Event-Triggered Control for Guaranteed-Cost Bipartite Formation of Multi-Agent Systems

WEI WANG\textsuperscript{1}, LI WANG\textsuperscript{1}, AND CHI HUANG\textsuperscript{2}, (Member, IEEE)

\textsuperscript{1}College of Data Science, Taiyuan University of Technology, Taiyuan 030024, China
\textsuperscript{2}School of Economic Information and Engineering, Southwestern University of Finance and Economics, Chengdu 610074, China

Corresponding author: Li Wang (wangli@tyut.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61872260, and in part by the Fundamental Research Funds for the Central Universities under Grant JBK190502.

\textbf{ABSTRACT} This paper is concerned with event-triggered guaranteed-cost bipartite formation control of multi-agent systems with antagonistic interactions. In order to save the limited network communication bandwidth of multi-agent systems, event-triggered sampled-data transmission strategy is adopted. Event conditions are designed for both leader and followers to reduce the frequency of state transmission more effectively. According to the event-triggered sampled-data of the leader and followers, bipartite formation controllers are designed for followers. By using the event conditions and the Lyapunov method, the sufficient conditions for the realization of bipartite formation and guaranteed-cost bipartite formation are obtained in terms of linear matrix inequality, respectively. Finally, the effectiveness of the theoretical results is demonstrated by numerical examples.

\textbf{INDEX TERMS} Multi-agent systems, guaranteed-cost, bipartite formation, antagonistic interactions, event-triggered sampling.

\textbf{I. INTRODUCTION}

Due to the wide application in different fields, cooperative control of multi-agent systems has attracted extensive attention. An agent is used to represent a dynamic system, which can be an aircraft, a mobile robot, a satellite, a ground/underwater vehicle, etc. The main research fields of cooperative control of multi-agent systems include flocking control [1], consensus control [2], [3], tracking control [4] and formation control [5] and so on. As one of the typical research issues of multi-agent systems, formation control aims to make agents form and maintain a given geometric structure. It is widely used in such fields as unmanned aerial vehicles, underwater vehicles, and multi-robot systems.

In the past decade, formation control has been investigated as an important subject, and many good research results have been obtained. The formation control problem involves various forms of systems, including fractional-order, first-order, second-order, and higher-order systems, as well as linear, nonlinear, continuous systems and discrete systems. Constraints such as time delay [6], [7], communication constraint [8] and disturbance [9]–[11] are also considered in formation control. Hua et al. studied the finite-time time-varying formation tracking problem for high-order multi-agent systems in [12]. In [13], time-varying output formation-containment control of general linear multi-agent systems was considered. Cluster formation of second-order multi-agent systems with fixed and switching topologies was considered in [14]. Circular formation flight control problem was considered for unmanned aerial vehicles with external disturbance in [9]. Fixed-time event-triggered time-varying formation control of a class of non-linear multi-agent systems with uncertain disturbances was studied in [11]. A necessary and sufficient condition was obtained for the formation of discrete-time multi-agent systems with one sample period delay in [7]. In [15], time-varying formation control problem for fractional-order linear multi-agent systems was addressed.

In natural or engineering scenarios, it is inevitable that agents not only cooperate but also compete. For example, a robot will usually cooperate with its teammate and compete with the enemy [16]. In an ecosystem, foragers that cooperate are friends, and those that compete are enemies [17]. In social networks [18]–[21], two individuals can be friendly or hostile to each other. In order to model this situation, signed graph is used to describe the cooperation and competition between...
agents, where there are positive and negative weights of edges. Altafini [22] studied the consensus problem of systems described by a signed graph and found that all agents can converge to a consensus value that is the same for all agents except for signs. This consensus mode has been denoted as bipartite consensus. Based on this work, the problem of bipartite consensus has received a lot of attention, and some good results have been obtained. Bipartite output consensus of heterogeneous multi-agent systems over signed graphs was discussed in [23]. Bipartite consensus and tracking control problems for networked systems subject to nonidentical matching uncertainties were studied in [24]. In [25], leader-following bipartite consensus was studied over a signed directed graph where all the followers are subjected to mismatched unknown disturbances. As an extension of the research on consensus, formation control has also been discussed for multi-agent systems with antagonistic interactions [26]. Bipartite formation of second-order nonlinear multi-agent systems with hybrid impulses was investigated in [27]. In [28], fixed-time bipartite output formation-containment tracking of multi-agent systems with multiple leaders was investigated. Moreover, as far as I know, there is still not much research on the problem of bipartite formation control of multi-agent systems.

In order to make agents realize the desired dynamic goals, information exchange between agents plays a key role. In general, it is assumed that the transfer of states between agents is continuous. However, in real systems, this assumption is sometimes impractical, and continuous transmission of information is unnecessary and energy consuming. Combined with the application requirements in practical operation, the states of agents are regularly sampled to update the distributed controller in [29]–[31]. For the controller based on sampling states, an event-triggered condition can be designed to optimize the sampling strategy. In an event-triggered sampled-data transmission strategy, states collected at certain instants become effective only when a state-dependent triggering condition is violated. State-dependent and time-dependent event-triggered protocols were proposed for the synchronization of nonlinear multi-agent systems in [32]. A distributed event-triggered mechanism [33] was constructed for output consensus problem of high-order nonlinear multi-agent systems. In [34], event-triggered sampled-data synchronization-based passivity of partially coupled neural networks was discussed. In order to avoid unnecessary information transmissions among agents, event-triggered condition with dynamically adjustable threshold parameter was introduced for the distributed control of multi-agent systems in [35].

Event-triggered control strategy has also been used in the bipartite control problem of multi-agent systems [36]–[39]. Bipartite event-triggered output consensus issue for heterogeneous linear multi-agent systems was discussed in [38]. Bipartite consensus of multi-agent systems with switching communication topologies was studied in [39]. Observer-based event-triggered fuzzy adaptive bipartite containment control of multi-agent systems was considered in [36]. In [37], finite-time bipartite consensus of multi-agent systems was studied via event-triggered strategy. However, the event-triggered conditions described in the bipartite control problems need to be continuously judged, so the Zeno behavior needs to be discussed. The event-triggered sampling state transition strategy can naturally avoid Zeno phenomenon, but as far as we know, it is rarely applied to bipartite control, especially bipartite formation control.

In the formation control of multi-agent systems, it is beneficial to design an appropriate event-triggered control mechanism to save limited energy. It is also necessary to consider that in the actual application system control process, there are usually some performance constraints that need to be considered. Therefore, the guaranteed performance control of multi-agent systems has received much attention recently [40]. Robust $H_{\infty}$ guaranteed cost time-varying formation tracking problem for high-order multi-agent systems was considered in [41]. In [42], guaranteed-performance time-varying formation control for swarm systems was investigated. There are also some event-triggered guaranteed performance results [43]–[47]. In [43], the problem of event-triggered guaranteed cost consensus of discrete-time multi-agent systems was studied. In [44]–[46], the event-triggered guaranteed cost consensus problem was considered for continuous-time multi-agent systems, but the Zeno phenomenon needs to be considered. Performance guaranteed sampled-data event-triggered consensus algorithm for multi-agent systems was proposed in [47], however, the objective function does not involve the energy consumption of control input.

Motivated by the above discussions, this paper investigates the guaranteed-cost bipartite formation of multi-agent systems with event-triggered sampled-data transmission strategy. The main challenges of this study are as follows: (i). The formation target of the second-order multi-agent systems has requirements on the position and velocity states of agents, so how to design the corresponding event conditions? In addition, if the leader has limited communication ability and needs to adopt an event-triggered sampling transmission strategy, how to design the corresponding event condition? (ii). In order to achieve the bipartite formation goal of multi-agent systems while keeping the total energy consumption of the control input within a certain range, appropriate performance functions need to be designed. (iii). Sufficient conditions of bipartite formation and guaranteed-cost bipartite formation should be constructed for second-order multi-agent systems.

The main contributions of this paper are threefold. Firstly, an event-triggered sampling control protocol is designed for the bipartite formation of leader-following multi-agent systems. Different from the bipartite formation control of [27], the state of the leader can be used to describe the distance between the followers and the target formation. Different from the event-triggered bipartite control in [36]–[39], the event-triggered sampling strategy is adopted in this paper.
to reduce the burden of the communication media and avoid the occurrence of Zeno phenomenon. Secondly, more optimized event conditions are designed both for followers and leader. Different from the event-triggered condition in [14], which relies on the sum of the state formation error and velocity formation error, the event condition designed in this paper requires that the distributed controller be updated as long as one of the state formation error and velocity formation error is large enough. Based on the sampling data of the leader and the neighboring followers, state feedback controllers for followers are designed. Thirdly, this paper discussed guaranteed-cost bipartite formation control problem, where the guaranteed-cost problem more complex. With the aid of Lyapunov theory and LMI approach, sufficient conditions for bipartite formation and guaranteed-cost bipartite formation of second-order nonlinear systems are proposed. Finally, examples are given to demonstrate the theoretical results.

