Comparing a few distributions of transverse momenta in high energy collisions

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Abstract: Transverse momentum spectra of particles produced in high energy collisions are very important due to their relations to the excitation degree of interacting system. To describe the transverse momentum spectra, one can use more than one probability density functions of transverse momenta, which are simply called the functions or distributions of transverse momenta in some cases. In this paper, a few distributions of transverse momenta in high energy collisions are compared with each other in terms of plots to show some quantitative differences. Meanwhile, in the framework of Tsallis statistics, the distributions of momentum components, transverse momenta, rapidities, and pasudorapidities are obtained according to the analytical and Monte Carlo methods. These analyses are useful to understand carefully different distributions in high energy collisions.

Keywords: Transverse momentum spectra, different distributions, high energy collisions

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1 Introduction

In high energy hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions, transverse momentum spectra of secondary particles are one of the “first day” measurable quantities. The transverse momentum spectra are expected to reflect the excitation degree of interacting system, which is useful to understand the properties of particle production and system evolution. To describe the transverse momentum spectra, one can use more than one probability density functions of transverse momenta. Strictly speaking, the probability density function and the distribution function are different concepts in statistics. We simply call the probability density function the distribution or function in some cases in the present work.

Three types of distributions will be compared with respective modified forms in the present work. Firstly, we compare the Hagedorn function with its modified forms which are suitable to fit the spectra in high transverse momentum region. Secondly, we compare the simplest standard distribution with its modified forms which are suitable to fit the spectra in low transverse momentum region. Thirdly, we compare the simplest Boltzmann distribution with its modified forms which are also suitable to fit the spectra in low transverse momentum region, though the Boltzmann distribution is one of the standard distribution.

After comparisons for transverse momentum distributions, we discuss an application of the Monte Carlo method according to the (transverse) momentum distribution and the assumption of isotropic emission in the subsequent part of the present work. In particular, in the framework of Tsallis statistics, the distributions of momentum components, transverse momenta, rapidities, and pasudorapidities are obtained according to the analytical and Monte Carlo methods.

2 Formalism and method

i) The Hagedorn function and its modified forms
The Hagedorn function and its modified forms are suitable to describe the transverse momentum ($p_T$) spectra of heavy flavor particles which are expectantly produced from the hard scattering process and distributed usually in a wider $p_T$ range. In general, the wider $p_T$ range is from 0 to the maximum $p_T$.

In refs. [1, 2], an inverse power-law

$$f_1(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A_1 p_T \left(1 + \frac{p_T}{p_1}\right)^{-n_1}$$

(1)

that is an empirical formula inspired by quantum chromodynamics (QCD) is used, where $N$ denotes the number of particles, $p_1$ and $n_1$ are the free parameters, and $A_1$ is the normalization constant. We call this type of inverse power-law the Hagedorn function [1].
In ref. [3], a modified Hagedorn function is shown as
\[ f_2(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A_2 \frac{p_T^2}{\sqrt{p_T^2 + m_0^2}} \left( 1 + \frac{p_T}{p_2} \right)^{-n_2}, \] (2)
where \( m_0 \) is the rest mass of considered particle, \( p_2 \) and \( n_2 \) are the free parameters, and \( A_2 \) is the normalization constant. We call Eq. (2) the first modified Hagedorn function.

In ref. [4–8], there is another inverse power-law
\[ f_3(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A_3 p_T \left[ 1 + \left( \frac{p_T}{p_3} \right)^2 \right]^{-n_3}, \] (3)
where \( p_3 \) and \( n_3 \) are the free parameters, and \( A_3 \) is the normalization constant. We call Eq. (3) the second modified Hagedorn function.

Even in ref. [9], there is the form
\[ f_4(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A_4 \left[ 1 + \left( \frac{p_T}{p_4} \right)^2 \right]^{-n_4}, \] (4)
where \( p_4 \) and \( n_4 \) are the free parameters, and \( A_4 \) is the normalization constant. We call Eq. (4) the third modified Hagedorn function.

ii) The simplest standard distribution and its modified forms

The simplest standard distribution and its modified forms are suitable to describe the \( p_T \) spectra of light flavor particles which are expectantly produced from the soft excitation process or thermal process and distributed mainly in a narrow \( p_T \) range. The narrow \( p_T \) range covers a range from 0 to around 2–3 GeV/c for pions produced in collisions at dozes of GeV. The boundary of narrow \( p_T \) range is changeable for different particles and at different energies.

