Due to the complex and harsh climate environment in Northwest China, the external insulation of power lines and high-speed train are exposed to the sandstorm environment all year round, which brings severe challenges to the external insulation design and protection of the electric systems. Based on the theories of streamer discharge, the plate-plate electrode was taken as the analysis object, and a theoretical physical model of the strong wind-sand dielectric discharge is established. In this model, the analytical expressions of electron ionization coefficient and positive ion generation coefficient in a wind-sand environment are deduced considering the combined effects of the airflow velocity, the distortion of the electric field caused by sand particles, and the electrons captured by sand particles in the discharge process; furthermore, the streamer breakdown criterion in wind-sand environment is proposed. Research results can provide an important theoretical basis for the quantitative calculation of the breakdown voltage and the external insulation protection in a strong wind-sand environment.

1. Introduction

In recent years, the power system has developed rapidly, which is the energy foundation support for the stable and rapid development of the country [1, 2]. However, the external insulation of transmission lines in Northwest China is exposed to the sandstorm environment for a long time, which is easy to cause abnormal flashover of the outdoor insulators and pose a serious threat to the safety and reliability of the external insulation for the northwest power grid [3, 4]. In addition, the strong wind-sand environment caused by the high-speed train from Lanzhou to Xinjiang running in the northwest region of China is the key factor to induce the abnormal breakdown or flashover of the external insulation of the train, which poses a serious threat to the safe operation of the train [5–7]. Therefore, the investigations of gas discharge characteristics and dielectric breakdown model in strong wind-sand environment are of great significance for external insulation design, insulation coordination, and insulation protection of the electric system.

Recently, researchers have performed a lot of research on gas discharge characteristics in wind-sand or airflow environment. In the area of air gap breakdown in wind-sand environment, He et al. [8] found that the breakdown voltage of the plate-plate gap in a wind-sand environment is related to the wind speed, gap distance, and sand concentration. Under the condition of pure airflow, the breakdown voltage increases with the increase of wind speed. At fixed wind speed, the breakdown voltage decreases with the increase of sand concentration, but the breakdown voltage-gap distance curve is lower than that under pure airflow. The simulation results show that the distortion of the local electric field caused by sand particles reduces the breakdown voltage. For the discharge of the needle-plate gap and in a wind-sand environment, [9] found that, with the increase of wind speed, the change trend of breakdown voltage in needle-plate gap increases first and then decreases. The experimental results of [10] showed that the breakdown voltage of rod-rod gaps decreases with increasing wind speed in both wind-sand and pure airflow environments, and the variation trends of the breakdown voltage with wind speed are the same under these two conditions. Ai Arainy et al. [11–14] also found that the breakdown voltage of rod-rod and rod-plate gaps in a sand dust environment is related to the gap distance, polarity of the impulse voltage, shape of the electrodes, and size of the sand particles. The above research
shows that the influence of wind and sand dust on gap breakdown voltage is closely related to electrode structure, voltage type, airflow velocity, sand dust concentration, and sand particles size. Besides, the distortion of the local electric field caused by sand particles is an important factor affecting gap discharge characteristics.

In the area of insulation flashover in a wind-sand and airflow environment, Sima et al. [15] studied the effects of wind speed, charge deposited on sand, and sand quality in flashover process. The results show that the flashover voltage of insulators increases with the increase of wind speed, and the flashover voltage in wind-and-sand condition is lower than that in wind-and-no-sand condition. The distortion of the electric field caused by sand particles is the main reason for this phenomenon. Some researchers have also studied the influence of wind speed on flashover path and discharge patterns of insulators in a wind-sand environment [16, 17]. Additionally, charged sand dust will also have an important impact on insulator flashover process. Yu et al. [18] found that charged sand particles will lead to the decrease of insulator flashover voltage, and the degree decrease increases obviously with the increase of charge amount of sand particles.

In the area of gas-solid or gas-liquid two-phase discharge, Marquard et al. [19] found that the influence of particles on corona discharge is related to the particle concentration. When the particle concentration is high, the corona starting voltage increases and the corona current decreases. By studying the corona discharge current density of gas-solid two-phase medium with micron particles, Xu et al. [20, 21] concluded that particles reduce the corona current, and, with the decrease of particle size, its influence on the corona current is more significant; that is, the smaller size of particles and higher concentration particles are easier to restrain the discharge process. The experimental results of Yao et al. [22] also show that the size of particles under DC voltage plays a decisive role in the breakdown voltage and the choice of breakdown path of two-phase mixture. Li et al. [23] carried out numerical simulation of rod-plate gap streamer discharge in 50~50% SF6/N2 gas mixtures based on ETG-FCT method and extended ETG scheme to gas discharge problems. Based on the above research results, it can be concluded that the main factor affecting the discharge of solid/liquid particles is the serious distortion of the local electric field in the mixture. Therefore, solving the local electric field in the mixture has become an important content to analyse its influence on the discharge process. In this regard, Ye et al. [24–26] have proposed the calculation methods of local electric field in the mixture such as dipole-enhanced model and displaced dipole model and presented a dielectric breakdown model for static gas-solid two-phase mixture, but the model did not consider the influence of airflow on discharge. Kang et al. [27–29] found that the flowing gases have a significant impact on the discharge process. The frequent collision between gas molecules and electrons or ions causes the change of discharge path and space charge, and the gas discharge model in the flowing gases environment was established. Guo et al. [30] improved a two-dimensional numerical model by considering the effect of wind velocity and wind direction on the movement of the corona charges. However, the dielectric breakdown process in strong wind-sand environment is the comprehensive effects of airflow and sand particles. How to obtain the correlation between the discharge parameters and the comprehensive effects of airflow or sand dust is the key to establish the dielectric breakdown model of strong wind-sand environment.

In this work, on the basis of streamer discharge theory, we attempt to present a theoretical model of strong wind-sand dielectric discharge considering the combined effects of airflow and sand particles on the discharge process, and the breakdown criterion for strong wind-sand dielectric discharge is obtained, which can realize the quantitative calculation and effective prediction of the breakdown voltage in a strong wind-sand environment.

2. Model of Strong Wind-Sand Dielectric Streamer Discharge

2.1. Basic Preconditions. Compared with Townsend's discharge theory, streamer discharge theory is more suitable for higher pd value conditions and considers the significant role of space charge in the discharge process, which is more suitable for the construction of discharge model under the strong wind-sand condition. According to the streamer discharge theory, establishing the relationship between airflow or sand particle and ionization coefficient and space charge (positive ion generation coefficient) is the key to establish the model of strong wind-sand dielectric discharge based on the streamer discharge theory.

