ON THE CAPACITY REGION OF COGNITIVE MULTIPLE ACCESS OVER WHITE SPACE CHANNELS

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Abstract

By exploiting white spaces in the already crowded spectrum, the Cognitive Radio (CR) technology has the potential to meet the rapidly increasing demand for broadband wireless communications. The cognitive multiple access channels (CogMAC) are commonly seen in many applications, such as the Cognitive Radio Sensor Networks (CRSN), and the 802.22 cognitive Wireless Regional Area Networks (WRAN). In this paper, the primary user (PU) activities are treated as on/off side information, which can be obtained causally or non-causally. The CogMAC is then modeled as multi-switch channels and their rate regions are characterized in some scenarios. Explicit forms of outer and inner bounds of the rate regions are derived by assuming additional side information, and they are shown to be tight in some special cases. An optimal rate and power allocation scheme that maximizes the sum rate is also proposed. The numerical results show the importance of side information in enhancing the capacity, the impact of channel correlation and PU activity, and the effectiveness of our rate allocation scheme.

Index Terms

Cognitive MAC channel, Capacity of Cognitive Radio Networks, Optimal Rate allocation, Three-switch Cognitive Channel, Cognitive Radio Sensor Network, 802.22 WRAN.

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I. Introduction

A. Motivation

To solve the dilemma between the ever increasing bandwidth demand and the actual under-utilization of spectrum resource [1], FCC has allowed unlicensed devices to access the temporarily unoccupied TV spectrum, namely white spaces [2]. Therefore, Cognitive Radio (CR) techniques that adopt the “sense and access” paradigm, in which the Secondary Users (SUs) identify the activity of the Primary User (PU) before accessing the white space channels, receive great research interest recently.

In particular, cognitive multiple access (MAC) communications over white spaces can be found in several practical systems, such as the Cognitive Radio Sensor Networks (CRSN) [3] and 802.22 cognitive Wireless Regional Area Networks (WRAN) [4]. In both networks, SUs are grouped in cells or clusters according to their locations and common channels [5]. The cluster members (CMs) transmit to the cluster head (CH) through a cognitive MAC in the uplink.

Therefore, the study on the capacity region of cognitive MAC over white space channels will allow us to better understand the performance of cognitive radio systems. Equipped with intrinsic spectrum sensing capability, SUs can obtain the states of PU activities, either causally or non-causally. Thus, PU activities can be viewed as side information about the channel state at both cognitive transmitters and receivers, and information theoretic results on channel capacity with side information [19] may be explored to reveal the fundamental limit and various tradeoffs.

B. Related Works

The existing studies on the capacity of cognitive channels are mainly conducted in the context of interference channels [6], further divided into the underlay [7] and overlay [8], [9] cases. The former assumes that the SU has the channel knowledge and can control its transmission power to restrict the interference to the PUs. In contrast, the overlay approach models the coexistent communications as the “interference channel with degraded message sets (IC-DMS)” [10], [11], and employs intricate coding schemes for capacity enhancement.

The capacity of the cognitive MAC fading channel is studied recently in [12], in which the ergodic sum-rate is obtained under the interference-power constraint. In [13] the impact of multiuser interference diversity is further exploited on the the capacity regions of various CR networks, including the cognitive MAC channel.
Recent works on the throughput of cognitive networks are fueled by the seminal work [14]. In particular, it has been shown that both the primary and secondary network can achieve the throughput scaling as two standalone networks [15] [16] [18] under various situations.

In the above study, concurrent transmission of the primary and secondary systems is allowed. To effectively control the interference, either channel gains of the SU-PU links or PU messages are assumed known at the secondary transmitter. However, many practical cognitive systems only determine the existence of PU transmissions through spectrum sensing, and access the white space channels to avoid undesired interference. In these applications, locally sensed PU activities become the major factors influencing the rate regions of SUs.

C. Summary of Contributions

To the best of our knowledge, the pioneering work [20] is the first to consider the “sense and access” diagram, in which the PU activities sensed at the cognitive transmitter and receiver are modeled as on/off side information, and the capacity of a two-switch cognitive channel is explored. Motivated by this work, in this paper we extend the study to a cognitive multiple access scenario. The contributions of this paper are summarized below:

1) Achievable Rate Regions of the Cognitive MAC Channel: By viewing the primary user activities around the transmitters and receiver as side information, we model the memoryless cognitive MAC channel as a three-switch MAC channel. The achievable regions of the three-switch MAC channel are derived for two scenarios with independent causal and non-causal side information at the transmitters, and a special case in which the receiver has strong spectrum sensing capacity.

2) Outer and Inner Bounds: The capacity regions of the three-switch MAC channel derived are intractable in general. We further obtain explicit outer and inner bounds of the capacity region by assuming additional side information at the transmitters or receiver. It is found that the outer and inner bounds coincide in two special cases: when the side information between the transmitters and receiver are highly correlated, or when the states of PU signals change slowly.

3) Sum-rate Optimal Rate/Power Allocation: We optimize the rate and power allocation between transmitters when both the transmitters and receiver have global side information, and analyze the impact of two parameters, PU occupation probability and correlation in
side information, on the sum rate.

4) Extension to the fading scenario and general \( m \)-user cases: We further analyze a general model with fading and interference, in which the receiver is active all the times. Furthermore, extension to the general \( m \)-user case is also discussed.

The rest of the paper is organized as follows. In Section II, we introduce the memoryless cognitive MAC channel and model it as a three-switch MAC channel. The main results of rate expressions, bounds and rate/power allocation are listed in Section III, with relevant derivations and analysis given in Section IV. Numerical results are presented in Section V, and the whole paper is concluded in Section VI.

II. PROBLEM FORMULATION AND SYSTEM MODEL

The cognitive MAC channel with neighboring primary activities is illustrated in Fig. 1. We follow [20] and model the memoryless cognitive MAC channel as a three-switch equivalent channel, as in Fig. 2. The states of switches at the two cognitive transmitters CT1, CT2 and the cognitive receiver CH are denoted respectively as \( S_{T_1} \), \( S_{T_2} \) and \( S_R \), taking values of either 1(on) or 0(off). When the cognitive users detect (strong enough) interference from PU signals, the switch is turned off to avoid collisions. Otherwise, the switch is turned on for opportunistic communications. Based on this model, the input and output of the channel is related as:

\[
Y = (S_{T_1}X_1 + S_{T_2}X_2 + Z) S_R,
\]

\( S_{T_1}, S_{T_2}, S_R \in \{0, 1\} \),

where \( X_1 \) and \( X_2 \) are the transmitted symbols of CT1 and CT2 with average power constraint \( \mathbb{E}[|X_i|^2S_{T_i}] \leq P_i, i = 1, 2 \), and \( Z \) is the AWGN noise with unit variance.

In this work, we assume perfect spectrum sensing for ease of discussion. Then, the state of switch is actually controlled by the PU occupation, which is regarded as a type of side information to the cognitive users. The side information is said to be causal if the transmitters or receiver only has knowledge of the past/current states of the corresponding switch, and non-causal if the future states are also known.

