The remaining parts for the long-standing $J/\psi$ polarization puzzle

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Based on the non-relativistic quantum chromodynamics factorization formalism, the polarization parameters $\lambda_\theta$ and $\lambda_\phi$ of $J/\psi$ hadroproduction are analyzed in helicity frame and calculated at QCD next-to-leading order for the first time. For prompt $J/\psi$ production, we take into account the feeddown contributions from $\chi_{cJ}$ and $\psi(2S)$ decays. The theoretical predictions for the polarization parameters $\lambda_\theta$ and $\lambda_\phi$ of $J/\psi$ are presented. With the theoretical results we have done the fit to the experimental measurements on yield and polarization for $J/\psi$ hadroproduction simultaneously, and found that the results are coincide with the experimental measurements at the LHC quite well.

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Introduction — Nonrelativistic quantum chromodynamics (NRQCD) \cite{1} is one of the most successful effective theory to describe the decay and production of heavy quarkonium (as a review, see e.g. \cite{2, 3}). By separating the processes related to heavy quarkonium as short-distance coefficients (SDC) and the long-distance matrix elements (LDMEs), NRQCD allows one to organize perturbative calculations as double expansions in both the coupling constant $\alpha_s$ and the heavy quark relative velocity $v$. In the past decade, great improvements are made in the next-to-leading order (NLO) QCD correction calculation \cite{4-10}. The NLO corrections to color-singlet $J/\psi$ hadroproduction have been investigated in Ref. \cite{4, 5}, and its transverse momentum ($p_t$) distribution is found to be enhanced by 2-3 orders of magnitude at high $p_t$ region and the $J/\psi$ polarization changes from transverse into longitudinal at NLO \cite{5}. The NLO corrections to $J/\psi$ production via S-wave color-octet (CO) states are first studied in Ref. \cite{6} and the corrections to $p_t$ distributions of $J/\psi$ yield and polarization are small. Ref. \cite{7-10} performed the complete NLO calculation for prompt $J/\psi$ hadroproduction and their results can fit the $p_t$ distributions of the experimental measurements at Tevatron and LHC.

Despite all the successes, we cannot overlook the challenges it is facing. In the $J/\psi$ case, the determination of CO long-distance matrix elements (LDMEs) suffers ambiguity by the freedom of fitting method. With different fitting strategies, quite different values of LDMEs are obtained, which lead to different descriptions of the polarization distribution. Three groups \cite{11, 12} have made great efforts to proceed the calculation of $J/\psi$ polarization $\lambda_\theta$ to QCD NLO, but none of their CO LDMEs can reproduce the experimental measurements form LHC \cite{14, 15} with good precision for low and high $p_t$ range of $J/\psi$ simultaneously. The big uncertainty on the related LDMEs still remains, and it is even more complicated that only one \cite{15} of the three groups includes the $\chi_{cJ}$ and $\psi(2S)$ feeddown from direct calculation and the result can be compared with the experimental measurement on prompt $J/\psi$ polarization. Thereafter, the $\eta_c$ hadroproduction measured by LHCb Collaboration \cite{16} provides another laboratory for the study of NRQCD. Ref. \cite{17} considers it as a challenge to NRQCD, while Refs. \cite{18, 19} found these data are consistent with the $J/\psi$ hadroproduction measurements. Further, with the constraint on the LDMEs obtained in Ref. \cite{18, 19}, the authors found a special way to reduce the LDMEs uncertainty for the $J/\psi$.

The $J/\psi$ polarization, encoded in the angular distributions of the lepton pair, is described by

$$
\frac{d^2N}{d\cos\theta d\phi} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin(2\theta) \cos \phi + \lambda_{\phi} \sin^2 \theta \cos(2\phi) \tag{1}
$$

