A NOTE ON QUANTIZATION OF COMPLEX SYMPLECTIC MANIFOLDS

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Abstract. To a complex symplectic manifold $X$ we associate a canonical quantization algebroid $\tilde{E}_X$. This is modeled on the algebras $\bigoplus_{\lambda \in \mathbb{C}} \rho_\ast \mathcal{E} e^{\lambda h - 1}$, where $\rho$ is a local contactification, $\mathcal{E}$ is an algebra of microdifferential operators and $h \in \mathcal{E}$ is such that $\text{Ad}(e^{\lambda h - 1})$ is the automorphism of $\rho_\ast \mathcal{E}$ corresponding to translation by $\lambda$ in the fibers of $\rho$. Our construction is similar to that of Polesello-Schapira’s deformation-quantization algebroid. The deformation parameter $\hbar$ acts on $\tilde{E}_X$ but is not central. If $X$ is compact, the bounded derived category of regular holonomic $\tilde{E}_X$-modules is a $\mathbb{C}$-linear Calabi-Yau triangulated category of dimension $\dim X + 1$.

Introduction

We construct here a canonical quantization algebroid on a complex symplectic manifold. Our construction is similar to that of the deformation-quantization algebroid in [10], which was in turn based on the construction of the microdifferential algebroid on a complex contact manifold in [5]. Let us briefly recall these constructions.

Let $Y$ be a complex contact manifold. By Darboux theorem, the local model of $Y$ is an open subset of a projective cotangent bundle $P^*M$. A microdifferential algebra on an open subset $V \subset Y$ is a $\mathbb{C}$-algebra locally isomorphic to the ring $\mathcal{E}_M$ of microdifferential operators on $P^*M$. Let $(\mathcal{E}, *)$ be a microdifferential algebra endowed with an anti-involution. Any two such pairs $(\mathcal{E}', *)$ and $(\mathcal{E}, *)$ are locally isomorphic. Such isomorphisms are not unique, and in general it is not possible to patch the algebras $\mathcal{E}$ together in order to get a globally defined microdifferential algebra on $Y$. However, the automorphisms of $(\mathcal{E}, *)$ are all inner and are in bijection with a subgroup of invertible elements of $\mathcal{E}$. As shown in [5], this is enough to prove the existence of a microdifferential algebroid $\mathcal{E}_Y$, i.e. a $\mathbb{C}$-linear stack locally represented by microdifferential algebras.

Let $X$ be a complex symplectic manifold. On an open subset $U \subset X$, let $(\rho, \mathcal{E}, *, h)$ be a quadruple of a contactification $\rho: V \rightarrow U$, a microdifferential algebra $\mathcal{E}$ on $V$, an anti-involution $*$ and an operator $h \in \mathcal{E}$ such that $\text{Ad}(e^{\lambda h - 1})$ is the automorphism of $\rho_\ast \mathcal{E}$ corresponding to translation by $\lambda \in \mathbb{C}$ in the fibers of $\rho$. One could try to mimic the above construction in order to get an algebroid from the algebras $\rho_\ast \mathcal{E}$. This fails because the automorphisms of $(\rho, \mathcal{E}, *, h)$ are not all inner, an outer automorphism being given by $\text{Ad}(e^{\lambda h - 1})$. There are two natural ways out.
The first possibility, utilized in [10], is to replace the algebra $\rho_*\mathcal{E}$ by its subalgebra $\mathcal{W}$ of operators commuting with $\hbar$. Then the action of $\text{Ad}(e^{\lambda\hbar^{-1}})$ is trivial on $\mathcal{W}$, and these algebras patch together to give the deformation-quantization algebroid $\mathcal{W}_X$. This is an alternative construction to that of [9], where the parameter $\hbar$ is only formal (note however that the methods in loc. cit. apply to general Poisson manifolds).