III. MAIN RESULTS

A. Achievable Rate and Capacity Regions of Cognitive MAC Channel

We first explore the rate regions of the memoryless cognitive MAC channel with independent transmitter side information, either causal or non-causal. Note that the causality of the receiver
side information does not matter in the study, as the receiver can decode after the transmission is finished. In the special case that the transmitter side information is also known at the receiver, we can derive the capacity region. These results are natural extension of those in [20] concerning the capacity of a single-link two-switch channel.

1) Causal Side Information at the Transmitters.

**Theorem 1**: For the three-switch channel with independent causal side information $S_{T_1}$ and $S_{T_2}$ at the transmitters and side information $S_R$ at the receiver, coding can be performed directly on the input alphabets (i.e., $U_1 = X_1$, $U_2 = X_2$) and an achievable rate region is given by:

$$R_{causal}^{S_{T_1}, S_{T_2}, S_R} = \bigcup \left\{ (R_1, R_2) : \begin{align*} R_1 &\leq I(X_1; Y, S_R | X_2) \\ R_2 &\leq I(X_2; Y, S_R | X_1) \\ R_1 + R_2 &\leq I(X_1, X_2; Y, S_R) \end{align*} \right\},$$

for all $p(X_1, X_2 | S_{T_1}, S_{T_2}) = p(X_1 | S_{T_1}) p(X_2 | S_{T_2})$, where $\bigcup$ denotes the convex hull of all rate pairs.

2) Non-causal Side Information at the Transmitters.

**Theorem 2**: For the three-switch channel with independent non-causal side information $S_{T_1}$ and $S_{T_2}$ at the transmitters and side information $S_R$ at the receiver, coding can be performed directly on the input alphabets (i.e., $U_1 = X_1$, $U_2 = X_2$) and an achievable rate region is given by:

$$R_{non-causal}^{S_{T_1}, S_{T_2}, S_R} = \bigcup \left\{ (R_1, R_2) : \begin{align*} R_1 &\leq I(X_1; Y, S_R | X_2) - I(X_1; S_{T_1}) \\ R_2 &\leq I(X_2; Y, S_R | X_1) - I(X_2; S_{T_2}) \\ R_1 + R_2 &\leq \left\{ \begin{array}{ll} I(X_1, X_2; Y, S_R) \\ -I(X_1; S_{T_1}) - I(X_2; S_{T_2}) \end{array} \right\} \end{align*} \right\},$$

for all $p(X_1, X_2 | S_{T_1}, S_{T_2}) = p(X_1 | S_{T_1}) p(X_2 | S_{T_2})$.

3) Strong Spectrum Sensing Capability at the Receiver.

**Theorem 3**: For the three-switch channel, if the transmitters’ side information $S_{T_1}$ and $S_{T_2}$ are known to the receiver, i.e. $(S_{T_1}, S_{T_2}) = f(S_R)$, no matter whether the transmitters’ side information is causal or non-causal, the channel capacity regions are the same, as given by:

$$C_{causal, non-causal}^{(S_{T_1}, S_{T_2})=f(S_R)} = \bigcup \left\{ (R_1, R_2) : \begin{align*} R_1 &\leq I(X_1; Y, S_R | X_2) \\ R_2 &\leq I(X_2; Y, S_R | X_1) \\ R_1 + R_2 &\leq I(X_1, X_2; Y, S_R) \end{align*} \right\}.$$

\(^1\)Here, we consider the cases where the side information of two transmitters are independent. Note this is reasonable when PU signal power is relatively low and the cognitive transmitters keep a non-trivial distance with each other.
for all \( p(X_1, X_2 | S_{T_1}, S_{T_2}) = p(X_1 | S_{T_1}) p(X_2 | S_{T_2}) \).

**B. Outer bounds and Inner bounds for Gaussian Switch Channel**

Since the achievable rate regions derived are generally intractable, we further explore explicit outer and inner bounds of the capacity region with the help of additional side information. We restrict to the Gaussian case to obtain the optimal results [20].

**Definition 1:** To facilitate the analysis, we define six events (a to f) related to primary states \( S_{T_1}, S_{T_2} \) and \( S_R \), together with their probabilities of occurrence as follows:

\[
a : \{ S_R = S_{T_1} = 1, S_{T_2} = 0 \}, \quad p_a \triangleq p(S_R = S_{T_1} = 1, S_{T_2} = 0);
b : \{ S_R = S_{T_2} = 1, S_{T_1} = 0 \}, \quad p_b \triangleq p(S_R = S_{T_2} = 1, S_{T_1} = 0);
c : \{ S_R = S_{T_1} = S_{T_2} = 1 \}, \quad p_c \triangleq p(S_R = S_{T_1} = S_{T_2} = 1);
d : \{ S_{T_1} = 1, S_{T_2} = 0 \}, \quad p_d \triangleq p(S_{T_1} = 1, S_{T_2} = 0);
e : \{ S_{T_1} = 0, S_{T_2} = 1 \}, \quad p_e \triangleq p(S_{T_1} = 0, S_{T_2} = 1);
f : \{ S_{T_1} = S_{T_2} = 1 \}, \quad p_f \triangleq p(S_{T_1} = S_{T_2} = 1).
\]

**Theorem 4:** For the three-switch MAC channel, *outer bound 1* of the capacity region can be obtained by assuming global side information at both the transmitters and the receiver:

\[
C^\ast\ast (P_1, P_2) = \bigcup_{p_a p_1^* + p_c p_2^* \leq P_1, p_b p_2^* + p_c p_1^* \leq P_2} \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq p_a \log (1 + P_1^*) + p_c \log (1 + P_2^*) \\
R_2 \leq p_b \log (1 + P_2^*) + p_c \log (1 + P_1^*) \\
R_1 + R_2 \leq p_a \log (1 + P_1^*) + p_c \log (1 + P_2^*) + p_c \log (1 + P_2^*) \end{array} \right\}. \tag{5}
\]

**Theorem 5:** For the three-switch MAC channel, *outer bound 2* of the capacity region can be obtained by assuming full side information only at the receiver:

\[
C_{S_{T_1}, S_{T_2}, S_R} (P_1, P_2) = \bigcup_{p_1^* p_1^* + p_2^* p_2^* \leq P_1, p_2^* p_2^* + p_2^* p_2^* \leq P_2} \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq p_a \log (1 + P_1^*) + p_c \log (1 + P_1^*) \\
R_2 \leq p_b \log (1 + P_2^*) + p_c \log (1 + P_2^*) \\
R_1 + R_2 \leq p_a \log (1 + P_1^*) + p_c \log (1 + P_1^*) + p_c \log (1 + P_2^*) \end{array} \right\}. \tag{6}
\]

**Theorem 6:** For the three-switch MAC channel (with either causal or non-causal side information), an *inner bound* of the capacity region can be obtained as follows:

\[
C_{S_{T_1}, S_{T_2}, S_R}^{\text{inner}} = \bigcup_{p_1^* p_1^* + p_2^* p_2^* \leq P_1, p_2^* p_2^* + p_2^* p_2^* \leq P_2} \left\{ (R_1, R_2) : \begin{array}{l}
R_1 \leq R_1^* - \Delta R_1 \\
R_2 \leq R_2^* - \Delta R_2 \\
R_1 + R_2 \leq R_1^* + R_2^* - \Delta (R_1 + R_2) \end{array} \right\}. \tag{7}
\]
where \((R_1^*, R_2^*)\) denotes the rate pair in outer bound 2, and \(0 \leq \Delta R_1 \leq p_h H(\mathbf{S}_{T_1}|\mathbf{S}_R)\), \(0 \leq \Delta R_2 \leq p_b H(\mathbf{S}_{T_2}|\mathbf{S}_R)\), and \(0 \leq \Delta (R_1 + R_2) \leq p_c H(\mathbf{S}_{T_1}, \mathbf{S}_{T_2}|\mathbf{S}_R)\) denote the rate gaps.

