Limits on the monopole magnetic field from measurements of the electric dipole moments of atoms, molecules and the neutron

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Abstract

A radial magnetic field can induce a time invariance violating electric dipole moment (EDM) in quantum systems. The EDMs of the Tl, Cs, Xe and Hg atoms and the neutron that are produced by such a field are estimated. The contributions of such a field to the constants, $\chi$ of the T,P-odd interactions $\chi_{e}N \cdot s/s$ and $\chi_{N}N \cdot I/I$ are also estimated for the TlF, HgF and YbF molecules (where $s$ ($I$) is the electron (nuclear) spin and $N$ is the molecular axis). The best limit on the contact monopole field can be obtained from the measured value of the Tl EDM. The possibility of such a field being produced from polarization of the vacuum of electrically charged magnetic monopoles (dyons) by a Coulomb field is discussed, as well as the limit on these dyons. An alternative mechanism involves chromomagnetic and chromoelectric fields in QCD.

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I. INTRODUCTION

Dirac \cite{1} considered magnetic monopoles and derived a quantization rule for magnetic charge $M$: $eM = \frac{\hbar c}{2}k$, where $k$ is an integer (below we put $\hbar = c = 1$). Zwanziger and Schwinger \cite{2,3} generalized this condition for dyons, which carry both electric ($q$) and magnetic charges: $q_1 M_2 - q_2 M_1 = \frac{k}{2}$. This formula may be derived heuristically by the quantization of the angular momentum (half-integer or integer) of the electromagnetic field in a system consisting of two dyons \cite{4–8}. E. Purcell and N. Ramsey in ref. \cite{9} and N. Ramsey \cite{10} discussed the possibility of there being elementary particle and nuclear EDMs due to the existence of magnetic monopoles. Later, E. Witten \cite{11} showed that ‘t Hooft-Polyakov magnetic monopoles carry small electric charges due to CP violating interactions.

In the case of the $\theta$-term (the interaction $\theta \frac{e^2}{42\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}$) the electric charge of the monopole is $q = -\frac{e\theta}{2\pi}$. V. Sokolov developed nonrelativistic Lagrangian and Hamiltonian formalisms for the interaction of electric and magnetic charges which do not involve “strings” \cite{7,8}.

It is possible that monopoles may appear in particle-antiparticle pairs in which the point-like positive and negative magnetic charges could be very hard or impossible to separate. Recall that a “supercritical” electric charge $Ze$ with $Z > 1/\alpha$ must be screened down to the value $Z = 1/\alpha$ due to spontaneous $e^+e^-$ pair production. The strength of the interaction in this case ($Ze^2 = Z\alpha \sim 1$) is still smaller than $M^2 \sim 1/\alpha$ (using the quantization rule for magnetic charge). Also, Nambu showed that in the standard electroweak model the classical solution is a monopole-antimonopole pair connected by a $Z^0$-field string \cite{12}. This could explain the absence of free monopoles. However even in this case one can still search for the effects of virtual monopole-antimonopole pairs and this will be discussed in the present paper.

In ref. \cite{13} it was pointed out that an electric dipole moment (EDM) of an atom (or any quantum system) can be induced by the interaction of the electrons with a radial magnetic field ($B \propto r$). The existence of such a field would contradict both time reflection invariance (T) and Gauss’s Law ($\oint B \cdot da = 0$). The limit on such a field can be of interest by itself.
as a very precise test of electrodynamics at small distances since the accuracy of EDM measurements is very high now. It was also pointed out in [13] that a radial magnetic field can be produced due to a monopole-antimonopole pair contribution to the magnetic moment (if there is no magnetic string) and a time invariance violating interaction which polarizes nucleon spins along the radial direction.

The mechanisms of radial magnetic field and EDM creation can be much simpler if the magnetic monopole has an electric charge or is subject to the strong interaction. The simplest example would be for a magnetic charge to be captured by an atomic nucleus and hence produce a “Coulomb” magnetic field $B = \frac{Mr}{r^3}$. It is easy to find the exact solution of this problem (an electron in the field of the dyon) in the nonrelativistic case. All stationary states in this problem possess an EDM. Another example is a dyon-antidyon system with nonzero orbital angular momentum. Similarly to the way in which orbiting electric charges produce a magnetic moment, orbiting magnetic charges produce an EDM [9].

