THREE TYPE I X-RAY BURSTS FROM CYGNOUS X-2: APPLICATION OF ANALYTICAL MODELS FOR NEUTRON STAR MASS AND RADIUS DETERMINATION

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ABSTRACT

We study the spectral and temporal properties of three type I X-ray bursts observed from Cygnus X-2 with the Rossi X-Ray Timing Explorer. Despite the short time durations (~5 s), these bursts show a radial expansion on the order of several neutron star (NS) radii. We apply the analytical models of spectral formation during the expansion and contraction stages to derive physical conditions for the matter in the burning zone close to the surface of the NS as well as to derive the NS's mass-radius relation. Our results, combined with statistical errors, show that the central object is a compact star with a mass of ~1.4 M⊙ and a radius of ~9 km. Our results favor the softer equation of state for NS matter.

Subject headings: accretion, accretion disks — radiative transfer — stars: fundamental parameters — stars: individual (Cyg X-2) — stars: neutron — X-rays: bursts

1. INTRODUCTION

The strong energy outbursts in the atmospheres of neutron stars (NSs) in low-mass X-ray binary (LMXB) systems are commonly referred to as X-ray bursts. Soon after their discovery by Grindlay & Heise (1975), X-ray bursts were separated into two classes: type I and type II bursts. Type I bursts are currently considered to be due to thermonuclear explosions on an NS surface, and type II bursts are believed to result from the instability of the gravitational energy release in the accretion column. In the latter case, the radiation pressure has a substantial influence on the radiative transfer problem during the burst, covering a wide range of values for the Eddington ratio λ. During the expansion stage to the observational spectral formation during the expansion and contraction stages to derive physical conditions for the matter in the burning zone close to the surface of the NS as well as to derive the NS’s mass-radius relation. Our results, combined with statistical errors, show that the central object is a compact star with a mass of ~1.4 M⊙ and a radius of ~9 km. Our results favor the softer equation of state for NS matter.

Most X-ray bursts are observed in LMXB sources whose color-color diagrams cause them to be classified as atoll sources. However, the two Z sources GX 17+2 and Cygnus X-2 are exceptions to this rule. The observational data of X-ray missions prior to the Rossi X-Ray Timing Explorer (RXTE) suffered from poor statistical and time resolution properties and thus could not resolve the nature of burstlike events detected from these sources. The first incontestable evidence of a type I burst from Cyg X-2 was reported by Smale (1998, hereafter S98). The analysis provided by S98 indicates that the burst color temperature increases with the transition to the contraction stage and that it goes down during a burst decay. In this Letter, we concentrate our efforts on the study of the spectral properties of the bursts from the source Cyg X-2. We have surveyed the RXTE public data archive and find more than 20 burst events in which the count rate increased by a factor of 3 on a timescale of 1–2 s. Temporal analysis of this set of bursts for which appropriate data configurations are available shows that for only three bursts (including the burst reported by S98) can the photospheric radius expansion be well established. As long as no other burst can be qualified as a type I, we summarize the results provided by the three bursts with radial expansion. In § 2, we present the results of the analytical models for X-ray spectral formation in the burst atmospheres along with the methodology that we implement in our data analysis. In § 3, we describe the observational data and reduction procedure. Section 4 contains a report of our results, which is followed by discussion and conclusions in § 5.

2. MODELS FOR A SPECTRAL FORMATION DURING EXPANSION AND CONTRACTION STAGES

T94 presented analytical approximations for the solution of the radiative transfer problem during the burst, covering a wide range of values for the Eddington ratio λ. In the case of a hydrostatic equilibrium atmosphere (λ < 0.9), the density profile is exponential. The solution for the color factor Tα as a function of the luminosity l, the helium abundance YHe, and the gravitational acceleration g is obtained in the form (T94)

\[ T_\alpha = \frac{TTT_{\text{eff}}}{1.32K(l, Y_{\text{He}}, g)\Lambda_5^{4/5}} \]

where

\[ K(l, Y_{\text{He}}, g) = \frac{l^{3/20}(1 - 5Y_{\text{He}}/8)^{2/15}(1 - Y_{\text{He}}/2)^{-1/6}l^{-16/5}}{g^{16/5}} \]

and

\[ \Lambda_5 = 0.14 \ln (8a) + 0.1 \exp (-5.5a) + 0.6, \quad a = \frac{l}{g} \]

\[ l = \frac{l_{\text{bol}}}{\epsilon_{\text{bol}}} \]

