Semi-inclusive Decay $B \to \phi X_s$: Rate and Momentum Spectrum of $\phi$

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Abstract

We study the rate and $\phi$ momentum distribution in semi-inclusive decays $B \to \phi X_s$ induced by the quark level processes $b \to \phi s$ and $b \to \phi sg$, in which the gluon is radiated from the internal charm quark loop or emitted from the virtual gluon of the strong penguin (inner bremsstrahlung). We find $\mathcal{B}(b \to \phi s) = 6.7 \times 10^{-5}$ and $\mathcal{B}(b \to \phi sg) = 3.8 \times 10^{-5}$. The momentum spectrum of $\phi$ produced by $b \to \phi sg$ is very broad. With the cut $|k_{\phi}| \geq 2.0$ GeV, $\mathcal{B}(b \to \phi s) = 6.1 \times 10^{-5}$ (where the Fermi motion of the $b$-quark in the $B$-meson is described by a Gaussian), and $\mathcal{B}(b \to \phi sg) = 1.0 \times 10^{-5}$. Due to the special nature of $\phi$, many difficulties which hindered a reliable theoretical prediction for $B \to \eta' X_s$ decay are absent in the process $B \to \phi X_s$. Therefore, theoretical predictions for $B \to \phi X_s$ are relatively clean. Moreover, the clear experimental signature of the $\phi$ is of great help. Data for $B \to \phi X_s$, both the branching ratio and the $\phi$ momentum distribution, would teach us about the strength of strong penguins which might be of great importance in the search for CP violation and for new physics at $B$ factories.

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1 Introduction

Understanding of pure penguin decays of $B$ mesons, is of utmost importance. Penguins have been serving as powerful probes for the Standard Model (SM) and for beyond the SM scenarios. Measurements of the electromagnetic penguin process $b \rightarrow s\gamma$ by CLEO [1], which agrees with the SM, has provided very stringent constraints on new physics. The strong penguin processes $B \rightarrow \eta' X_s$, $B \rightarrow \eta' K^{(*)}$, $B \rightarrow \phi K^{(*)}$ have been observed by CLEO [2], BABAR [3, 4] and BELLE [5, 6]. As is well known, it is quite difficult to provide theoretical estimates of exclusive nonleptonic decays, although there has been some progress on this topic. In particular we refer to the QCD factorization [7] and pQCD approaches [8] developed recently. To explain the large yields of $\eta'$, we encounter unknown parameters like the content of $\eta'$, mixing-angles, the $gg - \eta'$ coupling and so on, which have hindered reliable theoretical predictions. There are also suggestions that new physics could enhance the magnitude of the strong penguin. At present we cannot conclude how large the window is for new physics or whether it is required to explain the data. The semi-inclusive $B \rightarrow \phi X_s$ decay is theoretically cleaner than $B \rightarrow \eta' X_s$, since $\phi$ is almost a pure $s\bar{s}$ state with mass larger than $2M_K$ and does not couple to two gluons. In addition, $\phi$ coupling to three gluons is highly suppressed by the OZI rule.

There is a number of studies of $B \rightarrow K^{(*)}\phi$, using different approaches [9, 10, 11, 12]. Unfortunately the results do not converge. QCD factorization predictions [10, 11] are smaller than those of pQCD [12]. Compared with exclusive decays, the theoretical predictions for inclusive and semi-inclusive decay rates of $B$ mesons rest on more solid grounds. The semi-inclusive decay $B \rightarrow \phi X_s$ was studied a few years ago [13]. Recently it has caught renewed interest [14, 15] by generalizing the QCD factorization formalism [7] to semi-inclusive processes. In the present paper, we will study both the branching ratio and the $\phi$ momentum distribution, hereafter denoted by $|k_\phi|$. We take into account the Fermi motion of the $b$-quark for $b \rightarrow s\phi$ and the $b \rightarrow sg^*g$ strong penguin effects arising from the inner bremsstrahlung processes in which a gluon is emitted from the charm loop or from the virtual gluon in $b \rightarrow sg^*$.

Possible large $b \rightarrow sgg$ contribution to inclusive $B$ meson charmless decays was discussed in Refs. [13, 16, 17, 18, 19]. Furthermore, while studying penguin effects, Gerard and Hou [13, 14] found that the higher order processes $b \rightarrow sgg$ and $b \rightarrow sq\bar{q}$ dominate over $b \rightarrow sg$. Subsequently, it was found by Simma and Wyler [17], and independently by Liu and Yao [18],
that $b \rightarrow sgg$ is considerably suppressed as compared to $b \rightarrow sqq$. Both groups found that the large form factor $F_1(x)$ is absent when both gluons are on-shell, while if one of the two gluon goes off-shell, $F_1(x)$ survives.

In the present paper we work within the framework of an effective low energy theory with five active quarks which is obtained by integrating out heavy top and heavy gauge bosons [20]. We also use the QCD factorization framework to deal with the hadronic dynamics of $\phi$ formation. We find that the $b \rightarrow \phi sg$ contribution is very significant, $\mathcal{B}(b \rightarrow \phi sg) = 3.8 \times 10^{-5}$. After imposing a momentum cut $|k_\phi| \geq 2.0$ GeV, $\mathcal{B}(b \rightarrow \phi sg)$ is reduced to $1.0 \times 10^{-5}$, which is about 16% of the fast $\phi$ production due to $b \rightarrow \phi s$. To understand this "unusual" large contribution, we note that the amplitude for $b \rightarrow \phi sg$ is characterized by a factor $C_1 \frac{g^3}{16 \pi^2} \simeq 0.029$ which is numerically comparable to $C_6 = -0.041$, known to be the largest coefficient of the strong penguin four Fermion operators. It is interesting to note that similar large next-to-leading (NLO) corrections have also been found for $b \rightarrow sg$ [19] and $B \rightarrow K^*\gamma$ [21].

