Neutrino Mass Models in Extra Dimensions

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Abstract

Neutrinos play a crucial role in many areas of physics from particle physics at very short distances to astrophysics and cosmology. It is a long held believe that they are good probes of physics at the GUT scale. Recent developments have made it clear that they can also be of fundamental importance for the physics of extra dimensions if these exist. Here we pedagogically review the construction of neutrino mass models in extra dimensions within the brane scenarios. These models are usually nontrivial generalization of their four dimensional counter parts. We describe the theoretical tools that have been forged and the new perspectives gained in this rapidly developing area. In particular we discuss the issues involve with building models without the use of right-handed singlets. It is very difficult to directly test the origin of neutrino masses in the different models be it in four or more five dimensions. We point out that different models give very different indirect signatures at the TeV region and precision measurements.

I Introduction

We have now convincing evidence that the three active neutrinos of the standard model have different masses and they mix with each other. The results of the SuperK [1], SNO [2] and Kamland [3] experiments have now narrow the \( \nu \) mass pattern to one of three possibilities

- Inverted mass hierarchy (IMH)
- Normal mass hierarchy (NMH)
- Almost degenerate masses

The first two cases are depicted in Fig. 1 and we concentrate on them. The notations in the figure are standard. However, the overall mass scale is not known. We only have upper bounds of \(< 1\text{eV} \text{ from WMAP} \) [4] and \(< 2.2\text{eV} \text{ from tritium end point experiments} \) [5]. If neutrinos have Majorana masses
then a better limit of < .3 eV comes from neutrinoless double beta decay (∆ββ)_0ν experiments \[6\]. This latter result is model dependent and is subject to uncertainties in nuclear matrix element calculations as well as the implicit assumption that neutrino mass is the dominant effect leading to the decay. There are many excellent reviews on these are other questions we list the most recent ones in \[7\].

As with other fermions when neutrinos are massive their mass eigenstates and weak eigenstates do not coincide. In general they are related via a mixing matrix \(U_{PMNS}\) \[8\]. Analysis of the data yield

\[
U_{PMNS} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix} = \begin{pmatrix}
0.79 - 0.86 & 0.50 - 0.61 & .0 - 0.16 \\
0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\
0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77
\end{pmatrix}.
\]

In a gauge theory \(U_{PMNS}\) is a product of two mixing matrices:

\[
U_{PMNS} = U_l U_\nu
\]
where $U_l$ diagonalizes the charged lepton mass matrix and $U_\nu$ does the same for neutrinos. Often one chooses the mass eigenbasis for the charged leptons and $U_{PMNS}$ is just the neutrino mixing matrix. Furthermore, Eq. (1) is a low energy solution. It is typical that gauge models are formulated at some high energy scale and reach the above via renormalization group running. Whether this has a large effect is highly model dependent. In most of our discussions we assume that this is not large.

Now we return to the mixing matrix. It is clear that, unlike the down quark mixing, neutrino mixing are bi-large and the recent SNO data also rule out bi-maximal. One can reconstruct the neutrino mass matrix in the weak basis $M_\nu$ via $M_\nu = U_{PMNS}^T M_D U_{PMNS}$ where $M_D$ is the diagonal mass matrix with eigenvalues ($m_1$, $m_2$, $m_3$). It is $M_\nu$ that is of the greatest interest to theorists. In general $m_{1,2,3}$ are complex numbers for Majorana neutrinos. Customarily one phase is put into $U_{PMNS}$ and two are left in $M_D$. With the values of mass differences depicted in Fig. 1 and Eq. (1) the leading mass patterns are given by

1. IMH

   (a) 
   \[
   M_\nu \sim m_0 \begin{pmatrix} 1 & \times & \times \\ \times & 1/2 & 1/2 \\ \times & 1/2 & 1/2 \end{pmatrix} \]
   \noindent (2)

   (b) 
   \[
   M_\nu \sim m_0 \begin{pmatrix} \times & 1 & 1 \\ 1 & \times & \times \\ 1 & \times & \times \end{pmatrix} \]
   \noindent (3)

2. NMH

   \[
   M_\nu \sim m_0 \begin{pmatrix} \times & \times & \times \\ \times & 1/2 & -1/2 \\ \times & -1/2 & 1/2 \end{pmatrix} \]
   \noindent (4)

where $m_0$ is the unknown overall mass scale and $\times$ denotes some small number. The challenge to theorists is to construct viable models that give rise to one of the above structures. At the same time one gets a more or less natural understanding of why the overall neutrino mass scale is so much smaller than the charged fermions; i.e. what physics sets the scale of neutrino masses.
II Are Neutrinos Dirac or Majorana Particles?

