A Proposal on the Search for the Hybrid with $I^G(J^{PC}) = 1^{-}(1^{-})$ in the Process $J/\psi \rightarrow \rho \omega \pi \pi$ at Upgraded BEPC/BES

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Abstract We give the moment expressions for the boson resonances $X$ with spin-parity $J_X^{PC} = 0^{++}, 1^{--}, 1^{++}$ and $2^{++}$ possibly produced in the process $J/\psi \rightarrow \rho X$, $X \rightarrow b_1(1235)\pi$, $b_1 \rightarrow \omega \pi$ in terms of the generalized moment analysis method. The resonance with $J_X^{PC} = 1^{-+}$ can be distinguished from other resonances by means of these moments except for some rather special cases. The suggestion that the search for the hybrid with $I^G(J^{PC}) = 1^{-}(1^{-})$ can be performed in the decay channel $J/\psi \rightarrow \rho \omega \pi \pi$ at upgraded BEPC/BES is presented.

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1 Introduction

Apart from the ordinary $q\bar{q}$ mesons, the new hadronic states such as glueballs ($gg/\gamma g$), hybrids ($q\bar{g}$) and four-quark states ($qqgq$) also exist according to the predictions of QCD. Discovery and confirmation of any one of these new hadronic states would be the strong support to the QCD theory. Therefore, the search for or identifying these new hadronic states is a very exciting and attractive research subject both theoretically and experimentally.

These new hadronic states can have the same quantum number $J^{PC}$ as the ordinary $q\bar{q}$ mesons, what is more, they can also have the exotic quantum numbers $J^{PC}$ which are not allowed in the quark model such as $1^{-+}$, and thus they cannot mix with the ordinary mesons. Experimentally, GAMS collaboration, G179 collaboration at KEK, VES group, E852 collaboration at BNL, and Crystal Barrel all claimed that the evidence for the exotic state with $J^{PC} = 1^{-+}$ was observed. The observed $\rho \tau$, $\eta \pi$ and $\eta' \pi$ couplings of this state qualitatively support the hypothesis that it is a hybrid meson, although other interpretations cannot be eliminated.

In terms of the predictions of the lattice QCD, the lowest lying glueball with $J^{PC} = 1^{-+}$ has a higher mass than $J/\psi$. Bag model calculations predict that the lowest lying $qq$ states do not carry exotic quantum numbers and form nonets carrying the same quantum numbers as $qq$ nonets, and that most $qqg$ states can fall apart into two mesons and thus have a decay width in the order of their mass, which leads to that most $qqg$ states are expected to be essentially unconfined and will not be observed as resonance peaks with reasonably narrow widths. Therefore, the search for the glueballs and four-quark states with $J^{PC} = 1^{-+}$ at BEPC/BES could be disappointing.

However, lattice QCD predicts that the mass of the hybrid with $J^{PC} = 1^{-+}$ is $1.2 \sim 2.5$ GeV. In addition, the naive estimate of pQCD predicts that the $J/\psi$ hadronic decay processes are favorable to the production of hybrids. So, if the hybrids exist, the search for hybrid with $J^{PC} = 1^{-+}$ at BEPC/BES should be fairly hopeful.

H. Yu and Q.X. Shen have already discussed the possibility of the search for the hybrid with $J^{PC} = 1^{-+}$ in the processes $J/\psi \rightarrow \rho X$, $X \rightarrow \eta \pi (\eta' \pi, \rho \pi)$. For the decay modes of the hybrid with $I^G(J^{PC}) = 1^{-}(1^{-})$, according to the symmetrization selection rule, the $\eta \pi$, $\eta' \pi$ modes are strongly suppressed. A possible mechanism to explain why the $1^{-+}$ state was observed in the above suppressed decay channels is planned for a separate publication. The $\rho \tau$ mode is allowed, but this is a $P$-wave mode and thus the $\rho \tau$ mode should not be a dominant decay mode. The dominant decay mode should be the $b_1(1235)\pi$. Therefore, the probability of discovering the hybrid with $J^{PC} = 1^{-+}$ in the processes $J/\psi \rightarrow \rho \pi X$, $X \rightarrow b_1 \pi$, in principle, should be higher than that in the processes $J/\psi \rightarrow \rho X$, $X \rightarrow \eta \pi (\eta' \pi, \rho \pi)$. Also, since the dominant decay mode of $b_1$ is $\omega \pi$, compared with the study on the two-step two-body decay process of $J/\psi$ in Refs [9] and [12], the study on the three-step two-body decay process $J/\psi \rightarrow \rho \pi X$, $X \rightarrow b_1 \pi, b_1 \rightarrow \omega \pi$ perhaps could present more information to the experimentalists. In this work, we shall consider the process $J/\psi \rightarrow \rho X$, $X \rightarrow b_1 \pi, b_1 \rightarrow \omega \pi$.

