Effect of current on wave resistance of an Air Cushion Vehicle

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Abstract. The unsteady movement of the Air Cushion Vehicle on an ice cover in the presence of uniform current is considered. The ice cover is modelled as an elastic plate, floating on the surface of water of finite depth. Air Cushion Vehicle is modelled by the area of pressure moving with a variable speed. The effect of current on wave resistance of Air Cushion Vehicle is analyzed. It is received that the transverse current has little effect on the wave resistance. The longitudinal current leads to significant change of wave resistance on the modes of the accelerated movement for attainment of critical speed and decelerating from critical speed to a full stop. For uniform motion of vehicle the critical velocity increases for following current and decreases for opposing current.

1. Introduction
In recent decades, there has been a significant interest in the problem of body movement on an ice cover or on large floating structures. Among the most considerable works devoted to this subject it is possible to call [1-3]. In particular, works [4-6] are devoted to influence of a current on characteristics of flexural-gravity waves. In [5] it was obtained that, the water flow reoriented the flexural–gravity wave pattern relative to the direction of the moving load. The mathematical analysis of the blocking phenomenon due to the interaction of opposing current was first described by Peregrine [7] using linear water wave theory for free surface. Das et al [8] has analysed the dispersion relation associated with flexural gravity wave motion in the presence of compressive force to understand the dynamics of blocking of flexural gravity waves with/without current.

In development of [4, 5] in the present paper the unsteady motion of Air Cushion Vehicle (ACV) on the ice cover in the presence of current is considered. The joint impact of acceleration (deceleration) and current on the wave resistance of load is analyzed.

2. Mathematical statement
The hydrodynamic problem of ACV moving over continuous ice is considered. ACV is modeled by a system of surface pressures $q$ [9, 10], moving with velocity $u(t)$. The ice cover is modelled by a floating elastic plate of thickness $h$ and density $\rho_1$. It is assumed that water is an ideal incompressible liquid of depth $H$ and density $\rho_2$ and that, the motion of the liquid is potential. The coordinate system $Ox_1y_1z_1$ is located as follows: the plane $x_1Oy_1$ coincides with the unperturbed ice–water interface, the $x_1$ direction coincides with the direction of motion of the vehicle, and the $z_1$ axis is directed vertically upward. Moreover, it is assumed that there is a uniform flow of constant velocity $V(U,V,0)$. Velocity potential $\Phi(x_1,y_1,z_1,t)$ satisfies the three-dimensional Laplace equation and boundary and initial conditions:

$$\Delta \Phi = 0, \quad -H \leq z_1 \leq 0, \quad -\infty < x_1 < \infty, \quad -\infty < y_1 < \infty$$

(1)
\[
DV^4 w + \rho h \frac{\partial^2 w}{\partial t^2} + \rho_2 \left( \frac{\partial \Phi}{\partial t} + U \frac{\partial \Phi}{\partial x_1} + V \frac{\partial \Phi}{\partial y_1} \right) + \rho_2 g w = -q \quad \text{on} \quad z_1 = 0; \tag{2}
\]

\[
\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x_1} + V \frac{\partial w}{\partial y_1} = \frac{\partial \Phi}{\partial z_1} \quad \text{on} \quad z_1 = 0; \tag{3}
\]

\[
\frac{\partial \Phi}{\partial z_1} = 0 \quad \text{on} \quad z_1 = -H; \tag{4}
\]

\[
U \frac{\partial w}{\partial x_1} + V \frac{\partial w}{\partial y_1} = \frac{\partial \Phi}{\partial z_1} \quad \text{on} \quad z_1 = 0, \quad \text{at} \quad t = 0; \tag{5}
\]

\[
\rho h \frac{\partial^2 \Phi}{\partial z_1^2} + \rho_2 \left( \frac{\partial \Phi}{\partial t} + U \frac{\partial \Phi}{\partial x_1} + V \frac{\partial \Phi}{\partial y_1} \right) = 0 \quad \text{on} \quad z_1 = 0, \quad \text{at} \quad t = 0. \tag{6}
\]

Here \(D\) is the flexural rigidity of plate, \(D = \frac{Eh^3}{12(1-\nu^2)}\); \(E, w, \) and \(\nu\) are respectively the elastic modulus, deflection, and Poisson’s ratio of the plate.