The remainder of this paper is organized as follows. Section II shows some preparation knowledge about structurally balanced graph. Bipartite formation controllers are designed for followers based on the event-triggered sampled-data transmission strategy, and bipartite formation error systems are obtained. Section III derives the sufficient conditions of bipartite formation and guaranteed-cost bipartite formation in the form of linear matrix inequality, respectively. Numerical simulations are present to demonstrate theoretical results in Section IV. Section V contains some concluding remarks.

Notations: Throughout this paper, $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space. $\mathbb{R}^{n \times n}$ is the set of $n \times n$ real matrices. $\mathbb{N} = \{1, 2, \ldots \}$ denotes the set of all natural numbers. $[\cdot, \cdot]$ denotes the Euclidean norm for vector or the spectral norm of matrix. diag$[\cdot, \cdot, \cdot]$ represents the diagonal matrix. “$\ast$” in a matrix represents the elements below the main diagonal of a symmetric matrix. $\mathbf{1} = [1, 1, \cdots, 1]^T$ is a vector of appropriate dimensions. If not explicitly stated, matrices are assumed to have compatible dimensions.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider multi-agent systems consisting of $N$ followers and a leader, in which the leader is not affected by other agents, while the follower may be affected by the leader and other followers. The communication topology of $N$ followers can be described by a signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$. $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ denotes nodes of the graph, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ describes the edges between nodes, and the adjacency matrix $G = [a_{ij}] \in \mathbb{R}^{n \times n}$ is defined as: $a_{ij} \neq 0 \Leftrightarrow (v_i, v_j) \in \mathcal{E}$ and otherwise, $a_{ij} = 0$. $\mathcal{N}(i) = \{j : (v_i, v_j) \in \mathcal{E}\}$ is the index set of the neighbors of $v_i$. The in-degree matrix of $G$ is denoted as $D = \text{diag}[d_1, d_2, \ldots, d_N]$, $d_i = \sum_{\mathcal{N}(i)} a_{ij}, i = 1, 2, \ldots, N$. The Laplacian matrix of graph $\mathcal{G}$ is defined as $L = D - G$. The signed graph $\mathcal{G}$ is assumed to has no self-loops, i.e. $a_{ii} = 0, \forall i = 1, 2, \ldots, N$. Define a corresponding auxiliary matrix $G^+ = [a_{ij}] \in \mathbb{R}^{N \times N}$, which can be regarded as the adjacency matrix of graph $\mathcal{G}$. The Laplacian matrix of graph $G^+$ is defined as $L^+ = D^+ - G^+$.

Definition 1 ([22]): A signed graph $\mathcal{G}(A)$ is said structurally balanced if nodes $\mathcal{V}$ can be divided into two disjoint sets: $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, and $a_{ij} \geq 0$, for $v_i, v_j \in \mathcal{V}_q (q \in \{1, 2\})$, $a_{ij} \leq 0$ for $v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q, p \neq q (p, q \in \{1, 2\})$.

Lemma 1 ([22]): If a signed graph $\mathcal{G}(A)$ is structurally balanced, then $\exists D \in \mathbb{D}$ such that $DAD$ has all nonnegative entries, where $\mathbb{D} = \{\text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N) | \sigma_i \in \{1, -1\}\}$.

Suppose that the dynamic of the $i$th follower can be described by a second-order differential equation as follows:

$$\begin{align*}
\dot{x}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= f(v_i(t), t) + u_i(t),
\end{align*}$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ are the position and velocity of the $i$th follower; $u_i(t) \in \mathbb{R}^n$ is the feedback controller; $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a continuously differentiable vector-valued function. The dynamic of leader is given by:

$$\begin{align*}
\dot{x}_0(t) &= v_0(t), \\
\dot{v}_0(t) &= f(v_0(t), t).
\end{align*}$$

Assumption 1: For nonlinear function $f(x(t)) = [f_1(x_1(t)), \ldots, f_q(x_q(t))]$ and $\sigma_i \in \{1, -1\}$, $i = 1, 2, \ldots$, there always exist positive constants $\rho_k, k = 1, \ldots, n$ such that

$$|\sigma_j f_j(x(t)) - \sigma_2 f_2(y(t))| \leq \rho_k |\sigma_1 x(t) - \sigma_2 y(t)|$$

hold for any $x(t), y(t) \in \mathbb{R}^n$. Let $\rho = \max\{\rho_1, \rho_2, \ldots, \rho_n\}$.

The target formation is denoted as $x^* = [x_1^T, x_2^T, \ldots, x_N^T]$. For given $S > 0, T_1 > 0$ and $T_2 > 0$ with appropriate dimensions, the corresponding quadratic performance function of the formation control protocol for multi-agent systems is given by

$$J = J_x + J_u < \tilde{J}$$

where

$$J_x = \sum_{i=1}^{N} \int_{0}^{T_1} \sum_{j=1}^{N} [a_{ij}] [\sigma_j (x_j(t) - x_j^*) - \sigma_i (x_i(t) - x_i^*)] dt$$

$$x_i^* = \left[ x_1^*, x_2^*, \ldots, x_N^* \right]^T,$$

$$J_u = \sum_{i=1}^{N} \int_{0}^{T_2} u_i^T(t) S u_i(t) dt.$$
protocol \( u(t) \) if the position states and velocity states of agents satisfy
\[
\lim_{t \to \infty} \|x_i(t) - x_i^* - x_0(t)\| = 0, \quad i \in V_1,
\]
\[
\lim_{t \to \infty} \|x_i(t) - x_i^* + x_0(t)\| = 0, \quad i \in V_2,
\]
\[
\lim_{t \to \infty} \|v_i(t) - v_0(t)\| = 0, \quad i \in V_1,
\]
\[
\lim_{t \to \infty} \|v_i(t) + v_0(t)\| = 0, \quad i \in V_2,
\]
and there exists a constant \( \bar{J} \) such that \( J \leq \bar{J} \), \( \bar{J} \) is said to be a guaranteed cost.

In this paper, the periodic sampling method is adopted and the sampling period is denoted as \( h > 0 \). In addition, whether the sampled states are used to update the controllers and transmitted to the adjacent agents depends on event conditions. The purpose of event condition design is to avoid excess energy consumption and channel occupancy caused by the frequent transfer of unnecessary states when the agent’s states do not change or have little change for a period of time. Only if the event conditions based on the sampled data are violated will the sampled data be used to update the controllers of itself and the adjacent agents.

Describe the sampling position and velocity states of \( i \)th follower as \( \hat{x}_i(t) \) and \( \hat{v}_i(t) \), and the sampling position and velocity states of leader as \( \hat{x}_0(t) \) and \( \hat{v}_0(t) \), respectively. For the bipartite formation problem, a sampled-data based control strategy is designed for the \( i \)th follower as follows.

\[
u_i(t) = \sum_{j=1}^{N} a_{ij} \| \text{sgn}(a_{ij})(\hat{x}_j(t) - x_j^*) - (\hat{x}_i(t) - x_i^*) \|
+ \gamma (\text{sgn}(a_{ij})\hat{v}_j(t) - \hat{v}_i(t)) - b_1 (\hat{x}_i(t) - x_i^*)
- \sigma x_0(t) + \gamma (\hat{v}_i(t) - \sigma \hat{v}_0(t))]
\]

where \( \gamma \) is a positive scalar representing the velocity damping gain and \( b_1 \geq 0 \) are control gains from leader to the \( i \)th follower. \( \text{sgn}() \) is a symbolic function.