where $\theta$ is the polar angle between the direction of the positive lepton and chosen polarization axis, and $\phi$ is the azimuthal angle, measured with respect to the production plane. While all the three coefficients provide independent information, almost all the theoretical studies of $J/\psi$ polarization are restricted to $\lambda_\theta$. The parameter $\lambda_\phi$ has only been studied at QCD NLO work in Ref. \cite{11} with a few experimental data points measured by ALICE Collaboration \cite{21}. For $\lambda_{\theta\phi}$, there is no theoretical prediction at all. On the other side, there are experimental measurements on $\lambda_{\theta\phi}$ and $\lambda_\phi$ for $J/\psi$ and $\psi(2S)$ polarization from the LHCb \cite{14, 15} and the CMS \cite{22}. Based on the results from all the theoretical studies, it is believed that a combined fit includes $p_t$ distribution on $J/\psi$ production rate and polarization parameter $\lambda_\theta$ can be achieved. But it does not mean that the $J/\psi$ polarization puzzle is solved. There are still two parameters $\lambda_{\theta\phi}$ and $\lambda_\phi$ from experimental measurements without theoretical predictions to compare with. Therefore, theoretical study on these two polarization parameters is certainly needed. Are the theoretical predictions on these two parameters coincide with the experimental measurements? Otherwise, could the uncertainty on the related...
LDMEs be reduced by fitting on these measurements together with previous data fit? These are very important issue to settle down for the long-standing $J/\psi$ polarization puzzle.

In this Letter, we perform a theoretical analysis on the property of polarization parameters, and finish the calculation on $\lambda_{q\bar{q}}$ and $\lambda_\phi$ for $J/\psi$ and $\psi(2S)$ polarization in helicity frame based on NRQCD at QCD NLO. The results are obtained for the first time. By performing a combined fit, they coincide with the experimental measurements at the LHC quite well. Therefore, the last two pieces for the $J/\psi$ polarization are successfully explained. It means that the long-standing $J/\psi$ polarization puzzle is settled down completely.

** Calculation—** The three polarization parameters $\lambda_\theta$, $\lambda_{q\bar{q}}$ and $\lambda_\phi$ in Eq. [1] are defined as

$$
\lambda_\theta = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_{q\bar{q}} = \frac{\sqrt{2} R_{\text{red}} \sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_\phi = \frac{d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}},
$$

where

$$
d\sigma_{\lambda\lambda'}(\lambda, \lambda' = 0, \pm 1) = \text{spin density matrix of } J/\psi (\psi(2S)) \text{ hadroproduction, which following the NRQCD factorization, can be expressed as}
$$
\begin{align*}
d\sigma_{\lambda\lambda'} &= \int \frac{d^3p}{(2\pi)^3} f_{a/p}(x_1) f_{b/p}(x_2) \\
&= \frac{d\sigma_{\lambda\lambda'}}{d^3p} = \frac{d\sigma_{\lambda\lambda'}}{d^3p} (a \to (\sigma)_{\lambda\lambda'}(\lambda, \lambda' = 0, \pm 1))
\end{align*}

where $p$ is proton, the index $a, b$ run over the gluon $(g)$ and the light quarks $(u, d)$ and anti-quarks $(\bar{u}, \bar{d})$. $n$ denotes the color, spin and angular momentum states of the $c\bar{c}$ intermediate states ($3S_1^1$, $1S_0^1$, $3S_1^1$, $3P_1^1$, $3P_2^1$) for $J/\psi$ and $\psi(2S)$, or ($3P_1^1$, $3S_1^1$) for $\chi_{cJ}$. The functions $f_{a/p}(x_1)$ and $f_{b/p}(x_2)$ are the parton distribution functions (PDFs) for the incoming protons for parton types $a$ and $b$. The SDC $d\sigma$ can be perturbatively calculated and the LDMEs $\langle O_n^H \rangle$ are governed by nonperturbative QCD effects.

For inclusive $J/\psi$ production at the LHC, P parity is invariant for initial states protons and it involve only QCD interaction which is also P parity invariant. Based on P parity invariance for the production density matrix of the $J/\psi$ hadroproduction, together with definition of the helicity frame, a symmetry (asymmetry) relations can be deduced as (the detail is presented in Ref. [24]):

$$
\frac{d\sigma_{\lambda\lambda'}}{d\sigma} \big|_{y=a} = n_{\lambda\lambda'} \frac{d\sigma_{\lambda\lambda'}}{d\sigma} \big|_{y=-a}, \quad n_{\lambda\lambda'} = \begin{cases} 1 & \lambda = \pm \lambda' \\ -1 & \lambda = \pm 1, \lambda' = 0 \end{cases}
$$

Then the conclusions are obtained as $\lambda_{q\bar{q}} = 0$ for experiment with symmetry rapidity range ($a < |y| < b$) like that at the CMS and ATLAS, $\lambda_{q\bar{q}} \neq 0$ for half rapidity range ($y > b$) such as the case at the LHCb, and $\lambda_\theta, \lambda_\phi$ is symmetric for positive and negative rapidity in helicity frame.

