Elementary particles in the early Universe

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Abstract. The high-temperature limit of the Standard Model generated by the contractions of gauge groups is discussed. Contraction parameters of gauge group SU(2) of the Electroweak Model and gauge group SU(3) of Quantum Chromodynamics are taken identical and tending to zero when the temperature increases. Properties of the elementary particles change drastically at the infinite temperature limit: all particles lose masses, all quarks are monochromatic. Electroweak interactions become long-range and are mediated by neutral currents. Particles of different kind do not interact. It looks like some stratification with only one sort of particles in each stratum. The Standard Model passes in this limit through several stages, which are distinguished by the powers of the contraction parameter. For any stage intermediate models are constructed and the exact expressions for the respective Lagrangians are presented. The developed approach describes the evolution of the Standard Model in the early Universe from the Big Bang up to the end of several nanoseconds.

Keywords: particle physics - cosmology connection, physics of the early universe

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1 Introduction

Modern theory of elementary particles known as the Standard Model (SM) consists of the Electroweak Model (EWM), which unified electromagnetic and weak interactions, as well as Quantum Chromodynamics (QCD), describing their strong interactions. The Standard Model gives a good description of the experimental data and has been recently confirmed by the discovery of Higgs boson at LHC. The Standard Model is a gauge theory with SU(3) × SU(2) × U(1) gauge group, which is a direct product of simple groups.

The non-relativistic limit changes not only the physics but also the corresponding theoretical tools. In particular, the Lorentz invariance group of the special relativity space-time is transformed to the Galilei group when a dimensional parameter — the velocity of light $c$ — tends to the infinity and a dimensionless parameter tends to zero $\frac{v}{c} \to 0$. E. Wigner and E. Inönü were the first who formalized this operation and called it group contraction [1]. The conception of contraction (or limit transition) has been extended to quantum groups, supergroups and other algebraic structures, including fundamental representations of the unitary groups [2]. For a symmetric physical system the contraction of its symmetry group means a transition to some limit state. In the case of a complicated physical system the investigation of its limit states under the limit values of some of its parameters enables to better understand the system behavior.

For the modified Electroweak Model with the contracted gauge group SU(2; $j$) × U(1) it was demonstrated [3, 4], that the contraction parameter is connected with the system energy, so its zero limit corresponds to the low-energy limit of the Electroweak Model. The alternative rescaling of the gauge group and the field space gives the high-temperature limit of the Electroweak Model.

In the broad sense of the word deformation is an operation inverse to contraction. The non-trivial deformation of some algebraic structure generally means its non-evident generalization. Quantum groups [5], which are simultaneously non-commutative and non-cocommutative Hopf algebras, present a good example of similar generalization since previously Hopf algebras with only one of these properties were known. But when the contraction of some mathematical or physical structure is performed one can reconstruct the initial structure by the deformation in the narrow sense moving back along the contraction way.

We use this method in order to re-establish the evolution of the elementary particles and their interactions in the early Universe. We base on the modern knowledge of the particle
world which is concentrated in the Standard Model. In this paper we investigate the high-
temperature limit of the Standard Model generated by the contraction of the gauge groups
SU(2) and SU(3). Similar very high temperatures can exist in the early Universe after infla-
tion and reheating on the first stages of the Hot Big Bang [6]. At these times the elementary
particles demonstrate rather unusual properties. It appears that the SM Lagrangian falls into
a number of terms which are distinguished by the powers of the contraction parameter \( \epsilon \rightarrow 0 \).
As far as the temperature in the hot Universe is connected with its age, then moving forward
in time, i.e. back to the high-temperature contraction, we conclude that after the Universe
creation the elementary particles and their interactions pass a number of stages in their evo-
lution from the infinite temperature state up to the SM state. These stages of quark-gluon
plasma formation and color symmetries restoration are distinguished by the powers of the
contraction parameter and consequently by the time of its creation. From the contraction
of the Standard Model we can classify the stages in time as earlier-later, but we can not
determine their absolute date. To estimate the absolute date we use additional assumptions.

2 High-temperature Lagrangian of EWM

The Electroweak Model is a gauge theory with the gauge group SU(2) \( \times U(1) \) acting in boson,
lepton and quark sectors [9, 10]. Correspondingly its Lagrangian is the sum of boson, lepton
and quark Lagrangians

\[
L = L_B + L_L + L_Q. \quad (2.1)
\]

Lagrangian \( L \) is taken to be invariant with respect to the action of the gauge group in the
space of fundamental representation \( C_2 \):

\[
\text{SU(2)} : \quad z'' = Gz', \quad \begin{pmatrix} z'_1' \\ z'_2' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1,
\]

\[
\text{U(1)} : \quad z'' = e^{i\omega/2}z = e^{i\omega Y}z', \quad \omega \in \mathbb{R}. \quad (2.2)
\]

Generator \( Y \) of the group U(1) is proportional to unit matrix \( Y = \frac{1}{2}1 \). Generators
of SU(2)

\[
T_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2} \tau_1, \quad T_2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{1}{2} \tau_2,
\]

\[
T_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2} \tau_3, \quad (2.3)
\]

where \( \tau_k \) are Pauli matrices are subject to commutation relations

\[
[T_1, T_2] = iT_3, \quad [T_3, T_1] = iT_2, \quad [T_2, T_3] = iT_1 \quad (2.4)
\]

and form Lie algebra \( \text{su}(2) \).