Denote the \( m \)th broadcasting instant of follower \( i \) as \( t_m^i \), \( m \in \mathbb{N} \), the next broadcasting instant \( t_{m+1}^i \) is determined by
\[
t_{m+1}^i = t_m^i + \min_{l \geq 1} \left\{ l h | e_{x,i}^T(t_m^i + l h) \Phi_1 e_{x,i}(t_m^i + l h) \right\}
\]
\[
\geq \beta_{11} Y_{x,i}^T(t_m^i + l h) \Phi_1 Y_{x,i}(t_m^i + l h)
\]
\[
\text{or } e_{v,i}^T(t_m^i + l h) \Phi_2 e_{v,i}(t_m^i + l h)
\]
\[
\geq \beta_{21} Y_{v,i}^T(t_m^i + l h) \Phi_2 Y_{v,i}(t_m^i + l h) \right\}.
\]

where \( l \in \mathbb{N} \), \( \beta_{11}, \beta_{21} > 0 \), \( e_{x,i}(t_m^i + l h) = x_i(t_m^i + l h) - x_i(t_m^i) \), \( e_{v,i}(t_m^i + l h) = v_i(t_m^i + l h) - v_i(t_m^i) \), \( Y_{x,i}(t_m^i + l h) = \sum_{j=1}^{N} a_{ij} \| \text{sgn}(a_{ij})(x_j(t_m^i + l h) - x_j^*) - (x_i(t_m^i) - x_i^*) \|, \]
\[
Y_{v,i}(t_m^i + l h) = \sum_{j=1}^{N} a_{ij} \| \text{sgn}(a_{ij})v_j(t_m^i + l h) - v_i(t_m^i) \|,
\]
\[
\max \{ t_{m+1}^i \geq t_m^i, o \in \mathbb{N} \}, \quad \Phi_1 > 0 \quad \text{and } \quad \Phi_2 > 0 \quad \text{are weighting matrices.}
\]

In addition, the influence of leader on followers is also based on an event condition. Similarly, use \( t_{m+1}^0 \) to describe the \((m + 1)\)th broadcasting instant of leader, which is determined by
\[
t_{m+1}^0 = t_m^0 + \min_{l \geq 1} \left\{ l h | e_{x,0}^T(t_m^0 + l h) \Phi_0 e_{x,0}(t_m^0 + l h) \right\}
\]
\[
\geq \beta_{10} \sum_{i=1}^{N} Y_{x,i}^T(t_m^0 + l h) \Phi_0 Y_{x,i}(t_m^0 + l h)
\]
\[
\text{or } e_{v,0}^T(t_m^0 + l h) \Phi_0 e_{v,0}(t_m^0 + l h) \right\}.
\]

where \( \beta_{10} > 0 \), \( \beta_{20} > 0 \), \( e_{x,0}(t_m^0 + l h) = x_0(t_m^0 + l h) - x_0(t_m^0), \quad e_{v,0}(t_m^0 + l h) = v_0(t_m^0 + l h) - v_0(t_m^0), \quad Y_{x,0}(t_m^0 + l h) = \sigma (x_0(t_m^0 + l h) - x_0^*) - x_0(t_m^0 + l h), \quad Y_{v,0}(t_m^0 + l h) = \sigma (v_0(t_m^0 + l h) - v_0(t_m^0)), \quad \Phi_0 > 0 \quad \text{and } \quad \Phi_0 > 0 \quad \text{are weighting matrices.}
\]

Remark 1: According to (9) and (10), only if the measurement errors \( e_{x,i}(kh) \) or \( e_{v,i}(kh) \) are large enough will the \( i \)th follower broadcasting instant \( t_m^i \) be updated, that is, the \( i \)th follower’s states will be used to update its own and neighboring agents’ controllers. Similarly, only when the measurement errors of leader are large enough will the new states of leader be transmitted to followers. The measurement errors reflect the changes in the current sampling states and the transmitted states, thus avoiding the energy loss and channel occupancy caused by the agent unnecessarily transmitting information with little state change.

Based on this event-triggered sampling mechanism, controller (8) can be described as follows:

\[
u_i(t) = \sum_{j=1}^{N} a_{ij} \| \text{sgn}(a_{ij})(x_j(t_{m(i)} - x_j^*) - (x_i(t_{m(i)}) - x_i^*) \|
+ \gamma (\text{sgn}(a_{ij})v_j(t_{m(i)}) - v_i(t_{m(i)})) - b_1 |x_i(t_{m(i)}) - x_i^*)
- \sigma x_0(t_{m(i)}) + \gamma (v_i(t_{m(i)}) - \sigma \hat{v}_0(t_{m(i)}))]
\]

\[
= \sum_{j=1}^{N} a_{ij} \| \text{sgn}(a_{ij})(x_j(kh) - e_{x,i}(kh) - x_j^*)
\]
\[
- (x_i(kh) - e_{x,i}(kh) - x_i^*) + \gamma (\text{sgn}(a_{ij})(v_j(kh) - e_{v,i}(kh)) - e_{v,i}(kh) - (v_i(kh) - e_{v,i}(kh)))
- b_1 |x_i(kh) - e_{v,i}(kh) - x_i^* - \sigma (x_0(kh) - e_{v,i}(kh))\]
\[
+ \gamma (v_i(kh) - e_{v,i}(kh) - \sigma (v_0(kh) - e_{v,i}(kh)))\|
\]
\[
t \in [kh, (k + 1)h].
\]

Based on (9) and (10), the event-triggered sampling bipartite formation algorithm is presented in Algorithm 1.

Remark 2: The sampling period \( h \) and trigger condition parameters \( \beta_{11}, \beta_{21}, \beta_{10}, \beta_{20} \) need to meet the LMI conditions. The large the sampling period is, the smaller the sampling frequency and the controller update frequency will be. The parameters \( \beta_{11}, \beta_{21}, \beta_{10}, \beta_{20} \) affects the number of times the event is triggered and the resulting performance value. The smaller the parameter is, the closer the event triggering.
**Algorithm 1** Event-Triggered Sampling Bipartite Formation Algorithm for Followers and Leader

**Input:** Adjacency matrix $G$, agents’ dynamics, control gain matrix $B$, total duration of experiment $T$, positive scalar $\gamma$, desired formation $x^s = [x^s_1, x^s_2, \cdots, x^s_N]^T$, event condition parameters $h$, $\Phi_1$, $\Phi_2$, $\Phi_0$, $\beta_1$, $\beta_2$, $\beta_10$, $\beta_20$ (the controller of the system needs to satisfy bipartite formation condition LMI’s (15) which will be given below).

**Output:** The trajectory of agents and triggered instances for agents.

**Begin** $k = 0$; $m_i = 0$; $t^i_0 = 0$; $m_0 = 0$; $i^0_0 = 0$;

Follower $i$ sent $x(t^i_0)$ to its neighbors; Leader sent $x(t^i_0)$ to its neighbors;

Update the controller $u(t)$ based on (11);

for $k = 1, 2, \cdots, T$

if $e^f_i(h) \Phi e_u(kh) \geq \beta_1 Y^i_k(h) \Phi_1 Y_k u(kh)$ or $e^f_i(h) \Phi_2 e_u(kh) \geq \beta_2 Y^i_k(h) \Phi_2 Y_k u(kh)$

then $t^i_{m_k+1} = kh$ and follower $i$ sent $x(t^i_{m_k+1})$ to its neighbors

end if

if $e^f_0(kh) \Phi_0 e_u(kh) \geq \beta_10 \sum_{i=1}^N Y^i_0(kh) \Phi_1 Y^i_0(kh)$ or $e^f_0(kh) \Phi_2 e_u(kh) \geq \beta_20 \sum_{i=1}^N Y^i_0(kh) \Phi_2 Y^i_0(kh)$

then $t^0_{m_k+1} = kh$ and leader sent $x(t^0_{m_k+1})$ to its followers

end if

if follower $i$ received a state

then update the controller $u(t)$ based on (11)

end if

End

Agents’ trajectories vary with time; Collect the triggered instances of the agents;

Update the measurement error $e_{u}(kh), e_{v}(kh), e_{0}(kh), e_{i0}(kh)$;

**End**

controller is to the periodic sampling controller, and the smaller the final performance value will be. Tradeoff can be made between triggered times and performance constraints based on actual requirements.

Let $\delta x_i(t) = \sigma_i x_i(t) - x^s_i - x_0(t)$, $\delta v_i(t) = \sigma_i v_i(t) - v_0(t)$.

For $t \in [kh, (k + 1)h)$, one can deduce from (1), (2) and (11) that

$$
\delta x_i(t) = \delta v_i(t),
\delta v_i(t) = \sigma_i f(v_i(t), t) - f(v_0(t), t)
+ \sum_{j=1}^N \sigma_0 |a|_i [zg(x_i) + e_i - e_j - e_i - x^s_i] + e_i - e_j
+ y_i (zg(x_i) + e_i - e_j - e_i - e_j)
- \sigma_i b_i [x_i - e_i - x^s_i - x_i - x^s_i] + e_i - e_j
- e_i - e_j + y_i (v_i(t) - e_i - e_j - e_i - e_j).
$$

Notice that $\sigma_i^2 = 1$ and $\sigma_i \operatorname{sign}(a_{ij}) |a_j - \sigma_j| = 1$ for $a_{ij} \neq 0, \forall i, j = 1, 2, \cdots, N$, then one has

$$
\delta x_i(t) = \delta v_i(t),
\delta v_i(t) = \sigma_i f(v_i(t), t) - f(v_0(t), t)
+ \sum_{j=1}^N |a|_i [\delta x_j(kh) - e_i - e_j - e_i - e_j]
- e_i - e_j
+ \gamma_i (\delta x_i(kh) + e_i - e_j - e_i - e_j)
- \sigma_i b_i [x_i - e_i - x^s_i - x_i - x^s_i] + e_i - e_j
- e_i - e_j + \gamma v_i(t) (v_i(t) - e_i - e_j - e_i - e_j).
$$

**Denote** $\delta x(t) = [\delta x_1^T(t), \delta x_2^T(t), \cdots, \delta x_N^T(t)]^T$, $\delta v(t) = [\delta v_1^T(t), \delta v_2^T(t), \cdots, \delta v_N^T(t)]^T$,

$$
\begin{equation}
\begin{aligned}
\dot{\delta x}(t) &= \delta v(t),
\dot{\delta v}(t) = \Delta f(v(t), t) - (L^+ + B)(\delta x(kh) + \gamma \delta v(kh))
+ (L^+ + B) \Sigma e_i(kh) + \gamma v_i(kh))
- B e_i(kh) + \gamma \delta v(kh))
\end{aligned}
\end{equation}
$$

The following Lemmas are needed for the derivation of main results in this paper.

