SO(10) Unified Theories and Cosmology

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We review the status of a class of gauge unified models based on SO(10) group. After a pedagogical introduction to SO(10) gauge theories, we discuss the main phenomenological implications of these models. The upper limit on proton lifetime are obtained and the prediction for neutrino masses are compared with the astrophysical and cosmological constraints coming from solar neutrino data and dark matter problem. Possible scenarios for the production of the baryon asymmetry of the universe required by primordial nucleosynthesis are also discussed.

This paper is dedicated to the memory of Prof. Roberto Stroppolini, whose friendship and deep humanity enriched the authors. We will not forget his rigorous approach to physics and his invaluable effort and enthusiasm in over forty years of teaching activity.

1 Introduction

We will review the present status of a class of non SUSY Grand Unified Theories (GUT’s) based on the simple gauge group SO(10).

The unification of electromagnetic and weak interactions, achieved in the framework of Glashow-Salam-Weinberg $SU(2)_L \otimes U(1)_Y$ model, has been experimentally tested with a remarkable precision. Similarly, Quantum Chromodynamics, based on $SU(3)_c$ gauge group, is well established as the theory of strong interactions, though infrared slavery prevents us from a clear understanding of low energy phenomena and still the mechanism of confinement remains obscure. The Standard Model (SM) of elementary interactions, the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theory, is without any doubt already a piece of history of science. GUT’s represent, along the same ideological line, a further effort towards a simplified picture of the elementary particle world. It is worth

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reminding that the idea that all interactions can be described by a simple group gauge theory at very high energies, more than a theoretical prejudice, relies upon the fact that the three SM running coupling constants converge towards a common value as the energy scale increases, suggesting that at some scale $M$, interactions may undergo a phase transition to a different behaviour characterized by a larger gauge symmetry.

One may wonder how such a new scenario could be ever tested experimentally if the scale $M$, as suggested by many arguments, is so high ($10^{15} \div 10^{16}$ GeV) to be far away from the detection possibility of present and future accelerators. However, its presence would indirectly affect low energy physics by very tiny effects, proportional to some negative power of $M$. The most famous example is the prediction of proton instability, which actually is a peculiar signature of GUT’s. In this case the decay rates for typical channels as $p \rightarrow e^+\pi^0$ or $\pi^+\bar{\nu}_\mu$ are expected very small, being proportional to $M^{-4}$. Present and future experiments as Super-Kamiokande or ICARUS will test the interesting region for proton decay channel rates of $10^{32} \div 10^{33}$ years. In this respect, the minimal GUT, based on SU(5), is already at variance, in its minimal version, with the precise measurements of the SM coupling constants at the $Z^0$ mass scale and the present bound on proton lifetime.

In SO(10) GUT the interplay between low energy phenomena and large scales also may show up in neutrino physics. On the basis of the see-saw mechanism, (almost) left-handed neutrinos acquire masses of the order $m^2/M$, with $m$ of the order of the up quark mass of the same generations. As we will discuss, if $M$ represents an intermediate symmetry scale of the order of $10^{11}$ GeV, this would predict $m_{\nu_e} \ll m_{\nu_\mu} \sim 10^{-3}$ eV, $m_{\nu_\tau} \sim 10$ eV. The $\mu$ neutrino mass is actually in the range to explain, in the framework of the MSW mechanism, the solar neutrino flux deficit observed by many experiments. Masses for $\nu_\tau$ larger than 1 eV will be instead observed by CHORUS and NOMAD Collaborations and would render $\nu_\tau$ the main contribution to the hot component of dark matter (DM).