Remarks: When the PU states change very slowly, or are highly correlated among transmitters and receiver, it is shown that the outer bounds and inner bound coincide. In the case with global side information, we can employ rate and power allocation strategies to improve the system’s overall performance. Specifically, if we impose power constraints on both transmitters, an optimal rate and power allocation scheme is obtained to maximize the sum rate. We also analyze the effect of correlation in side information and PU occupation probability on the sum rate.

IV. Analysis

A. Preliminaries

The memoryless cognitive MAC channel is modeled as a three-switch channel, so that existing results on the memoryless MAC channel with transmitter and receiver side information [19] can be employed to derive the cognitive rate regions, which are listed as the following lemmas:

Lemma 1. Causal Case: An achievable rate region of the discrete memoryless MAC channel with receiver side information and independent causal transmitter side information is given by the convex closure of the rate pairs satisfying:

\[
\bigcup_{p_{\text{causal}}} \left\{ (R_1, R_2) : \begin{align*}
R_1 &\leq I(U_1; Y, \mathbf{S}_R | U_2) = I(U_1; Y | U_2, \mathbf{S}_R) \\
R_2 &\leq I(U_2; Y, \mathbf{S}_R | U_1) = I(U_2; Y | U_1, \mathbf{S}_R) \\
R_1 + R_2 &\leq I(U_1, U_2; Y, \mathbf{S}_R) = I(U_1, U_2; Y | \mathbf{S}_R)
\end{align*} \right\},
\]

where the message is contained in the mutually independent auxiliary random variables \(U_1\) and \(U_2\), and the causality is embodied in the following conditional probability distribution:

\[
p_{\text{causal}} = \left\{ \begin{array}{l}
p(U_1, U_2, X_1, X_2 | \mathbf{S}_{T_1}, \mathbf{S}_{T_2}) \\
= p(U_1, X_1 | \mathbf{S}_{T_1}) p(U_2, X_2 | \mathbf{S}_{T_2}) \\
= p(U_1) p(X_1 | U_1, \mathbf{S}_{T_1}) p(U_2) p(X_2 | U_2, \mathbf{S}_{T_2})
\end{array} \right\}.
\]

Lemma 2. Non-causal Case: An achievable rate region of the discrete memoryless MAC channel with receiver side information and independent non-causal transmitter side information is given by the convex closure of the rate pairs satisfying:

\[
\bigcup_{p_{\text{non-causal}}} \left\{ (R_1, R_2) : \begin{align*}
R_1 &\leq I(U_1; Y, \mathbf{S}_R | U_2) - I(U_1; \mathbf{S}_{T_1}) \\
R_2 &\leq I(U_2; Y, \mathbf{S}_R | U_1) - I(U_2; \mathbf{S}_{T_2}) \\
R_1 + R_2 &\leq I(U_1, U_2; Y, \mathbf{S}_R) - I(U_1; \mathbf{S}_{T_1}) - I(U_2; \mathbf{S}_{T_2})
\end{align*} \right\},
\]

where \((R_1^*, R_2^*)\) denotes the rate pair in outer bound 2, and \(0 \leq \Delta R_1 \leq p_h H(\mathbf{S}_{T_1}|\mathbf{S}_R)\), \(0 \leq \Delta R_2 \leq p_b H(\mathbf{S}_{T_2}|\mathbf{S}_R)\), and \(0 \leq \Delta (R_1 + R_2) \leq p_c H(\mathbf{S}_{T_1}, \mathbf{S}_{T_2}|\mathbf{S}_R)\) denote the rate gaps.
where the message is contained in the mutually independent auxiliary random variables $U_1$ and $U_2$, and the non-causality is embodied in the following conditional probability distribution:

$$p_{\text{non-causal}} = \begin{cases} 
  p(U_1, U_2, X_1, X_2 | S_{T_1}, S_{T_2}) \\
  = p(U_1, X_1 | S_{T_1}) p(U_2, X_2 | S_{T_2}) \\
  = p(U_1 | S_{T_1}) p(X_1 | U_1, S_{T_1}) p(U_2 | S_{T_2}) p(X_2 | U_2, S_{T_2}) 
\end{cases}.$$ 

**Lemma 3.** If the transmitters’ side information can be expressed as a function of the receiver side information, i.e. $\{S_{T_1}, S_{T_2}\} = f(S_R)$, the memoryless MAC channel capacity regions are the same for causal and non-causal cases, i.e. $C_{\text{causal}} = C_{\text{non-causal}}$, given by

$$\bigcup \left\{ (R_1, R_2) : \begin{array}{l}
  R_1 \leq I(X_1; Y | S_R, X_2) \\
  R_2 \leq I(X_2; Y | S_R, X_1) \\
  R_1 + R_2 \leq I(X_1, X_2; Y | S_R) \end{array} \right\},$$

with the conditional probability distribution $p(X_1, X_2 | S_{T_1}, S_{T_2}) = p(X_1 | S_{T_1}) p(X_2 | S_{T_2})$.

**B. Achievable Rate and Capacity Regions with Causal and Non-causal Side Information**

1) **Proof for Theorem 1:** We know from [23] that the transmitted symbols $X_1, X_2$ are actually deterministic functions of causal side information $S_{T_1}, S_{T_2}$ and auxiliary random variables $U_1, U_2$, i.e., $X_1 = f_1(U_1, S_{T_1})$ and $X_2 = f_2(U_2, S_{T_2})$. Since $S_{T_1}, S_{T_2} \in \{0, 1\}$, we define

$$X_1 = \begin{cases} 
  g_1(U_1), & \text{for } S_{T_1} = 1, \\
  h_1(U_1), & \text{for } S_{T_1} = 0, 
\end{cases} \quad X_2 = \begin{cases} 
  g_2(U_2), & \text{for } S_{T_2} = 1, \\
  h_2(U_2), & \text{for } S_{T_2} = 0. 
\end{cases}$$

When $S_{T_1}$ or $S_{T_2}$ is zero, $Y$ is not influenced by $X_1$ or $X_2$. Therefore, we can assume

$$X_1 = g_1(U_1), \quad \text{for } S_{T_1} = 0, 1,$$

$$X_2 = g_2(U_2), \quad \text{for } S_{T_2} = 0, 1.$$  

Note that $X_1(X_2)$ is the deterministic function of $U_1(U_2)$, and the distribution of $U_1(U_2)$ is independent of $S_{T_1}(S_{T_2})$, thus decoding $X_1(X_2)$ is sufficient for decoding $U_1(U_2)$, and $U_1$ and $X_1(U_2$ and $X_2$) are equivalent in terms of the amount of mutual information (with other variables). Without loss of generality, the rate region of cognitive MAC channel can be formulated by replacing $U_1$ and $U_2$ with $X_1$ and $X_2$ respectively in Lemma 1. 