First, in order to describe the effect of airflow or sand particles on discharge process, we put forward the basic preconditions for the modeling of strong wind-sand dielectric discharge. Previous studies have shown that the deflection of discharge path, the blowing away of some electrons and ions, the change in gas density, distorting the electric field, and the capture of charge caused by sand particles are the main factors affecting dielectric discharge process in strong wind-sand environment. Therefore, the basic preconditions for the effects of airflow or sand particles on discharge process are presented as follows [27, 31]:

1. The discharge path deflects an angle along the gas flow direction, that is, the motion path of electrons and ions participating in the discharge deflects an angle along the gas flow direction
2. Some electrons and ions are blown away by the gas flow, including the electrons and ions completely blown away and deviated from the main discharge path
3. Gas is a compressible fluid; that is, the gas density decreases with the increase of gas flow velocity
4. The sand particles are spheres of the same size and are evenly distributed in the electrode space
5. An equivalent electric field is used instead of the original electric field to consider the interaction among sand particles
(6) During the discharge process, the sand particles capture some electrons or ions for charging.

Based on the above six preconditions, streamer discharge in strong wind-sand environment with horizontal airflow velocity and short gap in uniform electric field can be described as a physical process shown in Figure 1.

In order to explain clearly the modeling thinking of this paper, the modeling process of strong wind-sand dielectric streamer discharge is shown in Figure 2.

The parameters defined in this paper are shown in Table 1.

2.2. Calculation of Electron Ionization Coefficient and Positive Ion Generation Coefficient. On the basis of the above preconditions, the Townsend ionization coefficient and positive ion generation coefficient will be solved step by step from the four following aspects.

2.2.1. Local Electric Field of Sand Particles. The local electric field near one sand particle is not only affected by the self-polarization of the sand particles but also affected by other sand particles. For the interaction between sand particles, the dipole-enhanced model is used to calculate the local electric field around sand particles [24, 25]. At this time, it can be considered that the sand is in an equivalent electric field $E_{\text{eq}}$ and the configuration diagram of the equivalent electric field is shown in Figure 3.

The equivalent electric field $E_{\text{eq}}$ can be calculated as

$$E_{\text{eq}} = \frac{E}{1 - \left[\left(\varepsilon_i - \varepsilon_e\right) / \left(\varepsilon_i + 2\varepsilon_e\right)\right] (2R/h)^3},$$

where $\varepsilon_i$ is the permittivity of the sand particles, $\varepsilon_e$ is the permittivity of the environment, $R$ is the radius of the sand particles, $E$ is the applied electric field, and $h$ is the distance between sand particles.

By solving the Laplace equation in the spherical coordinate system, it can be obtained that the external electric field $E_{\text{out}}$ of sand particles caused by the equivalent electric field is

$$E_{\text{out}} = r_0 \left( E_{\text{eq}} \cos \theta + \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} \frac{2E_{\text{eq}}R^3 \cos \theta}{r^3} \right) + \theta_0 \left( -E_{\text{eq}} \sin \theta + \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} \frac{E_{\text{eq}}R^3 \sin \theta}{r^3} \right).$$

According to (2), we can obtain the maximum electric field on the surface of one sand particle as follows:

$$E_{\text{max}} = \frac{3\varepsilon_i}{\varepsilon_i + 2\varepsilon_e} E_{\text{eq}}.$$

In addition, when the sand captures the electrons or ions, it will further affect the local electric field distribution. In order to calculate the local electric field after the sand is charged, we first assume that the saturation charge (i.e., the maximum charge) captured by every sand particle and the amount of saturated charge captured by the sand particles are related to the charging process of the sand particles. When the electric field generated by the charged sand particles is equal to the maximum electric field on the surface of sand particle caused by the equivalent electric field, the amount of charge captured by the sand particle reaches saturation [32]. At this time, the charging process of sand particles ends, and the saturated charging balance relationship of sand particles can be characterized by the following formula:

$$\frac{3\varepsilon_i}{\varepsilon_i + 2\varepsilon_e} E_{\text{eq}} = \frac{q_0}{4\pi\varepsilon_0 R^2},$$

where $q_0$ is the saturation charge of sand particles.

Solving (4), the saturation charge of sand particles is derived as

$$q_0 = \frac{12\pi\varepsilon_0 R^2 E}{\varepsilon_i + 2\varepsilon_e - (2R/h)^3 (\varepsilon_i - \varepsilon_e)}.$$

In practice, if the charge coefficient of the sand particles is expressed by $f$, the charge captured by sand particles is $q = f q_0$. According to the field charging model [32], we have

$$f = \frac{t}{t + \tau},$$

where $t$ is the charging time, with $t = 2R/v_e$, and $\tau$ is charge time constant, with $\tau = 4\varepsilon_i E_{\text{eq}} / J$, wherein $J$ is the electron current density and $J = ne v_e$, with $n$ being the number density of electrons before the sand particle is charged and $e$ is the absolute value of the charge carried by the electron. $v_e$ is the drift velocity of electrons.

Substitute $J$ into $\tau$ and expand it:

$$\tau = \frac{4\varepsilon_i E_{\text{eq}}}{J} = \frac{4\varepsilon_i E_{\text{eq}}}{nev_e} = \frac{4\varepsilon_i E_{\text{eq}}}{ne K_e E_{\text{eq}}} = \frac{4\varepsilon_i}{K_e ne},$$

where $K_e$ is the electron mobility, $K_e = v_e / E_{\text{eq}}$.

Substituting (7) into (6), we can obtain

$$f = \frac{t}{t + \tau} = \frac{ne R}{ne R + 2e_e E_{\text{eq}}}.$$

Therefore, the charge $q$ can be calculated as

$$q = f q_0 = \frac{ne R}{ne R + 2e_e E_{\text{eq}}} \frac{12\pi\varepsilon_0 R^2 E_{\text{eq}}}{\varepsilon_i + 2\varepsilon_e}.$$
Figure 1: Streamer discharge in uniform field under strong wind-sand environment.

Figure 2: Modeling flow chart of strong wind-sand dielectric streamer discharge.

