Thermal Mechanical Response of Elastic Half-Plane with Infinite Row of Parallel Cracks under Uniform Heat Flux

Sei UEDA** and Junpei ANDO***

In this study, thermal singular stresses in an elastic half-plane containing an infinite row of parallel cracks perpendicular to the boundary is considered. The half-plane is subjected to a uniform heat flux and a uniform mechanical load. The crack surfaces and free surface of the half-plane are maintained at uniform temperatures. The Fourier transform techniques are used to formulate the problem in terms of singular integral equations. The singular integral equations are solved by using the Gauss-Jacobi integration formula. Both the cases of an internal crack and an edge crack are studied. Numerical calculations are carried out, and the effects of the geometric parameters on the temperature-thermal stress distributions and the thermal stress intensity factors are shown graphically.

Key Words: Elasticity, Fracture Mechanics, Thermal Stress, Stress Intensity Factor, Uniform Heat Flux, Integral Transform

1. Introduction

In recent years, machines and structures have been moving towards high performance and high power, and are used in high-temperature environments. Therefore, these components are subjected to severe thermal loads which give rise to intense thermal stresses in the components, particularly around cracks and other kinds of defects. The concentration of stresses around defects often results in catastrophe(1). Also, stress-corrosion cracking (SCC), which requires further attention for geothermal energy extraction(2), involves the interaction between stress and a corrosive environment. The environment, in which a crack in the material is heated and cooled by the working fluid, is considered to influence SCC(3), (4). Then, the thermal singular stresses in an elastic half-plane containing a crack perpendicular to the boundary and subjected to a uniform heat flux were analyzed in our previous study(5).

On the other hand, in a material with many cracks, the stress field is affected by the interaction of cracks, and it is important to examine the effects of thermoelastic interactions between cracks on the fracture behavior. Several analytical studies concerning elastic interactions of many cracks have been reported(6) – (8).

In this study, an elastic half-plane containing an infinite row of parallel cracks perpendicular to the boundary under thermomechanical load is considered. The thermal singular stresses when the crack surfaces are maintained at uniform temperature are analyzed. The Fourier transform techniques are used to formulate the problem in terms of singular integral equations(9), (10). The singular integral equations are solved by using the Gauss-Jacobi integration formula(11). Numerical calculations are carried out, and the effects of the crack location, the crack spacing, and the thermal and mechanical loads on the thermal stress intensity factors of internal or edge cracks are shown graphically.

2. Description of the Problem

Consider an elastic half-plane containing an infinite row of parallel cracks perpendicular to the boundary, as shown in Fig. 1. A crack of length $2c = b-a$ ($0 \leq a < b < \infty$) is located along $y = \pm 2nd$ ($n = 0, 1, 2, \cdots$) with reference to the rectangular coordinate system $(x, y, z)$. Generally, the thermal and elastic analysis can be considered independently. However, it is also important to investigate the effect of thermoelastic interactions on the fracture behavior. Then, we consider a problem under thermal and mechan-

---

* Received 23rd November, 2005 (No. 04-1015). Japanese Original: Trans. Jpn. Soc. Mech. Eng., Vol.71, No.706, A (2005), pp.960–967 (Received 16th September, 2004)
** Department of Mechanical Engineering, Faculty of Engineering, Osaka Institute of Technology, 5–16–1 Omiya, Asahi-ku, Osaka 535–8585, Japan
E-mail: ueda@med.oit.ac.jp
*** Graduate Student, Major in Mechanical Engineering, Graduate School, Osaka Institute of Technology, 5–16–1 Omiya, Asahi-ku, Osaka 535–8585, Japan
ical loads, that is, the half-plane with cracks is subjected to a uniform heat flux \( q_0 \) in the \( x \)-direction and a uniform mechanical load \( \sigma_0 \) in the \( y \)-direction. The crack surfaces and free surface are maintained at uniform temperatures \( \Phi_c \) and \( \Phi_0 \), respectively. Also, there are relationships among \( \Phi_c, \Phi_0 \), and \( q_0 \). In following, the subscripts \( x, y, z \) will be used to refer to the direction of coordinates.