The most interesting possibility would be for the radial magnetic field and EDM to be induced by the virtual production of dyon-antidyon pairs. The mechanism could be the following. The electric field of an atomic nucleus polarizes the vacuum of dyon pairs and creates corrections to the radial electric field, $\delta E$. The ratio of the magnetic to the electric field produced by these dyons is $\frac{B}{E} = \frac{M}{q}$. Thus dyon vacuum polarization produces the radial magnetic field $B = \frac{M}{q} \delta E$. The interaction of atomic electrons with this magnetic field produces an atomic EDM. A similar mechanism could produce 3–5 orders of magnitude bigger P,T-odd effects in diatomic polar molecules. We also consider the contribution of this mechanism to the neutron EDM in the constituent quark model. Note that the electric field can be replaced by the strong field if the monopole interacts strongly. Moreover, it seems that “chromodyons”, which could exist in a generalization of QCD, could produce similar effects to that of “electromagnetic” dyons. Thus the problem considered in the present work could be related to the recent ideas about the role of monopole condensate in quark confinement (see e.g. the recent review in [14] and references therein).

In this paper we calculate the possible effects of time invariance violation in atoms,
molecules and the neutron produced by a radial magnetic field. We also estimate the field produced by the polarization of the dyon vacuum according to the mechanism discussed above. An accurate calculation of the monopole effects requires the solution of numerous complicated problems such as the large value of the magnetic charge, “strings”, the finite size of the classical monopole solution, etc. We stress that in the present work we are mainly trying to avoid these problems rather than to solve them. In fact we explore an approach using simple heuristic arguments and perturbation theory which allows us to estimate and compare the values of T,P-odd effects in different quantum systems. We must add that our attempt to use the results of two-vector potential theory for the dyon electrodynamics [15] (see also [16,17]) has lead to the conclusion that the magnetic field due to dyon vacuum polarization seems to vanish within this theory. (This contradicts the simple and natural picture discussed above! Note however that the theory [15] includes dyons only and the introduction of the usual charges into this theory leads to serious complications.) An alternative approach is to use the theories with ’t Hooft-Polyakov magnetic monopoles. We have not done any calculations within these theories. Our more simple “heuristic” calculation of the radial magnetic field due to dyon vacuum polarization (see below) is not based on a complete consistent theory and strictly speaking does not prove the existence of the effect. However, the calculations of the effects of this field in sections [11,15] apply to any contact radial magnetic field; they are not restricted to fields produced by dyon vacuum polarization. Thus, the results in these sections can be used in general by using the equations in terms of $B_0$ ($B_0$ is defined below).

II. THE RADIAL MAGNETIC FIELD DUE TO DYON VACUUM POLARIZATION

Let us start from an estimate of the radial magnetic field which could be produced due to polarization of the dyon vacuum by an electric charge. The correction to the electrostatic potential $\frac{\Psi}{r}$ due to the vacuum polarization of spin $\frac{1}{2}$ particles can be found in any textbook
on quantum electrodynamics (see e.g. [18]):
\[
\Phi(r) = e_1 \frac{2q^2}{r} \int_1^\infty e^{-2mrc} \left( 1 + \frac{1}{2\zeta^2} \right) \frac{\sqrt{\zeta^2-1}}{\zeta^2} \, d\zeta ,
\]
where \( q \) and \( m \) are the electric charge and mass of the dyons. (In fact, \( 2m \) in eq. (1) is the threshold of production of a dyon pair. In the case of a bound pair (connected e.g. by a \( Z^0 \)-string) we could substitute the mass of the two-dyon system instead.) The magnetic field due to the dyons can be expressed in terms of the corresponding electric field:
\[
B = E \cdot \frac{M}{q} = -\nabla \Phi \cdot \frac{M}{q},
\]
where \( M \) is the magnetic charge. If the dyons are heavy then the potential can be written as \( \Phi(r) = \text{constant} \cdot \delta(r) \), where the constant can be found by the integration of eq. (1) over \( r \). Thus polarization of the dyon vacuum produces the following radial magnetic field around the point-like charge \( e \):
\[
B \equiv B_0 \cdot \nabla \delta(r) ,
\]
with
\[
B_0 = -\frac{4}{15} \frac{eqM}{m^2} .
\]
Note that T-invariance in this case can be restored if there is one more dyon with the same mass but the opposite sign of the product \( qM \).

