A hydrodynamic analog of superradiant emission

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Superradiance is a quantum phenomenon that occurs when a collection of atoms exhibits cooperative, spontaneous emission of photons at a rate that exceeds that of its component parts [9]. Superradiance is of both fundamental and practical interest [22], as it has applications in various fields, including quantum information technologies [1,3,21], cryptography [10], and narrow linewidth lasers [19,4,24]. Here, we reveal a similar phenomenon in a hydrodynamic system consisting of a pair of vibrationally-excited cavities, coupled through their common wavefield, that spontaneously emit droplets via interfacial fracture. The amplified droplet emission rate varies sinusoidally with distance between the two cavities, establishing a hydrodynamic analog of superradiant photon emission. The emission events are random, but their probability depends non-linearly on the amplitude of the wavefield, a result reminiscent of Born’s rule. This classical system thus captures certain key aspects of the statistical structure of quantum mechanics.

When a group of $N$ quantum emitters (e.g. excited atoms) interact coherently with a common electromagnetic field, they may collectively emit photons at a rate that is amplified relative to that corresponding to the sum of the individual emitters. In quantum optics, this phenomenon is known as superradiance [9]. When the separation between the atoms is much smaller than the
emission wavelength, superradiance can be understood classically by picturing each atom as a tiny antenna emitting electromagnetic waves [31, 28]. A more puzzling type of superradiance occurs when the separation between the atoms is comparable to the emission wavelength, in which case quantum mechanics is required to account for the phenomenon [22, 15]. According to the quantum mechanical description, this type of superradiance occurs when at each absorption event, a single photon is stored in a cloud of N atoms of the same kind. The atoms interact with each other through the electromagnetic field, creating collective non-separable states that can radiate the photon faster or slower than if the photon were stored in a single atom, corresponding to super- or subradiant emission states, respectively [23]. The most elementary example of quantum superradiance was demonstrated using a pair of trapped ions whose separation distance, $d$, was varied gradually [8]. Experiments revealed sinusoidal oscillations of the spontaneous emission rate $\Gamma(d)$ of the two-ion crystal, in accord with detailed quantum mechanical theoretical analysis [18]. This type of superradiance is considered to be a purely quantum phenomenon with no known classical analog.

Fluid mechanics has produced laboratory-scale physical analogs for phenomena as disparate as the wave nature of light [30], black holes [29], the Casimir effect [7] and the Aharonov-Bohm effect [2]. The relatively recent discovery of a pilot-wave hydrodynamic system [6] has led to a new class of hydrodynamic quantum analogs [5] that includes analogs of quantized orbital states [12, 20], quantum corrals [17, 25], Friedel oscillations [26] and spin lattices [27]. We here present a hydrodynamic analog of superradiance, in a system of vibrationally excited hydrodynamic cavities that spontaneously emit droplets via interfacial fracture. The cavities are deep circular wells spanned by a thin layer of oil that allows for their coupling through a common wavefield (see Fig. 1). We demonstrate that the wavefield in each cavity is influenced by the presence of its neighbor. Specifically, the neighboring cavity may amplify the local oscillation amplitude, resulting in an increased chance of interfacial fracture and thus an amplified
Figure 1: The experimental setup. (a) A schematic illustration of a circular bath with two cavities spanned by a thin layer of fluorinated oil. The bath is vertically oscillated by an electromagnetic shaker, resulting in the emission of droplets from the two cavities. (b) A rare generation event in which two droplets are about to be created simultaneously. Scale bar, 3 mm.

Figure 1 shows a schematic representation of our experimental set-up. A bath of fluorinated oil has two 6-mm-deep circular wells that serve as hydrodynamic cavities. The cavities, each with diameter 7 mm, are separated by a center-to-center distance $d$ that is varied between experiments, from 8 mm to 12 mm, in 0.5 mm increments. In the shallow layer spanning the wells, the depth is $0.75 \pm 0.05$ mm. The system is subjected to vertical vibration by an electromagnetic shaker with forcing $F(t) = \gamma \cos(2\pi ft)$, where $\gamma = 1.75g$ and $f = 39 \text{ Hz}$ are the peak driving acceleration and frequency, respectively. A more detailed description of the experimental setup
is provided in the supplementary information (SI).