The remainder of the paper is organized as follows: In Sec.2 we study the momentum spectrum of $\phi$ resulting from $b \rightarrow \phi s$ by taking into account the effect of Fermi motion. Sec.3 contains a calculation of the $b \rightarrow \phi sg$ contributions. We present and discuss our results in Sec.4. Some useful formulas are given in three Appendices.

2 $B \rightarrow \phi X_s$ induced by $b \rightarrow \phi s$

In the low energy effective theory of the SM, the relevant effective Hamiltonian is written as follows [20]

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \left[ V_{cb} V_{cs}^* \sum_{i=1}^{2} C_i O_i - V_{tb} V_{ts}^* \left( C_9 O_9 + \sum_{j=3}^{10} C_j O_j \right) \right].$$

(1)

The operators $O$ are listed in App.A. The Wilson coefficients evaluated at scale $\mu = m_b$ are [20]

$$C_1 = 1.082, \quad C_2 = -0.185, \quad C_3 = 0.014,$$
$$C_4 = -0.035, \quad C_5 = 0.009, \quad C_6 = -0.041,$$
$$C_7 = -0.002/137, \quad C_8 = 0.054/137, \quad C_9 = -1.292/137,$$
$$C_{10} = 0.262/137, \quad C_g = -0.143.$$  

(2)
Figure 1: Leading diagram for \((B \to \phi X_s)_2\) defined in Eq. (3). \(O_i\) are strong penguin operators in which top penguin effects are embedded.

In the naive factorization approach, \(b \to \phi s\) decay is a color-suppressed process depicted in Fig. 1. The amplitude can be easily written as

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} \gamma_\mu (1 - \gamma_5) b \cdot \epsilon_\phi^b f_\phi m_\phi A_p, \tag{3}
\]

where

\[
A_p = -V_{tb} V_{ts}^* \left( a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right), \tag{4}
\]

and \(a_{2n-1} = C_{2n-1} + C_{2n}/N_c, a_{2n} = C_{2n} + C_{2n-1}/N_c, (n = 2, 3, 4, 5).\) The branching ratio is

\[
\mathcal{B}(b \to \phi s) = \tau_B \frac{G_F^2 f_\phi^2 m_b^3}{16\pi} |A_p|^2 \left( 1 - \frac{m_\phi^2}{m_b^2} \right) \left( 1 + 2 \frac{m_\phi^2}{m_b^2} \right). \tag{5}
\]

Let us denote the two body contribution to \(B \to \phi X_s\) by

\[
\mathcal{B}[(B \to \phi X_s)_2] \equiv \mathcal{B}(b \to \phi s). \tag{6}
\]

Using \(f_\phi = 233\) MeV, \(m_b = 4.8\) GeV, and the Wolfenstein parameterization \(V_{tb} = 1, V_{ts} = -A\lambda^2\) with \(A = 0.817\) and \(\lambda = 0.2237\) from Ref. [22], we have

\[
\mathcal{B}[(B \to \phi X_s)_2] = 4.9 \times 10^{-5} \quad \text{(naive factorization)}. \tag{7}
\]

It is believed that naive factorization works very well for color-allowed processes since Bjorken’s color transparency argument [23] applies, while for color-suppressed processes non-factorizable effects could play an important role. As demonstrated by Cheng and Soni [14],
the nonfactorization effects depicted in Fig.2 could be calculated in the BBNS QCD factorization framework. The calculation of the effects of Fig.2 was carried out in Refs. [14, 24].

Incorporating the nonfactorizable contributions, \( A_p \) in Eq.6 is modified to

\[
A'_p = V_{ub} V_{us}^* \left[ a^u_3 + a^u_4 + a^u_5 - \frac{1}{2} (a^u_7 + a^u_9 + a^u_{10}) + a^u_{10a} \right]
+ V_{cb} V_{cs}^* \left[ a^c_3 + a^c_4 + a^c_5 - \frac{1}{2} (a^c_7 + a^c_9 + a^c_{10}) + a^c_{10a} \right],
\]

(8)

where the coefficients \( a^q_i \)’s can be found in Ref. [24]. We subtract the contribution of gluon-spectator interactions (\( f_{II} \) in [24]), choose \( \gamma = 54.8^\circ \) and present the numerical results in Table 1. From Table 1, we can see that our results agree with those by Cheng and Soni [14]. We have

\[
B[(B \to \phi X_s)_2] = 6.7 \times 10^{-5} \quad \text{(QCD factorization).}
\]

(9)

This large branching ratio enhances the feasibility of measuring the strength of the strong penguin to test the SM by studying this decay mode at BABAR and BELLE.

To study the momentum spectrum, we employ the ACCMM model [25] (for an earlier version see [26]). In this model the bound state corrections to free \( b \)-quark decays are incorporated by attributing to the \( b \)-quark Fermi motion within the meson. The spectator quark is handled as
Table 1: Numerical values for $a_i^p$ (in units of $10^{-4}$) in QCD factorization and in naive factorization. Our results are given for $\gamma = 54.8^\circ$. Cheng and Soni’s results [14] are displayed for comparison.

