On the Poincaré and Gauge symmetry of a model where vector and axial vector interaction get mixed up with different weight

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A (1+1) dimensional model where vector and axial vector interaction get mixed up with different weight is considered with a generalized masslike term for gauge field. Through Poincaré algebra it has been made confirm that only a Lorentz covariant masslike term leads to a physically sensible theory as long as the number of constraints in the phase space is two. With that admissible masslike term, phase space structure of this model with proper identification of physical canonical pair has been determined using Diracs’ scheme of quantization of constrained system. The bosonized version of the model remains gauge non-invariant to start with. Therefore, with the inclusion of appropriate Wess-Zumino term it is made gauge symmetric. An alternative quantization has been carried out over this gauge symmetric version to determine the phase space structure in this situation. To establish that the Wess-Zumino fields allocates themselves in the un-physical sector of the theory an attempts has been made to get back the usual theory from the gauge symmetric theory of the extended phase-space without hampering any physical principle. It has been found that the role of gauge fixing is crucial for this transmutation.

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I. INTRODUCTION

In terms of fundamental interaction, Quantum Electrodynamics (QED) in (1+1) dimension can be categories in two different classes. The first way of description that came in the literature was originated from vector type of interaction between matter and gauge fields. The models which belong to this class are well known vector Schwinger model [1] and Thiring-Wess model [2]. The other way of description originated from chiral interaction between matter and gauge fields. Chiral Schwinger model [3] along with its different variants [4-8] and Chiral Thiring-Wess model [9, 10] are the example of this class. In the Chiral Schwinger model [3] and in its different variants [4-8], we find that Vector and axial vector interaction get mixed up with equal weight. Few years ago, the authors in [11] presented a model where unlike Chiral Schwinger model, vector and axial vector interactions did not mix up with equal weight. Few extensions over this model is also found in [12, 13]. The mixing of interaction with different weight may be regarded as a generalized version of QED (GVQED) which covers all the fundamentally different interaction and their mixing [11, 12]. The beauty of this model is that it is capable of interpolating both the QED and chiral QED. Both the Schwinger model [1] and the Chiral Schwinger model [3] can be achieved through the different choice its mixing weight factor of interaction. For unit weight factor it describes the Chiral Schwinger model and for vanishing weight it describe the vector Schwinger model [1].

Standard quantization scheme furnishes that these two models are fundamentally different so far theoretical spectrum and confinement aspect of fermion are concerned [15-17]. Needless to mention that Schwinger model [1] and its chiral generation, e.g., Chiral Schwinger model [3, 11], and the GVQED as presented by in [11-13], which covers the both into its own are of considerable interest because of their ability to describe different physical aspects which are found to exist even in (3+1) dimension. Schwinger model acquired popularity not only for its ability of describing mass generation via dynamical symmetry breaking [1, 15], but also it can describe the confinement property of fermion in lower dimension [1, 15] which is a real (3+1) dimensional aspect of QCD. On the other hand, Chiral Schwinger model is capable of describing mass generation as well like vector Schwinger model [3], however fermions are found to get liberated here which can be considered as lower dimensional de-confining state of fermion [3-5]. Since the GVQED presented in [11, 12], interpolates both the Schwinger model and Chiral generation of that, it is natural that all the surprises involved in the Schwinger model and Chiral Schwinger models lies significantly in this GVQED. All these

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models along with the GVQED [11, 12] are so rich in describing, different surprises like dynamical mass generation, confinement and de-confinement aspects of fermion, that till now investigation over these models are carried out [8, 10, 13, 37] and these models still remains as a fertile field to carry out further investigations. Our objective in this work is to carry out few investigations over the GVQED coined in [11] concerning the Poincaré and gauge symmetry. An attempt is also made here to single out the real physical canonical pairs embedded within the phase space of the system. It is true that a systematic quantization of this model is available in [11], however the definite identification of real physical canonical pairs lying within the phase space is found to be absent. In order to make it a compliment to the quantization part of the work [11], again quantization of this model has been carried out using Diracs’ scheme of quantization of constrained system. Besides, the gauge current of the model is anomalous which leads to a gauge non-invariant structure. So quantization of the gauge invariant version of this model would also be of considerable interest. In this context, gauge symmetric construction is made here extending the phase space with the inclusion of appropriate Wess-Zumino action [38] and in presence of that Wess-Zumino term an extension towards an alternative quantization is also made here to determine the canonical pair of fields which describe the Fock-space. As a natural corollary, it is shown that the physical contents of the theory remains identical, even after the extension of phase space by the use of the technique available from the work of Falck and Kramer in [39]. It is shown here explicitly that an appropriate gauge fixing is capable of mapping the Wess-Zumino added action onto the initial gauge non-invariant effective action. The plan of the paper is as follows.

In Sec. II, through the Poincaré algebra investigation has been carried out towards the search of the appropriate structure of masslike term which will be able to lead to a physically sensible theory starting from a very generalized masslike term. Sec. III, deals with the Diracs’ scheme of quantization of constrained system where we have attempted to single out the the real physical canonical pairs in a transparent manner to make this part a complement to the work [11]. An alternative quantization of the gauge invariant version of this model is made in Sec. IV. In Sec. V, it is shown that an appropriate gauge fixing can map the gauge invariant theory of the extended phase space onto the usual gauge non-invariant structure of it.

