Analysis of the Train in Tunnel Paradox Based on Special Theory of Relativity

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Abstract. The train in tunnel paradox has always been a topic of debate in the scientific community, in order to allow readers to better understand its basic principles, and it needed to combine the basic knowledge of the theory of relativity to outline the train tunnel paradox. Taking Einstein's special theory of relativity as the starting point of the argument, this paper elaborates on Einstein's theory of relativity, focuses on the simultaneous relativity and the theory of physical motion shape change in the special theory of relativity, and clarifies the famous train in tunnel paradox. Based on the results of different reference systems, the author analyzes the paradox principle of trains and tunnels. Through the analysis, it can be seen that the special theory of relativity is meaningful in real life, and the learners should pay attention to the cultivation of their own logical reasoning ability when learning, and emphasize the understanding of the spirit of scientific criticism, so as to improve the learning effect of the special theory of relativity.

1. Introduction
In 1905, Einstein dropped a "blockbuster" in the seemingly peaceful world of physics-special relativity. Due to the many puzzling conclusions brought about by the assumption of the constant speed of light, doubts are still heard today. Coupled with various paradoxes to fuel the flames, the already complicated problem is even more inconsistent. To answer all kinds of doubts, it is necessary to deeply understand what the theory of relativity is telling, the precise meaning of abstract concepts (time expansion, length contraction, and simultaneous relativity) and the scope of application [1].

At the present, the two basic principles of special relativity, the relativity of time and space, and the conclusions of relativity speed transformation formula, relativistic mass, mass-energy equation, etc. have been incorporated into high school physics textbooks. Einstein's special theory of relativity made people full of infinite reveries about the magical high-speed world, the colorful starry sky, and the mysterious and infinite universe [2]. At the same time, the special theory of relativity is also known for its profound and difficult to understand. It is meaningful to quantum mechanics and other fields, but many engineers or students have insufficient understanding of the special theory of relativity, which leads to insufficient depth of learning. Therefore, this paper will explain the basic principles of Einstein's special theory of relativity, and conduct research and analysis based on the famous train in tunnel paradox for the learning and reference of related personnel.

2. Two basic principles of Einstein's theory of relativity

2.1 Principle of Relativity
Einstein pointed out in his published paper that, "Examples such as this, and the failure to prove the
experimental motion of the earth relative to the 'optical medium', have caused such a speculation that the concept of absolute static is not in mechanics and electrodynamics. It conforms to the characteristics of the phenomenon. It should be considered that all the coordinate systems applicable to the mechanical equations are also applicable to the laws of electrodynamics and optics and this conjecture should be promoted to an axiom [3], which is the famous principle of relativity. In this regard, the content of the principle of relativity can be briefly summarized as, the laws of physics have the same form in different inertial reference frames. Among them, the inertial reference frame refers to a reference frame that is stationary or linearly moving at a constant speed.

Before the theory of relativity was put forward, people believed that the laws of mechanics were unchanged. However, the form of Maxwell's equations in electromagnetics is changed after changing different reference systems, that is, people thought that Maxwell's equations were not covariant in different reference systems. Einstein believed that Maxwell's equations should also be covariant in different reference systems. The form of Maxwell's equations changed in different reference systems because the time is actually different in different reference systems.

2.2 Principle of constant speed of light
In his thesis, Einstein also pointed out that "also introduce another postulate that seems incompatible with it on the surface, light always propagates in empty space at a certain speed, and this speed is the same as that of emission. The state of the body is irrelevant [4], that is to say, the speed of light remains constant (about $3\times10^8$ m/s) under any inertial reference frame in vacuum, and it has nothing to do with whether the observer is in motion or stationary during the observation process.

3. Discussion of the results of special relativity

3.1 Simultaneous relativity
Einstein believed that all judgments related to time are always connected with the concept of "simultaneity". According to the theory of relativity, two events that occur at the same time in a certain frame of reference do not necessarily occur at the same time in another frame of reference that moves relative to it. This conclusion is called simultaneous relativity, which property can be directly derived from the principle of constant speed of light.

3.2 Physical motion shape change theory
The conclusion that motion causes the length of the object to shrink was first proposed by Lorenz based on the Michelson-Morley experiment. Einstein first deduce it to the general situation, rather than a special situation that people think. The shortening of an object by movement means that the length of the moving object will shrink in the direction of the movement, that is, when the object moves in the forward and backward direction, it will shrink back and forth, but there will be no shrinkage in the up and down direction. The degree of shrinkage is expressed as,

$$L = L_0 \sqrt{1 - \left(\frac{u}{c}\right)^2}$$

Among them, $L_0$ is the intrinsic length, that is, the length of the object in the stationary reference frame. For example, the length of a ruler observed by a person on a train is the intrinsic length of the ruler; $L$ is the extrinsic length. When the ruler is on a moving train, the length seen by the observer on the ground is the extrinsic length. It is shortened; $c$ is the speed of light and $u$ is the relative speed. Obviously, the greater the relative speed, the more the object will shrink as it approaches the speed of light. It can be supposed that there is a sphere moving forward at high speed. When viewed on the sphere, it is still a sphere (the shape remains unchanged); when viewed on the ground, it is a sphere that shrinks back and forth and becomes an ellipsoid. The conclusion that motion shortens the length of an object can also lead to paradoxes, such as the train in tunnel paradox [5].
4. The train and tunnel paradox

4.1 Basic content
In the stationary reference frame, the train and the tunnel have exactly the same length, and the intrinsic length is $L_0$. The core problem of this paradox is that in the case of high-speed train movement, whether the length of the train or the tunnel will become shorter.

