Seesaw and Lepton Flavour Violation in SUSY $SO(10)$

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Abstract

That $\mu \to e, \gamma$ and $\tau \to \mu, \gamma$ are sensitive probes of SUSY models with a see-saw mechanism is a well accepted fact. Here we propose a ‘top-down’ approach in a general SUSY $SO(10)$ scheme. In this framework, we show that at least one of the neutrino Yukawa couplings is as large as the top Yukawa coupling. This leads to a strong enhancement of these leptonic flavour changing decay rates. We examine two ‘extreme’ cases, where the lepton mixing angles in the neutrino Yukawa couplings are either small (CKM-like) or large (PMNS-like). In these two cases, we quantify the sensitivity of leptonic radiative decays to the SUSY mass spectrum. In the PMNS case, we find that the ongoing experiments at the B-factories can completely probe the spectrum up to gaugino masses of 500 GeV (any $\tan \beta$). Even in the case of CKM-like mixings, large regions of the parameter space will be probed in the near future, making these two processes leading candidates for indirect SUSY searches.

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I. INTRODUCTION

It is known that Flavour Changing Neutral Current (FCNC) processes play an important role in the search for indirect signals of new physics \[1\]. On the other end the accumulating concordance between the Standard Model (SM) expectations and the vast range of experimental results in FCNC and $CP$ violation point towards a low energy new physics which is flavour blind. For instance considering Supersymmetric (SUSY) extensions of the SM all this phenomenological evidence could be seen as a consequence of the fact that the mechanism that breaks SUSY and conveys SUSY breaking to the observable sector is completely flavour blind. If this is indeed the case, it might be difficult to see any deviation at all from the SM predictions through FCNC \[2\]. However, we already know that the SM is not enough in that it has to be supplemented by a mechanism to provide non-zero neutrino masses and mixings. An appealing example of this is the seesaw mechanism \[3\] where we introduce neutrino Yukawa couplings $h_\nu$, and heavy right-handed neutrino Majorana masses $M_R$.

In spite of the possible flavour-blindness of SUSY breaking, the Supersymmetrization of the Seesaw leads to new SUSY sources of Lepton Flavour Violation (LFV) \[4\]. The point is that in the running of the slepton mass matrices down to the electroweak scale, contributions to these matrices proportional to $h_\nu^* h_\nu^\dagger$ give rise to a mismatch in the diagonalisation of the lepton-slepton mass matrices and hence the appearance of flavour changing gaugino-lepton-slepton vertices.

This effect was already noticed several years ago. What is new at this moment is the improvement in our knowledge of neutrino physics and prospects of better bounds from LFV physics. However, as shown in several recent studies \[5, 6, 7\], the low energy data on neutrino masses and mixings by themselves cannot predict the decay rates of $l_j \rightarrow l_i \gamma$ for a given set of SUSY breaking parameters. This ambiguity can be traced to the fact that the unknown parameters of $h_\nu$ and $M_R$ cannot be completely fixed even after knowing all the three masses and three mixing angles of the neutrino sector. Thus any “bottom-up” approach to the study of these lepton flavour violating processes suffers a large ambiguity which is encoded in an arbitrary orthogonal matrix $R$ \[5, 6, 7\] relating these low energy
neutrino parameters to the unknown Seesaw parameters $h^\nu$ and $M_R^1$. Several interesting works have studied this problem in detail for various neutrino spectra - hierarchical, inverse-hierarchical, degenerate - and under various assumptions on the mass eigenvalues of the right handed neutrino mass matrix. The bottom line of these analysis is that $l_j \to l_i \gamma$ and neutrino oscillation experiments provide complementary pieces of information to determine the seesaw parameter space.

In this paper, we take an alternative point of view namely, we use a ‘top-down’ approach introducing a high energy framework which eliminates the ambiguity on the Seesaw parameters. We consider a SUSY $SO(10)$ model where all fermions in a generation are included in a single representation. The crucial find of our analysis is that in a generic $SO(10)$ model at least one of the neutrino Yukawa couplings is of the same order as the large top Yukawa coupling. By generic we mean here that the result holds irrespective of the representations of the Higgs fields used to generate masses for the fermions. However, as expected the complete structure of $h^\nu$ remains unpredictable in such a general case. Essentially, the specification of the Grand Unified gauge group is not sufficient to uniquely determine the mixings appearing in the diagonalisation of $h^\nu h^\nu\dagger$ relevant for the analysis of $l_j \to l_i, \gamma$ decays.

Motivated by simple $SO(10)$ models, we propose that the mixings diagonalising the combination $h^\nu h^\nu\dagger$ would have magnitudes within the two ‘extreme’ limits - namely, the CKM angles of quark mixing and the PMNS angles of the leptonic mixing. We show semi-complete $SO(10)$ models where these two ‘extreme’ cases can be realised. We then demonstrate that phenomenologically viable seesaw mechanisms can be realised in both these cases by a suitable choice of the right handed neutrino Majorana mass matrix. We believe that these extreme cases would serve as “benchmark” scenarios for the seesaw induced lepton flavour violation within the context of SUSY $SO(10)$.