2) **Proof for Theorem 2:** We first derive the rate of $R_1$ based on Lemma 2.

$$R_1 \leq I(U_1; Y, S_R | U_2) - I(U_1; S_{T_1})$$
\[ I(U_1,X_1;Y,S_R|U_2,X_2) - I(U_1,X_1;S_{T_1}) \]
\[ = \left( I(X_1;Y,S_R|U_2,X_2) + I(U_1;Y,S_R|X_1,U_2,X_2) \right) \]
\[ - (I(X_1;S_{T_1}) + I(U_1;S_{T_1}|X_1)) \]
\[ \leq I(X_1;Y,S_R|X_2) - I(X_1;S_{T_1}), \]

where (a) is due to \( X_1 = f_1(U_1) \), for \( S_{T_1} = 0,1 \), and \( X_2 = f_2(U_2) \), for \( S_{T_2} = 0,1 \);

(b) holds as \( S_{T_1} \) and \( S_{T_2} \) are independent, thus \( U_2, X_2 \) are independent of \( U_1, X_1 \);

(c) is obtained as \( (U_1|X_1 = x_1) \rightarrow (S_{T_1}|X_1 = x_1) \rightarrow (Y,S_R|X_1 = x_1) \) forms a Markov Chain. Therefore \( I(U_1;Y,S_R|X_1) - I(U_1;S_{T_1}|X_1) \leq 0 \), and the equality is achievable by setting \( U_1 = X_1 \).

By symmetry we can similarly obtain \( R_2 \leq \max_{n(X_2|S_{T_2})} I(X_2;Y,S_R|X_1) - I(X_2;S_{T_2}) \).

For the sum rate of \( R_1 + R_2 \), we have [19]

\[ R_1 + R_2 \leq I(U_1,U_2;Y,S_R) - I(U_1;S_{T_1}) - I(U_2;S_{T_2}) \]

\[ \overset{(a)}{=} I(U_1,X_1,U_2,X_2;Y,S_R) - I(U_1,X_1;S_{T_1}) - I(U_2,X_2;S_{T_2}) \]

\[ = \left( I(X_1,X_2;Y,S_R) + I(U_1,U_2;Y,S_R|X_1,X_2) \right) \]
\[ - (I(X_1;S_{T_1}) + I(U_1;S_{T_1}|X_1)) \]
\[ - (I(X_2;S_{T_2}) + I(U_2;S_{T_2}|X_2)) \]

\[ \overset{(b)}{=} \left( I(X_1,X_2;Y,S_R) - I(X_1;S_{T_1}) - I(X_2;S_{T_2}) \right) \]
\[ + I(U_1,U_2;Y,S_R|X_1,X_2) - I(U_1;S_{T_1}|X_1,X_2) - I(U_2;S_{T_2}|X_1,X_2) \]

\[ \overset{(c)}{=} \left( I(X_1,X_2;Y,S_R) - I(X_1;S_{T_1}) - I(X_2;S_{T_2}) \right) \]
\[ + I(U_1,U_2;Y,S_R|X_1,X_2) - I(U_1,U_2;S_{T_1},S_{T_2}|X_1,X_2) \]

\[ \overset{(d)}{\leq} I(X_1,X_2;Y,S_R) - I(X_1;S_{T_1}) - I(X_2;S_{T_2}), \]
$x_2$) forms a Markov chain, so does $(U_2|x_1, X_2 = x_2) \rightarrow (S_{T_2}|X_1 = x_1, X_2 = x_2) \rightarrow (Y, S_R|x_1, X_2 = x_2)$. Hence, $(U_1, U_2|x_1, X_2 = x_2) \rightarrow (S_{T_1}, S_{T_2}|X_1 = x_1, X_2 = x_2) \rightarrow (Y, S_R|x_1, X_2 = x_2)$ also forms a Markov Chain. Therefore, $I(U_1, U_2; Y, S_R|x_1, X_2) - I(U_1, U_2; S_{T_1}, S_{T_2}|X_1, X_2) \leq 0$. Finally, the equality in (12) is achievable by setting $U_1 = X_1$ and $U_2 = X_2$ in (11).

3) **Proof for Theorem 3:** Since the three-switch MAC channel is a special case of the memoryless channel with side information, only that the side information here is binary, we can directly employ the result of Lemma 3 to complete the proof.

**Remarks:** In practise, the assumption in Theorem 3 implies that CH (or BS) can not only sense the primary activities at its side, but also the primary activities near the transmitters. This assumption is reasonable in certain scenarios, since CHs or BSs are usually equipped with more powerful hardware and abundant energy supplies.

C. **Outer and Inner Bounds**

Since the rate regions derived are intractable in general, we further explore explicit outer and inner bounds of the capacity region with the help of additional side information. We restrict to the Gaussian case to obtain the optimal results [20].

We consider two kinds of additional side information at the cognitive transmitters and/or receiver, and derive the corresponding outer bounds. In **Case 1**, both the transmitters and the receiver have full knowledge of all side information (i.e., $S_{T_1}$, $S_{T_2}$, and $S_R$). In **Case 2**, only the receiver has full side information. For the case when only the transmitters (both) have full side information, either of them transmits only when its own switch is on and $S_R = 1$. We assume that the receiver can infer the states of transmitters through signal detection \(^2\), thus this case coincides with **Case 1**. Note that when the receiver also has transmitter side information, there is no differences between the causal and non-causal case [20]. Also, arbitrary correlation among side information may be considered in these genie-aided scenarios.

1) **Outer bounds 1 - Global Side Information:** With global side information, the transmissions occur when $S_R = 1$ and $S_{T_1} + S_{T_2} \geq 1$, which can be further categorized into three subcases.

\(^2\)With the assumption that the receiver knows the codebooks of both transmitters, the receiver can distinguish when there is one or two active transmitters.
When $S_R = S_{T_1} = 1, S_{T_2} = 0$, the MAC channel degrades into a point-to-point channel, in which Gaussian input is optimal. Assuming CT1’s transmission power to be $P_1^a$, the achievable rate region corresponding to event $a$ is $C_{a,*,*,}(P_1^a, 0) = \bigcup \left\{ \begin{array}{lc} R_1^a \leq \log(1 + P_1^a) \\ R_2^a = 0 \end{array} \right\}$. Similarly, when $S_R = S_{T_2} = 1, S_{T_1} = 0$, assuming CT2’s transmission power to be $P_2^b$, the achievable rate region corresponding to event $b$ is $C_{b,*,*,}(0, P_2^b) = \bigcup \left\{ \begin{array}{lc} R_1^b = 0 \\ R_2^b \leq \log(1 + P_2^b) \end{array} \right\}$. When $S_R = S_{T_1} = S_{T_2} = 1$, the channel is the traditional MAC channel. Assuming the transmission power of CT1 and CT2 to be $P_1^c$ and $P_2^c$, respectively, the achievable rate region corresponding to event $c$ is:

$$C_{c,*,*,}(P_1^c, P_2^c) = \bigcup \left\{ \begin{array}{lc} R_1^c \leq \log(1 + P_1^c) \\ R_2^c \leq \log(1 + P_2^c) \\ R_1^c + R_2^c \leq \log(1 + P_1^c + P_2^c) \end{array} \right\}.$$  

Taking into account the power constraints: $p_a P_1^a + p_c P_1^c \leq P_1$, and $p_b P_2^b + p_c P_2^c \leq P_2$, outer bounds 1 can be derived as in (5).

