Research on automatic parking path optimization based on Ackerman model

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Abstract: In modern society, cars have been widely used. Every driver will face the problem of reversing. Experienced drivers can park the car to the designated position quickly and accurately. However, most drivers, especially some novices who have just obtained their driver's license, are very worried about the problem of parking. It is often difficult to meet both accuracy and speed. At this time, the automatic parking system appeared in front of the public. As one of the representative systems in the era of vehicle intelligence, the automatic parking system has become the main way for major automobile companies to show their strength in vehicle intelligence; In today's increasingly scarce parking spaces and shrinking parking space, automatic parking system has gradually become a "standard accessory" of vehicles and one of the main reference items for consumers to buy cars. Based on theory and practice, this paper analyzes the possible situations of automatic parking and solves the following problems to improve the automatic parking system. The acceleration model is solved by differential method. The unmanned vehicle travels along a straight line for 1.381m and reaches the maximum limit speed of 20km/h. When unmanned vehicle turns, it is necessary to establish the model of minimum curvature path and minimum path length to express the relationship between them. The initial position of the unmanned vehicle is at the intersection of the parking lot. Three parking situations will be considered, namely vertical parking space, parallel parking space and inclined parking space. A visual trajectory graph will be established to indicate the speed, path, acceleration and other parameters of each point, which vividly shows that the parking can be completed safely and quickly without conflict and collision.

Keywords: Ackerman steering model, Dijkstra algorithm, VISSIM model

1. Introduction

Automatic parking is one of the most landing scenes in automatic driving technology. For many drivers, in-line parking is a painful experience. The parking space in big cities is limited, and driving the car into a narrow space has become a necessary skill. It is rare to park a car without any trouble. Parking can lead to traffic jams, nerve fatigue and bumpers being bent. Fortunately, the development of technology provides a solution, which is the automatic parking function. Imagine finding an ideal parking place. You don't have to toss back and forth, but just start the button, sit down and relax, and everything else can be completed automatically. Automatic parking technology is also applicable to the active collision avoidance system, and finally realize the automatic driving of the vehicle. Generally speaking, in the future, parking lots and other infrastructure will be intelligent. In particular, after establishing a local road allocated computing cloud, it will communicate, interact and cooperate with all intelligent vehicles on the road. The future of automatic parking will involve collisions between many fields, which makes the intelligent background system and sports terminal start to play the whole intelligent future.

2. Study on the minimum turning radius of automobile

2.1. Based on the minimum turning radius model of automobile

The minimum turning radius refers to the radius of the track circle where the center of the outer wheel rolls on the support plane when the steering wheel turns to the limit position and the vehicle turns at the lowest stable speed. It represents the ability of the car to pass through narrow curved areas or bypass insurmountable obstacles. The smaller the turning radius, the better the mobility of the car. As shown in Figure 1: R is the minimum turning radius of the vehicle.
Figure 1: the driving track of the car when the steering wheel reaches the limit position to the left

Ackermann's principle was put forward by German vehicle Engineer lankensperger in 1817. It refers to that when the vehicle turns, each wheel rotates around the same center, so as to ensure that there is no sliding friction between the tire and the ground and is in the pure rolling state with the minimum friction. According to the geometric principle of Ackerman steering, as shown in Figure 2, when the four-wheel vehicle is steering, the deflection angle of the inner steering wheel is adjusted by using the characteristics of steering trapezoid and meeting the ideal steering condition equation (1) \( \beta \) Deflection angle of outer steering wheel \( \alpha \) It should be large, so that the center of the driving track of the four wheels can approximately intersect a certain instantaneous center point on the extension line of the rear axle of the vehicle, so as to ensure that the four wheels are pure rolling as far as possible, so as to reduce the wear of the tire.

\[
\cot \alpha = \cot \beta \pm \frac{B}{h_m}
\]

(1)

Ackerman steering principle is that under the assumption that the front wheel positioning angle of the vehicle is 0, the vehicle driving system is rigid and there is no lateral force during vehicle driving, the four wheels make pure rolling movement around the same circle center. The simplified steering principle of this model is shown in Figure 2.

