A Rough Approximation of Fuzzy Soft Set-Based Decision-Making Approach in Supplier Selection Problem

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**ABSTRACT**

Nowadays, supplier selection process, a multicriteria decision-making problem, has become one of the most indispensable parts for every purchasing sector for the improvement of performances of business operations. Most of the literatures in this field have considered only the opinion of decision-makers. But in fact, each company has its own opinion about the suppliers. The purpose of this paper is to select the best supplier by integrating the opinions of both decision-makers and company's stake holders. In this literature, these opinions are taken as fuzzy soft sets. These two fuzzy soft sets are then integrated by the rough approximation theory. The attributes in this literature are taken in the form of linguistic variable. At the end of this paper, a case study is given to illustrate the proposed method for selecting the best supplier.

**1. Introduction**

As information technology develops and economic market faces globalisation, a well-planned and well-executed supply chain management (SCM) system plays an important role in amplifying the competitive advantage. Basically, SCM is the integration of key business process from the end user to the original supplier, and it provides product service and information that add value for customers. Therefore, it is very important for all companies to have long relationship with few reliable suppliers. The success of a company is highly dependent on the selection of proper suppliers. So, supplier selection problem (SSP) is an important part of SCM. Selecting right suppliers extensively reduces the material purchasing cost and improves corporate competitiveness. Dickson [1] in his literature identified 23 criteria that have been considered by purchasing managers in different situations. The SSP involves trade-off between multiple criteria that are both qualitative and quantitative in nature. Hence, supplier’s selection problem is a multiple criteria decision problem and it is necessary to make a trade-off between conflicting tangible and intangible factors to find the best suppliers.
Researchers have proposed various methods for selecting the suppliers. The most commonly used model for SSP is data envelopment analysis (DEA). Assessing the efficiencies of alternatives (preferences) is the main concept of the DEA model. The DEA model was introduced by Liu et al. [2] to choose the supplier having greater supply diversity by evaluating three input criteria and two output criteria. This model and its enhanced form were carried out in agricultural, construction equipment manufacturing company, nuclear power station and electronics manufacturing company. But, researchers may have some problems with the application of the DEA model, such as they are confused while dealing with the number of input and output criteria. Also, the classification of the qualitative criteria is based on the intrinsic choice.

The quantitative criteria were taken into account by mathematical programming (MP) models. In this context, Talluri and Narasimhan [3] is the first group of researchers to consider the performance changeability measure. They developed two linear programming models to maximise and minimise the performance of a supplier against the best target measure set by the buyer. In his research work, Ng [4] developed a weighted linear programming model for the supplier selection problem with an objective of maximising the supplier score. Talluri [5] developed a binary integer linear programming model to choose a group of bids in respect to a company’s limitations. Hong et al. [6] presented a mixed-integer linear programming model for the SSP, based on the maximisation of the revenue. Ghodsypour and O’Brien [7] formulated a mixed-integer non-linear programming model by considering three constraints and the aim of their work was to determine the optimal allocation of products to suppliers so that the total annual purchasing cost could be minimised. A goal programming model to supplier selection problem was presented by Karpak et al. [8]. The aim of the model containing three goals, including cost, quality and delivery reliability, was to determine the optimal amount of products ordered, while subjecting it to buyer’s demand and supplier’s capacity constraints. Narasimhan et al. [9] and Wadhwa et al. [10] modelled the SSP as a multi-objective programming problem to select the optimal suppliers and determine the optimal order quantity.

A web-based analytic hierarchy process (AHP) method was presented by Akarte et al. [11] in an automobile casting concerning 18 criteria to choose suppliers. Muralidharan et al. [12] proposed a five-step AHP-based model to assist decision-makers in rating and selecting suppliers with respect to nine criteria in a bicycle manufacturing company. An interactive selection model with AHP was developed by Chan [13] to alleviate decision-makers in selecting the suppliers. His proposed model incorporated a method called chain of interaction, which was arranged to determine the relative importance of evaluating criteria without subjective human judgement. A similar work was done by Liu and Hai [14]. To determine the relative importance ratings among the criteria and sub-factors, they used Noguchi’s voting and ranking method in place of AHP’s pairwise comparison. In 2007, Chan and Kumar [15] developed a decision-making problem containing 14 criteria for SSP based on AHP and a sensitivity analysis was performed by them.

Choy et al. [16] presented a model based on the case-based reasoning technique for the supplier selection process. In this work, various evaluating criteria were grouped into three categories: technical capability, quality system and organisational profile. Only by changing the supplier’s evaluating factors, a similar work was done by many researchers such as Choy et al [17–21]. Considering the internal interdependency between supplier evaluating
factors, Sarkis and Talluri [22] applied the analytic network process (ANP) method for the supplier selection process. Bayazit [23] and Gencer et al. [24] implemented an ANP model to tackle the supplier selection problem.

Chen et al. [25] introduced a hierarchy model based on the fuzzy set theory and in their model they used a linguistic variable to assess the ratings and weights for the supplier evaluating factors. To tackle the imprecision involved in numerous subjective characteristics of suppliers, Sarkar and Mohapatra [26] used the fuzzy set approach and adopted a hypothetical case to illustrate how the two best suppliers were selected with respect to four performance-based and ten capability-based factors. Kaharaman et al. [27] applied a fuzzy AHP to select the best supplier and used a linguistic variable for evaluating suppliers’ criteria. A fuzzy AHP and the fuzzy synthetic extent analysis method were used by Chan and Kumar [28]. In their work, Bottani and Rizzi [29] used a fuzzy logic and developed an integrated approach involving the cluster analysis and AHP to group and rank alternatives for the supplier selection process. To assist the expert to define a complete rule set for evaluating the supplier performance, Jain et al. [30] suggested a fuzzy-based approach and used a genetic algorithm for the supplier selection process. Ramanathan [31] in his work suggested that DEA could be used to evaluate the performance of suppliers using both quantitative and qualitative information obtained from the total cost of ownership and AHP. Sevkli et al. [32] also applied an integrated AHP-DEA approach for the supplier selection process. In this approach, AHP was first used to evaluate the performance of suppliers with respect to quantitative factors. Then, the quantitative criteria along with the scores for each supplier calculated by AHP were passed to DEA and artificial neural network to measure the performance efficiency of each supplier.

