Graphene transparency in weak magnetic fields

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Abstract

We carry out an explicit calculation of the vacuum polarization tensor for an effective low-energy model of monolayer graphene in the presence of a weak magnetic field of intensity \( B \) perpendicularly aligned to the membrane. By expanding the quasiparticle propagator in the Schwinger proper time representation up to order \( (eB)^2 \), where \( e \) is the unit charge, we find an explicitly transverse tensor, consistent with gauge invariance. Furthermore, assuming that graphene is irradiated with monochromatic light of frequency \( \omega \) along the external field direction, from the modified Maxwell equations we derive the intensity of transmitted light. Corrections to this quantity, both calculated and measured, are of the order of \( (eB)^2/\omega^4 \). Our findings generalize and complement previously known results reported in the literature regarding the light absorption problem in graphene from the experimental and theoretical points of view, with and without external magnetic fields.

Keywords: graphene, magnetic fields, vacuum polarization

(Some figures may appear in colour only in the online journal)
1. Introduction

A decade has gone by since the groundbreaking experiments performed by Andrei Geim and Konstantin Novoselov [1] (Nobel Laureates in Physics in 2010) to isolate single layer membranes of graphite, graphene. Soon afterwards, theoretical [2] and experimental [3] groups highlighted the properties of charge carriers in this material, which strongly resemble ultrarelativistic electrons, thus establishing a bridge between solid state and particle physics (see, for instance, [4, 5]). Graphene has given rise to the new era of Dirac materials, with potential applications in nanotechnology, but also offering an opportunity to test the core of fundamental physics in a condensed matter environment. Mechanical, thermal and electronic properties of this two-dimensional crystal locate it among the best candidates to replace silicon in nanotechnological devices, basically due to its hardness, yet flexibility, high electron mobility and thermal conductivity [6].

The crystal structure of graphene consists of a honeycomb array of tightly packed carbon atoms, thus allowing an accurate tight-binding description. At low energies, such a description becomes in the continuous limit the Lagrangian of massless quantum electrodynamics in (2 + 1) dimensions, QED$_3$, for the charge carriers restricted to move along the membrane [4], but in which the ‘photon’ is allowed to move throughout space in such a way that the static Coulomb interaction is still described by a potential that varies as the inverse of the distance on the plane of motion of electrons. In this form, the low-energy dynamics of graphene is in accordance with the spirit of brane-world scenarios of fundamental interactions (see, for instance, [7]), where the gauge field (photon) is allowed to move throughout the bulk (full space), but matter fields are restricted to a brane (the graphene layer).

As expected, quantum field theoretical methods have been developed to describe phenomena in graphene which have been theorized in the high energy physics realm, but that would appear enhanced in this material due to the ratio of the speed of light in vacuum and the Fermi velocity of its charge carriers, $c/v_F \approx 300$. Theoretical objects such as the effective action in external electromagnetic fields have been calculated by several authors in connection with the Schwinger mechanism for pair production and the issue of minimal conductivity [8], ideas that have been generalized to the multilayer case [9]. Other ‘relativistic’ effects discussed in the literature include the Klein paradox [10], Casimir effect [11] and dynamical formation of a mass gap from excitonic condensates [12]. Graphene properties have been handled also from the perspective of non-commutative quantum mechanics [13].

A remarkable feature of graphene is the visual transparency of the membranes. Its opacity has been measured [14] to be roughly 2.3%, with almost negligible reflectance. This observation has opened the possibility of using single layers of this crystal in combination with biomaterials to produce clean hydrogen by photocatalysis [15] with visible light. The problem of light absorption in graphene can be addressed with quantum field theoretical methods [16]. Several authors have considered the Dirac picture for its charge carriers in terms of the degrees of freedom of QED$_3$ under different assumptions. Parity violating effects were considered in [17], whereas the influence of a strong magnetic field was considered in [18] in connection with the Faraday effect. Measurements of magneto-optical properties of epitaxial graphene have been reported in [19], in particular the polarization rotation and light absorption. Quantum Faraday and Kerr rotations have also been experimentally determined [20], and a full framework based on the equations of motion was presented in [21] to describe such effects. All the above mentioned results seem to be in accordance with the ‘relativistic’ behavior of charge carriers for a range of values of the external magnetic field intensity between 0.5 and 7 T [18]. For discussion of these results, the structure of the vacuum polarization tensor is the cornerstone. This operator has been calculated by several authors in
the presence of a strong magnetic field perpendicularly aligned with the graphene membrane [22]. In this work, we continue the discussion, but in our considerations the external magnetic field is weak in intensity. The article is organized as follows. We start modeling the low-energy behavior of graphene from the massless QED$_3$ subjected to an external magnetic field perpendicular to the membrane; namely, we consider the full space, but restrict the dynamics of charge carriers in graphene to an infinite plane where the third spatial component is set to zero. Expanding the quasiparticle propagator in the weak field regime, we calculate the vacuum polarization tensor to the leading order in the external field intensity in section 2. In section 3, we introduce the polarization operator in the modified Maxwell equation to describe the propagation of electromagnetic waves in space. From the matching conditions we calculate the transmission coefficient, and from there the intensity of transmitted light. We discuss our findings and conclude in section 4. Some details of the calculation of the polarization tensor are presented in an appendix.

