New QCD Sum Rule for $D(0^+)$

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We derive a new QCD sum rule for $D(0^+)$ which has only the $D\pi$ continuum with a resonance in the hadron side, using the assumption similar to that has been successfully used in our previous work to the mass of $D_s(0^+)(2317)$. For the value of the pole mass $M_c = 1.38$ GeV as used in the $D_s(0^+)$ case we find that the mass of $D(0^+)$ derived from this sum rule is significantly lower than that derived from the sum rule with the pole approximation. Our result is in agreement with the experimental data from Belle.

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I. INTRODUCTION

BaBar Collaboration discovered in 2003 a positive-parity scalar charm strange meson $D_{sJ}(2317)$ with a very narrow width [1], which was confirmed by CLEO later [2]. In the same experiment CLEO [2] observed the $1^+$ partner state at 2460 MeV. Since these two states lie below $DK$ and $D^*K$ threshold respectively, the potentially dominant $s$-wave decay modes $D_{sJ}(2317) \to D_sK$ etc are kinematically forbidden. Thus the radiative decays and isospin-violating strong decays become dominant decay modes. Therefore they are very narrow.

The discovery of these two states has inspired great interest in their nature in literature. The key point is to understand their low masses. The $D_{sJ}(2317)$ mass is significantly lower than the expected values in the range of 2.4 – 2.6 GeV in quark models [3]. The model using the heavy quark mass expansion of the relativistic Bethe-Salpeter equation predicted a lower value 2.369 GeV [4], which is still 50 MeV higher than the experimental data.

Later Belle [5] observed the wide $D(0^+)$ resonance with mass $M_0 = 2.308 \pm 0.0017 \pm 0.0015 \pm 0.0028$ GeV and width around $\Gamma = 276$ MeV. Another puzzle arises from this result: why are these two resonances nearly degenerate in mass while $D_s(0^-)$ is 100 MeV higher than $D(0^-)$?

The earlier results for the mass of $D_{sJ}(0^+)$ from QCD sum rules are either significantly larger than the experimentally observed mass of $D_{sJ}(2317)$ [6] or consistent with it within theoretical uncertainty but with significantly larger central value [5]. The results from lattice QCD calculations are similar, see [5] and [6, 10].

Recently, there have been two investigations on this problem using sum rules in full QCD including the $O(\alpha_s)$ corrections. In Ref. [11] the value of the charm quark pole mass $M_c = 1.46$ GeV is used and $0^+\bar{c}s$ is found to be 100 – 200 MeV higher than the experimental data. On the other hand, in Ref. [12] $M_c \sim 1.33$ GeV is used and the central value of the results for the $0^+\bar{c}s$ mass is in good agreement with the data. As commented by the author, the uncertainty in the value of $M_c$ is large.

The difficulty with the $\bar{c}s$ interpretation leads many authors to speculate that $D_{sJ}(2317)$ is a $\bar{c}qs\bar{q}$ four quark state [13, 14] or a strong $D\pi$ atom [15]. However, quark model calculations show that the mass of the four quark state is much larger than the $0^+\bar{c}s$ state [16, 17]. Furthermore, the four quark system has two $0^+$ states. Only one has been found below 2.8 GeV in the experimental search, consistent with the $\bar{c}s$ interpretation.

It was suggested in Ref. [18] that the low mass of $D_{sJ}(2317)$ could arise from the mixing between the $0^+\bar{c}s$ state and the $DK$ continuum. It was pointed out in [7] that in the formalism of QCD sum rules the physics of mixing with $DK$ continuum resides in the contribution of $DK$ continuum in the sum rule and including this contribution should render the mass of $D_{sJ}(0^+)$ lower.

Usually the contribution of the two-particle continuum is neglected in the QCD sum rules. In a recent work [19] we argued that because of the large $s$ wave coupling of $D_s(0^+)DK$ [20, 21] and the adjacency of the $D_s(0^+)$ mass to the threshold this contribution may not be neglected. We calculated this contribution under the assumption that the form factor for the coupling of the scalar current to the two-particle continuum in the low energy region is dominated by the bubble diagrams formed by the coupling of the $0^+$ to the two particles. We found that including this term in the sum rule indeed renders the mass and decay constant of $D_{sJ}(0^+)$ significantly lower. Using the pole mass value $M_c = 1.38$ GeV the mass of $D_s(0^+)$ is found to be in good agreement with experimental data.

