High-gain erbium silicate waveguide amplifier and a low-threshold, high-efficiency laser

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Abstract: Erbium-doped materials have played an important role in the fabrication of light sources used in silicon photonics. Recent studies demonstrated that erbium silicate nanowire had a high net gain attributable to its high erbium concentration and excellent material quality. We establish a more accurate and comprehensive theoretical model of erbium silicate nanowire, analyze the modeled nanowire’s properties, and optimize a high-gain erbium silicate waveguide amplifier and low-threshold, high-efficiency laser by considering upconversion, energy transfer, and amplified spontaneous emission. The simulation results and previous experimental data reported in reference showed some agreement. A proposed waveguide amplifier, based on the optimized design, displayed a gain greater than 20 dB/mm. Then, a 3.3 mW low-threshold laser with a maximum power-conversion efficiency of 50% was modeled by choosing the optimized resonator cavity and reflector. The results indicate that erbium silicate compound materials with large optical gains can serve as potential candidates for inclusion in scale-integrated amplifiers and other applications requiring lasers.

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1. Introduction

In recent years, silicon photonic technology has found application in many important functions [1-3]. Silicon-based light sources, however, continue to present puzzles to many researchers because of the indirect band properties of silicon [4]. Erbium (Er) ions emit light at 1530 nm, the standard communications wavelength; this frequency is transparent to planar silicon waveguides [5]. Consequently, Er\(^{3+}\)-based materials have inspired significant research exploring various approaches to silicon-based light sources, especially ones employing erbium-doped waveguide amplifiers in integrated optical circuits [5]. High gain is one of the most important features of devices incorporating Er\(^{3+}\)-based light sources. One of the most effective ways to produce high gain is to increase the concentration of Er\(^{3+}\). Such increased concentrations require overcoming the limitations to Er\(^{3+}\) solid solubility. Therefore, erbium silicate has recently attracted great interest for uses in small-size, high-optical-gain waveguide amplifiers because it contains an Er\(^{3+}\) concentration (\(\sim 10^{22}\) cm\(^{-3}\)) two to three orders of magnitude higher than other Er\(^{3+}\)-doped material (\(\sim 10^{16}\) cm\(^{-3}\) to \(10^{20}\) cm\(^{-3}\)) [6-12]. In addition, yttrium (Y) and ytterbium (Yb) cations, because of their similar ionic radii, are usually added to the erbium silicate structure to substitute for Er\(^{3+}\) in the silicate lattice and to prevent neighboring Er\(^{3+}\) from causing cooperative upconversion (CU) [11-15]. Furthermore, Yb\(^{3+}\) can act as a sensitizer because the capacity of Yb\(^{3+}\) to pump light is one order of magnitude higher than the absorption cross section of Er\(^{3+}\) at a 980 nm pump wavelength [13].

Accordingly, Suh \textit{et al.} [12] used the ion-beam sputter method to prepare an Er–Y silicate thin-film optical-waveguide amplifier, which produced population inversion of approximately 60\% and a 0.5 dB/cm net gain at a laser-excitation wavelength of 1480 nm. Wang \textit{et al.} [11], [13] demonstrated a two-orders-of-magnitude photoluminescence enhancement of Er–Yb/Y silicate by optimizing the concentration of Er, Yb, and Y using the sol–gel method. Strip-loaded, channel, and hybrid-structure Er–Yb/Y silicate-compound waveguide amplifiers, prepared with attention to their material components and structure optimization, achieved internal gains of 1.9 dB [16-19]. Research teams have not yet produced the high-gain optical-waveguide amplifiers the theories predict. In principle, so-called quenching and energy transfer upconversion effects caused by such high Er\(^{3+}\) concentration result in the reduction of the photostable Er\(^{3+}\) population and the loss of the pump, ultimately affect the luminous efficiency. And in technology, two main technical factors account for this difficulty. The first is the presence of large waveguide transmission losses up to 8 dB/cm [13]. The Er–Yb/Y silicate thin films usually assume the polycrystalline state after annealing, which results in large light-transport scattering losses. Side-wall roughness also is generated in the waveguide etching process. The second factor is that current pump lasers have difficulty in producing the high pump-power density required for high-density Er\(^{3+}\) population inversion.

Systematic studies, taking into account these research results, have examined a novel single-crystal core-shell-structure erbium chloride silicate (ECS) nanowire [20-25], Pan \textit{et al.} [20] first adopted a bottom-up chemical-vapor-deposition method to prepare single-crystal ECS nanowires. These nanowires, because of their single-crystal characteristics, were almost free of defects. Furthermore, the nanowire structure did not require waveguide etching.
Therefore, transmission losses could be reduced greatly. In contrast, these ECS nanowire samples presented the longest fluorescence lifetimes of all Er\(^3+\) compound materials with Er\(^3+\) concentrations higher than \(10^{22}\) cm\(^{-3}\) [21]. Such lifetimes provided significantly lower required pumping densities to achieve population inversion and high gain. In 2017, Sun \textit{et al.} [25] produced a single-crystal ECS nanowire that exhibited a large unit net gain of 124.5 dB/cm at 1532 nm, which was two orders of magnitude higher than existing reported values. The high-quality single-crystalline ECS structures allow the reduction of quenching effects despite high erbium-concentration, so leading to such large net material gain. Most reported work has focused on experimental results without providing systematic theoretical support. In addition, the important advance of employing these materials in a laser has yet to be achieved.

This paper presents a comprehensive theoretical model of ECS nanowires, tests of the model, and systematic analysis of the experimental results. The model takes into account upconversion, energy transfer, and amplified spontaneous emission (ASE) to make the simulation more accurate. Several analyses of the amplification properties of ECS nanowires show the basic consistency of the simulation and previous experimental results in [25]. Then, a proposed Er–Yb silicate nanowire amplifier, based on the optimization of various parameters, exhibits a high gain of over 20 dB/mm. Finally, the design of a 3.3 mW low-threshold laser with a maximum power-conversion efficiency of 50% incorporates the optimized resonator cavity and reflector. The comprehensive theoretical model provides theoretical support and optimization guidance for the future production of high-performance erbium silicate amplifiers and lasers. The possibilities presented by amplifiers and lasers using Er/Er-Yb silicate nanowire indicate that erbium silicate compound materials with significant optical gains are candidates for future scale-integrated light-source applications.

