Apostolov, Vestislav; Huang, Hongnian

A splitting theorem for extremal Kähler metrics. (English) J. Geom. Anal. 25, No. 1, 149-170 (2015).

The main result of this paper is the following splitting theorem for extremal Kähler metrics:

Let $X$ be the product of polarized complex projective manifolds, $(X_i, L_i)$, $i = 1, \ldots, r$, then any extremal Kähler metric on $X$, in the Kähler class $2\pi c_1(\bigotimes_{i=1}^r L_i)$, is the Riemannian product of extremal Kähler metrics on the factors $X_i$, in the Kähler classes $2\pi c_1(L_i)$, under at least one of the following assumptions:

1) $(X, L)$ is asymptotically Chow semi-stable; 2) for at most one $i$ the group $\text{Aut}_0(X_i, L_i)$ has a center of positive dimension.

The proof makes use of a result of G. Tian about approximation of any Kähler metric, in the class $2\pi c_1(L)$, with Fubini-Study metrics induced from the projective inclusion of $(X, L)$ and, moreover, generalizations of Donaldson and Mabuchi. This result generalizes a theorem of S.-T. Yau [Commun. Anal. Geom. 1, No. 3, 473–486 (1993; Zbl 0842.53035)] regarding the splitting of a Kähler-Einstein metric on the product of compact complex manifolds.

Reviewer: Antonella Nannicini (Firenze)

MSC:

53C55 Global differential geometry of Hermitian and Kählerian manifolds

Keywords:

Kähler geometry; extremal metrics; Chow stability

Full Text: DOI arXiv Link

References:

[1] Apostolov, V.; Calderbank, D.M.J.; Gauduchon, P.; Tønnesen-Friedman, C., Hamiltonian 2-forms in Kähler geometry. III. extremal metrics and stability, Invent. Math., 173, 547-601, (2008) · Zbl 1145.53055 · doi:10.1007/s00222-008-0126-x

[2] Bando, S.; Mabuchi, T., Uniqueness of Einstein Kähler metrics modulo connected group actions, Sendai, 1985, Amsterdam · Zbl 0641.53065

[3] Bourguignon, J.-P.; Li, P.; Yau, S.-T., Upper bound for the first eigenvalue of algebraic submanifolds, Comment. Math. Helv., 69, 199-207, (1994) · Zbl 0814.53040 · doi:10.1007/BF02564482

[4] Calabi, E.: Extremal Kähler metrics, seminar on differential geometry, pp. 259-290. Princeton Univ. Press, Princeton (1982)

[5] Calabi, E.; Chavel, I. (ed.); Farkas, H.M. (ed.), Extremal Kähler metrics, II, (1985), Berlin

[6] Catlin, D., The Bergman kernel and a theorem of Tian, Katata, 1997, Boston · Zbl 1182.32009 · doi:10.1007/s10240-008-0013-4

[7] Chen, X.X.: Space of Kähler metrics (IV)-on the lower bound of the $K$-energy. arXiv:0809.4081 · Zbl 1086.53101

[8] Chen, X.X.; He, W.Y., On the Calabi flow, Am. J. Math., 130, 539-570, (2008) · Zbl 1204.53050 · doi:10.1353/ajm.2008.0018

[9] Chen, X.X.; Tian, G., Geometry of Kähler metrics and holomorphic foliation by discs, Publ. Math. IHÉS, 107, 1-107, (2008) · Zbl 1182.32009 · doi:10.1007/s10240-008-0013-4

[10] Donaldson, S.K.: Eliashberg, Y. (ed.); etal., Symmetric spaces, Kähler geometry and Hamiltonian dynamics, 13-33, (1999), Providence · Zbl 0972.53025

[11] Donaldson, S.K., Scalar curvature and projective embeddings, I, J. Differ. Geom., 59, 479-522, (2001) · Zbl 1052.32017

[12] Donaldson, S.K., Scalar curvature and stability of toric varieties, J. Differ. Geom., 62, 289-349, (2002) · Zbl 1074.53059

[13] Donaldson, S.K., Conjectures in Kähler geometry, No. 3, 71-78, (2004), Providence · Zbl 1161.32010

[14] Donaldson, S.K., Scalar curvature and projective embeddings. II, Q. J. Math., 56, 345-356, (2005) · Zbl 1159.32012 · doi:10.1093/qmath/hah044

[15] Donaldson, S.K., Lower bounds on the Calabi functional, J. Differ. Geom., 70, 453-472, (2005) · Zbl 1149.53042

[16] Fujiki, A., On automorphism groups of compact Kähler manifolds, Invent. Math., 44, 225-258, (1978) · Zbl 0367.32004 · doi:10.1007/BF01403162
[17] Futaki, A., Asymptotic Chow semi-stability and integral invariants, Int. J. Math., 15, 967-979, (2004) · Zbl 1074.53060 · doi:10.1142/S0129167X04002612

[18] Helgason, S.: Differential Geometry, Lie Groups and Symmetric Spaces. Graduate Studies in Mathematics, vol. 34. American Mathematical Society, Providence (2001) · Zbl 0993.53002

[19] Huang, H.; Zheng, K., The stability of the Calabi flow near an extremal metric, Ann. Sc. Norm. Super. Pisa, Cl. Sci. (5), 11, 167-175, (2012) · Zbl 1246.53088

[20] Gauduchon, P.: Fibrés hermitiens à endomorphisme de Ricci non-négatif. Bull. Soc. Math. Fr. 113-140 (1977) · Zbl 0382.53045

