Phenomenology of light Higgs bosons in supersymmetric left-right models

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Abstract

We carry out a detailed analysis of the light Higgs bosons in supersymmetric left-right models (SLRM). This includes models with minimal particle content and those with additional Higgs superfields. We also consider models with non-renormalizable higher-dimensional terms. We obtain an upper bound on the mass of the lightest $CP$-even neutral Higgs boson in these models. The upper bound depends only on the gauge couplings, and the vacuum expectation values of those neutral Higgs fields which control the spontaneous breakdown of the $SU(2)_L \times U(1)_Y$ gauge symmetry. We calculate the one-loop radiative corrections to this upper bound, and evaluate it numerically in the minimal version of the supersymmetric left-right model. We consider the couplings of this lightest $CP$-even Higgs boson to the fermions, and show that in a phenomenologically viable model the branching ratios are similar to the corresponding branching ratios in the minimal supersymmetric standard model (MSSM). We then study the most promising particle for distinguishing the SLRM from other models, namely the doubly charged Higgs boson. We obtain the mass of this doubly charged Higgs boson in different types of supersymmetric left-right models, and discuss its phenomenology.

Key words: Supersymmetry; left-right symmetry; Higgs boson; doubly charged Higgs boson

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\footnote{Permanent address}
1 Introduction

One of the central problems of particle physics is to understand how the electroweak scale associated with the mass of the W boson is generated, and why it is so small as compared to the Planck scale associated with the Newton’s constant. In the Standard Model (SM) the electroweak scale is generated through the vacuum expectation value (VEV) of the neutral component of a Higgs doublet [1]. Apart from the fact that this VEV is an arbitrary parameter in the SM, the mass parameter of the Higgs field suffers from quadratic divergences, making the weak scale unstable under radiative corrections. Supersymmetry is at present the only known framework in which the weak scale is stable under radiative corrections [2], although it does not explain how such a small scale arises in the first place. As such, considerable importance attaches to the study of supersymmetric models, especially the Minimal Supersymmetric Standard Model (MSSM), based on the gauge group $SU(2)_L \times U(1)_Y$, with two Higgs doublet superfields. It is well known that, because of underlying gauge invariance and supersymmetry (SUSY), the lightest Higgs boson in MSSM has a tree level upper bound of $M_Z$ (the mass of Z boson) on its mass [3]. Although radiative corrections [4] to the tree level result can be appreciable, these depend only logarithmically on the SUSY breaking scale, and are, therefore, under control. This results in an upper bound of about $125 - 135$ GeV on the one-loop radiatively corrected mass [5] of the lightest Higgs boson in MSSM[7]. Because of the presence of the additional trilinear Yukawa couplings, such a tight constraint on the mass of the lightest Higgs boson need not a priori hold in extensions of MSSM based on the gauge group $SU(2)_L \times U(1)_Y$ with an extended Higgs sector. Nevertheless, it has been shown that the upper bound on the lightest Higgs boson mass in these models depends only on the weak scale, and dimensionless coupling constants (and only logarithmically on the SUSY breaking scale), and is calculable if all the couplings remain perturbative below some scale $\Lambda$ [7–13]. This upper bound can vary between 150 GeV to 200 GeV depending on the Higgs structure of the underlying supersymmetric model. Thus, nonobservation of such a light Higgs boson below this upper bound will rule out an entire class of supersymmetric models based on the gauge group $SU(2)_L \times U(1)_Y$.

The existence of the upper bound on the lightest Higgs boson mass in MSSM (with arbitrary Higgs sectors) has been investigated in a situation where the underlying supersymmetric model respects baryon ($B$) and lepton ($L$) number conservation. However, it is well known that gauge invariance, supersymmetry and renormalizibility allow $B$ and $L$ violating terms in the superpotential of the MSSM [14]. The strength of these lepton and baryon number violating terms is, however, severely limited by phenomenological [15,16], and cosmological [17] constraints. Indeed, unless the strength of baryon-number violating term is less than $10^{-13}$, it will lead to contradiction with the present lower limits on the lifetime of the proton. The usual strategy to prevent the appearance of $B$ and $L$ violating

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2 The two-loop corrections to the Higgs boson mass matrix in the MSSM are significant, and can reduce the lightest Higgs boson mass by up to $\sim 20$ GeV as compared to its one-loop value [6].
couplings in MSSM is to invoke a discrete $Z_2$ symmetry [18] known as matter parity, or R-parity. The matter parity of each superfield may be defined as

$$\text{(matter parity)} \equiv (-1)^{3(B-L)}. \quad (1)$$

The multiplicative conservation of matter parity forbids all the renormalizable $B$ and $L$ violating terms in the superpotential of MSSM. Equivalently, the R-parity of any component field is defined by $R_p = (-1)^{3(B-L)+2S}$, where $S$ is the spin of the field. Since $(-1)^{2S}$ is conserved in any Lorentz-invariant interaction, matter parity conservation and R-parity conservation are equivalent. Conservation of R-parity then immediately implies that superpartners can be produced only in pairs, and that the lightest supersymmetric particle (LSP) is absolutely stable.

Although the Minimal Supersymmetric Standard Model with R-parity conservation can provide a description of nature which is consistent with all known observations, the assumption of $R_p$ conservation appears to be ad hoc, since it is not required for the internal consistency of MSSM. Furthermore, all global symmetries, discrete or continuous, could be violated by the Planck scale physics effects [19]. The problem becomes acute for low energy supersymmetric models, because $B$ and $L$ are no longer automatic symmetries of the Lagrangian, as they are in the Standard Model. It is, therefore, more appealing to have a supersymmetric theory where R-parity is related to a gauge symmetry, and its conservation is automatic because of the invariance of the underlying theory under this gauge symmetry. Fortunately, there is a compelling scenario which does provide for exact R-parity conservation due to a deeper principle. Indeed, $R_p$ conservation follows automatically in certain theories with gauged $(B-L)$, as is suggested by the fact that matter parity is simply a $Z_2$ subgroup of $(B-L)$. It has been noted by several authors [20,21] that if the gauge symmetry of MSSM is extended to $SU(2)_L \times U(1)_F \times U(1)_{B-L}$, or $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the theory becomes automatically R-parity conserving. Such a left-right supersymmetric theory (SLRM) solves the problems of explicit $B$ and $L$ violation of MSSM, and has received much attention recently [22–28]. Of course left-right symmetric theories are also interesting in their own right, for among other appealing features, they offer a simple and natural explanation for the smallness of neutrino mass through the so called see-saw mechanism [29].

Such a naturally R-parity conserving theory necessarily involves the extension of the Standard Model gauge group, and since the extended gauge symmetry has to be broken, it involves a “new scale”, the scale of left-right symmetry breaking, beyond the SUSY and $SU(2)_L \times U(1)_Y$ breaking scales of MSSM. In [30] we showed that in the SLRM with minimal particle content the upper bound on the mass of the lightest neutral Higgs boson depends only on gauge couplings and those VEVs which break the $SU(2)_L \times U(1)_Y$ symmetry. Here we will present a detailed analysis of the Higgs sector of the left-right supersymmetric models, and consider some of the distinguishing features of the lightest Higgs boson in these models. In the SLRMs there are typically also other light Higgs particles. The light doubly charged Higgs boson, which we will consider in detail as well, should provide a clear signal in experiments.
The plan of the paper is as follows. In section 2, we review the various left-right supersymmetric models that we consider in this paper. In section 3, we obtain the tree level upper bound on the mass of the lightest CP-even Higgs boson for the various SLRMs considered in section 2. We use a general procedure to obtain this (tree-level) upper bound on the mass of the lightest CP-even Higgs boson in models with extended gauge groups, such as SLRMs. We show that the upper bound so obtained in the renormalizable models is independent of the supersymmetry breaking scale, as well as the left-right breaking scale. In the case of models containing non-renormalizable terms, although the upper bound depends on the left-right breaking scale, the dependence is extremely weak, being suppressed by powers of Planck mass.

In section 4 we calculate the radiative corrections to the upper bound on the mass of the lightest Higgs boson, and show that the most important radiative corrections arising from quark-squark loops are of the same type as in the MSSM based on $SU(2)_L \times U(1)_Y$. The radiatively corrected upper bound so obtained is numerically considerably larger than the corresponding bound in the MSSM, but for most of the parameter space is below 200 GeV. In section 5 we consider the branching ratios of the lightest neutral Higgs boson, and find that its couplings are similar to the corresponding Higgs couplings in the Standard Model in the decoupling limit.

In section 6 we discuss the mass of the lightest doubly charged Higgs boson and the possibility of its detection at colliders. In section 7, we present our conclusions. The full scalar potential of the minimal supersymmetric left-right model is presented in the Appendix A.

2 The Higgs sector of the left-right supersymmetric models

In this section we briefly review the minimal supersymmetric left-right model, and then discuss models which have an extended Higgs sector, and finally discuss models with non-renormalizable interaction terms in the superpotential.