The standard distribution has different forms. In the case of including rapidity and chemical potential, the simplest form can be written as [10]
\[ f_1(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_1 p_T \times \int_{y_{\text{min}}}^{y_{\text{max}}} \left[ \exp \left( \frac{\sqrt{p_T^2 + m_0^2} \cosh y - \mu}{T_1} \right) + S \right]^{-1} dy, \] (5)
where \( y_{\text{min}} \) and \( y_{\text{max}} \) denote the minimum and maximum \( y \) respectively, \( \mu \) denotes the chemical potential, \( T_1 \) is the free parameter of temperature, and \( C_1 \) is the normalization constant. In particular, \( S = 1, 0, \) and \(-1\) denote the Fermi-Dirac, Maxwell-Boltzmann, and Bose-Einstein statistics, respectively. This form is inconsistent with the classical ideal gas model, though it has many applications.

A modified form of the simplest standard distribution is \([10, 11]\)
\[ f_2(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_2 p_T \sqrt{p_T^2 + m_0^2} \int_{y_{\text{min}}}^{y_{\text{max}}} \cosh y \times \left[ \exp \left( \frac{\sqrt{p_T^2 + m_0^2} \cosh y - \mu}{T_2} \right) + S \right]^{-1} dy, \] (6)
where \( T_2 \) is the free parameter of temperature and \( C_2 \) is the normalization constant. We call Eq. (6) the first modified the simplest standard distribution. This form is consistent with the classical ideal gas model, i.e. it is close to Rayleigh distribution at low energy.

Another modified form of the simplest standard distribution is \([12]\)
\[ f_3(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_3 p_T^2 \times \int_{y_{\text{min}}}^{y_{\text{max}}} \left[ \exp \left( \frac{\sqrt{p_T^2 + m_0^2} \cosh y - \mu}{T_3} \right) + S \right]^{-1} dy, \] (7)
where \( T_3 \) is the free parameter of temperature and \( C_3 \) is the normalization constant. We call Eq. (7) the second modified the simplest standard distribution. This form is also inconsistent with the classical ideal gas model.

iii) The simplest Boltzmann distribution and its modified forms

In some cases, we can neglect chemical potential and/or spin effect, and/or consider only mid-rapidity, in the simplest standard distribution and its modified forms. In the case of neglecting simultaneously chemical potential and spin effect, and considering only mid-rapidity, we have simpler forms of the above Eqs. (5)–(7) to be
\[ f_1(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_1 p_T \exp \left( - \frac{\sqrt{p_T^2 + m_0^2}}{T_1} \right), \] (8)
\[ f_2(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_2 p_T \sqrt{p_T^2 + m_0^2} \exp \left( - \frac{\sqrt{p_T^2 + m_0^2}}{T_2} \right), \] (9)
and
\[ f_3(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_3 p_T^2 \exp \left( - \frac{\sqrt{p_T^2 + m_0^2}}{T_3} \right), \] (10)
respectively. We call Eqs. (9) and (10) the first and second modified the simplest Boltzmann distribution respectively. Only Eq. (9) is consistent with the classical ideal gas model at low energy.
Further, the velocity components are due to $\phi$ and $y$ according to definitions of $y$ and some concerned quantities and their distributions. In particular, if the analytic expression is difficult to obtain, we can use the Monte Carlo method to obtain some concerned distributions. In the Monte Carlo method [13], let $R_{1,2,3}$ denote random numbers distributed evenly in $[0,1]$. Some discrete values of $p_T$ can be obtained due to the following limitation.

$$\int_0^{p_T} f(p_T) dp_T < R_1 < \int_0^{p_T+\delta p_T} f(p_T) dp_T,$$

(11)

where $\delta p_T$ denote a small shift relative to $p_T$.

