Wess-Zumino actions for IIA D-branes
and their supersymmetries

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Abstract

We present Wess-Zumino actions for general IIA D-p-branes in explicit forms. We perform the covariant and irreducible separation of the fermionic constraints of IIA D-p-branes into the first class and the second class. A necessary condition which guarantees this separation is discussed. The generators of the local supersymmetry (kappa symmetry) and the kappa algebra are obtained. We also explicitly calculate the conserved charge of the global supersymmetry (SUSY) and the SUSY algebra which contains topological charges.

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1 Introduction

D-brane dynamics play an important role in the non-perturbative superstring physics. The characteristic feature of the D-branes is their non-zero Ramond/Ramond (RR) charges [1]. The RR coupling is realized through the Wess-Zumino action [2, 3, 4, 5]. The D-branes are representation of the global supersymmetry (SUSY) algebra with central extension whose origin is the Wess-Zumino action [1]. The Wess-Zumino action is essential for the D-brane dynamics showing rich structures of various dualities, and its explicit expression will be important especially for the D-brane quantization. Wess-Zumino actions of the superstring theories are required from the kappa invariance [7]. The actions for D-p-branes with the kappa symmetry have been proposed in references [8, 9, 4]. In these references a differential equation of the Wess-Zumino action is obtained from the requirement of the kappa symmetry, and explicit expressions of the Wess-Zumino actions are presented only for small p cases. In order to figure out a suitable treatment of fermionic variables, one needs an explicit expression of the fermionic constrains. In a previous paper we have given a concrete expression of the Wess-Zumino action by solving the differential equation for general IIB D-p-branes [10]. It has a compact and closed form and enables us to confirm algebraic properties of the local and the global symmetries. In this paper we have completed this program for IIA D-p-branes.

In section 2, we derive the Wess-Zumino action for general IIA D-p-branes in an explicit form. In section 3, the IIA D-p-brane actions are analyzed in canonical formalism. Covariant and irreducible separation of the fermionic constraints into the first class and the second class can be performed. We examine necessary conditions which guarantee this separation. The generators of the local supersymmetry (kappa symmetry) and the kappa algebra are obtained. In section 4, we calculate explicitly the conserved charge of the global supersymmetry (SUSY) and the SUSY algebra which contains topological charges. The physical interpretations of topological charges are discussed in the last section.

2 Wess-Zumino action for type IIA D-branes

A Wess-Zumino action for a D-p-brane is obtained as a $p + 1$ form part of a symbolic sum of differential forms

$$ L^{WZ} = T \, C \, e^{\mathcal{F}}, $$

(2.1)

where $C$ is the RR gauge field and $T$ is the D-brane tension [2, 3, 4, 5]. $\mathcal{F}$ is the supersymmetric extension of the Born-Infeld U(1) field strength,

$$ \mathcal{F} = dA - B $$

(2.2)

where $B$ is the NS-NS(NS) 2-form. Denoting $H$ as the field strength of $B$, $H = dB$, $R$ is the field strength of RR potential $C$,

$$ R = dC - H \, C. $$

(2.3)
It satisfies the Bianchi identity,
\[ dR - HR = 0. \tag{2.4} \]

Then the Wess-Zumino action satisfies
\[ dL^{WZ} = \mathcal{T}RE^F. \tag{2.5} \]

The form of the curvature \( R \) is determined by the requirement of kappa invariance of the total action. On the other hand the form of the RR potential \( C \) is not unique but determined up to RR gauge transformations
\[ C \rightarrow C' = C + d\Lambda - H\Lambda. \tag{2.6} \]

Under the RR transformation, \( R \) is invariant while the Wess-Zumino action changes by an exact form,
\[ L^{WZ} \rightarrow L^{WZ'} = L^{WZ} + d[\Lambda e^F]. \tag{2.7} \]

The Wess-Zumino action is obtained by solving the differential equation (2.5), or equivalently by solving (2.3) for the RR potential \( C \). The type IIA super D-p-brane is described by worldvolume fields; the 10-dimensional coordinates \( X^m \), 32 components Majorana fermion \( \theta \) and the U(1) gauge field \( A_\mu \). The kappa invariance requires the form of the equation (2.5)
\[ dL^{WZ} = \mathcal{T}RE^F, \quad R = d\bar{\theta}C_Ad\theta, \tag{2.8} \]

where \( C_A \) is introduced in the ref.[8] together with \( S_A \),
\[ C_A(\mathbb{H}) = \sum_{\ell=0} \left( \Gamma_{11}\right)^{\ell+1} \frac{\mathbb{H}^{2\ell}}{(2\ell)!} = \Gamma_{11} + \frac{\mathbb{H}^2}{2!} + \Gamma_{11} \frac{\mathbb{H}^4}{4!} + ... , \tag{2.9} \]
\[ S_A(\mathbb{H}) = \sum_{\ell=0} \left( \Gamma_{11}\right)^{\ell+1} \frac{\mathbb{H}^{2\ell+1}}{(2\ell+1)!} = \Gamma_{11}\mathbb{H} + \frac{\mathbb{H}^3}{3!} + \Gamma_{11} \frac{\mathbb{H}^5}{5!} + ... . \tag{2.10} \]

