Cooperative transport control by a multicopter system

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Abstract
To date, multicopters have mainly been used in the entertainment and information-gathering fields, but can potentially be used to transport payloads. Specifically, multicopters are more suited for manoeuvring tough terrains and delivering payloads to areas inaccessible by other means. The authors propose a control method to achieve cooperative transport with a plural multicopter system and address the problem of delivering a payload being suspended via cables from multicopters. In cooperative transport using a multicopter system, disturbance (weight, wind, and inertia force) may cause destabilization. A consensus algorithm is used to coordinate flight of helicopters in a formation and to ensure that the payload follows a virtual leader. Disturbances (such as wind and cable tension) are primary causes of destabilization and, in the multicopter setting, they are suppressed by an adaptive robust control, Robust Integral of Sign of the Error control. It is shown that the proposed control design can achieve not only convergence under the ideal setting but also robust operation under significant disturbances. In addition, additional control objectives such as cable tension, payload attitude, and collision avoidance can be integrated and achieved. As an auxiliary method, cable tension control, maintaining a horizontal payload attitude, and collision avoidance are considered. The usefulness of the proposed method was verified by simulations and laboratory flight testing demonstrations.

1 | INTRODUCTION

In recent years, the research and development of unmanned aerial vehicles (UAVs), such as multicopters, have been remarkable in the fields of searching missing person, surveillance, photography, transportation, and so on. There is no doubt that UAVs’ utilization will become more important to our lives in the future. The introduction of UAVs to the transportation industry has been attempted, and their capability of delivering rescue supplies during a disaster has been verified. However, the long-distance transportation of heavy payloads remains problematic.

Some methods have been invented to solve this problem by increasing the size of UAVs, which, on the other hand, brings new issues of cost and flexibility. Therefore, instead of using large and expensive UAVs, applying a multi-agent system with small and cheap ones might be a better solution.

UAVs are categorized into the stationary blade type and rotor blade type. This study focussed on multicopters, which are a type of rotary wing aircraft since we considered that the rotary wing-type UAV will be more suitable for cooperative transport. Hereafter, UAV will refer to multicopters. Also, we considered the situation of achieving the formation flight using multiple UAVs that follow a virtual leader. Commercially available multicopters can realize hovering and horizontal/vertical, separation/independent control using a flight controller module. Therefore, the multicopter can be considered as a nearly linear control system.

The studies of transferring a payload using UAVs have mainly been conducted with a single UAV [1–6]. Over the last few years, some mechanical approaches have been developed for the multi-UAV system. In [7], researchers proposed a solution by connecting the UAVs with joints and grippers, which can help UAVs to wrap the payload, and the researchers in [8, 9] connected the transported object directly to the UAVs. Similarly, Tagliabue et al. developed a solution using grippers with spherical joints, which does not require communication between the agents [10]. However, the above rigid joint methods will limit the size of UAVs and payload. In addition, some cooperative methods for transferring a payload fixed with cable by multiple UAVs [11, 12, 14, 15] was proposed, which has a high degree of
freedom in selecting the number and size of UAVs and payload. These research methods mainly use PD control for basic control. Geng, J. and Langelaan, J.W. proposed a human-in-the-loop control system, which treats it as the leader instead of a disturbance [13]. However, this method may sacrifice accessibility in real life. In the method of suspending the payload using cables, the UAVs and payload are affected through the cables in some complex influence. Therefore, inertia compensation and vibration suppression are important. Approaches to inertia compensation and vibration suppression are roughly classified into two types. The first is a method of accurately modelling and controlling a control object, which requires information such as attitude and acceleration, cable angle, and payload mass [11, 12, 14]. The second method is to use an observer to compensate for disturbance. In research using this method, cooperative transport has been realized using a simple model and actual UAV, position, and velocity information [15]. However, the unknown load disturbances from the payload and wind are considered constant.

Here, we considered UAV as a secondary linear systems of acceleration input by a flight controller, and the payload is suspended by a cable. Our research purpose is to realize a cooperative transportation method that can be implemented even if the operator does not precisely understand parameters such as the payload weight, cable length, and installation position. The proposed method only needs information on the position and speed of the UAV and the payload. Flocking and consensus algorithms are simple and widely used in formation methods for multi-agent systems. The researchers in [16] proposed a method applying a leader–follower system during payload transport. Here, it was easy to set an arbitrary formation shape using a consensus algorithm [17]. Although collaborative formation flight for multiple UAVs using a consensus algorithm has been previously investigated [18–20], only few studies have focussed on cooperative transportation [22].

Since multicopters are unstable systems and vulnerable to disturbances, they must overcome inertial forces and load disturbances caused by the UAVs and payload, and wind disturbance to avoid falling. The Kalman filter and disturbance observer [23] have been established as disturbance compensation methods. However, it is difficult to apply the Kalman filter for these non-linear continuous disturbances. Additionally, the disturbance observer requires a separate reference model design and low-pass filter. In [24], researchers proposed a control method to overcome wind disturbances for fixed-wing UAVs. The study in [25] shows a second-order multi-agent helicopter system that can deal with disturbances, but no actual machine experiments have been performed.

Dixon et al. motivated by the integral robust control [26], have proposed RISE (robust integral of signs of the error) [27, 28], which uses a non-linear switching function. This method can compensate for unknown time-varying disturbances. We regard the wind, payload weight changes, and inertia from the payload adding to the UAVs as unknown acceleration disturbances, addressed using the existing [21] method that applies the RISE to the multi-agent system.

In addition to the method of maintaining a horizontal payload attitude, we also propose an auxiliary control method that consists of correcting the looseness of the cable and avoiding obstacles approaching the payload, such as the ground, walls, and trees. With regard to the collision avoidance method, we adopted the conventional potential method and ensured safety by realizing avoidance behaviour when an obstacle is approached.

