Distance priors from Planck final release

Lu Chen, a,b Qing-Guo Huang a,b,c,d and Ke Wang e

aCAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
bSchool of Physical Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, China
cSynergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, 36 Lushan Lu, Changsha 410081, China
dCenter for Gravitation and cosmology, College of Physical Science and Technology, Yangzhou University, 88 South University Ave., Yangzhou 225009, China
eNational Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Beijing 100012, China

E-mail: chenlu@itp.ac.cn, huangqg@itp.ac.cn, wangke@itp.ac.cn

Received December 5, 2018
Revised January 28, 2019
Accepted February 1, 2019
Published February 14, 2019

Abstract. We present the distance priors from the finally released Planck TT, TE, EE + lowE data in 2018. The uncertainties are around 40% smaller than those from Planck 2015 TT+lowP. In order to check the validity of these new distance priors, we adopt the distance priors to constrain the cosmological parameters in different dark energy models, including the ΛCDM model, the wCDM model and the CPL model, and conclude that the distance priors provide consistent constraints on the relevant cosmological parameters compared to those from the full Planck 2018 data release.

Keywords: cosmological parameters from CMBR, dark energy theory

ArXiv ePrint: 1808.05724
1 Introduction

Since two supernova surveys reported the discovery of cosmic acceleration independently in 1998 [1, 2], a new component except from matter and radiation, named dark energy (DE) [3], is required assuming the general relativity (GR) remains correct for our universe. DE is the mathematically simplest explanation to the accelerating expansion of the universe, but its nature is still a puzzle. Several methods can be used to give constraints on the properties of DE. A straight-forward method is the distance measurement, such as the direct determination of $H_0$ [4], surveys on Type Ia Supernovae (SNe) [5, 6] and the baryon acoustic oscillation (BAO) measurements [7]. They provide absolute or relative distance measurements in a narrow range of redshift with percent level uncertainties. Obviously, the narrow detecting range of redshift restricts the validity of exploring the evolution of DE in full range of redshift. Moreover, the uncertainties increase as the redshift gets higher. DE is a component with negative pressure, which produces a force of repulsion, and affects the galaxy clustering. Then we can explore it using gravitational lensing [8], clusters of galaxies [9], redshift-space distortions (RSD) [10] and the Alcock-Paczynski (AP) effect [11]. However, our limited knowledge about the structure formation brings a big challenge in the specific processing. We can also use the cosmic microwave background (CMB) [12, 13] to constrain DE properties because DE plays an important role in the matter constitution and leaves footprints on the late-time power spectra of CMB. This method breaks the limitation of redshifts and possesses high-accuracy. Combining other measurements, a spatially flat $\Lambda$CDM model remains a convincing model. But it also has limitations that it requires the full Boltzmann analysis [14–16], which is really a very time-consuming process, especially as the CMB data set has become even larger with the final release. More importantly, equations for linear density perturbations in some DE models are difficult to build [17, 18]. As a result, the method of distance priors [19–21] are proposed to be a compressed likelihood to substitute the full Boltzmann analysis of CMB.

Since the first data release in 2013 [22], Planck satellite provides CMB data with high accuracy. Although the preliminary observations of $TE, EE$ power spectrum at high multipoles were released in Planck 2015 [23], this data release laid emphasis on the temperature power spectrum. Recently, Planck collaboration release the final data of the CMB anisotropies (hereafter Planck 2018) [13]. Since improved measurements of low-$l$ polarization allow the reionization optical depth to be determined with higher precision compared
to Planck 2015, there are significant gains in the precision of some parameters which are correlated with the reionization optical depth. Due to improved modelling of the high-l polarization, moreover, there are more robust constraints on many parameters which will be affected by residual modelling uncertainties only at the 0.5σ level. The constraints on the distance priors given in “Planck Blue Book” [24] are about 50% smaller than those given by Planck 2015 TT+lowP [25]. All in all, it is meaningful to update the distance priors with the full-mission Planck measurement of CMB.

Following the previous work in [26], we update the distance priors with Planck 2018 and present the constraints on several DE models with these new distance priors. This paper is organized as follows. In section 2.1, we show our methodology to reconstruct the distance priors from Planck 2018 chains. Then the new distance priors are presented in section 2.2. In section 2.3, we check our results in several different DE models. Concretely, we constrain the equation of state of DE from distance priors and compare our results with those by fitting the full data of Planck 2018 release. A brief summary is given in section 3. In addition, we provide a note on how to use the distance priors in the CosmoMC package in the appendix which should be quite useful for the readers.

2 Distance priors from Planck 2018 data and constraints on DE models

2.1 Methodology

The distance priors provide effective information of CMB power spectrum in two aspects: the acoustic scale $l_A$ characterizes the CMB temperature power spectrum in the transverse direction, leading to the variation of the peak spacing, and the “shift parameter” $R$ influences the CMB temperature spectrum along the line-of-sight direction, affecting the heights of the peaks.

