Hypersensitive tunable Josephson escape sensor for gigahertz astronomy

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Sensitive photon detection in the gigahertz band constitutes the cornerstone to study different phenomena in astronomy [1], such as radio burst sources [2], galaxy formation [3], cosmic microwave background [4], axions [5, 6], comets [7], gigahertz-peaked spectrum radio sources [8] and supermassive black holes [9]. Nowadays, state of the art detectors for astrophysics are mainly based on transition edge sensors [10–12] and kinetic inductance detectors [13–15]. Overall, all sensible nanobolometers so far are superconducting detectors [16] showing a noise-equivalent power (NEP) as low as $\sim 2 \times 10^{-20}$ W/Hz$^{1/2}$ [17]. Yet, fast thermometry at the nanoscale was demonstrated as well with Josephson junctions through switching current measurements [18, 19]. In general, detection performance is set by the fabrication process and limited by used materials. Here, we conceive and demonstrate an innovative tunable Josephson escape sensor (JES) based on the precise current control of the temperature dependence of a fully superconducting one-dimensional nanowire Josephson junction. The JES might be at the heart of future hypersensitive in situ–tunable bolometers or single-photon detectors working in the gigahertz regime. Operated as a bolometer the JES points to a thermal fluctuation noise (TFN) NEP$_{TFN}$ $\sim 1 \times 10^{-25}$ W/Hz$^{1/2}$, which as a calorimeter bounds the frequency resolution above $\sim 2$ GHz, and solving power below $\sim 40$ at 50 GHz as deduced from the experimental data. Beyond the obvious applications in advanced ground-based [20] and space [21] telescopes for gigahertz astronomy, the JES might represent a breakthrough in several fields of quantum technologies ranging from sub-THz communications [22] and quantum computing [23] to cryptography [24] and quantum key distribution [25].

Many features of the universe are hidden in infrared and microwave faint signals [1]. In particular, the study of cosmic microwave background polarization [4, 26] and galaxy expansion [3] benefits from ultrasensitive gigahertz bolometers, while the existence of axion-like-particles [5, 6] might be proven through the revelation of microwave single-photons (see Fig. 1-a). To improve photon detection sensitivity, novel superconducting sensors have been developed by miniaturizing the active region [27, 28], and drastically lowering their operation temperature via the Josephson coupling in complex nanostructures [17, 29–35]. Their properties are thus defined during the fabrication process and cannot be tuned during the operation. In analogy to the widespread transition edge sensor (TES), the JES exploits the change of resistance of a superconductor when transitioning to the dissipative state: either the absorption of radiation or the change of temperature [18, 19] trigger the passage from the superconducting to the normal regime yielding a sizable signal from the sensor. In contrast to TESs, the JES benefits from the possibility to finely tune in situ its working temperature and sensitivity.

The JES operation principle is based on a fully superconducting one-dimensional (1D) Josephson junction (JJ), i.e., two superconducting leads coupled by a superconducting nanowire with lateral dimensions smaller than its coherence length ($\xi$). The transition to the dissipative state can be understood to be due to $2\pi$ phase slips, qualitatively similar to the tilted washboard potential model (WP) of JJs, where a phase particle moves in the WP under action of friction forces [36, 37]. The effective WP profile strongly depends on both bias current ($I$) through the junction and Josephson energy ($E_J$) [3] (see left panels of Fig. 1-b and c). In particular, for a 1D nanowire JJ the escape barrier can be written as [3]

$$U(I, E_J) \sim 2E_J \left(1 - I/I_C\right)^{5/4},$$

where $E_J = \Phi_0 I_C/2\pi$, $\Phi_0 \simeq 2.067 \times 10^{-15}$ Wb is the flux quantum, and $I_C$ is the JJ critical current. Equation 6 shows that the phase particle escape from a potential minimum (and, therefore, the corresponding transition of the junction to the dissipative state) can be finely controlled either by rising $I$ or by suppressing $I_C$.

Thanks to the exploitation of a 1D JJ, the JES benefits from a two-fold advantage. On the one hand, transverse dimensions smaller than $\xi$ ensure a constant superconducting wave function along the wire cross section leading to uniform superconducting properties. On the other hand, a nanowire width ($w$) much smaller than the
London penetration depth ($\lambda_L$) guarantees the supercurrent density in the JJ to be homogeneous when current-biased the JES, and a uniform penetration of the film by an external magnetic field. For comparison, in a sufficiently shunted overdamped limit, we can calculate the resistance ($R$) versus temperature ($T$) characteristics of a JJ for different values of $I$. Both the transition temperature and its width decrease by rising $I$. c, Left: Schematics of the washboard model versus phase difference across a JJ for different values of $I$. Right: $R$ versus $T$ characteristics of a JJ for different values of $E_J$ at $I = 0$. Black dotted curves are calculated by varying $I$ in order to have the same transition temperature.

FIG. 2. Realization of the JES. a, False-color scanning electron micrograph of a typical JES. The nanosensor is AC current-biased (amplitude $I$) and the voltage drop across the wire ($V_{out}$) is measured via a voltage pre-amplifier connected to a lock-in amplifier. $R_L$ is a high-impedance load resistor. Inset: blow-up of the core of the JES showing the Al/Cu nanowire (red) in contact with thick Al leads (yellow). b, Back-and-forth DC current vs voltage ($I - V$) characteristics of a typical JES measured at different temperatures $T$ ranging from 20 mK to 160 mK in steps of $\sim 20$ mK. The curves are horizontally shifted for clarity. c, Temperature evolution of the critical ($I_C$, blue) and retrapping ($I_R$, green) current. Dashed black line: critical current prediction of the Bardeen model [42] (see Methods for details). Green line is a guide to the eye for the retrapping current. Inset: the nanowire switches from the superconducting to the normal state at $I_C$, while the transition from the resistive to the dissipationless regime occurs at $I_R$. The hysteretic behavior stems from Joule heating when transitioning from the normal to the superconducting state [41]. $I_R$ is almost $T$-independent, and it is $\simeq 26.6$ nA.
several applications, since it typically broadens the superconducting transition [40]. As we shall show, tuning the nanowire JJ through current injection will prove to be an excellent strategy to achieve near-to-ideal nanosensors with ultimate performance.