The rest of this paper is structured as follows: the model and network structure of the UAV and payload are described in Section 2. Section 3 presents the cooperative transport control method, wherein the payload follows a virtual leader according to a consensus algorithm. Additionally, we describe methods for maintaining a horizontal payload attitude, correcting cable slackness, collision avoidance, and disturbance compensation using RISE. Section 4 analyses the convergence of the proposed control method. Sections 5 and 6 present the simulation results and the actual quad-rotor experiments, which were conducted for validation.

## 2 MODELLING

In this section, the UAV and suspended payload dynamical models are explained. Additionally, the information network structure, which handles multi-agent control, is also described.

### 2.1 Dynamical model of UAV

Typical quad-rotor dynamics models are a complex non-linear model, wherein the motion of one axis affects those motions of other axes. Ref. [18] proposes a model that can be treated as a horizontal quaternary system and vertical secondary system under certain conditions, and we use it. The symbols are defined in Table 1.

| Symbols | Definition |
|---------|------------|
| $x, y, z$ (m) | Position $(x, y, z)$ $a$ |
| $\dot{x}, \dot{y}, \dot{z}$ (m/s) | Velocity $(x, y, z)$ $b$ |
| $\phi, \theta, \psi$ (rad) | Attitude angle (Roll, Pitch, Yaw) |
| $p, q, r$ (rad/s) | Angular velocity (Roll, Pitch, Yaw) |
| $I_{xx}, I_{yy}, I_{zz}$ (kg m$^2$) | Inertia moment (Roll, Pitch, Yaw) |
| $m$ (kg) | UAV mass |
| $g$ (m/s$^2$) | Gravity constant |
| $M_x, M_y, M_z$ (Nm) | Moment command |
| $T_{total}$ (N) | Total thrust command |

*a* ground coordinate system  
*b* body coordinate system

First, we will explain the quad-rotor dynamical model with the next assumption. $M$ and $T_{total}$ are the input commands.
Assumption 1. The actuator dynamics are relatively fast and, for the purpose of flight control design, they are neglected.

Under Assumption 1, the dynamics of the quad-rotor can be expressed as

\[
\begin{bmatrix}
\dot{r}_x \\
\dot{r}_y \\
\dot{r}_z \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
q_v y - r p_v & -s \phi & -c \phi \tau & 0 & 0 \\
q_v z - r p_v & -s \phi & -c \phi \tau & 0 & 0 \\
q_v x - r p_v & -s \phi & -c \phi \tau & 0 & 0 \\
c \phi \tau & s \phi & c \phi \tau & 0 & 0 \\
c \phi \tau & s \phi & c \phi \tau & 0 & 0 \\
c \phi \tau & s \phi & c \phi \tau & 0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
r \\
r \\
\phi \\
\theta \\
\psi
\end{bmatrix}
+ \begin{bmatrix}
-g & c \phi & s \phi & 0 & 0 \\
-c \phi & g & 0 & 0 & 0 \\
c \phi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

In the equation, \( \sin \) is abbreviated as \( S \), \( \cos \) is abbreviated as \( C \), and \( \tan \) is abbreviated as \( T \). The kinematics of the quad-rotor can be shown as

\[
\begin{bmatrix}
x \\
y \\
z \\
\phi \\
\theta \\
\psi
\end{bmatrix} =
\begin{bmatrix}
C \phi C \theta & S \phi C \theta & -S \phi S \theta & C \phi S \theta & S \phi C \theta & -C \phi S \theta \\
C \phi S \theta & S \phi S \theta & C \phi C \theta & -S \phi C \theta & C \phi S \theta & S \phi C \theta \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
r \\
r \\
\phi \\
\theta \\
\psi
\end{bmatrix}
\]

As can be seen, this is a non-linear model wherein the motion of one axis affects the other axes. The following assumptions are made:

Assumption 2. The quad-rotor flies at a sufficiently low speed with a sufficient thrust margin.

Under this assumption, the following steps are used to linearize the horizontal and vertical state equations of the quad-rotor and decouple each axis.

- The state of hovering at a constant altitude is considered as the equilibrium point.
- The second-order or higher minute terms of minute displacements from the equilibrium point are ignored.
- Motion in the yaw direction does not occur.
- Normalization is applied by variable transformation.

The state equations in the horizontal and vertical directions are summarized in Equations (1) and (2).

\[
\text{Horizontal} : \dot{b} = (I_2 \otimes A_2) b + (I_2 \otimes B_2)(M + \tau), \quad (1)
\]

\[
\text{Vertical} : \dot{r} = A_r r + B_r (f + \omega), \quad (2)
\]

Control input : \( M \in \mathbb{R}^{2n} \), \( f \in \mathbb{R}^n \),

Disturbance : \( \tau \in \mathbb{R}^{2n} \), \( \omega \in \mathbb{R}^n \),

where \( \otimes \) is the Kronecker product. Equation (1) summarizes the \( x, y \) components of \( b = [b_x, b_y]^T \), \( M = [M_x, M_y]^T \), \( \tau = [\tau_x, \tau_y]^T \). The other UAV variables are defined in the following expression. To control the actual vertical direction of UAV, a constant input \( V_r \) is required for hovering, in addition to the acceleration control input \( J \), but its description is not included.

\[
\begin{bmatrix}
b_x \\
b_y
\end{bmatrix} = [x^{(1)} x^{(2)} x^{(3)}]^T,
\begin{bmatrix}
r
\end{bmatrix} = [\xi \dot{\xi} 2]^T,
\]

\[
A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_r = [0 \ 0 \ 0 \ 1]^T,
\]

\[
\begin{bmatrix}
x^{(1)} \\
x^{(2)} \\
x^{(3)}
\end{bmatrix} = g \phi, \quad M_x = \frac{2}{3} M_2, M_y = \frac{1}{3} M_2, \quad \xi = -r_x, j = \frac{b_{xy}}{b_{xx}}.
\]

The above model provides input position and velocity, attitude angle and attitude angular velocity in the horizontal and vertical directions, and attitude angular acceleration and acceleration in the vertical direction, respectively.