The strong evidence of neutrino oscillations and mixing has certainly shattered the concept of separately conserved electron, muon or tau lepton number; however, the question of whether total lepton number is conserved is still open. The usual way of stating this is in terms of whether neutrinos are Dirac or Majorana particle. If neutrinos were Majorana then they are own anti-particle and lepton number violating reactions can be expected to proceed. Otherwise they are Dirac particles. Experimentally this is a very difficult question to answer. The favorite process is $\beta\beta$ decays. If one such decay were observed then it will be clear evidence that lepton number is not conserved and it will natural to conclude that neutrinos are Majorana or at least a Majorana mass term is not forbidden. On the other hand to prove that neutrinos are Dirac particles will be very difficult.

The nature of neutrinos has deep theoretical significance is modern particle physics. The active neutrino in the SM is a left-handed Weyl particle; i.e. is a helicity eigenstate. Due to gauge invariance it has no bare mass term. To give it a mass one usually introduces a right-handed fermion $N_R$ which is a SM singlet. Whether the $N_R$ exists and what is its nature are two of the most fundamental question in neutrino physics. Certainly if $N_R$ exists how many are there? What is its mass? Is it Dirac or Majorana particle? In the SM this is put in by hand and so the same is true $SU(5)$ GUT models. It is customary to use at least one to generate masses for some of the active neutrinos and three if we believe in some family symmetry. In $SO(10)$ GUT models $N_R$ naturally exists since the fundamental representation has 16 fermions which is just the right number to accommodate each SM family plus a $N_R$. Here it is natural to have three of them.

There are many theoretical reasons to pursue GUT models and understand neutrino masses in this context (see [9] for an up to date review); moreover, we wish to study this in a wider setting. As is well known $N_R$ being a singlet can have a Majorana mass term , $(N_R)^cN_R$ as well as a Dirac mass term coupling given by $y\bar{\nu}_L N_R H^0$ where $H^0$ is the SM Higgs boson and $y$ is the Yukawa coupling. After symmetry breaking the neutrino masses for one family is given by

$$\begin{pmatrix} \nu^c_L & N^c_R \end{pmatrix} \begin{pmatrix} m_{\nu} & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix}$$

For three families each entry in the above is a $3 \times 3$ matrix.

In the SM $m_{\nu} = 0$ and $m_D \sim \text{GeV}$. If $N_R$ is a Dirac fermion then $M_R = 0$ and we have Dirac masses for the active neutrinos. This is the
simplest extension of the SM. In this case it is very difficult to understand
the smallness of the neutrino masses and extreme fine tuning of the Yukawa
coupling \( \sim 10^{-11} \) to get it below the experimental bound. In GUT models
it is natural to take \( M_R \sim 10^{14} \) GeV and we have the seesaw mechanism for
generating neutrino masses.

### III \( N_R \) as a bulk fermion

Recently new perspectives in neutrino physics arise from the so call brane
world scenario. In the simplest form this makes use of the possibility that
there are more then 4 dimensions. The number of extra dimensions \( \delta \)
is between 1 and 7 as hinted by string theory. They are taken to be spatial
and can be relatively large [10] which enables one to solve the hierarchy
problem. Fields with SM quantum numbers are confined on a \( 1 + 3 \) hyper
surface whereas SM singlets such as the graviton can propagate in the full
\( 4 + \delta \) dimensional bulk. The extra dimensions are compactified in tori of
radii \( R_i \) where \( i = 1, \ldots , 6 \). Introducing \( N_R \) as a bulk field was done in [11].
As is well known compactification leads to 4D Kaluza-Klein(KK) excitations
of \( N_R \) with masses \( n/R, n = 0, 1, 2, \ldots \) where we have taken all radii to be
equal. The zero mode can now couple to the active neutrino on the brane
and the SM Higgs in the usual way. If \( N_R \) is taken to be Dirac then a small
Dirac neutrino mass can be generated given by

\[
m_D = \frac{y v}{(2\pi R M_\star)^{\delta/2}} \quad v = 245\text{GeV}
\]

without excessive fine tuning of \( y \). This is due to the volume dilution fac-
tor of \( RM_\star \) where \( M_\star \sim 10\text{TeV} \) is the fundamental scale for the theory
we can obtain \( m_D \sim 10^{-4}\text{eV} \). Even with this simple model and only one
family there is an important consequence for neutrino oscillation. The KK
excitations form an infinite tower of sterile neutrinos. One would expect an
effective active-sterile oscillation [12] in the Superk data. However, there is
no evidence in the data. This may indicate that the mixing between the
active neutrinos and the KK modes are small and/or \( R \) cannot be taken
too large. Other effects in nuclear and astrophysics of this simple model are
discussed in [13].