We can obtain the radial magnetic field of the nucleus by replacing \( \delta(r) \) by the proton density distribution, \( \rho_p(r) \approx Z\rho_0 \cdot \theta(R - r) \):
\[
B = B_0 \cdot \nabla \rho_p(r) \approx -Z\rho_0 B_0 \delta(r - R) \mathbf{n} ,
\]
where \( \mathbf{n} = r/r \) is a radial unit vector and \( \rho_0 = (\frac{4}{3}\pi R^3)^{-1} \). Here we took into account the fact that the nuclear density varies in a small interval around the nuclear radius \( R \).

III. THE INTERACTION BETWEEN AN ELECTRON AND THE RADIAL MAGNETIC FIELD

Consider now the interaction between an atomic electron and the contact radial magnetic field. A radial magnetic field cannot be described by a nonsingular vector potential \( \mathbf{A} \).
Therefore, we will avoid using the vector potential. Let us start from the nonrelativistic problem. The interaction between the spin magnetic moment and the magnetic field does not contain the vector potential:

\[ V = -\mu \sigma \cdot B = Z\mu B_0 \rho \sigma \cdot n \delta(r - R), \tag{5} \]

where \( \sigma \) are Pauli matrices. For the electron \( \mu = -\frac{e}{2m_e} \), with \( e > 0 \). Note that \( B_0 \) in the above equation is only given by eq. (3) for the dyon vacuum polarization mechanism, and can otherwise be considered as a more general parameter.

The orbital contribution to the interaction seems to vanish due to a cancelation between the contributions of the radial magnetic field of the nucleus and that of the electron. Let us first calculate the force acting on the electron from the radial magnetic field of the nucleus, in the latter’s rest frame:

\[ \mathbf{F}_1 = -e(\mathbf{v} \times \mathbf{B})/c, \tag{6} \]

where \( \mathbf{v} \) is the electron’s velocity. To calculate the second force acting on the electron, we first calculate the force acting on the nucleus from the magnetic field of the electron \( \mathbf{B}_e \). Since the electron is in motion, this magnetic field will be transformed into an electric field in the rest frame of the nucleus:

\[ \mathbf{E} = -(\mathbf{v} \times \mathbf{B}_e)/c \tag{7} \]

(to terms of order \( v/c \)).

Let us assume for now that the radial magnetic field around a particle is proportional to its electric charge. This is obviously true for the magnetic field due to the dyon vacuum polarization mechanism in eqs. (2) and (3), however this discussion is intended to be more general. (An argument for this proportionality that is based on angular momentum quantization will be given below.) Under this assumption the force on the nucleus from the electric field (7) is exactly equal to \( \mathbf{F}_1 \) in eq. (6). According to Newton’s third law the corresponding force acting on the electron from the nucleus is \( \mathbf{F}_2 = -\mathbf{F}_1 \), i.e. the net force
acting on the electron is \[ \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = 0. \] One can say that a cancelation occurs between the force \( \mathbf{F}_1 \) acting on the electron from the vacuum monopole distribution near the nucleus and the force \( \mathbf{F}_2 \) acting on the vacuum monopole distribution near the electron due to the electric field of the nucleus.

There is a second argument in favor of the cancelation of the orbital contribution. As is known \[ [4-8] \], the radial magnetic field of a magnetic charge and the radial electric field of an electric charge together produce a nonzero angular momentum around the axis connecting the charges (\( \sim \mathbf{E} \times \mathbf{B} \)). In Quantum Mechanics angular momentum is quantized. This requirement implies the quantization of the product of the magnetic and the electric charges. For two dyons this condition is

\[ K = q_1 M_2 - q_2 M_1 = \frac{k}{2}, \]

where \( K = \mathbf{J} \cdot \mathbf{n} \) is the projection of the angular momentum \( \mathbf{J} \) onto the axis connecting these particles and \( \mathbf{n} \) is a unit vector along this axis. Recall that the system’s orbital angular momentum is orthogonal to the connecting axis and does not contribute to \( K \). We see that for \( k = 0 \) the electromagnetic field angular momentum is zero due to the cancelation between the contributions of these two dyons.

