ESSENTIAL ISSUES ON SOLVING OPTIMAL POWER FLOW PROBLEMS USING SOFT-COMPUTING

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Abstract. Optimal power flow (OPF) problems are important optimization problems in power systems which aim to minimize the operation cost of generators so that the load demand can be met and the loadings are within the feasible operating regions of the generators. This brief paper emphasizes two essential issues related to solving the OPF problems and which are rarely addressed in recent research into power systems: 1) the necessity to validate operational constraints on OPF, which determine the feasibility of power systems designed for the OPF problems; and 2) the necessity to develop conventional methods for solving OPF problems which can be more effective than the commonly-used heuristic methods.

1. Introduction. For the past few decades, modern control centers of power systems have been equipped with computational tools to perform complex and extensive off-line studies in order to provide electrical power with minimum costs and minimum power interruptions, since the amount of power supply is more demanding and more stable supplies are required in the competitive environment \cite{23}. It is necessary to maintain the power systems operating at a minimum cost, and to ensure a satisfactory power supply to all users. This can be transformed into the

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commonly-known optimal power flow (OPF) problem which aims to determine optimal control variables for an efficient and robust power system [2, 3, 7, 10, 24]. The OPF problems generally consist of a cost function and a set of constraint functions. The cost function aims to achieve an optimal outcome for a specific objective such as fuel cost or network loss, by setting the system control variables. The constraint functions are intended to ensure that the generated power supply is adequate for all users and to compensate for the power losses due to the transmissions, while satisfying all constraints imposed by operational and physical limitations of the power system. However, research has mostly focused on the minimization of the cost functions, but very little attention has been paid to satisfying the constraint functions which are introduced to ensure the generated power satisfactorily meets the user demands and the power losses [1, 2, 3, 5, 7, 10, 11, 12, 21, 24]. This can lead to an undesirable situation, where a small generation cost can be achieved but unsustainable power is produced. Although this is an essential issue to consider when solving the OPF problems, it is rarely addressed. Also, the OPF problems are generally non-convex due to the presence of the valve-point loading effects in generators and the involvement of the flexible alternating current transmission systems [24]. This is why the heuristic algorithms such as evolutionary algorithms have commonly been used to solve the OPF problems [1, 3, 5, 12, 21, 24], since such approaches can be easily applied when solving difficult optimization problems [9]. More recent research also showed that particle swarm optimization (PSO) [4] is a more robust and efficient method than genetic algorithms when solving the OPF problems [2, 7, 10, 11]. They use heuristic operators in order to obtain the global optimal solution, since conventional gradient-based methods can find only a local optimal solution [24]. However, heuristic methods are also local search methods with no guarantee that the solution obtained will be either a global optimal solution or even a local optimal solution [17, 19]. Hence, they might not be the most suitable approaches for solving the OPF problems. Conventional gradient-based methods could perform more effectively and efficiently in solving OPF problems. However, recent research on solving the OPF problems tends to focus on the development of advanced heuristic methods, and research on the development of local search methods is rarely conducted. The development of local search methods is another essential issue when solving the OPF problems. This paper aims to discuss the two essential issues that were rarely addressed in the recent research on solving the OPF problems. Section 2 provides an overview of OPF problems and one particular OPF problem, namely the economic dispatch (ED) problem, is introduced. Then, an effective local search method namely Sequential Quadratic Programming (SQP) with active set strategy [14] is applied to solve the ED problem. Section 3 presents and compares the results obtained by the applied local search method and the heuristic method. A conclusion is given in Section 4.

2. Overview of optimal power flow problems. The OPF problem aims to optimize the performance of the steady state power system with respect to a cost function, which is the total generation cost for active and reactive power dispatch. It could represent the total generation cost or the total network loss.

Generally, an OPF problem can be formulated as [8]:

\[ \text{minimize } \text{cost function} \]

subject to

\[ \text{constraint functions} \]

\[ \text{system control variables} \]
\[
\begin{align*}
\text{min } & f(x) \\
\text{subject to } & g_i(x) = 0, i = 1, \ldots, N_E \\
& h_i(x) \leq 0, i = 1, \ldots, N_I
\end{align*}
\] (1a)

where \( x = [x_1, \ldots, x_n]^{\top} \) is the OPF decision vector with its components being those variables such as active/reactive power of swing buses, voltage angle/magnitude of swing and load buses, tap position of LTCs. In (1a), \( f \) is the cost function which is continuously differentiable with respect to all its arguments. \( g_i, i = 1, \ldots, N_E \) and \( h_i, i = 1, \ldots, N_I \) are continuously differentiable with respect to all their arguments. The equality constraints (1b) are the nodal power constraints which are the operational constraints on the specified power flow conditions, such as the requirements on the load demands and system losses. The inequality constraints (1c) are the bounds of the decision variables \( x = [x_1, \ldots, x_n]^{\top} \).

