THE UNITED PROOFS FOR THREE $q$-EXTENSIONS OF DOUGALL’S $2H_2$ SUMMATION FORMULA

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Abstract. In terms of the analytic continuation method, we give the united proofs for three $q$-extensions of Dougall’s $2H_2$ summation formula. Some related results are also discussed in this paper.

1. Introduction

For a complex number $x$, define the gamma function by Euler’s integral

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt \quad \text{with} \quad \Re(x) > 0.$$ 

Then the shifted factorial can be expressed as

$$(x)_n = \frac{\Gamma(x + n)}{\Gamma(x)},$$

where $n$ is an arbitrary integer.

Following Slater [21], define the bilateral hypergeometric series to be

$$rH_s\left[ \begin{array}{c} a_1, a_2, \cdots, a_r \\ b_1, b_2, \cdots, b_s \end{array} \right| z \right] = \sum_{k=-\infty}^{\infty} \frac{(a_1)_k(a_2)_k\cdots(a_r)_k}{(b_1)_k(b_2)_k\cdots(b_s)_k} z^k.$$

In 1907, Dougall [11] derived the beautiful identity

$$2H_2\left[ \begin{array}{c} a, b \\ c, d \end{array} \right| 1 \right] = \frac{\Gamma(1-a)\Gamma(1-b)\Gamma(c)\Gamma(d)\Gamma(c+d-a-b-1)}{\Gamma(c-a)\Gamma(c-b)\Gamma(d-a)\Gamma(d-b)},$$

provided $\Re(c+d-a-b) > 1$, according to the contour integral method. Different proofs of it can be found in [1] Section 2.8, [9], [20], [21] Section 6.1, and [22].

Subsequently, define the $q$-shifted factorial by

$$(x; q)_\infty = \prod_{i=0}^{\infty} (1-xq^i), \quad (x; q)_n = \frac{(x; q)_\infty}{(xq^n; q)_\infty},$$

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where \( x, q \) are complex numbers satisfying the condition \( |q| < 1 \) and \( n \) is an arbitrary integer. For convenience, we shall adopt the two simplified notations:

\[
\begin{align*}
(x_1, x_2, \cdots, x_r; q)_\infty &= (x_1; q) (x_2; q)_\infty \cdots (x_r; q)_\infty, \\
(x_1, x_2, \cdots, x_r; q)_n &= (x_1; q)_n (x_2; q)_n \cdots (x_r; q)_n.
\end{align*}
\]

Following Gasper and Rahman [12], define the bilateral basic hypergeometric series to be

\[
\begin{align*}
r\psi_s \left[ \begin{array}{c}
a_1, a_2, \cdots, a_r \\ b_1, b_2, \cdots, b_s
\end{array} \right| q; z &= \sum_{k=-\infty}^{\infty} \frac{(a_1, a_2, \cdots, a_r; q)_k}{(b_1, b_2, \cdots, b_s; q)_k} \left( (-1)^k q^{k^2} \right)^n z^k.
\end{align*}
\]

Thus Ramanujan’s \( 1 \psi_1 \) summation formula (cf. [12 Appendix II.29]) can be stated as

\[
1 \psi_1 \left[ \begin{array}{c}
a \\ c
\end{array} \right| q; z = \frac{(q, c/a, az, q/az; q)_\infty}{(c, q/a, az, q/az; q)_\infty},
\]

provided \( |c/a| < |z| < 1 \). Equation (2) is very important in the theory of special functions. Several beautiful proofs of it can be enjoyed in the papers [2, 3, 4, 8, 17].

Theorem 1. Let \( a, b, c, d \) be complex numbers. Then

\[
2 \psi_2 \left[ \begin{array}{c}
a, b \\ c, d
\end{array} \right| q; z &= \frac{(az, c/b, a/qd/abz; q)_\infty}{(z, d, q/b, cd/abz; q)_\infty} 2 \psi_2 \left[ \begin{array}{c}
a, abz/d \\ c, az
\end{array} \right| q; \frac{d}{a},
\]

where \( \max \{|z|, |cd/abz|, |d/a|, |c/b| \} < 1 \).

