ANISOTROPIC FLOW FROM AGS TO RHIC

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Abstract

The recent results on anisotropic flow in ultrarelativistic nuclear collisions along with recent methodical developments and achievements in the understanding of the phenomena, are reviewed. The emphasis is given to the elliptic flow results.

1 Introduction

In recent years, the subject of anisotropic flow in ultrarelativistic nuclear collisions attracts an increasing attention of heavy ion community. One of the main reasons for that is the sensitivity of anisotropic flow, and in particular elliptic flow [1], to the evolution of the system at the very early times [2].

Anisotropic flow is defined as azimuthal asymmetry in particle distribution with respect to the reaction plane (the plane spanned by the beam direction and the impact parameter). It is called flow for it is a collective phenomena, but it does not necessarily imply hydrodynamic flow. It is convenient to characterize the magnitude of this asymmetry using Fourier decomposition of the azimuthal distributions. Then the first harmonic describes so-called directed flow, and the second harmonic corresponds to elliptic flow; non-zero higher harmonics can be also present in the distribution. The corresponding Fourier coefficients, $v_n$ are used to quantify the effect [3].

The two reasons for anisotropic flow are the original asymmetry in the configuration space (non-central collisions !) and rescatterings. In a case of elliptic flow the initial “ellipticity” of the overlap zone is usually characterized by the quantity $\varepsilon = (\langle y^2 - x^2 \rangle) / (\langle y^2 + x^2 \rangle)$, assuming the reaction plane being $xz$-plane. With the system expansion the spatial anisotropy decreases. This is the reason for high sensitivity of elliptic flow to the evolution of the system in the very early times [2, 4], of the order of 2–5 fm/c, independent of the model.

Due to the lack of space we will not discuss in detail all recent developments regarding directed flow. Briefly mention a couple. In [5] a very interesting qualitative prediction is given: it is shown that the radial flow (isotropic expansion in the transverse plane) and an incomplete baryon stopping should
lead to a “wiggle” in the rapidity dependence of baryon directed flow; \(v_1(y)\) should change its sign three times with rapidity! This effect was also observed in a hydro model \([3]\). Once found it would be a strong evidence for the space-momentum correlations caused by radial flow. In a recent paper \([4]\) the important question of the role of the momentum conservation in directed flow measurements are discussed. Concrete recommendations for the analysis have been worked out.

Recently, many new results regarding elliptic flow have been obtained in all directions: experimental measurements, improving analysis methods, and theoretical understanding of the underlying physics of the phenomena. I will try to mention the most important results in all three directions, but concentrate mostly on the last two questions. A more complete picture of the recent experimental results can be found in \([5]\).

2 Improving the methods

A significant progress in theoretical description of anisotropic flow demands the accuracy in measurements. Thus the corresponding methods are evolving in the directions of improving both, the statistical uncertainties, and understanding systematics in the measurements. For the first, improving on the statistical errors, we mention extensive use of proper weights (the best would be \(w \propto v_n(y, p_t)\)), which leads to the improvement of the reaction plane resolution by 10%-20% and reduction of the statistical errors, and using the scalar-product approach \([9]\). In the scalar-product method the flow is given by:

\[
v_n(y, p_t) = \frac{\langle Q_n \cdot u(y, p_t) \rangle}{2\sqrt{\langle Q_n^a \cdot Q_n^b \rangle}},
\]

where \(u \equiv \cos(n\phi), \sin(n\phi)\) is a unit vector associated with a particle of a given rapidity and transverse momentum, and \(Q\) are flow vectors for the “full” event and subevents “a” and “b”:

\[
Q_n = \sum_i u_i,
\]

where sum is over all particles in an (sub)event. This method (which is also easy to implement and analyze) in addition to the flow angle takes into account the flow effects on the magnitude of the flow vector and as a result gives smaller statistical errors.

In is not possible to determine the reaction plane in the collision directly. Therefore, any measurement of the anisotropy in particle production with respect to the reaction plane is based on the measurements of particle azimuthal
correlations among themselves. Those correlations to a different degree (depending on what exactly is analyzed) include the contribution from the correlations that are not related to the orientation of the reaction plane (e.g. resonance decays), and often called non-flow contribution. For a reliable interpretation of the results the non-flow contribution should be estimated or, better, measured.