The second possibility, which we exploit here, is to make $\text{Ad}(e^{\lambda\hbar^{-1}})$ an inner automorphism. This is obtained by replacing the algebra $\rho_*\mathcal{E}$ by the algebra $\mathcal{E} = \bigoplus_{\lambda \in \mathbb{C}} \rho_*\mathcal{E} e^{\lambda\hbar^{-1}}$ (or better, a tempered version of it). We thus obtain what we call the quantization algebroid $\mathcal{E}_X$, where the deformation parameter $\hbar$ is no longer central. The centralizer of $\hbar$ in $\mathcal{E}_X$ is equivalent to the twist of $\mathcal{W}_X \otimes_{\mathbb{C}} (\bigoplus_{\lambda \in \mathbb{C}} \mathbb{C} e^{\lambda\hbar^{-1}})$ by the gerbe parameterizing the primitives of the symplectic 2-form. One should compare this with the construction in [4], whose authors advocate the advantages of quantization (as opposed to deformation-quantization) and of the complex domain.

There is a natural notion of regular holonomic $\mathcal{E}_X$-module. In fact, for any Lagrangian subvariety $\Lambda$ of $X$ there is a contactification $\rho: Y \to X$ of a neighborhood of $\Lambda$ in $X$ and a Lagrangian subvariety $\Gamma$ of $Y$ such that $\rho$ induces a homeomorphism $\Gamma \tilde{\to} \Lambda$. Then, an $\mathcal{E}_X$-module is called regular holonomic along $\Lambda$ if it is induced by a regular holonomic $\mathcal{E}_Y$-module along $\Gamma$.

One of the main features of our construction is that, if $X$ is compact, the bounded derived category of regular holonomic $\mathcal{E}_X$-modules is a $\mathbb{C}$-linear Calabi-Yau category of dimension $\dim X + 1$.

1. Stacks and algebroids

Let us briefly recall the notions of stack and of algebroid (refer to [3, 9, 1]).

A prestack $\mathcal{A}$ on a topological space $X$ is a lax analogue of a presheaf of categories, in the sense that for a chain of open subsets $W \subset V \subset U$ the restriction functor $\mathcal{A}(U) \to \mathcal{A}(W)$ coincides with the composition $\mathcal{A}(U) \to \mathcal{A}(V) \to \mathcal{A}(W)$ only up to an invertible transformation (satisfying a natural cocycle condition for chains of four open subsets). The prestack $\mathcal{A}$ is called separated if for any $U \subset X$ and any $p, p' \in \mathcal{A}(U)$ the presheaf $U \supset V \mapsto \text{Hom}_{\mathcal{A}(V)}(p|_V, p'|_V)$ is a sheaf. We denote it by $\text{Hom}_\mathcal{A}(p, p')$. A stack is a separated prestack satisfying a natural descent condition.

Let $\mathcal{R}$ be a commutative sheaf of rings. For $\mathcal{A}$ an $\mathcal{R}$-algebra denote by $\text{Mod}(\mathcal{A})$ the stack of left $\mathcal{A}$-modules. An $\mathcal{R}$-linear stack is a stack $\mathcal{A}$ such that for any $U \subset X$ and any $p, p' \in \mathcal{A}(U)$ the sheaves $\text{Hom}_{\mathcal{A}}(p', p)$ have an $\mathcal{R}|_U$-module structure compatible with composition and restriction. The stack of left $\mathcal{A}$-modules $\text{Mod}(\mathcal{A}) = \text{Fct}_\mathcal{R}(\mathcal{A}, \text{Mod}(\mathcal{R}))$ has $\mathcal{R}$-linear functors as objects and transformations of functors as morphisms.

An $\mathcal{R}$-algebroid $\mathcal{A}$ is an $\mathcal{R}$-linear stack which is locally non empty and locally connected by isomorphisms. Thus, an algebroid is to a sheaf of algebras what a gerbe is to a sheaf of groups. For $p \in \mathcal{A}(U)$ set $\mathcal{A}_p = \text{End}_\mathcal{A}(p)$. Then $\mathcal{A}|_U$ is represented by the $\mathcal{R}$-algebra $\mathcal{A}_p$, meaning that $\text{Mod}(\mathcal{A}|_U) \simeq \text{Mod}(\mathcal{A}_p)$. An $\mathcal{R}$-algebroid $\mathcal{A}$ is called invertible if $\mathcal{A}_p \simeq \mathcal{R}|_U$ for any $p \in \mathcal{A}(U)$. 

