Single-flavour and two-flavour pairing in three-flavour quark matter

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We study single-flavour quark pairing (“self-pairing”) in colour-superconducting phases of quark matter, paying particular attention to the difference between scenarios where all three flavours undergo single-flavour pairing, and scenarios where two flavours pair with each other (“2SC” pairing) and the remaining flavour self-pairs. We perform our calculations in the mean field approximation using a pointlike four-fermion interaction based on single gluon exchange. We confirm the result from previous weakly-coupled-QCD calculations, that when all three flavours self-pair the favored channel for each is colour-spin-locked (CSL) pseudoisotropic pairing. However, we find that when the up and down quarks undergo 2SC pairing, they induce a colour chemical potential that disfavors the CSL phase. The strange quarks then self-pair in a “polar” channel that breaks rotational invariance, although the CSL phase may survive in a narrow range of densities.

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I. INTRODUCTION

Matter at high density and sufficiently low temperature is expected to form a colour superconducting condensate of quark pairs [1, 2, 3, 4] (for reviews, see Ref. [5, 6, 7, 8, 9]). Such a phase of matter may exist in the cores of compact stars [10] or be created during low energy heavy-ion collisions [11]. At high density, quarks will fill up the available energy states to form a Fermi surface. Since two quarks in the antisymmetric colour antitriplet channel experience an attractive interaction, we expect a BCS pairing instability [12] at the Fermi surface. The instability is resolved via the opening of a energy gap in the quasiparticle spectrum. The quark pairing breaks the SU(3)colour gauge symmetry of QCD, justifying the name colour superconductor.

The strong interaction is most attractive between quarks in the colour-antisymmetric spin-0 channel, but Fermi-Dirac statistics then require flavour-antisymmetric pairing, involving two different flavours. Well-studied candidates include the 2SC [4, 5, 13, 14, 15] and CFL [14, 15, 16] phases of quark matter. At asymptotically high density, the favored phase of quark matter is the CFL phase.

However, the cores of compact stars are not asymptotically dense and it becomes necessary to consider real-world effects such as a non-zero strange quark mass, electrical and colour charge neutrality and β-equilibrium. Taken together, these constraints act to separate the Fermi surfaces of the different quark flavours. This means that as density decreases from the asymptotic regime it becomes harder and harder to maintain the CFL condensate, and we expect transitions to other phases. In such a context it is natural to look for single-flavour pairing patterns, and many different ones have been found [17, 18, 19, 20, 21]. Calculations in weak-coupling QCD indicate that for a single isolated flavour the colour-spin-locked (CSL) phase has the lowest free energy. However, in the realistic context of quark matter in neutron stars there are three flavours present. In this paper we explore the single-flavour phases in such a context, using a Nambu-Jona-Lasinio model. We find that if the up and down quarks undergo two-flavour (2SC) pairing then this typically induces a non-zero colour chemical potential that disfavors CSL pairing for the remaining flavour, which then self-pairs in a different “transverse polar” channel instead, which we will simply label as “1SC”.

The 1SC/polar channel was studied in the NJL context in Ref. [22], where we surveyed the pairing patterns that factorize into a product of colour, flavour, and Dirac (spin) structures (this ansatz excludes the CSL phase, where colour and spin are intertwined). We used an NJL model with various plausible interactions, and found that for a single flavour the most attractive channel was the (3+3,1,+) (Cγ3) channel, where the notation translates as (colour antisymmetric 3, flavour symmetric 6, spin 1, parity even), with Dirac structure Cγ3. Pairing between quarks of the same flavour in this channel was found to have a gap parameter of \( \sim 1 \) – 10 MeV. Unlike the CSL phase, this condensate breaks rotational symmetry. In weak-coupling QCD calculations this is called the “transverse polar” phase [22].

In this paper we do not attempt a comprehensive study of all the phases that have been suggested for quark matter. Our purpose is to show that the presence of 2SC pairing can affect the single-flavour pairing of the remaining flavour, so we restrict our study to polar and CSL pairing of individual flavours, and 2SC pairing of
The third colour of each flavour forms \((1SC)\) Cooper pairs (1SCu, 1SCd and 1SCs). The third colour of each flavour forms \((6SCS,0,+)(C\gamma_0\gamma_5)\) pairs.

FIG. 1: Pictorial representation of simple single-flavour pairing in neutral quark matter. This will be referred to as the \((1SC)^3\) phase in the text. The requirement of electric neutrality and a nonzero strange quark mass forces the Fermi momenta of the three flavours apart. Two colours of each flavour form \((3A,3S,1,+)\) Cooper pairs (1SCu, 1SCd and 1SCs). The third colour of each flavour forms \((6SCS,0,+)(C\gamma_0\gamma_5)\) pairs.

FIG. 2: Pictorial representation of simultaneous 2SC and 1SC pairing in neutral quark matter. This will be referred to as the 2SC+1SCs phase in the text. This will only occur if the condensation energy of the 2SC pairing is strong enough to offset the cost of dragging the red and green \(u\) and \(d\) Fermi momenta away from the values dictated by electrical neutrality and the strange quark mass, to a common value.

FIG. 3: Pictorial representation of CSL pairing in neutral quark matter. This will be referred to as the \((1SC)^3\) phase in the text. It is composed of up, down and strange quark CSL condensates, labelled CSLu, CSLd and CSLs respectively. The requirement of electric neutrality and a nonzero strange quark mass forces the Fermi momenta of the three flavours apart. The red, green and blue colours of each flavour pair in a colour-antisymmetric channel.

FIG. 4: Pictorial representation of 2SC and CSL pairing in neutral quark matter. This will be referred to as the 2SC+CSLs phase in the text. Colour neutrality will create a small \(\mathcal{O}(\Delta^2/\mu)\) splitting between the Fermi momenta of the blue \(s\) quarks and \(s\) quarks of the other two colours. If this splitting is sufficiently large it will prevent the CSLs condensate from forming.

