MAGNETOHYDRODYNAMICS OF POPULATION III STAR FORMATION
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ABSTRACT
Jet-driving and fragmentation processes in a collapsing primordial cloud are studied using three-dimensional MHD nested grid simulations. Starting from a rotating magnetized spherical cloud with a number density of \( n_c \approx 10^7 \) cm\(^{-3} \), we follow the evolution of the cloud until the adiabatic core (or protostar) formation epoch, \( n_c \approx 10^{22} \) cm\(^{-3} \). We calculate 36 models parameterizing the initial magnetic \( \gamma_0 \) and rotational \( \beta_0 \) energies. The evolution of collapsing primordial clouds is characterized by the ratio of the initial rotational energy to the magnetic energy, \( \gamma_0/\beta_0 \). The Lorentz force significantly affects cloud evolution when \( \gamma_0 > \beta_0 \), while the centrifugal force dominates the Lorentz force when \( \beta_0 > \gamma_0 \). When the cloud rotates rapidly with an angular velocity of \( \Omega_0 > 10^{-17} \) (\( n_c/10^3 \) cm\(^{-3} \)) s\(^{-1} \) and \( \beta_0 > \gamma_0 \), fragmentation occurs before protostar formation, but no jet appears after protostar formation. On the other hand, when the initial cloud has a magnetic field of \( B_0 > 10^{-9} \) (\( n_c/10^3 \) cm\(^{-3} \)) G and \( \gamma_0 > \beta_0 \), a strong jet appears after protostar formation without fragmentation. Our results indicate that Population III protostars frequently show fragmentation and protostellar jets. Population III stars are therefore born as binary or multiple stellar systems; as in present-day star formation, they can drive strong jets that disturb the interstellar medium significantly, and thus they may induce the formation of next-generation stars.

Subject headings: binaries: general — cosmology: theory — early universe — ISM: jets and outflows — MHD — stars: formation

1. INTRODUCTION
Magnetic fields play an important role in present-day star formation. Observations indicate that molecular clouds have magnetic field strengths of the order of \( \mu G \) and magnetic energies comparable to their gravitational energies (Crutcher 1999). These strong fields affect star formation processes significantly. For example, protostellar jets, which are ubiquitous in star-forming regions, are considered to be driven from protostars by the Lorentz force (Blandford & Payne 1982; Pudritz & Norman 1986). Jets influence gas accretion onto protostars and distort the ambient medium. In addition, the angular momentum of the cloud is removed by magnetic braking and protostellar jets. Tomisaka (2000) showed using a two-dimensional magnetohydrodynamics (MHD) calculation that 99.9% of the angular momentum was transferred from the center of the cloud by magnetic effects. The removal of angular momentum makes protostar formation possible in a parent cloud that has a much larger specific angular momentum than a protostar. Nevertheless, magnetic effects in primordial gas clouds have been ignored in many studies, because magnetic fields in the early universe are supposed to be extremely weak. However, recent studies indicate that magnetic fields of moderate strength existed even in the early universe. Ichiki et al. (2006) showed that cosmological fluctuations produced magnetic fields before the epoch of recombination. These fields were sufficiently large to seed the magnetic fields in galaxies. Langer et al. (2003) proposed a generation mechanism for magnetic fields at the epoch of reionization. They found that magnetic fields in intergalactic matter are amplified up to \( \sim 10^{-11} \) G. These fields can therefore increase up to \( \sim 10^{-7} \)–\( 10^{-5} \) G in the first collapsed object having number density \( n \approx 10^3 \) cm\(^{-3} \). These fields may influence the evolution of primordial gas clouds and formation of Population III stars.

Under spherical symmetry including hydrodynamical radiative transfer, many authors have carefully investigated both the present-day (e.g., Larson 1969; Masunaga & Inutsuka 2000) and primordial (e.g., Omukai & Nishi 1998) star formation processes. A significant difference between present-day and primordial star formation exists in the thermal evolution of the collapsing gas cloud because of differences in the abundance of dust grains and metals. In present-day star formation, the gas temperature in molecular clouds is \( \sim 10 \) K. These clouds collapse isothermally with polytropic index \( \gamma \approx 1 \) (\( P \propto \rho^\gamma \)) for \( n_c \lesssim 10^{11} \) cm\(^{-3} \), where \( n_c \) denotes the central number density of the collapsing cloud. Then the gas becomes adiabatic (\( \gamma \approx 7/5 \)) at \( n_c \approx 10^{12} \) cm\(^{-3} \) and an adiabatic core (or the first core) is formed. When the number density reaches \( n_c \approx 10^{16} \) cm\(^{-3} \), molecular hydrogen is dissociated (\( \gamma \approx 1.1 \)), and the cloud begins to collapse rapidly again. When the number density reaches \( n_c \approx 10^{20} \) cm\(^{-3} \), the equation of state becomes hard again (\( \gamma \approx 5/3 \)). The protostar forms at \( n_c \approx 10^{21} \) cm\(^{-3} \). On the other hand, primordial gas clouds have temperatures of \( \sim 200–300 \) K at \( n_c \approx 10^3 \) cm\(^{-3} \) (Omukai 2000; Omukai et al. 2005; Bromm et al. 2002; Abel et al. 2002; Yoshida et al. 2006). These clouds collapse, keeping \( \gamma \approx 1.1 \) for a long range of \( 10^5 \) cm\(^{-3} \) to \( 10^{16} \) cm\(^{-3} \). After the central density reaches \( n_c \approx 10^{16} \) cm\(^{-3} \), the thermal evolution of the primordial collapsing cloud begins to coincide with that of a present-day cloud (for details, see Fig. 1 of Omukai et al. 2005). The difference in thermal evolution between present-day and primordial clouds arises when \( n_c \approx 10^{16} \) cm\(^{-3} \).

In present-day star formation, two distinct flows (molecular outflow and optical jets) are frequently observed in star-forming regions. The driving mechanism of these flows is not yet understood, while recent numerical simulations (e.g., Tomisaka 2002;...
Banerjee & Pudritz 2006; Machida et al. 2005a, 2006a, 2008a) showed two distinct flows driven from the respective cores (the first core and the protostar), a low-velocity flow with a wide opening angle driven from the adiabatic core (the first core) and a well-collimated high-velocity flow driven from the protostar. They expected that the former corresponds to the molecular outflow, and the latter to the optical jet. In addition, many numerical simulations have shown fragmentation of the adiabatic core and thus possible formation of binary or multiple stellar systems. In contrast, primordial protostars are formed without the prior formation of an adiabatic core (i.e., the first core), because the thermal pressure increases smoothly, keeping $\gamma \simeq 1.1$ for $n_c \lesssim 10^{16}$ cm$^{-3}$. Even when the primordial cloud is strongly magnetized, the wide-angle outflow corresponding to the present-day molecular outflow (driven from the adiabatic core formed at $n_c \simeq 10^{11}$ cm$^{-3}$) may not appear, but a well-collimated jet (driven from the protostar) is ejected at $n_c \gtrsim 10^{21}$ cm$^{-3}$. In addition, it is expected that, in the collapsing primordial cloud, fragmentation occurs rarely for $n_c \lesssim 10^{16}$ cm$^{-3}$, because the cloud continues to collapse and the perturbation-inducing fragmentation cannot grow sufficiently.