Figure 2: Ackerman steering schematic

In Figure 2, O is the center of the steering circle, L is the vehicle wheelbase, and i is the distance from the wheel to the frame, \( \sigma \) Is the steering angle of the vehicle, \( R_0 \) is the turning radius of the center of mass, \( B \) is the wheelbase of the front (rear) axle, \( R_x \) is the distance from the steering center to the rear wheel, where \( B \) and \( L \) are vehicle parameters, which are set directly, \( \delta \) It is input by the driver and collected by the controller. \( \alpha \) Is the maximum rotation angle of the inner wheel, in degrees; \( \beta \) Is the maximum angle of...
the outer wheel, in degrees, calculated according to the input angle signal. $R$ is the minimum turning radius. According to the known parameters: the maximum steering wheel angle is $47^\circ$, and the transmission ratio of steering wheel to front wheel angle is $16:1$, so we can get $\delta = 29.375^\circ$.

$$\tan \delta = 0.5628$$ (2)

The outer wheel angle is $^{[2]}$

$$\alpha = \tan^{-1} \left( \frac{2 \cdot L \cdot \tan \sigma}{2 \cdot L + C \cdot \tan \sigma} \right) = 25.64^\circ$$ (3)

The inner wheel angle is $^{[2]}$

$$\beta = \tan^{-1} \left( \frac{2 \cdot L \cdot \tan \sigma}{2 \cdot L - C \cdot \tan \sigma} \right) = 34.12^\circ$$ (4)

According to the automobile design, the calculation formula of the minimum turning diameter is as follows$^{[1]}$, Calculated according to the outer wheel angle:

$$R_{\text{min}1} = \frac{L}{\sin \alpha} + l = 6.5209\text{m}$$ (5)

Calculated according to the angle of inner wheel:

$$R_{\text{min}2} = \sqrt{\left( \frac{L}{\sin \alpha} \right)^2 + K^2 + 2 \cdot \frac{L \cdot k}{\tan \beta}} + l = 4.8588\text{m}$$ (6)

Due to the steering system wheel angle of the real vehicle $\alpha$, $\beta$ It does not fully comply with Ackerman geometry, so that $r_{\text{min}1}$ and $r_{\text{min}2}$ are not completely consistent, and the actual minimum turning radius is the middle value.

$$R = \frac{R_{\text{min}1} + R_{\text{min}2}}{2} = 5.6894\text{m}$$ (7)

2.2. Based on vehicle driving curvature model

Definition of acceleration: it is the change rate or first derivative of acceleration vector to time. As shown in Figure 3, if the particle moves to point A at time $t$, its acceleration is $a_1$, and moves to point B at time $t + \Delta t$, its acceleration is $a_2$, then the average change rate of acceleration of the particle in the time interval of $\Delta t$ is

$$\text{Jerk} = a_2 - a_1 / \Delta t = \Delta a / \Delta t$$ (8)

![Figure 3: acceleration increment](image-url)
The average acceleration only provides the direction and average speed of acceleration change in a period of time, but it cannot accurately describe the change of acceleration direction and the details of fast and slow acceleration of particles in this period of time. Obviously, the shorter the observation time, the more accurately the average acceleration can reflect the acceleration change. As shown in Figure 4, applying the concept of limit, when $\Delta t \to 0$, then $\Delta a \to 0$, and the ratio $\Delta a / \Delta t$ will be infinitely close to a certain value. At the same time, it can be seen from the figure that the direction of $\Delta a / \Delta t$ is infinitely close to the tangent of the acceleration vector end point curve at time $t$. And point to the direction corresponding to the increase of $T$, so it is defined that the instantaneous acceleration of the particle at time $t$ is $f = \lim_{\Delta t \to 0} \frac{\Delta a}{\Delta t}$, that is, the instantaneous acceleration of the particle is equal to the change rate or first derivative of the acceleration vector to time. In Si, the unit of instantaneous acceleration is m / s$^3$.