Theorem 2. Let \( a, b, c, d \) be complex numbers. Then

\[
2 \psi_2 \left[ \begin{array}{c}
a, b \\ c, d
\end{array} \right| q; z = \frac{(q, b, c/a, d/a, az, q/az; q)_\infty}{(c, d, q/a, b/a, z, q/z; q)_\infty} 2 \psi_1 \left[ \begin{array}{c}
qa/c, qa/d \\ qa/b
\end{array} \right| q; \frac{cd}{abz},
\]

+ idem\((a/b)\),

where the convergent condition is \( \max \{|z|, |cd/abz|\} < 1 \) and the symbol idem\((a/b)\) after an expression signifies that the front expression is repeated with \( a \) and \( b \) interchanged.

Theorem 3. Let \( a, b, c, d \) be complex numbers. Then

\[
2 \psi_2 \left[ \begin{array}{c}
a, b \\ c, d
\end{array} \right| q; z = \frac{(q, c/b, q/d, abz/d, q/d/abz; q)_\infty}{(q/a, q/b, c/az, d/abz; q)_\infty} 2 \psi_1 \left[ \begin{array}{c}
qd/az \\ qd/abz
\end{array} \right| q; \frac{qb}{d},
\]

\- \frac{(q, b, q/d, qc/d, d/a, az/q^2/az; q)_\infty}{(q/a, c/d, q^2/d, q/b, d/az/d, qd/az; q)_\infty} 2 \psi_1 \left[ \begin{array}{c}
qa/d, qb/d \\ qc/d
\end{array} \right| q; z,
\]

provided \( \max \{|z|, |cd/abz|, |qb/d| \} < 1 \).

In 1950, Bailey [6] established Theorem 1 by applying the method of comparing coefficients to the product of \( 2 \psi_2 \). For the semi-finite form of it, the reader is referred to Chen and Fu [8]. Gasper and Rahman [12 Section 5.4] have shown that there exist two expansions of an \( r \psi_r \) series by means of \( r \psi_{r-1} \) series (cf. [12 Equations (5.4.4) and (5.4.5)]). The case \( r = 2 \) of the former is exactly Theorem 2 when \( d = b \), it becomes

\[
1 \psi_1 \left[ \begin{array}{c}
a \\ c
\end{array} \right| q; z = \frac{(q, c/a, az, q/az; q)_\infty}{(c, q/a, z, q/az; q)_\infty} 1 \psi_0 \left[ \begin{array}{c}
qa/c \\ c/az
\end{array} \right| q; \frac{c}{az}.
\]
Evaluating the \( \phi_0 \) series on the right hand side by \( q \)-binomial theorem (cf. [12] Appendix II.3):

\[
y_0 \left[ \begin{array}{c} a, b \\ q^{1+m}, d \\ q; z \end{array} \right] = \frac{(az;q)_\infty}{(z;q)_\infty},
\]

we get Ramanujan’s \( \psi_1 \) summation formula [2]. Theorem [2] was deduced by Chens and Gu [7] in accordance with Cauchy’s method. Interestingly, this theorem reduces directly to (2) when \( d = a \). Recently, the research of \( q \)-congruence associated with summation and transformation formulas for the unilateral basic hypergeometric series attracts several mathematicians. Some nice results can be seen in the papers [13, 14, 15, 16].

A property of the analytic function (cf. [19], p.90; see also [5, 17, 23]), which plays a central role in this paper, can be displayed as the following lemma.

**Lemma 4.** Let \( U \) be a connected open set and \( f, g \) be analytic on \( U \). If \( f \) and \( g \) agree infinitely often near an interior point of \( U \), then we have \( f(z) = g(z) \) for all \( z \in U \).

The structure of the paper is arranged as follows. By the utilization of Lemma 4, we shall supplies the united proofs of Theorems 1 in Section 2. Some related results are also discussed in Section 3.