The rest of this paper is organized as follows. In Sec. 2, we give the moment expressions for the resonances $X$ with the above spin-parity in the process $J/\psi \rightarrow \rho X$, $X \rightarrow b_1 \pi$, 

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b₁ − ωπ in terms of the generalized moment analysis method. In Sec. 3, we discuss how to identify the resonances X with different spin-parity. Our conclusion is given in Sec. 4.

2 Moment Analysis

We consider the process

e₊ + e₋ → J/ψ → ρ + X, X → b₁ + π, b₁ → ω + π. (1)

The S-matrix element of the process (1) can be written as

\[ \langle ρ \lambda_ρ, \omega \lambda_ω | S - 1 | e₊^r e₋^r \rangle \propto \langle ρ \lambda_ρ | T | e₊^r e₋^r \rangle \]

\[ \langle ρ \lambda_ρ, X_{λ_ρ} | T | ψ_{λ_ψ} \rangle \langle b₁ \lambda_{b₁} π | T_{2} | X_{λ_χ} \rangle (ω_{λ_ω} π | T_{3} | b₁ \lambda_{b₁}) , \] (2)

where

\[ \langle ψ_{λ_ψ} | T | e₊^r e₋^r \rangle \propto \epsilon_{λ_ψ}^{α_π} (\bar{p}_\pi) \bar{v}_\psi (p_π), \] (3)

\[ \langle ρ \lambda_ρ, X_{λ_ρ} | T | ψ_{λ_ψ} \rangle \propto A_{λ_ρ, λ_ψ}^{X_{λ_χ}} D_{λ_ρ, λ_χ}^{λ_ψ} (0, θ_ψ, 0), \] (4)

\[ \langle b₁ \lambda_{b₁} π | T_{2} | X_{λ_χ} \rangle \propto B_{λ_χ}^{J_{X}} D_{λ_χ, λ_{λ_χ}}^{J_{X}} (φ_1, θ_1, -φ_1), \] (5)

\[ \langle ω_{λ_ω} π | T_{3} | b₁ \lambda_{b₁} \rangle \propto C_{λ_ω} D_{λ_ω, λ_{λ_ω}}^{λ_χ} (φ_2, θ_2, -φ_2), \] (6)

and λ₁, λ₂, λ₃, λ₄, λ₅, and λ₆ are the helicities of J/ψ, ρ, X, b₁ and ω, respectively; r and r’ are the polarization indexes of the positron and electron, respectively; \( \bar{p}_\pi, \bar{p}_+, \bar{p}_- \) are the momenta of J/ψ, positron and electron in the c.m. system of e⁺e⁻, respectively; \( A_{λ_χ}^{J_{X}} \), \( B_{λ_χ}^{J_{X}} \) and \( C_{λ_ω} \) are the helicity amplitudes of the processes J/ψ → ρX, X → b₁π and b₁ → ωπ, respectively; \( \theta_ψ \) is the polar angle in the c.m. system of e⁺e⁻ in which z axis is chosen to be along the direction of the incident positron and the vector meson ρ lies in x-z plane; (θ₁, φ₁) describes the direction of the momentum of b₁ in the rest frame of X where the z₁ axis is chosen to be along the direction of the momentum of X in the c.m. system of e⁺e⁻. Similarly, (θ₂, φ₂) described the direction of the momentum of the vector mesons ω in the rest frame of b₁ where the z₂ axis is along the momentum of b₁ in the rest frame of X. The function \( D_{m,n}^{J} \) is the (2J + 1)-dimensional representation of the rotation group. Owing to the parity conservation for the process (1), these helicity amplitudes satisfy the following symmetry relations:

\[ A_{λ_χ,-λ_χ}^{J_{X}} = P_X (-1)^{J_{X}} A_{-λ_χ,λ_χ}^{J_{X}}, \]

\[ B_{λ_χ}^{J_{X}} = P_X (-1)^{J_{X}} B_{-λ_χ}^{J_{X}}, \]

\[ C_{-λ_ω} = C_{λ_ω}, \] (7)

where \( P_X \) is the parity of X.

