Ambipolar decay of magnetic field in magnetars and the observed magnetar activities

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Abstract

Magnetars are comparatively young neutron stars with ultra-strong surface magnetic field in the range $10^{14} - 10^{16}$ G. The old neutron stars have surface magnetic field somewhat less $\sim 10^8$ G which clearly indicates the decay of field with time. One possible way of magnetic field decay is by ambipolar diffusion. We describe the general procedure to solve for the ambipolar velocity inside the star core without any approximation. With a realistic model of neutron star we determine the ambipolar velocity configuration inside the neutron star core and hence find the ambipolar decay rate, associated time scales and the magnetic energy dissipated which is consistent with the magnetar observations.

Keywords: Neutron stars, magnetars, magnetic field

1 Introduction

The unique astrophysical objects neutron stars are mostly observed as radio pulsars. Though from neutron stars we receive radiation in all ranges of the electromagnetic spectrum, radiation in radio, X-ray and sometime $\gamma$-ray are dominating. Neutron stars are magnetized. Most of the radio pulsars are isolated neutron stars with rotation periods 100 ms to 10 s and typical surface magnetic field of the order $10^{10} - 10^{13}$ G. Few of them are in binary systems. Another group of pulsars have shorter periods and the pulsars with rotation periods less than 100 ms are known as milli-second pulsars. Most of them have typical surface magnetic field range $10^8 - 10^{10}$ G and are present in binary system. They are more aged than isolated radio pulsars. Another special kind of pulsars are anomalous X-ray pulsars. They are called “anomalous” because like ordinary pulsars they are neither rotation powered nor in binary system to be accretion powered. They are generally believed to have very strong surface magnetic field in the range of $10^{14} - 10^{16}$ G and their activity is due to magnetic energy dissipation. Another class of objects, the soft gamma repeaters, are also believed to come under the same category of this type of neutron stars with ultra-strong surface magnetic field. This class of objects are known as magnetars.
and are younger compared to other class of neutron stars. The emission from magnetars is believed to be caused by the magnetic field evolution inside magnetars. In this respect the internal magnetic field evolution is very important phenomenon to study.

The magnetic field evolution can occur through several processes: (a) Ohmic decay, (b) Ambipolar diffusion and (c) Hall-drift. The magnetar activity is generally explained by the field evolution inside the crust of the neutron star. In the core, the Ohmic decay time scale is generally very large because of the high conductivity of the matter in the core of a neutron star (Yakovlev and Shalybkov, 1991). Consequently, the field evolution is dominantly caused by the ambipolar diffusion of the charged particles in the core. The Hall drift does not dissipate the magnetic field, it only causes the change in field configuration inside the star. The field evolution due to ambipolar diffusion has been discussed in several recent publications [Gusakov et al. (2017); Passamonti et al. (2017); Castillo et al. (2017); Ofengeim and Gusakov (2018); Castillo et al. (2020)]. Most of them have studied the diffusion considering the present magnetic field as perturbation. In those studies the magneto-hydrodynamic equations have been treated with different approximations according to different physical situations suitable to neutron star core at different stages and accordingly the field evolution has been estimated. The crucial point to calculate the field evolution due to ambipolar diffusion is to have the ambipolar diffusion velocity profile inside the star core. The ambipolar velocity profile has been studied for normal and superfluid matter in non-rotating neutron star model by Passamonti et al. (Passamonti et al., 2017).

In the present work we focus on the field decay in the core of the star caused by the ambipolar diffusion by evaluating the ambipolar velocity of the charged particles in the core without considering any approximations. We consider the neutron star after the temperature reaches the values when neutrino can escape of the matter. Below this region of temperature at different temperature regions different damping factors to ambipolar diffusion is dominant. The procedure discussed here is free of constraints to any such specific temperature regime, for which some special type of damping of ambipolar diffusion is dominant. The way is general and applicable to all composition of star core at all temperature regimes below neutrino trapping temperature. Inside the magnetars, due to presence of strong magnetic field the neutron sperfluidity and protont superconductivity are quenched (Sinha and Sedrakian, 2014, 2015; Stein et al., 2016). Hence, we study the ambipolar diffusion velocity field and its effect on magnetic energy dissipation, time scale etc. generally with all terms present, extensively for the core with normal matter. In next section we describe the basic theoretical calculation for solving the ambipolar diffusion velocity in different regions of star. We then proceed to discuss our model for which we have calculated the ambipolar velocity in section 3. We report and discuss the result in the section 4 and finally we conclude in the last section (section 5).

