Experimental study on chaotic behaviors of a Chua’s circuit based on variable memristor

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Abstract: This paper presents experimental analysis of chaotic dynamics in a variable memristor-based Chua’s Circuit. The equivalent memristor used as a nonlinear resistor of the Chua’s circuit comprises operational amplifiers, multipliers, several passive elements, and variable resistor for its hysteresis loop control. Chaotic dynamics such as attractors and bifurcation were examined using a control resistor of the memristor as a variable resistor. Moreover, we showed the subtle dependence on the hysteresis loop of the memristor in Chua’s circuit. The circuit was experimentally implemented using an electronic circuit and measurements were noted. The implemented circuit showed intermittent chaotic dynamics from 4.3 kΩ to 16.1 kΩ in control resistor of the memristor. The simulated and measured results verified that the chaotic dynamics of the proposed circuit could be created and controlled using a variable memristor.

Keywords: Chua’s circuit, chaos, memristor, bifurcation, SPICE

Classification: Electron devices, circuits, and systems

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1 Introduction

Since Robert May [1] discovered chaotic behavior in the mathematical model for the prediction of biological populations in 1976, chaos research has been extensively employed in many fields of study during the past decades [2]. Chua’s circuit is one of the typical examples showing these chaotic phenomena in an electric circuit. After the advent of the Chua’s circuit [3], various mathematical models and theoretical analyses of its system have been frequently reported [4, 6]. In particular, the experimental study of Chua’s system was conducted on hybrid circuits with discrete components such as passive resistors, inductors, capacitors, and operational amplifiers [5, 6]. Recently, Chua’s circuits based on memristor [7], which is known as the fourth passive element, have received considerable research attention [8]. These circuits include several passive devices such as resistor, capacitors, inductor, and mimicked memristor as nonlinear resistor. These researches are mostly limited to demonstrating chaotic dynamics such as time series and attractors, not through experiments but through circuit analyses or mathematical methods [8]. So far, there are only a few real observations of the chaotic dynamics in memristor based Chua’s systems [9]. In particular, to the best of our knowledge, no study has reported on the dependence on the hysteresis loop of the mimicked memristor in the Chua’s circuit. In this paper, we have focused on the hardware implementation of the Chua’s circuit using the mimicked memristor and its experimental analysis of the chaotic behaviors using a control resistor.

2 Memristor-based Chua’s system with controllability

In general, the family of Chua’s circuit consists of five discrete components including several passive elements and a nonlinear resistor known as Chua’s diode [3, 7]. The first four elements are standard linear passive electrical components, namely, an inductance L, a resistance R, and two capacitances $C_1$ and $C_2$. The fifth element is the Chua’s diode, a component with negative resistance that is required for the generation of chaotic signals. Traditionally, these nonlinear resistors have been implemented using a combination of discrete op-amps (operational amplifiers) and linear passive resistors or diodes [7]. In this paper, we use an equivalent memristor based on Muthuswamy’s model [8] as a Chua’s diode. The memristor consists of three amplifiers, several fixed resistors, two multipliers, and a variable resistor for the control of the chaotic dynamics in the Chua’s circuit. The proposed memristor-based circuit for real observation of the Chua’s system is shown in Fig. 1. If the state variables in Fig. 1 are defined as $V_{c1}$, $V_{c2}$, and $I_1$, this circuit can be described using three ordinary differential equations [3, 7, 8];

$$C_2 \frac{dV_{c2}}{dt} = G(V_{c1} - V_{c2}) - g(V_m)$$

$$C_1 \frac{dV_{c1}}{dt} = G(V_{c2} - V_{c1}) + I_1$$

$$L \frac{dI_1}{dt} = -V_{c1}$$

(1)
where $G = 1/R$, $g(V_m)$ is the current $I_m$ of the nonlinear resistor, and $V_{c1}$ and $V_{c2}$ are the voltages across $C_1$ and $C_2$, respectively, and $I_1$ is the current through inductor.

In general, to realize the mimicking of the memristor with the hysteresis loop, complex circuit synthesis by active devices, such as the op-amp, and various passive elements is required. All these studies introduce nonlinearity in the I–V curves of the devices, which are based on the integration and multiplication of current and voltage using op-amps, multipliers, and other circuit elements. However, in these research efforts, the controllability of the nonlinear device has not been included. In our case, we leveraged Muthuswamy’s study [8], which is based on Zhong’s model [6] for a memristor equivalent. The variable equivalent memristor with a controllability circuit is shown in Fig. 1.

