Cooperative driving strategy based on naturalistic driving data and non-cooperative MPC

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Abstract: A cooperative driving strategy is proposed, in which the dynamic driving privilege assignment in real-time and the driving privilege gradual handover are realized. The first issue in cooperative driving is the driving privilege assignment based on the risk level. The risk assessment methods in 2 typical dangerous scenarios are presented, i.e. the car-following scenario and the cut-in scenario. The naturalistic driving data is used to study the behavior characteristics of the driver. TTC (time to collision) is defined as an obvious risk measure, whereas the time before the host vehicle has to brake assuming that the target vehicle is braking is defined as the potential risk measure, i.e. the time margin (TM). A risk assessment algorithm is proposed based on the obvious risk and potential risk. The naturalistic driving data are applied to verify the effectiveness of the risk assessment algorithm. It is identified that the risk assessment algorithm performs better than TTC in the ROC (receiver operating characteristic). The second issue in cooperative driving is the driving privilege gradual handover. The vehicle is jointly controlled by the driver and automated driving system during the driving privilege gradual handover. The non-cooperative MPC (model predictive control) is employed to resolve the conflicts between the driver and automated driving system. It is identified that the Nash equilibrium of the non-cooperative MPC can be achieved by using a non-iterative method. The driving privilege gradual handover is realized by using the confidence matrixes update. The simulation verification shows that the cooperative driving strategy can dynamically assign the driving privilege in real-time according to the risk level.

Key words: cooperative driving, risk assessment, naturalistic driving data, game theory

1. Introduction

Automotive intelligence has played an important role in reducing traffic accidents, improving traffic efficiency, and reducing driver operating load. However, many accidents related to intelligent vehicle in recent years have shown that it is very important to keep the driver in the control loop before the automated driving system is fully mature[1, 2]. When the vehicle is jointly controlled by the driver and automated driving system, the driving privilege handover is an important problem. In the driver assistance, the driving privilege is delivered between the driver and automated driving system without transition. The suddenly intervention of the automated driving system will make the driver fell uncomfortable. Moreover, it will be difficult for the driver to take over when the automated driving system suddenly withdraws. Therefore, the gradual transference of the driving privilege can increase the comfort and safety of the automated driving system.

Driver assistance or cooperative driving can be divided into two categories, i.e. the system driving human monitoring (SDHM) system and the human driving system monitoring (HDSM) system. The SDHM system includes ACC (adaptive cruise control) and LKA (lane-keeping assist), whereas the HDSM system includes AEB (autonomous emergency braking), FCW (forward collision warning) and LDW (lane departure warning). Firstly, the driver does not perform well when the operating load is excessively low and excessively high[3]. When the SDHM system is working, the driver will be more likely to be drowsiness or distraction due to the lack of operation requirements.

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Secondly, the automated driving system should not help with the simple and repetitive driving tasks, whereas the complex driving tasks are left to the driver. If the automated driving system only deals with the simple part of driving, the difficult part will be more difficult for the driver[4]. Therefore, HDSM is more helpful to the driver than SDHM before the automated driving system has good enough environment sensing system.

A cooperative driving strategy is proposed in this paper, in which the dynamic driving privilege assignment in real-time and the driving privilege gradual handover are realized. In normal driving, the vehicle is controlled by the driver to ensure that the driver is always in the control loop. When the vehicle enters the dangerous state, the driving privilege is gradually handed over to the automated driving system to assist the driver to avoid danger. When the vehicle returns to normal driving, the driving privilege is gradually handed back to the driver to ensure the successful takeover of the driver.

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The first issue in the cooperative driving strategy is the dynamic driving privilege assignment in real-time based on the risk level. Risk is a variable which is related to a lot of factors including the subjective feelings of the driver. Therefore, it is necessary to study the behavior characteristics of the driver in dangerous scenarios. Naturalistic driving studies can provide authentic driving behavior of the driver. A risk assessment algorithm based on the naturalistic driving data is proposed, and the driving assignment is implemented based on the risk level.

The second issue in the cooperative driving strategy is the driving privilege gradual handover. During the driving privilege gradual handover, the vehicle is jointly controlled by the driver and automated driving system. The game theory are employed to resolve the conflicts during driving privilege gradual handover. A driving privilege handover based on non-cooperative MPC (model predictive control) is proposed, and the driving privilege gradual handover between the driver and automated driving system is realized.

1.1 Risk Assessment

The common risk assessment method is to define a measure that characterizes the risk level in each scenario. TTC (time to collision) is a widely used risk measure[5-8], which is defined as the relative distance divided by the relative velocity. When the relative velocity is very small, TTC will be infinity. Hence, the reciprocal of TTC (1/TTC) is also...
used in risk assessment[9, 10]. However, TTC or 1/TTC cannot represent the risk level in all circumstances. Let’s consider 2 car-following cases. In case 1, the velocity of the host vehicle and target vehicle are 31m/s and 30m/s, respectively. And the relative distance is 10m. In case 2, the velocity of the host vehicle and target vehicle are 6m/s and 5m/s, respectively. And the relative distance is 10m. The TTC in these two cases are all 10s, but the driver will feel more dangerous in case 1. When the relative velocity is low, TTC will be very large. However, if the relative distance is too small in this circumstance, TTC will reduce rapidly and a rear-end collision risk will occur when the target vehicle brakes, even if TTC is very large in the current state. Too small relative distance and insufficient braking force are the main factors leading to danger in the car-following scenario[11]. Therefore, TTC cannot be used to evaluate the risk level when the relative velocity is small. THW (time headway) is also a commonly used risk measure[12], which is defined as the relative distance divided by the velocity of the host vehicle. However, the velocity of the target vehicle is not considered in THW, which is easy to obtain and is very important in risk assessment. Furthermore, the driver's choice of THW is affected by many other factors including the target vehicle type and road condition[13]. Hence, THW also cannot describe the danger in car-following accurately. Some new measure are introduced to describe the risk level, e.g. the weighted sum of 1/TTC and 1/THW[14], $T_{lab}[15]$. 

1.2 Dynamic Game

The non-cooperative dynamic game can be used to describe the problem that multiple decision makers act on the same dynamic system[16]. Hence, the non-cooperative dynamic game can be applied to deal with the conflict during driving privilege handover. The application of dynamic game in vehicle system is relatively limited. In [17, 18], the dynamic game is employed to study the vehicle evaluation method under worst-case. In [19, 20], the vehicle control strategies which take the driver behavior into consideration are constructed based on the closed-loop game. In [21], the game theory is used to model the control behavior of the driver and the active front steering controller.

2. Naturalistic Driving Data

The natural driving data used in this paper come from 3 databases, i.e. the China-FOT (China field operational test), the dangerous scenario database, and the OEM FOT.

2.1 China-FOT

![China-FOT](image)

Fig 2. The test vehicle, data acquisition system and carmera in the China-FOT

China-FOT is collected in Shanghai by Volvo, Tongji University, and Chalmers University of Technology. China-FOT lasts from July 2014 to December 2015. There are 8 test vehicles. The test vehicles are all Volvo S60L. The vehicle status information come from the CAN bus, and the surrounding traffic information are obtained by 4 cameras. 32 drivers participate in the test, including 25 men and 7 women. The age of the driver is between 28 and 39 (mean: 32.25; SD: 2.84). All drivers have their own vehicles before the test. The mileage of the driver range from 15,000km
to 240,000km (mean: 108,375; SD: 63,598). Hence, all drivers in China-FOT are not newbie. Each driver uses the test vehicle for about 3 months. Drivers can drive the test car to any place at any time during the test. China-FOT have collected 7,402 trips. The travel distance is 129,935 km.

