Composable security of delegated quantum computation

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Abstract

Delegating difficult computations to remote large computation facilities, with appropriate security guarantees, is a possible solution for the ever-growing needs of personal computing power. For delegated computation protocols to be usable in a larger context — or simply to securely run two protocols in parallel — the security definitions need to be composable. Here, we define composable security for delegated quantum computation, and prove that several known protocols are composable, including Broadbent, Fitzsimons and Kashefi’s Universal Blind Quantum Computation protocol.

We distinguish between protocols which provide only blindness — the computation is hidden from the server — and those that are also verifiable — the client can check that it has received the correct result. We show that the composable security definition capturing both these notions can be reduced to a combination of two distinct stand-alone security definitions.
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1 Introduction

1.1 Background

It is unknown in what form quantum computers will be built. One possibility is that large quantum servers may take a role similar to that occupied by massive superclusters today. They would be available as important components in large information processing clouds, remotely accessed by clients using their home-based simple devices. The issue of the security and the privacy of the computation is paramount in such a setting.

Childs [Chi05] proposed the first such delegated quantum computation (DQC) protocol, which hides the computation from the server, i.e., the computation is blind. This was followed by Arrighi and Salvail [AS06], who introduced a notion of verifiability — checking that the server does what is expected — but only for a restricted class of public functions, and many others, e.g., [BFK09, ABE10, Mor12, MDK10, DKL12, MF12, FK12].

However, with the exception of recent work by Broadbent, Gutoski and Stebila [BGS12], all previous DQC papers use stand-alone security definitions, i.e., none of them consider the composability of the protocol. This means that, even though they prove that the server cannot — from the information leaked during a single execution of the protocol in an isolated environment — learn the computation or produce a wrong output without being detected, they do not guarantee any kind of security in any realistic setting. In particular, if a server treats two requests simultaneously or if the delegated computation is used as part of a larger protocol (such as the quantum coins of Mosca and Stebila [MS10]), these works on DQC cannot be used to infer security.

To illustrate how insufficiently strong security notions can create a problem in a larger context, consider the task of computing a witness for a positive instance of an NP problem, and the (somewhat absurd) protocol in which the server simply picks a witness at random and sends it to the client. The protocol obviously does not leak any information about the input, since no information is sent from the client to the server. The client can also verify that the solution received is correct, and never accepts a wrong answer. But if the server ever learns whether the witness was accepted, he learns something about the input problem — if there are only two choices for the input with distinct witnesses, he learns exactly which one was used. These intuitive security notions are insufficient, and we have to take great care what security definitions are used.

What is more, problems are known which have stand-alone solutions, but for which no protocol which is composable exists. Coin expansion, in which two mutually distrustful players want to generate \( n \) random coin flips given \( m < n \) perfect coin flips, is such an example [HMU06]. Even though there exists a protocol which achieves this when run once, two parallel executions of the protocol allow a dishonest player to generate correlated coin flips.

1.2 Scope and security of DQC

A common feature of all DQC protocols is that the client, while not being capable of full-blown quantum computation, has access to limited quantum-

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\footnote{A \textit{composable security} framework must be used for a protocol to be secure in an arbitrary environment, see \textsection 1.3.}
enriched technology, which she needs to interact with the server. One of the key points upon which the different DQC protocols vary, is the complexity and the technical feasibility of the aforementioned quantum-enriched technology. In particular, in the proposal of Childs [Chi05], the client has quantum memory, and the capacity to perform local Pauli operations. The protocol of Arrighi and Salvail [AS06] requires the client to have the ability to generate relatively involved superpositions of multi-qubit states, and perform a family of multi-qubit measurements. Aharonov, Ben-Or and Eban [ABE10], for the purposes of studying quantum prover interactive proof systems, considered a secure DQC protocol in which the client has a constant-sized quantum computer. The blind DQC protocol proposed by Broadbent, Fitzsimons and Kashefi [BFK09] has arguably the lowest requirements on the client. In particular, she does not need any quantum memory and is only required to prepare single qubits in separable states randomly chosen from a small finite set analogous to the BB84 states. Alternatively, Morimae and Fujii [MF12] propose a DQC protocol in which the client only needs to measure the qubits she receives from the server to perform the computation.

A second important distinction between these protocols is in the types of problems the protocol empowers the client to solve. Most protocols, e.g., [Chi05, ABE10, BFK09, MF12, FK12], allow a client to perform universal quantum computation, whereas in [AS06] the client is restricted to the evaluation of random-verifiable functions.

Finally, an important characteristic of these protocols is the flavor of security guaranteed to the client. Here, one is predominantly interested in two distinct features: privacy of computation (generally referred to as blindness) and verifiability of computation. Blindness characterizes the degree to which the computational input and output, and the computation itself, remain hidden from the server. This is the main security concern of, e.g., [Chi05, BFK09, MF12]. Verifiability ensures that the client has means of confirming that the final output of the computation is correct. In addition to blindness, some form of verifiability is given by, e.g., [AS06, ABE10, FK12]. These works do however not concern themselves with the cryptographic soundness of their security notions. In particular, none of them consider the issue of composability of DQC. A notable exception is the recent work of Broadbent, Gutoski and Stebila [BGS12], who, independently from our work, prove that a variant of the DQC protocol of Aharonov, Ben-Or and Eban [ABE10] provides composable security.

The prospects of delegated quantum computation with suitable security properties go beyond the purpose of solving computational problems for clients. In [ABE10, AV12] verifiable quantum computation has been linked to quantum complexity theory, and to the fundamental problem of the feasibility of falsify-

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3This holds in the case of classical input and output. If quantum inputs and/or outputs are considered, then the client has to be able to apply a quantum one-time pad to the input state, and also decrypt a quantum one-time pad of the output state.

4The states needed by the protocol of [BFK09] are \((|0\rangle + e^{i k \pi/4} |1\rangle)/\sqrt{2}\) for \(k \in \{0, \ldots, 7\}\).

4Roughly speaking, a function \(f\) is random-verifiable if pairs of instances and solutions \((x, f(x))\) can be generated efficiently, where \(x\) is sampled according to the uniform distribution from the function’s domain.

5The work of Broadbent et al. [BGS12] is on one-time programs. Their result on the composability of DQC is obtained by modifying their main one-time program protocol and security proof so that it corresponds to the DQC protocol from [ABE10].
ing quantum mechanics [Vaz07]. The privacy properties of secure DQC have also been exploited in [MS10], where DQC is suggested as a component of the verification step of unforgeable quantum coins.

1.3 Composable security

The first frameworks for defining composable security were proposed independently by Canetti [Can01,Can05] and by Backes, Pfitzmann and Waidner [PW01] [BPW07] [BPW04], who dubbed them Universally Composable (UC) security and Reactive Simulatability, respectively. These security notions have been extended to the quantum setting by Ben-Or and Mayers [BM04] and Unruh [Unr04] [Unr10].

More recently, Maurer and Renner proposed a new composable framework, Abstract Cryptography (AC) [MR11,Man11]. Unlike its predecessors that use a bottom-up approach to defining models of computation, algorithms, complexity, efficiency, and then security of cryptographic schemes, the AC approach is top-down and axiomatic, where lower abstraction levels inherit the definitions and theorems (e.g., a composition theorem) from the higher level, but the definition or concretization of low levels is not required for proving theorems at the higher levels. In particular, it is not hard-coded in the security notions of AC whether the underlying computation model is classical or quantum, and this framework can be used equally for both.

Even though these frameworks differ considerably in their approach, they all share the common notion that composable security is defined by the distance between the real world setting, and an ideal setting in which the cryptographic task is accomplished in some perfect way. We use AC in this work, because it simplifies the security definitions by removing many notions which are not necessary at that level of abstraction. But the same results could have been proven using another framework, e.g., a quantum version of UC security [Unr10].

1.4 New results

In this paper, we define a composable framework for analyzing the security of delegated quantum computing, using the aforementioned AC framework [MR11,Man11]. We model DQC in a generic way, which is independent of the computing requirements or universality of the protocol, and encompasses to the best of our knowledge all previous work on DQC. We then define composable blindness and composable verifiability in this framework. The security definitions are thus applicable to any DQC protocol fitting in our model.

We study the relations between stand-alone security and composable security of DQC. We show that by strengthening the existing notion of stand-alone verifiability, we can close the gap between stand-alone and composable security of DQC. To do this we introduce the notion of independent verifiability. Intuitively, this captures the idea that the acceptance probability of the client should not depend on the input or computation performed, but rather only on the activities of the (dishonest) server. We then show that any protocol satisfying two simple definitions, stand-alone blindness and stand-alone independent verifiability, are always composable (see Corollary 6.8 on page 24 for the exact statement). This, for instance, implies that the protocol of Fitzsimons and Kashefi [FK12] provides composable blindness and verifiability.
Finally, we analyze the security of two protocols — Broadbent, Fitzsimons and Kashefi [BFK09] and Morimae and Fujii [MF12] — that do not provide any form of verifiability, so the generic reduction from composable to stand-alone security cannot be used. We prove that both these protocols provide perfect composable blindness (in Theorems 7.1 and 7.2 on pages 31 and 33).

1.5 Other related work

The blind DQC protocol of [BFK09] has recently been getting considerable attention in both the experimental and theoretical scientific community. Due to the relatively modest requirements on the client, a small-scale experimental realization of the protocol has already been demonstrated [BKB+12]. Various theoretical modifications of this protocol have been proposed. For instance, the settings where the client does only measurements [MF12], where the client uses weak coherent pulses [DKL12], or the server uses different types of computational resource states [MDK10] have been studied. Recently, a variant for continuous-variable quantum computation has been proposed as well [Mor12].

It is worth mentioning that the questions of secure delegated computation have initially been addressed in the context of classical client-server scenarios. Abadi, Feigenbaum and Kilian [AFK87] considered the problem of “computing with encrypted data”, where for a function $f$, an instance $x$ can be efficiently encrypted into $z = E_k(x)$ in such a way that the client can recover $f(x)$ efficiently from $k$ and $f(z)$ computed by the server. There they showed that no NP-hard function can be computed while maintaining unconditional privacy, unless the polynomial hierarchy collapses at the third level [AFK87]. The problem of securely delegating difficult and time-consuming computations was also studied in the framework of (computationally secure) public-key cryptography, essentially from its very beginnings [RAD78]. Even in this setting, this problem known as fully homomorphic encryption, was only solved recently [Gen09].

1.6 Structure of this paper

In Section 2 we introduce the AC framework that we use to model security. In Section 3 we then instantiated the abstract systems from Section 2 with the appropriate quantum systems and metrics used in this work. In Section 4 we explain delegated quantum computation, and model composable security for such protocols. In Section 5 we show that composable verifiability (which encompasses blindness) is equivalent to the distance between the real protocol and some ideal map that simultaneously provides both (stand-alone) blindness and verifiability. This map is however still more elaborate than existing stand-alone definitions. In Section 6 we break this map down into individual notions of blindness and independent verifiability, and prove that these are sufficient to achieve composable security. In Section 7 we prove that some existing protocols are composable blind, in particular, that of Broadbent, Kashefi and Fitzsimons [BFK09].
2 Abstract cryptography

2.1 Overview
To model security we use Maurer and Renner’s [MR11] Abstract Cryptography framework. The traditional approach to defining security can be seen as bottom-up. One first defines (at a low level) a computational model (e.g., a Turing machine or a circuit). Based on this, the concept of an algorithm for the model and a communication model (e.g., based on tapes) are defined. After this, notions of complexity, efficiency, and finally the security of a cryptosystem can be defined. The AC framework uses a top-down approach: in order to state definitions and develop a theory, one starts from the other end, the highest possible level of abstraction — the composition of abstract systems — and proceeds downwards, introducing in each new lower level only the minimal necessary specialization.

One may give the analogous example of group theory, which is used to describe matrix multiplication. In the bottom-up approach, one would start explaining how matrices are multiplied, and then based on this find properties of the matrix multiplication. In contrast to this, the AC approach would correspond to first defining the (abstract) multiplication group and prove theorems already on this level. The matrix multiplication would then be introduced as a special case of the multiplicative group.

On a high level of abstraction, a cryptographic protocol can be viewed as constructing some resource \( S \) out of other resources \( R \). For example, a one-time pad constructs a secure channel out of a secret key and an authentic channel; a quantum key distribution protocol constructs a shared secret key out of a classical authentic channel and an insecure quantum channel. If some protocol \( \pi \) constructs \( S \) out of \( R \) with error \( \varepsilon \), we write

\[
R \xrightarrow{\pi, \varepsilon} S.
\]  

(1)

For the construction to be composable, we need the following conditions fulfilled:

\[
R \xrightarrow{\pi, \varepsilon} S \text{ and } S \xrightarrow{\pi', \varepsilon'} T \implies R \xrightarrow{\pi \circ \pi', \varepsilon + \varepsilon'} T
\]

\[
R \xrightarrow{\pi, \varepsilon} S \text{ and } R' \xrightarrow{\pi', \varepsilon'} S' \implies R \parallel R' \xrightarrow{\pi \parallel \pi', \varepsilon + \varepsilon'} S \parallel S'
\]

where \( R \parallel R' \) is a parallel composition of resources, and \( \pi' \circ \pi \) and \( \pi|\pi' \) are sequential and parallel composition of protocols.

In Section 2.3 we provide a security definition which satisfies these conditions. Intuitively, the resource \( R \) along with the protocol \( \pi \) are part of the real or concrete world, and the resource \( S \) is some ideal abstraction of the resource we want to build. Eq. (1) is then satisfied if an adversary could, in an ideal world where the ideal resource is available, achieve anything that she could achieve in the real world. This argument involves, as a thought experiment, simulator systems which transform the ideal resource into the real world system consisting of the real resource and the protocol.

\footnote{It is important to point out that theorems proven at a certain (high) level of abstraction are completely precise (as they are mathematical theorems). This is true without instantiations of the lower levels, which is exactly the point of abstraction.}
2.2 Resources, converters and distinguishers

In this section we define (on a high level of abstraction) the elements present in Eq. (1) namely resources $R, S$, a protocol $\pi$, and a pseudo-metric allowing us to define the error $\varepsilon$.

