Image Quality Analysis of a Novel Histogram Equalization Method for Image Contrast Enhancement

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SUMMARY  The use of image contrast enhancement has become increasingly essential due to the need to better show the visual information contained within the image for all vision-based systems. This has lead to motivation for the design of a powerful and accurate automatic contrast enhancement for a digital image. Histogram equalization is the most commonly used contrast enhancement method. However, the conventional histogram equalization methods usually result in excessive contrast enhancement, which causes the unnatural look and visual artifacts of the processed image. In this paper, we propose a novel histogram equalization method using the automatic histogram separation along with the piecewise transformed function. The contrast enhancement results of the proposed method were not only analyzed through qualitative visual inspection and for quantitative accuracy, but are also compared to the results of other state-of-the-art methods.

key words: contrast enhancement, histogram equalization, quality analysis

1. Introduction

In the last decade, improvement of the visual image quality has been actively developed using the contrast enhancement techniques, which are treated as transforming one image to another so that the look and feel of an image can be improved for visual perception of human beings [1].

Contrast enhancement techniques have become widely available to provide a better transform representation for the most popular image processing systems not only software, but also hardware environments such as Photoshop [2], [3], mobile device, digital camcorder, digital TV, digital camera, and so on [4]–[7]. As a matter of fact, the use of contrast enhancement technology has extremely developed for a critical process [8], which facilitates the extensive research in areas including skull image and mammogram application [9], improvement of arterial visualization [10], tumor microcirculation [11], virtual restoration of ancient Chinese paintings [8], vision impairment estimation [12], recovery of underwater visibility [13], bas-relief generation [14], face recognition [15], and so on.

Naik et al. categorized image contrast enhancement method as contrast slicing, stretching and histogram equalization [16]. Histogram equalization is one of the most commonly used contrast enhancement methods on almost all types of images [17]. This popular method only requires the low computational complexity to facilitate the subsequent high-level operations such as detection and recognition [6], [18]. Generally speaking, histogram equalization methods can be categorized into two main types, global histogram equalization and local histogram equalization [5].

Global histogram equalization approach stretches the contrast of the high histogram region and compresses the contrast of the low histogram region via transform function. This is generated by using the histogram information of the entire input image [19]. Although global histogram equalization method is suitable for overall enhancement, it cannot adapt to local brightness features due to the use of whole histogram information. This causes the limitation in the ratio of contrast stretching in some parts of the image. That is to say, the method may not increase or it may even decrease the local contrast. The visibility of image textures are also reduced by condensing the data range in global view [3], [19]. To overcome such the drawback, many improved HE-based enhancement methods have been proposed [17], [20]–[22].

A direct extension of global histogram equalization is termed local histogram equalization. In order to efficiently improve the visibility of the small-scale, local histogram equalization method divides the input image into several sub-images, which are equalized independently by using the neighborhood histogram information. One common limitation of local histogram equalization is that the problem of over-enhancement in the local image and the noises enhancing artifacts [3], [19].

In this paper, a novel histogram equalization method is presented to enhance the contrast without losing the original histogram characteristics. It is expected to eliminate the above drawbacks of conventional global-based method and local-based method effectively. The organization of the proposed method is as follows:

1) Separation of histogram based on our proposed weighting mean function and automatic determination of recursion level.
2) Achieving contrast enhancement by equalizing sub-histogram respectively in small-scale detail.

Experimental results show that the proposed method performs more naturally than other state-of-the-art methods in a wide range of natural digital images. This will be demonstrated by qualitative and quantitative evaluations.

The rest of this paper is organized as follows. Our proposed method is reported in Sect. 2. In Sect. 3, the experimental results of our proposed method and other existing methods are presented and compared fairly. Section 4 con-
tains our concluding remarks.

2. Proposed Method

Contrast represents the measurement of intensity variation between the interesting region and the other parts of an image. Contrast enhancement can be achieved by adjusting the pixel intensity in the different regions of the image, so that the details in the interesting region can be brought out and revealed to the observers. For HE-based method, pixel intensity adjustment is performed by illustrating the transformed mapping curve in pixel by pixel fashion.