2. High-gain Erbium silicate amplifier

2.1 Theoretical models

Figure 1 shows the multiple-energy-levels model of an Er-Yb silicate nanowire system that uses 980 nm pumping. It can be concluded that the emission from \(4^F_7/2\), \(4^S_3/2\) or \(2^H_{11/2}\) and \(4^F_9/2\) are at a relative low level [26], so their radiative transition can be relatively negligible. Furthermore, \(4^F_{9/2}\), \(4^S_{5/2}\) and \(2^H_{11/2}\) energy levels are the intermediate levels of the upconversion effects, which have little influence on the luminescent levels. For simplification, the main consideration in the model concentrates on the initial and last transition energy levels, corresponding to the cooperative upconversion and the cross relaxation. In summary, this model uses Er\(^3+\) and Yb\(^3+\) structures with five and two energy levels, respectively (The virtual line representations are ignored). The five Er\(^3+\) levels—\(4^I_{15/2}\), \(4^I_{13/2}\), \(4^I_{11/2}\), \(4^I_{9/2}\), and \(4^F_{7/2}\)—are represented by \(N_1\), \(N_2\), \(N_3\), \(N_4\), and \(N_5\), respectively. Likewise, the average populations of the two Yb\(^3+\) levels—\(2^F_{7/2}\) and \(2^F_{5/2}\)—are represented by \(N_1^{\text{Yb}}\) and \(N_2^{\text{Yb}}\), respectively. In the chloride hosts, Er\(^3+\) can have longer upper state lifetimes because of low phonon energies. Therefore, for \(4^I_{9/2}\) and \(4^I_{11/2}\), not only non-radiative decays, but also radiative transitions play the role. There exists a branching ratio to describe the radiative and non-radiative lifetimes. The simulation involves two types of equation sets. The rate equation describes the population dynamics of Er and Yb ions on each level. The propagation equation describes the evolution of copropagating and contrapropagating components of signal, pump, and ASE along the waveguide. The rate and propagation equations serve primarily to form the theoretical foundation for waveguide amplifiers and lasers [27,28].
This explanation of the energy-levels diagram in Fig. 1 supports writing the steady-state rate equations for the Er-Yb silicate nanowire system. Thus, the equations for Er$^{3+}$ are

\[
\begin{align*}
\frac{dN_1}{dt} &= -R_{13}N_1 - W_{12}N_1 + W_{21}N_2 + A_{21}N_2 + C_2N_2^2 + C_3N_3 - C_{14}N_4N_4 + R_{31}N_3 \\
&\quad + A_{41}N_4 + A_{31}N_3 - K_uN_2^3N_1 = 0 \\
\frac{dN_2}{dt} &= W_{12}N_1 - W_{21}N_2 - A_{21}N_2 + A_{32}N_3 - 2C_2N_2^2 + 2C_{14}N_4N_4 = 0 \\
\frac{dN_3}{dt} &= R_{31}N_1 - A_{32}N_3 - 2C_3N_3^2 + A_{13}N_4 - R_{31}N_3 - A_{31}N_3 + K_uN_2^3N_1 = 0 \\
\frac{dN_4}{dt} &= -A_{41}N_4 + C_2N_2^2 + C_3N_3 - C_{14}N_4N_4 - A_{41}N_4 = 0 \\
\frac{dN_5}{dt} &= C_1N_1^2 - A_{11}N_1 = 0 \\
N_1 + N_2 + N_3 + N_4 + N_5 &= N_{Er}
\end{align*}
\]

The equations for Yb$^{3+}$ are

\[
\begin{align*}
\frac{dN_1^{Yb}}{dt} &= -R_{12}^{Yb}N_1^{Yb} + R_{21}^{Yb}N_2^{Yb} + A_{21}^{Yb}N_2^{Yb} + K_uN_2^{Yb}N_1 = 0 \\
\frac{dN_2^{Yb}}{dt} &= R_{12}^{Yb}N_1^{Yb} - R_{21}^{Yb}N_2^{Yb} - A_{21}^{Yb}N_2^{Yb} - K_uN_2^{Yb}N_1 = 0 \\
N_1^{Yb} + N_2^{Yb} &= N_{Yb}
\end{align*}
\]

Here, $\tau_{ij}$ represents the lifetime between levels $i$ and $j$, describes the spontaneous emission and nonradiative relaxation probability. $C_2$ and $C_3$ are the first- and second-order cooperative upconversion coefficients, $C_{14}$ is the Er$^{3+}$ cross-relaxation coefficient, $K_u$ is the Yb$^{3+}$-to-Er$^{3+}$ energy-transfer coefficient, and $N_{Er}$ and $N_{Yb}$ represent the Er$^{3+}$ and Yb$^{3+}$ concentrations, respectively. The stimulated emission and absorption transition rates of signal and pump wavelength, $W_{ij}$ and $R_{ij}$, respectively, are given by
\[
W_{12} = \frac{\sigma_{12}(v_j)}{A_s h v_s} \Gamma_s \phi_s(z) + \sum_{j=1}^{M} \frac{\sigma_{12}(v_j)}{A_s h v_j} \times \Gamma_s \left[ P_{\text{ASE}}^s(z, v_j) + P_{\text{ASE}}^s(z, v_j) \right],
\]
\[
W_{21} = \frac{\sigma_{21}(v_i)}{A_s h v_s} \Gamma_s \phi_s(z) + \sum_{j=1}^{M} \frac{\sigma_{21}(v_i)}{A_s h v_j} \times \Gamma_s \left[ P_{\text{ASE}}^s(z, v_j) + P_{\text{ASE}}^s(z, v_j) \right],
\]
\[
R_{31} = \frac{\sigma_{31}(v_i)}{A_s h v_p} \Gamma_p \phi_p(z),
\]
\[
R_{12}^{vb} = \frac{\sigma_{12}^{vb}(v_i)}{A_s h v_p} \Gamma_p \phi_p(z),
\]
\[
R_{21}^{vb} = \frac{\sigma_{21}^{vb}(v_i)}{A_s h v_p} \Gamma_p \phi_p(z),
\]