![Figure 4: average acceleration](image)

When the maximum acceleration of the trolley is 20 m / s$^3$

$$V = \frac{dx}{dt}$$  \hspace{1cm} (9)

$$a = \frac{dv}{dt}$$  \hspace{1cm} (10)

$$\text{jerk} = \frac{da}{dt} = 20 \text{ m / s}^3$$  \hspace{1cm} (11)

Given that the acceleration jerk is equal to 20 m / s$^3$, calculate the acceleration $a = 20t$ m/s$^2$, and substitute equation 11 into equation 10 to obtain $v = 10t^2$ m / s. From the meaning of the question, the final speed is 20 km / h, and the units are required to be consistent. When converted into m/s is 5.6 m/s, obtain $t$ equal to 0.748 s, and substitute $t$ into equation 9 to obtain $x = 1.381$ m. Therefore, after walking 1.381 m, the speed of the trolley reaches 20 km / h.

2.3. Based on vehicle driving curvature model

The curvature of the curve is defined by differentiation according to the rotation rate of the tangent direction angle of a point on the curve to the arc length, indicating the degree of deviation of the curve from the straight line. A numerical value that mathematically indicates the degree of curvature of a curve at a point. The greater the curvature, the greater the degree of curvature of the curve. The reciprocal of curvature is the radius of curvature.

The curvature path refers to the bending degree of the route. The greater the curvature, the more curved, and the smaller the curvature, the more straight. The required curvature is limited to the minimum value of curvature. To require the minimum curvature, we have to find the shortest path first. The starting point of the path is that the smaller the curvature, the greater the maximum speed the vehicle can reach, and the shorter the path, the local quantity (curvature, distance) can be found first. Therefore, we need to find the shortest path length when turning. The method is as follows:
When the vehicle speed is 20km / h, the rotation of the steering wheel determines the rotation of the wheel, and then determines the driving route of the car. From the driving route of the car, we can determine the curvature of the driving path of the car. The limitation of the transformation between the curvature on the driving path of the car and the path length can be seen from Figure 5.

![Figure 5: limitation of curvature and path length transformation](image)

The road is segmented by taking vertical segments with equal intervals on the contour centerline. \( P \) is a point on the vertical segment and \( W \) is the width of the vertical segment. Then, the coordinates of the control points on the road are shown as the relative position on the vertical line segment. For example, for the \( i \)th line segment in the above figure, assuming that the upper end point of the line segment is \( P_{i,1} \) and the lower end point is \( P_{i,0} \), let \( W_i = (P_{i,1}) - (P_{i,0}) \), then any point \( P_i \) on the line segment can be expressed as:

\[
p_i = p_{i,0} + \alpha W_i = (x_{i,0} + \alpha (x_{i,1} - x_{i,0}), y_{i,0} + \alpha (y_{i,1} - y_{i,0})) = (x_{i,0} + \alpha x_{i,1}, y_{i,0} + \alpha y_{i,1})
\]

(12)

3. Research on optimal target parking space of unmanned vehicle

The initial location of the unmanned vehicle is the garage entrance. In order to identify the optimal target parking space in the parking lot and quickly arrive and park safely according to the target parking space, it is necessary to drive to the destination through unmanned driving, corresponding to the three free parking spaces in Figure 2.1 (No. 10 vertical parking space, No. 82 parallel parking space and No. 31 inclined parking space (the inclination angle is 45 degrees)), as shown in the figure. When the unmanned car drives to the parking space For convenience, take a particle to observe the driving track of the unmanned vehicle. Here, the track refers to the track of a specific point in the vehicle body, that is, the control point; The control point is determined in advance. Therefore, the control point is located on the center of the rear axle of the unmanned vehicle, the position of the control point will coincide with the track point, and the speed direction at the control point will be consistent with the direction of the track point.

