Sensitivity measuring expected on the electromagnetic anomalous couplings in the $t\bar{t}\gamma$ vertex at the FCC-he

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Abstract

In this paper, we consider the electroweak production cross-section of a single anti-top-quark, a neutrino and a photon via charged current through the $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t} \nu_e \gamma \rightarrow \bar{t}(\rightarrow W^- \rightarrow (qq',l^-\bar{\nu}_l) + b)\nu_e \gamma$ signal. Further, we derived the sensitivity expected to the magnetic dipole moment ($\hat{a}_V$) and the electric dipole moment ($\hat{a}_A$) of the top-quark at the Future Circular Collider Hadron Electron (FCC-he). We present our study for $\sqrt{s} = 7, 0.7, 10 \, TeV$, $L = 50, 100, 300, 500, 1000 \, fb^{-1}$, $\delta_{sys} = 0, 3, 5 \%$ and $P_{e^-} = 0\%, 80\%, -80\%$, respectively. We find that the sensitivity estimated on dipole moments of the top-quark is of the order of magnitude $\mathcal{O}(10^{-1})$ for both hadronic and leptonic decay modes of $W^-$: $\hat{a}_V = [-0.2308, 0.2204]$, $|\hat{a}_A| = 0.2259$ at 95% C.L. in the hadronic channel with unpolarized electron beam $P_{e^-} = 0\%$. Our results with polarized electron beam for $P_{e^-} = 80\%$ and $P_{e^-} = -80\%$ are $\hat{a}_V = [-0.3428, 0.3321]$, $|\hat{a}_A| = 0.3371$ and $\hat{a}_V = [-0.2041, 0.1858]$, $|\hat{a}_A| = 0.1939$ at 95% C.L. in the hadronic channel. The corresponding results for the leptonic channel with $P_{e^-} = 0\%, 80\% - 80\%$ are $\hat{a}_V = [-0.3067, 0.2963]$, $|\hat{a}_A| = 0.3019$, $\hat{a}_V = [-0.4563, 0.4456]$, $|\hat{a}_A| = 0.4505$ and $\hat{a}_V = [-0.2695, 0.2512]$, $|\hat{a}_A| = 0.2592$, respectively. The results for $\hat{a}_V$ and $\hat{a}_A$ in the leptonic channel are weaker by a factor of 0.75 than those corresponding to the hadronic channel. Given these prospective sensitivities we highlight that the FCC-he is potential top-quark factory that is particularly well suited to sensitivity study on its dipole moments and with cleaner environments.

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I. INTRODUCTION

The characteristics of the top-quark, mainly the total decay width which is one of the fundamental property of top physics is measured with very good precision from the partial decay width $\Gamma(t \rightarrow Wb)$. In addition, its huge mass $m_t = 173.0 \pm 0.4 GeV$ [1], as well as its anomalous couplings to bosons in the $t\bar{t}\gamma$, $t\bar{t}Z$, $t\bar{t}g$, $Wtb$, $\bar{t}tH$ and $tq\gamma$ vertices, have turned the top-quark into one of the most attractive particles for new physics searches. Measurements of the properties of the top-quark offer an interesting probe to understanding the electroweak sector [2] and physics Beyond the Standard Model (BSM). These and other characteristics have led to developing your own physics program for the top-quark for present and future $pp$, $e^-p$ and $e^+e^-$ colliders. Therefore, top-quark physics is one of the most attractive topics at the Large Hadron Collider (LHC), as well at the High-Luminosity Large Hadron Collider (HL-LHC) and High-Energy Large Hadron Collider (HE-LHC). However, at the post LHC era a very attractive and interesting option to study the physics of the top-quark, mainly its anomalous couplings is through the future electron-proton ($e^-p$) hybrid colliders, such as the Future Circular Collider Hadron Electron (FCC-he) [3–8]. The $e^-p$ colliders will open up new perspective in the field of fundamental physics, especially for particle physics. Several potential features in favor of this type of electron-proton colliders are the following: 1) Would represent the high resolution collider, with the cleaner environment for exploring the substructure and dynamics inside matter, with unmatchable precision and sensitivity. 2) The center-of-mass energies are much higher than that of the future International Linear Collider (ILC) and the Compact Linear Collider (CLIC). 3) With concurrent $e^-p$ and $pp$ operation, the FCC-he would transform the LHC into an energy frontier accelerator facility. 4) With very precise strong and electroweak interaction measurements and with suppressed backgrounds from strong interactions, the $e^-p$ results would make the FCC-he a much more powerful search and measurement laboratory than present laboratories based on $pp$ collisions. 5) The joint $pp+e^-p$ facility can become a Higgs bosons and top-quark factory for study the physics of both with an unprecedented impact. 6) For it’s high-energy, high-luminosity and while maintaining a very clean experimental environment, the FCC-he has an outstanding opportunity to discover new physics BSM, such as Higgs sector, top-quark physics, exotic Higgs, dark matter, heavy neutrino, matter-antimatter asymmetry and possible discoveries in QCD. All these topics are being studied very actively. In conclusion, the physics program
of high-energy and high-luminosity $e^- p + pp$ collisions are vast and contain many unique opportunities. It provides high precision of Higgs boson, top-quark, QCD and electroweak physics complementary to $e^+ e^-$ colliders. Furthermore, $e^- p$ is an attractive, realistic option for a future energy frontier collider for particle physics. For an exhaustive study on the physics and detector design concepts see Refs. [3–8].

In the first instants of the creation of the universe, the Big Bang should have created equal amounts of matter and antimatter. However, there is an unexplained dominance of matter over antimatter observed in the universe. CP violation offers one explanation for the asymmetry in baryonic matter, however, there are not enough current observed sources of CP violation to account for the total matter-antimatter asymmetry. For this reason, it is necessary to study new sources of CP violation. About this topic, the Electric Dipole Moments (EDM) are very sensitive to CP violation in the quark and lepton sectors. EDM searches are then in the ideal situation of that an observation in the next generation of experiments would be a clear indication of new physics BSM. Under this perspective, the Anomalous Magnetic Dipole Moment (AMDM) and EDM of the top-quark are currently under intense scrutiny from the point of view: theoretical, phenomenological and experimental. Within the scope of this project, CP violation can be parameterized by the presence of anomalous couplings in the $t\bar{t}\gamma$ vertex of top quark production.

In this study, we focus on AMDM ($\hat{a}_V$) and the EDM ($\hat{a}_A$) of the top-quark. Since the AMDM and EDM of the top-quark is chirality changing it can be significantly enhanced compared to AMDM and EDM of light fermions by the large top coupling.

For the dominant $e^- p \rightarrow e^-\bar{b} \rightarrow \bar{t} \nu_e \gamma \rightarrow \bar{t} (\rightarrow W^- \rightarrow (q\bar{q'}, l^- \bar{\nu}_l) + b) \nu_e \gamma$ production channel considered here, we find that the proposed FCC-he with $\sqrt{s} = 7.07, 10 \, TeV$, $L = 50, 100, 300, 500, 1000 \, fb^{-1}$ can probe the dipole moments of the top-quark with good sensitivity. We focus on two different signals: (i) the hadronic channel with polarized electron beam for $P_{e^-} = -80, 0, 80\%$, and (ii) the leptonic channel with polarized electron beam for $P_{e^-} = -80, 0, 80\%$ in the $\bar{t} \nu_e \gamma$ final state. We show that sensitivity measuring expected on the electromagnetic anomalous couplings in the $t\bar{t}\gamma$ vertex can be probed at 95% Confidence Level (C.L.) with center-of-mass energies $\sqrt{s} = 7.07, 10 \, TeV$ and integrated luminosity $L = 1000 \, fb^{-1}$ at FCC-he.