The minimal SLRM is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The matter fields of this model consist of the three families of quark and lepton chiral superfields with the following transformation properties under the gauge group:

$$Q = \begin{pmatrix} U \\ D \end{pmatrix} \sim (3, 2, 1, \frac{1}{3}), \quad Q^c = \begin{pmatrix} D^c \\ U^c \end{pmatrix} \sim (3^*, 1, 2, -\frac{1}{3})$$

$$L = \begin{pmatrix} \nu \\ E \end{pmatrix} \sim (1, 2, 1, -1), \quad L^c = \begin{pmatrix} E^c \\ \nu^c \end{pmatrix} \sim (1, 1, 2, 1),$$

(2)
where the numbers in the brackets denote the quantum numbers under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The Higgs sector consists of the bidoublet and triplet Higgs superfields:

\[
\Phi = \begin{pmatrix} \Phi_1^0 & \Phi_2^- \Phi_1^+ \\ \Phi_2^- & \Phi_2^- \Phi_2^+ \end{pmatrix} \sim (1, 2, 2, 0), \quad \chi = \begin{pmatrix} \chi_1^0 & \chi_1^+ \\ \chi_2^- & \chi_2^0 \end{pmatrix} \sim (1, 2, 2, 0),
\]

\[
\Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta_R^- & \Delta_R^0 \\ \Delta_R^0 & -\frac{1}{\sqrt{2}} \Delta_R^- \end{pmatrix} \sim (1, 1, 3, -2), \quad \delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}} \delta_R^+ & \delta_R^{++} \\ \delta_R^{++} & -\frac{1}{\sqrt{2}} \delta_R^+ \end{pmatrix} \sim (1, 1, 3, 2),
\]

\[
\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta_L^- & \Delta_L^0 \\ \Delta_L^0 & -\frac{1}{\sqrt{2}} \Delta_L^- \end{pmatrix} \sim (1, 3, 1, -2), \quad \delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \delta_L^+ & \delta_L^{++} \\ \delta_L^{++} & -\frac{1}{\sqrt{2}} \delta_L^+ \end{pmatrix} \sim (1, 3, 1, 2). \quad (3)
\]

There are two bidoublet superfields in order to implement the $SU(2)_L \times U(1)_Y$ breaking, and to generate a nontrivial Kobayashi-Maskawa matrix. Furthermore, two $SU(2)_R$ Higgs triplet superfields $\Delta_R$ and $\delta_R$ with opposite $(B - L)$ are necessary to break the left-right symmetry spontaneously, and to cancel triangle gauge anomalies due to the fermionic superpartners. The gauge symmetry is supplemented by a discrete left-right symmetry under which the fields can be chosen to transform as

\[
Q \leftrightarrow Q^c, \quad L \leftrightarrow L^c, \quad \Phi \leftrightarrow -\tau_2 \Phi^T \tau_2, \quad \chi \leftrightarrow -\tau_2 \chi^T \tau_2, \quad \Delta_R \leftrightarrow \delta_L, \quad \delta_R \leftrightarrow \Delta_L. \quad (4)
\]

Thus, the $SU(2)_L$ triplets $\Delta_L$ and $\delta_L$ are needed in order to make the Lagrangian fully symmetric under $L \leftrightarrow R$ transformation, although these are not needed phenomenologically for symmetry breaking, or the see-saw mechanism.

The most general gauge invariant superpotential involving these superfields can be written as (generation indices suppressed)

\[
W_{\text{min}} = h_{\phi \phi} Q^T i \tau_2 \Phi Q^c + h_{\chi \chi} Q^T i \tau_2 \chi Q^c + h_{\phi \phi} L^T i \tau_2 \Phi L^c + h_{\chi \chi} L^T i \tau_2 \chi L^c + h_{\delta \delta} L^T i \tau_2 \delta L^c + h_{\Delta \Delta} L^T i \tau_2 \Delta L^c + \mu_1 \text{Tr}(i \tau_2 \Phi^T i \tau_2 \chi) + \mu_1' \text{Tr}(i \tau_2 \Phi^T i \tau_2 \Phi) + \mu_2 \text{Tr}(i \tau_2 \chi^T i \tau_2 \chi) + \text{Tr}(\mu_2 L \Delta_L \delta_L + \mu_2 R \Delta_R \delta_R). \quad (5)
\]

The scalar potential can be calculated from

\[
V = V_F + V_D + V_{\text{soft}}, \quad (6)
\]

where $V_F$, $V_D$, and $V_{\text{soft}}$ represent the contribution of $F$-terms, the $D$-terms, and the soft SUSY breaking terms, respectively. The full scalar potential can be found in Appendix A. The general form of the vacuum expectation values of the various scalar fields which preserve the $U(1)_{\text{em}}$ gauge invariance can be written as
the electromagnetic gauge invariance. The superpotential for this class of models obtains only one parity-breaking minimum, in contrast to the minimal model, which respects the second stage 

\[ \Delta_L \sim \neutrino mass in the see-saw mechanism is given by

\[ R \]

renormalizable level without spontaneous breaking of 

SU(2)_L \times SU(2)_R \times U(1)_{B-L} is broken to an intermediate symmetry group SU(2)_L \times U(1)_R \times U(1)_{B-L}, and at the second stage U(1)_R \times U(1)_{B-L} is broken to U(1)_Y at a lower scale. In this theory there is only one parity-breaking minimum, in contrast to the minimal model, which respects the electromagnetic-breaking scale. We note that the triplet vacuum expectation values

\[ v \]

We note that the triplet vacuum expectation values \( v_{\Delta R} \) and \( v_{\Delta L} \) represent the scale of \( SU(2)_R \) breaking and are, according to the lower bounds [31] on heavy W- and Z-boson masses, in the range \( v_{\Delta R}, v_{\Delta L} \gtrsim 1 \text{ TeV} \). These represent a new scale, the right-handed breaking scale. We note that \( \kappa'_1 \) and \( \kappa'_2 \) contribute to the mixing of the charged gauge bosons and to the flavour changing neutral currents, and are usually assumed to vanish. Furthermore, since the electroweak \( \rho \) parameter is close to unity, \( \rho = 0.9998 \pm 0.0008 \) [31], the triplet vacuum expectation values \( v_{\Delta L} \) and \( v_{\Delta L} \) must be small.

The Yukawa coupling \( h_{\chi L} \) is proportional to the neutrino Dirac mass \( m_D \). The light neutrino mass in the see-saw mechanism is given by \( \sim m_D^2/M, \) where \( m_M = h_{\Delta R} v_{\Delta R} \) is the Majorana mass. The magnitude of \( h_{\chi L} \) is not accurately determined given the present upper limit on the light neutrino masses. On the other hand the Yukawa coupling \( h_{\Phi L} \) is proportional to the electron mass and is, thus, small.

In the minimal model described above, parity cannot be spontaneously broken at the renormalizable level without spontaneous breaking of \( R \)-parity. This may be cured by adding more fields to the theory. In [24,32] it was suggested that a parity-odd singlet, coupled appropriately to triplet fields, be introduced so as to ensure proper symmetry breaking. This leads to a set of degenerate minima connected by a flat direction, all of them breaking parity. When soft SUSY breaking terms are switched on, the degeneracy is lifted, but the global minimum that results breaks \( U(1)_{em} \). Because of the flat direction connecting the minima, there is no hope that the fields remain in the phenomenologically acceptable vacuum, which rolls down to global minimum after SUSY is softly broken. The only option that is left is to have a relatively low \( SU(2)_R \) breaking scale, with spontaneously broken \( R \)-parity (\( \langle \nu^c \rangle \equiv \sigma_R \) is non-zero). We note that present experiments allow for a low \( SU(2)_R \) breaking scale.

There is an alternative to the minimal left-right supersymmetric model which involves the addition of a couple of triplet fields, \( \Omega_L(1,3,1,0) \) and \( \Omega_R(1,1,3,0) \), instead of a singlet Higgs superfield, to the minimal model [28]. In these extended models the breaking of \( SU(2)_R \) is achieved in two stages. In the first stage the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) is broken to an intermediate symmetry group \( SU(2)_L \times U(1)_R \times U(1)_{B-L} \), and at the second stage \( U(1)_R \times U(1)_{B-L} \) is broken to \( U(1)_Y \) at a lower scale. In this theory there is only one parity-breaking minimum, in contrast to the minimal model, which respects the electromagnetic-breaking gauge invariance. The superpotential for this class of models obtains additional terms involving the triplet fields \( \Omega_L \) and \( \Omega_R \):
such as Planck mass, $M_{\text{Planck}} \sim 10^{19}$ GeV, with the minimal field content ensures the correct pattern of symmetry breaking in the supersymmetric left-right model. In particular the scale of parity breakdown is predicted to be in the intermediate region $M_R \gtrsim 10^{10} - 10^{11}$ GeV, and $R$-parity remains exact. This theory contains singly charged and doubly charged Higgs scalars with a mass of order $M_R^2/M_{\text{Planck}}$, which may be experimentally accessible. However, what is different is the nature of see-saw mechanism. Whereas in the renormalizable version the see-saw mechanism takes its canonical form, in the non-renormalizable case it takes a form similar to what occurs in the non-supersymmetric left-right models, with the neutrino mass depending on the unknown parameters of the Higgs potential. This in general leads to different neutrino mass spectra, which can be experimentally distinguished.
3 The tree-level upper bound on the lightest Higgs mass