where \( A_s \) is the waveguide-core cross-sectional area, \( h \) is Planck’s constant, \( \sigma_{ij}(v_{s,p}) \) represents the absorption and emission cross sections for \( \text{Er}^{3+} \) and \( \text{Yb}^{3+} \) between levels \( i \) and \( j \) for the pump and signal frequencies \( (v_{s,p}) \), respectively. The ASE noise can be calculated by discretizing the \( \text{Er}^{3+} \) continuous absorption and emission spectra into \( M \) frequency slots having widths of \( \Delta v_j \) and centers at frequencies \( v_j \). In these expressions, \( P_s(z), P_p(z), \) and \( P_{\text{ASE}}^s(z, v_j) \) represent the signal, pump, and forward–backward ASE powers at the frequencies \( v_i, v_p, \) and \( v_j \), respectively. Also, \( \Gamma_s \) and \( \Gamma_p \), representing the confinement factors at the pump and signal wavelengths, are defined by
\[
\Gamma_{s,p} = \int \int \Psi_{s,p}(x,y) g_{s,p}(x,y) \, dx \, dy,
\]

where \( g_{s,p}(x,y) \) is the normalized \( \text{Er}^{3+} \) transverse density profile and \( \Psi_{s,p}(x,y) \) is the normalized transverse intensity distribution function of the signal and pump light. Here, \( g_{s,p}(x,y) \) is assumed to be 1 when \( \text{Er}^{3+} \) and \( \text{Yb}^{3+} \) are distributed uniformly in Er-Yb silicate. In this case, \( N_i \) is \( z \)-independent and changes with the direction of propagation in the \( z \) direction. Thus, \( N_i \) can be scaled down by \( N_i(z) \).

The emission cross section of \( \text{Er}^{3+} \) between \( ^4I_{15/2} \) and \( ^4I_{11/2} \) at the pump frequency is nearly zero, and the population of the \( \text{Er}^{3+} \) excited-state energy level \( ^4I_{11/2} (N_3) \) is far less than the \( \text{Er}^{3+} \) ground-state energy level \( ^4I_{15/2} (N_1) \). Therefore, the transitions between energy levels represented by the dotted lines in Fig. 1 can be neglected.

The propagation equations for the pump, signal, and forward–backward ASE powers are described by
They operate under the boundary conditions

\[ P_p(0) = P_{p0}, \]
\[ P_s(0) = P_{s0}, \]
\[ P_{\text{ASE}}(0, v_j) = 0, \]  \hspace{1cm} \text{and}  \hspace{1cm}
\[ P_{\text{ASE}}(L, v_j) = 0 \left( j = 1, 2, \ldots, M \right). \]  \hspace{1cm} \text{(10)}

where \( \alpha(v_{v_p}) \) are the propagation losses at the pump and signal wavelengths, respectively, \( L \) is the waveguide length, and \( m \) is the number of guided modes propagating at the signal wavelength. \( P_{p0} \) and \( P_{s0} \) are the input pump and signal power, respectively.

A combination of the rate and propagation equations allows the internal signal–optical gain \( G \), noise figure \( NF \), and total forward–backward ASE power for the Er-Yb silicate nanowire amplifier to be written as

\[ G(z)(dB) = 10 \log \left[ \frac{P(z)}{P(0)} \right], \]  \hspace{1cm} \text{(11)}
\[ NF(z)(dB) = 10 \log \left[ \frac{1}{G(z)} + \frac{P_{\text{ASE}}(z, v_j)}{G(z)h v_j \Delta v} \right], \]  \hspace{1cm} \text{(12)}

and

\[ P_{\text{ASE}}(z) = \sum_{j=1}^{M} P_{\text{ASE}}^s(z, v_j). \]  \hspace{1cm} \text{(13)}

### 2.2 Key parameters for Er/Er-Yb silicate nanowire simulations

Reference [25] reported only the experimental testing of the signal enhancement and transmission characteristics, including the absorption spectrum, of ECS nanowires. Some key parameters, such as the emission cross-sectional spectrum, Er\(^{3+}\) energy-levels lifetime, and Er\(^{3+}\) upconversion coefficient, remained unexamined in this research. The advancement of understanding of ECS nanowires would benefit from further theoretical and experimental data support. In the simulation, the McCumber theory and Forster–Dexter energy-transfer model,
combined with the experimental data of ECS absorption spectra from [25], were applied to support systematic calculations of the key parameters of ECS compounds.

Determinations of the energy-levels lifetime and emission and absorption cross sections of Er\(^{3+}\) are important for running ECS simulations. The true modal absorption coefficient \(\alpha_m\) of an ECS nanowire can be obtained by extrapolating the good linear dependence between the absorption coefficient and probe power down to the zero-power level. This extrapolation should use the experimental absorption spectrum of a single ECS nanowire obtained from the transmission experiment with a \(-15\) dBm launched infrared-signal power input [25]. The Er\(^{3+}\) absorption cross section \(\sigma_{12}\) can be extracted from \(\sigma_{12} = \alpha_m / \Gamma_s \ell_n\ell_p\), which corresponds to the modal absorption coefficient definition when the whole population is in the ground state. The resulting ECS nanowire absorption cross-sectional spectrum is shown in Fig. 2.

The Er\(^{3+}\) emission cross section \(\sigma_{21}\) can be calculated from the absorption cross section using the McCumber theory [29]. Thus,

\[
\sigma_{21} = \sigma_{12} e^{-2\pi \lambda_0 / 2c\ell_p / \hbar k_B T},
\]

where \(k_B\) is the Boltzmann constant, \(c\) is the velocity of light in a vacuum, \(T\) is the room temperature (300 K), and \(\lambda_0\) is the peak wavelength of the emission spectrum, which, in this case, was 1532 nm. The calculated ECS nanowire emission cross-sectional spectrum also appears in Fig. 2. The McCumber theory asserts that, near the peak wavelength (1525 nm to 1540 nm), the absorption cross section of Er\(^{3+}\) is basically equal to the emission cross section, as the inset of Fig. 2 shows. In contrast, however, the absorption cross section is larger than the emission cross section within the wavelength range of 1400 nm to 1525 nm, and the absorption cross section is smaller than emission cross section within the wavelength range of 1540 nm to 1650 nm.

