A NOTE ON INTUITIONISTIC FUZZY $\pi$-GENERALIZED SEMI IRRESOLUTE MAPPINGS

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ABSTRACT

In this paper a new class of mapping called intuitionistic fuzzy $\pi$-generalized semi irresolute mapping in intuitionistic fuzzy topological space is introduced and some of its properties are studied.

Keywords and Phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy $\pi$-generalized semi closed set, intuitionistic fuzzy $\pi$-generalized semi open set, intuitionistic fuzzy $\pi$-generalized semi irresolute mapping, intuitionistic fuzzy $\pi T_{1/2}$ space and intuitionistic fuzzy $\pi T_{1/2}$ space.

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1. INTRODUCTION

After the introduction of Fuzzy set (FS) by Zadeh [12] in 1965 and fuzzy topology by Chang [3] in 1967, several researches were worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov in 1983 as a generalization of fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy $\pi$-generalized semi irresolute mapping in intuitionistic fuzzy topological space and studied some of their properties. We provide some characterizations of intuitionistic fuzzy $\pi$-generalized semi irresolute mapping and established the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

1. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ and $B = \{ (x, \mu_B(x), \nu_B(x)) / x \in X \}$. Then

(1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(3) $A^c = \{ (x, \mu_A(x), \mu_A(x)) / x \in X \}$
(4) $A \cap B = \{ (x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)) / x \in X \}$
(5) $A \cup B = \{ (x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)) / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ instead of $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = (A / \mu_A, B / \mu_B, (A / \nu_A, B / \nu_B))$. The intuitionistic fuzzy sets $0 = \{ (x, 0, 1) / x \in X \}$ and $1 = \{ (x, 1, 0) / x \in X \}$ are respectively the empty set and the whole set of $X$.

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Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:
(a) 0, 1 ∈ τ,
(b) Gᵢ ∩ Gⱼ ∈ τ, for any Gᵢ, Gⱼ ∈ τ,
(c) Gᵢ ∈ τ for any arbitrary family {Gᵢ / i ∈ I} ⊆ τ.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $A'$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $\Lambda = \langle x, \mu, \nu \rangle$ be an IFS in X. Then, the intuitionistic fuzzy semi open and an intuitionistic fuzzy semi closed set are defined by
\[
\text{int}(A) = \cup \{ G / G \text{ is an IFOS in X and } G \subseteq A \},
\]
\[
\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in X and } A \subseteq K \}.
\]

Definition 2.5: [7] Let $f$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $f$ is said to be
(i) An intuitionistic fuzzy closed (IF closed in short) mapping if $f(A)$ is an IFCS A in Y for every IFCS A in X
(ii) An intuitionistic fuzzy α-closed (IFαCS in short) mapping if $f(A)$ is an IFαCS in Y for every IFCS A in X
(iii) An intuitionistic fuzzy semiclosed (IFS closed in short) mapping if $f(A)$ is an IFSCS in Y for every IFCS A in X
(iv) An intuitionistic fuzzy preclosed (IFP closed in short) mapping if $f(A)$ is an IFPCS in Y for every IFCS A in X.

Definition 2.6: [7] Let $f$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $f$ is said to be
(i) An intuitionistic fuzzy generalized closed (IFG closed in short) mapping if $f(A)$ is an IFRCS in Y for every IFRCS A in X
(ii) An intuitionistic fuzzy pre-regular closed (IFPR closed in short) mapping if $f(A)$ is an IFRCS in Y for every IFRCS A in X.

Definition 2.7: [7] Let $f$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $f$ is said to be an intuitionistic fuzzy almost closed (IFA closed in short) mapping if $f(A)$ is an IFCS in Y for each IFCS A in X

Definition 2.8: [8] A subset of A of a space (X, τ) is called:
(i) regular open if $A = \text{int}(\text{cl}(A))$
(ii) $\tau$ open if A is the union of regular open sets.