For both these “benchmark” scenarios, we have computed the $l_j \to l_i \gamma$ decay rates in a minimal supergravity (mSUGRA) or constrained MSSM (CMSSM) scenario, i.e., assuming completely universal soft SUSY breaking. We find that the present and future observational limits on $\mu \to e\gamma$ can significantly constrain large regions in the parameter space even in the above mentioned small mixing (CKM) case. In the more optimistic case of large mixing

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1 It is possible that mechanisms other than the seesaw can be more predictive for Lepton Flavour Violation. See, for an example, Ref. [8].
(PMNS) the decay rates can become very high. Indeed, if the new proposals to explore LFV, mainly the $\mu \rightarrow e\gamma$ decay at PSI [11], reach the planned sensitivity these LFV processes are going to be fully complementary with SUSY searches at LHC.

The rest of the paper is organised as follows. In the next section we show that in $SO(10)$ models at least one of the neutrino Yukawa couplings is forced to be as large as the top Yukawa. Semi-complete $SO(10)$ models are presented in section III for the minimal and maximal mixing scenarios. Results from numerical analysis and the constraints on SUSY parameter space are presented in section IV. Summarising remarks are presented in the last section.

II. $SO(10)$ AND NEUTRINO YUKAWA COUPLINGS

As mentioned in the introduction, the top-down approach makes it possible to predict the neutrino Yukawa couplings removing the ambiguity associated with the seesaw parameters relevant for lepton flavour violation. In first place, we must choose a gauge group for the Grand Unified Theory (GUT). $SO(10)$ is the natural choice as it is the minimal group that includes right handed neutrinos besides the rest of the SM fermions in a single representation. One would then have to consider a particular choice of the Higgs fields representations to generate fermion masses, which again introduces some sort of an ‘ambiguity’ at the high scale. To keep the discussion as general as possible, we do not resort to any particular model of fermion masses within $SO(10)$, but try to see how well one can predict $h^{\nu}$ in a generic scenario.

In the $SO(10)$ gauge theory, all the known fermions and the right handed neutrinos are unified in a single representation of the gauge group, the 16. To analyse the Yukawa matrices in this framework, we need to specify the superpotential. In principle, the superpotential can receive contributions both from renormalisable and non-renormalisable terms [12]. The product of two 16 matter representations can only couple to 10, 120 or 126 representations which can be formed either by a single Higgs field representation or a non-renormalisable product of representations of several Higgs fields. In either case, the Yukawa matrices resulting from the couplings to 10 and 126 are complex symmetric whereas they are anti-symmetric when the couplings are to the 120.

Therefore, the most general $SO(10)$ superpotential relevant for fermion masses can be
written as

$$W_{SO(10)} = h_{ij}^{10} 16_i 16_j 10 + h_{ij}^{126} 16_i 16_j 126 + h_{ij}^{120} 16_i 16_j 120,$$

where $i, j$ refer to the generation indices. In terms of the SM fields, the Yukawa couplings relevant for fermion masses are given by [13]:

\begin{align*}
16 &\ 16 \ 10 \ \supset \ 5 \ (uu^c + \nu \nu^c) + 5 \ (dd^c + ee^c), \\
16 &\ 16 \ 126 \ \supset \ 1 \ \nu^c \nu^c + 15 \ \nu \nu + 5 \ (uu^c - 3 \nu \nu^c) + 45 \ (dd^c - 3 \ nu \nu^c), \\
16 &\ 16 \ 120 \ \supset \ 5 \ \nu \nu^c + 45 \ uu^c + 5 \ (dd^c + ee^c) + 45 \ (dd^c - 3 \ ee^c),
\end{align*}

(2)

where we have specified the corresponding $SU(5)$ Higgs representations for each of the couplings and all the fermions are left handed fields. From above, it is clear that if only the $10$ and $126$ Higgs representations are present in the theory, the Yukawa matrices of the down quarks and charged leptons as well as the up quarks and neutrinos are deeply related. In fact, in a model where only the $10$-plets are present we would have exact quark–lepton Yukawa unification, not only among charged leptons and down quarks ($b$–$\tau$ unification) but also for up quarks and (Dirac) neutrinos. Similarly a dominant contribution from the $126$ representation would predict as well quark–lepton unification, although introducing the Georgi–Jarlskog factors of 3 [14]. If only one of these representations or any combination of them contributes, the Yukawa matrices would be exactly symmetric. These properties are broken by the introduction of the $120$ representation. The Yukawa couplings of this representation are anti-symmetric and break the quark–lepton unification because they can contribute independently to the quark and lepton Yukawa matrices. In general, both the symmetric and anti-symmetric contributions can be present leading to Yukawa matrices of generic nature.

The resulting mass matrices can be written as

\begin{align*}
M^u &= M_{10}^5 + M_{126}^5 + M_{120}^{45}, \\
M_{LR}^\nu &= M_{10}^5 - 3 M_{126}^5 + M_{120}^5, \\
M^d &= M_{10}^5 + M_{126}^{45} + M_{120}^5 + M_{120}^{45}, \\
M^e &= M_{10}^5 - 3 M_{126}^{45} + M_{120}^5 - 3 M_{120}^{45}, \\
M_{LL}^\nu &= M_{126}^{15}, \\
M_R^\nu &= M_{126}^1.
\end{align*}

(3)

(4)

(5)

(6)

(7)

(8)
We come now to discuss the main result of this section: *At least one of the Yukawa couplings in $h^\nu = v_u^{-1} M^\nu_{LR}$ has to be as large as the top Yukawa coupling.* This result holds true in general independently from the choice of the Higgses responsible for the masses in Eqs. (3, 4) provided that no accidental fine tuned cancellations of the different contributions in Eq. (4) are present. If contributions from the $10$'s solely dominate, $h^\nu$ and $h^u$ would be equal. If this occurs for the $126$'s, then $h^\nu = -3 \ h^u$. In case both of them have dominant entries, barring a rather precisely fine tuned cancellation between $M^5_{10}$ and $M^5_{126}$ in Eq. (4), we expect at least one large entry to be present in $h^\nu$. A dominant antisymmetric contribution to top quark mass due to the $120$ Higgs is phenomenologically excluded since it would lead to at least a pair of heavy degenerate up quarks. However, the $120$ can still provide a non negligible contribution to the up quark masses and in particular, to that of the top quark, if there is at the same time a large symmetric contribution. In this case the up quark and neutrino Yukawa matrices have both large symmetric and antisymmetric contributions. The above stated result holds also in this general situation (for a complete proof, see the Appendix).