The two tests for SO(10) GUT’s just mentioned demonstrate how important the interplay between particle physics, astrophysics and cosmology became in the last ten years, mainly due to an astonishing increase in precision of astrophysical measurements. This fact is particularly relevant, since effects which are instead proportional to $M$, i.e. which took place in the very early universe, can be tested by looking at the way they influenced the subsequent evolution of the universe. The baryon asymmetry, constrained by observation on primordial light nuclei abundances, the production of topological defects as monopoles or cosmic strings, finally the density perturbations caused by an inflationary epoch, provide a coherent set of severe constraints on GUT’s.
We hope we succeeded in communicating our strong feeling that, during next decade, observations of the universe will tell us many things about GUT’s, either confirming their role at high energy scales or ruling them out. This report is organized as follows: in section 2 we give a short pedagogical introduction to SO(10) GUT’s. Readers who are familiar with the subject can skip it and directly go to section 3, where we discuss a class of SO(10) models with $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R$ or $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ intermediate symmetry and which have been extensively studied in the past ten years.

Section 4 is devoted to a discussion of many phenomenological implications of SO(10) GUT’s and, in particular, of the models described in section 3. As far as the conclusions, we advice the reader to read again this Introduction and section 5.

2 An Introduction to SO(10) GUT’s

We will here shortly review the main features of SO(10) GUT’s. More detailed discussions can be found in.

A good starting point is perhaps to recall the classification of left-handed fermions in the SM; under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ we have

$$\psi_L \sim (1,1,1) \oplus \left( \begin{array}{c} 3,2,1 \\ \frac{1}{6} \end{array} \right) \oplus \left( \begin{array}{c} 3,1,-2 \\ \frac{2}{3} \end{array} \right) \oplus \left( 1,2,-\frac{1}{2} \right) \oplus \left( 3,1,\frac{1}{3} \right).$$

One may wonder if a simplification of this picture is possible, also allowing for a natural explanation of the electric charge quantization, by embedding the SM gauge group in a larger one G. If G is chosen to be a simple group, then the three independent gauge coupling would merge in one only. The first model realizing all this was proposed over twenty years ago by Georgi and Glashow, based on the group SU(5). In this case the number of fermion representations is reduced to two only,

$$\psi_L \sim 10 \oplus \overline{5}.$$  \hspace{1cm} (2)

The choice of 24 and $5 \oplus \overline{5}$ dimensional representations for the Higgs bosons gives the desired symmetry breaking pattern,

$$SU(5) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q.$$  \hspace{1cm} (3)

and the right quantization of hypercharge $Y$ and electric charge $Q$. There is also a beautiful prediction on fermion masses, due to the fact that, for each

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\textsuperscript{6} Actually SU(5) is the smallest group whose algebra contains $su(3) \otimes su(2) \otimes u(1)$ as a maximal subalgebra.
generation, down antiquark and lepton doublet are contained in one representation,

\[ m_b \sim 3m_\tau, \] (4)

once the masses are evolved down to low scales from the SU(5) unification scale. A glance to the vector gauge bosons, contained in the adjoint 24 representation,

\[ A \sim 24 \sim (8, 1, 0) \oplus (1, 3, 0) \oplus (1, 1, 0) \oplus \left( 3, 2, \frac{-5}{6} \right) \oplus \left( 3, 2, \frac{5}{6} \right), \] (5)

shows the presence of usual SM vector bosons as well as very massive leptoquarks, which allow for nucleon instability via processes like \( p \rightarrow e^+ \pi^0 \), whose rate is of the order of \( \alpha^2 m_p^5 M^{-4} \). The present enormous lower limit on proton lifetime for this channel, \( \tau_{p \rightarrow e^+ \pi^0} > 10^{33} \) years, therefore would require the SU(5) unification scale \( M \) to be larger than \( 10^{15} \) GeV.

Why abandon minimal SU(5)? Since SU(5) directly breaks down to SM, one expects that the three couplings \( \alpha_s \), \( \alpha_2 \) and \( \alpha_Y \) should meet at the unification scale \( M_{SU(5)} \). However, using the measurements of \( \alpha_i \) at the \( M_Z \) scale and assuming that only customary SM particles contribute to the renormalization group equations (RGE), the three couplings meet at three different points and only the scale at which \( \alpha_2 = \alpha_s \) is large enough (\( \gtrsim 10^{16} \) GeV) to be in agreement with the lower limit on proton lifetime. If this experimental evidence rules out minimal SU(5), it suggests on the other hand that unification may proceed through an intermediate symmetry stage. It has been observed, for example in the sixth reference of, that, if hypercharge receives a contribution from a generator of a non abelian group, as it is the case for SO(10) GUT’s, this would reconcile the experimental data with a GUT scheme.

