PRESERVATION OF DETERIORATING SEASONAL PRODUCTS WITH STOCK-DEPENDENT CONSUMPTION RATE AND SHORTAGES

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Abstract. In literature, many inventory studies have been developed by assuming deterioration of items as either a variable or constant. But in real life situation, deterioration of goods can be reduced by adding some extra effective capital investment in preservation technology. In this paper, a deteriorating inventory model with ramp-type demand under stock-dependent consumption rate by assuming preservation technology cost as a decision variable is formulated. Shortages are allowed and the unsatisfied demand is partially backlogged at a negative exponential rate with the waiting time. The objective of this study is to obtain the optimal replenishment and preservation technology investment strategies so that the total profit per unit time is maximum. Further, the necessary and sufficient conditions are considered to prove the existence and uniqueness of the optimal solution. Some numerical examples along with graphical representations are provided to illustrate the proposed model. Sensitivity analysis of the optimal solution with respect to major parameters of the system has been carried out and the implications are discussed.

1. Introduction. Now a days, deterioration of goods is a common phenomenon in every inventory sectors. Generally, we define deterioration as decay or damage of items, such as fruits, foods, vegetables, etc that can not be used for its original purpose. Highly volatile liquids like alcohol, turpentine, gasoline, radioactive materials, etc., deteriorate due to evaporation while kept in store. Therefore, in formulating inventory models, the deterioration of items should not be neglected. The investigation of deteriorating inventory began with Ghare and Schrader [17], who developed a classical inventory model with a constant rate of decay. Thereafter Covert and Philip [9], Sana, Goyal, and Chaudhuri [30], Dye, Chang, and Teng [13], Chung and

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Wee [7], Widyadana and Wee [54], Chung and Wee [6], Wee, Lee, Yu, and Wang [92], Sarkar [34], Sett, Sarkar, and Goswami [46] developed various type of inventory models by assuming deteriorating rate as either constant or time-dependent. By using the time-varying deterioration function, Sarkar [35] developed a deteriorating inventory model with trade-credit policy. Sarkar [36] derived a probabilistic deterioration model to find the integer number of deliveries and lot size with the help of an algebraical procedure.

Sarkar and Sarkar [40] developed an inventory model with time-varying demand and deterioration. They used Kuhn-Tucker optimization method to obtain the optimal replenishment cycle time and ordering quantity such that the total profit per unit time is maximum. Sarkar and Sarkar [41] discussed a control inventory problem with probabilistic deterioration. They solved the model with the help of Euler-Lagrange method. Sarkar, Sarkar, and Yun [42] derived retailer’s optimal strategy for deteriorating items with lifetime of products. Bouras and Tadj [4] developed a production model in a three-stock reverse-logistics system with deteriorating items under a continuous review policy. Sarkar, Saren, and Cárdenas-Barrón [44] formulated a trade-credit policy with variable deterioration for fixed lifetime products.

In the above mentioned papers the authors considered the deterioration rate as an exogenous variable, which is not subject to control. However, retailer can reduce the deterioration rate through various efforts like procedural changes and specialized equipment acquisition. For example, refrigeration equipments are used to reduce the deterioration rate of fruits, flowers, and sea foods in the supermarket. To highlight this type of phenomenal Hsu, Wee, and Teng [18] developed a solution procedure to determine the retailer’s replenishment and preservation technology investment policy for deteriorating items with a constant rate of deterioration. However, in their model the preservation technology cost was a fixed cost per inventory cycle. Dye and Hsieh [14] then extended model of Hsu, Wee, and Teng [18] by assuming the preservation technology cost as a function of the length of replenishment cycle. They also incorporate time-varying deterioration and reciprocal time-dependent partial backlogging rates in their model. Dye [15] considered an inventory model for a non-instantaneous deteriorating item to study the effect of preservation technology investment on inventory decisions. They developed several structural properties to find the optimal replenishment time and preservation technology strategies. They shown that if the preservation technology investment is profitable, a higher preservation technology investment leads to a higher optimal service rate. Zhang, Bai, and Tang [58] developed a optimal pricing policy for deteriorating items with preservation technology investment.

In real life marketing environment, we can often observe that many products of famous brand or fashionable commodities (e.g. shoes, hi-fi equipment, clothes etc.) may lead to a situation in which customer would like to backorder while shortage occur. Montgomery, Bazarra, and Keswani [24], Park [28], and Rosenberg [29] developed several inventory models with a mixture of backorders and lost sales. They considered that only a fixed fraction of demand during the stock out time is backlogged and the rest is lost. Abad [1] developed a pricing and lot-sizing problem for a product with a variable rate of deterioration with partial backorder. Chang and Dye [5] developed an inventory model in which the proportion of customers who would like to accept backlogging is the reciprocal of a linear function of the waiting time.
Papachristos and Skouri [27] established an inventory model for deteriorating item in which the backlogging rate decreases exponentially as the waiting time increases. Skouri and Papachristos [48] presented a multi-period inventory model using the negative exponential backlogging rate. Sometimes managers prefer to use planned backorders to reduce the total system cost. Cárdenas-Barrón [10] presented an economic production quantity (EPQ) model with rework process at a single-stage manufacturing system with planned backorders. Sana [31] investigated a deteriorating inventory model selling price dependent demand, time-varying deterioration, and partial backlogging over an infinite time horizon planning. Cárdenas-Barrón [11] presented inventory models with two backorders costs using analytic geometry and algebra. They used two types of backorder cost, one linear and other fixed backorders costs.

Widyadana, Cárdenas-Barrón, and Wee [55] developed a method without derivatives to solve an inventory model for deteriorating items with and without backorders. Sarkar and Sarkar [43] presented an improved inventory model with partial backlogging, time-varying deterioration, and stock-dependent demand. Wee, Huang, Wang and Cheng [53] developed an inventory model with partial backorder considering linear and fixed backorder costs. In their model they used time intervals rather than ordering size and backorder level to derive a closed form optimal solution. Sarkar [37] developed a supply chain coordination model with variable backorder, inspections, and discount policy for fixed lifetime products. Sarkar, Mandal, and Sarkar [39] developed a continuous review inventory model with backorder price discount under controllable lead time.

