Do organic and other exotic superconductors fail universal scaling relations?

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Universal scaling relations are of tremendous importance in science, as they reveal fundamental laws of nature. Several such scaling relations have recently been proposed for superconductors; however, they are not really universal in the sense that some important families of superconductors appear to fail the scaling relations, or obey the scaling with different scaling pre-factors. In particular, a large group of materials called organic (or molecular) superconductors are a notable example. Here, we show that such apparent violations are largely due to the fact that the required experimental parameters were collected on different samples, with different experimental techniques. When experimental data is taken on the same sample, using a single experimental technique, organic superconductors, as well as all other studied superconductors, do in fact follow universal scaling relations.

In spite of microscopic differences, all superconductors (SC) have one macroscopic property in common: they all conduct electricity without resistance. Therefore, it is not unreasonable to expect manifestations of universal behavior. We show here that when consistent experimental parameters are used, taken on the same sample, with a single experimental technique, all superconductors for which the data exists, indeed follow universal scaling relations 1,2.

Results

Our scaling plots shown in Figs. 1 and 2 currently include: elemental SC (such as Nb and Pb), cuprates (both along and perpendicular to the CuO 2 planes), iron-based SC (both along and perpendicular to iron-ar senic or iron-chalcogenide planes), organic SC (such as quasi-two-dimensional (BEDT-TTF) 2Cu[N(CN)] 2Br materials, alkali-doped fullerenes (such as K 2C 60 and Rb 2C 60), heavy-fermion SC CeCoIn 5, MgB 2, TiN, copper-free oxide SC Ba 1 -x K x BiO 3, negative-U induced SC in Tl,Pb 1 -x Te, Y 2CuJ 2, etc. Further measurements on different SC families, both conventional and unconventional, will serve as the ultimate test as to whether or not these scaling relations are truly universal in nature. (We note in passing that the only superconductor that significantly and systematically deviates from the scaling relations is the p-wave superconductor Sr 2RuO 4. At this moment it is not clear whether this violation is real, or it is due to material and/or experimental issues. It was shown 3 that superconductors in the clean limit do in fact fall to the right of the scaling line, and that might be the case with Sr 2RuO 4. However, we also note that the microwave surface impedance (MW SI) spectra of Sr 2RuO 4 were quite unusual 4, and to extract the penetration depth the authors had to modify the commonly-used fitting procedure. It remains to be seen if this modification also affected the absolute values of penetration depth (λ s).

Soon after superconductivity in the cuprates was discovered, Uemura et al. 5 proposed the first scaling law that related ab-plane superfluid density (or stiffness) ρ s to superconducting critical temperature T c as ρ s ∝ T c. This scaling works for underdoped cuprates, but fails for overdoped samples 6. Other deviations in the cuprates were also reported 7. Moreover, the scaling is not followed by other families of superconductors. Basov et al. 8, on the other hand, studied interplane (c-axis) response of the cuprates and showed that the zero-temperature c-axis effective penetration depth λ c is related to the c-axis DC conductivity just above T c, σ dc as

\[ \lambda_c \propto \sigma_{dc}^{-0.5}. \] (1)

This relation has been shown to be valid in a number of cuprate families.
Dordevic et al. extended this scaling relation [Eq. (1)] to other families of layered SC. What was found based on existing experimental data was that, similar to the cuprates, other layered SC followed similar scaling law, albeit with a different prefactor (Fig. 2 in Ref. 1). The prefactor was argued to be related to the energy scale from which the SC condensate was collected; in the cuprates the condensate was collected from an energy range two orders of magnitude broader than in other families. Alternatively, Schneider interpreted the observed scaling as due to quantum criticality.

Homes et al. proposed a modification to the scaling given by Eq. (1), to include the SC critical temperature $T_c$:

$$\rho_s = \frac{c^2}{\lambda_s} \propto T_c \sigma_{dc},$$

where $c$ is the speed of light. What was found was that all cuprate SC for which the data existed followed the scaling. Surprisingly, both the highly conducting copper-oxygen ($ab$) planes and nearly insulating out-of-plane ($c$ axis) properties followed the same universal scaling line. Moreover, several elemental SC, such as Nb and Pb, also followed the same scaling (Fig. 2 in Ref. 2). More recently, iron-based SC were also shown to follow the same scaling.