2 Theoretical description

2.1 Particle dynamics

The pioneer work on field decay mechanism is by Goldreich and Reisenegger in 1992 (Goldreich and Reisenegger, 1992). For the sake of simplicity we consider that the matter inside the neutron star is composed of nuclear matter with electrons in normal phase (not
in superfluid phase). Further, we consider the star is rotating with angular speed $\Omega$ and the particles inside the star are in dynamical equilibrium. Following (Goldreich and Reisenegger, 1992) we assume the Newtonian gravitational potential $\phi$ inside the star. Then the equilibrium conditions of all species in the presence of electromagnetic field $\mathbf{E}$ and $\mathbf{B}$ are respectively,

\[
0 = m_N \nabla U - \nabla \mu_n + \mathbf{F}^\text{int}_n - m_N 2\Omega \times \mathbf{v}_n \tag{1}
\]

\[
0 = m_N \nabla U - \nabla \mu_p + e(\mathbf{E} + \mathbf{v}_p \times \mathbf{B}) + \mathbf{F}^\text{int}_p - m_N 2\Omega \times \mathbf{v}_p \tag{2}
\]

\[
0 = m_e \nabla U - \nabla \mu_e - e(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) + \mathbf{F}^\text{int}_e - m_e 2\Omega \times \mathbf{v}_e \tag{3}
\]

where

\[
U = \nabla \left(\frac{r^2\Omega^2}{2} + \phi\right), \tag{4}
\]

$m_i$ is the mass in the medium inside the star, $v_i$ is the velocity, $\mu_i$ is the chemical potential of the $i$-th species and $\mathbf{F}^\text{int}_i$ is the force on the $i$-th particle due to interaction with other particles. Here we denote the species neutron ($n$), proton ($p$) and electron ($e$) by the subscript $i = n, p, e$ respectively. Here we assume the masses of nucleons are same as $m_N$.

Now multiplying the single particle equations of motion by number density of respective species, we write the equation for the neutrons as

\[
0 = m_N n_n \nabla U - n_n \nabla \mu_n + \mathbf{f}^\text{int}_n - m_N n_n 2\Omega \times \mathbf{v}_n \tag{5}
\]

and for charged particles together as

\[
0 = (m_N + m_e)n_c \nabla U - n_c(\nabla \mu_p + \nabla \mu_e) + \mathbf{f}^m + \mathbf{f}^\text{int}_p + \mathbf{f}^\text{int}_e - 2\Omega \times n_c(m_N \mathbf{v}_p + m_e \mathbf{v}_e) \tag{6}
\]

Here we have used the fact that $n_p = n_e = n_c$ due to charge neutrality condition where $n_i$ is number density of the $i$-th species. Here

\[
\mathbf{f}^m = \mathbf{j} \times \mathbf{B} \tag{7}
\]

is the magnetic force density with

\[
\mathbf{j} = \mathbf{j}_p + \mathbf{j}_e = n_c e(\mathbf{v}_p - \mathbf{v}_e), \tag{8}
\]

the total current density and

\[
\mathbf{f}^\text{int}_i = \Sigma_{j} n_{ij} \mathbf{F}^\text{int}_{ij}, \tag{9}
\]

with $n_{ij}$ number of collisions within unit volume between $i$-th and $j$-th particles, is the interaction force on $i$-th particle due to its interaction with others particle within unit volume $i.e.$ the interaction force density for $i$-th particle. Then interaction force density on neutrons is

\[
\mathbf{f}^\text{int}_n = n_{np} \mathbf{F}^\text{int}_{np} + n_{ne} \mathbf{F}^\text{int}_{ne} \tag{10}
\]

and on charged particles

\[
\mathbf{f}^\text{int}_p + \mathbf{f}^\text{int}_e = (n_{pn} \mathbf{F}^\text{int}_{pn} + n_{pe} \mathbf{F}^\text{int}_{pe}) + (n_{pe} \mathbf{F}^\text{int}_{pe} + n_{en} \mathbf{F}^\text{int}_{en}) = n_{pn} \mathbf{F}^\text{int}_{pn} + n_{en} \mathbf{F}^\text{int}_{en} \tag{11}
\]
Hence, we have
\[ f_{p}^{\text{int}} + f_{e}^{\text{int}} = -f_{n}^{\text{int}} = -f^{\text{int}} \text{ (say)} \]  
(12)

