INTEGRATED SUPPLY CHAIN DESIGN MODELS: A SURVEY AND FUTURE RESEARCH DIRECTIONS

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Abstract. Optimization models, especially nonlinear optimization models, have been widely used to solve integrated supply chain design problems. In integrated supply chain design, the decision maker needs to take into consideration inventory costs and distribution costs when the number and locations of the facilities are determined. The objective is to minimize the total cost that includes location costs and inventory costs at the facilities, and distribution costs in the supply chain. We provide a survey of recent developments in this research area.

1. Introduction. A supply chain is a system of facilities and activities that functions to procure, produce, and distribute goods to customers. Supply chain management is basically a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs (or maximize profits) while satisfying service level requirements [72].

The above definition assumes that the supply chain structure is given, and the objective is to minimize the system-wide cost (or to maximize profit) by best-planned movements of goods within the supply chain. However, the physical structure of a supply chain clearly will influence its performance, and it is very important to design an efficient supply chain to facilitate the movements of goods. Some major supply chain decisions include: Which suppliers should we use? How many factories and warehouses should we have and where should we locate them? How do we set the capacity at each location? What products should each factory produce? Given locations and capacities, supply chain decisions will then try to answer questions such as the following: what quantities should we produce and store at these locations? What quantities should be moved from location to location and at what time?

We can roughly classify the above decisions into three levels: strategic, tactical, and operational. In the supply chain design phase, strategic decisions, such as facility location decisions and technology selection decisions play major roles. Once the supply chain configuration is determined, the focus shifts to decisions at the
tactical and operational levels, such as inventory management decisions on raw materials, intermediate products, and end products; and distribution decisions within the supply chain.

The decisions at different levels are typically treated separately in the literature. The inventory theory literature tends to focus on developing and evaluating policies for supplying the distribution centers (DCs) and policies for filling retailer orders. These policies are evaluated based on the resulting service levels (percentage of retailer orders that are filled within the acceptable waiting period), shipping costs, inventory costs, and shortage costs (costs incurred when an order cannot be filled within the acceptable waiting period). See, for example, Hopp and Spearman [38], Nahmias [53] and Zipkin [84]. This line of research tends to incorporate demand uncertainty. On the other hand, the location theory literature tends to focus on developing models for determining the number of DCs and their locations, as well as the DC-retailer assignments. These decisions are evaluated based on resulting operational shipping costs and strategic location costs. With some notable exceptions, this work tends to ignore demand uncertainty. Daskin and Owen [21] provide an overview of facility location modeling as do the recent texts by Daskin [20] and Drezner [24]. See Owen and Daskin [56], Snyder [73], and Daskin, Snyder, and Berger [23] for reviews of facility location models in dynamic and uncertain environments.

As discussed above, the inventory literature tends to ignore the strategic location decision and its associated costs, whereas the location literature tends to ignore the operational inventory and shortage costs, as well as the demand uncertainty and the effects that reorder policies have on inventory and shipping costs. One reason for such disconnection is that the decision maker does not possess detailed information at the non-strategic level in the strategic design phase, thus the facility location decisions are usually made without many inputs regarding inventory and distribution costs. However, failure to take the related inventory and shipment costs into consideration when determining the locations of facilities can lead to sub-optimality. Thus, to achieve important cost savings, the supply chain should be optimized as a whole, that is, the major cost factors that can impact the performance of the supply chain should be considered jointly in the decision model. This is not only true for decisions at the same level (for instance, it is well known that the inventory management scheme and the transportation strategy should be integrated), but also applies to decisions at different levels.

One very important fact the decision makers should realize when making strategic decisions is that many decision parameters, such as demands and costs, may change dramatically from the decision time to the implementation time. Unfortunately, most location models treat all parameters as constants. Comparing with the more flexible and readily re-optimizable inventory management decisions and distribution decisions, facility location decisions are often fixed and difficult to change even in the intermediate term. This calls for models that address the inherent uncertainties in facility location problems. Shen, Coullard, and Daskin [68] propose integrating inventory risk pooling model with location models. By pooling the random demands, not only a better forecast of the demand can be obtained, other benefits, such as cost reduction, can also be achieved.

Strategic supply chain design and redesign have become a major challenge for firms as they simultaneously try to improve customer service and reduce operating costs. Although building a decision support system that integrates these cost
elements with customer service goals is a considerable undertaking for most businesses, doing so can provide a company with a tremendous competitive advantage in the marketplace. Bad locations of facilities, such as plants and DCs, can result in inefficiency and extra costs even if the production, inventory, and shipment plans are well optimized. In this paper, we only focus on models that include facility location decisions. Thus, other popular supply chain design or redesign concepts, such as postponement strategies [48], will not be covered in this paper. We refer the readers to Harrison [37] for a review of related research.

The remainder of the paper is organized as follows. In Section 2 we review some existing integrated decision making models in supply chain management. In Section 3 we describe the basic model from which the later models in this paper are further developed. We introduce a more general model in Section 4 where we also consider the impact of routing cost on supply chain decision making. Section 5 discusses the capacitated version of the basic model. Section 6 adds service considerations into the basic model and considers the tradeoff between minimizing cost and maximizing service. Section 7 introduces a model in which the decision maker has the flexibility to serve a subset of customers to maximize total profit. Section 8 considers a multi-commodity version of the basic model and Section 9 discusses the impact of unreliable supply on supply chain design decisions. Section 10 describes a model with parameter uncertainty and proposes a scenario-based solution algorithm. Finally, Section 11 concludes the paper with future research directions. In each section we only provide brief reviews of some related literature. For detailed reviews, we refer the readers to the papers discussed in each section.

2. Review of integrated decision making models. In the literature, we have seen many papers that study the integration and coordination of any two of the three important supply chain decisions. We will review these papers based on the following three categories: 1) location-routing (LR) models; 2) inventory-routing (IR) models; and 3) location-inventory (LI) models.

• Location-routing (LR) models:

Location and routing decisions are closely related. Both the location problem and the vehicle routing problem are NP-hard in general, which makes the integrated models even more complex. Most of the early studies on LR models focus on heuristic methods, which generally decompose the problem into three sub-problems on facility location, demand allocation, and vehicle routing. For a survey of earlier heuristics, we refer the readers to Laporte [45]. Nagy and Salhi [52] apply a “nested heuristic method” and a Tabu search to LR problems and solve problems with 400 demand points. A two-phase Tabu search approach is recently proposed by Tuzun and Burke [77]. Very recently, Wu, Low, and Bai [81] decompose the LR problem into a location-allocation problem and a vehicle routing problem, then use simulated annealing as the basis to develop search methods for both subproblems.

Laporte and Nobert [47] classify the exact algorithms into three categories: (i) direct tree search, (ii) dynamic programming, and (iii) integer programming algorithms. Berger, Coullard, and Daskin [7] provide a set-partitioning-based formulation for a LR problem with route-length constraints and describe a successful implementation.

LR problems have also been studied in systems with uncertainties. Laporte, Louveaux, and Mercure [46] study a class of stochastic LR problems with unknown demand. Decisions on depot location, fleet size and vehicle routes have to be made
before knowing the actual demand. A penalty will occur if the total demand of a
route exceeds the vehicle capacity (which is called ‘route failure’). Their objective
is to minimize the total costs while constraining the probability of route failure or
the total penalty associated with route failure. Berman, Jaillet, and Simchi-Levi
address location-routing problems in which the decision maker does not know
exactly the number and locations of retailers, instead, the only information available
is the related probability distributions. They suggest several models and provide
efficient heuristics for each model. A recent study is from Chan, Carter, and Burnes
[12], in which they apply the three-dimensional space-filling curve and the modified
Clarke-Wright heuristic to solve the problem.

Most studies mentioned above assume that the distribution system is only for
a single product. Bookbinder and Reece [10] formulate a multi-commodity capa-
citated LR model, which is solved as a generalized assignment problem within a
Benders decomposition based algorithm for the overall distribution/routing prob-
lem.

For reviews on LR problem, please refer to Balakrishnan, Ward, and Wong [4],
and Min, Jayaraman, and Srivastava [51]. Boffey and Karkazis [9] provide a good
survey on LR problems with obnoxious facilities or vehicles.

• Inventory-routing (IR) models:
The IR models involve inventory management and vehicle routing decisions.
There are four key characteristics associated with IR models: demands, which can
be either deterministic or stochastic; fleet size, i.e., the number of available vehicles,
which is either limited or unlimited; length of the planning horizon, which is either
long or short; and finally, number of demand points (e.g., retailers) visited on a
vehicle trip, which some models limit to be one while others allow multiple demand
points on a single route. Kleywegt, Nori and Savelsbergh [41] provide an excellent
survey and classification of IR models using the above four categories plus a fifth
category on the research contribution of the models. More recent literature review
can be found in Kleywegt, Nori and Savelsbergh [42] and Adelman [1].

• Location-inventory (LI) models:
There are many papers that study the location, inventory, and distribution co-
ordination issues, but most of these papers either ignore the inventory costs, or
approximate the nonlinear costs with linear functions. Recently there are a few
papers that consider nonlinear cost terms in their models. Erlebacher and Meller
[26] formulate a highly non-linear integer location/inventory model. They solve
the problem by using a continuous approximation as well as a number of construc-
tion and bounding heuristics. Computation times on a 600 node problem using
an exchange heuristic averaged 117 hours on a Sun Ultra Sparcstation. Shen [65],
Shen, Coullard and Daskin [68], Daskin, Coullard, and Shen [22] propose the joint
location/inventory model in which location, shipment and nonlinear safety stock
inventory costs are included in the same model. They develop an integrated ap-
proach to determine the number of DCs to establish, the location of the DCs, and
the magnitude of inventory to maintain at each center. We will provide detailed
reviews for these models and some recent developments in this research area.

