Optimal model of manufacturing control system

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Abstract. It is noted that the manufacturing control optimization is an effective way both in the production processes automation and in the application of the mode of an operational transition to the new products release. Joint consideration of the options from the standpoint of system analysis made it possible to form a methodology of structural-algorithmic modeling, on the basis of which a homogeneous mathematical method is proposed. Interesting results for technical and economic procedures with an aggregate consideration of operational procedures are presented. Results for operational procedures (with detailed representation) together with the theory of description are obtained simultaneously. It is noted that a more complete use of optimization requires a joint description of the specified subject areas. The proposed solution is built on the basis of cyber-physical systems, methodology of structural-algorithmic modeling and a homogeneous method of description. The illustration on a two-block model of the system is given. The features of operational procedures from the standpoint of an integral description are formulated, and a joint description of subject areas is presented.

1. Introduction
Interest in optimal production systems exists for a long time [1, 2]. When considering such systems, two options are distinguished: 1) automation of existing manual control systems; 2) systems with a quick transition to the new products release. The joint study of these options is considered in [3]. The systems are built on the basis of system analysis: structuring the system model, building the technology of mathematical description and software implementation. The methodology of structural-algorithmic modeling is proposed in [2], which made it possible to form a homogeneous method for the mathematical description of the system based on dynamic linear programming [2, 3].

This approach is one of the most efficient ways to solve the problem of digitization of the economy with the introduction of optimal planning that allows obtaining economic benefits and competitive advantages. Significant results for technical and economic procedures with an aggregate consideration of operational procedures are obtained [2]. On the other hand, the results for operational procedures (with a detailed representation) together with the theory of description are formulated [4].

2. Task definition
Separate scheduling of procedures does not allow the full benefit of optimization advantages, and therefore the integration of these procedures descriptions is urgently required.
The present work is dedicated to the assessment of the system mathematical description capabilities and study of the named procedures. The system construction of procedures fits with the modern concept of distributed information systems [5], which are called cyber-physical systems (CPS).

### 3. Task solution

A cyber-physical system (CPS) is an information technology concept that implies the integration of computing resources into physical entities of any kind, including biological and man-made objects [5]. CPS covers both technological (CPTS) and production (CPPS) processes and therefore CPS is increasingly used to build the digital economy. At the beginning, cyber-physical systems were developed for technological processes, and later extended to production systems.

In this case, the CPS is the concept of "assembling" the block system shown in Fig. 1. It is proposed to use the structural-algorithmic modeling as a methodology for local systems and the homogeneous method as a method itself.

The methodology involves the research goal definition, system structure identification and its mathematical description, and model application.

![Fig. 1. General structure of the system: 0 - workshops; 1 - managers of workshops; 2 - dispatcher; 3 - director; 0a - sites; 1a - managers of sites; 2a - chains of sites; 3a - shop manager](image)

The aim of this work is to study the cyber-physical system (CPS). The structure of such a system consists of two main blocks (blocks 1 and 2). Integration of blocks is required.

The following requirements are imposed on the mathematical methods of CPS description.

1. Optimal mode support.
2. Consideration of the system multilevel structure.
3. Clarity and simplicity of the planning process description together with a short time of calculations in the system.

4. Homogeneity of all levels description.

The description of block 1 is made using a homogeneous method and it is considered in detail in [2]. It is shown that this description is formed on the basis of the existing methods analysis and it satisfies the listed requirements. The static operation mode of element 1 is defined as follows.

\[
\begin{align*}
\sum_{i=0}^{l-1} D^m_k p_k(t_i) & \leq b^m(0), \\
\sum_{i=0}^{l-1} p_k(t_i) & \leq P(T), \\
D_k^m p_k(t_{i-1}) & \leq b^m_k(t_i), \\
D_k^m p_k(t_{i+1}) & \leq b^m_k(t_i), \\
b_k^m(t_i) & = b_k^m(t_{i-1}) + \Delta b_k^m(t_{i-1}), \\
G_k & = F_k P_k(T) \rightarrow \max,
\end{align*}
\]

where \( p \) is the column vector of the daily plan, \( R \) is the column vector of demand; \( D \) is the matrix of resources consumption rates; \( b \) is the column vector of the available amount of resources; \( b^m(0) \) is the vector of the amount of material resources of element 3 of block 1; \( \Delta b \) is the resource inflow; \( P \) is the column vector of the plan of element 3 of block 1; \( F \) is the row vector of profit from a unit of production release; \( t_0, T \) are the minimum time interval and the total simulation time, respectively; \( m = 1, M \) are the types of material resources; \( \psi = 1, \Psi \) are the types of other resources; \( i = 1, I \) are the time moments; \( k = 1, K \) are the numbers of elements 1 of block 1.

For block 2, the scheduling theory apparatus is used [6, 7]. To show the applicability of the homogeneous method, let us discuss the specifics of block 2. It is necessary to highlight the synthesis (design) and operation of the system. When system operates, it is necessary to coordinate three components [6]: ordering, distribution and coordination.

