DOA Based on Manifold Separation Technique and Dimensionality Reduction

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Abstract—In this paper, we investigate the high resolution of arbitrary array applied to wireless positioning system and propose a Direction of Arrival (DOA) estimation algorithm based on the Manifold Separation Technique (MST). In order to reduce the error of MST model caused by noise, this algorithm uses the MST to decompose the steering vector into sampling matrix (only describing the array itself) and Vandermonde form (only relating to the direction of arrival), the steering vector of arbitrary array is represented by combining the Effective Aperture Distribution Function (EADF). In order to reduce the complexity of Multiple Signal Classification (MUSIC) algorithm of two-dimensional array, the 2D spectral peak search is decomposed into two one-dimensional spectral peak searches using dimensionality reduction methods. Simulation results show that the algorithm not only reduces the computational complexity, but also has good performance of low signal to noise ratio (SNR).

1. INTRODUCTION
Direction of Arrival (DOA) positioning technique [1] is to analyze the incoming direction of the Radio Frequency (RF) signal by different antenna array. The most common directional antenna arrays include Uniform Linear Array [1], Uniform Rectangular Array [2], and Uniform Circular Array [3][4]. With a one-dimensional antenna array, only azimuth can be reliably measured. If a two-dimensional array is used, azimuth and elevation angles in three-dimensional space can be reliably measured. Among the many high-resolution DOA estimation algorithms with excellent performance, the subspace-based Multiple Signal Classification (MUSIC) [5] algorithm has high generalizability and a much smaller computational volume compared to the Maximum Likelihood (ML) [6] and Weighted Subspace Fitting (WSF) [7][8]. In array signal processing, it is often convenient to use an array of sensor structures with a Vandermond form. In order to map steering vectors that do not have a Vandermond form onto steering vectors that have the desired Vandermonde form, e.g. Beam-forming Transform [9], Array Interpolation [10], and Manifold Separation Technique (MST) have been developed. Array interpolation is the mapping of irregular arrays onto Uniform Array, usually affected by angle partitioning, number of correction directions, and sparsity of arrays, there are too many subjective factors in practical application. The MST is to decompose the steering vector into a product of the sampling matrix and the form of Vanderbilt, where noise causes errors in the MST model and affects angle estimates. To reduce the effect of measurement noise, the literature [12] gives the Effective Aperture Distribution Function (EADF) and the basis for the selection of the EADF matrix dimensions.
The performance of the MST-Dimension Reduction (DR)-MUSIC algorithm proposed in this paper is tested by using arbitrary array configurations in which the array form does not satisfy Vandermond form. The algorithm combines MST, EADF and reduced dimensional thinking to handle arbitrary array structures. The MST decomposes the array form into a Vandermond form, and the EADF allows a sufficiently accurate description of the defective array, represented by corrected measured Inverse Discrete Fourier Transform (IDFT). When arbitrary arrays are two-dimensional arrays, the application of MUSIC algorithm needs to perform two-dimensional peak search with high algorithm complexity, reducing the dimension needs to be done to reduce the algorithm complexity. The simulation results show that compared with the traditional 2D-MUSIC, the MST-DR-MUSIC algorithm proposed in this paper not only improves the computation speed but also has good low signal to noise ratio (SNR) performance in arbitrary array.

This paper is organized as follows: the first part presents the signal model, the second part presents the MST with the EADF to express an arbitrary array, the third part presents the derivation of the MST downscaling DOA algorithm, the fourth part presents the simulation results and discusses them, and the fifth part presents the conclusion.

2. SIGNAL MODEL

Assuming that there are $M$ sensors and $K$ ( $K$ less than $M$ ) non-coherent narrowband signals, the transmitting narrowband signal is $S$. While assuming that the signal is arriving at the sensor simultaneously and that the array output signal is $X$ in the $(M \times 1)$ dimension.

$$X = A(\theta, \phi)S + N \quad (1)$$

Where $S$ is the signal vector of the $(K \times 1)$ dimension and $N$ is the noise vector of the $(M \times 1)$ dimension. $A(\theta, \phi)$ is the directional matrix of the $(M \times K)$ dimension.

$$A(\theta, \phi) = [a(\theta_1, \phi_1), a(\theta_2, \phi_2), \ldots, a(\theta_K, \phi_K)] \quad (2)$$

Where $a(\theta_k, \phi_k)$ is the direction vector of the $k-th$ source and the direction vector of arbitrary array can be expressed as:

$$a(\theta_k, \phi_k) = \begin{bmatrix}
1 \\
\exp(j2\pi(x_1 \sin \theta_1 \cos \phi_1 + y_1 \sin \theta_1 \cos \phi_1) / \lambda) \\
\vdots \\
\exp(j2\pi(x_K \sin \theta_K \cos \phi_K + y_K \sin \theta_K \cos \phi_K) / \lambda)
\end{bmatrix} \quad (3)$$

where $\lambda$ is the wavelength. $\theta_k, \phi$ are the pitch and plane angles, respectively, and $(x_k, y_k)$ represents the coordinates of the $M-th$ array.