\( R \) is manifestly invariant under the SUSY transformation. The NS two form \( B \) and its field strength \( H \) are
\[ B = -\bar{\theta} \Gamma_{11} \Gamma \cdot (\Pi - \frac{1}{2}\partial\Gamma d\theta) \, d\theta, \tag{2.11} \]
\[ H = dB = -d\bar{\theta} \Gamma_{11} \Gamma \cdot \Pi \, d\theta. \tag{2.12} \]

In this section we will integrate (2.8) to get the Wess-Zumino action, or equivalently the RR potential \( C \), explicitly in an analogous manner to the type IIB case [10]. Several formulas of type IIA case are obtained from those of type IIB ones by replacing \( \tau_3 \) by \( \Gamma_{11} \). However there is a difference that \( \tau_3 \) commutes with \( \Gamma \) matrices while \( \Gamma_{11} \) anti-commutes with \( \Gamma \)'s. It makes the IIA formulas more involved.
In order to recognize the cyclic identity easier, it is convenient to introduce Γ matrix valued 10D vector for any type IIA spinors \( \psi \) and \( \phi \)

\[
(V^m_{\psi,\phi})_\beta^\alpha \equiv \delta^\alpha_\beta (\bar{\psi} \Gamma^m \phi) + (\Gamma_{11})^\alpha_\beta (\bar{\psi} \Gamma_{11} \Gamma^m \phi). \quad (2.13)
\]

We define "slashed" quantities by contracting with Γ from the right

\[
\bar{V}_{\psi,\phi} \equiv V^m_{\psi,\phi} \Gamma_m
\]

\( V^m \) contracted with \( \Gamma_m \) from the left defines \( \bar{V}^m \) as

\[
\bar{V}_{\psi,\phi} \equiv \Gamma_m V^m_{\psi,\phi} = \bar{V}^m_{\psi,\phi} \Gamma_m \equiv \{(\bar{\psi} \Gamma^m \phi) - \Gamma_{11}(\bar{\psi} \Gamma_{11} \Gamma^m \phi)\} \Gamma_m. \quad (2.15)
\]

Using this notation the cyclic identity holds as

\[
\bar{V}_{\psi,\theta} + \bar{V}_{\phi,\theta} \psi + \bar{V}_{\theta,\psi} \phi = 0 \quad (2.16)
\]

for three odd IIA spinors. It frequently appears an expression

\[
\bar{V}^m \equiv \bar{V}^m_{\theta,d\theta}. \quad (2.17)
\]

The cyclic identity (2.16) tells, for example,

\[
d\bar{V} \theta + 2 \bar{V} d\theta = 0 \quad \rightarrow \quad d\bar{V} \ d\theta = 0, \quad V \cdot dV = 0. \quad (2.18)
\]

The well-known IIA identities are rewritten in terms of \( \bar{V} \) variables. For example,

\[
\sum_{\text{perm} \psi,\phi,\theta,\chi} \left[ (\bar{\psi} \Gamma^m \phi) (\bar{\theta} \Gamma_{mn} \chi) + (\bar{\psi} \Gamma_{11} \phi) (\bar{\theta} \Gamma_{11} \Gamma_n \chi) \right]
= \sum_{\text{perm} \psi,\phi,\theta,\chi} \left[ (\bar{\psi} \Gamma^m \phi_1) (\bar{\theta} \Gamma_{mn} \chi_2 + \bar{\theta} \Gamma_{mn} \chi_1 + \eta_m \bar{\theta} \Gamma_{mn} \chi_2 - \eta_m \bar{\theta} \Gamma_{mn} \chi_1) + 1 \leftrightarrow 2 \right]
= \sum_{\text{perm} \psi,\phi,\theta,\chi} (\bar{\theta} \bar{V}_{\psi,\phi}) \Gamma_n \chi = 0, \quad (2.19)
\]

for type IIA spinors \( \psi, \phi, \theta, \chi \). In the last line of (2.19) permutation makes the bracket to vanish by the cyclic identity (2.16). More general IIA identities [14] are also rewritten using with the relation

\[
\Gamma_m \Gamma_{m_1 \cdots m_q} = \Gamma_{mn_1 \cdots m_q} + q \eta_m \Gamma_{m_1 \cdots m_q} , \quad O_{[m_1 \cdots m_q]} = \frac{1}{q!} \sum_{\text{perm } m_1 \cdots m_q} O_{m_1 \cdots m_q} , \quad (2.20)
\]