2.2 Dangerous Scenario Database
The dangerous scenario database are collected by using video drive recorder (VDR) installed on the vehicle. The VDR of Horiba with built-in velocity sensor and acceleration sensor is used. Brake deceleration equal to 0.4g is chosen as a trigger value, and the VDR only records the data within the period from 15s before to 5s after a trigger. About 4,000 trigger cases are collected during 4 years. The 4000 triggering case are manually screened, and the subjective judgment is used to eliminate the cases which are not dangerous. In the end, 500 dangerous cases with high risk level are obtained.

![Fig 3. The test vehicle, VDR and data processing system in the dangerous scenario database](image)

2.3 OEM FOT
The OEM FOT is derived from the field operational test of a anonymous OEM (original equipment manufacturer). All test vehicles are production passenger cars of the OEM. The vehicle status information come from the CAN bus, and the surrounding traffic information are obtained by Mobieye EyeQ3. Only part of the data in the OEM FOT are available. The travel distance is 1,220 km. The Mobieye EyeQ3 provides information of 5 targets in front, including target type, target width, relative distance, relative speed, relative acceleration, etc.

![Fig 4. The Mobileye EyeQ3 video information in the OEM FOT](image)

2.4 Scenario Extraction
Each of the 3 databases has its own advantages and disadvantages. The driving process of the driver is completely undisturbed in China-FOT. And China-FOT has long acquisition time, rich driving scenarios, and complete vehicle status information and video information. However, the surrounding traffic environment parameters in China-FOT are obtained through image identification, and the accuracy is not high. The dangerous scenario database mainly collects dangerous cases. The risk level of the dangerous case is high. However, the dangerous scenario database only records data within 20s for each cases. The OEM FOT has high quality surrounding traffic environment information,
but the amount of data is limited. Therefore, different scenarios are extracted by using different databases according to the features of these 3 databases.

a) The cut-in cases are extracted by using China-FOT, and 326 cut-in cases are obtained. These cut-in cases are classified into normal cut-in and dangerous cut-in by using the automatic detection method introduced in [22]. 249 normal cut-in cases and 77 dangerous cut-in cases are obtained.

b) The dangerous car-following cases are extracted by using the dangerous scenario database. The 500 dangerous cases are classified, and 75 dangerous car-following cases with high risk level are obtained.

c) The normal car-following cases are extracted by using the OEM FOT. Because the FOT is a control test for certain ADAS functions, the ADAS is working at certain times during the test. Firstly, the manual driving data are picked out by using the variables in the CAN bus which indicate the on/off of the ADAS. Next, the car-following cases are extracted by using the information collected by the Mobieye EyeQ3, including the target type, lateral and longitudinal relative distance, longitudinal relative velocity. In the last, manual screening is applied to remove the car-following cases which may be dangerous. 822 normal car-following cases are obtained.

3. Driving Privilege Assignment based on Risk Level
3.1 Obvious Risk and Potential Risk
As have mentioned before, there are some deficiencies in using TTC to describe the risk level. TTC or 1/TTC cannot accurately represent the risk level when the relative velocity is small. It will be very helpful to take the possible future braking operations of the target vehicle into account, e.g. the Mazda avoidance logic[23], the safety margin[24], and the responsibility sensitive safety (RSS)[25]. In this paper, TTC is defined as an obvious risk measure, whereas the time before the host vehicle has to brake assuming that the target vehicle is braking is defined as the potential risk measure, i.e. the time margin (TM).

![Fig 5. The collision avoidance process](image)

The collision avoidance process when the target vehicle brakes to stop with a constant deceleration is shown in Fig 5. $x_h$, $v_h$, $a_h$ are the position, velocity, and brake deceleration of the host vehicle, respectively. $x_t$, $v_t$, and $a_t$ are the position, velocity, and brake deceleration of the target vehicle, respectively. $a_h$ and $a_t$ take the the absolute value of the brake deceleration. $D$ is the relative distance. $t$ is the time from the current time until the host vehicle starts to decelerate. $t$ should contain 3 parts, i.e. the driver braking reaction time $\tau_1$, the braking system reaction time $\tau_2$, and the time $t_0$ which the driver can freely use. In order to avoid the collision when the target vehicle brakes, it should be

$$D + \frac{v_t^2}{2a_t} \geq v_h t + \frac{v_h^2}{2a_h}$$

That is

$$t_0 + \tau_1 + \tau_2 \leq \frac{D + \frac{v_t^2}{2a_t} - \frac{v_h^2}{2a_h}}{v_h}$$

When the target vehicle brakes with a constant deceleration, the maximum value of the sum of $\tau_1$, $\tau_2$ and $t_0$ is defined as the time margin (TM), i.e.
The brake deceleration of the host vehicle and the target vehicle is selected according to the friction limit of the vehicle, i.e. \( a_h = a_t = 7 \text{m/s}^2 \). TM indicates the reaction time left to the driver of the host vehicle if the target vehicle starts to brake. TTC indicates the risk level in the current state, whereas TM indicates the risk level if the target vehicle suddenly brakes. Therefore, TTC is defined as a obvious risk measure, and TM is defined as a potential risk measure. TM is mainly used to characterize the risk level when the relative velocity is small.

3.2 Risk Assessment in Car-following

A risk assessment algorithm is proposed by using the 75 dangerous car-following cases. The braking starting time in the dangerous car-following cases needs to be defined at first. The braking starting time is the moment when the driver feels the danger and responds to the danger. Therefore, the dangerous threshold is determined by using the TTC and TM at the braking starting time. Because the dangerous cases collected by the VDR do not include the brake and accelerator pedal information, the moment when the vehicle velocity begins to suddenly drop is defined as the brake starting time. 2 examples of the braking starting time identification are given in Fig 6. The vehicle velocity suddenly drops rapidly at point A. The section between A and B is defined as the emergency braking, and point A is defined as the brake starting time.

Since TTC may become very large when the risk level is low, 1/TTC is applied to define the obvious risk level. The 1/TTC at the braking start time is used to determine the obvious risk threshold. It is found that the 1/TTC at last-second braking onset is related to the velocity of the host vehicle[26]. Hence, it is necessary to discuss whether the velocity of the host vehicle has a significant influence on the 1/TTC at the braking start time. The relationship between the 1/TTC and velocity is shown in Fig 7. The regression coefficient test is used to verify whether the 1/TTC has a significant regression relationship with the velocity. The results are shown in Table 1. The Durbin-Watson test indicates that the residual has no significant autocorrelation. And the data is suitable for regression analysis. The regression coefficient test shows that 1/TTC and velocity have a significant regression relationship. Therefore, the impact of the velocity should be considered when 1/TTC is used to classify the risk level. The empirical regression coefficient between 1/TTC and velocity is -0.0717.

\[
TM = \frac{D + v_i^2}{v_h} - \frac{v_i}{2a_h}
\]  

(3)
The 5th, 50th, and 95th percentiles of the $1/\text{TTC}$ at the braking start time in 75 dangerous car-following cases are
\[
\begin{align*}
\text{ITTC}_{0.05} &= -0.0717v + 1.73 \\
\text{ITTC}_{0.50} &= -0.0717v + 1.18 \\
\text{ITTC}_{0.95} &= -0.0717v + 0.49
\end{align*}
\]