In this section, Automatic Weighting Mean-separated Histogram Equalization (AWMHE) method is proposed to perform the effective contrast enhancement of the digital image. Based on gray-level with our proposed weighting mean function, the AWMHE method separates an input image into the sub-images, which are determined precisely by using the global and local histogram information.

1) Automatic histogram separation: Suppose that an input image \( W \) is composed of \( V \) discrete gray levels and is denoted by \( \{G_0, G_1, \ldots, G_{V-1}\} \), the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) for an input image \( W \) is expressed as:

\[
PDF(G_h) = \frac{n^h}{n}, \quad \text{where} \quad h = 0, 1, \ldots, V - 1. \tag{1}
\]

\[
CDF(G_j) = \sum_{j=0}^{h} (PDF(G_j)), \tag{2}
\]

where \( n^h \) denotes the number of pixels that correspond to the value \( h \), and \( n \) is the total number of the pixels in the input image \( W \).

Then the weighting mean value \( X_t \) can be calculated by using our proposed weighting mean function, which can be expressed as follows:

\[
X_t = \frac{\sum_{i=a}^{b}(l \times CDF(l))}{\sum_{i=a}^{b}(CDF(l))}, \tag{3}
\]

where \([a, b]\) represents the sub-interval of histogram, \( l \) represents the corresponding gray-level, and \( t \) represents the recursion level. Notice that the sub-interval \([a, b]\) is initialized as \([0, 255]\) and \( t \) is initialized as 1.

In order to enhance the input image effectively, the appropriate number of sub-images should be determined accurately. Therefore, we consider that the existing subhistograms defined over a gray-level range \([X_a, X_{r+1}]\) at the recursion level \( r \) is \( 0 \leq r \leq t - 1 \). Suppose that the boundary \([X_0, X_t]\) is set at \([0, 256]\) and the range \([a, b]\) is defined as \([X_{a}, X_{r+1} - 1]\), we calculate each new mean value \( X_t \) for dividing the histogram by using our proposed weighting mean function (3). The optimum recursion level of histogram separation can be found out when the calculated maximum mean value \( X_t \) equals 255.

2) Piecewise transformed function: After the ideal weighting mean values are determined, we can directly decide the optimum number of sub-images based on the histogram separation, which can be expressed as follows:

\[
W_k = \{W(x, y)|X_k < W(x, y) \leq X_{k+1}, \forall W(x, y) \in W\}, \tag{4}
\]

where \( W_k \) represents each sub-image, and \( k = 0, 1, 2, \ldots, t - 1 \).

Then the relationship between gray-level \( G \) and each sub-image \( W_k \) is defined as the respective PDF.

\[
PDF_k(G_h) = \frac{n^h}{n_k}, \tag{5}
\]

where \( h = X_k + 1, X_k + 2, \ldots, X_{k+1} \).

Then the respective PDF is defined for each sub-image based on the respective PDF:

\[
CDF_k(G_h) = \sum_{j=X_k+1}^{h} (PDF_k(G_j)). \tag{6}
\]

The Sigma formula of function (6) can be removed according to the relationship between respective CDF and PDF. The function (6) can be re-written as:

\[
CDF_k(G_h) = CDF_k(G_{h-1}) + PDF_k(G_h), \tag{7}
\]

where \( h = X_k + 1, X_k + 2, \ldots, X_{k+1} \).

Finally, the piecewise transformed function is used to map the equalized image. This is characterized by utilizing CDF of sub-image \( W_k \) for \( k \) segments. Then the transformed function \( T_k \) is defined as below:

\[
T_k(G_h) = X_k + (X_{k+1} - X_k) \times CDF_k(G_h), \tag{8}
\]

where \( k = 0, 1, \ldots, t - 1 \) and \( h = X_k + 1, X_k + 2, \ldots, X_{k+1} \).

For achieving our objective of contrast enhancement, the flowchart of our proposed approach involves an automatic histogram separation stage and a piecewise transformed function stage as shown in Fig. 1.

As shown on the left-top in Fig. 1, the original image has insufficient contrast between human face and background.