2. A continuous model for graphene

The tight-binding approach to the description of monolayer graphene corresponds in the continuum to a massless version of quantum electrodynamics in (2 + 1) dimensions, but with a static Coulomb interaction which varies as the inverse of the distance, just as in ordinary space [4]. We adopt the conventions of [16–18] and consider an infinite graphene membrane immersed in a (3 + 1)-dimensional space oriented along the plane $z = 0$. The action for this model is expressed as

$$S = -\frac{1}{4} \int \! d^4xF_{\mu\nu}^2 + \int \! d^3x \vec{\mathcal{P}} \vec{\mathcal{A}},$$

(1)

with $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ and $\mathcal{P} = i\hat{p}^\mu(\partial_{\mu} + ieA_{\mu})$. In our considerations, circumflexed Greek indices $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ and so on take the values 0, 1, 2, 3; Greek indices $\mu, \nu, \gamma$, etcetera run from 0 to 2; and Latin indices $a, b$ and so on—which label the spatial coordinates on the graphene layer—take the values 1 and 2. Moreover, the re-scaled Dirac matrices are such that $\hat{\gamma}^0 = \gamma^0$, $\hat{\gamma}^{1,2} = v_F \gamma^{1,2}$, and for later convenience we also consider the matrix $\hat{\gamma}^3 = \gamma^3$, where $v_F$ is the Fermi velocity of quasiparticles in the crystal. In the natural units of the system (namely, when $v_F = 1$), the form of the action has been dubbed reduced QED and has been proposed in the context of brane-world scenarios [7].

Measuring the response of graphene to external electromagnetic fields amounts to calculating the effective action, which in turn is expressed through the vacuum polarization tensor $\Pi^{\hat{\mu}\hat{\nu}}$. Because in this case the dynamics of fermions is restricted to a plane, according to figure 1 we can express

$$\Pi^{\hat{\mu}\hat{\nu}}(p) = ie^2 \text{Tr} \left[ \int_{-\infty}^{\infty}dk\delta(k_z) \int \! \frac{d^3k}{(2\pi)^3} \hat{\gamma}^\mu S(k)\hat{\gamma}^\nu S(k + p) \right],$$

(2)

where the trace is over full space and then we set $\Pi^{\hat{\beta}\hat{\beta}} = \Pi^{\hat{\beta}\hat{0}} = 0$. Here, $S(p)$ represents the quasiparticle propagator (electric charge $-e$) and the double fermion line in the diagram specifies that the propagator is corrected by some classical external field. We consider the situation in which a uniform magnetic field is aligned perpendicularly to the graphene membrane. We think of this field as being weak in intensity. An estimate of the strength of such fields (see below) is of the order of millitesla. We expand the Schwinger propagator in the proper time representation [23],
\[ S(p) = \int_0^{\infty} ds \, e^{i \mu \left( n^2 + s \frac{\tan(eBs)}{n} + i \omega \right)} \times \left[ \gamma \cdot p_\parallel \left( 1 + \gamma^2 \tan(eBs) \right) + \gamma \cdot p_\perp \left( 1 + \tan^2(eBs) \right) \right], \tag{3} \]

