Mean and True Positions of Planets as Described in Gaṇitagannadī – A Karaṇa Text on Siddhāntic Astronomy in Kannada

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Mean and True Positions of Planets as Described in Gaṇitagannaḍi – A Karaṇa Text on Siddhāntic Astronomy in Kannaḍa

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1 INTRODUCTION

This is the second paper published in this journal concerning an astronomical manual (Sanskrit karana) of 1604 CE in Kannada, named Gaṇitagannaḍi, that is a commentary on Vārṣikatantra of Viddanācārya, written by Śaṅkaranārāyaṇa Jōisaru.¹ The earlier paper discussed the first chapter which was on the exact instant when the sun enters the sidereal Aries at the beginning of a given solar year (Sanskrit meṣaśankrānti) and the mean longitudes of all the planets at that instant. The present paper is on the mean positions of the planets corresponding to a count of civil days from the epoch (ahargaṇa) specified by the lunar phase (tithi) and lunar month of a given year. This is part of the first chapter itself. We include the second chapter giving the true positions which are obtained after the application of the first equation (manda) correction and the second equation (śīghra) correction for the five planets which look like stars (tārāgraha), i.e., Mercury, Venus, Mars, Jupiter and Saturn.

In the first part of this paper we provide a description of the procedure highlighting the technique used in the text. This will be followed by a diplomatic transcription of the text. The palm leaf manuscript includes both Sanskrit and Kannada languages and is written in the archaic script called Nandināgarī. We provide a translation of the commentary from Kannada. The suggestions on

¹ The earlier paper was Shylaja and Javagal 2020. For a description and examples of the karana genre, see Plofker 2009: §4.4.1.
possible corrections for scribal errors are also discussed at relevant places. As explained in the earlier paper, the Sanskrit verses are translated in to Kannada by rearranging phrases to follow the grammar of Kannada, where generally sentences commence with adjectives of the subject and end with the verb. It may be seen that words get rearranged to follow the grammar of Kannada. A small phrase may require explanation extending to several long sentences in the commentary. The rearranged phrases with intervening meanings read as complete sentences. This is the same method taught as “meaning according to word-sequence” (anvayānusārārtha) even today.

The introduction of classical texts of Sanskrit with commentaries in regional languages was already prevalent in Kerala by the seventeenth century, as described by Sarma (1972; 1985), who cites several examples of texts with prose or poem in Malayalam interlaced between Sanskrit verses. It would be interesting to study the evolution of such texts of astronomy in Kannada, with the very first example being offered by Gaṇitagannaḍi. Among the 466 manuscripts classified as astral sciences (jyotiṣam) in the Oriental Research Institute, Mysore (Malладевару 1983), the majority are devoted to predictive astrology. Five of them have Kannada translations, interlaced with Sanskrit verses, following the “syntactic sequence” (anvayānusāra) method. As pointed out by Gurevitch (2020), translations were initiated to bring texts within the reach of local populations. Considering the importance given to the routine astronomical calculations for Sringeri, which was a seat of knowledge, a text of this kind was perhaps in great demand.

This study presents translation at two levels. The Gaṇitagannaḍi itself is based on a translation. This translation of 1604 CE was aimed at providing an elaborate commentary with references to the original phrases as and when required. As the author mentioned in the introduction, it was for the benefit of beginners and students. He stated,

> ಇಲಿ್ಲ ಮುಂದೆಯೂ ಈ ಪ್ರ"ಾರದಲಿ್ಲ ಶಬ್ದಶಕಿ್ತಯ "ಾಣಿಸಿ ಅಥರ್ ಸಂಬಂಧ¬ಾಡಿ ಗ್ರಂಠರ್ವ ಹೇಳಿ್ಹೕನೆಂದರೆ | ಮುಂದೆ ಕೆಲವು ಬಳಿಗಳಲಿ್ಲ ಅನ್ವಯ ಯೋಜನೆಗಳು ಸಂಗತ°ಾ¦ಾವು | ಕೆಲವು ಬಳಿಗಳಲಿ್ಲ ಸಂಬಂಧ°ಾಗವು | ಅದೇಕೆಂದರೆ | ಇದು ಗಣಿತಪ್ರ¦ಾದ ಸಂ•ಾ್ಯ ±ಾಸ °ಾದ “ಾರಣ | ´ಾ–ಾದರೂ ಅದು ಅಲ್ಲದೇ | ಅಶಿ'nತ®ಾದ ªಾಲರಿಗೆ ಹೇಳು°ಾಗ ಪರಿ«ಾಷೆಯೇ ಪ್ರ¦ಾನ°ಾಗಿ ಶಿŊಯ ¬ಾಡಬೇ"ಾದ “ಾರಣದಿಂದಲೂ್ಲ | ಹೇಗೆ ಸುಸಂಗತ°ಾಗಿ ಅ¤ಾರ್ನುಸಂ¦ಾನವಹುದು | ಹೇಗೆ ªಾಲರಿಗೆ ತಿಳಿವುದು ´ಾಗೆ ಅಥರ್ವು ಹೇಳಲ್ಪಡುತಿದಿ್ಧೕತು | ಇಲಿ್ಲ ದುರನ್ವಯ ದುಯೋರ್ಜನೆ ಲಿಂಗ ವಚನ ವಿಭಕಿ್ತ ವ್ಯತ್ಯಯಗ¶ಾದವೆಂದು ಬಲ್ಲವರು ತಿಳಿಯ¯ಾಗದು.

Hereafter, the power of words is used to decipher the inner meaning of the work (grantha). Sometimes the interpretations are relevant. At some places they may not appear to be related. This is so because we are dealing with a mathematically oriented subject. When we are
teaching young students who are not yet competent [with the basics], the method of expression of the meanings of the words becomes very important. Scholars should not mistake that there is a misinterpretation or that the grammatical rules for gender and number (vacana) and case (vibhakti) are violated.

In this paper we have tried to follow a similar rationale, so that it will be understandable for present-day students and scholars who may not have previously been exposed to the texts and methods of teaching in the medieval period. The mathematical treatment of concepts is given priority. We have provided the original text with the translation so that any doubt on possible deviation from the original can be inspected immediately. We believe that this will be useful for readers who wish to understand the mathematical techniques. As we shall see later, the crisp and short phrases require a lengthy explanation, even to a person conversant with the tools of mathematics. Here is an example: the commentary for verse number 2.3 states,

The śīghrahara 10 [vyomendavaḥ] should be added to koṭiphala, if śīgrakendra is mrigādi and subtracted if it is karkyādi.

It is implied that a number 10, (called śīghrahara for a specific reason) should be added to the result obtained and called koṭiphala if the angle (called “centre” or kendra) with which we started the entire scheme of correction, is between 0 and 180; it should be subtracted if the angle is between 180 and 360 degrees. Thus, the English rendering requires a higher number of longer sentences. The difficulty posed by the absence of relevant diagrams in the original manuscript transmission is addressed by their introduction in this paper, in order to facilitate the mathematical treatment.

2 THE MEAN POSITIONS

Generally, all the texts (Siddhānta or Karana) start with the calculation of mean positions starting from the value of the ahargaṇa itself. Gaṇitagan-nadi too starts with a modified ahargaṇa or dyugaṇa which corresponds to the count from the midnight before the meṣa saṅkrānti (the date of entry of the sun in to Aries). For the date of calculation which is referred to as desired date (iṣṭa dina) and the average lunar day decided by the phase of moon (tithi) are known. The calculations are done to fix the tithi of the phase of the moon corresponding to the date of entry of sun in to Aries (saṅkrānti). Since the year is reckoned on the first day after new moon before the meṣa saṅkrānti, given as caitra śuddha pratipat, the meṣa saṅkrānti need not coincide with this. It is here that the method differs from that of other texts such as Karanakutūhala (Balachandra Rao and Uma 2008)
by Bhāskarācārya, where the ahargaṇa count is directly used to get the mean longitudes. In a later text, the Grahalāghava by Gaṇeśa Daivajña, the total number of civil days is regrouped in to cakras of 11 years and a modified number is used for deriving the mean longitudes of all planets (Balachandra Rao and Uma 2006).

For the Moon, from the described procedure, it is clear that the mean motion is taken to be \(12 + \frac{12}{68} + 1\) degrees per day. The procedure requires that the longitude of the moon obtained in units of rāśi, degrees, minutes and seconds, be converted in to one unit namely degrees. Since rāśi is 30 degrees, its count is multiplied by 30 and added to the degrees count. For example, if the longitude of the moon is 1 rāśi and 10 degrees, it is equivalent of 40 degrees. To get the tithi we have to divide this number by the motion of the moon \(12 + \frac{12}{68} + 1\) per day. The procedure states that division by 12 should suffice and the quotient is not needed. It should be noted that the words bhāga, aṃśa and bhāgi are used interchangeably for degrees. Thus conversion to degrees and division by 12 provides the sankrānti tithi, which is used for the exact calculation of dyugana. This provides the count of the month since the solar month starts from sankrānti tithi. Days from caitra śuddha pratipat to the date of interest are counted including the intercalary month if applicable. The number corresponding to that of sankrānti tithi is then subtracted. Here the idea of rtu (can be understood as season, a year has six rtus) is introduced to avoid one step of calculation. (Each rtu means 2 months). Therefore, dyugana count from the meṣa sankrānti is obtained.

Let us take an example. In the year śaka 1069 (corresponding to 1147 CE) the meṣa sankrānti occurred on 5th day after full moon in the month of Caitra based on a stone inscription (Shylaja and Geetha, 2016). This corresponds to March 24 as verified by another inscription recording a solar eclipse of the same year. Thus there is a difference of 20 days, which will be the carried on as the difference between dyugana and ahargaṇa (which starts from Caiтра śu 1) counts.

After getting the number of dyugana, its verification is done by the week day by dividing by 7. The remainder zero corresponds to Thursday, 1 corresponds to Friday and so on. The difference between dyugana and sāvana dhruva (it is the longitude for the beginning of the year for the planet as explained in the earlier paper) is called pada and is expressed in ghālige and vighalīge. The subtraction is explained step by step. Thus pada defines the number of days to the desired date as counted from the meṣa sankrānti, (defined as the First point of Aries in the current usage of spherical astronomy text books) effectively, the longitude expressed in units of days.

A quantity called pada was used in Vāsiṭṭasiddhānta as described by Shukla (2016, p502). It was coined as 1/248th part of the motion of the moon, equivalent to 1/9th of a day. Similar definitions existed for Jupiter and Saturn too, to derive the longitude. It referred to unequal divisions of the planets’ motion in a sidereal revolution. Here, in Vārṣiktantra, the definition itself is different. Lalla also
mean and true positions of planets

has defined similar divisions in Śisyadhīorddhidatantra (Chatterjee 1981). But that definition also is very different - the word used is pāda, which means a quarter. In the conventional methods (example Karanakutuhala) the ahargaṇa count is converted to the dhruvaka (longitude) and added to the dhruvāṃśa obtained earlier. Here the same procedure is adopted to get the dhruvaka using the quantity pada.