Boson sector \( L_B = L_A + L_\phi \) involves two parts: the gauge field Lagrangian

\[
L_A = \frac{1}{8g^2} \text{Tr}(F_{\mu\nu})^2 - \frac{1}{4} (B_{\mu\nu})^2 \\
= -\frac{1}{4} [(F_{\mu\nu}^1)^2 + (F_{\mu\nu}^2)^2 + (F_{\mu\nu}^3)^2] - \frac{1}{4} (B_{\mu\nu})^2 \quad (2.5)
\]
The new gauge fields \( Y = \frac{1}{2} (gA^3_\mu + g'B_\mu) \), \( A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g' A^3_\mu + g B_\mu) \), \( W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu) \)

are introduced instead of (2.8).
The quark Lagrangian is given by

$$L_Q = Q_l^\dagger i\tau_\mu D_\mu Q_l + u_r^\dagger i\tau_\mu D_\mu u_r + d_r^\dagger i\tau_\mu D_\mu d_r - h_d[d_r^\dagger(\phi^\dagger Q_l) + (Q_l^\dagger\phi)d_r] - h_u[u_r^\dagger(\tilde{\phi}^\dagger Q_l) + (Q_l^\dagger\tilde{\phi}) u_r],$$  

(2.15)

where the left quark fields form the SU(2)-doublet $Q_l = \begin{pmatrix} u_l \\ d_l \end{pmatrix} \in \mathbb{C}_2$, the right quark fields $u_r, d_r$ are the SU(2)-singlets, $\tilde{\phi}_i = \epsilon_{ik}\bar{\phi}_k$, $\epsilon_{00} = 1, \epsilon_{ii} = -1$ is the conjugate representation of SU(2) group and $h_u, h_d$ are constants. All fields $u_l, d_l, u_r, d_r$ are two component Lorentz spinors. The last four terms with the factors $h_d$ and $h_u$ specify the $d$- and $u$-quark mass. The covariant derivatives of the quark fields are given by

$$D_\mu Q_l = \left( \frac{\partial_\mu - ig}{2} \sum_{k=1}^{3} \tau_k A_{\mu k} - ig \frac{1}{6} B_{\mu} \right) Q_l,$$

$$D_\mu u_r = \left( \frac{\partial_\mu - ig}{2} B_{\mu} \right) u_r, \quad D_\mu d_r = \left( \frac{\partial_\mu + ig}{3} B_{\mu} \right) d_r.$$  

(2.16)

From the viewpoint of the electroweak interactions all known leptons and quarks are divided into three generations. The next two lepton generations are introduced in a way similar to (2.11). They are left SU(2)-doublets

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \in \mathbb{C}_2, \quad Y = -1$$  

(2.17)

and right SU(2)-singlets: $\mu_r, \tau_r, Y = -1$. In addition to $u$- and $d$-quarks of the first generation there is $(c, s)$ and $(t, b)$ quarks of the next generations, whose left fields

$$\begin{pmatrix} c_l \\ s_l \end{pmatrix} \in \mathbb{C}_2, \quad Y = 1$$  

(2.18)

are described by the SU(2)-doublets and the right fields are SU(2)-singlets: $c_r, t_r, Y = \frac{2}{3}$; $s_r, b_r, Y = -\frac{1}{3}$. Their Lagrangians are introduced in a way similar to (2.15). Full lepton and quark Lagrangians are obtained by the summation over all generations. In what follows we will often discuss only the first generations of leptons and quarks.

We consider a model where the contracted gauge group $SU(2; \epsilon) \times U(1)$ acts in the boson, lepton and quark sectors. The contracted group $SU(2; \epsilon)$ is defined by the consistent rescaling of the fundamental representation of the group SU(2) and the space $\mathbb{C}_2$

$$z'(\epsilon) = \begin{pmatrix} z_1' \\ \epsilon z_2' \end{pmatrix} = \begin{pmatrix} \alpha & \epsilon\beta \\ -\epsilon^* \beta & \alpha \end{pmatrix} \begin{pmatrix} z_1 \\ \epsilon z_2 \end{pmatrix} = u(\epsilon)z(\epsilon),$$

$$\det u(\epsilon) = |\alpha|^2 + \epsilon^2|\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1,$$  

(2.19)

when the real contraction parameter tends to zero $\epsilon \to 0$. The Hermite form

$$z^\dagger(\epsilon)z(\epsilon) = |z_1|^2 + \epsilon^2|z_2|^2$$  

(2.20)

remains invariant in the contraction limit.
The contracted group SU(2; $\epsilon = 0$) is isomorphic to Euclid group E(2). The space $C_2(\epsilon = 0)$ is a fiber space with one-dimensional base $\{z_1\}$ and one-dimensional fiber $\{z_2\}$. (A simple and the best known example of a fiber space is the non-relativistic space-time with one-dimensional base, which is interpreted as time, and three-dimensional fiber, which is interpreted as a proper space.) The action of the unitary group U(1) and the electromagnetic subgroup U(1)$_{em}$ in the space $C_2(\epsilon = 0)$ age given by the same matrices as in $C_2$.

The space $C_2(\epsilon)$ of the fundamental representation of SU(2; $\epsilon$) group can be obtained from $C_2$ by substitution of $z_2$ by $\epsilon z_2$. This substitution induces the other ones for Lie algebra generators $T_1 \rightarrow \epsilon T_1$, $T_2 \rightarrow \epsilon T_2$, $T_3 \rightarrow T_3$, with the new commutation relations

$$[T_1, T_2] = i\epsilon^2 T_3, \quad [T_3, T_1] = i T_2, \quad [T_2, T_3] = i T_1$$

(2.21)

for Lie algebra su(2; $\epsilon$). When $\epsilon \rightarrow 0$ the contracted algebra su(2; $\epsilon = 0$) has the structure of a semi-direct sum of Abelian translation subalgebra $t_2 = \{T_1, T_2\}$ and one-dimensional subalgebra u(1) = $\{T_3\}$.

As far as the gauge fields take their values in Lie algebra, we can substitute the gauge fields instead of transforming the generators, namely:

$$A^1_\mu \rightarrow \epsilon A^1_\mu, \quad A^2_\mu \rightarrow \epsilon A^2_\mu, \quad A^3_\mu \rightarrow A^3_\mu, \quad B_\mu \rightarrow B_\mu.$$  

(2.22)

Indeed, due to commutativity and associativity of multiplication by $\epsilon$ we have

$$su(2; \epsilon) \ni \{ A^1_\mu(\epsilon T_1) + A^2_\mu(\epsilon T_2) + A^3_\mu T_3 \}$$

$$= \{ (\epsilon A^1_\mu)T_1 + (\epsilon A^2_\mu)T_2 + A^3_\mu T_3 \}.$$  

(2.23)

For the standard gauge fields (2.14) these substitutions are as follows:

$$W^\pm_\mu \rightarrow \epsilon W^\pm_\mu, \quad Z_\mu \rightarrow Z_\mu, \quad A_\mu \rightarrow A_\mu.$$  

(2.24)

The left lepton $L_l = \left( \nu_l \atop e_l \right) \in C_2$, and quark $Q_l = \left( u_l \atop d_l \right) \in C_2$ fermionic fields are the SU(2)-doublets, so their components are transformed in the similar way as the components of the vector $z$, namely:

$$\nu_l \rightarrow \epsilon \nu_l, \quad e_l \rightarrow e_l, \quad u_l \rightarrow \epsilon u_l, \quad d_l \rightarrow d_l.$$  

(2.25)

The right lepton and quark fields are the SU(2)-singlets and therefore are not changed.