All the models that will be reviewed in this paper are built on the basic model
described in the next Section.
3. Basic model formulation. We consider the design of a three-tiered supply chain system consisting of one or more suppliers, DCs, and retailers. Each retailer has uncertain demand. The problem is to determine how many DCs to locate, where to locate them, which retailers to assign to each DC, how often to reorder at the DC and what level of safety stock to maintain, so as to minimize total location, shipment, and inventory costs, while ensuring a specified level of service.

We assume that location costs are incurred when DCs are established. Line-haul transportation costs are incurred for shipments from a supplier to the DCs. Local transportation costs are incurred in moving the goods from the DCs to the retailers. Inventory costs are incurred at each DC and consist of the carrying cost for the average inventory used over a period of time as well as safety stock inventory carried to protect against stockouts that might result from uncertain retailer demand. We assume that the non-DC retailers maintain only a minimal amount of inventory, which is ignored in the model below.

Inputs and Parameters

- \( I \): set of retailers
- \( J \): set of candidate DC locations
- \( \mu_i \): mean (daily) demand at retailer \( i \), for each \( i \in I \)
- \( \sigma^2_i \): variance of (daily) demand at retailer \( i \), for each \( i \in I \)
- \( f_j \): fixed (annual) cost of locating a DC at \( j \), for each \( j \in J \)
- \( d_{ij} \): cost of shipping a unit from DC \( j \) to retailer \( i \), for each \( i \in I \) and \( j \in J \)
- \( \alpha \): desired percentage of retailer orders satisfied (fill rate)
- \( \beta \): weight factor associated with the shipment cost
- \( \theta \): weight factor associated with the inventory cost
- \( z_\alpha \): standard normal deviate such that \( P(z \leq z_\alpha) = \alpha \)
- \( h \): inventory holding cost per unit of product per year
- \( F_j \): fixed administrative and handling cost of placing an order at DC \( j \), for each \( j \in J \)
- \( L \): DC order lead time in days
- \( g_j \): fixed shipment cost per shipment from the supplier to DC \( j \)
- \( \chi \): a constant used to convert daily demand into annual demand (e.g., 365 if demands occur every day of the year)

Decision Variables

\[
X_j := \begin{cases} 
1, & \text{if retailer } j \text{ is selected as a DC location, and 0} \\
0, & \text{otherwise, for each } j \in J 
\end{cases}
\]

\[
Y_{ij} := \begin{cases} 
1, & \text{if retailer } i \text{ is served by a DC based at location } j, \text{ and 0} \\
0, & \text{otherwise, for each } i \in I \text{ and each } j \in J 
\end{cases}
\]

To simplify notation, we assume all lead times are equal and the holding cost rates are the same at different DCs. The weight factors \( \beta, \theta \) will be used to adjust the relative proportion of different cost components. We next discuss how the working inventory cost and the safety stock cost are calculated.

\[^1\text{For discussion on how to deal with more than one supplier, we refer the readers to} \text{[22].} \]
3.1. Working Inventory Cost. In this subsection, we outline the inventory policy under which the system operates. A DC orders inventory from the supplier using an \((r, Q)\) policy with service level constraints. The frequency of orders and the order quantity at each DC is determined by the mean demand served by the DC which, in turn, is a function of the assignment of retailers to the DC.

Let \(S_j\) denote the set of retailers served by \(j\), \(D_j\) denote the total annual (expected) demand going through DC \(j\) \(\left(D_j = \sum_{i \in S_j} \mu_i = \sum_{i \in I} \mu_i Y_{ij}\right)\), and \(n\) be the number of shipments per year from the supplier. Then the average shipment size in one shipment from the supplier to DC \(j\) is \(D_j/n\), and the average working inventory cost at DC \(j\) is \(\theta h D_j/(2n)\). Assuming the delivery cost from the supplier to DC \(j\) can be calculated as: \(g_j + a_j D_j/n\), where \(g_j\) is the fixed cost of placing an order, then the total annual cost of ordering inventory from the supplier to DC \(j\) is given by

\[
F_j n + \beta(g_j + \mu_j D_j/n)n + \theta h D_j/(2n) \tag{1}
\]

It is easy to show that the optimal value of \(n\) that minimizes the above function is equal to \(\sqrt{\theta h D_j/(2(F_j + \beta g_j))}\). The corresponding total annual working inventory cost associated with DC \(j\) can be expressed as:

\[
\sqrt{2 \theta h D_j(F_j + \beta g_j)} + \beta \mu_j D_j \tag{2}
\]

3.2. Safety Stock Cost. Using Eppen’s risk-pooling result, the amount of safety stock required to ensure that stockouts occur with a probability of \(\alpha\) or less is:

\[
z_{\alpha} \sqrt{L \sum_{i \in I} \sigma_i^2}. \tag{3}
\]

The corresponding holding cost for the safety stock at DC \(j\) is \(\theta h z_{\alpha} \sqrt{L \sum_{i \in I} \sigma_i^2 Y_{ij}}\).

3.3. Model Formulation. Using the cost items described in earlier subsection, we can formulate the following supply chain design model:

Minimize

\[
\sum_{j \in J} \left\{ f_j X_j + \left[ \sum_{i \in I} (\beta \mu_i d_{ij} + \beta a_j \mu_i) \chi Y_{ij} \right] + \sqrt{2 \theta h (F_j + \beta g_j)} \sqrt{\sum_{i \in I} \mu_i \chi Y_{ij} + \theta h z_{\alpha} \sqrt{\sum_{i \in I} \sigma_i^2 Y_{ij}}} \right\}
\]

\[
= \sum_{j \in J} \left\{ f_j X_j + \left( \sum_{i \in I} d_{ij} \chi Y_{ij} \right) + K_j \sqrt{\sum_{i \in I} \mu_i \chi Y_{ij} + q \sqrt{\sum_{i \in I} \sigma_i^2 Y_{ij}}} \right\} \tag{4}
\]

Subject to

\[
\sum_{j \in J} Y_{ij} = 1, \quad \text{for each } i \in I \tag{5}
\]

\[
Y_{ij} - X_j \leq 0, \quad \text{for each } i \in I, j \in J \tag{6}
\]

\[
Y_{ij} \in \{0, 1\}, \quad \text{for each } i \in I, j \in J \tag{7}
\]

\[
X_j \in \{0, 1\}, \quad \text{for each } j \in J \tag{8}
\]
\[
\begin{align*}
\hat{d}_{ij} &= \beta \chi \mu_i (d_{ij} + a_j) \\
K_j &= \sqrt{2 \theta h \chi (F_j + \beta g_j)} \\
q &= \theta h z_w \\
\hat{\sigma}^2_i &= L \sigma^2_i
\end{align*}
\]

The first two terms of the objective function are structurally identical to those of the uncapacitated facility location (UFL) model, which include the fixed cost of locating facilities, and the delivery costs from the DCs to the retailers (represented by terms in \(d_{ij}\)) as well as the marginal cost of shipping a unit from a supplier to a DC (represented by terms in \(a_j\)). The last two terms are related to inventory costs, which are nonlinear in the assignment variables. \(K_j\) captures the inventory effects due to the fixed ordering costs at the DC as well as the fixed transport costs from a supplier to a DC. Finally, \(q\) captures the safety stock costs at the DCs [25]. The value of \(q\) depends on the desired service level. The constraints of the model are identical to those of the UFL location problem [5, 27, 43]. Thus the problem we are studying is more difficult than the standard UFL problem, which is already a notorious NP-hard problem. Constraint (5) stipulates that each retailer is assigned to exactly one DC. Constraint (6) states that retailers can only be assigned to candidate sites that are selected as DCs. Constraints (7) and (8) are standard integrality constraints.

Daskin et al. and Shen et al. [22, 68] have discovered some interesting properties of this model. For the traditional UFL location model, it is always optimal to assign demands to the facility that can serve the demands at lowest transportation cost (i.e., the facility \(j\) with the smallest value of \(d_{ij}\) for retailer \(i\)). However, in this new problem it may be optimal to assign retailers to a more remote DC. Furthermore, they construct examples in which it is optimal to locate a DC at a particular node, but for demands from that node to be assigned to a different DC if \(I = J\). The intuitive reason is that doing so may reduce the inventory and supplier-to-DC transport costs sufficiently enough to offset the increased local delivery costs from the DC to the retailers. In practice, it is unlikely that a supply chain manager would organize the distribution system in this way, even if doing so would result in small cost savings. Fortunately, [68] shows that this does not occur if the ratio of the demand variance to the mean demand is the same for all retailers (i.e., \(\sigma^2_i/\mu_i = \gamma, \forall i \in I\)).

With this assumption on demand, the objective function can be rewritten as

\[
\text{Minimize } \sum_{j \in J} \left\{ f_j X_j + \sum_{i \in I} \hat{d}_{ij} Y_{ij} + \hat{K}_j \sqrt{\sum_{i \in I} \mu_i Y_{ij}} \right\}
\]

(9)

where \(\hat{K}_j = K_j + q \sqrt{\gamma}\).

Shen [65] and Shen et al. [68] outline a column generation approach to minimize the above objective function, subject to the same constraints as those of the UFL problem. [22] proposes a Lagrangian relaxation approach for the same problem. Both the column generation and the Lagrangian relaxation approaches utilize a
low-order polynomial time algorithm for solving a subproblem of the following form:

\[
\text{Minimize } \sum_{i \in I} b_i Z_i + \sqrt{\sum_{i \in I} c_i Z_i}
\]

\[
\text{Subject to: } Z_i \in \{0, 1\} \forall i \in I
\]

where \(c_i \geq 0\). The algorithm involves sorting the retailers in increasing order of \(b_i/c_i\). Using the concavity of the square root operator, [68] shows that an optimal solution to the problem will consist of setting \(Z_i = 1\) for \(i = 1, \ldots, \hat{i}\) and \(Z_i = 0\) for \(i = \hat{i} + 1, \ldots, |I|\) for some \(\hat{i}\) that can be found in linear time. Thus, the complexity of the algorithm for each candidate location is \(O(|I| \log |I|)\). This subproblem must be solved for each candidate facility in each iteration of the Lagrangian relaxation approach and in the pricing problems of the column generation algorithm. Thus, the overall complexity of each iteration of the column generation and Lagrangian approach is \(O(|J||I| \log |I|)\).

