Inverting Sediment Bedforms for Exploring the Hazard of Volcanic Density Currents Directly in the Field

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Research Article

Keywords: Pyroclastic density, sediment bedforms, hazard of volcanic density, currents, wavelength and grain size

DOI: https://doi.org/10.21203/rs.3.rs-777070/v1

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Inverting sediment bedforms for exploring the hazard of volcanic density currents directly in
the field

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Abstract

Pyroclastic density currents are ground hugging gas-particle flows moving at high speed down the volcano slope. They are among the most hazardous events of explosive volcanism, causing devastation and deaths¹,². Because of the hostile nature they cannot be analyzed directly and most of their fluid dynamic behavior is reconstructed by the deposits left in the geological record, which frequently show peculiar structures such as bedforms of the types of ripples and dunes³,⁴. In this paper, we simplify a set of equations that link flow behavior to particle motion and deposition. This allows, for the first time, the build up of a phase diagram by which the hazard of dilute pyroclastic density currents can be explored easily and quickly by inverting bedforms wavelength and grain size.

Main

Geologists and engineers have always been fascinated by sediment bedforms. They are a natural beauty of practical importance and represent a primary resource in the reconstruction of ancient sedimentary environments⁵. They form both in fluvialite currents, turbiditic flows, snow avalanches and volcanic pyroclastic density currents (PDCs). When a current flowing over sediment exceeds the critical shear stress for motion, bedforms develop as a result of the interaction between sediment and fluid⁶. The first bedforms to develop are current ripples, which have wavelengths, \(W\), smaller than 60 cm⁶. Larger bedforms are called dunes⁷. It is widely recognized that the occurrence of ripples or dunes depends on hydrodynamic conditions and sediment characteristics. These are defined in phase diagrams⁸,⁹ where bedform characteristics as \(W\) and sediment median size, \(D\), are
related to flow parameters such as the densiometric Froude number $Fr'$ and the critical Shields number $\theta_t$. For symbols see table 1.

$Fr' = \frac{V}{\sqrt{g'H}}$ is a balance between inertial and gravitational effects, with $g' = g\left(\frac{\rho_{mix} - \rho_f}{\rho_f}\right)$ representing the reduced gravity, $g$ the gravity acceleration, $V$ the current velocity, $H$ the current depth.

$\rho_{mix} = \rho_s C + \rho_f (1 - C)$ (1)

is the density of the fluid-particle mixture with $\rho_s$ particle density, $\rho_f$ fluid density and $C$ particle volumetric concentration.

$\theta_t = \frac{\rho_{mix}u_*^2}{Dg(\rho_s - \rho_{mix})}$ (2)

is the threshold of initiation of motion of particles resting on the substrate and is a form of shear stress ($\tau = \rho_{mix}u_*^2$, where $u_*$ is the shear velocity) normalized to the sediment static load. It is a function of the Reynolds’ number of shear:

$Re_* = \frac{\rho_{mix}u_*D}{\mu}$ (3)

where $\mu$ is fluid viscosity.

PDCs form upon explosive eruptions when gases, fragments of magma and lithics, ranging in size from ash to blocks and bombs, are forced throughout the crater to form vertical eruption columns that collapse on the ground or are generated from gravitational failure of domes. They form flows that may spread around the volcano for many kilometers, causing devastation and death. The hazard potential of PDCs depends on impact parameters such as dynamic pressure:

$P_{dyn} = \frac{1}{2}\rho_{mix}V^2$ (4)

that contrasts the resistance of buildings to the flow, and the volumetric concentration of ash particles $C$, which represents a distinct source of hazard especially far from the volcano where the flow mechanical strength decays, but the current is still rich of ash in suspension. In fact, volcanic
ash in the air is very harmful to breath\textsuperscript{14}, even at temperatures lower than 200°C (that are typical of dilute PDCs), and can cause serious health issues and possibly death to a human being if flow duration, $t$, which is a proxy for exposure time, is longer than a couple of minutes\textsuperscript{14,15}.

