SAFETY ENGINEERING OF ANTHROPOGENIC OBJECTS

INFLUENCE OF THE BALLAST RESISTANCE ON THE STABILITY OF CONTINUOUS WELDED RAIL

Dominik POKROPSKI
Military University of Technology, Warsaw, Poland
ORCID: 0000-0001-5558-8946

Abstract

The article is about the issue of the influence of ballast resistance on the stability of the Continuous Welded Rail. The ballast resistance affects both the longitudinal and transverse displacements. It depends on the quality of the ballast, the degree of its compaction and contamination. The article contains an analysis of the impact of ballast resistance on the track based on the Finite Difference Method. The calculations showed that the resistance value directly affects the allowable critical force and the maximum temperature rise in the rail that does not endanger the safety of railway traffic.

Key words: Continuous Welded Rail, buckling of CWR, ballast, Finite Difference Method

INTRODUCTION

The development of rail transport creates new challenges in relation to this mode of transport. A big step towards improving the quality of transports is the widespread use of the Continuous Welded Rail (CWR). However, this solution also has some disadvantages besides advantages. The main problem is the occurrence of high thermal stresses which may cause the rail to break in winter or buckle in summer. For this reason, diagnostics of the contactless track is carried out to ensure safety. The analyzes described in the literature have shown that each of the pavement elements has an impact on the track stability. Nevertheless, the most important element is the ballast - its compaction, pollution and scattering of sleepers. The article shows the relationship between the ballast resistance and the longitudinal displacement and the maximum temperature rise in the rail.
1. **Ballast as a factor influencing track stability**

The resistance to the displacement of the track is very much dependent on the ballast. It depends on many parameters, especially the type and size of stone. The stability of the non-contact track is also dependent on the degree of compaction, layer thickness and maintenance condition. An important parameter is also the degree of scattering of the sleepers and the ambient temperature, especially whether the ballast is frozen.

The diameter of the aggregate grains used for the ballast ranges from 22.4 mm to 63 mm. Most often it is a 30 cm thick layer and, depending on the train speed, the width of pouring the sleepers is 40 cm for speed between 160 km/h and 50 cm speed more than 160 km/h. The side resistance of the ballast was determined on the basis of many years of measurements. They largely depend on the maintenance condition of the surface [1].

The resistance of the ballast affects the distribution of longitudinal forces in the track and the formation of stresses. It should be noted what influence the manufacturing technology has on the ballast resistance. In the studies carried out so far, it has been noticed that a clean and poorly compacted ballast has 2 to 3 times less resistance than a compacted ballast with noticeable contamination.

2. **Model of longitudinal track displacement**

The analysis covers the case of rails creep, i.e. their displacements along the track axis. The values of these displacements depended on the braking force of the rolling stock moving along the section in question [2]. The analyzes are limited only to longitudinal displacements along the track axis [3].

![Figure 1. Model of the CWR creep](image-url)
The figure shows:
q - rolling stock load,
t - unit rolling stock braking force,
k_0 - modulus of elasticity of the ballast,
l_t - train length.

For analysis, the characteristic of the resistance of the track grate was used, which takes into account the effect of vertical pressure. This nature of the track can be assumed based on the research carried out at the University of Technology in Delft [4].

**Figure 2. Creep characteristics for dynamic loads [1]**

For this model, in the longitudinal direction, the equilibrium equations take the following form [5]:

$$\frac{d^2 u}{dx^2} EA - \beta^2 EA = -t$$  \hspace{1cm} (1)

$$\frac{d^2 u}{dx^2} - \beta^2 = - \frac{t}{EA}$$  \hspace{1cm} (2)

where:

- t - unit braking force of the train
\[ \beta = \frac{k_0}{\sqrt{EA}} \]  

\( k_0 \) – ratio of the ballast elasticity

The displacement values resulting from the rolling stock load and the thermal load can be added together. Their value may not exceed the limit of elastic path displacement beyond which a permanent displacement will occur [1].

\[ u_{mech}(x) + u_t(x) \leq u_{tim}(x) \]  

where:
- \( u_{mech} \) – displacement due to rolling stock load,
- \( u_t \) – displacement caused by thermal forces,
- \( u_{tim} \) – maximal displacement where there is no permanent displacement.