as

\[
\sum_{\text{perm} \psi,\phi,\theta,\chi} \left[ (\bar{\psi} \Gamma^m \phi) (\bar{\theta} \Gamma_{mn_1 \cdots n_{2q-1}} \Gamma_{11})^{q-1} \chi + (2q - 1)(\bar{\psi} \Gamma_{11} \Gamma_{n_1} \phi) (\bar{\theta} \Gamma_{n_2 \cdots n_{2q-1}} \Gamma_{11})^{q} \chi \right]
= (-1)^{q-1} \sum_{\text{perm} \psi,\phi,\theta,\chi} (\bar{\theta} \bar{V}_{\psi,\phi}) \Gamma_{n_1 \cdots n_{2q-1}} \chi = 0, \quad q > 0. \quad (2.21)
\]
It is useful to have the following relations

\[ d \left( S_A e^\mathcal{F} \right) = \frac{1}{2} \left( d\mathcal{V} \tilde{\mathcal{C}}_A + C_A d\tilde{\mathcal{V}} \right) e^\mathcal{F}, \]

\[ d \left( C_A e^\mathcal{F} \right) = \frac{1}{2} \left( -d\mathcal{V} \Gamma_{11} \tilde{S}_A + S_A \Gamma_{11} d\mathcal{V} \right) e^\mathcal{F}, \]  

(2.22)

where \( \tilde{S}_A \) and \( \tilde{\mathcal{C}}_A \) are introduced by replacing \( \Gamma_{11} \rightarrow \left( -\Gamma_{11} \right) \) in \( S_A \) and \( C_A \) of (2.10).

Next we define j-form IIA spinor \( \Theta_j \) by

\[ \Theta_j \equiv \hat{\mathcal{V}}_j \Theta_{j-1}, \quad (j \geq 1), \quad \Theta_0 \equiv \theta, \]  

(2.23)

where

\[ \hat{\mathcal{V}}_j^m = \begin{cases} V^m & \text{for odd } j, \\ \tilde{V}^m & \text{for even } j. \end{cases} \]  

(2.24)

The concrete forms are

\[ \Theta_0 = \theta \]
\[ \Theta_1 = \mathcal{V} \theta \]
\[ \Theta_2 = \tilde{\mathcal{V}} \mathcal{V} \theta \]
\[ \Theta_3 = \mathcal{V} \tilde{\mathcal{V}} \mathcal{V} \theta \]
\[ \ldots \]  

(2.25)

It has the \( j + 1 \) parity and it holds a useful relation,

\[ d \Theta_j = \frac{2j+1}{2} d\hat{\mathcal{V}} \Theta_{j-1}, \quad (j = 1, 2, \ldots). \]  

(2.26)

The important quantities describing the D-branes are odd IIA spinor form \( \Theta_S \) and even one \( \Theta_C \) defined by

\[ \Theta_S \equiv - \sum_{n=0}^{\infty} \frac{1}{(4n+3)!!} \left( -\Gamma_{11} \right)^{n+1} \Theta_{2n+1} = \frac{1}{3!!} \Gamma_{11} \theta - \frac{1}{7!!} \Theta_3 + \ldots, \]

\[ \Theta_C \equiv \sum_{n=0}^{\infty} \frac{1}{(4n+1)!!} \left( \Gamma_{11} \right)^n \Theta_{2n} = \theta + \frac{1}{5!!} \Gamma_{11} \Theta_2 + \ldots. \]  

(2.27)

They satisfy, using (2.26)

\[ d\Theta_S = \frac{1}{2} \Gamma_{11} d\mathcal{V} \Theta_C, \]

\[ d\Theta_C = d\theta - \frac{1}{2} d\tilde{\mathcal{V}} \Theta_S. \]  

(2.28)
Now we are ready to integrate (2.8) to find the \(L^WZ\). We first write the right hand side of (2.8) as

\[
d\bar{\theta} \mathcal{C}_A e^\xi \ d\theta = dL_1 + I_1, \quad L_1 = d\bar{\theta} \mathcal{C}_A e^\xi \theta, \quad I_1 = -d\bar{\theta} d(C_A e^\xi) \theta .\]

(2.29)

Using (2.22), (2.18) and (2.26)

\[
I_1 = -\frac{1}{2} d\bar{\theta} S_A e^\xi \Gamma_{11} d\varphi \theta = -\frac{1}{3} d\bar{\theta} S_A e^\xi \Gamma_{11} d\Theta_1 \equiv dL_2 + I_2
\]

\[
L_2 = \frac{1}{3} d\bar{\theta} S_A e^\xi \Gamma_{11} \Theta_1 , \quad I_2 = -\frac{1}{3} d\bar{\theta} d(S_A e^\xi) \Gamma_{11} \Theta_1 . \quad (2.30)
\]

Repeating this procedure (actually it terminates at \((p+1)\)-th step for the \(p\) brane) and summing up \(L_j\)'s, we arrive at a compact expression for the Wess-Zumino action:

\[
L^WZ = T d\bar{\theta}(C_\Theta e^\xi + S_\Theta e^\xi) . \quad (2.31)
\]

The Wess-Zumino actions of the D-p-brane is the \((p+1)\) form part of (2.31). It is easy to check that (2.31) satisfies (2.8) by using the relations (2.22) and (2.28) with (2.18).