The realization of a typical JES is shown in the pseudo-color scanning electron micrograph displayed in Fig. 2-a. The nanosensor active region consists of a 1.5 – 5 μm-long and 100-nm-wide bilayer nanowire composed of aluminum (Al, with thickness $t_{Al} = 10.5$ nm) and copper (Cu, with thickness $t_{Cu} = 15$ nm), while the superconducting leads consist of 40-nm-thick Al banks (see Methods for fabrication details). Josephson transport in the nanosensor is highlighted by the DC current ($I$) vs voltage ($V$) characteristics shown in Fig. 2-b for bath temperatures ($T$) ranging from 20 mK to 160 mK. The wire normal-state resistance is $R_N \simeq 77$ Ω, and the typical heating-induced hysteretic behavior of the $I-V$ curves is observed [41]. On the one hand, the critical current obtains a maximum $I_C \simeq 575$ nA at $T \simeq 20$ mK (see Fig. 2-c), and monotonically decreases with $T$ following the prediction of Bardeen [42] (see Methods for details). On the other hand, the reтрapping current ($I_R \simeq 26.6$ nA) is constant in the whole temperature range.

As stated, the JES working principle is based on a 1D nanowire JJ. Indeed, the coherence length ($\xi \sim 220$ nm) and the London penetration depth ($\lambda_0 \sim 970$ nm) of the nanowire are much larger than the wire width ($w = 100$ nm) and total film thickness ($t_w = 25.5$ nm) thereby providing the frame of a 1D junction (see Methods). Yet, the nanowire length ($\sim 6.8 \xi$) reduces the influence of the superconducting proximity effect arising from the clean contact with the lateral Al leads [37]. We also note that the maximum magnetic field created by the critical current flow, $B_{max} \simeq 5$ μT, is negligibly small compared to the out-of-plane critical magnetic field of the wire ($B_C \simeq 21$ μT) thus implying a vanishing effect on the JES.

We investigated the behavior of the JES by recording the resistance versus temperature characteristics for several amplitudes $I$ of low-frequency AC bias current (see Fig. 3-a). The $R(T)$ characteristics monotonically shift towards low temperatures by increasing $I$, almost preserving the same shape up to the largest current amplitude. In particular, $I$ was varied between $\sim 3\%$ and $\sim 64\%$ of $I_C$. Note that although the transition curves

FIG. 3. Tuning the properties of the JES. a, Selected resistance ($R$) vs temperature ($T$) characteristics for different values of AC bias current amplitude $I$. Inset: Sketch of the temperature dependence of the JES resistance. The current-dependent escape temperature $T_e^*$ as well as the phase transition width $\delta T_C^*$ are indicated. $T_e^*$ monotonically decreases by increasing $I$ while the transition becomes sharper by increasing the biasing current. $T_C \simeq 130$ mK is the critical temperature of sample 1. b, Full behavior of $T_e^*$ vs $I$ for two different JESs. For large $I$, $T_e^*$ can be as small as $\simeq 20$ mK. c, Width of the phase transition $\delta T_C^*$ vs $I$ for two different JESs. $\delta T_C^*$ is suppressed by a factor of 4 at the largest biasing currents. Note the fine tunability of $T_e^*$ provided by the injection current. d, $R$ vs $T$ characteristics for different values of the perpendicular-to-plane magnetic field ($B$). The sizable widening of the phase transition is likely to stem from depairing in the nanowire induced by $B$ [40]. e, $T_e^*$ and $\delta T_C^*$ vs $B$. $T_e^*$ shifts towards lower values by rising $B$, but the transition becomes much broader at higher fields. Yet, $T_e^*$ is hardly tunable at large values of $B$. 

$\begin{align*}
T_e^* &\sim 120 \text{ mK,} \\
\delta T_C^* &\sim 3% \\
I_C &\sim 0.5 \text{ nA,} \\
R &\sim 26.6 \text{ nA.}
\end{align*}$
FIG. 4. **Performance of the JES.** a. Thermal model highlighting the predominant heat exchange mechanisms occurring in the nanosensor. $P_{in}$ is the power coming from the incident radiation, $P_{e-ph}$ is the heat exchanged between electrons in the nanowire residing at temperature $T_w$ (red box) and lattice phonons residing at bath temperature $T_b$ (gray box), and $P_{w-l}$ is the electron heat current flowing from the nanowire to the superconducting leads residing at $T_b$ (yellow boxes). b. Electronic temperature in the nanowire $T_w$ (blue dots) vs $P_{in}$ recorded at $T_b = 147.5$ mK. The red line represents the theoretical behavior (see SI). c. Deduced NEP$_{TFN}$ vs $I$ for sample 1 (triangles) and 2 (squares). d. Frequency resolution $\delta\nu_{TFN}$ vs $I$ for sample 1 (triangles) and 2 (squares). e. Resolving power $\nu/\delta\nu_{TFN}$ vs frequency $\nu$ calculated for sample 1. $\nu/\delta\nu_{TFN}$ increases by rising the bias current (from blue to green). f. Time constant $\tau$ vs $I$ for sample 1 (triangles) and 2 (squares). Dashed lines in panels c-f indicate the expected figures of merit when the sensor is operated at $T_C$ in TES mode for sample 1 (red) and sample 2 (yellow).

shift towards low $T$ by increasing $I$, the nanowire electronic temperature $T_w$ in the middle of the transition under current injection is not expected to coincide anymore with the bath temperature $T_b$. Indeed, when transitioning to the normal state, electrons in the nanowire are Joule overheated with respect to $T_b$ by the bias current (with final $T_w \leq T_C$), thus preventing the operation of the nanosensor as a conventional TES biased at those low bath temperatures, without additional shunting. By contrast, when operated in the dissipationless regime, i.e., as an escape sensor, $T_w$ coincides with $T_b$.