To include the feed-down contributions from $\psi(2S)$ and $\chi_{cJ}$ to $J/\psi$, we following the same treatment as in Ref. [13].

$$
\begin{align*}
d\sigma_{J/\psi}^{(2S)} &= \mathcal{B}(\psi(2S) \to J/\psi)d\sigma_{\lambda\lambda'}^{(2S)}, \\
d\sigma_{J/\psi}^{(J/\psi)} &= \mathcal{B}(\chi_{cJ} \to J/\psi) \sum_{J_z} \delta_{J_z - \lambda, J_z' - \lambda'} C_{J_z, J_z'}\lambda\lambda' \times d\sigma_{J_z, J_z'}^{(J/\psi)}.
\end{align*}
$$

where $C_{J_z, J_z'}\lambda\lambda'$ is the Clebsch-Gordan coefficient, and $\lambda, J_z$ are the quantum numbers of angular momentum.

To calculate the NRQCD prediction on the transverse momentum $p_t$ distribution of yield and polarization for heavy quarkonium hadroproduction at QCD NLO, we use the FDCHQHP package [25], which was based on the collection of the Fortran codes generated for all 87 parton level sub-processes by using FDC package [24] and implementation tool on job submission and numerical precision control. It is a very powerful tool for us to save a lot of work and finish this study. Even with it, we found that there were a few places had been improved on job submission and numerical precision control, and improved them. The updated version FDCHQHP [27] will be publicly available soon.

** LDMEs Strategy—** The CS LDMEs are estimated from the wave functions at the origin by

$$
\begin{align*}
\langle O^{2} \rangle_1 &= \frac{3N_c}{2\pi} |R_0(0)|^2, \\
\langle O^{3} \rangle_1 &= \frac{3}{4\pi} (2J + 1) |R_{J}(0)|^2.
\end{align*}
$$

where the wave function at the origin can be calculated via potential model [28], which gives $|R_{J}(0)|^2 = 0.81$ GeV$^3$, $|R_{\psi(2S)}(0)|^2 = 0.53$ GeV$^3$, and $|R_{J}(0)|^2 = 0.075$ GeV$^3$.

The CO LDMEs are extracted from the fit on experimental data with QCD NLO theoretical formula. However, different results for the LDMEs are obtained when different strategy are used in the fit. We briefly discuss different fit results and made a selection of them to represent the uncertainty on predictions induced by the LDMEs in following.

In the $J/\psi$ case, several groups of LDMEs [8, 10, 12, 13, 18, 19, 29] can be found. They are extracted by fitting the data of hadroproduction yield [8, 13], or combined with polarization [12] on $pp$ collisions. In their fits [8, 12, 13], the data with $p_t < 7$ GeV are excluded. Ref. [9] extracted the LDMEs with a wider set of data including the lower $p_t$ region ($p_t > 3$ GeV) hadroproduction and the production at $ep$ and $\gamma \gamma$ colliders with $p_t > 1$ GeV. By the assumption of heavy quark spin symmetry (HQS), the fit in Ref. [18] [19] took the $\eta_c$ measurement [16] ($p_t \geq 6$ GeV) into consideration, and they obtained consistent $J/\psi$ LDMEs. In Ref. [29], the authors incorporate the leading-power fragmentation corrections together with the usual QCD NLO corrections, which involves different SDC and results in different LDMEs. The group of Ref. [8] improved their analysis by taking into account the
feed-down contributions later \[30\], but no updated \(J/\psi\) LDMEs are presented. Among the LDMEs set mentioned above, only Ref. \[14\] fitted the prompt \(J/\psi\) hadroproduction by including the feed-down contributions from \(\chi_{cJ}\) and \(\psi(2S)\) which can be used to calculate these feed-down contribution to \(J/\psi\) polarization.

