Overview of Bachelors Theses 2021

Vitaly Aksenov, ITMO University
aksenov.vitaly@gmail.com
July 2021

1 Development of a Streaming Algorithm for the Decomposition of Graph Metrics to Tree Metrics

Student: Fafurin Oleg, ITMO University
External Supervisor: Michael Kapralov, EPFL

The embedding problem. We are given a graph $G$. We want to embed this graph onto some tree $T$, so that the shortest distance $d_G(u, v)$ between any pair of vertices $u$ and $v$ does not change much. In other words, we want to minimize

$$\max_{(u,v)} \frac{d_T(u,v)}{d_G(u,v)}.$$ 

This value is named the distortion. Obviously, the distortion is upper bounded by the maximal distortion of edges.

There exists an algorithm that embeds any graph on a tree with distortion $O(\log^2 n)$ in the streaming model, i.e., it can use only $O(n \cdot \text{polylog } n)$ memory. It consists of two parts.

In the first part, we insert edges one by one and if for a given edge $(u, v)$ the current distance is less than $t$ then we do not insert it. This algorithm, obviously, provides a distortion $O(t)$ for each edge and it can be proven that the total number of edges will not exceed $O(n^{1+\frac{1}{t}})$ [4]. Taking $t = \log n$, we get $O(\log n)$ distortion and $O(n \cdot \text{polylog } n)$.

In the second part, we use a streaming algorithm named FRT [5], it takes a graph with $O(n \cdot \text{polylog } n)$ edges and gets a tree with distortion $O(\log^2 n)$.

As the first result, we improved the distortion of this algorithm by taking $t$ to be $O(\log n)$ in the first part and, thus, giving $O(\frac{\log n}{\log \log n})$ distortion with $O(n \cdot \text{polylog } n)$ edges in the graph. So, in total, the algorithm gives $O(\frac{\log^2 n}{\log \log n})$ distortion.

The resulting distortion is the upper bound. We decided to find graphs for which the distortion matches that upper bound. The following two graphs satisfy.

Regular graph. We build a regular graph with degree $O(\frac{2 \log n}{\log \log n})$: at first, put all $n$ vertices on a cycle, and then connect each vertex with $O(\frac{\log n}{\log \log n})$ neighbours in both sides.
**Star.** Consider $0 < \alpha < 1$. One of the vertices is a center, from which there are $n^\alpha$ chains with length $n^{1-\alpha}$. Then, we take all the vertices on the distance at most $O\left(\frac{\log n}{\log \log n}\right)$ from the center and add all the edges between them.

Then, we implement the algorithm. The complexity of the first part appeared to be $O(m n \log n)$ where $m$ is the number of edges and $n$ is the number of vertices. The complexity of the second part is $O(n^2 \log n)$.

We run the resulting algorithm on several different open-source network graphs.

The following plot shows the distortion of paths after the first part of the algorithm on different graphs such as Facebook [7] and scale-free graphs [6].
The following plot shows the distortion of edges after the second part of the algorithm (FRT) on different scale-free graphs with different base.
2 Development of Memory-friendly Concurrent Data Structures

Student: Roman Smirnov, ITMO University
External Supervisor: Petr Kuznetsov, Telecom Paris

The main idea of this work is to implement the skip-list so that each node can store up to \( k \) elements instead of one. We designed and implemented the algorithm using locks. This thesis is mostly technical and the main results are the experiments.

At first, we chose the best \( k \)—it appeared to be 32. Then we compared our approach with two well-known concurrent data structures based on the skip-list: ConcurrentSkipListSet [1] from Java standard library and NonBlockingFriendlySkipListSet [11]. Please, note, that we compared sets and not maps. It can be seen as that our approach does not lose the performance much.

Then, we decided to replace Objects in the previous implementation by integers. For that, we rewrote our algorithm and ConcurrentSkipListSet. This improved the performance of our data structure almost 2 times since now \( k \) elements reside on the same cache line, while the results of ConcurrentSkipListSet barely changed.

As the result, we can say that the idea of batching the elements from different nodes into one seems to be a reasonable approach.
3 Theoretical Analysis of the Performance of Concurrent Data Structures

Student: Daniil Bolotov, ITMO University
External Supervisor: Petr Kuznetsov, Telecom Paris

In this work we tried to predict the performance of MCS lock [9] and Treiber stack [10]. The prediction is done in the similar manner as in [3].