A similar situation arises for the short-range induced magnetic field. If both the radial magnetic fields of the electron and the point-like nucleus are proportional to their charges then the angular momentum of the field will be zero due to a cancelation between the two contributions. (Actually, the nucleus is not point-like, therefore, strictly speaking, we should consider an electron-quark system here.) We stress that in the absence of such a cancelation there would be a problem: the size of the magnetic field region is very small (\( \sim m^{-1} \)), and so we cannot satisfy the condition of angular momentum quantization for nonzero \( K \) (this provides an argument for the proportionality of the radial magnetic fields to the electric charges).

Now there is a relation between the orbital contribution to the EDM of the two-particle system and \( K \). The angular wave function for the system is the Wigner \( D \)-function, \( D_{MK}^J(n) \), where \( M \) is the projection of \( \mathbf{J} \) onto the z-axis (compare e.g. with ref. \[ 7 \], where the charge-monopole solution was found). The orbital contribution to the EDM in such an angular
state is proportional to the angular integral \( \int |D_{MK}^J(n)|^2 n_z d\Omega = KM/[J(J+1)] \). This relation is especially simple for \( K = 0 \) since the \( D \)-function in this case coincides with the usual \( Y_{LM} \) angular function which gives an (orbital) EDM of zero. Thus, we can consider the zero value of the electromagnetic field angular momentum in the electron-nucleus case to be an argument for the absence of an orbital contribution to the electric dipole moment.

We must stress that there is no such cancelation between the contributions of the magnetic moments of the nucleus and the electron (see eq. (5)) since these magnetic moments are very different. For example, the nuclear magnetic moment can be zero.

In the relativistic case the interaction of a magnetic moment \( \mu \) with a magnetic field can be expressed in terms of the magnetic field only using the following well known identity (see e.g. [18]):

\[
j^\mu = \bar{\psi}_2 \gamma^\mu \psi_1 = \frac{1}{2m} \bar{\psi}_2 (p_1^\mu + p_2^\mu) \psi_1 - \frac{1}{2m} \bar{\psi}_2 \sigma^{\mu\nu} k_\nu \psi_1 ,
\]

where the first term in the r.h.s. is an orbital contribution to the electromagnetic current and the second term is a spin one and \( k_\nu = p_{2\nu} - p_{1\nu} \). Taking into account the fact that \( i(k \times A) = B \) we obtain (from \( \langle \psi_2 | V | \psi_1 \rangle = -e \langle \psi_2 | j^\mu A_\mu | \psi_1 \rangle \)) a relativistic expression for the interaction of a magnetic moment \( \mu \) with the radial magnetic field (4):

\[
V = -\mu \beta \Sigma \cdot B = Z \mu B_0 \rho_0 \begin{pmatrix}
\sigma \cdot n & 0 \\
0 & -\sigma \cdot n \\
\end{pmatrix} \delta(r - R) .
\]

Expression (9) can also be obtained from the quadratic form of the Dirac equation.

**IV. THE ATOMIC ELECTRIC DIPOLE MOMENT AND T,P-VIOLATION IN MOLECULES**

In this section we calculate the contribution of dyons to the atomic EDM, as well as their effect on molecules. The interaction (9) mixes atomic electron states of opposite parity, mostly \( s_{1/2} \) and \( p_{3/2} \) orbitals which are large at the nuclear surface.
ψ_{s_{1/2}}(R) = \left( \begin{array}{c} f_s \\ -i(\sigma \cdot n)g_s \end{array} \right) \frac{\chi}{\sqrt{4\pi}}, \quad \psi_{p_{1/2}}(R) = \left( \begin{array}{c} -(\sigma \cdot n)f_p \\ ig_p \end{array} \right) \frac{\chi}{\sqrt{4\pi}}

f_s f_p \approx N_0 \cdot \left[ Z^2 \alpha^2 + \frac{4ZR}{3a} \right]

g_s g_p \approx -N_0 Z^2 \alpha^2

N_o = \frac{Z R_r}{a^3(\nu_s\nu_p)^{3/2}}

R_r = \left[ \frac{2}{\Gamma(2\gamma + 1)} \left( \frac{a}{2ZR} \right)^{1-\gamma} \right]^2