From the $R$ vs $T$ curves we can specify a current-dependent temperature related to the resistive transition, i.e., the escape temperature [$T^\ast_{esc}(I)$]. The latter is the maximum value of $T$ providing a zero nanowire resistance (see the inset of Fig. 3-a and Methods). The $T^\ast_{esc}(I)$ characteristics for two JES samples are shown in Fig. 3-b. In particular, $T^\ast_{esc}$ is monotonically reduced by increasing $I$ with a maximum suppression $\sim 85\%$ (i.e., down to $\sim 20$ mK) of the nanowire intrinsic critical temperature, $T_C \sim 130$ mK. Moreover, the transition width ($\delta T^\ast_C$) narrows by increasing $I$ (see Fig. 3-c). In particular, $\delta T^\ast_C$ is suppressed by a factor of 4 at the largest current amplitude, mirroring the expected changes in the switching as shown in Fig. 1-b.

In addition, to prove the complementary tuning of the WP through the suppression of $I_C$ (see Eq. 6), we applied a perpendicular-to-plane magnetic field. The resulting shape of the corresponding transition degrades dramatically in the presence of $B$ (see Fig. 3-d). In particular, the $R$ vs $T$ characteristics appear to be scarcely tunable, while the onset and the width of the transition become comparable at high values of the magnetic field. $T^\ast_{esc}$ shows a stark variation at values of $B \rightarrow B_C$, and is joined to the outbreak of the transition width, as displayed in Fig. 3-e. The above results in a finite magnetic field validate therefore the bias current as an ideal tool to control the JES properties.

Insight into the behavior of the JES can be gained by considering the predominant heat exchange mechanisms occurring in the nanodevice, as schematically depicted in the thermal model of Fig. 4-a. The absorption of external incident radiation ($P_{in}$) leads to the increase of the nanowire electronic temperature $T_w$. Yet, the two lateral superconducting Al leads (residing at bath temperature $T_b$) serve as Andreev mirrors [43] thereby sup-
pressing heat out-diffusion ($P_{w-\ell}$) from the nanowire. As a consequence, the main thermal relaxation channel in the system stems from heat exchange with lattice phonons ($P_{e-ph}$) residing at $T_b$. For a normal metallic thin film, $P_{e-ph,n} = \Sigma_w n_T (T_w^0 - T_e^0)$ [10, 44], where $V_w$ is the nanowire volume, and $\Sigma_w$ is the electron-phonon coupling constant of the bilayer. $\Sigma_w$ was determined through energy-relaxation experiments [44] by injecting a known power, and by measuring the resulting steady-state electron temperature established in an ad hoc fabricated identical wire kept above its critical temperature. Figure 4-b shows the $T_w$ vs $P_w$ characteristic (blue dots) recorded at $T_b = 147.5$ mK along with a fit to the data which allows to extract the electron-phonon coupling constant in the Al/Cu nanowire, $\Sigma_w \approx 1.15 \times 10^9$ W m$^{-3}$K$^{-5}$ (see SI).

Yet, since the JES is operated in the superconducting state at $T_C$($I$), the latter can be substantially smaller than $T_C$ depending on the current amplitude. At sufficiently low temperature, the electronphonon heat exchange in a superconductor is exponentially suppressed with respect to the normal state owing to the presence of the energy gap, i.e., $P_{e-ph,s} \propto P_{e-ph,n} \exp\left[-\Delta_e/(k_BT_e)\right]$ [10], where $\Delta_e \approx 23$ $\mu$eV is the pairing potential in the nanowire (see SI). As we shall argue, the operation deeply in the superconducting state dramatically improves the JES key figures of merit for radiation detection.

In general, the performance of a bolometer can be quantified by the NEP that is the input power resolution per unit bandwidth. For the JES, the NEP is bounded by thermal fluctuations between the electron and phonon system in the nanowire [44]. Other limitations to the resolution can arise from the switching measurement, which we assume is optimized to be sub-dominant. From our experimental data we deduced the thermal fluctuation noise (TFN)-limited NEP$_{TFN}$, as shown in Fig. 4-c (see also SI). The NEP$_{TFN}$ monotonically decreases by increasing the current amplitude, and turns out to be in-situ finely controlled by tuning $I$. In particular, the JES points to noise values which are several orders of magnitude smaller than so far reported. Specifically, the best extracted NEP$_{TFN}$ obtains values as low as $\sim 1 \times 10^{-25}$ W/√Hz for $I = 370$ nA at $\sim 18$ mK. By contrast, in the normal state, the sensor is expected to provide a much higher NEP$_{TFN} \sim 6 \times 10^{-20}$ W/√Hz.

In pulsed detection mode, a relevant figure of merit of a radiation sensor is represented by the frequency resolution ($\nu/\delta\nu_{TFN}$), calculated vs incident radiation frequency in Fig. 4-e. $\nu/\delta\nu_{TFN}$ can reach $\sim 40$ at 50 GHz, and $\sim 800$ at 1 THz both for 370 nA.