For MCS lock, we consider a data structure that takes MCS lock, perform the critical section of size $C$, releases the lock, and then perform the parallel section of size $P$. Thus, we can get the following code that emulates such data structure.

```python
class Node:
    bool locked // shared, atomic
    Node next = null

tail = null // shared, global
threadlocal myNode = null // per process

operation():
    myNode = Node()
    myNode.locked = true
    pred = tail.getAndSet(myNode) // W or X
    if pred != null:
        pred.next = myNode
        while myNode.locked: // pass // Rf
            // CS started
            for i in 1..C: // C
                //nop
            // CS finished
        if myNode.next == null: // Rf
            if tail.CAS(myNode,null): // W or X
                return
            else:
                while myNode.next == null: // Rf
                    //pass
                myNode.next.locked = false // W
    //Parallel section
    for i in 1..P: // P
        //nop
```

By considering different schedules we can prove that the throughput is equal to:

$$
\begin{align*}
\frac{\alpha}{2Nf+C+2W} & , \text{ if } P + W \le (N-1) \cdot (2W + C + Rf) \\
\frac{\alpha}{(2W+C+Rf)+(P+W)} & , \text{ else }
\end{align*}
$$

where $C$ is the size of the critical section, $P$ is the size of the parallel section, $W$ is the cost of a write, $Rf$ is the cost of a read, and $N$ is the number of processes.

On Intel Xeon and 15 processes we get the following throughput, where red is the prediction and blue is the real execution:
Now, we consider Treiber stack. The pseudocode is the following:

```python
1. class Node:
2.     T data;
3.     Node next
4.   head = null //shared, atomic
5. push(data):
   6.     newHead = Node(data)
7.         while !success:
8.             oldHead = atomic_read(head) // M or X
9.                 newHead.next = oldHead
10.            success = head.compareAndSet(oldHead, newHead) // W
11.
12. pop():
   13.     Node oldHead
14.         while !success:
15.             oldHead = atomic_read(head) // M or X
16.                 if (oldHead == null) {
17.                     return DEFAULT_VALUE // corner case
18.                 }
19.             newHead = oldHead.next
20.             success = head.compareAndSet(oldHead, newHead) // W
21.         return oldHead.data
```

One can see that `push` and `pop` operations are similar and we can write them as one generic function as follows:

```python
1. pop_or_push_operation():
2.     while !success do
3.         current = atomic_read(head)
4.         new = critical_work(current)
5.         success = head.compareAndSet(current, new)
```

Then, we simulate the application of the Treiber stack: we take an element from the stack and then we perform an execution of size $P$. 
class Node:
  T data;
  Node next

head = null //shared, atomic

operation():
  newHead = Node(data)
  while !success:
    oldHead = atomic_read(head) // M or X
    newHead.next = oldHead
    success = head.compareAndSet(oldHead, newHead); // W
  for i in 1..P: //P
    //nop

By considering different schedules we can prove that the throughput is equal to:

\[
\begin{align*}
  \frac{\alpha}{M+W}, & \quad \text{if } P \leq (N-1) \cdot (M + W) \\
  \frac{\alpha \cdot N}{P+M+W}, & \quad \text{else}
\end{align*}
\]

On Intel Xeon and 15 processes we get the following results:

| Parallel work, cycles | Throughput, op/s |
|-----------------------|------------------|
| Threads: 5            |                  |
| 0 500                 | 1,000            |
| 1,000                 | 1,500            |
| 2,000                 | 2,000            |
| 2,500                 | 2,500            |
| 3,000                 | 3,000            |
| 3,500                 | 3,500            |
| 4,000                 | 4,000            |
| 4,500                 | 4,500            |
| 5,000                 | 5,000            |

| Threads: 10           |                  |
| 0 500                 | 1,000            |
| 1,000                 | 1,500            |
| 2,000                 | 2,000            |
| 2,500                 | 2,500            |
| 3,000                 | 3,000            |
| 3,500                 | 3,500            |
| 4,000                 | 4,000            |
| 4,500                 | 4,500            |
| 5,000                 | 5,000            |

| Threads: 15           |                  |
| 0 500                 | 1,000            |
| 1,000                 | 1,500            |
| 2,000                 | 2,000            |
| 2,500                 | 2,500            |
| 3,000                 | 3,000            |
| 3,500                 | 3,500            |
| 4,000                 | 4,000            |
| 4,500                 | 4,500            |
| 5,000                 | 5,000            |