![Figure 6: plan of parking lot](image)
3.1. mathematical model of unmanned vehicle driving to parking space 10

The initial position coordinates of the unmanned vehicle are unknown. The vehicle width is known to be 1.8m. Without touching the walls on both sides and pressing any parking line, the turning track of the vehicle's control point is the blue curve (A9-Q9) in Figure 7.

![Figure 7: control point trajectory 1](image)

After the unmanned vehicle turns right, the control point reaches Q_9 (9.59, 8.75). The unmanned vehicle drives along the straight line L, takes the end point of the parking line on the right side of garage 10 as the center P_{13} and half the vehicle width as the radius as the circle, randomly find a point Z_9 on the plane, take Z_9 as the center and 5m as the radius, and the circle intersects with the straight line L at D_{10}. And make circle p12 inscribed on circle Z_{9}, and drive the control point along line L to point D_{10} (33., 8.75), as shown in Figure 8. At this time, point D_{10} is the starting point for reversing into parking space 10 perpendicular to the road. At point D_{10}, turn the steering wheel to the right to the maximum angle 4700 to start reversing. When the control point reaches point A_{10}, the body of the unmanned vehicle is parallel to the parking line. At this time, return to the steering wheel, keep still, and continue to reverse until the body is completely poured into the parking space, as shown in Figure 9.

![Figure 8: after turning right, the trolley travels along a straight line](image)

![Figure 9: reverse warehousing](image)
The above process is the visual diagram of the trajectory of the unmanned vehicle control point from the initial position (parking lot entrance) to parking space 10. The motion trajectory of the control point can be divided into the following three stages:

Stage 1: the control point starts from the initial position A9 and turns to Q9 to the right. When the control point drives to Q9, the vehicle body is parallel to the vertical parking line for the next stage (A9-Q9).

Data: path length: 7.85m, speed: 2.8m/s, acceleration: 0m/s², angular velocity: 0.56rad/s, angular acceleration: 0rad/s², and the front direction faces the tangent direction of the path.

Stage 2: the control point starts from Q9 and drives forward along the straight line L. when it reaches D9 (i.e. the initial position of the next stage), it starts to reverse to parking space 10 (Q9-D9).

After analysis, the acceleration stage is divided into a. variable acceleration stage (acceleration increase), B. uniform acceleration stage, C. variable acceleration stage (acceleration decrease), D. uniform speed stage and E. deceleration stage.

4. Conclusion

In this paper, the parameters of passenger car with Ackerman structure are studied. The body is 4.9m long and 1.8m wide; The wheelbase of the car is 2.8m and the wheel spacing is 1.7m; The maximum accelerator acceleration is 3.0m/s², the limit maximum deceleration is -6.0m/s², the acceleration does not exceed 20.0m/s², the maximum steering wheel angle is 470 °, the transmission ratio between the steering wheel and the front wheel angle is 16:1, and the maximum steering wheel speed is 400m / s. The minimum turning radius of the vehicle is calculated according to the model parameters of the unmanned vehicle; If the maximum acceleration of the vehicle is limited to 20m /s², what is the shortest distance for the unmanned vehicle to accelerate to the maximum limited speed of 20km / h when driving along a straight line When the speed is 20km / h, what are the restrictions on the change rate of curvature on the path relative to the length of the path when the vehicle turns from a straight line.

As the initial position of the unmanned vehicle is the garage entrance, this paper establishes the mathematical model of unmanned parking, and draws the track from the initial position to the designated parking position (the parking space marked with red no parking in the figure has been occupied). The track includes the driving path length, vehicle orientation, speed, acceleration, acceleration, angular velocity, angular acceleration, etc. of the unmanned vehicle at each time, and gives the visual track diagram, It is required that there shall be no conflict or collision with other vehicles during parking. Three different parking spaces shall be considered respectively, namely (No. 10 vertical parking space, No. 82 parallel parking space and No. 31 inclined parking space (the inclination angle is 45 degrees)).

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