Definition 2.9: An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is called an
(a) intuitionistic fuzzy semi closed set [7] (IFSCS) if $\text{int}(\text{cl}(A)) \subseteq A$
(b) intuitionistic fuzzy $\alpha$-closed set [7] (IFαCS) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
(c) intuitionistic fuzzy pre-closed set [7] (IF PCS) if $\text{cl}(\text{int}(A)) \subseteq A$
(d) intuitionistic fuzzy regular closed set [7] (IFRCS) if $\text{cl}(\text{int}(A)) = A$
(e) intuitionistic fuzzy generalized closed set [9] (IF GCS) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
(f) intuitionistic fuzzy generalized semi closed set [8] (IFGSCS) if $\text{sc}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
(g) intuitionistic fuzzy $\alpha$ generalized closed set [8] (IFαGCS) if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
(h) intuitionistic fuzzy $\pi$-generalized semi closed set [8] (IF$\pi$GCS) if $\text{sc}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF$\pi$OS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy $\alpha$-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy $\alpha$ generalized open set and intuitionistic fuzzy $\pi$-generalized semi open set(IFOS, IFPOS, IFROS, IFGOS, IFGSCS, IFαGOS and IF$\pi$GOS) if the complement of $A'$ is an IFCS, IFαCS, IFRCS, IFPCS, IFGCS, IFGSCS, IFαGCS and IF$\pi$GCS respectively.

Result 2.10: [8] Every IFCS, IFSCS, IFGCS, IFRCS, IFαCS, IFGCS is an IF$\pi$GCS but the converses may not be true in general. (Every IFOS, IFPOS, IFGOS, IFGCS, IFαOS, IFGCS is an IF$\pi$GCS but the converses may not be true in general).

Definition 2.11: [5] Let $f$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $f$ is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IF(X)$ for every $B \in \sigma$. 

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Definition 2.12: [7] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be
(a) intuitionistic fuzzy semi continuous (IFS continuous in short) if \( f^{-1}(B) \in \text{IFSO}(X) \) for every \( B \in \sigma \)
(b) intuitionistic fuzzy \( \alpha \)-continuous (IF\( \alpha \)-continuous in short) if \( f^{-1}(B) \in \text{IF\( \alpha \)O}(X) \) for every \( B \in \sigma \)
(c) intuitionistic fuzzy pre continuous (IFP continuous in short) if \( f^{-1}(B) \in \text{IFPO}(X) \) for every \( B \in \sigma \)
(d) intuitionistic fuzzy completely continuous if \( f^{-1}(B) \in \text{IFRO}(X) \) for every \( B \in \sigma \).

Definition 2.13: [6] A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \gamma \) continuous (IF\( \gamma \) continuous in short) if \( f^{-1}(B) \) is an IF\( \gamma \)OS in \((X, \tau)\) for every \( B \in \sigma \).

Definition 2.14: [12] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if \( f^{-1}(B) \in \text{IFGCS(X)} \) for every IFCS \( B \) in \( Y \).

Result 2.15: [12] Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general.

Definition 2.16: [10] A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if \( f^{-1}(B) \) is an IFGCS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).

Definition 2.17: [9] A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \pi \)- generalized continuous (IF\( \pi \)GS continuous in short) if \( f^{-1}(B) \) is an IF\( \pi \)GS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).

Definition 2.18: [12] An IFTS \((X, \tau)\) is called an intuitionistic fuzzy \( T_{1/2} \) (IFT\( T_{1/2} \) in short) space if every IFCS \( B \) in \( X \) is an IFCS in \( X \).

Definition 2.19: [11] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if \( f^{-1}(B) \in \text{IFRO}(X) \) for every IFCS \( B \) in \( Y \).

Definition 2.20: [11] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be an intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if \( f^{-1}(B) \in \text{IFGCS(X)} \) for every IFGCS \( B \) in \( Y \).

Result 2.21: [8] Every IFGCS is an IF\( \pi \)GCS but not conversely.

Definition 2.22: [8] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \( \pi_2 T_{1/2} \) (IF\( \pi_2 T_{1/2} \) in short) space if every IF\( \pi \)GCS in \( X \) is an IFCS in \( X \).

Definition 2.23: [8] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \( \pi_2 T_{1/2} \) (IF\( \pi_2 T_{1/2} \) in short) space if every IF\( \pi \)GCS in \( X \) is an IFGCS in \( X \).

3. INTUITIONISTIC FUZZY \( \pi \)- GENERALIZED SEMI IRRESOLUTE MAPPINGS

In this section, we have introduced intuitionistic fuzzy \( \pi \)- generalized semi irresolute mappings and studied some of their properties.

