Massive fields dynamics in open bosonic string theory

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Abstract
We consider the theory of open bosonic string in massive background fields. The general structure of renormalization is investigated. A general covariant action for a string in background fields of the first massive level is suggested and its symmetries are described. Equations of motion for the background fields are obtained by demanding that the renormalized operator of the energy-momentum tensor trace vanishes.
1. Introduction

The theory of a string in massless background fields provides a possibility to consider string interactions in the low energy approximation [1-4]. Such a theory is remarkable due to its close connection with the two-dimensional quantum field theory and possible generalizations (see review [5]). Namely, this approach has led to the equations of string gravity which are now widely used for finding new cosmological solutions (see review [6]).

The crucial point of the string theory in massless background fields is the fundamental concept of Weyl invariance. In quantum theory it means that the renormalized operator of the energy-momentum tensor trace must vanish resulting in equations of motion for background fields. The general analysis of the renormalized operator for a bosonic string interacting with background metric, antisymmetric tensor and dilaton was performed in refs. [7, 8] (see also review [5]).

A natural development of the approach leads to the consideration of string in background fields which are connected with massive modes in the string spectrum. Unfortunately, construction of a consistent quantum theory in this case appears to be quite difficult. A string interacting with any finite number of massive background fields is non-renormalizable theory and requires infinite number of counterterms. So we have to deal with the theory containing infinite number of terms in classical action which describe interaction with background fields of all the massive modes. The only massive field that does not require infinite number of counterterms is the field of tachyon but in this case non-perturbative effects play a crucial role [9-13] (see also review [14]).

Recently several attempts were undertaken to describe string in massive background fields [12-23]. All these investigations (excluding work on the tachyon problem) concerned mainly only linear equations of motion for background fields. The linear approximation is of great importance since the equations for background fields in this approximation should correspond to the known equations defining the string spectrum. It was turned out that even at the linear level the whole clarity is absent and the very possibility of going beyond the linear approximation presents difficulties.

In ref. [24] we proposed an approach to the string theory in massive background fields that represents a direct generalization of the $\sigma$-model approach to the string theory in massless background fields. For a closed string interacting with background fields of all the massive modes we showed that the renormalization has a special structure. Namely, the renormalization of background fields of any massive level requires consideration only of background fields of this level and of all the lower ones, but is not affected by the infinite number of background fields belonging to higher levels. In principle, our approach allows to go beyond the linear approximation.

In the same paper [24] we examined in detail a closed bosonic string in background fields of the first massive level and received linear equations of...
motion, though the problem of agreement with the string spectrum was not solved completely. It is notable that the lagrangian of a closed bosonic string in massive background fields constructed in ref. [24] appeared then to be useful for the formulation of a generalized model of two-dimensional dilaton gravity [25].

This paper is devoted to further development of our approach [24] and to its application to the theory of open string in massive background fields. The theory of an open string is a field theory in space-time with boundary. Various aspects of quantum calculations in the theory of open strings were discussed in ref. [29]. As was noted in pioneer works [2], interaction of an open string with background fields corresponding to the open string spectrum is completely concentrated at the boundary of the world sheet. Detailed investigation of open string in massless background fields was conducted in refs. [26-28]. Questions of quantum field theory on a manifold with boundary were studied in recent works [30, 31].

The paper is organized as follows. Section 2 deals with the investigation of renormalization in the theory of a string interacting with arbitrary background fields both on the world sheet and its boundary. It is showed that the renormalization has the same structure as in ref. [24]. In section 3 we consider an open string in background fields of the first massive level, introduce the most general action, discuss its symmetries and conduct the renormalization of background fields. The renormalization of composite operators defining the energy-momentum tensor trace is carried out and linear equations of motion for background fields are derived in Section 4. Complete agreement with the equations specifying the string spectrum is established.

2 General analysis of renormalization

Our aim consists in construction of a \( \sigma \)-model type action describing interaction of an open string with massive background fields and deriving from the quantum Weyl invariance condition effective equations of motion for these fields. So we have to build up the renormalized operator of the energy-momentum tensor trace and to demand that it vanishes.