Here, \( j \) is assigned when considering a single UAV amongst \( n \) UAVs (\( j \in \{1, 2, \ldots, n\} \)). It is difficult to handle the above horizontal UAV model as it is. By using a flight controller, an actual UAV can treat horizontal control as a pseudo velocity input system or acceleration input system. Here, the horizontal model is also treated as the second-order integral system as in the vertical direction. The validity is confirmed in Section 6.1.

### 2.2 Suspended payload dynamics

The suspended payload carried by the UAVs is a type of pendulum, and can be considered as a complicated \( SO(3) \) problem [30]. Moreover, it is a non-linear and non-holonomic system wherein the UAV and payload movements affect each other. Particularly, in the case of using many UAVs, the slack of the cable cannot be ignored, depending on the relationship between the position and the posture of each UAV.

To simulate the swing of the payload suspended from the cable, the dynamic equations of the payload’s translation and rotation are derived using the Euler–Lagrange equations.

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial q_i} = Q_i, \quad L = T - U, \quad (3)
\]

where \( T \) is the physical energy, \( U \) is the potential energy, and \( Q \) is the external force. In this situation, \( D \) can be considered as zero. The symbols and coordinate systems (ground and body) are defined in Figure 1 and Table 2.

Here, \( SO(3) \) denotes the three-dimensional rotation group (special orthogonal group), and is \( \{ R \in \mathbb{R}^{3 \times 3} | R^T R = I, \ det (R) = 1 \} \). \( T_i \) is the cable tension, it is the element for
moving and rotating the payload and disturbance for UAVs. \( \mathbf{T}_i \) is set at each cable joint point and UAV’s centre of gravity.

### 2.2.1 Payload equation of motion

The translational motion equation of the payload in the reference coordinate system is expressed as follows:

\[
\dot{\mathbf{T}} = \frac{1}{2} \mathbf{I}_0 \dot{\mathbf{q}} \dot{\mathbf{q}}^T, \quad \mathbf{U} = \mathbf{M}_g \mathbf{x}_0 [0 \ 0 \ 1]^T, \quad \mathbf{Q} = \sum_{i=1}^{n} \mathbf{T}_i
\]

where \( \mathbf{I}_0 \) is the Hadamard product. The rotational motion equation in the body coordinate system is expressed as

\[
\mathbf{T} = \sum_{i=1}^{n} c_i \mathbf{T}_i',
\]

2.2.2 Derivation of cable tension

The tension of the next step \( \mathbf{T}_i \) is derived from the state of the UAV and payload. In a situation wherein the cable is loose, tension does not occur. The symbol \( b_i \in [0 \ 1] \) determines the presence or absence of tension. Moreover, \( \mathbf{C}_i = \mathbf{X}_i - (\mathbf{X}_p + \mathbf{R}'_i) \) is the distance vector of the UAV and cable joint point.

\[
b_i = \begin{cases} 
1 & |L_i| \geq C_i \\
0 & \text{else}
\end{cases}
\]

\[
\mathbf{T}_i = \mathbf{F}_{Gi} - \mathbf{F}_{Ri} - \mathbf{F}_{Ri}' + \mathbf{F}_{Qi},
\]

Equation (7) is the force exerted by the payload on each cable joint point. This tension consists of four types of force. The cable connects to the UAV’s centre of mass.

\[
\mathbf{F}_{Gi} \text{ of Equation (8) describes the force that the payload exerts on each cable joint point.} \quad \mathbf{F}_{Ri} \text{ of Equation (8) describes the inertia force from the rotational motion of the payload.} \quad \mathbf{F}_{Qi} \text{ of Equation (8) describes the inertia force exerted by the UAV’s translational motion.}
\]

Moreover, \( \mathbf{F}_{Ri}' \) is the force in the body coordinate system that can be converted to \( \mathbf{F}_{Ri} \) in the world coordinate system.

2.2.3 Disturbance applied to UAV

When considering UAV movement, the payload weight \( M_{pg} \) acts as a disturbance vector along the direction of the cable. When
2.2.4 Cable length constraint

The distance between the UAV and the cable joint position cannot exceed the length of the cable. When this length is over the cable length, this constraint can be satisfied by properly adjusting the position and attitude of the payload.

2.3 Network structure

The information network is used to describe the interactions amongst the UAVs (as followers), the virtual leader, and the payload. To consider the formation control problem, the network is represented by a graph based on graph theory [31].

2.4 Example of adjacency matrix

\[ a_{ij} = \begin{cases} 1 & \text{for agent } i \text{ connected from agent } j \\ 0 & \text{otherwise} \end{cases} \]

Here, 1 to \( n \) are assigned to the UAVs, \( n + 1 \) is assigned as the virtual leader, and \( n + 2 \) is assigned as the payload. An outline of the network structure is shown in Figure 2.

This is a concatenated undirected graph \( G \), which means that all followers can obtain the virtual leader’s information directly or indirectly.

The virtual leader’s information, and that of other agents, consists of position and velocity information, and is obtained using communication devices or sensors. The virtual leader’s position and velocity are specified by the operator as the reference trajectory.

The following assumption is made:

**Assumption 3.** The top \( n \)-by-\( n \) block of the adjacency matrix is symmetric, the last two rows are zeros, and the rest of the matrix are 1s.

3 PROPOSED CONTROL METHOD

The most important purpose of the control method proposed here is to make the payload follow the virtual leader [32]. UAV

is subject to unknown and time-varying disturbances such as wind and inertia of payload. In a multicopter system, perturbation rejection is important.

In actual operation, it is desirable that the operators can perform even if they do not accurately grasp parameters such as the payload weight, cable length, and installation position. Therefore, we investigated a method for stably carrying a payload, even if the above-mentioned parameters are unknown.

Let \( h_L = [x_L, y_L]^T, r_L = [z_L, b_L]^T, h_P = [x_P, y_P]^T, r_P = [z_P, b_P]^T \) for the position coordinates of the virtual leader and payload, respectively.