Because of the very hostile nature, the behavior of PDCs is difficult to analyze directly, and our understanding is primarily based on the information preserved in the sediments of past eruptions\textsuperscript{2,16,17}. laboratory to large-scale experiments\textsuperscript{18,19,20} and numerical modelling\textsuperscript{21,22}. All three methodologies are valid, particularly when they are integrated\textsuperscript{23}. The former, though, has the advantage of being directly linked to the ground truth when direct observations of the PDCs are not available, provided valid sedimentological models linking deposit characteristics to flow properties and their impact exist\textsuperscript{18}.

Bedforms of the types of dunes and ripples have been widely recognized in the deposits of dilute PDCs since the pioneering observations of Richards\textsuperscript{24}, Moore\textsuperscript{25} and Fisher and Waters\textsuperscript{26}. Further details on other types of bedforms are nowadays emerging from observation of recent eruptions\textsuperscript{27}.

Differently from what has been done for fluviatile and turbiditic currents, only very few attempts have been made to construct phase diagrams defining the stability fields of bedforms as a function of PDCs flow regimes. Only very recently Smith et al.\textsuperscript{17} have proposed a phase diagram for highly concentrated volcanic granular currents. Dellino et al.\textsuperscript{28}, basing on results of large-scale experiments, have proposed a phase diagram in which volcanic deposits are classified based on their sedimentation rate, $S_r$ and bedload transportation rate, $Q_b$. This agrees with the approach used in the field of sedimentary currents, for which it is widely recognized that the proportion of bedload to suspended load and the sediment size are the major controlling factors on bedforms formation\textsuperscript{9}.

$S_r$ is defined by Dellino et al.\textsuperscript{29}, as:

$$S_r = \left( \sum_{i=1}^{n} \rho \psi \frac{\phi_i}{\psi_i} \psi_i \frac{C}{\left(\left(10.065 \frac{P_n}{P_{ni}} + 0.1579\right) 0.7 + \left(10.065 \frac{P_n}{P_{ni}} + 0.1579\right) 0.3\right)} - 0.01 \right)$$

with the subscript $i$ referring to the $i$th particle-size class and $n$ being the number of size classes of the grain-size distribution of the sediment, where $P_n = w_0/k\nu$, is the Rouse number of the $i$th size fraction of the solid material suspended in the current, with $k$ the Von Karman constant (0.4) and $w_0$ the
terminal velocity of the $i_{th}$ size fraction. $P_{n^*} = P_{navg}/P_{nsusp}$ is the normalized Rouse number of the current, i.e. the ratio between the average Rouse number of the solid material in the current and the Rouse number at maximum suspension capacity. $\phi_i, \rho_{si}$ and $P_{ni}$ are the weight fraction, the density and the Rouse number of the $i_{th}$ grain-size fraction, respectively. The sedimentation rate was transformed in the sedimentation rate per unit width, $S_{rw}$ in order to make it comparable with $Q_b$ dimension. $Q_b$ is defined by Dellino et al. (modified from Wilcock and Crowe) as:

$$Q_b = \sum_i^n q_{bi} \quad (6)$$

where

$$q_{bi} = \frac{(\rho_s/\rho_{mix}^{1/3})g q_{bi}}{W_i^* \phi_i u_i^*}$$

and

$$W_i^* = \begin{cases} 
0.002\xi^{7.5} & \text{for } \xi < 1.35 \\
14 \left(1 - \frac{0.894}{\xi^{0.5}}\right) & \text{for } \xi \geq 1.35 
\end{cases} \quad (7)$$

$q_{bi}$ is the volumetric bedload transport rate of the $i_{th}$ size fraction per unit width of the flow, and $\xi = \tau/\tau_{ri}$ is the normalized shear stress, where $\tau_{ri}$ is the minimum shear stress needed to move the $i_{th}$ size fraction at bedload.

