Radiative heat transfer in plasma of pulsed high pressure caesium discharge

V F Lapshin
Department of Physics, St. Petersburg State Transport University, Moskovsky 9, St. Petersburg, 190031, Russia
E-mail: lapshinvf@mail.ru

Abstract. Two-temperature many component gas dynamic model is used for the analysis of features of radiative heat transfer in pulsed high pressure caesium discharge plasma. It is shown that at a sufficiently high pressure the radial optical thickness of arc column is close to unit ($\tau_R(\lambda) \sim 1$) in most part of spectrum. In this case radiative heat transfer has not local character. In these conditions the photons which are emitted in any point of plasma volume are absorbed in other point remote from an emission point on considerable distance. As a result, the most part of the electric energy put in the discharge mainly near its axis is almost instantly redistributed on all volume of discharge column. In such discharge radial profiles of temperature are smooth. In case of low pressure, when discharge plasma is optically transparent for its radiation in the most part of a spectrum ($\tau_R(\lambda) \ll 1$), the emission of radiation without reabsorption takes place. Radiative heat transfer in plasma has local character and profiles of temperature have considerable gradient.

1. Introduction
Pulsed high pressure caesium (HPC) discharge is of interest as an effective source of visible radiation of high power with a continuous spectrum [1-3]. The investigated discharge is realized in a sapphire tube with an internal radius $R \sim 2\div3$ mm and length $\sim 100$ mm. Temperature of walls has values $T_W \sim 1200\div1500$ K. In this work, the established mode is considered. In such mode the current pulse of given form $I(t)$ periodically (with a frequency $\nu \sim 1000$ Hz) is passed through the weakly ionized plasma which is cooling down after the previous current pulse or supported by the auxiliary discharge current $I_0 \sim 0,1\div1,0$ A. The duration of the current pulse has values $t_p \sim 0,1\nu^{-1}$ and amplitude $I_{max} \sim 10\div100$ A. The amount of caesium per unit length tubes $M_a$ is defined by its saturation pressure at the coldest part of walls. Results of theoretical and experimental study of pulsed HPC discharge are presented in works [4-9].

In the present work radiative heat transfer in plasma of pulsed HPC discharge is considered. The high pressure of plasma and, respectively, high speed of collisional processes in plasma, provide the local thermodynamic equilibrium (LTE) in discharge plasma. It means that Maxwell's distribution for electrons and Saha-Boltzmann equation take place with electron temperature. Let's note that temperatures of electrons and heavy particles can significantly differ. Existence of LTE significantly simplifies consideration of radiation transfer and allows to receive simple expressions for the
description of radiative heat transfer in axisymmetric plasma of HPC discharge. For research of HPC
discharge two-temperature many component gas dynamic model [5] is used.

2. Basic equations
In the absence of radiation scattering radiative transfer equation for the LTE plasma along the set
direction \( \hat{\Omega} \) takes the form:

\[
\frac{\partial}{\partial s} I_\lambda = k'_\lambda (I_{\lambda p} - I_\lambda)
\]

(1)

Here \( I_\lambda = I_\lambda (r, \hat{\Omega}) \) is the spectral intensity of radiation with wavelength \( \lambda \), \( r \) is the radial coordinate, \( s \) is the coordinate along a light beam, \( k'_\lambda \) is the absorption coefficient of caesium plasma [5], \( I_{\lambda p} = 2hc^2\lambda^{-5} \exp(hc/\lambda k_B T_e) - 1 \) is the Planck's black body intensity, \( T_e = T_e(r) \) is the electron temperature, \( \hat{\Omega}(\psi, \theta) \) is the unit length vector in the direction of radiation propagation (see figure 1).

![Figure 1. The plasma column geometry.](image)

The integrated solution of the equation (1) has the form:

\[
I_\lambda (s, \hat{\Omega}) = \int_{s_B}^{s} k'_\lambda (s') I_{\lambda p} (s') \exp \left(- \int_{s'}^{s} k'_\lambda (s'') ds'' \right) ds'.
\]

(2)

Here \( s_B \) is the coordinate of the point B lying on a plasma column surface (see figure 1). To carry out the integration, it is convenient to introduce a new variable \( l = (s_A - s') \cos \psi \). It allows to replace the integration along a beam \( \hat{\Omega} \) by integration along a projection of this beam to the plane which is perpendicular discharge axes:

\[
I_\lambda (r, \psi, \theta) = \int_{l_w}^{l} k'_\lambda (l') I_{\lambda p} \exp \left(- \int_{0}^{l'} k'_\lambda (l'') \frac{dl''}{\cos \psi} \right) \frac{dl'}{\cos \psi}.
\]

(3)

