The Phantom Term in Open String Field Theory

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Abstract

We show that given any two classical solutions in open string field theory and a singular gauge transformation relating them, it is possible to write the second solution as a gauge transformation of the first plus a singular, projector-like state which describes the shift in the open string background between the two solutions. This is the “phantom term.” We give some applications in the computation of gauge invariant observables.

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1 Introduction

Perhaps the most mysterious aspect of Schnabl’s analytic solution for tachyon condensation [1] is the so-called phantom term—a singular and formally vanishing term in the solution which appears to be solely responsible for the disappearance of the D-brane. Much work has since shed light on the term, either specifically in the context of Schnabl’s solution [1, 2, 3], or in some generalizations [4, 5, 6, 7, 8, 9], but so far there has been limited understanding of why the phantom term should be present.

In this paper we show that the phantom term is a consequence of a particular and generic property of string field theory solutions: Given any two solutions $\Phi_1$ and $\Phi_2$, it is always possible to find a ghost number zero string field $U$ satisfying

$$
(Q + \Phi_1)U = U\Phi_2.
$$

(1.1)

This is called a left gauge transformation from $\Phi_1$ to $\Phi_2$ [10]. The existence of $U$ implies that $\Phi_2$ can be expressed as a gauge transformation of $\Phi_1$ plus a singular projector-like state which encapsulates the shift in the open string background between $\Phi_1$ and $\Phi_2$. This is the phantom term. The phantom term is proportional to a star algebra projector called the boundary condition changing projector, which is conjectured to describe a surface of stretched string connecting two BCFTs [10]. One consequence of this description is that phantom terms, in general, do not vanish in the Fock space, as is the case for Schnabl’s solution.

This paper can be viewed as a companion to reference [10], to which we refer the reader for more detailed discussion of singular gauge transformations and boundary condition changing projectors. Our main goal is to show how the phantom term can be used...
to calculate physical observables, even for solutions where the existence of a phantom term was not previously suspected. We give three examples: The closed string tadpole amplitude [11] for identity-like marginal deformations [12]; the energy for Schnabl’s solution [1]; and the shift in the closed string tadpole amplitude between two Schnabl-gauge marginal solutions [13, 14]. The last two computations reproduce results which have been obtained in other ways [1, 15], but our approach brings a different perspective and some simplifications. This description of the phantom term will be useful for the study of future solutions.

2 The Phantom Term

To start, let’s review some concepts and terminology from [10]. Given a pair of classical solutions $\Phi_1$ and $\Phi_2$, a ghost number zero state $U$ satisfying

$$(Q + \Phi_1)U = U\Phi_2$$

(2.1)

is called a left gauge transformation from $\Phi_1$ to $\Phi_2$. If $U$ is invertible, then $\Phi_1$ and $\Phi_2$ are gauge equivalent solutions. $U$, however, does not need to be invertible. In this case, we say that the left gauge transformation is a singular gauge transformation.

It is always possible to relate any pair of solutions by a left gauge transformation. Given a ghost number $-1$ field $b$, we can construct a left gauge transformation from $\Phi_1$ to $\Phi_2$ explicitly with the formula:

$$U = Qb + \Phi_1 b + b\Phi_2$$

$$= Q_{\Phi_1\Phi_2}b.$$  

(2.2)

Here, $Q_{\Phi_1\Phi_2}$ is the shifted kinetic operator for a stretched string between the solutions $\Phi_1$ and $\Phi_2$. Equation (2.2) is not necessarily the most general left gauge transformation from $\Phi_1$ to $\Phi_2$. This depends on whether $Q_{\Phi_1\Phi_2}$ has cohomology at ghost number zero [10].

In the examples we have studied, the left gauge transformation $U$ has an important property: If we add a small positive constant to $U$, the resulting gauge parameter $U + \epsilon$ is invertible.\(^3\) This raises a question: If an infinitesimal modification of $U$ can make it invertible, why are $\Phi_1$ and $\Phi_2$ not gauge equivalent? The answer to this question is

\(^3\)This is true with the appropriate choice of sign for $U$. It would be interesting to better understand why this property holds.
The first term $\Phi_1(\epsilon)$ is a gauge transformation of $\Phi_1$, and the second term $\psi_{12}(\epsilon)$ is a remainder. Apparently, if $\Phi_1$ and $\Phi_2$ are not gauge equivalent, the remainder must be nontrivial in the $\epsilon \to 0$ limit:

$$\lim_{\epsilon \to 0^+} \psi_{12}(\epsilon) = X^\infty(\Phi_2 - \Phi_1).$$

This is the *phantom term*. The phantom term is useful because it gives an efficient method for computing gauge invariant observables from classical solutions. In the $\epsilon \to 0$ limit, the pure-gauge term $\Phi_1(\epsilon)$ effectively “absorbs” all of the gauge-trivial artifacts of the solution, leaving the phantom term to describe the shift in the open string background in a transparent manner.

In this paper we evaluate the on-shell action and the closed string tadpole amplitude [11]:

$$S[\Phi_2] = -\frac{1}{6} \text{Tr} [\Phi_2 Q \Phi_2], \quad \text{Tr}_V[\Phi_2] = \mathcal{A}_2(V) - \mathcal{A}_0(V).$$

Here we use the notation,

- $\mathcal{A}_0(V)$ = Disk tadpole amplitude for an on shell closed string
- $\mathcal{V} = c\bar{c}\mathcal{V}^m$ coupling to the reference BCFT
- $\mathcal{A}_2(V)$ = Same as $\mathcal{A}_0(V)$ but coupling to the BCFT of $\Phi_2$
- $\text{Tr}[\cdot]$ = 1-string vertex (the Witten integral)
- $\text{Tr}_V[\cdot]$ = 1-string vertex with midpoint insertion of $V$. 

\begin{align*}
\Phi_2 &= \frac{1}{\epsilon + U} [Q + \Phi_1](\epsilon + U) + \frac{\epsilon}{\epsilon + U}(\Phi_2 - \Phi_1), \\
\equiv \Phi_1(\epsilon) & \equiv \psi_{12}(\epsilon)
\end{align*}

which follows easily from the definition of $U$. The first term $\Phi_1(\epsilon)$ is a gauge transformation of $\Phi_1$, and the second term $\psi_{12}(\epsilon)$ is a remainder. Apparently, if $\Phi_1$ and $\Phi_2$ are not gauge equivalent, the remainder must be nontrivial in the $\epsilon \to 0$ limit:

$$\lim_{\epsilon \to 0^+} \psi_{12}(\epsilon) = X^\infty(\Phi_2 - \Phi_1).$$

This is the *phantom term*. The phantom term is proportional to a star algebra projector,

$$X^\infty = \lim_{\epsilon \to 0^+} \frac{\epsilon}{\epsilon + U},$$

called the *boundary condition changing projector* [10]. The boundary condition changing projector is a subtle object, and is responsible for some of the “mystery” of the phantom term. Based on formal arguments and examples, it was argued in [10] that the boundary condition changing projector represents a surface of stretched string connecting the BCFTs of $\Phi_2$ and $\Phi_1$.

