Mineral particle disintegration model

A G Oleinik and V V Biryukov
FIC Kola Science Center RAS, 24A, Fersmana Str., Apatity, 184209, Russia
E-mail: oleynik@iimm.ru, birukov@goi.kolasc.net.ru

Abstract. The study proposes a mathematical model for reducing the size of mineral particles in crushing and grinding operations, which is a form of the reflected normal Gaussian – Laplace distribution. The study considers the distribution density modification of mineral particles depending on the magnitude of the energy impact during grinding. Comparison of the results of computational experiments with the results of laboratory studies confirms the possibility of using the proposed model of disintegration of mineral particles in the development of software modules for the operational evaluation of crushing and (or) grinding of mineral raw materials.

1. Introduction
There is a need to develop new approaches to make research and implementation of ore dressing processes, the creation of energy and resource-saving technologies and devices. This will increase the competitiveness between industries dealing with mineral processing. The processing of solid minerals are considered the most energy-intensive in the processes of size reduction (crushing and grinding) [1]. Therefore, the issues of optimizing energy costs for changing the size of mineral raw materials is still relevant in the scientific industry around the world [2, 3].

Iron ore enterprises spend 50% of the total energy consumption for the ore grinding processes. One of the main directions of increasing the profitability of mining enterprises is to increase the efficiency of grinding processing operations in technological schemes for mineral processing.

The structure of the mineral dressing has the main influence on the effectiveness of the technological schemes. The selection of new grinding circuits is mainly based on the experience of a design engineer, knowledge of the industry, and what was previously used and worked well for ores with similar characteristics. However, the traditionally used methods for developing the structure of technological circuits of mineral dressing do not help to obtain the result expected. This issue is exacerbated by the development of objectively manifest tendencies of involvement in the processing of depleted ores and increased competition in the world mineral resource markets. Already in the late 20th century, the need for the use of mathematical modeling methods to improve approaches to the design of technological circuits of mineral dressing based on the results of laboratory studies became apparent.

Most Russian scientists made a great contribution to the formation and development of mathematical modeling of the separation of mineral component. I.T. Tikhonov developed the mathematical apparatus of diffuse mass transfer of narrow separation fractions under the action of forces of various nature. It identifies the analytical form of separation characteristics in various separation processes carried out in serial and promising enrichment devices. O.N. Tikhonov studied the energy processes of changing size [5] and the measurement of work indices for the "classical" laws of crushing [6]. L.A. Barsky particularly [7] analyzed the most important criteria for the effectiveness...
of the functioning of technological schemes for processing mineral raw materials. This becomes a basis for the development of cost-effective technologies for industrial production of minerals. Among foreign works, it should be noted the 1977 book A.J. Lynch [8], which presents approaches to modeling schemes for changing the size of minerals, as well as researches by A. Gupta and D. Yan. They present the mathematical apparatus and practical recommendations for the development of mineral processing equipment that provides cost-effective achieving maximum performance and product quality. The second expanded edition of the study was published in 2016 [9].

The growth of computing capabilities allowed the creation and practical use of resource-intensive computer models of multi-speed multiphase media [10–12].

Work on digital modeling of the separation processes of complex mineral complexes in relation to some designs of concentrators was developed at the Kola Scientific Center of the Russian Academy of Sciences. The work here was carried out in three main areas: the creation of an information and reference system to support research on the processes of separation of mineral components; development of simulation models of the separation processes used; development of logical and analytical models of ore processing [13, 14].

2. Modeling object and methods

The principle of block structure is applied in the development of models of structurally complex technological schemes of industrial design. The allocation of blocks during decomposition is carried out within expected stages and modes of the synthesized circuit based on the required description and visualization of the structure. In this regard, there are two basic principles of constructing the flow diagrams of mineral processing – cyclicity and staging. This allows for functional (in function), component (in the form of elements) and structural (in the form of reaction between elements) decomposition of the model of the mineral processing technological scheme, where the minimum the decomposition component (MCD) is the complex stage of the reduction in size and separation.

The inputs and outputs of the various components of the decomposition of the technological scheme are connected by material flows. Such a construction of a technological scheme establishes uniform input and output qualitative and quantitative technical and technological parameters based on the revealed patterns of their functioning.

The purpose of creating a mathematical apparatus for describing the process of disintegration of solid mineral particles is to develop a software module for the operational determination of the performance of technological equipment. This forms computer models of the minimum components of the decomposition of technological circuits in mineral processing.

There are several well-established theoretical approaches to the study and description of crushing and grinding processes. All these approaches are based on the laws of Rittinger, Bond, P.A. Rebinder and the law V.P. Kirpicheva and F. Kika [6, 15].

There are also examples of using the Fokker – Planck differential equation of diffusely convective mass transfer to describe the processes of reducing the size of mineral particles [16, 17]. Based on the Fokker-Planck partial differential equation the study describes the evolution of the probability density of the coordinates. As well as momentum of particles in processes one and two-parameter models of the flow structure in technological devices are constructed that perform predictive functions based on the use of conservation laws mass, momentum and energy. The disadvantages of such models include the complexity and cumbersomeness of numerical decision algorithms and the need for significant computing resources.