We adopt the popular definitions of the distance priors as follows [12]:

$$l_A = (1 + z_s) \frac{\pi D_A(z_s)}{r_s(z_s)}, \quad (2.1)$$

$$R(z_s) \equiv \frac{(1 + z_s)D_A(z_s)\sqrt{\Omega_m H_0^2}}{c}, \quad (2.2)$$

where $z_s$ is the redshift at the photon decoupling epoch. Here we use the values of $z_s$ given by the Planck 2018 chains. $r_s$ is the comoving sound horizon, defined by

$$r_s(z) = c \frac{1/(1+z)}{H_0} \frac{da}{a^2 E(a) \sqrt{3 \left(1 + \frac{3\Omega_b h^2}{4\Omega_m h^2} a\right)}},$$

$$\frac{3}{4\Omega_\gamma h^2} = 31500(T_{CMB}/2.7K)^{-4}, \quad T_{CMB} = 2.7255K. \quad (2.3)$$

And the angular diameter distance $D_A$ is given by

$$D_A = \frac{c}{(1 + z)H_0 \sqrt{|\Omega_k|}} \sinh \left[\Omega_k^{1/2} \int_0^z \frac{dz'}{E(z')}\right], \quad (2.4)$$
where \( \sinh(n(x)) \equiv \sin(n(x)), x, \sinh(n(x)) \) for \( \Omega_k < 0, \Omega_k = 0, \Omega_k > 0 \) respectively. Here \( E(z) \) is \( E(z) \equiv H(z)/H_0 \), i.e.

\[
E(z) = \left[ \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{de}\frac{\rho_{de}(z)}{\rho_{de}(0)} \right]^{\frac{1}{2}},
\]

(2.5)

where \( \Omega_r \) is the present fractional radiation density

\[
\Omega_r = \frac{\Omega_m}{1+z_{eq}}, \quad z_{eq} = 2.5 \times 10^4 \Omega_m h^2 (T_{CMB}/2.7K)^{-4}.
\]

For \( \Lambda \)CDM and \( w \)CDM models where \( w \) is a constant, \( \rho_{de}(z)/\rho_{de}(0) \) equals 1 and \((1+z)^{3(1+w)}\), respectively.

### 2.2 Results

In this section, we derive the distance priors in several different models using Planck 2018 TT, TE, EE + lowE which is the latest CMB data from the final full-mission Planck measurement [13]. We get the chains of \( l_A \) and \( R \) from the public Planck chains\(^1\) under the corresponding models released on 17 July 2018 with eqs. (2.1) and (2.2), then marginalize over the parameters except for \( \{R,l_A,\Omega_b h^2, n_s\} \). Distance priors are usually used to research the late-time universe expansion, so we present \( \Omega_b h^2 \) too. The scalar spectral index \( n_s \) is shown for the convenience of studying the matter power spectrum.

The main results are shown in table 1 where we list their 68% C.L. constraints and correlation matrices on the base \( \Lambda \)CDM model and its three 1-parameter extended models. From the results of the base \( \Lambda \)CDM model and the \( w \)CDM model, we can see that the distance priors are stable effective observables. Considering that the geometric degeneracy can be broken up significantly by the smoothing effect of CMB lensing on the power spectrum, which is scaled by the weak lensing parameter \( A_L \) [27], we also provide the constraints on the \( \Lambda \)CDM+\( \Omega_k \) model and the \( \Lambda \)CDM+\( A_L \) model. Comparing to the previous two models, the restrictions on \( R \) are weaker over 10% and those on \( A_L \) are slightly weaker too, which is consistent with the theoretical prediction.

In figure 1, we compare the constraints on the distance priors \( R, l_A \), as well as \( \Omega_b h^2 \) and \( n_s \) in the base \( \Lambda \)CDM model derived from Planck 2015 TT+lowP and Planck 2018 TT, TE, EE+lowE. Obviously, the constraints from the Planck final data release in 2018 are significantly improved. Actually, the errors from Planck 2018 TT, TE, EE+lowE are around 40% smaller than those from Planck 2015 TT+lowP.

### 2.3 Constraints on Dark Energy models using distance priors

In this section, we use the distance priors derived for the base \( \Lambda \)CDM model in the former subsection to constrain the cosmological parameters in the base \( \Lambda \)CDM model, the \( w \)CDM model and the CPL model [28, 29], and compare our results with the Planck 2018 results to test the validity of the distance priors given in this paper.