Apart from sharing the property that at least one eigenvalue of both $M^u$ and $M^\nu_{LR}$ has to be large, for the rest it is clear from (3) and (4) that these two matrices are not aligned in general, and hence we may expect different mixing angles appearing from their diagonalisation. This freedom is removed if one sticks to particularly simple choices of the Higgses responsible for up quark and neutrino masses. For instance, as long as one sticks to representations of the Higgs fields which preserve the quark-lepton Yukawa unification, like the $10$-plets (due to the underlying Pati-Salam symmetry), the mixing angles appearing in the diagonalisation of the up-quark mass matrix also appear in $h^\nu$. In this case $h^\nu$ can be completely predicted at the high scale. However, in a general scenario, this need not hold true. Large contributions from $120$ can break this alignment between up and the neutrino Yukawa matrices as is evident from Eqs. (4 - 6) and hence $h^\nu$ would be no longer predictable in this general situation.

Keeping with our philosophy to be as general as possible, we find two cases which would serve as ‘benchmark’ scenarios for seesaw induced lepton flavour violation in SUSY SO(10). The first one corresponds to a case where the mixing present in $h^\nu$ is small and CKM-like. This is typical of the models where fermions attain their masses through 10-plets. We will call this case, ‘the minimal case’. As a second case, we consider scenarios where the mixing
in \( h^\nu \) is no longer small, but large like the observed PMNS mixing. We will call this case the ‘the maximal case’. Within \( SO(10) \) this is possible in models with asymmetric Yukawa matrices. These two ‘benchmark’ cases, we believe, would span the range of lepton flavour violation generated by \( h^\nu h^\nu^\dagger \) at the weak scale in a fairly general way. We now proceed to study these constraints for the two cases.

III. \( SO(10) \) AND LEPTON FLAVOUR VIOLATION

As mentioned in the Introduction, the off-diagonal entries in the slepton mass matrices are generated by the product \( h^\nu h^\nu^T \) through renormalisation group evolution\(^2\). Having known the matrix \( h^\nu \) or even the product \( h^\nu h^\nu^T \) at the high scale, the off-diagonal entry \([m^2_\tilde{L}]_{ij}\) can be calculated at the weak scale. These entries would then contribute to the lepton flavour violating decays \( l_j \rightarrow l_i, \gamma \). A naive estimate of the branching ratios of these decays is given by (in standard notation) \(^{15}\):

\[
\text{BR}(l_j \rightarrow l_i, \gamma) \approx \frac{\alpha^3 \left( [m^2_\tilde{L}]_{ij} \right)^2}{G_F m^8_{\text{SUSY}} \tan^2 \beta}, \quad (9)
\]

where \( m_{\text{SUSY}} \) represents the typical soft Supersymmetric breaking mass. The present experimental limits on the branching ratios of these decays are given as \(^{16, 17}\)

\[
\begin{align*}
\text{BR}(\mu \rightarrow e, \gamma) & \leq 1.2 \times 10^{-11}, \quad (10) \\
\text{BR}(\tau \rightarrow \mu, \gamma) & \leq 5. \times 10^{-7}. \quad (11)
\end{align*}
\]

In future these bounds are expected to improve at least by a few orders of magnitude. In particular, in the proposed experiment at PSI, the limits on the \( \text{BR}(\mu \rightarrow e, \gamma) \) are expected to improve to \(^{11}\)

\[
\text{BR}(\mu \rightarrow e, \gamma) \leq 10^{-14}. \quad (12)
\]

These limits would now constrain the parameters governing the sparticle spectrum, namely \( m_0, M_{1/2}, A_0, \text{sg}(\mu) \) and \( \tan \beta \) in the mSUGRA with electroweak radiative breaking. It should be noted that the above mentioned RG effects would require the scale of Supersymmetry breaking to be higher than the scale of right handed neutrinos. Thus in models of Gauge

\(^2\) We have implicitly assumed mass matrices to be real for the following discussion and thus neglect any effects due to the presence of phases in the mass matrices.
Mediated Supersymmetry Breaking (GMSB) these effects would be absent. We now analyse these branching ratios in detail for the two cases mentioned at the end of the previous section, namely, the small mixing case with CKM angles and the large mixing case with PMNS angles in $h^\nu$.

