Synthesis of quasi-stationary inhomogeneous impedance body of arbitrary shape

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Abstract — Formulas were obtained for the reflection factors of a plane wave from the inhomogeneous impedance plane, which reflects the incident wave in the given direction. The problem of synthesis of impedance inhomogeneous rotation body was solved for an arbitrary shaped generatrix for the given scattering diagram for co- and cross polarizations. Impedance distributions were obtained in explicit form. Restrictions on the class of implementable scatterplots were formulated.

1. Introduction

An efficient way to control scattering characteristics of an object is to use a wide class of electrodynamics structures which physical characteristics are described by impedance boundary conditions [1], for example, the concentrated or distributed impedance loadings [2], [3]. The design of three-dimensional impedance bodies of arbitrary shape with large electrical sizes requires solving of inverse electromagnetic problems with approximate methods. To do so, we need to know the coefficients of wave reflection from inhomogeneous nonstationary impedance plane.

At the time, the paper authors have obtained reflection coefficients for the following cases of inhomogeneous impedance plane:
- the stationary impedance reflects the wave in nonspecular direction which lies in the incidence plane of a wave at the given polarization [4], [5];
- the stationary impedance reflects the wave in a direction at the given polarization [6];
- the quasi-stationary impedance reflects the wave in a direction at the given polarization with the specified frequency [7].

2. Problem statement

An impedance reflector in the form of rotation body (figure 1) is located in a free isotropic space. The reflector is limited by a piecewise-differentiable surface S with the meridian section \( \rho(\theta) \) contour. It’s excited by an electromagnetic wave with \( \omega_s \) frequency coming from an arbitrary direction. The impedance on the surface can be described with Leontovich impedance boundary conditions

\[
\mathbf{n} \mathbf{E} = -\hat{Z} \mathbf{n} \mathbf{H}.
\]

Let’s find the distribution of the passive surface impedance \( Z \) \((ReZ>0)\) that provides the object \( S \) with required scattering diagram \( F(\theta, \phi) \) with frequency \( \omega_s \), such as \( (\Delta \omega = \omega_s - \omega, << \omega_s) \).

3. Scattered Field

Let’s implement the impedance in a form of frequent orthogonal lattice impedance strips \( Z_E(x, y, t), Z_M(x, y, t) \) (figure 2). In this case
\[ E_u = Z_E H_v; \quad E_v = -Z_M H_u. \]

(2)

Figure 1. Problem statement.

*Figure 2. Problem statement.*

The orientation of the bands \( Z_E(x, y, t), Z_M(x, y, t) \) in the S plane is defined by \( \mathbf{u} \) and \( \mathbf{v} \) unit vectors (\( \mathbf{v} = [\mathbf{n}, \mathbf{u}], \cos \alpha = (\mathbf{u}, \mathbf{\tau}); \mathbf{n} - \) normal to S). \( \mathbf{u} = \mathbf{\tau} \cos \alpha + \eta \sin \alpha; \mathbf{v} = -\mathbf{\tau} \sin \alpha + \eta \cos \alpha \). Angle \( \alpha(x, y, t) \) is a function of time and coordinates on the S plane.

We can use the physical optics approximation to solve the problem. To do this, we need to use the reflection factors of a nonstationary inhomogeneous impedance plane which reflects an incident wave into required direction [7]. It’s electric and magnetic fields are orthogonal to the plane of incidence \( \Omega^i \) and reflection \( \Omega^s \) (figure 2) and can be described as:

\[ E_q^i = P_{i, 0} E_{q, 0} + P_{i, 2} H_{q, 2}^i; \quad H_q^i = P_{2, 0} E_{q, 2} + P_{2, 2} H_{q, 2}^i. \]

(3)

The scattered field in the far zone can be described by

\[ E^s_\theta = \frac{1}{4\pi R c} \int_S \frac{\partial}{\partial t} \left\{ \sin \phi \left[ J^{i}_{\theta}(q, t') \cos \theta - J^{m}_{\theta}(q, t') \right] + \cos \phi \left[ J^{i}_{\theta}(q, t') \cos \theta - J^{m}_{\theta}(q, t') \right] \right\} dS_q; \]

\[ E^s_\phi = \frac{1}{4\pi R c} \int_S \frac{\partial}{\partial t} \left\{ \sin \phi \left[ J^{i}_{\phi}(q, t') + J^{m}_{\phi}(q, t') \cos \theta \right] - \cos \phi \left[ J^{i}_{\phi}(q, t') + J^{m}_{\phi}(q, t') \cos \theta \right] \right\} dS_q, \]

(4)

where \( t' = t - R/c \); \( R \) – distance from the integration point to the observation point; \( c \) – light velocity.