Therefore as a first step, pada is converted to units of degrees, arc minutes and arc seconds by dividing by 70. All the steps for this are explained - the first division gives degrees. The remainder is multiplied by 60 and then divided by 70 to get ghalige. The remainder of this is again multiplied by 60 and divided by 70 to get vighalige. This gives the mean longitude of the sun (stated as Ravi). The rationale for this is explained in the next sentence - the mean gati (daily motion) of Ravi 59′ (arcmin) 8″ (arcsec), which is expressed as

\[ 1 \text{ deg} - 52 \text{ arcsec} = 1 - 52/3600 \text{ deg} \]

Now, 52/3600 is very close to 1/70. Hence the motion of the sun is taken to be \((1 - 1/70)\) degree per day. So, when the pada (in days, ghalige and vighalige) is divided by 70 and the ratio is subtracted from itself, the result would be the mean longitude of the sun in degrees, arcminutes and arcseconds, as the mean longitude is zero at the meṣa sankrānti.

For Mars, Mercury, Jupiter, Venus, Saturn, Moon’s nodes and the Moon’s apogee, the mean daily motions can be inferred to be: 4/229, (4/30 + 1/325), 1/361, 40/749, 1/897, 1/566 and 3/808 rāśis, respectively. The values in degrees per day are found by multiplying these by 30. The mean longitudes for any ahargaṇa would be

\[ \text{dhruvāṃśa} + (\text{pada} \times \text{gati}) \]

Thus for the remaining part of the verse gati is expressed as a ratio with the values of multiplier and divisor defined in the bhūtasankhya system for all planets. In case of Mars, the conversion in to units of degree is explained. If the pada is \(a^{b}c^{c}\), c is divided by 60 and added to \(b\), the sum is divided by 60 and added to \(a\). Thus the final value of pada is expressed in units of days. The mean motion of Mars is 4 rāśis or 120 degrees in 229 days here. Hence, numerator is 4 (in units of rāśis), expressed as kṛti, and denominator is 229 expressed as nidhi puṣa netra here, and (pada) multiplied by the ratio is the mean motion of Mars during a pada.

For Budha śīghrocca, the multiplier is not stated explicitly. Using the idea that a plural has been used for the multiplier, the same number as for the previous one (Mars) is employed. The divisor is 30. Apart from this, to this, one has to add 1 divided by 325. Thus the correction is in 2 steps.

For the Moon, from the described procedure, it is clear that the mean motion
is taken to be equal to

\[ 12 + \frac{12}{68} + 1 = 13.17647 \]

degrees per day.

The details of the multipliers and divisors for the planets, beginning with Mars are shown in Table 1. Table 2 compares the mean motions as derived from this text with those from Sūryasiddhānta, depicting the accuracy of the procedure.

| Name          | mult, div | Numerals in Bhūtasaṅkhyā system |
|---------------|-----------|---------------------------------|
| Kuja/Mars     | 4, 229    | Kṛti, nidhi pakṣa netra         |
| Budha/Mercury | (4), 30 ; 1, 325 | (kṛti), khāgni; Eka, paṅcarada - 2 steps |
| Guru/Jupiter  | 1, 361    | bhū, mahi ṣatkṛti               |
| Śukra/Venus   | 40, 749   | khābdi, tāna nāga               |
| Śani/Saturn   | 1, 897    | kṣiti, muni randhra nāga        |
| Rāhu/Node     | 1, 566    | ku, rasāngaiṣu                  |
| Candrocca/Apogee | 3, 808 | thri, vasu vyoma gaja           |

Table 1: The multipliers and divisors of the planets beginning with Kuja (Mars)

| The mean motion | Sūryasiddhānta | GG (this text) |
|-----------------|----------------|---------------|
| Candra/Moon     | 13.17635       | 13.17647      |
| Budha/Mercury   | 4.15210        | 4.0911        |
| Kuja/Mars       | 0.524019       | 0.524017      |
| Guru/Jupiter    | 0.08309        | 0.08310       |
| Śukra/Venus     | 1.602146       | 1.062136      |
| Śani/Saturn     | 0.033439       | 0.033444      |
| Rāhu/Node       | 0.052984       | 0.052240      |
| Candrocca/Apogee| 0.111383       | 0.111386      |

Table 2: The mean motions of the planets as derived in this text compared with the values from Sūryasiddhānta

Finally another correction for only the sun and the moon is specified. That is to add the result of pada divided by 150; the rationale for this is not explained here but is covered in the chapter called Chāyādhikāra.
The mean values for all planets are for the midnight of Laṅkā (equator). Here the central meridian is described as passing through Laṅkā, Ujjain (Avanti), Roh-tak, Mānasa Sarovar and another place called Svāminale mountain (which is not mentioned in the original Vārśiktantra). The correction for location of the observer requires the viṣuvadchāyā, which is the shadow length of a 12 aṅgula (inches) gnomon on the day of equinox. The lambajyā (R cosine) of this is multiplied by 5060 and divided by 120 to get the correction called yojanaphala.

The rationale is derived from Śūryasiddhānta (Bapu Deva Śastri 1861: vv. 1–59). The radius of earth is taken as 800 yojanas. Therefore its circumference is 5060 yojanas. Here the value for the ratio the circumference to the diameter of a circle, π, is taken as square root of 10. This calculation is needed to find the time difference between the observer’s place and the standard meridian just defined. The daily motion of each planet is different and therefore the time differences will have to be calculated individually. However, the observer is not on the equator but a certain latitude $\phi$. Therefore the circumference will be along a circle parallel to the equator, which is obtained by multiplying the radius by $\cos \phi$ as shown in Figure 1. Here we have the value of latitude from the gnomon shadow on equinoctial day. Therefore to get the cosine of that we have to use the sine tables (provided in the next chapter on true values) for an angle $(90 – \phi)$. This works out to be $116|27$. The number 5060 corresponding to the equator, is multiplied by $116|27$ and divided by 120 so that we svadeśabhūparidhi, (the circumference of the small circle at the latitude of the observer) for the given place.

The viṣuvad chāyā is 3 aṅgula; the lambajyā is $116|27$, can be understood as latitude, $\phi = \tan^{-1}(3/12) = 14^{\circ}2^{\prime}11^{\prime\prime}$

$$R \cos \phi = 116|27$$

From the Figure 1, the circumference of the earth at this latitude is

$$bhūparidhi = 5060 \times R \cos \phi / 120$$

Here, 5060, $= 2 \times 800 \times \sqrt{10}$, is taken from the Śūryasiddhānta

The $R$ sines are to be obtained from the sine tables provided in the next chapter with the value of $R$ as 120. This latitude of $14^{\circ}2^{\prime}11^{\prime\prime}$ refers to a location north of Śrīgeri (latitude 13°25′). However, since the author mentions the name Śrīgeri in the next chapter the Chāyādhikāra, this small difference may be attributed to his location in the outskirts of the town.

The next step is to get the mean values for the time of the day. This is achieved by taking the difference from midnight of the same day or the previous day. This is multiplied by the gati (or the daily motion) of the individual planets.

Thus we see that the technique offers a different approach as compared to the conventional methods (like those of the Karanakutūhala) in the determination of the mean positions. The multipliers for deriving the dhruvakās (longitudes) of planets have been modified suitably.
Figure 1: The circle parallel to equator at O is the svadeśabhūparidhi at latitude φ.

3 TRUE POSITIONS

The procedure is based on the Sūryasiddhānta but many details are not explicitly mentioned.

After getting the mean positions as explained in Section 2, the corrections to derive the true positions are performed in two steps. The first correction is called the manda correction and the second one is called śīghra. The very first verse introduces the reference points needed for the second correction, called śighrocca. The farthest point on the epicycle created for this correction also has the same name.

From the second verse onwards the procedure for the manda correction is described.

The positions of the mandocca (apogee) for all the planets are given. Then there is an explanation for how these numbers have been arrived at. As per the definitions provided in Sūryasiddhānta (1–41 and 42), the number of years since the epoch is multiplied by the number of revolutions in a mahāyuga or kalpa and is divided by the number of years in that period to get the mandocca in revolutions. The fractional part multiplied by 360 corresponds to the position on the ecliptic in degrees for the required date. He further states there can be an error of 1 or 2 liptis (arc minutes) from the epoch specified by the Ācārya and therefore he has added 1 degree to account for such small deviations.

The word kendra is used to indicate the angle between mandocca (or śighrocca) and the mean position (or position after manda correction). They are referred to by abbreviations manda (or śīghra). The corrections (as shown in the following discussion) derived using these are called mandaphala (or śīghraphala). It should
be noted that word *mṛgādi* is used here. All along the discussion used the zodiacal signs - here it becomes luni-solar *Mrga* corresponding to the month *Mārgaśīra*.

This correction can be understood with the help of Figure 2. The basic idea of the *manda* correction is to account for the elliptical orbit, which is achieved with another smaller circle moving along the mean circular orbit. (Bapu Deva Sastri 1861).

![Figure 2: Definition of Mandakendra](image)

In Figure 3, at the apogee A, the planet is farthest and at B it is the closest. The planet moves along the small circle so that the distance difference is achieved over half the orbit. There is always a phase difference between the true position (shown in red colour) and the mean position of the planet (shown in black).

Since the projection of the position on the radius vector is needed for the calculation we have to get the sine of the angle called *mandakendra*, shown in Figure 1. The correction is indicated by the dashed line in Figure 3.

The next verses describe how to get the *R* sine values. In all the astronomical texts the trigonometric sine ratio is treated as the arc *R* sine (angle), called as *jyā*. In this text the word *jīva* is also used. Here *R* is taken as 120. The arc itself is expressed in units of degree (*bhāga*) arc minutes (*kalā*) and arc seconds (*vikalā*). A table is provided for the calculation of *R* sine of any angle. Every 10 degrees is termed a *khaṇḍa* (section) and the value of the differences of *R* sine is provided. The value for any intermediate value is obtained by interpolation. The numbers are specified in *bhūtasāṅkhya*.
The author proceeds to explain how to get the $R$ sine for any angle. The angle should be divided by 10 to identify the $khaṇḍa$. All values preceding it are added up. The $jyā$ difference corresponding to the remainder after dividing by 10 is obtained between the successive $khaṇḍa$ and added to the earlier sum. This procedure will be clear with an example. If we want find the $R$ sine for 34 degrees, we look up the value for number 3, since $34/10$ has quotient 3 ($khaṇḍa$ number) and the remainder is 4. The sum of all $jyā$ values preceding 3 is $21 + 20 + 19 = 60$. Now we have to interpolate between $khaṇḍas$ 3 and 4 for the remainder 4, as $(\frac{17}{10} \times 4) = 6$ and remainder is 8. This is added to 60 as 66 and remainder 8 is multiplied by 60 to get $48'$. Thus the $R$ sine of 34 is $66°48'$.

The text also gives the same numbers in the reverse order as

2|5|9|12|15|17|19|20|21

The sums of all the preceding values of $jyā$, are provided in the next verse in the $bhūtasankhya$ system. These are termed $piṇḍikṛta jīva$.