A. The minimal Case: CKM mixings in $h^\nu$

The minimal Higgs spectrum to obtain phenomenologically viable mass matrices includes two 10-plets, one coupling to the up-sector and the other to the down-sector. In this way it is possible to obtain the required CKM mixing \[18\] in the quark sector. The $SO(10)$ superpotential is now given by

$$W_{SO(10)} = \frac{1}{2} h^u_{ij} 16_i 16_j 10_u + \frac{1}{2} h^d_{ij} 16_i 16_j 10_d + \frac{1}{2} h^R_{ij} 16_i 16_j 126.$$ \(13\)

We further assume the 126 dimensional Higgs field gives Majorana mass only to the right handed neutrinos. An additional feature of the above mass matrices is that all of them are symmetric. Without loss of generality we can rotate the 16-plet into a basis where the charged leptons and the down-type quarks are diagonal. In terms of the SM fields, we can rewrite the above as

$$W = h^u_{ij} Q_i u^c_j H_u + h^d_{ij} Q_i d^c_j H_d + h^e_{ij} L_i e^c_j H_d + h^\nu_{ij} L_i \nu^c_j H_u + \frac{1}{2} M_{Rij} \nu^c_i \nu^c_j.$$ \(14\)

Immediately we see that the following mass relations hold between the quark and leptonic mass matrices at the GUT scale:\[3:\]

$$h^u = h^\nu ; \quad h^d = h^e.$$ \(15\)

In the above basis, the symmetric $h^u$ is diagonalised by:

$$V_{CKM} h^u V_{CKM}^T = h^u_{\text{diag}}.$$ \(16\)

Hence from \[15\]:

$$h^\nu = V_{CKM}^T h^u_{\text{diag}} V_{CKM}.$$ \(17\)

\[3\] Clearly this relation cannot hold for the first two generations of down quarks and charged leptons. As usual, small corrections due to non-renormalisable operators or suppressed renormalisable operators \[14\] can be invoked.
According to Eq. \( \ref{eq:9} \), \( BR(\mu \rightarrow e\gamma) \) depends on:

\[
[h^{\nu}h^{\nu}]_{21} \approx h_t^2 V_{td} V_{ts} + O(h_c^2).
\] (18)

In this expression, the CKM angles are small but the presence of the large top Yukawa coupling compensates for such suppression. The large couplings in \( h^{\nu} \sim O(h_t) \) induce significant off-diagonal entries in \( m_L^2 \) through the RG evolution between \( M_{GUT} \) and the scale of the right-handed Majorana neutrinos \( ^4 \), \( M_R \). The induced off-diagonal entry relevant for \( \mu \rightarrow e, \gamma \) is of the order,

\[
[m_L^2]_{21} \approx -\frac{1}{8\pi^2} \left( 3m_0^2 + A_0^2 \right) h_t^2 V_{td} V_{ts} \log \left( \frac{M_{GUT}}{M_R} \right) + O(h_c^2).
\] (19)

The required right handed neutrino Majorana mass matrix consistent with both the observed low energy neutrino masses and mixings as well as with CKM like mixings in \( h^{\nu} \) is determined easily from the seesaw formula defined at the scale of right handed neutrinos as

\[
m_\nu = -h^{\nu T} M_R^{-1} h^{\nu} v_u^2, \tag{20}
\]

\[
= -h^{\nu} M_R^{-1} h^{\nu} v_u^2. \tag{21}
\]

where we have used the symmetric nature of the \( h^{\nu} \) in the second equation. Inverting Eq. \( \ref{eq:20} \), one gets:

\[
M_R = -h^{\nu} m_\nu^{-1} h^{\nu} v_u^2,
\]

\[
= V_{CKM} h_u^\nu \left[ V_{CKM}^T m_\nu^{-1} V_{CKM} \right] h_u^\nu V_{CKM}^T, \tag{22}
\]

where we have used Eq. \( \ref{eq:17} \) for \( h^{\nu} \). Furthermore, \( m_\nu^{-1} \) can be written as \( m_\nu^{-1} = U_{PMNS} \text{diag}[m_\nu^{-1}] U_{PMNS}^T \), whose entries are determined at the low scale from neutrino oscillation experiments. The structure of \( M_R \) can now be derived\(^5\) for a given set of neutrino masses and mixing angles. Neglecting the small CKM mixing in \( h^{\nu} \) we have

\[
M_R \approx v_u^2 \begin{pmatrix}
h_u^2 [m_\nu^{-1}]_{11} & h_u h_c [m_\nu^{-1}]_{12} & h_u h_t [m_\nu^{-1}]_{13} \\
h_u h_c [m_\nu^{-1}]_{12} & h_c^2 [m_\nu^{-1}]_{22} & h_c h_t [m_\nu^{-1}]_{23} \\
h_u h_t [m_\nu^{-1}]_{13} & h_c h_t [m_\nu^{-1}]_{23} & h_t^2 [m_\nu^{-1}]_{33}
\end{pmatrix}.
\] (23)

\(^4\) Typically one has different mass scales associated with different right handed neutrino masses.

\(^5\) The neutrino masses and mixings here are defined at \( M_{GUT} \). Radiative corrections can significantly modify the neutrino spectrum at the weak scale\(^19\). This is more true for the degenerate spectrum of neutrino masses\(^20\) and for some specific forms of \( h^{\nu} \)\(^21\). For our present discussion, with hierarchical neutrino masses and up-quark like neutrino Yukawa matrices, we expect these effects not to play a very significant role.
It is clear from above that the hierarchy in the $M_R$ mass matrix goes as the square of the hierarchy in the up-type quark mass matrix. Furthermore, for a hierarchical neutrino mass spectrum we have $m_{\nu_3} \approx \sqrt{\Delta m_{Atm}^2}$, $m_{\nu_2} \approx \sqrt{\Delta m_{\odot}^2}$ and $m_{\nu_1} \ll \sqrt{\Delta m_{\odot}^2}$ and for a nearly bi-maximal $U_{PMNS}$:

$$U_{PMNS} \approx \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix},$$  \quad (24)$$

it straightforward to check that all the right handed neutrino mass eigenvalues are controlled by the smallest left-handed neutrino mass.

