Turbulent meson condensation in quark deconfinement

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In a QCD-like strongly coupled gauge theory at large $N_c$, using the AdS/CFT correspondence, we find that heavy quark deconfinement is accompanied by a coherent condensation of higher meson resonances. This is revealed in non-equilibrium deconfinement transitions triggered by static, as well as, quenched electric fields even below the Schwinger limit. There, we observe a “turbulent” energy flow to higher meson modes, which finally results in the quark deconfinement. Our observation is consistent with seeing deconfinement as a condensation of long QCD strings.

Quark confinement is one of the most fundamental and challenging problems in elementary particle physics, left unsolved. Although quantum chromodynamics (QCD) is the fundamental field theory describing quarks and gluons, their clear understanding is limited to the deconfined phase at high energy or high temperature limits due to the asymptotic freedom. We may benefit from employing a more natural description of the zero temperature hadron vacuum. A dual viewpoint of quark confinement in terms of the “fundamental” degrees of freedom at zero temperature - mesons, is a plausible option.

The mesons appear in families: they are categorized by their spin/flavor quantum numbers, as well as a resonant excitation level $n$ giving a resonance tower such as $\rho(770), \rho(1450), \rho(1700), \rho(1900), \ldots$. In this Letter we find a novel behavior of the higher meson resonances, i.e., mesons with large $n$. In the confined phase, when the deconfined phase is approached, we observe condensation of higher mesons. In this state, macroscopic number of the higher meson resonances, with a characteristic distribution, are excited. The condensed mesons have the same quantum number as the vacuum. The analysis is done via the anti-de Sitter space (AdS)/conformal field theory (CFT) correspondence [1–3], one of the most reliable tools to study strongly-coupled gauge theories. By shifting our viewpoint from quark-gluon to meson degrees of freedom, we gain a simple and universal understanding of the confinement/deconfinement transition, with a bonus of solving mysteries in black holes physics through the AdS/CFT.

The system we study is the $\mathcal{N} = 2$ supersymmetric $SU(N_c)$ QCD which allows the simplest AdS/CFT treatment [4]. The deconfinement transition is induced by external electric fields [22]. In static fields, the confined phase becomes unstable in electric fields stronger than the Schwinger limit $E = E_{\text{Sch}}$ beyond which quarks are liberated from the confining force. We find that this instability is accompanied by the condensation of higher mesons. A striking feature is revealed for the case of an electric field quench: The kick from the quench triggers a domino-like energy transfer from low to high resonant meson modes. This leads to a dynamical deconfinement transition even below the Schwinger limit. The transfer we find resembles that of turbulence in classical hydrodynamics as higher modes participate; thus we call it a “turbulent meson condensation” and suggest it being responsible for deconfinement.

We remind that the $\mathcal{N} = 2$ theory is a toy model: The meson sector is confined and has a discrete spectrum while the gluon sector is conformal and is always deconfined. It resembles heavy quarkonia in a gluon plasma. Generically, quark deconfinement and gluon deconfinement can happen separately, as is known through charmonium experiments in heavy ion collisions. Here, we concentrate on the deconfinement of heavy quarks and not the gluons.

The higher meson resonances are naturally interpreted as long QCD strings, therefore our finding is consistent with interpreting deconfinement as condensation of QCD strings [4] (see [8,9,10]). Under the condensation, a quark can propagate away from its partner antiquark by reconnecting the bond QCD string with the background condensed strings. The gravity dual of the deconfined phase is with a black hole, so given the relation with long fundamental strings [8], our result may shed light on the issue of quantum black holes; In particular, our time-dependent analysis gives a singularity formation on the flavor D-brane in AdS, a probe-brane version of the Bizon-Rostworowski turbulent instability in AdS geometries [11].

Review: Meson effective action from AdS/CFT. — The effective field theory of mesons can be obtained for the $\mathcal{N} = 2$ supersymmetric QCD in the large $N_c$ $\lambda = N_c g_{YM}^2$ limits by the AdS/CFT correspondence [12, 13]. The meson action is nothing but a D7-brane action in the AdS$_5 \times S^5$ geometry:

$$S = -\frac{1}{(2\pi)^8 g_{YM}^2 l_s^8} \int d^8\xi \sqrt{-\det(g_{ab}[w]) + 2\pi l_s^2 F_{ab}(1)}$$
$$ds^2 = \frac{\rho^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{\rho^2} [d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\tilde{w}^2],$$
where \( r^2 \equiv \rho^2 + w^2 + \bar{w}^2 \), \( F_{ab} = \partial_a A_b - \partial_b A_a \), and the AdS\( _5 \) curvature radius is \( R \equiv (2\lambda)^{1/4} \). For the following calculations, it is convenient to define a rescaled gauge potential \( a_n \equiv 2\pi_s^2 R^{-2} a_n \). The D7-brane worldvolume fields are \( w(x^\mu, \rho) \) and \( a_n(x^\mu, \rho) \). (We set \( \bar{w}(x^\mu, \rho) = 0 \) consistently because of \( U(1) \)-symmetry in \((w, \bar{w})\)-plane.) We denote the location of the D7-brane at the asymptotic AdS boundary as \( w(x^\mu, \rho = \infty) = R^2 m \). Here, the constant \( m \) is related to the quark mass \( m_q \) as \( m_q = (\lambda/2\pi_s^2)^{1/2} m \). A static solution of the D7-brane in the \( \text{AdS}_5 \times S^5 \) geometry is given by \( w(x^\mu, \rho) = R^2 m \) and \( a_n(x^\mu, \rho) = 0 \). Using the AdS/CFT dictionary, normalizable fluctuations around the static solutions of \( w \) and \( a_n \) are interpreted as the infinite towers of scalar mesons \( \psi\bar{\psi} \) and vector mesons \( \psi\gamma^\mu\psi \), respectively. (We omit the pseudo scalar mesons which are irrelevant to our discussion.) In this paper, we focus on the meson condensation induced by an electric field along the \( x \)-direction. Thus, we only consider fluctuations of \( w \) and \( a_n \).

To derive the meson effective action, we use a coordinate \( z \) defined by \( \rho = R^2 m w \sqrt{1 - z^2} / z \) (where \( z = 0 \) is the AdS boundary, and \( z = 1 \) is the D7-brane center that is closest to the Poincaré horizon in the bulk AdS). The worldvolume is effectively in a finite box along the AdS radial direction, to give a confined discrete spectrum as we will see below. We expand the D7-brane action up to second order in the fluctuations \( \chi \equiv (R^{-2}w - m, a_n) \) as

\[
S = \int dt d^3 x \int_0^1 dz \frac{1 - z^2}{2z} \left[ \chi^2 - m^2 (1 - z^2) \chi^2 \right] + O(\chi^3),
\]

where \( \cdot \equiv \partial_\cdot \) and \( ' \equiv \partial_z \). An irrelevant overall factor is neglected. The equation of motion for \( \chi \) is

\[
(\partial_t^2 + \mathcal{H}) \chi = 0, \quad \mathcal{H} \equiv -m^2 z \left( \frac{1 - z^2}{1 - z^2} \right) \partial_z \left( \frac{1 - z^2}{z} \right) \partial_z.
\]  

The eigenfunction of \( \mathcal{H} \) is given by \( e_n(z) \equiv \sqrt{2/(2n+3)(n+1)(n+2)} z^F(n+3, -n, 2, z^2) \), with the eigenvalue \( \omega_n^2 = 4(n+1)(n+2)m^2 \), for the meson level number \( n = 0, 1, 2, \ldots \). Here \( F \) is the Gaussian hypergeometric function. The inner product is defined as \( \langle f, g \rangle \equiv \int_0^1 dz z^{-1}(1 - z^2)f(z)g(z) \), where \( \langle e_n, e_m \rangle = \delta_{nm} \) is satisfied. Note that an external electric field term, \( a_x = -Et \), satisfies Eq. (2) although it is non-normalizable. Expanding the scalar/vector fields as

\[
\chi = (0, -Et) + \sum_{n=0}^{\infty} c_n(t) e_n(z),
\]

we find an infinite tower of meson fields \( c_n(t) \) sharing the same quantum charge - higher meson resonances. Substituting Eq. (3) back to Eq. (1), we obtain the meson effective action

\[
S = \frac{1}{2} \int dt d^3 x \sum_{n=0}^{\infty} \left[ c_n^2 - \omega_n^2 \chi_n^2 \right] + \text{interaction},
\]

where we have omitted a constant term and total derivative terms, while higher order nonlinear terms give rise to meson-meson interactions. From the effective action, we obtain the energy \( \varepsilon_n \equiv \frac{1}{2} (\omega_n^2 c_n^2 + \omega_n^2 \chi_n^2) \) stored in the \( n \)-th meson resonance, and the linearized total energy \( \varepsilon = \sum_{n=0}^{\infty} \varepsilon_n \).