These 3 percentiles indicate that 5%, 50%, and 95% of the drivers brake when the $1/\text{TTC}$ reaches the corresponding threshold. When the obvious risk level is divided by using the threshold associated with the velocity, the thresholds will reach 0 as the velocity increases. A minimum value of the $1/\text{TTC}$ is needed for each obvious risk level. Consequently, the obvious risk level is
\[
\begin{align*}
\text{OR0: } 1/\text{TTC} &< \max(\text{ITTC}, \text{thr}_{1}) \\
\text{OR1: } \max(\text{ITTC}, \text{thr}_{1}) \leq 1/\text{TTC} &< \max(\text{ITTC}, \text{thr}_{2}) \\
\text{OR2: } \max(\text{ITTC}, \text{thr}_{2}) \leq 1/\text{TTC} &< \max(\text{ITTC}, \text{thr}_{3}) \\
\text{OR3: } 1/\text{TTC} &\geq \max(\text{ITTC}, \text{thr}_{3})
\end{align*}
\]

Where OR0 means no obvious risk. OR1, OR2 and OR3 indicate the level 1, level 2 and level 3 obvious risk level. \text{thr}_{1}, \text{thr}_{2}, and \text{thr}_{3} are the $1/\text{TTC}$ minimum values in the 3 risk levels, respectively. Refering to [27], the $1/\text{TTC}$ minimum values are set as \text{thr}_{1}=0.33s^{-1}, \text{thr}_{2}=0.66s^{-1}, and \text{thr}_{3}=1s^{-1}.

The TM at the brake starting time is used to determine the potential risk threshold. Similarly, the regression coefficient hypothesis test is used to discuss whether TM has a significant regression relationship with the velocity. The relationship between the TM and velocity is shown in Fig 8. The regression coefficient test is used to verify whether TM has a significant regression relationship with the velocity. The results are shown in Table 1. The Durbin-Watson test indicates that the residual has no significant autocorrelation. And the data is suitable for regression analysis. The regression coefficient test shows that TM and velocity have no significant regression relationship. Therefore, the influence of the velocity is not considered in the potential risk level. And the horizontal lines are employed to divide the TM thresholds.
The 5th, 50th, and 95th percentiles of TM at the brake starting time in 75 dangerous car-following cases are 1.4s, 0.5s, and 0, respectively. These 3 percentiles indicate that 5%, 50%, and 95% of the drivers brake when TM reaches the corresponding threshold. These 3 percentiles are applied to be the thresholds for the potential risk level, i.e.

\[
\begin{align*}
PR_0: & \quad TM > 1.4s; \\
PR_1: & \quad 0.5s < TM \leq 1.4s; \\
PR_2: & \quad 0 < TM \leq 0.5s; \\
PR_3: & \quad TM \leq 0.
\end{align*}
\]

Where PR0 means no potential risk. PR1, PR2 and PR3 represent the level 1, level 2 and level 3 potential risk level, respectively. Note that when TM<0, it means that the host vehicle cannot avoid a collision if the target vehicle suddenly brakes at the maximum deceleration. Hence, TM<0 is reasonable in some cases.

The risk assessment algorithm which considers the obvious risk and potential risk is
\[
\begin{align*}
RL0: & \; (PR0\&\&OR0)\|(PR1\&\&OR0)\|(PR0\&\&OR1) \\
RL1: & \; PR1\&\&OR1 \\
RL2: & \; (PR2\|OR2)\&(PR3\&\&OR3) \\
RL3: & \; OR3
\end{align*}
\] (7)

Where RL0 means no risk. RL1, RL2 and RL3 are the risk level 1, risk level 2 and risk level 3, respectively. && indicates logical and; || indicates logical or; ! indicates logical not.

PR1 and OR1 use the parameters after removing a small number of abnormal cases. If the driver is warned at PR1 or OR1, many false alarms will emerge. When PR1 and OR1 are simultaneously achieved, excessive false alarm can be avoided. Therefore, the RL1 is achieved when PR1 and OR1 are reached at the same time. PR3 and OR3 use the 95th percentile parameter, which is very urgent danger. Hence, RL3 is achieved when one of PR3 or OR3 is reached. The other situation between RL1 and RL3 is set as RL2.

4 example in the 75 dangerous car-following cases are picked out to demonstrate the evolution of obvious risk and potential risk in the car-following cases. The velocity of the host vehicle (v_h), the relative velocity (v_r), TM and 1/TTC of the 4 cases are shown in Fig 10 to Fig 13. Fig 10 is a danger that occurs shortly after the host vehicle starts. Fig 11 is a danger when the host vehicle approach a slowly moving target vehicle. The relative velocity is large in these 2 cases at the beginning. 1/TTC can detect the danger in the case with high relative velocity. TM can assist 1/TTC in defining the warning or intervention moment more precisely. Fig 12 is a stable car-following case in the urban elevated road. Fig 13 is a stable car-following case in the city road. The relative speed is very small in these 2 cases at the beginning, and the danger is caused by the sudden braking of the target vehicle. The 1/TTC is also very small when the relative velocity is low. However, TM has reached PR1 more than 10s before the driver of the host vehicle brakes. This indicates that the relative distance is too small, though the 1/TTC is very small. In the case of too small relative distance, the obvious risk level will increase rapidly if the target vehicle suddenly brakes. When the target vehicle starts to brake in the stable car-following case, the potential risk can help to detect the danger much more early.

![Fig 10. The velocity, TM and 1/TTC in the first car-following case](image-url)
The confusion matrix [28] are used to evaluate the effectiveness of the risk assessment algorithm. Many evaluation indicators can be obtained based on the confusion matrix, as shown in Table 3. The accuracy is a good evaluation.
indicator when the number of “positive” and “negative” are similar. When one type of data accounts for the majority, the accuracy will mainly consider the classification accuracy of the majority, and the classification accuracy of the minority will not have a significant impact on the result. The receiver operating characteristic (ROC) is not sensitive to data proportion[29]. Hence, the ROC is applied to compare the effectiveness of the TTC, TM, and risk assessment algorithms.

| Actual State | Positive | Negative |
|--------------|----------|----------|
| Detection State | Positive | False Positive (FP) |
|               | Negative | True Negative (TN) |

Table 3. Evaluation index based on confusion matrix

| Index               | Definition                  | Index          | Definition                  |
|---------------------|-----------------------------|----------------|-----------------------------|
| TP rate/sensitivity | $\frac{TP}{TP+FN}$          | FP rate        | $\frac{FP}{FP+TN}$          |
| FN rate             | $\frac{FN}{TP+FN}$          | Accuracy       | $\frac{TP+TN}{TP+FN+FP+TN}$ |
| TN rate             | $\frac{TN}{FP+TN}$          | Precision      | $\frac{TP}{TP+FP}$         |

Fig 14. The ROC of the TTC, TM and risk assessment algorithm

The TP rate of TTC and TM are verified by using the 75 dangerous car-following cases, and the FP rate of TTC and TM are verified by using the 822 normal car-following cases. The ROC curves of TTC and TM are shown in Fig 14. The TP rate and FP rate of the risk assessment algorithm are marked in Fig 14 with a symbol ‘+’. The value of $thr_1$ and the thresholds of PR1 have enormous effect on the TP rate and FP rate of the risk assessment algorithm. Hence, Fig 14 demonstrates the ROC curve of the risk assessment algorithm with different $thr_1$ when PR1 threshold is TM=1.4s and the ROC curve of the risk assessment algorithm with different PR1 thresholds when $thr_1=0.33s^{-1}$. The ROC curve of TTC is completely contained inside the ROC curve of the risk assessment algorithm when $thr_1$ is different, which indicates that the risk assessment algorithm is always better than TTC. The TP rate of the risk assessment algorithm cannot reach 1 when PR1 is different. This indicates that TTC have more influence on TP rate.
rather than TM. And TM mainly help with reducing the FP rate. TM makes up for the shortcomings of TTC that cannot describe the danger accurately in the cases with small relative velocity. Therefore, the risk estimation algorithm that considers both obvious risk and potential risk is better than TTC.

