Strings, Black Holes and the Extreme Energy Cosmic Rays

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ABSTRACT: In a large class of string and string inspired models the string excitation and black hole pictures invoked as an explanation of trans-GZK cosmic ray events are equivalent. Single particle inclusive distributions are asymptotically thermal at the Hagedorn temperature. The hadron multiplicities are reminiscent of multiplicities in heavy nucleus initiated interactions.

KEYWORDS: bhs, crp.
Primary cosmic rays of energies exceeding the Greisen-Zatsepin-Kuzmin (GZK) limit pose an exciting challenge to particle physics and/or astrophysics. In essence, there are no known sources within about 50 Mpc capable of producing hadrons of energies substantially exceeding energies of $5 \times 10^{19}\text{eV}$. Nevertheless, the trans-GZK cosmic rays produce air showers very similar in their structure to hadron induced ones. For a recent review of this puzzle, the reader may consult Sigl’s article [1]. In order to resolve the puzzle, we conjectured [2] that the primaries are neutrinos, which are able to penetrate the cosmic microwave background (CMBR) essentially uninhibited. In interactions with atmospheric nuclei, however, they have CMS energies such that $\sqrt{s}$ exceeds the characteristic scale, $M_*$, of a low scale string theory [3]. Consequently, interactions are unified and string excitations produce a large cross section, thus generating hadron-like air showers. As an alternative, Anchordoqui et al., ref. [4] proposed that ultra high energy neutrinos produce “mini black holes” as originally suggested by Feng and Shapere [5] and those are responsible for the trans-GZK cosmic ray events. The production cross section is assumed to be proportional to the Schwarzschild radius squared, cf. Dimopoulos and Landsberg [6].

In a more general context, the connection between string theories and black holes has been investigated over a number of years, following the original suggestion by Susskind [7]. A careful study of the black hole scenario was carried out by Ahn et al. [8]. In general, those investigations fall into two different categories. Some authors considered the statistical mechanics of gases made of strings, [9, 10]. Others, notably Amati and Russo [11] and Damour and Veneziano [12] performed explicit calculations within the framework of some specific string models and concluded that highly excited string states resemble (are identical to?) black holes.

In this article we suggest that the string excitation and mini black hole pictures of high energy reactions are, in essence, equivalent, provided that the black holes are treated quantum mechanically, as opposed to the semiclassical treatment given in refs. [4, 5]. The conjectured equivalence holds in a broad class of theories and we only need rather general features of them. This is fortunate: currently there is no really good candidate for a phenomenologically viable string model. In fact, it has been repeatedly suggested that string models are perhaps not directly relevant for physics: one should rather think about theories of a more general category, see, for instance, Johnson’s book [13] for a very lucid exposition. Nevertheless, for the sake of definiteness, we use the term “string” in what follows. Furthermore, we exploit the conjectured equivalence of the two pictures in order to predict single particle inclusive distributions in the high energy reactions induced by trans-GZK cosmic rays: in principle, this prediction is testable by following the development of trans-GZK showers induced in the atmosphere. Here we concentrate mostly on qualitative aspects of the subject: a more detailed, quantitative investigation is deferred to a future publication.

We illustrate our point on a simple example: the excitation of a high level string
state. The inclusive single particle distribution considered by Amati and Russo, loc. cit. can be dealt with similarly. The levels of a string are labeled by a positive integer $N$ such that the mass of the state is given by the formula

$$M^2 \sim NM_*^2 \quad (N \gg 1) \quad (1)$$

In eq. (1) $M_*$ stands for the characteristic string scale. In general, for $N \gg 1$ the state is a a highly degenerate one. Let now $\mathcal{O}$ be an operator which creates a string state of index $N$ from an initial state $\vert i \rangle$. (We have in mind, for instance, string excitation in $\nu$-quark interactions, as conjectured in ref. 2. In that case, $\vert i \rangle$ can stand for a quark state and $\mathcal{O}$ for the fermionic current corresponding to the neutrino.)

Omitting trivial factors, the transition probability is given by:

$$P \propto \sum_{\alpha} \langle i \vert \mathcal{O}^\dagger \vert N, \alpha \rangle \langle N, \alpha \vert \mathcal{O} \vert i \rangle \delta (E - NM_*) \quad (2)$$

In eq. (2) $\alpha$ stands for the collection of labels necessary for the full specification of states at level $N$: one has to sum over (most of) those, since the quantum numbers carried by $\vert i \rangle$ and $\mathcal{O}$ do not fully specify the substate at level $N$ to which the transition takes place.

