The Economics of Enlightenment: Time Value of Knowledge and the Net Present Value (NPV) of Knowledge Machines, A Proposed Approach Adapted from Finance

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1 Abstract

We formulate one methodology to put a value or price on knowledge using well accepted techniques from finance. We provide justifications for these finance principles based on the limitations of the physical world we live in. We start with the intuition for our method to value knowledge and then formalize this idea with a series of axioms and models. To the best of our knowledge this is the first recorded attempt to put a numerical value on knowledge. The implications of this valuation exercise, which places a high premium on any piece of knowledge, are to ensure that participants in any knowledge system are better trained to notice the knowledge available from any source. Just because someone does not see a connection does not mean that there is no connection. We need to try harder and be more open to acknowledging the smallest piece of new knowledge that might have been brought to light by anyone from anywhere about anything.
2 Knowledge for What Sake?

Despite the apparent lack of consensus as to what knowledge really is and a possibly demonstrable absence of well accepted reasons for maybe why we even need knowledge, most would agree that knowledge is valuable. (Russell 1948) is perhaps an ideal place to start examining the relation between individual experience and the general body of scientific knowledge; see also: (Kvanvig 2003; Pritchard 2009; Pritchard, Millar & Haddock 2010; Crevoisier 2016) (Boswell 1873; Toffler 1990) have a collection of quotes starting from the 1st century AD, stating the ancient belief that knowledge has worth. That the Wikipedia link, (Foot-note) about the value of knowledge, has material only from as recently as 2000 years ago, shows how limited our knowledge regarding the value history has placed on knowledge is. (Dancy, Sosa & Steup 2009; DeRose 2005; Figueroa 2016; Pritchard 2018; Hetherington 2018) are an excellent collection of articles on leading theories, thinkers, ideas, distinctions and

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1 As the name of (section 2 Knowledge for What Sake?) suggests, whatever knowledge might be if it can be traded for whichever sake, the Japanese drink that we might be able to ferret out, it might seem like a good exchange, or trade, to some of us. Such a trade or barter or willingness to pay for a commodity can help us establish the price or value for any good. The main issue with such an approach, though it can be helpful, is that it is an inaccurate estimate of the innate worth of the article being transacted since it involves subjective decisions being made by the participants including any dependencies on the particular situation under which the transaction was performed (circumstances that might have forced one of the participants to participate). Also, price estimation, which involves predicting future benefits, is highly uncertain and unreliable as evidenced by the enormous, growing and complex literature on asset pricing (Cochrane 2009). Hence, our approach to estimating the value of knowledge is much broader in scope than finding the price for an exchange, which involves monetary or utilitarian connotations (Foot-note). Price and value are used synonymously in daily life and in most financial transactions. But it should be clear that price and actual value are entirely distinct for knowledge since someone truly looking for a certain piece of knowledge will know that in some cases no amount of money can help obtain that piece of knowledge.

2 The utility maximization problem is the problem consumers face: "how should I spend my money in order to maximize my utility?" Within economics the concept of utility is used to model worth or value, but its usage has evolved significantly over time. Utility, Wikipedia Link

3 Knowledge is a familiarity, awareness, or understanding of someone or something, such as facts, information, descriptions, or skills, which is acquired through experience or education by perceiving, discovering, or learning. Knowledge, Wikipedia Link

4 The idea that knowledge has value is ancient. In the 1st century AD, Juvenal (55-130) stated “All wish to know but none wish to pay the price” (Highet 1961; Courtney 2013). In 1775, Samuel Johnson wrote: “All knowledge is of itself of some value.” These quotes are from this Wikipedia link: Knowledge Value, Wikipedia Link which has many more quotes. The first time we checked the link was around Oct-16-2017.

5 Epistemology is the study of the nature of knowledge, justification, and the rationality of belief. It is the branch of philosophy concerned with the theory of knowledge. Epistemology, Wikipedia Link
concepts in epistemology.

Summary 1. In later sections (and papers), we hope to pitch in to this effort to refocus what knowledge stands for and how best to spread it. Our contribution to the knowns and unknowns about knowledge is to formulate one methodology to put a value or price on knowledge (Foot-note 1) using well accepted techniques from finance. We start with the intuition for one method to value knowledge. We then formalize this idea with a series of axioms and models. To the best of our knowledge (limited as it is, which follows from our present understanding of knowledge and also from the definition of knowledge we use below, Definition 3), this is the first recorded attempt to put a numerical value on knowledge using well known valuation techniques. Our main result (Theorem 1) provides a lower bound for the value of knowledge. The implications of this valuation exercise, which places a high premium on any piece of knowledge, are to ensure that the participants in any knowledge system are better trained to notice the knowledge available from any source.

In a related paper, (Kashyap 2018), we discuss a process to maximize the efficiency of knowledge creation and show how our valuation method signifies that the best way for journals to select submissions would be randomly from a pool of papers meeting certain basic quality criteria. We specifically show that the best decision we can make with regards to the selection of articles by journals requires us to formulate a cutoff point, or, a region of optimal performance and randomly select from within that region of better results. The policy implication (for all fields) is to randomly select papers, based on publication limitations (journal space, reviewer load etc.), from an overall pool of submissions that have a single shred of knowledge (or one unique idea) and have the editors and reviewers coach the authors to ensure a better final outcome. The results in this present paper set the stage for the developments in (Kashyap 2018) where we compare existing publication processes with an alternative approach based on randomization.

(Section 3) lays down the framework required for our valuation exercise by providing the intuition, the corresponding assumptions and definitions along with a review of the relevant literature. (Section 4) builds upon this by making further assumptions and extends it to a formal approach to include axioms, notation, terminology and the actual results. We have tried to ensure that the bulk of the narrative within the main body of the paper is mostly self-contained so that it can be easily followed by a wider audience. But for those wishing to have more details and a deeper context we have provided a rich set of foot-notes that supplement the central arguments. We have provided a list of all the definitions and assumptions including a dictionary of notation and terminology in (appendices 6; 7).
3 Questions & Answers, Q&A, Definitions and Assumptions, D&A, in our DNA

It would not be entirely incorrect to state that the majority of the attempts at knowledge creation start with answering questions. In present day society we seem to be focused on answering questions that originate in different disciplines.

**Assumption 1.** *As a first step, we recognize that one possible categorization of different fields can be done by the set of questions a particular field attempts to answer. Since we are the creators of different disciplines, but we may or may not be the creators of the world (based on our present state of knowledge and understanding) in which these fields need to operate, the answers to the questions posed by any domain can come from anywhere or from phenomenon studied under a combination of many other disciplines.*

Hence, the answers to the questions posed under the realm of knowledge creation can come from seemingly diverse subjects such as: physics, biology, mathematics, chemistry, marketing, economics, finance and so on. This quest for answers is bounded only by our imagination (Calaprice 2000). As we linger on the topic of Questions & Answers, Q&A. The field that is most concerned with the valuation of assets is finance (for lack of knowledge of a better word, or terminology, on behalf of the authors, let us categorize knowledge under the umbrella of assets). Hence, it should not come as a surprise that finance can provide a very surprising answer to our main research question.

**Question 1. What is the value of knowledge in any field?**

Any answer we wish to seek would depend on some Definitions and Assumptions, D&A. But if we change those D&A we might get different Q&A.

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6 Answering questions can be also viewed as solving problems that arise in any facet of life. This suggests that we might be better off identifying ourselves with problems and solutions, which tacitly confers upon us the title Problem Solvers, instead of calling ourselves physicists, biologists, psychologists, marketing experts, economists and so on.

7 Maybe, we hold the lessons from the lives of every ancestor we have ever had, even telling us that Q&A and D&A might be in our very DNA, the biological one, which are always changing (Alberts, Johnson, Lewis, Raff, Roberts & Walter 2002; Alberts 2017). Evolution is constantly coding the information, compressing it and passing forward, what is needed to survive better and to thrive,
**Assumption 2.** Questions & Answers, Q&A, are important, but Definitions and Assumptions, D&A, are even more important since changing D&A could require us to consider different Q&A.

The implication of (assumption 2) is that all Q&A we consider here are to be viewed in the context of the D&A provided here. While we endeavor to provide as realistic and all encompassing a discussion as possible, a changing world (or changes in our understanding of it) might render our Q&A obsolete. A related question that comes up is: what is finance? The answer is that finance is a game where there are only three simple decisions to be made: Buy, Sell or Hold; the complication are mainly to get to these results (Kashyap 2015).

### 3.1 Related Literature

A recent attempt, (Martin 1996), in the context of the time periods discussed in (section 2), acknowledges that measuring the value of knowledge has not progressed much beyond an awareness that traditional accounting practices are misleading and can lead to wrong business decisions. Right at the outset, we distinguish between our knowledge valuation methodology and the valuation of patents since all patents have some knowledge associated with them, but not all knowledge might lead to a patent or an industrial application. A patent is defined as any new or non-obvious invention capable of industrial application (Pitkethly 1997). (Wu & Tseng 2006) provide a valuation technique based on real options. We also note that patents are mostly granted based on demonstrated novelty and the associated estimation of the future benefits of the corresponding idea or invention tend to be extremely difficult. The question of novelty with respect to knowledge and the distinction between old and new knowledge is considered further in (section 4).

