Generic modeling, identification and optimal feedforward torque control of induction machines using steady-state machine maps

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Abstract—A not yet available look-up table (LUT) based optimal feedforward torque control (OFTC) method for squirrel-cage induction machines (SCIMs) is presented. It is based on: (i) a generic transformer-like machine model in an arbitrarily rotating \((d,q)\)-reference frame, considering nonlinear flux linkages and iron losses in the stator laminations; (ii) machine identification by evaluating steady-state measurements over a grid of \((d,q)\) stator currents, producing frequency-dependent machine maps for e.g. flux linkages, torque, iron resistance and efficiency; and (iii) numerical optimization and extraction of OFTC lookup tables for optimal stator current references depending on reference torque and electrical frequency. In order to increase reproducibility, a feedback temperature controller is employed to keep the stator winding temperature constant. Moreover, throughout the identification, the electrical frequency is kept constant (per data set) by adapting the machine speed accordingly using a speed-controlled prime mover; this way the impact of iron losses becomes more balanced than for constant speed operation. The presented measurement results confirm that compared to constant flux operation or scalar V/Hz control, efficiency can be increased particularly in part-load operation by up to 7%.

Index Terms—machine identification, flux linkages, iron resistance, induction machine, efficiency, MTPA, MTPC, MTPL.

NOTATION

\(\mathbb{N}, \mathbb{R}\) : Natural, real numbers. \(\mathbf{x} := (x_1, \ldots, x_n)^\top \in \mathbb{R}^n\) : Column vector, \(n \in \mathbb{N}\) where "\(^\top\)" and "\(:=\" mean "transposed" and "is defined as", respectively. \(a^\top b := a_1b_1 + \cdots + a_nb_n\) : Scalar product of vectors \(a\) and \(b\). \(X \in \mathbb{R}^{n \times m}\) : Matrix with \(n\) rows and \(m\) columns. \(I_n \in \mathbb{R}^{n \times n}\) := \text{diag}(1, \ldots, 1) : Identity matrix. \(I^* = [\begin{bmatrix} 1 & 0 \end{bmatrix}]\) : Conjugate identity matrix. \(T_p(\phi_p) = \begin{bmatrix} \cos(\phi_p) & -\sin(\phi_p) \\ \sin(\phi_p) & \cos(\phi_p) \end{bmatrix}\) : Park transformation matrix with angle \(\phi_p \in \mathbb{R}\) and \(J := T_p(\pi/2) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\) : Counter-clock wise rotation matrix (by \(\pi/2\)).

I. MOTIVATION AND PROBLEM STATEMENT

Maximum efficiency operation (also known as loss minimizing control) of induction machines has been subject to extensive research in the past. Two main approaches are typically distinguished: (i) Offline calculation of optimal controller set points (e.g. [1–3]), and (ii) search-based online techniques (e.g. [4–7]).

For vector control systems, the objective is to compute optimal current set points, producing a given reference torque while complying with the physical machine and drive constraints (e.g. current and voltage limits). This problem is also known as the optimal feedforward torque control (OFTC) problem [8, 9]. Depending on the optimization goal, different torque control strategies are obtained, e.g. MTPC, minimizing the stator current magnitude and thus copper losses in the stator windings and rotor bars, or maximum torque per voltage (MTPV), minimizing the stator voltage magnitude. As an extension to the classical MTPC control strategy (for IMs see e.g. [1, 4, 10]), the maximum torque per loss (MTPL) [or maximum efficiency per torque (MEPT)] control strategy further considers frequency dependent iron losses, increasing the overall machine efficiency. As a consequence, modeling and identification of iron losses has been discussed in numerous publications [6, 7, 11, 12]. Apart from iron losses, the nonlinear flux characteristics need to be considered in the machine torque calculation. Therefore, the identification of flux linkage and inductance maps has also been investigated, e.g. in [2, 10, 13].

In order to cover the whole operation range, flux linkages as functions of the stator currents can be stored in look-up tables. In [1, 2], stator flux linkages are computed offline over a grid of \((d,q)\)-currents (stator flux orientation) using linear machine parameters. The obtained maps, in combination with an iron loss model (only [2]) are used for MTPL (MTPC) operation. A similar approach is presented in [3], where likewise offline calculation using experimentally identified or FEM-based machine parameter look-up tables is used. However, in the aforementioned papers, the offline calculation of flux linkages does not account for varying operating conditions such as temperature variations. Moreover, its impact on the identified flux linkage maps and \((d,q)\)-orientation is not considered. For synchronous machines (SMs), a solution was proposed in [14], where special emphasize was put on the compensation of temperature induced variations of the stator resistance. This is achieved by alternating between motor and generator mode measurements and exploiting the magnetic symmetry property of isotropic machines. However, iron losses are neglected in this approach. As opposed to [15], where the flux linkage imbalance due to iron losses is exploited in order to quantify iron losses. In addition, a simple temperature controller is employed, keeping constant the stator winding temperature. Again, only SMs are considered, though.