As presented in [2, 3, 5, 7, 10, 12, 21, 24, 25, 26, 7], the penalty function method is used to approximate Problem (1) as formulated by the following optimization problem:

\[
\begin{align*}
\text{min } & J_\lambda(x) = f(x) + \alpha \sum_{i=1}^{N_E} g_i(x)^2 \\
\text{subject to } & h_i(x) \leq 0, i = 1, \ldots, N_I
\end{align*}
\] (2a)

where \( \alpha \) is a sufficiently large penalty parameter, and \( J_\alpha(x) \) is known as the augmented cost function. Problem (2) is, in general, non-convex due to the presence of the valve-point loading effects in generators and the involvement of the flexible alternating current transmission systems.

2.1. Economic dispatch problem. Consider a common OPF problem, or equivalently called an economic dispatch (ED) problem, in the form of Problem (1) [8], where

\[
\begin{align*}
x &= [P_{G1}, \ldots, P_{GN_G}, Q_{G1}, \ldots, Q_{GN_G}, V_1, \ldots, V_{N_B}, \delta_1, \ldots, \delta_{N_B}, T_k, k = 1, \ldots, N_T]^{\top}
\end{align*}
\] (3)

The decision variables \( P_{Gi}, i = 1, \ldots, N_G, Q_{Gi}, i = 1, \ldots, N_G, V_j, j = 1, \ldots, N_B, \delta_j, j = 1, \ldots, N_B \) and \( T_k, k = 1, \ldots, N_T \), represent the real and reactive power generations, bus voltage magnitudes and angles, and transformer tap-settings, where \( N_G, N_B \) and \( N_T \) are the number of generators, buses and transformers, respectively.

The inequality constraints (1c) are specified by

\[
\begin{align*}
P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}}, & \quad Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}}, i = 1, \ldots, N_G \\
V_j^{\text{min}} \leq V_j \leq V_j^{\text{max}}, & \quad \delta_j^{\text{min}} \leq \delta_j \leq \delta_j^{\text{max}}, j = 1, \ldots, N_B \\
T_k^{\text{min}} \leq T_k \leq T_k^{\text{max}}, & \quad k = 1, \ldots, N_T.
\end{align*}
\] (4)

They can be expressed as:

\[
\begin{align*}
-P_{Gi} \leq -P_{Gi}^{\text{min}}, & \quad P_{Gi} \leq P_{Gi}^{\text{max}}, -Q_{Gi} \leq -Q_{Gi}^{\text{min}}, & \quad Q_{Gi} \leq Q_{Gi}^{\text{max}}, i = 1, \ldots, N_G, \\
-V_j \leq V_j^{\text{min}}, & \quad V_j \leq V_j^{\text{max}}, -\delta_j \leq -\delta_j^{\text{min}}, & \quad \delta_j \leq \delta_j^{\text{max}}, j = 1, \ldots, N_B \\
-T_k \leq T_k^{\text{min}}, & \quad T_k \leq T_k^{\text{max}}, k = 1, \ldots, N_T.
\end{align*}
\] (5)
The cost function (1a) is given by
\[ f(x) = C_t = \sum_{i=1}^{N_G} \left\{ a_i + b_i P_{Gi} + c_i P_{Gi}^2 + e_i |\sin(f_i P_{Gi})| \right\} \] (6)

where \( C_t \) is the generation cost; \( a_i, b_i, c_i, e_i \) and \( f_i \) are the cost coefficients of the \( i \)th generator. The equality constraints (1b) are specified below by the following equality constraints which represent the power generations, power loads and power losses through transmission:

\[ 0 = P_{Gi} - P_{Di} - V_i \sum_{j=1}^{N_B} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \] (7a)
\[ 0 = Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{N_B} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \] (7b)

where \( i = 1, 2, \ldots, N_B; P_{Di} \) (or \( P_{Gi} \)) and \( Q_{Di} \) (or \( Q_{Gi} \)) are the real and reactive power loads (or generations) at the bus \( i \), respectively; \( Y_{ij} \) and \( \theta_{ij} \) are the admittance and angle between the bus \( i \) and the bus \( j \), which are varied with respect to \( T_k \).