The lower right portion of the $S_{rw}$ vs $Q_b$ phase diagram of Dellino et al. represents the field of massive deposits due to highly concentrated flows, also known as pyroclastic flows. The upper portion of the diagram represents the field of stratified deposits with ripple and dune bedforms, which are related to highly expanded, fast-moving, dilute and turbulent PDCs. $W_i$ of bedforms that characterize dilute PDCs, shown in the diagram as the distance between two successive dunes or ripples (in cm), is inversely proportional to the $S_{rw}/Q_b$ ratio, with ripples having a ratio larger than 0.05 and dunes smaller than 0.05.

In this paper, we further populate the diagram in the portion of dilute PDCs by adding 88 points relative to various eruptions of Vesuvius, Campi Flegrei and Vulcano in Italy. With this addition, the new dataset consists of 98 deposits (Fig. 1) and covers a wide span of the $S_{rw}$ vs $Q_b$ space, allowing.
an analysis of bedforms in terms of a large range of flow parameters.

Fig. 1. $S_{aw}$ vs $Q_b$ diagram in which 88 points have been added to those of Dellino et al.²⁸. The $W_l$ of bedforms as a function of the $S_{aw}/Q_b$ ratio is inserted and also the legend of volcanoes from which deposits were analysed.

The bedform $W_l$ ranges from ripples (Fig. 2a), starting at 10 cm, to dunes (Fig. 2b), up to 250 cm.

Fig. 2. PDC deposits showing bedforms. a=ripples of PDC deposits at Vulcano. The curves enclose a ripple with $W_l = 40$ cm. b=a dune bedform of PDC deposits at Vesuvius. The curves enclose a dune with $W_l = 200$ cm.
We never found antidunes, in fact their interpretation has always been questioned in volcanic deposits.\textsuperscript{11,27}

The software PYFLOW 2.0 by Dioguardi and Mele\textsuperscript{31} has been used to plot data in Fig 1. It was implemented here so to obtain both the impact parameters of the current together with $S_{rw}$ and $Q_b$.

The software employs sediment data that result from time-consuming laboratory analyses, which involve technologies and calculation resources not available to all scientists (see the Method section).

The aim of this paper is to rearrange and simplify the dataset in order to construct a phase diagram by which to invert $W_l$ and $D$ of PDCs’ deposits bedforms and obtain the impact parameters directly in the field, without the need of the extra terms that require extensive work in the laboratory.

By means of regression analysis we obtained three fitting laws (Fig. 3a, b and c) that correlate just few of the many terms of the formulas of (5), (6) and (7) ($u^2C/Q_b$, $D^{0.5}u/\sqrt{u^2C}$, $u^{0.4}C^{0.62}/S_{rw}$, respectively), thus reducing the complexity of the original equations, and still guarantee high correlation coefficients, hence, a good approximation of the full PDCs impact and depositional models.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{regression_analysis}
\caption{Fig. 3. Fits resulting from the regression analysis. In the insets both the correlation coefficient, $r$, and the fitting equation are inset. a=parabolic relationship between $u^2C$ and $Q_b$, b=linear relationship between $D^{0.5}u$ and $\sqrt{u^2C}$. c=power-law relationship between $u^{0.4}C^{0.62}$ and $S_{rw}$, d=power-law relationship between $S_{rw}/Q_b$ and $W_l$.}
\end{figure}
A fourth fitting law between $S_{rw}/Q_b$ vs $W_l$ with a good correlation was obtained by selecting 32 deposits characterized by well exposed bedforms ranging between ripples and dunes (Fig. 3d).