In Fig. 1, OP1 and OP2 are the buffer and integrator blocks of the equivalent memristor, respectively, whereas OP3 is the current inverter. Further, two multipliers with a feedback loop for voltage multiplication are cascaded. In addition, in Fig. 1, a variable resistor $R_c$ enables control of the nonlinear hysteresis of the mimicked memristor.

Finally, the equations that represent the voltage driven memristor in Fig. 1 are as follows [8]:

$$I_m(t) = \left( -\frac{1}{R_3} + V_1^2 \times \frac{(R_4 + R_5)}{R_3 \times R_4 \times 100} \right) \times V_m(t)$$

(2)

where $I_m$ and $V_m$ represent the current and voltage of the equivalent memristor circuit, respectively. Equation (2) represents the I–V curve of the equivalent memristor circuit. In Fig. 1, the circuit parameters are $R_1 = R_2 = 2.2 \, \text{k} \Omega$, $R_3 = 1.5 \, \text{k} \Omega$, $R_4 = 3 \, \text{k} \Omega$, $R_5 = 32 \, \text{k} \Omega$, $R_6 = 4.7 \, \text{k} \Omega$, and $C = 47 \, \text{nF}$. We performed circuit analysis with a simulation program (SPICE) according to the resistance of the variable resistor.

3 Simulation results of the proposed Chua’s circuit

We carried out SPICE analysis for chaotic dynamics of the Chua’s circuit in Fig. 1. In this case, the values of the main circuit elements were optimized as $L = 15 \, \text{mH}$, $C_1 = 47 \, \text{nF}$, $C_2 = 4.7 \, \text{nF}$, and $R = 2 \, \text{k} \Omega$ so that chaotic phenomenon can be observed. SPICE simulation of the Chua’s system produces various chaotic
dynamics according to the changes of the nonlinear hysteresis of the memristor by a control resistor. We obtained the attractor and bifurcation by adjusting the resistance of the control resistor from 0 kΩ to 18 kΩ. Fig. 2(a) and (b) are simulated two-dimensional attractors of \( V_{c1} \) and \( V_{c2} \). In case of 15 kΩ, as shown in Fig. 2(a) the system has chaotic attractors that trace irregular patterns. If the \( R_c \) has changed to 18 kΩ, as shown in Fig. 2(b), the state variables \( V_{c1} \) and \( V_{c2} \) of the system show specific periodic patterns such as double scrolling. In conclusion, we can confirm from Fig. 3 that \( V_{c1} \) and \( V_{c2} \) of the system change to either the chaotic state or the periodic state depending on the variation of the hysteresis loop of the memristor caused by the control resistor.

\[
\lambda = \frac{1}{N} \sum_{n=0}^{N-1} \log_2 \left| \frac{\Delta x_n(t_{n+1})}{\Delta x_n(t_n)} \right|
\]  

(3)

Lyapunov exponents \( \lambda \) with 4700 points of the internal state for various values (0 kΩ < \( R_c \) ≤ 18 kΩ) of control resistor are calculated by the MATLAB program. The number of data of internal state \( x_i \) of time series affects the accuracy of the calculated Lyapunov exponent. We find that a positive Lyapunov exponent (>0) corresponds to a chaotic state and a negative Lyapunov exponent (<0) corresponds to a periodic or an equilibrium state. This is a very clear demonstration for the existence of the chaotic behavior in the proposed circuit according to the control resistor \( R_c \).
4 Electronic implementation and measurements

The Chua’s system in Fig. 1 was implemented in a hybrid electronic circuit for experimental observations of chaotic dynamics. Fig. 4 shows the implemented hardware board. The elements of the electronic circuit include an inductor, a standard resistor, two capacitors, and an equivalent memristor block that comprises two multipliers (AD633) [6] and several fixed resistors, three amplifiers (HA17711), and a variable resistor (50 kΩ) for controlling the hysteresis loop. The terminal configurations consist of the two power terminals for the operational amplifiers and multipliers, the state variable terminals, $V_{c1}$ and $V_{c2}$, and the tunable terminal for the control of the variable resistor. Measurements and analysis of the circuit were performed using an analog oscilloscope for the observation of the time series and attractor for the circuit, and a digital oscilloscope for the fast Fourier transform (FFT) frequency spectra analysis. Further, the current–voltage curve of the mimicked memristor block was measured using an analog oscilloscope.