The principle idea of recording nonlinear machine maps (e.g. stator flux linkage, torque or efficiency maps) depending on the stator currents can be transferred to SCIMs. However, while for synchronous machines the \((d,q)\)-reference frame is aligned with the rotor whose position can be measured, for asynchronous machines, there is no fixed rotor position to align with. Instead, a slip-based, arbitrarily rotating orientation can
be used, yielding a reproducible and unique reference frame (which does not necessarily coincide with the common rotor flux orientation). As temperature and frequency variations during the identification process deteriorate the reproducibility, active counter measures are required, though.

In this paper, an experimental machine identification method for induction machines is proposed. It is based on a generic (rotating) transformer-like machine model considering iron losses in the stator laminations and a nonlinear current-flux relation. Using steady-state measurements and the model equations, nonlinear machine maps over the \((d, q)\)-current plane—parametrized by the electrical frequency (and the temperature)—are obtained. Examples of such maps are stator flux linkage, machine torque or efficiency maps. Based on the machine maps, LUTs for various OFTC strategies such as MTPL, MTPC, constant flux (CF) and V/Hz control are available at different frequencies (including field-weakening). The SCIM model used throughout this work aims to be as accurate as possible. However, in order to draw relevant information from the measured quantities the following simplifying yet reasonable assumptions need to be imposed:

**Assumption 1** (Machine model). It is assumed that
(i) iron losses in the rotor are negligible,
(ii) stator and rotor stray flux linkages do not contribute to torque production, and
(iii) stray flux linkages in the stator iron core are negligible.

The two-phase SCIM electrical system in the arbitrarily rotating \((d, q)\)-reference frame—as illustrated in Fig. 1—is governed by the following set of voltage equations

\[
\begin{cases}
\dot{u}_s^{dq} = R_s^{dq}(\theta_s)\dot{i}_s^{dq} + \omega_p J\dot{\psi}_s^{dq} + \frac{d}{dt}\psi_s^{dq}, \\
0 = R_{s,Fe}^{dq}(\theta_s)\dot{i}_{s,Fe}^{dq} + \omega_p J\dot{\psi}_{s,Fe}^{dq} + \frac{d}{dt}\psi_{s,Fe}^{dq}, \\
0 = R_r^{dq}(\theta_r)\dot{i}_r^{dq} + (\omega_p - n_p \omega_m)(J\dot{\psi}_r^{dq} + \frac{d}{dt}\psi_r^{dq}),
\end{cases}
\]

where \(u_s^{dq}\) is the stator voltage, \(i_s^{dq}, i_{s,Fe}^{dq}\) and \(i_r^{dq}\) are the stator, stator iron and rotor currents, \(\psi_s^{dq} = \psi_{s,r}^{dq} + \psi_{s,Fe}^{dq}\) and \(\psi_r^{dq} = \psi_{r,\sigma}^{dq} + \psi_{r,m}^{dq}\) are the stator and rotor flux linkages with stator and rotor stray flux linkages \(\psi_{s,\sigma}^{dq}\) and \(\psi_{r,\sigma}^{dq}\) and main flux linkage \(\psi_{m}^{dq}\), \(\omega_p\) is the electrical (synchronous) frequency and \(\omega_m\) is the (mechanical) rotor angular velocity. Moreover, \(n_p\) denotes the number of pole pairs, \(R_s^{dq}\), \(R_{s,Fe}^{dq}\) and \(R_r^{dq}\) are the temperature-dependent stator, stator iron and rotor resistance matrices and \(\theta_s\) and \(\theta_r\) denote respective temperatures.

**Remark 1.** Since both stator iron and rotor circuit are not accessible by measurement equipment, exact knowledge of the currents and resistance values is not possible. However, only the power losses in said circuits are of interest. Therefore, the transmission ratios of both transformers can be chosen freely, such that the voltages at respective stator iron and rotor voltage sources in Fig. 1 are defined as stated in the equivalent circuit.

![Figure 1: Equivalent electrical circuit (including stator iron core) of the SCIM in the arbitrarily rotating \((d, q)\)-reference frame.](image)

The machine torque can be derived by stating the power balance: The instantaneous electrical (active) power \(p_s\) measured at the stator terminals must comprise all resistive power losses \(p_r\), the magnetizing power terms \(p_\psi\) and the mechanical power \(p_m\), i.e.

\[
p_s = p_r + p_\psi + p_m,
\]

where, for \(\kappa \in \{2/3, \sqrt{2}/3\}\) (Clarke transformation factor),

\[
\begin{align*}
\psi_s &= \frac{2}{3\kappa} (i_{s,Fc}^{dq})^\top u_s^{dq}, \\
p_r &= \frac{2}{3\kappa} (i_{r,Fc}^{dq})^\top R_r^{dq} (i_{r,Fc}^{dq}) + \frac{2}{3\kappa} (i_{s,Fc}^{dq})^\top R_{s,Fe}^{dq} (i_{s,Fc}^{dq}) + \frac{2}{3\kappa} (i_{r,Fc}^{dq})^\top R_{r,Fe}^{dq} (i_{r,Fc}^{dq}), \\
p_\psi &= \frac{2}{3\kappa} (i_{s,Fc}^{dq})^\top \frac{d}{dt}\psi_{s,Fe}^{dq} + 2\frac{1}{3\kappa} (i_{s,Fc}^{dq})^\top \frac{d}{dt}\psi_{s,Fc}^{dq} + \frac{2}{3\kappa} (i_{r,Fc}^{dq})^\top \frac{d}{dt}\psi_{r,Fc}^{dq}, \\
p_m &= \omega_m m_m.
\end{align*}
\]
torque can finally be stated as

\[ R_{\text{s},\text{Fe} s,\text{Fe}} x_{\text{s},\text{Fe}} = -\omega_p \psi_{\text{s},\text{Fe}} - \frac{d}{dt} \psi_{\text{s},\text{Fe}}, \]

\[ R_{r x x} x_{r x} = -(\omega_p - n_p \omega_m) \psi_{r x} - \frac{d}{dt} \psi_{r x}. \]