**Lemma 2 (Schur Complement [48]):** The LMI

$$
\begin{bmatrix}
A(x) & B(x) \\
B(x)^T & C(x)
\end{bmatrix} < 0
$$

is equivalent to one of the following conditions:

(i) $A(x) < 0$, $C(x) - B^T(x) A(x)^{-1} B(x) < 0$,

(ii) $C(x) < 0$, $A(x) - B(x) C(x)^{-1} B(x) < 0$,

where $A(x) = A^T(x)$ and $C(x) = C^T(x)$.

**Lemma 3 ([49]):** For positive definite matrix $W \in R^{m \times m}$, $r > 0$ and a vector valued function $v(t) : [0, r] \rightarrow R^m$, if the integrations concerned are well defined, the following inequality holds:

$$
\int_{t-r}^{t} v^T(s) W \delta v(s) ds \geq \frac{1}{r} \int_{t-r}^{t} v^T(s) ds W \int_{t-r}^{t} v(s) ds.
$$

**III. MAIN RESULTS**

In this part, sufficient conditions will be obtained for the multi-agent system (1) and (2) to realize bipartite formation, as well as the guaranteed bipartite formation conditions based on the performance function (4).

**Theorem 1:** If graph $G(A)$ is directed and structurally balanced and for any $h > 0$, $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_10 > 0$, $\beta_20 > 0$, there exist diagonal positive definite matrices $U_i, i = 1, 2, \ldots, 9$, positive definite matrices $P_1, P_2, Q_1,$
\( Q_2, R_1, R_2, X_1, X_2 \) and \( W_i, i = 1, 2, \ldots, 9 \) of appropriate dimensions, such that
\[
\begin{pmatrix}
\Sigma_1 & \Sigma_3 \\
* & \Sigma_2
\end{pmatrix} < 0
\]  
(15)
where \( \Sigma \), as shown at the bottom of the page

\[
\begin{align*}
\Sigma_1 &= \Sigma - \text{diag}(W_1^TU_1^{-1}W_1, W_2^TU_2^{-1}W_2, W_3^TU_3^{-1}W_3, \\
&\quad W_4^TU_4^{-1}W_4, W_5^TU_5^{-1}W_5, W_6^TU_6^{-1}W_6, \\
&\quad W_7^TU_7^{-1}W_7, W_8^TU_8^{-1}W_8, 0, 0, W_9^TU_9^{-1}W_9), \\
\Sigma_2 &= \text{diag}(-U_1, -U_2, -U_3, -U_4, -U_5, -U_6, -U_7, \\
&\quad -U_8, -U_9), \\
\Sigma_3 &= [\varepsilon_1 W_1^T, \varepsilon_2 W_2^T, \varepsilon_3 W_3^T, \varepsilon_4 W_4^T, \varepsilon_5 W_5^T, \varepsilon_6 W_6^T, \\
&\quad \varepsilon_7 W_7^T, \varepsilon_8 W_8^T, \varepsilon_{11} W_9^T], \\
\Pi_{11} &= Q_1 - R_1 + W_1^TU_1^{-1}W_1 + X_1, \\
\Pi_{13} &= -W_1^TU_1 - W_1 - \rho^2(U_1 + U_2 + \ldots + U_9) + X_2, \\
\Pi_{23} &= -W_2^TU_2^{-1}W_2, \\
\Pi_{24} &= -W_2^T, \\
\Pi_{27} &= W_2^T, \\
\Pi_{28} &= \gamma W_2^T, \\
\Pi_{33} &= +L^TU_1^T \otimes \Phi_1 + \Phi_0^T - W_3^T, \\
&\quad -Q_1 - (L^T + B)W_3 + W_3^TU_3^{-1}W_3, \\
\Pi_{34} &= -\gamma W_3^T, \\
\Pi_{35} &= -(L^T + B)W_4, \\
\Pi_{36} &= -(L^T + B)W_6, \\
\Pi_{37} &= W_4^T + W_5^T \otimes \Phi_1 + \Phi_0^T - (L^T + B)^T W_7, \\
&\quad -W_5^T - (L^T + B)W_7, \\
\Pi_{38} &= \gamma W_5^T, \\
\Pi_{311} &= -W_7^T, \\
\Pi_{44} &= L^T + B \otimes \Phi_2 - Q_2 + \beta_0 T^T \\
&\quad -\gamma (L^T + B)W_4^T - \gamma W_4^T, \\
&\quad + W_4 U_4^{-1} W_4, \\
\Pi_{45} &= -\gamma (L^T + B)^T W_4, \\
\Pi_{46} &= -\gamma (L^T + B)^T W_6, \\
\Pi_{47} &= W_4^T (L^T + B) \Sigma - (L^T + B)^T W_7, \\
\Pi_{48} &= L^T + T^T \Pi_2 \otimes \Phi_2 + \Sigma \otimes \Phi_0 - \gamma W_4^T (L^T + B) \Sigma \\
&\quad - \gamma (L^T + B)^T, \\
\Pi_{411} &= -\gamma (L^T + B)^T W_9, \\
\Pi_{55} &= -R_1 + W_5^TU_1^{-1}W_5, \\
\Pi_{57} &= W_5^T (L^T + B) \Sigma, \\
\Pi_{58} &= \gamma W_5^T (L^T + B) \Sigma, \\
\Pi_{66} &= -R_2 + W_6^T U_1^{-1}W_6, \\
\Pi_{67} &= W_6^T (L^T + B) \Sigma, \\
\Pi_{68} &= \gamma W_6^T (L^T + B) \Sigma, \\
\Pi_{77} &= \Sigma (L^T + B)^T W_8 + \gamma W_8^T (L^T + B) \Sigma \\
&\quad + W_8 U_8^{-1} W_8, \\
\Pi_{811} &= \gamma (L^T + B)^T W_9, \\
\Pi_{1111} &= h^2 R_2 - W_9 - W_9^T + U_9^T U_9^T, \\
\epsilon_i \text{ is a block entry matrix, for example } \varepsilon_2 &= [0, \ldots, \varepsilon_{11}], \\
&\text{then the multi-agent system can realize bipartite formation.}
\]