SO(10) GUT theories were proposed many years ago on the basis of completely independent motivations:

1. For each generation, all left-handed fermions are classified in only one irreducible representation, the 16-dimensional spinorial representation. Under SU(5) it decompose as \( 10 \oplus 5 \oplus 1 \), where the additional singlet, with respect to the SU(5) case, has the quantum numbers of \( \nu_L^c \). The presence of this state, sterile under the SM and SU(5) actions, is a consequence of the possibility to define in SO(10) a charge conjugation operator \( C \) (which is not the usual Dirac one) which is a linear combination of the

\( \frac{1}{2} \)It is worth pointing out that more complicated choices for the Higgs boson representation or SUSY SU(5) are in agreement with all available data and the following considerations do not apply.
algebra's generators. Under $\mathcal{C}$, the left-handed weak interacting neutrino state transforms into $\nu_L \xrightarrow{\mathcal{C}} \nu_L^c$.

2. Models based on SO(10) gauge group are naturally anomaly free. What can be regarded as a lucky circumstance in SU(5) model, because of their exact compensations for the 10 and $\bar{5}$ representations, is instead a general feature of orthogonal groups, with the only exception of SO(6).

3. There is an intriguing decomposition of the 16 under the Pati-Salam group $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$,

$$16 = (4, 2, 1) \oplus (\bar{4}, 1, 2),$$

which displays the quark-lepton universality of weak interactions.

Baryon number violation and proton instability is a feature of SO(10) GUT's as well. Among the gauge vector bosons, classified in the 45-dimensional representation, there are even more leptoquark states than in SU(5) case which can mediate nucleon decay. In particular, decomposing the SO(10) adjoint representation under $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ (this subgroup will play a relevant role as intermediate symmetry stage in the following) we have

$$45 = (8, 1, 1, 0) \oplus (1, 3, 1, 0) \oplus (1, 1, 3, 0) \oplus (1, 1, 1, 0) \oplus \left(3, 1, 1, \frac{4}{3}\right) \oplus \left(3, 1, 1, -\frac{4}{3}\right) \oplus \left(3, 2, 2, \frac{2}{3}\right) \oplus \left(3, 2, 2, -\frac{2}{3}\right).$$

The fact that the baryon and lepton number difference $B-L$ is gauged in SO(10) GUT's, and is eventually spontaneously broken at low scales, has important consequences for the production of a baryon asymmetry in the universe. We will come back to this point in section 4. It is also worth noticing that SO(10) embeds $SU(2)_L \otimes SU(2)_R$. The scales at which the two groups break down, say $M_L$ and $M_R$, are however quite distinct since $M_L$ is of the order of the electroweak scale while $M_R$ is expected to be very large ($\sim 10^{11}$ GeV). Actually baryon number generation is also a way to probe the difference $(M_R - M_L)/M_X$, where $M_X$ is the SO(10) breaking scale.

It is also remarkable the way weak hypercharge can be written in terms of right isospin $T^3_R$ and B-L,

$$Y = T^3_R + \frac{B - L}{2}.$$

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When $SU(2)_R$ gauge symmetry is restored, weak hypercharge coupling therefore receives a contribution in scale evolution by a non-abelian factor, which results into a change of the corresponding $\beta$ function from positive to negative. This may shift the $\alpha_Y - \alpha_2$ intersection point of $SU(5)$ prediction up to the larger scale when also $\alpha_2$ and $\alpha_3$ meet.

