Mass Generation and Gravity

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Abstract—Despite the success of the Higgs mechanism to account for the generation of masses of the Standard Model (SM) elementary particles, the ultimate nature and origin of “mass” remain open questions in contemporary physics. From a foundational perspective, mass should be fundamentally related to the gravitational interaction and, according to Mach, with the structure of the Universe. In the present letter, a fully dynamical mass generation mechanism induced by higher-order corrections in the GR Lagrangian is discussed. Notably, the vacuum energy plays a key role in this process, which applies to both vector bosons and fermions. We show that, at the classical level, the Higgs mechanism can be thought of as a particular case of the present mechanism.

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1. INTRODUCTION

The Brout-Englert-Higgs (BEH) mechanism\(^1\)–\(^4\) is considered as the key ingredient that brought consistency to the Standard Model (SM) of fundamental particles by rendering the underlying quantum field theories (QFTs) gauge invariant and renormalizable. The detection of a spinless bosonic particle with a mass of approximately 125 GeV and properties compatible with the SM Higgs boson\(^5,6\) has provided strong support for this model. Even though the BEH mechanism is generally regarded as the “fundamental” process responsible for the origin of masses of the SM fundamental particles, the physical mass of the Higgs boson is itself determined by a free parameter of the SM. Moreover, unresolved issues such as fine tuning and the hierarchy problems of the BEH theory, unanswered questions regarding a possible composite nature of the Higgs boson(s), issues concerning the origin of neutrino masses and the neutrino mass scale, and the cosmological constant problem, all indicate that the SM is far from being a complete picture of Nature.

It is universally recognized that the concept of “mass” is poorly understood in contemporary physics. First of all, mass is a strictly classical property that was imported to quantum mechanics with no further warrant. Second, if mass is indeed a quantized quantity, then the observed mass spectrum of the SM requires the corresponding mass operator to describe bound states of more fundamental constituents. By consistency, one would therefore be forced to assume that the ultimate fundamental constituents are massless, and that mass must have a strictly dynamical origin. Moreover, one can only hope that the theory capable of implementing this program will be essentially nonperturbative, UV-complete and dispense with the undesirable features of standard QFTs. Although some progress in these particular directions was made by string theories, all attempts to formulate a quantum theory in which the masses of all fundamental particles are dynamically generated have so far failed.

On the other hand, the idea that mass should have a strictly dynamical origin (already at the classical level) was first suggested a long time ago by Mach\(^7\). According to some interpretations of the notorious “Mach’s principle,” the mass-inertia of localized systems should somehow be determined by the global structure of the Universe\(^8\). A formal, mathematically precise expression of this principle, however, remains elusive. On the other hand, in the light of general relativity (GR), it is generally understood that mass should be fundamentally related with the gravitational interaction, which is also the dominant influence in the Universe at large scales. It is also generally believed that a more complete theory that supersedes the SM will necessarily incorporate (or be incorporated by) gravity. If we accept that GR is also the low energy limit of some UV-complete theory, then the nonminimal curvature-matter couplings among higher-order corrections in the corresponding effective Lagrangian suggest a bridge between the quantum realm and the Cosmos at the most funda-
In that spirit, a mass generation process mediated by gravity through nonminimal curvature-matter couplings was proposed in [9, 10], both for vector bosons and for fermions. Here we present a simpler way to achieve that same goal.

2. A REDUNDANT LAGRANGIAN FOR MAXWELL’S DYNAMICS

Consider a vector field $W^\mu$ coupled to gravity with dynamics determined by the Lagrangian

$$L_0 = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{\kappa} R + V(\Phi) R + \frac{1}{2} H(\Phi) \Phi^- \Phi^\nu + L_m,$$  (1)

where the tensor $W_{\mu\nu}$ is given in terms of the potential $\Phi = W_\mu W^\mu$. $L_m$ is the matter Lagrangian of the main component responsible for the geometry, $\kappa = 8\pi G$ is the gravitational constant, and the comma means an ordinary derivative, $\Phi^- = \partial^- \Phi = \Phi^- \Phi$. Here, we will consider only the Abelian case, for simplicity. The dynamical equation of the vector field is

$$W_{\mu\nu} + 2\Omega W^\mu = 0,$$  (2)

where the semicolon means a covariant derivative, $W_{\mu\nu} = \nabla_\nu W^\mu$, and $\Omega(\Phi) = RV' - \frac{1}{2} H' \Phi^\alpha \Phi^\alpha - H \Phi^\alpha$, a prime meaning a derivative with respect to $\Phi$. Variation of $g_{\mu\nu}$ yields