\text{Proof:} \text{ Let } t = t - kh \text{ for } t \in [kh, (k+1)h). \text{ Construct a Lyapunov candidate:}
\[
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),
\]
\text{where}
\[
V_1(t) = \delta x^T(t) P_1 \delta x(t) + \delta v^T(t) P_2 \delta v(t),
\]
\[
\Xi = \begin{pmatrix}
\Pi_{11} & P_1 & \Pi_{13} & \Pi_{14} & R_1 & 0 & 0 & \Pi_{17} & \Pi_{18} & -W_1^T B & -\gamma W_1^T B & -W_1^T \Sigma \\
* & \Pi_{22} & \Pi_{23} & \Pi_{24} & 0 & R_2 & 0 & \Pi_{27} & \Pi_{28} & -W_2^T B & -\gamma W_2^T B & -W_2^T \\
* & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & \Pi_{36} & \Pi_{37} & \Pi_{38} & \Pi_{39} & -W_3^T B & -\gamma W_3^T B & -W_3^T \\
* & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} & \Pi_{47} & \Pi_{48} & \Pi_{49} & -W_4^T B & -\gamma W_4^T B & -W_4^T \\
* & * & * & * & \Pi_{55} & 0 & \Pi_{57} & \Pi_{58} & \Pi_{59} & -W_5^T B & -\gamma W_5^T B & -W_5^T \\
* & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} & \Pi_{69} & -W_6^T B & -\gamma W_6^T B & -W_6^T \\
* & * & * & * & * & * & \Pi_{77} & \Pi_{78} & \Pi_{79} & -W_7^T B & -\gamma W_7^T B & -W_7^T \\
* & * & * & * & * & * & * & \Pi_{88} & \Pi_{89} & -W_8^T B & -\gamma W_8^T B & -W_8^T \\
* & * & * & * & * & * & * & * & \Pi_{1111}
\end{pmatrix}
\]
Taking the derivative of (16) along the trajectory (14) with respect to $t \in (kh, (k+1)h)$, one has
\[
\dot{V}_3(t) = \frac{h}{2} \delta x^T(t) \delta x(t) - h \int_{t-h}^{t} \delta x^T(s) R_1 \delta x(s) ds
+ h^2 \delta v^T(t) R_2 \delta v(t) - h \int_{t-h}^{t} \delta v^T(s) R_2 \delta v(s) ds,
\]
\[
\dot{V}_4(t) = \delta x^T(t) X_1 \delta x(t) + \delta v^T(t) X_2 \delta v(t) - \delta x^T(t-h) X_1 \delta x(t-h) - \delta v^T(t-h) X_2 \delta v(t-h).
\]
Based on Lemma 3, one has
\[
\dot{V}_3(t) \leq \frac{h}{2} \delta v^T(t) R_1 \delta v(t) + h^2 \delta v^T(t) R_2 \delta v(t)
- \int_{t-h}^{t} \delta x^T(s) R_1 \delta x(s) ds
- \int_{t-h}^{t} \delta v^T(s) R_2 \delta v(s) ds
= \delta v^T(t) \delta v(t) + h^2 \delta v^T(t) R_2 \delta v(t)
- [\delta x(t) - \delta x(t-h)]^T R_1 [\delta x(t) - \delta x(t-h)]
- [\delta v(t) - \delta v(t-h)]^T R_2 [\delta v(t) - \delta v(t-h)].
\]
According to the event condition (9), one has that
\[
e^T_{\alpha i}(kh) \Phi_1 e_{\alpha i}(kh)
< \beta_{1i} Y^T_{\alpha i}(kh) \Phi_1 Y_{\alpha i}(kh)
= \beta_{1i} \sum_{j=1}^{N} [a_{ij} (s_{\alpha ij} x_j(kh) - e_{\alpha ij}(kh) - x^*_j)]^T \Phi_1 [a_{ij} (s_{\alpha ij} x_j(kh) - e_{\alpha ij}(kh) - x^*_j)]
- [e_{\alpha ij}(kh) - x^*_j - (x_j(kh) - e_{\alpha ij}(kh) - x^*_j)]
= \beta_{1i} \sum_{j=1}^{N} [a_{ij} (\delta x_j(kh) - (\sigma_{\alpha ij} x_j(kh) - x^*_j))]
- \delta x_j(kh)]^T \Phi_1 [\beta_{1i} \sum_{j=1}^{N} [a_{ij} (\delta x_j(kh) - (\sigma_{\alpha ij} x_j(kh) - x^*_j))]
- [\delta x_j(kh) - (\sigma_{\alpha ij} x_j(kh) - x^*_j)]
\]
\[
= \beta_{2} \sum_{j=1}^{N} [a_{ij} (\delta x_j(kh) - (\sigma_{\alpha ij} x_j(kh) - x^*_j))]
- \delta x_j(kh)]^T \Phi_1 [\beta_{2} \sum_{j=1}^{N} [a_{ij} (\delta x_j(kh) - (\sigma_{\alpha ij} x_j(kh) - x^*_j))]
- [\delta x_j(kh) - (\sigma_{\alpha ij} x_j(kh) - x^*_j)].
\]
and
\[
e^T_{\alpha i}(kh) \Phi_2 e_{\alpha i}(kh)
< \beta_{2i} Y^T_{\alpha i}(kh) \Phi_2 Y_{\alpha i}(kh)
= \beta_{2i} \sum_{j=1}^{N} [a_{ij} (s_{\alpha ij} v_j(kh) - v_{\alpha ij}(kh)) - v_0(kh)]^T \Phi_2 [\beta_{2i} \sum_{j=1}^{N} [a_{ij} (s_{\alpha ij} v_j(kh) - v_{\alpha ij}(kh)) - v_0(kh)]
- [s_{\alpha ij} v_j(kh) - v_{\alpha ij}(kh) - v_0(kh)]
= \beta_{2i} \sum_{j=1}^{N} [s_{\alpha ij} v_j(kh) - v_0(kh)]^T \Phi_2 [\beta_{2i} \sum_{j=1}^{N} [s_{\alpha ij} v_j(kh) - v_0(kh)]
- [s_{\alpha ij} v_j(kh) - v_0(kh)].
\]
According to system (14), for any matrices $W_i, i = 1, 2, \ldots, 9$ of appropriate dimensions, one has the following identity
\begin{align*}
&2[-\dot{\nu}(t) + \Delta f(v(t), t) - (L^+ + B)(\delta x(kh) + \nu \delta v(kh)) \\
&+ (L^+ + B)\bar{\Sigma}(e_{\nu}(kh) + \nu e_{\delta v}(kh)) - B(\bar{e}_{\nu}(kh) \\
&+ \nu \bar{e}_{\delta v}(kh))]|(W_1 \dot{x}(t) + W_2 \delta v(t) + W_3 \delta x(kh)) | \\
&+ W_4 \delta v(kh) + W_5 \delta x(t - h) + W_6 \delta v(t - h) + W_7 e_{\nu}(kh) \\
&+ W_8 e_{\delta v}(kh) + W_9 \dot{\delta}(t)] = 0. \\
(28)
\end{align*}

It follows from Assumption 1 that
\begin{align*}
&2\Delta f(v(t), t)\dot{\delta}(t) \\
&\leq \rho^2 \delta v^T(t)U_1 \dot{\delta}(t) + \delta x^T(t)W_1^T U_1^{-1} W_1 \dot{x}(t). \\
(29)
\end{align*}

\begin{align*}
&2\Delta f(v(t), t)\dot{\delta}(t) \\
&\leq \rho^2 \delta v^T(t)U_2 \dot{\delta}(t) + \delta v^T(t)W_2^T U_2^{-1} W_2 \dot{\delta}(t). \\
(30)
\end{align*}

\begin{align*}
&2\Delta f(v(t), t)\dot{\delta}(t) \\
&\leq \rho^2 \delta v^T(t)U_3 \dot{\delta}(t) + \delta x^T(kh)W_3^T U_3^{-1} W_3 \dot{x}(kh). \\
(31)
\end{align*}

\begin{align*}
&2\Delta f(v(t), t)\dot{\delta}(t) \\
&\leq \rho^2 \delta v^T(t)U_9 \dot{\delta}(t) + \delta v^T(t)W_9^T U_9^{-1} W_9 \dot{\delta}(t). \\
(32)
\end{align*}

Setting $\phi(t) = \text{col}(\delta x(kh), \delta v(kh), e_{\nu}(kh), e_{\delta v}(kh), \bar{e}_{\nu}(kh), \bar{e}_{\delta v}(kh), \dot{\delta}(t)).$

From (16) to (32), one can obtain that
\begin{align*}
\dot{V}(t) \leq \phi^T(t)\Sigma \phi(t). \\
(33)
\end{align*}

It follows from (15) and Lemma 2 that $\dot{V}(t) < 0$ and $\lim V(t) = 0.$ Then error system (14) is asymptotically stable, multi-agent system realize bipartite formation. □

In the next part, the guaranteed-cost bipartite formation of multi-agent system (1) and (2) is discussed. It should be pointed out that guaranteed-cost bipartite formation control requires the network topology of followers to be undirected.

