Non-exponential decoherence and subdiffusion in atom-optics kicked rotor

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Quantum systems lose coherence upon interaction with the environment and tend towards classical states. Quantum coherence is known to exponentially decay in time so that macroscopic quantum superpositions are generally unsustainable. In this work, slower than exponential decay of coherences is experimentally realized in an atom-optics kicked rotor system subjected to non-stationary Lévy noise in the applied kick sequence. The slower coherence decay manifests in the form of quantum subdiffusion that can be controlled through the Lévy exponent. The experimental results are in good agreement with the analytical estimates and numerical simulations for the mean energy growth and momentum profiles of atom-optics kicked rotor.

Quantum systems undergo decoherence due to unavoidable interaction with the environment. As a consequence, the macroscopic quantum superpositions are strongly suppressed and classical behaviour emerges from the quantum regime [1, 2]. The physics at the borderline of classical and quantum regimes is not well understood yet and continues to attract attention [2]. Quantum systems coupled to the environment lose their coherence exponentially fast [3]. This is modelled by the decoherence factor of the form \( \exp(-t/t_c) \), where \( t_c \) is the coherence time that depends on the system parameters and the strength of coupling to the environment. In many applications, e.g., in quantum computers and in emerging quantum technologies [4], it is necessary to sustain quantum coherences for longer times which might be possible by tuning \( t_c \).

The experimental developments in quantum control and reservoir engineering [5–7] in the last two decades have allowed direct observation of decoherence dynamics. The experiments using single ions in harmonic traps coupled to an engineered reservoir of random electric fields [8], dispersively coupled atom and field in a cavity [9] and quantum localized states of ultracold atoms kicked by a noisy pulsed optical lattices [10, 11] have provided evidence for exponential decay of quantum coherence. These experiments allow tuning the coherence time \( t_c \) by changing a system parameter or the coupling to the environment. An alternative approach [12] to prolong \( t_c \) is to explore non-exponential or a relatively slow coherence decay rates of the form \( t^{-\alpha} \), where \( \alpha > 0 \) is the exponent. Such non-exponential decoherence has been theoretically shown for quantum kicked rotor [13, 14] and dissipative quantum two-level system [15] influenced by non-stationary noise. A novel feature in these is that they encompass a regime in which the mean coherence time diverges and the quantum system does not complete its transition to the classical regime. One challenge in experimental realization of non-exponential decoherence is that it will require quantum systems to be sensitive to non-stationary noise within the coherence time scales. In this work, experimental realisation of non-exponential decoherence in the atom-optics kicked rotor (AOKR) system is presented using an unusual form of non-stationary timing noise.

The standard AOKR system – cold atoms periodically pulsed (kicked) by the electromagnetic fields – corresponds to a fundamental model of Hamiltonian chaos [16]. In this system, classically chaotic dynamics leads to unbounded, diffusive mean energy growth whereas its quantized version suppresses the energy growth due to destructive quantum interferences. The resulting dynamical localization (DL) is a phase coherent effect analogous to the Anderson localisation in disordered periodic lattices [17–19] and was experimentally observed in one- [20], two- [21] and three-dimensions [22]. With its unambiguous and distinct signatures of energy growth in the classical and quantum regimes, AOKR is a suitable test-bed to study decoherence.

In AOKR, DL can be destroyed by inducing decoherence through (i) spontaneous emission of the atoms [16, 23], or (ii) addition of noise in the amplitude of the kicks [11] or in the periodicity of the kick [24] or in the phase of the periodic kicks [25]. In contrast to these approaches, we induce decoherence by suppressing kicks entirely at certain time instants dictated by the value of waiting time \( \tau \) between successive kicks drawn from Lévy distribution \( w(\tau) = \alpha \Gamma(\alpha)\Gamma(\alpha + 1)/\Gamma(\tau + \alpha + 1) \), where \( \alpha \) is the Lévy exponent [14]. For \( 0 < \alpha < 1 \), this represents a non-stationary noise with diverging mean waiting time \( \langle \tau \rangle \). We show through experiment and theory that this scenario leads to non-exponential decoherence rates and manifests as sub-diffusive quantum mean energy growth. Besides quantum decoherence, these results are relevant in the general context of transport and diffusion in chaotic quantum systems [26, 28] and disordered nonlinear lattices [29].