We want to highlight two key insights obtained using this model. First, we can compare the results obtained by the model outlined above with those of the UFL model. The UFL model considers only facility location costs and DC to retailer shipment costs. Using the same cost parameters for the two models, the UFL model consistently overestimates the number of facilities needed and underestimates the cost when compared with the model outlined above that incorporates inventory risk pooling effects and economies of scale in transport from the suppliers to the DCs. A network based on the number of facilities suggested by the UFL model would result in over-expenditures on facilities and a failure to capture the cost savings associated with inventory risk pooling and the economies of scale inherent in shipping and order placement represented in the model above. Our computational results show that the over-expenditure averaged nearly 7% above the minimum possible cost while the worst cost penalty was nearly 14%.

Second, when the fixed order costs are reduced significantly, as might occur as a result of such e-business innovations as online ordering and electronic data interchange, the total costs go down but the optimal number of DCs increases, thereby reducing the average DC to retailer distance. This is reassuring since e-business customers tend to be highly service sensitive. Having more DCs located closer to retailers will enable firms to serve such demanding customers better. In short, the model, even with its simplifications, can suggest the sort of network changes needed to capitalize fully on e-business technologies.

While the assumption that the demand variance-to-mean ratio is constant works for many common distributions (e.g., the Poisson distribution), it is restrictive in other contexts. Shu et al. [70] recently developed an algorithm to minimize objective function (4). The approach utilizes a low-order polynomial time algorithm for solving a subproblem of the following form:

\[
\text{Minimize } \sum_{i \in I} a_i Z_i + \sqrt{\sum_{i \in I} b_i Z_i} + \sqrt{\sum_{i \in I} c_i Z_i}
\]

\[
\text{Subject to: } Z_i \in \{0, 1\} \forall i \in I
\]

Shu et al. [70] show that the above problem can be solved efficiently through a corresponding concave minimization problem defined on a polyhedron. To get optimal solutions, it is sufficient to search all the extreme point of that polyhedron. By exploiting the special structures of the above problem, Shu et al. [70] design an algorithm to solve the problem in \(O(|I|^2 \log |I|)\) time.
A variable fixing technique can also be applied to the above solution approaches to speed up the computation. For the column generation approach, the key advantage of the variable fixing method is that once we determine that retailer \( j \in J \) will not be used as a DC in the optimal solution, then we do not need to solve the pricing problem corresponding to \( j \) anymore in the rest of the column generation procedure. In fact, all columns arising from using \( j \) as a DC (generated previously) can also be deleted from the LP. Computational results show that variable fixing techniques can help solve large-sized problems with several hundreds of retailers efficiently.

Results in this section are based on:
- Daskin, M., Coullard C. and Shen, Z.J. (2002). An Inventory-Location Model: Formulation, Solution Algorithm and Computational Results, *Annals of Operations Research* **110**, 83-106.
- Shen, Z.J., Coullard, C. and Daskin, M. (2003). A Joint Location-Inventory Model, *Transportation Science* **37**, 40-55.
- Shu, J., Teo, C.P. and Shen, Z.J. (2004). Stochastic Transportation-Inventory Network Design Problem, *Operations Research* **53**, 48-60.

4. Model with routing cost estimation. In this section, we consider a more realistic modeling of the shipment costs from DCs to retailers. Specifically, we consider a three-tiered supply chain system consisting of one or more suppliers, DCs, and retailers. We assume each retailer has uncertain demand that follows a certain probability distribution. The locations of the suppliers are known, but the exact locations of the retailers are not known. We assume the retailers are uniformly scattered in a connected area. We justify these assumptions by the fact that at the supply chain design stage there is no detailed information on the location and demand of each retailer. We assume the transportation cost exhibits economies of scale under which the average unit cost decreases as the travel distance increases. This realistic assumption will result in a nonlinear term in the formulation that we will present later.

4.1. Related Research. All the models we reviewed so far assume the locations of the retailers are given. However, during the supply chain design phase, usually not much information is given about retailer locations. Thus, they should not be fixed in the supply chain design stage. Continuous approximation models, which use continuous functions to represent distributions of retailer location and demand, have been developed to provide insights into complicated mathematical programming models. It is also widely recognized that continuous models should supplement mathematical programming models, not replace them (Geoffrion [28], Hall [44]). In this section, we plan to use continuous models to approximate the routing cost in our integrated supply chain model. For a review of continuous approximation models and their application in logistics, see Daganzo [18], Langevin, Mbaraga and Campbell [44], and Dasci and Verver [19].

Simchi-Levi (1992) considers a hierarchical planning model for stochastic distribution systems, in which the locations and demand of retailers are determined according to some probability distribution. Different decisions are grouped into three classes: strategic planning, tactical planning, and operational control. He believes that these classes are not independent and an integrated approach is required to avoid sub-optimization, but he did not pursue this approach and instead proposes a hierarchical approach in which the operational costs (location costs and routing
costs) of the probabilistic multi-depot distribution system are estimated first, then the service territories for each DC are designed, and finally, the routing strategy for the system is determined. The fixed cost of establishing a DC is assumed to be a constant. Also, no inventory related costs are included in his model.

We study an integrated stochastic supply chain design model in which we relax two major assumptions in the models from Section 3.3. Instead of assuming the decision maker knows exactly where the retailers are located, we assume the retailers are scattered in a connected region according to a certain distribution. Furthermore, we model the shipment from a DC to its retailers using a vehicle routing model instead of the linear direct shipping model.

4.2. Routing Cost Approximation. We assume each DC sends a truck to visit its retailers every day. We also assume that retailers are scattered in a region of area $A$ according to a spatial customer density (points per unit area) $\delta(a)$, and probability density function $f(a)$ of the customer coordinates $a = (a_1, a_2)$.

The vehicle routing problem (VRP) is an NP-hard problem; furthermore, since we focus on the design phase and only want to estimate the total expected routing costs as a result of different DC locations, we decide to approximate the optimal routing costs. Let $m$ be the total number of retailers served by a specific DC $j$, $q$ be the vehicle capacity, $d_{ij}$ be the distance between retailer $i$ and DC $j$, and $T^*_j$ be the length of the optimal tour that visits DC $j$ and the retailers it serves. Haimovich and Rinnooy Kan (1985) show that the optimal VRP distance $V_j$ can be approximated by the following formula:

$$V_j \approx 2\left(\sum_{i=1}^{m} \frac{\mu_i}{X} d_{ij}\right) / q + (1 - 1/q)T^*_j$$

(12)

Daganzo (1996) shows that if $m$ retailers are independently scattered in a region, then the approximate expected tour length $T_j$ can be expressed as:

$$T_j \approx \phi m E(\delta(a)^{-1/2}) = \phi m E\left((mf(a))^{-1/2}\right) = \phi \sqrt{m E\left(f(a)^{-1/2}\right)}$$

where $\phi = 0.75$ for Euclidean metrics.

Shen and Qi (2004) test the performance of the above formulation using data sets from Christofides, Mingozzi, and Toth (1979). They compare their solutions with those from a meta-heuristic (Agarwal, et al. 2004) that produces optimal or best solutions for the data sets. Their computational results show that when the number of customers is larger than 100, the above formulation produces solutions that are within 5% of those obtained by the meta-heuristic.

They also assume that there is a dedicated truck in each DC that delivers to the retailers every period using a certain route. It is reasonable to assume that, under
some conditions (e.g., the driver does not have to work overtime), the transportation cost related to this route is concave in the distance travelled. That is, the routing cost $RC_j(.)$ should be a concave function of $V_j$.

The integrated supply chain model can then be formulated as

$$ \min \sum_{j \in J} \left\{ f_j X_j + \alpha \sum_{i \in I} \mu_i Y_{ij} + RC_j \left( \sum_{i \in I} \hat{b}_{ij} Y_{ij} \right) + K_j \sqrt{\sum_{i \in I} \mu_i Y_{ij} + q \sqrt{\sum_{i \in I} \sigma_i^2 Y_{ij}}} \right\} $$

Subject to (5) − (8)

where $\hat{b}_{ij} = 2\mu_i d_{ij}/q + \chi(1 - 1/p)\phi \sqrt{\frac{n}{J}} \geq 0$.

Shen and Qi [69] develop a Lagrangian relaxation based solution algorithm. By exploiting the structure of the problem, they find an $O(|I|^2 \log(|I|))$ time algorithm for the following non-linear integer programming problem that must be solved in the Lagrangian relaxation subproblems.

$$ \text{Minimize} \quad \sum_{i \in I} a_i Y_{ij} + RC_j \left( \sum_{i \in I} b_i Y_{ij} \right) + K_j \sqrt{\sum_{i \in I} c_i Y_{ij}} + q \sqrt{\sum_{i \in I} d_i Y_{ij}} $$

Subject to $Y_{ij} \in \{0, 1\}$, for all $i \in I$

To show the effectiveness of their algorithm, Shen and Qi [69] present computational results for several instances of the problem with sizes ranging from 40 to 320 retailers. They also demonstrate the benefits of integrating location, inventory, and routing decisions in the same decision model by comparing the solutions from the integrated model with solutions from models that separate these decisions.

Results in this section are based on:

• Shen, Z.J. and Qi, L. (2004). Incorporating Inventory and Routing Costs in Strategic Location Models. To appear in European Journal of Operational Research.