A liquid bath of uniform depth subject to vertical vibration at a fixed frequency, undergoes two critical transitions as the driving amplitude is increased progressively. The first transition occurs as the vibrational acceleration, \( \gamma = 4\pi^2 f^2 A \), where \( A \) is the vibration amplitude, is increased beyond the Faraday threshold, \( \gamma_F \), at which point the initially flat free surface destabilizes into a pattern of standing Faraday waves \cite{11}. As the driving amplitude is increased further, the stabilizing influence of surface tension is exceeded by the destabilizing inertial forces associated with the bath vibration, and the interfacial fracture threshold, \( \gamma_B \), is crossed. Above this threshold, the Faraday waves break spontaneously, and millimetric droplets are emitted from the free surface in an irregular fashion \cite{14, 13}. Importantly, for shallow layers, both \( \gamma_F \) and \( \gamma_B \) depend strongly on the local depth of the liquid. We thus define \( \gamma_{cF} \) and \( \gamma_{sF} \) to be the Faraday threshold above the cavities and the shallow region, respectively, and likewise for \( \gamma_{cB} \).

With the increase of the driving acceleration, \( \gamma \), our variable-depth system undergoes the following evolution. First, as the acceleration crosses \( \gamma_{cF} \), Faraday waves appear above the cavities and propagate some distance into the surrounding shallow region. When \( \gamma > \gamma_{sF} \), Faraday waves emerge over the entirety of the bath surface, but are most vigorous above the cavities. Figure 2 illustrates the instantaneous wavefield near the cavities when \( \gamma_{cF} < \gamma < \gamma_{cB} \).

Figure 2a shows the wavefield of a single cavity, whereas figures 2b-d show the two-cavity wavefield for three different values of the center-to-center separation distance, \( d = 8, 10.5, 12 \) mm, respectively. Figure 2e depicts the resulting wavefield for two distant cavities, with \( d = 87 \) mm. Notably, even at such large separation distances, the perturbation wavefield can reach the other cavity, allowing for long-range interactions. The perturbation wavefield, recorded near the frequency of the most unstable Faraday mode, \( f/2 \), is shown in video SV1 in the SI.

When the acceleration is increased beyond the interfacial breaking threshold of the cavities, \( \gamma_{cB} < \gamma < \gamma_{sB} \), droplet emission sets in. Video SV2 in the SI shows the spontaneous droplet
emission from a pair of hydrodynamic cavities. The emission events occur unpredictably, as indicated by Fourier analysis shown in the SI, but arise exclusively within the cavities. We define a spontaneous emission rate, $\Gamma$, for the combined two-cavity system, as the average number of emission events per second, and the anomalous emission rate, $\Gamma_N(d) = (\Gamma(d) - 2\Gamma_0) / 2\Gamma_0$, where $\Gamma_0 = 1.47 \, \text{s}^{-1}$, is the measured emission rate of a single cavity in isolation.

In Figure 3(a), we present our experimental measurements of the dependence of the anomalous emission rate $\Gamma_N(d)$ on the separation distance $d$. An amplification of up to 46% relative
to $2\Gamma_0$ is evident. As in the trapped ion pair experiment [8], the amplified emission rate of our two-cavity system oscillates sinusoidally as a function of the distance between the cavities. Our system thus represents a hydrodynamic analog of superradiant emission. The observed oscillatory behaviour shows that the probability of the emission events is affected by the interference between the waves generated by the individual cavities. The dashed curve in Fig. 3(a) represents a simple fit for the anomalous emission rate, $\Gamma_N(d) = A \cos^2(2kd)$ for $A = 1.36$, and
$k = \frac{2\pi}{\lambda}$, with $\lambda = 6.60 \pm 0.05$ mm being the experimentally measured wavelength of the Faraday waves in the vicinity of the cavities. Here, we did not characterized the expected decay of the anomalous emission rate with increasing separation distance, $d$. However, we note that for sufficiently large values of $d$, the amplitude of the common wavefield will decay significantly due to viscous damping, as will the anomalous emission rate. Figure 3(b) depicts the time dependence of the emission events from a single cavity, showing the unpredictability of a single emission event, as is confirmed by the FFT analysis presented in the SI.