While it is not straightforward to draw a distinction between basic and applied research; many attempts at seeking knowledge are purely for the sake of understanding a concept; the immediate utility of such an undertaking is overlooked and perhaps not even recognized when such efforts are undertaken (Shepard 1956; Nelson 1959; Reagan 1967; Pavitt 1991). (Foray & Lundvall 1998) is a detailed discussion of the economic impact of knowledge. (Barnett 1999) mentions that even universities, which are solely meant to create and spread knowledge, might have lost their building what is essential right into our genes. For information storage in DNA and related applications see: (Church, Gao & Kosuri 2012; Lutz, Ouchi, Liu & Sawamoto 2013; Kosuri & Church 2014; Roy, Meszynska, Laure, Charles, Verchin & Lutz 2015).

Deoxyribonucleic acid (DNA) is a molecule composed of two chains (made of nucleotides) that coil around each other to form a double helix carrying the genetic instructions used in the growth, development, functioning and reproduction of all known living organisms and many viruses. DNA, Wikipedia Link

Electronic copy available at: https://ssrn.com/abstract=3282967
way and how they need a new sense of purpose. (Delanty 2001) is about the role of universities in the knowledge society.

(Bozeman & Rogers 2002) admit that determining the value of scientific and technical knowledge poses a great many problems (the value of knowledge shifts dramatically over time as new uses for the knowledge emerge; a related problem is that market-based valuation of knowledge is an inadequate index of certain types of scientific knowledge). They present an alternative framework for the value of scientific and technical knowledge, one based not on market pricing of information, but instead on the intensity and range of uses of scientific knowledge. Their churn model of scientific knowledge value emphasizes the distinctive properties of scientific and technical knowledge and focuses on the social context of its production. They consider the value of scientific and technical knowledge in enhancing the activities of the set of individuals who interact in the demand, production, technical evaluation, and application of scientific and technical knowledge.

It is worth noting that goods that are not actively traded pose many valuation challenges. There are many interesting techniques used to determine the value of assets, especially non-financial ones. (Zhao & Zhou 2011) consider status indicators that determine wine prices such as: individual wine tasting scores rated by critics and more general status indicators like classified appellation affiliation, extra designation on the label, or a winery’s organizational status; (Gustafson, Lybbert & Sumner 2016) present an experimental approach to measure consumer willingness to pay for wine attributes; (Costanigro, McCluskey & Mittelhammer 2007) provide empirical evidence that the wine market is differentiated into multiple segments or wine classes based on price ranges and find evidence that consumers value the same wine attributes differently across categories.

(Ortiz, Stone & Zissu 2017) develop a valuation model for a crowd funding initiative to finance a pipeline to transport beer, which takes into account the terms of the contract, the current and expected future price of beer, and the life expectancy of the investor; (Masset & Weisskopf 2015) examine the performance, selectivity, and market-timing abilities of wine fund managers; (Aznar & Guijarro 2007) discuss aesthetic variables in paintings; (Carmona 2015) is about jewelry appraisal; (De Groot, Wilson & Boumans 2002) present a conceptual framework and typology for describing, classifying and valuing ecosystem functions, goods and services; (Throsby 2003) is a discussion of the non-market value of cultural goods - literature, visual arts, music, theater, dance, etc.; (Lebreton, Jorge, Michel, Thirion & Pessiglione 2009) isolate brain regions that may constitute a system that automatically engages in valuating the various components of our environment so as to influence our future choices.

As we go about applying finance principles to assess the value of knowledge in all domains, we need to bear in mind that all valuations are subjective since they are done by social beings and a hallmark of the social sciences is the lack of objectivity. Here we assert that objectivity is with respect to comparisons done by different participants and that a comparison is a precursor to a decision. (Nagel 2012) clarifies the distinction between subjective and objective; (Little 1991; Manicas 1991; Rosenberg 2018) discuss the philosophy of the social sciences; (Kashyap
points out that the social sciences are objectively subjective; (Gerring 2011) is a detailed account of social science methods used to provide explanations of observed phenomena; (Harré 1985) explores the premise that knowledge is a basis for moral good and relates various views about the nature of science to different historical schools of philosophy; (Papineau 2002; O’hear 1993; Tweney, Doherty & Mynatt 1981) are detailed accounts of the philosophy and methodology of science.

3.2 Framework (D&A) for Knowledge Valuation

Assumption 3. Despite the several advances in the social sciences, we have yet to discover an objective measuring stick for comparison, a so called, True Comparison Theory, which can be an aid for arriving at objective decisions. Hence, despite all the uncertainty in the social sciences, the one thing we can be almost certain about is the subjectivity in all decision making.

For our present purposes, the lack of such an objective measure means that the difference in comparisons, as assessed by different participants, can give rise to different valuations for the same element of knowledge (or asset). We consider two extreme individuals and their perspectives, which would influence their valuations.

Definition 1. Type A person, who has All the known knowledge in the universe. So if any new knowledge becomes available he is desperate to have it, since without this new knowledge he is incomplete.

We have to mention here that a type A person would place an extremely high valuation on every element of knowledge he already has and any new knowledge that might come up.

Definition 2. Type Z person, who has no knowledge about anything in the universe. So he cares nothing about any knowledge, wants nothing and his valuation for all pieces of knowledge would be Zero.

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8 Philosophy of science is a sub-field of philosophy concerned with the foundations, methods, and implications of science. The central questions of this study concern what qualifies as science, the reliability of scientific theories, and the ultimate purpose of science. [Philosophy of Science, Wikipedia Link](https://en.wikipedia.org/wiki/Philosophy_of_science)

The scientific method is an empirical method of knowledge acquisition which has characterized the development of science since at least the 17th century. [Scientific Method, Wikipedia Link](https://en.wikipedia.org/wiki/Scientific_method)

The philosophy of social science is the study of the logic, methods, and foundations of social sciences such as psychology, economics, and political science. [Philosophy of Social Science, Wikipedia Link](https://en.wikipedia.org/wiki/Philosophy_of_social_sciences)
It is worth noting that there might be views expressed by people, (or, people with beliefs in this very world that we live in who do not seek anything not even knowledge), that once you stop looking for things you will have everything. This would be a contradiction to our definition, since in this case the type Z person is the one who wants nothing and would put a valuation of zero on any new knowledge, but since he wants nothing he would actually have everything and he becomes the type A person. Alternately, the type A person, has all the knowledge in the world which suggests that he wants nothing, or his valuation of everything including knowledge is zero, making him the type Z person. The scope of our discussion will be restricted to someone who is between Type A and Z, so that he has a non-zero valuation of every piece of knowledge.

The assumption made in finance regarding homogeneous expectations (Levy & Levy 1996; Chiarella & He 2001), especially in the derivation of many asset pricing models, investment analysis and portfolio management principles (Bodie, Kane & Marcus 2014), is stunningly sophisticated, yet seemingly simplistic. Most people would argue that no two people are alike, so this assumption does not seem validated (Valsiner 2007; Buss 1985; Plomin & Daniels 1987). Then again, rethinking this slightly might lead to the following modified assumption.

**Assumption 4.** *The homogeneous expectations assumption in finance is perhaps a very futuristic one where we are picking the best habits and characteristics from our fellow beings (maybe not just humans?); and the environment we live in and the external stimulus we receive tends to become more similar (or we start to perceive it as more alike?), and at some point in the future we might tend to have more in common with each other fulfilling this great assumption, which seems more of a prophecy.)*

There are many issues if we become too much like one another (Slatkin 1987; Frankham 1995; 1997). But with respect to finance we might evolve enough, so that one day we might have the same expectations with respect to our monetary concerns. This would also be the day when the Bid-Offer spread would cease to matter, or, we would be indifferent to it making every coffee shop, theater, street corner, pub, or everywhere . . . a venue for any product (Kashyap 2015).

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9 If we become too similar, then mother nature, or, evolution, will have less to work with. Since more differences tell her, which traits are better for certain conditions; and many possibilities create stronger survival potential. This is studied under the label inbreeding. Not to mention, if we all look alike, think alike and act alike the world would be a very mundane place.

Inbreeding is the production of offspring from the mating or breeding of individuals or organisms that are closely related genetically. [Inbreeding, Wikipedia Link](https://ssrn.com/abstract=3282967)
Assumption 5. Using a related concept from economics regarding equilibriums (Dixon 1990; Varian 1992) when we continue to evolve and evolve towards similarity, both the type A and type Z kind of person can be equilibriums, since they are (they become?) the same kind of person with respect to their views on identifying the value of elements around them.

To better understand knowledge, let us first start with what is not knowledge. Anything that we don’t know is not knowledge. We will further try to provide one definition for what knowledge might be keeping in mind that as generic as we want to make any definition, we need to be open to the possibility that the definition might need to be altered depending on the specifics of the situation.