Following the same treatment in Ref. \[14\], we refit the \(J/\psi\) LDMEs by using more experimental data where the data in \(p_t < 7\) GeV region are excluded as usual. In addition to the \(p_t\) distribution of the production yield data from the CDF \[31\] and the LHCb \[32\] in old fit, the polarization data of \(\lambda_\theta\), \(\lambda_{\theta\phi}\) and \(\lambda_\phi\) from the LHCb \[14\] and \(\lambda_\theta\) and \(\lambda_\phi\) from the CMS \[22\] are used. While the \(\lambda_{\theta\phi}\) is exactly zero in our calculation for the CMS so that it can not be fitted. The non-zero data on \(\lambda_{\theta\phi}\) for the CMS could be from P-parity broken, i.e. from the electro-weak production for \(J/\psi\). Therefore we calculated the leading contribution from \(3 S_1^{[1]}\), \(1 S_0^{[8]}\), \(3 S_1^{[8]}\), \(3 P_1^{[8]}\) and found that their production rate is smaller about 5 order in magnitude than the QCD production processes. So we suggest that \(\lambda_{\theta\phi}\) for the CMS should be constrained as zero in the experimental measurement.

To deal with the feed-down contributions from \(\psi(2S)\) and \(\chi_{cJ}\), we use the CO LDMEs from Ref. \[14\], namely \(\langle O^{(2S)}(1 S_0^{[8]}) \rangle = -1.2 \times 10^{-4}\) GeV\(^3\), \(\langle O^{(2S)}(3 S_1^{[8]}) \rangle = 3.4 \times 10^{-3}\) GeV\(^3\), \(\langle O^{(2S)}(3 P_1^{[8]}) \rangle / m_Q^2 = 4.2 \times 10^{-3}\) GeV\(^3\), and \(\langle O^{(2S)}(3 S_1^{[8]}) \rangle = 2.21 \times 10^{-3}\) GeV\(^3\).

By fitting the totally 86 data points of \(J/\psi\) and minimizing \(\chi^2\), we obtain

\[
\langle O^{J/\psi}(1 S_0^{[8]}) \rangle = (5.66 \pm 0.47) \times 10^{-2}\) GeV\(^3\),
\langle O^{J/\psi}(3 S_1^{[8]}) \rangle = (1.17 \pm 0.58) \times 10^{-3}\) GeV\(^3\), \langle O^{J/\psi}(3 P_1^{[8]}) \rangle / m_Q^2 = (5.4 \pm 0.5) \times 10^{-4}\) GeV\(^3\),

which will be taken as default values to present our results on polarization parameters.

To investigate the uncertainties from different set for the values of LDMEs, the other five sets of LDMEs in Table. \[4\] are also used to present the final numerical results.

**Numerical results**—In our numerical calculation, the parton distribution function CTEQ6M \[33\] and the corresponding two-loop QCD coupling constant \(\alpha_s\) are used. The charm-quark mass is chosen as \(m_c=M_H/2\) approximately, where the masses of relevant quarkonia \(M_H\) are 3.0 GeV, 3.5 GeV and 3.686 GeV for \(H=J/\psi\), \(\chi_{cJ}(J=0,1,2)\) and \(\psi(2S)\), respectively. The renormalization and factorization scales are chosen as \(\mu_f = \mu_t = \sqrt{4m_c^2 + p_t^2}\), while the NRQCD scale is \(\mu_L = m_c\). Branching ratios are \(B[J/\psi\to J/\psi]:0.61\) and \(B[\chi_{cJ}\to J/\psi]:0.0127, 0.339, 0.192\) for \(J = 0,1,2\) \[34\], respectively. Additionally, a shift \(p_H^t \approx p_t^H \times (M_H/M_H^t)\) is used while considering the kinematics effect in the feed-down from higher excited states.