The angular distribution for the process (1) is

\[ W(θ_ρ, θ_1, θ_2, φ_2) \propto \sum_{λ_J,A_J} \sum_{λ_{λ_χ}} \sum_{λ_{λ_ω}} \sum_{λ_χ} \sum_{λ_ω} I_{λ_J,λ_χ} A_{λ_χ,λ_χ}^{J_{X}} A_{λ_χ,λ_χ}^{J_{X}} \]

\[ \times D_{λ_χ, λ_{λ_χ}}^{λ_χ} (0, θ_ρ, 0) D_{λ_ω, λ_{λ_ω}}^{λ_ω} (φ_1, θ_1, -φ_1) D_{λ_ω, λ_{λ_ω}}^{λ_ω} (φ_2, θ_2, -φ_2) D_{λ_ω, λ_{λ_ω}}^{λ_ω} (φ_2, θ_2, -φ_2), \] (8)

where the density matrix elements \( I_{λ_J,λ_χ} \) are

\[ I_{λ_J,λ_χ} = \frac{1}{4} \sum_{r,r'} \langle ψ_{λ_χ} | T | e₊^r e₋^r \rangle \langle ψ_{λ_χ} | T | e₊^r e₋^r \rangle^* \propto 2|\bar{p}_π|^2 δ_{λ_J,λ_χ} δ_{λ_J,±1}. \]

The moments for the process (1) can be defined by

\[ M(j, L, M, ℓ, m) = \int dθ_ρ \sin θ_ρ dθ_1 sin θ_1 dφ_1 dθ_2 sin θ_2 dφ_2 W(θ_ρ, θ_1, θ_2, φ_2) \]

\[ \times D_{0-M}(0, θ_ρ, 0) D_{M,m}(φ_1, θ_1, -φ_1) D_{m,0}(φ_2, θ_2, -φ_2). \] (10)

Equation (10) can be reduced to

\[ M(j, L, M, ℓ, m) \propto \sum_{λ_J} \sum_{λ_{λ_χ}} \sum_{λ_{λ_ω}} \sum_{λ_χ} \sum_{λ_ω} A_{λ_χ,λ_χ}^{J_{X}} A_{λ_χ,λ_χ}^{J_{X}} B_{λ_χ}^{J_{X}} B_{λ_ω}^{J_{X}} C_{λ_ω}^{λ_χ} (1λ_J j 0|1λ_J) \times \langle 1(λ_ρ - λ_χ) j(-M) \]

\[ \times |1(λ_ρ - λ_χ) j⟩ \langle J_X λ_χ^L M|J_X λ_χ^R⟩ \langle J_X λ_ω^L Mm|J_X λ_ω^R⟩ (1λ_χ^L 0m|1λ_ω^R 0m|1λ_ω^L 0|1λ_χ^R 0) \], (11)

where \( j_{1m_{1}}j_{2m_{2}}j_{3m_{3}} \) is Clebsch-Gordan coefficients.

In the process X → b₁π, if we restrict ℓ ≤ 1, where ℓ is the relative orbital angular momentum between b₁ and π, the quantum numbers \( J^G(J_X^C) \) of X allowed by the parity-isospin conservation law in the process (1) are 1− (1−), 1− (0−), 1− (1+) and 1− (2+). For the resonances X with \( J_X^C = 0^+, 1^-, 1^+ \) and \( 2^+ \), the nonzero moment expressions derived from Eq. (11) are shown in Appendixes A, B, C and D.

There are four, twenty-one, sixteen, and thirty-one nonzero moment expressions for \( J_X^C = 0^+, 1^-, 1^+ \) and \( 2^+ \), respectively. In the following section, we shall discuss how to identify the X with the above \( J_X^C \).

3 Discussion

Since the helicity amplitudes \( |C_0|^2 \) and \( |C_1|^2 \) are independent of the spin-parity of the resonance X, we find that if \( |C_0|^2 \neq |C_1|^2 \) the moment expressions have the following characteristics: For \( J_X^C = 0^+ \), the moments are always
equal to zero in the case $L > 0$ or $M > 0$ or $m > 0$; For $J_X^{P_XC} = 1^{+}$, the nonzero moments with $L = 0, 1, 2, M = 1, 2$ and $m = 0, 2$ exist but the moments are zero in the case $m = 1$; For $J_X^{P_XC} = 1^{−}$, the nonzero moments with $L = 0, 1, 2, M = 1, 2$ and $m = 0, 2$ exist, the nonzero moments with $m = 1$ also exist; For $J_X^{P_XC} = 2^{+}$, apart from the nonzero moments with $L = 0, 1, 2, M = 0, 1, 2$ and $m = 0, 1, 2$, the nonzero moments with $L = 3, 4$ exist. Therefore, from these characteristics, we can easily identify the resonances $X$ with $J_X^{P_XC} = 0^{++}, 1^{−}, 1^{++}$ and $2^{++}$ experimentally.

