Phase-Sensitive Probes of Nuclear Polarization in Spin-Blockaded Transport

M. S. Rudner\textsuperscript{1}, I. Neder\textsuperscript{1}, L. S. Levitov\textsuperscript{2}, B. I. Halperin\textsuperscript{1}

\textsuperscript{(1)} Department of Physics, Harvard University, 17 Oxford St., Cambridge, MA 02138
\textsuperscript{(2)} Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139

Spin-blockaded quantum dots provide a unique setting for studying nuclear-spin dynamics in a nanoscale system. Despite recent experimental progress, observing phase-sensitive phenomena in nuclear spin dynamics remains challenging. Here we point out that such a possibility opens up in the regime where hyperfine exchange directly competes with a purely electronic spin-flip mechanism such as the spin-orbital interaction. Interference between the two spin-flip processes, resulting from long-lived coherence of the nuclear-spin bath, modulates the electron-spin-flip rate, making it sensitive to the transverse component of nuclear polarization. In a system repeatedly swept through a singlet-triplet avoided crossing, nuclear precession is manifested in oscillations and sign reversal of the nuclear-spin pumping rate as a function of the waiting time between sweeps. This constitutes a purely electrical method for the detection of coherent nuclear-spin dynamics.

Due to their long coherence times, nuclear spins offer unique opportunities to study quantum dynamics. While conventional techniques using RF fields probe macroscopic groups of spins, currently there is wide interest in the local dynamics of nanoscale groups of spins. In particular, semiconducting quantum dots have emerged as a platform for investigating the intriguing quantum many-body dynamics of coupled electron and nuclear spins \cite{1}–\cite{3}. Many experimental achievements in this field rely on the phenomenon of spin blockade, in which electrons must flip their spins in order to pass through the system in accordance with the Pauli exclusion principle \cite{3}. Such spin-blockaded transport is sensitive to the nuclear-polarization-dependent Overhauser field, which controls the energy splittings between electronic spin states. This sensitivity affords an exciting opportunity to electrically control and detect the states of nuclear spins \cite{10,11}.

In this Rapid Communication, we propose a method which can be used to electrically probe coherent nuclear phenomena, such as Larmor precession or Rabi oscillations, which are not easily probed by the Overhauser effect alone. We identify a regime where transport is sensitive to all three vector-components of the local nuclear polarization, due to coherent interplay between hyperfine coupling and an electron-only spin-coupling such as the spin-orbit interaction \cite{12,13}. As we show, systems with such sensitivity host a variety of new phenomena arising from coherent nuclear dynamics. Recently observed oscillations of the nuclear pumping rate controlled by nuclear Larmor precession \cite{14} indicate that this unexplored regime in now within experimental reach.

Here we investigate these phenomena in a two-electron double quantum dot in a uniform magnetic field. The two-electron singlet and m = ±1 triplet states are coupled via the difference of hyperfine fields due to transverse nuclear polarization in the two dots, ΔB_{nuc,⊥} \cite{10,11,13}. Without an additional coupling, the electron spin-flip rate depends only on |ΔB_{nuc,⊥}|, and not on the orientation of ΔB_{nuc,⊥} in the XY plane \cite{13}.

The situation becomes considerably more interesting when singlet-triplet transitions can occur due to either the spin-orbit or the hyperfine interaction (see Fig\textsuperscript{1}a). In this case, we find that the probability of electron spin-flip depends on the angle θ of the nuclear polarization measured in the XY plane relative to a fixed axis determined by the spin-orbit interaction. Such a dependence makes electron transport sensitive to the phase of nuclear spin precession. Below, we analyze this phenomenon for a Landau-Zener-type process, occurring when the electronic system is swept through a singlet-triplet level crossing, as employed e.g. in Refs.\textsuperscript{14,16} and depicted in Fig\textsuperscript{1b}. To make contact with experiment, we focus in particular on resulting signatures in nuclear spin polarization. Strikingly, we find robust oscillations in the

