The Critical Thinking Process of Senior High School Students in Problem-Solving of Linear Equations System

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Abstract. In this 21st century, having critical thinking skill is a must. This critical thinking skill will lead students to solve the problem they find. Problem solving, one of mathematical abilities, is an advance ability that need more complex thinking process, like critical thinking. The purpose of this study is describing senior high school students critical thinking process in solving problem of linear equations system. This is descriptive qualitative research with two tenth grade students of SMA Negeri 1 Klego as the subjects. In this study, purposive sampling method is used to choose the subjects of the research. To collect the data of student’s critical thinking process, used tests, interviews, and observations. The study result showed that the students’ critical thinking process had not fully worked out. Both subjects can go through the clarification step of critical thinking well, but not in the analysis and strategy step.

1. Introduction
Mathematics is a compulsory subject at every level of education. Learning mathematics in schools has certain goals, one of which is for students to have problem solving ability [1]. This problem solving ability has a role in preparing students to face their life's challenges and in developing their knowledge and common sense [2]. Problem solving is an advanced mathematical skill that requires more complex thinking processes [3]. One of the thinking processes needed in problem solving is critical thinking. Dwyer, Hogan, and Stewart posted in their research that critical thinking skills enhance the possibility of producing logical inference and conclusion to respond a problem or argument [4]. Moreover, critical thinking has been emphasized as one of the competencies of the 21st century that must be possessed by students [5].

Critical thinking means studying the ideas produced, deciding what steps to take to solve a problem or determining the right cause, then evaluating and refining the solution [6]. This critical thinking ability leads to the critical understanding required in problem solving [7]. Critical thinking also allows students to study problems in a systematic way, face challenges in an orderly manner, ask innovative questions and plan original solutions [8]. Thus, critical thinking is closely related to problem solving. One of the studies on critical thinking processes in solving problem was conducted by Chukwuyenum and the results include: 1) understanding and stating the meaning of various data; 2) identify and examine problem-solving ideas; 3) draw appropriate conclusions; 4) explain the results of reasoning with logical reasons; 5) looking for other alternative solutions [9].

In schools, only a few students are trained to think critically [10]. Students who find it difficult to face math problems tend to be given the answer right away rather than being encouraged to come up
with ideas that can make students answer themselves correctly. This makes students not accustomed to thinking critically when faced with problems. PISA, an assessment of mastery of mathematics especially in non-routine questions, shows that the results obtained by Indonesia in 2018 were only 379, far below the average of 489 [11]. The low mastery of problem-solving also occurred in SMA N 1 Klego Boyolali, especially in the material on Linear Equation Systems. It was described in the result of National Examination in the indicator of "solving problems in mathematics or in everyday life related to linear equation system in two variables" which is 30.30% lower than Boyolali Regency 32.62%, and Central Java Province. 33.07% [12]

Mathematics learning in schools still seems to put aside the development of critical thinking during the learning process. What is taught is more about mastery of the material so that it causes students to be unable to apply critical thinking processes in dealing with problems encountered in daily life. The problem is a situation faced by students and requires a solution to that situation [13]. Problem solving itself is using all the knowledge that has been previously owned to solve new problem situations [14]. Therefore, students need to think critically, so that they can apply their knowledge to solve the problems, especially in linear equations system.

Critical thinking consists of several stages, they are identifying problems, delivering reasons, deducting, doing induction, doing an evaluation, making decisions, and doing an action, these stages are similar to Polya’s problem solving steps, i.e. understanding the problem, making a plan of solving, doing the plan, and looking back [15]. The process of critical thinking consists of several steps and indicators that are shown in Table 1 [16].

| Critical Thinking Steps | Indicators |
|-------------------------|------------|
| Clarification           | ● Stating the known information of the problem appropriately  
                          ● Addressing the problem correctly |
| Analysis                | ● Identifying information relevant to the problem with logical reasons  
                          ● Planning troubleshooting steps  
                          ● Making conclusions |
| Strategy                | ● Evaluating the steps and the results of problem solving  
                          ● Finding alternative solutions to the problem |

Critical thinking is a multidimensional construct that consist of cognitive, dispositional, motivational, attitudinal, and metacognitive functions [17]. Therefore, it is possible for each student to have their own different thinking processes depends on their multidimensional construct factors, including critical thinking. Based on the problems previously described, this study will discuss students' critical thinking processes in solving linear equation system problems.