However, if $|C_0|^2 = |C_1|^2$, some of the preceding characteristics disappear, which leads to that the situations in the case $|C_0|^2 = |C_1|^2$ are more complex than those in the case $|C_0|^2 \neq |C_1|^2$. We will turn to the special case $|C_0|^2 = |C_1|^2$ below.

**Case 1** $|C_0|^2 = |C_1|^2$, $|B_0^1|^2 \neq |B_1^1|^2$ and $3|B_0^0|^2 \neq 4|B_1^2|^2$

In this case, only for $J_X^{P_XC} = 2^{++}$, there are four nonzero moments with $L = 4$, so the resonance with $J_X^{P_XC} = 2^{++}$ can be distinguished from other resonances. Then, for $J_X^{P_XC} = 0^{++}$, there are only two nonzero moments with $L = 0$, and for $J_X = 1$, there are four nonzero moments with $L = 2$, in addition to two nonzero moments with $L = 0$, hence the resonance with $J_X = 0$ can also be distinguished from that with $J_X = 1$. Finally, to distinguish the resonance with $J_X^{P_XC} = 1^{+}$ from that with $J_X^{P_XC} = 1^{−}$, we consider the following moment expression $H \equiv M(00000)/8 - M(02000)/4 - M(20000)/4 + 25M(22000)/2$ and find that the $H$ satisfies

$$H \propto \begin{cases} 0, & (J_X^{P_XC} = 1^{+}), \\ -\frac{15}{2}[|A_0^2|^2|B_1^1|^2|C_1|^2 < 0], & (J_X^{P_XC} = 2^{++}), \end{cases}$$

(12)

Using Eq. (12), we can still distinguish the resonance $X$ with $J_X^{P_XC} = 1^{−}$ from that with $J_X^{P_XC} = 1^{+}$.

**Case 2** $|C_0|^2 = |C_1|^2$, $|B_0^1|^2 \neq |B_1^1|^2$ and $3|B_0^0|^2 = 4|B_1^2|^2$

In this case, compared to the above case, the numbers of the nonzero moments for $J_X^{P_XC} = 0^{++}, 1^{−}$ and $1^{++}$ remain unchanged, but for $J_X^{P_XC} = 2^{++}$, the moments with $L = 4$ disappear. There are still only two nonzero moments with $L = 0$ for $J_X^{P_XC} = 0^{++}$ and six nonzero moments with $L = 0, 2$ not only for $J_X = 1$ but also for $J_X = 2$. In this case, owing to the unchanging of Eq. (12) and

$$H \propto -\frac{15}{2}[|A_0^2|^2|B_1^1|^2|C_1|^2 < 0], \quad (J_X^{P_XC} = 2^{++}),$$

(13)

the crucial point is to distinguish the resonance with $J_X^{P_XC} = 1^{−}$ from that with $J_X^{P_XC} = 2^{++}$. We also find $H_1 \equiv M(02000)/4 - 5M(22000)/2$ satisfies

$$H_1 \propto \begin{cases} 3(|A_0^2|^2 + |A_1^1|^2)|B_1^1|^2|C_1|^2 > 0, & (J_X^{P_XC} = 2^{++}), \\ \frac{9}{2} |A_1^1||B_1^1|^2|C_1|^2 > 0, & (J_X^{P_XC} = 1^{++}), \\ -\frac{9}{2}([A_0^2]^2 - |A_1^1|^2)(|B_1^1|^2 - |B_0^0|^2)|C_1|^2, & (J_X^{P_XC} = 1^{−}). \end{cases}$$

(14)

So, if it is determined experimentally that $H > 0$ or $H_1 \leq 0$, from Eqs (12) ~ (14), the $J_X^{P_XC}$ of $X$ must be $1^{−}$. However, if $H < 0$ or $H_1 > 0$, we cannot distinguish the resonance with $J_X^{P_XC} = 2^{++}$ from that with $J_X^{P_XC} = 1^{−}$.

