Development of Mathematical Model of Circular Grill of Piece-Smooth Profiles and Creation on Its Basis of Gas-Sucting Fans

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Abstract. Further intensification of mining operations, application of innovative technologies that ensure efficient extraction and processing of mineral raw materials is limited by the requirements to the air and gas dynamic safety system, one of the energy-intensive elements of which are gas-sucking fans characterized by insufficient adaptability and aerodynamic loading. Using the Christofel-Schwartz equation, taking into account the theory of attached vortices, the Chaplygin method of singular points and residues, the conformal mappings method was modified and an additive mathematical model of the circular lattice of "S"-shaped profiles with circulation control vortexes was developed. The uniqueness of the obtained solution is proved up to a constant for the given parameters of vortex sources. A technique for calculation of aerodynamic schemes of adaptive highly loaded circular gratings with "S"-shaped profiles and built-in vortex sources is proposed. A parametric series of patented block-modular gas-sucking ventilators was developed on the basis of the designed aerodynamic scheme of TS145-20, providing for the coverage of ventilation regimes for gas-abundant coal mines for a perspective up to 2025. The test results of the prototype gas-sucking fan BPVG-7 confirmed the increase in adaptability by more than 50% and aerodynamic loading by 35%.

1. Introduction

The energy intensity of existing eco-technologies of gas-abundant coal mines reaches 25%, while up to 40% of electric energy is spent inefficiently. Ensuring the quality of the main production, its energy efficiency often contradicts with the energy intensity of the auxiliary technological process that ensures the environmental safety. Moreover, the insufficient efficiency of aerogasdynamic safety restricts the introduction of new technologies forming integrated innovative subsoil use. [1]

The growth of load on stoping face combined with the requirement to ensure the aerogasdynamic safety actualizes the task of developing of methodology for design and creation of nature-like adaptive gas-sucking fans which adequately and at the same time economically reasonably creating the required fields of depression, realizing the concept of optimal environmental technology of subsoil use [2].

Large potential possibilities for increasing aerodynamic loading and adaptability of gas-sucking ventilators are incorporated in active energy methods of circulation control.

The cavities of the profiled blades of gas-sucking fan made in the form of a vortex chamber inscribed in their "S"-shaped outlet section are functioning as the vortex source for control the energy interaction of the impeller with the air flow and feedback with the parameters of the external network.
through the high-pressure cavity of the fan casing. [3-b]

The parameters of the vortex system being formed around the blade profile under the influence of the vortex chamber and determining the adaptability and aerodynamic characteristics of the gas-sucking fan are a function not only of the geometric parameters of the vortex chambers and blade profiles but also depend on its feedback with the characteristics of the external network contributing to the growth of the adaptability of turbo-machines.

In domestic and foreign literature there is insufficient data on the vortex control of flow passing around the blades of impellers of radial turbo-machines, taking into account the feedback of the dependence of the energy parameters of the vortex system forming around the profiles on the characteristics of the external network.

In this paper a method is proposed for construction of a mathematical model of a circular grill of piecewise smooth "S"-shaped profiles with vortex chambers in mutual connection with characteristics of an external network to provide air-gas dynamic safety while increasing the intensity of the main technological process.

Fig. 1 shows a profile of blade 1 with an "S"-shaped outlet portion 2 of the gas-sucking fan impeller, provided with a cylindrical vortex chamber 3 built into it, an inlet tangential channel 4 (drain), and outlet channels 5 (source). The positions 6, 7 designate the front and rear points of total flow deceleration, respectively. Depending on the energy parameters of the vortex chamber, the geometric point of full deceleration 6 is moved to the position of the actual effective point of complete deceleration 8. The position 9 corresponds to the corner point of conjugation of the blade 1 with its "S"-shaped outlet section 2.

The patented design of the gas-sucking fan is a qualitatively new stage in the development of radial turbo-machines with "S"-shaped blades, presenting a hydro-dynamic analog of a turbo-machine with "S"-shaped profiles of blades of variable curvature [5, 6].

![Image of a gas-sucking fan profile](image-url)

**Figure 1.** Profile of impeller blade of a gas-sucking fan with an "S"-shaped outlet section and a vortex built in it.

The practical significance of solving of an aerodynamics problem of turbo-machines with "S"-shaped profiles of variable curvature is explained by the necessity to establish general regularities for the considered class of flows in which under real conditions it is possible to ensure smooth flow around sections of the profile with large curvature, including acute angles, gas-sucking fans, thus providing high efficiency of operation in a wide range of parameters of external network.