\(^1\)From the Planck 2018 release, four sets of chains named as base_plikHM_TTTEEE_low_lowlowE, base_w_plikHM_TTTEEE_low_lowlowE, base_omegak_plikHM_TTTEEE_low_lowlowE and base_Alens_plikHM_TTTEEE_low_lowlowE are used in our paper to generate table 1.

And chains named base_plikHM_TTTEEE_low_lowlowE, base_w_plikHM_TTTEEE_low_lowlowE_BAO and base_wwa_plikHM_TTTEEE_low_lowlowE_BAO are used to generate the contours in section 2.3.
where $\tilde{z}$ matrix. Here we use the approximate formula of $C_{l}$ from the correlation matrix of $\Lambda$CDM model listed in table 1 [31]. ($l_x$ correlation matrix for from $\Lambda$CDM+ $\Lambda$CDM+$\Omega_b$ is given by $A_{\text{Planck}}$. The 68% C.L. limits for $\Lambda$CDM+$\Omega_b$ are set to their mean values and $C_{l}$ temperature Commander likelihood and the low-$l$ temperature Commander likelihood and the low-$l$ temperature Commander likelihood [13].

We modify the MCMC chains package CosmoMC [30] by adding $\chi^2_{\text{distance priors}}$, which is given by

$$
\chi^2_{\text{distance priors}} = \sum (x_i - d_i)(C^{-1})_{ij}(x_j - d_j),
$$

where $x_i = \{R(z_*), l_A(z_*), \Omega_b h^2\}$ are values predicted in different DE models, $d_i = \{R_{\text{Planck}}^{\Lambda}, l_A^{\text{Planck}}, \Omega_b h^2_{\text{Planck}}\}$ are set to their mean values and $C_{ij}$ is their covariance matrix derived from the correlation matrix of $\Lambda$CDM model listed in table 1 [31]. ($C^{-1})_{ij}$ means its inverse matrix. Here we use the approximate formula of $z_*$ to calculate $x_i$ [32]

$$
z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}],
$$

where

$$
g_1 = \frac{0.0738(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}},
$$

$$
g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}.
$$

| $\Lambda$CDM | Planck TT, TE, EE + lowE | $R$ | $l_A$ | $\Omega_b h^2$ | $n_s$ |
|-------------|------------------------|-----|-------|---------------|------|
| $1.7502 \pm 0.0046$ | $1.0$ | $0.46$ | $-0.66$ | $-0.74$ |
| $l_A$ | $0.301.471 \pm 0.089$ | $0.46$ | $1.0$ | $-0.33$ | $-0.35$ |
| $\Omega_b h^2$ | $0.02236 \pm 0.00015$ | $-0.66$ | $-0.33$ | $1.0$ | $0.46$ |
| $n_s$ | $0.9649 \pm 0.0043$ | $-0.74$ | $-0.35$ | $0.46$ | $1.0$ |

| $w$CDM | Planck TT, TE, EE + lowE | $R$ | $l_A$ | $\Omega_b h^2$ | $n_s$ |
|--------|------------------------|-----|-------|---------------|------|
| $1.7493 \pm 0.0046$ | $1.0$ | $0.47$ | $-0.66$ | $-0.71$ |
| $l_A$ | $0.301.462 \pm 0.089$ | $0.47$ | $1.0$ | $-0.34$ | $-0.36$ |
| $\Omega_b h^2$ | $0.02239 \pm 0.00015$ | $-0.66$ | $-0.34$ | $1.0$ | $0.44$ |
| $n_s$ | $0.9653 \pm 0.0043$ | $-0.72$ | $-0.36$ | $0.44$ | $1.0$ |

| $\Lambda$CDM+$\Omega_k$ | Planck TT, TE, EE + lowE | $R$ | $l_A$ | $\Omega_b h^2$ | $n_s$ |
|----------------|------------------------|-----|-------|---------------|------|
| $1.7429 \pm 0.0051$ | $1.0$ | $0.54$ | $-0.75$ | $-0.79$ |
| $l_A$ | $0.301.409 \pm 0.091$ | $0.54$ | $1.0$ | $-0.42$ | $-0.43$ |
| $\Omega_b h^2$ | $0.02260 \pm 0.00017$ | $-0.75$ | $-0.42$ | $1.0$ | $0.59$ |
| $n_s$ | $0.9706 \pm 0.0047$ | $-0.79$ | $-0.43$ | $0.59$ | $1.0$ |

| $\Lambda$CDM+$A_L$ | Planck TT, TE, EE + lowE | $R$ | $l_A$ | $\Omega_b h^2$ | $n_s$ |
|----------------|------------------------|-----|-------|---------------|------|
| $1.7428 \pm 0.0053$ | $1.0$ | $0.52$ | $-0.72$ | $-0.80$ |
| $l_A$ | $0.301.406 \pm 0.090$ | $0.52$ | $1.0$ | $-0.41$ | $-0.43$ |
| $\Omega_b h^2$ | $0.02259 \pm 0.00017$ | $-0.72$ | $-0.41$ | $1.0$ | $0.58$ |
| $n_s$ | $0.9707 \pm 0.0048$ | $-0.80$ | $-0.43$ | $0.58$ | $1.0$ |