AMDM and EDM searches of the top-quark are performed in the Standard Model (SM) and on a variety of physics processes. The sensitivity estimated on the AMDM and EDM
TABLE I: Sensitivities achievable on the electromagnetic dipole moments of the top-quark in the SM and in different processes and colliders.

| Model | Sensitivity of the SM | C. L. | References |
|-------|-----------------------|-------|------------|
| SM    | $a_t = 0.02, d_t < 10^{-30}(ecm)$ | 68%   | [9], [10–12] |

| Model | Theoretical sensitivity: $\hat{a}_V, \hat{a}_A$ | C. L. | Reference |
|-------|-----------------------------------------------|-------|-----------|
| Top-quark pair production at LHC | $(-0.041, 0.043), (-0.035, 0.038)$ | 68%   | [13]      |
| $t\bar{t}\gamma$ production at LHC | $(-0.2, 0.2), (-0.1, 0.1)$ | 90%   | [14]      |
| Radiative $b \to s\gamma$ transitions at Tevatron and LHC | $(-2, 0.3), (-0.5, 1.5)$ | 90%   | [15]      |
| Process $pp \to p\gamma^*\gamma^*p \to pt\bar{t}p$ at LHC | $(-0.6389, 0.0233), (-0.1158, 0.1158)$ | 68%   | [16]      |
| Measurements of $\gamma p \to t\bar{t}$ at LHeC | $(-0.05, 0.05), (-0.20, 0.20)$ | 90%   | [17]      |
| Top-quark pair production $e^+e^- \to t\bar{t}$ at ILC | $(-0.002, 0.002), (-0.001, 0.001)$ | 68%   | [18]      |

of the top-quark in the SM, as well as in different processes and colliders are reported in Table I. Other direct collider probes of the AMDM and EDM have been studied extensively [13, 19–33].

Plan of the article is as follows: In Section II, we introduce the top-quark effective electromagnetic interactions. In Section III, we sensitivity measurement on top-quark anomalous electromagnetic couplings through $e^-p \to e^-\bar{b} \to \bar{t}\nu_e\gamma \to \bar{t}(\to W^- \to (qq', l^-\bar{l}) + b)\nu_e\gamma$ signal. Finally, we present our conclusions in Section IV.

II. SINGLE TOP-QUARK PRODUCTION VIA THE PROCESS $e^-p \to e^-\bar{b} \to \bar{t}\nu_e\gamma$

A. Effective Lagrangian of $t\bar{t}\gamma$ interaction of the top-quark

The SM predicts CP violation outside the K, D and B meson systems is small to be observed. However, in some extensions of the SM, CP violation might be considerably enhanced, especially in the presence of heavy particles as the top quark. In particular, CP-violating EDM of the top-quark could be enhanced.

Single top-quark production processes are sensitive to the anomalous couplings in the
Furthermore, since the top-quark lifetime is shorter than the timescale of spin decoherence induced by QCD, its decay products preserve information of its polarization by the production mechanism. This provides additional powerful tools in the search for BSM physics in single top-quark studies.

On the other hand, due to the absence so far of any signal of new heavy particles decaying into top-quark, an attractive approach for describing possible new physics effects in a model-independent way is based on effective Lagrangian. In this approach, all the heavy degrees of freedom are integrated out leading to obtain the effective interactions between the SM particles. This is justified because the related observables have not shown any significant deviation from the SM predictions so far. The Lagrangian describing interaction of the anomalous $t\bar{t}\gamma$ coupling including the SM contribution and terms BSM which are related to new physics has the structure:

$$L_{\text{eff}} = L^{(4)}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_n \left[ C_n \mathcal{O}_n^{(6)} + C^*_n \mathcal{O}^\dagger_n^{(6)} \right].$$  

Here, $L_{\text{eff}}$ is the effective Lagrangian gauge-invariant which contains a series of dimension-six operators built with the SM fields, $L^{(4)}_{\text{SM}}$ is the renormalizable SM Lagrangian of dimension-four, $\Lambda$ is the scale at which new physics expected to be observed, $C_n$ are Wilson coefficients which are dimensionless coefficients and $\mathcal{O}_n^{(6)}$ represents the dimension-six gauge-invariant operator. The $\mathcal{O}_n^{(6)}$ operator and the unknown coefficients $C_n$, combined with $\Lambda^{-2}$, produce the non-standard coupling constants, that is generated anomalous contributions to the photon-top-quark interaction vertex which is similar in structure to radiative corrections in the SM.

The most general Lagrangian term that one can write for the $t\bar{t}\gamma$ coupling up to dimension-six gauge invariant operators \[14, 16, 18, 34, 35\] is:

$$L_{t\bar{t}\gamma} = -g_e Q_t t\bar{t}\gamma (\sigma^\mu A_\mu),$$  

this equation includes the SM coupling and contributions from dimension-six effective operators. $g_e$ is the electromagnetic coupling constant, $Q_t$ is the top-quark electric charge and the Lorentz-invariant vertex function $\Gamma^\mu_{t\bar{t}\gamma}$ is given by:

$$\Gamma^\mu_{t\bar{t}\gamma} = \gamma^\mu + \frac{i}{2 m_t} (\hat{a}_V + i \hat{a}_A \gamma_5) \sigma^{\mu\nu} q_\nu.$$  

\[3\]
$m_t$ and $q$ are the mass of the top-quark and the momentum transfer to the photon, respectively. The $\hat{a}_V$ and $\hat{a}_A$ couplings in Eq. (3) are real and related to the AMDM and the EDM of the top-quark. These couplings $\hat{a}_V$ and $\hat{a}_A$ are directly related to the $a_t$ and $d_t$, via the relations:

$$\hat{a}_V = Q_t a_t, \quad (4)$$
$$\hat{a}_A = \frac{2m_t}{e} d_t. \quad (5)$$

As shown in Refs. [31, 36–38], among operators of dimension-six there exist only two relevant for the $t\bar{t}\gamma$ interaction operator:

$$O_{uW}^{33} = \bar{q}_{L3} \sigma^\mu\nu \tau^a t_R \tilde{\phi} W^a_{\mu\nu} + \text{h.c.}, \quad (6)$$
$$O_{uB\phi}^{33} = \bar{q}_{L3} \sigma^\mu\nu t_R \tilde{\phi} B_{\mu\nu} + \text{h.c.}, \quad (7)$$

where the index 3 means the 3rd quark generation, $\bar{q}_{L3}$ is the quark field, $\sigma^{\mu\nu}$ are the Pauli matrices, $\tilde{\phi} = i\tau_2 \phi^*$, $\phi$ is the SM Higgs doublet, $W^a_{\mu\nu}$ and $B_{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ gauge field strength tensors which are defined as:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (8)$$
$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu, \quad (9)$$

