Light-Front Holography in QCD and Hadronic Physics

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Following the recent observation that for light quark masses a harmonic potential and linear Regge trajectories in the light-front form of dynamics correspond to a linear potential in the usual instant-form, and the realization that the light-front effective confinement potential can be obtained from an effective theory which encodes the fundamental conformality of the classical QCD Lagrangian, we describe a procedure to extend the light-front holographic approach to hadronic physics to include light-quark masses. The procedure allows us to extend the formalism of de Alfaro, Fubini and Furlan to the frame-independent light-front Hamiltonian theory in the approximation where the transverse dynamics is unchanged to first order without modifying the emerging confinement scale.

1 Introduction

In the ultrarelativistic limit of zero quark masses one can reduce the strongly correlated multi-parton light-front dynamical problem in QCD to an effective one-dimensional quantum field theory, which encodes the fundamental conformal symmetry of the classical QCD Lagrangian. This procedure leads to a semiclassical relativistic light-front wave equation for arbitrary spin which incorporates essential spectroscopic and non-perturbative dynamical features of hadron physics, similarly to the the Schrödinger and Dirac equations in atomic physics \textsuperscript{1,2,3}.

A key element in the search for a semiclassical approximation to QCD, in its nonperturbative regime, is the correspondence between the equations of motion in Anti–de Sitter (AdS) space and the light-front (LF) Hamiltonian equations of motion for relativistic light hadron bound-states in physical space-time \textsuperscript{1} inspired by the AdS/CFT correspondence \textsuperscript{4}. In fact, light-front holographic methods were originally introduced the matching of the electromagnetic \textsuperscript{5} and gravitational \textsuperscript{6} form factors in AdS space \textsuperscript{7,8} with the corresponding expressions derived from LF quantization in physical space time. This approach allows us to establish a precise relation between wavefunctions in AdS space and the LF wavefunctions (LFWFs) describing the internal structure of hadrons. However the actual form of the effective potential has remained unknown until very recently.

It was been realized \textsuperscript{3} that the form of the effective LF confining potential can be obtained from the framework introduced by V. de Alfaro, S. Fubini and G. Furlan (dAFF) \textsuperscript{9}, by extending the dAFF formalism to the frame-independent light-front Hamiltonian theory. It was shown by dAFF that a scale can appear in the Hamiltonian while retaining the conformal invariance of the action \textsuperscript{9}. This remarkable result is based on the isomorphism of the algebra of the one-dimensional conformal group $Conf(R^1)$ to the algebra of generators of the group $SO(2, 1)$. One of the generators of this group, the rotation in the 2-dimensional space, is compact and has

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therefore a discrete spectrum with normalizable eigenfunctions. As a result, the form of the evolution operator is fixed and includes a confining harmonic oscillator potential, and the time variable has a finite range. As pointed out by dAFF, the relation between the generators of the conformal group and the generators of $SO(2,1)$ suggests that the new scale may play a fundamental role. In fact, it was shown in Ref. 3 that there exists a remarkable threefold connection between the one-dimensional semiclassical approximation to light-front dynamics with gravity in a higher dimensional AdS space, and the constraints imposed by the invariance properties under the full conformal group in one dimension $Conf(R^1)$. This provides a new insight into the physics underlying confinement, chiral invariance, and the QCD mass scale.

It was also shown very recently that an effective harmonic potential in the light-front form of dynamics corresponds, for light quark masses, to a linear potential in the usual instant-form\cite{10}, a result which suggests that the Wilson area law for confinement is also valid for light quarks. Conversely, for a linear potential in the instant-form, the front-form is a harmonic oscillator, thus the prediction of linear Regge trajectories in the hadron mass square for small quark masses\cite{10}, in agreement with the observed spectrum for light hadrons.

Motivated by these recent results, we will discuss in this article how the light-front holographic ideas can be extended in a simple and consistent way to first order in the light-quark masses. In particular, we will show that in this approximation the results are stable; that is, described with identical values of the gap constant. To first order in the quark masses the transverse dynamics is unchanged, and the effective LF confining interaction is given by the effective one-dimensional quantum field theory.

