Large-distance behaviour of the graviton two-point function in de Sitter spacetime

Atsushi Higuchi\textsuperscript{1} and Spyros S. Kouris\textsuperscript{2}
Department of Mathematics, University of York
Heslington, York, YO10 5DD, United Kingdom
\textsuperscript{1}Email: ah28@york.ac.uk
\textsuperscript{2}Email: ssk101@york.ac.uk

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Abstract

It is known that the graviton two-point function for the de Sitter invariant “Euclidean” vacuum in a physical gauge grows logarithmically with distance in spatially-flat de Sitter spacetime. We show that this logarithmic behaviour is a gauge artifact by explicitly demonstrating that the same behaviour can be reproduced by a pure-gauge two-point function.

1 Introduction

De Sitter spacetime is a maximally symmetric solution of the vacuum Einstein equations with positive cosmological constant,

\[ R_{ab} - \frac{1}{2}g_{ab}R + \Lambda g_{ab} = 0. \]  \hspace{1cm} (1)

(See Ref. \textsuperscript{1} for a detailed description of de Sitter spacetime.) Physics in this spacetime has been studied extensively due to its relevance to inflationary cosmologies \textsuperscript{2}. The graviton two-point function has been of particular interest in this context. Ford and Parker analysed linearised gravity in spatially-flat de Sitter spacetime and found that the mode functions in a physical gauge are similar to those of minimally-coupled massless...
scalar field [3]. Since the latter theory exhibits infrared (IR) divergences similar to those of massless scalar field theory in two-dimensional Minkowski spacetime [4], one may suspect that there would be IR divergences in linearised gravity in de Sitter spacetime as well. However, it was shown that there are no physical IR divergences in the graviton two-point function in de Sitter spacetime which was obtained by analytic continuation from that on the 4-sphere [3]. It was also found that the behaviour of mode functions responsible for the apparent IR divergences in spatially-flat de Sitter spacetime is a gauge artifact [6], and it was shown explicitly that the IR divergences in the graviton two-point function in a physical gauge can be gauged away [7]. Thus, it has been established that there are no physical IR divergences in linearised gravity in de Sitter spacetime. However, there is another apparent problem which is closely related: the graviton two-point function grows logarithmically with distance. If this behaviour were physical, then it would have a significant effect on physics of inflationary cosmologies.

The aim of this paper is to show that this logarithmic growth is also a gauge artifact. Specifically, we show that the large-distance logarithmic behaviour of the physical graviton two-point function computed by Allen [7] can be gauged away by demonstrating that the same behaviour arises in the two-point function of pure-gauge form. The rest of the paper is organised as follows. In section 2 we present the graviton two-point function in a physical gauge and show that its logarithmically growing part can be gauged away. In section 3 we show that this logarithmic behaviour can be reproduced by a pure-gauge field obeying a relativistic field equation. We summarise our results and make some remarks in section 4. In Appendix A, we list some integrals used in this work. Appendix B contains details of the calculations in section 3. We adopt the metric signature \((-+++)\) and set \(\hbar = c = 16\pi G = 1\) throughout this paper.

## 2 The physical graviton two-point function

We work with the metric which covers half of de Sitter spacetime:

\[
ds^2 = \frac{1}{H^2 \lambda^2}(-d\lambda^2 + dx^2 + dy^2 + dz^2),
\]

where \(H^2 = \Lambda / 3\), i.e. \(g_{ab} = (H\lambda)^{-2}\text{diag}(-1, 1, 1, 1)\). In the expanding half of de Sitter spacetime, the parameter \(\lambda\) takes positive values and decreases from \(\infty\) to 0 towards the future. By letting \(g_{ab} = g_{ab}^{(0)} + h_{ab}\), where \(g_{ab}^{(0)}\) is the de Sitter metric (2), and linearising the Hilbert-Einstein Lagrangian density, we have for linearised gravity

\[
\mathcal{L} = \sqrt{-g^{(0)}} \left[ \frac{1}{2} \nabla_a h^{ac} \nabla^b h_{bc} - \frac{1}{4} \nabla_a h_{bc} \nabla^b h^{bc} + \frac{1}{4} \left( \nabla^a h - 2 \nabla^b h_{ab} \right) \nabla_a h \right]
\]
\[ -\frac{1}{2} H^2 \left( h_{ab} h^{ab} + \frac{1}{2} h^2 \right) \],

(3)

where the covariant derivative \( \nabla_a \) is compatible with the background metric \( g_{ab}^{(0)} \). Indices are raised and lowered by \( g_{ab}^{(0)} \) and \( h \) is the trace of \( h_{ab} \). (We will denote \( g_{ab}^{(0)} \) by \( g_{ab} \) from now on.) The Lagrangian density (3) yields the following Euler-Lagrange equation:

\[ \frac{1}{2} (\Box h_{ab} - \nabla_a \nabla_c h^c_b - \nabla_b \nabla_c h^c_a + \nabla_a \nabla_b h) + \frac{1}{2} g_{ab} (\nabla_c \nabla_d h^{cd} - \Box h) - H^2 \left( h_{ab} + \frac{1}{2} g_{ab} h \right) = 0, \]

(4)

where \( \Box = \nabla_a \nabla^a \). Equation (4) is invariant under the gauge transformations

\[ h_{ab} \to h_{ab} + \nabla_a \Lambda_b + \nabla_b \Lambda_a. \]

(5)

We will first fix the gauge completely at the classical level.

By imposing the gauge condition,

\[ \nabla_b h^{ab} - \frac{1}{2} \nabla^a h = 0, \]

(6)

we find from (4)

\[ \frac{1}{2} \Box h_{ab} - \frac{1}{4} g_{ab} \Box h - H^2 \left( h_{ab} + \frac{1}{2} g_{ab} h \right) = 0. \]

(7)

The trace \( h \) can be gauged away as follows. By taking the trace of (7) we have

\[ (\Box + 6 H^2) h = 0. \]

(8)

Using this equation, we can replace \( h_{ab} \) by a traceless field satisfying (8),

\[ \tilde{h}_{ab} = h_{ab} + \frac{1}{6 H^2} \nabla_a \nabla_b h, \]

(9)

which is gauge-equivalent to the original field \( h_{ab} \). Thus, the trace \( h \) can be gauged away.

The field \( \tilde{h}_{ab} \), which we will denote by \( h_{ab} \) from now on, is transverse-traceless, (i.e. it satisfies \( \nabla^b h_{ab} = h^c_c = 0 \)) and obeys the following equation:

\[ (\Box - 2 H^2) h_{ab} = 0. \]

(10)

The general solution of this equation can be found, e.g. in [6]. Now, this equation allows solutions which are pure gauge: the field

\[ h_{ab}^{(\xi)} = \nabla_a \xi_b + \nabla_b \xi_a \]

(11)
is transverse-traceless and satisfies (11) if \( \nabla^c \xi_c = 0 \) and
\[
(\Box + 3H^2)\xi_a = 0.
\] (12)

This gauge freedom allows us to fix the gauge further and in effect we can impose the following gauge conditions on the field \( h_{ab} \):
- traceless: \( h^c_c = 0 \);
- transverse: \( \nabla_b h^{ab} = 0 \);
- synchronous: \( t^a h_{ab} = 0 \),

where \( t^a \) is the future-pointing unit vector parallel to \( -\partial/\partial \lambda \).