Based on the specified values of displacements, a criterion can be developed that determines the risk of track creep. They can be represented by the following formula [6]:

\[ d_k = \frac{u_{max}(x)}{u_{tim}(x)} \]  

The phenomenon of creep may occur when \( d_k \geq 1 \).

3. **Lateral displacement**

When the ballast and sleepers do not ensure full track stability, the track grate buckling occurs.

The design model treats the path as a beam whose ends are points of change in the nature of the buckling curve. The beam rests on a resilient ground and its ends are free to move. Figure 3 shows the scheme that was adopted for further analysis. The inflection points between which the analyzed section is located are marked as "inflection point" [7].
To determine the value of the resistance $\beta(x)$, use the formula [8]:

$$\beta(x) = [1 - \frac{H}{2d}(1 + \sin \frac{\pi x}{l})]^2 k_0$$  \hspace{1cm} (6)

where:
H - track height,
d - sleeper bedding height,
l - track section length,
k_0 - unit ballast resistance.

The critical buckling force can be represented by the following relationship:

$$P_{kr} = \frac{\pi^2 EI}{l^2} + \frac{k_0 l^2}{\pi^2} \left[ 1 - \left( 1 + \frac{8}{3\pi} \right) \frac{H}{d} + \left( \frac{7}{16} + \frac{4}{3\pi} \right) \left( \frac{H}{d} \right)^2 \right]$$  \hspace{1cm} (7)

Finally, based on the principle of minimum potential energy, the critical force can be written as [9]:

$$P_{kr} = \frac{4\pi^2 EI}{l^2} + \frac{\beta l^2}{4\pi^2}$$  \hspace{1cm} (8)

Based on the theory of elasticity and the theory of small deformations, an algorithm has been developed to determine the values of forces at lateral displacement of the track for the
buckling phenomenon. Due to the lack of a spontaneous return to the original state of the buckled track, only the static character of the buckling can be considered. A track that buckles is not able to regain its operational capacity by itself without human intervention [10].

The ballast resistance $\beta$ depends on [11]:
- the height of pouring sleepers,
- precision of covering the head of sleepers,
- the state of contamination of the ballast,
- condition of railway sleepers,
- type and condition of fastenings.

The described equation of the non-contact track was used for further analyzes.

4. Analysis of longitudinal displacement

To perform the analysis using the finite difference method, the track was divided into sections of short length. The longitudinal displacement of the track was described by differential equations of the following form:

$$\frac{d^2 u}{dx^2} - \beta^2 u = -\frac{t}{EA}$$

(9)

Based on the Finite Difference Method, this equation can be written into a system of $n$ equations for each node of the rail track under consideration.

For the $i$-th node, the written equations have the following form [12]:

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2} - \beta^2 u = -\frac{t}{EA}$$

(10)

For the sake of simplicity, the following symbols have been adopted:

$$F = -(2 + \beta^2 \Delta x^2)$$

(11)

and:

$$Q = -\frac{t\Delta x^2}{EA}$$

(12)

On the basis of the above-described method of determining the deflection value in individual nodes, a matrix of numerical coefficients was developed.
Table 1. Matrix of numerical coefficients for a system of equations

After checking the convergence of the program for calculating longitudinal displacement (it was determined at \( x = 5 \) m), the longitudinal resistance values for the track with the 60E1 type rail were entered to the program and the longitudinal displacement were calculated.

