MULTIPLE COMMON DUE-DATES ASSIGNMENT AND OPTIMAL MAINTENANCE ACTIVITY SCHEDULING WITH LINEAR DETERIORATING JOBS

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Abstract. In this paper, we consider the multiple common due-dates assignment and machine scheduling with linear deteriorating jobs and optimal maintenance activity. The linear deteriorating jobs means job processing times are an increasing function of their starting times. The maintenance activity requires a fixed time interval. During the time interval, the machine is turned off and no job is processed. Once completing the maintenance, the machine will revert to its initial condition. The objective is to schedule the jobs, the due dates and the maintenance activity, so as to minimize the total cost including earliness, tardiness, and the due dates. We provide some properties of optimal sequence and introduce an efficient $O(n^2 \log n)$ algorithm to solve the problem.

1. Introduction. Production scheduling and planning preventive maintenances are the most common and significant problems faced by the manufacturing industry, and its importance for production enterprises and service organizations is widely recognized by both practitioners and management scientists; see books on maintenance [14,15], related dissertation about maintenance [1], and various papers [2,7,11,19]. In addition, the processing times are not constant but subject to various effects in the practical environment, such as steel production, medical procedure process, or labors production system [9,13]. Therefore, recent researches related to various effects on the actual processing times of the jobs and to machine maintenance are considerable and valuable. One major type of the effects is deteriorating effects, which mean that machine qualities may get worse, or a human operator may get tired. A survey [3] on scheduling with deteriorating and a book about deterioration [6] is commended to readers. Consequently, scheduling problem

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with deteriorating jobs and machine maintenance activity are the more realistic and challenging issues ([16,17]).

Another direction of research related to this study deals with scheduling problems with due-date assignment. In supply chain management, the job due dates quoted are the important decision by the supplier. Customers often demand that supplier meet contracted due dates or face large penalties. The jobs are to be completed neither too early nor too late; otherwise, they lead to an earliness penalty (holding cost) or a tardiness penalty (contractual penalties)[22]. Li et al. [12] consider the problem of scheduling deteriorating jobs and due date assignment on a single machine, they present some polynomial-time algorithms to solve the problem in the case of two popular due date assignment methods: CON and SLK. Kuo and Yang [10] study a single machine scheduling problem with deteriorating jobs in which processing times are an increasing function of their start times, they give a concise analysis of the problem and provide a polynomial time algorithm. However, these models neglect the effect of planning preventive maintenances for the deterioration emergence. Yang et al. [20] discuss a due date assignment and scheduling problems with a job-dependent aging effect under a deteriorating maintenance activity consideration simultaneously; they show that there exists a polynomial time solution for the problem. Hsu et al. [8] introduce linear deteriorating jobs in a single machine scheduling problem with due-date assignment and maintenance activity. They proved the problem can be solved optimally in polynomial time.

Moreover, some papers extends the common due date problem to allow for multiple common due dates. This extension allows for greater flexibility in modeling real-life problems [5]. Chand and Chhajed [4] introduce the problem of simultaneous determination of optimal due dates and optimal schedule for the single machine problem with multiple common due dates, they provide an efficient optimal algorithm to solve it. Wang and Wang [18] extend the study of Chand and Chhajed [4] to a case with learning effect. Yang et al. [21] extend the study of Chand and Chhajed [4] to a case that the processing time of a job is a function about its position and its resource allocation in a job sequence. However, they also neglect the preventive maintenances for the improvement of product flexibility and the effect of changing processing times.

In this paper, we consider a more complex scheduling setting with the consideration of multiple common due-dates assignment and deteriorating jobs under maintenance activity. Although the scheduling problems with deteriorating jobs and machine maintenance or deteriorating jobs and due-date assignment have been studied in the literature, however, they have scarcely been considered simultaneously. We try to establish a trade-off relationship between the due dates costs of all jobs under deterioration and the impact of maintenance activity in the planning horizon. We show that the weighted sum of earliness, tardiness and due-date assignment penalties can be solved in polynomial time by converting it to a weighted-bipartite matching problem.