3.3 Risk Assessment in Cut-in

The driving behavior of the driver in cut-in scenario is studied by using the 249 normal cut-in cases and 77 dangerous cut-in cases. Similarly, the time when the driver starts braking in the cut-in is the moment when the driver feels the danger and response to the danger. Therefore, the driver behavior at the brake starting time in the cut-in scenario is presented. The brake starting time is defined by the moment when the driver of the host vehicle steps the brake pedal, which can be distinguished by the video and the brake pressure in the CAN bus.

Fig 15. The lateral division of the lane

Fig 16. The lateral position of the target vehicle at brake starting time

The lane is divided into five sections laterally, and the dividing lines are represented by y0 to y4, respectively. The distance between each dividing line is 1/4 lane width. The lateral position of the target vehicle at the brake starting time is analyzed. An example of the cut-in case at brake starting time is given in Fig 16, in which the target vehicle is at the position y1. The lateral position of the target vehicle in the 249 normal cut-in cases and 77 dangerous cut-in cases are shown in Fig 17. In dangerous cut-in cases, the brake time of the host vehicle is earlier than that in normal cut-in cases. In both normal cut-in cases and dangerous cut-in cases, most drivers will start braking when the target
car reaches the lane line. Very few drivers start braking before the target car reaches the lane line. Therefore, the position that the target vehicle arrives at the lane line is chosen as the moment to start the risk assessment in the cut-in scenario. After the target vehicle reaches the lane line, the risk level is estimated by using the risk assessment algorithm in the car-following scenario.

3.4 Driving Privilege Assignment

The credibility of the driver’s operation is related to the risk level. When the vehicle is in the normal driving state, it indicates that the driver can make a correct judgment on the traffic environment and maintain good control of the vehicle. The driver's operation is highly reliable at this time. When the vehicle enters a dangerous state, it indicates that the driver already has a misjudgement of the traffic environment, or has incorrect operation, distraction, or sluggish operation. The credibility of the driver's operation is low at this time. When the vehicle enters a dangerous state from normal driving state, the driving privilege is gradually transferred from the driver to the automated driving system. When the vehicle returns from a dangerous state to the normal driving state, the driving privilege is gradually returned from the automated driving system to the driver. The weight factor of the driver is denoted as $\kappa_1$, and the weight factor of the automated driving system is denoted as $\kappa_2$. The weight factors vary according to a linear law in the transfer of the driving privilege. The weight factors of the driver and the automated driving system during the driving privilege handover can be expressed as

$$\kappa_i(k) = \begin{cases} a, k < k_0 \\ \frac{\beta - a}{K} (k - k_0) + a, k_0 \leq k \leq k_T \\ \beta, k > k_T \end{cases}$$ (8)

Where $A$ is the total assignable driving privilege. $a$ is the driving weight of the driver in the current state, and $\beta$ is the driving weight assigned to the driver after the handover. $\alpha, \beta \in [0, A]$. $k_0$ is the step that starts the driving privilege handover, and $k_T$ is the step at the end of the handover. $K$ is the total number of step in the entire driving privilege handover, $K= k_T - k_0$. If the driving right handover duration is $t$ and the time of each step is $T$, then $K = t/T$.

During the transfer of driving privilege from the driver to the automated driving system, 3 handover strategies are set according to the degree of danger. When the vehicle is in normal driving state, the driving right is completely allocated to the driver. When entering the risk level 1, the driving privilege is slowly transferred from the driver to the automated driving system within 3s. When entering the risk level 2 The driving privilege is handed over from the driver to the automated driving system within 1s. When entering the risk level 3, the driving privilege is handed over from the driver to the automated driving system within 0.5s. That is

$$
\begin{align*}
\text{RL1}: & \quad \kappa_i(k_0) = A, \kappa_2(k_T) = 0, \kappa_1(k_T) = A, t = 3s; \\
\text{RL2}: & \quad \kappa_i(k_0) = A, \kappa_2(k_T) = 0, \kappa_1(k_T) = A, t = 1s; \\
\text{RL3}: & \quad \kappa_i(k_0) = A, \kappa_2(k_T) = 0, \kappa_1(k_T) = A, t = 0.5s;
\end{align*}
$$ (9)

When the vehicle completes the collision avoidance, the driver needs to be reminded and the driving privilege is returned to the driver from the automated driving system. 2 handover modes are set according to the state of the driver. If the driver is ready to drive, the driving privilege is returned to the driver within 6s. If the driver is not ready to drive, the driving privilege is returned to the driver within 30s. That is

$$
\begin{align*}
\text{Mode 1}: & \quad \kappa_i(k_0) = 0, \kappa_2(k_T) = A, \kappa_1(k_T) = A, t = 6s; \\
\text{Mode 2}: & \quad \kappa_i(k_0) = 0, \kappa_2(k_T) = A, \kappa_1(k_T) = A, t = 30s;
\end{align*}
$$ (10)

4. Driving Privilege Gradual Handover based on Non-cooperative MPC
4.1 System Model

The joint lateral control of the vehicle steering by the driver and automated driving system is achieved by the cooperative driving lateral control model. The lateral control state variable is \( x_y = [y, v_y, \psi, \omega]^T \). Where \( y \) is the lateral displacement, \( v_y \) is the lateral velocity, \( \psi \) is the yaw angle, and \( \omega \) is the yaw rate. The state equation of cooperative driving lateral control can be expressed as

\[
\begin{align*}
\dot{x}_y &= A_{y,x} x_y + B_{y,1}(k) u_{y,D} + B_{y,2}(k) u_{y,A} \\
z_y &= C_y x_y
\end{align*}
\]

(11)

With

\[
A_{y,x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{11} & a_{12} \\ 0 & 0 & 0 \end{bmatrix}, \quad B_{y,1,a} = B_{y,2,a} = \begin{bmatrix} 0 \\ b_1 \\ b_2 \end{bmatrix}, \quad C_y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{11} = -\frac{2C_f + 2C_r}{m v_y}, \\
a_{12} = -\frac{2a C_f - 2b C_r}{I_y v_y}, \quad a_{21} = -\frac{2a^2 C_f + 2b^2 C_r}{I_y v_y}, \quad b_1 = \frac{2C_r}{m}, \quad b_2 = \frac{2a C_r}{I_y}.
\]

Where \( v_y \) is the longitudinal velocity of the vehicle. \( C_f \) and \( C_r \) are the cornering stiffness of each of the front and rear tires. \( a \) and \( b \) are the distances of the front and rear axles from the center of gravity of the vehicle. \( I_y \) is yaw inertia of the vehicle. \( m \) is the vehicle mass. \( u_{y,D} \) is the steering input of the driver, and \( u_{y,A} \) is the steering input of the automated driving system. \( z_y \) is the observable state variable.