Let the temperature and the components of the displacement vector in the \( x \)- and \( y \)-directions be labeled \( \phi(x, y) \) and \( u_x(x, y), u_y(x, y) \). The heat flow components \( q_x(x, y) \), \( q_y(x, y) \) and the stress components \( \sigma_{xx}(x, y), \sigma_{yy}(x, y), \sigma_{xy}(x, y) \) are

\[
q_x = -\lambda \frac{\partial \phi}{\partial x}, \quad q_y = -\lambda \frac{\partial \phi}{\partial y}
\]

\[
\sigma_{xx} = \frac{\mu}{k-1} \left[ (1+k) \frac{\partial u_y}{\partial y} + (3-k) \frac{\partial u_x}{\partial x} \right] - \frac{2\mu(1+\nu)}{1-2\nu} \alpha \phi
\]

\[
\sigma_{yy} = \frac{\mu}{k-1} \left[ (3-k) \frac{\partial u_x}{\partial x} + (1+k) \frac{\partial u_y}{\partial y} \right] - \frac{2\mu(1+\nu)}{1-2\nu} \alpha \phi
\]

\[
\sigma_{xy} = \mu \left[ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right]
\]

where, \( \lambda, \alpha, \sigma \) and \( \nu \) are the thermal conductivity, the coefficient of linear expansion and Poisson's ratio, and \( \mu \) and \( k = 3 - 4\nu \) are Lamé's constants.

The governing equations for the temperature and displacements are

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0
\]

\[
(k+1) \frac{\partial^2 u_x}{\partial x^2} + (k-1) \frac{\partial^2 u_x}{\partial y^2} + 2 \frac{\partial^2 u_y}{\partial x \partial y} - 4\alpha(1+\nu) \frac{\partial \phi}{\partial x} = 0
\]

\[
(k+1) \frac{\partial^2 u_y}{\partial y^2} + (k-1) \frac{\partial^2 u_y}{\partial x^2} + 2 \frac{\partial^2 u_x}{\partial x \partial y} - 4\alpha(1+\nu) \frac{\partial \phi}{\partial y} = 0.
\]

This problem can be solved using the superposition technique. First, we solve the disturbed temperature generated from a heat flux disturbed by an isothermal crack. Next, we obtain the thermal stresses generated by the disturbed temperature for the uncracked medium. Finally, by solving the isothermal crack problem with the equal and opposite thermal stresses on the crack surfaces, the singular stresses can be obtained. Also, because of the assumed symmetry of the geometry, it is sufficient to consider the problem only for \( 0 \leq x < \infty, 0 \leq y < d \).

### 3. Temperature Field Analysis

The symmetry and boundary conditions for the temperature field can be written as

\[
\begin{align*}
\phi(x, 0) &= \Phi_c \quad (a < x < b) \\
q_x(x, 0) &= 0 \quad (0 \leq x \leq a, b \leq x < \infty) \\
q_y(x, d) &= 0 \quad (0 \leq x < \infty) \\
\phi(0, y) &= \Phi_0 \quad (0 \leq y < d) \\
q_x(\infty, y) &= q_0 \quad (0 \leq y < d).
\end{align*}
\]

The general solution of the temperature field is obtained by solving governing equation (3) using the Fourier integral transform techniques:

\[
\phi(x, y) = \frac{2}{\pi} \int_0^\infty \left\{ A^\phi(s) \exp(-sy) + B^\phi(s) \exp(sy) \right\} \sin(sx) ds + \Phi_0 - \frac{q_0}{\lambda} x,
\]

where \( A^\phi(s) \) and \( B^\phi(s) \) are the unknown functions to be solved. The first condition in Eqs. (5) and (7) are equivalent to

\[
q_x(x, 0) = -\lambda \frac{\partial \phi}{\partial x}(x, 0) = 0 \quad (a < x < b) \quad (10)
\]

\[
q_y(0, y) = -\lambda \frac{\partial \phi}{\partial y}(0, y) = 0 \quad (0 \leq y < d).
\]

The problem may be reduced to a singular integral equation by defining the following new unknown function \( G^\phi(x) \):

\[
G^\phi(x) = \begin{cases} \frac{-\partial \phi}{\partial y}(x, 0) & (a < x < b) \\ 0 & (0 \leq x \leq a, b \leq x < \infty) \end{cases}
\]

The subsidiary condition is obtained from the second condition in Eq. (5).

\[
\int_a^b G^\phi(t) dt = 0
\]