2. Problem formulation. The following notations are used throughout this paper.

\( n \): the total number of jobs;

\( c \): the length of a maintenance activity;

\( m \): the number of due dates, \( 1 \leq m \leq n \);

\( D_i \): the due dates, \( i = 1, 2, \ldots, m \), and \( D_1 \leq D_2 \leq \cdots \leq D_m \);

\( I_i \): the set of jobs assigned to due date \( D_i \), \( i = 1, 2, \ldots, m \);
where \( \pi \) (in positions Lemma 3.1. lemma 3.1 and lemma 3.2 also hold. Thus, we omit the proofs. Making use of the analysis in the Chand and Chhajed [4], the following assignment problem, these properties still hold when a rate modifying activity is considered. For a given sequence \( \pi = (J_1, J_2, \cdots, J_n) \) in a sequence, \( n_i \) is the number of jobs assigned to due date \( D_1, |I_i| = n_i \) and \( n = \sum_{i=1}^{m} n_i \); 
\( a_j \): the basic processing time of job \( J_j, j = 1, 2, \ldots, n \); 
\( p_j \): the actual processing time for the job scheduled in position \( j \) in a sequence, \( j = 1, 2, \ldots, n \).

The problem under consideration can be described as follows: There are \( n \) independent jobs \( J = \{J_1, J_2, \cdots, J_n\} \) to be processed on a single machine; all the jobs are non-preemptive and available at time zero. The processing time \( p_j \) of job \( J_j \) is a linear increasing function of its starting time \( t \), that is \( p_j = a_j + bt \), where \( b \) is a constant growth rate and \( t \) is the starting time of job \( J_j \). For a given schedule \( \pi = (J_1, J_2, \cdots, J_n) \), let \( d_{[j]} \) be the due date for job \( J_{[j]} \); \( C_{[j]} \) represents the completion time for job \( J_{[j]} \), \( j = 1, 2, \ldots, n \); \( E_{[j]} = \max \{0, d_{[j]} - C_{[j]}\} \) and \( T_{[j]} = \max \{0, C_{[j]} - d_{[j]}\} \) represent the earliness and the tardiness for job \( J_{[j]} \), \( j = 1, 2, \ldots, n \), respectively. Assume that once the maintenance activity has been completed, machine will revert to its initial condition, it means that if job \( J_j \) is the first job scheduled after the maintenance activity, then its starting time is set to zero, i.e. \( t = 0 \).

We assume that the number of distinct due dates \( m \) to be assigned to the jobs is given, and there is no idle time between the processing of jobs, the vector \( (n_1, n_2, \cdots, n_m) \) of \( m \) positive integers such that \( n = \sum_{i=1}^{m} n_i \) is externally specified and \( |I_i| = n_i \) is known. We denote \( N_i = \sum_{k=1}^{i} n_k \) for \( i = 1, 2, \ldots, m \) with \( N_0 = 0 \); \( N_i \) is the total number of jobs assigned to the first \( i \) due dates.

The objective is to find the optimal \( D = \{D_1, D_2, \cdots, D_m\}, I = \{I_1, I_2, \cdots, I_m\} \), an optimal position of the job before which the maintenance activity is scheduled, and an optimal job sequence that minimize the function as follows:

\[
Z = \sum_{i=1}^{m} \sum_{j \in I_i} (\alpha E_{[j]} + \beta T_{[j]} + \gamma D_{[j]}),
\]

and \( d_j = D_i \) for \( j \in I_i \), where \( \alpha \geq 0, \beta \geq 0, \gamma \geq 0(\beta > \gamma) \) are the earliness, tardiness and due-date unit time penalties, respectively. Then, using the three-field notation, the considered problem can be denoted as \( lma, p_j = a_j + bt \sum_{i=1}^{m} \sum_{j \in I_i} (\alpha E_{[j]} + \beta T_{[j]} + \gamma D_{[i]}), \) where \( ma \) is the maintenance activity.