We recognize that the quantity in eq. (2),

$$\rho_M = \sum_{\alpha} \vert N, \alpha \rangle \langle N, \alpha \vert \delta (E - NM_*) \quad (3)$$

is just the (microcanonical) density matrix of the final state. (In statistical mechanics, a smoothing of the delta function is necessary in order to define a level density; in the present context, such a smoothing is automatic if the finite width of resonances is taken into account.) The summation over $\alpha$ leads to the information loss discussed by Amati [14].

It is well known that a large microcanonical ensemble can be well approximated by a canonical one, see, e.g. Huang’s textbook [13]. The argument given in textbooks runs as follows. One establishes that the fluctuations in a canonical ensemble asymptotically follow a normal distribution; the width of the distribution gets narrower as the energy increases. In the limit of infinite energy the normalized distribution becomes a $\delta$ function.

A more direct approach is the following. Let $E$ be the energy as before and $H$ be the Hamiltonian of the system. One can write the microcanonical density matrix in operator form as follows.

$$\rho_M = \delta (E - H) \quad (4)$$

In the present context, the energy is the invariant center of mass energy of the interaction described by eq. (2), $E = \sqrt{s}$ and $H$ is the mass operator of the string.
One then uses the following complex integral representation of the δ function:

$$\delta (E - H) = \frac{1}{2\pi i} \int_{L - i\infty}^{L + i\infty} dv \exp v (E - H),$$  \hspace{1cm} (5)

where an appropriate choice of $L$ gives a convergence factor. Next, one takes the trace of both sides to obtain the partition functions. (This is sufficient: by adding external source terms to $H$, one obtains the generating functional of the correlation functions.) One has then:

$$\exp S(E) = \frac{1}{2\pi i} \int_{L - i\infty}^{L + i\infty} dv \exp (vE) \exp (S_c(v));$$  \hspace{1cm} (6)

We used the relation between the partition function and entropy on both sides of the previous equation. Obviously, $\exp S_c(v)$ is the canonical partition function at inverse temperature $v$. Next, one evaluates the integral in the previous equation by means of the method of steepest descents. The location of the saddle point, $v = \beta$, is given by the equation:

$$E = -\left(\frac{\partial S_c}{\partial v}\right)_{v=\beta},$$  \hspace{1cm} (7)

which is the usual thermodynamic relation between temperature and energy. Using the preceding equation, the leading order contribution of the integral is given by

$$\exp(S_c(\beta) - E\beta) = \exp S(E).$$  \hspace{1cm} (8)

The quantity $S_c\beta - E\beta$ is recognized as the Legendre transform of the canonical entropy. Finally, by carrying out the Gaussian integration, one obtains a contribution to the entropy, which, up to an irrelevant additive constant equals to $-3/2 \ln E/M_*$. All these formulae are valid for $E/M_* \gg 1$.

An operational definition of a “large” microcanonical ensemble is that terms of $O(1/E)$ in the expression of the entropy are negligible\textsuperscript{1}.

Here we consider level densities of the form:

$$d(E) = \exp S(E) \sim C (E/M_*)^{-\gamma - 3/2} \exp \left(\alpha (E/M_*)^\delta\right);$$  \hspace{1cm} (9)

The quantities $C, \gamma, \alpha, \delta$ depend on the specific model considered. The contribution of the Gaussian integral to the entropy has been taken into account. (In known string models, $\delta = 1$.) The inverse temperature is:

$$\beta = \frac{\partial S}{\partial E} \sim -\frac{\gamma + 3/2}{E} + \alpha \delta \left(\frac{E}{M_*}\right)^{\delta - 1} M_*^{-1}$$  \hspace{1cm} (10)

\textsuperscript{1}Amati and Russo loc. cit. use an adaptation of the Darwin-Fowler method and discuss some terms of $O(1/N)$ in the framework of specific string models. For $N \gg 1$, the asymptotic approach used here and the Darwin-Fowler method yield identical results.
Notice that for $\delta = 1$, the temperature is asymptotically constant and, hence, it may be identified with a Hagedorn temperature, $T_H$. Notice, however, that the expression of the inverse temperature, eq. (10) gives a temperature approaching $T_H$ from above. From a physical point of view, this is unacceptable: the Hagedorn temperature is supposed to be a maximal temperature. This is due to the fact that the coefficient of $1/E$ in eq. (10) is negative. The problem is corrected by taking into account the contribution of Kaluza-Klein excitations to the entropy. Apart from an additive constant, the contribution is proportional to $(E/M_\ast)^n$, $n$ being the number of compact dimensions. For instance, for an open superstring in the critical dimensionality, $n = 6$ and $\gamma = 9/2$, giving $-(\gamma + 3/2) + n = 0$. Thus, the approach to the Hagedorn temperature in this case is determined by higher order terms in the asymptotic expansion.