**Optimal Rate/Power Allocation:** When both the transmitters and receiver have global state information, we can further explore the optimal rate and power allocation.

Without loss of generality, we may take the optimal sum rate as our objective:

$$\begin{align*}
\text{maximize : } R_1 + R_2 &= p_a \log(1 + P_1^a) + p_b \log(1 + P_2^b) + p_c \log(1 + P_1^c + P_2^c), \\
\text{subjectto : } &\begin{cases}
p_a P_1^a + p_c P_1^c \leq P_1, \\
p_b P_2^b + p_c P_2^c \leq P_2, \\
P_1^a, P_2^b, P_1^c, P_2^c \geq 0.
\end{cases}
\end{align*}$$  \hspace{1cm} (13)

This is a convex optimization problem, and can be solved through KKT conditions.

$$\begin{align*}
\frac{\partial L(P_1^a, P_2^b, P_1^c, P_2^c)}{\partial P_1^a} &= 0, \\
\frac{\partial L(P_1^a, P_2^b, P_1^c, P_2^c)}{\partial P_2^b} &= 0.
\end{align*}$$

Substituting the constraints $P_1^c = \frac{p_a - p_a P_1^a}{p_c}$, $P_2^c = \frac{p_b - p_b P_2^b}{p_c}$, the solution is obtained as

$$\begin{align*}
P_1^a &= P_2^b = \frac{P_1 + P_2}{p_a + p_b + p_c}, \\
P_1^c &= \frac{(p_a + p_c) P_1 - p_a P_2}{(p_a + p_b + p_c) P_c}, \\
P_2^c &= \frac{(p_b + p_c) P_2 - p_b P_1}{(p_a + p_b + p_c) P_c}.
\end{align*}$$  \hspace{1cm} (14)

The corresponding optimal sum rate is given as follows.

**Corollary 1:** The maximal sum rate is $(R_1 + R_2)_{\text{max}} = (p_a + p_b + p_c) \log \left( 1 + \frac{P_1 + P_2}{p_a + p_b + p_c} \right)$. 

\[ \]
2) *Outer bounds 2 - Full Side Information at Receiver:* In Case 2, transmissions occur only when $S_{T_1} + S_{T_2} \geq 1$. This can be further categorized into three subcases, which correspond to the events $d$, $e$ and $f$ in Definition 1.

When $S_{T_1} = 1, S_{T_2} = 0$ or $S_{T_1} = 0, S_{T_2} = 1$, the MAC channel degrades into a point-to-point channel. We assume CT1 and CT2’s transmission power to be $(P^d, 0)$ and $(0, P^e)$, respectively. When $S_R = 1$, the achievable rate regions for event $d$ and $e$ are $C_{d,s_{T_1},s_{T_2},*}(P^d, 0) = \cup \left\{ \frac{R_1^d}{R_2^d} \leq \log \left(1 + P^d\right) - \frac{P^e}{R_2^e} = 0 \right\}$ and $C_{e,s_{T_1},s_{T_2},*}(0, P^e) = \cup \left\{ \frac{R_1^e}{R_2^e} \leq \log \left(1 + P^e\right) - \frac{P^d}{R_2^d} = 0 \right\}$.

When $S_{T_1} = S_{T_2} = 1$, the channel becomes the basic MAC channel. We assume the trans-
mission power of CT1 and CT2 to be $P_1^f$ and $P_2^f$, respectively. When $S_R = 1$, the achievable rate region for event $f$ is:

$$C_{f,s_{T_1},s_{T_2},*}(P_1^f, P_2^f) = \cup \left\{ \begin{array}{l}
\frac{R_1^f}{R_2^f} \leq \log \left(1 + P_1^f\right) \\
\frac{R_1^f}{R_2^f} \leq \log \left(1 + P_2^f\right) \\
\frac{R_1^f}{R_2^f} \leq \log \left(1 + P_1^f + P_2^f\right)
\end{array} \right\}.$$

The power constraints are $p_d P_1^d + p_f P_1^f \leq P_1$ and $p_e P_2^e + p_f P_2^f \leq P_2$.

Since the transmitters do not have full side information, they can not discriminate the three subcases and have to use the same transmission power, i.e. $P_1^d = P_1^f$, $P_2^e = P_2^f$. The transmission power is bounded as $P_1^d = P_1^f \leq \frac{P_1}{p_d + p_f}$ and $P_2^e = P_2^f \leq \frac{P_2}{p_e + p_f}$.

Although transmissions occur with the probabilities of $p_d$, $p_e$ and $p_f$, effective transmissions occur only when $S_R = 1$, with the probabilities of $p_a$, $p_b$ and $p_c$. Only effective transmissions contribute to system capacity, therefore outer bound 2 is derived as in (6).

3) *Inner bound:* The inner bound can not be calculated directly. However, we can obtain it with the help of Genie information [20]. Suppose a genie provides some additional information about the transmitter states to the receiver every channel use through a genie variable $G$. The idea of genie-assisted lowerbound is that “the improvement in capacity induced by the genie information $G$ cannot exceed the entropy rate of the genie information itself”, i.e., $R_{s_{T_1},s_{T_2},s_R} - R_{S_{T_1},s_{T_2}} \leq H (G|S_R)$.

When $S_{T_1} = 1, S_{T_2} = 0$ or $S_{T_1} = 0, S_{T_2} = 1$, the MAC channel degrades to a point-to-point channel. Following the idea of genie-assisted bound, the inner bounds for the two subcases are $R_1 \geq R_1^* - H (G_1|S_R)$ and $R_2 \geq R_2^* - H (G_2|S_R)$. When $S_{T_1} = 1, S_{T_2} = 1$, the lowerbound on sum rate is $R_1 + R_2 \geq R_1^* + R_2^* - H (G_1, G_2|S_R)$, where $(R_1^*, R_2^*)$ denotes the maximal rate pair.
in outer bound 2. To sum up, \( \Delta R_1 \leq p_a H (G_1 | S_R) \leq p_a H (S_{T_1} | S_R) \), \( \Delta R_2 \leq p_b H (G_2 | S_R) \leq p_b H (S_{T_2} | S_R) \), and \( \Delta (R_1 + R_2) \leq p_c H (G_1, G_2 | S_R) \leq p_c H (S_{T_1}, S_{T_2} | S_R) \). Thus, the overall inner bound is obtained as in Theorem 6.

Remarks: According to Theorem 6, in event \( d, e \) and \( f \), the inner bounds are \( H (G_1 | S_R) \), \( H (G_2 | S_R) \) and \( H (G_1, G_2 | S_R) \) bits per channel use lower than the outer bounds with full side information at the receiver, respectively.