![Image](261x147 to 273x162)

![Image](263x178 to 273x193)

Figure1 Arbitrary Array Mode
3. MST AND EADF REPRESENT ARBITRARY ARRAY

The MST is to decompose the steering vector of the array into the product of the sampling matrix and the Vandermonde form, and then we can extend the fast-DOA algorithm suitable for linear arrays to arbitrary arrays. The sampling matrix describes the array itself and is only related to the actual structure of the array. The Fourier basis of space in the form of Vandermonde is only related to the direction of arrival. In order to reduce the influence of noise, the sampling matrix of MST can be approximated by the EADF.

Combining the concepts of Effective Aperture Distribution Function and Manifold Separation Technique, we define the beam pattern of $M$ sensors of arbitrary array as $\mathbf{D}$ and the sampling matrix as $\mathbf{G}$, then the steering vector matrix of arbitrary array is expressed as:

$$\mathbf{A}(\phi) = \mathbf{GD}(\phi) + o(N_c)$$

(4)

The beam pattern $\mathbf{D}(\phi) = [d(\phi_1), \ldots, d(\phi_M)]$ is only related to the direction of arrival. $d(\phi_k)$ represents the Vandermonde structure of the $(M \times 1)$ signal.

Finally, complete content and organizational editing before formatting. Please take note of the following items when proofreading spelling and grammar:

In order to simplify the problem, it is assumed that the array elements are isotropic and there is no coupling between array elements. The specific method of determining the sampling matrix is as follows: first fix the elevation angle, and let the azimuth angle be scanned on $\phi \in [-\pi, \pi)$ to obtain the angle set: $\phi = [\phi_1, \phi_2, \ldots, \phi_Q]$.

If the angles do not overlap, the EADF matrix $\mathbf{G}$ can be obtained by calculating the IDFFT of the $Q$ point, and further simplify the sampling matrix can be truncated, if a certain level of truncation is appropriate, it will have no impact.

![Figure 2 EADF of Arbitrary Array](image)

As shown in Figure 2, it indicates the optimal truncation location, and it is important to note that the truncation length $N_c$ is kept to an odd number.
4. MUSIC BASED ON MST AND DIMENSION REDUCTION

Spatial spectrum estimation is mainly a method of analysis and processing based on the received data of (1). Assuming that the noise is a Gaussian zero mean process, the noise of each array element is independent, and the noise and the signal are also independent of each other, then the covariance matrix $R_{xx} \in \mathbb{C}^{M \times M}$ of the received signal is expressed as:

$$R_x = E(XX^H) = ARAR^H + \sigma_n^2 I$$  \hfill (6)

Where $\cdot^H$ represents the Hermitian operation, $R = E(SS^H)$ is the signal covariance matrix, and $\sigma_n^2$ is the noise power.

Since the signal power is significantly greater than the noise power, the feature vector corresponding to the larger eigenvalue is the vector corresponding to the incident signal. Feature decomposition of the covariance matrix of the received signal:

$$R_x = UUU^H + UUU^H$$  \hfill (7)

Where $A$ is the diagonal matrix corresponding to the covariance matrix, and the values on the diagonal approximate the signal power and noise power. $U_s$ is the signal subspace of the $(M \times P)$ dimension and $U_n$ is the noise subspace of the $M \times (M - P)$ dimension. Under ideal conditions the signal subspace and the noise subspace are orthogonal to each other. And the signal subspace is the same space as the space formed by the direction vector of the incident signal, so there:

$$A(\theta, \phi)U_n = 0$$  \hfill (8)

$$A(\theta, \phi) = U_s$$  \hfill (9)

The power spectrum of the MUSIC algorithm can be expressed as:

$$P_{MUSIC} = \frac{1}{A^H U_n U_s^H A}$$  \hfill (10)

The angle of arrival is the angle corresponding to the peak:

$$\left(\theta, \phi\right) = \arg\max(\frac{1}{A^H U_n U_s^H A})$$  \hfill (11)

The above is the derivation of the general form of the MUSIC algorithm, and then the MST algorithm combined with dimensionality reduction is derived.