II. TO FIND OUT THE LORENTZ TRANSFORMATION OF THE FIELDS AND EVALUATION OF THE REQUIREMENT TO BE PHYSICALLY SENSIBLE

A model where we find the mixing of both vector and axial vector interaction with different weight is given by the following generating functional

$$Z(A) = \int d\psi d\bar{\psi} \exp[i \int d^2 x L_F].$$

with

$$L_F = \bar{\psi} \gamma^\mu [i \partial_\mu + e \sqrt{\pi} A_\mu(1 - r \gamma_5)] \psi.$$

The integration over the fermionic degrees of freedom $\psi$ leads to a determinant which is singular in nature [11, 16, 17]. In order to remove the singularity we need to regularize the theory. After proper regularization if we express the fermionic determinant in terms of auxiliary scalar field $\phi$, we get

$$Z(A) = \int d\phi \exp[i \int d^2 x L_B].$$

with

$$L_B = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + e A^\mu (\partial_\mu + r \partial_\nu) \phi + \frac{e^2}{2} (\alpha A_0^2 + \beta A_0 A_1 + \gamma A_1^2),$$

where $\partial_\mu = \epsilon_{\mu\nu} \partial^\nu$ and $\epsilon^{01} = +1$. A generalized masslike term has been included here as counter term in place of standard $\frac{1}{2} ae^2 A_\mu A^\mu$ term since we are intended to study whether any other alternative masslike term can serve as a physically sensible counter term for regularization like the chiral Schwinger model [5, 8]. The parameters $\alpha, \beta$ and $\gamma$, therefore, stand as the regularization ambiguity parameter. Needless to mention that in this situation ambiguity emerged out during the process of regularization in order to remove the divergence of the fermionic determinant. If we now take into account the kinetic term of the back ground electromagnetic field the lagrange density then turns into

$$L_B = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + e A^\mu (\epsilon_{\mu\nu} \partial^\nu + r g_{\mu\nu} \partial^\nu) \phi + \frac{1}{2} \frac{e^2}{2} (\alpha A_0^2 + 2 \beta A_0 A_1 + \gamma A_1^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$
Starting with this generalized masslike term we now proceed to investigate which type of term leads to a physically sensible theory. The word physically sensible implies a structure that not only maintains physical Lorentz invariance but also leads to an exactly solvable nature at the same time. To this end, we would like to study the Lorentz transformation property of the fields and the Poincaré algebra of the theory in an explicit manner [5]. In this context, we need to calculate the momenta of the fields describing the theory. From the standard definition the momenta corresponding to the fields $\phi$, $A_0$ and $A_1$ are found out:

$$\pi_\phi = \dot{\phi} - eA_1 + erA_0,$$

(6)

$$\pi_0 = 0,$$

(7)

$$\pi_1 = A_1 - A'_0.$$

(8)

Here $\pi_\phi$, $\pi_0$, and $\pi_1$ are the momenta corresponding to the fields $\phi$, $A_0$ and $A_1$. For this theory $\Omega_1 = \pi_0 \approx 0$, is the primary constraint. A Legendre transformation leads to the following canonical Hamiltonian density.

$$\mathcal{H}_c = \frac{1}{2}(\pi_\phi^2 + \phi'^2 + \pi_1^2) + \pi_1 A'_0 + \frac{1}{2}e^2(A_1 - rA_0)^2 + e\pi_\phi (A_1 - rA_0) - e(A_0\phi' - rA_1\phi') - \frac{e^2}{2}(\alpha A_0^2 + 2\beta A_0 A_1 + \gamma A_1^2).$$

(9)

Time evolution of primary constraint with respect to the Hamiltonian gives the following secondary constraint:

$$\Omega_2 = \pi'_1 + e^2((\alpha - r^2)A_0 + (r + \beta)A_1) + e(r\pi_\phi + e\phi') \approx 0$$

(10)

The constraints are all weak conditions at this stage. To impose it as a strong condition into the system we need to have the expression of $A_0$. Equation (10), gives

$$A_0 = -\frac{1}{e^2(\alpha - r^2)}(\pi'_1 + e(r\pi_\phi + e\phi') + e^2(r + \beta)A_1).$$

(11)

Inserting the value of $A_0$ in equation (9) we get the following reduced Hamiltonian.

$$H_R = \frac{\pi_1^2}{2} + \frac{\pi_\phi^2}{2e^2(\alpha - r^2)} + \frac{1}{2} \frac{\alpha \pi_\phi^2}{(\alpha - r^2)} + \frac{e^2}{2}((1 - \gamma) + \frac{(\beta + r)^2}{(\alpha - r^2)})A'_0^2$$

+ \frac{(1 + \alpha - r^2)\phi'^2}{(\alpha - r^2)} + \frac{(\beta + r + \alpha r - r^3)}{(\alpha - r^2)}eA_1\phi' + \frac{(\alpha + \beta r)}{(\alpha - r^2)}eA_1\pi_\phi