in powers of \((eB)\), retaining terms up to order \(\lesssim eB^2\). We adopt a prescription where we split the transverse and parallel components—with respect to the magnetic field direction—of an arbitrary vector \(v^\mu\) defined on the graphene membrane according to \(v^\mu = (v_\parallel, v_\perp)\) such that \(v^2 \geq 0\). Any reference to the third spatial component has been taken into account in the \(\delta(k_z)\) integration in equation (2) and does not appear in what follows. Therefore, \(\tilde{\gamma} \cdot v = \tilde{\gamma} \cdot v_\parallel + \tilde{\gamma} \cdot v_\perp\) and \(v^2 = v_\parallel^2 + v_\perp^2\). Furthermore, for the relevant coordinates we take \(g^{\mu\nu} = \text{diag}(1, -1, -1) \equiv g_0^{\mu\nu} + g_1^{\mu\nu}\), such that \(g_0^{\mu\nu} = \text{diag}(1, 0, 0)\). Thus, in the weak field limit, the structure of the quasiparticle propagator becomes \([24]\)

\[ S(p) = S_0(p) + eBS_1(p) + (eB)^2S_2(p) \]

\[ \equiv \frac{\tilde{\gamma} \cdot v}{p^2} + i eB \frac{\tilde{\gamma} \cdot p_\parallel \gamma^2}{(p^2)^2} + \frac{2(eB)^2}{(p^2)^4} \left[ p_\perp^2 \tilde{\gamma} \cdot p_\parallel - p_\parallel^2 \tilde{\gamma} \cdot p_\perp \right]. \tag{4} \]

Here, the matrices \(\gamma^1\) and \(\gamma^2\) do not appear rescaled because the operators \(\mathcal{O}^\pm = (I \pm \gamma^2)/2\), with \(I\) the identity matrix, correspond to the (pseudo)spin projection operators \([24]\). With the above expansion (4), it is straightforward to verify that the structure of the vacuum polarization is

\[ \Pi^{\hat{\alpha}\hat{\beta}}(p) = \eta^{\hat{\alpha}} \left[ \Pi_0^{\hat{\alpha}\hat{\beta}}(p) + (eB)^2 \Pi_2^{\hat{\alpha}\hat{\beta}}(p) \right] \eta^{\hat{\beta}}, \tag{5} \]

where we have defined \(\eta^{\hat{\alpha}} = (1, v_\parallel, v_\perp, 1)\). Notice that the tensors \(\Pi_0^{\hat{\alpha}\hat{\beta}}\) and \(\Pi_2^{\hat{\alpha}\hat{\beta}}\) have the block matrix structure

\[ \Pi_0 = \begin{pmatrix} \Pi & 0 \\ 0 & 0 \end{pmatrix}, \tag{6} \]

where \(\Pi\) represent \(3 \times 3\) matrices corresponding to the dynamical coordinates of the quasiparticles. Explicitly, \(\Pi_0^{\hat{\alpha}\hat{\beta}}\) represents the polarization tensor in vacuum and \(\Pi_2^{\hat{\alpha}\hat{\beta}}\) stands for the quadratic order contribution to the polarization tensor. The linear correction in \((eB)\),

\[ \frac{\gamma \cdot p_\parallel \gamma^2}{(p^2)^2} \]

\[ \frac{2(eB)^2}{(p^2)^4} \left[ p_\perp^2 \tilde{\gamma} \cdot p_\parallel - p_\parallel^2 \tilde{\gamma} \cdot p_\perp \right] \]

\[ \equiv \frac{\tilde{\gamma} \cdot v}{p^2} + i eB \frac{\tilde{\gamma} \cdot p_\parallel \gamma^2}{(p^2)^2} + \frac{2(eB)^2}{(p^2)^4} \left[ p_\perp^2 \tilde{\gamma} \cdot p_\parallel - p_\parallel^2 \tilde{\gamma} \cdot p_\perp \right]. \tag{4} \]

Figure 1. Vacuum polarization diagram.

5 We emphasize that the Schwinger phase that accompanies the fermion propagator (3) in the proper time representation does not contribute to the vacuum polarization tensor, and thus we neglect it from the start.
\[ \Pi_{\alpha\beta}(0) \sim \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma^\alpha (\not{q} \cdot \not{k}) \gamma^\beta \gamma^2 (\not{q} \cdot (k + p)) \right] \]

\[ + \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma^\alpha \gamma^2 (\not{q} \cdot \not{k}) \gamma^\beta (\not{q} \cdot (k + p)) \right], \quad (7) \]

upon taking the traces, reduces to

\[ \Pi_{\alpha\beta}(0) \sim e^{\alpha\beta} \left[ \int \frac{d^4 k}{(2\pi)^4} \frac{k_y (k + p)_y}{k^2 (k + p)^2} \right] - \int \frac{d^4 k}{(2\pi)^4} \frac{k_y (k + p)_y}{k^2 (k + p)^2} \], \quad (8) \]

Using the identity

\[ \frac{1}{AB^q} = \frac{\Gamma(p + q)}{\Gamma(p)\Gamma(q)} \int_0^1 dx \frac{x^{p-1}(1-x)^{q-1}}{[Ax + B(1-x)]^{p+q}}, \quad (9) \]

followed by the change of variables \( k \rightarrow k - px \), it is straightforward to show that both the integrals in the squared bracket become identical and thus the correction of order \( eB \) vanishes, a consequence of our working assumption that parity is preserved. In other words, contributions to the polarization arising from a Chern–Simons term are not considered in this article.