Depending on what model of computing is instantiated at a lower level, a resource can be modeled as a random system in the classical case [Mau02, MPR07], or, if the underlying system is quantum, as a sequence of CPTP maps with internal memory (e.g., quantum combs [CDP09]). However, in order to define the security of a protocol, it is not necessary to go down to this level of detail, a resource can be modeled in more abstract terms. A resource is an (abstract) system with interfaces specified by a set $\mathcal{I}$ (e.g., $\mathcal{I} = \{A, B, E\}$). Each interface $i \in \mathcal{I}$ is accessible to a user $i$ and provides her or him with certain functionalities. Furthermore, a dishonest user might have access to more functionalities than an honest one, and these should be clearly marked as such (e.g., a filter covers these functionalities for honest player, and a dishonest user removes the filter to access them). We call these guaranteed and filtered functionalities. For example, a key distribution resource is often modeled as a resource which either produces a secret key or an error flag. This resource has no guaranteed functionalities at Eve’s interface, but may provide her with the filtered functionality of preventing a key being generated. Alice’s interface guarantees that she gets a secret key (or an error flag), but it may also provide her with the filtered functionality of choosing what key is generated.

A protocol $\pi = \{\pi_i\}_{i \in \mathcal{I}}$ is a set of converters $\pi_i$, indexed by the set of interfaces $\mathcal{I}$. A converter is an (abstract) system with only two interfaces, an outside interface and an inside interface. The outside interface is connected to the outside world, it receives the inputs and produces the outputs. The inside interface is connected to the resources used.

In Figure 1 we illustrate this by connecting a one-time pad protocol to a resource $R$ consisting of a secret key and an authentic channel. Let $\pi = (\pi_A, \pi_B, \pi_E)$ be a one-time pad protocol, and $\pi_A$ be Alice’s part of the protocol: $\pi_A$ is connected at the inner interface to a resource generating a secret key and to an authentic channel (for this example, we assume that neither the ideal key nor the authentic channel produce an error, they both always generate a key and deliver the message, respectively), both of which we combine together as the resource $R$. At the outer interface it receives some message $x$, it gets a key $k$ from the key resource, and sends $x \oplus k$ down the authentic channel. Bob’s part of the protocol $\pi_B$ receives $y$ from the authentic channel and $k$ from the key resource at its inner interface, and outputs $y \oplus k$ at the outer interface. Note that the protocol also specifies an honest behavior for Eve, $\pi_E$, which consists in not listening to the communication channel, i.e., it is a converter with no functionalities at the outer interface and which blocks the leaks from the authentic channel at the inner interface.

Converters connected to resources build new resources with the same interface set, and we write either $\pi_i R$ or $R \pi_i$ to denote the new resource with the...
The concrete setting of the one-time pad with Eve's honest protocol $\pi_{E}$. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. The converters ($\pi_A$, $\pi_B$, $\pi_E$) of the one-time pad protocol are connected to the resource $R$ consisting of a secret key and an authentic channel.

To measure how close two resources are, we define a pseudo-metric on the space of resources. We do this with the help of a distinguisher. For $n$-interface resources a distinguisher $D$ is a system with $n+1$ interfaces, where $n$ interfaces connect to the interfaces of a resource $R$ and the other (outside) interface outputs a bit. For a class of distinguishers $D$, the induced pseudo-metric, the distinguishing advantage, is

$$d(R, S) := \max_{D \in D} \Pr[D_R = 1] - \Pr[D_S = 1],$$

where $D_R$ is the binary random variable corresponding to $D$ connected to $R$.

If $d(R, S) \leq \varepsilon$, we say that the two resources are $\varepsilon$-close and sometimes write $R \approx \varepsilon S$; or $R = S$ if $\varepsilon = 0$.

### 2.3 Security

We now have introduced all the notions used in the generic security definition:

**Definition 2.1** (See [MR11, Mau11]). Let $R_\phi$ and $S_\psi$ be resources with interfaces $I$ and filters $\phi$ and $\psi$. We say that a protocol $\pi$ (securely) constructs $S_\psi$ out of $R_\phi$ within $\varepsilon$, and write $R_\phi \xrightarrow{\varepsilon} S_\psi$, if there exist converters $\sigma = \{\sigma_i\}_{i \in I}$ —

---

10 There is no mathematical difference between $\pi_{iR}$ and $R_{i\pi_i}$. It sometimes simplifies the notation to have the converters for some players written on the right of the resource and the ones for other players on the left, instead of all on the same side, hence the two notations.

11 In this work we study information-theoretic security, and therefore the only class of distinguishers that we consider is the set of all distinguishers.

12 In [MR11] this definition is given on a higher level of abstraction. However for the particular case of filtered resources, Definition 2.1 is equivalent.
which we call simulators — such that,

\[ \forall P \subseteq \mathcal{I}, \quad d(\pi_P \phi_P R, \sigma_{\mathcal{I}\setminus P} \psi_P S) \leq \varepsilon. \] (2)

We illustrate this definition in the case of the one-time pad. In this example, we wish to construct a secure channel \( S \), which is depicted in Figure 2 and defined as follows (for simplicity, we assume that Alice and Bob are always honest, and ignore their filtered functionalities): \( S \) takes a message \( x \) at the \( A \)-interface, leaks the message length \( |x| \) at the \( E \)-interface, and outputs \( x \) at the \( B \)-interface. This resource captures the desired notion of a secure channel, because it only leaks the message size, and does not provide the adversary with any functionality to falsify the message. We model explicitly that the message size leak at the \( E \)-interface is not a guaranteed functionality by depicting it in gray in Figure 2.

We additionally draw the filter converter \( \psi_E \), which covers the cheating interface and can be removed by a dishonest player. \( \psi_E \) has no functionalities at the outer interface, and blocks this message size leak at the inner interface. In the general case, these filters can be defined for all interfaces.\(^\text{13}\)

\[ \text{Secure channel } S \]

\[ \text{Alice} \quad x \quad \psi_E \quad |x| \quad \text{Eve} \quad \text{Bob} \]

\[ \text{Figure 2} \quad \text{– A secure channel from Alice to Bob. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. A filter } \psi_E \text{ covers Eve’s cheating functionality.} \]

The correctness of the protocol \( \pi \) is captured by measuring the distance between \( \pi R \), the combination of the entire honest protocol with the resources (Figure 1), and \( \psi S \), the ideal resource with all filtered functionalities obstructed (Figure 2). In the case of the one-time pad, we have \( d(\pi_A \pi_B \pi_E R, \psi_E S) = 0 \); since the resources \( \pi_A \pi_B \pi_E R \) and \( \psi_E S \) both simply take a message \( x \) as input at the \( A \)-interface and output the same message at the \( B \)-interface, no distinguisher can notice a difference.

If a player \( i \) cheats, she does not (necessarily) follow her protocol \( \pi_i \), but can interact arbitrarily with her interface. We thus remove the corresponding protocol converters from the real setting to model the resulting resource, which we depict for the one-time pad in Figure 3a. Security of the protocol in the presence of a cheating party \( i \) is achieved if this player is not able to accomplish more than what is allowed by her interface of the ideal resource with the filter removed. This is the case if there exists a simulator converter \( \sigma_i \) that, when plugged into the \( i \)-interface of the ideal resource \( S \), can convert between the interaction with the corrupt player (or distinguisher) and the filtered functionalities of the resource, such that the real and ideal worlds are indistinguishable.

\(^{13}\)We only denote Eve’s filter explicitly in the following, since Alice and Bob’s filters are trivial (the identity).
For example, in the case of the one-time pad and a dishonest Eve, a cipher $y$ is leaked at the $E$-interface, whereas in the ideal setting, only the message length is leaked. The simulator $\sigma_E$ therefore must recreate a cipher given the message length. It does this by simply generating a random string $y$ of the corresponding length and outputting it at its outer interface. This is illustrated in Figure 3b.

It is not hard to verify that with this simulator, $d(\pi_A\pi_B\mathcal{R},\sigma_E\mathcal{S}) = 0$, since the resources $\pi_A\pi_B\mathcal{R}$ and $\sigma_E\mathcal{S}$ both take a message $x$ at their $A$-interface, which they output at their $B$-interface, and output a completely random string of the same length at their $E$-interface.

(a) The concrete resource resulting from honest Alice and Bob running their one-time pad protocols ($\pi_A, \pi_B$) with a secret key and authentic channel.

(b) The ideal resource $\mathcal{S}$ constructed by the one-time pad for an honest Alice and Bob, and a simulator $\sigma_E$ plugged into Eve’s interface.

Figure 3 – The real and ideal settings for the one-time pad with a cheating Eve. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. Since these resources are indistinguishable, the one-time pad provides perfect security.

[Definition 2.1] requires $2^n$ inequalities to be satisfied in a model with $n$ players, i.e., one for every possible subset of dishonest players. In practice however, if we are only interested in modeling security when a given set of players is known to always be honest—e.g., Alice and Bob are honest in the one-time pad example—then it is sufficient to consider only the corresponding inequalities from Eq. (2). This is equivalent to giving those players arbitrary filtered functionalities, and reflects the fact that we do not place any restrictions on what these players might achieve, were they to be dishonest.
Remark 2.2. Abstract cryptography (AC) differs from universal composability (UC) in many conceptual and mathematical ways. In particular, the AC requirement that there exist distinct simulators at each interface instead of merging all dishonest players into one entity make it strictly more powerful than UC: this allows noncommunicating dishonest players to be modeled as a feature of the ideal resource, and thus directly capture notions such as coercibility [MR11].

However, in the special case of one dishonest player, Eq. (2) is equivalent to what one obtains by modeling the same problem with UC. Since the rest of this work deals with delegated quantum computation, a two-party protocol with one dishonest player, the same results could have been obtained using the UC framework.

3 Quantum systems

In [Section 2] resources and converters were introduced as abstract systems. Here we model them explicitly for the special case of two-party protocols considered in the rest of the work. In [Section 3.1] we first briefly define the notation and some basic concepts that we use. In [Section 3.2] we then model two-party protocols. And finally in [Section 3.3] we define several metrics which correspond to the distinguishing advantage for specific resources.

3.1 Notation and basic concepts

H always denotes a finite-dimensional Hilbert space. We denote by $L(\mathcal{H}_A, \mathcal{H}_B)$ the set of linear operators from $\mathcal{H}_A$ to $\mathcal{H}_B$, by $L(\mathcal{H})$ the set of linear operators from $\mathcal{H}$ to itself, and by $P(\mathcal{H})$ the subset of positive semi-definite operators. We define the set of normalized quantum states $S(\mathcal{H}) := \{ \rho \in P(\mathcal{H}) : \text{tr } \rho = 1 \}$ and the set of subnormalized quantum states $S_{\leq}(\mathcal{H}) := \{ \rho \in P(\mathcal{H}) : \text{tr } \rho \leq 1 \}$. We write $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ for a bipartite quantum system and $\rho_{AB} \in S_{\leq}(\mathcal{H}_{AB})$ for a bipartite quantum state. $\rho_A = \text{tr}_B(\rho_{AB})$ and $\rho_B = \text{tr}_A(\rho_{AB})$ denote the corresponding reduced density operators.

The set of feasible maps between two systems $A$ and $B$ is the set of all completely positive, trace-preserving (CPTP) maps $\mathcal{E} : L(\mathcal{H}_A) \to L(\mathcal{H}_B)$. By the Kraus representation, such a map can always be given by a set of linear operators $\{E_k \in L(\mathcal{H}_A, \mathcal{H}_B)\}_k$ with $\sum_k E_k^\dagger E_k = \mathbb{1}_A$. We then have $\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$. We also consider trace non-increasing maps—in particular, to describe the evolution of a system conditioned on a specific measurement outcome—i.e., maps with operators $E_k$ such that $\sum_k E_k^\dagger E_k \leq \mathbb{1}_A$. Though when unspecified, we always mean trace-preserving maps. For a quantum state $\rho \in S_{\leq}(\mathcal{H}_{AC})$ and a map $\mathcal{E} : L(\mathcal{H}_A) \to L(\mathcal{H}_B)$, $\mathcal{E}(\rho)$ is shorthand for $(\mathcal{E} \otimes \text{id}_C)(\rho)$, where $\text{id}_C$ is the identity on system $C$.

Throughout this paper we mostly use the standard notation for common quantum gates, for instance $X$ and $Z$ denote the Pauli-X and Pauli-Z operators. We will additionally often refer to the the parametrized phase gate $Z_\theta = |0\rangle \langle 0| + e^{i\theta} |1\rangle \langle 1|$, and the two-qubit controlled-Z gate $\text{ctrl-Z} = |00\rangle \langle 00| + |01\rangle \langle 01| + |10\rangle \langle 10| - |11\rangle \langle 11|$. 

\footnote{For a more detailed introduction to quantum information theory we refer to [NC00,Wat08].}
3.2 Two-party protocols

A two-party protocol can in general be modeled by a sequence of CPTP maps \( \{E_i : \mathcal{L}(\mathcal{H}_{AC}) \to \mathcal{L}(\mathcal{H}_{AC})\}_i \) and \( \{F_i : \mathcal{L}(\mathcal{H}_{CB}) \to \mathcal{L}(\mathcal{H}_{CB})\}_i \), where A and B are Alice and Bob’s registers, and C represents a communication channel. Initially Alice and Bob place their inputs in their registers, and the channel \( C \) is in some fixed state \( |0\rangle \). The players then apply successively their maps to their respective registers and the channel. For example, in the first round Alice applies \( E_1 \) to the joint system \( AC \), and sends \( C \) to Bob, who applies \( F_1 \) to \( CB \), and returns \( C \) to Alice. Then she applies \( E_2 \), etc. The messages sent on the channel \( C \) correspond to messages leaving the corresponding converter at the inner interface and being sent through a channel resource \( \mathcal{R} \) to the other player.

The inputs are initially received by the converters at the outer interfaces, and the final contents of the A and B registers is output at the outer interface once the last map of the protocol has been applied. This is illustrated in Figure 4.

\[ \text{Figure 4} – \text{A generic two-party protocol. Alice has access to the left interface and Bob to the right interface. The protocol} (\pi_A, \pi_B) \text{consists in sequences of maps. The channel resource} \mathcal{R} \text{simply transmits the messages between the players.} \]

For a protocol with \( N \) rounds, the resource \( \pi_i\mathcal{R} \), corresponding to one of the players’ protocol plugged into the channel resource \( \mathcal{R} \), has been called a quantum \( N \)-comb and thoroughly studied by Chiribella, D’Ariano and Perinotti [CDP09]. In particular, they give a concise representation of combs in terms of the Choi-Jamiolkowski isomorphism, and define the appropriate distance measure between combs, corresponding to the optimal distinguishing advantage, which we sketch in the next section.