**Case 3** $|C_0|^2 = |C_1|^2$ and $|B_0^1|^2 = |B_1^1|^2$

In this case, there are only two nonzero moments with $L = M = \ell = m = 0$ both for $J_X^{P_XC} = 0^{++}$ and $J_X^{P_XC} = 1^{−}$, two nonzero moments with $L = 0$ and four nonzero moments with $L = 2$ for $J_X^{P_XC} = 1^{++}$, and two nonzero moments with $L = 0$ and at least four nonzero moments with $L = 2$ for $J_X^{P_XC} = 2^{++}$. Therefore, the resonances with $J_X^{P_XC} = 1^{++}$ and $2^{++}$ can be distinguished from the resonance with $J_X^{P_XC} = 0^{++}$ (or $1^{−}$). But it is almost impossible to distinguish the resonance with $J_X^{P_XC} = 1^{−}$ from that with $J_X^{P_XC} = 0^{++}$ except in the radiative $J/\psi$ decay process. Because for the radiative $J/\psi$ decay process $e^+ + e^- \rightarrow J/\psi \rightarrow \gamma X, X \rightarrow b_1\pi, b_1 \rightarrow \omega\pi$, $A_0^0 = A_0^1 = A_1^0 = A_1^1$, we find

$$M(00000) - 10M(20000) \propto \begin{cases} 0, & (J_X^{P_XC} = 0^{++}), \\ 108 |A_1^1|^2|B_1^1|^2|C_1|^2, & (J_X^{P_XC} = 1^{−}). \end{cases}$$

(15)

Obviously, using Eq. (15) we can distinguish the $0^{++}$ state from the $1^{−}$ state in the radiative $J/\psi$ decay process.

From the above discussions, we get that if $|C_0|^2 \neq |C_1|^2$ we can easily identify the resonances $X$ with $J_X^{P_XC} = 0^{++}$, $1^{−}$, $1^{++}$ and $2^{++}$, but if $|C_0|^2 = |C_1|^2$ and $|B_0^1|^2 = |B_1^1|^2$ (or $|C_0|^2 = |C_1|^2$ and $3|B_0^0|^2 = 4|B_1^2|^2$), the identification of the resonances $X$ with $J_X^{P_XC} = 1^{−}$ and $0^{++}$ (or $2^{++}$) is very difficult. However, we also want to note the following two points: 1) Since the ratio of the helicity amplitudes for the process $b_1(1235) \rightarrow \omega\pi$, $|C_0|$ and $|C_1|$, can be measured
experimentally in other process such as $J/\psi \rightarrow b_1 \pi$, $b_1 \rightarrow \omega \pi$, the measurement of the ratio of $|C_0|$ and $|C_1|$ can be first performed in order to confirm whether $|C_0|^2$ is equal to $|C_1|^2$ or not; 2) Even though $|C_0|^2 = |C_1|^2$, one could expect that the probability of the simultaneous appearance of $|C_0|^2 = |C_1|^2$ and $|B_0|^2 = |B_1|^2$ (or $|C_0|^2 = |C_1|^2$ and $3|B_0|^2 = 4|B_1|^2$) would be fairly small.

It is worth while pointing out that the above moment expressions and the discussions are also valid for the process $J/\psi \rightarrow \gamma X$, $X \rightarrow b_1 \pi$, $b_1 \rightarrow \omega \pi$ provided $A_{00}^0 = A_{01}^0 = A_{10}^0 = A_{11}^0 = A_{20}^0 = A_{01}^1 = 0$.

4 Conclusion

The twenty-one nonzero moment expressions for $J_X^{PC} = 1^{-+}$ show that the possibility of the resonance $X$ with $J_X^{PC} = 1^{-+}$ produced in the process (1) exists. At the same time, we can easily distinguish it from other resonances except for some rather special cases. Therefore, generally speaking, if the 50 million $J/\psi$ events in the upgraded BEPC/BES are obtained, the search for the hybrid with $J_X^{PC} = 1^{-+}$ in the process $J/\psi \rightarrow pX$, $X \rightarrow b_1 \pi$, $b_1(1235) \rightarrow \omega \pi$ is feasible.