Table 1. The 68% C.L. limits for $R$, $l_A$, $\Omega_b h^2$ and $n_s$ in different cosmological models and their correlation matrix for from Planck 2018 TT, TE, EE + lowE. Notice that the Planck collaboration use Planck TT, TE, EE + lowE to represent the combination of the combined likelihood of TT, TE, EE spectra at $l \geq 30$, the low-$l$ temperature Commander likelihood and the low-$l$ SimAll EE likelihood [13].
In the base $\Lambda$CDM model, we constrain the set of parameters $\{\Omega_m, H_0, \Omega_b h^2\}$. In figure 2, we show the comparison of our results from distance priors and the global fitting results from Planck 2018. Clearly, the contours are almost overlapping, which indicates that the distance priors can take place of full Planck released data effectively.

To constrain the equation of state of DE better, we combine the distance priors for the base $\Lambda$CDM model with the low redshift Baryon Acoustic Oscillation (BAO) measurements. We use the 6dFGS [33], SDSS-MGS [34] and the final DR12 anisotropic BAO data [35] at $z = 0.106, 0.15, 0.38, 0.51, 0.61$. Constraints on the parameters set $\{\Omega_m, H_0, \Omega_b h^2, w\}$ in the $w$CDM model and $\{\Omega_m, H_0, \Omega_b h^2, w, w_a\}$ in the CPL model are shown in figure 3 and figure 4 respectively. We can see that our results from distance priors for the base $\Lambda$CDM model and BAO measurements are consistent with those from Planck 2018.
Figure 2. Constraints on parameters in the base $\Lambda$CDM model. The Planck 2018 chains gives the red dashed contours and the black solid ones are our results from distance priors for the base $\Lambda$CDM model.

3 Summary

In this work, we update the distance priors from the final release of the Planck collaboration in the base $\Lambda$CDM model, the $w$CDM model, the $\Lambda$CDM+$\Omega_k$ model and the $\Lambda$CDM+$A_L$ model. We give their mean values and the correlation matrices. Our new constraints on the distance priors are about 40% tighter than those from Planck 2015 TT+lowP [25]. Compared to our previous work [26] based on the Planck data release in 2015, our new results are slightly improved and $R$ in the base $\Lambda$CDM model gives the best improvement about 8%.

We also check our results in the base $\Lambda$CDM model with the distance priors and constrain the related parameters in the $w$CDM model and the CPL model combining the low redshift BAO measurements. In all of these three DE models, we obtain quite similar constraints compared to Planck 2018 release. It indicates that the distance priors derived from the base $\Lambda$CDM model can be used to replace the global fitting of full data released by Planck in 2018 for the other DE models. As for the other non-DE models, our conservative suggestion is that the distance priors derived from the base $\Lambda$CDM model is no longer the good substitute for the global fitting of full CMB data. For example, the distance priors derived from the base $\Lambda$CDM model are different from that of the other two extended models: $\Lambda$CDM+$\Omega_k$ and $\Lambda$CDM+$A_L$. 
Figure 3. Constraints on parameters in the $w$CDM model. The Planck 2018 chains gives the red dashed contours and the black solid ones are our results from distance priors for the base $\Lambda$CDM model.

In order to understand the nature of DE better, we can combine distance priors with more distance measurements. Another effective way to improve the results is to minimize the uncertainties of CMB experiments. We expect dramatic progress with the fourth-generation of CMB experiments. For example, CORe (Cosmic Origins Explorer) [37] is designed to determine the cosmological model about 2–3 times better than the Planck satellite.

Acknowledgments

We acknowledge the use of HPC Cluster of ITP-CAS. This work is supported by grants from NSFC (grant No. 11335012, 11575271, 11690021, 11747601), Top-Notch Young Talents Program of China, and partly supported by the Strategic Priority Research Program of CAS and Key Research Program of Frontier Sciences of CAS.
Figure 4. Constraints on parameters in the CPL model. The Planck 2018 chains gives the red dashed contours and the black solid ones are our results from distance priors for the base ΛCDM model.

A Note on using distance priors in the CosmoMC package

In this appendix, we give a note about how to use the distance priors in the CosmoMC package (version: April 2015). Taking figure 2 for example, the modified files are listed bellow.