| $a_i^q$ | Our results | Ref. [14] | Naive factorization |
|---------|-------------|-----------|---------------------|
| $a_3^{c,u}$ | 76.0+27.8i | $(a_3)$ 74+26i | 23 |
| $a_4^c$ | -375-71.6i | $(a_4)$ -353-58i | -303 |
| $a_4^u$ | -318-151i | | -303 |
| $a_5^{c,u}$ | -68.3-31.4i | $(a_5)$ -67-30i | -46 |
| $a_7^{c,u}$ | 1.37+0.3i | $(a_7)$ -0.89-1.13i | 1.16 |
| $a_9^{c,u}$ | -90.8-1.4i | $(a_9)$ -92.9-2.8i | -87.9 |
| $a_{10}^{c,u}$ | 1.95+7.23i | $(a_{10})$ 0.6+6.4i | 12.2 |
| $a_{10a}$ | -0.52-1.0i | | |

an on-shell particle with definite mass $m_{sp}$ and momentum $|p| = p$. Consequently, the $b$-quark is considered to be off-shell with a momentum dependent virtual mass $W(p)$

$$W^2(p) = M_B^2 + m_{sp} - 2M_B \sqrt{m_{sp}^2 + p^2},$$

(10)
in which energy-momentum conservation is imposed. The momentum of the $b$-quark is modeled by a Gaussian distribution function with a free parameter $p_F$

$$\phi_F(p) = \frac{4}{\sqrt{\pi}} \frac{1}{p_F^3} \exp \left(-\frac{p^2}{p_F^2}\right).$$

(11)

It is interesting to note that the parameter $p_F$ and the $b$-quark average mass $\langle m_b \rangle$ were obtained from a fit to the photon momentum distribution in $B \to X_s \gamma$ by CLEO [27], to read $p_F = 410$ MeV and $\langle m_b \rangle = 4.690$ GeV. Using these values and Eqs. [10, 11] we get $m_{sp} = 298$ MeV for $B_{u,d}$.

Now, the momentum spectrum of $\phi$ resulting from the decay $b \to \phi s$ of a $b$-quark of mass $W(p)$, is given by

$$\frac{d\Gamma(|k_\phi|, p)}{d|k_\phi|} = \frac{W(p)}{E_b} \frac{\Gamma_0(W(p))}{k_\phi^b - |k_{-\phi}|} \left[ \theta(|k_\phi| - |k_{b_-}|) - \theta(|k_\phi| - k_\phi^b) \right],$$

(12)

where $k_{b,\pm}$ provide the limits of the momentum range which results from the Lorentz boost, i.e.

$$k_{\phi,\pm}^b(p) = \frac{1}{W(b)}(E_bk_0 \pm pE_0)$$

(13)
Figure 3: Momentum spectrum of $\phi$ in $B \to \phi X_s$ decay. The solid curve is for $b \to \phi s$ including Fermi motion. The dotted curve is for $b \to \phi s g$.

with

$$k_0 = \frac{1}{2W(p)}(W(p)^2 - m_\phi^2), \quad E_0 = \sqrt{k_0^2 + m_\phi^2}, \quad E_b = \sqrt{W(p)^2 + p^2}. \quad (14)$$

Finally,

$$\Gamma_0(W(p)) = \frac{G_F^2 f_\phi^2 W(p)^3}{16\pi}|A'_p|^2 \left(1 - \frac{m_\phi^2}{W(p)^2}\right)^2 \left(1 + 2 \frac{m_\phi^2}{W(p)^2}\right). \quad (15)$$

To get the momentum spectrum of $\phi$ from the semi-inclusive decay of the $B$ meson, we have to fold in the $b$-quark momentum probability distribution as given by $\phi_F(p)$,

$$\frac{dB(B \to \phi X_s)}{d|k_\phi|} = \tau_B \int_0^{p_{\text{max}}} dp\phi_F(p)p^2 \frac{d\Gamma(|k_\phi|, p)}{d|k_\phi|}, \quad (16)$$

where $p_{\text{max}}$ is determined by $W(p)^2 \geq (m_\phi + m_s)^2$, leading to

$$p_{\text{max}} = \frac{1}{2M_B} \sqrt{\left(M_B^2 + m_{sp}^2 - (m_\phi + m_s)^2\right)^2 - 4M_B^2 m_{sp}^2}. \quad (17)$$

Our numerical results for the $\phi$ momentum spectrum are presented in Fig.3. We can see the spectrum peaking near $|k_\phi| = 2.4$ GeV which corresponds to $M_{X_s} = 1.15$ GeV. To suppress indirect $\phi$ production, one can impose a momentum cut $|k_\phi| \geq 2.0$ GeV. With this cut, the branching ratio is

$$B[(B \to \phi X_s)_2] = 6.1 \times 10^{-5} \quad \text{(QCD factorization + Fermi motion + $|k_\phi| \geq 2.0$ GeV).} \quad (18)$$
3 Contribution of the three body decay $b \to \phi sg$

The $b \to \phi sg$ contribution to the momentum spectrum of $\phi$ has been studied by Deshpande et al. \cite{13} for the case of gluon radiating from external quarks. They find that the effect is rather small, $\mathcal{B}(b \to \phi sg)/\mathcal{B}(b \to \phi s) \approx 3\%$. From now on we will neglect gluon emission from external lines, and study instead the effect of the gluon radiating from an internal quark or splitting off an internal off-shell gluon as shown by Fig.4. Both these processes are sometimes referred to as “inner bremsstrahlung”. The calculation of Fig.4.(a)-(b) is straightforward. However the calculation of Fig.4.(c) and (d) is tedious. A calculation of the charm loop for $b \to sg^*g$ has been carried out recently by Greub nad Liniger \cite{19}, in a detailed study of $\mathcal{O}(\alpha_s)$ corrections to $\Gamma(b \to sg)$. It gives

$$J_{\alpha\beta}^{AB} = T_{\alpha\beta}^+(q, k)\{T^A, T^B\} + T_{\alpha\beta}^-(q, k)[T^A, T^B].$$

(19)

$q$, $A$ and $\alpha$ denote the momentum, color and Lorentz index for the on-shell gluon, respectively, while $k$, $B$ and $\beta$ stand for the same attributes for the other, off-shell, gluon. The quantities $T_{\alpha\beta}^+(q, k)$ and $T_{\alpha\beta}^-(q, k)$ are \cite{19}

$$T_{\alpha\beta}^+(q, k) = \frac{g^2}{32\pi^2} \left[ E(\alpha,\beta,k)\Delta i_5 + E(\alpha,\beta,q)\Delta i_6 - E(\beta,k,q)\frac{k_\alpha}{q \cdot k}\Delta i_{23} \right]$$