4.2 Results based on different reference systems
For the Person A outside the tunnel and the person B on the train, the reference system is different, and the results obtained will be different.

Person A outside the tunnel, as the train is moving, the train has become shorter (see figure 1).

![Figure 1 Person A outside the tunnel](image1)

Person B on the train, the train is indeed moving, but according to the theory of relativity, the tunnel is also moving relative to the train. Therefore, it can be considered that the train is stationary, the length is still unchanged, and the tunnel is moving backward at high speed, so the tunnel will be shortened (see figure 2).

![Figure 2 Person B on the train](image2)

Person A outside the tunnel, he can prove that the train is shorter by experiment. There are gate A and gate B before and after the tunnel (see figure 3). After the train enters the tunnel, the two gates are closed at the same time to keep the train in the tunnel, which means that the train is shorter.
Person B on the train, he can also use experiments to prove that the tunnel has become shorter (see figure 4). First, when the train passes through the tunnel, the head of the train has passed through the tunnel, the rear of the train has not yet entered the tunnel. Second, one projectile A and projectile B are fired upwards at the front and rear ends of the train. Finally, it proves that the tunnel is shorter than the train.

4.3 Analysis of the paradox principle of train and tunnel

In fact, the above two experiments can be successful, and the conclusions of both are correct. In other words, for the tunnel, the train becomes shorter; for the train, the tunnel becomes shorter. However, in the perspectives of the other two experiments, the situation will change.

For trains, we can conduct an experiment in which the two gates of the tunnel are closed at the same time (see figure 5). From the perspective of the tunnel, the front and rear gates are closed at the same time, but on the train, the front and rear gates are not closed at the same time. Person B on the train thought that the tunnel had become shorter. Before the train had fully entered the tunnel, the tunnel closed gate A, which happened first; after a while, when the locomotive had already left the tunnel, the tunnel closed the gate B, which happened later. That is to say, the closing of the two gates occurred at the same time in the tunnel, and successively in the train.
Figure 5 The two gates of the tunnel are closed at the same time

As far as the tunnel is concerned, the experiment in which shells are fired at both ends of the train does not happen at the same time. The tunnel thought that the train had become shorter (see figure 6). Before the locomotive had left the tunnel, the rear-end shell B was fired, which happened first. After a while, the locomotive pulled out of the tunnel, and then shell A was fired. That is to say, the two shells fired at the same time from the train's perspective, but from the tunnel's perspective, did not happen at the same time.

Figure 6 The train had become shorter

It shows that some conclusions of the special theory of relativity are not all consistent with the phenomena in life. According to Einstein's principle of relativity similar to the train tunnel paradox, if the triggering of the alarm red light is regarded as a physical process, the result of whether the light is on or not will not change due to the different choices of the inertial system, that is, it is either A is not bright, or it is bright in B. The difference between different inertial systems is only the difference in the time, space and interval of optical signal transmission. Through the analysis of the legend, it can be seen that the apparently contradictory two sides of the paradox, in fact, one of the narratives must violate the scope of application of the theory of relativity, or the simultaneity is absolutized and transacted. Those who think that both A and B can happen in their own inertial system are wrong. Only by using the relativity of the same time correctly can we get self-consistent results.

5. Conclusion
The two basic principles of the special theory of relativity, the relativity of time and space, and the conclusions of the relativistic speed transformation formula, the relativistic mass, and the mass-energy equation have been widely used in scientific research. The so-called train in tunnel paradox only depends on the viewer, no one is wrong and there is no paradox. Einstein's special theory of relativity made people come up with infinite reveries about the magical high-speed world, the colorful starry sky,
and the mysterious and infinite universe. At the same time, the special theory of relativity is also known for its profound meaning, which is difficult to understand. The special theory of relativity is meaningful in real life. The learners should pay attention to the cultivation of their own logical reasoning ability when studying the basic knowledge of physics and application methods, and emphasize the understanding of the spirit of scientific criticism, so as to improve the learning effect of the special theory of relativity.

This paper mainly qualitatively expounds the train tunnel paradox based on the special theory of relativity, but there is no quantitative explanation. In the follow-up research, mathematical analysis will be used to further research and analyze the basic principles of the train tunnel paradox.

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