$khaṇḍa$ 1 2 3 4 5 6 7 8 9
$piṇḍikṛta jīva$ 21 40 60 77 92 104 113 118 120

Another interesting part introduced by the author is the table of $utkramajyā$. This trigonometric ratio $(1 - \cos)$ is not included along with the other three in the text.
books of today, although it has been named versine.

$|2|7|16|28|43|60|79|99|120|

For example if the angle is 60, its *utkramajyā* is $R (1 - \cos 60)$ which is 60. This is the number in the 6th *khanda*.

The next verse gives the values of divisors for *manda* corrections for the planets. Here the author follows a technique that is different from others, for example, *Karana kutihala*. In most Indian texts on astronomy including *Sūryasiddhānta*, the computation of *mandaphala* is based on an epicycle model (Figure 4).

![Diagram of mean and true positions of planets](image)

**Figure 4**: Derivation of the *manda* correction

$P_o$ is the mean position of the planet. $P$ is the position corrected for *manda*, referred to as *mandasphuta*. The angle $MOP_o$ is called $M$, *mandakendra*; the angle $POP_o$, $\Delta \theta$, is called *mandaphala*. Writing $r$ as the radius of epicycle and $R$ as the radius of deferent, we get from triangles OQP and PQP,$_o$,

$PQ = OP \sin \Delta \theta$ and also $PQ = r \sin M$ or

$$\Delta \theta = \frac{r \sin M}{R} \text{ (in radians)} = \frac{r \sin M}{R} \times \frac{3438}{60} \text{ (in degrees)},$$

(1)

where $r$ and $R$ are the radii of the epicycle and the deferent, respectively.
From the descriptive procedure we understand that the mandaphala is given as

\[
\frac{R \sin M \times 60}{x + \frac{R \sin M \times 60}{y}}
\]

where \(M\) is the mandakendra, and the denominator is called the corrected mandaccheda. Here \(x\) and \(y\) are specified for each planet. For instance, for the Sun, \(x = 3230\) (vyomāgnidanta) and \(y = 90\) (khāṅka). The first term in the denominator is much larger than the second term.

Therefore,

\[
\frac{R \sin M \times 60}{x + \frac{R \sin M \times 60}{y}}
\]

is approximated as

\[
\frac{R \sin M \times 60}{x + \frac{R \sin M \times 60}{y}} = \frac{R \sin M \times 60}{x} \left[1 - \frac{R \sin M \times 60}{xy}\right]
\]

In the Sūryasiddhānta, \(r\) is of the form

\[
x' - y' \sin M x' \left[1 - \frac{y'}{x'} \sin M\right]
\]

and \(R = 360\). For instance, for the Sun, \(x' = 14\) and \(y' = \frac{1}{3}\). In Table 3 the values for mandaphala from the two texts are compared.

Table 3 shows that the Ganitaganāḍi expression for the mandaphala would give very nearly the same results as the ones following from Sūryasiddhānt rules. For all planets the divisors are derived and provided using the siddhanatic values of the peripheries. The values from Karaṇakutūhala (Balachandra Rao and Uma 2008) are compared in Table 4.

The correction is explained in the next verse. It is called tātkālika, which can be interpreted as “as applicable for that instant.” The procedure to apply this correction appears to be have been devised by the author himself. The \(R\) sine of the mandakendra is converted to lipti (arcminutes) by multiplying the value in degrees by 60. Then the liptis are added so that the entire value is in liptis. These are to be divided by different numbers for each planet specified by the verse beginning with khāṅka, 90 for the sun and so on. The result is again added to the numbers specified earlier in the verse beginning with vyomāgnidantaḥ, to get the divisors. Dividing the \(R\) sine of mandakendra by this corrected divisor gives the mandaphala. The numbers are 90 (khāṅka), 490 (khatāna), 300, (viyadabhrarāma), 70 (khāsva) 170 (kha śailendu), 21 (indupakṣa), 380 (khūśāgni). Thus the correction extends to the fraction of a degree.
### Table 3: Comparison of mandaphalas in the Čaṇḍoguśī and the Sūryasiddhānta

| Planet      | mandaphala in Čaṇḍoguśī | mandaphala in Sūryasiddhānta | degrees | degrees |
|-------------|--------------------------|------------------------------|---------|---------|
| Ravi/Sun    | 3.7797 - 0.0003 sin M    | 2.2291 [1 - 0.0247 sin M]   | 117     | 114     |
| Śrītu/Venus | 3.7806 - 0.0003 sin M    | 2.2283 [1 - 0.0238 sin M]   | 117     | 114     |
| Črun/Jupiter| 5.2757 - 0.0003 sin M    | 5.0955 [1 - 0.0104 sin M]   | 130     | 127     |
| Buđha/Mercury| 4.7631 - 0.0003 sin M   | 4.7750 [1 - 0.0667 sin M]   | 130     | 127     |
| Kuja/Mars   | 11.9402 [1 - 0.0399 sin M] | 11.9375 [1 - 0.0400 sin M] | 130     | 127     |
| Śukra/Venus | 1.9103 - 0.0003 sin M    | 1.9100 [1 - 0.0833 sin M]   | 130     | 127     |
| Śani/Saturn| 7.8006 - 0.0003 sin M    | 7.7992 [1 - 0.0204 sin M]   | 130     | 127     |

Table provided by the anonymous referee.
Table 4: The ratios of circumferences of epicycles of planets used in this text, compared to those in the *Karaṇakutūhala* (KK)

| Name of planet | Mandacheda Divisor | Phrase as described | Circumference of epicycle | Circumference of epicycle (KK) |
|----------------|--------------------|---------------------|---------------------------|-------------------------------|
| *Ravi* / Sun   | 3230               | *vyomāgnidanta*     | 13 | 22                        | 13 | 40                        |
| *Candra* / Moon| 1413               | *śīkhirupāśaka*     | 30 | 34                        | 31 | 36                        |
| *Kuja* / Mars   | 603                | *purāmbarānga*      | 70 | 38                        | 70 | 38                        |
| *Budha* / Mercury| 1510              | *digartho candra*   | 28 | 36                        | 38 | 38                        |
| *Guru* / Jupiter| 1371              | *rūpāgaviśva*       | 31 | 30                        | 33 | 33                        |
| *Śukra* / Venus | 3769              | *ankarasādrirāma*   | 11 | 28                        | 11 | 11                        |
| *Śani* / Saturn | 923                | *tripakṣarandhra*    | 46 | 48                        | 50 | 50                        |

The final value after the correction is called *mandasphuṭa* (corrected for *manda*).

The next verse provides similar divisors *śīghracheda* for the second correction. Although the author declares it is on the same lines as done for *manda* correction, the procedure is not very clear as can be seen later. Prior to the discussion on *śīghra* correction, he summarises the procedure for *manda* in a single sentence, whose translation was also difficult. Here is the summary:

- Get the *mandakendra*, difference between *mandocca*, the apogee and the mean.
- Get the *R* sine of *mandakendra* and *koṭi* (*R* cosine) also. (*Koṭi* is not needed for *manda* correction).
- Convert the *R* sine in to arc minute by multiply by 60 and adding to the *lipti* component.
- Divide it by the appropriate number as given by the sequence stated in the verse starting with 90.
- Add the result to corresponding numbers provided by the sequence starting with 3230.
- Divide the product of 6 and *R* sine of *mandakendra* by the corrected divisor.

This last step, namely dividing it by 6, was not specified in the procedure earlier. This division is necessary because while, converting it into *lipti* we had multiplied it by 60. Essentially, the procedure can be written in the modern notations as an equation,

\[ a = \frac{6R \sin M}{\text{corrected divisor}} \] (2)
where,

$$\text{corrected divisor} = \text{number } 3230 + \frac{R \sin M(\text{in liptis})}{\text{Correction factor 90}}$$

for the sun.

Similar devisors are derived for other planets.

The śīghra correction takes the manda corrected position as the reference. Let us first see how the correction is achieved.

The procedure for śīghraphala, which has been very aptly clarified and compared with the procedure in Sūryasiddhānta by the referee is being reproduced here.²

The śīghrocca for the planets is the sun itself. In the Figure 2.4, the sun, the planet and the earth are represented by S, E and P. The relevant angles are marked as $\theta_{ms}$ mandasphuṭa, $\theta_s$ śīghra, $r$, radius of śīghra epicycle and $R$, the radius of deferent.

![Figure 5: Derivation for śīghraphala from Sūryasiddhānta](image)

$$\text{śīghrakendra} = \theta_{ms} - \theta_s = -M_{sk}$$

śīghraphala = $\Delta \theta$, is given by

$$R \sin \Delta \theta = \frac{r \sin(\theta_{ms} - \theta_s)}{\left\{\left[R + r \cos(\theta_{ms} - \theta_s)\right]^2 + r \sin(\theta_{ms} - \theta_s)^2\right\}^{1/2}}$$

$$\sin \Delta \theta = \frac{r/R \sin(M_{sk})}{\left\{\left[1 + r/R \cos(M_{sk})\right]^2 + r/R \sin(M_{sk})^2\right\}^{1/2}}$$

² The anonymous referee of has kindly provided a critical analysis and comparison of this formula with the one in Sūryasiddhānta.
Now, let us see the procedure in Gaṇitagannaḍi.

The epicycle of the śīghra is rather large although Figure 5 represents it as a small circle of radius $d$. $S'$ is the direction of śīghroccha, the conjunction of the planet with the sun. The manda corrected position is $P_a$. By the time the mean position has changed to $P_b$ from conjunction the projection on the epicycle would have moved from $J'$ to $J$. The corresponding shift on the orbit takes it to the point $P$ as the true position.

![Figure 6: Diagram for explanation of śīghraphala](image)

From the Figure 6, we can derive an expression for the angle $\theta$, the śīghraphala. The śīghraphala, $s$ is expressed as (Somayaji, 1971)

$$R \sin \theta = \frac{d}{k} R \sin m$$

where $k$ is called the calabāṇa, $EJ$, the distance of the planet from earth at the desired instant. (Bapu Deva Sastri 1861). The word caladbāṇa also is used.

From the properties of similar triangles we can show that

$$k_2 = \left[\frac{d}{a} R \sin m\right]^2 + \left[d + \frac{d}{a} R \cos m\right]^2$$

The procedure requires that $\left[\frac{d}{a} R \sin m\right]$ and $\left[\frac{d}{a} R \sin \cos m\right]$ be determined, these are termed bhujaphala and kotiphala respectively. The author has used a different technique to compute $k$, the calabāṇa. The term $d$ in the expression $\left[d + \frac{d}{a} R \cos m\right]$. 

**Figure 6:** Diagram for explanation of śīghraphala
has been fixed to 10. Accordingly kotiphala is added to 10 and its square is added to the square of bhujaphala, essentially getting the value of \( k^2 \). Its square root is the divisor for bhujaphala again to get śīghraphala.