The fitting laws allow interpreting $Q_b$ and $S_{rw}$ in terms of the deposit formation processes, and also to relate them to the current’s flow parameters. In Fig. 3a a relationship between $Q_b$ and $Cu_{\ast}^2$ is shown. Since $C$ is directly proportional to $\rho_{mix}$ and $\rho_{mix}u_{\ast}^2$ is the turbulent shear stress of the current$^{32}$, it means that $Q_b$ is proportional to the shear stress, which confirms the finding of sedimentary currents$^{29}$. The relationship between $Du_{\ast}$ and $Cu_{\ast}^2$ of Fig. 3b implies that shear stress is proportional to bedforms grain size, confirming what reported for sedimentary deposits$^{28}$. On Fig. 3c a relationship between the product of $C^{0.62}u_{\ast}^{0.4}$ and $S_{rw}$ is shown. Since the exponents of $C$ and $u_{\ast}$ are both lower than 1, while in the fitting with $Q_b$ of Fig. 3a they are 1 and 2 respectively, it means that with an increase of $C$ and $u_{\ast}$ the difference between $S_{r}$ and $Q_b$ increases, and $S_{rw}/Q_b$ decreases. This justifies that with the decrease of $S_{rw}/Q_b$, the bedform wavelength increases continuously, as it is shown by Fig. 3d. This outcome deserves an additional comment, because in sedimentary deposits ripples and dunes are not believed to represent a continuum, being them separate by a hydrodynamic discontinuity$^{33}$. This happens because ripples, being small, do not interfere with the upper current surface, while dunes, being larger, interfere with it. This discontinuity does not appear in the $S_{rw}/Q_b$ vs $W_l$ diagram of Fig. 3d, likely because a true interface between the current and the surrounding atmosphere does not exist in PDCs, which are instead characterized by a very gradual passage between the two$^{10}$ (see the method section for our model of dilute PDCs).

The four fitting laws make up a system of equations

\begin{align*}
\text{(8)} \quad u_{\ast}^{0.4}C^{0.62} & = 0.2168S_{r}^{0.2938} \\
S_{rw}/Q_b & = 52.92W_l^{-1.518}
\end{align*}

\begin{align*}
u^2C & = 1.5099Q_b^2 + 0.3874Q_b + 0.0011 \\
D^{0.5}u_{\ast} & = 205.02u_{\ast}^2C + 0.163
\end{align*}
that can be solved numerically, once $D$ and $W_l$ are specified, to obtain $u_*$, $C$, $S_{rw}$ and $Q_b$. Important information on the hazard of dilute PDCs can be obtained from the first three parameters. $C$ and $u_*$ serve for the calculation of $P_{dyn}$ (2), since $C$ is used for obtaining $\rho_{mix}$ in (2) by means of (1) and $u_*$ is used for the calculation of $V$ by means of the law of the wall of a turbulent boundary layer\(^{32}\)

$$V(y) = u_*(\frac{1}{k} \ln \frac{y}{k_s} + 8.5) \quad (9)$$

which is the physical model of PDCs that we employ (see the method section), where $V(y)$ is the velocity profile of the stratified flow\(^{32}\), and $k_s$ is the substrate roughness.

When comparing results obtained by (8) with those resulting from PYFLOW 2, the average absolute error of $u_*$ is 28% and that of $C$ is 30%. This means that a good approximation can be achieved for exploring the range of impact parameters by means of the simplified formulas, without the terms that involve extensive laboratory analysis.

The absolute error of $S_{rw}$ is about 45%. While it is larger than that of $u_*$ and $C$, we discuss also the role of $S_{rw}$ because it allows the calculation of flow duration, $t^{15}$, which is an important factor of hazard. The total time of aggradation is a proxy of flow duration, $t$, which is equal to deposit thickness, $H_{dep}$, divided by $A_r$, the aggradation rate. Sedimentation occurs by continuous aggradation during the passage of the current, and $A_r$ is equal to $S_{rw}$ divided by one meter, which is the reference width of the sedimentation rate per unit width, (see Dellino et al.\(^{28}\)). Therefore, flow duration, which approximates the time in which harmful concentrations of ash are suspended in the current to which a human being can be exposed, can be calculated by means of $S_{rw}$. With our model a reasonable approximation can be achieved also on such a relevant parameter of PDCs.