The operator specifies the position and velocity of the virtual leader. The controller achieves the control purpose by agreeing the position and velocity of the payload. The control objective is expressed by the following equation:

\[ \lim_{t \to \infty} (h_L - h_P) = 0, \quad \lim_{t \to \infty} (r_L - r_P) = 0. \]  

3.1 Coordinated transfer

To make a formation, the following variables are defined as shown in Figure 3. Here, \( d_{hi} \in \mathbb{R}^2 \) and \( d_{ri} \in \mathbb{R}^2 \) are vectors specifying the position of the UAV in relation to the reference formation point.

\[ b_i = b_i - d_{hi}, \quad r_i = r_i - d_{ri}. \]

The laws proposed for controlling the horizontal direction and vertical direction of UAVs \( i \in \{1, 2, \ldots, n\} \) are described below:

\[ M_i = -\sum_{j=1}^{n} a_{ij} \left( \delta_0 (b_i - b_{ij}) + \delta_1 (b_i - b_{ij}) \right) \\
+ a_{iL} a_{iP} \left( \delta_0 (h_L - h_P) + \delta_1 (h_L - h_P) \right) \\
- \ddot{r}_i + f_{hi}. \]  

\[ f_i = -\sum_{j=1}^{n} a_{ij} \left( \gamma_0 (r_i - r_{ij}) + \gamma_1 (r_i - r_{ij}) \right) \\
+ a_{iL} a_{iP} \left( \gamma_0 (r_L - r_P) + \gamma_1 (r_L - r_P) \right) \\
- \dot{\omega} + f_{ri}. \]
The purpose of the first line in Equations (12) and (13) is to control agents to form the formation, while the second line is used to assure the payload can follow the virtual leader. According to the studies in [19], the condition that control gains \( \delta_0 > 0, \delta_1 > 0, \gamma_0 > 0, \) and \( \gamma_1 > 0 \) must be satisfied, which can be derived by the Lyapunov stability theory.

Additionally, \( \hat{r}_i \) and \( \hat{\omega}_i \) are the control terms by RISE that corrects disturbances \( \tau_i \) and \( \omega_i \). \( f_y, f_z \) and \( f_{\beta} \) are collision avoidance terms. These values will be explained in more detail later in the text.

### 3.2 Disturbance compensation

RISE is mainly used to compensate for the disturbance from the payload that is carried by the UAVs. RISE is a type of adaptive control that eliminates the disadvantage, whereby the control input becomes discontinuous, while retaining the robust characteristics against model uncertainty and unknown disturbances. By deriving the compensation input such that the error between the state estimated from the control input by the observer and the actual state affected by the disturbance becomes zero, the state estimated from the control input by the observer can be derived by the Lyapunov stability theory.

The purpose of the first line in Equations (12) and (13) is to control agents to form the formation, while the second line is used to assure the payload can follow the virtual leader. According to the studies in [19], the condition that control gains \( \delta_0 > 0, \delta_1 > 0, \gamma_0 > 0, \) and \( \gamma_1 > 0 \) must be satisfied, which can be derived by the Lyapunov stability theory.

Moreover, \( \hat{\omega}_i \) is used in Equation (13). By setting the parameter gain according to Theorem 1, the disturbance effect can be cancelled as follows: \( \lim_{t \to \infty} (\omega_i(t) - \hat{\omega}_i(t)) = 0 \). With this method, it is possible to handle the unknown weight changes of the payload during flight. When applied to the horizontal control method, it can be expected that wind disturbances can be dealt with.

### 3.3 Maintain attitude

We consider keeping the attitude angle of the payload horizontal.

\[
\lim_{t \to \infty} \theta(t) = 0, \quad \lim_{t \to \infty} \psi(t) = 0.
\]

By extending the vertical component \( r_p \) of Equation (13) to the following expression, we can maintain a horizontal payload attitude.

\[
r_p = r_i - \delta_x - a_p \alpha_p \{ (h_{xy} - \alpha_p) \psi + (h_{yx} - y_p) \theta \},
\]

where \( \alpha_p \) and \( \beta_p \) are the PI control gain, \( \theta \) is the roll angle of the payload around the \( x \) axis, and \( \psi \) is the pitch angle around the \( y \) axis.

Consensus between the agents cannot be achieved using the method of directly controlling the thrust \( f \) instead of the position \( r_p \) of agent \( i \), because agent \( j \) performs consensus control with the position and velocity of agent \( i \) as a reference value.

### 3.4 Correcting slack in cable

To effectively achieve transport, cable slackness is undesirable. In other words, it is preferred that the tension \( T \) from the payload is added to all UAVs. Here, \( T_i \) is a downward (negative) vertical disturbance \( \omega \).

If the agent does not obtain tension \( T_i \) from the payload, the component \( r_{\beta} \) of Equation (13) is controlled towards the upward direction until tension occurs.

\[
r_{\beta} = \begin{cases} r_i - \delta_i + b_i \varepsilon_j \int_0^t d\tau, & \xi_j < T_i < 0 \\ r_i - \delta_i, & \text{otherwise}. \end{cases}
\]

\( \varepsilon_j \) is I gain and \( b_i \) is defined by Equation (7). The threshold for determining the presence or absence of tension \( T_i \) is \( \xi_j \).

If it is difficult to directly observe the tension \( T_i \) with an actual machine, the disturbance estimate \( \omega \) is used for correction. Note that the load disturbance by payload weight applied to the UAVs is downward.
3.5 Collision avoidance

This study proposes a stereoscopic avoidance method by using the potential method and setting the safety region of the sphere of the radius \( \Delta R \) shown in Figure 4 against the payload.

For the \( m \) obstacles and nearest terrain, the relative distance between the \( k \)th obstacle and the payload is expressed as follows:

\[
|l_j| = \sqrt{(x_{P} - x_j)^2 + (y_{P} - y_j)^2 + (z_{P} - z_j)^2},
\]

where \( x_{P}, y_{P}, z_{P} \) are the payload coordinates.