$$M_{R_1} \approx \frac{m_{\nu_1}^2}{4 m_{\nu_1}}; \quad M_{R_2} \approx \frac{m_{\nu_2}^2}{4 m_{\nu_1}}; \quad M_{R_3} \approx \frac{m_{\nu_3}^2}{2 m_{\nu_1}}.$$  \quad (25)$$

This implies that we can not choose an arbitrarily small neutrino mass if we want the right-handed neutrino masses to be below $M_{GUT}$. In our numerical examples in section IV, we choose $m_{\nu_3} = 0.05$ eV, $m_{\nu_2} = 0.0055$ eV, $m_{\nu_1} = 0.001$ eV.

At this point we have both $h^\nu$ and $M_R$ determined and we can now use the experimental bounds on $BR(\mu \to e, \gamma)$ to constrain the SUSY parameter space. The only unknowns are the SUSY breaking soft terms $m_0$, $M_{1/2}$, $A_0$ and $\text{sg}(\mu)$, $\tan \beta$. For instance, from Eqs. (9,19) we have

$$m_0^4 \geq \frac{3 \alpha^3}{8 \pi^2 G_F^2} \left| \text{log} \frac{M_X}{M_R} \right|^2 \frac{h_t^4 V_{td}^2 V_{ts}^2}{B} \tan \beta^2,$$  \quad (26)$$

where $B$ represents the experimental limit on the branching ratio $BR(\mu \to e, \gamma)$, $A_0$ is assumed to vanish and $m_0$ is identified with $m_{SUSY}$. Taking the futuristic limit of $B \leq 10^{-14}$, we see that $m_0$ can be probed up to 1 TeV for large $\tan \beta \sim 40$, bordering the limit that will be probed at LHC [22]. We quantify these results with a numerical analysis in the next section.

B. The maximal case: $PMNS$ mixing angles in $h^\nu$

The minimal $SO(10)$ model presented in the previous sub-section would inevitably lead to small mixing in $h^\nu$. In fact, with two Higgs fields in symmetric representations, giving masses to the up-sector and the down-sector separately, it would be difficult to avoid the small CKM like mixing in $h^\nu$. To generate mixing angles larger than CKM angles, asymmetric mass matrices have to be considered. In general, it is sufficient to introduce asymmetric textures
either in the up-sector or in the down-sector. In the present case, we assume that the down-sector couples to a combination of Higgs representations (symmetric and anti-symmetric) \( \Phi \), leading to an asymmetric mass matrix in the basis where the up-sector is diagonal. We have:

\[
W_{SO(10)} = \frac{1}{2} h_{ii}^{u,\nu} 16_i 16_j 10^u + \frac{1}{2} h_{ij}^{d,e} 16_i 16_j \Phi + \frac{1}{2} h_{ij}^R 16_i 16_j 126 ,
\]

where the 126, as before, generates only the right handed neutrino mass matrix. To study the consequences of these assumptions, we see that at the level of \( SU(5) \), we have

\[
W_{SU(5)} = \frac{1}{2} h_{ii}^u 10_i 10_i 5_u + h_{ii}^\nu \bar{5}_i 1_i 5_u + h_{ij}^d 10_i \bar{5}_j 5_d + \frac{1}{2} M_{1j}^R 1_i 1_j,
\]

where we have decomposed the 16 into 10 + 5 + 1 and 5_u and \( \bar{5}_d \) are components of 10_u and \( \Phi \) respectively. To have large mixing \( \sim U_{PMNS} \) in \( h^\nu \) we see that the asymmetric matrix \( h^d \) should now be able to generate both the CKM mixing as well as PMNS mixing. This is possible if

\[
V_{CKM}^T h^d U_{PMNS}^T = h^{d,\text{diag}}.
\]

This would mean that the 10 which contains the left handed down-quarks would be rotated by the CKM matrix whereas the \( \bar{5} \) which contains the left handed charged leptons would be rotated by the \( U_{PMNS} \) matrix to go into their respective mass bases. Thus we have, in analogy with the previous sub-section, the following relations hold true in the basis where charged leptons and down quarks are diagonal:

\[
\begin{align*}
  h^u &= V_{CKM} h^{u,\text{diag}} V_{CKM}^T, \\
  h^\nu &= U_{PMNS} h^{u,\text{diag}}.
\end{align*}
\]

Using the seesaw formula of Eq. 20 and Eq. 31 we have

\[
M_R = \text{Diag}\{ \frac{m_\nu^2}{m_{\nu_1}}, \frac{m_\nu^2}{m_{\nu_2}}, \frac{m_\nu^2}{m_{\nu_3}} \}.
\]

This would mean that this setup would require \( M_R \) to be diagonal at the \( SO(10) \) level in the basis of diagonal \( h^{u,\nu} \), Eq. 27. We now turn our attention to lepton flavour violation in the scenario. The branching ratio, \( \text{BR}(\mu \rightarrow e, \gamma) \) would now be dependent on:

\[
[h^\nu h^\nu^T]_{21} = h_i^2 U_{\mu_3} U_{e_3} + h_c^2 U_{\mu_2} U_{e_2} + \mathcal{O}(h_u^2),
\]