Based on automatic histogram separation, the original histogram is divided into eight sub-histograms. In order to preserve the mean brightness of original histogram characteristics, the mapping curve can be illustrated independently by using piecewise transformed function along with the resultant sub-histograms.

Next, the hybrid mapping curve can be illustrated by using piecewise transformed function and the separated sub-histograms. As shown on the right-bottom in Fig. 1, the solid line represents the mapping curve of the proposed method, and the dotted line represents the mapping curve.
of conventional HE. As can be seen, our method has a more smooth curve compared to the conventional HE. Note that $x$-coordinate represents the original intensity of the input image, and $y$-coordinate represents the transformed intensity after contrast enhancement is achieved.

Based on our proposed smooth mapping curve, the equalized image is produced. As a result shown on the left-bottom in Fig. 1, the equalized image has a more perceivable contrast between human face and background than that of the original image.

3. Experimental Results

In order to demonstrate the benefits of the proposed method, we have simulated the HE, brightness preserving bi-histogram equalization (BBHE) [17], dualistic sub-image histogram equalization (DSIHE) [20], and recursive sub-image histogram equalization (RSIHE) [2] with a variety of original digital images to illustrate the ability of the proposed approach. The enhancement results are analyzed in two ways:

1. Qualitative comparison of enhancement results of each method.
2. Quantitative evaluation for all test images through the use of three metrics.

As robust visual quality is extremely desirable for the contrast enhancement of the original digital image, some specific properties are indicative of an effective contrast enhancement method. These are the probability of erroneous associations due to noise and unsuitable illumination changes created by over-enhancement.

Figure 2 shows the original Einstein image and the corresponding contrast enhanced versions (Figs. 2 (b)-(f)). Figure 2 (b) is the traditional histogram equalized image of the original Einstein image. The contrast of the image is maximized at the expense of the unsuitable illumination changes. Brighter regions become even brighter, darker regions get even darker. BBHE [17], DSIHE [20], and RSIHE [2] use the local histogram information to reduce the effect of HE. However, their histogram equalized images still have some flavors of HE: the hair, and the clothing are still darker and the bright face and background region are unnatural look. This situation can be easily observed in images of Figs. 2 (c)-(e). Figure 2 (f) shows the enhancement result of the proposed AWMHE method. Compared to the other methods, our proposed AWMHE method gives a more natural brightness improvement on the hair and face in the circled region.

Figure 3 shows the original Astro image and the corresponding contrast enhanced versions (Figs. 3 (b)-(f)). Figure 3 (b) is the traditional histogram equalized image of Astro image shown in Fig. 3 (a). This image still has non-uniform illumination. Unnatural look of the histogram equalized images are also appeared using BBHE [17], DSIHE [20], and RSIHE [2]. The equalized images of these methods exhibit black noise around the ground. Figure 3 (f) shows the histogram equalized Astro image using AWMHE. Again, our proposed method preserves the mean brightness and achieves the natural brightness improvement of the original image. Thus, the equalized image has a more perceivable contrast between the neighborhood of the circled...
Fig. 2  The “Einstein” digital image: (a) is the original image; the remaining five images are equalized images generated using (b) the traditional HE method, (c) the BBHE method [17], (d) the DSIHE method [20], (e) the RSIHE method [2], and (f) the proposed AWMHE method.

Fig. 3  The “Astro” digital image: (a) is the original image; the remaining five images are equalized images generated using (b) the traditional HE method, (c) the BBHE method [17], (d) the DSIHE method [20], (e) the RSIHE method [2], and (f) the proposed AWMHE method.
prominence and the un-circled ground.

In addition to qualitative evaluation, quantitative accuracy is also evaluated for our AWMHE method and other state-of-the-art methods by using Peak Signal-to-Noise Ratio ($PSNR$) [22], Absolute Mean Brightness Error ($AMBE$) [22], and measure of enhancement ($EME$) [23]. The $PSNR$ is defined as follows:

$$PSNR = 20\log_{10} \frac{255}{RMS\ E},$$  \hspace{1cm} (9)

where $RMS\ E$ is root mean-squared error, defined as

$$RMS\ E = \sqrt{\frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (W(x,y) - \hat{W}(x,y))^2}{MN}}.$$  \hspace{1cm} (10)

Here $W$ and $\hat{W}$ are the original and enhanced images, of size $M \times N$, respectively. Note that higher $PSNR$ value represents greater image quality.

