Do we finally understand Quantum Mechanics?

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The ontology emerging from quantum field theory and the results following from Bell’s theorems allowed the development of an intuitive picture of the microscopic world described by quantum mechanics, that is, we can say that we understand this theory. However there remain several aspects of it that are still mysterious and require more work on the foundations of quantum mechanics.

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I. INTRODUCTION

Fifty years ago R. Feynman said “nobody understands quantum mechanics”[1] characterizing the intellectual mood of that time. Due to his deserved authority and the undeniable intelligence of the big number of scientist that failed to develop a definite interpretation of quantum mechanics, the pessimist idea that we would never understand it was established. Fortunately this pessimism is perhaps unfounded and today we may have reached a level of understanding sufficient for the development of an intuitive picture of the physical systems described by quantum theory.

The essential developments that allowed this understanding are the consequences of several ideas related to Bell’s theorems and the emergence of an ontology consistent with quantum field theory. In this work we will present the main features of this progress towards a final understanding of quantum mechanics. Much progress has been done in the last decades but there are still remaining mysteries to be understood therefore we can answer the question in the title by a “yes but…”

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Although most of the material presented in this work is well known for experts in quantum mechanics these issues are not usually present in textbooks and lectures. This work is therefore a useful complement in the teaching of quantum theory because it brings intuitive insights and also presents some unsolved questions as possible research subjects in the foundations of quantum mechanics.

II. EINSTEIN DILEMMA

In a pioneering contribution that started an important field of research in the foundations of quantum mechanics[2], A. Einstein, B. Podolsky and N. Rosen raised the question of the completeness of quantum mechanics. An actualized reformulation of this issue can be posed as the Einstein dilemma: are the quantum mechanical distributions of the values that an observable can take in a particular state of a system of an ontological or a gnoseological nature? Let us analyse these choices. For any observable $A$ of a system, position, momentum, angular momentum, etc., quantum mechanics provides a distribution with an expectation value $\langle A \rangle$ and a width $\Delta_A$ that characterizes the uncertainty or indeterminacy of the observable in the given state. We can take two options concerning the nature of this distribution: gnoseological or ontological; that is, are the uncertainties or indeterminacies in our knowledge of the system or in the system itself?

We can think that each observable has some definite and exact value in the system, called the putative value[3], that quantum mechanics is unable to predict in general. In this case the distribution represents our ignorance of the reality of the system: it is a problem of our knowledge and therefore the distribution is of a gnoseological nature. Since certainty is an attribute of knowledge, we can call $\Delta_A$ the uncertainty of the observable in the given state. If this is so, quantum mechanics is not a complete theory and we immediately ask if there is a better one, generically called hidden variable theories, that may predict these exact values assigned to the observables.

In the opposite interpretation, we assume that the observable doesn’t have a unique and sharp value and, instead, it is unprecise or diffuse by nature: it is not a problem
of our knowledge but of the system itself with blurred observables. Correspondingly \( \Delta_A \) characterizes an *indeterminacy* of the observable in the given state.

### III. FOR WHOM THE BELL TOLLS

For the classically oriented intuition, the gnoseological option is less traumatic. In fact, this was the choice of Einstein, Podolsky and Rosen, and an intense research activity in hidden variables began. However severe difficulties appeared with them. The first was a “no go” theorem by von Neumann showing that such theories could not exist, but this result was disproved by a counterexample by Bell.

The next important result was a theorem by Bell and another by Kochen and Specker that proved that *the existence of non contextual putative values is in contradiction with the formalism of quantum mechanics*.

In the proof of these theorems it is assumed that the putative values do not depend on the context. This requires a detailed explanation: in the description of a physical system we choose a set of commuting observables in order to fix the state, that is, we choose a context. For instance, the position of a particle along one direction \( X \) and the momentum along an orthogonal direction \( P_y \) or \( P_z \) (they commute with \( X \)) or total angular momentum \( J^2 \) and its projection along one arbitrary direction \( J_z \), or the position of one particle and the momentum of another particle. Any observable can belong to different contexts and it is a very reasonable assumption to think that the putative value assigned to it does not depend on the context. After all, the context is decided by the theoretical physicist in his office and this should have no effect on the reality of the system, that is, the putative value are also required to be context independent.

After the appearance of the original proofs by Bell and Kochen-Specker several examples of the incompatibility of quantum mechanics with the existence of putative values were presented involving spin systems in different Hilbert space dimension and also concerning position and momentum observables. An elegant and simple proof of high didactic value, based on the geometrical structure of the Hilbert space, was also produced.
We just saw that the formalism of quantum mechanics forbids the existence of definite values for the observables. A much more important result is that the existence of non contextual putative values is in contradiction with empirical reality. This result follows from the experimental violation of Bell’s inequalities. Assuming the existence of definite numerical values, even if they are unknown, Bell derived an inequality concerning statistical averages of such values. Several equivalent versions of this inequality, that could be tested experimentally, were derived and finally the result of several experiments violated these predictions based on the existence of non contextual putative values.