Besides the said methods, there are several new methods introduced by researchers in the field of SSP. Boran et al. [33] developed a three-step, multi-criteria model for evaluating the environmental performance of suppliers rank of the suppliers through the fuzzy TOPSIS (Technique for Ordered Performance by Similarity to Ideal Solution) method. Also, Awasthi et al. [34] proposed the combination of the TOPSIS method with intuitionistic fuzzy set for choosing a right supplier in the group decision-making procedure. Other extended work on the TOPSIS method was done by Jadidi et al. [35], Sreekumar et al. [36], Mukherjee et al. [37] and many other researchers. A more detailed review can be found in [38–40].

The supplier selection process is considered as a multi-criteria decision-making problem with informational uncertainties. To tackle this type of uncertainties, probability theory, fuzzy set theory [41], rough set theory [42], and other mathematical tools are well known and used in different works. But, each of these theories has its inherent difficulties as pointed out by Pawlak in [43]. In 1999, Molodstov [44] introduced the concept of soft sets, a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting the existing methods. It has been found that the mathematical tools dealing with uncertainties like soft set theory, fuzzy set theory, rough set theory are closely related concepts [45, 46]. Maji et al. [47] defined fuzzy soft sets, combining soft sets with fuzzy sets as an extension of Molodstov’s soft sets. This way of investigation was further explored by several researchers [48, 49]. The application of soft sets in decision-making problem was initiated in [50]. Roy and Maji [51] presented a novel method of object recognition from an imprecise multi-observer data to address fuzzy soft set-based decision-making problems. Feng et al. [52] proposed an adjustable approach to (weighted) fuzzy soft
set-based decision-making using level soft sets. This approach was further investigated in [53, 54]. A fuzzy soft set-based SSP was initiated by Xiao et al. [55]. Feng et al. [56] provided a framework to combine fuzzy sets, rough sets and soft sets all together, which give rise to several interesting new concepts such as rough soft set, soft rough set, soft rough fuzzy set and fuzzy soft rough set. There also exist some other works in which the rough set theory is used with the other methodology. In their work, Xu et al. [57] formulated an inventory decision-making model in fuzzy rough environments and investigated the effect of imperfect quality items in the inventory problem. A DEA model (data envelopment analysis) with rough parameters was developed in the literature of Xu et al. [58]. In the process of solving the rough DEA model, they used the $\alpha$-optimistic and $\alpha$-pessimistic values of rough variable to transfer the rough model into deterministic linear programming. In their study, Huang et al. [59] combined the grey theory and the rough set theory with the moving average autoregressive exogenous prediction model to create an automatic stock market forecasting and portfolio selection mechanism. Chang et al. [60] developed a decision-making rule in the supplier selection problem by using the rough set theory with the help of a set of questionnaire. In their literature, Banerjee et al. [61] proposed an intelligent and optimised scheme to solve the parking space problem for a small city (e.g. Mauritius) using a reactive search technique (named as Tabu Search) assisted by the rough set.

The main factor of each group decision-making is the preferences determined by decision-makers for alternatives concerning each criterion. The literature review explains that the opinions of experts are in the centre of many literatures. If some errors are involved in the expert’s comments, then that will influence the computational procedure of the method. In this context, our proposed method is new. The proposed method considers expert’s comments as well as company’s view on the alternatives.

The aim of this paper is to present a suitable method for the supplier selection using the rough approximation of a fuzzy soft set. Besides the decision of experts, the opinion of company’s stake holder is also considered here in this paper. This study attempts to present a useful approach that will be more compatible with the real world.

The rest of the paper is organised as follows: Section 2 is full of information about the basic of fuzzy set, soft set, fuzzy soft set and rough set. In Section 3, there are two subsections. In Section 3.1, rough approximations of fuzzy soft sets are defined and in Section 3.2, the defined approach is applied to the supplier selection problem. A numerical example is provided in Section 4 to illustrate the methodology. A comparison with the other relevant methods is given in Section 5 and finally a conclusion is given in Section 6.

### 2. Preliminaries

Let us have a look at the following definitions:

**Fuzzy number** :- Let $\mathbb{R}$ be a set of real numbers, the fuzzy number $\tilde{r}$ is a mapping $\mu_{\tilde{r}} : \mathbb{R} \rightarrow [0, 1]$ with the following properties:

1. $\mu_{\tilde{r}}$ is upper semi-continuous.
2. $\tilde{r}$ is a convex fuzzy set, i.e. $\mu_{\tilde{r}}(\lambda x + (1 - \lambda)y) \geq \min\{\mu_{\tilde{r}}(x), \mu_{\tilde{r}}(y)\}, \forall x, y \in \mathbb{R}, \lambda \in [0, 1]$.
3. $\tilde{r}$ is normal i.e. $\exists x_0 \in \text{such that} \mu_{\tilde{r}}(x_0) = 1$.
4. Closure of the support of $\tilde{r}$ is compact i.e. $\text{cl}(\text{supp}(\tilde{r}))$ is compact.
Triangular Fuzzy Number (TFN) :- A TFN $\tilde{A}$ is defined as $\tilde{A} = (a_1, a_2, a_3)$ and its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & x \in [a_1, a_2] \\
\frac{a_1-x}{a_3-a_2}, & x \in [a_2, a_3] \\
0, & \text{otherwise}
\end{cases}$$

The primary operations on TFN are as follows:

