Near-Extremal Kerr $AdS_2 \times S^2$ Solution and Black-Hole/Near-Horizon-CFT Duality

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Abstract

We study the thermodynamics of the near horizon of near extremal Kerr geometry (near – NHEK) within an $AdS_2/CFT_1$ correspondence. We do this by shifting the horizon by a general finite mass, which does not alter the geometry and the resulting solution is still diffeomorphic to NHEK, however it allows for a Robertson Wilczek two dimensional Kaluza-Klein reduction and the introduction of a finite regulator on the $AdS_2$ boundary. The resulting asymptotic symmetry group of the two dimensional Kaluza-Klein reduction leads to a non-vanishing quantum conformal field theory ($CFT$) on the respective $AdS_2$ boundary. The $s$-wave contribution of the energy-momentum-tensor of the $CFT$, together with the asymptotic symmetries, generate a Virasoro algebra with calculable center and non-vanishing lowest Virasoro eigen-mode. The central charge and lowest eigen-mode reproduce the near – NHEK Bekenstein-Hawking entropy via the statistical Cardy Formula and our derived central charge agrees with the standard Kerr/CFT Correspondence. We also compute the Hawking temperature of the shifted near – NHEK by analyzing quantum holomorphic fluxes of the Robinson and Wilczek two dimensional analogue fields.

Keywords: Black Hole Thermodynamics; Black-Hole/CFT Duality; Quantum Gravity.

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1 Introduction

Black hole thermodynamic quantities [1–3],

\[
\begin{align*}
T_H &= \frac{\hbar c}{2\pi} \quad \text{Hawking Temperature} \\
S_{BH} &= \frac{A}{4\hbar c} \quad \text{Bekenstein-Hawking Entropy}
\end{align*}
\]

provided an ample testing bed for most current competing theories of quantum gravity. It is widely believed that any viable ultraviolet completion of general relativity should reproduce some variant of (1.1), perhaps modulo some real finite parameter that would need to be fixed by experiment. To date there is a plethora of different approaches for arriving at (1.1), with string theories and loop quantum gravity the predominant competitors and no clear consensus of which approach should be preferred over the other.

1.1 Black Hole Temperature and Effective Action

Since Hawking’s original analysis of the density of quantum states in terms of Bogolyubov coefficients, analysis via effective actions and their associated energy momentum tensors of the semiclassical matter fields has been explored in various settings for arriving at \( T_H \) [4–12]. Of particular interest is the realization by Robinson and Wilczek (RW) that anomalous two dimensional chiral theories in the near horizon of black holes are rendered unitary by requiring the black hole to radiate at temperature \( T_H \) [13–33]. This procedure requires a dimensional reduction yielding two dimensional analogues for various types of four dimensional black holes (RW2DA), beyond the basic Schwarzschild case, coupled to two dimensional matter fields.

More recently Rodriguez and Yildirim showed by analyzing quantum conformal matter in the background of certain RW2DA black holes that the resulting energy momentum tensor (EMT)
is holomorphic in the horizon limit or at the asymptotic infinity boundary \([34, 35]\). Furthermore, this resulting holomorphic EMT is dominated by one component equaling the four dimensional Hawking flux of temperature \(T_H\) weighted by a central charge \(c\). A plausible interpretation of this result is that in the near horizon regime a four or higher dimensional spacetime metric exhibits a Kaluza-Klein reduction into two dimensional fields \(g^{(2)\mu \nu}, A_\mu\) and \(\Phi\). In this interpretation \(A_\mu\) and \(\Phi\) are of gravitational origin, yet mathematically behave as two dimensional \(U(1)\) gauge and conformal scalar fields. Thus a quantum field theoretic study of \(\Phi\) in two dimensions with respect to \(g^{(2)\mu \nu}\) and \(A_\mu\) will have quantum gravitational implications in the near horizon of four dimensional black holes. Depending on the asymptotic symmetry group of certain RW2DA, this may suggest a sort of \(AdS_2/CFT_1\) relationship for computing four dimensional black hole temperature.

### 1.2 Holographic Black Hole Entropy

The fact that quantum gravity may be dual to a \(CFT\) \([36]\) has spawned a surge, led by Strominger \([37, 38]\), Carlip \([39–42]\) and Park \([43–45]\), in applying \(CFT\) techniques to compute Bekenstein-Hawking Entropy of various black holes \([34, 35, 46–53]\). The most notable such example is by far the Kerr/\(CFT\) correspondence and its extensions \([38, 54–71]\), where the general idea is that the asymptotic symmetry group (ASG), preserving certain metric boundary or fall off conditions, is generated by a Virasoro algebra with calculable central extension:

\[
[Q_m, Q_n] = (m - n)Q_{m+n} + \frac{c}{12} m (m^2 - 1) \delta_{m+n,0}.
\]

The Bekenstein-Hawking entropy is then obtained from Cardy’s Formula \([72, 73]\) in terms of \(c\) and the proper normalized lowest eigen-mode via:

\[
S = 2\pi \sqrt{\frac{c}{6}} Q_0.
\]

In the Kerr/\(CFT\) case a thermal Cardy Formula is employed

\[
S_{BH} = \frac{\pi^2}{3} c_L T_L,
\]

which depends on the left Frolov-Thorne vacuum temperature \(T_L\) for generic Kerr geometry \([74]\). This is in part due to the vanishing surface gravity of the extremal Kerr geometry, which is usually employed in regulating the quantum charges of (1.2), thus leading to a finite \(Q_0^{L,R}\). However, a finite zero mode may be inferred by the identification:

\[
\frac{\partial S_{CFT}}{\partial Q_0} = \frac{\partial S_{BH}}{\partial Q_0} = \frac{1}{T} \Rightarrow Q_0 = \frac{\pi^2}{6} c T^2.
\]

Requiring that the ASG contains a proper \(SL(2,\mathbb{R})\) subgroup in the above equation, yields the general value \(T = \frac{1}{2\pi}\), which is the case for Kerr/\(CFT\), where \(T_L = \frac{1}{2\pi}\), \(T_R = 0\), and \(c_L = c_R = c = 12 J\) and the extremal Bekenstein-Hawking entropy is recovered via the thermal Cardy Formula:

\[
S_{BH} = \frac{\pi^2}{3} c(T_L + T_R) = 2\pi J.
\]

From the definition (1.5) we see that \(T\) in general should be unitless. An interesting choice is to employ the Hawking temperature scaled by the finite time regulator \(1/\kappa\) giving the general result \(T = \frac{1}{2\pi}\) and also extends smoothly to extremality\(^1\).