![Fig. 2. ECS nanowire absorption and emission cross-sectional spectra from 1400 nm to 1600 nm. The inset depicts a detailed view of the gray highlighted region of the spectra from 1525 nm to 1540 nm.](image)

The lifetime of a given energy level is the time constant describing the exponential decay of ions from that level. The inverse luminescent lifetime can be written as a sum of inverse lifetimes which represent radiative and non-radiative decay, as following equation:

\[
\frac{1}{\tau} = \frac{1}{\tau_{rad}} + \frac{1}{\tau_{non-rad}},
\]

(15)
The McCumber theory also supports the conclusion that the Er\(^{3+}\) energy-level \(4I_{13/2}\) radiative lifetime \(\tau\) can be calculated by integrating the emission cross-sectional spectra [30] using

\[
\frac{1}{\tau_{\text{rad}}} = \frac{8\pi n^2}{c^2} \int v^2 \sigma_{21}(v) dv,
\]

where \(n\) is the refractive index of the ECS nanowire. In addition, the probability of non-radiative decay depends on the phonon energy of the parent material. The phonon energy in the Er silicate material is low, resulting in the small probability of non-radiative decay. Thus, the calculated Er\(^{3+}\) energy level \(4I_{13/2}\) lifetime is nearly 5 ms. For upper state lifetimes, it can be inferred from [25] that the \(4I_{9/2}\) and \(4I_{11/2}\) lifetimes could not be very long, because of such high 1.53 \(\mu\)m gain (\(4I_{13/2}\) to \(4I_{15/2}\)) reported. Reference [5] showed that the \(4I_{9/2}\) and \(4I_{11/2}\) lifetimes is about \(-\mu\)s magnitude, 1 \(\mu\)s and 100 \(\mu\)s, respectively.

What’s more, it is known that the excited state lifetime is also effected by the quenching effect. However, the critical quenching concentration in ECS structures remains at a high level because the high-quality single-crystalline property allows the reduction of quenching effects despite high erbium-concentration. Therefore, the reduction of the excited state lifetime caused by concentration quenching effect can be relatively negligible in such structures.

The Er\(^{3+}\) upconversion coefficient is also a critical parameter in these simulations. Upconversion and cross-relaxation are well-known essential processes of energy transfer. The Forster–Dexter energy-transfer model for dipole–dipole interactions can be applied to determine the upconversion coefficient, cross-relaxation coefficient, and Yb\(^{3+}\)-to-Er\(^{3+}\) energy-transfer coefficient. Research has suggested that the dipole–dipole model was adequate for upconversion and Er–Yb energy-transfer cases [31, 32]. The transition probability rate can be described by

\[
P_{\text{da}} = \frac{1}{\tau_d} \frac{3\hbar c^4}{4\pi n^4 R_{\text{da}}} \int \frac{f_D(E) F_s(E)}{E^4} dE,
\]

where \(\tau_d\) is the radiative lifetime of the appropriate donor level, \(Q_a\) is the absorption cross section, \(R_{\text{da}}\) is the donor–acceptor separation, \(f_D(E)\) and \(F_s(E)\) are the normalized line shapes of the donor-emission and acceptor-absorption bands, respectively, and \(E\) is the dipole energy. Furthermore [33], defines \(R_0\), the critical interaction distance, such that, when \(R_{\text{da}} = R_0\). \(P_{\text{da}}\tau_d = 1\). The formula can be rewritten in terms of \(R_0\). Thus,

\[
P_{\text{da}} = \frac{R_0^6}{\tau_d R_{\text{da}}^6}.
\]

The simple case of the nearest-neighbor interaction could be used with the ECS nanowire; then, only those excited donors that had acceptors as their nearest neighbors were considered. In this case, \(R_{\text{da}}\) was chosen as the distance to the nearest neighbor with a probability of 1 – 1/e. Using a random distribution enables the energy-transfer coefficient to be written as

\[
C_y = \frac{17.6 R_0^6 (N_a + N_d)}{\tau_d},
\]

where \(N_a\) and \(N_d\) are the acceptor and donor concentrations, respectively. The emission and absorption spectra data can be used to determine \(R_0\) at a value of 1.02 nm for the Er\(^{3+}\)-to-\(4I_{13/2}\) upconversion energy transfer. Some experimental data in similar materials that could support the calculated values [27, 28, 34, 35]. The experimental values and calculated values are in the
same order of magnitude. To sum up, such approximate formula could basically calculate these energy transfer parameters.

The energy transfer upconversion at high concentrations mainly includes cooperative upconversion, excited state absorption (ESA), and cross relaxation. For these upconversions, the ESA can be relatively negligible because of the small Er$^{3+}$ absorption cross section for 980nm pump. And the cooperative upconversion plays a leading role for the related $^4I_{13/2}$ and $^4I_{11/2}$ level. To sum up, the factors considered in the model for energy transfer upconversion mainly based on the cooperative upconversion and cross relaxation.

Table 1 summarizes the parameters used for modeling Er/Er-Yb silicate nanowire in the preceding calculations and analysis.

The waveguide structure featured a single ECS nanowire waveguide with 1 µm diameter; the 1.4-µm-wide fiber supports single-mode HE$_{11}$ propagation at telecommunication wavelengths around 1530 nm. The refractive indices of the fiber and ECS nanowire were assumed to be 1.468 and 1.640, respectively. The use of COMSOL software to solve the fundamental modes of the ECS nanowire and fiber at both pump and probe signal wavelengths yielded calculated confinement factors of 0.972 and 1.03 for signal wavelengths of 1532 nm and 980 nm, respectively. The confinement factor for pump greater than 1 means that the mode gain is greater than material gain: because the propagation of the mode is slow, the waveguide mode does feel more gain than the plane wave in the uniform medium [36].