with $a, b, c = 1, 2, 3$. From the parametrization given by Eq. (3), and from the operators of dimension-six given in Eqs. (6) and (7) after replacing $<\tilde{\phi}> \rightarrow \frac{1}{\sqrt{s}}$ give rise to the corresponding CP even $\hat{a}_V$ and CP odd $\hat{a}_A$ couplings:

$$\hat{a}_V = \frac{2m_t}{e} \sqrt{\frac{2v}{\Lambda^2}} \text{Re} \left[ \cos \theta_W C_{uB\phi}^{33} + \sin \theta_W C_{uW}^{33} \right], \quad (10)$$
$$\hat{a}_A = \frac{2m_t}{e} \sqrt{\frac{2v}{\Lambda^2}} \text{Im} \left[ \cos \theta_W C_{uB\phi}^{33} + \sin \theta_W C_{uW}^{33} \right], \quad (11)$$

which are related to the AMDM and EDM of the top-quark. The $\hat{a}_V$ and $\hat{a}_A$ couplings contain $\nu = 246$ GeV, the breaking scale of the electroweak symmetry, $\sin \theta_W (\cos \theta_W)$, the sine(cosine) of the weak mixing angle and $\Lambda$ is the new physics scale.
B. Cross-section of the $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma \rightarrow \bar{t}(\rightarrow W^- \rightarrow (qq', l^-\bar{\nu}_l) + b)\nu_e\gamma$ signal

The FCC-eh project [3–8] offer $e^-p$ collisions at TeV-scale center-of-mass energy and luminosities of order 1000 times larger than that of Hadron-Electron Ring Accelerator (HERA), the first and to date the only lepton-hadron collider worldwide, providing fascinating probes of QCD and hadron structure as well as a novel configuration for Higgs boson, top-quark and BSM physics.

The physics highlights are available from combining the capabilities of these facilities they were already mentioned in the introduction. However, a general aspect is to maximize the BSM physics search potential at high energies by exploiting the unique capabilities of an $e^-p$ collider.

The most significant top-quark production processes at the $e^-p$ colliders are single top-quark, $t\bar{t}$, and associated $tW$ production. In Ref. [17] is shown the values of the associated cross-sections for these processes. The main source of production is single-top via the charged current $W$ t-channel [40], whereas for the signals, $t\bar{t}$ and $tW$, the cross-section is minor. In addition, taking into account the advantage of an experimental cleaner environment than the $pp$ colliders, we can anticipate a potential efficiency of these colliders to study the top-quark physics.

The deep inelastic $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma \rightarrow \bar{t}(\rightarrow W^- \rightarrow (qq', l^-\bar{\nu}_l) + b)\nu_e\gamma$ scattering process is measured at FCC-eh via the exchange of a $W^\pm$ boson in charged current scattering as shown in Figs. 1 and 2.

For the calculation of the charged current cross-section, we consider the CTEQ6L1 PDFs [41] and we apply the following detector acceptance cuts on the pseudorapidity of the photon and the transverse momentum of the photon and the neutrino, respectively, to reduce the background and to optimize the signal sensitivity:

$$|\eta^\gamma| < 2.5,$$
$$p^\gamma_T > 20\, GeV,$$
$$p^{(\nu)}_T > 20\, GeV. \quad (12)$$

The proposed FCC-he is well-suited for discovering physics BSM and for precisely unraveling the structure of the fundamental physics with unpolarized and polarized electron beam.
In particle physics, polarization refers to the extent to which a particle spin is aligned along a particular direction. The t-channel single-top-quarks of the process $e^- p \to e^- b \to \bar{t} \nu_e \gamma$ are produced with a strong degree of polarization along the direction of the momentum of the light spectator quark, whose direction then defines the top-quark spin axis. Regarding this subject, the physics in study can be maximized by the use of polarized beams. In this paper shows the important role of polarized beam and summarizes the benefits obtained from polarizing the electron beam. The polarized $e^-$ beam, combined with the clean experimental environment provided by the FCC-he, will allow to improve strongly the potential of searches for the dipole moments, which opens the possibility to resolve shortcomings of the SM. With these arguments, we consider polarized electron beam in our study. The formula for the total cross-section for an arbitrary degree of longitudinal $e^-$ beams polarization is given by [42]:

$$
\sigma_{e^->r} = \sigma_{e^->0} \cdot (1 - P_{e^-}), \quad \sigma_{e^->l} + \sigma_{e^->r} = 2\sigma_{e^->0},
$$

(13)

where $\sigma_{e^->r}$, $\sigma_{e^->l}$ and $\sigma_{e^->0}$ represent the right, left and without electron beam polarization, respectively and $P_{e^-}$ is the polarization degree of the electron.

We have implemented $t\bar{t}\gamma$ effective coupling corresponding given by the Lagrangian (2) in CalcHEP [43] to compute the tree level amplitudes relevant for the process. The partonic cross-section is convoluted with CTEQ6L1 PDFs [41]. Finally, we use CalcHEP to compute numerically the cross-section $\sigma(\sqrt{s}, \hat{a}_V, \hat{a}_A, P_{e^-})$ as a function of the center-of-mass energy and effective couplings. We displayed the 7.07 $TeV$ and 10 $TeV$ cross-section of the $2 \to 3$ process $e^- p \to e^- b \to \bar{t} \nu_e \gamma$ with $P_{e^-} = 0\%$, $P_{e^-} = -80\%$ and $P_{e^-} = 80\%$ in Eqs. (14)-(25) for the FCC-he:

i) Total cross-section for $\sqrt{s} = 7.07 TeV$ and $P_{e^-} = 0\%$:

$$
\sigma(\hat{a}_V) = \left[(0.0236)\hat{a}_V^2 + (0.0000489)\hat{a}_V + 0.737\right](pb), \quad (14)
$$

$$
\sigma(\hat{a}_A) = \left[(0.0236)\hat{a}_A^2 + 0.737\right](pb). \quad (15)
$$

ii) Total cross-section for $\sqrt{s} = 10 TeV$ and $P_{e^-} = 0\%$:
\[ \sigma(\hat{a}_V) = \left[ (0.0593)\hat{a}_V^2 + (0.000618)\hat{a}_V + 1.287 \right] (pb), \]
\[ \sigma(\hat{a}_A) = \left[ (0.0593)\hat{a}_A^2 + 1.287 \right] (pb). \]

\[ \sigma(\hat{a}_V) = \left[ (0.00472)\hat{a}_V^2 + (0.0000109)\hat{a}_V + 0.148 \right] (pb), \]
\[ \sigma(\hat{a}_A) = \left[ (0.00472)\hat{a}_A^2 + 0.148 \right] (pb). \]

\[ \sigma(\hat{a}_V) = \left[ (0.00472)\hat{a}_V^2 + (0.000127)\hat{a}_V + 0.257 \right] (pb), \]
\[ \sigma(\hat{a}_A) = \left[ (0.00472)\hat{a}_A^2 + 0.257 \right] (pb). \]

\[ \sigma(\hat{a}_V) = \left[ (0.0119)\hat{a}_V^2 + (0.000127)\hat{a}_V + 0.257 \right] (pb), \]
\[ \sigma(\hat{a}_A) = \left[ (0.0119)\hat{a}_A^2 + 0.257 \right] (pb). \]

\[ \sigma(\hat{a}_V) = \left[ (0.0423)\hat{a}_V^2 + (0.000417)\hat{a}_V + 1.328 \right] (pb), \]
\[ \sigma(\hat{a}_A) = \left[ (0.0423)\hat{a}_A^2 + 1.328 \right] (pb). \]

\[ \sigma(\hat{a}_V) = \left[ (0.107)\hat{a}_V^2 + (0.00196)\hat{a}_V + 2.315 \right] (pb), \]
\[ \sigma(\hat{a}_A) = \left[ (0.107)\hat{a}_A^2 + 2.315 \right] (pb). \]