According to the proposed mathematical model, an aerodynamic profile of the "S"-shaped impeller blade with a vortex chamber may be presented in a form of a piecewise-smooth profile with an angular point of conjugation in which a vortex source is located with energy parameters ensuring smooth flowing to the points of conjugation [5,6].

Taking into account the proposed assumptions, the suggested problem reduces to investigation of the flow by an unrestricted air stream of a circular grill of profiles in a form of an analytic polygon transformed into a piecewise smooth "S"-shaped profile consisting of two segments of logarithmic spirals with a corner point in a place of their conjugation and a schematized vortex source in a 4-
dimensional Riemannian domain $F_r$ (Fig. 2). Under the condition that the domain $F_r$ is simply connected, the function of the conformal mapping of the exterior of a disc of unit radius on $n_\sigma$-sheet Riemann surface in the region $F_\alpha$ to the exterior of a 4-sheet polygonal contour of a schematized circular lattice in the domain $F_r$ may be obtained on the basis of the Christoffel-Schwartz formula [7]:

$$r(\alpha) = \int_{\alpha} \frac{\prod_{n=1}^{n_\sigma} (\alpha - \tau_{y_n})^{\beta_{y_n} - 1} \prod_{k=1}^{n_\sigma} (\alpha - \tau_{k})^{\beta_{k} - 1}}{\prod_{n=1}^{n_\sigma} (\alpha - p_{n})^{\beta_{n} - 1}} d\alpha,$$

where: $r = ze^{i\nu}, \alpha = le^{i\theta}$ are complex coordinates of points in the regions $F_r$ and $F_\alpha$, respectively; $z, \nu$ are the radius and polar angle in the plane $F_r$ respectively; $l, \theta$ - radius and polar angle in the plane $F\alpha$, respectively; $P$ is the form parameter of the equivalent circular lattice of profiles in the form of segments of logarithmic spirals; $\tau_{y_n}$ $(n = 1, ..., n_\sigma)$ are points on the circle of unit radius corresponding to the corner points $y_n$ of the polygonal contour $\tau_k$ - $(k = 1, 2)$ - points on a circle of unit radius, corresponding to the jet flow channels and the source of the vortex chamber of the region $F_r$; $\beta_{y_n} = \pi \beta_{y_n}$; $\beta_{n} = \pi \beta_{n}$ are the outer corners of the 4-sheet polygonal contour of the circular grid of profiles, respectively, at the corner points $y_n$ and the points of the schematized vortex source with its drain and source, $n_\sigma$ is the number of blades of the impeller.

**Figure 2.** Circular grating of "S"-shaped profiles in the form of segments of a logarithmic spiral with an integrated vortex source at the corner point of their conjugation in the region $F_r$ and corresponding to it the circles in the region $F_\alpha$. $F_r$.

The number of angles of analytical polygon that schematizes the "S"-shaped profile of the vortex source is determined by the formula:

$$n_\sigma = n_y + 2.\quad (2)$$

The outer angles of the polygon contour, taking into account the single-connection in the region $F_r$, are calculated by the formula corresponding to the model of a univalent polygon:

$$\sum_{n=1}^{n_\sigma} \beta_{y_n} + \sum_{k=1}^{2} \beta_{k} = \pi(n_y + 4).\quad (3)$$

In accordance with (1) for each given 4-sheets contour of "S"-shaped profiles of blades in the form of segments of a logarithmic spiral with a vortex source at the corner point of their conjugation, it is necessary to calculate unknown values: $P, \tau_{y_n}, \tau_{k}$ the amount of which is $(n_\sigma + 3)$, that is, for the case of the "S"-shaped profile of the blade, taking into account (2) $n_\sigma = 3$. 

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In accordance with (1) the condition of single-valuedness of the mapping function \( r(\alpha) \) for a 4-sheet contour of "\( S \)"-shaped profiles of blades in the form of segments of a logarithmic spiral with a vortex source at the corner point of their conjugation of a circular lattice has the form:

\[
\sum_{n=1}^{y_n} (\overline{\beta}_{y_n} - 1) \tau_{y_n} + \sum_{k=1}^{2} (\overline{\beta}_k - 1) \tau_k = 0. \quad (4)
\]

Thus, we obtain a closed system of 4 equations for determination of 4 unknowns.

