A Double-channel iterative NFXLMS algorithm used in Horizontal Vibration Isolation

ZHANG Chi1, YIN WenSheng2

1The department of Mechanical Engineering, Tsinghua University, Beijing, China
2The department of Mechanical Engineering, Tsinghua University, Beijing, China
yinws@tsinghua.edu.cn

Abstract. In order to solve the multi-directional coupling problem of the vibration isolation platform caused by the partial load, a novel adaptive feed-forward control method is proposed. In the paper, we mainly used the Double-channel iterative NFXLMS algorithm to get a better performance of vibration isolation. And the control system is not sensitive to the error from the model identification. Then the algorithm is validated by the simulation and experiments. The results show that the performance of the vibration isolation system is mainly improved in the low frequency.

1. Introduction

1.1. Introduction
The performance of the precision motion control systems, such as lithography machines, are dramatically influenced by the external caused vibrations [1]. In order to preserve the high accuracy of these systems, precision vibration isolation system is widely used. And the active vibration isolation system can use sensors and controllers to achieve a better vibration isolation performance in the wide frequency band [2][3]. In the field of vibration isolation, the most commonly adopted control method is the feedforward algorithm and feedback algorithm. Various methods such as PID control [4], adaptive control [5], neural network [6], and Data-driven algorithm [7] have been applied in the active vibration isolation system to attenuate vibration. But most of these methods focus on the vertical vibration isolation.

Metal springs are widely used in vibration isolation platforms for its capture of small size, simple structure and low cost. On the other hand, the horizontal stiffness of the metal spring varies with the amount of compression [8], so that the control model change with it and are coupled with each other, which can cause the traditional model-based control methods fail. And the Non-model based algorithm may be too complex with high computational cost.

Based on the results of the horizontal dynamics modeling of the isolation platform, a novel adaptive feedforward control is proposed, using the FXLMS algorithm with dynamic objective function and variable step size. The algorithm not only can better adapt to the difference of system identification between the isolation modules, but also solve the problem of coupling.

1.2. Dynamics model of the platform
The simplified model of the existing metal spring isolator is shown in the Figure1. The platform is connected to the base by four modules. And each module is supported by a metal spring and equipped
with two motors that controls horizontal and vertical active control. Vibration is transmitted from the spring, forming the Primary Channel, and the motor model is the Secondary Channel.

![Figure 1. The simplified structure of isolation platform.](image)

Analyzing the x-direction movement of a random point A on the vibration isolation platform, its velocity can be expressed as follows

$$\dot{x}_A = \dot{x}_o + r_{oA} \times \dot{\gamma}$$  \hspace{1cm} (1)

Where $\dot{x}_o$ is the x-direction velocity of center of the rotation. $\dot{\gamma}$ is the rotational velocity of the platform around the Z axis. $r_{oA}$ is the distance from point A to the center of rotation. The second term in the equation is the component of the platform's rotational velocity in the x direction. When the partial load exists, the center of rotation and the center of mass do not coincide. And then the difference in the secondary channel of each module is amplified, which introduces rotation coupling. Therefore, it is necessary to consider both the rotation coupling and the x-direction vibration for the control.

The dynamic model of the vibration isolation system in the horizontal direction can be established according to H. X. Zeng [5]. Moreover, the transfer function of secondary channel of the horizontal x direction and the coupling channel between the x direction and the $\gamma$ direction are both second-order models similarly, as the following form

$$S(z) = \frac{a_1 z^2 + b_1 z + c_1}{a_2 z^2 + b_2 z + c_2}$$  \hspace{1cm} (2)

2. Design of Feedforward Control for the platform

2.1. FXLMS algorithm

The structure of the FXLMS feedforward algorithm applied in the field of vibration isolation [9] is shown in the Figure 2.

![Figure 2. The structure of LMS algorithm in vibration isolation.](image)

The main method is to add an adaptive FIR filter $W(z)$ in front of the secondary control channel $S(z)$, so the control signals $y'(n)$ can be formulated as follows:

$$y'(n) = W(z) \ast x(n) \ast S(z)$$  \hspace{1cm} (3)

Each parameter of the filter $W(z)$ can be adjusted online by the LMS algorithm to minimize the output vibration of the vibration platform after the feed-forward control. The platform vibration output
is the result of control signal combined with the vibration transmitted by the primary channel. The objective function optimized by the LMS algorithm is the 2-norm of the vibration output, as shown in equation (4)

\[ J = \frac{1}{2} e^2(n) = \frac{1}{2} (d(n) - y'(n))^2 \]  

(4)

It is very difficult to solve the optimal filter coefficients of minimize the objective function directly. Therefore, the main method is to dynamically update the filter coefficients by the steepest descent method. Also it has been proved in the related literature\(^{[10]}\) that as long as the phase error of the secondary channel model is not up to 90°, the identification accuracy of the secondary channel does not affect the convergence of the FXLMS algorithm.