Definition 3.1: A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \pi \)- generalized semi irresolute (IF\( \pi \)GS irresolute) mapping if \( f^{-1}(A) \) is an IF\( \pi \)GCS in \((X, \tau)\) for every IF\( \pi \)GCS \( A \) of \((Y, \sigma)\).

Theorem 3.2: If \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)GGS irresolute mapping, then \( f \) is an IF\( \pi \)GGS continuous mapping but not conversely.

Proof: Let \( A \) be any IFCS in \( Y \). Since every IFCS is an IF\( \pi \)GCS, \( A \) is an IF\( \pi \)GCS in \( Y \). Since \( f \) is an IF\( \pi \)GGS irresolute mapping, \( f^{-1}(A) \) is an IF\( \pi \)GGS in \( X \). Hence \( f \) is an IF\( \pi \)GGS continuous mapping.

Example 3.3: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \{ (x, (0.1u_1, 0.2v_1), (0.3u_2, 0.3v_2), 0.3u_3, 0.3v_3) \}, G_2 = \{ (x, (0.1u_1, 0.2v_1), (0.3u_2, 0.3v_2), 0.3u_3, 0.3v_3) \}, G_3 = \{ (x, (0.1u_1, 0.2v_1), (0.3u_2, 0.3v_2), 0.3u_3, 0.3v_3) \}, G_4 = \{ (x, (0.1u_1, 0.2v_1), (0.3u_2, 0.3v_2), 0.3u_3, 0.3v_3) \}, G_5 = \{ (x, (0.1u_1, 0.2v_1), (0.3u_2, 0.3v_2), 0.3u_3, 0.3v_3) \}, G_6 = \{ (x, (0.1u_1, 0.2v_1), (0.3u_2, 0.3v_2), 0.3u_3, 0.3v_3) \}, \text{ and } \sigma = \{ 0, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF\( \pi \)GGS continuous mapping. Let \( B = \{ y, (0.1u_1, 0.1v_1) \}, \)
Theorem 3.4: If \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IF\( \tau \)-GSCS irresolute mapping, then \( f \) is an IFGS continuous mapping but not conversely.

Proof: Let \( A \) be an IFCS in \( X \). Since every IFCS is an IF\( \tau \)-GSCS, \( A \) is an IF\( \tau \)-GSCS in \( Y \). By hypothesis, \( f^{-1}(A) \) is an IF\( \tau \)-GSCS in \( X \). This implies \( f^{-1}(A) \) is an IFGSCS in \( X \). Hence \( f \) is an IFGS continuous mapping.

Example 3.5: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \langle x, (0.2a, 0.4b), (0.5a, 0.4b) \rangle \), \( G_2 = \langle x, (0.1a, 0.3b), (0.3a, 0.4b) \rangle \), \( G_3 = \langle x, (0.1a, 0.3b), (0.3a, 0.4b) \rangle \), \( G_4 = \langle x, (0.4a, 0.4b), (0.3a, 0.5b) \rangle \). Then \( \tau = \{ 0.0, G_1, G_2, G_3, G_4, 1.0 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFGS continuous mapping. Let \( B = \langle y, (0.5a, 0.3b), (0.5a, 0.4b) \rangle \) is an IF\( \tau \)-GSCS in \( Y \). But \( f^{-1}(B) = \langle x, (0.5a, 0.3b), (0.5a, 0.4b) \rangle \) is not an IF\( \tau \)-GSCS in \( X \). Therefore \( f \) is not an IF\( \tau \)-GS irresolute mapping.

Theorem 3.6: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be any two IF\( \tau \) irresolute mappings. Then \( g \circ f : (X, \tau) \rightarrow (Z, \eta) \) is an IF\( \tau \)-GCS irresolute mapping.

Proof: Let \( A \) be an IFCS in \( X \). Since \( f \) is an IF\( \tau \)-GSCS irresolute mapping, \( f^{-1}(A) \) is an IF\( \tau \)-GSCS in \( X \). Hence \( f \) is an IFGCS continuous mapping.