Here integration is carried out along a segment AC in the direction from point A to point C, \( r = OA, l_w = AC = AB \cos \psi, l_w = l_w (r, \theta) = r \cos \theta + (R^2 - r^2 \sin^2 \theta)^{1/2} \). Now main spectral characteristics of radiative heat transfer, such as the radial radiation flux \( F_\lambda (W/m^2) \), volume density of energy radiation \( U_\lambda (J/m^3) \) and volume power of energy losses on the radiation \( W_\lambda (W/m^3) \), can be written by means of the equation (3) in a form

\[
F_\lambda (r) = \int_{(4\pi)} (\Omega \cdot \hat{e}_r)I_\lambda d\Omega = 4 \int_{0}^{\pi} d\theta \cos \theta \int_{0}^{\pi/2} d\psi \cos \psi \int_{l_w}^{l} k'_\lambda I_{\lambda p} \exp \left(- \int_{0}^{l} k'_\lambda \frac{dl'}{\cos \psi} \right) dl'.
\]

(4)
\[ U_\lambda(r) = \frac{1}{c} \int I_\lambda d\Omega = \frac{4\pi}{c} \int_{0}^{\pi/2} \int_{0}^{l_w} k_\lambda I_{\lambda P} \exp \left( -\int_{0}^{l} \frac{dl'}{\cos \psi} \right) dl', \]  
\[ W_\lambda(r) = \frac{1}{r} \frac{\partial}{\partial r} (RF_\lambda) = ck'_\lambda (U_{\lambda P} - U_\lambda). \]  

Here \( \hat{e}_r \) is the unit length vector in radial direction, \( U_{\lambda P} = 4\pi J d\lambda / c \) is the equilibrium density of radiation energy. Let's note here that the right part of (6) is the difference between the emission and absorption at a given point. Full losses of energy \( W_{rad} \) on radiation can be calculated integrating equation (6) over the area of wavelengths 100 nm - 4000 nm where the radiation contributes significantly to energy balance of the discharge.

To demonstrate features of radiative heat transfer in HPC caesium discharge separate components of \( W_{rad} \) corresponding to losses of energy in different parts of a spectrum are calculated:

\[ W_{rad}(r) = \int W_\lambda d\lambda = W_1 + W_2 + W_{nonl} = \int W_\lambda d\lambda + \int W_{nonl} d\lambda. \]  

Here \( R \), \( \tau_R(\lambda) = k'_\lambda dr \) is the radial optical thickness of the plasma discharge column, values \( W_1, W_2, W_{nonl} \) designate energy losses on radiation in those parts of a spectrum for which \( \tau_R(\lambda) < 0.3 \), \( \tau_R(\lambda) > 3 \) and \( 0.3 < \tau_R(\lambda) < 3 \) respectively. According to the equation (6), the value \( W_{nonl} \) we will present in the form:

\[ W_{nonl} = W_{nonl}^{(em)} - W_{nonl}^{(abs)} = \int_{0.3<\tau_R(\lambda)<3} ck'_\lambda U_{\lambda P} d\lambda - \int_{0.3<\tau_R(\lambda)<3} ck'_\lambda U_\lambda d\lambda. \]  

Here \( W_{nonl}^{(em)} \) is the radiation energy emitted from plasma, and \( W_{nonl}^{(abs)} \) is the energy absorbed by unit of plasma volume in a unit of time in the spectral range, for which \( 0.3 < \tau_R(\lambda) < 3 \). \( W_{nonl}^{(abs)} \) is the energy transferred by radiation to this point of plasma from all other points of plasma volume. This value characterizes not local transfer of energy in discharge plasma. The energy absorbed in the spectral range \( 0.3 < \tau_R(\lambda) < 3 \) by unit of length of arc column in a unit of time, is defined by formulae

\[ P_{nonl}^{(abs)} = 2\pi \int_{0}^{R} r W_{nonl}^{(abs)} dr. \]  

The ratio of \( P_{nonl}^{(abs)} \) to electric power \( P_j = IE \) put per unit length of arc (\( E \) is the longitudinal electric field strength in plasma), characterizes a role of not local radiative heat transfer in the discharge.

3. Results of calculations and discussion

The analysis of features of radiative heat transfer in plasma of HPC discharge is carried out for two modes of operation: the mode A (\( M_a = 0.007 \) mg/cm, \( \nu = 1350 \) Hz, \( t_p = 35 \) \( \mu s, \) \( l_{max} = 70 \) A) and the mode B (\( M_a = 0.06 \) mg/cm, \( \nu = 1000 \) Hz, \( t_p = 62.5 \) \( \mu s, \) \( l_{max} = 120 \) A). Calculations are carried out for triangular shape of current pulse \( I(t) = I_0 + (I_{max} - I_0)t/t_p, \) \( 0 < t < t_p, \) at \( R = 2,5 \) mm, \( I_0 = 0,2 \) A. Results of calculations are given in figures 2-6 for \( t/t_p = 0.5 \) (it corresponds to the middle of current pulse). As seen in figure 2, in mode A discharge plasma is optically transparent for own radiation (\( \tau_R(\lambda) <<1 \)) and in mode B \( \tau_R(\lambda) \sim 1 \) in large part of spectrum.