2.2. Double-channel iterative NFXLMS algorithm

We introduce a double iterative FXLMS algorithm to solve the coupling problem, and the control structure block diagram is shown in Figure 3.

![Figure 3. The structure of Double-channel iterative NFXLMS algorithm](image)

For the horizontal x-direction vibration, the active control is mainly implemented by two motors in a diagonal position of the platform. Because of the influence of the partial load, the motor signal control of each module not only controls the x-direction vibration of the centroid, by the Secondary channel \(S_1(z)\) \(S_2(z)\), but also introduces the rotational coupling, by which we called Coupling channel \(C_1(z)\) \(C_2(z)\). Then the signal that controls the movement of the centroid and that introduces the rotational coupling are formulated as

\[ U_x(n) = x(n) \cdot W_1(z)S_1(z) + x(n) \cdot W_2(z)S_2(z) \]  
\[ U_y(n) = x(n) \cdot W_1(z)S_1(z) + x(n) \cdot W_2(z)S_2(z) \]  

(5)  
(6)

Thus, the x-direction vibration of the platform is \(e_x = d(n) - U_x(n)\). And the rotation coupling error of the platform is \(e_y = U_y\). The vibration \(e_x\) is selected as the optimization objective function of the FIR filter No. 1, thus:

\[ J_1 = \frac{1}{2} e_x^2(n) = \frac{1}{2} (d(n) - U_x(n))^2 \]  

(7)

The design of adaptive FIR filter No. 1 is mainly used to suppress the x-direction vibration signal. By using the variable adaptive convergence factor, the design of FIR filter No. 2 aims to make the objective function as the following form

\[ J_2 = \frac{1}{2} e_z^2(n) = \frac{\sigma_z}{2} e_x^2(n) + \frac{\sigma_y}{2} e_y^2(n) \]  

(8)
In the equation, $\sigma_x, \sigma_\gamma$ are the proportional components of the rotation coupling and the x-direction vibration in the objective function. And $\sigma_x, \sigma_\gamma$ dynamically change with the vibration.

The FXLMS algorithm uses a reference model for the control channel, and it has been mentioned previously that the coupling channel has an approximate form to the transfer function of the secondary channel. So we set the reference function in the same second-order model $H(z)$. Then the two filter convergence factors were designed separately using the NLMS algorithm with iterative variable step size $\mu_1, \mu_2$.

$$W(n+1) = W(n) - \frac{\mu}{2} \nabla J(n) = W(n) + \frac{\mu}{x(n)x(n)^T + \delta} e(n)x(n)H(z)$$  \hspace{1cm} (9)

According to the fact that the noise in the vibration system is irrelevant, the autocorrelation estimation can be introduced to eliminate the noise interference. The specific method for updating the convergence factor $\mu_1(n)$ of the No. 1 Filter is as follows

$$p(n) = \alpha p(n-1) + \beta e_x(n-1)e_x(n)$$ \hspace{1cm} (10)
$$\mu_1(n) = \gamma \mu_1(n-1) + (1-\gamma)p^2(n)$$ \hspace{1cm} (11)

Where $p(n)$ is the autocorrelation estimation of the x-direction vibration of the platform, each parameter should be satisfied $0<\gamma, \alpha, \beta<1$. Therefore, a fast convergence of the step size factor can be achieved, and the steady-state error is small as well.

For the No. 2 adaptive channel, the instantaneous ratio of the rotational coupling and the translational vibration $\theta = e_\gamma/e_x$ is calculated using the online method, and the convergence factor $\mu_2(n)$ is formulated as follows

$$\mu_2(n) = \gamma \mu_2(n-1) + (1-\gamma)p^2(n)(1-\theta + \theta^2)$$ \hspace{1cm} (12)

Then the objective function $J_2$ of the No. 2 Filter is:

$$J_2 = \frac{1-\theta}{2} e_x^2(n) + \frac{\theta}{2} e_\gamma^2(n)$$ \hspace{1cm} (13)

When the ratio $\theta$ is larger, the coupling angle error is larger. And then the control component of the rotation in the objective function get a greater impact, so that a better fast convergence can be achieved. And when the power ratio $\theta$ is small, the controller can concentrate on the x-direction vibration.

Through the design of the above two FIR adaptive filters, the platform gets a better performance with smaller rotation coupling error and smaller x-direction vibration at the same time.