In Fig. 4, which was constructed by means of (8), the main flow variables and impact parameters are shown as a function of $D$ and $W_l$. The $W_l$ range was set between 10 and 300 cm. Bedforms with larger $W_l$ can be found in the geologic record of volcanic deposits, but this scenario is out of the range of applicability of our model. We are, in fact, considering bedforms that develop on an almost flat surface. Much larger bedforms, instead, typically develop as an interplay between the current’s flow
dynamics and large ground morphology elements\textsuperscript{27} (e.g. ridges, big obstacles). The range of $D$ of Fig. 4 was set between 4 and -2 phi (0.0064 mm and 4 mm respectively). We do not include coarser values because, in volcanic sediments, larger sizes (coarse lapilli and bombs) do not form dunes, but lenticular beds representing highly concentrated traction-carpets at the base of PDCs,\textsuperscript{34,35} to which our model does not apply.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Trends of flow variables and impact parameters of dilute PDCs as a function of bedform wavelengths. The various curves represent the behavior of different grain size $D$. Grain size is expressed in phi units (phi=-log\textsubscript{2}D, with D in mm).}
\end{figure}

In Fig. 4a the value of velocity $V$, as averaged in the first 1000 cm of the current, which was obtained by integrating (9) over flow height, with $k_s = 10$ cm, is plotted against $W_l$. We chose this depth-averaging height because, in dilute PDCs, the portion responsible for the dynamic impact is the
lowermost one (the shear flow) and 1000 cm represent a reasonable estimate of an average building height. \( V \) increases at increasing \( W_l \) but with different trends and rates depending on the particle grain size \( D \). In the diagram it ranges from about 10 m/s to about 130 m/s. The trends are significantly different for the finer grain sizes (1 to 4 phi) at the smallest \( W_l \) (up to 50 cm), which can be interpreted as the smaller it is \( W_l \), the smaller are \( Q_b \) and shear stress, therefore the higher the velocity required to develop bedforms with the finest \( D \). This is because for fine ash, due to the very low \( Re^* \), the initiation of motion at the bedload occurs at a very high \( \theta_t \). With larger \( W_l \), corresponding to larger \( Q_b \) and larger shear stress, higher velocities are needed with coarser particles, because \( \theta_t \) decreases down to the constant value characteristic of high \( Re^* \), which in turn implies that a larger shear stress is needed to move larger grain sizes. The volumetric concentration, \( C \), ranges from less than 0.001 to about 0.017 (Fig. 4b). It decreases as \( W_l \) increases, and it does so for all grain sizes, although at a rate that decreases at decreasing \( D \), because a higher concentration favors a higher \( S_{rw} \) (see (7)) and a larger \( S_{rw}/Q_b \) ratio, hence a smaller \( W_l \). The evident change in trend with decreasing grain size can be explained by the fact that the finer the particles, the lower the concentration required to develop bedforms with small wavelengths; the increase of shear stress and \( Q_b \) results in similar concentrations for all grainsizes. The density of the current (Fig. 4c) follows the trend of concentration, as it is calculated by means of (1) and fixing \( \rho_s = 2000 \text{ kg/m}^3 \) and \( \rho_f = 0.9 \text{ kg/m}^3 \) (which is reasonable if the fluid, made up of volcanic gas plus entrained cold atmosphere, is at about 200 °C) and varies from less than 2 kg/m\(^3\) to about 35 kg/m\(^3\). By using the values of density and velocity in (4) the trend of \( P_{dyn} \) is shown in Fig. 4d. It varies from less than 1 kPa with smaller \( W_l \) and finer \( D \), which is a value that does not cause severe damages to buildings, to almost 30 kPa with larger \( W_l \) and coarser \( D \), which can destroy even the more resistant, modern buildings of reinforced concrete. The sedimentation rate (Fig. 4e) increases as grain size coarsens, meaning that with finer sizes flow duration is longer, as it is expected since finer sizes result in a smaller settling velocity. As far as the wavelength is concerned, for the finest sizes, \( S_{rw} \) increases at increasing \( W_l \), meaning a decrease of...
flow duration with longer bedforms. With the coarsest sizes, instead, the sedimentation rate decreases as \( W_i \) increases, meaning a longer flow duration with longer bedforms.