2 Light-Front Semiclassical Approximation to QCD

In the front-form of relativistic dynamics\cite{11} the four-momentum generators $P^\mu$ of a hadron $P^\mu = (P^-, P^+, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, are constructed canonically from the QCD Lagrangian by quantizing the system on the light-front at fixed LF time $x^+, x^\pm = x^0 \pm x^3$. The LF Hamiltonian $P^-$ generates the LF time evolution $P^- |\phi\rangle = i \frac{\partial}{\partial x^+} |\phi\rangle$, whereas the LF longitudinal $P^+$ and transverse momentum $\mathbf{P}_\perp$ are kinematical generators.

Each hadronic eigenstate $|\psi\rangle$ is expanded in a Fock-state complete basis of non-interacting $n$-particle states $|n\rangle$ with an infinite number of components: $|\psi\rangle = \sum_n \psi_n |n\rangle$, where the LFWFs $\psi_n$ are boost invariant. In order to reduce the strongly correlated multi-parton bound-state dynamics to an effective one-dimensional problem it is crucial to identify the key dynamical variable which controls the bound state\cite{1}, the invariant mass of the constituents in each $n$-particle Fock state $M^2_n = (k_1 + k_2 + \cdots + k_n)^2$,

$$M^2_n = \sum_{i=1}^n \frac{k^2_{i\perp} + m_i^2}{x_i},$$

(1)

where $\sum_{i=1}^n x_i = 1$, and $\sum_{i=1}^n k_{i\perp} = 0$. In fact, the LF wave function is off-shell in $P^-$ and consequently in the invariant mass. Alternatively, it is useful to consider its canonical conjugate invariant variable in impact space. This choice of variable will also allow us to separate the dynamics of quark and gluon binding from the kinematics of constituent spin and internal orbital angular momentum\cite{1}.

For a $q\bar{q}$ bound state, the invariant mass (1), which is also the LF kinetic energy, is $M^2_{q\bar{q}} = \frac{k^2_{x(1-x)}}{x(1-x)}$. Similarly, in impact space the relevant variable is $\zeta^2 = x(1-x) b^2_1$, the invariant separation between the quark and antiquark. Thus, to first approximation, LF dynamics depends only on the boost invariant variable $M_n$ or $\zeta$, and the dynamical properties are encoded in the hadronic LF wave function $\phi(\zeta)$

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi}\zeta},$$

(2)
where we have factored out the longitudinal $X(x)$ and orbital dependence. The normalization of the transverse and longitudinal modes is given by $\langle \phi|\phi \rangle = \int d\zeta \phi(\zeta)^2 = 1$ and $\langle X|X \rangle = \int_0^{\infty} dx x^{-1}(1-x)^{-1}X^2(x) = 1$.

In the limit of zero quark masses the longitudinal modes decouple from the invariant LF Hamiltonian equation $H_{LF}|\phi \rangle = M^2|\phi \rangle$ with $H_{LF} = P^\mu P_\mu = P^+ - P_\perp^2$. We obtain the wave equation

$$\left( -\frac{d^2}{dx^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta,J) \right) \phi_{n,J,L} = M^2 \phi_{n,J,L},$$

(a relativistic single-variable LF Schrödinger equation, where $n$, the number of nodes in $\zeta$, $J$ the total angular momentum and $L$ the internal orbital angular momentum of the constituents. The effective potential $U$ acts on the valence sector of the theory and follows from the systematic expression of the higher Fock components as functionals of the lower ones. This method has the advantage that the Fock space is not truncated, and the symmetries of the Lagrangian are preserved. The effective interaction potential $U$ is instantaneous in LF time $x^+$, not instantaneous in ordinary time $x^0$, and it represents the complete summation of interactions obtained from the Fock state reduction.

If we compare the invariant mass in the instant-form in the hadron center-of-mass system, $P = 0$, $M_{qq}^2 = 4m_q^2 + 4p^2$, with the invariant mass in the front-form in the constituent rest frame, $k_q + k_{\bar{q}} = 0$ for equal quark-antiquark masses, we obtain the relation found in Ref.

$$U = V^2 + 2\sqrt{p^2 + m_q^2}V + 2V\sqrt{p^2 + m_q^2},$$

where we identify $p_\perp^2 = \frac{k_q^2}{4x(1-x)}$, $p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and $V$ is the effective potential in the instant-form. Thus, for small quark masses a linear instant-form potential $V$ implies a harmonic front-form potential $U$ and thus linear Regge trajectories. For large quark masses this relation is still valid for large $q\bar{q}$ separation, but the non-local mass terms in (4) become important. One can also show how the two-dimensional front-form harmonic oscillator potential for massless quarks takes on a three-dimensional form when the quarks have mass since the third space component is conjugate to $p_3$, which has an infinite range for $m \neq 0$.