The continuous state equation is discretized. The discrete state equation is

\[
\begin{align*}
x_y(k+1) &= A_y x_y(k) + B_{y,1} u_{y,D}(k) + B_{y,2} u_{y,A}(k) \\
z_y(k) &= C_y x_y(k)
\end{align*}
\]

(12)

Where \( A_y \) is the matrix corresponding to \( A_{y,x} \) after discretization. \( B_{y,1} \) and \( B_{y,2} \) are the matrixes corresponding to \( B_{y,1,a} \) and \( B_{y,2,a} \) after discretization.

The discrete state space equation is iterated. And the cooperative driving lateral control model can be expressed as

\[
Z_y(k) = \Psi_y x_y(k) + \Theta_y U_{y,1}(k) + \Theta_y U_{y,2}(k)
\]

(13)

With

\[
Z_y(k) = \begin{bmatrix} z_y(k+1) \\ z_y(k+2) \\ z_y(k+N_y -1) \\ z_y(k+N_y) \end{bmatrix}, \quad U_{y,1} = \begin{bmatrix} u_{y,D}(k) \\ u_{y,D}(k+1) \\ u_{y,D}(k+N_u -2) \\ u_{y,D}(k+N_u -1) \end{bmatrix}, \quad U_{y,2} = \begin{bmatrix} u_{y,A}(k) \\ u_{y,A}(k+1) \\ u_{y,A}(k+N_u -2) \\ u_{y,A}(k+N_u -1) \end{bmatrix}, \quad \Psi_y = \begin{bmatrix} C_y A_y \\ C_y A_y^{N_y-1} \\ \vdots \\ C_y A_y^{N_y-N_u} \end{bmatrix}, \\
\Theta_y = \begin{bmatrix} C_y B_{y,1} \\ C_y A_y B_{y,1} \\ \vdots \\ C_y A_y^{N_y-N_u} B_{y,1} \\ C_y B_{y,2} \\ C_y A_y B_{y,2} \\ \vdots \\ C_y A_y^{N_y-N_u} B_{y,2} \end{bmatrix}, \quad \Theta_y = \begin{bmatrix} C_y A_y B_{y,1} \\ C_y A_y^{N_y-N_u} B_{y,1} \\ \vdots \\ C_y A_y B_{y,2} \\ C_y A_y^{N_y-N_u} B_{y,2} \end{bmatrix}.
\]

Where \( N_y \) is the preview horizon. \( N_u \) is the control horizon.

The joint longitudinal control of the vehicle by the driver and the automated driving system is achieved by the
cooperative driving longitudinal control model. The longitudinal control state variable is \( x_c = [d, v_c]^T \). Where \( d \) is the relative distance between the host vehicle and the target vehicle. \( v_c \) is the longitudinal velocity. The state equation of cooperative driving longitudinal control can be expressed as

\[
\begin{align*}
    \dot{x}_c &= A_c x_c + B_{c,1} u_{s,D} + B_{c,2} u_{s,A} + B_{c,w} w_s \\
    z_s &= C_c x_c
\end{align*}
\]

With

\[
A_c = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad B_{c,1,c} = B_{c,2,c} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{c,w,c} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

Where \( w_s \) is the measurable disturbance. In the longitudinal control model, \( w_s \) is the target vehicle velocity, i.e. \( w_s = v_t \). \( u_{s,D} \) is the longitudinal control input of the driver, and \( u_{s,A} \) is the longitudinal control input of the automated driving system.

The continuous state equation is discretized. The discrete state equation is

\[
\begin{align*}
    x_c(k+1) &= A_c x_c(k) + B_{c,1} u_{s,D}(k) + B_{c,2} u_{s,A}(k) + B_{c,w} w_s(k) \\
    z_s(k) &= C_c x_c(k)
\end{align*}
\]

Where \( A_c \) is the matrix corresponding to \( A_{c,c} \) after discretization. \( B_{c,1} \) and \( B_{c,2} \) are the matrixes corresponding to \( B_{c,1,c} \) and \( B_{c,2,c} \) after discretization.

The discrete state space equation is iterated. And the cooperative driving longitudinal control model can be expressed as

\[
Z_s(k) = \Psi_s x_c(k) + \Theta_{s,1} U_{s,1}(k) + \Theta_{s,2} U_{s,2}(k) + \Xi_s D_s(k)
\]

With

\[
\begin{align*}
    Z_s(k) &= \begin{bmatrix} z_c(k+1) \\ z_c(k+2) \\ \vdots \\ z_c(k+N_w-1) \\ z_c(k+N_w-2) \end{bmatrix}, \\
    U_{s,1} &= \begin{bmatrix} u_{s,D}(k) \\ u_{s,D}(k+1) \\ \vdots \\ u_{s,D}(k+N_w-2) \\ u_{s,D}(k+N_w-1) \end{bmatrix}, \\
    U_{s,2} &= \begin{bmatrix} u_{s,A}(k) \\ u_{s,A}(k+1) \\ \vdots \\ u_{s,A}(k+N_w-2) \\ u_{s,A}(k+N_w-1) \end{bmatrix}, \\
    \Psi_s &= \begin{bmatrix} C_{A,c} \\ C_{A,c}^2 \\ \vdots \\ C_{A,c}^{N_w} \end{bmatrix}, \\
    \Theta_{s,1} &= \begin{bmatrix} C_{A,B_{s,1}} & 0 & \cdots & 0 \\ C_{A,B_{s,1}} & C_{A,B_{s,1}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{A,B_{s,1}} & C_{A,B_{s,1}} & \cdots & C_{A,B_{s,1}} \\ C_{A,B_{s,1}} & C_{A,B_{s,1}} & \cdots & C_{A,B_{s,1}} \\ C_{A,B_{s,1}} & C_{A,B_{s,1}} & \cdots & C_{A,B_{s,1}} \end{bmatrix}, \\
    \Theta_{s,2} &= \begin{bmatrix} C_{A,B_{s,2}} & 0 & \cdots & 0 \\ C_{A,B_{s,2}} & C_{A,B_{s,2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{A,B_{s,2}} & C_{A,B_{s,2}} & \cdots & C_{A,B_{s,2}} \\ C_{A,B_{s,2}} & C_{A,B_{s,2}} & \cdots & C_{A,B_{s,2}} \\ C_{A,B_{s,2}} & C_{A,B_{s,2}} & \cdots & C_{A,B_{s,2}} \end{bmatrix}, \\
    D_s(k) &= \begin{bmatrix} w_s(k) \\ w_s(k+1) \\ \vdots \\ w_s(k+N_w-2) \\ w_s(k+N_w-1) \end{bmatrix}, \\
    \Xi_s &= \begin{bmatrix} C_{A,B_{s,w}} & 0 & \cdots & 0 \\ C_{A,B_{s,w}} & C_{A,B_{s,w}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{A,B_{s,w}} & C_{A,B_{s,w}} & \cdots & C_{A,B_{s,w}} \\ C_{A,B_{s,w}} & C_{A,B_{s,w}} & \cdots & C_{A,B_{s,w}} \\ C_{A,B_{s,w}} & C_{A,B_{s,w}} & \cdots & C_{A,B_{s,w}} \end{bmatrix}.
\end{align*}
\]

According to (13) and (16), the system model in the cooperative driving can be uniformly expressed as
\[ Z(k) = \Psi x(k) + \Theta U_1(k) + \Theta U_2(k) + \Xi D(k) \]
\[ U_1 \in \mathbb{U}_1, U_2 \in \mathbb{U}_2 \]  
(17)

Where \( \mathbb{U}_1 \) and \( \mathbb{U}_2 \) are the sets of all feasible controls in the cooperative driving system model. For the lateral control model, the model parameters in (17) are the ones in the lateral control model, and \( D(k) = 0 \). For the longitudinal control model, the model parameters in (17) are the ones in the longitudinal control model. In this way, a uniform cooperative driving linear system model is obtained.