The demand of seasonal/fashionable products such as clothes, sporting goods, children’s toys, and electrical home appearances increases up to a certain moment as time increase and then stabilizes to a constant rate. To represent such type of demand pattern the term “ramp-type” is used. In this direction, Mandal and Pal [23] was the first author to incorporate ramp-type demand in inventory model. Wu [56] developed an EOQ model with ramp-type demand, Weibull distributed deterioration and partial backlogging. Wu, Ouyang, and Yang [57] discussed an inventory model for deteriorating items with a ramp-type demand under stock-dependent consumption rate. A similar type model with partial backlogging was considered by Skouri, Konstantaras, Papachristos, and Ganas [49]. Sarkar, Sett, Goswami, and Sarkar [45] considered a deteriorating inventory model with ramp-type demand and partial backlogging.

Most of the previous inventory models consider the demand rate as either constant or time-varying but independent of inventory level. It is common phenomenon of the customer is to buy a single item which is choosing from a large amount of items, i.e., the demand rate may go up or down if the on-hand inventory level increases or decreases. For this reason, the retailers have to store a large amount of items to their stored place. According to Levin, McLaughlin, Lemone, and Kottas [21], “At times the presence of inventory has a motivational effect on the people around it. It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more”. Silver and Peterson [47] also noted that the sales at the retail level tend to be proportional to the inventory displayed. Padmanabhan and Vrat [26] developed an inventory model for perishable items with stock-dependent selling rate. Liao, Tsai, and Su [22] discussed an inventory model with initial-stock-dependent consumption rate and permissible delay-in-payment. Dye and Ouyang [10] extended Padmanabhan and Vrat’s [26] model with linear
time proportional backlogging rate, and then established the unique optimal solution to the problem for non-profitable building up inventory. Alfares [8] found out an inventory model with stock-level dependent demand rate and variable holding cost.

Chung and Wee [32] developed the scheduling and replenishment plan for an integrated deteriorating inventory model with stock-dependent selling rate. Sana and Chaudhuri [33] presented a deterministic EOQ model with stock-dependent demand rate where a supplier gives a retailer both a credit period and a price discount. An EOQ model for perishable item with stock-dependent demand and price discount rate was presented by Sana [33]. Sarkar [38] developed an EOQ model with finite replenishment rate to investigate the retailer’s optimal replenishment policy under permissible delay-in-payment with stock-dependent demand.

Asghari, Abrishami, and Mahdavi [3] formulated a dynamic nonlinear programming model of reverse logistics network design with incentive dependent return. Damghani and Shahrokh [12] solved a new multi-product multi-period multi-objective aggregate production planning problem using fuzzy goal programming. Özyörük and Dönmez [25] developed a fuzzy multi-objective linear programming model for the supply chain of a company dealing with procurement, storage, filling, and distribution of liquefied petroleum gas (LPG). Kusukawa [19] discussed an optimal operation for a 2-stage-ordering-production system for a retailer and a manufacturer. Lee, Kim, and Opit [20] derived a mixed integer model for stock pre-positioning to support an emergency disaster relief response against the event of earthquake. Watanabe and Kusukawa [51] considered an optimal operational policy for a green supply chain (GSC) where a retailer pays an incentive for collection of used products from customers. Thongdee and Pitakaso [50] developed some algorithms using the Differential Evolution Algorithm (DE) to solve a multi-objective, sources and stages location-allocation problem.

We summarize our contribution compared with other models in Table 1.

| Author(s) | Ramp-type demand | Deterioration | Preservation | Stock-dependent consumption rate | Partial backlogging |
|-----------|------------------|---------------|--------------|----------------------------------|--------------------|
| Ghare and Schrader [17] | ✓ | | | | |
| Covert and Philip [9] | ✓ | | | | |
| Sana, Goyal, and Chaudhuri [30] | ✓ | | | | |
| Dye, Chang, and Teng [13] | ✓ | | | | ✓ |
| Chung and Wee [4] | ✓ | ✓ | | | |
| Widyadana and Wee [54] | ✓ | | | | |
| Chung and Wee [6] | ✓ | | | | |
| Lee, Yu, and Wang [52] | ✓ | | | | |
| Sarkar [34] | | ✓ | | | |
| Sett, Sarkar, and Goswami [46] | ✓ | | | | |
| Sarkar [35] | | ✓ | | | |
| Sarkar and Sarkar [36] | ✓ | | | | |
| Sarkar and Sarkar [40] | ✓ | | | | |
| Sarkar and Sarkar [41] | ✓ | ✓ | | | |
| Sarkar [42] | ✓ | | | | |

Table 1. Comparison between the contributions of different authors
In this proposed model, an effort has been made to develop an inventory model with controllable deterioration rate by assuming ramp-type demand under stock-dependent consumption rate. Shortages are allowed and partially backlogged of unfilled demand. The main purpose of this paper is to obtain the optimal replenishment policy and preservation technology investment strategies which maximized the total profit per unit time. The rest of the paper is designed as follows: In section 2, notation and assumptions are given. In section 3, the model is formulated in two