We wish to finally comment onto the JES time constant ($\tau$), which is one of the fundamental figures of merit for a radiation sensor. It is basically given by the ratio between the electron heat capacitance and the electron-phonon heat conductance in the nanowire [44], since heat conduction through the lateral Al electrodes is negligible in a JES. In pulsed detection mode $\tau$ determines the minimum speed of the read-out electronics (which has to be faster than $\tau$), and the minimum time separation for the independent detection of two photons. Figure 4-f shows the expected JES time constant vs bias current $I$, as deduced from the experimental data (see SI). In particular, $\tau$ increases monotonically by increasing $I$, and varies between $\sim 1 \mu$s at low current amplitude and $\sim 100$ ms at 370 nA.

In summary, we have conceived and demonstrated an innovative hypersensitive superconducting radiation sensing element supplied with the capability of in-situ fine tuning its performances by a current bias. Our nanosensor has the potential to drive radiation detection in the gigahertz regime towards unexplored levels of sensitivity by lowering the thermal fluctuation limitation to NEP down to $\sim 1 \times 10^{-25}$ W/√Hz, with a corresponding limit in frequency resolution at $\sim 2$ GHz. The JES is expected to have significant impact in radio astronomy [1-4, 7, 9, 20, 21], space spectroscopy [8] and dark matter search [5, 6], since its working mechanism could allow, in principle, the immediate replacement of TESs in already existing experiments and telescopes. Furthermore, the JES could have countless applications in several fields of quantum technology where extrasensitive photon detection is a fundamental task, such as sub-terahertz communication [22], quantum computation [23] and quantum cryptography [24, 25].

**METHODS**

**Fabrication**

The JESs were fabricated by electron-beam lithography and two angles shadow-mask electron-beam evaporation of metals onto an oxidized silicon wafer through a suspended resist mask. The 1D sensor active region consists of a bilayer of Al ($t_{Al} = 10.5$ nm) and Cu ($t_{Cu} = 15$ nm) evaporated at an angle of 0°. The total volume of the sensor active region is $V_w = V_{Al} + V_{Cu} \approx 3.83 \times 10^{-21}$ m$^{-3}$, with $V_{Al} \approx 1.58 \times 10^{-21}$ m$^{-3}$ and $V_{Cu} \approx 2.25 \times 10^{-21}$ m$^{-3}$. The lateral 40-nm-thick Al banks were then evaporated at an angle of 40°.
Measurements

All measurements were performed in a filtered He\(^3\)-He\(^4\) dry dilution refrigerator at different bath temperatures in the range 20 – 160 mK. The resistance \(R\) vs temperature characteristics of the JES and of the Al banks were obtained by conventional four-wire low-frequency lock-in technique at 13.33 Hz. To this end, AC excitation currents with typical root mean square amplitudes \(I \approx 15 – 380\) nA were imposed through the device. The current was generated by applying an AC voltage bias \((V_{ac})\) to a load resistor of impedance \((R_L)\) much larger than the sample resistance \((R_L = 100\) k\(\Omega\) \(\gg R)\). The critical temperature of the Al banks was measured with the same set-up. The \(I\) vs \(V\) characteristics of the nanowires were obtained by applying a low-noise DC biasing current, while the voltage drop was measured via a room-temperature battery-powered differential preamplifier.

Device parameters

The temperature dependence of the critical current of the nanowire can be fitted through the phenomenological equation \(I_C(T) = I_{C,0} \left[1 - \left(T/T_C\right)^2\right]^{3/2}\), where \(I_{C,0}\) is the zero-temperature critical current. The fit provides \(I_{C,0} \approx 615\) nA and a critical temperature \(T_{C,1dT} \approx 133\) mK, which is in good agreement with the experimental value obtained from the resistance versus temperature characteristics. From the nanowire normal-state resistance \((R_N \approx 77 \, \Omega)\) we determined the superconducting coherence length in the active region \(\xi = \sqrt{\hbar/\left[(I_{Al}N_{Al} + t_CuN_{Cu})R_\text{ex}^2\Delta_w\right]} \approx 220\) nm, where \(h\) is the reduced Planck constant, \(\epsilon\) is the electron charge, while \(N_{Al} = 2.15 \times 10^{17} \, J^{-1}m^{-3}\) and \(N_{Cu} = 1.56 \times 10^{17} \, J^{-1}m^{-3}\) are the density of states at the Fermi level of aluminum and copper, respectively. The superconducting energy gap of the bilayer, \(\Delta_w = 23\) \(\mu\)eV, has been determined by tunnel spectroscopy performed on \(ad\ hoc\) fabricated nominally identical wires equipped with Al tunnel probes. For further details, see SI. The London penetration depth was determined as \(\lambda_L = \sqrt{\hbar(t_{Al} + t_{Cu})\mu_0 R_N/(\pi\mu_0 l\Delta_w)} \approx 970\) nm, where \(\mu_0\) is the magnetic permeability of vacuum. The maximum magnetic field generated by the bias current at the wire surface reads \(B_{1,max} = \mu_0 I_{C,0}/(2\pi t_w) \approx 5\) \(\mu\)T, where \(I_{C,0}\) is the zero-temperature critical current, and \(t_w = t_{Al} + t_{Cu}\) is the total thickness of the JES active region. Finally, the critical temperature of the Al banks was \(T_{C,Al} \approx 1.3\) K.

The current-dependent escape temperature \([T^*_E(I)]\) is defined as the maximum value of temperature providing \(R(I) = 0\), i.e., when the JES is in the dissipationless state.

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AUTHOR CONTRIBUTIONS STATEMENT

F.P. and N.L. fabricated the samples. F.P., V.B. and G.G. performed the measurements. F.P., V.B. and G.G. analysed the experimental data with inputs from F.G.. P.V. developed the theoretical models with inputs from F.G.. F.G. and F.P. conceived the experiment. F.P. and V.B. and G.G. analysed the experimental data with inputs from F.G.. G.G. performed the measurements. F.P., V.B. and G.G. wrote the manuscript with inputs from all authors. All authors discussed the results and their implications equally at all stages.