Combining (2) and (3), invoking (4) and solving for \( m_m \) yields

\[ m_m = \frac{1}{3} \frac{2}{\omega_p \omega_m} \left( x_{d q} + x_{d q, \text{Fe}} + x_{d q} \right) \psi_{s d q} - \frac{2}{3} \omega_p \psi_{s d q} \]

where Assumption 1 is used, i.e.

\[ \left( \psi_{s d q} \right) \psi_{s d q} = \left( \psi_{s d q} \right) \psi_{s m}, \]

\[ \left( \psi_{s d q, \text{Fe}} \right) \psi_{s d q, \text{Fe}} = \left( \psi_{s d q, \text{Fe}} \right) \psi_{s m}, \]

\[ \left( \psi_{r x} \right) \psi_{r x} = \left( \psi_{r x} \right) \psi_{r x}. \]

as \((s_{s,\text{ref}}) = (s_{s,\text{ref}},\psi_{s,\text{ref}},\psi_{s,\text{ref}},\omega_{s,\text{ref}}) = 0\). Now, defining the virtual current \( m_m := x_{d q} + x_{d q, \text{Fe}} + x_{d q} \) and imposing \((s_{s,\text{ref}}) = 0\) helps with the imposed speed controller. The stator current controller is implemented in the arbitrarily rotating \((d, q)-\)reference frame.

\[ m_m = \frac{1}{3} \omega_p \psi_{s d q} \psi_{s d q} - \frac{2}{3} \omega_p \psi_{s d q} \psi_{s m}. \]

\[ B. \ Control \ system \]

The control system is depicted in Fig. 2: For some given reference torque \( m_m \) and electrical frequency \( \omega_p \), a look-up table (LUT) based optimal feedforward torque controller (OFTC) generates an optimal pair of reference currents \( i_{s,\text{ref}} \).

\[ \text{The reference torque itself can be commanded directly or by a superimposed speed controller. The stator current controller is implemented in the arbitrarily rotating} (d, q)-\text{reference frame. It guarantees that the reference currents are tracked properly, i.e.} \]

\[ i_{s,\text{ref}} - i_{d q} = 0_2 \] holds after finite time. In this work, for the sake of simplicity, a standard proportional-integral (PI) controller with decoupling (feedforward disturbance compensation) is used. However, any other type of current controller may be used instead.

For induction machines, magnetic isotropy is a commonly imposed assumption making the choice of the rotating reference frame arbitrary. However, the steady-state machine behaviour is dominated by two quantities, namely the slip frequency \( \omega_p = n_p \omega_m \) and the stator current magnitude \( \| i_{s,\text{ref}} \| \).

Assuming that the electrical frequency \( \omega_p \) can be chosen freely, a \textit{reproducible} and \textit{unique} reference frame is obtained by fixing the electrical frequency \( \omega_p \)—i.e. the time derivative of the electrical angle \( \phi_p \)-to

\[ \frac{d}{dt} \phi_p = \omega_p = n_p \omega_m + \frac{1}{T_{i,\text{lpf}} i_{d q,\text{lpf}}}, \]

where \( i_{s,\text{lpf}} \) are the low-pass filtered stator currents, i.e.

\[ \frac{d}{dt} i_{d q,\text{lpf}} = \frac{1}{T_{i,\text{lpf}}} (i_{s,\text{lpf}} - i_{d q,\text{lpf}}), \]

with filter time constant \( T_{i,\text{lpf}} \). The transformation angle \( \phi_p \) for the Park transformation is obtained by integrating \( \omega_p \) over time, i.e. \( \phi_p = \int \omega_p \, dt \).

\textbf{Remark 2.} Often \( T_{i,\text{lpf}} \) is selected to comply with the rotor time constant, yielding an approximate rotor flux orientation. However, it is important to note that the choice of \( T_{i,\text{lpf}} \) is arbitrary, as an accurate rotor flux orientation is not required.

\textbf{C. Steady-state machine properties}

In steady-state the derivatives of the flux linkages in (1) are zero. Assuming constant temperatures \( \theta_s, \theta_r \), it follows that the resistances \( R_{s}\) (\( \theta_s \)) and \( R_{r}\) (\( \theta_r \)) are constant, too, and the iron resistance depends on the frequency and the stator currents, i.e. \( R_{s,\text{Fe}} = R_{s,\text{Fe}}(\omega_p, i_{s}, \lambda_{s}) \).