Another major difference between present-day and primordial star formation exists in their magnetic evolution. In present-day star formation, neutral gas is well coupled with ions for $n_c \lesssim 10^{12}$ cm$^{-3}$ and $n_c \gtrsim 10^{15}$ cm$^{-3}$, while the magnetic field is dissipated by Ohmic dissipation for $10^{12}$ cm$^{-3} < n_c \lesssim 10^{15}$ cm$^{-3}$ (for details, see Nakano et al. 2002). Machida et al. (2007) showed that $\sim 99\%$ of the magnetic field in the collapsing cloud is dissipated for $10^{12}$ cm$^{-3} < n_c \lesssim 10^{15}$ cm$^{-3}$. On the other hand, magnetic evolution in a primordial gas cloud was carefully investigated by Maki & Susa (2004, 2007), who found that the magnetic field couples strongly with the primordial gas during all phases of the collapse, as long as the initial field strength is weaker than $B_0 \lesssim 10^{-3}(\alpha/10^{-3})^{0.35}$ G. Maki & Susa (2007) also showed that the ionization fraction is sufficiently high for the magnetic field to couple with the gas, even in the accretion phase after protostar formation. In summary, the magnetic field is largely dissipated by Ohmic dissipation before protostar formation in present-day clouds, while the magnetic field can continue to be amplified without dissipation in primordial clouds.

Recent cosmological simulations implied that a single massive star is formed without fragmentation in the first collapsed object formed at $n_c \simeq 10^3$ cm$^{-3}$ (Abel et al. 2000, 2002; Bromm et al. 2002; Yoshida et al. 2006). However, their simulations investigated cloud evolution only for $n_c \lesssim 10^{17}$ cm$^{-3}$. Since protostars form at $n_c \simeq 10^{21}$ cm$^{-3}$ (Omukai & Nishi 1998), protostars have not yet formed in their simulations. In addition, only one model (or a few models at most) can be calculated by their method, which calculates for the cloud with the lowest angular momentum because it collapses first (Bromm et al. 2002; Yoshida et al. 2006). Because of the lower angular momentum, fragmentation occurs rarely in these clouds. On the other hand, Machida et al. (2008a) showed that fragmentation occurs frequently after the equation of state becomes hard for $n_c \gtrsim 10^{17}$ cm$^{-3}$, indicating that binary or multiple stellar systems can also form in the early universe.

The magnetic field in the collapsing cloud is closely related to the fragmentation or formation of binary or multiple stellar systems. In present-day star formation, the magnetic field strongly suppresses rotation-driven fragmentation (Price & Bate 2007; Hennebelle & Teyssier 2008; Machida et al. 2008c). For primordial clouds, magnetic effects on fragmentation are still unknown.

In this study, we investigate the evolution of weakly magnetized primordial clouds and the formation of Population III stars using three-dimensional MHD simulations and show the driving condition of jets from proto–Population III stars and the fragmentation condition in collapsing primordial cloud cores. The structure of the paper is as follows. The framework of our models and the numerical method are given in § 2. The numerical results are presented in § 3. We discuss the fragmentation and jet-driving conditions in § 4 and summarize our results in § 5.

### 2. MODEL

Our initial conditions are almost the same as those of Machida et al. (2006c, 2008b). To study cloud evolution, we use the three-dimensional ideal MHD nested grid code. We solve the MHD equations including self-gravity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P - \frac{1}{4\pi} \mathbf{B} \times (\nabla \times \mathbf{B}) - \rho \nabla \phi, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (3)$$

$$\nabla^2 \phi = 4\pi G \rho, \quad (4)$$

where $\rho$, $\mathbf{v}$, $P$, $\mathbf{B}$, and $\phi$ denote the density, velocity, pressure, magnetic flux density, and gravitational potential, respectively. For the gas pressure, we use a barotropic relation that approximates the result of Omukai et al. (2005). They considered detailed thermal and chemical processes in collapsing primordial gas, adopting a simple one-zone model in which the core collapses at approximately the free-fall timescale and the size of the core is about the Jeans length, as in the Larson-Penston self-similar solution (Penston 1969; Larson 1969). We fit our function to the thermal evolution derived by Omukai et al. (2005) as a function of the density (see Fig. 1 of Machida et al. 2008b), and the pressure is calculated using the fitted function. Therefore, our equation of state is barotropic.

As the initial conditions of the clouds, we consider the density profile of the critical Bonnor-Ebert sphere (Ebert 1955; Bonnor 1956) with perturbations,

$$\rho(r, \varphi) = \begin{cases} 
 \rho_{BE}(r) \left[1 + \delta \rho(r, \varphi)\right], & \text{for } r < R_0, \\
 \rho_{BE}(R_0) \left[1 + \delta \rho(r, \varphi)\right], & \text{for } r \geq R_0,
\end{cases} \quad (5)$$

where $\rho_{BE}(r)$ denotes the density distribution of the critical Bonnor-Ebert sphere. The central (number) density of the Bonnor-Ebert sphere is set at $\rho_{BE}(0) = 2.1 \times 10^{-21}$ g cm$^{-3}$ ($n_{BE} = 10^3$ cm$^{-3}$). The factor $f$ denotes the density enhancement factor, and the density is increased by a factor $f = 1.86$ to promote the collapse. The initial cloud has therefore a central density of $n_0 = 1.86 \times 10^3$ cm$^{-3}$. The initial temperature is set at 250 K, and the radius of a Bonnor-Ebert sphere is $R_0 = 6.5$ pc, corresponding to the nondimensional radius of Bonnor-Ebert sphere, $\xi_{max} = 6.5$. Outside this radius, a uniform gas density of $n_{BE}(R_0) = 130$ cm$^{-3}$ is assumed. To promote fragmentation, a small $m = 2$ mode density perturbation is imposed on the spherical cloud, such as

$$\delta \rho = A_\varphi (r/R_0)^2 \cos 2\varphi, \quad (6)$$

where $A_\varphi$ is the amplitude of the perturbation and $A_\varphi = 0.01$ is adopted in all models. The total mass contained inside $R_0$ is $M_c = 1.2 \times 10^4 \, M_\odot$. The density enhancement factor $f$ specifies the ratio of the thermal to the gravitational energy $\alpha_0 (= E_{th}/|E_{grav}|)$, where $E_{th}$ and $E_{grav}$ are the thermal and gravitational...
energies of the initial cloud. The initial cloud has \( \alpha_0 = 0.5 \) when \( f = 1.86 \) and \( A_\psi = 0 \).

The initial cloud rotates around the z-axis at a uniform angular velocity of \( \Omega_0 \) and has a uniform magnetic field \( B_0 \) parallel to the z-axis (or rotation axis). The initial model is characterized by two nondimensional parameters: the ratios of the rotational energy to the gravitational energy \( \beta_0 (= E_{rot}/E_{grav}) \) and of the magnetic energy to the gravitational energy \( \gamma_0 (= E_{mag}/E_{grav}) \), where \( E_{rot} \) and \( E_{mag} \) are the rotational and magnetic energies.

We made 36 models using different values for these two parameters. All the models examined here and their parameters (\( \beta_0 \) and \( \gamma_0 \)) are summarized in Table 1. These tabulated values are calculated in the cases of spherical clouds, i.e., \( \beta_0 = 0.1 \). We examine a large parameter range of \( \beta_0 = 10^{-6}, 10^{-1}, \) \( \gamma_0 = 0 - 0.2 \). For convenience, \( \beta_0 \) is referred to as the “initial rotational energy,” and \( \gamma_0 \) as the “initial magnetic energy.” The angular velocity \( \Omega_0 \) and the magnetic field \( B_0 \) are also summarized in Table 1. In addition, we estimate the mass-to-flux ratio, 

\[
M/\Phi = \frac{M}{\pi R_0^2 B_0}, \tag{7}
\]

where \( M \) and \( \Phi \) denote the mass contained within the cloud radius \( R_0 \) and the magnetic flux threading the cloud, respectively.

