Platonic Scattering Cancellation for Bending Waves in a Thin Plate

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We propose an ultra-thin elastic cloak to control the scattering of bending waves in isotropic heterogeneous thin plates. The cloak design makes use of the scattering cancellation technique applied, for the first time, to the biharmonic operator describing the propagation of bending waves in thin plates. We first analyze scattering from hard and soft cylindrical objects in the quasistatic limit, then we prove that the scattering of bending waves from an object in the near and far-field regions can be suppressed significantly by covering it with a suitably designed coating. Beyond camouflaging, these findings may have potential applications in protection of buildings from earthquakes and isolating structures from vibrations in the motor vehicle industry.

In the past few years, metamaterials have become an attractive research focus for physicists and engineers due to their ability to control electromagnetic, acoustic, and elastic waves in ways that cannot be achieved using "regular" materials. Consequently, exciting applications of metamaterials ranging from sub-wavelength imaging to cloaking have been reported²¹–²². Soon after the advent of negative refraction and sub-wavelength imaging¹, it has been shown that dielectric or conducting objects could be made "invisible" to electromagnetic waves by coating them with plasmonic materials²³–²⁵. A promising route to electromagnetic invisibility relies on the scattering cancellation phenomenon, where the fields scattered from a coating with negative (local) polarizability are specifically designed to cancel the fields scattered from the object²⁶–²⁷. The scattering cancellation technique has been proven relatively robust to changes in the design parameters, geometry, losses, and frequency of operation²⁸,²⁹. It has also been used to make electrodes or metal films transparent to THz³⁰ or GHz³¹ radiation. Experimental evidence of such invisibility devices has recently been demonstrated in the microwave regime³²–³⁴. An alternative route to designing invisibility devices makes use of coordinate transformations³⁵,³⁶ to mold the wave flow in desired manners³⁷–³². It should also be mentioned here that with their possible applications in stealth technology, noninvasive probing, and sensing, electromagnetic invisibility cloaks open promising avenues for new technologies in medicine, defense, and telecommunications³³–³⁴.

Cloaking ideas and designs for acoustic waves have also been developed. The possibility of two-dimensional acoustic cloaking for pressure waves in transversely anisotropic fluids has recently been shown³⁵. Subsequent investigation of this cloaking idea for concentric layers of solid lattices behaving as artificially anisotropic fluids in the homogenization limit has been provided³⁶. Following a similar approach, a micro-structured metallic structure has been designed as a cloak for surface liquid waves; the design has been validated experimentally around 10 Hz³⁷. Additionally, three-dimensional acoustic cloaking for pressure waves in fluids has been envisaged; this cloak makes use of the fact that the scalar wave equation retains its form under geometric transforms³⁸,³⁹. Like in electromagnetic cloaking, the scattering cancellation technique offers an alternative path towards acoustic cloaking⁴⁰–⁴². However, cloaking fully coupled pressure and shear elastodynamic waves in solids is a more elusive task, since the Navier-Stoke's equations do not retain their form under geometric transforms⁴³,⁴⁴. Although the theoretical foundation of these cloaking effects is well established⁴⁵,⁴⁶, the inherent requirement of singular bulk metamaterial properties that can hardly be met in practice still hinders the practicality of cloaking devices⁴⁷,⁴⁸.

In this Report, a biharmonic wave equation with appropriate boundary conditions, which describes the propagation of bending waves, is derived from the generalized elasticity theory⁴⁹,⁵⁰ and is used in designing a cloak that operates in thin plates. The idea behind the cloak design is to choose a coating material with density \( \rho_c \) satisfying \( \rho_c < \rho_0 \) or \( \rho_c < 0 \) and to conceal an object with density \( \rho_s > \rho_0 \). Here, \( \rho_0 \) represents the density of the surrounding medium.
The parameters of the cloak are deduced using the scattering cancellation technique applied to a platonic metamaterial coating. Note that the term "platonic" was coined by McPhedran in reference to thin plates while investigating wave propagation in complex thin plate elastic artificial materials. Recent advances in the field of high frequency homogenization in platonics provide a theoretical foundation for understanding frequency dependent effective mass density taking negative or less than unity values.