From control protocol (8), the performance function can be expressed as
\begin{align*}
J_u &= \sum_{i=1}^{N} \int_{0}^{\infty} u_i^T(t)Su_i(t)dt \\
&= \sum_{i=1}^{N} \left[ \int_{0}^{h} u_i^T(t)Su_i(t)dt + \cdots + \int_{(k-1)h}^{kh} u_i^T(t)Su_i(t)dt \right] \\
&+ \int_{(k-1)h}^{kh} u_i^T(t)Su_i(t)dt + \cdots + \int_{kh}^{\infty} u_i^T(t)Su_i(t)dt \\
&= \sum_{i=1}^{N} \left[ \int_{0}^{h} u_i^T(t)Su_i(t)dt + \cdots + \int_{(k-1)h}^{kh} u_i^T(t)Su_i(t)dt \right] \\
&+ \int_{(k-1)h}^{kh} u_i^T(t)Su_i(t)dt + \cdots + \int_{kh}^{\infty} u_i^T(t)Su_i(t)dt \\
&= \int_{0}^{h} \phi^T(0)(\Omega \otimes S)\phi(0)dt + \cdots \\
&+ \int_{(k-1)h}^{kh} \phi^T((k-1)h)(\Omega \otimes S)\phi((k-1)h)dt \\
&+ \int_{kh}^{\infty} \phi^T((k-1)h)(\Omega \otimes S)\phi((k-1)h)dt \\
&+ \int_{0}^{\infty} \phi^T(kh)(\Omega \otimes S)\phi(kh)dt. \\
(34)
\end{align*}

\begin{align*}
J_x &= \sum_{i=1}^{N} \sum_{j=1}^{N} [a_{ij}][\delta x_j(t) - \delta x_i(t)]^T J_1[\delta x_j(t) \\
&- \delta x_i(t)]dt + \sum_{i=1}^{N} \sum_{j=1}^{N} [a_{ij}][\delta v_j(t) - \delta v_i(t)]^T J_2 [\delta v_j(t) - \delta v_i(t)]dt \\
&= \int_{0}^{\infty} \delta x^T(t)(L^+ \otimes T_1)\delta x(t) + \delta v^T(t)(L^+ \otimes T_2) \delta v(t)dt, \\
(35)
\end{align*}

where $\phi(t) = \text{col}(\delta x(kh), \delta v(kh), e_{\nu}(kh), e_{\delta v}(kh), \bar{e}_{\nu}(kh), \bar{e}_{\delta v}(kh), \dot{\delta}(t))$ for $t \in [kh, (k + 1)h), L_B = L^+ + B, \Omega,$ as shown at the bottom of the next page.

Thus,
\begin{align*}
J &= J + \int_{0}^{\infty} \dot{V}(t)dt - V(t)|_{t \rightarrow \infty} + V(0) \\
&= \int_{0}^{h} \phi^T(0)(\Omega \otimes S)\phi(0) + 2\delta x^T(t)(L^+ \otimes T_1)\delta x(t) \\
&+ 2\delta v^T(t)(L^+ \otimes T_2) \delta v(t) + \cdots \\
&+ \int_{(k-1)h}^{kh} \phi^T((k-1)h)(\Omega \otimes S)\phi((k-1)h) \\
&+ 2\delta x^T(t)(L^+ \otimes T_1)\delta x(t) \\
&+ 2\delta v^T(t)(L^+ \otimes T_2) \delta v(t)dt \\
&+ \cdots + \int_{kh}^{\infty} \phi^T(kh)(\Omega \otimes S)\phi(kh) \\
&+ 2\delta x^T(t)(L^+ \otimes T_1)\delta x(t) \\
&+ 2\delta v^T(t)(L^+ \otimes T_2) \delta v(t)dt \\
&+ \sum_{k=1}^{N} \left[ \int_{0}^{h} \dot{V}(t)dt + \cdots + \int_{(k-1)h}^{kh} \dot{V}(t)dt \right] \\
&+ \int_{kh}^{\infty} \dot{V}(t)dt - V(t)|_{t \rightarrow \infty} + V(0) \\
&= \int_{0}^{h} \phi^T(0)\Sigma \phi(0)dt + \cdots + \int_{(k-1)h}^{kh} \phi^T(t)\Sigma \phi(t)dt \\
&+ \int_{kh}^{\infty} \phi^T((k-1)h)\Sigma \phi((k-1)h)dt \\
&+ \int_{0}^{\infty} \phi^T(kh)\Sigma \phi(kh)dt. \\
(36)
\end{align*}
\[\vec{\Pi}_{38} = \gamma W_7^T (L^T + B) \bar{\Sigma} - (L^T + B)^T W_8 - \gamma (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{39} = -W_3^T B + (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{40} = -\gamma W_3^T B + \gamma (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{44} = L^T \beta_2 L^T \otimes \Phi_2 - Q_2 + \beta_{20} (I \otimes \Phi_0) - \gamma (L^T + B)W_4^T - \gamma W_4 (L^T + B)^T W_4 + U_4^{-1}W_4 + \gamma^2 (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{47} = W_4^T (L^T + B) \bar{\Sigma} - \gamma (L^T + B)^T W_7 - \gamma (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{48} = L^T \beta_2 L^T \otimes \Phi_2 + \gamma (L^T + B)^T W_8 - \gamma^2 (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{49} = -W_4^T B + \gamma (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{50} = -\gamma W_4^T B + \gamma^2 (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{77} = \bar{\Sigma} (L^T + B)^T W_7 + \bar{\Sigma} (L^T + B)^T W_7 + \gamma \bar{\Sigma} (L^T + B) \bar{\Sigma} + W_8 U_7^{-1} W_8 + \gamma^2 \bar{\Sigma} (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{78} = \bar{\Sigma} (L^T + B)^T W_8 + \gamma \bar{\Sigma} (L^T + B) \bar{\Sigma} + \bar{\Sigma} (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{79} = -W_7^T B - \Sigma (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{80} = -\gamma W_7^T B - \gamma \Sigma (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{88} = \bar{\Sigma} (L^T + B)^T \otimes \Phi_2 + \gamma \bar{\Sigma} (L^T + B) \bar{\Sigma} + \bar{\Sigma} (L^T + B)^2 \Sigma \otimes S,\]
\[\vec{\Pi}_{89} = W_8^T B - \gamma \Sigma (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{90} = -\gamma W_8^T B - \gamma^2 \Sigma (L^T + B)B \otimes S,\]
\[\vec{\Pi}_{99} = -\frac{1}{N} \Phi_0 + B^2 \otimes S,\]
\[\vec{\Pi}_{100} = -\frac{1}{N} \Phi_0 + \gamma^2 B^2 \otimes S.\]

Based on the proof of Theorem 1, one has the following results.

**Theorem 2:** If graph \(G(A)\) is undirected and structurally balanced and for any \(h > 0, \beta_{ii} > 0, \beta_{ii} > 0, \beta_{ii} > 0\), there exist diagonal positive definite matrices \(U_i, i = 1, 2, \ldots, 9\), positive definite matrices \(P_1, P_2, Q_1, Q_2, R_1, R_2, X_1, X_2\) and \(W_i, i = 1, 2, \ldots, 9\) of appropriate dimensions, such that

\[
\begin{pmatrix}
\tilde{\Sigma}_1 \\
\Sigma_3
\end{pmatrix} < 0
\]  
(37)

where

\[
\tilde{\Sigma}_1 = \tilde{\Sigma} - \text{diag}(W_1^T U_1^{-1} W_1, W_2^T U_2^{-1} W_2, W_3^T U_3^{-1} W_3, W_4^T U_4^{-1} W_4, W_5^T U_5^{-1} W_5, W_6^T U_6^{-1} W_6, W_7^T U_7^{-1} W_7, W_8^T U_8^{-1} W_8, 0, 0, W_9^T U_9^{-1} W_9),
\]
\[
\Sigma_3 = \begin{bmatrix}
\epsilon_1 W_1^T, \\
\epsilon_2 W_2^T, \\
\epsilon_3 W_3^T, \\
\epsilon_4 W_4^T, \\
\epsilon_5 W_5^T, \\
\epsilon_6 W_6^T, \\
\epsilon_7 W_7^T, \\
\epsilon_8 W_8^T, \\
\epsilon_9 W_9^T
\end{bmatrix},
\]
\[
\Sigma_2 = \text{diag}(-U_1, -U_2, -U_3, -U_4, -U_5, -U_6, -U_7, -U_8, -U_9),
\]

then the multi-agent system can realize guaranteed-cost bipartite formation. Meanwhile, the guaranteed cost satisfies

\[
\bar{J} = \delta x^T (0) P_1 \delta x(0) + \delta v^T (0) P_2 \delta v(0)
\]

\[
+ h \int_{-h}^{0} \int_{-\theta}^{\theta} \delta x^T (s) R_1 \delta x(s) ds d\theta
\]

\[
\Omega = \begin{pmatrix}
L_B^2 & \gamma L_B^2 & -L_B^2 \Sigma & -\gamma L_B^2 \Sigma & L_B \Sigma & \gamma L_B \Sigma \\
* & \gamma^2 L_B^2 & -\gamma L_B^2 \Sigma & -\gamma L_B^2 \Sigma & \gamma L_B \Sigma & \gamma^2 L_B \Sigma \\
* & * & \Sigma L_B^2 & \gamma \Sigma L_B^2 & \gamma^2 L_B \Sigma & \gamma^2 L_B \Sigma \\
* & * & * & \Sigma L_B^2 & -\Sigma L_B \Sigma & -\Sigma L_B \Sigma \\
* & * & * & * & B^2 & \gamma B^2 \\
* & * & * & * & * & \gamma^2 B^2
\end{pmatrix}
\]

