The Ultraviolet and Infrared Behavior of an Abelian Proca Model From the Viewpoint of a One-Parameter Extension of the Covariant Heisenberg Algebra

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Abstract

Recently a one-parameter extension of the covariant Heisenberg algebra with the extension parameter $l$ ($l$ is a non-negative constant parameter which has a dimension of $[\text{momentum}]^{-1}$) in a $(D + 1)$-dimensional Minkowski space-time has been presented [G. P. de Brito, P. I. C. Caneda, Y. M. P. Gomes, J. T. Guaitolini Junior and V. Nikoofard, Effective models of quantum gravity induced by Planck scale modifications in the covariant quantum algebra, Adv. High Energy Phys. 2017 (2017) 4768341]. The Abelian Proca model is reformulated from the viewpoint of the above one-parameter extension of the covariant Heisenberg algebra. It is shown that the free space solutions of the above modified Proca model satisfy the modified dispersion relation

$$\frac{p^2}{1 + l^2 p^2} = m^2 c^2$$

where $\Lambda = \hbar l$ is the characteristic length scale in our model. This modified dispersion relation describes two massive vector particles with the effective masses $M_\pm (\Lambda) = \frac{2m}{1 + \sqrt{1 - 2m^2 \Lambda^2}}$. Numerical estimations show that the maximum value of $\Lambda$ in a four-dimensional space-time is near to the electroweak length scale, i.e., $\Lambda_{\text{max}} \approx l_{\text{electroweak}} \approx 10^{-18} \, m$. We show that in the infrared/large-distance domain the modified Proca model behaves like an Abelian massive Lee-Wick model which has been presented by Accioly and his co-workers [A. Accioly, J. Helayel-Neto, G. Correia, G. Brito, J. de Almeida and W. Herdy, Interparticle potential energy for D-dimensional electromagnetic models from the corresponding scalar ones, Phys. Rev. D 93 (2016) 105042].

Keywords: Classical field theories; Gauge field theories; Nonlinear or nonlocal theories and models; Higher derivatives; Canonical formalism, Lagrangians, and variational principles; Gauge bosons; Characteristic length scale

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1 Introduction

Although quantum field theory is a very successful theory which describes the fundamental interactions at the microscopic level, the study of short-distance (high-energy) behavior of fundamental interactions in quantum field theory leads to ultraviolet divergences [1-3]. Today we know that these ultraviolet divergences in quantum field theories can be removed by using the standard renormalization techniques. One of these renormalization techniques which is very close to the Pauli-Villars regularization technique is the addition of higher-order derivative terms to the Lagrangian density of a quantum field theory [1-3]. On the other hand, this idea that there is a minimal length scale in the measurement of space-time distances of the order of the Planck length is predicted by different theories of quantum gravity such as string theory, loop quantum gravity and non-commutative geometry [4-6]. The existence of this minimal length scale in quantum gravity leads to the following generalized uncertainty principle:

\[ \Delta X \sim \frac{\hbar}{\Delta P} + \alpha \frac{\Delta P}{\hbar}, \]  

where \( l_s = \sqrt{\frac{\alpha'}{c}} \approx 10^{-32} \text{ cm} \), \( l_s \) is the string length, and \( \frac{\hbar}{\alpha'} \) is the string tension [4]. The generalized uncertainty principle (1) implies the existence of a nonzero minimal length scale which is given by

\[ \Delta X_{\text{min}} = 2l_s. \]

It should be noted that the reformulation of the quantum field theory in the presence of a minimal length scale is another way for obtaining a divergence free quantum field theory [4-6]. In 2006, C. Quesne and V. M. Tkachuk introduced a \((\beta,\beta')\)-two-parameter extension of the covariant Heisenberg algebra in a \((D+1)\)-dimensional Minkowski space-time which is described by the following modified commutation relations:

\[
\begin{align*}
[X^\mu, P^\nu] &= -i\hbar \left[ (1 - \beta P^2) \eta^{\mu\nu} - \beta' P^\mu P^\nu \right], \\
[X^\mu, X^\nu] &= i\hbar \frac{2\beta - \beta' - (2\beta + \beta')\beta P^2}{1 - \beta P^2} (P^\mu X^\nu - P^\nu X^\mu), \\
[P^\mu, P^\nu] &= 0,
\end{align*}
\]