| Length of track [m] | Load [kN/m] | Longitudinal resistance [kN/m²] | Longitudinal displacement [µm] |
|---------------------|-------------|-------------------------------|-------------------------------|
| 200                 | 80          | 20                            | 49,99                         |
|                     |             | 30                            | 33,33                         |
|                     |             | 40                            | 25,00                         |
|                     |             | 50                            | 20,00                         |
|                     |             | 60                            | 16,67                         |
|                     |             | 70                            | 14,29                         |
|                     |             | 80                            | 12,50                         |
|                     |             | 90                            | 11,11                         |
|                     |             | 100                           | 10,00                         |

| Node number/ displacement | \( \mathbf{u}_1 \) | \( \mathbf{u}_2 \) | \( \mathbf{u}_3 \) | \( \mathbf{u}_4 \) | \( \mathbf{u}_5 \) | ... | \( \mathbf{u}_{n-4} \) | \( \mathbf{u}_{n-3} \) | \( \mathbf{u}_{n-2} \) | \( \mathbf{u}_{n-1} \) | \( \mathbf{u}_n \) |
|--------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1                        | 0                 | 2                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 |
| 2                        | 1                 | F                 | 1                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 |
| 3                        | 0                 | 1                 | F                 | 1                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 |
| 4                        | 0                 | 0                 | 1                 | F                 | 1                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 |
| 5                        | 0                 | 0                 | 0                 | 1                 | F                 | 1                 | 0                 | 0                 | 0                 | 0                 | 0                 |
| 6                        | 0                 | 0                 | 0                 | 0                 | 1                 | F                 | 1                 | 0                 | 0                 | 0                 | 0                 |
| ...                      | 1                 | F                 | 1                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 |
| n-4                      | 0                 | 0                 | 0                 | 0                 | 0                 | 1                 | F                 | 1                 | 0                 | 0                 | 0                 |
| n-3                      | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 1                 | F                 | 1                 | 0                 | 0                 |
| n-2                      | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 1                 | F                 | 1                 | 0                 |
| n-1                      | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 1                 | F                 | 1                 |
| n                        | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 0                 | 1                 | 2                 |

Table 2. Matrix of numerical coefficients for a system of equations
Figure 5 shows the dependence of the longitudinal displacement depending on the value of the longitudinal resistance coefficient of the ballast and the train length. It can be noticed that the displacement values for low values of the longitudinal drag coefficient do not change significantly when the length of the moving train changes. Such changes are noticeable in the range of 60 kN/m² to 80 kN/m². It is a range in which the coefficient values are included, corresponding to the values obtained immediately after the construction or modernization of the route. This may be due to the fact that the new route is not affected by the impact of ballast pollution, rail wear or degradation of sleepers. At lower k₀ values, i.e. for pavements after long service life, non-contact track creep occurs regardless of the train length. It is related to the degradation of the pavement elements and the loss of load-bearing capacity of the reinforcement layers. The maneuvers of even a short train set have a large impact on the track, and thus cause a large longitudinal displacement.
5. Analysis of lateral displacement

The value of the critical force for transverse track displacement can be described using the equation [9]:

\[ P_{cr} = \frac{4\pi^2EI}{l^2} + \frac{\beta l^2}{4\pi^2} \]  \hspace{1cm} (13)

where:

- \( \beta \) - lateral resistance of the ballast, value in the range from 10 kN/m\(^2\) to 200 kN/m\(^2\),
- \( l \) - distance between the inflection points of the buckling curve, calculated according to the formula [9]:

\[ l = \sqrt{\frac{4\pi^2 EI}{\beta}} \]  \hspace{1cm} (14)

After substituting, the value of the maximum possible temperature difference \( \Delta T_0 \) was obtained for the given design and operation conditions. The equation determining the value of \( \Delta T_0 \) was derived as:

\[ \Delta T_0 = \frac{4\pi^2 l}{A\alpha l^2} + \frac{\beta l^2}{4A\alpha E\pi^2} \]  \hspace{1cm} (15)

The analysis was performed for several construction and operational variants. It was made with the use of MATLAB.

The program analyzes the dependence of the maximum allowable rail temperature change in relation to the neutral temperature for several variants. Due to the experimental determination of the value of the ballast resistance, the calculations were made for the values most frequently given in the literature, with different degrees of compaction and contamination of the ballast.