Making use of boundary condition (10) with Eqs. (8) and (11), the following singular integral equation for the determination of unknown function \( G^\phi(t) \) can be obtained:

\[
\int_a^b \left\{ \frac{1}{t-x} + \frac{1}{t+x} + M(t, x) \right\} G^\phi(t) dt = \frac{q_0}{\lambda},
\]

where the kernel function \( M(t, x) \) is

\[
M(t, x) = \frac{\pi}{2d} \left[ \frac{\sinh[\pi(t-x)/d]}{\cosh[\pi(t-x)/d]-1} + \frac{\sinh[\pi(t+x)/d]}{\cosh[\pi(t+x)/d]-1} \right] - \frac{1}{t-x} + \frac{q_0}{\lambda x}.
\]

Once \( G^\phi(t) \) is obtained, the temperature field can be easily calculated as
\[
\phi(x, y) = \phi'(x, y) + \Phi_0 - \frac{q_0}{A} \left( \frac{u}{c^2} \right)
\]

where the disturbed term \( \phi'(x, y) \) is

\[
\phi'(x, y) = \frac{1}{\pi} \int_0^\infty \frac{4 \cosh(sy)}{s[\exp(2sd) - 1]} \sin(sx) \sin(s) ds + \ln \left( \frac{(x + t)^2 + y^2}{(x - t)^2 + y^2} \right) \left( \int_0^b G^\beta(t) dt \right)
\]

\( (1 \leq (a + b)/2c < \infty) \). (17)

Therefore, the temperature of the free surface \( \Phi_0 \) is found as

\[
\Phi_0 = \Phi_e - \frac{1}{\pi} \int_0^\infty \frac{4 \cosh(sy)}{s[\exp(2sd) - 1]} \sin(sx_e) \sin(s) ds + \ln \frac{x + t}{x - t} \int_0^b G^\beta(t) dt + \frac{q_0}{A} x_e
\]

\( (a < x_e < b) \). (18)

Also, \( \Phi_0 \) is \( \Phi_e \) for \( (a + b)/2c = 1.0 \), and the temperature field for \( (a + b)/2c \rightarrow \infty \) can be obtained as

\[
\phi'(x', y) = \frac{1}{\pi} \left[ \int_0^\infty \frac{4 \cosh(sy)}{s[\exp(2sd) - 1]} \sin(sx') \sin(sy) ds + \ln \left( \frac{(x' + t)^2 + y^2}{(x' - t)^2 + y^2} \right) \left( \int_0^b G^\beta(t) dt \right) \right]
\]

\( \Phi_0 \rightarrow \infty \), (19)

where

\[ x' = x - \frac{a + b}{2} \]. (20)

4. Thermal Stress Analysis

The symmetry and boundary conditions for the uncracked elastic medium can be written as

\[
u^T_{xx}(x, 0) = 0 \quad (0 \leq x < \infty)
\]

\( (21) \)

\[
u^T_{yy}(x, 0) = 0 \quad (0 \leq x < \infty)
\]

\( (22) \)

\[
u^T_{tx}(0, y) = 0 \quad (0 \leq y < d)
\]

\( (23) \)

\[
u^T_{ty}(0, y) = 0 \quad (0 \leq y < d)
\]

\( (24) \)

\[
u^T_{xy}(x, d) = 0 \quad (0 \leq x < \infty)
\]

\( (25) \)

\[
\frac{\partial}{\partial x} u^T_{xy}(x, d) = 0 \quad (0 \leq x < \infty),
\]

\( (26) \)

where the superscript \( T \) indicates the physical quantities induced by the temperature change in the medium without any cracks. The components of the displacement vector, \( u^T_{x}(x, y) \) and \( u^T_{y}(x, y) \), are given by

\[
u^T = \frac{3}{2} u^T_{x}, \quad u^T_{y} = \frac{1}{2} u^T_{y}
\]

\( (27) \)

where \( u^{T1}_{x}(x, y) \), \( u^{T1}_{y}(x, y) \) are the particular solutions of Eq. (4), and \( u^{T2}_{x}(x, y) \), \( u^{T2}_{y}(x, y) \) \( (i = 2, 3) \) are the general solutions of homogeneous equations obtained by setting \( \phi = 0 \) in Eq. (4). To obtain the particular solutions \( u^{T1}_{x}(x, y) \), \( u^{T1}_{y}(x, y) \), the thermoelastic displacement potential \( F(x, y) \) is introduced as follows:

\[
u^T_{x} = \frac{\partial F}{\partial x}, \quad \nu^T_{y} = \frac{\partial F}{\partial y}.
\]

