The Development of a Manufacturing Flow Model of Garments by Graphs Transformation

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Abstract. Expanding the functionality of Computer-Aided Process Planning systems (CAPP systems) at garment enterprises is an actual problem. In these systems the manufacturing flows are modeled using a tabular form to represent information about the manufacture of the product. However, the table in which records are interleaved sequentially does not represent the positions of the elements of the technological process and their interrelations. The aim of this article is to improve the quality of design solutions obtained by modeling the manufacturing flow of garments using CAPP systems. The study of this problem is based on the use of concepts and models of graph theory. The development of a manufacturing flow model of garments is represented iterative transformation of graphs. The graph of the flow of garments manufacturing model receive from the initial tree of the technological process by the operations of elementary contraction. The ways of arrangement are revealed depending on the positions and interrelations of the vertices on the graph: trivial, consecutive and adjacent, consecutive and non-adjacent, parallel ways. Technological operations are completed in the manufacturing operation by the specified ways of arrangement. Modeling of the flow is accompanied by the transformation of the adjacency matrixes defining the graphs. The structure of each manufacturing operation from the flow scheme is represented by a subgraph received by operations of the vertices removal from the initial tree of technological process. The requirement to completing the production operations of the manufacturing flow is formulated: the number of connected components of the subgraph of the production operation should be minimized. The contents of this article may be useful for the development of specialized software applications for solving problems of modeling manufacturing flow of garments.

1. Introduction
In modern conditions the design of manufacturing flows of garments is carried out at a high organizational and technical level. The clothing companies introduce computer-aided process planning systems, for example, "Julivi" (company "SAPRLEGPROM", Kiev, Ukraine), "Stylon. Garment manufacture" (LLC "Stilon", St. Petersburg, Russia), "Effect plus "(the company "Effect", Vladimir, Russia) [1-3], software products of other companies. The use of these systems contributes to the effectiveness of clothing production by improving the quality of design solutions and shortening the execution of work. In the modules "Technological sequence", "Division scheme of labor" for input, processed and output information about the manufacture of products used tabular form of data presentation.
According to the theory, the initial information about the manufacture of a garment to be launched into production is represented by a graph in the form of a tree [4]. The model of manufacturing flow of garments is developed by the completing of technological operations meeting the compatibility requirements [4-8]. Fulfillment of these requirements ensures the effective functioning of the manufacturing flow. The functional model of a flow of garments manufacturing is represented by a scheme which in a form is also the graph [4-7, 9]. The problem of modeling the manufacturing flow of garments should be considered accordingly in the aspect of transforming graphs.

With the graph vertices the characteristics of the elements (technological, manufacturing operations) of the manufacturing process of the garment model are linked. Unlike tables, the graph shows the position and interrelationships of these elements, parallel processing of parts, the order of assembly of the product.

2. Literature review

Production time is used as a characteristic of the production rate in the manufacturing flow. The production time shows the average time interval through which a single product is periodically launched into the manufacturing flow or released from it [10]. Thus, the distribution of work between performers or workstations in the manufacturing flow is carried out in accordance with the production time.

Balancing the production line is the key principle by which manufacturing flows models of garments are developed. Line balancing is provided by the completing of technological operations, which is implemented on the basis of a specific method of combinatorics or heuristics [11].

For different methods of heuristics, the corresponding form of presentation of the original data is typical. For example, according to the method of ranked positional weights, a table of technological operations, ordered according to the data calculated from the precedence matrix, is used [12]. According to the largest-candidate rule, lists of technological operations are formed in order of decreasing time, indicating the immediate predecessors [13]. By the Kilbridge-Wester's method, the time expenditure of the technological operations represented by the priority diagram [14] is considered. In the listed forms, the initial data is systematized according to the positions and interrelationships of the technological operations on the graph reflecting the manufacturing process of the product.

Completing of technological operations by the method of branches and boundaries is carried out according to the technological sequence of manufacturing the product. This document is presented in the form of a table. As a result of the completing procedure the output document "Division of labor scheme" is also drawn up in the form of a table. In essence, the division scheme of labor reflects the functional model of the manufacturing flow of garments.