We then analyze the response of the platonic cloak concealing a cylindrical obstacle in the presence of a plane wave elastic excitation (harmonic vibration of the plate in the vertical z-direction). It is assumed that the out-of-plane dimension of the obstacle is negligible compared to its in-plane dimensions. We show that in the quasi-static limit, i.e. for $k_0 a_s \ll 1$, where $k_0$ is the bending wavenumber in the surrounding medium and $a_s$ is the in-plane dimension of the scatterer, the scattering is dominated by the zeroth-order multipole, unlike in the electrodynamics case where the first significant order is the dipolar one. This is not the only fundamental difference between electrodynamic/acoustic and elastic wave scattering phenomena: the fourth order biharmonic partial differential equation, which typically describes the propagation of bending waves in ultra-thin plates, is not equivalent to the vector/scalar wave equations that describe electromagnetic or acoustic wave propagation. For instance note that propagating and evanescent waves coexist even in homogeneous thin plates. Consequently, one can anticipate that a new cloaking method and new relevant physics are introduced following this route.

**Results**

It has been recently shown that the elasticity equations are not invariant under coordinate transformations. Indeed, equations describing the propagation of elastodynamic waves under a time harmonic excitation $\psi$:

$$U^{\text{inc}}(r, \theta) = \sum_{n=0}^{\infty} \epsilon_n \hat{F}_n(k_0 r) \cos n\theta$$

(3)

Similarly, fields inside the shell and the object are expressed as

$$U^{\text{circ}}(r, \theta) = \sum_{n=0}^{\infty} \epsilon_n \hat{F}_n(k_0 r) \cos n\theta, \quad \text{for } r < a_s.$$  (4)

Figure 1 | Geometry under investigation: An object at the center of a thin elastic plate lies on the trajectory of a planar bending wave, incident from right to left in presence of a cylindrical shell that may act as a platonic cloak. Inset shows a top view of the cloak and object.

$$\sigma^{\text{scat}} = \frac{4}{k_0} \sum_{n=0}^{\infty} |A_n|^2.$$  (5)

Note that coefficients $B_n$ are absent in Eq. (5) since the modified Bessel functions $K_n(k_0 r)$ have no contribution to the scattered field as $r \to \infty$. Generally speaking, the possibility for an observer to detect the object in the far-field is determined by the value of $\sigma^{\text{scat}}$. As a result, minimizing or completely canceling $\sigma^{\text{scat}}$ would lead to the undetectability (invisibility) of the object in the far-field, independent of the observer’s position. This can be achieved by canceling the coefficients $A_n = \hat{A}_n / \epsilon_n$ that significantly contribute to the scattering. Here, $\hat{A}_n$ is given by the determinant:

$$\epsilon_n = \begin{cases} 1 & \text{for } n = 0, \\ 2 & \text{for } n \neq 0. \end{cases}$$
where

\[ S_x(k, r) = D_0 [n^2 (1 - v_0) (k r)^2] Z_0(k, r) - D_0 (1 - v_0) k r Z_0(k, r), \]

\[ T_x(k, r) = D_0 [n^2 (1 - v_0) Z_0(k, r) - D_0 (1 - v_0) (k r)^2] k r Z_0(k, r), \]

where \( k \in \{ k_x, k_y, k_z \} \) and \( Z_0(n) = \{ j \theta_n, Y_1(n), H^{(1)}_0(n), K_0(n), L_0(n) \}. \) Upper and lower signs in Eq. (7) should be selected for \( Z_0(n) \in \{ j \theta_n, Y_1(n), H^{(1)}_0(n) \} \) and \( Z_0(n) \in \{ K_0(n), L_0(n) \}, \) respectively. The expression of \( a_n \) could be obtained from Eq. (6) by replacing \( f \) by \( H_n(1) \) in the first column (see supplemental materials\(^4\)). Given the general complexity of this expression, it is instructive to analyze the low frequency limit corresponding to elastically small obstacles and shells, i.e. \( k a_x < 1, \) \( k a_y < 1, \) \( k a_z < 1, \) and \( k a_z < 1. \) Note that with the parameters in our study, it is sufficient to only keep \( k a_x < 1 \) and \( k a_z < 1. \) Under this assumption, \( \sigma = \sigma_0 \) in Eq. (5) is dominated by the monopole term \( A_0 \):