Next, the potential field \( U_k \) generated when an obstacle approaches the safe area is defined as follows:

\[
U_k = \begin{cases} 
\eta \left( \frac{1}{|l_j| + 1} - \frac{1}{\Delta R + 1} \right)^2 & |l_j| \leq \Delta R \\
0 & \text{otherwise},
\end{cases}
\]

where \( \eta \) is a positive effect. This potential field decreases as the payload and obstacles move apart, and smoothly becomes zero at the boundary of the safety area. The total potential field generated by all \( m \) obstacles in a safe place is expressed as follows:

\[
U_{total} = \sum_{i=1}^{m} U_i
\]

The collision avoidance term \( f \in \mathbb{R}^3 \), which is added to all agents, can be determined by partially differentiating Equation (23) with respect to the \( l_i \) coordinates.

\[
f = -\nabla U_{\text{total}} = -\sum_{i=1}^{n} \frac{\partial U_i}{\partial l_i},
\]

where

\[
\frac{\partial U_i}{\partial l_i} = \frac{\partial U_i}{\partial |l_j|} \frac{\partial |l_j|}{\partial l_i},
\]

4 CONVERGENCE ANALYSIS

In this section, the description of the horizontal direction is omitted, and the vertical direction will be described.

Thus, it is shown that the payload can be transported using the method of following a virtual leader. The payload trajectory is to follow that of a virtual leader in the presence of disturbances, which satisfy the following assumption.

Assumption 4. The vertical disturbances \( \omega(t) \) and their first and second derivatives \( \dot{\omega}(t) \) and \( \ddot{\omega}(t) \) are bounded.

\[
\omega(t), \dot{\omega}(t), \ddot{\omega}(t) \in \mathcal{L}_\infty,
\]

\[
\|\dot{\omega}\| \leq c_1, \|\ddot{\omega}\| \leq c_2,
\]

where \( c_1 \) and \( c_2 \) are positive constants.

4.1 Convergence condition of disturbance estimation error in vertical direction

In multi-agent systems, the Graph Laplacian concept \( \mathcal{L} \) is widely used. This study used the \( \mathcal{N} \) matrix and focussed only on the network structure between the agents. The previously described control input to each agent can be collectively described in the entire system. Figure 5 explains the definition of \( \mathcal{N} \) by considering the adjacency matrix shown in Figure 2 as an example.

Using \( \mathcal{N} \), Equation (13) for the control input of each UAV, and Equation (14) for the state estimation value can be
where $r$ to this system, and selecting the gains $\alpha_r$ and $\beta_r$ according to Assumptions 3 and 4.

### Theorem 1

A multi-agent system consisting of $n \geq 2$ machines satisfies Assumptions 3 and 4.

When applying the control rule expressed by Equations (13) and (17) to this system, and selecting the gains $\alpha_r$ and $\beta_r$, which satisfy Equations (32) and (33), the disturbance estimated in the vertical direction $\hat{\omega}$ asymptotically converges to the actual disturbance value $\omega$.

This means that the disturbance estimation error $\hat{\omega}$ converges to zero $\lim_{t \to \infty} \hat{\omega}(t) = \lim_{t \to \infty} (\omega(t) - \hat{\omega}(t)) = 0$.

\[
\alpha_r \mathcal{N} < \frac{2}{\gamma_1} \xi < 4 \frac{y_0}{\gamma_1}, \quad \beta_r > \gamma_1 + \frac{1}{\gamma_1 \| \mathcal{N} \| \epsilon_r}. \tag{32} \tag{33}
\]

**Proof.** The derivative of $\epsilon_r$ in Equation (16) becomes the following equation from Equations (15), (28), and (30).

\[
\dot{\epsilon}_r = \dot{\gamma} - \dot{\hat{\omega}} = (\dot{\gamma} - \hat{\omega}) = \left( f + \omega - \frac{y_0}{\gamma_1} \right) \tag{34}
\]

From Equation (34), the disturbance estimation error and its derivative can be expressed as follows:

\[
\hat{\alpha} = \dot{\epsilon}_r + \gamma_1 \mathcal{N} \epsilon_r, \tag{35}
\]

\[
\dot{\hat{\alpha}} = \dot{\epsilon}_r + \hat{\alpha} + \alpha_r \gamma_1 \mathcal{N} \epsilon_r = \alpha_r \gamma_1 \mathcal{N} \epsilon_r - \alpha \hat{\alpha} - \beta_r \sigma(\epsilon_r).
\]

Additionally, $y(t)$ is defined as follows:

\[
y(t) = [\epsilon_r(t) \quad \epsilon_r(t) \quad \epsilon_r(t) \quad \sqrt{\xi_w} \epsilon_r(t)]^T \in \mathbb{R}^{3e+1}. \tag{37}
\]

Consider the candidate Lyapunov function $V(y(t)) \in \mathbb{R}$ expressed by Equation (38).

\[
V(y(t)) = \frac{1}{2} \dot{\gamma}^T \gamma + \frac{1}{2} \dot{\hat{\omega}}^T \hat{\omega} + \frac{1}{2} \mathcal{N}^T \mathcal{N} \epsilon_r - \frac{1}{2} \epsilon_r^T \epsilon_r - \frac{1}{2} \xi_w \epsilon_r^T \epsilon_r \tag{38}
\]

When $\beta_r$ satisfies Equation (33), the auxiliary variable $\xi_w$ is non-negative and $V(y(t)) > 0$. The proof of $\xi_w \geq 0$ is provided in Appendix A.1.

Thus, $V(y(t))$ is satisfied.

\[
\Omega_{Lw}(y) \leq V(y(t)) \leq \Omega_{Hw}(y), \tag{40}
\]

where $\Omega_{Lw}(y), \Omega_{Hw}(y) \in \mathbb{R}$ is a continuous and positive definite function.

