Time-Dependent Hartree-Fock Solution of Gross-Neveu models: Twisted Kink Constituents of Baryons and Breathers

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We find the general solution to the time-dependent Hartree-Fock problem for the Gross-Neveu models, with both discrete (GN) and continuous (NJL) chiral symmetry. We find new multi-baryon, multi-breather and twisted breather solutions, and show that all GN \( N \) baryons and breathers are composed of constituent twisted kinks of the NJL model.

Self-interacting fermion systems describe a wide range of physical phenomena in particle, condensed matter and atomic physics [1–10]. Applications include solitons, excitons, polaritons, breathers and inhomogeneous phases in superconductors, conducting polymers, liquid crystals, particle physics, and cold atomic gases, and also illustrate phenomena in strong interaction particle physics and condensed matter physics [1, 13]. In the 't Hooft limit, \( N \to \infty \), \( Ng^2 = \text{constant} \), semiclassical methods become exact, as pioneered in this context by Dashen, Hasslacher and Neveu [14]. Classically, the GN model has a discrete chiral symmetry, while the NJL model has a continuous chiral symmetry. At finite temperature and density, and at large \( N \), these models exhibit inhomogeneous phases with crystalline condensates, directly associated with chiral symmetry breaking [16]. The basic physics of these GN phases is the Peierls effect of condensed matter physics [3, 17–19]. This analysis of equilibrium thermodynamics is based on exact spatially inhomogeneous solutions to the gap equation, or equivalently the Hartree-Fock problem, which solves the Dirac equation subject to constraints on the scalar and pseudoscalar condensates [16, 20]. Here we extend these results to the complete exact solution of the \textit{time-dependent} Hartree-Fock (TDHF) problem, relevant for scattering processes, transport phenomena and non-equilibrium physics:

\begin{equation}
\mathcal{L}_{\text{GN}} = \overline{\psi} i \partial_t \psi + \frac{g^2}{2} (\overline{\psi} \psi )^2 \tag{1}
\end{equation}

\begin{equation}
\mathcal{L}_{\text{NJL}} = \overline{\psi} i \partial_t \psi + \frac{g^2}{2} \left[ (\overline{\psi} \overline{\psi} )^2 - (\overline{\psi} i \gamma_5 \psi )^2 \right] \tag{2}
\end{equation}

These are soluble paradigms of symmetry breaking phenomena in strong interaction particle physics and condensed matter physics [1, 13]. In the 't Hooft limit, \( N \to \infty \), \( Ng^2 = \text{constant} \), semiclassical methods become exact, as pioneered in this context by Dashen, Hasslacher and Neveu (DHN) [14, 15]. Classically, the GN model has a discrete chiral symmetry, while the NJL model has a continuous chiral symmetry. At finite temperature and density, and at large \( N \), these models exhibit inhomogeneous phases with crystalline condensates, directly associated with chiral symmetry breaking [16]. The basic physics of these GN phases is the Peierls effect of condensed matter physics [3, 17–19]. This analysis of equilibrium thermodynamics is based on exact spatially inhomogeneous solutions to the gap equation, or equivalently the Hartree-Fock problem, which solves the Dirac equation subject to constraints on the scalar and pseudoscalar condensates [16, 20]. Here we extend these results to the complete exact solution of the \textit{time-dependent} Hartree-Fock (TDHF) problem, relevant for scattering processes, transport phenomena and non-equilibrium physics:

\begin{equation}
\text{GN}_2 : (i \partial_t - S(x, t)) \psi_\alpha = 0 ; \quad S = -g^2 \sum_\beta \overline{\psi}_\beta \psi_\beta \tag{3}
\end{equation}

\begin{equation}
\text{NJL}_2 : (i \partial_t - S(x, t) - i \gamma_5 P(x, t)) \psi_\alpha = 0 ; \quad S = -g^2 \sum_\beta \overline{\psi}_\beta \psi_\beta , \quad P = -g^2 \sum_\beta \overline{\psi}_\beta i \gamma_5 \psi_\beta \tag{4}
\end{equation}

We solve these TDHF problems in full generality, describing the dynamics, including scattering, of non-trivial topological objects such as kinks, baryons and breathers. Surprisingly, we found that the most efficient strategy is to solve the (apparently more complicated) NJL \( N \) model first, and then obtain \( N \) solutions by imposing further constraints on these solutions. This reveals, for example, that the \( N \) baryons and breathers found by Dashen, Hasslacher and Neveu [14] are in fact bound objects of twisted NJL \( N \) kinks, and that the scattering of \( N \) baryons and breathers can be deduced from the scattering of twisted kinks. This includes new breather and multi-breather solutions in NJL \( N \), as well as new multi-baryon and multi-breather solutions for \( N \).