3. Optimal solution. Chand and Chhajed [4] provided and showed that several properties of an optimal solution for the original multiple common due dates assignment problem, these properties still hold when a rate modifying activity is considered. Making use of the analysis in the Chand and Chhajed [4], the following lemma 3.1 and lemma 3.2 also hold. Thus, we omit the proofs.

**Lemma 3.1.** For any given \( D = \{D_1, D_2, \cdots, D_m\} \) and given schedule \( \pi \), there exists an optimal \( I \) such that \( I_i = \{\pi[N_{i-1}+1], \pi[N_{i-1}+2], \cdots, \pi[N_i]\} \) for \( i = 1, 2, \ldots, m \), where \( \pi_{[r]} \) is the job in position \( r \) in sequence \( \pi \).

This lemma says that there is an optimal solution such that \( n_i \) consecutive jobs (in positions \( N_{i-1} + 1 \) to \( N_i \)) in \( \pi \) are assigned to due date \( D_i \).

**Lemma 3.2.** For a given sequence \( \pi \), there exits an optimal \( D \) such that for any \( i \in \{1, 2, \cdots, m\}, D_i = C_{[k_i]}, \) where \( k_i = N_{i-1} + \lceil \frac{n_i(\beta - \gamma)}{(\alpha + \beta)} \rceil \).
Lemma 3.3. Let there be two sequences of numbers $x_i$ and $y_i$, the sum of $\sum_{i=1}^n x_i y_i$ is the least (largest) if the sequences are monotonic in the opposite (same) sense.

For a given position $h$ for the maintenance activity and a sequence $\pi = (J_{[1]}, J_{[2]}, \ldots, J_{[h]}, J_{[h+1]}, \ldots, J_{[n]})$, then the actual processing time of jobs can be written as follows:

$$p_{[1]} = a_{[1]},$$
$$p_{[2]} = a_{[2]} + bt = a_{[2]} + ba_{[1]},$$
$$p_{[3]} = a_{[3]} + b(a_{[2]} + (1 + b)a_{[1]}),$$
$$\ldots,$$
$$p_{[k]} = a_{[k]} + b(a_{[h-1]} + (1 + b)a_{[h-2]} + \cdots + (1 + b)^{h-k} a_{[1]}),$$
$$p_{[h+1]} = a_{[h+1]},$$
$$p_{[h+2]} = a_{[h+2]} + bt = a_{[h+2]} + ba_{[h+1]},$$
$$\ldots,$$
$$p_{[n]} = a_{[n]} + b(a_{[n-1]} + (1 + b)a_{[n-2]} + \cdots + (1 + b)^{n-h} a_{[1]}).$$

By lemma 3.2, the optimal position of due dates $D_i (i = 1, 2, \ldots, m)$ can be determined. If the maintenance activity is performed before the due date $D_i (1 \leq l \leq m)$ i.e. $N_{i-1} + 1 \leq h < k_l$, then the total cost (denoted by $Z_1$) associated with jobs in $(I_1, I_2, \ldots, I_{l-1})$ is given by

$$Z_1 = \sum_{i=1}^{l-1} [\gamma C_{[k_i]} + \sum_{j=N_{i-1}+1}^{k_i} \alpha (C_{[k_i]} - C_{[j-1]}) + \sum_{j=k_i+1}^{N_{i-1}} \beta (C_{[j]} - C_{[k_i]}))] = \sum_{j=1}^{N_{i-1}} w_{j} p_{[j]},$$

where for $i = 1, 2, \ldots, l - 1$,

$$w_{j} = \begin{cases} 
\alpha(j - 1 - N_{i-1}) + \gamma(n - N_{i-1}), & j = N_{i-1} + 1, \ldots, k_i, \\
\beta(N_{i-1} - j + 1) + \gamma(n - N_{i-1}), & j = k_i + 1, \ldots, N_{i-1}.
\end{cases}$$