Alternative possible rescaling of the group SU(2; $j$) and the space $C_2(j)$ in the form

$$z'(j) = \left( \begin{array}{c} jz'_1 \\ jz'_2 \end{array} \right) = \left( \begin{array}{cc} \alpha & j\beta \\ -j\beta & \bar{\alpha} \end{array} \right) \left( \begin{array}{c} jz_1 \\ z_2 \end{array} \right) = u(j)z(j),$$

$$\det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1.$$  

(2.26)

in the contraction limit $j \rightarrow 0$ gives the same contracted group E(2), but a different fiber space $C_2(j = 0)$: now with the base $\{z_2\}$ and the fiber $\{z_1\}$. From the mathematical point of view it is not important if the first or the second Cartesian axis forms the base of splitting and in this sense the constructions (2.26) and (2.19) are equivalent. But the doublet components are interpreted as certain physical fields, therefore the fundamental representations (2.26) and (2.19) of the same contracted unitary group lead to different limit cases of the Electroweak Model.
In the second case (2.26) the complete Lagrangian is given by the sum

\[ L(j) = L_0 + j^2 L_f, \]

(2.27)

where \( L_0 \) is the limit Lagrangian and \( L_f \) is its tending to zero part. It was shown in [3, 4] that masses of all particles involved in the Electroweak Model remain the same under contraction \( j^2 \to 0 \). In this limit the contribution \( j^2 L_f \) of neutrino, \( W \)-boson and \( u \)-quark fields as well as their interactions with the other fields will be vanishingly small in comparison to the contribution of electron, \( d \)-quark and the remaining boson fields. So Lagrangian (2.27) describes very weak interaction of neutrino fields with the matter. On the other hand, the contribution of the neutrino part \( j^2 L_f \) to the complete Lagrangian rises when the parameter \( j^2 \) is increased, that again corresponds to the experimental facts. Therefore the contraction parameter can be phenomenologically connected with the neutrino energy and rescaling (2.26) corresponds to the low energy limit of the Electroweak Model. In this limit the first components of the lepton and quark doublets become infinitely small in comparison to their second components. On the contrary, when the energy increases the first components of the doublets become greater than their second ones. In the infinite energy (temperature) limit the second components of the lepton and quark doublets will be infinitely small as compared to their first components. This case is described by (2.19).

The next reason for inequality of the first and second doublet components is the special mechanism of spontaneous symmetry breaking, where small field excitations \( v + \chi(x) \) with respect to the second component of the vacuum vector (2.10) are regarded. So Higgs boson field \( \chi \) and constant \( v \) are multiplied by \( \epsilon \). As far as masses of all particles are proportionate to \( v \) we obtain the following transformation rule

\[ \chi \to \epsilon \chi, \quad v \to \epsilon v, \quad m_p \to \epsilon m_p, \quad p = \chi, W, Z, e, u, d. \]

(2.28)

After transformations (2.24), (2.25), (2.28) and spontaneous symmetry breaking the boson Lagrangian (2.5), (2.6) can be represented in the form

\[ L_B(\epsilon) = -\frac{1}{4} Z^2_{\mu\nu} - \frac{1}{4} F^2_{\mu\nu} + \epsilon^2 L_{B,2} + \epsilon^3 g W^+_{\mu} W^-_{\nu} \chi + \epsilon^4 L_{B,4}, \]

(2.29)

where

\[ L_{B,4} = \frac{1}{2} F_{\mu\nu}^2 \]

(2.30)

and

\[ L_{B,2} = \frac{1}{2} (\partial_{\mu} \chi)^2 + \frac{1}{2} m_Z^2 (Z_{\mu})^2 - \frac{1}{2} W^+_{\mu} W^-_{\mu} + \frac{g m_z}{2 \cos \theta_W} (Z_{\mu})^2 \chi + \frac{g^2}{8 \cos^2 \theta_W} (Z_{\mu})^2 \chi^2 - 2 i g (W^+_{\mu} W^-_{\nu} - W^-_{\mu} W^+_{\nu}) \left( \mathcal{F}_{\mu\nu} \sin \theta_W + Z_{\mu\nu} \cos \theta_W \right) - \frac{i}{2} e \left[ A_{\mu} (W^+_{\mu} W^-_{\nu} - W^-_{\mu} W^+_{\nu}) + i \epsilon A_{\nu} (W^+_{\mu} W^-_{\mu} - W^-_{\mu} W^+_{\mu}) \right] - \frac{i}{2} g \cos \theta_W \left[ Z_{\mu} (W^+_{\mu} W^-_{\nu} - W^-_{\mu} W^+_{\nu}) - Z_{\nu} (W^+_{\mu} W^-_{\mu} - W^-_{\mu} W^+_{\mu}) \right] - \frac{e^2}{4} \left\{ (W^+_{\mu})^2 + (W^-_{\mu})^2 \right\} (A_{\nu})^2 - 2 (W^+_{\mu} W^+_{\nu} + W^-_{\mu} W^-_{\nu}) A_{\mu} A_{\nu} + \left[ (W^+_{\mu})^2 + (W^-_{\mu})^2 \right] (A_{\mu})^2 - \frac{g^2}{4} \cos \theta_W \left\{ (W^+_{\mu})^2 + (W^-_{\mu})^2 \right\} (Z_{\mu})^2 - \frac{1}{4} (\partial_{\mu} \chi)^2 \]

(2.31)
\[ -2 \left( W^+ W^+ - W^- W^- \right) Z_{\mu} Z_{\nu} + \left[ (W^+)^2 + (W^-)^2 \right] (Z_{\mu})^2 \] -
\[ - \epsilon g \cos \theta_W \left[ W^+ W^- A_{\mu} Z_{\nu} + W^+ W^- A_{\mu} Z_{\nu} - \frac{1}{2} (W^+ W^- + W^+ W^-) (A_{\mu} Z_{\nu} + A_{\nu} Z_{\mu}) \right]. \] (2.31)