Then the value of \( a \) is adjusted as per the ratio \( d/a \). For, example for Mars, the ratio is known to be 1.5 (given as the ratio of radii of peripheries with 360). If \( d \) is 10 the value of \( a \) will \( d/1.5 \). However the ratio \( d/a \) will not change. It is to be noted that the coefficients of \( R \sin m \) and \( R \cos m \) are same. By this adjustment the coefficient of numerator in (3) also will be the same. To take care of the trijā, multiplication by 120 also is necessary. Let us call the ratio of \( d/a \) as \( y \). Since \( d \) is fixed at 10 the value of \( a \) is \( 10/y \). The śīghracheda is 720 \( y \) which we can write as \( A \).

\[
bhujaphala = \frac{yR \sin m \times 60}{\text{śīghracheda}} = \frac{120 \sin m \times 60}{A} \quad \frac{10}{10} = \frac{y \sin m}{A} \tag{5}
\]

Similarly the coefficient of \( R \cos m \) also is adjusted by dividing by \( A \). This looks very tricky but we can see that it is devised to get rid of several steps such as division by 60 and 120. Thus the same Bhujaphala and Kotiphala (with \( R = 120 \)) are

\[
\text{Bhujaphala} = BP = \frac{R \sin (M_{sk})}{\text{śīghracheda}} \times 60
\]

\[
\text{Kotiphala} = KP = \frac{R \cos (M_{sk})}{\text{śīghracheda}} \times 60
\]

Here, the divisor, śīghracheda is provided for all planets (e.g., for Mars it is 1110).

The hypotenuse calabāṇa is defined as

\[
\text{calabāṇa} = ([10 + KP]_2 + BP^2)_2
\]

\[
\text{śīghraphala} = \Delta \theta, \quad \text{is given by}
\]

\[
R \sin \Delta \theta = 120 \times BP \times R/\text{calabāṇa}
\]

this is same as equation (3) above and is further reduced to

\[
\sin \Delta \theta = \frac{720 \sin (M_{sk})}{\text{śīghracheda} \left[ 10 + \left( \frac{120 \cos (M_{sk})}{\text{śīghracheda}} \times 60 \right)^2 + \left( \frac{120 \sin (M_{sk})}{\text{śīghracheda}} \times 60 \right)^2 \right]^{1/2}} \tag{6}
\]

Thus if we identify \( r/R \) of (3) with \( 720/\text{śīghracheda} \) of (4); they are identical.

Table 6 lists the numbers and the implied ratios, which is in agreement with the values currently in use. Thus Ganitagannadi (GG) has the same procedure from the Sūryasiddhānta (SS) to aid calculations. (śīghrakendra as \( M_{sk} \), can have any value from 0 to 90). The agreement to second decimal place implies 6′. The values of the ratios of the radii of the planets are concealed in these numbers (A) provided as śīghracheda.
This procedure has a great advantage in computations, since only bhujaphala and koṭiphala are to be read out from the sine tables and the constants take care of the conversions. It can be summarized as follows:

1. Calculate the calabāṇa and śīghrakendra for the individual planet
2. Get the bhujaphala and koṭiphala putting the corresponding śīghracheda
3. Calculate śīghraphala putting using \((6)\)

Thus if we identify \(r/R\) of \((4)\) with \(720/\text{śīghracheda}\) of \((3)\); they are identical.

The comparison of the ratios are in the Table 6 below.

| Name of planet | Śīghracheda Divisor | Phrase bhūtasaṅkhyā implied ratio of orbit radii |
|----------------|---------------------|-----------------------------------------------|
| Kuja/ Mars     | 1110                | Digīśvara                                      | 1.54 |
| Budha/ Mercury | 1956                | tarka śarāṅka candra                          | 0.37 |
| Guru/ Jupiter  | 3651                | ku arthānga rāma                               | 5.07 |
| Śukra/ Venus   | 993                 | jvalanāṅka nanda                               | 0.72 |
| Śani/ Saturn   | 6562                | dvāṅgaiśu tarka                                | 9.11 |

Table 5: The values of śīghracheda for five planets and implied ratio of radii

| Planet        | sg (GG) 720/sg r in (SS) | r/R for M = 0 | r/R for M = 90 |
|---------------|--------------------------|---------------|---------------|
| Kuja / Mars   | 1110 0.6486 235 − 3|sinM| 0.6317 | 0.6389 |
| Budha / Mercury | 1956 0.3681 133 − |sinM| 0.3694 | 0.3667 |
| Guru / Jupiter | 3651 0.1972 70 + 2|sinM| 0.1944 | 0.1999 |
| Śukra / Venus | 993 0.725 262 − 2|sinM| 0.7277 | 0.7222 |
| Śani / Saturn | 6562 0.1097 39 + |sinM| 0.1083 | 0.1111 |

Table 6: Ratios compared from Gaṇitagannaḍi (GG) and Sūryasiddhānt (SS). (śīghracheda is abbreviated as sg, mandakendra as M).

The next step is to get the sine inverse form the same sine tables which is quite straightforward and explained already.

The next verse describes the procedure for getting the sphaṭagati, the true motion of the planet. We will see that the concept of calabāṇa has been utilized here also to lessen the steps of calculations. It is assumed that the reader is aware of the procedure and the steps are mentioned very briefly. The average value of the gati obtained as an average for one revolution is called the mean. The first step of mandasphuṭa correction uses the value of koṭiphala arrived above. This procedure is not discussed here.
The correction in the second step requires the śīghra corrected value and the calabāṇa, earth planet distance, to get the sphuṭtagati or the true motion. In case of the sun and the moon the second step is not needed. Here only the second step is explained. The planet earth distance which was termed calabāṇa is being used again here.

The difference between the gati of the śīghrocca (U) and that of the planet (V) is multiplied by a quantity which we shall call \( q \), defined as the difference of the śīghrahara as per catuḥpratinyāya. This phrase is not explained and the meaning is not very clear. But we try to understand the procedure and interpret. After multiplication it is divided by the same śīghrahara used for getting calabāṇa. This is added to or subtracted from the gati obtained after manda sphuṭa correction.

The sphuṭtagati consists of three components - the mean motion of the planet, the mean motion after the manda correction and the mean motion after the śīghra correction. The last quantity is given by

\[
d m = U - (U - V)R \cos \theta/k
\]

where \( \theta \) is the śīghraphala and \( k \) caladbāṇa, is the earth-planet distance. Here \( U \) represents the mean motion of śīghrocca and \( V \) is the mean motion of the planet. In the case of planets śīghrocca is the sun itself. Therefore \((U - V)\) is a measure of the difference in speeds of sun and planet. The difference between the two becomes substantial as the planet - earth distance and the sun - planet distances have a larger range as compared to the sun or the moon. The statement above can be expressed as an equation as given in the text as

\[
d m = U - \frac{(U - V) \times q}{\text{calabāṇa}}
\]

Thus we can interpret that the quantity \( q \) is \( R \cos \theta \). The meaning of catuḥpratinyāya perhaps is discussed elsewhere and assumed to be known to the reader. It implies \( 10 - \) (the śīghrahara added value) = \( 10 - (10 + R \cos \theta) \), which is \( \cos \theta \), itself. The phrase used is “caladbāṇa harāntareṇa.” The word bāṇa refers to the term \( (R + R \cos \theta) \). Then, śīghrahara is \( 10 \), so the difference will be \( R \cos \theta \).

The calabāṇa is converted to arc seconds and subtracted from mandasphuṭagati if calabāṇa is smaller; that gives the sphuṭtagati.

The equation (8) also shows the effect of the difference of speeds as seen from the earth. The projection of difference of speeds in the line of sight is achieved by the multiplication by \( R \cos \theta \). If \( dm \) is negative the difference implies the vakragati - the apparent reversal in the direction of motion. This idea is used to fix the onset of retrograde motion for these five planets.

The next verse mentions a correction to be done for the sun and the moon. This is described in the Sūryasiddhānta as per the verse quoted in the text. (This
verse is included in the appendix) This is called the bhujāntara correction and is needed because of the non-uniform motion of the sun. As can be guessed this is a direct consequence of the elliptical orbit.

All these computations are for midnight at Ujjain. The time difference will be determined with reference to a uniform motion of 360 degrees a day or 21600 arcminutes per day. The word cakralipti, number of liptis (arcminutes) in a cakra (circle), is used for 21600. This is the only place where kaṭapayādi system has been used to denote this as anantapura.

The procedure here is as follows:- the R sine of the sun is converted to lipti and divided by 27; the result in liptis is added to the sun and the moon. Addition or subtraction is decided by bhujaphala (as positive or negative). That gives Ravibhujasamānśkritacandra - which means moon corrected for Ravibhuj. This correction is to be done for all planets. But for all the others it is quite small and therefore the author states that he specifically applies it for the moon. This correction should be done for all planets. This is as per the verse in Sūryasiddhānta (II - 46) - the gati of planets should be multiplied by Ravibhujaphala in kala and divided by 21600; the result is positive or negative as is the case for the Sun. The mean gati of the Moon is 791. Dividing 21600 by 791 gives 27. Therefore the author gives the rule as divide by 27.

This completes the second chapter called Grahasphuṭādhikāra. The colophon is identical to the one for the first chapter, with identical adjectives.

This chapter for calculation of true positions of planets has used procedures which render computations easy and simplified. The rationale for the procedures has been explained. The ratios of planetary distances and the epicycle radii are compared with those given in Karaṇakutūhala. The constants used here have been modified by the author himself and small corrections also have been incorporated.

Finally a note on the colophon: the author has attributes “like a full moon for the ocean of nectar, and, who, to the ignorant astronomers is like Garuḍa (Brahmin kite, the mythological enemy for snakes) to snakes.” The corresponding translation can have two interpretations in the absence of the specific case endings:

- Dēmaṇajyotisāṅgānāya-sudhārṇava-pūrṇacandra – can be a single phrase meaning “like the full moon for the nectar ocean of Dēmaṇa who is an expert astronomer.”
- Vāsavguru Dēmaṇa – can be one phrase comparing Dēmaṇa to the guru of the Gods. Now agragānya gets attributed to the ocean so that the implied meaning is “like the full moon for the nectar ocean of expert astronomers.”

Generally the ocean and full moon metaphor is used to signify happiness - akin to the high tides associated with full moon. Here the ambiguity arises with the
word *dēmanajyotisagraṇaṇya* leading to the above two possibilities. This is by treating the expression as a descriptive compound (*karmadhāraya*) as suggested by the referee and K. R. Ganesha (personal communication).

There is yet another interpretation as provided by Mahesh and Seetharama Javagal (2020) in the context of edition of *Karaṇābharaṇa* by the same author, Śaṅkaranārāyaṇa Joyisa. The translation of the same phrase reads

Composed by Śaṅkaranārāyaṇa Joyisa who is “a falcon to the serpents of unaccomplished astronomers,”
and
the full moon emerged from the nectar-ocean of the foremost astronomer Dēmaṇa Joyisa, the one who is equivalent to the guru of Indra.