\( (28) \)

The thermal stresses may only be induced by the disturbed temperature field \( \phi'(x, y) \), and the governing equation for \( F(x, y) \) becomes

\[
\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = \beta \phi'.
\]

\( (29) \)

where

\[
\beta = \frac{1 + \nu}{1 - \nu} \alpha.
\]

\( (30) \)

The particular solution of Eq. (29) can be obtained as

\[
F(x, y) = \frac{\beta}{\pi} \int_0^\infty \frac{\sinh(y) \sin(nx) \sin(st) ds}{s^2 \sinh(sd) \sin(\pi y)} (31)
\]

The general solutions \( u^{T1}_{x}(x, y) \) and \( u^{T1}_{y}(x, y) \) \( (i = 2, 3) \) can be expressed as

\[
u^T_{x} = 2 \int_0^\infty \frac{[A^T(s) + syD^T(s)] \sinh(sy)}{\sinh(\pi y)} ds + \frac{B^T(s)}{\sinh(\pi y)} \cos(sx) ds
\]

\( (32) \)

\[
u^T_{y} = 2 \int_0^\infty \frac{[A^T(s) + syD^T(s) - \kappa D^T(s)] \sinh(sy)}{\sinh(\pi y)} \sin(\pi y) ds
\]

\( (33) \)

\[
u^T_{x} = \sum_{n=1}^\infty \frac{n \pi}{d} \left[ A^T_n + \frac{n \pi}{d} x B^T_n \right] \exp\left(-\frac{n \pi}{d} x\right) \sin(\pi y)
\]

\( (34) \)

\[
u^T_{y} = \sum_{n=1}^\infty \frac{n \pi}{d} \left[ A^T_n + \frac{n \pi}{d} x B^T_n \right] \exp\left(-\frac{n \pi}{d} x\right) \sin(\pi y)
\]

\( (35) \)

where \( A^T(s), B^T(s), D^T(s), \kappa D^T(s), A^T_n, B^T_n \) \( (n = 1 ~ \infty) \) are the unknowns to be solved. By substituting Eqs. (28), (32)–(35) into boundary conditions (21)–(26) with Eqs. (2), (27) and (31), the thermal stress component \( \sigma_{xy}(x, y) \) is obtained. For the case of \( y = 0 \), \( \sigma_{xy}(x, 0) \) are reduced as follows:

\[
\sigma_{xy}(x, 0) = -\frac{1}{\pi} \beta \mu \int_0^b \ln \left( \frac{x + t}{x - t} \right) \frac{G^\beta(t) dt}{x - t} - \frac{2}{\pi} \beta \mu \left[ 1 - \sum_{n=1}^\infty \frac{n \pi}{d} \right] \exp\left(-\frac{n \pi}{d} x\right)
\]

\[
\times \int_0^b G^\beta(t) dt - \frac{1}{\pi} \beta \mu \int_0^\infty \frac{1}{s \sinh(3sd)} \sin(\pi y) dt
\]

\[
- \exp(-2sd) \sin(\pi y) ds \int_0^b G^\beta(t) \sin(\pi x) dt
\]

\( (1 \leq (a + b)/2c < \infty) \) (36)
\[ \sigma_{yy}(x', 0) = -\frac{1}{\pi} \mu \int_{0}^{\infty} \ln \left| \frac{x' + t}{x' - t} \right| G^s(t) dt \]

\[ = -\frac{1}{\pi} \beta \int_{0}^{\infty} \frac{1}{s \sinh^2 (sd)} [1 + 2sd - \exp(-2sd)] \sin(sx') s \int_{0}^{\infty} G^s(t) \sin(st) dt \]

\[ ((a + b)/2c \to \infty). \quad (37) \]

### 5. Isothermal Crack Problem

The symmetry and boundary conditions can be written as:

\[ \sigma_{yy}(x, 0) = -\left[ \sigma_0 + \sigma_{yy}(x, 0) \right] \quad (a < x < b) \quad (38) \]

\[ u_y(x, 0) = 0 \quad (0 \leq x \leq a, b \leq x < \infty) \quad (39) \]

\[ \sigma_{xy}(x, 0) = 0 \quad (0 \leq x < \infty) \quad (40) \]

\[ u_y(x, d) = 0 \quad (0 \leq x < \infty) \quad (41) \]

\[ \sigma_{xy}(x, d) = 0 \quad (0 \leq x \leq \infty) \quad (42) \]

\[ \sigma_{xx}(0, y) = 0 \quad (0 \leq y < d) \quad (43) \]

\[ \sigma_{yy}(0, y) = 0 \quad (0 \leq y < d). \quad (44) \]