The dimensionless Hamiltonian for AOKR, i.e., two-level atoms in a pulsed standing wave of near-resonant
FIG. 1. (Color online) (a) Experimental schematic shows the 1-D optical lattice and the absorption imaging system below the lattice. Off-resonant laser light after passing through an acousto-optic modulator, single-mode fibre and beam expansion creates a 1-D standing wave. (b) Position in time axis at which potential kicks are applied for periodic and various Lévy exponents \( \alpha \). Kicks are skipped as dictated by the Lévy distributed waiting times between successive kicks.

light, subjected to Lévy noise is given by

\[
H = \frac{p^2}{2} + K \cos x \sum_{n=1}^{N} (1 - g_n) \phi_{sq}(t-n). \tag{1}
\]

where \( g_n \) is a telegraph stochastic process that randomly switches between 0 and 1 [30]. If \( g_n = 1 \), then no kick acts at that instant and if \( g_n = 0 \) then a scaled kick strength of amplitude \( K \) acts at that time instant. The waiting time between the successive occurrences of 0 is taken from \( w(\tau) \). The rectangular pulse \( \phi_{sq}(t) \) is of unit amplitude and \( K \) determines whether the dynamics is chaotic, regular or mixed. The noise-free limit, with \( g_n = 0 \) for all \( n \), represents the standard AOKR. In this limit, the classical system is integrable for \( K = 0 \). If \( 0 < K < 1 \), the dynamics is mixed as the regular and chaotic regions coexist in phase space. When \( K > 1 \), the system becomes increasingly chaotic. For \( K \approx 5.8 \) used in this work, the classical phase space is largely chaotic over the experimentally accessible energy range.

The experimental set-up to realise the system in Eq. 1 is shown in Fig. 1(a). Approximately \( 10^7 \) atoms of \(^{87}\text{Rb} \) are loaded into a standard magneto-optic trap and laser cooled to 30 \( \mu \)K [22]. They are then transferred into a crossed optical dipole trap (wavelength \( \lambda = 1064 \) nm) for further forced evaporative cooling to 3 \( \mu \)K. A far detuned 1-D optical lattice is superimposed on this cold sample of atoms and pulses. The lattice laser is \(-6.8\) GHz detuned from 5 \( S_{1/2}^F = 1 \rightarrow 5F_{3/2}^F = 2 \) transition of \(^{87}\text{Rb} \). For the periodically kicked AOKR, the on-time of the pulse is 220 ns and the off-time is 10.6 \( \mu \)s such that the kick period is \( T = 11.02 \) \( \mu \)s. For the experimental parameters used in this work, the kick strength is \( K \approx 5.8 \) with 10% uncertainty. The scaled Planck’s constant is \( \hbar_s = \hbar \omega_s T \), where \( \omega_s \approx 24 \) KHz is the recoil frequency of the lattice beam. This yields \( \hbar_s \approx 2.09 \).

FIG. 2. (Color online) Measured mean energy with \( K \approx 5.8 \) for periodic kick sequence (circles) and for \( \alpha = 0.75 \) (triangles). Dashed lines are the analytical results, and solid lines are from numerical calculations. Error bars represent the standard deviation of energy measurement over 14 Lévy noise realizations. Black solid line is the simulated energy growth for classical kicked rotor with \( \alpha = 0.75 \). Inset shows the classical stroboscopic section for \( \alpha = 0.75 \). Momentum profiles corresponding to labels (a,b,c) are shown in Fig. 3. Vertical line marks the break-time \( t_b \).

Fig. 2(b) displays periodic kick sequence, a crucial ingredient necessary to maintain the quantum coherence and dynamical localization. We induce decoherence by using a kick sequence with waiting time, a random integral multiple of \( T \), drawn from Lévy distribution \( w(\tau) \) with exponent \( \alpha \). Asymptotically \( w(\tau) \) decays as \( \tau^{-\alpha-1} \). The number of kicks actually imparted in any finite time interval is a random variable and its mean is dependent on \( \alpha \) (see Fig. 2(b)). This represents the source of noise for the periodically kicked AOKR. When \( \alpha \) is larger, the number of kicks is larger too. Properties of Lévy distribution relevant for our purposes are reviewed in Ref. [14]. As shown in Fig. 1(a), the radio-frequency applied to the acousto-optic modulator (AOM) is pulsed in accordance with \( w(\tau) \), thereby pulsing the lattice beam which acts on the cold atomic cloud during pulse on-times. The momentum distribution of the cloud is measured after a fixed time-of-flight by absorption imaging. The experimental data reported in this paper, for each value of \( \alpha \), are averaged over 14 realizations of Lévy distributed waiting times.

To compare with the experimental results, we performed numerical simulations of the noisy AOKR system in Eq. 1 by replacing the square pulses with delta kicks, and the parameters were closely matched with the experimental ones. The corresponding Floquet operator is repeatedly applied on an initial state \( \psi(x,0) \), taken to be of Gaussian form. The waiting time obtained from \( w(\tau) \) with exponent \( \alpha \) is incorporated into the Floquet operator. The simulation results represent an average over 900 realizations of Lévy distributed waiting times.