1. Import the results about the distance priors in table 1.

   (i) Add the path of the distance priors parameter file named distance.ini in the CosmoMC input file test.ini.
(ii) Create the distance priors parameter file distance.ini. The mean values of distances priors and $\Omega_b h^2$ should be included.

```
~/cosmomc/batch2/distance.ini:
```

```bash
use_distance=T
redo_no_new_data =T
redo_add=T
redo_likeoffset = 0
prior_name = base_plikHM_TTTEEE_lowE
r=1.750235
la=301.4707
omegabh2=0.02235976

class [omegam]= 0.31 0 1 0.01 0.005
param[H0] = 68 20 100 0.1 0.1
param[omegak] = 0
param[mnu] = 0.06
param[w] = -1
param[wa] = 0
param[mmbh]= 3.046
param[ombh2] = 0.0222 0.005 0.1 0.0001 0.0002
num_massive_neutrinos=1
distance_invcov_file = %DATASETDIR%Distance_invcov.txt
highL_theory_cl_template = %DATASETDIR%HIGH_lensedCls.dat
```

(iii) Create a file of the inverse matrix of the covariance matrix and put it in the data folder of CosmoMC package.

```
~/cosmomc/data/Distance_invcov.txt:
```

```
# PLA 2018 base_plikHM_TTTEEE_lowE
# R 1_a a%omegabh2
 94392.3971 -1360.4913 1664517.2916
-1360.4913 161.4349 3671.6180
1664517.2916 3671.6180 79719182.5162
```

2. Set parameterization=background because we will run chains without the CMB calculation of the package and modified the background parameters as follows.

(i) ```
~/cosmomc/source/driver.F90:
```

```f90
!call Setup%Config%SetTheoryParameterization(Ini, BaseParams% NameMapping,'theta')
call Setup%Config%SetTheoryParameterization(Ini, BaseParams% NameMapping,'background')
```

(ii) Specify the list of the background parameters.

```
~/cosmomc/paramnames/params_background.paramnames:
```

| Parameter | Description |
|-----------|-------------|
| omegam    | \Omega_m   |
| H0        | H_0        |
| omegak    | \Omega_K   |
| mnu       | \Sigma m_\nu |
| w         | w           |
| wa        | w_a        |
| nnu       | N_\{\rm eff\} |
| ombh2     | \Omega_b h^-2 |

(iii) Define the background parameters in the subroutine `BK_ParamArrayToTheoryParams` in the same order of the file `params_background.paramnames`.