\[
\sigma_0 = 4 \pi |A_0|^2 = \frac{\pi^2 (k_0 a_x)^4}{16 k_0} |\rho_r - \rho_0 + \gamma (\rho_s - \rho_0)|^2, \quad \gamma = a_x / a_c. \tag{8}
\]

Note that here, terms scaling with \((k a_x)^m, m \geq 3\), in expressions of \( A_0, n \gg 0, \) are safely assumed to be zero for \( k a_x < 1. \) It is clear from Eq. (8) that \( \sigma_0 \) of elastically small obstacles in a thin plate scales with \( k_0 a_x^4 \). Also, as \( \gamma \to 1 \) and \( \rho_s \to \rho_0 \) (no shell, only bare object), Eq. (8) reduces to

\[
\sigma_0 = 4 \pi |A_0|^2 = \frac{\pi^2 (k_0 a_x)^4}{16 k_0} |\rho_r - \rho_0|^2. \tag{9}
\]

By enforcing \( A_0 = 0, \) i.e. \( \sigma_0 = 0, \) in Eq. (8), we can derive the quasistatic design rule for cloaking:

\[
\gamma = \frac{\rho_r - \rho_0}{\rho_s - \rho_0}, \tag{10}
\]

which relates \( \rho_s, \rho_r, a_x, \) and \( a_c. \) It is found that, to satisfy Eq. (10), \( \rho_s \) must take negative values for \( \rho_s > \rho_0 \) and values larger than \( \rho_0 \) for \( \rho_s < \rho_0. \)

For larger objects, retardation effects become important and numerical calculations using the full \( 8 \times 8 \) matrix in Eq. (6) are necessary to analyze the problem.

It is interesting to investigate two additional types of obstacles, namely when a “clamped” and a stress-free boundary conditions are set on the outer boundary of the system \( (r = a_x), \) while the same conditions of continuity of \( U, \partial U/\partial r, M, \) and \( V, \) as before are hold on the outer boundary \( (r = a_x). \) For the first case, we have \( U = \partial U/\partial r = 0 \) whereas \( M, \) and \( V, \) are unconstrained. The \( 8 \times 8 \) system in Eq. (6) reduces thus to a \( 6 \times 6 \) system (see supplementary materials\(^6\)). Applying the analysis described before to this case reveals that the monopole scattering coefficient \( A_0 = -1 \) in the quasistatic limit regardless of the cloak’s parameters. This means that the resulting scattering amplitude \( f(\theta) = -2 / \sqrt{\pi k} \) (or the scattering cross-section \( \sigma = 4 \pi k \)) becomes singular in the quasistatic limit (all other scattering coefficients tend to zero). This is unique to elastic waves, because a very small object would scatter infinitely and it has no equivalent in electromagnetics or acoustics\(^6\). This discussion shows that it is not possible to use shells of tailored density, in the quasistatic limit, to cancel the monopole scattering of a clamped obstacle since their corresponding coefficients differ by a factor \((k a_z)^2\).

For the stress-free boundary conditions, \( M, \) and \( V, \) are set to zero on \( r = a_x \) whereas the same continuity conditions hold for the other parameters. We obtain here also a \( 6 \times 6 \) system (see supplementary materials\(^7\)). By applying the analysis above, we show that the monopole scattering coefficient of a stress-free hole of radius \( a_z \) cloaked with a shell of density \( \rho_s \) and radius \( a_z = a_z, \) is given by

\[
A_0 = -\frac{\pi}{8} \{ 1 + \gamma^2 (v + 1) / (v - 1) + (\gamma^2 - 1) \rho_r / \rho_0 \}. \tag{11}
\]

