Numerical simulation of the effect of laser radiation on matter in an external magnetic field

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Abstract. Numerical study of the influence of magnetic field on the plasma compressed by powerful source is carried out. Numerical experiments for the magnetized plasma behavior during and after laser pulse are presented. Convective and thermal parts of a model were tested on several tasks. Spatial distribution of the plasma pressure, temperature, velocity, density, magnetic pressure and laser radiation are obtained in the presence of the external magnetic field.

1. Introduction
The experimental study of thermonuclear plasma should be accompanied by the construction of multilevel radiation and gas dynamic computational modeling that adequately describe the processes in the active zone of fusion reactor. Thus, calculation-theoretical methods are an important element in the development process of the concept of magneto-inertial fusion. A magneto-inertial fusion system is a pulsed thermonuclear device, where a cylindrically or spherically symmetric target is placed in the seed magnetic field and compressed by powerful laser beams or shells of all kinds, including gas, liquid and metal shells, plasma liners formed by the merger of high-speed plasma jets, etc. It is obvious that the developed mathematical models and methods of computing target plasma dynamics of magneto-inertial fusion require in this case obligatory verification on the basis of comparing to the reliable computation and experimental data of the physical experiment.

In this theoretical study, we have calculated the behavior of plasma, due to the interaction of laser pulse with magnetized plasma [1-7]. Nonlinear stability of an axial electric field was taken into account [8], modeling and simulation of laser beams and plasma jet by numerical methods for different applications [9, 10] have been done previously. However, the influence of magnetic fields on power source-matter interaction still requires consideration.

We developed a high accuracy method (eighth order accuracy) for numerical simulation of the laser-matter interaction in the external magnetic field. The effects of magnetic field in plasma compressed by lasers, namely the influence of laser power on magnetized target, are studied.

A numerical method for high resolution developed in this work has improved dissipative and dispersive properties (especially for modeling of turbulent flows) when the multidimensional convective flux discretization is implemented. Quasi-monotone numerical methods do not cause the appearance of false extrema in the spatial distributions (unphysical oscillations developing over time).
The advantage of higher resolution is that high spatial accuracy is obtained with a relatively small number of points of the spatial grid. This approach describes well the sharp and monotone spatial profiles of all gas dynamic parameters. Also note that the developed method is applicable for numerical solving the problem of compression of magnetized plasma by high speed plasma jets.

2. Mathematical model of laser-target interaction with external magnetic field

In general, the subjects of magneto-inertial fusion research are spatial flows from the plasma involving heat and mass transfer, electromagnetic fields, and nuclear reactions. Solving these problems requires the development of new plasmadynamic mathematical models that describe the processes proceeding in thermonuclear plasma, as well as the creation of high-precision numerical methods for solving them, that will allow carrying out with adequate numerical simulation. At the same time, it is expedient to base initial studying of the main physical relationships of magneto-inertial fusion on system of simplified one-dimensional mathematical models.

We will perform the description of the one-dimensional mathematical model of compressing the thermonuclear magneto-inertial fusion targets process for the centrally-symmetric coordinate system. The mathematical model of magneto-inertial fusion targets presented in this work is based on one-dimensional equations of radiation plasma dynamics: Euler system of equations (1), the transfer equation of own broadband radiation (2), the equation of magnetic induction (3), the transfer equation of laser radiation, computational methods for the equations of state and absorption coefficients for laser radiation. This model determines the conditions of creating and achieving self-sustaining fusion reaction:

\[
\frac{\partial \rho}{\partial t} + \text{Div}(\rho u) = F_\rho, \quad F_\rho = -\rho u \frac{(v-1)}{r},
\]

\[
\frac{\partial (\rho u)}{\partial t} + \text{Div}(\rho u^2 + P) = F_{\rho u} + f_r, \quad F_{\rho u} = -\rho u^2 \frac{(v-1)}{r}, \quad f_r = \frac{1}{c} [\vec{j} \times \vec{H}]_r,
\]

\[
\frac{\partial (\rho E)}{\partial t} + \text{Div}(\rho Eu + Pu + q_\Sigma) = F_E + q_r + Q_{\text{Fus}}^e, \quad q_r = j_E, \quad q_\Sigma = q_e + q_i + q_{\text{laz}}.
\]

\[
F_E = -(\rho Eu + Pu) \frac{(v-1)}{r}, \quad P = P_e + P_i, \quad \text{Div}(\ ) = \frac{1}{J} \frac{\partial (J )}{\partial \xi}, \quad J = r^{(v-1)},
\]