```
~/cosmomc/source/CosmologyParameterizations.f90:
```

```fortran
subroutine BK_ParamArrayToTheoryParams (this , Params , CMB)
  class ( BackgroundParameterization ) :: this
  real ( mcp ) Params (:)
  class ( TTheoryParams ), target :: CMB
  real ( mcp ) omegam , h2
  select type ( CMB )
    class is ( CMBParams )
      omegam = Params (1)
      CMB%H0 = Params (2)
      CMB%omk = Params (3)
      CMB%omnuh2=Params (4)/neutrino_mass_fac*(standard_neutrino_neff/3)**0.75_mcp
      CMB%w = Params (5)
      CMB%wa = Params (6)
      CMB%nnu = Params (7)
      CMB%h=CMB%H0/100
      h2 = CMB%H**2
      CMB%Yhe=0.24
      CMB%omnu = CMB%omnuh2/h2
      CMB%omegam =omegam
      CMB%omegamh2=CMB%omegam*h2
      CMB%ombh2=Params (8)
      CMB%omb=CMB%ombh2/h2
      CMB%omc= CMB%omegam - CMB%omnu - CMB%omb
      CMB%omch2 = CMB%omc*h2
      CMB%zre=0
      CMB%tau=0
      CMB%omdmh2 = CMB%omch2+ CMB%omnuh2
      CMB%omdm = CMB%omdmh2/h2
      CMB%omv = 1- CMB%omk - CMB%omb - CMB%omdm
      CMB%nufrac=CMB%omnuh2/CMB%omdmh2
      CMB%reserved=0
      CMB%fdm=0
      CMB%iso_cdm_correlated=0
      CMB%Alens=1
  end select
end subroutine BK_ParamArrayToTheoryParams
```
3. Add the likelihood of the distance priors and call it in the program.

   (i) Create a new likelihood file named distance.f90 in the source folder. It is the main file including reading the mean values of distance priors and the inverse matrix mentioned before, as well as calculating $\chi^2_{\text{distance priors}}$.

   ~/cosmomc/source/distance.f90:

   module distance
   use CosmologyTypes
   use MatrixUtils
   use LikelihoodCosmology
   implicit none
   private

   type, extends(TCosmoCalcLikelihood) :: DistanceLikelihood
   real(mcp) :: R, la, omegabh2, ns
   real(mcp), allocatable, dimension(:, :) :: distance_invcov
   contains
   procedure :: LogLikeTheory => Distance_LnLike
   end type DistanceLikelihood

   public DistanceLikelihood, DistanceLikelihood_Add
   contains

   subroutine DistanceLikelihood_Add(LikeList, Ini)
   class(TLikelihoodList) :: LikeList
   class(TSettingIni) :: ini
   Type(DistanceLikelihood), pointer :: this
   character(LEN=:), allocatable :: distance_invcov_file
   if (Ini%Read_Logical('use_distance', .false.)) then
     allocate(this)
     this%LikelihoodType = 'distance'
     this%name = Ini%Read_String('prior_name')
     this%R = Ini%Read_Double('r')
     this%la = Ini%Read_Double('la')
     this%omegabh2 = Ini%Read_Double('omegabh2')
     this%needs_background_functions = .true.
     call LikeList%Add(this)
     allocate(this%distance_invcov(3,3))
     this%distance_invcov = 0
   end if
   if (Ini%HasKey('distance_invcov_file')) then
     write(*,*) 'start to read distance_invcov_file'
     distance_invcov_file = Ini%ReadFileName('distance_invcov_file')
     call File%ReadTextMatrix(distance_invcov_file, this%distance_invcov)
     write(*,*) 'successfully read distance_invcov_file'
   else
     write(*,*) 'ERROR: distance_invcov_file'
   end if
   end subroutine DistanceLikelihood_Add

   real(mcp) function Distance_LnLike(this, CMB)
   use constants, only : c, const_pi
   Class(DistanceLikelihood) :: this

   -- 11 --
Class (CMBParams) CMB
real (mcp), dimension(3,3) :: invC
real (mcp), dimension(3) :: x, d
real (mcp) :: R, l_a, z_star, g_1, g_2

g_1 = 0.0783 D0 *( CMB % ombh2 ) **( -0.238 D0) /(1.0 D0 + 39.5 D0 *( CMB % ombh2 ) **0.763 D0)
g_2 = 0.560 D0 /(1.0 D0 + 21.1 D0 *( CMB % ombh2 ) **1.81 D0)
z_star = 1048.0 D0 *(1.0 D0 + 0.00124 D0 *( CMB % ombh2 ) **( -0.738 D0)) *(1.0 D0 + g_1 *( CMB % omegamh2 )** g_2)
R = CMB % H0 *( CMB % omegam ) **(0.5 d0) * this % Calculator % AngularDiameterDistance(z_star) *(1.0 D0 + z_star )/c *1000.0 D0
l_a = const_pi / this % Calculator % CMBToTheta(CMB)
invC = this % distance_invcov
d (1) = this % R
d (2) = this % l_a
d (3) = this % omegabh2
!d (4) = this % ns
x (1) = R
x (2) = l_a
x (3) = CMB % ombh2
Distance_LnLike = DOT_PRODUCT ((x-d), MATMUL (invC ,(x-d))) /2.0 d0
end function Distance_LnLike
end module distance

(ii) Call the likelihood of distance priors in DataLikelihoods.f90.
~/cosmomc/source/DataLikelihoods.f90:
... use distance ...
call DistanceLikelihood_Add(DataLikelihoods, Ini) ...

(iii) Modified the Makefile in the source folder and include the new likelihood to make sure the program can be compiled successfully.