Since $p_{[j]} = a_{[j]} + bt$, the total cost associated with jobs in $(I_1, I_2, \ldots, I_{l-1})$ can be written as follows:

$$Z_1 = w_{[1]} a_{[1]} + w_{[2]} (a_{[2]} + ba_{[1]}) + w_{[3]} (a_{[3]} + b(a_{[2]} + (1 + b)a_{[1]})) + \cdots + w_{N-1} (a_{N_{i-1}} + b(a_{N_{i-2}} + \cdots + (1 + b)^{N_{i-2}} a_{[1]}))$$

$$= W_1 a_{[1]} + W_2 a_{[2]} + \cdots + W_{N_{i-1}} a_{[N_{i-1}]},$$

where,

$$W_1 = w_{1} + w_{2} b + w_{3} b(1 + b) + \cdots + w_{N_{i-2}} b(1 + b)^{N_{i-2}} - 2,$$
$$W_2 = w_{2} + w_{3} b + w_{4} b(1 + b) + \cdots + w_{N_{i-3}} b(1 + b)^{N_{i-3}} - 3,$$
$$\ldots,$$
$$W_{N_{i-1}} = w_{N_{i-1}} - 1 + w_{N_{i-1}} b,$$

$$W_{N_{i-1}} = w_{N_{i-1}}.$$

The earliness cost (denoted by $Z_j$) associated with job $j$, $j = k_l, k_l-1, \ldots, N_{i-1} + 1$, is given by

$$Z_{k_l} = 0,$$
$$Z_{k_l-1} = \alpha p_{[k_l]},$$
$$Z_{k_l-2} = \alpha (p_{[k_l]} + p_{[k_{l-1}]}),$$
$$\ldots,$$
\[ Z_{h+1} = \alpha(p_{[k_1]} + p_{[k_{l-1}]} + p_{[k_{l-2}]} + \cdots + p_{[h+2]}), \]
\[ Z_h = \alpha(p_{[k_1]} + p_{[k_{l-1}]} + p_{[k_{l-2}]} + \cdots + p_{[h+2]} + p_{[h+1]} + c), \]
\[ Z_{h-1} = \alpha(p_{[k_1]} + p_{[k_{l-1}]} + p_{[k_{l-2}]} + \cdots + p_{[h+2]} + p_{[h+1]} + c + p_{[h]}), \]
\[ \ldots, \]
\[ Z_{N_l-1+1} = \alpha(p_{[k_1]} + p_{[k_{l-1}]} + p_{[k_{l-2}]} + \cdots + p_{[h+2]} + p_{[h+1]} + c + p_{[h]} + \cdots + p_{[N_l-1+2]}). \]

The tardiness cost (denoted by \( Z_j \)) associated with job \( j, j = k_l+1, k_l+2, \ldots, N_l \), is given by
\[ Z_{k_l+1} = \beta p_{[k_l+1]}, \]
\[ Z_{k_l+2} = \beta(p_{[k_l+1]} + p_{[k_{l+1}]}), \]
\[ \ldots, \]
\[ Z_{N_l} = \beta(p_{[k_l+1]} + p_{[k_{l+1}]} + \cdots + p_{[N_l]}). \]

The due date cost (denoted by \( Z_{d_l} \)) is given by
\[ Z_{d_l} = n_l \gamma(p_{[1]} + p_{[2]} + \cdots + p_{[h]} + c + p_{[h+1]} + \cdots + p_{[k_l-1]} + p_{[k_l]}). \]