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2. Quantization of contact symplectic manifolds

Let \( Y \) be a complex contact manifold. In this section we describe a construction of the microdifferential algebroid \( E_Y \) of [5] and recall some results on regular holonomic \( E_Y \)-modules.

By Darboux theorem, the local model of \( Y \) is an open subset of the projective cotangent bundle \( P^* M \) with \( M = \mathbb{C}^{\frac{1}{2}(\dim Y + 1)} \). By definition, a microdifferential algebra on \( Y \) is a \( \mathbb{C} \)-algebra locally isomorphic to the ring of microdifferential operators \( \mathcal{E}_M \) on \( P^* M \) from [11].

Consider a pair \( p = (\mathcal{E}, * ) \) of a microdifferential algebra \( \mathcal{E} \) on an open subset \( V \subset Y \) and an anti-involution \( * \), i.e. an isomorphism of \( \mathcal{C} \)-algebras \( * : \mathcal{E} \to \mathcal{E}^{op} \) such that \( ** = \text{id} \). Any two such pairs \( \mathcal{E}' \) and \( p \) are locally isomorphic, meaning that there locally exists an isomorphism of \( \mathbb{C} \)-algebras \( f : \mathcal{E}' \to \mathcal{E} \) such that \( f^* \circ \mathcal{E} \). Moreover, by [5, Lemma 1] the automorphisms of \( p \) are all inner and locally in bijection with the group

\[
\{ b \in \mathcal{E}^\times; \; b^* b = 1, \; \sigma(b) = 1 \},
\]

by \( b \mapsto \text{Ad}(b) \). Here \( \sigma(b) \) denotes the principal symbol and \( \text{Ad}(\mathcal{A})(a) = aba^{-1} \).

**Definition 2.1.** The microdifferential algebroid \( E_Y \) is the \( \mathbb{C} \)-linear stack on \( Y \) whose objects on an open subset \( V \) are pairs \( p = (\mathcal{E}, * ) \) as above. Morphisms \( p' \to p \) are equivalence classes \( [a, f] \) of pairs \( (a, f) \) with \( a \in \mathcal{E} \) and \( f : \mathcal{E}' \to \mathcal{E} \). The equivalence relation is given by \( (ab, f) \sim (a, \text{Ad}(\mathcal{A})(f) \) for \( b \) as in (2.1). Composition is given by \( [a, f] \circ [a', f'] = [af(a'), ff'] \). Linearity is given by \( [a_1, f_1] + [a_2, f_2] = [a_1 + a_2 b, f_1] \) for \( b \) as in (2.1) with \( f_2 f_1^{-1} = \text{Ad}(\mathcal{A}) \).

**Remark 2.2.** For \( M \) a complex manifold, denote by \( \Omega_M \) the sheaf of top-degree forms. The algebra \( \mathcal{E}_{\Omega^1_M/2} = \Omega^{1/2}_M \otimes_{\mathcal{O}_M} \mathcal{E}_M \otimes_{\mathcal{O}_M} \Omega^{-1/2}_M \) has a canonical anti-involution \( * \) given by the formal adjoint at the level of total symbols. The pair \( (\mathcal{E}_{\Omega^1_M/2}, *) \) is a global object of \( E_{\text{P}^* M} \) whose sheaf of endomorphisms is \( \mathcal{E}_{\Omega^1_M/2} \). Thus \( E_{\text{P}^* M} \) is represented by \( \mathcal{E}_{\Omega^1_M/2} \).

As the algebroid \( E_Y \) is locally represented by a microdifferential algebra, it is natural to consider coherent or regular holonomic \( E_Y \)-modules. Denote by \( \text{Mod}_{\text{coh}}(E_Y) \) and \( \text{Mod}_{\text{rh}}(E_Y) \) the corresponding stacks. For \( \Lambda \subset Y \) a Lagrangian subvariety, denote by \( \text{Mod}_{\Lambda, \text{rh}}(E_Y) \) the stack of regular holonomic \( E_X \)-modules with support on \( \Lambda \).

Denote by \( \mathcal{C}_{\Omega^1_M/2} \) the invertible \( \mathbb{C} \)-algebroid on \( \Lambda \) such that the twisted sheaf \( \Omega^{1/2}_\Lambda \) belongs to \( \text{Mod}(\mathcal{C}_{\Omega^1_M/2}) \).