The magnetic field independent vacuum polarization tensor \( \Pi_{\alpha\beta}(0) \) has been calculated by many authors \[16, 25\]. It is of the form

\[ \Pi_{\alpha\beta}(0) = 4\pi \alpha \Pi_{\text{vac}}(p) \left( g^{\alpha\beta} - \frac{\not{p} \gamma^\alpha}{p^2} \right), \quad (10) \]

with \( \alpha = e^2/4\pi \) and \( \alpha = e^2/(4\pi) \) as usual. Moreover, \( \not{p} \) is the magnitude of the momentum vector with components \( \not{p} = \eta^\alpha \not{p}_\alpha \), with the third spatial component set to zero, and the polarization scalar

\[ \Pi_{\text{vac}}(p) = \frac{i}{8} \not{p}. \quad (11) \]

This vacuum contribution is transverse, as demanded by gauge invariance.

On the other hand, the quadratic correction has two contributions,

\[ \Pi_{\alpha\beta}^{(2)} = \Pi_{\alpha\beta}^{(2)-11} + 2\Pi_{\alpha\beta}^{(2)-20} \]

\[ = \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma^\alpha S_1(k) \gamma^\beta S_1(k + p) \right] \]

\[ + 2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma^\alpha S_2(k) \gamma^\beta S_0(k + p) \right], \quad (12) \]

with a suggestive notation that the \( \Pi_{\alpha\beta}^{(2)-11} \) contribution comes from each of the quasiparticle propagators being dressed at the first order in the external field, whereas \( \Pi_{\alpha\beta}^{(2)-20} \) has one propagator without field, whereas the second one is dressed at order \( (eB)^2 \). The factor of 2 is a symmetry factor. Evaluation of these integrals is cumbersome, but straightforward. Our procedure was the following: we started by inserting the expansion in equation (4) into each of the contributions to the polarization tensor in equation (12). Then, with the aid of the
identity in equation (9), followed by the shift of variables \( k \rightarrow k - p \), taking the traces over full space and performing the remaining contractions, we obtain
\[
\Pi_{(2)}^{2g-20} = \frac{3i\alpha}{\pi^2} \epsilon^{ab} \left[ t_{13}^{(1)}(\tilde{p}) - \tilde{p}_\parallel^2 t_{03}^{(1)}(\tilde{p}) \right],
\]
\[
\Pi_{(21)}^{2g-20} = \frac{4i\alpha}{\pi^2} \left[ \left( \epsilon^{ab} - \epsilon^{\alpha\beta} \right) \left( t_{13}^{(1)}(\tilde{p}) + \tilde{p}_\parallel^2 t_{03}^{(1)}(\tilde{p}) \right) + \epsilon^{\alpha\beta} \left( t_{13}^{(1)}(\tilde{p}) + \tilde{p}_\parallel^2 t_{03}^{(1)}(\tilde{p}) \right) + \left( \tilde{p}_\parallel^a \tilde{p}_\parallel^\beta + \tilde{p}_\parallel^\alpha \tilde{p}_\parallel^\beta \right) \left( t_{13}^{(1)}(\tilde{p}) + \tilde{p}_\parallel^2 t_{03}^{(1)}(\tilde{p}) \right) \right].
\] (13)