Then dividing the eq. (5) by neutron mass density \( \rho_{n} \) and eq. (6) by proton mass density \( \rho_{p} \) we have

\[
0 = \nabla U - \nabla \left( \frac{\mu_{n}}{m_{N}} \right) + \frac{f^{\text{int}}}{\rho_{n}} - 2\Omega \times \mathbf{v}_{n} \\
0 = \nabla U - \nabla \left( \frac{\mu_{p} + \mu_{e}}{m_{N}} \right) + \frac{f_{m}}{\rho_{p}} - \frac{f^{\text{int}}}{\rho_{p}} - 2\Omega \times \mathbf{v}_{p} 
\]  
(13)

(14)
as \( m_{e} \ll m_{N} \).

Then from the difference of these equations we get the combined equation of motion as

\[
0 = x_{p} \nabla \beta + \frac{f^{\text{int}}}{\rho_{n}} + x_{p} 2\Omega \times \mathbf{V}_{p} - \frac{f_{m}}{\rho} 
\]  
(15)

where \( \beta = (\mu_{p} + \mu_{e} - \mu_{n})/m_{N} \) and \( \mathbf{V}_{p} = \mathbf{v}_{p} - \mathbf{v}_{n} \) is the proton velocity in neutron rest frame. Then in absence of rotation the magnetic force is balanced by force due to (1) Transfusion and (2) Interaction.

### 2.1.1 Transfusion

When due to motion of charged particles the chemical equilibrium is disturbed, the weak reaction

\[
n \longleftrightarrow p + e
\]

tries to restore the chemical equilibrium. In this case, the number of particles generated or destroyed per unit time is given by reaction rate \( \Gamma = \lambda \beta \). The charged particles follow the equation of continuity

\[
\frac{\partial n_{i}}{\partial t} + \nabla \cdot (n_{i} \mathbf{v}_{i}) = -\lambda \beta
\]  
(16)

and as \( n_{p} = n_{e} = n_{c} \) we have [Goldreich and Reisenegger, 1992]

\[
\nabla \cdot (n_{c} \mathbf{w}) = -\lambda \beta
\]  
(17)

where

\[
\mathbf{w} = \frac{\mathbf{v}_{p} + \mathbf{v}_{e}}{2}
\]  
(18)

### 2.1.2 Interaction

In absence of superfluidity and superconductivity, the interaction between species are basically mechanical collision between them and that can be expressed as proportional to their relative velocity as

\[
\mathbf{F}_{ij}^{\text{col}} = -D_{ij}(\mathbf{v}_{i} - \mathbf{v}_{j})
\]  
(19)

where \( D_{ij} \) is the collision coefficient for collision between \( i \)-th and \( j \)-th species.
Hence,
\[ f^{\text{int}} = f^{\text{int}} = n_{np}D_{np}V_p + n_{ne}D_{ne}V_e = n_{np}m_n\tau_{np}V_p + n_{ne}m_n\tau_{ne}V_e = D_{np}V_p + D_{ne}V_e = (D_{np} + D_{ne})V \]  

(20)

where, \( D_{ij} = n_{ij}m_n\tau_{ij} \) with \( \tau_{ij} \) the collision rate between \( i \)-th and \( j \)-th species, and \( V_i \) is velocity of \( i \)-th species in rest frame of neutron \( v_i - v_n \). Here we define the ambipolar velocity
\[ V = \frac{D_{np}V_p + D_{ne}V_e}{D_{np} + D_{ne}}. \]  

(21)

### 2.2 Field evolution

Now in a plasma due to ambipolar diffusion (Goldreich and Reisenegger, 1992) the magnetic field evolution is given by
\[ \frac{\partial B}{\partial t} = \nabla \times (V \times B) \]  

(22)

and the decay in magnetic energy is given by
\[ \frac{\partial E_B}{\partial t} = -\frac{1}{4\pi} \int d^3x V \cdot f^m \]  

(23)

where \( V \) is the ambipolar diffusion velocity.