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{(Color online) Coherent interplay of hyperfine and spin-orbit mediated transitions. a) Interdot tunneling in the triplet state is suppressed by Pauli exclusion, but can be mediated by the hyperfine and spin-orbit interactions which do not conserve electron spin. b) The singlet and triplet levels |S⟩ and |T⟩ exhibit an avoided crossing with splitting |v_{SO}| = |v_{SO} + v_{HF}e^{iθ}| when exchange energy compensates Zeeman energy. c) Behavior near the S – T crossing, controlled by |v_{SO}|, is sensitive to the relative phase θ between spin-orbit and hyperfine matrix elements. d) Phase-dependent transition probability, Eq.\textsuperscript{3}, with v_{SO} = 0.4\sqrt{\hbar g}, and v_{HF} = 0.6\sqrt{\hbar g}.}
\end{figure}
nuclear spin pumping rate which result from nuclear pre-
cession and survive even after averaging over the distribu-
tion of random initial nuclear spin states. Note that
other types of electron-only coupling such as Zeeman cou-
pling to the nonuniform field of a micromagnet[17] can
produce analogous effects.

The origin of the spin-angle dependence can be un-
derstood heuristically by analyzing the avoided crossing
that opens near the degeneracy of the triplet and singlet
levels $|T_v\rangle$ and $|S\rangle$ in nonzero magnetic field, circled
in Fig[1]. Due to the spin-orbit interaction, tunneling
between dots is accompanied by a rotation of electron
spin [18] which introduces a nonzero spin-flip amplitude.

In addition, the hyperfine interaction between electron
and nuclear spins allows electronic transitions accompa-
nied by nuclear spin flips, described by the effective
Hamiltonian $H_{\text{HF}} = A^+ | S \rangle \langle T_v | + A^- | T_v \rangle \langle S |$, with
$A^\pm = \sum_{i=1}^{N_{\text{nuc}}} g_i I_i^\pm$. The magnitude and sign of each cou-
pling constant $g_i$ depends on the location of nucleus $i$
(positive in dot 1 and negative in dot 2).

Taking advantage of the fact that the number of nu-
cleii interacting with the electrons is very large, $N_{\text{nuc}} \approx O(10^9)$, we note that the commutator $[A^+, A^-]$ is typi-
ically smaller than $A^+ A^-$ by a factor of order 1/$N_{\text{nuc}}^{1/2}$. This
allows us to treat $A^+$ as a nearly classical variable,
$A^+ \approx v_{\text{HF}} e^{i\theta}$. Here $v_{\text{HF}}$ is proportional to the magnitude
of the transverse hyperfine difference field $\Delta B_{\text{nuc},\perp}$, and
$\theta$ describes its orientation in the XY plane.

Together, the spin-orbit and hyperfine pieces provide
a matrix element between the singlet and triplet states:

$$v_\theta \equiv \langle S | H | T_v \rangle = v_{\text{SO}} + v_{\text{HF}} e^{i\theta}. \quad (1)$$

Evolution in the $\{ | S \rangle, | T_v \rangle \}$ subspace near the level
crossing is described by the $2 \times 2$ Hamiltonian

$$H_{ST_v} = \begin{pmatrix} \frac{1}{2} \Delta & v_\theta \\ -v_\theta & -\frac{1}{2} \Delta \end{pmatrix}, \quad \Delta(t) = \varepsilon_{T_v}(t) - \varepsilon_S(t), \quad (2)$$

where $\varepsilon_{T_v}$ and $\varepsilon_S$ are the energies of the diabatic $|S\rangle$ and
$|T_v\rangle$ states. The level detuning $\Delta(t)$ can be controlled
by electrostatic gates and/or magnetic field.

