INSTANTON EFFECTS IN SUPERGRAVITY THEORIES*

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Abstract

Non-perturbative effects of instanton-like solutions are studied within the framework of supergravity theories with field-dependent gauge functions. Fermionic zero modes are constructed and some typical correlation functions are evaluated. The effects of instantons are very similar to those in globally supersymmetric theories: they preserve supersymmetry while breaking a chiral $U(1)$ symmetry. Non-perturbative amplitudes receive corrections which are suppressed at large distances.

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Instanton solutions of Euclidean field equations exist in globally supersymmetric gauge theories \cite{1,2} as well as in some more complicated theories which appear as low-energy limits of superstring compactifications \cite{3,4}. Non-perturbative effects of supersymmetric instantons have been studied mainly within the framework of *global* supersymmetry \cite{1,2}. It has been also asserted that some similar non-perturbative effects are important in supergravity theories, particularly in the context of the gaugino condensation mechanism of supersymmetry breaking \cite{5}. More recently, gaugino condensation has attracted considerable attention in phenomenological applications of superstring theory \cite{3}. The reason is that the two basic ingredients of this supersymmetry breaking mechanism have a very natural basis in superstring theory: 1) gauge groups with “hidden” gauginos are typical for heterotic superstring compactifications, 2) gauge couplings are functions of neutral scalar fields *e.g.* of the dilaton and of the moduli.

In this letter, we study non-perturbative effects of instantons in supergravity theories with field-dependent gauge functions. Our main motivation has been to elaborate on the relation between instantons and gaugino condensation in superstring-type supergravity theories. This is the reason why we focus our attention on a specific supergravity Lagrangian, with the gauge coupling constant determined by the vacuum expectation value (VEV) of the dilaton field. We should mention already at this point that our strategy can be readily applied to some more complicated cases, as discussed at the end of this letter.

Our starting point is the supergravity Lagrangian describing a general four-dimensional heterotic superstring compactification at the string tree level. In the standard supergravity formalism \cite{4}, it is specified by: 1) the gauge function $f = kS$, where $S$ is the chiral dilaton superfield and $k$ is the Kač-Moody level of the algebra that generates the gauge group, 2) the Kähler potential $K = - \ln(S + \bar{S}) + G$, and 3) the superpotential $W$. The functions $G$ and $W$ are model-dependent. However, their forms are not important for the following considerations, except for the fact that both these functions do not depend on the dilaton. Since we are interested in classical solutions with $W = 0$, hereafter we neglect the superpotential.
In order to analyze classical field equations, it is convenient to represent the dilaton and its supersymmetric partners by a linear multiplet. The linear dilaton multiplet $L$ contains the scalar dilaton $l$, the dilatino $\chi$, and the vector $h^m_0$ which is dual to the field-strength of the antisymmetric tensor field $b_{pq}$: $h^m_0 = \frac{1}{2} \epsilon^{mnpq} \partial_n b_{pq}$. In the linear formulation, the full Lagrangian, including gauge kinetic terms, originates from the d-density of the function

$$L = \left( \tilde{L}/2 \right)^{-1/2} e^{-\alpha'_G/2},$$

with $\tilde{L} = L - 2\alpha'k\Omega$, where $\Omega$ is the Chern-Simons superfield and $\alpha'$ is a constant of length dimension 2. In the context of string theory, $\alpha'$ corresponds to the string scale. The component form of this Lagrangian has been written explicitly in [9]. In particular, one can see that the Kalb-Ramond field is always contained in the vector

$$h^m = h^m_0 - \alpha'k\omega^m,$$

where $\omega^m$ is the gauge topological current ($\partial_n \omega^m = F^{(a)}_{mn} \tilde{F}^{(a)}_{mn}$, where $a$ denote indices of the adjoint representation).

