Anomalous Lorentz transformation and side jump of a massive fermion

Feng Li$^{1,*}$ and Shuai Y.F. Liu$^{2,3,†}$

$^1$Lanzhou University, Lanzhou, Gansu, 730000, China
$^2$Quark Matter Research Center, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, Gansu, 730000, China
$^3$University of Chinese Academy of Sciences, Beijing, 100049, China

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The side jump in the anomalous Lorentz transformation, arising from the spin-orbit interactions, plays important roles in various intriguing physics, such as chiral vortical effects and spin polarization. In this work, the side jump of the spin-half massive particles, which has rarely been discussed, is visualized and evaluated for the first time. A compact analytical expression describing such side jumps is derived, and found approaching the one describing the chiral fermions in the massless limit. It is further demonstrated that the covariance of the total angular momentum, which would be broken by a normal Lorentz transformation, is restored after the obtained side jumps are taken into account.

The side-jumps, i.e., the anomalous displacements arising from the spin-orbit interactions, are first introduced for dealing with the scattering processes in the anomalous Hall effects [1–3]. Given the relativistic nature of the spin-orbit interactions, it is demonstrated in Ref [4–8] that the side jumps in the scattering processes are related to the anomalous Lorentz transformation, where the space-time coordinates of the chiral particles transform as $x'^\mu = \Lambda^\mu_\nu x^\nu + \Delta^\mu$, with the spin dependent displacement $\Delta^\mu$ being essential for preserving the covariance of the total angular momentum [4, 7]. As a part of the orbital quantity, i.e., the particle position $x$, $\Delta^\mu$ connects the orbital angular momentum with the spin, and is therefore the bridge via which the orbital and the spin angular momenta communicate under a Lorentz transformation. Consequently, the side jumps play a key role in the microscopic explanation of the chiral vortical effect [9–12], i.e., the macroscopic polarization occurring in a vortical fluid due to the spin-orbit interaction. Such theory insights have recently been implemented, for the first time, in the transport model [13], which respects the angular momentum conservation law, for simulating the heavy-ion collisions. As expected, the chiral vortical effect is dynamically generated in the simulation. It also provides a microscopic and promising perspective for understanding the spin puzzle in the heavy-ion collisions, i.e., the mismatch between the thermal model prediction [14] and the experimental measurements [15, 16] of the local $\Lambda$ polarization.

Although the side jumps of the massless particles have been extensively studied [4–8], those of the massive fermions have rarely been investigated, while the latter is essential for a more realistic simulation where the massive partons are involved, and could be crucial for a more rigorous description on the local polarization of the $\Lambda$ baryon which possesses a massive strange quark. In this letter, we derive the side jumps occurring in the anomalous Lorentz transformation of the massive fermions, for the first time. Despite the complication arising from a new dynamical degree of freedom, i.e., the spin orientation, which would be enslaved to the momentum direction in the massless case, the final expression of the obtained $\Delta^\mu$ is incredibly compact. Our derivation starts from an intuitive visualization, which explicitly demonstrate the existence and the properties of the side jumps. Guided by the intuition from the visualization, we calculate the side jump accordingly by evaluating the expectation value of the coordinates of the fermionic wave-package in the boosted frame. It will then be shown that the obtained side jumps coincide with those in Ref [4, 7] in the massless limit, and the covariance, and hence the conservation law, of the total angular momentum is preserved under an arbitrary boost once the side jumps are taken into consideration.

To visualize this side-jump effect, we plot the probability density, or the charge density, defined as $\psi^\dagger \psi$, of a spinor wave-package polarized in the positive $z$-direction in both the particle rest frame and the frame boosted in the positive $x$-direction with a rapidity equal to 4 in the left and right panels of Fig. 1, respectively. The spinor