Definition 3. Knowledge is a connection between elements of this universe. The elements could be many (more than two), two, or in some cases a link from one element to the same element and all other combinations. This requires us to clarify what is an element. We suggest that the element discussed here is anything that belongs to this universe and any characteristic of that element, as

10 Equilibriums should perhaps be more aptly named Quasi-Equilibriums since we never know what a true equilibrium is; but perhaps any system fluctuates between multiple equilibriums, somewhat like a see-saw: (Mantzicopoulos & Patrick 2010; Stocker 1998).

A seesaw (also known as a teeter-totter or teeter-board) is a long, narrow board supported by a single pivot point, most commonly located at the midpoint between both ends; as one end goes up, the other goes down. [Seesaw, Wikipedia Link]

11 Many times what we don’t know is scary or can cause confusion or frustration. Confusion and Frustration, though, scary and ugly to begin with, can be powerful motivators, as long as, we don’t let them bother us. This is because:

• Confusion is the beginning of Understanding.

• Necessity, is the mother of all creation / innovation / invention, but the often forgotten father, is Frustration, which is sometimes, even more necessary, than necessity herself. Simply put, some amount of frustration can be highly stimulating and lead to great possibilities.

• What we learn from the story of, Beauty and the Beast (De Beaumont 1804), is that, we need to love the beasts to find beauty. Hence, if we start to love these monsters (Confusion and Frustration), we can unlock their awesomeness and find truly stunning solutions.

• Beauty and the Beast (French: La Belle et la Bête) is a fairy tale written by French novelist Gabrielle-Suzanne Barbot de Villeneuve and published in 1740 in The Young American and Marine Tales (French: La Jeune Américaine et les contes marins). Her lengthy version was abridged, rewritten, and published first by Jeanne-Marie Leprince de Beaumont in 1756. [Beauty and the Beast, Wikipedia Link]
observable in this universe.12

Remark 1. These connections can also be viewed as answers to appropriately posed questions governed by suitable assumptions and definitions. Our present endeavors in knowledge creation, or scientific research, can be understood in this way.

Remark 2. Another possibility, which we consider in more detail in (section 4.4), is that knowledge, or the connections between elements, exists with or without our observation of those links. Many times our cognizance or understanding could be incomplete or even incorrect and progressively gets better even though the actual phenomenon itself has not changed.

Remark 3. (Section 4.6) considers the dimensions to which we are (seem to be?) presently restricted to and the possibility that any understanding of our universe is with respect to the limitations imposed by the dimensions we are able to perceive. The possibility of higher dimensions means the possibility of better comprehension or it could simply be an altered view of perception from a different number of dimensions (Kashyap 2019).

Remark 4. It is important to emphasize here that the connection between elements is not just at any particular point in time, but inter-temporally and even across other higher dimensions. This also tells us that knowledge from one time period is valuable for other time periods since they could be linked. Knowledge across time periods can be useful should there be a possibility of the same connection reoccurring, or, by using the connection we know about to create a modified or new connection; that is the value of knowledge is enhanced if we are able to use (or reuse) existing connections discovered from another time period.

Definition 4. Knowledge machines are elements themselves that look to create, or discover, or record connections between the various elements. They are people, research journals, books, music, robots and everything else that fulfills the property of being part of the efforts to add to the collective pool of knowledge. We can also term them knowledge seekers.

12 We emphasize the word universe since our definition would need to be modified once we establish (there is already speculation regarding this eventuality) the possibility of other universes (Carr 2007; Weinberg 2007).

The multiverse is a hypothetical group of multiple universes including the universe in which humans live.  

Electronic copy available at: https://ssrn.com/abstract=3282967
4 Switching on Knowledge Machines

4.1 Money Machines that Run Secretly

In finance (Cochrane 2009), we talk about something called as a “Money Machine” which will get turned off, as soon as people step in to take advantage of it. This is also known as arbitrage (Shleifer & Vishny 1997; Brown & Werner 1995) and it is possible when the law of one price is violated (Isard 1977; Crouhy-Veyrac, Crouhy & Melitz 1982; Protopapadakis & Stoll 1983; Goodwin, Grennes & Wohlgenant 1990; Froot, Kim & Rogoff 1995). This is nothing but buying the asset that we think is cheaper than what it should be and selling the asset that we think is more expensive than what it should be. This is based on the price of the asset, or, the market assessment of the asset in an applicable market, as of today. When no price is available either due to the lack of a corresponding market or participants, we can use the expectation of discounted future cash flows.

There is a generally accepted concept in finance theory called the time value of money (Ross, Westerfield & Jaffe 2002; Bierman Jr & Smidt 2012; Delaney, Rich & Rose 2016; Petters & Dong 2016; Marty 2017; Olson &

13 If payoff \( A \) is always at least as good as payoff \( B \), and sometimes \( A \) is better, then the price of \( A \) must be greater than the price of \( B \). If two portfolios (assets or goods) have the same payoffs (in every state of nature), then they must have the same price. A few other subtleties can arise; Arbitrage is possible when one of three conditions is met:

- The same asset does not trade at the same price on all markets ("the law of one price").
- Two assets with identical cash flows do not trade at the same price.
- An asset with a known price in the future does not trade today at its future price discounted at the risk-free interest rate (or, the asset has significant costs of storage; as such, for example, this condition holds for grain but not for securities).

In economics and finance, arbitrage is the practice of taking advantage of a price difference between two or more markets: striking a combination of matching deals that capitalize upon the imbalance, the profit being the difference between the market prices. When used by academics, an arbitrage is a (imagined, hypothetical, thought experiment) transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, it is the possibility of a risk-free profit after transaction costs. For example, an arbitrage opportunity is present when there is the opportunity to instantaneously buy something for a low price and sell it for a higher price. [Arbitrage, Wikipedia Link]

14 The law of one price (LOOP) states that in the absence of trade frictions (such as transport costs and tariffs), and under conditions of free competition and price flexibility (where no individual sellers or buyers have power to manipulate prices and prices can freely adjust), identical goods sold in different locations must sell for the same price when prices are expressed in a common currency. This law is derived from the assumption of the inevitable elimination of all arbitrage. [Law of One Price, Wikipedia Link]
This idea can be simply stated as the fact that most people (and perhaps even animals with non-monetary rewards: Hayden 2016) would rather have a certain sum of money now rather than the same sum of money later. This intuitively makes sense to most people, since we are not sure whether we will receive that certain sum in the future, due to the main uncertainties that the future holds. Though, it can be easily seen that for people that can travel through time, money would be the same whether now or later. As unlikely or likely the possibility of time travel might be, it is mainly being used here to illustrate further the notion of why money has different values at different periods of time (section 4.6).

4.2 Present Value and Future Value of Money

For completeness, and to act as a reference point, we summarize the formula for the time value of money. More mathematically advanced treatments can use the concept of discount functions in both discrete, or, continuous time with exponential or hyperbolic discounting, among other possibilities, including the usage of differential equations; see: (Thaler 1981; Frederick, Loewenstein & O’donoghue 2002; Marzilli Ericson, White, Laibson & Cohen 2015; Cruz Rambaud & Ventre 2017; Figure 2)

The essence of the below equations (eqns: 1; 2; 3; 4), with regard to money are that money in the future decreases in value when it is measured in the present, or brought into the present, since the interest rate is usually non-negative, \( i \geq 0 \); to be precise, let us term this discounting. Likewise,

\[ g(D) = \frac{1}{1 + kD} \]

where \( g(D) \) is the discount factor that multiplies the value of the reward, \( D \) is the delay in the reward, and \( k \) is a parameter governing the degree of discounting. This is compared with the formula for exponential discounting: \( f(D) = e^{-kD} \). Hyperbolic Discounting. Wikipedia Link

Compounding or Discounting arises due to two possibilities (which we can also view as principles in this case) one of which is direct and the other is indirect.

- When we compound we are accumulating interest, which is the direct return we get that justifies this increasing value we assign to money that we have today, which we could potentially invest. Likewise, when we discount a sum of money we are likely to
money from the present when it is to be valued in the future, or taken into the future, increases in value; again for precision sake, let us call it compounding.

\[ PV = \frac{FV}{(1+i)^n} \iff FV = PV(1+i)^n \quad (1) \]

Here, \( PV \) (present value) is the value of money at \( time = 0 \) or at the present moment. \( FV \) (future value) is the value of money at \( time = n \) or in the future. \( n \) is the number of time periods (not necessarily an integer). \( i \) is the rate (interest rate) at which money compounds each period. The cumulative present value of multiple future cash flows can be calculated by summing the contributions of \( FV_t \), the value of future cash flows at time \( t \),

\[ PV = \sum_{t=1}^{n} \frac{FV_t}{(1+i)^t} \quad (2) \]

Expressed using discount functions, \( f(\cdots) \), which for money is less than one, that is \( f(i,n) = 1/(1+i)^n \leq 1 \), we can write this as,

\[ PV = FV f(i,n) \iff FV = \frac{PV}{f(i,n)} \quad (3) \]

We could include the growth rate of money, \( g \), to depict any increase or decrease in value not captured by interest rates. That is \( g \) would be the growth rate of money over each time period. To get sensible results, we usually require \( g < i \) though \( g \) can be positive or negative. This would change the discount function, \( f(\cdots) \), as follows: \( f(i,g,n) = (1+g)^n/(1+i)^n \leq 1 \)

\[ PV = FV f(i,g,n) \iff FV = \frac{PV}{f(i,g,n)} \quad (4) \]

receive in the future, we are taking away interest, which upholds our rule to decrease the value of money.