The expression of the Hagedorn temperature in terms of $M_\ast$ is somewhat model dependent: in the calculations cited above, one has, $T_H = M_\ast/(a\pi)$, with $a$ ranging, approximately, between 2 and 4 for various string models. For the purposes of numerical estimates in this work, we adopt $a = 3$: this is equivalent, of course, to fixing the parameter $\alpha$ in eq. (10). Equation (10) establishes the connection between the string and (microscopic) black hole pictures: in fact, it means that – in a sense – a large and highly degenerate isolated system serves as its own thermal reservoir. Consequently, the terms “string” and “black hole” are used interchangeably in what follows.

The asymptotic estimate given here has far reaching consequences. In particular, the inclusive single particle distribution can be calculated using elementary statistical mechanics of a canonical ensemble as it was carried out in detail for specific string models by Amati and Russo, ref. [11]. We have in the rest frame of a decaying resonance:

$$dn = \frac{d\sigma}{\sigma} = \frac{1}{\exp (k_0/T) \pm 1} \frac{d^3k}{k_0 L^2}, \quad (11)$$

where $k_\mu$ stands for the four momentum of the particle observed in the decay and, asymptotically, we may put $T = T_H$. The sign $\pm$ is valid for Fermi/Bose statistics of the observed particle, respectively. The quantity $L$ is a characteristic length of the system. Tentatively we put it equal to a compactification radius in a symmetric toroidal compactification scheme. In what follows, we also put $k^2 = 0$. This is a permissible approximation except perhaps in the far infrared of the observed particle which, however, is not very relevant for the development of observable showers.

We now note that the expression of $dn$ is Lorentz invariant. Consequently, one can replace the rest frame expression of $k_0$ by a Lorentz invariant quantity reducing to $k_0$ in that frame. This is elementary. Use the invariant:

$$K = \frac{(P \cdot k)}{\sqrt{P^2}} \quad (12)$$
in place of $k_0$ in eq. (11). Here, $P$ stands for the total four momentum of the decaying resonance. In a scattering process, such as
\[ \nu + q \rightarrow \text{resonance} \rightarrow (k) + X, \]
one can approximate $\sqrt{P^2} \approx \sqrt{s}$. (Here, "$(k)$" stands for the observed particle of four momentum $k$.) The inclusive distribution, eq. (11) with the replacement $k_0 \rightarrow K$ shows the expected properties if evaluated in the laboratory frame. In the high energy limit, fermions and bosons have to be treated somewhat differently, due to their different statistics.

We evaluate the invariant $K$ in the laboratory frame, where the target quark of an effective mass $m$ is approximately at rest and the incident neutrino has energy $E_\nu$. In the limit $E_\nu \gg m$ we have:
\[ K \sim k_{0,L} \sqrt{E_\nu} \left( \frac{m}{E_\nu} + 2 \sin^2 (\theta/2) \right). \tag{13} \]
Here, $k_{0,L}$ stands for the energy of the observed particle evaluated in the laboratory frame; $\theta$ is the angle between the incident neutrino and the observed particle.

There is a forward peak ($\theta = 0$) both for a boson and a fermion observed. For a boson, we have
\[ \frac{1}{\sigma} \int d^3k \frac{1}{k_0} \exp \left( \frac{k_0}{T_H} \right) - 1 = \frac{L^2}{12} T_H^2, \tag{14} \]
whereas for a fermion the term $\propto m/E_\nu$ can be neglected in eq. (13) and one finds that the forward peak remains finite:
\[ \frac{1}{\sigma} \int d^3k \frac{1}{k_0} \left| \theta = 0 \right| \sim \frac{L^2}{2}. \]

The different behavior of observed bosons and fermions is due, of course, to their different statistics. Obviously, $dn$ is exponentially small for a finite value of the emission angle $\theta$, as expected: this is just the Boltzmann limit of the thermal distribution.

One can evaluate the total multiplicities by means of a straightforward integration. As stated before, neglecting the rest masses of the observed particles is a fair approximation. With that, one has:
\[ I \equiv L^2 \int \frac{d^3k}{k_0} \frac{1}{\exp(k_0/T_H) - 1} = \frac{L^2}{12} T_H^2, \tag{15} \]
The corresponding integral for fermions is smaller by a factor of $1/2$.

