Cosmic microwave background constraints on a decaying cosmological term related to the thermal evolution

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Abstract

We constrain the thermal evolution of the universe with a decaying cosmological term by using the method of the analysis for the Wilkinson Microwave Anisotropy Probe (WMAP) observation data. The cosmological term is assumed to be a function of the scale factor that increases toward the early universe, and the radiation energy density is lower compared to that in the model with the standard cosmological constant (ΛCDM). The decrease in the radiation density affects the thermal history of the universe; e.g. the photon decoupling occurs at higher-z compared to the case of the standard ΛCDM model. As a consequence, a decaying cosmological term affects the cosmic microwave background (CMB) anisotropy. Thanks to the Markov-Chain Monte Carlo method, we compare the angular power spectrum in the decaying ΛCDM model with the CMB data, and we get severe constraints on parameters of the model.

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I. INTRODUCTION

Recent astronomical observations such as high redshift type Ia supernovae (SNIa) [1, 2], cosmic microwave background (CMB) anisotropy [3], suggest the existence of dark energy strongly. Although many researchers have investigated dark energy, its nature is still unknown. In the proposed models or the equation of state of dark energy [4], the cosmological constant cannot be excluded [2].

If we assume that the dark energy is equivalent to a cosmological constant Λ, there rises again so called a cosmological constant problem [5]: the present value of Λ is extraordinarily small compared with an inferred vacuum energy during the Planck time. To solve this problem, it is natural to consider that Λ decreases from a large value at the early epoch to the present value. Many functional forms of Λ have been suggested: for instance decaying-Λ has been introduced as a function of a scalar field in the Brans-Dicke gravitational theory [6]. The evolution of the universe under various models of Λ which are included in the energy-momentum tensor has been investigated analytically [7].

In addition, interacting Λ with other kinds of energy has been also discussed. The vacuum energy of Λ coupled with baryon could be ruled out, because baryon-antibaryon created through vacuum decay causes pair-annihilation. The produced high energy gamma ray flux is contradicted with the observations [8]. On the other hand, the vacuum energy decayed into the photon could affect the cosmological evolution significantly. Assuming that the ratio of the vacuum energy to the radiation is constant at the radiation dominated era \( (z > 10^5) \), Freese et al. [8] investigated the effects on primordial nucleosynthesis and obtained a limit of vacuum to photon energy ratio, which is less than 0.07. Furthermore, it is pointed out that observational constraints from the CMB intensity put the limit on the ratio of the vacuum to the radiation energy to be \( \sim 10^{-3} \) [9].

From the thermodynamical point of view, the temperature-redshift relation is modified by including adiabatic photon creation due to vacuum decay [10, 11]. A phenomenological decaying-Λ has been found to affect the cosmological evolution after the recombination [12, 13, 14]. In models having Λ terms as a function of the scale factor, the radiation and matter temperatures would be significantly lower compared to the standard cold dark matter model with a constant Λ (SΛCDM) [12]: the molecular formation is occurred at earlier epoch by \( \Delta z < 10^3 \) [13]. Furthermore, it is shown that in some parameter regions, the radiation
temperature could become higher compared with the SACDM model, which is found to be consistent with the observational result of $z < 4$ [14].

Related to the recent observations, the first star formation that occurred at the end of the dark age has been investigated progressively [15]. In decaying Λ models, using a cooling diagram, the first star formation was estimated to occur at an earlier epoch by $\Delta z \sim 20$ with its mass $\sim 10^6 M_\odot$ [13]. In the meantime, from an observational approach, the CMB polarization observed by the WMAP satellite predicts via measured reionization redshift with use of the SACDM model that a first object was formed around $z = 10$ [3, 16]. Since the CMB anisotropies give severe constraints on parameter regions concerning the cosmological evolution, we can also estimate the era of the first star formation in the decaying-Λ cosmology using the CMB anisotropy.

In the present paper, we constrain the parameter regions that determine the thermal history of the universe with a Λ decaying into the photon (hereafter we call it DΛCDM). In Sec. II we describe briefly the thermal evolution and clarify the effects on photon decoupling in the DΛCDM model. In Sec. III we examine the consistency between DΛCDM and the CMB anisotropy data of Winlinson Microwave Anisotropy Probe (WMAP) using the Markov-Chain Monte Carlo (MCMC) method. Summary and discussion are given in Sec. IV.