3.3 Distance measures

The trace distance between two states \( \rho \) and \( \sigma \) is given by \( D(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_{\text{tr}} \), where \( \| \cdot \|_{\text{tr}} \) denotes the trace norm and is defined as \( \| A \|_{\text{tr}} := \text{tr} \sqrt{A^\dagger A} \). If \( D(\rho, \sigma) \leq \varepsilon \), we say that the two states are \( \varepsilon \)-close and often write \( \rho \approx_\varepsilon \sigma \). This corresponds to the distinguishing advantage between two resources \( \mathcal{R} \) and \( \mathcal{S} \), which take no input and produce \( \rho \) and \( \sigma \), respectively, as output: the

\[ \text{One could consider a more general two-party setting, where the players have access to other resources than a channel, e.g., public randomness. But since in the rest of this work we are interested only in protocols where the players have no other resource than a channel, we also consider only this case here.} \]
probability of a distinguisher guessing correctly whether he holds $R$ or $S$ is exactly \( \frac{1}{2} + \frac{1}{2} D(\rho, \sigma) \). In Appendix A we define the generalized trace distance and the purified distance, which are more appropriate for characterizing the distance between subnormalized states.

Another common metric which corresponds to the distinguishing advantage between resources of a certain type is the diamond norm. If the resources $R$ and $S$ take an input $\rho \in S(H_A)$ and produce an output $\sigma \in S(H_B)$, the distinguishing advantage between these resources is the diamond distance between the correspond maps $E, F : L(H_A) \to L(H_B)$. A distinguisher can generate a state $\rho_{AR}$, input the $A$ part to the resource, and try to distinguish between the resulting states $E(\rho_{AR})$ and $F(\rho_{AR})$. We have $d(R, S) = \diamond(E, F) = \frac{1}{2} \| E - F \|_\diamond$, where $\| \Phi \|_\diamond := \max \{ \| (\Phi \otimes \text{id}_R)(\rho) \|_{tr} : \rho \in S(H_{AR}) \}$ is the diamond norm. Note that the maximum of the diamond norm can always be achieved for a system $R$ with $\text{dim} H_R = \text{dim} H_A$. Here too, we sometimes write $E \approx_\epsilon F$ if two maps are $\epsilon$-close.

If the resources considered are halves of two player protocols, say $\pi_iR$ or $\pi_jR$, the above reasoning can be generalized for obtaining the distinguishing advantage. The distinguisher can first generate an initial state $\rho \in S(H_{AR})$—which for convenience we define as a map on no input $\rho := D_0()$—and input the $A$ part of the state into the resource. It receives some output $\rho_{CR}$ from the resource, can apply some arbitrary map $D_1 : L(H_{CR}) \to L(H_{CR})$ to the state, and input the $C$ part of the new state in the resource. Let it repeat this procedure with different maps $D_i$ until the end of the protocol, after which it holds one of two states: $\phi_{AR}$ if it had access to $\pi_iR$ and $\varphi_{AR}$ if it had access to $\pi_jR$. The trace distance $D(\phi_{AR}, \varphi_{AR})$ defines the advantage the distinguisher has of correctly guessing whether it was interacting with $\pi_iR$ or $\pi_jR$, and by maximizing this over all possible initial inputs $\rho_{AR} = D_0()$, and all subsequent maps $\{ D_i : L(H_{CR}) \to L(H_{CR}) \}$, the distinguishing advantage between these resources becomes

$$d(\pi_iR, \pi_jR) = \max_{\{ D_i \}} D(\phi_{AR}, \varphi_{AR}).$$

This has been studied by Chiribella et al. \cite{CDP09}, and we refer to their work for more details.

4 Delegated quantum computation

In the (two-party) delegated quantum computation (DQC) model, Alice asks a server, Bob, to execute some quantum computation for her. Intuitively, Alice plays the role of a client, and Bob the part of a computationally more powerful server. Alice has several security concerns. She wants the protocol to be blind, that is, she wants the server to execute the quantum computation without learning anything about the input other than what is unavoidable, e.g., an upper bound on its size, and possibly whether the output is classical or quantum. She may also want to know if the result sent to her by Bob is correct, which we refer to as verifiability.
In Section 4.1 we model the ideal resource that a DQC protocol constructs and the structure of a generic DQC protocol. And in Section 4.2 we apply the generic AC security definition (Definition 2.1) to DQC.

4.1 DQC model

4.1.1 Ideal resource

To model the security (and correctness) of a delegated quantum computation protocol, we need to model the ideal delegated computation resource \( S \) that we wish to build. We start with an ideal resource that provides blindness, and denote it \( S^b \).

The task Alice wants to be executed is provided as an input to the resource \( S^b \) at the \( A \)-interface. It could be modeled has having two parts, some quantum state \( \psi_A \) and a classical description \( \Phi_A \) of some quantum operation that she wants to apply to \( \psi \), i.e., she wishes to compute \( \Phi(\psi) \). This can alternatively be seen as applying a universal computation \( U \) to the input \( \psi_A \otimes |\Phi\rangle \langle \Phi|_A \).

We adopt this view in the remainder of this paper, and model the resource as performing some fixed computation \( U \). There is however no reason to enforce that the input be a pair of a quantum state and classical operation, so we generalize it to any quantum input \( \psi_A \).

The ideal resource \( S^b \) thus takes this input \( \psi_A \) at its \( A \)-interface, and, if Bob does not activate his filtered functionalities — which can be modeled by a bit \( b \), set to 0 by default, and which a simulator \( \sigma_B \) can flip to 1 to signify that it is activating the cheating interface — \( S^b \) outputs \( U(\psi_A) \). This ensures both correctness and universality (in the case where \( U \) is a universal computation). Alternatively, \( S^b \) can be restricted to work for inputs corresponding to a certain class of computational problems, if we desire a construction only designed for such a class.

If the cheating \( B \)-interface is activated, the ideal resource outputs at this interface all the “permitted leaks” that are intentionally made public by a protocol, e.g., the number of qubits and steps that Bob will need to perform the computation, and possibly whether the final result of the computation that Bob should deliver is a quantum state or a classical string.

Bob also has another filtered functionality, one which allows him to tamper with the final output. The most general operation he could perform is to give \( S^b \) a quantum state \( \psi_B \) — which could be entangled with Alice’s input \( \psi_A \) — along with the description of some map \( \mathcal{E} : \mathcal{L}(\mathcal{H}_{AB}) \to \mathcal{L}(\mathcal{H}_A) \), and ask it to output \( \mathcal{E}(\psi_{AB}) \) at Alice’s interface. Since \( S^b \) only captures blindness, but says nothing about Bob’s ability to manipulate the final output, we define it to perform this operation and output any \( \mathcal{E}(\psi_{AB}) \) at Bob’s request. This is depicted in Figure 5a with the filtered functionalities in gray.

**Definition 4.1.** The ideal DQC resource \( S^b \) which provides both correctness and blindness takes an input \( \psi_A \) at Alice’s interface, but no honest input at Bob’s interface. Bob’s filtered interface has a control bit \( b \), set by default to 0, which he can flip to activate the other filtered functionalities. The resource \( S^b \) then outputs the permitted leaks at Bob’s interface, and accepts two further

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10If a protocol requires the input to have a certain structure for correctness, e.g., part classical part quantum, this can embed in \( U \), e.g., \( U \) first measures the part of the input expected to be classical.
Blind DQC resource $S^b$

$$\rho_A = \begin{cases} 
U(\psi_A) & \text{if } b = 0, \\
E(\psi_{AB}) & \text{if } b = 1.
\end{cases}$$

(a) $S^b$ provides blindness — it only leaks the permitted information at Bob’s interface — but allows Bob to choose Alice’s output.

Secure DQC resource $S^{bc}$

$$\rho_A = \begin{cases} 
U(\psi_A) & \text{if } c = 0, \\
|\text{err}\rangle \langle \text{err}| & \text{if } c = 1.
\end{cases}$$

(b) $S^{bc}$ provides both blindness and verifiability — in addition to leaking only the permitted information, it never outputs an erroneous computation result.

**Figure 5** — Ideal DQC resources. The client Alice has access to the left interface, and the server Bob to the right interface. The double-lined input flips a bit set by default to 0. The functionalities provided at Bob’s interface are grayed to signify that they are accessible only to a cheating server. If Bob is honest, this interface is obstructed by a filter, which we denote by $\perp_B$ in the following.

inputs, a state $\psi_B$ and map description $[E][E]$. If $b = 0$, it outputs the correct result $U(\psi_A)$ at Alice’s interface; otherwise it outputs Bob’s choice, $E(\psi_{AB})$.

A DQC protocol is verifiable if it provides Alice with a mechanism to detect a cheating Bob and output an error flag $\text{err}$ instead of some incorrect computation. This is modeled by weakening Bob’s filtered functionality: an ideal DQC resource with verifiability, $S^{bc}$, only allows Bob to input one classical bit $c$, which specifies whether the output should be $U(\psi_A)$ or some error state $|\text{err}\rangle$, which by construction is orthogonal to the space of valid outputs. The ideal resource thus never outputs a wrong computation. This is illustrated in Figure 5b.

**Definition 4.2.** The ideal DQC resource $S^{bc}$ which provides correctness, blindness and verifiability takes an input $\psi_A$ at Alice’s interface, and two filtered control bits $b$ and $c$ (set by default to 0). If $b = 0$, it simply outputs $U(\psi_A)$ at Alice’s interface. If $b = 1$, it outputs the permitted leaks at Bob’s interface, then reads the bit $c$, and conditioned on its value, it either outputs $U(\psi_A)$ or $|\text{err}\rangle$ at Alice’s interface.

4.1.2 Concrete setting

In the concrete setting, the only resource that Alice and Bob need is a (two-way) communication channel $\mathcal{R}$. Alice’s protocol $\pi_A$ receives $\psi_A$ as an input on its outside interface. It then communicates through $\mathcal{R}$ with Bob’s protocol $\pi_B$, and produces some final output $\rho_A$. For the sake of generality we assume that the operations performed by $\pi_A$ and $\pi_B$, and the communication between them, are all quantum. Of course, a protocol is only useful if Alice has very few quantum operations to perform, and most of the communication is classical. However, to model security, it is more convenient to consider the most general case possible, so that it applies to all possible protocols.

As described in Section 3.2 their protocols can be modeled by a sequence of CPTP maps $\{E_i : \mathcal{L(H_{AC})} \to \mathcal{L(H_{AC})}\}_{i=1}^N$ and $\{F_i : \mathcal{L(H_{CB})} \to \mathcal{L(H_{CB})}\}_{i=1}^{N-1}$. We illustrate a run of such a protocol in Figure 6. This is a special case of
Figure 4 in which Bob has neither input nor output. The entire system consisting of the protocol \((\pi_A, \pi_B)\) and the channel \(R\) is a map which transforms \(\psi_A\) into \(\rho_A\). If both players played honestly and the protocol is correct, this should result in \(\rho_A = U(\psi_A)\).

\[
\begin{align*}
\psi_A & \xrightarrow{E_1} \rho_A \\
\psi_A & \xrightarrow{E_2} \rho_A \\
\vdots & \\
\psi_A & \xrightarrow{E_N} \rho_A
\end{align*}
\]

\[
\begin{align*}
\pi_A & \xrightarrow{F_1} \pi_B \\
\pi_A & \xrightarrow{F_2} \pi_B \\
\vdots & \\
\pi_A & \xrightarrow{F_{N-1}} \pi_B
\end{align*}
\]

Figure 6 – A generic run of a DQC protocol. Alice has access to the left interface and Bob to the right interface. The entire system builds one CPTP operation which maps \(\psi_A\) to \(\rho_A\).

4.2 Security of DQC

Since we are interested in modeling a cheating server Bob, but do not care what happens if the client Alice does not follow her protocol, it is sufficient to take from Definition 2.1 the equations corresponding to an honest Alice. Applying this to the DQC model from the previous section, we get that a protocol \(\pi\) constructs a blind quantum computation resource \(S^b\) from a communication channel \(R\) within \(\varepsilon\) if there exists a simulator \(\sigma_B\) such that

\[
\begin{align*}
\pi_A R \pi_B & \approx_\varepsilon S^b \perp_B \\
\pi_A R & \approx_\varepsilon S^b \sigma_B,
\end{align*}
\]

where \(\perp_B\) is a filter which obstructs Bob’s cheating interface. The first condition in Eq. (4) captures the correctness of the protocol, and we say that a protocol provides \(\varepsilon\)-correctness if this condition is fulfilled. The second condition, which we illustrate in Figure 7, measures the security. If it is fulfilled, we have \(\varepsilon\)-blindness. If \(\varepsilon = 0\) we say that we have perfect blindness.

Likewise in the case of verifiability, the ideal resource \(S^{bv}\) is constructed by \(\pi\) from \(R\) if there exists a simulator \(\sigma_B\) such that,

\[
\begin{align*}
\pi_A R \pi_B & \approx_\varepsilon S^{bv} \perp_B \\
\pi_A R & \approx_\varepsilon S^{bv} \sigma_B,
\end{align*}
\]

\(\varepsilon\)-blindness, designed specially for the protocol of [BFK09] was also introduced in [DKL12]. There, \(\varepsilon\) quantifies the distance between the original protocol, and a realization of the protocol where Alice’s quantum devices are not perfect. Since we prove in Section 7.1 that this protocol is blind with error \(\varepsilon = 0\), by the triangle inequality, a realization of a protocol which is \(\varepsilon\)-blind w.r.t. the definition in [DKL12] is \(\varepsilon\)-blind w.r.t. the definition introduced in this paper.
The first condition from Eq. (5) is identical to the first condition of Eq. (4) and captures $\varepsilon$-correctness. Since this ideal resource provides both blindness and verifiability, we say that a protocol is $\varepsilon$-secure if the second condition in Eq. (5) (also illustrated by Figure 7) is satisfied. If $\varepsilon = 0$ we say that we have perfect security.

Note that the exact metrics used to distinguish between the resources from Eqs. (4) and (5) have been defined in Section 3.3. $\pi_A R \pi_B$ and $\mathcal{S} \mathcal{\sigma}_B$—as can be seen from their depictions in Figures 6 and 5 (with a filter blocking the cheating interface of the latter)—are resources which implement a single map, so the diamond distance corresponds to the distinguishing advantage. $\pi_A R$ and $\mathcal{S} \mathcal{\sigma}_B$ are half of two-party protocols, so the distinguishing metric corresponds to the distance between quantum $N$-combs introduced by Chiribella et al. [CDP09] and described in Section 3.3.

## 5 Composable security of DQC

Finding a simulator to prove the composability of a protocol can be challenging. In this section we reduce the task of proving that a DQC protocol provides composable security (blindness and verifiability) to proving that the map implemented by the protocol is close to some ideal map that intuitively provides both (stand-alone) blindness and verifiability. The converse also holds: any protocol which provides composable security must be close to this ideal map.