We consider the case where the system is initialized to
the “$(0, 2)$” singlet state $|S\rangle$ with both electrons resid-
ing in the right dot (large positive $\Delta$), and then swept
through the avoided crossing to the “$(1, 1)$” charge regime
with one electron on each dot (see Fig[1]). For a con-
stant sweep rate $\beta = |d\Delta/dt|$, the electron spin flip in
such a model is interpreted as a Landau-Zener transition
occurring with probability (see Fig[1])

$$P_{\text{LZ}} = 1 - \exp \left( -2\pi |v_{\text{SO}} + v_{\text{HF}} e^{i\theta}|^2 / \hbar \beta \right), \quad (3)$$

where $P_{\text{LZ}}$ is the probability to remain in the ground
state. The explicit dependence on the phase $\theta$ of nuclear
polarization, which enters through the matrix element
$v_\theta$, shows the singlet/triplet transition probability’s sen-
sitivity to the transverse nuclear polarization $v_{\text{HF}}$.

Model [2] provides a useful heuristic for understanding
electron spin dynamics, in particular for situations where
the nuclear spin state is characterized by a well-defined
azimuthal angle $\theta$. However, to understand the behav-
ior with more general initial states and to account for
the effects of back-action on the nuclei, we must examine
the quantum many-body dynamics of coherently coupled
electron and nuclear spins. By solving this problem be-
low, we will further justify the form of Eq.(1) and will
obtain the electron and nuclear spin-flip rates.

A key new feature here is the relaxed selection rule gov-
erning nuclear pumping: due to spin-orbit coupling, elec-
tron spin flips may occur with or without a compensating
nuclear spin flip. Although these two processes appar-
ently lead to orthogonal final states, long-lived coherence
of the nuclear spin bath allows interference between dif-
f erent transition sequences (see Fig[2]). Moreover, a single
electron can change the total nuclear polarization by an
amount $\Delta m$ that can be larger than 1, and of either sign.

At this point it is convenient to map the problem
onto the bipartite 1-dimensional quantum walk shown in
Fig[2], where each unit cell is labeled by $m$, the $z$-
projection of the total nuclear spin $I_z = \sum_i I_i^z$. In doing
so, $[I_z, A^\pm]$ remains nonzero. Intracell hopping between
internal states $T$ and $S$, characterized by $\Delta m = 0$, de-
scribes a spin-orbit transition occurring with amplitude
$v_{\text{SO}}$. Inter-cell hopping, characterized by $\Delta m = \pm 1$, de-
cribes a hyperfine transition that occurs with the ampli-
tude $v_{\text{HF}} = \langle T, \ m - 1 \ | H_{\text{HF}} | S, m \rangle$. Initially, we consider
sweeps which are fast compared to the nuclear Larmor
period, and so neglect the nuclear Zeeman energy.

Because $N_{\text{nuc}}$ is large, the value of $v_{\text{HF}}$ will change very
little during the course of a few sweeps, and we treat it
here as a constant. However, the initial values of $v_{\text{HF}}$
may differ from one run to another, with a statistical dis-
tribution of the form $p(v) \propto v e^{-v^2/s^2}$, where $s$ is a constant.
Additionally, because the typical values of total nuclear
spin will be large, of order $\sqrt{N_{\text{nuc}}}$, we may take
the ladder of allowed values of $m$ to be infinite while the
total number of sweeps is not too large.

The state of the system evolves according to

$$i\hbar \dot{\psi}_m = v_{\text{SO}} \psi_m^S + v_{\text{HF}} \psi_{m+1}^S + \frac{1}{2} \Delta(t) \psi_m^T,$$

$$i\hbar \dot{\psi}_m^T = v_{\text{SO}} \psi_m^T + v_{\text{HF}} \psi_{m-1}^T - \frac{1}{2} \Delta(t) \psi_m^S. \quad (4)$$