The distribution component determines the location of the processed products by the items of equipment. It is specific only to processing at the sites level.

The coordination assumes the joint work of separate sites and it is used both during processing and assembly. The concept of coordination has two varieties [2]: presence of links between the sites; coordination of the economic interests of the sites using the target functions. In this publication, we will restrict ourselves to the first variety.

The components must be integrated. Let us consider the characteristics of separate components from the standpoint of system.

The most difficult component is "ordering" that can be seen as lining up products in a queue for processing.

The "distribution" component is presented in detail in [6, 7]. Here, apparently, an attempt to systematically connect the components is made for the first time. A fairly general description of the distribution for the processing procedure is given in [4]. The maximized functional for this model (element 1a in Fig. 1) determines the greatest profit from the selected parts processing:

\[
f = \alpha_1 \sum_{i=1}^l c_i x_i - \alpha_2 \sum_{i=1}^l \sum_{j=1}^m d_{ij} t_{ij},
\]

where \( x_i \) (\( i = 1, L \)) are the variables, \( \alpha 1 \) and \( \alpha 2 \) are the weight factors, \( d_{ij} \) is the cost of preparation and installation of one set of technological equipment and tools for processing of the i-th type parts by the j-th equipment.
Balance time limitations are defined as:

\[
\sum_{i=1}^{L} x_i \left( t_{ij} + z_{ij} r_{ij} \right) + t_j^q \leq y_j T, \quad j = 1, 2, \quad (8)
\]

where \( t_{ij} \) is the processing time of the \( i \)-th part by the \( j \)-th equipment, \( ^{\wedge} t_j \) are the auxiliary parameters of the downtime, \( r_{ij} \) is the changeover time of the \( j \)-th equipment for processing the \( i \)-th parts, \( z_{ij} \) is the number of \( j \)-th equipment that are used for processing the \( i \)-th type parts, \( y_j \) is the number of \( j \)-th units of equipment that are part of the production site, \( T \) is the time interval. The values \( z_{ij} \) and \( y_j \) are specified.

If there are \( l \) types \((l = 1, M_j)\) of equipment \( j \), then the task dimension grows and a superscript \( l \) is added to the variables. The same description can be used for the batch processing of parts. Expressions (7), (8) may include the cost of the equipment, production of tooling, warehouses, storages, and pallets.

In [7], the choice of equipment for the design and modernization of the system is carried out on the basis of expressions (7), (8).

Note that in the "coordination" component a linear programming task can be used during processing [2]. A more complex and accurate version utilizing the apparatus of Petri nets is required for the assembly process.

The review [7] analyzes the following solution methods of the "ordering" component: combinatorial analysis, mathematical programming, statistical modeling, and directed search by the branch and bound method. Of these, the most common are combinatorial analysis and branch and bound search. However, the most promising method is the combination of combinatorial and optimization methods. Let us call this method combinatorial-optimization.

In this method, the following stages on the basis of a matrix with elements \( a_{ij} \) \((i = 1, m), (j = 1, n)\) corresponding to the norms for processing parts \( i \) on equipments \( j \) are distinguished.

1. Necessary conditions for permutations of adjacent pairs are determined, Hamiltonian paths of columns \( i \) in the graph and \( i \) in the processing sequence are selected, and the corresponding graph of the named pairs of columns is constructed.

2. Sufficient conditions for permutations with the graph construction are revealed.

3. Hamiltonian paths in the graph are distinguished.

4. For them, the total processing time is calculated and the path with the least value is selected.

The conducted research allows concluding that the component "distribution" is considered for the processing case, the "coordination" component is not actually discussed and for the assembly only a verbal model is provided, the combinatorial-optimization method is the most suitable for the "ordering" component.

Further a systematic description of the blocks (Fig. 1) is given. Elements 1 and 3 of block 1 can be represented by expressions similar to (1) - (6). In [2] it is noted that the expressions are valid for processing procedures. At the same time, if the matrix of resource consumption norms \( D \) is replaced by the matrix of applicability ("node - product" or "resources - node"), then a description of the assembly procedure can be obtained.

Element 2 of block 1 can be defined as:

\[
D_k^{m} p_k(t_{i+}) \leq p_k(t_i), \quad k=2, K, \quad (9)
\]

\[
G = \sum_{k=1}^{K} G_k \rightarrow \text{max}, \quad (10)
\]

The advantage of a description in the form (1) - (6) consists in taking into account the dynamics of the planning process, for example, during the operational transition to the new products release:

\[
z(t_i) = A_{iz}(t_{i+}) + B_{iz}p(t_{i+}), \quad z(0) = z_0, \quad (11)
\]
\[ p_i(t_i) = F_i z_i(t_{i-1}), \]  
\[ \sum_{i=0}^{N-1} D_i^m p_{1k}(t_i) \leq b_i^m(0), \]  
\[ \sum_{i=0}^{N-1} p_k(t_i) \leq P(T), \]  
\[ D_k^m p_i(t_{i+1}) \leq b_k^m(t_i), \]  
\[ D_k^m p_i(t_{i+1}) \leq b_k^m(t_i), \]  
\[ b_k^m(t_i) = b_k^v(t_{i-1}) + \Delta b_k^v(t_{i-1}), \]  
\[ G_s = f_s p_s \rightarrow \text{max}, \]  
\[ D_r p_r + S_r \leq y_r T - T_{1r}, \]  

where \( z \) is the column vector of the (planned) incomplete production; \( p_i \) is the column vector of dimension \( J \) of the resource sets launching into production, \( P \) is the column vector of the plan of element 1 of block 1; \( A, B, C \) are the identity matrices of the corresponding dimensions.