For the corresponding version of Problem (2), \( f \) is specified by equation (6). \( g_i \), with \( i = 1, \ldots, N_E \), is specified by equation (7a) and \( h_i \), with \( i = 1, \ldots, N_I \), is specified by equation (7b), where \( N_E = 2N_G \) and \( N_I = 4N_G + 4N_B + 2N_T \).

The objective of the cost function (6) is to minimize the generation cost, \( C_t \), by optimizing the generated real powers, \( P_{G1}, \ldots, P_{GN_G} \). The equality constraint (7a) aims to ensure that all the generated real power, \( P_{G1}, \ldots, P_{GN_G} \), can adequately supply all the demands, \( P_{D1}, \ldots, P_{DN_T} \), and compensate for the transmission losses, while the equality constraint (7b) aims to ensure that the generated reactive powers are satisfactory. Therefore, it is necessary to find a feasible solution in order to satisfy both the equality constraints (7a) and (7b). Although some infeasible solutions may achieve small generation costs, only unsustainable power can be produced since the solution cannot satisfy the equality constraints (7a) and (7b) and is therefore infeasible.

3. Proposed local search algorithm. To solve the augmented cost function \( J_\lambda(x) \) in (2), the conventional gradient-based methods could be used. However, the augmented cost function \( J_\lambda(x) \) is non-differentiable due to the presence of the terms, \( e_i |\sin(f_i P_{Gi})|, i = 1, \ldots, N_G \) in (6). Therefore, it is necessary to transform the non-differentiable terms into a differentiable estimate. Based on the approach on page 185 of [20], the terms, \( |\sin(f_i P_{Gi})| \) with \( i = 1, \ldots, N_G \), can be approximated by \( L_{i,\rho}(\sin(f_i P_{Gi})) \), where

\[ L_{i,\rho}(y) = \begin{cases} |y|, & |y| > \rho \\ \left( y^2 + \rho^2 \right) / 2\rho, & |y| \leq \rho \end{cases} \] (8)

Using the approximation formulated in (8), the augmented cost function (2) can be transformed as the following corresponding approximate augmented cost function:

\[ J(x, \alpha, \beta, \rho) = f_\rho(x) + \alpha \sum_{i=1}^{N_E} g_i(x)^2 + \beta \sum_{j=1}^{N_I} h_j(x)^2 \] (9)
where

\[ f_\rho(x) = C_t + \sum_{i=1}^{N_G} a_i + b_i P_{Gi} + c_i P_{Gi}^2 + \sum_{i=1}^{N_G} e_i L_{i,\rho}(\sin(f_i P_{Gi})) \]

It is clear that \( J(x, \alpha, \beta, \rho) \) is differentiable and its gradient can be readily obtained. Thus, by making the penalty parameters \( \alpha \) and \( \beta \) sufficiently large, and the smoothing parameter \( \rho \) sufficiently small, this approximate augmented cost function \( J(x, \alpha, \beta, \rho) \) is minimized subject to the inequality constraints specified by (5) and the equality constraints specified by (7). Problem (9) can be solved by using the sequential quadratic programming algorithm with active set strategy (namely SQP algorithm), which has attracted the interest of many mathematicians and engineers when solving real-world problems involving nonlinear constrained optimization [14, 15, 16]. To perform the algorithm, Problem (9) is formulated as the following quadratic programming sub-problem, namely \( P_k \) by (10) as,

\[
\begin{align*}
\min \quad & \frac{1}{2} (d^k)^\top H_k d^k + \nabla f_\rho(x^k)^\top d^k \\
\text{subject to} \quad & \nabla h_j(x^k)^\top d^k + h_j(x^k) \leq 0, \quad j = 1, \ldots, N_I, \\
& \nabla g_i(x^k)^\top d^k + g_i(x^k) \leq 0, \quad j = 1, \ldots, N_E
\end{align*}
\]

where \( x^k \) is the \( k \)-th iteration of the decision variable represented by (3); \( \lambda^k = [\alpha, \beta] \) is the associated multipliers in (9); \( H_k \) is the positive-definite approximation of the Hessian of the Lagrangian function; and \( N_I \) and \( N_E \) are the numbers of inequality and equality constraints represented by (5) and (7) respectively.