II. THERMAL EVOLUTION WITH A DECAYING COSMOLOGICAL TERM

Using the Friedmann-Robertson-Walker metric, the Einstein equation and/or the energy-momentum conservation law are written as follows:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \bar{\rho} a^2 - K,$$

(1)

$$\dot{\bar{\rho}} = -3 \frac{\dot{a}}{a} (\bar{\rho} + \bar{p}),$$

(2)

where $a, K$ and $G$ are the cosmic scale factor, the curvature and the gravitational constant, respectively. We note that bars such as $\bar{\rho}$ and $\bar{p}$ indicate the average values during the cosmological evolution. We choose the unit such that the velocity of light $c = 1$. Note that dots in Eqs. (1) and (2) indicate the derivative concerning a conformal time $\tau$. The total energy density $\bar{\rho}$ and the pressure $\bar{p}$ are written as

$$\bar{\rho} = \bar{\rho}_m + \bar{\rho}_\gamma + \bar{\rho}_\nu + \bar{\rho}_\Lambda,$$

$$\bar{p} = \bar{p}_\gamma + \bar{p}_\nu + \bar{p}_\Lambda,$$

(3)
where the subscripts \( m, \gamma, \nu, \) and \( \Lambda \) indicate the nonrelativistic matter (baryon plus cold dark matter), photon, neutrino, and a cosmological term, respectively. The equation of states \( \bar{p}/\bar{\rho} \) for individual components are written as,

\[
\frac{\bar{p}}{\bar{\rho}} = \begin{cases} 
1/3 & \text{relativistic particles}, \\
0 & \text{non-relativistic particles}, \\
-1 & \text{cosmological term}.
\end{cases}
\]  

(4)

Here the energy densities of matter and neutrino vary as \( \bar{\rho}_m = \bar{\rho}_{m0}a^{-3} \) and \( \bar{\rho}_\nu = \bar{\rho}_{\nu0}a^{-4} \), where the subscript 0 means the present value.

From Eqs. (2), (3) and (4), we get the evolution equation of the photon energy density after the epoch of electron-positron pair-annihilation:

\[
\frac{d\Omega_\gamma}{da} + 4\frac{\Omega_\gamma}{a} = -\frac{d\Omega_\Lambda}{da} ,
\]  

(5)

with the density parameter \( \Omega_i \)

\[
\Omega_i = \frac{\bar{\rho}_i}{\rho_{\text{crit}}}, \quad \rho_{\text{crit}} = \frac{3H_0^2}{8\pi G} ,
\]

where \( H_0 \) is the Hubble constant in units of km/sec/Mpc.

In the DΛCDM model, the evolution of the photon is affected by the time-dependent cosmological term. In this work, we assume a functional form of \( \Lambda \) as follows \[12, 13, 14, 17\]:

\[
\Omega_\Lambda = \Omega_{\Lambda1} + \Omega_{\Lambda2}a^{-m} ,
\]  

(6)

where \( \Omega_{\Lambda1}, \Omega_{\Lambda2} \) and \( m \) are constants. Note that the present value of \( \Omega_\Lambda \) is expressed by \( \Omega_{\Lambda0} = \Omega_{\Lambda1} + \Omega_{\Lambda2} \). Formalism of this paper is based on previous studies in Refs. \[12, 13\].

Integration of Eq. (5) with (6) and Stefan-Boltzmann’s law, \( \bar{\rho}_\gamma \propto T_\gamma^4 \), leads to the following photon temperature as a function of the scale factor \[14\],

\[
T_\gamma = \frac{T_{\gamma0}}{a} \times \begin{cases} 
1 + \frac{\alpha}{\Omega_{\gamma0}}(a^{4-m} - 1) & (m \neq 4) , \\
\left(1 + 4\frac{\Omega_{\Lambda2}}{\Omega_{\gamma0}} \ln a\right)^{1/4} & (m = 4) .
\end{cases}
\]  

(7)

where \( T_\gamma \) is the present photon temperature and \( \alpha \equiv m\Omega_{\Lambda2}/(4 - m) \). The present photon energy density \( \Omega_{\gamma0} = 2.471 \times 10^{-5}h^{-2}(T_{\gamma0}/2.725 \text{ K})^4 \) with the normalized Hubble constant \( h \)
FIG. 1: Upper panel: the evolution of the photon temperature in DΛCDM (m = 0 − 1.2) with ΩΛ2 = 10^{-4} after the hydrogen recombination era. Lower panel: the ratios of Tγ to that of SΛCDM. (H_0 = 100 h km/sec/Mpc). Second terms in Eq. (7) are characteristic ones in the DΛCDM model; the evolution of the photon is modified by those terms.