Another wide class of grinding models is based on the use of patterns of modification of the distribution function of the dispersed composition of particles in the processes of particle size reduction. Distribution functions can be constructed on the basis of either the lognormal or the Rosin-Rammmer (Weibull) distribution. The disadvantages of such models include the lack of a clear algorithm for the formation of initial and final distribution functions and the difficulty of interpreting the results.
In this paper, there is possibility of constructing an analytically simple model of the disintegration of mineral particles in MCD. The researchers do not examine the actual grinding of mineral particles, but only consider their proposed approach to the mathematical description of this process. This is planned to be used to model the functioning of the MCD of the mineral processing technological scheme.

3. Disintegration model

As the object of modeling, we consider the change in the granulometric composition of the feed sample, given by the continuous distribution function of the outputs. It was gained from magnetite quartzites in the range [0; 3] mm. The initial data were obtained as a result of laboratory tests of the sample using a laser diffractometer ANALYSETTE 22 NANOTEC from FRITSCH. In this case, both integrated (cumulative) distribution graphs (curves in Figure 1) and there is discrete output density distribution curves of the studied particle size class before and after grinding the test sample obtained (Figure 1).

According to the Kirpichev-Kik law, energy consumption for reducing the size of mineral particles is proportional to their volume or mass. Then the specific energy (the energy spent on the destruction of a unit mass of mineral particles belonging to the same size class) is described:

\[
\frac{d_1}{d_2} = \exp(kE) \implies E = \frac{1}{k} \cdot \ln \frac{d_1}{d_2},
\]

where

\[
d_1 \text{ and } d_2 \text{ are the particle sizes before and after grinding, } k \text{ is the coefficient of proportionality, depending on the properties of the material being ground and the particular type of equipment, } E \text{ is the energy spent on grinding.}
\]

This stage of the study revealed the result of grinding particles belonging to one class of particle size, it does not depend on the result of grinding particles belonging to another class of particle size. It is proposed to use the reflected normal distribution (Gaussian – Laplace distribution) of the form for describing the distribution density of the newly formed during grinding of mineral particles:

\[
f(d) = \frac{1}{\sqrt{2\pi} \sigma} \left[ \exp \left( -\frac{(d - d_2)^2}{2\sigma^2} \right) + \exp \left( -\frac{(d + d_2)^2}{2\sigma^2} \right) \right],
\]

where

\[
\sigma = d_1 \left( \exp \left( -\frac{E}{2} \right) - \exp \left( -E \right) \right), \quad d_2 = \frac{d_1}{\exp(E)}.
\]

4. Results and discussion

The particle size reduction model was successively applied to all particle size classes identified in the sample feed the grinding operation with a sequential increase in energy exposure. As the energy
applied for grinding is increased, the type of graph changes (Figure 2). There is a change in the type of density distribution graphs of the outputs of several model particle sizes: 2 mm; 1 mm; 0.5 mm.

Figure 2. The calculated graphs of the density of the distribution of the output for various particle sizes (E5> E4> E3> E2> E1)

As the applied energy increases, the mathematical expectation of the distribution shifts to the left and the shape of the distribution density graph becomes asymmetric (the Gauss – Laplace distribution). Physically, this corresponds to the gradual transition of particles from “large” classes to smaller, and to the accumulation of the yield of ground material near the “0” scale of particle size.

Analysis of the dispersed composition using a laser diffractometer also showed a shift and change in the shape of the distribution functions and the distribution density of the outputs in the fineness classes of magnetite quartzites (Figure 1).
Using the proposed model of particle size reduction based on the Gaussian – Laplace distribution, a computational experiment was conducted with five model classes of particle size – 1, 0.9, 0.8, 0.7, 0.6 mm. The experiment assumed that the outputs of all model classes are the same and the proportion of each is 0.2. The total yield of all classes is 1. The grinding energy equal to 0.4 conventional units was experimented on these size classes.

Figure 3 presents the distribution density curves for various particle size classes obtained as a result of modeling the process of their disintegration. The presented summary graph of the distribution density accurately reproduces the shape and behavior of the actual grinding (Figure 1, b).

![Figure 3. Calculated graphs change in the density form of the output distribution for different size classes (1; 0.9; 0.8; 0.7; 0.6 mm)](image)

5. Conclusion

The model of disintegration of mineral particles uses the $E$ coefficient of energy costs. The model will further be developed to determine the dependence of this coefficient on the physic-mechanical properties of the crushed minerals – modulus, hardness, strength. In addition, the $E$ coefficient depends on the type of equipment used for disintegration.

Comparison of the obtained results of laboratory measurements suggests, the proposed model of disintegration of mineral particles can be used in the development of a software module for evaluating the performance of equipment for crushing and (or) grinding of mineral raw materials. However, the materials should be taken in the minimum components of the decomposition of mineral processing technological circuits. The proposed model can also be applied autonomously to specific processes of particle size reduction (type, energy intensity of equipment, physical and mechanical properties of solid particles) for the operational forecasting of the results of mineral raw materials.