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In the subalgebra of wedge states with insertions, the 1-string vertex \( \text{Tr}[\cdot] \) is equivalent to a correlation function on the cylinder [16] whose circumference is determined by the total wedge angle (cf. appendix A of [8]). The shift in the action and the tadpole between the solutions \( \Phi_1 \) and \( \Phi_2 \) can be conveniently expressed using the phantom term:

\[
S[\Phi_2] - S[\Phi_1] = \frac{1}{6} \text{Tr} \left[ \psi_{12}(\epsilon)Q\psi_{12}(\epsilon) \right] - \frac{1}{3} \text{Tr} \left[ \psi_{12}(\epsilon)Q\Phi_2 \right].
\]

(2.7)

and

\[
\mathcal{A}_2(V) - \mathcal{A}_1(V) = \text{Tr}_V \left[ \psi_{12}(\epsilon) \right].
\]

(2.8)

These equations are exact for any \( \epsilon \), though they are most useful in the \( \epsilon \to 0 \) limit. It would be interesting to see whether the phantom term can also be useful for computing the boundary state [17].

Note that the phantom term is a property of a pair of solutions and a singular gauge transformation relating them. In this sense, a solution by itself does not have a phantom term. That being said, some solutions—like Schnabl’s solution—seem to be naturally defined as a limit of a pure gauge configuration subtracted against a phantom term. Other solutions, like the “simple” tachyon vacuum [8] and marginal solutions, can be defined directly without reference to a singular gauge transformation or its phantom term. It would be interesting to understand what distinguishes these two situations, and why.

3 Relation to Schnabl’s Phantom Term

The phantom term defined by equations (2.3) and (2.4) is different from the phantom term as it conventionally appears in Schnabl’s solution [1] or some of its extensions [4, 5, 9]. The standard phantom term can be derived from the identity

\[
\Phi_2 = \frac{1 - X^N}{U}(Q + \Phi_1)U + X^N\Phi_2.
\]

(3.1)

where we define \( X \):

\[
U \equiv 1 - X.
\]

(3.2)

In the \( N \to \infty \) limit, the second term in (3.1) is the phantom term:

\[
\lim_{N \to \infty} X^N\Phi_2 = X^\infty\Phi_2.
\]

(3.3)
This is different from (2.4), though both phantom terms are proportional to the boundary condition changing projector. The major difference between the identities (2.3) and (3.1) is that \( \Phi_1(\epsilon) \) is exactly gauge equivalent to \( \Phi_1 \) for all \( \epsilon > 0 \), whereas the corresponding term in (3.1),

\[
\frac{1 - X^N}{U} (Q + \Phi_1) U,
\]

is not gauge equivalent to \( \Phi_1 \), or even a solution, for any finite \( N \). In this sense (2.3) is a more natural, and this is the definition of the phantom term we will use in subsequent computations. However, the phantom term can be defined in many ways using many different identities similar to (2.3) and (3.1), and for certain purposes some definitions may prove to be more convenient than others.

To make the connection to earlier work, let us explain how the identity (3.1) leads to the standard definition of Schnabl’s solution as a regularized sum subtracted against a phantom term. We can use Okawa’s left gauge transformation

\[
U = 1 - \sqrt{\Omega} cB\sqrt{\Omega},
\]

(3.5)
to map from the perturbative vacuum \( \Phi_1 = 0 \) to Schnabl’s solution

\[
\Phi_2 = \Psi = \sqrt{\Omega} cB \Omega \frac{KB}{1 - \Omega} c\sqrt{\Omega}.
\]

(3.6)

Substituting these choices into (3.1) gives the expression

\[
\Psi = -\sum_{n=0}^{N-1} \psi'_n + \sqrt{\Omega} c\Omega^N \frac{KB}{1 - \Omega} c\sqrt{\Omega},
\]

(3.7)

where \( \psi'_n \equiv \frac{d}{dn} \psi_n \), \( \psi_n \equiv \sqrt{\Omega} cB\Omega^n c\sqrt{\Omega} \).

(3.8)

To simplify further, expand

\[
\frac{K}{1 - \Omega} = \sum_{n=0}^\infty \frac{B_n}{n!} (-K)^n,
\]

(3.9)

inside the second term of (3.7), where \( B_n \) are the Bernoulli numbers. The expansion in powers of \( K \) is equivalent to the \( \mathcal{L}^- \) level expansion [9, 19],\(^5\) which will play an important

\(^4\)See \([2, 18]\) and appendix A of \([8]\) for explanation of the algebraic notation for wedge states with insertions which we employ.

\(^5\)\( \mathcal{L}^- \) is the BPZ odd component of Schnabl’s \( \mathcal{L}_0 \), the zero mode of the energy momentum tensor in the sliver coordinate frame \([1, 19]\). It is a derivation and a reparameterization generator, and computes scaling dimension of operator insertions in correlation functions on the cylinder. See \([9]\) for discussion of the \( \mathcal{L}^- \) level expansion.
role in simplifying correlators involving the phantom term. The upshot in the current context is that the higher powers of $K$ in (3.9) can usually be ignored in the $N \to \infty$ limit [5], so we can effectively replace the sum by its first term

$$\frac{K}{1 - \Omega} \to 1. \quad (3.10)$$

Then the $N \to \infty$ limit of (3.1) reproduces the usual expression for Schnabl’s solution

$$\Psi = \lim_{N \to \infty} \left[ - \sum_{n=0}^{\infty} \psi_n' + \psi_N \right], \quad (3.11)$$

where $\psi_N$ is the phantom term.