Figure 4: Diagrams for $\mathcal{B}[(B \to \phi X_s)_3]$ defined in Eq.23.
The dimensionally regularized expressions for the $\Delta i$ functions are given by Greub and Liniger \cite{19}. To reduce the difficulty in numerical calculations of multiple integrals, we specialize their functions to $d = 4$ and integrate out the two Feynman parameters. For $d = 4$, $E(\alpha, \beta, \gamma) = -i\epsilon_{\alpha\beta\gamma\mu}\gamma^\mu\gamma_5$, in the Bjorken-Drell conventions. The analytical expressions for the functions $\Delta i$ are collected in App.B, where we have used the $\overline{\text{MS}}$ scheme.

The dimensionless variables $s$, $t$ and $u$ are defined as

$$s = \frac{(p_{\phi} + p_s)^2}{m_b^2}, \quad t = \frac{(p_{\phi} + q)^2}{m_b^2}, \quad u = \frac{(q + p_s)^2}{m_b^2}. \quad (22)$$

We write the amplitude of the three body contribution to $B \to \phi X_s$ as

$$\mathcal{B}[(B \to \phi X_s)_3] \equiv \mathcal{B}(b \to \phi sg). \quad (23)$$

Now, neglecting gluon bremsstrahlung from external quarks, the inner bremsstrahlung diagrams (see Fig.4), lead to the matrix element

$$\mathcal{M}_3 = \mathcal{M}_{O_g} + \mathcal{M}_{O_c} + \mathcal{M}_{\Delta^+} + \mathcal{M}_{\Delta^-}, \quad (24)$$

and its square reads

$$|\mathcal{M}_3|^2 = |\mathcal{M}_{O_g}|^2 + |\mathcal{M}_{O_c}|^2 + |\mathcal{M}_{\Delta^+}|^2 + |\mathcal{M}_{\Delta^-}|^2 + 2\Re(\mathcal{M}_{O_g}\mathcal{M}_{O_c}^\dagger) + 2\Re(\mathcal{M}_{\Delta^+}\mathcal{M}_{\Delta^-}^\dagger)
+ 2\Re(\mathcal{M}_{\Delta^+}\mathcal{M}_{O_c}^\dagger) + 2\Re(\mathcal{M}_{\Delta^-}\mathcal{M}_{O_g}^\dagger) + 2\Re(\mathcal{M}_{\Delta^-}\mathcal{M}_{O_g}^\dagger). \quad (25)$$

The explicit forms for the terms in the amplitude squared are given in App.C.

The decay distribution of the $\phi$ in $\mathcal{B}[(B \to \phi X_s)_3]$ is

$$\frac{d\Gamma(b \to \phi sg)}{du \, dt} = \frac{1}{256\pi^3}m_b \frac{G_F^2}{2} |V_{ts}|^2 \frac{1}{6} |\mathcal{M}_3|^2; \quad (26)$$

where the factor $\frac{1}{6}$ is due to the average of spin and color of the $b$-quark. In the phase space integration, we impose the cut $(p_s + q)^2 \geq m_K^2$. 

\[ -E(\alpha, k, q)\frac{k^\beta}{q \cdot k} \Delta i_{25} - E(\alpha, k, q)\frac{q^\beta}{q \cdot k} \Delta i_{26} \] \[ L, \quad (20) \]

\[ T_{\alpha\beta}(q, k) = -\frac{g^2}{32\pi^2}\left[k g_{\alpha\beta} \Delta i_{12} + \frac{q g_{\alpha\beta}}{q \cdot k} \Delta i_{13} + \gamma^\beta k^\alpha \Delta i_{18} + \gamma^\alpha k^\beta \Delta i_{11} + \gamma^\alpha q^\beta \Delta i_{12} + \frac{k^\alpha k^\beta}{q \cdot k} \Delta i_{15} + \frac{k^\alpha q^\beta}{q \cdot k} \Delta i_{17} + \frac{q^\alpha k^\beta}{q \cdot k} \Delta i_{19} + \frac{q^\alpha q^\beta}{q \cdot k} \Delta i_{21} \right] L. \quad (21) \]
4 Results and discussion

Our results for the contribution of $b \to \phi sg$ to the $\phi$ spectrum, are displayed in Fig.3 by the dotted curve, which is much broader than that due to $b \to \phi s$. One can improve the predictions for the spectrum of the $\phi$ mesons by adding Fermi motion to the three body processes, including radiation from external quarks. The contribution of $b \to \phi sg$ is quite substantial,

$$B[(B \to \phi X_s)_{3}] = 3.8 \times 10^{-5},$$

where the left hand side of the last equation was defined in Eq.23. Now, after applying a momentum cut $|k_\phi| \geq 2.0$ GeV, the branching ratio is reduced to

$$B[(B \to \phi X_s)_{3}] = 1.0 \times 10^{-5} \quad (|k_\phi| \geq 2.0 \text{ GeV}).$$

which is about 16% of $B(b \to \phi s)$. Interestingly, such a large NLO correction was also found in $b \to sg$ [13] and $B \to K^*\gamma$ [21]. In Ref. [19], Greub and Liniger have found that the next-to-leading logarithmic result $B_{NLL}(b \to sg) = (5.0 \pm 1.0) \times 10^{-5}$ is more than a factor of two larger than the leading logarithmic result $B_{LL}(b \to sg) = (2.2 \pm 0.8) \times 10^{-5}$. The reasons for this enhancement also apply here.

