Numerical Simulation of Two Dimensional Flows in Yazidang Reservoir

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Abstract. This paper studied the problem of water flow in the Yazidang reservoir. It built 2-D RNG turbulent model, rated the boundary conditions, used the finite volume method to discrete equations and divided the grid by the advancing-front method. It simulated the two conditions of reservoir flow field, compared the average vertical velocity of the simulated value and the measured value nearby the water inlet and the water intake. The results showed that the mathematical model could be applied to the similar industrial water reservoir.

1. Introduction
Located in the middle and upper reaches of the Yellow River and the connecting zone of deserts and Loess Plateau in the north-central area of China, Ningxia has a continental sub-humid and semi-arid climate. With the rainy season mainly concentrated from June to September, it lacks snow and rain, is dry and has strong, highly dusty winds. All these features have determined that the industrial and agricultural water in Ningxia is mainly supplied by the Yellow River. Located far away from the Yellow River, Ningdong Energy-chemical Industry Base of Lingwu City, Ningxia, has to meet its water demand by building a reservoir to pump water from the Yellow River. Yazidang Reservoir can supply industrial, agricultural and domestic water for Ningdong Energy-chemical Industry Base. The long-term safe and effective operation of Yazidang Reservoir is of vital importance for the base. Therefore, studying the runoff and sediment transport pattern, the water pollution level and sediment thickness is of great importance to extend the service life of the reservoir. This paper mainly studies the water flow movement pattern of Yazidang Reservoir, before which a thorough understanding of the topography of the reservoir is needed to be obtained. Firstly, Google Earth software is used to obtain the regional image of the reservoir, and multiple local images are combined into a complete wide-field-of-view of the reservoir by means of image stitching [1-3]. Secondly, the edge-detection
method [4-7] is used to detect the water edge in the image. Then the water edge information is extracted and saved as relevant files. At last, this research loads the file to Google Earth software to extract the longitude and latitude information about the water edge, which, combined with the measured data, can accurately reflect the topography of Yazidang Reservoir. This research combines the topographic data of Yazidang Reservoir with the data of the 17 cross sections measured in 2014 to reflect the initial topography and sectional distribution of Yazidang Reservoir, with the results shown in Figure 1.

Figure 1. Yazid Ang reservoir section diagram.

After the topographic data of the reservoir are obtained, related mathematic models need to be built for numerical simulation of water flow movement. The horizontal scale of Yazid Ang Reservoir is much larger than its vertical scale. Therefore, a depth-averaged plane 2-D turbulence mathematical model would be more proper. This research proposes to build the depth-averaged plane 2-D turbulence mathematical model for numerical simulation of the water flow movement of Yazid Ang Reservoir so as to find out the laws in water flow movement and provide reference for the reasonable operation of the reservoir.

2. Mathematical Model

2.1. Basic Equation

The depth-averaged plane 2-D RNG $\kappa$ – $\varepsilon$ turbulence mathematical model contains two modules, namely, the flow module and the sediment module. This research mainly studies the water flow movement in Yazid Ang Reservoir, and the water module contains the following two equations.

Water flow continuity equation:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0$$  \hspace{1cm} (1)

$x$ -scale momentum equation:

$$\frac{\partial (hu)}{\partial t} + \frac{\partial (huu)}{\partial x} + \frac{\partial (huv)}{\partial y} = \frac{\partial}{\partial x} \left[ (v + v_e) h \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial y} \left[ (v + v_e) h \frac{\partial u}{\partial y} \right] - gh \frac{\partial \zeta}{\partial x} - \tau_{bx}$$  \hspace{1cm} (2)

$y$ -scale momentum equation:

$$\frac{\partial (hv)}{\partial t} + \frac{\partial (hvu)}{\partial x} + \frac{\partial (hvv)}{\partial y} = \frac{\partial}{\partial x} \left[ (v + v_e) h \frac{\partial v}{\partial x} \right] + \frac{\partial}{\partial y} \left[ (v + v_e) h \frac{\partial v}{\partial y} \right] - gh \frac{\partial \zeta}{\partial y} - \tau_{by}$$  \hspace{1cm} (3)

Turbulent kinetic energy $k$ equation:
\[
\frac{\partial (hk)}{\partial t} + \frac{\partial (huk)}{\partial x} + \frac{\partial (hvk)}{\partial y} = \frac{\partial}{\partial x} \left( \alpha_k (v + v_\tau) h \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( \alpha_k (v + v_\tau) h \frac{\partial k}{\partial y} \right) + S_k
\]  

(4)

Turbulent kinetic energy dissipation rate \( \varepsilon \) equation:

\[
\frac{\partial (he)}{\partial t} + \frac{\partial (hue)}{\partial x} + \frac{\partial (hve)}{\partial y} = \frac{\partial}{\partial x} \left( \alpha_\varepsilon (v + v_\varepsilon) h \frac{\partial \varepsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \alpha_\varepsilon (v + v_\varepsilon) h \frac{\partial \varepsilon}{\partial y} \right) + S_\varepsilon
\]  