![FIG. 1. Probability density of a massive spinor polarized in the positive z-direction in the particle rest frame (left) and the frame boosted in the positive x-direction (right)](image-url)
wave-package is conventionally written as [17]
\[ \psi(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\varepsilon_k}} a(k) u(k) e^{-i(\xi_k - k \cdot x)} \] (1)
where
\[ u(k) = \sqrt{m} S(-\eta_k) \left( \xi \right) \] (2)
is the solution to the Dirac equation in the momentum representation with \( \eta_k \) being related with \( k \) by \( k = m \sinh |\eta_k| \) and
\[ S(\eta) = [\Lambda_{\eta/2}]^\mu(1,0)\nu^\gamma \gamma_{\mu} \] (3)
being the transformation matrix which is not only used in Eq. (2), but also employed for transforming a spinor wave-package as
\[ \psi'(x) = S(\eta) \psi(\Lambda_{\eta}^{-1} x) \] (4)
under a boost with rapidity \( \eta \), where
\[ \Lambda_{\eta} = \begin{pmatrix} \cosh \eta & -\sinh \eta \hat{n} \\ -\sinh \eta \hat{n} & \cosh \eta - 1 \end{pmatrix} I + (\cosh \eta - 1) \hat{n} \hat{n} \]. (5)
The polarization direction of the wave-package in its rest frame is determined by \( \xi \), which satisfies \( \xi^T \sigma \xi = \hat{n} \) with \( \hat{n} \) being the spin direction of the fermionic wave-package in its rest frame, and \( \sigma \) being the Pauli matrices. \( \hat{n} \) is chosen along the z-axis when we plot Fig. 1. \( a(k) = a_0(k) e^{i\phi(k)} \) is the c-number amplitude for given momentum \( k \) which satisfies the normalization condition that
\[ \int d^3x \psi^\dagger \psi = \int \frac{d^3k}{(2\pi)^3} |a(k)|^2 = 1, \] (6)
and is chosen Gaussian, i.e., \( N e^{-|k|^2/2\sigma^2} \), for plotting Fig. 1, where the mass and the smearing width \( \sigma \) are taken as 0.5 and 1 fm\(^{-1} \), respectively. All the above and the following calculations are under the Weyl representation, while the physics should be independent of the choice of the representation. As shown in Fig. 1, the wave-package in the boosted frame is shifted in the positive y-direction, which is perpendicular to both the spin direction in the particle rest frame and the direction of the boost as well, indicating the existence of a side jump \( \Delta \propto \hat{n} \times \eta \). The side jump of a massive fermion is thus confirmed and visualized, and will be evaluated as follows.
Under the assumption that \( \lim_{|k'| \to \infty} a(k') = 0 \), the expectation value of the spatial coordinate of a boosted spinor wave-package, evaluated using the \( \psi'(x) \) defined in Eq. (4), is
\[ \bar{x}_{\eta}(t) = \int d^3x \psi^\dagger(x) \psi'(x) \]
\[ = i \int \frac{d^3k'}{(2\pi)^3} \bar{\psi}^\dagger(k', t) \partial_{k'} \psi'(k', t) \] (7)
where
\[ \bar{\psi}'(k', t) = \int d^3x \psi'(x) e^{-i k' \cdot x} \]
\[ = S(\eta) \frac{\varepsilon_k}{\varepsilon_{k'}} \frac{a(k)}{\sqrt{2\varepsilon_k}} u(k) e^{-i k' \cdot t} \] (8)
with \( k \) being related with \( k' \) by a Lorentz transformation, namely, \( k' = \Lambda_{\eta} k \). Under the replacement that \( k' \to k \), the integration measure \( d^3k'/2\varepsilon_{k'} = d^3k/2\varepsilon_k \) keeps invariant, and Eq. (7) can thus be written, after a lengthy calculation, as
\[ \bar{x}_{\eta}(t) = \bar{v}_\eta t + i \int \frac{d^3k}{(2\pi)^3} \left[ \frac{a^*(k)}{\sqrt{2\varepsilon_k}} u^\dagger(k) S^2(\eta) \partial_{k'} \left( \frac{\varepsilon_k a(k)}{\varepsilon_{k'}} \right) \right] 
\[ = \bar{x}_0 + \bar{v}_\eta t + \delta \bar{x}_{\eta}, \] (9)
where
\[ \bar{x}_0 \equiv - \int \frac{d^3k}{(2\pi)^3} |a(k)|^2 \partial_{k'} \phi(k) \] (10)
is the initial position of the wave-package which will be taken as zero, corresponding to a constant \( \phi(k) \), for simplicity in the following discussion,
\[ \bar{v}_\eta \equiv \int \frac{d^3k}{(2\pi)^3} |a(k)|^2 \frac{k'}{\varepsilon_{k'}} \] (11)
is the velocity of the charge center of the wave-package, and
\[ \delta \bar{x}_{\eta} \equiv \int \frac{d^3k}{(2\pi)^3} |a(k)|^2 \left[ \frac{\sinh \eta (\hat{n} \times \eta)}{\varepsilon_k + m (\hat{n} \times k) + \frac{\cosh \eta - 1}{\varepsilon_k + m} (\hat{n} \times k) \cdot \eta} \right] \] (12)
is a time-independent anomalous displacement which is related to the spin direction, and therefore unique to the fermionic wave-package. It needs to be clarified that \( \delta \bar{x}_{\eta} \) is not the side jump we are looking for, since \( \delta \bar{x}_{\eta=0} \) is in general nonzero, while the side jump vanishes by definition at \( \eta = 0 \), corresponding to the case where no boost is carried out at all. The true side jump, which turns out to be a superposition of both the \( \delta \bar{x}_{\eta} \) and \( \delta \bar{x}_{\eta=0} \), will be derived in the following section.
Due to the appearance of \( \delta \bar{x}_{\eta} \), the world-line in the boosted frame \( (t, \bar{x}_{\eta}(t)) \) does not intersect the one in the original frame \( (t, \bar{x}_{\eta=0}(t)) \), indicating that any point on either the world-line can be transformed to the other not by a homogeneous Lorentz transformation, but rather by a Poincare transformation, i.e.,
\[ \left( \begin{array}{c} t' \\ \bar{x}_{\eta}(t') \end{array} \right) = \Lambda_{\eta}^* \left( \begin{array}{c} t \\ \bar{x}_{\eta=0}(t) \end{array} \right) + \Delta, \] (13)
where \( \Lambda_{\eta}^* \) stands for the Lorentz transformation bringing the 4-velocity \( (\bar{\gamma}_{\eta=0}, \bar{\gamma}_{\eta=0} \bar{v}_{\eta=0}) \) to \( (\bar{\gamma}_{\eta}, \bar{\gamma}_{\eta} \bar{v}_{\eta}) \) with \( \bar{\gamma}_{\eta} \)
vanishes as well, which makes the additional displacement $\Delta$ is the so-called side jump. Under the gauge employed in Ref \[4, 7\], the side jump is considered purely spatial, and therefore expressed, according to Eq. (13), as