All these results related with Bell’s contribution support then the ontological interpretation of quantum mechanic distributions, although the existence of context dependent putative values can not be excluded on logical ground, even though they are very unlikely. In the particular case of the position observable, if we accept that its distribution is ontological, then we must abandon the image of a point particle (except when the particle is in an eigenstate of the position operator, that is, with $\Delta x = 0$) and think of it as an extended object: the “particle” is more something like a “field” with all particle properties extended in physical space. An electron in a hydrogen atom is not a point-like particle located with some probability in a region called an “orbital” but is the orbital itself. However, in a spatial interaction or in a measurement the electron collapses (see later) and presents a “point-like” characteristic. Anyway, the localization or the “point-likeness” has a limit because one can show by a (dangerous) heuristic argument involving the uncertainty position-momentum relation that the location indeterminacy can not be smaller than the Compton length of the particle $\lambda = h/mc$.

The view of particles as extended objects is compatible with, and suggested by, quantum field theory, perhaps the most successful theory in physics.

IV. QUANTUM FIELD THEORY ONTOLOGY

Quantum field theory, conciliating quantum mechanics and special relativity, was originally developed as a quantum electrodynamical theory and was later generalized
to all interactions (except gravity) to achieve a successful description of the time evolution and interaction of all the particles in the Standard Model: quarks, leptons and intermediary bosons. There are many good books at the advanced undergraduate and graduate level for his theory [16]. However for the purpose of this work we don’t need all mathematical details, that sometimes blur the essential features of the theory, and a “minimal quantum field theory” [17] is sufficient. The particle “wave function” of quantum mechanics $\psi(x, t)$ becomes in quantum field theory the amplitude for the creation of such a particle at the space-time point $(x, t)$. A characteristic feature of this theory is the expansion of all amplitudes in terms of creation and annihilation operators of all possible particle properties.

These expansions allow, and suggest, an ontology where the quantum field of a particle system is build by the creation, propagation and annihilation of real entities —virtual particles— with ephemeral existence because they don’t satisfy the energy-momentum relations of permanent particles $E^2 - P^2 = m^2$ (they are off the “mass shell”). These virtual particles are not only off the mass shell but they also violate causality, because they can propagate in space-like trajectories outside the light cone. One of the beauties of quantum field theory is the restoration of global causality in the quantum field that requires the existence of antiparticles.

The terms of the expansions can be represented by Feynman diagrams; however, in this ontology, these are not only term in a perturbation expansion but represent really occurring processes. Even the vacuum becomes dynamical features with creation, propagation and annihilation of all possible virtual particles and there are empirical facts (Lamb shift and Casimir effect) supporting this dynamical vacuum. The physical vacuum is different from “nothing”. An argument, of historical interest, that can be related with the dynamic vacuum was produced by Johannes Kepler four centuries before its discovery. Kepler argued that in vacuum there is nothing to oppose the propagation of light and therefore its speed should be unlimited. The argument is correct, but the premise is false: in vacuum there is something—a sea of virtual particles—to oppose the propagation of light.

The *Quantum Field* of a particle system is then a physical entity extended and evolving in space-time according to specific equations of motion (Schrödinger, Dirac,
Klein-Gordon) made by an infinite set of virtual particles.

V. INDIVIDUALITY LOSS

One of the fundamental features of reality discovered by quantum mechanics is the \textit{individuality loss}. In our perception of macroscopic objects we take for granted that their individuality is conserved: if we look at a stone, close our eye for a second, and observe it again, we never doubt that we are dealing with \textit{the same} stone. This anthropocentric conviction can not be extrapolated to the microscopic world. Identical \textit{classical} systems have an individuality that is conserved through the time evolution and interaction with other systems. So classical systems, even when they are “identical”, can be assigned an individual identity that is conserved: they can have a name, an ID number, a licence plate. Quantum mechanics requires a drastic conceptual change: \textit{the individuality loss}. A set of five identical “classical” atoms is countable (five in total) and numerable (the atom number one, the number two, . . . ) but real atoms, necessarily described by quantum mechanics, are countable but not numerable: if we artificially assign a number to each atom, that is, if we assign an individuality, we must correct this error by considering also all possible permutations in the assignment. The individuality of a particle is entangled with the individuality of all other identical ones in the universe (although “for all practical purposes” a cluster decomposition isolating a particular system from the rest is possible to an extremely good approximation\cite{18}).