At time $t_0$, the system is initialized to the singlet state
with polarization $m = m_0$: $\psi_m^S = \delta_{m, m_0}$, $\psi_m^T = 0$. The
expected change of polarization,

$$\langle \Delta m \rangle = \sum_m (m - m_0) P_m: \quad P_m = |\psi_m^S|^2 + |\psi_m^T|^2, \quad (5)$$

is determined by the probabilities $\{P_m\}$ to find the nu-
clear spin with $z$-projection $m$ in the final state at time
$t_F$ after the sweep. For convenience we take the (non-
classical) initial nuclear state to be an eigenstate of $I_z$,
but the framework can be applied for more general states.
In the Fourier representation $|\psi_m\rangle = \frac{1}{2\pi} \int d\theta e^{-im\theta} |\psi_\theta\rangle$, where $|\psi_m\rangle$ and $|\psi_\theta\rangle$ are two-component spinors $(\psi^T, \psi^S)^T$ and $(\psi^m_\theta, \psi^0_\theta)^T$, the dynamics for different $\theta$ components decouple. We obtain

$$i\hbar \frac{d}{dt} |\psi_\theta\rangle = H_{ST_\theta} |\psi_\theta\rangle,$$

(6)

where $H_{ST_\theta}$ is given in Eq. (2). Here the parameter $\theta$ plays the role of the azimuthal angle of $\Delta B_{\text{nucl}}$ in the classical problem, Eq. (1). Indeed, a state $\psi^{S,T}_\theta \propto \delta(\theta - \theta_0)$ is a coherent state concentrated near $\theta_0$.

To calculate $\langle \Delta m \rangle$, we make use of the relation $m |\psi_m\rangle = \frac{i}{2\pi} \int d\theta \frac{d}{d\theta} (e^{-im\theta}) |\psi_\theta\rangle$. Integrating by parts to move the derivative onto $|\psi_\theta\rangle$, we find

$$\langle \Delta m \rangle = \frac{1}{2\pi \beta} \int d\theta \langle \psi_\theta | \frac{d}{d\theta} |\psi_\theta\rangle,$$

(7)

where without loss of generality we have taken $m_0 = 0$.

To evaluate expression (7), we must solve for the two-level dynamics of $|\psi_\theta\rangle$ under the time dependent Hamiltonian (2). We can write the evolution operator as

$$U(\theta) = \begin{pmatrix} a_\theta & b_{\theta} e^{i\varphi_0(\theta)} \\ -b_\theta^* e^{-i\varphi_0(\theta)} & a_\theta^* \end{pmatrix}, \quad |a_\theta|^2 + |b_\theta|^2 = 1,$$

(8)

where $\varphi_0(\theta) = \arg |v_\theta|$ (see Fig. 1). With this parametrization, we have $a_\theta = a_{-\theta}$, and $b_\theta = b_{-\theta}$.

The initial state $|\psi^{(0)}_{\theta}\rangle = (0, 1)^T$ evolves into $|\psi_\theta\rangle = (b_{\theta} e^{i\varphi_0(\theta)}, a_\theta^*)^T$. Because $a_\theta$ and $b_\theta$ are even functions of $\theta$, the only term that contributes to $\langle \Delta m \rangle$ is that generated when the derivative acts on $e^{i\varphi_0(\theta)}$ in Eq. (7), giving

$$\langle \Delta m \rangle = -\frac{1}{2\pi} \int d\theta \left( \frac{d\varphi_0}{d\theta} \right) |b_{\theta}|^2,$$

(9)

where $|b_{\theta}|^2$ is the singlet-triplet transition probability in the $\theta$ channel. Comparing this result to that for the net electron spin-flip probability, given by $P_{\text{el}} = \frac{1}{2\pi} \int |b_{\theta}|^2 d\theta$, we note that, unlike the situation where only the hyperfine interaction is present, here there is no simple relationship between the electron and nuclear spin-flip rates.

The analysis leading to Eq. (9) is valid for arbitrary time dependence $\Delta(t)$, and in particular can even describe cases involving multiple $S - T$ crossings. Remarkably, for very slow sweeps the change in polarization becomes sharply quantized. Indeed, since $|b_{\theta}|^2 \to 1$ in the adiabatic limit, the integral (9) is equal to a winding number: $\langle \Delta m \rangle = -\frac{1}{\beta} \oint d\varphi_0$. Thus $\langle \Delta m \rangle = -1$ or 0 if $v_\theta$ does or does not wind around the origin as $\theta$ traverses the Brillouin zone, depending on the relative magnitudes of $v_{\text{HF}}$ and $v_{\text{SO}}$. The sign is negative because electron transitions from $S$ to $T$ flip nuclear spins from up to down.

