Analysis and Measurement for the Optical Error of the Cat’s Eye Retro-Reflector

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Abstract. To enhance the coordinates measuring accuracy of the multi-beam laser tracking 3D coordinates measuring system, measurement for the optical error of the cat’s eye retro-reflector is necessary. For this purpose, measurement method for the optical error of the cat’s eye retro-reflector is proposed. The main sources of the optical error of cat’s eye retro-reflector are analysed first, and associated experiment setup and measuring data processing method are described. Experiment results show that the maximum optical error of the measured cat’s eye retro-reflector is approximately 4μm.

1. Introduction
Cat’s eye retro-reflector (called “cat’s eye” for short) works as an objective mirror in the multi-beam laser tracking measuring system (as shown in figure 1). When it works, cat’s eye moves along with the measured object whose position is associated with the 3D coordinates of objective point. The 3D coordinates can be obtained through calculating the relative length variations from the cat’s eye retro-reflector to the base point B, which are measured by the laser tracking interferometer.

![Figure 1. Multi-beam laser tracking 3D coordinates measuring system.](image)

Optical length that the laser beam propagates in the ideal cat’s eye with the variant incidence angle should be constant. Actually, because of the optical errors of cat’s eye, optical length that the laser beam propagates in the cat’s eye varies with the incidence angle, which leads that the relative length variation measured by the laser tracking interferometer can not reflect the actual changes of the
target’s positions, furthermore causes the coordinates measurement errors. Therefore, a measurement and data processing method of the optical error of cat’s eye is presented in this paper, and processed data can be used for the correction of the optical error of cat’s eye in 3D coordinates measurement process.

2. The main sources of the optical error of the cat’s eye
Cat’s eye consists of the front and rear hemispheres made of optical glass (figure 2), and its main technical parameters include radius of front hemisphere \( r_1 \) and rear hemisphere \( r_2 \), optical glass reflective index \( n \), adhesive layer’s thickness \( s \). For improving retro-reflective characteristic of the cat’s eye, generally the function of \( r_1 \) and \( r_2 \) is \( r_2 = r_1/(n-1) \). When the laser beam propagates in the cat’s eye, the difference between actual and ideal state of the technical parameters of the cat’s eye results in the additional optical length called optical error of cat’s eye. The main optical error sources of the cat’s eye include several factors.

![Figure 2. Cat’s eye retro-reflector.](image)

(1) Hemisphere roundness errors
Suppose the roundness errors of the front and rear hemispheres are \( \Delta r_1 \) and \( \Delta r_2 \) respectively, then the additional optical length caused by roundness errors is

\[
\Delta_1 = 2\sqrt{(n\Delta r_1 - \Delta r_1)^2 + (n\Delta r_2)^2}
\]

(2) Heterogeneity of the hemisphere material reflective index
Suppose the heterogeneity of the hemisphere material reflective index in all direction is \( \Delta n \), then the additional optical length caused by the heterogeneity is

\[
\Delta_2 = 2\Delta n(r_1 + r_2)
\]

(3) Eccentricity between the front and rare hemispheres
The additional optical length caused by the eccentricity of the front and rear hemispheres has relation with the eccentricity \( e_h \), eccentric angle \( \alpha_h \) and laser beam incidence angle \( \theta \).

\[
\Delta_3 \approx 2(n-1)e_h\cos(\alpha_h + \theta)
\]

(4) Deviation of the adhesive layer’s thickness
Deviation of the adhesive layer’s thickness causes the eccentricity between the front and rare hemispheres, and this effect can be considered with equation (3) synchronously.

With a made cat’s eye, its optical error is a fixed variable systematic error (a function of incidence angle of laser beam), and compensation value of different azimuth angles can be obtained by the measurement of the optical error of the cat’s eye, which can be used in some programs in system self-calibration and coordinates measurement process for compensating the distance variation value measured by the laser tracking system.

3. Measurement of the optical errors of the cat’s eye retro-reflector
In order to make the measuring state consistent with the working state of the cat’s eye, a measuring principle shown as figure 3 has been designed. The cat’s eye and the measurement fixture are located
on a rotary worktable, and adjust the cat eye’s position to make its optical center in line with the vertical axis of the worktable. The incidence of light from the dual-frequency laser interferometer propagates through the optical center of the cat’s eye in order to measure the relative change of the optical length. By the cat’s eye rotating the horizontal axis and the worktable rotating the vertical axis, the laser beam can propagate through the cat’s eye in different angles.

Firstly, with the angle $\varphi$ fixed, alter the angle $\theta$ to measure the changes of the optical length when the laser beam propagates through the corresponding measurement section (shown as figure 3). Change the angle $\varphi$ by rotating the measurement fixture and alter the angle $\theta$ by rotating the worktable. Set the reading of the laser interferometer to zero at the beginning of measuring every section, and then record the reading of laser interferometer when the worktable rotates to different angles. All raw readings will be processed in the later procedure.

![Figure 3. Measuring principle.](image)

Figure 3 shows the experiment setup used for measuring the optical errors of the cat’s eye. Generally, the optical error of the cat’s eye is only about several micrometers.

![Figure 4. Experiment setup.](image)

In the experiment setup, HP5528 dual-frequency laser interferometer (the resolution is 0.01µm and the precision is ±0.1ppm) is used for measuring the relative variation of optical length to ensure the measurement precision; the high precision worktable of Talyrond200 roundness measuring instrument works as the rotary worktable. Furthermore, eccentricity between the optical center of cat’s eye and
the rotary axis of worktable should be limited in several dozens of micrometers by reasonable design of geometric position accuracy of the measurement fixture and making use of automatic centering setup of roundness measuring instrument worktable.