5. Model with capacitated DCs. Ozsen, Daskin, and Coullard [57] introduce a capacitated version of the model in Section 3. They assume that the DCs have capacity restrictions. The capacity constraints are defined based on how the inventory is managed. Thus, the relationship between the capacity of a DC and the inventory levels are embedded in the model. Their model is capable of evaluating the tradeoff between having more DCs to ensure sufficient system capacity versus ordering more frequently through the definition of capacity.

Assume the inventory is managed by a $(r, Q)$ model with a type-I service level constraint at each DC. Define the following notation for each $j \in J$:

• $C_j$: the capacity of DC $j$
• $r_j$: the reorder point at DC $j$
• $Q_j$: the reorder quantity at DC $j$

It is easy to see that the inventory at DC $j$ reaches its maximum when there is no demand during the lead time. Thus, the maximal accumulation at DC $j$ equals $Q_j + r_j$, and $r_j = \text{safety stock} + E[\text{demand during lead time}]$. Let $D_j$ be the expected annual demand of retailers served by DC $j$. We can write the capacity constraint for DC $j$ as:
Or equivalently,
\[ Q_j + r_j \leq C_j \]

Adding this constraint to Problem (4)-(8) in Section 3.3, we obtain a new model with nonlinear terms in both the objective function and the constraints.

Ozsen, Daskin, and Coullard [57] apply a Lagrangian relaxation solution algorithm to solve this problem. The Lagrangian subproblem is also a non-linear integer program, and they propose an efficient algorithm for the continuous relaxation of this subproblem. They conduct an extensive computational study to demonstrate the effectiveness of their algorithm. They also show that although typically the number of DCs open increases as more and more DCs have limited capacity, however, the increase is not dramatic.

Results in this section are based on:
• Ozsen, L., M. S. Daskin and C. R. Coullard (2003). “Capacitated Facility Location Model with Risk Pooling,” submitted.

6. Model with service considerations. When designing supply chains, firms are often faced with the competing objectives of improving customer service and reducing cost. We extend the basic model in Section 3 to include a customer service element and develop practical methods for quick and meaningful evaluation of cost/service tradeoffs. Service is measured by the fraction of all demands that are located within an exogenously specified distance of the assigned DC.

In all the models we reviewed so far, a retailer may be assigned to a DC that is very far away if doing so reduces the total costs. This may not be desirable in a highly competitive business environment. Many companies consider service time, defined as how long it takes to transport the products (services) to the customer site when they are needed, to be a critical performance metric. For example, General Motors Corporation developed a program in Florida to reduce the amount of time that Cadillac buyers wait for new cars. New cars are held at a DC and are made available to the dealers on order. A 24-hour delivery standard is used to deliver new cars from DCs to dealers (Wall Street Journal, 1995). This program has since been expanded to include areas in Maryland and California. Another example comes from the PC industry. To provide high quality service, IBM has implemented a parts stocking plan to support a time-based service strategy. Specifically, IBM wants to keep the response time within a threshold level, say 2 hours to one set of customers, 8 hours to another set of customers, and 24 hours for the rest of the customers [49]. In both of the above environments, determining the locations of the DCs that will hold the inventory (e.g., vehicles, parts) is critical, since it will impact both the total costs (facility location costs, inventory costs, and transportation costs) and the customer responsiveness. There is a clear need to evaluate trade-offs between the total cost and customer service. In this section, we incorporate customer responsiveness into a supply chain design model and analyze such trade-offs.

6.1. Related Research. Ross and Soland [62], in one of the earliest papers on multi-objective location problems, argue that practical problems involving the location of public facilities should be modeled as multi-objective problems. Cost and service are the typical objectives, although there exist several distinct objectives in each of those two categories: fixed investment cost, fixed operating cost, variable
operating cost, total operating cost, and total discounted cost are all reasonable cost objectives to consider; and both demand served and response time (or distance traveled) are appropriate objectives for service measurement. They treat such multi-objective problems in the framework of a model for selecting a subset of M sites where public facilities are established to serve client groups located at N distinct points. They use an interactive approach that involves solving a finite sequence of generalized assignment problems. For a review of other multi-objective location models, we refer the reader to Shen [66].

Nozick and Turnquist [55] present an optimization model which is closely related to ours: it minimizes cost and maximizes service coverage. They adopt the simple \((S - 1, S)\) inventory policy and use a linear function to approximate the safety stock inventory cost function, which is then embedded in a fixed-charge facility location model.

6.2. Model Formulation. Model (4)-(8) in Section 3.8 captures important facility location, transportation and inventory costs. Some retailers may be served very well, in the sense that they are located very close to the DCs to which they are assigned, while other retailers may be served very poorly by this criterion. The maximal covering location problem (Church and ReVelle [15]) maximizes the number of customers that can be covered by a fixed number of facilities. Customer \(i\) is covered if node \(i\) is assigned to a facility that is within \(d_c\) of node \(i\), where \(d_c\) is the coverage distance. Instead of maximizing covered demand volume, for fixed total demand, we can minimize the uncovered demand volume. In a manner similar to Daskin [20], we can then formulate a model that simultaneously minimizes the fixed costs of the facilities and a weighted sum of the uncovered demand volume as follows:

\[
\min \sum_{j \in J} f_j X_j + W \sum_{j \in J} \sum_{i \in I} \tilde{d}_{ij} Y_{ij} 
\]

Subject to (5) – (8)

where \(W\) is the weight on the uncovered demand volume and

\[
\tilde{d}_{ij} = \begin{cases} 
\chi \mu_i, & \text{if } d_{ij} > d_c \\
0, & \text{if not}
\end{cases} 
\]

Objectives (4) and (14) can be combined as follows:

\[
\min \sum_{j \in J} f_j X_j + \sum_{j \in J} \sum_{i \in I} \tilde{d}_{ij} Y_{ij} + K_j \sqrt{\sum_{i \in I} \mu_i Y_{ij}} + q \sqrt{\sum_{i \in I} \sigma_i^2 Y_{ij}} 
\]

where \(\tilde{d}_{ij} = \tilde{d}_{ij} + W \tilde{d}_{ij}\).

This model is structurally identical to (4). The only difference is that we penalize all assignments of demand nodes to DCs that are more than \(d_c\) away from the DC. By varying the weight \(W\) on uncovered demand volume, we can trace out an approximation to the set of non-inferior solutions to the tradeoff between location-inventory costs and customer responsiveness. Very small values of \(W\) correspond to minimizing the total location-inventory cost; very large values of \(W\) are equivalent to minimizing the total location-inventory cost subject to the constraint that all demands are covered within a distance of the DC to which they are assigned.
Shen and Daskin [75] propose two solution approaches, one based on the weighting method [16] and the other based on genetic algorithms, to solve this multi-objective model. The genetic algorithm performs very well compared to the weighting method, and it is the only feasible approach for large-sized problem instances, since the weighting method requires excessive computational time in such cases.

Through tests of the model on datasets ranging in size up to 263 demand nodes, they show that it is important for a company to find the right tradeoff between supply chain cost and service. The cost difference between the cost-minimization solution and the service-maximization solution can be quite large. In their tests, the cost of the service-maximization solution ranged from 6 to 20 times larger than that of the cost-minimization solution. They also show that significant improvements in customer service can often be achieved at relatively small cost. While this often entails locating additional DCs, the cost of the incremental facilities is largely offset by reduced outbound transportation costs. Furthermore, they find that it can be very time consuming to find the optimal cost-service tradeoff curve for large problem instances. In this case, their proposed genetic algorithm can be a very good alternative to solve the problem since it can generate high quality solutions quickly. Furthermore, the number of non-dominated solutions found by the genetic algorithm greatly exceeds the optimal number of supported solutions found by the Lagrangian procedure. Similar algorithms can be developed for more general supply chain design problems.

Results in this section are based on

- Shen, Z.J. and Daskin, M. (2005) Trade-offs Between Customer Service and Cost in Integrated Supply Chain Design, *M&SOM*, 7, 188-207.

7. **Profit-maximizing model with demand choice flexibility.** In this Section, we use “customer” instead of “retailer” to denote the party that requests service from DCs.

Minimizing total cost has been the primitive objective in most of the supply chain network design models. These models typically require that every customer’s demand has to be satisfied. However, for a profit-maximizing business, it may not always be optimal to satisfy all potential demands, especially if the additional cost is higher than the additional revenue associated with servicing these customers. Furthermore, if the company is facing competition, sometimes it might be more profitable to lose some potential customers to competitors since the cost of maintaining these customers can be prohibitively high. Most supply chain related costs, such as location, transportation, and inventory related costs of an item depend on total demand volume, and no clear method exists for determining a customer’s profitability a priori, based solely on the characteristics of this customer.

Shen [67] addresses this problem by proposing a profit-maximizing supply chain design model in which a company can choose whether to satisfy a customer’s demand. Specifically, he considers a company that produces a single product. After production, this product will be shipped to several DCs for distribution. The cost of establishing a DC is a concave function of the annual demand flow through the DC [31]. Delivering the product from the production site to a DC results in a fixed cost and a variable per unit cost. The company can set different sale prices for different regions (defined by the DCs) depending on customers’ willingness to buy in each region. A delivery cost will also be charged to the customer, based on the distance between the customer and the DC that serves the customer. Shen [67]
further assumes that every customer has a reserve price, and that the company will lose a customer if the total price charged by the company (sale price plus the delivery cost) is higher than the customer’s reserve price.

7.1. Related Research. The literature on integrating cost-minimizing supply chain network design models with profit-maximizing microeconomic theory is limited. Hansen et al. [35] and Hansen et al. [36] provide surveys on profit-maximizing location models.