The energy costs of the disintegration of mineral particles are considered as one of the main performance criteria. This is used in the development of information technology for the structural synthesis of multistage technological process schemes using the example of the synthesis of mineral
processing schemes. The structural synthesis on elements with limited compatibility is solved as the problem of satisfying constraints (CSP – Constraint satisfaction problem). This makes to use methods and heuristics developed in the theory of satisfaction of constraints, to jointly process both quantitative and qualitative limitations of the subject area [18].

The study proposes the presented model as part of the creation of this technology within its interfacing with databases (raw materials, technologies, equipment) and the forms some restrictions for use in the procedures for deriving effective schemes of technological processes.

The assumption presented at the present stage of model development is still to be studied and verified.

The model presented in this paper based on the Gaussian – Laplace statistical distribution does not require large computational resources, but it is possible to determine the particle size distribution with sufficient accuracy finished material according to the characteristics of raw materials and equipment.

The scope of the model is an operational modeling and optimization of grinding technology.

An important aspect of the model development are the processes associated with the discovery of mineral grains. When studying the grinding process, the size of the solid particles as well as other physical properties (density, magnetic susceptibility, floatability, etc.), have their influence for the results of further separation processes.

Acknowledgment
The study presented is partially supported by the RFBR grant No. 19-07-00359-a

References
[1] Pokrajcic Z and Lewis-Gray E 2010 Advanced comminution circuit design – essential for industry The AusIMM Bull. 38–42 Retrieved from: http://www.ceecthefuture.org/wp-content/uploads/2013/08/2010_aug_buliten_038advanced_comminution_circuit_design.pdf
[2] Ballantyne G R and Powell M S October 2014 Benchmarking comminution energy consumption for the processing of copper and gold ores Minerals Engineer. 65 109–14
[3] Arsentiev V A, Baranov V F and Weisberg L A 2007 Current State and Prospects for the Development of Crushing and Grinding of Mineral Raw Materials Mining J. 2 10–4
[4] Pokrajcic Z 2010 A methodology for the design of energy efficient comminution circuits Doctoral Dissertation (University of Queensland) 320 p Retrieved from: https://espaces.library.uq.edu.au/view/UQ:211463
[5] Tikhonov O N 2008 Calculation of the energy of crushing and grinding, taking into account the characteristics of fineness Ore dress. 3 10–4
[6] Tikhonov O N 2008 Methods of measuring work indices for the laws of Rittinger, Kik-Kirpichev and Bond Ore dress. 5 10–4
[7] Barsky L A and Kozin V Z 1978 System analysis in mineral processing (Moscow: Nedra) 487 p
[8] Lynch A J 1977 Mineral crushing and grinding circuits: their simulation, optimization, design, and control (New York: Elsevier Sci.) 342 p
[9] Gupta A and Yan D S 2016 Mineral Processing Design and Operations: An Introduction 2nd ed 882 p
[10] Sarhan A R, Naser J and Brooks G 2016 CFD simulation on influence of suspended solid particles on bubbles' coalescence rate in flotation cell Int. J. of Mineral Proc. 146 54–64
[11] Narasimha M, Sripriya R and Banerjee P K 2005 CFD modeling of hydrocyclone – prediction of cut size Int. J. of Mineral Proc. 75 53–68
[12] Leahy M J, Schwarz M P and Davidson M R November 2006 An air sparging CFD model of heap bioleaching of chalcocite Appl. Mathem. Model. 30(11)1428–44
[13] Skorokhodov V F, Khokhulya M S, Opalev A S et al May 2013 Computational fluid dynamics methods in research and analysis of mineral separation J. of Mining Sci. 49(3) 507–13
[14] Biryukov V V, Skorokhodov V F, Nikitin R M and Oleinik A G 2017 Formation of models of technological schemes for processing rare-earth mineral raw materials using methods of system analysis Proc. of the Kola Sci. Center of the Russ. Acad. of Sci.: Inform. Technol. 8(3) 124–34

[15] Plov P I and Pryadko N S 2012 Modeling of closed grinding cycles on the basis of the Rittinger hypothesis Zagabachenya cinnamon kopaln, VIP 51(92) 98–107

[16] Manita O A, Romanov M S and Shaposhnikov S V November 2015 On Uniqueness of Solutions To Nonlinear Fokker-Planek-Kolmogorov Equations Nonlinear Analysis. Theory Methods Appl. 128 199–226

[17] Vasiliev P V 2012 Numerical solution of the equation of kinetics of disintegration and disclosure of polycrystalline minerals Sci. reports of Belgorod State Univer. Ser. Econ. Comp. sci. 7(126) 92–100

[18] Zuenko A and Oleynik Y 2019 Constraint Programming Based on Matrix-Like Representation of Qualitative Constraints Frontiers in Artific. Intellig. and Applicat. 312 264–75