\(^1\)A similar identification can be found in \([39, 75]\).
The general Kerr entropy may be obtained by analyzing the Frolov-Thorne vacuum in the near extremal case, where

\[ T_L = \frac{(GM)^2}{2\pi J} \quad \text{and} \quad T_R = \frac{\sqrt{(GM)^4 - (GJ)^2}}{2\pi J}. \]  

(1.7)

Using these values and \( c = 12J \) in (1.6) yields the standard area law

\[ S_{BH} = 2\pi \left( GM^2 + \sqrt{G^2 M^4 - J^2} \right). \]  

(1.8)

Yet, the above result requires combining quantities derived separately at extremality and near-extremality. It is also not obvious that the combination of temperatures in (1.7), \( T = T_L + T_R \), yields the value \( \frac{1}{2\pi} \) except in the extremal limit. However recasting the values of \( c, T_L, T_R \) into more general variables we see that:

\[ c = \frac{3A}{2G}, \quad T_L = \frac{4(GM)^2}{A} \quad \text{and} \quad T_R = \frac{4\sqrt{(GM)^4 - (GJ)^2}}{A}. \]  

(1.9)

substituting these values into (1.4) yields \( S_{BH} = \frac{A}{4GM} \) and assuming they smoothly extend back to non-extremality we have:

\[ T = T_L + T_R = \frac{4(GM)^2}{A} + \frac{4\sqrt{(GM)^4 - (GJ)^2}}{A} = \frac{1}{2\pi}, \]  

(1.10)

which would provide a more wholesome computation of the near-extremal Kerr black hole entropy. This is precisely the aim of this note, to construct a near horizon CFT dual for the near-NHEK spacetime and compute its corresponding entropy within a statistical Cardy formula (1.3), without mixing result derived separately at extremality and near-extremality. We will do this within an AdS/CFT\(_2\) correspondence by performing a RW two dimensional reduction of the near-NHEK geometry in a specific finite mass gauge following similar constructions as found in [35, 76].

In [35] Button, Rodriguez, Whiting and Yildirim showed that the RW2DA of non-extremal Kerr-Newman-AdS is asymptotically AdS\(_2\), for a specific choice of metric and gauge field fall off conditions, with effective near horizon functional derived via the RW dimensional reduction procedure. Evaluating the resulting effective EMT on the AdS\(_2\) boundary and computing its response to a total symmetry transformation yields a central charge:

\[ \frac{c}{24\pi} = \frac{r_+^2 + a^2}{4\pi G\Xi} \Rightarrow c = \frac{3A}{2\pi G}. \]  

(1.11)

which is in agreement with the Kerr/CFT correspondence within the appropriate limits, i.e.

\[ \lim_{\ell \to \infty, \quad Q \to 0, \quad a \to GM} c = 12J. \]  

(1.12)

Computation of the asymptotic symmetry algebra yields the normalized zero mode

\[ Q_0 = \frac{A}{16\pi G}, \]  

(1.13)

which together with the central charge and (1.3) yields the Bekenstein-Hawking Entropy of the KNAdS black hole. It is interesting to note that the lowest Virasoro eigen-mode satisfies

\[ Q_0 = GM^2_{rr}, \]  

(1.14)

where \( M^2_{rr} \) is the irreducible mass of the KNAdS black hole and agrees with (1.9). This suggests a possible addition to the AdS/CFT dictionary, that the eigen-value of the CFT’s Hamiltonian is proportional to the irreducible mass of its black hole dual.
An additional distinct example of reducing near horizon dynamics to two dimensional physics was introduced by Castro and Larsen [76] by recasting the NHEK [77] into two dimensional fields $g^{(2)}_{\mu\nu}, \psi$ and $A_{\mu}$
\[
ds^2 = \frac{1 + \cos^2 \theta}{2} \left[ ds^2_{(2)} + e^{-2\psi} \ell^2 d\theta^2 \right] + e^{-2\psi} \ell^2 \frac{2 \sin^2 \theta}{1 + \cos^2 \theta} (d\phi + A)^2
\]
and evaluating the Einstein-Hilbert action over (1.15) and integrating out angular degrees of freedom. A careful study of the asymptotic boundary currents of the resulting functional, within a well defined variational principle, yields a one dimensional quadratic two form with transformation Law:
\[
\delta_{\epsilon + \Lambda} T_{tt} = T_{tt} \epsilon'(t) + \xi(t) T_{tt}' + \frac{12J}{12} \xi'''(t) + O \left( e^{-\rho/\ell} \right)
\]
in Gauss normal coordinates. This closely resembles the transformation law for the energy momentum tensor of a conformal field with center $c = 12J$, in agreement with the Kerr/CFT correspondence [38]. However, a computation of the asymptotic symmetry algebra within this work was not performed.

We will proceed in Section 2, by reviewing the relevant near-NHEK geometry and introduce the finite mass gauge by solving the vacuum Einstein field equations for generic four dimensional Einstein-Hilbert Theory with a specific initial ansatz. In Section 3 we perform the RW dimensional reduction to the geometry of interest and derive a two dimensional near horizon Liouville-like CFT in terms of the resulting RW2DA fields. Analysis of the asymptotic symmetries of this theory leads to black hole thermodynamics of the near-extremal Kerr throat. Next, in Section 4, we derive a normalized AdS$_2$ effective action via off shell analysis of the Einstein-Hilbert action directly within the finite mass gauge. The resulting boundary counter term contribution to the effective action yields a precise one dimensional quadratic two form with calculable central charge, reproducing the results of the previous section. Finally in Section 5 we close with a discussion of our results and hint towards future work.

## 2 Geometry

We are interested in studying four dimensional vacuum solutions which in the near horizon have the form:
\[
ds^2 = K_1(\theta) g^{(2)}_{\mu\nu} dx^\mu dx^\nu + K_2(\theta) e^{-2\varphi} d\theta^2 + K_3(\theta) e^{-2\varphi} [d\phi + A]^2,
\]
The above two dimensional field splitting provides a robust platform for constructing CFT duals for relevant classical spacetimes with non vanishing surface gravity (near-extremal), by analyzing the ASG of the Kaluza-Klein fields $g^{(2)}_{\mu\nu}, A$ and $\varphi$. The NHEK is a four dimensional vacuum solution, which is derived by taking the extremal near horizon limit:
\[
r = GM + \lambda U, \ t' = \frac{t}{\lambda}, \ \phi' = \phi + \frac{t}{2GM\lambda}, \ \lambda \to \infty
\]
of the generic Kerr metric:
\[
ds^2_{\text{Kerr}} = -\frac{\Sigma \Delta}{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta} dt'^2 + \Sigma \left[ \frac{dr^2}{\Delta} + d\theta^2 \right] + \left( \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \right) \sin^2 \theta \left[ d\phi' + \frac{2rGMa}{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta} dt' \right]^2,
\]
where
\[
\begin{align*}
\Sigma &= r^2 + a^2 \cos^2 \theta, \\
\Delta &= (r - r_+) (r - r_-), \\
r_\pm &= GM \pm \sqrt{(GM)^2 - a^2}, \\
a &= \frac{J}{M},
\end{align*}
\]
which yields:
\[
\begin{align*}
ds^2_{NHEK} &= \frac{1 + \cos^2 \theta}{2} \left[ -\frac{U^2}{\ell^2} dt^2 + \frac{\ell^2}{U^2} du^2 + \ell^2 d\theta^2 \right] + \ell^2 \frac{2 \sin^2 \theta}{1 + \cos^2 \theta} \left( d\phi + \frac{U}{\ell^2} dt \right)^2. 
\end{align*}
\]
\hspace{1cm} (2.5)

The above metric is of the form (2.1), and may be tuned to near-extremality via the finite temperature gauge yielding:
\[
\begin{align*}
ds^2_{near-NHEK} &= \frac{1 + \cos^2 \theta}{2} \left[ -\frac{U^2 - \epsilon^2}{\ell^2} dt^2 + \frac{\ell^2}{U^2 - \epsilon^2} du^2 + \ell^2 d\theta^2 \right] + \ell^2 \frac{2 \sin^2 \theta}{1 + \cos^2 \theta} \left( d\phi + \frac{U}{\ell^2} dt \right)^2, 
\end{align*}
\]
\hspace{1cm} (2.6)

where \( \epsilon = \frac{1}{2\ell} (r_+ - r_-) \) is a finite excitation above extremality and \( \ell^2 = 2G^2 M^2 \).

