Central Coulomb Effects on Pion Interferometry

D. Hardtke and T.J. Humanic

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Department of Physics, The Ohio State University, Columbus, OH 43210.

Abstract

Using a simple final-state rescattering model coupled with a simple Coulomb model, the effects of the central Coulomb potential on pion interferometry measurements in 158 GeV/nucleon Pb+Pb collisions are explored. Although the central Coulomb potential does not introduce correlations between pions, it does prevent an accurate measurement of the momentum difference. This momentum difference smearing effect leads to a reduction in the measured correlation radii and lambda parameters. These distortions are important in 158 GeV/nucleon Pb+Pb collisions because of large source sizes and the strong central Coulomb potential.

1 INTRODUCTION

Pion interferometry is a useful tool for probing the freeze-out conditions in relativistic heavy-ion collisions, but in order to interpret the measured correlation radii it is very important to understand any final state interactions that may distort the measurements. Recent NA44 measurements of the transverse mass ($m_T = \sqrt{p_T^2 + m^2}$) dependence $\pi^-/\pi^+$ ratio in 158 GeV/nucleon Pb+Pb collisions [1] show a noticeable enhancement in the $\pi^-/\pi^+$ ratio at low $m_T$. This enhancement has been interpreted as evidence for a strong Coulomb interaction between the pion emitting source and the outgoing pions [1, 2]. The strong central Coulomb potential is created by the primary protons in the projectile and target nuclei.

This work explores the influence of this central Coulomb potential on two-pion Bose-Einstein correlation measurements. The single particle data [1] suggest that central Coulomb effects will be important only for low-$p_T$ pion correlation measurements, but these measurements have been shown to be most sensitive to the lifetime of the pion emitting source [3]. It is thus important to understand the central Coulomb effects in order to extract useful information from pion interferometry measurements.

Although the Au+Au system at 10.8 GeV/c is not studied explicitly in this work, the qualitative conclusions should be applicable at these energies since a similar enhancement in the $\pi^-/\pi^+$ ratio is seen at these energies [1, 3].
2 Rescattering+Coulomb Model

As input to these calculations, 158 GeV/nucleon Pb+Pb events from a simple rescattering model were used. A brief summary of the rescattering calculation used is given below. More information about this method can be found elsewhere \[9, 10, 11\]. Rescattering is simulated using a Monte Carlo cascade calculation which assumes strong binary collisions between hadrons. Besides more common hadrons such as pions, kaons, and nucleons, the calculation also includes the $\rho, \omega, \eta, \eta', \phi, \Delta$, and $K^*$ resonances. Resonances can be present at hadronization and also can be produced as a result of rescattering. Relativistic kinematics is used throughout. Isospin-averaged scattering cross sections are taken from Prakash et al \[9\].

The HBT source sizes extracted from this model are in reasonable agreement with the measured source sizes form S+Pb and S+Ag collisions and preliminary Pb+Pb data \[7, 8\]. The rescattering process generates strong position-momentum correlations, leading to a $p_T$ dependence of the extracted radius parameters that matches the experimental observations for the S induced collisions. The position-momentum correlations produced in the rescattering stage are important for the Coulomb calculations.

This model uses isospin averaged cross-sections. Thus, for example, all $\pi^0$, $\pi^+$ and $\pi^-$'s are treated as generic $\pi$'s during the rescattering stage. Coulomb interactions are assumed not to be important during the strong rescattering stage. The effects of the Coulomb interaction are added after freeze-out using an analytic correction to the particle momenta.

The model produces a list of particle freeze-out positions and momenta. Coulomb interactions are added by assuming that these particles are produced in a classical Coulomb potential\[10\]. The average classical Coulomb potential is

$$V_s = \frac{Z_{eff} e^2}{r_a},$$

where $Z_{eff}$ is the effective charge and $r_a$ is the average freeze-out radius. Of course, in a realistic model, each particle is produced in a different Coulomb potential that depends on freeze-out time and position. Using the average Coulomb potential, however, is computationally much simpler and should not change the qualitative conclusions.