(5)

In the equation, \( t \) is time, \( \xi \) is water level, \( h \) is water depth, \( u, v \) are the components of average vertical flow velocity in \( x, y \) directions, \( \nu \) is the viscosity coefficient of water flow movement, \( \nu_\tau \) is the viscosity coefficient of flow turbulence, \( g \) is gravitational acceleration, \( k \) is turbulent kinetic energy, \( \varepsilon \) is turbulent kinetic energy dissipation rate, \( \alpha \) is recovery saturation coefficient, \( \tau_{xx}, \tau_{yy} \) are \( x \)-scale and \( y \)-scale bottom friction items, \( S_k, S_\varepsilon \) are the source items of \( k \) equation and \( \varepsilon \) equation, with the formulas as follows:

\[
\tau_{xx} = \frac{g n^2 h u'^2}{u'^2 + v'^2}
\]  

(6)

\[
\tau_{yy} = \frac{g n^2 v u'^2}{u'^2 + v'^2}
\]  

(7)

\[
S_k = h(P_k + P_{kv} - \varepsilon)
\]  

(8)

\[
S_\varepsilon = h \left[ \frac{\varepsilon}{k} (C_{\varepsilon} P_k - C_{\varepsilon} \varepsilon) + P_{\varepsilon} \right]
\]  

(9)

\[
P_k = 2(v + v_\nu) \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + (v + v_\nu) \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2
\]  

(10)

\[
\nu_\nu = C_{\nu} \frac{k^2}{\varepsilon}
\]  

(11)

\[
P_{kv} = C_{\nu} \frac{u'^3}{h}
\]  

(12)

\[
P_{\varepsilon} = C_{\varepsilon} \frac{u'^4}{h^2}
\]  

(13)

\[
u_\nu = \sqrt{C_f (u'^2 + v'^2)}
\]  

(14)

\[
C_{\nu} = \frac{1}{C_f^2}
\]  

(15)
\[ C_h = \frac{3.6C_{2e}C_\mu^{\frac{1}{4}}}{C_f^{\frac{1}{4}}} \]  
\[ C_{1\varepsilon}^\ast = C_{1\varepsilon} - \frac{\eta_0(\eta_0 - \eta)}{\eta_0(1 + \beta_1\eta_0^2)} \]  
\[ \eta_i = \frac{P_k - k}{\sqrt{v_t/\varepsilon}} \]  

\( n \) is the Manning coefficient, with relevant parameter values in the model shown as Table 1.

**Table 1.** Relevant Parameter Values of Plane 2-D RNG \( \kappa - \varepsilon \) Turbulence Mathematical Model.

| C_\mu | C_{1\varepsilon} | C_{2\varepsilon} | \eta_0 | C_f | \beta_1 | \alpha_k | \alpha_\varepsilon |
|-------|-----------------|-----------------|--------|-----|--------|---------|-----------------|
| 0.0845 | 1.42 | 1.68 | 4.377 | 0.003 | 0.012 | 1.39 | 1.39 |

2.2. **Equation Dissipation and Definite Condition**

Unstructured grid finite volume method [8-10] is used for the dissipation of the equation, in which the array of forward difference is used for the instantaneous items, with power-law scheme for the convection items, central difference for the dissipation item, and linear processing for the source item. The quantity of flow is given for the inlet boundary, the turbulent kinetic energy is \( k = 0.01U_{in}^2 \), \( U_{in} \) is the mean velocity on the inlet boundary, which can be computed with the flow rate and cross-section average velocity profile formula. The turbulent kinetic energy dissipation rate is \( \varepsilon = 0.09k^{1.5}/(0.05H) \), \( H \) is the water depth at the inlet, which can be given on the basis of the measured value; the water level and flow are given for the outlet boundary; the near-shore solid boundary is under non-slip treatment.

2.3. **Computational Domain and Meshing**

According to the results of 2-d flow programming analog computation of Yazidang Reservoir based on the above mathematical model, the initial water level of the reservoir is 1, 247 m, and the locations of the computational domain, the inlet, and intake, etc. are shown in Figure 1. There are 17 cross sections from the inlet to the intake, of which sections 4-5 and sections 9-13 are divided into two parts, the left part and the right part by the reservoir bank curve. Advancing front technique [11-12] is used to generate a triangular net in the computational domain which consists of 7, 267 nodes and 11, 919 cells, as shown in Figure 2.