The time derivative of $V(y(t))$ is expressed as follows:

\[
\dot{V}(y(t)) = \epsilon_r^T \dot{\epsilon}_r + \frac{1}{2} \dot{\hat{\omega}}^T \hat{\omega} + \frac{1}{2} \dot{\epsilon}_r^T \epsilon_r
\]

\[
\quad - \frac{1}{2} \mathcal{N}^T \mathcal{N} \epsilon_r - \alpha_r \hat{\alpha}^T \hat{\alpha} - \epsilon_r^T \epsilon_r
\]

\[
\quad - \frac{1}{2} \dot{\gamma}^T \gamma - \frac{1}{2} \dot{\hat{\omega}}^T \hat{\omega} - \beta_r \sigma(\epsilon_r) \tag{41}
\]

Design parameters are introduced such that $\xi_w \in \mathbb{R} > 0$. 

\[
0 \leq \left( \sqrt{\xi_w \dot{\gamma}^T \gamma - \frac{1}{2} \epsilon_r^T \epsilon_r} \right)^T \left( \sqrt{\xi_w \dot{\gamma}^T \gamma - \frac{1}{2} \epsilon_r^T \epsilon_r} \right),
\]

\[
\dot{\gamma}^T \gamma \leq \frac{1}{2} \epsilon_r^T \epsilon_r + \frac{1}{4} \lambda_w \epsilon_r^T \epsilon_r. \tag{42}
\]

By substituting Equation (42) into Equation (41), the following inequality can be obtained:

\[
\dot{V} \leq -\left( \frac{y_0}{\gamma_1} - \xi_w \right) \dot{\gamma}^T \gamma - \epsilon_r^T \left( \frac{1}{2} \gamma_1 \mathcal{N}^T - \frac{1}{2} \lambda_w \right) \epsilon_r
\]

\[
- \alpha_r \gamma_1 \mathcal{N}^T \epsilon_r - \alpha \hat{\alpha}^T \hat{\alpha} - \beta_r \sigma(\epsilon_r)
\]

\[
- \frac{1}{2} \epsilon_r^T \epsilon_r - \alpha, \hat{\alpha}^T \hat{\alpha} - \beta_r \sigma(\epsilon_r). \tag{43}
\]
First, to obtain $V'(y(t)) < 0$, the following relationship must be satisfied:

$$
\frac{\dot{y}_0}{y_1} - \xi_w > 0, \quad (44)
$$

$$
\frac{1}{2}y_1N - \frac{1}{4\xi_w}l_N > 0, \quad (45)
$$

$$
\alpha_r l_N - \frac{1}{2}y_1\alpha_r^2N > 0. \quad (46)
$$

The three equations above are introduced into the next equation, as follows:

$$
4\dot{y}_0N > 4\xi_w y_1N > 2l_N > y_1\alpha_r N. \quad (47)
$$

The arbitrary design parameters $\xi_w$ for deriving the stable condition are expressed by the following equation (47).

Next, since $V'(y(t))$ is semi-negative, $V'(y(t)) - V'(0, y(0)) = \int_0^t V'(y(\tau))d\tau \leq 0$ is satisfied. Thus, $V'(y(t)) \leq V'(0, y(0)) \in L_\infty$ holds. From $V'(y(t)) \in L_\infty$ and Equation (38), we can see that there exists $\varepsilon_r, \hat{r}, \bar{w} \in L_\infty$. Additionally, $\tilde{r}, \hat{r}, \bar{r}, \hat{r}, \tilde{r}$, and $\tilde{r} \in L_\infty$ can be derived because, from these relationships and Equations (28), (29), (30), and (34), it is known that $\tilde{r}, \hat{r}, \bar{r}, \hat{r}, \tilde{r}$, and $\tilde{r} \in L_\infty$, which is also consistent with Assumption 1.

From the above considerations, $V'(y(t))$ is bounded by the semi-positive definite function $-\Omega_w(y)$, which is equal to the right side of Equation (43).

$$
\Omega_w(y) = \left(\frac{\dot{y}_0 - \xi_w}{y_1}\right)\hat{r}^T \hat{r} + \varepsilon_r^T \left(\frac{1}{2}y_1N - \frac{1}{4\xi_w}l_N\right) \varepsilon_r
$$

$$+ \bar{w}^T \left(\alpha_r l_N - \frac{1}{2}y_1\alpha_r N\right) \bar{w}
$$

$$+ \frac{1}{2}(\varepsilon_r - \alpha_r \bar{w})^T N (\varepsilon_r - \alpha_r \bar{w}). \quad (48)
$$

From Equations (44) to (46), $\varepsilon_r = 0$, $\hat{r} = 0$, $\bar{w} = 0$ can be obtained if $V' \equiv 0$. Additionally, at this time, $\hat{r} = 0$, $\tilde{r} = 0$ is established from Equations (28) and (30). This Lyapunov function is asymptotically stable at the origin, according to LaSalle’s invariance principle. Because $t \rightarrow \infty$ becomes $\bar{w} \rightarrow 0$ and $\tilde{r} \rightarrow 0$, each agent achieves an accurate disturbance estimation.

5 | SIMULATION

The proposed control method inputs are Equations (12) and (13), combined with Equations (19), (20), (24), and (17).

For comparison, the case not using the proposed method and expressed by the following equation was verified. With this equation, the UAVs make a formation and follow the virtual leader.

$$
M_j = -\sum_{j=1}^{s+1} a_{ij} \left\{ \delta_0(b_{fi} - b_{fj}) + \delta_1(b_{fi} - b_{fj}) \right\}, \quad (49)
$$

$$
J_j = -\sum_{j=1}^{s+1} a_{ij} \left\{ \gamma_0(r_{fi} - r_{fj}) + \gamma_1(r_{fi} - r_{fj}) \right\}. \quad (50)
$$

5.1 | Condition setting

Through simulation, we confirmed that the swinging payload accurately follows the virtual leader and that the posture can be maintained horizontally when using the proposed method. The dynamical model was taken from Section 2, and the simulation conditions are listed in Table 3. For the first 3 s, when the UAVs take off and create a formation, the virtual leader stops, then moves 1 m forward, and then turn left. After 10 s, the weight of the payload is doubled. We set one UAV cable to be longer than the other cable, and set the conditions such that they did not contribute to the transport.