$$\left(\frac{1}{\kappa} + V\right) \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = -E_{\mu\nu} - \tau_{\mu\nu} - T_{\mu\nu} - \frac{\lambda}{\kappa} g_{\mu\nu},$$  (3)

where

$$E_{\mu\nu} = W_\mu \Lambda W_{\alpha\nu} + \frac{1}{4} W_{\alpha\beta} W^{\alpha\beta} g_{\mu\nu},$$  (4)

is the energy-momentum tensor of the vector field, $T_{\mu\nu}$ is the energy-momentum tensor of matter,

$$\lambda g_{\mu\nu} = \kappa( T_{\mu\nu})_{\text{vac}} = \kappa \rho_{\text{vac}} g_{\mu\nu},$$  (5)

and

$$\tau_{\mu\nu} = V_{\mu\nu} - \nabla_\mu V(\Phi) g_{\nu\nu} + \frac{1}{4} H(\Phi) \Phi_\mu \Phi_\nu,$$

$$- \frac{1}{4} H(\Phi) \Phi_\mu \Phi_\nu g_{\mu\nu} + \Omega(\Phi) W_\mu W_\nu.$$  (6)

Lemma 2.1. Let us consider a vector field with dynamics determined by the Lagrangian (1) and such that the functions $V$ and $H$ are given by

$$V(\Phi) = \beta \Phi,$$  (7)

where $\beta$ is a dimensionless constant.

$$H(\Phi) = -\frac{3\beta^2}{(1 + \beta \kappa \Phi)},$$  (8)

where $\beta$ are dimensionless constants, one for each vector field. Then, the dynamical equation (2) takes the form

$$W_{\alpha\nu} + 2\beta(\kappa T_{\mu\nu} + 2\lambda) W^\alpha = 0,$$  (9)

where $T_{\mu\nu} = T_{\mu\nu}^{\text{vac}}$ is the trace of the energy-momentum tensor of matter.

We can now state the following theorem that establishes a gravitational mass generation mechanism for vector fields:

Theorem 2.2. When $T_m$ vanishes and $\lambda \neq 0$, the vector field acquires a mass term, one for each vector field according to the value of the specific nonminimal curvature-matter coupling constant $\beta$, and the dynamical equation for the field $W^\alpha$ becomes

$$W_{\alpha\nu} + 4\beta \lambda W^\alpha = 0.$$  (10)

Corollary 2.3. In the case where the matter energy-momentum tensor is traceless and $\lambda$ vanishes, the field $W^\alpha$ satisfies the dynamical equation

$$W_{\alpha\nu} = 0.$$  (11)

Note that, although Eq. (11) has the same form as Maxwell’s ordinary equation in curved space-time, the metric field contained in the covariant derivative $\nabla_\mu$ is a solution of the modified field equation (3), which is not gauge-invariant.

We summarize the main features of the gravitational mass generation process for vector bosons:

- The free $W^\mu$ field obeys Maxwell’s dynamics determined by the Lagrangian $L = -(1/4)W_{\mu\nu} W^{\mu\nu}$.

- There is a nonminimal coupling of the field with gravity controlled by the scalar $\Phi$ coupled to the gravitational metric.

- The state of matter is represented by a traceless energy-momentum tensor and the constant $\lambda$.

- As a consequence of this coupling, the vector field acquires a mass.

- The value of the mass does not depend on Newton’s gravitational constant.
The existence of massless vector fields in Nature is understood, in the present framework, as meaning that only the photon and the gluons couple minimally to gravity. The dynamics of all other vector fields is determined by the Lagrangian (1).

The vacuum energy density plays a key role in the present mechanism and could, in principle, be used to constrain the values of the couplings $\beta$. Unfortunately, there is no reliable estimate of the vacuum energy contribution from the SM fields, which depends on the cut-off scale one adopts and ignores the influence of gravity. For a cut-off of the order of the Planck mass, a standard calculation yields the value $\rho_{\text{vac}} \sim 10^{71}$ GeV$^4$, which implies $\lambda \sim 10^{32}$ GeV$^2$. In this case, taking $m_{W} \approx 80$ GeV as the mass term of the Proca equation (10), we obtain

$$\beta|_{\text{Planck scale}} = \frac{m_{W}^2}{4\lambda} \sim 10^{-30}. \quad (12)$$

On the other hand, at the SM scale the vacuum energy density is of the order $\rho_{\text{vac}} \sim 10^8$ GeV$^4$ [11, 12], which in its turn implies $\lambda \sim 10^{-30}$ GeV$^2$. In this case, for the same mass of the $W$ boson, we obtain $\beta|_{\text{SM scale}} \sim 10^{33}$. Finally, for the observational value of $\rho_{\text{vac}} \sim 10^{-47}$ GeV$^4$ [13], we get $\beta|_{\text{observ.}} \sim 10^{58}.$