The complex velocity on the 4-sheets Riemann surface of the contour of the "\( S \)"-shaped profiles of the blades in the form of segments of a logarithmic spiral with a vortex source at the corner point of their conjugation of the schematized circular grating, taking into account [7], equation (1) and the condition that \( \frac{dR[r(\alpha)]}{d\alpha} = \frac{dR(r)}{dr} \) we obtain in the form:

\[
\frac{dR}{dr} = -\kappa_{pq} \prod_{m=1}^{3} (\alpha - \tau_{0m})(\alpha - \alpha_{03})(\alpha - \frac{1}{\alpha_{03}}) \prod_{n=1}^{3} (\alpha - \beta_{yn}) \prod_{k=1}^{2} (\alpha - \tau_k) \prod_{k=1}^{2} \left( \frac{1}{\alpha_{0k}} \right),
\]

where \( \kappa_{pq} = k_{pq} + \frac{p_2^2 - q_2^2}{p_2 q_2} \)-coefficient characterizing the flow regime in the circular grill of profiles as a function of parameters of the vortex source; \( k_{pq} \)-coefficient characterizing the flow regime at the entrance to the circular grill of the profiles; \( R[r(\alpha)] \) is the complex flow potential outside the circle of unit radius on the 4-sheets Riemann surface in the region \( F_k \).

It should be noted that since the width of the schematized jet channels in the neighborhood of the points \( k=1,2 \) has a finite value, its walls are parallel, that is, \( \beta_{1,2} = 0 \).

From (5) it follows that the presence of a branch point \( (\tau_{0k} = \tau_{y_n}) \) and a return point at the corner point of the polygon contour, \( \beta_{1,2} = 2 \) leads to reducing of corresponding factors in the numerator and denominator, that is, at the point of return the corner point and the branch point disappear. Thus, taking into account the equation of critical points for a given class of flows \( m = k \) [7], when the vortex chamber is located at the corner point of the profile, we get:

\[
\frac{dR}{dr} = -\kappa_{pq} \prod_{m=1}^{3} (\alpha - \tau_{0m})(\alpha - \alpha_{03})(\alpha - \frac{1}{\alpha_{03}}) \prod_{k=1}^{2} (\alpha - \tau_k) \prod_{k=1}^{2} \left( \frac{1}{\alpha_{0k}} \right). \quad (6)
\]

In accordance with the uniqueness theorem for the solution of the Dirichlet-Neumann problem for the given parameters of the vortex source and the flow at the entrance to the circular lattice of profiles, the equation (6) corresponds to the unique solution up to a constant [7].

The regularity obtained in accordance with formula (6) is of great practical interest, since it means that in the presence of a vortex in the angular point of the circular grating profile with the intensity of \( \rho_k \) that is, a feature characterizing the return point, and the source with a \( q_i \) flow, at this corner point of branching flow, the fixing of a local vortex source on the profile may be achieved and, therefore, smooth flowing around it is ensured.

Dependence of the energy parameters of the vortex on the characteristics of the external network, explained by the aerodynamic coupling through the high-pressure cavity of the gas-sucking fan body, makes it possible to conclude that the angular point of the "\( S \)"-shaped profile of the blade is being smoothly flown around in a wide range of external condition changes.

The equations for circulation around the "\( S \)"-shaped profile with a vortex chamber of a rotating radial lattice with allowance for [7,8], see Fig. 2, we obtain in the form:
\[ \rho_0 + \rho_n = \rho \sum_n = q_u \frac{\sin(\theta_c - \theta_3 - \delta\theta_a)}{1 - \cos(\theta_c - \theta_3 - \delta\theta_a)} - 4P(1 - 1) \frac{\sin(\theta_3 + \delta\theta_a)}{P^2 + 2p\cos(\theta_3 + \delta\theta_a) + 1} - 4Pq \frac{(P^2 + 1)\sin(\theta_{2,3} + \delta\theta_a)}{n_s(P^2 + 2p\cos(\theta_{2,3} + \delta\theta_a) + 1)(P^2 - 1)} - 4P\rho_0 \frac{\cos(\theta_{2,3} + \delta\theta_a)}{n_s(P^2 + 2p\cos(\theta_{2,3} + \delta\theta_a) + 1)} + (7) \]

