CUSPIDAL EDGES WITH THE SAME FIRST FUNDAMENTAL FORMS ALONG A COMMON CLOSED SPACE CURVE

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Abstract. Along an embedded space curve $C$, local existence of four distinct cuspidal edges with the same first fundamental forms was shown in the authors’ previous work. Here, if $C$ is closed, we show the existence of infinitely many cuspidal edges with the same first fundamental forms.

1. Results

For the purpose of starting our results, let us first introduce some terminology. By ‘$C^r$-differentiable’ we mean $C^\infty$-differentiability if $r = \infty$ and real analyticity if $r = \omega$. We denote by $S^1 := \mathbb{R}/\mathbb{Z}$ the 1-dimensional torus of length $l > 0$ and by $\mathbb{R}^3$ the Euclidean 3-space. We fix a closed $C^r$-embedded curve $\gamma : S^1 \to \mathbb{R}^3$ with positive curvature function, and denote by $C$ the image of $\gamma$. Let $\mathcal{F}(C)$ be the set of $C^r$-cuspidal edge germs along $C$, that is

$$\mathcal{F}(C) := \left\{ f : S^1 \times (-\varepsilon, \varepsilon) \to \mathbb{R}^3 ; \ f(t, 0) \text{ is a cuspidal edge along } C \text{ for each } t \in S^1 \right\}.$$ 

We let $n(t)$ (resp. $b(t)$) be the unit principal normal (resp. bi-normal) vector of $\gamma(t)$. For each $f \in \mathcal{F}(C)$, there exists a unique parametrization $(t, v)$ of $f$ (called Fukui’s formula, cf. [2] and [3]) such that for sufficiently small $\varepsilon > 0$

$$(1.1) \quad f(t, v) := \gamma(t) + (v^2, v^3 \beta(t, v)) \begin{pmatrix} \cos \theta(t) & -\sin \theta(t) \\ \sin \theta(t) & \cos \theta(t) \end{pmatrix} \begin{pmatrix} n(t) \\ b(t) \end{pmatrix} \quad (t \in S^1, \ |v| < \varepsilon),$$

where

- $t$ is an arc-length parameter of $\gamma$,
- $\beta(t, v)$ is a $C^r$-function, and
- for each $t \in S^1$, the map $(-\varepsilon, \varepsilon) \ni v \mapsto (v^2, v^3 \beta(t, v)) \in \mathbb{R}^2$ is a cusp at $v = 0$ with the normalized half-arc-length parameter (see [3, Appendix A] and also [5]). In particular $\beta(t, 0) \neq 0$ for each $t \in S^1$.

We call this expression the normal form of $f$. Here, the angle $\theta(t)$ in (1.1) is called the cuspidal angle at $\gamma(t)$. We denote by $\kappa(t)$ the curvature function of $\gamma(t)$. In this situation, the singular curvature $\kappa_s(t)$ and the limiting normal curvature $\kappa_\nu(t)$ (cf. [4]) along the singular set of $f \in \mathcal{F}(C)$ are given by (cf. [3])

$$(1.2) \quad \kappa_s(t) = \kappa(t) \cos \theta(t), \quad \kappa_\nu(t) = \kappa(t) \sin \theta(t).$$

Here, we prepare a lemma:

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Lemma 1.1. Let \( f, g \in \mathcal{F}'(C) \) be two cuspidal edges along \( C \). Suppose that the image of \( f \) coincides with that of \( g \) and \( f(t, 0) = g(t, 0) \) holds for \( t \in S^1 \). Then \( f \) coincides with \( g \) as map germs along \( C \) in the normal form.