If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two TFNs then

(i) Addition of two fuzzy numbers is defined as

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

(ii) Multiplication of two fuzzy numbers is defined as

$$\tilde{A} \otimes \tilde{B} \approx (a_1b_1, a_2b_2, a_3b_3)$$

(iii) Multiplication of a scalar and a fuzzy number is defined as

$$c \otimes \tilde{A} = (ca_1, ca_2, ca_3)$$

Defuzzification :- In the decision-making problem to reach a certain point in respect of a decision, a crisp number must be needed. The process in which fuzzy numbers are transformed into a crisp real number is called defuzzification. There are many methods of defuzzification. But in this literature, the signed distance method [62] is used for its simplicity. If $\tilde{A} = (a_1, a_2, a_3)$ is a TFN, then by the signed distance method defuzzification of $\tilde{A}$ is given by

$$d(\tilde{A}) = \frac{1}{2} \int_0^1 [\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)]d\alpha, \text{ where } \alpha \in [0, 1]$$

$$\tilde{A}_L(\alpha) = a_1 + (a_2 - a_1)\alpha, \quad \tilde{A}_R(\alpha) = a_3 - (a_3 - a_2)\alpha$$

so, $d(\tilde{A}) = \frac{1}{4}(a_1 + 2a_2 + a_3)$

Linguistic variable :- Some variables come front in the process of decision-making. They may be quantitative or qualitative. By taking numeric quantities, the quantitative variables can be handled. But for qualitative variables, precise numeric values cannot be used as uncertainty exists therein. To tackle such type of variables, linguistic variable are used in many literatures. Linguistic variable is a function whose elements of the domain are linguistic terms such as very poor, poor, medium, good, very good and whose elements of the range are a real number with a specific range i.e. the elements may be in the form of triangular fuzzy number, trapezoidal fuzzy number, interval valued fuzzy number, grey number etc. Let us consider the linguistic variable ‘Quality of the material’ supplied by the suppliers in a tabular form depicted in Table 1.

Soft sets and fuzzy soft set :- In order to deal with many complicated problems in the fields of engineering, social science, economics and medical science, etc. involving uncertainties, Molodstov [44] pointed out that probability theory, Fuzzy set theory, intuitionistic
Table 1. Representation of linguistic variable.

| Linguistic term       | Representation of linguistic term as TFN |
|-----------------------|------------------------------------------|
| Very poor             | (10,10,20)                               |
| Poor                  | (20,30,40)                               |
| Medium poor           | (30,40,50)                               |
| Fair                  | (40,50,60)                               |
| Medium good           | (50,60,70)                               |
| Good                  | (60,70,80)                               |
| Very good             | (80,100,100)                             |

Table 2. Representation of the soft sets.

|           | \(e_2\) | \(e_3\) | \(e_4\) |
|-----------|---------|---------|---------|
| \(H_1\)  | 1       | 0       | 1       |
| \(H_2\)  | 1       | 1       | 0       |
| \(H_3\)  | 0       | 0       | 1       |
| \(H_4\)  | 0       | 1       | 1       |
| \(H_5\)  | 1       | 0       | 1       |

fuzzy set theory, rough set theory etc., which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. The reason for these difficulties is possibly the inadequacy of the parameterisation tool of the theory. To deal with the uncertainties, he proposed a new theory, soft set theory, which is claimed to be free of the earlier difficulties.

A pair \((F,E)\) is called a soft set (over the Universe of discourse \(U\)) if and only if \(F\) is a mapping of \(E\) into the set of all subsets of \(U\).

In other words, the soft set is a parameterised family of subsets of the set \(U\). Every set \(F(\varepsilon), \varepsilon \in E\), from this family may be considered as the set of \(\varepsilon\) elements of the set \((F,E)\), or as the set of \(\varepsilon\) approximate elements of the soft set.

Example 2.1: Let \(U = \{H_1, H_2, H_3, H_4, H_5\}\) be the set of five households in a rural community and \(E = \{e_1\) (poor household status), \(e_2\) (poor monetary status), \(e_3\) (poor clothing status), \(e_4\) (poor housing status)\} be the set of parameters and \(A = \{e_2, e_3, e_4\}\).

Then \((F,A) = \{F(e_2) = \{H_1, H_2, H_5\}, F(e_3) = \{H_2, H_4\}, F(e_4) = \{H_1, H_3, H_4, H_5\}\}\) is the soft set representing the ‘Poverty of a Household’ in that community. This soft set can be represented in a tabular form as shown in Table 2.

A pair \((F,A)\) is called a fuzzy soft set over \(U\) where \(F: A \rightarrow \tilde{P}(U)\) is a mapping from \(A\) to \(\tilde{P}(U)\) and \(\tilde{P}(U) = \) set of all fuzzy subsets of \(U\).

Example 2.2: Let us consider the previous example.

Then \((F,A) = \{F(e_2) = \{H_1 /0.25, H_2 /0.35, H_3 /0.50, H_4 /0.90, H_5 /0.15\}, F(e_3) = \{H_1 /0.50, H_2 /0.50, H_3 /0.25, H_4 /0.60, H_5 /0.50\}, F(e_4) = \{H_1 /0.25, H_2 /0.10, H_3 /0.50, H_4 /0.30, H_5 /0.90\}\}\) is the fuzzy soft set representing the poor households in the community.

Rough sets and rough approximation :- The rough set theory is regarded as an alternative mathematical tool for imperfect data analysis. As we all are aware of the fact that the problem of imperfect knowledge has been tackled for a long time by philosophers, logicians
and mathematicians, now it has also been a crucial issue for computer scientists, particularly in the area of artificial intelligence (AI). The rough set theory, proposed by Pawlak [43], presents still another attempt to this problem. The theory has attracted attention of many researchers and practitioners all over the world, who contributed essentially to its development and applications.

From theoretical point of views, the rough set theory has an overlap with many other theories. However, we will desist to discuss these connections here. But, it may be considered as the independent discipline in its own rights.

It is based on the assumption that with every object of the Universe of discourse, some information is associated. Objects are indiscernible or similar in view of the same information. The indiscernibility relation generated in this way is the mathematical basis of the rough set theory. The set of such objects is called an elementary set. It forms a basic granule of knowledge about the Universe of discourse. Any union of the elementary sets is referred to as a crisp set; otherwise, the set is treated as rough. In contrast to the fuzzy set theory, the rough set theory expresses vagueness, not by means of membership, but employing a boundary region of a set. If the boundary region of a set is empty, it means that the set is crisp, otherwise the set is rough (inexact). Non-empty boundary region of a set means that the available knowledge about the set is not sufficient to define the set precisely.