Corresponding to the fact that the lagrangian is determined up to total divergence, the RR potential \(C\) is obtained from (2.31) up to RR gauge transformations (2.6). For a D-2-brane the Wess-Zumino action given from (2.31) is

\[
L^WZ = T \left[ \frac{1}{2} d\bar{\theta} \Pi^2 \theta - \frac{1}{3} d\bar{\theta} \Pi \Theta_1 + \frac{1}{15} d\bar{\theta} \Theta_2 + d\bar{\theta} \Gamma_{11} \theta \mathcal{F} \right] . \quad (2.32)
\]

It differs from, for example, one in the ref. [14] by a RR transformation (2.6) with \(\Lambda\)

\[
\Lambda = \frac{1}{90} d\bar{\theta} \Theta_2 + \frac{1}{12} (\Pi m - \frac{1}{3} d\bar{\theta} \Gamma^m \theta) \bar{\theta} \Gamma_m \Theta_1 . \quad (2.33)
\]

It is noted that \(\Lambda\) does not have explicit dependence on \(X\) and \(A\) but on \(\Pi\) and \(dA\).

We summarize our results by adding the previous results for type IIB [11] in Table 1.
Table 1: Summary of type IIA and IIB D-branes

|        | IIA                                      | IIB                                      |
|--------|------------------------------------------|------------------------------------------|
| $B$    | $\bar{\theta} \Gamma_{11} \Gamma \cdot (\Pi - \frac{1}{2} \bar{\theta} \Gamma d\theta) \, d\theta$ | $\bar{\theta} \tau_3 \Gamma \cdot (\Pi - \frac{1}{2} \bar{\theta} \Gamma d\theta) \, d\theta$ |
| $H$    | $- d\bar{\theta} \Gamma_{11} \Gamma \cdot \Pi \, d\theta$ | $- d\bar{\theta} \tau_3 \Gamma \cdot \Pi \, d\theta$ |
| $C$    | $d\bar{\theta} ( C_A \Theta_C + S_A \Theta_S )$ | $d\bar{\theta} \tau_1 ( S_B \Theta_C + C_B \Theta_S )$ |
| $R$    | $d\bar{\theta} C_A \, d\theta$ | $d\bar{\theta} S_B \tau_1 \, d\theta$ |
| $C_{A,B}$ | $\sum_{\ell=0} (\Gamma_{11})^{\ell+1} \frac{\Pi^{\ell}}{(2\ell)!}$ | $\sum_{\ell=0} (\tau_3)^{\ell+1} \frac{\Pi^{\ell}}{(2\ell)!}$ |
| $S_{A,B}$ | $\Gamma_{11} \sum_{\ell=0} (\Gamma_{11})^{\ell} \frac{\Pi^{\ell+1}}{(2\ell+1)!}$ | $\sum_{\ell=0} (\tau_3)^{\ell} \frac{\Pi^{\ell+1}}{(2\ell+1)!}$ |
| $\Theta_C$ | $\sum_{n=0} \frac{1}{(4n+1)!!} (\Gamma_{11})^n \Theta_{2n}$ | $\sum_{n=0} \frac{1}{(4n+1)!!} (-\tau_3)^n \Theta_{2n}$ |
| $\Theta_S$ | $- \sum_{n=0} \frac{1}{(4n+3)!!} (-\Gamma_{11})^{n+1} \Theta_{2n+1}$ | $\sum_{n=0} \frac{1}{(4n+3)!!} (-\tau_3)^{n+1} \Theta_{2n+1}$ |

3Local supersymmetry (kappa symmetry)

In this section we derive constraint equations in the canonical formalism, and examine the correct treatment of the fermionic constraints. The lagrangian density of the system is

$$\mathcal{L} = \mathcal{L}^{DBI} + \mathcal{L}^{WZ},$$

$$\mathcal{L}^{DBI} = - T_{(p)} \sqrt{- \text{det}(G_{\mu\nu} + \mathcal{F}_{\mu\nu})},$$

$$\mathcal{L}^{WZ} = [L^{WZ}]_{p+1}, \quad \mathcal{L}^{WZ} = T_{(p)} \, d\bar{\theta} \, (C_A \Theta_C + S_A \Theta_S) e^F.$$

where the Wess-Zumino action has been determined in (2.31) and $[ \ ]_{p+1}$ means p+1-form coefficient.