Proof. Since the two surfaces share the same image as map germs, their cuspidal angle functions and sectional cusps must coincide, so we get the assertion. \( \square \)

![Figure 1. Cuspidal edges along closed space curves \( \gamma_2 \) (left) and \( \gamma_3 \) (right)](image)

Imitating the definition of ‘isomer’ as a local map germ given in [3], we give the following:

Definition 1.2. A cuspidal edge \( g \in \mathcal{F}'(C) \) is called a \( C \)-isomer of \( f \in \mathcal{F}'(C) \) if

- there exist \( \varepsilon > 0 \) and a \( C^r \)-diffeomorphism \( \varphi \) on \( U_{\varepsilon}(S^1) := S^1 \times (-\varepsilon, \varepsilon) \) such that the pull-back of the first fundamental form of \( g \) by \( \varphi \) coincides with that of \( f \) on \( U_{\varepsilon}(S^1) \) and,
- the image of \( f \) does not coincide with that of \( g \).

For \( f \in \mathcal{F}'(C) \), the singular curvature function \( \kappa_s : S^1 \to \mathbb{R} \) is defined along its singular set. By (1.2), \( \kappa_s(t) \leq \kappa(t) \) holds, and \( \kappa_s(t) \) depends only on the first fundamental form of \( f \) (cf. (2.1)). We then consider the condition

\[
\max_{t \in S^1} |\kappa_s(t)| < \min_{t \in S^1} |\kappa(t)|, \tag{1.3}
\]

and define the subclass

\[
\mathcal{F}'_s(C) := \{ f \in \mathcal{F}'(C) ; f \text{ satisfies } (1.3) \}
\]

of \( \mathcal{F}'(C) \). For example, two cuspidal edges belonging to \( \mathcal{F}'_s(C) \) in Figure 1 are obtained from (1.1) by substituting \( \beta(t, v) = 1 \) and \( \cos \theta = 1/4 \) if \( m = 2 \) and \( \cos \theta = 1/2 \) if \( m = 3 \) along the curve

\[
\gamma_m(t) := \left( (2 + \cos mt) \cos t, (2 + \cos mt) \sin t, \sin mt \right) \quad (t \in \mathbb{R}/2\pi \mathbb{Z}) \tag{1.4}
\]

with a rotational symmetry of \( 2\pi/m \)-radians. As shown in the authors’ work [3], each \( f \in \mathcal{F}'_s(C) \) has three locally distinct \( C \)-isomers at each neighborhood of a point on \( C \). It should be remarked that cuspidal edges with constant Gaussian curvature satisfies \( \kappa_\nu = 0 \). In particular, such surfaces do not belong to \( \mathcal{F}'(C) \) (see [1] for such a case).

Definition 1.3. We say that \( C \) has a non-trivial symmetry if there exists an isometry \( T \) of the Euclidean space \( \mathbb{R}^3 \) such that \( T(C) = C \) and there is a point \( P \in C \) such that \( T(P) \neq P \). On the other hand, a \( C^r \)-function \( \mu : S^1(= \mathbb{R}/\mathbb{Z}) \to \mathbb{R} \) is said to have a symmetry if there exists a constant \( c \in [0, l] \) such that \( \mu(t) = \mu(t + c) \) \((c \neq 0)\) holds for \( t \in \mathbb{R} \) or \( \mu(t) = \mu(c - t) \) holds for \( t \in \mathbb{R} \).

If \( C \) has a non-trivial symmetry, then \( \kappa(t) \) also admits a symmetry. The main result is as follows:
Theorem 1.4. Let $C$ be the image of a closed $C^\omega$-curve embedded in $\mathbb{R}^3$ and $f$ a cuspidal edge belonging to $C$. Then there are four continuous 1-parameter families of real analytic cuspidal edges $\{f^i_t\} \in \mathcal{F}_\omega$ (i = 1, 2, 3, 4) satisfying the following properties:

(a) $f = f^1_t$ or $f = f^2_t$, and all of $f^i_t$ ($b \in S^1$) for $i = 1, 2, 3, 4$ have the same first fundamental form as $f$.