Surface currents \( \mathbf{J}^e = \mathbf{\tau} J^e_{\theta} + \eta J^e_{\phi} \) and \( \mathbf{J}^m = \mathbf{\tau} J^m_{\theta} + \eta J^m_{\phi} \) can be determined using the reflection coefficients (3), obtained in the work [7].

4. Impedance synthesis

The required impedance distribution can be found from the boundary conditions (2):

\[ Z_E = \frac{[\mathbf{t}^i, \mathbf{u}] e^{i\phi} + [\mathbf{t}^s, \mathbf{u}] H_0^i e^{i\phi}}{[\mathbf{h}^i, \mathbf{v}] e^{i\phi} + [\mathbf{h}^s, \mathbf{v}] H_0^i e^{i\phi}}; \quad Z_M = -\frac{[\mathbf{t}^i, \mathbf{v}] e^{i\phi} + [\mathbf{t}^s, \mathbf{v}] H_0^i e^{i\phi}}{[\mathbf{h}^i, \mathbf{u}] e^{i\phi} + [\mathbf{h}^s, \mathbf{u}] H_0^i e^{i\phi}}, \]

(5)

where the incident and the scattered fields are: \( \mathbf{H}^i = e^{i\phi}; \mathbf{E}^s = e^{i\phi}; \mathbf{H}^s = e^{i\phi}; \mathbf{E}^i = e^{i\phi}; \Phi_{i,s} = -k_{i,s} (\mathbf{k}^{i,s}, \mathbf{R}); \mathbf{R} \) - radius-vector; in indexes of exponents \( i = \sqrt{-1} \).
The conditions of absolutely reactive impedance implementation can be found from (5):

$$\text{Re}(Z_E) = 0; \text{Re}(Z_M) = 0.$$ 

They define the restrictions on the class of realized scattering diagrams:

$$h_x^s = \frac{\cos \gamma \cos(\alpha + \beta)}{\sin^2(\alpha + \beta) + \cos^2(\alpha + \beta)}; H = \frac{\cos^2 \gamma \cos(\alpha + \beta) + \sin^2(\alpha + \beta)}{\cos \gamma} \quad (6)$$

Absolutely reactive impedance, according to (6), performs full reflection of the waves only in a cone \( \theta_1 = \theta_s \) with amplitude \( H \) and polarization \( h_x^s \). If we also set the polarization of the reflected wave, for example \( l_x^s = 0; h_x^s = 1 \), then (6) implies that a complete reflection is only possible in the plane of incidence of the wave \( (\alpha + \beta = \pm \pi) \). This means, as shown by studies [4] - [7], that, in addition to the beam in the given direction at the given frequency and polarization, there is a specular beam at the co-polarization with initial frequency \( \omega_i \).

So the system of parallel strips \( (\alpha = 0; Z_M = 0; Z_{22} = Z_E; Z_{12} = Z_{21} = 0) \), reflecting the wave in one plane:

$$Z_{22} = i \cos \theta_t \tan [0.5 \{ k_s x \left( \omega / \omega_i \right) \sin \theta_t + \cos \varphi_s \sin \theta_j - \Delta \alpha \}]; \quad (7)$$

creates the scattered field with the following normalized scattering diagram:

$$F_0(\theta, \varphi) = F_0^i(\theta, \varphi) + F_0^s(\theta, \varphi) \quad (8)$$

where

$$F_0^i(\theta, \varphi) = \cos \varphi(\cos \theta_i + \cos \theta_j) \frac{\cos \theta - \cos \theta_i}{4 \cos \theta_t} \left[ \frac{k_i b \sin \varphi \sin \theta_i}{k_j b \sin \varphi \sin \theta_j} \sin \left[ k_s x a (\cos \varphi \sin \theta + \sin \theta_t) \right] \right];$$

$$F_0^s(\theta, \varphi) = \cos \varphi(\cos \theta_i + \cos \theta_j) \frac{\cos \theta + \cos \theta_i}{4 \cos \theta_t} \left[ \frac{k_i b \sin \varphi \sin \theta_i}{k_j b \sin \varphi \sin \theta_j} \sin \left[ k_s x a (\cos \varphi \sin \theta - \cos \varphi_s \sin \theta_j) \right] \right].$$