Allen considered quantisation of linearised gravity in this gauge, which we call the physical gauge, and computed the symmetrised two-point function. Here, we present essentially the same results for the unsymmetrised two-point function, \( G_{aba'b'}(x, x') = \langle 0| h_{ab}(x) h_{a'b'}(x')|0 \rangle \), where the state \( |0 \rangle \) is the so-called Euclidean vacuum [8]. The unprimed indices refer to the spacetime point \( x \), whereas the primed indices refer to the spacetime point \( x' \).

Following Allen, we define the projection operator \( P_{ab} = g_{ab} + t_a t_b \) at point \( x \), which projects tensors onto the flat spatial section of constant \( \lambda \). The tensor \( P_{ab} \) is the metric tensor on this spatial section. In our coordinate system it has the form
\[
P_{ab} = (H\lambda)^{-2} \text{diag}(0, 1, 1, 1).
\] (13)

We define \( P_{a'b'} \) to be the same projection operator at point \( x' \). We define a bi-covector \( P_{ab'} \) by
\[
P_{ab'} = (H^2 \lambda \lambda')^{-1} \text{diag}(0, 1, 1, 1).
\] (14)

Next we define the comoving spatial separation \( r \) of two points \( x = (\lambda, x_1, x_2, x_3) \) and \( x' = (\lambda', x'_1, x'_2, x'_3) \) as
\[
r(x, x') = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + (x_3 - x'_3)^2}.
\] (15)

For any given two points \( x \) and \( x' \) the vectors \( V^a \) and \( V^{a'} \) are defined in components as
\[
V^a = \frac{H\lambda}{r}(0, x - x'), \quad V^{a'} = \frac{H\lambda'}{r}(0, x' - x).
\] (16)

The vector \( V^a \) and \( V^{a'} \) are the unit vectors at points \( x \) and \( x' \), respectively, which is parallel to the projection of the tangent vector to the geodesic joining two points \( x \) and \( x' \) onto the constant \( \lambda \) hypersurface.
The field $h_{ab}(\lambda, \mathbf{x})$ has the following mode expansion:

$$h_{ab}(\lambda, \mathbf{x}) = \int d^3k \sum_{s=1}^{2} \left[ b^{(s)}(k) \frac{H}{4\sqrt{2\pi}} \xi^{3/2} H^{(1)}_{3/2}(k\lambda) \hat{H}_{ab}^{(k,s)} e^{ik\cdot x} + \text{h.c.} \right],$$  \hspace{1cm} (17)

where the symmetric traceless tensors $\hat{H}_{ab}^{(k,s)}$ satisfy $\hat{H}_{ab}^{(k,s)*} \hat{H}^{(k,s)}_{ab} = \delta^{ss'}$ and $t^a \hat{H}_{ab}^{(k,s)} = 0$. We have defined $k = ||\mathbf{k}||$. Here, $d^3k = dk_1dk_2dk_3$ and $\mathbf{k} \cdot \mathbf{x} = k_1x_1 + k_2x_2 + k_3x_3$. The Hankel function $H_{3/2}(z)$ is given by

$$H_{3/2}^{(1)}(z) = \sqrt{\frac{2}{\pi}} \left( -\frac{i}{z^{3/2}} - \frac{1}{z^{1/2}} \right) e^{iz}.$$  \hspace{1cm} (18)

The operators $b^{(s)}(k)$ and $b^{(s)*}(k)$ satisfy the commutation relations

$$[b^{(s)}(k), b^{(s')}\,(k')^\dagger] = \delta^{ss'} \delta^3(k - k'),$$  \hspace{1cm} (19)

with all other commutators vanishing. The Euclidean vacuum $|0\rangle$ is defined by $b^{(s)}(k)|0\rangle = 0$ for all $\mathbf{k}$ and $s$. By using the mode expansion (17) and remembering that the two-point function is a maximally symmetric bi-tensor in the spatial sections, we find

$$G_{aba'b'}(x, x') = f_1(\lambda, \lambda', r) \theta^{(1)}_{aba'b'} + f_2(\lambda, \lambda', r) \theta^{(2)}_{aba'b'} + f_3(\lambda, \lambda', r) \theta^{(3)}_{aba'b'}$$  \hspace{1cm} (20)

where the bi-tensors $\theta^{(i)}_{aba'b'}$ are given by

$$\theta^{(1)}_{aba'b'} = (V_aV_b - \frac{1}{3} P_{ab})(V_{a'}V_{b'} - \frac{1}{3} P_{a'b'})$$  \hspace{1cm} (21)

$$\theta^{(2)}_{aba'b'} = P_{aa'}P_{bb'} + P_{ba'}P_{ab'} - \frac{2}{3} P_{ab}P_{a'b'}$$  \hspace{1cm} (22)

$$\theta^{(3)}_{aba'b'} = 4V_aV_bV_{a'}V_{b'} + P_{aa'}V_bV_{b'} + P_{ba'}V_aV_{b'} + P_{ab'}V_aV_{a'} + P_{bb'}V_aV_{a'}.$$  \hspace{1cm} (23)

The functions $f_1$, $f_2$ and $f_3$ are given by

$$f_1 = \frac{H^2}{8\pi^2} \left[ \frac{3}{4} V^2(V^2 - 3) \psi_2 - \frac{1}{5} \left( 15V^4 - 40V^2 - 12 \right) \right]$$  \hspace{1cm} (24)

$$+ \frac{H^2\lambda\lambda'}{8\pi^2r^2} \left[ \frac{3}{4} (5V^2 - 9) \psi_2 + \frac{15V^4 - 37V^2 + 16}{1 - V^2} \right],$$

$$f_2 = \frac{H^2}{8\pi^2} \left[ -\frac{1}{5} \psi_1 + \frac{1}{20} V^2(V^2 + 5) \psi_2 - \frac{1}{75} (15V^4 + 80V^2 - 32) \right]$$  \hspace{1cm} (25)

$$+ \frac{H^2\lambda\lambda'}{8\pi^2r^2} \left[ \frac{1}{4} (V^2 + 3) \psi_2 + \frac{3V^4 + 7V^2 - 4}{3(1 - V^2)} \right],$$

$$f_3 = \frac{H^2}{8\pi^2} \left[ \frac{1}{4} V^2(V^2 + 1) \psi_2 - \frac{1}{15} (15V^4 + 20V^2 + 8) \right]$$  \hspace{1cm} (26)

$$+ \frac{H^2\lambda\lambda'}{8\pi^2r^2} \left[ \frac{1}{4} (5V^2 + 3) \psi_2 + \frac{15V^4 - V^2 - 8}{3(1 - V^2)} \right].$$
where
\[ V = \frac{\lambda - \lambda' + i\epsilon}{r} \]  
(27)

and
\[ \psi_1 = \log[\alpha^4 r^4(1 - V^2)^2] + 4\gamma, \]
(28)
\[ \psi_2 = V \log\left(\frac{V + 1}{V - 1}\right)^2. \]
(29)

Here, \( \gamma \) is Euler's constant and \( \alpha(>0) \) is an infrared cut-off. Since these results are essentially the same as those in Ref. [7] — recall that we have set \( 16\pi G = 1 \) — we omit the details of the calculations. The method is similar to that used in the next section.

