Vehicle vibration safety estimation area

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Annotation. It is known that the problem of mathematical modeling of the car vibration isolation system can be solved in the frequency and time domain. As the primary vehicle vibration isolation system has non-linear elements, the question arises: how does the linearized dynamic system solution in the frequency domain corresponding to the calculations of the accepted parameters in the time domain? The problem is solved at random kinematic perturbation from the road surface. Therefore, when working in the time domain to estimate the adequacy of solutions, it is necessary to make a choice of the method of statistical linearization from the known in practice design of automatic control systems. Four methods of statistical linearization are considered, using which calculations in the frequency domain have been carried out. For the chosen dynamic system with initial and statistically linearized nonlinear elements similar actions in the time domain were carried out. It is shown that the first method of statistical linearization is the most flexible, according to the amplitude-frequency response of the system. Such calculations were carried out for two surfaces corresponding to the cobblestone and subsoil roads at different speeds of the vehicle movement. The results of the research are separate-frequency and integral parameters. The last ones do not give any priority in the choice of the calculation field, under the condition of vehicle movement safety, i.e. there is no tire contact loss with the supporting surface.

Introduction

The solution of the problem of determining the parameters and characteristics of nonlinear dynamic systems uses two approaches to its realization depending on the chosen area of solution. There are two such areas - frequency \([1-4]\) and time domain \([5-10]\). The choice of the area of solution in the assessment of the quality of the analyzed system of the object is an important task, which is important in terms of the calculation accuracy and the possibility of using its equivalent characteristics for the research and choice of its optimal parameters. It should be noted that in the analysis of dynamic systems both estimation areas of determined values are used. The methods of statistical linearization are used to solve problems in the frequency domain. Simulation of the analyzed system can also be carried out in the time domain \([11-17]\) with the obtained data. The deviation of the calculated parameters may be incorrect when solving problems in the frequency domain. This is proved by the formulation of problems of modernization of methods of statistical linearization \([18-24]\).

This research focuses on a dynamic system that is equivalent to a vehicle vibration isolation system with non-linear characteristics of the primary suspension system and tires. The perturbation is a random process determined by the microprofile of the road surface \([25]\) and the vehicle speed. The problem of estimating the convergence of the obtained results is set when the research is carried out in the frequency and time domain. For this purpose, the efficiency of four methods of statistical linearization was compared with the results of calculations of the initial dynamic system in the time
domain. The choice of the method of statistical linearization is carried out in order to estimate the adequacy of the determined spectral characteristics of the system and the parameters of vibration safety in the frequency and time domain. To confirm the conformity of the obtained data, the dynamic system is tested according to the calculations of the analyzed characteristics in the frequency and time domain. The test parameters are the own and the resonant frequencies of this system. The unequivacity of the carried out actions is provided by estimation of calculation accuracy of considered spectral characteristics of the analyzed object.

**Mathematical model of non-linear vibration isolation system**

To solve this problem, a non-linear dynamic system equivalent to the primary system of suspension, vibration isolation of vehicle components and constructions is considered (Fig. 1). This dynamic system was considered when calculating the required spectral characteristics as a result of modeling in the frequency and time domain.

The generalized coordinates are the center of mass displacements and angular displacements in the longitudinal and transverse planes relatively to the axes passing through the center of mass of systems I, II, III and V, respectively. Each system IV and VI has one degree of freedom - the vertical displacement of their mass centers.

**Fig. 1.** Dynamic system scheme equivalent to the vehicle's vibration isolation system:
I - vehicle body, II - engine, III - driver's cabin, IV - driver's seat, V - undercarriage, VI - unsprung mass, 1, 2, 3, 4, 5, 6 - elements (elastic and damping) of the vibration isolation system of the body, engine, driver's cab, seat, undercarriage and tire, respectively.

The solution to this problem is to determine the spectral density of the analyzed vibration signal power, for example, vibration acceleration on the driver's seat. In the frequency domain, we solve this problem by transforming the initial linear system of differential equations into a system of algebraic equations using the Laplace transformation. Using the generalized Kramer rule, we determine the vector of the frequency characteristics of the dynamic system using generalized coordinates. This
allows for the spectral characteristics of the perturbation to estimate the characteristics of the analyzed vibration signals to determine the parameters of vibration safety and coefficients of statistical linearization. In this case, four known methods of statistical linearization are considered. They allow calculating constant coefficients in three variants and frequency response of a nonlinear element. In the time domain, the considered problem was solved by the numerical method using the recurrent difference equations. The spectral densities of the vibration signals were calculated using the Fourier transform algorithm at the final range.