The quantity $\langle \Delta m \rangle$ exhibits interesting dependence on the sweep speed, which can be analyzed most straightforwardly for linear sweeps $\Delta(t) = -\beta t$, when $|b_{\theta}|^2$ is given by the Landau-Zener formula [19]. Due to the absence of dynamics for very fast sweeps, and since $\langle \Delta m \rangle = 0$ for very slow sweeps in the region $|v_{\text{SO}}| > |v_{\text{HF}}|$, the polarization efficiency is a non-monotonic function of sweep rate (see Fig. 2). In the limit of weak hyperfine interaction, $|v_{\text{HF}}| \ll |v_{\text{SO}}|$, expanding the exponential in Eq. (3), we find

$$\langle \Delta m \rangle \approx (v_{\text{HF}}/v_{\text{SO}})^2 \lambda e^{-\lambda}, \quad \lambda = 2\pi v_{\text{SO}}^2/\hbar \beta.$$

(10)

This expression attains its maximum value $|\langle \Delta m \rangle|_{\text{max}} = v_{\text{HF}}^2/(v_{\text{SO}}^2)$ at the optimal sweep rate $\beta_{\text{opt}} = 2\pi v_{\text{SO}}^2/\hbar$.

Next we consider a generalization to multiple sweeps and show that dynamical polarization is sensitive to nuclear Larmor precession between sweeps. We focus on a sequence of two identical gate sweeps, where each individual sweep is short on the scale of the nuclear Larmor time, but with a waiting time $\Delta t$ between sweeps long enough to allow nuclear spin precession. Individual sweeps may pass through the $S - T$ crossing one or more times. Assuming that only one nuclear species contributes to the hyperfine field, the change in nuclear polarization due to the two sweeps is given by

$$\langle \Delta m \rangle = \frac{1}{2\pi} \int \frac{d\theta}{\Delta t} (U^{-1} \partial_\theta U), \quad U = U(\theta + \Delta \theta) W U(\theta),$$

(11)

where $U(\theta)$ is the evolution matrix for a single sweep, given by (8), $W$ is the electronic evolution operator for the waiting interval $\Delta t$, the expectation value is taken over the state $|\psi^{(0)}_{\theta}\rangle = (0, 1)^T$, and $\Delta \theta = \omega_L \Delta t$ is the precession angle, with $\omega_L$ the nuclear Larmor frequency. Due to the periodicity of $U(\theta)$ in $\theta$, Eq. (11), $\langle \Delta m \rangle$ exhibits periodic oscillations in $\Delta \theta$ which are a direct manifestation of coherent nuclear polarization dynamics.

The analysis is simple when the detuning $|\Delta| \approx \pi$ is very large in the interval between sweeps. Then $W \approx \exp(\text{i} \chi \sigma_3)$, where the phase $\chi$ is large and very sensitive to any fluctuations in the waiting time $\Delta t$ or other
parameters of the system. The resulting randomness of \( \chi \) completely suppresses coherence in the electronic state between sweeps, while nuclei remain coherent. For demonstration, below we choose the particular protocol \( \Delta(t) = \beta(t_n - t) \), for \(|t - t_n| < \tau \), and \( \Delta = \infty \) otherwise, where \( n = 1, 2 \), and \( \tau \ll \Delta t = t_2 - t_1 \) with \( t_1 \) and \( t_2 \) the times of passage through the avoided crossing.