The up and down quarks. We do not take into account the CFL phase, nor its gapless variant gCFL, nor crystalline (LOFF) pairing, and so on. Our study is relevant to regions of the phase diagram where three flavours of quark are present, but there is either no cross-flavour pairing, or only pairing between the up and down quarks (“2SC+s” in the nomenclature of Ref. \([15]\)). Such regions have been found to exist in studies of the QCD phase diagram using NJL models. For example Ref. [28] finds a 2SC phase with strange quarks for stronger diquark coupling \((G_D = G_S)\) at \(T = 0\), over the chemical potential range \(398\ MeV < \mu < 412\ MeV\) (see their Figs. 2 and 6). Ref. [28] also finds such a phase (see their Fig. 7). It is also interesting to note that there is some chance our considerations might be relevant to the early life of the star, when \(T \approx 1\ MeV\), neutrinos are trapped, and the resultant lepton number chemical potential can favor a 2SC phase with strange quarks \([31]\). However some studies only find up and down quarks at this time \([31]\). Moreover, single flavour phases will only be relevant if they have large enough gaps to survive at these temperatures. We find that their critical temperatures are of order 1 MeV (see Fig. 4), but this result is sensitive to the cutoff (see end of Section \(V\)) so it remains an open question whether they can survive during the era of neutrino trapping.

The paper has the following structure. Sections II sums- marizes the 1SC and CSL pairing patterns. Sections III and IV discuss the model and approximations used to calculate the free energy of the colour superconducting phases that are of interest here. In all cases, electrical and colour neutrality constraints are imposed. Section \(V\) shows the behaviour of the 1SC and CSL gap parameters as functions of \(\mu\) and \(T\). The gaps are then used in a free energy calculation, the results of which are presented in Section \(VI\) as a sequence of \((\mu, T)\) phase diagrams for neutral three flavour quark matter for different values of \(M_s\). Section \(VII\) presents conclusions and directions for further research.
II. 1SC AND CSL PAIRING

The forms of zero-temperature pairing that will be compared in this paper are shown in Figs. 1-4. In the "(1SC)\( \times \) channel (Fig. 1), the Fermi momenta of the three flavours are so different that only single-flavour pairing is possible. Two colours of each flavour form \((3_A,3S,1,+)\) Cooper pairs (1SCu, 1SCd and 1SCs). The third colour of each flavour forms \((6_s,3_S,0,+)\) pairs. In the 2SC+1SCs phase (Fig. 2), the up and down quarks are able to pair with each other in the 2SC channel (which only involves two colours), but the strange quarks undergo single-flavour pairing. Again, the third colour of each flavour forms \((6_s,3_S,0,+)\) pairs.

Figures 3 and 4 are the equivalents of Figures 1 and 2, where the 1SC pairing of individual flavours is replaced by CSL pairing.

We give the explicit form of the pairing patterns mentioned so far. The basic two-colour single-flavour spin-1 ISM pairing (the "polar phase" in Ref. [24]), described here as \((3_A,3S,1,+)\), is

\[
\Delta_{a\beta}^{ij} = \Delta_{1SC}(C\gamma_3)^{ab}(\lambda_2)_{a\beta}\delta_{ij}.
\]

(1)

The colour indices are \(a, \beta\), and \(\lambda_2\) is an antisymmetric Gell-Mann matrix in colour space. The Dirac indices are \(i, j\), and \(C = i\gamma_0\gamma_2\) is the usual charge conjugation matrix. The flavour indices \(i, j\) are included for completeness: in this single-flavour channel \(i = j = 1\). This condensate breaks rotational invariance, as the explicit occurrence of the \(\gamma_3\) matrix indicates. Only two colours (typically taken as red and green) of quark are involved in the pairing meaning that the remaining (blue) quark is required to find an alternative channel in which to pair. The \(SU(3)\) colour symmetry is broken down to an \(SU(2)\) subgroup.

The single-colour single-flavour \((6_s,3_S,0,+)\) pairing is

\[
\Delta_{a\beta}^{ab} = \Delta_{05}(C\gamma_0\gamma_5)^{ab}\delta_{a\beta}\delta_{ij}.
\]

(2)

This channel is attractive in an NJL model with gluon-exchange-type interaction, but the pairing disappears as the quark masses go to zero [25]. This means it may be somewhat suppressed for up and down quarks, depending on how much of their vacuum constituent mass survives at the relevant densities [26, 32, 33]. Our earlier NJL study estimated the pairing gap in the \(C\gamma_0\gamma_5\) channel to be of the order 1-100 eV. This is much smaller than the typical gaps in the 2SC, 1SC, and CSL channels, so when we perform free energy calculations we will neglect the \((6_s,3_S,0,+)\) pairing, treating those quarks as unpaired.

CSL pairing is

\[
\Delta_{a\beta}^{ab} = \Delta_{CSL}(C\gamma_A)^{ab}\lambda_A^{a\beta}\delta_{ij},
\]

(3)

where we sum over \(A = 1, 2, 3, \gamma_A = (\gamma_1, \gamma_2, \gamma_3)\) and \(\lambda_A = (\lambda_7, \lambda_5, \lambda_2)\) which are the antisymmetric Gell-Mann matrices in colour space. The symmetry breaking associated with the CSL phase is

\[
SU(3)_{\text{colour}} \times SO(3)_{\text{J}} \times U(1)_B \rightarrow SO(3)_{\text{colour}+J} \times Z_2,
\]

(4)

where \(SU(3)_{\text{colour}}\) is gauged, \(SO(3)_{\text{J}}\) is the rotation group and \(U(1)_B\) is baryon number. The unbroken rotational symmetry \(SO(3)_{\text{colour}+J}\) corresponds to a locking between the colour and spin degrees of freedom. This can be thought of as associating a direction in configuration space with a linear combination of directions in colour space, i.e. red and green quarks pair in the \(z\)-direction, red and blue quarks in the \(y\)-direction and green and blue quarks in the \(x\)-direction. Therefore, unlike the polar phase, CSL pairing remains isotropic. The locking is similar to that which is found in the CFL phase between colour and flavour. The CSL phase is the colour superconducting equivalent of the \(B\)-phase of superfluid helium-3 [34].

In general, one would expect that the CSL condensate could involve a linear combination of \(C\gamma_i\) and \(C\sigma_{0i}\) with a colour structure that is correlated with the spatial direction. However, Table I in Ref. [20] shows that the colour antisymmetric \(C\sigma_{03}\) channel gives no contribution when using the mean-field approximation within an NJL model with a full gluon interaction vertex. This is precisely the interaction that we use, so only the \(C\gamma_i\) term is included in the CSL ansatz of Equation (3).