4.2 Cost Function

The 2 players in the non-cooperative MPC (driver and automated driving system) expect to minimize the cost functions for their own goals

\[
\begin{align*}
\min_{V_1(k)} & \quad V_1(k) \\
\text{s.t.} & \quad Z(k) = \Psi x(k) + \Theta U_1(k) + \Theta U_2(k) \\
\end{align*}
\]

Where \( V_1(k) \) is the cost function of the driver and \( V_2(k) \) is the cost function of the automated driving system.

The cost functions of the 2 players are defined as

\[
\begin{align*}
V_1(k) &= \|Z(k) - T_1(k)\|_2^2 + \|U_1(k)\|_2^2 \\
V_2(k) &= \|Z(k) - T_2(k)\|_2^2 + \|U_2(k)\|_2^2 \\
\end{align*}
\]

With

\[
T_1(k) = \begin{bmatrix} t_1(k) & t_1(k-N_p+2) & \cdots & t_1(k-N_p+1) \\
0 & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots \\
0 & \cdots & \ddots & \ddots \\
0 & \cdots & \ddots & 0 \\
0 & \cdots & 0 & \ddots \\
\end{bmatrix}, \quad T_2(k) = \begin{bmatrix} t_2(k) & t_2(k-N_p+2) & \cdots & t_2(k-N_p+1) \\
0 & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots \\
0 & \cdots & \ddots & \ddots \\
0 & \cdots & \ddots & 0 \\
0 & \cdots & 0 & \ddots \\
\end{bmatrix}, \quad R_1 = \begin{bmatrix} r_1 & 0 & \cdots & 0 \\
0 & r_1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & \ddots & r_1 \\
0 & \cdots & 0 & r_1 \\
\end{bmatrix}, \quad R_2 = \begin{bmatrix} r_2 & 0 & \cdots & 0 \\
0 & r_2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & \ddots & r_2 \\
0 & \cdots & 0 & r_2 \\
\end{bmatrix}
\]

\[
Q_1(k) = \begin{bmatrix} q_1(k+1) & 0 & \cdots & 0 \\
0 & q_1(k+2) & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & \ddots & q_1(k+N_p) \\
0 & \cdots & 0 & q_1(k+N_p) \\
\end{bmatrix}, \quad Q_2(k) = \begin{bmatrix} q_2(k+1) & 0 & \cdots & 0 \\
0 & q_2(k+2) & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & \ddots & q_2(k+N_p) \\
0 & \cdots & 0 & q_2(k+N_p) \\
\end{bmatrix}
\]

\[
q_1(k) = \begin{bmatrix} \kappa_1(k) & 0 \\
0 & \lambda_1(k) \end{bmatrix}, \quad q_2(k) = \begin{bmatrix} \kappa_2(k) & 0 \\
0 & \lambda_2(k) \end{bmatrix}, \quad t_1(k) = \begin{bmatrix} y_1(k) \\
y_2(k) \end{bmatrix}, \quad t_2(k) = \begin{bmatrix} y_3(k) \\
y_4(k) \end{bmatrix}
\]

\( q_1(k) \) is confidence matrix of the the driver and \( q_2(k) \) is the confidence matrix of the automated driving system. \( \kappa_1(k) \) and \( \kappa_2(k) \) are the weight factors, which are related to the driving privilege assignation weight. \( \lambda_1(k) \) and \( \lambda_2(k) \) are the dynamic factors, which are related to the dynamic characteristics of the driving privilege assignation. Both \( Q_1(k) \) and \( Q_2(k) \) are semi-positive definite matrixes. And both \( R_1 \) and \( R_2 \) are positive definite matrixes. \( T_1(k) \) and \( T_2(k) \) are the local target trajectories of the 2 players. \( T_1(k) \) and \( T_2(k) \) need to be updated before each non-cooperative MPC optimization. The update equations are

\[
T_1(k+1) = GT_1(k) + HT_1(k+1) \\
T_2(k+1) = GT_2(k) + HT_2(k+1)
\]

(20)

With
Where $I_j$ is $j$-dimensional unit matrix. And $j$ is the number of state variables.

For the lateral system model, $t_1(k)$ and $t_2(k)$ are

$$t_1(k) = \begin{bmatrix} y_D(k) \\ \psi_D(k) \end{bmatrix}, \quad t_2(k) = \begin{bmatrix} y_A(k) \\ \psi_A(k) \end{bmatrix}$$

Where $y_D(k)$ and $\psi_D(k)$ are the desired lateral displacement and the desired yaw angle of the driver. $y_A(k)$ and $\psi_A(k)$ are the desired lateral displacement and the desired yaw angle of the automated driving system.

For the longitudinal system model, $t_1(k)$ and $t_2(k)$ are

$$t_1(k) = \begin{bmatrix} d_{D,(k)} \\ v_{x,D}(k) \end{bmatrix}, \quad t_2(k) = \begin{bmatrix} d_{A,(k)} \\ v_{x,A}(k) \end{bmatrix}$$

Where $d_{D,(k)}$ and $v_{x,D}(k)$ are the desired relative distance and the desired longitudinal velocity of the driver. $d_{A,(k)}$ and $v_{x,A}(k)$ are the desired relative distance and the desired longitudinal velocity of the automated driving system.

4.3 Nash Equilibrium

2 error variables are defined as

$$e_i(k) = T_i(k) - \Psi\xi(k) - \Theta_i U_i(k) - \Xi D(k)$$

$$e_2(k) = T_2(k) - \Psi\xi(k) - \Theta_2 U_2(k) - \Xi D(k)$$

(21)

The cost functions can be transformed according to the error variables

$$V_i(k) = \|\Theta_i U_i(k) - e_i(k)\|_Q + \|U_i(k)\|_R, \quad i = 1, 2$$

(22)

The partial derivative of $V_i(k)$ to $U_i(k)$ is

$$\frac{\partial V_i(k)}{\partial U_i(k)} = -2\Theta_i^T Q_i(k) e_i(k) + 2[\Theta_i^T Q_i(k) \Theta_i + R_i] U_i(k), \quad i = 1, 2$$

(23)

Since $Q_i(k)$ is semi-definite matrix and $R_i$ is positive definite matrix, the second-order partial derivative is always larger than 0. Therefore, the solution of the first-order partial derivative equal to 0 is the minimum value of the cost function of the $i$-th player, i.e.

$$U_i^*(k) = F_i(k) e_i(k)$$

$$U_i^*(k) \in U_i^*, \quad i = 1, 2$$

(24)

With

$$F_i(k) = [\Theta_i^T Q_i(k) \Theta_i + R_i]^\top \Theta_i^T Q_i(k)$$

Where $U_i^*(k)$ represents the optimal control sequence that minimizes the cost function of the $i$-th player. $U_i^*$ is the set of all optimal control of the $i$-th player.

The non-cooperative MPC is solved by using a iterative method in previous studies[21, 30]. The following theorem shows that the Nash equilibrium solution for the non-cooperative MPC can be achieved by a non-iterative method.