where the master integral
\[
I_{\text{fin}}^{(2)}(\tilde{p}) = \int_0^1 x^4(1-x)^8 \int d^3k \left( \frac{k_0^2}{k^2 + \tilde{p}_\parallel^2 x(1-x)} \right)^n,
\]
\[
= (-1)^{m+n-r} \frac{i\pi}{(\tilde{p}^2)^{m+n-r-3/2}} B(n+1, r-n-1)
\]
\[
\times B \left( m + \frac{1}{2}, r - m - n - \frac{3}{2} \right)
\]
\[
\times B \left( f - r - m - n + \frac{5}{2}, g - r - m - n + \frac{5}{2} \right). \] (14)
is written in terms of beta functions \( B(x, y) \) and whose explicit evaluation is presented in the appendix. Making use of the master integral, the quadratic correction in the external field to the polarization tensor can be written as
\[
\Pi_{(2)}^{ab} = \Pi_0(\tilde{p}) \epsilon^{ab} + \Pi_\parallel(\tilde{p}) \epsilon^{\alpha\beta}, \] (15)
with the transverse tensors
\[
\epsilon^{ab} = \left( \epsilon^{\alpha\beta} - \frac{\tilde{p}_\parallel^a \tilde{p}_\parallel^\beta}{\tilde{p}_\parallel^2} \right), \quad \epsilon^{\alpha\beta} = \left( \epsilon^{ab} - \frac{\tilde{p}_\parallel^a \tilde{p}_\parallel^b}{\tilde{p}_\parallel^2} \right), \] (16)
and the polarization scalars
\[
\Pi_0 = \frac{i}{8\rho^3} \left( 1 - \frac{5\rho_\parallel^2}{\rho^2} \right), \quad \Pi_\parallel(\tilde{p}) = \frac{i}{4\rho^3} \left( 1 - \frac{\rho_\parallel^2}{\rho^2} \right). \] (17)

Thus, the final expression for \( \Pi^{ab} \) becomes
\[
\Pi^{ab}(p) = 4\kappa A \left[ \left( \Pi_{\text{vac}}(\tilde{p}) + (eB)^2 \Pi_0(\tilde{p}) \right) \epsilon^{ab} + (eB)^2 \Pi_\parallel(\tilde{p}) \epsilon^{\alpha\beta} \right]. \] (18)

The above result, equation (18), substituted back into equations (5) and (6) comprises the main result of this section and is the basis for our discussion below. Before proceeding, a few comments are in order.
3. Light absorption

From the action of our model, equation (1), we can describe the propagation of electromagnetic waves throughout space according to the modified Maxwell equations

$$\partial_\mu F^{\mu\nu} + \delta(z)\Pi^{\mu\nu}A_\mu = 0,$$

which fulfill the conditions

$$A_\mu|_{z=0^+} - A_\mu|_{z=0^-} = 0,$$

$$\left(\partial_z A_\mu\right)|_{z=0^+} - \left(\partial_z A_\mu\right)|_{z=0^-} = \Pi^{\mu\nu}_z A_\nu|_{z=0^-}.$$  (20)

Following [16–18], we interpret the delta function in equation (19) as a current along the graphene plane. Thus, from Ohm’s law,

$$j_\mu = \sigma_{ab}E_b.$$  (21)

Assuming a varying electric field with frequency $\omega$ expressed in a temporal gauge $A_0 = 0$, namely, $E_b = i\omega A_b$, and noticing, from the generalized Maxwell equations (19), that $j_\mu \approx \Pi_{ab}A_b$, we can identify the conductivity tensor as

$$\sigma_{ab} = \frac{\Pi_{ab}}{i\omega}.$$  (22)

From the symmetric character of the polarization tensor, the transverse conductivity vanishes identically. For the problem of light absorption, let us consider a plane wave of frequency $\omega$, which travels along the $z$-direction from below the graphene layer with a linear polarization along the $\hat{e}_x$ direction. Moreover, considering that the wave propagates perpendicularly to the graphene plane, the reflected and transmitted waves can be described as

$$A = e^{-i\omega t}\begin{cases}
\hat{e}_z e^{ikz} + (r_{xx} \hat{e}_x + r_{xy} \hat{e}_y) e^{-ikz}, & z < 0 \\
(t_{xx} \hat{e}_x + t_{xy} \hat{e}_y) e^{ikz}, & z > 0
\end{cases}$$  (23)

where $\hat{e}_{x,y}$ are the unit vectors along the directions $x$ and $y$ on the membrane. Thus, from the general form of the vacuum polarization tensor, the boundary conditions (20) simplify to

$$A_\mu|_{z=0^+} - A_\mu|_{z=0^-} = 0,$$

$$\left(\partial_z A_\mu\right)|_{z=0^+} - \left(\partial_z A_\mu\right)|_{z=0^-} = \alpha\Psi(\omega)\delta^{ab}A_b|_{z=0^-}.$$  (24)
where

\[ \Psi(\omega) = a \left[ \Pi_{\omega\omega}(\omega) + (eB)^2\Pi_b(\omega) \right]. \]  

(25)