A malicious server Bob will not apply the CPTP maps assigned to him by the protocol, but his own set of cheating maps $\{F_i : \mathcal{L}(\mathcal{H}_{CB}) \rightarrow \mathcal{L}(\mathcal{H}_{CB})\}_{i=1}^{N-1}$. A protocol provides stand-alone blindness if the final state held by Bob could have been generated by a local map on his system—say, $F_i$—independently from Alice’s input, but which naturally depends on his behavior given by the maps $\{F_i\}$. It provides stand-alone verifiability if the final state held by Alice is either the correct outcome or some error flag. Combining the two gives an ideal map of the from $U \otimes F_{ok} + E_{err} \otimes F_{err}$, where $F_{ok}$ and $F_{err}$ break $F$ down in two maps which result in the correct outcome and an error flag, respectively.

**Definition 5.1 (Stand-alone blind verifiability).** We say that a DQC protocol provides stand-alone $\varepsilon$-blind verifiability, if, for all adversarial behaviors $\{F_i\}$, there exist two completely positive, trace non-increasing maps $F_{ok}^B$ and $F_{err}^B$. 

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19 We provide formal definitions of stand-alone blindness and verifiability in Section 6.1.
such that
\[ \mathcal{P}_{AB} \approx_\varepsilon \mathcal{U}_A \otimes \mathcal{F}^\text{ok}_B + \mathcal{E}^\text{ct}_A \otimes \mathcal{F}^\text{err}_B, \]
where \( \mathcal{P}_{AB} : \mathcal{L}(\mathcal{H}_{AB}) \rightarrow \mathcal{L}(\mathcal{H}_{AB}) \) is the map corresponding to a protocol run with Alice behaving honestly and Bob using his cheating operations \( \{ F_i \} \), and \( \mathcal{E}^\text{ct} \) discards the A system and produces an error flag \( | \text{err}\rangle | \text{err}\rangle \) orthogonal to all possible valid outputs. We say that the protocol provides stand-alone \( \varepsilon \)-blind verifiability for a set of initial states \( B \), if Eq. (6) holds when applied to these states, i.e., for all \( \psi_{ABR} \in B \),
\[ \mathcal{P}_{AB}(\psi_{ABR}) \approx_\varepsilon (\mathcal{U}_A \otimes \mathcal{F}^\text{ok}_B + \mathcal{E}^\text{ct}_A \otimes \mathcal{F}^\text{err}_B)(\psi_{ABR}). \]

Remark 5.2. For simplicity, this definition assumes the allowed leaks (e.g., input size, computation size) to be fixed, and applies to all protocols \( \mathcal{P}_{AB} \) tailored for inputs with an identical leak (e.g., identical size). These leaks could be explicitly modeled by allowing the maps \( \mathcal{F}^\text{ok}_B \) and \( \mathcal{F}^\text{err}_B \) to depend on them.

We now prove that [Definition 5.1] is equivalent to composable security.

**Theorem 5.3.** Any protocol which provides stand-alone \( \varepsilon \)-blind verifiability is \( 2\varepsilon \)-secure. And any protocol which is \( \varepsilon \)-secure provides stand-alone \( \varepsilon \)-blind verifiability.

**Proof.** We start by showing that blind verifiability implies composable security, i.e., there exists a simulator \( \sigma_B \) such that the two resources in Figure 7 are \( \varepsilon \)-close. To do this, we define \( \sigma_B \) to work as follows. It sets the bit \( b = 1 \), receives the permitted leaks from \( S^{\text{bv}}_B \), picks an input \( \psi_B \) compatible with this information, and runs the protocol \( \pi_A \) on this input with its internal register, which we denote by \( B \). After the last step, it projects the state it holds in \( B \) on \( | \text{err}\rangle | \text{err}\rangle \) and \( I - | \text{err}\rangle | \text{err}\rangle \), and sends \( c = 0 \) to \( S^{\text{bv}}_B \) if no error was detected, otherwise it sends \( c = 1 \). As defined in [Definition 4.2], \( S^{\text{bv}}_B \) then either outputs the correct result or an error flag depending on the value of \( c \).

As described in Section 3.3, the most general operation the distinguisher can perform to distinguish between the resources \( \pi_A R \) and \( S^{\text{bv}}_B \), is to choose some initial state \( \psi_{AR} \), send \( \psi_A \) to the system with which it is interacting, apply some operations \( \{ F_i : \mathcal{L}(\mathcal{H}_{CR}) \rightarrow \mathcal{L}(\mathcal{H}_{CR}) \}_{i=1}^N \) each time it receives some message on the channel \( C \), and return each time the new state in \( C \).

Let \( \rho_{AR}^{\psi} \) be the final state when the distinguisher is interacting with \( \pi_A R \). By Eq. (6) (with \( B \) renamed \( R \)), this state is \( \varepsilon \)-close to
\[ \gamma_{AR}^{\psi} = (\mathcal{U} \otimes \mathcal{F}^\text{ok})(\psi_{AR}) + | \text{err}\rangle | \text{err}\rangle \otimes \mathcal{F}^\text{err}(\psi_R), \]
for some \( \mathcal{F}^\text{ok} \) and \( \mathcal{F}^\text{err} \) which depend only on \( \{ F_i \} \), not on \( \psi_{AR} \).

When the distinguisher is interacting with \( S\sigma_B \) and using the same operations \( \{ F_i \} \), and initial state \( \psi_{AR} \), let \( \varphi^{\psi}_{ARB} \) be the state of the system at the end of the subroutine \( \pi_A \) and before sending the bit \( c \) to \( S^{\text{bv}}_B \). Then, using Eq. (6) (with \( B \) renamed \( R \), \( A \) renamed \( B \), and a new extension \( A \)), we find that \( \varphi^{\psi}_{ARB} \) is \( \varepsilon \)-close to
\[ \gamma_{ARB}^{\psi} = (\text{id}_A \otimes \mathcal{F}^\text{ok} \otimes \mathcal{U})(\psi_{AR} \otimes \psi_B) + (\text{id}_A \otimes \mathcal{F}^\text{err})(\psi_{AR}) \otimes | \text{err}\rangle | \text{err}\rangle. \]

The final operation performed by \( S^{\text{bv}}_B \) to generate the output can be seen as a map \( S \), which conditioned on \( B \) being an error, deletes \( B \) and overwrites \( A \) with
an error, and conditioned on $B$ being a valid output, deletes $B$ and applies $\mathcal{U}$ to the system $A$. Since a map can only decrease the distance between two states, the final state of the system after this operation, $\phi_{AR} := S(\varphi_{ARB})$, is $\varepsilon$-close to $\mathcal{S}(\gamma_{ARB}) = \tau_{AR}$. By the triangle inequality we thus have $\rho_{AR} \approx 2\varepsilon \phi_{AR}$.

We now prove the converse. If the protocol is $\varepsilon$-secure, there exists a simulator $\sigma_B$ such that $\pi_A \approx \delta \sigma_B$. A distinguisher interacting with one of the two systems chooses an initial state $\psi_{AR}$, and applies operations $F_i : \mathcal{L}(H_{CR}) \rightarrow \mathcal{L}(H_{CR})$ to the messages received on the channel $C$ and the system $R$.

Consider now the interaction of the simulator and the distinguisher. Since the simulator deletes its internal memory when it terminates, and outputs only a single bit $c$ notifying the ideal resource to output the correct result or an error flag, the combined action of the two can be seen as a CPTP map $F : \mathcal{L}(H_R) \rightarrow \{0, 1\} \times \mathcal{L}(H_R)$. Conditioning on the output $\{0, 1\}$, we explicitly define two trace non-increasing maps $F^{ok}, F^{err} : \mathcal{L}(H_R) \rightarrow \mathcal{L}(H_R)$.

Since the ideal blind and verifiable DQC resource outputs the correct result upon receiving 0, and an error flag otherwise, the joint map of ideal resource, simulator and distinguisher is given by $\mathcal{U} \otimes F^{ok} \otimes \mathcal{E}^{err} \otimes F^{err}$. And this map must be $\varepsilon$-close to the real map, otherwise the distinguisher would have an advantage greater than $\varepsilon$.

\section{Reduction to stand-alone security}

Although the notion of blind verifiability defined in the previous section captures composable security in a simple way, it is still more elaborate than existing definitions found in the literature, that treat blindness and verifiability separately. In Section 6.1, we provide definitions for these separate notions of blindness and verifiability, and strengthen the latter by requiring the test of correctness to be independent from the input. In Section 6.2, we show that in the case where the server Bob does not hold an entanglement of the input (e.g., when the input is classical), these notions are sufficient to obtain stand-alone blind verifiability (and hence composable security) with a similar error parameter. In the case where Bob’s system is entangled to Alice’s input, we show that the same holds, albeit with an error increased by a factor $(\dim H_A)^2$.

This implies that the protocol of Fitzsimons and Kashefi [FK12], which satisfies these individual stand-alone definitions, provides composable security (see Appendix C for a proof sketch).

\subsection{Blindness and independent verifiability}

Stand-alone blindness can be seen as a simplification of blind verifiability, in which we ignore Alice’s outcome and only check that Bob’s system could have been generated locally, i.e., is independent from Alice’s input (and output).

\begin{definition}[Stand-alone blindness] A DQC protocol provides stand-alone $\varepsilon$-blindness, if, for all adversarial behaviors $\{F_i\}_i$, there exists a CPTP map $F : \mathcal{L}(H_B) \rightarrow \mathcal{L}(H_B)$ such that

$$\text{tr}_A \circ P_{AB} \approx \varepsilon \mathcal{F} \circ \text{tr}_A,$$

where $\circ$ is the composition of maps, and $P_{AB} : \mathcal{L}(H_{AB}) \rightarrow \mathcal{L}(H_{AB})$ is the map corresponding to a protocol run with Alice behaving honestly and Bob using

\end{definition}
his cheating operations \(\{F_i\}\). We say that the protocol provides stand-alone \(\varepsilon\)-blindness for a set of initial states \(B\), if Eq. (7) holds when applied to these states, i.e., for all \(\psi_{ABR} \in B\),

\[
\text{tr}_A \circ \mathcal{P}_{AB}(\psi_{ABR}) \approx_{\varepsilon} \mathcal{F} \circ \text{tr}_A(\psi_{ABR}).
\]

Likewise, stand-alone verifiability can also be seen as a simplification of blind verifiability, in which we ignore Bob’s system and only check that Alice holds either the correct outcome or an error flag \(|\text{err}\rangle\), which by construction is orthogonal to any possible valid output. In the following we define verifiability only for the case where Bob’s system is not entangled to Alice’s input, since otherwise the correct outcome depends on Bob’s actions, and cannot be modeled by describing Alice’s system alone\(^{20}\).

**Definition 6.2** (Stand-alone verifiability). A DQC protocol provides stand-alone \(\varepsilon\)-verifiability, if, for all adversarial behaviors \(\{F_i\}\) and all initial states \(\psi_{AR_1} \otimes \psi_{R_2B}\), there exists a \(0 \leq p_{\psi} \leq 1\) such that

\[
\rho_{\psi_{AR_1}} \approx_{\varepsilon} p_{\psi}(U \otimes \text{id}_{R_1})(\psi_{AR_1}) + (1 - p_{\psi})|\text{err}\rangle \langle \text{err}| \otimes \psi_{R_1},
\]

where \(\rho_{\psi_{AR_1}}\) is the final state of Alice and the first part of the reference system. We say that the protocol provides stand-alone \(\varepsilon\)-verifiability for a set \(B\) of initial states in product form, if Eq. (8) holds for all \(\psi_{AR_1} \otimes \psi_{R_2B} \in B\).

As mentioned in Section 1, stand-alone blindness and stand-alone verifiability together are strictly weaker than composable blindness and verifiability. This seems to be because the verification procedure can depend on the input (as in the example from Section 1.1), and thus if Bob learns the result of this measurement, he learns something about the input. This motivates us to define a stronger notion, in which Alice’s measurement is independent of her input. To do this, we divide Alice’s internal system in two subsystems \(A\) and \(\bar{A}\), such that \(A\) contains Alice’s output, and \(\bar{A}\) is the register that she measures to decide if the output is correct, i.e., she applies some projection \(\{P_{\text{ok} \bar{A}}, P_{\text{err} \bar{A}}\}\), and conditioned on the outcome, either outputs \(A\) or deletes \(A\) and outputs \(|\text{err}\rangle\). The notion of verifiability is strengthened by additionally requiring that leaking this system \(\bar{A}\) to the adversary does not provide him with more information about the input, i.e., Bob could (using alternative maps) generate the system \(\bar{A}\) on his own.

Before writing up the definition, we need to introduce some extra notation. Let the state of the system just before Alice’s final measurement be given by \(\phi^\psi_{AABR}\), and define

\[
\begin{align*}
\phi^\text{ok}_{AABR} &:= P_{\text{ok}} A \phi^\psi_{AABR} \bar{A} \quad \phi^\text{err}_{AABR} := P_{\text{err}} A \phi^\psi_{AABR} \bar{A}
\end{align*}
\]

The final state of the system after Alice generates the error flag is then

\[
\rho^\psi_{ABR} = \text{tr}_A(\phi^\text{ok}_{AABR}) + |\text{err}\rangle \langle \text{err}| \otimes \text{tr}_{A \bar{A}}(\phi^\text{err}_{AABR}).
\]

\(^{20}\)The resulting definition is equivalent to that of FK12 and stand-alone authentication definitions BCG+02, which bound the probability of projecting the outcome on the space of invalid results.
Definition 6.3. A DQC protocol provides stand-alone $\varepsilon$-independent $\varepsilon$-verifiability, if, in addition to providing stand-alone $\varepsilon$-verifiability, for all adversarial behaviors $\{F_i : \mathcal{L}(\mathcal{H}_{CB}) \to \mathcal{L}(\mathcal{H}_{CB})\}_i$, there exist alternative maps $\{F'_i : \mathcal{L}(\mathcal{H}_{CB}) \to \mathcal{L}(\mathcal{H}_{CB})\}_i$ (for an initially empty system $\bar{B}$), such that
\[
\text{tr}_A \circ Q_{ABB} \approx_{\varepsilon} \text{tr}_A \circ Q'_{ABB},
\]
where $\circ$ is the composition of maps, and $Q_{ABB} : \mathcal{L}(\mathcal{H}_{AB}) \to \mathcal{L}(\mathcal{H}_{ABB})$ and $Q'_{ABB} : \mathcal{L}(\mathcal{H}_{AB}) \to \mathcal{L}(\mathcal{H}_{ABB})$ are the maps corresponding to runs of the two protocols until Alice generates and measures the register $\bar{A}$ with the accept/reject bit [Eq. (9)], but before generating the final outcome [Eq. (10)].