It follows that the total multiplicity of a particle of any kind is proportional to its statistical weight, viz.
\[ n_B = \frac{1}{12} L^2 T_H^2 g_B, \tag{15} \]
respectively. In the asymptotic limit the multiplicities are independent of the incident energy: an observable property of this scheme.

As far as unconfined particles ($\gamma$, W, Z, leptons) are concerned, eqs. (15, 16) contain the final answer. However, for quarks and gluons, one has to consider their fragmentation into observable hadrons. Instead of carrying out a detailed analysis of the fragmentation, we adopt here an approximate procedure, which, however, should give us an idea about the order of magnitude of hadrons of any flavor produced.

We notice that popular analytical fits to parton fragmentation functions, on the average, give about $\langle n_H \rangle \approx 3$ hadrons produced from each parton, see [17] for a typical reference. Consequently, in order to obtain the multiplicities of hadrons produced, we just multiply the quark and gluon multiplicities by 3.

Below, we list the statistical weights for each kind of particle considered, taking three families into account for quarks and leptons.

\[
g_q + g_q = 72 \\
g_g = 16 \\
g_t + g_t = 24 \\
g_W + g_W + g_Z + g_\gamma = 8
\] (17)

In compiling the statistical weights, we assumed that in the string regime symmetries are unbroken; in particular, $W$ and $Z$ are transverse. For the time being, we did not consider the various superpartners which may be produced in such reactions.

Finally, we have to determine the characteristic length, $L$, occurring in eq. (11). We use the ADD formula [18], assuming, for the sake of simplicity a symmetric toroidal compactification:

\[
\frac{1}{L} = 2\pi M_s \left( \frac{M_s}{M_P} \right)^{2/n}
\] (18)

In eq. (18), $M_P$ stands for the four dimensional Planck mass and $n$ is the number of compactified dimensions. Previously we presented evidence that $M_s$ should be around 80 TeV or so, [19]. In what follows, we use that value of $M_s$ for the purposes of numerical estimates. However, given the uncertainties at the present stage, it is reasonable to conjecture $50\text{TeV} \leq M_s \leq 100\text{TeV}$.

Using these ingredients, we estimate the total number of hadrons generated in the initial interaction. One has the estimate:

\[
N_{\text{had}} \approx 3 \times \left( g_g + 1/2 (g_q + g_\gamma) \right) I
\] (19)

It is instructive to compare the multiplicity obtained from eq. (19) at a reasonable number of extra dimensions ($n = 6, 7, 8$) with what an incident proton or nucleus
would produce\(^2\). In Table 1 we summarize the hadron multiplicities estimated in the black hole picture, using \(M_* = 80\text{TeV}\).

The number of prompt leptons can be calculated analogously.

Let us recall that according to available data, the trans-GZK air showers are “hadron-like”: the showers develop similarly to hadron induced ones at lower energies, see Sigl, *loc. cit*. Let us consider a primary energy, \(E_L = 3 \times 10^{11}\text{GeV}\), about the highest energy observed, \([1]\). Assuming no new physics between present accelerator energies and that energy, one can extrapolate the multiplicity of secondary hadrons in various reactions (\(pp, p\bar{p}, e\bar{e}\)). The energy dependence of the multiplicity is similar in all those reactions, *cf.* the tables in the Particle Data Group, \([21]\). Lumping all the data together, one obtains a simple fit:

\[
N_{ch} = 2.60 - 0.18 f + 1.23 f^2,
\]

where \(f = s/1\text{GeV}^2\). Extrapolated to \(E_L = 10^{11}\text{GeV}\) in a \(pp\) interaction, one gets \(N_{ch} \approx 170\). The total multiplicity, relevant for shower development, is estimated by multiplying this number by \(3/2\), since most of the secondaries are pions.

Clearly, the hadronic multiplicities produced by the microscopic black holes are larger; thus, one may conclude that the event in question was initiated by a heavy nucleus. Adopting a simple superposition model of nuclear interactions, see *e.g.* Gaisser \([22]\) one can estimate an “effective atomic number” of the reaction by taking the ratio of the multiplicity produced by a black hole to that of the extrapolated hadronic multiplicity. All such formulae are based on a number of assumptions and cannot be used for obtaining quantitative estimates.

In the following Table we summarize the effective atomic numbers, \(A_{\text{eff}}\), for the number of compactified dimensions considered here.

Given thecrudeness of the approximations used in obtaining the entries of the preceding Table, this is merely a hint to be followed up by more careful calculations. The analysis of future data from detectors presently under construction will, eventually, bear out the behavior of showers induced by trans-GZK cosmic rays.