It is well known that the strength of strong penguins is an important issue of current interest, relevant to CP violation, tests of the SM and to searches for new physics signals at BABAR and BELLE. The first evidence for a strong penguin was found by CLEO in 1997 by measuring $B \to K\eta'$ and $B \to \eta'X_s$ [29], which turned out be "unexpectedly large". Recently both BABAR [3] and BELLE [5] have confirmed the CLEO measurement with improved precision. Theoretically, there are many models to explain the yield of $\eta'$ in $B$ decays. The measurement of the $\eta'$ momentum spectrum by CLEO [30] has ruled out models with a $b \to c\bar{c}s$ enhancement through a possible $c\bar{c}$ content in the $\eta'$ wave function. Up to now, there are two surviving explanations for the high yield of $\eta'$ in $B$ decays: A model in which new physics enhances the strong penguin $b \to sg$ [31] and a model incorporating the QCD anomaly coupling $g - g - \eta'$ [32, 33]. The $\eta'$ is a very complicated object in QCD. There are unsolved puzzles involving the $\eta'$, such as its large mass, mixing between flavor singlet and octet, gluonic content, decay constants and its mixing with glueballs, which inhibit reliable theoretical predictions for $B \to \eta'K^{(*)}$ and $B \to \eta'X_s$. The aforementioned difficulties are absent for $\phi$ and theoretical predictions for $B \to \phi X_s$ are
rather clean. Furthermore, $\phi$ has a clear experimental signature. We can therefore conclude that the theoretical study and experimental measurement of $B \to \phi X_s$ will provide quite a clean ground for testing the SM and searching for new physics.

In summary, we have studied the semi-inclusive decays $B \to \phi X_s$. We have calculated the contributions of $b \to \phi s$ and of the $b \to \phi sg$ subprocesses with the gluon radiated from the charm loop or from the off-shell gluon of $b \to sg^*$. These effects are found to be quite large, giving $B(b \to \phi sg) = 3.8 \times 10^{-5}$. Cutting the $\phi$ momentum $|k_\phi| \geq 2.0$ GeV, results in $B(b \to \phi sg) = 1.0 \times 10^{-5}$. Adding this contribution to the dominant process $b \to \phi s$, we predict

$$B(B \to \phi X_s) = 10.5 \times 10^{-5} \quad (29)$$

and

$$B(B \to \phi X_s) = 7.1 \times 10^{-5} \quad (|k_\phi| \geq 2.0 \text{ GeV}). \quad (30)$$

This large decay rate and its clear signature render detailed studies of $B \to \phi X_s$ at BABAR and BELLE feasible in the near future. These will shed light on the strength of strong penguins.

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**Appendix A: The operators $O_i$**

The operators in Eq. 1 read

\[
\begin{align*}
O_1 &= \bar{c}_\alpha \gamma^\mu L b_\alpha \cdot \bar{s}_\beta \gamma_\mu L c_\beta, \\
O_2 &= \bar{c}_\alpha \gamma^\mu L b_\beta \cdot \bar{s}_\beta \gamma_\mu L c_\alpha, \\
O_3 &= \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \bar{s}_\beta \gamma_\mu L s_\beta, \\
O_4 &= \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \bar{s}_\beta \gamma_\mu L s_\alpha, \\
O_5 &= \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \bar{s}_\beta \gamma_\mu R s_\beta, \\
O_6 &= \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \bar{s}_\beta \gamma_\mu R s_\alpha, \\
O_7 &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot e_s \bar{s}_\beta \gamma_\mu R s_\beta, \\
O_8 &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\beta \cdot e_s \bar{s}_\beta \gamma_\mu R s_\alpha, \\
O_9 &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot e_s \bar{s}_\beta \gamma_\mu L s_\beta, \\
O_{10} &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\beta \cdot e_s \bar{s}_\beta \gamma_\mu L s_\alpha, \\
O_g &= \frac{g_5}{16\pi^2} \bar{s}_\alpha \sigma^{\mu\nu} R T^a_{\alpha\beta} m_b b_\beta G^a_{\mu\nu}.
\end{align*}
\]
Where $\alpha$ and $\beta$ are the $SU(3)$ color indices and $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$.

Appendix B: Functions for the charm loop

Using the notations $r_1 = \frac{2k}{m_c}$, $r_2 = \frac{k^2}{m_c}$, $q$ for the momentum of the on-shell gluon in the final state and $k$ for the momentum of the off-shell gluon, we have

$$\Delta i_5 = -2 + \frac{16 + 2r_2}{r_1} (G_0(r_2) - G_0(r_1 + r_2)) - \frac{12}{r_1} (G_-(r_2) - G_-(r_1 + r_2)), \quad (32)$$

$$\Delta i_6 = \frac{7}{4} + \frac{7}{r_1} + \frac{r_2}{2r_1} - \frac{r_2^2}{3r_1} + \frac{r_2}{3r_1} - \frac{4r_2(1 + r_2 - r_1 r_2)}{r_1^2} G_0(r_2) + \frac{2(r_1 + 3r_1^2 + r_2 + r_1 r_2)}{r_1^2} G_0(r_1 + r_2) - \frac{4(r_1 - 3r_2)}{r_1^2} G_-(r_2) + \frac{4(1 - 3r_2)}{r_1} G_-(r_1 + r_2)$$

$$+ \frac{5(4 - r_2) r_2}{r_1^2} T_0(r_2) + \frac{5(r_1^2 + 2r_1 (r_2 - 2) + r_2 (r_2 - 4))}{r_1^2} T_0(r_1 + r_2), \quad (33)$$

$$\Delta i_{23} = -2 + \frac{4}{r_1} (G_-(r_2) - G_-(r_1 + r_2)) - \frac{2r_2}{r_1} (G_0(r_2) - G_0(r_1 + r_2)), \quad (34)$$