As shown in ref. [24], to make the theory renormalizable in the case of a closed string one has to consider an action comprising infinite set of terms describing interaction with all possible background fields. After rescaling string coordinates \( x^\mu \to \sqrt{\alpha'} x^\mu \) the total action takes the form

\[
S = \int_{M^2} d^2 z \sqrt{g} \sum_{n=0}^{\infty} (\alpha')^n \sum_{i_n=1}^{N_n} \mathcal{O}^{(n)}_{i_n}(z, \partial x(z)) B^{(n)}_{i_n}(x(z))
\]

(1)

\( B^{(n)}_{i_n} \) are background fields corresponding to the \( n \)-th level in the closed string spectrum, \( \mathcal{O}^{(n)}_{i_n} \) are constructed from \( g^{ab}, e^{ab}, \partial x, D\partial x, \ldots, R^{(2)}, \partial R^{(2)}, D\partial R^{(2)} \),
Here the derivative $D_a$ is covariant under reparametrizations both on the world sheet and in the D-dimensional space-time:

$$D_a \partial_b x^\mu = \partial_a \partial_b x^\mu - \Gamma^\mu_{ab}(g) \partial_c x^\mu + \Gamma^\mu_{\lambda \rho} (G) \partial_a x^\lambda \partial_b x^\rho.$$  \hspace{1cm} (2)

For each $n$ dimension of all $O^{(n)}_{i_n}$ in two-dimensional derivatives equals $2n + 2$. $N_n$ is the total number of all independent $O^{(n)}_{i_n}$ belonging to the $n$-th level. The integral in (3) is taken over the whole world sheet, and we use a euclidian metrics.

In case of an open string there appears a possibility to introduce interaction at the boundary of the world sheet $\partial M$ and the total action should be of the form

$$S = \int_{M^2} d^2 z \sqrt{g} \sum_{n=0}^{\infty} (\alpha')^n \sum_{i_n=1}^{N_n} O^{(n)}_{i_n}(z, \partial x(z)) B^{(n)}_{i_n}(x(z)) +$$

$$+ \int_{\partial M} dt e^{\sum_{k=0}^{\infty} (\alpha')^{k/2}} \sum_{i_k=1}^{N_k} O^{(k)}_{i_k}(t, x(t)) B^{(k)}_{i_k}(x(t))$$  \hspace{1cm} (3)

Here $B^{(k)}_{i_k}$ are background fields belonging to the $n$-th massive level of the open string, $O^{(k)}_{i_k}$ are constructed from $\dot{x}^\mu = \frac{dx^\mu}{dt}$, $\ddot{x}^\mu$, ..., the external curvature of the boundary $K(t) = e^{-2n_a} \left( \frac{d^2 z^a}{dt^2} + \Gamma^a_{bc} \frac{dz^b}{dt} \frac{dz^c}{dt} \right)$ and its derivatives. $t$ is a parameter along the boundary, $n_a$ is a unit vector normal to it and $e^2(t) = g_{ab}(z(t)) \frac{dz^a}{dt} \frac{dz^b}{dt}$ is one-dimensional metrics. $O^{(k)}_{i_k}$ may contain any powers of derivatives with respect to $t$, that is why the second integral in (3) is expanded in powers of $\alpha'^{1/2}$.

To construct the renormalized operator of the energy-momentum tensor trace one has to renormalize both the background fields $B_{i_k}$ and the composite operators $O_{i_k}$. Renormalization of fields is constructed by demanding that divergences of the quantum effective action vanish. In one-loop approximation it appears as

$$\Gamma^{(1)} = S + \frac{1}{2} Tr ln ||H_{\mu \nu}||,$$  \hspace{1cm} (4)

where $S$ is the classical action and $H_{\mu \nu}$ represents its second functional derivative:

$$H_{\mu \nu} = \frac{\delta^2 S}{\delta x^\mu \delta x^\nu}.$$  \hspace{1cm} (5)