ADDITIONAL INFORMATION

The authors declare no competing financial interests.
SUPPLEMENTARY INFORMATION

I. MODEL OF THE TUNABLE JOSEPHSON ESCAPE SENSOR

The Josephson escape sensor (JES) is composed of two superconducting leads interrupted by a one-dimensional superconducting nanowire (both length and width are shorter than its superconducting coherence length $\xi$ and its London penetration depth $\lambda_L$), that is a 1D fully superconducting Josephson junction (JJ). The electronic transport of such a system can be described through the overdamped resistively shunted junction (RSJ) model [1]. Here, the transition to the dissipative state is attributed to a $2\pi$ phase-slip of a phase particle moving in a tilted washboard potential (WP) under the action of a friction force. Within the RSJ model, the dependence on the bias current ($I$) of the stochastic phase difference $[\varphi(t)]$ over the JJ is given by

$$2\varepsilon \frac{\varphi(t)}{\hbar} R_N \sin \varphi(t) = I + \delta I_{th}(t),$$  \hspace{1cm} (2)

where $h$ is the reduced Planck constant, $I_C$ is the junction critical current and $\delta I_{th}(t)$ is the thermal noise generated by the shunt resistor $R_N$, with $k_B$ the Boltzmann constant and $T$ the temperature.

A. General solution

The 1D nature of the JJ composing the JES entails the homogeneous flow of the bias current across the nanowire section. Under the assumption of phase-slips induced only from sources outside the junction, the voltage drop across the JJ reads [2]

$$V(I, E_J, T) = R_N \left( I - I_{C,0} \right) \text{Im} \frac{\mathcal{I}_1 - iz \left( \frac{E_J}{k_B T} \right)}{\mathcal{I}_0 - iz \left( \frac{E_J}{k_B T} \right)},$$  \hspace{1cm} (3)

where $I_{C,0}$ is the JJ zero-temperature critical current, $\mathcal{I}_\mu(x)$ is the modified Bessel function with imaginary argument $\mu$, $E_J = \Phi_0 I_C / 2\pi$ (with $\Phi_0 \approx 2.067 \times 10^{-15}$ Wb the flux quantum) is the Josephson energy, and $z = \frac{E_J}{k_B T} I_C$. Thus, the current and Josephson energy dependent $R(T)$ characteristics of the JES can be calculated by

$$R(I, E_J, T) = \frac{dV(I, E_J, T)}{dT}.$$  \hspace{1cm} (4)

In the limit of $I = 0$ the junction resistance can be simplified as [1]

$$R(I = 0, E_J, T) = R_N \frac{1}{\mathcal{I}_0 \left( \frac{E_J}{k_B T} \right)^2},$$  \hspace{1cm} (5)

where $\mathcal{I}_0$ is the zero-order modified Bessel function.

B. Simplified solutions for $I \to 0$ and $I \to I_C$

A simplified picture of the dependence of $R(T)$ on bias current and Josephson energy can be provided by considering the influence of $I$ and $E_J$ on the WP. In fact, the escape barrier for a JJ takes the form [3]

$$U(I, E_J) \sim 2E_J \left( 1 - I / I_C \right)^{5/4},$$  \hspace{1cm} (6)

where the exponent $5/4$ stems from the nanowire nature of the constriction. Equation 6 shows that $I$ and $E_J$ control the escape of a phase particle from a potential minimum of the WP. The resulting thermal fluctuation induced voltage at low temperature reads [4]

$$V(I, E_J, T) \sim R_N I_C a(I / I_C) e^{-U(I, E_J) / (k_B T)} \left( 1 - e^{-\pi h I / (e k_B T)} \right),$$  \hspace{1cm} (7)

where $a(x) \sim 1$.

From this, we get the simplified results at low bias current ($I \to 0$)

$$R(I \to 0, E_J, T) \sim R_N \frac{U(I \to 0, E_J)}{k_B T} e^{-U(I \to 0, E_J) / (k_B T)},$$  \hspace{1cm} (8)

whereas at high bias current ($I \to I_C$)

$$R(I, E_J, T) \sim R_N f(I / I_C) \frac{U(I, E_J)}{k_B T} e^{-U(I, E_J) / (k_B T)},$$  \hspace{1cm} (9)

where $f \propto U''(I, E_J) / U(I, E_J) \propto I_C / (I_C - I)$.

The increase of $I$ and reduction of $E_J$ have similar effects on $U(I, E_J)$ (see Eq. 6), therefore the exponentials in Eqs. 8 and 9 produce comparable consequences to the $R(T)$ for $I \to I_C$ and $E_J \to 0$. On the contrary, we observe that the $R(T)$ rises faster for $I \to I_C$, because of the divergent prefactor $f(I / I_C)$ (see Eq. 9). The latter arises from the dependence of the potential barrier $[U(I, E_J)]$ on the WP tilt. On the one hand, $I \to I_C$ and $E_J \to 0$ decrease the superconducting-to-dissipative transition temperature similarly to $E_J$ reduction. On the other hand, the increase of bias current improves the sharpness of the transition more than the suppression of Josephson energy. Therefore, we can conclude that bias current injection is the best strategy to reach ultimate sensing performance from a JES.

II. PROPERTIES OF THE AL/CU BILAYER

In order to provide a full description of the characteristics of the JES active region, we measured the energy gap ($\Delta_w$) and the electron-phonon energy relaxation constant ($\Sigma_w$) of the Al/Cu bilayer.
A. Auxiliary device design and fabrication

The false-color scanning electron micrograph of the auxiliary devices (ADs) is shown in Fig. 5-a. A typical AD consists of the same Al/Cu bilayer (w, red) of the JES contacted by two Al banks (S, yellow). In addition, the device is equipped with two Al tunnel probes (TP, blue) directly coupled to the wire allowing to perform both spectroscopy (for the measurement of \( \Delta_w \)) and thermometry (for the determination of \( \Sigma_w \)).

