Controlled switching of discrete solitons in waveguide arrays

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We suggest an effective method for controlling nonlinear switching in arrays of weakly coupled optical waveguides. We demonstrate the digitized switching of a narrow input beam for up to eleven waveguides in the engineered waveguide arrays. © 2022 Optical Society of America

OCIS codes: 190.0190, 190.4370, 190.5530
Discrete optical solitons were first suggested theoretically as stationary nonlinear localized modes of a periodic array of weakly coupled optical waveguides. Because the use of discrete solitons promises an efficient way to control multi-port nonlinear switching in a system of many coupled waveguides, this field has been extensively explored theoretically. More importantly, the discrete solitons have also been generated experimentally in fabricated periodic waveguide structures.

The most common theoretical approach to study discrete optical solitons in waveguide arrays is based on the decomposition of the electric field in the periodic structure into a sum of weakly coupled fundamental modes excited in each waveguide of the array. According to this approach, the wave dynamics can be described by an effective discrete nonlinear Schrödinger (DNLS) equation, that possesses spatially localized stationary solutions in the form of localized modes of a lattice model. Many properties of the discrete optical solitons can be analyzed in the framework of the DNLS equation.

One of the major problems for achieving the controllable multi-port steering of discrete optical solitons in waveguide arrays is the existence of an effective periodic potential which appears due to the lattice discreteness, known as the Peierls-Nabarro (PN) potential. It represents the energy cost associated with a shift of a nonlinear localized mode by a half of the waveguide spacing. Its magnitude can be roughly estimated as $\sim |A|^4$, where $A$ is the soliton amplitude. As a consequence of this potential, a narrow large-amplitude discrete soliton does not propagate in the lattice and it becomes trapped by the array. Several ideas to exploit the discreteness properties of the array for the optical switching were suggested, including the demonstration of
the output channel selection for the multi-port devices. However, the soliton steering and switching is well controlled only in the limit of broad beams whereas the soliton dynamics in highly discrete arrays has been shown to be more complicated. In this Letter, we suggest a “discreteness engineering” approach and demonstrate how to achieve highly controllable multi-port soliton switching in the arrays by a desired integer number of waveguides, the so-called “digital soliton switching”.

We consider a standard model of the waveguide arrays with a modulated coupling described by the normalized DNLS equation of the form,

\[ i \frac{du_n}{dz} + V_{n+1}u_{n+1} + V_{n-1}u_{n-1} + \gamma |u_n|^2 u_n = 0, \]  

(1)

where \( u_n \) is the effective envelope of the electric field in the \( n \)-th waveguide and \( z \) is the propagation distance. Unlike the standard models, the coupling \( V_n \) between two neighboring guides is assumed to vary, either through the effective propagation constant or by changing the spacing between neighboring guides. The parameter \( \gamma = \omega_n n_2/(cA_{\text{eff}}) \) is the effective waveguide nonlinearity associated with the Kerr nonlinearity of the core material.

The steering and trapping of discrete solitons in the framework of the model have been analyzed in many studies. Being kicked by an external force, the discrete soliton can propagate through the lattice for some distance, but then it gets trapped due to the effect of discreteness. For a larger force, the output soliton position fluctuates between two (or more) neighboring waveguides making the switching uncontrollable. Here, we suggest to modulate the waveguide coupling in order to achieve a controllable output and to engineer the switching results. The key idea is
to break a symmetry between the beam motion to the right and left at the moment of trapping; this allows the elimination of chaotic trapping observed in homogeneous arrays. In this way, we achieve a controllable digitized switching where the continuous change of the input beam amplitude results in a quantized displacement of the output beam by an integer number of waveguides.

We have tested different types of modulation in the array parameters and the corresponding structures of array super-lattices. An example of one of the optimized structures is shown in Fig. 1, where we modulate the coupling parameter $V_n$ in a step-like manner. We also notice that the use of a linear ramp potential (e.g. of the form $V_n = an$) for this purpose does not lead to an effective switching but, instead, makes the soliton switching even more chaotic due to the phenomenon of Bloch oscillations which become randomized in the nonlinear regime.