(b) If $g \in \mathcal{F}_\omega$ is a $C$-isomer of $f$, then $g$ coincides with a cuspidal edge belonging to one of these four families as a map germ.

(c) Suppose that the singular curvature function $\kappa_s$ of $f$ is non-constant and $C$ is not a circle. Then for each choice of $f^i_t$ ($i \in \{1, 2, 3, 4\}$, $c \in [0, l]$),

$$\Lambda^i_c := \{(j, a) \in \{1, 2, 3, 4\} \times S^1; f^i_j \text{ is congruent to } f^i_t\}$$

is a finite set. In particular, there are mutually non-congruent uncountably many $C$-isomers of $f$.

(d) Suppose that $C$ does not lie in a plane and has no symmetry. If the singular curvature function $\kappa_s$ is non-constant, then the set $\Lambda^i_c$ is a one-point-set for each $(i, b) \in \{1, 2, 3, 4\} \times S^1$.

To construct $f^i_t$ along the closed curve $C$, the real analyticity of $f$ is required, since we need to apply the Cauchy-Kovalevsky theorem inductively. It should be remarked that condition (1.3) is also needed to apply the Cauchy-Kovalevsky theorem.

Example 1.5. There exists a cuspidal edge $f$ satisfying condition (c) as follows: We can take a closed convex $C^\omega$-regular curve $C$, which has no non-trivial symmetry and is lying in the 2-plane $\mathbb{R}^2 := \{(x, y, 0); x, y \in \mathbb{R}\}$ in $\mathbb{R}^3$. Considering the approximation of $C$ by Fourier series, by Lemma [4.1] in the appendix, we may assume that $C$ is real analytic. Let $\pi : S^2 \setminus \{(0, 0, 1)\} \to \mathbb{R}^2$ be the stereographic projection. We let $\gamma(t)$ ($0 \leq t \leq l$) be the arc-length parametrization of $C$ and set

$$\tilde{\gamma}_s(t) := \frac{\pi^{-1}(s \gamma(t)) + (0, 0, 1)}{s},$$

which is a real analytic 1-parameter deformation of $\tilde{\gamma}_0 := \gamma$ for $s \in \mathbb{R}$. We denote by $C_s$ the image of $\tilde{\gamma}_s(t)$. Then $C_0 = C$. Since the length $L(s)$ of $C_s$ depends real analytically by $s$, it is a real analytic function of $s$. So we set $\gamma_s(t) := \tilde{\gamma}_s(t)/L(s)$, and then $\gamma_0 = \gamma$ and $t$ can be taken as an arc-length parameter $t$ of $\gamma_s(t)$ of period $l$ for each $s$. Since $C$ has no symmetric plane curve, its curvature function also has no symmetry. Since the curvature functions $\kappa_s(t)$ of $\gamma_s(t)$ are $l$-periodic, $\kappa_s(t)$ has no symmetry for sufficiently small $s$ (cf. Lemma [4.1]). In particular, the image of $\gamma_s$ also has no symmetry. Using Fukui’s formula (cf. [1.1]), we can construct a $C^\omega$- cuspidal edge $f$ with constant cuspidal angle with $\beta(t, v) = 1$. If $C$ has no symmetry, then neither does its singular curvature. So $f$ satisfies (c) of Theorem 1.4.

2. Proof of Theorem 1.4

A positive semi-definite $C^\omega$-metric $ds^2 = Edt^2 + 2Fdtdv + Gdv^2$ defined on $U_2(S^1)$ is called a periodic Kossowski metric if it satisfies the following:

(i) $E_2(t, 0) = 2F_2(t, 0)$ and $G_2(t, 0) = G_2(t, 0) = 0$.

(ii) There exists a $C^\omega$-function $\lambda$ defined on $U_2(S^1)$ satisfying $EG - F^2 = \lambda^2$.

