Mapping Coupled Time-series Onto Complex Network

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Abstract
For the sake of extracting hidden mutual and coupled information from possibly uncoupled time-series, we explored the profound measures of network science on time-series. Alongside common methods in time-series analysis of coupling between financial and economic markets, mapping coupled time-series onto networks is an outstanding measure to provide insight into hidden aspects embedded in couplings intrinsically. In this manner, we discretize the amplitude of coupled time-series and investigate relative simultaneous locations of the corresponding amplitudes (nodes). The transmissions between simultaneous amplitudes are clarified by edges in the network. In this sense, by segmenting magnitudes, the scaling features, volatilities’ size and also the direction of the coupled amplitudes can be described. The frequency of occurrences of the coupled amplitudes is illustrated by the weighted edges, that is to say, some coupled amplitudes in the time-series can be identified as communities in the network. The results show that despite apparently uncoupled joint probabilities, the couplings possess some aspects which diverge from random Gaussian noise. Thereby, with the aid of the network’s topological and statistical measurements, we distinguished basic structures of coupling of cross-market networks. Meanwhile, it was discovered that even two possibly known uncoupled markets may possess coupled patterns with each other. Thereby, those markets should be examined as coupled and weakly coupled markets!

Keywords: Coupled Time-series, Complex Networks, Financial Markets

1. Introduction
It is intriguing to study coupled markets by mapping their coupling onto a network. The reasoning behind this includes the numerous measures introduced by network science. In addition to previously applied measures by time-series analysis, network analysis measures help scholars to explore deeper characteristics of economic and financial networks. In the present study, we show that coupling of time-series (which may be apparent so far, or highly coupled with each other) contains information which may have been remained uncovered. This discovery is demonstrated by measures of describing the features of a network obtained from the coupling of two time-series.

In order to extract more hidden information from time series, network science has been utilized for analyzing the extraction of information about the time series of a wide variety of fields through the analysis of the derived network [1–5]. Considering that network science [6, 7] has become highly applicable in time series analysis and multi-agent based models [8], such as financial markets [9–15] to clarify the intrinsic structure of the couplings between the price volatility structure [16] and macroeconomic measures [17, 18], the economic recovery plans [19] relating to financial and economic crises [20] and systemic risk [21–25], risk measurement in financial directed causality networks [26], multivariate financial time-series [27], analysis of nonlinear time-series [28], analysis of time-series by mapping to weighted and directed networks [29], analysis of linear regression patterns in a non-stationary time-series [30], lagged effects of a dynamical system by network science [31]. Moreover, these methodologies have successfully been applied for chaotic time-series [32], higher-order network analysis [33, 34], and also tourism management [35]. Along with giving us the ability to reason about network topology and community structures [36], the internal interactions and information transmissions in magnitude-wise aspects are crucial. Hence, as a multi-scale approach in mapping time-series onto a complex network, a novel algorithm in transmission of regression patterns between two time-series was developed in a complex network viewpoint [30]. Other proposed methods include mapping multi-variate time-series onto multiplex [37, 38] networks to develop multi-dimensional signal processing [27] to assess financial instability [1, 39, 40] by visibility graphs [41], machine learning algorithms by visibility graphs to multiplex learning networks [42] and financial minimum spanning tree [36, 43]. At the heart of the above applications are measures for reveal-
ing the structural topology of networks of coupled time-series whether the time-series are random, periodic (ordered) or fractal which will cause the mapping process to result in a random network, regular network and scale-free network, respectively [1].

Cross-correlation in financial time-series intrinsically contains scaling behaviors [44, 45]. Those scaling behaviours not only emerge in temporal aspects, but also, they emerge in higher statistical moments of price return distribution—correlation coefficient. In this context, the present study casts lights into the behavior of couplings between financial time-series by applying novel measures of network science. In respect to the cross-correlation networks [46], the network properties such as clustering coefficient, efficiency, the cross-correlation degree of cross-correlation interval and also modularity of dynamic states have been investigated [32].

In particular, we are supposed to capture/estimate temporal/dynamic behaviors of the financial time-series by mapping onto a network perspective as follows, by:

I. introducing the mapping algorithm from coupled time-series onto a network;
II. constructing of the networks obtained by coupling of two financial time-series, and;
III. the networks obtained by fractional Gaussian noise (fGns) coupled by their corresponding 1-step lag with a diverse range of Hurst exponents;
IV. comparing the obtained networks and extracting the hidden features of couplings.

Hurst exponent is a criterion to perceive to what extent two signals are coupled in various time-scales. We observed that according to some features both coupled and uncoupled time-series are different. On the other hand, some features of those couplings are significantly close to the networks obtained by fGns. However, there exist features where none of the networks converge to a definite value. Based on the segregation of those networks, the information transitions and common features among couplings are revealed.