Given the fact that the Higgs sector of SLRM models contain a large number of Higgs multiplets, and the VEVs of some of the Higgs fields involve possibly large mass scales compared to the electroweak and SUSY breaking scales, it is important to ask what is the mass of the lightest Higgs boson in these models. The upper bound on the lightest Higgs boson mass in the minimal model was derived in [30] in the limit when $\kappa'_1, \kappa'_2, \sigma_L, v_{\Delta L}, v_{\delta L} \to 0$, using the fact that for any Hermitean matrix the smallest eigenvalue must be smaller than that of its upper left corner $2 \times 2$ submatrix. In the basis in which the first two indices correspond to $(\Phi_0^1, \chi_0^2)$, we find for matrix elements $m_{11}^2, m_{22}^2, m_{12}^2$ (see Appendix A)

$$m_{11}^2 = -m_{\Phi \chi}^2 \frac{\kappa_2}{\kappa_1} + \frac{1}{2}(g_L^2 + g_R^2)\kappa_1^2,$$

$$m_{22}^2 = -m_{\Phi \chi}^2 \frac{\kappa_1}{\kappa_2} + \frac{1}{2}(g_L^2 + g_R^2)\kappa_2^2,$$

$$m_{12}^2 = m_{\Phi \chi}^2 - \frac{1}{2}(g_L^2 + g_R^2)\kappa_1\kappa_2. \quad (10)$$

It follows that the upper bound on the lightest Higgs boson mass in the minimal supersymmetric left-right model can be written as [30]:

$$m_h^2 \leq \frac{1}{2}(g_L^2 + g_R^2)(\kappa_1^2 + \kappa_2^2)\cos^2 2\beta = \left(1 + \frac{g_R^2}{g_L^2}\right)m_{W_L}^2\cos^2 2\beta, \quad (11)$$

where $\tan \beta = \kappa_2/\kappa_1$. The upper bound (11) is not only independent of the supersymmetry breaking parameters (as in the case of supersymmetric models based on $SU(2)_L \times U(1)_Y$), but it is also independent of the $SU(2)_R$ breaking scale, which, a priori, can be large. The upper bound is controlled by the weak scale vacuum expectation value, $\kappa_1^2 + \kappa_2^2$, and the dimensionless gauge couplings ($g_L$ and $g_R$) only. Since the former is essentially fixed by the electroweak scale, the gauge couplings $g_L$ and $g_R$ determine the bound.

We will see below that even when $\kappa'_1, \kappa'_2, \sigma_L, v_{\Delta L}, v_{\delta L}$ are non-zero, the upper bound on the lightest Higgs mass at the tree level does not depend on either the right-handed breaking scale or the SUSY breaking scale. A general method to find an upper limit for the lightest Higgs mass in models based on $SU(2)_L \times U(1)_Y$ and maximally quartic potentials was presented in [35]. We will apply this method to the case of SLRMs, with possible nonrenormalizable terms, in an appropriate manner.

Consider a set of scalar fields $\Phi_j$ transforming under $SU(2)_L$, and define a discrete transformation $P : \Phi_j \to (-1)^{2T_j}\Phi_j$ of these fields, where $2T_j + 1$ is the dimension of $SU(2)_L$ representation$^{|3}$. By setting the $P$-even fields to their VEVs, a normalized field can be

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$^3$ One could as well choose any other spontaneously broken $U(1)$ [35], but $P$ provides the best limit in the case of SLRM.
defined in the direction of $SU(2)_L$ breaking, $\phi = \frac{1}{v_0} \sum_{\text{odd}} v_i \Phi_i$, where $v_0^2 = \sum_i v_i^2$, and all the fields orthogonal to $\phi$ have zero VEVs. Since the original lagrangian is invariant under the transformation $P$, the potential must be invariant under $P$, so that the potential contains only even powers of $\phi$,

$$V(\phi) = V(0) - \frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4 + \frac{1}{6}A\phi^6 + \ldots$$  \hspace{1cm} (12)

We take into account only the leading nonrenormalizable terms and thus $\phi^6$ is the largest power in the potential $V(\phi)$. If $\phi$ were the mass eigenstate, then by using the minimization condition the Higgs mass would be $\lambda\phi v^2_0 + 4Av_0^4$. In the general case, this expression provides an upper bound on the mass of the lightest Higgs boson,

$$m_h^2 \leq \lambda\phi v^2_0 + 4Av_0^4.$$  \hspace{1cm} (13)

Since only the $SU(2)_L$ doublet fields are relevant for obtaining this bound, adding extra singlets or triplets in the model has no effect on the bound. Thus, there are three separate cases to be considered for deriving an upper bound on the mass of the lightest Higgs boson in the models discussed in the last section, namely: (A) $R$-parity is broken (sneutrinos get VEVs), (B) $R$-parity is conserved because there are additional triplets, and (C) $R$-parity is conserved because there are nonrenormalizable terms.

Let us first consider the situation in the minimal model, case (A). We define a new neutral scalar field in the direction of the breaking in the $SU(2)_L$ doublet space:

$$\Phi^0 = \frac{1}{v} \left( \kappa_1 \text{Re} \Phi_1^0 + \kappa_1' \text{Re} \Phi_2^0 + \kappa_2 \text{Re} \chi_1^0 + \kappa_2' \text{Re} \chi_2^0 + \sigma \text{Re} \tilde{\nu} \right),$$  \hspace{1cm} (14)

where

$$v^2 = \kappa_1^2 + \kappa_1'^2 + \kappa_2^2 + \kappa_2'^2 + \sigma_L^2.$$  \hspace{1cm} (15)

All the $SU(2)_L$ doublet fields which are orthogonal to $\Phi^0$ have vanishing VEVs. We calculate next the quartic term $(\Phi^0)^4$ in the potential. It has contributions from both the F-terms and the D-terms, and we find that the upper bound on the mass of the lightest Higgs boson in the minimal SLRM is

$$m_h^2 \leq \frac{1}{2v^2} \left[ g_L^2 (\omega^2_\kappa + \sigma_L^2)^2 + g_R^2 \omega^4_\kappa + g_{B-L}^2 \sigma_L^4 + 8(h_{\Phi L} \kappa_1' + h_{\chi L} \kappa_2')^2 \sigma_L^2 + 8h_{\Delta L} \sigma_L^4 \right],$$  \hspace{1cm} (16)

where

$$\omega^2_\kappa = \kappa_1^2 - \kappa_2^2 - \kappa_1'^2 + \kappa_2'^2.$$  \hspace{1cm} (17)
As is evident, the upper bound (16) is independent of SUSY and right-handed breaking scales, and depends only on the dimensionless gauge and Yukawa couplings, and vacuum expectation values which are determined by the weak scale:

\[ m_{W_L}^2 = \frac{1}{2} g_L^2 \left( \kappa_1^2 + \kappa_2^2 + \kappa_1' \right) + 2 v_{\Delta L}^2 + 2 v_{\delta L}^2 + O \left( \frac{\kappa_1' m_{W_L}^2}{m_{W_R}^2} \right). \]  

(18)

The triplet VEVs \( v_{\Delta L} \) and \( v_{\delta L} \) must be small in order to maintain \( \rho \simeq 1 \). In the limit when \( \kappa_1', \kappa_2', \sigma_L, v_{\Delta L}, v_{\delta L} \to 0 \), the bound (16) reduces to the upper bound (11).

It is obvious that the addition of extra triplets does not change this bound. Thus, the bound for the case (B), the SLRM with additional triplets to ensure that \( R \)-parity is not spontaneously broken, can be obtained from (16) by taking the limit \( \sigma_L \to 0 \).

The total number of nonrenormalizable terms in case (C) is rather large. All the coefficients in nonrenormalizable terms are proportional to inverse powers of a large scale. Thus the largest contribution comes from those terms which have the smallest number of large scales and the largest number of potentially large VEVs from \( SU(2)_L \) triplets. We recall that the terms of the form \( (\Phi_0^0)^4 \) and \( (\Phi_0^0)^6 \) in the potential are needed to determine the contributions to the mass bound. The nonrenormalizable terms have no effect on the contribution from \( D \)-terms. The leading terms in the superpotential which can give a \( (\Phi_0^0)^4 \) and \( (\Phi_0^0)^6 \) type F-term contribution are of the type

\[ W_{NR} = A \text{Tr}(i \tau_2 \Phi^T i \tau_2 \chi) \text{Tr}(\Delta_R \delta_R) + B \text{Tr}(i \tau_2 \Phi^T i \tau_2 \chi)^2, \]  

(19)

i.e. one term with two and another with four bidoublet fields. Here \( A, B \sim 1/M_{\text{Planck}} \).

With \( SU(2)_L \) singlets fixed to their VEVs, the corresponding \( F \)-terms are

\[ V_{NR} = O(AB v_{\Delta R} v_{\delta R})(\Phi_0^0)^4 + O(B^2)(\Phi_0^0)^6. \]  

(20)

The contribution to the Higgs mass bound from these nonrenormalizable terms is

\[ O(v_R^2/M_{\text{Planck}}^2)(\Phi_0^0)^2 + O(1/M_{\text{Planck}}^2)(\Phi_0^0)^4. \]  

(21)

If the VEV \( v_R \sim 10^{10} \) GeV in these models, the contribution is numerically negligible. Therefore the upper bound for this class of models is essentially the same as in the case (B).