| Author(s) | Ramp-type demand | Deterioration | Preservation | Stock-dependent consumption rate | Partial backlogging |
|-----------|------------------|---------------|--------------|---------------------------------|---------------------|
| Bouras and Tadj [4] | | √ | | | |
| Sarkar, Saren, and Cárdenas-Barrón [44] | | √ | | | |
| Hsu, Wei, and Teng [18] | | √ | √ | | |
| Dye and Hsieh [14] | | √ | √ | | |
| Dye [15] | | √ | √ | | |
| Zhang, Bai, and Tang [68] | | √ | √ | | |
| Montgomery, Bazaraa, and Keswani [24] | | | | | √ |
| Park [28] | | | | | √ |
| Rosenberg [29] | | | | | √ |
| Abad [1] | | √ | | | |
| Chang and Dye [5] | | √ | | | |
| Papachristos and Skouri [27] | | √ | | | |
| Skouri and Papachristos [48] | | √ | | | |
| Cárdenas-Barrón [10] | | | | | √ |
| Sana [31] | | √ | | | |
| Cárdenas-Barrón [11] | | √ | | | |
| Widyadana, Cárdenas-Barrón, and Wee [55] | | √ | | | |
| Sarkar and Sarkar [40] | | √ | √ | | |
| Wee, Huang, Wang and Cheng [53] | | √ | | | |
| Sarkar [37] | | | | | |
| Sarkar, Mandal, and Sarkar [39] | | | | | |
| Mandal and Pal [23] | | √ | | | |
| Wu [50] | | √ | | | |
| Wu, Ouyang, and Yang [57] | | √ | | √ | |
| Skouri, Konstantaras, Papachristos, and Ganas [49] | | √ | | | |
| Sarkar, Sett, Goswami, and Sarkar [45] | | √ | | √ | |
| Levin, McLaughlin, Lemone, and Kottas [21] | | | | | |
| Silver and Peterson [47] | | | | | |
| Padmanabhan and Vrat [29] | | √ | | √ | |
| Liao, Tsai, and Su [22] | | √ | | | |
| Dye and Ouyang [16] | | | | | |
| Alfares [2] | | | | | |
| Chung and Wee [8] | | | | | |
| Sana and Chaudhuri [32] | | | | | |
| Sana [33] | | | | | |
| Sarkar [38] | | | | | |
| This paper | | √ | √ | √ | √ |

Table 1. Comparison between the contributions of different authors
cases. Numerical experiments and sensitivity analysis are presented to illustrate the model in section 4. Finally, conclusions are made in section 5.

2. Notation and assumptions. To derive the model, the following notation and assumptions are used

2.1. Notation.
- $Q$: order quantity per cycle (units)
- $\theta$: deterioration rate
- $\delta(t)$: backlogging rate
- $s$: selling price per unit ($/unit$)
- $A$: ordering cost ($/order$)
- $h$: inventory holding cost ($/unit/week$)
- $p$: purchasing cost ($/unit$)
- $b$: backorder cost ($/unit$)
- $l$: lost sell cost ($/unit$)
- $m(\tau)$: reduced deterioration rate, a function of $\tau$
- $\tau$: preservation technology cost per unit time for reducing the deterioration rate in order to preserve the products, $0 \leq \tau \leq w$, where $w$ is the maximum cost of investment in preservation technology
- $t_1$: length of time in which the inventory level falls to zero (week)
- $I(t)$: on-hand inventory level at time $t$
- $T$: fixed length of each ordering cycle (week)
- $\Pi_1(t_1)$: total profit for Model 1 ($/unit/week$)
- $\Pi_2(t_1)$: total profit for Model 2 ($/unit/week$)

2.2. Assumptions.
1. The model is considered for a single item.
2. Deterioration rate $\theta$ is constant and there is no replacement or repair of deteriorated units during the period under consideration.
3. The reduced deterioration rate $m(\tau)$ is assumed to be a strictly increasing function of the preservation technology cost $\tau$, where $\lim_{\tau \to \infty} m(\tau) = \theta$.
4. The theoretical demand rate $R(t)$ is assumed to be a ramp-type function of time, i.e.,
   
   $$ R(t) = D_0 [t - (t - \mu)H(t - \mu)], \quad D_0 > 0 $$

   where $H(t - \mu)$ is the Heaviside’s function as follows:
   
   $$ H(t - \mu) = \begin{cases} 
   1 & \text{if } t \geq \mu \\
   0 & \text{if } t < \mu 
   \end{cases} $$

5. $D(t)$ is the selling rate at time $t$, and it is influenced by the theoretical demand rate and the on-hand inventory according to relation
   
   $$ D(t) = \begin{cases} 
   R(t) + \alpha I(t), & I(t) > 0 \\
   R(t), & I(t) \leq 0 
   \end{cases} $$

   where $\alpha$ is positive constant and $I(t)$ is the on-hand inventory level at time $t$.
6. Shortages are allowed and partially backlogged at a rate $\delta(t) = e^{-\sigma t}$, which is a decreasing function of time with $0 \leq \delta(t) \leq 0, \delta(0) = 1$ and $\lim_{t \to \infty} \delta(t) = 0$. The cases with $\delta(t) = 1$ (or) $0$ for all $t$ correspond to complete backlogging (or complete lost sales) models.
7. Lead time is considered as negligible.
3. Model formulation. Here, an inventory model for deteriorating items with ramp-type demand and stock-dependent selling rate is considered. The replenishment at the beginning of the cycle brings the inventory level up to $I_{\text{max}}$. The inventory level decreases during the time interval $[0,t_1]$ due to demand and deterioration of items, and falls to zero at $t = t_1$. Thereafter shortages occur during the period $(t_1, T)$, which are partially backlogged. The inventory level $I(t), 0 \leq t \leq T$ satisfies the following differential equations

$$\frac{dI(t)}{dt} + [\theta - m(\tau)]I(t) = -D(t), \ 0 \leq t \leq t_1, \ I(0) = I_{\text{max}}$$ (1)

$$\frac{dI(t)}{dt} = -D(t)\delta(T - t), \ t_1 \leq t \leq T, \ I(t_1) = 0$$ (2)

The solutions of these differential equations depend on the real selling rate. Two cases are considered in this paper: (a) $t_1 \leq \mu$ (b) $t_1 \geq \mu$. The fluctuation of the inventory level for the two cases is depicted in Figure 1 and Figure 2, respectively.