In the large-\( r \) limit these functions become
\[ f_1 = \frac{3H^2}{10\pi^2} + O(r^{-2}), \]
(30)
\[ f_2 = \frac{H^2}{40\pi^2} \left(\frac{32}{15} - 2 \log \alpha^2 r^2 - 4\gamma\right) + O(r^{-2}), \]
(31)
\[ f_3 = -\frac{H^2}{15\pi^2} + O(r^{-2}). \]
(32)

Notice that the two-point function \( G_{aba'b'} \) is IR divergent and grows logarithmically with distance due to the behaviour of the function \( f_2 \). However, this behaviour can be a gauge artifact because linearised gravity has gauge invariance [5]. The two-point function \( G_{aba'b'} \) is physically equivalent to \( G_{\text{mod}aba'b'} \) if
\[ G_{aba'b'}(x, x') = G_{\text{mod}aba'b'}(x, x') + \nabla_a \nabla_a' T_{bb'} + \nabla_b \nabla_a T_{ab'} + \nabla_a \nabla_b T_{ba'} + \nabla_b \nabla_b T_{aa'}. \]
(33)

If there is a field \( T_{aa'} \) such that \( \nabla_a \nabla_a' T_{bb'} + \nabla_b \nabla_a T_{ab'} + \nabla_a \nabla_b T_{ba'} + \nabla_b \nabla_b T_{aa'} \) contains a term proportional to \( -(H^2/20\pi^2) \log \alpha^2 r^2 \times \theta^{(2)}_{aba'b'} \), then the modified two-point function \( G_{\text{mod}aba'b'} \) will be IR finite and has no logarithmic growth with distance \( r \). Allen found that the IR divergence could be gauged away in a similar manner. However, it is not difficult to show that the logarithmic growth with distance \( r \) can be gauged away together with the IR divergence. Indeed we find
\[ \log \alpha^2 r^2 \times \theta^{(2)}_{aba'b'} = -\frac{1}{6} \left( \nabla_a \nabla_a' K_{bb'} + \nabla_a \nabla_b' K_{ba'} + \nabla_a \nabla_a' K_{ab'} + \nabla_b \nabla_b' K_{aa'} \right) \]
\[ + \frac{1}{3} \left( P_{aba'b'} V_a V_b + P_{a'b'} V_a V_b + P_{aba'b'} V_a V_b + P_{b'b'} V_a V_b \right. \]
\[ + 4P_{aba'b'} V_a V_b + 4P_{a'b'} V_a V_b - 4P_{aba'} V_{bb'} - 4P_{ab'} V_{ab'} \]
\[ - 8V_a V_b' V_a V_b' \right) \]
(34)

with
\[ K_{aa'} = \frac{r^2}{H^2\lambda \nu} (V_a V_a' + 2P_{aa'}) \log \alpha^2 r^2. \]
(35)
Therefore we have
\[
G_{aba'b'}(x, x') = G_{aba'b'}^{\text{mod}}(x, x') + \frac{H^2}{120\pi^2} (\nabla_a \nabla_a' K_{bb'} + \nabla_b \nabla_a' K_{ab'} + \nabla_a \nabla_b' K_{ba'} + \nabla_b \nabla_b' K_{aa'}),
\]
(36)

where \(G_{aba'b'}^{\text{mod}}(x, x')\) does not grow logarithmically as the function of the distance between the two points \(x\) and \(x'\) and is IR finite.

This proves that the log \(r\) behaviour of the two-point function \(G_{aba'b'}\) is a gauge artifact. However, it may be desirable to use the two-point function \(\langle 0 | \nabla_a \xi_b(x) \nabla_a' \xi_{b'}(x') | 0 \rangle\) of a vector field \(\xi_a\) for gauging away the log \(\alpha^2 r^2\) term in order to have a better understanding of the logarithmic growth. In the next section we show how this can be done.

### 3 Pure-gauge two-point function

Recall that the tensor \(h^{(\xi)}_{ab} = \nabla_a \xi_b + \nabla_b \xi_a\) given by (11) satisfies \(\nabla^b h^{(\xi)}_{ab} = h^{(\xi)\,a}_{\,\,a} = 0\) and \((\Box - 2H^2) h^{(\xi)}_{ab} = 0\). These are the equations satisfied by \(h_{ab}\) in the physical gauge. Hence, it is natural to expect that the two-point function of \(h^{(\xi)}_{ab}\) has a structure similar to \(G_{aba'b'}\). We will find that this is indeed the case and that the log \(\alpha^2 r^2\) term in the two-point function \(G_{aba'b'}\) can be gauged away, in the manner described at the end of the previous section, using the two-point function of \(h^{(\xi)}_{ab}\) with an additional condition \(t^a \xi_a = 0\).

First we note that the transverse solutions to equation (12) satisfying the condition \(t^a \xi_a = 0\) are
\[
\xi^{(s)}_\alpha(k, x, \lambda) = \frac{H}{4\sqrt{2}\pi} \lambda^{3/2} H^{(1)}_{5/2}(k\lambda) e^{ik \cdot x} \hat{h}^{(k,s)}_{\alpha},
\]
(37)

where the polarisation vectors \(\hat{h}^{(k,s)}_{\alpha}\), \(s = 1, 2\), are orthogonal to \(k^a\) and \(t^a\) and satisfy
\[
P^{ab} h^{(k,s)}_{\alpha} h^{(k,s')}_{\beta} = \delta_{ss'}.\]
(38)

The Hankel function \(H^{(1)}_{5/2}(z)\) is given by
\[
H^{(1)}_{5/2}(z) = \sqrt{\frac{2}{\pi}} \left( -\frac{3i}{z^{5/2}} - \frac{3}{z^{3/2}} + \frac{i}{z^{1/2}} \right) e^{iz}.\]
(39)

We expand the field \(\xi_a(x)\) as
\[
\xi_a(x, \lambda) = \sum_{s=1}^{2} \int d^3k \left[ c^{(s)}(k) \xi^{(s)}_\alpha(k, x, \lambda) + c^{(s)}(k) \dagger \xi^{(s)}_\alpha(k, x, \lambda) \right].\]
(40)
We then impose the commutation relations

\[
[c^{(s)}(k), c^{(s')}(k')^{\dagger}] = \delta^{ss'} \delta^3(k - k'),
\]

(41)

with all other commutators being zero. (Since the field \(\xi_a\) is a fictitious field introduced to rewrite the two-point function \(G_{aba'b'}\), there is no need to derive these commutation relations.) Requiring that \(a^{(s)}(k)|0\rangle = 0\) for all \(s\) and \(k\), we have the two-point function of \(\xi_a\),

\[
M_{aa'}(x, x') = \langle 0|\xi_a(x)\xi_{a'}^{\dagger}(x')|0\rangle = \int d^3k \sum_{s=1}^{2} \xi_{a}^{(s)}(k, x, \lambda)\xi_{a'}^{(s)}(k, x', \lambda').
\]

(42)