4 Radiative corrections

Since it is known that the radiative corrections to the lightest Higgs mass are significant in the MSSM, as well as its extensions based on the \( SU(2)_L \times U(1)_Y \), it is important
to consider the radiative corrections to the upper bound on the lightest Higgs boson mass obtained in the previous section. In this section we discuss the one-loop radiative corrections to the upper bound on the lightest Higgs mass in the minimal supersymmetric left-right model, which was obtained in last section. We shall use the method of one-loop effective potential [36] for the calculation of radiative corrections, where the effective potential may be expressed as the sum of the tree-level potential plus a correction coming from the sum of one-loop diagrams with external lines having zero momenta,

\[ V_{\text{1-loop}} = V_{\text{tree}} + \Delta V_1, \]  

where \( V_{\text{tree}} \) is the tree level potential (6) evaluated at the appropriate running scale \( Q \), and \( \Delta V_1 \) is the one loop correction given by

\[ \Delta V_1 = \frac{1}{64\pi^2} \sum_i (-1)^{2J_i}(2J_i + 1)m_i^4 \left( \ln \frac{m_i^2}{Q^2} - \frac{3}{2} \right), \]  

where \( m_i \) is the field dependent mass eigenvalue of the \( i \)th particle of spin \( J_i \). The dominant contribution to (23) comes from top-stop (\( t - \tilde{t} \)) system. However, under certain conditions the contribution of bottom-sbottom (\( b - \tilde{b} \)) can be nonnegligible. We shall include both these contributions in our calculations of the radiative corrections.

In order to evaluate the contributions of top-stop and bottom-sbottom to (23), we need the stop and sbottom mass matrices for the SLRM. From (A.1), (A.2) and (A.3), it is straightforward to calculate squark mass matrices [27]. Ignoring the interfamily mixing, the part of the potential containing the stop and sbottom mass terms can be written as

\[ V_{\text{squark}} = \left( U_L^* U_R^* \right) \tilde{M}_U \left( U_L U_R \right) + \left( D_L^* D_R^* \right) \tilde{M}_D \left( D_L D_R \right), \]  

where the mass matrix elements for the stop are

\[
(\tilde{M}_U)_{UU} = \tilde{m}_Q^2 + m_u^2 + \frac{1}{4} g_L^2 \omega_\kappa^2 - \frac{1}{6} g_B^2 \omega_L^2, \\
(\tilde{M}_U)_{UR} = h_{\phi Q} A_{\phi Q} \kappa_1' + h_{\chi Q} A_{\chi Q} \kappa_2 - \mu_1 (h_{\phi Q} \kappa_2' + h_{\chi Q} \kappa_1) - 2 h_{\phi Q} \mu_1 \kappa_1 - 2 h_{\chi Q} \mu_1' \kappa_2' + (h_{\phi L} h_{\phi Q} + h_{\chi L} h_{\chi Q}) \sigma_L \sigma_R, \\
= \left[ (\tilde{M}_U)_{UU} \right]^*, \\
(\tilde{M}_U)_{RU} = m_{\tilde{Q}_c}^2 + m_u^2 + \frac{1}{4} g_R^2 (\omega_\kappa^2 - 2 \omega_R^2) + \frac{1}{6} g_B^2 \omega_R^2, \]  

while for the sbottom these are
where top and sbottom squared masses are given by \((h_Q Q_{2})^2\) and \((h_{Q Q_{1}})^2\), respectively, \(m_{Q}^2, m_{Q_{c}}, A_{Q}, A_{Q_{c}}\) and \(A_{\chi Q}\) are soft supersymmetry breaking parameters (see eq. (A.3)), and

\[
\omega_R^2 = v_{\Delta R}^2 - v_{\delta R}^2 - \frac{1}{2} \sigma_R^2, \quad \omega_R^2 = \kappa_1^2 + \kappa_2^2 - \kappa_2^2 - \kappa_1^2.
\] (27)

In order that \(SU(3)_C \times U(1)_em\) is unbroken, none of the physical squared masses of squarks can be negative. Necessarily then all the diagonal elements of the squark mass matrices should be non-negative. Combining the diagonal elements of the stop and sbottom mass matrices leads to the inequality

\[
m_{Q}^2 + m_{Q_{c}}^2 \geq \frac{1}{2} g_R^2 \omega_R^2 = \frac{1}{2} g_R^2 (v_{\Delta R}^2 - v_{\delta R}^2 - \frac{1}{2} \sigma_R^2).
\] (28)

where we have ignored terms which are of the order of the weak scale or less.

The eigenvalues \(m_{i_1,2}^2, m_{b_1,2}^2\) of the stop and sbottom mass squared matrices are given by \((m_{i_1}^2 > m_{i_2}^2, m_{b_1}^2 > m_{b_2}^2)\)

\[
m_{i_1,2}^2 = m_i^2 \pm \Delta_i^2,
\]

\[
m_{b_1,2}^2 = m_b^2 \pm \Delta_b^2,
\] (29)

where

\[
m_i^2 = \frac{1}{2} \left[ m_Q^2 + m_{Q_{c}}^2 + 2m_i^2 + \frac{1}{4} (g_L^2 + g_R^2) \omega_R^2 - \frac{1}{2} g_R^2 \omega_R^2 \right],
\]

\[
\Delta_i^2 = \frac{1}{2} \left[ \left[ m_Q^2 - m_{Q_{c}}^2 + \frac{1}{4} (g_L^2 - g_R^2) \omega_R^2 + \frac{1}{2} g_R^2 \omega_R^2 - \frac{1}{3} g_{B-L}^2 \omega_R^2 \right]^2
\]

\[
+ 4 \left[ h_{Q Q} \mu_{Q_{k} \kappa_2} - h_{Q Q_{1}} \mu_{Q_{k} \kappa_{1}} \right]^2 \right]^\frac{1}{2},
\] (30)

and

\[
m_b^2 = \frac{1}{2} \left[ m_Q^2 + m_{Q_{c}}^2 + 2m_b^2 - \frac{1}{4} (g_L^2 + g_R^2) \omega_R^2 + \frac{1}{2} g_R^2 \omega_R^2 \right],
\]

\[
\Delta_b^2 = \frac{1}{2} \left[ \left[ m_Q^2 - m_{Q_{c}}^2 + \frac{1}{4} (g_L^2 - g_R^2) \omega_R^2 + \frac{1}{2} g_R^2 \omega_R^2 - \frac{1}{3} g_{B-L}^2 \omega_R^2 \right]^2
\]

\[
+ 4 \left[ h_{Q Q} \mu_{Q_{k} \kappa_2} - h_{Q Q_{1}} \mu_{Q_{k} \kappa_{1}} \right]^2 \right]^\frac{1}{2},
\] (31)
Here we have defined

\[ \Delta \equiv \frac{1}{2} \left\{ \left[m_Q^2 - m_{Q'}^2 - \frac{1}{4}(g_L^2 - g_R^2)\omega_\kappa^2 - \frac{1}{2}g_R^2\omega_R^2 - \frac{1}{3}g_{B-L}^2\omega_R^2 \right]^2 \right. 
+ 4 [h_{\Phi Q} A_\delta \kappa_1 - h_{\Phi Q} \bar{\mu}_b \kappa_2]^2 \left\}^{\frac{1}{2}}. \] 

(31)

Using eqs. (29), (30) and (31) in (23), we have calculated the radiatively-corrected expressions for the matrix elements of the upper left corner 2 × 2 submatrix of the 10 × 10 CP-even Higgs mass matrix. After imposing the appropriate one-loop minimization conditions, we find the following form for the radiatively corrected upper left corner 2 × 2 submatrix of CP-even Higgs mass matrix:

\[
\begin{pmatrix}
\frac{1}{2}(g_L^2 + g_R^2)\kappa_1^2 & -\frac{1}{2}(g_L^2 + g_R^2)\kappa_1\kappa_2 \\
-\frac{1}{2}(g_L^2 + g_R^2)\kappa_1\kappa_2 & \frac{1}{2}(g_L^2 + g_R^2)\kappa_2^2
\end{pmatrix} + \begin{pmatrix}
\tan\beta & -1 \\
-1 & \cot\beta
\end{pmatrix} (\Delta \begin{pmatrix} \Delta_1 & \Delta_2 \\ \Delta_2 & \Delta_2 \end{pmatrix}),
\]

where

\[
\Delta = \left[ -2m_{\Phi X}^2 + \frac{3}{32\pi^2} \left( \frac{g_L^2}{\sin^2\beta m_W^2} \right) A_t \bar{\mu}_t \frac{f(m_{t_1}^2) - f(m_{t_2}^2)}{m_{t_1}^2 - m_{t_2}^2} \\
+ \frac{3}{32\pi^2} \left( \frac{g_L^2}{\cos^2\beta m_W^2} \right) A_b \bar{\mu}_b \frac{f(m_{b_1}^2) - f(m_{b_2}^2)}{m_{b_1}^2 - m_{b_2}^2} \right],
\]

(34)

\[ f(x^2) = 2x^2(\ln(x^2/Q^2) - 1), \]

(35)

and

\[
\Delta_{11} = \frac{m_b^4}{\cos^2\beta} \left[ \ln \left( \frac{m_{b_1}^2 m_{b_2}^2}{m_b^4} \right) + \frac{2A_b (A_b - \bar{\mu}_b \tan\beta) \ln \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right)}{m_{b_1}^2 - m_{b_2}^2} \\
+ \frac{m_b^4 A_b^2 (A_b - \bar{\mu}_b \tan\beta)^2 g(m_{b_1}^2, m_{b_2}^2) + m_b^4 (A_t - \bar{\mu}_t \cot\beta)^2 \bar{\mu}_b^2 g(m_{t_1}^2, m_{t_2}^2)}{\sin^2\beta (m_{t_1}^2 - m_{t_2}^2)^2} \right],
\]