There exists a critical value of \( M/\Phi \) below which a cloud is supported against gravity by the magnetic field. For a cloud with a uniform density, Mouchovis & Spitzer (1976) derived a critical mass-to-flux ratio,

\[
\left( \frac{M}{\Phi} \right)_{\text{crit}} = \frac{\zeta}{3\pi} \left( \frac{5 \sqrt{B_{\text{norm}}}}{G} \right)^{1/2}, \tag{8}
\]

where \( \zeta = 0.53 \) for uniform spheres (see also Mac Low & Klessen 2004). For convenience, we use the mass-to-flux ratio normalized by the critical value,

\[
\left( \frac{M}{\Phi} \right)_{\text{norm}} \equiv \left( \frac{M}{\Phi} \right) / \left( \frac{M}{\Phi} \right)_{\text{crit}}, \tag{9}
\]

which is summarized in Table 1. In our conditions, the normalized mass-to-flux ratio is related to the initial magnetic energy \( \gamma_0 \) as

\[
\left( \frac{M}{\Phi} \right)_{\text{norm}} = 1.3 \gamma_0^{1/2}. \tag{10}
\]
A detailed discussion of the effect of the magnetic field can be found in Machida et al. (2005b, 2007), where the importance of the mass-to-flux ratio at the center is emphasized.

We adopted the nested grid method (for details, see Machida et al. 2005b, 2006b) to obtain high spatial resolution near the center. Each level of a rectangular grid has the same number of cells (=128; 128; 64), with the cell width \( h(l) \) depending on the grid level \( l \). The cell width is halved with every increment of the grid level. The highest level of the grid changes dynamically; a new finer grid is generated whenever the minimum local Jeans length \( k_J \) falls below \( 8h(l_{\text{max}}) \), where \( h \) is the cell width. The maximum level of grids is restricted to \( l_{\text{max}} = 30 \). Since the density is highest in the finest grid, generation of a new grid ensures the Jeans condition of Truelove et al. (1997) with a safety factor of 2.

We begin our calculations with three grid levels (\( l = 1–3 \)). The box size of the initial finest grid \( l = 3 \) is chosen to be \( 2R_0 \), where \( R_0 \) is the radius of the critical Bonnor-Ebert sphere. The coarsest grid (\( l = 1 \)) then has a box size of \( 2^3R_0 \). Mirror symmetry with respect to \( z = 0 \) is imposed. A boundary condition is imposed at \( r = 2^3R_0 \), where the magnetic field and ambient gas rotate at an angular velocity of \( \Omega_0 \) (for details, see Matsumoto & Tomisaka 2004).

3. RESULTS

3.1. Typical Jet Model

First, we show cloud evolution with a clear jet from a non-fragmented core. Figure 1 shows the cloud’s evolution before protostar formation \( n_e < 10^{21} \text{ cm}^{-3} \) from the initial stage for model 23. We define the protostar formation epoch as the stage of \( n_e = 10^{21} \text{ cm}^{-3} \). Model 23 has parameters of \((\beta_0, \gamma_0) = (10^{-4}, 2 \times 10^{-3})\), and the initial cloud has a magnetic field strength of \( B_0 = 10^{-6} \text{ G} \) and an angular velocity of \( \Omega_0 = 2.3 \times 10^{-16} \text{ s}^{-1} \).

Figure 1a shows the initial spherical cloud (i.e., the Bonnor-Ebert sphere) threaded by a uniform magnetic field. Figures 1b–1d show the cloud structure around the center when the central density reaches \( n_e = (b) 4.1 \times 10^{17} \text{ cm}^{-3}, (c) 7.0 \times 10^{18} \text{ cm}^{-3}, \) and \( (d) 5.5 \times 10^{18} \text{ cm}^{-3} \), respectively. The density contours projected on the sidewall in these figures indicate that the central region becomes oblate as the cloud collapses because of the magnetic field and rotation, both of which are amplified by the cloud collapse. Figures 1e–1d also show that the magnetic field lines gradually converge toward the center as the central density increases.

Figure 1e shows the cloud structure at \( n_e = 9.2 \times 10^{17} \text{ cm}^{-3} \), where a thin disk is formed at the center of the cloud. In this
The disk is threaded by magnetic field lines that converge strongly toward the center of the cloud. The cloud structure at $n_c = 3.3 \times 10^{18} \text{ cm}^{-3}$ is shown in Figure 1f, in which the magnetic field lines are strongly converged, but slightly twisted. In Figures 1e and 1f, clear shocks are seen above and below the disk. Figures 1a–1f show that the magnetic field lines are hardly twisted before protostar formation ($n_c \lesssim 10^{21} \text{ cm}^{-3}$), because the collapse timescale is shorter than the rotational timescale.

Figure 2 shows cloud evolution after protostar formation ($n_c > 10^{21} \text{ cm}^{-3}$). The top and middle rows show the density and velocity distributions on the $z = 0$ and $y = 0$ planes, respectively, and the bottom row shows the cloud structures and configurations of the magnetic field lines in three dimensions. Figures 2a–2c show the cloud structure 1.4 days after protostar formation. In this model, when the central density reaches $n_c \simeq 7.7 \times 10^{21} \text{ cm}^{-3}$, a protostar surrounded by a strong shock is formed near the center of the cloud. The protostar has a mass of $M = 7.6 \times 10^{-3} M_\odot$ and...
Fig. 3.—Time sequence of model 21 [(β0, γ0) = (10^{-4}, 2 \times 10^{-7})]. The density distribution (false color and contours) and velocity vectors (arrows) on the z = 0 plane are plotted in the top row. The magnetic field lines (black and white streamlines) and high-density region (red isodensity surface) in three dimensions are plotted in the bottom row. In each panel in the bottom row, the density contours (false color and contour lines) and velocity vectors (thin arrows) are also projected on each wall surface. The central number density $n_c$, elapsed time $t$, and elapsed time after protostar formation $t_p$ are denoted in each panel of the top row. The grid level ($l$) is shown in the top left corner of each panel.

a radius of $r = 1.1 R_\odot$ at its formation epoch. The shock front corresponding to the protostellar surface is seen in Figure 2a ($r \approx 0.005$ AU; contours of $n \sim 10^{21}$ cm$^{-3}$). The magnetic field lines are strongly converged to the center of the cloud, but slightly twisted, at this epoch. The jet begins to be ejected 1.68 days after protostar formation. Figures 2d–2f show the cloud structure 2.82 days after protostar formation. The jet ejected near the protostar is indicated by the red contour in Figure 2e and the red transparent surface in Figure 2f, which mark the boundary between the inflow ($v_r < 0$) and jet ($v_r > 0$) regions. The magnetic field lines are twisted significantly inside the jet region because of the rapid rotation of the protostar. The jet affects the density distribution; the butterfly-shaped density distribution is shown in Figures 2e and 2f (see contour lines near the protostar projected on the right sidewall). Strong shock waves are generated at the upper and lower ends of the jet, reflecting a strong mass ejection.