### 2.1 Finite Mass Gauge

To aid in our CFT dual construction we want to endow the near-NHEK geometry with a finite ADM mass and angular momentum parameter \( M \) and \( a \). We will do this by solving the Einstein Field Equations directly for specific boundary conditions, as opposed to implementing a parameter tuning process in the transformations of (2.2). Starting with (2.1) as our ansatz, we impose the following symmetry conditions:

#### Constraints 1 (Black Hole Symmetries).

- The horizon is topologically \( S^2 \), i.e.
  \[
  K_2 K_3 = \sin^2 \theta \text{ and } K_1 = K_2.
  \]
  \hspace{1cm} (2.7)

- The metric is maximally isometric in \( t \) and \( \phi \), which implies
  \[
  g^{(2)}_{\mu\nu} = g^{(2)}_{\mu\nu}(r), \quad \varphi = \varphi(r) \text{ and } A = A_\mu(r) dx^\mu.
  \]
  \hspace{1cm} (2.8)

- The metric is axially symmetric in four dimensions and spherically symmetric in two:
  \[
  ds^2 = g^{(2)}_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{dr^2}{f(r)} \text{ and } A = A_t(r) dt.
  \]
  \hspace{1cm} (2.9)

Implementing these conditions within the four dimensional vacuum Einstein field equations we
obtain the relatively simple set of coupled differential equations:

\[
\frac{\varphi'(r)K_2'(\theta)}{K_2(\theta)} = 0
\]

\[
K_2(\theta) (-2e^{-2\varphi(r)} f'(r) \varphi'(r) + f(r) (4e^{-2\varphi(r)} \varphi'(r)^2 - 2e^{-2\varphi(r)} \varphi''(r)) - 2) + K_2''(\theta) - 3 \cot(\theta) K_2'(\theta) = 0
\]

\[
\sin(\theta) (e^{-4\varphi(r)} \sin^2(\theta) A_1'(r)^2 + K_2'(\theta)^2) + \sin(\theta) K_2(\theta)^2 (-2e^{-2\varphi(r)} f'(r) \varphi'(r) + f(r) (4e^{-2\varphi(r)} \varphi'(r)^2 - 2e^{-2\varphi(r)} \varphi''(r)) - 2) - K_2(\theta) (\sin(\theta) K_2''(\theta) + \cos(\theta) K_2'(\theta)) = 0
\]

\[
e^{-6\varphi(r)} \sin^2(\theta) A_1'(r)^2 - e^{-4\varphi(r)} K_2(\theta)^2 f''(r) - e^{-2\varphi(r)} (K_2(\theta)^2 (2f(r) (4e^{-2\varphi(r)} \varphi'(r)^2 - 2e^{-2\varphi(r)} \varphi''(r)) - 2e^{-2\varphi(r)} f'(r) \varphi'(r)) - K_2'(\theta)^2 + K_2(\theta) (K_2''(\theta) + \cot(\theta) K_2'(\theta)) + 4f(r) e^{-4\varphi(r)} K_2(\theta)^2 \varphi'(r)^2 = 0
\]

(2.10)

where prime denotes derivation with respect to the argument of the given function. A general solution to the above field equations is given by:

\[
\begin{align*}
\varphi(r) &= \frac{\varphi}{C_1(1-\cos(\theta))^{3/2} \sqrt{\cos^2(\theta)-1}} - \frac{C_2(1-\cos(\theta)) \cos(\theta) \sqrt{\cos^2(\theta)-1}}{(\cos(\theta)-1) \sqrt{\cos(\theta)+1}} \\
K_2(\theta) &= \frac{\sqrt{C_1^2 - 4C_1C_2}}{e^{-2\varphi}} + C_3 \\
A_1(r) &= \frac{r^2}{e^{-2\varphi}} + rC_5 + C_4 \\
f(r) &= \frac{r^2}{e^{-2\varphi}} + rC_5 + C_4
\end{align*}
\]

(2.11)

where \(C_i\)'s are integration constants and the full line element is given in (2.1) and reads:

\[
ds^2 = \left( \frac{C_1(1-\cos(\theta))^{3/2} \sqrt{\cos^2(\theta)-1}}{\sqrt{\cos(\theta)+1}} - \frac{C_2(1-\cos(\theta)) \cos(\theta) \sqrt{\cos^2(\theta)-1}}{(\cos(\theta)-1) \sqrt{\cos(\theta)+1}} \right) dr^2 + \left( -\left( \frac{r^2}{e^{-2\varphi}} + rC_5 + C_4 \right) dt^2 + \frac{1}{\sqrt{e^{-2\varphi}} + rC_5 + C_4} d\theta^2 \right) + \frac{\sin^2(\theta)}{\sqrt{\cos(\theta)+1}} - \frac{C_2(1-\cos(\theta)) \cos(\theta) \sqrt{\cos^2(\theta)-1}}{(\cos(\theta)-1) \sqrt{\cos(\theta)+1}} e^{-2\varphi} \left[ \frac{r^2 \sqrt{C_1^2 - 4C_1C_2}}{e^{-2\varphi}} + C_3 \right] dt^2
\]

(2.12)

Next, we impose the final set of boundary conditions to fix the integration constants in the solution above:

**Constraints 2.** [Finite Mass Gauge]

- The inner and outer horizons are located at:
  \[
f(r_{\pm}) = 0 \text{ and } r_{\pm} = GM \pm \sqrt{(GM)^2 - a^2}.
\]

(2.13)

- The ADM mass is non zero and equal to the parameter \(M\):

- The total angular momentum is given by \(J = \frac{a^2 + r^2}{2GM}\).

Applying these conditions yields the final solution to (2.10)

\[
\begin{align*}
& e^{-2\varphi} = r_+^2 + a^2 \\
& K_2(\theta) = \frac{1 + \cos^2 \theta}{r_+^2 + a^2} \\
& A_t(r) = \frac{-2GM}{r_+^2 + a^2} \\
& f(r) = \frac{r^2 - 2rGM + a^2}{r_+^2 + a^2},
\end{align*}
\]
(2.14)

with line element

\[
\begin{align*}
& ds^2 = 1 + \frac{\cos^2 \theta}{2} \left[ -\frac{r^2 - 2rGM + a^2}{r_+^2 + a^2} dt^2 + \frac{r_+^2 + a^2}{r^2 - 2rGM + a^2} dr^2 + \left( \frac{r - 2GM}{r_+^2 + a^2} \right) d\theta^2 \right] \\
& \quad + \frac{2\sin^2 \theta}{1 + \cos^2 \theta} \left( r_+^2 + a^2 \right) \left[ d\phi + \left( \frac{r - 2GM}{r_+^2 + a^2} \right) dt \right]^2.
\end{align*}
\]
(2.15)

The above line element clearly exhibits global $AdS_2 \times S^2$ topology and is diffeomorphic to (2.5) and (2.6) as we will demonstrate shortly. However, the above line element is written in Boyer-Lindquist type coordinates and exhibits two physical parameters $M$ and $a$, which will be useful for tuning purposes in our CFT construction.