The main lobe of the diagram \( F_0^i(\theta, \varphi) \) \( (\cos \varphi \sin \theta + \sin \theta_t = 0) \) for \( \omega_i \) frequency of the incident wave is oriented in specular direction \( (\varphi = \pi; \sin \theta - \sin \theta_t = 0; \theta = \theta_s) \). For \( \theta = \theta_s \) it disappears, i.e. there is a complete reflection of the wave \( F_0^s(\theta, \varphi) = F_0^s(\theta, \varphi) \) for a given frequency \( \omega_j \) in the direction \( \cos \varphi \sin \theta - \cos \varphi_s \sin \theta_j = 0 \).

If the quasi-stationary impedance \( Z_{22}(t) \) depends on time only, then when a wave incident from a direction different from the normal \( \theta \neq 0 \), the scattered field at the frequency \( \omega = \omega_1 + \Delta \omega \) has a main lobe with a maximum in nonspecular direction \( \theta = \theta_s = \arcsin (\omega_i \sin \theta_t / \omega_s) \).

Figure 3 shows the RP \( F_0(\theta, \varphi) = F_0^i(\theta, \varphi) + F_0^s(\theta, \varphi) \) of an antenna \( (\varphi = 0) \) with cone-shaped impedance reflector \( \rho(\theta) = -h / \sin(\theta + \beta) \); with vertex angle of \( 2 \beta = 120^\circ \) and base radius of \( a = 100 \text{mm} \); the distance from the cone vertex to the rectilinear radiator is \( h = 100 \text{mm} \).

Impedance simulates a parabolic reflector, which aperture coincides with the cone base (the maximum radiation is directed along the axis). The curve 1 is the RP of an equivalent reflector antenna, the curve 2 is the radiation pattern (RP) of the impedance antenna \( F_0^i(\theta, \varphi) \) at the given frequency \( \omega_s = \omega_1 + \Delta \omega \) and the curve 3 is the RP of the impedance antenna \( F_0^s(\theta, \varphi) \) at the frequency of the irradiator \( \omega_i \). In this case \( F_0^s(\theta, \varphi) \) coincides with the pattern of Kirchhoff’s "black" body model scat-
tered field. \( F_\theta^0(\theta, \varphi) \) in the area of main and the first side lobes is almost identical to the RP of the equivalent parabolic antenna.

The calculations above are given for the physical optics approximation. For the verification of carried out research the figure 4 shows the scattering diagrams of impedance reflector in the form of a disk with the radius of \( a = 100 \text{mm} \), reflecting the incident wave \( \theta_i = 0 \) (\( \lambda = 30 \text{mm} \)) at \( \theta_i = 45^\circ \).

![Figure 3. Radiation patterns.](image1)

![Figure 4. Bistatic RCS diagrams.](image2)

Stationary impedance (7) (\( \Delta \omega = 0 \): \( Z_{22} = i \cos \theta_i g(0.5k_i x_0 \sin \theta_i + \cos \varphi_i \sin \theta_i) \)) implemented by a ribbed structure with a period of 2.5 mm and the width of the grooves of 2 mm. Calculations were performed with Ansoft HFSS. The curve 1 describes the scattering diagram of the impedance disk, the curve 2 describes the scattering diagram of the equivalent perfectly conducting disk, turned away from the direction of incidence of the wave at an angle \( \theta_i/2 = 22.5^\circ \). Precise calculations using HFSS (after 14th steps, when the construction was divided on 731945 tetrahedral, the archived precision was 0.005) demonstrated good coincidence of given (curve 2) and implemented (curve 1) diagrams.

5. Conclusion
The problems of synthesis of isotropic and anisotropic, stationary and quasi-stationary impedance reflectors and antennas with the reflector of arbitrary shape are solved in an explicit form. A class of implemented scattering diagrams is found. The numerical results confirm the validity of the derived formulas. These formulas (5) - (7) allow to solve the synthesis problem with high accuracy.

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