This is based on the mythological story that the moon was churned out of the ocean (*amṛta manthana*). The simile classified as *rupakālankāra* describes the happiness of the father provided by the genius of the son. Here the fact that Dēmaṇa is the father has been utilized although not specified and *Garuḍa* is translated as falcon.

These titles are not found in the earlier works of Śaṅkaranārāyaṇa Joyisa, namely *Tantradarpaṇa* (1601 CE) and *Karaṇābharaṇam* (1603 CE) where it reads

...composed by Śaṅkaranārāyaṇa Joyisa, the son of Dēmaṇa Joyisa, the astronomer, who is equivalent to the guru of Indra, a resident of Śṛṅgapūri. (Mahesh and Seetharama Javagal 2020)

Perhaps he was bestowed with the titles in 1604 CE. Or, did he crown himself, or, did he become more poetic?

4 GRAHAMADHYĀDHIKĀRA TRANSLATION

This chapter, a continuation of the verses explained in our earlier paper (Shylaja and Javagal 2020), explains getting the mean positions of all planets. Since there is an ending note stating that *Dhruvādhiṅkāra* is concluded and *Grahamadhyādhiṅkāra* is commencing, we may consider this as a sub-section of the first chapter. For degrees the words *bhāga* and *bhāgi* are used interchangeably. We have retained the usage in English as well.

The text is provided in the next section as is given in the manuscript which has the text in both the languages, Sanskrit and Kannaḍa. As mentioned earlier the script is *Nandināgarī* and here we have put both languages in Kannada script. Translation and the verses from 1 to 10 of *Dhruvādhiṅkāra* have been already provided in the earlier paper. It is to be noted that the translation is provided only for the ṭīke or commentary in Kannaḍa not for the *mūla*, the original Sanskrit verses. Very long phrases have been split to shorter sentences.
Now the procedure to derive the mean values for the required date will be explained from the number of dyuganas, whose derivation, based on the parameters like saṅkrānti and tīthi is explained.

VERSE || 11 ||

[This is to get the tīthi of saṅkrānti.] The dhruvāṃśa, obtained earlier [for the beginning of the year], of the moon is considered. The rāśi part is multiplied by 30 and added to the degrees part so that we have it expressed in degrees. This is divided by 12 to get the quotient as the saṅkrānti tīthi. The remainder is of no consequence here.

Saṅkrānti tīthi is defined as the number of civil days intervening from caitra śuddha pratipat to meṣa saṅkrānti for the sun [to cover it] with its mean daily motion.

[Now the calculation of dyugana] Number of days from caitra śuddha pratipat is counted – this should include the intercalary month if needed. The number of saṅkrānti tīthis is subtracted. Every rtu, season, has two months. The number of rtus elapsed are subtracted to get the dyurāśi. The dyugana is obtained by removal of saṅkrānti tīthis and rtus; this is the number of tīthis from the beginning of the solar year [meṣa saṅkrānti].

VERSE || 12 ||

As per [the verse starting with] “nāgāpta śiṣṭa”, the dyugana thus obtained is divided by 7. The remainder is added to the vāra of sāvanadhruva. 7 is subtracted from it if it is more than 7.

As per [the verse starting with] “vārapatirniśīthe”, the remainder obtained is the weekday number starting from Friday. If the number is 1 it is midnight of Friday; 2 implies midnight of Saturday [and so on].

FIRST LINE OF VERSE || 13 ||

This method gives the weekday correction of one day more, or one day less which can be applied to the dyugana-count (so that the calculated and the actual week-day are the same). The sāvana dhruva should be subtracted from the corrected dyugana in units of ghalige and vīghalige. This is how you can do it. Take out one dyugana [which is equal to 60 ghalige] and [write it] as 59 ghalige. Place the dyugana and take one from it and bring (it as) 60 (ghalige). From this, with (1 ghalige further written as) 59 (ghalige and 60 vīghalige), the sāvanadhruva with ghalige and vīghalige is subtracted. The result is called a pada expressed in units of day, ghalige and vīghalige. (“pada” is the time-interval between the meṣa saṅkrānti
MEAN AND TRUE POSITIONS OF PLANETS

(beginning of the solar year) and the beginning of the desired day) [This is the number] to be used for all the planets.

Now consider the pada twice – as per [the verse starting with] “khāgamśa hīnena phalam”, divide the second one by 70 to get degrees. The remainder is multiplied by 60 and divided by 70 to get lipti and similarly vilipti. These values are subtracted from the (first) pada to get the mean sun in bhāga units. That should be divided by 30 to get rāsi. The remainder is bhāga, thus you get the mean sun for the midnight of the desired day in units of bhāga, lipti and vilipti.

The rule applied is – for one day the mean gati is 59 lipti 8 vilipti. The mathematical explanation of this is that the value gets lesser by 1 bhāgi in 70 days.

SECOND LINE OF VERSE || 13 ||

[The same pada is used now for the moon.] It is multiplied by 12 specified as arkanighnam (arka 12, nighnam, multiplication) in the verse. Two copies of the product are kept; the lower copy is divided by 68 to get the product in units of bhāga, and added to the upper copy. This is added to one pada. Divide it by 30; if the result is more than 12 divide it by 12; discarding the quotient, the remainder is the rāsi, and the lower units are bhāga, lipti and vilipti. As per the verse “dhruvesu yojya”, this quantity in rāsi and other units, is added to the dhruva for the beginning of the year (obtained earlier) for the moon to get the mean moon for midnight of the desired day.

VERSES || 14 || AND || 15 ||

To get the mean Kuja (Mars): All the three copies of the pada are multiplied by 4 as per (the verse) “kṛtaghnāt.” The lower two are divided by 60 and (and the square of 60 respectively) and added back. The sum is divided by nidhi, pakṣa, netra that is 229, to get bhāgi. The reminder is multiplied by 60 and again divided by 229 to get lipti and same way to get vilipti. The rāsi and sub units, obtained this way is added to the dhruva of Kuja (obtained earlier) to get mean Kuja.

Here kṛti [corresponds to] 4 rāsi and nidhi, pakṣa, netra [corresponds to] 229 days, arrived at as completion of 4 rāsi s by the mean Kuja. Therefore the rule of three used is [as follows] 229 days correspond to 4 rāsi – therefore how many rāsi for the desired number of days? It is the same procedure for all the planets [hereafter].

Now the (determination of) śīghroccha, higher apsis of the epicycle, of Buddha (Mercury).

The divisor has been specified as 30, from “jñāh khāgnībhiḥ”, in plural, but not the multiplier. Based on the context we consider the multiplier gunaka is the same as that for Mars namely 4. The pada is multiplied by 4 as before and divided by 30 and expressed as rāsi which is kept aside. Multiplying the pada by one [you
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get back \( \text{the same } \text{pada} \). This is divided by \( 325 (\text{pancaradaih}) \) to get the \( \text{rāśi} \) units and added to the earlier obtained \( \text{rāśi} \). This is finally added to \( \text{varṣa dhruva} \) to get Budha \( \text{śighroccha} \).

Now the procedure for mean Guru (Jupiter) – the multiplier is 1, specified by \( \text{bhu} \). This is to be divided by 361 to get Guru. This \( \text{converted to units of } \text{rāśi} \) is added to \( \text{dhruva} \) for the year to get mean Guru.

Now the \( \text{śigrocca} \) of Śukra (Venus) – \( \text{pada} \) is multiplied by 40 and divided by 749. The result \( \text{converted to } \text{rāśi} \) units is added to the \( \text{dhruva} \) obtained earlier.

To get the mean Śani (Saturn) – \( \text{pada} \) is multiplied by 1, and divided by 897; the product \( \text{converted to units of } \text{rāśi} \) is added to the \( \text{dhruva} \) to get mean Śani.

The multiplier for, Rāhu (Moon’s ascending node) is 1. It is divided by 566. This is subtracted from the \( \text{Dhruva} \) as per \( \text{tamasah pratipa} \), to get mean Rāhu. Adding 6 \( \text{rāśis} \) will fetch Ketu.

For getting the \( \text{candrocca} \), (Moon’s apogee) \( \text{pada} \) is multiplied by 3 and divided by 808; the result \( \text{converted to } \text{rāśis} \) is added to previously obtained \( \text{dhruva} \).

**VERSE || 16 ||**

\( \text{Dyugana} \) is divided by 150; the result expressed in \( \text{lipti} \), \( \text{vilipti} \) is subtracted from the mean values for the sun and the moon. Thus all the mean positons of all planets are obtained for the midnight of Lanka. Lanka is to be understood as the south of \( \text{mahāmeru} \).

**VERSE || 17 ||**

Getting the \( \text{lambajyā} \) of the place of observation (\( \text{svadeśa} \)) is explained later in the chapter \( \text{chāyādhyāya} \); this is multiplied by 5060 and divided by the \( \text{trijyā} \) 120, which is defined in \( \text{sphuṭādhyaya} \). This is the \( \text{svadeśabhūparidhi} \), the circumference of the small circle at the observer’s latitude. For a place with an equinoctial shadow of 3 \( \text{aṅgula} \), the \( \text{lambajyā} \) is \( 116/27 \). The derivation of 5060 is explained as per \( \text{Sūryasiddhānta} \) in this verse.

**Quotation from Sūryasiddhānta**

Multiply the square of the earth’s diameter (1600) by 10 and its square root is the circumference in yojanas.

The \( \text{bhūmadhyarekha} \) stretches from Lanka to Meru Mountain. Rouhitaka country, Svāmimale, Avanti is Ujjaini, Amarādri sāra is Mānasa Sarovara. The north south axis, \( \text{ sutra} \), passes through these and is called \( \text{bhūmadhyarekhā} \).
The distance in *yojana* of the place of observation from the *bhūmadhyarekhā* to the east or west is to be determined. This number is multiplied by the mean *gati* [in *lipti*] of all the planets and divided by the circumference, *svadeśabhūparidhi* obtained earlier. This is subtracted from the mean values of the respective planets, if *svadeśa*, the place of observation, is to the east, or added [if it is] to the west. This is the correction [called] *yojanasanskāra*.

Now the procedure for getting all the mean planets at midnight of the *iṣṭakāla* desired date.

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One has to get the time interval in *ghalie viḥalige*, ahead of or lagging behind, from midnight; this time interval is multiplied by the *madhya gati* (mean rate of motion) in *lipti*, *vilipti* and divided by 60. If the *iṣṭakāla* (time of interest) is before midnight, the values in [*lipti*, *vilipti*] have to be subtracted; if it is after midnight [they have] to be added. The mean positions of all planets are now available for the desired time.

This completes the first chapter called *Grahamadhyādhikāra* of the book called *Gaṇitagannaḍi*, a commentary of *Vārṣiktantra* in the language of *Karṇāṭa* written by Śaṅkaranārāyaṇa Jyōtiṣi, who, is like Garuda (mythological enemy of snakes, the Brahmīny kite) to snake-like ignorant astronomers, and who, akin to a full moon for the ocean of nectar [and] of *Brhaspati* - like Dēmaṇa, an eminent astronomer and a resident of Śṛṅgapura.