+ \frac{(\beta + r)}{(\alpha - r^2)}\pi'_1 A_1 + \frac{r}{(\alpha - r^2)}\phi'\pi_\phi + \frac{\phi'\pi'_1}{e(\alpha - r^2)} + \frac{r}{e(\alpha - r^2)}\pi_\phi\pi'_1].$$

(12)

For this reduced Hamiltonian the ordinary Poisson’s brackets become inadequate [40]. So it becomes essential to calculate the Dirac brackets between the fields describing the Hamiltonian to proceed further. The Dirac bracket [40] between the two variables $A$ and $B$ is defined by

$$[A(x), B(y)]^* = [A(x), B(y)] - \int [A(x)\omega_i(\eta)]C^{-1}_{ij}(\eta, z)[\omega_j(z), B(y)]d\eta dz,$$

(13)

where $C^{-1}_{ij}(x, y)$ is given by

$$\int C^{-1}_{ij}(x, z)[\omega_i(z), \omega_j(y)]dz = 1.$$  

(14)

Here $\omega_i$’s represents the second class constraints that remains embedded within the phase space of the theory. The matrix $C^{-1}(x, y)$ for the theory under consideration is

$$C^{-1}(x, y) = \frac{1}{e^2(\alpha - r^2)} \begin{pmatrix} 0 & \delta(x-y) \\ -\delta(x-y) & 0 \end{pmatrix},$$

(15)
Our task becomes little easier since it is found that the Dirac brackets between the fields remains canonical.

\[
[A_1(x), \pi_1(y)]^* = \delta(x - y),
\]

(16)

\[
[\phi(x), \pi_\phi(y)]^* = \delta(x - y),
\]

(17)

\[
[A_1(x), \phi(y)]^* = 0.
\]

(18)

The reduced Hamiltonian can be expressed in the following form after few steps of algebra:

\[
H_R = \int dx \left[ \frac{1}{2} \left( \pi_1^2 + \pi_\phi^2 + \phi'^2 \right) + \frac{1}{2} e^2 (1 - \gamma) A_1^2 + \frac{1}{2e^2(\alpha - r^2)} (\xi^2 + 2\partial_1 (\xi \pi_1)) + eA_1(\pi_\phi + \phi') \right].
\]

(19)

The total momentum and the boost generator in (1 + 1) are defined by

\[
P = \int dx [\pi_1 A_1' + \pi_\phi A_0' + \pi_\phi \phi'],
\]

(20)

and

\[
M = t(\pi_\phi \phi' + \pi_1 A_1' + \pi_0 A_0') + \int dx [xH_R + \pi_1 A_0 + \pi_0 A_1].
\]

(21)

In the reduced phase space that is in the constrained subspace the equation (20) and (21) reads

\[
P_R = \int dx [\pi_1 A_1' + \pi_\phi \phi'],
\]

(22)

\[
M_R = tP_R + \int dx [xH_R - \frac{1}{e^2(\alpha - r^2)} \pi_1 \xi]
\]

(23)

where

\[
\xi = \pi_1' + e(r \pi_\phi + \phi') + e^2 (r + \beta) A_1,
\]

(24)

and the total Hamiltonian \(H_R\) and the Hamiltonian density \(\mathcal{H}_R\) are related by \(H_R = \int dx \mathcal{H}_R\). The momentum operator \(P_R\) transform the fields within the constrained subspace. Similarly, the Hamiltonian operator \(H_R\) generate the time translation of the same. The time translation of the fields are given by

\[
\dot{\phi} = [\phi(x), H_R(y)] = \pi_\phi + eA_1 + \frac{r}{e(\alpha - r^2)} \xi,
\]

(25)

\[
\dot{A}_1 = [A_1(x), H_R(y)] = \pi_1 - \frac{1}{e^2(\alpha - r^2)} \xi'.
\]

(26)

However, the most interesting one is the action of the Lorentz-boost generator \(M_R\) on the fields in the constrained subspace. We now turn to observe that. Let us now see how the fields get transformed under the Lorentz-boost. Calculating the Poisson brackets of the fields \(\phi\) and \(A_1\) with the Lorentz-boost and expressing these in terms of \(\dot{\phi}\) and \(\dot{A}_1\) using equation (25) and (26), we find the expected transformation of the fields \(\phi\) and \(A_1\) under the Lorentz-boost.

\[
[\phi, M_r] = t\phi' + x\dot{\phi}.
\]

(27)

\[
[A_1, M_R] = tA_1' + x\dot{A}_1 + A_0.
\]

(28)

With the use of the above transformation rules (27) and (28), and the Dirac brackets (16), (17) and (18), it is straightforward to see that the following Poincaré algebra

\[
[H_R, M_R]^* = P_R.
\]