As the main flux linkages \( \psi \) connects the three electric circuits (stator, stator iron, rotor), the rotor currents \( i_{r} \), the stator iron currents \( i_{s,\text{Fe}} \) and the rotor stray flux linkages \( \psi_{r,\sigma}(i_{r}, \psi_{r,\sigma}) \) can be considered functions of the stator currents \( i_{s} \), the electrical frequency \( \omega_p \) and the mechanical speed \( \omega_m \), only. By defining \( \omega_p \) as in (8), an additional degree of freedom is removed and the machine state is entirely described by any three of the aforementioned quantities. Hence, the fundamental idea is that—in steady-state—each combination of stator currents and frequency \((i_{s}, i_{s,\text{lpf}}, \omega_p)\) represents a reproducible and unique operating point.

\textbf{Remark 3.} Note that this is rather intuitive for permanent magnet and reluctance synchronous machines with two states only and a rotor fixed \( d \)-axis. For (isotropic) induction machines, however, by default there is no fixing point for the \((d, q)-\)reference frame. Moreover, with the rotor currents (or flux linkages) there are two additional states to be considered.

\section{III. \ Machinery \ Identification}

Assuming basic parameter knowledge (e.g. acquired by no-load and locked-rotor tests) and the control system implemented as described in Sec. II-B, the proposed identification can be conducted. The diagram in Fig. 3 illustrates the steps of the identification process.

\textbf{A. \ Laboratory \ setup}

The drive setup is depicted in Fig. 4: The current controlled device under test (DUT), a star-connected SCIM with ratings as listed in Table I, is mechanically coupled with a speed controlled prime mover, a more powerful permanent magnet synchronous machine (PMSM). Although, here, the machine torque \( m_m \) is measured for validation purposes, a torque sensor is \textit{not} mandatory. The machines are supplied by two voltage source inverters (VSIs), whose mutual DC link voltage \( u_{dc} \) is measured. By choosing a back-to-back configuration of the VSIs, energy circulates during the identification and only losses must be provided by the grid. The stator voltages \( u_{abc} \) of the DUT are reconstructed from the current controller output \( u_{\text{abc,ref}} \), requiring good knowledge of the modulation strategy (e.g. regular sampled, symmetric PWM), whereas the stator currents \( i_{s,abc} \) and machine angular velocity \( \omega_m \) are measured directly. In addition, the stator winding temperature \( \theta_s \) is measured using PT100 resistance temperature detectors (RTDs). The control system is implemented on a dSPACE real-time system with DS1007 processor board and various I/O cards, running at a fixed sampling rate of 8 kHz.
Figure 2: Overview of the control system for optimal feedforward torque control (OFTC) operation and machine identification (reference current generation, RCG).

Figure 3: Machine identification process chart.

Figure 4: Laboratory setup comprising (A) induction machine (DUT), (B) torque sensor, (C) prime mover (PMSM), (D) inverter, (E) real-time system and (F) host PC.

### B. Reference current generation

As the prime mover controls the speed \( \omega_{\text{m}} \), the SCIM stator currents \( i^d_s, i^q_s \) and frequency \( \omega_p \) can be set freely. Given a reference frequency \( \omega_{\text{p,ref}} \), the objective is to gather machine data for each pair of stator currents \( (i^d_s, i^q_s) \in \mathbb{I}_s \) within the feasible (rated) set \( \mathbb{I}_s := \{(i^d_s, i^q_s) | i^d_s + i^q_s \leq i^2_{s,\text{R}} \} \). This is achieved by sampling the current plane over a regular grid as illustrated in Fig. 5, showing the first (I) and fourth (IV) quadrant of the current plane (here rotated by \( \pi/2 \) for illustrative purposes). For reasons of redundancy, only the positive \( d \)-axis is covered. Note that in order to account for the frequency dependency, measurements need to be recorded at various frequencies. While a minimum amount of excitation current \( i_{s,\text{min}}^d \) is required for the \( (d, q) \)-orientation (the denominator in (8) must not be zero), magnetic saturation sets in for high \( i^d_s \), \( i^q_s \) values of \( i_{\text{m},\text{R}}^d, i_{\text{m},\text{R}}^q \) must not be zero), magnetic saturation sets in for high values of \( i^d_s, i^q_s \), which motivates for an upper limit \( i_{s,\text{max}}^d \), \( i_{s,\text{max}}^q \) (see Fig. 5). In turn, the absolute value of the \( q \)-current is varied from zero to some defined maximum value \( i_{s,\text{max}}^q \), \( i_{s,\text{max}}^q \) (see Fig. 5). The corresponding grid points are given by

\[
\bar{i}_d^d = (i_{s,\text{min}}^d, \ldots, i_{s,\text{max}}^d) \quad \text{and} \quad \bar{i}_q^q = (-i_{s,\text{max}}^q, \ldots, 0, \ldots, i_{s,\text{max}}^q) \quad \text{with} \quad 0 \leq m \leq 2^{16}, \quad 0 \leq n \leq 2^{16}.
\]

The arguments \( m \) and \( n \) define the grid size \( m \times n \) and represent a trade-off between accuracy and effort. Note that the grid corners may be located outside of \( \mathbb{I}_s \) (light green area) as

![Figure 2: Overview of the control system for optimal feedforward torque control (OFTC) operation and machine identification (reference current generation, RCG).](image1)

![Figure 3: Machine identification process chart.](image2)

![Figure 4: Laboratory setup comprising (A) induction machine (DUT), (B) torque sensor, (C) prime mover (PMSM), (D) inverter, (E) real-time system and (F) host PC.](image3)
Figure 5: First (I) and fourth (IV) quadrant of the current grid (rotated by $\pi/2$), with grid vectors $i^d_s$ and $i^q_s$.

indicated by gray dots in Fig. 5. The associated current pairs are skipped.