Fermion masses in $SO(10)$ GUT’s may be produced via the usual symmetry breaking mechanism and Yukawa couplings to Higgs bosons of the form

$$10_H \cdot (16_F \otimes 16_F)_{10}, \ 126_H \cdot (16_F \otimes 16_F)_{126}, \ 120_H \cdot (16_F \otimes 16_F)_{120}. \quad (9)$$

All fermion Dirac masses are expected to be generated at the very last stage, when SM breaks down to $SU(3)_c \otimes U(1)_Q$. The presence of both $\nu_L$ and $\nu_L^c$ in the 16 representation however, along with a Dirac term,

$$m^D \nu_L^T \sigma_2 \nu_L, \quad (10)$$

allows for a Majorana mass,

$$m^M = \nu_L^T \sigma_2 \nu_L^c, \quad (11)$$

which appears when both $SU(2)_R$ and $U(1)_{B-L}$ are spontaneously broken.

The neutrino mass matrix therefore takes the form, up to radiative corrections,

$$m_\nu = \begin{pmatrix} 0 & m^D \\ m^D & m^M \end{pmatrix}. \quad (12)$$

Because the scale $M_R$ at which $SU(2)_R$ is broken is much higher than $M_L$, it follows that the two mass eigenvalues are approximately equal to $m^M$ and $(m^D)^2/m_M \ll m_M$. Thus, this see-saw mechanism for neutrino masses predicts an (almost) right-handed very heavy neutrino and a very light (almost) left-handed one, much lighter, for a factor $m^D/m^M \sim M_L/M_R$, than the charged lepton or quarks of the same generation. This beautiful prediction of SO(10) GUT’s may explain why weak interacting neutrinos are expected very light (though till now they could be well massless states!). If $M_R$ is of the order of $10^{11} \text{ GeV}$, as noticed in [12][19][20], neutrino masses may be in the right range to explain the solar neutrino problem in the MSW scheme and to account for the hot component of DM (see section 4).

We close this short summary of SO(10) GUT’s with some remarks on the symmetry breaking pattern. In general the pattern from SO(10) down to the SM gauge group depends on the Higgs boson representations which

$^d$A similar Majorana term could in principle be added for $\nu_L$ states but the addition of a $SU(2)_L$ Higgs triplet of high mass would change the ratio $M_Z/M_W$. 

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are considered. There is in fact quite a large variety of models, leading to different results for the unification scale. A common feature of all these models is, however, that the symmetry breaking takes place via an intermediate stage, with group symmetry $G' \subset SO(10)$,

$$SO(10) \xrightarrow{M_X} G' \xrightarrow{M_R} G_{SM} \xrightarrow{M_{EW}} SU(3)_c \otimes U(1)Q.$$  

This result holds for all models based on Higgs chosen in the low dimensional representations $(10, 16, 45, 54, 120, 126, 210)$, since in all these cases the components invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ have little groups larger than the SM group. We have indicated the second symmetry breaking scale with $M_R$, since it typically corresponds with the breaking of $SU(2)_R$, though this is not always the case.

3 A Class of SO(10) Models

In this section we discuss in more details a class of models of SO(10) GUT’s with $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ or $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ intermediate symmetry group $[4]$. In table 1 are reported the four possible intermediate groups $G'$ along with the Higgs representation used to break SO(10). In all cases the second symmetry breaking at the scale $M_R$ is realized using the $126 \oplus 126$ bispinorial representations. Actually this could be achieved using spinorial representation $16$ as well, but this would result in too small Majorana masses for right-handed neutrinos (and via the see-saw mechanism, too large masses for left-handed ones).