Assuming that the initial elevation angle is $\theta_0$, (4) can be rewritten as:

$$A(\phi) = G(\theta_0)D(\phi) + \sigma(N_s)$$  \hfill (12)

Among them, $G(\phi)$ is generated by fixing the elevation angle $\theta_0$, and scanning the azimuth angle at $\phi \in [-\pi, \pi]$ to obtain the field source steering vector $A(\phi)$ and the wave beam pattern $D(\phi)$, and $G(\phi)$ can be obtained according to the least square method. Substituting (12) into the signal model (1), so that the received signal matrix is rewritten as:

$$X = G(\theta_0)D(\phi)S + N$$  \hfill (13)

The new covariance matrix is now expressed as:

$$R_x = E(xx^H) =$$

$$G(\theta_0)D(\phi)RR(\phi)D(\phi)G^H(\theta_0) + \sigma_n^2 I$$  \hfill (14)

Applying the MST to dimensionality reduction and MUSIC algorithm, $\phi$ the azimuth $\phi$ spectral peak search function becomes:

$$P(\phi)_{MUSIC} = \frac{1}{D^H(\phi)G^H(\theta_0)U_s U_s^H G(\theta_0)D(\phi)}$$  \hfill (15)

Azimuth $\phi$ to be estimated as:

$$\phi = \arg\max(P(\phi)_{MUSIC})$$  \hfill (16)
At this point, the azimuth $\hat{\varphi}$ has been obtained. Return it to the spectral function and perform a spectral peak search to find the elevation $\hat{\theta}$:

$$\hat{\theta} = \arg \max \left( \frac{1}{D^H(\hat{\varphi})G^H(\hat{\theta})U_n^H G(\hat{\theta}) D(\hat{\varphi})} \right)$$ (17)

5. SIMULATION AND RESULT

Experiment I: Comparison of 2D-MUSIC and dimensionality reduction MUSIC. The experimental parameters were set as: 200 sampling points, number of array elements $M=6$, SNR = 20dB, search step length 1 degree, angle of incidence (80,20). Figure 4 represents the results of the dimensionality reduction MUSIC simulation, Figure 4(a) represents the peak performance of the azimuthal angle, and Figure 4(b) represents the peak search of the elevation angle. The simulation results from Figure 4 show that the descending-dimensional MUSIC and the 2D-MUSIC search results from Figure 3 are consistent, which indicates that the descending-dimensional MUSIC estimation angle is feasible. To further compare the performance of descending-dimensional MUSIC and 2D-MUSIC, 500 Monte Carlo experiments were performed at different signal-to-noise ratios.

$$\frac{1}{K} \sum_{k=1}^{K} \sqrt{\frac{1}{S} \sum_{s=1}^{S} \left( \hat{\theta}_{k,s} - \hat{\theta} \right)^2 + \left( \hat{\varphi}_{k,s} - \varphi \right)^2}$$ (18)

Where $\hat{\theta}_{k,n}$ and $\hat{\varphi}_{k,n}$ represent the estimated values of $\theta$ and $\varphi$ in the $n$th simulation, respectively.

Figure 3 Spectrum of 2D-MUSIC

(a) Spectrum peak of Azimuth  (b) Spectrum of Elevation

Figure 4 Spectrum in reduced dimension
The simulation results in Figure 5 show that 2D-MSUIC and the dimensionality reduction MUSIC have similar performance curves. From experiment I, it is concluded that the dimensionality reduction not only does not affect the search dimensionality reduction accuracy of the MUSIC algorithm, but also reduces the complexity of the algorithm. According to the experiment I, this dimensionality reduction method not only does not affect the search accuracy of the MUSIC algorithm, but also reduces the complexity of the algorithm.

Experiment II: Under the same input signal, compare the 2D-MUSIC algorithm with the dimensionality reduction MST algorithm proposed in this paper. The experiment was performed in an arbitrary array configuration with a sampling length of 200 times. Simulation Figure 6 represents the root-mean-square error of the elevation and azimuthal conjunction in the SNR variation curves of this algorithm and the conventional 2D-MUSIC algorithm. It can be seen from Figure 6 that this algorithm has a smaller error than 2D-MUSIC algorithm at low SNR. The simulation results in Figure 7 show that the algorithm still applies to multiple signals.
In summary, the algorithm proposed in this paper first reduces the model error of the MST using the EADF function and then uses the reduced dimensional thought to reduce the complexity of the spectral peak search. Comparison of multiple sets of experiments shows that the algorithm has good low SNR performance, as well as lower algorithm complexity.

6. CONCLUSION
The DOA based on MST and dimensionality reduction can be effectively applied to arbitrary arrays and has good low SNR performance. This paper describes the working mechanism of the MUSIC algorithm in detail, and uses MST and EADF to represent arbitrary array’s steering vector. In order to reduce the complexity of the 2D-MUSIC algorithm, a descending dimension algorithm based on MST is introduced. This is a combination of the advantages of MST, EADF and dimension reduction to achieve an algorithm with low SNR performance and low complexity. The next work focuses on real arrays, such as inter-coupling between sensors, antenna fabrication errors, and sensor orientation, to further improve the algorithm by analyzing the real arrays.

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