In (10), \( d^k \) is the solution of Problem \( P_k \), which is solved based on the active set strategy algorithm (detailed on pages 427 to 434 in [18]). With \( d^k \), the new iterates, \( x^{k+1}, \lambda^{k+1} \) and \( H^{k+1} \), can be determined by

\[ x^{k+1} = x^k + \sigma_k d^k \]

\[ \lambda^{k+1} = \lambda^k + \sigma_k (\lambda^k - \lambda^k) \]

\[ H^{k+1} = H^k + \gamma_k (\gamma_k)^\top - \frac{H^k s^k (s^k)^\top H^k}{(s^k)^\top H^k s^k} \]

where

\[ s^k = x^{k+1} - x^k, \]

\[ \gamma_k = \nabla_x J(x^{k+1}, \lambda^{k+1}, \rho)^\top - \nabla_x J(x^k, \lambda^k, \rho)^\top \]

\( \sigma_k \in (0, 1] \) is the step-length parameter and \( \tilde{\sigma}^k \) is the corresponding multipliers. Hence, \( P_k \) can be constructed and be solved consequently. By repeating this process, the original constrained nonlinear optimization problem can be efficiently solved. The following SQP algorithm is proposed to solve \( P_k \):

**SQP algorithm**

**Step 1:**
Set \( k = 0 \); Choose a starting point and a positive definite matrix \( H^0 \) for the sub-problem \( P_0 \).

**Step 2:**
Use the active set strategy algorithm to obtain \( d^k \) by solving the sub-problem \( P_j \).
Step 3:
If $d^k = 0$, $x^k$ is the KKT point and go to Step 7

Step 4:
Update $x^{k+1}$ based on (11).

Step 5:
Update $\lambda^{k+1}$ and $H^{K+1}$ by (12) and (13) respectively.

Step 6:
$k = k + 1$. Go to Step 2.

Step 7:
Return $x^k$ as the optimal solution namely $x^{opt}$.

The optimization procedure is carried out by the optimization toolbox in MATLAB. The penalty parameter and in Problem (9) are set as $10^6$. When the optimal solution of Problem (9), $x^{opt}$, is obtained, we can substitute $x^{opt}$ into the original augmented cost function (6) to obtain the actual optimal cost (i.e. the generation cost). Also, we can substitute $x^{opt}$ to the equality constraints (7a) and (7b) to check whether or not $x^{opt}$ is a feasible solution (i.e. to check whether sustainable power can be generated).

4. Experimental results. The effectiveness of the proposed SQP algorithm was evaluated by solving the OPF problems which are involved in the design of small, medium and large scale power systems, namely WSCC 9 bus-system (with 3 generators), IEEE 30 bus-system (with 6 generators) and Poly-system (a power system with 36 generators). The numbers of decision variables in WSCC 9 bus-system, IEEE 30 bus-system and Poly-system are 26, 76 and 304 respectively. The results obtained by the SQP algorithm were compared with those obtained by an effective heuristic method, namely advanced PSO [2], which has been developed to solve OPF problems and non-convex parametrical problems.

The advanced PSO is intended to overcome the limitation of the standard PSO [6] which usually converges to a near-optimal solution. It is integrated with the genetic mutation in order to help to obtain the optimum. Similar to the standard PSO algorithm, the advanced PSO starts by randomly generating a swarm of particles, and moves the particle positions iteratively based on the operations of genetic mutation and swarm movement. [2] has demonstrated that better results can be obtained when solving the OPF problems and some non-convex parametrical problems, when comparing the standard PSO with the other evolutionary algorithms. Therefore, the advanced PSO is used in this research for comparison with the proposed SQP algorithm.

Both the proposed SQP algorithm and advanced PSO were developed using the Matlab R2011b, whereby the Matlab subroutine fmincon is used to determine the optimum of the power flow problems. The mechanism and parameters used for the advanced PSO are identical to those used in [2], and Problem (9) was used as the fitness function on the advanced PSO. For the numbers of computational evaluations, both the advanced PSO and the SQP algorithm used 10000 computational evaluations for solving the OPF on the two smaller scaled systems, WSCC 9 bus-system and the IEEE 30 bus-system. They both used 50000 computational evaluations for the larger scale system, China-system. Thirty runs were performed using both
methods for each power system; the initial swarm used in the advanced PSO and the initial starting point used in the SQP algorithm were generated randomly for each run.