Once the correct spontaneous symmetry breaking pattern is realized, the main goal is to obtain informations on the unification scales $M_X$ and $M_R$. This is done by evolving the SM coupling constants, experimentally known at the $M_Z$ scale $[3]$.

$$\sin^2(\theta_W) = 0.2315 \pm 0.0002,$$
$$\alpha_s = 0.120 \pm 0.005,$$
$$\alpha_{em} = (127.9 \pm 0.9)^{-1},$$  

with the energy scale using the RGE,

$$\mu \frac{d}{d\mu} \alpha_i(\mu) = \beta_i(\alpha_i(\mu)).$$  

The scales $M_X$ and $M_R$ are then obtained by requiring that SO(10) or $G'$ symmetries are restored. The main problem in this procedure is that there is

\[\text{The only exception is the 144-dimensional representation.}\]
Table 1: Four possible intermediate symmetry groups in SO(10) GUT's. D is the left-right discrete symmetry, $\omega_{ab}$ is a second-rank traceless symmetric tensor; $\Phi_{abcd}$ is a fourth-rank antisymmetric tensor, and the indices 1...6 correspond to $SO(6) \sim SU(4)_{PS}$, whereas 7...0 correspond to $SO(4) \sim SU(2)_L \otimes SU(2)_R$.

| $G'$ | Higgs direction | Repr. |
|------|----------------|------|
| A $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \times D$ | $\omega_L = \frac{2(\omega_{11} + ... + \omega_{99})}{\sqrt{54}}$ | 54 |
| B $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$ | $\Phi_L = \frac{\Phi_{1214} + \Phi_{2356} + \Phi_{1456}}{\sqrt{3}}$ | 210 |
| C $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ | $\Phi_T = \Phi_{7890}$ | 210 |
| D $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ | $\Phi(\theta) = \cos \theta \Phi_L + \sin \theta \Phi_T$ | 210 |

usually a huge number of Higgs scalars which contributes to RGE as soon as the scale becomes larger than their mass. It is customary to adopt a simplifying assumption, the Extended Survival Hypothesis (ESH) \cite{23}, i.e. to consider in RGE only those scalars which are required to drive symmetry breaking at $M_R$ and at the electroweak scale. In this case one gets \cite{12} (the results are updated for the new values on SM gauge couplings)

| $M_X/10^{15}$ GeV | $M_R/10^{11}$ GeV | $M_X/10^{15}$ GeV | $M_R/10^{11}$ GeV |
|----------------|----------------|----------------|----------------|
| A 0.6 | 460 | C 4.7 | 2.8 |
| B 1.6 | 0.7 | D 9.5 | 0.067 |

The phenomenological implications of these results will be discussed in section 4. We only notice here that models with the left-right $D$ symmetry \cite{24} give smaller values for $M_X$ and so shorter proton lifetime. The physical content of the models with intermediate symmetry containing $SU(2)_L \otimes SU(2)_R$ and $D$ broken at the highest scale was first stressed in \cite{25}.

The ESH may be too drastic since in the 210 and 126 representations there are multiplets with high quantum numbers, which may substantially contribute to RGE. However, the mass spectrum of scalars depends on the coefficients of the non trivial SO(10) invariants which appear in the scalar potential, which can be constrained by requiring that the potential absolute minimum is in
the desired direction to give the considered symmetry breaking pattern. This fact results in rather restrictive conditions on scalar contributions to RGE. For details see last reference quoted in [12] or, for a summary of results, ref. [26].

4 Phenomenology of SO(10) GUT's

We discuss here three main phenomenological features of SO(10) GUT’s and, related to that, what experiments tell us on these models: proton instability, neutrino masses and baryon asymmetry of the universe. In particular we will only consider the models described in section 3. There are actually others fascinating issues, as SO(10) inflationary models and topological defect production, which however will not be covered here for brevity (see on these topics for example [27] and [28]).