We stress that while it is well known that the classical equations for the GN \( N \) and NJL \( N \) models are closely related to integrable models [21], this fact is only directly useful for the solution of the TDHF problem for the simplest case of kink scattering in the GN \( N \) model, which reduces to the integrable Sinh-Gordon equation. The more general self-consistent TDHF solutions to (3, 4) involving twisted kinks, baryons and breathers, do not satisfy the Sinh-Gordon equation; instead we find a general “master equation” [see Eq. (8)], whose solution reduces to a finite algebraic problem solvable in terms of determinants.

We also stress that these more general solutions require a self-consistency condition relating the filling fraction of valence fermion states to the parameters of the condensate solution, as for the static GN \( N \) baryon [14], the static twisted kink [22], and the GN \( N \) breather [14]. For our time-dependent solutions, this important fact means that during scattering processes there is non-trivial back-reaction between fermions and their associated condensates and densities [23]. Kink scattering in the GN \( N \) model, described by Sinh-Gordon solitons [24, 25], is much simpler as there is no self-consistency condition or back-reaction.

With Dirac matrices, \( \gamma^0 = \sigma_1, \gamma^1 = i \sigma_2, \gamma_5 = -\sigma_3 \), and light-cone coordinates (note \( \bar{z} \) is not the complex conjugate of \( z \)): \( z = x - t, \bar{z} = x + t, \partial_0 = \bar{\partial} - \partial, \partial_1 = \bar{\partial} + \partial \), the Dirac equation in (4) is:

\begin{equation}
2 i \bar{\partial} \psi_2 = \Delta \psi_1 , \quad 2 i \partial \psi_1 = -\Delta^* \psi_2 , \quad \Delta = S - i P \tag{5}
\end{equation}
Write the complex potential $\Delta$ and continuum spinor $\psi_\zeta$:

$$\Delta = \frac{N}{D}, \quad \psi_\zeta = \frac{e^{i(\zeta \bar{z} - z/\zeta)/2}}{D \sqrt{1 + \zeta^2}} \left( \zeta N_1 - \zeta N_2 \right)$$

(6)

where $D$ is real, and the complex light-cone spectral parameter $\zeta$ is related to the energy $E$ and momentum $k$ as: $k = \frac{1}{2} (\zeta - \frac{1}{\zeta})$, $E = \frac{1}{2} (\zeta + \frac{1}{\zeta})$, in units of $m$, the dynamically generated fermion mass. The ansatz for $\psi_\zeta$ anticipates the fact that the potential $\Delta$ is transparent.

We solve (5), and associated TDHF consistency conditions, using an ansatz method, positing a decomposition with a finite number $n$ of simple poles:

$$\mathcal{N}_{1,2}(\zeta) = \mathcal{N}_{1,2}^{(0)} + \sum_{i=1}^{n} \frac{1}{\zeta - \zeta_i} \mathcal{N}_{1,2}^{(i)}$$

(7)

Matching powers of $\zeta$ we learn that $D$ and $\mathcal{N}$ must satisfy the “master equation”

$$4 \partial \bar{\partial} \ln D = 1 - |\Delta|^2$$

(8)

in addition to various sum rules obeyed by the residues $\mathcal{N}_{1,2}^{(i)}$ [26]. Furthermore, the following equations must hold for all $i = 1, \ldots, n$,

$$2i(D \partial - \bar{D} \partial) \mathcal{N}_{2}^{(i)} - \zeta_i(D \mathcal{N}_{2}^{(i)} - \mathcal{N}_{1}^{(i)}) = 0$$

$$2i\zeta_i(D \partial - \bar{D} \partial) \mathcal{N}_{1}^{(i)} + D \mathcal{N}_{1}^{(i)} - N^* \mathcal{N}_{2}^{(i)} = 0$$

(9)