Figure 1 illustrates the evolution of the photon temperature in the DΛCDM model. It can be seen that Tγ in DΛCDM is lower compared to that in SΛCDM. For 0 < m < 4, the photon evolves as Tγ ∝ a^{-1} at the early epoch and the slope of Tγ against a decreases due to the contribution of a^{4-m} near the present epoch. For m > 4, the opposite results are obtained. Therefore, decaying-Λ affects the thermal history at around the present epoch of z < 10^3 and particularly after the hydrogen recombination era.

If ΩΛ2 and/or m is very large, the solution with present Tγ0 fixed indicates that the total energy density becomes negative for some epoch of z > 0. As a consequence, these parameters were constrained as mΩΛ2 ≤ 10^{-3} [12]. In our analysis, the photon temperature
for large \( m \) and/or \( \Omega_{\Lambda_2} \) becomes also negative at some epoch of \( a < 1 \). By excluding this kind of solution, we obtain the upper limits on both \( \Omega_{\Lambda_2} \) and \( m \) from Eq. (7):

\[
\alpha < \Omega_{\gamma_0} \quad (m < 4).
\]  

(8)

In the case of \( m \geq 4 \), we assume \( T_\gamma > 0 \) until the primordial nucleosynthesis epoch, \( a = 10^{-10} \), and obtain the limits

\[
\Omega_{\gamma_0} \gtrsim 92\Omega_{\Lambda_2}, \quad (m = 4)
\]

(9)

\[
\Omega_{\gamma_0} > -10^{10(m-4)}\alpha \quad (m > 4)
\]

(10)

On the other hand, for \( \Omega_{\Lambda_2} < 0 \) or \( m < 0 \), we find that \( T_\gamma \) becomes negative at some time of \( a > 1 \). Therefore we impose the two conditions of \( \Omega_{\Lambda_2} \geq 0 \) and \( m \geq 0 \).

In the DΛCDM model, the cosmological term decreases from the early time to the present, because the second term in Eq. (6) becomes larger than the first one at the early epoch of \( z > 0 \). Since the first term in Eq. (6) dominates near the present epoch, the \( \Lambda \) term is nearly constant for low-\( z \). Although cosmological models with the \( \Lambda \) term have been tightly constrained from the luminosity-redshift relation of SNIa, effects on the expansion rate are negligible in the DΛCDM model.

In the SΛCDM model, the ratio of the photon to the neutrino temperature is \( T_\gamma / T_\nu = (11/4)^{1/3} \) after electron-positron annihilation, because \( T_\gamma \) and \( T_\nu \) evolve as \( \propto a^{-1} \). In the DΛCDM model, since a decaying-\( \Lambda \) alters the evolution of the photon, the ratio of the photon-to-neutrino depends on time. Nonetheless if we set \( T_\gamma / T_\nu = (11/4)^{1/3} \) at \( a = 10^{-10} \) in the DΛCDM model, the present neutrino temperature is lower than that in SΛCDM as seen in the second column of Table I. Although recent observational studies put constraints on properties of the cosmic neutrino background such as the neutrino species or masses (e.g. \[18\]), there is no observation about its temperature (or the energy density). Therefore, DΛCDM seems to have no problems for the lower neutrino temperature.

CMB temperature at \( z = 0 \) was measured accurately by the Far Infrared Absolute Spectrophotometer of Cosmic Background Explorer: \( T = 2.725 \pm 0.002 \) K at 2\( \sigma \) C.L. \[19\]. On the other hand, CMB temperature observation at \( z > 0 \) are reported in literature \[20, 21\]. Consistency of the temperature evolution in the DΛCDM model with these observational results has been discussed \[14\]. When \( m \) and/or \( \Omega_{\Lambda_2} \) take a large value, \( T_\gamma \) in the DΛCDM model is not consistent with the temperature observation. Puy \[14\] has put constraints only
on the $m - \Omega_{A2}$ plane from the temperature observation of $z < 1$ \cite{21}:

$$|m| \leq 1, |\Omega_{A2}| \leq 10^{-4}. \tag{11}$$

These limits are obtained by comparing the observational temperature including 1σ error with the temperature given in Eq. (7). Thus, we perform the extended analysis to constrain the parameters using the available observations as precisely as possible. Figure 2 shows constraints on the $m - \Omega_{A2}$ plane from the observational temperature using the same analysis in Ref. [14] under the theoretical request of $T_\gamma > 0$. Constraints from $T_\gamma$ at $z > 1$ are similar for $m > 4$ as shown in the upper panel of Fig.2 and those from $T_\gamma$ at $z < 1$ have large uncertainty as shown in the lower panel of Fig.2. These limits of $m$ or $\Omega_{A2}$ obtained from the observed temperatures are consistent with the excluded region by Eqs.(8), (9) and (10). Since obtained constraints are rather rough, we put further severe limits using CMB anisotropy as shown in Sec.III.

Now we estimate the epoch of photon decoupling by calculating the visibility function $g(t)$ which has a peak at the epoch of the last-scattering surface:

$$g(t) = -\dot{\tau} e^{-\tau}, \quad \tau = \sigma_T \int n_e dt,$$

where $\sigma_T$ is the Thomson-scattering cross-section and $n_e$ is the number density of the free electrons that depends on the recombination history of the universe.

Figure 3 illustrates the visibility function as a function of redshift in the SΛCDM and DΛCDM models. The epoch of photon decoupling shifts to higher-$z$ as shown in the third column of Table I. When we take the upper limits obtained from Eq. (8), ($\Omega_{A2}, m$) = ($10^{-4}, 1.2$), the photon decoupling occurs at $z_{dec} = 2040$ that is earlier by $\Delta z_{dec} \sim 950$ compared to the case of the SΛCDM. In addition, we find that photon temperature at the last-scattering surface is about 0.1% lower than that in the SΛCDM model, as shown in the fourth column of Table I. These effects should be further constrained by the cosmological observations such as CMB anisotropy as shown in the following sections.

III. EFFECTS ON CMB ANISOTROPY AND CONSTRAINTS BY MARKOV-CHAIN MONTE CARLO ANALYSIS

Before discussing effects of a decaying-Λ on the CMB power spectrum, let us formulate the Boltzmann equation for the photon in the DΛCDM model based on the cosmological
FIG. 2: Constraints on the $m - \Omega_{\Lambda 2}$ plane from observational temperatures. The black-solid line shows the upper limits of parameters by Eqs. (8), (9), and (10), and the other lines indicate the upper limits obtained from the same analysis in Ref. [14]. Upper panel: constraints from the temperature at $z > 1$. Lower panel: constraints from the temperature at $z < 1$. 
FIG. 3: Visibility function vs the redshift for a fixed parameter $\Omega_{\Lambda 2} = 10^{-4}$. Peaks of $g$ show the epoch of photon decoupling. As the values of the parameter $m$ increase, the photon decoupling occurs at an earlier epoch.

perturbation theory. The line element in the synchronous gauge with the flat space is written as

$$ds^2 = a(\tau) \left[ -d\tau^2 + (\delta_{ij} + h_{ij}) \, dx^i \, dx^j \right], \quad (12)$$

where $h_{ij}$ is the metric perturbation. We introduce two fields $h(k, \tau)$ and $\eta(k, \tau)$ in the Fourier $k$-space and write the scalar mode of $h_{ij}$ as the Fourier integral:

$$h_{ij}(x, \tau) = \int d^3k e^{ik \cdot x} \left[ \hat{k}_i \hat{k}_j h(k, \tau) + \left( \hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} \right) 6\eta(k, \tau) \right],$$
TABLE I: Present neutrino temperature \((z = 0)\), redshift, and photon temperature at the last-scattering surface \((z_{dec})\) in the decaying-\(\Lambda\) model with \(\Omega_{\Lambda} = 10^{-4}\).

| Parameter \(m\) | \(T_\nu\) [K] \((z = 0)\) | \(z_{dec}\) | \(T_\gamma\) [K] \((z_{dec})\) |
|----------------|-----------------|--------|-----------------|
| \(m = 0.0\)   | 1.945           | 1087   | 2965            |
| \(m = 0.5\)   | 1.774           | 1188   | 2957            |
| \(m = 1.0\)   | 1.416           | 1480   | 2939            |
| \(m = 1.2\)   | 1.022           | 2043   | 2921            |

where \(k_j = k\hat{k}_j\) with the unit vector \(\hat{k}_j\). Components of the energy-momentum tensor with perturbed parts are given by

\[
T^0_0 = - (\bar{\rho} + \delta \rho),
\]
\[
T^0_j = -T^j_0 = (\bar{\rho} + \bar{p}) v_j,
\]
\[
T^i_j = (\bar{\rho} + \delta \rho) \delta^i_j + \Sigma^i_j,
\]

where \(\delta \rho, \delta p, v_i\) and \(\Sigma^i_j\) are perturbed parts of the energy density, pressure, velocity of fluids, and anisotropic stress, respectively.