### 4 Example 1: Identity-like Marginals

We start with a simple example: computing the shift in the closed string tadpole amplitude between the identity-like solution for the tachyon vacuum [8, 20, 21],

$$\Phi_1 = c(1 - K), \quad (4.1)$$

and the identity-like solution for a regular marginal deformation [12],

$$\Phi_2 = cV, \quad (4.2)$$

where $V$ is a weight 1 matter primary with regular OPE with itself. Both these solutions are singular. For example, we cannot evaluate the tadpole directly because

$$\text{Tr}_V[cV] \quad (4.3)$$

requires computing a correlator on a surface with vanishing area. However, with the phantom term, we can circumvent this problem with a few formal (but natural) assumptions.

We can relate the above solutions with a left gauge transformation

$$U = Q_{\Phi_1 \phi_2} B$$

$$= 1 + Bc(K + V - 1). \quad (4.4)$$
The shift in the open string background is described by the phantom term:\(^6\)

\[
\psi_{\Phi_1, \Phi_2}(\epsilon) = \frac{\epsilon}{\epsilon + \bar{\epsilon}U}(\Phi_2 - \Phi_1) \quad (\bar{\epsilon} \equiv 1 - \epsilon)
\]

\[
= \left( -\epsilon c + \frac{\epsilon \bar{\epsilon}}{\epsilon + \bar{\epsilon}(K + V)} Bc\partial_c \right) (1 - K - V)
\]

\[
= \left( -\epsilon c + \bar{\epsilon} \int_0^\infty dt e^{-t} \Omega_V^{\bar{\epsilon} t/\epsilon} Bc\partial_c \right) (1 - K - V), \quad (4.5)
\]

where in the third line we defined the states

\[
\Omega_V^t \equiv e^{-t(K+V)}. \quad (4.6)
\]

These are wedge states whose open string boundary conditions have been deformed by the marginal current \(V\) [22]. In the \(\epsilon \to 0\) limit the phantom term is

\[
\lim_{\epsilon \to 0} \psi_{\Phi_1, \Phi_2}(\epsilon) = \Omega_V^\infty Bc\partial_c(1 - K - V), \quad (4.7)
\]

Note that the phantom term corresponds to a nondegenerate surface with the boundary conditions of the marginally deformed BCFT, and so will naturally reproduce the expected coupling to closed strings. This is in spite of the fact that both solutions we started with were identity-like. Also note that the phantom term vanishes in the Fock space (since \(B\) kills the sliver state), but still it is nontrivial.

Now we can use the phantom term to compute the shift in the closed string tadpole amplitude:

\[
\text{Tr}_V[\psi_{\Phi_1, \Phi_2}(\epsilon)]. \quad (4.8)
\]

Since the amplitude vanishes around the tachyon vacuum, only the marginally deformed D-brane should contribute. Plugging (4.5) in, we find

\[
\text{Tr}_V[\psi_{\Phi_1, \Phi_2}(\epsilon)] = \text{Tr}_V \left[ \epsilon c B(K + V - 1)c + \frac{\epsilon}{\epsilon + \bar{\epsilon}(K + V)} Bc\partial_c \right]. \quad (4.9)
\]

\(^6\)In the following examples the dependence on \(\epsilon\) simplifies if we make a reparameterization \(\epsilon \to \epsilon/\bar{\epsilon}\) relative to (2.3), where by definition \(\bar{\epsilon} \equiv 1 - \epsilon\).
The first term in the trace formally vanishes.\(^7\) Then
\[
\text{Tr}_V[\psi_{\Phi_1, \Phi_2}(\epsilon)] = \text{Tr}_V \left[ \frac{\epsilon}{\epsilon + \epsilon(K + V)} Bc\partial c \right] = \int_0^\infty dt \ e^{-t} \text{Tr}_V[\Omega_V^{\epsilon t/\epsilon} Bc\partial c]. \tag{4.10}
\]

With the reparameterization we can scale the deformed wedge state inside the trace to unit width:

\[
\text{Tr}_V[\Omega_V^{\epsilon t/\epsilon} Bc\partial c] = \text{Tr}_V \left[ \left( \frac{\epsilon t}{\epsilon} \right)^{1/2} \Omega_V Bc\partial c \right] = \text{Tr}_V[\Omega_V Bc\partial c]. \tag{4.11}
\]

Integrating over \(t\) gives
\[
\text{Tr}_V[\psi_{\Phi_1, \Phi_2}(\epsilon)] = \text{Tr}_V[\Omega_V Bc\partial c]. \tag{4.12}
\]

Note that this is manifestly independent of \(\epsilon\). In the general situation, explicitly proving \(\epsilon\)-independence requires much more work than is needed to compute the result, and it is easier to assume gauge invariance and take the \(\epsilon \to 0\) limit. At any rate, further simplifying (4.12), we can replace the ghost factor \(Bc\partial c\) in the trace with \(-c^\perp\).\(^8\) Mapping from the cylinder to the unit disk gives
\[
\text{Tr}_V[\psi_{\Phi_1, \Phi_2}(\epsilon)] = -\frac{1}{2\pi i} \exp \left[ \int_0^{2\pi} d\theta V(\epsilon^i \theta) \right] V(0)c(1)_{\text{disk}}. \tag{4.14}
\]

This is exactly the closed string tadpole amplitude for the marginally deformed D-brane, as defined in the conventions of [11].

5 Example 2: Energy for Schnabl’s Solution

In this section we compute the energy for Schnabl’s solution,
\[
\Psi = \sqrt{\Omega}c \frac{KB}{1 - \Omega} c\sqrt{\Omega}. \tag{5.1}
\]

\(^7\)This first term in (4.9) formally vanishes because it is the trace of a state which has negative scaling dimension plus a state which is BRST exact with respect to the BRST operator of the marginally deformed BCFT. However, rigorously speaking the trace is undefined, since computing it requires evaluating a correlator on a surface with vanishing area. This is a remnant of the fact that our marginal solution and tachyon vacuum are too “identity-like.” However, since the offending term is proportional to \(\epsilon\), and (4.8) is formally independent of \(\epsilon\), we will set \(\epsilon = 0\) and ignore this term.

\(^8\)This follows from the fact that the derivation \(B^-\) annihilates the 1-string vertex[8], and
\[
-\frac{1}{2}B^- (\Omega_V c\partial c) = \Omega_V Bc\partial c + \Omega_V c. \tag{4.13}
\]
The original computation of the energy, based on the expression (3.11), was given in [1] (see also [2, 3]). Our computation will be quite different since we define the phantom term in a different way.

