Thermal fluctuations of bouncing cosmology revisited

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In this note, we revisit the thermal fluctuations generated during bouncing cosmology, taking Unruh effect into account. We find that due to the additional effect on temperature, the dependence of power spectrum on $k$ will get corrected with an indication of blue tilt at large $k$ region, which is in consistent with the case of vacuum initial conditions.

INTRODUCTION

The well-known observational data of Cosmic Microwave Background anisotropy as well as Large Scale Structure [1] shows that the structure of our universe, such as galaxies, temperature fluctuations and so on, comes from the primordial perturbation at early times with a nearly scale invariant power spectrum, as successfully predicted by Standard Big Bang Theory [2,3]. In common sense, this perturbation was initiated from vacuum quantum fluctuations, which was then stretched out of the horizon. It was noticed, however, that the vacuum initial condition is by no means the only way to generate cosmic perturbations and form structure. Thermal fluctuation, of which fluctuations comes from a thermal ensemble, has been argued to be the alternative origin to source the cosmic structure [7].

Unfortunately, as was concluded in [7] and summarized in the following paper [8], that the scale invariant power spectrum can not be generated in normal inflationary stage. Due to this reason, people begin to find new physics in order to save the scale invariance. Among which are string gas cosmology [9,10], non-commutative inflation scenario [11], holographic cosmology [12], loop quantum cosmology [13] and so on. See also [14–17].

Recently, people paid more and more attention to the scenario of non-singular bouncing cosmology, which has the advantage of getting rid of the initial singularity which plagued the Standard Big Bang or inflation theory of cosmology [18–31], also see [32] for a review and the references therein). Starting from a large and cold initial state, a bounce scenario can also lead to scale invariant power spectrum of perturbation. It was then argued in [33] that thermal fluctuations is not only possible to occur, but also able to obtain scale invariant power spectrum as the case of vacuum initial conditions. The fluctuations can either be ensemble of particle gas, or holographic radiation, or even string gas. Imprints of non-Gaussianities could also been obtained due to thermal fluctuations.

In this paper, we revisit the thermal fluctuations generated during bouncing cosmologies, taking for example that the fluctuations as normal particle radiation of which the equation of state $w_\gamma = \frac{4}{3}$. When the fluctuation moves with varying velocity, i.e. with an acceleration (which is a very common case), then an Unruh effect should be taken into account. It will cause an additional “Unruh” temperature which may give a correction to the original temperature $T_r$. We have calculated that it will scale as $T_r$ times some power law of $k$. For the original case which can get scale invariant power spectrum, the power law index is positive and will give an enhancement at large $k$ region (small scales). It will cause an indication of blue tilt in the power spectrum, which is in consistent with what happens in the case of vacuum initial conditions [31].

The paper will be organized as follows: we first remind the reader the main steps in calculating thermal fluctuations in bounce cosmology in the next section, and then give our results on correction to the spectrum index by taking the Unruh effect into account. The last paragraph is the summary. We leave the basic conjecture for background evolution and the formulae for perturbation in the Appendix A.

THERMAL FLUCTUATIONS IN BOUNCING COSMOLOGY

In this section, we review the thermal fluctuations in bouncing cosmology, as is already mentioned in [31,33]. In order to obtain the spectrum at Hubble crossing at the expanding phase, we need to calculate it first in the contracting phase, then transform it into expanding phase using matching conditions at the bouncing point. We put the general argument of the metric perturbation evolution and matching conditions to Appendix A. First of all, from a perturbed Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = a(\eta)^2[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)d\tau^2]$$

we write down the first perturbed Einstein Equation as:

$$-3\dot{\mathcal{H}}(\Phi' + \mathcal{H}\Phi) + \nabla^2 \Phi = 4\pi G a^2 \delta \rho.$$  

In the above two formulae, $a(\eta)$ is the scale factor in terms of the conformal time $\eta \equiv \int \frac{dt}{a(t)}$, $\Phi$ is the Newtonian potential, $\mathcal{H} \equiv \frac{\dot{a}}{a(t)}$ is the Hubble parameter in the