where \(\rho\) is the density, \(u\) is the speed along the radial coordinate \(r\), \(t\) is the time, \(P = P(\rho, \varepsilon)\) is the static pressure, \(\varepsilon\) is the specific internal energy, \(E = (\varepsilon + u^2/2)\) is the total energy of gas stream, \(\vec{F} = (F_\rho, F_{\rho u}, F_E)\) is the vector of sources, \(F_\rho\) is the mass flow density, \(F_{\rho u}\) is the momentum flux density, \(F_E\) is the laser fluence, \(q, q_r\) are total and spectral radiant fluxes, respectively, \(T_e, T_i\) are the electron and ion temperatures in plasma volume \((T = T_e = T_i)\), respectively, \(\chi_e\) is the spectral absorption constant, \(f_r\) is the electromagnetic force, \(q_r\) is the energy flow in the electromagnetic field, \(q_e = -\lambda_e \text{grad} T_e, \quad q_i = -\lambda_i \text{grad} T_i, \quad \lambda_e\) and \(\lambda_i\) are the thermal conductivity coefficients of electrons and ions, \(j_r\) is the specific current, \(\vec{H}(r)\) is the magnetic induction vector, \(P_e\) is the electron pressure, \(P_i\) is the ion pressure, index \(v = (1,2)\) obeys plane and axial symmetry.

The contribution of local (thermonuclear) energy release \(Q_{\text{Fus}}^e\) to the electronic component of plasma due to the energy transfer to electrons from thermonuclear \(\alpha\)-particles can be determined using an approximation formula [11]:

\[
Q_{\text{Fus}}^e = 8.483 \times 10^{29} \rho^2 \left(1 + 0.232 \cdot T_i^{3/4}\right) \exp \left(-20/T_i^{1/3}\right), \quad \left(\text{erg cm}^{-3} \cdot \text{s}\right).
\]

The broadband radiation transfer can be considered using multigroup diffusive approximation [12]:
\[ \frac{1}{r^n} \frac{d}{dr} \left( r^n q_r \right) + \frac{\chi_r c U_r}{c} = \chi_e 4\sigma T^4 + \frac{c}{3} \frac{d}{dr} U_r + \chi_e q_e = 0, \tag{2} \]

where \( q_r \) and \( U_r \) are the spectral flux and apparent density of broadband radiation, respectively; \( c \) is the speed of light, \( \nu \) is the frequency group number, \( \chi_r \) is the spectral absorption constant, \( n=0 \) refers to the flat layer, \( n=1 \) is applied to the infinite one-dimensional cylinder. Here \( q_v \) refers to the radiation flux in the axis \( r \) direction.

The equation of magnetic induction taking into account the continuity equation and the conservation law \( \text{div} (\mathbf{B}) = 0 \) can be written as [13]:

\[ \frac{\partial (B_z / \rho)}{\partial t} + \frac{1}{\mu_0} \frac{\partial J (\mathbf{B}_z / \rho)}{\partial r} = \frac{c^2}{4\pi \mu_0} \frac{\partial}{\partial r} \left( \frac{J_r \partial B_r}{\sigma} \right), \tag{3} \]

The electrical conductivity is determined by the Spitzer formula [14] taking into account possible plasma magnetization:

\[ \sigma(z) = 12.06 \times 10^{13} \cdot \tau^{3/2} \cdot \frac{1}{\ln \lambda} \cdot \sum_i n_i z_i^2 \left( 1 + (\Omega_e \tau_e)^2 \right)^2, \]

where \( z_i \) and \( n_i \) are the average charge and ion density, respectively, \( \Omega_e = e|\mathbf{B}|/m_e c \) is the electron gyrofrequency, \( m_e \) and \( M_i \) are the electron and ion masses, respectively.

The thermal conductivity coefficients of electrons and ions \( \lambda_{\text{e,i}} \), in case plasma is magnetized, can be calculated using the formulas [14].

Parameters of laser radiation along an axis \( r \) are found out on the basis of the solution of the transfer equation of laser radiation:

\[ \frac{d^2 q_{\text{Las}}}{dz} - \chi_{\omega} q_{\text{Las}} = 0, \tag{4} \]

where \( \tau \) is the half-width at half-maximum pulse duration. An absorption constant of laser radiation \( \chi_{\omega} \) is determined using a continuum absorption mechanism that is opposite to the braking electron radiation machine in the conditions of the local thermodynamic equilibrium (LTE):

\[ \chi_{\omega} = \left[ \frac{4.97 \cdot 10^3}{\lambda^2} \cdot \frac{\sum_{n_e} n_e}{\sum_{n_i} n_i} \cdot \frac{1}{\sqrt{1 - n_e/n_c}} \right], \quad n_e < n_c, \tag{5} \]

where \( \lambda \) is the wavelength of laser radiation (\( \mu m \)), \( n_e \) and \( n_i \) are electron and ion densities (\( \text{cm}^{-3} \)), \( kT_e \) is the electron temperature (\( \text{keV} \)), \( g \) is the Gaunt factor [14, 15].