Under the assumption that $\lambda_2(t, 0) > 0$, the singular curvature $\kappa_s$ of $ds^2$ is defined by (cf. [3] Remark 3.5)

$$\kappa_s(t) := \frac{-F_2(t, 0)E_2(t, 0) + 2E(t, 0)F_2(t, 0) - E(t, 0)F_2(t, 0)}{2E^{3/2}(t, 0)\lambda_2(t, 0)}.$$


for each \( t \in S^1 \). If \( ds^2 \) is the first fundamental form of \( f \in \mathcal{F}^0(C) \), it is a periodic Kossowski metric (cf. [3, Lemma 2.9]), and the singular curvature of \( f \) has the above expression.

**Lemma 2.1.** Let \( ds^2 \) be a periodic Kossowski metric on \( U_\varepsilon(S^1) \) satisfying (i), (ii) and (iii). We also let \( C \) be a closed \( C^2 \)-curve embedded in \( \mathbb{R}^3 \). Then, for each parametrization \( \gamma(t) \) of \( C \), there exists a unique cuspidal edge \( f_+ \in \mathcal{F}_+^0(C) \) (resp. \( f_- \in \mathcal{F}_-^0(C) \) satisfying:

1. \( f_+(t, 0) = \gamma(t) \) (resp. \( f_+(t, 0) = \gamma(t) \)) for each \( t \in S^1 \),
2. the first fundamental forms of \( f_+ \) (resp. \( f_- \)) is \( ds^2 \), and
3. the function \( \sqrt{\kappa^2 - \kappa_s^2} \) (resp. \( -\sqrt{\kappa^2 - \kappa_s^2} \)) is the limiting normal curvature of \( f_+ \) (resp. \( f_- \)).

**Proof.** We can find a partition \( 0 = t_0 < t_1 < \cdots < t_n = l \) of \( S^1 = \mathbb{R}/l\mathbb{Z} \) such that there exist local coordinate systems \((U_i; x_i, y_i) \) \( (i = 1, \ldots, n) \) of \( U_\varepsilon(C) \) containing \( [t_{i-1}, t_i] \times \{0\} \), where the metric has the expression \( ds^2 = E_i(dx_i)^2 + G_i(dy_i)^2 \) on \( U_i \).

For each \( i = 1, 2, \ldots, n \), we can apply [3, Theorem 12], and obtain two maps \( g_{\pm,i} : U_i \to \mathbb{R}^3 \) satisfying (2) and (3) such that \( g_{\pm,i}(t, 0) = \gamma(t) \) for \( t \in (t_{i-1} - \delta, t_i + \delta) \). Since conditions (2) and (3) do not depend on coordinates, the uniqueness of such two maps yields that \( g_{\pm,i} = g_{\pm,i-1} \) holds on a neighborhood of \( (t_{i-1} - \delta, t_i + \delta) \times \{0\} \).

Finally, we get \( g_{\pm,n} = g_{\pm,1} \) on \( U_n \cap U_1 \). In fact, if not, we have \( g_{\pm,n} = g_{\mp,1} \) and the cuspidal angle function \( \theta(t) \) takes different sign at \( t = 0 \) and \( t = l \). Then by the continuity of \( \theta(t) \), it has a zero, a contradiction. So the desired two cuspidal edges are obtained.

**Lemma 2.2.** Suppose \( f, g \in \mathcal{F}^0(C) \) are written in the normal form. If they have the same image as a map germ, then there exist \( c \in [0, l] \) and \( \sigma \in \{1, -1\} \) such that \( g(t, v) = f(\sigma t + c, v) \).

**Proof.** Since the singular set images of \( f \) and \( g \) coincide with \( C \), there exist a constant \( c \) and \( \sigma \in \{1, -1\} \) such that \( g(t, 0) = f(\sigma t + c, 0) \). Then the conclusion follows by Lemma 2.1.