To integrate block 1 with block 2, three components should be considered in the latter: distribution, coordination, and ordering.

It is convenient to take expressions (7), (8) as the basis to describe the "distribution" component of block 2. We represent them in matrix-vector form for elements 1a.

\[ G_r = f_r p_r \rightarrow \text{max}, \]  
\[ D_r p_r + S_r \leq y_r T - T_{1r}, \]  

where \( D_r \) is the matrix of the processing time of the \( j \)-th equipment, \( T_1 = \{ t_q \} \) is the downtime vector, \( S_r \) is the changeover time matrix, \( r_{ij} \) is the standard changeover time of the \( q \)-th equipment that are used for processing the \( j \)-th type parts, \( Y = \{ y_q \} \) is the vector of the number of \( q \)-th units of equipment that are part of the production site, \( p_r = (p_j) \) is the number of processed parts; \( T \) is the time interval; \( r \) is the number of the structural element of the site.

Imposed restrictions on material resources are defined as:

\[ D_r^m p_r \leq b_r^m, \]  

where \( f \) is the row vector of profit; \( p \) is the column vector of the output; \( r (r = 1, R) \) is the site number; \( D_r^m \) is the matrix of material resources consumption rates; \( b_r^m \) is the amount of material resources; \( m = 1, M \) are the types of material resources. If \( D_r^m \) is the identity matrix, then the processing procedure is referred, otherwise the assembly procedure is considered.

Expression (21) shows the connection between the "distribution" component and the "coordination" component.

\[ D_r^m p_r \leq p_{r-1}, \]  

\[ G = \sum_{r=1}^{R} G_r \rightarrow \text{max}, \]  

Expressions (22), (23) can be applied to display the group processing option without changing the groups composition.

The groups reformatting case can be presented in the following way. The matrix \( V_k = V_{k1}(gr_k \times b_k) \) \( V_{k2}(b_k \times gr_k + 1) \) is inserted between the elements \( k \) and \( (k + 1) \), where \( gr_k \) are the groups of the corresponding elements, \( b_k \) are the initial resources of the element \( k \).
For the assembly procedure, the applicability matrix $D_{rm}$ (for example, parts - node) that defines the set allowing the assembly beginning is considered. If the set includes $s$ parts $d$, then the time value in the matrix should be multiplied by $s$.

The above description can rather be called quasi-optimal. Full optimality can be achieved using the combinatorial optimization method of ordering. It is valid for the processing procedure.

Expressions (19)-(23) represent the idea of the integral model, and the numerous variants are possible.

For block 2, it is possible to use a description of the dynamics similar to expressions (11)-(18).

4. Conclusion
Thus, a multi-level block model of the manufacturing control system is formed. The integration of technical, economic and operational control processes is completed. The analysis of operational management process components is carried out. The connection between the components of distribution, coordination and ordering is shown. A joint mathematical description of the processing and assembly procedures is given. The technologies of synthesis (design) and analysis (use) of the model of a multi-level describing integrated manufacturing control system are considered. Preconditions for various features of the systems are created.

Further applied task solution requires a separate consideration of the issues of generating numerical data necessary for the debugging procedure of the operating system model and obtaining data from the real system.

Optimal models are an essential aid to leaders in making difficult decisions.

References
[1] Haseeb M., Hussain H., Slusarczyk B. and Jermsittiparsert K. 2016 Industry 4.0: A Solution towards Technology Challenges of Sustainable Business Performance (socsci-08-00154 9 January).

[2] Chertovskoy V D 2019 The basics of the theory of adaptive automated production control systems Proc. of the All-Russian Meeting on Control Problems (VSPU-2019, Meeting) (Moscow: Institute of control problems) pp 2676-79

[3] Sagygaliev K S 1988 Planning reconciliation in a three-tier active system Automation, Remote Control 3 80-91

[4] Pavlov K S and Khobotov E N 2015 Models of equipment selection and replacement in industrial systems of machine-building enterprises Automation, Remote Control 12 105-43

[5] Golenko-Ginzbur D., Kac V, Sinjakovsky S and Ickovich E L 2000 Three-level man-machine system control Automation, Remote Control 5 166–84

[6] Ugolnikov G A and Usov A B 2010 Three-level environmental management system Control problems 126-32

[7] Novikov D A 2007 Organizational Systems Management Theory (Moscow: Fizmatgiz) p 584