In Figs. 2 and 3, we illustrate the central results for the case \( \alpha = 0.75 \). The classical stroboscopic section (Fig. 2) obtained by simulations for the AOKR in Eq. 1 with
Lévy exponent $\alpha = 0.75$ shows that the phase space is largely chaotic for momenta up to $p = 60$. Corresponding energy growth displayed as solid (black) line is consistent with approximately quasi-linear growth proportional to $t$. On the other hand, the measured mean energy $\langle E \rangle$ for $\alpha = 0.75$ (see Fig. 2), averaged over Lévy noise, displays sub-diffusive growth. This is in good agreement with the quantum simulations (blue, solid line) and also with the theoretical result (green, dashed line) $\langle E \rangle \approx A_0 t + A_1 e^{\alpha t}$. Regression performed on the observed $\langle E \rangle$ with this analytical form gives $A_0 = 0.123$ and $A_1 = 2.6(2)$ and $\alpha = 0.75(2)$. For stationary noise imposed on the kicking sequence, exponential decoherence would have resulted in the experimental data (blue triangles) having similar trend as the classical mean energy growth (black solid line). At $\alpha = 0.75$, the noise is non-stationary and the signature of slower than exponential decoherence rate comes from this observation that AOKR with Lévy distributed inter-kick intervals display sub-diffusion rather than normal diffusion expected under conditions of complete decoherence. In a related theoretical work on kicked rotor with amplitude noise applied with Lévy distributed waiting times [13][14][32], subdiffusion resulting from non-exponential decoherence was noted. We emphasize that the observed sub-diffusion in this experiment does not arise from spontaneous emission of photons (at less than 1% per kick) or the amplitude noise (about 1%), both of which are highly suppressed. This is confirmed by the fact that noise-free AOKR with periodic kicks displays energy saturation and DL (red circles in Fig. 2) beyond break-time $t_b \approx 3$, in agreement with the established results [20][33].

Figure 3(a,b,c) displays, respectively, the measured momentum profile $f(p)$ corresponding to the labels (a,b,c) indicated in Fig. 2. For periodically kicked AOKR, $f(p)$ at $t = 70 \approx 20t_b$, obtained from the absorption image shown in Figure 3(a) displays expected exponential profile. The momentum distribution for $\alpha = 0.75$ at $t = 14 \approx 4t_b$ shown in Fig. 3(b), extracted from the corresponding absorption image, maintains a reasonable exponential profile indicating that coherence is preserved. However, at $t = 70 \approx 20t_b$ the momentum distribution in Fig. 3(c) is well approximated by a Gaussian profile implying loss of coherence. The numerically simulated momentum profile is also in good agreement with the measured profiles.

The starting point for theoretical analysis is the modified delta-kicked rotor Hamiltonian of the form

$$H = \frac{p^2}{2} + K \cos x \sum_n (1 - g_n) \delta(t - n). \quad (2)$$

As before, we take the waiting time between successive kicks from Lévy distribution $w(\tau)$ characterised by exponent $\alpha$. The corresponding Floquet operator is

$$\hat{F}(K) = e^{-\frac{\pi^2 p^2}{2} t} e^{-\frac{\pi}{2} K \cos x} e^{-\frac{\pi}{2} K' \cos x}, \quad (3)$$

where $K' = Kg$. Following Ref. [34] and its extension to non-perturbative regime in Ref. [13][14], we relate the decoherence factor to the survival probability of quasi-energy eigenstates $|s\rangle$ of the Floquet operator $\hat{U} = e^{-\frac{\pi^2 p^2}{2} t} e^{-\frac{\pi}{2} K \cos x}$ to obtain an expression for $\langle p^2(t) \rangle$. An important ingredient is the use of random phase approximation to estimate the contribution $\langle s | e^{iK'_n \cos x/h_s} | s \rangle$ when the quasi-energy state responds to noise by transiting from state $|s\rangle$ to $|s'\rangle$. For our noise scenario, we obtain the decoherence factor $D(t, 0)$ for the quantum evolution from $n = 0$ until $n = t$ to be

$$D(t, 0) \sim e^{-(1-q^2)t} E_{\alpha} \{ (1 - q^2)^{\frac{\sin \pi \alpha}{\pi \alpha}} t^\alpha \}, \quad (\alpha < 1), \quad (4)$$

$$D(t, 0) \sim e^{-(1-q^2)(1-1/\bar{\tau})t}, \quad (\alpha > 1), \quad (5)$$

where $\bar{\tau}$ is the mean waiting time between kicks, the function $q \left(K'_t/h_s\right)$ is given by

$$q \left(K'_t/h_s\right) = 1 - \frac{K'^2}{2h_s^2} \cos^2 x + \frac{K'^4}{4h_s^4} \cos^4 x + \ldots . \quad (6)$$