We see then that it is possible to find values of the density of the shell \( \rho_s \) and ratio \( \gamma \) that make \( A_0 \) vanish thereby canceling the scattering from the cloaked object in the quasistatic regime. This condition is found to be

\[
\rho_s / \rho_0 = 1 + \gamma^2 (v + 1) / (v - 1) \gamma^2 - 1 \rho_r / \rho_0. \tag{12}
\]

In all the results presented in the rest of the Report, in-plane dimension of the object is set to \( a_y = 1 \) m. Figure 2 shows the dependence of the scattering amplitude \( |f(\theta)| \) on \( k a_z \) for \( \rho_s / \rho_0 = 0.1 \) [Figs. 2(a), 2(b) and 2(c)] and \( \rho_s / \rho_0 = 10 \) [Figs. 2(d), 2(e) and 2(f)]. We show here that curves of scattering amplitude \( |f(\theta)| \) vs. \( \theta \) in logarithmic scale contain angular information about the scatterer. [Figs. 2(a), 2(b) and 2(d), 2(e)] and curves of normalized backward scattering amplitude \( f(\theta) / / \sqrt{\theta} \) vs. \( k a_z \) [Figs. 2(c) and 2(f)] give insight into the spectral dependence of the object visibility for an observer placed at the angular position of the illumination (monostatic scattering). The simulations are carried out for the first scenario (soft object e.g. soil) for \( k a_z = 0.1 \) (quasistatic limit) and \( k a_z = 1 \) (Mie scattering). It is clearly seen that, for the soft object with \( \rho_s / \rho_0 = 0.1 \) [Figs. 2(a) and (d)] and the rigid object with \( \rho_s / \rho_0 = 10 \) [Figs. 2(b) and 2(e), the scattering amplitude \( |f(\theta)| \) is circularly symmetric for \( k a_z = 0.1 \). This is due to the fact that scattering is dominated by the monopole term \( A_0 \). The scattering behavior of soft and rigid objects is quite different for different values of \( k a_z \). For the soft object [Figs. 2(a), 2(b), solid red lines] \( A_0 \) is still dominant for \( k a_z = 1 \), and no significant angular dependence is observed. For the rigid object, [Figs. 2(e), solid red line], higher order multipoles start to contribute quite significantly to the overall scattering when the size is increased to \( k a_z = 1 \). This has consequences on cloaking behavior. Consider the soft and rigid objects cloaked with shells with \( \rho_s = 1.3 \rho_0 \) and \( \rho_s = -2 \rho_0 \), respectively. For both shells, \( \gamma = 0.2 \). Note that these values satisfy Eq. (10). Scattering amplitude \( |f(\theta)| \) and normalized backward scattering amplitude \( f(\theta) / / \sqrt{\theta} \) of the cloaked objects are shown in Fig. 2 (dashed blue lines). Scattering reduction is clearly observed for both objects in the long wavelength limit while in the Mie scattering regime only the soft object gets cloaked in every direction.

Also, curves of \( f(\theta) / / \sqrt{\theta} \) vs. \( k a_z \), in Figs. 2(c) and 2(f) show that the scattering from the rigid object undergoes a more dynamic variation in \( k a_z \)-range [0, 1], contrary to the scattering from the soft object, where one broad maximum is observed in the same \( k a_z \)-range. This is associated with the fact that higher densities correspond to smaller effective wavelengths in the object, thus causing several internal resonances even for moderately sized objects. In addition, scattering reduction is more pronounced and broader for cloaking of the soft object [Fig. 2(c) and 2(f)].