### 3 Model

We consider a star composed of normal nucleons and electrons. We assume the magnetic field of the star is purely poloidal as \( B = B_\zeta \hat{\zeta} + B_z \hat{z} \) in cylindrical coordinate system with \( \hat{\zeta} \) as the unit vector in the direction of cylindrical radius. We choose the rotation axis as the positive \( z \) axis and the system has axial symmetry. With this choice of configuration we solve the dynamical equilibrium equation for ambipolar velocity \( V \) as
\[ 0 = x_p \nabla \beta + \frac{D}{\rho_n}V + x_p 2\Omega \times (V + Aj) - \frac{f^m}{\rho}. \]  

(24)

where \( D = D_{np} + D_{ne} \).

To arrive at the above equation we express proton and electron velocities \( (v_p, v_e) \) in the eq. (18) in terms of ambipolar velocity \( V \) and current density \( j \) with
\[ A = \frac{D_{ne}}{n_{ce}(D_{np} + D_{ne})}. \]  

(25)

Expressing the velocities of species in terms of ambipolar velocity \( V \) we can express the chemical in-equilibrium as
\[ \nabla \beta = -\frac{1}{\lambda} \nabla \{\nabla \cdot (n_c V)\} \]  

(26)
from eq. (17). Substituting this in eq. (24), we get

$$0 = -\frac{x}{\lambda} \nabla \{ \nabla \cdot (n_e \mathbf{V}) \} + \frac{D}{\rho_n} \mathbf{V} + x_p 2\Omega \times (\mathbf{V} + \mathbf{A}_j) - \frac{f_m}{\rho} \cdot$$  \hspace{1cm} (27)

Eq. (27) is an inhomogeneous second order differential equation in $\mathbf{V}$ for a rotating neutron star with a magnetic field $\mathbf{B}$. We try to solve this equation with boundary conditions for ambipolar velocity and its gradient to vanish at the centre of the star. To obtain its solution, we assume the present magnetic field configuration to be completely poloidal which can be obtained from the magnetic vector potential

$$\mathbf{A} = \frac{1}{2} \zeta B \sin \theta \hat{\phi}$$  \hspace{1cm} (28)

where the magnitude of the magnetic field $B$ varies inside the star with the density profile \cite{bandyopadhyay1997}, parametrised as:

$$B \left( \frac{n_b}{n_0} \right) = B_s + B_c \left[ 1 - \beta \left( \frac{n_b}{n_0} \right)^\gamma \right],$$  \hspace{1cm} (29)

where $n_0$ is the nucleon saturation density, $n_b$ the baryonic number density and $B_s$ and $B_c$ are the magnitude of the magnetic field at the surface and core of the star respectively.

To obtain the solution of ambipolar velocity from eq. (27) further we consider a model star with matter inside the star composed of only nucleons with electrons and equation of state of the matter constructed within relativistic mean field model with GM1 parametrisation \cite{glendenning1991}. To be specific we consider a star with mass $2.217 \, M_\odot$ and radius $10.614 \, \text{km}$. With the magnetic field configuration described above the field profile inside the star is shown in figure 1. To get this profile the value of $\beta$ and $\gamma$ are set to 0.01 and 2.0 respectively.

We now solve for the ambipolar diffusion velocity ($\mathbf{V}$) in the core of the star taking into consideration the above magnetic field. The reaction rate and the collision coefficients...
are assumed constant throughout the star. To evaluate these constants, we consider a typical matter density of 2.2$n_0$. The corresponding charged particle densities, neutron density and their fractions are shown in table 1. However, taking the actual values of these quantities at different points of the star, the solution for ambipolar velocity profile can also be obtained by a more general numerical method.