$$\Delta i_{25} = -2 (G_0(r_2) - G_0(r_1 + r_2)), \quad (35)$$

$$\Delta i_2 = -\frac{8}{3} \ln \frac{\mu}{m_c} + \frac{22}{9} + \frac{16 + 2r_2}{3r_1} G_0(r_2) - \frac{16 - 4r_1 + 2r_2}{3r_1} G_0(r_1 + r_2)$$

$$- \frac{4}{r_1} (G_-(r_2) - G_-(r_1 + r_2)), \quad (36)$$

$$\Delta i_3 = 4 \ln \frac{\mu}{m_c} - \frac{85}{36} + \frac{19 - 3r_2}{r_1} + \frac{r_2}{9r_1} \left( \frac{1}{r_1} - 1 \right) + \frac{4r_2}{3r_1} \left( 1 - r_2 - \frac{3}{r_1} \right) G_0(r_2)$$

$$- \left( \frac{10}{3} + \frac{2}{3r_1} - \frac{14r_2}{3r_1^2} + \frac{2r_2}{r_1} - \frac{4r_2^2}{3r_1^2} \right) G_0(r_1 + r_2) + \frac{4}{r_1} \left( 1 + \frac{2r_2}{r_1} \right) G_-(r_2)$$

$$- \frac{4}{r_1} \left( 1 + \frac{3r_2}{r_1} - r_2 \right) G_-(r_1 + r_2) + \frac{9r_2^2}{r_1^2} (4 - r_2) T_0(r_2)$$

$$+ 9 \left( 1 - \frac{4}{r_1} - \frac{4r_2}{r_1^2} + \frac{2r_2}{r_1} + \frac{r_2^2}{r_1^2} \right) T_0(r_1 + r_2), \quad (37)$$

$$\Delta i_8 = \frac{16}{3} \ln \frac{\mu}{m_c} - \frac{32}{9} + \frac{8}{3r_1} (r_2 - 4) G_0(r_2) - \frac{8}{3} \left( 1 - \frac{4}{r_1} + \frac{r_2}{r_1} \right) G_0(r_1 + r_2), \quad (38)$$

$$\Delta i_{11} = -\frac{8}{3} \ln \frac{\mu}{m_c} + \frac{22}{9} + \frac{4}{3} G_0(r_2) + \frac{2 (8 + r_1 + r_2)}{3r_1} (G_0(r_2) - G_0(r_1 + r_2))$$

$$- \frac{4}{r_1} (2G_-(r_1) - G_-(r_2) - G_-(r_1 + r_2)), \quad (39)$$

$$\Delta i_{12} = -\frac{16}{3} \ln \frac{\mu}{m_c} + 4 + \frac{24 - 2r_2}{r_1} - \frac{2}{r_1^2} \left[ (1 + 2r_1) r_2 G_0(r_2) - (r_1^2 + r_2 + 2r_1 r_2) G_0(r_1 + r_2) \right]$$

$$+ \frac{8r_2}{r_1^2} (G_-(r_2) - G_-(r_1 + r_2)) \quad (40)$$
whose explicit form is given by the equations above we used $G_{\pm}(r_1, r_2)$ are defined by

$G_{\pm}(r_1, r_2) = \left[ 1 + \frac{2}{r_1} \right] G_0(r_2) + \frac{4}{3} \left[ 1 + \frac{2}{r_1 + r_2} \right] G_0(r_1 + r_2),$

$\Delta i_{15} = \frac{4}{3} \left( 1 + \frac{2}{r_2} \right) G_0(r_2) + \frac{4}{3} \left( 1 + \frac{2}{r_1 + r_2} \right) G_0(r_1 + r_2),$

$\Delta i_{17} = \frac{2}{3} \left( \frac{8 + r_2}{3r_1} \right) G_0(r_2) + \frac{2}{3} \left( \frac{8 + r_2}{r_1 + r_2} \right) G_0(r_1 + r_2)$

$+ \frac{4}{r_1} (G_{-}(r_2) - G_{-}(r_1 + r_2)),$

$\Delta i_{19} = \frac{2}{3} \left( 3 + \frac{r_2}{r_1} - \frac{10}{r_1} \right) (G_0(r_2) - G_0(r_1 + r_2)) + \frac{4}{3(r_1 + r_2)} G_0(r_1 + r_2)$

$+ \frac{4}{r_1} (G_{-}(r_2) - G_{-}(r_1 + r_2)),$

$\Delta i_{21} = \frac{2}{3} + \frac{16}{r_1} + \frac{8r_2}{3r_1} - \frac{2r_2(24r_1 + 3r_1^2 + 20r_2 + 7r_1r_2 + 4r_2^2)}{3r_1^2(r_1 + r_2)} G_0(r_1 + r_2)$

$+ \frac{2r_2}{3r_1^2} (20 + 3r_1 + 4r_2) G_0(r_2) + \frac{4}{r_1^2} [(r_1 + 4r_2)(G_{-}(r_1 + r_2) - G_{-}(r_2))$}

$+ 2r_2(4 - r_2) T_0(r_2) + 2(r_1 + r_2)(r_1 + r_2 - 4) T_0(r_1 + r_2)].$

The function $G_i(t)(i = -1, 0, 1, 2)$ are defined as

$G_i(t) = \int_0^1 dx x^i \ln[1 - tx(1 - x)] - i\epsilon.$

$G_{1,2}(t)$ can be simplified to $G_0$. The explicit form of $G_{-1,0}(t)$ could be found in Ref. [19]. In the equations above we used $G_{-}(t) \equiv G_{-1}(t)$.