## 2.2 Surface Gravity

To compute the surface gravity of the gauged near $-NHEK$ $(gnNHEK)$ we will exploit the fact that it is maximally isometric with respect to the coordinates $t$ and $\phi$ and define the general Killing vector:

\[
\xi = \xi_t(t) + \Omega_H \xi_\phi(\phi),
\]
(2.16)

where $\Omega_H$ is the minimum of the function

\[
\frac{d\phi}{dt} = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \sqrt{\left( \frac{g_{t\phi}}{g_{\phi\phi}} \right)^2 - \frac{g_{tt}}{g_{\phi\phi}}}
\]
(2.17)

evaluated on the horizon $r_+$. Given this above Killing vector and making use of the geodetic equation to rearrange Frobenius’ theorem

\[
\nabla_{[\alpha} \xi_{\mu} \xi_{\nu]} = 0
\]
(2.18)

for hypersurface orthogonal congruences of null generators, we obtain:

\[
\kappa^2 = -\frac{1}{2} \nabla^\mu \xi^\nu \nabla_{\mu} \xi_{\nu} |_{r_+}.
\]
(2.19)

Evaluating this over the connection of (2.15) yields:

\[
\kappa_{gnNHEK} = \frac{1}{2} f'(r_+) = \frac{r_+ - GM}{r_+^2 + a^2},
\]
(2.20)

which leaves us with a finite non zero value, but may be tuned to zero for the case when $a \to GM$ i.e. when we approach extremality. We will also note the horizon area of the $gnNHEK$ black hole, which comes from evaluating the integral

\[
A = \int d^2 x \sqrt{\gamma} = 4\pi \left( r_+^2 + a^2 \right),
\]
(2.21)

where $\gamma_{ab}$ are the metric degrees of freedom left over in (2.15) after setting $dr = dt = 0.$
2.3 *NHEK*-Map

As mentioned above, the solution (2.15) is a *near*- NHEK solution which can be mapped into (2.5) via a similar coordinate relationship between $U$ and $r$ as in [77]. Starting with (2.15) and applying the coordinate redefinitions:

$$r = GM + \lambda U, \; t = \frac{i}{\lambda}, \; \phi = \tilde{\phi} + \frac{\bar{i}}{2GM\lambda}$$

and in stead of taking the limit as $\lambda \to 0$ we set $a \to GM$ resulting in the NHEK solution:

$$ds^2 = \frac{1 + \cos^2 \theta}{2} \left[ -\frac{U^2}{\ell^2} dt^2 + \frac{\ell^2}{U^2} dU^2 + \ell^2 d\theta^2 \right] + \ell^2 \frac{2 \sin^2 \theta}{1 + \cos^2 \theta} \left( d\tilde{\phi} + \frac{U}{\ell^2} dt \right)^2.$$  \hspace{1cm} (2.23)

In fact, as discussed in [80–82], there always exists a coordinate mapping between a *near*- NHEK and a NHEK type solution as evident in the form of their Kretschmann invariants:

$$R_{\mu\nu\beta\gamma}R^{\mu\nu\beta\gamma} = \frac{1536 \sin^2 \theta (-52 \cos(2\theta) + \cos(4\theta) - 45)}{(\cos(2\theta) + 3)^6 \ell^4} \begin{cases} \ell^2 = 2G^2M^2 & \text{NHEK} \\ \ell^2 = 2G^2M^2 & \text{near} - \text{NHEK} \\ \ell^2 = r_+^2 + a^2 & \text{gnNHEK} \end{cases}$$

Notice, that for the choice $a = GM$ all invariants exhibit $\ell^2 = 2G^2M^2$, i.e. the extremal limit of (2.15).

3 Quantum Fields in *gnNHEK* Spacetime

We will now study the resulting near horizon matter theory via the RW dimensional reduction procedure. Our goal, for this section, will be to apply our previous techniques from [34,35] to study the resulting thermodynamics of the *gnNHEK* within an AdS$_2$/CFT$_1$ formalism.

3.1 RW Dimensional Reduction

In our initial ansatz leading to the *gnNHEK* solution we assumed a specific decomposition of our four dimensional spacetime into two dimensional black hole and matter fields. However, we have not shown that these fields are the correct RW2DA useful in a holographic study of the quantum spacetime in the near horizon regime.

Let us consider a single free scalar field in the background of (2.15) with functional:

$$S_{free} = \frac{1}{2} \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_{\mu} \phi \partial^\mu \phi \right)$$

$$= -\frac{1}{2} \int d^4x \left( \partial_{\mu} \left( \sqrt{-g} \partial^\mu \partial_{\nu} \phi \right) \right) \phi$$

$$= -\frac{1}{2} \int d^4x \left( \partial_t \left( -\sin \theta \left( a^2 + r_+^2 \right) \frac{r_+^2 + a^2}{r^2 - 2rGM + a^2} \partial_t \right) + \partial_r \left( \sin \theta \left( a^2 + r_+^2 \right) \frac{r^2 - 2rGM + a^2}{r_+^2 + a^2} \partial_r \right) + \partial_{\theta} \left( \sin \theta \left( a^2 + r_+^2 \right) \frac{r^2 - 2rGM + a^2}{r_+^2 + a^2} \partial_{\theta} \right) + \frac{\cos(2\theta) + 3}{16 \sin \theta} \partial_{\phi} \right)$$

$$+ 2\partial_t \left( \sin \theta \left( a^2 + r_+^2 \right) \frac{r^2 - 2rGM + a^2}{r_+^2 + a^2} \frac{r^2 - 2rGM + a^2}{r_+^2 + a^2} \partial_{\phi} \right) \phi.$$  \hspace{1cm} (3.1)
The above functional is reduced to a two dimensional theory by expanding the four dimensional scalar field in terms of spherical harmonics

\[ \varphi(t, r, \theta, \phi) = \sum_{lm} \varphi_{lm}(r, t) Y_{lm}(\theta, \phi), \]  

(3.2)

where \( \varphi_{lm} \) is a complex interacting two dimensional scalar field, and integrating out angular degrees of freedom. Transforming to tortoise coordinates defined as:

\[ \frac{dr^*}{dr} = \frac{1}{f(r)} \]  

(3.3)

and considering the region very close to \( r_+ \) we find the two dimensional action is much reduced since all interaction, mixing and potential terms (\( \sim l(l + 1) \ldots \)) are weighted by a factor of \( f(r(r^*)) \sim e^{2kr^*} \), which vanishes exponentially fast as \( r \to r_+ \). This leaves us with an infinite collection of massless charged scalar fields in the very near horizon region, with \( U(1) \) gauge charge \( m \) and remnant functional:

\[ S = -\frac{r^2 + a^2}{2} \int d^2x \varphi_{lm}^* \left[ \frac{1}{f(r)} (\partial_t - imA_t)^2 + \partial_r f(r) \partial_r \right] \varphi_{lm}. \]  

(3.4)

Thus, we arrive at the RW2DA for the \( ngNHEK \) solution given by:

\[ g^{(2)}_{\mu\nu} = \begin{pmatrix} -f(r) & 0 \\ 0 & \frac{1}{f(r)} \end{pmatrix} \]  

(3.5)

and \( U(1) \) gauge field

\[ A = A_t dt. \]  

(3.6)

Given the initial ansatz (2.1), it is not surprising that the only relevant physical fields in the region \( r \sim r_+ \) are the RW2DAs above and makes the holographic statement that we may learn much about the quantum nature of spacetime in the near horizon regime via the semiclassical analysis of \( g^{(2)}_{\mu\nu}, A \) and \( \varphi_{lm} \).