We select the input profile in the form of a narrow sech-like beam localized only on a few waveguides,

$$u_n(0) = A \text{sech}\left[ A(n - n_c)/\sqrt{2}\right] e^{-ik(n-n_c)}, \quad (2)$$

for $n - n_c = 0, \pm 1$, and $u_n = 0$, otherwise. For the particular results presented below, we select the array of 101 waveguides and place the beam at the middle position, $n_c = 50$. The maximum normalized propagation distance used in our simulations is $z_{\text{max}} = 45$ (in units of the coupling length).

Parameter $k$ in the ansatz (2) has the meaning of the transverse steering velocity of the beam, in analogy with the continuous approximation. It describes the value of an effective kick of the beam in the transverse direction at the input, in order
to achieve the beam motion and shift into one of the neighboring (or other desired) waveguide outputs.

In our simulations, we control the numerical accuracy by monitoring two conserved quantities of the model (2), the soliton power $P = \sum_n |u_n(z)|^2$, and the system Hamiltonian, $H = \sum_n \{V_n(u_n u_{n+1}^* + u_{n+1}^* u_n) + (\gamma/2)|u_n|^4\}$.

The input condition (2) is not an exact stationary solution of the discrete equation (1) even for $k = 0$, and as the input kick ($k \neq 0$) forces the soliton to move to the right ($k < 0$) or left ($k > 0$), the motion is accompanied by some radiation. The effective lattice discreteness can be attributed to an effective periodic PN potential. Due to both the strong radiation emission and the PN barrier which should be overtaken in order to move the beam, the discrete soliton gets trapped at one of the waveguides in the array, as shown in Fig. 2. In most cases, the shift of the beam position to the neighboring waveguide is easy to achieve, as shown in many studies. However, the soliton switching becomes rather complicated and even chaotic when the kicking force becomes stronger.

We have studied many different regimes of the soliton multi-port switching in the array and revealed that the most effective switching in a desired waveguide position (i.e. desired output) can be achieved by varying the coupling between waveguides, either through the effective propagation constant or by changing the spacing between neighboring guides, as shown in Fig. 1. This $V_n$ profile was obtained after performing a numerical sweep in $V_n$ and $A$ for fixed momentum $k$. In this case, the selection of a finite value of the steering parameter $k$ allows to switch the whole beam into a neighboring waveguide, as shown in Fig. 2, with only a small amount of radiation. By
decreasing the amplitude of the input pulse at a fixed value of the steering parameter, fixed to be say $k = \pm 0.9$, it is possible to achieve self-trapping of the soliton beam at some (short) distance from the initial center at different waveguide position. Due to the step-like modulated coupling, we create a selection between the beam motion to the right and left at the moment of trapping thus suppressing or eliminating the chaotic trapping observed in homogeneous waveguide arrays. In this way, we achieve a controllable digitized nonlinear switching where the continuous change of the amplitude of the input beam results in a quantized displacement of the output beam by an integer number of waveguides. Consequently, for the parameters discussed above we observe almost undistorted switching up to eleven waveguides, and Fig. 3 shows an example of the digital soliton switching to the eleventh waveguide.

Figure 4 gives a summary of the results for the parameters discussed above; it shows the discrete position of the soliton at the output as a function of the input beam amplitude, for two fixed values of the steering parameter $k = \pm 0.9$. In a remarkable contrast with other studies, the coupling modulation allows a controllable digitized switching in the array with very little or no distortion. The figure also shows a slight asymmetry in the final displacement, depending on whether the beam is kicked uphill or downhill.

If we were to use five guides instead of three for the input beam, one could expect a smaller amount of radiation emitted. However, this would imply a longer distance before the beam gets trapped by one of the waveguides in the array due to the effective Peierls-Nabarro potential. Also, this means that one could in principle switch the soliton beam to any desired waveguide in the waveguide array, no matter
how far; it would be just a matter of choosing an initial beam wide enough, i.e., closer to the continuum limit (in addition to the optimization of the coupling in a step-wise manner) by removing the random selection between the directions and suppressing the beam random switching.