This theory finds many interesting applications. The rough set approach seems to be of fundamental importance to AI and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, and decision analysis, knowledge discovery from databases, expert systems, inductive reasoning and pattern recognition.

The main advantage of the rough set theory in data analysis is that it does not need any preliminary or additional information about data – like probability in statistics, or basic probability assignment in the Dempster–Shafer theory, grade of membership or the value of possibility in the fuzzy set theory.

Similar to the fuzzy set theory, it is not an alternative to the classical set theory, but it is embedded in it. The rough set theory can be viewed as a specific implementation of Frege’s [63] idea of vagueness, i.e. imprecision in this approach is expressed by a boundary region of a set, and not by a partial membership, like in the fuzzy set theory. In contrast to crisp sets, the rough set theory cannot be characterised by means of information about their elements. Basically, they are exemplified by some decision rules – certain and uncertain. Roughly, it can be argued that the certain decision rules describe lower approximation of decisions and the uncertain decision rules refer to the boundary region of the decisions.

The rough set concept can be defined quite generally by means of topological operations, interior and closure, called approximations. Suppose, we are given a set of objects $U$ called the universe and an indiscernibility relation $R \subseteq U \times U$, representing the lack of knowledge about elements of $U$. For the sake of simplicity, let us assume that $R$ is an equivalence relation. Let $X$ be a subset of $U$. The aim is to characterise the set $X$ with respect to $R$. To this end, we will need the basic concepts of the rough set theory given below.

Let $U$ be the universe of discourse of non-empty finite set of objects and $A$ be the non-empty finite set of attributes, then the pair $I = (U, A)$ is said to be an information system. For any $B \subseteq A$, there exists an equivalence (indiscernibility) relation $R(B)$ such that $R(B) = \{(x, y) \in U \times U : \text{for every } a \in B, a(x) = a(y)\}$, where $a(x)$ denotes the value of attribute $a$ for the element $x$. $R(B)$ is called the B-indiscernibility relation having equivalence classes $[x]_B$. 

For an information system \( I(U, A) \) any \( B \subseteq A, X \subseteq U \) can be approximated based on information contained in \( B \) into \( B \)-lower approximation and \( B \)-upper approximation (Pawlak [43]) of \( X \) which are represented as \( B(X) \) and, respectively, where

\[
B(X) = \{ x : [x]_B \subseteq X \} \quad \text{and} \quad \bar{B}(X) = \{ x : [x]_B \cap X \neq \emptyset \}
\]

It can be noted that the objects in \( B(X) \) are characterised as the certain members of \( X \) based on the knowledge contained in \( b \), whereas objects in \( \bar{B}(X) \) are classified as the possible members of \( T \) based on the knowledge contained in \( \bar{B} \). The \( B \)-boundary region of \( X \) is expressed as

\[
BN_B (X) = \bar{B}(X) - B(X)
\]

If the underlying set is crisp in nature, then the boundary region of the set is a null set, since the corresponding lower and upper approximation set of the crisp set are equal sets. A set is a rough set if the boundary region is nonempty, i.e. if \( BN_B(X) \neq \emptyset \), then \( X \) is said to be a rough with respect to \( B \).

3. Methodology

In this section, at first rough approximations of fuzzy soft sets are introduced and the approach is applied to a supplier selection problem.

3.1. Rough Approximation of Fuzzy Soft Sets

In this portion of the literature, rough approximations of fuzzy soft sets having membership values in linguistic term represented by the triangular fuzzy number are introduced.

Let \( H = \{ H_1, H_2, H_3, \ldots, H_n \} \) be a set of alternatives and \( C = \{ A_1, A_2, A_3, \ldots, A_m \} \) be a set of parameters and \( A = \{ A_1, A_2, \ldots, A_k \} \) be a subset of \( C, k < m \).

Let \( F : A \rightarrow \bar{P}(H) \) be a mapping, then the fuzzy soft set \((F, A)\) will be in the form of \((F, A) = \{ F(A_1) = \{ H_1/v_{11}, H_2/v_{12}, H_3/v_{13}, \ldots, H_n/v_{1n} \}, F(A_2) = \{ H_1/v_{21}, H_2/v_{22}, H_3/v_{23}, \ldots, H_n/v_{2n} \}, \ldots, F(A_k) = \{ H_1/v_{k1}, H_2/v_{k2}, H_3/v_{k3}, \ldots, H_n/v_{kn} \} \} \). where \( v_{ij} \) is the linguistic variable represented by a triangular fuzzy number \( (v_{ij}^1, v_{ij}^2, v_{ij}^3) \), \( i = 1, 2, \ldots, k; j = 1, 2, \ldots, n \).

Now let us consider a set of decision-makers \( E = \{ E_1, E_2, E_3, \ldots, E_p \} \), appointed to express their decision on the different alternatives in the linguistic form by considering all the parameters.

Let \( G : E \rightarrow \bar{P}(H) \) be a mapping, then the fuzzy soft set \((G, E)\) can be expressed as \((G, E) = \{ G(E_1) = \{ H_1/u_{11}, H_2/u_{12}, H_3/u_{13}, \ldots, H_n/u_{1n} \}, G(E_2) = \{ H_1/u_{21}, H_2/u_{22}, H_3/u_{23}, \ldots, H_n/u_{2n} \}, \ldots, G(E_p) = \{ H_1/u_{p1}, H_2/u_{p2}, H_3/u_{p3}, \ldots, H_n/u_{pn} \} \} \). where \( u_{ij} \) is the linguistic variable represented by a triangular fuzzy number \( (u_{ij}^1, u_{ij}^2, u_{ij}^3) \), \( i = 1, 2, \ldots, p; j = 1, 2, \ldots, n \).