C. Measurement details

Suppose the unique operating point $(i^d_{s,ref}, i^q_{s,ref}, \omega_{p,ref})$ is to be recorded. While the stator current references are tracked by the current control system, the mechanical speed $\omega_m$ needs to be adapted by the speed controller of the prime mover, in order to reach the target frequency $\omega_{p,ref}$ (recall (8)). Moreover, the stator winding temperature $\vartheta_s$ should be kept constant throughout the measurement.

1) Temperature control: The temperature dependency of the resistive components is taken care of by pre-heating the machine to an intermediate temperature, slightly below the rated load temperature. A hysteresis temperature controller is employed, regulating the stator winding temperature after each recording: If the measured temperature is outside a defined temperature band $\vartheta_s,ref \pm \vartheta_{\varepsilon}$ around the reference temperature $\vartheta_{s,ref}$, the machine is either heated up by using the maximum (negative) q-axis current $i^{q}_{s,ref} = -i^{q}_{s,max}$, or cooled down by using zero q-axis current $i^{q}_{s,ref} = 0$; meanwhile, the d-axis current is set to its minimum value $i^{d}_{s,ref} = i^{d}_{s,min}$, i.e.

$$ i^{d}_{s,ref} = i^{d}_{s,min} \quad i^{q}_{s,ref} = \begin{cases} 0 & \vartheta_{s} > \vartheta_{s,ref} + \vartheta_{\varepsilon} \\ -i^{q}_{s,max} & \vartheta_{s} < \vartheta_{s,ref} - \vartheta_{\varepsilon} \end{cases} $$

(10)

In order to reduce the temperature control time, the current reference vectors (sequence of all feasible grid points) are sorted by magnitude and alternated between low and high magnitude values.

2) Constant frequency operation: The dependency of iron losses on the frequency $\omega_s$ is typically modelled by a frequency (and current) dependent iron loss resistance $R_{s,Fe}(\omega_s, i^{dq})$. Assuming that $R_{s,Fe}$ is not known a priori, it is reasonable to make sure that it does not vary with the electrical frequency during the measurement. With the current references being variable parameters it follows that constant electrical frequency operation is realized only by adapting the mechanical speed accordingly, i.e.

$$ \omega_{m,ref} = \frac{1}{n_p} \left( \omega_{p,ref} - \frac{1}{T_{i,lpf}} \frac{i^q_{s,ref}}{i^{q}_{s,ref}} \right). $$

(11)

Here $\omega_{m,ref}$ is the reference speed for the speed controlled prime mover, given the current references $i_{s,ref}^{d}$ and $i_{s,ref}^{q}$ at the respective operating point and the desired electrical frequency of the current map recording $\omega_{p,ref} = \text{const}$.

3) Examplary operating point: An examplary measurement window is shown qualitatively in Fig. 6 for two different operating points, (OP1, dashed) low generating $q$-current and (OP2, dotted) high motoring $q$-current. From top to bottom, (1) the temperature, (2) the electrical frequency and mechanical speed references, and (3) the stator current references are shown. In order to guarantee smooth operation, even at the operating limits (i.e. voltage limit), the current references are ramped-up slowly. At first the $d$-current is ramped up and held constant at its target value $i^{d}_{s,ref}$ for some time. As the rotor flux linkage builds up slowly, the voltage limit may only be exceeded after some time. During that time the temperature tends to fall, as the current magnitude is low. Next, the $q$-current is ramped up to its target value $i^{q}_{s,ref}$. For operating point (OP2), the temperature rises as the current magnitude is high (dashed line), whereas it drops further (dotted line) for operating point (OP1). Note that changing the $q$-current requires adaption of the mechanical speed according to (11). When both, $i^{d}_{s,ref} = i^{d}_{s,min}$ and $i^{q}_{s,ref}$ are set, the test data is recorded. The length of the measurement window is a trade-off between temperature variation and the amount of data acquired and available for averaging. Once, the test data is recorded, the $\vartheta$-controller is activated. If the temperature $\vartheta_{s}$ drops below the lower limit $\vartheta_{s,ref} - \vartheta_{\varepsilon}$, the machine is heated up (OP2). In turn, if the temperature exceeds the upper limit $\vartheta_{s,ref} + \vartheta_{\varepsilon}$, the machine is cooled down (OP1). Immediately after the temperature band is reached again, the measurement is continued with the next operating point. Note that the length of the $\vartheta$-control period varies and depends on both reference temperature $\vartheta_{s}$ and width of the temperature band $\vartheta_{\varepsilon}$. In practice, an intermediate reference temperature lower than the rated conditions equilibrium temperature has proven a good compromise reducing the overall $\vartheta$-control time.