Table 1 lists the $PSNR$ values of each method for 15 test images. Due to our proposed AWMHE method enhances the image by equalizing the optimum subhistograms in small-scale detail, the enhanced image of AWMHE method produces neither noise artifacts nor over-enhancement. Therefore, our proposed method achieves the highest $PSNR$ values. The performance of brightness preservation is rated by an objective measurement $AMBE$. It is defined as the absolute differential gray-level mean between the original image and enhanced image.

$$AMBE = |W_e - \hat{W}_e|.$$  \hspace{1cm} (11)

$W_e$ and $\hat{W}_e$ denote the gray-level mean of the original and enhanced image, respectively. Note that lower $AMBE$ value indicates that the brightness is better preserved.

Table 2 lists the $AMBE$ values of each method for 15 test images. The proposed AWMHE method enhances the image using the piecewise transformed function along with the entire mean brightness preservation. Therefore, our method achieves the lowest $AMBE$ values among all.

In order to further verify the effectiveness of our proposed method, we use $EME$ metric to evaluate the local contrast of the enhanced image. For $EME$ metric [23], the difference between the maximum intensity and minimum intensity within each local region is calculated as a natural log ratio by using non-overlapping block separation. The $EME$ metric is defined as follows:

Table 1: The comparison of $PSNR$ Values between each method.

| Test images | HE   | BBHE | DSIHE | RSIHE | AWMHE |
|-------------|------|------|-------|-------|-------|
| Einstein    | 15.0754 | 15.2986 | 15.6117 | 19.6596 | 29.7015 |
| Astro       | 14.2673 | 14.2722 | 14.2754 | 18.1372 | 32.2136 |
| Elaine      | 18.6226 | 18.7900 | 18.8154 | 23.7119 | 38.7714 |
| Peppers     | 19.2244 | 19.7440 | 19.7234 | 23.7150 | 33.5822 |
| Lena        | 16.6746 | 19.6195 | 19.3112 | 24.4033 | 36.8307 |
| Sea         | 16.5857 | 19.8882 | 19.1558 | 22.3420 | 39.5756 |
| Boats       | 17.9821 | 18.0720 | 18.0801 | 21.7637 | 29.0152 |
| Bee         | 21.8731 | 24.7989 | 24.2499 | 26.8737 | 36.7929 |
| Woman       | 17.8270 | 17.7926 | 18.3150 | 22.6265 | 30.9228 |
| Donna       | 14.3606 | 16.2868 | 16.8542 | 21.2863 | 32.6153 |
| House       | 15.1822 | 18.8001 | 18.1392 | 23.6839 | 32.9613 |
| Tulips      | 20.7408 | 22.0821 | 21.8099 | 29.8857 | 38.2261 |
| Vacas       | 16.3450 | 18.1387 | 18.2614 | 23.4952 | 34.6954 |
| Road        | 15.0727 | 14.7096 | 15.0935 | 19.6049 | 26.1478 |
| City        | 15.3858 | 16.7364 | 16.0510 | 20.7480 | 31.9631 |

Table 2: The comparison of $AMBE$ Values between each method.