Under the assumption of isotropic emission in the rest frame, we have the momentum components to be

$$p_x = p_T \cos \phi, \quad p_y = p_T \sin \phi, \quad p_z = p_T / \tan \theta,$$

(12)

where

$$\phi = 2\pi R_2, \quad \theta = 2 \arcsin \sqrt{R_3}$$

(13)

due to $\phi$ and $\theta$ satisfy the distributions

$$f_\phi(\phi) = \frac{1}{2\pi}, \quad f_\theta(\theta) = \frac{1}{2} \sin \theta$$

(14)

respectively [13].

The momentum $p$ and energy $E$ can be obtained by

$$p = \sqrt{p_x^2 + p_y^2}, \quad E = \sqrt{p^2 + m_0^2}.$$  

(15)

Further, the velocity components are

$$\beta_x = \frac{p_x}{E}, \quad \beta_y = \frac{p_y}{E}, \quad \beta_z = \frac{p_z}{E}.$$  

(16)

The rapidity $y$ and pseudorapidity $\eta$ are [14]

$$y \equiv \frac{1}{2} \ln \left( \frac{E + p_x}{E - p_x} \right), \quad \eta \equiv -\ln \tan \left( \frac{\theta}{2} \right).$$  

(17)

According to definition of $y$, we can define $y_1$ by $E$ and $p_x$, and $y_2$ by $E$ and $p_y$, to be

$$y_1 \equiv \frac{1}{2} \ln \left( \frac{E + p_x}{E - p_x} \right), \quad y_2 \equiv \frac{1}{2} \ln \left( \frac{E + p_y}{E - p_y} \right).$$  

(18)

Combining with the distribution of $\theta$ and the definition of $\eta$, we have the distribution of $\eta$ to be

$$f_\eta(\eta) = \frac{1}{2 \cosh^2 \eta}$$

(19)

which satisfies approximately the Gaussian distribution with the width of $\sigma_\eta \approx 0.91$ [15]. The distribution of $y$ is expected to obey the Gaussian distribution with the width of $\sigma_y < \sigma_\eta$.

According to $p_T$ distribution and isotropic assumption, many quantities can be obtained. In fact, the scatter plots of particles in the three-dimensional momentum ($p_x$-$p_y$-$p_z$), velocity ($\beta_x$-$\beta_y$-$\beta_z$), and rapidity ($y_1$-$y_2$-$y$) spaces can be obtained based on the above discussions. We shall not discuss the scatter plots of particles due to they being beyond the focus of the present work.

v) Analytical and Monte Carlo calculations based on momentum distribution

Although we can obtain other distributions based on $p_T$ distributions and the assumption of isotropic emission, consistent $p_T$ and $y$ distributions should be obtained from the momentum ($p$) distribution and the assumption of isotropic emission. There are various $p$ distributions which may be from the Fermi-Dirac, Maxwell-Boltzmann, Bose-Einstein, or Tsallis statistics, etc. As an example, we use the the $p$ distribution in the Tsallis statistics.

In the Tsallis statistics, one has [10, 16–18]

$$f_\mu(p) = \frac{1}{N} \frac{dN}{dp} = C p^{q-1} \left[ 1 + \frac{q-1}{T} \sqrt{p^2 + m_0^2} \right]^{-\frac{q}{q-1}},$$  

(20)

where $T$ is the temperature, $q$ is the entropy index, $C$ is the normalization constant, and $\mu$ and $S$ are neglected for convenient treatment. The invariant $p$ distribution is

$$E \frac{d^3N}{dp^3} = C \sqrt{p^2 + m_0^2} \left[ 1 + \frac{q-1}{T} \sqrt{p^2 + m_0^2} \right]^{-\frac{q}{q-1}}.$$  

(21)

The distributions of unit $p_T$ and $y, p_T, y$, and $p_x$ are

$$f_{p_T,y}(p_T, y) = \frac{1}{N} \frac{d^2N}{dp_T dy} = C p_T \sqrt{p_T^2 + m_0^2} \cosh y \left[ 1 + \frac{q-1}{T} \sqrt{p_T^2 + m_0^2} \cosh y \right]^{-\frac{q}{q-1}},$$  