\[
\tilde{\Sigma} = \begin{pmatrix}
\tilde{\Pi}_{11} & P_1 & \Pi_{13} & \Pi_{14} & R_1 & 0 & \Pi_{17} & \Pi_{18} & -W_1^T B & -\gamma W_1^T B & -W_1^T \\
* & \Pi_{21} & P_2 & P_3 & 0 & R_2 & \Pi_{27} & \Pi_{28} & -W_2^T B & -\gamma W_2^T B & P_2 - W_2^T \\
* & * & \Pi_{33} & \tilde{\Sigma}_{34} & P_{35} & P_{36} & \tilde{\Sigma}_{37} & \tilde{\Sigma}_{38} & \tilde{\Pi}_{39} & \Pi_{310} & \Pi_{311} \\
* & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} & \Pi_{47} & \Pi_{48} & \Pi_{49} & \Pi_{410} & \Pi_{411} \\
* & * & * & * & \Pi_{55} & 0 & \Pi_{57} & \Pi_{58} & -W_3^T B & -\gamma W_3^T B & -W_3^T \\
* & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} & -W_4^T B & -\gamma W_4^T B & -W_4^T \\
* & * & * & * & * & * & \Pi_{77} & \Pi_{78} & \Pi_{79} & \Pi_{710} & \Pi_{711} \\
* & * & * & * & * & * & * & \Pi_{88} & \Pi_{89} & \Pi_{810} & \Pi_{811} \\
* & * & * & * & * & * & * & * & \Pi_{99} & \Pi_{910} & -B^T W_9 \\
* & * & * & * & * & * & * & * & * & \Pi_{1010} & -\gamma W_9 \\
\end{pmatrix}
\]
Remark 3: According to the performance function shown in (36), followers can realize the guaranteed property on the basis of the realization of bipartite formation by adding a positive definite term to the left side of the linear matrix inequality (15). So if condition (37) is satisfied, condition (15) must also be met. In other words, as long as the undirected graph satisfies condition (37), bipartite formation can be realized. Correspondingly, when verifying the guaranteed-cost bipartite formation through numerical simulation, only the feasible solution of (37) is needed.

Remark 4: To ensure the feasibility of the LMI conditions, appropriate Lyapunov candidate is defined and some free matrices are introduced by the identity (28). By increasing the number of unknown matrices to be larger than the dimension of LMI, the feasibility of the LMI conditions is increased. Schur’s complement lemma is used to deal with the nonlinear problems in initial matrix $\mathcal{Z}$. Finally, the feasible solution is obtained through the LMI toolbox in Matlab.

Remark 5: In this paper, the guaranteed cost is related to the initial state value of agents, so the initial value can be adjusted to achieve a prescribed performance. Thus, the guaranteed-cost control problem for a given performance can be solved to a certain extent.

In this paper, the conditions of bipartite formation and guaranteed bipartite formation are obtained by using Lyapunov theory and matrix inequality theory. Since the flexibility of Lyapunov candidate selection, bipartite formation conditions may be somewhat conservative. The method of enhancing the feasibility of the condition by defining the appropriate Lyapunov function as far as possible has been used in many literatures [44], [47], [50]. In addition, in the study of guaranteed-cost control problem, this paper requires that the connective topology of followers be undirected, which is also a kind of conservatism. In order to deal with performance functions with adjacency strength, such an assumption is required in many literature [40], [44], [51]. To study the guaranteed-cost control problem of directed graphs, it is necessary to define a relatively simple performance function like in [50].

More generally, the control problem of multi-agent systems with switching topologies is also widely concerned by scholars [5], [14], [40], [51]. [5] studied formation control problems for unmanned aerial vehicle swarm systems with switching interaction topologies, in which the formation condition involves the minimum nonzero eigenvalue of all communication topologies. In [51], guaranteed-cost consensus for high-order nonlinear multi-agent networks with switching topologies is considered, the minimum nonzero eigenvalue and maximum eigenvalue of all communication topologies are included in the consensus condition. Similar to the [14], the formation condition in this paper involves the Laplace matrix of connection topologies. Since the Lyapunov candidate (16) does not dependent on the network topology, it can be directly used in the switching topology. So for the formation control problem of switched systems, all interaction topologies in the switching set need to satisfy the LMI condition (15). In addition, since the event conditions and performance function $J_x$ involve the adjacency relationship between agents, event conditions and the performance function of the switched system should be based on the switched adjacency matrix. The switching system’s guaranteed-cost formation condition is that all interaction topologies in the switching set satisfy the LMI condition (37).
IV. SIMULATIONS

In this section, examples are designed to demonstrate the effectiveness of the proposed approach.

Example 1: Consider a multi-agent system with a leader and 6 followers whose communication topology is shown in Fig. 1. From Fig. 1, $D = \text{diag}(1, 1, 1, -1, -1, -1)$.

Let the feedback control gain $\gamma = 0.02$ and the control gain matrix from leader is $B = \text{diag}(0, 4, 2, 5, 4, 2)$. For vector $u = [u_1, u_2]^T$, the nonlinear function is denoted by $f(u) = 0.01 \begin{bmatrix} |u_1 + 1| - |u_1 - 1| \\ |u_2 + 1| - |u_2 - 1| \end{bmatrix}^T$. It can be easily verified that $f(\cdot)$ satisfies Assumption 1 with constant $\rho = 0.01$. The desired geometric formation is $x^* = [0, 20, 20\sqrt{3}, 0, 20\sqrt{3}, 0, 20, 20\sqrt{3}, 0, 20\sqrt{3}, 40]^T$, where the desired geometric formation of two groups are set to be the same. Suppose the sampling period $h = 0.005$. The parameters involved in the event condition are denoted as $\beta_{1i} = 0.002, \beta_{2i} = 0.003, i = 1, 2, 3, 4, 5, 6, \beta_{10} = 0.02I_6, \beta_{20} = 0.04I_6, \Phi_1 = 0.1I_2, \Phi_2 = 0.1I_2, \Phi_{01} = 0.3I_2, \Phi_{02} = 0.3I_2$. With the help of the LMI toolbox in Matlab, the feasible solution of (15) is obtained. Under the action of the controller (11) with event conditions (9) and (10), the trajectories of $\delta x_i(t)$ and $\delta v_i(t), i = 1, 2, \ldots, 6$ of 6 followers are shown in Fig. 2 and Fig. 3, respectively. It can be found that the error systems are asymptotically stable.

The trajectories of 6 followers are shown in Fig. 4. The position snapshots and the desired formation of followers at different times are shown in Fig. 5. Asterisks and circles indicate the positions of the two groups of agents, while green lines indicate the target formation shape. As can be seen from Fig. 4 and Fig. 5, the two groups of followers gradually form and maintain their own formation respectively. Since the target formation was set the same, the two groups of followers were symmetrically positioned on both sides of the target formation. That is to say, the multi-agent system realizes bipartite formation. Fig. 6 shows the event-triggered instants of 6 followers and the leader during the time period $[0, 5]$. The states of followers and leader are propagated and used to update the controller only at instants shown in Fig. 6.

Example 2: Consider that the communication topology of followers is undirected and shown in Fig. 7. The remaining parameters are the same as in Example 1.

The performance function $J$ and a guaranteed cost $\bar{J}$ are shown in Fig. 8. We can see that the function converges.
to a value that satisfies $J < \bar{J}$. In order to analyze the relationship between parameters $\beta_1$, $\beta_2$ and the guaranteed cost $J$, the performance changes with different values of $\beta_1$ and $\beta_2$ are shown in the Fig. 9. $J_1$, $J_2$ and $J_3$ represent the performance when “$\beta_1 = 0.002, \beta_2 = 0.003$”, “$\beta_1 = 0.002, \beta_2 = 0.01$” and “$\beta_1 = 0.015, \beta_2 = 0.01$” respectively. It can be found that the smaller the event condition parameter $\beta_1$, $\beta_2$, the smaller the performance. This means that the tighter the event conditions, the better the system can meet the performance constraints.

Example 3: Consider a multi-agent system with 6 followers whose communication topology is represented by an undirected graph depicted in Fig. 10. Let the desired geometric formation is $x^* = [-20, -20, 20, -20, 20, 0, 20 \sqrt{3} - 20, -20, -20, 20]$. Applying the event-triggered controller given in this paper, the snapshots of followers and the desired formation are shown in Fig. 11, one can conclude that followers realize bipartite formation. The $\delta x_i(t)$ in [27] is represented by $e_{xi}(t)$, whose consensus reflects the bipartite formation of followers. With the same initial values of followers, under the controller designed in [27] and this paper, The trajectories of $e_{xi}(t)$, $i = 1, 2, \cdots, 6$ and the trajectories of $\delta x_i(t), i = 1, 2, \cdots, 6$ are described in Fig. 12 and Fig. 13, respectively. With this comparison, the controller designed in this paper can realize the goal of bipartite formation faster.