| Parameters                                    | Symbol | Values                                |
|-----------------------------------------------|--------|---------------------------------------|
| Er$^{3+}$ concentration                        | $N_{E3}$ | $(0.1 - 2) \times 10^{22}$ cm$^{-3}$ |
| Yb$^{3+}$ concentration                        | $N_{Yb}$ | $(0 - 1.44) \times 10^{22}$ cm$^{-3}$ |
| signal wavelength                             | $\lambda_s$ | 1532 nm                             |
| pump wavelength                                | $\lambda_p$ | 980 nm                              |
| Er$^{3+}$ emission cross section at 1532 nm    | $\sigma_{21}$ | $1.24 \times 10^{-20}$ cm$^2$      |
| Er$^{3+}$ absorption cross section at 1532 nm  | $\sigma_{12}$ | $1.24 \times 10^{-20}$ cm$^2$      |
| Er$^{3+}$ absorption cross section at 980 nm   | $\sigma_{13}$ | $2.58 \times 10^{-21}$ cm$^2$      |
| Yb$^{3+}$ emission cross section at 980 nm    | $\sigma_{Yb}^{21}$ | $1.2 \times 10^{-20}$ cm$^2$      |
| Yb$^{3+}$ absorption cross section at 980 nm   | $\sigma_{Yb}^{12}$ | $1.2 \times 10^{-20}$ cm$^2$      |
| Er$^{3+}$ emission lifetime at $^4I_{13/2}$ level | $\tau_{21}$ | 5 ms                                 |
| Er$^{3+}$ nonradiative lifetime at $^4I_{11/2}$ level | $\tau_{32}$ | 100 µs                              |
| Er$^{3+}$ radiative lifetime at $^4I_{11/2}$ level | $\tau_{31}$ | 100 µs                              |
| Er$^{3+}$ nonradiative lifetime at $^4I_{9/2}$ level | $\tau_{43}$ | 1 µs                                |
| Er$^{3+}$ radiative lifetime at $^4I_{9/2}$ level | $\tau_{41}$ | 1 µs                                |
| Yb$^{3+}$ emission lifetime at $^2F_{5/2}$ level | $\tau_{Yb}^{21}$ | 2 ms                                |
| first-order cooperative upconversion coefficient of Er$^{3+}$ | $C_2$ | $(0.13 - 1.14) \times 10^{-16}$ cm$^3$ / s |
| second-order cooperative upconversion coefficient of Er$^{3+}$ | $C_3$ | $(0.13 - 1.14) \times 10^{-16}$ cm$^3$ / s |
| Er$^{3+}$ cross-relaxation coefficient         | $C_{14}$ | $(0.91 - 7.98) \times 10^{-15}$ cm$^3$ / s |
| Yb$^{3+}$-to-Er$^{3+}$ energy-transfer coefficient | $K_{uf}$ | $2.257 \times 10^{-16}$ cm$^3$ / s       |
| propagation loss                              | $\alpha(v_{s,p})$ | nearly 0                             |
The energy model and parameters were determined. Then, the simulation and experimental values of the gain and threshold pump power of the 56.2-µm-long ECS nanowire waveguide amplifier were compared. The results, appearing in Table 2, show that the simulation and experimental data display basic agreement. These results verify the correctness of the parameter determination and modeling.

| Parameters                  | Simulation Data                | Experimental Data in [25] |
|-----------------------------|-------------------------------|---------------------------|
| unit gain                   | 12.5 dB/mm at 75.6 mW         | 12.45 dB/mm at 75.6 mW    |
| threshold pump power        | 50 mW                         | 55 mW                     |

2.3 Analyses and Optimizations

Figure 3 shows the signal gain of the ECS nanowire waveguide amplification as a function of waveguide propagation distance for a 1532 nm input signal with power of 4 µW pumping at different pump powers for a wavelength of 980 nm. The signal gain of ECS nanowire increased nearly linearly with the increase of waveguide transmission length before reaching a maximum value, as shown in Fig. 3(a). Then, the gain began to decrease with further increases in transmission length, as shown in Fig. 3(b). These results support the conclusion that, for the given pump power, the maximum gain of the amplifier corresponds to an optimum pump length. When the value of \( L \) exceeded this optimized value, the gain dropped very quickly because, in this case, the nanowire completely absorbed the pump. Here, too, the remainder of the nanowire was not pumped but, rather, absorbed the amplified signal while there was no population inversion. As a result, the optimum pump length depended on the pump power and increased with increases in pump power. In contrast, the signal gain also could be enhanced further with an increase in pump power at the same transmission length because of enhanced population inversion. Figure 3(a) illustrates that the signal gain was 0.7 dB at a pump power of 75.6 mW when the propagation distance reached 56.2 µm. Thus, the unit net gain was calculated to be 12.50 dB/mm, which accords with the previous experimental data of 12.45 dB/mm reported in [25]. Figure 3(b) shows that the optimum pump length was nearly 600 µm at a pump power of 75.6 mW. Thus, the experimental ECS nanowire should be lengthened to 600 µm. The gain, at this length, could reach 4 dB, which is almost six times higher than 0.7 dB. Furthermore, the gain improved to 10 dB at 1.1 mm when the pump power was increased to 100 mW.

![Fig. 3. Signal net gain vs (a) short propagation distance and (b) long propagation distance for different input pump powers from 50 mW to 100 mW. The inset in (b) shows signal gain vs propagation distance with pump power of 75.6 mW. The optimum pump length is 600 µm. The gain can reach 4 dB when \( N_{th} = 1.62 \times 10^{22} \text{cm}^{-3} \) and \( P_p = 75.6 \text{ mW} \).](image-url)
Figure 4 shows the signal gain of the ECS nanowire waveguide amplification as a function of input pump power at the waveguide length of 56.2 μm. The results show that the amplifier gain first increased steeply with the pump power, but as the pump power continued to increase, the gain increased slowly or even saturated. These results suggest that the signal light power increases significantly with increases in pump power and that the increase in signal power further enhances $\text{Er}^{3+}$ stimulated radiation. Thus, this effect causes the $\text{Er}^{3+}$ concentration in the excited state to reduce more rapidly, which, in turn, restrains the further amplification of the signal, resulting in gain saturation. Therefore, the pump power must be chosen carefully. Moreover, demonstrations can produce a threshold pump power of about 50 mW in an ECS nanowire; this result is close to the previous experimental data (near 55 mW) reported in [25].