By using (37) and (39), we find

\[
\sum_{s=1}^{2} \xi_{a}^{(s)}(k, x, \lambda)\xi_{a'}^{(s)}(k, x', \lambda') = \frac{H^2 E(k, \lambda, \lambda')}{16\pi^3\lambda\lambda'}e^{ik(-\lambda'-\lambda)x}e^{ik((x-x')^2/2)}\sum_{s=1}^{2} \hat{h}_{a}^{(s)}(k)\hat{h}_{a'}^{(s)}(k),
\]

(43)

where

\[
E(k, \lambda, \lambda') = 9k^{-5} - 9i(\lambda - \lambda')k^{-4} + [9\lambda\lambda' - 3(\lambda^2 + \lambda'^2)]k^{-3}
\]

\[
-3i\lambda\lambda'(\lambda - \lambda')k^{-2} + \lambda^2\lambda'^2k^{-1}.
\]

(44)

Using the properties of \(\hat{h}_{a}^{(s)}(k)\), we get

\[
\sum_{s=1}^{2} \hat{h}_{a}^{(s)}(k)\hat{h}_{a'}^{(s)}(k) = \frac{1}{H^2\lambda\lambda'} \left( \tilde{\delta}_{aa'} - \frac{\tilde{k}_a\tilde{k}_{a'}}{k^2} \right),
\]

(45)

where \(\tilde{\delta}_{aa'} = \lambda\lambda'P_{aa'}\). (In components, \(\tilde{\delta}_{11} = \tilde{\delta}_{22} = \tilde{\delta}_{33} = 1\) and all other components vanish.) We have also defined the spatial part of \(k_a\) as \(\tilde{k}_a = P_a\tilde{k}_b\). It is also convenient to define the spacelike projections of the partial derivatives as \(P_a\partial_e = \tilde{\partial}_a\) and \(P_{a'}\partial_{e'} = \tilde{\partial}_{a'}\). Combining equations (12)–(13), we obtain

\[
M_{aa'}(x, x') = \frac{1}{16\pi^3\lambda^2\lambda'^2} \int d^3k E(k, \lambda, \lambda')e^{ik((\lambda-\lambda')x+i\lambda')x} \left( \tilde{\delta}_{aa'} - \frac{\tilde{k}_a\tilde{k}_{a'}}{k^2} \right)
\]

\[
= \frac{1}{16\pi^3\lambda^2\lambda'^2} \int dk k^2 E(k, \lambda, \lambda')e^{ik\Lambda} \int d\Omega_k e^{ik(x-x')} \left( \tilde{\delta}_{aa'} - \frac{\tilde{k}_a\tilde{k}_{a'}}{k^2} \right),
\]

(46)
where the convergence factor $e^{-k\epsilon}$ has been introduced to make the integral converge for large wave numbers $k$. We have also defined $\Delta \lambda = \lambda - \lambda' + i\epsilon$, and $\Omega_k$ is the solid angle in the $k$ space. Noting that $\tilde{\delta}_a e^{ik(x-x')} = i\tilde{\kappa}_a e^{ik(x-x')}$ and

$$
\int d\Omega_k e^{ik(x-x')} = \frac{4\pi \sin kr}{kr},
$$

we obtain

$$
M_{a'a'}(x, x') = \frac{1}{4\pi^2 \lambda^2 \lambda'^2} \int dk k^2 E(k, \lambda, \lambda')e^{ik\Delta \lambda} \times \left(\delta_{aa'} - \frac{\tilde{\delta}_a \tilde{\delta}_{a'}}{k^2}\right) \frac{\sin kr}{kr}.
$$

We will show that the pure-gauge two-point function,

$$
T_{a'b'} \equiv 4\langle 0 | \nabla_{(a'b)} \nabla_{(a'b')} | 0 \rangle,
$$

has the same log $\alpha^2 r^2$ term as $G_{a'b'}$. We first note that

$$
\nabla_a \xi_b + \nabla_b \xi_a = L_{\xi} g_{ab} = \xi^c \partial_c g_{ab} + g_{ac} \partial_b \xi^c + g_{bc} \partial_a \xi^c,
$$

where $L_{\xi}$ is the Lie derivative with respect to the vector $\xi^a$. Since the vector field we consider (i.e., $\xi^a$) satisfies $t^a \xi_a = 0$, we have $\xi^c \partial_c g_{ab} = 0$. [Recall that the metric $g_{ab}$ depends only on time $\lambda$ and that $t^a$ is proportional to $-\partial/\partial\lambda$.] Hence we find

$$
T_{a'b'} \equiv 4\langle 0 | \nabla_{(a'b)}(x) \nabla_{(a'b')} (x') | 0 \rangle = \nabla_a \nabla_{a'} M_{bb'} + \nabla_a \nabla_{b'} M_{ba'} + \nabla_b \nabla_{a'} M_{ab'} + \nabla_b \nabla_{b'} M_{aa'} = g_{ac} g_{a'c'} \partial_b \partial_{b'} (0|\xi^c \xi^{c'}| 0) + g_{ac} g_{b'c'} \partial_a \partial_{a'} (0|\xi^c \xi^{c'}| 0) + g_{bc} g_{a'c'} \partial_a \partial_{b'} (0|\xi^c \xi^{c'}| 0) + g_{bc} g_{b'c'} \partial_a \partial_{a'} (0|\xi^c \xi^{c'}| 0).
$$

Define $T_{a'b'}^{A}$ to be the spacelike projection of $T_{a'b'}$,

$$
T_{a'b'}^{A} = P^c_a P^d_{a'} P^e_{b'} P^d_{b'} T_{cde'd'}.
$$

Then, remembering that $t^a \xi_a = 0$, we find

$$
T_{a'b'}^{A} = \tilde{\partial}_b \partial_{b'} M_{aa'}(x, x') + \tilde{\partial}_b \partial_{a'} M_{bb'}(x, x') + \tilde{\partial}_b \partial_{b'} M_{ba'}(x, x') + \tilde{\partial}_a \partial_{a'} M_{bb'}(x, x'),
$$

where $M_{a'a'}$ is given by (48). The components with one timelike index can be represented by

$$
T_{a'b'}^{B} = P^c_a t^d_{a'} P^e_{b'} P^d_{b'} T_{cde'd'} = -\frac{H}{\lambda} \partial_{b'} \partial \lambda^2 M_{aa'}(x, x') - \frac{H}{\lambda} \partial_{a'} \partial \lambda^2 M_{bb'}(x, x').
$$
(The minus sign arises due to the fact that the parameter $\lambda$ decreases towards the future.) We define $T_{aba'}^B$, as the tensor obtained by interchanging the primed and unprimed indices in $T_{aad'}^B$. The components with two timelike indices can be represented by

$$T_{ab'}^C \equiv P_a^{c\ell}l^{c'}P_{b'}^{\ell'}l'_{d'e'}=\frac{H^2}{\lambda^2} \frac{\partial^2}{\partial \lambda \partial \lambda'} [\lambda^2 \lambda'^2 M_{ab'}(x,x')] .$$

The pure-gauge two-point function $T_{aba'}$ is given by

$$T_{aba'} = T_{aba'}^A + t_b T_{aa'b'}^B + t_a T_{ba'b'}^B + t_a T_{ba'}^B$$

$$+ t_a t_a T_{bb'}^C + t_b t_a T_{ab'}^C + t_b t_a T_{ab'}^C + t_b T_{ab'}^C .$$

We will find that $T_{aba'}^A$ contains a term proportional to $\log \alpha^2 r^2 \times \theta^{(2)}_{aba'}$, whereas the other two tensors $T_{aba'}^B$ and $T_{aba'}^C$ contain no terms which grow with the distance $r$.