(22)

$$f_{p_T}(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C p_T \sqrt{p_T^2 + m_0^2} \int_{y_{\min}}^{y_{\max}} \cosh y \left[ 1 + \frac{q-1}{T} \sqrt{p_T^2 + m_0^2} \cosh y \right]^{-\frac{q}{q-1}} dy.$$  

(23)
\[
f_{y}(y) = \frac{1}{N} \frac{dN}{dy} = C \cosh y \int_{0}^{\infty} p T \sqrt{p_{T}^{2} + m_{0}^{2} \cosh y}^{\frac{q - 1}{T}} dp T, \tag{24}
\]
\[
f_{p_{T}}(p_{T}) = \frac{1}{N} \frac{dN}{dp_{T}} = \frac{1}{2 \pi} \int_{-\infty}^{\infty} \frac{1}{\sqrt{p_{T}^{2} + p_{y}^{2}}} f_{p T} \left( \sqrt{p_{T}^{2} + p_{y}^{2}} \right) dp y
\]
\[
= C \frac{1}{2 \pi} \int_{-\infty}^{\infty} p_{T}^{2} + p_{y}^{2} + m_{0}^{2} \int_{y_{\min}}^{y_{\max}} \cosh y \times
\]
\[
\left[ 1 + q - 1 \right] \sqrt{p_{T}^{2} + p_{y}^{2} + m_{0}^{2} \cosh y}^{\frac{q - 1}{T}} dp y, \tag{25}
\]
respectively, where \( C \) in the above equations may be different from each other.

In the Monte Carlo method [13], some discrete values of \( p \) can be obtained due to the following limitation
\[
\int_{0}^{p} f_{p}(p') dp' < R_{1} < \int_{0}^{p + \delta p} f_{p}(p') dp', \tag{26}
\]
where \( \delta p \) denote a small shift relative to \( p \). Then
\[
p_{T} = p \sin \theta = p \sin \left( 2 \arcsin \sqrt{R_{3}} \right). \tag{27}
\]
Other quantities have the same expressions as those in subsection iv).

3 Results and discussion

The transverse momentum spectra obtained from the Hagedorn function and its modified forms are presented in Fig. 1. The solid, dashed, dotted, and dot-dashed curves represent the results from Eqs. (1)–(4), respectively, with \( p_{1} = p_{2} = p_{3} = p_{4} = 10 \text{ GeV}/c \) and \( n_{1} = n_{2} = n_{3} = n_{4} = 5 \). In particular, the dashed curves with marks I and II corresponding to \( m_{0} = 0.139 \) (pion) and 0.938 GeV/c\(^{2} \) (proton) in Eq. (2) respectively. One can see that the effect of rest mass in the first modified form is very small due to large \( p_{T} \). The other two modified forms describe narrow \( p_{T} \) range with high probability density in low \( p_{T} \) region. In particular, the third modified form has the maximum probability density at \( p_{T} = 0 \), which is not correct comparing with general experimental data.

The transverse momentum spectra obtained from the simplest standard distribution and its modified forms are shown in Fig. 2. The solid, dashed, and dotted curves represent the results from Eqs. (5)–(7), respectively, with \( T_{1} = T_{2} = T_{3} = 0.2 \text{ GeV} \), \( \mu = 0.1 \text{ GeV} \), \( y_{\min} = -0.5 \), and \( y_{\max} = 0.5 \). The upper [(a)(b)], middle [(c)(d)], and lower [(e)(f)] panels correspond to \( S = 1, 0, \) and \( -1 \), respectively; and the left [(a)(c)(e)] and right [(b)(d)(f)] panels correspond to \( m_{0} = 0.139 \) and 0.938 GeV/c\(^{2} \) respectively. One can see that the modified forms contribute a wider \( p_{T} \) range than the simplest standard distribution, though a lower probability density in very-low \( p_{T} \) region in the modified forms appears due to the limitation of normalization. For pion \( p_{T} \) distribution, the effect of \( S \) is obvious, which should be considered in the calculation due to small mass. For proton \( p_{T} \) distribution, the effect of \( S \) is very small, which can be neglected in the calculation due to large mass.