In Fig. \[1\], \(\lambda_{\theta\phi}\) is presented for each \((3 S_1^{[1]}, 3 S_1^{[8]}, 3 P_1^{[8]}))\) channel with rapidity range \(y > 0, y < 0\) and \(|y| > 0,\) and it shown that \(\lambda_{\theta\phi}\) is of asymmetry for rapidity range just as the conclusion from Eq.\[3\]

By using the default LDMEs set in Eq. \[7\], \(\lambda_{\theta\phi}\) at the CMS is presented in Fig. \[2\]. It shown that final numerical results are of asymmetry within very good numerical precision and the theoretical prediction coincide with the experimental measurements at the CMS quite well for both \(J/\psi\) and \(\psi(2S)\).

To investigate the uncertainties from different sets of LDMEs, \(\lambda_\theta, \lambda_{\theta\phi}\) and \(\lambda_\phi\) are presented by using the LDMEs sets in TABLE. \[4\]. In Fig. \[3\], the prediction of prompt \(J/\psi\) polarization can explain the CMS data in a wide \(p_t\) region \((10\) GeV \(< p_t < 70\) GeV) in both \(|y| < 0.6\) and \(0.6 < |y| < 1.2\) rapidity bins. The newly fitted LDMEs provides an excellent description of the \(\lambda_\theta\) at CMS window at transverse momentum range 14 GeV \(< p_t < 70\) GeV \[22\]. All the six fits scheme provide a good description of the \(\lambda_{\theta\phi}\) and \(\lambda_\phi\). As we have mentioned in the above context, the SDC \(\text{Re}(d\sigma/d\Omega)\) is exact zero in the symmetry rapidity region (e.g. \(a < |y| < b\)) for all the channels. So the value of LDMEs would not alter the \(\lambda_{\theta\phi}\) from zero for CMS data. The CMS data of \(\lambda_\phi\) are
covered by the uncertainty from six LDMEs fit schemes. In Fig. 3, $\lambda_0$, $\lambda_{\theta\phi}$ and $\lambda_{\phi}$ in five rapidity bins at the LHCb are presented from the top to the bottom rows. For the different rapidity bins, the $p_t$ distribution of $\lambda_0$, $\lambda_{\theta\phi}$ and $\lambda_{\phi}$ behave in a similar way. At low $p_t$ region ($p_t < 4$ GeV), the different LDMEs sets provide large uncertainty, while $p_t$ increasing, the results from the different LDMEs sets converge to zero. It clearly shown that our new fit coincide with the experimental measurements at the LHCb quite well for $J/\psi$ production.

**Summary and conclusion** — In this Letter, we finished calculation on $\lambda_{\phi}$, $\lambda_{\theta\phi}$ and $\lambda_0$ for $J/\psi$ and $\psi(2S)$ polarization in helicity frame based on NRQCD at QCD NLO. The results are obtained for the first time and they coincide with the experimental measurements at the LHC quite well. Therefore, the last two pieces for the $J/\psi$ polarization are successfully explained. It means that the long-standing $J/\psi$ polarization puzzle is settle down completely.

By applying P parity invariance analysis, we obtained the conclusions in helicity frame that $\lambda_{\phi} = 0$ for experiment with symmetry rapidity range ($a < |y| < b$) like that at the CMS and ATLAS, and $\lambda_{\theta\phi} \neq 0$ for half rapidity range ($y > b$) such as the case at the LHCb for polarization of $J/\psi$ and $\psi(2S)$ hadroproduction at the LHC. However, the electro-weak production for $J/\psi$ can break P-parity, our calculation shown that the leading contribution from $3^1S_1$, $1^3S_0$, $1^3S_1$, $3P^1$ is smaller about 5 order in magnitude than the QCD production processes. Therefore this contribution can be ignored. This conclusion could be applied in the CMS or ATLAS experimental measurement to keep $\lambda_{\theta\phi} = 0$ in each iteration of fitting to improve the measurements. It also indicate that the polarization measurement could be performed in half rapidity range with $y > 0$ or $y < 0$ separately, in this way, $\lambda_{\theta\phi} \neq 0$ could be achieved.