By using (48) and (53) we have

$$T_{aba'}^A = \frac{1}{4\pi^2 \lambda^2 \lambda'} \int dk k^2 E(k, \lambda, \lambda') e^{i k \Delta \lambda}$$

$$\times \left(\delta_5 \delta_5 \delta_{a'a'} + \delta_5 \delta_5 \delta_{a'b'} + \delta_5 \delta_5 \delta_{b'a'} + \delta_5 \delta_5 \delta_{b'b'} - \frac{4}{k^2} \delta_5 \delta_5 \delta_{a'b'} \delta_5 \delta_5 \right)$$

$$\times \frac{\sin kr}{kr} .$$

This tensor can be evaluated by using the integration formulas in Appendix A. The result is

$$T_{aba'}^A = T^{(1)} \times \theta^{(1)}_{aba'}^A + T^{(2)} \times \theta^{(2)}_{aba'}^A + T^{(3)} \times \theta^{(3)}_{aba'}^A ,$$

where the bi-tensors $\theta^{(i)}_{aba'}$ are defined in section 2, while the $T^{(i)}$ are given as follows:

$$T^{(1)} = -\frac{H^4}{4\pi^2} \left\{ \frac{36}{5} + 3V^2(4 + 3V^2(4 - \psi)) \right. + 3\frac{\lambda \lambda'}{r^2} \left[ -\frac{12V^2}{1-V^2} + 8 + 15V^2(4 - \psi) \right]$$

$$+ 3\frac{\lambda^2 \lambda'^2}{r^4} \left( 15(4 - \psi) + \frac{4(-7 + 5V^2)}{(1-V^2)^2} \right) \right\} ,$$

$$T^{(2)} = -\frac{H^4}{4\pi^2} \left\{ \frac{18}{5} \gamma - \frac{108}{25} + \frac{1}{5} V^2[4 + 3V^2(4 - \psi)] \right. + \frac{\lambda \lambda'}{r^2} \left[ -2 + 3V^2(4 - \psi) - \frac{6V^2}{1-V^2} \right]$$

$$+ \frac{\lambda^2 \lambda'^2}{r^4} \left[ \frac{4V^2 - 2}{(1-V^2)^2} + 3(4 - \psi) \right] + \frac{9}{5} \log \alpha^2 r^2(1-V^2) \right\} ,$$

$$T^{(3)} = -\frac{H^4}{4\pi^2} \left\{ \frac{3}{5} + V^2(4 + 3V^2(4 - \psi)) - \frac{3V^2(1 + V^2V^2 - 2)}{(1-V^2)^3} \right\} ,$$

$$= -\frac{H^4}{4\pi^2} \left\{ \frac{3}{5} + V^2(4 + 3V^2(4 - \psi)) - \frac{3V^2(1 + V^2V^2 - 2)}{(1-V^2)^3} \right\} .$$
\[+ \frac{\lambda \lambda'}{r^2} \left[-1 + 15V^2(4 - \psi) - \frac{3[-1 + V^2(11 + V^2(7V^2 - 17))]}{(1 - V^2)^3}\right]
\[+ \frac{\lambda^2 \lambda'}{r^4} \left[15(4 - \psi) - \frac{4[9 + V^2(5V^2 - 12)]}{(1 - V^2)^3}\right] \right]. \quad (61)

To compute \(T_B^{aa'b'}\) we use (48) and (54) and find
\[T_B^{aa'b'} = -\frac{H}{4\pi^2 \lambda \lambda'} \int dk k^2 \frac{\partial^2}{\partial \lambda \partial \lambda'} \left[E(k, \lambda, \lambda') e^{ik\Delta \lambda}\right] \times \left(\tilde{\delta}_{aa'} \tilde{\partial}_{b'} + \tilde{\delta}_{ab'} \tilde{\partial}_{a'} - 2 \frac{\tilde{\partial}_{b'} \tilde{\partial}_{a'}}{k^2} + \frac{\sin kr}{kr}\right). \quad (62)

Then, using the integral formulas in Appendix A, we have
\[T_B^{aa'b'} = \tilde{T}^{(1)} \beta^{(1)}_{aa'b'} + \tilde{T}^{(2)} \beta^{(2)}_{aa'b'} \quad (63)

with
\[\beta^{(1)}_{aa'b'} = P_{aa'} V_{b'} + P_{ab'} V_{a'} + 2V_a V_{a'} V_{b'}, \quad (64)
\[\beta^{(2)}_{aa'b'} = P_{a'b'} V_a - P_{aa'} V_{b'} - P_{ab'} V_{a'} - 5V_a V_{a'} V_{b'}, \quad (65)

and
\[\tilde{T}^{(1)} = \frac{H^4}{4\pi^2} \left\{ \frac{3\lambda}{r} \frac{1}{1 - V^2} + \frac{2\lambda \lambda'}{r^3} \frac{(3\lambda - \lambda')}{(1 - V^2)^2} \right. \]
\[+ \frac{1}{r^4} \frac{8\lambda^2 \lambda'^2 V}{(1 - V^2)^3} \right\}. \quad (66)
\[\tilde{T}^{(2)} = \frac{H^4}{4\pi^2} \left\{ \frac{2\lambda}{r} + \frac{1}{r^3} \left[ \frac{2\lambda \lambda'(-3\lambda + \lambda')}{1 - V^2} + 6\lambda^2(\lambda + \lambda') \right] \right. \]
\[+ \frac{1}{r^4} \left[ \frac{2\lambda^2 \lambda'^2 V(3V^2 - 5)}{(1 - V^2)^2} - \frac{3}{2} \lambda^4 \psi_2 \right] \}\). \quad (67)

We have defined \(\psi_2 = V^{-1}\psi\).

Next we examine the tensor \(T_C^{ab'}\) by using (55) and (18). We first find
\[T_C^{ab'} = \frac{H^2}{4\pi^2 \lambda \lambda'} \int dk k^2 \frac{\partial^2}{\partial \lambda \partial \lambda'} E(k, \lambda, \lambda') e^{ik\Delta \lambda} \times \left(\tilde{\delta}_{ab'} \tilde{\partial}_{\lambda'} - \frac{\tilde{\partial}_{a'} \tilde{\partial}_{\lambda'}}{k^2} + \frac{\sin kr}{kr}\right). \quad (68)

By integrating over \(k\), we have
\[T_C^{ab'} = S^{(1)} P_{ab'} + S^{(2)} V_a V_{a'}, \quad (69)\]
where the $S^{(1)}$ and $S^{(2)}$ are given by

$$S^{(1)} = -\frac{\lambda \lambda' H^4}{2\pi^2} \left[ \frac{1}{r^2 (1 - V^2)^2} + \frac{1}{r^4} \left( \frac{2\lambda \lambda' V^2}{(1 - V^2)^2} + \frac{4\lambda \lambda' V^2}{(1 - V^2)^3} \right) \right],$$  \hspace{1cm} (70)

$$S^{(2)} = -\frac{\lambda \lambda' H^4}{2\pi^2} \left[ \frac{1}{r^2 (1 - V^2)^2} + \frac{1}{r^4} \lambda \lambda' (1 - V^2)^3 \right].$$  \hspace{1cm} (71)

Details of the calculations presented here can be found in Appendix B.