Thus, the transmission coefficients can be straightforwardly obtained [16–18]

\[ t_{xx} = \frac{2\alpha}{\omega \Psi_N(\omega) + 2\alpha}, \quad t_{xy} = 0, \]  

(26)

with \( \Psi_N(\omega) = N\Psi(\omega) \), accounting for the degrees of freedom of charge carriers. Therefore, the intensity of transmitted light is

\[ I = |t_{\omega\omega}|^2 \simeq 1 + \frac{\alpha \text{Im} \Psi_N(\omega)}{\omega^2} + \mathcal{O}(\alpha^2). \]  

(27)

In terms of the conductivity tensor \( \sigma \), \( I \) can be expressed as

\[ I = 1 - \text{Re} \sigma_{xx} + \mathcal{O}(\alpha^2). \]  

(28)

Substituting the explicit form of the polarization scalars, we finally arrive at the main result of this article, namely,

\[ I = 1 - \alpha x \left( 1 + 4 \frac{(eB)^2}{\omega^4} \right), \]  

(29)

which is plotted in figure 2 as a function of the frequency of incident light \( \omega \) for several values of the external magnetic field. To have an idea of the scales involved, for visible light, within the estimate of 85–97% of transparency of graphene samples, the magnetic field intensities are of a few millitesla. Comparing with the measured universal absorption rate \( I \simeq \alpha x = 2.3\% \) [14], we conclude that, in the weak field limit, the intensity of transmitted light is corrected by factors \( (eB)^2/\omega^4 \), in consistency with the experimental and theoretical findings for these quantities in the absence of external fields as well as in the presence of a strong magnetic field [16, 19–22].
4. Final remarks

In this work, we have calculated the vacuum polarization tensor in a low energy effective model of graphene based on the massless QED$_3$. We have considered a uniform magnetic field aligned perpendicularly to the graphene membrane and expanded the charge carrier propagator in the weak field regime. The Passarino–Veltman types of integral involved in the calculation of the polarization operator were obtained after a lengthy, but straightforward, procedure from a single master integral that yields a transverse $\Pi^{\mu\nu}$, equation (18), in every order of expansion on the intensity of the external field. One part of this object is inherited from the form of the polarization tensor in vacuum and receives a leading correction of order $(eB)^2$, whereas the second part is transverse in the coordinates on the graphene membrane and vanishes in the absence of the field. Direct calculation does not always render a manifestly transverse polarization operator [29], for instance in ordinary QED. Spurious terms might arise as a consequence of a regularization procedure. Nevertheless, careful treatment of the regulators ensures that gauge invariance is preserved for arbitrary magnetic field strength. QED$_3$, being superrenormalizable, lacks UV-regularization issues. We have presented an alternative calculation to the standard representation of the polarization tensor as a double proper time integral [23, 26–28], which manifestly preserves gauge invariance.

As an application of the vacuum polarization tensor, we have estimated the light absorption in graphene. We observe a deviation of the form $(eB)^2/\omega^4$ as compared to the vacuum result for graphene opacity. Our findings are in agreement with previously reported theoretical calculations [16–18, 21] as well as the experimental light absorption of 2.3% per graphene membrane [14]. Further applications of the polarization tensor in weak magnetic fields presented here and the effective action derived from it are under scrutiny and will be presented elsewhere.

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Appendix

In this appendix, we compute the master integral in equation (14). For this purpose, we write

$$I_{\mu\nu}^{fg} = \int_0^1 x^f(1-x)^g J_{\mu\nu}(x; p),$$

(A.1)

with

$$J_{\mu\nu}(x; p) = \int d^3k \frac{k_{\mu}^2 m k_{\nu}^{2n}}{k^2 + p^2 x(1-x)}.$$ 

(A.2)

After Wick rotating to Euclidean space, writing $\pi_{\mu\nu} = \parallel \perp \perp \perp$, and with the aid of the identity
we immediately obtain

$$J_{\text{int}}(x; p) = (-1)^{m+n-r} \pi B((n + 1, r - n - 1)B\left( r + \frac{1}{2}, r - m - n - \frac{3}{2} \right)$$

$$\times \frac{1}{p^2 x(1 - x)^{-m-n-3/2}}. \quad (A.4)$$

Then, the remaining integral over $x$ in equation (A.1) can be performed from the definition of the beta function

$$B(x, y) = \int_0^1 dt \, t^{x-1}(1 - t)^{y-1}, \quad (A.5)$$

which finally lead us to the result (14).

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