The ranges of \( W_i \) and \( D \) used in (8) for obtaining the trends of Fig. 4 replicate the ranges in our dataset, and result in parameters that span from currents that do not impact severely on structures, to values of devastating effects. Such a range well represents the situation of large-scale PDCs whose strength decreases along runout\(^{15}\), and change from totally destructive flows around the volcano to residual currents that in the distal outreach do not possess a high strength but can still be rich of ash. Such fine glassy material can be highly dangerous to breath even at concentrations lower than 0.001\(^{38}\) if flow duration \( t \), which can be calculated by means of \( S_{rw} \), lasts more than a couple of minutes. Thus, with our model it is possible to invert bedforms of past eruptions, and follow the different aspects of PDCs hazard as they evolve along flow runout.

In order to help scientists not availing of numerical resources to take advantage of our results, we solved (8) at discrete intervals of \( D \) and \( W_i \) and constructed a phase diagram where the stability fields of \( P_{dyn} \), \( C \) and \( S_{rw} \) are represented inside a grid (Fig. 5). The values are averaged among the four neighboring grid points and the uncertainty is expressed in terms of one standard deviation. \( P_{dyn} \) is calculated by considering the average value obtained by integration over the first 1000 cm of the current, and setting \( k_s =10 \) cm and \( \rho_s = 2000 \) kg/m\(^3\).
Fig. 5. Phase diagram in which the stability fields of the impact parameters $P_{\text{dyn}}, C$ and $S_{\text{rw}}$, are expressed as a function of $W_l$ and $D$ of bedforms. The values inside the grid represent the average between the four neighboring grid points and the uncertainty is expressed as the standard deviation. $K_s = 10$ cm, $\rho_s = 2000$ kg/m$^3$.

In the supplementary Information, additional diagrams with $k_s = 10$ cm and $\rho_s = 1000$ kg/m$^3$; $k_s = 30$ cm and $\rho_s = 2000$ kg/m$^3$; and $k_s = 10$ cm and $\rho_s = 1000$ kg/m$^3$ are included (Supp. Fig. 1,2 and 3 respectively), and a table is also provided (Supp. Tab.1) where the values of $u^*$, $C$ and $S_r$ are set at half phi intervals of $D$ in relation to $W_l$. By means of these data, and specifying in (4) and (9) the value of $k_s$, $\rho_s$, and $H$ at which to integrate $V$, more precise data of the impact parameters can be obtained.

With our diagrams and tables at hand it is thus possible for every scientist working on hazardous volcanoes to make an exploratory hazard assessment by means simply of the wavelength and grain size of bedforms. It is true that bedforms are not always well exposed in their complete longitudinal profile, because of truncations due to erosion. Sometimes they are also difficult to measure precisely,
because a direct access to the deposit is hard. Anyway, our experience tells that dilute PDCs most always leave well-preserved bedforms as a trace of their passage. Scientists working on active volcanoes are encouraged to look for good outcrops where bedforms can be measured. By means of our phase diagrams, now they have a tool for exploring the behavior of hazardous pyroclastic density currents directly in the field.

Method

The reconstruction of the impact parameters of PDCs is based on a flow mechanical model that starts with the assumption that the turbulent current is velocity and density stratified\textsuperscript{12,40}. In the stratified multiphase gas-particle current, the basal part is a shear flow that moves attached to the ground and has a density higher than atmosphere (Fig. 6). The upper part is buoyant, because particle concentration decreases with height down to a value that, combined with the effect of gas temperature, makes the mixture density lower than the surrounding atmosphere.