**Proof of Theorem 1.4.** We may assume \( f(t, v) \) is expressed in a normal form. Let \( ds^2 \) be the first fundamental form of \( f \). By Lemma 2.1, for each \( b \in [0, l] \), there exist four \( C \)-isomers \( f^i_b \) of \( f \) \( (i = 1, 2, 3, 4) \) such that

- \( f^i_0(t, 0) = \gamma(t + b) \) \( (t \in S^1) \) holds for \( i = 1, 2, \) and \( f^i_0(t, 0) = \gamma(-t + b) \) \( (t \in S^1) \) holds for \( i = 3, 4 \),
- the absolute value of the limiting normal curvature of \( f^i_b \) is equal to
  \[ \sqrt{\kappa(\sigma' t + b)^2 - \kappa_s(t)^2}, \]
where \( \sigma'_i = 1 \) if \( i = 1, 2 \) and \( \sigma'_i = -1 \) if \( i = 3, 4 \), and
- the sign of the limiting normal curvature of \( f^i_b \) is \( +1 \) if \( i = 1, 3 \) and \( -1 \) if \( i = 2, 4 \).

By Lemma 2.1 (a) is obvious. We prove (b): If \( g \) is a \( C \)-isomer of \( f \), then there exist \( b \in [0, l] \) and \( \sigma \in \{1, -1\} \) such that \( g(t, 0) = f(\sigma t + b, 0) \) for \( t \in S^1 \). By Lemma 2.2, \( g \) coincides with \( f^i_b \) for some \( i \in \{1, 2, 3, 4\} \) as a map germ.

We next prove (c): Fix \( f_0 := f^0_b \) \( (i \in \{1, 2, 3, 4\}) \) and \( b \in [0, l] \). We let \( f_n = f^{i_n}_b \) \( (n = 1, 2, \ldots) \) be mutually distinct \( C \)-isomers of \( f \) which are congruent to \( f_0 \), where \( i_n \in \{1, 2, 3, 4\} \) and \( b_n \in [0, l] \). Since the possibilities of \( i_n \) are at most four, we may set \( j := j_n \). In particular \( \{b_n\} \) consists of distinct elements.
By Lemma 2.2, there exist an isometry \( T_n \) of \( \mathbf{R}^3 \), \( c_n \in [0, l) \) and \( \sigma_n \in \{1, -1\} \) such that \( f_n(t, v) = T_n \circ f_0(\sigma_n t + c_n, v) \) holds. Since the possibilities of \( \sigma_n \) are only two for each \( n \), we may set \( \sigma = \sigma_n \) for all \( n \). Since \( f_n \) \((n \geq 1)\) has the same first fundamental form as \( f_0 \), we have \( \kappa_n(t) = \kappa_n(\sigma t + c_n) \). If \( \{c_n\} \) contains infinite elements, then by real analyticity, the singular curvature function \( \kappa_n(t) \) is constant, a contradiction.

So we may assume \( c_n \) does not depend on \( n \), and we set \( c := c_n \). Then we have
\[
f_n(t, v) = T_n \circ T_n^{-1} \circ f_1(t, v) \quad (n \geq 2).
\]
Substituting \( v = 0 \), the fact \( f_n = f'_n b_n \) yields that \( \gamma(\sigma' t + b_n) \) is congruent to \( \gamma(\sigma' t + b_1) \), where \( \sigma' = 1 \) if \( j = 1, 2 \) and \( \sigma' = -1 \) if \( j = 3, 4 \). In particular,
\[
k(\sigma' t + b_1) = \kappa(\sigma' t + b_1), \quad \sigma''_{\tau}(\sigma' t + b_n) = \tau(\sigma' t + b_1)
\]
hold, where \( \sigma''_{\tau} \in \{1, -1\} \) and \( \tau(t) \) is the torsion function of \( \gamma(t) \). Substituting \( t = 0 \), we have
\[
k(b_1) = \kappa(b_1), \quad \sigma''_{\tau}(b_n) = \tau(b_1).
\]
Since the sequence \( \{b_n\} \) consists of distinct elements, this accumulates to a value \( b_n \in [0, l) \). Since \( \kappa(t) \) and \( \tau(t) \) are real analytic functions, they must be constant.