The transverse momentum spectra obtained from the simplest Boltzmann distribution and its modified forms are displayed in Fig. 3. The solid, dashed, and dotted curves represent the results from Eqs. (8)–(10), respectively, with \( T_{1} = T_{2} = T_{3} = 0.2 \text{ GeV} \). The left [(a)] and right [(b)] panels correspond to \( m_{0} = 0.139 \) and 0.938 GeV/c\(^{2} \) respectively. One can see that the modified forms contribute a wider \( p_{T} \) range than the simplest Boltzmann distribution, though a lower probability density in very-low \( p_{T} \) region in the modified forms appears due to the limitation of normalization. For pion \( p_{T} \) distribution, the effect of mass on the two modified forms is obvious, which does not need to be distinguished clearly in the calculation. For proton \( p_{T} \) distribution, the effect of mass on the two modified forms is obvious, which should be distinguished in the calculation.

In the framework of Tsallis statistics with \( q = 1.05 \), Figure 4 shows the distributions of (a) \( p_{z} \) for pions and protons at \( T = 0.15 \text{ GeV} \), (b) \( p_{T} \) for pions and protons at given \( T \), (c) \( y \) (\( \eta \)) for pions at three \( T \) shown in the panel, and (d) \( y \) (\( \eta \)) for protons at three \( T \), respectively. The curves represent the results calculated from Eqs. (25), (23), (24), and (19), respectively, in the analytical calculation. The symbols represent the results obtained from Eqs. (12), (27), and (17) in the Monte Carlo calculation. One can see the natural result that the analytical and Monte Carlo calculations are consistent with each other. This also confirms that our calculations are correct. Another observation is that the distribution of \( y \) is closer to that of \( \eta \) at higher \( T \), in particular for lighter particle.

From the above discussions one can see that the trends of modified functions show large departures from that of original function in some cases. Because of the limitation of normalization, the increase (decrease) of probability in low \( p_{T} \) region results in the decrease (increase) of probability in very-low \( p_{T} \) region. The modified functions, Eqs. (2) and (3), do not cause large
Fig. 1. Transverse momentum spectra obtained from the Hagedorn function and its modified forms. The solid, dashed, dotted, and dot-dashed curves represent the results from Eqs. (1)–(4), respectively, with \( p_1 = p_2 = p_3 = p_4 = 10 \, \text{GeV/c} \) and \( n_1 = n_2 = n_3 = n_4 = 5 \). In particular, the dashed curves with marks I and II corresponding to \( n_0 = 0.139 \) and 0.938 GeV/\( c^2 \) in Eq. (2) respectively. The modified functions, Eqs. (6) and (7) [Eqs. (9) and (10)], result in larger probability in low \( p_T \) region and smaller probability in very-low \( p_T \) region comparing with the original function, Eq. (5) [Eq. (8)]. In particular, for a given \( p_T \) spectrum, the modified functions, Eqs. (6) and (7) [Eqs. (9) and (10)], “measure” lower temperatures than the original function, Eq. (5) [Eq. (8)]. For example, if we use the modified functions to “measure” (fit) the spectra (solid curves) of original function in Figs. 2 and 3, the modified temperatures for the spectra of pions and protons are smaller than 0.2 GeV which is the temperature of the spectra of original function. Contrarily, if we use the original function to “measure” the spectra (dashed and dotted curves) of modified functions in Figs. 2 and 3, the original temperatures for the spectra of pions and protons are greater than 0.2 GeV which is the temperature of the spectra of modified functions.

Figures 5 and 6 show the situations of the modified functions “measuring” the spectra (solid curves) of the original one in Figs. 2 and 3 respectively. The values of related temperature parameters are shown in each panel. Other parameters for Fig. 5 are the same as for Fig. 2. One can see that the modified temperatures \( (T_2 \text{ and } T_3) \) for the spectra of pions and protons are smaller than the original temperature \( (T_1) \), though the modified functions does not fit the original one. This inconsistent results render that the modified functions may be necessary. Contrarily, in the case of the modified functions fitting the original one, one can obtain consistent results which mean that the modified functions are not necessary.