### 2. Proofs of Theorems 1-3

#### §1. Proof of Theorem 1

For a positive integer \( m \), we have the relation

\[
2\psi_2 \left[ \begin{array}{c} a, b \\ q^{1+m}, d \\ q; z \end{array} \right] = \sum_{k=-m}^{\infty} \frac{(a, b; q)_k}{(q^{1+m}, d; q)_k} z^k = \sum_{k=0}^{\infty} \frac{(a, b; q)_{k-m}}{(q^{1+m}, d; q)_{k-m}} z^{k-m}
\]

Using (3) and Heine’s transformation formula between two \( 2\phi_1 \) series (cf. [12] Appendix III.2):

\[
2\phi_1 \left[ \begin{array}{c} a, b \\ c, z; q \end{array} \right] = \frac{(c/b, bz; q)_\infty}{(c; z; q)_\infty} 2\phi_1 \left[ \begin{array}{c} abz/c, b \\ bz \\ q; \frac{c}{b} \end{array} \right]
\]

we gain

\[
2\psi_2 \left[ \begin{array}{c} a, b \\ q^{1+m}, d \\ q; z \end{array} \right]
= \frac{(a, b; q)_{m-z^m}}{(q^{1+m}, d; q)_{-m}} \frac{(d/a, azq^{-m}; q)_\infty}{(azq^{-m}; q)_\infty} 2\phi_1 \left[ \begin{array}{c} abzq^{-m}/d, aq^{-m} \\ azq^{-m} \\ q; \frac{d}{a} \end{array} \right]
\]

\[
= \frac{(a, b; q)_{m-z^m}}{(q^{1+m}, d; q)_{-m}} \frac{(d/a, azq^{-m}; q)_\infty}{(azq^{-m}; q)_\infty} \sum_{k=-m}^{\infty} \frac{(abzq^{-m}/d, aq^{-m}; q)_{k+m}}{(q, azq^{-m}; q)_{k+m}} \frac{d}{a}^{k+m}
\]

\[
= \frac{(az, q^{1+m}/b, d/a, qd/abz; q)_\infty}{(z, d/q, b, q^{1+m}/d/abz; q)_{-m}} 2\psi_2 \left[ \begin{array}{c} a, abz/d \\ q^{1+m}, az \\ q; \frac{d}{a} \end{array} \right].
\]
Split the 2ψ2 series on both sides into two parts to achieve
\[
\sum_{k=0}^{\infty} \frac{(a, b; q)_k}{(q^{1+m}, d; q)_k} z^k + \sum_{k=1}^{\infty} \frac{(q/d; q)_k}{(q/a, q/b; q)_k} \prod_{i=1}^{k} \left( q^{1+m} - q^i \right) \left( \frac{d}{abz} \right)^k \\
= \frac{(az, q^{1+m}/b, d/a, qd/abz; q)_\infty}{(z, d/q, b, q^{1+m}/abz; q)_\infty} \times \left\{ \sum_{k=0}^{\infty} \frac{(a, abz/d; q)_k}{(q^{1+m}, az; q)_k} \left( \frac{d}{a} \right)^k + \sum_{k=1}^{\infty} \frac{(q/az; q)_k \prod_{i=1}^{k} (q^{1+m} - q^i)}{(q/a, qd/abz; q)_k} \left( \frac{1}{b} \right)^k \right\}. 
\]

Define two functions \( f(c) \) and \( g(c) \) by
\[
f(c) = \sum_{k=0}^{\infty} \frac{(a, b; q)_k}{(c, d; q)_k} z^k + \sum_{k=1}^{\infty} \frac{(q/d; q)_k}{(q/a, q/b; q)_k} \prod_{i=1}^{k} (c - q^i) \left( \frac{d}{abz} \right)^k,
\]
\[
g(c) = \frac{(az, c/b, d/a, qd/abz; q)_\infty}{(z, d/q, b, cd/abz; q)_\infty} \times \left\{ \sum_{k=0}^{\infty} \frac{(a, abz/d; q)_k}{(c, az; q)_k} \left( \frac{d}{a} \right)^k + \sum_{k=1}^{\infty} \frac{(q/az; q)_k \prod_{i=1}^{k} (c - q^i)}{(q/a, qd/abz; q)_k} \left( \frac{1}{b} \right)^k \right\}.
\]

Then (4) shows that
\[
f(c) = g(c) \tag{5}
\]
for \( c = q^{1+m} \). According to Lemma 2, (3) is correct for all \( |c| < \min\{1, |abz/d|\} \). By the analytic continuation, the restriction on \( c \) can be relaxed. This completes the proof of Theorem 1.