Since \( C \) is a closed curve, it must be a circle lying in a plane, a contradiction. Finally, we show (d): Suppose \( f_1 = f'_1 b_1 \) is a cuspidal edge which is congruent to \( f_0(= f'_0) \), where \( j \in \{1, 2, 3, 4\} \) and \( b_1 \in [0, 1) \). Like as in the case of (b), there exist an isometry \( T \) of \( \mathbf{R}^3 \), \( c \in [0, l) \) and \( \sigma \in \{1, -1\} \) such that \( f_1(t, v) = T \circ f_0(\sigma t + c, v) \). Since \( C \) has no symmetry, \( T \) is the identity map and \( (\sigma, c) \) must be \((1, 0)\). So \( f_1(t, 0) = f_0(t, 0) \). By Lemma 1.4 \( f_1 \) coincides with \( f_0 \). Hence \( A_b^1 \) is a one-point-set. \( \square \)

Appendix A. A Property of Non-Symmetric Functions

We prove the following assertion:

**Lemma A.1.** Let \( \{\mu_s(t)\}_{s \in \mathbb{R}} \) be a continuous one-parameter family of \( C^\infty \)-functions on \( \mathbf{R} \) satisfying \( \mu_s(t + l) = \mu_s(t) \) for each \( s \). If \( \mu_0 \) has no symmetry (cf. Definition 1.3), then \( \mu_s \) also has no symmetry for sufficiently small \( s > 0 \).

**Proof.** If the assertion fails, then there exists a monotone decreasing sequence \( \{s_n\} \) converging to \( 0 \) such that \( \tilde{\mu}_n := \mu_{s_n} \) has a certain symmetry, that is, there exist a constant \( c_n \in [0, l) \) and a sign \( \sigma_n \in \{1, -1\} \) such that
\[
\tilde{\mu}_n(\sigma_n t + c_n) = \mu_n(t).
\]
Since \( \sigma_n = \pm 1 \), by replacing \( \{s_n\} \) by some subsequence if necessary, we may assume that \( \sigma := \sigma_n \) does not depend on \( n \). Since \( \mathbf{R}/\mathbb{Z} \) is compact, replacing \( \{s_n\} \) by some subsequence if necessary, we may assume that \( c_n \) converges to \( c_0 \). Then taking the limit \( n \to \infty \), we have \( \mu_0(\sigma t + c_0) = \mu_0(t) \). Since \( \mu_0 \) is non-symmetric, we have \( \sigma = 1 \) and \( c_0 \in \{0, 1\} \). Then, we may assume that \( c_0 = 0 \) without loss of generality. If \( c_n \) is an irrational number, then (A.1) yields that \( \mu_n \) is a constant function. Since \( \mu_0 \) has no symmetry, we may assume that \( c_n \) is a rational number for sufficiently large \( n \), and can write \( c_n := q_n/p_n \), where \( p_n \) and \( q_n \) are relatively prime integers. Then there is a pair \((a, b)\) of integers such that \( ap_n + bq_n = 1 \) and
\[
\mu_n(t) = \mu_n(t + bc_n) = \mu_n\left(t + \frac{1 - ap_n}{p_n}\right) = \mu_n\left(t + \frac{1}{p_n}\right).
\]
Fix an irrational number \( x_0 \in (0, 1) \), and then there exists an integer \( r_n \) such that \( x_n := r_n/p_n \) \((n = 1, 2, 3, ...\) converges to \( x_0 \). Since \( \mu_n(t) = \mu_n(t + x_n) \) by (A.2), taking the limit \( n \to \infty \), we have \( \mu_0(t) = \mu_0(t + x_0) \), contradicting the assumption that \( \mu_0(t) \) has no symmetry. \( \square \)
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