Dynamics of the rotor of the centrifugal pump with not opposed scheme impellers

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Annotation
The neo-positive arrangement of the impellers is sometimes found in centrifugal pumps with hydrostatic bearings. Such a scheme is due to the high requirements for the dimensions of modern pumps. The most critical problem arising in such pumps is the problem of stable start dynamics. This article discusses the mathematical model of starting the rotor of a centrifugal pump with hydrostatic bearings. The resulting mathematical model allows us to determine the time of the output to the steady-state mode and set the angular velocity of the rotor.

Introduction
Low-consumption centrifugal pumps are used in many industries. As a result, there is a lot of literature\cite{1-4} that describes the calculations (associated with finding the various parameters of these pumps) in detail. However, there is still a lot of room for research, in particular, issues related to finding the steady-state angular velocity of the pump rotor.

To solve such problems, we often use methods of hydrodynamic modeling \cite{5-9}. However, the application of those methods is difficult due to a large number of calculations. Thus, we need to find other approaches.

The literature \cite{10-16} reviews other methods of modeling processes occurring in centrifugal pumps, which is usually either a numerical solution of the Navier-Stokes equations or reliance on empirical methods. We need a way to solve the start problem without empirical formulas and numerical solution of the Navier-Stokes equations, which is what we attempt to do in this paper.

Methods
In this paper, we study the dynamics of the low-inertia rotor of a low-flow centrifugal cantilever pump with hydrostatic bearings at low vibrations. Figure 1 shows the scheme of the pump.

Let’s examine the torques affecting the pump shaft (Figure 2).

On the figure, $M_{ik}$ is torque from the impeller, $M_{\tau}$ is viscous friction torque, and $M_{sh}$ is torque from the engine. The dry friction force is not considered, because there is always a thin layer of oil with thickness $\delta_0$ between the shaft and the bearing.
Figure 1. Pump scheme. 1 – pump impeller, 2 – hydrostatic bearings, 3 – stator, 4 – rotor, 5 – inlet pipe, 6 – outlet pipe, 7 – bearing supply channels.

Figure 2. Torques affecting the shaft.

Let's make the equation of moments relative to the axis of the shaft:

\[ J \cdot \frac{d\omega}{dt} = M_{dv}(t) - 2 \cdot M_{rh}(t) - 2 \cdot M_{vt}(t). \] (1)
Here, \( J \) is the moment of inertia of the rotor relative to its axis, \( \omega \) is the angular velocity of rotation of the pump shaft, t is the time from the start of the pump, \( g \) is the acceleration of gravity.

The formula below describes the moment generated by the engine.

\[
M_{\text{dr}}(t) = K - K_1 \cdot \omega(t). \tag{2}
\]

Here, \( K \) and \( K_1 \) are the coefficients determined by the design of this particular motor.

The moment on the impeller is equal to:

\[
M_{rk} = M_c + M_{df}. \tag{3}
\]

Here, \( M_c \) is the centrifugal moment, \( M_{df} \) is the disk friction moment.

The centrifugal torque is calculated [1]:

\[
M_c = \rho \cdot Q \cdot R_2 \cdot C_{2u}. \tag{4}
\]

Here, \( \rho \) is the density of the liquid, \( Q \) is the flow rate, \( R_2 \) is the outer radius of the impeller.

Since the valve is closed, \( Q \) equals 0, respectively, \( M_c = 0 \).

The moment of disk friction is:

\[
M_{df}(t) = \frac{\omega(t) \cdot \pi \cdot \mu \cdot R_2^4}{a}. \tag{5}
\]

Here, \( a \) is the axial clearance between the impeller and the pump housing, \( \mu \) is the dynamic viscosity of the working fluid.

Thus, the torque of the impeller is equal to:

\[
M_{rk}(t) = \frac{\omega(t) \cdot \pi \cdot \mu \cdot R_2^4}{a}. \tag{6}
\]

Here, \( l \) is the length of the bearing pocket, \( d \) is the diameter of the pump shaft, \( \varphi \) is the angle, \( y \) is the size of the gap (for small oscillations we take \( y = \delta_0 \) - the thickness of the liquid film at the start of the start), \( \delta \) is the average gap between the bearing and the shaft.

We substitute these values in equation (1) and obtain a linear inhomogeneous differential equation of the second order, the solution of which is the solution of the Cauchy problem under a known initial condition \( \omega' + B/A \cdot \omega = K/A; \omega(0) = 0 \)

The linear nonuniform differential equation corresponds to a linear homogeneous differential equation: \( \omega' + B/A \cdot \omega = 0 \)

The solution of the equation is: \( \omega(t) = C \cdot e^\omega(-B/A \cdot t) \)

To find the solution of the inhomogeneous equation, we use the method of variation of the constant, i.e., we consider it with the function \( t \).

We obtain the solution in the form: \( \omega(t) = K/B + C_1 \cdot e^\omega(-B/A \cdot t) \).

From the initial conditions we find the constant \( C_1 \).

Thus, the solution of the equation looks like this: \( \omega(t) = K/B \cdot (1 - e^\omega(-B/A \cdot t)) \)

\section*{Results}

This technique will be tested on the pump having the parameters:

\begin{align*}
K &= 0.9 \text{ N*s}, \quad a = 0.001 \text{ mm}, \quad \mu = 0.000276 \text{ PA*s}, \quad R_2 = 0.033 \text{ mm}, \quad J = 0.0001 \text{ N*m*S}^2, \\
K_1 &= 0.0015 \text{ N*m*s}, \quad l = 0.02 \text{ mm}, \quad d = 0.01 \text{ mm}.
\end{align*}

We obtain a graph of the angular velocity of the pump shaft on time at these values of parameters (Figure 3).
The graph shows that the steady-state angular velocity of the pump shaft is $\omega(t)=599.515$ rad/s.

**Summary**

Thus, the obtained technique makes it possible to find the dependence of the angular velocity on time and to predict the steady-state rotor speed, under the condition of small oscillations of the rotor in the inner ring of the bearing.

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