The ADs were fabricated by electron-beam lithography and 3-angles shadow mask evaporation of metals onto a silicon wafer covered with 300 nm SiO\(_2\) carried out in a ultra-high vacuum electron-beam evaporator. The 13-nm-thick Al probes were evaporated at an angle of -40° and then oxidized by exposition to 200 mtorr of O\(_2\) for 5 minutes (we call the AlOx layer as \( I \)). The Al/Cu bilayer \( t_{Al} = 10.5 \text{ nm} \) and \( t_{Cu} = 15 \text{ nm} \) was evaporated at an angle of 0°. Finally, the 40-nm-thick Al electrodes were evaporated at an angle of 40°.

B. Measurement of the energy gap \( \Delta_w \)

The energy gap of Al/Cu bilayer was measured by considering the temperature dependence of the current-voltage \( (I-V) \) characteristics of the \( S-w-I-TP \) tunnel Josephson junction (JJ), as sketched in Fig. 5-a. Within this configuration, a quasiparticle tunneling current is seen for \( V > (\Delta_w + \Delta_{TP})/e \) [5], where \( \Delta_{TP} \) is the energy gap of \( TP \), and \( e \) is the electron charge. Fig. 5-b, shows the \( I-V \) characteristics measured in the voltage range \( \pm 350 \text{ mV} \) at two different bath temperatures, \( T_b = 20 \text{ mK} \) (blue line) and \( T_b = 250 \text{ mK} \) (red line). As expected, by rising the temperature the critical voltage for switching to the resistive state is decreased.

In order to quantify \( \Delta_w \), we zoomed the \( I-V \) characteristics around the switching point at positive voltage bias (see Fig. 5-c). The difference between the blue and the red line is due to the full suppression of the energy gap of the Al/Cu bilayer. In fact, \( \Delta_{TP}(T) = \Delta_{0,TP} \) (where \( \Delta_{0,TP} \) is the zero-temperature energy gap of the tunnel probe) in the complete temperature range, since in the worst case we have \( T_b = 250 \text{ mK} < 0.4T_{C,Al} \) [1], where \( T_{C,Al} \approx 1.3 \text{ K} \) is the measured critical temperature of the Al probes. Therefore, the difference of the projections of the linear part of the \( I-V \) curves (to avoid the quasiparticle sub-gap conduction) at \( I = 0 \) represents exactly \( \Delta_w \). By repeating the measurements ten times, we obtained \( \Delta_w = 23 \pm 3 \mu \text{eV} \).

C. Measurement of the electron-phonon coupling constant \( \Sigma_w \)

In order to obtain the electron-phonon coupling constant of the Al/Cu bilayer (\( \Sigma_w \)), we injected a power \( P_{in} \) directly into the wire and measured its electronic temperature (\( T_w \)). To this end, we used a voltage-biased \( S-w-I-TP \) tunnel junction as heater and a current-biased \( S-w-I-TP \) JJ as thermometer [3], as shown in Fig. 6-a. Since we were interested in the normal-state coupling constant, we performed the experiments at \( T_b = 147.5 \text{ mK} \) (\( T_w \geq T_b \geq T_{C,Al} \)). The resulting \( T_w \) versus \( P_{in} \) characteristics are shown in Fig. 6-b. As expected, the electronic temperature of the wire monotonically increases by rising the injected power.

In order to analyze the experimental data, we have considered the thermal model shown in Fig. 6-c: the inward contribution of the heater (\( P_{in} \)) is balanced by the outward power flowing through the thermometer (\( P_{w,ITT} \)) and the electron-phonon relaxation channel (\( P_{e-ph} \)). The outward contributions through the Al banks can be neglected thanks to the Andreev mirror effect [6, 7], since \( T_w \ll T_{C,Al} \approx 1.3 \text{ K} \). As a consequence, the energy bal-
**FIG. 6. Measurement of the electron-phonon coupling constant of the nanowire.** (a) False-color scanning electron micrograph of the AD used to determine \( \Sigma_w \): Al banks (yellow) and probes (blue) are made of Al and the wire active region (red) is composed of an Al/Cu bilayer. The experimental setup used to obtain \( \Sigma_w \) is shown as well: a S-w-I-TP JJ is used as heater to inject \( P_w \), while the other S-w-I-TP JJ acts as thermometer. (b) Electronic temperature of the active region (\( T_w \)) as a function of the inward power (\( P_{in} \)): experimental data for three different set of measurements (colored dots and triangles), and fitting curve obtained by solving Eq. (10) (gray line). The electron-phonon parameter \( \Sigma_w = (1.15 \pm 0.02) \times 10^9 \text{ W/m}^3\text{K}^5 \) is the resulting fit parameter. (c) Thermal model of the active region: the input power (\( P_{in} \)) is balanced by the outward contributions due to out-diffusion through the thermometer (\( P_{w-TP} \)) and electron-phonon relaxation (\( P_{e-ph} \)).

The resistance equation of our system reads

\[ P_{in} = P_{w-TP} + P_{e-ph}. \]  

By solving the Eq. 12 for \( T_w \), we computed the expected temperature of the wire active region as a function of \( P_{in} \). Since all the other device parameters are known, we fit our experimental data with Eq. 10 and extracted the values of \( \Sigma_w \). The resulting fitting curve is represented by the gray line in Fig. 6-b obtained for \( \Sigma_w = (1.15 \pm 0.02) \times 10^9 \text{ W/m}^3\text{K}^5 \). Finally, we notice that, the extracted value of \( \Sigma_w \) is in good agreement with the average of the coupling constants of Cu (\( \Sigma_{Cu} = 2.0 \times 10^9 \text{ W/m}^3\text{K}^5 \)) and Al (\( \Sigma_{Al} = 0.5 \times 10^9 \text{ W/m}^3\text{K}^5 \)) [5], weighted on the ratio between the volumes of the two layers (\( \Sigma_{w,\text{theo}} = 1.3 \times 10^9 \text{ W/m}^3\text{K}^5 \)).