Table 1: The parameters defined in this paper.

| Parameters | Unit symbol | Meaning                                        | Value            |
|------------|-------------|------------------------------------------------|------------------|
| $E$        | V/m         | Applied electric field                          |                  |
| $R_e$      | m           | Electrode radius                                |                  |
| $d$        | m           | Gap distance                                    |                  |
| $v_e$      | m/s         | Electron velocity                               |                  |
| $v$        | m/s         | Gas flow velocity                               |                  |
| $\theta$   | °           | Deflection angle                                |                  |
| $K_e$      | m$^2$/(V·s) | Mean electron mobility                          |                  |
| $\lambda_e$| m           | Electron free path                              |                  |
| $e$        | C           | Electronic charge                               |                  |
| $V_i$      | V           | Atomic ionization potential                     |                  |
| $\alpha$   | cm$^{-1}$   | Townsend’s first ionization coefficient          |                  |
| $t$        | s           | Time                                            |                  |
| $\mu$      |             | Blowing away factor                             |                  |
| $\rho$     | kg/m$^3$    | Gas density                                     |                  |
| $\rho_0$   | kg/m$^3$    | Static gas density                              |                  |
| $\gamma_a$ |             | Specific heat ratio                             |                  |
According to (9), it can be obtained that the charged electric field $E_q$ generated on the surface of sand particles after charging is

$$E_q = \frac{q}{4\pi \varepsilon_0 R^2}. \tag{10}$$

The maximum electric field $E_{\text{max}}''$ on the surface of the charged sand particles consists of the charged electric field $E_q$ on the surface of the sand particles and the maximum electric field $E_{\text{max}}'$ on the surface of the sand particles, which can be calculated as

$$E_{\text{max}}'' = E_{\text{max}}' + E_q = (1 + f) - \frac{3\varepsilon_i}{\varepsilon_i + 2\varepsilon_e} E_{\text{0c}}. \tag{11}$$

$$= \left(1 + \frac{neR}{neR + \varepsilon_e E_{\text{0c}}} \right) \frac{3\varepsilon_i}{\varepsilon_i + 2\varepsilon_e} E_{\text{0c}}.$$  

### 2.2.2. Deflection of the Breakdown Path

In the discharge process of the strong wind-sand dielectric, charged particles obtain electric field acceleration in the direction of electric field; that is, the charged particles obtain a drift velocity. At the same time, charged particles are frequently collided by neutral molecules in the direction of airflow to obtain another horizontal velocity, namely, airflow velocity. Therefore, under the combined action of electric field force and horizontal collision force, the average moving path of charged particles will deflect an angle along the airflow direction, which will affect the ionization coefficient. For the specific influence process, please refer to the results previously published in our article in [27].

Combined with the change of spatial electric field in wind-sand environment and considering the path deflection effect of charged particles, Townsend’s first ionization coefficient in wind-sand environment is deduced as

$$\alpha_a = \frac{1}{\lambda_e \cos \left( \arctan \left( \frac{v}{K_e E_{\text{0c}}} \right) \right)} \exp \left\{ -\frac{V_i}{E_{\text{0c}} \lambda_e \cos \left( \arctan \left( \frac{v}{K_e E_{\text{0c}}} \right) \right)} \right\}, \tag{12}$$

where $\theta$ is the deflection angle of electron moving path along the horizontal direction, $v$ is the gas flow velocity, $K_e$ is the electron mobility, $\lambda_e$ is the electron mean free path, and $V_i$ is the ionization potential.

### 2.2.3. Some Electrons and Ions Are Blown Away by Airflow

Due to frequent collisions between neutral molecules and electrons and ions in the direction of airflow, some electrons and ions involved in discharge will be blown away or deviated from the path of main discharge by airflow, which will affect the discharge process. Please refer to the article in [27] for the specific impact process. Combined with the spatial electric field change of wind-sand environment and comprehensively considering the discharge path deflection and blowing away effects, Townsend’s first ionization coefficient in wind-sand environment can be deduced as

$$\alpha_{\text{ab}} = \exp \left( -\mu_1 \rho \left( \frac{\nu}{E_{\text{0c}}} \right) \right) \alpha_a, \tag{13}$$
where $\mu_1$ is the electron blowing away coefficient and $\rho$ is the gas density.

In the process of streamer discharge, the generation and development of positive ions have an important influence on the process of streamer discharge. In static air, the number of electrons generated by collision ionization per unit distance of electron moving is equal to the number of ions; that is, the positive ion generation coefficient is equal to the electron ionization coefficient. However, in wind-sand environment, in addition to considering the change of electron ionization coefficient, the blowing away effect of positive ions needs to be considered. For this purpose, the positive ion generation coefficient in wind-sand environment is defined as the number of positive ions generated by collision ionization per unit distance of electrons moving along the direction of electric field and not blown away by airflow. According to literature [27] and (13), the positive ion generation coefficient in wind-sand environment can be calculated as

$$\alpha_i = \exp\left(-\mu_2 \frac{\rho v}{E_{0c}}\right) \alpha_{ob}, \quad (14)$$

where $\mu_2$ is the positive ion blowing away coefficient.

2.2.4. Change of the Gas Density. Based on the previous research [27], the relationship between the mean electron free path and the airflow velocity can be deduced as follows:

$$\lambda_e = \frac{4M}{n_d \rho_0 S N_A \rho_0 \{1 + (\gamma_s - 1/2) \frac{v^2}{s^2}\}^{1/(\gamma_s - 1)}, \quad (15)$$

where $\gamma_s$ is the specific heat ratio of a gas, Mach’s number $Ma = v/s$, $s$ is the speed of sound, $d_m$ is the effective molecular diameter, $N_A$ is Avogadro’s number, $M$ is the relative molecular weight, and parameter $\rho_0$ represents the corresponding value at a stationary point, which is taken as the reference value.

Therefore, Townsend’s ionization coefficient considering the spatial electric field of sand particles, the deflection of discharge path, the blowing away of some electrons and ions, and the change of gas density can be calculated as

$$\alpha = A(v, E_{0c}) \exp\left(-\mu_1 \rho v + B(v, E_{0c})\right), \quad (16)$$

where

$$A(v, E_{0c}) = \frac{1}{\lambda_e \cos[\arctan(v/K_{E_{0c}})]}, \quad (17)$$

$$B(v, E_{0c}) = A(v, E_{0c})V.$$

At this time, the positive ion generation coefficient is calculated as

$$\alpha_i' = A(v, E_{0c}) \exp\left(-\mu_2 \rho v + B(v, E_{0c})\right), \quad (18)$$

where $\mu = \mu_1 + \mu_2$.

2.2.5. Some Electrons or Ions Are Captured by the Sand Particles. In the process of gas discharge in strong wind-sand environment, in addition to considering the above four factors, the electrons or ions will be captured by sand particles for charging in the process of migration along the electric field direction, resulting in the reduction of the number of electrons or ions. At the same time, the electric field generated by the charged sand particles will further distort the local electric field and affect the ionization process.