- The indirect possibility is that our investment could become more valuable not just due to the interest we receive but due to other factors wherein our investments could intrinsically do well becoming more valuable (other benefits such as good will, reduced taxes etc. can occur due to making investments), which could act as indirect returns to us when we have ownership in the investment. Similarly, when we discount we are taking into account the risk that we might never receive that sum of money in the future, which makes the value of money lesser in an indirect way.

- For simplicity, the general practice is to club together all these possibilities into one number called the discount rate, which is related to interest rates, inflation and risk among other things. This also tells us that the rate for compounding and discounting could be different.
4.3 Once On, Never Off

When an opportunity to make money is known and if many people become aware of this opportunity, it usually disappears (Brealey, Myers, Allen & Mohanty 2012; Foot-notes 13 [14]). *In contrast to money (making), the most wonderful thing about knowledge (creation) is that if more people know about it, the more switched on it will be.* Once we become acquainted with a connection, (either ourselves, or due to the guidance of someone else), we generally see other links; we either put a spin on the connection by relating it to other elements, (new connections), or, we find other characteristics of the same connection, which by our (definition 3) can be viewed as new connections. Hence as more people become aware of any piece of knowledge, they add more pieces of knowledge to it making the overall body of knowledge grow with time.

That being said, the dissimilarities between money and knowledge do not end there. *Money is not like knowledge in many ways.* For most of us money is quantifiable but knowledge is not. It is hard to individuate or shred knowledge into bits and pieces in a way that is convincing to most. Money requires collective agreement and it has value due to this consensus; true knowledge does not require acceptance, though recognition of it does.

That knowledge has value is usually not questioned, but to persuade everyone that knowledge is like money is a tough ask; except perhaps for die-hard classical economists, who can will themselves to believe that everything has monetary value (Sandel 2012).

To overcome some of these objections, we start by looking at the current mechanisms through which we capture,
represent and store knowledge. Knowledge is collected by doing research (Creswell 2008) and the most fundamental units or tools of research can be thought of as questions and answers. Knowledge is represented as concepts, ideas or in a more rigorous manner as mathematical theorems. Knowledge is deposited in journals, books, articles and the like. To provide a monetary analogy, this would be like the different currencies, denominations (dollars, cents, etc) and forms (credit card, cheques, cash, gold, etc.) we use for money and store it in banks, safes at home, under our mattress or wherever at times.

This simplification allows us to count (introduce numbers) or use numerical methods to measure knowledge that we have accumulated in familiar places. The impact of this knowledge or its worth would be in how many new connections it will spawn or how it will be connected to other elements of knowledge as time passes. This is a conservative approach as it would under-count and undervalue the knowledge we have gathered. We are ignoring the links to knowledge outside our stockpiles since we do not explicitly consider this knowledge outside our

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19 Research has been defined in a number of different ways, and while there are similarities, there does not appear to be a single, all-encompassing definition that is embraced by all who engage in it. [Research, Wikipedia Link]

• One definition of research is used by the OECD, "Any creative systematic activity undertaken in order to increase the stock of knowledge, including knowledge of man, culture and society, and the use of this knowledge to devise new applications.'

• Another definition of research is given by John W. Creswell (Creswell 2002), who states that 'research is a process of steps used to collect and analyze information to increase our understanding of a topic or issue'. It consists of three steps: pose a question, collect data to answer the question, and present an answer to the question.

• The Merriam-Webster Online Dictionary defines research in more detail as "studious inquiry or examination; especially : investigation or experimentation aimed at the discovery and interpretation of facts, revision of accepted theories or laws in the light of new facts, or practical application of such new or revised theories or laws'

• These definitions of research ignore the possibility that research can be undertaken without the aim of devising new applications or consciously collecting data and outside the defined boundaries of our understanding of experiments and investigative processes; and surely in ways we don’t know how research can be done.

• There is a ton of literature on how to make research economically useful (Pavitt 1991); what are benefits and costs of research collaboration at the individual, firm and international levels in the past, present and future (Katz & Martin 1997; Melin 2000; Beaver 2001; Miotti & Sachwald 2003);

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20 For brevity we ignore knowledge outside the boundaries of our knowledge stores but available everywhere; for example: growing on trees, hanging on statues, and so on, both literally and figuratively. We justify this preclusion by stating that for most of us there are no easy ways to snatch and store knowledge present in kaleidoscopic forms; for those of us unable to spot knowledge handed to us in papers (perhaps, sent to us for review), that do not match our exact templates, page limits, artificial discipline boundaries we have imposed and so no, it would be a hard ask to find knowledge lying under a paddy field. It would be even harder to assess the impact over time of this knowledge that has not yet been seen.
familiar stash. We would like to highlight that we are not trying to put an exact monetary value on knowledge. For example, if someone has come up with a new theorem and published it in an article, our valuation is not the exact amount of money to pay for their contribution; though our techniques are a numerical approach at putting a value, they will not provide a literal price for any knowledge transaction (our main result, theorem 1, makes this clear).

**Another core problem is to attempt to value knowledge only in terms of time elapsed.** But if anything, as time passes the breadth covered by any piece of knowledge, how well it integrates into its related topics (or seemingly unrelated fields; assumption 1) and how much support it has, will increase as new connections will get added. With additional breadth, value will increase. This can be measured by $g$, the growth rate of knowledge over each time period. Even by ignoring breadth we are putting a lowest possible estimate on the value of knowledge. It is possible that we have not yet uncovered additional connections, that increase the breadth of applicability of this knowledge. If we have reasons to believe that certain knowledge gets stale over time, (section 4.4) considers the plausibility of this alternative.

### 4.4 The Relativity of Generally New, But Not So Specially New Theories of Knowledge

We consider the possibility that new knowledge could replace old knowledge making it obsolete. (Shapere 1980; 1989) are fascinating accounts of scientific change; (Fuchs 1993) considers a sociological theory of scientific change that can address many problems including the relationship between the natural and social sciences; (Laudan, Laudan & Donovan 1988), which is housed in (Donovan & Laudan 2012), a larger collection of empirical studies on scientific change, subject key claims of some of the theories of scientific change to the kind of empirical scrutiny that has been so characteristic of science itself; (Laudan, ... & Wykstra 1986) consider the role of historians and their views on evaluating and improving theories of scientific change; also relevant would be the many discussions on scientific realism: (Bhaskar 1998; Psillos 2005; Bhaskar 2009; Smart 2014; Sankey 2016).

It is certainly tempting to think that newer more accurate theories would render the older less accurate theories covering the same domain as less useful. Newtonian mechanics (Halliday & Resnick 1967) was incredibly useful

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21 Scientific realism is the view that the universe described by science is real regardless of how it may be interpreted. [Scientific Realism, Wikipedia Link](https://en.wikipedia.org/wiki/Scientific_realism)

22 Classical or Newtonian mechanics provides extremely accurate results when studying large objects that are not extremely massive and speeds not approaching the speed of light. When the objects being examined have about the size of an atom’s diameter, it becomes necessary to introduce the other major sub-field of mechanics: quantum mechanics (Shankar 2012). To describe velocities that are not small compared to the speed of light, special relativity is needed. In case that objects become extremely massive, General relativity

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and valuable but seems to have become less so once Einstein’s theory of special relativity (Einstein 1956) replaced it to cover the cases of objects moving at speeds closer to the speed of light.

- **First, we would like to distinguish between theory and knowledge.**

We will continue with our mechanics example, but it will become clear that this distinction applies more generally. A theory is a model or a simplification of reality, based on various assumptions and valid for a limited range of conditions (for discussions about the use and abuse of models, see: Morgan & Morrison 1999; Derman 2011; Rescigno, Beck & Thakur 1987). In Physics the models are usually mathematical that apply under certain physical criteria. Newton’s laws are mathematical models that are limited to non-relativistic speeds (speeds much lower than the speed of light) and low gravitational fields, and within those limits they provide exceptionally accurate answers.