- Tor after dumping

For a contactless track along the working platform, available for the value of the transverse resistance of the ballast \( \beta = 80 \text{ kN/m}^2 \). So the value is low value, but only in the event that a densified DGS stabilizer is not provided. For such a low value, the critical force calculated from the use of the program is \( P_{kr} = 1429 \text{ kN} \), which will give the following value:

\[ \Delta T_0 = 38,3^\circ \text{C} \]
The obtained value is not fully sufficient to safely operate the track in polish climatic conditions. However, it should be remembered that even in the absence of the stabilizer passage, after a certain period of operation, the track becomes compacted and the ballast resistance increases, resulting in an increase in the value of $\Delta T_0$ [13].

- Track during exploitation, no pollution

After running with the dynamic stabilizer of the track (DGS), or after about two weeks of train traffic at a reduced speed, the track obtains the maximum degree of ballast compaction. Then also the lateral resistance of the ballast reaches the highest values, up to approx (150-160) kN/m$^2$. This value gradually decreases with increasing pollution and the movement of rolling stock. For the value of the lateral resistance of the ballast $\beta = 160$ kN/m$^2$, the calculated critical force is $P_{kr} = 2020$ kN, and the value of the maximum temperature change to:

$$\Delta T_0 = 54,8^\circ C$$

This value allows the track to be exploited without the risk of lateral displacement in polish climatic conditions.

- Track with contaminated ballast

After a longer period of operation, in the event that the trail has not been cleaned of gravel, the value of the transverse resistance $\beta$ systematically decreases, reaching values of even (30-40) kN/m$^2$. For such a contaminated track, the value of the critical force is equal to $P_{kr} = 875$ kN, which is associated with a decrease in the value:

$$\Delta T_0 = 23,4^\circ C$$

Low value carries a very high risk of track buckling, and thus the need for continuous monitoring of the track condition and limiting the speed of train movement. On such a polluted trail, revitalization works must be commissioned.
Figure 6. Dependence between maximum change of the rail temperature and the lateral resistance of the ballast

The relationship presented in the graph is described by a quadratic function. For the smallest values presented in the tests carried out to date (value $\beta = 10 \text{kN/m}^2$), $\Delta T_0$ reaches the value of 13.5°C, which excludes any possibility of track operation. The maximum values presented in the diagram are only theoretical, as the track machines used in construction and modernization do not allow to obtain $\beta$ values of 200 kN/m$^2$. For the line manager, values of interest may range from about 40 kN/m$^2$ in the case of poorly maintained tracks, to 160 kN/m$^2$ in the case of modernized tracks.

CONCLUSION

After performing the calculations with the use of MATLAB and the Finite Difference Method and obtaining the results and diagrams, the parameters influencing the longitudinal displacements were obtained. The first is the value of the longitudinal resistance of the ballast. Increasing this value causes a proportional decrease in the value of rail displacements. The charts presented in the article show how the value of rail creep changes with the decrease in longitudinal resistance below the range of 60 kN / m$^2$ - 80 kN / m$^2$. This perfectly shows the impact of the ballast bedding on the safe operation of the line. An important parameter is
also the length of the train whose maneuvers cause creeping. Here, however, the change of the parameter did not show such a drastic increase or decrease in longitudinal displacements.

In the case of lateral displacement, the critical forces in the rails and the maximum temperature changes of the rails were calculated. The relationship between the ballast resistance and the calculated values is visible. For contaminated crushed stone, the maximum temperature change is only 23.4°C, which indicates a high risk of railway track buckling. The highest value of the maximum temperature increase was calculated for the track after the boost and stabilization of DGS. It amounts to 54.8°C, which guarantees the safe use of the railway line despite changing climatic conditions.

It can be stated that the parameter determining the value of the longitudinal displacement is the ballast resistance ratio. For this reason, it is very important to control the state of contamination, drainage and quality of backfilling of railway sleepers.

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