In order to analyse this process, it is assumed that the sand particles are spherical and evenly distributed in the gap space, the radius of sand particles is $R$, the permittivity of sand particles is $\varepsilon_p$, the ambient permittivity is $\varepsilon_a$, the streamer discharge is filamentous channel discharge, and the channel path is mostly gap sand path. It is assumed that the sand particles are evenly distributed on this discharge channel path. With every sand particle as the centre, the electrode space is divided into multiple cubic units with the same volume. The length of each unit is $h$, the volume is $h^3$, the cross-sectional area of the cube unit is $S = h^2$, and the volume of spherical sand particles is $4\pi R^3/3$; then the volume fraction of sand particles is $V_p = (4\pi R^3/3)/h^3$ (i.e., the volume of sand particles in unit space volume).
Each equivalent unit is shown in Figure 4, where EFGH is the unit area (GH length is \( h \)), and ABCD is the influence area of sand particles for local electric field (BD length is \( 2d_0 \)). Under the action of the local electric field force, electrons will be attracted by the electric field from the EG surface into the AB surface and then move from the AB surface to the \( O'KO \) surface. Except that some electrons are captured by sand particles, other electrons continue to through the sand particles and participate in the discharge.

In region EABG, the electrons do not enter the influence range of the local electric field of sand particles, and the ionization in this region is only affected by the equivalent electric field \( E_{0c} \). In region \( AO'KOB \), the electrons are affected by the local electric field of the uncharged sand particles. The ionization in this region needs to consider the effect of the distorting electric field of the sand particles, and it is assumed that the number of electrons reaching the particles \( DZ_h \), the ionization coefficient in this region is \( \alpha_i \). In region \( O'MO DC \), the sand particles have been charged, and the electrons will be affected by the local electric field of the charged sand particles. For ionization in this region, the role of the charged electric field caused by charged sand particles needs to be further considered, and it is assumed that the number of electrons from surface \( O'MO \) to CD surface is approximately equal to the number of electrons generated by the average electric field along the axis of region \( C DC' D' \) on CD surface. We assume that the ionization coefficient in the left half of area ABCD is \( \alpha' \), that in the right half of area ABCD is \( \alpha'' \), and that outside of area ABCD is \( \alpha \).

From the above analysis, it can be seen that the area ABEG and area CDHF outside area ABCD are not affected by sand particles, and the ionization coefficient in this area can be calculated as

\[
\alpha = A(v,E_{0c}) \exp\left( \frac{-\mu_1 \rho v + B(v,E_{0c})}{E_{0c}} \right). \tag{19}
\]

In this region, the positive ion generation coefficient is

\[
d'_1 = A(v,E_{0c}) \exp\left( \frac{\mu \rho v + B(v,E_{0c})}{E_{0c}} \right). \tag{20}
\]

When electrons enter the AB surface, they will be affected by the local electric field of sand particles. According to the dipole-enhanced model [24], taking \( \theta = 0 \), we can get that the external electric field of the sand sphere is

\[
E_{\text{out}} = \left( E_{0c} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + 2\varepsilon_2} \frac{2E_{0c}R^3}{r^3} \right). \tag{21}
\]

From (21), it can be obtained that the average electric field in region \( ABA'B' \) is

\[
E_1 = \int_{r} d_{0} \frac{E_{\text{out}} dr}{d_0 - R} = \int_{r} d_{0} \left( E_{0c} + \frac{2E_{0c}R^3}{r^3} \right) dr \tag{22}
\]

Let \( k = \varepsilon_1 - \varepsilon_2 \); according to Figure 3, we can derive

\[
\int_{r} d_{0} \left( E_{0c} + k \frac{2E_{0c}R^3}{r^3} \right) dr = E_{0c} \left( d_0 - R \right) + 2kE_{0c}R^3 \int_{r} d_{0} \frac{1}{r^3} dr = \left( d_0 - R - kR \frac{R^2 - d_0^2}{d_0^2} \right) E_{0c}. \tag{23}
\]

We then substitute (23) into (22) and derive

\[
E_1 = \left( 1 + \frac{kR(R + d_0)}{d_0^2} \right) E_{0c}. \tag{24}
\]

Thus, the ionization coefficient in the region \( ABA'B' \) can be derived as

\[
\alpha' = A(v,E_1) \exp\left( -\mu_1 \rho \frac{v}{E_1} \right) \exp\left( \frac{B(v,E_1)}{E_1} \right) \frac{kR(R + d_0)(B(v,E_{0c}) + \mu_1 \rho v)}{(d_0^2 + kR(R + d_0))E_{0c}} \tag{25}
\]

In this region, the positive ion generation coefficient is
\[ \begin{align*}
\bar{\alpha}_i &= A(\nu, \bar{E}_i) \exp \left( -\mu \frac{\nu}{E_i} \right) \exp \left( -\frac{B(\nu, \bar{E}_i)}{E_i} \right) \\
&= \alpha \exp \left( \frac{kRD_0 (R + d_0) (\nu + \mu \nu)}{(d_0^2 + kR(R + d_0)E_{\text{ac}})} \right). 
\end{align*} \tag{26} \]

The number of electrons produced by initial electrons \( n_0 \) moving from the curved surface \( O'KO \) to the curved surface \( O'OO' \) is

\[ \begin{align*}
\bar{\alpha}_i &= \alpha \left[ 1 - \frac{2d_0}{h} + \frac{2(d_0 - R)}{h} \right] \exp \left( \frac{kR(R + d_0)(B(\nu, E_{\text{ac}}) + \mu \nu)}{(d_0^2 + kR(R + d_0)E_{\text{ac}})} \right). 
\end{align*} \tag{28} \]

After the sand particles are charged, the average electric field in region \( C'D'C'D \) is

\[ 
\bar{E}_2 = \left[ \frac{\int_{d_0}^{d_1} (E_{out} + E_1') dr}{d_0 - R} \right] \\
= \bar{E}_1 + \frac{\int_{d_0}^{d_1} d\rho/4\pi \varepsilon_0 e^{-2} dr}{d_0 - R} \\
= \left( 1 + \frac{kR(R + d_0)}{d_0^2} + \frac{3fL_0 R}{(\varepsilon_1 + 2\varepsilon_0)d_0} \right) E_{\text{ac}}. 
\] \tag{29} \]

According to (28), the average ionization coefficient in region \( C'D'C'D \) is

\[ \begin{align*}
\bar{\alpha}_i &= \alpha \left[ 1 - \frac{2d_0}{h} + \frac{2(d_0 - R)}{h} \right] \exp \left( \frac{kR(R + d_0)(B(\nu, E_{\text{ac}}) + \mu \nu)}{(d_0^2 + kR(R + d_0)E_{\text{ac}})} \right). 
\end{align*} \tag{30} \]