An interesting metaphoric tool for a didactic presentation of the identity entanglement in quantum mechanics is provided by some short stories by Julio Cortázar\cite{19} where the identity of some characters are swaped.

In the ontology suggested by quantum field theory the individuality loss is very natural because in this interpretation we are not dealing with one, or two, or many particles as individual entities. For instance, the quantum field for a one electron system, or for several electrons system, is made up by the creation, propagation and annihilation of virtual particles that are not assigned to any of the individual electrons of the system: in a two electron field there is no way to differentiate one
electron from the other because they are both simultaneously made by an active background of ephemeral virtual particles with a mean value of two for the particle number observable, but each virtual component of the field is not assigned to any one of the electrons.

VI. DECOHERENCE IN THE CLASSICAL LIMIT

Quantum mechanics is also applicable to macroscopic systems that do not exhibit indeterminacies and other astonishing features of microscopic quantum systems. Besides treating systems with ontological indeterminacies, quantum theory can also describe ensembles of systems with gnoseological uncertainties. The appropriate tool is the statistical operator $W$, also called the density matrix. Let us assume an observable $A$ with eigenvalues $\{a\}$ and eigenvectors $\varphi_a$. A state $\psi$ where the observable has an ontological indeterminacy is given by

$$ \psi = \sum_a f_a \varphi_a . \quad (1) $$

Now we can think on an ensemble of systems where each member of the set is in some state $\varphi_a$. If we have only a statistical knowledge on how often these states are realized, that is, we know the occupation probability $\lambda_a$ for each state, the state of the ensemble is described by the statistical operator

$$ W = \sum_a \lambda_a P_a , \quad (2) $$

where $P_a$ is a projector in the state $\varphi_a$. Notice that the Pure State (1) corresponds to an ontological indeterminacy of the observable whereas the Mixed State (2) implies a gnoseological uncertainty of the ensemble. (Mixed states are also required to describe the state of a subsystem of a system in a known pure state.)

Macroscopic systems are almost never found in a superposition state like (1) because if they are forced in such a state, in an extremely short time, estimated by $\frac{\hbar}{E}$, where $E$ is the total macroscopic energy of the system (a large value), there is a transition from the superposition state $\psi$ to the mixed state $W$. This transition is called decoherence and it explains why quantum effects are not observed in macroscopic systems.
This also solves some misunderstanding that appear when we ignore decoherence and transfer a microscopic state to a macroscopic system. The most famous example of this error is the Schrödinger cat argument: assume a quantum system in a superposition of two states, spin up or down, or atom decayed or not, or particle at right or left, etc. Assume also some amplifying mechanism that couples one of these two states with the release of poison that kills a cat. Clearly, to think that a real cat is in a superposition of live-death is absurd and this will never appear in an experiment. Due to decoherence, at all reasonable times in this cruel experiment the cat will be alive or dead and never alive and dead. The incorrect, but popular, result that the cat is in some live-death superposition is due to a misuse of quantum mechanics and perhaps this motivated Stephen Hawking to say “When I hear of Schrödinger’s cat, I reach for my gun” paraphrasing the opinion about “culture” attributed to several Nazi leaders but with origin in a play of Hanns Johst.

Quantum mechanics is also applicable to macroscopic systems and their state is the result of the decoherence of the superposition states: the ontological indeterminacies characteristic of quantum mechanics become gnoseological uncertainties in macroscopic systems.

VII. WHAT REMAINS MYSTERIOUS

The image that quantum field theory suggests for a particle in space-time is quite intuitive and is also compatible with many quantum features that where once considered counterintuitive. There remain however several features of quantum mechanics that resist an intuitive explanation. Some of them are the measurement problem, the position-momentum relation, the quantization of rotations, and several other. It is an open question whether unexpected future developments will produce an explanation of these features or they will just become familiar by getting used to them although no deep understanding may never appear. After all, “understanding” is a human mental state conditioned by our brain that has reached its state after a few million years of evolution but it might not necessarily be adequate for the microscopic world.
A. measurement

The essential difficulty in understanding the process of measurement is the mechanism by which the ontological indeterminacy of an observable in a physical system becomes a gnoseological uncertainty: the “collapse”. This collapse is acausal and indeterministic and we don’t know if this is a fact of nature, difficult to accept for our “classical” mind, that we must just accept or if we will sometimes be able to explain it.

Let us assume a quantum system in a state $\psi$ given in terms of the eigenvectors of an observable $A$ as in Eq.(1). Now, if we decide to measure $A$, we put the system in interaction with a measurement apparatus that is necessarily a macroscopic system. As it happens with Schrödinger’s cat, the hole system decoheres: the quantum system goes to one of the eigenstates $\varphi_a$ and the display of the apparatus shows the eigenvalue $a$. It is impossible to predict which one of the states will result (indeterminacy) and we can only give a probability for this, $\lambda_a = |f_a|^2$, neither can we give a hamiltonian that describes the time evolution during the collapse (acausality).