Zhang [83] considers a profit-maximizing location model, where a firm needs to decide where to locate a single DC to serve the customers and to set the price for its product, so as to maximize its total profit. He shows that under certain assumptions on the complexity counts, a special case of this problem can be solved in polynomial time. However, he does not consider any cost related terms in his model, not even the DC location costs.

Another location model is proposed by Meyerson [50], where he considers a profit-earning facility location problem for a given set of demand points, and the decision maker needs to open some facilities such that every demand may be satisfied from a local facility and the total profit is maximized. Instead of incurring a cost, opening a facility can gain a certain profit which is a function of the amount of demand the facility satisfies. Since this profit-earning problem is NP-hard, Meyerson develops an approximation algorithm that is based on linear program rounding.

Demand choice flexibility in inventory models is proposed by Geunes, Shen, and Romeijn [30]. Their models address economic ordering decisions when a producer can choose whether to serve multiple markets. They assume the per unit revenue from serving a specific customer is exogenously given, not a decision variable. Furthermore, their models are pure inventory models; supply chain design related decisions, such as location decisions, are absent from the models.

Shen [67] considers not only location decisions and customer-DC assignment decisions, but also production and inventory related decisions within a supply chain network. It is assumed that after production, the product is shipped to several DCs, and then from DCs to each customer the company has decided to serve. Contrary to Meyerson’s model, [67] assumes opening a facility incurs a certain cost, however, by serving the customers assigned to this facility, a certain amount of profit will be gained, which is also a function of the total demand served by the facility.

7.2. Model Formulation. The profit-maximizing supply chain network design problem can be described as following: Given a production site \( o \), a set \( I \) of customers and a set \( J \) of candidate DC locations, the decision maker wants to determine (i) the number and locations of the DCs, (ii) the assignments of customers to DCs, (iii) the inventory replenishment strategy at each DC, and (iv) the sale price of the product at each region, so that the total profit is maximized. The profit is equal to the revenue minus the total cost, which includes the DC location cost, inventory holding cost at DCs, and delivery cost to DCs.

To model this problem, we define the following additional notation:
\( f_j(x) \) (annual) cost of locating and operating a DC at \( j \), for each \( j \in J \). It is a concave function of the annual demand \( x \) flowing through DC \( j \).

\( K \) fixed cost of delivering product from the production site to a DC

\( d_{oj} \) distance from the production site \( o \) to DC \( j \), \( j \in J \)

\( d_{ij} \) distance from the DC \( j \) to customer \( i \), for \( i \in I, j \in J \)

\( c(d_{oj}) \) delivery cost per unit from the production site \( o \) to DC \( j \).

It has the following structure:

\[
c(d_{oj}) = \begin{cases} 
  c_1, & \text{if } 0 < d_{oj} \leq d_1 \\
  c_2, & \text{if } d_1 < d_{oj} \leq d_2 \\
  \vdots \lec
  c_t, & \text{if } d_{t-1} < d_{oj} \leq d_t 
\end{cases}
\]

It is typically assumed that \( c_1 \leq c_2 \leq \ldots \leq c_t \).

\( p_j \) sale price per unit at the region served by DC \( j \)

\( u_i \) customer \( i \)'s reserve price

\( g(d_{ij}) \) delivery cost per unit charged to customer \( i \) if delivery is made from DC \( j \). It has the following structure:

\[
g(d_{ij}) = \begin{cases} 
  v_1, & \text{if } 0 < d_{ij} \leq d_1 \\
  v_2, & \text{if } d_1 < d_{ij} \leq d_2 \\
  \vdots \lec
  v_t, & \text{if } d_{t-1} < d_{ij} \leq d_t 
\end{cases}
\]

It is typically assumed that \( v_1 \leq v_2 \leq \ldots \leq v_t \).

Shen [67] assumes the company will lose a customer if the total price a customer has to pay is higher than the customer’s reserve price, i.e., if \( u_i < p_j + g(d_{ij}) \). He defines

\[
r_i(p_j, d_{ij}) = \begin{cases} 
  p_j + g(d_{ij}) - c(d_{oj}), & p_j + g(d_{ij}) \leq u_i \\
  0, & p_j + g(d_{ij}) > u_i 
\end{cases}
\]

Shen [67] formulates the decision problem as a set-covering model, and presents a branch-and-price algorithm – a variant of branch-and-bound in which the nodes are processed by solving linear-programming relaxations via column-generation – to solve this model. To solve the resulting pricing problem, it suffices to find, for every DC \( j \in J \), a maximum-reduced-cost set \( R^*_j \subset I \), that uses \( j \) as the designated DC. Thus, the pricing problem reduces to finding \( R^*_j \) for each \( j \in J \). To find \( R^*_j \), the following integer programming problem, \( P_j \), must be solved:

\[
P_j : \text{Maximize } \sum_{i \in S} a_i Y_i - \sqrt{2K\sigma_1} \sqrt{\sum_{i \in S} \mu_i Y_i} - h z_\alpha \sqrt{\sum_{i \in S} \sigma_i^2 Y_i} - f_j \left( \sum_{i \in S} \mu_i Y_i \right) \quad \text{subject to} \quad Y_i \in \{0,1\}, \quad \forall i \in S, S \subseteq \Omega \] (16)

If the variance-to-mean ratio at each retailer is identical for all retailers, that is, if \( \mu_i = \gamma \sigma_i^2 \forall i \in I \), then the objective function of \( P_j \) can be written as
\[ P_j : \text{Maximize} \ \sum_{i \in S} a_i(p_j)Y_i - \sqrt{2K\hat{h} \sum_{i \in S} \mu_i Y_i - f_j \left( \sum_{i \in S} \mu_i Y_i \right)} \]  
subject to \[ Y_i \in \{0,1\}, \ \forall i \in S, S \subseteq \Omega \]

where \( a_i(p_j) \equiv r_i(p_j, d_{ij})\mu_i - \bar{\pi}_i \), and \( \hat{K} = K + q\sqrt{L\gamma} \).

Since the objective function (17) of \( P_j \) is a convex, solving the relaxed version (with the integrality restriction on the \( Y_i \) variables relaxed, i.e., with feasible region \( 0 \leq Y_i \leq 1 \)) results in an extreme point solution.

Shen [67] shows that for a given \( p_j \) value, problem \( P_j \) can be solved in time \( O(|I|\log|I|) \). However, the sale price \( p_j \) is also a decision variable in problem \( P_j \). Let \( P \) denote the set of values that \( p_j \) can take, he then shows that to find the optimal sale price \( p_j \), one only needs to consider at most \( n \) different values.

**Lemma 1.** There are at most \( m_1 \leq |I| \) different \( p_j \) values that we need to search to find the price \( p_j \) that maximizes (17).

The author wants to emphasize the importance of using profit maximization as the primary objective instead of cost minimization for many businesses. Maximizing profit is not the same as minimizing total cost when the revenue is not fixed, which is true for most businesses nowadays. We hope this problem will attract more attention from the research community in the near future. Important additional research can and should flow from this work.

Results in this section are based on
- Shen, Z. J. (2006). A Profit Maximizing Supply Chain Network Design Model with Demand Choice Flexibility, *Operations Research Letters*, 34, pp. 673-682.

8. **Model with multiple commodities.** In this section, we study a multi-commodity supply chain design model. Here a “commodity” can either represent a specific product or a product category.

Nowadays, retailers carry thousands of different products and the amount of data involved in a supply chain design model can be overwhelming. It is not necessary, and may not be possible, to account for all distinct products in the strategic supply chain design phase. Aggregated information at the product category level should be used instead (Simchi-Levi, Kaminsky, Simchi-Levi [72]).

Warszawski and Peer [80] and Warszawski [79] are among the first to study the multi-commodity location problem. These models consider fixed location costs and linear transportation costs, and assume that each warehouse can be assigned at most one commodity.

Geoffrion and Graves [29] consider the capacitated version of the multi-commodity location problem in which they impose capacity constraints on the suppliers and the DCs. They also assume that each customer must be served with all the products it requires from a single DC or directly from a supplier.

The costs of many economic activities in a supply chain, such as production and carrying inventory, exhibit economies of scale; however, because of the difficulty associated with solving the resulting nonlinear model, such economies of scale have been ignored in most of the location literature. In this section, we propose a supply chain design model that considers multiple products (or product categories) and economies of scale cost terms. We show the multi-commodity extension of many well-studied problems can be treated as special cases of our model.
Later in this section, we use the term *facility* to denote a physical site whose cost exhibits economies of scale. Such a facility can be a plant that produces a certain type of product, or a warehouse or a DC that keeps inventory. Our multi-commodity supply chain design model can be stated as follows: We are given a set of alternative facility locations, a set of retailers, a set of different products, and a certain activity whose cost can be modelled using a concave function. The objective is to design a supply chain system that can serve outside demand at minimum cost.

We use the following additional notation in this section:

- \( L \): set of commodities. A commodity can either represent a specific product or a product category;
- \( I \): set of retailers that have demand for commodity \( l \in L \);
- \( \mu_{il} \): mean annual demand from retailer \( i \) for commodity \( l \).

When facility \( j \) is used to serve the retailers in set \( S \subset I \), the associated total cost is given by the following cost components.

- \( \sum_{i \in S, l \in L} d_{ijl} \mu_{il} \): the term \( d_{ijl} \mu_{il} \) is linear in \( \mu_{il} \) where \( d_{ijl} \) is a constant;
- \( \sum_{l \in L} G_{jl}(\sum_{i \in S} \mu_{il}) \): the term \( G_{jl}(\sum_{i \in S} \mu_{il}) \) is concave and non-decreasing in the total mean demand for commodity \( l \) at facility \( j \).