Theorem 3.7: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \tau \)-GS irresolute mapping and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) be any two IF\( \eta \) irresolute mappings. Then \( g \circ f : (X, \tau) \rightarrow (Z, \eta) \) is an IF\( \eta \)-GS irresolute mapping.

Proof: Let \( A \) be an IF\( \tau \)-GSCS in \( X \). Then \( f^{-1}(A) \) is an IF\( \tau \)-GSCS irresolute mapping, \( f^{-1}(A) \) is an IF\( \tau \)-GS continuous mapping. Let \( B = \langle y, (0.5a, 0.3b), (0.5a, 0.4b) \rangle \) is an IF\( \tau \)-GSCS in \( Y \). But \( f^{-1}(B) = \langle x, (0.5a, 0.3b), (0.5a, 0.4b) \rangle \) is not an IF\( \tau \)-GSCS in \( X \). Therefore \( f \) is not an IF\( \tau \)-GS irresolute mapping.

Theorem 3.8: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \tau \)-GS irresolute mapping and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) be any two IF\( \eta \) continuous mappings. Then \( g \circ f : (X, \tau) \rightarrow (Z, \eta) \) is an IF\( \eta \)-GS continuous mapping.

Proof: Let \( A \) be an IF\( \tau \)-GCS in \( Z \). Then \( f^{-1}(A) \) is an IF\( \tau \)-GCS irresolute mapping, \( f^{-1}(A) \) is an IF\( \tau \)-GS continuous mapping. Let \( B = \langle y, (0.5a, 0.3b), (0.5a, 0.4b) \rangle \) is an IF\( \tau \)-GCS in \( Y \). But \( f^{-1}(B) = \langle x, (0.5a, 0.3b), (0.5a, 0.4b) \rangle \) is not an IF\( \tau \)-GCS in \( X \). Therefore \( f \) is not an IF\( \tau \)-GS continuous mapping.

Theorem 3.9: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \tau \)-GS irresolute mapping, then \( f \) is an IF\( \tau \) irresolute mapping if \( X \) is an IF\( \tau \)-T\(_{1/2} \) space.

Proof: Let \( A \) be an IFCS in \( Y \). Since every IFCS is an IF\( \tau \)-GSCS, \( A \) is an IF\( \tau \)-GSCS in \( Y \). By hypothesis, \( f^{-1}(A) \) is an IF\( \tau \)-GSCS in \( X \). Hence \( f \) is an IFGSCS in \( X \). Therefore \( f^{-1}(A) \) is an IF\( \tau \)-GCS in \( X \). That is \( f^{-1}(A) \) is an IF\( \tau \)-GS in \( X \). Hence \( f \) is an IF\( \tau \)-GS continuous mapping.

Theorem 3.10: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping from an IFTS \( X \) into an IFTS \( Y \). Then the following conditions are equivalent if \( X \) and \( Y \) are IF\( \tau \)-T\(_{1/2} \) spaces:

(i) \( f \) is an IF\( \tau \)-GSCS irresolute mapping
(ii) \( f^{-1}(B) \) is an IF\( \tau \)-GSOS in \( X \) for each IF\( \tau \)-GSOS \( B \) in \( Y \)
(iii) \( \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) \) for each IFS \( B \) of \( Y \).

Proof: (i) \( \Rightarrow \) (ii): Obviously true.

(ii) \( \Rightarrow \) (iii): Let \( B \) be any IFS in \( Y \). Clearly \( B \subseteq \text{cl}(B) \). Then \( f^{-1}(B) \subseteq f^{-1}(\text{cl}(B)) \). Since \( \text{cl}(B) \) is an IFCS in \( Y \), \( \text{cl}(B) \) is an IF\( \tau \)-GCS in \( Y \). Therefore \( f^{-1}(\text{cl}(B)) \) is an IF\( \tau \)-GSCS in \( X \), by hypothesis. Since \( X \) is an IF\( \tau \)-T\(_{1/2} \) space, \( f^{-1}(\text{cl}(B)) \) is an IFGSCS in \( X \). Hence \( \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) = f^{-1}(\text{cl}(B)) \). That is \( \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) \).