After research the graphs a group of scientists have identified and described in the book [3] four arrangement ways of the technological operations that form manufacturing operations: consecutive and adjacent, consecutive and on-adjacent, parallel and adjacent, parallel and non-adjacent ways. Definitions for ways of arrangement are given in the view of the manufacturing operation as a set of several technological operations. The differences of the listed ways are established on the basis of graph theory. However, manufacturing operations have also more complex structures and also to be trivial on structure.

According to the compatibility requirements of technological operations in a manufacturing operation should strive to ensure parallel processing of parts, observe the principle of processing units, adhere to the order of assembly of the product. In addition, it is also required to reduce the distances between workstations and the time periods for processing the parts, improve the flow function, simplify the process, group the units of equipment. A list of these problems belongs to the classical problems of modeling of manufacturing flow. The method of solving these problems is Production Flow Analysis [15].

Thus, in Computer-Aided Process Planning systems of garment manufacture it is necessary to represent flow models through graphs.
3. Methodological framework

3.1. Transformation of graphs by operations of elementary contraction

The technological process of manufacturing a garment is expediently represented by a tree [4-6, 9]. Showing the target function of the technological process, the tree is represented by a directed graph.

A successive transformation of the tree graph results in a different arrangement solution when developing a manufacturing flow scheme. In solving the problem posed, only the operations of vertex identification and edge contraction in the set of operations performed over the elements of the graph is applicable.

Elementary contraction in the graph is carried out on the basis of operation of identifying its vertices [16] which essence consists in the following. The pair of vertices \( u \) and \( v \) in the graph is deleted with the replacement by a new vertex \( w \) and adjacent to the same vertices of the graph as the remote vertices \( u, v \)/figure 1, a)/.

The identification of adjacent vertices \( u \) and \( v \) is performed with their replacement by a new vertex \( w \) and by contracting the edge \( (u, v) \)/figure 1, b)/. This operation is called edge contraction [16].

The technologist implements the completing procedures observance of the conditions of compatibility of technological operations in manufacturing operations [4-8]. At the same time the list and the initial order of the execution of technological operations in the scheme of manufacturing flow unchanged.

The vertices of the initial graph correspond to the technological operations of manufacturing the model of the garment. From the initial graph is obtained a finite graph by operations of elementary contraction that show the completing of operations. The vertices of the final graph are manufacturing operations which are approximately equal to or equal to production time by the duration of execution [6].

Technological relations on the initial graph are transformed in two ways with their division into internal and external relations for each manufacturing operation. Internal relations in manufacturing operations on a finite graph do not show, they are removed when the edges are contracted. External relations represent the production and technological relations between the elements of the flow.

3.2. Arrangement ways of technological operations on the graph

The manufacturing flow of garments is modelled on the basis of the concept of group technology [17].

First of all, consider the possibility of combining technological operations on individual branches of the graph. This direction allows to form in a stream of a group of workers specializing in the manufacture of an assembly unit. Completing of operations along a branch answers the principle of parallelism of fabrication [7] of details and assembly units of a garment in manufacturing flow.

If the formation of the manufacturing operation is unattainable in the specified direction, then the selection of technological operations located on several branches is carried out. Search of the decision is conducted on two branches, then on three branches, and so on. But at the same time, they pursue the goal of forming in the flow a group of performers specialized now in the fabrication of several assembly units.

In an extreme situation carry out completing, following along chords concerning branches of a tree of technological process. However, the latter direction does not contribute to the creation in the stream of groups specialized in the fabrication of assembly units of the garment.

Completing of operations is carried out step by step.

Initially any manufacturing operation is an empty set. Consequently, the notion of the compatibility of technological operations has no value yet.

The trivial way of arrangement in the graph is realized by including one single technological operation in the new manufacturing operation /figure 2, a)/. For this purpose, a technological operation from the set of uncompleted elements in the initial graph is randomly selected. The way doesn't demand performance of any operations on the elements of the graph.
At this stage of completing the manufacturing operation structure is presented by a trivial subgraph which predetermined the arrangement way name. The formation of this production operation can be completed if the time parameter of the selected manufacturing operation is within the allowable range of its deviation from the flow’s production time. Otherwise the procedure will continue.