\( ^6 \) The couplings of \( \Phi \) in the superpotential can be either renormalisable or non-renormalisable. See for a non-renormalisable example.
where $U_{fi}$ are elements of the $U_{PMNS}$ matrix. It is immediately clear from the above that in contrast to the CKM case here the dominant contribution to the off-diagonal entries depends on the unknown magnitude of the element $U_{e3}$. If $U_{e3}$ is very close to its present limit $\sim 0.24$, the first term on the RHS of the Eq. (33) would dominate. Moreover, this would lead to large contributions to the off-diagonal entries in the slepton masses with $U_{\mu 3}$ of $O(1)$. We have

$$[m_{\tilde{L}}^2]_{21} \approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2) h_t^2 U_{\mu 3} U_{e 3} \log\left(\frac{M_{GUT}}{M_R}\right) + O(h_c^2).$$  (34)

The above contribution is large by a factor $(U_{\mu 3} U_{e 3})/ (V_{td} V_{ts}) \sim 140$ compared to the CKM case. From Eq. (33) we see that it would mean about a factor $10^4$ times larger than the CKM case in BR($\mu \rightarrow e, \gamma$). In case $U_{e3}$ is very small, i.e., either zero or $\lesssim h_c^2/h_t^2 U_{e 2} \sim 4 \times 10^{-5}$, the second term $\propto h_t^2$ in Eq. (33) would dominate. However the off-diagonal contribution in slepton masses, now being proportional to charm Yukawa could be much smaller, in fact, even smaller than the CKM contribution by a factor

$$\frac{h_c^2 U_{\mu 2} U_{e 2}}{h_t^2 V_{td} V_{ts}} \sim 7 \times 10^{-2}.$$  (35)

If $U_{e3}$ is close to it’s present limit, the current bound on BR($\mu \rightarrow e, \gamma$) would already be sufficient to produce stringent limits on the SUSY mass spectrum. Indeed from:

$$m_0^4 \geq \frac{3}{8} \alpha^3 \pi^2 G_F^2 \left|\log \frac{M_X}{M_R}\right|^2 h_t^4 U_{\mu 3}^2 U_{e 3}^2 B \tan^2 \beta^2 \ (A_0 = 0),$$  (36)

$B \leq 10^{-11}$ probes $m_0$ at the TeV level even for small $\tan \beta$. We make these statements more concrete in the next section with our results from numerical analysis.

IV. $SO(10)$ AND SUSY MASS SPECTRUM

As mentioned in the introduction we have chosen to work within a mSUGRA framework with flavour blind universal soft breaking terms at the GUT scale. It is known [12] that if the soft breaking terms are hard at scales above $M_{GUT}$ and if the perturbative RG approach can be safely used in such energy interval above $M_{GUT}$, then the large value of $h_t$ can lead to relevant radiative contributions to the mass of the stau, spoiling the slepton mass universality at $M_{GUT}$. Here we are interested in the effects produced by the large $h'\nu$ coupling in the running of the $m_{\tilde{L}}^2$ down to the $M_R$ scales which are below $M_{GUT}$. For simplicity,
we assume the soft masses to be universal at $M_{\text{GUT}}$, hence neglecting the possible above mentioned effects which would further enhance the stringent bounds that we obtained here.

In our numerical analysis, we have considered MSSMRN (MSSM + Right handed neutrinos) RGE from the scale $M_{\text{GUT}}$ down to the scale of the different RH neutrinos, which we integrate out in several steps until the scale of the lightest RH neutrino, $M_{R_1}$. From $M_{R_1}$ to $M_Z$ the standard MSSM RGE are used \[21\]. As a result of these renormalisation group effects, the slepton mass matrices which were diagonal at $M_{\text{GUT}}$ to start with are now non-diagonal at the weak scale. At this latter scale, we have numerically diagonalised these mass matrices and the corresponding eigenvalues and mixing matrices are found. We used the complete calculations of Hisano et. al \[15\] to compute the branching ratios of both $\mu \rightarrow e, \gamma$ and $\tau \rightarrow \mu, \gamma$.

We produce scatter plots of $\text{BR}(\mu \rightarrow e, \gamma)$ and $\text{BR}(\tau \rightarrow \mu, \gamma)$ vs. $M_{1/2}$ for small and large values of $\tan \beta$. We restrict the allowed SUSY parameter space by imposing i) experimental constraints from direct sparticle searches, in particular requiring $m_{\tilde{t}}$ and $m_{\chi^\pm}$ to be above $M_Z$ and $m_{\tilde{g}} \geq 33$ GeV; ii) the LSP to be neutral and iii) $2 \times 10^{-4} \leq \text{BR}(b \rightarrow s, \gamma) \leq 4 \times 10^{-4}$. In each scatter plot, for a given $\tan \beta$ and $sg(\mu)$, the rest of the parameters are allowed to vary within the ranges:

a) $90 \text{ GeV} \leq m_0 \leq 900 \text{ GeV}$ ;

b) $90 \text{ GeV} \leq M_{1/2} \leq 700 \text{ GeV}$,

c) $A_0$ is parameterised as $A_0 = A_1 m_0$ with $A_1$ lying between (-3, 3). For the neutrino mass eigenvalues, we have used the values specified at the end of section IIIA. We have chosen $\sin^2 2\theta_{\text{solar}} \approx 0.71$; $\sin^2 2\theta_{\text{atm}} \approx 1$ and $U_{e3} \approx 0.15$.