We take the reference solution to be the perturbative vacuum, and map to Schnabl’s solution using Okawa’s left gauge transformation

\[ U = 1 - \sqrt{\Omega} cB \sqrt{\Omega} \]

\[ = Q_{0\Psi} \left( B \frac{1 - \Omega}{K} \right). \] (5.2)

The regularized phantom term is

\[ \psi_{0\Psi}(\epsilon) = \frac{\epsilon}{\epsilon + \epsilon U} \Psi = \sqrt{\Omega} cB \frac{\epsilon}{1 - \epsilon \Omega} \frac{K}{1 - \Omega} c\sqrt{\Omega}. \] (5.3)

In the \( \epsilon \to 0 \) limit the ratio \( \frac{\epsilon}{1 - \epsilon \Omega} \) approaches the sliver state (see later), so we can replace the factor \( \frac{K}{1 - \Omega} \) with its leading term in the \( \mathcal{L}^- \) level expansion. Then (2.3) gives a regularized definition of Schnabl’s solution:

\[ \Psi = \lim_{\epsilon \to 0^+} \left[ \Psi_\epsilon + \sqrt{\Omega} cB \frac{\epsilon}{1 - \epsilon \Omega} c\sqrt{\Omega} \right]. \] (5.4)

Note that \( \Psi_\epsilon \) here is precisely the pure gauge solution discovered by Schnabl [1]:

\[ \Psi_\epsilon = \epsilon \sqrt{\Omega} cB \frac{KB}{1 - \epsilon \Omega} c\sqrt{\Omega}. \] (5.5)

Using (3.8) we can express this regularization in the form

\[ \Psi = \lim_{\epsilon \to 0^+} \sum_{n=0}^{\infty} \epsilon^n \left[ -\psi_n' + \epsilon \psi_n \right]. \] (5.6)

Clearly this is different from the standard definition of Schnabl’s solution, (3.11).

To calculate the action we use (2.7):

\[ S = \frac{1}{6} \text{Tr} \left[ \psi_{0\Psi}(\epsilon) Q \psi_{0\Psi}(\epsilon) \right] - \frac{1}{3} \text{Tr} \left[ \psi_{0\Psi}(\epsilon) Q \Psi \right]. \] (5.7)

A quick calculation shows that the second term can be ignored in the \( \epsilon \to 0 \) limit, essentially because the phantom term vanishes when contracted with well-behaved states. Therefore

\[ S = \frac{1}{6} \lim_{\epsilon \to 0^+} \text{Tr} \left[ \psi_{0\Psi}(\epsilon) Q \psi_{0\Psi}(\epsilon) \right]. \] (5.8)
Substituting the phantom term (5.3) gives an expression of the form
\[ \text{Tr} \left[ \psi_0 \Phi (\epsilon) Q \psi_0 \Phi (\epsilon) \right] = - \text{Tr} \left[ C_1 \frac{\epsilon}{1 - \epsilon \Omega} C_2 \frac{\epsilon}{1 - \epsilon \Omega} \right] + \text{Tr} \left[ C_3 \frac{\epsilon}{1 - \epsilon \Omega} C_4 \frac{\epsilon}{1 - \epsilon \Omega} \right], \quad (5.9) \]
where
\[
C_1 = \left[ c, \Omega \right] \frac{KB}{1 - \Omega}, \quad (5.10)
\]
\[
C_2 = c \Omega \partial \epsilon \frac{K}{1 - \Omega}, \quad (5.11)
\]
\[
C_3 = c K \left[ c, \Omega \right] \frac{KB}{1 - \Omega}, \quad (5.12)
\]
\[
C_4 = c \Omega \frac{K}{1 - \Omega}. \quad (5.13)
\]
To understand what happens in the \( \epsilon \to 0 \) limit, note that the factor \( \frac{\epsilon}{1 - \epsilon \Omega} \) inside the trace (5.9) approaches the sliver state:
\[ \lim_{\epsilon \to 0^+} \frac{\epsilon}{1 - \epsilon \Omega} = \Omega^\infty. \quad (5.14) \]
To prove this, expand the geometric series
\[ \frac{\epsilon}{1 - \epsilon \Omega} = \epsilon \sum_{n=0}^{\infty} \epsilon^n \Omega^n. \quad (5.15) \]
and expand the wedge state in the summand around \( n = \infty \):
\[ \Omega^n = \Omega^\infty + \frac{1}{n + 1} \Omega_{(1)} + \frac{1}{(n + 1)^2} \Omega_{(2)} + \ldots, \quad (5.16) \]
where \( \Omega_{(1)}, \Omega_{(2)}, \ldots \) are the coefficients of the corrections in inverse powers of \( n + 1 \) (actually \( \Omega_{(2)} \) is the first nonzero correction in the Fock space). Plugging (5.16) into the geometric series and performing the sums gives
\[ \frac{\epsilon}{1 - \epsilon \Omega} = \Omega^\infty + \frac{\epsilon \ln \epsilon}{\epsilon} \Omega_{(1)} + \frac{\epsilon \text{Li}_2 \epsilon}{\epsilon} \Omega_{(2)} + \ldots. \quad (5.17) \]
Only the sliver state survives the \( \epsilon \to 0 \) limit. This means that for small \( \epsilon \) equation (5.9) is dominated by correlation functions on the cylinders with very large circumference. In
this limit, it is useful to expand the fields \( C_1, \ldots, C_4 \) into a sum of states with definite scaling dimension (the \( L_0 \) level expansion). To leading order this expansion gives

\[
C_1 = \sqrt{\Omega} (\partial c B) \sqrt{\Omega} + \ldots, \quad (L_0 = 1),
\]
\( (5.18) \)

\[
C_2 = -\frac{1}{2} \sqrt{\Omega} (c \partial c \partial^2 c) \sqrt{\Omega} + \ldots, \quad (L_0 = 0),
\]
\( (5.19) \)

\[
C_3 = \sqrt{\Omega} (K c \partial c B) \sqrt{\Omega} + \ldots, \quad (L_0 = 1),
\]
\( (5.20) \)

\[
C_4 = -\sqrt{\Omega} (c \partial c) \sqrt{\Omega} + \ldots, \quad (L_0 = -1).
\]
\( (5.21) \)