FIG. 3. (Color online) (a,b,c) Experimentally obtained momentum profiles corresponding to labels (a,b,c), respectively, in Fig. 2. The insets show the optical density of the absorption images from which momentum profiles have been extracted. Solid lines in (b,c) are obtained from quantum simulation. Dashed lines in (b) are shown as a guide to the eye.
and $E_a(.)$ is the Mittag-Leffler function. The decoherence rate is non-exponential for $\alpha < 1$ and exponential for $\alpha > 1$. Using the results in Eqs. 4 and 5 for $\alpha < 1$ and $t >> 1$, we obtain the mean energy growth as

$$\langle E \rangle \approx A_0 t + A_1 t^\alpha,$$

and for $\alpha > 1$ and $t >> 1$, we get

$$\langle E \rangle \approx A_2 t.$$

where the constants $A_0$, $A_1$ and $A_2$ depend on the break-time $t_b$ for the corresponding standard kicked rotor system, $\hbar$, and $\alpha$. These results are in good agreement with the experimental data and simulations presented in Figs. 2 and 4. In order to compare the analytical results for $\langle E \rangle$ with the experimental data, we use $A_0$, $A_1$ and $A_2$ as fitting parameter. In Refs. 10, 11, sub-diffusive mean energy growth appears as the signature of decoherence due to spontaneous emission (SE) or amplitude noise (AN). We emphasize that the observed sub-diffusion in this work results from non-exponential decohering effect of Levy noise as opposed to SE or AN (which are in any case suppressed) because the best fit value of $\alpha$ is close to that used in generating kick sequences with Levy noise, as predicted by Eqs. 4 and 5. Such physical relevance for the sub-diffusive growth exponent is absent in case of SE or AN induced decoherence.

A broader picture of the results in Fig. 4 displays the experimentally measured mean energy growth for values of Lévy exponent $\alpha = 0.25, 0.5, 0.75$ and 2.0. For comparison, this figure also shows the observed $\langle E \rangle$ for AOKR with random kicks or stationary timing noise (STN), i.e., the kick period is $T + \delta$, where $-\Delta \leq \delta \leq \Delta$ is a uniformly distributed random variable with noise strength $\Delta \approx 20\%$. The quantum simulations performed for the AOKR system using experimental parameters agree with the measurements. For $\alpha < 1$, subdiffusion is clearly visible, implying non-exponential decoherence. In contrast, for STN, approximately normal diffusion is indicative of exponential decoherence. In Fig. 4 the experimental results are also compared with the analytical results in Eq. 7 and we obtain a good agreement between the two. In all these cases, subdiffusion of quantum mean energy growth is highly pronounce for $\alpha < 1$, the regime in which $\langle \tau \rangle$ diverges as well. This differs considerably from normal diffusion exhibited by AOKR with STN. The classical stroboscopic plots corresponding to parameters used in Fig. 4 have predominantly chaotic features for the energies accessed by these experiments. It is indeed surprising that the AOKR system 'feels' the Lévy distributed waiting times in few tens of kicks and clearly distinguishes it from the case of STN through different energy growth profiles.

The measured momentum profiles provide further evidence for slower decoherence but do not provide access to phase information. Instead, decay of occupation probability $f(p = 0)$ against time in log-log plot. (Inset) $f(0)$ for numerically simulated AOKR with stationary amplitude noise with noise strengths of 16% and 25%. All the solid lines (black, green, blue, red) are to guide the eye.
of Fig. 3 though associated with subdiffusive growth for $\langle E \rangle$. Thus, in spite of energy growth being qualitatively similar for stationary and non-stationary noise in AOKR, they can be distinguished by their qualitatively different coherence decays.

In summary, an experimental realization of slower than exponential decoherence in a noisy atom-optics kicked rotor system is presented in which the waiting times $\tau$ between subsequent kicks are chosen from Lévy distribution characterised by the exponent $\alpha$. For $0 < \alpha < 1$, the noise induced in AOKR is nonstationary and the accompanying slower decoherence manifests as subdiffusive quantum mean energy growth. Remarkably, the AOKR system can 'feel' the non-stationary Lévy distributed kick sequence in few tens of kicks. By tuning $\alpha$, mean coherence time can be prolonged and it is possible to access the regime of non-exponential decoherence rates in experiments. The analytical expressions obtained for subdiffusive energy growth are in good agreement with the experimental and simulation results of AOKR system.

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