Research on Region Division of Large Workpiece Based on Dexterous Workspace of Robot

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Abstract. In order to complete the processing of large-scale workpieces, a region division of large-scale workpieces based on robot dexterous workspace is studied. The linkage coordinate system of the robot is established by D-H method, and the forward kinematics equation of the robot is obtained; Monte Carlo method is used to analyze the workspace of the robot, and MATLAB is used to program to draw the workspace and task space of the robot; Taking an example part as the object, the feature of the part is studied, and the process of determining the task space area of the workpiece in the dexterous workspace of the robot is given, and the region of the workpiece is divided based on the size of the task space and the geometric features of the workpiece.

Keywords: Robot; Working space; Area division; MATLAB.

1. Introduction
At present, the main processing methods for large parts are segmented processing method and integral processing method. Segmented processing method is to process each small section of workpiece, and finally splice it by welding, riveting or threaded connection. The integral machining method uses large-scale numerical control machine tools or mobile robots to process the whole[1]. This paper mainly studies the overall processing of large workpiece by robot. Because the robot's workspace is limited, it can't process large-scale workpiece completely in one area. Therefore, it is necessary to rotate the workpiece to realize the overall processing of large workpiece. Every time the workpiece is rotated once, its processing area will change, so it is necessary to divide the large workpiece area to ensure the integrity and feasibility of processing[2]. In this paper, the range of dexterous workspace of robot is studied[3-4], According to the robot's dexterous work space, the maximum segmentation area of the large workpiece can be determined. According to the geometric features of the part and the maximum segmentation region, the region division methods of regular geometric features and multiple small faces are studied respectively[5-6]. The research results of this paper can provide reference for the regional division of large-scale workpieces of the same type.

2. Establishment of Robot Kinematics Model
This paper takes MOTOMAN UP50 robot as the research object, which has six links and six joints and is a six degree of freedom robot. Firstly, the robot linkage coordinate system is constructed by DH parameters, as shown in figure 1. Analyze the established connecting rod coordinate system, and obtain the DH parameters according to the meaning of the DH connecting rod parameters, as shown in Table 1.
### Table 1. Robot D-H parameters and joint variables.

| Joint $i$ | Connecting rod length $a_{i-1}$ | Torsion angle $\alpha_{i-1}$ | Link offset $d_i$ | Joint angle $\theta_i$ | Joint variables $\theta_i$ |
|-----------|---------------------------------|-----------------------------|-----------------|----------------------|-------------------------|
| 1         | 0                               | 0                           | $d_1$           | $\pi i$              | $\theta_1$             |
| 2         | $a_1$                           | $-\pi/2$                    | 0               | $-\pi/2$             | $\theta_2$             |
| 3         | $a_2$                           | 0                           | 0               | 0                    | $\theta_3$             |
| 4         | $a_3$                           | $-\pi/2$                    | $d_4$           | 0                    | $\theta_4$             |
| 5         | 0                               | $\pi/2$                     | 0               | 0                    | $\theta_5$             |
| 6         | 0                               | $-\pi/2$                    | $d_6$           | 0                    | $\theta_6$             |

Among them, $a_1=145\text{mm}$, $a_2=870\text{mm}$, $a_3=110\text{mm}$; $d_1=540\text{mm}$, $d_4=1025\text{mm}$, $d_6=175\text{mm}$.

![DH coordinate system of robot.](image)

**Figure 1.** DH coordinate system of robot.

Through $a_{i-1}$, $\alpha_{i-1}$, $d_i$ and $\theta_i$ get the general formula of connecting rod transformation $^i\mathbf{T}_1$:

$$^i\mathbf{T}_1 = \text{Rot}(x, a_{i-1})\text{Trans}(x, a_{i-1})\text{Rot}(z, \theta_i)\text{Trans}(z, d_i)$$

$$

to
n

$$^i\mathbf{T}_1 =
\begin{bmatrix}
  c\theta_i & -s\theta_i & 0 & a_{i-1} \\
  s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\
  s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\
  0 & 0 & 0 & 1
\end{bmatrix}
$$

(1)

Multiply the homogeneous transformation of each link to obtain the transformation matrix of the end coordinate system relative to the base coordinate system $^6\mathbf{T}_0$:

$$^6\mathbf{T}_0 = ^0\mathbf{T} \cdot ^1\mathbf{T} \cdot ^2\mathbf{T} \cdots ^5\mathbf{T} =
\begin{bmatrix}
  n_x & a_x & a_x & p_x \\
  n_y & a_y & a_y & p_y \\
  n_z & a_z & a_z & p_z \\
  0 & 0 & 0 & 1
\end{bmatrix}
$$