Now the novel approach, rough approximation of fuzzy soft set \((G, E)\) has been done by using the fuzzy soft set \((F, A)\) and this process gives two fuzzy soft sets named as lower approximation \((G_L, E)\) and upper approximation \((G_U, E)\) and they are defined as follows:

Lower approximation \( G_L = \{ G_L(E_1) = \{ H_1/\alpha_{11}, H_2/\alpha_{12}, H_3/\alpha_{13}, \ldots, H_n/\alpha_{1n} \}, \ldots, G_L(E_p) = \{ H_1/\alpha_{p1}, H_2/\alpha_{p2}, H_3/\alpha_{p3}, \ldots, \} \}

...
Let us consider the example considered in Example 2.2 with the linguistic variable from Table 1.

Example 3.1: Let us consider the example considered in Example 2.2 with the linguistic variable from Table 1 \((F, A) = (F(e_2) = \{H_1/P, H_2/MP, H_3/M, H_4/F, H_5/VP\}, F(e_4) = \{H_1/P, H_2/VP, H_3/F, H_4/P, H_5/FG\})\) is the fuzzy soft set representing the poor households in the community.

By considering all the parameters, let an expert \(E_1\) express his/her opinion as a fuzzy soft set represented as \((G, E_1) = (G(E_1) = \{H_1/F, H_2/MP, H_3/M, H_4/G, H_5/F\})\). \((G, E_1)\) by using \((F, A)\) the two fuzzy soft sets will be \((G_L, E) = (G_L(E_1) = \{H_1/\gamma_1, H_2/\gamma_2, H_3/\gamma_3, H_4/\gamma_4, H_5/\gamma_5\})\).

where

\[
\begin{align*}
\gamma_1 & = \bigcup \{(20, 30, 40), (40, 50, 60)\} = (20, 40, 60). \\
\gamma_2 & = \bigcup \{(10, 10, 20), (30, 40, 50)\} = (10, 25, 50). \\
\gamma_3 & = \bigcup \{(20, 30, 40)\} = (20, 30, 40). \\
\gamma_4 & = \bigcup \{(40, 50, 60), (50, 60, 70), (20, 30, 40)\} = (20, 70, 70). \\
\gamma_5 & = \bigcup \{(10, 10, 20), (40, 50, 60)\} = (10, 30, 60). \\
\end{align*}
\]

\((G^U, E) = (G^U(E_1) = \{H_1/\delta_1, H_2/\delta_2, H_3/\delta_3, H_4/\delta_4, H_5/\delta_5\})\).

where

\[
\begin{align*}
\delta_1 & = \bigcup \{(20, 30, 40), (40, 50, 60)\} = (20, 40, 60). \\
\delta_2 & = \bigcup \{(30, 40, 50), (50, 60, 70)\} = (30, 50, 70). \\
\delta_3 & = \bigcup \{(40, 50, 60)\} = (40, 50, 60). \\
\delta_4 & = \bigcup \{(40, 50, 60), (50, 60, 70)\} = (40, 55, 70). \\
\delta_5 & = \bigcup \{(40, 50, 60)\} = (40, 50, 60). \\
\end{align*}
\]

Some properties of the lower and upper fuzzy soft approximations are now discussed and are illustrated by a simple example.

Let us first consider two linguistic terms having a triangular fuzzy number representation \(T_a = (a^1, a^2, a^3)\) and \(T_b = (b^1, b^2, b^3)\).
Now if $a^3 > b^3$, then $T_a > T_b$.
If $a^3 = b^3$, then we say that $T_a > T_b$ if $a^1 > b^1$.
and finally $a^3 = b^3$ and $a^1 = b^1$, then $T_a > T_b$ if $a^2 > b^2$.

**Property 3.1:** The lower approximation of an alternative $A_j$ with respect to an Expert $E_l$ will be unaltered from the corresponding decision of the expert if $u_{lj} \leq \min\{v_{1j}, v_{2j}, \ldots, v_{lj}\}$.

In the example, the alternative $H_3$ has received the decision $P$ from $E_1$ and the company has rated $H_3$ with respect to all the attributes by $F, P, F$. Clearly $P = \min\{F, P, F\}$ as discussed in this section. So, it will get the lower approximation value as the TFN of $P$, i.e. $(20, 30, 40)$.

**Property 3.2:** The upper approximation of an alternative $A_j$ with respect to an expert $E_l$ will be unaltered from the corresponding decision of the expert $E_l$ if any of the following conditions hold:

(i) If $v_{1j}^1 \leq u_{lj}^1$, then we must have $v_{3j}^3 \leq u_{lj}^1$ (where $j$ is fixed and $i$ varies over 1 to $k$).
(ii) If $v_{1j}^1 \leq u_{lj}^1$, then we must have $v_{1j}^1 \leq u_{lj}^3$ (where $j$ is fixed and $i$ varies over 1 to $k$).
(iii) $v_{ij} = u_{lj}$ (where $j$ is fixed and $i$ varies over 1 to $k$).

In the given example, the alternative $H_5$ has received the decision $F$ from $E_1$ and the company has rated $H_4$ with respect to all the attributes by VP, F, VG. Here VP satisfies condition (i), VG satisfies condition (ii) and F satisfies condition (iii). So, it will get the upper approximation value as the TFN of $F$ i.e. $(40, 50, 60)$.

**Property 3.3:** Two same valued entries in a column of the matrix of experts opinion must gather same result in the matrix of the lower approximation, i.e. $u_{lj} = u_{lj} \Rightarrow \alpha_{lj} = \alpha_{lj}$.

It follows from the definition of lower approximation.

**Property 3.4:** Two same valued entries in a column of the matrix of experts opinion must gather the same result in the matrix of the upper approximation, i.e. $u_{lj} = u_{lj} \Rightarrow \beta_{lj} = \beta_{lj}$.

It follows from the definition of upper approximation.

**Property 3.5:** The value of the lower approximation of an alternative with respect to an expert is always less than or equal to the corresponding value of the Upper approximation, i.e. $\alpha_{lj} \leq \beta_{lj} \forall l, j$.