We can calculate the binding energy of the CSL channel, following Ref. [21], and compare it with the results for the polar 1SC channel, taken from the same reference. The result is shown in Table I where for a given condensate of gap parameter \(\Delta\) the binding energy via interaction \(I\) is given by \(E = -S I\Delta^2\), so larger values of \(S\) imply stronger binding. \(I\) can be "inst" (instanton interaction), "mag" (magnetic gluon interaction), or "elec" (electric gluon interaction). The table shows that the CSL condensate is more tightly bound than the polar condensate, leading one to expect that, other things being equal, CSL pairing will be favored over polar pairing.

III. OUTLINE OF CALCULATION

The aim of this paper is to map out the \((\mu, T)\) phase diagram for the 2SC, unpaired, (1SC)\(^3\), (CSL)\(^3\), 2SC+1SCs, and 2SC+CSLs phases of neutral three flavour quark matter at densities relevant for compact stars \((\mu \sim 500\,\text{MeV})\), for different values of \(M_s\). The phase diagram is obtained by calculating the free energy of each phase, imposing neutrality conditions and equilibration under the weak interactions, and minimizing the free energy with respect to the pairing strengths \(\Delta_i\) in the various phases. In section IV we will discuss the NJL model within which the free energy is calculated. In this section we assume that the free energy is known.

To obtain neutral matter we introduce chemical potentials coupled to the gauged charges. (In full QCD
TABLE I: Binding strengths of CSL (1st row) and 1SC (2nd row) channels in an NJL model in the mean-field approximation. The 1SC results are taken from Table 1 in [20]. We see that CSL is more strongly bound.

| phase | colour flavour | j | parity | Dirac | BCS enhancement | $S_{\text{inst}}$ | $S_{\text{elec}}$ | $S_{\text{mag}}$ |
|-------|----------------|---|--------|-------|----------------|-----------------|----------------|----------------|
| CSL   | $3_A$          | $3_S$ | 1
|       |                |     | S      | Cγ4 | LR            | O(1)            | 0              | +96             | +48             |
| “Polar” (1SC) | $3_A$ | $3_S$ | 1
|       |                |     | S      | Cγ3 | LR            | O(1)            | 0              | +32             | +16             |

Finding the free energy of a phase therefore corresponds to finding a stationary point of $\Omega$ in the parameter space of chemical potentials and gap parameters. If we work in the neutral subspace, fixing $\mu_e$, $\mu_3$, $\mu_8$ as functions of the $\Delta_\rho$, then the relevant stationary point will always be a minimum with respect to the gap parameters.

We can now anticipate some simplifications that will streamline our calculation.

Firstly, as is clear from Figs. 1, 2, 3, 4, we will always be concerned with colour superconducting condensates that are at least invariant under the $SU(2)$ colour subgroup that rotates red and green quarks into each other. This means that $\mu_3$ will always be zero.

Secondly, it is never necessary to solve the full set of coupled gap and neutrality equations. The single-flavour pairing is so weak that its back-reaction on the chemical potentials via the neutrality condition can be neglected. Thus for the 2SC+1SCs and 2SC+CSLs phases one can first treat the strange quarks as free, determine $\mu_e$ and $\mu_8$ by solving for 2SC pairing, and then use those values in the gap equations for the strange quark pairing. Similarly, for the (1SC)$^2$ and (CSL)$^2$ phases one can use the values of $\mu_e$ and $\mu_8$ that would ensure neutrality in totally unpaired quark matter (in the case of $\mu_8$ this value is zero).

Actually, the only phase where the exact value of $\mu_8$ is important is the 2SC+CSLs phase, where the nonzero $\mu_8$ puts a stress on the CSLs pairing. Following the standard argument [23], we expect that CSL pairing can occur as long as the cost of breaking a CSL pair is greater than the gain in energy from converting a blue quark to a red/green one, i.e.,

$$|\mu_8| < 2\Delta_{\text{CSL}}.$$  \hspace{1cm} (8)

The phase diagrams will be presented in section VII will show this behaviour.

IV. MODEL AND APPROXIMATIONS

The NJL-type Lagrangian that will be used in the following calculations is,

$$\mathcal{L}_{\text{NJL}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}}$$
$$= \bar{\psi} \left(i\gamma_\mu \partial^\mu - \mu \right) \psi + \mathcal{L}_{\text{int}},$$  \hspace{1cm} (9)

$$\mathcal{L}_{\text{int}} = \frac{1}{2} \bar{\psi} \gamma_\mu (\partial^\mu - \mu_\nu) \gamma_\nu \psi + \frac{1}{4} e^2 \bar{\psi} \gamma_\mu \gamma_\nu \psi \gamma^\mu \gamma^\nu \psi + \frac{1}{2} \bar{\psi} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \psi \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \psi + \frac{1}{2} \bar{\psi} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \psi \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \psi.$$  \hspace{1cm} (10)
where $\mu = \mu_0$ is a diagonal matrix in colour-flavour space, with entries given by $\mu_a$. The up and down quarks are treated as massless, and the strange quark mass is only taken into account to leading order, as an effective chemical potential for strangeness. The interaction we use is abstracted from single-gluon exchange by replacing the gluon propagator with a four-fermion coupling constant,

$$\mathcal{L}_{\text{int}} = -\frac{3}{8} G(\bar{\psi} \Gamma_\mu^a \psi)(\bar{\psi} \Gamma_\mu^a \psi),$$  \hfill (11)

where

$$\Gamma_\mu^a = \gamma_\mu \lambda^a,$$  \hfill (12)

where the $\lambda^a$’s are the standard Gell-Mann matrices ($a = 1 \ldots 8$), giving the interaction the quantum numbers of single quark. In order to study diquark pairing and colour superconductivity, we Fierz transform the interaction into the form

$$\mathcal{L}_{\text{int}} = G \sum \alpha_\rho \, (\bar{\psi} \Gamma_\rho^{qq} \psi)(\psi^T \Gamma_\rho^{qq} \psi),$$  \hfill (13)

where $\rho$ labels the different diquark pairing channels, $\Gamma_\rho^{qq}$ specifies the form of the diquark pairing in channel $\rho$ and $\Gamma_\rho^{qq}$ is defined by

$$\Gamma_\rho^{qq} = \gamma_0(\bar{\Gamma}_\rho^{qq})^+ \gamma^T.$$

(14)

Note that we sum up the terms for all the different pairing channels, even though no single Fierzing of the original interaction would give the sum of all of them. This is legitimate because there are no cross-terms between different channels in the binding energy.