For an invertible \( \mathcal{C} \)-algebroid \( R \), denote by \( \text{LocSys}(R) \) the full substack of \( \text{Mod}(R) \) whose objects are local systems (i.e. have microsupport contained in the zero-section).

By [5, Proposition 4] (see also [2, Corollary 6.4]), one has

**Proposition 2.3.** For \( \Lambda \subset Y \) a smooth Lagrangian submanifold there is an equivalence

\[
\text{Mod}_{\Lambda, \text{rh}}(E_Y) \simeq p_1_* \text{LocSys}(p_1^{-1} \mathcal{C}_{\Omega^1_M/2}),
\]

where \( p_1 : \Lambda \times \mathbb{C}^\times \to \Lambda \) is the projection.
Recall that a \( \mathbb{C} \)-linear triangulated category \( T \) is called Calabi-Yau of dimension \( d \) if for each \( M, N \in T \) the vector spaces \( \text{Hom}_T(M, N) \) are finite dimensional and there are isomorphisms

\[
\text{Hom}_T(M, N)^\vee \simeq \text{Hom}_T(N, M[d]),
\]

where \( H^\vee \) denotes the dual of a vector space \( H \).

Denote by \( D^b_{\text{rh}}(E_Y) \) the full triangulated subcategory of the bounded derived category of \( E_Y \)-modules whose objects have regular holonomic cohomologies.

The following theorem is obtained in \cite{7} as a corollary of results from \cite{6}.

**Theorem 2.4.** If \( Y \) is compact, \( D^b_{\text{rh}}(E_Y) \) is a \( \mathbb{C} \)-linear Calabi-Yau triangulated category of the same dimension as \( Y \).

3. Quantization of symplectic manifolds

Let \( X \) be a complex symplectic manifold. In this section we describe a construction of the deformation-quantization algebroid \( W_X \) of \cite{10}, which we also use to introduce the quantization algebroid \( \hat{E}_X \). We then discuss some results on regular holonomic \( \hat{E}_X \)-modules.

By Darboux theorem, the local model of \( X \) is an open subset of the cotangent bundle \( T^*M \) with \( M = \mathbb{C}^{2\dim X} \). A contactification \( \rho: V \to U \) of an open subset \( U \subset X \) is a principal \( \mathbb{C} \)-bundle whose local model is the projection

\[
P^*(M \times \mathbb{C}) \supset \{ \tau \neq 0 \} \xrightarrow{\xi} T^*M
\]
given by \( \rho(x, t; \xi, \tau) = (x, \xi/\tau) \). Here, the \( \mathbb{C} \)-action is given by translation \( t \mapsto t + \lambda \).

Note that the outer isomorphism of \( \rho_*\mathcal{E}_M \times \mathbb{C} \) given by translation at the level of total symbols is represented by \( \text{Ad}(e^{\lambda \hbar}) \).

Consider a quadruple \( q = (\rho, \mathcal{E}, \ast, \hbar) \) of a contactification \( \rho: V \to U \), a microdifferential algebra \( \mathcal{E} \) on \( V \), an anti-involution \( \ast \) and an operator \( \hbar \in \mathcal{E} \) locally corresponding to \( \partial_\hbar^{-1} \). Any two such quadruples \( q' \) and \( q \) are locally isomorphic, meaning that there locally exists a pair \( \tilde{f} = (\chi, f) \) of a contact transformation \( \chi: \rho' \to \rho \) over \( U \) and a \( \mathbb{C} \)-algebra isomorphism \( f: \chi_*\mathcal{E}' \to \mathcal{E} \) such that \( f\ast' = \ast f \) and \( f(\hbar') = \hbar \). Moreover, by \cite{10} Lemma 5.4] the automorphisms of \( q \) are locally in bijection with the group

\[
\mathbb{C}_U \times \{ b \in \rho_*\mathcal{E}^\times; [h, b] = 0, b^*b = 1, \sigma_0(b) = 1 \},
\]

by \( (\mu, b) \mapsto (T_\mu, \text{Ad}(be^{\lambda \hbar^{-1}})) \). Here \( [h, b] = \hbar b - bh \) is the commutator and \( T_\mu \) denotes the action of \( \mu \) on \( V \).