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**5 SPHUȚĀDHIKARA – TRANSLATION**

**SECOND CHAPTER Grahasphuṭādhikāra**, getting the true position of planets.

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Ravi, the sun, is the foremost of all planets is reckoned as the *śīghrocca* for *Guru*, *Kuja* and *Śani* [for calculations from their] mean positions [which are known]. For *Budha* and *Śukra* the mean Sun (Ravi) is the mean [position] and the *śīghrocca* are themselves.

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The *śīghrocca* was told first; now *mandocca* is being told. For the sun it is 78 (*vasvādri*), for *Māṅgala*, 130 (*khaviśva*), for *Budha*, 221 (*rūpākṛti*), for *Guru*, 172 (*dvadrindavaḥ*), for *Śukra*, 80 (*khāṣṭa*) and for *Śani*, 237 (*agāgnidasrāḥ*). This is
obtained from the mandocca bhagaṇas [number of revolutions of the apogee, mandocca] stated in the Sūryasiddhānta in verses (I: 41 and 42) starting with “prāggate sūryamandasya” up to “gognayaḥ sani mandasya”, [these numbers are] multiplied by the number of years up to the desired year, and divided by the number of years in a kalpa. The current mandocca will be deficient by a few liptis from the date provided by Ācārya and one bhāgi has been added to account for this as Dhruva (constants).

VERSE || 3 ||
Kendra of a planet is obtained in raśi [and its subunits] after subtracting the śīghrocca or mandocca from the mean planet. If the kendra is tulādi (starting with tulā, the angle is between 180 and 360 degrees) the calculated śīghrabhuṭaphala or mandabhujaphala should be added to the mean planet. If the kendra is meśādi (the angle is between 0 and 180 degrees) it should be subtracted. Later the method of getting the śīghraphala will be told [where] the śīghrahara 10 [vyomendavaḥ] should be added to koṭiphala, if śīghrakendra is mrigādi and subtracted if it is karkyādi.

VERSE || 4 ||
Now the procedure to get bhuja and koṭi.* (In a right angled triangle, bhuja is the opposite side of the right-triangle and koṭi is the adjacent side of the right-triangle). In the odd quadrant Bhuja is determined by the current angle. Koṭi is (yet to be) covered. In the even (yugma) quadrant it is the opposite of this. That means - the angle to be covered determines the bāhu and the angle covered determines the koṭi. The bhuja and koṭi are for three rāśis (90 deg). For 12 rāśis there are four padas; there are two odd (oja) quadrants. For rāśis 0, 1 and 2 are the same as for rāśis 6, 7 and 8. Here bhuja is determined by angle covered and koṭi by the angle yet to be covered. For rāśis 3, 4 and 5 and also for 9, 10, 11 which are even quadrants koṭi is determined by angle covered and bhuja by angle to be covered. Thus the bhuja and koṭi (found) for three rāśis repeat for the others. From bhuja, koṭi can be determined by subtracting by 3 rāśis.

(* We are thankful to the anonymous referee for pointing out the confusion in the original work itself. The corrections as per convention have been incorporated here.)

VERSE || 5 ||
Now to get the Rsine for bhuja and koṭi: The rāśi number is multiplied by 30 and added to bhāgi [degrees], divide this total [in degrees] bhāgi by 10. The quotient is the number of the khaṇḍajīvā covered all ready. The corresponding jīvā is written down. The value of the next khaṇḍaj jīvā is divided by 10 and multiplied by the
remainder whose līpti and vīlīpti have been converted to bhāga and added back to the bhāga value. The result (quotient) is added to the earlier obtained jīvā. The remainder (in this step) is multiplied by 60 and divided by 10, converted to līpti, vīlīpti and added to the jīvā. This is the jīvā or koṭi derived for the desired angle. It should be noted that this is in [units of] bhāgādi (degrees).

VERSE || 6 ||

The nine khaṇḍas of the jīvās, (expressed in bhūtasāṅkhya) in the direct order are stated:

21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2

and

2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |

VERSE || 7 ||

The successive jyākhaṇḍas are added to get the pīṇḍikṛtajīvā. They are 21, 41, 60, 77, 92, 104, 113, 118, and 120.

The utkramapīṇḍas are also provided. They are 2, 7, 16, 28, 43, 60, 79, 99, and 120.

VERSE || 8 ||

The mandacheda [divisors to get mandaphala] is being told [in words]. (vyomāgni-danta) 3230, (śikhirupāsakra) 1413, 603, (purāmbarāṅga) 1510, (digarthacandra) 1371, (rūpāgaviśva) 3769 (aṅkarasādrirāma) and 923 (tripaksarandhra), for the planetary bodies starting from the sun (Ravi). These are the values of the divisors [for getting the mandaphala].

VERSE || 9 ||

To get the correction for the instant, Rsine is multiplied by 60 and added to līpti. The sum is divided by the numbers 90 and others for the respective planets as prescribed in the verse starting with khāṅkaiḥ. (2.9) The result is added to the numbers mentioned earlier as vyomāgni-danta and so on {((490 (khatāṇa), 300 (vīyad abhrarāma), 70 (khaśva), 170 (khaśailendu), 21(indupakṣa), 380(khāstāgni)) to get the corrected divisor sphuṭamandacheda.

Now the sīghra cheda for the five planets starting from Kuja.

VERSE || 10 ||

1110 (dīgīśvara), 1956 (tarkaśarāṅkacandra), 3651 (koarthāṅgarāma), 993 (jvalanāṅkananda), 6562 (dvāṅgaiṣutarka). These sīghrachedas have been de-
vised similar to mandacheda as specified in [Sūrya]śiddhänta.

**VERSE || 11 ||**

To get the mandaphala and śīghraphala, one needs bhujaphala and kotiphala. The bhujā and koṭijīvas are kept in two places; multiply them by 60 and add to the litpi in lower place. Then they are divided by the respective manda cheda and śīghra cheda to get bhujaphala and kotiphala in degrees etc. As stated in the verse the manda arises because of only bhujaphala and therefore there is no need of kotiphala for the manda correction. The jyā of bhujā is multiplied by 60 and added to the lipti part and divided by the divisors as specified by vyomāgni etc. which are made true by correcting with khankaih etc., for the planets beginning with the Sun for the respective planets. The result in bhāga units [degrees] is the [first correction] mandaphala. This is negative if the kendra obtained by subtracting mean from mandocca is in karkyādi, (starting from karka, between 0 and 180) positive if it is tulādi (between 180 and 360). This gives the [longitude] after the [first] manda correction, mandasphuṭa.

**VERSES || 12 ||, || 13 || AND || 14 ||**

The mean motion madhyagati of the planets are being told in lipti, vilipti. For Ravi it is 59|8, for the moon 790|35 for Kuja 31|26, for Budha 245|32, for Guru 5|0, for Śukra 96|8, for Śani 2|0, for Rāhu 3|11, for candrocca 6|41. While making the mandasphuṭa correction, for getting the R sine, the khanda corresponding to the eṣya [to be covered part], is multiplied by the madhyagati in lipti, vilipti and multiplied by 6 (rasaghna) and divided by the appropriate divisor and added if it is karkyādi, subtracted for makarādi. This gives mandasphuṭagati.

The derivation of the (mean motion) madhyagati is done by dividing the bhagana (number of revolutions) as specified in Sūryasiddhānta by the bhūsāvanadina (number of days). It is [done] like this. Number of revolutions of Ravi is 4320000. The number of sāvana days are 1577917828. When this is divided [by number of revolutions] we get o rāsi, o bhāga, lipti 59 and vilipti 8. This is done for all planets. The meaning of madhyagati is the number of liptis covered in a day.

Although the procedure for mandasphuṭa is explained, I am summarising it again. It is like this. After obtaining the mean planets, take the difference with respective mandoccas, get the R sine by using the rule as bhūgāstayoḥ kenduhṛta, (verse number 2.5 above) multiply by 60 and add the lipti. Consider the numbers specified by the verse khāṅka and so on, added to the original divisors specified by vyomāgni, and divide the jyā, which is already multiplied by 60 by the revised divisor, and take the result in bhāga. When the mean planet is corrected with this, by subtraction, if it is meṣādi, and by addition, if it is tulādi, the mandasphuṭa
is obtained. Here it should be remembered that the sun and the moon are \textit{sphuṭa} by this correction. The five planets \textit{Kujādi} will be \textit{spaṣṭa} after the two corrections, namely, \textit{manda} and \textit{śīghra}. Thus after completing the explanation for \textit{manda}, I proceed to explain \textit{śīghraphala}.

\textbf{VERSE \| 15 \|}

Now, the procedure for \textit{śīghraphala} for the planets starting from \textit{Kuja}. The \textit{śīghraocca} subtracted from the \textit{mandasphuṭa} corrected planet is the \textit{kendra}. Both the \textit{bhuja} (R sine) and \textit{koṭi} (R cosine) are obtained. As per the verse \textit{do koṭṭh five kharasaṁ niḥatyāt}, (verse 2.11 above) the \textit{bhujājīva} and \textit{kotijīva} are multiplied by 60, the remainder is added back. These are divided by the appropriate divisors as specified by the verse starting with \textit{digiśvara}. (verse 2.10 above) The result from \textit{bhuja} is \textit{bhujaphala}; the result from \textit{koṭi} is \textit{koṭiphala}. \textit{Vyomendu 10}, is the \textit{śīghrahara}. The \textit{koṭiphala} obtained is added to or subtracted from this \textit{hara} (10) as per \textit{mrigādi} or \textit{karkādi}. The square of this sum or difference is obtained. Next, as stated by \textit{dorjyaphala varga yogīt}, the square of \textit{bhujaphala} is obtained. The two squares are added and the square root is the \textit{phala} called the \textit{calabāṇa}.

\textbf{VERSE \| 16 \|}

The \textit{bhujaphala} is multiplied by the trijya 120, divided by \textit{calabāṇa}. The inverse sine, \textit{cāpa}, of this is the \textit{śīghraphala} in \textit{bhāga} (degrees). For \textit{Kuja}, Budha, Guru, Šukra and Šani this is applied as positive or negative as mentioned earlier. Thus we get all the true planets.

\textbf{VERSE \| 17 \|}

The procedure to get the inverse sine, \textit{cāpa} (the arc of the angle). The arc has to be obtained (from the \textit{jyā}). Subtract as many \textit{khaṇḍajīvās} as possible from the \textit{jīvā}. Keep aside the number of \textit{khaṇḍajīvās} subtracted. The remainder is multiplied by 10 and divided by the \textit{khaṇḍajīvā} which is the \textit{khaṇḍa} which comes after the subtracted ones. When this is added to 10 times the number (of \textit{khaṇḍajīvās}) kept aside [earlier], that (sum) is the desired arc in (degrees). The result is the inverse sine, \textit{cāpa}.