![Graphical presentation of the inventory system (Case 1: $\mu \geq t_1$)](image)

**Figure 1**: Graphical presentation of the inventory system (Case 1: $\mu \geq t_1$)
3.1. **Model 1** $t_1 \leq \mu$. In this case, the real selling rate $D(t)$ is

$$D(t) = \begin{cases} 
D_0 t + \alpha I(t), & 0 \leq t \leq t_1 \\
D_0 t, & t_1 \leq t \leq \mu \\
D_0 \mu, & \mu \leq t \leq T
\end{cases}$$

Therefore, (1) becomes

$$\frac{dI(t)}{dt} + [\theta - m(\tau)] I(t) = -[D_0 t + \alpha I(t)], \quad 0 \leq t \leq t_1$$

(3)

Solving (3) with the boundary condition $I(0) = I_{max}$, one has

$$I(t) = I_{max} e^{-\eta t} - \frac{D_0}{\eta^2} \left(e^{-\eta t} + \eta t - 1\right)$$

(4)

where $\eta = \alpha + \theta - m(\tau)$

Using the boundary condition $I(t_1) = 0$ and (4), the maximum inventory level for

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**Figure 2**: Graphical presentation of the inventory system (Case 2: $\mu \leq t_1$)
each cycle can be obtained as
\[ I_{max} = \frac{D_0}{\eta^2} (\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1) \]  
(5)

Using the value of \( I_{max} \), (4) becomes
\[ I(t) = \frac{D_0}{\eta^2} \left( \eta t_1 e^{\eta(t_1-t)} - e^{\eta(t_1-t)} + 1 \right) - \frac{D_0 t}{\eta} \]  
(6)

Again (2) leads to the following two equations
\[ \frac{dI(t)}{dt} = -\frac{D_0 \eta}{\sigma^2 e^{\sigma T}} (e^{\sigma t_1} - e^{\sigma t}) \quad t_1 \leq t \leq \mu \quad \text{with} \quad I(t_1) = 0 \]  
(7)
\[ \frac{dI(t)}{dt} = -\frac{D_0 \mu}{\sigma e^{\sigma T}} (e^{\sigma T} - e^{\sigma t}) \quad \mu \leq t \leq T \quad \text{with} \quad -I(T) = S \]  
(8)

Solving (7) and (8) with the boundary conditions, one has
\[ I(t) = \frac{D_0}{\sigma^2 e^{\sigma T}} \left( e^{\sigma t_1} (\sigma t_1 - 1) - e^{\sigma t} (\sigma t - 1) \right) \]  
(9)
\[ I(t) = \frac{D_0 \mu}{\sigma e^{\sigma T}} (e^{\sigma T} - e^{\sigma t}) - S \]  
(10)

Considering the continuity of \( I(t) \) at \( t = \mu \), we obtain the maximum amount of demand backlogged per cycle from (9) and (10) as
\[ S = \frac{D_0 \mu}{\sigma e^{\sigma T}} (e^{\sigma T} - e^{\sigma \mu}) - \frac{D_0}{\sigma^2 e^{\sigma T}} \left( e^{\sigma t_1} (\sigma t_1 - 1) - e^{\sigma \mu} (\sigma \mu - 1) \right) \]  
(11)

Putting the value of \( S \), (10) reduces to
\[ I(t) = \frac{D_0 \mu}{\sigma e^{\sigma T}} (e^{\sigma \mu} - e^{\sigma t}) + \frac{D_0}{\sigma^2 e^{\sigma T}} \left( e^{\sigma t_1} (\sigma t_1 - 1) - e^{\sigma \mu} (\sigma \mu - 1) \right) \]  
(12)

Now the order quantity \( Q \) is
\[ Q = I_{max} + S = \frac{D_0}{\eta^2} (\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1) \]
- \[ \frac{D_0}{\sigma^2} \left( e^{(\mu - T)\sigma} - \sigma \mu - e^{(t_1 - T)\sigma} (1 - \sigma t_1) \right) \]

Next, the total cost per cycle consists of the following four values
(a) Ordering cost per cycle = \( A \)
(b) Purchase cost per cycle = \( pQ \)
\[ = p \left\{ \frac{D_0}{\eta^2} (\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1) + \frac{D_0 \mu}{\sigma e^{\sigma T}} (e^{\sigma T} - e^{\sigma \mu}) \right. \]
- \[ \frac{D_0}{\sigma^2 e^{\sigma T}} \left( e^{\sigma t_1} (\sigma t_1 - 1) - e^{\sigma \mu} (\sigma \mu - 1) \right) \}

(c) Holding cost per cycle
\[ = h \int_0^{t_1} I(t)dt = hD_0 \left( \frac{\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1}{\eta^3} - \frac{t_1^2}{2\eta} \right) \]
(d) Backlogging cost per cycle

\[ b \int_{t_1}^{T} [-I(t)] dt \]

\[ = \frac{bD_0}{\sigma^2 e^{\sigma T}} \left[ \mu e^{\sigma T} - (T - \mu) e^{\sigma \mu} - \frac{2}{\sigma} (e^{\sigma \mu} e^{\sigma t_1}) + e^{\sigma t_1} \{ T(1 - \sigma t_1) - t_1(2 - \sigma t_1) \} \right] \]

(e) Lost sale cost per cycle

\[ = l \left( \int_{t_1}^{\mu} D_0 [1 - \delta(T - t)] dt + \int_{t_1}^{T} D_0 [1 - \delta(T - t)] dt \right) \]

\[ = l D_0 \left[ \frac{1}{\sigma^2} \left\{ e^{(\mu - T)\sigma} - \sigma \mu - e^{(\mu T - T)\sigma} (1 - \sigma t_1) \right\} + (\mu^2 - t_1^2)/2 + \mu(T - \mu) \right] \]

(f) Sales revenue per cycle

\[ = s \left( \int_{0}^{t_1} D(t) dt + \int_{t_1}^{T} D(t) \delta(T - t) dt \right) \]

\[ = s D_0 \left[ \alpha \left( \frac{\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1}{\eta^3} - \frac{t_1^2}{2\eta} \right) - \frac{e^{(\mu - T)\sigma} - \sigma \mu - e^{(\mu T - T)\sigma} (1 - \sigma t_1)}{\sigma^2} + \frac{t_1^2}{2} \right] \]

(g) Preservation technology cost = \tau T.