3. Theoretical research

It is assumed that the elastic restoring force is much stronger than the inertial force. Using equations (1) and (3), the linearized plate-covered boundary condition free surface (2) in the presence of uniform flow is obtained as

\[
D \frac{\partial^4 \Phi}{\partial z_1^4} + \rho_2 \left( \frac{\partial \Phi}{\partial t} + U \frac{\partial \Phi}{\partial x_1} + V \frac{\partial \Phi}{\partial y_1} \right)^2 \Phi + \rho_2 g \frac{\partial \Phi}{\partial z_1} = -\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1} + V \frac{\partial}{\partial y_1} \right) q \quad \text{on} \quad z_1 = 0; \tag{7}
\]

We introduce the moving coordinate system attached to the vehicle:

\[
x = x_1 - s; \quad y = y_1; \quad z = z_1;
\]

here \(s\) is the distance traveled by the vehicle, \(s = \int_0^t u(\tau) d\tau\). In the moving coordinate system, boundary condition (7) becomes:

\[
D \frac{\partial^4 \Phi}{\partial z_1^4} + \rho_2 \left( \frac{\partial \Phi}{\partial t} - u \frac{\partial \Phi}{\partial x_1} + U \frac{\partial \Phi}{\partial x_1} + V \frac{\partial \Phi}{\partial y_1} \right)^2 \Phi + \rho_2 g \frac{\partial \Phi}{\partial z_1} = -\left( \frac{\partial}{\partial t} - u \frac{\partial}{\partial x_1} + U \frac{\partial}{\partial x_1} + V \frac{\partial}{\partial y_1} \right) q \quad \text{on} \quad z = 0; \tag{8}
\]

Equations (1), (4)-(6), (8) are non-dimensionalized with the use of characteristic length \(H\) and the characteristic speed \((gH)^{1/2}\). Then the problem is solved using the Fourier and Laplace transforms similar to [10].

According to [9, 10] the wave resistance acting on the ACV is calculated by the formula:

\[
R = \int \int q \frac{\partial w}{\partial x} dx dy, \tag{9}
\]

where the plate deflection \(w\) is given by:

\[
w = \frac{1}{4\pi^2} \int kdk \int d\theta \int_0^\infty \frac{(l-1)q}{1+k\xi^2} \exp \left\{ ik \left[ (l-\xi) \cos \theta + (y-\eta) \sin \theta \right] \right\} d\xi d\eta; \tag{10}
\]
\[ I = \exp(\sigma s - nt) \int_0 f(\tau) \cos(\beta^{1/2} (t - \tau)) \, d\tau + n \sin(\beta^{1/2} t) \beta^{-1/2}, \]

\[ f(\tau) = \exp\left[ -\sigma s (\tau) + nt \right] \left[ -\sigma u(\tau) + n \right], \quad \kappa = D \left( \rho_2 gH^3 \right), \]

\[ n = \sigma U + \sigma_y V; \quad \sigma = ik \cos \theta; \quad \sigma_y = ik \sin \theta; \quad \beta = (1 + k k^4) k \tanh k. \]

The speed of the vehicle is calculated by the formula:

\[ u = u_1 \tanh(\mu t) + (u_2 - u_1) \tanh(\mu_t (t - t_2)) + \tanh(\mu t_2) \]

\[ \frac{2}{2}, \quad (11) \]

where \( u_1 \) and \( u_2 \) are the successive values of the vehicle speed; \( \mu_1 \) and \( \mu_2 \) are coefficients that characterize the acceleration (deceleration) of the vehicle; the value \( t_2 \) is the time at which the vehicle acceleration is \( \mu_2(u_2 - u_1)/2 \). Equation (11) allows to investigate the following modes of change in the vehicle speed (modes of motion) with time: 1) acceleration \( \rightarrow \) motion at a specified speed; 2) acceleration \( \rightarrow \) motion at a specified speed \( \rightarrow \) deceleration to a full stop.