In order to observe the electrical characteristics of the memristor block, a small test resistor was inserted into the circuit. Fig. 5(a) and (b) show the measured

![Simulated bifurcation diagram of the state variable $V_{c1}$ and Lyapunov exponents according to the control resistor $R_c$ as the variable.](image)

![Implemented hardware of the variable memristor-based Chua’s circuit](image)
hysteresis loops of the current and voltage of the circuit under the conditions of an input sine wave voltage of the 1-V amplitude and 1-kHz frequency. The current–voltage curves show variations in the pinched hysteresis loop by changing values of the control resistor.

Further, we measured the time series of the Chua’s circuit with a control resistor $R_c$ using an analog oscilloscope. Fig. 6 shows the transient responses of $V_{c1}$ and $V_{c2}$. In Fig. 6(a)–(d), the measured transient waveforms of $V_{c1}$ (top waveform) and $V_{c2}$ (bottom waveform) demonstrate that the circuit generated chaotic signals or periodic pulses under specific conditions of the control resistor. In case of $R_c = 6 \text{k}\Omega$ and $15 \text{k}\Omega$, as shown in both of Fig. 6(a) and (c), respectively, the system possesses a chaotic state showing irregular time series. If the $R_c$ is changed to $8.5 \text{k}\Omega$ and $18 \text{k}\Omega$, as shown in Fig. 6(b) and (d), respectively, the state variables $V_{c1}$ and $V_{c2}$ of the system show specific periodic time waveforms.

Second, we examined the frequency spectra of the time waveforms under the same conditions as those used to obtain the time waveform in Fig. 6. Fig. 7 shows the frequency spectra of $V_{c1}$, which is one of the state variables of the circuit in Fig. 6. With a control resistor of $R_c = 6 \text{k}\Omega$ and $15 \text{k}\Omega$ in Fig. 7(a) and (c) which
correspond on Fig. 6(a) and (c), respectively, the peaks become wider and more spread out in the range of 0–20 kHz, respectively. The spectra are almost continuous, which clearly shows one of the chaotic characteristics. Moreover, in the case of Fig. 7(b) and (d), which correspond to Fig. 6(b) and (d), respectively, there are several spikes showing the periodic state.

![Fig. 7. Measured frequency spectra of the state variable $V_{c1}$ according to the control $R_c$.](image)

Also, we measured the attractors of the state variables between $V_{c1}$ and $V_{c2}$. Fig. 8(a)–(d) present attractor between $V_{c1}$ and $V_{c2}$ under the same conditions as used to create the simulated results in Fig. 2. These figures were measured in the Lissajous mode with an analog oscilloscope. In Fig. 8(a) and (c) present the measured attractors at $R_c = 6$ kΩ and $R_c = 15$ kΩ, respectively. Two figures show irregular traces, which is characteristic of a chaotic signal. Fig. 8(b) and (d) show

![Fig. 8. Measured attractor of the Chua’s circuit between $V_{c1}$ (y-axis) and $V_{c2}$ (x-axis) according to the control resistor.](image)
that for control resistances \( R_c \) of 8.5 k\( \Omega \) and 18 k\( \Omega \), respectively, periodic waveforms have particular periodic shapes such as double scroll. It is clearly evident that these measurement results and the simulated results in Fig. 2 are in good agreement.

Finally, we measured the bifurcation plot of the proposed circuit for various to the control resistor \( R_c \). The measured plot shows a behavior of the circuit under a wide range of the control resistor with non-chaos and chaos. The bifurcation of Fig. 9 is closed to the simulated diagram of Fig. 3.

5 Conclusion

In summary, we explored the chaotic behaviors of a variable memristor-based Chua’s system. The circuit has a mimicked memristor with a control resistor that makes the dynamics of the Chua’s circuit control. In the Chua’s system, SPICE simulations for attractor and bifurcation diagrams were performed. We confirmed that the Chua’s circuit can produce and control chaotic signals or periodic signals in controlled conditions. Further, the proposed circuit was implemented in an electronic circuit and the data was experimentally measured. From the simulated data and measured results, including the time series waveforms, frequency spectra, and attractor, the proposed circuit exhibited various chaotic dynamics based on the control resistor of the memristor.

Measurement results of the implemented circuit showed good agreement with the simulated results. We think that the proposed Chua’s chaotic circuit can be applied in various fields such as chaotic neural networks and chaos communication.

Acknowledgments

This work was supported by the Human Resource Training Program for Regional Innovation and Creativity through the Ministry of Education and National Research Foundation of Korea (NRF-2014H1C1A1066686).