Therefore, total profit per unit time under the condition \( t_1 \leq \mu \) is

\[ = \Pi_1(t_1, \tau) \]

\[ = \frac{1}{T} \left[ \text{sales revenue-ordering cost-holding cost-backlogging cost-lost sale cost} \right. \]

\[ - \text{purchase cost-preservation technology cost} \]

\[ = \frac{D_0}{T} \left[ (s\alpha - h) \left( \frac{\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1}{\eta^3} - \frac{t_1^2}{2\eta} \right) + (s + l) \left( \frac{t_1^2}{2} \right) \right. \]

\[ - \frac{p}{\eta^2} (\eta t_1 e^{\eta t_1} - e^{\eta t_1} + 1) - \frac{s + l - p}{\sigma^2} \left\{ e^{(\mu - T)\sigma} - (1 - \sigma t_1) e^{(\mu T - T)\sigma} - \sigma \mu \right\} \]

\[ - l \left( \frac{\mu^2}{2} + \mu(T - \mu) \right) - \frac{1}{D_0} (A + \tau T) - \frac{b}{\sigma^2 e^{\sigma T}} \left\{ \mu e^{\sigma T} - (T - \mu) e^{\sigma \mu} \right\} \]

\[ - \frac{2(e^{\mu \sigma} - e^{\mu t_1})}{\sigma} + e^{\sigma t_1} \{ T - T \sigma t_1 - 2t_1 + \sigma t_1^2 \} \] \hspace{1cm} (13)

3.2. Model 2 \( t_1 \geq \mu \). In this case the selling rate \( D(t) \) is In this case the selling rate \( D(t) \) is

\[ D(t) = \begin{cases} 
D_0 t + \alpha I(t), & 0 \leq t \leq \mu, \\
D_0 \mu + \alpha I(t), & \mu \leq t \leq t_1, \\
D_0 \mu, & t_1 \leq t \leq T.
\end{cases} \]

Hence, (1) reduce to the following equations

\[ \frac{dI(t)}{dt} + [\theta - m(\tau)] I(t) = -[D_0 t + \alpha I(t)], \quad 0 \leq t \leq \mu \] \hspace{1cm} (14)

\[ \frac{dI(t)}{dt} + [\theta - m(\tau)] I(t) = -[D_0 \mu + \alpha I(t)], \quad \mu \leq t \leq t_1 \] \hspace{1cm} (15)
Solving (14) and (15) with the boundary conditions $I(0) = I_{\text{max}}$ and $I(t_1) = 0$, one can find the inventory level $I(t)$ as

$$I(t) = I_{\text{max}} e^{-\eta t} - \frac{D_0}{\eta^2} (e^{-\eta t} + \eta t - 1), \quad 0 \leq t \leq \mu$$

(16)

$$I(t) = \frac{D_0 \mu}{\eta} (e^{\eta(t_1-t)} - 1), \quad \mu \leq t \leq t_1$$

(17)

Considering the continuity of $I(t)$ at $t = \mu$, the maximum inventory level $I_{\text{max}}$ from (16) and (17) is

$$I_{\text{max}} = \frac{D_0 \mu}{\eta} e^{\eta t_1} - \frac{D_0}{\eta^2} (e^{\eta \mu} - 1)$$

(18)

Putting the value of $I_{\text{max}}$ into (16), the inventory level $I(t)$ becomes

$$I(t) = \frac{D_0 \mu}{\eta} e^{\eta(t_1-t)} - \frac{D_0}{\eta^2} \left( e^{\eta \mu} + \eta t_1 - 1 \right)$$

(19)

Again (2) becomes

$$\frac{dI(t)}{dt} = -D_0 \mu \delta(T-t), \quad t_1 \leq t \leq T$$

(20)

Solving (20) with the boundary condition $I(t_1) = 0$, the inventory level $I(t)$ can be obtained as

$$I(t) = \frac{D_0 \mu}{\sigma \epsilon T} (e^{\sigma t_1} - e^{\sigma T}), \quad t_1 \leq t \leq T$$

(21)

Putting $t = T$ in (21), the maximum amount of demand backlogged per cycle is

$$S \equiv -I_3(T) = \frac{D_0 \mu}{\sigma \epsilon T} (e^{\sigma T} - e^{\sigma t_1})$$

Now the order quantity $Q$ is

$$Q = I_{\text{max}} + S = D_0 \mu \left( \frac{e^{\eta t_1}}{\eta} - \frac{e^{\eta \mu}}{\mu \eta^2} + \frac{e^{\sigma T} - e^{\sigma t_1}}{\sigma e T} \right)$$

Next, the total profit per cycle consists of the following values

(a) Ordering cost per cycle = $A$

(b) Purchase cost per cycle = $pQ$

$$= D_0 \mu p \left( \frac{e^{\eta t_1}}{\eta} - \frac{e^{\eta \mu}}{\mu \eta^2} + \frac{e^{\sigma T} - e^{\sigma t_1}}{\sigma e T} \right)$$

(c) Holding Cost per cycle

$$h \int_0^{t_1} I(t) = D_0 \mu h \left[ \frac{e^{\eta \mu}}{\mu \eta^3} (e^{-\eta \mu} + \eta \mu - 1) + \frac{1}{\eta^2} (e^{\eta t_1} - e^{\eta \mu} - \eta (t_1 - \mu)) \right] - \frac{\mu}{2 \eta}$$