§2. Proof of Theorem 2

By means of (3) and Watson’s transformation formula for three 2ψ1 series (cf. [12, Appendix III.32]):
\[
2\psi_1 \left[ \begin{array}{c} a, b \\ c \\ | q; z \end{array} \right] = \frac{(b, c/a, az, q/az; q)_\infty}{(c, b/a, z, q/z; q)_\infty} 2\psi_1 \left[ \begin{array}{c} a, qa/c \\ qa/b \\ | q; \frac{qc}{abz} \end{array} \right] + \text{idem}(a; b),
\]
we attain
\[
2\psi_2 \left[ \begin{array}{c} a, b \\ q^{1+m}, d \\ | q; z \end{array} \right] = \frac{(a, b; q)_m z^{-m} (bq^{-m}, d/a, azq^{-m}, q^{1+m}/az; q)_\infty}{(q^{1+m}, d; q)_m} \frac{dq^{-m}, b/a, z, q/z; q}_\infty \times 2\psi_1 \left[ \begin{array}{c} aq^{-m}, qa/d \\ qa/b \\ | q; q^{1+m}d \\ abz \end{array} \right] + \text{idem}(a; b)
\]
\[
= \frac{(q, b, d/a, az, q/az, q^{1+m}/a; q)_\infty}{(q^{1+m}, d, b/a, z, q/z, q/a; q)_\infty} 2\psi_1 \left[ \begin{array}{c} aq^{-m}, qa/d \\ qa/b \\ | q; q^{1+m}d \\ abz \end{array} \right] + \text{idem}(a; b).
\]

Split the 2ψ2 series on the left hand side into two parts to obtain
\[
\sum_{k=0}^{\infty} \frac{(a, b; q)_k}{(q^{1+m}, d; q)_k} z^k + \sum_{k=1}^{\infty} \frac{(q/d; q)_k \prod_{i=1}^{k} (q^{1+m} - q^i)}{(q/a, q/b; q)_k} \left( \frac{d}{abz} \right)^k \\
= \frac{(q, b, d/a, az, q/az, q^{1+m}/a; q)_\infty}{(q^{1+m}, d, b/a, z, q/z, q/a; q)_\infty} \sum_{k=0}^{\infty} \frac{(qa/d; q)_k \prod_{i=1}^{k} (q^{1+m} - qa^i)}{(q/a, qa/b; q)_k} \left( \frac{d}{abz} \right)^k + \text{idem}(a; b).
\]
Define two functions $f(c)$ and $g(c)$ by

\[
f(c) = \sum_{k=0}^{\infty} \frac{(a, b; q)_k}{(c, d; q)_k} c^k + \sum_{k=1}^{\infty} \frac{(q/d; q)_k \prod_{i=1}^{k} (c - q^i) \left( \frac{d}{abz} \right)^k}{(q/a, q/b; q)_k},
\]

\[
g(c) = \frac{(q, b, d/a, a, z, q/a, c/a; q)_\infty}{(c, d, b/a, z, q/z, q/a; q)_\infty} \sum_{k=0}^{\infty} \frac{(qa/d; q)_k \prod_{i=1}^{k} (c - aq^i) \left( \frac{d}{abz} \right)^k}{(q, qa/b; q)_k}
+ \text{idem}(a; b).
\]

Then (9) gives that

\[f(c) = g(c)\quad (7)\]

for $c = q^{1+m}$. In accordance with Lemma 3, (7) is true for all $|c| < 1$. Through the analytic continuation, the restriction on $c$ could be relaxed. This finishes the proof of Theorem 2.