In our case $H_{\mu \nu}$ is an operator of the following form:

$$H \sim -D^2 + \sum_{k=0}^{\infty} P^{a_1...a_2k} D_{a_1} ... D_{a_{2k}} +$$
\[ + \sum_{k=0}^{\infty} V_k \left( \frac{dz^b}{edt} D_b \right)^k \]

where each term of expansions depends only on fields of the given massive level.

\[ D^2 = g^{ab} D_a D_b \]

Coefficients \( P^{a_1 \ldots a_{2k}} \) and \( V_k \) are functions of background fields and can be presented as series in powers of \( \alpha' \):

\[ P = \sum_{n=0}^{\infty} (\alpha')^n P^{(n)}, \quad V = \sum_{m=0}^{\infty} (\alpha')^{m/2} V^{(m)}, \]

Divergences in (4) appear from the expression

\[ \text{Tr} \ln \mathcal{H} \sim \text{Tr} \ln (-D^2) \]

\[ - \sum_{l=1}^{\infty} \frac{1}{l} \text{Tr} \left( \sum_{k=0}^{\infty} P^{a_1 \ldots a_{2k}} D_{a_1} \ldots D_{a_{2k}} \frac{1}{D^2} \right) + \]

\[ + \sum_{k=0}^{\infty} V_k \left( \frac{dz^b}{edt} D_b \right)^k \frac{1}{D^2} \]

Being local constructions, the divergences must be expanded in the same set of \( \mathcal{O}_{i_n} \) as the action (3)

\[ \sum_{n=0}^{\infty} (\alpha')^n \int \frac{d^2z}{M^2} \prod_{i_n=1}^{N_n} \mathcal{O}_{i_n}^{(n)} T_{i_n}^{(n)}(B) + \]

\[ + \sum_{k=0}^{\infty} \alpha'^{k/2} \int \frac{dte}{\partial M} \prod_{i_k=1}^{N^B_k} \mathcal{O}_{i_k}^{B(k)} T_{i_k}^{B(k)}(B), \]

where \( T_{i_n}^{(n)}(B), T_{i_k}^{B(k)}(B) \), are some dimensionless functions of background fields.

It is obvious from dimensional considerations that counterterms of some given power in \( \alpha' \) can depend only on background fields of the corresponding massive level and of all the lower ones. Therefore to renormalize background fields of the \( n \)-th massive level of the closed string it is sufficient to calculate divergences generated by background fields from the \( n \)-th and all the lower levels of the closed string spectrum and by fields from the \( 2n \)-th and all the lower levels of the open string spectrum. Similarly, renormalization of \( k \)-th level fields of the open string requires to consider the open string spectrum fields of the \( k \)-th and all the lower levels and the closed string spectrum fields of the \( \lfloor k/2 \rfloor \)-th and all the lower levels (\( \lfloor \cdot \rfloor \) means an integer part of a number).

For example, to renormalize the fields of the first massive open string level one is to study divergences generated by these fields and by the massless ones.
Moreover, if we are interested merely in linear approximation it is sufficient to restrict ourselves to the study of divergences generated by fields of the only given massive level. Contributions of all other levels contain products of several background fields and so are beyond the linear approximation.

3 Action, symmetries, renormalization

In this section we will consider an open string interacting with all the massive fields of the first level, describe its symmetries and construct one-loop renormalization of background fields. The action should be the sum of the free string action $S_0$ and the action $S_I$ describing interaction with the fields of the first massive level and containing all possible terms of the second order in derivatives:

$$S[x] = S_0[x] + S_I[x]$$

$$S_0[x] = \frac{1}{4\pi} \int_{M^2} d^2z \sqrt{g_{ab}} \partial_a x^\mu \partial_b x^\nu \delta_{\mu\nu}$$

$$S_I[x] = \frac{\alpha'^{1/2}}{2\pi} \int dte(t) \left[ A_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu + B_\mu(x) \ddot{x}^\mu + K^2 \varphi_1(x) + \dot{K} \varphi_2(x) + K \dot{x}^\mu \varphi_\mu(x) \right]$$

(10)

Massless fields do not contribute to renormalization of massive fields in the linear approximation and so are not considered here. A theory similar to (10) was investigated in [16], but it did not contain all possible background fields and calculations were not explicitly covariant.