The Lagrangian can be continued to Euclidean space by following the usual prescriptions [1]. In particular, the vector field $h^m$ is replaced by $ih^m$ due to its dependence on the totally antisymmetric $\epsilon$ tensor. The bosonic part of the Lagrangian becomes

$$\mathcal{L}_B = \frac{1}{2\alpha'} \mathcal{R} + \frac{1}{4\alpha'^2} \partial_m l \partial^m l + \frac{1}{4\alpha'^2} h^m h^m + \frac{k}{4l} F^{(a)}_{mn} F^{(a)}_{mn} + G_{ij} D_m z^i D^m \bar{z}^j,$$

where $z^i$ are scalar components of chiral superfields and $D_m$ are gauge covariant derivatives.

We will consider nonperturbative effects due to gauge fields which do not couple to these scalars – a pure gauge “hidden” sector in superstring terminology. We also neglect auxiliary components of vector supermultiplets which vanish for supersymmetric backgrounds.

We are interested in the solutions of classical field equations which generalize the usual self-dual (or anti-self-dual) gauge configurations. Due to the dilaton-dependence of gauge kinetic terms, such solutions must necessarily involve both gauge fields and a space-time dependent dilaton. The configurations

$$F^{(a)}_{mn} = \pm \tilde{F}^{(a)}_{mn} \quad (3a)$$

$$h_m = \pm \partial_m l \quad (3b)$$
do indeed satisfy all field equations in a flat vierbein background $e^m_\mu = \delta^m_\mu$, with constant (possibly zero) VEVs of scalars $z^i$. Specific solutions of eqs.(3) have already appeared in the context of superstring solitons [3, 4, 10]. In particular, the one-instanton configuration of size $\rho$, centered at position $x_0$,

$$A^{(a)}_m = 2 \frac{\eta_{amn}(x-x_0)^n}{(x-x_0)^2 + \rho^2},$$

where $\eta_{amn}$ is the 't Hooft symbol [11], is accompanied by the dilaton configuration [10]

$$l(x) = l_0 + 4\alpha' k \frac{(x-x_0)^2 + 2\rho^2}{((x-x_0)^2 + \rho^2)^2}.$$

The Lagrangian $\mathcal{L}_B$ becomes a total divergence when evaluated on a self-dual (and anti-self-dual) solution of eqs.(3). The instanton action

$$S = \int d^4x \sqrt{g} \mathcal{L}_B = \frac{8\pi^2 k}{l_0}$$

depends on the asymptotic value $l_0$ of the dilaton only. This is exactly the same action as for a single instanton configuration in a Yang-Mills system without a dilaton, with the gauge coupling constant identified with $\sqrt{l_0/k}$. The instanton size $\rho$ and its position $x_0$ are the bosonic zero modes of this action, in addition to the usual rotational and gauge zero modes. The non-perturbative amplitudes will involve the usual suppression factor of $e^{-S}$.

In order to compute non-perturbative contributions to correlation functions, we need the fermionic zero modes in the instanton background. The fermionic field equations following from the Lagrangian of ref.[9] involve terms which are linear and trilinear in fermions. We will argue later on that the trilinear part vanishes for solutions of the linearized equations. The linearized field equations in the bosonic background under consideration are

$$\bar{\sigma}^n \partial_n \chi + \frac{3h_n - 4\partial_n l}{4l} \bar{\sigma}^n \chi + i\alpha' k F^{(a)}_{mn} \bar{\sigma}_{mn} \bar{\chi}^{(a)} - \frac{1}{2} (h_n + \partial_n l) \bar{\sigma}^m \sigma^n \psi_m = 0$$

$$\sigma^n D_n \bar{\chi}^{(a)} + \frac{h_n - 2\partial_n l}{4l} \sigma^n \bar{\chi}^{(a)} + i \frac{2l}{4l} F^{(a)}_{mn} \sigma^n \chi + \frac{1}{2} [F^{(a)}_{mn} + \tilde{F}^{(a)}_{mn}] \sigma^m \bar{\psi}^n = 0$$

$$\epsilon^{mnpq} \sigma_n \partial_p \bar{\psi}_q + \frac{h_n - \partial_n l}{4l^2} \sigma_n \bar{\sigma}_m \chi + \frac{\alpha' k}{2l} [F^{(a)}_{mn} - \tilde{F}^{(a)}_{mn}] \sigma_n \bar{\lambda}^{(a)} = 0$$
where $\chi$ is the dilatino, $\lambda^{(a)}$ are the gauginos, and $\psi_n$ are the gravitinos. In the equations above, we neglected other fermions which can be set zero for constant (possibly zero) VEVs of their scalar superpartners. In the equations conjugate to (7-9), $h_n$ is replaced by $-h_n$ and $\tilde{F}_{mn}^{(a)}$ by $-\tilde{F}_{mn}^{(a)}$.