Consider the quantization algebra

\[
\tilde{\mathcal{E}} = \bigoplus_{\lambda \in \mathbb{C}} (C^\infty_{\hbar} \rho_*\mathcal{E}) e^{\lambda \hbar^{-1}},
\]

where \( C^\infty_{\hbar} \rho_*\mathcal{E} = \{ a \in \rho_*\mathcal{E}; \text{ad}(\hbar)^N(a) = 0, \exists N \geq 0 \} \) locally corresponds to operators in \( \rho_*\mathcal{E}_M \times \mathbb{C} \) whose total symbol is polynomial in \( t \). Here \( \text{ad}(\hbar)(a) = [h, a] \) and the product in \( \tilde{\mathcal{E}} \) is given by

\[
(a \cdot e^{\lambda \hbar^{-1}})(b \cdot e^{\mu \hbar^{-1}}) = a \text{Ad}(e^{\lambda \hbar^{-1}})(b) \cdot e^{(\lambda + \mu)\hbar^{-1}}.
\]

One checks that \( \tilde{\mathcal{E}} \) is coherent.

\footnote{The statement in \cite{7} Theorem 9.2 (ii) is not correct. It should be read as Theorem 2.4 above.}
**Definition 3.1.** The quantization algebroid $\tilde{E}_X$ is the $\mathbb{C}$-linear stack on $X$ whose objects on an open subset $U$ are quadruples $q = (\rho, E, *, h)$ as above. Morphisms $q' \to q$ are equivalence classes $[\tilde{a}, \tilde{f}]$ of pairs $([\tilde{a}, \tilde{f}])$ with $\tilde{a} \in \tilde{E}$ and $\tilde{f}: q' \to q$. The equivalence relation is given by $(\tilde{a}b, \tilde{f}) \sim (\tilde{a}, \text{Ad}(\tilde{b})\tilde{f})$ for $\tilde{b} = be^{\mu h^{-1}}$ with $(\mu, b)$ as in (3.3). Here $\text{Ad}(\tilde{b}) = (T_{\mu \chi}, \text{Ad}((\chi)))$. Composition and linearity are given as in Definition 2.1.

A similar construction works when replacing the algebra $\tilde{E}$ by its subalgebra $\mathcal{W} = C^0_0 h, \rho, E$ of operators commuting with $h$. Locally, this corresponds to operators of $\rho, \mathcal{E} \mathcal{M} \times \mathbb{C}$ whose total symbol does not depend on $t$. Then the action of $\text{Ad}(e^{\mu h^{-1}})$ is trivial on $\mathcal{W}$, and these algebras patch together to give the deformation-quantization algebroid $\mathcal{W}_X$ of $[10]$.

The parameter $h$ acts on $\tilde{E}_X$ but is not central. The centralizer of $h$ in $\tilde{E}_X$ is equivalent to the twist of $\mathcal{W}_X \otimes \mathbb{C} (\bigoplus_{\lambda \in \mathbb{C}} C_{\mathbb{C}} e^{\mu h^{-1}})$ by the gerbe parameterizing the primitives of the symplectic 2-form.

If $X$ admits a global contactification $\rho: Y \to X$ one can construct as above a $\mathbb{C}$-algebroid $E_{[\rho]}$ on $X$ locally represented by $\rho, \mathcal{E}$. Then there are natural functors
\[
(3.3) \quad \rho^{-1} E_{[\rho]} \to E_Y, \quad E_{[\rho]} \to \tilde{E}_X.
\]

**Remark 3.2.** For $M$ a complex manifold, the algebra $\mathcal{E} \mathcal{M}^{1/2}$ on $T^* M$ has an anti-involution $*$ and a section $h = \partial_t^{-1}$ on the open subset $\tau \neq 0$. For $\rho$ as in (3.1), the quadruple $(\rho, \mathcal{E} \mathcal{M}^{1/2}, *, h)$ is a global object of $\tilde{E}_{T^*M}$ whose sheaf of endomorphisms is $\mathcal{E}_{\mathcal{M}^{1/2}}$. Thus $\tilde{E}_{T^*M}$ is represented by $\mathcal{E}_{\mathcal{M}^{1/2}}$.