The lepton Lagrangian (2.11) in terms of electron and neutrino fields takes the form
\[ L_L(\epsilon) = L_{L,0} + \epsilon^2 L_{L,2} = \nu_l^1 i \tilde{\tau}_\mu \partial_\mu \nu_l + \epsilon \nu_l^1 i \tilde{\tau}_\mu \partial_\mu \nu_l + g' \sin \theta_w \epsilon \nu_l^1 \tilde{\tau}_\mu Z_{\nu} e_r - \] \[ g' \cos \theta_w \epsilon \nu_l^1 \tilde{\tau}_\mu A_{\mu} e_r + g \frac{2}{2 \cos \theta_w} \nu_l^1 \tilde{\tau}_\mu Z_{\nu} \nu_l + \] \[ + \frac{\epsilon^2}{\sqrt{2}} \left[ g \cos \theta_w \epsilon \nu_l^1 \tilde{\tau}_\mu W_{\mu}^+ e_l + \epsilon \nu_l^1 \tilde{\tau}_\mu W_{\mu}^- \nu_l \right]. \] (2.32)

The quark Lagrangian (2.15) in terms of u- and d-quarks fields can be written as
\[ L_Q(\epsilon) = L_{Q,0} - \epsilon m_u (u^1 \nu_l + u^4 \nu_r) + \epsilon^2 L_{Q,2}, \] (2.33)
where
\[ L_{Q,0} = d^1_l i \tilde{\tau}_\mu \partial_\mu d_r + u^1_l i \tilde{\tau}_\mu \partial_\mu u_r + u^4_l i \tilde{\tau}_\mu \partial_\mu u_r - \] \[ - \frac{1}{3} g' \cos \theta_w d^1_l \tilde{\tau}_\mu A_{\mu} d_r + \epsilon \nu_l^1 i \tilde{\tau}_\mu Z_{\nu} \partial_\mu \nu_l + \] \[ + \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u^1_l \tilde{\tau}_\mu Z_{\nu} \nu_l + \frac{2}{3} \epsilon \nu_l^1 \tilde{\tau}_\mu A_{\mu} u_r - \frac{2}{3} g' \sin \theta_w u^4_l \tilde{\tau}_\mu Z_{\nu} u_r, \] (2.34)
\[ L_{Q,2} = d^1_l i \tilde{\tau}_\mu \partial_\mu d_l - m_d (d^1_l d_l + d^4_l d_r) - \frac{\epsilon}{3} d^1_l \tilde{\tau}_\mu A_{\mu} d_l - \] \[ - \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) d^1_l \tilde{\tau}_\mu Z_{\nu} d_l + \frac{g}{\sqrt{2}} \left[ u^1_l \tilde{\tau}_\mu W_{\mu}^+ d_l + d^4_l \tilde{\tau}_\mu W_{\mu}^- u_l \right]. \] (2.35)

The complete Lagrangian of the modified model is given by the sum \( L(\epsilon) = L_B(\epsilon) + L_L(\epsilon) + L_Q(\epsilon) \) and can be written in the form
\[ L(\epsilon) = L_{\infty} + \epsilon L_1 + \epsilon^2 L_2 + \epsilon^3 L_3 + \epsilon^4 L_4. \] (2.36)

The contraction parameter is monotonous function \( \epsilon(T) \) of the temperature with the property \( \epsilon(T) \to 0 \) for \( T \to \infty \). Very high temperatures can exist in the early Universe just after its creation.

It is well known that to gain a better understanding of a physical system it is useful to investigate its properties for the limiting values of physical parameters. It follows from the decomposition (2.36) that there are five stages in the evolution of the Electroweak Model after the creation of the Universe which are distinguished by the powers of the contraction parameter \( \epsilon \). This offers an opportunity for construction of intermediate limit models. One can take the Lagrangian \( L_{\infty} \) for the initial limit system, then add \( L_1 \) and obtain the second
limit model with the Lagrangian $L_1 = L_\infty + L_1$. After that one can add $L_2$ and obtain the third limit model $L_2 = L_\infty + L_1 + L_2$. The last limit model has the Lagrangian $L_3 = L_\infty + L_1 + L_2 + L_3$.

In the infinite temperature limit ($\epsilon = 0$) Lagrangian (2.36) is equal to

$$L_\infty = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{4}F_{\mu\nu}^2 + \nu_l^\dagger i\tau_\mu \partial_\mu \nu_l + u_l^\dagger i\tau_\mu \partial_\mu u_l + e_r^\dagger i\tau_\mu \partial_\mu e_r + d_r^\dagger i\tau_\mu \partial_\mu d_r + u_l^\dagger i\tau_\mu \partial_\mu u_r + L_{\text{int}}^\dagger(A_\mu, Z_\mu),$$

(2.37)

where

$$L_{\text{int}}^\dagger(A_\mu, Z_\mu) = -\frac{g}{2\cos\theta_w} \nu_l^\dagger \tau_\mu Z_\mu \nu_l + \frac{2e}{3} u_l^\dagger \tau_\mu A_\mu u_l + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r +$$

$$+ \frac{g}{\cos\theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \tau_\mu Z_\mu u_l - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r -$$

$$- \frac{1}{3} g' \cos \theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin \theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \frac{2}{3} g' \cos \theta_w u_l^\dagger \tau_\mu A_\mu u_r -$$

$$- \frac{2}{3} g' \sin \theta_w u_l^\dagger \tau_\mu Z_\mu u_r.$$ 

(2.38)

We can conclude that the limit model includes only massless particles: photons $A_\mu$ and neutral bosons $Z_\mu$, left quarks $u_l$ and neutrinos $\nu_l$, right electrons $e_r$ and quarks $u_r, d_r$. This phenomenon has a simple physical explanation: the temperature is so high, that the particle mass becomes a negligible quantity as compared to its kinetic energy. The electroweak interactions become long-range because they are mediated by the massless neutral $Z$-bosons and photons. Let us note that $W_{\mu \pm}$-boson fields, which correspond to the translation subgroup of Euclid group $E(2)$, are absent in the limit Lagrangian $L_\infty$ (2.37).