The $\text{Er}^{3+}$ concentration has a large impact on the optimization of the ECS nanowire amplifier. The process of $\text{Er}^{3+}$ cooperative upconversion become increasingly obvious for high $\text{Er}^{3+}$ concentrations. This trend merits attention. Figure 5 shows the signal gain of the ECS nanowire waveguide amplification as a function of $\text{Er}^{3+}$ concentration at 75.6 mW pump power. These results demonstrate that the gain increases with increases in $\text{Er}^{3+}$ concentration at low levels and decreases with further increases in $\text{Er}^{3+}$ concentration. This outcome occurs because the $\text{Er}^{3+}$ cooperative upconversion effect intensifies at high $\text{Er}^{3+}$ concentrations and the energy exchange between the neighboring $\text{Er}^{3+}$ generates nonradiative transitions that result in $\text{Er}^{3+}$ concentration quenching. Therefore, the population of optically active $\text{Er}^{3+}$ is reduced, which limits optical amplification. The optimum $\text{Er}^{3+}$ concentration in ECS nanowire obtained from the simulation analysis produces a maximum gain of 0.97 dB when the $\text{Er}^{3+}$ concentration is reduced to $2.1 \times 10^{21} \text{ cm}^{-3}$. The optimal unit gain of 17.3 dB/mm is nearly 40% higher than the unit gain of experimental ECS nanowires reported by [25] to be 12.45 dB/mm.
The gain of amplifiers may be improved further by introducing Yb$^{3+}$ to form Er-Yb silicate nanowires in which the total concentration, pump light, and transmission length remain unchanged. This configuration presents two advantages. First, the introduction of Yb$^{3+}$ dilutes the Er$^{3+}$ concentration and suppresses the cooperative upconversion process of Er$^{3+}$. Second, Yb functions as an effective sensitizer for Er$^{3+}$ at the 980 nm excitation wavelength because the absorption cross section of Yb$^{3+}$ to pump light is one order of magnitude higher than that of Er$^{3+}$. Therefore, the introduction of Yb$^{3+}$ increases the absorption efficiency of pump light. Both the Yb and Er concentrations and the ratio of the Yb and Er concentrations make important contributions to the amplifier gain optimization of Er-Yb silicate nanowires.

Figure 6(a) shows that the gain first increases and then decreases with increases in the Yb:Er ratio. The inhibition of upconversion is not obvious when the Yb$^{3+}$ concentration is too low, and the luminous efficiency of Er$^{3+}$ is reduced when the Yb$^{3+}$ concentration is too high. As a result, the ratio of Er$^{3+}$ to Yb$^{3+}$ must be set near an optimum value. The optimum Yb:Er ratio is about 2.2:1. The unit net gain at this level can reach 21.0 dB/mm (1.19 dB for 56.2 μm), which is nearly 70% higher than the 12.45 dB/mm reported in [25] for experiments with ECS nanowires. In addition, Fig. 6(b) shows that the optimum Yb:Er ratio varies with the propagation distance. These variations occur because the absorption efficiency of nanowires to pump light differs for different components. When the Yb:Er ratio is near 2.2:1, the conversion efficiency of nanowires to pump light is the highest. Thus, the gain changes fastest with propagation distance (pump length), as shown in the inset, but the optimum pump length is also relatively short because the pump light is absorbed more quickly. The general length, taking into account the small optimum device sizes, should be controlled within 600 μm. Thus, in this case, the Yb:Er ratio of 2.2:1 can be the optimal choice. It is worth noting that the Yb$^{3+}$ in the excited state will transfer its energy to another Er$^{3+}$ in the excited state in the Er and Yb co-doped system. Such energy transfer upconversion that Yb$^{3+}$ directly participate involves Er$^{3+}$ $^4I_{13/2}$, $^4I_{11/2}$ and $^4F_{9/2}$ levels. Here, the theoretical optimized Yb$^{3+}$ doping concentration is at a relative low level (2.2:1). In this case, the energy exchange between Er$^{3+}$ and Yb$^{3+}$ still based on the Er$^{3+}$ in the ground state, so the energy transfer upconversion that Yb$^{3+}$ directly participate is not obvious to be taken consideration.
Fig. 6. Signal net gain vs Yb:Er ratio (a) and propagation distance (b). (a) The optimum Yb:Er ratio is 2.2:1. The gain can be improved to 1.19 dB (210 dB/cm), where $L = 56.2 \, \mu m$, $P_p = 75.6 \, mW$, and $N_{total} = 1.62 \times 10^{22} \, cm^{-3}$. (b) Signal net gain vs propagation distance at Yb:Er ratios from 1:0 to 1:5 at input pump power of 75.6 mW. The inset in (b) shows a magnified view of the red dashed region from 0 μm to 60 μm.

3. Low-threshold and high-efficiency erbium silicate laser

Integrated lasers in silicon photonics platforms have been the objects of intensive research because silicon itself is a poor light emitter [37,38]. Er$^{3+}$-doped material lasers have the advantage of offering straightforward, monolithic fabrication that yields high-performance narrow-linewidth lasers [40]. However, as previously noted, Er silicate lasers, like Er$^{3+}$-doped material lasers, are rarely investigated because high-gain optical Er silicate waveguide amplifiers, outlined in theory, have not been produced for use. Here, using the optimization results of amplifiers, we design and simulate a small-size, low-threshold Er–Yb silicate compound nanowire laser. The output power of this model can be as high as 50 mW with a power-conversion efficiency near 50%. The threshold pump power can be as low as 3.3 mW, which is nearly five times lower than the values currently reported.

3.1 Theoretical models

The radial size of Er silicate compound nanowires has been approximately equal to the 1530 nm lasing wavelength of the near-infrared nanowire laser. The limiting conditions of the light and electric field within the nanowire, reflectivity of the end surface of the nanowire, and surface scattering will affect the loss of the resonator directly. Then, this loss will affect the lasing threshold and output power. Therefore, the design parameters of effective structures, such as Fabry–Perot (F–P) resonators and distributed Bragg reflector/distributed feedback (DBR/DFB) microcavities, are critical for obtaining effective optical feedback, which is the key to lasing [38]. A low-threshold Er–Yb silicate laser can be designed and simulated using the aforementioned optimization results for determining Er–Yb silicate nanowire amplifier parameters.
The schematic configuration of the Er-Yb silicate nanowire waveguide laser is shown in Fig. 7. The resonator is considered to be a single nanowire attached to dielectric mirrors M1 and M2 that exhibit partial reflectance at the signal and pump wavelengths, namely, the 1532 nm and 980 nm schemes, respectively. The pump is injected from the right side, and laser output occurs on the left side. L is the cavity length. The cavity has been modeled to be double-pass. Thus, \( P^+ \) and \( P^- \), the copropagating and contrapropagating components for signal and pump in the resonator, respectively, represent the overall forward and backward transmission of the optical evolution.