Now consider the following: If a cylinder of circumference \( L \) has insertions of total scaling dimension \( h \) separated parametrically with \( L \), rescaling the cylinder down to unit circumference produces an overall factor of \( L^{-h} \), which vanishes in the large circumference limit if \( h \) is positive. Since the sum of the lowest scaling dimensions of \( C_1 \) and \( C_2 \) is positive, the corresponding term in (5.9) must vanish. The sum of the lowest scaling dimensions of \( C_3 \) and \( C_4 \) is zero, so the corresponding term in (5.9) is nonzero and receives contribution only from the leading \( L_0 \) level of \( C_3 \) and \( C_4 \). Therefore the action simplifies to

\[
S = -\frac{1}{6} \lim_{\epsilon \to 0} \epsilon^2 \sum_{L=2}^{\infty} \frac{\epsilon L}{L_0} \left[ K c \partial c B \frac{\epsilon \Omega}{1 - \epsilon \Omega} \frac{\epsilon \Omega}{1 - \epsilon \Omega} \right].
\]
\( (5.22) \)

Expanding the geometric series gives

\[
S = -\frac{1}{6} \lim_{\epsilon \to 0} \epsilon^2 \sum_{L=2}^{\infty} \epsilon L \sum_{k=1}^{L-1} \frac{\epsilon \Omega}{1 - \epsilon \Omega} \left[ K c \partial c B \Omega^{L-k} c \partial c \Omega^k \right].
\]
\( (5.23) \)

Scaling the total wedge angle inside the trace to unity,

\[
S = -\frac{1}{6} \lim_{\epsilon \to 0} \epsilon^2 \sum_{L=2}^{\infty} L \epsilon L \left( \frac{1}{L} \sum_{k=1}^{L-1} \frac{\epsilon \Omega}{1 - \epsilon \Omega} \left[ K c \partial c B \Omega^{L-k} c \partial c \Omega^k \right] \right).
\]
\( (5.24) \)

Expanding the factor in parentheses around \( L = \infty \), the sum turns into an integral:

\[
\frac{1}{L} \sum_{k=1}^{L-1} \frac{\epsilon \Omega}{1 - \epsilon \Omega} \left[ K c \partial c B \Omega^{L-k} c \partial c \Omega^k \right] = \int_0^1 dx \left[ K c \partial c B \Omega^{1-x} c \partial c \Omega^x \right] + \mathcal{O} \left( \frac{1}{L} \right) + \ldots.
\]
\( (5.25) \)

The order \( 1/L \) terms and higher do not contribute in the \( \epsilon \to 0 \) limit, as explained in (5.17). Therefore

\[
S = -\frac{1}{6} \left( \lim_{\epsilon \to 0} \epsilon^2 \sum_{L=2}^{\infty} L \epsilon L \right) \int_0^1 dx \left[ K c \partial c B \Omega^{1-x} c \partial c \Omega^x \right]
\]

\[
= -\frac{1}{6} \int_0^1 dx \left[ \Omega^{1-x} B c \partial c Q (\Omega^x B c \partial c) \right].
\]
\( (5.26) \)
A moment’s inspection reveals that this integral is precisely the action evaluated on the “simple” solution for the tachyon vacuum [8], expressed in the form [23]

$$\Psi_{\text{simp}} = c - \frac{B}{1 + K} c \partial c.$$  \hfill (5.27)

Therefore

$$S = -\frac{1}{6} \text{Tr}[\Psi_{\text{simp}} Q \Psi_{\text{simp}}] = \frac{1}{2\pi^2},$$  \hfill (5.28)

in agreement with Sen’s conjecture.

6 Example 3: Tadpole Shift Between Two Marginals

In this section we use the phantom term to compute the shift in the closed string tadpole amplitude between two Schnabl-gauge marginal solutions [13, 14]:

$$\Phi_1 = \sqrt{\Omega} c V_1 \frac{B}{1 + \frac{1}{\kappa} V_1} c \sqrt{\Omega},$$

$$\Phi_2 = \sqrt{\Omega} c V_2 \frac{B}{1 + \frac{1}{\kappa} V_2} c \sqrt{\Omega},$$  \hfill (6.1)

where $V_1$ and $V_2$ are weight 1 matter primaries with regular OPEs with themselves (but not necessarily with each other).\footnote{For example, we could choose $V_1 = e^{\sqrt{\alpha'} X^0}$ to be the rolling tachyon deformation, and $V_2 = e^{-\sqrt{\alpha'} X^0}$ to be the “reverse" rolling tachyon; or we could choose $V_1$ and $V_2$ to be Wilson line deformations along two independent light-like directions. In both these examples the $V_1$-$V_2$ OPE is singular.} Our main interest in this example is to understand how the boundary condition changing projector works when connecting two distinct and nontrivial BCFTs; In this case the projector has a rather nontrivial structure and possible singularities from the collision of matter operators at the midpoint [10]. This is the first example of a phantom term which does not vanish in the Fock space (at least in the case where the $V_1$-$V_2$ OPE is regular). This example also gives an independent derivation of the tadpole amplitude for Schnabl-gauge marginals, which previously proved difficult to compute [15]. Another computation of the tadpole for the closely related solutions of Kiermaier, Okawa, and Soler [24] appears in [25].

We will map between the marginal solutions $\Phi_1$ and $\Phi_2$ using the left gauge transfor-
This choice of $U$ is natural to the structure of the marginal solutions, since it factorizes into a product of left gauge transformations through the Schnabl-gauge tachyon vacuum [10]. The regularized boundary condition changing projector is

\[
\frac{\epsilon}{\epsilon + \bar{\epsilon}U} = \epsilon + \frac{\bar{\epsilon}}{1 + \frac{1 - \Omega}{K} - \bar{\epsilon} \Omega} \sqrt{\Omega} + \frac{\epsilon}{1 + \frac{1 - \Omega}{K} V_2 - \epsilon \Omega} Bc \sqrt{\Omega} \equiv P_1
\]

\[
+ \frac{\epsilon^2}{1 + \frac{1 - \Omega}{K} V_2 - \epsilon \Omega} B[c, \Omega] \frac{1}{1 + \frac{1 - \Omega}{K} V_1 - \epsilon \Omega} \sqrt{\Omega} \equiv P_2
\]

\[
+ \frac{\epsilon^2}{1 + \frac{1 - \Omega}{K} V_2 - \epsilon \Omega} B[c, \Omega] \frac{1}{1 + \frac{1 - \Omega}{K} V_1 - \epsilon \Omega} \sqrt{\Omega} \equiv P_{12}
\]