Possibility to add an arbitrary total derivative to the lagrangian in (10) yields to the symmetry of the theory under the following transformations:

$$\left\{ \begin{array}{l} \delta A_{\mu\nu} = \partial_{(\mu} \Lambda_{\nu)} \\ \delta B_\mu = \Lambda_\mu \\ \delta \varphi_\mu = \partial_\mu \Lambda \\ \delta \varphi_2 = \Lambda \end{array} \right.$$  

(11)

where $\Lambda_\mu(x)$ and $\Lambda(x)$ are arbitrary functions playing the role of transformation parameters. As one can see, the field $B_\mu(x)$ and $\varphi_2(x)$ are Stuckelberg ones and the symmetry allows to choose them to be equal to zero. Thus the essential background fields are $A_{\mu\nu}(x), \varphi_\mu(x), \varphi_1(x)$.

To renormalize background fields it is necessary to calculate divergences of the action

$$\frac{1}{2} Tr \ln(S_{\alpha\beta} + \frac{\alpha'^{1/2}}{2\pi} V_{\alpha\beta}),$$

(12)

where we denote

$$S_{\alpha\beta} \equiv \frac{\delta^2 S_0[x]}{\delta x^\alpha \delta x^\beta}, \quad \frac{\alpha'^{1/2}}{2\pi} V_{\alpha\beta} \equiv \frac{\delta^2 S_I[x]}{\delta x^\alpha \delta x^\beta}$$

(13)
is expanded into series in powers of \( (\alpha')^{1/2} \):

\[
\frac{1}{2} Tr \ln (S_{0 \alpha \beta} + \frac{\alpha'^{1/2}}{2\pi} V_{\alpha \beta}) = \frac{1}{2} Tr \ln S_{0 \alpha \beta} + \frac{\alpha'^{1/2}}{2} Tr V_{\alpha \gamma} G^{\alpha \beta} + O(\alpha'),
\]

where Green function \( G^{\alpha \beta} \) of a free string is determined by the equation

\[
2\pi S_{0 \alpha \beta} G_{\beta \gamma} = \delta_{\gamma \alpha}.
\]

Divergences of the first term in (14) are cancelled by the corresponding contribution of the ghosts providing that \( D = 26 \). Terms \( O(\alpha') \) contribute to renormalization of background fields of the second and all the higher levels and so will be omitted.

Choose coordinates \((t, y)\) on the world sheet so that \( t \) be a parameter along the boundary \( \partial M \) and \( y \) equal the distance between the point \( z^a = (t, y) \) and the boundary along a geodesic line targent to the internal normal vector \( n_a \).

The metrics in terms of these coordinates is

\[
ds^2 = e^{2(t, y)} dt^2 + dy^2.
\]

Specifying the coordinate \( t \) so that \( e(t, y)|_{y=0} = 1 \) we get the following expansion

\[
e(t, y) = 1 - K(t) y - \frac{1}{4} R(t, 0) y^2 + O(y^3)
\]

Calculation of \( V_{\alpha \beta} \) in such coordinates gives

\[
V_{\alpha \beta}(t, y; t', y') = \delta_{\partial M}(z) \delta_{\partial M}(z') \sum_{k=0}^{2} V_{(k)\alpha \beta} \frac{d^k \delta(t-t')}{dt^k}
\]