It can be seen that the positive ion generation coefficient in this region is

\[ \begin{align*}
\bar{\alpha}_i &= \alpha \left[ 1 - \frac{2d_0}{h} + \frac{2(d_0 - R)}{h} \right] \exp \left( \frac{kR(R + d_0)(B(\nu, E_{\text{ac}}) + \mu \nu)}{(d_0^2 + kR(R + d_0)E_{\text{ac}})} \right). 
\end{align*} \tag{31} \]

The number of electrons produced by initial electrons \( n_0 \) moving from the curved surface \( O'MO \) to the FH plane can be calculated as

\[ \begin{align*}
n_R &= n_t \exp \left[ \alpha \left( \frac{h}{2} - d_0 \right) + \alpha'' (d_0 - R) \right] \\
&= n_t \exp \left[ \frac{h}{2} \left( \alpha \left( 1 - \frac{2d_0}{h} \right) + \alpha'' \frac{2(d_0 - R)}{h} \right) \right]. 
\] \tag{32} \]
According to (32), the average ionization coefficient in region $FHOO'$ is

$$
\alpha_2 = a \left[ \frac{1 - 2d_0}{h} + a'' \frac{(d_0 - R)}{h} \right]
$$

$$
= a \left[ 1 - \frac{2d_0}{h} + \frac{2(d_0 - R)}{h} \right] \exp \left( \frac{kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2}{(d_0^3(\epsilon_i + 2\epsilon_e) + kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2)} \right). \tag{33}
$$

The average positive ion generation coefficient in this region is

$$
\alpha_2 = a' \left[ 1 - \frac{2d_0}{h} + \frac{2(d_0 - R)}{h} \right] \exp \left( \frac{kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2}{(d_0^3(\epsilon_i + 2\epsilon_e) + kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2)} \right). \tag{34}
$$

Thus, the total number of electrons generated by the initial electron $n_0$ moving from the EG plane to the FH plane is

$$
n_R = n_0 \exp \left[ \left( \frac{h}{2} - d_0 \right) + a'' (d_0 - R) \right]
$$

$$
= n_0 \exp \left[ \frac{h}{2} \left( a \left( \frac{1 - 2d_0}{h} + a'' \frac{(d_0 - R)}{h} \right) \right) \right] \tag{35}
$$

According to (35), the average ionization coefficient under strong wind-sand environment can be derived as

$$
\alpha = \alpha' \left[ 1 - \frac{2d_0}{h} + \frac{(d_0 - R)}{h} \right] \exp \left( \frac{kR(R + d_0)(\mu \rho v + B(v, E_{0c}))}{(d_0^2 + kR(R + d_0))E_{0c}} \right)
$$

$$
+ \exp \left( \frac{kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2}{(d_0^3(\epsilon_i + 2\epsilon_e) + kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2)} \right). \tag{36}
$$

Then the average positive ion generation coefficient under strong wind-sand environment can be calculated as

$$
\alpha_i = \alpha_i' \left[ 1 - \frac{2d_0}{h} + \frac{(d_0 - R)}{h} \right] \exp \left( \frac{kR(R + d_0)(\mu \rho v + B(v, E_{0c}))}{(d_0^2 + kR(R + d_0))E_{0c}} \right)
$$

$$
+ \exp \left( \frac{kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2}{(d_0^3(\epsilon_i + 2\epsilon_e) + kRd_0(R + d_0)(\epsilon_i + 2\epsilon_e) + 3f\epsilon_iRd_0^2)} \right). \tag{37}
$$
From equation (28), we can find that $\pi, \pi_1$, and $\pi_2$ satisfy $\pi = (\pi_1 + \pi_2)$, and $\pi, \pi_1$, and $\pi_2$ satisfy $\pi_i = (\pi_{i1} + \pi_{i2})$, which can be further simplified as

$$\pi = 2A(v, E_{0v}) \frac{(d_0 - R)}{h} \exp \left\{ - (\mu \rho v + B(v, E_{0v})) \times \frac{(1 + kR(R + d_0)/d_0^2)^2 + 3f (\epsilon_i + 2\epsilon_e) d_0 \times kR(R + d_0)/d_0^4}{2(1 + kR(R + d_0)/d_0^4)(1 + kR(R + d_0)/d_0^4 + 3f (\epsilon_i + 2\epsilon_e) d_0)} \right\}.$$  

(38)

The general form of (38) with ionization coefficient can be rewritten as

$$\pi = A' \exp \left( \frac{B'}{E} \right),$$  

(39)

where

$$A' = 2A(v, E_{0v}) \frac{d_0 - R}{h}.$$  

(40)

And

$$B' = (\mu_1 \rho v + B(v, E_{0v})) \left[ 1 - \frac{\epsilon_i - \epsilon_e}{\epsilon_i + 2\epsilon_e} \left( \frac{2R}{h} \right)^3 \right] \times \frac{(1 + kR(R + d_0)/d_0^4)^2 + 3f (\epsilon_i + 2\epsilon_e) d_0 \times kR(R + d_0)/d_0^4}{2(1 + kR(R + d_0)/d_0^4)(1 + kR(R + d_0)/d_0^4 + 3f (\epsilon_i + 2\epsilon_e) d_0)}.$$  

(41)

The general form of positive ion generation coefficient can be written as

$$\pi_i = A' \exp \left( \frac{B_i}{E} \right),$$  

(42)

where

$$B_i' = (\mu \rho v + B(v, E_{0v})) \left[ 1 - \frac{\epsilon_i - \epsilon_e}{\epsilon_i + 2\epsilon_e} \left( \frac{2R}{h} \right)^3 \right] \times \frac{(1 + kR(R + d_0)/d_0^4)^2 + 3f (\epsilon_i + 2\epsilon_e) d_0 \times kR(R + d_0)/d_0^4}{2(1 + kR(R + d_0)/d_0^4)(1 + kR(R + d_0)/d_0^4 + 3f (\epsilon_i + 2\epsilon_e) d_0)}.$$  

(43)

So far, the general forms of electron ionization coefficient and positive ion generation coefficient in strong wind-sand environment have been solved.

2.3. Streamer Breakdown Criteria. On the basis of streamer discharge theory, combined with the calculation results of electron ionization coefficient and positive ion generation coefficient, we attempt to establish the analytical solution of streamer breakdown criterion to quantitatively calculate the dielectric breakdown voltage in uniform field in strong wind-sand environment.