As an example, we discuss the estimate of non-flow contribution in STAR elliptic flow measurements [10]. An important observation for that is on the centrality dependence of the non-flow effects. The azimuthal correlation between two particles can be written as

$$\langle u_{\text{n,1}} u_{\text{n,2}}^* \rangle \equiv \langle e^{i n \phi_1} e^{-i n \phi_2} \rangle = v_n^2 + \delta_n ,$$

where $n$ is the harmonic, and the average is taken over all pairs of particles. The $\delta_n$ represents the contribution to the pair correlation from non-flow effects. Then, the correlation between two subevent flow angles is

$$\langle \cos(2(\Psi_{2}^{(a)} - \Psi_{2}^{(b)})) \rangle \propto M_{\text{sub}}(v_2^2 + g) \propto M_{\text{sub}} v_2^2 + \bar{g},$$

where $M_{\text{sub}}$ is the multiplicity for a sub-event. Here, we have taken into account that the strength of the non-flow correlations scale in inverse proportion to the multiplicity: $g \propto 1/M_{\text{sub}}$. What is important is that the non-flow contribution to $\langle \cos(2(\Psi_{2}^{(a)} - \Psi_{2}^{(b)})) \rangle$ is approximately independent of centrality. The typical shape of $\langle \cos(2(\Psi_{2}^{(a)} - \Psi_{2}^{(b)})) \rangle$, see, for example, Fig. 1, is peaked at mid-central events due to the fact that for peripheral collisions, $M_{\text{sub}}$ is small, and for central events, $v_2$ is small. In the estimates [11] of the systematic errors, the
authors set the quantity \( \tilde{g} = 0.05 \). The justification for this value was the observation of similar correlations for the first and higher harmonics (it has been investigated up to the sixth harmonic). One could expect the non-flow contribution to be of similar order of magnitude for all these harmonics, and model simulations support this conclusion. Given the value \( \tilde{g} = 0.05 \), one simply estimates the contribution from non-flow effects to the measurement of \( v_2 \) from the plot of \( \langle \cos(2(\Psi_2^{(a)} - \Psi_2^{(b)})) \rangle \) using Eq. (4) (see circle-point error-bars in Fig. 6). The relative contribution of non-flow effects is largest for very central and very peripheral bins (where, the reaction plane resolution is smallest!).

Anisotropic flow is a genuine multiparticle phenomena (which justifies use of the term collective flow). It means that if one considers many-particle correlations instead of two-particle correlations, the relative contribution of non-flow effects (due to few particle clusters) would decrease. Considering many-particle correlations, one has to subtract the contribution from correlations in the lower-order multiplets and use cumulants instead of simple correlation functions \[11\]. For example, correlating four particles, one gets

\[
\langle u_{n,1}u_{n,2}u_{n,3}^*u_{n,4}^* \rangle = v_n^4 + 2 \cdot 2 \cdot v_n^2 \delta_n + 2 \delta_n^2. \tag{5}
\]

In this expression, two factors of “2” in front of the middle term correspond to the two ways of pairing \((1,3)(2,4)\) and \((1,4)(2,3)\) and account for the possibility to have non-flow effects in the first pair and flow correlations in the second pair and vice versa. The factor “2” in front of the last term is due to the two ways of pairing. The pure four-particle non-flow correlation is omitted from this expression (see below on the possible magnitude of such a contribution). If one subtracts from the expression (5) twice the square of the expression (3), one is left with only the flow contributions

\[
\langle \langle u_{n,1}u_{n,2}u_{n,3}^*u_{n,4}^* \rangle \rangle \equiv \langle u_{n,1}u_{n,2}u_{n,3}^*u_{n,4}^* \rangle - 2\langle u_{n,1}u_{n,2}^* \rangle^2 = -v_n^4, \tag{6}
\]

where the notation \( \langle \langle ... \rangle \rangle \) is used for the cumulant. A very elegant way of calculating cumulants in flow analysis with the help of the generating function is proposed in \[11\]. The simulations \[27\], see Fig. 2, confirm that using 4-particle cumulants reliably removes non-flow contributions. It also shows that even in the presence of genuine 4-particle correlations (due to clusters decaying into 4-particles, such were introduced into simulations) those correlations are combinatorially suppressed compared to real flow correlations and there is no real need for use of higher order cumulants in flow analysis.