§3. **Proof of Theorem** 6. In terms of (3) and the transformation formula involving three $\phi_1$ series (cf. [12] Appendix III.31):

\[
2\phi_1 \left[ \frac{a, b}{c} \mid q; z \right] = \frac{(abz/c, q/c; q)_\infty}{(az/c, q/a; q)_\infty} 2\phi_1 \left[ \frac{c/a, qc/abz}{qc/az} \mid q; \frac{qb}{c} \right]
\]

\[- \frac{(b, q/c, c/a, az/q, q^2/az; q)_\infty}{(c/q, qb/c, q/a, az/c, qc/az; q)_\infty} 2\phi_1 \left[ q/a/c, qb/c \mid q^2/c; q; z \right],
\]

we get

\[
2\psi_2 \left[ \frac{a, b}{q^{1+m}, d} \mid q; z \right]
= \frac{(a, b; q)_m z^{-m} (abzq^m/d, q^{1+m}/d; q)_\infty}{(az/d, q^{1+m}/a; q)_\infty} 2\phi_1 \left[ \frac{d/a, q^{1+m}d/abz}{qd/az} \mid q; \frac{qb}{d} \right]
\]

\[- \frac{(a, b, q)_m z^{-m} (bq^{-m}, q^{1+m}/d, d/a, azq^{-m-1}, q^{2+m}/az; q)_\infty}{(az^{-m-1}, qb/d, q^{1+m}/a, az/d, qd/az; q)_\infty} 2\phi_1 \left[ q/a/d, qb/d \mid q^{2+m}/d; q; z \right].
\]

Split the $2\psi_2$-series on the left hand side into two parts to gain

\[
\sum_{k=0}^{\infty} \frac{(a, b; q)_k}{(q^{1+m}, d; q)_k} z^k + \sum_{k=1}^{\infty} \frac{(d/a; q)_k \prod_{i=1}^{k} (q^{1+m} - q^i) \left( \frac{d}{abz} \right)^k}{(q/a, q/b; q)_k}
\]

\[
= \frac{(q, q^{1+m}/b, q/d, abz/d, qd/abz; q)_\infty}{(q/a, q/b, q^{1+m}, az/d, q^{1+m}/d/abz; q)_\infty} 2\phi_1 \left[ \frac{d/a, q^{1+m}d/abz}{qd/az} \mid q; \frac{qb}{d} \right]
\]

\[- \frac{(q, b, q/d, q^{2+m}/d, d/a, az/q, q^2/az; q)_\infty}{(q/a, q^{1+m}, d/q, q^{2+m}/d, qb/d, az/d, qd/az; q)_\infty} 2\phi_1 \left[ q/a/d, qb/d \mid q^{2+m}/d; q; z \right].\quad (8)
\]
Define two functions $f(c)$ and $g(c)$ by
\[
f(c) = \sum_{k=0}^{\infty} \frac{(a,b;q)_k}{(c,d;q)_k} z^k + \sum_{k=1}^{\infty} \frac{(q/d;q)_k \prod_{i=1}^{k}(c-q^i)^{k}}{(q/a,q/b;q)_k} \left( \frac{d}{abz} \right)^k,
\]
\[
g(c) = \frac{(q,c/b,q/d,abz/d,cd/abz;q)_{\infty}}{(q/a,q/b,c,az,d,cd/abz;q)_{\infty}} 2\phi_1 \left[ \begin{array}{c} \frac{cd/abz,d/a}{q/a,b/a,z,q/z} \\ q/a,b/a \\ q \end{array} \right],
\]
\[
-(q,b,q/d,qc/d,d/a,az/q,q^2/az;q)_{\infty} 2\phi_1 \left[ \begin{array}{c} qa/d,qb/d \\ qc/d \\ q \end{array} \right].
\]
Then (9) offers that
\[
f(c) = g(c)
\]
for $c = q^{1+m}$. According to Lemma [11, [39] is right for all $|c| < \min\{1,|d/q|,|abz/d|\}$. Via the analytic continuation, the restriction on $c$ can be relaxed. This completes the proof of Theorem [3].

3. Some related discuss

The iteration of Theorem 1 produces another transformation formula between two $2\psi_2$ series due to Bailey [10]:
\[
2\psi_2 \left[ \begin{array}{c} a,b \\ c,d \\ q,z \\ q/a,b \\ q \end{array} \right] = \frac{(az,bz,qc/abz,qd/abz;q)_{\infty}}{(q/a,q/b,c,d;q)_{\infty}} 2\psi_2 \left[ \begin{array}{c} abz/c,abz/d \\ az,bz \\ q \end{array} \right],
\]
where $\max\{|z|,|cd/abz|\} < 1$.