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conformal time, $\delta \rho$ is the perturbation of energy density, and prime denotes for derivative with respect to $\eta$. Assumption is taken that we are in conformal Newtonian gauge and there is no anisotropic stress \cite{34}. At Hubble radius crossing all the three terms on the left-hand side of the above equation are of the same order of magnitude, so modulo a constant of the order one, we can write down the Fourier space correlation function of $\Phi$ as:

$$|\Phi(k)|^2 = 16\pi^2G^2k^{-4}a^{-4} < \delta \rho(k)^2 > .$$  \hspace{1cm} (3)

In the system of thermal equilibrium, the energy density fluctuations are determined by thermodynamics. The correlation function of $\delta \rho$ in momentum space are correlated to that in position space by the relation $< \delta \rho(k)^2 > \sim k^{-3} < \delta \rho(x)^2 >$, where the latter can be represented as:

$$< \delta \rho(x)^2 > = CV(R)\frac{T^2}{H^6},$$  \hspace{1cm} (4)

where $CV(R) \equiv \frac{\partial < \delta \rho >}{\partial T}$ is the heat capacity in a sphere of radius $R$.

We assume that the fluctuations consist of an ensemble of hot particle gas with constant equation of state $w_r$. Then from the continuation function and the second thermodynamics law, one can derive the energy density and temperature which scale as:

$$\rho_r \sim a^{-3(1+w_r)} ,$$  \hspace{1cm} (5)

$$T_r \sim a^{-3w_r} ,$$  \hspace{1cm} (6)

respectively. This gives

$$\rho_r \sim T_r^3 ,$$  \hspace{1cm} (7)

and thus

$$< \delta \rho(x)^2 > = \frac{T^2}{R^3} \frac{\partial \rho_r}{\partial T_r}$$

$$\sim R^{-3}T_r^{2+\frac{1}{w_r}}$$

$$\sim t^{-3(1+p)-6pw_r} ,$$  \hspace{1cm} (8)

where in the last formulation we have used $R = \frac{1}{H}$. Combining Eqs. 8 and 9, one can obtain the dependence on $t$ and $k$ of $|\Phi|^2$ that is:

$$|\Phi|^2 \sim \frac{k^3}{H^4} < \delta \rho(x)^2 > \sim k^{-3}t^{1-3p-6pw_r} ,$$  \hspace{1cm} (9)

and

$$|\Phi| \sim k^{-\frac{3}{2}}t^{\frac{1-3p-6pw_r}{2}} .$$  \hspace{1cm} (10)

At the Hubble crossing, where $k_* = t_*^{\frac{1}{2}}$, we have

$$|\Phi| \sim k_*^{\frac{1-3p(1+2pw_r)}{P}} .$$  \hspace{1cm} (11)

thus the power spectrum at Hubble crossing time $t_*$ will be (See Appendix A for more details):

$$\mathcal{P}_\Phi(k_*) \equiv \frac{k^3}{2\pi} |\Phi|^2 \sim k_*^{1-3p(1+2pw_r)} .$$  \hspace{1cm} (12)

From above we can see that the condition for the scale invariance of the spectrum is

$$1 - 3p - 6pw_r = 0 ,$$  \hspace{1cm} (13)

and for the case that the fluctuation behaves like normal radiation, $w_r = \frac{1}{3}$, it is straightforwardly obtained that $p = \frac{1}{3}$ and the equation of state of the background $w = \frac{2}{3}$.

\textbf{CORRECTION FROM UNRUH EFFECT}

The Unruh effect, which was first described by S. Fulling in 1973, P. Davies in 1975 and B. Unruh in 1976 \cite{35}, predicts that an accelerating observer will observe non-zero temperature where an initial observer would see none. In other words, from the viewpoint of the observer, particles which is accelerating will look like a warm gas which is in thermal equilibrium with non-zero temperature \cite{36}. As a consequence, it will cause the black body radiation as Unruh radiation, which can be viewed as the near-horizon form of Hawking radiation (see e. g. \cite{37} for a comprehensive review and references therein).