4 Conclusion

It can be seen from the results of the previous section that $T^B_{ab'b'}$ and $T^C_{ab'}$ are of order $r^{-1}$ and of order $r^{-2}$, respectively, and that they are both IR finite. Hence, we have from (56)

$$T_{ab'b'} = T^{A}_{ab'b'} + O(r^{-1})$$

$$= -\frac{H^4}{4\pi^2} \left( \frac{18}{5} \gamma - \frac{108}{25} + \frac{9}{5} \log \alpha^2 r^2 \right) \times \theta^{(2)}_{ab'b'}$$

$$- \frac{36}{5} \times \theta^{(1)}_{ab'b'} + \frac{3}{5} \times \theta^{(3)}_{ab'b'} + O(r^{-1}).$$  \hspace{1cm} (72)

By comparing this result and the physical two-point function $G_{ab'b'}$ given by (20) with (30)–(32), we find

$$G_{ab'b'}(x, x') = G^{\text{mod}}_{ab'b'}(x, x') + \frac{4}{9H^2} \langle 0 | \nabla_{(a} \xi_{b)}(x) \nabla_{(a'} \xi_{b')}(x') | 0 \rangle,$$  \hspace{1cm} (73)

where the two-point function $G^{\text{mod}}_{ab'b'}(x, x')$, which is related to $G_{ab'b'}(x, x')$ by a gauge transformation, is IR finite and does not grow logarithmically with $r$. Thus, we have shown that the logarithmic behaviour in $G_{ab'b'}(x, x')$ can be gauged away by a two-point function of a pure-gauge field.

Recently, Hawking, Hertog and Turok \[9\] have found that the physical graviton two-point function is well-behaved for large distances in open de Sitter spacetime. Now, gauge-invariant correlation functions must be the same in the de Sitter invariant vacuum, whether we use the spatially-flat or open coordinate system. Hence, their result implies that there cannot be any physical effects due to the logarithmic term in the two-point function (20). This is consistent with our result. Finally, it will be interesting to investigate the implication of our result for two-point functions in covariant gauges (see, e.g. Ref. \[3, 10\]).
Acknowledgement

We thank Bernard Kay for useful discussions. Some of the calculations in this paper were performed using Maple V Release 5.1.
Appendix A. Some useful integrals

We first obtain
\[ \int_\alpha^\infty \frac{dk}{k} e^{ik(x+i\epsilon)} = \int_\alpha^{\infty} \frac{d\kappa}{\kappa} \cos \kappa + i \int_\alpha^{\infty} \frac{d\kappa}{\kappa} \sin \kappa = -\gamma - \log \alpha (x + i\epsilon) + \frac{\pi i}{2}, \quad (A1) \]
where the terms of order \( \alpha \) or higher are neglected. By differentiating this formula with respect to \( x \) we find
\[ \int_0^{\infty} dk e^{ik(x+i\epsilon)} = \frac{i}{x + i\epsilon}. \quad (A2) \]
\[ \int_0^{\infty} dk k^2 e^{ik(x+i\epsilon)} = \frac{2i}{(x + i\epsilon)^3}. \quad (A3) \]
\[ \int_0^{\infty} dk k e^{ik(x+i\epsilon)} = -\frac{1}{(x + i\epsilon)^2}. \quad (A4) \]

We define \( K = \frac{\pi i}{2} - \gamma \) and use integration by parts to find the following formulas:
\[ \int_\alpha^{\infty} \frac{dk}{k^2} \frac{e^{ikx}}{k^2} = \frac{1}{\alpha} + ix[K + 1 - \log \alpha x], \quad (A5) \]
\[ \int_\alpha^{\infty} \frac{dk}{k^3} \frac{e^{ikx}}{k^3} = \frac{1}{2\alpha^2} + \frac{ix}{\alpha} - \frac{x^2}{2} \left[ K + \frac{3}{2} - \log \alpha x \right], \quad (A6) \]
\[ \int_\alpha^{\infty} \frac{dk}{k^4} \frac{e^{ikx}}{k^4} = \frac{1}{3\alpha^3} + \frac{ix}{2\alpha^2} - \frac{x^2}{2\alpha} - \frac{ix^3}{6} \left[ K + \frac{11}{6} - \log \alpha x \right], \quad (A7) \]
\[ \int_\alpha^{\infty} \frac{dk}{k^5} \frac{e^{ikx}}{k^5} = \frac{1}{4\alpha^4} + \frac{ix}{3\alpha^3} - \frac{x^2}{4\alpha^2} - \frac{ix^3}{6 \alpha} + \frac{x^4}{24} \left[ K + \frac{25}{12} - \log \alpha x \right], \quad (A8) \]
\[ \int_\alpha^{\infty} \frac{dk}{k^6} \frac{e^{ikx}}{k^6} = \frac{1}{5\alpha^5} + \frac{ix}{4\alpha^4} - \frac{x^2}{6\alpha^3} - \frac{ix^3}{12 \alpha^2} + \frac{x^4}{24 \alpha} - \frac{ix^5}{120} \left[ K + \frac{137}{60} - \log \alpha x \right]. \quad (A9) \]

Appendix B. Details of the calculation of the pure-gauge two-point function

B1. Spatial components

To calculate \( T^A_{a b a'} b' \) we start from the quantity
\[ A_{a b a'b'} = \frac{4\pi}{(\lambda \lambda)^2} \left( \tilde{\delta}_{a a'} \tilde{\partial} a \tilde{\partial} b' + \tilde{\delta}_{b a'} \tilde{\partial} a \tilde{\partial} b' + \tilde{\delta}_{a a'} \tilde{\partial} b \tilde{\partial} a' + \tilde{\delta}_{b a'} \tilde{\partial} b \tilde{\partial} a' \right. \]
\[ \left. - \frac{4}{k^2} \tilde{\partial} a \tilde{\partial} b \tilde{\partial} a' \tilde{\partial} b' \right) \sin kr \frac{kr}{k} \quad (B1) \]
in equation (57). By using

$$\tilde{\partial}_a f(r) = \frac{\partial f}{\partial r} r, \quad \tilde{\partial}_a r_{a'} = -\delta_{aa'},$$

(B2)

and using symmetries of $A_{aba'b'}$ we find

$$A_{aba'b'} = \frac{1}{(\lambda')^2} \left[ A^{(1)} \frac{r_{a'b'} r_{a'a'}}{r^4} + A^{(2)} \frac{\tilde{\delta}_{aa'} r_{a'b'} + \tilde{\delta}_{bb'} r_{a'a'}}{r^2} + A^{(3)} \frac{\tilde{\delta}_{aa'} r_{a'b'} + \tilde{\delta}_{bb'} r_{a'a'}}{r^2} + A^{(4)} \tilde{\delta}_{ab'} \tilde{\delta}_{aa'} + \tilde{\delta}_{bb'} \tilde{\delta}_{aa'} \right],$$