~/cosmomc/source/Makefile:
... DATAMODULES = $( PLANCKLIKEFILES ) $( OUTPUT_DIR )/mpk.o $( OUTPUT_DIR )/wigglez.o \ $( OUTPUT_DIR )/bao.o $( SUPERNOVAE ) $( SZ ) $( OUTPUT_DIR )/supernovae.o $( OUTPUT_DIR )/HST.o $( OUTPUT_DIR )/CMB.o $( OUTPUT_DIR )/CMBlikes.o $( OUTPUT_DIR )/ElementAbundances.o $( OUTPUT_DIR )/distance.o ...
$( OUTPUT_DIR )/distance.o: $( OUTPUT_DIR )/Likelihood_Cosmology.o ...

4. Sometimes people want or have to combine the distance priors with BAO measurements. In these cases, $r_s(z_d)$ is given by following code where the baryon drag epoch $z_d$ is given by Eisenstein & Hu [36], instead of camb.

(i) ~/cosmomc/source/bao.f90:
... real (mcp) function get_rs_drag(this , CMB, Theory)
class (TBAOLikelihood) :: this
class (CMBParams) CMB
Class (TCosmoTheoryPredictions), target :: Theory
if (BAO_fixed_rs > 0) then
    ! this is just for use for e.g. BAO 'only' constraints
    get_rs_drag = BAO_fixed_rs
else
    ! get_rs_drag = Theory%derived_parameters( derived_rdrag )
    get_rs_drag = this%Calculator%distanceR(CMB)*148.92D0/153.017D0
end if
end function

(ii) ~/cosmomc/source/Calculator_Cosmology.f90:

... procedure :: ...
procedure :: distanceR
procedure :: ...

real (mcp) function distanceR(this, CMB)
class (TCosmologyCalculator) :: this
class (CMBParams) CMB
call this%ErrorNotImplemented('distanceR')
distanceR = 0
end function distanceR

(iii) ~/cosmomc/source/Calculator_CAMB.f90:

... procedure :: ...
procedure :: distanceR => CAMBCalc_distanceR
procedure :: ...

function CAMBCalc_distanceR(this, CMB) result (distanceR)
use ModelParams
class (CAMB_Calculator) :: this
class (CMBParams) CMB
real (mcp) distanceR
distanceR = distanceOfR()
end function CAMBCalc_distanceR

(iv) ~/cosmomc/camb/modules.f90:

module ModelParams
...
function distanceOfR()
real (dl) zdrag, adrag, atol
real (dl) distanceOfR
real (dl) obh2, omh2, b1, b2
real (dl) rombint
external rombint
obh2=CP%omegab*(CP%h0/100.0d0)**2
omh2=(CP%omegab+CP%omegac+CP%omegan)*(CP%h0/100.0d0)**2
b1 = 0.313D0*omh2**(-0.419D0)*(1.0D0+0.607D0*omh2**0.674D0)
b2 = 0.238D0*omh2**0.223D0
zdrag = 1291.0D0*omh2**0.251D0*(1.0D0+b1*obh2**b2)/(1.0D0+0.659D0*omh2**0.828D0)
adrag = 1.0D0/(1.0D0+zdrag)
atol = 1e-6

- 13 -
distanceOfR = rombint (dsound_da, 1d-8, adrag, atol)
end function distanceOfR
end module ModelParams

Now you can compile the program and run './cosmomc test.ini' for test.

References

[1] Supernova Search Team collaboration, Observational evidence from supernovae for an accelerating universe and a cosmological constant, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201] [inSPIRE].

[2] Supernova Cosmology Project collaboration, Measurements of Ω and Λ from 42 high redshift supernovae, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133] [inSPIRE].

[3] P.J.E. Peebles and B. Ratra, The Cosmological constant and dark energy, *Rev. Mod. Phys.* **75** (2003) 559 [astro-ph/0207347] [inSPIRE].

[4] A.G. Riess et al., A 2.4% Determination of the Local Value of the Hubble Constant, *Astrophys. J.* **826** (2016) 56 [arXiv:1604.01424] [inSPIRE].

[5] SNLS collaboration, Supernova Constraints and Systematic Uncertainties from the First 3 Years of the Supernova Legacy Survey, *Astrophys. J. Suppl.* **192** (2011) 1 [arXiv:1104.1443] [inSPIRE].

[6] Supernova Cosmology Project collaboration, The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above z > 1 and Building an Early-Type-Hosted Supernova Sample, *Astrophys. J.* **746** (2012) 85 [arXiv:1105.3470] [inSPIRE].

[7] 2dfGRS collaboration, The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications, *Mon. Not. Roy. Astron. Soc.* **362** (2005) 565 [astro-ph/0501174] [inSPIRE].

[8] DES collaboration, The dark energy survey, *AIP Conf. Proc.* **878** (2006) 213 [inSPIRE].

[9] LSST Dark Energy Science collaboration, LSST and the Dark Energy Science collaboration, *PoS (EPS-HEP2017) 045* (2017) [inSPIRE].

[10] H. Gil-Marín et al., The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: RSD measurement from the LOS-dependent power spectrum of DR12 BOSS galaxies, *Mon. Not. Roy. Astron. Soc.* **460** (2016) 4188 [arXiv:1509.06386] [inSPIRE].

[11] C. Alcock and B. Paczynski, An evolution free test for non-zero cosmological constant, *Nature* **281** (1979) 358 [inSPIRE].