Our results given by Eqs. (14)-(25) show the effect of taking $-80\%$ beam polarization for electron, which results in the enhancement of the SM and non-SM single-top production cross-section as the cross-section scales as $(1 + P_{e^{-}})$, $P_{e^{-}}$ being the degree of polarization of the electron.

The variation of the single top-quark production cross-section with the effective $t\bar{t}\gamma$ couplings, $\hat{a}_V$ or $\hat{a}_A$ and taking one anomalous coupling at a time are shown in Figs. 3-6.
The curves depict the cross-section for $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma$ from the 80%, −80% polarized and unpolarized $e^-$ beam, respectively, and we fixed the energy of the $e^-$ beam $E_e = 250\,GeV$, 500\,GeV and the energy of the $p$ beam $E_p = 50\,TeV$. These figures have shown a stronger dependence of the cross-section on the anomalous coupling $\hat{a}_V(\hat{a}_A)$ in the range allowed by these parameters. Our results indicate that considering the proposed 10\,TeV energy, detector acceptance cuts (see Eq. (12)) and −80% electron polarization, the cross-sections as a function of $\hat{a}_V$ or $\hat{a}_A$ are higher. For instance, the cross-section projected is $\sigma(\hat{a}_V, -80\%) = (1.66)\sigma(\hat{a}_V, 0\%)$ for $\sqrt{s} = 7.07\,TeV$, while $\sigma(\hat{a}_V, -80\%) = (1.78)\sigma(\hat{a}_V, 0\%)$ for $\sqrt{s} = 10\,TeV$, that is, there is an improvement in the cross-section by a factor of 1.66 (1.78) for the polarized case with respect to the case unpolarized. Similar results are obtained for $\sigma(\hat{a}_A, 80\%)$.

The cross-sections for the energies $\sqrt{s} = 7.07\,TeV$ and 10\,TeV are shown in Figs. 7 and 8. As can be seen from Figs. 7-8, the surfaces $\sigma(e^-p \rightarrow \bar{t}\nu_e\gamma)$ as functions of $\hat{a}_V$ and $\hat{a}_A$ have extreme points: maximum and minimum. The minimum value corresponds to the SM, while the maximum value corresponds to the anomalous contribution, which is consistent with Eqs. (14)-(25). In both figures, the cross-section depends significantly on the observables $\hat{a}_V$ and $\hat{a}_A$.

With the purpose of comparison and analysis, we compare our results for the anomalous couplings $\hat{a}_V$ and $\hat{a}_A$ with those quoted in the papers [15, 16] (see also Table I). The authors of Ref. [15], specifically discussed the bounds on the AMDM and EDM of the top-quark that can be obtained from measurements of the semi-inclusive decays $B \rightarrow X_s\gamma$, and of $t\bar{t}\gamma$ production at the Tevatron and the LHC. Performing their analysis they find that the AMDM is bounded by $-2 < \kappa < 0.3$ whereas the EDM is bound by $-0.5 < \tilde{\kappa} < 1.5$, respectively. For our case, we consider the process of single anti-top-quark production through charged current with the $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma \rightarrow \bar{t}(\rightarrow W^- \rightarrow (qq', l^-\bar{\nu}_l) + b)\nu_e\gamma$ signal. We based our results on the data at $\sqrt{s} = 10\,TeV$, $L = 1000\,fb^{-1}$, $\delta_{sys} = 0\%$, $P_{e^-} = 0\%$ and 95\% C.L., we obtain $\hat{a}_V = (-0.2308, 0.2204)$, $\hat{a}_A = |0.2259|$ and $\hat{a}_V = (-0.3067, 0.2963)$, $\hat{a}_A = |0.3019|$ for the hadronic and leptonic modes. Although the conditions for the study of the dipole moments of the top-quark through the $b \rightarrow s\gamma$ transitions at Tevatron and LHC, and $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma \rightarrow \bar{t}(\rightarrow W^- \rightarrow (qq', l^-\bar{\nu}_l) + b)\nu_e\gamma$ are different, our results are competitive with respect to the results reported in Ref. [15]. More recently, using the process $pp \rightarrow p\gamma^*\gamma^*p \rightarrow p\bar{t}\bar{t}p$, a detailed study on the top-quark anomalous couplings $\hat{a}_v$ and $\hat{a}_A$ for
LHC at 14 $TeV$ with 300 $fb^{-1}$ of data [16] is done. The 68% C.L. bounds that they have obtained are found to be in the intervals of $\hat{a}_V = (0.6389, 0.0233)$ and $\hat{a}_A = (0.1158, 0.1158)$.

From the comparison of our study via the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he, with respect to the process $pp \rightarrow p\gamma^* \gamma^* p \rightarrow pt \bar{t} p$ at the LHC, our results indicate a significant improvement in the measurements for $\hat{a}_V$ and $\hat{a}_A$.

Additionally, it is noteworthy that with our process the total cross-sections is a factor $O(10^3)$ between $pp \rightarrow p\gamma^* \gamma^* p \rightarrow pt \bar{t} p$ at the FCC-he, with respect to the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he, indicating that our results project 3 orders of magnitude more higher than the reported in Ref. [16]. These predictions indicate that the sensitivity on the anomalous couplings $\hat{a}_V$ and $\hat{a}_A$ can be measured better at the FCC-he by a few orders of magnitude in comparison with the predictions of the LHC.

### III. MODEL-INDEPENDENT SENSITIVITY ESTIMATES ON THE $\hat{a}_V$ AND $\hat{a}_A$

In Tables II-VII is shown the results for the model-independent sensitivity achievable at 95% C.L. for the non-standard couplings $\hat{a}_V$ and $\hat{a}_A$ obtained from an analysis of the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he. At the FCC-he, we assume the center-of-mass energies $\sqrt{s} = 7.07, 10\, TeV$ and luminosities $\mathcal{L} = 50, 100, 300, 500, 1000\, fb^{-1}$ with unpolarized and polarized electron beam $P_{e^-} = -80\%, 0\%, 80\%$. Additionally, we impose the acceptance cuts for the FCC-he given by Eq. (12) and take into account the systematic uncertainties $\delta_{sys} = 0\%, 3\%, 5\%$.