The simplest way to obtain solutions of eqs.(7-9) is to perform supersymmetry and superconformal transformations on the bosonic background. The transformation rules are

$$\delta \chi = -\frac{i}{2} (h_n + \partial_n l) \sigma^n \bar{\epsilon} - 2i l \zeta, \quad \text{(10)}$$

$$\delta \bar{\lambda}^{(a)} = \frac{1}{2} F_{mn}^{(a)} \bar{\sigma}^{mn} \bar{\epsilon}, \quad \text{(11)}$$

$$\delta \bar{\psi}_n = (\partial_n - \frac{h_n}{4l}) \bar{\epsilon} - \bar{\sigma}_n \zeta, \quad \text{(12)}$$

where $\epsilon$ and $\zeta$ are the supersymmetry and superconformal transformation parameters, respectively. The transformation rules for the conjugate fermions $\bar{\chi}$, $\lambda^{(a)}$ and $\bar{\psi}_n$ are obtained by formal complex conjugation of eqs.(10-12), with $h_n \rightarrow -h_n$.

We begin with a self-dual (one-instanton) configuration $F_{mn}^{(a)} = \tilde{F}_{mn}^{(a)}$, $h_m = \partial_m l$. We choose the supergravity gauge $\sigma^n \bar{\psi}_n = \bar{\sigma}^n \psi_n = 0$. A supersymmetry transformation with $\epsilon = 0$, $\bar{\epsilon} = l^{1/4} \bar{\theta}$, where $\bar{\theta}$ is a constant spinor, yields

$$\chi = -il^{1/4} \partial_n l \sigma^n \bar{\theta}, \quad \text{(13a)}$$

$$\bar{\lambda}^{(a)} = \frac{l}{2} l^{1/4} F_{mn}^{(a)} \bar{\sigma}^{mn} \bar{\theta}, \quad \text{(13b)}$$

while leaving zero all other fermions, including the gravitino. This fermion configuration does indeed satisfy eqs.(7-9). Another solution with a vanishing gravitino field can be generated by a supersymmetry transformation with $\epsilon = 0$, $\bar{\epsilon} = l^{1/4} x_n \bar{\sigma}^n \theta$, where $\theta$ is a constant spinor, followed by a superconformal transformation with $\zeta = l^{1/4} \theta$, $\bar{\zeta} = 0$. This solution is not normalizable though with respect to any reasonable norm, since $\chi(x) \rightarrow -2i l_0^{5/4} \theta$ at $x \rightarrow \infty$. A normalizable solution can be obtained by subtracting $\chi_0(x) = -2i l_0 l^{1/4} \theta$, a trivial solution of eqs.(7-9) with all other fermions set zero, which can be thought of as the
fermionic superpartner of the constant dilaton mode \( l_0 \). The final form of the corresponding normalizable solution is

\[
\chi = -i l^{1/4} x_n \partial_m l \sigma^m \bar{\sigma}^n \theta - 2i l^{1/4} (l - l_0) \theta
\]

(14a)

\[
\bar{\chi}^{(a)} = -l^{1/4} x^m F_{mn} \bar{\sigma}^n \theta
\]

(14b)

with all other fermions equal to zero.