(36)
\[ \Delta_{22} = \frac{m_t^4}{\sin^2 \beta} \ln \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) + \frac{2A_t}{m_{t_1}^2 - m_{t_2}^2} \ln \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) \]
\[ + \frac{m_t^4}{\sin^2 \beta} \frac{A_t^2}{m_{t_1}^2 - m_{t_2}^2} \cos \beta \ln \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) + \frac{m_b^4}{\cos^2 \beta} \frac{A_b - \mu_b \tan \beta}{m_{b_1}^2 - m_{b_2}^2} \frac{\mu_b^2}{m_{b_1}^2 - m_{b_2}^2} \ln \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right), \]
\[ (37) \]

\[ \Delta_{12} = \frac{m_t^4}{\sin^2 \beta} \frac{A_t - \mu_t \cot \beta}{m_{t_1}^2 - m_{t_2}^2} \times \left[ \ln \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) + \frac{A_t}{m_{t_1}^2 - m_{t_2}^2} g(m_{t_1}^2, m_{t_2}^2) \right] \]
\[ + \frac{m_b^4}{\cos^2 \beta} \frac{A_b - \mu_b \tan \beta}{m_{b_1}^2 - m_{b_2}^2} \times \left[ \ln \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right) + \frac{A_b}{m_{b_1}^2 - m_{b_2}^2} g(m_{b_1}^2, m_{b_2}^2) \right], \]
\[ (38) \]

with
\[ g(m_{1}^{2}, m_{2}^{2}) = 2 - \frac{m_{1}^{2} + m_{2}^{2}}{m_{1}^{2} - m_{2}^{2}} \ln \left( \frac{m_{1}^{2}}{m_{2}^{2}} \right). \]
\[ (39) \]

We have neglected D-terms in the squark masses, because these are small, and, since we are including only the quark-squark contributions to \( \Delta V_1 \), in order to gain approximate independence of the renormalization scale \( Q \) (see also the inequality \( (28) \)).

Using eqs. \( (33) \) and \( (34) \), we obtain the one-loop radiatively corrected upper bound on the lightest Higgs boson mass in the SLRM:

\[ m_h^2 \leq \frac{1}{2} \left[ (g_L^2 + g_R^2) \left( \kappa_1^2 + \kappa_2^2 \right) \cos^2 \beta \right. \]
\[ + \frac{3g_L^2 m_{W_L}^2}{8\pi^2m_{W_L}^2} \left. \left( \Delta_{11} \cos^2 \beta + \Delta_{22} \sin^2 \beta + \Delta_{12} \sin 2\beta \right) \right] \]
\[ (40) \]

For \( \tan \beta \lesssim 20 \), one can neglect the \( b \)-quark contribution in the radiative corrections. Then, in the approximation \( [37,38] \)
\[ |m_{t_1}^2 - m_{t_2}^2| \ll |m_{t_1}^2 + m_{t_2}^2|, \]
\[ (41) \]

the upper bound \( (40) \) on the lightest Higgs mass reduces to

\[ m_h^2 \leq \frac{1}{2} \left[ (g_L^2 + g_R^2) \left( \kappa_1^2 + \kappa_2^2 \right) \cos^2 \beta \right. \]
\[ + \frac{3g_L^2 m_{W_L}^2}{8\pi^2m_{W_L}^2} \left. \left( \ln \left( \frac{m_{t_1}^2 m_{t_2}^2}{m_t^4} \right) + 2 \frac{A_t^2}{M_s^2} \left( 1 - \frac{\tilde{A}_t^2}{12M_s^2} \right) - \frac{\mu^4}{3M_s^4} \right) \right] \]
\[ (42) \]
where $\tilde{A}_t = A_t - \mu_1 \cot \beta$, and $M_s$ is the supersymmetry breaking scale ($2M_s^2 = m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2$).

In this limit the upper bound eq. (42) on the lightest Higgs mass in the supersymmetric left-right model differs in form from the corresponding MSSM upper bound only because of $\mu''_1$ being nonzero. The upper bound is maximised by

$$|\tilde{A}_t| = (\sqrt{6})M_s$$

(43)

for a given value of $\mu''_1$.

The radiatively corrected upper bound (40) on the mass of the lightest Higgs boson is plotted in Fig.1 as a function of the large scale $\Lambda$ up to which the supersymmetric left-right model remains perturbative. The upper bound comes from the requirement that all the gauge couplings of the SLRM remain perturbative below the scale $\Lambda$. We have taken into account the dominant radiative corrections coming from the quark and squark loops in our calculations. In Fig.1 we have taken two values of $\tan \beta = 2$ and $\tan \beta = 20$. In the figure the upper bound is shown for two different values of the $SU(2)_R$ breaking scale, $M_R = 10$ TeV and $M_R = 10^{10}$ GeV, respectively, and for two values of soft supersymmetry breaking mass parameter, $M_s = 1$ TeV and $M_s = 10$ TeV. It is seen from this figure that if the difference between the $SU(2)_R$ breaking scale and the large scale $\Lambda$ is more than two orders of magnitude, the radiatively corrected upper bound on the mass of the lightest Higgs boson remains below 250 GeV. For large values of $\Lambda$ the upper bound is below 200 GeV. The upper bound increases with increasing $M_R$ and with increasing soft supersymmetry breaking parameters. It is considerably larger than the corresponding upper bound in the MSSM.
Fig. 2. The branching ratio of the lightest neutral CP even Higgs boson $h$ to fermion pairs in SLRM (solid curves) as a function of the second lightest Higgs $H$ mass. The tree level mass of the lightest Higgs is $\sim 78$ GeV and $\tan \beta = 50$. The dotted curves correspond to MSSM.

5 Couplings of the lightest neutral Higgs to fermions and the decoupling limit

In order to study the phenomenology of the lightest Higgs boson in the SLRM, we must obtain its couplings to the fermions and see how these differ from the corresponding MSSM couplings. The major difference between the two models arises due to the triplet Higgs couplings to leptons. As discussed earlier, the lightest neutral Higgs is composed mainly of the bidoublet fields, and we do not expect the triplet couplings to have a large effect on the lightest neutral Higgs branching ratios. Another difference in the Higgs couplings in the SLRM and the MSSM arises because of the mixing between the Higgs (ino) and lepton sectors in models where $R$-parity is spontaneously broken. Since the lightest chargino contains gaugino and higgsino admixture, one might expect that in some region of the parameter space the couplings are significantly different from the couplings in the MSSM. We will take the lightest chargino to correspond to the $\tau$-lepton. It will be denoted by $\tau$ in the following. Thus, it is important to study in detail the couplings of the lightest Higgs boson to the $\tau$.

In Figure 2 we have plotted the branching ratio of the lightest neutral Higgs boson to $b\bar{b}$, $c\bar{c}$, and $\tau^+\tau^-$ pair both in the MSSM and in the SLRM for large $\tan \beta$ and a tree-level mass $m_h \sim 78$ GeV as a function of the second lightest neutral Higgs boson mass. We note that for the second lightest neutral Higgs with mass $\sim 160$ GeV, the branching ratio to bottom pair almost vanishes while the branching ratio to a $\tau$ pair is enhanced. This behaviour occurs in a transition region where the composition of the light Higgs bosons change rapidly, and the lightest Higgs coupling to $b\bar{b}$ becomes negligible. Similar
phenomenon occurs for small $\tan \beta$.

It is well known that in the left-right symmetric models problems with FCNC are expected if several light Higgs bosons exist [39]. In the supersymmetric case, the problem could be expected to be more severe, since the potential is more constrained. Furthermore, in the four Higgs-doublet model [40], which is also the number of doublets in the supersymmetric left-right model, it was found that the potential is always real, thus preventing spontaneous violation of CP. Strong limits on the FCNC Higgs were obtained in [41] assuming that the only source for CP violation is the complex phase in the Kobayashi-Maskawa matrix and that the model is manifestly left-right symmetric. If these conditions are relaxed, for example by defining the symmetry transformation by Eq. (4), a weakened lower limit for second lightest neutral Higgs is obtained from the neutral meson mass difference. The limit is less severe for large values of $\tan \beta$, but remains of the same order than in the nonsupersymmetric left-right model. Taking into account the uncertainties in calculating the mass difference, $m_{H_{\text{FCNC}}} \gtrsim O(1 \, \text{TeV})$. Thus the relevant limit to discuss is the one in which all the neutral Higgs bosons, except the lightest one, are heavy.