![Figure 2. Meshing map of Yazid Ang reservoir.](image-url)
3. Contrastive Analysis of the Measured Results of Water Flow and the Simulated Results

Based on the actual operation mode of Yazidang Reservoir, operation condition 1 is designed as follows: when the water level of the reservoir drops to 1, 247 m, the reservoir will be fed by pumping the water from of the Yellow River through a pump station, with a daily inflow of 600, 000 cubic meters and a daily intake of 400, 000 cubic meters, until the water level rises to 1, 249. 5 m, with the flow field shown in Figure 3. As the industrial water, agricultural water and domestic water in Ningdong Energy-chemical Base increases, the operating condition 2 is designed as follows: when the water level drops to 1, 247 m, the water will be replenished, with a daily inflow of 800, 000 cubic meters and a daily intake of 600, 000 cubic meters, until the reservoir level rises to 1, 249. 5 m, with the flow field shown in Figure 4.

![Figure 3. Water Flow Field Chart for Operating Condition 1.](image)

![Figure 4. Water Flow Field Chart for Operating Condition 2.](image)

According to the analysis of Figure 3 and Figure 4, with the increase of the water inflow and water intake, the flow fields under both operating conditions fit well with the actual situations; two reflux zones have formed on both sides of the water intakes, the reflux zone on the downside of the water intake larger than that on the upside; with the increase of the inflow, the flow velocity near the water inlet increases, and the water flow that enters the reservoir area has a greater effect on the flow field of the upper part of the reservoir. As the middle part of the reservoir is long and shallow, it has a restriction and mitigation effect on the flow field in the upper part of the reservoir, while the water flow that enters the reservoir area through the water intake has a relatively small effect on the flow field of the intake; with the increase of the water intake, the flow velocity near the intake also increases, which has a greater effect on the flow field of the two reflux zones on both the upper part and the lower parts of the intake; the waterway near the water inlet is narrow, while the water area near the water outlet is wide, thus leading to high flow rate near the inlet and much lower flow rate near the outlet; due to the relatively high flow rate near the water inlet, the cross section near the water inlet is repeatedly washed, and the sediment is accumulated in a long and shallow area in the middle of the reservoir, which has reduced the water flow rate in the middle part of the reservoir and further affected the flow rate in the intake area.

In 2014, Acoustic Doppler Current Profilers, RTK, among other instruments were used for the actual measurement of the 17 cross sections of the reservoir. The data obtained during the period of actual measurement are selected for numerical simulation, and the initial water level obtained in the
analog simulation is 1, 247 m. According to the analog simulation based on the above mathematical model, the water level rises to 1, 249.5 m after 24d. A comparative analysis was conducted on the measured values and the simulated values of section 3 near the water inlet, the right part of section 4 near the water inlet and section 14 and section 15 near the water intake, with the results shown in Figure 5.

![Figure 5. Comparison Chart of the Mean Velocity of Cross Section.](image)

According to the analysis of Figure 5, the simulation value agrees quite well with the measured values, indicating the pretreatment of the reservoir topography is appropriate and the mathematical model is effective; it can be seen from the measured average vertical velocity of the right part of cross section 3, cross section 4 and other cross sections that as the right bank of the cross section is close to the water inlet, the flow rate near the right bank of the cross section is higher than that on the left, and as the distance between the cross section and the water inlet increases, the flow rate near the right bank of the cross section gradually decreases, while the flow rate on the left is largely unaffected; it can be seen from the measured average vertical velocity of cross section 14, cross section 15 and other cross sections that since the left bank of the cross section is close to the water inlet, the flow rate near the left bank of the cross section is higher than that near the right, and as the distance between the cross section and the water intake increases, the flow rate on the left bank of the cross section decreases rapidly, while the flow rate on the right bank is largely unaffected.

4. Conclusion
With Yazidang Reservoir of Lingwu City, Ningxia as the research object, this paper has constructed the initial topography and sectional distribution of Yazidang Reservoir. On such basis, this research builds a plane 2-d RNG $\kappa - \varepsilon$ turbulence mathematical model and dissipated relevant equations with
finite volume methods to determine reasonable initial conditions. This research uses advancing-front method for triangular meshing of the computational domain and designs two operating conditions based on the actual modes of operation of Yazidang Reservoir, thus realizing the distribution of the water area flow field of Yazidang Reservoir and completing the contrastive analysis of the measured results and the simulated results of the cross-section average vertical flow rate of Yazidang Reservoir. The results indicate that the 2-d flow field distribution and cross-section average vertical flow rate agree well with the measured data, suggesting that the built mathematical is reasonable, the non-structured mesh finite volume method is appropriate and the meshing with advancing-front method is accurate enough to simulate the water flow movement of Yazidang Reservoir. This has laid a foundation for further study of the sediment transport pattern, water pollution level and sediment thickness of Yazidang Reservoir, shed light on how to extend the service life of the reservoir, providing a reference for the reasonable operation of similar reservoirs.

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