Evaluation of analytical model for magnetic field distribution in magnetizing fixture study

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Abstract. A simple analytical model for magnetic field of a circular current was evaluated for the case of 4-multilayer solenoid of single-pole magnetizing fixture. The exact solution of the analytical model was computationally simulated and presented as 2D spatial distribution of magnetic field strength as functions of applied current and the position of each single coil in the solenoid. The results were compared with experimental measurements performed on commercial magnetizing fixture in order to gauge the effectiveness of the model in real application conditions. Qualitatively, the model shows an excellent agreement to the general trend of spatial distribution of magnetic field patterns obtained from experimental data despite of a considerable percentage error between the model and experimental results.

1. Introduction
Bonded and sintered permanent magnets such as ferrite and NdFeB, respectively, are commonly molded into various shapes (blocks, rings, discs arcs, segments, etc.) and volumes to meet diverse applications in home appliances, speakers, office automation equipment, medical laboratory diagnostic test equipment, and more [1–3]. In the last stage of manufacturing process, those magnets are magnetized using a specialized device known as magnetizing fixture, which is an integral part of impulse current generator that could be used to magnetize or demagnetize permanent magnets. Currently, magnetizing fixtures that available in the market can be categorized as single pole and multipolar fixtures. Single pole fixtures are mostly used to generate simple 2-pole magnets (axial and diametrical orientations), while multipolar fixtures are applied to produce multipole magnets with lateral, multipole, and radial magnetic field alignments. These two types of fixtures are mostly customized by request and made dedicated solely for specific magnetic material requirements and/or particular custom-made magnetizer. Since designing both fixture and impulse magnetizer from scratch requires a prior information regarding magnetic flux generated inside the cavity of the fixture, acquiring basic knowledge in both analytical and numerical methods in calculating magnetic flux has become mandatory skills for those who want to design and build their own fixture. For this reason, we have extended a simple mathematical model [4,5] that originally derived for a circular current loop into a model that could measure spatial distribution of magnetic field strength of 4-multilayer solenoid. It should be note that the model that employed in this present study inherently adopts steady current approximation. Accordingly, it by default excludes self- and mutual inductances effects that may contribute significantly to the total magnetic flux that being generated during discharging process. In order to measure the margin error in the model, experimental data obtained from measurement on commercial 4-multilayer magnetizing fixture was used as reference to verify the calculation results.

2. Methods

2.1. Mathematical model
The vector potential $\mathbf{A}$ at point $P$ as shown in figure 1a where the current $I$ flows within circular loop of radius $R$ that located in the $x$-$y$ plane and centered at the origin is given by [4,6]:

$$A_\phi (r, z, \phi) = \frac{\mu_0 I}{4\pi} \frac{d\phi}{d} = \frac{\mu_0 I}{2\pi} \frac{r}{(R^2 + r^2 + z^2)^{3/2}} R \cos \phi d\phi \psi$$

Substituting equation (1) with $\phi = \pi + 2\xi$ so that $d\phi = -2d\xi$ and $\cos \phi = 2 \sin \xi - 1$, and performing some algebraic manipulation, one would obtain:

$$A_\phi (r, z) = \frac{\mu_0 I}{\pi k} \sqrt{\frac{r}{R}} \left[ \left( 1 - \frac{k^2}{2} \right) K - E \right]$$

where $K$ and $E$ are the complete elliptic integral of first and second kinds [7], respectively, and

$$k^2 = \frac{4Rr}{(R+r)^2 + z^2}$$

The magnetic field $\mathbf{B}$ components in cylindrical coordinate system for single circular current loop are

$$B_r (r, z) = -\frac{\partial}{\partial z} A_\phi = \frac{\mu_0 I}{2\pi} \frac{z}{r[(R+r)^2 + z^2]^{3/2}} \left[ (-K) + \frac{R^2 + r^2 + z^2}{(R+r)^2 + z^2} E \right]$$

$$B_\phi (r, z) = \frac{\partial}{\partial r} A_r - \frac{\partial}{\partial z} A_z = 0 \quad \text{(because $A_r$ and $A_z$ are zero)}$$

$$B_z (r, z) = \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) = \frac{\mu_0 I}{2\pi} \frac{1}{[(R+r)^2 + z^2]^{1/2}} \left[ K + E \left( \frac{R^2 - r^2 - z^2}{(R+r)^2 + z^2} \right) \right]$$