At the same temperature \( T \), the distributions of \( p_x \) and \( p_T \) for pions are much narrower than those for protons due to the fact that the distribution width increases with the increase of mass as indicated in the ideal gas model based on the Maxwell-Boltzmann statistics. With the increase of \( T \), the distribution of \( y \) is closer to that of \( \eta \) for not only pions but also protons. The degree of closeness for pion spectrum is much more than that for proton spectrum at a given temperature. This is a natural conclusion due to the definitions of \( y \) and \( \eta \), though we obtain this conclusion in the framework of Tsallis statistics and the assumption of isotropic emission by the analytical and Monte Carlo calculations.

Before conclusions, as an example of the applications of the above distributions, Figure 7 present some comparisons with the \( p_T \) spectra of (a)(b) positive pions (\( \pi^+ \)) and (c)(d) protons (\( p \)) produced in (a)(c) central (0–5%) and (b)(d) peripheral (80–92%) gold-gold (Au-Au) collisions at 200 GeV. The symbols represent the experimental data measured by the PHENIX Collaboration [19]. The solid, dashed, dotted, and dot-dashed curves represent the fitting results by Eqs. (1), (6), (9), and (23), respectively. In the calculation, from panels (a) to (d), we take in proper order \( n_1 = 40, 39, 27, \) and 35 in Eq. (1) with \( p_1 = 10 \, \text{GeV/c}; \, T_2 = 0.16, \)
Fig. 2. Transverse momentum spectra obtained from the simplest standard distribution and its modified forms. The solid, dashed, and dotted curves represent the results from Eqs. (5)–(7), respectively, with $T_1 = T_2 = T_3 = T = 0.2$ GeV, $\mu = 0.1$ GeV, $y_{\text{min}} = -0.5$, and $y_{\text{max}} = 0.5$. Panels (a)(b), (c)(d), and (e)(f) correspond to $S = 1, 0, \text{ and } -1$, respectively; and panels (a)(c)(e) and (b)(d)(f) correspond to $m_0 = 0.139$ and $0.938$ GeV/c$^2$ respectively.
Fig. 3. Transverse momentum spectra obtained from the simplest Boltzmann distribution and its modified forms. The solid, dashed, and dotted curves represent the results from Eqs. (8)–(10), respectively, with \( T_1 = T_2 = T_3 = T = 0.2 \) GeV. Panels (a) and (b) correspond to \( m_0 = 0.139 \) and 0.938 GeV/c\(^2\) respectively.

Fig. 4. Distributions of (a) \( p_x \) for pions and protons at \( T = 0.15 \) GeV, (b) \( p_T \) for pions and protons at given \( T \), (c) \( y(\eta) \) for pions at three \( T \) shown in the panel, and (d) \( y(\eta) \) for protons at three \( T \), respectively, in the framework of Tsallis statistics with \( q = 1.05 \). The curves and symbols represent the results obtained from analytical and Monte Carlo calculations.
Fig. 5. Transverse momentum spectra obtained from the simplest standard distribution and its modified forms which fit the former. The solid, dashed, and dotted curves represent the results from Eqs. (5)–(7) with $T_1$, $T_2$, and $T_3$, respectively. Meanwhile, $\mu = 0.1$ GeV, $y_{\text{min}} = -0.5$, and $y_{\text{max}} = 0.5$. Panels (a)(b), (c)(d), and (e)(f) correspond to $S = 1$, 0, and $-1$, respectively; and panels (a)(c)(e) and (b)(d)(f) correspond to $m_0 = 0.139$ and 0.938 GeV/c$^2$, respectively.
Fig. 6. Transverse momentum spectra obtained from the simplest Boltzmann distribution and its modified forms which fit the former. The solid, dashed, and dotted curves represent the results from Eqs. (8)–(10) with $T_1$, $T_2$, and $T_3$, respectively. Panels (a) and (b) correspond to $m_0 = 0.139$ and 0.938 GeV/c² respectively.