\textbf{VERSE \| 18 \|}

The \textit{gati} of the \textit{mandasphuṭa} is subtracted from the \textit{gati} of the difference of \textit{śīghraocca} and \textit{graha}. What remains is multiplied by the difference between the \textit{śīghrahara} and the \textit{calabāṇa} as per the [rule of] \textit{catuḥpratinyāya}. This is divided by the \textit{calabāṇa} and the result in \textit{liptadi} [sundivisions of arcminutes] is added to the \textit{mandasphuṭa} if the \textit{bāṇa} is greater than the \textit{hara}, and subtracted from it if the \textit{bāṇa} is
less than the hara. The result is the sphaṭagati (true rate of motion). If the (earlier) result is greater than the mandagati, the mandagati is subtracted from the (earlier) result, and what remains is the vakragati (retrograde rate of motion).

**VERSE || 19 ||**

The $R \sin e$ of the sun (Ravi) is converted to lipti as per the verse uspāṁśu dor-japhalam and divided by 27. The result in liptis is added to or subtracted from the moon, Candra, as per the correction to the sun. If the bhujaphala is positive it is added to the moon. If it is negative for the sun it should be subtracted from the moon also. That gives Ravibhujasanśkṛtacandra - moon corrected for Ravibhujana. This correction is to be done for all planets as stated in the [Sūrya]siddhānta. But for all the others it is quite small and therefore I told it specifically for the moon. This correction should be done for all planets.

**VERSE || 20 || SŪRYASIDDHĀNTA (II - 46)**

This is as per the statement in [Sūrya]siddhānta - the gati of planets should be multiplied by Ravibhujaphala in kala (arc minutes) and divided by cakralipti, that is 21600; the result is positive or negative as is the case for the sun. The mean gati of the moon is 791 lipti. Adripakṣa, 27 is the result when 21600 is divided by this. Therefore I made the rule for division by 27.

This completes the second chapter called Grahasphuṭādhikāra of the book called Ganitagannadi, a commentary of Vārṣikatāra in the language of Karṇāṭa written by Śaṅkaranārāyaṇa Jyōtiṣi, who, to the ignorant astronomers, is like Garuḍa to snakes and who, akin to a full moon for the ocean of nectar [and] of Bṛhaspati – like Dēmaṇa, an eminent astronomer and a resident of Śṛṅgapura.

**6 TEXTS**

Here we give the text from the original palm leaf manuscript for the second half of first chapter and the second chapter (covered in this paper) which has the verses in Sanskrit and commentary in Kannada. As mentioned earlier the script is Nandināgarī and here we have put both languages in Kannada script.

**CONTINUATION OF CHAPTER 1**

 우리나라

모두 이용해서 의심적이고 대답으로 응답하는 기술적 육성의

이래서 환영을 바시고 다음과 같이 입증해보자 || ||
MEAN AND TRUE POSITIONS OF PLANETS

ಇಂದುಧು್ರ°ಾಂ±ಾ ರವಿಭಿವಿರ್ಭ"ಾ್ತಃ ಸಂ”ಾ್ರಂತಿಸಂŌಾಸಿ್ತಥಯೋ ಭವಂತಿ |
£ಾಭಿವಿರ್ಶು¥ಾ್ಧಶ್ಚ ಗತತುರ್ಹೀ§ಾಶೆ್ಚŒ£ಾ್ರದಿ”ಾ�³ಾ್ಯತಿ್ತಥಯೋ ದು್ಯ®ಾಶಿಃ || 11 ||

ಇಂದುಧು್ರ°ಾಂ±ಾ ಯೆಂದು ಮುಂನ ಬಂದ ಚಂದ್ರನ ವಷರ್ಧು್ರವವನಿಕಿ್ಕಕೊಂಡು | ಅದಂ®ಾಶಿ ತಿ್ರಂಶದು್ಗಣಿತಂ «ಾಗಯುತಂ ಯೆಂಬ ಪರಿ«ಾಷೆಯಿಂದ ®ಾಶಿ³ಾ್ಥನವಂ 30 ರಿಂಗು-|
ಣಿಸಿ ಕೆಳಗಿದ್ದ «ಾಗಿಯಂ ಕೂಡಿ ಹೀಗೆ «ಾಗೀಕರಿಸಿಕೊಂಡು | ರವಿಭಿವಿರ್ಭ"ಾ್ತಃ ಯೆಂದು 12 ರಿಂಬಂಗಿಸಿ ಬಂದ ಲಬ್ಧವೇ ಸಂ”ಾ್ರಂತಿ ತಿಥಿಯೆಂಬ ಸಂ೦ಯನುಳ್ಳದಹುದು | ಶೇಷ-|
ದಿಂದ ಪ್ರಯೋಜನವಿಲ್ಲ | ಸಂ”ಾ್ರಂತಿ ತಿಥಿಯೆಂದರೆ ಚೈತ್ರ ಶುದ್ಧ ¨ಾಡ್ಯ°ಾರಭ್ಯ ಸೂಯರ್ನು ಮಧ್ಯಚಾರದಿಂದ ಮೇಷ®ಾಶಿಗೆ ಪ್ರವೇಶವಹ ಪಯರ್ಂತರ ಮಧ್ಯದಲಿ್ಲ ಉಂಟಾದ ತಿಥಿ ಸಂ೦ಯುಳ್ಳದಹುದು | ಇಂನು ತಂನ ಇಷ್ಟ ದಿನಕೆ್ಕ ಚೈತ್ರ ಶುದ್ಧ ¨ಾಡ್ಯ ಆರಭ್ಯ°ಾಗಿ ಸಂದ|
ದಿನವ ಲೆಕಿ್ಕಸಿ ಇರಿಸಿಕೊಂಡು | ಆ ಮಧ್ಯದಲಿ್ಲ ಅಧಿಕ¬ಾಸವುಂಟಾದರೆ ಅದನೂ ಸಹ°ಾಗಿ ಕೂಡಿಕೊಂಡು | ಅವರೊಳಗೆ | £ಾಭಿವಿರ್ಶು¦ಾ್ಯಃ ಯೆಂದು ಈ ಮೊದಲು ಬಂದ ಸಂ-|
”ಾ್ರಂತಿ ತಿಥಿಗಳಂ ಕಳದು | ಅಲಿ್ಲ ಗತತುರ್ಹೀ§ಾಯೆಂದು ಚೈ£ಾ್ರದಿ ಯೆರಡೆರಡು ¬ಾಸಕೆ್ಕ|
ವೊಂದೊಂದು ಋತುವೆಂದು ಇಷ್ಟದಿಂದ ಹಿಂದೆ ಸಂದ ಋತು ಸಂಖೆಯಂ ಕಳದುಳಿದದು | ಚೈ£ಾ್ರದೀಷ್ಟ ತಿಥಿಗಳೇ³ೌರವ²ಾರ್ದಿ­ಾದ ತಂನ್ನ ಇಷ್ಟ ದಿ-|
ನಕೆ್ಕ ಬಂದ ದು್ಯಗಣವಹುದು ||

ನ–ಾಪ್ತಶಿಷೊ್ಟೕ ಧು್ರವ°ಾರಯುಕೊ್ತೕ ದು್ಯಸಂಚಯೋ °ಾರಪತಿನಿರ್ಶೀಥೇ |
ಅಹಗರ್ಣಃ³ಾವನ§ಾಡಿಕೋನಃ ಪದಂ ಗ್ರ´ಾಸ್ತತ್ರ ಭವಂತಿ ಸೂ­ಾರ್ತ್ || 12 ||
ಆ ದು್ಯಗಣವಂ ನ–ಾಪ್ತಶಿ²ಾ್ಟ ಯೆಂದು 7 ರಿಂ «ಾಗಿಸಿ ಮಿಕ್ಕ ಶೇಷಕೆ್ಕ ಧು್ರವ°ಾರಯು-|
ತಿಥ ಯೆಂದು ³ಾವನಧು್ರವೆಯ °ಾರವಂ ಕೂಡಿ ಯೇಳರಿಂದಧಿಕ°ಾದರೆ ಯೇಳಂ|
ಕಳದುಳಿದದು |
°ಾರಪತಿನಿರ್ಶೀಥೇ ಯೆಂದು ಶುಕ್ರ°ಾ®ಾದಿ­ಾಗಿ ಇಷ್ಟದಿನಕೆ್ಕ ಬಂದ °ಾರ ಸಂಖೆಯಹು-|
ದು | ಇಲಿ್ಲ ವೊಂದು ಉಳಿದರೆ ಶುಕ್ರ°ಾರ ಮಧ್ಯ®ಾತೆ್ರಗೆ ಬಂದದು | ಯೆರಡು ಉಳಿದರೆ|
ಶನಿ°ಾರ ಮಧ್ಯ®ಾತಿ್ರಗೆ ಬಂದದು ಯೆಂದರಿವುದು |||

ಇಂತು °ಾರವನರಿತುಕೊಂಡು ತಂನ ಇಷ್ಟದಿನಕೆ್ಕ ಬಂದ ದು್ಯಗಣದಲಿ್ಲ ವೊಂದು ಹೆಚಿ್ಚದರೂ|
ವೊಂದು ಕುಂ¥ಾದರೂ ವೊಂದು ಕಳದು ಕೂಡಿ ಸರಿದಂದುಕೊಂಬುದು | ಇಂ¤ಾ ದು್ಯಗಣ-|
ದಲಿ್ಲ | ಅಹಗರ್ಣಾ ³ಾವನ§ಾಡಿಕೋನಃ ಯೆಂದು ³ಾವನ ಧು್ರವದ ಘಳಿಗೆ ವಿಘಳಿಗೆಗಳಂ|
ಕಳವುದು | ಅದೆಂತೆಂದರೆ | ದು್ಯಗಣವನಿಕಿ್ಕಕೊಂಡು ಅಲಿ್ಲಂದ ವೊಂದಂ ತೆಗೆದುಕೊಂಡು|
ಕೆಳಗೆ ಅರುವತ್ತನಿಕಿ್ಕಕೊಂಡು | ಅಲಿ್ಲ ಕ್ರಮದಿಂದ ದು್ಯಗಣದ ಕೆಳಗಿದ್ದ 59 ರಲು್ಲ 60 ರಲು್ಲ ³ಾವನ ಧು್ರವದ|
್ಾದ ಮೂರು ಪ್ರತಿಯನು-|
ಳ್ಳದಹುದು | ಅದಕೆ್ಕ ಪದವೆಂಬ ಸಂ೦ಯಹುದು | ಆ ಪದದಲಿ್ಲಯೇ ಸೂ­ಾರ್ದಿ ಗ್ರಹರೆ-|
¯ಾ್ಲ ಉತ್ಪಂನರಹರು ||