Similar high energies can exist in the early Universe after inflation and reheating on the first stages of the Hot Big Bang [6, 11] in the pre-electroweak epoch. However the Universe evolution is more interesting and the limit Lagrangian $L_\infty$ can be considered as a good approximation after the Big Bang just as the nonrelativistic mechanics is a good approximation of the relativistic one at low velocities.

From the explicit form of the interaction part $L_{\text{int}}^\dagger(A_\mu, Z_\mu)$ it follows that there are no interactions between particles of different kind, for example neutrinos interact only with each other by neutral currents. All other particles are charged and interact with particles of the same sort by massless $Z_\mu$-bosons and photons. Particles of different kind do not interact. It looks like some stratification of the Electroweak Model with only one sort of particles in each stratum.

From the contraction of the Electroweak Model we can classify events in time as earlier-later, but we can not determine their absolute time without additional assumptions. At the level of classical gauge fields we can already conclude that the $u$-quark first restores its mass in the evolution of the Universe. Indeed the mass term of the $u$-quark in the Lagrangian (2.36) $L_1 = -m_u (u_l^\dagger u_l + u_r^\dagger u_r)$ is proportional to the first power $\epsilon$, whereas the mass terms of $Z$-boson, electron and $d$-quark are multiplied by the second power of the contraction parameter

$$\epsilon^2 \left[ \frac{1}{2} m_Z^2 (Z_\mu)^2 + m_e (e_r^\dagger e_r) + m_d (d_r^\dagger d_r) \right].$$ 

(2.39)

At the same time massless Higgs boson $\chi$ and charged $W$-boson appear. They restore their masses after all other particles of the Electroweak Model because their mass terms are multiplied by $\epsilon^4$. 

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The electroweak interactions between the elementary particles are restored mainly in the epoch which corresponds to the second order of the contraction parameter. There is one term in Lagrangian (2.29) $L_3 = g W_\mu^+ W_\mu^- \chi$ proportionate to $\epsilon^3$. The final reconstruction of the electroweak interactions takes place at the last stage ($\approx \epsilon^4$) together with the restoration of mass of all particles.

Two other generations of leptons and quarks develop in a similar way: for the infinite energy there are only massless right $\mu$- and $\tau$-muons, left $\mu$- and $\tau$-neutrinos, as well as massless left and right quarks $c_l, c_r, s_r, t_l, t_r, b_r$, $c$- and $t$-quarks first acquire their mass and after that $\mu$-, $\tau$-muons, $s$, $b$-quarks become massive.

3 QCD with contracted gauge group

Strong interactions of quarks are described by QCD. Like the Electroweak Model QCD is a gauge theory based on the local color degrees of freedom [12]. The QCD gauge group is SU(3), acting in three dimensional complex space $\mathbb{C}_3$ of color quark states. The SU(3) gauge bosons are called gluons. There are eight gluons in total, which are the force carrier of the theory between quarks. The QCD Lagrangian is taken in the form

$$\mathcal{L} = \sum_q \bar{q}^i (i \gamma^\mu) (D_\mu)_{ij} q^j - \frac{1}{4} \sum_{\alpha=1}^{8} F^\alpha_{\mu\nu} F^{\mu\nu\alpha},$$  (3.1)

where $D_\mu q$ are covariant derivatives of quark fields $q = u, d, s, c, b, t$

$$D_\mu q = \left( \partial_\mu - i g_s \left( \frac{\lambda^\alpha}{2} A_\mu^\alpha \right) \right) q, \quad q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \in \mathbb{C}_3, \quad \lambda^a$$

$g_s$ is the strong coupling constant, $t^a = \lambda^a/2$ are the generators of SU(3), $\lambda^a$ are Gell-Mann matrices in the form

$$\lambda^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -2 \end{pmatrix},$$  (3.3)

gluon stress tensor

$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g_s f^{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu,$$  (3.4)

with the nonzero structure constant antisymmetric on all indices of the gauge group:

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = \frac{1}{2},$$

$$f^{156} = f^{367} = -\frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2},$$  (3.5)
where $[\ell^\alpha, \ell^\beta] = if^{\alpha\beta\gamma} \ell^\gamma$, $\alpha, \beta, \gamma = 1, \ldots, 8$. The mass terms $-m_{q_i}q_i^2$ are not included as far as they are present in the electroweak Lagrangian.

The choice of Gell-Mann matrices in the form (3.3) fix the basis in SU(3). This enable us to write out the covariant derivatives (3.2) in the explicit form

$$D_\mu = iD_\mu - ig_8 \frac{q}{2} \begin{pmatrix} A_\mu^R + \frac{1}{\sqrt{3}}A_\mu^3 & A_\mu^1 - iA_\mu^2 & A_\mu^4 - iA_\mu^3 & A_\mu^5 - iA_\mu^5 \\ A_\mu^1 + iA_\mu^2 & \frac{1}{\sqrt{3}}A_\mu^8 - A_\mu^3 & A_\mu^6 - iA_\mu^6 & -\frac{2}{\sqrt{3}}A_\mu^8 \\ A_\mu^4 + iA_\mu^3 & A_\mu^6 + iA_\mu^6 & \frac{1}{\sqrt{3}}A_\mu^8 & A_\mu^1 - iA_\mu^1 \\ A_\mu^5 - iA_\mu^5 & -\frac{2}{\sqrt{3}}A_\mu^8 & A_\mu^1 - iA_\mu^1 & \frac{1}{\sqrt{3}}A_\mu^8 \end{pmatrix}$$

where

$$A_\mu^R = \frac{1}{\sqrt{3}}A_\mu^8 + A_\mu^3, \quad A_\mu^G = \frac{1}{\sqrt{3}}A_\mu^8 - A_\mu^3, \quad A_\mu^B = -\frac{2}{\sqrt{3}}A_\mu^8,$$

$$A_\mu^R + A_\mu^G + A_\mu^B = 0,$$

$$A_\mu^{GR} = A_\mu^1 + iA_\mu^2 = \tilde{A}_\mu^R,$$

$$A_\mu^{BG} = A_\mu^6 + iA_\mu^7 = \tilde{A}_\mu^G.$$

(3.7)

Let us note, that in QCD the special mechanism of spontaneous symmetry breaking is absent, therefore gluons are massless particles.