The fermionic field configurations of eqs.(13) and (14) satisfy not only the linearized eqs.(7-9), but also the full equations which involve additional trilinear terms. The reason is that the Lagrangian is invariant under global \( U(1) \) chiral symmetry transformations, with charge +1 fermions \( \chi, \bar{\chi}^{(a)} \) and \( \bar{\psi}_n \), and charge \(-1\) conjugate fermions. The trilinear terms in fermionic field equations have charges +1 or \(-1\), hence they always involve some fermions of opposite charges whereas in solutions (13) and (14) all non-vanishing fermions have the same charges. Eqs.(13) and (14) provide therefore 4 fermionic zero modes of the action in a self-dual instanton background. The zero modes in an anti-instanton background are obtained by formal complex conjugation of the instanton modes. If the gauge group is larger than \( SU(2) \), additional zero modes can be constructed by following the usual procedures of instanton calculations. Without losing generality, we restrict our attention to the minimal case of \( SU(2) \), with 8 real bosonic zero modes and 4 fermionic zero modes given by eqs.(13) and (14).

The space-time-dependent part of the dilaton field, the antisymmetric tensor field and the dilatino component of the fermionic zero modes are all of order \( O(\alpha') \). The integration measure over the zero modes may also contain corrections of order \( O(\alpha'/\rho^2) \) to the standard instanton measure with the coupling constant \( g = \sqrt{l_0/k} \). These corrections are not important though for the following computations of non-perturbative amplitudes, where we will limit ourselves to extracting the leading order behaviour in \( \alpha' \). The supersymmetric \( SU(2) \) instanton zero mode measure is

\[
d\mu e^{-S} = \kappa \Lambda^6 g^{-4} d\rho^2 d^4 x_0 l_0^{-1} d^2 \theta d^2 \bar{\theta}
\]

(15)
where $\kappa$ is a known constant $[1]$, and the classical factor $e^{-S}$ has been incorporated into the measure. In eq.(13), $g$ has been identified with the gauge coupling constant at the energy scale $(\alpha')^{-1/2}$, and $\Lambda = (\alpha')^{-1/2} \exp(-4\pi^2/3g^2)$ is the strong interaction scale.

In order for a correlation function to receive a non-vanishing instanton contribution, it must contain at least 4 fermions which can saturate the fermionic zero modes of eqs.(13) and (14). As an example, we compute the following two-point correlation functions:

\[ \langle \bar{\lambda}^2(x)\bar{\lambda}^2(y) \rangle \equiv \langle \bar{\lambda}^{(a)}(x)\bar{\lambda}^{(a)}(x) \bar{\lambda}^{(a)}(y)\bar{\lambda}^{(a)}(y) \rangle, \quad (16) \]

\[ \langle \bar{\lambda}^2(x)\chi^2(y) \rangle \equiv \langle \bar{\lambda}^{(a)}(x)\bar{\lambda}^{(a)}(x) \chi(y)\chi(y) \rangle. \quad (17) \]

Only the zero mode components contribute to these amplitudes, giving

\[ \langle \bar{\lambda}^2(x)\bar{\lambda}^2(y) \rangle = \int d\mu e^{-S} l^{1/2}(x) F^2(x) (x-y)^2 l^{1/2}(y) F^2(y) \frac{\theta^2\bar{\theta}^2}{16}, \quad (18) \]

\[ \langle \bar{\lambda}^2(x)\chi^2(y) \rangle = \int d\mu e^{-S} l^{1/2}(x) F^2(x) (x-y)^{-2} l^{1/2}(y) \left( \frac{\partial}{\partial y} [l(y) - l_0] (x-y)^2 \right)^2 \frac{\theta^2\bar{\theta}^2}{4}. \quad (19) \]

As a result of the zero mode integration we obtain

\[ \langle \bar{\lambda}^2(x)\bar{\lambda}^2(y) \rangle = \frac{(64\pi)^2}{20} \kappa \Lambda^6 g^{-4} \{ 1 + O[\frac{\alpha'}{(x-y)^2}] \}, \quad (20) \]

\[ \langle \bar{\lambda}^2(x)\chi^2(y) \rangle = \frac{191(64\pi)^2}{75} \kappa \Lambda^6 g^{-4} \alpha' k^2 \frac{\alpha'}{(x-y)^2} \{ 1 + O[\frac{\alpha'}{(x-y)^2}] \}. \quad (21) \]

All zero mode integrations are convergent, as in the globally supersymmetric case $[1]$.