For this experimentally relevant case, it is helpful to think of the matrix-products defining \( \mathcal{U} \) and \( \mathcal{U}^{-1} \) in Eq.\((11)\) as sums over amplitudes associated with all possible histories of \( S - T \) transitions. Fast electron decoherence between sweeps suppresses the contribution of terms arising from non-identical histories in \( \mathcal{U} \) and \( \mathcal{U}^{-1} \), leaving behind a sum of classical \textit{probabilities} of all trajectories. Starting with the singlet initial state and summing over all final states, we find

\[
\langle \Delta m \rangle = \left[ -(1, 1) \partial_\theta \frac{d}{2\pi} \mathcal{M}(\theta + \Delta \theta) \mathcal{M}(0, 1)^T \right]_{\eta=0}
\]

\[
\Rightarrow \mathcal{M}(\theta) = \begin{pmatrix} 1 - p(\theta) & p(\theta)e^{i\frac{\Delta}{2}} \\ p(\theta)e^{-i\frac{\Delta}{2}} & 1 - p(\theta) \end{pmatrix}
\]

(12)

where \( \eta \) is an auxiliary variable introduced for bookkeeping, and \( p(\theta) = |v_0|^2 \) is the transition probability in the \( \theta \) channel for a single sweep. Here the phase \( \varphi(\theta) \) is the sum of the geometric part \( \varphi_0(\theta) = \arg[v_0] \), the Stokes phase \( \varphi_s(\theta) \), see e.g. Ref.\[20\], and the dynamical phase \( \varphi_{\text{ad}}(\theta) = \frac{1}{\hbar} \int_{-\tau}^{\tau} E(s, \theta) dt \) associated with adiabatic evolution on a single branch of the two-level spectrum of Hamiltonian \( H_{ST+} \), Eq.\((2)\), with \( E(\Delta, \theta) = -\sqrt{(\Delta/2)^2 + |v_0|^2} \). Equation \((12)\) then gives:

\[
\langle \Delta m \rangle = -\oint \frac{d\theta}{2\pi} \left\{ \frac{d\varphi}{d\theta} p(\theta) + \frac{d\varphi'}{d\theta} (1 - 2p(\theta))p'(\theta) \right\}, \tag{13}
\]

where \( p'(\theta) = p(\theta + \Delta \theta), \varphi'(\theta) = \varphi(\theta + \Delta \theta) \).

The two terms count the contributions from the two sweeps. The factor \( 1 - 2p(\theta) \) accounts for the fact that a transition from \( S \) to \( T \) on the first sweep reverses the direction of nuclear spin flips in the second sweep. Oscillations and sign reversal of \( \langle \Delta m \rangle \) as a function of the precession angle \( \Delta \theta \) are clearly visible in Fig.\[3\] where we have used direct numerical integration to find the transition amplitudes \( b(\theta) \) needed to evaluate expression \((13)\).

The dynamical phase contributes to \( \langle \Delta m \rangle \) through

\[
\frac{d\varphi_{\text{ad}}}{d\theta} = -\int_{-\tau}^{\tau} \frac{dt}{\hbar} \partial_\theta E(s, \theta) \partial_\theta = \frac{4\text{SO}_{HF}}{\hbar^3} \sin \theta \sinh^{-1} \left| \frac{\Delta_0}{2|v_0|} \right| \tag{14}
\]

where \( d\varphi_{\text{ad}}/d\theta \) grows logarithmically with the range of detunings \( 2\Delta_0 \) spanned by the sweep. Note that \( \partial E/\partial \theta \) is the group velocity in the Bloch band corresponding to the 1D quantum walk, Fig.\[2\].

This contribution describes DNP buildup at times long after the level crossing. The in-plane component of electron spin maintains a nonzero average value due to the presence of the spin-orbit interaction. While this remnant electron spin polarization can be small, its influence on nuclear polarization accumulates over a long time, as indicated by the \( 1/\beta \) dependence. As a result, this contribution to DNP typically dominates over that of the geometric phase \( \varphi_0(\theta) \), see Fig.\[2\].

Equation \((11)\) can be readily extended to arbitrary series of non-identical sweeps. If we take into account classical noise in the detuning \( \Delta(t) \) during each sweep, the primary effect is to flatten the function \( p(\theta) \), causing it to saturate toward the value 1/2 for all \( \theta \). Because the qualitative form of \( p(\theta) \) is preserved, however, oscillations resulting from the geometric phase contribution \( dp(\theta)/d\theta \) survive under quite general circumstances.