The total earliness, tardiness and due-date cost (denoted by \( Z_2 \)) associated with jobs in \( I_l \) is given by
\[ Z_2 = \sum_{j=N_l-1+1}^{N_l} Z_j + Z_{d_l} \]
\[ = \alpha \sum_{j=N_l-1+1}^{k_l} (j - 1 - N_{l-1})p_{[j]} + (h - N_{l-1})\alpha c + \beta \sum_{j=k_l+1}^{N_l} (n_l - j + 1)p_{[j]} \]
\[ + n_l \gamma(p_{[1]} + p_{[2]} + \cdots + p_{[h]} + c + p_{[h+1]} + \cdots + p_{[k_l-1]} + p_{[k_l]}), \]
\[ = \sum_{j=1}^{N_{l-1}} n_l \gamma p_{[j]} + \alpha \sum_{j=N_{l-1}+1}^{k_l} [(j - 1 - N_{l-1}) + n_l \gamma]p_{[j]} + \beta \sum_{j=k_l+1}^{N_l} (n_l - j + 1)p_{[j]} \]
\[ + [(h - N_{l-1})\alpha + n_l \gamma]c, \]
\[ = \sum_{j=1}^{N_l} \bar{w}_j p_{[j]} + [(h - N_{l-1})\alpha + n_l \gamma]c, \]

where for \( i = l, \)
\[ \bar{w}_j = \begin{cases} n_l \gamma, & j = 1, \ldots, N_{l-1}, \\ \alpha(j - 1 - N_{l-1}) + n_l \gamma, & j = N_{l-1} + 1, \ldots, k_l, \\ \beta(n_l - j + 1), & j = k_l + 1, \ldots, N_l. \end{cases} \]

Since \( p_{[j]} = a_{[j]} + bt \), the total cost associated with jobs in \( I_l \) can be written as follows:
\[ Z_2 = \bar{W}_1 a_{[1]} + \bar{W}_2 (a_{[2]} + ba_{[1]}) + \bar{W}_3 (a_{[3]} + b(a_{[2]} + (1 + b)a_{[1]})) + \cdots \]
\[ + \bar{W}_h (a_{[h]} + (1 + b)a_{[h-1]} + (1 + b)a_{[h-2]} + \cdots + (1 + b)^{h-2}a_{[1]})) + \bar{W}_{h+1} a_{[h+1]} \]
\[ + \bar{W}_{h+2} (a_{[h+2]} + ba_{[h+1]}) + \cdots + \bar{W}_{N_l} (a_{[n]} + b(a_{[n-1]} + (1 + b)a_{[n-2]} + \cdots + (1 + b)^{n-h}a_{[h+1]})) \]
\[ + [(h - N_{l-1})\alpha + n_l \gamma]c \]
\[ = \bar{W}_1 a_{[1]} + \bar{W}_2 a_{[2]} + \cdots + \bar{W}_{N_l} a_{[N_l]} + [(h - N_{l-1})\alpha + n_l \gamma]c, \]
\[ (4) \]
where,
\[
\tilde{W}_1 = \tilde{w}_1 + \tilde{w}_2 b + \tilde{w}_3 b(1 + b) + \cdots + \tilde{w}_h b(1 + b)^{h-2}, \\
\tilde{W}_2 = \tilde{w}_2 + \tilde{w}_3 b + \tilde{w}_4 b(1 + b) + \cdots + \tilde{w}_h b(1 + b)^{h-3}, \\
v, \quad \tilde{W}_{h-1} = \tilde{w}_{h-1} + \tilde{w}_h b, \\
\tilde{W}_h = \tilde{w}_h, \\
\tilde{W}_{h+1} = \tilde{w}_{h+1} + \tilde{w}_{h+2} b + \tilde{w}_{h+3} b(1 + b) + \cdots + \tilde{w}_{N_i} b(1 + b)^{N_i-h-2}, \\
\tilde{W}_{h+2} = \tilde{w}_{h+2} + \tilde{w}_{h+3} b + \tilde{w}_{h+4} b(1 + b) + \cdots + \tilde{w}_{N_i} b(1 + b)^{N_i-h-3}, \\
v, \\
\tilde{W}_{N_i-1} = \tilde{w}_{N_i-1} + \tilde{w}_{N_i} b, \\
\tilde{W}_{N_i} = \tilde{w}_{N_i}.
\]