Saying that Newton was proved wrong by Einstein would be incorrect in any sense. What relativity did was to expand the range of physical conditions over which we can explain the movement of objects. Special relativity extended the range to include high speeds, and general relativity extended it again to include high gravitational fields where Newtonian mechanics becomes applicable. [Newtonian Mechanics, Wikipedia Link](https://en.wikipedia.org/wiki/Newtonian_mechanics)

Quantum mechanics (also known as quantum physics, quantum theory, the wave mechanical model, or matrix mechanics), including quantum field theory, is a fundamental theory in physics which describes nature at the smallest scales of energy levels of atoms and subatomic particles. [Quantum Mechanics, Wikipedia Link](https://en.wikipedia.org/wiki/Quantum_mechanics)

General relativity (also known as the general theory of relativity) is the geometric theory of gravitation published by Albert Einstein in 1915 and the current description of gravitation in modern physics. [General Relativity, Wikipedia Link](https://en.wikipedia.org/wiki/General_relativity)

Special relativity (also known as the special theory of relativity) is the generally accepted and experimentally well-confirmed physical theory regarding the relationship between space and time. [Special Relativity, Wikipedia Link](https://en.wikipedia.org/wiki/Special_relativity)

(Diallo, Padilla, Bozkurt & Tolk 2013) make the case that theory can be captured as a model.

A theory is a contemplative and rational type of abstract or generalizing thinking, or the results of such thinking. [Theory, Wikipedia Link](https://en.wikipedia.org/wiki/Theory)

A conceptual model is a representation of a system, made of the composition of concepts which are used to help people know, understand, or simulate a subject the model represents. [Conceptual Model, Wikipedia Link](https://en.wikipedia.org/wiki/Conceptual_model)

(Greca & Moreira 2002) discuss the relationships among physical, mathematical, and mental models in the process of constructing and understanding physical theories.
fields. At present, we do not seem to need theories that explain motion above the speed of light\(^{27}\) and even general relativity is not applicable everywhere because it fails at singularities like the center of black holes (Iyer & Bhawal 2013).\(^{28}\) To summarize, as new theories emerge the criteria under which we can apply them to obtain answers, or knowledge, becomes much larger.

- **Second, we would like to emphasize that theory is not knowledge.**

From (definition 3), Knowledge exists independent of our awareness of it. Connections might exist between objects without our knowledge of those relationships. Theory only helps us to uncover or sense these connections. Then again, new knowledge only becomes more useful by knowing the old knowledge or connection, which shows how the new knowledge might be better and in which situations the new theory would apply. If old knowledge is forgotten or lost, new knowledge, might need to rediscover the old connections, before its value becomes enhanced.

- **Lastly, if any theory is shown to be completely incorrect, it was never true knowledge in the first place. But old incorrect theories are necessary since they will help us recognize what new knowledge cannot be.**

When any theory becomes known to be wrong, we cannot discard it entirely. We still need to know the old erroneous theory as we search for a better understanding of the cosmos using the new theory that has replaced the old theory as our tool. This is because, as we build upon the new theory to overcome any limitations it might have and come up with newer theories, we need to know what theories will not work. This means, new knowledge becomes more valuable in yielding newer knowledge when used together with old knowledge, even if it is now shown to be wrong since it tells us what not to do. This can also be compared to trial and error learning. We try and fail many times,\(^{27}\) There is some awareness and much speculation about objects that can travel faster than light. A tachyon or tachyonic particle is a hypothetical particle that always moves faster than light. Most physicists believe that faster-than-light particles cannot exist because they are not consistent with the known laws of physics (Bilaniuk, Deshpande & Sudarshan 1962; Randall 2006). But surely, beliefs change as unknown laws become known or as our knowledge increases.\(^{28}\) A black hole is a region of space-time exhibiting such strong gravitational effects that nothing, not even particles and electromagnetic radiation such as light, can escape from inside it. At the center of a black hole, as described by general relativity, lies a gravitational singularity, a region where the space-time curvature becomes infinite.

\[\text{Electronic copy available at: https://ssrn.com/abstract=3282967}\]
but at last when we succeed we have avoided the many ways in which we have failed earlier to succeed this time. An example of this would be about finding a way (or connection) from one landmark in a city to another. It is easier to find newer better paths if we know older worse paths that tell us which routes not to take.

We will require newer and better theories as our understanding of the world improves or as our knowledge increases. (Question 2) and associated remarks briefly consider the limiting case or the asymptotics of increasing knowledge. The discussion above (sections 4.3; 4.4) suggests that the discount function for knowledge should always be one or greater than one, stated as the below axiom.

**Axiom 1.** **Knowledge never decreases in value.** If it decreases in value, it was never knowledge to begin with. This gives the following discount function (or weight function), \( h(k,n) \), for knowledge. \( n \) is the number of time periods (not necessarily an integer). \( k \) is the rate (knowledge rate) at which knowledge compounds each period.

\[
h(k,n) = (1 + k)^n \geq 1 \Rightarrow k \geq 0
\]  

(5)

**Remark 5.** Another way to look at change in knowledge is by considering the connections any element of knowledge is likely to establish with other elements of knowledge as time passes. The growth rate in the number of connections to any element of knowledge can be expressed using the function in (Eq. 5). The discount function then tells us how this piece of knowledge will link up with other elements of knowledge. The discount function can be easily modified to include the growth rate of knowledge, \( g \), over each time period.

Instead of a discount function, since by the arguments in (sections 4.3; 4.4; axiom 1) knowledge never decreases in value (irrespective of whether it moves forward or backward in time), we will use the term weight. The weights to be used can be based on the following assumptions:

29 Such unwanted outcomes creep up because we live in a world that requires around 2000 IQ points, to consistently make correct decisions; but the smartest of us has only a fraction of that (Ismail 2014). Hence, we need to rise above the urge to ridicule the seemingly obvious blunders of others, since without those marvelous mistakes the path ahead will not become clearer for us. As Taleb explains, “it is trial with small errors that leads to progress”. That being said, if there are big errors that might incapacitate the person trying the trial from further trials; as long as someone else has observed the attempts with huge errors, the rest of society benefits from it; assuming, of course, that the big blow up has left a non-trivial portion of society intact, or at-least not too shaken up. (Ismail 2014) mentions the following quote from Taleb, “Knowledge gives you a little bit of an edge, but tinkering (trial and error) is the equivalent of 1,000 IQ points. It is tinkering that allowed the industrial revolution”. **Nassim Taleb and Daniel Kahneman discuss Trial and Error IQ Points, among other things, at the New York Public Library on Feb 5, 2013.**
Assumption 6. *Past knowledge is important the older it is. That is knowledge from long ago is more valuable than knowledge yesterday.*

* This is because the knowledge we have today is built on the connections we have created or discovered in the past. So the foundation for today is the knowledge from long ago. The more knowledge we have from the past the more consistent the knowledge we have today (that it is connected to more elements making it important and crucial to new elements that will get added); otherwise any alternate connections from the past could render a large body of knowledge obsolete, but then again, only by knowing alternate chains of connections, which though fallow, we can create better connections. So knowledge is never wasted and it never loses value.

* A subtle point that we need to understand is that knowledge or the connections between elements could stay the same over time, while acknowledging that the awareness of the connections could be time varying. Time is the dimension we do not have control over as per our current understanding and methods of navigating the cosmos (section 4.6 has more details). As we move through time we uncover new connections. This implies that someone could discover all the connections between various elements at some point in time, but another person could come to know about these same connections at another point in time. Their overall knowledge is the same, but this awareness has happened at different points in time, which is a constraint due to the physical limitations we presently have in our Universe. The assumptions we make here is with respect to the total recorded knowledge that is presently accumulated by all knowledge seekers.

* We can make a related assumption that knowledge becomes more important at an increasing rate, going back from today (Rosser & Lis 2016; Courant, Robbins & Stewart 1996; Wagenaar & Sagaria 1975; Meadows, Meadows, Randers & Behrens 1972; Leike 2001; Foot-note 30 Figure 3). When any piece of knowledge becomes most useful, its weight at that time can be captured as the position of the center of the peak of a Gaussian function, or the mean of a normal distribution and the weight at other times could taper off similar to the normal density function: (Rao 1973; Cramér 2016; Foot-note 31 Figure 4).

Assumption 7. *Future knowledge is important the closer it is to us. That is knowledge that is far away from today is less valuable than knowledge about tomorrow.*

* If future knowledge cannot be immediately connected to the knowledge we have now, or at any point in time, we fail to appreciate its value. For example, if we took the knowledge of differential equations to the past
before we had knowledge of algebra or before zero was being used, it would be very hard to see the importance of calculus.

• While this assumption seems counter intuitive, let us clarify with an example: If someone tells us (correctly is another assumption which assumes they are using the right theory capable of making this prediction) that the world will end in 10 seconds versus 10 years. It would seem that it does no good to know that the world will end in 10 seconds, but knowing exactly when it will end in 10 years is incredibly valuable. In this latter case, it would seem that future knowledge is more important the “further” away from us it is, because there’s more that we can do in preparation for the end of the world. But the knowledge that the world will end in 10 years has to be accompanied by the knowledge that it will not end in every smallest fraction of time before those ten years. Without this knowledge, it is no good that we prepare for ten years only to realize the world is gone well before those ten years and will reestablish itself, perhaps in the same or in an altered form. It is even likely that this could happen multiple times in those ten years and hence all the knowledge within those ten years is essential and to be more precise, more essential than the knowledge ten years away.