For numerical purpose, equation (4) and equation (6) are expanded and rewritten as multiple summation of total magnetic field with respect to their respective cylindrical coordinate components $(r, z)$ as follows

$$B_{rn}(r_l, z_j) = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} \sum_{l=1}^{n_4} B_r (R_k, r_l, z_j)$$

$$B_{zn}(r_l, z_j) = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} \sum_{l=1}^{n_4} B_z (R_k, r_l, z_j)$$
The multiple summation in the equation (7) and equation (8) describe the \( n \)-th wire loop at \((k, l)\) position will induce magnetic field at \((i, j)\) position (see figure 1b). Here, \( z_{jl} = z_l - z_i \) defines magnetic field at index \( j \) position is calculated with respect to \( n \)-th coil at index \( l \) position along \( z \)-direction. The maximum integer values of \( n_3 \) and \( n_4 \) are approximated by \( L_1/D \) and \( L_2/D \), respectively, where \( D \) represents a distance between two centers of nearest neighbor coils. In addition, the maximum values of \( n_1 \) and \( n_2 \) are proportional to \( n_1 = 2b/(\text{number of grid mesh in radial direction}) \) and \( n_2 = L_2/(\text{number of grid mesh in } z\text{-direction}) \). The number of grid mesh is arbitrary, but it should produce smoother mesh transition with low computational cost. Lastly, the magnitude of total magnetic field that found within the cavity of the solenoid is simply

\[
B^2_{total} = B^2_{rn} + B^2_{zn}
\]  

(9)

2.2. Experimental methods

Based on standard measurement procedures [8,9], we have developed our own in-house magnetizing fixture testing method where the setup details are given as follows. Measurement of the distribution of magnetic field strength within the cavity of MPØ52 was carried out using impulse current magnetizer K-Series Magnet-Physik Dr. Steingroever GmbH with incorporated single-pole magnetizing fixture MPØ52 (figure 2a) at applied voltage 500 V and applied current 1800 A. Data sampling was performed using DC Gaussmeter model GM2 AlphaLab with ST Axial typed probe (range:0.1–30 kG) on coverage area of 30 mm \( \times \) 22.5 mm over \( r\text{-}z \) plane as indicated in figure 2b. The ST Axial probe was mounted onto customized millimeter gauge measuring tool where its measuring head position can be rolled upside-down or move to left or right direction. Moreover, the 4-multilayer MPØ52 has a closely packed 64 wire loops that being forged in 16 wire loops configuration per layer. The body area between the 60 mm height solenoid and the cylindrical cavity of radius 26 mm is separated by a 4 mm thick insulator. The copper wire used in the fixture has an average diameter of 3 mm. Furthermore, the measurement was taken only at one-half upper area of the fixture because of fixture’s highly symmetric geometry around its \( z \)-axis.

3. Results and discussion

Using specialized mathematical software, the exact solutions of equation (7), equation (8), and equation (9) were each calculated and presented in figure 3. The 2D spatial distribution patterns of
those $B_m(r,z)$, $B_{mz}(r,z)$, and $B_{total}(r,z)$ obtained in our calculations are in excellent agreement with the 3D representative model that previously reported by Lane and his coworkers [10]. From figure 3a, the magnetic field $B_{mz}(r,z)$ is clearly concentrated near the center of the solenoid with a predominant high magnetic flux distribution resides at regions lying between $z = (-26, 0)$ and $(26,0)$. Meanwhile, the magnetic field $B_{m}(r,z)$ depiction in figure 3b is presumed to featuring four quadrants area with two quadrants on the upper right and bottom left represent positive magnetic field strength, while the other quadrants on the opposite side suggest negative magnetic field strength. However, one should remember that $B_{m}(r,z)$ is an odd function. Hence, its real physical interpretation should be concluded based on the absolute value $|B_{mz}(r,z)|$ instead of $B_{m}(r,z)$ where one would find that all quadrants exert positive magnetic field with equal strength (no negative values).

Moreover, based on careful examination on figures 3a and figure 3c, there is a noticeable difference that indicates the yellow-colored region in $B_{total}(r,z)$ is somehow narrower as compared to the one in $B_{m}(r,z)$. This difference is caused by insignificant magnetic field contribution of $B_{m}(r,z)$ to $B_{total}(r,z)$ in the perimeter of $z$ and $r$ axes. This narrowing region (yellow area) brings further consequence in which magnetic material samples that will be magnetized should have dimensions not greater than the area confined by dashed virtual box as depicted in figure 3c. Otherwise, the magnetized magnetic materials would tend to possess varying magnetic strength distribution over its surface. For instance, if someone wants to manage magnetic flux that enters a magnetic material.

**Table 1.** Magnetic field values (in Tesla) obtained from measurement on MPØ52 along $r$-$z$ plane. The simulation data at similar positions are given in bolded italic numbers.

| $r$ (mm) | 0   | 5   | 10  | 15  | 20  |
|----------|-----|-----|-----|-----|-----|
| $z$ (mm) |     |     |     |     |     |
| 0        | 1.53(1.68) | 1.55(1.69) | 1.58(1.73) | 1.61(1.79) | 1.69(1.87) |
| 5        | 1.45(1.66) | 1.44(1.67) | 1.50(1.70) | 1.58(1.76) | 1.66(1.85) |
| 10       | 1.46(1.59) | 1.39(1.60) | 1.50(1.63) | 1.53(1.69) | 1.61(1.78) |
| 15       | 1.35(1.48) | 1.34(1.49) | 1.33(1.52) | 1.45(1.57) | 1.53(1.66) |
| 20       | 1.29(1.34) | 1.29(1.34) | 1.32(1.37) | 1.32(1.41) | 1.38(1.48) |
| 25       | 1.16(1.17) | 1.11(1.18) | 1.16(1.19) | 1.16(1.22) | 1.24(1.26) |
| 30       | 0.99(1.00) | 0.98(1.01) | 1.00(1.01) | 0.98(1.02) | 1.00(1.03) |