Figures 2g–2l show the cloud structure 3.48 days after protostar formation, the last stage of this calculation. Until the last stage, the jet continues to extend, as seen in Figures 2e and 2h. In Figure 2i, the isodensity surface appears as a shallow cone on the disk, which corresponds to the outer shock in Figure 2g. The magnetic field lines inside the jet region are more strongly twisted with time, as shown in Figures 2f and 2i. At the end of the calculation, the jet has a maximum speed of $v_{\text{max}} = 66.3$ km s$^{-1}$, and it extends out to 0.05 AU. The mass of the protostar reaches $M_{\text{ps}} = 8.5 \times 10^{-3} M_\odot$, while the mass of the outflowing gas is $M_{\text{out}} = 1.1 \times 10^{-3} M_\odot$. About 10% of the accreting matter is therefore ejected from the center of the cloud via the protostellar jet.

3.2. Typical Fragmentation Model

Next, we describe cloud evolution exhibiting fragmentation. Figure 3 shows cloud evolution for model 21 with $(\beta_0, \gamma_0) = (10^{-4}, 2.5 \times 10^{-7})$, which corresponds to a magnetic field strength of $B_0 = 10^{-6}$ G and an angular velocity of $\Omega_0 = 2.3 \times 10^{-16}$ s$^{-1}$ at the initial stage. Model 21 has the same rotational energy as model 23 (described above), but a magnetic energy $10^{-4}$ times that of model 23. Figures 3a and 3b show the cloud structure just before protostar formation $(n_c = 1.0 \times 10^{17}$ cm$^{-3}$). As in model 23, a thin disk forms at the center of the cloud before the protostar forms. The disk maintains an almost axisymmetric structure in the plane perpendicular to the rotation axis. Figure 3b also shows an hourglass configuration of the magnetic field lines, similar to model 23.

Figures 3c and 3d show the cloud structure 2.4 days after protostar formation. The central region is deformed from a disk structure into a ring at $n_c \approx 8 \times 10^{18}$ cm$^{-3}$, and the ring breaks into fragments. Fragments are located at $(x, y) \approx (\pm 0.035$ AU, $\pm 0.055$ AU), with long spiral tails that are the remnants of the ring (Fig. 3c). The magnetic field lines are distributed strongly along the spiral tails (Fig. 3d). Figures 3e and 3f show the cloud structure 8.8 days after protostar formation. The fragments appear rounded in Figure 3e. The central density of the fragments reaches $n_c \approx 1.8 \times 10^{22}$ cm$^{-3}$ at this epoch. Fragments are surrounded by their respective disks with spiral arms, and the disks grow with time (compare Figs. 3c and 3e). At the end of the calculation, each fragment (i.e., each protostar) has a mass...
of $\sim 1.3 \times 10^{-2} M_\odot$ and a radius of $\sim 1.4 R_\odot$. The separation between fragments is $0.89$ AU. As shown in Figure 3f, the magnetic field lines are distributed along the spiral tails, indicating that the vertical component of the magnetic field is weaker than the other components (i.e., $B_z \ll B_r, B_\theta$). This is due to the weak magnetic field around the protostars, and as a result, the magnetic field lines easily follow the orbital motion of the fragments.

This fragmentation model does not exhibit jet formation at the end of the calculation, despite having the same initial rotation speed as the previous jet model. This is also due to the weak magnetic field. Note that the calculation was halted 8.8 days after the protostellar core formation, and we do not reject the possibility that a jet appears at a later stage.

### 3.3. Fragmentation and Magnetic/Rotational Energy

Figure 4 shows the final stage in the plane of the initial magnetic ($\gamma_0$, $x$-axis) and rotational ($\beta_0$, $y$-axis) energies for every model. The density distribution in the $z = 0$ plane is shown in each panel. Models are classified into four types: fragmentation (blue background), nonfragmentation (red background), merger (green background), and no-collapse (gray background). The number density at

![Image](image-url)
fragmentation epoch ($n_f$) and separation between furthest fragments ($R_f$) for each model are listed in columns (9) and (10) of Table 1. In fragmentation models, calculations were stopped when the Jeans condition was violated in any grid except the finest. This occurs either when fragments escape from the finest grid or when gas far from the center becomes denser than that in the finest grid. In nonfragmentation models, we stopped calculations after we confirmed that fragmentation was not likely to occur around the protostar. The fragmentation reproduced here is therefore restricted to cases in which fragmentation occurs near the central region and just after protostar formation. In merger models $[\gamma_0, \alpha_0] = (10^{-2}, 2 \times 10^{-3})$, ($10^{-3}, 2 \times 10^{-3}$)], fragments merge to form a single core at the center of the cloud after fragmentation. The merged core did not undergo fragmentation again, although we performed calculations with the merger models for a sufficiently long time. In the no-collapse model of $\gamma_0 = (0.1, 0.2)$, the cloud oscillates without collapse because this model has the large magnetic and rotational energies.

In Figure 4, fragmentation models are distributed in the top left, indicating that a cloud tends to fragment when the initial cloud has a weaker magnetic field and faster rotation: a magnetic field suppresses fragmentation, while rotation promotes it. This effect of the magnetic field and rotation on fragmentation is similar to that in present-day star formation (Hosking & Whitworth 2004; Machida et al. 2004, 2005a, 2008c; Price & Bate 2007; Hennebelle & Teyssier 2008). In fragmentation models, those with larger rotational energies undergo fragmentation in the earlier evolutionary phases with wider separation. For example, in the fourth column in Figure 4, model 4 ($\beta_0 = 0.1$) fragments at $n_c = 9.5 \times 10^{17} \text{cm}^{-3}$ with a separation of $R_{\text{sep}} \approx 4.3 \text{AU}$, while model 22 ($\beta_0 = 10^{-4}$) does so at $n_c = 1.6 \times 10^{19} \text{cm}^{-3}$ with a separation of $R_{\text{sep}} \approx 0.2 \text{AU}$. Machida et al. (2008b) also showed that a cloud with a faster rotation exhibits wider separation. On the other hand, when clouds have the same rotational energies at the initial stage, models with a weaker magnetic field undergo fragmentation at earlier evolutionary phases with wider separation. For example, in the fourth row in Figure 4, model 20 ($\gamma_0 = 2 \times 10^{-9}$) fragments at $n_c = 7.9 \times 10^{18} \text{cm}^{-3}$, while model 22 ($\gamma_0 = 2 \times 10^{-5}$) does so at $n_c = 1.6 \times 10^{19} \text{cm}^{-3}$. Moreover, the models with much larger magnetic energies (models 23 and 24) produce a single compact core at the center of the cloud.

Fragmentation and nonfragmentation models are clearly separated in Figure 4. In addition, merger models are located at the boundary between the two. Of the fragmentation models, those with weaker magnetic fields and faster rotation tend to have wider separation. In nonfragmentation models, clouds have more compact cores in models with stronger magnetic fields and slower rotations. These features indicate clearly that rotation promotes fragmentation and the magnetic field suppresses fragmentation in primordial, as well as present-day star formation.

The models presented here have a small amplitude of the non-axisymmetric density perturbation ($A_x = 0.01$) at the initial stage. If a larger amplitude was set at the initial stage, the boundary between the fragmentation and nonfragmentation models might change slightly. Machida et al. (2008e) discussed the fragmentation condition for a present-day magnetized cloud and indicated that it does not depend qualitatively on the initial amplitude of the nonaxisymmetric perturbation. Moreover, Machida et al. (2008b) showed that the fragmentation condition for a primordial nonmagnetized cloud depends slightly on the initial amplitude of the nonaxisymmetric perturbation. We therefore expect that this initial amplitude hardly affects fragmentation, even in primordial magnetized collapsing clouds.