3 Conformal Invariance and Light-Front Hamiltonian Dynamics

When extended to light-front holographic QCD, the dAFF framework gives important insights into the QCD confining mechanism. It turns out that it is possible to introduce a scale by a redefinition of the quantum mechanical evolution operator while leaving the action conformally invariant, and consequently to a redefinition of the corresponding evolution parameter $\tau$, the range of which is finite. Remarkably this procedure determines uniquely the form of the light-front effective potential and correspondingly the modification of AdS space.

One starts with the one-dimensional action $S = \frac{1}{2} \int dt \left( \dot{Q}^2 - \frac{d^2}{dx^2} \right)$, which is invariant under conformal transformations in the variable $t$. In addition to the Hamiltonian $H$ there are two more invariants of motion for this field theory, namely the dilatation operator $D$ and $K$, corresponding to the special conformal transformations in $t$. Specifically, if one introduces the new variable $\tau$ defined through $d\tau = dt/(u+v+t+w)$ it then follows that the operator $G = uH + vD + wK$ generates the quantum mechanical evolution in $\tau$ $G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle$. In the Schrödinger representation

$$G = \frac{1}{2} \left( -\frac{d^2}{dx^2} + \frac{g}{x^2} \right) + \frac{i}{4} \left( x \frac{d}{dx} + \frac{d}{dx} x \right) + \frac{1}{2} wx^2,$$

is the superposition of the ‘free’ Hamiltonian $H$, the generator of dilatations $D$ and the generator of special conformal transformations $K$ in one dimension, the generators of $Conf(R^1)$; namely $G = uH + vD + wK$. The conformal group $Conf(R^1)$ is locally isomorphic to $SO(2,1)$, the
Lorentz group in 2+1 dimensions. Since the generators of $Conf(R^4)$ have different dimensions, their relations with the generators of $SO(2,1)$ imply a scale, which here plays a fundamental role, as already conjectured in\(^9\).

Comparing the dAFF Hamiltonian (5) with the light-front wave equation (3) and identifying the variable $x$ with the light-front invariant variable $\zeta$, we have to choose $u = 2$, $v = 0$ and relate the dimensionless constant $g$ to the LF orbital angular momentum, $g = L^2 - 1/4$, in order to reproduce the light-front kinematics. Furthermore $w = 2\lambda^2$ fixes the confining light-front potential to a quadratic dependence\(^9\), $U \sim \lambda^2 \zeta^2$, and thus from (4) to a linear potential for massless quarks.

4 AdS Gravity and Light-Front Dynamics

Anti-de Sitter AdS\(_5\) is a five-dimensional space with negative constant curvature and a 4-dimensional boundary, Minkowski space-time. In the AdS/CFT correspondence, the consequence of the $SO(2,4)$ isometry of AdS\(_5\) is the conformal invariance of the dual field theory. Recently we have derived wave equations for hadrons with arbitrary spin starting from a dilaton-modified effective action in AdS space\(^2\). An essential element is the mapping of the higher-dimensional equations to the LF Hamiltonian equation found in Ref.\(^1\). This procedure allows a clear distinction between the kinematical and dynamical aspects of the LF holographic approach to hadron physics. Accordingly, the non-trivial geometry of pure AdS space encodes the kinematics, and the additional deformations of AdS encode the dynamics, including confinement\(^2\), and determine the form of the LF effective potential from the precise holographic mapping to light-front physics\(^1,2\). The variable $z$ of AdS space is identified with the LF boost-invariant transverse-impact variable $\zeta$\(^1,5,6\) thus giving the holographic variable a precise definition in LF QCD. The LF mapping also provides a precise relation between the bound-state amplitudes in AdS space and the boost-invariant light-front wavefunctions describing the internal structure of hadrons in physical space-time. One finds from the dilaton-modified AdS action the effective LF potential\(^2,14\)

$$U(\zeta, J) = \frac{1}{2} \varphi''(\zeta) + \frac{1}{4} \varphi'(\zeta)^2 + \frac{2J - 3}{2\zeta} \varphi'(\zeta),$$

(6)

provided that the product of the AdS mass $\mu$ and the AdS curvature radius $R$ are related to the total and orbital angular momentum, $J$ and $L$ respectively, according to $(\mu R)^2 = -(2 - J)^2 + L^2$.