For $x_p \geq 0.11$, $\beta$-reactions would be driven by direct Urca (dUrca) processes (Lattimer et al., 1991). For this process, the reaction rate is given by (Haensel and Schaeffer, 1992)

$$\lambda = 3.5 \times 10^{36} \frac{m_n^*}{m_n m_p} m_p^* T_8^4 \left( \frac{\rho}{\rho_{nuc}} \right)^{\frac{4}{3}} \text{erg}^{-1}\text{cm}^{-3}\text{s}^{-1} \quad (30)$$

The collision coefficients are obtained from (Yakovlev and Shalybkov, 1991), who calculated the collision frequencies and proved that the neutron-proton scattering is much more frequent than the neutron-electron scattering

$$\tau_{np} = 6.6 \times 10^{16} T_8^2 \rho_{14}^{-\frac{2}{3}} \text{s}^{-1} \quad (31)$$

Here one should note that as an example of the solution procedure we have taken here a typical density of matter and other necessary quantities corresponding to that density. However, in actual calculation every input parameter required to solve should be taken as function of density. However, the the nature of result does not differ much except nominal quantitative corrections.

The angular velocity is calculated using the time-period of rotation of a magnetar which is obtained from Olausen and Kaspi (Olausen and Kaspi, 2014). We consider the value:

$$\Omega = 0.628 \text{ rad/s} \quad (32)$$

## 4 Results

With the above described model of star with matter equation of state within relativistic mean field model with GM1 parametrization (Glendenning and Moszkowski, 1991) and the density dependent purely poloidal magnetic field, a numerical solution of Eq. (27) is carried out to obtain the profile of the ambipolar velocity $V$. We carry out our calculations for a particular temperature $T = 8 \times 10^8$ K which falls within the range of typical temperature inside neutron star core in the early age of the stars. The radial and axial components of the velocity vector has been shown in figure. 2 and figure. 3 for upper and lower hemisphere respectively. The azimuthal component has the same profile (a consequence of the poloidal nature of magnetic field) as the radial component, its magnitude being negligible. The total velocity profile in a quadrant part of the neutron star core is shown in figure. 4.
Figure 2: The radial and axial components of ambipolar diffusion velocity in the upper hemisphere of the neutron star core ($z > 0$). (a): the radial component of ambipolar diffusion velocity, (b): The axial component of ambipolar diffusion velocity.

Figure 3: Same as in figure 2 in the lower hemisphere of the neutron star core ($z < 0$).

Figure 4: The ambipolar diffusion velocity vector in the core of the neutron star. The circular arcs represents the region of different radius inside the neutron star core. The arrow represents the direction of the velocity vector.
The time-scale of magnetic field evolution as caused by ambipolar diffusion can be estimated as

\[ t = \frac{L}{\langle V \rangle} \tag{33} \]

where \( L \) is the characteristic length over which magnetic field varies and \( \langle V \rangle \) is the volume average of the magnitude of the diffusion velocity. For a characteristic length of 8 kms, we get a timescale of \( \approx 9.42 \times 10^4 \) years. The energy dissipation rate due to ambipolar diffusion can be found by substituting \( V \) in eq. (23). For the velocity vector shown in figure, we obtain the magnetic energy dissipation rate

\[ \frac{dE_B}{dt} = -1.22 \times 10^{36} \text{ ergs/s}. \tag{34} \]

The negative sign indicates the decrease in magnetic energy.

5 Discussion

In the present work, we outline the general method to obtain the magnetic energy decay rate due to ambipolar diffusion in the core of a compact star. In some of the existing literature \cite{Gusakov2017, Castillo2017, Ofengeim2018, Castillo2020}, the field evolution has been computed using perturbation method and discussed the importance of ambipolar diffusion process in the magnetic field evolution. With this approach the background baryon velocity completely depends on boundary condition of magnetic field profile which in turn depends on rate of field evolution and that is unknown a priori. Moreover, the ambipolar velocity has been determined as an estimate from the baryon background velocity \cite{Ofengeim2018, Castillo2020}. They consider the chemical potential variation inside the star as it is in equilibrium, with no variation of the chemical potential due to drift of charged particles \cite{Gusakov2017, Castillo2017} arguing that chemical imbalance is not caused by the \( \beta \)-equilibrium reactions, rather controlled by magnetic field profile only, which is quite debatable. Moreover, the approximation of equality of all particle velocity has been considered which is not required to consider in present calculation. On the other hand, \cite{Castillo2020} considers the low temperature limit only in which the transfusion can be neglected. In addition, the true nature of the matter considering the presence of strong interaction has not been taken into account \cite{Castillo2020}.