In order to introduce the notion of regular holonomic $\tilde{E}_X$-modules we need some geometric preparation.

**Proposition 3.3.** Let $\Lambda$ be a Lagrangian subvariety of $X$. Up to replacing $X$ with an open neighborhood of $\Lambda$, there exists a unique pair $(\rho, \Gamma)$ with $\rho: Y \to X$ a contactification and $\Gamma$ a Lagrangian subvariety of $Y$ such that $\rho$ gives a homeomorphism $\Gamma \xrightarrow{\sim} \Lambda$.

Let us give an example that shows how, in general, $\Gamma$ and $\Lambda$ are not isomorphic as complex spaces.

**Example 3.4.** Let $X = T^* \mathbb{C}$ with symplectic coordinates $(x; u)$, and let $\Lambda \subset X$ be a parametric curve $\{(x(s), u(s)); s \in \mathbb{C}\}$, with $x(0) = u(0) = 0$. Then $Y = X \times \mathbb{C}$ with extra coordinate $t$, $\rho$ is the first projection and $\Gamma$ is the parametric curve $\{(x(s), u(s), -f(s)); s \in \mathbb{C}\}$, where $f$ satisfies the equations $f'(s) = u(s)x'(s)$ and $f(0) = 0$. For $x(s) = s^3, u(s) = s^7 + s^8$ we have $f(s) = s^3 + s^7 + s^8$. This is an example where $f$ cannot be written as an analytic function of $(x, u)$. In fact, $s^{11} = \frac{1}{15} x(s)u(s) - \frac{1}{15} f(s)$ and $s^{11} \not\in \mathbb{C}[[s^3, s^7 + s^8]]$.

One checks as in [11] that the functors induced by (3.3)
\[
\text{Mod}_{T^*\mathbb{C}, \text{coh}}(E_{[\rho]}) \xleftarrow{\Phi} \text{Mod}_{T^*\mathbb{C}, \text{coh}}(E_{[\rho]}) \xrightarrow{\Phi} \text{Mod}_{\Lambda, \text{coh}}(\tilde{E}_X),
\]
are fully faithful. Denote by $\text{Mod}_{T^*\mathbb{C}, \text{rh}}(E_{[\rho]})$ the full abelian substack of $\text{Mod}_{T^*\mathbb{C}, \text{coh}}(E_{[\rho]})$ whose essential image by $\Phi$ consists of regular holonomic $E_{\mathbb{V}}$-modules. Denote by $\text{Mod}_{\Lambda, \text{rh}}(\tilde{E}_X)$ the essential image of $\text{Mod}_{T^*\mathbb{C}, \text{rh}}(E_{[\rho]})$ by $\Psi$. 

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Definition 3.5. The stack of regular holonomic \( \tilde{\mathcal{E}}_X \)-modules is the \( \mathbb{C} \)-linear abelian stack defined by

\[
\text{Mod}_{\text{rh}}(\tilde{\mathcal{E}}_X) = \lim_{\Lambda} \text{Mod}_{\text{rh}}(\tilde{\mathcal{E}}_X).
\]

As a corollary of Proposition 2.3 we get

Proposition 3.6. If \( \Lambda \subset X \) is a smooth Lagrangian submanifold, there is an equivalence

\[
\text{Mod}_{\text{rh}}(\tilde{\mathcal{E}}_X) \simeq p_1^* \text{LocSys}(p_1^{-1} C_{\Omega/\Lambda}^{1/2}),
\]

where \( p_1 : \Lambda \times \mathbb{C}^\times \to \Lambda \) is the projection.

Remark 3.7. When \( X \) is reduced to a point, the category of regular holonomic \( \mathcal{E}_X \)-modules is equivalent to the category of local systems on \( \mathbb{C}^\times \).

Finally, as a corollary of Theorem 2.4 we get

Theorem 3.8. If \( X \) is compact, \( D^b_{\text{rh}}(\tilde{\mathcal{E}}_X) \) is a \( \mathbb{C} \)-linear Calabi-Yau triangulated category of dimension \( \dim X + 1 \).

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