The residues of $\psi_\zeta$ at the poles $\zeta = \zeta_i$ provide normalizable bound state spinor solutions:

$$\psi^{(i)} = \frac{1}{D \partial_i} \left( \zeta_i \mathcal{N}_1 - \mathcal{N}_2 \right) \quad V_i = e^{-i(\zeta_i \bar{z} - z/\zeta_i)/2}$$

(10)

An alternative set of normalizable bound state spinors comes from $\psi_\zeta$ at the complex conjugate poles $\zeta_j^*$:

$$\phi^{(i)} = \frac{1}{\bar{D} \partial_i} \left( \zeta_j^* \mathcal{N}_1^* - \mathcal{N}_2^* \right)$$

(11)

These two sets of bound states are linearly related, $\psi^{(i)} = \sum_j \Omega_{ij} \phi^{(j)}$. The condition that $D$ is real and has no zeroes restricts the matrix $\Omega$ to the form

$$\Omega_{ij} = i \bar{\Omega}_{ij}/(\zeta_j^*)^2$$

(12)

where $\bar{\Omega}$ is a positive definite hermitean matrix. Together with (7) we obtain a finite dimensional algebraic system:

$$\mathcal{N}_1(\zeta_i^*) + \sum_{i,k} \frac{1}{\zeta_i^*} \bar{V}_i \Omega_{ik} V_k^* \bar{\zeta}_k \mathcal{N}_1(\zeta_k^*) = D$$

$$\mathcal{N}_2(\zeta_i^*) + \sum_{i,k} \frac{1}{\zeta_i^*} \bar{V}_i \Omega_{ik} V_k^* \mathcal{N}_2(\zeta_k^*) = N$$

(13)

We have found a remarkably simple solution to this algebraic system, which yields a compact determinant expression for all the ansatz quantities in the TDHF solution:

$$D = \det(\omega + B), \quad N = \det(\omega + A)$$

$$\mathcal{N}_1(\zeta) = \det(\omega + C), \quad \mathcal{N}_2(\zeta) = \det(\omega + D)$$

(14)

with matrices

$$B_{ij} = \frac{i}{\zeta_j - \zeta_i} A_{ij}$$

$$C_{ij} = \frac{\zeta_j - \zeta_i}{\zeta_j - \zeta_i^*} B_{ij} = \frac{\zeta_i}{\zeta_j} A_{ij}$$

(15)

where $\omega$ is a positive definite hermitean matrix: $\omega_{ij} = \zeta_i \overline{\Omega}_{ij}^{-1} \zeta_j$. This gives the complete solution to the Dirac equation for time-dependent transparent (complex) potential $\Delta$, and via the master equation (8) gives a natural relativistic generalization of the Kay-Moses general transparent static Schrödinger potential [27]

$$V(x) = -\partial_x^2 \ln \det(1 + A), \quad A_{ij} = \sqrt{a_i a_j} e^{(\kappa_i + \kappa_j)x}$$

(16)

and its time-dependent Schrödinger generalization [28]. It also provides a new closed-form solution to the finite algebraic problem, found recently in [20], for the static transparent NJL2 Dirac equation.

We now show that this solution also gives a self-consistent solution to the fully quantized TDHF problem (4), provided certain filling-fraction conditions are satisfied by the combined soliton-fermion system, generalizing the conditions already found by DHN, Jackiw-Rebbi, and Shei [11, 14, 22]. Consider first the induced fermion density in the Dirac sea. Introducing a cut-off scale $\Lambda$,

$$\rho_{\text{ind}} = \frac{1}{2\pi^2} \int_{1/\Lambda}^\infty \frac{d \zeta}{2 \pi} \left( \psi_i^\dagger \psi_i - 1 \right)$$

(17)

$$\rho_{\text{ind}} = \frac{1}{2\pi^2} \int_{1/\Lambda}^\infty \frac{d \zeta}{2 \pi^2} \left( \zeta^2 (|\mathcal{N}_1|^2 D - D^2) + |\mathcal{N}_2|^2 D - D^2 \right)$$

(18)

The pole ansatz (7), a partial fraction decomposition, and the known asymptotic behavior of the ansatz functions, combine to show that the linear and logarithmic divergent terms cancel, leading to the finite result:

$$\rho_{\text{ind}} = \frac{i}{4\pi} \sum_{i,j} \zeta_i \zeta_j \mathcal{N}_1^{(i)}(\zeta_j) \mathcal{N}_2^{(j)}(\zeta_j) (\bar{\Omega}^{-1})_{ij} \ln \frac{\zeta_j}{\zeta_i}$$