(d) Backlogging cost per cycle

$$b \int_{t_1}^T -I(t) e^{-\eta t} dt = \frac{D_0 \mu b}{\alpha^2} \left[ 1 - e^{\sigma (T-t)} [1 - \sigma (t_1 - T)] \right]$$

(e) Lost sell cost per cycle

$$l \int_{t_1}^T D_0 \mu [1 - \delta(T-t)] dt = \frac{D_0 \mu l}{\sigma} \left[ e^{\sigma (t_1 - T)} - \sigma (t_1 - T) - 1 \right]$$
(f) Sale revenue per cycle

\[
= s \left[ \int_0^{t_1} D(t) dt + \int_{t_1}^T D(t) \delta(T - t) dt \right]
\]

\[
= D_0 \mu s \left\{ \alpha \left( \frac{e^{\eta t_1}}{\mu \eta} (e^{-\eta t_1} + \eta - 1) + \frac{1}{\eta^2} (e^{\eta t_1} - e^{\eta t_1} - \eta(t_1 - \mu)) - \frac{\mu}{2\eta} \right) 
\right. \\
+ \left. t_1 - \frac{\mu}{2} + \frac{1}{\sigma} \left( 1 - e^{\sigma(t_1 - T)} \right) \right\}
\]

(g) Preservation technology cost = \tau T.

Therefore, total profit per unit time under the condition \( t_1 \geq \mu \) is

\[
= \Pi_2(t_1, \tau)
\]

\[
= \frac{1}{T} \left[ \text{sales revenue-ordering cost-holding cost-backlogging cost-lost sale cost} \\
- \text{purchase cost-preservation technology cost} \right]
\]

\[
= \frac{D_0 \mu}{T} \left[ s \left( t_1 - \frac{\mu}{2} \right) + l(t_1 - T) + \frac{s + l - p}{\sigma} \left( 1 - e^{\sigma(t_1 - T)} \right) + (\alpha s - h) \right.
\]

\[
\left. - \frac{e^{\eta t_1}}{\mu \eta^3} (e^{-\eta t_1} + \eta - 1) + \frac{1}{\eta^2} (e^{\eta t_1} - e^{\eta t_1} - \eta(t_1 - \mu)) - \frac{\mu}{2\eta} \right]
\]

\[
- \frac{b}{\sigma^2} \left( 1 - e^{\sigma(t_1 - T)} \right) \left( 1 - \sigma(t_1 - T) \right) - \frac{1}{\sigma^2} \left( e^{\eta t_1} - e^{\eta t_1} - 1 \right)
\]

\[
- \frac{D_0}{T} \left( A + \tau T \right)
\]

(22)

The total profit function of the system over \([0, T]\) takes the form

\[
\Pi(t_1, \tau) = \left\{ \begin{array}{ll}
\Pi_1(t_1, \tau) & \text{if } t_1 \leq \mu \\
\Pi_2(t_1, \tau) & \text{if } t_1 \geq \mu
\end{array} \right.
\]

(23)

It is easy to check that this function is continuous at \( \mu \).

3.3. Solution procedure. In this section for any given \( \tau \), the results which ensure the existence and uniqueness of the optimal solution that maximized the total profit is developed.

Taking first order derivative of (13) for \( t_1 \leq \mu \), one has

\[
\frac{d\Pi_1(t_1|\tau)}{dt_1} = \frac{t_1 D_0}{T} F(t_1)
\]

(24)

where

\[
F(t_1) = s + l - b(t_1 - T)e^{\sigma(t_1 - T)} + \frac{1}{\eta} (e^{\eta t_1} - 1)(s \alpha - h) \\
- (s + l - p)e^{\sigma(t_1 - T)} - pe^{\eta t_1}
\]

(25)

On the other hand we have

\[
F(0) = (s + l - p)(1 - e^{-\sigma T}) + be^{-\sigma T} > 0
\]
and 

\[ F(T) = \frac{(e^{\eta T} - 1)}{\eta} [s\alpha - h - \eta p] < 0 \]

Again it can be noticed that 

\[ F'(t_1) = \alpha s e^{\eta t_1} - e^{\eta t_1} [h + \eta p] - e^{(t_1 - T)\sigma} [b + \sigma (t_1 - T)b] - \sigma e^{(t_1 - T)\sigma} [s + l - p] < 0 \] (26)

which implies that \( F(t_1) \) is a strictly monotone decreasing function of \( t_1 \). Therefore, the equation 

\[ F(t_1) = 0 \] (27)

has a unique root \( t_1^* \in (0, T) \) for which 

\[ \frac{d^2 \Pi_1(t_1 | \tau)}{dt_1^2} \bigg|_{t_1 = t_1^*} = \frac{D_0 t_1^*}{T} F'(t_1^*) < 0 \] (28)

Next, we study the conditions under which the optimal preservation technology cost \( \tau \) not only exists but is unique. For any given feasible \( t_1^* \), taking first and second order derivative of (13) with respect to \( \tau \), we obtain 

\[ \frac{d \Pi_1(\tau|t_1^*)}{d\tau} = \frac{D_0}{T} \left[ (\alpha s - h)m'(\tau) \left\{ \frac{3}{\eta^4} \left( 1 - e^{\eta t_1^*} + \eta t_1^* e^{\eta t_1^*} \right) - \frac{t_1^*}{2\eta^2} \right\} \right] 
   - pm'(\tau) \frac{\eta^3}{2} \left\{ 2(1 - e^{\eta t_1^*} + \eta t_1^* e^{\eta t_1^*}) - \eta^2 t_1^* e^{\eta t_1^*} \right\} \] (29)

and 

\[ \frac{d^2 \Pi_1(\tau|t_1^*)}{d\tau^2} = \frac{D_0}{T} \left[ \frac{1 - e^{\eta t_1^*} + \eta t_1^* e^{\eta t_1^*}}{2\eta^4} \left\{ \frac{24(\alpha s - h) - 12\eta p|m'(\tau)|^2}{\eta^4} \right\} \right] 
   + \frac{6\eta (\alpha s - h) - 4\eta^2 p|m''(\tau)|}{\eta^3} \cdot \frac{t_1^*}{2\eta^2} \left( 1 + 2e^{\eta t_1^*} - \eta t_1^* e^{\eta t_1^*} \right) 
   - \frac{m'(\tau)}{\eta^2} \left( \frac{1 + 2e^{\eta t_1^*} - \eta t_1^* e^{\eta t_1^*}}{\eta^2} \right) \right] \] (30)