where

\[
V_{(2)\alpha \beta} = B_{\alpha \beta} + B_{\beta \alpha} - 2A_{\alpha \beta}
\]
\[
V_{(1)\alpha \beta} = K \Phi_{[\alpha \beta]} + 2C_{\alpha(\beta \mu)} \dot{x}^\mu
\]
\[
V_{(0)\alpha \beta} = K^2 \varphi_{1, \alpha \beta} + K \Phi_{\alpha \beta} + K \Phi_{[\alpha \mu], \beta} \dot{x}^\mu + C_{\alpha(\mu \nu), \beta} \dot{x}^\mu \dot{x}^\nu + H_{(\mu \alpha), \beta} \dot{x}^\mu
\]

and

\[
H_{\alpha \beta} \equiv B_{\alpha \beta} + B_{\beta \alpha} - 2A_{\alpha \beta},
\]
\[
C_{\mu(\alpha \beta)} \equiv A_{\alpha \beta, \mu} - A_{\mu \alpha, \beta} - A_{\mu \beta, \alpha} + B_{\mu, \alpha \beta}
\]
\[
\Phi_{[\mu \alpha]} = \varphi_{\alpha, \mu} - \varphi_{\mu, \alpha}
\]
\[
\Phi_\mu = \varphi_{2, \mu} - \varphi_\mu.
\]
Here the function $\delta_{\partial M}(z)$ is defined as follows

$$\frac{\delta x^\mu(t)}{\delta x^\nu(z')} = \delta(t - t') \delta_{\partial M}(z') \delta^\mu_\nu. \tag{21}$$

Considering (18), the divergences of (14) are contained in the expression

$$\frac{\alpha_{1/2}^\alpha}{2} \sum_{k=0}^{2} \int dt V(\kappa \alpha \beta)(t) \frac{d^k G^\alpha(t, 0; t', 0)}{dt^k} \bigg|_{t' \to t} = -\mu^\epsilon \frac{\alpha^{1/2}}{2\pi} \int dt V(0) \alpha \beta(t, 0; t', 0) \bigg|_{t' \to t} \tag{22}$$

where we used that the divergences of Green function in coincident points in the framework of dimensional renormalization are [30, 31]:

$$G^{\mu \nu}(t, 0; t', 0) |_{t' \to t} = -\mu^\epsilon \frac{\alpha^{1/2}}{\pi \epsilon} \eta^{\mu \nu},$$

$$\frac{d}{dt} G^{\mu \nu}(t, 0; t', 0) |_{t' \to t} = \frac{d^2}{dt^2} G^{\mu \nu}(t, 0; t', 0) |_{t' \to t} = 0. \tag{23}$$

Omitting in (22) total derivatives we arrive at the following one-loop effective action:

$$\Gamma^{(1)} = \frac{\alpha_{1/2}^\alpha \mu^\epsilon}{2\pi} \int dt \left(\dot{x}^\mu \dot{x}^\nu (\ddot{\circ} A_{\mu \nu} - \frac{1}{\epsilon} \Box \ddot{\circ} A_{\mu \nu}) + \ddot{\circ} B_{\mu} - \frac{1}{\epsilon} \Box \ddot{\circ} B_{\mu} + K^2 (\ddot{\circ} \varphi_1 - \frac{1}{\epsilon} \Box \ddot{\circ} \varphi_1) + \ddot{\circ} K (\ddot{\circ} \varphi_2 - \frac{1}{\epsilon} \Box \ddot{\circ} \varphi_2) + K \dot{x}^\alpha (\ddot{\circ} \varphi_\alpha - \frac{1}{\epsilon} \Box \ddot{\circ} \varphi_\alpha) \right) + (\text{fin}),$$

$$\Box \equiv \eta^{\alpha \beta} \partial_\alpha \partial_\beta,$$

where $\circ$ denotes bare background fields and $(\text{fin})$ stands for a finite part of the one-loop correction.

To cancel the divergences in (24) renormalization of all the background fields should be of the form:

$$\circ \Phi = \mu^{-\epsilon}(\Phi + \frac{1}{\epsilon} \Box \Phi) \tag{25}$$

where $\Phi = (A_{\mu \nu}, B_{\mu}, \varphi_1, \varphi_2, \varphi_\mu)$. 