The essential difference between a measurement in a macroscopic classical system and a corresponding one in a quantum system is that in the classical case, the measurement informs us about a preexistent property of the system, whereas in a quantum system, there is no preexistent value for the observable and the measurement forces the system in one of the eigenstates of the observable.

B. space-time and energy-momentum relation

There are two fundamental perspectives in the consideration of physical reality: the space-time and the energy-momentum view, that is, kinematics and dynamics. Whereas space and time are immediately related to our sense perception and are therefore intuitive, energy-momentum require a definition and, in classical physics, are related to the concept of matter in movement. So we define $E = \frac{1}{2}mv^2$ and $p = mv$ and their relativistic extension $E^2 - P^2 = m^2$ and $P = \gamma mv$. These intuitive relations are retained and are compatible with the mathematically more abstract
formulations where momentum is given as the Legendre transformation in the transition from the lagrangian to the hamiltonian as well as the Poisson bracket relation and the view of energy-momentum as generators of space-time translations. The intuitive components are retained in the powerful abstract mathematical formalism of classical physics. In quantum mechanics position and momentum become somehow incompatible, they “don’t commute”. The intuitive view as matter in movement is lost, but the abstract commutation relations, or equivalent, the generators of the group of translation are retained. Energy-momentum are then related to space-time by the Fourier transformations and it would be important to recover an intuitive explanation for this mathematical formalism. The proof that Fourier transformation is compatible with a postulated position-momentum (being-becoming) symmetry principle\[24\] shreds some light to the problem but is not conclusive enough to make it intuitive.

C. quantization of energy and rotations

The quantization of energy and rotations are an outstanding feature of quantum mechanics that we have accepted by accustomation, we just got used to it, but remain counterintuitive and are not really understood. Of course, they are an unavoidable consequence of the mathematical formalism of the theory and have their root in the position-momentum commutation relations; however this does not give an intuitive understanding.

A long hope, inspired by general relativity, is to find an explanation of quantum mechanics based on a particular structure of space-time. This “pregeometry” should explain the quantization of rotations, and the resulting energy quantization, as self-consistent possibility.

D. state determination

Position and momentum are the unique observables of a spinless particle moving in space, all other observables are functions of them. Therefore it came as a surprise, indicative of some missing understanding of the reality of the system, that the
complete knowledge of the position distribution $|\psi(x)|^2$ and momentum distribution $|\phi(p)|^2$ (where $\phi$ and $\psi$ are related by Fourier transformation) is not sufficient in order to determine the state of the system. This question, initially raised by Pauli\cite{25}, triggered an intense investigation on the necessary and sufficient information needed for an unambiguous state determination, an unsolved problem in quantum mechanics\cite{26–31}. It has been conjectured, but unproved, that the correlation observable $C = XP + PX$ might provide the missing information for complete state determination\cite{32}.

E. compatibility with general relativity

The greatest cultural debt of theoretical physics is quantum gravity, a theory enclosing quantum mechanics and general relativity. There are several approaches to such a theory but they have not reached sufficient development for an established theory. Quantum mechanics and general relativity are incompatible at a very fundamental level and therefore the new theory will bring profound, perhaps revolutionary, concepts and ideas and certainly a different understanding of quantum mechanics. The incompatibility of quantum mechanics and general relativity arises from the fact that general relativity requires a precise, sharp, definition of energy-momentum at every precise space-time point, that is, the energy-momentum tensor that determines the space-time structure through Einstein’s equations. However the uncertainty principle of quantum mechanics forbids the simultaneous and precise definition of these quantities.

Quantum gravity is necessary for the description of physical systems in unusual extreme conditions where quantum fluctuations become self devouring black holes, like in the first $10^{-43}$s (Planck time) of the universe or the region $10^{-35}$m (Planck length) around the center of a black hole “the undiscovered country from hose bourns no traveller returns”\cite{33}. Although the applicability of such a theory is extremely small, the cultural gap is big and physics will not rest until quantum gravity is found.
VIII. CONCLUSION

Progress made in the second half of the XX century have clarified several aspects of quantum theory and we can today imagine intuitively the microscopic world. So we can think of a hydrogen atom as an extended field for the electron—the orbitals—with virtual photons binding it to the proton, that in its turn is a field made of three quark fields in a sea of gluons and other intermediary bosons. All this in a fascinating and beautiful swarm of virtual particles.

There are however many unsolved questions that make the research in the foundations in quantum mechanics a relevant scientific activity.

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