Assuming that the number of initial electrons in the cathode is $n_0$ and the number of sand particles along the radial direction of the streamer discharge circuit is $m$, the total number of electrons generated by the initial electrons moving from the cathode to the anode along the electric field direction can be calculated as

$$n_{em} = n_0 \exp(\pi d) - \frac{q}{e} \exp(\pi mh) - \frac{q}{e} \exp(\pi (m - 1)h) - \frac{q}{e} \exp(\pi (m - 2)h) - \cdots - \frac{q}{e} \exp(\pi h)$$

$$= n_0 \exp(\pi d) - \frac{q}{e} \left[ \exp(\pi h + d) - \exp(\pi h) \right] \frac{\exp(\pi h)}{\exp(\pi h) - 1}.$$  

(44)
From (44), the following equation is defined:
\[
\frac{q \exp (\alpha (h + d)) - \exp (\alpha h)}{e \exp (\alpha h) - 1} = \gamma n_0 \exp (\alpha d),
\]
(45)
where \( \gamma \) is a constant, which takes a value between (0, 1). Let \( \exp (\alpha h) = x \). Solving (45) results in
\[
\frac{q \exp (\alpha (h + d)) - \exp (\alpha h)}{e \exp (\alpha h) - 1} = \gamma n_0 \left( \frac{\gamma}{\gamma - cf} \right)^m,
\]
(46)
where \( c = q_0/n_0 \), \( f = q/q_0 \).
Substituting (46) into (44), the total number of electrons reaching the anode is calculated as
\[
n_{em} = n_0 \exp (\alpha d) - \gamma n_0 \left( \frac{\gamma}{\gamma - cf} \right)^m.
\]
(47)

Based on the streamer theory [33], the space charge field formed by positive ions is calculated. Assuming that the ion set at the head of the electron avalanche is spherical, the positive ion density in the spherical ion set is
\[
n' = n_{em} \frac{\pi dx}{4\pi r^2} = n_{em} \frac{\pi x}{4r^2}.
\]
(48)
Then the space charge field formed by positive ions can be calculated as
\[
E_{i} = \frac{en'}{4\pi \varepsilon_{0}r} \left( \frac{4}{3} \pi r^3 \right)
\]
(49)
where \( E_{i} \) is the space charge field and \( r \) is the electron diffusion radius.

In the process of dielectric discharge under strong wind-sand environment, electrons are affected by the horizontal collision force of gas molecules and the electric field force. The ratio of mean electron velocity \( \overline{v}_e \) to electron drift velocity \( \overline{v}_E \) in the electric field direction is equal to the ratio of mean electron free path \( \overline{\lambda}_e \) to electron free path \( \overline{\lambda}_E \) in the electric field direction, so we can obtain
\[
\frac{\overline{v}_e}{\overline{v}_E} = \frac{\overline{\lambda}_e}{\overline{\lambda}_E}
\]
(50)
According to [33], energy \( eE\overline{\lambda}_e \) obtained by an electron in a uniform field is equal to the energy lost by the collision between an electron and a gas molecule. Thus, we have
\[
eE\overline{\lambda}_e = g \frac{\overline{v}_e}{\overline{v}_E} \frac{1}{2} m_e \overline{v}_e^2.
\]
(51)
The mean electron free path is given by
\[
\overline{\lambda}_e = \frac{m_e e}{c^2 K_e}.
\]
(52)
where \( g \) is the electron collision energy loss coefficient, \( K_e \) is the electron mobility, \( \nu_e \) is the electron random velocity, and \( m_e \) is the electron mass.

Combining (46) and (47), we can get
\[
\frac{\nu_e}{\overline{v}_E} = \frac{2.45}{g}.
\]
(53)
Assuming that \( \nu_e \) is submitted to Maxwell velocity distribution, we can obtain
\[
\frac{\nu_e}{\overline{v}_E} = 1.44 \frac{1}{\sqrt{g}}
\]
(54)
Substituting (49) into (46), let \( 1/2 m_e \nu_e^2 = eV \), and we can get
\[
\frac{V}{\overline{v}_E} = \frac{\overline{\lambda}_e}{1.44 \sqrt{g}}
\]
(55)
Therefore, the electron diffusion radius can be calculated as
\[
r = \sqrt{\frac{2V x_e}{\overline{\lambda}_e}}
\]
(56)
Substituting (56) into (49), we can obtain the space charge field as follows:
\[
E_{i} = \frac{e\pi (n_{em} \exp (\alpha d) - \gamma n_0 (\gamma/\gamma - cf)^m) \sqrt{1.5\sqrt{g}}}{3\pi \varepsilon_{0} \overline{\lambda}_E x_e}
\]
(57)
Substitute the empirical value \( g = 0.025 \) and let \( n_0 = 1, e = 1.6 \times 10^{-6} \), and \( \varepsilon_{0} = 8.85 \times 10^{-12} \); (57) can be written as
\[
E_{i} = 6.6 \times 10^{-10} \overline{\lambda}_e (\exp (\alpha d) - \gamma (\gamma/\gamma - cf)^m). \sqrt{\overline{\lambda}_E x_e}
\]
(58)
According to the streamer discharge theory, when the space charge field \( E_{i} \) and the external equivalent electric field \( E_{0c} \) are of the same order of magnitude, the streamer will be formed. Therefore, the streamer breakdown criterion can be expressed as
\[
E_{i} = KE_{0c},
\]
(59)
where \( K \) is the scale factor.
Substituting (58) into (59), we can get
\[
\overline{\pi} x_e = \ln \left( 3.11 \times 10^{-4} \frac{KE_{0c} \sqrt{\overline{\lambda}_E x_e}}{\overline{\lambda}_e} + \gamma (\gamma/\gamma - cf)^m \right).
\]
(60)
Then substituting (39) and (42) into (60), we can obtain
\[ A\exp\left(\frac{B'}{E}\right) x_E = \ln\left(3.11 \times 10^{-4} \frac{KE_\infty \sqrt{x_E}}{A\exp\left(-B'/E\right)} + y\left(\frac{\gamma}{\gamma - cf}\right)^m\right). \]  

In the above equation, \( x_E \) represents the spatial distance required for streamer formation.

\[ A\exp\left(-\frac{dB'}{U}\right) d = \ln\left(3.11 \times 10^{-4} \frac{KE_\infty \sqrt{x_E}}{A\exp\left(-dB'/U\right)} + y\left(\frac{\gamma}{\gamma - cf}\right)^m\right). \]  

We now substitute \( U/d \) for \( E \) in (61), and then the streamer breakdown criterion under uniform field in strong wind-sand environment can be deduced as

So far, the analytical form of streamer breakdown criterion in uniform field under strong wind-sand environment has been obtained as in (62), the breakdown voltage is taken as the implicit function of airflow and sand particles related parameters in strong wind-sand environment, and its corresponding relationship can be solved by numerical method.