(18)

For consistency with axial current conservation, this must be cancelled by the contribution from the discrete bound states [26]. The physical bound state spinors are in general a (orthonormal) superposition of the basis bound states (10), $\tilde{\psi}_i = \sum_j C_{ij} \psi_j$, where we find that the matrix $C$ is directly related to the matrix $\bar{\Omega}$ as:
The density from the bound states (with occupation fractions $\nu_k$) is

$$\rho_b = \sum_{i,j} \frac{\zeta_i^{*} \nu_{i} N_{j}^{(i)*} N_{j}^{(j)}}{D^2 V_{i}^{*} V_{j}} \sum_{k} \nu_k C_{k1} C_{k2} \tag{19}$$

Then the condition $\rho_{nd} + \rho_b = 0$ leads to a consistency condition for the filling fractions $\nu_k$ which can be written

$$\nu_k = \frac{1}{2\pi} \text{eigenvalues of } \left( C^{\dagger} M \hat{\Omega}^{-1} C^{-1} + h.c. \right) \tag{20}$$

where $M$ is the diagonal matrix $M_{ij} = -i \delta_{i,j} \ln(-\zeta_j^*)$. Having found a candidate solution with vanishing fermion density, we now consider the TDHF self-consistency condition (20) holds. This proves full TDHF self-consistency is satisfied provided the previously found filling-fraction condition (20) holds. This proves full TDHF self-consistency for the NJL$_2$ system (4). For GN$_2$ we impose reality of the condensate $\Delta$ and relax the consistency condition on the pseudoscalar condensate, as discussed below.

We illustrate the TDHF solution (14, 15, 20) with some examples. We write $\zeta_j$ in terms of phase and boost parameters: $\zeta_j = -e^{-i\phi_j/\eta_j}$. With just one pole, $B = e^{2x \sin \phi}$, $A = e^{-2i\phi_b}$, and $\Delta = (1 + e^{-2i\phi} e^{2x \sin \phi})/(1 + e^{2x \sin \phi})$, which is Shei’s twisted kink for the NJL$_2$ model [22], with filling fraction $\nu = \phi/\pi$. When $\phi = \mp \pi/2$ we get a real solution of the GN$_2$ model, the usual kink/anti-kink. With two poles we obtain real $\Delta$, for GN$_2$, either by choosing $\phi_1 = \phi_2 = \pi/2$, which gives

$$\Delta = \frac{1 - U_1 - U_2 + \left( \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \right)^2 U_1 U_2}{1 + U_1 + U_2 + \left( \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \right)^2 U_1 U_2}, \quad U_i \equiv \frac{|\eta_i| V_i}{2 \sin \phi_i} \tag{22}$$

describing scattering of 2 kinks, or alternatively by choosing $\zeta_1 = -\zeta_2$, which means $\phi_1 = \pi - \phi_1$ and $\eta_1 = \eta_2 (= 1$ for rest frame). Then $V_2 = V_1^*$ and

$$B = \left( \frac{U_1 - i e^{i\phi_1} V_1^2}{2 U_1^2} \right), \quad A_{ij} = B_{ij} e^{i(\phi_1 + \phi_2)} \tag{23}$$

Choosing $\omega = 1$ we obtain the DHN GN$_2$ baryon [14]

$$\Delta = \frac{1 + 2 \cos^2(\phi_1) U_1 + \cos^2(\phi_2) U_1^2}{1 + 2U_1 + \cos^2(\phi_2) U_1^2}$$

$$= 1 + y \tan(y \bar{x} - b) - y \tan(y \bar{x} + b) \tag{24}$$

where $y = \sin \phi_1 = \tanh(2b)$, and the $x$ origin has been shifted. The GN$_2$ consistency condition leads to filling fractions $\nu_1 = \frac{2\eta_1}{\pi}$, $\nu_2 = 1$. This shows that the DHN GN$_2$ baryon is in fact a bound object of two twisted kinks with filling fractions (which also determine the baryon size) related to the twist angle. Furthermore, the mass of the DHN baryon is related to the masses of the constituent twisted kinks as: $M = M_{\text{kink}}(\phi_1) + M_{\text{kink}}(\pi - \phi_1) = 2N_x \sin \phi_1$. Choosing instead an off-diagonal mixing matrix