$T_{0,1}$ are defined by

$T_i = \int_0^1 dx \frac{x^i}{1 - x(1 - x)t - i\epsilon}$

Their explicit forms are given by

$T_0(t) = \begin{cases} 
\frac{4 \arctan \sqrt{\frac{t}{t - 4}}}{\sqrt{t(t - 4)}}; & 0 \leq t \leq 4 \\
\frac{2i\pi + 2\ln(\sqrt{\tau - (t - 4)} - 2\ln(\sqrt{\tau + (t - 4)}))}{\sqrt{t(t - 4)}}; & t > 4.
\end{cases}$

The one loop two-point charm penguin function is well known in the form of

$G(t) = -\frac{2}{3} + \frac{4}{3} \ln \frac{\mu}{m_c} - \int_0^1 d\xi 4\xi(1 - \xi) \ln[1 - \xi(1 - \xi)t + i\epsilon],$

whose explicit form is given by

$G(t) = -\frac{8}{9} + \frac{4}{3} \ln \frac{\mu}{m_c} - \frac{2}{3} \left( 1 + \frac{2}{t} \right) G_0(t).$
Appendix C: Squares and interferences of amplitudes

Now we write down the terms in Eq.23 needed for calculating the inclusive decays

$$|\mathcal{M}_{O_s}|^2 = \left(\frac{g_s^3}{8\pi^2}\right)^2 \left(\frac{f_\phi}{4}\right)^2 |C_s|^2 2m_b^2 \left[ 4|F_1|^2st^2(s + t) + 2\Re(F_0 F_1^*)tu(-7s^2 + 6st + 5t^2) + |F_0|^2u\left(12st(t + u) + 4s^2(7t + u) + t^2(t + 11u)\right) \right].$$

$$|\mathcal{M}_{O_c}|^2 = \left(\frac{g_s^3}{8\pi^2}\right)^2 \left(\frac{f_\phi}{4}\right)^2 |C_c|^2 2m_b^2 \left[ |F_3|^2(s^3 - 4t^2u + s^2(u - 2t) + st(t + 5u)) - 2\Re(F_3 F_5^*)s(s - t) + |F_5|^2s(s + t) \right].$$

For the charm loop penguin contributions, we get

$$|\mathcal{M}_{\Delta^+}|^2 = \frac{49}{81} \left(\frac{g_s^3}{16\pi^2}\right)^2 \left(\frac{f_\phi}{4}\right)^2 |C_1|^2 2m_b^2 \left[ 4s^2(s + u)|J_{i51}^0|^2 + 4t\left(tu + s(t + 2u)\right)|J_{i6}^0|^2 + tu^2(2st + u(s + t))|J_{i231}^0|^2 + t(2s^3 - st^2 - t^3)|J_{i23}^1|^2 + \Re\left[ 4J_{i51}^0(2s^2(s + t)J_{i51}^1 + 2s(tu + st + su)J_{i6}^0 + stu(s + u)J_{i23}^0 + s^3uJ_{i23}^1 - s^2t(s + t)J_{i23}^0 + 4sJ_{i51}^1(2t(s + t)J_{i6}^0 - (s^2 - st - t^2)uJ_{i23}^0 + st(s + t)J_{i23}^1) + 4J_{i6}^0\left(u(t^2u - s(t + u)(s - t))J_{i23}^0 + 2s^2tuJ_{i23}^1 - st^2(s + t)J_{i23}^2 + 2suJ_{i23}^0\left(s(tu - st - su)J_{i23}^1 + t(s^2 - st - t^2)J_{i23}^2 - 2s^2t^2(s + t)J_{i23}^2 \right) \right] \right).$$

$$|\mathcal{M}_{\Delta^-}|^2 = \left(\frac{g_s^3}{16\pi^2}\right)^2 \left(\frac{f_\phi}{4}\right)^2 |C_1|^2 2m_b^2 \left[ 2s^2(s + u)|J_{i23}^0|^2 + 2st(t + u)|J_{i3}^0|^2 + 2tu(s + t)|J_{i12}^0|^2 + 4st\Re\left( sJ_{i23}^0J_{i2}^0 - uJ_{i8}^0J_{i12}^0 \right) - 4s^2(s + t)\Re\left( sJ_{i8}^0J_{i17}^0 - 2s^2tu\Re\left[ J_{i8}^0J_{i17}^1 - J_{i8}^0J_{i12}^1 + J_{i8}^0J_{i17} - J_{i8}^0J_{i12}^1 \right] + stu\Re\left[ 2(s + u)J_{i12}^0J_{i17}^0 + 2tJ_{i12}^0J_{i17}^0 + s(s + u)J_{i17}^1 - J_{i8}^0J_{i12}^1 \right] - s^2t^2u\Re\left[ J_{i17}^1J_{i12}^0 + J_{i117}^1J_{i21}^1 \right] \right] \right).$$

The interference terms are read as

$$\Re(\mathcal{M}_{O_s}\mathcal{M}_{O_s}^\dagger) = -\left(\frac{g_s^3}{8\pi^2}\right)^2 |C_1C_b|^2 \left(\frac{f_\phi}{4}\right)^2 8m_b^2 \Re\left\{ 2st[F_3(s - t) - F_5(s + t)]F_1^* - F_0^*\left[ 8F_4s^2(s + t) + F_3u(2s^2 - 9st - 3t^2) + F_5u(3st + t^2 - 2s^2) \right] \right\};$$