It is assumed that in the specified moving coordinate system, the pressure \( q(x, y) \) does not depend on time. The system of moving pressures is given by a function \( q(x, y) \) in the form [9], [10]:

\[ q = q_0 \left[ \tanh(\alpha_1(x + L/2)) - \tanh(\alpha_1(x - L/2)) \right] \cdot \left[ \tanh(\alpha_2(y + B/2)) - \tanh(\alpha_2(y - B/2)) \right] / 4, \]

where \( q_0 \) is the nominal pressure, \( L \) is the vehicle width, \( B \) is the vehicle length, \( \alpha_1 \) and \( \alpha_2 \) are parameters that characterize the deviation of the pressure distribution in the longitudinal and transverse directions from a rectangular shape. The larger the values of \( \alpha_1 \) and \( \alpha_2 \), the closer the pressure distribution to a rectangular shape. For \( \alpha_1 \rightarrow \infty \) and \( \alpha_2 \rightarrow \infty \), the pressure \( q \) is equivalent to the pressure \( q_0 \) distributed uniformly over a rectangle.

4. Results and discussions

We investigate the dimensionless wave-resistance coefficient \( A \):

\[ A = R \rho_2 gH / (Pq_0), \quad P = q_0 LB. \]

Numerical calculations using equations (9)-(12) are performed for the following parameter values:

\[ \rho_2 = 1000 \text{ kg} \cdot \text{m}^{-3}; \quad \rho_1 = 900 \text{ kg} \cdot \text{m}^{-3}; \quad E = 5 \cdot 10^9 \text{Pa}; \quad \nu = 1/3; \]

\[ L = 10 \text{ m}; \quad B = 5 \text{ m}; \quad \alpha_1 = \alpha_2 = 1 \text{ m}^{-1}. \]

Let us consider uniform motion of the vessel with a constant speed of \( u_i \) at \( u_i = u_2 \); \( \mu_i = 0.5 \text{s}^{-1} ; t = 60 \text{s} \). Figures 1-3 show the wave resistance of ACV versus speed of uniform motion \( u_i \) for different values of basin depth \( H \), plate thickness \( h \), and current. We will refer to the critical value of the \( u^* \) for which the wave resistance reaches the maximum value. In the absence of current \( U = V = 0 \) m/s values of critical speeds of \( u^* \) lie between \( \sqrt[3]{gH} \) and \( u_{\text{min}} = 2 \left( D g^3 / 27 \rho_2 \right)^{1/6} \), where \( u_{\text{min}} \) is the minimum phase speed for the liquid of infinite depth. From Figures 1-3 it follows that in case of a longitudinal current \( (U \neq 0, V = 0) \), values of critical speeds \( u^* \) change at a value \( U \). Thus, we obtain that \( u^* = u_0^* + U \), where \( u_0^* \) is the critical speed of uniform motion in the absence of current \( (U = V = 0) \). Therefore, the critical velocity \( u^* \) increases for following current and decreases for opposing current. In case of transverse current \( (U = 0, V \neq 0) \), the value of critical speed \( u^* \) is a little less than \( u_0^* \). We note that the longitudinal current does not change value of an absolute maximum of wave resistance, and the transverse current reduces it a little.
Figure 1. Wave resistance of ACV versus speed \( u_1 \) for \( t=60 \) s, \( u_1=u_2; \mu_1=0.5s^{-1}; H=10m; h=0.2m \).

Figure 2. Wave resistance of ACV versus speed \( u_1 \) for \( t=60 \) s, \( u_1=u_2; \mu_1=0.5s^{-1}; H=10m; h=2m \).

Figure 3. Wave resistance of ACV versus speed \( u_1 \) for \( t=60 \) s, \( u_1=u_2; \mu_1=0.5s^{-1}; H=50m; h=0.2m \).

Figures 4 and 5 represent the wave resistance \( A \) versus time \( t \) for acceleration and deceleration, respectively, at \( H=10m \) and \( h=0.2m \). Moreover, curves 1 and 2 in Figure 6 correspond to speed of vehicle \( u(t) \) for Figures 4 and 5, respectively. We note, that the value of speed 9m/s is close to value \( u_0^* \). Thus, Figure 4 corresponds to acceleration and attainment of the critical speed. Figure 5 corresponds to motion at critical speed and subsequent deceleration. From Figures 2 and 3, it follows that current changes wave resistance at acceleration and deceleration of vehicle. For longitudinal current \((U\neq0, V=0)\), even the small speed of current of \( U=\pm1m/s \) is capable to reduce wave resistance several times. Transverse current \((V\neq0, U=0)\), has weaker influence on wave resistance, than longitudinal current at acceleration and deceleration.