The transmitters only need to send one bit notification to the receiver when there is a change of PU states. We assume that, on average, the PU activities at the transmitters change every \( N \) time slots. Then the genie information rate is at most \( 1/N \) bit per time slot for each transceiver pair. Formally, the gap between outer bound 2 and inner bound is restricted as \( H (G_1 | S_R) = H (G_2 | S_R) \leq H (G_1) = H (G_2) = \frac{1}{N} \) and \( H (G_1, G_2 | S_R) \leq H (G_1, G_2) = \frac{2}{N} \).

Moreover, when the side information between the transmitters and receiver are highly correlated, \( H (S_{T_1} | S_R) \), \( H (S_{T_2} | S_R) \) and \( H (S_{T_1}, S_{T_2} | S_R) \) also converge to zero, and so do \( H (G_1 | S_R) \), \( H (G_2 | S_R) \) and \( H (G_1, G_2 | S_R) \). Therefore, we conclude that the outer bounds and inner bound coincide when the states of PU signals change very slowly or the side information between the transmitters and receiver are highly correlated.

D. Effect of correlation and PU occupation rate

We take the optimized sum rate in Corollary 1 as an example, and analyze how it is influenced by two system parameters within a specific probabilistic model. We assume the PU occupation rates at both transmitters and the receiver to be the same, \( \mu = 1 - E [S_{T_1}] = 1 - E [S_{T_2}] = 1 - E [S_R] \); and the mutual correlation coefficient between the three states also to be the same, as \( \rho \).

It is relatively straightforward to extend the study to the more general scenario regarding channel correlation and PU activity. According to the definition of correlation coefficient, the joint probability distribution is denoted in Table I, where \( p_0 = \mu [\mu^2 + \rho (1 - \mu^2)] \), \( p_1 = (1 - \rho^2) (1 - \mu) \mu^2 \), \( p_2 = (1 - \rho) \mu [(1 - \mu)^2 + \rho (\mu - \mu^2)] \), and \( p_3 = [(1 - \mu)^2 + \rho (\mu - \mu^2)] (1 - \mu + \mu \rho) \). The probabilities for the six events in Definition 1 can be expressed as functions of \( \mu \) and \( \rho \):

\[
\begin{aligned}
  p_a &= p_b = p_2, \\
  p_c &= p_3, \\
  p_d &= p_e = p_1 + p_2, \\
  p_f &= p_2 + p_3.
\end{aligned}
\]

Combining (14) and (15), the sum rate can be re-written in the forms of \( \rho \) and \( \mu \):

\[
(R_1 + R_2)_{\text{max}} = p (\mu, \rho) \log \left( 1 + \frac{P_1 + P_2}{p (\mu, \rho)} \right),
\]

(16)
where \( p(\mu, \rho) = (1 + \mu - \mu \rho) [(1 - \mu)^2 + \rho(\mu - \mu^2)] \).

We find that (16) is monotonically increasing with \( p(\mu, \rho) \). Moreover, since \( 0 < \mu, \rho < 1 \),

\[
\frac{\partial p(\mu, \rho)}{\partial \mu} = \mu(1 - \rho)^2 (3\mu - 2) - 1 < 0, \\
\frac{\partial p(\mu, \rho)}{\partial \rho} = 2\mu^2 (1 - \rho) (1 - \mu) > 0.
\]

Thus (16) is a monotonically decreasing function of \( \mu \) and monotonically increasing function of \( \rho \). The insights here are that the sum rate increases when the PU is less active and when the correlation among the side information is stronger.

E. Extension to Interference and Fading model

We first consider a natural extension of (1), in which the secondary receiver keeps on all the time. It is reasonable as the receiving process will not cause interference to the PU system. Our intension is to examine how much gain we can obtain by allowing the secondary receiver to remain receiving even in the presence of PU interference. Thus the received signal falls in one of the following two categories:

\[
Y = \begin{cases} 
S_T_1 X_1 + S_T_2 X_2 + Z, & S_R = 1, \\
S_T_1 X_1 + S_T_2 X_2 + I, & S_R = 0,
\end{cases}
\]

where \( I \) denotes the PU interference plus noise, which is modeled as a Gaussian variable with variance \( P_I \). \( S_T_1, S_T_2 \) and \( S_R \) are defined the same as in the three-switch model (1).

Following the same line in Corollary 1 (Section IV.C), the maximal sum rate for cognitive MAC channel with interference is obtained after solving the convex optimization problem:

\[
(R_1 + R_2)_{\text{max}} = \max_{P'_1 + P'_2 + P''_1 + P''_2 \leq P_1 + P_2} \left[ p_{ni} \log \left( 1 + \frac{P'_1 + P'_2}{p_{ni}} \right) + p_i \log \left( 1 + \frac{P''_1 + P''_2}{p_i P_I} \right) \right],
\]

where \( p_{ni} \triangleq p_a + p_b + p_c \) and \( p_i \triangleq p_d + p_e + p_f - (p_a + p_b + p_c) \) are the probabilities with and without PU interference at the receiver, respectively. \( P'_1 + P'_2 \) is the power spent when \( S_R = 1 \), and \( P''_1 + P''_2 \) is the power spent when \( S_R = 0 \), both of which are under the average power constraints. It can be shown that the above maximum is achieved when

\[
P''_1 + P''_2 = \left( \frac{p_i (P_1 + P_2) - p_{ni} P_i (P_I - 1)}{p_{ni} + p_i} \right) ^+, \\
P'_1 + P'_2 = P_1 + P_2 - (P''_1 + P''_2).
\]

From (19), we observe that when \( p_{ni} (P_I - 1) > P_1 + P_2 \), the sum-rate optimal strategy is to avoid transmission when PU is active, and this is identical to the three-switch model. Also as
expected, it suggests that when $P_I$ is large or $p_i$ is small, performance gain through decoding in the presence of PU would be very limited. Numerical results in Section V validate our analysis, where marginal performance improvement is observed even when $P_I$ is not large.

We continue to include channel fading into consideration, and explore the following model

\[
Y = \begin{cases} 
S_T^1 \sqrt{H_1} X_1 + S_T^2 \sqrt{H_2} X_2 + Z, & S_R = 1, \\
S_T^1 \sqrt{H_1} X_1 + S_T^2 \sqrt{H_2} X_2 + I, & S_R = 0,
\end{cases} \quad (20)
\]

where $\sqrt{H_1}$ and $\sqrt{H_2}$ are the channel gains between the cognitive transmitters and receiver.

By letting $\sqrt{h_1} = \frac{S_T^1 \sqrt{H_1}}{S_R - (S_R - 1) \sqrt{P_I}}$ and $\sqrt{h_2} = \frac{S_T^2 \sqrt{H_2}}{S_R - (S_R - 1) \sqrt{P_I}}$, the channel model can be normalized as $Y = \sqrt{h_1} X_1 + \sqrt{h_2} X_2 + Z$.

Define $h = [h_1, h_2]$ as the normalized channel gain vector and consider all fading possibilities, whose cdf and pdf are denoted by $F_i(h_i)$ and $f_i(h_i)$, respectively. The achievable rate region of the cognitive MAC fading channel [24] can be expressed as:

\[
C_{\text{fading}}(P_1, P_2) = \bigcup_{P \in \mathcal{F}} \left\{ \begin{array}{l}
R_1 \leq E_h \left[ \log (1 + h_1 P_1 (h)) \right] \\
R_2 \leq E_h \left[ \log (1 + h_2 P_2 (h)) \right] \\
R_1 + R_2 \leq E_h \left[ \log \left( 1 + \sum_i h_i P_i (h) \right) \right]
\end{array} \right\}, \quad (21)
\]

where $\mathcal{F} \equiv \{ P : E_h [P_i(h)] \leq P_i, \forall i \in 1, 2 \}$ is the set of all possible power allocation schemes within the power constraints.