With the interaction expressed as in Eq. (13) it is straightforward to do a mean field calculation of the free energy is now in a suitable form to allow the free energy to be calculated. This is done by performing a bosonisation of the Fierz transformed Lagrangian via a Hubbard-Stratonovich transformation. The general expression for the thermodynamic potential of a colour superconducting condensate is

$$\Omega = -T \sum_n \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} \text{Tr} \ln \left( \frac{S^{-1}(i\omega_n, \vec{p})}{T} \right) + \frac{\Delta_\rho \Delta_\rho}{4\alpha' G},$$

(15)

where $S^{-1}(i\omega_n, \vec{p})$ is the full inverse quark propagator. $\omega_n = (2n - 1)\pi T$ are the Matsubara frequencies and the trace is over colour, flavour, and spinor indices. The Matsubara summation is performed using the identity

$$T \sum_n \ln \left( \frac{\omega_n^2 + \epsilon_a(\vec{p})^2}{T^2} \right) = |\epsilon_a(\vec{p})| + 2T \ln(1+e^{-|\epsilon_a(\vec{p})|/T}).$$

(16)

The functions $\epsilon_a(\vec{p})$ are the dispersion relations for the fermionic quasiparticles. They are not explicitly $T$-dependent, but they depend upon the gap and quark chemical potentials which are $T$-dependent.

For convenience, we use neutral unpaired quark matter as the zero of free energy, so we subtract it from the value computed above. This gives a physically meaningful quantity without ultraviolet divergences:

$$\Omega = -\int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} \sum_a \left( |\epsilon_a(\vec{p})| + 2T \ln(1+e^{-|\epsilon_a(\vec{p})|/T}) \right) + \frac{\Delta_\rho \Delta_\rho}{4\alpha' G} - \frac{1}{12\pi^2} \left( \mu_e^4 + 2\pi^2 T^2 \mu_e^2 + \frac{7\pi^4}{15} T^4 \right) - \Omega_{\text{free}}(\mu, M, \mu_{unp}, T),$$

(17)

where the electron contribution to the free energy has been included. The electron chemical potential in neutral unpaired quark matter is

$$\mu_{unp} = \frac{M_e^2}{4\mu}.$$  \hfill (18)

The quasiquark dispersion relations $\epsilon_a(\vec{p})$ are the values of the energy at which the propagator diverges, i.e.

$$\det S^{-1}(\epsilon_a(\vec{p}), \vec{p}) = 0.$$  \hfill (19)

The calculation of $\Omega$ then depends upon the calculation of the determinant of the matrix $S^{-1}$. This matrix can be block-diagonalized in the colour-flavour space, with one block for each independent pairing channel. For each colour and flavour there are eight dispersion relations (four Dirac branches including left-handed and right-handed, particle and antiparticle, doubled by the Nambu-Gorkov formalism). In calculating the free energy of quark matter with three flavours and three colours the index $a$ in (19) will therefore vary from 1 to 72.

A. Dispersion relations

For each unpaired colour and flavour of quark, there two branches of the dispersion relation (particle and antiparticle) each with multiplicity 4:

$$E_{\text{free}}^\pm = p \pm \mu.$$  \hfill (20)

For each condensate we now give the dispersion relation of the quasiquarks (with their multiplicity) and the binding energy parameter $\alpha'$.

1. 2SC

For 2SC pairing of the red and green quarks of up and down flavour, the dispersion relations are

$$E_{\text{2SC}} = \sqrt{(p \pm \mu)^2 + \Delta^2} \pm \delta \mu,$$  \hfill (21)

each with multiplicity 4 (spin and Nambu-Gorkov), where

$$\bar{\mu} = \frac{1}{2}(\mu_{ru} + \mu_{gd}) = \frac{1}{2}(\mu_{ru} + \mu_{gd}),$$

$$\mu = \bar{\mu} + \frac{\mu_8}{3},$$

$$\delta \mu = \frac{1}{2}(\mu_{rd} - \mu_{ru}) = \frac{1}{2}(\mu_{rd} - \mu_{ru}) = \frac{\mu_8}{2}.$$  \hfill (22)
The binding energy parameter is
\[ \alpha' = \frac{1}{4}. \quad (23) \]

If \( \Delta_{2SC} < \delta \mu \), then the 2SC condensate is in the gapless 2SC state \[46, 47].

2. 1SC

For 1SC pairing of red and green quarks of some flavour \( f \) (assumed to have common chemical potential \( \bar{\mu} \)), the dispersion relations are
\[ E_{1SC}^2 = p^2 + m^2 + \bar{\mu}^2 + \Delta^2 \pm 2\sqrt{\bar{\mu}^2(p^2 + m^2) + \Delta^2 p_1^2}, \quad (24) \]
each with multiplicity 4. The binding energy parameter is
\[ \alpha' = \frac{1}{8}. \quad (25) \]

3. CSL

For CSL pairing of three colours of a single massless flavour, in the case \( \mu_8 = 0 \), we can obtain a simple closed expression for the determinant of the inverse propagator,
\[
\det S^{-1}(p_0, p) = \left( \Delta^2 + \mu^2 - (p - p_0)^2 \right)^2 \\
\times \left( \Delta^2 + \mu^2 - (p + p_0)^2 \right)^2 \\
\times \left( 4\Delta^4 + \mu^4 + (p^2 - p_0^2)^2 - 2\mu^2(p^2 + p_0^2) \\
+ \Delta^2(5\mu^2 + (3p - 5p_0)(p + p_0))^2 \right) \\
\times \left( 4\Delta^4 + \mu^4 + (p^2 - p_0^2)^2 - 2\mu^2(p^2 + p_0^2) \\
+ \Delta^2(5\mu^2 + (3p + 5p_0)(p - p_0))^2 \right), \quad (26)
\]
which has dimension 24, as expected. From this the dispersion relations of the 24 quasiparticles can be obtained. Note that there are 4 gapless modes (first line of the determinant), and the remaining 20 are gapped. When we include a nonzero quark mass all the quark modes are gapped, which is different from the behaviour in weak-coupling QCD \[22\]. For a discussion of this point see Refs. \[48, 49\]. In the general case \( \mu_8 \neq 0, m > 0 \), the determinant can be readily obtained using a symbolic mathematics program such as Mathematica, but is too complicated to display here.