We first show in Lemma 6.6 that in the special case of initial states which are not entangled between Alice and Bob’s systems (e.g., the input is classical), stand-alone blindness and independent verifiability are sufficient to achieve blind verifiability, i.e., composable security. In Theorem 6.7 we then generalize this to any initial state.

Remark 6.4. By the triangle inequality, if a protocol provides both stand-alone $\varepsilon$-blindness and stand-alone $\varepsilon$-independent $\varepsilon'$-verifiability, then there exists a map $F' : \mathcal{L}(\mathcal{H}_B) \to \mathcal{L}(\mathcal{H}_{AB})$ such that
\[
\text{tr}_A \circ Q_{ABB} \approx_{\varepsilon} F' \circ \text{tr}_A.
\]

### 6.2 Composability from stand-alone security

We first show in Lemma 6.6 that in the special case of initial states which are not entangled between Alice and Bob’s systems (e.g., the input is classical), stand-alone blindness and independent verifiability are sufficient to achieve blind verifiability, i.e., composable security. In Theorem 6.7 we then generalize this to any initial state.

Remark 6.5. The two proofs in this section only hold for DQC protocols that implement unitary transformations. This is however the case of the universal computation of \cite{BFK09, MF12, FK12}, which, by appending the classical part of the input to the output, implement a unitary transformation on a classical-quantum system.

Lemma 6.6. If a DQC protocol implementing a unitary transformation provides stand-alone $\varepsilon_{\text{bl}}$-blindness and stand-alone $\varepsilon_{\text{ind}}$-independent $\varepsilon_{\text{ver}}$-verifiability for any pure initial state of the form $\psi_{AR_1} \otimes \psi_{RB}$, then the protocol provides stand-alone $\delta$-blind verifiability with $\delta = 2\sqrt{2}\varepsilon_{\text{ver}} + \varepsilon_{\text{bl}} + \varepsilon_{\text{ind}}$ for these initial states in product form.

Proof. In this proof, we use several times the following simple equality. For two states $\rho = |0\rangle\langle 0| \otimes \rho_0 + |1\rangle\langle 1| \otimes \rho_1$ and $\sigma = |0\rangle\langle 0| \otimes \sigma_0 + |1\rangle\langle 1| \otimes \sigma_1$, we have
\[
D(\rho, \sigma) = D(\rho_0, \sigma_0) + D(\rho_1, \sigma_1).
\]

In Remark 6.4 we combined the conditions of blindness and the independence of the verifiability mechanism into one new formula, Eq. (12). It is thus sufficient to prove that if [Eq. (12) and Eq. (8)] are satisfied for any pure product initial state $\psi_{AR_1} \otimes \psi_{RB}$, then we have blind verifiability, i.e.,
\[
\rho_{AR_1, R_2 B} \approx_{\delta} (U \otimes \text{id}_{R_1} \otimes F_{\text{cl}})(\psi_{AR_1} \otimes \psi_{RB})
+ \|\text{err}\|_1 \otimes \psi_{R_1} \otimes (\text{id}_{R_2} \otimes F_{\text{err}})(\psi_{R_2 B}),
\]
for some $F^{ok}$ and $F^{err}$.

Since $|err\rangle$ is orthogonal to any valid output, both the RHS of $\text{Eq. (14)}$ and LHS (given in $\text{Eq. (10)}$) are a linear combination of orthogonal states on the same subspaces. And thus by $\text{Eq. (13)}$ to show that $\text{Eq. (14)}$ holds for some $\delta$, it is sufficient to find maps $F^{ok}$ and $F^{err}$, and $\delta_1$ and $\delta_2$ with $\delta_1 + \delta_2 = \delta$, such that

$$
\phi^{ok}_{AR,R_2B} \approx \delta_1 (U \otimes \text{id}_{R_2} \otimes F^{ok})(\psi_{AR_1} \otimes \psi_{R_2B}), \quad (15)
$$

$$
\phi^{err}_{R_2B} \approx \delta_2 (\text{id}_{R_2} \otimes F^{err})(\psi_{R_2B}). \quad (16)
$$

Let $\mathcal{F}' : \mathcal{L}(\mathcal{H}_B) \rightarrow \mathcal{L}(\mathcal{H}_{AB})$ be the map guaranteed to exist by the combination of blindness and independent verifiability ($\text{Eq. (12)}$), and let $\mathcal{P}^{ok}_A$ and $\mathcal{P}^{err}_A$ be the maps corresponding to the projections performed by the protocol on system $A$ to detect if Bob was dishonest. We define

$$
\mathcal{F}^{ok}_B := \text{tr}_A \circ \mathcal{P}^{ok}_A \circ \mathcal{F}',
$$

$$
\mathcal{F}^{err}_B := \text{tr}_A \circ \mathcal{P}^{err}_A \circ \mathcal{F}'.
$$

Note that w.l.o.g., we can take $\mathcal{F}'$ to generate a linear combination of two orthogonal states, one in the $\mathcal{P}^{ok}_A$ subspace and one in the $\mathcal{P}^{err}_A$ subspace. Thus, applying $\text{Eq. (11)}$ to the initial state $\psi_{AR_1} \otimes \psi_{R_2B}$ and using $\text{Eq. (13)}$ we find that there exist $\varepsilon_1$ and $\varepsilon_2$ with $\varepsilon_1 + \varepsilon_2 = \varepsilon_{\text{ind}} + \varepsilon_{\text{bl}}$ such that

$$
\phi^{ok}_{R_2B} \approx \varepsilon_1 (\text{id}_{R_2} \otimes \mathcal{F}^{ok}_B)(\psi_{R_2B}), \quad (17)
$$

$$
\phi^{err}_{R_2B} \approx \varepsilon_2 (\text{id}_{R_2} \otimes \mathcal{F}^{err}_B)(\psi_{R_2B}). \quad (18)
$$

Note that $\text{Eq. (18)}$ is exactly one of the conditions we need to find, namely $\text{Eq. (15)}$.

We take the definition of verifiability, $\text{Eq. (8)}$, again, both the RHS and LHS (defined in $\text{Eq. (10)}$) are linear combinations of orthogonal states on the same subspaces, hence there exist $\varepsilon_1$ and $\varepsilon_2$ with $\varepsilon_1 + \varepsilon_2 = \varepsilon_{\text{ver}}$, such that

$$
\phi^{ok}_{AR_1} \approx \varepsilon_1 p^\psi(U \otimes \text{id}_{R_1})(\psi_{AR_1}), \quad (19)
$$

$$
\text{tr} (\phi^{err}_{R_2B}) \approx \varepsilon_2 1 - p^\psi. \quad (20)
$$

From $\text{Eq. (20)}$ we have that $\text{tr} (\phi^{ok}_{AR_1}) = 1 - \text{tr} (\phi^{err}_{R_2B}) \approx \varepsilon_2 p^\psi$. The generalized trace distance (see Appendix A) between the two states from $\text{Eq. (19)}$ is thus bounded by $D(\phi^{ok}_{AR_1}, p^\psi U(\psi_{AR_1})) \leq \varepsilon_1 + \varepsilon_2 = \varepsilon_{\text{ver}}$. From $\text{Lemma A.1}$ we can upper bound the purified distance with the generalized trace distance, and get $P(\phi^{ok}_{AR_1}, p^\psi U(\psi_{AR_1})) \leq \sqrt{2\varepsilon_{\text{ver}}}$. We can now apply Uhlmann’s theorem to the purified distance (see $\text{Lemma A.2}$) and find that since $U(\psi_{AR_1})$ is a pure state, there exists a $\sigma_{R_2B}$ such that $P(\phi^{ok}_{AR_1 R_2 B}, p^\psi U(\psi_{AR_1}) \otimes \sigma_{R_2B}) = P(\phi^{ok}_{AR_1}, p^\psi U(\psi_{AR_1}))$. Hence by $\text{Lemma A.1}$ $\text{Eq. (17)}$ and the triangle inequality,

$$
D(\phi^{ok}_{AR_1 R_2 B}, U(\psi_{AR_1}) \otimes \mathcal{F}^{ok}_B(\psi_{R_2B}))
\leq D(\phi^{ok}_{AR_1 R_2 B}, p^\psi U(\psi_{AR_1}) \otimes \sigma_{R_2B})
+ D(p^\psi U(\psi_{AR_1}) \otimes \sigma_{R_2B}, U(\psi_{AR_1}) \otimes \mathcal{F}^{ok}_B(\psi_{R_2B}))
\leq \sqrt{2\varepsilon_{\text{ver}}} + D(p^\psi \sigma_{R_2B}, \phi^{ok}_{R_2B}) + D(\phi^{ok}_{R_2B}, \mathcal{F}^{ok}_B(\psi_{R_2B}))
\leq 2\sqrt{2\varepsilon_{\text{ver}}} + \varepsilon_1.
$$

Combining this with our bound for $\text{Eq. (16)}$ we prove the lemma. \(\blacksquare\)
In the following theorem we consider the case of an initial state arbitrarily entangled between Alice and Bob. We reduce this case to the separable state case treated in Lemma 6.6 with an increase of the error by a factor of \((\dim \mathcal{H}_A)^2\).

**Theorem 6.7.** If a DQC protocol implementing a unitary transformation provides stand-alone \(\varepsilon_{bl}\)-blindness and stand-alone \(\varepsilon_{ind}\)-independent \(\varepsilon_{ver}\)-verifiability, then it provides stand-alone \(\delta\)-blind verifiability with \(\delta = N^2(2\sqrt{2\varepsilon_{ver}} + \varepsilon_{bl} + \varepsilon_{ind})\), for \(N = \dim \mathcal{H}_A\).

**Proof.** For \(n = \log \dim \mathcal{H}_A\) and an initial state \(\psi_{ABR}\) we define the state 

\[
\psi'_{QABRS} := |\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n} \otimes \psi'_{BRS},
\]

where \(|\Phi^+\rangle = ((00) + (11))/\sqrt{2}\) is an EPR pair and \(\psi'_{BRS} = \psi_{BRA}^R\). For any map \(\mathcal{E}_{AB}: \mathcal{L}(\mathcal{H}_{AB}) \to \mathcal{L}(\mathcal{H}_{AB})\) we have

\[
\mathcal{E}_{AB}(\psi_{ABR}) = 2^{2n} \text{tr}_{QS}\left(|\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n} \mathcal{E}_{AB}(\psi_{QABRS}') |\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n}\right).
\]

The projection on \(|\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n}\) can be seen as a teleportation of the system \(S\) into \(A\) with a post-selection on the branch where no bit or phase corrections are necessary.

Let \(\mathcal{Q}_{AB}: \mathcal{L}(\mathcal{H}_{AB}) \to \mathcal{L}(\mathcal{H}_{AB})\) be the map corresponding to a run of the protocol with Alice behaving honestly and Bob using his cheating strategy. Furthermore, let \(\mathcal{F}': \mathcal{L}(\mathcal{H}_B) \to \mathcal{L}(\mathcal{H}_{AB})\) be the map guaranteed to exist by the combination of blindness and independent verifiability [Eq. (12)], and let \(\mathcal{P}^\text{ok}_A\) and \(\mathcal{P}^\text{err}_A\) be the maps corresponding to the projections performed by the protocol on system \(A\) to detect if Bob was dishonest. We define

\[
\mathcal{F}^\text{ok}_B := \text{tr}_A \circ \mathcal{P}^\text{ok}_A \circ \mathcal{F}', \\
\mathcal{F}^\text{err}_B := \text{tr}_A \circ \mathcal{P}^\text{err}_A \circ \mathcal{F}', \\
\mathcal{R}_{AB} := \mathcal{U} \otimes \mathcal{F}^\text{ok} + \mathcal{E}^\text{err} \otimes \mathcal{F}^\text{err},
\]

where \(\mathcal{U}\) is the map implemented by the DQC protocol and \(\mathcal{E}^\text{err}\) deletes the contents of \(A\) and outputs the error flag \(|\text{err}\rangle\).

We then have for all \(\psi_{ABR}\),

\[
\begin{align*}
D(\mathcal{Q}_{AB}(\psi_{ABR}), \mathcal{R}_{AB}(\psi_{ABR})) & = 2^{2n} D\left(\text{tr}_{QS}\left(|\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n} \mathcal{Q}_{AB}(\psi_{QABRS}') |\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n}\right), \\
& \quad \text{tr}_{QS}\left(|\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n} \mathcal{R}_{AB}(\psi_{QABRS}') |\Phi^+\rangle\langle\Phi^+|_{QS}^{\otimes n}\right)\right) \\
& \leq 2^{2n} D(\mathcal{Q}_{AB}(\psi_{QABRS}'), \mathcal{R}_{AB}(\psi_{QABRS}')).
\end{align*}
\]

Note that the state \(\psi_{QABRS}'\) is in product form w.r.t. the systems \(QA\) and \(BRS\). This allows us to apply Lemma 6.6 from which we get

\[
D(\mathcal{Q}_{AB}(\psi_{QABRS}'), \mathcal{R}_{AB}(\psi_{QABRS}')) \leq 2\sqrt{2\varepsilon_{ver}} + \varepsilon_{bl} + \varepsilon_{ind}. \quad \square
\]

If part of the subsystem \(A\) of the initial state \(\psi\) is classical, the proof can be trivially modified to reduce the factor \((\dim \mathcal{H}_A)^2\) to only the quantum part of \(A\). This is done by leaving this classical part in the \(A\) subsystem of \(\psi'\), and only teleporting the quantum part of the state from \(S\) to \(A\).

**Corollary 6.8.** If a DQC protocol implementing a unitary transformation provides stand-alone \(\varepsilon_{bl}\)-blindness and stand-alone \(\varepsilon_{ind}\)-independent \(\varepsilon_{ver}\)-verifiability, then it is (composably) \(\delta\)-secure for classical inputs and \(\delta N^2\)-secure for quantum inputs, where \(\delta = 4\sqrt{2\varepsilon_{ver}} + 2\varepsilon_{bl} + 2\varepsilon_{ind}\) and \(N = \dim \mathcal{H}_A\).
Proof. Immediate by combining Theorem 5.3 and Lemma 6.6 in the case of a classical input, and Theorem 5.3 with Theorem 6.7 for a quantum input.

7 Composable blindness of some protocols

We prove in this section that two different DQC protocols proposed in the literature construct the ideal blind quantum computation resource $S_b$ given in Definition 4.1. To show this, we need to prove that both conditions from Eq. (4) are satisfied for $\varepsilon = 0$. In Appendix B we show that stand-alone correctness is equivalent to composable correctness, and thus the first part of Eq. (4) is immediate from existing literature. In the following sections, we prove that these protocols also provide perfect blindness. Note that they do not provide verifiability, we therefore cannot use the generic results from Section 6 to prove that they are blind.