• We can make a related assumption that knowledge becomes less important at an increasing rate going forward from today (Figure 3), similar to what we discussed in the point above, (assumption 6), for past knowledge when any piece of knowledge becomes most useful its weight at that time can be represented as the position of the center of the peak of a Gaussian function, or the mean of a normal distribution and the weight at other

30 Exponential growth is exhibited when the rate of change—the change per instant or unit of time—of the value of a mathematical function is proportional to the function’s current value, resulting in its value at any time being an exponential function of time, i.e., a function in which the time value is the exponent. [Exponential Growth, Wikipedia Link](https://en.wikipedia.org/wiki/Exponential_growth) [Exponential Growth, Mathworld Link](https://mathworld.wolfram.com/ExponentialGrowth.html) The exponential function \( x(t) = x(0)e^{kt} \), \( k > 0 \), satisfies the linear differential equation:

\[
\frac{dx}{dt} = kx
\]

saying that the change per instant of time of \( x \) at time \( t \) is proportional to the value of \( x(t) \), and \( x(t) \) has the initial value \( x(0) \). In the above differential equation, if \( k < 0 \), then the quantity experiences exponential decay.

A quantity is subject to exponential decay if it decreases at a rate proportional to its current value. [Exponential Decay, Wikipedia Link](https://en.wikipedia.org/wiki/Exponential_decay) [Exponential Decay, Mathworld Link](https://mathworld.wolfram.com/ExponentialDecay.html) Symbolically, this process can be expressed by the following differential equation, where \( N \) is the quantity and \( \lambda \) (lambda) is a positive rate called the exponential decay constant:

\[
\frac{dN}{dt} = -\lambda N
\]

The solution to this equation is:

\[
N(t) = N_0e^{-\lambda t}
\]

where \( N(t) \) is the quantity at time \( t \), and \( N_0 = N(0) \) is the initial quantity, i.e. the quantity at time \( t = 0 \).
times could taper off similar to the normal density function: \( \text{(Figure 4)} \)

- It is of course possible that knowledge from different times in the far away future, that can shed light on connections we are working with today, can become valuable; so there can be weights for future knowledge that have multiple regions of significant value. This is expressed by the use of multi-modal probability distributions: \( \text{(Cramér 2016; Figure 5)} \)

**Assumption 8.** Certain knowledge could be useful at certain points in time, and then decays around a certain point of high importance.

- We view this as impulse functions over an instantaneous period of time or as impulse response functions

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\[ f(x) = ae^{-\frac{(x-b)^2}{2c^2}} \]  \( \text{(9)} \)

for arbitrary real constants \( a, b \) and non zero \( c \). It is named after the mathematician Carl Friedrich Gauss. The graph of a Gaussian is a characteristic symmetric 'bell curve' shape. [Gaussian Function, Wikipedia Link][1] [Gaussian Function, Mathworld Link][2]

In probability theory, the normal (or Gaussian or Gauss or Laplace–Gauss) distribution is a very common continuous probability distribution. The normal distribution is sometimes informally called the bell curve. However, many other distributions are bell-shaped (such as the Cauchy, Student’s t, and logistic distributions). [Normal Distribution, Wikipedia Link][3] The probability density of the normal distribution is:

\[ f(x | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]  \( \text{(10)} \)

where \( \mu \) is the mean or expectation of the distribution (and also its median and mode), \( \sigma \) is the standard deviation, and \( \sigma^2 \) is the variance.

---

\[ f(x; \alpha, \beta) = \text{constant} \cdot x^{\alpha-1}(1-x)^{\beta-1} \]

\[ = \frac{1}{\text{B}(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1} \]  \( \text{(11)} \)

where the beta function, \( \text{B} \), is a normalization constant to ensure that the total probability is 1. In the above equations \( x \) is a realization, an observed value that actually occurred, of a random process \( X \). The beta function, also called the Euler integral of the first kind, is a special function defined by

\[ \text{B}(x, y) = \int_{0}^{1} t^{x-1}(1-t)^{y-1} \, dt \quad ; x, y \text{ can be complex numbers with their real part greater than zero.} \]  \( \text{(12)} \)

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[1]: https://en.wikipedia.org/wiki/Gaussian_function
[2]: https://mathworld.wolfram.com/GaussianFunction.html
[3]: https://en.wikipedia.org/wiki/Normal_distribution

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whose effect persists over slightly longer intervals of time around the time of the shock. That is when certain knowledge becomes extremely valuable over a localized duration of time and lesser in value (negligible in comparison to its value when it is highest) as we move away that region of time (Nikodým 1966; Hassani 2009; Hamilton 1994; Figure 6; 7).

33 In mathematics, the Dirac delta function (δ function or impulse function) is a generalized function or distribution introduced by the physicist Paul Dirac. It is used to model the density of an idealized point mass or point charge as a function equal to zero everywhere except for zero and whose integral over the entire real line is equal to one. The Dirac delta can be loosely thought of as a function on the real line which is zero everywhere except at the origin, where it is infinite.

\[ \delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \] (13)

and which is also constrained to satisfy the identity

\[ \int_{-\infty}^{\infty} \delta(x) \, dx = 1 \] (14)

1. This is merely a heuristic characterization. The Dirac delta is not a function in the traditional sense as no function defined on the real numbers has these properties. The Dirac delta function can be rigorously defined either as a distribution or as a measure.

2. A closer look at the impulse function should tells us that we can also consider suitable impulse response functions to capture the weight we are assigning to how the value of any particular knowledge might change over time.

In signal processing, the impulse response, or impulse response function, of a dynamic system is its output when presented with a brief input signal, called an impulse. Impulse response functions describe the reaction of endogenous macroeconomic variables such as output, consumption, investment, and employment at the time of the shock and over subsequent points in time. A heuristic technique (Ancient Greek: "find" or "discover"), often called simply a heuristic, is any approach to problem solving, learning, or discovery that employs a practical method, not guaranteed to be optimal, perfect, logical, or rational, but instead sufficient for reaching an immediate goal. Distributions (or generalized functions) are objects that generalize the classical notion of functions in mathematical analysis. Distributions or Generalized Functions, Wikipedia Link. In mathematical analysis, a measure on a set is a systematic way to assign a number to each suitable subset of that set, intuitively interpreted as its size. In this sense, a measure is a generalization of the concepts of length, area, and volume. Technically, a measure is a function that assigns a non-negative real number or \(+\infty\) to (certain) subsets of a set. Measure, Wikipedia Link.
4.5 Knowledge Valuation

With the above framework, which consists of various assumptions, definitions, notation and terminology (all of which are summarized in appendices 6, 7) we present the answer to our primary research question (question 1) as (theorem 1) below.

**Theorem 1.** The value of all knowledge, or the value of any knowledge at any point in time is growing. This means the past, present and future value of every piece of knowledge is infinity.

\[
PRVK \triangleq FUVK (k, n) = \infty \iff FUVK \triangleq PRVK \bar{h} (\bar{k}, n) = \infty
\]

\[
PRVK \triangleq PAVK (k, n) = \infty \iff PAVK \triangleq PRVK \bar{h} (\bar{k}, n) = \infty
\]

\[
PAVK \triangleq FUVK h (k, n) = \infty \iff FUVK \triangleq PAVK \bar{h} (\bar{k}, n) = \infty
\]

Here, \(PAVK, PRVK\) and \(FUVK\) denote the past (PA), present (PR) and future (FU) Value of any element of Knowledge. The notation \(t \triangleq m\) means “\(t\) is equal by definition (under certain conditions) to \(m\)”. \(n\) is the number of time periods (not necessarily an integer). \(k, \bar{k}\) are the rates (knowledge rates) at which knowledge compounds each period. \(h (k, n)\) and \(\bar{h} (\bar{k}, n)\) are the weight functions and they satisfy the property given in (axiom 1) as below,

\[
h (k, n) = (1 + k)^n \geq 1 \Rightarrow k \geq 0
\]

\[
\bar{h} (\bar{k}, n) = (1 + \bar{k})^n \geq 1 \Rightarrow \bar{k} \geq 0
\]

**Proof.** Appendix 8.

Remark 6. This result provides a lower bound for the value of knowledge. The implications of this valuation exercise, which places a high premium on any piece of knowledge, are to ensure that participants in any knowledge system are better trained to notice the knowledge available from any source.

We clarify further as to what \(PAVK, PRVK\) and \(FUVK\) mean. \(PAVK\) or the Past Value of Knowledge, would refer to the value in the past of any knowledge that we have today or we are likely to find in the future. \(PRVK\) would refer to the present value of any knowledge that we had from the past or we are likely to find in the future.
future. $FUVK$ would refer to the future value of any knowledge that we have today or we had from the past.

**Remark 7.** When a decision maker is confronted with two or more choices or options, all of which provide infinite value, he is indifferent between the choices and hence a random selection from the options available would suffice.

The policy implications based on this lower bound for all efforts regarding knowledge creation (which is the one of the main purposes or perhaps even the sole purpose of all journals, researchers, etc.) are discussed in (Kashyap 2018). When there are constraints (time and other resources) a random selection from many qualified papers with inputs from reviewers to improve the randomly selected papers will be a more fruitful outcome for everyone involved and for society as well. (Kashyap 2017) has a discussion of how this randomizing approach is part of a bigger series of solution techniques to counter the uncertainty we encounter in our lives.