3. Simulation and Discussion

The proposed physical model is calculated to obtain the variation characteristic curve of breakdown voltage with various parameters of airflow and sand through simulation. For air, we let the initial air density \( \rho_0 = 1.205 \text{kg/m}^3 \), the permittivity of the environment medium \( \varepsilon_r = 1 \), \( \gamma_s = 1.4 \), the mean free path of air molecules \( \lambda_m = 1.03 \times 10^{-5} \text{ m} \), ionization potential \( V_i = 15 \), the speed of sound \( s = 340 \text{ m/s} \), gap distance \( d = 0.01 \text{ m} \), the electron blowing away coefficient \( \mu_1 = 15 \), and the positive ions blowing away coefficient \( \mu_2 = 15000 \). For sand particles, we let the permittivity of the sand particles \( \varepsilon_i = 6 \), \( f \) is the charging coefficient of sand particles, which is the ratio of the real charge \( q \) captured by sand particles to the saturation charge \( q_0 \) of sand particles, and its range of value is within \([0, 1] \). Let the attraction coefficient of sand particle \( c = 0.06 \) and the capture coefficient of sand particle \( \gamma = 0.1 \); \( R \) is the radius of sand particles; the average size range of sand particles used in simulation is within \([0, 200] \mu\text{m} \) (referring to the operation environment of Lanzhou-Xinjiang Railway, the actual size distribution of sand particles in the surrounding environment of Lanzhou-Xinjiang Railway is within \([0, 200] \mu\text{m} \)); \( h \) represents the length of each unit of the discharge gap, which is taken to be within \([0, d] \); \( m = d/h \), indicating the number of sand particles in the discharge gap, which can reflect the concentration of sand particle; \( 2R/h \) represents the ratio of sand diameter to unit length in each unit of discharge gap, which can reflect the size of sand volume fraction \( V_p \), which is taken to be within \([0, 1] \). According to the values and ranges of the above parameters, the model is simulated and calculated to obtain the relationship curve of dielectric breakdown voltage with airflow velocity under different parameters in strong wind-sand environment. The calculation results are shown in Figures 5–7.

Figure 5 shows the variation curves of dielectric breakdown voltage with airflow velocity under different sand particle size \( R \) when the airflow velocity increases from 0 m/s to 300 m/s, the gap distance \( d = 0.01 \text{ m} \), the charging coefficient \( f = 1 \) (i.e., saturated charging), and the number of sand particles \( m = 20 \). It can be seen from Figure 4 that when the condition is in the pure airflow environment, the breakdown criterion obtained from equation (62) will transform into the streamer breakdown criterion in the pure airflow environment. The breakdown voltage gradually decreases with the increase of airflow velocity. The calculation results are consistent with the experimental results of our previous articles [27, 28]. When the charging coefficient \( f \), gap distance \( d \), and parameter \( m \) (that can represent sand concentration) remain unchanged, the breakdown voltage increases with the decrease of sand particle size \( R \). For example, when \( R = 50 \mu\text{m} \), the breakdown voltage is about 22 kV/cm at the airflow speed \( v = 30 \text{ m/s} \), while when \( R \) is 100 \mu\text{m}, 150 \mu\text{m}, and 200 \mu\text{m}, the breakdown voltage is about 17 kV/cm, 15 kV/cm, and 11 kV/cm, respectively, at the same airflow speed, which is, respectively, about 77\%, 68\%, and 50\% of the breakdown voltage value at \( R = 50 \mu\text{m} \). These results show that when \( m \) is fixed (the sand concentration remains unchanged), the sand particle size has a significant impact on the dielectric breakdown voltage in wind-sand environment, and the breakdown voltage decreases with the increase of sand particle size. The variation trend of breakdown voltage with sand particle size shows a trend similar to that in the experimental results obtained by relevant scholars in [34, 35]. The effect of sand particles on gap discharge is mainly the combined effect of the local electric field and the capture of charge. It can be seen from the simulation results that the breakdown voltage of large-size sand particles is lower than that of small-size sand particles; that is, the comprehensive impact of small-size sand particles on the breakdown voltage in strong wind-sand environment is weaker than that of large-size sand particles. The main reason for this result is that small-size sand particles have less impact on the local electric field (including before and after charging) than large-size sand particles. The impact of small-size sand particles on the breakdown voltage through the local electric field is weaker than that through the capture of charge. Therefore, the change in charge of sand particles
plays a dominant role in small-size sand particles case. The large-size sand is just the opposite; that is, the impact of large-size sand particles on the breakdown voltage through local electric field is stronger than that through capture of charge. Therefore, the distortion of local electric field caused by sand particles plays a dominant role in large-size sand particles case.

Figure 6 shows the variation curves of dielectric breakdown voltage under different $m$ (sand concentration) when the airflow velocity increases from 0 m/s to 300 m/s, the gap distance $d = 0.01$ m, the charging coefficient $f = 1$ (i.e., saturated charging), and the volume fraction $2R/h = 0.3$. As can be seen from Figure 5, when the volume fraction of sand particles $2R/h$ remains unchanged, the breakdown voltage increases with the increase of sand concentration $m$. When $m = 50$, the breakdown voltage is about 26 kV/cm at the airflow speed $v = 30$ m/s, while when $m = 40, 30$, and 10, the breakdown voltage is about 24 kV/cm, 22 kV/cm, and 17 kV/cm, respectively, at the same airflow speed, which is, respectively, about 92%, 84%, and 65% of the breakdown voltage value at $m = 50$. These results show that when the volume fraction is fixed, the sand concentration also has a significant effect on the dielectric breakdown voltage in wind-sand environment. The effect of sand concentration on the dielectric breakdown voltage is the combined effect of the local electric field of charged particles and the capture of charge. It can be seen from the simulation results that when the volume fraction remains unchanged, the dielectric breakdown voltage in high-concentration sand particles case is higher than that in low-concentration sand particles case; that is, the comprehensive impact of high-concentration sand particles on the breakdown voltage in strong wind-sand environment is stronger than that of low-concentration sand particles. The main reason for this result is that, with the increase of sand concentration, the size of sand particle decreases in the same proportion when the volume fraction remains unchanged, which can be seen from equations (1), (3), and (11). At this time, the equivalent electric field and the maximum electric field remain unchanged, but the maximum charged electric field or local electric field of charged sand particles decreases, which will reduce the local ionization coefficient of sand and suppress the occurrence of discharge. In addition, although the reduction of sand particle size decreases the charging capture capacity of a single sand particle, the increase of sand concentration rapidly increases the number of sand particles, which increases the overall charging capture capacity of sand particles, reduces the total number of discharge effective electrons, and then suppresses the development of discharge. Therefore, when the volume fraction remains unchanged, the increase of sand concentration also reduces sand particles size, which decreases the local electric field of charged sand particles and increases the overall amount of charge captured by sand particles. This comprehensive effect jointly suppresses the development of discharge and increases the breakdown voltage. Therefore, the breakdown voltage in wind-sand environment with high-concentration and small-size sand particles is much higher than that with low-concentration and large-size sand particles; that is to say, the sand dust environment with high-concentration and small-size sand particles makes it more difficult to form streamer discharge and has stronger discharge suppression action and higher breakdown voltage, which is in good agreement with the experimental results of relevant scholars in [22, 36].