Another observation is that the sech-like initial profile is not really fundamental. A (kicked) nonlinear impurity-like profile of the form $u_n(0) = A[(1 - A^2)/(1 + A^2)]^{n-n_c}/2 \exp[-ik(n-n_c)]$ will also show similar behavior, as our additional computations show. The reason for this behavior seems to rest on the observation that, for any system with local nonlinearity, a narrow initial profile will effectively render the system into a linear one containing a small nonlinear cluster (or even a single site); the bound state will therefore strongly resemble the one corresponding to a nonlinear impurity.\footnote{13}

In conclusion, we have suggested a novel approach to achieve a digitized switching in waveguide arrays by using the concept of discrete optical solitons. Our approach involves a weak step-like modulation of the coupling strength (or, equivalently, distance between the waveguides) in the arrays with the period larger than the waveguide spacing. Such a super-lattice structure allows the modification of trapping properties of the waveguide array due to discreteness which in turn permits engineering of the strength of the effective trapping potential. We have demonstrated numerically the controlled switching up to eleven waveguides in the arrays by using very narrow input beams localized on three waveguides only.

R. A. Vicencio acknowledges support from a Conicyt doctoral fellowship. M.I. Molina and Yu. S. Kivshar acknowledge support from Fondecyt grants 1020139 and
7020139. In addition, Yu.S. Kivshar acknowledges the warm hospitality of the Department of Physics of the University of Chile.
References

1. D. N. Christodoulides and R. I. Joseph, Opt. Lett. 13 794 (1988).

2. Yu. S. Kivshar, Opt. Lett. 18, 1147 (1993).

3. W. Krolikowski and Yu. S. Kivshar, J. Opt. Soc. Am. B 13, 876 (1996).

4. A. B. Aceves, C. De Angelis, T. Peschel, R. Muschall, F. Lederer, S. Trillo, and S. Wabnitz, Phys. Rev. E 53, 1172 (1996).

5. For a recent overview, see A.A. Sukhorukov, Yu.S. Kivshar, H.S. Eisenberg, and Y. Silberberg, IEEE J. Quantum Electron. 39, 31 (2003).

6. H. S. Eisenberg, Y. Silberberg, R. Morandotti, A. R. Boyd, and J. S. Aitchison, Phys. Rev. Lett. 81, 3383 (1998).

7. Y. Silberberg and G.I. Stegeman, In: Spatial Solitons, Eds: S. Trillo and W.E. Torruellas (Springer-Verlag, Berlin, 2001), p. 37.

8. The existence of the PN potential was confirmed experimentally in R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg and Y. Silberberg, Phys. Rev. Lett. 83, 2726 (1999).

9. A.B. Aceves, C. De Angelis, S. Trillo, and S. Wabnitz, Opt. Lett. 19, 332 (1994).

10. O. Bang and P.D. Miller, Opt. Lett. 21, 1105 (1996).

11. R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg and Y. Silberberg, Phys. Rev. Lett. 83, 4756 (1999).

12. The use of a symmetric $V_n$ around $n = n_c$ allows to reach eleven guides on either side, thus making possible switching up to twenty-two guides.

13. M. I. Molina, Mod. Phys. Lett. B 17, 111 (2003); M. I. Molina, presented at the
XIII Symposium of the Chilean Physics Society, Concepción, Chile, 13-15 Nov. 2002.
Figure Captions

Fig. 1. Example of the optimized modulation of the propagation constant $V_n$ in the waveguide array.

Fig. 2. One-site switching of a discrete soliton in the waveguide array with the modulated coupling shown in Fig. 1.

Fig. 3. Discrete switching by eleven sites in the waveguide array modulated according to Fig. 1.

Fig. 4. Soliton switching in a waveguide array with an optimized coupling. Shown is the soliton output displacement as a function of the input beam amplitude (A). A step size of 0.0005 separates consecutive points.
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