The mechanism responsible for the superradiant emission of droplets is the wave coupling between the two cavities. We quantify this coupling by measuring the correlations between the emission events in the two-cavity system, as detailed in the SI. We see that the two cavities are strongly anti-correlated, with the correlation values varying from $C = -0.30$ (for $d = 11.5$ mm) to $C = -0.41$ (for $d = 10$ mm). These anticorrelations, together with the amplification of the combined emission rate, suggest that the two cavity system cannot be factored into distinct states, as the probabilities of emission events in the two cavities are coupled. Acting on one of the cavities of this coupled system, by, for example, changing its position or depth, would affect the emission rate of its neighbouring cavity. The possibility thus arises of altering the system’s global emission rate by a local operation on one of its individual components, thereby creating a new platform for probabilistic computational operations in fluid mechanics.

In our system, the probability of random discrete events, specifically drop ejection, is prescribed by a continuous wavefield resulting from two interfering sources. While such a probabilistic structure is ubiquitous in the microscopic, quantum realm, it has not previously been reported in a classical system. In QM, probabilities are calculated in accordance with Born’s rule from the squared amplitude of the wave function. Here, the only possible source of the sinusoidal oscillations shown in Fig. 3(a) is the wavefield, and the probability of an emission event is related non-linearly to the amplitude of that field in each cavity. Our system thus not
only represents a hydrodynamic analog of superradiant emission, but exhibits a probabilistic structure reminiscent of Born’s rule.

It is also worth considering the relation between the system introduced here and pilot-wave hydrodynamics [5]. In the latter, the notion of an analog photon is more nebulous: when the system jumps between quantized states (e.g. the walking droplet transitions from one orbit to another), energy is dumped into or extracted from the bath. In the system considered here, droplets are generated by breaking waves, their appearance representing a discrete transition event, an analog of photon emission from an excited state. We note that in our current experiments, we used fluorinated oil in order to facilitate the rapid reabsorption of the emitted droplets into the bath. However, this reabsorption can be minimized by using a relatively low density silicon oil, in which case the generated particles may persist on the surface, bounce and self-propel, thereby providing a possible link between pilot-wave hydrodynamics and the new class of analog systems established here.

We have introduced the first hydrodynamic analog of quantum superradiance. While some aspects of superradiance can be described classically [31, 28], the sinusoidal dependence of the amplified emission rate on separation distance (see Fig. 3b) has never been reported in a classical system. Moreover, our study suggests that particle creation through interfacial fracture may provide a valuable new platform for exploring hydrodynamic quantum analogs, and for inspiring new physical pictures of quantum processes. Most notably, it provides a physical picture of the probabilistic structure, of the continuous-to-discrete transition, associated with the measurement process in quantum systems.

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Author contributions

V.F. conceptualized the research. V.F. and K.P. designed the experimental system, performed the experiments, analyzed the data and prepared the figures. V.F., K.P. and J.B. wrote and revised the manuscript.

Competing interests

The authors declare no competing interests.

Data and materials availability

All data are available in the main text or the supplementary materials.
Supplementary Information

Materials and Methods

A circular stainless steel base has the geometry shown in Fig. S1. A 6 mm-thick acrylic plate, with two circular openings that serve as the hydrodynamic cavities, is inserted into the bath and submerged under a $0.75 \pm 0.05$ mm deep layer of fluorinated oil of density $\rho = 1855$ kg m$^{-3}$, viscosity $\nu = 2.2$ cSt and surface tension $\sigma = 16$ mN m$^{-1}$. The circular cavities are both 7 mm in diameter and are separated by a center-to-center distance $d$ that is varied from 8 mm to 12 mm, in 0.5 mm increments.

Figure S1: A schematic of the experimental setup: (a) An oblique perspective view of the setup, showing the two camera angles from which the experiment was filmed. (b) A plan view of the oil bath, with two cavities at its center. (c) A side view of the vibrational apparatus [16].

The system was mounted on an optical table and oscillated vertically by an electromagnetic shaker with forcing $F(t) = \gamma \cos(2\pi ft)$, with $\gamma$ and $f$ being the peak vibrational acceleration and frequency, respectively. To insure a spatially uniform vibration, we connected the shaker
to the bath by a steel rod coupled with a linear air bearing. We monitor the forcing with two accelerometers placed on opposite sides of the bath. A closed-loop feedback control system then insures a constant vibrational acceleration amplitude to within ±0.002 g [16]. All experiments presented here were conducted at a fixed driving frequency of $f = 39$ Hz, chosen to be close to the natural frequency of the cavities. This natural frequency was determined experimentally by performing a frequency sweep, and identifying the frequency that yielded the maximal emission rate for a fixed amplitude. The amplitude was fixed at 1.75 g, which we determined as optimal to ensure a high droplet production rate while avoiding the vibration-induced drainage of the cavities that arises at higher amplitudes.