We start in Section 7.1 with the DQC protocol of Broadbent, Fitzsimons and Kashefi [BFK09], which we describe in detail in Section 7.1.1. In this protocol, Alice hides the computation by encrypting it with a one-time pad. The core idea used to construct the simulator can also be used to prove the security of the one-time pad. In Section 7.1.2 we thus first sketch the security proof of the one-time pad, and in Section 7.1.3 we prove that the DQC protocol of Broadbent, Fitzsimons and Kashefi provides perfect blindness.

Morimae and Fujii [MF12] proposed a DQC protocol with one-way communication from Bob to Alice, in which Alice simply measures each qubit she receives, one at a time. We show in Section 7.2 that the general class of protocols with one-way communication is perfectly blind.

7.1 DQC protocol of Broadbent, Fitzsimons and Kashefi

7.1.1 The protocol

This protocol [BFK09] was originally called Universal Blind Quantum Computation (UBQC), and in the following we use this name. For an overview of the UBQC protocol, we assume familiarity with measurement-based quantum computing, for more details see [RB01, DKP07]. Suppose Alice has in mind a unitary operator $U$ that is implemented with a measurement pattern on a brickwork state $G_{n \times (m+1)}$ (Figure 8) with measurements given as multiples of $\pi/4$ in the $(X,Y)$ plane with overall computation size $S = n \times (m + 1)$. Note that measurement based quantum computation, where the measurements are restricted in the sense above is approximately universal, so there are no restrictions imposed on $U$ [BFK09].

This pattern could have been designed either directly in MBQC or generated from a circuit construction. Each qubit in $G_{n \times (m+1)}$ is indexed by a column $y \in \{0, \ldots, m\}$ and row $x \in \{1, \ldots, n\} = [n]$. Thus each qubit is assigned a measurement angle $\phi_{x,y}$, and two seta $D_{x,y}, D'_{x,y} \subseteq [n] \times \{0, \ldots, y-1\}$ which we call $X$-dependencies and $Z$-dependencies, respectively.

The dependency sets comprise subsets of the set of the two-coordinate indices. They reflect the fact that in measurement-based quantum computation, to ensure a correct and deterministic computation, the measurement angles which define the computation may have to be modified for each qubit depending on
Figure 8 – The brickwork state, $G_{n \times m}$, a universal resource state for measurement-based quantum computing requiring only single qubit measurement in the $(X,Y)$ plane [BFK09]. Qubits $|\psi_{x,y}\rangle$ ($x = 1, \ldots, n, y = 1, \ldots, m$) are arranged according to layer $x$ and row $y$, corresponding to the vertices in the above graph, and are originally in the $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ state. Controlled-$Z$ gates are then performed between qubits which are joined by an edge. The rule determining which qubits are joined by an edge is as follows: 1) Neighboring qubits of the same row are joined; 2) For each column $j = 3 \mod 8$ and each odd row $i$, the qubits at positions $(i, j)$ and $(i + 1, j)$ and also on positions $(i, j + 2)$ and $(i + 1, j + 2)$ are joined; 3) For each column $j = 7 \mod 8$ and each even row $i$, the qubits at positions $(i, j)$ and $(i + 1, j)$ and also on positions $(i, j + 2)$ and $(i + 1, j + 2)$ are joined. The quantum input is usually placed in the leftmost column of the brickwork state, whereas the output is generated in the rightmost column by sequential single qubit measurements. The qubits are usually measured from top to bottom per column, where the order of columns is from left to right.

some of the prior measurement outcomes. In particular, here we assume that the dependency sets $D_{x,y}$ and $D'_{x,y}$ are obtained via the flow construction [DK06].

During the execution of the computation, the adapted measurement angle $\phi'_{x,y}$ is computed from $\phi_{x,y}$ and the previous measurement outcomes in the following way: let $s_{x,y}^X = \oplus_{i \in D_{x,y}} s_i$ be the parity of all measurement outcomes for qubits in $D_{x,y}$ and similarly, $s_{x,y}^Z = \oplus_{i \in D'_{x,y}} s_i$ be the parity of all measurement outcomes for qubits in $D'_{x,y}$ (the index $i$ here is a two coordinate index, an element of $[n] \times \{0, \ldots, m\}$). Then,

$$\phi'_{x,y} = (-1)^{s_{x,y}^X} \phi_{x,y} + s_{x,y}^Z \pi. \quad (21)$$

This will be used in a protocol, where the first column of the brickwork state is a one-time pad encryption of the input. The measurement angles of the first two columns then have to be updated to compensate for (bit) flips $i_x$ performed by the encryption, namely

$$\phi'_{x,0} = (-1)^{i_x} \phi_{x,0} \quad \text{and} \quad \phi'_{x,1} = \phi_{x,1} + i_x \pi. \quad (22)$$

21In UBQC with a quantum input, the input is initially encoded with a variant of the quantum one-time pad by Alice, to preserve her privacy. The operators implementing the one-time pad that Alice applies to the input may include an arbitrary rotation within the $XY$ plane of the Bloch sphere (a $Z_\theta$ rotation), and a Pauli-$X$ operator. Because of the commutation relation $(X \otimes \text{id})\text{ctrl-Z} = \text{ctrl-Z}(X \otimes Z)$ between the Pauli-$X$ operator and the controlled $Z$ entangling operation, this component of the one-time pad must be accounted for in the measurement angles for the neighbours of the input layer, as in Eq. (22).
Protocol 1 Universal Blind Quantum Computation

Alice’s input:
- An $n$-qubit unitary map $U$, represented as a sequence of measurement angles $\{\phi_{x,y}\}$ of a one-way quantum computation over a brickwork state of the size $n \times (m+1)$, along with the $X$ and $Z$ dependency sets $D_{x,y}, D'_{x,y}$, respectively.
- An $n$-qubit input state $\rho_{\text{in}}$

Alice’s output (for an honest Bob):
- The $n$-qubit quantum state $\rho_{\text{out}} = U\rho_{\text{in}}U^\dagger$

The protocol

1. State preparation
   1.1. For each $x \in [n]$, Alice applies $X_{i_x}Z_{\theta_{x,0}}$ to the $x$th qubit of the input $\rho_{\text{in}}$, where the binary values $i_x$ and the angles $\theta_{x,0} \in \{k\pi/4\}_{k=0}^7$ are chosen uniformly at random for each $x$. This is equivalent to encrypting it with a quantum one-time pad. The result is sent to Bob.
   1.2. If $i_x = 1$, Alice updates the measurement angles $\phi_{x,0}$ and $\phi_{x,1}$ to compensate for the introduced bit flip (see Eq. (22)).
   1.3. For each column $y \in [m-1]$, and each row $x \in [n]$, Alice prepares the state $|+\rangle_{x,y} := \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta_{x,y}}|1\rangle)$, where the defining angle $\theta_{x,y} \in \{k\pi/4\}_{k=0}^7$ is chosen uniformly at random, and sends the qubits to Bob.
   1.4. Bob creates $n$ qubits in the $|+\rangle$ state, which are used as the final output layer, and entangles the qubits received from Alice and this final layer by applying $\text{ctrl}-Z$ operators between the pairs of qubits specified by the pattern of the brickwork state $G_{n \times (m+1)}$.

2. Interaction and measurement
   For $y = 0, \ldots, m-1$, repeat
   For $x = 1, \ldots, n$, repeat
   2.1. Alice computes the updated measurement angle $\phi'_{x,y}$ (see Eq. (21)), to take previous measurement outcomes received from Bob into account.
   2.2. Alice chooses a binary digit $r_{x,y} \in \{0, 1\}$ uniformly at random, and computes $\delta_{x,y} = \phi'_{x,y} + \theta_{x,y} + \pi r_{x,y}$.
   2.3. Alice transmits $\delta_{x,y}$ to Bob, who performs a measurement in the basis $\{|+\delta_{x,y}\rangle, |-\delta_{x,y}\rangle\}$.
   2.4. Bob transmits the result $s_{x,y} \in \{0, 1\}$ to Alice.
   2.5. If $r_{x,y} = 1$, Alice flips $s_{x,y}$; otherwise she does nothing.

3. Output Correction
   3.1. Bob sends to Alice all qubits in the last (output) layer.
   3.2. Alice performs the final Pauli corrections $\{Z^{s_{x,y}}X^{r_{x,y}}\}_{x=1}^n$ on the received output qubits.
Protocol 1 implements a blind quantum computation for an input $\psi_A = \rho_{in} \otimes |U\rangle \langle U|^2$. It was shown in [BFK09] that this protocol is correct, i.e., if both Alice and Bob follow the steps of the protocol then the final output state is $\rho_{out} = U \rho_{in} U^\dagger$.

7.1.2 One-time pad proof sketch

The basic idea behind the construction of the simulator required for the proof of composable security of the UBQC protocol can be used in the case of a simpler protocol— the Quantum One-Time Pad (QOTP). The QOTP ensures confidentiality, but not authenticity, of the exchange of quantum messages over an untrusted quantum channel.

The ideal confidentiality resource $S$, which we wish to construct, has three interfaces, $\ A$ (Alice, the sender), $\ B$ (Bob, the receiver) and $\ E$ (Eve, the eavesdropper). Alice inputs a message $\rho_{in}^A$, Eve only learns the message size—though for simplicity, we assume that the message size is fixed, and do not model it explicitly in the following— but can arbitrarily modify or replace the message. Similarly to the blind DQC ideal resource (Definition 4.1), the eavesdropper’s capacity to arbitrarily manipulate the message is captured by allowing some arbitrary state $\rho_{in}^E$ and a description of a map $\mathcal{E} : \mathcal{L}(\mathcal{H}_{AE}) \rightarrow \mathcal{L}(\mathcal{H}_B)$ to be input at the $E$-interface of the ideal resource, which then outputs $\mathcal{E}(\rho_{AE})$ at the $B$-interface. This is depicted in Figure 9 with Eve’s functionalities grayed to signify that they are only accessible to a cheating player.

| Confidential channel $S$ |
|--------------------------|
| $\rho_{in}^A$ $\rightarrow$ $\rho_{out}^A$ $\rightarrow$ $\rho_{out}^B$ |
| $\mathcal{E}$ |
| $\rho_{in}^E$ $\rightarrow$ $\rho_{out}^E$ |

**Figure 9** – A confidential channel. Alice and Bob have access to the left and right interface, respectively, and Eve accesses the lower interface. This channel guarantees that Eve does not learn Alice’s input $\rho_{in}^A$, but allows her to modify what Bob receives. If Eve does not activate her cheating interface, the state $\rho_{out}^A$ is output at Bob’s interface.

The resources $\mathcal{R}$ available to the QOTP protocol $(\pi_A, \pi_B)$ are a shared secret.

---

22The particular variant of the UBQC protocol we present assumes a quantum input and a quantum output, however the Protocol is easily modified to take classical inputs and/or produces classical outputs, see [BFK09]. In the classical input case, the quantum input is simply not sent, and the preparation of the classical input is assumed to be encoded in the computation itself. For the classical output, the server would simply measure out the final column of qubits as well, which produces a one-time padded version of the computation result. The quantum input-output setting is more general than other variants, and the security of this variant implies the security of the classical input/output versions. Also, the quantum one-time pad of the input states used in this protocol could be replaced with a standard quantum one-time pad which uses only the local $X$ and $Z$ gates, instead of the $X$ and the parametrized $Z_\theta$ gate, as presented here. In this case Bob would teleport the input state onto the brickwork state built out of the pre-rotated $|+\rangle$ qubits, and the protocol would continue as we have presented (but taking into account the teleportation outcomes reported by Bob).
key and an insecure quantum channel, which simply outputs at the \( E \)-interface anything which Alice inputs, and forwards to the \( B \)-interface anything which Eve inputs. \( \pi_A \) applies bit and phase flips (conditioned on the bits of the secret key) to Alice’s input and sends the result down the insecure channel, and \( \pi_B \) decrypts by applying the same flips to whatever it receives. This is illustrated in Figure 10.

![Figure 10](image)

**Figure 10** – The concrete setting of the QOTP, with Alice accessing the left interface, Bob the right one and Eve the lower interface. The QOTP encrypts a message \( \psi \) by applying bit and phase flips, \( \rho := Z^x X^z \psi X^z Z^z \), and decrypts by applying the reverse operation, \( \psi' := X^x Z^z \rho' Z^z X^z \).

To prove that this protocol constructs the ideal confidentiality resource, we need to find a simulator \( \sigma_E \) that, when plugged into the \( E \)-interface of the ideal resource, emulates the communication on the insecure quantum channel and finds the appropriate inputs \( \rho_{in}^E \) and \( \mathcal{E} \) that correspond to Eve’s tampering, so that ideal and concrete cases are indistinguishable. In other words, we need to find a \( \sigma_E \) such that

\[
\pi_A \mathcal{R} \pi_B = \mathbb{S} \sigma_E. \tag{23}
\]

In the concrete setting, the distinguisher accessing \( \pi_A \mathcal{R} \pi_B \) can choose an arbitrary input \( \rho_{in}^{AR} \), apply an arbitrary map \( \mathcal{D} \) to the state on the quantum channel (output at the \( E \)-interface) and its own system \( R \), and put the result back on the quantum channel. After decryption by \( \pi_B \), it ends up with the final state \( \rho_{out}^{BR} \). We depict this for one-qubit messages in Figure 11 by rearranging Figure 10 as a circuit with the addition of the purifying system \( R \) and map \( \mathcal{D} \).

![Figure 11](image)

**Figure 11** – Interaction of the distinguisher and the QOTP.

In the ideal setting, the simulator \( \sigma_E \) needs to simulate the quantum channel and provide the ideal resource \( \mathbb{S} \) with information allowing it to generate the same output \( \rho_{out}^{BR} \) as in the concrete case. It does this by outputting half an EPR pair (for every qubit of the message) at its outer interface, and transmitting the other half along with any state it received at its outer interface to the ideal.
resource. It also provides the ideal resource with the “instructions” $E$ to gate teleport the real input through the map $D$ of the distinguisher, i.e., it teleports the input using the EPR half, registers the possible bit and phase flips, and outputs the second state received after having corrected the bit and phase flips from the teleportation. Plugging this simulator into the $E$-interface of Figure 9 along with the distinguishers input $\rho_{AE}^{in}$ and map $D$, and rewriting it as a circuit for one-qubit messages results in Figure 12.

![Figure 12](image1.png)

**Figure 12** – Interaction of the ideal confidentiality resource $S$ and the simulator $\sigma_E$ with the distinguisher. $S$ does not leak any information to the adversary, it receives inputs from Alice ($\rho_{in}^{in}$) and the simulator, and transmits some state to Bob. $\sigma_E$ — which does give information to the adversary — has no access to the confidential message $\rho_{in}^{in}$.