**Figure 3.** Magnetic field strength and its spatial distribution area for the cases of (a) $B_{mz}(r,z)$, (b) $B_{m}(r,z)$, and (c) $B_{total}(r,z)$ obtained from equation (7), equation (8), and equation (9), respectively, at $j=1800$ A and fixture dimensions $r = 26$ mm, $D = 3$ mm, and $L_z = 60$ mm. Dashed virtual box shows a coverage area of magnetic flux with uniform magnetic field strength.
sample within ± 1% error gradient based on the information provided in figure 3c, the sample dimension should not exceed 22 mm and 10 mm in length/diameter and thickness, respectively.

To evaluate the accuracy of the model in predicting spatial distribution of magnetic field generated by real single-pole magnetizing fixture, both experimental and simulation data that obtained using similar setting criteria are tabulated in table 1. In general, the values from calculation results could closely match and also follow the increasing and decreasing trends that constructed by experimental data. This can be clearly seen from table 1 that magnetic field strength increases radially outward and also monotonically decreases as it move away from the centre of the cavity (z = 0) along axial symmetry z-axis. Therefore, the model is considered to have a good qualitative analysis in making proper estimation of spatial magnetic strength distribution in single-pole magnetizing fixture.

Finally, the value differences between experimental and simulation data is calculated and presented in figure 4. It is obvious that there is a relatively large error gap between experimental and simulation data. However, a careful analysis shows that calculated magnetic field strength at z ≥ 20 mm tend to have error gap ≤ 7 % while for the cases of z ≤ 20 mm the error gap varies between 7 % – 14 %. There are several possible causes that may lead to discrepancy between experimental and numerical studies. First, please note that the mathematical model employed in this present study is time independent equations while the phenomenon occurs in the experiment could only precisely calculated by time-dependent ones. Second, apart from the model excluding self and mutual inductances, we have treated the winding coils as 1-dimensional point lines that advertently ignore the surface area of the wires. Unfortunately, a fully mathematical treatment that takes into account 3-dimensional geometrical structure of the wires could only be carried out by using numerical finite element method [11,12]. Hence, a moderate margin error between experimental and calculation results using analytical approach might be expected. Therefore, in our next future study the mathematical model would be modified for time varying current case using more robust computational technique such as finite element method.

4. Conclusions
In this study, we have demonstrated that the expanded mathematical model could give a qualitative insight regarding the spatial distribution of magnetic field inside the cavity of single-pole magnetizing fixture. In addition, the information obtained from the simulation results could also be used as a guidance to determine the dimension requirements of magnetic materials prior to magnetizing process in order to ensure magnetic flux with uniform magnetic field strength enter the materials homogeneously. While the model is proven to be highly reliable for providing coarse estimation of spatial distribution of magnetic field, a large percentage error between experimental and calculation results confirms that the steady current based model is less accurate for real application conditions and further improvement to the model by incorporating time varying current phenomenon is highly required.
Acknowledgements
The authors would like to thanks to Research Center for Physics, Indonesian Institute of Sciences (LIPI) for financial support of this work.

References
[1] Eckert P R, Goltz E C and Filho A F F 2014 Sensors 14 13070–87
[2] Lee K M and Son H 2007 IEEE Trans. Magn. 43 3904–13
[3] Petruska A J and Abbott J J 2011 IEEE Trans. Magn. 49 811–9
[4] Smythe W R 1989 Static and Dynamics Electricity ed C. E. Baum (Boca Raton: Taylor and Francis) p 290
[5] Simpson J, Lane J, Immer C and Youngquist R 2001 Simple analytic expressions for the magnetic field of a circular current loop NASA Tech. Rep. Server (NTRS)
[6] Jackson J D 1998 Classical Electrodynamics (Hoboken: John Wiley and Sons) pp 181–3
[7] Byrd P F and Friedman M D 1971 Handbook of Elliptic Integrals for Engineers and Scientists (New York: Springer-Verlag) p 8
[8] ASTM International 2001 Standard Test Method for DC Magnetic Properties of Materials Using Ring and Permeameter Procedures with DC Electronic Hysteresigraphs (West Conshohocken: ASTM International) ASTM A773/A773M-01
[9] ASTM International 1997 Standard Test Method for Magnetic Properties of High-Coercivity Permanent Magnet Materials Using Hysteresigraphs (West Conshohocken: ASTM International) ASTM A977/A977M-02
[10] Lane J E, Youngquist R C, Immer C D and Simpson J C 2001 Magnetic Field, Force, and Inductance Computations for an Axially Symmetric NASA Tech. Rep. Server (NTRS)
[11] Kim P S 2001 Proc. IEEE International Symposium Industrial Electronics Pusan, South Korea, June 2001 (Pistacaway: IEEE Xplore Digital Library) pp 531–6
[12] Nakata T and Takahashi N 1986 IEEE Trans. Magn. 5 526–8