In order to extract the expected sensitivity at 95% C.L. on the effective operators couplings $\hat{a}_V$ and $\hat{a}_A$, we compute $\sigma_{BSM}(\hat{a}_V, \hat{a}_A)$ cross-section of the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ as function of the effective couplings as discussed in the previous section, and we assume the measured cross-section to coincide with the SM predictions and we construct the following $\chi^2$ function:

$$\chi^2(\hat{a}_V, \hat{a}_A) = \left( \frac{\sigma_{SM} - \sigma_{BSM}(\sqrt{s}, \hat{a}_V, \hat{a}_A, P_{e^-})}{\sigma_{SM} \sqrt{(\delta_{st})^2 + (\delta_{sys})^2}} \right)^2. \quad (26)$$

$\sigma_{SM}$ is the cross-section of the SM and $\sigma_{BSM}(\sqrt{s}, \hat{a}_V, \hat{a}_A, P_{e^-})$ is the total cross-section containing contributions from the SM and BSM, while $\delta_{st} = \frac{1}{\sqrt{N_{SM}}}$ and $\delta_{sys}$ are the statistical and systematic uncertainties, respectively. The number of events $N_{SM}$ for the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ is calculated by $N_{SM} = \mathcal{L}_{int} \times \sigma_{SM} \times BR(\bar{t} \rightarrow W^- b) \times BR(W^- \rightarrow q\bar{q}'(l^- \nu_l)) \times$
\[ \epsilon_{b-tag}, \text{ where } \mathcal{L}_{int} \text{ is the integrated FCC-he luminosity and } b\text{-jet tagging efficiency is } \epsilon_b = 0.8 \]  

The top-quark decays weakly and almost 100% to a \( W \) boson and \( b \) quark, specifically \( t \rightarrow \bar{b}W^- \), where the \( W \) boson decays to either hadronically (\( W \rightarrow qq' \)) or leptonically (\( W \rightarrow l^-\nu_l \)), with a Branching Ratio of: \( BR(W \rightarrow qq') = 0.674 \) for hadronic decay, \( BR(W \rightarrow l\nu_l)(l = e, \mu) = 0.213 \) for light leptonic decays and \( BR(W \rightarrow \tau\nu_\tau) = 0.113 \). \[44\]  

Next, we present the sensibility measurement for the anomalous couplings \( \hat{a}_V \) and \( \hat{a}_A \) as is shown in Tables II-VII which are obtained for \( \sqrt{s} = 7.07, 10 TeV, \mathcal{L} = 50 - 1000 \text{ fb}^{-1} \) and \( P_{e^-} = -80\%, 0\%, 80\% \), where only one coupling at a time is varied.  

From Tables II-VII, the results for the dipole moments \( \hat{a}_V \) and \( \hat{a}_A \), for specific values of \( \sqrt{s} = 10 TeV, \mathcal{L} = 1000 \text{ fb}^{-1}, P_{e^-} = -80\%, 0\%, 80\% \) and \( \delta_{sys} = 0\% \) are as follows:  

i) Sensitivity on \( \hat{a}_V \) and \( \hat{a}_A \) for \( \sqrt{s} = 10 TeV, P_{e^-} = -80\% \) and \( BR(W^- \rightarrow \text{hadronic}) \):  

\[ -0.2041 \leq \hat{a}_V \leq 0.1858, \quad 95\% \text{ C.L.} \]  

\[ -0.1939 \leq \hat{a}_A \leq 0.1939, \quad 95\% \text{ C.L.} \]  

ii) Sensitivity on \( \hat{a}_V \) and \( \hat{a}_A \) for \( \sqrt{s} = 10 TeV, P_{e^-} = -80\% \) and \( BR(W^- \rightarrow \text{leptonic}) \):  

\[ -0.2695 \leq \hat{a}_V \leq 0.2512, \quad 95\% \text{ C.L.} \]  

\[ -0.2592 \leq \hat{a}_A \leq 0.2592, \quad 95\% \text{ C.L.} \]  

iii) Sensitivity on \( \hat{a}_V \) and \( \hat{a}_A \) for \( \sqrt{s} = 10 TeV, P_{e^-} = 0\% \) and \( BR(W^- \rightarrow \text{hadronic}) \):  

\[ -0.2308 \leq \hat{a}_V \leq 0.2204, \quad 95\% \text{ C.L.} \]  

\[ -0.2259 \leq \hat{a}_A \leq 0.2259, \quad 95\% \text{ C.L.} \]  

iv) Sensitivity on \( \hat{a}_V \) and \( \hat{a}_A \) for \( \sqrt{s} = 10 TeV, P_{e^-} = 0\% \) and \( BR(W^- \rightarrow \text{leptonic}) \):  

\[ -0.3067 \leq \hat{a}_V \leq 0.2963, \quad 95\% \text{ C.L.} \]  

\[ -0.3019 \leq \hat{a}_A \leq 0.3019, \quad 95\% \text{ C.L.} \]  

v) Sensitivity on \( \hat{a}_V \) and \( \hat{a}_A \) for \( \sqrt{s} = 10 TeV, P_{e^-} = 80\% \) and \( BR(W^- \rightarrow \text{hadronic}) \):
\[-0.3428 \leq \hat{a}_V \leq 0.3321, \quad 95\% \text{ C.L.}, \]  
\[-0.3371 \leq \hat{a}_A \leq 0.3371, \quad 95\% \text{ C.L.}. \]  

(35) \hspace{1cm} (36)

vi) Sensitivity on $\hat{a}_V$ and $\hat{a}_A$ for $\sqrt{s} = 10 \text{ TeV}$, $P_{e^-} = 80\%$ and $\text{BR}(W^- \rightarrow \text{leptonic})$:

\[-0.4563 \leq \hat{a}_V \leq 0.4456, \quad 95\% \text{ C.L.}, \]  
\[-0.4505 \leq \hat{a}_A \leq 0.4505, \quad 95\% \text{ C.L.}. \]  

(37) \hspace{1cm} (38)

A direct comparison of Eqs. (27)-(38) for $\hat{a}_V$ and $\hat{a}_A$ shown that the sensitivity is increases up to 12% for the case with $P_{e^-} = -80\%$ and $\text{BR}(W^- \rightarrow \text{hadronic, leptonic})$ that for the case with $P_{e^-} = 0\%$ and $\text{BR}(W^- \rightarrow \text{hadronic, leptonic})$. While from Eqs. (27)-(30) and (35)-(38) the sensitivity is increases up to 67% for the case with $P_{e^-} = -80\%$ and $\text{BR}(W^- \rightarrow \text{hadronic, leptonic})$ with respect to the case with $P_{e^-} = 80\%$ and $\text{BR}(W^- \rightarrow \text{hadronic, leptonic})$, respectively.