Analysis of the mathematical modeling results
During the first modeling stage, a dynamic system with seventeen degrees of freedom equivalent to the vehicle’s vibration isolation system was tested with a linear problem set. The solution of this problem was carried out on the spectrum of natural frequencies and amplitude-frequency response of the system (resonance frequencies). It showed full matching of the obtained results in frequency and time areas. In addition, the accuracy of the analyzed characteristics was estimated. It is shown that for the spectral density of the process power at linear and nonlinear systems the normalized root-mean-square error is 9%. The amplitude-frequency response of the analyzed system is determined according to the expression

\[ H_{qy} (\omega) = \frac{G_{qy} (\omega)}{G_{q} (\omega)} \]

where \( G_{qy} (\omega) \) — cross-spectral density of vibration signals \( q(t) \) and \( y_c(t) \); \( q(t) \) — input vibration signal - disturbance from road surface microprofile; \( y_c(t) \) — output vibration signal - vibration acceleration in the driver's seat; \( G_{q} (\omega) \) — spectral density of power perturbation \( q(t) \).

In this case, the normalized random error of the frequency response of the system is calculated as follows

\[ \varepsilon_{ff} (\omega) = \left[ \frac{1 - \gamma^2_{qy} (\omega)}{\gamma_{qy} (\omega)} \right] \frac{1}{\sqrt{2n_d}} \]

where \( \gamma_{qy} (\omega) \) — vibration coherence function \( q(t) \) and \( y_c(t) \).

We obtain that in the considered frequency range, i.e. in the range of intensive vibration output signal 0-50 (60) sec\(^{-1}\), the calculated error of amplitude-frequency response is 8-14%. It should be noted that the initial spectral characteristics for the analysis were determined taking into account the features of the digital spectral analysis. In this case, the function of coherence in the analyzed frequency range changes for the vibration signal output in the range 0,6-1,0 depending on the conditions of the vehicle movement. The calculations are carried out the variants of perturbation from the microprofile of the cobblestone and subsoil roads with the mean-square deviations of the road heights of 20 mm and 30 mm in the speed range of 20-80 km/h.

Therefore, according to the amplitude-frequency response of the system, the choice of the method of statistical linearization was made, with the use of which we have the smallest error of linearized and nonlinear system equivalence. This system analysis was carried out when the vehicle moved along the cobblestone road at a speed of \( v_e=40 \text{ km / h} \). At this "resonance" speed, the low-frequency resonance areas of the dynamic system and the maximum perturbation power are combined. This results in a significantly non-linear system. According to the calculations (Fig. 2), it is possible to conclude that the first method of statistical linearization is significant.

The system was therefore tested over the full range of vehicle movements in order to avoid errors in analysis and to ensure the validity of the data obtained. These results show that a significant difference in the amplitude-frequency response of the nonlinear and linearized dynamic system is detected only in the mode of "resonance speed" when the vehicle is moving on a dirt road. Thus, there
is a problem of calculations accuracy in the frequency domain. For this case, we turn to the parameter which describes the tire's contact with the supporting surface. The possibility of this effect is defined as following

\[ P_{hu} = 0.5 \cdot \left(1 - \Phi(\mu)\right), \quad \Phi(\mu) = \frac{2}{\sqrt{\pi}} \int_0^\mu e^{-v^2} dv, \quad \mu = \frac{h_u}{\sqrt{2 \cdot D_{hu}}}, \]

where \( \Phi(\mu) \) - probability integral, \( h_u \) - static tire deflection, \( D_{hu} \) - tire deflection variance.

Fig. 2. Amplitude-frequency response of the dynamic vehicle vibration isolation system on the driver's seat. Cobblestone road, \( v_a = 40 \) km/h:
1 - non-linear vibration isolation system; 2, 3, 4, 5 - statistically linearized system of the 1st, 2nd, 3rd, 4th method of statistical linearization

In this case there is a high probability of up to 35% loss of tire contact with the ground surface when the vehicle moves along the dirt road (Fig. 3). Thus, there is a safety problem with the vehicle, and the estimation of vibration safety is incorrect. The following diagrams show that when the vehicle moves along the cobblestone road at a speed of 40 km/h and the probability of contact loss of 20% is rather high, the results of calculations are comparable. Confirmation of the obtained results is provided
by separate-frequency and integral estimation of vibration safety parameters (Fig. 4). In this case, as well as in the analysis of amplitude-frequency response, the greatest difference between the estimated parameters occurs when the vehicle moves along the dirt road at a speed of 40 km/h.

Fig.3. Possibility of loss of contact between the tyre and the ground surface. Dirt road: n, non-linear, l- linearized dynamic system

Conclusions
The results of the calculations of spectral characteristics for the nonlinear system of vehicle vibration isolation in the frequency and time domain, as well as the parameters of vibration safety allow to make the following conclusion:

- created methods of analysis of dynamic systems in the frequency and time domain make it possible to carry out an estimation of the object vibration state;
- the best approximation of the results of calculations of nonlinear and linearized systems is possible using the first method of statistical linearization;
- the deviation from the adaptability of calculations in the frequency and time domain takes place at a significant probability of loss of contact of the tire with the ground surface;
- the research shows that vibration safety can be estimated in the frequency and time domain.
Fig. 4. Octave spectra of vibration acceleration in the driver's seat. Dirt road:
\(a, b\), - \(v_a = 40\) kph, \(v_a = 60\) kph, IA-integrated adjusted estimation; I - integrated estimation

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