Figures 2a, 2b and 2c illustrate the averaged results obtained by both the advanced PSO and the SQP algorithm among the 30 runs. Figure 1a illustrates the averaged results for the augmented cost $J_\lambda(x)$ formulated in Problem (2a). Figures 1b and 1c illustrate the generation cost $f_\rho(x)$ and the sum of equality constraint values $\sum_{i=1}^{N_g} g_i(x)^2$ formulated in Problem (9), respectively. Figure 1a shows that the averaged augmented costs obtained by the SQP algorithm are smaller than those obtained by the advanced PSO. Figure 1b shows that the SQP algorithm outperforms the advanced PSO for which smaller generation cost among all tested systems can be obtained except IEEE 30-bus system. Also, Figure 1c shows that the SQP algorithm outperforms the advanced PSO in handling the equality constraints for all the systems where the sum of the equality constraint values obtained by the SQP algorithm are smaller.

These numerical results show the effectiveness of the SQP algorithm, where better power systems requiring small generation costs are produced. Hence, the power systems which are optimized by the SQP algorithm have lower fuel costs. Also, the equality constraints can be met by the SQP algorithm, but not by the advanced PSO. Addressing the equality constraints is important since it ensures that the generated power can adequately supply all users and compensate the power loss due to the transmissions. This can lead to an undesirable situation if those equality constraints cannot be satisfied. However, the advanced PSO performed poorly when addressing this.

Furthermore, Figures 2a, 2b and 2c illustrate the variances obtained by both the advanced PSO and the SQP algorithm for the 30 runs. The three figures illustrate the variances for the augmented cost $J_\lambda(x)$ in Problem (2a), the generation cost $f_\rho(x)$ in Problem (9) and the sum of equality constraint values $\sum_{i=1}^{N_g} g_i(x)^2$ in Problem (9). They show that, in general, the variances obtained by the SQP algorithm are smaller than those obtained by the advanced PSO. Therefore, these results indicate that the SQP algorithm can produce more robust solution quality compared with the advanced PSO.

The t-test is then used to evaluate the significance of the two algorithms, Advanced PSO and SQP algorithm for the three systems in terms of the total cost, the generation cost and the constraints. Table 1 shows that all t-values between the two algorithms for all cases are higher than 2.15. Based on the normal distribution table, if the t-value is higher than 2.15, the significance is 98% confident. Therefore, SQP algorithm significantly outperforms Advanced PSO with 98% confidence in the three power systems for all cases.

|                  | WSCC9  | IEEE 30 | Poly   |
|------------------|--------|---------|--------|
| Total cost       | 4.2053e+006 | 16.452  | 2.0122e+007 |
| Generation cost  | 4.2457 | 7.0609  | 8.1759 |
| Constraints      | 4.1611 | 16.454  | 20.122 |

Table 1. t-values between the two algorithms, Advanced PSO and SQP algorithm
5. Conclusion. This paper addresses two essential issues on solving OPF problems: 1) It is necessary to validate the satisfaction of equality constraints, which
are seldom addressed in OPF research. Without validation, infeasible power systems are likely to be generated. Hence, the generated power might not be adequate to supply all users and compensate for the power loss due to the transmissions. 2)
The experimental results showed that the tested conventional method can produce better solution quality and more robust solutions to the OPF problems. Hence, apart from developing heuristic algorithms for solving the OPF problems, the development of conventional methods should not be overlooked.

REFERENCES

[1] A. G. Bakirtzis, P. N. Biskas, C. E. Zoumas and V. Petridis, Optimal power flow by enhanced genetic algorithm, *IEEE Transactions on Power Systems*, 17 (2002), 229–236.

[2] K. T. Chatuverdi, Manjaree Pandit and L. Srivastava, Self-organizing hierarchical particle swarm optimization for nonconvex economic dispatch, *IEEE Transactions on Power Systems*, 23 (2008), 1079–1087.

[3] C. L. Chiang, Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels, *IEEE Transactions on Power Systems*, 24 (2005), 1690–1699.