2. Mapping Single Time-series Onto Network

Considering scaling features of amplitude-wise financial correlations [45, 47], alongside with the fact that correlation coefficients just reveal the linear co-behaviors of the time-series, there exists a vital need to consider the effects of the direction and the size of amplitudes which may consist of nonlinear behaviors. In this way, without the need for necessarily linear relation, the couplings are defined. This procedure can be explored by temporal-interval [32] and amplitude-interval points of view [2]. To explore amplitude intervals (amplitude bins), we generate a method to couple the amplitude of a time-series with its corresponding 1-step lag. Hence, we make a segmentation on the amplitudes and convert the amplitudes onto several bins [2]—from now on, we consider these amplitude bins as nodes in a certain network. For illustrative purposes, Fig. 1, top subfigure, shows the way we design the process.

The Hurst exponent of a system implies how two time-series—also one single time-series and its lags—are coupled (uncoupled). In this regard, financial time-series contain some structural and intrinsic information whether they are a developed or an emerging market [44, 50, 51]. Initially, we generate some fractional Gaussian noise (fGns), and we will further compare the characteristics of real-world time-series. To cast light into the effects of Hurst exponents in the behavior of a certain fractional Gaussian noise, in Fig. 2, the auto-correlation matrices pertaining to the Hurst exponents are depicted. It is notable that for higher Hurst exponents (as an identification of stronger coupling), we observe higher correlations around the diameter of the correlation matrix.

From Fig. 2, one observes that a high Hurst exponent leads to a high elongation around its main diameter of joint probabilities, refer to Eq. 1.

For the sake of quantifying the elongation of couplings, we introduce a deformation parameter, R, which widely clarifies the couplings behavior. To quantify this deformation, we introduce a deformation parameter based on the standard deviations along diameters of joint probabilities matrix, R, as Eq. 1 illustrates;

\[ R = \frac{\sigma_i - \sigma_j}{\max[\sigma_i, \sigma_j]} \tag{1} \]

where, \( \sigma \) denotes the standard deviations along diameters of the joint probability matrix. Further, the relationship between the individual parameter \( R \) relative to the corresponding Hurst

\[ \text{Figure 1: The algorithm of mapping is demonstrated. This figure depicts the way links in the network are generated. As shown, when the relative amplitudes corresponding to two time-series are located in the same amplitude bins (nodes), a weighted loop is considered. On the contrary, when the amplitudes are not located in the same bin, two nodes connecting with an weighted edge are generated. The term weight, here, implies the frequency of this directed situation and can be applied with the timing resolution considerations and the weighting threshold sampling [48] and the persistence of the edges [49]. Thereby, the outcome will be a temporal coupling network.} \]
exponents will be clarified in Fig. 2, bottom subfigure.

For now, according to Fig. 2 and applying Eq. 1, one is able to estimate the couplings’ Hurst exponent corresponding to each cross-market based on its $R$ parameter—which has already obtained by the cross-market joint probabilities.

3. Mapping Coupled Time-series Onto Network

In the following, we map the coupling of cross-market time-series onto a network. The algorithm which is applied here is the same as the previous one, but two signals with the same chronological time-stamp (no lag) is considered. Next, the outcome will be compared with the fGns which are mapped onto the network before. This process is showed in Fig. 3. By focusing on this phenomenon for cross-markets joint probabilities, the extent to which the strength of couplings causes elongation is carried out in [53]. Besides our efforts on considering the Hurst exponent of the couplings of cross-market time-series [47, 54], in the present study, we will investigate the cross-market couplings in a financial network approach.

From Fig. 1, it should be highly emphasized that in addition to considering the positive and negative directions of amplitudes, we account for the differences between the size and the locations in the amplitude bins by generating edges. In this regard, the placement of amplitudes in the same amplitude bins leads to a loop. Meanwhile the placement of amplitudes in different amplitude bins leads to an edge. In this regard, the direction of edges stands for emphasizing on the difference between whether the first signal is in bin A and the other one in B, as opposed to whether the first signal in bin B and the second one in A.

Here, the role of the Hurst exponent should be highlighted—as an intrinsic structure of time series—during the process of mapping the coupling onto the joint probability matrix.

In this research we generate discrete intervals to evaluate the coupled amplitude of the markets by mapping the time series onto coupled networks. Smaller amplitude bins lead to higher amount of bins and consequently more noise detection in the couplings. Accordingly, wider amplitude bins imply more similar events between the markets, and, smaller amplitude bins yield to more noise detection among amplitudes.

Results and Discussion

- Firstly, it is notable that fGn with Hurst=0.5 (shown in dashed gray line in the radar plot in Fig. 3) is the indication of lack of information in the coupling. Hence, convergence of any measurements to this value illustrates insignificant information embedded in the coupling. It is worth noticing that, a Gaussian random noise is the symbol of no coupling information. In this regard, DJIA-SSEC’s coupling is close to an uncoupled situation, which is shown in Fig. 3.

- Deformation Ratio, $R$. According to Fig. 3, the process of mapping coupling onto a network is clarified. As shown from the comparison between joint probability matrices of DJIA-SSEC and DJIA-S&P500, the strength of couplings are visually showing that the coupling of DJIA-S&P500 is stronger than that of DJIA-SSEC. This feature is quantified based on Eq. 1 with the $R$ parameter which is considered in the radar plot in Fig. 3.