A consecutive and adjacent way of arrangement is carried out by joining a technological operation that immediately precedes or directly follows the technological operation included in the manufacturing operation earlier. Respectively over elements of the graph carry out operation of the edge contraction /figure 2, b)/. Consecutive and adjacent way is the most effective way of arrangement since it is aimed at minimizing the duration of the production cycle due to the parallel fabrication of parts in the manufacturing flow.

A consecutive and non-adjacent way of arrangement is characterized by accession of the technological operation that does not have a direct connection to an already completed technological operation, but is embodied on the same path in the directed graph /figure 2, c)/. In this situation the vertex identification operation is performed. Completing in this way preserves the possibility of parallel processing of parts in the manufacturing flow.

The parallel method of arrangement is realized by joining to the technological operation from the manufacturing operation another technological operation located on another branch of the graph. The transformation of the initial graph occurs with the operation of vertex identification /figure 2, d)/. With this way of arrangement, the duration of the production cycle for the manufacture of the product increases due to the consecutive processing of different details in one workplace.

Subsequent completing is carried out by means of one or another way of arrangement considered with respect to one of the technological operations completed earlier. By applying noted ways, a manufacturing operation of an arbitrary structure is obtained which is limited only to the tree.

Figure 3 shows an example of the outcome and result of transformation of directed graphs namely the directed rooted tree /figure 3, a)/ of the technological process of garment manufacturing into the multigraph /figure 3, b)/ which represents the manufacturing flow model. To transformation the initial tree, all four arrangement ways of technological operations are used: trivial, consecutive and adjacent, consecutive and non-adjacent, parallel ways.

3.3. Transformation of adjacency matrixes of graphs
The graph is determined to within isomorphism by adjacency of vertices which is described by means of matrices [16, 18].

The adjacency matrix $A(D)$ of the digraph $D$ is represented by the square matrix $[a_{ij}]$ of order $p$ ($i,j = 1, 2, ..., p$) defined by the number of vertices. Both the row number $i$ and the column number $j = i$ of the matrix are associated with the index of the vertex $v_i$ of the digraph. The matrix element $a_{ij} = 1$ if $\{v_i, v_j\}$ is the directed edge of the digraph $D$. Otherwise $a_{ij} = 0$.

From the values of the elements of the adjacency matrix it is determined that the vertex $v_i$:
- is a leaf that starts the branch of the tree if the sum of the elements in the column is zero. $\sum_{i=1}^p a_{ij} = 0$.
- serves as a tree root if the sum of the elements in the row is zero. $\sum_{j=1}^p a_{ij} = 0$.

Figure 4 shows the adjacency matrix for the example of the process tree in Figure 3, a).

On values of elements from a matrix find also a path which the detail of garment passes in technological process until the readiness of a product. According to the technique [9] before the construction of the process tree the details of the garment are coded using sequence numbers. In the example /see figure 3, a)/ three details are denoted by the codes $r = 1, 2, 3$. The path search begins with the initial vertex of the graph branch ($\sum_{i=1}^p a_{ij} = 0$). In the row of the matrix associated with this vertex an element with a unit value is searched. The label of the head of the arrow is determined by number of a column in which the found element is located. Then pass into the row that is associated with the arrow head. These actions are repeated many times until they reach the tree root. By means of
the adjacency matrix defining the directed root tree three paths are found that start at the vertices \( v_1, v_4 \) and \( v_6 \).

\[
p = 9 \Rightarrow \begin{cases} 
\sum_{i=1}^{p} a_{ij} = \sum_{i=1}^{9} a_{i1} = 0; \\
\sum_{i=1}^{p} a_{ij} = \sum_{i=1}^{9} a_{i4} = 0; \\
\sum_{i=1}^{p} a_{ij} = \sum_{i=1}^{9} a_{i6} = 0;
\end{cases}
\Rightarrow \begin{cases} 
v_1 v_2 v_3 v_7 v_9 v_9 - 1st \text{ path}; \\
v_4 v_5 v_7 v_9 v_9 - 2nd \text{ path}; \\
v_6 v_9 v_9 - 3rd \text{ path}.
\end{cases}
\]

With each completing iteration is performed the operation of elementary contraction in the digraph which leads to the transformation of the adjacency matrix.