where \( \mu, \nu = 0, 1, 2, \cdots, D \), \( \beta \) and \( \beta' \) are two non-negative constant parameters with dimension of \([\text{momentum}]^{-2}, [X^\mu, P^\nu] \) are the modified position and momentum operators, \( P^2 = (P^0)^2 - \sum_{i=1}^{D} (P^i)^2 = (P^0)^2 - \vec{P}^2 \), and \( \eta_{\mu\nu} = \eta^{\mu\nu} = \text{diag}(+1, -1, -1, \ldots, -1) \) is the flat Minkowski metric [6]. The Quesne-Tkachuk algebra (2) predicts the existence of an isotropic minimal length scale which is given by

\[ (\Delta X^i)_0 = \hbar \sqrt{(D\beta + \beta') \left[ 1 - \beta ((P^0)^2) \right]}, \quad \forall i \in \{1, 2, \ldots, D\}. \]

In recent years, reformulation of Maxwell electrodynamics in the presence of a minimal length scale and the study of short-distance behavior of Maxwell theory has attracted a considerable attention among
researchers in quantum field theory [7-10]. In Ref. [9], it has been shown that in minimal length electro-
statics the classical self-energy of a point charge has a finite value. A free massless spin-2 field \( h_{\mu\nu}(x) \) in a \((D + 1)\)-dimensional Minkowski space-time is described by the Pauli-Fierz action as follows:

\[
S_{PF} = \frac{c^3}{16\pi G_N(D+1)} \int d^{D+1}x \, L_{PF},
\]

\[
L_{PF} = \frac{1}{4} \left( \partial_\lambda h^{\mu\nu}(x) \partial^4 h_{\mu\nu}(x) - 2\partial_\mu h^{\mu\nu}(x) \partial^4 h_{\nu\lambda}(x) + 2\partial_\mu h^{\mu\nu}(x) \partial_\lambda h_{\nu\lambda}(x) - \partial_\mu h^{\nu\lambda}(x) \partial^4 h_{\nu\lambda}(x) \right).
\]

The reformulation of the Pauli-Fierz theory from the viewpoint of the Quesne-Tkachuk algebra has been studied in details in Ref. [11]. In Ref. [12], the following non-local model for electrodynamics has been presented

\[
L = - \frac{1}{4\mu_0} \left( F_{\mu\nu}(x) \right) \left( e^{\frac{i2}{c^2} \Box} F^{\mu\nu}(x) \right) - J^\mu(x)A_\mu(x),
\]

where \( l_* \) is a constant parameter which has a dimension of [length], \( A^\mu = (\phi, A_x, A_y, A_z) \) is the potential four-vector, \( J^\mu = (c\rho, J_x, J_y, J_z) \) is the current four-vector, and \( \Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \) is the d’Alembertian operator in a four-dimensional flat space-time. The authors of Ref. [12] have shown that the classical self-energy of a point charge in Eq. (4) has a finite value (see Eq. (44) in Ref. [12]). In a recent paper, a one-parameter extension of the covariant Heisenberg algebra in a \((D + 1)\)-dimensional Minkowski space-time has been suggested [13]. This algebra is a covariant generalization of the Kempf-Mangano algebra [14]. In the present paper the ultraviolet/short-distance and infrared/large-distance behavior of an Abelian Proca model in the framework of the covariant Kempf-Mangano algebra are studied analytically.

This paper is organized as follows. In Sect. 2, the structure of one-parameter extension of the covariant Heisenberg algebra in a \((D + 1)\)-dimensional flat space-time is introduced according to Ref. [13]. In Sect. 3, Lagrangian reformulation of the Abelian Proca model from the viewpoint of one-parameter extension of the covariant Heisenberg algebra in a \((D + 1)\)-dimensional space-time is presented. In Sect. 4, we show that the free space solutions of the modified Proca model in Sect. 3 describe two massive vector particles. Our calculations show that there is a characteristic length scale \( \Lambda \) whose maximum value is near to the electroweak length scale, i.e., \( \Lambda_{\text{max}} \sim l_{\text{electroweak}} \sim 10^{-18} \) m. In summary and conclusions, we show that in the infrared region the modified Proca theory in Sect. 3 behaves like an Abelian massive Lee-Wick model. SI units are used throughout this paper.