**Higher meson condensation and deconfinement.**— The confined phase becomes unstable in strong static electric fields. Here, we examine this from the viewpoint of meson condensation. In Eq. (1), the meson couplings depend on the external electric field \( E \) nonlinearly. Mesons in a single flavor theory is neutral under \( E \), but it can polarize, and non-linear \( E \) may cause a meson condensation. We first solve the equations of motion obtained from the full nonlinear D-brane action (1) with an external electric field, and then decompose the solution \( \chi(t, z) \) as Eq. (3). In this way, we can study how the infinite tower of mesons \( c_n(t) \) behave towards the deconfinement transition.

The static D7-brane solution in the presence of a constant electric field introduced by \( a_x = -Et \) was obtained...
in Refs. \[14,16\]. Also, the Schwinger limit \( E = E_{\text{Sch}} = 0.5759 m^2 \) beyond which the first order phase transition to deconfinement occurs was found \[15,16\]. Fig. 1 shows the shape of the D7-brane, which is the scalar field configuration \( \omega(\rho) \), for \( E/E_{\text{Sch}} = 0.5, 0.8, 0.95, 1 \) and the critical embedding. At the critical embedding which is a confinement/deconfinement phase boundary although the solution itself is thermodynamically unfavorable, the brane becomes conical and an effective horizon starts emerging on the worldvolume. As the solution approaches the critical embedding, the D7-brane bends more toward the Poincaré horizon of AdS; the brane with the sharpest bending is for the critical embedding.

The bending leads to meson condensation, where sharp bending means that higher meson modes are excited. In order to clarify this, we decompose the scalar field by the meson eigenmodes and plot the ratio \( |c_n|/|c_0| \) as a function of \( n \) in Fig. 2, where we define \( |c_n| = \sqrt{|\text{scalar}|^2 + |\text{vector}|^2} \) for illustration. As the electric field increases, the higher meson condensate \( |c_n| (n \gg 1) \) compared to the lowest \( |c_0| \) grows rapidly. Note that vector mesons are not excited in static electric fields since the gauge potential is always given by \( a_x = -Et \) (no higher modes). This implies that the condensed mesons have the same quantum number as the vacuum. The geometrical reason of the condensation of the higher mesons is simple; The D7-brane bends singularly near the Poincaré horizon of AdS due to the nature of the metric and higher eigenmodes are necessary to reproduce it. It is similar to Fourier-decompose a delta-function (narrow Gaussian).

“Flow” of energy from low to high resonant meson modes takes place at the same time. For static solutions, the energy stored by the \( n \)-the meson mode is given by \( \varepsilon_n = \omega_n^2 c_n^2/2 \). In Fig. 3 the meson energy distribution in static \( E \) shows enhancement at higher modes as increasing \( E \), that is, energy “flow” to higher meson resonances. The exponential behavior of the energy distribution in Fig. 4 for \( E_{\text{Sch}} \geq E \) provides a well-defined effective temperature \( T \). At the critical embedding it turns to a power-law, which exhibits a Hagedorn behavior, and reminds us of the Kolmogorov scaling.

We conclude for the static case that just before the quark deconfinement induced by an applied electric field, meson resonances condense coherently.

**Meson turbulence in quenched electric fields.—** The higher meson condensation seems to be a sufficient cause of quark deconfinement. This is clearly seen in a time-dependent, electric field quench that we study below. Starting from the \( E = 0 \) vacuum in the confined phase, we turn on the electric field smoothly to reach a final value \( E_f \) in the duration \( \Delta V \). In our previous work \[3\], we found that the system deconfines with an emergence of a strongly redshifted region on the D7-brane toward a naked-singularity formation. This is interpreted as an instability toward deconfinement, which happens, to our surprise, even when the final field strength is below the Schwinger limit. In the following, we choose a weak electric field (\( E_f/E_{\text{Sch}} = 0.2672 \)) and a switch-on duration \( m \Delta V = 2 \), in which sub-Schwinger-limit deconfinement is realized.