Fig. 7. Transverse momentum spectra of (a)(b) $\pi^+$ and (c)(d) $p$ produced in (a)(c) 0–5% and (b)(d) 80–92% Au-Au collisions at 200 GeV. The symbols represent the experimental data measured by the PHENIX Collaboration [19]. The solid, dashed, dotted, and dot-dashed curves represent the fitting results by Eqs. (1), (6), (9), and (23), respectively.
0.13, 0.29, and 0.19 GeV in Eq. (6) with \( \mu = 0 \) at high energy and \( y_{\text{min}} \approx -0.35 \) and \( y_{\text{max}} \approx 0.35 \) in the experiment; \( T_2 = 0.15, 0.12, 0.29, \) and 0.19 GeV in Eq. (9); \( T = 0.14, 0.12, 0.27, \) and 0.19 GeV in Eq. (23) with \( q = 1.01 \). One can see that the mentioned distributions describe partly the spectra of \( \pi^+ \) and \( p \) produced in central and peripheral Au-Au collisions at 200 GeV. The temperature in central collisions is larger than that in peripheral collisions. The temperature for the spectra of pions is smaller than that for the spectra of protons.

Generally, Eq. (1) describes the spectra in high \( p_T \) region due to the hard scattering process. Eqs. (6), (9), and (23) describes the spectra in low \( p_T \) region due to the soft excitation process. In particular, Eqs. (6), (9), and (23) are harmonious in thermodynamics. To describe the spectra in whole \( p_T \) region, a superposition of Eq. (1) and one of Eqs. (6), (9), and (23) should be used. There are two types of superpositions,

\[
f_0(p_T) = \frac{1}{N} \frac{dN}{dp_T} = k f_S(p_T) + (1 - k) f_H(p_T) \tag{28}
\]

and

\[
f_0(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A_1 \theta(p_1 - p_T) f_S(p_T) + A_2 \theta(p_T - p_1) f_H(p_T), \tag{29}
\]

where \( f_S(p_T) \) denotes one of the soft components, Eqs. (6), (9), and (23); \( f_H(p_T) \) denotes the hard component, Eq. (1); \( k \) denotes the contribution fraction of the soft component in Eq. (28); \( A_1 \) and \( A_2 \) are constants which result in the two components to be equal to each other at \( p_T = p_1 \); and \( \theta(p_1 - p_T) \) and \( \theta(p_T - p_1) \) are the usual step function.

In our recent works [20, 21], to extract the kinetic freeze-out temperature and transverse flow velocity, the two types of superpositions are used respectively, where the soft component is described by the blast-wave model with Boltzmann-Gibbs statistics [22–24] and with Tsallis statistics [25–27]. There are small differences (< 5%) in the parameters extracted by the two superpositions. The first superposition can obtain a smooth curve easily, and the parameters are entangled in the extraction process. The second superposition has no entanglement in the extraction process of the parameters, and the curves are possibly not smooth in the point of split joint, \( p_1 \).

4 Conclusions

To conclude, the transverse momentum spectra obtained from different functions or distributions are compared. For the Hagedorn function, the effect of rest mass in the first modified form is very small due to large transverse momentum. The other two modified forms describe narrow transverse momentum range. For the simplest standard and Boltzmann distributions, the modified forms contribute a wider transverse momentum range than the original distributions, though a lower probability density in very-low transverse momentum region appears in the modified forms.

For a given transverse momentum spectrum, the modified forms “measure” lower temperature comparing with the simplest standard and Boltzmann distributions. Comparing with the original function with its modified forms, it is hard to say that which one is better. Based on the Tsallis momentum distribution and the isotropic assumption, the distributions of momentum components, transverse momenta, rapidities, and pseudorapidities for pions and protons are obtained by the analytical and Monte Carlo methods. It is natural that the rapidity distribution is closer to the pseudorapidity one at higher temperature and with smaller mass.

Comparing with the experimental data measured by the PHENIX Collaboration, Eq. (1) is confirmed to fit the spectra in high transverse momentum region. As the harmonious distributions in thermodynamics, Eqs. (6), (9), and (23) are confirmed to fit the spectra in low transverse momentum region. To fit the spectra in whole transverse momentum region, two types of superpositions, Eqs. (28) and (29), which combine Eq. (1) and one of Eqs. (6), (9), and (23) are suitable. In the superpositions, Eqs. (2)–(4) can replace Eq. (1), Eqs. (5) and (7) can replace Eq. (6), and Eqs. (8) and (10) can replace Eq. (9).

Data Availability

The data used to support the findings of this study are quoted from the mentioned references. As a phenomenological work, this paper does not report new data.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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