Changes of the energy parameters of the vortex source \( q_u + i\rho_\psi \), being analog of hydrodynamic vortex chamber, leads to a shift of the point of branching flow \( \theta_5 \) at the outlet from circular grating in relation to the rear critical point of bodily analytical profile \( \theta_2 \), which increases the effective curvature of the profile and results in increasing of the effective angle of the flow outlet from the circular grill of profiles with an integrated vortex source.

The coefficient of theoretical pressure created by the circular grid of profiles is connected with circulation around the profile by the relation:

\[ \psi_T = 4n_a \omega \rho \sum_n. \]  

Taking into account (7, 8), the equation of the ideal aerodynamic characteristic of a rotating radial grill of "S"-shaped profiles in the form of logarithmic spirals with angular points of conjugation and vortex control of circulation, we write in the form:

\[ \psi_{r, T} = \psi_{r, T} + K_q + K_\rho \rho_n, \]  

where: \( K_q = \frac{2\sin(\theta_c - \theta_3)}{1 - \cos(\theta_c - \theta_3)} + \frac{2p\sin\theta_5}{P^2 - 1} \) – the coefficient of influence of the flow of a vortex source onto the theoretical pressure developed by the grill; \( K_\rho = \frac{2\sin\delta\theta_a}{1 - \cos\delta\theta_a} + 4P \frac{\sin\theta_5}{P^2 - 1} \) – coefficient of influence of circulation of the vortex source onto the theoretical pressure developed by the grill; \( \psi_{r, T} \) – coefficient of theoretical pressure of a circular grill with classical profiles.

The Fig. 3 shows the specific ideal aerodynamic characteristic of a rotating circular grill of "S"-shaped profiles with vortex chambers. From the analysis of the Fig. 3 it may be seen that the "S"-shape lattice of profiles with vortex sources makes it possible to adjust the theoretical pressure coefficient in a wide range and, what is more important, the functional dependence of the increase of the theoretical pressure coefficient on the flow coefficient.

In the circular grill of "S"-shaped profiles with vortex sources, when the intensity of the vortex sources changes at the end points of the profiles, the angle of the exit of the flow from the circular grill changes at a fixed value of its flow rate, which significantly improves the adaptability of the gas-sucking fans.

As the intensity of the vortex source increases, the main flow at the exit from the circular grill of the "S"-shaped profiles rotates in the direction of its rotation, which, according to the Euler equation for the theoretical turbo-machine [7], leads to increase in the theoretical pressure coefficient \( \psi_{T} \), i.e. to the mode of supercirculation.
Figure 3. Aerodynamic characteristics of a rotating circular grill of profiles with a vortex source: 1 – classical theoretical profile; 2 – profile with a positive vortex source; 3 – profile with a negative vortex source; 4 – profile with an alternating vortex source.

Figure 4. The field of design of gas exhausting ventilation modes of gas-abundant coal mines and its overlapping by zones of economical operation of gas-sucking fans: 1 – BRVG-7; 2 – BRVG-9; 3 – BRVG-5; 4 – BRVG-20.

On the basis of carried out theoretical and experimental studies, taking into account the fields of designed gas-sucking modes, based on the designed, radial aerodynamic scheme of TS 145-20, a standard series of gas-suction fans of block-modular design is proposed, the parameters of which completely cover the required and predicted ventilation modes. The Fig. 4 shows the field of designed ventilation modes.

The tests of the prototype of the gas-sucking ventilator BRVG-7 with the power regulator have confirmed the increase in the depth of economic regulation by 50% and the aerodynamic loading by 35%.

2. Conclusions
The suggested grapho-analytical model of a circular grill with "S"-shaped profiles and vortex circulation control allows to create aerodynamic schemes of increased adaptability and loading.

The feedback of the energy parameters of the vortex source "S"-shaped profiles of the blades with parameters of external network substantially increase the adaptability of gas-sucking fans.

The patented structure and way of adaptability and aerodynamic loading increase is the basis for designing of a new generation of gas-sucking fans.

3. References
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