The electric field induces nontrivial dynamics in the meson sector. We decompose the time-dependent solutions of the meson fields \[3\] and calculate the energy \( \varepsilon_n \) of the meson resonances. In Fig. 5, the time evolution of the condensate \( |c_n| \) is shown for several time slices \( mt = 15, 40, 49.3 \), while the time \( mt = 49.3 \) is just before deconfinement. Figure 5 plots \( \varepsilon_n/\varepsilon \). We find that the condensate and the energy are transferred to higher meson modes during the time evolution. This tendency is similar to the static case in which they are “transferred” more in stronger electric fields. This indicates that higher meson condensation is universally related to quark confinement.

The observed time evolution of the distribution suggests that turbulence is taking place in the meson sector. This is because higher modes have smaller wave lengths, and the transfer of energy and momentum indicates that smaller structure are being organized during the time evolution toward deconfinement. Our finding can be considered as a probe-brane version of the turbulent instability of the AdS spacetime \[11\] in which a non-linear evolution of a perturbed AdS spacetime causes a high-momentum instability resulting in a black hole formation.

**Conclusion and discussions.—** In this work we found, via the AdS/CFT correspondence, that higher meson resonances become condensed near the deconfinement transition caused by electric fields. This was confirmed for both the static electric fields and the time-dependent quenched electric field, in \( N = 2 \) supersymmetric QCD which mod-
The dissociation of the QCD string is described by the electric field quench. Data for times $mt = 15, 40$, and $49.3$ are shown. We set $E_f/E_{\text{Sch}} = 0.2672$ and $m\Delta V = 2$. A clear (non-thermal) growth at large $n$ is found along the time evolution.

![Graph](image1.png)

**FIG. 4:** (Color online) Meson condensate induced by an electric field quench. Data for times $mt = 15, 40$, and $49.3$ are shown. We set $E_f/E_{\text{Sch}} = 0.2672$ and $m\Delta V = 2$. A clear (non-thermal) growth at large $n$ is found along the time evolution.

The meson turbulence obviously share some features with the AdS turbulence $[11, 21]$. It may have relation with thermalization and numerical simulations of glasma $[22–24]$. Our observation is also valid in a temperature-driven deconfinement $[25]$. Thus, we conjecture that quark deconfinement is universally associated with a condensation of higher meson resonances.

**Acknowledgment.** K. H. would like to thank A. Buchel, H. Fukaya, D.-K. Hong, N. Iqbal, K.-Y. Kim, J. Maldacena, S. Sugimoto, S. Yamaguchi and P. Yi for valuable discussions, and APCTP focus week program for its hospitality. This research was partially supported by the RIKEN iTHEIS project.

els heavy quarkonia in a gluon plasma. No internal symmetry is broken during this process since the mesons that participates in the deconfinement with their condensation have the same quantum numbers as the vacuum and the external force.

The physics in the gravity dual side is simple. The gravity-dual of the scalar meson condensation is the deformation of the probe D7-brane. The electric field breaks the supersymmetries of the brane configuration making the Poincaré horizon to attract the D7-brane and to bend it. The tip of the D7-brane becomes sharper as the electric field increases, and finally when it exceeds the critical value, deconfinement transition occurs. At the verge of deconfinement (at which we can use the meson terminology), higher meson resonances condense coherently.

A single meson can be regarded as quarks connected by a QCD string and higher meson resonances as its coherent fluctuation. At the deconfinement transition, one needs to dissociate the QCD string to liberate the quarks. The dissociation of the QCD string is described by the coherent dynamics of the excited mesons. Our finding is consistent with this view, and in particular with deconfinement as QCD string condensation $[6–10]$. Our result seems further consistent with Hagedorn transition in string theory $[17, 18, 28]$ and the black hole / string correspondence $[19, 20]$.

In quark models, the higher resonant meson naively corresponds to a state $\bar{\psi}(x)U(x, y)\psi(y)$, where $U(x, y) \equiv P \exp[i \int_x^y G]$, which can be expanded as $\bar{\psi}(x)\Box^n \psi(x)$ where $\Box$ is the covariant Laplacian. Our conjecture waits for a confirmation by lattice QCD on condensation of this operator.

The meson condensation is found along the time $t = 15, 40, 49$. A single meson can be regarded as quarks connected by a QCD string and higher meson resonances as its coherent fluctuation. At the deconfinement transition, one needs to dissociate the QCD string to liberate the quarks. The dissociation of the QCD string is described by the coherent dynamics of the excited mesons. Our finding is consistent with this view, and in particular with deconfinement as QCD string condensation $[6–10]$.

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