In the DΛCDM model, the energy density of the photon in the background part is obtained from Eq. (5). The perturbed parts of the equation of the energy-momentum conservation that correspond to the first order perturbation reduce to the following equations,

\[
\dot{\delta}_\gamma = - \frac{4}{3} \theta_\gamma - \frac{2}{3} \dot{h} + \frac{\dot{\rho}_\Lambda}{\bar{\rho}_\gamma} \delta_\gamma,
\]
\[
\dot{\theta}_\gamma = \frac{1}{4} k^2 \delta_\gamma - k^2 \sigma_\gamma + \frac{\dot{\rho}_\Lambda}{\bar{\rho}_\gamma} \theta_\gamma,
\]

where \(\delta_\gamma = \delta \rho_\gamma / \bar{\rho}_\gamma\) and \(\theta_\gamma\) is the divergence of the fluid velocity, \(\theta_\gamma \equiv ik^j v_j\). \(\sigma_\gamma\) is defined by

\[(\bar{\rho}_\gamma + \bar{p}_\gamma) \sigma_\gamma \equiv - \left( \dot{\mathbf{k}}_i \cdot \hat{\mathbf{k}}_j - \frac{1}{3} \delta_{ij} \right) \Sigma^i_j.\]

Equations (14) and (15) are the continuity and Euler equations, respectively. Note that we take into account the interaction terms proportional to \(\dot{\rho}_\Lambda\) in these equations and the effects will be small since the ratio \(\rho_\Lambda / \rho_\gamma\) at \(z = 10^3\) is less than \(10^{-3}\).

Moreover, to construct the perturbed evolution equation of the photon, we need the contribution of the higher multipole moments. The Boltzmann equation for a relativistic
particle in $k$-space is written as follows [22]:
\[
f_0 \frac{\partial \Psi}{\partial \tau} + \Psi \frac{\partial f_0}{\partial \tau} + ik\mu f_0 \Psi + \frac{d \ln f_0}{d \ln q} \left( \dot{\eta} + \frac{\dot{h} + 6\dot{\eta}}{2}\mu^2 \right) = \left( \frac{\partial f}{\partial \tau} \right)_{\text{col}},
\]
where $\mu = \hat{k} \cdot \hat{n}$, and $q_i = qn_i$ is the 3-dimensional momentum. The right-hand side of Eq. (16) is the collision term. The distribution function expressed by the convolution of zeroth-order and the perturbed part is written as
\[
f(x^i, q, n_j, \tau) = f_0(q, \tau)(1 + \Psi(x^i, q, n_j, \tau)).
\]
In the SΛCDM model, the zeroth-order distribution function, $f_0$, of the photon is described as
\[
f_0^\gamma(q) = \frac{1}{h_p \exp(q/k_B T^\gamma) - 1},
\]
where $h_p$ and $k_B$ are the Planck and Boltzmann constants, respectively. However, in models with a created photon by decaying vacuum energy, the spectral distribution of CMB is a function of both temperature and the comoving number of photon, where $f_0$ takes a generalized Planckian form [11]. Therefore, we cannot drop the second term in the left-hand side in Eq. (16), which is the time derivative of $f_0$.