(2)

3. **Workspace Analysis of Robot**

Robot workspace refers to the set of space points that can be reached by reference points that can be described by robot end effector. The description is the reachable working space of the robot, and the reachable space is defined as:

$$W_R = \{p(\theta) : \theta \in Q\} \subset \mathbb{R}^3$$

(3)

Because the robot needs to process the workpiece, it needs to consider the robot's position and pose, that is, to find the dexterous workspace of the robot:
Smart workspace is a subset of reachable workspace. The rear three joints of the MOTOMAN UP50 robot intersect at one point, the latter three joints determine the posture of the robot's wrist, and the first three joints determine the position of the robot's wrist. Therefore, only the space range that can be reached by the first three joints is required.

Monte Carlo method takes probability and statistics as numerical calculation method, and solves complex calculation problems by generating multiple random numbers. The steps to get the workspace of the robot by Monte Carlo method are as follows:

- According to the kinematic model established in section 1, the position vector of the end effector is obtained.
- Within the scope of each joint, N random numbers are sequentially produced, and these N random numbers can obtain N sets of variable value combinations.
- Substituting each group of joint variables into $^5T_\theta$ in turn, obtain N groups of $P = [P_x, P_y, P_z]^T$, this value is the position vector of the end effector.
- The obtained points are drawn in MATLAB, that is, the workspace of the robot is obtained. The resulting workspace is shown in the figure below:

According to the above figure, the spatial coordinate range of x-axis is: $-2.1884 \sim 2.197$; The space coordinate range of Y axis is: $-2.1994 \sim 2.187$; The space coordinate range of Z axis is: $-0.9092 \sim 3.2327$.

**4. Region Segmentation Method for Large Workpiece**

**4.1. Smart Working Space and Task Space**

This paper takes an example part as the research object, as shown in figure 6:
Study the area division method of the part. According to the angle range of each joint of the robot:

\[
\theta_1 = [-180°, 180°], \quad \theta_2 = [-90°, 135°], \quad \theta_3 = [-160°, 280°], \quad \theta_4 = [-360°, 360°], \quad \theta_5 = [-125°, 125°], \quad \theta_6 = [-360°, 360°].
\]

And the robot connecting rod parameters can get the schematic diagram of the cross-sectional profile of the X-Z plane and X-Y plane of the robot, as shown in figure 7 and figure 8:

**Figure 6.** Drawing of parts to be processed.

The dotted line in figure 8 represents the cross-section of the lower surface of the workpiece, and the ‘+’ line represents the cross-sectional profile of the X-Y plane. It can be seen from figure 7 that when the workpiece coordinate system is at the position of the robot base coordinate system \((x_0, y_0, z_0) = (1.7, 0, -1.2)\), Only part of the task space of the workpiece is in the robot's dexterous workspace, Therefore, the workpiece needs to be rotated so that the workpiece can be processed as a whole.

**4.2. Expression of Workpiece Coordinate System in Robot Base Coordinate System**

In order to determine the size of the area divided space and the number of rotations of the workpiece, It is necessary to express the size parameters of the workpiece in the workpiece coordinate system in the robot base coordinate system.

**Figure 7.** X-Z surface outline of dexterous workspace and task space.

The dotted line in figure 8 represents the cross-section of the lower surface of the workpiece, and the ‘+’ line represents the cross-sectional profile of the X-Y plane. It can be seen from figure 7 that when the workpiece coordinate system is at the position of the robot base coordinate system \((x_G, y_G, z_G) = (1.7, 0, -1.2)\), Only part of the task space of the workpiece is in the robot's dexterous workspace, Therefore, the workpiece needs to be rotated so that the workpiece can be processed as a whole.

**Figure 8.** X-Y surface outline of dexterous workspace and task space.

**Figure 9.** Definition of coordinate system.

The description of the tool coordinate system \(\{T\}\) relative to the robot base coordinate system \(\{B\}\) can be expressed by the product of the transformation matrix, as follows:
The homogeneous transformation matrix of the robot wrist coordinate system relative to the robot base coordinate system has been obtained in the robot forward kinematics, namely:

\[
^B_T = ^{W^B_T} T
\]

(5)

The coordinate value \(p(x,y,z)\) of the boundary point of the area under the workpiece coordinate system, through the matrix \(^B_T\), the coordinates of point \(p\) in the workpiece coordinate system can be transformed into the coordinates of point \(p\) in the robot base coordinate system, as shown in equation 8:

\[
^B_p = ^{G^B_T} G^p = \begin{bmatrix} p_x2 \\ p_y2 \\ p_z2 \\ 1 \end{bmatrix}
\]

(8)

By comparing \(^Bp\) position \((p_{x2}, p_{y2}, p_{z2})\) and the boundary coordinates of the robot working area\((p_x, p_y, p_z)\). If \(^Bp\) is not in the robot workspace, rotate the workpiece so that the processing area is in the robot workspace.