Appendix A: The Nonzero Moments for $J_X^{PC} = 0^{++}$

$$M(00000) \propto 2(|A_{00}^0|^2 + 2|A_{10}^0|^2)|B_0|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(00020) \propto \frac{1}{2}(|A_{00}^0|^2 + 2|A_{10}^0|^2)|B_0|^2(|C_0|^2 - |C_1|^2),$$
$$M(20000) \propto -2(|A_{01}^0|^2 - |A_{10}^0|^2)|B_0|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(20020) \propto -\frac{1}{2}(|A_{00}^0|^2 - |A_{10}^0|^2)|B_0|^2(|C_0|^2 - |C_1|^2),$$

Appendix B: The Nonzero Moments for $J_X^{PC} = 1^{-+}$

$$M(00000) \propto 2(|A_{00}^0|^2 + 2|A_{10}^0|^2 + 2|A_{11}^0|^2)|B_0|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(00020) \propto \frac{1}{2}(|A_{00}^0|^2 + 2|A_{10}^0|^2 + 2|A_{11}^0|^2)|B_0|^2 - |B_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(02000) \propto \frac{1}{2}(|A_{00}^0|^2 - |A_{10}^0|^2 + 2|A_{11}^0|^2)|B_0|^2 - |B_1|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(02020) \propto -\frac{1}{2}(|A_{00}^0|^2 - |A_{10}^0|^2 + 2|A_{11}^0|^2)|B_0|^2 - |B_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(21121) \propto \frac{1}{2}(|A_{00}^0|^2 - |A_{10}^0|^2 + 2|A_{11}^0|^2)(|C_0|^2 - |C_1|^2),$$
$$M(22000) \propto -\frac{1}{2}(|A_{00}^0|^2 - |A_{10}^0|^2 - |A_{11}^0|^2)(|B_0|^2 + |B_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(22020) \propto -\frac{1}{2}(|A_{00}^0|^2 + |A_{10}^0|^2 - 2|A_{11}^0|^2)|B_0|^2 + |B_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22121) \propto -\frac{1}{2}(|A_{00}^0|^2 + |A_{10}^0|^2 - 2|A_{11}^0|^2)|B_0|^2 + |B_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22200) \propto -\frac{1}{2}(|A_{00}^0|^2 - |B_0|^2)(|B_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(22220) \propto -\frac{1}{2}(|A_{00}^0|^2 - |B_0|^2)(|B_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(22221) \propto -\frac{1}{2}(|A_{00}^0|^2 - 2|B_0|^2 + |B_1|^2)(|C_0|^2 - |C_1|^2),$$

No. 1
A Proposal on the Search for the Hybrid with $J_X^{PC} = 1^{-+}$ in · · ·

47
$M(22222) \propto -\frac{16}{125} |A_{01}^2| |B_1|^2 (|C_0|^2 - |C_1|^2)$.

Appendix C: The Nonzero Moments for $J_{K^+C}^{++} = 1^{++}$

$M(00000) \propto 8(|A_{10}^2|^2 + |A_{11}^2|^2 + |A_{12}^2|^2) |B_1|^2 (|C_0|^2 + 2|C_1|^2)$,
$M(00020) \propto -\frac{8}{5} (|A_{01}^2|^2 + |A_{10}^2|^2 + |A_{11}^2|^2) |B_1|^2 (|C_0|^2 - |C_1|^2)$,
$M(00200) \propto \frac{4}{5} (|A_{01}^2|^2 - 2|A_{10}^2|^2 + |A_{11}^2|^2) |B_1|^2 (|C_0|^2 + 2|C_1|^2)$,
$M(02000) \propto -\frac{4}{25} (|A_{01}^2|^2 - 2|A_{10}^2|^2 + |A_{11}^2|^2) |B_1|^2 (|C_0|^2 - |C_1|^2)$,
$M(00000) \propto \frac{4}{25} (|A_{01}^2|^2 + |A_{10}^2|^2 - 2|A_{11}^2|^2) |B_1|^2 (|C_0|^2 + 2|C_1|^2)$,
$M(02000) \propto -\frac{4}{25} (|A_{01}^2|^2 + |A_{10}^2|^2 - 2|A_{11}^2|^2) |B_1|^2 (|C_0|^2 - |C_1|^2)$,
$M(02000) \propto -\frac{4}{25} (|A_{01}^2|^2 - 2|A_{10}^2|^2 - 2|A_{11}^2|^2) |B_1|^2 (|C_0|^2 + 2|C_1|^2)$,
$M(22000) \propto -\frac{4}{25} (|A_{01}^2|^2 - 2|A_{10}^2|^2 - 2|A_{11}^2|^2) |B_1|^2 (|C_0|^2 - |C_1|^2)$,
$M(22200) \propto -\frac{6}{125} (|A_{01}^2|^2 - 2|A_{10}^2|^2 - 2|A_{11}^2|^2) |B_1|^2 (|C_0|^2 - |C_1|^2)$,
$M(22100) \propto -\frac{6}{125} \text{Re}(A_{11}^2 A_{10}^*) |B_1|^2 (|C_0|^2 + 2|C_1|^2)$,
$M(22122) \propto -\frac{16}{125} \text{Re}(A_{11}^2 A_{10}^* |B_1|^2 (|C_0|^2 - |C_1|^2)$,
$M(22200) \propto -\frac{6}{25} |A_{01}^2|^2 |B_1|^2 (|C_0|^2 + 2|C_1|^2)$,
$M(22222) \propto -\frac{6}{25} |A_{01}^2|^2 |B_1|^2 (|C_0|^2 - |C_1|^2)$.