## 2 One-Parameter Extension of the Covariant Heisenberg Algebra

In 2017, G. P. de Brito and co-workers introduced a one-parameter extension of the covariant Heisenberg algebra [13]. This algebra in a \((D + 1)\)-dimensional Minkowski space-time is characterized by the
following modified commutation relations:

\[
[X^\mu, P^\nu] = -i\hbar \left( \frac{1}{2} \sqrt{1 - 2l^2 P^2} \eta^{\mu\nu} - l^2 P^\mu P^\nu \right),
\]

(5)

\[
[X^\mu, X^\nu] = 0,
\]

(6)

\[
[P^\mu, P^\nu] = 0,
\]

(7)

where \( l \) is a non-negative constant parameter which has a dimension of \([\text{momentum}]^{-1}\). The modified position and momentum operators \( X^\mu \) and \( P^\mu \) in the above algebra have the following exact coordinate representation [13]

\[
X^\mu = x^\mu, \quad P^\mu = \frac{1}{1 + \frac{e^2}{2}\vec{p}^2} p^\mu,
\]

(8)

(9)

where \( x^\mu \) and \( p^\mu = i\hbar \partial^\mu \) are the position and momentum operators which satisfy the following usual covariant Heisenberg algebra

\[
[x^\mu, p^\nu] = -i\hbar \eta^{\mu\nu},
\]

(10)

\[
[x^\mu, x^\nu] = 0,
\]

(11)

\[
[p^\mu, p^\nu] = 0.
\]

(12)

In Eq. (9) \( \vec{p}^2 = p_\alpha p^\alpha = -\hbar^2 \Box \).

According to Eqs. (8) and (9) in order to reformulate a quantum field theoretical model in the framework of a one-parameter extension of the covariant Heisenberg algebra, the usual position and derivative

\[1\text{It must be emphasized that this algebra is a relativistic generalization of the following algebra}\]

\[
[X^i, P^j] = i\hbar \left( \frac{\vec{p}^2}{\sqrt{1 + 2\beta^2 \vec{p}^2}} \delta^{ij} + \vec{p}^i p^j \right),
\]

\[
[X^i, X^j] = 0,
\]

\[
[P^i, P^j] = 0, \quad i, j = 1, 2, \cdots, D,
\]

which was introduced previously by Kempf and Mangano in Ref. [14].
operators \( (x^\mu, \partial_\mu) \) must be replaced by the modified position and derivative operators \( (X^\mu, \nabla_\mu) \) as follows:

\[
\begin{align*}
  x^\mu & \longrightarrow X^\mu = x^\mu, \\
  \partial_\mu & \longrightarrow \nabla_\mu := \frac{1}{1 - \frac{\hbar l^2}{2 \Box}} \partial_\mu.
\end{align*}
\]

(13) 
(14)

Note that the quantity \( \hbar l \) in Eq. (14) defines a characteristic length scale \( \Lambda := \hbar l \) in our calculations. For the space-time distances very greater than \( \Lambda \) the space-time algebra becomes the usual covariant Heisenberg algebra (Eqs. (10)-(12)), while for the space-time distances very smaller than \( \Lambda \) the structure of space-time must be described by Eqs. (5)-(7).

3 Reformulation of the Abelian Proca Model in the Framework of One-Parameter Extension of the Covariant Heisenberg Algebra

The Abelian Proca model for a massive spin 1 vector field \( A^\mu = (\phi, A^1, \ldots, A^D) = (\phi, A) \) in the presence of an external current \( J^\mu = (c\rho, J^1, \cdots, J^D) = (c\rho, J) \) in a \((D + 1)\)-dimensional Minkowski space-time is [15-17]

\[
\mathcal{L} = - \frac{1}{4\mu_0} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2\mu_0} \left( \frac{mc}{\hbar} \right)^2 A_\mu(x) A^\mu(x) - J^\mu(x) A_\mu(x),
\]

(15)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic field tensor and \( m \) is the mass of the gauge particle. If we use Eq. (13) together with the transformation rule for a covariant vector, we will obtain the following results

\[
A_\mu(x) \longrightarrow B_\mu(X) = \frac{\partial x^\nu}{\partial X^\mu} A_\nu(x)
\]

(16)

\[
J_\mu(x) \longrightarrow j_\mu(X) = \frac{\partial x^\nu}{\partial X^\mu} J_\nu(x)
\]