It follows from the definitions of upper approximation and lower approximation.

### 3.2. Rough Approximation of Fuzzy Soft Sets Approach to Supplier Selection Problem

In this section, the rough approximation of fuzzy soft sets approach introduced in the previous section is applied to a supplier selection problem.
Let us consider $S = \{S_1, S_2, \ldots, S_n\}$ be a set of $n$ suppliers, $X = \text{set of all attributes}$ and let $A = \{A_1, A_2, \ldots, A_m\} \subseteq X$ be a set of $m$ attributes on which decision has to be made. To rank the suppliers, the following steps are to be used in this literature.

Step 1
At first company forms the original Linguistic terms as fuzzy soft variables like ‘Effective Suppliers’ $U = (F, A)$ over $S$ from the data they have from different suppliers and previous records. Let $F(A_i)$ denote the component fuzzy soft set for the $i$th attribute. So, a total number of $m$ such component fuzzy soft sets are formed and are arranged in a tabular form in Table 3. The entries of this table are TFN representing the linguistic variables assigned by the company to each supplier for each attribute. If $\tilde{a}_{ij}$ is the $(i,j)$th entry of the table, then $\tilde{a}_{ij}$ = linguistic variable given for $i$th attribute for $j$th supplier by the company and is represented as $\tilde{a}_{ij} = (\tilde{a}_{1ij}, \tilde{a}_{2ij}, \tilde{a}_{3ij})$, a TFN.

Step 2
An expert group $E = \{E_1, E_2, E_3, \ldots, E_p\}$ containing $p$ experts is appointed to evaluate the elements of $S$ by considering the members of $A$. Thus, the decision is a fuzzy soft set $V = (G, E)$ over $S$. Let the component fuzzy soft set for the $i$th expert is $G(E_i)$ and so $p$ numbers of such component of fuzzy soft sets are formed and are arranged in a tabular form in Table 4. The entries of this table are also TFN representing the linguistic variables assigned by the experts to the suppliers. If $\tilde{e}_{ij}$ is the $(i,j)$th entry of the table, then $\tilde{e}_{ij}$ = the linguistic variable decided by $i$th expert for $j$th supplier and is represented as $\tilde{e}_{ij} = (\tilde{e}_{1ij}, \tilde{e}_{2ij}, \tilde{e}_{3ij})$, a TFN. For simplicity each evaluation result of experts is considered with equal importance.

Step 3
To reach at a concrete conclusion and to avoid the sole dependency on experts, the information given by the company and the decision of the experts are both considered here and a novel method is formed. A rough approximation is made on fuzzy soft sets $V$ using the fuzzy soft set $U$. By the rough approximation of fuzzy soft sets, two new fuzzy soft sets are evaluated and they are $V_L = (G_L, E)$ and $V_U = (G_U, E)$. If $G_L(E_i)$ and $G_U(E_i)$ are the component fuzzy soft sets for the $i$th expert for the lower approximation and upper approximation,
Table 5. Lower approximation.

| $E_1$ | $\tilde{\beta}_{11}$ | $\tilde{\beta}_{12}$ | $\tilde{\beta}_{13}$ | $\tilde{\beta}_{1n}$ |
|-------|----------------------|----------------------|----------------------|----------------------|
| $E_2$ | $\tilde{\beta}_{21}$ | $\tilde{\beta}_{22}$ | $\tilde{\beta}_{23}$ | $\tilde{\beta}_{2n}$ |
| $E_3$ | $\tilde{\beta}_{31}$ | $\tilde{\beta}_{32}$ | $\tilde{\beta}_{33}$ | $\tilde{\beta}_{3n}$ |
| \vdots | $\tilde{\beta}_{p1}$ | $\tilde{\beta}_{p2}$ | $\tilde{\beta}_{p3}$ | $\tilde{\beta}_{pn}$ |

Table 6. Upper approximation.

| $E_1$ | $\tilde{\alpha}_{11}$ | $\tilde{\alpha}_{12}$ | $\tilde{\alpha}_{13}$ | $\tilde{\alpha}_{1n}$ |
|-------|-----------------------|-----------------------|-----------------------|-----------------------|
| $E_2$ | $\tilde{\alpha}_{21}$ | $\tilde{\alpha}_{22}$ | $\tilde{\alpha}_{23}$ | $\tilde{\alpha}_{2n}$ |
| $E_3$ | $\tilde{\alpha}_{31}$ | $\tilde{\alpha}_{32}$ | $\tilde{\alpha}_{33}$ | $\tilde{\alpha}_{3n}$ |
| \vdots | $\tilde{\alpha}_{p1}$ | $\tilde{\alpha}_{p2}$ | $\tilde{\alpha}_{p3}$ | $\tilde{\alpha}_{pn}$ |

respectively, then a total number of $p$ such components are formed for each approximation and are represented in Tables 5 and 6. If $\tilde{\alpha}_{ij}$ and $\tilde{\beta}_{ij}$ are the $(i, j)^{th}$ entry of Tables 5 and 6, respectively, then $\tilde{\alpha}_{ij}$ and $\tilde{\beta}_{ij}$ can be calculated as explained in Equations (1) and (2) in the previous section.

$\tilde{\alpha}_{ij} \in G_L(E_i) = \bigcup_{\lambda} \{\tilde{a}_{ij} \in F(A_{2}); \tilde{a}_{ij} < \tilde{e}_{ij} \in G(E_i)\}, \quad i = 1, 2, \ldots, p, \quad j = 1, 2, \ldots, n$

$\tilde{\beta}_{ij} \in G_U(E_i) = \bigcup_{\eta} \{\tilde{a}_{ij} \in F(A_{2}); \tilde{a}_{ij} \cap \tilde{e}_{ij} \neq \phi\}, \quad i = 1, 2, \ldots, p, \quad j = 1, 2, \ldots, n$

and are represented by TFN as $\tilde{a}_{ij} = (\tilde{a}_{ij}^1, \tilde{a}_{ij}^2, \tilde{a}_{ij}^3)$, $\tilde{\beta}_{ij} = (\tilde{\beta}_{ij}^1, \tilde{\beta}_{ij}^2, \tilde{\beta}_{ij}^3)$.