The calculation of the optical \( \chi_{\omega}(T, \rho) \) and thermodynamic \( e(T, \rho) \), \( P(T, \rho) \) parameters of the operating environment was carried out in the framework of LTE approximation using the computer system ASTEROID developed by RAS correspondent member S.T. Surzhikov [16]. This system works on Tomas-Fermi’s model with quantum and exchange corrections [17, 18] and on average charge model [19-21].

A computational domain and target of magneto-inertial fusion consist of the central part and one coaxial layer. They have a cylindrical shape with the following range of values of initial parameters of the target and medium. The central part of the target (inner radius \( R = 0.05 \text{ cm} \)) is filled with \( D-T \) mixture with density \( \rho = 5 \times 10^2 \text{ g/cm}^3 \) and temperature of \( T = 297 \text{ K} \). It was surrounded by the coaxial layer (outer radius \( R_0 = 0.1 \text{ cm} \)) consisting of metal (Al) with density \( \rho = 2.7 \text{ g/cm}^3 \) and temperature of \( T = 297 \text{ K} \). The computational domain has an outer radius \( r = 0.2 \text{ cm} \). Thermodynamic parameters of external rarefied medium (Ar) are defined: \( T = 297 \text{ K}, \rho = 1.29 \times 10^{-3} \text{ g/cm}^3 \).

3. Results
For a reasonable quantitative assay of physical processes, which can proceed in plasma of a magneto-inertial fusion target, it is necessary to carry out verification of the recommended mathematical model and numerical methods and test the results on the model problems.

Convective parts of a computer model of magneto-inertial fusion targets are tested on the one-dimensional Riemann problem (for some particular initial data, called the Soda problem) about decay of an unstable fracture configuration. The comparison of exact and approximate solutions shows that the difference is no more than one percent.

The "thermal" part of the model was tested on some tasks allowing accurate analytical solutions: heating of the continuous medium [13], filling a flat semi-bounded space \( r>0 \), by heat flow through the left-hand fixed boundary \( r=0 \).

Numerical calculations presented in this paper are carried out for the pulsed Nd laser with radiation flux density being within the range from \( 10^{12} \) to \( 10^{14} \) W/cm\(^2\). Here is a brief description of the obtained results, which geometrical representation is shown in figures 1–4. These results correspond to the following parameters of the mathematical model: \( Q^e_{\text{Fus}} = 0 \), laser radiation power density \( q_{\text{Las}} = 2 \cdot 10^{14} \) W/cm\(^2\), and exposure time \( t_{\text{Las}} \in [0,10] \) ns.

**Figure 1.** Spatial distribution of the pressure (left) and temperature (right) at \( t=2.14 \) ns (\( t_{\text{Las}}=5 \) ns).

**Figure 2.** Distribution of plasma velocity (left) and plasma density (right) at \( t=2.14 \) ns along the radial coordinate.
The distributions of static $P$ and magnetic $P_{\text{mag}}$ pressure, plasma temperature $T$, velocity $V$, density $\rho$ and laser flux $q_{\text{Las}}$, that correspond to the first "initial" compression stage ($t=2.14$ ns) of a magneto-inertial fusion target placed in an external magnetic field are given in figures 1-3. The laser pulse duration is 5 ns, the laser radiation flux density is $2 \times 10^{14}$ W/cm$^2$. A thin metallic cylindrical shell material is Al. The target (outer) radius is 0.1 cm, and the inner radius is 0.05 cm.

The plasmodynamic parameters of the collapse stage are defined by the shock wave reflected from the axis of symmetry, which moves towards the outer boundary of the computational domain. At the same time the maximum values of plasma pressure and temperature are observed along the axis of symmetry of the system.

The distributions of the magnetic pressure $P_{\text{mag}}$ and combined flux $q$ of self-radiation plasma formation are shown in figures 4-6. These distributions correspond to the "expansion" stage of plasma formation ($t=5.63$ ns).
Figure 5. Plasma velocity (left) and density (right) at $t=5.63$ ns as a function of radial coordinate.

Figure 6. Distribution of the magnetic pressure (left) and laser radiation (right) at $t=5.63$ ns ($t_{\text{Las}}=5$ ns).

4. Conclusion
We developed a high accuracy method (eighth order accuracy) for numerical simulation of the laser-matter interaction in the external magnetic field. Quasi monotone compact higher order finite difference scheme provides the improved dissipative and dispersive properties. Numerical study of the influence of magnetic field on plasma compressed by powerful source is carried out. The effects of magnetic field in plasma compressed by lasers, namely the influence of laser power on magnetized target, are studied. Numerical experiments for the magnetized plasma behavior during and after laser pulse are presented.

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