### 3.2 Effective Gravitational Action and Asymptotic Symmetries

We would like to interpret (3.4) as a useful action for gravity in the near horizon of classical four dimensional spacetime. This can be done by only considering the \( s-wave \) contribution and making a field redefinition rendering the scalar field unitless [84–87]. The \( s-wave \) approximation is sensible in this scenario, since we will interpret \( \varphi_{lm} \) as a component of the gravitational field and hence it should be real and unitless. Most of the interesting gravitational dynamics seem to be contained in this region or approximation [88]². Motivated by these arguments we make the field redefinition

\[ \varphi_{00} = \sqrt{\frac{6}{G}} \psi, \]  

(3.7)

where \( \psi \) is now unitless and the \( \sqrt{6} \) was chosen to recover the Einstein coupling \( \frac{1}{16\pi G} \) in the quantum gravitational effective action of (3.4) within the \( s-wave \) approximation. Applying the field redefinition (3.7) to (3.4) yields:

\[ S^{(2)}[\psi, g] = \frac{3(r^2 + a^2)}{G} \int d^2x \sqrt{-g^{(2)}} \psi \left[ D_\mu \left( \sqrt{-g^{(2)}} g^{(2)}_{\mu\nu} D_\nu \right) \right] \psi, \]  

(3.8)

²In [35] it was shown that \( \varphi_{lm} \) dies exponentially fast in time by analyzing the asymptotic behavior of its field equation. However we find the statement relating \( \varphi_{lm} \) to a real gravitational field component, a stronger justification to neglect higher order terms in \( l \) and \( m \).
where $D_\mu$ is the gauge covariant derivative. In addition to redressing the scalar field, our choice of field redefinition has also rendered the effective coupling unitless, hinting towards a finite quantum theory. The effective action of this quantum theory, which may be extracted via zeta-function regularization of the functional determinant in (3.8), is given by the sum of two functionals [14,89]:

$$\Gamma = \Gamma_{\text{grav}} + \Gamma_{U(1)},$$

(3.9)

where

$$\Gamma_{\text{grav}} = \frac{(r_+^2 + a^2)}{16 \pi G \Xi} \int d^2x \sqrt{-g^{(2)}} \frac{1}{\Box_{g^{(2)}}} R^{(2)}$$

and

$$\Gamma_{U(1)} = \frac{3e^2(r_+^2 + a^2)}{\pi G \Xi} \int \mathcal{F} \frac{1}{\Box_{g^{(2)}}} \mathcal{F}.$$  

(3.10)

Next, we introduce the auxiliary scalars $\Phi$ and $B$ satisfying:

$$\Box_{g^{(2)}} \Phi = R$$

and

$$\Box_{g^{(2)}} B = \epsilon^{\mu \nu} \partial_\mu A_\nu,$$

(3.11)

which transforms the functional (3.9) into a Liouville CFT of the form:

$$S_{\text{NHCFT}} = \frac{(r_+^2 + a^2)}{16 \pi G \Xi} \int d^2x \sqrt{-g^{(2)}} \left\{ -\Phi \Box_{g^{(2)}} \Phi + 2\Phi R^{(2)} \right\}$$

$$+ \frac{3e^2(r_+^2 + a^2)}{\pi G \Xi} \int d^2x \sqrt{-g^{(2)}} \left\{ -B \Box_{g^{(2)}} B + 2B \frac{\epsilon^{\mu \nu}}{\sqrt{-g^{(2)}}} \partial_\mu A_\nu \right\}.$$  

(3.12)

Now, we turn our attention to computing the ASG of (3.8). The behavior of the RW2DA fields at large $r$ is defined by

$$g^{(0)}_{\mu \nu} = \begin{pmatrix}
-\frac{r^2}{\ell^2} + \frac{2GM}{\ell^2} - \frac{r^2}{\ell^2} + O \left( \left( \frac{1}{r} \right)^3 \right) & 0 \\
0 & \frac{r^2}{\ell^2} + O \left( \left( \frac{1}{r} \right)^3 \right)
\end{pmatrix},$$

(3.13)

$$A^{(0)}_t = \frac{r}{\ell^2} - \frac{2GM}{\ell^2} + O \left( \left( \frac{1}{r} \right)^3 \right),$$

(3.14)

which yield an asymptotically $AdS_2$ configuration with Ricci Scalar, $R = -\frac{4}{\ell^2} + O \left( \left( \frac{1}{r} \right)^1 \right)$, where $\ell^2 = r_+^2 + a^2$. In addition, we impose the following metric and gauge field fall-off conditions:

$$\delta g_{\mu \nu} = \begin{pmatrix}
O \left( \left( \frac{1}{r} \right)^3 \right) & O \left( \left( \frac{1}{r} \right)^0 \right) \\
O \left( \left( \frac{1}{r} \right)^0 \right) & O \left( \left( \frac{1}{r} \right)^0 \right)
\end{pmatrix}$$

and

$$\delta A = O \left( \left( \frac{1}{r} \right)^0 \right),$$

(3.15)

which imply the following set of asymptotic metric preserving diffeomorphisms:

$$\xi_n = \xi_1(r) \frac{e^{ink(\pm r^*)}}{\kappa} \partial_t + \xi_2(r) \frac{e^{ink(\pm r^*)}}{\kappa} \partial_r,$$

(3.16)

where $r^*$ is the tortoise coordinate defined by $dr^* = \frac{1}{f(r)} dr$,

$$\xi_1 = Cr e^{inkr^*}, \quad \xi_2 = \frac{irC(r - GM)}{\kappa n (r^2 - 2rGM + a^2)}.$$  

(3.17)
Quantum Fields in gnNHEK Spacetime

$C$ is an arbitrary normalization constant and $\kappa$ is the surface gravity of the gnNHEK black hole. Under this set of diffeomorphisms the gauge field transforms as:

$$\delta_\xi A_\mu = \left( \mathcal{O} \left( \frac{1}{r} \right)^0, \mathcal{O} \left( \frac{1}{r} \right)^1 \right) \quad (3.18)$$

and thus $\delta_\xi$ may be elevated to a total symmetry of the action, i.e.