Step 4

From three fuzzy soft sets $V_L$, $V$ and $V_U$, three fuzzy sets $\tilde{W}_L$, $\tilde{W}$ and $\tilde{W}_U$ are formed and the corresponding membership functions of the fuzzy sets are defined as

$\mu_{\tilde{W}_L}(S_k) = M_L(S_k)$

$\mu_{\tilde{W}}(S_k) = M(S_k)$

$\mu_{\tilde{W}_U}(S_k) = M_U(S_k), \quad k = 1, 2, \ldots, n$

where

$M_L(S_k) = (\min_x \{\tilde{a}_{xk}^1\} \frac{1}{p} \sum_{y=1}^{p} \tilde{a}_{yk}^2, \max_z \{\tilde{a}_{zk}^3\})$

$M(S_k) = (\min_x \{\tilde{e}_{xk}^1\} \frac{1}{p} \sum_{y=1}^{p} \tilde{e}_{yk}^2, \max_z \{\tilde{e}_{zk}^3\})$

and $M_U(S_k) = (\min_x \{\tilde{\beta}_{xk}^1\} \frac{1}{p} \sum_{y=1}^{p} \tilde{\beta}_{yk}^2, \max_z \{\tilde{\beta}_{zk}^3\})$

$x, z = 1, 2, \ldots, p$
Step 5
From the three fuzzy sets $\tilde{W}_V$, $\tilde{W}_V$ and $\tilde{W}_V$, a fuzzy soft set $H = (\nu, C)$ is constructed over $S$, where $C = \text{Alternative with low confidence (ALC), Alternative with medium confidence (AMC), Alternative with high confidence (AHC)}$ is a set of attributes and $\nu_{\text{ALC}} = \mu_{\tilde{W}_V}, \nu_{\text{AMC}} = \mu_{\tilde{W}_V}, \nu_{\text{AHC}} = \mu_{\tilde{W}_V}$.

Now three weights $w_1$, $w_2$, and $w_3$ so that $w_1 + w_2 + w_3 = 1$ are estimated for ALC, AMC and AHC, which will be associated with $\mu_{\tilde{W}_V}$, $\mu_{\tilde{W}_V}$ and $\mu_{\tilde{W}_V}$, respectively.

Step 6
At this stage finally the score or weight of $k$th supplier $S_k$ is defined as

$$\xi_k = \xi(S_k) = w_1 \times \nu_{\text{ALC}} + w_2 \times \nu_{\text{AMC}} + w_3 \times \nu_{\text{AHC}}$$

(7)

Step 7
After getting the weight of each supplier in the form of TFN, the defuzzified technique [62] is used to make that crisp i.e. $D(\xi_k) = \tau_k$.

4. Case Study
In this section, the proposed approach is applied to a real case study, for a typical supplier selection problem.

ABP limited is one of the largest printing media companies in India. Daily newspapers (Anandabazar Patrika, The Telegraph) and different magazines (Sananda etc.) are main products of this company. We are focusing our view on the raw material (paper) needed for the newspapers and magazines. For the raw materials, the company needs suppliers. Four paper mills quoted their tenders for this. They are Emami paper mills (Andhra Pradesh, India), Jeangu paper mills (Korea), Andhra Pradesh paper Mills (Andhra Pradesh, India) and Pansia Paper (Newzealand). The supplied paper must be of 42 g and the dimension width $\times$ Diameter should be 700 $\times$ 1015. The printing company needs the best supplier that suits well with the four attributes: Product Quality ($A_1$), Service Quality ($A_2$), Delivery in time ($A_3$) and Price ($A_4$). A group of experts ($E_1$, $E_2$, $E_3$) have been appointed by the company and they have been asked to submit their decision in terms of linguistic terms on the rating of the suppliers. Also, the information, based on different attributes of the suppliers have been collected from the company’s stake holders.

Let the suppliers be $S_1$, $S_2$, $S_3$, $S_4$ and the scale used for linguistic variables for supplier rating is in Table 1.

Step 1
The company’s information in linguistic terms about the suppliers against the attributes is collected and is shown in Table 7.

| $S_k$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|-------|-------|-------|-------|-------|
| $S_1$ | H     | F     | MH    | H     |
| $S_2$ | F     | H     | MH    | F     |
| $S_3$ | MH    | H     | F     | VH    |
| $S_4$ | ML    | MH    | H     | F     |
Step 2

A group of three experts $E_1, E_2, E_3$ are appointed to evaluate the suppliers by considering the different attributes. The decision of the experts about the acceptance level of the suppliers is shown in Table 8.

Step 3

By combining the decision of experts and the information given by the company, two fuzzy soft set lower approximation and upper approximation are formed. The elements of these two fuzzy soft sets are calculated with the help of (1) and (2) and are depicted in Tables 9 and 10, respectively.

Step 4

From Tables 8–10, three fuzzy sets are formed by using (3)–(6) and are as follows:

$$
\mu(\tilde{W}_{V_{L}}) = \{(S_1, (40, 51.67, 70)), (S_2, (30, 49.16, 80)), (S_3, (40, 58.33, 80)), (S_4, (30, 45, 70))\} \\
\mu(\tilde{W}_{V}) = \{(S_1, (40, 53.33, 70)), (S_2, (30, 53.33, 80)), (S_3, (50, 66.67, 80)), (S_4, (30, 50, 70))\} \\
\mu(\tilde{W}_{V_{U}}) = \{(S_1, (40, 62.5, 80)), (S_2, (40, 56.11, 80)), (S_3, (40, 66.67, 100)), (S_4, (30, 53.33, 80))\}
$$

(8)