The propagation equations must be changed in two directions because of the copropagation and contrapropagation for signal and pump in the resonator. Thus,

\[
\frac{dP^+}{dz} = \mp \Gamma_s \left[ \sigma_{21} N_2(z) - \sigma_{12} N_1(z) \right] P^+ + \mp \alpha(v_s) P^+ (z) \\
\frac{dP^-}{dz} = \pm \Gamma_s \left[ \sigma_{21} N_2(z) - \sigma_{12} N_1(z) \right] P^- + \pm \alpha(v_s) P^- (z) \\
\frac{dP_{\text{ASE}}}{dz} = \pm \Gamma_s \sigma_{21} (v_j) N_2(z) P_{\text{ASE}}(z, v_j) \\
\pm mhv_j \Delta v_s \Gamma_s (v_j) \sigma_{21} (v_j) N_2(z) (j = 1, 2, \ldots, M) \\
\tag{20}
\]

The coupled rate and propagation equations must be solved simultaneously with the boundary conditions governing the input and output characteristics of the resonator. The boundary conditions are used to govern the input–output characteristics of the optical cavity. Thus,

\[
\begin{align*}
P^+ (0) &= R_{p_1} P^+_p (0) \\
\Gamma_s (L) &= R_{2s} P^+ (L) \\
\Gamma_s (L) &= R_{2p} P^- (L) \\
P^- (0) &= R_{p_1} P^- (0) + T_{1p} P^+_p 0 \\
\Gamma_s (L) &= T_{2s} P^+ (L) \\
\end{align*}
\tag{21}
\]

where, at mirrors M1 and M2, \( R_{1s}, R_{2s}, R_{1p}, \) and \( R_{2p} (R_{1,2,s,p}) \) are the reflectivity of signal and pump, respectively, and \( T_{1s}, T_{2s}, T_{1p}, \) and \( T_{2p} (T_{1,2,s,p}) \) are the transmissivity of signal and pump, respectively. \( P_{\text{out}} \) is the laser output. The approximation formula

\[
T_{1,2,s,p} \approx 1 - R_{1,2,s,p} 
\tag{22}
\]
is applied.

The 1 µm diameter Er-Yb silicate nanowire channel waveguide was pumped with a wavelength of 980 nm at an Er–Yb total concentration $N_{\text{total}}$ of $1.62 \times 10^{22}$ cm$^{-3}$. The Yb:Er ratio was set at 2.2:1. The cavity was formed by using a high reflector (M1: $T_p = 92\%$ at 980 nm, $R_{1s} = 99.8\%$ at 1532 nm) attached to the entrance end-face and a partially reflective output coupler (M2: $R_{2p} = 99.8\%$ at 980 nm, $R_{2s} = 95\%$ at 1532 nm) at the other end [39]. The stable Er-Yb silicate nanowire laser output was considered at $\lambda_s = 1532$ nm.

3.2 Results and discussion

Figure 8 shows the output power of the Er-Yb silicate nanowire laser as a function of cavity length for different pump powers. It indicates that, in a steady laser oscillation, the light wave exhibits no power loss during the round trip. The threshold gain $g_{\text{th}}$ for optical resonators can be defined so that the photon steady-state oscillation condition is

$$\Gamma g_{\text{th}} = \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right).$$

Thus, for settled $R_1$, $R_2$, and $\Gamma$, a threshold cavity length can be found, as indicated in Fig. 8(a). Here, the threshold resonator length is about 12 µm, which permits reducing the size of the laser and supports scale-integration requirements. The threshold cavity length increases with decreases in pump power because the oscillations require longer distances for light amplification at low pump powers. Figure 8(b) indicates that the output signal power increases with increases in resonant cavity lengths shorter than the optimum cavity length; at longer cavity lengths, the output signal power tends to decrease. Longer resonant cavity lengths lead to insufficient pumping distances, thus causing the absorption of the signal light. Therefore, the optimum resonant cavity length increases with increases in pump power. The output signal power can reach 50 mW at a 330 µm cavity length with a pump power of 100 mW; the power conversion efficiency in this configuration approaches 50%.

Figure 9 shows the output power of the Er-Yb silicate nanowire laser as a function of pump power at different cavity lengths. The results in Fig. 9(a) show that the output signal power increases with increases in pump power and gradually saturates. This result occurs because increases in signal power further enhance Er$^{3+}$ stimulated radiation, thus causing

![Fig. 8. Output power variations by cavity length. (a) Output power for short cavity lengths at varying pump powers (1–80 mW). The threshold resonator length is about 12 µm at a pump power of 75.6 mW. (b) Output power for long cavity lengths at varying pump powers (1–100 mW). The optimum cavity lengths were approximately 130 µm, 170 µm, 250 µm, 290 µm, and 330 µm for pump powers of 20 mW, 40 mW, 60 mW, 80 mW, and 100 mW, respectively.](image)
more rapid reductions in the Er\textsuperscript{3+} concentration of the excited state. This reduction, in turn, restrains the further amplification of the signal, resulting in laser output saturation that slows as the cavity length increases.

Figure 9(b) illustrates the results from applying the optimum resonant cavity length reported above to the Er-Yb silicate nanowire laser. Thus, a threshold pump power producing oscillation can be found. The laser output power increases with increases in the resonator length, but the threshold pump power of the laser also increases. The threshold pump power was approximately 3.3 mW when the resonator length was below 60 μm. Then, as the inset to Fig. 9(b) shows, the threshold pump power reached approximately 5 mW, 7 mW, 11 mW, 13 mW, and 15 mW for resonator lengths under 130 μm, 170 μm, 250 μm, 290 μm, and 330 μm, respectively. The laser power-conversion efficiency and threshold pump power of this configuration, considered together, are competitive. The need for compromises will arise when design requirements are set: smaller-size lasers can be used to reduce the input pump power, and larger-size lasers can be used to improve the power conversion efficiency.