$$\Delta^i = (\bar{\psi}^i_\eta \Lambda^*_{\eta j} - \Lambda^*_{\eta j} \bar{\psi}^i_\eta) \delta x^j_{\eta=0} + \delta x^j_{\eta=0}. \quad (14)$$

In the following sections, we shall evaluate Eq. (14) in two simple cases, where the amplitude $|a(k)|^2$ either depends only on $|k|$, or is chosen as the $\delta$-function centered at $p$.

In the case where $|a(k)|^2 = |\alpha(|k|)|^2$, $\bar{v}_{\eta=0}$ vanishes, indicating that the wave-package is stationary in the original frame. It is the case where Fig. 1 is plotted. $\delta x_{\eta=0}$ vanishes as well, which makes $\Delta$ equal to $\delta x_p$. After neglecting the integral terms vanishing due to the symmetry in $k$-space in Eq. (12), we obtain

$$\Delta = \int \frac{d^3k}{(2\pi)^3} |a(k)|^2 \frac{m \sinh \eta - k \cdot \bar{n} \cosh \eta}{2(m + \varepsilon_k)\varepsilon_k} (\bar{n} \times \eta), \quad (15)$$

perpendicular to both the polarization direction in the particle rest frame and the boost direction, which agrees with the observation from the visualization shown in the previous section. In Fig. 2, we plot the side jumps obtained using a Gaussian c-number amplitude $a(|k|)$ with different masses and smearing width $\sigma$s. Readers can compare the shift of the charge center of the wave packages shown in Fig. 1 with the side jump read off from the long-dashed line in the upper-left panel of Fig. 2, and find that they are consistent. We see, from Fig. 2, three features of the side jumps. First, they saturate for large $\eta$, which is true for an arbitrary choice of $a(|k|)$, since, as $\sinh \eta$ being close to $\cosh \eta$, the right hand side of Eq. (15) approaches