To obtain the Boltzmann equation for photon, we expand the angular dependent part of the perturbation in a series of Legendre polynomials $P_l(\hat{k} \cdot \hat{n})$ as follows:
\[
F^\gamma\left( k, \hat{n}, \tau \right) = \int_0^\infty dq q^2 f_0(q, \tau) \Psi = \sum_{l=0}^{\infty} (-i)^l (2l + 1) F^\gamma_l(\mu) P_l(\mu).
\]
We integrate Eq. (16) multiplied by $q^3 dq f_0$ over the whole $p$ space and divide it by $\int q^3 dq f_0$. Then we obtain the following Boltzmann equation for the CMB photon in $k$-space:
\[
\dot{F}^\gamma + ik\mu F^\gamma + \frac{4}{3} \left( \dot{h} + 6\dot{\eta} \right) P_2(\mu) + \frac{2}{3} \dot{h} - \frac{\dot{\rho}_\Lambda}{\rho_\gamma} F^\gamma = \left( \frac{\partial F^\gamma}{\partial \tau} \right)_{\text{col}}.
\]
The last term of the left-hand side in Eq. (17) corresponds to a new one that appeared in the DΛCDM model. The collision term in the right-hand side is described as Thomson scattering [22]:
\[
\left( \frac{\partial F^\gamma}{\partial \tau} \right)_{\text{col}} = a n_e x_e \sigma_T \left[ -F^\gamma + F^\gamma_0 + 4\dot{\hat{n}} \cdot \mathbf{v}_e - \frac{1}{2} \left( F^\gamma + G^\gamma_0 + G^\gamma_2 \right) P_2(\mu) \right],
\]
where $G^\gamma_2$ is the difference of the two linear polarization components.

Substituting the Legendre expansion for $F^\gamma$, and using the orthonormality of the Legendre polynomial with the recursion relation $(l + 1)P_{l+1}(\mu) = (2l + 1)\mu P_l(\mu) - lP_{l-1}(\mu)$, we get
the Boltzmann equations for the photon in the DΛCDM model as follows:

\[
\begin{align*}
\dot{\theta}_\gamma & = \frac{1}{4} k^2 \delta_\gamma - k^2 \sigma_\gamma - a n_e x_e \sigma_T (\theta_\gamma - \theta_b) + \frac{\dot{\rho}_\Lambda}{\rho_\gamma} \theta_\gamma, \\
\dot{\sigma}_\gamma & = \frac{4}{15} \theta_\gamma - \frac{3}{10} k F_{\gamma 3} + 2 \frac{2}{15} (\dot{h} + 6 \dot{\eta}) - \frac{9}{10} a n_e x_e \sigma_T + \frac{1}{20} a n_e x_e \sigma_T (G_{\gamma 0} + G_{\gamma 2}) + \frac{\dot{\rho}_\Lambda}{\rho_\gamma} \theta_\gamma, \\
\dot{F}_{\gamma l} & = \frac{k}{2 l + 1} [l F_{\gamma l-1} - (l + 1) F_{\gamma l+1}] - a n_e x_e \sigma_T F_{\gamma l} + \frac{\dot{\rho}_\Lambda}{\rho_\gamma} F_{\gamma l} \quad (l \geq 3),
\end{align*}
\]

where

\[
\delta_\gamma = F_{\gamma 0}, \quad \theta_\gamma = \frac{3}{4} k F_{\gamma 1}, \quad \sigma_\gamma = \frac{F_{\gamma 2}}{2}.
\]

We calculate the CMB power spectrum by modifying the CAMB code \cite{23} based on the CMBFAST code \cite{24}, where we include the modified Boltzmann equation of Eqs. (14), (19)–(21). Figure 4 shows the effects of \( m \) on the angular power spectrum with the following cosmological parameters: the baryon density parameter \( \Omega_b h^2 = 0.0223 \), the cold dark matter (CDM) density parameter \( \Omega_{CDM} h^2 = 0.104 \), \( K = 0 \), \( h = 0.73 \), and the reionization is neglected. We find that a decaying-Λ modifies the CMB power spectrum as follows: if \( m \) and/or \( \Omega_\Lambda^2 \) is small, the amplitude of the power spectrum decreases. If we take larger values of \( m \) and/or \( \Omega_\Lambda^2 \), the first and third peaks of the power spectrum increase due to the large baryon density relative to the photon energy density. Furthermore, the CMB power spectrum shifts toward higher-\( l \), because the photon last-scattering occurs at an earlier epoch as seen in Fig. 3 and Table I. We have found that the new term in the DΛCDM model increases the angular power spectrum at \( l > 20 \) by about \( 10^{-3} \) percents. Therefore, the observational constraints are the same even if these terms are not included.

The CMB angular power spectrum is rather sensitive to other cosmological parameters. For instance, baryon and CDM densities affect the amplitude of CMB anisotropy. Therefore, we need to carry out the MCMC approach \cite{25} to constrain the possible parameters: \( \Omega_b h^2 \), \( \Omega_{CDM} h^2 \), \( h \), the reionization redshift \( z_{\text{re}} \), the scalar spectral index \( n_s \), the amplitude of density fluctuation \( A_s \), and two parameters in DΛCDM (\( \Omega_\Lambda^2 \) and \( m \)). We note that we do not assume flat universe.