3. A REDUNDANT LAGRANGIAN FOR DIRAC’S DYNAMICS

As in the previous case, we may consider a fermion field $\Psi$ coupled to gravity with dynamics determined by the Lagrangian

$$L = L_D + \frac{1}{\kappa} R + \frac{1}{\kappa} S(N)R$$
$$+ \frac{1}{2\kappa} B(N)N^\mu N_\mu + L_m, \quad (13)$$

where $L_D$ is Dirac’s Lagrangian, $N$ is given by $N = \bar{\Psi} \Psi$, and $m$ is the matter Lagrangian. Variation in $g_{\mu\nu}$ yields

$$\frac{1}{\kappa} (1 + S) \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right)$$
$$\quad = - T^D_{\mu\nu} - \tau_{\mu\nu} - T^m_{\mu\nu}, \quad (14)$$

where $T^D_{\mu\nu}$ is the energy-momentum tensor of the fermion field,

$$T^D_{\mu\nu} = \frac{1}{4} \left( \bar{\Psi} \gamma_{\mu} \nabla_{\nu} \Psi - \nabla_{\mu} \bar{\Psi} \gamma_{\nu} \Psi \right) + \text{symm}(\mu \leftrightarrow \nu),$$

$T^m_{\mu\nu}$ is the energy-momentum tensor of matter, and

$$\kappa \tau_{\mu\nu} = S_{\mu\nu} - \Box S_{\mu\nu} + BN_{\mu} N_\nu$$
$$\quad - \frac{1}{2} BN_{\alpha} N^\alpha g_{\mu\nu}. \quad (15)$$

Using the trace of Eq. (14), the dynamical equation for $\Psi$ takes the form

$$i\gamma^\mu \nabla_\mu \Psi + \Theta \Psi = 0, \quad (16)$$

where $\Theta = R S^I - B' N_\alpha N^\alpha - B \Box N$, and the prime means derivative with respect to $N$.

Following the same steps as in the previous section, we can now state the following theorem that establishes a mass generation mechanism for a spinor field:

**Theorem 3.1.** Let us consider a spinor field controlled by the Lagrangian (13) and such that the functions $S$ and $B$ are given by

$$S(N) = a + bN,$$  \hspace{1cm} (17)

$$B(N) = \frac{3b^2}{(1 - a - bN)}, \quad (18)$$

where $a$ and $b$ are constants. In this case, the dynamical equation for $\Psi$ becomes

$$i\gamma^\mu \nabla_\mu \Psi - m\Psi = 0, \quad (19)$$

where the mass depends only on constants $a$, $b$ and the trace of $T^m_{\mu\nu}$:

$$m = \frac{b}{(1 - a)} T_m. \quad (20)$$

**Corollary 3.2.** In the case the matter energy-momentum tensor is traceless, Eqs. (16) and (14) imply that the field $\Psi$ satisfies Dirac’s dynamics,

$$i\gamma^\mu \nabla_\mu \Psi = 0. \quad (21)$$

Let us summarize the main features of the gravitational mass generation mechanism for fermions:

- The free $\Psi$ field obeys Dirac’s dynamics driven by the Lagrangian $L_D$.
- There is a non-minimal coupling of the field with gravity controlled by the scalar $N = \bar{\Psi} \Psi$ coupled to the gravitational metric.
- The state of matter is represented by an energy-momentum tensor with constant trace.
- As a consequence of this coupling, the spinor field acquires mass.
- The value of the mass does not depend on Newton’s gravitational constant.
4. CONCLUSION

Let us point out that in the BEH mechanism the mass appears as a consequence of the vacuum energy of the Higgs field, represented by an energy-momentum tensor of the form $T_{\mu\nu} = U_0 g_{\mu\nu}$, where the constant $U_0$ is given by the extremum of a potential $U$. This implies that, at the classical level, the BEH mechanism can be considered as a particular case of the present mechanism. We also note that although in the BEH mechanism the Higgs field must be in direct interaction with the vector field, this is not necessary in the present mechanism, as the mass appears through the mediation of the gravitational field. In other words, gravity is a catalyst of the process of mass generation.

It should be noted that nonminimal curvature-matter couplings lead to violations of the Strong Equivalence Principle (SEP) that could, in principle, be detected in strong field regimes. Although the SEP is strongly constrained at the solar system scale [14], constraints at larger scales are, however, poorly studied [15].

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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