$$(\bar{n} \times \eta) \int \frac{d^3k}{(2\pi)^3} |a(k)|^2 \frac{m + \varepsilon_k - k \cdot \eta}{2(m + \varepsilon_k)(\varepsilon_k - k \cdot \eta)},$$

which is independent of $\eta$. Second, the side jump decreases with mass, and under this particular Gaussian choice of $a(|k|)$, it is almost proportional to $m^{-1}$ if the ratio $\sigma/m$ is fixed. However, the side jump does not necessarily diverge when $m$ approaches zero. As shown in the lower-right panel of Fig. 2, the side jump is finite with a finite smearing width $\sigma$ even though the mass parameter is set equal to zero, which is reasonable, since the total energy of the Gaussian wave-package in the particle rest frame is finite as long as $\sigma > 0$, and increases with the smearing widths $\sigma$, indicating that the smearing width plays a similar role to the mass parameter. Consequently, the side jump decreases with the smearing width, as well, which is the third feature of the side jumps.

In the case where $|a(k)|^2 = (2\pi)^3 \delta^3(k - p)$, $\bar{v}_{\eta=0} = \bar{p}/\varepsilon_p$, and $v_{\eta} = p'/\varepsilon_{p'}$, indicating that the charge center of the wave-package moves with the speeds $p/\varepsilon_p$ and $p'/\varepsilon_{p'}$ in the original and the boosted frame, respectively, which implies that $\Lambda^x_{\eta}$ and $\Lambda^x_{\eta}$ are identical. We thus obtain from Eq. (14) that

$$\Delta = \sinh \eta \left[ \frac{m}{\varepsilon_p} (\bar{n} + (1 - \frac{m}{\varepsilon_p})(\bar{n} \cdot \bar{p})\bar{p}) \right] \times \eta$$

$$= \frac{\sinh \eta}{\varepsilon_{p'}} W \times \eta \quad (16)$$

where

$$W^\mu = \frac{1}{2m} \varepsilon^{\mu\alpha\beta\gamma} \tilde{u}(p) \Sigma_{\alpha\beta} u(p) \gamma_{\mu}$$

$$= \frac{1}{2} (\bar{n} \cdot p, m\bar{n} + (\varepsilon_p - m)(\bar{n} \cdot \bar{p})\bar{p}) \quad (17)$$

is the Pauli-Lubanski vector \[18-20\], which is covariant and equal to $(0,m\bar{n}/2)$ in the particle rest frame, with $\Sigma_{\alpha\beta}$ being the spin tensor operator, defined as $\frac{i}{2} [\gamma^\alpha, \gamma^\beta]$ for the Dirac spinor. In the massless limit, $W = \lambda p$ \[20\], where $\lambda$ is the helicity of the particle, Eq. (16) thus gives the side jump coinciding with the one written in Ref \[7\]. In the following section, we shall re-derive Eq. (16) based on the covariance of the total angular momentum, and express it in the same covariant form as the one given by Ref \[7\]. The derivation can be regarded in return as a proof that the covariance of the total angular momentum tensor is preserved in the anomalous Lorentz transformation with the side jump expressed in Eq. (16).

The total angular momentum $J^{\mu\nu}$ is the conserved charge corresponding to the $SO(1,3)$ symmetry, and
therefore equal to
\[ \int d^3x J^{\mu\nu}, \] (18)

where \( J \) is the corresponding Noether current, which, obtained using the conventional Lagrangian for the Dirac spinor \([17]\), is
\[ J^{\alpha\beta\gamma\delta} = \bar{\psi} i \gamma^\alpha (x^\mu \partial^\nu - x^\nu \partial^\mu - i \Sigma^{\mu\nu}) \psi. \] (19)