$$\delta_\xi \rightarrow \delta_{\xi + \Lambda}, \quad (3.19)$$
in accordance with (3.15). Switching to light cone coordinates $x^\pm = t \pm r$, we see that the set $\xi_\pm^\pm$ is well behaved on the $r \rightarrow \infty$ boundary and form a centerless Witt or $Diff(S^1)$ subalgebra:

$$i \{ \xi_\pm^m, \xi_\pm^n \} = (m - n) \xi_\pm^{m+n}. \quad (3.20)$$

3.3 Energy Momentum and The Virasoro algebra

We define the energy momentum tensor and $U(1)$ current of (3.12) in their usual ways:

$$\langle T_{\mu\nu} \rangle = \frac{2}{\sqrt{-g^{(2)}}} \frac{\delta S_{NHCFT}}{\delta g^{(2)\mu\nu}}$$

$$= \frac{r^2 + a^2}{8\pi G} \left\{ \partial_\mu \Phi \partial_\nu \Phi - 2 \nabla_\mu \partial_\nu \Phi + g^{(2)\mu\nu} \left[ 2R^{(2)} - \frac{1}{2} \nabla_\alpha \Phi \nabla^\alpha \Phi \right] \right\}$$

$$+ \frac{6e^2(r^2 + a^2)}{\pi G} \left\{ \partial_\mu B \partial_\nu B - \frac{1}{2} g_{\mu\nu} \partial_\alpha B \partial^\alpha B \right\} \quad (3.21)$$

$$\langle J^\mu \rangle = \frac{1}{\sqrt{-g^{(2)}}} \frac{\delta S_{NHCFT}}{\delta A_\mu} = \frac{6e^2(r^2 + a^2)}{\pi G} \frac{1}{\sqrt{-g^{(2)}}} \epsilon^{\mu\nu} \partial_\nu B$$

Next, solving the equation of motions for the auxiliary fields:

$$\Box g^{(2)} \Phi = R^{(2)}$$

$$\Box g^{(2)} B = \epsilon^{\mu\nu} \partial_\mu A_\nu \quad (3.22)$$

using the metric (3.5) and gauge field (3.6) and employing modified Unruh Vacuum boundary conditions (MUBC) [90]

$$\left\{ \begin{array}{l} \langle T_{++} \rangle = \langle J_+ \rangle = 0 \quad r \rightarrow \infty, \quad \ell \rightarrow \infty \\ \langle T_{--} \rangle = \langle J_- \rangle = 0 \quad r \rightarrow r_+ \end{array} \right., \quad (3.23)$$

we determine all relevant integration constants of (3.21) and (3.22). For large $r$ and to $\mathcal{O}(\frac{1}{r})^2$, which we will denote as the single limit $r \rightarrow \infty$ in the remainder of this section, the resulting energy momentum tensor is dominated by one holomorphic component, $\langle T_{--} \rangle$. We are interested in the response of the energy momentum tensor and the $U(1)$ current to a total symmetry $\delta_{\xi_+ + \Lambda}$, which after expanding in terms of the boundary fields (3.13) and (3.14) we obtain:

$$\left\{ \begin{array}{l} \delta_{\xi_- + \Lambda} \langle T_{--} \rangle = \xi_-^n \langle T_{--} \rangle' + 2 \langle T_{--} \rangle' \langle \xi_- \rangle' + \frac{r^2_+ + a^2}{4\pi G} \langle \xi_- \rangle'' + \mathcal{O} \left( \frac{1}{r} \right)^3 \\ \delta_{\xi_- + \Lambda} \langle J_- \rangle = \mathcal{O} \left( \frac{1}{r} \right)^3 \end{array} \right. \quad (3.24)$$

$^3$Large $r$ behavior will be synonymous with large $x^+$ behavior.
Quantum Fields in $\text{gNHEK}$ Spacetime

From the above we see that $\langle T_{\mu\nu} \rangle$ transforms asymptotically as the energy momentum tensor of a one dimensional CFT with center:

$$\frac{c}{24\pi} = \frac{r_+^2 + a^2}{4\pi G} \Rightarrow c = \frac{3A}{2\pi G}. \quad (3.25)$$

We should also note that the above central charge is in congruence with the 2-dimensional conformal/trace anomaly [91]:

$$\langle T_{\mu\mu} \rangle = -\frac{c}{24\pi} R^{(2)} \quad (3.26)$$

Next, we define the quantum generators via the conserved charge:

$$Q_n = \lim_{r \to \infty} \int dx^- \langle T_{\mu} \rangle \xi_n, \quad (3.27)$$

Whose algebraic structure is revealed by computing its response to a total symmetry, while compactifying the $x^-$ coordinate to a circle from $0 \to 2\pi/\kappa$:

$$\delta_{\xi_{m+n}} Q_n = [Q_m, Q_n] = (m-n)Q_n + \frac{c}{12} m (m^2 - 1) \delta_{m+n,0}, \quad (3.28)$$

from the above we see that the quantum symmetry generators form a centrally extended Virasoro algebra with regulated/normalized zero-mode $Q_0 = \frac{A}{16\pi G}$. 

### 3.4 $\text{AdS}_2/CFT_1$ and Entropy of Near Extremal Kerr

By employing the finite mass gauge, we have now shown that the near-extremal Kerr throat is holographically dual to a CFT with center

$$c = \frac{3A}{2\pi G} \quad (3.29)$$

and lowest Virasoro eigen-mode

$$Q_0 = \frac{A}{16\pi G}. \quad (3.30)$$

We are now free to use the above results within the statistical Cardy Formula (1.3):

$$S = 2\pi \sqrt{\frac{c Q_0}{6}} = \frac{A}{4G} = 2\pi \left( GM^2 + \sqrt{G^2 M^4 - (GJ)^2} \right), \quad (3.31)$$

which is in agreement with the area law (1.8), however derived without mixing results computed separately at near-extremality and extremality. Also the $c$ and $Q_0$ extend smoothly to extremality in the limit as $a \to GM$ yielding:

$$\lim_{a \to GM} c = 12J \text{ and } \lim_{a \to GM} Q_0 = J/2 \quad (3.32)$$

which is the same value of the left central charge obtained in the Kerr/CFT correspondence [38] and together in the statistical Cardy formula the above values reproduce the extremal Kerr entropy. In addition, our derived zero-mode in (3.29) is in accordance with [35] and relates to the irreducible mass of its black hole dual via:

**Assertion 1.** The lowest Virasoro eigen-mode of a quantum CFT is proportional to the irreducible mass of its black hole dual via the form

$$Q_0 = GM_{\text{prim}}^2 \quad (3.33)$$

This may be a general statement with a large avenue of application, though we are not aware of a rigorous proof at this time.
3.5 Near Extremal Black Hole Temperature

To extract the temperature of the $gnNHEK$ horizon, we will focus on the gravitational part of (3.12), i.e.

\[ S_{\text{grav}} = \frac{r^2 + a^2}{16\pi G} \int d^2x \sqrt{-g^{(2)}} \left\{ -\Phi \Box_{g^{(2)}} \Phi + 2\Phi R^{(2)} \right\}. \]  

(3.34)