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| Reference            | $\langle O^{J/\psi}(1^3S_0^{[1]}) \rangle$ | $\langle O^{J/\psi}(1^3S_1^{[8]}) \rangle$ | $\langle O^{J/\psi}(3^1S_1^{[8]}) \rangle$ | $\langle O^{J/\psi}(3P_0^{[8]}) \rangle/m_T^2$ |
|----------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-----------------------------------------------|
| butenschoen et al. (2011) | 1.32                                      | $3.04 \times 10^{-2}$                     | $1.68 \times 10^{-3}$                     | $-4.04 \times 10^{-3}$                       |
| Chao et al. (2012)     | 1.16                                      | $8.9 \times 10^{-2}$                     | $3.0 \times 10^{-3}$                      | $5.6 \times 10^{-3}$                         |
| Gong et al. (2013)     | 1.16                                      | $9.7 \times 10^{-2}$                     | $-4.6 \times 10^{-3}$                     | $-9.5 \times 10^{-3}$                        |
| Bodwin et al. (2014)   | 0                                         | $9.9 \times 10^{-2}$                     | $1.1 \times 10^{-2}$                      | $4.9 \times 10^{-3}$                         |
| Zhang et al. (2015)    | 0.645                                     | $0.78 \times 10^{-2}$                    | $1.0 \times 10^{-2}$                      | $1.7 \times 10^{-2}$                         |

*TABLE I: The values of LDMEs for $J/\psi$ (in units of GeV$^3$).*
[1] G. T. Bodwin, E. Braaten, and G. P. Lepage, Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, Phys.Rev. D51 (1995) 1125–1171, arXiv:hep-ph/9407339.

[2] N. Brambilla et al., Heavy quarkonium: progress, puzzles, and opportunities, Eur. Phys. J. C71 (2011) 1534, arXiv:1010.5827.

[3] N. Brambilla et al., QCD and Strongly Coupled Gauge Theories: Challenges and Perspectives, Eur. Phys. J. C74 (2014) 2981, arXiv:1404.3723.

[4] J. M. Campbell, F. Maltoni, and F. Tramontano, QCD corrections to J/ψ and Upsilon production at hadron colliders, Phys.Rev.Lett. 98 (2007) 252002, hep-ph/0703113.

[5] B. Gong and J.-X. Wang, Next-to-leading-order QCD corrections to J/ψ polarization at Tevatron and Large-Hadron-Collider energies, Phys.Rev.Lett. 100 (2008) 232001, arXiv:0802.3727.

[6] B. Gong, X. Q. Li, and J.-X. Wang, QCD corrections to J/ψ production via color octet states at Tevatron and LHC, Phys.Lett. B673 (2009) 197–200, arXiv:0805.4751.

[7] M. Butenschoen and B. A. Kniehl, Reconciling J/ψ production at HERA, RHIC, Tevatron, and LHC with NRQCD factorization at next-to-leading order, Phys.Rev.Lett. 106 (2011) 022003, arXiv:1009.5662.

[8] Y.-Q. Ma, K. Wang, and K.-T. Chao, J/ψ(ψ′) production at the Tevatron and LHC at O(αs^n_f) in nonrelativistic QCD, Phys.Rev.Lett. 106 (2011) 042002, arXiv:1009.3655.

[9] M. Butenschoen and B. A. Kniehl, World data of J/ψ production consolidate NRQCD factorization at NLO, Phys.Rev. D84 (2011) 051501, arXiv:1105.0820.

[10] Y.-Q. Ma, K. Wang, and K.-T. Chao, A complete NLO calculation of the J/ψ and ψ′ production at hadron colliders, Phys. Rev. D84 (2011) 114001, arXiv:1012.1030.

[11] M. Butenschoen and B. A. Kniehl, J/ψ polarization at Tevatron and LHC: Nonrelativistic-QCD factorization at the crossroads, Phys.Rev.Lett. 108 (2012) 172002, arXiv:1201.1872.

[12] K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, and Y.-J. Zhang, J/ψ Polarization at Hadron Colliders in Nonrelativistic QCD, Phys. Rev. Lett. 108 (2012) 242004, arXiv:1201.2875.

[13] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, Polarization for Prompt J/ψ, ψ(2S) production at the Tevatron and LHC, Phys. Rev. Lett. 110 (2013) 042002, arXiv:1205.6682.