The contracted special unitary group SU(3; $\kappa$) is defined by the action

$$q'(\kappa) = \begin{pmatrix} q_1' \\ k_1 q_2' \\ k_1 k_2 q_3' \end{pmatrix} = \begin{pmatrix} u_{11} & \kappa_1 u_{12} & \kappa_1 \kappa_2 u_{13} \\ k_1 u_{21} & u_{22} & k_2 u_{23} \\ k_1 k_2 u_{31} & k_2 u_{32} & u_{33} \end{pmatrix} \begin{pmatrix} q_1 \\ \kappa_1 q_2 \\ \kappa_1 \kappa_2 q_3 \end{pmatrix} = U(\kappa)q(\kappa), \quad \det U(\kappa) = 1, \quad U(\kappa)U^\dagger(\kappa) = 1$$

(3.8)

on the complex space $C_3(\kappa)$ in such a way that the hermitian form

$$q^\dagger(\kappa)q(\kappa) = |q_1|^2 + \kappa_1^2 \left(|q_2|^2 + \kappa_2^2 |q_3|^2\right)$$

(3.9)

remains invariant, when the contraction parameters tend to zero: $\kappa_1, \kappa_2 \to 0$. Transition from the classical group SU(3) and space $C_3$ to the group SU(3; $\kappa$) and space $C_3(\kappa)$ is given by the substitution

$$q_1 \to q_1, \quad q_2 \to \kappa_1 q_2, \quad q_3 \to \kappa_1 \kappa_2 q_3, \quad A_\mu^{GR} \to k_1 A_\mu^{GR}, \quad A_\mu^{BG} \to k_2 A_\mu^{BG}, \quad A_\mu^{BR} \to \kappa_1 \kappa_2 A_\mu^{BR},$$

(3.10)

and diagonal gauge fields $A_\mu^{RR}, A_\mu^{GG}, A_\mu^{BB}$ remain unchanged.

Substituting (3.10) in (3.1), we obtain the quark part of the Lagrangian in the form

$$\mathcal{L}_q(\kappa) = \sum q \left\{ i\bar{q}_1 \gamma^\mu \partial_\mu q_1 + \frac{g_8}{2} |q_1|^2 \gamma^\mu A_\mu^{RR} + \kappa_1^2 \left[i\bar{q}_2 \gamma^\mu \partial_\mu q_2 + \frac{g_8}{2} \left(|q_2|^2 \gamma^\mu A_\mu^{GG} + q_1 \bar{q}_2 \gamma^\mu A_\mu^{GR} + \bar{q}_1 q_2 \gamma^\mu A_\mu^{GR}\right) \right] + \right.$$
The gluon part of Lagrangian is as follows

\[ + \kappa_1^2 \kappa_2^2 \left[ i \bar{q}_3 \gamma^\mu \partial_\mu q_3 + \frac{g_a}{2} \left( |q_3|^2 \gamma^\mu A^B_\mu + q_1 \bar{q}_3 \gamma^\mu A^{BR}_\mu + q_1 q_3 \gamma^\mu A^{BR}_\mu + q_2 \bar{q}_3 \gamma^\mu A^{BG}_\mu \right) \right] = L_q^\infty + \kappa_1^2 L_q^{(2)} + \kappa_1^2 \kappa_2^2 L_q^{(4)}. \] (3.11)

Let us introduce the notations

\[ \partial A^k \equiv \partial_\mu A^k_\mu - \partial_\nu A^k_\nu, \quad [k, m] \equiv A^k_\mu A^m_\nu - A^m_\mu A^k_\nu, \] (3.12)

then the gluon tensor has the following components

\[ F_{\mu \nu}^1 = \kappa_1 \left\{ \partial A^1 + \frac{g_a}{2} \left( 2[2, 3] + \kappa_2^2 ([4, 7] - [5, 6]) \right) \right\}, \]
\[ F_{\mu \nu}^2 = \kappa_1 \left\{ \partial A^2 + \frac{g_a}{2} \left( -2[1, 3] + \kappa_2^2 ([4, 6] + [5, 7]) \right) \right\}, \]
\[ F_{\mu \nu}^3 = \partial A^3 + \frac{g_a}{2} \left( \kappa_2^2[1, 2] - \kappa_2^2[6, 7] + \kappa_2^2 \kappa_2^2[4, 5] \right), \]
\[ F_{\mu \nu}^4 = \kappa_1 \kappa_2 \left\{ \partial A^4 - \frac{g_a}{2} \left( [1, 7] + [2, 6] + [3, 5] - \sqrt{3}[5, 8] \right) \right\}, \]
\[ F_{\mu \nu}^5 = \kappa_1 \kappa_2 \left\{ \partial A^5 + \frac{g_a}{2} \left( [1, 6] - [2, 7] + [3, 4] - \sqrt{3}[4, 8] \right) \right\}, \]
\[ F_{\mu \nu}^6 = \kappa_2 \left\{ \partial A^6 + \frac{g_a}{2} \left( \kappa_2^2 ([2, 4] - [1, 5]) + [3, 7] + \sqrt{3}[7, 8] \right) \right\}, \]
\[ F_{\mu \nu}^7 = \kappa_2 \left\{ \partial A^7 + \frac{g_a}{2} \left( \kappa_1^2 ([1, 4] + [2, 5]) - [3, 6] - \sqrt{3}[6, 8] \right) \right\}, \]
\[ F_{\mu \nu}^8 = \partial A^8 + \frac{g_a \sqrt{3}}{2} \kappa_2^2 \left( \kappa_2^2[4, 5] + [6, 7] \right). \] (3.13)