(29)
\[ [P_R, M_R]^* = H_R, \]  
\[ [P_R, H_R]^* = 0. \]

is satisfied iff \( \beta = 0 \) and \( \alpha = -\gamma \). We should mention here that it is valid only for the very structure of the constraints which are given in equation (7) and (10). If we set \( \alpha = r^2 \), the constraint structure will get altered and in that case total scenario will be different. In fact, the number of constraint will be greater than two in this situation like the Faddeevian \([41, 42]\) class of regularization of Chiral Schwinger model \([5–8]\). To study the aforesaid situation let us set \( \alpha = r^2 \) and carry out the Poincar’e algebra for this special case. The constraint \( \omega_2 \) now takes the form

\[ \omega_2 = \pi_1' + e^2(r + \beta)A_1 + er\pi_\phi + e\phi'. \]  

The effective Hamiltonian of this theory in the present situation can be written down as

\[ H_{eff} = H + v\omega_2 + u\omega_1. \]

The consistency of \( \omega_2 \) with time requires \( \dot{\omega}_2 = 0 \), which fixes the velocity \( v \). The velocity \( v \) is found out to

\[ v = A_0 + \frac{\gamma + r^2}{2\beta}A_1. \]

With this velocity \( v \) the \([\omega_2(x), H(y)]\) gives birth of a new constraint

\[ \omega_3 = (r + \beta)\pi_1 + 2\beta A_0' + (\gamma + r^2)A_1'. \]

So in the present situation, three constraints are embedded in the phase space of the theory and the constraints are

\[ \bar{\omega}_1 = \pi_0, \]
\[ \bar{\omega}_2 = \pi_1' + e^2(r + \beta)A_1 + er\pi_\phi + e\phi' \]
\[ \bar{\omega}_3 = (r + \beta)\pi_1 + 2\beta A_0' + (\gamma + r^2)A_1'. \]

The matrix constructed out of the Poisson’s brackets within the constraints is

\[
C_{ij} = \begin{pmatrix}
0 & 0 & \frac{2\beta\partial_1}{2(r + \beta)(r^2 + \gamma)}
0 & -2e^2\beta\partial_1 & (r^2 + \gamma)\partial_1^2 + e^2(r + \beta)^2
\frac{2\beta\partial_1}{2(r + \beta)(r^2 + \gamma)} & (r^2 + \gamma)\partial_1^2 + e^2(r + \beta)^2
\end{pmatrix} \delta(x - y)
\]

The Hamiltonian in the reduced phase space in this situation reads

\[ H_r = \int dx \left[ \frac{(1 + r^2)}{r^2} \phi'^2 + \frac{\pi_1^2}{2} + \frac{1}{2e^2r^2}\pi_1' - e^2(\gamma)A_1^2 + \frac{1}{2}(\beta^2 - \gamma)A_1^2 + e(r + \beta)\phi' + \frac{1}{er^2}\phi'\pi_1 + \frac{\beta}{r^2}\pi_1'A_1 \right]. \]

The dirac brackets of the fields with which the reduced Hamiltonian is constituted with are computed as follows.

\[ [A_1(x), A_1(y)]^* = \frac{1}{2e^2\beta} \delta'(x - y), \]
\[ [A_1(x), \pi_1(y)]^* = \frac{(\beta + r)}{2\beta} \delta(x - y), \]
\[ [\pi_1(x), \pi_1(y)]^* = \frac{e^2(r + \beta)^2}{4\beta} \epsilon(x - y), \]
\[ [\phi(x), \pi_1(y)]^* = -\frac{er}{4\beta}(r + \beta)\epsilon(x - y), \]
\[ [\phi(x), \phi(y)]^* = \frac{r^2}{4\beta} e(x - y). \] (45)

Let us now proceed to calculate the Poincaré algebra for this special situation. There are three elements in this algebra like the previous situation. One of the elements of course, is \( H_r \), which is given in equation (41), and the rest of the two are the total momentum and the boost generator. These two respectively are

\[ P = \int dx[\pi_1 A'_1 + \pi_0 A'_0 + \pi_0 \phi'], \] (46)

and

\[ M = t(\pi_0 \phi' + \pi_1 A'_1 + \pi_0 A'_0) + \int dx[\pi H_R + \pi_1 A_0 + \pi_0 A_1]. \] (47)

In the constrained subspace space these two reduce to

\[ \bar{P}_r = \pi_0 \phi' + \pi_1 A'_1, \] (48)

and

\[ \bar{M}_r = t(\pi_0 \phi' + \pi_1 A'_1) + \int x H_R dx - \pi_1 \left[ \frac{1}{2\beta} (r + \beta) \partial^{-1} \pi_1 + \frac{(\gamma + r^2)}{2\beta} A_1 \right]. \] (49)

respectively. The the action of the Lorentz-boost generator \( \bar{M}_R \) on the fields in the constrained subspace for this case is

\[ [\phi, \bar{M}_r] = t \phi' + x \phi. \] (50)

\[ [A_1, \bar{M}_R] = t A'_1 + x A'_1 + A_0. \] (51)

\[ [A_0, \bar{M}_R] = t A'_0 + x A'_1 + A_1. \] (52)

With the use of the above transformation rules (50) and (51), and the Dirac brackets (41), (42), (43), (44) and (45), a little algebra shows that the following Poincaré algebra

\[ [H_R, M_R]^* = P_R, \] (53)

\[ [P_R, M_R]^* = H_R, \] (54)

\[ [P_R, H_R]^* = 0. \] (55)

holds if the conditions \( r^2 = 1 \) and \( 2\beta + r(1 + \gamma) = 0 \) are satisfied simultaneously. This result agrees with result available in [8] for weight factor \( r = -1 \) with the choice of parameters \( \beta = -1 \) and \( \gamma = -3 \). The result also reminds the result obtained in [10]. At this point we would like to end up our the investigation through Poincaré algebra on this model and would like to proceed with the lorentz covariant mass like term for the gauge field (which of course is result obtained from the Poincaré algebra) and carry out investigation to shed light on some of the important facts those which would be of orth unravelling for this model.