(B3)

where

$$A^{(1)} = 4\pi \left( -\frac{4k \sin kr}{r} - \frac{40 \cos kr}{r^2} + \frac{180 \sin kr}{k r^3} + \frac{420 \cos kr}{k^2 r^4} - \frac{420 \sin kr}{k^3 r^5} \right),$$

(B4)

$$A^{(2)} = 4\pi \left( -\frac{k \sin kr}{r} - \frac{7 \cos kr}{r^2} + \frac{27 \sin kr}{k r^3} + \frac{60 \cos kr}{k^2 r^4} - \frac{60 \sin kr}{k^3 r^5} \right),$$

(B5)

$$A^{(3)} = 4\pi \left( \frac{4 \cos kr}{r^2} - \frac{24 \sin kr}{k r^3} - \frac{60 \cos kr}{k^2 r^4} + \frac{60 \sin kr}{k^3 r^5} \right),$$

(B6)

$$A^{(4)} = 4\pi \left( -\frac{2 \cos kr}{r^2} + \frac{6 \sin kr}{k r^3} + \frac{12 \cos kr}{k^2 r^4} - \frac{12 \sin kr}{k^3 r^5} \right),$$

(B7)

$$A^{(5)} = 4\pi \left( \frac{4 \sin kr}{k r^3} + \frac{12 \cos kr}{k^2 r^4} - \frac{12 \sin kr}{k^3 r^5} \right).$$

(B8)

Using the definitions for $P_{ab}$ and $V_a$, we can express $A_{aba'b'}$ in terms of $P_{ab}$ and $V_a$ as

$$H^{-4} A_{aba'b'} = A^{(1)} \times V_a V_b V_a' V_b' + A^{(2)} \times (P_{aa'} V_b V_b' + P_{ba'} V_a V_b' + P_{ab} V_b V_a' + P_{bb'} V_a V_a') + A^{(3)} \times (P_{a'b'} V_a V_b + P_{ab} V_a' V_b') + A^{(4)} \times (P_{a'b'} P_{ab} + P_{bb'} V_a V_a') + A^{(5)} \times P_{ab} P_{a'b'}.$$

(B9)

The identity $\tilde{\delta}_{ab} A_{aba'b'} = 0$ implies that $A^{(1)} - 4A^{(2)} + 3A^{(3)} = A^{(4)} + 3A^{(5)} = 0$. These equations are indeed satisfied. They allow us to eliminate $A^{(1)}$ and $A^{(5)}$. Thus, we obtain

$$A_{aba'b'} = -3A^{(3)} \times \theta^{(1)}_{aba'b'} + A^{(4)} \times \theta^{(2)}_{aba'b'} + A^{(2)} \times \theta^{(3)}_{aba'b'},$$

(B10)

where the $\theta^{(i)}$ are defined in section 3. By substituting this in equation (57) and noting that

$$\frac{\cos kr}{k^n} e^{ik \Delta \lambda} = \frac{1}{2} \left[ e^{ik(r+\Delta \lambda)} + e^{ik(-r+\Delta \lambda)} \right],$$

(B11)

$$\frac{\sin kr}{k^n} e^{ik \Delta \lambda} = \frac{1}{2i} \left[ e^{ik(r+\Delta \lambda)} - e^{ik(-r+\Delta \lambda)} \right],$$

(B12)
we can calculate $T^A_{aba'}$, using the integrals in Appendix A. Thus we obtain

$$T^A_{aba'} = T^{(1)} \times \theta^{(1)}_{aba'} + T^{(2)} \times \theta^{(2)}_{aba'} + T^{(3)} \times \theta^{(3)}_{aba'}$$  \hspace{1cm} (B13)$$

where

$$T^{(1)} = -\frac{H^4}{4\pi^2} \left\{ -\frac{36\lambda\lambda'(\Delta \lambda)^2}{r^2(r-\Delta \lambda)(r+\Delta \lambda)} - \frac{60\lambda^2\lambda'^2(\Delta \lambda)^2}{r^2(r-\Delta \lambda)^2(r+\Delta \lambda)^2} \right. \\
-\frac{84\lambda^2\lambda'^2}{(r-\Delta \lambda)^2(r+\Delta \lambda)^2} + \frac{36}{5} \frac{36h(\lambda, \lambda')}{r^4} + \frac{12(\lambda^2 + \lambda'^2)}{r^2} \\
\left. -\frac{18(\lambda^5 - \lambda'^5)}{r^5} \log \frac{r + \Delta \lambda}{-r + \Delta \lambda} \right\}, \hspace{1cm} (B14)$$

$$T^{(2)} = -\frac{H^4}{4\pi^2} \left\{ \frac{18}{5} \gamma - \frac{108}{25} - \frac{2\lambda\lambda'(3(\Delta \lambda)^2 + 2\lambda')}{r^2(r-\Delta \lambda)(r+\Delta \lambda)} - \frac{4\lambda^2\lambda'^2}{(r-\Delta \lambda)^2(r+\Delta \lambda)^2} \right. \\
+ \frac{4}{5} \frac{(\Delta \lambda)^2}{r^2} + \frac{8\lambda'}{r^2} + \frac{12h(\lambda, \lambda')}{r^4} \\
\left. + \frac{9}{5} \log \alpha^2(r^2 - (\Delta \lambda)^2) - \frac{6}{5} \frac{\lambda^5 - \lambda'^5}{r^5} \log \frac{r + \Delta \lambda}{-r + \Delta \lambda} \right\}, \hspace{1cm} (B15)$$

$$T^{(3)} = -\frac{4\pi H^4}{4\pi^2} \left\{ 3((\Delta \lambda)^2 - \lambda\lambda')r^6 + (33\lambda\lambda'(\Delta \lambda)^2 - 6(\Delta \lambda)^4 - 36\lambda^2\lambda'^2)r^4 \\
+ (-51\lambda\lambda'(\Delta \lambda)^4 - 48\lambda^2\lambda'^2 + 3(\Delta \lambda)^6)r^2 \\
+ 20\lambda^2\lambda'^2(\Delta \lambda)^4 + 21\lambda\lambda'(\Delta \lambda)^6]/r^2(r + \Delta \lambda)^3(r - \Delta \lambda)^3 - \frac{3}{5} \\
\left. + \frac{-\lambda\lambda' + 4(\Delta \lambda)^2}{r^2} + \frac{12h(\lambda, \lambda')}{r^4} - \frac{6(\lambda^5 - \lambda'^5)}{r^5} \log \frac{r + \Delta \lambda}{-r + \Delta \lambda} \right\}. \hspace{1cm} (B16)$$

with

$$h(\lambda, \lambda') = \lambda^4 + \lambda^3\lambda' + \lambda^2\lambda'^2 + \lambda\lambda'^3 + \lambda'^4. \hspace{1cm} (B17)$$

The expression \((\ref{eq:11})\) follows by reexpressing these in terms of $V$ and $\psi$.