With the identification of the vertices \( v_\alpha \) and \( v_\beta \) (\( \alpha \neq \beta \quad \alpha, \beta \in \{1, p\} \)) the row and column of the matrix with a smaller number \( \alpha (\alpha < \beta) \) is associated with the new vertex \( v_{\alpha, \beta} \). Then in each column the sum of the values of the elements from the \( \alpha \) and \( \beta \)-th rows is determined. The received sum is appropriated to the element in the row which is associated again \( (i = \alpha; \quad \alpha \leftrightarrow \alpha, \beta) \). The values of the elements starting with \( (\beta + 1) \)-th to the final row of the matrix move one position upward to the predecessor row. Further perform similar actions but with the elements in the columns of the matrix. As a result the order of the square matrix is reduced by one.

\[
G - v_\alpha - v_\beta + v_{\alpha, \beta} : \quad a_{ij} = \begin{cases} 
\alpha_{ij}, & \text{if } (1 \leq i < \alpha) \land (\alpha < i < \beta); \\
\alpha_{ij} + \alpha_{ij}, & \text{if } i = \alpha; \\
\alpha_{(i+1)j}, & \text{if } \beta \leq i < p; \\
\alpha_{ij}, & \text{if } (1 \leq j < \alpha) \land (\alpha < j < \beta); \\
\alpha_{ij} + \alpha_{ij}, & \text{if } j = \alpha; \\
\alpha_{(i+1)j}, & \text{if } \beta \leq j < p; \\
\alpha_{ij}, & \text{if } (1 \leq j < \alpha) \land (\alpha < j < \beta); \\
\alpha_{ij} + \alpha_{ij}, & \text{if } j = \alpha; \\
\alpha_{(i+1)j}, & \text{if } \beta \leq j < p;
\end{cases}
\Rightarrow \begin{cases} 
(i = \alpha) \leftrightarrow v_{\alpha, \beta} \Rightarrow a_{ij} = \begin{cases} 
\alpha_{ij}, & \text{if } (1 \leq i < \alpha) \land (\alpha < i < \beta); \\
\alpha_{ij} + \alpha_{ij}, & \text{if } j = \alpha; \\
\alpha_{(i+1)j}, & \text{if } \beta \leq j < p; \\
\alpha_{ij}, & \text{if } (1 \leq j < \alpha) \land (\alpha < j < \beta); \\
\alpha_{ij} + \alpha_{ij}, & \text{if } j = \alpha; \\
\alpha_{(i+1)j}, & \text{if } \beta \leq j < p;
\end{cases}
\end{cases}
\Rightarrow \begin{cases} 
(j = \alpha) \leftrightarrow v_{\alpha, \beta} \Rightarrow p = p - 1.
\end{cases}
\]

The contraction operation of the edge \( (v_\alpha, v_\beta) \) to be directed from vertex \( v_\alpha \) is written as \( G - v_\alpha - v_\beta + v_{\alpha, \beta} \). In the implementation of this operation the same actions are performed as in the operation of vertex identification \( v_\alpha \) and \( v_\beta \). In addition, the element is zeroed at the intersection of the row and column that are associated with the new vertex \( v_{\alpha, \beta} \).

\[
(i = \alpha \land j = \alpha) \leftrightarrow v_{\alpha, \beta} \rightarrow a_{\alpha \alpha} = 0.
\]

The adjacency matrix in figure 5 defines the final graph from the example (see figure 3, b).

With consecutive performance of operations of elementary contraction is simultaneously transformed the adjacency matrix. As a result a final directed graph is obtained that displays the scheme of a manufacturing flow.

3.4. Representation of structure of production operation by a subgraph

The structure of the created production operation should be represented a subgraph with one or several connected components.