We use expressions for the electron wave functions at the nuclear surface from [19]. In the notation used in this book $E = -\frac{e^2}{2a\nu_s^2}$ is the electron energy, $a = (m_e e^2)^{-1}$ is the Bohr radius, $\Gamma$ is the gamma function, $\gamma = \sqrt{1 - Z^2 \alpha^2}$ and $\chi$ is the electron spinor. Using $(\sigma \cdot n)^2 = 1$ we obtain the matrix element of the interaction of the electron magnetic moment with the radial magnetic field $B$ (11):

$\langle s_{1/2} | V | p_{1/2} \rangle = \frac{3e}{4\pi} \frac{Z(Z\alpha) R_r B_0}{a^2(\nu_s\nu_p)^{3/2}} \left[ \frac{Z^2 \alpha^2}{R} + \frac{2Z}{3a} \right].$

Note that the first term in the brackets is much larger in heavy atoms and the second term is necessary for the correct nonrelativistic limit as $Z\alpha \to 0$. (It is interesting that the “relativistic enhancement factor” in this case can exceed several hundred.)

The electric dipole moment of an atom with one external electron (e.g. Tl, Cs, Fr) generated by the interaction (11) can be calculated using perturbation theory in $V$:

$d_A = \langle \tilde{\psi} | -e r | \tilde{\psi} \rangle = \frac{2}{3} e \sum_n \frac{r_{0n} V_{n0}}{E_0 - E_n},$

where $\tilde{\psi}$ is the perturbed wave function, $r_{0n}$ is the radial integral ($\langle s_{1/2} | r_z | p_{1/2} \rangle = -\frac{1}{3} r_{sp}$) and $V_{n0}$ is the matrix element of $V$ between the $|s_{1/2}\rangle$ and $|p_{1/2}\rangle$ orbitals ($|0\rangle = |6s_{1/2}\rangle$ and $|n\rangle = |np_{1/2}\rangle$ in Cs, $|0\rangle = |6p_{1/2}\rangle$, $|n\rangle = |ns_{1/2}\rangle$ in Tl). There is a simpler way to obtain numerical results for the atomic EDM: to use existing calculations of the electron EDM enhancement factor in atoms and molecules [19–22] or calculations of the atomic EDM produced by the T,P-odd electron-nucleon interaction $V_{ps} = i \frac{G}{\sqrt{2}} k_{1N} \overline{\psi}_e \gamma_5 \psi_e \overline{\psi}_N \psi_N$, where $k_{1N}$ is a dimensionless
interaction constant \((k_1p\) or \(k_{1n}\)) \cite{19,21,23}. Comparison of the matrix element of the effective interaction between the electron EDM \(d_e\) and the atomic electric field \(E\) \((-d_e(\beta - 1)\Sigma \cdot E\) \cite{19,20\}) with expression (11) as well as a similar comparison with the matrix element of the T,P-odd interaction \(V_{ps}\) give the following substitutions in the expressions for the atomic and molecular EDM (and linear energy shifts in external electric fields):

\[
\frac{d_e}{e \cdot \text{cm}} \rightarrow \left(\frac{\text{TeV}}{m}\right)^2 \times -3.92 \cdot 10^{-25} \frac{Z R_\gamma (4 \gamma^2 - 1)}{A^4} \approx \left(\frac{\text{TeV}}{m}\right)^2 \begin{cases} 
-6 \cdot 10^{-23} \text{ for Tl, Hg, TlF, \ldots} \\
-2.6 \cdot 10^{-23} \text{ for Cs, Xe, \ldots}
\end{cases}
\]

\[
k_{1p} \rightarrow -0.87 \cdot 10^{-3} \frac{Z}{\gamma A^4} \left(\frac{\text{TeV}}{m}\right)^2 \approx \left(\frac{\text{TeV}}{m}\right)^2 \begin{cases} 
-1.5 \cdot 10^{-2} \text{ for Tl, Hg, TlF, HgF, \ldots} \\
-1.0 \cdot 10^{-2} \text{ for Cs, Xe, \ldots}
\end{cases}
\]

\[
\tilde{m}^2 = \frac{m^2}{qM} = -\frac{4e}{15B_0}.
\]