In the present work we would like to show that the same assumption for the form factor for the coupling of the scalar current to the two-particle continuum also leads to the mass value of $D(0^+)$ in agreement to the experimental data, thus, explains the nearly degeneracy of these two states.

In Section II we derive the $D\pi$ continuum contribution...
and write down the full QCD sum rule for \( D(0^+) \). The numerical results of the mass of \( D(0^+) \) are collected in Section III.

II. THE QCD SUM RULE FOR THE SCALAR CHARM MESON

We consider the scalar correlation function

\[
\Pi(q^2) = i \int d^4x \exp[iqx] \langle 0 | T \{ J(x), J^\dagger(0) \} | 0 \rangle
\]

where \( J(x) = \bar{c}(x) d(x) \) is the interpolating current for the charged scalar charm meson. \( \Pi(q^2) \) satisfies the following dispersion relation

\[
\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\text{Im} \Pi(s)}{s - q^2 + i\epsilon}.
\]

The leading terms of the imaginary part \( \text{Im} \Pi(s) \) at the quark gluon level and its \( \alpha_s \) correction have been calculated in \[12\] [23]. After making the Borel transformation to suppress the contribution of higher excited states and invoking the quark-hadron duality, one arrives at the sum rule

\[
\int dt \frac{\text{Im} \Pi^H(t)}{\pi} \exp[-\frac{t}{M_B^2}] = \frac{M_c^2}{(m_c - m_d)^2} \{ \int dt \left[ \frac{1}{8\pi^2} 3t(1 - \frac{M_c^2}{t})^2 + \frac{4\alpha_s}{3\pi} G(\frac{M_c^2}{t}) \right] + M_c \langle \bar{d}d \rangle \exp[-\frac{M_c^2}{2M_B^2}] + \frac{3}{2} - \frac{M_c^2}{2M_B^2} \langle G^2 \rangle \} \frac{M_c^2}{2M_B^2} \exp[-\frac{M_c^2}{2M_B^2}].
\]

where \( m_c \) and \( m_d \) are the charm and down quark current mass. The charm quark pole mass is defined as \[23\]

\[
M_c = m_c(p^2)[1 + \left( \frac{4}{3} + \ln \left( \frac{p^2}{M_c^2} \right) \right)].
\]

\( \langle \bar{d}d \rangle \) is the down quark condensate, \( \langle \alpha_s G^2 \rangle \) is the gluon condensate, \( \langle g_s \bar{d} d \cdot G d \rangle \) is the quark gluon mixed condensate. \( M_B \) is the Borel mass. The radiative correction function reads

\[
G(x) = \frac{9}{4} + 2 \text{Li}_2(x) + \log(x) \log(1 - x) - \frac{3}{2} \log\left( \frac{1}{x} - 1 \right) - \log(1 - x) + x \log\left( \frac{1}{x} - 1 \right) - \frac{x}{1 - x} \log(x).
\]

The above equation is of the same form as that for \( D_s(0^+) \). The difference between the two cases is the following. \( D_s(0^+) \) is a very narrow resonance below the \( DK \) threshold. Neglecting the small iso-spin violating interaction it can be represented by a pole term in the sum rule. On the other hand, \( D(0^+) \) is a very broad resonance in the \( D\pi \) spectra and there is no pole term in the hadron spectra. In the traditional treatment of QCD sum rules the very broad resonance is still approximated by a pole term solely for simplicity and without justification. Moreover, the remaining two-particle continuum is completely neglected.

According to this procedure the spectral density at the hadronic level is taken to be the pole term plus the continuum starting from some threshold which is identified with the QCD continuum.

\[
\frac{\text{Im} \Pi^H(t)}{\pi} = \left( \frac{f_0 M_B^2}{m_c - m_d} \right)^2 \delta(t - M_0^2) + \text{QCD continuum} \times \theta(t - s_0)
\]

where \( f_0 \) is the vector current decay constant of \( 0^+ \bar{c}d \) particle analogous to \( f_\pi = 132 \text{ MeV} \), \( M_0 \) is the mass of this particle, and \( s_0 \) is the continuum threshold above which the hadronic spectral density is modelled by that at the quark gluon level. The recent work \[12\] also uses the above ansatz. The results from the above sum rule with \( M_c = 1.38 \text{ GeV} \) are shown in Fig. 1. This may not be a good approximation. The importance of \( D\pi \) continuum in the \( D(0^+) \) channel was first emphasized by Blok, Shifman and Uraltsev in Ref. \[23\] from the consideration of duality.