Now, we discuss the optimal rate/power allocation under fading and interference when the PU states and channel gains are known to both the transmitters and receiver. According to [24], when global state information is available, the maximum sum rate $R_1 + R_2$ can be optimized over all possible power allocation schemes. The solution has the form of:

\[
P_1^* (h, \lambda) = \begin{cases} 
\left( \frac{1}{2 \lambda_1} - \frac{1}{h_1} \right)^+, & h_1 \geq \frac{\lambda_2}{\lambda_1} h_2, \\
0, & \text{else},
\end{cases}
\]

\[
P_2^* (h, \lambda) = \begin{cases} 
\left( \frac{1}{2 \lambda_2} - \frac{1}{h_2} \right)^+, & h_2 \geq \frac{\lambda_1}{\lambda_2} h_1, \\
0, & \text{else},
\end{cases}
\]

where the constant vector $\lambda = [\lambda_1, \lambda_2]$ is determined by the following average power constraints:

\[
\int_0^\infty \left( \frac{1}{2 \lambda_1} - \frac{1}{h} \right)^+ F_1 \left( \frac{\lambda_1 h}{\lambda_2} \right) f_1 (h) dh \leq P_1,
\]
\[ \int_0^\infty \left( \frac{1}{2\lambda_2} - \frac{1}{h} \right)^+ F_2 \left( \frac{\lambda_2}{\lambda_1} h \right) f_2(h) \, dh \leq P_2. \]

**Remarks:** From (22), we can see that for the on/off cognitive MAC channel with PU interference and fading, the optimal rate/power allocation is given by the generalized time-domain water-filling, which is performed on the generalized parameters \( h_1 \) and \( h_2 \), taking into account the factors of the PU states and PU interference.

When \( h_1 \) and \( h_2 \) are identically distributed, \( \lambda_1 = \lambda_2 \) by symmetry. An observation from (22) is that when the channel gain of one SU transmitter is worse than another, the transmission will be turned off to save power. In other words, the two SU transmitters will transmit simultaneously with equal power only when their channel gains are the same. Therefore, the cognitive MAC channel with fading and interference can be further simplified into the following model:

\[
Y = \begin{cases} 
S'_{T_1} X_1 + S'_{T_2} X_2 + Z', & S_R = 1, \\
S'_{T_1} X_1 + S'_{T_2} X_2 + I', & S_R = 0,
\end{cases} \quad (23)
\]

where \( S'_{T_1} = \begin{cases} 1, & S_{T_1} = 1 \text{ and } H_1 \geq H_2, \\
0, & S_{T_1} = 0 \text{ or } H_1 < H_2, \end{cases} \) and \( S'_{T_2} = \begin{cases} 1, & S_{T_2} = 1 \text{ and } H_1 \leq H_2, \\
0, & S_{T_2} = 0 \text{ or } H_1 > H_2, \end{cases} \) are the generalized binary transmitter side information, \( Z' = \frac{Z}{\max\{\sqrt{H_1}, \sqrt{H_2}\}} \) and \( I' = \frac{I}{\max\{\sqrt{H_1}, \sqrt{H_2}\}} \) are the generalized noise and interference. Note that (23) has the same form as (17), so similar methods can be employed to obtain the maximum sum rate.

**F. Extension to the m-user Case**

Throughout this paper, we focus on the three-switch channel where we only consider two transmitters. However, the results and analysis can be generalized to the \( m \)-user case. Here we provide a brief discussion. The \( m \)-user multi-switch model can be expressed as (c.f. (1))

\[
Y = \left( \sum_{i=1}^m S_{T_i} X_i + Z \right) S_R. \quad (24)
\]

We denote \( M \subseteq \{1, 2, \cdots, m\} \) as the set of transmitters under consideration, and \( M^c \) as its complement. Let \( R(M) = \sum_{i \in M} R_i \) be the sum rate of set \( M \). Let \( U(M) = \{ U_i : i \in M \} \), \( X(M) = \{ X_i : i \in M \} \) and \( S_T(M) = \{ S_{T_i} : i \in M \} \). The other definitions are the same as in the two-user case.

The achievable region with independent general side information is given by:

\[
\bigcup_{P_{\text{causal}}} \{ R : R(M) \leq I(U(M); Y, S_R | U(M^c)) \}.
\]
for the causal case, where \( p_{\text{causal}} = \prod_{i=1}^{m} p(U_i)p(X_i|U_i,S_{T_i}); \) and

\[
\bigcup_{p_{\text{non-causal}}} \left\{ R : R(M) \leq I(U(M);Y,S_R|U(M^c)) - \sum_{i \in M} I(U_i;S_{T_i}) \right\}
\]

for the non-causal case, where \( p_{\text{non-causal}} = \prod_{i=1}^{m} p(U_i|S_{T_i})p(X_i|U_i,S_{T_i}) \) for non-causal case.

Following the same lines in the proofs of Theorems 1-2, the achievable region for the \( m \)-user cognitive MAC channel with independent on/off side information can be expressed as:

\[
\bigcup \{ R : R(M) \leq I(X(M);Y,S_R|X(M^c)) \}
\]

for the causal case, and

\[
\bigcup \left\{ R : R(M) \leq I(X(M);Y,S_R|X(M^c)) - \sum_{i \in M} I(X_i;S_{T_i}) \right\}
\]

for the non-causal case.

Following the same lines in the proofs of Theorem 4-6, the outer and inner bounds for the \( m \)-user case can be obtained. We define \( S_T = \{ S_{T_1}, S_{T_2}, \ldots, S_{T_m} \} \in S_T \) as the \( m \)-length binary state vector of the \( m \) transmitters, where \( S_T : |S_T| = 2^m \) is the set of all possible transmitter states. We also define \( M_{S_T} = \{ i : S_{T_i} = 1 | S_T \} \) as the set of active transmitters when the state vector is \( S_T \).