The magnetic energy dissipation due to ambipolar diffusion is related to ambipolar velocity of charged particles. In this context we have constructed the equations from simple microscopic idea of particle dynamics in a normal matter composed of proton-electron plasma in the background of neutrons and in the presence of external magnetic field. Equations contain terms which are sensitive to temperature and density. To get a complete picture in the star core interior for different temperature regime, it is necessary to solve the equations for a wide range to temperature and matter density with different possibilities of matter properties and composition which is to be addressed in near future. To get the general idea of solution procedure, at some specific matter density compatible to the core of the compact stars, we solve the dynamical equilibrium equation to get ambipolar diffusion velocity profile inside a comparatively young neutron star core with typical temperature of \( \sim 8 \times 10^8 \) K. As temperature is high inside the core, it is quite
natural for the matter in the core to be in non-superfluid phase. Moreover, in presence of ultra strong magnetic field inside the magnetars, the neutron superfluidity and proton superconductivity are quenched (Sinha and Sedrakian, 2014, 2015; Stein et al., 2016).

From this point of view we consider the normal matter in this current calculation. From the profile of the ambipolar velocity we can get the idea of magnetic energy dissipation due to ambipolar diffusion, the typical time scale of magnetic field decay and also the amount of heating due to field decay. We have shown that with a realistic model, the energy dissipation rate is $\sim 10^{36}$ ergs $s^{-1}$ as expected (Thompson and Duncan, 1995). We obtain a value of magnetic energy dissipation as $\sim 3.64 \times 10^{48}$ ergs during the timescale of evolution which reproduce the order of total magnetic energy budget as mentioned in literature by (Enoto et al., 2019). The time scale of magnetic field decay is $\sim 10^4 - 10^5$ yrs, which is compatible with the observed lifetime of soft gamma repeaters activity. One should note that our model considers the core to be composed of normal (non-superfluid) matter which is very possible scenario if the star posses very strong magnetic field like magnetars (Sinha and Sedrakian, 2014, 2015; Stein et al., 2016). We take an approximation by considering a purely poloidal magnetic field. An additional toroidal magnetic field will not only change the profile of azimuthal component of $V$, but will also increase the magnitude of the radial and the axial components. The increased magnitude of $V$ shall decrease the time-scale of evolution possibly to $10^3 - 10^4$ yrs.

In this work we have not taken any approximation considering the temperature regime. We have discussed the result for a particular temperature but the solution procedure is not specific to that temperature. The procedure is general which can be employed for any temperature. Ambipolar velocity profile will be different for different temperature region because of temperature dependence of various parameters involved in the differential equation of ambipolar velocity. In this respect, this work shows a general procedure to solve the ambipolar diffusion problem to get an exact picture of the field decay rate and heating due to field decay with different external realistic conditions relevant at different thermal and temporal stage of the stars. The work shows a general procedure to get the ambipolar velocities without any approximation or assumptions of special case and a solution with a typical model of star containing normal matter as an example.

Natural extension of the work is to get field decay in all possible scenario associated with the neutron star observations to date. The detail calculation of decay rate at different stages of neutron star during its thermal evolution considering different phases of matter inside the core of the star requires extensive study of the subject keeping in mind the different external conditions. It includes the cases, superfluidity of neutrons, superconductivity of protons and in some cases of magnetars with the quenching of superconductivity. All these conditions together will determine the thermal and field evolution and hence the persistent luminosity of some class of neutron stars. Finally we should emphasize that the obtained numerical values of different physical quantities are not final because we have not considered real system as a whole, though the procedure is general and to get the real picture we need extensive computational work considering the whole system together with proper input of temporal evolution.
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