The high precision results available in modern high statistics and large acceptance experiments become sensitive to another effect usually neglected in
flow analysis, namely, event-by-event flow fluctuations. The latter can have two different origins: “real” flow fluctuations – fluctuations at fixed impact parameter and fixed multiplicity (see, for example, [12, 13]), and impact parameter variations among events from the same centrality bin in a case where flow does not fluctuate at fixed impact parameter. Note that these fluctuations affect any kind of analysis, including the “standard” one based on pair correlations. The reason is that any flow measurements are based on correlations between particles, which are sensitive only to certain moments of the distribution in $v_2$. In the pair correlation approach with the reaction plane determined from the second harmonic, the correlations are proportional to $v^2$. Averaging over events gives $\langle v^2 \rangle$, which in general is not equal to $\langle v \rangle^2$. The 4-particle cumulant method involves the difference between 4-particle correlations and (twice) the square of the 2-particle correlations. It is usually assumed that this difference comes from non-flow correlations. Note, however, that this difference ($\langle v^4 \rangle - \langle v^2 \rangle^2 \neq 0$) could be due to flow fluctuations. Let us consider an example where the distribution in $v$ is flat from $v = 0$ to $v = v_{\text{max}}$. Then, a simple calculation would lead to the ratio of the flow values from the standard 2-particle correlation method and 4-particle cumulants as large as

$$\frac{\langle v^2 \rangle^{1/2}}{\langle v^2 \rangle^2 - \langle v^4 \rangle}^{1/4} = 5^{1/4} \approx 1.5.$$ 

3 The physics of elliptic flow

Many important developments in this area: better understanding of the transverse momentum dependence of anisotropic flow in low $p_t$ region with the help of the “blast wave” model [14, 15], attempts [18] to describe $v_2(p_t)$ in high $p_t$ region accounting for the parton energy loss – “jet quenching”, calculation in a parton cascade model [19], analysis of the anisotropies in Color Glass Condensate [13], and detailed analysis of the elliptic flow in the hydro models [23, 24, 25].

A very interesting development in this field is an attempt to calculate elliptic flow in Color Glass Condensate – classical field approach to describe ultrarelativistic nuclear collisions. One of the important consequences in this approach [13] is strong event-by-event fluctuations in $v_2$. As it has been already discussed, such fluctuations would manifest themselves in the difference of the flow results derived from 2- and 4-particle correlations.
3.1 Transverse momentum dependence

I would like to mention here what is usually referred to as “hydro inspired”, “blast wave” or “expanding shell”, models. Such models consider particle production from a thermal source in a form of an expanding shell with the radial expansion velocity having some azimuthal modulation. The case of directed flow was discussed in \[16\]. That model was used to fit E877 data and gave quite reasonable results \[16\]. The model was generalized for the elliptic flow case in \[14\]. Later it was used to fit STAR data and was further generalized for the case of the elliptic shape shell \[13\]. In this approach:

\[
v_2(p_t) = \frac{\int_0^{2\pi} d\phi_b \cos(2\phi_b) I_2(\alpha_t) K_1(\beta_t)(1 + 2s_2 \cos(2\phi_b))}{\int_0^{2\pi} d\phi_b I_0(\alpha_t) K_1(\beta_t)(1 + 2s_2 \cos(2\phi_b))},
\]

where \(I_0, I_2,\) and \(K_1\) are modified Bessel functions, and where \(\alpha_t(\phi_b) = (p_t/T_f) \sinh(\rho(\phi_b)),\) and \(\beta_t(\phi_b) = (m_t/T_f) \cosh(\rho(\phi_b)).\) The assumptions of this model are boost-invariant longitudinal expansion and freeze-out at constant temperature \(T_f\) on a thin shell, which expands with a transverse rapidity exhibiting a second harmonic azimuthal modulation, \(\rho(\phi_b) = \rho_0 + \rho_a \cos(2\phi_b).\) Here, \(\phi_b\) is the azimuthal angle (measured with respect to the reaction plane) of the boost of the source element on the freeze-out hyper-surface \[14\], and \(\rho_0\) and \(\rho_a\) are the mean transverse expansion rapidity \((v_0 = \tanh(\rho_0))\) and the amplitude of its azimuthal variation, respectively. In Fig. 3, the fit to the minimum-bias data with \(s_2 = 0\) is shown as the dotted lines. The relatively poor fit led the authors to introduce a spatially anisotropic freeze-out hyper-surface, with one extra parameter, \(s_2,\) describing the variation in the azimuthal density of the source elements, \(\propto 2s_2 \cos(2\phi_b).\) This additional parameter leads to a good description of the data, shown as the solid lines in Fig. 1. A positive value of the \(s_2\) parameter would mean that there are more source elements moving in the direction of the reaction plane. The model predicts a specific dependence of the elliptic flow on the particle mass. This mass-dependent effect is larger for lower temperatures \((T_f)\) and larger transverse rapidities \((\rho_0).\)