We consider the Unruh effect by taking the velocity part of the fluctuation into account. The $0-i$ component of the perturbed Einstein Equations is:

$$(\Phi' + \mathcal{H}\Phi)_i = 4\pi Ga (\rho + P) \delta u_i ,$$  \hspace{1cm} (14)

where “,$i$” denotes the covariant derivative with respect to the 3-dimensional metric $\gamma_{ij}$, and $\delta u_i$ is the velocity of perturbation in $i$ direction. Ignoring the anisotropy of the perturbation and assuming $\delta u_1 = \delta u_2 = \delta u_3 = \delta u$, and presenting the perturbation in its Fourier form, we have:

$$|\delta u(k)| = \frac{k(\Phi'(k) + \mathcal{H}\Phi(k))}{4\pi Ga (\rho + P)} .$$  \hspace{1cm} (15)

It is useful to define the comoving curvature perturbation:

$$\zeta = \Phi + \frac{\mathcal{H}}{H^2 - \mathcal{H}'} (\Phi' + \mathcal{H}\Phi) ,$$  \hspace{1cm} (16)

which satisfies the equation

$$\zeta'' + 2\frac{z'}{z} \zeta + c_s^2 k^2 \zeta = 0 ,$$  \hspace{1cm} (17)

where $z \equiv \frac{aH}{\sqrt{H^2 - \mathcal{H}^2}}$. Furthermore, by assuming that in the whole process the non-adiabatic perturbation can be
At the Hubble crossing, we replace ignored 1, we have:

\[ \zeta' = -\frac{k^2(\Phi' + H\Phi)}{12\pi Ga^2(\rho + P)} , \]  

(18)

which shows that \( \zeta \) is conserved at large scales.

From 15 and 15, and considering the velocity perturbation in position space, \( \delta u(x) \sim k^2 \delta u(k) \), we have

\[ |\delta u(x)| = \frac{3a k^2 |\zeta'|}{2\pi} , \]  

(19)

which relates the dynamics of the fluctuations to the metric perturbation. In general, the fluctuation is not moving uniformly and with an acceleration \( a_u \). Thus from above, we can define the Unruh temperature of the perturbation to be:

\[ T_u = \frac{a_u}{2\pi} \left| \frac{\delta u(x)}{2\pi} \right| \sim \frac{1}{2\pi} \left(3a k^2 |\zeta'| \right) , \]  

(20)

where a dot means derivative with respect to cosmic time \( t \) and we used \( \frac{d}{dt} = \frac{d}{d\eta} \).

Since all the terms in 10 are of the same order of magnitude, then up to a coefficient of order one, we have:

\[ T_u = \frac{1}{2\pi} \left(3a^2 k^2 |\Phi'| \right) , \]

\[ \sim \frac{1}{2\pi} \left(3(\frac{\rho}{\rho + P}) k^2 \left(k^{-\frac{3}{2}} t^\frac{1}{3} \frac{1 - 3w_p}{2} \right) \right) , \]  

(21)

where Eq. 10 is taken into account. Then we have:

\[ T_u \sim k^{-1} t^\frac{1 - 3w_p}{-3w_p} \sim k^{-1} t^\frac{1 - 3w_p}{-3w_p} T_r . \]  

(22)

At the Hubble crossing, we replace \( t \) by \( \frac{1}{k^3} \) to get:

\[ T_u \sim k^{\frac{1 + 3w_p}{1 - 3w_p}} T_r . \]  

(23)

This is the relation between the temperature of the thermal gas fluctuation and its Unruh correction, and the relationship depends on the equation of state of the background\(^2\). For example, for the \( w = \frac{7}{3} \) case of which a scale invariant power spectrum can be obtained, we have \( p = \frac{2}{3(1 + w)} = \frac{1}{3} \) and thus \( T_u \sim k^{\frac{4}{3}} T_r \).