Resonant effects at the nuclear Larmor frequency also can be seen directly in the time dependence of electron transport properties. As the transverse polarization precesses in time, the matrix element \( v_0 \) traces out the dashed circle shown in Fig.\[1\], leading to a modulation of \( |v_0| \) at the nuclear Larmor frequency. This effect can be detected as an ac modulation of conductance (current) in the system, or as a correlation between the electron spin flip probabilities on successive sweeps through the avoided crossing. Such measurements would constitute a purely electrical detection of nuclear spin dynamics.

In summary, we have shown that the coherent evolution of nuclear spins in quantum dots can be observed through oscillations and sign reversal of the nuclear spin pumping rate, which occur due to the coherent interplay between hyperfine and spin-orbit couplings. Recent observations indicate that this interesting regime is now within experimental reach, opening new possibilities to explore many-body spin dynamics in the solid state.

We thank H. Bluhm, H.-A. Engel, S. Foletti, M. B. Hastings, J. J. Krich, E. I. Rashba, and A. Yacoby for helpful discussions, and acknowledge support from W. M. Keck Foundation Center for Extreme Quantum Information Theory, and NSF grants DMR-0906475 and DMR-0757145.

![FIG. 3: (Color online) Polarization due to a round trip through the S – T crossing with \( v_{HF} = 0.6, v_{SO} = 0.4, \Delta_0 = 50, \) and precession \( \Delta t = \omega_L \Delta t \) between forward and return sweeps, Eq.\[13\]. Here \( \omega_L \) is the nuclear Larmor frequency and \( \Delta t \) is the waiting time. Dashed lines show \( \langle \Delta m \rangle \) calculated using \( \varphi_0(\theta) \) and the dynamical phase, Eq.\[14\], neglecting Stokes phase. The oscillation phase shifts by \( \pi/2 \) for faster sweeps where \( \frac{d\varphi_{\text{ad}}}{d\theta} \) is small (see inset). Units are chosen with \( \hbar = 1 \).](https://example.com/figure3.png)
[1] R. Hanson et al., Rev. Mod. Phys. 79, 1217 (2007).
[2] A. V. Khaetskii, D. Loss, and L. Glazman, Phys. Rev. Lett. 88, 186802 (2002); Phys. Rev. B 67, 195329 (2003).
[3] J. M. Taylor, C. M. Marcus, and M. D. Lukin, Phys. Rev. Lett. 90, 206803 (2003).
[4] W. A. Coish and D. Loss, Phys. Rev. B 70, 195340 (2004).
[5] S. I. Erlingsson and Y. V. Nazarov, Phys. Rev. B 70, 205327 (2004).
[6] W. M. Witzel, R. de Sousa, S. Das Sarma, Phys. Rev. B 72, 161306(R) (2005).
[7] W. Yao, R-B. Liu, L. J. Sham, Phys. Rev. B 74, 195301 (2006).
[8] G. Chen, D. L. Bergman, L. Balents, Phys. Rev. B 76, 045312 (2007).
[9] K. Ono et al. Science 297, 1313 (2002).

[10] J. R. Petta et al. Science 309, 2180 (2005).
[11] F. H. L. Koppens et al. Science 309, 1346 (2005).
[12] K. C. Nowack et al. Science 318, 1430 (2007).
[13] A. Pfund, I. Shorubalko, K. Ensslin, R. Leturcq, Phys. Rev. B 79, 121306(R) (2009).
[14] S. Foletti, et al. arXiv:0801.3613
[15] O. N. Jouravlev, Y. V. Nazarov, Phys. Rev. Lett. 96, 176804 (2006).
[16] J. R. Petta et al. Phys. Rev. Lett. 100, 067601 (2008).
[17] M. Pioro-Ladrière et al. Nat. Phys. 4, 776 (2008).
[18] Y. Meir, Y. Gefen, O. Entin-Wohlman, Phys. Rev. Lett. 63, 798 (1989)
[19] M. S. Rudner and L. S. Levitov, Phys. Rev. Lett. 102, 065703 (2009).
[20] Y. Kayanuma, Phys. Rev. A 55, R2495 (1997).