5.2 | Transport of payload

The results of the simulation are presented in Figures 6 to 12. With the proposed method, we can see that the payload follows the virtual leader, and we can suppress the vibration of the payload. Specifically, the payload does not sink because of the load in the vertical direction, and follows the leader.
In other words, Equations (10) and (18) are satisfied. According to Figure 11, if there is no proposed method, the UAV No. 1 does not contribute to the transfer of the payload due to the long cable. It can be seen that the load in the vertical direction is distributed to all UAVs by the proposed method of Equation (20). Also, as a comparison between the RISE
and regular PID controller, we made a simulation for PID controller in the vertical direction (see Figure 10), and it can be seen that the RISE controller has great advantages in terms of both tracking accuracy and overshoot.

5.3 Disturbance and disturbance compensation by RISE

Figures 11 and 12 were compared with regard to the vertical disturbance applied to the UAVs. After 10 s, we set the payload’s weight doubled.

As can be seen, all UAVs contributed to the payload transport by bearing the load and satisfying the threshold $\zeta$ of Equation (20) as much as possible.

The disturbance estimated by RISE is presented in Figure 12(a). Compared with Figure 11(b), we can see that the disturbance was estimated accurately ($\lim_{t \to \infty} \{\omega_i(t) - \hat{\omega}_i(t)\} = 0$). Additionally, the state estimation error shown in Figure 12(b) converged to zero as time elapsed ($\lim_{t \to \infty} \{r_i(t) - \hat{r}_i(t)\} = 0$).

5.4 Collision avoidance

The situation of two obstacles in the simulation was set assuming avoidance of obstacles and terrain. Two cylinder obstacles were installed beside and under the virtual leader route (Figure 13, 14). The payload route and position of the obstacle are shown in Figure 15. Because parameter $\Delta R'(=0.3 \text{ m})$ was set, the UAV was able to move while maintaining a distance from the obstacle.

6 EXPERIMENT

In this experiment, HUBSON X-4 toy drone produced by G-FORCE was selected, which is unstable and weak for unpredictable disturbances under its default settings. Also, a plastic bottle was used as the payload (Figure 22).

The overall experimental setting is shown in Figure 16. We implemented a control method to obtain information regarding the position and attitude of the UAV and payload using

![FIGURE 13](image13.png) Trajectory (collision avoidance): (a) horizontal and (b) three-dimensional

![FIGURE 14](image14.png) Trajectory (vertical): (a) X–Z axis and (b) Y–Z axis

![FIGURE 15](image15.png) State (collision avoidance): (a) trajectory and (b) distance between payload and obstacles

![FIGURE 16](image16.png) Experimental system

![FIGURE 17](image17.png) Experimental device
OptiTack, which is a motion capture system, and then provided the information as feedback to a C++ environment. Because the SDK is unpublished, it was remodelled to disassemble the attached remote control and directly apply the control signal (voltage) to the joystick (variable resistor) (Figure 17). The main specifications of the experimental apparatus are presented in Table 4. The experimental device was configured using easily obtained equipment. The update period was 120 Hz or more, and the measured time delay is about 20 ms, which was sufficient for verifying the proposed method. We also uploaded the video to YouTube [34].

6.1 Confirmation of linear characteristics of UAV

To confirm whether HUBSON X-4 can fit the dynamical model described in Section 2.1, we measured its responses, which are presented in Figure 18. It can be seen that the UAV shows linear characteristics in both horizontal and vertical directions. Additionally, after measuring the position change by adding a step input to the UAV in the hovering state, it was confirmed that the control input (voltage) was applied, and the acceleration response of the UAV was approximately linear and proportional. The coefficient was $G_h = 2.2 \text{ [m/s}^2\text{/Volt]}$ in the horizontal direction, and $G_v = 1.9 \text{ [m/s}^2\text{/Volt]}$ in the vertical direction.

6.2 RISE characteristics confirmation using single UAV

The effect of RISE was verified on one UAV. Wind was applied to confirm the characteristics in the horizontal direction (Figures 19 and 21(a)). Two weights have been added to check the characteristics in the vertical direction (Figures 20 and 21(b)). The experimental parameters are shown in Table 5.

Under any conditions, when we did not use RISE, the deviation from the target position occurs due to the influence of disturbance. When we used RISE, it has successfully reached the target position. Vibration occurs in the horizontal case, which is considered to be the influence of considering the UAV horizontal model as second-order system.
6.3 | Cooperative transport

The control method used in the verification is expressed by Equations (51) and (53), combined with Equations (24) and (17).

\[
M_i = -\sum_{j=1}^{n+1} a_{ij} \{ \delta_0 (b_j - b_i) + \delta_1 (b_j - b_i) \} + f_i, \quad (51)
\]

\[
f_i = -\sum_{j=1}^{n+1} a_{ij} \{ \gamma_0 (r_j - r_i) + \gamma_1 (\dot{r}_j - \dot{r}_i) \} - a_{ij} a_{ip} \{ \gamma_0 (r_p - r_i) + \gamma_1 (\dot{r}_p - \dot{r}_i) \} - \hat{\omega}_i + f_i, \quad (52)
\]

For the UAV used in this experiment, it was difficult to accurately adjust the trim in the horizontal direction. Thus, the main proposed method was mounted in the vertical direction.

The parameters implemented according to the simulation are listed in Table 6. Starting from the hovering state, the virtual leader moved by 1 m in approximately 10 s and went straight 1 m towards the left direction in the next 10 s.