The first two terms, $P_1$ and $P_2$, are regularized boundary condition changing projectors from $\Phi_1$ to the tachyon vacuum (Schnabl’s solution), and from the tachyon vacuum to $\Phi_2$, respectively. These terms vanish in the Fock space in the $\epsilon \to 0$ limit. The third term $P_{12}$ is the nontrivial one: In the $\epsilon \to 0$ limit it approaches the sliver state in the Fock space, with the boundary conditions of $\Phi_2$ on its left half and the boundary conditions of $\Phi_1$ on its right half; it represents the open string connecting the BCFTs of $\Phi_2$ and $\Phi_1$ [10]. If $V_1$ and $V_2$ have regular OPE, $P_{12}$ is a nonvanishing projector in the Fock space. If the OPE is singular, $P_{12}$ may be vanishing or divergent because of an implicit singular conformal transformation of the boundary condition changing operator between the BCFTs of $\Phi_2$ and $\Phi_1$ at the midpoint. Part of our goal is to see how this singularity is resolved when we compute the overlap. The phantom term is

\[
\psi_{12}(\epsilon) = (\epsilon + P_1 + P_2 + P_{12})(\Phi_2 - \Phi_1).
\]

First let us consider the contribution to the tadpole from $P_{12}$:

\[
\text{Tr}_Y[\psi_{12}(\Phi_2 - \Phi_1)]
\]
Plugging everything in gives

\[
\text{Tr}_V[P_{12}(\Phi_2 - \Phi_1)] = \text{Tr}_V \left[ \frac{\bar{\epsilon}^2 \epsilon}{1 + \frac{1-\Omega}{K}} V_2 - \bar{\epsilon} \Omega \right] B[c, \Omega] \left[ \frac{1}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} \left( V_2 \frac{1}{1 + \frac{1-\Omega}{K} V_2} - V_1 \frac{1}{1 + \frac{1-\Omega}{K} V_1} \right) \epsilon \Omega \right] = \bar{\epsilon} \epsilon \text{Tr}_V \left[ B c \Omega \left( \frac{1}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} \left( V_2 \frac{1}{1 + \frac{1-\Omega}{K} V_2} - V_1 \frac{1}{1 + \frac{1-\Omega}{K} V_1} \right) \Omega \right) \frac{1}{1 + \frac{1-\Omega}{K} V_2 - \bar{\epsilon} \Omega} \right].
\]

\[(6.6)\]

In the second step we inserted a trivial factor of \(cB\) next to the commutator \([c, \Omega]\), which allows us to remove the \(c\) ghost from the difference between the solutions. Now let’s look at the factor above the braces. Re-express it with a few manipulations

\[
\bar{\epsilon} \Omega \left( V_2 \frac{1}{1 + \frac{1-\Omega}{K} V_2} - V_1 \frac{1}{1 + \frac{1-\Omega}{K} V_1} \right) \Omega
\]

\[
= -\bar{\epsilon} \Omega \left( \frac{1}{1 + V_1 \frac{1-\Omega}{K}} \frac{1-\Omega}{K} \right) \left( \frac{K \Omega}{1 - \Omega} + \bar{\epsilon} \frac{K \Omega}{1 - \Omega} \left( \frac{1 - \Omega}{K} V_2 \frac{1}{1 - \frac{1-\Omega}{K} V_2} - 1 \right) \right) \Omega
\]

\[
= \bar{\epsilon} \Omega \frac{1}{1 + V_1 \frac{1-\Omega}{K}} - \bar{\epsilon} \frac{K \Omega}{1 - \Omega} \frac{1}{1 - \frac{1-\Omega}{K} V_2} \Omega
\]

\[
= -\left( 1 - \bar{\epsilon} \Omega \frac{1}{1 + V_1 \frac{1-\Omega}{K}} \right) \frac{K \Omega}{1 - \Omega} + \frac{K \Omega}{1 - \Omega} \left( 1 - \bar{\epsilon} \frac{1}{1 - \frac{1-\Omega}{K} V_2} \right).
\]

\[(6.7)\]

Express the factors on either side of the underbrace in equation (6.6) in the form:

\[
\frac{1}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} = \frac{1}{1 + V_1 \frac{1-\Omega}{K}} \left( \frac{1}{1 - \bar{\epsilon} \Omega \left( \frac{1}{1 + V_1 \frac{1-\Omega}{K}} \right)} \right)
\]

\[
\frac{1}{1 + \frac{1-\Omega}{K} V_2 - \bar{\epsilon} \Omega} = \left( \frac{1}{1 - \bar{\epsilon} \frac{1}{1 + \frac{1-\Omega}{K} V_2} \Omega} \right) \frac{1}{1 + \frac{1-\Omega}{K} V_2}.
\]

\[(6.8)\]

Plugging everything into (6.6), the factors in parentheses above cancel against the factors.
in parentheses in (6.7). Thus the contribution to the tadpole from $P_{12}$ simplifies to

$$\text{Tr}_V[P_{12} (\Phi_2 - \Phi_1)] = \bar{\epsilon} \text{Tr}_V \left[ \frac{\epsilon}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} \frac{K \Omega}{1 - \Omega} \frac{1}{1 + \frac{1-\Omega}{K} V_2} B c \Omega c \right] \rightarrow \frac{K}{1-\Omega} \Omega V_1^\infty$$

$$= -\bar{\epsilon} \text{Tr}_V \left[ B c \Omega c \frac{1}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} \frac{K \Omega}{1 - \Omega} \frac{1}{1 + \frac{1-\Omega}{K} V_2} \right] \rightarrow \Omega V_2^\infty \frac{K}{1-\Omega}. \quad (6.9)$$