| Test images | HE   | BBHE | DSIHE | RSIHE | AWMHE |
|-------------|------|------|-------|-------|-------|
| Einstein    | 21.0754 | 17.2332 | 11.7608 | 8.9768 | 0.0604 |
| Astro       | 1.2535  | 6.5153 | 3.5019 | 2.8664 | 1.0503 |
| Elaine      | 8.1194  | 4.9531 | 4.3490 | 2.9212 | 0.3986 |
| Peppers     | 12.7117 | 5.8611 | 5.9838 | 5.7416 | 1.1190 |
| Lena        | 29.6081 | 12.7515 | 14.1876 | 6.3683 | 1.1365 |
| Sea         | 49.3977 | 14.6684 | 20.2889 | 11.6449 | 1.8507 |
| Boats       | 2.0395  | 18.8947 | 10.3940 | 8.6490 | 1.2506 |
| Bee         | 26.4450 | 7.8383 | 11.3143 | 7.0931 | 1.4856 |
| Woman       | 15.3867 | 15.6674 | 11.0657 | 2.2673 | 1.2600 |
| Donna       | 41.1844 | 27.0488 | 23.6191 | 7.9339 | 2.1020 |
| House       | 30.7102 | 8.1792 | 10.8071 | 1.8356 | 1.2388 |
| Tulips      | 10.8185 | 1.6746 | 3.4691 | 1.2110 | 0.0062 |
| Vacas       | 27.2247 | 14.8441 | 14.0099 | 4.4963 | 1.4093 |
| Road        | 12.6274 | 24.9643 | 11.7955 | 7.2578 | 1.2733 |
| City        | 12.3907 | 15.8077 | 0.5446 | 2.7738 | 1.5040 |
where $W_p$ represents the $p$-th separated image block, \( \max(W_p) \) represents the local maximum intensity value of the $p$-th separated image block, \( \min(W_p) \) represents the local minimum intensity value of the $p$-th separated image block, and $Q$ represents the total number of the image blocks. With respect to the smooth contrast enhancement, lower $EME$ value represents greater enhancement quality.

Table 3 lists the $EME$ values of each method for 15 test images. Based on the proposed smooth mapping curve, our AWMHE method achieves the lowest $EME$ values compared to other methods for all test images.

As shown on Fig. 2 (b) and Fig. 3 (b), the extreme over-enhancement and noise artifacts exist in the enhanced images generated by using HE. This is because HE method can not adapt to pixel intensity of the local region due to the use of global probability and cumulative distribution. However, this method is very simple and powerful based on the use of global histogram information.

BBHE [17] method further improves the overall extreme over-enhancement caused by the use of global histogram of conventional HE based on the local probability and cumulative distribution. As shown on Fig. 2 (c) and Fig. 3 (c), the extreme over-enhancement and noise artifacts still exist. This is because BBHE [17] method only divides the image histogram into two sub-histograms for the local histogram equalization.

As DSIHE [20] method is similar to BBHE [17], it divides the image histogram into two sub-histograms according to the median intensity. However, the enhanced images shown on Fig. 2 (d) and Fig. 3 (d) are also involved with the extreme over-enhancement and noise artifacts.

Figure 2 (e) shows the enhanced image using RSIHE [2] method for Einstein image, and Fig. 3 (e) shows the enhanced image using RSIHE [2] method for Astro image. As can be seen, the extreme contrast enhancement of the human face on Fig. 2 (e), and the black noise artifacts of the ground on Fig. 3 (e) are both reduced. This is because RSIHE [2] method uses the recursive version of DSIHE [20] to divide the image histogram into 2’ sub-histograms for the more detailed local histogram equalization, where $r$ is experimentally set to 2 according to [2]. However, the evident unnatural looking effects still exist in the enhanced images generated by RSIHE [2] method.

As the results of Einstein and Astro images, AWMHE method produces the enhanced images with neither noise artifacts nor over-enhancement shown on Fig. 2 (f) and Fig. 3 (f). This is because our proposed AWMHE method can automatically divide the image histogram into appropriate number of sub-histograms based on the weighting mean function and optimum recursion level procedure. For all test images, our proposed method achieves the highest PSNR values, the lowest AMBE values, and the lowest $EME$ values listed in Tables 1-3. Compared to other state-of-the-art methods, our proposed method can obtain the better enhancement results due to the achievement of local histogram equalization along with appropriate and automatic histogram separation in small-scale detail.

### 4. Conclusions

We have presented a novel histogram equalization method for image contrast enhancement. First the histogram is divided into several sub-histograms automatically by using our proposed weighting mean function. Second, each piece of the resultant sub-histograms is equalized with the piecewise transformed function to achieve complete contrast enhancement in small-scale detail. Experimental results show that the proposed method produces effective and robust enhancement in a variety of test images, and achieves the best quality through qualitative visual inspection and quantitative accuracies of $PSNR$, $AMBE$, and $EME$ compared to other state-of-the-art methods.

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