To avoid confusion note that the limits are usually presented for \(C^{sp} \equiv k_1 = 0.4k_{1p} + 0.6k_{1n}\).

Using equation (13) and the results of numerical atomic and molecular calculations of T,P-odd effects induced by the electron EDM or the interaction \(V_{ps}\) (see e.g. \cite{19,23}) one can easily calculate the contribution of dyons to the atomic EDM and the constants, \(\chi\) of the T,P-odd interactions \(\chi_eN \cdot s/s\) and \(\chi_NN \cdot I/I\) for molecules (here \(s\) is the electron and \(I\) the nuclear spin and \(N\) is the molecular axis). See tables \[\] and \[\]. Note that for the atoms and molecules with closed electron shells (Hg, Xe, TlF) the effect is proportional to the hyperfine interaction. For these we present rough estimates based on the expression for \(k_{1p}\) in eq. (13). However a more accurate calculation is possible using the approach of ref. \cite{24}.

One can also use eq. (13) to calculate the limit on \(\tilde{m}^2\) from the known limits on the electron EDM \(d_e\) and \(k_{1p}\). At present the best limit follows from the measurement of the EDM of the Tl atom \cite{27}: \(d_A(\text{Tl}) = [-1.05 \pm 0.70 \pm 0.59] \cdot 10^{-24}e \cdot \text{cm}\) or \(d_e = [1.8 \pm 1.2 \pm 1.0] \cdot 10^{-27}e \cdot \text{cm}\). Using eq. (13) or table \[\] we obtain the following limits for the dyon mass \(m\) and the radial magnetic field produced by a particle with charge \(e\) (\(B = B_0 \nabla \delta(r)\)):

\[
\frac{1}{\tilde{m}^2} = \frac{qM}{m^2} = -15B_0 \frac{4e}{[100 \text{TeV}]^2} = \frac{1}{[0.35 \pm 0.23 \pm 0.20]},
\]

\[\]
i.e. \( |\tilde{m}| = \left| \frac{\tilde{m}}{\sqrt{qM}} \right| > 100 \text{ TeV}. \) According to Dirac \[1\] \( eM = \frac{1}{2} \) and if the dyon charge \( q \sim e \) the dyons would be very heavy. The situation is different if the product \( qM \) is proportional to the strength of the T-violating interaction. Recall that according to \[1\] \( |qM| \approx \frac{\theta}{4 \pi} \) and the present limit is \( |\theta| < 4 \cdot 10^{-10} \) \[28,29\]. In this case \( qM < 30 \cdot 10^{-12} \) and so \( m > 100 \text{ MeV} \cdot \sqrt{qM} \cdot 10^{12} \) is not necessarily large. Note also that according to \[12\] the mass of a monopole-antimonopole pair (connected by a \( Z^0 \)-string) in the standard model is in the TeV range.