### 3.4. Jets and Magnetic/Rotational Energy

Figure 5 shows the final states in the plane of the initial magnetic ($\gamma_0$, $x$-axis) and rotational ($\beta_0$, $y$-axis) energies, contrasting the jets at almost the same evolutionary stage of $n_c \approx 10^{22} – 10^{23} \text{cm}^{-3}$. Each panel shows the density and velocity distribution in the $y = 0$ plane, and a thick red contour denotes the boundary between inflow ($v_y < 0$) and outflow ($v_y > 0$) regions. The gas flows out of the central region inside the red contour. As listed in Table 1, jets appear in 13 of 36 models. The maximum speed of the jet ($v_{\text{jet}}$) is also listed in Table 1. We call these “jet models” and the others “nonjet models.” Panels without thick red contours indicate nonjet models. In all the jet models, a jet appears only after a protostar forms. After protostar formation, the rotation timescale becomes shorter than the collapse timescale because of the accumulation of angular momentum and the hardness of the equation of state at the cloud center, and a strong centrifugal force drives the jet via disk wind mechanisms (Tomisaka 2002; Banerjee & Pudritz 2006; Machida et al. 2008a). The jet disturbs the density distribution near the protostar. As shown in Figure 5, the jet supplies gas above and below the disk, and these regions become denser for the jet models, while they remain less dense for the nonjet models.

The jet models appear in the bottom right in Figure 5, indicating that a jet is driven in a strongly magnetized but slowly rotating cloud. It should also be noted that the nonjet models coincide with the fragmentation models, except models 31 and 32. In other words, almost all the clouds experience either jet formation or fragmentation. In the fragmentation models, the angular momentum of the parent cloud is distributed to both the orbital and spin angular momenta of the fragments, and fragmentation therefore reduces the spin angular momentum of the protostar. This reduction in the spin angular momentum of the fragments suppresses jet formation.

Two mechanisms can drive the jet, a disk wind, and a magnetic pressure-driven wind (Machida et al. 2008a). In model 28, the protostar manages to drive the jet, while the initial cloud has a quite small magnetic field ($\beta_0 = 2 \times 10^{-5}$). When the cloud has a weak magnetic field, a jet is considered to be driven by the magnetic pressure gradient force, not by the disk wind mechanism. The jet structure in model 28 is similar to the magnetic bubble as reproduced by Tomisaka (2002) and Banerjee & Pudritz (2006). Tomisaka (2002) showed that when the magnetic field around the central object is extremely weak ($\beta_p \gg 1$, where $\beta_p$ is the plasma beta), the magnetic field is amplified by the spin of the central object, and the magnetic pressure drives a jet. On the other hand, a prominent jet appears in model 29, which is adjacent to model 28. Since the initial magnetic field in model 29 is stronger than that in model 28, the jet in model 29 is considered to be driven mainly by the disk wind mechanism.

Although the initial clouds have strong magnetic fields in models 11, 30, and 36, very weak jets appear. The weakness of the jet in models 30 and 36 is due to insufficient rotation to drive a strong jet, and that in model 11 is due to a low amplification of the magnetic field during the collapse; magnetic field amplification is related to rotation, and spin-up is also related to magnetic field strength as discussed in § 4.1. Therefore, a moderate rotation speed, as well as a moderate magnetic field strength, is necessary to drive a strong jet. In summary, a jet appears in models distributed in the bottom right (i.e., models with stronger magnetic field and slower rotation), and models showing strong jets are limited to the range of $10^{-5} \leq \beta_0 \leq 10^{-3}$ and $10^{-3} \leq \gamma_0 \leq 10^{-1}$.

Figure 6 shows the configuration of magnetic field lines (black and white streamlines) and the structure of the jet (transparent
red isovelocity surface) for each model. In the jet models, the magnetic field lines are twisted inside the jet region. Nearly axisymmetric jets are seen in models 18, 23, 24, 28, 29, 34, and 35. These jets have hourglass-like configurations of magnetic field lines, where the poloidal component is more dominant than the toroidal component, as shown in Figures 1f and 2c. These configurations of the magnetic field lines can easily drive a jet by the disk wind mechanism (e.g., Blandford & Payne 1982). Nonaxisymmetric jets are seen in models 17, 30, and 36. They are caused by a nonaxisymmetric density distribution at the protostar formation epoch. Such a distribution is also reflected by nonaxisymmetric circumstellar disks, represented by a red isodensity surface in Figure 6. For example, model 30 exhibits a nonaxisymmetric jet (transparent red isovelocity surface) driven from the barlike density distribution (red isodensity surface) at the root of the jet.

In the fragmentation models, the configurations of the magnetic field lines are disturbed, and the toroidal field tends to be more dominant than the poloidal field, as shown in models 16

Fig. 5.—Final states on the $y = 0$ plane plotted against parameters $\gamma_0$ and $\beta_0$. The model numbers are shown in the top left corner outside each panel. Density (false color and contours) and velocity vectors (arrows) are plotted in each panel. The red line represents the border between inflow and outflow. The central number density $n_c$, grid level ($l$), grid scale, and velocity unit are shown in each panel.
These configurations of the magnetic field lines indicate a weak field, insufficient to drive a jet. The weak poloidal field is attributed to oblique shocks on the surface of the disk envelope, and the disturbance of the field lines is attributed to the orbital motion and spin of the fragments. The spin of a fragment winds the magnetic fields around its rotation axis, and the field strength is amplified. In further stages, the magnetic pressure may drive a jet.

4. DISCUSSION

4.1. Magnetic Flux–Spin Relation

Figure 7 shows the evolutionary track of the magnetic field strength (x-axis) and angular velocity (y-axis) from the initial state for some models. The x-axis indicates the square root of the magnetic pressure $B_0^2/(8\pi)^{1/2}$ divided by the square root of the thermal pressure $P_{th}^{1/2}$ [hereafter the normalized magnetic field,
The ratio of the magnetic energy to the thermal energy (or the inverse square root of the plasma beta, $\beta_p^{-1/2}$), which corresponds to the ratio of the magnetic flux density and angular velocity at the cloud center are plotted. The $x$-axis indicates the square root of the magnetic pressure $[B_c/(8\pi P_c)^{1/2}]$ divided by the square root of the thermal pressure ($P_c$). The $y$-axis represents the angular speed $\Omega_c$ divided by the free-fall rate $[(4\pi G \rho_c)^{1/2}]$. The symbols represent the magnetic field and the angular velocity at the initial state (asterisks) and at each central density (diamonds; $n_c = 10^5, 10^6, 10^7, \ldots, 10^{19}$ cm$^{-3}$). Each line denotes the evolutionary path from the initial state ($n_0 = 1.86 \times 10^3$ cm$^{-3}$). The arrows show the direction of evolution. The thick gray band denotes the magnetic flux–spin relation $\Omega_c^2/[(0.25)^2 \times 4\pi G \rho_c] + B_c^2/[(0.2)^2 \times 8\pi P_c] = 1$ (see eq. [15]).