In Fig. 1a) and 1b) we show the scatter plots for $\text{BR}(\mu \rightarrow e, \gamma)$ for the CKM case and $\tan \beta = 2$ and $\tan \beta = 40$ respectively. Similar figures for $\text{BR}(\tau \rightarrow \mu, \gamma)$ and the same values of $\tan \beta$ are presented in Figs. 1c) and 1d). All these plots are calculated with $\mu > 0$ but the results do not change significantly with negative $\mu$. These plots reflect an interesting correlation between the branching ratios and the GUT value of the universal gaugino mass. This is due to the fact that the gaugino mass fixes the chargino and neutralino masses at $M_{1/2}$ and, to a small extent it also influences the slepton masses through RGE. However, for a fixed $M_{1/2}$ the different values of $m_0$ and $A_0$ can change the value of the BR within a range of 3 orders of magnitude. Nevertheless, this fact has still important consequences. For instance, for $\tan \beta = 40$ reaching a sensitivity of $10^{-14}$ for $\text{BR}(\mu \rightarrow e\gamma)$ would allow us to
probe completely the SUSY spectrum up to $M_{1/2} = 300$ GeV (notice that this corresponds to gluino and squark masses of order 750 GeV) and would still probe a large regions in parameter space up to $M_{1/2} = 700$ GeV. In the case of smaller values of $\tan \beta$ the BR scales as $(\tan \beta)^2$ and therefore for $\tan \beta = 2$ only a small part of the parameter space with $M_{1/2} \leq 300$ GeV can be probed with this sensitivity. Similarly in the $\tau \rightarrow \mu \gamma$ decay a sensitivity of $6 \times 10^{-8}$ which could be reached in the B factories in the near future [17, 25], would allow to probe a sizeable piece of the parameter space for large $\tan \beta$.

In the PMNS scenario Figs. 2a) and 2b) show the plots for BR($\mu \rightarrow e, \gamma$) for $\tan \beta = 2, 40$, whereas Figs. 2c and 2d are for BR($\tau \rightarrow \mu, \gamma$). As we said in the previous section (see Eq. (34)) in the PMNS case, the results concerning BR($\mu \rightarrow e, \gamma$) strongly depend on the unknown value of $U_{e3}$. In the plots 2a and 2b, the value of $U_{e3}$ chosen is very close to the present experimental upper limit [24]. As long as $U_{e3} \gtrsim 4 \times 10^{-5}$, the plots scale as $U_{e3}^2$, while for $U_{e3} \lesssim 4 \times 10^{-5}$ the term proportional to $m^2_\tau$ in Eq. (34) starts dominating and then, the result is insensitive to the choice of $U_{e3}$. Here we see that with the present limit on BR($\mu \rightarrow e, \gamma$), all the parameter space would be completely excluded up to $M_{1/2} = 700$ GeV for $U_{e3} = 0.15$ for any value of $\tan \beta$. For instance, a value of $U_{e3} = 0.01$ would reduce the BR by a factor of 225 and still most of the parameter space for $\tan \beta = 40$ would be completely excluded. For $\tan \beta = 2$ this would probe approximately half the parameter space up to $M_{1/2} = 300$ GeV. Contrary to expectations the present bound of the $\tau \rightarrow \mu \gamma$ starts exploring the SUSY parameter already for low $\tan\beta = 2$. For larger $\tan \beta = 40$ even the present bound rules out significant regions of the parameter space. The main advantage of this decay mode is that it does not depend on the value of $U_{e3}$ and therefore provides an important constraint on the parameter space of the model for any value of $U_{e3}$.

It is important to emphasise that these radiative leptonic decays are a much more powerful probe on the SUSY parameter space than the very similar $b \rightarrow s \gamma$ decay. This can be traced to the fact that slepton and chargino/neutralino masses are a factor $\sqrt{6}$ smaller that the corresponding gluino and squark masses for the same GUT initial parameters due to RGE effects. Therefore, these decays will become in the near future the most stringent constraints on the SUSY parameter space and offer an excellent opportunity for SUSY searches.
V. DISCUSSION OF THE RESULTS AND FINAL REMARKS

As we have mentioned, $SO(10)$ is an interesting and predictive scheme for $BR(l_j \to l_i, \gamma)$ in so that one of the neutrino Yukawa couplings has to be of the $\mathcal{O}(h_t)$. In our plots of Figs 1 and 2 we exhibited results of the two ‘extreme’ cases of “small ” (CKM-like) and “large ” (PMNS -like) lepton mixing angles. We see that the constraints coming from $BR(\mu \to e, \gamma)$ and $BR(\tau \to \mu, \gamma)$ are very significant. In particular for the PMNS case, even the present bounds on these two branching ratios (Eqs. (10, 11)) are able to exclude large regions of the SUSY $SO(10)$ parameter space which are still allowed by all the present SUSY accelerator tests.

Needless to say, improving the experimental sensitivity of those two processes, would place lepton radiative decays in the forefront of the indirect searches of SUSY signals. Indeed, reaching $\mathcal{O}(10^{-14})$ for $BR(\mu \to e, \gamma)$ would probe larger regions of the SUSY $SO(10)$ parameter space than that probed by $b \to s, \gamma$ even in the less optimistic case of the small CKM lepton angles. Obviously, this statement becomes stronger when one moves to the PMNS case unless $U_{e3}$ is really very small, say $U_{e3} \lesssim 10^{-3}$. If this latter circumstance occurs, then $\tau \to \mu, \gamma$ becomes more powerful SUSY probe than $\mu \to e, \gamma$ and again, reaching sensitivity for $BR(\tau \to \mu, \gamma)$ of $\mathcal{O}(10^{-8})$ we could do better than $b \to s, \gamma$ in constraining the SUSY $SO(10)$ parameter space.