On AMD Opteron and 15 processes we get the following results:

| Parallel work, cycles | Throughput, op/s |
|-----------------------|------------------|
| Threads: 5            |                  |
| 0 500                 | 1,000            |
| 1,000                 | 1,500            |
| 2,000                 | 2,000            |
| 2,500                 | 2,500            |
| 3,000                 | 3,000            |
| 3,500                 | 3,500            |
| 4,000                 | 4,000            |
| 4,500                 | 4,500            |
| 5,000                 | 5,000            |

| Threads: 10           |                  |
| 0 500                 | 1,000            |
| 1,000                 | 1,500            |
| 2,000                 | 2,000            |
| 2,500                 | 2,500            |
| 3,000                 | 3,000            |
| 3,500                 | 3,500            |
| 4,000                 | 4,000            |
| 4,500                 | 4,500            |
| 5,000                 | 5,000            |

| Threads: 15           |                  |
| 0 500                 | 1,000            |
| 1,000                 | 1,500            |
| 2,000                 | 2,000            |
| 2,500                 | 2,500            |
| 3,000                 | 3,000            |
| 3,500                 | 3,500            |
| 4,000                 | 4,000            |
| 4,500                 | 4,500            |
| 5,000                 | 5,000            |

As a result, we get pretty good theoretical approximation of the throughput.
4 Parallel Batched Interpolation Search Tree

Student: Alena Martsenyuk, MIPT

In this thesis, we show how to design parallel batched implementation of Interpolation Search Tree [8]. “Parallel batched” means that we ask the data structure to apply multiple operations together in parallel.

We developed the data structure that applies a batch of $m$ operations in $O(m \log \log n)$ work and $O(\log m \log \log n)$ span, where $n$ is the current size of the tree.

For experiments, we used an Intel Xeon machine with 16 threads. On this plot, you can see how much time (OY-axis) it takes to apply $m$ (OX-axis) operations using different number of processes into a tree of size $2.5 \cdot 10^7$.

On this plot, you can see how much time (OY-axis) it takes to apply $10^6$ operations using different number of processes into a tree of size $n$ (OX-axis).
Finally, we insert $10^6$ elements into the tree of size $5 \cdot 10^7$ and check the speedup. The speedup is approximately 11 on 16 processes.
5 Parallel Batched Self-adjusting Data Structures

Student: Vitalii Krasnov, MIPT

In this thesis, we show how to design parallel batched self-adjusting binary search tree. We based our data structure on CBTree data structure [2].

We proved that the resulting data structure is static-optimal, i.e., the total work is equal to $O\left(\sum x \cdot \frac{c_x}{m} \right)$ where $m$ is the total number of operations from the start of the existence of the data structure and $c_x$ is the number of times $x$ is requested. The span of the algorithm is $\frac{m}{C}$ where $C = \min_{x} c_x$.

For experiments, we used an Intel Xeon machine with 16 threads. All our experiments has the following construction: we continuously add $10^3$ elements to the same tree until it becomes very large—so, the tree is always the same but growing. On the first plot, one can see how much time (OY-axis) it takes to apply batches of size $10^3$ into a growing tree (OX-axis). The speedup is approximately 9 on 12 processes.

On the second plot, one can see how much time (OY-axis) it takes to apply batches of size $10^3$ taken from a normal distribution into a growing tree (OX-axis).
Also, our data structure outperforms the set data structure from the standard C++ library in the sequential setting.

Сравнение времени работы std::set и самоподстраивающегося дерева
6 Parallel Batched Persistent Binary Search Trees

Student: Ildar Zinatulin, MIPT

In this thesis, we show how to design a persistent parallel batched binary search tree. We consider persistence in the sense of versions. Suppose we are asked to apply operations \( op_1, op_2, \ldots, op_m \). A result of any operation is the new version of the tree, and operations should be applied in some “sequential” order \( op_{\pi(1)}, \ldots, op_{\pi(m)} \), i.e., a version of the tree after operation \( op_{\pi(j)} \) should be the initial tree after an application of all first \( j \) operations \( op_{\pi(1)}, \ldots, op_{\pi(j)} \).

We designed a persistent binary search tree that applies the operations in the order of their arguments. The idea is a little bit complicated and is similar to the scan function — we make two traversals from top to bottom. The work of the resulting algorithm is \( O(m \log n) \) and the span is \( O(\log n \log m) \).

For experiments, we used an Intel Xeon machine with 16 threads. We performed only one experiment — the speedup of an application of a batch with size \( 10^5 \) to a tree with size \( 10^6 \). As for the binary search tree we used Treap. The blue dot on the plot is the sequential algorithm for the persistent Treap.
References

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