14, with dashed lines for even $K$ and solid for odd. The darker lines are for $K = 13$ and $K = 14$; the lower-$K$ lines converge to these two. In (b), the vertical bars span four extrapolations to infinite resolution obtained from separate quadratic and cubic fits to even and odd $K$.

Let us discuss the behavior of the correlator at small, large, and intermediate $r$. First, at small $r$, the graphs of $f$ for different $K$ approach 0 as $K$ increases. This follows Eq. (25), which gives the form $f = 1 - 1/K$. Second, at large $r$, obviously, the behavior is different for even and odd $K$. However, the difference gets smaller as $K$ gets larger. The reason for this difference is that there are different finite-dimensional representations of the super-algebra [10] for even and odd $K$, which only coincide at infinite resolution. Looking at the detailed information of the computation of the correlator at larger $r$, we found that for even $K$ there are massless states that contribute to the correlator, while there is no massless state that makes any contribution for odd $K$. Instead, it is the lowest massive states that contribute the most for odd $K$, and these states become massless as the resolution approaches infinity.
Figure 2: Same as Fig. 1, except for the $\mathcal{N} = (8, 8)$ theory with $K = 10$ and 11 the largest values of the resolution.

The results for the $\mathcal{N} = (2, 2)$ theory are shown in Fig. 1, and the results for the $\mathcal{N} = (8, 8)$ theory in Fig. 2. The results for the $\mathcal{N} = (2, 2)$ theory are obtained for higher resolution and more clearly show the convergence of the two representations. We are now able to extend our calculation of the $\mathcal{N} = (2, 2)$ SYM theory from $K = 11$ [10] to $K = 14$. To calculate continuum results in SDLCQ, it is customary to extrapolate the finite resolution results to infinite resolution. We have done this for the curves for the $\mathcal{N} = (2, 2)$ and $(8, 8)$ theories by performing the extrapolation at successive values of $r$. We fit the even and odd resolution points with separate quadratic and cubic functions of $1/K$ to perform four extrapolations. Typical results of these calculations are shown in Fig. 3 for the $\mathcal{N} = (8, 8)$ theory at $\log_{10}(r) = 0.2$ and at $\log_{10}(r) = 0.5$, where $r$ is measured in units of $\sqrt{\pi/g^2 N_c}$.

In the intermediate-$r$ region for the $\mathcal{N} = (2, 2)$ theory, clearly the behavior of the correlation in Fig. 1(a) changes, and there is a flat region. The behavior is even more apparent in Fig. 1(b), the extrapolation to infinite resolution. In [10] we first saw the flat region for the $\mathcal{N} = (2, 2)$ SYM theory, indicating that the correlator in this case behaves like $1/r^{4.7}$. Note that the region of flattening extends farther out as $K$ gets larger, for both odd and even $K$, implying again that the even and odd representations appear to agree as $K$ goes to infinity. Unfortunately, there is currently no known prediction for the behavior of the correlator in the intermediate-$r$ region this theory or even any reason to believe that the correlator should have a special behavior at intermediate values of $r$.

In the intermediate-$r$ region for the $\mathcal{N} = (8, 8)$ theory, there is a prediction that the correlation should have a special behavior. This predicted behavior is very different from the behavior we found in the $\mathcal{N} = (2, 2)$ SYM theory. We expect from Eq. (20) that the behavior is $1/r^5$. In [8] we found that the correlator may be approaching this behavior, and we indicated that conclusive evidence would be a flat region in the
Figure 3: Sample quadratic (solid) and cubic (dashed) fits for extrapolations to infinite resolution in the $\mathcal{N} = (8, 8)$ theory. The log-log derivative $d \log_{10}(f)/d \log_{10}(r)$ of the scaled correlation function $f$, as defined in Eq. (27) of the text, is plotted versus $1/K$ for (a) $\log_{10}(r) = 0.2$ and (b) $\log_{10}(r) = 0.5$, where $r$ is measured in units of $\sqrt{\pi/g^2N_c}$. Computed values of the log-log derivative are marked by circles for even $K$ and triangles for odd $K$.

The derivative of the scaled correlator at a value of $-1$. In Fig. 2(b) we see that as $r$ increases, the slope approaches $-1$. It overshoots $-1$ and then appears to approach $-1$ from below and to remain consistent with that value for an extended range of $r$. Beyond this point, we see that the errors increase significantly. Thus, there is a finite range of intermediate values of $r$ over which the numerical solution of strong-coupling SYM theory is consistent with the $1/r^5$ prediction of weak-coupling supergravity.

6 Discussion

We have presented numerical results for the two-point correlation function of the stress-energy tensor, using SDLCQ in $1 + 1$ dimensions in the large-$N_c$ approximation, for $\mathcal{N} = (2, 2)$ SYM theory up to resolution $K = 14$ and for $\mathcal{N} = (8, 8)$ SYM theory up to resolution $K = 11$. There are two distinct classes of representations for these SYM theories, one for the resolution $K$ even and one for $K$ odd, and these representations become identical as $K \to \infty$. The two-point correlators of the stress-energy tensor behave like $1/r^4$ in the UV (small $r$) and IR (large $r$, $K$ even) regions. The large-$r$ behavior for $K$ odd, on the other hand, has an exponential decay, since in the odd $K$ representations the massless states become massless only as $K \to \infty$.

While there is no known prediction for the correlator of the stress energy tensor at intermediate values of $r$ for the $\mathcal{N} = (2, 2)$ theory, it is interesting that we find the dominant power law behavior $1/r^{4.7}$. There is clear convergence over a wide range of intermediate values of $r$. 



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The results presented here also include a correction to our earlier work on the $\mathcal{N} = (2, 2)$ theory. We recently found a sign error in one term in our numerical calculations. This correction does not change the qualitative behavior found in our previous calculation; the quantitative change is quite small.

In $\mathcal{N} = (8, 8)$ SYM theory in $1 + 1$ dimensions, the correlator is expected to behave like $1/r^5$ in the intermediate region. We were able to confirm this power-law behavior with a flat region in the derivative of the scaled correlator. We have done this by calculating the correlator at successive values of the resolution and extrapolating the function to infinite resolution. The maximum resolution we reached was $K = 11$. At that resolution the total number of basis states in the SDLCQ approximation is $3 \times 10^{12}$. Using all of the symmetries of the $\mathcal{N} = (8, 8)$ theory, including some that are specific to the sector where the correlator is non-vanishing, we were able to reduce the number of basis states that we needed to consider to $3 \times 10^7$.

While just a few years ago it seemed inconceivable that one could reach these resolutions in SDLCQ, advances in our understanding of the problem as well as advances in software and hardware have allowed us to do so. We remain confident that we can, in fact, reach even higher resolutions. At this point we believe that the most important direction is to increase the number of fields that we consider. In particular, we are interested in considering theories with both extended supersymmetry and fundamental matter.

Acknowledgments

This work was supported in part by the U.S. Department of Energy and the Minnesota Supercomputing Institute. One of the authors (U.T.) would like to thank the Research Corporation for supporting his work.

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