The critical value $J = L = 0$ corresponds to the lowest possible stable solution, the ground state of the LF Hamiltonian, in agreement with the AdS stability bound $(\mu R)^2 \geq -4$\(^15\), where $R$ is the AdS radius. The correspondence between the LF and AdS equations thus determines the LF confining interaction $U$ in terms of the effective modification of the infrared region of AdS space. The choice of the dilaton profile $\varphi(z) = \lambda z^2$ introduced in\(^16\) thus follows from the requirements of conformal invariance. This specific form for $\varphi(z)$ leads through (6) to the effective LF potential $U(\zeta, J) = \lambda^2 \zeta^2 + 2\lambda(J - 1)$, and corresponds to a transverse oscillator in the light-front. The term $\lambda^2 \zeta^2$ is determined uniquely by the underlying conformal invariance of classical QCD incorporated in the one-dimensional effective theory, and the constant term $2\lambda(J - 1)$ by the embedding space\(^2,14\). For $\lambda > 0$, the wave equation (3) has eigenfunctions

$$\phi_{n,L}(\zeta) = \lambda^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\lambda^2 \zeta^2/2} L_n^L(\lambda \zeta^2),$$

(7)

and eigenvalues

$$M^2_{n,J,L} = 4\lambda \left( n + \frac{J + L}{2} \right),$$

(8)

an important result also found in Ref.\(^17\). This result not only implies linear Regge trajectories, but also a massless pion and the relation between the $\rho$ and $a_1$ mass usually obtained from the Weinberg sum rules\(^18\).
5 Inclusion of Light Quark Masses

The partonic shift in the hadronic mass from small quark masses follows from the computation of the hadronic matrix element \( \langle \psi(P')|P_\mu P^\mu|\psi(P) \rangle = M^2 \langle \psi(P')|\psi(P) \rangle \) expanding the initial and final hadronic states \( |\psi \rangle \) in terms of their Fock components following the same steps as in Ref.\(^1\), but keeping the quark mass in the kinetic energy terms of the LF Hamiltonian. The result is

\[
\Delta M^2 = \left\langle \psi \left| \sum_a \frac{m_a^2}{x_a} \right| \psi \right\rangle, \tag{9}
\]

where \( \Delta M^2 = M^2 - M_0^2 \) is the hadronic mass shift. Here \( M_0^2 \) is the value of the hadronic mass computed in the limit of zero quark masses, given by Eq. (8). This expression is identical to the Weisberger result for a partonic mass shift \(^19\). Notice that this result is exact to first order in the light-quark mass if the sum in (9) is over all Fock states \( n \) the light-quark mass if the sum in (9) is over all Fock states \( n \).

The partonic shift in the hadronic mass from small quark masses amounts to the replacement \( m_q \in (10) \) of the "current" quark masses, i.e., the quark masses appearing in the QCD Lagrangian.

The longitudinal factor \( X(x) \) in the LFWF (2) can be determined in the limit of massless quarks from the precise mapping of light-front amplitudes for arbitrary momentum transfer \( Q^2 \). Its form is \( X(x) = x^\frac{1}{2}(1 - x)^{\frac{1}{2}} \). This expression of the LFWF gives a divergent expression for the partonic mass-shift (10), and, evidently, realistic effective two-particle wave functions have to be additionally suppressed at the end-points \( x = 0 \) and \( x = 1 \). As pointed out in \(^20\), the essential dynamical variable which controls the bound state wave function in momentum space is the invariant mass (1). Thus, for the effective two-body bound state the inclusion of light quark masses amounts to the replacement

\[
M_{q\bar{q}}^2 = \frac{k^2}{x(1 - x)} \rightarrow \frac{k^2}{x(1 - x)} + \frac{m_q^2}{x(1 - x)} + \frac{m_{\bar{q}}^2}{1 - x}, \tag{11}
\]

in the LFWF in momentum space. This is in fact the correct prescription, since it preserves the invariant properties of the LFWF.