Table 3 compares the laser properties of the Er-Yb silicate nanowire laser with values obtained from other Er-doped-materials lasers in [40-46] operating at wavelengths near 1530 nm. Purnawirman et al. [40] reported results for integrated Al\textsubscript{2}O\textsubscript{3}:Er\textsuperscript{3+} lasers with high-resolution Si\textsubscript{x}N\textsubscript{4} cavities patterned in a state-of-the-art 300 mm silicon wafer line. They obtained optical power above 5 mW ($P_{th} = 44$ mW) from a DBR cavity at 1561 nm. Later, they demonstrated high-power, stable, Er-doped DFB lasers compatible with complementary metal–oxide–semiconductors having a maximum output power of 75 mW ($P_{th} = 31$ mW) [41]. Belt et al. [42] also reported the experimental demonstration of both integrated Al\textsubscript{2}O\textsubscript{3}:Er\textsuperscript{3+} waveguide DBR (2.1 mW output power, $P_{th} = 11$ mW) and DFB (0.27 mW output power, $P_{th} = 21$ mW) lasers on a silicon nitride ultra-low-loss waveguide platform. Lancaster et al. [43] reported efficient 1550 nm laser operations in a waveguide-chip femtosecond laser inscribed in bulk fluorozirconate glass. The ternary-doped glass comprised Er as the active laser ion, Yb as the sensitizer, and cerium (Ce) to enhance the branching ratio from the pump to the erbium upper-laser state. The maximum efficiency of this configuration reached 25% with a 100 mW threshold. In 2017, Purnawirman et al. [44] reported ultra-narrow-linewidth Al\textsubscript{2}O\textsubscript{3}:Er\textsuperscript{3+} DFB lasers with a wavelength-insensitive silicon-compatible waveguide design. They compared the performance of DFB lasers in both discrete quarter-phase-shifted (QPS) and distributed phase-shifted (DPS) cavities and obtained output powers of 0.41 mW ($P_{th} = 55$ mW) at 1536 nm and 5.43 mW ($P_{th} = 14$ mW) at 1565 nm in QPS-DFB and DPS-DFB.
laser configurations, respectively. The same year, Brüske et al. [46] demonstrated efficient Er:Ti:LiNbO$_3$ ridge-waveguide lasers operating at 1531 nm and 1561 nm with efficiencies of 19% ($P_{th} = 116$ mW) and 33% ($P_{th} = 41$ mW), respectively.

Here, we designed a single-crystal core-shell structure Er-Yb silicate nanowire laser, based on experimental ECS nanowire amplifiers, to operate at 1532 nm. The minimum threshold pump power of this design was as low as 3.3 mW, and the maximum output power reached 50 mW. The power conversion efficiency was near 50%. The performance measures clearly show that an Er-Yb silicate nanowire laser can obtain higher power-conversion efficiency and lower thresholds at smaller sizes because Er-Yb silicate has a higher optical material gain. The power conversion efficiency can be improved several times over, and the produced threshold pump power can be more than five times lower than the values reported in references.

| Year | Material | Laser Wavelength | Cavity Length | Output Power | Power Conversion Efficiency | Threshold Pump Power | Source |
|------|----------|------------------|---------------|--------------|-----------------------------|----------------------|--------|
| 2013 | Al$_2$O$_3$:Er$^{3+}$ | 1561 nm | 20 mm | > 5 mW at 250 mW | 2% | 44 mW | [40] |
| 2014 | Al$_2$O$_3$:Er$^{3+}$ | 1563 nm | 23 mm | 75 mW at > 1 W | 7.5% | 31 mW | [41] |
| 2014 | Al$_2$O$_3$:Er$^{3+}$ | 1535 nm | 20 mm | 0.528 mW at 55 mW | 0.96% | 38 mW | [42] |
| 2014 | Al$_2$O$_3$:Er$^{3+}$ | 1534 nm | 21.5 mm | 0.369 mW at 55 mW | 0.67% | 25 mW | [42] |
| 2016 | Al$_2$O$_3$:Er$^{3+}$ | 1563 nm | 2 cm | > 1 mW at 80 mW | 1.3% | 20 mW | [43] |
| 2016 | Er$^{3+}$ Active Yb$^{3+}$Ce$^{3+}$ Co-Doped Fluorozirconate Er$^{3+}$ Active Yb$^{3+}$Ce$^{3+}$ Co-Doped Fluorozirconate | 1558-1568 nm | 14.5 mm | 108 mW at 540 mW | 20% | 60-145 mW | [44] |
| 2016 | Er$^{3+}$ Active Yb$^{3+}$Ce$^{3+}$ Co-Doped Fluorozirconate Er$^{3+}$ Active Yb$^{3+}$Ce$^{3+}$ Co-Doped Fluorozirconate | 1560-1587 nm | 28.4 nm | 55 mW at 510 mW | 10.8% | 90-130 mW | [44] |
| 2017 | Al$_2$O$_3$:Er$^{3+}$ | 1536 nm | 2 cm for QPS-DFB | 0.41 mW at 180 mW | 0.23% | 55 mW | [45] |
| 2017 | Al$_2$O$_3$:Er$^{3+}$ | 2 cm for DPS-DFB | 5.43 mW at 180 mW | 3% | 14 mW | [45] |
| 2017 | Er:Ti:LiNbO$_3$ | 1531 nm | 6.8 cm | 37.5 mW at 316 mW | 11.9% | 116 mW | [46] |
| 2017 | Er:Ti:LiNbO$_3$ | 1561 nm | 6.8 cm | 50 mW at 200 mW | 25% | 41 mW | [46] |
| 2018 | ErYbCS | 1532 nm | 60 μm | 13 mW at 100 mW | 13% | 3.3 mW | this work |
| 2018 | ErYbCS | 1532 nm | 330 μm | 50 mW at 100 mW | 50% | 15 mW | this work |

4. Conclusion

A more accurate and systematic theoretical model of Er/Er-Yb silicate nanowires was established to inform the design and simulation of silicon-based waveguide amplifiers and lasers. Several key parameters, such as emission cross section, Er$^{3+}$ energy-levels lifetime, upconversion coefficients, and Er–Yb energy-transfer coefficients, were calculated systematically and combined with the experimental data for ECS absorption spectra. The amplifier and laser properties of Er/Er-Yb silicate nanowires were analyzed numerically at different lengths, pump powers, concentrations, and Er:Yb ratios. The simulation results of
ECS and the previous experimental results reported in [25] matched basically, demonstrating the accuracy of the model. The influences of waveguide length, pump power, concentration, and material components were analyzed. The simulation results provided the basis for the theoretical research and optimum design of Er/Er-Yb silicate nanowire amplifiers and lasers. The gain can be increased by selecting the optimum pump length, properly increasing the pump power, effectively reducing the Er$^{3+}$ concentration, adding Yb$^{3+}$, and adjusting the Er:Yb ratio. Finally, the model projects the viability of a high-gain waveguide amplifier (unit gain over 20 dB/mm) and a low-threshold and high-efficiency laser (threshold pump power as low as 3.3 mW and power-conversion efficiency near 50%). These results not only provide support and guidance for the construction of high-performance Er-silicate amplifiers and lasers but also indicate the great prospects for applications of Er silicate compound materials in scale-integrated light sources.

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