Structures and algebras of p+1 bosonic constraints are determined only by the Dirac-Born-Infeld part and do not depend on the form of the Wess-Zumino action;

$$\begin{aligned}
H & \equiv \frac{1}{2} [ \bar{p}^2 + \bar{E}^a G_{ab} \bar{E}^b + T_{(p)}^2 \mathbf{G}_F ] = 0, \\
T_a & \equiv \bar{p} \Pi_a + \bar{E}^b \mathcal{F}_{ab} = 0, \quad (a = 1, 2, \ldots p)
\end{aligned}$$

where $p_m, \zeta, E^\mu$ are canonical momenta conjugate to $X^m, \theta$ and $A_\mu$ respectively and
\[ G_P = \det(G + F)_{ab}. \]  
\[ \tilde{E}^a \]  
\[ \tilde{p}_m \]  
\[ E^a = \frac{\partial L^{WZ}}{\partial F_{0a}} = \frac{\partial L^{DBI}}{\partial F_{0a}} \] 
(3.3)

where \( \mathcal{L} \) is regarded as a function of \( \Pi_\mu, F \) and \( \theta \). In addition there appear bosonic U(1) constraints, \( E^0 = 0 \), and \( \partial_a E^a = 0 \). The fermionic constraint follows from the definition of the momentum \( \zeta \),

\[
F \equiv \zeta + p_\ell (\bar{\theta} \Gamma^\ell) \\
+ E^a \left[ \bar{\theta} \Gamma_{11} \Gamma_{a} \right. + \frac{1}{2} \bar{\theta} \Gamma^\ell \partial_\ell \theta) - \frac{1}{2} \bar{\theta} \Gamma^\ell (\bar{\theta} \Gamma_{11} \Gamma_{a} \theta) - (\frac{\partial L^{WZ}}{\partial \theta})
\]
(3.4)

Both structure and the algebra of the fermionic constraints are governed by the Wess-Zumino action.

The Poisson bracket of \( F \)’s is calculated as

\[
\{ F_\alpha(\sigma), F_\beta(\sigma') \} = \int d^p \sigma \left[ 2 \left( c \Xi \right)_{\alpha \beta} + (\partial_a E^a) \left( \bar{\theta} \Gamma(\alpha) \cdot (\bar{\theta} \Gamma_{11} \Gamma)_{\beta} \right) \right] \delta^p(\sigma - \sigma').
\]
(3.5)

Here \( c \) is the charge conjugation matrix and the symmetric bracket is defined as \( A_{(\alpha \beta)} = \frac{1}{2} (A_{\alpha \beta} + A_{\beta \alpha}) \). The second term is the Gauss law constraint and \( \Xi \) is given by

\[
\Xi \equiv \tilde{\gamma} + \Gamma_{11} \Pi_a \tilde{E}_a + T_{(p)} \left[ \mathcal{C}_A e^F \right]_p.
\]
(3.6)

Here \( [...)_p \) is a spatial p form coefficient (coefficient of \( d\sigma^1 \ldots d\sigma^p \)) of the expression \( [...]. \) Unlike IIB case the matrix \( \Xi \) in (3.6) is not nilpotent, instead there is a zero eigenmatrix;

\[
\tilde{\Xi} \equiv \tilde{\gamma} - \Gamma_{11} \Pi_a \tilde{E}_a - T_{(p)} \left[ \tilde{\mathcal{C}}_A e^F \right]_p
\]
(3.7)

satisfying

\[
\Xi \tilde{\Xi} = 2H + 2\tilde{\tau}^a T_a \approx 0
\]
(3.8)

with

\[
\tilde{\tau}^a = \tilde{E}_a \Gamma_{11} - T_{(p)} \Gamma_{11} \left[ d\sigma^a S_A e^F \right]_p
\]
(3.9)

From (3.8) it follows that the rank of \( \Xi \) is one half of 32, reflecting the fact that a half of the fermionic constraints are the first class and the remaining half are second class.
The first class constraint set for IIA D-p-branes is obtained, using with the zero eigenmatrix $\tilde{\Xi}$ in (3.7),

\[
\begin{align*}
\tilde{H} &= H + F \tilde{\tau}^a \partial_a \theta, \\
\tilde{T}_a &= T_a + F \partial_a \theta = p \partial_a x + \zeta \partial_a \theta + E^b F_{ab}, \\
\tilde{F} &= F \tilde{\Xi}.
\end{align*}
\]

(3.10)

$\tilde{H}$ and $\tilde{T}_a$ generate $(1 + p)$ dimensional diffeomorphism. $\tilde{F}$ is the generator of the kappa symmetry,

\[
\{\tilde{F}_\alpha(\sigma), \tilde{F}_\beta(\sigma')\} = -4(C\tilde{\Xi} L)_{\alpha\beta} \delta(\sigma - \sigma') + \cdots, \quad L_{\alpha\beta} = \delta_{\alpha\beta} \tilde{H} + \tilde{\tau}^a_{\alpha\beta} \tilde{T}_a
\]

(3.11)

where $\cdots$ contains $\tilde{F}$ and Gauss law constraint. In case of the Green-Schwarz superstring the constraints $(\tilde{F}_1, L_{11})$ commute with $(\tilde{F}_2, L_{22})$, so that right and left moving modes are independent. The D-p-branes are not the case.