Results of calculations of power of Joule heating \( \sigma E^2 \) and energy losses because of radiation (\( W_{rad} \)), electron heat conductivity (\( \text{div} q_e \)) and collisions with ions (\( Q_{ea} \)) and neutral atoms (\( Q_{ea} \)) are shown in...
Figure 2. Radial optical thickness of the plasma discharge column $\tau_R(\lambda)$: 1 - the mode A ($p(t) = 90$ Torr), 2 - the mode B ($p(t) = 680$ Torr).

Figure 3. Here $\sigma$ is the electrical conductivity of plasma, $Q_{eh} = \frac{3m_e}{m_a} n_e k_B (T_e - T_h)$, $\tau_{eh}^{-1}$ is the effective frequency of collisions of electrons with heavy particles, $q_e = -\lambda_e \frac{\partial T_e}{\partial r}$, $\lambda_e$ is the thermal conductivity of electrons, $n_e$ is the concentration of electrons, $m_e$ and $m_a$ are the mass of electrons and atoms of caesium, $T_h$ is the temperature of heavy particles. As shown in figure 3, electric energy is put in the discharge, mainly, to the hottest central region where electrical conductivity of plasma is maximum. At a high pressure (figure 3b) the most part of the electric energy enclosed in the discharge is radiated by plasma. On the periphery of an arc, in its coldest part, $W_{rad} < 0$.

Figure 3. The various mechanisms of energy exchange in plasma: 1 - $\sigma E^2$, 2 - $W_{rad}$, 3 - $Q_{eh}$, 4 - $Q_{ca}$, 5 - div$q_e$. Discharge parameters: (a) – mode A, (b) – mode B.
It means that in this area of discharge the plasma heats through absorption of radiation. Let's note that other mechanisms of loss of electron energy play a noticeable role only in cold area near the wall. At the relatively low pressure, when $\tau_R(\lambda) \ll 1$, loss of energy $W_{\text{rad}}$ (see figure 3a) is significantly less. In this case energy losses through collision mechanism and heat conductivity play significantly large role.

Losses of energy on radiation in various parts of a spectrum are compared in figure 4. As is seen from figure 4a, at the relatively low pressure (mode A) the main contribution to radiation losses of energy $W_{\text{rad}}$ introduce the quantity $W_1$ corresponding to emission without reabsorption. In this case heat transfer in plasma has local character. The part of energy absorbed in plasma due to not local heat transfer is small. The ratio $P_{\text{nonl}}^{(abs)} / P_J$ is equal 0.17 only. In this case, heating of plasma is non-uniform and temperature profiles have considerable gradients (see figure 5).

![Figure 4](image-url)

**Figure 4.** Radial distribution of energy losses to radiation in various parts of a spectrum: 1 – $W_{\text{rad}}$, 2 – $W_{\text{nonl}}$, 3 – $W_1$, 4 – $W_2$. Discharge parameters: (a) – mode A, (b) – mode B.

At a sufficiently high pressure (mode B) radiation losses of energy $W_{\text{nonl}}$ in spectral area for which $0.3 < \tau_R(\lambda) < 3$ has a dominant role (see figure 4b). As the length of photon free path in this spectral area is comparable with sizes of the plasma column, radiative heat transfer has in the mode B not local character. In this case, points, in which the photon is radiated and absorbed, may be retired one from the other. In the considered mode the ratio $P_{\text{nonl}}^{(abs)} / P_J$ is equal 0.76. Thus, the most part of the electric energy put in the discharge mainly near its axis thanks to not local radiation heat exchange is almost instantly redistributed on all volume of a discharge column. As a result, radial profiles of temperature are smooth. Formation of steep temperature fronts in these conditions is impossible.

The contribution in heating of plasma through reabsorption of radiation from spectral area for which $\tau_R(\lambda) \sim 1$ is shown in figure 6 for the mode B. As illustrated in figure 6, in external, colder part of an arc, reaches the maximum value of $W_{\text{nonl}}^{(abs)}$. It means that not local heat transfer plays the same role in heating of electrons here, as well as Joule heating.
4. Conclusion

In the present work calculation of radiative heat transfer in plasma pulsed high pressure caesium discharge is carried out. It is shown that at a sufficiently high pressure the radial optical thickness of an arc column $\tau_R(\lambda) \sim 1$ for considerable part of a spectrum. In this case not local radiation heat transfer takes place in the discharge. In these conditions the electric energy put in the discharge is almost instantly redistributed on all volume of plasma. In other cases, when $\tau_R(\lambda) << 1$ or $\tau_R(\lambda) >> 1$, radiation heat exchange has local character.

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