(iii) \( \Rightarrow \) (i): Let \( B \) be an IF\( \tau \)-GSCS in \( Y \). Since \( Y \) is an IF\( \tau \)-T\(_{1/2} \) space, \( B \) is an IFCS in \( Y \) and \( \text{cl}(B) = B \).
Hence \( f^{-1}(B) = f^{-1}(\text{cl}(B)) \supseteq \text{cl}(f^{-1}(B)) \), by hypothesis. But clearly \( f^{-1}(B) \subseteq \text{cl}(f^{-1}(B)) \). Therefore, \( \text{cl}(f^{-1}(B)) = f^{-1}(B) \).

This implies \( f^{-1}(B) \) is an IFCS in \( X \) and hence it is an IF\( \pi \)GS irresolute mapping.

**Theorem 3.11:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF continuous mapping and \( (Y, \sigma) \) is an IF \( \pi_{T_{1/2}} \) space. Then the following statements are equivalent:

(i) \( f \) is an IF\( \pi \)GS irresolute mapping

(ii) \( f \) is an IF\( \pi \)GS continuous mapping

**Proof:** (i) \( \Rightarrow \) (ii): Follows from the theorem 3.2.

(ii) \( \Rightarrow \) (i): Let \( f \) be an IF\( \pi \)GS continuous mapping . Let \( A \) be an IF\( \pi \)GSCS in \( (Y, \sigma) \). Since \( (Y, \sigma) \) is an IF \( \pi_{T_{1/2}} \) space, \( A \) is an IFCS in \( (Y, \sigma) \) and by hypothesis \( f^{-1}(A) \) is an IF\( \pi \)GSCS in \( (X, \tau) \). Therefore \( f \) is an IF\( \pi \)GS irresolute mapping.

**Theorem 3.12:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a mapping from an IFTS \( X \) into IFTS \( Y \). Then the following conditions are equivalent.

(i) \( f \) is an IF\( \pi \)GS irresolute mapping

(ii) \( f^{-1}(B) \) is an IF\( \pi \)GSOS in \( X \) for every IF\( \pi \)GSOS \( B \) in \( Y \).

**Proof:** (i) \( \Rightarrow \) (ii): Let \( B \) be an IF\( \pi \)GSOS in \( Y \), then \( B^c \) is an IF\( \pi \)GSCS in \( Y \). Since \( f \) is an IF\( \pi \)GS irresolute mapping, \( f^{-1}(B^c) \) is an IF\( \pi \)GSSCS in \( X \). But \( f^{-1}(B^c) = (f^{-1}(B))^c \), implies \( f^{-1}(B) \) is an IF\( \pi \)GSOS in \( X \).

(ii) \( \Rightarrow \) (i): Let \( B \) be an IF\( \pi \)GSCS in \( Y \). By our assumption \( f^{-1}(B^c) \) is an IF\( \pi \)GSOS in \( X \) for every IF\( \pi \)GSOS \( B^c \) in \( Y \). But \( f^{-1}(B^c) = (f^{-1}(B))^c \), which implies \( f^{-1}(B) \) is an IF\( \pi \)GSCS in \( X \). Hence \( f \) is an IF\( \pi \)GS irresolute mapping.

**Theorem 3.13:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)GS irresolute mapping and \( g: (Y, \sigma) \rightarrow (Z, \delta) \) is an IF\( \alpha \) continuous mapping, then \( g \circ f: (X, \tau) \rightarrow (Z, \delta) \) is an IF\( \pi \)GS continuous mapping.

**Proof:** Let \( A \) be an IFCS in \( Z \). Then \( g^{-1}(A) \) is an IF\( \alpha \)CS in \( Y \), Since \( g \) is an IF\( \alpha \) continuous. Since every IF\( \alpha \)CS is an IF\( \pi \)GSCS, \( g^{-1}(A) \) is an IF\( \pi \)GSCS in \( Y \). But \( f \) is an IF\( \pi \)GS irresolute mapping. Therefore \( f^{-1}(g^{-1}(A)) \) is an IF\( \pi \)GSCS in \( X \). Hence \( g \circ f \) is an IF\( \pi \)GS continuous mapping.

**Theorem 3.14:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)GS irresolute mapping and \( g: (Y, \sigma) \rightarrow (Z, \delta) \) is an IF\( \alpha \)G continuous mapping, then \( g \circ f: (X, \tau) \rightarrow (Z, \delta) \) is an IF\( \pi \)GS continuous mapping.