ಪದಂ ಸ್ವ•ಾ–ಾಂಶಫಲೇನ ಹೀನಂ «ಾ–ಾದಿಕೋ ಮಧ್ಯದಿ°ಾಕರಃ ³ಾ್ಯತ್ |
ಪದವಂ ಬೇರೊಂದು ಪ್ರತಿಯನಿರಿಸಿಕೊಂಡು •ಾ–ಾಂಶಫಲೇನ ಹೀನಂ ಯೆಂದು 70 ರಿಂ |
«ಾಗಿಸಿ ಬಂದ ಲಬ್ಧ «ಾಗಿ | ಆ ಶೇಷವಂ 60 ರಿಂಗುಣಿಸಿ ಕೆಳಗಣ ಘಳಿಗೆಯಂ ಕೂಡಿ 70
ರಿಂದ ಬಂದದು ಲಿಪಿಕ್ಕೆ ತಹುದು ಇದಂ ಮುಂದಿನ ಪದದೊಳಗೆ ಕ್ರಮದಿಂದ ಕಳೆಯಲು ಉಳಿದದು. ಮಾಯದಿತ್ಯನಹನು ಆಗಿಯ ಥಾನಮಂ 30 ರಿಂದೆತಿಕ್ಕೆ ಬಂದ ಲಬ್ಧವಂ ಮೇಲಣ ಪ್ರತಿಯೊಳು ಕೂಡಿ | ಅದೇ ಶೇಷವೇ ಆಗಿ | ಮೊದಲವೇ ಲಿಪಿಕ್ಕೆ ವಿಲಿಪಿಕ್ಕೆ. ಇಂತು ಶಿಶಿ «ಾಗಿ ಲಿಪಿಕ್ಕೆ ವಿಲಿ¨ಟಮಂ ನೋಡುವ ದಿನದ ಮಧ್ಯ ¢ಟಕದಿಂದಬಂದ ಮಧ್ಯಚಂದ್ರನಹನು. ಇಲಿಲ ದಿನ 1ಕೆಮಧ್ಯಗತಿ ಲಿಪಿಕ್ಕೆ 59 ವಿಲಿಪಿಕ್ಕೆ 8 | ಈ ಲೆಕ್ಕದಲಿಲ 70 ದಿನಕೆಕ ವೇದು «ಾಗಿ ಕಡಮೆ ಯಹುದೆಂಬದೀಗ ಗಣಿ ¢ಟಸನೆ.ಅಂ±ದಿಕೇಂದೋ ಪದಮಕರ್ಣಿಘ್ನಂ ¢ಟವ²ಟಂಗ «ಾಗೇನ ಪದೇನ ಯುಕ್ತಂ || 13 || ಪದಂ ಪದವನು | ಅಕರ್ನಿಘ್ನಂ ¢ಟಪುದು ಯೆಂದು 12 ರಿಂಗುಣಿಸಿ ¢ಟವ²ಟಂಗ «ಾಗೇನ ಯುಕ್ತಂ ¢ಟಪುದು ಯೆಂದು | ಆ ಹನೆರಡರಿಂದಧಿಕ ¢ಟಗಿದ್ದರೆ 12 ರಿಂದೆತಿಕ್ಕೆ ಬಂದ ಲಬ್ಧವಂ ಮೇಲಣ ಪ್ರತಿಯಂ ನಿಧಿಪಕ್ಷನೇತ್ರ ಯೆಂದು 229 ರಿಂದೆತಿಕ್ಕೆ ಬಂದವು ಲಿಪಿಕ್ಕೆ ವಿಲಿಪಿಕ್ಕೆ. ಇಂತು ¢ಟದ ¢ಟದಿಯಂ ಕುಜನ ವಷರ್ ಧು್ರವದೊಳು ಕೂಡಲು ನೋಡುವ ದಿನದ ಮಧ್ಯ ¢ಟಕದಿಂದಬಂದ ಮಧ್ಯಚಂದ್ರನಹನು ||

ಇಲಿಲ ಕೃತ ¢ಟದ 4 ¢ಟಾಶಿ | ನಿಧಿಪಕ್ಷನೇತ್ರ ¢ಟದ 229 ದಿನ || ಈ ಇನೂ್ನರಿಪ್ಪತೊ್ತಂ-ತು್ತ ದಿನಕೆಮಧ್ಯಗತಿ ವಶದಿಂದ 4 ¢ಟಾಶಿ ಬಹುದು | ಅದರಿಂದ ನಿಧಿಪಕ್ಷನೇತ್ರ ¢ಟದಿಂದ ¢ಟಪುದು ಯೆಂದು ¢ಟವ²ಟಂಗ ಹೇಳಿ ಗುಣಕವ ಪೇಳಿದದಲ್ಲ ¢ಟಗಿ ಪ್ರಕರಣಬಲದಿಂದ ಕುಜಗೆ ಪೇಳಿದ ಕೃತ ಯೆಂಬುದೇ ಗುಣಕವೆಂದು | ಪದವಂ ಮೂರು ¢ಟಾರವೆಂದರಿವುದು ||

ಬುಧ ಶೀಘೊ್ರೕಚ್ಚವಂ ತಹರೆ ||

ಜ್ಞಃ ¢ಟಗಿ ಥಾನಮಂ ¢ಟವ²ಟಂತ್ತರಂ ಘಾನಿಂದ ¢ಟಪುದು ಯೆಂದರಿಗೆ ¢ಟಗಿ ¢ಟಗಿ ಪೇಳಿದ ¢ಟಾಂತರಂ ಯೆಂದು ¢ಟವ²ಟಂಗ ಹೇಳಿ ಗುಣಕವ ಪೇಳಿದದಲ್ಲ ¢ಟಗಿ ಪ್ರಕರಣಬಲದಿಂದ ಕುಜಗೆ ಪೇಳಿದ ಕೃತ ಯೆಂಬುದೇ ಗುಣಕವೆಂದು | ಪದವಂ
ಯೋಜಿ ಶೈನೂಡ ಭೂಕಣೋರ್ ದಿವಗುಣಾನಿ ತುದ್ವಗರ್ತೋ ದಶಗುಣಾತ್ಪದಂ ಭೂಪರಿಧಿಭರ್ವೇತ್ || (I-59)

ಅಂದರಿಂದ, ಭೂಮಧ್ಯರೇಖೆಯೆಂದರೆ ಮೇರುಪವರ್ದಕು ಲಂಕೆಗೂ ಸೂತ್ರವ ಹಿಡಿಯಾಗಿದೆ. ಹೀತ್ಕವೆಂದರೆ ದೇಶವಿಶೇಷ, ಅಂಬಲಿ, ಅವಂತಿಯೆಂದರೆ ಉಜೆಣರಿಂದ ಇವರ ಮೇಲೆ ಆ ದ್ನಣೋತ್ತರ ಸೂತ್ರವಿದ್ದದರಿಂದ ಈ ಪ್ರದೇಶಗಳು ಭೂಮಧ್ಯರೇಖೆಯೆಂದು ಮುನ್ನನಿಸಿದ್ದಾಗಿ ಬಂದ ಲಿಪಿಯಂಗರಿಸಿದ್ದ ಲಿಪಿಯಂಗರಲಿ್ಲ ಕಾವು ಮೂಡದರೆ ಕಳವುದು | ಪಡುವದರೆ ಕೂಡುವದು || ಇದು ಯೋಜನಸಂಖ್ಯಾಪ್ರಾರ್ಥನೆಗಳು.
MEAN AND TRUE POSITIONS OF PLANETS

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ಬ.ಸ.ಶ್ರೀಲಾಜಾ ಅವರು ಸೀತಾರಾಮ್ ಜಯಗಳು

| ಯಕ್ಷಗಾಲಗಳ ಸ್ವಿತ್ತರ ಜೀವನದಲ್ಲಿ ಬರುವ ಬಾರಿ | ಯಕ್ಷಗಾಲಗಳ ಸ್ವಿತ್ತರ ಜೀವನದಲ್ಲಿ ಬರುವ ಬಾರಿ | ಯಕ್ಷಗಾಲಗಳ ಸ್ವಿತ್ತರ ಜೀವನದಲ್ಲಿ ಬರುವ ಬಾರಿ | 2.7 |}
| ಕೋಟಿ ಉತ್ತರಾಮ ಜೀವೆಗಳು | ಕೋಟಿ ಉತ್ತರಾಮ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಮೂರು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಮೂರು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಮೂರು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |}
| ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |}
| ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |}
| ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |}
| ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |}
| ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |}
| ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | ಪಿಂಡಿಕೃತ ಜೀವೆಗಳು | 2 | 7 | 16 | 28 | 43 | 60 | 79 | 99 | 120 |}
| ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು ಎರಡು ಪಾಕತಮಲುಗಳು ಮೊದಲು | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 | 1 | 120 |}
| ಇವೀಗಳು | 2 | 5 | 9 | 12 | 15 | 17 | 19 | 20 | 21 |
2.8

| 2.9 |

2.10

| 2.11 |

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ಕುಧಿಗಳಿಗೆ ಶೀಘ್ರಫಲವಂ ತಹರೆ | ತಂದು ಮಂದಸು್ಫಟ ಗ್ರಹದೊಳಗೆ ತಂದು ಶೀಘೊ್ರೌಚ್ಚಮಂ ಕಳದು ಕೇಂದ್ರವಿಟ್ಟ | ಭು›ಾಕೋಟಿಗಳೆರಡಂ ಪಡದು ಬೇರೆ ಬೇರೆ ಜೀವೆಗೊಟು್ಟ | ದೋಃ ಕೋಟಿಜೀವೇ ಖರಸೈಃ ನಿಹ§ಾ್ಯತ್ ಯದು ಭು›ಾಜೀವೆಯಂ ಕೋಟಿಜೀವೆಯಂ 60 ರಿಂ ಗುಣಿಸಿ ಶೇಷವಂ ಕೂಡಿ ದಿಗೀಶ್ವ®ಾ ಏನೂದಿದ | ಕೋಟಿಜೀವೆಯಂ ಬಂದಿ ಕೋಟಿಫಲವಹುದು | ಈ ಹರದಲಿ್ಲದ ಕೋಟಿಫಲಯಂ ಕೂಡಿದು¥ಾಗಲಿ ಕಳದು¥ಾಗಲಿಅದಂ ವಗರ್ಂ ಗೊಂಡು ಇರಿಸಿ ದೋ›ಾ್ಯರ್ಫಲಗರ್ಯೋ–ಾತ್ ಯದು ಭು›ಾಫಲವಂವಗರ್ಂ ಗೊಂಡು ಇವೆರಡು ವಗರ್ವನು ಕೂಡಿ | ಮೂಲಂ ಗೊಳಲು | ಬಂದ ಫಲವಂ |ಚಲ¥ಾ್ಬಣಮು¥ಾಹರಂತಿ | ಚಲªಾಣದಿಂ «ಾಗಿಸಿ ಬಂದ «ಾ–ಾದಿ ಫಲ | ಅದಕೆ್ಕ ಚಾಪಂ ಗೊಂಡರೆ «ಾ–ಾದಿ ಶೀಘ್ರಫಲ¬ḣಾದ | ಕುಜಬುಧಗುರುಭೃಗುಶನಿಗಳಿಗೆ ಮುಂನಿನಂತೆ ಋಣ ಧನವರಿತು ಮಂದಸು್ಫಟ-ಗ್ರಹದೊಳಗೆ ಸಂಸ್ಕರಿಸುವುದು | ಅವರು ಸು್ಫಟವಹರು ||

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