For $P_{e^-} = 0\%$ and the hadronic channel of the W boson, Figs. 9-10 show the prospects of the sensitivity on the electromagnetic anomalous couplings $\hat{a}_V$ and $\hat{a}_A$ at the FCC-he. Also, in order to obtain the plots we have assumed that $\sqrt{s} = 7.07, 10 \text{ TeV}$ and $\mathcal{L} = 50, 250, 1000 \text{ fb}^{-1}$ at the 95% C.L.. Of these contours plots, our forecast for the future sensitivity of the observables $\hat{a}_V$ and $\hat{a}_A$ are based on the process $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma$, as well as in the future projections of the FCC-he for $\sqrt{s}$ and $\mathcal{L}$. The regions allowed for the top-quark AMDM, EDM and collider sensitivity are colored in pink, blue, purple, respectively, while the prediction corresponds to the SM can be obtained from Eqs. (14)-(25). The 95% C.L. regions for each of these couplings separately are $\hat{a}_V \in [-0.25, 0.25]$, $\hat{a}_A \in [-0.29, 0.29]$ for $\sqrt{s} = 7.07 \text{ TeV}$ and $\mathcal{L} = 250 \text{ fb}^{-1}$. In addition, $\hat{a}_V \in [-0.20, 0.20]$, $\hat{a}_A \in [-0.20, 0.20]$ for $\sqrt{s} = 10 \text{ TeV}$ and $\mathcal{L} = 1000 \text{ fb}^{-1}$. These results are consistent with those shown in Tables IV and V. It is worth mentioning that, the results obtained in Tables IV and V, as well as the corresponding results obtained through the contours Figs. 9 and 10, in some cases are more sensitive than the reported ones in Table I. In particular, an improvement is reachable in comparison with the constraints obtained from the radiative $b \rightarrow s\gamma$ transitions at Tevatron and LHC [15] and $pp \rightarrow p\gamma^*\gamma^*p \rightarrow p\bar{t}\bar{t}p$ [16] searches mentioned in Table I and subsection B.
IV. CONCLUSIONS

As mentioned above, due to its sizable $t\bar{t}\gamma$ coupling the top-quark is one of the most attractive particles and provides one of the most convincing alternatives to probe new physics BSM, such as AMDM ($\hat{a}_V$) and EDM ($\hat{a}_A$). Furthermore, the EDM is particularly interesting because it is very sensitive to possible new sources of CP violation in the quark and lepton sectors.

In this paper, we have studied the potential of the FCC-he for sensitivity measuring expected on the electromagnetic anomalous couplings in the $t\bar{t}\gamma$ vertex. We consider the single top-quark production mode $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma$ with $W$ boson exchange via the $t$-channel, that is through charged current production. This channel has the largest cross-section and is hence the dominant production mode of single top-quark [45]. Our study is based on the projections for the center-of-mass energies $\sqrt{s}$, the integrated luminosity $L$ and the polarization electron beam of the FCC-he. Additionally, we take into account kinematic cuts and systematic uncertainties $\delta_{\text{sys}} = 0\%, 3\%, 5\%$. The cut based optimization at $7.07\, TeV$ and $10\, TeV$, involves a set of selection cuts in various kinematic variables, carefully chosen with the criterion of not being built with kinematic properties of one or part of the decay products of the top-quark, that could in principle bias the sensitivity to the anomalous couplings. These kinematic variables selected for these cuts include $\eta$, $p_T^{\gamma}$ and $p_T^{(\nu)}$ as defined in Eq. (12). The final results of this optimization for the resulting cross-section of the $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t}\nu_e\gamma$ signal and prospective sensitivities for the dipole moments $\hat{a}_V$ and $\hat{a}_A$ indicated in Figs. 3-10 and Tables II-VII as well as in Eqs. (14)-(25) and (27)-(38) imply that the process $e^-p \rightarrow \bar{t}\nu_e\gamma$ at FCC-he is an excellent option for probing the physics of the top-quark. This makes a future $e^-p$ collider an ideal tool to study the electromagnetic properties of the top-quark through the $t\bar{t}\gamma$ vertex.

From Tables III, V and VII, the sensitivity estimated on dipole moments of the top-quark are $\hat{a}_V = [-0.2308, 0.2204]$, $|\hat{a}_A| = 0.2259$ at 95\% C.L. in the hadronic channel with unpolarized electron beam $P_{e^-} = 0\%$. In the case with polarized electron beam for $P_{e^-} = 80\%$ and $P_{e^-} = -80\%$ are $\hat{a}_V = [-0.3428, 0.3321]$ with $P_{e^-} = 0\%$, 80\% – 80\% are $\hat{a}_V = [-0.3067, 0.2963]$, $|\hat{a}_A| = 0.3019$, $\hat{a}_V = [-0.4563, 0.4456]$, $|\hat{a}_A| = 0.4505$ and $\hat{a}_V = [-0.2695, 0.2512]$, $|\hat{a}_A| = 0.2592$, respectively. The results for $\hat{a}_V$ and $\hat{a}_A$ in
the leptonic channel are weaker by a factor of 0.75 than those corresponding to the hadronic channel. From these results, we find that the sensitivity estimated on dipole moments of the top-quark are of the same order of magnitude as those reported in Table I and Refs. [13, 19–33]. In particular, of the comparison with the constraints obtained from the radiative $b \rightarrow s\gamma$ transitions at Tevatron and LHC [13] and the process $pp \rightarrow p\gamma^*\gamma^*p \rightarrow pt\bar{t}p$ at LHC [16], our results are more sensitive. Given these prospective sensitivities, we highlight that the FCC-he is the potential top-quark factory that is particularly well suited to sensitivity study on its dipole moments with cleaner environments.

Summarizing, the FCC-he offers us significant opportunities to study the anomalous couplings of the quark-top. However, more extensive studies on the theoretical, phenomenological and experimental level they are needed. These new possibilities for investigating the electromagnetic properties of the top-quark will eventually open new avenues in the understanding of the quark-top physics, as well as new physics BSM.

Acknowledgments

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TABLE II: Sensitivities on the AMDM $\hat{a}_V$ and the EDM $\hat{a}_A$ of the top-quark through the process $e^- p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he.

| $\sqrt{s}$ (fb$^{-1}$) | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
|------------------------|----------------|------------|--------------|------------|--------------|
|                        |                | Hadronic   | Leptonic     |            |              |
| 50 0%                  |                | [-0.6599, 0.6578] | 0.6587       | [-0.8815, 0.8794] | 0.8802       |
| 50 3%                  |                | [-1.3758, 1.3737] | 1.3742       | [-1.4138, 1.4118] | 1.4123       |
| 50 5%                  |                | [-1.7606, 1.7585] | 1.7589       | [-1.7792, 1.7772] | 1.7776       |
| 100 0%                 |                | [-0.5551, 0.5530] | 0.5539       | [-0.7414, 0.7393] | 0.7401       |
| 100 3%                 |                | [-1.3666, 1.3645] | 1.3651       | [-1.3864, 1.3843] | 1.3849       |
| 100 5%                 |                | [-1.7563, 1.7542] | 1.7546       | [-1.7657, 1.7636] | 1.7640       |
| 300 0%                 |                | [-0.4221, 0.4199] | 0.4208       | [-0.5636, 0.5615] | 0.5624       |
| 300 3%                 |                | [-1.3604, 1.3583] | 1.3589       | [-1.3672, 1.3651] | 1.3656       |
| 300 5%                 |                | [-1.7534, 1.7513] | 1.7517       | [-1.7565, 1.7545] | 1.7549       |
| 500 0%                 |                | [-0.3715, 0.3695] | 0.3704       | [-0.4962, 0.4941] | 0.4949       |
| 500 3%                 |                | [-1.3591, 1.3571] | 1.3576       | [-1.3632, 1.3612] | 1.3617       |
| 500 5%                 |                | [-1.7528, 1.7507] | 1.7511       | [-1.7547, 1.7526] | 1.7530       |
| 1000 0%                |                | [-0.3126, 0.3105] | 0.3115       | [-0.4174, 0.4153] | 0.4162       |
| 1000 3%                |                | [-1.3582, 1.3561] | 1.3567       | [-1.3602, 1.3582] | 1.3587       |
| 1000 5%                |                | [-1.7523, 1.7503] | 1.7507       | [-1.7533, 1.7512] | 1.7516       |