- Degree Measurements. The measurements corresponding to degrees, such as mean squared out-degrees $<k_{\text{out}}^2>$, mean squared in-degrees $<k_{\text{in}}^2>$, mean squared total-degrees $<k_{\text{total}}^2>$, mean out-degrees $<k_{\text{out}}>$, mean in-degrees $<k_{\text{in}}>$ and mean total-degrees $<k_{\text{total}}>$, significantly contain the power of proving the segregation among cross-markets and fGn with Hurst=0.5. The mentioned features in the radar plot in Fig 3 contain significant coupling information among fGns and also the cross-market coupling mapped onto a network.
Also, the standard deviation of total-degree $k_{std}$ turns up to identify cross-market couplings.

- **Clustering Measurements** [55]. When it comes to clustering features, the standard deviation of global clustering coefficient $Cl.Coeff_{std}$ is capable of exploring the difference between cross-market couplings. Moreover, the undirected local clustering coefficient $Cl.Coeff_{local}$ can distinguish among the networks of coupled and uncoupled cross-markets. It is shown that this feature converges to fGn with Hurst=0.5 for the network extracted from uncoupled cross-market. Also, the directed local clustering coefficient $Cl.Coeff_{directed}$ is different for uncoupled and coupled outcomes. Whereas, the global clustering coefficient $Cl.Coeff_{global}$ for fGns, uncoupled and coupled cross-markets are approximately similar.

- **Length (Shortest Path Between Pair-wise Vertices) Measurements**. It is striking that the directed mean length $L_{directed}$ and the undirected mean length $L_{undirected}$ significantly explore the differences between coupled cross-markets from uncoupled cross-markets and the fGns.

- **Assortativity Measurements** [56]. The variance of scalar assortativity coefficient $Sc.Ass.Coeff_{var}$. for fGns, uncoupled and coupled cross-markets are approximately similar. Conversely, the assortativity coefficient variance $Ass.Coeff_{var}$. assortativity coefficient $Ass.Coeff$. scalar assortativity coefficient $Sc.Ass.Coeff$. markedly distinguish among coupled cross-markets from fGns and uncoupled cross-markets.

- **Modularity Measurements** [57, 58]. By extracting community structures, the modularity measurements enable scholars to distinguish the dynamic states of the network. Thereby, following the approach of mapped networks [32], we find it highly useful to investigate the coupling networks structures by comparing their modularity, as shown in Fig. 3. As depicted, notwithstanding that out-degree modularity enables one to identify the cross-markets from fGn, the total-degree modularity $Modularity_{total-degree}$ is highly capable of showing the divergence between uncoupled and coupled cross-markets. This implies, that based on the out-degree modularity measurement, there exists mutual information among markets and it is not uncoupled in this manner. In this regard, markets are coupled or weakly coupled (not necessarily uncoupled)!

To further explore the payoffs and to assess the differences, DJIA-S&P500’s coupling is adequately far from Gaussian random noise (fGn with Hurst = 0.5) and DJIA-SSEC’s coupling. The flipside of the coin is that DJIA-SSEC’s coupling is closer
to Gaussian random noise (fGn with Hurst \( H = 0.5 \)) rather than DJIA-S&P500, but it is still totally different. In such a situation we can conclude that although the joint probabilities of DJIA and SSEC time-series show that they are uncoupled (refer to deformation ratio, \( R \), in Eq. 1 and Fig. 3), by mapping coupled time-series onto a complex network perspective, more hidden properties are revealed and the obtained results illustrate that according to some other network criteria, those markets still possess coupling information! Hence, being a market contributes to being coupled with others! Thus, it is better to use the term, weakly coupled markets rather than the term, uncoupled markets. As depicted in Fig. 3, the joint probability matrices can be considered as the adjacency matrices corresponding to the network of couplings. Along with giving us the ability to reason about the couplings formation, Fig. 3 will extend our knowledge toward realistic simulations in intrinsic coupled structures in the network.

4. Conclusion

Taken together, mapping cross-correlation of time-series onto a network contributes to defining a coupling network. Topological and statistical parameters along with the deformation ratio of adjacency matrix—regarding the cross-correlations—are able to reveal the coupling information which has previously been beyond the reach of researchers. Not only comparing fGns with cross-market coupled networks proves pair-wise interconnectedness, but also it clarifies the diverse structure of the coupling. The reasoning behind this claim is that couplings with different Hurst exponents show a diverse range of behaviors (anti-persistent, random, persistent) which can be reflected in a so-obtained mapped network. In the present study, cross-correlation of some time-series are mapped onto a network. Accordingly, a mapped coupling network is explored. Meanwhile, a deformation parameter which is extracted from a relation for the standard deviations alongside both diameters of the directed weighted adjacency matrix (joint probability matrix) is introduced. The above mentioned approach is applied on cross-markets coupling networks such as DJIA-SSEC and DJIA-S&P500.

With the aid of the network topological and statistical measurements, we distinguished a basic structure of coupling of cross-market networks. Meanwhile, it was discovered that even two previously known uncoupled markets may possess coupled aspects with each other. Therefore, those markets should be examined as coupled and weakly coupled markets!

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