Due to high complication in (30), the straightforward proof does not exist. If 

\[ \frac{d^2 \Pi_1(\tau|t_1^*)}{d\tau^2} < 0 \]

then \( \Pi_1(t_1^*, \tau) \) is a concave function of \( \tau \) for a given \( t_1^* \), hence that value of \( \tau \) obtained from (29) is unique.

Thus we can conclude that for any given feasible \( t_1 \) there exists a unique \( \tau^* \) such that \( \Pi_1(\tau|t_1^*) \) is maximum. Because \( \tau \) is bounded over \([0, w]\), the above derivation also indicates that the optimal \( \tau^* \) should be selected to satisfy 

\[ \frac{d \Pi_1(\tau|t_1)}{d\tau} = 0, \text{ otherwise } \tau^* = \left\{ \begin{array}{ll}
0 & \text{if } \frac{d \Pi_1(\tau|t_1)}{d\tau} \bigg|_{\tau=0} < 0 \\
& \text{if } \frac{d \Pi_1(\tau|t_1)}{d\tau} \bigg|_{\tau=w} > 0 
\end{array} \right. \]
For the second model taking the first and second order derivative of (22) with respect to \( t \), one has
\[
\frac{d\Pi_2(t_1|\tau)}{dt_1} = \frac{\mu D_0}{T} F(t_1) \tag{31}
\]
and
\[
\frac{d^2\Pi_2(t_1|\tau)}{dt_1^2} = \frac{\mu D_0}{T} F'(t_1) < 0 \tag{32}
\]
where \( F(t_1) \) is given by (25). Now the inequality in (32) follows from (28) implies that \( \Pi_2(t_1|\tau) \) is strictly concave function. Hence \( F(t_1) = 0 \) has a unique root \( t_1^* \in (0, T) \).

Next, we study the conditions under which the optimal preservation technology cost \( \tau \) not only exists but is unique. For any given feasible \( t_1^* \), taking first and second order derivative of (22) with respect to \( \tau \) we obtain
\[
\frac{d\Pi_2(\tau|t_1^*)}{d\tau} = \frac{D_0 \mu}{T} \left[ (s \alpha - h) m'(|\tau|) \left\{ 3 + e^{\mu\eta}(\mu\eta - 3) + 2\mu^2e^{\eta t_1^*} - \mu t_1^*\eta^2(1 + e^{\eta t_1^*}) \right\} + \frac{\eta^2\mu^2}{2} \right] + \frac{p m'(\tau)}{\mu^2} \left\{ 2(e^{\mu\eta} - 1) - \mu\eta(e^{\eta t_1^*} + e^{\eta m}) + \mu t_1^*\eta^2e^{\eta t_1^*} \right\} \tag{33}
\]
and
\[
\frac{d^2\Pi_2(\tau|t_1^*)}{d\tau^2} = \frac{D_0 \mu}{T} \left[ \left( \frac{12}{\eta^2} + 6(e^{\mu\eta} + e^{\eta t_1^*}) + \mu(1 - e^{\mu\eta}) \right) - 2\eta t_1^*(1 + 2e^{\eta t_1^*}) + \eta^2 t_1^* e^{2\eta t_1^*} \right] \left[ m'(\tau) \right]^2 - \left( e^{\eta t_1^*}(\eta t_1^* - 2) - e^{\eta m} \right) + \frac{\eta t_1^* - \eta m - 3(1 - e^{\eta m})}{\eta^2} e^{\eta m'}(\tau) \left\{ m''(\tau) \right\} \right] - \frac{p m'(\tau)}{\mu^2} \left\{ 2(1 - e^{\mu\eta}) + \mu\eta(e^{\eta t_1^*} + e^{\eta m}) - \mu\eta^2 t_1^* e^{\eta t_1^*} \right\} \tag{34}
\]
Due to high complication in (34), the straightforward proof does not exist. If \( \frac{d^2\Pi_2(\tau|t_1^*)}{d\tau^2} < 0 \), then \( \Pi_2(\tau|t_1^*, \tau) \) is a concave function of \( \tau \) for a given \( t_1^* \), hence that value of \( \tau \) obtain from (33) is unique.

Thus we can conclude that for any given feasible \( t_1 \) there exists a unique \( \tau \) such that \( \Pi_2(\tau|t_1) \) is maximum. Because \( \tau \) is bounded over \([0, w]\), the above derivation also indicates that the optimal \( \tau^* \) should be selected to satisfy
\[
\frac{d\Pi_2(\tau|t_1)}{d\tau} = 0, \text{ otherwise } \tau^* = \begin{cases} 0 & \text{if } \left. \frac{d\Pi_2(\tau|t_1)}{d\tau} \right|_{\tau=0} < 0 \\ w & \text{if } \left. \frac{d\Pi_2(\tau|t_1)}{d\tau} \right|_{\tau=w} > 0 \end{cases}
\]

4. Numerical experiments. This section develops some numerical examples to check the existence and uniqueness of our solution. Some input parametric values are considered to obtain the optimum results.
4.1. **Example 1.** Let us consider the following parametric values:

\[ \begin{align*}
D_0 &= 1000, \\
A &= \$50/\text{order}, \\
s &= \$35/\text{unit}, \\
h &= \$3/\text{unit/week}, \\
p &= \$15/\text{unit}, \\
b &= \$5/\text{unit}, \\
l &= \$8/\text{unit}, \\
w &= \$200, \\
\mu &= 0.9 \text{ week,} \\
\alpha &= 0.1, \\
\gamma &= 0.01, \\
\sigma &= 0.2, \\
\theta &= 0.2, \\
T &= 1 \text{ week,} \\
m(t) &= \theta(1 - e^{-\gamma t}), \gamma \geq 0 \text{ (See for instance Dye and Hsieh [14]).}
\end{align*} \]

The optimal solution is \( t_1 = 0.87(\mu < \mu) \text{ week}, \tau = \$197.20/\text{week} \) and the corresponding total profit is \( \Pi_1(t_1, \tau) = \$9249.33/\text{unit} \) (See Figure 3).