By means of the vertex removal operation that is executed repeatedly, the directed rooted tree of the technological process is subsequently converted into a subgraph of the manufacturing operation. Vertices of the received subgraph display technological operations only from structure of the considered manufacturing operation. Thus, those tree vertices that are mapped to the technological operations in the compositions of other manufacturing operations of the flow are deleted.

Under the operation of vertex removal \( v_\alpha \) we mean an exception from the graph \( G \) of the vertex \( v_\alpha \) itself along with the edges incident to it [16]. Accordingly, from the adjacency matrix, both the \( \alpha \)-th
row and the $\alpha$-th column previously matched to the remote vertex $v_\alpha$ are deleted. With the removal of each vertex, the order of the adjacency matrix determining the subgraph decreases by one.

$$G - v_\alpha; \begin{cases} a_{ij} = \{a_{ij}, \text{if } 1 \leq i < \alpha; \\ a_{(i+1)j}, \text{if } \alpha \leq i < p; \} \land \begin{cases} a_{ij} = \{a_{ij}, \text{if } 1 \leq j < \alpha; \\ a_{(i+1)(j+1)}, \text{if } \alpha \leq j < p; \} \Rightarrow \\ p = p - 1. \end{cases} \end{cases}$$

The number of connectivity components of a subgraph of a particular manufacturing operation is also determined from the adjacency matrix.

Let as a result of completing of $p$ technological operations of the process $m$ manufacturing operations forming the manufacturing flow of garments are formed. Let also the symbol with the index $p_z$ denote the number of vertices of the subgraph. The variable $p_z$ shows the number of technological operations that make up a particular $z$-th manufacturing operation in the flow, $z = 1, 2, ..., m$. It is obvious that $\sum_{z=1}^{m} p_z = p$.

Before counting the number $K_z$ of connectivity components of the $z$-th subgraph of the manufacturing operation is equated to zero, $K_z = 0$. Then the values of the element sum in the first row, in the second row, and further, up to the last row of the adjacency matrix are considered. The zero value of the elements sum in the current row indicates the presence of one connectivity component. The total number of subgraph components is equal to the number of rows of the adjacency matrix with zero sum value.

$$K_z = 0; \ i = 1 \Rightarrow \forall i = 1, 2, ..., p_z,$$

$$\left[ \sum_{j=1}^{p_z} a_{ij} = 0 \rightarrow K_z = K_z + 1 \right] \Rightarrow (i < p_z \rightarrow i = i + 1).$$

In the example given the set of vertices $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ of the final graph corresponds to subgraphs of manufacturing operations having one component. The set of vertices $\{v_4, v_5, v_7, v_9\}$ corresponds to subgraphs containing two connected components.

When developing the scheme of a manufacturing flow minimize communications between its elements that leads to decrease in duration of a technological cycle of manufacturing of a product [19, 20]. When developing a manufacturing flow scheme, the connections between its elements are minimized which leads to a reduction in the duration of the technological cycle of the product manufacturing. $(K_z - 1) \rightarrow \min$. The optimization criterion of the structure should be considered in addition to the known criteria adopted for selecting the composition of the manufacturing operation, coordinated with the production time: homogeneity of technological operations in the specialty, according to the equipment used, according to the category.

4. Discussions

Professor Murygin V. E. researched and outlined in the thesis [4] a representation of the technological process of making a garment and a functional model of a manufacturing flow of garments in the form of graphs. These data representation models serve as input and output information for simulating the manufacturing flows of garments performed by means of an Computer-Aided Process Planning system.

Scientists of the Moscow State University of Design and Technology Serova T. M., Afanas’eva A. I., Illarionova T. I., Dell’ R. A. researched the procedures for completing technological operations in developing a manufacturing flow model in the technological aspect of the problem [6].

In this paper, the transformation of graphs as a solution of mathematical problems for the purpose of obtaining models of manufacturing flows of garments is presented for the first time.