Let $k$ denote the summation index of the $2\psi_2$ series in Theorem 2. Replace $k$ by $-k$ to achieve
\[
2\psi_2 \left[ \begin{array}{c} q/c,q/d \\ q/a,q/b \\ q \end{array} \right] = \frac{(q,b,c/a,d/a,az,q/az;q)_{\infty}}{(c,d/a,b/a,z,q/z;q)_{\infty}} 2\phi_1 \left[ \begin{array}{c} qa/c,q/a,d \\ qa/b \\ q \end{array} \right] + \text{idem}(a;b).
\]
Employing the substitutions $a \rightarrow q/c$, $b \rightarrow q/d$, $c \rightarrow q/a$, $d \rightarrow q/b$, $z \rightarrow cd/abz$ in the last equation, we attain the case $r = 2$ of [12, Equation (5.4.5)]:
\[
2\psi_2 \left[ \begin{array}{c} a,b \\ c,d \\ q,z \\ q/a,b \\ q \end{array} \right] = \frac{(q,q/d,a,c,b,abz/a,qd/abz;q)_{\infty}}{(q/a,q/b,c,d,cd/abz,qabs/cd;q)_{\infty}} 2\phi_1 \left[ \begin{array}{c} qa/c,qb/c \\ qa/b \\ q \end{array} \right] + \text{idem}(c;d),
\]
provided $\max\{|z|,|cd/abz|\} < 1$.

Performing the replacements $a \rightarrow b$, $b \rightarrow a$, $z \rightarrow cd/qab$ in Theorem 4 we have
\[
2\psi_2 \left[ \begin{array}{c} a,b \\ c,d \\ q \end{array} \right] = \frac{(c/a,c/q,q^2/c,q/d;q)_{\infty}}{(c,c/q,a,q/b;q)_{\infty}} 2\phi_1 \left[ \begin{array}{c} d/b,q \\ q^2a/c \\ q \end{array} \right] - \frac{(q,a/b,q/d,qc/d,cd/qa,c/qa,q^2a/cd;q)_{\infty}}{(q/b,c,q^2/d,qa/d,c/qa,q^2a/c;qa)_{\infty}}
\]
\[
\times 2\phi_1 \left[ \begin{array}{c} qa/d,qb/d \\ qc/d \\ q \end{array} \right].
\]
Calculating the second $2\phi_1$ series on the right hand side by $q$-Gauss summation formula (cf. [12, Appendix II.8]):
\[
2\phi_1 \left[ \begin{array}{c} a,b \\ c \\ q \end{array} \right] = \frac{(c/a,c/b;q)_{\infty}}{(c,c/ab;q)_{\infty}}
\]
we obtain
\[
2\psi_2 \left[ \begin{array}{c|c} a, b & cd \\ c, d & qab \end{array} \right] = \frac{(c/a, c/q, q^2/c, q/a, q/d; q)_\infty}{(c, c/qa, q/a, q/b, q; q)_\infty} 2\phi_1 \left[ \begin{array}{c} d/b, q \\ q^2a/c & q; d \end{array} \right]
+ \frac{q, a, c/b, d/b, cd/qa, q^2a/cd}{c, d, q/b, qa/c, qa/d, cd/qab} \left[ \begin{array}{c|c} q \\ q \end{array} \right]_\infty.
\]

Applying another form of Heine’s transformation (cf. [12, Appendix III.1])
\[
2\phi_1 \left[ \begin{array}{c} a, b \\ c \\
\end{array} \right] = \frac{(b, az; q)_\infty}{(c, q/z; q)_\infty} 2\phi_1 \left[ \begin{array}{c} c/b, qz \\ az \\
\end{array} \right]_\infty
\]
to the last equation, we recover the following formula due to Chu [10]:
\[
2\psi_2 \left[ \begin{array}{c|c} a, b & cd \\ c, d & qab \end{array} \right] = \frac{(a, c/b, d/b, cd/qa, q^2a/cd)}{c, d, q/b, qa/c, qa/d, cd/qab} \left[ \begin{array}{c|c} q, qa/b, q/c, q/d \\ qa/c, qa/d, qa/a, qa/b \\
\end{array} \right]_\infty 2\phi_1 \left[ \begin{array}{c} qa/c, qa/d \\ qa/b \\
\end{array} \right]_\infty
\]
where the convergent condition is \(|cd/qab| < 1\).