D. Extraction of dq-machine maps

Having recorded data sets for different values of $\omega_{p,ref}$, machine maps are extracted from the measured time series data for each data set by averaging over the test data window at each valid operating point. Note that due to slow transients, parts of the test data window need to be discarded, as only steady-state data is required. If the steady-state stator currents do not match their references (e.g. because the voltage limit was exceeded) the respective operating point is discarded. Finally, the raw data maps are refined by increasing the resolution using the loess curve fitting method in MATLAB R2019b.

1) Flux linkage maps: An estimate of the stator flux linkage is obtained from the steady-state stator voltage equation in (1),
The stator resistance is adapted using the measured stator winding temperature, i.e.

\[
\psi_s^{dq} = \left(1 + \frac{\vartheta}{\omega} J^{-1} (u_{s}^{dq} - R_s(\vartheta_s) i_s^{dq}) \right).
\]

The resulting two quadrant nonlinear stator flux linkage maps are shown, exemplary for $\omega_{p,ref} = 0.5 \omega_{p,R}$, in Fig. 7. It can be seen in the $d$-component (see Fig. 7a), that it saturates strongly in $i_s^d$-direction, whereas in $i_s^q$-direction it is almost constant and symmetric (motorizing and generating mode). Moreover, it is observed in the $q$-component (see Fig. 7b), that its absolute value decreases towards the $i_s^q$-axis. Only for higher loads (i.e. $q$-currents), there is an angle difference between the rotor and stator flux linkages, which causes a significant $q$-component of the stator flux linkage. Unlike the $d$-component, the $q$-stator flux linkage is antisymmetric with respect to the $i_s^d$-axis.

2) Machine torque map: Since rotor current and flux linkage are not measured, the machine torque has to be either measured using a torque sensor, or calculated using (7).

If a torque sensor is available, the measured torque (load torque) $\tilde{m}_m$ is typically biased due to friction $m_f$, i.e. $\tilde{m}_m = m_m + m_f$. Unlike machines with constant excitation (e.g. permanent magnet machines), where the separation of iron losses and frictional losses is non-trivial, for induction machines, only frictional losses occur at zero currents $i_s^{dq} = (0, 0)^T$. However, as $i_s^d \geq i_s^{d,\min} > 0$, the relevant data point is not recorded. Since torque maps are recorded for various frequencies (and thus speeds), the following solution is proposed: For each torque map recording the $d$-axis torque values (i.e. $i_s^q = 0$) are extrapolated to the origin, i.e. $m_f(\omega_m) = m_m(0, 0, \omega_{p,ref})$. The resulting value is added to a friction look-up table for the respective speed $\omega_m = \omega_{p,ref}/n_p$. Finally, friction can be compensated for by using this look-up table (see Fig. 8). The corresponding (measured) torque map for $\omega_{p,ref} = 0.5 \omega_{p,R}$ is shown in Fig. 9a.

![Figure 6: Exemplary measurement window for two different operating points (indicated by dashed and dotted lines).](image1)

![Figure 7: Stator flux linkage maps at $\omega_{p,ref} = 0.5 \omega_{p,R}$.](image2)

![Figure 8: Friction look-up table.](image3)
approximate machine torque \( \hat{m}_m = \frac{2}{3} \eta_p (i_{d}^{dq})^\top J \psi_s^{dq} \). However, particularly for higher frequencies, this approximation becomes rather inaccurate.

An alternative approach is based on the symmetry property of the stator flux linkages in motor and generator mode, respectively (see also the vector diagrams in e.g. [2, 14]). Let \( \Gamma \) denote the matrix equivalent to the complex conjugate operator for complex numbers. In the absence of iron losses the flux symmetry

\[
\psi_s^{dq}(i_s^{dq}) = \Gamma^* \psi_s^{dq}(\Gamma^* i_s^{dq})
\]

holds, i.e. the flux linkage vectors in motor and generator mode (conjugate current vector \( \Gamma^* i_s^{dq} \)) have identical \( dq \)-axis components and \( q \)-components of equal amplitude but opposing signs. If, however, iron losses are considered, this symmetry property is perturbed. Assuming all quantities in (12) are known exactly, the perturbation is caused by iron losses only and, hence, the amount of perturbation implies the amount of iron losses (see similar approach in [15]). The mean (symmetric) part of the measured flux linkage \( \bar{\psi}_s \), and the perturbation \( \delta \psi_s^{dq} \) (deviation from mean) are defined as

\[
\begin{align*}
\bar{\psi}_s^{dq}(i_s^{dq}) &= \frac{1}{2} \left( \psi_s^{dq}(i_s^{dq}) + \Gamma^* \psi_s^{dq}(\Gamma^* i_s^{dq}) \right), \\
\delta \psi_s^{dq}(i_s^{dq}) &= \frac{1}{2} \left( \psi_s^{dq}(i_s^{dq}) - \Gamma^* \psi_s^{dq}(\Gamma^* i_s^{dq}) \right).
\end{align*}
\]