In Figure 7, all arrows point in the same (top right) direction, indicating that the normalized magnetic field and angular velocity in each model increase as the central density increases. For comparison, the relationship

$$\frac{\Omega_c}{(4\pi G \rho_c)^{1/2}} \propto \frac{B_c}{(8\pi P_c)^{1/2}}$$

is plotted with the solid line in the bottom right corner of Figure 7, and all the models evolve in a direction almost parallel to this line. This implies that both the normalized magnetic fields and angular velocities are amplified during the early phase of the collapse, with almost the same growth rate as shown in equation (11). In Figure 7, each evolutionary track is almost parallel to the solid line, but the gradient of each track is slightly larger than that of the solid line. This indicates that the normalized angular velocity is growing at a slightly higher rate than the normalized magnetic field. The growth rates of the magnetic field and angular velocity depend on the geometry of the collapse (spherical or disklike), as shown in Machida et al. (2005b, 2006b, 2007). When the cloud collapses spherically, the growth rates of the magnetic field and angular velocity are

$$B, \Omega \propto \rho^{2/3}.$$  

Thus, the growth rate of the normalized angular velocity is

$$\frac{\Omega_c}{(4\pi G \rho_c)^{1/2}} \propto \rho^{1/6}.$$
On the other hand, the growth rate of the normalized magnetic field is different from the right-hand side of equation (13), because the thermal pressure also increases as the cloud collapses. Since we numerically fitted the thermal evolution of a collapsing protostellar cloud using a spherical symmetry calculation (see Fig. 1 of Machida et al. 2007), the polytropic index $\gamma$ ($P \propto \rho^{1+\gamma}$) cannot be expressed in a simple manner. However, the polytropic index $\gamma$ can be approximated at $\gamma \approx 1.1$ ($P \propto \rho^{1.1}$) for $10^4 \text{ cm}^{-3} \leq n \leq 10^{18} \text{ cm}^{-3}$. Using this expression, the growth rate of the normalized magnetic field is described as

$$\frac{B_c}{(8\pi P_c)^{1/2}} \propto \rho^{-0.12}. \quad (14)$$

Thus, the growth rate of the normalized angular velocity (eq. [13]) is slightly larger than that of the normalized magnetic field (eq. [14]). In a collapsing cloud, when angular velocity is transferred by magnetic braking, the growth rate of the normalized angular velocity may be smaller than that in equation (13). However, magnetic braking becomes effective only after protostar formation, because the magnetic field lines begin to twist during this phase, as shown in $\S$ 3.1. Thus, before protostar formation, magnetic braking is ineffective, and the growth rate of the normalized angular velocity is considered to be well described by equation (13).

As the cloud collapses, the normalized angular velocities converge to $\Omega_c/(4\pi GP_c)^{1/2} \approx 0.25$, and the normalized magnetic fields converge to $B_c/(8\pi P_c)^{1/2} \approx 0.2$, as shown in Figure 7. Thus, the normalized magnetic field and angular velocity converge to the so-called flux-spin relation,

$$\frac{\Omega_c^2}{(0.25)^24\pi G\rho_c} + \frac{B_c^2}{(0.2)^28\pi P_c} = 1. \quad (15)$$

This relation is represented by the gray band in Figure 7. The numerical factors in equation (15) are determined empirically. Equation (15) is an extension of the flux-spin relation of Machida et al. (2005b, 2006b) for primordial star formation. Figure 7 indicates that the magnetic field strengths and angular velocities converge to the flux-spin relation represented by equation (15), even in primordial magnetized clouds. If an initial stage is located at the bottom left side of Figure 7, points indicating the two normalized variables move toward the top right according to equations (13) and (14) in the nearly spherical collapse phase. When the point reaches the gray band of equation (15), the geometry of the cloud’s collapse changes from a sphere to a disk. After the transition, the point remains in the vicinity of the gray band with small oscillations.

When an evolutionary track converges to the flux-spin relation, the central cloud deforms from a sphere to a disk, because the central angular velocity and/or magnetic field have been amplified during the nearly spherical collapse phase. For example, model 23, shown in Figure 1, produces a thin disk at $n_c \approx 10^{13} \text{ cm}^{-3}$, at which point the evolutionary track converges to the gray band in Figure 7. After the evolutionary track converges to the flux-spin relation, it oscillates around its convergence value, as shown in Figure 7. We also noticed that strong accretion shocks are generated on the surface of the disk when the model converges to the flux-spin relation. Shock generation is also related to the oscillation of the evolutionary track after convergence. When the evolutionary track oscillates around the flux-spin relation, a new shock is generated intermittently inside the collapsing disk. This intermittent shock generation results in a nested structure of the shocks. Such nested shocks were first reproduced in the simulated collapse of a rotating isothermal disk by Norman et al. (1980).

The convergence to the flux-spin relation indicates a power-law amplification of angular velocity and magnetic field during the disklike collapse phase, as well as the spherical collapse phase. In the case of homologous collapse of an isothermal disk, this power-law amplification is expressed as $\Omega_c, B_c \propto \rho_c^{1.25}$ (Machida et al. 2005b). For a polytropic gas, the growth rate is modified as $\Omega_c, B_c \propto \rho_c^{1.05}$. The normalized variables are also modified as $\Omega_c/(4\pi GP_c)^{1/2} \propto \rho_c^{(1.1-1)/2}$ and $B_c/(8\pi P_c)^{1/2} = \text{const}$ (see also Machida et al. 2008b). In our case of $\gamma \approx 1.1$, the normalized angular velocity would increase slightly in proportion to $\rho_c^{0.05}$, and such a small increase is barely seen in Figure 7, because of the significant amplitude of the oscillation.

### 4.2. Fragmentation/Jet Conditions

All the models examined in this paper exhibit convergence to the flux-spin relation of equation (15) as shown in Figure 7. The convergence point within the flux-spin relation depends on the initial angular velocity and magnetic field and characterizes the fate of a cloud. A model with initially fast rotation and a weak magnetic field starts its evolutionary track at a point in the top left, and the evolutionary track converges to the horizontal gray band. Such a model is called a “rotation-dominated model.” On the other hand, a model with initially slow rotation and a strong magnetic field starts its evolutionary track at a point in the bottom right, and the evolutionary track converges to the vertical gray band. Such a model is called a “magnetically dominated model.” We classified all the models into rotation ($M$) and magnetically ($B$) dominated models, as summarized in column (12) of Table 1.

Only the rotation-dominated models exhibit fragmentation, because the rotation is sufficiently amplified to promote fragmentation, and magnetic braking is insufficient to prevent fragmentation. Machida et al. (2008b) examined fragmentation of nonmagnetized protostellar clouds and showed that fragmentation occurs in the case in which the normalized angular velocity at the center converges to

$$\frac{\Omega_c}{\sqrt{4\pi G\rho_c}} \approx 0.25 \quad (16)$$

before protostar formation. A magnetically dominated model never converges to equation (16), because the evolutionary track never reaches the horizontal band in Figure 7. A rotation-dominated model satisfies equation (16) when the initial normalized angular velocity is

$$\frac{\Omega_0}{\sqrt{4\pi G\rho_0}} \approx 0.25 \left(\frac{n_0}{n_{cr}}\right)^{1/6}, \quad (17)$$

where $n_{cr} \approx 10^{17} \text{ cm}^{-3}$ and $n_0 = 1.86 \times 10^3 \text{ cm}^{-3}$ denote the upper bound of the number density and the initial central density, respectively. For $n \lesssim n_{cr}$, the equation of state is approximated by the polytrope with $\gamma \approx 1.1$. We used the relationship $\Omega_c \propto \rho_c^{-0.25}$ for deriving equation (17). Examining the initial conditions of the rotation-dominated models, all except models 31–33 satisfy equation (17), and these models exhibit fragmentation, consistent with the fragmentation condition. The fragmentation condition of equation (16) is therefore valid even for magnetized primordial clouds.
Even though the fragmentation condition (eq. [16]) is realized, fragmentation does not occur in rotation-dominated models 31–33. This is because the evolutionary tracks in these models manage to reach the condition of equation (16) before protostar formation. Fragmentation can occur after a thin disk forms, as shown in Machida et al. (2008b). Thus, fragmentation does not occur in these models, because an adequate thin disk has not yet formed. In these models, fragmentation may occur when the initial cloud has a lower central density of $n_0 < 10^5 \text{ cm}^{-3}$, because the cloud can collapse for a long period, and an adequate thin disk forms before protostar formation.