In addition, different from the bipartite formation control in [27], the study of bipartite formation control problem...
in this paper allows the connected topology to be directed graph.

V. CONCLUSION

In this paper, event-triggered guaranteed-cost bipartite formation control problem for second-order nonlinear multi-agent systems has been addressed. Based on the sampling states of the leader and the neighboring followers, bipartite formation controllers have been designed for followers. According to the definition of bipartite formation, the error system of bipartite formation has been obtained. Using Lyapunov’s stability theory, sufficient condition for the realization of bipartite formation of multi-agent system has been obtained. In addition, for the given quadratic performance function with the controller information and bipartite formation error, a sufficient condition for multi-agent systems to realize guaranteed-cost bipartite formation has been given. Simulation and experimental results show the effectiveness of proposed methods.

REFERENCES

[1] Z. Xu, H. Liu, and Y. Liu, “Fixed-time leader-following flocking for nonlinear second-order multi-agent systems,” IEEE Access, vol. 8, pp. 86262–86271, 2020.
[2] T. Wang, H. Zhang, and Y. Zhao, “Consensus of multi-agent systems under binary-valued measurements and recursive projection algorithm,” IEEE Trans. Autom. Control, vol. 65, no. 6, pp. 2678–2685, Jun. 2020.
[3] C. Huang, X. Zhang, H.-K. Lam, and S.-H. Tsai, “Synchronization analysis for nonlinear complex networks with reaction-diffusion terms using fuzzy-model-based approach,” IEEE Trans. Fuzzy Syst., early access, Feb. 20, 2020, doi: 10.1109/TFUZZ.2020.2974143.
[4] Y. Liu, D. Yao, H. Li, and R. Lu, “Distributed cooperative formation control for platoon of vehicles with adaptive NN,” IEEE Trans. Cybern., early access, Jan. 11, 2021, doi: 10.1109/TCYB.2020.304883.
[5] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, “Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying,” IEEE Trans. Ind. Electron., vol. 64, no. 6, pp. 5014–5024, Jun. 2017.
[6] H. Kang, W. Wang, C. Yang, and Z. Li, “Leader-following formation control and collision avoidance of second-order multi-agent systems with time delay,” IEEE Access, vol. 8, pp. 142571–142580, 2020.
[7] G. Xu, C. Huang, and G. Zhai, “A necessary and sufficient condition for designing formation of discrete-time multi-agent systems with delay,” Neurocomputing, vol. 315, pp. 48–58, Nov. 2018.
[8] P. Xu, J. Wen, C. Wang, and G. Xie, “Distributed circle formation control over directed networks with communication constraints,” IFAC-PapersOnLine, vol. 52, no. 3, pp. 108–113, 2019.

[9] Y. Chen, R. Yu, Y. Zhang, and C. Liu, “Circular formation flight control for unmanned aerial vehicles with directed network and external disturbance,” IEEE/CIAA J. Autom. Sinica, vol. 7, no. 2, pp. 505–516, Mar. 2020.
[10] J. Wang, Y. Xu, Y. Xu, and D. Yang, “Time-varying formation for high-order multi-agent systems with external disturbances by event-triggered integral sliding mode control,” Appl. Math. Comput., vol. 359, pp. 333–343, Oct. 2019.
[11] Y. Cai, H. Zhang, Y. Wang, J. Zhang, and Q. He, “Fixed-time time-varying formation tracking for nonlinear multi-agent systems under event-triggered mechanism,” Inf. Sci., vol. 564, pp. 45–70, Jul. 2021.
[12] Y. Hua, X. Dong, L. Han, Q. Li, and Z. Ren, “Finite-time time-varying formation tracking for high-order multiagent systems with mismatched disturbances,” IEEE Trans. Syst. Man, Cybern. Syst., vol. 50, no. 10, pp. 3795–3803, Oct. 2020.
[13] S. Zuo, Y. Song, F. L. Lewis, and A. Davoudi, “Time-varying output formation containment of general linear homogeneous and heterogeneous multiagent systems,” IEEE Trans. Control Netw. Syst., vol. 6, no. 2, pp. 537–548, Jun. 2019.
[14] W. Wang, C. Huang, J. Cao, and F. E. Alsaadi, “Event-triggered control for sampled-data cluster formation of multi-agent systems,” Neurocomputing, vol. 267, pp. 25–35, Dec. 2017.
[15] Y. Gong, G. Wen, Z. Peng, T. Huang, and Y. Chen, “Observer-based time-varying formation control of fractional-order multi-agent systems with general linear dynamics,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 67, no. 1, pp. 82–86, Jan. 2020.
[16] K. Hosoda, T. Takuma, A. Nakamoto, and S. Hayashi, “Biped robot design powered by antagonist pneumatic actuators for multi-modal locomotion,” Robot. Auto. Syst., vol. 56, no. 1, pp. 46–53, Jan. 2008.
[17] C. M. Synk, B. F. Kim, C. A. Davis, J. Harding, V. Rogers, P. T. Hurley, M. R. Emery, and K. E. Nachman, “Gathering Baltimore’s bounty: Characterizing behaviors, motivations, and barriers of foragers in an urban ecosystem,” Urban Forestry Urban Greening, vol. 28, pp. 97–102, Dec. 2017.
[18] Q. He, X. Wang, Z. Lei, M. Huang, Y. Cai, and L. Ma, “TIFIM: A two-stage iterative framework for influence maximization in social networks,” Appl. Math. Comput., vol. 354, pp. 338–352, Aug. 2019.
[19] Q. He, L. Sun, X. Wang, Z. Wang, M. Huang, B. Yi, Y. Wang, and L. Ma, “Positive opinion maximization in signed social networks,” Inf. Sci., vol. 558, pp. 34–49, May 2021.
[20] Q. He, X. Wang, C. Zhang, M. Huang, and Y. Zhao, “IIOMF: An iterative framework to settle influence maximization for opinion formation in social networks,” IEEE Access, vol. 6, pp. 49654–49663, 2018.
[21] Q. He, X. Wang, F. Mao, J. Lü, Y. Cai, M. Huang, and Q. Xu, “CAOM: A community-based approach to tackle opinion maximization for social networks,” Inf. Sci., vol. 513, pp. 252–269, Mar. 2020.
[22] C. Altavini, “Consensus problems on networks with antagonist interactions,” IEEE Trans. Autom. Control, vol. 58, no. 4, pp. 935–946, Apr. 2013.
[23] E. Li, Q. Ma, and G. Zhou, “Bipartite output consensus for heterogeneous linear multi-agent systems with fully distributed protocol,” J. Franklin Inst., vol. 356, no. 5, pp. 2870–2884, Mar. 2019.
[24] M. Liu, X. Wang, and Z. Li, “Robust bipartite consensus and tracking control of high-order multi-agent systems with matching uncertainties and antagonistic interactions,” IEEE Trans. Syst. Man, Cybern. Syst., vol. 50, no. 7, pp. 2541–2550, Jul. 2020.
[25] S. Bhowmick and S. Panja, “Leader–follower bipartite consensus of linear multiagent systems over a signed directed graph,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 66, no. 8, pp. 1436–1440, Aug. 2019.
[26] W.-L. Zou and G. Li, “Formation behaviors of networks with antagonistic interactions of agents,” Int. J. Distrib. Sensor Netw., vol. 13, no. 8, Aug. 2017, Art. no. 155017772629.
[27] W. Wang, C. Huang, C. Huang, J. Cao, J. Lu, and L. Wang, “Bipartite formation problem of second-order nonlinear multi-agent systems with hybrid impulses,” Appl. Math. Comput., vol. 370, Apr. 2020, Art. no. 124926.
[28] Y. Cai, H. Zhang, Y. Wang, Z. Gao, and Q. He, “Adaptive bipartite fixed-time time-varying output formation-containment tracking of heterogeneous linear multiagent systems,” IEEE Trans. Neural Netw. Learn. Syst., early access, Mar. 3, 2021, doi: 10.1109/TNNLS.2021.3079573.
[29] D. Zhang, Z. Xu, H. R. Karimi, Q. Wang, and L. Yu, “Distributed $H_\infty$ output-feedback control for consensus of heterogeneous linear multiagent systems with aperiodic sampled-data communications,” IEEE Trans. Ind. Electron., vol. 65, no. 5, pp. 4145–4155, May 2018.
[30] Y. Liu and H. Su, “Some necessary and sufficient conditions for containment of second-order multi-agent systems with intermittent sampled data,” ISA Trans., vol. 108, pp. 154–163, Feb. 2021.