The total cost (denoted by \( Z_3 \)) associated with jobs in \((I_{l+1}, I_{l+2}, \cdots, I_m)\) is given by
\[
Z_3 = \sum_{i=l+1}^{m} [n_i \gamma C_{[k_i]} + \sum_{j=N_i-1}^{k_i} \alpha (C_{[k_i]} - C_{[j]}) + \sum_{j=k_i+1}^{N_i} \beta (C_{[j]} - C_{[k_i]})] = \sum_{j=1}^{N_m} \tilde{w}_j p[j],
\]
where for \( i = l + 1, l + 2, \cdots, m, \)
\[
\tilde{w}_j = \begin{cases} 
  n - N_i, & j = 1, \cdots, h, \\
  \gamma (n - N_i), & j = h + 1, \cdots, N_i-1, \\
  \gamma (n - N_i - 1) + \alpha (j - 1 - N_i-1), & j = N_i-1 + 1, \cdots, k_i, \\
  \beta (N_i - j + 1), & j = k_i + 1, \cdots, N_i. 
\end{cases}
\]

Therefore the total cost associated with jobs in \((I_{l+1}, I_{l+2}, \cdots, I_m)\) can be written as follows:
\[
Z_3 = \tilde{W}_1 a_{[1]} + \tilde{W}_2 a_{[2]} + \tilde{b}_{a_{[1]}} + \tilde{w}_3 (a_{[3]} + b(a_{[2]} + (1 + b)a_{[1]}))) + \cdots \\
+ \tilde{w}_3 (a_{[h]} + b(a_{[h-1]} + (1 + b)a_{[h-2]} + \cdots + (1 + b)^{h-2} a_{[1]})) \\
+ \tilde{w}_{h+1} a_{[h+1]} + \tilde{w}_{h+2} a_{[h+2]} + \tilde{w}_{h+3} (a_{[h+1]} + b(a_{[h+1]}))) + \cdots \\
+ \tilde{w}_n (a_{[n]} + b(a_{[n-1]} + (1 + b)a_{[n-2]} + \cdots + (1 + b)^{n-h-2} a_{[h+1]}))) + \gamma (n - N_i)c \\
= \tilde{W}_1 a_{[1]} + \tilde{W}_2 a_{[2]} + \cdots + \tilde{W}_n a_{[n]} + \gamma (n - N_i)c,
\]
where,
\[
\tilde{W}_1 = \tilde{w}_1 + \tilde{w}_2 b + \tilde{w}_3 b(1 + b) + \cdots + \tilde{w}_h b(1 + b)^{h-2}, \\
\tilde{W}_2 = \tilde{w}_2 + \tilde{w}_3 b + \tilde{w}_4 b(1 + b) + \cdots + \tilde{w}_h b(1 + b)^{h-3}, \\
\cdots, \\
\tilde{W}_{h-1} = \tilde{w}_{h-1} + \tilde{w}_h b, \\
\tilde{W}_h = \tilde{w}_h, \\
\tilde{W}_{h+1} = \tilde{w}_{h+1} + \tilde{w}_{h+2} b + \tilde{w}_{h+3} b(1 + b) + \cdots + \tilde{w}_{N_i} b(1 + b)^{N_i-h-2}, \\
\cdots, \\
\tilde{W}_{n-1} = \tilde{w}_{n-1} + \tilde{w}_n b, \\
\tilde{W}_n = \tilde{w}_n.
\]
Consequently, the total cost (for given \(N_{i-1} + 1 \leq h < k_i\)) is given by

\[
Z = W_1 a_{[1]} + W_2 a_{[2]} + \cdots + W_{N_{i-1}} a_{[N_{i-1}]} + \tilde{W}_1 a_{[1]} + \tilde{W}_2 a_{[2]} + \cdots + \tilde{W}_n a_{[n]} + (h - N_{i-1})a + n h c + W_1 a_{[1]} + W_2 a_{[2]} + \cdots + W_n a_{[n]} + \gamma(n - N_i)c. \tag{8}
\]