The gluon part of Lagrangian is as follows

\[ L_{gl}(\kappa) = -\frac{1}{4} F_{\mu \nu}^\alpha F^{\mu \nu \alpha} \]
\[ = -\frac{1}{4} \left\{ H_3^3 + H_8^2 + \kappa_1^2 \left( F_1^2 + F_2^2 + 2H_3F_3 \right) + \kappa_2^2 \left( G_6^3 + G_7^2 + 2H_3G_3 - 2\sqrt{3}H_8G_3 \right) + \kappa_1^4 P_3^2 + \kappa_1^2 \kappa_2^2 G_3^2 + \kappa_1^2 \kappa_2^2 \left[ P_4^2 + P_5^2 + 2 \left( F_1G_1 + F_2G_2 + F_3G_3 + F_6G_6 + F_7G_7 + \sqrt{3}H_8P_3 \right) \right] + \kappa_1^2 \kappa_2^4 \left( G_1^2 + G_2^2 - 4G_3P_3 \right) + \kappa_1^2 \kappa_2^2 \left( F_6^2 + F_7^2 + 2F_3P_3 \right) + \kappa_1^4 \kappa_2^2 P_3^2 \right\}. \] (3.14)
where

\[
F_1 = \partial A^1 + g_s[2, 3], \quad F_2 = \partial A^2 - g_s[1, 3], \\
G_1 = \frac{g_s}{2} ([4, 7] - [5, 6]), \quad G_2 = \frac{g_s}{2} ([4, 6] + [5, 7]), \\
H_3 = \partial A^3, \quad F_3 = g_s[1, 2], \quad G_3 = -\frac{g_s}{2}[6, 7], \\
P_4 = \partial A^4 - \frac{g_s}{2} \left([1, 7] + [2, 6] + [3, 5] - \sqrt{3}[5, 8]\right), \\
P_5 = \partial A^5 + \frac{g_s}{2} \left([1, 6] - [2, 7] + [3, 4] - \sqrt{3}[4, 8]\right), \\
P_6 = \partial A^6 + \frac{g_s}{2} \left([3, 7] + \sqrt{3}[7, 8]\right), \\
P_7 = \partial A^7 - \frac{g_s}{2} \left([3, 6] + \sqrt{3}[6, 8]\right), \\
P_8 = \partial A^8.
\]

(3.15)

In the framework of Cayley-Klein scheme \[2\] the gauge group SU(3;\(\kappa\)) has two one-parameter contractions \(\kappa_1 \to 0, \kappa_2 = 1\) and \(\kappa_2 \to 0, \kappa_1 = 1\), as well as one two-parameter contraction \(\kappa_1, \kappa_2 \to 0\). We consider the following contraction: \(\kappa_1 = \kappa_2 = \kappa = \epsilon \to 0\), which corresponds to the infinite temperature limit of QCD. The quark part of Lagrangian (3.11) is represented as a sum of terms proportional to zero, the second and forth powers of the contraction parameter \(\epsilon\)

\[
\mathcal{L}_q(\epsilon) = L_q^\infty + \epsilon^2 L_q^{(2)} + \epsilon^4 L_q^{(4)}
\]

(3.16)

and gluon part is represented as a sum

\[
\mathcal{L}_{gl}(\epsilon) = L_{gl}^\infty + \epsilon^2 L_{gl}^{(2)} + \epsilon^4 L_{gl}^{(4)} + \epsilon^6 L_{gl}^{(6)} + \epsilon^8 L_{gl}^{(8)}.
\]

(3.17)

In the infinite temperature limit \(\kappa = \epsilon \to 0\) most parts of gluon tensor components are equal to zero and the expressions for two nonzero components are simplified

\[
F_{\mu \nu}^3 = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu = \frac{1}{2} \left(F_{\mu \nu}^{RR} - F_{\mu \nu}^{GG}\right), \quad F_{\mu \nu}^8 = \partial_\mu A^8_\nu - \partial_\nu A^8_\mu = \frac{\sqrt{3}}{2} \left(F_{\mu \nu}^{RR} + F_{\mu \nu}^{GG}\right)
\]

(3.18)

so we can write out the QCD Lagrangian \(\mathcal{L}_\infty\) in this limit explicitly

\[
\mathcal{L}_\infty = L_q^\infty + L_{gl}^\infty
\]

\[
= \sum_q \left\{ i \bar{q}_R \gamma^\mu \partial_\mu q_R + g_s \frac{2}{\sqrt{3}} |q_R|^2 \gamma^\mu A_R^{RR} \right\} - \frac{1}{4} \left(F_{\mu \nu}^{RR}\right)^2 - \frac{1}{4} \left(F_{\mu \nu}^{GG}\right)^2 - \frac{1}{4} F_{\mu \nu}^{RR} F_{\mu \nu}^{GG}.
\]

(3.19)

From \(\mathcal{L}_\infty\) we conclude that only the dynamic terms for the first color component of massless quarks survive under the infinite temperature, which means that the quarks are monochromatic, and the terms also survive, which describes the interactions of these components with \(R\)-gluons. Besides \(R\)-gluons there are also \(G\)-gluons, which do not interact with the quarks.

Similarly to the Electroweak Model starting with \(\mathcal{L}_q(\epsilon)\) (3.16) and \(\mathcal{L}_{gl}(\epsilon)\) (3.17) one can construct a number of intermediate models for QCD, which describe the gradual restoration of color degrees of freedom for the quarks and the gluon interactions in the Universe evolution.

It follows from Lagrangian \(\mathcal{L}(\epsilon) = \mathcal{L}_q(\epsilon) + \mathcal{L}_{gl}(\epsilon)\), that the total reconstruction of the quark color degrees of freedom will take place after the restoration of all quark masses (\(\approx \epsilon^2\)) at the same time with the reestablishment of all electroweak interactions (\(\approx \epsilon^4\)). Complete color interactions start to work later because some of them are proportionate to the eighth power \(\epsilon^8\).
4 Estimation of boundary values

As it was mentioned the contraction of the gauge group of QCD gives an opportunity to order in time different stages of its development, but does not make it possible to bear their absolute date. Let us try to estimate this date with the help of additional assumptions. The equality of the contraction parameters for QCD and the EWM is one of these assumptions.

Then we use the fact that the electroweak epoch starts at the temperature $T_4 = 100\text{ GeV}$ (1 GeV $= 10^{13} K$) and the QCD epoch begins at $T_8 = 0, 2\text{ GeV}$. In other words we assume that complete reconstruction of the EWM, whose Lagrangian has minimal terms proportional to $\epsilon^4$, and QCD, whose Lagrangian has minimal terms proportional to $\epsilon^8$, take place at these temperatures. Let us denote by $\Delta$ the cutoff level for $\epsilon^k$, $k = 1, 2, 4, 6, 8$, i.e. for $\epsilon^k < \Delta$ all the terms proportionate to $\epsilon^k$ are negligible quantities in the Lagrangian. At last we suppose that the contraction parameter inversely depends on temperature

$$\epsilon(T) = \frac{A}{T},$$

where $A$ is constant.