**Theorem 1**: For the system model defined in (17) and the cost function defined in (18), the dynamic game has a unique Nash equilibrium solution if and only if $I-L(k)$ is reversible. And the Nash equilibrium solution of the non-
cooperative MPC is

\[
\begin{bmatrix}
U^*_1(k) \\
U^*_2(k)
\end{bmatrix} = K(k) \begin{bmatrix} T_1(k) \\
T_2(k)
\end{bmatrix} - \begin{bmatrix} \Psi^T \\
\Xi^T
\end{bmatrix} \begin{bmatrix} x(k) \\
D(k)
\end{bmatrix}
\]

With

\[
K(k) = (I-L(k))^{-1} M(k), \quad M(k) = \begin{bmatrix} F_1(k) & 0 \\
0 & F_2(k)
\end{bmatrix}, \quad L(k) = \begin{bmatrix} 0 & -F_1(k)\Theta \\
-F_2(k)\Theta & 0
\end{bmatrix}.
\]

**Proof:**

(1) Existence and uniqueness. In non-cooperative MPC, all the players only know the initial state \(x(1)\) in each optimization step. And the remaining states \(x(2), \ldots, x(N_p)\) are state predictions. The information set of the \(i\)-th player is \(\eta_i(k) = \{x(1)\}\). Hence, The non-cooperative MPC is an open-loop dynamic game. When the system model is linear and the cost function is quadratic, the dynamic game is a linear quadratic game. The existence and uniqueness of the Nash equilibrium solution of the open-loop linear quadratic game can be discriminated by the reversibility of a predefined matrix \([31]\). The non-cooperative MPC has a unique Nash equilibrium solution if and only if \(P(k)\) is reversible. Where

\[
P(k) = \begin{bmatrix} \Theta^T Q_i(k)\Theta_1 + R_i & \Theta^T Q_i(k)\Theta_2 \\
\Theta^T Q_i(k)\Theta_1 & \Theta^T Q_i(k)\Theta_2 + R_i
\end{bmatrix}
\]

The \(P(k)\) can be transformed into

\[
P(k) = \begin{bmatrix} \Theta^T Q_i(k)\Theta_1 + R_i & 0 \\
0 & \Theta^T Q_i(k)\Theta_2 + R_i
\end{bmatrix} \begin{bmatrix} I - L(k) \\
\end{bmatrix}
\]

Since \(Q_i(k)\) is a semi-definite matrix and \(R_i\) is a positive definite matrix, \(P(k)\) is reversible if and only if \(I-L(k)\) is reversible. Hence, the non-cooperative MPC has a unique Nash equilibrium solution if and only if \(I-L(k)\) is reversible.

(2) Construction. The optimal response function of the non-cooperative MPC can be represented in a matrix form as

\[
\begin{bmatrix}
U^*_1(k) \\
U^*_2(k)
\end{bmatrix} = M(k) \begin{bmatrix} T_1(k) \\
T_2(k)
\end{bmatrix} - \begin{bmatrix} \Psi^T \\
\Xi^T
\end{bmatrix} \begin{bmatrix} x(k) \\
D(k)
\end{bmatrix} + L(k) \begin{bmatrix} U_1(k) \\
U_2(k)
\end{bmatrix}
\]

(25)

When \(I-L(k)\) is reversible, the non-cooperative MPC has a unique Nash equilibrium solution. The Nash equilibrium solution is denoted as \((U^*_1(k), U^*_2(k))\). The Nash equilibrium solution should satisfy the optimal response function, i.e.

\[
\begin{bmatrix}
U^*_1(k) \\
U^*_2(k)
\end{bmatrix} = M(k) \begin{bmatrix} T_1(k) \\
T_2(k)
\end{bmatrix} - \begin{bmatrix} \Psi^T \\
\Xi^T
\end{bmatrix} \begin{bmatrix} x(k) \\
D(k)
\end{bmatrix} + L(k) \begin{bmatrix} U_1(k) \\
U_2(k)
\end{bmatrix}
\]

(26)

If \(I-L(k)\) is reversible, (26) has a solution. The Nash equilibrium solution of the non-cooperative MPC is

\[
\begin{bmatrix}
U^*_1(k) \\
U^*_2(k)
\end{bmatrix} = (I-L(k))^{-1} M(k) \begin{bmatrix} T_1(k) \\
T_2(k)
\end{bmatrix} - \begin{bmatrix} \Psi^T \\
\Xi^T
\end{bmatrix} \begin{bmatrix} x(k) \\
D(k)
\end{bmatrix}
\]

(27)

This completes the proof. □

In MPC, a local optimal solution within the preview horizon is solved at each step. And the preview horizon will recede after each optimization. Only the first control input in each step works. Therefore, the feedback gains of the 2 player in the non-cooperative MPC are
\[
\begin{bmatrix}
K_1(k) \\
K_2(k)
\end{bmatrix} =
\begin{bmatrix}
I_l & 0 & \cdots & 0 \\
0 & I_l & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\Psi & \Xi \\
\Psi & \Xi
\end{bmatrix}
\begin{bmatrix}
x(k) \\
D(k)
\end{bmatrix}
\]

Where \(I_l\) is \(l\)-dimensional unit matrix. And \(l\) is the number of control variables.

The control inputs of 2 players (driver and automated driving system) in the non-cooperative MPC can be expressed as

\[
u_1^*(k) = K_1(k) \begin{bmatrix} T_1(k) \\ T_2(k) \end{bmatrix} \begin{bmatrix} \Psi & \Xi \\ \Psi & \Xi \end{bmatrix} \begin{bmatrix} x(k) \\ D(k) \end{bmatrix}
\]
\[
u_2^*(k) = K_2(k) \begin{bmatrix} T_1(k) \\ T_2(k) \end{bmatrix} \begin{bmatrix} \Psi & \Xi \\ \Psi & \Xi \end{bmatrix} \begin{bmatrix} x(k) \\ D(k) \end{bmatrix}
\]

The Nash equilibrium inputs of the driver and the automated driving system in the driving privilege gradual handover strategy is obtained by using (29). Although the process of solving the optimal solution by using (24) is convenient and clear, the calculation result of (24) may be numerical instability. Therefore, the method introduced in [32] is applied to avoid the numerical instability of \(F_i(k)\), i.e.

\[
F_1(k) = \begin{bmatrix}
S_{\theta_1}(k)\Theta_1 \\
S_{\theta_2}
\end{bmatrix}
\begin{bmatrix}
S_{\theta_1}(k) \\
0
\end{bmatrix}
\]
\[
F_2(k) = \begin{bmatrix}
S_{\theta_1}(k)\Theta_2 \\
S_{\theta_2}
\end{bmatrix}
\begin{bmatrix}
S_{\theta_1}(k) \\
0
\end{bmatrix}
\]

\[
\begin{cases}
S_{\theta_1}(k)^TS_{\theta_1}(k) = Q_1(k) \\
S_{\theta_2}^TS_{\theta_2} = R_1
\end{cases}
\]
\[
\begin{cases}
S_{\theta_1}(k)^TS_{\theta_1}(k) = Q_2(k) \\
S_{\theta_2}^TS_{\theta_2} = R_2
\end{cases}
\]

Where \(A^+\) is the generalized inverse matrix of \(A\).