4.2. **Example 2.** All the parametric values are identical to Example 1 except \( \mu = 0.5 \). The optimal solutions are \( t_1 = 0.66(> \mu) \text{ week}, \tau = \$105.89/\text{week} \), and the corresponding total profit per unit time is \( \Pi_2(t_1, \tau) = \$1182.56/\text{unit} \) (See Figure 4).

4.3. **Sensitivity analysis.** This section studied the effects of changes in parameters such as \( A, s, p, h, b, \) and \( l \) on optimal total profit. The sensitivity analysis is performed by changing each of the parameters by \(-50\%, -25\%, +25\%, \) and \( +50\% \) taking one parameter at a time while keeping the remaining parameters unchanged. The results of Example 1 and Example 2 are presented in Table 2.

| Parameters | Changes (in %) | Model 1 | Model 2 |
|-----------|----------------|---------|---------|
| \( D_0 \) | \(-50\% \) | +00.27  | +00.76  |
| \( A \)   | \(-25\% \) | +00.14  | +00.38  |
|           | \(+25\% \) | -00.14  | -00.38  |
|           | \(+50\% \) | -00.27  | -00.76  |
| \( s \)   | \(-50\% \) | -95.65  | -97.17  |
|           | \(-25\% \) | -48.39  | -49.19  |
|           | \(+25\% \) | +49.34  | +49.97  |
|           | \(+50\% \) | +99.48  | +100.52 |
| \( p \)   | \(-50\% \) | -42.97  | -44.83  |
|           | \(-25\% \) | -21.34  | -22.18  |
|           | \(+25\% \) | -21.10  | -21.84  |
|           | \(+50\% \) | -41.96  | -43.41  |
| \( h \)   | \(-50\% \) | +04.81  | +03.57  |
|           | \(-25\% \) | +02.12  | +01.63  |
|           | \(+25\% \) | -01.69  | -01.37  |
|           | \(+50\% \) | -03.04  | -02.53  |
| \( b \)   | \(-50\% \) | +00.22  | +00.42  |
|           | \(-25\% \) | +00.11  | +00.18  |
|           | \(+25\% \) | -00.08  | -00.14  |
|           | \(+50\% \) | -00.15  | -00.25  |
| \( l \)   | \(-50\% \) | +00.06  | +00.11  |
|           | \(-25\% \) | +00.03  | +00.05  |
|           | \(+25\% \) | -00.03  | -00.05  |
|           | \(+50\% \) | -00.06  | -00.09  |

On the basis of the sensitivity analysis of different parameters, the following features are observed:

* Increasing value of ordering cost increases the material cost, shipping cost, placing order’s cost; as a result the total profit decreases. From Table 2, it is observed that this parameter is slightly sensitive cost parameter for both
models. For the positive change and negative change in total cost for this parameter, it follows symmetrical change.

* Increasing value of selling price increases the total profit. From Table 2, one can observe that this parameter is highly sensitive on total profit for both models.

**Figure 3:** Graphical presentation of total profit function versus time and preservation technology cost (Example 1)

**Figure 4:** Graphical presentation of total profit function versus time and preservation technology cost (Example 2)
From Table 2, it is noticed that slight change in purchasing cost results larger change in total profit for both models. Increasing value of holding cost, backorder cost, and lost sell cost decrease the total profit. For both models, these two parameters are slightly sensitive on total profit.

5. Conclusions. The effect of deterioration cannot be ignored in inventory system because there are sufficient amounts of goods deteriorate during the normal storage period. Increasing deterioration rate decreases the total profit. Thus it is very essential to control the deterioration of goods. Deterioration rate can be control by adding some extra effective capital investment in preservation technology. In this direction the proposed study considered the concept of preservation technology investment to reduce the deterioration rate. It is a very common phenomenon that any customer would like to choose a single type of item from a large amount of stock. Thus to maintain higher sells, retailers have to store huge amount of stocks. In this regard, this model considered stock-dependent consumption rate. In this marketing environment, when a new brand of consumer goods are launched, the demand of goods increases quickly to a certain moment and after sometime it stabilizes. Finally, it becomes almost constant. Keeping in mind this type of demand pattern, ramp-type function was considered. Hence, this model was described with the combination of stock-dependent consumption rate, ramp-type demand, and a time-varying backorder rate with preservation technology investment. A useful solution procedure was given to obtain the optimal replenishment policy and preservation technology investment. This model used the concept of preservation technology for deteriorating items to reduce the deterioration. Therefore, the model is very practical for the retailers who would like to use preservation technology in their warehouses to control the deterioration rate under other assumptions of this model. The industry manager can decide, based on cost, that they would use the preservation of products or not. This model can further be extended by taking more realistic assumptions such as finite replenishment rate, stochastic demand or demand with uncertainty, linear increasing demand, price, and advertising-dependent demand or power-demand. The can model can be extended with multi-item with budget and space constraints. Then the control theory can be used for that model’s solution procedure. There are several extensions of this work that could constitute future research related in this field. One another extension can be considered in the direction of green supply chain model.

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