Figure 7 shows the variation curves of dielectric breakdown voltage under different charging coefficient $f$ when the airflow velocity increases from 0 m/s to 300 m/s, the gap distance $d = 0.01$ m, the number of sand particles $m = 20$, and the volume fraction $2R/h = 0.2$. As can be seen from Figure 6 that the breakdown voltage increases with the increase of the charging coefficient $f$, which is similar to the change trend of literature [31], $f$ only reflects the amplitude of the breakdown voltage and does not affect the change trend of the breakdown voltage with the airflow velocity. The change trend of the breakdown voltage with the airflow velocity is basically the same under different $f$ values. When $f = 1$, the breakdown voltage is about 22 kV/cm at the airflow velocity in different sand particles sizes ($f = 1, m = 20$).
speed $v = 30$ m/s, while when $f$ is 0.8, 0.5, and 0.2, the breakdown voltage is about 20 kV/cm, 18 kV/cm, and 12 kV/cm, respectively, at the same airflow speed, which is, respectively, about 91%, 82%, and 55% of the breakdown voltage value at $f = 1$. These results show that when the sand volume fraction and concentration remain unchanged, the dielectric breakdown voltage in wind-sand conditions is affected by the charging coefficient. The influence of the charging coefficient on the dielectric breakdown voltage is mainly reflected in the comprehensive influence of the local electric field caused by charged sand particles and the amount of captured charge in the discharge space. When the sand charging coefficient increases, the captured charge by sand particles increases and the local electric field increases, which results in the increase of ionization coefficient and promotes discharge. At the same time, with the increase of sand charging coefficient, the number of electrons captured by sand particles also increases, which will reduce the total number of effective electrons and inhibit the discharge. It can be seen from the calculation results that the larger the charging coefficient, the higher the breakdown voltage; that is to say, the increase of sand charging coefficient has a stronger impact on the increase of breakdown voltage caused by the decrease of number of effective electrons than the decrease of breakdown voltage caused by the increase of local electric field caused by charged sand particles.

The above discussion shows that the dielectric breakdown voltage in strong wind-sand environment is significantly different from that in static environment. The breakdown voltage is comprehensively affected by the airflow velocity $v$, sand particle size $R$, sand particle concentration $m$, and sand charging coefficient $f$.

### 4. Conclusion

In this work, based on streamer discharge theory, we presented a physical model of wind-sand dielectric streamer discharge by analysing the influence of airflow and sand particles on discharge process in strong wind-sand environment. Six factors that influence the streamer discharge process are considered: the deflection of the major discharge path, the blowing away of some electrons and ions, the decline in gas density, the distortion of the electric field caused by sand particles, the capture of the electrons, and the electric field formed by charged sand particles. The analytical solutions of electron ionization coefficient and positive ion generation coefficient are obtained, and the dielectric breakdown criterion is proposed. The analytical form of the presented model can be used to quantitatively calculate the dielectric breakdown voltage in strong wind-sand environment. The following conclusions are drawn:

1. The variation trend of breakdown voltage with airflow velocity is in good agreement between strong wind-sand environment and pure airflow environment. The breakdown voltage decreases with the increase of airflow velocity, and the variation curve of breakdown voltage with airflow velocity under wind-sand condition is lower than that under pure airflow condition. Airflow is the main factor affecting the variation trend of breakdown voltage, and sand particles will reduce the amplitude of breakdown voltage, which is related to the sand particle size, concentration, and charging coefficient.

2. When the sand concentration is constant, the dielectric breakdown voltage increases with the decrease of sand particle size. The effect of sand particle size on dielectric breakdown voltage is the comprehensive effect of the local electric field and the capture of charge. When sand particle size is small, the capture of charge is the dominant factor, which results in the fact that the comprehensive effect is to inhibit discharge and increase the breakdown voltage. When sand particle size is large, the distortion of local electric field caused by sand particles is the dominant factor, which results in the fact that the comprehensive effect is to promote the discharge and reduce the breakdown voltage.

3. When the sand volume fraction remains unchanged, the dielectric breakdown voltage increases with the increase of sand concentration; that is, the breakdown voltage is higher at high-concentration and small-size sand particle. At this time, the influence of sand particles on the dielectric breakdown voltage is the combined effect of the local electric field of charged sand particles and the capture of charge. At high-concentration and small-size sand particles, the electric field generated by charged sand particles decreases, and the overall amount of charge captured by sand particles increases, both of which inhibit the discharge and make the breakdown voltage increase. Therefore, in wind-sand environment, the breakdown voltage with high-concentration and small-size sand particles is higher; that is, the high-concentration and small-size sand particle can restrain the development of streamer discharge in strong wind-sand environment.
(4) The dielectric breakdown voltage in wind-sand environment increases with the increase of sand charging coefficient. The influence of sand charging coefficient on dielectric breakdown voltage is the comprehensive effect of the local electric field of charged sand particles and the amount of captured charge. When the charging coefficient is small, the electric field generated by charged sand particles is the dominant factor, and the comprehensive effect is to promote discharge and reduce the breakdown voltage. When the charging coefficient is large, the amount of captured charge is the dominant factor, and the comprehensive effect is to restrain the development of discharge and increase the breakdown voltage.

(5) The presented streamer discharge model and corresponding breakdown criteria can be applied to realize the quantitative analysis and calculation of dielectric breakdown voltage in the wind-sand environment. This paper extends the classical streamer discharge theory for static gases to the gas-solid two-phase flow environment. The proposed model in this paper can provide a theoretical reference for the design and protection of external insulation in gas-solid two-phase flow environment.

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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