(17)
Using Eqs. (14) and (16) the modified electromagnetic field tensor $G_{\mu\nu}(X)$ becomes

$$F_{\mu\nu}(x) \rightarrow G_{\mu\nu}(X) = \nabla_\mu B_\nu(X) - \nabla_\nu B_\mu(X) = \frac{1}{1 - \frac{\Lambda^2}{2} \Box} F_{\mu\nu}(x).$$

(18)

If we use Eqs. (15)-(18), we will obtain the modified Lagrangian density for an Abelian Proca model in the ultraviolet region as follows:

$$L = -\frac{1}{4\mu_0} G_{\mu\nu}(X) G^{\mu\nu}(X) + \frac{1}{2\mu_0} \left(\frac{mc}{\hbar}\right)^2 B_\mu(X) B^\mu(X) - j^\mu(X) B_\mu(X)$$

$$= -\frac{1}{4\mu_0} \left(\frac{1}{1 - \frac{\Lambda^2}{2} \Box} F_{\mu\nu}(x)\right) \left(\frac{1}{1 - \frac{\Lambda^2}{2} \Box} F^{\mu\nu}(x)\right) + \frac{1}{2\mu_0} \left(\frac{mc}{\hbar}\right)^2 A_\mu(x) A^\mu(x) - J^\mu(x) A_\mu(x)$$

$$= \sum_{n=0}^\infty \Lambda^{2n} L_n,$$

(19)

where

$$L_0 = -\frac{1}{4\mu_0} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2\mu_0} \left(\frac{mc}{\hbar}\right)^2 A_\mu(x) A^\mu(x) - J^\mu(x) A_\mu(x),$$

(20)

$$L_1 = -\frac{1}{4\mu_0} F_{\mu\nu}(x) \Box F^{\mu\nu}(x).$$

(21)

The Lagrangian density (19) describes an infinite derivative Abelian massive gauge field $A_\mu(x)$. It should be noted that the expression $L_0 + \Lambda^2 L_1$ in the above equations is the Lagrangian density of an Abelian massive Lee-Wick model [18].

For a classical field theory which is described by the following Lagrangian density:

$$L = L(A_\lambda, \partial_\nu A_\lambda, \partial_\nu \partial_\nu A_\lambda, \partial_\nu \partial_\nu \partial_\nu A_\lambda, \ldots),$$

(22)

the Euler-Lagrange equation for the gauge field $A_\lambda$ becomes [21,22]

$$\frac{\partial L}{\partial A_\lambda} - \left(\frac{\partial L}{\partial A_{\lambda,\nu_1}}\right)_{\nu_1} + \left(\frac{\partial L}{\partial A_{\lambda,\nu_1 \nu_2}}\right)_{\nu_1 \nu_2} + \ldots + (-1)^k \left(\frac{\partial L}{\partial A_{\lambda,\nu_1 \nu_2 \ldots \nu_k}}\right)_{\nu_1 \nu_2 \ldots \nu_k} + \ldots = 0,$$

(23)

where

$$A_{\lambda,\nu_1 \nu_2 \ldots \nu_k} := \partial_\nu_1 \partial_\nu_2 \ldots \partial_\nu_k A_\lambda,$$

(24)

$$\frac{\partial A_{\lambda,\nu_1 \nu_2 \ldots \nu_k}}{\partial A_{\lambda,\mu_1 \mu_2 \ldots \mu_k}} = \delta^\nu_1^\mu_1 \delta^\nu_2^\mu_2 \ldots \delta^\nu_k^\mu_k.$$
If we insert Eq. (19) into Eq. (23), we will obtain the inhomogeneous infinite derivative Proca equation as follows:

\[
\frac{1}{(1 - \Lambda^2 \Box)^2} \partial_\mu F^{\mu\nu}(x) + \left(\frac{mc}{\hbar}\right)^2 A^\nu(x) = \mu_0 J^\nu(x). \tag{26}
\]

In the limit \( \Lambda \to 0 \), the modified Proca equation in Eq. (26) becomes the usual Proca equation, i.e.,

\[
\partial_\mu F^{\mu\nu}(x) + \left(\frac{mc}{\hbar}\right)^2 A^\nu(x) = \mu_0 J^\nu(x). \tag{27}
\]