### III. Singling out of the real physical canonical pair using Dirac quantization scheme

Putting \( \beta = 0 \) and \( \alpha = -\gamma, \) (a condition for maintenance of Lorentz invariance) and setting \( \alpha = a \), we get a lorentz covariant mass-like term for gauge field and the reduced Hamiltonian with this setting reads

\[
H_R = \frac{\pi_1^2}{2} + \frac{\pi_1^2}{2(a - r^2)} + \frac{a \pi_0^2}{2 (a - r^2)} + \frac{e^2 a(1 + a - r^2)}{2(a - r^2)} A_1^2
+ \frac{(1 + a - r^2)}{(a - r^2)} \phi^2
+ \frac{a \phi' \pi_0}{e(a - r^2)} + \frac{\phi' \pi_1^2}{e(a - r^2)} + \frac{r \pi_0 \pi_1^2}{e(a - r^2)}
+ e r \frac{(1 + a - r^2)}{(a - r^2)} A_1 \phi' + \frac{a}{(a - r^2)} A_1 \pi_0 + \frac{r \pi_1 A_1}{(a - r^2)}. \] (56)
Using Dirac brackets (16), (17) and (18), we get the following first order differential equations of motion for the fields describing the theory in the constrained subspace.

\[ \dot{A}_1 = \pi_1 - \frac{1}{e^2(a-r^2)} \pi''_1 - \frac{r}{(a-r^2)} A'_1 - \frac{\phi''}{e(a-r^2)} - \frac{r \pi'_\phi}{e(a-r^2)}, \]  
\[ (57) \]

\[ \ddot{\pi}_1 = -e^2 a \frac{(1 + a - r^2)}{(a-r^2)} A_1 - e r \frac{(1 + a - r^2)}{(a-r^2)} \dot{\phi} - e \frac{a}{(a-r^2)} \pi'_\phi - \frac{r}{(a-r^2)} \pi'_1, \]  
\[ (58) \]

\[ \ddot{\pi}_\phi = \frac{(1 + a - r^2)}{(a-r^2)} \phi'' + e r \frac{(1 + a - r^2)}{a - r^2} A'_1 + \frac{r}{a - r^2} \pi'_\phi + \frac{\pi''_1}{e(a-r^2)}, \]  
\[ (59) \]

\[ \dot{\phi} = \frac{a}{(a-r^2)} \pi_\phi + e \frac{a}{(a-r^2)} A_1 + \frac{r}{e a} \pi'_1 + \frac{r}{(a-r^2)} \phi'. \]  
\[ (60) \]

A little algebra converts the above four equations (57), (58), (59), and (60), to the following second order differential equations.

\[ \Box + e^2 a \frac{(1 + a - r^2)}{(a-r^2)} \pi_1 = 0, \]  
\[ (61) \]

\[ \Box [\phi + e \frac{1}{(1 + a - r^2) \pi_1}] = 0, \]  
\[ (62) \]

\[ \Box + e^2 a \frac{(1 + a - r^2)}{(a-r^2)} (A_1 + \frac{r}{e a} \phi') = 0, \]  
\[ (63) \]

\[ \Box (\pi_\phi + \frac{r}{e a} \pi'_1) = 0. \]  
\[ (64) \]

Now a careful look reveals that within the above four equations (61), (62), (63), and (64), the theoretical spectra are hidden in a significant manner. Note that, equation (61), describes a massive boson with square of the mass \( m^2 = e^2 a \frac{(1+a-r^2)}{(a-r^2)} \) and equation (62) describes a massless boson which is equivalent to a free fermion in \((1 + 1)\) dimension. So unlike the Schwinger model, fermions gets de-confined here. We have noticed that equation (63) and (64), describe the Klein-Gordon type equations for a massive and a massless excitation respectively. The fields describing equations (63) and (64), can be considered as the momenta corresponding to the fields satisfying equation (61) and (62). Note that the fields satisfying equation (61) and (63), satisfy canonical poisson brackets between themselves. Similarly, the fields satisfying equation (62) and (64), satisfy the same canonical condition. So our description gives a transparent picture not only for the theoretical spectrum but also for the physical canonical pairs of the phase space. So this section, will certainly complement the quantization part of the work reported in [11]. Let us we end up the discussion related to the theoretical spectra and identification of the real canonical pair of the gauge non-invariant version of the GVQED and proceed to deal with the gauge invariant version of the GVQED in the next two sections.