### 4.6 The Limits, The Physics and The Motivation for Time Travel

The concept of time value of money (or the time value of anything) arises because we (most of us?) do not have a way to travel across time. Physics provides us with a theoretical basis for moving across time (Lewis 1976; Deutsch & Lockwood 1994; Woodward 1995; Nahin 2001; Gott 2002; Arntzenius & Maudlin 2002; Kaku 2009; Nahin 2017), which is one of the dimensions of the physical world we live in. We are four dimensional creatures: latitude, longitude, height and time are our dimensions since we need to know these four co-ordinates to fully specify the position of any object in our universe. This is perhaps best made clear to lay audiences with regards to physics, such as many of us, by the movie Interstellar (Thorne 2014). Also (Sagan 2006) has a mesmerizing account of many physical aspects, including how objects or beings can transform between worlds that are governed by higher or lower dimensions and change their shapes or their physical form when they enter a lower dimensional world though their shape is unchanged in the higher dimensional world. As they move from the higher dimensions into the lower dimensions, they would need to obey the physical laws of the lower dimension, or take the physical shape or be limited by the properties prescribed by the lower dimensional world. This also implies that changing the number of dimensions based on which we sense our environment could alter our understanding of what is around us.

Time is our last dimension since it is the one which we cannot control or move around in. But we can change the other three co-ordinates (the first three co-ordinates are the co-ordinates of space, and we can change where we are in space) and hence we have three degrees of freedom. This points to a possibility that, beings from a higher dimension can travel through time and enter our three dimensional world; but they would need to be limited by the physical rules as dictated by the dimensions of our world. This also suggests that one way to enter our dimension from a higher dimensional universe (or even a lower dimension) would be to be born in our world and
likewise, the way to leave our universe to go into a higher (or lower) dimension, might be to die. And while it might be hard to take physical material from one universe to another. Thoughts, or knowledge, might be mobile across dimensions (Suddendorf & Corballis 2007; Suddendorf & Busby 2003; Suddendorf & Busby 2005; Boyer 2008; Botzung, Denkova & Manning 2008; Suddendorf, Addis & Corballis 2011). And birth (or rebirth) would be one way to travel across time. We could speculate further that the reason for this cycle of births and deaths might be to collect the most precious commodity that we can think of, which is knowledge, since it is the only entity that gains in value as it moves across time.

Now getting back to knowledge, knowledge now or later would still be valuable (Theorem 1). If anything, for a time traveler that travels around time collecting knowledge instead of money, knowledge from different time periods would hold more value, in comparison to money, which would have the same value at any point in time for this time traveler.34

Question 2. What is the ultimate goal of seeking knowledge?

Remark 8. The motivation to be knowledge seekers is to obtain an understanding of the connections between everything in this universe, through time and other possible higher dimensions. This state of possessing complete knowledge could be termed as “Enlightenment” (for a discussion from recent history, see: Kant 1784; Foucault 1984; Norris 1994; Kapferer 2007).35

34 The money a time traveler has would not require an adjustment for simple things like inflation or interest rates since the time traveler will travel to the point in time where he could get the maximum value for his money. Say he wants coffee and he has one dollar, he will go to the date, when he can get the coffee that maximizes his utility (Jehle & Reny 2011; Foot-note 2) both in terms of quantity and quality.

35 Enlightenment is the "full comprehension of a situation". Enlightenment, Wikipedia Link

The other implications of being a knowledge seeker are:

- Being aware of the possibility of attaining complete knowledge, is that attempts at change might appear highly overrated and overstated. We are not saying that change is unnecessary, just that trying to change something without understanding it completely (that is, before attaining enlightenment) might be a recipe for a disaster. If we understood something and then wish to change it that might be warranted, but trying to change something without knowing it fully is ridiculous at best.

- Definitions suggest that everything in this universe has knowledge associated with it and / or it is contributing to the efforts at understanding everything else. This makes everything important, worthy of respect and infinitely valuable (Kashyap 2017; 2018 consider this in the case of publications, job interviews and school admissions).

- An unintended yet welcome consequence of our efforts to understand everything in the universe could be that we might just end up understanding one another better, perhaps, becoming more tolerant in the process; making us wonder whether the true
5 Conclusion

We have formalized a methodology to estimate the value of knowledge and established a lower bound using well established principles from finance. In a related paper (Kashyap 2018), we show that our methodology has significant policy implications for all fields of research and also for all efforts aimed at the creation and dissemination of knowledge. If one of the main purposes or perhaps even the sole purpose of all journals, researchers, etc. is knowledge creation, our valuation suggests that the best way for journals to select submissions would be randomly from a pool of papers meeting certain basic quality criteria. Such a random selection from many qualified papers, (after factoring in constraints such as reviewer load, number of possible publications etc.), with inputs from reviewers to improve the randomly selected papers will be a more fruitful outcome for everyone involved and for society as well. In such a scenario, the editors and reviewers are not looking for ways to reject a paper instead they are coaching the authors to ensure a better final end product.

Despite any rough edges in our discussion\(^{36}\), our only hope is that this paper represents the first step towards a formal mechanism for putting the value on something, which we deem valuable, but fail to recognize it in most places we see it (Foot-note \(^{29}\)). Just because we do not see a connection does not mean that there is no connection. Newton discovered gravity, which has existed since time immemorial, only when an apple fell on his head. Though the truth behind this myth can be debated, the metaphor is relevant for us, since we sometimes only notice things that hit us: (McKie & De Beer 1951; Fara 1999)\(^{37}\) We need to try harder and be more open to acknowledging the purpose of all knowledge creation might be to simply make us more tolerant. If someone is completely ignorant and still highly tolerant then they have achieved the same end goal as someone who goes about understanding everything and then becomes tolerant. This would also be the case when our type A and type Z person end up in identical states.

\(^{36}\) To remind ourselves about one thing on closing: "The source of most (all) human conflict (and misunderstanding) is not because of what is said (written) and heard (read), but is partly due to how something is said and mostly because of the difference between what is said and heard and what is meant and understood . . ." Part of the problem is also due to the use of a very imprecise language like English for communication. If the only paper written thus far in recorded history (to the best of our knowledge) to attempt to find the value of knowledge using numerical techniques, does not find a place in every journal it is submitted to, then journals (and researchers) have lost touch with their roots which is to find and disseminate knowledge (no matter what form and shape it comes in).

\(^{37}\) Squirreled away in the archives of London’s Royal Society is a manuscript containing the truth about the apple. It is the manuscript for what would become a biography of Newton entitled Memoirs of Sir Isaac Newton’s Life written by William Stukeley, an archaeologist and one of Newton’s first biographers, and published in 1752 (Stukeley 1936). Newton told the apple story to Stukeley, who relayed it as such: “After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some apple trees... he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion’d by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself...” [Newton Gravity Apple Linke]
smallest piece of new knowledge that might have been brought to light by anyone from anywhere about anything and when there are constraints (time and other resources) a random selection from many qualified papers will lead to better utilization of resources and greater productivity for everyone involved (Kashyap 2018).

Success is a very relative term. In the extreme case, which we study a bit about in finance as well, one person’s success (profit) could be someone else’s failure (loss). That being said, to triumph in creating a valuation for knowledge and almost everything else, it is important to know where we are and start the journey towards where we want to be. A consequence (perhaps unintended) of taking the first step on a journey means that the percentage progress we have made, in terms of the distance travelled, shoots up to infinity. So once we start the trip it becomes manageable immediately. The subjectivity in how we compare things (assumption 3) means that the benchmark (assumption 2) can be used if benchmarks can be understood to fall under the category of definitions and assumptions) for knowledge valuation might be constantly changing; though our results show that, whatever the measuring stick (or at least a wide array of benchmarks), the value of every bit of knowledge will remain infinite, which means that we need to keep on learning till there is nothing left to learn or we no longer need to learn anything.

6 Appendix: List of All Definitions and Assumptions within the Paper

Assumption. As a first step, we recognize that one possible categorization of different fields can be done by the set of questions a particular field attempts to answer. Since we are the creators of different disciplines, but we may or may not be the creators of the world (based on our present state of knowledge and understanding) in which these fields need to operate, the answers to the questions posed by any domain can come from anywhere or from phenomenon studied under a combination of many other disciplines.

Question. What is the value of knowledge in any field?

Assumption. Questions & Answers, Q&A, are important, but Definitions and Assumptions, D&A, are even more

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38 Knowledge is synonymous to light in many cultures across the world; for someone that is able to differentiate between darkness and light, a single ray of light can be the greatest ally in keeping hope alive; MacMillan 2008; Newman 2017; One of the most popular festivals of Hinduism, Diwali the festival of lights, symbolizes the spiritual “victory of light over darkness, good over evil and knowledge over ignorance”. [Deepavali, Festival of Lights, Wikipedia Link]

39 Kashyap (2019b) provides an infinite progress benchmark to be successful and has a detailed discussion.
important since changing D\&A could require us to consider different Q\&A.