After taking divergence of both sides of Eq. (26) and using the relations \( \partial_\nu \partial_\mu F^{\mu\nu}(x) = 0 \) and \([\partial_\nu, \Box] = 0\), we obtain

\[
\partial_\mu J^\mu(x) = \frac{1}{\mu_0} \left(\frac{mc}{\hbar}\right)^2 \partial_\nu A^\nu(x). \tag{28}
\]

Note that the above equation is a consequence of the modified field equation (26). If we substitute (28) in (26), we will obtain

\[
\frac{1}{(1 - \Lambda^2 \Box)^2} \Box A^\nu(x) + \left(\frac{mc}{\hbar}\right)^2 A^\nu(x) = \mu_0 \left[ J^\nu(x) + \left(\frac{\hbar}{mc}\right)^2 \partial_\nu \left(\partial_\mu J^\mu(x)\right)\right]. \tag{29}
\]

In the next section, we will study the free space solutions of the inhomogeneous infinite derivative Proca equation.

### 4 Free Space Solutions of the Infinite Derivative Abelian Proca Model

In free space (i.e., \( J^\mu = (0, 0, \ldots, 0) \)), the infinite derivative field equation (29) can be written as follows:

\[
\frac{1}{(1 - \frac{\Lambda^2}{2} \Box)^2} \Box A^\nu(x) + \left(\frac{mc}{\hbar}\right)^2 A^\nu(x) = 0. \tag{30}
\]

The modified field equation (30) has the following plane wave solution

\[
A^\nu(x) = A \, e^{-\frac{\hbar}{mc} \cdot \mathbf{p} \cdot x} \, \epsilon^\nu(p), \tag{31}
\]

where \( \epsilon^\nu(p) \) is the polarization vector and \( A \) is the amplitude of the vector field. If we insert (31) in (30), we will obtain the following modified dispersion relation:

\[
\frac{p^2}{(1 + \frac{\Lambda^2}{2\hbar} p^2)^2} = m^2 c^2. \tag{32}
\]
The modified dispersion relation (32) leads to the following modified energy-momentum relations:

\[ E^2_s(\Lambda) = M^2_s(\Lambda)c^4 + c^2 \vec{p}^2, \]
\[ E^2_s(\Lambda) = M^2_s(\Lambda)c^4 + c^2 \vec{p}^2, \]  

where the effective masses \( M_s(\Lambda) \) and \( M_s(\Lambda) \) are defined as follows:

\[ M_s(\Lambda) := \frac{2m}{1 - \sqrt{1 - 2 \left( \frac{mc}{\hbar} \right)^2}}, \]  
\[ M_s(\Lambda) := \frac{2m}{1 + \sqrt{1 - 2 \left( \frac{mc}{\hbar} \right)^2}}. \]

In order to avoid imaginary masses in (35) and (36) the characteristic length scale \( \Lambda \) must satisfy the following relation

\[ \Lambda \leq \frac{1}{\sqrt{2}} \lambda_c, \]  

where \( \lambda_c = \frac{\hbar}{mc} \) is the reduced Compton wavelength of the particle \( m \).

According to Eq. (37), the maximum value of the characteristic length \( \Lambda \) is

\[ \Lambda_{\text{max}} = \frac{1}{\sqrt{2}} \lambda_c. \]

Now, let us study the low-energy (large-distance) behavior of the effective masses \( M_s(\Lambda) \) for \( \Lambda \to 0 \). For \( \Lambda \to 0 \) the effective masses \( M_s(\Lambda) \) in Eqs. (35) and (36) have the following low-energy expansions:

\[ M_s(\Lambda) = \left[ 2m \lambda^2_c \left( \frac{1}{\Lambda^2} - m - \frac{m}{2\lambda^2_c} \Lambda^2 + O(\Lambda^4) \right) \right], \]
\[ M_s(\Lambda) = m \left[ 1 + \frac{\Lambda^2}{2\lambda^2_c} + \frac{\Lambda^4}{2\lambda^4_c} + O(\Lambda^6) \right]. \]  

Equations (39) and (40) show that the low-energy limit of our model contains two massive vector particles, one with the usual mass \( m \) and the other a heavy-mass particle of mass \( \frac{2m\lambda^2_c}{\Lambda^2} \). \[ ^3 \]

\[ ^3 \]For \( \Lambda = \frac{1}{\sqrt{2}} \lambda_c \) both effective masses \( M_s(\Lambda) \) and \( M_s(\Lambda) \) have the same value \( M_s(\Lambda) = M_s(\Lambda) = 2m. \)

\[ ^4 \]It is necessary to note that the appearance of such heavy-mass particles in higher-order derivative quantum field theories leads to an indefinite metric (for a review, see Refs. [23-25]).
5 Summary and Conclusions