IV. AN ALTERNATIVE QUANTIZATION OF THE GAUGE INVARIANT VERSION OF THE THEORY IN THE EXTENDED PHASE SPACE

The standard way of expressing a theory into its gauge invariant version is to extend the phase space with the inclusion of Wess-Zumino field [38]. So by adding the appropriate Wess-Zumino action to the action of the usual
bosonized gauge non-invariant action we get a gauge invariant theory of the same and the lagrangian of which is given by

\[
L = \int dx \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + e e^{\mu\nu} A_\mu \partial_\nu \phi + e r g^{\mu\nu} A_\mu \partial_\nu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right. \\
+ \frac{ae^2}{2} A_\mu A^\mu + \frac{1}{2} (a - r^2) \partial_\mu \theta \partial^\mu \theta - e r g^{\mu\nu} A_\mu \partial_\nu \theta \\
+ e (a - r^2) g^{\mu\nu} A_\mu \partial_\nu \theta + B \partial^\mu A_\mu + \left. \frac{\tilde{\alpha}}{2} B^2 \right].
\]

The last two terms of the lagrangian (65), imply the lorentz type gauge fixing term at the action level. It is needed for quantization in the alternative manner [43–46]. The Euler-Lagrange equations of motion of the fields (of both the usual and extended phase space) are

\[
\Box \phi = -e \tilde{\partial}_\mu A^\mu - e r \partial_\mu A^\mu, \quad (66)
\]

\[
\Box \theta = \frac{e r}{(a - r^2)} \tilde{\partial}_\mu A^\mu - e \partial_\mu A^\mu, \quad (67)
\]

\[
\partial_\mu A^\mu + \tilde{\alpha} B = 0, \quad (68)
\]

\[
\partial_\mu F^{\mu\nu} - \partial^\nu B + J^\nu = 0, \quad (69)
\]

where \( J^\mu \) is the electromagnetic current which is defined by

\[
J^\mu = e e^{\mu\nu} \partial_\nu \phi + e r g^{\mu\nu} \partial_\nu \phi - e r c^{\mu\nu} \partial_\nu \theta + e (a - r^2) g^{\mu\nu} \partial_\nu + e^2 A^\mu. \quad (70)
\]

The equation (66), (67), (68) and (69), agrees with the following exact solution of the fields \( \phi, \theta \) and \( A_\mu \).

\[
\phi = \frac{(a - r^2)}{ea(1 + a - r^2)} F + \frac{h}{a} + r \eta, \quad (71)
\]

\[
\theta = -\frac{r}{ea(1 + a - r^2)} F - \frac{r}{a(a - r^2)} h + \eta, \quad (72)
\]

\[
A_\mu = \frac{1}{e^2 a} \left[ \frac{(a - r^2)}{(1 + a - r^2)} \tilde{\partial}_\mu F + \partial_\mu B + e \partial_\mu h - e \partial_\mu \eta \right], \quad (73)
\]

where the fields \( h, B, \eta, F \) are Fock-space fields. Equations (66), (67), (68), (69), (70), (71), (72) and (73) after a few steps of algebra lead to the following differential equations.

\[
(\Box + m^2) \Box F = 0, \quad (74)
\]

\[
\Box h = 0, \quad (75)
\]

\[
\Box B = 0, \quad (76)
\]

\[
\Box \eta = \tilde{\alpha} e B, \quad (77)
\]

where square of the mass is \( m^2 \) is given by

\[
m^2 = \frac{e^2 a (1 + a - r^2)}{(a - r^2)}. \quad (78)
\]
which is identical to the physical mass as we have obtained in Sec. III, during the quantization of the system in its
gauge non-invariant version. The Fock-space fields has the following relation with canonical variables of the physical
system.

\[
\eta = \frac{a - r^2}{a} \phi + \frac{r}{a} \phi,
\]

(79)

\[
h = (a - r^2)(\phi - r \theta) - \frac{1}{e(1 + a - r^2 \pi)}.
\]

(80)

\[
B = \pi_0.
\]

(81)

\[
F = \Box^{-1} \pi_1
\]

(82)

Since \( \pi_1 = -\epsilon^{\mu \nu} \partial_\nu A_\mu = -\tilde{\partial}^\mu A_\mu \). It is straightforward to see that the equal time commutator of the Fock-space fields
are

\[
[\eta(x), \eta(y)] = i \frac{1}{a} \delta(x - y),
\]

(83)

\[
[F(x), F(y)] = im^2 \delta(x - y),
\]

(84)

\[
[h(x), h(y)] = i \delta(x - y),
\]

(85)

\[
[B(x), \eta(y)] = ie \delta(x - y).
\]

(86)

This completes the quantization of the gauge invariant version of the theory in the extended phase space. When the
phase space of a theory is extended in order to restore the gauge symmetry it is expected that the fields needed for
the extension will allocate themselves in the un-physical sector the theory. So it would be interesting if get back the
usual gauge non-invariant version from the gauge symmetric one of the extended phase space peeping the physical
principles intact. We will now turn towards that.