\[ \epsilon_{\text{bol}} = 1 \]
$K^{15/2}(l, Y_{\text{He}}, g)$, and $g_{14} = g/10^{14}$ cm s$^{-2}$. The radiative transfer equation in the $l \sim 1$ case also admits a semianalytical treatment, which is presented in detail in Shaposhnikov & Titarchuk (2002). Here we present the final results, obtained for the color factor $T_{\text{c}}$ and color temperature $kT$:

$$T_{\text{c}} = 0.37m^{0.174}r_{\text{NS}}^{-0.28}T_{b}^{-0.64}(2 - Y_{\text{He}})^{-0.074},$$

$$kT = 0.76m^{0.42}r_{\text{NS}}^{-0.78}T_{b}^{-0.64}(2 - Y_{\text{He}})^{-0.324} \text{keV},$$

where $r_{\text{NS}}$ is the NS radius in units of 10$^6$ cm, $m = M_{\text{NS}}/M_{\odot}$ is the NS mass in units of solar mass, and $T_{b}$ is the temperature at the bottom of the burst atmosphere in units of 10$^8$ K. With equations (1)–(3) in hand, we apply our formalism to the Proportional Counter Array (PCA) burst data. The outcome of the fitting procedure implemented to X-ray burst data is that we obtain the color temperature of radiation $kT_{\text{c}}$ and the bolometric flux $F_{\text{bol}}$ with corresponding statistical errors. Subscripts $c$ and $b$ denote that these values are detected by a distant observer. Renormalization to the real condition at the photospheric surface should account for both dilution and gravitational redshift. The dilution is described by the relation $L_{\text{c}} = 4\pi d^2\xi_{\text{c}}F_{\text{bol}}$, where $\xi_{\text{c}}$ is the anisotropy factor determined by the emission anisotropy of the burst. The local luminosity $L$ and temperature $kT$ are connected with those observed at infinity by

$$L_{\text{c}} = L/(z + 1)^2 \text{ and } kT_{\text{c}} = kT/(z + 1),$$

where the redshift factor is $(z + 1) = (1 - 2GM_{\text{c}}/R_{\text{NS}}c^2)^{-1/2} = (1 - 0.297m/r_{\text{NS}})^{-1/2}$. From the definition of the effective temperature and color factor, we can write

$$4\pi R_{\text{NS}}^2\sigma T_{\text{eff}}^4 = 4\pi R_{\text{NS}}^2\sigma (T_{\text{c}})^4 = L.$$  

After the atmospheric touchdown occurs (or when no substantial photospheric expansion occurs), we rewrite $L$ as $L_{\text{edd}}$, where

$$L_{\text{edd}} = 4\pi cGM_{\text{NS}}(z + 1)/k_{\text{NS}}(2 - Y_{\text{He}}) = L_{\text{edd, c}}(z + 1),$$

with $k_{\text{NS}} = 0.2$ cm$^2$ s$^{-1}$. Substituting this into equation (5), after introducing the general relativistic corrections in equation (4), we obtain

$$r_{\text{NS}}^2 = 19.5(T_{\text{c}}/kT_{\text{c}})^4lm/(2 - Y_{\text{He}})(z + 1)^3.$$  

The dimensionless luminosity $l$ measured by a local observer can be expressed in terms of observed fluxes and gravitational redshift as

$$l = L/L_{\text{edd}} = F_{\text{bol}}(z + 1)/F_{\text{bol,edd}},$$

where $F_{\text{bol,edd}} \approx L_{\text{edd, c}}/(4\pi d^2\xi_{\text{c}})$ is the observed burst peak bolometric flux (we neglect the relativistic corrections in evaluating $F_{\text{bol,edd}}$). Using the above expression for $F_{\text{bol,edd}}$ and equation (6) for $L_{\text{edd, c}}$, we can rewrite this equation as

$$l = 0.476\xi_{\text{c}}d_{10}^{-2}F_{\text{bol}}(2 - Y_{\text{He}})(z + 1)/m,$$

where $d_{10} = d/10$ kpc and $F_{\text{bol}} = F_{\text{bol,edd}}/10^{-8}$ ergs cm$^{-2}$ s$^{-1}$. Because $T_{\text{c}}$ and $z + 1$ are dependent on $r_{\text{NS}}$, we can find $r_{\text{NS}}$ as a numerical solution of equation (7) for a particular set of the parameters $m$ and $Y_{\text{He}}$. Furthermore, we can determine the NS mass $m$ using the Eddington limit (eq. [6]), provided that the distance to the source is known. Finally, we are able to test the conditions at the burning zone during the expansion stage by virtue of equation (3).  