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References

[1] G.E. Andrews, R. Askey, R. Roy, Special Functions, Cambridge University Press, Cambridge, 2000.
[2] G.E. Andrews, On Ramanujan’s summation of \(1\psi_1(a; b; z)\), Prop. Amer. Math. Soc. 22 (1969), 552–553.
[3] G.E. Andrews, On a transformation of bilateral series with applications, Proc. Amer. Math. Soc. 25 (1970), 554–558.
[4] G.E. Andrews, R. Askey, A simple proof of Ramanujan’s summation of \(1\psi_1\), Aequationes Math. 18 (1978), 333–337.
[5] R. Askey, M.E.H. Ismail, The very well poised \(6\psi_6\), Proc. Amer. Math. Soc. 77 (1979), 218–222.
[6] W.N. Bailey, On the basic bilateral hypergeometric series \(2\psi_2\), Quart. J. Math. (Oxford) 1 (1950), 194–198.
[7] V.Y.B. Chen, W.Y.C. Chen, N.S.S. Gu, On the bilateral series \(2\psi_2\), preprint, arXiv: math/0701062v1 [math.CO], 2007.
[8] W.Y.C. Chen, A.M. Fu, Semi-finite forms of bilateral basic hypergeometric series, Prop. Amer. Math. Soc. 134 (2006), 1719–1725.
[9] W.C. Chu, Asymptotic method for Dougall’s bilateral hypergeometric sums, Bull. Sci. Math. 131 (2007), 457–468.
[10] W.C. Chu, \(q\)-extensions of Dougall’s bilateral \(2H_2\)-series, Ramanujan J. 25 (2011), 121–139.
[11] J. Dougall, On Vandermonde’s theorem and some more general expansions, Proc. Edinburgh Math. Soc. 25 (1907), 114–132.
[12] G. Gasper, M. Rahman, Basic Hypergeometric Series (2nd edition), Cambridge University Press, Cambridge, 2004.
[13] V.J.W. Guo, M.J. Schlosser, Some \(q\)-supercongruences from transformation formulas for basic hypergeometric series, preprint, arXiv: 1812.06324v1 [math.NT], 2018.
[14] V.J.W. Guo, M.J. Schlosser, A \(q\)-analogue of the (I,2) supercongruence of Van Hamme, Int. J. Number Theory 15 (2019), 29–36.
[15] V.J.W. Guo, W. Zudilin, A q-microscope for supercongruences, Adv. Math. 346 (2019), 329–358.
[16] V.J.W. Guo, W. Zudilin, On a q-deformation of modular forms, J. Math. Anal. Appl. 475 (2019), 1636–1646.
[17] M.E.H. Ismail, A simple proof of Ramanujan’s 1ψ1 sum, Proc. Amer. Math. Soc. 63 (1977), 185–186.
[18] F. Jouhet, Some more Semi-finite forms of bilateral basic hypergeometric series, Ann. Combin. 11 (2007), 47–57.
[19] S. Lang, Complex Analysis (4th edition), Springer, New York, 1999.
[20] M.J. Schlosser, A simple proof of Bailey’s very-well-poised 6ψ6 summation, Proc. Amer. Math. Soc. 130 (2002), 1113–1123.
[21] L.J. Slater, Generalized Hypergeometric Functions, Cambridge University Press, Cambridge, 1966.
[22] C.A. Wei, A short proof for Dougall’s 2H2-series identity, Discrete Math. Soc. 312 (2012), 2997–2999.
[23] J.M. Zhu, Generalizations of a terminating summation formula of basic hypergeometric series and their applications, J. Math. Anal. Appl. 436 (2016), 740–747.