[4] M. Clerc and J. Kennedy, The particle swarm explosion, stability, and convergence in a multidimensional complex space, *IEEE Transactions on Evolutionary Computations*, 6 (2002), 58–73.

[5] D. Devaraj and B. Yegnanarayana, Genetic-algorithm-based optimal power flow for security enhancement, *IEEE Proceedings: Generation, Transmission and Distribution*, 152 (2005), 899–905.

[6] R. Eberhart and J. Kennedy, A new optimizer using particle swarm theory, in *Proceedings 6th International Symposium on Micro Machine and Human Science*, IEEE Service Center, Nagoya, 1995, 39–43.

[7] A. A. A. Esmin, G. L. Torres and A. C. Zamhroni, A hybrid particle swarm optimization applied to loss power minimization, *IEEE Transactions on Power Systems*, 20 (2005), 859–866.

[8] L. K. Kirchmayer, *Economic Operation of Power Systems*, Wiley, New York, 1958.

[9] K. F. Man, K. S. Tang and S. Kwong, Genetic algorithm: concepts and applications, *IEEE Transactions on Industrial Electronics*, 43 (1996), 519–534.

[10] K. Meng, H. G. Wang, Z. Y. Dong and K. P. Wong, Quantum inspired particle swarm optimization for valve point economic load dispatch, *IEEE Transactions on Power Systems*, 25 (2010), 215–222.

[11] N. Mo, Z. Y. Zou, K. W. Chan and G. T. Y. Pong, Transient stability constrained optimal power flow using particle swarm optimization, *IET Proceedings Generation, Transmission and Distribution*, 1 (2007), 476–483.

[12] S. R. Paranjothi and K. Anburaja, Optimal power flow using refined genetic algorithm, *Electric Power Components and Systems*, 30 (2002), 1055–1063.

[13] J. Pilgrim, F. Li and R. K. Aggarwal, Genetic algorithms for optimal reactive power compensation on the national grid system, Proceedings of IEEE Power Engineering Society Transmission and Distribution Conference 2000, 524–529.

[14] M. J. D. Powell, A fast algorithm for nonlinearly constrained optimization calculations, *Numerical Analysis*, 630 (1977), 144–157.

[15] M. J. D. Powell, Algorithms for nonlinear constraints that use Lagrangian functions, *Mathematical Programming*, 14 (1978), 224–248.

[16] M. J. D. Powell, The convergence of variable metric methods for nonlinearly constrained optimization calculations, in *Proceedings of Nonlinear Programming* 3 (1978), 27–63.

[17] C. R. Reeves, Genetic algorithms and neighbourhood search, *Evolutionary Computing: AISB Workshop*, 1994, 115–130.

[18] W. Y. Sun and Y. X. Yuan, *Optimization Theory and Methods: Nonlinear Programming*, Springer, 2006.

[19] R. J. M. Vaessens, E. H. L. Aarts and J. K. Lenstra, A local search template, *Proceedings of parallel problem-solving from nature*, 2 (1992), 65–74.

[20] K. L. Teo, C. J. Goh and K. H. Wong, *A Unified Computational Approach to Optimal Control Problems*, New York: Longman Scientific & Technical, 1991.

[21] M. Todorovski and D. Rajicic, An initialization procedure in solving optimal power flow by genetic algorithm, *IEEE Transactions on Power Systems*, 21 (2006), 480–487.

[22] R. J. M. Vaessens, E. H. L. Aarts and J. K. Lenstra, A local search template, *Proceedings of Parallel Problem-Solving from Nature*, 2 (1992), 65–74.
[23] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, New York, John Wiley & Sons (2nd edition), 1996.
[24] J. Yuryevich and K. P. Wong, Evolutionary programming based optimal power flow algorithm, *IEEE Transactions on Power Systems*, 14 (1999), 1245–1250.
[25] C. J. Yu, K. L. Teo, L. S. Zhang and Y. Q. Bai, On a refinement of the convergence analysis for the new exact penalty function method for continuous inequality constrained optimization problem, *Journal of Industrial Management and Optimization*, 8 (2012), 485–491.
[26] C. J. Yu, K. L. Teo, L. S. Zhang and Y. Q. Bai, A new exact penalty function method for continuous inequality constrained optimization problems, *Journal of Industrial Management and Optimization*, 6 (2010), 895–910.

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