The general solutions of the elastic field are obtained by solving governing equation (4), in the absence of a temperature term, by Fourier integral transform technique:

\[ u_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \{A(s) + (|s|y - \kappa)B(s)\} \exp(-|s|y) + \{C(s) - (|s|y + \kappa)D(s)\} \exp(|s|y) \exp(-i\pi y) \right] ds \]

\[ + \sum_{n=1}^{\infty} \frac{n\pi}{d} \left( A_n + \frac{n\pi}{d} + \kappa \right) B_n \exp \left( -\frac{n\pi}{d} y \right) \cos \left( \frac{n\pi}{d} y \right) \]

\[ u_y = -\frac{i}{2\pi} \int_{-\infty}^{\infty} \left[ \{A(s) + |s|yB(s)\} \exp(-|s|y) + \{C(s) - |s|yD(s)\} \exp(|s|y) \exp(-i\pi y) \right] ds \]

\[ + \sum_{n=1}^{\infty} \frac{n\pi}{d} \left( A_n + \frac{n\pi}{d} + \kappa \right) B_n \exp \left( -\frac{n\pi}{d} y \right) \sin \left( \frac{n\pi}{d} y \right) \quad (45) \]

where \( A(s), B(s), C(s), D(s), A_n, B_n, (n = 1 \sim \infty) \) are unknowns to be solved. The problem may be reduced to a singular integral equation by defining the unknown functions \( G^m(x) \) and \( G^t(x) \):

\[ -\frac{d}{dx} u_y(x, 0) = G^m(x) + G^t(x), \quad (46) \]

The numerical results of the temperature and thermal stress distributions, and thermal stress intensity factors are calculated. For the numerical calculations, examination of the convergence reveals that the semi-infinite integrals and infinite series of Eqs. (17), (19), (50) and (51) give satisfactory accuracy \((< 10^{-10})\). Also, no more than forty terms in the Gauss-Jacobi integration formula are found to be necessary in obtaining highly accurate values of stress intensity factors. Tables 1 and 2 show the stress intensity factors generated by the mechanical load, together
with the numerical solutions obtained in previous studies \(^{12,13}\). The numerical solutions are found to be in good agreement.

First, we consider the disturbed temperature field \(\phi'(x,y)\). Figures 2–4 indicate the effect of the normalized crack spacing \(d/c\) on the normalized temperature field \((\phi'(x,y), \phi'(x',y))/(q_0c/\lambda)\) on the plane \(y/c = 0\) for the normalized crack locations \((a+b)/2c = 1.5, 1.0\) and \(\infty\), respectively. For the case of \((a+b)/2c = 1.5\) (Fig. 2), it is noted that the increase in \(d/c\) leads to a decrease in \(\phi'(x,0)/(q_0c/\lambda)\) and \(\phi'(x,0)/(q_0c/\lambda)\) for \(d/c \to \infty\) approaches that in the case of a single crack \(^{5}\). For the case of \((a+b)/2c = 1.0\) (Fig. 3), the value of \(\phi'(x,0)/(q_0c/\lambda)\) changes linearly between \(x/c = 0.0\) and 2.0 independent of \(x/c\) and \(d/c\). For the case of \((a+b)/2c \to \infty\) (Fig. 4), \(\phi'(x',0)/(q_0c/\lambda)\) has an opposite symmetry for \(x' = 0\).

Next, we study the thermal stress distribution for the elastic medium in the absence of cracks. Figures 5–7 indicate the normalized thermal stress component \((\sigma_{yy}^T(x,y), \sigma_{yy}^T(x',y))/(\beta\mu q_0c/\lambda)\) on the plane \(y/c = 0\) generated by the temperature field shown in Figs. 2–4, respectively. The thermal stresses increase with decreasing \(d/c\).