We now show that the circuits from Figure 11 and Figure 12 are identical, hence Eq. (23) holds. The argument generalizes straightforwardly to multiple qubit messages. We first rearrange Figure 12 by grouping the state preparation (performed by $\sigma_E$) and the actual teleportation (performed by $S$). This results in Figure 13.

![Figure 13](image2.png)

**Figure 13** – Reformulation of Figure 12 by grouping the simulator and the teleportation step of the ideal confidentiality resource. The circuit in the dashed box simply encrypts the input with a random bit and phase flip, and therefore corresponds to $\pi_A$.

The circuit in the dashed box of Figure 13 teleports the input from the first wire to the third wire (without correcting the random flips). This is equivalent to simply performing a random bit and phase flip on the input, which is exactly what is done by the QOTP in Figure 11.
7.1.3 Security

In this section we prove that the UBQC protocol (Protocol 1) provides perfect blindness, i.e., we find a simulator \( \sigma_B \) such that the two interactive boxes in Figure 7 are indistinguishable. Similarly to the one-time pad proof sketch from Section 7.1.2, we construct a simulator which sends only EPR pair halves and random strings, then transmits the other halves and the transcript to the ideal blind DQC resource. Whenever a one-time padded quantum state should have been sent, the ideal resource teleports it using the EPR half, and uses the bit and phase flips of the teleportation as one-time pad key. And whenever a random string \( r \) was sent instead of some one-time padded string \( s \), the ideal resource sets \( r \oplus s \) as the random key used to encrypt and send \( s \).

To prove that the real and ideal settings are identical, we replace steps of the protocol by equivalent steps, until we end up with the desired simulator and ideal resource.

Protocol 1 does not explicitly model the information that is intentionally allowed to leak. This information consists in the size of the brickwork state (which leaks upper bounds on the input state size and computation size), and whether the last column of the brickwork state should be measured, i.e., whether the output of the protocol is classical or quantum. It is simply assumed that this information is known by the server (Bob), otherwise it could not perform the desired computation. For simplicity we also avoid modeling this information in the following. The protocol and proof can however be trivially changed to include it.

**Theorem 7.1.** The DQC protocol described in Protocol 1 provides perfect blindness.

**Proof.** To prove that \( \pi_A^R = S^b \sigma_B \), we successively modify the protocol \( \pi_A \), replacing some steps with equivalent steps that implement the same map, resulting in several intermediary protocols, until we achieve a version which is equivalent to \( S^b \sigma_B \).

The first intermediary protocol is given by Protocol 2. Compare Step 1.1 of Protocol 1 and Step 1.1 of Protocol 2. In the former, Alice picks random values \( \theta_{x,0} \) and \( i_x \) and performs corresponding phase and bit rotations on the \( x \)th input qubit. In the latter, she performs a random \( \theta'_{x,0} \) phase rotation, and teleports the resulting state. For teleportation outcomes \( i_x \) and \( r_{x,0} \), and setting \( \theta_{x,0} := \theta'_{x,0} + \pi r_{x,0} \), Bob holds exactly the same state. Since the different values of \( i_x \) and \( \theta_{x,0} \) occur with the same (uniform) probabilities in both protocols, these implement identical maps.

Likewise, compare Step 1.3 of Protocol 1 and Step 1.3 of Protocol 2. In the former Alice sends a state \( \lvert +_{\theta_{x,0}} \rangle \) to Bob; in the latter Bob ends up holding the state \( \lvert +_{\theta'_{x,0} + \pi r_{x,0}} \rangle \). If Alice sets \( \theta_{x,0} := \theta'_{x,0} + \pi r_{x,0} \) in her internal memory, all states of the systems are identical for both protocols.

Finally, the only other difference between these protocols is in Steps 2.2 of the two protocols, respectively. In the former, Alice sends Bob the angle \( \phi'_{x,0} + \theta_{x,0} + \pi r_{x,0} \), for some randomly picked bit \( r_{x,0} \); in the latter, she sends \( \phi'_{x,0} + \theta'_{x,0} \). But as we’ve already established, these two angles are identical, and occur with the same (uniform) probabilities.

Now, compare Protocol 2 and Protocol 3. The main difference is between Step 2.2 of Protocol 2 and Step 2.2 of Protocol 3. In the former, Alice had picked...
Protocol 2  UBQC, equivalent protocol for Alice, first version

The protocol

1. State preparation

1.1. For each \( x \in [n] \), Alice prepares an EPR pair \((\mid 00 \rangle + \mid 11 \rangle) / \sqrt{2}\) and sends half to Bob. She picks an angle \( \theta'_{x,0} \in \{k\pi/4 \}_{k=0}^7 \) uniformly at random, and applies \( Z_{\theta'_{x,0}} \) to the \( x \)th qubit of the input \( \rho_m \). She then teleports the resulting qubit using her half of the EPR pair, and registers the values of the bit and phase flips resulting from the teleportation in \( i_x \) and \( r_{x,0} \), respectively.

1.2. If \( i_x = 1 \), Alice updates the measurement angles \( \phi_{x,0} \) and \( \phi_{x,1} \) (see Eq. (22)).

1.3. For each column \( y \in [m-1] \), and each row \( x \in [n] \), Alice prepares an EPR pair \((\mid 00 \rangle + \mid 11 \rangle) / \sqrt{2}\) and sends half to Bob. She then picks an angle \( \theta'_{x,y} \in \{k\pi/4 \}_{k=0}^7 \) uniformly at random, performs a \( Z_{\theta'_{x,y}} \) rotation followed by a Hadamard \( H \) on her half of the pair, and measures it in the computational basis. She stores the result in \( r_{x,y} \).

2. Interaction and measurement

For \( y = 0, \ldots, m-1 \), repeat

For \( x = 1, \ldots, n \), repeat

2.1. Alice computes the updated measurement angle \( \phi'_{x,y} \) (see Eq. (21)).

2.2. Alice computes \( \delta_{x,y} = \phi'_{x,y} + \theta'_{x,y} \) and transmits this to Bob.

2.3. Alice receives a bit \( s_{x,y} \in \{0,1\} \) from Bob.

2.4. If \( r_{x,y} = 1 \), Alice flips \( s_{x,y} \); otherwise she does nothing.

3. Output correction

3.1. Alice receives \( n \) qubits from Bob, and performs the final Pauli corrections \( \{Z^{s_{x,y}}X^{r_{x,y}}\}_{x=1}^n \) on these qubits.

\[ \theta'_{x,y} \] uniformly at random, and sends Bob \( \delta_{x,y} \), a one-time padded version of \( \phi'_{x,y} \) with \( \theta'_{x,y} \) as the key; hence \( \delta_{x,y} \) is uniformly distributed. In the latter protocol, Alice instead picks \( \delta_{x,y} \) uniformly at random (in Step 2.2), then computes \( \theta'_{x,y} := \delta_{x,y} - \phi_{x,y} \) (in Step 2.3) to get the value of the uniform key used to encrypt \( \phi_{x,y} \).

In Protocol 2 Alice used the value of \( \theta'_{x,y} \) in Steps 1.1 and 1.3. Since \( \theta'_{x,y} \) is not available at those stages of Protocol 3 the corresponding steps are delayed until this value is available. Hence Step 1.1 of Protocol 3 only consists in performing the first part of the teleportation (which commutes with the \( Z_{\theta'_{x,0}} \) rotation) and in Step 1.3 Alice only sends half an EPR pair. In Step 2.3 after computing \( \theta'_{x,y} \), Alice completes those two steps by performing the missing operations.

Protocol 4 consists in exactly the same steps as Protocol 3 but their order has been rearranged, and the different parts have been renamed “simulator” and “ideal resource”. The ideal blind DQC resource constructed meets the requirements of Definition 4.1 we have \( \pi_A R = S^b \sigma_B \) and conclude the proof.

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Protocol 3 UBQC, equivalent protocol for Alice, second version

The protocol

1. State preparation
   1.1. For each $x \in \mathbb{Z}_n$, Alice prepares an EPR pair $|00⟩ + |11⟩/\sqrt{2}$ and sends half to Bob. She performs the first measurement of a teleportation that determines the bit flip, i.e., for each $x$ she performs a CNOT on the corresponding EPR half using the input qubit as control, and measures the EPR half in the computational basis. She records the outcome in $i_x$.
   1.2. If $i_x = 1$, Alice updates the measurement angles $\phi_{x,0}$ and $\phi_{x,1}$ (see Eq. (22)).
   1.3. For each column $y \in \mathbb{Z}_{m-1}$, and each row $x \in \mathbb{Z}_n$, Alice prepares an EPR pair $(|00⟩ + |11⟩)/\sqrt{2}$ and sends half to Bob.

2. Interaction and measurement

   For $y = 0, \ldots, m-1$, repeat
   For $x = 1, \ldots, n$, repeat
   2.1. Alice computes the updated measurement angle $\phi'_{x,y}$ (see Eq. (21)).
   2.2. Alice picks an angle $\delta_{x,y} \in \{k\pi/4\}_{k=0}^7$ uniformly at random, and sends it to Bob.
   2.3. Alice receives a bit $s_{x,y} \in \{0, 1\}$ from Bob.
   2.4. Alice computes $\theta'_{x,y} = \delta_{x,y} - \phi'_{x,y}$. She then applies $Z^{\theta'_{x,y}}$, followed by a Hadamard $H$ and a measurement in the computational basis to the $x$th qubit of the input $\rho_m$ if $y = 0$, and to the corresponding EPR half if $y > 0$. She stores the result in $r_{x,y}$.
   2.5. If $r_{x,y} = 1$, Alice flips $s_{x,y}$; otherwise she does nothing.

3. Output Correction

   3.1. Alice receives $n$ qubits from Bob, and performs the final Pauli corrections $\{Z^{s_{x,y}}X^{s_{x,y}}\}_{x=1}^n$ on these qubits.

7.2 One-way communication

If a protocol only requires one-way communication from Bob to Alice, the protocol model described in Section 4.1.2 can be simplified: it only consists in two operations. Bob generates a state $\tau$, which he sends to Alice on the channel $C$. She then applies some operation $\mathcal{E} : \mathcal{L}(H_{AC}) \to \mathcal{L}(H_A)$ to her input and $\tau$, and outputs the contents of her system $A$.

Theorem 7.2. Any DQC protocol $\pi$ with one-way communication from Bob to Alice provides perfect blindness.

Proof. The simulator $\sigma_B$ works as follows. It receives some state $\psi_C$ from the distinguisher, and provides it to the abstraction $\mathcal{S}^b$ along with a description of the map $\mathcal{E}$ that is used by $\pi_A$. Alice’s output is thus $\mathcal{E}(\psi_{AC})$, and we immediately have $d(\pi_A R, S \sigma_B) = 0$.

This proof does not mention the permitted leaks at the $B$-interface. This is
Protocol 4 UBQC, simulator and ideal resource

The simulator

1. For each column $y \in \{0, \ldots, m - 1\}$, and each row $x \in [n]$, the simulator prepares an EPR pair $(|00\rangle + |11\rangle)/\sqrt{2}$ and outputs half at its outer interface.

2. For each column $y \in \{0, \ldots, m - 1\}$, and each row $x \in [n]$, the simulator picks an angle $\delta_{x,y} \in \{k\pi/4\}_{k=0}^{7}$ uniformly at random, and outputs it at its outer interface. It receives some response $s_{x,y} \in \{0,1\}$.

3. The simulator receives $n$ qubits, which correspond to the last (output) layer.

4. The simulator transmits all EPR pair half, all angles $\delta_{x,y}$, bits $s_{x,y}$ and output qubits to the ideal blind delegated quantum computation resource, along with instructions to perform the operations described hereafter.

The ideal blind DQC resource

1. The blind DQC resource receives the input $\rho_{in}$ and a description of the computation given by angles $\phi_{x,y}$ at its $A$-interface, and all the information described in Step 4 above at its $B$-interface.

2. For each $x \in [n]$, it performs the first measurement of a teleportation of the input, i.e., for each $x$ it performs a CNOT on the corresponding EPR half using the input qubit as control, and measures the EPR half in the computational basis. It records the outcome in $i_x$.

3. If $i_x = 1$, it updates the measurement angles $\phi_{x,0}$ and $\phi_{x,1}$ (see Eq. (22)).

4. For $y = 0, \ldots, m - 1$, repeat
   For $x = 1, \ldots, n$, repeat
   4.1. It computes the updated measurement angle $\phi'_{x,y}$ (see Eq. (21)).
   4.2. It computes $\theta'_{x,y} = \delta_{x,y} - \phi'_{x,y}$. It then applies $Z_{\theta'_{x,y}}$, followed by a Hadamard $H$ and a measurement in the computational basis to the $x$th qubit of the input $\rho_{in}$ if $y = 0$, and to the corresponding EPR half if $y > 0$. It stores the result in $r_{x,y}$.
   4.3. If $r_{x,y} = 1$, it flips $s_{x,y}$; otherwise it does nothing.

5. The ideal blind DQC resource performs the final Pauli corrections \( \{Z_{s_{x,m}}^x X_{s_{x,m}}^x\}_{x=1}^n \) on the received output qubits, and outputs the result at its $A$-interface.
because protocols with one-way communication make the (implicit) assumption that this information is known to the server. Alternatively, one could include a single message from Alice to Bob containing this information, and adapt the proof above accordingly.

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Appendices

A Distance measures for subnormalized states

In Section 3.3 we introduced the trace distance $D(\rho, \sigma)$ between two quantum states. Another widely used measure is the fidelity, defined as

$$F(\rho, \sigma) := \text{tr} \left( \sqrt{\rho^{1/2} \sigma \rho^{1/2}} \right).$$

When dealing with subnormalized states, we need to generalize these measures to retain their properties. The following distance notions are treated in detail in [TCR10], and we refer to that work for more information.

For any two subnormalized states $\rho, \sigma \in S_{\leq} (\mathcal{H})$, we define the generalized trace distance as

$$\bar{D}(\rho, \sigma) := D(\rho, \sigma) + \frac{1}{2} |\text{tr} \rho - \text{tr} \sigma|,$$

and the generalized fidelity as

$$\bar{F}(\rho, \sigma) := F(\rho, \sigma) + \sqrt{(1 - \text{tr} \rho)(1 - \text{tr} \sigma)}.$$

The (generalized) fidelity has a useful property, known as Uhlmann’s theorem (see [NC00] or Lemma A.2 here below), which states that for any two states $\rho, \sigma$, there exist purifications of these states which have the same fidelity. We define a metric, the purified distance, based on the fidelity, so as to retain this property:

$$P(\rho, \sigma) := \sqrt{1 - F^2(\rho, \sigma)}.$$

This metric coincides with the generalized distance for pure states, and is larger otherwise.