Appendix D: The Nonzero Moments for $J_{K^+C}^{++} = 2^{++}$

$M(00000) \propto 2(|A_{00}^2|^2 + 2|A_{01}^2|^2 + 2|A_{10}^2|^2 + 2|A_{11}^2|^2 + 2|A_{12}^2|^2) (|B_0|^2 + 2|B_1|^2) (|C_0|^2 + 2|C_1|^2)$,
$M(00200) \propto \frac{4}{5} (|A_{00}^2|^2 + 2|A_{01}^2|^2 + 2|A_{10}^2|^2 + 2|A_{11}^2|^2 + 2|A_{12}^2|^2) (|B_0|^2 + 2|B_1|^2) (|C_0|^2 - |C_1|^2)$,
$M(02000) \propto \frac{4}{5} (|A_{00}^2|^2 + |A_{01}^2|^2 + 2|A_{10}^2|^2 + |A_{11}^2|^2 - 2|A_{12}^2|^2) (|B_0|^2 + 2|B_1|^2) (|C_0|^2 + 2|C_1|^2)$,
$M(02000) \propto \frac{4}{5} (|A_{00}^2|^2 + |A_{01}^2|^2 + 2|A_{10}^2|^2 + |A_{11}^2|^2 - 2|A_{12}^2|^2) (2|B_0|^2 - |B_1|^2) (|C_0|^2 - |C_1|^2)$,
$M(02011) \propto \frac{4 \sqrt{7}}{25} (|A_{00}^2|^2 + |A_{01}^2|^2 + 2|A_{10}^2|^2 + |A_{11}^2|^2 - 2|A_{12}^2|^2) \text{Re}(B_0^2 B_0^* (|C_0|^2 - |C_1|^2)$,
$M(02022) \propto \frac{16}{125} (|A_{00}^2|^2 + |A_{01}^2|^2 + 2|A_{10}^2|^2 + |A_{11}^2|^2 - 2|A_{12}^2|^2) B_1^2 (|C_0|^2 - |C_1|^2)$,
$M(04000) \propto \frac{4}{5} (3|A_{00}^2|^2 - 4|A_{01}^2|^2 + 6|A_{10}^2|^2 - 4|A_{11}^2|^2 + 3|B_0|^2 - 4|B_1|^2) (|C_0|^2 + 2|C_1|^2)$,
$M(04020) \propto \frac{8}{25} (3|A_{00}^2|^2 - 4|A_{01}^2|^2 + 6|A_{10}^2|^2 - 4|A_{11}^2|^2 + 1|A_{12}^2|^2 + 2|A_{10}^2|^2 + 2|B_1|^2) (|C_0|^2 - |C_1|^2)$,
$M(04021) \propto \frac{4 \sqrt{7}}{25} (3|A_{00}^2|^2 - 4|A_{01}^2|^2 + 6|A_{10}^2|^2 - 4|A_{11}^2|^2 + 1|A_{12}^2|^2) \text{Re}(B_0^2 B_0^* (|C_0|^2 - |C_1|^2)$,
$M(04022) \propto \frac{8 \sqrt{7}}{125} (3|A_{00}^2|^2 - 4|A_{01}^2|^2 + 6|A_{10}^2|^2 - 4|A_{11}^2|^2 + 1|A_{12}^2|^2) B_1^2 (|C_0|^2 - |C_1|^2)$,
$M(02000) \propto -\frac{2}{5} (|A_{00}^2|^2 - |A_{01}^2|^2 + 2|A_{10}^2|^2 - 2|A_{11}^2|^2 - 2|A_{12}^2|^2) (|B_0|^2 + 2|B_1|^2) (|C_0|^2 + 2|C_1|^2)$,
$M(02020) \propto -\frac{2}{25} (|A_{00}^2|^2 - |A_{01}^2|^2 + 2|A_{10}^2|^2 - 2|A_{11}^2|^2 - 2|A_{12}^2|^2) (B_0^2 + B_1^2) (|C_0|^2 - |C_1|^2)$,
$M(21121) \propto \frac{2}{5} [3 \text{Im}(A_{10}^2 A_{10}^* + A_{11}^2 A_{11}^* + \sqrt{6} \text{Im}(A_{12}^2 A_{11}^*))] \text{Im}(B_0^2 B_0^*) (|C_0|^2 - |C_1|^2)$,
$M(22200) \propto -\frac{2}{25} (2|A_{00}^2|^2 - |A_{01}^2|^2 + 2|A_{10}^2|^2 + 2|A_{11}^2|^2 + 2|A_{12}^2|^2) (|B_0|^2 + B_1^2) (|C_0|^2 + 2|C_1|^2)$,
$M(22220) \propto -\frac{2}{125} (2|A_{00}^2|^2 - |A_{01}^2|^2 + 2|A_{10}^2|^2 + 2|A_{11}^2|^2 + 2|A_{12}^2|^2) (B_0^2 + B_1^2) (|C_0|^2 - |C_1|^2)$.
\[ M(20021) \propto -\frac{2}{\sqrt{3}} \langle A_{01}^2 \rangle - |A_{11}^2|^2 \langle A_{11}^2 \rangle + |A_{12}^2|^2 \langle A_{12}^2 \rangle \ Re \left( B_{2}^* B_{0}^* \right) |(C_0^2 - |C_1|^2) , \]