**Proof:** Let \( A \) be an IFCS in \( Z \). By assumption, \( g^{-1}(A) \) is an IF\( \alpha \)GCS in \( Y \). Since every IF\( \alpha \)GCS is an IF\( \pi \)GSCS, \( g^{-1}(A) \) is an IF\( \pi \)GSCS in \( Y \). But \( f \) is an IF\( \pi \)GS irresolute mapping, implies \( f^{-1}(g^{-1}(A)) \) is an IF\( \pi \)GSCS in \( X \). Hence \( g \circ f \) is an IF\( \pi \)GS continuous mapping.

**Theorem 3.15:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)GS irresolute mapping and \( g: (Y, \sigma) \rightarrow (Z, \delta) \) is an IFG continuous mapping, then \( g \circ f: (X, \tau) \rightarrow (Z, \delta) \) is an IF\( \pi \)GS continuous mapping.

**Proof:** Let \( A \) be an IFCS in \( Z \). By assumption, \( g^{-1}(A) \) is an IFGCS in \( Y \). Since every IFGCS is an IF\( \pi \)GSCS, \( g^{-1}(A) \) is an IF\( \pi \)GSCS in \( Y \). But \( f \) is an IF\( \pi \)GS irresolute mapping. Therefore \( f^{-1}(g^{-1}(A)) \) is an IF\( \pi \)GSCS in \( X \). Hence \( g \circ f \) is an IF\( \pi \)GS continuous mapping.

**Theorem 3.16:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)GS irresolute mapping and \( g: (Y, \sigma) \rightarrow (Z, \delta) \) is an IFGS continuous mapping, then \( g \circ f: (X, \tau) \rightarrow (Z, \delta) \) is an IF\( \pi \)GS continuous mapping.

**Proof:** Let \( A \) be an IFCS in \( Z \). Then \( g^{-1}(A) \) is an IFGCS in \( Y \). Since \( g \) is an IFGS continuous. Since every IFGCS is an IF\( \pi \)GSCS, \( g^{-1}(A) \) is an IF\( \pi \)GSCS in \( Y \). Since \( f \) is an IF\( \pi \)GS irresolute mapping, \( f^{-1}(g^{-1}(A)) \) is an IF\( \pi \)GSCS in \( X \). Hence \( g \circ f \) is an IF\( \pi \)GS continuous mapping.

**Definition 3.17:** Let \( A \) be an IFS in an IFTS \( (X, \tau) \). Then \( \pi \)-generalized Semi closure of \( A \) (\( \pi \text{gscl}(A) \) in short) and \( \pi \)-generalized semi interior of \( A \) (\( \pi \text{gsint}(A) \) in short) are defined by
Proposition 3.18: If \( A \) is an IFS in \( X \), then \( A \subseteq \pi_{\text{gscl}}(A) \subseteq \cl(A) \).

Proof: The result follows from the definition.

Theorem 3.19: If \( A \) is an IF\(\pi\)GSCS in \( X \) then \( \pi_{\text{gscl}}(A) = A \).

Proof: Since \( A \) is an IF\(\pi\)GSCS, \( \pi_{\text{gscl}}(A) \) is the smallest IF\(\pi\)GCS which contains \( A \), which is nothing but \( A \). Hence \( \pi_{\text{gscl}}(A) = A \).

Theorem 3.20: If \( A \) is an IF\(\pi\)GSOS in \( X \) then \( \pi_{\text{gsint}}(A) = A \).

Proof: Similar to above theorem.

Proposition 3.21: Let \((X, \tau)\) be any IFTS. Let \( A \) and \( B \) be any two intuitionistic fuzzy sets in \((X, \tau)\). Then the intuitionistic fuzzy \(\pi\)-generalized Semi closure operator satisfies the following properties.

(i) \( A \subseteq \pi_{\text{gscl}}(A) \)
(ii) \( \pi_{\text{gsint}}(A) \subseteq A \)
(iii) If \( A \subseteq B \Rightarrow \pi_{\text{gscl}}(A) \subseteq \pi_{\text{gscl}}(B) \)
(iv) If \( A \subseteq B \Rightarrow \pi_{\text{gsint}}(A) \subseteq \pi_{\text{gsint}}(B) \)

Theorem 3.22: If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IF\(\pi\)GS irresolute mapping, then \( f(\pi_{\text{gscl}}(A)) \subseteq \cl(A) \) for every IFS \( A \) of \( X \).