Correspondingly, outer bound 1 which assumes global side information is

\[
C_{\text{outer 1}}^{\text{m}}(P_1,P_2,\ldots,P_m) = \bigcup_{C_1} \left\{ R : R(M) \leq \sum_{S_T \in S_T} p(S_T,S_R = 1) \log \left( 1 + \sum_{i \in M \cap M_{S_T}} p_i^{S_T,S_R = 1} \right) \right\},
\]

where \( C_1 \) is the power constraint given by

\[
C_1 \triangleq \left\{ P : \sum_{S_T \in S_T} p(S_T,S_R = 1)p_i^{S_T} \leq P_i, \forall i \in \{1,2,\ldots,m\} \right\}.
\]

Outer bound 2 which assumes full side information at the receiver is

\[
C_{\text{outer 2}}^{\text{m}}(P_1,P_2,\ldots,P_m) = \bigcup_{C_2} \left\{ R : R(M) \leq \sum_{S_T \in S_T} p(S_T,S_R = 1) \log \left( 1 + \sum_{i \in M \cap M_{S_T}} p_i^{S_T} \right) \right\},
\]

where \( C_2 \) is the power constraint expressed as

\[
C_2 \triangleq \left\{ P : \sum_{S_T \in S_T} p(S_T)p_i^{S_T} \leq P_i, \forall i \in \{1,2,\ldots,m\} \right\}.
\]

Similarly, the inner bound is obtained as

\[
C_{\text{inner}}^{\text{m}}(P_1,P_2,\ldots,P_m) = \{ R : R(M_{S_T}) = R_{\text{outer 2}}(M_{S_T}) - \Delta R(M_{S_T}) \},
\]
where $R_{\text{outer}}^2(M_{ST})$ is the maximal sum rate of transmitter set $M_{ST}$ in outer bound 2, and $\Delta R(M_{ST}) \leq \rho(S_T, S_R = 1) H(S_T(M) | S_R)$ is the capacity gap.

V. Numerical Results

A. Outer and Inner Bounds

To plot the outer and inner bounds, we travel through all possible power pairs which satisfy the power constraints, and compute the corresponding rate pairs. As for the inner bound, we calculate the gap between outer bound 2 and inner bound and subtract it from outer bound 2.

Fig. 3 to Fig. 6 show the outer bound 1, 2 and the inner bound under different PU occupation rates and PU states correlation coefficients $\rho$. The parameters are $(\mu = 0.1, \rho = 0)$, $(\mu = 0.1, \rho = 0.9)$, $(\mu = 0.5, \rho = 0)$ and $(\mu = 0.5, \rho = 0.9)$ for the four figures, respectively. All four cases are under the power constraints of $P_1 \leq 1$ and $P_2 \leq 1$. Based on these parameters, we calculate $p_a$ to $p_f$ according to (15), and obtain the bounds. We also plot the sum rate of outer bound 1 with respect to PU occupation rate $\mu$ and correlation coefficient $\rho$ on Fig. 7 to Fig. 8, in which the sum rate corresponds to the corner points in Fig. 3 to Fig. 6.

The insights obtained from these results are summarized below.

1) Correlation coefficient $\rho$: By comparing Fig. 3 with Fig. 4, and Fig. 5 with Fig. 6, we find gaps between the outer and inner bounds become closer as $\rho$ grows. On the one hand, outer bound 2 approaches outer bound 1 as $\rho$ grows, as the transmitters in Case 2 gradually have global side information, and Case 2 evolves to Case 1. On the other hand, inner bound approaches outer bound 2 because $H(S_{Ti}(t) | S_R(t))$ decreases as $\rho$ grows. In addition, Fig. 8 also demonstrates that the maximal sum rate increases with $\rho$, which indicates more correlation improves the overall performance.

2) PU occupation rate $\mu$: By comparing Fig. 3 with Fig. 5, and Fig. 4 with Fig. 6, we observe that as $\mu$ decreases, the overall system throughput grows. This is further verified in Fig. 7. When $\mu = 0$, the PUs keep silent, and the cognitive MAC rate region evolves to the traditional MAC rate region. When $\mu = 1$, the spectrum is always occupied by PUs, the cognitive MAC rate degrades to zero.

3) State changing rate: Fig. 3 to Fig. 6 show that, the inner bound are closer to the outer bounds when the $N$ increases. This indicates that, in the case where the PU states change slowly, the inner bound approaches outer bound 2.
4) Transmitter Side information: Compared with Case 1, the transmitters in Case 2 lack global side information. Hence, the big gap between outer bound 1 and 2 implies the importance of the transmitter side information. Moreover, this gap is larger in Fig. 5 than in Fig. 3, which indicates that the transmitter side information is more valuable in busy channels.

B. Optimized Sum Rate in Interference Model

We now provide numerical results for the cognitive MAC channel when the receiver is exposed to PU interference (c.f. (17)), which is discussed in Section VI.E. The joint probabilities are shown in Table I. In addition, we assume that the power constraints are $P_1 \leq 1$ and $P_2 \leq 1$. The maximal sum rates with respect to the PU interference power $P_I$, PU occupation rate $\mu$ and the correlation coefficient $\rho$ are plotted in Fig. 9 and Fig. 10, which are computed according to (15), (18), and (19).

Suppose that the receiver employs an energy detector with a threshold $2W$, through which it determines if a PU is present. Fig. 9 shows that the advantage of keeping the receiver always on (and decoding) over the original three-switch model is significant only for small $P_I$ values and when $\mu$ is large (i.e., the PU is active). Of course, the performance of model (17) is no worse than that of (1) concerning the optimal sum rate, due to the optimal rate/power allocation, which essentially turns off transmission in the face of severe interference.

In Fig. 10, $P_I$ is selected as $2W$, $5W$ and $10W$ and the following two insights on the additional benefit brought by allowing the secondary receiver to remain receiving even in the presence of PU interference are further confirmed.

1) It is rewarding to turn on the receiver’s switch only when the PUs are very active: when $\mu$ is large, the additional time period that the receiver can decode is longer, which results in more benefit. As seen, the additional benefit is obvious only when $\mu > 0.5$.

2) More opportunities can be exploited under lower correlation among PU states: this is due to the fact that when $\rho$ is large, the transmitters are likely to keep silent when the receiver is under PU interference.

VI. Conclusion

In this paper, the cognitive MAC channel is modeled as a three-switch channel, and the achievable rate regions are obtained when viewing PU activities as causal/non-causal on/off side
information. The closed form outer and inner bounds are derived, which are shown to be tight under some special cases. A rate allocation scheme is also proposed to maximize the sum rate with global side information, and the effect of correlation in side information and PU activities is analyzed. The extension to the fading scenario and general $m$-user case is discussed. The numerical results show the importance of transmitter side information in enhancing the capacity and the effectiveness of our rate allocation scheme.

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### TABLE I

**Joint Probability Distribution under Non-fading and Fading Models**

| $S_R = 0$ | 
|---|---|
| $S_{T_1}$ | $S_{T_2}$ | 0 | 1 |
| 0 | $p_0$ | $p_1$ |
| 1 | $p_1$ | $p_2$ |

| $S_R = 1$ | 
|---|---|
| $S_{T_1}$ | $S_{T_2}$ | 0 | 1 |
| 0 | $p_1$ | $p_2$ |
| 1 | $p_2$ | $p_3$ |

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Fig. 1. Memoryless Cognitive MAC Channel

Fig. 2. Three Switch MAC Channel
Fig. 3. Capacity region bounds when $\mu = 0.1, P = 1, \rho = 0$

Fig. 4. Capacity region bounds when $\mu = 0.1, P = 1, \rho = 0.9$
Fig. 5. Capacity region bounds when $\mu = 0.5, P = 1, \rho = 0$

Fig. 6. Capacity region bounds when $\mu = 0.5, P = 1, \rho = 0.9$
Fig. 7. Effect of PU activities on sum rate

Fig. 8. Effect of side information’s correlation on sum rate
Fig. 9. Sum rate in interference model v.s. $P_I$

Fig. 10. Sum rate in interference model v.s. $\rho$