We can perform the irreducible and covariant separation of the fermionic constraints into the first class and the second class as was done for the D-string case [13]:

\[
\begin{align*}
\text{first class constraints} &: \quad \tilde{F}_1 \equiv F \tilde{\Xi}^{1+\Gamma_{11}}_2 = 0 \\
\text{second class constraints} &: \quad F_2 \equiv F^{1-\Gamma_{11}}_2 = 0
\end{align*}
\]

(3.12)

The covariant first class constraints in (3.12) make the covariant gauge fixing possible; for example one of the chirality components of $\theta$ to be zero [8, 12]. By using with the second class constraints $F_2 = 0$ in (3.12), the Dirac bracket is defined as

\[
\{A, B\}_D = \{A, B\} - \{A, F_2\} \frac{\Xi_{22} c^{-1}}{2 T^2 G_F} \{F_2, B\}, \quad \Xi_{22} = \frac{1 + \Gamma_{11}}{2} \Xi \frac{1 - \Gamma_{11}}{2}
\]

(3.13)

The Dirac bracket, or equivalently the separation (3.12), it well defined if

\[
(G_F) = \det(G + F)_{ab} \neq 0
\]

(3.14)

This is the condition for the covariant quantization discussed in [12]. If a conformally flat metric can be assumed, $-G_{00} = G_{11} = \cdots = G_{pp}$, $G_F$ is written as

\[
G_F = (G_{11})^p + (F_{ab})^2 (G_{11})^{p-2} + \cdots + (F_{a_1 a_2} \cdots F_{a_{p-1} a_p})^2
\]

(3.15)

The necessary condition of the separation (3.14) is satisfied by following cases;

\[
\begin{align*}
\text{(i)} \quad & G_{11} > 0 \\
\text{(ii)} \quad & G_{11} = 0, \quad (F_{a_1 a_2} \cdots F_{a_{p-1} a_p})^2 > 0 \quad \Rightarrow \quad G_F > 0
\end{align*}
\]

(3.16)

(i) are cases in which the static gauge can be taken. The case (ii) would corresponds to massless particle solutions, in which the non-zero worldvolume magnetic field, $(F_{a_1 a_2} \cdots F_{a_{p-1} a_p})^2 > 0$, guarantees the separation (3.12).
There is alternative necessary condition; the condition of the DBI action to be well defined, \(-\det(G + F)_{\mu\nu} \geq 0\). For D-string case this necessary condition adding to (3.14) reduces to that: if \(E^1 \neq 0\) then \(G_{11} \neq 0\) [3]. In other words, \(E^1 \neq 0\) guarantees both the separation and the static picture of a D-string. However for IIA D-p-branes for \(p > 1\) the situation is different from the D-string, the massless solutions can not be excluded only by the condition of the DBI action \(-\det(G + F)_{\mu\nu} \geq 0\).

4 Global supersymmetry (SUSY)

The global supersymmetry transformations of \(X, \theta\) and \(A\) are

\[
\delta_\epsilon \theta = \epsilon, \quad \delta_\epsilon X^m = \epsilon \Gamma^m \theta \\
\delta_\epsilon A = (\epsilon \Gamma_{11} \Gamma_\theta)_{\mu} dX^m - \frac{1}{6} \epsilon \Gamma_{11} V \cdot \Gamma \theta,
\]

so that \(d\theta, \Pi\) and \(F\) are SUSY invariant. The canonical supersymmetry generator is

\[
Q_\epsilon = \int d\sigma^p (p_m \delta_\epsilon X^m + \zeta \epsilon \delta_\epsilon \theta + E^a \delta_\epsilon A_a) - \int d\sigma^p U_\epsilon^0.
\]

\(U_\epsilon^0\) is determined from the surface term of the SUSY variation of the Wess-Zumino lagrangian,

\[
\delta_\epsilon L^{WZ} \equiv d(U_\epsilon) \quad , \quad U_\epsilon^0 = [U_\epsilon]_p .
\]

Under the SUSY transformation the RR potential transforms as

\[
\delta_\epsilon C = (d D_\epsilon) e^B
\]

where \(D_\epsilon\) is even form for IIA. The Wess-Zumino action transforms by an exact form with

\[
U_\epsilon = D_\epsilon e^{dA}.
\]

Explicit expression of \(D_\epsilon\) is calculated as

\[
D_\epsilon = \{\tilde{\Theta}_C (C_A \delta_\epsilon \tilde{\Theta}_C + S_A \delta_\epsilon \tilde{\Theta}_S) + \tilde{\Theta}_S (-\tilde{S}_A \delta_\epsilon \tilde{\Theta}_C + \Gamma_{11} \tilde{C}_A \delta_\epsilon \tilde{\Theta}_S)\} e^B.
\]

The conserved SUSY charges are obtained completely from (4.2), (4.3), (4.4), (4.5).