We have seen that small Dirac neutrino masses are quite natural in extra
dimensional model; however, to obtain the hierarchy indicated by the data is
more difficult. One way is to make use of family symmetry as in 4D models.
Another solution is purely extra dimensional which can also incorporate the
masses of quarks and charged leptons. This is the split fermion scenario \cite{14} where the left-handed and right-handed fields are localized at different positions in the extra dimensional space. For example, the $(\nu_e, e)_L$ doublet is at $z = 0$ and $e_R$ is at $z = z_1$ where $z$ denotes the extra dimension. This is repeated for other fermions. The gauge and Higgs fields propagate in the bulk. The chiral fermions are also bulk fields and their zero modes, denoted by $\Psi$, are given Gaussian profiles in $z$. For a fermion located at $z_i$ its wave function is given by

\[
\Psi_i(x, z) \sim \frac{1}{\pi^{\frac{1}{2}} \sigma^\frac{1}{2}} \Psi_i(x)e^{-\frac{(z-z_i)^2}{2\sigma^2}}.
\tag{7}
\]

where $\sigma$ is a free parameter that is taken to be universal for all fermions for simplicity. The product of two fermion fields can be approximately replaced by

\[
\Psi_i(x, z) \Psi_j(x, z) \sim \exp\left(-\frac{(\Delta_{ij})^2}{4\sigma^2}\right)\delta(z - \bar{z}_{ij}) \bar{\Psi}_i(x)\Psi_j(x) \tag{8}
\]

where $\bar{z}_{ij} = (y_i + y_j)/2$ is their average positions and $\Delta_{ij} = z_i - z_j$. After integrating out $z$ it is the overlap of the two different chiralities of a fermion that determines its mass. Explicitly it is given by $\Delta_{ij}$. The origin of the large mass for fermion is due to the proximity of its left and right-handed components in the extra dimension. Similarly, if the chiral components of a fermion is far apart in $z$ it will be light. Hence, one does not need to fine tune Yukawa couplings but instead the positions $z_i$ are used to fit the data. It is found that realistic quark and lepton mass matrices and an acceptable CKM matrix can be be found \cite{15}. In this scenario the fermion masses becomes a geographical problem in the extra dimension. Stating it differently we need a mechanism that puts the fermions in their correct positions instead of putting it in by hand. As a result, although it is non-trivial to be able to reproduce the observed masses and mixing, these models suffer from a lack of predictive power.

**IV What if there is no $N_R$?**

As can be seen from Eq.(5) that neutrinos will be massless in the SM and can have only a Majoring mass via radiative effects. This is first done in \cite{16} within the SM gauger group by adding a $SI(2)$ singlet but $U(1)$ charged scalar field. The model also requires an additional doublet Higgs field $S^+$. Due to Fermi statistics $S$ can only couple to leptons of two different families. The Feynman diagram generating neutrino mass is given below
The resulting mass matrix is symmetric and takes the form

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix} \quad (9)$$

where $m_0$ is an overall mass scale set by the mass of $S$. In Eq. (9) $a$, $b$, and $c$ are functions of lepton masses and Higgs masses. The characteristics of the Zee mass matrix are the vanishing diagonal elements. This structure leads to bi-maximal mixing [17] and does not agree with the latest SNO data [2]. Furthermore, the mass matrix is intriguing and can serve as the leading order to the correct mass matrix. To do so one has to find a natural way of filling in some of the zero’s in the Zee mass matrix.

This can be achieved in 5D unification models which possess gauge symmetries in the bulk. The components of the the 5D fields that form representations of the symmetry can have different boundary values on the 4D brane. This reduces the symmetry that particles resides on the brane sees. This is known as orbifold symmetry breaking (OSB) mechanism.