Finally, we consider the effects of \((a+b)/2c\) and \(d/c\) on the normalized stress intensity factors \((K_{Ia}, K_{Ib})/\sigma_0(\pi c)^{1/2}\) and thermal stress intensity factors \((K_{T,Ia}, K_{T,Ib})/\sigma_T^0(\pi c)^{1/2}\). Figure 8 shows the influence of \((a+b)/2c\) on \((K_{Ia}, K_{Ib})/\sigma_0(\pi c)^{1/2}\) of an internal crack \((a+b)/2c\).
Fig. 6 Thermal stress distribution for \((a+b)/2c = 1.0\) and \(y/c = 0.0\).

Fig. 7 Thermal stress distribution for \((a+b)/2c \to \infty\) and \(y/c = 0.0\).

Fig. 8 Stress intensity factors for \((a+b)/2c > 1.0\) for various values of \(d/c\). It is seen that the values of \((K_{la}, K_{lb})/\sigma_0(\pi c)^{1/2}\) increase with decreasing \((a+b)/2c\), and the effect of \((a+b)/2c\) on \(K_{la}/\sigma_0(\pi c)^{1/2}\) is greater than that on \(K_{lb}/\sigma_0(\pi c)^{1/2}\). Also, the values of \((K_{la}, K_{lb})/\sigma_0(\pi c)^{1/2}\) and the effect of crack location on \((K_{la}, K_{lb})/\sigma_0(\pi c)^{1/2}\) decrease with decreasing \(d/c\). Figures 9 and 10 show the influence of \(d/c\) on \((K_{la}^T, K_{lb}^T)/\sigma_0(\pi c)^{1/2}\) for various values of \((a+b)/2c\). The values of \((K_{la}^T, K_{lb}^T)/\sigma_0(\pi c)^{1/2}\) decrease with increasing \((a+b)/2c\). Also, the thermal stress intensity factors increase with decreasing \(d/c\), attain maxima, and then decrease.

Figure 11 shows the influence of \(d/c\) on \(K_{lb}/\sigma_0(2\pi b)^{1/2}\) and \(K_{lb}^T/\sigma_0(2\pi b)^{1/2}\) for an edge crack \((a+b)/2c = 1.0\). In general, the negative stress intensity factor \(K_I\) has no meaning. The value of \(K_{lb}^T/\sigma_0(2\pi b)^{1/2}\) is negative, which means that the crack surfaces will come in contact. When \(d/c \to \infty\), \(K_{lb}/\sigma_0(2\pi b)^{1/2} = 1.1215^{(13)}\). On the other hand, \((K_{lb}^T)/\sigma_0(2\pi b)^{1/2}\) is affected by the interaction of compressive and tensile stresses shown by the dotted line in Fig. 6, resulting in \(K_{lb}^T/\sigma_0(2\pi b)^{1/2} = 0.0\).

Figure 12 indicates the influence of \(d/c\) on the com-
combined stress intensity factor \( (K_{ia} + K_{ib}^T)/\sigma_0(\pi c)^{1/2} \) for various values of the normalized loading combination parameter \( \sigma_0^T/\sigma_0 \) at \((a+b)/2c = 1.5\). It is noted that the increase in \( \sigma_0^T/\sigma_0 \) leads to an increase in \( (K_{ia} + K_{ib}^T)/\sigma_0(\pi c)^{1/2} \). Also, \( (K_{ia} + K_{ib}^T)/\sigma_0(\pi c)^{1/2} \) decreases with decreasing \( dc \) when \( \sigma_0^T/\sigma_0 = 0.0, 0.5 \). However, \( (K_{ia} + K_{ib}^T)/\sigma_0(\pi c)^{1/2} \) increases with decreasing \( dc \) when \( \sigma_0^T/\sigma_0 = 1.0, 1.5, 2.0 \) attains a maximum, and then decreases. Figure 13 shows the influence of \( dc \) on \( (K_{ia} + K_{ib}^T)/\sigma_0(\pi c)^{1/2} \) for \( q_{li} \) \( \leq 0 \). \( (K_{ia} + K_{ib}^T)/\sigma_0(\pi c)^{1/2} \) decreases with increasing \( \sigma_0^T/\sigma_0 \). Then, the stress intensity factor near the free surface decreases upon applying thermal load to the material. From this, it is found that the crack propagation is restrained.

7. Conclusions

In this study, the thermal singular stresses in an elastic half-plane containing an infinite row of isothermal parallel cracks perpendicular to the boundary under a uniform heat flux and a uniform mechanical load were analyzed. Numerical calculations were carried out, and the effects of the crack location, the crack spacing, and the thermal and mechanical loads on the thermal stress intensity factors of internal or edge cracks were shown graphically. Moreover, we considered the effects of the thermomechanical interaction on the fracture behavior.