The poisson bracket of (4.2) is calculated as

\[
\{Q_\epsilon, Q_{\epsilon'}\} = \int d\sigma^p \left[ 2 (\epsilon \Gamma \epsilon') p + 2 (\epsilon \Gamma_{11} \Gamma \epsilon') \partial_a X E^a \\
- \frac{1}{2} (\partial_a E^a) \left\{ (\bar{\theta} \Gamma_{11} \Gamma \epsilon') \cdot (\bar{\theta} \Gamma \epsilon') - (\bar{\theta} \Gamma_{11} \Gamma \epsilon') \cdot (\bar{\theta} \Gamma \epsilon') \right\} \\
- 2\Gamma_{(p)} \int d\sigma^p \bar{\epsilon} [C_A (dX)e^{dA}]_{(p)} \epsilon' \right].
\]
In the last term we left only contribution which could remain for the non-trivial topological configuration of $X$ and $A$.

In 10-dimensions there are $10 + 1 + 10 + 45 + 210 + 252 = \frac{32 \times 33}{2} = 528$ independent symmetric Gamma matrices;

$$c\Gamma_M = \{ c\Gamma_m, c\Gamma_{11}, c\Gamma_m\Gamma_{11}, c\Gamma_{m_1m_2} \Gamma_{m_4\Gamma_{11}}, c\Gamma_{m_1m_2m_3m_4m_5} \},$$

(4.8)

Using this basis (4.7) are decomposed to define the central charges,

$$\{ Q_\alpha, Q_\beta \} = -2 (c\Gamma_m)_{\alpha\beta} \mathcal{P}^m$$

$$-2 (c\Gamma_{11}\Gamma_m)_{\alpha\beta} \int d\sigma^p \left( \partial_a X^m E^a \right)$$

$$+ \int d\sigma^p (\partial_a E^a) \left[ -\frac{10}{32} (c\Gamma_{11})_{\alpha\beta} (\bar{\theta}\theta) + \frac{6}{32} (c\Gamma_{m_1m_2})_{\alpha\beta} (\bar{\theta}\Gamma_{11}\Gamma^{m_1m_2}\theta) ight. 
- \left. \frac{2}{32} (c\Gamma_{m_1m_2m_3m_4}\Gamma_{11})_{\alpha\beta} (\bar{\theta}\Gamma_{m_4m_3m_2m_1}\theta) \right]$$

$$-2T(c\Gamma_M)_{\alpha\beta} Z^M,$$

(4.9)

with

$$Z^M = \frac{1}{32} \text{tr} \int \left( \Gamma^M [C_A(dX)e^{dA}]_P \right).$$

(4.10)

Here $\Gamma^M$ is inverse of $\Gamma_M$, $\text{tr}(\Gamma^N c^{-1} c\Gamma_M) = 32 \delta^N_M$ and $\mathcal{P}^m$ in the first term is the total momentum. The SUSY central charges contain membrane charges and topological charges for the worldvolume gauge field. The possible interpretation of the central charges have been given in [14]. We discuss it further in the next section. The SUSY central charges, $Z^M$, are listed in the table 2. The vector indices of $Z^M$ are abbreviated, for examples $Z^{[2]}$ in place of $Z^{m_1m_2}$.

Table 2: SUSY central charges: $Z^M$

| $Z^{11}$ | $Z^{[2]}$ | $Z^{[4]}$ |
|---|---|---|
| D-0 | 1 | 0 |
| D-2 | $dA$ | $(dX)^2$ | 0 |
| D-4 | $(dA)^2/2!$ | $(dX)^2 dA$ | $(dX)^4$ |
| D-6 | $(dA)^4/3!$ | $(dX)^4 (dA)^2/2$ | $(dX)^4 (dA) + * (dX)^6$ |
| D-8 | $(dA)^4/4!$ | $(dX)^4 (dA)^2/3! + * (dX)^8$ | $(dX)^4 (dA)^4/2 + * (dX)^6 dA$ |

In the rest frame IIA D-p-branes the SUSY algebra (4.9) is written as

$$\{ Q_\alpha, Q_\beta \} = -2 \left( (c\mathcal{F})_{\alpha\beta} \otimes 1_{AB} + \sum_{i=1}^{3} (a_i)_{\alpha\beta} \otimes (\tau_i)_{AB} \right)$$

(4.11)
with
\[
\begin{align*}
    a_1 &= Tc\gamma_2 Z^2 \\
    a_2 &= TicZ^{11} + Tic\gamma_4 Z^{[4]}_{11} \\
    a_3 &= c\gamma_m \int \partial_a X^m E^a
\end{align*}
\]