![FIG. 1: \( M_0 vs M_B^2 \) from the sum rule with pole approximation for \( D(0^+) \). Curves from top to bottom correspond to \( s_0 = 8.0, 7.5, 7.0 \text{ GeV}^2 \) respectively.](image)

In our previous work \[19\] it was pointed out that the strong s wave coupling of the \( D_s(0^+) \) with the two particle DK state and the adjacency of the \( D_s(0^+) \) mass to the DK continuum threshold result in large coupling channel effect which corresponds to the configuration mixing in the formalism of the quark model. Therefore, the two-particle continuum can not be neglected. In the present case of \( D(0^+) \) the two-particle term is the whole hadron spectra needed in the sum rule. We calculate this term with the same assumption used in \[19\]. Besides giving a more accurate sum rule for the mass of \( D(0^+) \), this also constitutes a check for the approximation used in \[19\] for \( D_s(0^+) \).

The contribution of the \( D\pi \) continuum to the hadronic...
spectral density reads

\[
\text{Im} \Pi^H(t) = \frac{3}{32\pi^2} \left( \sqrt{1 - \frac{(M_D + m_\pi)^2}{t}} - \sqrt{1 - \frac{(M_D - m_\pi)^2}{t}} \right) F(t)^2 \theta(\sqrt{t} - M_D - m_\pi) \theta(s_0 - t) + \text{QCD continuum} \times \theta(t - s_0),
\]

where \( F(t) \) is the form-factor defined by

\[
F(t) = \langle 0 | \bar{c}(0) d(0) | D \pi \rangle.
\]

Using the relation between \( s, t \) and the scattering angle \( \theta \) one finds that the latter is an analytic function of \( t \) with only a short cut. Therefore, this term can be approximated by a pole form \( \frac{g^2}{t - t_0} \), where

\[
t_0 = \frac{1}{2} \left( 2M_D^2 + 2m_\pi^2 - M_0^2 + \frac{(M_D^2 - m_\pi^2)^2}{M_0^2} \right),
\]

\[
c = -\frac{2M_D^2 + 2m_\pi^2 - M_0^2}{\frac{(M_D^2 - m_\pi^2)^2}{M_0^2} + t_0 - 2M_D^2 - 2m_\pi^2}.
\]

Different from the case of \( D_s(0^+) \), the crossing term is actually small for experimental values of the masses in the present case of \( D(0^+) \). But we shall still keep it in the formulae. In the chiral lagrangian the interaction between \( D \) and \( \pi \) through exchanging two pions is suppressed by \( (E/2\sqrt{2\pi f_\pi})^4 \) for low energy \( E \) of \( \pi \). Hence its contribution can be neglected. Summing the chain of the bubble diagrams, we have

\[
F(t) = \frac{\lambda}{(t - M_0^2)[1 - \frac{1}{(t - M_0^2) + \frac{c}{t - t_0}] \Sigma(t)}
\]

where

\[
\Sigma(q^2) = -\frac{3}{2g^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2} \frac{1}{(q - k)^2 - M_D^2} + \text{subtraction}
\]

From Cutkosky’s cutting rule,

\[
\text{Im} \Sigma(t) = \frac{3g^2}{16\pi} \sqrt{t} \theta(\sqrt{t} - M_D - m_\pi),
\]

and we have taken into account two intermediate states of different charge. From the dispersion relation,

\[
\Sigma(t) = \Sigma(M_0^2) + \frac{1}{\pi} \int_{(M_D + m_\pi)^2}^{\infty} dt' \frac{3g^2}{16\pi} \sqrt{t'} \frac{(t' - M_0^2)}{(t' - M_D^2)(t' - t)}
\]

where \( M_0 \) is the renormalized mass of the \( D(0^+) \) meson. The mass renormalization condition leads to \( \text{Re} \Sigma(M_0^2) = 0 \). Therefore we have