The integration in Eq. (18) is again evaluated under the assumption that \( |a(k)|^2 \) is the delta function centered at \( p \), and \( J^{\mu\nu} \) is thus expressed as a summation of the orbital and the spin components, i.e.,
\[ J^{\mu\nu} = L^{\mu\nu} + S^{\mu\nu} \] (20)

where
\[ L^{\mu\nu} = \tilde{x}^\mu p^\nu - \tilde{x}^\nu p^\mu, \quad S^{\mu\nu} = \frac{\varepsilon^{\mu\nu\alpha\beta} W_\alpha \bar{u}_\beta}{p \cdot \tilde{u}} \] (21)

with \( \tilde{u} \) being \((1, 0)\) in all the frames and therefore not a covariant vector. It needs to be clarified that both \( L^{\mu\nu} \) and \( S^{\mu\nu} \) stand for only the real part of the expectation values of \( i(x^\mu \partial^\nu - x^\nu \partial^\mu) \) and \( \Sigma^{\mu\nu} \), respectively, while the imaginary part of the latter two expectation values are of equal magnitudes but opposite signs, and therefore cancel with each other in Eq. (19). Also, the decomposition of \( J^{\mu\nu} \) is not unique, it can also be carried out with a pseudo-gauge transformation \([21]\).

The Lorentz covariance of \( S^{\mu\nu} \) is explicitly broken by \( \tilde{u} \) in Eq. (21), while the total angular momentum \( J^{\mu\nu} \) should be covariant. We therefore construct an anomalous Lorentz transformation for \( \tilde{x} \), under which \( \tilde{x} \) transforms as \( \tilde{x} \rightarrow \tilde{x}' + \Delta \), so that the covariance of the total angular momentum \( J^{\mu\nu} \) is preserved. In the following discussion, any primed 4-vector, such as \( \tilde{a}' \), stands for a new 4-vector equal to the original one multiplied by the Lorentz transformation matrix, such as \( \Lambda a \), regardless whether the 4-vector is covariant or not, so \( \tilde{x}' \) stands for \( \Lambda \tilde{x} \). Under a boost, the total angular momentum \( J^{\mu\nu} \) transforms as
\[ J^{\mu\nu} \rightarrow (\tilde{x}^\mu + \Delta^\mu) p^\nu - (\tilde{x}'^\nu + \Delta^\nu) p'^\mu + \frac{\varepsilon^{\mu\nu\alpha\beta} W_\alpha \bar{u}'_\beta}{p' \cdot \tilde{u}}. \] (22)

Meanwhile, the covariance of \( J^{\mu\nu} \) could be explicit if we pretend both the \( \tilde{x} \) and \( \tilde{u} \) to be covariant and to transform as \( \tilde{x} \rightarrow \tilde{x}' \) and \( \tilde{u} \rightarrow \tilde{u}' \equiv \Lambda \tilde{u} = (\cosh \eta, -\sinh \eta \tilde{\eta}), \) respectively. Under such a hypothesis, the total angular momentum \( J^{\mu\nu} \) should transform as
\[ J^{\mu\nu} \rightarrow \tilde{x}^\mu p'^\nu - \tilde{x}'^\nu p^\mu + \frac{\varepsilon^{\mu\nu\alpha\beta} W_\alpha' u'_\beta}{p' \cdot \tilde{u}'}. \] (23)

We learn, by comparing Eq. (22) with Eq. (23), that the total momentum \( J^{\mu\nu} \) is covariant as long as the anomalous displacement \( \Delta \) satisfies
\[ \Delta^\mu p'^\nu - \Delta^\nu p'^\mu = \frac{\varepsilon^{\mu\nu\alpha\beta} W_\alpha' u'_\beta}{p' \cdot \tilde{u}'} - \frac{\varepsilon^{\mu\nu\alpha\beta} W_\alpha \bar{u}_\beta}{p \cdot \tilde{u}}. \] (24)

We therefore obtain \( \Delta \), which is again assumed to be purely spatial, by multiplying \( \tilde{u}' \) on both the sides of Eq. (24), and it is
\[ \Delta^\mu = \frac{\varepsilon^{\mu\nu\alpha\beta} W'_{\alpha \beta}}{(p' \cdot u')(p' \cdot \tilde{u})}, \] (25)