**B2. Components with one time index**

We first find a simpler form for the expression \((\ref{eq:12})\). We note that

$$\frac{4\pi}{H^3\lambda \lambda'^2} \left( \delta_{aa'} \delta_{b'} - \frac{1}{k^2} \delta_{b'} \delta_{a'} \delta_{aa'} \right) \frac{\sin kr}{kr} \\
= 4\pi \left( \frac{\cos kr}{r} - \frac{\sin kr}{kr^2} \right) \times P_{aa'} V_{b'} \\
+ 4\pi \left( \frac{\sin kr}{kr^2} + \frac{3 \cos kr}{k^2 r^3} - \frac{3 \sin kr}{k^3 r^4} \right) \times (-P_{aa'} V_{b'} + P_{a'b'} V_a - P_{ab'} V_a) \\
+ 4\pi \left( \frac{\cos kr}{r} - \frac{6 \sin kr}{k^2 r^3} - \frac{15 \cos kr}{k^3 r^4} + \frac{15 \sin kr}{k^3 r^4} \right) V_a V_{a'} V_{b'}. \hspace{1cm} (B18)$$
We also find
\[
\frac{k^2}{\lambda} \frac{\partial}{\partial \lambda} \left[ E(k, \lambda, \lambda') e^{i k (\lambda - \lambda')} \right] = \left[ 3 \lambda k^{-1} - 3i \lambda \Delta \lambda + \lambda \lambda' (-\lambda' + 3 \lambda) k + i \lambda^2 \lambda'^2 k^2 \right] e^{i k \Delta \lambda}.
\] (B19)

Therefore we have
\[
\frac{H \partial}{\lambda \partial \lambda} \tilde{\partial}_{\nu} [\lambda^2 M_{a' a}(x, x')] = \frac{H^4}{16 \pi^3} \int dk \left\{ 3 \lambda k^{-1} - 3i \lambda \Delta \lambda + \lambda \lambda' (-\lambda' + 3 \lambda) k + i \lambda^2 \lambda'^2 e^{i k \Delta \lambda} \right\}
\times \left\{ 4 \pi \left( \frac{\cos kr}{r} - \frac{\sin kr}{k r^2} \right) \times P_{a a' \nu'} V_{b' a'} + 4 \pi \left( \frac{\sin kr}{k r^2} + 3 \cos kr \frac{3 \sin kr}{k^2 r^3} \right) \times (- P_{a a' \nu' b' a'} + P_{a b' \nu' a'} - P_{a b' \nu' a'} V_{a a'}) \right\}. \] (B20)

We obtain the analogous expression for \((\frac{H}{\lambda} \partial / \partial \lambda) \tilde{\partial}_{a'} [\lambda^2 M_{a b'} (x, x')]\) by interchanging \(a'\) with \(b'\). Thus, we have
\[
T^{B}_{a a' b'} = - \frac{H \partial}{\lambda \partial \lambda} \tilde{\partial}_{a'} [\lambda^2 M_{a a'} (x, x')] - \frac{H \partial}{\lambda \partial \lambda} \tilde{\partial}_{a'} [\lambda^2 M_{a b'} (x, x')]
= - \frac{H^4}{16 \pi^3} \int dk \left[ 3 \lambda k^{-1} - 3i \lambda \Delta \lambda + \lambda \lambda' (-\lambda' + 3 \lambda) k + i \lambda^2 \lambda'^2 k^2 \right] e^{i k \Delta \lambda} \times D_{a a' b'}, \] (B21)

where \(D_{a a' b'}\) is given by
\[
D_{a a' b'} = D^{(1)} \beta^{(1)}_{a a' b'} + D^{(2)} \beta^{(2)}_{a a' b'} \] (B22)

with
\[
D^{(1)} = 4 \pi \left( \frac{\cos kr}{r} - \frac{\sin kr}{k r^2} \right),
\]
\[
D^{(2)} = 8 \pi \left( \frac{\sin kr}{k r^2} + \frac{3 \cos kr}{k^2 r^3} - \frac{3 \sin kr}{k^3 r^4} \right) \] (B23)

and
\[
\beta^{(1)}_{a a' b'} = P_{a a'} V_{b'} + P_{a b'} V_{a'} + 2 V_{a} V_{a'} V_{b'} , \] (B24)
\[
\beta^{(2)}_{a a' b'} = P_{a b'} V_{a} - P_{a a'} V_{b'} - P_{a b'} V_{a'} - 5 V_{a} V_{a'} V_{b'} . \] (B25)

Again, by performing the \(k\) integration using the formulas in Appendix A we find the result in (63).
\section*{B3. Components with two time indices}

We begin by noting that

\begin{equation}
\frac{4\pi}{H^2\lambda\lambda'} \left( \delta_{ab'} - \frac{\partial a \partial b'}{k^2} \right) \frac{\sin kr}{kr} = 4\pi \left( \frac{\sin kr}{kr} + \frac{\cos kr}{k^2 r^2} - \frac{\sin kr}{k^3 r^3} \right) P_{aa'} + 4\pi \left( \frac{\sin kr}{kr} + \frac{3 \cos kr}{k^2 r^2} - \frac{3 \sin kr}{k^3 r^3} \right) V_a V_{a'}
\end{equation}

and

\begin{equation}
k^2 \frac{\partial^2}{\partial \lambda \partial \lambda'} \left[ E(k, \lambda, \lambda') e^{ik\Delta \lambda} \right] = \left( \lambda\lambda' k - i\lambda\lambda' \Delta \lambda k^2 + \lambda^2 \lambda'^2 k^3 \right) e^{ik\Delta \lambda}.
\end{equation}

Therefore we find

\begin{equation}
T_{ab'} = \frac{H^2}{\lambda\lambda'} \frac{\partial^2}{\partial \lambda \partial \lambda'} \left[ (\lambda\lambda')^2 M_{ab'}(x, x') \right] = \frac{H^4}{16\pi^2} \int dk \left( \lambda\lambda' k - i\lambda\lambda' \Delta \lambda k^2 + \lambda^2 \lambda'^2 k^3 \right) e^{ik\Delta \lambda} \times E_{ab'},
\end{equation}

where $E_{ab'}$ is given by

\begin{equation}
E_{ab'} = E^{(1)} P_{ab'} + E^{(2)} V_a V_{b'},
\end{equation}

with

\begin{align}
E^{(1)} &= 4\pi \left( \frac{\sin kr}{kr} + \frac{\cos kr}{k^2 r^2} - \frac{\sin kr}{k^3 r^3} \right), \\
E^{(2)} &= 4\pi \left( \frac{\sin kr}{kr} + \frac{3 \cos kr}{k^2 r^2} - \frac{3 \sin kr}{k^3 r^3} \right).
\end{align}

We integrate over $k$ using the formulas in Appendix A and obtain

\begin{equation}
T_{ab'} = S^{(1)} P_{ab'} + S^{(2)} V_a V_{b'},
\end{equation}

with

\begin{align}
S^{(1)} &= -\frac{H^4 \lambda\lambda'}{2\pi^2} \left[ \frac{\lambda^2 + \lambda'^2}{(r - \Delta \lambda)^2(r + \Delta \lambda)^2} + \frac{4\lambda\lambda'(\Delta \lambda)^2}{(r + \Delta \lambda)^3(r - \Delta \lambda)^3} \right], \\
S^{(2)} &= -\frac{H^4 \lambda\lambda'}{2\pi^2} \left[ \frac{1}{(r - \Delta \lambda)^2(r + \Delta \lambda)^2} + \frac{4\lambda\lambda'}{(r + \Delta \lambda)^3(r - \Delta \lambda)^3} \right].
\end{align}

By expressing these in terms of $V$ and powers of $\lambda\lambda'$ we find (71).
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