[21] Gauduchon, P.: Calabi’s extremal metrics: an elementary introduction. In preparation · Zbl 0432.53009

[22] Kobayashi, S.: Transformation Groups in Differential Geometry. Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 70. Springer, New York (1972) · Zbl 0246.53031 · doi:10.1007/978-3-642-61981-6

[23] Kobayashi, S., First Chern class and holomorphic tensor fields, Nagoya Math. J., 77, 5-11, (1980) · Zbl 0432.53049

[24] Kobayashi, S.: Differential Geometry of Complex Vector Bundles. Publications of the Mathematical Society of Japan, vol. 15. Princeton University Press/Iwanami Shoten, Princeton/Tokyo (1987). Memorial Lectures 5 · Zbl 0678.53002 · doi:10.1515/9781400856882

[25] LeBrun, C.R.; Simanca, S., Extremal Kähler metrics and complex deformation theory, Geom. Funct. Anal., 4, 298-336, (1994) · Zbl 0801.53050 · doi:10.1007/BF01896244

[26] Luo, H., Geometric criterion for geiseler-Mumford stability of polarized manifolds, J. Differ. Geom., 49, 577-599, (1998) · Zbl 1006.32022

[27] Mabuchi, T., Stability of extremal Kähler manifolds, Osaka J. Math., 41, 563-582, (2004) · Zbl 1076.32017

[28] Mabuchi, T., An energy-theoretic approach to the Hitchin-Kobayashi correspondence for manifolds. I, Invent. Math., 159, 225-243, (2005) · Zbl 1118.53047 · doi:10.1007/s00222-004-0387-y

[29] Mabuchi, T., An energy-theoretic approach to the Hitchin-Kobayashi correspondence for manifolds. II, Osaka J. Math., 46, 115-139, (2009) · Zbl 1209.53032

[30] Mabuchi, T., Uniqueness of extremal Kähler metrics for an integral Kähler class, Int. J. Math., 15, 531-546, (2004) · Zbl 1058.53021 · doi:10.1142/S0129167X04002429

[31] Mabuchi, T.: SKS-stability of constant scalar curvature polarization. Preprint (2008), arXiv:0812.4903 · Zbl 1077.53068

[32] Mabuchi, T.: A stronger concept of SKS-stability. Preprint (2009), arXiv:0910.4617 · Zbl 0801.53050

[33] Mabuchi, T., Asymptotics of polybalanced metrics under relative stability constraints, Osaka J. Math., 48, 845-856, (2011) · Zbl 1229.14037

[34] Ono, H.; Sato, Y.; Yotsutani, N., An example of asymptotic Chow unstable manifold with constant scalar curvature, Ann. Inst. Fourier, 62, 1245-1287, (2012) · Zbl 1255.53057 · doi:10.5802/afst.1272

[35] Phong, D.; Sturm, J., Scalar curvature, moment maps, and the Deligne pairing, Am. J. Math., 126, 693-712, (2004) · Zbl 1077.53068 · doi:10.1007/s00222-004-0387-y

[36] Stoppa, J., $K$-stability of constant scalar curvature Kähler manifolds, Adv. Math., 221, 1397-1408, (2009) · Zbl 1181.53060 · doi:10.1016/j.aim.2009.02.013

[37] Stoppa, J.; Székelyhidi, G., Relative $SKS$-stability of extremal metrics, J. Eur. Math. Soc., 13, 899-909, (2011) · Zbl 1230.53069 · doi:10.4171/JEMS/270

[38] Székelyhidi, G.: Extremal metrics and SKS-stability. Ph.D. thesis, arXiv:math/0610002 · Zbl 1204.53050

[39] Székelyhidi, G., Extremal metrics and SKS-stability, Bull. Lond. Math. Soc., 39, 1-17, (2007) · Zbl 1111.53057

[40] Tian, G., On a set of polarized Kähler metrics on algebraic manifolds, J. Differ. Geom., 32, 99-130, (1990) · Zbl 0706.53036

[41] Tian, G., Kähler-Einstein metrics with positive scalar curvature, Invent. Math., 130, 1-37, (1997) · Zbl 0892.53017 · doi:10.1007/s002220050017

[42] Tosatti, V.; Weinkove, B., The Calabi flow with small initial energy, Math. Res. Lett., 14, 1033-1039, (2007) · Zbl 1230.53069 · doi:10.4310/MRL.2007.v14.n6.a11

[43] Wang, X., Moment maps, Futaki invariant and stability of projective manifolds, Commun. Anal. Geom., 12, 1037-1067, (2004) · Zbl 1086.53101 · doi:10.4310/CAG.2004.v12.n5.a2

[44] Yau, S.-T., Nonlinear analysis in geometry, Geneva, 1986

[45] Yau, S.-T., Open problems in geometry, Los Angeles, CA, 1990, Providence

[46] Yau, S.-T., A splitting theorem and an algebraic geometric characterization of locally Hermitian symmetric spaces, Commun. Anal. Geom., 1, 473-486, (1993) · Zbl 0924.11055

[47] Zelditch, S., Szegő kernel and a theorem of Tian, Int. Math. Res. Not., 6, 317-331, (1998) · Zbl 0922.58082 · doi:10.1155/S107379289800021X

[48] Zhang, S., Heights and reductions of semi-stable varieties, Compos. Math., 104, 77-105, (1996) · Zbl 0924.11055

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.