In a PDC, particles are mainly transported by turbulent suspension and sedimentation is controlled by a balance between flow shear velocity $u_*$, which is controlled by fluid turbulence and favors suspension, and particle settling velocity, $w_t = (4gD(\rho_s - \rho_{mix})/3C_d\rho_{mix})^{0.5}$, which favors sedimentation, where $C_d$ is drag coefficient. During sedimentation, it is assumed that particles of different composition, i.e. crystals and glass, settle at the same aerodynamic conditions, e.g., with the same

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6}
\caption{Sketch of the model of a pyroclastic density current used in this paper}
\end{figure}
terminal velocity\textsuperscript{15}. Therefore, by equating the settling velocity of the glass and crystal components
in the deposit, and assuming that sedimentation starts when $P_n = 2.5$, hence when $w_t = u^*$, flow shear
velocity and density $\rho_{sf}$ of the shear flow can be calculated after $D$, $\rho_s$ and $C_d$ are measured in the
laboratory\textsuperscript{36}. These are the main input data in the PYFLOW\textsubscript{2.0} code\textsuperscript{31}, which allows reconstructing
the current parameters.

The code is based on a model that assumes PDCs behave as turbulent boundary layer shear flows
moving over a rough surface\textsuperscript{37}, which velocity profile is given by (9). The model has been validated
by experiments\textsuperscript{18} and already applied to other eruptions\textsuperscript{40,41}. Here it is summarized as adapted from
Dellino et al.\textsuperscript{15}.

The maximum volumetric concentration of particles that can be transported in turbulent suspension,
i.e. the maximum current capacity, is a function of the Rouse number of the particulate mixture taken
in suspension. The profile of volumetric concentration over current height is regulated by the Rouse
model\textsuperscript{42}.

\begin{equation}
C(y) = C_0 \left(\frac{y_0}{H-y_0}\right)^{P_n} \tag{10}
\end{equation}

where $C_0$ is the particle volumetric concentration at a reference height $y_0$ and $H$ is the total current
thickness. Assuming steady sedimentation, $H$ is obtained by the ratio $H_{dep}/C_{sf}$ where $H_{dep}$ is deposit
thickness and $C_{sf}$ is the depth-averaged concentration in the basal shear flow, which can be calculated
by $\rho_{sf} = \rho_s C_{sf} + \rho_f (1-C_{sf})$, when $\rho_{sf}$ and $\rho_f$ are known.

The shear-flow height and density are obtained by solving the system of (11) and (12), which is valid
for a turbulent current

\begin{align*}
\tau &= (\rho_{sf} - \rho_f) g \sin \alpha H_{sf} \tag{11} \\
\tau &= \rho_{sf} u_*^2 \tag{12}
\end{align*}

where $\tau$ is the shear-driving stress of the flow moving down an inclined slope of angle $\alpha$.

The density profile, which is a function of concentration, particle density and gas density, is:

\begin{equation}
\rho_{mix}(y) = \rho_f + C_0 \left(\frac{y_0}{H-y_0}\right)^{P_n} (\rho_s - \rho_f) \tag{13}
\end{equation}
The gas density and Rouse number are obtained by solving numerically the following system:

\[ \rho_a(y) = \rho_f + C_0 \left( \frac{y_0}{H-y_0} \right)^{\eta_f} \left( \rho_s - \rho_f \right) \] (14)

\[ \rho_{sf} = \frac{1}{H_{sf}-y_0} \int_{y_0}^{H_{sf}} \left( \rho_f + C_0 \left( \frac{y_0}{H-y_0} \right)^{\eta_f} \left( \rho_s - \rho_f \right) \right) dy \] (15)

Equation (14) states that atmospheric density, \( \rho_a \), is reached at the top of the shear flow, \( H_{sf} \), and equation (15) states that the average density of the shear flow, \( \rho_{sf} \), refers to the part of the flow that goes from the reference level, \( y_0 \), to the shear flow top height, \( H_{sf} \).