To start the MCMC calculations, we assume the priors on the cosmological parameters as follows: \( 0.5 \leq n_s \leq 1.5 \), \( \Omega_b h^2 = 0.022\pm0.0022 \) at 1σ C.L. (BBN prior), \( 0.01 \leq \Omega_{CDM} h^2 \leq 0.99 \), \( -0.3 \leq \Omega_K (\equiv -K/H_0^2) \leq 0.3 \), \( 0 \leq \Omega_\Lambda \leq 1.0 \), and \( 10 \text{ Gyr} < t_0 < 20 \text{ Gyr} \) (age of the universe).
FIG. 4: Comparison of the angular power spectrum in the decaying Λ model with the WMAP observation data. The solid line is the result of SACDM. The dashed, dot-dashed, and dotted lines are those of DΛCDM with \((\Omega_{\Lambda 2}, m) = (10^{-4}, 0.5), (10^{-4}, 0.8), \text{and} (10^{-4}, 1.0)\), respectively.

We constrain the relation between \(\Lambda_2 (m)\) and other parameters from the recent CMB observations of WMAP [16, 26], BOOMERanG [27], CBI [29], and Acber [28]. Our results are shown in Figs. 5 and 6. Figure 5 shows the constraints on the \(m - \Omega_{\Lambda 2}\) plane and our constraint is severer than that from the observed radiation temperature in Fig 2. In our analysis, we obtain upper limits of \(m\) and \(\Omega_{\Lambda 2}\) such as \(m \leq 4.2\) and \(\Omega_{\Lambda 2} \leq 1.7 \times 10^{-4}\) at the 95.4% confidence levels, respectively. Therefore, we cannot find the clear evidence of a decaying-\(\Lambda\). Figure 6 shows the contours between \(\Omega_{\Lambda 2}\) and other cosmological parameters \((\Omega_b h^2, \Omega_{\text{CDM}} h^2, n_s, z_{re} H_0)\) at the 68.3% and 95.4% confidence levels. Parameters in the DΛCDM model have no degeneracy with other cosmological parameters; the parameters are
FIG. 5: Contours of the $m - \Omega_{\Lambda 2}$ plane from CMB. Solid lines indicate 68.3% and 95.4% confidence levels. The dotted-line is the upper-limit from Eqs. (8), (9), and (10) with $h = 0.73$.

independent on other parameters. Table II shows the comparison of cosmological parameters between the SΛCDM and DΛCDM model obtained from our MCMC analysis. As expected from Figs. 5 and 6 differences in the cosmological parameters are as small as some percents. However, the value of $\Omega_\Lambda$ differs around 10 percents, which should be further constrained by future observations.

IV. SUMMARY AND DISCUSSION

We have investigated the possible difference in the thermal evolution of the universe with a decaying-$\Lambda$ term as a function of the cosmic scale factor that reduces the photon energy density. Although the energy density of a $\Lambda$-term is increasing at the early era, the effects of the decaying $\Lambda$ on the cosmic expansion rate can be ignored at the $\Lambda$ dominate
FIG. 6: Constraints on $\log \Omega_{\Lambda 2}$ against $\Omega_b h^2$ (left top panel), $\Omega_{CDM} h^2$ (right top panel), $n_s$ (left middle panel), $z_{re}$ (right middle panel), $\Omega_{\Lambda 1}$ (left bottom panel), and $H_0$ (right bottom panel) from WMAP three year results. Two curves in each panel correspond to the 68.3% and 95.4% confidence levels, respectively.

epoch. On the other hand, a decaying-$\Lambda$ term has been insisted to alter the evolution of the photon temperature $^{[12, 13, 14]}$. Depending on parameters in the DΛCDM model, the photon energy density could be lower or higher compared with that in the SΛCDM model at
\[ z \geq 0 \]. However, the second case should not occur, because the photon temperature becomes negative at some epoch of \( z < 0 \). We estimate the epoch of the last-scattering surface by calculating the visibility function, and find that \( z_{\text{dec}} = 2040 \) for \( m = 1.2 \) and \( \Omega_{\Lambda_2} = 10^{-4} \) in the DΛCDM model, which indicate that the decoupling occurs earlier by \( \Delta z_{\text{dec}} = 950 \) compared to the case of the SACDM.