We now analyze how introducing a suitably designed negative density shell can drastically lower the overall scattering. We schematically indicate the scattering, in analogy with the polarization in the electromagnetic case, with vertical arrows of opposite direction: the positive (solid) arrow stands for ‘positive’ scattered wave, whereas the negative (dashed) one corresponds to scattering with opposite phase [inset of Fig. 3(b)]. Figure 3(a) shows the contours of \( |\rho_r / \rho_0| \) (in logarithmic scale) that fulfill Eq. (10) for varying values of \( \rho_r / \rho_0 \) and \( \gamma \). The solid blue line in the figure represents the case \( \rho_r = 0 \) and + or - sign indicates whether the required shell density is positive or negative. Let \( e^{\gamma s} \) represent the scattering efficiency computed by the ratio of scattering cross-sections of the cloaked and bare objects. Figure 3(b) plots \( e^{\gamma s} \) vs. \( \rho_r / \rho_0 \) for a cloaked object with \( \rho_r / \rho_0 = 10 \) for various values of \( \gamma \) and for \( k a_z = 0.1 \) (the quasistatic limit). Figure 3
clearly shows that for certain values of $r_c$, significant scattering reduction is achieved, and this is also obtained for different values of $c$. We notice that, for an ultrathin cloak (in the limit of $c \ll 1$), there is no dip in $e_{sc}$; instead the scattering reduction can be quite broadband for thicker shells. For instance, the shell with $c = 0.5$ and $r_c/r_0 = 0.52$ significantly suppresses the scattering. This agrees well with the result given by Eq. (8), which considers only the zeroth-order multipole. As schematized in the inset of Fig. 3(a), a platonic cover with relative density less than unity may induce an out-of-phase monopolar mode with respect to the local elastic displacement field, thus permitting dramatic cancellation of the field scattered from the object. For other values of $c$, different values of $r_c/r_0$ ranging from negative values to low positive (less than unity), are required to achieve significant cancellation of scattering.

In reality, while modeling the frequency response of the cloaking phenomenon, one needs to take into account the inherent dispersion characteristics of the shell material since it is not possible to have a negative and frequency independent density due to causality considerations. A negative density metamaterial may be achieved in acoustics by mimicking the way we achieve negative permittivity in electromagnetics with low frequency homogenization in high contrast periodic media, or high frequency and high order homogenization.

**Figure 2** | Scattering amplitude $|f|/\theta|$ in logarithmic scale for the soft object ($\rho_s/\rho_0 = 0.1$) with (a) $k_0 a_s = 0.1$ and (b) $k_0 a_s = 1$ and for the rigid object ($\rho_s/\rho_0 = 10$) with (d) $k_0 a_s = 0.1$ and (e) $k_0 a_s = 1$. The dashed blue line represents the cloaked scenario whereas the solid red line stands for the bare object. Plots in (c) and (f) are the normalized backward scattering amplitude $|f|/\sqrt{\omega_0}$ vs. $k_0 a_s$ for soft and rigid objects, respectively.

**Figure 3** | (a) Contour plot of the solutions of Eq. (10). (b) Scattering efficiency $e_{sc}$ of the cloaked object with $\rho_s/\rho_0 = 10$ vs. $\rho_s/\rho_0$ for various values of $\gamma$ and $k_0 a_s = 0.1$. The inset gives the sketch of cloaked object with arrows marking opposite phase of scattered wave in core and shell.
genization\(^5\) in moderate contrast media. For elasticity, spring mass or inertial resonator metamaterials could lead to the desired effect near the resonance frequencies\(^5\). We suppose in the following that the material comprising the shell follows a classic Drude-like model of the type

\[
\frac{\rho_\text{eff}}{\rho_0} = \frac{k_0^4}{k_0^4 + i\gamma_c}
\]

[notice the fourth order dependence on \(k_0\) stemming from the nature of the biharmonic equation], where \(\rho_\text{eff} = 1\), and \(k_0\) and \(\gamma_c\) are the plasma wavenumber and Ohmic loss, respectively. This model can be obtained via a homogenization applied to a composite shell with high contrast inclusions, and is in all ways analogous to that derived in\(^5\). This Drude dispersion is plotted in Fig. 4(a) with parameters \(k_0a_0 = 0.15\) and \(\gamma_c = 10^{-2}k_0^2\). Figure 4 (b) plots scattering efficiency \(\varepsilon_{\text{sc}}\) of an object cloaked with a shell made of Drude material vs. \(k_0a_0\) for various values of \(\gamma_c\) and \(\gamma = 0.5\).