Using simple energy and momentum conservation, the momentum of the particle at freeze-out can be related to the momentum of the particle at the detector using

$$E(p) = E(p_i) + V_s,$$

where $p$ is the measured momentum of the particle, $p_i$ is the momentum of the particle at freeze-out, $E(p) = (p^2 + m^2)^{1/2}$, and $V_s$ is the effective average Coulomb potential. Note that $V_s$ is positive for positive particles and negative for negative particles. Using this equation, the momentum shift for each particle is calculated:

$$\Delta p = p - p_i.$$  \hspace{1cm} (3)

Up to this point the calculation is identical to \[10\]. In \[10, 11\], the $\pi^+ / \pi^-$ ratio at mid-rapidity is calculated to be,

$$\frac{N(\pi^+)}{N(\pi^-)} = \frac{\sqrt{p_T^2 - 2E(p_i)|V_s| + V_s^2 (E(p_i) - |V_s|)}}{\sqrt{p_T^2 + 2E(p_i)|V_s| + V_s^2 (E(p_i) + |V_s|)}},$$  \hspace{1cm} (4)
The effect of the central Coulomb potential on the measured momentum difference is calculated to be,

\[ Q \approx Q_i (1 + \frac{V_s}{p_i}) , \] (5)

where \( Q_i \) is the momentum difference at freeze-out and \( Q \) is the measured momentum difference. This formula predicts that negatively charged particle pairs will give a measured source size that is larger than the actual source size and that positively charged pairs will give a measured source size that is smaller than the actual source size.

The simple analytical calculation \[10\] does not include any directional dependence. Correlation functions are typically measure experimentally in three dimensions and parameterized by \[12\]

\[ C_2(\vec{Q}, \vec{k}) = 1 + \lambda \exp(-R_{to}^2(\vec{k})Q_{to}^2 - R_{ts}^2(\vec{k})Q_{ts}^2 - R_{tl}^2(\vec{k})Q_{tl}^2), \] (6)

where \( \vec{k} = \vec{p}_1 + \vec{p}_2 \) is the total momentum of the pair, \( Q_t \) is the momentum difference in the beam direction, \( Q_{to} \) is the momentum difference in the direction transverse to beam and parallel to the total transverse momentum of the pair, and \( Q_{ts} \) is the momentum difference in the direction transverse to beam and perpendicular to the total transverse momentum of the pair. In this work, the directional dependence is explicitly considered. \( \Delta p \) is calculated analytically using equation (3), and then the directional dependence is added using

\[ \vec{p} = \vec{p}_i + \Delta p \hat{x}, \] (7)

where \( \hat{x} \) is the unit vector in the direction of the freeze-out position \( \vec{x} \). This approach assumes that the central Coulomb charge is spherically symmetric.

The procedure used in these calculations is the following. Events were generated using the simple rescattering model. This produces a list of pion freeze-out momenta and positions. The Coulomb potential is added using equations (2),(3), and (7). The correlation functions are then calculated using the procedure described in \[6, 7\]. All correlation functions are evaluated in the LCMS frame where \( p_{z1} + p_{z2} = 0 \). The only free parameter in the calculation is magnitude of the average Coulomb potential, \( V_s \).

### 3 Results and Discussion

The average Coulomb potential \( V_s \) is found by fitting equation (4) to the NA44 data \[1\]. The value of \( V_s \) extracted from the single particle data is \( \approx 5 \) MeV. This is actually a lower limit on the Coulomb potential since equation (4) is the mid-rapidity limit \( (y = 2.9) \) and the NA44 data was measured over the rapidity range \( 3.1 < y < 4.1 \). This value of \( V_s \) is still large enough to have effects on the measured pion correlation functions.