To appreciate this a little more we consider a 5D theory with the extra dimension compactified in a circle; i.e. $z \in [-\pi R, \pi R]$. If we impose a parity of $Z_2$ such that a field $A(x, z)$ transform as

$$P : z \leftrightarrow -z \quad PA(z) = \pm A(z). \quad (10)$$

Clearly $z = 0$ and $z = \pi R$ are fixed points and the background geometry is $S_1/Z_2$. Now we impose a second parity by defining $z' = z - \pi R/2$ such that

$$P' : z' \leftrightarrow -z' \quad (11)$$

The fixed points are at $z = 0$ and $z = \frac{\pi R}{2}$ and the geometry is $S_1/(Z_2 \times Z_2')$. The fields can have different parities under $P$ and $P'$. In particular the
Fourier decomposition of a bulk field into their KK modes that conform to the parities assignments are listed in Table I.

| $(P, P')$ | form | mass | $z = 0$ | $z = \frac{\pi R}{2}$ |
|-----------|------|------|---------|----------------------|
| $(+, +)$  | $\frac{1}{\sqrt{2\pi R}} [A_0(x) + \sqrt{2} \sum_{n=1} A_{2n+}^+(x) \cos \frac{2n\pi z}{R}]$ | $\frac{2n}{R}$ | $\checkmark$ | $\checkmark$ |
| $(+, -)$  | $\frac{1}{\sqrt{\pi R}} \sum_{n=0} A_{2n+1}^-(x) \cos \frac{(2n+1)\pi z}{R}$ | $\frac{(2n+1)}{R}$ | $\checkmark$ | $X$ |
| $(-, +)$  | $\frac{1}{\sqrt{\pi R}} \sum_{n=0} A_{2n+1}^+(x) \sin \frac{(2n+1)\pi z}{R}$ | $\frac{(2n+1)}{R}$ | $X$ | $\checkmark$ |
| $(-, -)$  | $\frac{1}{\sqrt{\pi R}} \sum_{n=0} A_{2n+2}^-(x) \sin \frac{(2n+2)\pi z}{R}$ | $\frac{(2n+2)}{R}$ | $X$ | $X$ |

Table I. KK decomposition of a bulk field $A(x, z)$ with parities $(P, P')$

The zero modes have $(++)$ parity and are identified as SM fields. As an example consider $SU(5)$ with the Higgs field in the 5 representation. We can assign $(++)$ parities to the $SU(2)$ components and different parities to the $SU(3)$ color Higgs components. The former is just the SM Higgs field and couples to SM matter fields which are placed on the $z = 0$ brane whereas the colored Higgs are KK excitations and will have masses $\frac{2n+1}{R} \ n = 1 \ldots$. They latter will be heavy if the compactification radius $R$ is small and of the order of the unification scale. This provide a new natural solution to the doublet-triplet splitting problem in $SU(5)$ GUT [18]. For a review of orbifold GUT models see [19]. With this technique we can now shape new tools for investigating the fermion mass problem.

In Ref.(20) and (21) this has been applied to study neutrino mass generation without the benefit of $N_R$ fields for $SU(3)$ and $SU(5)$ unification models. The first model is particularly simple and illustrates the clearly physics involved and extends previous work on this model [22]. The $SU(3)$ symmetry is in the bulk and acts on a background geometry of $S - 1/Z_2 \times Z_2'$. It only unifies the $SU(2) \times U(1)$ symmetry to $SU(3)_W$ and has obvious built in lepton number number violation when the leptons are placed in the fundamental 3 i.e. $(\nu_L \ e_L \ e_R^c)^T$. The unified value of the weak mixing angle is well known to be given by $\sin^2(\theta_W) = \frac{1}{4}$ [23] and one can expect unification to occur around $\sim 1 - 10$ TeV after renormalization group considerations. The lepton triplet is placed on the $z = 0$ brane and $SU(3)$ is broken by orbifolding the gauge fields and not by the Higgs mechanism which is reserved for SM symmetry breaking. We require a bulk Higgs field in the 3 denoted by $\phi_3$ and a symmetric antisextet $\bar{6}$ to give realistic charged lepton masses. We denote fields in $\bar{6}$ by $\phi_6$. Due to the requirement of $Z_2 \times Z_2'$ invariance we also introduce a second Higgs triplet $\phi'_3$ for the construction of the necessary 3 6 3 coupling.
In this model the parities $P$ and $P'$ are represented by $3 \times 3'$ matrices:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (12)$$