For instance, when \( d_{ijl} \) corresponds to unit transportation cost for commodity \( l \) between facility \( j \) and retailer \( i \), \( \sum_{i \in S, l \in L} d_{ijl} \mu_{il} \) captures the total transportation cost if facility \( j \) provides the retailers in set \( S \) with commodity \( l \). The term \( G_{jl}(\sum_{i \in S} \mu_{il}) \) can be interpreted as the economies of scale cost term within the supply chain. For example, it can represent the facility operation and inventory replenishment cost, or the cost of purchasing and operating a technology at a plant.

**Decision Variables**

\[ X_j := 1, \text{ if } j \text{ is selected as a facility location, and } 0 \text{ otherwise}, \quad \text{for each } j \in J \]
\[ Y_{ijl} := 1, \text{ if the demand for commodity } l \text{ of retailer } i \text{ is served by } j, \text{ and } 0 \text{ otherwise}, \quad \text{for each } i \in I, j \in J, l \in L \]

The problem can be formulated as:

\[
\min \sum_{j \in J} \left\{ f_j X_j + \sum_{l \in L} \left[ \sum_{i \in I} (d_{ijl} \mu_{il}) Y_{ijl} + G_{jl}(\sum_{i \in I} \mu_{il} Y_{ijl}) \right] \right\}
\]

Subject to

\[ \sum_{j \in J} Y_{ijl} = 1, \quad \text{for each } i \in I, l \in L \]
\[ Y_{ijl} - X_j \leq 0, \quad \text{for each } i \in I, j \in J, l \in L \]
\[ Y_{ijl} \in \{0,1\}, \quad \text{for each } i \in I, j \in J, l \in L \]
\[ X_j \in \{0,1\}, \quad \text{for each } j \in J \]

Shen [66] proposes Lagrangian relaxation embedded in a branch and bound algorithm to solve the above problem. The Lagrangian subproblem \( P(j) \) can be decomposed into \( L \) smaller problems for each commodity \( l = 1, 2, \ldots, |L| \), and \( P(j) \) can be solved efficiently after solving these \( L \) smaller problems.

The proposed model includes many well-studied problems as special cases, for example, the plant location and technology acquisition problem (Dasci and Vertre [19]), the multi-commodity version of the location-inventory problem (Shen [65]).
Shen, Coullard, Daskin [68], Daskin, Coullard, Shen [22], the safety stock optimization problem (Agarwal and Palekar [2]), and the market selection inventory problem (Geunes, Shen, Romeijn [30]).

Shen [66] also discusses an interesting scenario where the objective function contains not only product-dependent cost terms, but also some cost terms that are concave functions of the total demand for different products. By utilizing the results from Shu, Teo, and Shen [70] and Shen and Qi [69], problems whose objective functions have two or three concave terms can also be solved efficiently.

Results in this section are based on

- Shen, Z. J. (2005). Multi-commodity Supply Chain Design Problem. *IIE Transactions*, 37, 753-762.

9. **Model with unreliable supply.** A supply chain network is a complex system in which there are many uncertainties, such as demands from the customers, yields of the suppliers, and delivery reliability. Because most supply chain design decisions (for example, facility location) are irreversible, it is not reasonable to treat these uncertain factors in a supply chain as deterministic parameters.

Qi and Shen [60] consider the following multi-period problem: in each period, multiple retailers order a specific product from a supplier, and the supplier ships the product to some intermediate facilities selected from a set of candidate locations. Some assembly and packaging activities may be performed to satisfy orders from different retailers. Due to a certain service requirement, some amount of final product inventory must be kept in these facilities, and be ready for delivery to retailers at the beginning of each period (They assume that any retailer can be served by more than one facility). However, the amount of final product delivered on time to a retailer may not be exactly the amount this retailer requests from the supplier, because of the quality issues resulting from different production/assembly capabilities in different facilities, mistakes made during the assembly/packaging operations, the weather or other factors that may impact the on-time delivery from facilities to retailers. Thus, the decision maker needs to take this unreliability issue into consideration when designing the supply chain.

The amount of goods delivered from a facility to a retailer is modeled by the product of the order quantity from this retailer and a random variable associated with this facility, which is called the **reliability coefficient**. This method is prevalent in the random yield literature (for a review, see [82]).

9.1. **Related Literature.** Supply disruptions have been considered in inventory models [6, 32, 54, 58, 59]. These papers assume there is only one supplier. [17, 54, 70] consider two or more suppliers in their models, but with only one retailer.

Vidal and Goetschalckx [78] give a qualitative discussion of global supply chain design with a very general large-scale MIP that incorporates the reliability of suppliers into the constraints. A more efficient solution algorithm is proposed in [63]. Bundschuh, Klabjan, and Thurston [11] study a model for a multi-stage supply chain network, where each node in the network is the supplier of the nodes on the next stage. They also consider the reliability of suppliers by adding constraints to the model.

9.2. **Model Formulation.** Qi and Shen [60] consider facility location costs, working inventory costs and safety stock costs at facilities, as well as the penalty costs and transportation costs associated with retailers. The retail price of the product
at each retailer is given, and the objective of our model is to maximize the expected annual profit. They define the following additional notation for this problem:

**Additional Parameters**
- $c$: purchasing price from the supplier per unit of product
- $\chi$: number of periods per year
- $R_j$: the reliability coefficient associated with facility $j$, $j \in J$, which is a random variable between 0 and 1. Let $\theta_j = E(R_j)$ and $\tau^2_j = Var(R_j)$
- $p_i$: retail price at retailer $i$ per unit of product, $i \in I$
- $\pi_i$: penalty cost of lost goodwill at retailer $i$ per unit of product, $i \in I$
- $v_i$: salvage value at retailer $i$ per unit of product, $i \in I$

**Decision Variables**
- $X_j$: \(\begin{cases} 1 & \text{facility } j \in J \text{ is open} \\ 0 & \text{otherwise} \end{cases}\)
- $Q_{ij}$: order quantity at facility $j \in J$ from retailer $i \in I$ in each period (We assume that the order quantity from retailer $i$ to facility $j$ is the same in each period.)

Let $X$ denote the $1 \times m$ matrix $(X_j, j \in J)$, and $Q$ denote the $n \times m$ matrix $(Q_{ij}, i \in I, j \in J)$. $R_jQ_{ij}$ represents the actual quantity retailer $i$ receives from facility $j$ in each period if retailer $i$ orders $Q_{ij}$ from facility $j$, and the reliability coefficient associated with facility $j$ is $R_j$.

**Profit at Retailer $i \in I$**

They assume that open facilities only deliver products to retailers at the beginning of every period, and that each retailer acts like a “newsboy” in the Newsboy problem and maintains only a minimal amount of inventory. The holding cost of the inventory at retailers can therefore be ignored in the integrated model.

Retailer $i$’s ($i \in I$) inventory problem can be formulated as a Newsboy problem, which is proposed and studied by Dada, Petruzzi, and Schwarz [17]:

Maximize \( T_i(Q) \equiv E\bigl\{ p_iD_i + v_i\left[ \sum_{j \in J} R_jQ_{ij} - D_i \right]^+ - (p_i + \pi_i)[D_i - \sum_{j \in J} R_jQ_{ij}]^+ \bigr\} \)

\( = \sum_{j \in J} \left\{ f_jX_j + c\chi \sum_{i \in I} Q_{ij} + a_j\chi \sum_{i \in I} Q_{ij} + K_j \sqrt{\sum_{i \in I} Q_{ij}} \right\} + \chi \sum_{i \in I} T_i(Q) \)  

s.t. \( Q_{ij} \geq 0 \quad j \in J \)

**Integrated Model**

Qi and Shen [60] assume that the per-unit purchase and transportation costs are based on the quantity ordered, not the quantity actually received by the retailers. Based on this assumption, they formulate the following integrated model:

**Problem P:**

Maximize \( -\sum_{j \in J} \left\{ f_jX_j + c\chi \sum_{i \in I} Q_{ij} + a_j\chi \sum_{i \in I} Q_{ij} + K_j \sqrt{\sum_{i \in I} Q_{ij}} \right\} + \chi \sum_{i \in I} T_i(Q) \)

\( = \phi(Q) - \sum_{j \in J} f_jX_j \)

s.t. \( 1 - e^{-\beta Q_{ij}} \leq X_{ij} \leq 1, j \in J \)
\( Q_{ij} \geq 0 \quad i \in I, j \in J \)
\( X_{ij} \in \{0, 1\} \quad j \in J \)
where \( \phi(Q) \equiv -\sum_{j \in J} \left( (c + a_j) \chi \sum_{i \in I} Q_{ij} + K_j \sqrt{\sum_{i \in I} Q_{ij}} \right) + \chi \sum_{i \in I} T_i(Q) \).

The objective of Problem P is to maximize the expected annual profit of the entire system including all facilities and retailers. In the objective function, the first term represents the facility location cost for opening facilities and the second term is the annual purchasing cost from the supplier. The third and forth terms represent the working inventory cost and the safety stock cost associated with each facility, respectively. And the last term is the profit earned at retailers.

The first constraint stipulates that retailers can only order from open facilities. An exponential function is used to formulate this restriction, instead of other commonly used methods such as the big-M method, because of the quick convergence property of the exponential function. The positive constant \( \beta \) in the first constraint is used to expedite the convergence.

Let \((X^*, Q^*)\) denote the optimal solution to Problem P. Since Problem P is a highly nonlinear and mixed-integer optimization problem, with an objective function neither convex nor concave, it is very difficult to be solved directly by any standard algorithm. Qi and Shen [60] first study the relationship between Problem P and its Lagrangian Dual problem by relaxing the first constraint.