We conclude with a few remarks.

- We demonstrated the equivalence of a model of neutrino induced string excitations and one involving (microscopic) black holes, both purporting to explain trans-GZK cosmic rays. The equivalence simplifies the calculation of single particle inclusive distributions within such a scheme. In fact, at least asymptotically, the inclusive distributions can be calculated in a simple way, using

\(^2n = 6, 7\) are theoretically motivated: superstrings and \(n = 11\) SUGRA, respectively. We added \(n = 8\) in order to keep possible future models in mind. Low numbers of extra dimensions are probably excluded, see \([20]\).
Lorentz invariance and elementary statistical mechanics. It is to be pointed out that the asymptotic equivalence of the “excited string” and “mini black hole” pictures is independent of the tree approximation used by Amati and Russo, loc. cit. It is, nevertheless, pleasing that an explicit calculation bears out the the black body nature of a highly excited string.

- While we concentrated on qualitative aspects of our discussion, it is likely that this can be followed up by a more quantitative study, involving a MC simulation of the string/black hole induced trans-GZK showers.

- There remain some questions to be clarified. In particular, the leading asymptotic estimates are largely model independent, but one has little knowledge about model dependent corrections of $O(1/E)$. Very often, however, leading terms in an asymptotic expansion yield quantitatively correct estimates$^3$.

The idea of the research reported here arose in discussions with Zoltán Kunszt during the authors’ visit at ETH in Zurich: we are grateful to Professor Kunszt for the hospitality extended to us and a fruitful exchange of ideas. We further thank Luis Anchordoqui, Will Burgett and Haim Goldberg for several enjoyable discussions on the puzzle of trans-GZK cosmic rays.

References

[1] G. Sigl, Annals of Physics 303, 117 (2003).

[2] G. Domokos and S. Kovési-Domokos, Phys. Rev. Lett. 82, 1366 (1999).

[3] P. Horava and E. Witten, Nuc. Phys. B475, 94 (1996) J. Lykken, Phys.Rev. D54, 3693 (1996). N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys.Lett. B429, 263 (1998). I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys. Lett. B4336, 257 (1998).

[4] L.A. Anchordoqui and Haim Goldberg, Phys. Rev. D65, 047502 (2002).

[5] J.L. Feng and A.D. Shapere, Phys.Rev. D65, 124027 (2002).

[6] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87 161602 (2001).

[7] L. Susskind, hep-th/9309145 (unpublished).

[8] E.-J. Ahn, M. Ave, M. Cavaglia and A.V. Olinto, Phys. Rev. D68, 043004 (2003).

[9] P. Salomonson and B.-S. Skagerstam, Nuclear Physics B268, 349 (1986.)

$^3$A famous example is the evaluation of $\Gamma(z)$ by means of the Stirling formula even for $|z|$ not much larger than 1.
[10] M.J. Bowick and L.C.R. Wijewardhana, Phys.Rev. Letters 54, 2485 (1985).

[11] D. Amati and J.G. Russo, Phys. Letters 454, 207 (1999).

[12] Th. Damour and G. Veneziano, Nuc. Phys. B568, 93 (2000).

[13] Clifford V. Johnson, “D-Branes”. Cambridge Monographs on Mathematical Physics, Cambridge University Press (2003).

[14] D. Amati, in Proceedings of The Abdus Salam Memorial Meeting, edited J. Ellis, F. Hussain, T. Kibble, G. Thompson and M. Virasoro. World Scientific Publishing Co. Singapore (1997).

[15] K. Huang, “Statistical Mechanics”, second edition. Wiley & Sons, 1987.

[16] D. Horn and F. Zachariasen, “Hadron Physics at Very High Energies”, Ch. 6. Benjamin, New York (1973). References to the original articles are given in this work.

[17] V.D. Barger and R.J.N. Phillips, “Collider Physics”, Ch. 6. Addison-Wesley (1987).

[18] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263 (1998).

[19] W.S. Burgett, G. Domokos and S. Kovesi-Domokos, hep-ph/0209162. (This is an expanded version of a paper presented at ICHEP 2002, Amsterdam.)

[20] S. Hannestad and G.G. Raffelt, Phys. Rev. Lett. 88:071301 (2002).

[21] K. Hagiwara et al., Phys.Rev. D66, 010001 (2002). (http://pdg.lbl.gov)

[22] T.K. Gaisser, “Cosmic Rays and Particle Physics”, Ch. 16. Cambridge University Press, Cambridge (1990).

[23] D.J. Bird et al., Astrophys. J. 441, 144 (1995)