5. Conclusion

Considering the completing as an iterative process performed over the graph, a trivial way of arrangement the technological operation in the structure of the manufacturing operation is highlighted. Starting in a trivial way, the subsequent formation of the manufacturing operation is continued by one of three ways of arrangement the technological operations: consecutive and adjacent, consecutive and
non-adjacent, parallel. Further the manufacturing operation is formed by alternating the listed ways in a given order. At the same time the successive execution of elementary contraction operations transforms the initial tree of process into a connected graph (possibly into a multigraph) that displays the manufacturing flow scheme.

The structure of each of the completed manufacturing operations according to the flow scheme is a separate subgraph of the initial graph with one or more connectivity components of the subgraph. The number of connected components of the subgraph of the manufacturing operation is required to be minimized.

6. Recommendations

The above conversions of the adjacency matrices performed with operations on graph elements are in essence a mathematical support of a specialized software application. Representing the flow model in the form of an oriented graph will allow the technologist to visualize the sequence of actions on the way to obtaining the optimal design solutions.

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Appendix

Figure A1. Operations over the elements of graph:
   a) vertex identification, b) edge contraction

Figure A2. Arrangement ways of technological operations on the graph:
   a) trivial, b) consecutive and adjacent, c) consecutive and non-adjacent, d) parallel
Figure A3. Transformation of the directed rooted tree (a) to the focused multigraph (b).

| Vertex | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | The sum on a row |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----------------|
| $v_1$  | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 1               |
| $v_2$  | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     | 1               |
| $v_3$  | 0     | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 1               |
| $v_4$  | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 1               |
| $v_5$  | 0     | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 1               |
| $v_6$  | 0     | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 1               |
| $v_7$  | 0     | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 1               |
| $v_8$  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 1               |
| $v_9$  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0               |

The sum on a column

0 1 1 1 0 1 0 2 2 1 8

Figure A4. Adjacency matrix of a tree of technological process.

| Vertex | $v_{1,2}$ | $v_{3,5}$ | $v_{4,6}$ | $v_{7,9}$ | $v_8$ | Summ on a row |
|--------|-----------|-----------|-----------|-----------|-------|--------------|
| $v_{1,2}$ | 0         | 1         | 0         | 0         | 0     | 1            |
| $v_{3,5}$ | 0         | 0         | 0         | 2         | 0     | 2            |
| $v_{4,6}$ | 0         | 1         | 0         | 0         | 1     | 2            |
| $v_{7,9}$ | 0         | 0         | 0         | 0         | 1     | 1            |
| $v_8$    | 0         | 0         | 0         | 1         | 0     | 1            |

Summ on a column

0 2 0 3 2 7

Figure A5. Adjacency matrix of a multigraph.
Figure A6. Adjacency matrices of subgraphs that map the structures of production operations to the vertices of a finite graph:

a) $v_1_2$, b) $v_4_6$, c) $v_3_5$, d) $v_7_9$, e) $v_8$

| Vertex | $v_1$ | $v_2$ | Summ on a row |
|--------|------|------|----------------|
| $v_1$  | 0    | 1    | 1              |
| $v_2$  | 0    | 0    | 0              |
| **Summ on a column** | **1** | **0** | **1**         |

| Vertex | $v_4$ | $v_6$ | Summ on a row |
|--------|------|------|----------------|
| $v_4$  | 0    | 0    | 0              |
| $v_6$  | 0    | 0    | 0              |
| **Summ on a column** | **0** | **0** | **0**         |

| Vertex | $v_3$ | $v_5$ | Сумма по a row |
|--------|------|------|----------------|
| $v_3$  | 0    | 0    | 0              |
| $v_5$  | 0    | 0    | 0              |
| **Summ on a column** | **0** | **0** | **0**         |

| Vertex | $v_7$ | $v_9$ | Summ on a row |
|--------|------|------|----------------|
| $v_7$  | 0    | 0    | 0              |
| $v_9$  | 0    | 0    | 0              |
| **Summ on a column** | **0** | **0** | **0**         |

| Vertex | $v_8$ | Summ on a row |
|--------|------|----------------|
| $v_8$  | 0    | 0              |
| **Summ on a column** | **0** | **0**         |