The fragmentation condition can be rewritten in terms of the rotational energy. Assuming that the center of the cloud undergoes homologous collapse with rigid rotation, the parameter $\beta(=E_{\text{rot}}/|E_{\text{grav}}|)$ is expressed as $\beta = \Omega^2/(4\pi G \rho)$. Equation (16) therefore indicates that fragmentation occurs when the rotational energy reaches ~6% of the gravitational energy, due to spinning of the cloud center.

All the magnetically dominated models presented here show jet formation. The condition for jet formation coincides with the flux-spin relation of the magnetically dominated models,

$$\frac{B_c}{\sqrt{8\pi P_c}} \simeq 0.2.$$  \hspace{1cm} (18)

When the central magnetic field is amplified to the level shown in equation (18) before protostar formation, a jet appears just after a protostar forms. This condition is expressed as

$$\frac{B_0}{\sqrt{8\pi P_0}} \geq 0.2 \left(\frac{n_0}{n_{\text{cr}}}\right)^{0.12},$$  \hspace{1cm} (19)

using the power-law amplification of the magnetic field during the spherical collapse (eq. [14]). Equation (18) can be rewritten as $\beta_p \simeq 25$, since $B_c/(8\pi P)^{1/2} = \beta_p^{-1/2}$. This indicates that a jet is driven by the protostar when the magnetic field is amplified and the plasma beta reaches $\beta_p \simeq 25$ before protostar formation. Equation (18) cannot be realized in rapidly rotating clouds, because the evolutionary track converges to the horizontal gray band in Figure 7 and never reaches the vertical band.

Although the magnetically and rotation-dominated models are clearly distinct (see Figs. 4 and 5), some models on the border between them present mixed features. Models 11 and 17, for example, exhibit both fragmentation and jets. In these models, fragments merge to form a single core after fragmentation, while other fragmentation models do not show merger until the end of the calculation. After the merger, the jet begins to be driven from the merged core in these models.

In the fragmentation models, we do not observe a jet driven from either fragment. This indicates that fragmentation occurs in rotation-dominated models where the magnetic field is too weak to drive the jet. However, it may be possible that the magnetic field amplified by the spin of the protostars drives the jet in later stages. We conclude that the condition of equation (18) is valid for jets formed within a few tens of days after protostar formation.

### 4.3. Magnetic Fields and Rotation Periods of Proto–Population III Stars

In our calculations, both the normalized angular velocity and magnetic field never exceeded conditions shown in equations (16) and (18) (i.e., the gray band in Fig. 7) in any model. Therefore, for a given central density, the maximum angular velocity and magnetic field strength are expressed as

$$\Omega_{\text{max}} \leq 0.25 \sqrt{4\pi G \rho_c},$$

$$B_{\text{max}} \leq 0.2 \sqrt{8\pi P_c}.$$  \hspace{1cm} (20) (21)

When the protostar is formed at $n_c = 10^{21} \text{ cm}^{-3}$, it has a maximum angular velocity of

$$\Omega_{\text{ps}} = 1.1 \times 10^{-5} \text{ s}^{-1},$$  \hspace{1cm} (22)

which corresponds to a rotation period of $P > 6.6$ days. The maximum magnetic field strength is given by

$$B_{\text{ps}} = 2.6 \times 10^4 \text{ G}.$$  \hspace{1cm} (23)

at the protostar formation epoch ($n = 10^{21} \text{ cm}^{-3}$), when the thermal pressure adopted in our calculation is used. We measured the magnetic field strengths ($B_{\text{ps}}$) and angular velocities ($\Omega_{\text{ps}}$) at the protostar formation epoch for every model, and the resulting values are listed in columns (7) and (8) of Table 1. For models with parentheses, we cannot follow cloud evolution to the protostar formation epoch, because fragmentation occurs in relatively early phases. For these models, the magnetic field and angular velocity just before fragmentation are listed. As listed in Table 1, protostars at their formation epoch have angular velocities in the range $3.3 \times 10^{-7} \text{ s}^{-1} < \Omega_{\text{ps}} < 1.1 \times 10^{-5} \text{ s}^{-1}$, which corresponds to rotation periods of 6.6 days $< P < 220$ days. These angular velocities are well restricted by the value shown in equation (22). The magnetic fields are in the range $0.01 \text{ G} < B_{\text{ps}} < 3.1 \times 10^4 \text{ G}$ and are also restricted by the value shown in equation (23).

Observations indicate that present-day protostars have magnetic fields of at most ~1 kG (Johns-Krull et al. 1999a, 1999b, 2001; Bouvier et al. 2007). According to our simulations, Population III protostars can possess magnetic fields about 10 times stronger than those of present-day protostars. The smallness of the magnetic field in present-day protostars is attributed to magnetic dissipation during the protostellar collapse phase. Machida et al. (2007) studied the collapse of magnetized clouds in the present-day star formation and showed that protostellar magnetic fields are at most ~kG, because the magnetic field is largely dissipated by Ohmic dissipation in the late phase of collapse, $10^{11} \leq n \leq 10^{15} \text{ cm}^{-3}$ (see also Nakano et al. 2002). On the other hand, the magnetic field dissipation is not effective in primordial collapsing clouds, as shown in Maki & Susa (2004, 2007). Population III stars therefore possess stronger magnetic fields than present-day stars, if they are formed via the processes of the magnetically dominated models.

The magnetic fields and angular velocities listed in Table 1 are the values at the moment of protostar formation, at which the mass of protostars is only $\sim 10^{-3} - 10^{-2} \text{ M}_{\odot}$. Since stars acquire a large fraction of their mass in a subsequent accretion phase, the magnetic field strength and angular velocity may change as the stars evolve further. Angular momentum is transferred by magnetic interaction between the protostar and circumstellar disk. The magnetic field can be amplified by convection inside the protostar. However, since the purpose of this paper is to investigate magnetic effects in collapsing primordial clouds, we do not discuss subsequent evolution of the magnetic field and angular velocity. To determine the magnetic field and angular velocity of
Population III stars, further long-term calculations are necessary, including a model of stellar evolution.