The binding energy parameter is
\[ \alpha' = \frac{1}{8}. \quad (27) \]

V. RESULTS

The following sections present the results of solving the gap and neutrality equations and calculating the free energies for the (1SC)\(^3\), (CSL)\(^3\), 2SC+1SCs, and 2SC+CSLs condensates. The momentum cutoff was fixed at \( \Lambda = 600 \text{ MeV} \), with the value of the coupling \( G \) calibrated to give \( \Delta_{2SC}(\mu = 500, M_s = 0) = 31.1 \text{ MeV} \), as in Ref. \[22\]. This corresponds to \( \Delta_{CS}(\mu = 500, M_s = 0) = 25 \text{ MeV} \). We also performed some calculations at \( \Lambda = 800 \text{ MeV} \) with the same calibration, to check for cutoff sensitivity.

If single-flavour pairing were ignored, the phase diagram would contain a transition from unpaired quark matter to 2SC quark matter at \( \mu = M_s^2/(2\mu) \). In principle this critical value also depends on temperature, but at the temperatures of interest to us (\( T \lesssim 1 \text{ MeV} \)) this can be neglected. Thinking of this as a background, we can then include the possibility of 1SC and CSL single-flavour pairing, and we expect to find that in the low-temperature parts of the unpaired region there are regions of (1SC)\(^3\) or (CSL)\(^3\) phases, and in the low-temperature parts of the 2SC region there are regions of 2SC+CSLs or 2SC+1SCs. This is indeed what we ultimately find.

As mentioned in section \[IV\] we can calculate \( \mu_c \) and \( \mu_8 \) in the 2SC/unpaired background, before including the effects of single flavour pairing. We show these results in the next subsection. Then we go on to solve the gap equations for the various forms of single flavour pairing, and compare their free energies.

A. Chemical potentials with no single flavour pairing

Figure \[6\] shows (for four different strange quark masses) the values of \( \mu_c \) and \( \mu_8 \) in neutral unpaired/2SC matter at \( T \approx 0 \) over a range of quark chemical potential \( 450 \text{ MeV} < \mu < 550 \text{ MeV} \). This required simultaneously solving the 2SC gap equation and the two neutrality equations, and choosing the phase with the lowest free energy at each value of \( \mu \).

At \( \mu \approx M_s^2/(2\mu) \) we see the transition from unpaired to 2SC, at which \( \mu_c \) jumps from about \( M_s^2/(4\mu) \) to about \( M_s^2/(2\mu) \) \[50\]. In the unpaired phase, \( \mu_8 \) is zero because nothing picks out any direction in colour space. In the 2SC phase, however, the red and green light quarks pair, and this means that a nonzero \( \mu_8 \) is required to maintain colour neutrality. At moderate densities \( \mu_8 \) is negative because the number of red and green quarks is enhanced by the 2SC pairing (there is more phase space above the Fermi surface than below it), and a negative value of \( \mu_8 \) restores the colour balance by favoring blue quarks (\( T_K = \text{diag}(1, 1, -2) \)). As the quark chemical potential approaches the cutoff, however, the situation is reversed: degrees of freedom above the cutoff are excluded, so the condensate contains more red and green holes below the
Fermi surface than red and green quarks above it. Thus to \( 2\text{SC} \) at high and the cutoff, \( \Lambda \). Depending on the values of the strange quark mass \( M_s \), we find that \( \mu_s \) goes through zero and becomes positive at \( \mu > \Lambda \). For a different cutoff, this would have occurred at a different density.

We now proceed to solve the gap equations for the single-flavour-pairing phases. Figure 6 shows how the single-flavour pairing gap parameters vary over the quark chemical potential range \( 450 \text{ MeV} < \mu < 550 \text{ MeV} \).

\[ \Delta^d > \Delta^u > \Delta^s. \]  

This follows from the electrical-neutrality-induced ordering of Fermi momenta in unpaired quark matter \( p_F^u > p_F^d > p_F^s \) and the fact that quarks with larger Fermi momentum have more phase space at their Fermi surface, and hence stronger pairing. We also see that \( 1\text{SC} \) pairing gives a larger gap than CSL pairing, but it will turn out that CSL has a larger condensate binding energy (see Table I) and so, other thing being equal, we expect CSL to be favoured over \( 1\text{SC} \). This is confirmed by the free energy calculation and can be seen by the presence of the CSL phase in the phase diagrams of section VI.

In the \( 2\text{SC}+\text{CSLs} \) and \( 2\text{SC}+1\text{SCs} \) phases only the strange quarks undergo single-flavour pairing, so there is only one gap parameter, \( \Delta^s \). This is shown by the solid lines in Fig. 6. In the top left panel of the figure, the solid line (\( 2\text{SC}+1\text{SCs} \)) ends at \( \mu \approx 510 \text{ MeV} \), because the \( 2\text{SC} \) phase ceases to be favored below that point. Notice that in general the value of \( \Delta^s \) in the presence of \( 2\text{SC} \) pairing of the up and down quarks is slightly higher than its value when all three flavours undergo single-flavour pairing. This is because \( 2\text{SC} \) pairing pulls the up and down Fermi surfaces together, which drags the strange Fermi momentum up a little in order to maintain electrical neutrality. This increases the phase space at the strange quark Fermi surface, boosting the gap by a small amount.

The gap parameters for the CSLs condensate in the \( 2\text{SC}+\text{CSLs} \) phase show a non-monotonic behaviour (solid lines in lower two panels of Fig. 6). That is because CSL pairing is very sensitive to the value of the \( \mu_s \) chemical potential, which tries to split apart the Fermi momentum of the blue quarks from those of the red and green quarks, and the background \( 2\text{SC} \) pairing induces a non-zero \( \mu_s \) to maintain colour neutrality. CSL pairing is disfavored when its gap is less than a critical value of order \( \mu_s \). For the cutoff that we used, \( \Lambda = 600 \text{ MeV} \), \( \mu_s \) goes through zero at \( \mu \approx 530 \text{ MeV} \) (see Sect. VI A), so the CSL gap is boosted there. For a different cutoff, this would have occurred at a different density.