The commercial cat’s eye CER75 made by Leica was measured by the above experiment setup. During the measurement, we made $\varphi$ equate $30^\circ$ and measured six sections; $\theta$ equates $10^\circ$, 13 points were measured in every measuring section from $-60^\circ$ to $+60^\circ$ (the work range of the cat’s eye). Table 1 shows the raw measuring data.

4. Data processing

In the table of the raw measuring data, the errors include not only the optical error of the cat’s eye, but also the effects of assembled eccentricity which should be separated during the measurement. On the other hand, the additional optical length caused by the eccentricity of the front and rear hemispheres and the one caused by assembled eccentricity are both sine (cosine) function of $\theta$. From the viewpoint of employing the cat’s eye, both should be separated from the original measuring data synchronously. Therefore, establish a fitting function for processing the measurement data as follows:

$$\Delta(\theta) = d + e \cos(\alpha + \theta)$$

Where $d$——a constant associated with the initial measured points;

$e$, $\alpha$——synthetical eccentricity of hemisphere and assembled eccentricity, synthetical eccentricity angle.

$\hat{\Delta}(\theta_j) (j = 1,2,\ldots,13)$——reflects the influence made by two kinds of eccentricity and the initial measurement zero position. When $\hat{\Delta}(\theta_j)$ is determined, the optical errors can be obtained according the following equation.

$$\Delta_e(\theta_j) = R(\theta_j) - \hat{\Delta}(\theta_j) \quad (j = 1,2,\ldots,13)$$

Where $R(\theta)$ is the raw measuring data of every measured point.

Because of the independence of the data from every measuring section, a definite method is required to associate them together. Considering the common point of $\theta=0^\circ$ (the peak of hemisphere) on every measuring section, the value of $\Delta_e(\theta=0^\circ)$ should be equal at this point. Subtracting $\Delta_e(\theta=0^\circ)$ from $\Delta(\theta)$ of each section, the benchmark of measuring data is unified. Table 1 shows the optical error of the cat’s eye retro-reflector at each measuring point $\Delta(\theta, \varphi) \quad (i=1,2,\ldots,6; j=1,2,\ldots,13)$.

| $\theta$ | 0° | 30° | 60° | 90° | 120° | 150° | 0° | 30° | 60° | 90° | 120° | 150° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\varphi$ | 0° | 30° | 60° | 90° | 120° | 150° | 0° | 30° | 60° | 90° | 120° | 150° |
| -60° | -0.7 | -0.5 | +2.5 | -0.7 | -1.6 | +0.8 | -1.07 | +0.65 | +2.05 | +0.87 | +0.27 | +0.69 |
| -50° | -3.9 | +3.1 | +4.3 | +2.6 | +2.5 | +26.9 | -0.03 | -0.57 | -1.90 | -0.95 | -1.93 | -0.28 |
| -40° | -10.2 | +7.3 | +11.9 | +6.6 | +6.1 | +56.3 | -0.93 | -0.80 | +0.22 | -1.60 | -1.65 | +1.59 |
| -30° | -16.6 | +11.6 | +15.7 | +10.9 | +6.1 | +82.3 | -0.94 | -0.41 | -1.00 | -1.34 | -1.91 | +0.43 |
| -20° | -23.5 | +14.7 | +20.6 | +15.0 | +3.7 | +107.9 | -0.66 | -0.57 | -0.54 | -0.55 | -1.47 | +0.06 |
| -10° | -31.5 | +18.2 | +25.0 | +18.2 | -0.9 | +132.1 | -0.90 | +0.40 | +0.16 | +0.17 | -0.25 | +0.27 |
| 0° | -38.7 | +19.5 | +27.7 | +19.6 | -9.3 | +151.1 | 0 | 0 | 0 | 0 | 0 | 0 |
| +10° | -46.5 | +20.4 | +30.9 | +20.4 | -20.6 | +172.0 | +0.39 | +0.07 | +1.27 | +0.19 | -0.10 | +0.98 |
| +20° | -55.4 | +20.1 | +31.3 | +19.9 | -35.5 | +186.0 | -0.47 | -0.17 | +0.73 | +0.05 | -1.58 | +0.98 |
| +30° | -63.7 | +19.0 | +31.7 | +17.5 | -50.6 | +195.1 | -1.13 | -0.31 | +1.20 | -1.03 | -1.45 | +0.40 |
| +40° | -71.5 | +17.4 | +27.9 | +15.4 | -67.1 | +200.2 | -1.92 | -0.09 | -1.51 | -0.89 | -1.37 | +0.44 |
| +50° | -77.6 | +14.7 | +26.1 | +12.4 | -84.8 | +200.4 | -1.86 | -0.15 | -1.24 | -0.78 | -1.66 | +0.37 |
| +60° | -79.8 | +11.1 | +25.1 | +9.1 | -101.3 | +196.0 | +1.08 | -0.39 | +0.75 | -0.22 | -0.43 | +0.47 |
Table 1 shows that the maximum optical error of the measured cat’s eye retro-reflector is approximately 4µm.

5. Conclusion
Through the experiment, the optical error of the cat’s eye can be obtained, which can be used in the compensation of the measuring result of laser tracking interferometer and improve the precision of laser tracking 3D coordinates measuring system. Experiment result shows that the maximum optical error of the measured cat’s eye retro-reflector is approximately 4µm which is close to the result 3µm reported on foreign literature.

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