Similarly, the total cost (for given \(k_i \leq h \leq N_i\)) is given by

\[
Z = W_1 a_{[1]} + W_2 a_{[2]} + \cdots + W_{N_{i-1}} a_{[N_{i-1}]} + \tilde{W}_1 a_{[1]} + \tilde{W}_2 a_{[2]} + \cdots + \tilde{W}_n a_{[n]} + (N_i - h)\beta c + \tilde{W}_1 a_{[1]} + \tilde{W}_2 a_{[2]} + \cdots + \tilde{W}_n a_{[n]} + \gamma(n - N_i)c. \tag{9}
\]

By combining Eq. (8) and Eq. (9), the total cost is given by

\[
Z = Z_A + Z_B + Z_C + M,
\]

where,

\[
Z_A = (W_1 + \tilde{W}_1 + \tilde{W}_1 a_{[1]} + (W_2 + \tilde{W}_2 + \tilde{W}_2 a_{[2]} + \cdots + (W_{N_{i-1}} + \tilde{W}_{N_{i-1}} + \tilde{W}_{N_{i-1}} a_{[N_{i-1}]}),
\]

\[
Z_B = (W_{N_{i-1}+1} + \tilde{W}_{N_{i-1}+1} a_{[N_{i-1}+1]} + (\tilde{W}_{N_{i-1}+2} + \tilde{W}_{N_{i-1}+2} a_{[N_{i-1}+2]} + \cdots + (\tilde{W}_{N_i} + \tilde{W}_{N_i}) a_{[N_i]},
\]

\[
Z_C = \tilde{W}_{N_{i+1}} a_{[N_{i+1}]} + \tilde{W}_{N_{i+2}} a_{[N_{i+2}]} + \cdots + \tilde{W}_n a_{[n]},
\]

\[
M = \begin{cases} \alpha(h - N_{i-1} + n\gamma)c + \gamma(n - N_i)c, & 1 \leq h < k_i, \\ \beta(N_i - h)c + \gamma(n - N_i)c, & k_i \leq h \leq N_{i-1}. \end{cases}
\]

and \(W_j (j = 1, 2, \cdots, N_{i-1})\) are determined by Eq. (3), \(\tilde{W}_j (j = 1, 2, \cdots, N_i)\) are determined by Eq. (5) and \(W_j (j = 1, 2, \cdots, n)\) are determined by Eq. (7).

**Theorem 3.4.** The \(1|ma, p_j = a_j + bt\sum_{i=1}^{m} \sum_{j \in I_i} (\alpha E_{[j]} + \beta T_{[j]} + \gamma D_{[i]} \) problem can be solved in \(O(n^2 \log n)\) time.

**Proof.** Once the position of maintenance activity has been determined by the lemma 3.3, \(Z_A, Z_B, Z_C\) can be solved in \(O(n \log n)\) time, respectively. Then \(Z\) can be solved in \(O(n \log n)\) time, since the maintenance activity can be scheduled \(m\) different positions and \(1 \leq m \leq n\), then the problem can be solved in \(O(n^2 \log n)\) time. \(\square\)

4. **Conclusions.** Scheduling problems with due date assignment have received increasing attention. In this paper, we focus on a single machine scheduling with linear deteriorating jobs and multiple common due-date assignment under an optimal maintenance activity considered where the objective is to determine the optimal job sequence, the due dates and the optimal maintenance activity locations to minimize the total cost including earliness, tardiness, and the due dates. We show that the problem can be solved optimally in polynomial time. Future research may focus on similar problems with multiple maintenance activities or multiple due windows. It would also be worthwhile to investigate an extension of the problem to the general job processing times in other machine settings.