$$\omega = \left( \begin{array}{cc} \sec \chi & \tan \chi \\ \tan \chi & \sec \chi \end{array} \right) \tag{25}$$

leads to the DHN GN$_2$ breather [14]

$$\Delta = \frac{1 + 2 \sec \chi \cos \phi_1 U_1 - 2 \tan \chi \sin \phi_1 \cos(2\tan \phi_1 U_1) + \cos^2 \phi_1 U_1^2}{1 + 2 \sec \chi U_1 + 2 \tan \chi \sin \phi_1 \cos(2\tan \phi_1 U_1) + \cos^2 \phi_1 U_1^2}$$

with filling fractions $\nu_{1,2} = \frac{1}{\pi}((\phi_1 + \phi_2) \pm (\phi_1 - \phi_2) \sec \chi)$. Thus the DHN GN$_2$ breather is also a bound object of two twisted kinks, with filling fractions related to the twist angles.

Using the off-diagonal form (25) for $\omega$, but not imposing the reality condition, $\phi_1 + \phi_2 = \pi$, we obtain a new twisted breather solution to the NJL$_2$ model, shown in Fig. 1. At the three pole level, we find the scattering of three kinks if $\omega = 1$. These are GN$_2$ kinks if $\phi_1 = \phi_2 = \phi_3 = \pi/2$, and twisted NJL$_2$ kinks otherwise. The scattering of a GN$_2$ baryon and a kink is obtained by choosing $\zeta_1 = -\zeta_2$, and the scattering of a breather with a kink is obtained by choosing the off-diagonal form (25) for a $2 \times 2$ sub-block of $\omega$. New breather solutions are obtained by choosing $\eta_1 = \eta_2 = \eta_3$, and a more general $3 \times 3$ off-diagonal form of $\omega$. At the four pole level, in addition to the scattering of 4 (in general twisted) kinks, we can combine the spectral parameters of the kinks pairwise, e.g. as $\zeta_1 = -\zeta_2$ and $\zeta_3 = -\zeta_4$, to obtain the scattering of two baryons [23]. Further choosing the cor-
respective $2 \times 2$ sub-blocks of the mixing matrix $\omega$ to have the breather form (25), we obtain the scattering of two $\text{GN}_2$ breathers [29]. Relaxing the pairwise reality conditions, $\phi_1 + \phi_2 = \pi$ and $\phi_3 + \phi_4 = \pi$, we obtain another new solution, the scattering of two twisted $\text{NJL}_2$ breathers, as shown in Fig. 2. Choosing equal boost parameters $\eta_i$, and a more general off-diagonal mixing matrix $\omega$, we obtain a novel 4-breather solution in which all 4 twisted kink constituent kinks “breathe”. The pattern should now be clear. Choosing different boost parameters $\eta_i$ gives a solution describing scattering of twisted kink constituents. If some of the $\eta_i$ are equal, the solution describes bound combinations of twisted kink constituents, which are baryons if $\omega$ is diagonal, and breathers if $\omega$ is off-diagonal. The fermion filling fraction consistency condition (20) can always be solved for any given choice of spectral parameters $\zeta$ and mixing matrix $\omega$.

In the special case where all $\phi_i = \frac{\pi}{2}$, the $V_i$ are real, and we have $B_{ij} = \eta_i \eta_j V_i V_j / (\eta_i + \eta_j)$, and $A_{ij} = -B_{ij}$, which agrees with the known multi-kink scattering solutions of $\text{GN}_2$ [24], and whose non-relativistic limit agrees with the Schrödinger results of Kay-Moses and Nogami-Warke [27, 28]. For $\text{NJL}_2$, taking all $\eta_i = 1$ we obtain the static solution $\Delta = \det(1 + A) / \det(1 + B)$, with

$$\hat{B}_{ij} = \frac{e^{(x-x_i) \sin \phi_i + (x-x_j) \sin \phi_j}}{2 \sin \left(\frac{\phi_i + \phi_j}{2}\right)} , \quad \hat{A}_{ij} = \frac{\hat{B}_{ij}}{e^{(\phi_i + \phi_j)}}$$

where we have removed phases from the determinant, which is possible for diagonal $\omega$. This is a compact solution for the algebraic system found recently in [20] for self-consistent static multi-twisted kinks in $\text{NJL}_2$. Taking the $\eta_i$ different, our solution (14, 15, 20) gives the full time-dependent generalization.

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