Lemma A.1 (See [TCR10] Lemma 6). Let $\rho, \sigma \in S_{\leq} (\mathcal{H})$. Then

$$D(\rho, \sigma) \leq P(\rho, \sigma) \leq \sqrt{2} \bar{D}(\rho, \sigma).$$
Uhlmann’s theorem restated for the purified distance is as follows.

**Lemma A.2** (See [TCR10, Lemma 8]). Let \( \rho, \sigma \in S_\leq(\mathcal{H}_A) \) and \( \varphi \in S_\leq(\mathcal{H}_{AR}) \) be a purification of \( \rho \). Then there exists a purification \( \psi \in S_\leq(\mathcal{H}_{AR}) \) of \( \sigma \) such that \( P(\rho, \sigma) = P(\varphi, \psi) \).

**B Stand-alone and composable correctness**

A protocol provides stand-alone correctness if, when Bob behaves honestly, Alice ends up with the correct output. This must also hold with respect to a purification of the input.

**Definition B.1.** A DQC protocol provides stand-alone \( \varepsilon \)-correctness, if, when both parties behave honestly, for all initial states \( \psi_{AR} \), the final state of Alice’s system and the reference system \( \rho_{AR}^{\psi} \) is

\[
\rho_{AR}^{\psi} \approx_\varepsilon (U \otimes \text{id}_R)(\psi_{AR}). \tag{24}
\]

It is straightforward, that this also implies composable correctness, as defined in Eqs. (4) and (5) in Section 4.2.

**Lemma B.2.** A DQC protocol which provides stand-alone \( \varepsilon \)-correctness is also compositely \( \varepsilon \)-correct.

**Proof.** The resources \( \pi_A R \pi_B \) and \( S_B \) have only one input and output, both on the A-interface. By **Definition B.1**, for any initial state \( \psi_{AR} \), the output of \( \pi_A R \pi_B \) is \( \varepsilon \)-close to \( (U \otimes \text{id}_R)(\psi_{AR}) \), whereas the output of \( S_B \) is exactly \( (U \otimes \text{id}_R)(\psi_{AR}) \). So the two resources are \( \varepsilon \)-close. \( \square \)

**C DQC protocol of Fitzsimons and Kashefi**

Fitzsimons and Kashefi [FK12] extend the DQC protocol of [BFK09] to include verifiability. They do this by inserting *trap qubits* at random positions in the brickwork state, that are not relevant for computing the correct outcome. They prove that if the measurement results of these qubits are not what the client Alice expects, she knows that the server is cheating, and if no traps are triggered, Alice can be sure (up to some error \( \varepsilon \)) that the server is running the correct protocol.

Fitzsimons and Kashefi [FK12] use stand-alone security in their proofs, and from the results in **Section 6** we know that this is sufficient to obtain composable security. The notion of *independent verifiability* had however not been proposed prior to the current work, and their security definitions are expressed differently than those from **Section 6**. So it is not possible to directly apply **Theorem 6.7** to their results, and for completeness we include here a proof sketch that the protocol of Fitzsimons and Kashefi does provide \( \varepsilon \)-security.

**Lemma C.1.** If the protocol of [FK12] is run with parameters such that it has error \( \varepsilon \), then it is \( 4\sqrt{2}e^{1/4} \)-secure for classical inputs and \( 4\sqrt{2}e^{1/4}N^2 \)-secure for quantum inputs, where \( N \) is the dimension of Alice’s input register.
Proof sketch. The protocol of [FK12] is an extension of the UBQC protocol of [BFK09] analyzed in Section 7.1 and also provides perfect blindness.

The verifiability definition used in [FK12] is expressed differently from that of Definition 6.2. For a pure input $|\psi_{AR}\rangle$, the correct output is $|U\psi_{AR}\rangle := U \otimes \text{id}_R |\psi_{AR}\rangle$. The projector

$$\Pi := \text{id}_AR - |U\psi_{AR}\rangle \langle U\psi_{AR}| - |\text{err}\rangle \langle \text{err}| \otimes \text{id}_R$$

defines the space where an erroneous output is accepted, and the verifiability criterion of [FK12] can be reduced to

$$\text{tr} (\Pi \rho_{AR}) \leq \varepsilon,$$

(25)

where $\rho_{AR}$ is the state of Alice and the reference system at the end of the protocol. Note that the output can always be written as a linear combination of the error flag and some accepted output,

$$\rho_{AR} = p\sigma_{AR} + (1 - p)|\text{err}\rangle\langle \text{err}| \otimes \psi_R.$$

Plugging this in the two definitions of stand-alone verifiability we find that Definition 6.2 is equivalent to requiring $pD(\sigma_{AR}, |U\psi_{AR}\rangle) \leq \varepsilon$ and Eq. (25) is equivalent to having $p(1 - F^2(\sigma_{AR}, |U\psi_{AR}\rangle)) \leq \varepsilon$, where $D(\cdot, \cdot)$ is the trace distance [Section 3.3] and $F(\cdot, \cdot)$ is the fidelity [Appendix A]. Using standard bounds between the trace distance and fidelity, we find that any protocol which respects Eq. (25) for all pure AR inputs provides stand-alone $\sqrt{\varepsilon}$-verifiability.

The verification mechanism is independent of the input, so the probability of generating an error depends only on the dishonest server’s behavior. For any input a dishonest server Bob can generate the final state $\rho_{AB}$ on his own — where $\tilde{A}$ is a copy of the system Alice measures to decide if she accepts or rejects the final output — by disregarding the communication with Alice, picking an input himself, and running the protocol on his own. The protocol thus provides perfectly independent $\sqrt{\varepsilon}$-verifiability.

Putting this together with Corollary 6.8 concludes this proof. \qed

References

[ABE10] Dorit Aharonov, Michael Ben-Or, and Elad Eban. Interactive proofs for quantum computations. In Proceedings of Innovations in Computer Science, ICS 2010, pages 453–469, 2010. [arXiv:0810.5375].

[AFK87] Martín Abadi, Joan Feigenbaum, and Joe Kilian. On hiding information from an oracle. In Proceedings of the 19th symposium on theory of computing, STOC ’87, pages 195–203. ACM, 1987. [doi:10.1145/28395.28417].

[AS06] Pablo Arrighi and Louis Salvail. Blind quantum computation. International Journal of Quantum Information, 4(05):883–898, 2006. [doi:10.1142/S0219749906002171] [arXiv:quant-ph/0309152].

[AV12] Dorit Aharonov and Umesh Vazirani. Is quantum mechanics falsifiable? A computational perspective on the foundations of quantum mechanics. eprint, 2012. [arXiv:1206.3680].
[BCG+02] Howard Barnum, Claude Crépeau, Daniel Gottesman, Adam Smith, and Alain Tapp. Authentication of quantum messages. In *Proceedings of the 43rd symposium on foundations of computer Science, FOCS ’02*, pages 449–458. IEEE, 2002. [arXiv:quant-ph/0205128].

[BFK09] Anne Broadbent, Joseph Fitzsimons, and Elham Kashefi. Universal blind quantum computation. In *Proceedings of the 50th symposium on foundations of computer science, FOCS ’09*, pages 517–526. IEEE Computer Society, 2009. [doi:10.1109/FOCS.2009.36].

[BGS12] Anne Broadbent, Gus Gutoski, and Douglas Stebila. Quantum one-time programs. eprint, 2012. [arXiv:1211.1080].

[BKB+12] Stefanie Barz, Elham Kashefi, Anne Broadbent, Joseph F. Fitzsimons, Anton Zeilinger, and Philip Walther. Demonstration of blind quantum computing. *Science*, 335(6066):303–308, January 2012. [doi:10.1126/science.1214707, arXiv:1110.1381].

[BM04] Michael Ben-Or and Dominic Mayers. General security definition and composability for quantum & classical protocols. eprint, 2004. [arXiv:quant-ph/0409062].

[BPW04] Michael Backes, Birgit Pfitzmann, and Michael Waidner. A general composition theorem for secure reactive systems. In *Proceedings of the First Theory of Cryptography Conference, TCC ’04*, pages 336–354. Springer, 2004. [doi:10.1007/978-3-540-24638-1_19].

[BPW07] Michael Backes, Birgit Pfitzmann, and Michael Waidner. The reactive simulatability (RSIM) framework for asynchronous systems. Cryptology ePrint Archive, Report 2004/082, 2007. Extended version of [PW01]. [IACR e-print: 2004/082].

[Can01] Ran Canetti. Universally composable security: A new paradigm for cryptographic protocols. In *Proceedings of the 42nd Symposium on Foundations of Computer Science, FOCS ’01*, page 136. IEEE, 2001.

[Can05] Ran Canetti. Universally composable security: A new paradigm for cryptographic protocols. Cryptology ePrint Archive, Report 2000/067, 2005. Updated version of [Can01]. [IACR e-print: 2000/067].

[CDP09] Giulio Chiribella, Giacomo Mauro D’Ariano, and Paolo Perinotti. Theoretical framework for quantum networks. *Phys. Rev. A.*, 80:022339, Aug 2009. [doi:10.1103/PhysRevA.80.022339, arXiv:0904.4483].

[Chi05] Andrew M. Childs. Secure assisted quantum computation. *Quantum Information & Computation*, 5(6):456–466, 2005. [arXiv:quant-ph/0111046].

[DK06] Vincent Danos and Elham Kashefi. Determinism in the one-way model. *Physical Review A.*, 74(5):052310, Nov 2006. [doi:10.1103/PhysRevA.74.052310, arXiv:quant-ph/0506062].
[DKL12] Vedran Dunjko, Elham Kashefi, and Anthony Leverrier. Blind quantum computing with weak coherent pulses. *Physical Review Letters*, 108:200502, May 2012. [doi:10.1103/PhysRevLett.108.200502, arXiv:1108.5571].

[DKP07] Vincent Danos, Elham Kashefi, and Prakash Panangaden. The measurement calculus. *Journal of the ACM*, 54(2), April 2007. [doi:10.1145/1219092.1219096, arXiv:0704.1263].

[FK12] Joseph Fitzsimons and Elham Kashefi. Unconditionally verifiable blind computation. eprint, 2012. [arXiv:1203.5217].

[Gen09] Craig Gentry. Fully homomorphic encryption using ideal lattices. In *Proceedings of the 41st symposium on theory of computing*, STOC ’09, pages 169–178. ACM, 2009. [doi:10.1145/1536414.1536440].

[HMU06] Dennis Hofheinz, Jörn Müller-Quade, and Dominique Unruh. On the (im)possibility of extending coin toss. In *Advances in Cryptology, Proceedings of EUROCRYPT ’06*, volume 4004 of Lecture Notes in Computer Science, pages 504–521. Springer, 2006. [IACR e-print: 2006/177].

[Mau02] Ueli Maurer. Indistinguishability of random systems. In Lars Knudsen, editor, *Advances in Cryptology, Proceedings of EUROCRYPT ’02*, volume 2332 of Lecture Notes in Computer Science, pages 110–132. Springer, May 2002. [doi:10.1007/3-540-46035-7_8].

[Mau11] Ueli Maurer. Constructive cryptography — a new paradigm for security definitions and proofs. In *Proceedings of Theory of Security and Applications, TOSCA 2011*, pages 33–56. Springer, 2011. [doi:10.1007/978-3-642-27375-9_3].

[MDK10] Tomoyuki Morimae, Vedran Dunjko, and Elham Kashefi. Ground state blind quantum computation on AKLT state. eprint, 2010. [arXiv:1009.3486].

[MF12] Tomoyuki Morimae and Keisuke Fujii. Blind quantum computation for Alice who does only measurements. eprint, 2012. [arXiv:1201.3966].

[Mor12] Tomoyuki Morimae. Continuous-variable blind quantum computation. *Physical Review Letters*, 109:230502, Dec 2012. [doi:10.1103/PhysRevLett.109.230502, arXiv:1208.0442].

[MPR07] Ueli Maurer, Krzysztof Pietrzak, and Renato Renner. Indistinguishability amplification. In Alfred Menezes, editor, *Proceedings of the 27th Annual International Cryptology Conference on Advances in Cryptology, CRYPTO ’07*, volume 4622 of Lecture Notes in Computer Science, pages 130–149. Springer, August 2007. [doi:10.1007/978-3-540-74143-5_8].
[MR11] Ueli Maurer and Renato Renner. Abstract cryptography. In Proceedings of Innovations in Computer Science, ICS 2010, pages 1–21. Tsinghua University Press, 2011.

[MS10] Michele Mosca and Douglas Stebila. Quantum coins. In Error-Correcting Codes, Finite Geometries and Cryptography, volume 523 of Contemporary Mathematics, pages 35–47. American Mathematical Society, 2010. [arXiv:0911.1295].

[NC00] Michael A. Nielsen and Isaac L. Chuang. Quantum Computation and Quantum Information. Cambridge University Press, 2000.

[PW01] Birgit Pfitzmann and Michael Waidner. A model for asynchronous reactive systems and its application to secure message transmission. In IEEE Symposium on Security and Privacy, pages 184–200, 2001. doi:10.1109/SECPRI.2001.924298.

[RAD78] Ronald L. Rivest, Leonard M. Adleman, and Michael L. Dertouzos. On data banks and privacy homomorphisms. In Foundations of Secure Computation, pages 169–177. Academic Press, 1978.

[RB01] Robert Raussendorf and Hans J. Briegel. A one-way quantum computer. Physical Review Letters, 86:5188–5191, May 2001. doi:10.1103/PhysRevLett.86.5188.

[TCR10] Marco Tomamichel, Roger Colbeck, and Renato Renner. Duality between smooth min- and max-entropies. IEEE Transactions on Information Theory, 56(9):4674–4681, 2010. doi:10.1109/TIT.2010.2054130, arXiv:0907.5238.

[Unr04] Dominique Unruh. Simulatable security for quantum protocols. eprint, 2004. [arXiv:quant-ph/0409125].

[Unr10] Dominique Unruh. Universally composable quantum multi-party computation. In Advances in Cryptology, Proceedings of EUROCRYPT ’10, pages 486–505. Springer, 2010. doi:10.1007/978-3-642-13190-5_23, arXiv:0910.2912.

[Vaz07] Umesh Vazirani. Computational constraints on scientific theories: insights from quantum computing, 2007. Workshop on the Computational Worldview and the Sciences, http://www.cs.caltech.edu/~schulman/Workshops/CS-Lens-2/cs-lens-2.html.

[Wat08] John Watrous. Theory of quantum information, 2008. Lecture Notes, http://www.cs.uwaterloo.ca/~watrous/quant-info/