The parities of the components of Higgs fields are engineered according to phenomenological needs and are

$$\phi_3 = \begin{pmatrix} \phi_3^{-} (++) \\ \phi_3^{0} (++) \\ h_3^{+} (-+) \end{pmatrix}, \quad \phi_3' = \begin{pmatrix} \phi_3'^{-} (-+) \\ \phi_3'^{0} (-+) \\ h_3'^{+} (++) \end{pmatrix}, \quad (13)$$

and

$$\phi_6 = \begin{pmatrix} \phi_{11}^{++} (+-) & \phi_{12}^{--} (--) & \phi_{13}^{00} (++) \\ \phi_{12}^{++} (++) & \phi_{22}^{--} (--) & \phi_{23}^{00} (++) \\ h_3^{-} (--) & h_3^{0} (--) & \phi_{33}^{--} (++) \end{pmatrix}. \quad (14)$$

With these ingredients 1 loop neutrino masses can be generated via

The dominant contribution comes from (a) and leads to a Zee-like neutrino mass matrix. The next two diagrams give the necessary correction to this. In contrast to 4D models these terms exist as part of the 5D model. We do not have to put them in by hand. Two parameters central to all extra dimension models are the compactification radius $R$ and the cutoff scale $M_*$ at which the field theory becomes strong. A lower limit on $R^{-1} > 1 \text{TeV}$ can be obtained from the non observation of KK modes and $RM_* \sim 100$. 

![Figure 3: Neutrino Mass Generation in SU(3) Model](image)
Without fine tuning of Yukawas other than that required to get the charged lepton masses and using $R \sim 2$ TeV one can obtain

$$
M_\nu \sim \begin{pmatrix}
0.420 & 1.0 & 0.922 \\
1.0 & 0.097 & -0.464 \\
0.922 & -0.464 & 0.006
\end{pmatrix} \times 0.0441 \text{ (eV)}.
$$

which gives a good fit to the SuperK and SNO data. This solution is of the IMH type and interestingly predicts a detectable neutrinoless double beta decay rate in the next round of experiments. Furthermore, the model has very rich phenomenology at the TeV scale which is the compactification scale here. In particular double charged gauge boson are predicted for this model, which can be searched for in linear colliders. Rare decays such as $\mu \rightarrow 3e$ and similar $\tau$ decays are also sensitive probes of the model \[24\].

In the $SU(3)$ model it is known that quarks cannot be accommodated and has to be placed at the other fixed point. It may be more natural to place quarks and leptons on the same footing as in the $SU(5)$ model. To generate neutrino masses with only the fifteen SM fermions per family requires the use of either 15 or 10 bulk Higgs field. The procedure is similar to the case of $SU(3)$ exploiting OSB. The addition of exotic Higgs exacerbates the gauge unification problem in $SU(5)$. This can be solved by additional fermions (see \[24\]). After the dust settles unification is found to take place at $10^{15}$ GeV. An interesting result emerges for this model. It is more natural to get the NMH with the 15 Higgs and IMH solutions can only be found with 10 Higgs fields.

V Conclusion

We have reviewed the construction of neutrino mass models in extra dimensions. The possibility of large extra dimension, i.e $1/R \ll M_{\text{Planck}}$ has resulted in gaining very different perspectives in model building compared similar exercises in 4D. These models also have different low energy phenomenology than their 4D counterparts. In the Dirac neutrino case one expects a sterile component in neutrino oscillation pattern. In the radiative mass mechanism we expect KK modes of gauge bosons that couples to bileptons. The 4D models on the other hand have only exotic scalar particles. These models has a serious down side. Thus far all of them has many free parameters and is no better or worse than the 4D models. We also encounter very deep theoretical questions. What physics determines the compactification scale remains unanswered. For the orbifold models the origin of parities of the various fields is a mystery.
Due to the lack of space we have not discuss neutrino mass in models that implement the seesaw mechanism in extra dimensions. This is studied for the $SO(10)$ case in [28]. Omitted are also discussions of neutrino masses in models with warp extra dimensions [25]. We will only mention that this has been explored in the case of Dirac neutrinos [26] and [27]. Within the context of unification models the $SU(5)$ case was studied in [29].

The many beautiful neutrino experiments have given us the first hint of physics beyond the SM. The bilarge neutrino mixing came as real surprise to theorists. They have stimulated us to explore scenarios which we have not ventured before. We can look forward for more data from the ongoing neutrino experiments and perhaps even more unexpected results. Happily there will be more data to come both from high energy experiments with the anticipated turning on of LHC as well as many precision measurements at lower energies. Before the new physics becomes clear it behooves us to keep an open mind and be alert of new signatures.

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