The energy momentum is given by:

\[ \langle T_{\mu\nu} \rangle = \frac{2}{\sqrt{-g^{(2)}}} \frac{\delta S_{\text{NHCFT}}}{\delta g^{(2)\mu\nu}} \]

\[ = -\frac{r^2 + a^2}{8\pi G} \left\{ \partial_\mu \Phi \partial_\nu \Phi - 2\nabla_\mu \partial_\nu \Phi + g^{(2)\mu\nu} \left[ 2R^{(2)} - \frac{1}{2} \nabla_\alpha \Phi \nabla^\alpha \Phi \right] \right\}. \]  

(3.35)

and following the steps (3.21) through (3.23), however focusing on the horizon limit $r \to r_+$, we are left with one holomorphic component given by:

\[ \langle T_{++} \rangle = -\frac{r^2 + a^2}{32\pi G} f' \left( \frac{r^+}{r_+} \right)^2. \]  

(3.36)

This is precisely the value of the Hawking Flux of the $gnNHEK$ metric weighted by the central charge (3.25):

\[ \langle T_{++} \rangle = cHF = -\frac{\pi}{12} \left( T_H \right)^2 \Rightarrow, \]  

(3.37)

with Hawking temperature [18,92]:

\[ T_H = \frac{f' \left( \frac{r^+}{r_+} \right)}{4\pi}. \]  

(3.38)

This is an interesting result, which hints that the $AdS_2/CFT_1$ correspondence constructed here contains information about both black hole entropy and temperature. Though the $\langle T_{++} \rangle$ component in the horizon limit is not precisely the Hawking Flux of the four dimensional parent black hole, but given prior knowledge of the central extension, it is possible to read off or extract the relevant information from the correspondence.

4 $gnNHEK$ and $AdS_2$ Quantum Gravity

We will now turn our attention to the extremal case $a = GM$ in (2.15) for which the surface gravity vanishes identically. In other words, in contrast to the previous section we are interested in a formalism allowing $a = GM$ from the outset of the calculation, which is where the finite mass gauge will come in handy since in this limit the line element (2.15) still exhibits an interesting Boyer-Lindquist structure. This specific case is cumbersome, as the Charge regulators of the previous section depended on $\kappa$ thus we will follow the seminal work of [76,93–97]. The convenient decomposition of (2.15) into two dimensional fields should allow for an off-shell analysis of the Einstein-Hilbert action and integrating out angular degrees of freedom should leave us with an alternate effective two dimensional theory. As already seen in the previous section, the resulting effective theory should hold computational significance relevant to the near horizon regime of the near-extremal and extremal Kerr black hole.
4.1 Bulk Action

Writing (2.15) in terms of two dimensional fields and making the field redefinition \( e^{-2\varphi(r)} \rightarrow (r^2_+ + a^2) \) we have

\[
ds^2 = \frac{1 + \cos^2 \theta}{2} \left[ g^{(2)}_{\mu\nu} dx^\mu dx^\nu + e^{-2\varphi(r^2_+ + a^2)} d\theta^2 \right] + \frac{2 \sin^2 \theta}{1 + \cos^2 \theta} e^{-2\varphi(r^2_+ + a^2)} [d\phi + A]^2
\]

and substituting into the Einstein-Hilbert action

\[
S_{EH} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} R
\]

and integrating out angular degrees of freedom we obtain

\[
S = \frac{2\pi}{16\pi G} \int d^2x \left\{ 2e^{-4\varphi(r)}(r^2_+ + a^2)^2 A_\nu A^{\nu} - 2e^{-2\varphi(r)}(r^2_+ + a^2) \left[ (f''(r) - 4f'(r)\varphi'(r) + f(r) \left( 6\varphi'(r)^2 - 4\varphi''(r) \right) \right] + 2 \right\},
\]

which may be recast into covariant form

\[
S = \frac{(r^2_+ + a^2)}{4G} \int d^2x \sqrt{-g^{(2)}} \left\{ e^{-2\varphi} R^{(2)} + \frac{1}{(r^2_+ + a^2)} + 2\nabla_\mu e^{-\varphi} \nabla^\mu e^{-\varphi} - \frac{(r^2_+ + a^2)}{2} e^{-4\varphi} R^{2} \right\} + \int d^2x \{ \text{Total Derivative Terms} \cdots \}.
\]

Setting \( \ell^2 = (r^2_+ + a^2) \) and dropping total derivatives we obtain the AdS2 action:

\[
S_{AdS_2} = \frac{\ell^2}{4G} \int d^2x \sqrt{-g^{(2)}} \left\{ e^{-2\varphi} R^{(2)} + \frac{1}{\ell^2} + 2\nabla_\mu e^{-\varphi} \nabla^\mu e^{-\varphi} - \frac{\ell^2}{2} e^{-4\varphi} R^2 \right\}.
\]

This theory is a classical effective gravity theory and exhibits regularization via a suitable choice of boundary counterterms. This process is equivalent to renormalizing the theory on the CFT side to ensure finite energy momentum and current [97]. (4.5) exhibits three equations of motion obtained by variation with respect to \( g^{(2)}_{\mu\nu}, A \) and \( \varphi \). Remembering that for any two dimensional Riemannian metric \( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \) we obtain:

\[
T_{\mu\nu} = 0 \quad \text{Einstein,} \quad (4.6)
\]

\[
\nabla_\mu F^{\mu\nu} = 0 \quad \text{Maxwell,} \quad (4.7)
\]

\[
R^{(2)} - e^{-2\varphi} \ell^2 F^2 = 0 \quad \text{constant Scalar.} \quad (4.8)
\]

We will be interested in constructing the boundary counterterms to our above derived AdS2 action by considering general solutions to (4.6)-(4.8). A general set, as written in Gauss normal form, is given by

\[
ds^2_{(2)} = e^{-2\varphi} dp^2 + \gamma_{tt} dt^2 \quad \text{and} \quad A = A_\rho d\rho + A_t dt,
\]

where

\[
\gamma_{tt} = -\frac{1}{4} e^{-2\varphi} \left( e^{\rho/t} - h(t) e^{-\rho/t} \right)^2,
\]

\[
A_t = \frac{1}{2t} e^{\rho/t} \left( 1 - \sqrt{h(t)} e^{-\rho/t} \right)^2,
\]

\[
A_\rho = 0,
\]
and includes a free function \( h(t) \). The \( KLBH \) solution is recovered for the choice

\[
h(t) = \frac{G^2 M^2 - a^2}{\ell^2}
\]

and coordinate redefinition:

\[
(r - GM) = \frac{\ell}{2} e^{\rho/\ell} \left( 1 + \frac{G^2 M^2 - a^2}{\ell^2} e^{-2\rho/\ell} \right).
\]

The extremal case corresponds to the case \( h(t) = 0 \).