4.3. Research on Workpiece Area Division

![Figure 10. x-z projection of the workpiece to be processed.](image)

![Figure 11. x-y projection of the workpiece to be processed.](image)

Robot processing is mainly to process the side surface of the part, as shown in figure 10, the upper and lower shadow parts are regular geometry, and the middle part is divided into several small face areas. When dividing the middle part of the area, it is necessary to consider the integrity of the small area in the dexterous workspace of the robot.

First, the region with a single feature is divided. Take the z-axis as the projection axis to obtain the projection surface of the x-y plane of the workpiece to be processed. According to the working range of the robot, the workpiece is divided into \(n\) identical working areas, as shown in figure 11. The coordinate \(^6p\) of point A in the workpiece coordinate system is \((-0.6,0.8,0)\), The coordinates \(^Bp\) in the robot base coordinate system are \((1.1,-0.8,-1.2)\).

\[
^B_p = ^{G^B_T} G^p = \begin{bmatrix} 1 & 0 & 0 & 1.7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.6 \\ 0.8 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0.8 \\ -1.2 \\ 1 \end{bmatrix}
\]

(9)

The coordinates of point A in the robot base coordinate system obtained by formula 9 are compared
with the coordinates of point A measured in the robot base coordinate system. Verify the accuracy of coordinate transformation.

In order to improve the processing efficiency of the robot, the number of divisions of the processing area should be as few as possible. Therefore, the area ABCD is the maximum value in the dexterous workspace of the robot.

\[ a_{\text{max}} = 1.2 \]  

\[ \theta = 2 \sin^{-1} \left(\frac{a}{d_s}\right) = 74^\circ \]  

\[ n = \frac{360}{\theta} \approx 5 \]  

For the shadow part, the workpiece needs to be rotated 5 times to complete the machining. When dividing the non-shaded area in figure 10, because the area is divided into multiple small areas by the beam. When dividing, it is not only necessary to consider the processing integrity of the entire large area, but also consider the integrity of each small area in the robot's dexterous work space. The straight line section AB in figure 7 is the maximum diameter \( d_0 = 2.1 \text{m} \) of the processed part. Project the workpiece to the y-z plane of the workpiece coordinate system. The center coordinates of the maximum working diameter of the robot in the workpiece coordinate system are \((y_0,z_0) = (0,1.2)\). The radius from the vertex of each small surface to the center of the circle can be determined by formula 13, by comparing \( r \) and \( r_0 \) to determine which small surface can be divided into a region.

\[ r = \sqrt{(y-y_0)^2 + (z-z_0)^2} \]  

The coordinates of vertex C, D, E and F in figure 10 are \((-0.35,1.98), (-0.51,1.05), (-0.41,0.46), (-0.76,0.46)\). Calculate the radius value of each vertex separately by formula 13: \( r_C = 0.86\text{m}, r_D = 0.53\text{m}, r_E = 0.85\text{m}, r_F = 1.06\text{m} \). Comparing with \( r_0 = 1.05\text{m} \), it can be seen that point F exceeds the smart working space of the robot. Therefore, only two small surfaces at the bottom can be divided into one area, and there are 12 small surfaces at the bottom. Therefore, it is necessary to rotate the workpiece 6 times, each rotating 60 degrees. The small surface in the middle is assigned 2 surfaces to each area, and the small surface in the upper layer is assigned 4 surfaces to each area. The dot-dashed area in figure 10 is the divided area, which realizes the complete processing of the entire part.

5. Conclusion
Establish the robot linkage coordinate system through the DH parameter method, and obtain the homogeneous transformation matrix of the robot end coordinate system relative to the base coordinate system. The Monte Carlo method is used to obtain the working space range of the robot. Determine the task space area of the workpiece in the dexterous workspace of the robot, and based on the size of the task space and the geometric characteristics of the workpiece, the workpiece is divided into regions, which can realize the complete processing of the workpiece.

Establish the maximum dexterous workspace when the workpiece is in a station, plan the small surface in the dexterous space, make the workpiece pass through the minimum number of transposition, realize the complete processing of the workpiece, and improve the processing efficiency of the robot.

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