To visualize the wave field (as in Fig. 2), we placed a semi-reflective mirror at 45° between the bath and a charge-coupled device (CCD) camera that was mounted directly above the setup. We illuminated the bath with a diffuse-light lamp facing the mirror horizontally, yielding images with bright regions corresponding to horizontal parts of the surface, extrema or saddle points. The droplet generation events were recorded at 480 fps with a regular video camera (One Plus 6, 16MP camera, Sony IMX 519 sensor) placed beside the bath, as shown in Fig.1. An event was classified as a generation event only when the droplet was fully detached from the liquid thread. The emission events were counted manually, and each event was labeled with the time of its occurrence (see Fig. S2(a) as an example). Each data point in Fig. 3(a) represents an average over a time interval of 300 seconds, corresponding to roughly $950 - 1300$ droplet emission events.

To estimate the vertical error bars in Fig. 3(a), we defined a running average for the emission rate $\Gamma(d)$ as $S_d(T) = N/T$ where $N$ represents the number of emission events up to time $T$. As $T$ approaches infinity, $S_d(T)$ approaches the steady state emission rate $\Gamma(d)$, and the vertical error bars are then calculated as two standard deviations of the sequence $S_d(T)$ from its average value $\Gamma(d)$. 

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The horizontal error bars in Fig. 3(a) in the manuscript, represent the mechanical precision error of the laser cutter used to fabricate the hydrodynamic cavities.

**Unpredictability of emission events**

To confirm that our discrete drop generation events were random, we performed a fast Fourier transform (FFT) on the measured time series of the ejection events. In particular, using the timestamps that we assigned to the events during the measurements, we define a binary drop generation function by assigning the values of 1 and 0 to emission events and non-events, respectively. Figure S2 shows a typical result for a 5-minute-long measurement of droplets generated in a single cavity. Fig. S2a shows a plot of the binary drop generation function defined using the timestamps. Fig. S2b indicates that no dominant Fourier frequency components arise in the FFT, and so confirms that the events are random for the purposes of our analysis.

**Inter-cavity coupling**

To quantify the coupling between the two cavities, we define a characteristic time window \( \tau(d) \), for a given separation distance \( d \), to be the average time it takes for a single cavity to produce a single droplet. This allows us to define the following emission states. The state \(|00\rangle\) represents the case where no droplets are generated during the time interval \( \tau \), \(|01\rangle\) and \(|10\rangle\) represent the case where a single droplet is generated in one of the two cavities, and \(|11\rangle\) represents the case where both cavities produce a droplet during \( \tau \). We then proceed to define \( N_{ij}, \ i, j \in 0, 1 \) as the sum of all \(|ij\rangle\) events during the running time of the experiment, and the correlation function \( C \) as

\[
C = \frac{N_{00} - N_{01} - N_{10} + N_{11}}{N_{00} + N_{01} + N_{10} + N_{11}}.
\]

For two uncorrelated cavities, one expects \( C \approx 0 \), as arises for sufficiently distant cavities (see Fig. S3). We observe anti-correlation \( (C < 0) \) between the two coupled cavities over the full
Figure S2: (a) Time series of the drop generation events from a single cavity over 300 seconds. (b) FFT of (a) reveals that these events are effectively random.

range of separation distances $d$, indicating that when an emission event occurs in one cavity, it tends to suppress emission in the other. The correlation between the cavities varies from $C = -0.30$ (for $d = 11.5$ mm) to $C = -0.41$ (for $d = 10$ mm). We note that these values depend weakly on our choice of the time window $\tau$. For example, doubling the time window increases the maximal correlation to $C = -0.45$, and decreases the minimal correlation to $-0.25$. 

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Figure S3: Measured correlation, $C$, of the drop emission event in the two cavities as a function of their separation distance $d$, as defined in the main text. The upper dashed line represents two uncorrelated cavities.