Proof: Let \( A \) be an IFCS of \( X \). Then \( \cl(f(A)) \) is an IFCS of \( Y \). Since every IFCS is an IF\(\pi\)GSCS, \( \cl(f(A)) \) is an IF\(\pi\)GSCS in \( Y \). Clearly \( A \subseteq \cl(f(A)) \).

Therefore \( \pi_{\text{gscl}}(A) \subseteq \pi_{\text{gscl}}(f^{-1}(\cl(A))) = f^{-1}(\cl(f(A))) \). Hence \( f(\pi_{\text{gscl}}(A)) \subseteq \cl(f(A)) \) for every IFS \( A \) of \( X \).

Theorem 3.23: If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is IF\(\pi\)GS irresolute, then \( \pi_{\text{gscl}}(f^{-1}(B)) \subseteq f^{-1}(\cl(B)) \) for every IFS \( B \) of \( Y \).

Proof: Let \( B \) be an IFS of \( Y \). Then \( \cl(B) \) is an IFCS of \( Y \). Since every IFCS is an IF\(\pi\)GSCS, \( \cl(B) \) is an IF\(\pi\)GSCS in \( Y \). By hypothesis, \( f^{-1}(\cl(B)) \) is IF\(\pi\)GSCS in \( X \). Clearly \( B \subseteq \cl(B) \) implies \( f^{-1}(B) \subseteq f^{-1}(\cl(B)) \). Therefore, \( \pi_{\text{gscl}}(f^{-1}(B)) \subseteq \pi_{\text{gscl}}(f^{-1}(\cl(B))) = f^{-1}(\cl(B)) \). Hence \( \pi_{\text{gscl}}(f^{-1}(B)) \subseteq f^{-1}(\cl(B)) \) for every IFS \( B \) of \( Y \).

Theorem 3.24: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a mapping from an IFTS \( X \) into IFTS \( Y \). Then the following conditions are equivalent

(i) \( f \) is an IF\(\pi\)GS irresolute mapping
(ii) \( f^{-1}(B) \) is an IF\(\pi\)GSOS in \( X \), for each IF\(\pi\)GSOS in \( Y \)
(iii) \( f^{-1}(\pi_{\text{gsint}}(B)) \subseteq \pi_{\text{gsint}}(f^{-1}(B)) \)
(iv) \( \pi_{\text{gscl}}(f^{-1}(B)) \subseteq f^{-1}(\cl(B)) \) for every IFS \( B \) of \( Y \).

Proof:

(i) \( \Rightarrow \) (ii): is obviously true.

(ii) \( \Rightarrow \) (iii): Let \( B \) be an IF\(\pi\)GSCS in \( Y \) and \( \pi_{\text{gsint}}(B) \subseteq B \). Then \( f^{-1}(\pi_{\text{gsint}}(B)) \subseteq f^{-1}(B) \). Since \( \pi_{\text{gsint}}(B) \) is an IF\(\pi\)GSOS in \( Y \), \( f^{-1}(\pi_{\text{gsint}}(B)) \) is an IF\(\pi\)GSOS in \( X \), by hypothesis. Hence \( f^{-1}(\pi_{\text{gsint}}(B)) \subseteq \pi_{\text{gsint}}(f^{-1}(B)) \).

(iii) \( \Rightarrow \) (iv): is obvious by taking complement in (iii).

(iv) \( \Rightarrow \) (i): Let \( B \) be an IF\(\pi\)GSCS in \( Y \) and \( \pi_{\text{gscl}}(B) = B \). Hence \( f^{-1}(B) = f^{-1}(\pi_{\text{gscl}}(B)) \subseteq \pi_{\text{gscl}}(f^{-1}(B)) \). Therefore, \( \pi_{\text{gscl}}(f^{-1}(B)) = f^{-1}(B) \). This implies \( f^{-1}(B) \) is an IF\(\pi\)GSCS in \( X \). Thus \( f \) is an IF\(\pi\)GS irresolute mapping.
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