\[
F(t) = \lambda \left( t - M_0^2 - \frac{3g^2}{16\pi} (1 + \frac{c(t - M_D^2)}{t - t_0}) \right)
\]

\[
\times P \int_{(M_D + m_\pi)^2}^{\infty} dt' \frac{k(t')}{\sqrt{t'}} \frac{(t' - M_0^2)}{(t' - M_D^2)(t' - t)} - i \frac{3g^2}{16\pi} \frac{c(t - M_D^2)}{t - t_0} \theta(\sqrt{t} - M_D - m_\pi)\right)^{-1},
\]

where \( P \) denotes the principal part of integration.

### III. The Mass of \( D(0^+) \)

In the numerical analysis, we use \( m_d = 8 \text{ MeV}, m_c = 1.18 \text{ GeV}, M_c = 1.38 \text{ GeV}, \langle dd \rangle = -(0.243)^2 \text{ GeV}^3, \langle dg \rangle \).
\( Gd \) = 0.8 GeV\(^2 \) \( \langle \bar{d}d \rangle \), \( \alpha_s G^2 \) = 0.06 GeV\(^4 \), \( \Lambda_{\text{QCD}} \) = 0.325 GeV, \( M_D = 1.869 \) GeV, \( m_\pi = 0.140 \) GeV, \( f_D = 0.2 \) GeV \cite{22}, \( f_\pi = 0.132 \) GeV \cite{23}.

The \( g \) value has not been determined very well theoretically. It was found to be in the interval \( g = 7.5 - 5.1 \) GeV in Ref. \cite{20}. Inclusion of the contribution of \( D\pi \) channel may lower the continuum in the sum rule analysis of the scalar current.

Since we cannot do wave function renormalization for large \( M \), we determine it from the experimental width \( \Gamma \) of \( D \) as the input. For the experimental central value \( \Gamma = 276 \) MeV \cite{5}, we have to input a "trial" value of \( M_0 \) and require that the ratio of the hadron side (the left hand side or LHS) to the quark-gluon side (the right hand side or RHS) of the sum rule is equal to 1 in the middle of the working range of \( M_B^2 \). The results are shown in Fig. 3, 4, 5 for \( s_0 = 8.0, 7.5, 7.0 \) respectively.

Comparing these results to those obtained in the pole approximation shown in Fig. 2 one finds that the results for the mass of \( D(0^+) \) are lower by 80 – 40 MeV. For \( M_e = 1.38 \) GeV, \( s_0 = 8.0 - 7.0 \) GeV\(^2 \) we find \( M_0 = 2.30 - 2.27 \) GeV in agreement with the experimental data. The curve is very sensitive to the trial value of \( M_0 \), hence the uncertainty of our results from different values of \( M_B \) for fixed values of other parameters are small. Therefore, with the same approximation for the two-particle continuum and same values of \( M_e \) and \( s_0 \) we obtain simultaneously the correct values of the masses of the narrow resonance \( D_s(0^+) \) below the threshold and the broad resonance \( D(0^+) \) above the threshold.

Under our approximation for the two-particle continuum and acceptable value of \( g \), there is no solution for the sum rule for \( D(0^+) \) containing a below-threshold pole term and the two-particle continuum in the hadron side as that used in the \( D_s(0^+) \) case.

The good convergence of the OPE series and dominance of the \( D\pi \) term over the QCD continuum beyond \( s_0 \) constraint the Borel mass in a region depending on \( M_e \) and \( s_0 \). For \( M_e = 1.38 \) GeV, \( s_0 = 8.0, 7.5, 7.0 \) GeV\(^2 \), \( M_B^2 \in [1.15, 2.5], [1.15, 2.25], [1.15, 2.2] \) GeV\(^2 \) respectively. Since the hadron side of the sum rule depends on the unknown value \( M_0 \) in a complicated way, we have to input a "trial" value of \( M_0 \) and require that the ratio of the hadron side (the left hand side or LHS) to the quark-gluon side (the right hand side or RHS) of the sum rule is equal to 1 in the middle of the working range of \( M_B^2 \). The results are shown in Fig. 3, 4, 5 for \( s_0 = 8.0, 7.5, 7.0 \) respectively.

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