In the limit of zero quark masses the effective LFWF for a two-parton ground state in impact space is

\[
\psi(x, \zeta) \sim \sqrt{x(1 - x)} e^{-\frac{1}{2} \lambda \zeta^2}, \tag{12}
\]

where the invariant transverse variable \( \zeta^2 = x(1 - x)b_\perp^2 \) and \( \lambda > 0 \). The longitudinal factor \( \sqrt{x(1 - x)} \) is determined from the precise holographic mapping of transition amplitudes in the limit of massless quarks. The Fourier transform of (12) in momentum-space is

\[
\psi(x, k_\perp) \sim \frac{1}{\sqrt{x(1 - x)}} e^{-\frac{k^2}{2x\lambda(x - 1)}}, \tag{13}
\]

where the explicit dependence of the wavefunction in the LF off shell-energy is evident.

For the effective two-body bound state the inclusion of light quark masses amounts to the replacement in (13) of the \( q - \bar{q} \) invariant mass (11), the key dynamical variable which describes the off energy-shell behavior of the bound state \(^20\),

\[
\psi(x, k_\perp) \sim \frac{1}{\sqrt{x(1 - x)}} e^{-\frac{1}{2x}\left(\frac{k^2}{x(1 - x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1 - x}\right)}. \tag{14}
\]
Its Fourier transform gives the LFWF in impact space including light-quark masses\textsuperscript{20},

\[
\psi(x, \zeta) \sim \sqrt{x(1-x)} e^{-\frac{x}{2} \left(\frac{m_u^2 + m_d^2}{x + \frac{1}{x}}\right)} e^{-\frac{1}{2} \lambda \zeta^2},
\]

(15)

which factorizes neatly into transverse and longitudinal components. The holographic LFWF (15) has been successfully used in the description of diffractive vector meson production at HERA\textsuperscript{21}, in $B \to \rho \gamma$\textsuperscript{22} and $B \to K^* \gamma$\textsuperscript{23} decays as well as in the prediction of $B \to \rho$\textsuperscript{24} and $B \to K^*$\textsuperscript{25} form factors. The LFWF has also been used in Ref.\textsuperscript{26} to compute the spectrum of light and heavy mesons.

For excited meson states we can follow the same procedure by replacing the key invariant mass variable in the polynomials in the LFWF using (11). An explicit calculation shows, however, that the essential modification in the hadronic mass, from small quark masses, comes from the shift in the exponent of the LFWF. The corrections from the shift in the polynomials accounts for less than 3\%. This can be understood from the fact that to first order the transverse dynamics is unchanged, and consequently the transverse LFWF is also unchanged to first order. Thus our expression for the LFWF

\[
\psi_{n,L}(x, \zeta) \sim \sqrt{x(1-x)} e^{-\frac{x}{2} \left(\frac{m_u^2 + m_d^2}{x + \frac{1}{x}}\right)} \zeta^2 e^{-\frac{1}{2} \lambda \zeta^2} L_n^L(\lambda \zeta^2),
\]

(16)

and from (10) the hadronic mass shift $\Delta M^2$ for small quark masses\textsuperscript{7}

\[
\Delta M_{mq, mq}^2 = \frac{\int_0^1 dx \, e^{-\frac{x}{2} \left(\frac{m_u^2 + m_d^2}{x + \frac{1}{x}}\right)} \left(\frac{m_u^2}{x} + \frac{m_d^2}{1-x}\right)}{\int_0^1 dx \, e^{-\frac{x}{2} \left(\frac{m_u^2 + m_d^2}{x + \frac{1}{x}}\right)}},
\]

(17)

which is independent of $L$, $S$ and $n$. For light quark masses, the hadronic mass shift is the longitudinal $1/x$ average of the square of the effective quark masses, i.e., the effective quark masses from the reduction of higher Fock states as functionals of the valence state\textsuperscript{13}. The final result for the hadronic spectrum of mesons modified by light quark masses is thus

\[
M_{n, L, mq, mq}^2 = \Delta M_{mq, mq}^2 + 4\lambda \left(n + \frac{J + L}{2}\right),
\]

(18)

with identical slope $4\lambda$ from the limit of zero quark masses. In particular, we obtain from (18) the spectral prediction for the $J = L + S$ strange meson mass spectrum

\[
M_{n, L, S}^2 = M_{K^+}^2 + 4\lambda \left(n + L + \frac{S}{2}\right),
\]

(19)

where $M_{K^\pm} \approx 494$ MeV.