In order to facilitate comparison to future experimental data, the pion correlation functions are calculated for the low \( p_T \) NA44 acceptance \[12\]. The rapidity and \( p_T \) range are \( 0 < p_T < 400 \) MeV/c and \( 3.1 < y < 4.1 \), respectively, and the mean \( p_T \) is approximately 150 MeV/c. The correlation function is calculated with three values of \( V_s \): \( V_s = 0 \) (i.e., no Coulomb effects), \( V_s = 5 \) MeV (i.e., \( \pi^+\pi^- \)), and \( V_s = -5 \) MeV (i.e., \( \pi^-\pi^- \)).

Figure 1 compares the momentum difference shifts in \( Q_{ts}, Q_{to} \) and \( Q_t \) for the positive pion pairs (\( V_s = 5 \) MeV) and the negative pion pairs (\( V_s = -5 \) MeV). These momentum
Table 1: The mean and RMS deviations of the momentum difference shifts caused by the central Coulomb potential for negative and positive pion pairs.

|       | $\pi^-\pi^-$ | $\pi^+\pi^+$ |
|-------|-------------|-------------|
| mean(MeV) | RMS(MeV) | mean(MeV) | RMS(MeV) |
| $\Delta Q_{ts}$ | -2.4 | 6.4 | 2.5 | 6.3 |
| $\Delta Q_{to}$ | -1.6 | 6.0 | 1.7 | 5.9 |
| $\Delta Q_l$ | -1.3 | 5.0 | 1.5 | 4.8 |

Table 2: The fitted results of the gaussian parameterization of the correlation functions for pions without the central Coulomb potential, with a negative Coulomb potential ($\pi^-\pi^-$), and with a positive Coulomb potential ($\pi^+\pi^+$).

| $V_s$(MeV) | $R_{ts}$(fm) | $R_{to}$(fm) | $R_l$(fm) | $\lambda$ |
|-----------|-------------|-------------|-----------|-----------|
| 0.        | 5.55 ± 0.12 | 6.37 ± 0.26 | 8.13 ± 0.63 | 0.63 ± 0.03 |
| -5.0 ($\pi^-\pi^-$) | 5.53 ± 0.09 | 6.17 ± 0.06 | 6.73 ± 0.13 | 0.54 ± 0.01 |
| 5.0 ($\pi^+\pi^+$) | 5.33 ± 0.10 | 6.14 ± 0.16 | 7.38 ± 0.22 | 0.62 ± 0.02 |

difference shifts are then characterized by a mean momentum shift and an RMS deviation of the momentum shift, and the values are listed in Table 1. Figure 1 and Table 1 show that the mean shift in all three components of the momentum difference is small. This is what would be expected since pions of similar momentum should experience roughly the same Coulomb impulse. The RMS deviations of the momentum shift, however, are significantly larger than the mean shifts. This means that although the net momentum difference shift is small, the direction of the momentum difference vector is changed appreciably by the central Coulomb potential.

The projections of the correlation functions in the NA44 acceptance with and without the central Coulomb potential are shown in Figure 2, and the extracted fit parameters are listed in Table 2. The extracted radii are smaller for both the positive and negative Coulomb potentials compared to the extracted radii without the Coulomb potential, and the reduction is largest for $R_L$. The magnitude of the reduction in $R_L$ and $\lambda$ is largest for the negative pions. The transverse radius parameters ($R_{ts}$ and $R_{to}$) are affected only slightly.

There are two effects due to the central Coulomb potential that lead to a distortion of the correlation function. The central Coulomb potential introduces a net shift in the momentum difference. This net shift was pointed out in [10], [13]. The net shift in the momentum difference, however, has only a small effect compared to the momentum difference smearing effect. The momentum difference smearing effect, characterized by the RMS of the momentum difference shift distributions, is the more important of the two effects and leads to a reduction of the measured radii and lambda parameters. The effect is similar to the well understood effect of the experimental momentum resolution on the correlation function. The momentum difference smearing due to the central Coulomb potential is similar for $Q_{ts}$, $Q_{to}$ and $Q_l$, but, due to the larger radius in the longitudinal direction, the change in the extracted radius is largest for $R_l$.