As far as the minimal terms in the QCD Lagrangian are proportional to $\epsilon^8$ and QCD is completely reconstructed at $T_8 = 0, 2\text{ GeV}$, we have the equation $\epsilon^8(T_8) = A^8 T_8^{-8} = \Delta$ and obtain $A = T_8^\Delta 1/8 = 0, 2\Delta 1/8 \text{ GeV}$. The minimal terms in the EWM Lagrangian are proportional to $\epsilon^4$ and it is reconstructed at $T_4 = 100\text{ GeV}$, so we have $\epsilon^4(T_4) = A^4 T_4^{-4} = \Delta$, i.e. $T_4 = A \Delta^{-1/4} = T_8^\Delta 1/8 \Delta^{-1/4} = T_8 \Delta^{-1/8}$ and we obtain the cutoff level $\Delta = (T_8 T_4^{-1})^8 = (0, 2 \cdot 10^{-2})^8 \approx 10^{-22}$, which is consistent with the typical energies of the Standard Model.

From the equation $\epsilon^k(T_k) = A^k T_k^{-k} = \Delta$ we obtain

$$T_k = \frac{A}{\Delta^{1/k}} = \frac{T_8 \Delta^{1/8}}{\Delta^{1/k}} = T_8 \Delta^{k/8} \approx 10^{88 - 15k} \text{ GeV}.$$  

Simple calculations give the following estimations for the boundary values of the temperature in the early Universe (GeV): $T_1 = 10^{18}$, $T_2 = 10^7$, $T_3 = 10^3$, $T_4 = 10^2$, $T_6 = 1$, $T_8 = 2 \cdot 10^{-1}$. The obtained estimation for the “infinity” temperature $T_1 \approx 10^{18} \text{ GeV}$ is comparable with Planck energy $\approx 10^{19} \text{ GeV}$, where the gravitation effects are important. So the developed evolution of the elementary particles does not exceed the range of the problems described by the electroweak and strong interactions.

It should be noted that for the power function class $\epsilon(T) = BT^{-p}$ the estimations for temperature boundary values are very weakly dependent on power $p$. So practically we obtain the same $T_k$ for $p = 10$ as for the simplest function (4.1) with $p = 1$.

5 Conclusion

We have investigated the high-temperature limit of the SM which was obtained from the first principles of the gauge theory as the contraction of its gauge group. It was shown that the mathematical contraction parameter is proportional to the temperature and its zero limit corresponds to the infinite temperature limit of the Model. In this limit the SM passes through several stages, which are distinguished by the powers of the contraction parameter, what gives the opportunity to classify them in time as earlier-later. To determine the absolute date of these stages the additional assumptions were used, namely: the inverse dependence of $\epsilon$ on the temperature (4.1) and the cutoff level $\Delta$ for $\epsilon^k$. The unknown parameters are determined with the help of the QCD and EWM typical energies.
The exact expressions for the respective Lagrangians for any stage in the SM evolution are presented. On the base of decompositions (2.36), (3.16), (3.17) the intermediate models $L_k$ for any temperature scale are constructed. It gives an opportunity to draw conclusions on the interactions and properties of the elementary particles in each of the considered epochs. The presence of several intermediate models in the interval from Plank energy $10^{19}$ GeV up to the EWM typical energy $10^2$ GeV instead of only one model automatically takes away the so-called hierarchy problem of the SM [12].

At the infinite temperature limit ($T > 10^{18}$ GeV) all particles including vector bosons lose their masses and the electroweak interactions become long-range. Monochromatic massless quarks are exchanged by only one sort of $R$-gluons. Besides $R$-gluons there are also $G$-gluons, which do not interact with quarks. It follows from the explicit form of Lagrangians $L_{\inf}(A_{\mu}, Z_{\mu})$ (2.38) and $L_{\inf}$ (3.19) that only the particles of the same sort interact with each other. Particles of different sorts do not interact. It looks like some stratification of leptons and quark-gluon plasma with only one sort of particles in each stratum.

At the level of classical gauge fields it is already possible to give some conclusions on the appearance of the elementary particles mass on the different stages of the Universe evolution. In particular we can conclude that half of quarks ($\approx \epsilon, 10^{18}$ GeV $> T > 10^7$ GeV) first restore their mass. Then $Z$-bosons, electrons and other quarks become massive ($\approx \epsilon^2, 10^7$ GeV $> T > 10^3$ GeV). Finally Higgs boson $\chi$ and charged $W^\pm$-bosons restore their masses because their mass terms are multiplied by $\epsilon^4$ ($T < 10^2$ GeV).

In a similar way it is possible to describe the evolution of particle interactions. Self-action of Higgs boson appears with its mass restoration. At the same epoch interactions of four $W^\pm$-bosons, as well as of two Higgs and two $W$-bosons (2.30) start. The only one term in the Lagrangian, which is proportional to the third power of $\epsilon$, describes the interaction of Higgs boson with charged $W^\pm$-bosons ($T < 10^3$ GeV). The rest of the electroweak particle interactions appear in the second order of the contraction parameter ($10^7$ GeV $> T > 10^3$ GeV).

Some part of color interactions between quarks in Lagrangian (3.11) is proportional to $\epsilon^2$ ($T < 10^7$ GeV) and the rest part is proportional to $\epsilon^4$ ($T < 10^2$ GeV). Therefore the complete restoration of quark color degrees of freedom takes place after the appearance of quark masses ($\approx \epsilon^2, T < 10^7$ GeV) (2.39) together with the restoration of all electroweak interactions ($\approx \epsilon^4, T < 10^2$ GeV). Complete color interactions start later because they are proportional to $\epsilon^8$ ($T < 10^{-1}$ GeV).

The evolution of the elementary particles and their interactions in the early Universe obtained with the help of the contractions of the gauge groups of the SM does not contradict the canonical one [6], according to which the QCD phase transitions take place later then the electroweak phase transitions. The developed evolution of the SM present the basis for a more detailed analysis of different phases in the formation of leptons and quark-gluon plasma.

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