In order to consider in detail the neutral Higgs couplings to the $\tau$-leptons, we'll need the interaction Lagrangian. We have seen that at the tree level the lightest Higgs can be written as

$$h = \frac{1}{v} \sum_k \left( 1 + \mathcal{O} \left( \frac{m_h^2}{m_{H_2}^2} \right) \right) \langle \phi_k \rangle \phi_k + \sum_k \mathcal{O} \left( \frac{m_h}{m_{H_2}} \right) \psi_k,$$

(44)

where $\phi_k$ are the scalar bidoublet fields and $\psi_k$ are all the other fields. In the decoupling limit $m_{H_2} \gg m_h$, the relevant Higgs-fermion interaction Lagrangian can be written as

$$L = -\frac{1}{v} \psi_i^+ v K_{ijk} \psi_j^- h + h.c.,$$

(45)

where $\psi^T = (-i \lambda^L, -i \lambda^R, \phi_2^-, \Delta^-, \tau^T)$ and $\psi'^+ T = (-i \lambda^L, -i \lambda^R, \phi_1^+, \chi^+ N, \delta^R, \tau^T)$. The mass eigenstates $\chi^\pm$ are found by diagonalizing the mass matrices by unitary matrices $U, V$ [2],

$$\chi^+ = V \psi^+, \quad \chi^- = U \psi^-.$$

(46)

The diagonal chargino mass matrix is

$$M_{\chi^{\pm}}^2 = VX^\dagger VX^\dagger = U^* X X^\dagger U^T.$$

(47)

We decompose the mass matrix $X$ and the diagonalizing matrices $V$ and $U$ as

4 When R-parity is broken, the ground state can violate CP via the soft breaking terms involving sneutrinos [42].
5 For a more general treatment, see [43].
\[ X = X_0 + \epsilon X_1, \quad V = (1 + \epsilon Y)V_0, \quad U = (1 + \epsilon Z)U_0, \]  
\[ (48) \]

where the contribution of the terms proportional to \( \epsilon \) in determining the \( \tau \) lepton eigenstate is small. One can solve for the matrix elements to the order \( O(\epsilon^2) \),

\[ Y_{jk} = \begin{cases} 
0, & j = k, \\
\frac{[V_0(X_0^+X_1+X_1X_0)V_0^+)_k}{(M_0^2)_{jj} - (M_0^2)_{kk}}, & j \neq k
\end{cases} \]  
\[ (49) \]

\[ Z_{jk} = \begin{cases} 
0, & j = k, \\
\frac{[U_0^*(X_0X_1^+X_1X_0^+U_0^T)_k}{(M_0^2)_{jj} - (M_0^2)_{kk}}, & j \neq k
\end{cases} \]  
\[ (50) \]

where we have denoted \( M_0^2 \equiv U_0^*X_0X_0^+U_0^T \).

In terms of the mass eigenstates, the interaction Lagrangian can be written as

\[ L_{\text{int}} = -\frac{1}{v}(\chi^- U^*)_i v_k y_{ijk} (V^+ \chi^+)_j h + h.c. \]  
\[ (51) \]

Here we denote \( C \equiv \frac{1}{v} U^* v_k y_{ijk} V^+ \), where \( C \) is the coupling matrix for charginos. Thus the diagonal couplings to \( \tau \) are given by

\[ C_{11} \sim \frac{1}{v} \sum_{jk} U_{01j} X_{jk} V_{01k}. \]  
\[ (52) \]

The chargino mass matrix can be written as \( X = X_0 + \epsilon X_1 \), where

\[ X_0 = \begin{pmatrix}
 m_L & 0 & 0 & 0 & 0 & 0 \\
 0 & m_R & 0 & 0 & \sqrt{2}g_R v_R & g_R \sigma_R \\
 0 & 0 & 0 & \mu_1 & 0 & 0 \\
 0 & 0 & \mu_1 & 0 & 0 & 0 \\
 0 & -\sqrt{2}g_R v_{\Delta R} & 0 & 0 & \mu_2 & -\sqrt{2}h_\Delta \sigma_R \\
 0 & 0 & -h_{\phi L} \sigma_R & -h_{\chi L} \sigma_R & 0 & 0 \\
\end{pmatrix} \]  
\[ (53) \]

and
We have here assumed for simplicity that $\mu_1' = \mu_1'' = 0$, and $\sigma_L = v_{\delta_L} = v_{\Delta_L} = 0$. The matrix $X_0$ has one zero eigenvalue corresponding to the $\tau$ mass, $(M_0)^2_{11} = 0$. The $\tau$ mass is

$$m_\tau = (U^*_0XV^\dagger)_11 = (Z^*_1U^*_0X_0V^\dagger_0 + U^*_0X_0V^\dagger_0Y_1 + U^*_0X_1V^\dagger_0)_11.$$  \hspace{1cm} (55)$$

For the SM one has $vC_{11} = (U^*_0X_1V^\dagger_0)_11 = m_\tau$. Thus, in the decoupling limit, we obtain the following result for the ratio of the Yukawa couplings $y_{SM}$ and $y_{SLRM}$:

$$\frac{y_{SLRM}}{y_{SM}} = 1 + \frac{(Z^*_1M_0 - M_0Y_1)_11}{(U^*_0X_1V^\dagger_0)_11} = 1,$$  \hspace{1cm} (56)$$

since the matrix $M_0$ is diagonal. Even if the $\tau$'s contained large fraction of gauginos or higgsinos, the couplings responsible for Higgs decays to the charged leptons or quarks are the same as in the Standard Model, since the physically relevant parameter region is close to the decoupling limit.

Contrary to the MSSM, one might have the lightest Higgs decaying to a $\tau$-lepton and a heavier chargino. Whether this decay mode is kinematically possible is much more model dependent than the decay to leptons, and we cannot say anything general about it.

6  The lightest doubly charged Higgs boson

We have seen that the neutral Higgs sector contains one relatively light Higgs boson, which however may be heavier as compared to the lightest Higgs in the MSSM, and which has MSSM like couplings to the fermions of the model. Thus, one cannot tell from the properties of the lightest Higgs boson, about the nature of the model. If a light Higgs is found before any supersymmetric particles are observed, one would not even know whether it is the Higgs boson of a supersymmetric model.

Altogether there are four doubly charged Higgs bosons in the SLRM, of which two are right-handed and two left-handed. The mass matrices of the left-handed triplets depend
Fig. 3. The mass $m^{++}$ of the lightest doubly charged Higgs boson as a function of the soft trilinear coupling $A_\Delta$ for different values of the right-handed sneutrino VEVs $\sigma_R$. We have varied $\sigma_R$ in its allowed range of 100 GeV to 8.45 TeV as indicated in the figure.

The mass matrix for the right-handed scalars depends on the right-triplet VEV instead of the left-triplet VEV. Nevertheless, it was noticed in [26] that in the SLRM with broken R-parity the lightest doubly charged scalar tends to be light. Also, it was shown in [44] that in the nonrenormalizable case it is possible to have light doubly charged Higgs bosons. On the other hand, in the nonsupersymmetric left-right model all the doubly charged scalars tend to have a mass of the order of the right-handed scale [45]. This is also true in the SLRM with enlarged particle content [33]. Thus a light doubly charged Higgs would be a strong indication of a supersymmetric left-right model with minimal particle content.

In the case of broken R-parity, the mass matrix of the right-handed doubly charged scalars can be obtained from the potential in appendix A. The allowed parameter space is strongly constrained by demanding that all the eigenvalues of the mass squared scalar matrices remain positive. In order to find the relevant parameter region, we have studied the bounds on some of the masses in the model. Although these may not be important as actual bounds, they restrict the parameter space. In the following we shall assume that $\tan \beta > 1$. From the bound obtained from submatrix $(\Phi^0_{2r}, \chi^0_{1r})$, we find the following constraint

$$-\frac{1}{2}g^2_L(\kappa_2^2 - \kappa_1^2) - \frac{1}{2}g^2_R D - (h^2_{LX}\kappa_2^2 - h^2_{\phi L}\kappa_1^2)\sigma_R^2 \geq 0,$$  \hspace{1cm} (57)$$

where $D = 2\nu_{\Delta R}^2 - 2\nu_{\delta R}^2 - \sigma_R^2 + \kappa_2^2 - \kappa_1^2$. If the terms with bidoublet lepton Yukawa couplings
are ignored, and $\tan \beta > 1$, Eq. (57) indicates that $D < 0$ and thus $2v_{\delta R}^2 + \sigma_R^2 > 2v_{\delta R}^2$. Other useful constraints follow from $(\Delta_R^-, \delta_R^{++})$ and $(\Delta_R^0, \delta_R^0)$ mass matrices,

$$
(A_{\Delta} v_{\Delta R} - 4h_{\Delta R}^2 v_{2R} + h_{\Delta R} \mu_{2R} v_{\delta R}) \sigma_R^2 + g_R^2 D(v_{\delta R}^2 - 2v_{\Delta R}^2) \geq 0, \\
A_{\Delta} v_{\Delta R} + h_{\Delta R} \mu_{2R} v_{\delta R} \geq 0.
$$

(58)

We have studied in Figure 3 an example with the soft masses and right-handed breaking scale, as well as the $\mu_{2R}$ parameter, of the order of 10 TeV. The maximum Majorana Yukawa coupling allowed by positivity of the mass eigenvalues in this case is $h_{\Delta} \sim 0.4$. For $h_{\Delta} = 0.4$ we have plotted the allowed doubly charged Higgs mass $m^{++}$ as a function of $A_{\Delta}$ for fixed $\sigma_R = 100$ GeV, . . . , 8.45 TeV. It is seen that relatively narrow bands of $A_{\Delta}$ and $\sigma_R$ are allowed. Even in the maximal case the mass of the doubly charged scalar $m^{++} \sim 1$ TeV and from the figure we see that the lightest of the doubly charged scalars can be as light or even lighter than the lightest neutral Higgs boson. The mass of the dangerous flavour changing Higgs boson is $\sim \sqrt{2} g_R \sqrt{D}$.