In 2017, G. P. de Brito and his collaborators proposed a covariant generalization of the Kempf-Mangano algebra in a \((D + 1)\)-dimensional Minkowski space-time [13]. In this paper, after reformulation of the Abelian Proca model from the viewpoint of the covariant generalization of the Kempf-Mangano algebra, we showed that our modified Proca model describes two massive vector particles (see Eqs. (35) and (36)). We proved that there is a characteristic length scale \(\Lambda = \hbar l\) in the modified Proca theory (Eq. (19)) whose upper limit is given by Eq. (38), i.e., \(\Lambda_{\text{max}} = \frac{1}{\sqrt{2}} \lambda_c\). According to our calculations, for \(\Lambda \to 0\) the modified Proca theory in Eq. (19) behaves like an Abelian massive Lee-Wick model, i.e.,

\[
L_{\text{Abelian massive Lee-Wick model}} = L_0 + \Lambda^2 L_1 + O(\Lambda^4)
= -\frac{1}{4\mu_0} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{1}{4\mu_0} \Lambda^2 F_{\mu\nu}(x) \Box F^{\mu\nu}(x)
+ \frac{1}{2\mu_0} \left(\frac{mc}{\hbar}\right)^2 A_\mu(x) A^\mu(x) - J^\mu(x) A_\mu(x) + O(\Lambda^4).
\]

(41)

Now, let us estimate the numerical value of \(\Lambda_{\text{max}} = \frac{1}{\sqrt{2}} \lambda_c = \frac{1}{\sqrt{2}} \frac{\hbar}{mc}\) in Eq. (38).

The usual Proca equation (27) plays a fundamental role in nuclear and low-energy particle physics [26,27]. The four-dimensional Proca wave equation for a neutral \(Z^0\) boson in a nucleus is

\[
\left[\frac{1}{c^2 \partial^2} - \nabla^2 + \left(\frac{MC}{\hbar}\right)^2\right] \phi_\mu(\vec{x}, t) = \frac{1}{\epsilon_0} \rho_\mu(\vec{x}, t),
\]

(42)

where \(\rho_\mu(\vec{x}, t)\) is the neutral weak-charge density (see page 10 in Ref. [26]). The mass of the \(Z^0\) boson is [26]

\[
M_z = (91.187 \pm 0.007) \text{ GeV}/c^2 \sim 100 \text{ proton masses}.
\]

(43)

According to Eq. (38) the maximum value of the characteristic length scale \(\Lambda\) in our paper is proportional to \(\lambda_c = \frac{\hbar}{mc}\), i.e.,

\[
\Lambda_{\text{max}} \sim \lambda_c.
\]

(44)

Inserting (43) into \(\lambda_c = \frac{\hbar}{mc}\), we find

\[
\lambda_z = \frac{\hbar}{M_z c} \approx 2 \times 10^{-18} \text{ m}.
\]

(45)

A comparison between Eqs. (44) and (45) shows that the maximum value of the characteristic length scale \(\Lambda\) in this research is

\[
\Lambda_{\text{max}} \sim 10^{-18} \text{ m}.
\]

(46)

It is interesting to note that the numerical value of \(\Lambda_{\text{max}}\) in Eq. (46) is near to the electroweak length scale [27,28], i.e.,

\[
\Lambda_{\text{max}} \sim l_{\text{electroweak}} \sim 10^{-18} \text{ m}.
\]

(47)
On the other hand, the numerical estimation of $\Lambda_{\text{max}}$ in Eq. (46) is about three orders of magnitude smaller than the nuclear scale of $10^{-15} \text{ m}$ [28], i.e.,

$$\Lambda_{\text{max}} \sim 10^{-3} l_{\text{nuclear scale}}.$$  (48)

We showed that in the infrared region, the usual Proca model is recovered (Eq.(15)), while in the ultraviolet region the Abelian Proca model must be described by an infinite derivative Lagrangian density (Eq. (19)). In our future works, we will study the interparticle potential energy for the modified Proca model which has been presented in this paper.

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