Using (15) the stator flux linkage can be rewritten as

\[
\psi_s^{dq}(i_s^{dq}) = \bar{\psi}_s^{dq}(i_s^{dq}) + \delta \psi_s^{dq}(i_s^{dq}).
\]

Assuming that the torque detuning caused by the iron loss current term in (7) can likewise be described using the flux imbalance, i.e. \( (\bar{\psi}_s^{dq})^\top J \psi_s^{dq} = (\bar{\psi}_s^{dq})^\top J \delta \psi_s^{dq} \), the machine torque can be approximated by

\[
\hat{m}_m = \frac{2}{3} \eta_p (i_s^{dq})^\top J \bar{\psi}_s^{dq}.
\]

The corresponding torque estimation error \( \Delta m_m = m_m - \hat{m}_m \) is shown in Fig. 9b for both estimation methods. The opaque surface shows the error without flux symmetrization, whereas the solid surface refers to (17). It can be seen that a good approximation of the machine torque is obtained, with an error of less than 4%. For the simple approximation the error reaches values of up to 8%.

3) Iron resistance map: If the machine torque is measured, iron losses can be determined using the steady-state voltage equation of the iron circuit (1), multiplied by the transpose of the iron current, yielding

\[
p_{s,Fe} = \frac{\omega_p}{n_p} m_m - \frac{2}{3} \eta_p (i_s^{dq})^\top J \psi_s^{dq}.
\]

Assuming a scalar iron resistance \( R_{s,Fe} = R_{s,Fe} I_2 \) allows for determining

\[
R_{s,Fe} = \frac{\omega_p^2 (\psi_s^{dq})^\top \psi_s^{dq}}{p_{s,Fe}}.
\]

The corresponding iron resistance map is depicted in Fig. 11a. Moreover, the opaque surface indicates the iron resistance if it was calculated using the estimated torque (17).

4) Efficiency map: Considering the electrical and mechanical power terms as defined in (3), the machine efficiency can be calculated as the ratio of output over input power. If \( p_s \geq 0 \) (per definition, passive sign convention), the machine operates in motor mode, while, for \( p_s < 0 \), it operates in generator mode. Hence, the efficiency \( \eta \) is defined as

\[
\eta := \begin{cases} 
\frac{p_m}{p_s}, & \text{for } |p_s| \geq |p_m| \\
\frac{p_m}{|p_s|}, & \text{for } |p_s| < |p_m|,
\end{cases}
\]

where \( \text{sign } p_s = \text{sign } p_m \) is assumed. Note that the case \( \text{sign } p_s \neq \text{sign } p_m \) exists for low power operation, i.e. when both machines operate in motor mode, providing just enough power to compensate losses on both sides.

An efficiency estimate \( \hat{\eta} \) is obtained by using the estimated torque \( \hat{m}_m \) for the mechanical power and subsequent efficiency calculation. Measured and estimated efficiency maps for \( \omega_{p,ref} = 0.5 \omega_{p,R} \) are shown in Fig. 10. At zero \( q \)-current (zero load), the efficiency is zero per definition. As expected the measured efficiency (see Fig. 10a) is clearly lower than the estimated efficiency (see Fig. 10b), which neglects iron losses.

5) V/Hz map: Although the voltage-over-frequency ratio is not a relevant parameter for the efficiency optimization, it is required for the commonly used constant V/Hz (or V/f) control method. In order to allow for a quantitative comparison of the vector controlled maximum efficiency control method and the scalar V/Hz control method, a V/Hz map is computed. The V/Hz ratio—in the following denoted by \( \xi \)—is defined by

\[
\xi := \frac{2 \pi}{\omega_p} \sqrt{(i_{d}^{dq})^2 + (i_{q}^{dq})^2}.
\]

The corresponding map is shown in Fig. 11b.
E. Look-up table generation

In general, the mapping of machine torque $m_m$ to stator currents $i_{s}^{dq}$ is ambiguous in the sense that different pairs of stator currents may produce the same torque. Therefore, various OFCT strategies may be defined (e.g. MTPL) being subject to either an equality constraint or an optimization problem (see e.g. [8]). For each strategy, LUTs are generated for both $d$- and $q$-currents. The LUTs are generated as follows: (i) Compute torque contour lines for reference torque $m_{m,ref} \in [-m_{m,R}, m_{m,R}]$, (ii) use the resulting $d$- and $q$-currents for looking up the secondary variable from the respective map (e.g. $\eta$ or $\xi$) and (iii) evaluate the equality constraint or the optimization problem on those values; it is recommended to interpolate in-between values on the resulting contour lines as to increase resolution and to obtain a more smooth result. The corresponding $i_{s,ref}^{d}$ and $i_{s,ref}^{q}$ values are stored in the LUTs $\mathcal{L}_{s,ref}^{M}$ and $\mathcal{L}_{s,ref}^{M}$, where superscript ‘M’ is replaced by the respective OFCT strategy. Repeating this procedure for machine maps of different efficiencies yields the 2D LUTs $\mathcal{L}_{s,ref}^{M}$ and $\mathcal{L}_{s,ref}^{M}$.