**Problem LR:**

\[
v(\lambda) = \text{Maximize} \quad \phi(Q) - \sum_{j \in J} f_j X_j + \sum_{j \in J} \sum_{i \in I} \lambda_{ij} \left( X_j - 1 + e^{-\beta Q_{ij}} \right)
\]

s.t.
\[
Q_{ij} \geq 0 \quad i \in I, j \in J
\]
\[
X_j \in \{0, 1\} \quad j \in J
\]

Let \( \lambda \) denote the \( n \times m \) matrix \((\lambda_{ij}, \quad i \in I, j \in J)\), and rewrite the objective function of LR as
\[
-\sum_{j \in J} (f_j - \sum_{i \in I} \lambda_{ij}) X_j + \phi(Q) + \sum_{j \in J} \sum_{i \in I} \lambda_{ij} (e^{-\beta Q_{ij}} - 1),
\]

which we denote by \( L(\lambda, X, Q) \).

Since \( X \) and \( Q \) are independent in Problem LR, the optimal solutions for \( X \) and \( Q \) can be determined separately. Since \( X_j \in \{0, 1\}, \forall j \in J \), its optimal solution can be directly determined by its corresponding coefficient, \(-f_j + \sum_{i \in I} \lambda_{ij}\). If the coefficient of \( X_j \) is positive, then \( X_j = 1 \); if it is negative, \( X_j = 0 \). If the coefficient of \( X_j \) is equal to 0, they use the following rule to get a feasible solution: \( X_j = 0 \) when \( Q_{ij} = 0 \), and \( X_j = 1 \) otherwise, \( \forall i \in I \). Qi and Shen [60] propose an algorithm based on the bisection search and the outer approximation algorithm [39-40]. They show that their algorithm is more efficient than the outer approximation algorithm when being applied to the above problem.

Results in this section are based on:

- Qi, L. and Shen, Z. J. (2005). A Supply Chain Design Model with Unreliable Supply, Working paper, University of California at Berkeley.

10. **Model with parameter uncertainty—scenario based approach.** The earlier models assume the decision maker knows the demand parameters. That is, the mean \( \mu_i \) and the standard deviation \( \sigma_i \) of retailer \( i \)'s demand are known parameters. Snyder, Daskin, and Teo [74] present a stochastic version of the model in Section 10 that explicitly handles parameter uncertainty by allowing parameters to be described by discrete scenarios, each with a specified probability of occurrence. The goal is to choose DC locations, assign retailers to DCs, and set inventory levels.
at DCs to minimize the total systemwide cost. To model this problem, we need to define the following additional notation:

**Set**
- $S$: set of scenarios, indexed by $s$.

**Parameters**
- $\mu_{is}$: mean daily demand at retailer $i$ in scenario $s$, for $i \in I$, $s \in S$
- $\sigma^2_{is}$: variance of daily demand at retailer $i$ in scenario $s$, for $i \in I$, $s \in S$
- $d_{ij}s$: per-unit cost to ship from DC $j$ to retailer $i$ in scenario $s$, for $i \in I$, $j \in J$, $s \in S$
- $q_s$: probability that scenario $s$ occurs, for $s \in S$

**Decision Variables**
- $x_j := 1$, if $j$ is selected as a facility location, and 0 otherwise, for each $j \in J$
- $y_{ijs} := 1$, if retailer $i \in I$ is served by DC $j \in J$ in scenario $s \in S$, and 0 otherwise

Now the above problem can be modeled as follows:

Minimize $\sum_{s \in S} \sum_{j \in J} \left( f_j X_j + \left( \sum_{i \in I} \hat{d}_{ijs} Y_{ijs} \right) + K_j \sqrt{\sum_{i \in I} \mu_{is} Y_{ijs} + q \sqrt{\sum_{i \in I} L_j \sigma^2_{is} Y_{ijs}}} \right)$

Subject to
- $\sum_{j \in J} Y_{ijs} = 1$, for each $i \in I, s \in S$
- $Y_{ijs} - X_j \leq 0$, for each $i \in I, j \in J, s \in S$
- $Y_{ij} \in \{0, 1\}$, for each $i \in I, j \in J, s \in S$
- $X_j \in \{0, 1\}$, for each $j \in J$

where

$$\hat{d}_{ijs} = \beta \chi_{is} (d_{ijs} + a_j)$$
$$q = \theta h z_\alpha$$

Snyder et al. [74] present a Lagrangian-relaxation based solution algorithm for the above model. They show the Lagrangian subproblem is a non-linear integer program, but it can be solved by a low-order polynomial algorithm. They present quantitative and qualitative computational results on problems with up to 150 nodes and 9 scenarios, and describe both algorithm performance and solution behavior as key parameters change. One of their major findings is that the stochastic (i.e., min-expected-cost) solutions and the individual scenario solutions differ substantially in their choices of DC locations. This suggests that each of the single-scenario solutions would perform poorly in long-run expected cost. Furthermore, they observe that implementing the stochastic solution will entail roughly 8% regret on average and nearly 25% regret in the worst case. Finally, they note that on average half of the retailers are assigned to different DCs in different scenarios, indicating the value of allowing retailer assignments to be scenario dependent.

Results in this section are based on:
Conclusion and future research. In this paper, we reviewed recent developments in the area of integrated supply chain design. The algorithms reviewed in this paper can also be applied to a wide range of other concave cost minimization problems. We believe there are many interesting future research problems in this area, especially the following four directions.

11.1. Capacity Modeling. Except for the model in Section 5, most models reviewed in this paper are designed for de novo planning in which one can often allow the capacity of each DC to be as large as necessary to satisfy the demands assigned to the DC. In many practical contexts, the candidate DC sites may have limited capacity. It is interesting to investigate how the methodology presented in Section 5 can be applied to other models in this paper.

11.2. Large-scale Systems. In many supply chain design models, the number of demand points can be hundreds or even thousands, and for such a large data set, it is very difficult, if not impossible, to solve the models in this paper. It is important to design efficient heuristics that can provide good solutions quickly.

11.3. Supply Chain Competition. All the models reviewed in this paper assume all parties involved have the same objective: minimize system total cost or maximize system profit. In practice, there are likely more than one company, each of which may try to design their own supply chain using some of the models reviewed in this paper. It is interesting to study the competition among supply chains and how competition impacts their supply chain design decisions. For instance, one can expand the model in Section 7 to include two supply chains and each of them need to decide a subset of customers to serve while maximizing their individual profits. It’s an open question whether there exists any equilibrium strategy in the competition game.

11.4. Robustness, Reliability, and Risk Management. Supply chain network design decisions are usually strategic and once implemented they are difficult to reverse. During the time when the design decisions are in effect, many decision parameters, such as demands and costs, may change dramatically. This calls for models that address the inherent uncertainties of facility location problems.

Chen et al. [13] apply the conditional value-at-risk (CVaR) idea (Rockafeller and Uryasev [61]) to a location model. CVaR approximately equals the average of the worst case $\alpha$% scenarios. Their model minimizes the expected regret with respect to an endogenously selected subset of worst-case scenarios whose collective probability of occurrence is at most 1-$\alpha$. This new model, the $\alpha$-reliable Mean-excess regret model, or Mean-excess model for short, has demonstrated significant improvements over the $\alpha$-reliable $p$-median Minimax regret model in numerical tests. In addition, they present a heuristic which efficiently solves the $\alpha$-reliable $p$-median Minimax regret model by solving a series of Mean-excess sub-problems. We plan to apply the CVaR concept to the supply chain design models described in this paper.

Unlike the traditional facility location models that implicitly assume the facilities will never fail, Snyder and Daskin [75] propose location models in which facilities do fail from time to time due to poor weather, disasters, changes of ownership, or
other factors. Such failures may lead to increased costs due to retailer reassignments. They present models for choosing facility locations to minimize cost, while also taking into account the expected transportation cost after failures of facilities. The goal is to choose facility locations that are both inexpensive under traditional objective functions and also reliable. They apply this reliability concept to the median problem and uncapacitated facility location problem and present a Lagrangian relaxation algorithm. It is interesting to apply the same reliability concept to the supply chain design models we reviewed in this paper.

REFERENCES

[1] Adelman, D., A price-directed approach to stochastic inventory/routing, Working Paper, University of Chicago, (2003).
[2] Agarwal, A. and U.S. Palekar, A Branch-and-price algorithm for multi-echelon distribution system design, Working paper, University of Illinois, (2003).
[3] Agarwal, R., R. K. Ahuja, G. Laporte, and Z. J. Shen, A composite very large-scale neighborhood search algorithm for the vehicle routing problem, to appear in “Handbook of Scheduling: Algorithms, Models, and Performance Analysis” (Ed. J. Y-T. Leung), to be published by ChapmanHall/CRC, 2004.
[4] Balakrishnan, A., J. E. Ward and R. T. Wong, Integrated facility location and vehicle routing models: recent work and future prospects, American Journal of Mathematical and Management Sciences, 7(1987), 35-61.
[5] Balinski, M., Integer programming: methods, uses, computation, Management Science, 13(1965), 253-313.
[6] Berk, E. and A. Arreola-Risa, Note on “Future supply uncertainty in EOQ models”, Naval Research Logistics, 41(1994), 129-132.
[7] Berger, R. T., C. R. Couillard and M. S. Daskin, Modeling and solving location-routing problems with route-length constraints, Working Paper, Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA, 1995.
[8] Berman O., P. Jaillet, and D. Simchi-Levi, Location-routing problems with uncertainty, Chapter 18 in “Facility Location” (Ed. Drezner Z.), Springer, New York, 1995.
[9] Boffey, B. and J. Karkazis, Location, routing and the environment, Chapter 19 in “Facility Location” (Ed. Drezner Z.), Springer, New York, 1995.
[10] Bookbinder, J. H. and K. E. Reece, Vehicle routing considerations in distribution system design, European Journal of Operational Research, 37(1988), 204-213.
[11] Bundschuh, M., D. Klabjan, and D. L. Thurston, Modeling robust and reliable supply chains, Working paper, University of Illinois at Urbana-Champaign, 2003.
[12] Chan, Y., W. B. Carter, M. D. Burns, A Multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands, Computers & Operations Research, 28(2001), 803-826.
[13] Chen, G., M. Daskin, Z.-J. Shen and S. Uryasev, A new model for stochastic facility locations, Naval Research Logistics, (2004).
[14] Christofides, N., A. Mingozzi, and P. Toth, The vehicle routing problem, in “Combinatorial Optimization” (eds: N. Christofides, A. Mingozzi, P. Toth, and C. Sandi), Wiley, Chichester, 1979.
[15] Church, R. L. and C. S. ReVelle, The maximal covering location problem, Papers of the Regional Science Association, 32(1974), 101-118.
[16] Cohon, J. L., “Multiobjective Programming and Planning,” Academic Press, New York, 1978.
[17] Dada, M., N. Petruzzi, and L. B. Schwarz, A newsvendor model with unreliable suppliers, in review with Manufacturing & Service Operations Management, (2003).
[18] Daganzo, C., Logistics systems analysis, in “Lecture Notes in Economics and Mathematical Systems” (eds: M. Beckmann and W. Krelle),Springer Verlag, Berlin, 1996.
[19] Dasci, A. and V. Verter, A continuous model for production-distribution system design, European Journal of Operational Research, 129(2001), 287-298.
[20] Daskin, M. S., “Network and Discrete Location: Models, Algorithms, and applications,” Wiley-Interscience, New York, 1995.
[21] Daskin, M. S. and S. H. Owen, Location models in transportation, in “Handbook of Transportation Science” (eds R. Hall), Kluwer Academic Publishers, Norwell, MA, 1999, 311-360.