(4.12)

for chiral gamma matrices \( \gamma_m \). \( a_1 \) and \( \tau_i \) are \( 2 \times 2 \) matrices whose indices represent chiralities. The above SUSY algebra (4.11) can be diagonalized because of symmetric indices. In order to examine the bound of the SUSY algebra, we assume \([a_3, a_1 + ia_2] = 0\) and use the fact of \( \text{tr}[a_1, a_2] = 0 \), and also assume that the second part of the right hand side of (4.11) can be diagonalized as \( \lambda \tau_3 \). The assumption \([a_3, a_1 + ia_2] = 0\) reduces to \( E^a = 0 \). For a situation with \( \partial_\mu X^m = \delta_\mu^m \), \( \theta = 0 \) and constant \( U(1) \) gauge fields, the SUSY algebra (4.11) becomes in the rest frame with \( c = i\Gamma_0 \)

\[
\langle P_0 (1 + \lambda \tau_3) \mid \rangle \geq 0, \quad \lambda^2 = -\frac{1}{16} \text{tr} \sum_i (a_i)^2 = V^2_{(p)} T^2 G_F
\]

(4.13)

analogous to the D-string case [13]. \( V_{(p)} \) is the p-brane volume. By using the \( \tau \) diffeomorphism constraint of the D-p-brane (3.10), \( P_0 = \lambda \) is shown under the same assumption,

\[
\int d^p \sigma \tilde{H} = -\frac{1}{2} \frac{1}{V_{(p)}} \left[-(P_0)^2 + \lambda^2 \right] = 0
\]

(4.14)

This confirms the BPS saturated state (4.13). It suggests that groundstates of IIA D-p-branes, which are BPS saturated states, satisfy the above assumptions; \( \partial_\mu X^m = \delta_\mu^m \), \( \theta = 0 \), \( F_{ab} = \text{const.} \) and \( E^a = 0 \). In the BPS saturated state, (4.13) becomes the projection operator \( \lambda(1 + \tau_3) \), then it leads to that only \( N=1 \) SUSY survived.

5 Summary and Discussions

In this paper we obtained the Wess-Zumino action for IIA D-p-branes in an explicit form. A compact expression is obtained by choosing the spinor variables (2.27) analogous to IIB case [10]. The fermionic constraints can be separated into the first class and the second class in a covariant and irreducible way. The first class constraints generate the local supersymmetry (kappa symmetry). In contrast with the case of the fundamental superstring, D-p-brane picture has massive ground states and allow the static gauge. This assumption of the D-brane leads to the covariant and irreducible kappa generator. We have derived the necessary condition of the well-defined separation in (3.14), \( G_F \neq 0 \) [12]. For the super-D-string we have shown that \( (F_{01})^2 > 0 \) guarantees both the static string ground state and the well-defined separation: [13]. This quantity \( F_{01} \) (\( E^1 \) in canonical variable) is interpreted as non-trivial winding of the \( S^1 \) [13]. In other words, the winding of \( S^1 \) makes groundstates to be massive and string-like, and also makes the well-defined separation possible. On the other hand for IIA D-branes the role of \( E^a \) may be different from the one for the D-string.
The global supersymmetry (SUSY) charges and SUSY algebra are calculated for general IIA case. The SUSY algebra contains topological charges with a typical form

$$Z[q] \sim \int (dX)^q (dA)^{p-q}$$

(5.1)

The $q = p$ term is a membrane charge and measures the topologically non-trivial worldvolume. $q < p$ charges correspond to the topological charges for the gauge field. For $p = 2$, $q = 0$ (5.1) allows Dirac monopole configuration in $2 + 1$ dimension [14]. For $q = 0$ (5.1) allows monopole configuration in $p + 1$ dimensions if the gauge group $G$, which gives nontrivial homotopy class $\Pi_p(G)$ [17], is induced by the Chan-Paton factor [1, 16]. It is interesting to consider the case where the worldvolume gauge fields get masses by some mechanism [1, 2]. In such cases for $q = p - 2$, (5.1) represents a $q$-dimensional defect for D-$p$ brane. For example, a surface defect for D-4 brane, a 4-dimensional defect for D-6 brane, and so on. It will also represents $q$-dimensional defects if non-Abelian $A \in G$ is induced where $G$ belongs to non-trivial homotopy class $\Pi_{p-q-1}(G)$. These defects may be interpreted as solitonic configurations of brane coordinates arising from intersections [18, 19, 2].

In this paper we have considered D-$p$-branes only in the trivial background. In case of type IIA there exists an alternative background, namely massive IIA supergravity [20]. The massive IIA supergravity background allows the Chern-Simon terms [3, 5, 21]. The mass parameter is interpreted as the square root of the cosmological constant [22]. The coefficient of the Chern-Simon term is quantized as usual, and so is the cosmological constant. The quantization of the cosmological constant is consistent with the duality argument [9]. The RR gauge-invariant field strength for this case has the following mass dependence [22],

$$dL^{WZ+CS} = T \cdot R e^F, \quad R = dC - HC + me^B$$

(5.2)

The mass dependent term in (5.2) gives the Chern-Simon term. Although $R$ has mass dependence, it again satisfies the same Bianchi identity as the one without mass term (2.4). The previous works related to the Chern-Simon term have been done in the superspace language. It is interesting to examine how the Chern-Simon term is incorporated with the kappa symmetry and supergravity in the component language. We leave this issue for future investigation.

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