Collective effects in traffic on bi-directional ant-trails

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Abstract

Motivated by recent experimental work of Burd et al., we propose a model of bi-directional ant-traffic on pre-existing ant-trails. It captures in a simple way some of the generic collective features of movements of real ants on a trail. Analyzing this model, we demonstrate that there are crucial qualitative differences between vehicular- and ant-traffic. In particular, we predict some unusual features of the flow rate that can be tested experimentally. As in the uni-directional model a non-monotonic density-dependence of the average velocity can be observed in certain parameter regimes. As a consequence of the interaction between oppositely moving ants the flow rate can become approximately constant over some density interval.

keywords: cellular automata, ant trails, collective effects, computer simulations.

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1 Introduction

At first sight, traffic on ant trails might look rather similar to human traffic. There are, however, important differences due to the cooperative nature of ant traffic as we will point out in the following. An important form of communication between ants is chemotaxis: Ants deposit pheromones on the substrate as they move forward (Hölldobler and Wilson 1990); other following ants are attracted to it and follow the trail.

Understanding the basic principles governing the formation of the ant-trails and identifying the factors that influence the movements of ants on such trails are of fundamental importance in population biology of social insect colonies (Wilson 1971). Moreover, insights gained from these fundamental studies are finding important applications in computer science (Dorigo et al.1999), communication engineering (Bonabeau et al.2000), artificial “swarm intelligence” (Bonabeau et al.1999) and “micro-robotics” (Krieger et al.2000) as well as in management (Bonabeau and Meyer 2001). Furthermore, ant-trail is an example of systems of interacting elements driven far from equilibrium; the collective spatio-temporal organizations in such systems (Camazine et al.2001, Anderson et al.2002, Mikhailov and Calenbuhr 2002, Chowdhury et al.2004) are of current interest in statistical physics (Schmittmann and Zia 1995, Schütz 2000). Our investigation is intended to provide insight into the effects of the mutual interactions of the ants on their collective mass flow on the trails.

In this paper we focus attention on the ant-traffic on pre-existing ant-trails rather than addressing the question of the formation of such trails (Ermentrout and Edelstein-Keshet 1993, Watmough and Edelstein-Keshet 1995, Edelstein-Keshet et al.1995, Nicolis and Deneubourg 1999, Rauch et al. 1995, Millonas 1992), which is an interesting example of self-organization (Camazine et al.2001, Anderson et al.2002). In other words, we study ant-traffic on trails which persist for very long time because of the availability of extensive or renewable resources.

The similarities between ant-traffic and vehicular traffic have inspired some recent experimental investigations (Burd et al. 2002, Burd and Aranwela 2003) as well as theoretical modelling (Couzin and Franks 2003) of collective movements of ants on trails. Our aim is to develop simple models of ant-traffic along the lines of discrete models of vehicular traffic (Chowdhury et al.2000, Helbing 2001). Our recent idealized model of uni-directional ant-traffic (Chowdhury et al.2002, Nishinari et al.2003) does not correspond to the most commonly observed ant-trails. Usually in natural trails the traffic is bi-directional with out-bound ants proceeding towards the resources to be collected and the nest-bound ants carrying the cargo. Therefore, in this paper we propose a new model of bi-directional ant-traffic.

The paper is organized as follows. For the convenience of readers, we begin with a brief introduction to the general modelling strategy. Next, in order to provide insight into the role of pheromone-induced indirect interactions among the ants, we briefly review the most important unusual features of our earlier model of idealized uni-directional ant-traffic (Chowdhury et al.2002, Nishinari et al. 2003). As we shall show, the uni-directional counterpart helps in identifying the physical origins of some of the observed features of the bi-directional ant-traffic. Then we introduce our new model of bi-directional ant-traffic, present the results of our computer simulations and interpret the results physically. In the concluding section we summarize our main theoretical predictions and point out the current difficulties in comparing these predictions with the experimental data available in the literature (Burd et al.2002).
General modeling strategy

First we describe the general modeling strategy for a uni-directional ant-trail model. Later we will generalize this to take into account counterflow.

In order to describe the motion of ants one can, in principle, write differential equations which would be analogs of Newton’s equations. However, in practice, particularly for numerical studies with computer simulations, it is much simpler to work with discretized models (Ermentrout and Edelstein-Keshet 1993) that also take into account the self-organized nature of motion.