7
4 Renormalization trace of energy-momentum tensor and equations of motions

In classical theory trace of the energy-momentum tensor for the theory \([10]\) on the \(2 + \epsilon\)-dimensional world sheet is

\[
T(z) = g_{ab}(z) \frac{\delta S}{\delta g_{ab}(z)} = 
\]

\[
= \frac{\epsilon}{8\pi} g^{ab}(z) \partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu} + \frac{\alpha^{1/2}}{8\pi} H_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \delta_{\partial M}(z) - 
\]

\[
- \frac{\alpha^{1/2}}{4\pi} K^2 \varphi_1 \delta_{\partial M}(z) + \frac{\alpha^{1/2}}{2\pi} K \varphi_1 \delta_{\partial M}(z) + 
\]

\[
+ \frac{\alpha^{1/2}}{4\pi} K \dot{x}^\mu \Phi_\mu \delta_{\partial M}(z) - \frac{\alpha^{1/2}}{4\pi} \dot{x}^\mu \Phi_\mu S'_{\partial M}(z) + O(\epsilon \alpha^{1/2}). \tag{26}
\]

Terms \(O(\epsilon \alpha^{1/2})\) will not contribute to the renormalized trace of energy-momentum tensor.

To calculate the trace of the energy-momentum tensor in quantum theory we should define renormalized values for composite operators. Consider, for example, the vacuum average for one of these operators:

\[
< H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu > = \int Dxe^{-S[x]} H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu \bigg/ \int Dxe^{-S[x]}. 
\]

Making the shift \(x^\mu = \bar{x}^\mu + \zeta^\mu\) in the functional integral (\(\bar{x}^\mu\) are solutions of the classical equations of motion) and using (23) we get in linear approximation:

\[
< H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu > = \mu \epsilon \bar{x}^\mu \bar{x}^\nu (H_{\mu\nu}(\bar{x}) - \frac{1}{\epsilon} \square H_{\mu\nu}(\bar{x})) + \text{(fin)}. \tag{27}
\]

Renormalized operators should have a finite average value

\[
< [H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu] > = H_{\mu\nu}(\bar{x}) \dot{x}^\mu \dot{x}^\nu + \text{(fin)} \tag{28}
\]

hence

\[
(H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu) = \mu \epsilon \bar{x}^\mu \bar{x}^\nu (H_{\mu\nu}(\bar{x}) - \frac{1}{\epsilon} \square H_{\mu\nu}(\bar{x})) + \text{(fin)}, \tag{29}
\]

or, using (23),

\[
(H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu) = [H_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu] \tag{30}
\]

In the same way one can get

\[
(\varphi_1) = [\varphi_1], \quad (\Phi_\mu \dot{x}^\mu) = [\Phi_\mu \dot{x}^\mu] \tag{31}
\]
Similar but more tedious calculation gives the following renormalization of the operator $g^{ab}(z)\partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu}$:

$$(g^{ab}(z)\partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu})_o = [g^{ab}(z)\partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu}] +$$

$$+ \frac{\alpha'^{1/2} \mu}{2\pi} [V_{(2)} \delta M(z) + K V_{(2)} \delta M(z)]$$

$$- \frac{1}{2} R V_{(2)} \delta M(z) - K^2 V_{(2)} \delta M(z) +$$

$$+ \frac{\alpha'^{1/2} \mu}{2\pi} \frac{d^2}{dt^2} V_{(2)} \delta M(z) + 4 V_{(0)} \delta M(z)],$$

$$V_{(2)} = V_{(2)\mu\nu} \eta^{\mu\nu}, \quad V_{(0)} = V_{(0)\mu\nu} \eta^{\mu\nu}. \quad (32)$$