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TABLE III: Sensitivities on the AMDM $\hat{a}_V$ and the EDM $\hat{a}_A$ of the top-quark through the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he.

| $\mathcal{L} (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
|-------------------------|---------------|-------------|-------------|-------------|-------------|
| $\sqrt{s} = 10$ TeV, $P_{e^-} = 0\%$, 95% C.L. | **Hadronic** | | | **Leptonic** | |
| 50 | 0% | [-0.4824, 0.4719] | 0.4777 | [-0.6428, 0.6324] | 0.6384 |
| 50 | 3% | [-1.1429, 1.1326] | 1.1391 | [-1.1618, 1.1514] | 1.1579 |
| 50 | 5% | [-1.4667, 1.4563] | 1.4632 | [-1.4757, 1.4653] | 1.4722 |
| 100 | 0% | [-0.4065, 0.3961] | 0.4017 | [-0.5414, 0.5309] | 0.5368 |
| 100 | 3% | [-1.1386, 1.1281] | 1.1347 | [-1.1482, 1.1378] | 1.1443 |
| 100 | 5% | [-1.4647, 1.4542] | 1.4612 | [-1.4692, 1.4588] | 1.4657 |
| 300 | 0% | [-0.3101, 0.2997] | 0.3052 | [-0.4126, 0.4022] | 0.4079 |
| 300 | 3% | [-1.1356, 1.1252] | 1.1317 | [-1.1388, 1.1284] | 1.1349 |
| 300 | 5% | [-1.4633, 1.4529] | 1.4598 | [-1.4648, 1.4544] | 1.4613 |
| 500 | 0% | [-0.2735, 0.2631] | 0.2686 | [-0.3638, 0.3534] | 0.3589 |
| 500 | 3% | [-1.1349, 1.1246] | 1.1311 | [-1.1369, 1.1265] | 1.1330 |
| 500 | 5% | [-1.4629, 1.4526] | 1.4595 | [-1.4639, 1.4535] | 1.4604 |
| 1000 | 0% | [-0.2308, 0.2204] | 0.2259 | [-0.3067, 0.2963] | 0.3019 |
| 1000 | 3% | [-1.1345, 1.1241] | 1.1306 | [-1.1355, 1.1251] | 1.1316 |
| 1000 | 5% | [-1.4628, 1.4524] | 1.4593 | [-1.4632, 1.4528] | 1.4597 |

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TABLE IV: Sensitivities on the AMDM $\hat{a}_V$ and the EDM $\hat{a}_A$ of the top-quark through the process $e^- p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he.

\[
\sqrt{s} = 7.07 \text{ TeV}, \quad P_e = -80\%, \quad 95\% \text{ C.L.}
\]

| $\mathcal{L} (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
|---|---|---|---|---|---|
| 50 | 0% | [-0.5756, 0.5574] | 0.5696 | [-0.7694, 0.7512] | 0.7612 |
| 50 | 3% | [-1.3809, 1.3627] | 1.3685 | [-1.4029, 1.3848] | 1.3904 |
| 50 | 5% | [-1.7727, 1.7545] | 1.7582 | [-1.7832, 1.7651] | 1.7687 |
| 100 | 0% | [-0.4835, 0.4654] | 0.4789 | [-0.6469, 0.6288] | 0.6401 |
| 100 | 3% | [-1.3757, 1.3575] | 1.3633 | [-1.3869, 1.3688] | 1.3746 |
| 100 | 5% | [-1.7702, 1.7521] | 1.7558 | [-1.7756, 1.7574] | 1.7511 |
| 300 | 0% | [-0.3659, 0.3477] | 0.3639 | [-0.4910, 0.4729] | 0.4864 |
| 300 | 3% | [-1.3722, 1.3540] | 1.3599 | [-1.3760, 1.3579] | 1.3636 |
| 300 | 5% | [-1.7686, 1.7504] | 1.7541 | [-1.7704, 1.7522] | 1.7559 |
| 500 | 0% | [-0.3209, 0.3027] | 0.3203 | [-0.4316, 0.4134] | 0.4280 |
| 500 | 3% | [-1.3715, 1.3533] | 1.3592 | [-1.3738, 1.3556] | 1.3615 |
| 500 | 5% | [-1.7683, 1.7501] | 1.7538 | [-1.7693, 1.7512] | 1.7549 |
| 1000 | 0% | [-0.2678, 0.2496] | 0.2694 | [-0.3618, 0.3436] | 0.3599 |
| 1000 | 3% | [-1.3709, 1.3528] | 1.3586 | [-1.3721, 1.3539] | 1.3598 |
| 1000 | 5% | [-1.7680, 1.7499] | 1.7536 | [-1.7686, 1.7504] | 1.7541 |

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TABLE V: Sensitivities on the AMDM $\hat{a}_V$ and the EDM $\hat{a}_A$ of the top-quark through the process $e^-p \rightarrow e^-\bar{b} \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he.

| $\sqrt{s}$ = 10 TeV, $P_e = -80\%$, 95% C.L. | Hadronic | Leptonic |
|---------------------------------------------|----------|----------|
| $\mathcal{L} (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
| 50 | 0% | [-0.4210, 0.4027] | 0.4103 | [-0.5595, 0.5411] | 0.5482 |
| 50 | 3% | [-1.1423, 1.1239] | 1.1287 | [-1.1529, 1.1346] | 1.1394 |
| 50 | 5% | [-1.4679, 1.4496] | 1.4531 | [-1.4729, 1.4546] | 1.4581 |
| 100 | 0% | [-0.3556, 0.3372] | 0.3450 | [-0.4719, 0.4536] | 0.4609 |
| 100 | 3% | [-1.1399, 1.1215] | 1.1263 | [-1.1453, 1.1269] | 1.1317 |
| 100 | 5% | [-1.4668, 1.4484] | 1.4520 | [-1.4693, 1.4509] | 1.4545 |
| 300 | 0% | [-0.2724, 0.2541] | 0.2621 | [-0.3609, 0.3425] | 0.3503 |
| 300 | 3% | [-1.1382, 1.1198] | 1.1246 | [-1.1400, 1.1216] | 1.1264 |
| 300 | 5% | [-1.4660, 1.4477] | 1.4512 | [-1.4669, 1.4485] | 1.4520 |
| 500 | 0% | [-0.2409, 0.2226] | 0.2307 | [-0.3187, 0.3004] | 0.3083 |
| 500 | 3% | [-1.1379, 1.1195] | 1.1243 | [-1.1389, 1.1206] | 1.1254 |
| 500 | 5% | [-1.4659, 1.4475] | 1.4510 | [-1.4664, 1.4480] | 1.4515 |
| 1000 | 0% | [-0.2041, 0.1858] | 0.1939 | [-0.2695, 0.2512] | 0.2592 |
| 1000 | 3% | [-1.1376, 1.1192] | 1.1240 | [-1.1382, 1.1198] | 1.1246 |
| 1000 | 5% | [-1.4657, 1.4474] | 1.4509 | [-1.4660, 1.4476] | 1.4512 |