$$\Re(\mathcal{M}_{\Delta^+}\mathcal{M}_{\Delta^-}^\dagger) = \frac{7}{9} \left(\frac{g_s^3}{16\pi^2}\right)^2 C_1^2 \left(\frac{f_\phi}{4}\right)^2 2m_b^2 \Re\left\{ 2sJ_{i51}^0\left[ 2s^2(J_{i51}^0 + J_{i8}^0) + 2st(J_{i51}^1 + J_{i8}^1) + 2u(s + t)J_{i8}^0 + stu(J_{i17}^1 - J_{i12}^1) \right] \right\};$$
\(-2s^2J^i_{15}[2s^2(t+u)J^0_{18} + tu(J^0_{17} - J^0_{12})] + 4J^0_{16}[(s^2 + t^2)J^0_{13} + (t+u)J^0_{17} + t(J^1_{17} - J^1_{12})] \\
+tuJ^0_{12} + st(s+u)J^0_{12} + 2stuJ^0_{06}(sJ^0_{17} + tJ^0_{17} - J^1_{17}) \\
+uJ^0_{12}[(s^2 - 2t^2)sJ^0_{18} - 2t(s^2 - su + tu)J^0_{12} - st(s^2 + tu)J^1_{12}] \\
-2st^2J^0_{3} - st(s + su - ut)J^0_{12} - 2st(s + u)J^0_{12} - J^1_{12}[2s^2(t + s)J^1_{18}] \\
-2su(s^2 + t^2)J^0_{18} + 2t^2u(2s + t)J^0_{12} + stu(s^2 - tu)J^0_{12} \\
+s^2t^2u(J^1_{17} - J^1_{12}) + stu(tu + st + su)J^0_{12} \right) \right), (55)\\
\mathcal{R}(\mathcal{M}_{\Delta_s M^1_{\Omega_s}}) = \frac{7}{9} \frac{g^6}{128\pi^4} C^2 \left[ \frac{f_\phi}{4} \right]^2 4m^2 \mathcal{R}\left\{ -F^*_3 \left[ (s^2 + 2tu + su - st)J^0_{15} + tJ^0_{16} \right] \\
+2stu(J^0_{12} + tJ^1_{12}) + J^0_{12} \left[ (s + u) - s^2 - 2tu \right] \\
+2F^*_4 \left[ (s + t)J^0_{15} + 2tJ^1_{16} + suJ^0_{16} \right] \\
+F^*_5 \left[ 2sJ^0_{15} + 2tJ^0_{16} - tuJ^0_{16} \right] \right) \right), (56)\\
\mathcal{R}(\mathcal{M}_{\Delta_s M^1_{\Omega_s}}) = -\frac{g^6}{128\pi^4} C^2 \left[ \frac{f_\phi}{4} \right]^2 3m^2 \mathcal{R}\left\{ -F^*_3 \left[ (s^2 + 2tu - st)J^0_{12} - tu(2s + t)J^0_{12} - 2tu(2s + t)J^1_{12} + 2stuJ^0_{18} \\
-8s^2(s + t)J^1_{18} - 2stu(5s + 2t)J^1_{12} - stu(3s + t)J^1_{17} + tu(4s^2 + st - t^2)J^1_{12} \\
-t^2uJ^0_{12} \right] \\
-F^*_5 \left[ 4sJ^0_{12} + 4tJ^0_{13} - 3suJ^0_{17} - 3tuJ^0_{12} \right] \right) \right), (58)\\
\mathcal{R}(\mathcal{M}_{\Delta_s M^1_{\Omega_s}}) = -\frac{g^6}{128\pi^4} C^2 \left[ \frac{f_\phi}{4} \right]^2 3m^2 \mathcal{R}\left\{ F^*_3 \left[ s^2(t - s - u)J^0_{12} + st(t + u - s)J^0_{13} \\
-tu(2s + t)J^0_{12} - s(2J^0_{18} + tJ^0_{12} + s + u)J^0_{17} \right] \\
-F^*_4 \left[ 2sJ^0_{18} + tu(J^0_{17} - J^0_{12}) \right] \\
+F^*_5 \left[ 2sJ^0_{12} + tJ^0_{13} \right] \right) \right). (59)\\

The functions \( F_i \) are defined by

\[
F_0 = \int_0^1 dx \frac{\phi(x)m^2}{k_0^2 k_2^2}; (60)\\
F_1 = \int_0^1 dx \frac{x\phi(x)m^2}{k_0^2 k_2^2}; (61)\\
F_3 = \int_0^1 dx \frac{\phi(x)m^2}{k_2^2} G(r_3), (62)\]
\[ F_4 = \int_0^1 dx \frac{x \phi(x) m_b^2}{k^2} G(r_3), \]  
\[ F_5 = \int_0^1 dx \frac{\phi(x) m_b^2}{k_0^2} G(r_3), \]  
(63)  
(64)

and \( r_3 = \frac{k_0^2}{m_c^2} \). \( \phi(x) \) is the distribution function of \( \phi \) meson, which can be found in Ref. 28. In numerical calculation we take \( \phi(x) = 6x(1-x) \), which kills the IR pole of the gluon propagator \( \frac{1}{k^2} \). The other gluon is always time-like \( k_0^2 > 0 \), so there is no IR divergence in our calculations.

The \( J \) functions are

\[ J_{i2}^0 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_2}{r_2}, \]  
\[ J_{i3}^0 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_3}{r_2}, \]  
(65)  
(66)

\[ J_{i5}^0 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_5}{r_2}, \]  
\[ J_{i5}^1 = \int_0^1 dx_0 \frac{x \phi(x) \Delta i_5}{r_2}, \]  
(67)  
(68)

\[ J_{i6}^0 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_6}{r_2}, \]  
\[ J_{i8}^0 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_8}{r_2}, \]  
(69)  
(70)

\[ J_{i12}^0 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_{12}}{r_2}, \]  
\[ J_{i11}^1 = \int_0^1 dx_0 \frac{\phi(x) \Delta i_{11}}{r_2}, \]  
(71)  
(72)

\[ J_{i17}^1 = \int_0^1 dx_0^2 \frac{2x \phi(x) \Delta i_{17}}{r_1 r_2}, \]  
\[ J_{i21}^0 = \int_0^1 dx_0^2 \frac{2 \phi(x) \Delta i_{21}}{r_1 r_2}, \]  
(73)  
(74)
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