C. Non-BCS temperature dependence

The temperature dependence of the single-flavour gap parameters is shown in Fig. 7. We see that the gap parameters drop with increasing temperature, and that each vanishes at a second-order phase transition at some critical temperature \( T_{\text{crit}} \). The critical temperatures deviate slightly from the BCS prediction \( T_{\text{crit}} \approx 0.57 \Delta/(T = 0) \).
FIG. 6: Variation of the 1SC (upper) and CSL (lower) gaps with quark chemical potential at $T = 0.03$ MeV and for $M_s = 250, 175$ MeV. Solid lines are for the strange quarks, in the case where up and down quarks are undergoing 2SC pairing. The various broken lines are for the three flavours of quark in the case where all of them undergo single-flavour pairing.

FIG. 7: Variation of the 1SC (upper) and CSL (lower) gaps with temperature at $\mu = 540$ MeV and for $M_s = 250, 175$ MeV. Solid lines are for the strange quarks, in the case where up and down quarks are undergoing 2SC pairing. The various broken lines are for the three flavours of quark in the case where all of them potentially undergo single-flavour pairing.

$$T_{\text{crit}}^{1\text{SC}} \approx \Delta_{\text{1SC}}^f(T = 0), \quad \Delta_{\text{1SC}}(T = 0),$$

$$T_{\text{crit}}^{\text{CSL}} \approx \Delta_{\text{CSL}}^f(T = 0).$$

Such deviation was also found in weak-coupling-QCD studies of single-flavour pairing, and non-BCS critical temperatures have also been seen in gapless phases and crystalline phases.

It is interesting to note that because each flavour has a different critical temperature for its self-pairing, we expect a series of phase transitions as the system cools (imagining these phases to be in the core of a neutron star, for example), with single-flavour pairing appearing first for the down quarks, then for the up quarks, and finally for the strange quarks.

In the absence of any 2SC pairing, we also see that the critical temperature for 1SC pairing of any given flavour is approximately equal to the critical temperature for CSL pairing of the same flavour, even though the 1SC phase has a larger gap. This means that if we heat up a (CSL)$^3$ phase, we do not expect it to turn into (1SC)$^3$. However, in the presence of 2SC pairing, we find that the critical temperature for 2SC+CSLs is smaller than that for 2SC+1SCs, so we expect to find 2SC+CSLs $\rightarrow$ 2SC+1SCs $\rightarrow$ 2SC as we heat up the system, and this is seen in the phase diagrams of section VI.

We repeated our calculations for a higher cutoff $\Lambda =$
FIG. 8: Phase diagrams for one- and two-flavour pairing in neutral three flavour quark matter with values of $M_s$ between 175 and 250 MeV, calculated in an NJ model with cutoff $\Lambda = 600$ MeV. For the 2SC+1SCs and 2SC+CSLs phases see Figs. 2 and 4. The phase labelled “CSL” involves CSL pairing of some or all of the $u$, $d$, and $s$ quarks.

800 MeV, calibrating the coupling to give the same 2SC gap, and we found that the single-flavour pairing gaps dropped by a factor of $\sim 8$, but the relations were still valid.

VI. DISCUSSION OF PHASE DIAGRAMS

Section IV describes how we calculate the free energy for each phase as a function of $(\mu, T, M_s)$. At each value of $(\mu, T)$ the favored phase is the one with the lowest free energy. Fig. 8 presents this information as a series of phase diagrams.

The crude structure of the diagrams involves the expected transition from unpaired quark matter at low density to two-flavour-paired quark matter (2SC) at high density, at a critical chemical potential $\mu_{\text{crit}} \approx M_s^2/(2\mu)$. In the $\mu$-$T$ diagram this transition occurs along a line that comes down very steeply at $\mu = \mu_{\text{crit}}$, because we are interested in low temperatures $T \lesssim 1$ MeV, so $T \ll \Delta_{2SC}$. As the strange quark mass drops, this line moves to the left, and 2SC occupies more and more of the phase space.

In both the unpaired and 2SC regions of phase space, we find that, at sufficiently low temperatures, single-flavour pairing occurs. Below $\mu_{\text{crit}}$ at zero temperature, the quark matter is in the (CSL)$^3$ phase in which all three flavours undergo CSL pairing, as depicted in Fig. 3. Since the three flavours have different Fermi momenta, they have different CSL gaps, and different critical temperatures, so the region we have labelled in the phase diagrams as “CSL” actually contains three bands, one where just down quarks undergo CSL pairing, then below that one where down and up quarks undergo CSL pairing, and finally at the lowest temperatures one where all three flavours undergo CSL pairing. We do not expect any 1SC pairing in this region because the 1SC phases do not have higher critical temperatures than the CSL phases (Fig. 4).
Above $\mu_{\text{crit}}$, the phase structure is more complicated. A colour chemical potential $\mu_{\text{c}}$ is generated in order to keep the 2SC phase colour neutral, and this acts to split the blue quark Fermi momentum apart from the red and green quark Fermi momenta. The value of $\mu_{\text{c}}$ is typically of order a few MeV, which is easily large enough to disrupt any attempt at CSL pairing, which itself only has a gap on the order of one MeV. As a result, the favored phase at low temperature for $\mu > \mu_{\text{crit}}$ is typically the 2SC+1SCs phase, in which the red and green strange quarks form a spin-1 condensate (Fig. 4). At a particular value of $\mu$, however, $\mu_{\text{c}}$ may pass through zero, and in that region of phase space, where $|\mu_{\text{c}}| < 2\Delta_{\text{CSL}}$, the strange quarks of all three colours are able to pair in a 2SC+CSLs phase (Fig. 4), which produces an “island” of 2SC+CSLs in the 2SC+1SCs band. For our particular choice of NJL model and cutoff, this occurs at $\mu \approx 530$ MeV for $M_s = 175, 200, 225$ MeV. As we emphasized in section Sect. V A, however, the position of such an island is sensitive to details of the cutoff, and is not a robust prediction of the model.

VII. CONCLUSIONS

In this paper we have shown that single flavour pairing is strongly affected by what is happening to other flavours. In particular, two-flavour pairing of the light flavours induces large enough colour chemical potentials to change the nature of the favored single-flavour pairing pattern for the strange quarks. Our results, shown in Fig. 8, were obtained using a NJL model in the mean field approximation with a pointlike four-fermion interaction whose index structure is based on single gluon exchange. We treated the up and down quarks as massless, and treated the strange quark mass as a fixed parameter. Some of the interesting features, such as the 2SC+CSLs “island”, are cutoff-dependent, but we found that the basic structure of the diagram is a robust prediction of the model. At high temperatures ($T \gg 1$ MeV), only flavour-antisymmetric pairing such as 2SC can survive. At sufficiently low temperatures we find either CSL pairing of some or all flavours, or 2SC+1SCs pairing. The critical temperature for the formation of these condensates is not reliably predicted by our model, but appears to be of order 1 MeV.