However, the so-called organic (or molecular) superconductors failed to provide a convincing data set for the scaling Eq. (2), and were not included in the original plot (Fig. 2 in Ref. 2). Several other families of superconductors, such as dithiocarbamides and heavy fermions, were also not considered for the same reason. It has been argued that organic SC in their most conducting planes follow different scaling laws, such as $T_c \propto \lambda_s^2$.

Below we show that these discrepancies stem mostly from the fact that the required experimental data for Eqs. (1) and (2), namely $T_c$, $\sigma_{dc}$ and $\lambda_s$, were collected on different samples, and more importantly, using different experimental techniques. This introduced significant scatter in data points, and gave the impression that some families of SC did not follow the scaling relations. The superconducting transition temperature $T_c$ is extracted from either DC resistivity or magnetization measurements and its values are fairly reliable and accurate. On the other hand, the experimental values of $\sigma_{dc}$ and $\lambda_s$ can be quite problematic. The values of DC conductivity at the transition $\sigma_{dc}$ and the zero-temperature penetration depth $\lambda_s$ (or alternatively the superfluid density $\rho_s$) can be extracted from a variety of experimental techniques, and in many cases those values are significantly different from each other. These problems seem to be most pronounced in highly-anisotropic SC, such as the cuprates and organic SC.

The DC conductivity at the transition is most directly obtained from transport (resistivity) measurements, but it can also be obtained from infrared (IR) and MW SI measurements, in the $\omega \rightarrow 0$ limit. The values obtained from these spectroscopic techniques are in some cases significantly different from the ones obtained from transport measurements. For example, for the organic compound (TMTSF)$_2$ PF$_6$ along the most conducting $a$ axis Dressel et al. report values obtained from both transport and IR measurements (Table 1 in Ref. 15). The value obtained from the IR measurements is $\sigma_{dc} (\omega \rightarrow 0) \approx 1,100$ $\Omega^{-1}$ cm$^{-1}$, whereas the DC value of conductivity is $\sigma_{dc} \approx 80,000$ $\Omega^{-1}$ cm$^{-1}$, i.e. it is more than 72 times higher. This is an extreme example, but the values for other compounds also show large discrepancies (Table 1 in Ref. 15). Especially challenging are the IR measurements on systems with very small and very large conductivities, and one expects large error bars associated with them.

Similar problems occur with the superfluid density. This quantity can be extracted from optical spectroscopies (IR and MW SI), as well as muon spin resonance ($\mu$SR) measurements. The superfluid density in layered systems along their least conducting direction is usually very small, which is also challenging for IR spectroscopy. Similar to $\sigma_{dc}$, the values of $\lambda_s$ obtained from different experimental techniques can differ significantly. For example, the values for the underdoped $\La_{2-x}\Sr_x\CuO_4$ reported by Panagopoulos et al. were several times smaller that those reported by IR spectroscopy. For for the $x = 0.08$ sample the $\mu$SR value of the penetration depth is 9.2 $\mu$m, whereas the value obtained using IR on the sample with nominally the same doping level is 24.2 $\mu$m (Table 1 in Ref. 17). In this case the IR penetration depth is 2.6 times smaller, which results in superfluid density which is almost 7 times larger [Eq. (2)]. Similar discrepancies are seen in other samples characterized by large anisotropy.

The above examples illustrate the need for consistent data sets, i.e. data obtained on the same sample, with a single experimental technique. Therefore, in our current plots we include only such data points. The only two experimental techniques that can deliver both $\sigma_{dc}$ and $\lambda_s$ simultaneously are IR and MW SI. Whenever possible, we used the data from IR spectroscopy, although in some cases, especially for systems with low $T_c$, as well as systems with very low and very large conductivities, we were forced to use the MW SI data.

In Fig. 1 we re-plot the scaling from Eq. (1), but we now keep only the data points taken on the same sample, with a single experimental

**Figure 1** | Basov scaling plot, Eq. (1). The gray stripe corresponds to $\lambda_s = (45 \pm 25) \sigma_{dc}^{0.5}$. Only data points obtained from optical spectroscopies (IR and MW SI) are included in the plot. The data points are from: cuprates $ab$-plane$^{2,13,25}$, cuprates $c$-axis$^5$, pnictides$^{11,54}$, elements$^2$, TIN$^{32}$, $\Ba_{1-x}\K\BiO_3$$^{35}$, $\MgB_2$$^{28,29}$, organic SC$^{1,19,25}$, fullerenes$^{26}$, heavy fermion $\CeCoIn_5$$^{37}$, negative-U induced SC $\Tl\Pb_{1-x}\Te$ and $\Y_2\C_2\I_2$$^{31}$.