[22] Daskin, M., C. Coulard and Z. -J. Shen, An inventory-location model: formulation, solution algorithm and computational results, Annals of Operations Research, 110(2001), 83-106.

[23] Daskin, M. S., L. V. Snyder and R. T. Berter, Facility location in supply chain design, forthcoming in logistics systems, in “Design and Optimization” (Eds. A. Langevin and D. Riopel), Kluwer, 2003.

[24] Drezner, Z., “Facility Location: A Survey of Applications and Methods,” Springer, 1995.

[25] Eppen, G., Effects of centralization on expected costs in a multi-location newsboy problem, Management Science, 25(1979), 498-501.

[26] Erlebacher, S. J. and R. D. Meller, The interaction of location and inventory in designing distribution systems, IIE Transactions, 32(2000), 155-166.

[27] Erlenkotter, D., A dual-based procedure for uncapacitated facility location, Operations Research, 14(1978), 361-368.

[28] Geoffrion, A. M., The purpose of mathematical programming is insight, not numbers, Interfaces, 7(1976), 81-92.

[29] Geoffrion, A. M. and G. W. Graves, Multicommodity distribution system design by benders decomposition, Management Science, 20(1974), 822-844.

[30] Geunes, J., Z. -J. Shen, H. E. Romeijn, Economic ordering decisions with market choice flexibility, Naval Research Logistics, 51(2004), 117-136.

[31] Ghiani, G., G. Laporte, and R. Musmanno, “Introduction to Logistics Systems Planning and Control,” John Wiley & Sons Ltd, Chichester, England, 2004.

[32] Gupta, D., The (q, r) inventory system with an unreliable supplier, INFOR, 34(2)(1996), 59-76.

[33] Haimovich, M. and A. H. G. Rinnooy Kan, Bounds and heuristics for capacitated routing problems, Math. Oper. Res., 10(1985), 527-542.

[34] Hall, R. W., Discrete models/continuous models, Omega, Int. J. of Mgmt Sci., 14(1986), 213-220.

[35] Hansen, P., M. Labbe, D. Peeters, and J.-F Thisse, Facility location analysis, Fundamentals of Pure and Applied Economics, 22(1987), 1-70.

[36] Hansen, P., D. Peeters, and J.-F Thisse, The profit-maximizing weber problem, Location Science, 3(1995), 67-85.

[37] Harrison, T., “Principles for the Strategic Design of Supply Chains,” Kluwer Academic Publishers, 2003, Chapter 1.

[38] Hopp, W. and M. L. Spearman, “Factory Physics: Foundations of Manufacturing Management,” Irwin, Chicago, 1996.

[39] Horst, R., P. Pardalos, and N. V. Thoai, “Introduction to Global Optimization,” Kluwer Academic Publishers, Boston, 2000.

[40] Horst, R. and H. Tuy, “Global Optimization,” Springer-Verlag, New York, 1990.

[41] Kleywegt, A., V. S. Nori, M.W.P. Savelbergh, The stochastic inventory routing problem with direct deliveries, Transportation Science, 36(2002), 94-118.

[42] Kleywegt, A., V. S. Nori, M.W.P. Savelbergh, Dynamic programming approximations for a stochastic inventory routing problem, Working paper, Georgia Inst. of Technology, (2002).

[43] Korkel, M., On the exact solution of large-scale simple plant location problems, European Journal of Operations Research, 39(1989), 157-173.

[44] Langevin, A., P. Mbaraga, and J. F. Campbell, “Continuous approximation models in freight distribution: an overview,” Transportation Research, 30B(1996), 163188.

[45] Laporte, G., Location-routing problems, in “Vehicle Routing: Methods and Studies” (eds: Golden, B.L. and A. A. Assad), North-Holland, Amsterdam, Holland, 1988, 163-197.

[46] Laporte, G., F. Louveaux, and H. Mercure, Models and exact solutions for a class of stochastic location-routing problems, European Journal of Operational Research, 39(1989), 71-78.

[47] Laporte, G. and Y. Nobert, Exact algorithms for the vehicle routing problem, Surveys in “Combinatorial Optimization” (eds S. Martello et al.), North-Holland, Amsterdam, 1987.

[48] Lee, H., Effective inventory and service management through product and process redesign, Operations Research, 44(1996), 151-159.

[49] , Ma, Y.-H. and G. R. Wilson, “Neighborhoods: A new service parts stock plan,” INFORMS Annual Meeting, San Jose, CA, 2002.

[50] Meyerson, A., “Profit-Earning Facility Location,” STOC 2001, Hersonissos, Crete, Greece, 2001.
[51] Min, H., V. Jayaraman, and R. Srivastava, Combined location-routing problems: a synthesis and future research directions, European Journal of Operational Research, 108(1998), 1-15.

[52] Nagy, G. and S. Salki, Nested heuristic methods for the location-routing problem, The Journal of the Operational Research Society, 47(1996), 1166-1174.

[53] Nahmias, S., “Production and Operations Management,” Irwin, Chicago, 1997.

[54] Parlar, M., Continuous-review inventory problem with random supply interruptions, European Journal of Operational Research, 99(1997), 366-385.

[55] Nozick, L. and M. Turnquist, Inventory, transportation, service quality and the location of distribution centers, European Journal of Operations Research, 129(2001), 362-371.

[56] Owen, S. H. and M. S. Daskin, Strategic facility location: a review, European Journal of Operational Research, 111(1998), 423-447.

[57] Parlar, M. and D. Berkin, Future supply uncertainty in EOQ models, Naval Research Logistics, 38(1991), 107-121.

[58] Parlar, M. and D. Perry, Analysis of a (q, r, t) inventory policy with deterministic and random yields when future supply is uncertain, European Journal of Operational Research, 84(1995), 431-443.

[59] Qi, L. and Z. -J. Shen, A supply chain design model with unreliable supply, Working paper, University of California at Berkeley, (2005).

[60] Ross, G.T. and R. M. Soland, A multicriteria approach to location of public facilities, European Journal of Operational Research, 4(1980), 307-321.

[61] Santoso, T., S. Ahmed, M. Goetschalckx, and A. Shapiro, “A Stochastic Programming Approach for Supply Chain Network Design under Uncertainty, Working paper,” School of Industrial & Systems Engineering, Georgia Institute of Technology, 2003.

[62] Sheffi, Y., Supply chain management under the threat of international terrorism, The International Journal of Logistics Management, 12 (2)(2001), 1-11.

[63] Shen, Z. -J., “Approximation Algorithms for Various Supply Chain Problems,” Ph.D Thesis, Dept. of Industrial Engineering and Management Sciences, Northwestern University, 2000.

[64] Shen, Z. -J., A multi-commodity supply chain design problem, IIE Transactions, 37(2005), 753-762.

[65] Shen, Z. -J., A profit maximizing supply chain network design model with demand choice flexibility, Operations Research Letters, to appear.

[66] Shen, Z. -J., C. R. Coullard and M. S. Daskin, A Joint location-inventory model, Transportation Science, 37(2003), 40-55.

[67] Snyder, L. V. and M. S. Daskin, Reliability models for facility location: the expected failure cost case, Transportation Science, 39(2005), 400-416.

[68] Tomlin, B. T., Coping with supply chain disruptions: dual sourcing, volume flexibility, and contingency planning, Working paper, University of North Carolina at Chapel Hill, 2003.

[69] Vidal, C. J. and M. Goetschalckx, Modeling the effect of uncertainties on global logistics systems, Journal of Business Logistics, 21 (1)(2000), 95-120.
[80] Warszawski, A. and S. Peer, *Optimizing the location of facilities on a building site*, Operational Research Quarterly, 24(1973), 35-44.
[81] Wu, T. H., C. Low, J. W. Bai, *Heuristic solutions to multi-depot location-routing problems*, Computers & Operations Research, 29(2002), 1393-1415.
[82] Yano, C. A. and H. L. Lee, *Lot sizing with random yields: a review*, Operations Research, 43(1994), 311-334.
[83] Zhang, S., *On a profit maximizing location model*, Annals of Operations Research, 103(2001), 251-260.
[84] Zipkin, P. H., “Foundations of Inventory Management,” Irwin, Burr Ridge, IL, 1997.

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