[12] WMAP collaboration, Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, *Astrophys. J. Suppl.* **180** (2009) 330 [arXiv:0803.0547] [inSPIRE].

[13] Planck collaboration, Planck 2018 results. VI. Cosmological parameters, arXiv:1807.06209 [inSPIRE].

[14] R. Bean and O. Dore, Probing dark energy perturbations: The Dark energy equation of state and speed of sound as measured by WMAP, *Phys. Rev. D* **69** (2004) 083503 [astro-ph/0307100] [inSPIRE].

[15] J. Weller and A.M. Lewis, Large scale cosmic microwave background anisotropies and dark energy, *Mon. Not. Roy. Astron. Soc.* **346** (2003) 987 [astro-ph/0307104] [inSPIRE].
[16] H. Li, J.-Q. Xia, G.-B. Zhao, Z.-H. Fan and X. Zhang, On using the WMAP distance priors in constraining the time evolving equation of state of dark energy, *Astrophys. J.* 683 (2008) L1 [arXiv:0805.1118] [inSPIRE].

[17] G.R. Dvali and G. Gabadadze, Gravity on a brane in infinite volume extra space, *Phys. Rev. D* 63 (2001) 065007 [hep-th/0008054] [inSPIRE].

[18] K. Koyama, Structure formation in modified gravity models alternative to dark energy, *JCAP* 03 (2006) 017 [arXiv:0805.1118] [inSPIRE].

[19] J.R. Bond, G. Efstathiou and M. Tegmark, Forecasting cosmic parameter errors from microwave background anisotropy experiments, *Mon. Not. Roy. Astron. Soc.* 291 (1997) L33 [astro-ph/9702100] [inSPIRE].

[20] G. Efstathiou and J.R. Bond, Cosmic confusion: Degeneracies among cosmological parameters derived from measurements of microwave background anisotropies, *Mon. Not. Roy. Astron. Soc.* 304 (1999) 75 [astro-ph/9807103] [inSPIRE].

[21] Y. Wang and P. Mukherjee, Observational Constraints on Dark Energy and Cosmic Curvature, *Phys. Rev. D* 76 (2007) 103533 [astro-ph/0703780] [inSPIRE].

[22] PLANCK collaboration, Planck 2013 results. XVI. Cosmological parameters, *Astron. Astrophys.* 571 (2014) A16 [arXiv:1303.5076] [inSPIRE].

[23] PLANCK collaboration, Planck 2015 results. XIII. Cosmological parameters, *Astron. Astrophys.* 594 (2016) A13 [arXiv:1502.01589] [inSPIRE].

[24] PLANCK collaboration, The Scientific programme of Planck, *astro-ph/0604069* [inSPIRE].

[25] PLANCK collaboration, Planck 2015 results. XIV. Dark energy and modified gravity, *Astron. Astrophys.* 594 (2016) A14 [arXiv:1502.01590] [inSPIRE].

[26] Q.-G. Huang, K. Wang and S. Wang, Distance priors from Planck 2015 data, *JCAP* 12 (2015) 022 [arXiv:1509.00969] [inSPIRE].

[27] E. Calabrese, A. Slosar, A. Melchiorri, G.F. Smoot and O. Zahn, Cosmic Microwave Weak lensing data as a test for the dark universe, *Phys. Rev. D* 77 (2008) 123531 [arXiv:0803.2309] [inSPIRE].

[28] M. Chevallier and D. Polarski, Accelerating universes with scaling dark matter, *Int. J. Mod. Phys. D* 10 (2001) 213 [gr-qc/0009008] [inSPIRE].

[29] E.V. Linder, Exploring the expansion history of the universe, *Phys. Rev. Lett.* 90 (2003) 091301 [astro-ph/0208512] [inSPIRE].

[30] A. Lewis and S. Bridle, Cosmological parameters from CMB and other data: A Monte Carlo approach, *Phys. Rev. D* 66 (2002) 103511 [astro-ph/0205436] [inSPIRE].

[31] Y. Wang and S. Wang, Distance Priors from Planck and Dark Energy Constraints from Current Data, *Phys. Rev. D* 88 (2013) 043522 [Erratum ibid. D 88 (2013) 069903 [arXiv:1304.4514] [inSPIRE].

[32] W. Hu and N. Sugiyama, Small scale cosmological perturbations: An Analytic approach, *Astrophys. J.* 471 (1996) 542 [astro-ph/9510117] [inSPIRE].

[33] F. Beutler et al., The 6dF Galaxy Survey: Baryon Acoustic Oscillations and the Local Hubble Constant, *Mon. Not. Roy. Astron. Soc.* 416 (2011) 3017 [arXiv:1106.3366] [inSPIRE].

[34] A.J. Ross, L. Samushia, C. Howlett, W.J. Percival, A. Burden and M. Manera, The clustering of the SDSS DR7 main Galaxy sample — I. A 4 per cent distance measure at $z = 0.15$, *Mon. Not. Roy. Astron. Soc.* 449 (2015) 835 [arXiv:1409.3242] [inSPIRE].

[35]
[35] BOSS collaboration, *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample*, Mon. Not. Roy. Astron. Soc. **470** (2017) 2617 [arXiv:1607.03155] [arXiv].

[36] D.J. Eisenstein and W. Hu, *Baryonic features in the matter transfer function*, Astrophys. J. **496** (1998) 605 [astro-ph/9709112] [arXiv].

[37] CORE collaboration, *CORE (Cosmic Origins Explorer) A White Paper*, arXiv:1102.2181 [arXiv].