In such discretized models, continuous space is replaced by a discretized mesh of cells; the mesh is usually referred to as a lattice. The size of each of these cells is such that it can accommodate at most one ant at a time. Therefore, the positions of the ants can change only by discrete amounts which can only be integral multiples of the cell size. Depending on the ant species under consideration, the typical linear size of these cells can be of the order of 0.1 cm to 1 cm.

Moreover, time is also assumed to increase in discrete steps; the duration of each step may be taken as the average reflex time of an ant. Consequently, the sequence of successive states is like a sequence of snapshots of the evolving system. Since the measured average speeds of ants in free-flow (i.e., unhindered by any other ant) has been observed (Burd et al.2002) to be of the order of 1 cm/sec, the real time associated with each time step of our model can be of the order of 1 sec. Furthermore, each element of the system (e.g., ant, pheromone, etc) can take one of the few allowed discrete states; thus, not only the positions but also the velocities of the ants are restricted to a few discrete values. In the following we will study the simplest case that ants are allowed to move to nearest neighbour cells only, i.e. the maximal velocity is 1 cell/time step.

The dynamics of the system, including the movements of the ants, are governed by well defined prescriptions that are usually referred to as the “update rules”. Given the state of the system at some arbitrary time step \( t \), the update rules decide the corresponding state of the system at the time step \( t + 1 \). These rules capture the essential behavioral features of individual ants, i.e., their responses to their immediate local environment (which include their interactions with the other neighboring ants). The models with parallel (i.e., synchronous) updating rules are usually referred to as cellular automata (CA) (Chopard and Droz 1998). The rules need not be deterministic and therefore in computer simulations one has to average over different realizations of the dynamics. In the context of traffic models, the individual-to-individual variations of behavioral patterns are usually captured through stochastic rules with appropriate probabilities (Chowdhury et al.2000).

For the sake of simplicity of theoretical calculations, we impose periodic boundary conditions which reduces boundary effects. In such a discrete model system of \( L \) cells with a total of \( N \) ants the average speed \( v \) of the ants is computed as follows: the instantaneous average speed \( v(t) \) (i.e., speed of the ants averaged over the entire population) is given by \( n(t)/N \) where \( n(t) \) is the number of ants that move forward to the next cell at the time step \( t \). However, in general, \( v(t) \) fluctuates with time; the average of \( v(t) \) over sufficiently long period of time in the steady state of the system can be identified with the average speed \( v \). The average number of ants passing through a detector location per unit time is called the flux, \( F \); it is related to the density \( c = N/L \) of the ants and their average speed \( v \) by the relation \( F = cv \).
All the data reported in this paper have been generated for trails as large as \( L = 10^3 \), in the units of cell size, i.e., about 1 to 10 meters. In all our computer simulations of the model, we begin with random initial conditions and let the system evolve following the update rules specified below. Long after the system reaches its steady state (typically, \( 10^4 - 10^5 \) time steps) we begin our computations of the steady-state properties like, for example, flux, etc. The data are averaged over the next \( 10^5 - 10^6 \) time steps as well as sufficient number of runs, each starting with different initial conditions.

**Brief review of uni-directional ant-traffic model**

In the model of uni-directional circular ant-traffic the trail consists of one row of cells for the ants and a parallel lattice of cells for the pheromones (fig.1). Ants are allowed to move only in one direction (say, clockwise). The state of the system is updated at each time step in two stages (see fig.1). In stage I ants are allowed to move while in stage II the pheromones are allowed to evaporate. In each stage the stochastic dynamical rules are applied in parallel to all ants and pheromones, respectively.

**Stage I: Motion of ants**

An ant in a cell cannot move if the cell immediately in front of it is also occupied by another ant. However, when an ant finds that the cell immediately in front of it is not occupied by any other ant, the likelihood of its forward movement to the ant-free cell is \( Q \) or \( q \), per unit time step, depending on whether or not the target cell contains pheromone. Thus, \( q \) (or \( Q \)) would be the average speed of a free ant in the absence (or presence) of pheromone. To be consistent with real ant-trails, we assume \( q < Q \), as presence of pheromone increases the average speed.

**Stage II: Evaporation of pheromones**

Trail pheromone is volatile. So, the deposited pheromone will gradually decay unless reinforced by the following ants. In order to capture this process, we assume that each cell occupied by an ant at the end of stage I also contains pheromone. On the other hand, pheromone in any ‘ant-free’ cell is allowed to evaporate; this evaporation is also assumed to be a random process that takes place at an average rate of \( f \) per unit time.