4.1 Proton lifetime and $M_X$ GUT scale

The stronger lower limit on proton lifetime comes from the channel $p \rightarrow e^+\pi^0$ [14].

$$\tau_{p \rightarrow e^+\pi^0} = \frac{\tau_{p}}{Br(p \rightarrow e^+\pi^0)} > 0.55 \cdot 10^{33} \text{years}.$$  \hspace{1cm} (16)

Further improvements on this value, as well as on partial mean lifetimes for many other channels, are expected in the next few years from Super-Kamiokande [3]. The ICARUS project [4] should further increase the present limits as well, in particular for exotic channels, such as $p \rightarrow e \nu \nu$, up to the range $10^{32} \div 10^{33}$ years. The bound on $\tau_{p \rightarrow e^+\pi^0}$ can be translated into a lower limit on $M_X$, which is the scale at which leptoquarks take mass [12, 29].

$$M_X = \left[ \frac{\tau_{p \rightarrow e^+\pi^0}}{10^{32} \text{years}} \right]^{\frac{1}{4}} 10^{15} \text{GeV} \gtrsim 1.5 \cdot 10^{15} \text{GeV}.$$  \hspace{1cm} (17)

From this lower bound one is therefore led to the conclusion that the two models with D symmetry are ruled out (actually model B is at the very limit of compatibility with experiments), while the ones based on $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R$ or $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ intermediate symmetry are still in good shape, predicting partial lifetimes for the $e^+\pi^0$ channel of the order of $9 \cdot 10^{35}$ and $5 \cdot 10^{34}$ years respectively.

4.2 Neutrinos and the $M_R$ breaking scale

There are two experimental facts which are somehow suggesting that neutrinos may be massive particles. On one hand the reduction of the observed flux of
solar neutrinos\textsuperscript{9} with respect to the one predicted by solar models\textsuperscript{12} may be explained in terms of MSW neutrino oscillations\textsuperscript{8}.
Furthermore evidences for DM in galactic halos and at supercluster scales, together with studies on structure formations, are in agreement with a massive neutrino with mass of few eV\textsuperscript{31}.

The idea of neutrino oscillations was proposed long ago in pioneering works by Bruno Pontecorvo\textsuperscript{32} and has received a tremendous revival in the last ten years, after it was realized that neutrino flux may undergo resonant $\nu_e \leftrightarrow \nu_\mu$ or $\nu_e \leftrightarrow \nu_\tau$ transitions when passing through matter and in particular the solar interior\textsuperscript{8}. This effect has been widely advocated as a solution for the \textit{solar neutrino problem} which we mentioned above. Actually helioseismology\textsuperscript{33} constraints so much solar models that it appears unlikely to reconcile the deficit in neutrino flux by an even slight change in the sun central temperature. Furthermore the different observed reductions for $^7\text{Be}$ and $^8\text{B}$ neutrinos are at variance with the fact that both originate from the same parent $^7\text{Be}$ nuclei. Present observations\textsuperscript{9} require the following ranges for neutrino squared mass difference and mixing angle (for oscillations of $\nu_e$ into a $\nu_\mu$ or $\nu_\tau$):

\begin{align}
\Delta m^2 &\sim 10^{-6} \div 10^{-5} \text{eV}^2, \quad \sin^2 2\theta \sim 10^{-3} \div 10^{-2} \quad \text{(small angle solution)} \\
\Delta m^2 &\sim 10^{-5} \div 10^{-4} \text{eV}^2, \quad \sin^2 2\theta \sim 0.2 \div 1 \quad \text{(large angle solution)}.
\end{align}

(18)

Before comparing these results with SO(10) GUT predictions let us also briefly review what cosmological DM may tell us on neutrino masses. Evidence for the existence of galactic DM was found as early as 1922 by J. H. Jeans\textsuperscript{34}.

From observations on galactic rotation curves one gets, for the actual to critical density parameter $\Omega = \rho/\rho_C$,

$$\Omega \geq 0.1,$$

while, looking at larger structures, cluster or superclusters\textsuperscript{35},

$$\Omega_{\text{cluster}} > 0.2 \pm 0.3.$$

(20)

Not all matter contributing to $\Omega$ is likely to be baryonic since the baryon contribution $\Omega_b$ is strongly constrained by primordial nucleosynthesis to be\textsuperscript{36}

$$\Omega_b < 0.1.$$

(21)