In the $\epsilon \rightarrow 0$ limit, we claim that the factors above the braces approach the sliver state with boundary conditions deformed by the corresponding marginal current, multiplied by the factor $\frac{K}{1-\Omega}$. If $V_1$ and $V_2$ have regular OPE, we can expand the factors outside the braces in the $L^-$ level expansion and pick off the leading term in the $\epsilon \rightarrow 0$ limit. This gives precisely the difference in the closed string tadpole amplitude between the two solutions. Unfortunately this argument does not work when $V_1$ and $V_2$ have singular OPE, since contractions between $V_1$ and $V_2$ produce operators of lower conformal dimension which make additional contributions. It is not an easy task to see what happens in this case in the $\epsilon \rightarrow 0$ limit, but there is no reason to believe that (6.9) should calculate the shift in the tadpole amplitude. This is a remnant of the midpoint singularity of the boundary condition changing projector when the boundary condition changing operator between the BCFTs of $\Phi_2$ and $\Phi_1$ has nonzero conformal weight. To fix this problem we need to account for the “tachyon vacuum” contributions to the phantom term. Let us focus on the contribution from $P_1$:

$$\text{Tr}_V[P_1 (\Phi_2 - \Phi_1)] = \bar{\epsilon} \text{Tr}_V \left[ \frac{\epsilon}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} \Omega V_2 \frac{1}{1 + \frac{1-\Omega}{K} V_2} B c \Omega c \right] - \text{Tr}_V[P_1 \Phi_1]$$

$$= \bar{\epsilon} \text{Tr}_V \left[ \frac{\epsilon}{1 + V_1 \frac{1-\Omega}{K} - \bar{\epsilon} \Omega} \frac{K \Omega}{1 - \Omega} \left( -\frac{1}{1 + \frac{1-\Omega}{K} V_2} + 1 \right) B c \Omega c \right] - \text{Tr}_V[P_1 \Phi_1]. \quad (6.10)$$

Note that this precisely cancels the problematic contractions between $V_1$ and $V_2$ in the first term in (6.9). A similar thing happens for the second term when we consider the
contribution of $P_2$. Therefore, in total the overlap is

$$\text{Tr}_V[\psi_{12}(\epsilon)] = \epsilon \text{Tr}_V[\Phi_2 - \Phi_1] - \text{Tr}_V[P_1\Phi_1] + \text{Tr}_V[P_2\Phi_2]$$

$$+ \epsilon \text{Tr}_V \left[ \frac{\epsilon}{1 + V_1 \frac{1 - \Omega}{K} - \bar{\epsilon} \Omega} \frac{K \Omega}{1 - \bar{\epsilon} \Omega} Bc\Omega c \right] - \bar{\epsilon} \text{Tr}_V \left[ Bc\Omega c \frac{K \Omega}{1 - \Omega} \frac{1}{1 + \frac{1 - \Omega}{K} V_2 - \epsilon \Omega} \right].$$

(6.11)

Note that we have not yet taken the $\epsilon \rightarrow 0$ limit, so this formula is valid for all $\epsilon$. Now we simplify further by taking $\epsilon \rightarrow 0$, and a short calculation shows that the first three terms vanish. Therefore consider the contribution from the fourth term:

$$\bar{\epsilon} \text{Tr}_V \left[ \frac{\epsilon}{1 + V_1 \frac{1 - \Omega}{K} - \bar{\epsilon} \Omega} \frac{K \Omega}{1 - \bar{\epsilon} \Omega} Bc\Omega c \right] = \epsilon \bar{\epsilon} \sum_{L=0}^{\infty} \epsilon^L \text{Tr}_V \left[ \left( \frac{\Omega - \frac{1}{\epsilon} V_1 \frac{1 - \Omega}{K}}{K \Omega} \right)^L \frac{K \Omega}{1 - \Omega} Bc\Omega c \right].$$

(6.12)

Expanding the summand perturbatively in $V_1$,

$$\text{Tr}_V \left[ \left( \frac{\Omega - \frac{1}{\epsilon} V_1 \frac{1 - \Omega}{K}}{K \Omega} \right)^L \frac{K \Omega}{1 - \Omega} Bc\Omega c \right]$$

$$= \text{Tr}_V \left[ \Omega^L \frac{K \Omega}{1 - \Omega} Bc\Omega c \right]$$

$$- \text{Tr}_V \left[ \left( \sum_{k=0}^{L-1} \Omega^{L-1-k} \left[ \frac{1}{\epsilon} V_1 \frac{1 - \Omega}{K} \right]^k \Omega^k \right) \frac{K \Omega}{1 - \Omega} Bc\Omega c \right]$$

$$+ \text{Tr}_V \left[ \left( \sum_{k=0}^{L-2} \sum_{l=0}^{L-2-k} \Omega^{L-2-k-l} \left[ \frac{1}{\epsilon} V_1 \frac{1 - \Omega}{K} \right]^k \Omega^l \right) \frac{K \Omega}{1 - \Omega} Bc\Omega c \right]$$

$$- \ldots.$$

(6.13)

To derive the $\epsilon \rightarrow 0$ limit we expand this expression around $L = \infty$. Then each term in the perturbative expansion is a correlator on a very large cylinder, and we can pick out the leading $\mathcal{L}^-$ level of every field in the trace whose total width is fixed in the $L \rightarrow \infty$
limit. This gives:

\[
\text{Tr}_V \left( \left( \Omega - \frac{1}{\epsilon} V_1 \frac{1 - \Omega}{K} \right) \frac{L}{1 - \Omega} Bc\Omega c \right) 
\]

\[
= - \text{Tr}_V[\Omega L Bc\partial c] + \frac{1}{\epsilon} \text{Tr}_V \left[ \sum_{k=0}^{L-1} \Omega^{L-1-k} V_1 \Omega^k Bc\partial c \right]
\]

\[
- \frac{1}{\epsilon^2} \text{Tr}_V \left[ \sum_{k=0}^{L-2} \sum_{l=0}^{L-2-k} \Omega^{L-2-k-l} V_1 \Omega^k V_1 \Omega^l Bc\partial c \right] + ... + \mathcal{O} \left( \frac{1}{L} \right).
\] (6.14)

Scaling the circumference of the cylinders with \( \frac{1}{L} \), the sums above turn into integrals which precisely reproduce the boundary interaction of the marginal current [24]:

\[
\text{Tr}_V \left( \left( \Omega - \frac{1}{\epsilon} V_1 \frac{1 - \Omega}{K} \right) \frac{L}{1 - \Omega} Bc\Omega c \right) 
\]

\[
= - \text{Tr}_V[\Omega Bc\partial c] + \frac{1}{\epsilon} \int_0^1 dx \text{Tr}_V[\Omega^{1-x} V_1 \Omega^x Bc\partial c] 
\]

\[
- \frac{1}{\epsilon^2} \int_0^1 dx \int_0^{1-x} dy \text{Tr}_V[\Omega^{1-x-y} V_1 \Omega^x V_1 \Omega^y Bc\partial c] + ... + \mathcal{O} \left( \frac{1}{L} \right)
\]

\[
= - \text{Tr}_V[e^{-(K+V_1)} Bc\partial c] + \mathcal{O} \left( \frac{1}{L} \right). \] (6.15)