For the sake of simplicity, we consider only the presence or absence of pheromone in a cell. However, it will be straightforward to generalize the model to describe different levels of strength of the pheromone as well as their gradual evaporation and even diffusion. Moreover, in real ant trails, the extent of reinforcement of the trail by dropping of pheromone may depend on the existing local strength of pheromone. But, these details are not incorporated in our model since we do not explicitly treat the different levels of strength of the pheromone. Besides, incorporation of these details is not expected to affect the qualitative features of the results. The total amount of pheromone on the trail can fluctuate although the total number of the ants is independent of time because of the periodic boundary conditions.

In the two special cases \( f = 0 \) and \( f = 1 \) this model of uni-directional ant traffic becomes identical to one of the special cases of the Nagel-Schreckenberg model (Nagel and Schreckenberg 1992) of vehicular traffic with the corresponding likelihoods of forward movements \( Q \) and \( q \), respectively. In vehicular traffic, usually, the inter-vehicle interactions tend to hinder each other’s
motion so that the average speed of the vehicles decreases monotonically with the increasing density of the vehicles. In contrast, in our model of uni-directional ant-traffic the average speed of the ants varies non-monotonically with their density over a wide range of small values of $f$ (see fig.2 left) because of the coupling of their dynamics with that of the pheromone. This uncommon variation of the average speed gives rise to the unusual dependence of the flux on the density of the ants in our uni-directional ant-traffic model (fig.2 right).

The model of bi-directional ant-traffic

We develop the model of bi-directional ant-traffic by extending the model of uni-directional ant-traffic described in the previous section.

The model

In the models of bi-directional ant-traffic the trail consists of two lanes of cells for the ants (see fig.3). These two lanes need not to correspond to physically separate rigid lanes in real space; these are, however, convenient for describing the movements of ants in two opposite directions. In the initial configuration, a randomly selected subset of the ants move in the clockwise direction in one lane while the others move counterclockwise in the other lane. However, ants are allowed neither to take U-turn nor to change lane. Thus, the ratio of the populations of clockwise-moving and anti-clockwise moving ants remains unchanged as the system evolves with time. All the numerical data presented in this paper have been obtained by Monte Carlo simulations of the symmetric case where an equal number of ants move in the two directions. Therefore, the average flux is identical for both directions. In all the graphs we plot only the flux for clockwise-moving ants.

The rules governing the deposition and evaporation of pheromone in the model of bi-directional ant-traffic are identical to those in the model of uni-directional traffic. The common pheromone trail is created and reinforced by both the clockwise and counterclockwise moving ants. The probabilities of forward movement of the ants in the model of bi-directional ant-traffic are also natural extensions of the similar situations in the uni-directional traffic. When an ant (in either of the two lanes) does not face any other ant approaching it from the opposite direction the likelihood of its forward movement onto the ant-free cell immediately in front of it is $Q$ or $q$, respectively, depending on whether or not it finds pheromone ahead. Finally, if an ant finds another oncoming ant just in front of it, as shown in fig.3 it moves forward onto the next cell with probability $K$; in such situations, pheromone is present in both the cells.

Since in reality ants do not segregate in perfectly well defined lanes, head-on encounters of oppositely moving individuals occur quite often although the frequency of such encounters and the lane discipline varies from one species of ants to another (Burd et al. 2002). In reality, two ants approaching each other feel the hindrance, turn by a small angle to avoid head-on collision (Couzin and Franks 2003) and, eventually, pass each other. At first sight, it may appear that the ants in our model follow perfect lane discipline and, hence, the model dynamics is unrealistic. However, that is not true. The violation of lane discipline and head-on encounters

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5 Although U-turn of foragers is not uncommon (Beckers et al. 1992), U-turn of followers on pre-existing trails is very rare.
of oppositely moving ants is captured, effectively, in an indirect manner by assuming $K < Q$. Since, as in the uni-directional case, we have $q < Q$ this implies the existence of two different parameter regimes, $q < K < Q$ and $K < q < Q$. It is worth mentioning here that even in the special limit $K = Q$ the traffic dynamics on two lanes would remain coupled because the pheromone dropped by the ants on one lane also influences the ants moving on the other lane in the opposite direction.