The mass of the lightest doubly charged Higgs, $m_{H^{++}}$, in the case of nonrenormalizable couplings has been considered in [44,46,33]. A general statement in these works is that the mass is $m_{H^{++}} \sim v_{2R}^2 / M$, where $M$ is the scale of the nonrenormalizable terms. Relevant constraints on the parameters follow again from the positivity of the mass bounds. From the submatrix $(\Delta_R^-, \delta_R^{++})$, a bound on $m_{H^{++}}$ can be obtained and we get

$$
0 < \frac{1}{2} g_R^2 (v_{\delta R}^2 - v_{\Delta R}^2)(\kappa_2^2 - \kappa_1^2).
$$

(59)

This gives a condition $v_{\delta R} / v_{\Delta R} > 1$. From the bound on $m_h$ from $(\phi_{2R}^0, \chi_{1R}^0)$ submatrix, we find

$$
0 < \frac{1}{2} [-g_R^2 (\kappa_2^2 - \kappa_1^2) - g_R^2 (2v_{\Delta R}^2 - 2v_{\delta R}^2 - \kappa_2^2 + \kappa_1^2)]
$$

(60)

Thus the $D$-term, $D = 2v_{\Delta R}^2 - 2v_{\delta R}^2 + \kappa_2^2 - \kappa_1^2$ has to be negative. From the mass bound on the doubly charged Higgses, we get

$$
0 < m_{H^{++}}^2 \\
< [g_R^2 (v_{\Delta R}^2 - v_{\delta R}^2) D] / (v_{\Delta R}^2 + v_{\delta R}^2) + \frac{1}{M} 8b_R v_{\Delta R} v_{\delta R} \mu_{2R} + \frac{1}{M^2} 4b_R (2a_R + b_R) v_{\Delta R}^2 v_{\delta R}^2.
$$

(61)

We see that the $b_R$-parameter must necessarily be nonvanishing.

In Fig. 4 we have plotted the mass of the lightest doubly charged Higgs boson as a function of the nonrenormalizable $b_R$-parameter for two values of the ratios, $v_{2R}^2 / M =
Fig. 4. Mass of the doubly charged Higgs as a function of the nonrenormalizable $b_R$-parameter. In a) $v_R^2/M = 10^4$ GeV and $D = m_{soft}^2$, while in b) $v_R^2/M = 10^2$ GeV and $D = (3 \text{ TeV})^2$ (solid line), except for $m_{soft} = 10$ TeV also $D = (10 \text{ TeV})^2$ is shown (dashed line). The soft supersymmetry breaking parameters are marked in the figure, as well as the $b_R$ parameters. The curves are brought closer to each other for convenience. In both figures $\tan \beta = 50$, $M = 10^{10}$ GeV, $\mu_{2R} = 1$ TeV and $\mu_1 = \mu'_1 = \mu''_1 = 500$ GeV. In a) $M_R = 10^7$ GeV and in b) $M_R = 10^6$ GeV.

$10^2$ GeV and $v_R^2/M = 10^4$ GeV. One should note that at low energies $b_R$ is the only nonrenormalizable parameter, which is necessarily nonvanishing. In particular one can find solutions with $a_R = 0$. In Appendix B we give examples of mass spectra using the tree level potential for the renormalizable and nonrenormalizable models. One can expect that radiative corrections will change the spectrum somewhat, but qualitative features should remain unchanged. The value of $b_R$ needed to produce a proper minimum depends on the other parameters in the model. In Fig. 4 a) $v_R^2/M = 10^4$ GeV and $\sqrt{D}$ is the same as the soft supersymmetry breaking parameters. The mass of the flavour changing Higgs is roughly proportional to the value of $\sqrt{D}$. For larger values of the $D$-term, larger $b_R$’s are needed to retain positive doubly charged Higgs mass, as seen from Eq. (61). It is seen that the mass of the doubly charged Higgs rises rapidly with $b_R$ to multi-TeV region as is expected for large $v_R^2/M$. In Fig.4 b) $v_R^2/M = 10^2$ GeV, and $\sqrt{D} = 3$ TeV (solid lines) for all the values of the soft supersymmetry breaking parameters that are plotted. This shows the dependence on the soft mass parameters. For the soft mass parameter of 10 TeV we have plotted for comparison also the dependence on $b_R$ when $\sqrt{D} = 10$ TeV (dashed line). It is seen that increasing the soft parameter, while the $D$-term remains constant, smaller $b_R$’s are needed to produce positive masses. In Fig. (4) we have taken $\tan \beta$ to be 50. Large $\tan \beta$ somewhat increases the masses of the neutral and pseudoscalar Higgses and decreases the mass of the lightest doubly charged Higgs. With both values of $v_R^2/M$ it is possible to obtain a doubly charged Higgs, which is light enough to be detected in future experiments, but $m_{H^{++}}$ increases with $b_R$ much faster for large $v_R^2/M$. 

22
The collider phenomenology of the doubly charged scalars has been actively studied, since they appear in several extensions of the Standard Model, can be relatively light and have clear signatures. Here we will discuss the production of the doubly charged scalar in colliders [47,48]. If the light doubly charged Higgs $H^{++}$ is lighter than any of the supersymmetric partners, the dominant decay mode of $H^{++}$ is to like sign leptons.

Kinematically, production of a single doubly charged scalar would be favoured. If the couplings between electrons and triplet Higgses are large enough, the doubly charged particles can be detected in $e^+e^-$ linear colliders in single production almost up to the kinematical limit. The $l^-l^-$ colliders are especially useful in studying the doubly charged Higgs, since for nonzero triplet couplings it can be produced as an s-channel resonance. The possibilities for single production in hadron colliders in $WW$ fusion does not depend on the coupling to leptons, but it does depend, in addition to $m_{H^{--}}$, on the VEV of the triplet to which the doubly charged scalar belongs. If $m_{W_R} \sim 1$ TeV, one can detect doubly charged Higgs bosons with $m_{H^{++}} \sim 1$ TeV at the LHC [48].

The advantage of pair production of the doubly charged scalars compared to the single production is that it is relatively model independent. It can occur even if $W_R$ is very heavy, as in the nonrenormalizable case, or the triplet Yukawa couplings are very small. The doubly charged Higgses can be produced in $f\bar{f} \to \gamma^*, Z^* \to H^{++}H^{--}$ both at lepton and hadron colliders, if kinematically allowed. At LHC this cross section falls off rapidly close to $m_{H^{++}} \sim 500$ GeV [48]. The pair production cross section at Tevatron and at LEP II are given in [46]. For $e^+e^-$ linear colliders the pair production cross section for $\sqrt{s} = 500$ GeV is $106 (78)$ fb for $m_{H^{++}} \lesssim 230 (240)$ GeV. Thus the detection of doubly charged scalars in pair production is possible close to the kinematical limit.

7 Conclusions

Supersymmetric left-right models are well motivated extensions of the MSSM, since they conserve $R$-parity as a consequence of gauge invariance. We have made a detailed study of the Higgs sector in these models. The lightest CP even Higgs boson can be considerably heavier than in the MSSM, but its couplings to fermions remain similar to the couplings of the Standard Model Higgs boson. If a Higgs, which nevertheless is too heavy to be the Higgs boson of the MSSM, is detected, one should consider extended supersymmetric models such as the ones studied in this paper.

In the SLRM with the minimal particle content one has typically also a light doubly charged Higgs boson. If this particle is found, it is a strong indication of the SLRM with minimal particle content.

We wish to emphasize the importance of studying the full mass matrices, including the soft mass parameters in determining the low energy mass spectrum, especially the masses of the light scalar particles.
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A Appendix: Scalar potential of the minimal SLRM

The different components of the scalar potential (6) for the minimal left-right supersymmetric model can be written as follows \((g_L, g_R, g_{B-L})\) are the three gauge couplings:

\[
V_F = |h_{\Phi L} i \tau_2 \Phi L^c + h_{\chi L} i \tau_2 \chi L^c + 2 h_{\delta L} L^T i \tau_2 \delta L|^2 \\
+ |h_{\Phi L} L^T i \tau_2 \Phi + h_{\chi L} L^T i \tau_2 \chi + 2 h_{\Delta R} L^T i \tau_2 \Delta_R|^2 \\
+ |h_{\Delta R} L^T L^T (i \tau_2) + \mu_2 R \delta_R|^2 + |h_{\delta L} L L^T (i \tau_2) + \mu_2 L \Delta_L|^2 \\
+ |h_{\Phi Q} Q^c Q^T (i \tau_2) + h_{\Phi L} L^T L^T (i \tau_2) + \mu_1 (i \tau_2) \chi^T (i \tau_2) + 2 \mu_1' (i \tau_2) \Phi^T (i \tau_2)|^2 \\
+ |(i \tau_2)(h_{\Phi Q} \Phi + h_{\chi Q} \chi) Q^c|^2 + |Q^T (i \tau_2)(h_{\Phi Q} \Phi + h_{\chi Q} \chi)|^2 \\
+ |\mu_2 L \Delta_R|^2 + |\mu_2 L \delta_R|^2, \tag{A.1}
\]