If massive, light neutrinos would contribute to $\Omega$ as\textsuperscript{36}

$$\Omega_\nu h^2 = \frac{m_\nu}{90 \text{eV}}, \quad h \sim 0.54 \div 0.73;$$

(22)

so $\nu_\mu$ and $\nu_\tau$ could easily give the inflation desired prediction $\Omega = 1$ without violating experimental limits on their masses ($m_{\nu_\mu} < 0.17 \text{MeV}$, $m_{\nu_\tau} < 24 \text{MeV}$).
However, neutrinos are hot DM, i.e. they were relativistic when galaxy formation started and structure formation models based on inflationary schemes predict that hot DM only generates too few old galaxies. Better agreement with data is obtained in case of mixed hot + cold (non relativistic) scenario, with $\Omega_{\text{hot DM}} \simeq 0.25$ and $\Omega_{\text{cold DM}} \simeq 0.7$.

Going back to SO(10) GUT’s, we already mentioned that light neutrinos are predicted, in general, to get a mass via the see-saw mechanism,

$$ m_{\nu_i} = \left( \frac{m_\tau}{m_b} \right)^2 \frac{m_{\nu_i}^2 g_{2R}^2}{M_R h_i}, \quad (23) $$

where $u_i$ is the up quark of the same generation, $g_{2R}$ is the $SU(2)_R$ coupling at the $M_R$ scale and $h_i$ the Yukawa couplings of the $i$-th fermion generation to Higgs responsible for the symmetry breaking at $M_R$. Using $m_\tau = 1.777\, GeV$, $m_b = 4.3\, GeV$, and $m_c = 1.3\, GeV$, one gets

$$ m_{\nu_\mu} \sim 2.9 \frac{1}{M_R(10^{11}\, GeV)} \frac{g_{2R}}{h_2} 10^{-3}\, eV, \quad m_{\nu_\tau} \sim 5 \frac{1}{M_R(10^{11}\, GeV)} \frac{g_{2R}}{h_3} 10\, eV. \quad (24) $$

For the models C and D of section 3, which passed the proton lifetime test, we therefore get

- model C : $m_{\nu_\mu} \sim \frac{g_{2R}}{h_2} 10^{-3}\, eV$, $m_{\nu_\tau} \sim \frac{g_{2R}}{h_3} 17.5\, eV$;
- model D : $m_{\nu_\mu} \sim \frac{g_{2R}}{h_2} 4.3 \cdot 10^{-2}\, eV$, $m_{\nu_\tau} \sim \frac{g_{2R}}{h_3} 750\, eV. \quad (25)$

If $g_{2R}/h_2$ is order of the unity, the model C with $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R$ intermediate symmetry gives a value of $\Delta m^2 = m_{\nu_\mu}^2 - m_{\nu_\tau}^2 \sim m_{\nu_\tau}^2$ of the right order of magnitude required by MSW solution to solar neutrino problem with $\nu_e - \nu_\mu$ oscillation. In this case $\nu_\tau$ would contribute to $\Omega$ with a fraction $\Omega_{\nu_\tau} h^2 \sim 0.2$, slightly larger than what desired in the hot + cold scenario, which requires $m_{\nu_\tau} \sim 10\, eV$. The predictions of model D seem less satisfactory: if the value for $m_{\nu_\mu}$ is still in mild agreement with the large angle MSW solution (which is however theoretically disfavoured) a too heavy $\nu_\tau$ is predicted, even incompatible with the Cowlsik and McClelland bound $\sum_i m_{\nu_i} \lesssim 100\, eV$. It should also be mentioned that if one releases ESH approximation one can get in the case of model D with $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ intermediate symmetry the upper bound $M_R \lesssim 5 \cdot 10^{11}\, GeV$ which turns into the lower bounds for light neutrino masses

- model D (without ESH): $m_{\nu_\mu} \gtrsim \frac{g_{2R}}{h_2} 6 \cdot 10^{-3}\, eV$, $m_{\nu_\tau} \gtrsim \frac{g_{2R}}{h_3} 100\, eV. \quad (26)$
which are both at the boundary of values in agreement with MSW solution and $\Omega_\nu < 1$.