This is our main result in this paper. We conclude that the Unruh Temperature caused by thermal fluctuations scales as the original temperature times a term that has positive power law of \( k \). This means that in the region of large \( k \), the Unruh effect will get larger and have a magnificant backreaction on the fluctuation.

Next We will roughly estimate how the backreaction would be. From the above relation (23) between \( T_r \) and \( T_u \), we can similarly parameterize \( T_u \sim a^{-3w_p} \), where one can calculate easily that at the Hubble crossing \( w_r' = w_r - \frac{1 - 3w_p}{4p} \). At the large \( k \) region where \( T_u \) dominates, one can get the dependence of the power spectrum on \( k \) as what has been done at the above, \( P_{\Phi}(k) \sim k^{-\frac{1 - 3w_p}{2 - 3w_p}} \). For our case \( p = \frac{1}{3} \) and \( w_r = \frac{1}{3} \), we can get \( w_r' = \frac{1}{3} \), and thus \( P_{\Phi}(k) \sim k^{\frac{2}{3}} \). From this we can see, at the large \( k \) region, the power spectrum will get an indication of blue tilt, which is in consistent with the case of vacuum initial conditions.

**SUMMARY**

In this short note, we reinvestigated the thermal fluctuations generated during bouncing cosmology, taking into account the corrections from Unruh effect. Our calculation shows that for the original case of scale invariant power spectrum, the additional correction leads to a blue tilt at large \( k \) region, which is in consistent with the case of vacuum initial condition as previously discussed.

Unruh effect is of great importance in its own right and has been applied as a tool to investigate other phenomena such as the thermal emission of particles from black holes and cosmological horizons. However, its applications on thermodynamics of cosmological perturbations is not clear yet. Whether it will give rise to more corrections or modifications to the early universe is interesting and worth studying. It can also been applied to the Holographic Cosmology, cf. the recently released paper by E. Verlinde who argues that the Einstein Gravity is emergent from thermodynamics and holographic principle. These extensions of this project are expected to be done in the future work.

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Appendix A

The appendix aims at clarifying the basic foundations of background and perturbation evolution for our analysis. In the framework of Friedmann-Robertson-Walker (FRW) geometry and taking the assumption of some constant equation of state \( w \) for the background, the scale factor of the universe scales as:

\[
a(t) \sim t^p ,
\]

where

\[
p = \frac{3(1+w)}{2},
\]

and the Hubble parameter becomes:

\[
H = \frac{p}{t} .
\]

Thus from the conditions for the comoving wave mode \( k \) at Hubble crossing, we have

\[
k_* = a(t_*)H(t_*) \sim t_*^{-1} .
\]

Then we briefly review the perturbation evolution through the bounce. From the perturbed Einstein equation, we can get the equation for \( \Phi \) as:

\[
\Phi'' + \frac{k^2}{a^2} \Phi + \frac{4H''H - 4H'H'}{H^2 - H'} \Phi + \frac{2H'}{H^2 - H'} \Phi = 0 .
\]

The equation has a general solution of \( \Phi(k, \eta) \) which can be parameterized as:

\[
\Phi(k, \eta) = D(k)_+ + S(k)_+ \eta^{-2\nu} ,
\]

where \( \nu = \frac{5 + 3w}{2(1+w)} \) and \( \pm \) refers to modes in the expanding (contracting) phase. Applying the Hwang-Vishniac [42] (Dueruelle-Mukhanov [43]) conditions which match modes before and after the bounce, we can get the dominant mode for the expanding phase turns out to be [44–47]:

\[
D_+ = O(1)D_- + O(1)(\frac{k}{k_*})^2 S_- .
\]

The spectrum in the expanding phase can only be given by the \( D_- \) mode [33], which indicates that

\[
\mathcal{P}_\Phi(k) \sim \mathcal{P}_{D_-}(k) .
\]

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