The disturbance value estimated by RISE is shown in Figure 23. The case not using the proposed control method and the case using the proposed control method are both shown in Figures 24 to 26. By implementing the proposed method, it was confirmed that the payload could follow the virtual leader.

Figure 23 is a graph that estimates the payload weight. The unit of \( \hat{\omega} \) of the disturbance estimation value is voltage, but converted to load using \( G_r \) of Figure 18. The black line is the sum of the disturbance estimates of the three UAVs, and roughly estimates the payload weight of 0.025 g.

Although the payload considered in the experiment cannot be lifted by one UAV, the proposed method enabled its transport by multiple UAVs. By setting the control gains of Table 6 and the trajectory of the virtual leader, the operator could achieve coordinated transport without having to know the details of the control target, such as the cable length, joint point, and payload weight.

6.4 | Collision avoidance

A cylindrical obstacle was installed (Figure 27) beside the path and the UAV was moved forward by 1 m. The route of the payload and the position of the obstacle are shown in Figure 28. Because the parameter \( \Delta R(= 0.4 \text{ m}) \) was set, the UAV was able to move while maintaining a distance from the obstacle.

7 | CONCLUSION

A control algorithm is proposed for cooperatively transporting an object suspended by cables from multicopters. Its design employs the method of decoupling motion along the axes and
parametrizing dynamics. Several features are added to make the proposed method practical. External disturbances are suppressed by robust control RISE. Control of cable tension is included to evenly distribute the payload weight among multicopters, and potential slack of the cable is addressed. Stability and convergence under the proposed method are analytically shown. Effectiveness of the proposed control is shown by both simulation and laboratory flight demonstrations. In the latter, the operator succeeded in transporting a payload whose weight could not be carried using any one of the UAVs, and the control inputs are determined by setting simple parameters and by piloting the trajectory of the virtual leader.

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APPENDIX

A.1 Non-negative proof of auxiliary variable $z_w$

Here, we explain when the auxiliary variable $z_w$ of the Lyapunov function candidate of Equation (37) is non-negative.

**Lemma A1.** If the gain $\beta_r$ satisfies the condition of Equation (53), $z_w \in \mathbb{R}$ in Equation (54) is non-negative.

$$
\beta_r > \gamma_1 + \frac{1}{\gamma_1 \|N\|} \gamma_2, \quad (A1)
$$

$$
z_w = \beta_r \epsilon_r^T (0) \text{sgn}(\epsilon_r (0)) - \epsilon_r^T (0) \dot{\omega}(0)
- \int_0^t \tilde{\omega}^T (\omega - \beta_r \text{sgn}(\epsilon_r)) \, d\omega. \quad (A2)
$$

**Proof.** The third term on the right side of Equation (54) is expressed as follows, based on Equation (36).

$$
\int_0^t \tilde{\omega}^T (\omega - \beta_r \text{sgn}(\epsilon_r)) \, d\omega
= \int_0^t (\epsilon_r + \gamma_1 N \epsilon_r)^T (\omega - \beta_r \text{sgn}(\epsilon_r)) \, d\omega
= \int_0^t \epsilon_r^T \omega - \beta_r \epsilon_r^T \text{sgn}(\epsilon_r) + \gamma_1 \epsilon_r^T N \dot{\omega}
- \gamma_1 \beta_r \epsilon_r^T N \text{sgn}(\epsilon_r) \, d\omega
= [\epsilon_r^T \tilde{\omega}]_0^t - \int_0^t \epsilon_r^T \omega \, d\omega - [\beta_r \epsilon_r^T \text{sgn}(\epsilon_r)]_0^t
+ \int_0^t \left( \gamma_1 \epsilon_r^T N \dot{\omega} - \gamma_1 \beta_r \epsilon_r^T N \text{sgn}(\epsilon_r) - \epsilon_r^T \tilde{\omega} \right) \, d\omega
+ \epsilon_r^T \dot{\omega} - \epsilon_r^T (0) \dot{\omega}(0) - \beta_r \epsilon_r^T (0) \text{sgn}(\epsilon_r)
+ \beta_r \epsilon_r^T (0) \text{sgn}(\epsilon_r)). \quad (A3)
$$

Here, using the following relational expression, we derive the conditional expression for the magnitude relationship consisting of only scalars.

$$
a \cdot b \leq \|a\| \|b\|. \quad (A4)
$$

$$
\|e_r\| \leq \epsilon_r^T \text{sgn}(\epsilon_r), \quad (A5)
$$

$$
\|N\| = \sqrt{\lambda_{\max}(N^T N)}, \quad (A6)
$$

where $\lambda_{\max}(\bullet)$ is the largest eigenvalue of $\bullet$. Finally, the third term on the left side of Equation (54) is expressed as follows:

$$
\int_0^t \tilde{\omega}^T (\omega - \beta_r \text{sgn}(\epsilon_r)) \, d\omega
\leq \int_0^t \|e_r\| \cdot \left\{ -\gamma_1 \beta_r \|N\| + \gamma_1 \|N\| \|\dot{\omega}\| + \|\tilde{\omega}\| \right\} \, d\omega
+ \|e_r\| \cdot \left\{ \|\tilde{\omega}\| - \beta_r \right\} - \epsilon_r^T (0) \dot{\omega}(0)
$$
\[ + \beta_w \epsilon_r^T(0)\text{sgn}(\epsilon_r(0)). \] 

(A7)

Therefore, \( z_w \) is non-negative because Equation (54) is \( z_w \geq 0 \) when the following condition is satisfied:

\[
\begin{aligned}
\{ -\gamma_1 \beta_w \| \mathcal{N} \| + \gamma_1 \| \mathcal{N} \| \| \dot{\omega} \| &+ \| \ddot{\omega} \| < 0 \\
\| \dot{\omega} \| - \beta_r &< 0. 
\end{aligned}
\]

(A8)

If Equation (60) is rearranged using Equation (27), then, Equation (53) can finally be derived. When \( \epsilon_r = 0, z_w = 0 \). \( \square \)