which, after the substitution that \( W' \rightarrow \lambda p' \) corresponding to the massless limit, is exactly the same side jump as the one provided in Ref. [7]. The obtained side jump also contributes to the particle number current in the same way demonstrated in Ref. [7], with the \( \Delta \) in the literature being replaced with Eq. (25), and leads to anomalous currents. Given the identity that \( W \times \tilde{\eta} = W \times \eta \), it is easy to show that the side jumps obtained in both the approaches, i.e., Eq. (16) and Eq. (25), are identical. Therefore, we have proved that the covariance of the total angular momentum is preserved, after the anomalous Lorentz transformation, with the side jumps given either by Eq. (16) or by Eq. (25), is employed, and the angular momentum conservation law, which would be in general broken with a normal Lorentz transformation, is thus restored. Furthermore, since the spin representation is not specified in the definition of the Pauli-Lubanski vector, both Eq. (16) and Eq. (25) might hold not only for the spin-half fermions but also for any spinning particles, which is worthwhile to check in future.

In summary, we visualize and derive the side jumps embedded in the anomalous Lorentz transformations, arising from the spin-orbit interactions, for the massive fermions. The side jumps are found to be perpendicular to both the boosting direction and the Pauli-Lubanski vector. They decrease with the particle mass, increase with the boosting rapidity, and saturate at the large rapidity. We show that the obtained side jumps coincide with those provided by Ref [7] in the massless limit, and the covariance, and hence the conservation law, of the total angular momentum is preserved under an arbitrary boost after the side jumps are taken into consideration. The side jumps given either by Eq. (16), or equivalently by Eq. (25), can be implemented in the transport models where spins are treated as the dynamical degrees of freedom, like positions and momenta. Such transport models can be employed to investigate various intriguing phenomena, including the spin polarization in the heavy-ion collisions, which is remained as a part of our future plan. Meanwhile, it would also be interesting to investigate the connection between the side-jump mechanism proposed in this letter with the other theoretical approaches concerning the related topic \([21–26]\).
[2] N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald, and N. P. Ong, Rev. Mod. Phys. 82, 1539 (2010).
[3] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Rev. Mod. Phys. 87, 1213 (2015).
[4] J.-Y. Chen, D. T. Son, M. A. Stephanov, H.-U. Yee, and Y. Yin, Phys. Rev. Lett. 113, 182302 (2014).
[5] C. Duval, M. Elbistan, P. A. Horvthy, and P. M. Zhang, Phys. Lett. B 742, 322 (2015).
[6] M. Stone, V. Dwivedi, and T. Zhou, Phys. Rev. Lett. 114, 210402 (2015).
[7] J.-Y. Chen, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. 115, 021601 (2015).
[8] Y. Hidaka, S. Pu, and D.-L. Yang, Phys. Rev. D 95, 091901 (2017).
[9] A. Vilenkin, Phys. Rev. D 20, 1807 (1979).
[10] D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009).
[11] D. E. Kharzeev and D. T. Son, Phys. Rev. Lett. 106, 062301 (2011).
[12] D. E. Kharzeev, J. Liao, S. A. Voloshin, and G. Wang, Prog. Part. Nucl. Phys. 88, 1 (2016).
[13] S. Y. F. Liu, Y. Sun, and C. M. Ko, (2019), arXiv:1910.06774 [nucl-th].
[14] F. Becattini and I. Karpenko, Phys. Rev. Lett. 120, 012302 (2018).
[15] T. Niída (STAR), Nucl. Phys. A 982, 511 (2019).
[16] J. Adam et al. (STAR), Phys. Rev. Lett. 123, 132301 (2019).
[17] M. E. Peskin and D. V. Schroeder, An Introduction to quantum field theory (Addison-Wesley, Reading, USA, 1995).
[18] J. K. Lubański, Physica 9, 310 (1942).
[19] J. K. Lubanski, Physica 9, 325 (1942).
[20] S. Weinberg, The Quantum theory of fields. Vol. 1: Foundations (Cambridge University Press, 2005).
[21] F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019).
[22] W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, Phys. Rev. C 97, 041901 (2018).
[23] N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, Phys. Rev. D 100, 056018 (2019).
[24] J.-H. Gao and Z.-T. Liang, Phys. Rev. D 100, 056021 (2019).
[25] D.-L. Yang, K. Hattori, and Y. Hidaka, (2020), arXiv:2002.02612 [hep-ph].
[26] S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. Ryblewski, (2020), arXiv:2002.03937 [hep-ph].