V. APPROPRIATE GAUGE FIXING TO LAND ONTO THE GAUGE NON-INARIANT MODEL
FROM ITS GAUGE INVARIANT VERSION

In [39], we have found a technique how to get back the usual gauge non-invariant theory from a gauge symmetric
theory of the extended phase space. We would like to make an extension for this GVQED following the guideline
available in [39] towards getting back the original gauge non-invariant theory. Let us see how this technique respond
to GVQED. Lagrangian of GVQED when added with the Wess-Zumino term in order to restore the local gauge
symmetry turns into

\[
L_e = \int dx [\frac{1}{2} (\dot{\phi}^2 - \phi'^2) + \frac{1}{2} a \phi^2 (A_0^2 - A_1^2) + \frac{1}{2} (A_1 - A_0')^2
+ e (A_0 \phi' - A_1 \dot{\phi}) + er (A_0 \phi - A_1 \phi') + \frac{1}{2} (a - r^2) (\dot{\theta}^2 - \theta'^2)
- er (A_0 \theta' - A_1 \dot{\theta}) + e (a - r^2) (A_0 \dot{\theta} - A_1 \theta')].
\]

(87)

Let us now proceed to calculate the momenta corresponding to the field \( A_0, A_1, \phi, \)and \( \theta \). From the standard definition,
the momenta corresponding to the fields \( A_0, A_1, \phi, \)and \( \theta \) are found out:

\[
\pi_0 = 0,
\]

(88)

\[
\pi_1 = \dot{A}_1 - A_0'.
\]

(89)
These two gauge fixing conditions are as it has been found in [39], we too have introduced two gauge conditions to get back the gauge non-invariant theory.

\[ \pi_\phi = \dot{\phi} - eA_1 + erA_0, \]

\[ \pi_\theta = (a - r^2)\dot{\theta} + e(a - r^2)A_0 + erA_1. \]  

Using (88), (89), (90) and (91), canonical Hamiltonian in this situation is found out.

\[
H_{ce} = \int dx \left[ \frac{1}{2}(\pi_\phi^2 + \pi_\theta^2 + \phi'^2) + \pi_1 A'_0 + \frac{e^2a(1 + a - r^2)}{2(a - r^2)}A_1^2 + e\phi'(rA_1 - A_0) \\
+ e\pi_\phi(A_1 - rA_0) + \frac{1}{2}(a - r^2)\theta'^2 + e\theta'((a - r^2)A_1 - A_0) + \frac{1}{2(a - r^2)}\pi_\theta^2 \\
+ \frac{e}{(a - r^2)}(rA_1 + (a - r^2)A_0)\pi_\theta \right].
\]

Equation (88) is independent of velocity. So it is the primary constraint of the theory as usual. The time evolution of the primary constraint (88), with respect to the Hamiltonian is

\[ [\pi_0, H_{ce}] = \pi_1' + e(er\pi_\phi + \phi') - e(\pi_\theta - r\theta') \approx 0, \]

which gives the secondary constraint of the theory. It is found that the Poisson bracket of the secondary constraint \( \hat{\omega}_2 \) with the Hamiltonian vanishes. So there lies only two constraints in the phase space of the theory. Following are those two.

\[ \hat{\omega}_1 = \pi_0 \approx 0, \]

\[ \hat{\omega}_2 = \pi_1' + e(r\pi_\phi + \phi') - e(\pi_\theta - r\theta') \approx 0. \]

As it has been found in [39], we too have introduced two gauge conditions to get back the gauge non-invariant theory. These two gauge fixing conditions are

\[ \hat{\omega}_3 = \theta' \approx 0, \]

\[ \hat{\omega}_4 = \pi_\theta + e((a - r^2)A_0 + rA_1) \approx 0. \]

Inserting the conditions (96) and (97) as strong condition into \( \hat{\omega}_2 \) and \( H_{ce} \), we find that \( \hat{\omega}_2 \) and \( H_{ce} \), reduce to the following

\[ \hat{\omega}_{2R} = \pi_1' + e^2(a - r^2)A_0 + e^2rA_1 + e(r\pi_\phi + \phi') \approx 0, \]

\[ \hat{H}_R = \frac{1}{2}(\pi_1^2 + \phi'^2) + \pi_1 A'_0 + \frac{1}{2}ae^2(A_1^2 - A_0)^2 \\
+ e\phi'(rA_1 - A_0) + \frac{1}{2}[\pi_\phi + e(A_1 - rA_0)]^2. \]

Note that equation (98) and (99), is identical to the equations (100) and (101), respectively, when \( \alpha = -\gamma = a \) and \( \beta = 0 \). For this \( \hat{H}_R \), the ordinary poisson brackets become adequate [40]. So we need to evaluate the dirac brackets among the fields. It necessitates the computation of the matrix formed out of the poisson brackets between the constraints along with the gauge fixing conditions themselves. The constraints along with the gauge fixing conditions gives the following matrix when Poisson brackets among themselves are evaluated.

\[
C_{ij} = \begin{pmatrix}
0 & 0 & 0 & -e^2(a - r^2) \\
0 & -e\delta_1 & 0 & e\delta_1 \\
e^2(a - r^2) & 0 & e\delta_1 & 0
\end{pmatrix} \delta(x - y).
\]