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TABLE VI: Sensitivities on the AMDM $\hat{a}_V$ and the EDM $\hat{a}_A$ of the top-quark through the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t}\nu_e \gamma$ at the FCC-he.

| $\sqrt{s}$ (fb$^{-1}$) | Hadronic L$(fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $\hat{a}_A$ | Leptonic $\hat{a}_V$ | $\hat{a}_A$ |
|-------------------------|----------------------|---------------|-------------|-------------|----------------|-------------|
| 50 0%                   | [-0.9861, 0.9838]    | 0.9842        | [-1.3174, 1.3150] | 1.3152 |
| 50 3%                   | [-1.4429, 1.4406]    | 1.4406        | [-1.5905, 1.5882] | 1.5881 |
| 50 5%                   | [-1.7936, 1.7915]    | 1.7913        | [-1.8772, 1.8749] | 1.8746 |
| 100 0%                  | [-0.8294, 0.8271]    | 0.8276        | [-1.1079, 1.1056] | 1.1059 |
| 100 3%                  | [-1.4019, 1.3996]    | 1.3997        | [-1.4874, 1.4851] | 1.4852 |
| 100 5%                  | [-1.7731, 1.7708]    | 1.7705        | [-1.8177, 1.8153] | 1.8151 |
| 300 0%                  | [-0.6305, 0.6282]    | 0.6289        | [-0.8421, 0.8398] | 0.8404 |
| 300 3%                  | [-1.3725, 1.3702]    | 1.3702        | [-1.4046, 1.4023] | 1.4024 |
| 300 5%                  | [-1.7588, 1.7565]    | 1.7563        | [-1.7744, 1.7721] | 1.7719 |
| 500 0%                  | [-0.5550, 0.5527]    | 0.5535        | [-0.7413, 0.7389] | 0.7396 |
| 500 3%                  | [-1.3663, 1.3640]    | 1.3641        | [-1.3861, 1.3838] | 1.3839 |
| 500 5%                  | [-1.7559, 1.7536]    | 1.7534        | [-1.7654, 1.7630] | 1.7628 |
| 1000 0%                 | [-0.4669, 0.4646]    | 0.4654        | [-0.6236, 0.6212] | 0.6219 |
| 1000 3%                 | [-1.3617, 1.3594]    | 1.3595        | [-1.3718, 1.3695] | 1.3696 |
| 1000 5%                 | [-1.7537, 1.7514]    | 1.7512        | [-1.7585, 1.7562] | 1.7559 |

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TABLE VII: Sensitivities on the AMDM $\hat{a}_V$ and the EDM $\hat{a}_A$ of the top-quark through the process $e^- p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$ at the FCC-he.

| $\sqrt{s}$ (fb$^{-1}$) | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
|-------------------------|----------------|------------|-------------|------------|-------------|
| 50 0%                   | [-0.7189, 0.7083] | 0.7129     | [-0.9589, 0.9482] | 0.9526     |
| 50 3%                   | [-1.1770, 1.1663] | 1.1703     | [-1.2567, 1.2460] | 1.2499     |
| 50 5%                   | [-1.4835, 1.4728] | 1.4765     | [-1.5256, 1.5149] | 1.5185     |
| 100 0%                  | [-0.6054, 0.5947] | 0.5995     | [-0.8072, 0.7965] | 0.8011     |
| 100 3%                  | [-1.1563, 1.1456] | 1.1497     | [-1.2003, 1.1896] | 1.1936     |
| 100 5%                  | [-1.4733, 1.4627] | 1.4663     | [-1.4953, 1.4846] | 1.4882     |
| 300 0%                  | [-0.4613, 0.4506] | 0.4555     | [-0.6146, 0.6039] | 0.6087     |
| 300 3%                  | [-1.1419, 1.1312] | 1.1352     | [-1.1576, 1.1469] | 1.1509     |
| 300 5%                  | [-1.4665, 1.4558] | 1.4594     | [-1.4739, 1.4633] | 1.4669     |
| 500 0%                  | [-0.4067, 0.3959] | 0.4009     | [-0.5416, 0.5308] | 0.5357     |
| 500 3%                  | [-1.1389, 1.1282] | 1.1323     | [-1.1485, 1.1378] | 1.1419     |
| 500 5%                  | [-1.4651, 1.4544] | 1.4581     | [-1.4696, 1.4589] | 1.4626     |
| 1000 0%                 | [-0.3428, 0.3321] | 0.3371     | [-0.4563, 0.4456] | 0.4505     |
| 1000 3%                 | [-1.1367, 1.1259] | 1.1300     | [-1.1415, 1.1309] | 1.1348     |
| 1000 5%                 | [-1.4640, 1.4533] | 1.4570     | [-1.4663, 1.4556] | 1.4593     |

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FIG. 1: A schematic diagram for the single top-quark production through charged currents at $e^-p$ colliders via the process $e^-p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$. 

FIG. 2: Feynman diagrams contributing single top-quark production through charge current at $e^-p$ collider via the subprocess $e^- b \rightarrow \bar{t} \nu_e \gamma$. 

(1) 

(2) 

(3) 

(4)
FIG. 3: The total cross sections of the process $e^- p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$ as a function of $\hat{a}_V$ for center-of-mass energies of $\sqrt{s} = 7.07 \, TeV$ at the FCC-he.

FIG. 4: The total cross sections of the process $e^- p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$ as a function of $\hat{a}_A$ for center-of-mass energies of $\sqrt{s} = 7.07 \, TeV$ at the FCC-he.
FIG. 5: Same as in Fig. 3, but for $\sqrt{s} = 10 \, \text{TeV}$.

FIG. 6: Same as in Fig. 4, but for $\sqrt{s} = 10 \, \text{TeV}$.
FIG. 7: The total cross sections of the process $e^- p \rightarrow e^- \bar{b} \rightarrow \bar{t} \nu_e \gamma$ as a function of $\hat{a}_V$ and $\hat{a}_A$ for center-of-mass energy of $\sqrt{s} = 7.07 \, TeV$ at the FCC-he.

FIG. 8: Same as in Fig. 7, but for center-of-mass energy of $\sqrt{s} = 10 \, TeV$. 


FIG. 9: Sensitivity contours at the 95% C.L. in the $\hat{a}_V - \hat{a}_A$ plane through the process $e^- p \rightarrow e^- b \rightarrow \bar{t} \nu_e \gamma$ for $\sqrt{s} = 7.07$ TeV at the FCC-he.

FIG. 10: Same as in Fig. 9, but for $\sqrt{s} = 10$ TeV.