Our analysis, referring in all generality to the framework of a relevant class of unified SUSY models, once again emphasises the importance and need for a strenuous effort in pursuing the challenging experimental road of lepton radiative decays.

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Appendix I

In this appendix we discuss in detail the proof that the large hierarchy in $M^u$ necessarily requires that the symmetric Yukawa couplings include a large entry of $\mathcal{O}(m_t)$. Let us decompose a generic mass matrix in symmetric and anti-symmetric parts as follows:

$$M = M_S + M_A,$$  \hspace{1cm} (37)
The parameter $c$

The eigenvalues of these matrix can be found solving the cubic equation:

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0$$  \hspace{1cm} (40)

with the coefficients,

$$a = \text{Tr}[M^u M^u^T]$$

$$= a_{11}^2 + a_{22}^2 + a_{33}^2 + 2 (a_{12}^2 + a_{13}^2 + a_{23}^2 + b_{12}^2 + b_{23}^2 + b_{13}^2),$$  \hspace{1cm} (41)

$$b = \mathcal{M}_{11}[M^u M^u^T] + \mathcal{M}_{22}[M^u M^u^T] + \mathcal{M}_{33}[M^u M^u^T]$$  \hspace{1cm} (42)

where $\mathcal{M}_{ij}[M^u M^u^T]$ is the minor obtained by omitting the $i$ row and $j$ column from the determinant. These are given as

$$\mathcal{M}_{11} = (a_{12}^2 + (b_{12} - a_{12})^2 + (a_{23} + b_{23})^2) \left( (b_{13} - a_{13})^2 + a_{33}^2 + (b_{23} - a_{23})^2 \right)$$

$$- (a_{22} (a_{23} - b_{23}) + (a_{23} + b_{23}) a_{33} + (b_{12} - a_{12}) (b_{13} - a_{13}))^2$$  \hspace{1cm} (43)

$$\mathcal{M}_{22} = (a_{11}^2 + (b_{12} + a_{12})^2 + (a_{13} + b_{13})^2) \left( (b_{13} - a_{13})^2 + a_{33}^2 + (b_{23} - a_{23})^2 \right)$$

$$- (a_{11} (a_{13} - b_{13}) + (a_{13} + b_{13}) a_{33} + (b_{12} + a_{12}) (a_{23} - b_{23}))^2$$  \hspace{1cm} (44)

$$\mathcal{M}_{33} = (a_{11}^2 + (b_{12} + a_{12})^2 + (a_{13} + b_{13})^2) \left( (b_{12} - a_{12})^2 + a_{23}^2 + (b_{23} + a_{23})^2 \right)$$

$$- (a_{11} (a_{12} - b_{12}) + (a_{12} + b_{12}) a_{12} + (a_{13} + b_{13}) (a_{23} + b_{23}))^2$$  \hspace{1cm} (45)

The parameter $c$ is given as follows:

$$c = \text{Det}[M^u M^u^T]$$

$$= \left( b_{23}^2 a_{11} - 2 b_{13} b_{23} a_{12} + 2 b_{12} b_{23} a_{13} + b_{13}^2 a_{22} - a_{13}^2 a_{22} - 2 b_{12} b_{13} a_{23} ight. + 2 a_{12} a_{13} a_{23} - a_{11} a_{23}^2 + b_{12}^2 a_{33} - a_{12}^2 a_{33} + a_{11} a_{22} a_{33} \right)^2$$  \hspace{1cm} (46)
To generate a hierarchical spectrum, as required by the up-quark sector, $a$, $b$ and $c$ have to satisfy the following conditions:

\begin{align*}
a &= m_u^2 + m_c^2 + m_t^2 \\
b &= m_u^2 m_t^2 + m_c^2 m_t^2 + m_u^2 m_c^2 \\
c &= m_t^2 m_c^2 m_u^2
\end{align*}

We want to prove that at least one of the elements in the symmetric matrix must be of order $m_t$. To do this we show that the case where $m_t$ has its origins solely from the anti-symmetric part i.e, the elements $b_{ij}$ is not consistent with the observed spectrum. In such a case at least one of the $b_{ij}$ is as large as $m_t$. However from Eqs. (41-46) we see that such an assumption would be in conflict with the condition in Eq. (48) as it leads to $b \sim m_t^4$ through one of the $M_{ij}$. For ex: if $b_{23}^2 \sim m_t^2$; $M_{11} \sim m_t^4$. This is in fact true for any of the $b_{ij}$. Therefore a large $b_{ij}$ would lead to a degenerate spectrum rather than a hierarchical spectrum required. Thus $m_t$ cannot have its origins solely from the anti-symmetric part.

The only allowed possibilities would then be, either $m_t$ has its origins solely in the symmetric part or through elements of both $M_S$ and $M_A$, when some elements in both matrices are of comparable magnitude $\sim O(m_t)$. In either case, it is clear that $M_S$ would contain at-least one element as large as $m_t$. 

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FIG. 1: The scatter plots of branching ratios of rare leptonic radiative decays vs. $M_{1/2}$ are shown for the (minimal) CKM case for two specific values of $\tan \beta$. Results do not alter significantly with the change of sign($\mu$).
FIG. 2: The scatter plots of branching ratios of rare leptonic radiative decays vs. $M_{1/2}$ are shown for the (maximal) PMNS case for two specific values of $\tan \beta$. Results do not alter significantly with the change of sign($\mu$).

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