These curves are obtained using Eqs. (2) and (3), consistent with rigorous elastic scattering theory\(^4\) and validated against results obtained using a finite elements commercial software\(^5\). It is clear that the presence of a cloak with negative effective density may allow for a drastic reduction of the overall scattering at a desired wavenumber (\(k_0a_0 = 0.11\) in this example), independently of the angle of observation. It is stressed here that the scattering cross-section can be reduced by over five orders of magnitude compared to the uncloaked scenario and by six orders of magnitude compared to the case of an obstacle of the same size and density \(\rho_\text{eff}/\rho_0 = 10\). A slight broadening of the cloaking dip and a corresponding deterioration of the effect may be noticed when elastic loss \(\gamma_c\) is increased.

**Discussion**

We have put forward a technique to cloak objects from elastic waves governed by the scalar fourth order biharmonic equation. The functionality of the obtained cloak could be seen in Fig. 5, where we plot the amplitude distribution of the scattered elastic displacement field \(U\) in the presence of cloaked [Fig. 5(a)] and uncloaked [Fig. 5(b)] objects. When it is surrounded by the negative density platon shell (following the Drude-like model described above), the field amplitude is constant and equal to unity everywhere in space, in contrast to the considerably perturbed fields in the uncloaked case. The scatter-
ing reduction can be attributed to the proper choice of mass density of the platonic shell, in view of the scattering reduction predicted in Figs. 3 and 4.

In summary, we proposed a design of an elastic cloak based on the scattering cancellation technique, inspired by earlier applications in electrodynamics and acoustic scenarios. Here, however, the challenge is associated with the fact that a different, more complex analytical form governs bending waves. The cloaking mechanism introduced here presents significant advantages in comparison with transformation acoustic designs: there is no need of anisotropy and inhomogeneity of the material parameters. Using a homogeneous isotropic platonic shell with low or negative density we have been able to greatly reduce the scattering from soft and highly rigid objects as well as from stress-free holes. It should also be noted here that the proposed technique cannot be used to cloak clamped objects in the quasistatic limit. However, cloaks made of layered shells have the potential to overcome this problem at higher frequencies (where scattering from clamped objects is finite) by adding extra degrees of freedom to cancel more than one scattering coefficient, as previously demonstrated in electromagnetic cloak designs.

Experimental realization of this idea may be within reach in the near future (note for instance that lensing of bending waves via negative refraction was theoretically predicted using the biharmonic plate model in and experimentally confirmed in a thin Duralumium plate in and a theoretical proposal for cloaking via geometric transforms in the biharmonic operator was also experimentally confirmed in
to thin plates in the motor vehicle and airplane industry, in which the scattering of bending waves in thin plates generated by engines may be totally suppressed, or smart secure buildings, protecting them from bending and other types of seismic vibration damages caused by scattering from neighboring buildings.

**Methods**

Analytical methods based on scattering Mie theory of cylindrical objects in thin elastic plates are used to obtain the numerical simulations in this Report (for further details and physical insight, see supplementary material). The vertical displacement of the plate is the solution of the fourth order partial differential equation of Kirchhoff. We proceed, as is usually done, by expanding the impinging plane waves and the scattered fields in terms of Bessel and Hankel functions in polar coordinate system centered with the object to be cloaked. We then apply four elasticity boundary conditions on each cylindrical interface in order to obtain the scattering coefficients for waves, which uniquely determine the displacement fields everywhere. The displacement field distributions and scattering cross sections are computed using Bessel developments and respectively. In the quasistatic limit, where the size of the elastic core sphere is much smaller than the wavelength and only the lowest-order Mie coefficient is important, an analytical formula is obtained [Eq. (10)] and it gives similar results to the full-wave simulations. Proper convergence for all the results is reached. The simulations given in Fig. 5 are obtained using the commercial finite-elements software COMSOL Multiphysics.

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Author contributions
M.F., P.-Y.C. and A.A. conceived the theoretical idea. M.F. conducted the numerical simulations and wrote the manuscript text. P.-Y.C., H.B., S.E., S.G. and A.A. discussed the implementation of the concept and reviewed the manuscript.

Additional information
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