[14] LHCb, R. Aaij et al., Measurement of J/ψ polarization in pp collisions at √s = 7 TeV, Eur. Phys. J. C73 (2013) 2631, arXiv:1307.6379.

[15] LHCb, R. Aaij et al., Measurement of ψ(2S) polarization in pp collisions at √s = 7 TeV, Eur. Phys. J. C74 (2014) 2872, arXiv:1403.1359.

[16] LHCb, R. Aaij et al., Measurement of the ηc(1S) production cross-section in proton-proton collisions via the decay ηc(1S) → pp, Eur. Phys. J. C75 (2015) 311, arXiv:1409.3612.

[17] M. Butenschoen, Z.-G. He, and B. A. Kniehl, ηc production at the LHC challenges nonrelativistic-QCD factorization, Phys.Rev.Lett. 114 (2014) 092004, arXiv:1411.5287.

[18] H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, and K.-T. Chao, ηc production at LHC and indications on the understanding of J/ψ production, Phys.Rev.Lett. 114 (2015) 092005, arXiv:1411.7350.

[19] H.-F. Zhang, Z. Sun, W.-L. Sang, and R. Li, Impact of ηc hadroproduction data on charmonium production and polarization within NRQCD framework, Phys.Rev.Lett. 114 (2015) 092006, arXiv:1412.0508.

[20] Z. Sun and H.-F. Zhang, Reconciling charmonium production and polarization data in the midrapidity region at hadron colliders within the nonrelativistic QCD framework, Chin. Phys. C42 (2018) 043104, arXiv:1505.02675.

[21] ALICE, B. Abelev et al., J/ψ polarization in pp collisions at √s = 7 TeV, Phys. Rev. Lett. 108 (2012) 082001, arXiv:1111.1630.

[22] CMS, S. Chatrchyan et al., Measurement of the prompt J/ψ and ψ(2S) polarizations in pp collisions at √s = 7 TeV, Phys. Lett. B727 (2013) 381–402, arXiv:1307.6070.

[23] M. Beneke, M. Kramer, and M. Vantinnen, Inelastic photoproduction of polarized J/ψ/p, Phys.Rev. D57 (1998) 4258–4274, hep-ph/9709376.

[24] Y. Feng and J.-X. Wang, In preparing.

[25] L.-P. Wan and J.-X. Wang, FDCHQHP: A Fortran package for heavy quarkonium hadroproduction, Comput.Phys.Commun. 185 (2014) 2939–2949, arXiv:1405.2143.

[26] J.-X. Wang, Progress in FDC project, Nucl.Instrum.Meth. A534 (2004) 241–245, hep-ph/0407058.

[27] Y. Feng and J.-X. Wang, In preparing.

[28] E. J. Eichten and C. Quigg, Quarkonium wave functions at the origin, Phys.Rev. D52 (1995) 1762–1768, hep-ph/9503356.

[29] G. T. Bodwin, H. S. Chung, U.-R. Kim, and J. Lee, Fragmentation contributions to J/ψ production at the Tevatron and the LHC, Phys.Rev.Lett. 113 (2014) 022001, arXiv:1403.3612.

[30] H. S. Shao, H. Han, Y. Q. Ma, C. Meng, Y. J. Zhang, and K. T. Chao, Yields and polarizations of prompt J/ψ and ψ(2S) production in hadronic collisions, JHEP 05 (2015) 103, arXiv:1411.3300.

[31] CDF, D. Acosta et al., Measurement of the J/ψ meson and b–hadron production cross sections in pp collisions at √s = 1960 GeV, Phys. Rev. D71 (2005) 032001, hep-ex/0412071.

[32] LHCb, R. Aaij et al., Measurement of J/ψ production in pp collisions at √s = 7 TeV, Eur. Phys. J. C71 (2011) 1645, arXiv:1103.0423.

[33] J. Pumplin, D. Stump, J. Huston, H. Lai, P. M. Nadolsky, et al., New generation of parton distributions with uncertainties from global QCD analysis, JHEP 02 (2002) 012, hep-ph/0201195.

[34] Particle Data Group, C. Patrignani et al., Review of Particle Physics, Chin. Phys. C40 (2016) 100001.