\[ M(20022) \propto -\frac{6}{\sqrt{5}} \langle A_{01}^2 \rangle - |A_{11}^2|^2 \langle A_{11}^2 \rangle + |A_{12}^2|^2 \langle A_{12}^2 \rangle \ Re \left( B_{2}^* B_{0}^* \right) |(C_0^2 - |C_1|^2) , \]
\[ M(22100) \propto -\frac{\sqrt{2}}{\sqrt{3}} \sqrt{6} \ Re \left( A_{01}^2 A_{00}^2 - A_{11}^2 A_{10}^2 + 6 \ Re \left( A_{22} A_{21}^2 \right) \right) |(B_0^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(22120) \propto -\frac{\sqrt{2}}{\sqrt{3}} \sqrt{2} \ Re \left( A_{01}^2 A_{00}^2 - A_{11}^2 A_{10}^2 - 3\sqrt{2} \ Re \left( A_{22} A_{21}^2 \right) \right) |(B_0^2 - |B_1|^2)^2 |(C_0^2 - |C_1|^2) , \]
\[ M(22121) \propto -\frac{\sqrt{2}}{\sqrt{3}} \sqrt{6} \ Re \left( A_{01}^2 A_{00}^2 - A_{11}^2 A_{10}^2 + \sqrt{6} \ Re \left( A_{22} A_{21}^2 \right) \right) |(B_1^2 + B_0^2)^2 |(C_0^2 - |C_1|^2) , \]
\[ M(22222) \propto -\frac{\sqrt{2}}{\sqrt{3}} \sqrt{6} \ Re \left( A_{01}^2 A_{00}^2 - A_{11}^2 A_{10}^2 + 6 \ Re \left( A_{22} A_{21}^2 \right) \right) |B_1^2 |(C_0^2 - |C_1|^2) , \]
\[ M(23121) \propto -\frac{\sqrt{2}}{\sqrt{3}} \sqrt{2} \ Im \left( A_{11}^2 + A_{12}^2 \right) + \sqrt{6} \ Im \left( A_{11}^2 A_{12}^2 \right) |(B_1^2 B_0^2)^2 |(C_0^2 - |C_1|^2) , \]
\[ M(23221) \propto -\frac{2}{\sqrt{3}} \langle A_{01}^2 \rangle + 2\sqrt{6} \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24000) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 2 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24020) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 2 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24021) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 2 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24022) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 2 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24100) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24120) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24121) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24200) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24220) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24221) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]
\[ M(24222) \propto -\frac{2}{\sqrt{5}} \langle A_{01}^2 \rangle + 3 \ Re \left( A_{12} A_{10}^2 \right) |(B_0^2)^2 + |B_1|^2)^2 |(C_0^2 + 2|C_1|^2) , \]

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