### 3. Observations

The Z source Cyg X-2 has been observed extensively by the PCA on board the RXTE throughout the mission. We reviewed the entire Cyg X-2 archived data set in an attempt to locate thermonuclear bursts. When a burst was found, we determined what Experimental Data System (EDS) configuration was used for that observation. We then chose those bursts for which the EDS configuration was well suited to our analysis. Among the more than 20 burstlike events, we chose three for our analysis: burst 1 was detected on 1996 March 27 (observation identification [obsid] 10066-01-01-00; S98), burst 2 was detected on 1998 August 10 (obsid 2004-01-05), and burst 3 was detected on 1998 September 12 (obsid 30046-01-01-00). The first two bursts can be classified confidently as type I X-ray bursts. The nature of burst 3 is uncertain, as we discuss below. During these observations, four additional EDS modes were utilized along with two standard modes: the binned mode (0–35 PCA channels in 16 energy bins, with a 2 ms time resolution), the event mode (36–255 PCA channels in 64 bins, with a 125 μs time resolution), and two single-bit modes with 125 μs time resolution covering the 0–13 and 14–35 PCA energy channels correspondingly. Burst 1 was observed during PCA gain epoch 2, while bursts 2 and 3 were observed during PCA gain epoch 3, resulting in a different instrumental response across the PCA channels. For our temporal burst analysis, we are particularly interested in binned and event modes. For the data reduction, we use the procedure adopted by S98 in that we extract a sequence of 0.5 s spectral slices during each burst. Then we extract the spectra of persistent emission prior to each burst. The time duration for the persistent emission spectra is determined by the requirement that the persistent flux does not undergo substantial systematic changes during the interval. All spectra produced are dead time–corrected according to RXTE Data Reduction Recipes.  

### 4. Data Analysis and Results

The best fits for persistent flux background spectra were obtained using the XSPEC comptt+bbody model (Titarchuk 1994b). Best-fit parameters for bursts 1, 2, and 3 are, respectively, as follows: soft photon temperature = 1.09 ± 0.06, 1.04 ± 0.02, and 1.04 ± 0.05 keV; plasma temperature = 3.1 ± 0.1, 4.2$^{+1.2}_{-0.8}$, and 3.0 ± 0.2 keV; plasma optical depth = 4.7 ± 0.3, 2.1$^{+0.6}_{-0.4}$, and 3.9 ± 0.4; and blackbody temperature = 0.56 ± 0.03, 0.47 ± 0.03, and 0.47 ± 0.05 keV. For each burst, we fitted each half-second spectral slice over the energy range of 2.8–30 keV, using the blackbody model that best describes the background-subtracted burst emission. To obtain the burst emission, we subtracted the persistent component as given by the analysis of preburst data. The radiated energy from the burst can affect the persistent flux both in magnitude and in its spectral properties, causing systematic trends in our results. Specifically, if the burst results in an increase of the persistent component, our analytical approach will result in a slight overestimation of the NS radius.  

$^3$ At this point, because of the quality of data and the fact that no theoretical model for interaction of burst and persistent energy flux has been developed, we cannot refine the method of persistent emission subtraction.
best-fit values of the color temperatures $kT$, the reduced $\chi^2_{\nu,\text{red}}$ values of the blackbody spectral fits to the data, accompanied by the luminosities $L$ and the inferred NS radii, are plotted on Figure 1. The typical behavior of a thermonuclear runaway process is most pronounced in the first burst; after a quick rise, the color temperature gradually hardens from 1 to 2 keV and then decreases during the decay phase. The situation is less obvious for burst 3, where the temperature stays rather high throughout the decay. We speculate that the persistent emission is affected by the burst itself. In fact, the photons generated by the bursts can be intercepted by the accretion disk and reflected. This results in an apparent anisotropy of the burst radiation (see Lapidus, Sunyaev, & Titarchuk 1986 and Popham & Sunyaev 2001 for details).