Assumption. Despite the several advances in the social sciences, we have yet to discover an objective measuring stick for comparison, a so called, True Comparison Theory, which can be an aid for arriving at objective decisions. Hence, despite all the uncertainty in the social sciences, the one thing we can be almost certain about is the subjectivity in all decision making.

Definition. Type A person, who has All the known knowledge in the universe. So if any new knowledge becomes available, he is desperate to have it, since without this new knowledge he is incomplete.

Definition. Type Z person, who has no knowledge about anything in the universe. So he cares nothing about any knowledge, wants nothing and his valuation for all pieces of knowledge would be Zero.

Assumption. The homogeneous expectations assumption in finance is perhaps a very futuristic one where we are picking the best habits and characteristics from our fellow beings (maybe not just humans?) and the environment we live in and the external stimulus we receive tends to become more similar (or we start to perceive it as more alike?), and at some point in the future, we might tend to have more in common with each other, fulfilling this great assumption, which seems more of a prophecy.

Assumption. Using a related concept from economics regarding equilibriums (Dixon 1990; Varian 1992; Foot-note 10), when we continue to evolve and evolve towards similarity, both the type A and type Z kind of person can be equilibriums, since they are the same kind of person with respect to their views on identifying the value of elements around them.
Definition. *Knowledge is a connection between different elements of this universe. The elements could be many (more than two), two or in some cases, a link from one element to the same element and all other combinations.* This requires us to clarify what is an element. We suggest that the element discussed here is anything that belongs to this universe and any characteristic of that element, as observable in this universe (Foot-note 12).

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**Definition.** Knowledge machines are elements themselves that look to create, or discover, or record, connections between the various elements. They are people, research journals, books, music, robots and everything else that fulfills the property of being part of the efforts to add to the collective pool of knowledge. We can also term them knowledge seekers.

**Axiom.** Knowledge never decreases in value. If it decreases in value, it was never knowledge to begin with. This gives the following discount function, \( h(k, n) \), for knowledge.

\[
h(k, n) = (1 + k)^n \geq 1 \Rightarrow k \geq 0 \quad (15)
\]

**Assumption.** Past knowledge is important the older it is. That is knowledge from long ago is more valuable than knowledge yesterday.

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**Assumption.** Future knowledge is important the closer it is to us. That is knowledge that is far away from today is less valuable than knowledge about tomorrow.

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**Assumption.** Certain knowledge could be useful at certain points in time, and then decays around a certain point of high importance.
7 Appendix: Dictionary of Notation and Terminology

- **PV** (present value) is the value of money at time \( t = 0 \) or at the present moment.
- **FV** (future value) is the value of money at time \( t = n \) or in the future.
- **n** is the number of periods (not necessarily an integer).
- **i**, **k** are the rates (of money or knowledge, known as interest rate or knowledge rate) at which the corresponding amount (of money or knowledge respectively) compounds each period.
- \( \bar{k} \) is also a knowledge rate.
- **FV\_t**, is the value of cash flows at time \( t \).
- **g** is the growth rate of money or knowledge over each time period.
- \( f(\cdots) \), is the discount function, which for money is less than one, that is \( f(i,n) = 1/(1+i)^n \leq 1 \) or \( f(i,g,n) = (1+g)^n/(1+i)^n \leq 1 \) when the growth rate \( g \) is included.
- \( h(k,n) \) or \( \bar{h}(\bar{k},n) \) are the discount (or weight) functions for knowledge which are greater than one, that is, \( h(k,n) = (1+k)^n \geq 1 \Rightarrow k \geq 0 \). Strictly speaking since the weight functions for knowledge are greater than one we need to write this as \( h(k,n) = (1+k)^n > 1 \Rightarrow k > 0 \).
- **PAVK**, **PRVK** and **FUVK** denote the past (PA), present (PR) and future (FU) Value of any element of Knowledge.
- \( \triangleq \) is sometimes used in mathematics (and physics) for a definition. The notation \( t \triangleq m \) (often) means “\( t \) is defined to be \( m \)” or “\( t \) is equal by definition to \( m \)” (often under certain conditions).

8 Appendix: Proof of Theorem 1

**Proof.** We first try to establish the present value of a particular piece of knowledge that will become known in the future.

Relationship between **PRVK** and **FUVK**

We divide the time between the future and the present into many intervals. The possibility of some knowledge becoming known in the future will depend on the knowledge we have today and how we will add to it in the
intervals between now and later as we approach the future time when the new knowledge will become known. The new knowledge rests on the many connections it has with the knowledge we know today and the knowledge that we will accumulate from now till the future point in time. Even if the new knowledge does not depend on any of the existing knowledge and its connections from now to the future, we would still need all this knowledge from now to the future to be able to establish that this new knowledge is independent of the other knowledge elements. We can then consider that this new knowledge will have to be linked (or still requires connections) to many pieces of knowledge in the intervals between now till later or we need to be aware of all this knowledge to establish that there is no relationship between the new future knowledge and the knowledge in between. This can be viewed as the change in the connections the new knowledge will have or the growth in the new knowledge based on the connections it will have, as given by the weight functions (assumptions 6, 7, 8)

\[ PRVK \triangleq FUVK h(k, n) \]

We then consider the limit as the number of intervals goes to infinity, that is as \( n \to \infty \). The present value is then given by,

\[ PRVK \triangleq \lim_{n \to \infty} FUVK h(k, n) \]

Using the property of the weight function,

\[ h(k, n) = (1 + k)^n \geq 1 \Rightarrow k \geq 0 \]

Also noting that since the knowledge across all those time intervals, between the future and now, needs to be connected as discussed in Assumption 7, the growth rate of knowledge is strictly greater than zero or, \( k > 0 \). By Axiom 1 every piece of knowledge has value and hence the value of the piece of knowledge that becomes known in the future when it becomes known is greater than zero, \( FUVK > 0 \). We then have,

\[ PRVK \triangleq \lim_{n \to \infty} FUVK (1 + k)^n \]

\[ PRVK \triangleq \infty \]

Next, we consider the future value of any piece of knowledge that we have today,

Relationship between \( FUVK \) and \( PRVK \)
Using a similar method of dividing the time between the present and future into many intervals, we consider
the connections the piece of knowledge we have today is likely to establish with other elements of knowledge as
time passes. As before, there will be direct connections between the knowledge today and the knowledge we will
accumulate based on it; and to know that the knowledge today does not depend on other knowledge will still require
knowing about the other knowledge elements. This can be viewed as the growth in this knowledge or change in this
knowledge based on the connections it will have as given by the weight functions (assumptions 6\textsuperscript{6}, 7\textsuperscript{7}, 8\textsuperscript{8})

\[
F_{UVK} \triangleq PRVK \tilde{h}(\bar{k}, n)
\]

We then consider the limit as the number of intervals goes to infinity, that is as \( n \to \infty \). The future value is then
given by,

\[
F_{UVK} \triangleq \lim_{n \to \infty} PRVK \tilde{h}(\bar{k}, n)
\]

Using the property of the weight function,

\[
\tilde{h}(\bar{k}, n) = (1 + \bar{k})^n \geq 1 \Rightarrow \bar{k} \geq 0
\]

Also noting that since the knowledge across all those time intervals, between now and the future, needs to be
connected as discussed in Assumption 7, the growth rate of knowledge is strictly greater than zero or, \( k > 0 \). By
Axiom 1 every piece of knowledge has value and hence the value of the piece of knowledge that we have today is
greater than zero, \( PRVK > 0 \).

\[
F_{UVK} \triangleq \lim_{n \to \infty} PRVK (1 + k)^n
\]

\[
F_{UVK} \triangleq \infty
\]

The relationships between the present value and past value; the past value and present value; the past value and
future value; and future value and past value can be established using similar arguments as above. To summarize,
the below relationships can be shown to give the results in the statement of (Theorem 1).

Relationship between \( PRVK \) and \( PAVK \)

Relationship between \( PAVK \) and \( PRVK \)

Relationship between \( PAVK \) and \( FUVK \)

Relationship between \( FUVK \) and \( PAVK \)
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Figure 1: Time Value of Money: Discounting and Compounding

Figure 2: Discount Functions: Hyperbolic and Exponential

10 Figures
Figure 3: Exponential Growth and Decay

Figure 4: Gaussian Weight Functions

Figure 5: Multi-Modal Weight Functions
Figure 6: Impulse Functions
**Impulse Function**

- Derivative of $u(t)$ at $t=0$ doesn’t exist
  - Discontinuous function
- Assume function varies linearly across the discontinuity
  - As $\varepsilon \to 0$ abrupt discontinuity occurs at origin
  - Derivative between $\varepsilon$ & $-\varepsilon$ is $\frac{0.5}{\varepsilon}$
  - Derivative for $t > \varepsilon$ is $-ae^{-a(t-\varepsilon)}$

Figure 7: Impulse Function with Linear Discontinuity