### III. Reproducibility of the Josephson Escape Sensor

We investigated the behavior of a second JES. The device is nominally identical to the one presented in the main text. The samples were fabricated simultaneously (same electron beam lithography and angle resolved evaporation steps). Therefore, the differences in the critical current (\( I_{C,0-1} \approx 575 \text{ nA} \) while \( I_{C,0-2} \approx 385 \text{ nA} \)) can be ascribed to small dissimilarities in the nanowires width.

The resistance (\( R \)) versus temperature (\( T \)) characteristics of the second JES are shown in Fig. 7 for different values of bias current (\( I \)). The device shows a normal state resistance \( R_N \approx 81 \Omega \), similar to that of the sample presented in the main text (\( \approx 77 \Omega \)). In agreement with the JES showed in the main text (see Fig. 3-a), by rising \( I \) the transition from the superconducting to the dissipative state shifts towards lower temperature. In particular, the transition temperature shows a variation from about 140 mK at \( I = 15 \text{ nA} \) to about 23 mK at \( I = 380 \text{ nA} \). Furthermore, the superconducting-to-dissipative state transition preserves the same appearance for every value of
bias current.

The quantitative analysis of the dependence of the effective escape temperature \( T^*_e \) and transition width \( \delta T^*_C \) on the bias current \( I \) is reported in the main text. In particular, the \( T^*_e \) and \( \delta T^*_C \) dependence on the normalized current \( I/I_C \) of the two JESs is in good agreement, as shown in Fig. 3 of the main text.

IV. PERFORMANCE OF THE JES AND TES OPERATION

The thermal model of the JES is depicted in Fig. 8. The input power \( P_m \) increases the electronic temperature of the one-dimensional JJ \( T_w \). The two superconducting Al leads (kept at the bath temperature \( T_b \)) serve as Andreev mirrors \([6, 7]\), since \( T_w, T_b < 0.2 T_{C, Al} \approx 1.3 \) K. Therefore, the electronic heat out-diffusion \( P_{w,1} \) is exponentially damped by the superconducting energy gap of Al and can be neglected. As a result, the only thermalization channel for the quasiparticles is the electron-phonon coupling \( P_{e-ph} \).

The JES is operated at \( T^*_e(I) \) that can be much smaller than the intrinsic wire critical temperature \( T_C \): the sensor operates strongly in the superconducting state for high values of \( I \). Therefore, the electron-phonon coupling is exponentially damped by the presence of the superconducting gap \([10, 11]\). On the contrary, in TES operation the nanowire is almost in the normal-state \( T \approx T_C \), so that \( \Delta_w = 0 \), therefore the electron-phonon thermalization can be described by means of the normal metal thin film relation \([5]\).

In the following we will show all the relations used to extract from our experimental data the principal figures of merit for the JES and the device operated as a conventional TES at about \( T_C \). In particular, we will calculate the thermal fluctuations limited noise equivalent power \( \text{NEP}_{TFN} \), the frequency resolution \( \delta \nu \) and the thermal time constants \( \tau_0 \) and \( \tau_{eff} \).

A. Figures of merit for the TES-mode operation

For a bolometer the NEP is limited by the thermal fluctuation noise \([5]\). In case of TES operation \( (T \approx T_C) \), the noise equivalent power can be written \([9]\)

\[
\text{NEP}_{TFN,TES} = \sqrt{4\Gamma G_{th,TES}k_B T_C^2},
\]

where \( \Gamma = n/(2n+1) \) describes the effect of the temperature gradient across the thermal link (with \( n = 5 \) for a pure metal), \( G_{th,TES} \) is the thermal conductance of heat losses, and \( k_B \) is the Boltzmann constant.

In our device the only channel for thermal losses is the electron phonon coupling \( (P_{e-ph}) \). Since at \( T_C \) the JJ is partially dissipative, the electron-phonon coupling of a normal-metal diffusive thin film \( (P_{e-ph,n}) \) described by Eq. 12 can be considered \([5]\).

The thermal conductance for a TES \( (G_{th,TES}) \) is obtained by calculating the derivative of \( P_{e-ph,n} \) with respect to the electronic temperature \( (T_w) \)[12]

\[
G_{th,TES} = \frac{dP_{e-ph,n}}{dT_w} = 5\Sigma_w V_w T_w^4,
\]

where \( \Sigma_w = 1.15 \times 10^8 \text{W m}^{-3}\text{K}^{-5} \) is the value derived from experimental measures, \( V_w = V_{Al} + V_{Cu} = 3.83 \times 10^{-21} \text{m}^{-3} \) is the total volume of the Al/Cu bilayer, with \( V_{Al} \) and \( V_{Cu} \) the aluminum and copper volumes, respectively.