2. **Methods**

This research is descriptive analysis research with a qualitative method. The purpose of qualitative research to know the phenomenon experienced by research subjects holistically and in the form of words and language [18]. The purpose of this study is identifying and describing students' critical thinking processes in problem solving of linear equation system.

This study conducted in SMAN 1 Klego on December 2019. The research subjects were two tenth grade students chosen by using purposive sampling method. Subject selection was based on several criteria, 1) students who have high scores in mathematics and 2) students who have good communication skills. The data were collected using students’ critical thinking tests, interviews, and observations. The test was developed based on the stages of critical thinking which include clarification, analysis, and strategy in linear equation system problems. The test is in the form of story questions that require a good
and deep understanding so that it can lead students to think critically in solving problems. Also, interviews were done based on an interview guide sheet on several indicators of critical thinking steps. Subjects did the critical thinking test of linear equations system before the researcher interviewed and observed them. All results of the test, interviews, and observation were analyzed qualitatively. The aim of that analysis is to describe and to know the students’ characteristics of their critical thinking process in solving linear equations system problem. To analyze the data, the researcher used the steps of data reduction, data display, and verification [19].

3. Results and Discussion

The results of this research consist of critical thinking processes, interviews, and observation. There were found two students with high competency in mathematics as subjects, they’re called S-01 for subject 1 and S-02 for subject in the next discussion. After that, to know their critical thinking process, they were given a test about finding the length of the sides of a triangle problem using a linear equations system. After they completed the test, an interview was conducted by observing critical thinking steps to know the critical thinking processes they went through.

3.1. Analysis responses of Subject 1 (S-01)

S-01 can perform all stages of the critical thinking process well. The following explanation is an analysis of S-01’s answers and interviews in each step of the critical thinking process.

3.1.1. Clarification step.

Figure 1. The responses of S-01 in clarification step

In the clarification step, S-01 can identify the information contained in the questions given. He also knew what asked in the question, but he did not write down on the sheet. When S-01 was asked why he didn't write it down on the sheet, he said that he was not used to writing down what was asked when working on the questions. Even though writing down what is known and what is asked in this matter is important, because it will serve as a guide for us to make the right answers and conclusions to the problem. S-01 can also create a mathematical model of a given problem.

3.1.2. Analysis step.

Figure 2. The responses of S-01 in analysis step

With the information provided in the questions, S-01 plans to solve the problem using the substitution and elimination methods. According to S-01, this elimination substitution method will be easier to use
than using just elimination or substitution alone. Every step of the work up to the conclusion of S-01 can do it well and correctly. S-01 makes conclusions by returning to the original problem, which is the length of the sides of the triangle.

3.1.3. **Strategy step.** At this stage, S-01 rechecks every step of the answer sheet from the beginning till the end to find out whether there is an error or not. He did not find any mistakes in his work. Besides evaluating the solution steps, there is an indicator to find an alternative solution to the problems given. S-01 does not try other ways to find solutions to the problem because according to him the solution he used was the easiest compared to other solutions. From the interview with S-01, found that he knew another method that could be used, namely Crammer. However, he did not use it because he thought the method was more suitable for linear equations with more variables.

3.2. **Analysis responses of Subject 2 (S-02)**
S-02 has carried out the critical thinking process stage, but it is still not thoroughly running well, especially in the analysis and strategy stage. The following explanation is an analysis of the answers and the S-02 interview at each stage of critical thinking.

3.2.1. **Clarification step.**

![Figure 3](image3.png)

**Figure 3.** The response of S-02 in clarification step

At the clarification step, the response of S-02 is similar to S-01. He could determine what was known and asked in the problem but he did not write it down. He also created a mathematical model of the given problem, but he did not explain the variables he used.