### 4.2 Boundary Counterterms

Following \([76,93,94]\), we now determine the boundary counterterms within a well defined variational principle for (4.5). The boundary contribution has the local form:

\[
S_{ct} = \frac{\ell^2}{2G} \int dt \sqrt{-\gamma} \left\{ e^{-2\varphi} K + \alpha e^{-\varphi} + \beta e^{-3\varphi} A_a A^a \right\},
\]

where the first term above is just the standard Gibbons-Hawking term for extrinsic curvature \( K = \frac{1}{2} \gamma^{tt} \sqrt{g_{tt}} \partial_t \gamma_{tt} = e^\varphi/\ell \), and \( \alpha \) and \( \beta \) are yet to be determined constants. Considering full variations including boundary terms of (4.5) we are left with solving the constraint equations:

\[
\begin{align*}
\pi_{ab} \delta \gamma^{ab} &= 0, \\
\pi_a \delta A_a &= 0, \\
\pi_c \delta \varphi &= 0,
\end{align*}
\]

i.e. we require vanishing canonical momenta on the boundary. Expanding the momenta on the asymptotic \( AdS_2 \) boundary defined by the zeroth order fields:

\[
\begin{align*}
\gamma^{(0)}_{tt} &= -\frac{1}{4} e^{-2\varphi(0)} e^{2\rho/\ell}, \\
A^{(0)}_t &= \frac{1}{2\ell} e^{\rho/\ell}, \\
\varphi^{(0)} &= \text{constant},
\end{align*}
\]

and solving the constraint equations (4.14) we find:

\[
\alpha = -\frac{1}{2\ell} \quad \text{and} \quad \beta = \frac{\ell}{2}.
\]

Substituting these values back into (4.13) and summarizing we obtain the total renormalized action:

\[
S^t_{AdS_2} = \frac{\ell^2}{4G} \int d^2x \sqrt{-g^{(2)}} \left\{ e^{-2\varphi} R^{(2)} + \frac{1}{\ell^2} + 2 \nabla_\mu e^{-\varphi} \nabla^\mu e^{-\varphi} - \frac{\ell^2}{2} e^{-4\varphi} F^2 \right\} + \frac{\ell^2}{2G} \int dt \sqrt{-\gamma} \left\{ e^{-2\varphi} K - \frac{1}{2} e^{-\varphi} + \frac{\ell}{2} e^{-3\varphi} A_a A^a \right\}.
\]

The above action is nearly identical to the one derived in [76], but differs in the coupling \( \frac{\ell^2}{4G} \).
4.3 Boundary Currents, Asymptotic Symmetries and Central Extension

The boundary energy momentum tensor and $U(1)$ current are defined as

$$
T_{tt} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\text{AdS}_2}^{ct}}{\delta g_{tt}} = -\frac{\ell^2}{4G} \left( e^{-\varphi} \gamma_{tt} + \ell e^{-3\varphi} A_t A_t \right) \\
J_t = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\text{AdS}_2}^{ct}}{\delta A_t} = \frac{\ell^2 e^{-3\varphi}}{2G} (-e^{-\varphi} \ell^2 n^\mu j_{\mu t} + \ell A_t),
$$

where $n^\mu$ is the radial ($\rho$) normal. We are interested in total symmetries on the $\text{AdS}_2$ boundary (4.15) preserving the conditions:

$$
\delta g_{\rho\rho} = \delta g_{t\rho} = 0, \quad \delta g_{tt} = 0, \quad 2\rho/\ell, \quad \delta A_t = 0,
$$

and gauge $A_\rho = 0$. A general set of diffeomorphisms preserving the above is given by:

$$
\epsilon = \left[ \xi(t) + 2\ell^2 \left( e^{2\rho/\ell} - h(t) \right)^{-1} \xi''(t) \right] \partial_t - \ell \xi'(t) \partial_\rho
$$

$$
\Lambda = -2\ell e^{-\rho/\ell} \left( 1 + \frac{G^2 M^2 - a^2 \ell^2}{\ell^2} e^{-\rho/\ell} \right)^{-1} \xi''(t),
$$

where $\xi(t)$ is an undetermined function of time. Expanding the boundary energy momentum tensor of (4.18) in terms of boundary fields (4.15) and computing its response to a total symmetry we find:

$$
\delta \epsilon + \Lambda T_{tt} = 2T_{tt} \xi'(t) + \xi(t) T_{tt}' + \frac{c}{12} \ell \xi'''(t) + \sqrt{h} \cdot \mathcal{O} \left( e^{\rho/\ell} \right), \quad \text{where}
$$

$$
c = \frac{6\ell^2}{G},
$$

which is precisely the transformation law of an energy momentum tensor of a one dimensional CFT\footnote{The factor $\frac{1}{12}$ in the anomaly is dependent upon the choice of conformal coordinates and normalization of $Q$. For tortoise light cone coordinates and unit normalization, as in Section 3, the the factor is $\frac{1}{24\pi} [91,98]$.}. We should note that the factor of $\ell$ is separated in the anomalous term to ensure proper units of the central charge given by:

$$
c = \frac{6 \left( r_+^2 + a^2 \right)}{G} = \frac{3A}{2\pi G},
$$

in accord with (3.25). Implementing Assertion 1 we have

$$
c = \frac{3A}{2\pi G},
$$

$$
Q_0 = \frac{A}{16\pi G},
$$

which together inside (1.3) gives

$$
S = \frac{A}{4G}
$$

reproducing the standard area law.
Conclusion and Comments

In addition, we have avoided any regulators depending on factors of $\sim 1/\kappa$ or $\sim 1/h$ and thus, the above analysis may be repeated for the specific case $a = GM$ from the outset, for which (4.21) becomes:

$$\delta_{\epsilon+\Lambda} T_{tt} = 2T_{tt}\xi'(t) + \xi(t)T_{tt}' + \frac{c}{12}\xi''(t), \quad \text{where} \quad c = 12J.$$

The above result is precisely the left central charge of the Kerr/CFT correspondence [38] obtained from the $gnNHEK$ solution in the limit $a = GM$.

5 Conclusion and Comments

To conclude, we have analyzed quantum near-extremal Kerr black hole properties in the near horizon regime via the construction of an $AdS_2/CFT_1$ correspondence of the $gnNHEK$ metric, as outlined in Table 1, and extending our previous analysis [34,35] to a new spacetime. The main results of our work includes the central charge $c = \frac{14GM}{J\pi}$, which was computed via a Lagrangian analysis of conserved currents of two different near horizon theories.

| CFT | Black Hole |
|-----|------------|
| Conformal Group | Asymptotic Symmetry Group |
| center | $\frac{14GM}{J\pi}$ |
| Hamiltonian eigen-value | $GM_{tt}$ |
| Regulator | $\kappa_{gnNHEK}$ |

Table 1: Black-Hole/Near-Horizon-CFT Duality

It is conceivable that other $AdS_2 \times S^2$ gauges, exhibiting the field splitting of (2.1), exist with relevance and physical connections to other classical near-extremal solutions. Many analogues to the NHEK for charged rotating black holes with negative and positive cosmological consents have been shown to exist, see [71] for a comprehensive review, which suggests similar such analogues to the $gnNHEK$ solution and similar analysis of this note should be applicable. In particular, Assertion 1 may be a useful tool in the asymptotic symmetry analysis of other extremal black holes, which have zero surface gravity and hence the need for a thermal Cardy formula (1.4). However extremal black holes in general have well defined horizons, which leads to finite non zero irreducible mass, thus allowing the implementation of a standard statistical Cardy formula (1.3) thus leading to the Bekenstein-Hawking entropy.

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