As a result, the renormalized operator of the energy-momentum tensor trace has the form

$$[T] = \alpha'^{1/2} \delta(y)[RE^{(0)}(x) + \tilde{x}^\mu \tilde{x}^\nu E^{(1)}_{\mu\nu}(x) + \tilde{x}^\mu E^{(2)}_{\mu}(x) +$$

$$+ K \tilde{x}^\mu E^{(3)}_{\mu} + K^2 E^{(4)}(x) + K E^{(5)}(x)] +$$

$$+ \alpha'^{1/2} \delta''(y)[\tilde{x}^\mu E^{(6)}_{\mu}(x) + K E^{(7)}(x)] +$$

$$+ \alpha'^{1/2} \delta''(y) E^{(8)}(x) \quad (33)$$

with

$$E^{(0)}(x) = -\frac{1}{8\pi}(B_{\alpha,\alpha} - A_{\alpha}^\alpha)$$

$$E^{(1)}_{\mu\nu}(x) = \frac{1}{4\pi}(2\Box A_{\mu\nu} - 2A_{\mu\nu} - A_{\alpha,\mu\nu} - 2A_{\mu,\alpha\nu} - 2A_{\nu,\alpha\mu} +$$

$$+ 3B_{\mu\nu,\alpha} + B_{\mu,\nu} + B_{\nu,\mu})$$

$$E^{(2)}_{\mu}(x) = \frac{1}{4\pi}(2\Box B_{\mu} + 3B_{\alpha,\mu} - A_{\alpha,\mu} - 4A_{\mu,\alpha})$$

$$E^{(3)}_{\mu}(x) = \frac{1}{2\pi}(\Box \varphi_{2,\mu} - \varphi_{\mu} + \Box \varphi_{\mu} - \varphi^\alpha_{\mu,\alpha})$$

$$E^{(4)}(x) = \frac{1}{4\pi}(2\Box \varphi_{1} - B_{\alpha}^\alpha + \varphi_{1}^\alpha - 2\varphi_{1})$$

$$E^{(5)}(x) = \frac{1}{2\pi}(\Box \varphi_{2} - \varphi_{\alpha}^\alpha)$$

$$E^{(6)}_{\mu}(x) = \frac{1}{2\pi}(\Box \varphi_{2,\mu} - \varphi_{\mu})$$

$$E^{(7)}(x) = \frac{1}{4\pi}(B_{\alpha}^\alpha - A_{\alpha}^\alpha + 4\varphi_{1})$$

$$E^{(8)}(x) = \frac{1}{4\pi}(B_{\alpha}^\alpha - A_{\alpha}^\alpha) \quad (34)$$

From the requirement of quantum Weyl invariance it follows that all the coefficients $[34]$ at the linear independent operators in $(33)$ should be equal to zero.
Therefore equations of motion for the background fields of the first massive level are \( E(x) = 0 \).

Using the symmetry of the theory under the transformations (11) to make \( B_\mu = 0 \) and \( \varphi_2 = 0 \) and returning to the dimensional string coordinates \( x^\mu \rightarrow \alpha'^{-1/2} x^\mu \) one can rewrite these equations as

\[
B_\mu = 0, \quad \varphi_2 = 0, \quad \varphi_1 = 0, \quad \varphi_\mu = 0, \\
\Box A_{\mu\nu} - m^2 A_{\mu\nu} = 0, \quad A_\mu^\mu = 0, \quad \partial_\mu A_\mu^\nu = 0, \tag{35}
\]

where \( m^2 \) is the mass of the first level.

So the equations of motion for the background fields of the first massive level of the open string are equivalent to the equations describing a massive field with spin 2. As is well known, the analysis of the spectrum of open string physical states gives the same result.

5 Summary

We have considered the general approach to the theory of an open string interacting with massive background fields. Our analysis has shown that there exist a consistent description of string models in background fields corresponding to a finite number of the massive string modes. We have proposed the most general model describing interaction of an open string with background fields of the first massive level. The theory is invariant under symmetry transformations of background fields.

The renormalized operator of the energy-momentum tensor trace has been constructed. In linear approximation it has been shown that the requirement of quantum Weyl invariance leads to equations of motion for background fields which are consistent with the structure of the open string spectrum at the first massive level. The approach proposed opens up possibilities for deriving equations of motion for massive fields of higher spins in the framework of the string theory.

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