Steps 5 and 6

Now, in this particular case considering the weights 0.25, 0.5 and 0.25 for ALC, AMC and AHC, respectively, the scores of each supplier after defuzzification are as follows:

$$
\tau(S_1) = 56.25 \\
\tau(S_2) = 54.61 \\
\tau(S_3) = 64.79 \\
\tau(S_4) = 50.42
$$

---

**Table 8. Experts’ decision on acceptance level of suppliers.**

|      | $E_1$ | $E_2$ | $E_3$ |
|------|-------|-------|-------|
| $S_1$ | F     | MF    | F     |
| $S_2$ | F     | ML    | H     |
| $S_3$ | H     | MH    | H     |
| $S_4$ | ML    | F     | MH    |

**Table 9. Lower approximation of experts’ decision.**

|      | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|------|-------|-------|-------|-------|
| $E_1$ | (40,50,60) | (40,50,60) | (40,60,80) | (30,40,50) |
| $E_2$ | (40,55,60) | (30,40,50) | (40,55,70) | (30,45,60) |
| $E_3$ | (40,50,60) | (40,57.5,80) | (40,60,80) | (30,50,70) |

**Table 10. Upper approximation of experts’ decision.**

|      | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|------|-------|-------|-------|-------|
| $E_1$ | (40,62.5,80) | (40,57.5,80) | (40,70,100) | (30,50,70) |
| $E_2$ | (40,62.5,80) | (30,53.33,70) | (40,60,80) | (30,55,80) |
| $E_3$ | (40,62.5,80) | (40,57.5,80) | (40,70,100) | (30,55,80) |
So, the suppliers can be ranked as $S_3 > S_1 > S_2 > S_4$

Thus, $S_3$ be the preferable supplier for any company followed by $S_1$, $S_2$, $S_4$.

**Classification of the Suppliers**

It is also a challenging task to determine the certain line of separation on or above which, a supplier should be considered as non-poor and below which should be treated as poor. In the literature if we merely look into the approaches where soft computing techniques have been applied we find only few attempts for the same. The methodology is illustrated below.

In this method, we discard the concept of only one separation and propose for three lines of separation which lead to six disjoint intervals for Totally Poor (TP), Poor (P), Not so Poor (NSP), Good (G), Very Good (VG) and Excellent (E) groups of Suppliers. The advantage of considering these six groups of suppliers lies in the fact that the company can make proper decision for screening in their future correspondences. Otherwise, all suppliers of the groups TP, P, NSP, G, VG and E would lie in the same group and would be treated in the same manner by the company. Thus from the computational point of view, we require four intervals of values of $\tau_k$ for the six above-mentioned groups. To obtain these intervals, the maximum of each interval is computed.

At first, to obtain the maximum value of the $\tau_k$ of the group TP, a fictitious Supplier $S_{TP}$ is considered whose decisions with respect to all the attributes provided by the Company are assumed to be {P, P, P, P}. Also the decision vector for TP provided by the group of Experts is assumed to be {P, P, P}. This is based on the hypothesis that a totally poor Supplier can mostly get these ratings from both the company and the Experts. Here $\tau_k(S_{TP}) = 27.35$. Similarly, the maximum of the interval for the group Poor is calculated by considering the fictitious Supplier $S_P$ whose conditions with respect to the attributes are assumed to be {MP, MP, MP, MP} and {MP, MP, MP}. Here $\tau_k(S_P) = 38.11$. Again to find the maximum of the interval for the group Not So Poor, a fictitious Supplier $S_{NSP}$ is considered whose conditions are expressed by the decision vector {F, F, F, F} and {F, F, F}. Here $\tau_k(S_{NSP}) = 49.77$. In a similar way, the decision vectors for the fictitious suppliers G are assumed to be {MH, MH, MH, MH} and {MH, MH, MH} and those for the fictitious supplier VG are assumed to be {H, H, H} and {H, H, H}. Thus we obtain $\tau_k(S_G) = 56.83$ and $\tau_k(S_{VG}) = 72.81$. The maximum value of the Excellent group of suppliers is undoubtedly 100 as the maximum value of $\tau_k$ is 100. Thus we obtain the required intervals by this simple methodology. In our example, we observe that the suppliers $S_1$, $S_2$ and $S_4$ are in the group of ‘Good Suppliers’ and $S_3$ is in the group of ‘Very Good Suppliers’. This methodology can be applied for supplier selection problems where the alternatives are needed to be classified in different groups.

5. Comparative analysis

Before we conclude, let us compare the discussed method with some of the existing ones. We follow up some criteria for the evaluation process and see whether these approaches fulfil them or not. The method discussed here is claimed to be better in both computational simplicity and logical considerations. Normally most of the group decision-making techniques impose the ranking preferences determined by decision-makers for different alternatives about each criterion. In the literature review, it is quiet vivid that the decisions of the experts are the only important factors to be considered. Particularly in this
Table 11. Comparison with some other methods.

| Authors               | Whether Company’s information is considered or not | Whether classification of suppliers is included or not |
|-----------------------|-----------------------------------------------------|------------------------------------------------------|
| Muralidharan et al. [12] | No                                                  | No                                                   |
| Liu et al. [14]       | No                                                  | No                                                   |
| Mukherjee et al. [37] | No                                                  | No                                                   |
| Xiao [55]             | Yes                                                 | No                                                   |
| Chang et al. [60]     | No                                                  | No                                                   |
| Proposed method       | Yes                                                 | Yes                                                  |

circumstance, our proposed method is more effective as it studies the experts’ comments as well as the company’s view on the alternatives. In Table 11, such a comparison is displayed. The obtained results cannot be compared with the others as none of them has included the information or decision vector provided by the company about the suppliers.

6. Conclusion

Supplier selection is one of the most important problems in each manufacturing company. Companies, in order to increase the competitive advantage, must have an efficient supplier selection system. To this end, a novel supplier selection method based on rough approximation of fuzzy soft set was proposed in this literature in order to show the importance of decisions of experts and that of stakeholders of company. A case study of the selection problem of suppliers is used to illustrate the proposed approach. This proposed approach can help in more accurate selections in other field also, such as management and economic and other decision-making problems.

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