In study [10] it was shown that wave resistance reaches the greatest values at the movement with a critical speed or at attainment of supercritical speed with the minimum possible acceleration. Figure 7 shows curves of wave resistance versus time for attainment of supercritical speed with low acceleration. The dependence of speed on time is set by the equation (11), and curve 3 in Figure 6 illustrates this dependence. It is visible that the current slightly changes wave resistance in this case.
Figure 4. Wave resistance of ACV versus time for acceleration \((u_1=u_2=9\,\text{m/s};\,\mu_1=0.1\,\text{s}^{-1})\).

Figure 5. Wave resistance of ACV versus time for deceleration \((u_1=9\,\text{m/s};\,u_2=0\,\text{m/s};\,\mu_1=5\,\text{s}^{-1};\,\mu_2=0.1\,\text{s}^{-1},\,t_2=40\,\text{s})\).

Figure 6. Vehicle speed \(u\) versus time \(t\): curve 1 refers to acceleration at \(u_1=u_2=9\,\text{m/s};\,\mu_1=0.1\,\text{s}^{-1}\); curve 2 refers to deceleration at \(u_1=9\,\text{m/s};\,u_2=0\,\text{m/s};\,\mu_1=5\,\text{s}^{-1};\,\mu_2=0.1\,\text{s}^{-1},\,t_2=40\,\text{s}\); curve 3 refers to acceleration at \(u_1=u_2=20\,\text{m/s};\,\mu_1=0.05\,\text{s}^{-1}\); curve 4 refers to \(u_1=u_2=13\,\text{m/s};\,\mu_1=0.1\,\text{s}^{-1}\); curve 5 refers to \(u_1=u_2=20\,\text{m/s};\,\mu_1=0.1\,\text{s}^{-1}\).

Figures 8 and 9 depict the wave resistance for acceleration of ACV for thick ice plate and deep water, respectively. In this case, curves 4 and 5 in Figure 6 show speed dependences. It is to be noted that here the critical speed \(u_0^*\approx13\,\text{m/s}\) for parameters of Figure 8, and \(u_0^*\approx20\,\text{m/s}\) for parameters of Figure 9. Thus, Figures 8 and 9 correspond to acceleration and attainment of the critical speed. Let us compare results of Figures 8, 9 and 4. We can see that, increase in thickness of ice and basin depth reduces influence of current on wave resistance at acceleration and attainment of the critical speed.
Figure 7. Wave resistance of ACV versus time for attainment of supercritical speed with low acceleration ($u_1=20\text{m/s}; \mu_1=0.05\text{s}^{-1}$).

Figure 8. Wave resistance of ACV versus time for acceleration ($u_1=13\text{m/s}; \mu_1=0.1\text{s}^{-1}; H=10\text{m}; h=2\text{m}$).

Figure 9. Wave resistance of ACV versus time for acceleration ($u_1=20\text{m/s}; \mu_1=0.1\text{s}^{-1}; H=50\text{m}; h=0.2\text{m}$).

Summing up the result of all figures, it is possible to conclude that the effect of current can be significant at unsteady moving of ACV on ice cover. The received results can be useful also to studying of deflections of VLFS located in the field of tidal currents.

5. Conclusions

Effect of current on wave resistance of unsteady moving Air Cushion Vehicle was investigated. It is received that the transverse current has little effect on the wave resistance. The longitudinal current leads to significant change of wave resistance on the modes of the accelerated movement for attainment of critical speed and decelerating from critical speed to a full stop. For uniform motion of vehicle the critical velocity increases for following current and decreases for opposing current. The increase in thickness of ice and basin depth reduces influence of current on wave resistance. The received results can be useful for resonance breaking of ice cover by Air-Cushion Vehicles.

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