In order to determine the performances of a sensor in single-photon detection, the frequency resolution \( \delta \nu_{TES} \) is the most used figure of merit. For a TES it reads \([12]\)

\[
\delta \nu_{TES} = \frac{2.36}{h} \sqrt{\frac{4}{\alpha} \sqrt{\frac{n}{2}} k_B T_C^2 C_{e,TES}},
\]

where \( h \) is the Planck constant, \( \alpha = \frac{dR}{dT} \frac{T}{R} \) is the electrothermal parameter accounting for sharpness.
FIG. 8. Thermal model of the JES. The input power \( P_{in} \) is balanced by the outward contributions due to out-diffusion through Al leads \( (P_{w-l}) \) and electron-phonon relaxation \( (P_{e-ph}) \). Since the electronic temperature of the wire always respects \( T_w < 0.2T_{C,Al} \), we can consider the two Al leads as perfect Andreev Mirrors and neglect the two \( P_{w-l} \) terms.

of the phase transition from the superconducting to the normal-state [12], \( n = 5 \) is the electron-phonon coupling for a pure metal and \( C_{e,TES} \) is the electron heat capacitance. It is interesting to note the strongly dependence on \( \alpha \) value which determines the negative electrothermal feedback (NETF) mechanism [12]. The electron heat capacitance is written

\[
C_{e,TES} = (\gamma_C V_{Cu} + \gamma_A V_{Al}) T_C,
\]

where \( \gamma \) is the Sommerfeld coefficient \( (\gamma_C = 70.5 \text{ JK}^{-2}\text{m}^{-3}, \gamma_{Al} = 91 \text{ JK}^{-2}\text{m}^{-3}) \) for copper and aluminum, respectively. Moreover, for a TES the temperature variation after energy absorption is calculated by solving the time dependent energy balance equation that takes in account all the exchange mechanisms [5]. In particular, the re-thermalization of the quasiparticles to \( T_b \) shows an exponential dependence on time with constant \( \tau = \frac{C_{e,TES}}{G_{th,TES}}. \) (17)

This is the intrinsic recovery time of the film which does not consider the Joule heating due to the current flowing through the sensor. Instead, including the heating term in the negative NETF configuration, the pulse recovery time becomes [12]

\[
\tau_{eff} = \frac{\tau}{1 + \frac{\alpha}{n}}
\]

which depends on \( \alpha \) (i.e. the main parameter of the NETF). When the pulse recovery time is much shorter than the intrinsic time constant \( (\tau_{eff} < \tau) \), the energy into the sensor is removed by decreasing its overheating, i.e., compensating for the initial temperature variation (NETF), instead of being dissipated through the substrate.

B. Figures of merit for the JES

The JES operates at the escape temperature \( T_e^* \), defined as the maximum temperature measured in the superconducting state before the transition to the dissipative state. Since the current injection does not change the energy gap of the wire \( (\Delta_w \sim \text{const}) \), the detector works deeply in the superconducting state since \( T_e^*(I) \ll T_C \) for high values of \( I \). Therefore, all figures of merit have to be calculated deeply in the superconducting state.

The thermal fluctuations limited noise equivalent power can be written

\[
\text{NEP}_{TFN,JES} = \sqrt{4Y_{G_{th,JES}} k_B T_e^*}. \quad (19)
\]

The thermal conductance in the superconducting state is described by [11]

\[
G_{th,JES} \approx \frac{\sum_{x} V_{th,JES} T_e^*}{96\zeta(5)} \left[ f_1 \left( \frac{1}{\tilde{\Delta}} \right) \cosh(\tilde{\eta}) e^{-\tilde{\Delta}} + \pi \tilde{\Delta} f_2 \left( \frac{1}{\tilde{\Delta}} \right) e^{-2\tilde{\Delta}} \right], \quad (20)
\]

where the first and the second terms refer to electron-phonon scattering and recombination processes, respectively. Here, \( \zeta(5) = 1.0369 \) is the Riemann zeta function, \( \tilde{\Delta} = \Delta_w/k_BT \) (with \( \Delta_w = 23 \mu eV \) the experimental gap of the wire), \( \tilde{\eta} = \hbar/k_BT \) is exchange field (0 in this case), \( f_1(x) = \sum_{n=0}^{3} C_n x^n \) with \( C_0 \approx 440, C_1 \approx 500, C_2 \approx 1400, C_3 \approx 4700 \) and \( f_2(x) = \sum_{n=0}^{3} B_n x^n \) with \( B_0 = 64, B_1 = 144, B_2 = 258. \)

Operating as a calorimeter, the frequency resolution \( \delta \nu_{JE} \) of a superconducting thermal sensor can be computed from [13]

\[
\delta \nu_{JES} = \frac{4}{h} \sqrt{2 \ln 2 k_B T_e^* C_{e,JES}}, \quad (21)
\]

where \( C_{e,JES} \) is the electron heat capacitance calculated at the escape temperature \( T_e^* \) considering the damping
term $\Theta_{Damp}$ typical in a BCS superconductor

\[ C_{e,\text{JES}} = (\gamma_{Cu} V_{Cu} + \gamma_{Al} V_{Al}) T_e^* \Theta_{Damp}. \]  

(22)

The low temperature exponential suppression $\Gamma_{Damp}$ with respect to the normal state heat capacitance, which leads to the JES high detection sensitivity, is written as

[14]

\[ \Theta_{Damp} = \frac{C_s}{1.34(\gamma_{Cu} + \gamma_{Al}) T_e^*}. \]  

(23)

The electronic heat capacitance is given by

\[ C_s = 1.34(\gamma_{Cu} + \gamma_{Al}) T_{C} \left( \frac{\Delta_w}{k_B T_e^*} \right)^{3/2} e^{\Delta_w/k_B T_e^*}, \]  

(24)

where the critical temperature $T_{C,w} \approx 150 \text{ mK}$ is related to the measured gap $\Delta_w \approx 23 \text{ \mu eV}$. Considering the relaxation time in the weak link, the predominant thermalization mechanism is due to the electron-phonon interaction, which defines the sensor relaxation half-time $\tau_{1/2}$ [13]

\[ \tau_{1/2} = \tau \ln 2, \]  

(25)

where $\tau$ is the thermal time constant (see Eq. 17) considering $C_{e,\text{JES}}$ and $G_{th,\text{JES}}$ for a Josephson escape sensor.

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