Our analysis was focused on the 2SC/1SC/CSL comparison. We ignored other single-flavour pairing patterns such as those with Dirac structure $C_{\sigma 03}$ and $C_{\gamma 07\gamma 5}$, since for an NJL model with our interaction they have much smaller gap parameters, and hence either make a negligible contribution to the free energy, or are disfavored relative to the 1SC ($C_{\gamma 3}$) phase. (It is interesting to note that weak-coupling QCD calculations find that a transverse planar phase is the strongest competitor to CSL; this phase has not yet been studied in the NJL context.) More importantly, we also explicitly ignored other possible competing phases such as the CFL phase and crystalline (LOFF) pairing, and we also ignored chiral condensates that would give larger (density dependent) quark masses. One natural direction in which to extend this work is to perform a simultaneous comparison of all known phases, along the lines of Refs. 28, 53, although a definitive calculation is not yet possible because we still do not know what true ground state underlies the unstable gapless phases 28, 47, 54, 55, 56, 57.

Another topic for future work is the possible ramifications of our results for the phenomenology of quark matter in neutron stars. The phase diagrams of Fig. 8 show that a core of quark matter inside a neutron star could have a complicated radial structure of different phases, each with its own special properties. Moreover, this structure would change over time as the star’s interior cooled. In our calculation the single-flavour phases had gaps of order 1 MeV, so the phase structure would settle down less than a year after the supernova, but it should be noted that we completely ignored the very weak single-flavour single-colour pairing that accompanies most of these phases, whose gap parameter could be as small as 1 eV, leading to the emergence of further phase structure after millions of years.

For neutron star phenomenology, we are interested in the transport properties of the different phases, in particular their permeability to magnetic fields, as well as thermal properties, neutrino emissivity and mean free path, viscosity, and so on. These are sensitive to the low-energy degrees of freedom, such as Goldstone bosons and ungapped quark modes. We postpone a detailed study and now sketch some of the obvious features. First, some general points: a massive flavour with CSL pairing is completely and isotropically gapped with an energy gap of approximately $m\Delta/\mu$ for $m, \Delta \ll \mu$, so some of the quasiquarks become gapless in the massless limit (see discussion after Eq. (21)). A massive flavor with ISG pairing has a direction-dependent gap, and in the massless limit becomes ungapped at two points on the Fermi surface. However we must remember that, like 2SC pairing, ISG pairing only involves two of the three colours, so there may be other gapless modes involving the third colour. Focussing on the phases that we have discussed here, the most completely gapped phase is (CSL)$^3$, in which all quark modes are gapped, although if the up and down flavours have low constituent masses there will be modes with rather small gaps. (From Ref. 28, Fig. 4, we expect $m_u, m_d \sim 20$ MeV at $\mu \approx 400$ MeV, so assuming a CSL gap parameter of 1 MeV we find an energy gap of about 50 keV.) Next comes the 2SC+CSLs phase, which has blue up and down quarks that are effectively gapless (they may have an eV-scale gap due to pairing in some very weak channel such as $C_{\gamma 07\gamma 5}$, which is suppressed for light quarks). Finally, the 2SC+1SCs phase has in addition the blue strange quarks (which can generate a bigger $C_{\gamma 07\gamma 5}$ self-pairing gap because the strange quark is heavier). The 2SC+1SCs phase is also special because rotational invariance is broken by the ISG pairing, and for the red and green strange quarks the gap varies over
the Fermi surface, but is always of order $\Delta_{1SC} \vartriangleleft 10$. The full cooling phenomenology of these phases has not yet been worked out, although such questions have already been discussed for isolated single flavours 49 and for two-flavour quark matter 48.

The magnetic properties of these phases are also expected to show some variety. In unpaired quark matter there is no Meissner effect, so magnetic flux is not expelled or confined to flux tubes. In an idealized 2SC phase the four participating quark species are uniformly gapped, but the remainder are gapless. The colour $SU(3)$ and electromagnetic $U(1)$ gauge symmetries are broken down to a colour $SU(2)$ (giving masses to five of the gluons) and a rotated electromagnetic $U(1)_G$. In the (CSL) phase all the gauge bosons have Meissner masses. The gluon masses are of order $g\Delta_{CSL}$, and the photon mass is of order $e\Delta_{CSL}$ 52. The magnetic properties of the 2SC+CSLs phase have not yet been calculated: we expect mixing of the photon and the eighth gluon, with Meissner masses that depend on $g$ as well as $e$. In the 2SC+1SCs phase, under the $U(1)_G$ which survived the formation of the 2SC condensate, the red, green, and blue strange quarks have charges $-1, 0, 0$ respectively, so the additional 1SCs pairing breaks the $U(1)_G$ symmetry, giving the $\tilde{Q}$ photon a mass of order $e\Delta_{1SC}$. The gluons have much larger masses of order $g\Delta_{2SC}$. It remains to be seen whether this phase is a type-I or type-II superconductor for the $\tilde{Q}$ photon.

We conclude that although single-flavour pairing makes a very small contribution to the free energy of quark matter, it contributes interesting extra structure to the low-temperature part of the quark matter phase diagram. Single-flavour pairing determines the symmetries and low-energy-excitations in this region, making it directly relevant for the study of signatures of quark matter in neutron stars.