Similar computations can be performed for other correlation functions. The common property of all these instanton-induced amplitudes is their non-vanishing $U(1)$ charge equal +4 (or −4 for anti-instantons); the $U(1)$ symmetry is broken down to its discrete $\mathbb{Z}_4$ subgroup. For gauge groups larger than $SU(2)$ the unbroken subgroup is $\mathbb{Z}_n$, where $n$ is the number of fermionic zero modes (e.g. $n = 2M$ for $SU(M)$, $n = 60$ for $E_8$ etc.). Instantons do not contribute non-zero VEVs for the $U(1)$-invariant four-fermion operators present in the Lagrangian, nor do they induce a scalar potential. Local supersymmetry is preserved by instanton solutions.
The superstring-type supergravity under consideration becomes a standard globally supersymmetric theory in the limit $\alpha' \to 0$. The dilaton $l(x)$ is frozen then at its VEV $l_0$. Out of all instanton-induced amplitudes, only the pure gaugino correlation function of eq.(20) remains non-zero in this limit. The result agrees with ref.[1]. As expected, the supergravity corrections are negligible at distances $(x - y)^2 \gg \alpha'$.

We are in a position now to discuss the relation between instanton-induced amplitudes and a gaugino condensate $\langle \bar{\lambda}^2(x) \rangle$. In the limit $(x - y)^2 \to \infty$ the gaugino amplitude (20) goes to a constant, as expected on the basis of unbroken supersymmetry [1]. One can argue now in the spirit of ref.[1] that the amplitude factorizes in this limit into $\langle \bar{\lambda}^2(x) \rangle \langle \bar{\lambda}^2(y) \rangle$. Self-consistency of the theory, more precisely factorization and clustering, imply then the existence of yet another non-perturbative effect that gives rise to gaugino condensation – instantons have too many fermionic zero modes to produce a two-gaugino condensate. Note that this effect should not produce a dilatino condensate, as seen from the large $(x - y)^2$ behaviour of the amplitude (21). In pure supersymmetric Yang-Mills theory, supersymmetry is protected by Witten’s index theorem [12]. This may be different though in the context of supergravity. The full supergravity transformation of the dilatino contains a term of the form given in eq.(10), with $\zeta(x) = \alpha' k l^{-1} \bar{\lambda}^2(x) \varepsilon(x)/8$, where $\varepsilon(x)$ is the local supersymmetry transformation parameter. Gaugino condensation $\langle \bar{\lambda}^2(x) \rangle \neq 0$ gives rise to a constant term of order $O(\alpha')$ in the supergravity transformation for the dilatino. One can argue that local supersymmetry is dynamically broken, with the dilatino identified as the goldstino [5].

Although our analysis has been restricted so far to a supergravity theory with the gauge kinetic terms depending on the dilaton in a way dictated by superstring theory in the tree approximation, our strategy can be readily extended to more complicated cases. It can be shown that self-dual solutions satisfying conditions similar to eqs.(3) exist for a supergravity theory defined by the d-term of $f(\hat{L}) e^{-\alpha'G/2}$, where $f$ is an arbitrary real function. They also exist in the presence of the so-called Green-Schwarz term [13] which appears at the one-loop level as a result of integrating out the heavy string states [14]. In all these cases the zero modes can be constructed in a very similar way, and the calculation of non-perturbative
amplitudes is straightforward.

The results of this work are not surprising. The fact that supersymmetry is gauged in supergravity theory turns out to be without much importance for instantons. They preserve supersymmetry while breaking a chiral $U(1)$ symmetry. Non-perturbative amplitudes receive corrections and some new correlation functions appear. However, all of them are suppressed at large distances. Hence the presence of instantons does not trigger dynamical supersymmetry breaking in supergravity theories, although such a breaking may occur as a result of other non-perturbative effects.

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