V. THE NEUTRON ELECTRIC DIPOLE MOMENT

Another way to search for T-violation is by neutron EDM measurement. We can easily estimate the neutron EDM using eqs. (2)–(12) and the constituent quark model in a 3D-oscillator potential. The magnetic moment of the neutron can be reproduced if we assume that the quarks have magnetic moments \( \mu_q = \frac{e_q}{2m_q} \), masses \( m_q = \frac{m_n}{3} \) and are in 1s states with the total spin of the d-quarks equal to 1 (SU(6) model). The matrix element \( \langle s_\frac{1}{2} | V | p_\frac{1}{2} \rangle \) in eq. (12) can be easily estimated using the values of the oscillator wave functions at \( r = 0 \), \( \psi_s(0) \) and \( \psi_p(0) \). \( V \) in this case is the interaction between the magnetic moment of one quark and the radial magnetic field produced by the two other quarks (with total charge \( e_2 + e_3 = -e_1 \)). The sum over the excited states in eq. (12) is saturated by the nearest state \( 1p_{\frac{1}{2}} \) in the oscillator model and the result can be expressed in terms of one parameter, \( a_n = \sqrt{\frac{\hbar}{2m_n \omega}} \), which is in fact the size of the nucleon \( (a_n \sim \frac{1}{m_n}, \text{ where } m_n \text{ is the } \pi \text{-meson mass}) \). After summation over the 3 quarks the EDM of the neutron is:

\[
d_n \sim \sum_{q=1}^{3} \frac{2e_q^3 \cdot (B_0/e)}{3(2\pi)^{\frac{1}{2}} a_n} \cdot \langle \sigma_z^{(q)} \rangle = 1.7 \cdot 10^{-3} \frac{\alpha e}{m^2 a_n} \sim 3 \cdot 10^{-26} e \cdot \text{cm} \left( \frac{\text{TeV}}{m} \right)^2.
\] (15)

Of course this is only an order of magnitude estimate which is however good enough to say that the neutron EDM is \( 10^5 \) times smaller than that of a heavy atom with nonzero electron angular momentum (see table \[1\]). The electron EDM will be even smaller since it should be proportional to \( \frac{m_e}{m^2} \) and \( m_e \sim 10^{-3} m_n \).
A neutron EDM can also be made up of “intrinsic” quark EDMs. This quark EDM (as well as the electron’s EDM) could appear due to radiative corrections with dyon loops. See Fig. 1. Fig. 1a corresponds to the vacuum transformation of a homogeneous electric field to a magnetic field ($F_{\mu\nu}F_{\mu\nu}$ term) and should be omitted. Diagram 1b is forbidden by the Furry theorem (which however must be checked for dyons). Other diagrams such as that in Fig. 1c are of a higher order in the electromagnetic interaction. They are not necessarily small (since $qM$ could be $\sim 1$) but they hardly can exceed the lower order contribution of Fig. 1d calculated in eq. (15).

We stress once more that a similar mechanism can possibly produce an EDM of nucleons and nuclei if we consider “chromomagnetic” monopoles instead of the “usual” monopoles. All that is required is to replace electric ($q$) and magnetic ($M$) charges in the expression for $\tilde{m}^2 = \frac{m^2 qM}{qM}$ by “chromoelectric” (color) and “chromomagnetic” charges. An atomic EDM in this case can be generated by the interaction between electrons and nuclear T,P-odd moments.

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FIG. 1. Diagrams showing the production of a quark EDM due to radiative corrections with dyon loops (a–c) and the diagram showing the mechanism calculated in eq. 15 (d).
### TABLE I. The contribution of a radial magnetic field (due to virtual dyons) to various atomic EDMs.

| Atom   | $\frac{d_A}{e\cdot cm} \cdot (\frac{m}{\text{TeV}})^2$ |
|--------|---------------------------------------------------------|
| Tl     | $3 \cdot 10^{-20}$                                    |
| Cs     | $-3 \cdot 10^{-21}$                                   |
| Hg     | $\sim 3 \cdot 10^{-24}$                               |
| Xe     | $\sim 1.5 \cdot 10^{-25}$                             |
| Xe ($^3P_2$) | $-3 \cdot 10^{-21}$                               |

### TABLE II. The contribution of a radial magnetic field (due to virtual dyons) to the constants $\chi$ for molecules.

| Molecule       | $\chi \cdot (\frac{m}{\text{TeV}})^2$ (Hz) |
|----------------|----------------------------------------------|
| TlF ($\chi_N$) | $\sim -0.1$                                   |
| HgF ($\chi_e$) | 1500                                         |
| YbF ($\chi_e$) | 300                                          |

TABLE I. The contribution of a radial magnetic field (due to virtual dyons) to various atomic EDMs.

TABLE II. The contribution of a radial magnetic field (due to virtual dyons) to the constants $\chi$ for molecules.