3.2.2. **Analysis step.**

![Figure 4](image4.png)

**Figure 4.** The response of S-02 in analysis step

To solve the given problem, S-02 uses the elimination substitution method. The completion steps carried out by S-02 are also correct. However, in concluding, S-02 did not make conclusions following the problem in question. It should deduce how long the sides of the triangle are, not the side proportions. After being traced, students experienced errors in remembering what was asked. In the previous step, he did not write down what was asked.
3.2.3. **Strategy step.** In the strategy stage, S-02 re-checks the steps of his work to a conclusion, but he is not aware of the mistakes in making his conclusions. He only realized this mistake during the interview. S-02 also did not try to do it any other way, because according to him only the elimination-substitution method could be used in the problem. S-02 doesn't know if other methods could be used. Knowledge of other alternative solutions is important so that students can compare which method is more effective and efficient.

Based on the explanation above, a summary of the critical thinking process comparison of S-01 and S-02 can be made as in Table 2.

| Critical Thinking Steps | Subject 1 (S-01)                                                                                                                                                                                                 | Subject 2 (S-02)                                                                                                                                                                                                 |
|------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Clarification          | ● The subject can explain all information about what is known and ask questions correctly. Subject drew a triangle to illustrate the unknown sides of the triangle.                                                | ● The subject can determine what the information is and what is asked after reading it several times. But the subject didn't write it down on the answer sheet.                                                   |
|                        | ● Subjects can make mathematical models correctly according to the given problem.                                                                                                                                 | ● The subject can change the problem into a mathematical model correctly.                                                                                                                                 |
| Analysis               | ● The subject determines the substitution and elimination methods to determine the lengths of the unknown sides of the triangle.                                                                                  | ● The subject can determine the substitution and elimination methods to be used in solving the problem.                                                                                                           |
|                        | ● The subject carries out the completion plan correctly.                                                                                                                                                      | ● The subject can carry out the completion plan correctly.                                                                                                                                                        |
|                        | ● The subject concludes by returning to the original problem appropriately.                                                                                                                                     | ● The subject makes conclusions by returning to the context of the original problem, but the conclusion made is not correct.                                                                                  |
| Strategy               | ● The subject checks the work steps from the beginning to the conclusion. He also checks the solutions obtained by substituting the solutions into linear equations.                                                | ● The subject checks the steps of the process, but he does not check the solutions he gets. The subject does not try to enter the solution obtained into existing linear equations. Also, the subject is not aware of the mistake in taking his conclusions. |
|                        | ● The subject knows that there are other ways to solve the given linear equation problem, but according to him an easier way to solve the problem is to use the elimination and substitution method. | ● The subject also did not try other solutions, because according to him the elimination and substitution methods were the only ways that could be used to solve the given linear equation problems. |

From the explanation in Table 2 above, the two subjects had the same thinking process at the clarification step. The differences occur at the analysis stage. Both subjects use elimination and substitution methods to solve the given system of linear equations problems. However, in concluding, S-01 and S-02 make different conclusions. S-02 made inaccurate conclusions when returning to the context of the problem. Besides, S-01 and S-02 differ in the reasons for using the solutions they use. According to S-01, there are other ways to solve the given problem, but choosing to use the elimination substitution method because it is easier. On the other hand, S-02 uses the substitution elimination method because according to him there is no other way to solve the problem other than using the substitution elimination method.
As previously mentioned, the critical thinking process includes mentioning information and problems appropriately, identifying relevant information, planning solutions, making conclusions, evaluating, and finding other alternative solutions to problems [16]. Both subjects have already carried out the stages of the thinking process, but not all stages have been passed properly. Difficulties are experienced at the analysis and strategy stage, especially in making conclusions and determining other alternatives in solving problems. This is because students are not trained to make conclusions and only arrive at procedural work. Also, students are accustomed to working with one formula only so they do not master other formulas or alternative solutions. Thinking critically means achieving the goals efficiently. The efficiency requires not only cognitive but also non-cognitive dimensions that are shaped from the learning process, for without them, critical thinking is not possible [20]. Therefore, it is necessary to develop a design or learning method that is following the conditions of students to be affected in the learning process [21]. By applying these designs or learning methods, students can practice and develop their critical thinking skills.

4. Conclusion
The students' critical thinking process in solving linear equation system problems has not been going well. This can be seen from students who have not been able to find the right answers and conclusions to the problem given. Students' critical thinking processes have similarities in the clarification step and differences in the analysis and strategy steps. The first subject could think critically better than the second subject. The researchers suggest that students' critical thinking must be trained in the learning process, especially in solving problems. For this reason, future researchers are encouraged to develop methods or strategies that can improve students' ability to think critically.

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