Appendix

The functions \( A(s), B(s), C(s) \) and \( D(s) \), and the constants \( A_n \) and \( B_n (n = 1 \sim \infty) \) are given as

\[
A(s) = \frac{\kappa + 1 + (4|s|d - \kappa - 1)\exp(-2|s|d)}{|s|^{(\kappa + 1)}[1 - \exp(-2|s|d)]^2} \int_a^b [G(t) + G(t')] \exp(i\pi sl)dt
\]

(56)

\[
B(s) = \frac{2}{|s|^{(\kappa + 1)}[1 - \exp(-2|s|d)]} \int_a^b [G(t) + G(t')] \exp(i\pi sl)dt
\]

(57)

\[
C(s) = \frac{4|s|d + \kappa + 1 - (\kappa + 1)\exp(-2|s|d)}{|s|^{(\kappa + 1)}[1 - \exp(-2|s|d)]^2} \exp(-2|s|d) \int_a^b [G(t) + G(t')] \exp(i\pi sl)dt
\]

(58)

\[
D(s) = \frac{2}{|s|^{(\kappa + 1)}[1 - \exp(-2|s|d)]} \exp(-2|s|d) \int_a^b [G(t) + G(t')] \exp(i\pi sl)dt
\]

(59)

\[
A_n = \frac{1}{(n\pi d)^2} \int_a^b [G(t) + G(t')]
\]

(60)

\[
B_n = \frac{2}{(n\pi d)^2} \int_a^b [G(t) + G(t')]
\]

(61)

References

(1) Takeuchi, Y., Analyses of Thermal Stresses, (in Japanese), (1989), Nisshin Publication.

(2) Shibuya, Y., Sekine, H., Takahashi, Y. and Abe, H., Extension of Multiple Geothermal Cracks during Extraction of Heat, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol.51, No.464, A (1985), pp.1066–1072.

(3) Takemoto, M. and Nagata, J., Thermal Stress Corrosion Cracking of Type 304 Steel under Natural Convective Heat Transfer, Kagaku Kogaku Ronbunshu, (in Japanese), Vol.12, No.6 (1986), pp.675–680.

(4) Otani, R. and Komai, K., Kankyo-Kouon Kyodogaku, (in Japanese), No.7 (1984), Ohmsya, p.7.

(5) Ueda, S. and Junpei, A., Thermal Singular Stresses in an Elastic Half-Plane with a Vertical Crack under a Uniform Heat Flux, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol.70, No.698, A (2004), pp.1420–1426.

(6) Ishida, M. and Igawa, H., Equally Spaced Equal Collinear and Parallel Cracks under Various Loads: Some Asymptotic Behavior and Formulae of Stress Intensity Factors, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol.59, No.561, A (1993), pp.1262–1269.

(7) El-Fattah, A. and Rizk, A., Transient Stress Intensity Factors for Periodic Array of Cracks in a Half-Plane.
due to Convective Cooling, J. Thermal Stresses, Vol.26 (2003), pp.443–456.

(8) Noda, N., Tsuru, M. and Oda, K., Stress Intensity Factors of Double and Multiple Edge Cracks, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol.62, No.598, A (1996), pp.1361–1367.

(9) Sneddon, I.N. and Lowengrub, M., Crack Problems in the Classical Theory of Elasticity, (1969), John Wiley & Sons, Inc., New York.

(10) Gupta, G.D., A Layered Composite with a Broken Laminate, Int. J. Solids Struct., Vol.9 (1973), pp.1141–1154.

(11) Erdgan, F., Gupta, G.D. and Cook, T.S., Methods of Analysis and Solution of Crack Problems, Edited by Sih, G.C., (1972), Noordhoff, Leyden.

(12) Nishitani, H., Suematsu, M. and Saito, K., Tensile of Half-Plane with a Infinite Row of Ellipse Holes (or Cracks) and Infinite Plane with Two Infinite Rows of Ellipse Holes, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol.39, No.324, A (1973), pp.2323–2330.

(13) Nishitani, H., Interference Effects among Cracks or Notches in Two-Dimensional Problems, Proc. Int. Conf. Frac. Mech. Tech., No.2, (1977), pp.1127–1142.