\[
V_D = \frac{1}{8} g_L^2 \sum_a \left[ \text{Tr}(\Phi^T \tau_a \Phi) + \text{Tr}(\chi^T \tau_a \chi) + 2 \text{Tr}(\Delta^T \tau_a \Delta) + 2 \text{Tr}(\delta^T \tau_a \delta) \right] \\
+ L^T \tau_a L + Q^T \tau_a Q \right] + \frac{1}{8} g_R^2 \sum_a \left[ - \text{Tr}(\Phi \tau_a \Phi^T) - \text{Tr}(\chi \tau_a \chi^T) \right] \\
+ 2 \text{Tr}(\Delta^T \tau_a \Delta_R) + 2 \text{Tr}(\delta^T \tau_a \delta_R) + L^T \tau_a L^c + Q^T \tau_a Q^c \right] \right] \\
+ \frac{1}{8} g_{B-L}^2 \left[ \frac{1}{2} \text{Tr}(\Delta_R^T \Delta_R + \delta_R^T \delta_R - \Delta_L^T \Delta_L + \delta_L^T \delta_L) \\
- L^T \tau = L^T L^c + \frac{1}{3} Q^T Q - \frac{1}{3} Q^c Q^T \right]^2, \tag{A.2}
\]

\[
V_{\text{soft}} = m_{\Phi}^2 \text{Tr}[\Phi]^2 + m_{\chi}^2 \text{Tr}[\chi]^2 - (m_{\Phi \chi}^2 \text{Tr}(i \tau_2 \Phi^T i \tau_2 \chi) + m_{\Phi \Phi}^2 \text{Tr}(i \tau_2 \Phi^T i \tau_2 \Phi) \\
+ m_{\phi}^2 \text{Tr}(i \tau_2 \Phi^T i \tau_2 \chi) + \text{h.c.}) + m_{\Delta R}^2 |\Delta_R|^2 + m_{\delta R}^2 |\delta_R|^2 - (m_{\Delta \Phi}^2 \text{Tr}\Delta \Phi + \text{h.c.}) \\
+ m_{\Delta L}^2 |\Delta_L|^2 + m_{\delta L}^2 |\delta_L|^2 - (m_{\Delta \chi}^2 \text{Tr}\Delta \chi + \text{h.c.}) + m_L^2 |L|^2 + m_{L^c}^2 |L^c|^2 \\
+ (L^T i \tau_2 (A \Phi + A \chi) L^c + A_{\Delta R} L^c i \tau_2 \Delta_R L^c + A_{\delta L} L^T i \tau_2 \delta_L L + \text{h.c.}) \\
+ m_Q^2 |Q|^2 + m_{Q^c}^2 |Q^c|^2 + (Q^T i \tau_2 (h_{\Phi Q} A \Phi + h_{\chi Q} A \chi) Q) + \text{h.c.}). \tag{A.3}
\]
From (6), (A.1), (A.2) and (A.3) it is straightforward to derive the mass matrix for the CP-even Higgs scalars, whose eigenvalues will provide the masses of the physical scalar Higgs bosons.

B Appendix: Examples of mass spectra

Here we give one example each of the mass spectrum of Higgs bosons in renormalizable and nonrenormalizable supersymmetric left-right models. In table B.1 we have shown the spectrum in the renormalizable model, and in table B.2 the spectrum for the nonrenormalizable model with minimal particle content. In both examples the soft supersymmetry breaking parameters are $\sim 10$ TeV. In table B.1, the right handed scale is 10 TeV, and in table B.2 $v_R^2/M = 100$ GeV. We have denoted $h = H^0_1$ and $H = H^0_2$. 
| particle | mass (TeV) | composition |
|----------|-----------|-------------|
| $H_{10}^0$ | 22.7 | $-0.3\nu_R^r + 0.7\Delta_R^0 - 0.6\delta_R^0$ |
| $H_9^0$ | 20.2 | $0.98\nu_L^r - 0.2\phi_2^{0r}$ |
| $H_8^0$ | 12.7 | $\phi_1^{0r}$ |
| $H_7^0$ | 11.9 | $-0.2\nu_R^r - 0.98\phi_2^{0r} + 0.1\chi_1^{0r}$ |
| $H_6^0$ | 10.3 | $0.1\Delta_L^0 - \delta_L^0$ |
| $H_5^0$ | 9.70 | $-\Delta_L^0 - 0.1\delta_L^0$ |
| $H_4^0$ | 6.60 | $-0.6\nu_R^r + 0.3\Delta_R^0 + 0.7\delta_R^0$ |
| $H_3^0$ | 3.53 | $-0.1\phi_2^{0r} - \chi_1^{0r}$ |
| $H_2^0$ | 1.95 | $0.7\nu_R^r + 0.6\Delta_R^0 + 0.4\delta_R^0$ |
| $H_1^0$ | 0.096 | $\chi_2^{0r}$ |
| $A_8$ | 22.3 | $-0.9\nu_R^i - 0.5\Delta_R^0 - 0.2\delta_R^0$ |
| $A_7$ | 20.2 | $0.98\nu_L^i + 0.2\phi_2^{0i}$ |
| $A_6$ | 12.7 | $\phi_1^{0i}$ |
| $A_5$ | 12.4 | $0.5\nu_R^i - 0.5\Delta_R^0 - 0.7\delta_R^0$ |
| $A_4$ | 11.9 | $0.2\phi_L^r - 0.98\phi_2^{0i} - 0.1\chi_1^{0i}$ |
| $A_3$ | 10.3 | $-0.1\Delta_L^0 \delta_L^0$ |
| $A_2$ | 9.70 | $-\Delta_L^0 - 0.1\delta_L^0$ |
| $A_1$ | 3.53 | $0.1\phi_2^{0i} - \chi_1^{0i}$ |
| $H_8^+$ | 20.2 | $0.98\bar{c}_L + 0.2\phi_1^+$ |
| $H_7^+$ | 19.0 | $-0.4\bar{c}_R + 0.7\Delta_R^+ - 0.5\delta_R^+$ |
| $H_6^+$ | 12.7 | $\phi_2^+$ |
| $H_5^+$ | 11.9 | $-0.2\bar{c}_L + 0.98\phi_1^+$ |
| $H_4^+$ | 10.3 | $0.1\Delta_L^+ - \delta_L^+$ |
| $H_3^+$ | 9.70 | $-\Delta_L^+ - 0.1\delta_L^+$ |
| $H_2^+$ | 9.42 | $-0.8\bar{c}_R - 0.1\Delta_R^+ + 0.5\delta_R^+$ |
| $H_1^+$ | 3.53 | $-0.1\phi_1^+ + \chi_2^+$ |
| $H_{4+}$ | 14.7 | $0.8\Delta_R^{++} - 0.6\delta_R^{++}$ |
| $H_{3+}$ | 10.3 | $0.1\Delta_L^{++} - \delta_L^{++}$ |
| $H_{2+}$ | 9.70 | $-\Delta_L^{++} - 0.1\delta_L^{++}$ |
| $H_{1+}$ | 0.169 | $-0.6\Delta_R^{++} - 0.8\delta_R^{++}$ |

Table B.1
Mass spectrum of a renormalizable model with minimum particle content.
| particle | mass (TeV) | composition |
|----------|-----------|-------------|
| $H^0_8$  | 1570      | $0.7(\delta^0_R - \Delta^0_R)$ |
| $H^0_7$  | 53.8      | $0.1\phi^0_2 + 0.3\phi^2_2 - 0.9\chi^0_2$ |
| $H^0_6$  | 52.9      | $0.9\phi^0_2 - 0.1\chi^0_1 + 0.3\phi^0_1$ |
| $H^0_5$  | 11.7      | $\Delta^0_L$ |
| $H^0_4$  | 9.24      | $-0.1\phi^0_2 - \chi^0_1$ |
| $H^0_3$  | 8.0       | $\delta^0_L$ |
| $H^0_2$  | 0.284     | $-0.7(\delta^0_R + \Delta^0_R)$ |
| $H^0_1$  | 0.091     | $\phi^0_1 + 0.1\chi^0_2$ |
| $A^0_6$  | 53.8      | $0.1\phi^0_1 + 0.3\phi^0_2 + 0.9\chi^0_2$ |
| $A^0_5$  | 52.9      | $-0.9\phi^0_2 - 0.1\chi^0_1 + 0.3\chi^0_2$ |
| $A^0_4$  | 14.1      | $-0.7(\delta^0_1 + \Delta^0_1)$ |
| $A^0_3$  | 11.7      | $\Delta^0_L$ |
| $A^0_2$  | 9.24      | $0.1\phi^0_2 - \chi^0_1$ |
| $A^0_1$  | 8.0       | $\delta^0_L$ |
| $H^+_6$  | 929       | $0.7(\delta^+_R - \Delta^+_R)$ |
| $H^+_5$  | 53.8      | $-0.1\chi^+_1 - 0.3\chi^+_2 - 0.9\phi^+_2$ |
| $H^+_4$  | 52.9      | $-0.9\chi^+_2 - 0.1\phi^+_1 + 0.3\phi^+_2$ |
| $H^+_3$  | 11.7      | $\Delta^+_L$ |
| $H^+_2$  | 9.24      | $-0.1\chi^+_2 + \phi^+_1$ |
| $H^+_1$  | 8.0       | $\delta^+_L$ |
| $H^{++}_4$ | 19.6   | $-0.98\delta^++_R + 0.2\Delta^++_R$ |
| $H^{++}_3$ | 11.7 | $\Delta^+_L$ |
| $H^{++}_2$ | 8.0    | $\delta^+_L$ |
| $H^{++}_1$ | 0.202 | $-0.2\delta^++_R - 0.98\Delta^+_R$ |

Table B.2
Mass spectrum of a nonrenormalizable model with minimum particle content.

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