We examine qualitatively the effects of a decaying-\( \Lambda \) term on CMB angular power spectrum. We obtain the modified Boltzmann equation of photons in the DΛCDM model based on the cosmological perturbation theory and calculate the CMB angular power spectrum. We find that a decaying-\( \Lambda \) could alter the CMB angular power spectrum significantly due to the following reasons: large baryon energy density relative to the photon density causes to boost up the first and third peaks; the early photon decoupling shifts CMB spectrum to higher multipoles.

Finally, using the Markov-Chain Monte Carlo analysis, we can put constraint on \( m \), \( \Omega_{\Lambda_2} \), and cosmological parameters. We obtain the upper limits of parameters in DΛCDM: \( m < 4.2 \) and \( \Omega_{\Lambda_2} < 1.7 \times 10^{-4} \). In our analysis, the upper limit of \( m \) is close to 4. A decaying-\( \Lambda \) might affect primordial nucleosynthesis because the \( \Lambda \)-term evolves as radiation (photon, neutrino, and electron-positron) at the early universe. However, if \( m \) is large, \( \Lambda_2 \) tends to be small as shown in Fig.5. As the result, \( \Lambda \)-term becomes smaller than radiation components. In fact, at \( T = 10^9 \) K, the ratio of the \( \Lambda \)-term and radiation density is \( 10^{-5} \) for \( m = 4.2 \) and \( \Omega_{\Lambda_2} = 10^{-10} \) (upper limits from WMAP results). Therefore, we can say that effects of the early universe such as nucleosynthesis is small. Therefore, the effects of the \( \Lambda \)-term on the physical processes in the early universe, such as on the nucleosynthesis, is negligible.

Interestingly, there is no degeneracy between the two parameters in DΛCDM and other cosmological parameters. From these constraints, the contribution of a decaying-\( \Lambda \) term to the cosmic thermal evolution should be extremely small, since the best-fit values of \( m, \Omega_{\Lambda_2} \) are nearly zero.

We assume a cosmological term as a function of scale factor for simplicity. Even if we parameterize reasonably the evolution of the cosmological term or the equation of state of dark energy, our results would not change qualitatively.

On the other hand, we find that the reionization occurs at \( z_{\text{re}} = 11 \) in the DΛCDM model, which suggests that a first object could be formed at around this epoch. We should note
TABLE II: Comparison of cosmological parameters between the SACDM and DACDM models obtained from Markov-Chain Monte Carlo analysis.

| Parameter            | DACDM             | SACDM             |
|----------------------|-------------------|-------------------|
| \( \Omega_{\Lambda} \) | \( \Omega_{\Lambda} \) | \( \Omega_{\Lambda} \) |
| \( \Omega_{\Lambda} \) | \( \Omega_{\Lambda} \) | \( \Omega_{\Lambda} \) |
| \( m \)               | < 1.7 \times 10^{-4} | < 4.2             |
| \( \Omega_b h^2 \)    | 0.0221^{+0.0019}_{-0.0028} | 0.0223 \pm 0.0007 |
| \( \Omega_{CDM} h^2 \) | 0.103^{+0.021}_{-0.018} | 0.1037 \pm 0.0081 |
| \( \log_{10}(10^{10} A_s) \) | 2.991^{+0.019}_{-0.015} | 3.156 \pm 0.056 |
| \( z_{re} \)           | 10.6^{+6.2}_{-8.0} | 10.9^{+2.6}_{-2.7} |
| \( n_s \)              | 0.945^{+0.050}_{-0.031} | 0.951 \pm 0.016 |
| \( H_0 \)             | 70.7 \pm 19.7 | 71 \pm 3         |
| \( \Omega_\Lambda \)   | 0.757^{+0.083}_{-0.026} | 0.763 \pm 0.034 |
| \( \Omega_m \)         | 0.249^{+0.261}_{-0.099} | 0.233^{+0.033}_{-0.034} |

that we assume that the reionization history can be described by step-function as discussed in Ref. [3]. The next CMB satellite, Plank, is expected to determine detailed reionization history. Then a variable \( \Lambda \)-term model such as DACDM should be constrained further.

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[32] Recently, WMAP five year results have been released [31], where the third peak of the spectrum is clearly determined. However differences in the first peak of WMAP five year compared to the three year results are within 1σ level, and those in the third peak are still more than 1σ level. Therefore if we reanalyze with the new WMAP data, our results would not be affected within the present observational error of WMAP five year