The second burst has a peak flux that is approximately 25% higher than that of the other two bursts. This discrepancy can again be explained by considering the disk reflection and/or the dynamical evolution of the NS–accretion disk geometry.\footnote{The area of the emitting NS surface exposed to the observer can change during bursts with radial expansion. A strong indication that this may occur has recently been found by the authors in X-ray bursts from 4U 1728–34. We will discuss this point in a forthcoming paper.}

We solve equation (10) for each pair of values $kT_a$ and $F_{b,8}$ and a given set of parameters $Y_{\text{He}}$ and $m$ to obtain the radius of the emitting surface $r_a$ and the Eddington ratio $l$. It should be noted that a solution exists in the narrow parameter space for $r_a$, $m$, and $Y_{\text{He}}$ only. In Figure 1, the values of the inferred characteristics are shown along with a plot of color temperatures, obtained using blackbody fits to the data. After the luminosity peak, the photospheric radius drops quickly, while $l$ remains close to unity. After about 4 s, the photosphere reaches touchdown, the radius levels off at constant value, and the Eddington ratio drops rapidly. Because the thickness of the atmosphere (scale height $H \approx kT/mg \sim 10^7$ cm) is negligible with respect to the NS radius $R_{\text{NS}}$, we assume the photospheric radius, $R_{\text{ph}}$, after touchdown, is equal to the $R_{\text{NS}}$. Then, for a particular distance, we determine the NS mass assuming the Eddington luminosity at the peak of the burst. Figure 2 shows two contours for distances of 9 and 11 kpc and a cosmic elemental abundance, along with the $R$, $\pi$, and $\pi'$ equations of state for NS matter from Baym & Pethick (1979). The filled squares with error bars on masses and radii represent the NS mass-radius determinations inferred from the average peak (Eddington) luminosity and the contraction stage spectra.

from the observations:

$$r_a = 9.28 \xi_d d_{10}^2 F_{b,8} (T_b/kT_a)^4 (1 - 0.297 m/r_a).$$

$$\xi_d = 9.28 \xi_d d_{10}^2 F_{b,8} (T_b/kT_a)^4 (1 - 0.297 m/r_a).$$

$$d = 11 \text{ kpc, } m = 1.44 \pm 0.06$$

$$d = 9 \text{ kpc, } m = 0.96 \pm 0.04$$

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5. DISCUSSION AND CONCLUSIONS

In light of the recently discovered millisecond variability in the X-ray flux from a number of LMXBs, the task of independent determination of masses and radii of NSs in these systems becomes very important. Kilohertz quasi-periodic oscillations (kHz QPOs) are seen in the power density spectrum of many LMXBs as twin peaks with frequencies within the range of 500–1200 Hz. A number of models were put forward to explain these high-frequency patterns. The first, and most obvious, was the suggestion of an interaction between the Keplerian frequency at the NS surface (or last stable orbit) and the frequency of the NS rotation (the beat-frequency model [BFM]; Alpar & Shaham 1985; Miller, Lamb, & Psaltis 1998). Despite some discrepancies between the model predictions and the observational results, (the peak separation is not constant when the twin QPO frequencies vary with time; see, e.g., van der Klis 2000), the BFM became fashionable and has been applied to infer information on the NS masses (Zhang, Strohmayer, & Swank 1997). Because of general relativistic (GR) effects, no stable particle motion is allowed for a circular orbit radius of less than $R_{\text{isco}} = \frac{6GM}{c^2} = 8.9m$ km. The corresponding frequency of the orbital motion (assumed close to the QPO frequency) is $\nu_{\text{orb}} \approx 2210/m$ Hz. The NS masses in the LMXB exhibiting kHz QPO phenomena should be equal to or even exceed $2M_\odot$ if the highest observed kHz QPO is interpreted as the frequency of the innermost stable circular orbit. For Cyg X-2, the corresponding values are 1005 Hz for the maximum QPO frequency (Wijnands & van der Klis 1998) and, consequently, $2.2M_\odot$ for the NS inferred mass, which is clearly too high. Our analysis of the burst spectra, which takes into account all corrections due to the GR and electron scattering effects, is in disagreement with the mass-radius constraints obtained using the kHz QPO frequency values evaluated within the BFM frameworks.

It is worth noting that in a recent study of the optical and UV light curves of Cyg X-2 by Orosz & Kuulkers (1999), who elaborate independent constraints on the NS mass in Cyg X-2, $M_{\text{NS}} = 1.78 \pm 0.23 M_\odot$, which is consistent with our above-mentioned mass determination. Sco X-1 and Cyg X-2 have many similar features in terms of their timing and spectral properties, and they have almost the same upper limit for the bolometric flux (see Bradshaw, Fomalont, & Geldzahler 1999). Thus, we expect that the similar mass-radius values inferred for Cyg-2 (mass = $1.44 \pm 0.06$, radius = $9.0 \pm 0.5$ km) should also be applied to Sco X-1.

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