5. Simulation Verification

5.1 Parameter Specification

The vehicle parameters are shown in Table 4. The preview horizon is chosen as \(N_p=10\), and the control horizon is chosen as \(N_u=10\). The time step is \(T=0.01\)s. Similar to optimal control, only the relative values of \(q_1(k), q_2(k)\) and \(r_1, r_2\) have influence on the control result in non-cooperative MPC. Therefore, \(r_1=1\) and \(r_2=1\) are set in the simulation.

| Symbol | Unit | Value |
|--------|------|-------|
| \(a\)  | m    | 1.0   |
| \(b\)  | m    | 1.5   |
| \(m\)  | kg   | 1270  |
| \(I_z\)| kg·m² | 1443.1 |
| \(C_f\)| kN/rad | 30    |
| \(C_r\)| kN/rad | 30    |

5.2 Lane-change Scenario

Firstly, the cooperative driving strategy is verified by using the lane-change scenario. The driver of the host vehicle desires a left lane-change, but the driver does not notice a target vehicle in the rear is approaching in the target lane. The automated driving system considers that it is not suitable for lane-change after risk assessment, and the automated
driving system expects lane keeping. There is a conflict between the intention of the driver and the intention of the automated driving system. The cooperative driving strategy carries out a driving privilege handover at the appropriate time based on the result of risk assessment.

Fig 18. The conflict in lane-change scenario

The driver desire to change the lane at 2s. The length of the lane-change trajectory is 80m and the width is 3.5m. The lane-change trajectory uses a fifth-order polynomial trajectory[33]. The velocity of the host vehicle is 20m/s. In lane-change case 1, the target vehicle is 30m behind, and the velocity of the target vehicle is 23m/s. In lane-change case 2, the target vehicle is 40m behind, and the velocity of the target vehicle is 23m/s. \( \lambda_1(k) \) and \( \lambda_2(k) \) are set to be constant in these 2 cases, i.e. \( \lambda_1(k)=2 \), \( \lambda_2(k)=2 \). In lane-change case 1, the risk level change from RL0 to RL2 at 3.2s. \( \kappa_1(k)=0.1 \) and \( \kappa_2(k)=0 \) at the beginning. At 3.2s, \( \kappa_1(k) \) linearly decreases to 0 and \( \kappa_2(k) \) linearly increases to 0.1 within 1s. In lane-change case 2, the risk level change from RL0 to RL2 at 4.9s. \( \kappa_1(k)=0.1 \) and \( \kappa_2(k)=0 \) at the beginning. At 4.9s, \( \kappa_1(k) \) linearly decreases to 0 and \( \kappa_2(k) \) linearly increases to 0.1 within 1s.

Fig 19. The weight factor in the lane-change scenario
The $\kappa_1(k)$ and $\kappa_2(k)$ in these 2 cases are shown in Fig 19. The simulation results are shown in Fig 20. When the driving privilege is transformed during the lane-change, the vehicle can smoothly return to the initial lane. The driving privilege gradual handover strategy can realize the gradual transition of the driving privilege between the driver and the automated driving system when the risk level is raised, so that the vehicle returns to the lane. The cooperative driving strategy can assist the driver to avoid the danger and can ensure that the intervention of the automated driving system is not too abrupt. A stable trajectory can be planned in these 2 cases. The front wheel angle are kept in a small range, which is very beneficial for maintaining vehicle stability.

5.3 Cut-in Scenario
Secondly, the cooperative driving strategy is verified by using the cut-in scenario. When the host vehicle goes
straight, the target vehicle in the lane beside cuts in. The driver of the host vehicle did not notice the cut-in of the target vehicle, and no action was taken. Hence, the driver's intention is velocity keeping. The automated driving system starts the risk assessment after the target vehicle crosses the lane line. And the automated driving system makes a decision to decelerate to the same velocity of the target vehicle based on the result of risk assessment. The conflict arises at this time. The cooperative driving strategy transfer the driving privilege from the driving to the automated driving system based on the result of risk assessment.

![Fig 21. The conflict in cut-in scenario](image1)

![Fig 22. The weight factor in the cut-in scenario](image2)

In cut-in case 1, the initial velocity of the host vehicle is 8m/s. The target vehicle start to cut in 10m before the host vehicle, and the velocity of the target is 5m/s. In cut-in case 2, the initial velocity of the host vehicle is 12m/s. the target vehicle start to cut in 10m before the host vehicle, and the velocity of the target is 10m/s. \( \lambda_1(k) \) and \( \lambda_2(k) \) are set to be constant in these 2 cases, i.e. \( \lambda_1(k)=100, \lambda_2(k)=100 \). In cut-in case 1, the risk level change from RL0 to RL1 at 0.5s. \( \kappa_1(k)=0.1 \) and \( \kappa_2(k)=0 \) at the beginning. At 0.5s, \( \kappa_1(k) \) linearly decreases to 0 and \( \kappa_2(k) \) linearly increases to 0.1
within 3s. In cut-in case 2, the risk level change from RL0 to RL2 at 0.6s. $\kappa_1(k)=0.1$ and $\kappa_2(k)=0$ at the beginning. At 0.6s, $\kappa_1(k)$ linearly decreases to 0 and $\kappa_2(k)$ linearly increases to 0.1 within 1s.

The $\kappa_1(k)$ and $\kappa_2(k)$ in these 2 cases are shown in Fig 22. The simulation results are shown in Fig 23. When the target vehicle cuts in, the risk level increases as the relative distance decreases. The cooperative driving strategy begins to transfer the driving privilege from the driver to the automated driving system when the risk level reaches the corresponding threshold. The brake deceleration is small when the risk level is low. The comfort is satisfied while the safety is ensured. When the risk level is high, the brake deceleration is increased to ensure safety. The automated driving system in the cooperative driving strategy can gradually intervene, and the intervention strategy can be adjusted according to the risk level. Therefore, the cooperative driving strategy can better balance comfort and safety.

Fig 23. The simulation results of the cut-in scenario
6. Conclusions

The cooperative driving strategy is decomposed into 2 issues, i.e. the driving privilege assignment and the driving privilege handover. The driving privilege assignment in real-time is proposed based on the risk level, whereas the driving privilege gradual handover is realized by using the dynamic game. Since the risk level is related to the subjective feeling of the driver, the naturalistic driving data is used to study the behavior characteristics of the driver in typical dangerous scenarios, i.e. the car-following scenario and the cut-in scenario. The dangerous and normal car-following cases and cut-in cases are extracted by using the naturalistic driving data. TTC is defined as the obvious risk measure, whereas the reaction time left to the driver if the target vehicle starts to brake is defined as the potential risk measure, i.e. time margin (TM). A risk assessment algorithm is proposed based on the obvious risk and potential risk. The dangerous and normal car-following cases are applied to verify the effectiveness of the risk assessment algorithm. It is identified that the risk assessment algorithm performs better than TTC in ROC. The braking moment of the driver in the cut-in scenario is studied by using the dangerous and normal cut-in cases. The results show that most drivers start braking when the target vehicle reaches the lane line. Therefore, the moment when the target vehicle reaches the lane line is taken as the time that the risk estimation is started in the cut-in scenario. In order to avoid the uncomfortable caused by the sudden intervention of the automated driving system and the difficulties in taking over caused by the sudden withdrawal of the automated driving system, the driving privilege gradual handover is proposed. During the driving privilege gradual handover, the vehicle is jointly controlled by the driver and automated driving system. The non-cooperative MPC are employed to deal with conflicts between the driver and automated driving system. The system model and cost function are constructed, and the Nash equilibrium solution of non-cooperative MPC is obtained. The driving privilege gradual handover is realized through the update of the confidence matrix. The simulation verification shows that the cooperative driving strategy can realize the driving privilege gradual handover in the dangerous process. The safety can be ensured while the comfortable is maintained.

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