Results and interpretations

Figs. 4 and 5 show the variations of flux and average speeds with density of ants in our model for the two relevant cases $q < K < Q$ and $K < q < Q$ and different values of the evaporation probability $f$. In both cases, the non-monotonic variation of the average speed with density gives rise to the unusual shape of the flux-versus-density diagram over a range of values of $f$. This feature of the model of bi-directional traffic is similar to that of the uni-directional ant-traffic (compare the figs 4 and 5 with fig. 2). It results from the formation of a ‘loose cluster’ (Nishinari et al. 2003), i.e. a localized region of increased density where the ants move almost coherently.

Another interesting phenomenon related to the cluster formation is coarsening. At intermediate time usually several loose clusters are formed. However, the velocity of a cluster depends on the distance to the next cluster ahead. This velocity is controlled by the survival probability $p_s$ of the pheromone created by the last ant of the previous cluster. Obviously, $p_s$ decreases with increasing distance. Therefore clusters with a small headway move faster than those with a large headway. This induces a coarsening process such that after long times only one loose cluster survives.

For both parameter regimes, the influence of ants in the counterdirection is determined by the evaporation probability $f$. In the regime $q < K < Q$ this influence is not necessarily repulsive. For $f \ll 1$ the ants move predominantly with the hopping probability $K$ or $Q$. The hopping probability is decreased by ants moving in the opposite direction and therefore the coupling is repulsive. But at high evaporation rates $f \approx 1$ the coupling becomes attractive for $q < K$. In the regime $K < q < Q$ one finds in principle the same mechanism. For high evaporation rates the coupling is repulsive since $K < q$ and becomes even more repulsive as the evaporation probability decreases such that more ants move with probability $Q$. 
An additional feature of the density-dependence of the flux in the bi-directional ant-traffic model is the occurrence of a plateau region. This plateau is more pronounced for \( K < q < Q \) (fig.5) than for \( q < K < Q \) (fig.4). Such plateaus in the flux-versus-density diagram have been observed earlier (Janowsky and Lebowitz 1992, Tripathy and Barma 1997) in models related to vehicular traffic where randomly placed bottlenecks (‘defects’) slow down the traffic in certain locations along the route. Here the mechanism is similar although in principle we have dynamical defects. Note that in fig.4 the plateaus appear only in the two limits \( f \to 0 \) and \( f \to 1 \) but not for an intermediate range of values of \( f \). In the limit \( f \to 0 \), most often the likelihood of the forward movement of the ants is \( Q = 0.75 \) whereas they are forced to move with a smaller probability \( K = 0.5 \) at those locations where they face another ant immediately in front approaching from the opposite direction (like the situations depicted in fig.3). Thus, such encounters of oppositely moving ants have the same effect on ant-traffic as that of the bottlenecks on vehicular traffic.

But why do the plateaus re-appear in fig.4 also in the limit \( f \to 1 \)? At sufficiently high densities, oppositely moving ants facing each other move with probability \( K = 0.5 \) rather than \( q = 0.25 \). In this case, locations where the ants have to move with the lower probability \( q \) will be, effectively bottlenecks and hence the re-appearance of the plateau. As \( f \) approaches unity there will be larger number of such locations and, hence, the wider will be the plateau. This is consistent with our observation in fig.4.

**Summary**

In this paper we have introduced a model of bi-directional ant-traffic. The two main theoretical predictions of this model are as follows:

(i) The average speed of the ants varies non-monotonically with their density over a wide range of pheromone evaporation rates. This unusual variation of average speed with density gives rise to the uncommon shape of the flux-versus-density diagrams and has already been observed in the uni-directional model.

(ii) Over some regions of parameter space, the flux exhibits plateaus when plotted against density. This new feature is characteristic for the bi-directional model and has its origin in the mutual hindrance of ants moving in opposite directions.

In principle, it should be possible to test these theoretical predictions by comparing with the corresponding experimental data. Interestingly, various aspects of locomotion of individual ants have been studied in quite great detail (Lighton et al.1987, Zollikofer 1994, Weier et al. 1995). However, traffic is a collective phenomenon involving a large number of interacting ants. Surprisingly, to our knowledge, the results published by Burd et al.(2002) on the leaf-cutting ant *Atta Cephalotes* are the only set of experimental data available on the density-dependence of flux of ants on trails. Unfortunately, the fluctuations in the data are too high to make any direct comparison with our theoretical predictions.