As an example, the predictions for the $J = L + S$ light vector mesons are compared with experimental data in Fig. 1. The data is from PDG\textsuperscript{27}. The spectrum is well reproduced with identical values for the mass scale $\sqrt{\lambda} = 0.54$ GeV for the light vector sector. The model predictions for the $K^*$ sector shown in Fig. 1 (b) is very good. However the states $K^*_0(1430)$ and $K^*_2(1430) – which belong to the $J = 0$, $J = 1$ and $J = 2$ triplet for $L = 1$, are degenerate. This result is in contradiction with the spin-orbit coupling predicted by the LF holographic model for mesons; a possible indication of mixing of the $K^*_0$ with states which carry the vacuum quantum numbers. Fitting the quark masses to the observed masses of the $\pi$ and $K$ we obtain for $\sqrt{\lambda} = 0.54$ MeV the average effective light quark mass $m_q = 46$ MeV, $q = u, d$, and $m_s = 357$ MeV, values between the current \textit{MS} Lagrangian masses normalized at 4 GeV and typical constituent masses. With these values one obtains $\Delta M_{mq, mq}^2 = 0.067\lambda$, $\Delta M_{mq, ms}^2 = 0.837\lambda$, $\Delta M_{ms, ms}^2 = 2.062\lambda$, for $\sqrt{\lambda} = 0.54$ MeV.
For heavy mesons conformal symmetry is strongly broken and there is no reason to assume that the LF potential in that case is similar to the massless one. Indeed, a simple computation shows that the model predictions for heavy quarks (without introducing additional elements in the model) is not satisfactory. In fact, the data for heavy mesons can only be reproduced at the expense of introducing vastly different values for the scale $\lambda$. Another important point are the leptonic decay widths. For light quarks the quark masses have little influence on the result, only about 2 % for the $\pi$ meson and 5 % for the $K$ meson, but using the formalism also for the $B$ and $D$ mesons leads to widely different values when compared with experiment. For large quark masses the form of the LF confinement potential $U$ cannot be obtained from the conformal symmetry of the effective one-dimensional quantum field theory. In this case an important dependence on the heavy quark mass is expected, as suggested by the relation given by Eq. (4) between the effective potentials in the front-form and instant-form of dynamics.

6 Conclusions

The connection of light-front dynamics, its holographic mapping to gravity in a higher dimensional space, and the procedure introduced by de Alfaro, Fubini and Furlan provides new insights into the physics underlying confinement, chiral invariance, and the origin of the QCD mass scale. This threefold connection leads to effective one-dimensional quantum field theory, which encodes the fundamental conformal symmetry of the classical QCD Lagrangian. A mass gap and a confinement scale arise when one extends the formalism of dAFF to frame-independent light-front Hamiltonian theory, thus leading to emerging confinement. The resulting light-front potential has a unique form of a harmonic oscillator in the front-form of dynamics and correspond to a linear potential in the usual instant-form. The result is a relativistic light-front wave equation for arbitrary spin which incorporates essential spectroscopic and dynamical features of hadron physics. We have shown how the procedure can be extended to light quarks without modifying to first approximation the traverse dynamics and the universality of the Regge slopes. As an example we show the new results for the $K^*$ radial and orbital excitations.

Recent discussions of light-front holographic predictions are given Refs. [29,30,31].
Acknowledgements

Based on an invited talk, presented by GdT at the Rencontres de Moriond, QCD and High Energy Interactions (2014). We thank Joshua Erlich, Stanislaw Glazek and Arkadiusz Trawiński for helpful conversations and collaborations. GdT wants to thank the organizers for the great hospitality at La Thuile. This research was supported by the Department of Energy, contract DEAC0276SF00515. SLAC-PUB-15954.

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