In [10] only the net momentum difference shifts were considered. This leads to the conclusion that positively charged pairs should measure a source size smaller than the true
source size and negatively charged pairs should measure a source size larger than the true source size. The present work suggests that both negative and positive pairs measure a source size smaller than the true source size due to the momentum difference smearing effect.

In [13], the influence of the nuclear Coulomb charge on the measured transverse correlation radii is much larger than in the present work and qualitatively different. This is probably due to the different approach used here. In [13], the Klein-Gordon equation is used to find the wave function for a particle originating in a Coulomb potential parameterized by

\[ U = \pm Ze^2 \Phi(r/\sqrt{2R_0})/r, \]  

(8)

where \( Z \) is the charge number, \( R_0 \) is the source radius, and \( \Phi \) is the error function. In [13], the calculation is for Au+Au collisions at 1 GeV/nucleon. \( R_0 \) is estimated to be 4.5 fm and \( Z \) is taken to be 160. Equation (8) reduces to equation (1) in the limit \( r > R_0 \). In this work, however, equation (1) is not actually used to calculate the value of the average Coulomb potential \( V_s \), but instead a value of \( V_s \) is extracted from experimental data. This leads to a smaller value for the average Coulomb potential than would be extracted from equation (8).

If equation (1) is evaluated with \( Zeff = 160 \) and \( ra = 4.5 \) fm, a value of \( V_s \approx 50 \) MeV is extracted, which should be compared to value \( V_s = 5 \) MeV extracted from the experimental data and used for these calculations. In addition, it is assumed in this work that the net nuclear Coulomb charge is spherically symmetric, whereas [13] only looks at the effect of the Coulomb field in the transverse direction. The current model has in fact been examined for the case in which the Coulomb field is assumed to have only a transverse component, and in that case it was found that the transverse radii \( R_{ts} \) and \( R_{ts} \) are reduced much more strongly than for the central potential used in the calculations presented here. Even in the limit where the Coulomb field is assumed to have only a transverse component, the current work disagrees qualitatively with [13] in that the Coulomb potential reduces all measured correlation radii whereas [13] predicts that the measured sideward radius \( R_{ts} \) should be larger than the true source size for low-\( p_T \) \( \pi^- \pi^- \) pair measurements.

The problem of the central Coulomb potential on the measured pion correlation functions was also addressed in Ar + KCl collisions at 1.8 GeV/nucleon [14]. The approach used, however, was much different than that used here. In [14], an attempt was made to correct the experimental data for the central Coulomb distortions. The momentum shift of each pion due to the central Coulomb charge was calculated using a formalism correct in both the classical and quantum mechanical limit [15]. It was found that the central Coulomb effect on the measured correlation function was small compared to the experimental error on the measured radii. The measured radii in [14] were much smaller than the predicted radii in Pb+Pb collisions at 158 GeV/nucleon, so the momentum difference smearing effect should be much less important.

### 4 Conclusions

Using a simple rescattering model and a classical Coulomb model, it was shown that the central Coulomb potential in 158 GeV/nucleon Pb+Pb collisions causes a reduction in the the measured HBT radii and lambda parameters for charged pions. This effect should be considered when comparing theoretical predictions to experimental data.
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6 FIGURE CAPTIONS

Figure 1: The shifts in $Q_{ts}$, $Q_{to}$, and $Q_{t}$ due to the central Coulomb potential for negative and positive pion pairs. The average Coulomb potential $V_s = 5$ MeV.

Figure 2: The projections of the correlation functions (C2) in $Q_{ts}$, $Q_{to}$, and $Q_{t}$ for positive and negative pions (open triangles) compared with the projections without the central Coulomb potential (open circles). The fit functions are also included. The projections are over the lowest 20 MeV in the other momentum difference directions.

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