By combining the velocity and density profiles, the dynamic pressure profile is finally obtained. The profiles of the flow parameters are expressed in terms of a probability density function that depends on the variance of particle characteristics.

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| Symbol | Description                                                                 | Dimension |
|--------|------------------------------------------------------------------------------|-----------|
| $A_r$  | Aggradation rate per unit width                                             | ms$^{-1}$ |
| $C$    | Particle volumetric concentration                                           | -         |
| $C_0$  | Reference known concentration (0.75)                                        | -         |
| $C_{sf}$ | Depth-averaged concentration in the basal shear flow                        | -         |
| $C_d$  | Particle drag coefficient                                                   | -         |
| $D$    | Sediment median size                                                        | mm        |
| $Fr'$  | Froude number                                                               |           |
| $g$    | Gravity acceleration (9.81)                                                 | ms$^{-2}$ |
| $H$    | Current depth                                                               | cm        |
| $H_{dep}$ | Deposit thickness                                                        | cm        |
| $k$    | Von Karman constant (0.4)                                                   | -         |
| $k_s$  | Substrate roughness                                                         | cm        |
| $P_{dyn}$ | Dynamic pressure                                                        | Pa        |
| $P_n$  | Particle Rouse number                                                      | -         |
| $P_n^*$ | Normalized Rouse number                                                    | -         |
| $P_{navg}$ | Average Rouse number of solid material                                    | -         |
| $P_{ni}$ | Rouse number of the $i$th particle-size class                              | -         |
| $P_{n_{sup}}$ | Rouse number at maximum suspension capacity                           | -         |
| $Q_b$  | Bedload transportation rate                                                 | m$^2$s$^{-1}$ |
| $q_{bi}$ | Volumetric bedload transport rate of the $i$th particle-size class         | m$^2$s$^{-1}$ |
| $Re_*$ | Reynolds’ number                                                            | -         |
| $S_r$  | Sedimentation rate                                                          | kgm$^{-2}$s$^{-1}$ |
| $S_{rw}$ | Sedimentation rate per unit width                                         | m$^2$s$^{-1}$ |
| $t$    | Flow duration                                                                | s         |
| $u_*$  | Shear velocity                                                               | ms$^{-1}$ |
| $V$    | Current velocity                                                            | ms$^{-1}$ |
| $W_{i,r}$ | Dimensionless transport rate of the $i$th particle-size class              | -         |
| $W_{i}$ | wavelength                                                                  | cm        |
| $w_i$  | Particle terminal velocity                                                  | ms$^{-1}$ |
| $w_{ti}$ | Terminal velocity of the $i$th particle-size class                         | ms$^{-1}$ |
| $y$    | Flow vertical coordinate                                                    | cm        |
| $y_0$  | Specific height of $C_0$                                                    | -         |
| $\alpha$ | Slope angle                                                                | $^\circ$ |
| $\phi$ | Unit of grain-size distribution ($\phi= -\log_2 d; d$ is in mm)            | -         |
| $\phi_i$ | Weight fraction of the $i$th size class                                     | Weight%   |
| $\theta$ | Shield’s number                                                            | -         |
| $\mu$  | Fluid viscosity                                                              | Pas       |
| $\rho_f$ | Fluid density                                                              | kgm$^{-3}$ |
| $\rho_{mix}$ | Density of the fluid-particle mixture                                       | kgm$^{-3}$ |
| $\rho_i$ | Particle density                                                            | kgm$^{-3}$ |
| $\rho_{sf}$ | Density of shear flow                                                       | kgm$^{-3}$ |
| $\rho_{ni}$ | Density of the $i$th particle-size class                                    | kgm$^{-3}$ |
| $\tau$ | Shear stress at the base of the current                                    | Pa        |
| $\tau_i$ | Minimum shear of the $i$th size fraction                                   | Pa        |
| $\xi$  | Normalized shear stress                                                     | -         |

Table 1. List of Symbols, with description and physical dimension.
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