3. Control Simulation and Experiment

3.1. Simulation in Matlab

The algorithm is simulated in Matlab and Simulink. The both adaptive FIR filters are designed in 3 orders. And the Secondary channel $S(z)$, the Primary channel $P(z)$, and the actual identification channel $H(z)$ all adopt a second-order model. The models are set as follows

$$S(z) = \frac{4es}{ms^2 + 4cs + 4k} \hspace{1cm} P(z) = \frac{4cs + 4k}{ms^2 + 4cs + 4k} \hspace{1cm} H(z) = \frac{4es}{ms^2 + 4cs + 4k}$$ \hspace{1cm} (14-16)

Where $m, k, c, e$ denote the mass, stiffness coefficient, damping coefficient and the coefficient of actuator of the vibration isolation platform. And $\dot{m}, \dot{k}, \dot{c}, \dot{e}$ denote the corresponding identification parameter. The models are simplified for the simulation and the values of parameters are shown in the Table. 1.
Table 1. Value of the Parameters

| Parameter | Value    |
|-----------|----------|
| m         | 30kg     |
| k         | 11500N/m |
| e         | 0.5N/ V  |

The input signal is a 1-100HZ sweep signal and more low frequency signal components are added to ensure adequate excitation. Then the simulating result of the x-direction error combined with the rotation coupling output is obtained. The control results are shown in Figure 4 and Figure 5.

![Figure 4. The output signal with control and without control.](image)

![Figure 5. Transmissibility curve with feedforward and feedback control.](image)

It can be seen from the figure that the convergence speed of the novel algorithm is fast while the steady-state error is small. Further, initial vibration isolation frequency of the vibration transmissibility curve is down to about 7 Hz.

3.2. The experiment in the platform

The feedforward control method proposed in this paper has been applied to a mental spring active vibration isolation system, which is shown in Figure 6. And we focus on horizontal vibration in experiments.

![Figure 6. The mental spring active vibration isolation platform](image)

![Figure 7. The diagram of control experiments](image)

Simplified diagram of the experiment is shown in the Figure 7. During experiments, feedforward controller parameters are updated online based on ground vibration and platform vibration measured by velocity sensor. Then the controller yields the corresponding control signal driving voice coil motor to achieve vibration isolation. The results of vibration isolation from 2Hz-100Hz are shown in Figure 8. As the figure shown, vibration is attenuated at lower frequencies. The maximum vibration attenuation is near 10Hz, about -25 dB.
4. Conclusion
A double-channel iterative NFXLMS algorithm is proposed in the paper to solve the coupling problem of the vibration isolation platform. It is based on the FXLMS algorithm and it can simultaneously suppress the rotation coupling and x-direction error of the platform. Finally, the method is effectively implemented on the vibration isolation table.

References
[1] D. Karnopp, M. J. Crosby, & R. A. Harwood. (1974). Vibration control using semi-active force generators. Transactions of the ASME Journal of Engineering for Industry, 96(2), 619-626.
[2] Kang, M. S. (2003). Optimal feedforward control of active magnetic bearing system subject to base motion. Control Applications, 2003. CCA 2003. Proceedings of 2003 IEEE Conference on (Vol.1, pp.748-753 vol.1). IEEE.
[3] C. Collette, S. Janssens, & K. Artoosand C. Hauviller. (2010). Active vibration isolation of high precision machines. Diamond Light Source Proceedings, 1(MEDSI-6), 451-456.
[4] Yoshioka H, Murai N. AN ACTIVE MICROVIBRATION ISOLATION SYSTEM. Proceedings of the 7th International Workshop on Accelerator Alignment, 2002: 388-401.
[5] H. X. Zeng, (2014). Research of an active vibration control method with position stabilization function. M.S. thesis, Dept. Mech. Eng., Tsinghua Univ., Beijing, China.
[6] Zhou Zhenhua, Chen Xuedong, Zhou Bo. Feedforward compensation in vibration isolation system subject to base motion. Journal of Vibration and Control, 2015, 21(6): 1201-1209.
[7] Zhang, Z., & Yin, W. (2017). Data-driven feedforward control on active vibration isolation system. International Conference on Control, Automation and Systems, 2017.
[8] Yang, G., Xiao, S., & Zhang, W. (2010). Analysis on the lateral stiffness of the helical circle spring. China Railway Science.
[9] Pu, H., Luo, X., Jiang, W., Dong, K., & Chen, X. (2010). Modelling and control of hybrid vibration isolation system for high-precision equipment. IEEE International Conference on Control and Automation (pp.2152-2157). IEEE.
[10] Kuo, S. M., Morgan, D. R. Active noise control: a tutorial review. Proceedings of the IEEE 2006, 87(6): 943 – 973.