4.3 Baryogenesis via leptogenesis in SO(10) GUT's

The value of $\Omega_\nu$ previously mentioned, obtained from the comparison of experimental data on light nuclei abundances in the universe and theoretical predictions on primordial nucleosynthesis, also gives another important parameter, the present baryonic asymmetry normalized to photon density,

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 3 \cdot 10^{-10}. \quad (27)$$

Starting from big bang (likely) symmetric conditions $n_B = n_{\bar{B}}$, it is clear that the value for $\eta$ requires at some stage of universe evolution baryon number violating processes. Actually it was Sakharov who pointed out thirty years ago the necessary conditions for the production of a baryon asymmetry:

1. Baryon number violating interactions.

2. C and CP violation.

3. Non equilibrium conditions.

It was soon realized that GUT’s may be the natural framework for the production of a finite value for $\eta$. In the standard scenario, it is generated by out of equilibrium decays of heavy Higgs or gauge bosons. However, it was pointed out by several authors that anomalous B+L violating processes mediated by $SU(2)_L \otimes U(1)_Y$ sphaleronic configurations completely wash out any asymmetry produced at GUT scales, unless a finite asymmetry is also present in the difference B-L. This cannot be achieved in minimal SU(5) theory, for which B-L is a global symmetry, but can be easily implemented in SO(10) GUT’s, when $U(1)_{B-L}$ is spontaneously broken at $M_R$.

This possibility has been studied in ref. for the model D of section 3, but a similar result for $\eta$ can be obtained for the favorite case with $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ intermediate symmetry. The mechanism is based on out of equilibrium decays of Higgs bosons of the 210 representation into Majorana neutrinos at $M_R$. The resulting lepton number asymmetry is then converted into baryon number at low scales via the shuffling effects of sphalerons, giving a result for $\eta$ compatible with the value required by nucleosynthesis. This baryogenesis via leptogenesis scenario was first considered for heavy Majorana neutrino decays in.
There are actually several baryogenesis models (for a recent review see ref. 38), but it is nevertheless worth pointing out the fact that the production of an asymmetry in B-L is a rather natural and unavoidable prediction of SO(10) GUT’s.

5 Conclusions

We have reviewed the status of a class of SO(10) GUT’s, constraining the predictions for the unification scales $M_R$ and $M_X$ with the most recent available data on proton lifetime and SM gauge couplings.

What we may conclude in short is that the two models with D parity seem to be excluded since they predict a too low value for $\tau_{p \to e^+\pi^0}$ while the ones with $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R$ (model C) and $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ (model D) intermediate symmetry are in this respect still in good shape.

We have also pointed out that there is a quite huge low energy phenomenology ranging from neutrino masses to cosmological observations of light nuclei which may provide further and quite stringent constraints on GUT’s and are certainly providing a hint for what to look for beyond the SM.

By considering neutrino mass predictions within the see-saw mechanism, we get rather intriguing values for $\nu_\mu$ and $\nu_\tau$ for model C,

$$m_{\nu_\mu} \sim \frac{g_{2R}}{h_2} \frac{g_{2R}}{h_3} 10^{-3} eV, \quad m_{\nu_\tau} \sim \frac{g_{2R}}{h_3} 17.5 eV,$$

which, though the Yukawa couplings $h_i$ are unknown parameters, are of the order of magnitude to fit in the MSW solution to solar neutrino problem and to generously contribute to the hot component of dark matter. Model D seems instead to provide a much larger value for $m_{\nu_\tau}$ in the Extended Survival Hypothesis and so it is a bit disfavoured, though it cannot presently be ruled out because of the poor knowledge of SO(10) GUT Yukawa sector. For both models a prediction for baryon asymmetry in agreement with the value known for $\eta$ seems unavoidable. Next decade experiments will hopefully provide new informations to either ruling out SO(10) GUT’s or confirming their many low energy and cosmological predictions.

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