The behavior of \(v_2(p_t)\) at large transverse momenta is also very interesting. One of the possibilities is that the anisotropy at such transverse momenta is due to path length dependent nuclear modification of the parton fragmentation function (jet quenching). High \(p_t\) parton produced in the direction of long axis of the overlapping region exhibits more inelastic (in addition to elastic) collisions than that emitted along the short axis. It results in smaller probability to fragment into high \(p_t\) hadron. The effect depends on the density of the media and thus the observed anisotropy could serve as a measure on
this very density \[18\] (and features of the energy loss itself). The transverse momenta, where \(v_2(p_t)\) saturates could also help in understanding the origin of the particles in the region of 2–5 GeV/c: do they acquire their transverse momentum due to multiple scattering or they come from a fragmentation of even higher \(p_t\) parton? The preliminary STAR data \[28\], Fig. 4, support the idea of flow saturation at high \(p_t\).

3.2 Hydro and low density limits

The values of elliptic flow measured at RHIC are comparable to that in hydrodynamic models. There was a clear disagreement at lower energies. As it has been mentioned in the introduction, for elliptic flow one needs rescatterings. Denser the matter and more rescattering means higher elliptic flow. The current understanding is that in limit of zero mean free path, the hydro limit, one gets the largest possible values of elliptic flow.

Interesting that the flow values obtained in parton cascade calculations \[19\] at different transport opacities could in principle significantly exceed the flow values from hydro calculations. It raises a question about validity of the assumption that the largest values of flow can be reached in hydro model. Two lines shown in Fig, 6 correspond to the results of this model for two values of transport opacity, corresponding to 35 (upper line) and 13 times higher gluon density as given by HIJING model. Note, however, that this opacity was calculating assuming 1 mb gluon transport cross section and assuming the hadronization picture when the number of gluons equals the number of hadrons. If one would, for example, consider a system as a constituent quark gas, one would have to increase the cross section approximately by 3 times and
quark density by approximately 2 to 3 times. That would give the opacities very close to that needed to describe the data.

In the hydro limit elliptic flow is basically proportional to the original spatial ellipticity of the nuclear overlapping region \[ v_2 \propto \varepsilon. \] In the opposite limit, usually called the low density limit \[ v_2 \propto \varepsilon dN/dy / S, \] where \( S \) is the area of the overlapping zone. It results in a different centrality dependencies of the elliptic flow in these two limits. The comparison of the results on elliptic flow from this point of view was first done in \[ 21 \]. In this picture, the transition to the deconfinement would lead to some wiggles in \( v_2/\varepsilon \) dependence, ("kinks") \[ 22, 20, 21 \].

One indication that at RHIC energies flow is still proportional to the par-
article density can be seen from Fig. 7 (taken from [26]) which show that \( v_2(\eta) \)
closely follows \( dN_{ch}/d\eta \). The 3D hydro calculations [23] also cannot describe
the pseudorapidity dependence of elliptic flow, once more indicating that the
hydro description could be not correct in spite of the large values of \( v_2 \) mea-
sured at RHIC.

Taking the recent (year 1, \( \sqrt{s_{NN}} = 130 \) GeV) STAR results on elliptic
flow from 4-particle cumulants [27] (no non-flow effects) and plotting them
along preliminary NA49 [8] and E877 results, Fig. 8, also suggests that even
at RHIC energies elliptic flow continue to rise with particle density. (Note
possible systematic errors in Fig. 8 of the order of 10-20% due to uncertain-
ties in centrality measurements. However, this uncertainty does not alter the
general trend). Decisive measurements could be already at full RHIC ener-
gies of \( \sqrt{s_{NN}} = 200 \) GeV. If \( v_2 \) would continue to rise with particle density
(which increases for about 15% in this energy range), it could give difficulties
to hydrodynamic interpretation.

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