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[1] B. C. Barrois, “Nonperturbative Effects In Dense Quark Matter,” Ph.D. thesis UMI 79-04847.
[2] D. Bailin and A. Love, Phys. Rept. 107, 325 (1984).
[3] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B 422, 247 (1998) arXiv:hep-ph/9711395.
[4] R. Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998) arXiv:hep-ph/9711396.
[5] K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011331.
[6] M. G. Alford, Ann. Rev. Nucl. Part. Sci. 51, 131 (2001) arXiv:hep-ph/0102047.
[7] M. G. Alford, Nucl. Phys. Proc. Suppl. 117 (2003) 65 arXiv:hep-ph/0209287.
[8] G. Nardulli, Riv. Nuovo Cim. 25N3 (2002) 1 arXiv:hep-ph/0202037.
[9] T. Schafer, arXiv:hep-ph/0304281.
[10] N. K. Glendenning, “Compact stars: Nuclear physics, particle physics, and general relativity”, Springer-Verlag, New York, USA (1997).
[11] P. Senger, J. Phys. G 31, S1111 (2005).
[12] J. Bardeen, L.N. Cooper, J.R. Schrieffer, Phys. Rev. 106, 162 (1957).
[13] M. Buballa and M. Oertel, arXiv:hep-ph/0205027.
[14] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537, 443 (1999) arXiv:hep-ph/9804403.
[15] M. G. Alford, J. Berges and K. Rajagopal, Nucl. Phys. B 558 (1999) 219 arXiv:hep-ph/9903502.
[16] M. Buballa and M. Oertel, Nucl. Phys. A 703 (2002) 770 arXiv:hep-ph/0109095.
[17] M. Iwasaki and T. Iwado, Phys. Lett. B 350, 163 (1995). M. Iwasaki and T. Iwado, Prog. Theor. Phys. 94, 1073 (1995). M. Iwasaki, S. Ishiiawa and T. Tanaka, Prog. Theor. Phys. 104, 1041 (2000).
[18] T. Schafer, Phys. Rev. D 62, 094007 (2000) arXiv:hep-ph/0006034.
[19] M. Buballa, J. Hosek and M. Oertel, Phys. Rev. Lett. 90 (2003) 182002 arXiv:hep-ph/0204127.
[20] M. G. Alford, J. A. Bowers, J. M. Cheyne and G. A. Cowan, Phys. Rev. D 67 (2003) 054018 arXiv:hep-ph/0210106.
[21] A. Schmitt, arXiv:nucl-th/0405076.
[22] A. Schmitt, Phys. Rev. D 71 (2005) 054016 arXiv:hep-ph/0412033.
[23] M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. Lett. 92 (2004) 222001 arXiv:hep-ph/0311286.
[24] M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. D 71, 054009 (2005) arXiv:hep-ph/0406137.
[25] M. Alford, J. Bowers and K. Rajagopal, Phys. Rev. D 63, 074016 (2001) hep-ph/0008208.
[26] J. A. Bowers and K. Rajagopal, Phys. Rev. D 66, 065002 (2002) hep-ph/0204079.
[27] J. A. Bowers, arXiv:hep-ph/0305301.
[28] S. B. Ruster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, Phys. Rev. D 72, 034004 (2005) arXiv:hep-ph/0503184.
[29] H. Abuki and T. Kunihiro, arXiv:hep-ph/0509172.
[30] A. W. Steiner, S. Reddy and M. Prakash, Phys. Rev. D 66 (2002) 094007 arXiv:hep-ph/0205201.
[31] S. B. Ruster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, arXiv:hep-ph/0509073.
[32] M. Huang, P. f. Zhuang and W. q. Chao, Phys. Rev. D 65 (2002) 076012 arXiv:hep-ph/0112124.
[33] D. Blaschke, M. K. Volkov and V. L. Yudichev, arXiv:hep-ph/0301065.
[34] D. Vollhardt and P. Wölfle, The Superfluid Phases of Helium 3, Taylor and Francis, (1990).
[35] A. Gerhold and A. Rebhan, Phys. Rev. D 68 (2003) 011502 arXiv:hep-ph/0305108.
[36] D. D. Dietrich and D. H. Rischke, Prog. Part. Nucl. Phys.
[37] A. Gerhold, arXiv:hep-ph/0411086.
[38] M. Buballa and I. A. Shovkovy, Phys. Rev. D 72, 097501 (2005) arXiv:hep-ph/0508197.
[39] P. F. Bedaque and T. Schafer, Nucl. Phys. A 697, 802 (2002) arXiv:hep-ph/0105150.
[40] F. Neumann, M. Buballa and M. Oertel, Nucl. Phys. A 714 (2003) 481 arXiv:hep-ph/0210078.
[41] S. Reddy and G. Rupak, Phys. Rev. C 71, 025201 (2005) arXiv:nucl-th/0405054.
[42] K. Kikkawa, Prog. Theor. Phys. 56, 947 (1976).
[43] D. Ebert, L. Kaschluhn and G. Kastelewicz, Phys. Lett. B 264 (1991) 420.
[44] H. Reinhardt, Phys. Lett. B 244 (1990) 316.
[45] J. Berges and K. Rajagopal, Nucl. Phys. B 538 (1999) 215 arXiv:hep-ph/9804233.
[46] I. Shovkovy and M. Huang, Phys. Lett. B 564 (2003) 205 arXiv:hep-ph/0302142.
[47] M. Huang and I. Shovkovy, Nucl. Phys. A 729 (2003) 835 arXiv:hep-ph/0307273.
[48] D. N. Aguilera, D. Blaschke, M. Buballa and V. L. Yudichev, Phys. Rev. D 72, 034008 (2005) arXiv:hep-ph/0503288.
[49] A. Schmitt, I. A. Shovkovy and Q. Wang, arXiv:hep-ph/0510347.
[50] M. Alford and K. Rajagopal, JHEP 0206 (2002) 031 [hep-ph/0204001].
[51] A. Schmitt, Q. Wang and D. H. Rischke, Phys. Rev. D 66 (2002) 114010 [arXiv:nucl-th/0209050].
[52] J. A. Bowers, J. Kundu, K. Rajagopal and E. Shuster, Phys. Rev. D 64 (2001) 014024 [arXiv:hep-ph/0101067].
[53] D. Blaschke, S. Fredriksson, H. Grigorian, A. M. Oztas and F. Sandin, Phys. Rev. D 72, 065020 (2005) arXiv:hep-ph/0503194.
[54] M. Huang and I. A. Shovkovy, Phys. Rev. D 70, 094030 (2004) [hep-ph/0408268].
[55] R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli and M. Ruggieri, Phys. Lett. B 605, 362 (2005) [Erratum-ibid. B 615, 297 (2005)] [hep-ph/0410401].
[56] I. Giannakis and H. C. Ren, Phys. Lett. B 611, 137 (2005) [hep-ph/0412015].
[57] K. Fukushima, hep-ph/0506080.
[58] A. Schmitt, Q. Wang and D. H. Rischke, Phys. Rev. Lett. 91 (2003) 242301 [arXiv:nucl-th/0301090].
[59] A. Schmitt, Q. Wang and D. H. Rischke, Phys. Rev. D 69 (2004) 094017 [arXiv:nucl-th/0311006].