Fig. 12 shows the resulting LUTs over frequency $\omega_p \in [0.1, 1.5]$ · $\omega_{p,R}$ and reference torque $m_{m,ref} \in [0, 1] · m_{m,R}$. Four different OFCT strategies have been evaluated: (i) maximum torque per loss (‘M’ = ‘MTPL’), minimizing iron losses and copper losses in both stator windings and rotor bars [see Figs. (a) and (e)], (ii) maximum torque per current (‘M’ = ‘MTPC’), minimizing copper losses in the stator windings [see Figs. (b) and (f)], (iii) constant flux (‘M’ = ‘CF’), where $i_{s}^{d} = \text{const}$. holds over the whole speed and torque range [see Figs. (c) and (g)], and (iv) constant voltage-over-frequency ratio (‘M’ = ‘VHz’) with $\xi_{R} = 2\pi u_{s,R}/\omega_{p,R}$ [see Figs. (d) and (h)]. For MTPC and MTPL, respectively, LUTs have been calculated based on the measured torque $m_m$ and the estimated torque $\hat{m}_m$; the results in Fig. 12 refer to the measured torque.

It can be observed that for all OFCT strategies, except for VHz, $\mathcal{L}_{s,ref}^{M}$ does not vary with $\omega_p$ and is almost found in $m_{m,ref}$ in the rated frequency range; differences are found mainly in $\mathcal{L}_{s,ref}^{d}$. For MTPL and MTPC [see Figs. (a) and (b)], a strongly nonlinear relation in $m_{m,ref}$ is observed. For CF [see Fig. (c)], the LUT is perfectly constant, whereas for VHz [see Fig. (d)], it is weakly nonlinear, even though the excitation is supposed to be constant. For MTPL even a slight variation in $\omega_p$ direction is observed. Beyond rated frequency, which is only feasible for MTPC and MTPL, $i_{s,ref}^{d}$ clearly decreases, whereas $i_{s,ref}^{q}$ increases.

IV. RESULTS AND DISCUSSION

The measured efficiencies of MTPL, MTPC, CF and VHz are compared for different electrical frequency and torque values in Fig. 13: (i) The left column shows 2D contour plots of the machine efficiency for first quadrant operation (motoring mode). Additionally, the respective OFCT trajectories are laid over the contour plots. (ii) The center column shows the efficiencies of the different control strategies plotted versus the machine torque. (iii) In the right column, the surface area...
under the respective efficiency-over-torque plots (integral of efficiency over torque) is shown as a bar plot, in order to provide a performance measure over the entire torque range. The enclosed areas are normalized with respect to the enclosed area by the MTPL curve which serves as benchmark OFTC strategy. Each row of Fig. 13 shows results for one specific electrical frequency (i.e. $\omega_p \in [0.2, 0.4, 0.6, 0.8, 1.0] \cdot \omega_{p,R}$). MTPL and MTPC trajectories are shown for both, measured torque (argument $m_m$) and estimated torque (argument $\tilde{m}_m$).

A first observation is that the efficiency generally increases with the frequency. Looking at the contour efficiency plots (left column), it is observed that the MTPC($m_m$), MTPL($\tilde{m}_m$) and MTPL($\tilde{m}_m$) trajectories do not change significantly with the frequency. In contrast, the MTPL($m_m$) trajectory is shifted in $-i_{dq}$-direction with increasing frequencies, resulting in a deviation of the MTPL($m_m$) and MTPL($\tilde{m}_m$) trajectories. Looking at the efficiency-over-torque plots (center column), the previous observation of increasing efficiency for higher electrical frequencies is confirmed. Naturally, the MTPL($m_m$) curve marks the upper limit for all curves. Nevertheless, the difference between MTPC and MTPL is rather low (for the considered SCIM) for $m_m > 0.5 m_{m,R}$. For lower torques, both CF and V/Hz, become significantly worse than MTPL and MTPC. Finally, looking at the enclosed areas (normalized with respect to the enclosed area by the MTPL curve), the previous statements are quantified. The average efficiency of V/Hz is decreased by 14% for low $\omega_p = 0.2 \omega_{p,R}$, and about 7% for higher (up to rated) frequencies. For CF, a difference of 4% to 6% is observed, depending on the frequency. The difference between MTPL and MTPC can be considered negligible at rated frequency. However, this statement cannot be generalized, as for other induction machines, higher iron losses might occur.

V. CONCLUSION

An experimental machine identification for SCIMs was presented resulting in (i) flux linkage, torque and efficiency maps over a grid of stator $dq$-currents, and (ii) OFTC LUTs, allowing for maximum efficiency operation at any operating point. It was confirmed that MTPL (and MTPC) operation increases efficiency compared to CF or VHz control by 4% to 7% on average. Main differences were observed in part-load operation. If a torque sensor is available, iron losses can be quantified exactly and MTPL operation is recommended. If not, it is advisable to use MTPC instead, as an almost equal performance is achieved (at least in the rated speed range).

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Figure 13: Efficiency comparison at different electrical frequencies ($\omega_p \in [0.2, 0.4, 0.6, 0.8, 1] \cdot \omega_{p,R}$, from top to bottom), with efficiency contour plots in current locus (left column), efficiency-over-torque plots (middle column) and overall performance bar plots (right column).
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