**Figure 2** | Homes’ scaling plot, Eq. (2). The gray stripe corresponds to $\rho_s = (110 \pm 60) T_c \sigma_{dc}$. The data points are the same as in Fig. 1.
Table 1 | Parameters of organic (molecular) superconductors used in Figs. 1 and 2. The data shown is for quasi-2D organic SC, both along their most conducting planes (1–5) and perpendicular to them (6 and 7). Also shown are the parameters for new data points in Figs. 1 and 2.

| label | organic SC | \(\sigma_{dc} \text{ [}\Omega^{-1}\text{cm}^{-1}]\) | \(\lambda_s \text{ [\mu m]}\) | \(T_c \text{ [K]}\) | technique | Reference |
|-------|------------|-------------------|-----------------|-----------------|-----------|-----------|
| 1     | (BEDT-TTF)_2Cu(NCS)_2 | 3800              | 0.8             | 8.6             | MW SI (35 GHz) | 18        |
| 2     | (BEDT-TTF)_2Cu(NCS)_2 | 3700              | 1.4             | 8.3             | MW SI (60 GHz) | 18        |
| 3     | (BEDT-TTF)_2Cu(NCN)_2Br | 4000              | 1.5             | 11.3            | MW SI       | 18        |
| 4     | (BEDT-TTF)_2Cu(NCN)_2Br | 13150             | 0.322           | 11              | MW SI       | 19        |
| 5     | (BEDT-TTF)_2Cu(NCN)_2Br | 25                | 6               | 8               | IR         | 25        |
| 6     | (BEDT-TTF)_2Cu(NCN)_2Br | 6.4               | 38              | 11.3            | MW SI       | 18        |
| 7     | (BEDT-TTF)_2Cu(NCN)_2S | 4                 | 40              | 8.3             | MW SI       | 18        |

Table 1 is followed by references to the literature, which are not included in this text. The table includes several parameters for organic superconductors, such as the superfluid density \(\lambda_s\), the superconducting transition temperature \(T_c\), and the technique used for the measurements. The data is for both quasi-2D organic SC parallel to the conducting planes and perpendicular to them.

In summary, the data shown is for quasi-2D organic SC, both along their most conducting planes (1–5) and perpendicular to them (6 and 7). Also shown are the parameters for new data points in Figs. 1 and 2.

Discussion

Possible theoretical explanation of the observed scaling is a work in progress, but some existing proposals are worth mentioning. Tallon et al. argued that the scaling can be explained using a dirty limit picture in which the energy gap scales with \(T_c^{2/3}\). However, it is well known that many superconductors on the scaling plot are not in the dirty limit. In fact, many of them are in the clean limit, and some of them have even shown quantum oscillations. This issue of "dirtiness" in superconductors has been discussed before. Zaanen argued that the superconducting transition temperature in cuprates is high because the normal state in these systems is as viscous as is allowed by the laws of quantum mechanics. Zaanen also introduced the notion of Plankian dissipation in the cuprates. However, this proposal does not explain why all superconductors, not just the cuprates, follow the same scaling. Imry et al. demonstrated that the scaling may be recovered in an inhomogeneous superconductor in the limit of small intergrain resistance in a simple granular superconductor model. The scaling relation Eq. (2) has also been derived using the gauge/gravity duality for a holographic superconductor.

In summary, we have shown that when consistent data sets are used, all superconductors for which the data sets exist do indeed follow universal scaling relations that span more than seven orders of magnitude. Future experiments on other (exotic) SC will serve as important test of validity of scaling relations, and will verify if they are truly universal.

Methods

Data points shown in Figs. 1 and 2 are collected from different literature sources, either IR or MW SI measurements. Those two experimental techniques can simultaneously deliver the two parameters needed for scaling Eq. (2), namely the optical conductivity at \(T_c\), \(\sigma_{dc} = \sigma_0(\omega 
olimits\rightarrow 0)\), and the superfluid density \(\rho_s\) (or the penetration depth \(\lambda_s\)). This selection assures that the required parameters were collected on the same sample, in a single measurement, without the use of contacts.

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S.V.D. supervised the project and wrote the manuscript; D.N.B. and C.C.H. helped write the manuscript.

Additional information
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