5. SUMMARY

In this study, we calculated cloud evolution from the stage of \( n_c = 10^5 \text{ cm}^{-3} \) until a protostar forms (\( \approx 10^{22} \text{ cm}^{-3} \)) for 36 models, parameterizing the initial magnetic field strength and rotation, to investigate the effect of magnetic fields in collapsing primordial clouds. In Figure 8, the fates of all the clouds are plotted in the plane of the parameters \( \gamma_0 \) and \( \beta_0 \). The top and right axes show the mass–to–magnetic flux ratio \((M/\Phi)_{\text{norm}}\) normalized by its critical value (see eq. [9]). The circles, squares, triangles, and crosses indicate models showing fragmentation, jets, both fragmentation and jets, and neither fragmentation nor jets, respectively. In models located in the top right (diamond), the cloud oscillates around the initial state without collapse, because a strong magnetic field and rapid rotation suppress the collapse of the cloud. The solid line in Figure 8 represents \( \beta_0 = \gamma_0 \), indicating that magnetic energy dominates the rotational energy in models distributed above the line, and vice versa. All the models showing fragmentation lie above this line, while almost all the models showing jets lie below it. The solid line clearly separates the models; fragmentation occurs, but no jet appears when \( \beta_0 > \gamma_0 \), and a jet appears after protostar formation without fragmentation when \( \beta_0 < \gamma_0 \). In addition, the solid line almost coincides with the boundary between the rotation-dominated and the magnetically dominated models (see Table 1 for which category each model falls into). As a result, the evolution of the collapsing primordial cloud is controlled more by the centrifugal force than by the Lorentz force when \( \beta_0 > \gamma_0 \), while the Lorentz force dominates when \( \gamma_0 > \beta_0 \).

The top and right axes of Figure 8 show the magnetic field strength and angular velocity, respectively, of the initial cloud.
They can be scalable at any initial density as \((n_0/10^3 \text{ cm}^{-3})^{2/3}\), assuming spherical collapse, which is approximated well in the early phase. A jet is driven when the initial cloud has a magnetic field of \(B_0 \gtrsim 10^{-9}(n_0/10^3 \text{ cm}^{-3})^{2/3} \text{ G}\) if the cloud rotates slowly with \(\Omega \lesssim 4 \times 10^{-17}(n_0/10^3 \text{ cm}^{-3})^{-2/3} \text{ s}^{-1}\). This condition corresponds to that in Machida et al. (2006c) and is also consistent with the condition of equation (19).

A strong jet is expected in primordial star formation. The power of a jet, e.g., its mass ejection rate, is considered to be controlled by the accretion rate, as indicated in present-day star formation; the mass ejection rate of a jet is 1/10 of the mass accretion rate onto the protostar. The accretion rate of primordial star formation is expected to be considerably larger than that of present-day star formation and produces a stronger jet. The lifetime of the jet also seems to be controlled by accretion in the present-day star formation; the jet stops when mass accretion stops. For Population III stars, gas accretion continues during their entire lifetimes (Omukai & Palla 2001, 2003). Therefore, a jet may also continue during the entire lifetime of the protostar, and the strong jet propagates to disturb the surrounding medium significantly. The disturbance of the medium could trigger subsequent star formation, as frequently observed in present-day star formation.

On the other hand, when the initial cloud’s magnetic field is weaker than \(10^{-9} \text{ G}\), magnetic effects can be ignored, at least before protostar formation. Assuming power-law growth of \(B_0 \propto n_c^{2/3}\), the critical strength of the magnetic field, \(B_0 = 10^{-9} \text{ G}\) at \(n_c = 10^3 \text{ cm}^{-3}\), corresponds to \(B_0 = 5 \times 10^{-13} \text{ G}\) at \(n_c = 0.01 \text{ cm}^{-3}\), which is much stronger than the background magnetic field of \(10^{-18} \text{ G}\) derived by Ichiki et al. (2006). However, when the magnetic field is amplified to \(B \sim 10^{-9}(n_c/10^3 \text{ cm}^{-3})^{-2/3} \text{ G}\) by some mechanism, the magnetic field can affect the collapse of the primordial cloud. Even if a cloud has a magnetic field weaker than the critical strength \(B_0 = 10^{-9} \text{ G}\), the magnetic field may play an important role after protostar formation. Tan & Blackman (2004) analytically studied the evolution of accretion disks around the first stars, suggesting that magnetic fields amplified in the circumstellar disk eventually gave rise to protostellar jets during the protostellar accretion phase.

Rotation promotes fragmentation when the first collapsed object has an angular velocity of \(\Omega_0 \gtrsim 10^{-17}(n_0/10^3 \text{ cm}^{-3})^{2/3} \text{ s}^{-1}\), as shown in Figure 8. This condition coincides with that given by equation (17). Fragmentation is expected to produce binary or multiple stellar systems. When a multiple stellar system forms, some stars can be ejected by close encounters. At the protostar formation epoch, the protostar has a mass of \(M \sim 10^{-3} M_\odot\). The ejected proto–Population III stars may evolve to metal-free brown dwarfs or low-mass stars. When a binary component in a multiple stellar system is ejected from the parent cloud by protostellar interaction, a low-mass metal-free binary may also appear in the early universe. Suda et al. (2004) indicated that the recently discovered extremely metal-poor ([Fe/H] \(<-5\) stars (Christlieb et al. 2002; Frebel et al. 2005) formed as binary components from metal-free gas and were then polluted by companion stars during stellar evolution. Komiyama et al. (2007) showed that the binary frequency in Population III stars is comparable to or larger than that of present-day stars. In order to confirm the ejection scenario, further long-term calculations are required.

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REFERENCES

Machida, M. N., Matsumoto, T., Hanawa, T., & Tomisaka, K. 2005a, MNRAS, 362, 382
———. 2006b, ApJ, 645, 1227
Machida, M. N., Tomisaka, K., & Hanawa, T. 2005b, MNRAS, 362, 369
Machida, M. N., Omukai, K., Matsumoto, T., & Inutsuka, S. 2008b, ApJ, 677, 813
Machida, M. N., Tomisaka, K., & Matsumoto, T. 2004, MNRAS, 348, L1
Machida, M. N., Tomisaka, K., Matsumoto, T., & Inutsuka, S. 2008c, ApJ, 677, 327
Maki, H., & Sasa, H. 2004, ApJ, 609, 467
Masunaga, H., & Inutsuka, S. 2000, ApJ, 531, 350
Matsumoto, T., & Tomisaka, K. 2004, ApJ, 616, 266
Mouschovias, T. Ch., & Spitzer, L. 1976, ApJ, 210, 326
Nakano, T., Nishi, R., & Umebayashi, T. 2002, ApJ, 573, 199
Nakano, M. L., Wilson, J. R., & Barton, R. T. 1980, ApJ, 239, 968
Okumai, K. 2000, ApJ, 534, 809
Okumai, K., & Nishi, R. 1998, ApJ, 508, 141
Okumai, K., & Palla, F. 2001, ApJ, 561, L55
———. 2003, ApJ, 589, 677
Okumai, K., Tsuribe, T., Schneider, R., & Ferrara, A. 2005, ApJ, 626, 627
Penston, M. V. 1969, MNRAS, 144, 425
Price, D. J., & Bate, M. R. 2007, MNRAS, 377, 77
Pudritz, R. E., & Norman, C. A. 1986, ApJ, 301, 571
Suda, T., Aikawa, M., Machida, M. N., Fujimoto, M. Y., & Iben, I. J. 2004, ApJ, 611, 476
Tan, J. C., & Blackman, E. G. 2004, ApJ, 603, 401
Tomisaka, K. 2000, ApJ, 528, L41
———. 2002, ApJ, 575, 306
Truelove, J. K., Klein, R. I., McKee, C. F., Holliman, J. H., Howell, L. H., & Greenough, J. A. 1997, ApJ, 489, L179
Yoshida, N., Omukai, K., Hernquist, L., & Abel, T. 2006, ApJ, 652, 6