Plugging this back into (6.12) and taking the \( \epsilon \to 0 \) limit gives

\[
\lim_{\epsilon \to 0} \epsilon \text{Tr}_V \left[ \frac{\epsilon}{1 + V_1 \frac{1 - \Omega}{K} - \epsilon\Omega} \frac{K\Omega}{1 - \Omega} Bc\Omega c \right] = - \text{Tr}_V[e^{-(K+V_1)} Bc\partial c]. \] (6.16)

A similar argument for the final term in (6.11), and removing the \( B \) ghost following (4.13) gives

\[
\lim_{\epsilon \to 0} \text{Tr}_V[\psi_{12}(\epsilon)] = - \text{Tr}_V[e^{-(K+V_2)}] + \text{Tr}_V[e^{-(K+V_1)}] 
\]

\[
= \mathcal{A}_2(V) - \mathcal{A}_1(V). \] (6.17)

which is the expected shift in the closed string tadpole amplitude between the two marginal solutions.
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References

[1] M. Schnabl, “Analytic solution for tachyon condensation in open string field theory,” Adv. Theor. Math. Phys. 10, 433 (2006) [arXiv:hep-th/0511286].

[2] Y. Okawa, “Comments on Schnabl’s analytic solution for tachyon condensation in Witten’s open string field theory,” JHEP 0604, 055 (2006) [arXiv:hep-th/0603159].

[3] E. Fuchs and M. Kroyter, “On the validity of the solution of string field theory,” JHEP 0605, 006 (2006) [arXiv:hep-th/0603195].

[4] T. Erler, “Split string formalism and the closed string vacuum. II,” JHEP 0705, 084 (2007) arXiv:hep-th/0612050.

[5] T. Erler, “Tachyon Vacuum in Cubic Superstring Field Theory,” JHEP 0801, 013 (2008) [arXiv:0707.4591 [hep-th]].

[6] I. Y. Aref’eva, R. V. Gorbachev, D. A. Grigoryev, P. N. Khromov, M. V. Maltsev and P. B. Medvedev, “Pure Gauge Configurations and Tachyon Solutions to String Field Theories Equations of Motion,” JHEP 0905, 050 (2009) [arXiv:0901.4533 [hep-th]].

[7] I. Y. Aref’eva, R. V. Gorbachev and P. B. Medvedev, “Pure Gauge Configurations and Solutions to Fermionic Superstring Field Theories Equations of Motion,” J. Phys. A A 42, 304001 (2009) [arXiv:0903.1273 [hep-th]].

[8] T. Erler and M. Schnabl, “A Simple Analytic Solution for Tachyon Condensation,” JHEP 0910, 066 (2009) [arXiv:0906.0979 [hep-th]].

[9] T. Erler, “Exotic Universal Solutions in Cubic Superstring Field Theory,” JHEP 1104, 107 (2011) [arXiv:1009.1865 [hep-th]].

[10] T. Erler and C. Maccaferri, “Connecting Solutions in Open String Field Theory with Singular Gauge Transformations,” arXiv:1201.5119 [hep-th].
[11] I. Ellwood, “The Closed string tadpole in open string field theory,” JHEP 0808, 063 (2008). [arXiv:0804.1131 [hep-th]].

[12] I. Ellwood, “Rolling to the tachyon vacuum in string field theory,” JHEP 0712, 028 (2007) [arXiv:0705.0013 [hep-th]].

[13] M. Kiermaier, Y. Okawa, L. Rastelli and B. Zwiebach, “Analytic solutions for marginal deformations in open string field theory,” JHEP 0801, 028 (2008) [arXiv:hep-th/0701249].

[14] M. Schnabl, “Comments on marginal deformations in open string field theory,” Phys. Lett. B 654, 194 (2007) [arXiv:hep-th/0701248].

[15] I. Kishimoto, “Comments on gauge invariant overlaps for marginal solutions in open string field theory,” Prog. Theor. Phys. 120, 875 (2008) [arXiv:0808.0355 [hep-th]].

[16] L. Rastelli, A. Sen and B. Zwiebach, “Boundary CFT construction of D-branes in vacuum string field theory,” JHEP 0111, 045 (2001) [hep-th/0105168].

[17] M. Kiermaier, Y. Okawa and B. Zwiebach, “The boundary state from open string fields,” [arXiv:0810.1737 [hep-th]].

[18] T. Erler, “Split string formalism and the closed string vacuum,” JHEP 0705, 083 (2007) [arXiv:hep-th/0611200].

[19] L. Rastelli and B. Zwiebach, “Solving open string field theory with special projectors,” JHEP 0801, 020 (2008) arXiv:hep-th/0606131.

[20] E. A. Arroyo, “Generating Erler-Schnabl-type Solution for Tachyon Vacuum in Cubic Superstring Field Theory,” J. Phys. A 43, 445403 (2010) [arXiv:1004.3030 [hep-th]].

[21] S. Zeze, “Regularization of identity based solution in string field theory,” JHEP 1010, 070 (2010) [arXiv:1008.1104 [hep-th]].

[22] M. Kiermaier, Y. Okawa and P. Soler, “Solutions from boundary condition changing operators in open string field theory,” JHEP 1103, 122 (2011) [arXiv:1009.6185 [hep-th]].
[23] L. Bonora, C. Maccaferri and D. D. Tolla, “Relevant Deformations in Open String Field Theory: a Simple Solution for Lumps,” JHEP 1111, 107 (2011) [arXiv:1009.4158 [hep-th]].

[24] M. Kiermaier, Y. Okawa and P. Soler, “Solutions from boundary condition changing operators in open string field theory,” JHEP 1103, 122 (2011) [arXiv:1009.6185 [hep-th]].

[25] T. Noumi and Y. Okawa, “Solutions from boundary condition changing operators in open superstring field theory,” JHEP 1112, 034 (2011) [arXiv:1108.5317 [hep-th]].