We hope our predictions will motivate more experimental measurements. The new experiments should be carried out, preferably, on a circular trail to mimic the periodic boundary conditions. Since we assumed all the ants to have identical size, the experiments should be done with only one type of ants, although polymorphism is quite common. Furthermore, in order to see the effect of \( f \) on the flux, the experiments should be repeated with different species of
ants whose trail pheromones evaporate at significantly different rates. The typical magnitudes of $f$, for which the non-monotonic variation of the average speed with density is predicted, correspond to pheromone lifetimes in the range from few minutes to tens of minutes.

In the models developed so far we do not distinguish between the characteristic features of outbound and nest-bound ants although, small variations in their free-flow velocities is expected in real ant-trails (Burd et al. 2002). We are now investigating the effects of this difference in the average speeds of nest-bound and outbound ants as well as that of open boundary conditions on the flux. The results of these studies, together with the results for unequal populations of the outbound and nestbound ants, will be reported in detail elsewhere (John et al. 2004).

For a direct application to ant trails the use of open boundary conditions (with ‘reservoirs’ representing e.g. nest and food source) would be more realistic. Investigations of this situation are currently being carried out (John et al. 2004). However, from previous experiences with nonequilibrium systems one expects that the behaviour of the open system, e.g. the phase diagram, will be determined by that of the periodic system studied here (Schütz 2000).

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Figure 1: Schematic representation of typical configurations of the uni-directional ant-traffic model with two different rows of cells for ants and the pheromones. The symbols \( \bullet \) indicate the presence of pheromone. This figure also illustrates the update procedure. Top: Configuration at time \( t \), i.e. before stage I of the update. The non-vanishing probabilities of forward movement of the ants are also shown explicitly. Middle: Configuration after one possible realisation of stage I. Two ants have moved compared to the top part of the figure. The open circle with dashed boundary indicates the location where pheromone will be dropped by the corresponding ant at stage II of the update scheme. Also indicated are the existing pheromones, that may evaporate in stage II of the updating, together with the average rate of evaporation. Bottom: Configuration after one possible realization of stage II. Two drops of pheromones have evaporated and pheromones has been dropped/reinforced at the current locations of the ants.
Figure 2: The average speed (left) and the flux (right) of the ants, in the \textit{uni-directional} ant-traffic model, are plotted against their densities for the parameters $Q = 0.75$, $q = 0.25$. The discrete data points corresponding to $f = 0.0001(\circ)$, $0.0005(\bullet)$, $0.001(\blacksquare)$, $0.005(\triangle)$, $0.01(\diamond)$, $0.05(\triangleleft)$, $0.10(\triangledown)$, $0.25(\triangleplus)$ have been obtained from computer simulations; the lines connecting these data points merely serve as the guide to the eye. The cases $f = 0$ and $f = 1$ are also displayed by the uppermost and lowermost curves (without points); these are exact results. Curves plotted with filled symbols have unusual shapes.

Figure 3: A typical head-on encounter of two ants moving in opposite directions in our model of \textit{bi-directional} ant-traffic. This is a totally new process which does not have any analog in the model of uni-directional ant-traffic. The mesh of cells in the middle corresponds to the common pheromone trail.
Figure 4: The average speed (left) and the flux (right) of the ants in the model of bi-directional traffic are plotted as functions of their density for the case $q < K < Q$ and several different values of the pheromone evaporation probability $f$. The parameters are $Q = 0.75, q = 0.25$ and $K = 0.5$. The symbols $\circ, \bullet, \blacksquare, \triangle, \ast, +, \triangledown, \diamond$ and $\triangleleft$ correspond, respectively, to $f = 0, 0.0005, 0.005, 0.05, 0.075, 0.1, 0.25, 0.5$ and $1$. The lines are merely guides to the eye.

Figure 5: The average speed (left) and the flux (right) of the ants in the model of bi-directional ant traffic are plotted as functions of their density for the case $K < q < Q$ and several different values of the pheromone evaporation probability $f$. The parameters are $Q = 0.75, q = 0.50$ and $K = 0.25$. The symbols $\circ, \square, \blacklozenge, \triangle, \blacklozenge, \triangleleft$ and $\triangledown$ correspond, respectively, to $f = 0, 0.0005, 0.005, 0.05, 0.5$ and $1$. The lines are merely guides to the eye. The inset in the left diagram is a magnified re-plot of the same data, over a narrow range of density, to emphasize the fact that the unusual trend of variation of flux with density in this case is similar to that observed in the fig.4.