The determinant of \( C_{ij} \) is non vanishing. So it is invertible and its inverse is

\[
C^{-1}_{ij} = \frac{1}{e^2(a - r^2)} \begin{pmatrix}
0 & \delta(x - y) & 0 & \delta(x - y) \\
0 & 0 & -\frac{e}{2\epsilon}(x - y) & 0 \\
-\delta(x - y) & 0 & 0 & 0
\end{pmatrix}
\]

\[ e(x - y) \]
where the constant \( c = e^2(a - r^2) \). Therefore, from the definition of Dirac brackets \(^{[13]}\) the Dirac brackets between the field variables can now be computed in a straightforward manner.

\[
[A_0(x), A_1(y)]^*= \frac{1}{e^2(a - r^2)} \delta'(x - y),
\]

(102)

\[
[A_0(x), \phi(y)]^*= \frac{r}{e(a - r^2)} \delta(x - y),
\]

(103)

\[
[A_0(x), \pi_\theta(y)]^*= \frac{1}{2e} \epsilon(x - y),
\]

(104)

\[
[A_0(x), \pi_\phi(y)]^*= \frac{1}{e(a - r^2)} \delta'(x - y),
\]

(105)

\[
[A_1(x), \pi_\theta(y)] = -\frac{1}{2e} \epsilon(x - y),
\]

(106)

\[
[A_0(x), \pi_1(y)]^*= \frac{r}{(a - r^2)} \delta(x - y),
\]

(107)

\[
[\pi_\phi(x), \pi_\theta(y)]^* = r \delta'(x - y),
\]

(108)

\[
[A_1(x), \pi_1(y)]^* = \delta(x - y),
\]

(109)

\[
[\phi(x), \pi_\phi(y)]^* = \delta(x - y).
\]

(110)

Note that the role of gauge fixing is very crucial role to gate back the usual theory since the other choice of valid gauge fixing certainly exists, but that will lead to a different effective theory which may not help to get back to the usual theory in a straightforward manner.

### VI. CONCLUSION

We have considered the GVQED coined in \(^{[11]}\), with a generalized masslike term for gauge fields. It is added as a counter term to remove the divergence of the fermionic determinant. In this context, we should mention that all possible masslike term are not admissible as it gets restricted in order to be physically sensible, however masslike term may take some generic shape. It may even take a structure which looks Lorentz non-covariant however it does not stand as a hindrance in the way of the theory to be exactly physical Lorentz invariant \(^{[5–8]}\). In this respect, an investigation through the Poincaré algebra has been carried out using a generalized masslike term for the gauge field. The algebra has imposed some condition on the parameters used in the generalized masslike term and on the weight factor of mixing. In fact, we have found two possibilities. In the first case it does not put any restriction on the weight factor of mixing, however it suggests a restriction that admits the Lorentz covariant structure of the masslike term. No other masslike term is admissible for this theory as long as its phase space contains two constraints. In the second case, i.e., when \( \alpha = r^2 \), it imposes restrictions on both the weight factor and the parameters within the masslike term simultaneously. We have found that the number of constraint in this situation is more than two and the masslike term is of Lorentz non-covariant in nature. It is worth mentioning here that the mixing weight \( r \neq 1 \) fails provide any physically sensible theory having Lorentz non-covariant masslike term.

With the admissible masslike term in the first possibility as obtained from the Poincaré algebra, we quantize the theory using the Dirac’s scheme of quantization of constrained system. The result though was known from the work available in \(^{[11]}\) that the theoretical spectrum contains a massive and massless boson, nevertheless a more transparent calculation has been presented here with the identification of real canonical pairs of the phase space. Massive boson as usual can be considered as photons acquire mass via a dynamical symmetry breaking. On the other hand, the massless boson of the theoretical spectrum may be considered as free fermion. So fermion gets liberated here which
can be thought of as de-confinement in lower dimension. So the model may be useful to study the lower dimensional QGP phenomena. The quantization of the theory with masslike term as obtained in the second possibility can get a ready idea form the work of one od us [10], with few redefinitions of the parameter used there.

The bosonized version of the model with the admissible masslike term for the gauge fields obtained from both the possibilities are found to be gauge nonsymmetric. The first possibility is considered here and it has been made gauge invariant with the inclusion of Wess-Zumino field [38], and an alternative quantization of this gauge invariant version of the model is carried out using lorentz gauge. The spectrum here too suggests the appearance of the same massive and a mass less boson. Equation (74) and (75), represents the massive and massless field respectively. The extra equations (76) and (77), appears because of the presence of the auxiliary field \( \mathcal{B} \) in the Lorentz type gauge fixing term at the action level. This type of investigation can be carried out for the second possibilities also which we would like to carry out in our future works.

When phase space has been extended with the inclusion of Wess-Zumino field to restore the gauge symmetry of the theory generated from the fist possibility, it has been found that the fields inserted for this extension allocate themselves in the un-physical sector of the theory without disturbing the physical sector at all. Using the method developed by Falck and Kramer in [39], it has been found that an appropriate gauge fixing maps the gauge invariant theory of the extended phase space onto the usual gauge non-invariant theory. The role of gauge fixing is found to be crucial here.

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