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REFERENCES

[1] M. A. Bajestani, Integrating Maintenance Planning and Production Scheduling, Making Operational Decisions with a Strategic Perspective, Ph.D thesis, University of Toronto in Toronto, 2014.
[2] W. W. Cui, Z. Q. Lu and E. Pan, Integrated production scheduling and maintenance policy for robustness in a single machine Computers and Operations Research, 47 (2014), 81–91.
[3] T. C. E. Cheng, Q. Ding and B. M. T. Lin, A concise survey of scheduling with time-dependent processing times European Journal of Operational Research, 152 (2004), 1–13.
[4] S. Chand and D. Chhajed, A single machine model for determination of optimal due dates and sequence Operations Research, 40 (1992), 596–602.
[5] B. Dickman and Y. Wilamowsky, Multiple common due dates Naval Research Logistics, 48 (2001), 293–298.
[6] S. Gawiejnowicz, Time-dependent Scheduling, Springer, Berlin, 2008.
[7] M. Gopalakrishnan, S. L. Ahire and D. M. Miller, Maximizing the effectiveness of a preventive maintenance system: an adaptive modeling approach Management Science, 43 (1997), 827–840.
[8] C. J. Hsu, C. J. Yang and D. L. Yang, Due-date assignment and optional maintenance activity scheduling with linear deteriorating jobs, Journal of Marine Science and Technology, 19 (2011), 97–100.
[9] M. A. Kubzin and V. A. Strusevich, Two-machine flow shop no-wait scheduling with machine maintenance 4OR: A Quarterly Journal of Operations Research, 3 (2005), 303–313.
[10] W. H. Kuo and D. L. Yang, A note on due-date assignment and single-machine scheduling with deteriorating jobs, Journal of the Operational Research Society, 59 (2008), 857–859.
[11] I. Kacem and E. Levner, An improved approximation scheme for scheduling a maintenance and proportional deteriorating jobs Journal of Industrial and Management Optimization, 12 (2016), 811–817.
[12] S. S. Li, C. T. Ng and J. J. Yuan, Scheduling deteriorating jobs with CON/SLK due date assignment on a single machine International Journal of Production Economics, 131 (2011), 747–751.
[13] G. Mosheiov, Scheduling jobs under simple linear deterioration Computers and Operations Research, 21 (1994), 653–659.
[14] D. Nyman and J. Levitt, Maintenance Planning, Scheduling and Coördination, 2nd edition, Industrial Press, New York, 2010.
[15] D. Palmer, Maintenance Planning and Scheduling Handbook, 2nd edition, McGraw Hill, New York, 1999.
[16] K. Rustogi and V. A. Strusevich, Single machine scheduling with general positional deterioration and rate-modifying maintenance Omega, 40 (2012), 791–804.
[17] K. Rustogi and V. A. Strusevich, Combining time and position dependent effects on a single machine subject to rate-modifying activities Omega, 42 (2014), 166–178.
[18] J. B. Wang and M. Z. Wang, Single machine multiple common due dates scheduling with learning effect Computers and Mathematics with Applications, 60 (2010), 2998–3002.
[19] X. Y. Yu, Y. L. Zhang and G. Steiner, Single-machine scheduling with periodic maintenance to minimize makespan revisited Journal of Scheduling, 17 (2014), 263–270.
[20] S. J. Yang, C. J. Hsu and D. L. Yang, Single-machine scheduling with due-date assignment and aging effect under a deteriorating maintenance activity consideration, International Journal of Information and Management Sciences, 21 (2010), 177–195.
[21] S. J. Yang, H. T. Lee and J. Y. Guo, Multiple common due dates assignment and scheduling problems with resource allocation and general position-dependent deterioration effect The International Journal Advanced Manufacturing Technology, 67 (2013), 181–188.
[22] C. L. Zhao, Y. Q. Yin, T. C. E. Cheng and C. C. Wu, Single-machine scheduling and due date assignment with rejection and position-dependent processing times Journal of Industrial and Management Optimization, 10 (2014), 691–700.

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